Mechanical Stability of a Strongly-Interacting Fermi Gas of Atoms

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A strongly-attractive, two-component Fermi gas of atoms exhibits universal behavior and should be mechanically stable as a consequence of the quantum mechanical requirement of unitarity. This requirement limits the maximum attractive force to a value smaller than that of the outward Fermi pressure. To experimentally demonstrate this stability, we use all-optical methods to produce a highly degenerate, two-component gas of $^6$Li atoms in an applied magnetic field near a Feshbach resonance, where strong interactions are observed. We find that the gas is stable at densities far exceeding that predicted previously for the onset of mechanical instability. Further, we provide a temperature-corrected measurement of an important, universal, many-body parameter which determines the stability—the mean field contribution to the chemical potential in units of the local Fermi energy.

Strongly interacting Fermi systems are expected to exhibit universal behavior \[1\]. In atomic gases, such strong forces can be produced in the vicinity of a Feshbach resonance, where a bound molecular state in a closed exit channel is magnetically tuned into coincidence with the total energy of a pair of colliding particles \[2\]. In this case, the zero energy scattering length $a_S$, which characterizes the interactions at low temperature, can be tuned through $\pm \infty$. For very large values of $|a_S|$, the important properties of the system (e.g., the effective mean field potential, the collision rate, the superfluid transition temperature, etc.) are predicted to lose their dependence on the magnitude and sign of $a_S$, and instead become proportional to the Fermi energy with different universal proportionality constants. For this reason, tabletop experiments with strongly interacting atomic Fermi gases can provide measurements that are relevant to all strongly interacting Fermi systems \[1, 8\], thus impacting theories in intellectual disciplines outside atomic physics, including materials science and condensed matter physics (superconductivity), nuclear physics (nuclear matter), high-energy physics (effective theories of the strong interactions), and astrophysics (compact stellar objects).

In a gas of degenerate fermions, compression of the gas is resisted by the Pauli exclusion principle, leading to an effective pressure known as the Fermi pressure or degeneracy pressure. The Fermi pressure plays an important role in nature, providing, for example, the outward force which stabilizes neutron stars against gravitational collapse. Because of the Fermi pressure, a confined cloud of degenerate fermions is always larger than a cloud of bosons with an equivalent temperature and particle number. This effect has been directly observed in two elegant experiments \[4, 5\].

An important question is: At what point, if at all, do strong attractive interactions overcome the Fermi pressure? Previously, it was predicted that an atomic Fermi gas could become mechanically unstable for sufficiently large attractive interactions \[10\]. If true, this prediction presents a possible roadblock to attempts to use a Feshbach resonance to produce a high-temperature superfluid \[6, 8, 9\], as the gas should be unstable in the required regime. Recently, however, it has become apparent that the gas may indeed be stable. Heiselberg has estimated the maximum ratio of the attractive potential to the local Fermi energy for a strongly interacting Fermi gas. Using a self-consistent many-body approach, he finds that a two-component Fermi gas is mechanically stable, although a gas with more than two components is not \[11\]. A similar conclusion can also be drawn from an examination of the compressibility of a strongly attractive two-component gas \[8\].

In this Letter, we first show heuristically that the maximum inward force arising from attractive interactions in a two-state gas of atomic fermions is limited by quantum mechanics to a value less than the outward Fermi pressure. We then demonstrate this idea experimentally by directly cooling a two-component Fermi gas of $^6$Li atoms to high degeneracy in an optical trap at magnetic fields near a Feshbach resonance, where strong interactions are observed \[10\]. The highest densities obtained in the experiments far exceed those predicted previously \[8\] for the onset of mechanical instability. Imaging the cloud both in the trap and after abrupt release, we find no evidence of instability. To quantitatively describe the stability, we present a measurement of the relevant universal many-body parameter $\beta$—the mean-field contribution to the chemical potential in units of the local Fermi energy. In Ref. \[10\], we provided a first estimate of $\beta$ assuming a zero temperature gas. Here, we present a method to properly account for the non-zero temperature of the gas. This revised measurement of $\beta$ is in good agreement with a prediction of \[1\].

