Unified Length Scale of Spray Structure by Unlike Impinging Jets

Chihiro INOUE,1)† Yuta TAKEUCHI,2) Koji NOZAKI,2) Takehiro HIMENO,2)
Toshinori WATANABE,2) Go FUJI,3) and Yu DAIMON4)

1)Department of Aeronautics and Astronautics, Kyushu University, Fukuoka, Fukuoka 819–0395, Japan
2)Department of Aeronautics and Astronautics, The University of Tokyo, Bunkyo-ku, Tokyo 113–8656, Japan
3)Research Unit II, JAXA, Tsukuba, Ibaraki 305–8505, Japan
4)Research Unit III, JAXA, Tsukuba, Ibaraki 305–8505, Japan

In bi-propellantthrusters, impinging type injectors are widely used to deliver propellants to a combustion chamber. By impinging the jet streams of fuel and oxidizer, the spray spreads while the two liquids mix. To design the injectors, several correlations related to injection conditions have been proposed (e.g., Rupe factor), and practically utilized over the last half-century. However, the physical meanings of the past correlations are not well understood, because the essential scale of the spray structure is elusive. In this paper, we derive the global length scale of the spray produced by impinging injectors of unlike doublet, fuel-oxidizer-fuel triplet, and oxidizer-fuel-oxidizer triplet in a consistent manner. The unified length scale is found representing the spray width ratio of oxidizer to fuel evidenced by comprehensive cold-flow tests including several past studies, covering various parameters such as injector types, nozzle diameters, physical properties of working liquids, and injection velocities. Finally, we clearly provide the physical meaning based on practical correlations in a phenomenological sense.

Key Words: Bi-Propellant Thruster, Impinging Atomization, Spray, Mixing, Length Scale, Patternator

Nomenclature

| Symbol | Definition |
|--------|------------|
| D      | diameter of injector [m] |
| FR     | mass flow rate ratio to total flow rate |
| MR     | mixture ratio of oxidizer to fuel |
| N      | number of fuel nozzles per oxidizer nozzle in an injector element |
| q      | local mass flow rate [kg/s] |
| V      | injection velocity [m/s] |
| x, y, z | coordinate system [m] |
| Y      | absolute value of center of gravity in the y-direction [m] |
| ∆      | section width in a mechanical patternator [m] |
| θ      | angle between fuel and oxidizer jets [deg] |
| ρ      | density [kg/m³] |

Subscripts

| Subscript | Description |
|-----------|-------------|
| f         | fuel        |
| o         | oxidizer    |
| local     | local value at each patternator section |

1. Introduction

Bi-propellant thrusters in space propulsion systems that utilize storable hypergolic propellants have continuously demonstrated high reliability and good performance in past missions for several decades. Most of the bi-propellant thrusters have spray elements incorporating impinging injectors to deliver the propellants to the combustion chamber.

The impingement of fuel and oxidizer jet streams results in liquid sheet extension and spray formation (mixing process), liquid-phase reactions, droplet evaporation, and gas-phase reactions (reaction process), which all occur in a sequential fashion. The overall thrust performance of the chamber is primarily dependent on the combinations of fuel and oxidizer, and their mixing properties. Some correlations related to injection conditions were therefore proposed in the past to evaluate the mixing properties downstream. A practical indicator, the so-called Rupe factor (RF), was derived as a function of the injection momentum flux ratio and nozzle diameter ratio from comprehensive cold-flow tests for unlike doublet injectors.1,2)

\[ RF = \frac{\rho_f V_f^2 D_f}{\rho_o V_o^2 D_o} \] (1)

The value of unity achieves the uniform mixture ratio and optimum performance. Elverum and Morey3) proposed a similar formulation (EM) for triplets as

\[ EM = \frac{N \rho_f V_f^2 D_f^{0.5}}{\rho_o V_o^2 D_o^{0.5}}, \] (2)

in which \( N = 2 \) for the Fuel-Oxidizer-Fuel (FOF) element and \( N = 0.5 \) for the Oxidizer-Fuel-Oxidizer (OFO) element. EM \( \approx 1 \) is the recommended criteria for FOF triplet. The two correlations of RF and EM are determined by the injector types of \( N \), the configuration of \( D \), the liquid density of \( \rho \) and the injection velocity of \( V \). The simple formulae are widely accepted for the practical injector design. This fact also indicates that cold-flow tests are effective approaches for predicting the performance of actual thrusters lately confirmed by several studies.4–8) However, the physical meaning...
of the correlations has been rarely discussed so far.

