Supersymmetry With Prejudice: Fitting the Wrong Model to LHC Data

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Abstract: We critically examine interpretations of hypothetical supersymmetric LHC signals, fitting to alternative wrong models of supersymmetry breaking. The signals we consider are some of the most constraining on the sparticle spectrum: invariant mass distributions with edges and end-points from the golden cascade decay chain $\tilde{q}_L \rightarrow q\chi_0^0 (\rightarrow \tilde{l^\pm} l^\mp q) \rightarrow \chi_0^0 l^\pm l^- q$. We assume a CMSSM point to be the ‘correct’ one, but fit the signals instead with minimal gauge mediated supersymmetry breaking models (mGMSB) with a neutralino quasi-stable lightest supersymmetric particle, minimal anomaly mediation (mAMSB) and large volume string compactification models (LVS). mAMSB and LVS can be unambiguously discriminated against the CMSSM for the assumed signal and 1 fb$^{-1}$ of LHC data at $\sqrt{s} = 14$ TeV. However, mGMSB would not be discriminated on the basis of the kinematic end-points alone, and would require further, more detailed investigation. The best-fit points of mGMSB and CMSSM look remarkably similar, making experimental discrimination at the LHC appear unlikely by any means.

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1 Introduction

The Large Hadron Collider (LHC) is currently actively engaged in searches for new physics, including supersymmetry (SUSY). No signal has yet been found, and the CMS and ATLAS experiments have significantly extended previous exclusion limits [1, 2]. In the near future, as more data is collected by the experiments, the observation of a supersymmetric signal is quite plausible. In the event of a signal, it will be important to extract as much empirical information as possible about the sparticle spectrum, since it contains clues about the mechanism of supersymmetry breaking. We may hope to rule out one mechanism in favour of another. One will want to bring all of the data that robustly constrain the supersymmetry breaking mechanism to bear in order to separate different models empirically. However, the usual search variables (number of events past certain cuts or total cross-sections), while perfectly suited to searching for supersymmetry, are blunt instruments when it comes to measuring supersymmetric masses in detail: they give only gross information about the overall mass scale of the supersymmetric particles. Since this is typically described by some parameter in the SUSY breaking mechanism, such measurements will not tend to be very good at disentangling models. One needs to measure observables which are sensitive to the mass spectrum of the sparticles, reasonably accurate, and robust with respect to experimental systematics such as how well one has parameterised one’s detector. Arguably the best examples of such observables come from SUSY cascade decays. SUSY cascade decay chains give specific kinematics to the final state particles, and particular
kinematic variables have been shown to contain a wealth of information about the sparticle masses. Maxima and minima of invariant mass distributions, if observed, have several advantages in the inference of sparticle masses. They can be essentially Standard Model background-free, particularly if flavour subtracted. Also, although the shape [3] of the distributions themselves are subject to significant detector corrections, which may require a lot of integrated luminosity to model well, the end-points of the distributions are expected to be much less sensitive to such effects. For example, the golden decay chain

$$\tilde{q}_L \rightarrow \chi^0_2 \left( \rightarrow \tilde{t}^\pm \tilde{\tau}^\mp q \right) \rightarrow \chi^0_1 l^+ l^- q,$$

shown in Figure 1 has been shown to be most useful [4]. The presence of this cascade leads to events with two opposite-sign same-flavour (OSSF) leptons, jets and missing energy. The end-points yield useful information coming from the invariant mass distributions of the di-leptons $m_{ll}$, from the jet and lepton pair $m_{llq}$ and from each lepton and the jet $m_{lj}$. Despite the fact that one obtains highly correlated mass measurements from such end-points, considering the measurements in parallel helps discriminating different models of supersymmetry breaking [5]. Even if additional decay chains are identified in the data, they are not expected to add significant discriminatory power over the dominant golden chain. It is by no means guaranteed that the golden decay chain is present however, for instance it only exists in about a quarter of the parameter space [6] of the constrained minimal supersymmetric standard model (CMSSM). In the case that the golden chain is not present, one would use kinematic edge data from all chains that one can identify. The resulting information is then likely to be less constraining on the sparticle spectrum than the golden chain. We then view studies assuming the observation of the golden decay chain to be the most optimistic cases as far as model discrimination goes. If two models cannot be experimentally discriminated with this assumption, it is extremely unlikely that they will be discriminated between without the golden cascade. The kinematic data have been further combined with cross-section information in order to improve the precision of mass measurements within particular models with more parameters than the CMSSM [7].

Combining the power of the LHC and a linear collider leads to much more information about the model than is possible from LHC measured kinematic endpoints, and constitutes a significant improvement on the information obtained from the LHC alone [8]. Using SUSY signal measurements from both a linear collider and the LHC in order to measure a large part of the MSSM spectrum may be possible, allowing checks of unification relations in various models [9–11]. The additional information coming from linear collider data would
be ideal to include in order to discriminate models, but in this paper we restrict ourselves to potential LHC data, since the linear collider is not yet built.

Kinematic edge predictions resulting from golden chain decays have been examined in the literature to see if there could be model discrimination coming from their measurement. In Ref. [12], it was seen whether the ratios of the measurements would discriminate the CMSSM, an intermediate-scale string model and a mirage unification model. The parameters of the models were all scanned over, but no experimental errors were taken into account. In any case, it was concluded that there was no clear separation between the models from using the edge variables, even for infinitely precise measurements. We go beyond this work by examining different models, and by fixing a benchmark model such that we can use the experimental resolutions estimated by ATLAS, assuming a certain integrated luminosity. In Ref. [13], the golden decay chain was used in fits to the CMSSM. Hypothetical invariant mass end-points were fit using different sparticle spectrum calculators in order to examine the differences between them, quantifying the theoretical error. The best-fit values of each spectrum calculator were within 95% confidence level (C.L.) limits of each other, assuming a huge LHC luminosity (300 fb$^{-1}$). A number of other fitting groups have investigated the effects of LHC data on global fits to the CMSSM, including the Fittino Collaboration [14], SFitter [15] and Refs. [16, 17]. Those works focused more on the constraining power of the LHC data on CMSSM fits. In Ref. [18], current indirect data on $B$ decays, electroweak observables and the dark matter relic density were combined with direct sparticle search limits in fits to the CMSSM, mAMSB, LVS and mGMSB models (to be introduced below) in order to examine whether current data show any preference for the model of supersymmetry breaking. It was found that current indirect data is too weak to select any of the models. On the other hand, end-point data taken from the golden cascade would be enough to robustly constrain the CMSSM in 1 fb$^{-1}$ of integrated luminosity, at a particular benchmark point studied by ATLAS, called SU3 [16]. Such robustness is signalled by prior independence in Bayesian fits, indicating that the data is sufficiently powerful to constrain the model hypothesised. Ref. [19] also examined fits from the SU3 point golden cascade fits on the CMSSM (with and without including cosmological data) as well as models with more free parameters than the CMSSM. Model comparison between the non-universal higgs model, the CMSSM and the CMSSM but with non-universal gaugino masses was examined using Bayesian techniques. Some non-robustness in the non-CMSSM models with respect to changing the priors was discovered: there was not enough power in the data to properly constrain the models with larger parameter spaces.

Since the CMSSM may be robustly constrained by the end-point data, but models with more parameters may not, in the present paper we answer the following question: Is kinematic edge data from 1 fb$^{-1}$ of the 14 TeV LHC constraining enough to allow us to distinguish between simpler different models of supersymmetry breaking (i.e. with fewer parameters in the CMSSM)? This question will require a numerical statistical analysis: even if it is clear analytically that a model can be chosen such that its mass spectrum is close to the CMSSM, the question is: can it be made close enough in terms of the errors on the observables to provide a viable fit? Conversely, even if two models cannot exactly reproduce the same mass spectrum, are the errors on the observables small enough
such that the two models are discriminated? We shall test robustness by looking for a lack of prior dependence in the hypothesis testing, and agreement between Bayesian and frequentist inferences.