The mechanical stability of the gas can be understood through a heuristic discussion of the forces acting on a gas of fermions in a 50-50 mixture of two spin components. The equation of state for a normal, zero-temperature Fermi gas is \[11\]

$$\epsilon_F(x) + U_{MF}(x) + U_{\text{trap}}(x) = \mu, \quad (1)$$
where $\mu$ is the chemical potential, $\epsilon_F(x)$ is the local Fermi energy, $U_{\text{MF}}(x)$ is the mean field contribution to the chemical potential, and $U_{\text{trap}}(x)$ is the trap potential. In the local density approximation, $\epsilon_F(x) = \hbar^2 k_F^2(x)/(2M)$, where $k_F$ is the local Fermi wavevector, which is related to the density according to $n(x) = k_F^3(x)/(\pi^2)$.  

For each spin component, there is an effective outward force arising from the Fermi pressure $P_{\text{Fermi}} = 2n(x)\epsilon_F(x)/5$, even in the absence of interatomic interactions. The outward force per unit volume is $-\nabla P_{\text{Fermi}}$. The corresponding force per particle is $-(\nabla P_{\text{Fermi}})/n$. The local force per particle is then easily shown to be

$$F_{\text{Fermi}} = -\nabla \epsilon_F(x),$$  

(2)

Since the density decreases with distance from the center, $F_{\text{Fermi}}$ is outward.

Two body scattering interactions make a contribution $U_{\text{MF}}$ to the chemical potential, which is given in the mean field approximation by,

$$U_{\text{MF}}(x) = \frac{4\pi \hbar^2 a_{\text{eff}}}{M} n(x),$$  

(3)

where $a_{\text{eff}}$ is an effective scattering length, which generally depends on the local thermal average of a momentum-dependent scattering amplitude. The potential is attractive when $a_{\text{eff}} < 0$, and repulsive when $a_{\text{eff}} > 0$. The local force per particle arising from the mean field potential is just

$$F_{\text{MF}} = -\nabla U_{\text{MF}}(x).$$  

(4)

If we assume that $a_{\text{eff}}$ is energy-independent and equal to $a_S$ for $a_S < 0$, it is easy to show that the gas becomes unstable for suitably large values of $a_S$. In this case, the inward force from the mean-field potential, $F_{\text{MF}} \propto \nabla n(x)$ while the outward force from the Fermi pressure, $F_{\text{Fermi}} \propto \nabla n^{2/3}(x)$. Then the inward force exceeds the outward when $|F_{\text{MF}}| > |F_{\text{Fermi}}|$, i.e., when $k_F |a_{\text{eff}}| > \pi/2$. The corresponding density for mechanical instability satisfies $n > n_0$ where $n_0 = \pi/(48 |a_S|^3)$. This result matches the previous prediction of Heiselberg which was derived via a more rigorous calculation.

The assumption that $a_{\text{eff}}$ is energy-independent, however, is only valid if $k_F |a_S| \ll 1$. Outside that regime, this assumption violates the quantum mechanical requirement of unitarity. At intermediate densities, where $a_S \gg k_F^{-1} \propto n^{-1/3} \gg R$, with $R$ the range of the collision potential, two-body scattering is dominant, but the mean field interaction is proportional to a momentum-dependent two-body T-matrix element, which in turn is proportional to the scattering amplitude $f(k)$. It is well known that $f$ has a magnitude which is limited by unitarity to a maximum of $1/k$. Hence, one expects that in a zero-temperature Fermi gas, one should use an effective scattering length in Eq. 3 with a maximum magnitude on the order of $a_{\text{eff}} = 1/k_F$. For $a_{\text{eff}} = -1/k_F$, it is easy to show that the mean field contribution to the chemical potential is

$$U_{\text{MF}}(x) = \beta \epsilon_F(x),$$  

(5)

where the universal parameter $\beta$ has a maximum value of $-4/(3\pi) = -0.42$ in this heuristic treatment. In this limit, both the inward force, $F_{\text{MF}}$ and the outward force, $F_{\text{Fermi}}$ are proportional to the same power of the atomic density. The ratio of their magnitudes, however, is given by $|F_{\text{MF}}|/|F_{\text{Fermi}}| = |\beta| < 1$. Thus, one finds that a two-component Fermi gas is mechanically stable as a result of the quantum mechanical requirement of unitarity. This is in contrast to attractive Bose gases [12] and Bose-Fermi mixtures [17] which exhibit dramatic instabilities.