In this paper, we emphasize a new essential length scale representing the spray structure in a unified manner for unlike impinging doublet, FOF triplet, and OFO triplet jets, as evidenced by comprehensive cold-flow experiments. We show that both RF and EM are derived forms from the newly defined length scale, deepening the understanding of fundamental meaning of the two practical correlations in a phenomenological sense.

2. Length Scale of Spray Structure

We consider three types of unlike impinging injectors: doublet, FOF triplet, and OFO triplet. Each single element is illustrated in Fig. 1. The origin of the coordinate system is located at the stagnation point. The symbol $\bullet$ shows the “absolute” central gravity in the $x$-$y$ plane at $z = L$. The oxidizer nozzle diameter is usually larger than the fuel nozzle. By the impingement of jets ejected from the nozzles aligned along the $x$-axis, the spray extends in the $y$-direction. The center of gravity of the symmetric spray with respect to the $x$-axis is obviously located at $y = 0$; whereas, the absolute position of the center of gravity ($Y$), or the absolute primary moment, is calculated in a non-dimensional form using the distribution of $\dot{q}$ in the $x$–$y$ plane at $z = L$ for both oxidizer and fuel.

$$\frac{Y_o}{L} = \frac{\sum (|y| \cdot \dot{q}_o)}{\sum \dot{q}_o}$$

$$\frac{Y_f}{L} = \frac{\sum (|y| \cdot \dot{q}_f)}{\sum \dot{q}_f}$$

An obvious length scale of the impinging jets is $D$. A global spray structure, e.g. spreading width, produced by the impinging jets, as shown in Fig. 2, is governed by the momentum ratio of impinging jets. In Fig. 2, the two nozzles have the same diameter of $D = 1 \text{ mm}$. The working liquid is water. An instantaneous image (white) is overlapped with a time-integrated image (black). Each black line indicates the trajectory of a single drop. Despite the presence of the sheet, we confirmed that all lines start from the impingement point (stagnation point). Therefore, a non-dimensional global length scale of the spray defined as $\lambda$ is given by Eq. (4) as the momentum ratio, because capillarity and viscosity is negligible for the global scale under high Weber number and high Reynolds number conditions.

$$\lambda \sim \sqrt{\rho V^2 D^2}$$

Here, $^*$ indicates a non-dimensional value or the ratio of the fuel value to the oxidizer value. The square root on the right-hand-side is for matching the order of length on both sides. We will consider the net momentum ratio at the impact point (see Fig. 3). Since the streams collide obliquely with an angle of $\theta$, the length in the $x$-direction in Fig. 3 is an approximate value. In Fig. 3(a), the net impact area of fuel is $\approx D_f^2$, and that of the oxidizer is $\approx D_o^2$. The area ratio of fuel to oxidizer is $D_f/D_o$. In Fig. 3(b), the net impact area ratio is $\approx D_f^2/(D_oD_f/2) = 2D_f/D_o$ for FOF, and it is $D_f^2/2D_o$ for OFO. Since the net impact area ratio is approximately equal to $N \cdot D_f/D_o$, Eq. (5) is deduced by substituting to $D^2$ in Eq. (4) for all injector types in a consistent man-
which coincides with the square root of RF at \( N = 1 \). In the case of \( D_f \) being infinitely larger or smaller than \( D_o \), \( D_f/D_o \) is no longer a scale factor, and \( \lambda \) becomes the so-called effective velocity ratio, a representative length scale of a jet in cross-flow matter.\(^{10,11}\)

### 3. Cold-Flow Test Conditions

To identify a general relationship between \( Y \) and \( \lambda \) under various conditions of injector type, nozzle diameter, working liquids (density ratio and miscibility), and injection velocity, we examine the results of cold-flow experiments using a conventional two-dimensional mechanical patternator. The nozzle configurations are denoted in Table 1. There are 11 injector types, with a total of 80 test cases created by varying the injection velocities.