1.1 SUSY Breaking Models

In this subsection, we summarise the alternative hypotheses of SUSY breaking that we shall use. The parameters of the CMSSM are: a flavour blind SUSY breaking scalar mass $m_0$, a common gaugino mass $M_{1/2}$, a flavour blind SUSY breaking scalar trilinear coupling $A_0$ and tan $\beta$, the ratio of the minimal supersymmetric standard model (MSSM) Higgs vacuum expectation values (VEVs). Below a grand unification theory (GUT) scale of $M_{GUT} \sim 2 \times 10^{16}$ GeV, the SUSY breaking terms of different flavours evolve separately to the weak scale. In anomaly mediated SUSY breaking [20] SUSY-breaking is communicated to the visible sector via the super-Weyl anomaly. In its original manifestation, pure anomaly mediation suffers from negative slepton mass squared parameters, signalling a scalar potential minimum inconsistent with a massless photon. Minimal AMSB (mAMSB) assumes the existence of an additional contribution to scalar masses $m_0$ at $M_{GUT}$ giving it a total of three parameters: the VEV of the auxiliary field in the supergravity multiplet representing the overall sparticle mass scale, $m_{aux}$, $m_0$ and tan $\beta$. As advertised above, minimal gauge mediated SUSY breaking (mGMSB) [21] also has three continuous parameters: the overall messenger mass scale, $M_{mess}$, a visible sector soft SUSY-breaking mass scale, $\Lambda$ and tan $\beta$. It also contains an additional discrete parameter, namely $N_{mess}$, the number of SU(5) $5 \oplus \overline{5}$ representations of mediating fields. The example of a moduli mediated model which we consider is the Large Volume Scenario (LVS) derived in the context of $IIB$ flux compactification [22–25], whose two extra-SM parameters can be parametrised by a universal scalar mass $m_0$ and tan $\beta$. At an intermediate scale of $10^{11}$ GeV, the LVS has a universal gaugino mass $M_{1/2} = \sqrt{3} m_0$ and a universal trilinear scalar coupling $A_0 = -\sqrt{3} m_0$.

In Section 2 following, we detail the predictions of the golden cascade edges, as well as the expected precision that would come from LHC measurements. We also specify the SU3 CMSSM benchmark. In Section 3, we summarise the statistics we shall use to perform hypothesis testing on the different SUSY breaking models, defining parameter ranges for the fits. The results of the hypothesis tests are given in Section 4. We show that mGMSB cannot be discriminated from SU3 by the edge data alone. It is then examined in more detail. We sum up and conclude in Section 5.

2 Kinematic Edges at SU3

The ATLAS collaboration has published a series of studies on reconstructing SUSY benchmark points in the Supersymmetry section of [26]. We are specifically interested in the study of the CMSSM SU3 benchmark point and associated mass reconstruction using kinematic end points from golden cascades. The input parameters for the SU3 point are shown in Table 1. SU3 is a point in the bulk region of the parameter space with $m_{\tilde{\chi}_1^0} = 118$ GeV and $m_{\tilde{g}} = 720$ GeV. Its spectrum contains the mass ordering $m_{\tilde{\chi}_1^0} < m_{\tilde{q}} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0}$ or
$m_{\chi_1} < m_{\chi_2} < m_{\tilde{t}} < m_{\tilde{q}}$ so that the golden decay chain is active (in the latter case, the $\chi^0_2$ decay is three-body as the $\tilde{t}_R$ is off-shell). We note that the SU3 point has recently been ruled out by the ATLAS experiment’s jets plus zero lepton missing transverse momentum search [1, 27]. This does not matter for the purposes of the present paper: one must simply bear in mind that a heavier point will have decreased statistics, and consequently will require more luminosity to discriminate against other models.

In the golden decay chain in Fig. 1, one may construct several Lorentz invariant quantities from the four momenta of the visible particles: the quark and leptons. These are predicted to have various maxima and minima, each predicted by the theory to be related to the masses of the supersymmetric particles involved in the cascade decay. We shall now detail this dependence, which differs depending on whether the $\chi^0_2$ decays through a two body decay with an on-shell slepton ($m_{\chi^0_2} > m_{\tilde{t}_R}$) or a three body decay ($m_{\chi^0_2} < m_{\tilde{t}_R}$).

We now detail each case in turn, collecting the edge predictions from Refs. [5, 28] for completeness.

### 2.1 Prediction of kinematic edges with an on-shell slepton

One kinematic maximum that we use is the di-lepton mass edge. In terms of the sparticle masses, it is predicted to be

$$m_{\ell\ell}^{\text{edge}}^2 = \frac{(m_{\tilde{q}}^2 - m_{\chi_2}^0)(m_{\tilde{l}}^2 - m_{\chi_1}^0)}{m_{\tilde{l}}^2}. \quad (2.1)$$

There are two $lq$ edges, in ascending order $m_{lq}^{\text{low}}$ and $m_{lq}^{\text{high}}$, respectively. They are defined to be the maximum or minimum of various quantities for $m_{\chi_2}^0 > m_{\tilde{t}_R}$:

$$m_{lq}^{\text{high}} = \max \left[ m_{lq}^{nr}, m_{lq}^{far} \right], \quad (2.2)$$

$$m_{lq}^{\text{low}} = \min \left[ m_{lq}^{nr} (\text{max}), m_{lq}^{far} (\text{max}), m_{lq} (\text{max}) \right], \quad (2.3)$$

where the quantities on the right hand side are defined to be:

$$m_{lq}^{nr2} (\text{max}) = \frac{(m_{\tilde{q}}^2 - m_{\chi_2}^0)(m_{\tilde{l}}^2 - m_{\chi_2}^0)}{m_{\chi_2}^0}, \quad (2.4)$$

$$m_{lq}^{far2} (\text{max}) = \frac{(m_{\tilde{q}}^2 - m_{\chi_2}^0)(m_{\tilde{l}}^2 - m_{\chi_1}^0)}{m_{\chi_1}^0}, \quad (2.5)$$

$$m_{lq}^2 (\text{max}) = \frac{(m_{\tilde{q}}^2 - m_{\chi_1}^0)(m_{\tilde{l}}^2 - m_{\chi_1}^0)}{2m_{\tilde{l}}^2 - m_{\chi_1}^0}. \quad (2.6)$$

The $llq$ edge is defined as

$$m_{llq}^{\text{edge}}^2 = \max \left[ \frac{(m_{\tilde{q}}^2 - m_{\chi_2}^0)(m_{\tilde{l}}^2 - m_{\chi_1}^0)}{m_{\chi_2}^0}, \frac{(m_{\tilde{q}}^2 - m_{\chi_2}^0)(m_{\tilde{l}}^2 - m_{\chi_1}^0)}{m_{\chi_1}^0}, \frac{(m_{\tilde{q}}^2 m_{\tilde{l}}^2 - m_{\chi_2}^0 m_{\chi_1}^0)(m_{\tilde{q}}^2 - m_{\chi_1}^0)}{m_{\chi_2}^0 m_{\chi_1}^0} \right]. \quad (2.7)$$
Table 1. Input parameters of the CMSSM SU3 benchmark point.