A possible criticism of our heuristic argument is that the two-body relative wavevector $k$ lies in the range $0 \leq k < k_F$. Since $f$ increases as $k$ decreases, $f(k)$ should be averaged over $W(k)$, the probability distribution for $k$, where $f dk W(k) = 1$. For a noninteracting gas at zero temperature, one can show

$$W(k) = \frac{6}{\pi k_F^3} \left[1 - \frac{3}{2} \frac{k}{k_F} + \frac{1}{2} \left(\frac{k}{k_F}\right)^3\right] \Theta(k_F - k),$$  

(6)

where $k = |k|$. Near a Feshbach resonance, where $|a_S| \gg R$, we assume that $f(k)$ takes the form expected for a zero energy resonance, and that the effective scattering length is determined by the real part of $-f$, i.e., $a_{\text{eff}} = \langle a_S/(1+k^2 a_S^2) \rangle$, where $\langle ... \rangle$ denotes averaging with Eq. 5. We find that $|a_{\text{eff}}|$ has a maximum value of $1.05/k_F$ when $k_F |a_S| = 2$, and decreases slowly for $k_F |a_S| > 1$, reaching $0.6/k_F$ at $k_F |a_S| = 10$.

Our heuristic estimate, which neglects the effects of the interactions on the free particle wavefunctions, yields a value of $\beta$ comparable to that estimated by Heiselberg. In contrast to our heuristic approach, the self consistent many-body approach of Ref. [1] also predicts that $\beta$ is independent of the sign and magnitude of $k_F a_S$ in the intermediate density limit where $k_F |a_S| \gg 1$. Hence, for strongly interacting fermions, $\beta$ is always negative and the effective mean field interaction should always be attractive.

To demonstrate that a strongly attractive, two spin-component Fermi gas of atoms is stable, we employ a 50-50 mixture of the two lowest hyperfine states of $^6$Li, i.e., where $\langle F = 1/2, M = \pm 1/2 \rangle$ states in the low field basis. This mixture has a predicted broad Feshbach resonance at 860 G [18, 19]. A magnetic field of 910 G is applied to the two-component Fermi gas of atoms is stable, we employ a 50-50 mixture of the two lowest hyperfine states of $^6$Li, i.e., $\langle F = 1/2, M = \pm 1/2 \rangle$ states in the low field basis. This mixture has a predicted broad Feshbach resonance at 860 G [18, 19]. A magnetic field of 910 G is applied to

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The gas mixture is prepared and rapidly cooled to degeneracy at 910 G by forced evaporation in our ultra-stable CO$_2$ laser trap, as described for our previous experiments [14]. For our trap, $\omega_x = 2\pi \times (6625 \pm 50$ Hz), $\omega_z = 2\pi \times (230 \pm 20$ Hz), and $\omega \equiv (\omega_x^2 \omega_z)^{1/3} = 2\pi \times$
(2160 ± 65 Hz). Following evaporation, the trap is recom-
pressed to full depth over 0.5 s, and allowed to remain 
for an additional 0.5 s, to ensure thermal equilibrium. 
The CO$_2$ laser power is then extinguished and the gas 
is imaged as described in our previous paper [10]. Af-
fter release, the gas expands hydrodynamically, rapidly 
increasing in the transverse dimension while remaining 
nearly stationary in the axial direction [10].

We determine the number of atoms by numerically in-
tegrating the column density (Table I). For a typical 
number, $N \approx 8 \times 10^4$ atoms per state, the correspond-
ing Fermi density for the experiments is calculated to 
be $n_F \approx 4.8 \times 10^{13}$/cm$^3$ per state [12]. The density 
n$_F$, can be compared to the density $n_0 = \pi/(48 |a_S|^3)$ 
predicted for the onset of mechanical instability, assum-
ing a momentum-independent scattering length. Using 
the best available molecular potentials, which are con-
strained by measurements of the zero crossing in the s-
wave scattering length [10, 20], the zero energy scat-
tering length $a_S$ is estimated to be $\approx -10^4 a_0$ at 910 G 
($a_0 = 0.53 \times 10^{-8}$ cm), within a factor of two [21]. Then 
n$_0 = 4.4 \times 10^{11}$/cm$^3$, showing that the density $n_F$ exceeds 
n$_0$ by one to two orders of magnitude.

