Interpretation of Axial Resonances in $J/\psi \phi$ at LHCb

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We suggest that the $J/\psi \phi$ structures observed by LHCb can be fitted in two tetraquark multiplets, the $S$-wave ground state and the first radial excitation, with composition $[cs][\bar{cs}]$. When compared to the previously identified $[cq][\bar{c}\bar{q}]$ tetraquark multiplet, the observed masses agree with what expected for a multiplet with $q \rightarrow s$. We propose the $X(4274)$, fitted by LHCb with a single $1^{++}$ resonance, to correspond rather to two, almost degenerate, unresolved lines with $J^{PC} = 0^{++}$, $2^{++}$. Masses of missing particles in the $1^S$ and $2^S$ multiplets are predicted.

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The LHCb Collaboration has reported [1] the observation of four $J/\psi \phi$ structures, $X(4140)$, $X(4274)$, $X(4500)$, $X(4700)$. These can be fitted with single Breit-Wigner resonances with: $J^{PC} = 1^{++}$ ($X(4140)$, $X(4274)$) and $J^{PC} = 0^{++}$ ($X(4500)$, $X(4700)$). We propose these structures to be interpreted as $S$-wave tetraquarks, with $[cs][\bar{cs}]$ diquark-antidiquark composition. Related considerations can be found in [2].

As we shall see shortly, masses and mass differences lead to classify the lowest lying structures, $X(4140)$ and $X(4274)$, in the ground state (1S) multiplet, while the two heavier ones, $X(4500)$ and $X(4700)$ are attributed to the first radially excited (2S) multiplet.

Ground level and first radial excitation multiplets have been considered in [3, 4] for the classification of $X(3872)$, $Z(3900)$, $Z'(4020)$ and of $Z(4430)$ as $[cq][\bar{c}\bar{q}]$ $(q,q' = u,d)$ tetraquarks in $1S$ and $2S$ states, respectively. In the present note we adopt the pattern of spin-spin couplings within tetraquarks introduced in [5].

Members of a tetraquark multiplet in $S$-wave differ for the arrangement of quark and antiquark spins and the spectrum is determined by spin-spin interactions, with couplings to be determined phenomenologically, as it happens for $q\bar{q}$ or $qqq$ hadrons.

We denote by $[cq]_{s=0,1}[\bar{c}\bar{q}]_{\bar{s}=0,1}$ the $S$-wave tetraquarks with all possible spin quantum numbers. In the $|s,\bar{s},J\rangle$ basis we have the following states (we restrict to electrically neutral ones for simplicity$^1$)

\begin{align}
J^{PC} = 0^{++} & \quad X_0 = |0,0\rangle, \quad X'_0 = |1,1\rangle \quad (1) \\
J^{PC} = 2^{++} & \quad X_2 = |1,1\rangle_2 \quad (2) \\
J^{PC} = 1^{++} & \quad X = \frac{1}{\sqrt{2}} (|1,0\rangle_1 + |0,1\rangle_1) \quad (3) \\
J^{PC} = 1^{+-} & \quad X^{(1)} = \frac{1}{\sqrt{2}} (|1,0\rangle_1 - |0,1\rangle_1) \quad (4) \\
X^{(2)} & = |1,1\rangle_1 \quad (5)
\end{align}

In the case of $[cq][\bar{c}\bar{q}]$ states, $X$ was identified with $X(3872)$, and $X^{(1,2)}$ with $Z(3900)$ and $Z(4020)$, respectively [5]. It was shown in [3] that the ordering of the $Z(3900)$ and $Z(4020)$ masses could be simply explained with the hypothesis that the dominant spin-spin interactions in tetraquarks are those inside the diquark or the antidiquark. The ansatz explains why the $Z$ state not degenerate with the $X(3872)$ is the heaviest. In fact, under this hypothesis, the Hamiltonian simply counts the number of spin 1 in each diquark, and it is seen from 4 and 5 that $X$ and $X^{(1)}$ have one spin 1 while $X^{(2)}$ has two spins 1 and therefore it is heavier.

A further (still untested) consequence is that all states originating from the $|1,1\rangle$ configurations, namely $X^{(2)}$, $X'_0$ and $X_2$, should be degenerate in mass.