| Parameter | $m_0$ | $m_{1/2}$ | $A_0$ | $\tan\beta$ | $\text{sgn}\mu$ |
|-----------|-------|-----------|-------|--------------|------------------|
| Value     | 100 GeV | 300 GeV   | -300 GeV | 6            | +1               |

unless $m_q^4 < m_{\tilde\chi}^2 m_{\tilde\chi}^0 < m_{\tilde\chi}^4$, and $m_q^4 m_{\tilde\chi}^2 m_{\tilde\chi}^0 < m_{\tilde\chi}^2 m_{\tilde\chi}^4$, in which case it the right-hand side is equal to $(m_{\tilde{q}} - m_{\tilde{\chi}^1})^2$. For the $llq$ threshold variable, the prediction is

$$m_{lq}^{\text{thr}} = \frac{1}{4 m_{\tilde{\chi}_2}^2} \left[ 2 m_{\tilde{q}}^2 (m_{\tilde{\chi}^0}^2 - m_{\tilde{\chi}_2}^2)(m_{\tilde{\chi}^0}^2 - m_{\tilde{\chi}_1}^2) + (m_{\tilde{q}}^2 + m_{\tilde{\chi}^0}^2)(m_{\tilde{\chi}^0}^2 - m_{\tilde{\chi}_1}^2)(m_{\tilde{\chi}^2}^2 - m_{\tilde{\chi}_1}^2) - (m_{\tilde{q}}^2 - m_{\tilde{\chi}^0}^2) \sqrt{(m_{\tilde{\chi}^0}^2 + m_{\tilde{\chi}_2}^2)^2 (m_{\tilde{\chi}^0}^2 + m_{\tilde{\chi}_2}^2)^2 - 16 m_{\tilde{\chi}_2}^2 m_{\tilde{\chi}^0}^2 m_{\tilde{\chi}^2}^2} \right]. \tag{2.8}$$

This edge is the $m_{lq}$ minimum for all events for which $\frac{1}{\sqrt{2}} \leq m_{ll}/m_{ll}(\text{max}) \leq 1$.

2.2 Prediction of kinematic edges with three-body $\chi_{2}\tilde{\chi}^0$ decay

When $m_{\tilde{\chi}_2} < m_{\tilde{l}^R}$, the $\chi_{2}^0$ decays via a virtual $\tilde{l}^R$ into leptons and $\chi_{1}^0$, and in this case the above Eqs. 2.1-2.8 should be altered to the following:

$$m_{l}^{\text{edge}} = (m_{\tilde{\chi}_2} - m_{\tilde{\chi}_1})^2, \tag{2.9}$$

$$m_{lq}^{\text{edge}(\text{high})} = \frac{(m_{\tilde{q}}^2 - m_{\tilde{\chi}_2}^2)(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)}{2 m_{\tilde{\chi}_2}^2}, \tag{2.10}$$

$$m_{lq}^{\text{edge}(\text{low})} = \frac{m_{lq}^{\text{edge}(\text{high})}}{\sqrt{2}}, \tag{2.11}$$

$$m_{llq}^{\text{edge}} = \begin{cases} (m_{\tilde{q}}^2 - m_{\tilde{\chi}_2}^2)^2 & \text{if } m_{\tilde{q}}^2 > m_{\tilde{\chi}_2} m_{\tilde{\chi}_1}, \\ (m_{\tilde{q}}^2 - m_{\tilde{\chi}_2}^2)(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)/m_{\tilde{\chi}_2}^2 & \text{otherwise}, \end{cases} \tag{2.12}$$

$$m_{lq}^{\text{thr}} = \frac{(m_{\tilde{\chi}_2} - m_{\tilde{\chi}_1})^2}{2} + \frac{m_{lq}^2 - m_{\tilde{\chi}_2}^2}{4 m_{\tilde{\chi}_2}^2} \left( 3 m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2 - 2 m_{\tilde{\chi}_2} m_{\tilde{\chi}_1} - \sqrt{m_{\tilde{\chi}_2}^2 + m_{\tilde{\chi}_1}^2 + 4 m_{\tilde{\chi}_2} m_{\tilde{\chi}_1} (m_{\tilde{\chi}_2}^2 + m_{\tilde{\chi}_1}^2) - 10 m_{\tilde{\chi}_2}^2 m_{\tilde{\chi}_1}^2} \right). \tag{2.13}$$

There is obviously less information than in the case where the slepton is on-shell, because there are less constraints coming from 4-momentum conservation. In particular, we see that $m_{\tilde{l}^R}$ does not feature in the equations, and there is no information on its mass held in the kinematic edges.

2.3 ATLAS reconstruction of the edges

ATLAS have calculated the expected positions of the $m_{l}^{\text{edge}}$, $m_{llq}^{\text{edge}}$, $m_{lq}^{\text{thr}}$, $m_{lq}^{\text{edge}(\text{low})}$ and $m_{lq}^{\text{edge}(\text{high})}$ mass distributions. We re-calculate these using the spectrum obtained for the SU3 point from S0FTSUSY3.1.7 [29]. We take into account the possibility that $m_{\tilde{l}} > m_{\tilde{\chi}_2}$. 


leading to a three-body decay \cite{28}. Since it is not possible to reconstruct the individual squark masses or flavour, we consider $m_{\tilde{q}_L}$ to be the average of the masses of the $\tilde{u}_L$ and $\tilde{d}_L$ squarks, as do ATLAS. In Table 2 we show the positions of the edges as calculated by ATLAS, and those which we obtain from \texttt{SOFTSUSY3.1.7}. For the di-lepton edges, the \texttt{SOFTSUSY3.1.7} values are approximately 4 GeV higher than those given by ATLAS, and for the edges and thresholds involving quarks the discrepancy is larger, around 20-30 GeV.

We have also checked that all the models possess the necessary mass ordering for all edges to exist simultaneously in at least some part of their parameter space. For instance, in mAMSB this can be achieved when $m_{\text{aux}}/m_0 \sim 10$.

With the SU3 spectrum, ATLAS simulated 1 fb$^{-1}$ of LHC data at $\sqrt{s} = 14$ TeV centre of mass energy and simulated the reconstruction of the positions of the edges and thresholds. Full details are available in \cite{26}. The results of this reconstruction are shown in column three of Table 2, which shows the central values of the reconstructed edges and an estimate of the total error which is arrived at by combining in quadrature the estimated statistical, systematic and jet energy scale (JES) errors. For each edge, we further assume a theoretical error on the \texttt{SOFTSUSY3.1.7} prediction of the edge of half of the difference between \texttt{SOFTSUSY3.1.7} prediction and the number under the ATLAS theory column of the table. We fit the four SUSY breaking models listed in Section 1.1 to the reconstructed end-points in Table 2. We have thus neglected the correlations in JES and other systematic errors. This should be a reasonable approximation for our purposes, and is conservative in the sense that including the correlations would actually decrease the total error volume. Thus, if we conclude that two models may be discriminated by including the errors independently, we may conclude that the would also be discriminated by including the measurement correlations.

Table 2. This table shows the position of the endpoints and thresholds for the SU3 CMSSM point in GeV. The column labelled ‘ATLAS theory’ is as predicted by ISAJET7.75 \cite{30} and used in the experiment’s simulations. The simulations of SUSY signal events in 1 fb$^{-1}$ of 14 TeV LHC collisions yielded the values marked in the reconstruction column. The final column shows the SU3 values predicted by \texttt{SOFTSUSY3.1.7}.

| Mass Distribution | ATLAS theory | reconstruction | \texttt{SOFTSUSY3.1.7} |
|-------------------|--------------|----------------|------------------------|
| $m_{\ell L}^\text{edge}$ | 100.2 | 99.7 $\pm$ 1.4 | 103.9 |
| $m_{\tilde{q}_L}^\text{edge}$ | 501 | 517 $\pm$ 33.7 | 532 |
| $m_{\tilde{l}L}$ | 249 | 265 $\pm$ 23.7 | 265 |
| $m_{\tilde{l}L}(\text{low})$ | 325 | 333 $\pm$ 11.7 | 344 |
| $m_{\tilde{l}L}(\text{high})$ | 418 | 445 $\pm$ 19.0 | 446 |

\footnote{The presence of the two-body versus the three-body decay can affect the shape of the distribution of the di-lepton invariant mass. We do not take this into account into our fits, considering only the position of the edge and not its shape.}
3 Inference and Fit Details