We conduct experiments for the doublet (cases D1 and D2) and FOF triplet (cases T1–T5) injectors using simulant liquids of dyed water/water (\( \rho_o/\rho_f = 1 \)). Methylene-blue dye is added to the oxidizer simulant water in order to distinguish oxidizer and fuel. The visualized image of dyed water/water jets is shown in Fig. 4. The miscible liquids form a sheet and drops while mixing. The drops downstream are divided through the sections of the patternator (Fig. 5(a)), whose resolution in space is \( 10 \text{ mm} \times 10 \text{ mm} \) in the \( x-y \) plane (\( \Delta = 10 \text{ mm} \)), and collected into the test tubes (Fig. 5(b)) over a certain period. For each test tube, we measure the total weight and calculate the value of \( (\dot{q}_o + \dot{q}_f) \). The local flow rate normalized by the total injection flow rate is given by \( \text{FR}_{\text{local}} = (\dot{q}_o + \dot{q}_f)/\sum (\dot{q}_o + \dot{q}_f) \). The local mixture ratio, \( \text{MR}_{\text{local}} = (\dot{q}_o/\dot{q}_f) \), is measured using an absorbance spectrometer (Kyoritsu ABS-G525, wavelength of 525 nm) according to a previously calibrated equation for the absorbance value to \( \text{MR}_{\text{local}} \), as shown in Fig. 6. Here, dyed water (blue) represent the oxidizer simulant and water represent the fuel simulant. The weight ratio is principally proportional to the absorbance. Experimental results of \( \lambda \) show a clear linearity. The gradient of the fit line depends on the experimental cases. We obtain \( \text{FR}_{\text{local}} \) and \( \text{MR}_{\text{local}} \) with relative errors of 1% and 2%, respectively. We also calculate \( Y \) following Eq. (3) using the results of \( \dot{q} \) under each injection condition of \( \lambda \).

![Fig. 4. Visualization result of dyed water/water impinging jets (The two nozzles have the same diameter of \( D = 1 \text{ mm} \).)](image)

![Fig. 5. Experimental apparatus for the cold-flow tests.\(^{9}\)](image)

![Fig. 6. A calibration result to calculate \( \text{MR}_{\text{local}} \) from absorbance.](image)
Sato et al.\textsuperscript{12)} conducted tests for other doublet injectors (cases D3 and D4) using an immiscible combination of trichloroethylene (TCE) / water ($\rho_o/\rho_f = 1.3$). The patternator resolution space was $\Delta = 7$ mm. Tamura et al.\textsuperscript{13)} conducted tests for FOF and OFO triplet injectors (cases T6 and T7) using an immiscible combination of water/kerosene ($\rho_o/\rho_f = 1.3$), with the patternator space of $\Delta = 11$ mm. For each test tube, the total weight and the respective weights of oxidizer simulant and fuel simulant were measured to calculate FR$_{local}$ and MR$_{local}$ after the two liquids separated. Spray was collected in the $x$–$y$ plane at $L = 80$ mm in cases D1 and D2, $L = 50$ mm in cases D3 and D4, $L = 60$ mm in cases T1–T5, and $L = 100$ mm in cases T6 and T7. Since drops spread linearly from the impinging point (see Fig. 2), the difference of $L$ can be normalized.\textsuperscript{9)}

4. Results and Discussion

By accumulating the experimental results of the two-dimensional distributions of FR$_{local}$ and MR$_{local}$ in the $x$-direction, we obtain one-dimensional distributions along the $y$-direction. Hereafter, we discuss the distributions in the $y$-direction (spray width direction). Figure 7 shows the results of FR$_{local}$ and MR$_{local}$ for doublet injectors with the same injector configurations and different liquid properties. At each $\lambda$, the results of case D1 and case D4 show similarity, indicating that the dominant factor is $\lambda$ and the density ratios of the liquid can be normalized. Liquid miscibility has little effects on the distributions. Even when $\lambda$ increases, the distributions of FR$_{local}$ do not change. However, MR$_{local}$ at the spray edge increases, because the penetrating force of fuel jet strengthens and the fuel jet pushes the oxidizer to the outer edge of the spray. Figure 8 shows the results for FOF triplet and OFO triplet injectors. By comparing Fig. 7(b) and Fig. 8(c) at $\lambda = 1.0$, the spray of the doublet injector widens more than the spray of the triplet injector. The distribution of FR$_{local}$ in the FOF type is easily affected by $\lambda$, while the OFO type is robust. As $\lambda$ increases, MR$_{local}$ at the spray edge tends to increase as in the doublet injector cases.