Of course, the spin-spin couplings referring to different diquarks are not expected to vanish exactly, as indicated by the fact that $X(3872)$ and $Z(3900)$ are not exactly degenerate. Improving over the simple picture just described will however have to wait the identification of other members of the multiplet, to fix the subdominant spin-spin couplings$^2$. It would be interesting to obtain information on spin-spin couplings from non-perturbative

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$^1$ Considering a pair of charge-conjugated bosons (diquark-antidiquark) each with spin $s$ and total spin $J$, the total wavefunction has to be completely symmetric under exchange of coordinates, spins and charges.\textsuperscript{4}$

$^2$ Spin-spin interactions are expected to be proportional to the overlap probability $|\psi(0)|^2$ of the two quarks/antiquarks in-
QCD methods, e.g., from lattice QCD studies like those in presented in [9].

It is not difficult to see that the spectrum of the $S$-wave $1S$ ground states is characterised by two quantities, Fig. [1] the diquark mass, $m_{cq}$ (or $m_{cs}$) for $J/\psi \phi$ resonances), and the spin-spin interaction inside the diquark or the antidiquark, $\kappa_{cq}$ ($\kappa_{cs}$). The first radially excited, $2S$-states are shifted up by a common quantity, the radial excitation energy, $\Delta E_r$, which is expected to be mildly dependent on the diquark mass [10]; we expect $E_r(cq) \sim E_r(cs)$.

For the $X(3872)$ multiplet, as derived in [3], we have

\begin{align}
M_{X(3872)} &= M_{Z(3900)} = 2m_{cq} - \kappa_{cq} \\
M_{X(4020)} &= 2m_{cq} + \kappa_{cq}
\end{align}

The radial excitation gap is equal to the mass difference $Z(4430) - Z(3900)$. From the experimental masses we obtain the parameters of the $[cq][\bar{c}q']$ multiplets, to wit

\begin{align}
m_{cq} &= 1980 \text{ MeV} \\
\kappa_{cq} &= 67 \text{ MeV} \\
\Delta E_r(cq) &= 530 \text{ MeV}
\end{align}

For the $[cs][\bar{c}s]$ multiplets, we use as input the masses of $X(4140)$, ($J^{PC} = 1^{++}$) and of $X(4500)$, $X(4700)$, ($J^{PC} = 0^{++}$), attributing the latter to the $2S$-multiplet. Generalising previous formulae, see Fig. [1] we have

\begin{align}
M_{X(4140)} &= 2m_{cs} - \kappa_{cs} \\
M_{X(4500)} &= 2m_{cs} + \Delta E_r(cs) - 3\kappa_{cs} \\
M_{X(4700)} &= 2m_{cs} + \Delta E_r(cs) + \kappa_{cs}
\end{align}

and we obtain the very reasonable values of the parameters

\begin{align}
m_{cs} &= m_{cq} + \Delta m_s = m_{cq} + 130 \text{ MeV} \\
\kappa_{cs} &= 50 \text{ MeV} \\
\Delta E_r(cs) &= 460 \text{ MeV}
\end{align}

With this, we can predict all particles in the $1S$ and $2S$ multiplets.

![Diagram](Image)

**FIG. 1:** Mass spectrum of the states in Eqs. (1)-(5) as given by a Hamiltonian of spin-spin interactions confined inside diquarks. Under this assumption $|1,1\rangle$ states are degenerate, see text. The assignment of $X(4140)$ and our hypothesis on $X(4274)$ structure? is shown.

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M_{X(4020)} &= 2m_{cq} + \kappa_{cq}
\end{align}

We predict the lower $0^{++}$ and higher $0^{++}$ states to be at

\begin{align}
m_{0^{++}}(1S) &= 3450 \text{ MeV} \\
m_{0^{++}}(1S) &\approx m_{2^{++}}(1S) = 4240 \text{ MeV}
\end{align}

The lower $0^{++}$ state would not show up in the LHCb spectrum, being below the $J/\psi \phi$ threshold.

The higher mass $1S$ states, $0^{++}$ and $2^{++}$, are close to the structure observed by LHCb at 4274 MeV. LHCb fits the 4274 structure with a single resonance and finds $J^{PC} = 1^{++}$ at $5\sigma$ [1]. This attribution is not compatible with the tetraquark model, which admits only one $J^{PC} = 1^{++}$ state. Rather, we would like to propose three alternative options for the structure at 4274 MeV

1. $J^{PC} = 0^{++}$
2. $J^{PC} = 2^{++}$
3. two unresolved, approximately degenerate, lines with $J^{PC} = 0^{++}$ and $J^{PC} = 2^{++}$

What we prefer in the third option is that, in that case, LHCb would have seen all the accessible $C = +1$, $1S$ states. A further experimental study of the structure at 4274 MeV, with respect to the three options presented above, would add valuable information.