Assuming some model hypothesis $H$, Bayesian statistics helps update a probability density function (PDF) $p(m|H)$ of model parameters $m$ with data. The prior encodes our knowledge or prejudices about the parameters. Since $p(m|H)$ is a PDF in $m$, \( \int p(m|H)dm = 1 \), which defines a normalisation of the prior. One talks of priors being ‘flat’ in some parameters, but care must be taken to refer to the measure of such parameters. A prior that is flat between some ranges in a parameter $m_1$ will not be flat in a parameter $x \equiv \log m_1$, for example. The impact of the data is encoded in the likelihood, or the PDF of obtaining data set $\mathcal{d}$ from model point $\mathcal{m}$: $p(\mathcal{d}|\mathcal{m}, H) \equiv \mathcal{L}(\mathcal{m})$. The likelihood is a function of $\chi^2$, i.e. a statistical measure of how well the data are fit by the model point. One useful quantity is the posterior: the PDF of the model parameters $m$ given some observed data $\mathcal{d}$ and assuming hypothesis $H$: $p(m|\mathcal{d}, H)$. Bayes’ theorem states that

\[
p(m|\mathcal{d}, H) = \frac{p(\mathcal{d}|m, H)p(m|H)}{p(\mathcal{d}|H)},
\]

where $p(\mathcal{d}|H) \equiv Z$ is the Bayesian evidence, the probability density of observing data set $\mathcal{d}$ integrated over all model parameter space. The Bayesian evidence is given by:

\[
Z = \int \mathcal{L}(\mathcal{m})p(m|H) \, dm
\]

where the integral is over $N$ dimensions of the parameter space $m$. We note that the evidence depends upon the ranges of $m$ assumed.

In order to select between two models $H_0$ and $H_1$ one needs to compare their respective posterior probabilities given the observed data set $\mathcal{d}$, as follows:

\[
\frac{p(H_1|\mathcal{d})}{p(H_0|\mathcal{d})} = \frac{p(\mathcal{d}|H_1)p(H_1)}{p(\mathcal{d}|H_0)p(H_0)} \frac{Z_1 p(H_1)}{Z_0 p(H_0)},
\]

where $p(H_1)/p(H_0)$ is the prior probability ratio for the two models, which we set to unity as we adopt the position that no mechanism of mediation is a priori more likely than any other. It can be seen from Eq. 3.3 that Bayesian model selection revolves around the evaluation of the Bayesian evidence. As the average of likelihood over the prior, the evidence automatically implements Occam’s razor. A theory with fewer parameters has a higher prior density since it integrates to 1 over the whole space. Indeed, a theory with the same number of parameters, but larger a priori parameter ranges will have a correspondingly smaller evidence, for a similar reason, provided both ranges cover the high likelihood region. There is thus a preference for fewer parameters and smaller ranges, unless the data strongly require there be more. Evaluation of the evidence is a computationally intensive task, and specific algorithms are required to make it practically possible. We use the nested sampling approach of [31] to evaluate the evidence. A by-product of this approach is that it also produces posterior inferences. This method is implemented by the MultiNest algorithm of [32, 33] which we use in this paper.
The natural logarithm of the ratio of posterior model probabilities quantifies the level of discrimination between two models:

$$\Delta \log Z = \log \left[ \frac{p(H_1|d)}{p(H_0|d)} \right] = \log \left[ \frac{Z_1 p(H_1)}{Z_0 p(H_0)} \right].$$

(3.4)

We summarise the convention we use in this paper in Table 3.

In Bayesian model selection the results will always depend to some extent on the priors. Rather than seeking a unique ‘right’ prior, one should check the independence of conclusions with respect to a reasonable variation of the priors. Such a sensitivity analysis is required to ensure that the resulting model comparison is not overly dependent on a particular choice of prior and the associated metric in parameter space, which controls the value of the integral involved in the computation of the Bayesian evidence. Prior dependence has been studied in the CMSSM fitted to indirect data in [34], where it was demonstrated that the indirect data was not constraining enough to allow a prior-independent determination of the preferred regions of the parameter space. Prior dependence in parameter estimation was also treated in [35, 36], and in evidence evaluation in [18, 37].

We have considered two different prior PDFs in this analysis. The first is the standard “linear prior” where $p(m_1) = p(m_2)$ for $m_1, m_2$ being two different points in the parameter space of one of the models under consideration. We shall contrast the results with linear priors versus those with log priors: each parameter $m$ with dimensions of mass has a prior whose distribution is flat in $\log(m)$, except for $A_0$ in the CMSSM. $A_0 = 0$ requires a different treatment because of the singularity at 0 in $\log A_0$: we choose a prior that is flat in $\log(|A_0| + C)$. For this particular study, we pick $C = 60$ GeV, but the results are not at all sensitive to the value chosen (indeed, we shall see that they are not sensitive to the choice of log or flat priors - a much larger change).

Before proceeding, we specify the parameter ranges over which we sample for the different models. We consider only the positive sign of $\mu$, as it is well known that the kinematical edges we consider do not have the power to distinguish the sign of $\mu$. It is unlikely that the LHC will have enough data to distinguish different signs of $\mu$: given current search constraints where soft SUSY breaking terms are expected to be heavy, the sign of $\mu$ may only have a fairly small effect on aspects of the spectrum. It affects heavier chargino and neutralino masses and mixings, and the third family sfermion mixings, all of which will be difficult to measure accurately at the LHC (but which may well be accurately
measured at a future linear collider). The ranges over which we vary the continuous model parameters are shown in Table 4.

We bound $\tan \beta$ from below by 2, as values lower than this are in contravention of LEP2 Higgs searches, and from above by 62, since such large values lead to non-perturbative Yukawa couplings below the GUT scale and calculability is lost. In mGMSB the discrete parameter $N_{mess}$, the number of messenger multiplets, is varied between 1 and 8. Higher values of $N_{mess}$ lead to problems with perturbativity of gauge interactions at the GUT scale [21]. We wish to avoid possible contributions from gravity mediation in our GMSB fits. Gravity mediated contributions will always be present and of order $F/M_{Pl}$, where $\sqrt{F}$ is the supersymmetry breaking scale, and we require these contributions to the soft masses to be less than 1 GeV. This implies a maximum value of $F$ of around $10^{19}$ GeV. Since the mass scale $\Lambda = F/M_{mess} \sim 10^5$ GeV, we restrict $M_{mess}$ to be less than $10^{14}$ GeV. In the CMSSM the unification scale is the standard GUT scale $M_{GUT} \approx 2 \times 10^{16}$ GeV, while for the LVS the soft terms are defined at the intermediate string scale $m_s \approx 10^{11}$ GeV as in [25].

The constraints we use are all shown in Table 2. We treat the measurements $D_i$ of the observables as independent. We also assume Gaussian errors on all measurements. The pull of observable $i$ is calculated by

$$s_i = \frac{|c_i - p_i|}{\sigma_i},$$

where $c_i$ is the central experimental value of observable $i$, $p_i$ is the prediction of it by the model point and hypothesis assumed and $\sigma_i$ is the standard deviation incorporating both experimental and theoretical uncertainties, added in quadrature. The pull is a measure of how far the prediction is from the central experimental value in comparison to the error. In the limit of large statistics, where the experimental measurements have Gaussian probability distributions, $\chi^2 = \sum_i s_i^2$ follows a well-known (\'\chi^2\') distribution. The log likelihood of a prediction $p_i$ of an observable $i$ is given by

$$\log \mathcal{L}_i = -\frac{s_i^2}{2} - \frac{1}{2} \log(2\pi) - \log(\sigma_i)$$

Table 4. Ranges for the parameters in mGMSB and the Large Volume Scenario. In mGMSB we also vary the discrete parameter $N_{mess}$ between 1 and 8. For all models, $2 \leq \tan \beta \leq 62$. 