From the results of cases D1–D4 and cases T1–T7, we obtain the values of $Y_o$ and $Y_f$. In the doublet injector cases, $Y_o$ increases as $\lambda$ increases due to penetration of the fuel jet into the oxidizer jet at the impinging point, consistent with results shown in Fig. 7. In the FOF triplet injector cases, $Y_o$ increases as $\lambda$ increases. In the OFO case (T7), however, $Y_o$ remains constant and $Y_f$ decreases as $\lambda$ increases. Hence, the trend between $Y_o$ and $Y_f$ differs depending on the injector type. In order to find a unified relationship, we plot the ratio of $Y_o$ to $Y_f$ against $\lambda$ in Fig. 10. We find clear linearity as
which is valid within ±20% error (gray region) independent of the injection conditions for all injector types. In Fig. 10, the experimental data of $Y_o$ and $Y_f < \Delta/2$ shown in Fig. 9 are excluded to avoid division by zero in Eq. (6). The reasons of ±20% error are attributed to the bent spray shape in the $x$–$y$ plane typically at $\lambda$ of non-unity and due to the injector misalignment. We also confirm that the properties of the working liquids, e.g. density ratio and miscibility, are well normalized.

A reasonable definition for the half width of spray is $2Y$ in the $y$-direction. Then, $\lambda$ corresponds to the spray width ratio of oxidizer to fuel. Figure 11 denotes the respective relation-
ship between \( Y_o/Y_f \) and \( \lambda \), RF and EM in a log-log plot. We are convinced that \( \lambda \) is exactly the same as \( Y_o/Y_f \), as mentioned above. RF and EM are equivalent to the square of the spray width ratio in doublet and triplet injectors, respectively.

\[
\left( \frac{Y_o}{Y_f} \right)^2 \approx RF \tag{7}
\]

\[
\left( \frac{Y_o}{Y_f} \right)^2 \approx EM \tag{8}
\]

Therefore, the past correlations are revealed to be related to the spray width. Their recommended value of unity corresponds to the same spray width between the oxidizer and fuel, indicating that spray distribution in the width direction (y-direction) is more important for the thrust performance than distribution in the thickness direction (x-direction). Since \( \lambda \) is directly proportional to \( Y_o/Y_f \) rather than the square as RF and EM, \( \lambda \) is identified as the unified length scale representing the global spray structure independent of the injector type. We can use \( \lambda \) independent of injector configurations such doublet, FOF-triplet, and OFO-triplet, and obtain a clear understanding of the phenomenological meaning. A new formulation is yielded as

\[
\lambda = 1, \tag{9}
\]

which is consistently valid as the best performance condition for doublet and triplet injectors.

5. Conclusions

We conducted comprehensive cold-flow tests and successfully clarified the unified length scale consistent for the impinging jet spray from unlike doublet, and FOF and OFO triplet injectors, widely employed in bi-propellant thrusters. The cold-flow tests, including some from several past studies, covered various parameters of injector types, nozzle configurations, physical properties of working liquids and injection conditions. All results denoting the spray width ratio of oxidizer to fuel were equivalent to the unified length scale independent of the injection parameters. Past correlations proposed by Rupe and Elveram with Morey were found to correspond to the square of the spray width ratio of doublet and triplet injectors, respectively, showing that their value of unity produced the same spray width between the oxidizer and fuel. Since the tractable length scale is linearly proportional to the physical phenomenon, we are convinced that the present length scale of \( \lambda \) represents the global spray structure as the unified formulation of past correlations. We conclude that \( \lambda \) of unity is the unified criterion for designing the high-performance injectors used in bi-propellant thrusters.

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Associate Editor