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**TABLE I:** Input and predicted masses for $1S$ and $2S \, cs$ tetraquarks.

| Radial | Particle | $J^{PC}$ | Input | Predicted | Notes |
|--------|----------|----------|-------|-----------|-------|
| 1S     | $X_0$    | $0^{++}$ | 4140  | 3450      | below $J/\psi \phi$ threshold |
| 1S     | $X$      | $1^{++}$ | 4140  | 4274      | decays in $\chi_c \phi$ |
| 1S     | $X^{(1)}$| $1^{--}$ | 4700  | 4700      | part of 4274 structure? |
| 1S     | $X^{(2)}$| $2^{++}$ | 4240  | 4240      | part of 4274 structure? |
| 2S     | $X_0$    | $0^{++}$ | 4500  | 4700      | $S_{cs} = 1$ decays in $\chi_c \phi$ |
| 2S     | $X$      | $1^{++}$ | 4600  | 4700      | $S_{cs} = 0$ decays in $\eta \phi$ |
| 2S     | $X^{(1)}$| $1^{++}$ | 4600  | 4700      | $S_{cs} = 1$ decays in $\chi_c \phi$ |
| 2S     | $X^{(2)}$| $2^{++}$ | 4700  | 4700      | $S_{cs} = 0$ decays in $\eta \phi$ |

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A simple explanation of the dominance of inter-diquark interaction could be that diquarks and antidiquarks are at such relative distance in the hadron, as to suppress the overlap probability, unlike what happens, e.g., in the usual baryons.
To complete the picture, we list the predicted masses of the $1S, X^{(1,2)}, C = -1$ states

$$m(X^{(1)}) \approx 4140 \text{ MeV}$$

$$m(X^{(2)}) \approx 4240 \text{ MeV}$$

As for the $2S$ multiplet, we have

$$m_{2++}(2S) \approx 4700 \text{ MeV}$$

$$m_{1++}(2S) \approx 4600 \text{ MeV}$$

and two $C = -1, 2S$ states

$$m_{1+-}(2S) \approx 4600 \text{ MeV}$$

$$m_{1+-'}(2S) \approx 4700 \text{ MeV}$$

The situation is summarized in Tab. I.

\section{Conclusions}

In conclusion, the $J/\psi \phi$ structures observed by LHCb can be fitted in two tetraquark multiplets, the $S$-wave ground state and the first radial excitation. When compared to the previous $[cq][\bar{c}\bar{q}]$ multiplet, the observed masses agree well$^3$ with what expected for a multiplet with $q \to s$.

The hypothesis is however inconsistent with the attribution of the $X(4274)$ structure to a single $1^{++}$ resonance. Rather we propose this structure to correspond to two, almost degenerate, unresolved lines with $J^P C = 0^{++}, 2^{++}$, an hypothesis which may not be in conflict with the present analysis. If this solution would be supported by a more detailed analysis, LHCb would have seen, in a single experiment, all possible $1S$-wave states with $C = +1$ (since the lowest $0^{++}$ is predicted to be below threshold) and the beginning of the $2S$ multiplet.

In addition

1. Two $1^{+-}$ states should be observed very close in mass to $X(4140)$ and $X(4274)$ respectively.

2. Radial excitations with $1^{+-}$ quantum numbers should follow by the assignment of the observed $X(4500)$ and $X(4700)$ as the radial excitations of $X(4140)$ and $X(4274)$ respectively.

The discovery of $C = +1$ structures calls for an exploration of $C = -1$ channels and of other $C = +1$ channels, to survey different options of the heavy quark spin, $S_{c\bar{c}}$. Channels of choice could be

$$C = -1 \quad \chi_{cJ} \phi (S_{c\bar{c}} = 1), \eta_c \phi (S_{c\bar{c}} = 0)$$

$$C = +1 \quad h_c \phi (S_{c\bar{c}} = 0)$$

An alternative view on $C = -1$ states is found in \cite{11}.

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