| CMSSM                  | mAMSB                  | mGMSB                  | LVS |
|------------------------|------------------------|------------------------|-----|
| 1 GeV $\leq m_0 \leq$ 2 TeV | 1 GeV $\leq m_0 \leq$ 2 TeV | 1 GeV $\leq m_0 \leq$ 2 TeV |     |
| 60 GeV $\leq m_{1/2} \leq$ 2 TeV | 20 TeV $\leq m_{aux} \leq$ 100 TeV | 20 TeV $\leq m_{aux} \leq$ 100 TeV |     |
| 20 TeV $\leq $ 4 TeV $\leq A_0 \leq$ 4 TeV | $10^4$ GeV $\leq \Lambda \leq$ $10^6$ GeV | $10^5$ GeV $\leq M_{mess} \leq$ $10^{14}$ GeV | 1 GeV $\leq m_0 \leq$ 2 TeV |
The combined log likelihood is the sum of the individual log likelihoods,

$$\log L^\text{tot} = \sum_i \log L^i.$$  \hspace{1cm} (3.7)

We do not use any indirect observables in this article. If an edge or threshold is not present due to the mass ordering in the spectrum, the likelihood of that point is set to zero. Eq. 3.7 amounts to assuming that the measurements of each end-point are independent. This is not strictly true: jet-energy scale errors, for instance, will tend to correlate $m_{lq}^\text{edge}$, $m_{lq}^\text{thr}$, $m_{lq}^\text{low}$ and $m_{lq}^\text{high}$, for instance. However, this is not expected to be a large effect, and neglecting the resulting correlation should yield a reasonable approximation. Correlations between the sparticle masses coming from the measurements are automatically taken into account by Eqs. 2.1-2.8.

Aside from the Bayesian evidence, we shall evaluate the comparative quality of fit of each model via the $p$-value of their best-fit points. For a given model, the best-fit point in parameter space is defined to be the one with the lowest $\chi^2$. The $p$-value is constructed as follows: it is the probability of obtaining $\chi^2$ at least as large as the one actually observed $\chi^2_0$, assuming the best-fit point of the hypothesised model:

$$p = \int_{\chi^2_0}^{\infty} \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2} \, dx,$$  \hspace{1cm} (3.8)

where $k$ is the number of degrees of freedom: the number of observables minus the number of parameters in the model. $p$-values do not depend upon priors. However, in common problems, the interpretation of the $p$-value is problematic because of the identification of the number of degrees of freedom. One could always add additional observables that are insensitive to the value of the model parameters at the best-fit point, changing the value of $p$, for instance. Also, the presence of physical boundaries may spoil the interpretation of $p$ as calculated in Eq. 3.8 [38]. Nevertheless, we use $p$-values as a qualitative estimator of the overall quality of the fit in each case: a small $p$-value indicates the fact that the model is not able to fit the data well, and a $p$-value closer to unity indicates that the model may fit it. We calculate the $p$-value by minimising the $\chi^2$ function using the minimiser MINUIT [39] (a particular configuration of MultiNest has also shown to be able to perform this task [40]). We use the point sampled by MultiNest during the evidence calculation with the highest likelihood as a starting seed.

Since we are assuming that the LHC measurements discover missing transverse momentum like signals, which yield SUSY signals leading to the endpoints detailed in Table 2, we require a neutral MSSM particle that is stable, at least on time-scales required for it to traverse an LHC detector. Therefore, in mAMSB, the CMSSM and the LVS the neutralino must be the lightest supersymmetric particle (LSP), or else we set the likelihood to zero. In mGMSB the gravitino $\tilde{G}$ is the LSP, and the collider signatures are to a large part determined by the identity of the next-to-lightest supersymmetric particle (NLSP). If the stau is the NLSP we reject the point, assigning it a zero likelihood. If the neutralino is the NLSP we consider its decay length. If the (bino-like) neutralino decays inside the detector, then the classic di-photon and missing transverse energy of low scale mGMSB is
realised, in contradiction to the signals that we assume from Ref. [26]. We therefore ensure that in mGMSB the NLSP is the neutralino and that it is stable on detector time-scales. Specifically, we calculate the decay length of the neutralino according to [41], where

$$L_{\text{decay}} = \frac{1}{\kappa_\gamma} \left( \frac{100 - \text{GeV}}{m_{NLSP}} \right)^5 \left( \frac{\Lambda}{100 - \text{TeV}} \right)^2 \left( \frac{M_{\text{mess}}}{100 - \text{TeV}} \right)^2 10^{-4} \text{ m.}$$ \hspace{1cm} (3.9)

where $\kappa_\gamma$ is the photino component of the neutralino, since in mGMSB the neutralino NLSP is predominantly photino-like. If the decay length is less than ten metres we reject the point. We also apply some simple direct search bounds, adapted from [18, 42, 43]. If a sparticle mass falls these bounds, the corresponding point is assigned a zero likelihood.

To calculate the MSSM spectrum we use SOFTSUSY3.1.7 which calculates the spectrum of the CMSSM, mAMSB and mGMSB. By modifying the unification scale from $M_{GUT}$ to $m_{\text{string}} \sim 10^{11}$ GeV and by not enforcing gauge coupling unification, SOFTSUSY3.1.7 can also provide the spectrum in the LVS case. Parameter space points which do not break electroweak symmetry correctly or have tachyonic sparticles are assigned zero likelihood. However, this disallowed part of parameter space is included in our calculation of the prior volume and so will consequently reduce the evidence. Points which have a charged LSP are rejected.

4 Fits to Edge Data

4.1 Hypothesis Testing

The Bayesian evidence values calculated for the different models and priors are shown in Table 5. Although there is a small dependence of the evidence upon the prior, there is a much larger difference between the evidences of some of the models and so we may expect to reliably discriminate between them on that basis. One would strongly discriminate against LVS and mAMSB in favour of SU3 based either on the Jeffreys’ scale of Bayesian evidence differences or on the $p-$values. However, we see that we would not discriminate between mGMSB and the CMSSM using the evidence. Reassuringly, the $p-$values point in the same direction as the Bayesian evidence: mAMSB and LVS would be discriminated against, but mGMSB and the CMSSM could not be distinguished on the basis of edge data alone. The agreement of the interpretation of the naive frequentist ($p-$value) and Bayesian (evidence) measures of hypothesis test is another signal that the fits are fairly robust, together with their approximate prior independence.

4.2 Best Fit Points

The best-fit points along with their $\chi^2$ values divided by the number of degrees of freedom ($\chi^2$/d.o.f) and the associated $p-$value are shown in Table 6. The table illustrates that SOFTSUSY3.1.7 is able to fit the $\mu > 0$ CMSSM to the assumed edge variables extremely

Due to the small neutralino-chargino splitting in mAMSB we must reject any points that would violate the long-lived charged stable particle bounds from Tevatron, which requires $\Delta m = m^\chi^+_{1} - m^\chi^0_{1} > 50$ MeV. In practice, we find that this bound does not constrain the mAMSB parameter space since mAMSB predicts larger splittings [44].
Table 5. Hypothesis testing statistics for the different models. The columns labelled $Z$ show the Bayesian evidence for either linear or logarithmic priors. The error on each entry of the Bayesian $\log Z$ delivered by MultiNest is $\pm 0.1$.

$$\begin{array}{|l|c|c|c|}
\hline
\text{Model} & \log Z(\text{linear}) & \log Z(\text{logarithmic}) & p\text{-value} \\
\hline
\text{CMSSM} & -28.1 & -25.1 & 0.64 \\
\text{mGMSB} & -27.1 & -25.8 & 0.83 \\
\text{mAMSB} & -55.7 & -54.1 & <10^{-10} \\
\text{LVS} & -47.0 & -47.0 & 1.4 \times 10^{-9} \\
\hline
\end{array}$$

Table 6. Best-fit points (defined as having the highest likelihood) for each model, along with the associated value of $\chi^2$/d.o.f and $p$-value. We have assumed that $\mu > 0$ for each point.