Although the molecular potentials are constrained by the 
measured zero crossing in the scattering length [10, 
20], the precise location of the Feshbach resonance still 
may be incorrect. Since the scattering length has not 
been directly measured, it is possible that the attractive 
potential is not as large as expected. However, a simple 
argument based on our previous experiments shows that 
a_S must be large [10]. We observe similar hydrodynamic 
expansion after release from either full trap depth $U_0$ or 
from a reduced trap depth of $U_0/100$. In the latter case, 
we estimate that $k_F^{-1} = 4000 a_0$. Since the observed 
hydrodynamic expansion appears to be independent of the 
trap depth, as expected for a unitarity limited interac-
tion, we conclude that in the shallow trap we must have 
$|k_F a_S| > 1$. Then we must have $|a_S| > 4000 a_0$ and 
n$_0 \leq 6.9 \times 10^{12}$/cm$^3$.

Thus, we have clearly demonstrated that the gas is 
mechanically stable in the intermediate density regime, 
where $|a_S|$ is large compared to the interparticle spac-
ing. The remainder of this paper provides a revised esti-
mate of the universal parameter $\beta$ which quantitatively 
determines the stability of the gas. In our previous esti-
mate [10], we assumed a zero temperature gas and cal-
culated $\beta$ from an estimate of the release energy. There, 
we obtained a value of $\beta$ substantially smaller in magni-
tude than predicted [1]. Here, we determine $\beta$ from the 
transverse spatial widths of the expanding gas and prop-
erly include both finite temperature and hydrodynamic 
scaling effects.

The one dimensional transverse spatial profiles after 
expansion for 0.4-0.8 ms are very well fit by normal-
ized finite-temperature Thomas-Fermi (T-F) distribu-
tions which determine the width $\sigma_x$ as well as the ra-
tio of the temperature to the Fermi temperature, $T/T_F$. 
We expect that $T/T_F$ is approximately constant during 
the expansion, since we observe hydrodynamic scaling of the 
transverse radii consistent with an effective poten-
tial $\propto n^{2/3}$ [10], suggesting an adiabatic process. The 
T-F shape is not unreasonable despite a potentially large 
mean field interaction, since the mean field contribution to 
the chemical potential, Eq. 5 is proportional to the lo-
cal Fermi energy. In this case, assuming Eq. 4 for a nor-
mal degenerate gas at zero temperature [11], it is easy to 
show that the mean field should simply scale the Fermi 
ergy of the trapped cloud without changing the shape from 
that of a T-F distribution [10]. The zero temperature 
spatial distribution then corresponds to that of a 
harmonic oscillator potential, with the frequencies scaled 
so that $\omega'_i = \omega_i/\sqrt{1 + \beta}$, $i = x, y, z$. Hydrodynamic 
expanion then preserves the shape of the trapped atom 
spatial distribution [10, 11]. Assuming that $T/T_F$ is 
small, one expects that a finite temperature spatial dis-
tribution for a harmonic potential is a reasonable approx-
imation. A Sommerfeld expansion [22] of the normalized 
density for $|x| \leq \sigma_x$ yields

$$n(x)/N = \frac{16}{5\pi\sigma_x} \left[f_0(x) + 5\pi^2 (T/T_F)^2 f_2(x)\right],$$

where $f_0(x) = g^3(x)/3$, $f_2(x) = g(x)/8 - g^3(x)/6$, and 
g(x) = \sqrt{1 - x^2}/\sigma_x$.

Eq. 4 is used to fit the measured transverse spatial dis-
tributions, Fig. 1. We obtain the results given in Table 
I for four trials each at expansion times $t$ of 0.6 ms 
and 0.8 ms. Note that Eq. 4 begins to break down near 
$|x| \approx \sigma_x$ for $T/T_F > 0.15$. However, an exact treatment 
using polylogarithm functions yields similar results even 
for our highest temperatures where $T/T_F \approx 0.18$.