$$\begin{array}{|l|l|c|c|}
\hline
\text{Model} & \text{parameters} & \chi^2/\text{d.o.f} & p\text{-value} \\
\hline
\text{CMSSM} & m_0 = 92.1 \text{ GeV}, m_{1/2} = 300.6 \text{ GeV} \\
& A_0 = 984 \text{ GeV}, \tan \beta = 12.3 \\
& 0.22/1 & 0.64 \\
m\text{AMSB} & m_{\text{aux}} = 28.46 \text{ TeV}, m_0 = 255.5 \text{ GeV} \\
& \text{tan } \beta = 22.4 \\
& 52/2 & <10^{-10} \\
m\text{GMSB} & M_{\text{mess}} = 1.0 \times 10^{14} \text{ GeV}, \Lambda = 1.78 \times 10^4 \text{ GeV} \\
& N_5 = 5, \text{tan } \beta = 22.2 \\
& 0.36/2 & 0.83 \\
\text{LVS} & m_0 = 359 \text{ GeV}, \tan \beta = 4.75 \\
& 44.2/3 & 1.4 \times 10^{-9} \\
\hline
\end{array}$$

well, despite the fact that they were produced by a different SUSY spectrum calculator. This is implied by the statement that there are only small differences in the masses of sparticles appearing in the golden decay chain between the spectrum calculators anyway, as Ref. [13] shows. Performing another fit for $\mu < 0$, we confirm our earlier assertion that the edges we study are not sensitive to the sign of $\mu$, obtaining a total $\chi^2$ of 0.14 and a $p$-value of 0.71. Similar fits are obtained for the other models under study for $\mu < 0$ as for $\mu > 0$, and so we simply show results of the fits for $\mu > 0$. Non-LHC data may separate the two signs of $\mu$: famously, the anomalous magnetic moment of the muon is sensitive to it (and prefers $\mu > 0$ in the CMSSM). Also, linear collider measurements of neutralinos and charginos may accurately constrain all of the parameters appearing in their mixing matrices, including $\mu$ [45]. While we display only the absolute best-fit point in the table for mGMSB, there are in fact best-fit points for $N_5 = 3, 4$ and 6 which have $p$-values larger than 0.05, indicating that one would not necessarily discriminate against mGMSB with these values of $N_5$ either.

We plot the spectra of the CMSSM and mGMSB best-fit points in Fig. 2. The decays were calculated with SDECAY 1.3b [46], and we display only those decays whose branching ratios are higher than 10%. The figure shows that the two best-fit spectra and decays are remarkably similar, and could prove difficult to discriminate. Although the heavier third generation squarks are somewhat heavier in mGMSB, they may be difficult to access experimentally because decays to them from the gluino are phase-space suppressed. Although,
Figure 2. Spectra and decays in the best-fit points of the CMSSM (top panel) and mGMSB (bottom panel). Here, the super partners are displayed with tildes, unlike in the rest of the paper.

in the mGMSB panel, the decay of $\chi^0_1$ to gravitino (ejecting a photon) is shown, the neutralinos are actually quasi-stable and so this decay will not show up in the experiment. We find that the decay length of the neutralino for the best-fit point is about 12.5AU, due to the very high messenger scale. The splitting between gluino and first two generations of squark (denoted $\tilde{q}_L$ and $\tilde{q}_R$ respectively, in the figure) are smaller for mGMSB, which could potentially make one of the jets from gluino decay softer, so there potentially could be a potential discriminator in the hardness of this jet, or indeed the multiplicity from gluino decays, if the jet is too soft to make it past jet cuts. A feasibility study of experimental separation between these two models would require a detailed study, and is beyond the
Figure 3. Renormalisation of CMSSM and mGMSB best-fit points. We show the most relevant DR mass parameters as a function of the renormalisation scale \( \mu \) for each model. The CMSSM model curves continue to \( \ln(\mu/\text{GeV}) \approx 37 \), whereas the mGMSB model curves terminate at \( \ln(\mu/\text{GeV}) \approx 32 \).

What leads to the similarities between the mGMSB and CMSSM best-fit points’ spectra? In the CMSSM the soft-terms run from the GUT scale, while in mGMSB they run from the messenger scale \( M_{\text{mess}} \). We observe that the messenger scale of the mGMSB best-fit point is as close as possible to the GUT scale given the range assumed in Table 4, \( M_{\text{mess}} = 1 \times 10^{14} \text{ GeV} \). Working to one-loop order, since the ratio of each MSSM group’s gaugino mass \( M_i \) to its gauge coupling squared \( g_i^2 \) does not run, if there exists a renormalisation scale \( \mu = \mu_0 \) for which

\[
\frac{M_3(\mu)}{g_3(\mu)^2} = \frac{M_2(\mu)}{g_2(\mu)^2} = \frac{M_1(\mu)}{g_1(\mu)^2},
\]

then Eq. 4.1 applies for any \( \mu \), in particular at the weak scale. In the CMSSM, Eq. 4.1 is satisfied because \( M_3(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_1(M_{\text{GUT}}) \) as well as \( g_3(M_{\text{GUT}}) = g_2(M_{\text{GUT}}) = g_1(M_{\text{GUT}}) \), whereas the mGMSB soft SUSY breaking boundary conditions are \( M_i(M_{\text{mess}}) = N_5 A g_i^2(M_{\text{mess}}) f / (16\pi^2) \) [41], where \( f \) is a dimensionless number depending upon parameter space (but not on the gauge group \( i \)). The mGMSB gaugino masses thus explicitly satisfy Eq. 4.1 in a different way to the CMSSM at \( \mu = M_{\text{mess}} \). Numerically, substituting \( \mu = M_Z \) into Eq. 4.1 leads to the approximate pattern \( M_3 : M_2 : M_1 \sim 6 : 2 : 1 \) for the weak-scale gaugino masses, which applies to both mGMSB and the CMSSM.

The high-scale scalar mass boundary conditions in mGMSB have more complicated expressions than in the CMSSM, as they depend on the quadratic Casimir operators and \( g_i \). They are not universal at the GUT scale. We find that the SUSY breaking right-handed slepton mass parameter for the best-fit mGMSB point at the GUT scale is 92.8 GeV, close to the CMSSM value of 92.4 GeV. The left-handed slepton mass parameters are somewhat larger as they are charged under \( SU(2) \), but at the weak scale it is the right-handed sleptons which are lightest and whose mass parameters we use to calculate the edge positions. This
difference therefore does not affect the quality of the fit. The mGMSB squark masses at the messenger scale are significantly different to the CMSSM squark masses which are given by $m_0$. However, during the renormalisation group running the squark masses are renormalised by the contributions from the gluino, and thus at the low scale the squark masses for both model points are similar to the gluino mass. Finally, the trilinear $A$-terms differ for the CMSSM and mGMSB best-fit points, but they affect the end-points by less than 1%. We display the renormalisation of the most relevant mass parameters in Fig. 3.

Since $\chi^0_1$ and $\chi^0_2$ are approximately bino and wino-dominated respectively, tuning $\Lambda$ allows mGMSB to match both gaugino masses to the ones required by our benchmark CMSSM point in the $2:1$ ratio that applies to both models. The other messenger scale scalar masses are fixed, but we may then tune $M_{mess}$ to get one of them (say, $m_{\tilde{\tau}_R}(M_{mess})$) to match with its equivalent value in the CMSSM benchmark. The other mass (in this case $m_{\tilde{g}_R}(M_{mess})$) is then predicted by mGMSB, and must renormalise (within an accuracy dictated by the measurement errors) to the tree-level value in the CMSSM benchmark model. We see from Fig. 3, that this is indeed the case.

The pulls from each observable $s_i$ are displayed in Fig. 4 for the best-fit mGMSB and CMSSM models. We see a similar pattern for each of the observables except for $m_{ll}^{\text{edge}}$, which is larger for mGMSB. However, it is clear that each of the observables is well-fit by each best-fit model, with no one observable dominating the $\chi^2$. Note that, even though mGMSB has a higher value of $\chi^2$, it has a slightly higher $p$-value because it has less free continuous parameters, and therefore a larger number of degrees of freedom.

We note that the edge information is not the only information one would collect about the models to use to discriminate them. Before sufficient statistics have been collected to constrain the kinematic edges, we would have rate data on the number of signal events passing cuts in missing transverse momentum type searches. The production rates of supersymmetric particles at the LHC typically dominantly depend upon the squark and gluino masses, since these are the strongly interacting particles with the largest direct production cross sections. They then decay in various ways into different channels. The rates for the individual channels do have a complicated dependence on the detailed MSSM parameters, but still: all channels are proportional to the total SUSY production cross section, which is a function of squark and gluino masses only, to a good approximation. Therefore the
total SUSY production cross-section is a function of squark and gluino masses, and we compare them at the best-fit points of the CMSSM and mGMSB models in Table 7. We also show the total next-to-leading order SUSY production cross-section as calculated by PROSPINO [47]. This is the cross-section without cuts or acceptance corrections, so the measurable cross-section will be some factor times smaller (around 30 in some examples). We see from Table 7 that the CMSSM and mGMSB have similar squark and gluino masses, resulting in a similar total SUSY LHC production cross-section. Thus, the models would likely require other more detailed empirical information to tell them apart. We have shown the gluino and squark masses obtained in the LVS best-fit model, because they are not yet ruled out at 95% confidence level by 165 pb$^{-1}$ of LHC data [48], unlike the best fit mGMSB and CMSSM points. If we scaled up the masses of all sparticles at the mGMSB and CMSSM points so that squarks and gluino masses are similar to the LVS best-fit point, we would have a total SUSY cross-section of around 1/10th of the value that SU3 has. If the number of events past cuts just scaled with the total SUSY cross-section, we would then expect to require 10 fb$^{-1}$ of LHC data in order to achieve similar fractional precisions on the end-points as the ones assumed in the present paper. Of course, a dedicated simulation of LHC collisions would be required to calculate this number more exactly and to verify that for heavier spectra mGMSB is indeed able to emulate the CMSSM spectrum.

### 4.3 Posterior Distributions for CMSSM and mGMSB

We now discuss some features of the posterior distributions for the models that are difficult to discriminate: the CMSSM and mGMSB. We do not present the frequentist bounds upon the parameters using $\Delta \chi^2$ because it has poor coverage properties [38]. Figure 5 shows the 2D posterior for log priors in the $m_0$-$m_{1/2}$ plane for the correct hypothesis for SUSY breaking, the CMSSM. It also shows the 95% Bayesian confidence interval contours for both sets of priors. The posterior is a localised single mode distribution, and the two contours lie on top of one another, demonstrating prior independence in this plane. This is not the case for the trilinear couplings $A_t$ which are not well constrained by the edges, because these parameters have only a small effect on the mass spectrum to which our fits are sensitive. Our posteriors are in agreement with previous fits of the CMSSM using kinematic invariants [7, 16, 19].

Turning to mGMSB, Figure 6 shows 1D posteriors for the mass scale $\Lambda$ and the logarithm of the messenger scale $\log_{10}(M_{mess})$ in GeV. We see from the left-hand panel that,

|     | CMSSM | mGMSB | LVS  |
|-----|-------|-------|------|
| $m_{\tilde{q}}$/GeV | 716   | 686   | 1116 |
| $m_{\tilde{g}}$/GeV | 662   | 662   | 1019 |
| $\sigma_{NLO}$/pb   | 22    | 25    | 1.7  |

**Table 7.** Mass spectra and total next-to-leading order SUSY 14 TeV LHC production cross-section $\sigma_{NLO}$ of the best-fit points for the CMSSM, mGMSB and LVS models in GeV. $m_{\tilde{q}}$ is an averaged first family squark mass.
in contrast to the CMSSM, the posterior is strongly multi-modal, irrespective of prior. This is because the physical masses in mGMSB are proportional to \( \Lambda \), and \( N_{mess} \) is a discrete parameter. Each peak in the posterior for \( \Lambda \) corresponds to a different value for \( N_{mess} \), with lower values of \( \Lambda \) being associated with higher values of \( N_{mess} \), since their product must be the mass scale given by the edge measurements. In the right-hand panel, we display the posterior of \( M_{mess} \) separated according to different values of \( N_{5} \), as well as summed (‘Total’). The \( M_{mess} \) posterior extends down to \( 10^{11} \) GeV, having some substructure due to overlapping modes. There is a positive correlation between \( M_{mess} \) and \( N_{mess} \). From this we can infer that value of \( N_{mess} \) larger than five would only be favoured with unfeasibly high messenger scales. Low values of \( M_{mess} \) require lower values of \( N_{mess} \) in order to fit the data. Indeed, the MultiNest algorithm identifies modes with \( N_{mess} = 1, 2 \), but these modes are of poor fit quality compared with those of intermediate messenger number \( N_{mess} = 3 − 6 \). This is a salutary lesson that fitting a low dimensional model to constraining data can still lead to a complicated mode structure in the posterior.

5 Conclusions

We have evaluated the ability of the LHC, through the measurement of kinematic end points in supersymmetric signals, to distinguish between different models of supersymmetry breaking with a small number of parameters. We find that the mAMSB and LVS models can be unambiguously discriminated from our CMSSM benchmark model by the end-points with just 1 fb\(^{-1}\) of data. However, kinematic edges could not discriminate between the best-fit CMSSM and mGMSB models, the spectra of which turn out to be very similar (except for the gravitino mass, which is irrelevant for LHC signals because the lightest neutralino is quasi-stable). Reassuringly, one reaches these conclusions whether or not one uses Bayesian
or frequentist statistics to perform the hypothesis test. This is additional confirmation that the sparticle spectrum is sufficiently constrained by the measurements in these models, and is confirmation of the fact that if a fit has sufficient data, a Bayesian interpretation will be approximately prior independent and give the same results as a frequentist interpretation. A previous study [19], found a significant prior dependence in models of SUSY breaking that have more parameters than the CMSSM. This is not so surprising given that the number of parameters would outnumber the number of experimental constraints. In that case, we would not even be able to calculate the $p$-value, since the number of degrees of freedom would be negative, and the system is under constrained.

The best-fit mGMSB and CMSSM spectra look remarkably similar, and a dedicated analysis is required to see if LHC data can tell them apart, which looks a priori unlikely. It should be possible to use a future direct detection of dark matter consistent with the CMSSM lightest neutralino mass in order to discriminate against mGMSB, whose gravitino LSP predicts zero direct detection cross-section because it interacts too weakly\textsuperscript{3}. Another possible future extension of this work is to perform a simulated experimental study of the best-fit mGMSB and CMSSM models, in order to see if there are any observables that could discriminate between their rather close spectra and decays. One could also attempt to answer the question: what sub-space of the CMSSM parameter space predicts observables that are close to those of mGMSB? We would not expect mGMSB to be able to mimic a focus-point spectrum with large $m_0$ but $m_{1/2}$ moderate for example, since this would result in a rather hierarchical mass pattern, which the relatively compressed spectra of mGMSB may find hard to reproduce. It is also true however, that the focus point does

\textsuperscript{3}A recent study showed that forthcoming ton scale direct detection experiments will probe the majority of the CMSSM parameter space that currently fits indirect data well [49]
not possess the golden decay chain and so different observables to the ones studied here would have to be examined.

Kinematic end-points of cascade decays are arguably the best tool for discriminating different SUSY breaking models from LHC data, since they are sensitive to the sparticle spectrum and do not require several hundred fb$^{-1}$ of integrated luminosity in order to parameterise the detector response well. In the case that other cascades than the golden one assumed here are present and identifiable, one would include their data. The fit is still likely to be dominated by the constraints coming from the golden cascade, however. The golden cascade utilised here may not be present, even in the event of a SUSY signal at the LHC. However, in that case other, less constraining cascades will be used but are unlikely to provide the discriminating power that the golden one does. This study is therefore an estimate of the maximum discriminatory power one could have.