The value of the parameter $\beta$ can be determined 
from the transverse radii of the trapped cloud, $\sigma_x(0) = 
\sqrt{2c'_F/M\omega_x^2}$, where $c'_F = \hbar\omega'(6N)^{1/3}$ is the Fermi energy 
including the mean field contribution. Then, $\sigma_x(0) = 
(1 + \beta)^{1/4} \sigma_{xF}$ [23], where $\sigma_{xF} = \sqrt{2\epsilon_F/(M\omega_x^2)}$, and

![Transverse spatial profile of the expanding cloud at 600 \(\mu\)s after release.](image-url)
TABLE I: Value of \( \beta \): \( t \) is the expansion time, \( N \) is the number of atoms per state, \( \sigma_{xF} \) is Fermi radius without the mean field interaction, \( \sigma_x(t) \) is the measured Fermi radius obtained from the finite temperature fit to the data using Eq. 4.

| \( t (\mu s) \) | \( N \) | \( \sigma_{xF} (\mu m) \) | \( T/T_F \) | \( \sigma_x(t) (\mu m) \) | \( \beta \) |
|----------------|------|-----------------|----------|----------------|------|
| 600            | 66,000 | 3.49            | 0.128    | 99            | -0.237 |
| 86,400         | 3.65  | 0.144           | 102      | -0.220        |
| 3.49           | 84,300 | 3.36            | 0.140    | 101           | -0.234 |
| 80,200         | 3.60  | 0.141           | 101      | -0.208        |
| 800            | 67,200 | 3.50            | 0.146    | 133           | -0.176 |
| 87,000         | 3.65  | 0.179           | 131      | -0.344        |
| 70,400         | 3.53  | 0.151           | 130      | -0.273        |
| 82,000         | 3.62  | 0.183           | 128      | -0.382        |

\( \epsilon_F = \hbar \omega (6N)^{1/3} \) is the Fermi energy in the absence of interactions. As we have pointed out previously, the observed anisotropic expansion can arise from unitarity-limited collisional hydrodynamics or superfluid hydrodynamics. In either case, the effective potential is \( \propto \eta^{2/3} \), and we can assume \( \sigma_x(t) = \sigma_x(0) b_x(t) \). Hence,

\[
\beta = \left( \frac{\sigma_x(t)}{b_x(t) \sigma_{xF}} \right)^4 - 1, \tag{8}
\]

For our trap parameters, \( b_x(0.6 \text{ ms}) = 29.74 \) and \( b_x(0.8 \text{ ms}) = 39.88 \). The scale factor \( b_x(t) \) properly includes the spatial anisotropy of the expansion and the correct hydrodynamic scaling. We obtain from Table I an average value \( \beta = -0.26 \pm 0.07 \).

This result is consistent with that obtained from the measured transverse release energy. The release energy can be initially calculated from the zero temperature T-F fits or calculated numerically from second moment of the density. Since \( T/T_F \approx 0.15 \) from Table I, the measured release energy is larger than the zero temperature release energy by a factor \( \eta = 1 + (2\pi^2/3)(T/T_F)^2 \approx 1.15 \). Reducing the release energy by a factor \( \eta \) and using the method of Ref. 14 yields an average value of \( \beta \) consistent with that given above.

The measured average value of \( \beta = -0.26 \pm 0.07 \) can be compared with predictions of the energy per particle for the conditions of our experiment, where \( k_F a_S = -7.4 \). For our equation of state, the sum of the local kinetic and mean field energy per particle is \((3/5)(1 + \beta)\epsilon_F(x)\). In Ref. 1, Heiselberg provides two estimates for the total energy per particle. His Eq. 14 based on the Galitski equations, yields \( \beta_{calc} = -0.54 \). Using the Wigner-Seitz cell approximation, his Eq. 14 provides an alternative estimate, \( \beta_{calc} = -0.33 \). An independent calculation by Steede using effective field theory yields \( \beta = -0.46 \). The theoretical predictions are for zero temperature, and the second is in reasonable agreement with our measurements including the temperature correction to the spatial distributions. However, the predictions do not include the additional temperature dependence of \( \beta \) which arises from a thermal average. Since this important universal many-body parameter can now be experimentally measured, further refinement of both the experimental measurements and the theory is worthwhile and may permit the observation of the superfluid corrections to \( \beta \). In addition, mixtures of the three lowest hyperfine states of \(^6\text{Li}\) may permit observation of predicted mechanical instabilities in a three-component Fermi gas.

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Note that $6\pi/35$ in the denominator of Eq. 11 of Ref. \[1\] should read $6/(35\pi)$, H. Heiselberg, private communication.