In summary, although kinematic end-point data deliver important information on the nature of SUSY breaking (discriminating against mAMSB, LVS and the CMSSM), there still may exist degeneracies between some models (for example mGMSB and the CMSSM SU3 benchmark point). It would be interesting to see if a future linear collider with a sufficient centre of mass energy could help separate the models. Refs. [9–11] demonstrate that using LHC and linear collider data leads to accurate measurements of most of the SUSY spectrum, as long as the relevant sparticles are kinematically accessible at the linear collider. It was demonstrated how bottom-up renormalisation to high energies allows checks on unification relations in different models. Given that top-down model discrimination is much more constrained than the bottom-up analysis, and has many less parameters, it seems plausible that model discrimination could be reached by including the precise linear collider data. We leave a confirmation of this to a future study.

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References

[1] Atlas Collaboration Collaboration, G. Aad et. al., Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in sqrt(s) = 7 TeV proton-proton collisions, arXiv:1102.5290.

[2] CMS Collaboration Collaboration, V. Khachatryan et. al., Search for Supersymmetry in pp Collisions at 7 TeV in Events with Jets and Missing Transverse Energy, Phys.Lett. B698 (2011) 196–218, arXiv:1101.1628.

[3] D. Miller, P. Osland, and A. Raklev, Invariant mass distributions in cascade decays, JHEP 0603 (2006) 034, [hep-ph/0510356].

[4] I. Hinchliffe, F. Paige, M. Shapiro, J. Soderqvist, and W. Yao, Precision SUSY measurements at CERN LHC, Phys.Rev. D55 (1997) 5520–5540, [hep-ph/9610544].

[5] B. C. Allanach, C. G. Lester, M. A. Parker, and B. R. Webber, Measuring sparticle masses in non-universal string inspired models at the LHC, JHEP 09 (2000) 004, [hep-ph/0007009].
[6] B. K. Gjelsten, D. J. Miller, and P. Osland, Measurement of SUSY masses via cascade decays for SPS 1a, JHEP 12 (2004) 003, [hep-ph/0410303].

[7] C. G. Lester, M. A. Parker, and M. J. White, 2, Determining SUSY model parameters and masses at the LHC using cross-sections, kinematic edges and other observables, JHEP 01 (2006) 080, [hep-ph/0508143].

[8] LHC/LC Study Group Collaboration, G. Weiglein et. al., Physics interplay of the LHC and the ILC, Phys.Rept. 426 (2006) 47–358, [hep-ph/0410364].

[9] G. Blair, W. Porod, and P. Zerwas, Reconstructing supersymmetric theories at high-energy scales, Phys.Rev. D63 (2001) 017703, [hep-ph/0007107].

[10] G. Blair, W. Porod, and P. Zerwas, The Reconstruction of supersymmetric theories at high-energy scales, Eur.Phys.J. C27 (2003) 263–281, [hep-ph/0210058].

[11] G. Blair, W. Porod, and P. Zerwas, Reconstructing supersymmetric theories by coherent LHC / LC analyses, [hep-ph/0403133].

[12] B. Allanach, D. Grellscheid, and F. Quevedo, Selecting supersymmetric string scenarios from sparticle spectra, JHEP 0205 (2002) 048, [hep-ph/0111057].

[13] B. Allanach, S. Kraml, and W. Porod, Theoretical uncertainties in sparticle mass predictions from computational tools, JHEP 0303 (2003) 016, [hep-ph/0302102].

[14] P. Bechtle, K. Desch, M. Uhlenbrock, and P. Wienemann, Constraining SUSY models with Fittino using measurements before, with and beyond the LHC, Eur. Phys. J. C66 (2010) 215–259, [arXiv:0907.2589].

[15] R. Lafaye, T. Plehn, M. Rauch, and D. Zerwas, Measuring Supersymmetry, Eur. Phys. J. C54 (2008) 617–644, [arXiv:0709.3985].

[16] L. Roszkowski, R. Ruiz de Austri, and R. Trotta, Efficient reconstruction of CMSSM parameters from LHC data - A case study, Phys. Rev. D82 (2010) 055003, [arXiv:0907.0594].

[17] H. K. Dreiner, M. Kramer, J. M. Lindert, and B. O’Leary, SUSY parameter determination at the LHC using cross sections and kinematic edges, JHEP 04 (2010) 109, [arXiv:1003.2648].

[18] S. S. AbdusSalam, B. C. Allanach, M. J. Dolan, F. Feroz, and M. P. Hobson, Selecting a Model of Supersymmetry Breaking Mediation, Phys. Rev. D80 (2009) 035017, [arXiv:0906.0957].

[19] A. Fowlie and L. Roszkowski, Reconstructing ATLAS SU3 in the CMSSM and relaxed phenomenological supersymmetry models, [arXiv:1106.5117].

[20] L. Randall and R. Sundrum, Out of this world supersymmetry breaking, Nucl. Phys. B557 (1999) 79–118, [hep-th/9810155].

[21] G. F. Giudice and R. Rattazzi, Theories with gauge-mediated supersymmetry breaking, Phys. Rept. 322 (1999) 419–499, [hep-ph/9801271].

[22] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications, JHEP 03 (2005) 007, [hep-th/0502058].

[23] J. P. Conlon, F. Quevedo, and K. Suruliz, Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking, JHEP 08 (2005) 007, [hep-th/0505076].

[24] J. P. Conlon, S. S. Abdussalam, F. Quevedo, and K. Suruliz, Soft SUSY breaking terms for
chiral matter in IIB string compactifications, JHEP 01 (2007) 032, [hep-th/0610129].

[25] B. C. Allanach, M. J. Dolan, and A. M. Weber, Global Fits of the Large Volume String Scenario to WMAP5 and Other Indirect Constraints Using Markov Chain Monte Carlo, JHEP 08 (2008) 105, [arXiv:0806.1184].

[26] The ATLAS Collaboration, G. Aad et. al., Expected Performance of the ATLAS Experiment - Detector, Trigger and Physics, arXiv:0901.0512.

[27] M. J. Dolan, D. Grellscheid, J. Jaeckel, V. V. Khoze, and P. Richardson, New Constraints on Gauge Mediation and Beyond from LHC SUSY Searches at 7 TeV, arXiv:1104.0585.

[28] C. G. Lester, M. A. Parker, and M. J. White, 2, Three body kinematic endpoints in SUSY models with non-universal Higgs masses, JHEP 10 (2007) 051, [hep-ph/0609298].

[29] B. C. Allanach, SOFTSUSY: A C++ program for calculating supersymmetric spectra, Comput. Phys. Commun. 143 (2002) 305–331, [hep-ph/0104145].

[30] F. E. Paige, S. D. Protopopescu, H. Baer, and X. Tata, ISAJET 7.69: A Monte Carlo event generator for pp, anti-p p, and e+e- reactions, hep-ph/0312045.

[31] J. Skilling, Nested Sampling, in American Institute of Physics Conference Series (R. Fischer, R. Preuss, and U. V. Toussaint, eds.), pp. 395–405, Nov., 2004.

[32] F. Feroz and M. P. Hobson, Multimodal nested sampling: an efficient and robust alternative to MCMC methods for astronomical data analysis, arXiv:0704.3704.

[33] F. Feroz, M. P. Hobson, and M. Bridges, MultiNest: an efficient and robust Bayesian inference tool for cosmology and particle physics, arXiv:0809.3437.

[34] R. Trotta, F. Feroz, M. P. Hobson, L. Roszkowski, and R. Ruiz de Austri, The Impact of priors and observables on parameter inferences in the Constrained MSSM, JHEP 12 (2008) 024, [arXiv:0809.3792].

[35] B. C. Allanach, Naturalness priors and fits to the constrained minimal supersymmetric standard model, Phys. Lett. B635 (2006) 123–130, [hep-ph/0601089].

[36] B. C. Allanach, K. Cranmer, C. G. Lester, and A. M. Weber, Natural Priors, CMSSM Fits and LHC Weather Forecasts, JHEP 08 (2007) 023, [arXiv:0705.0487].

[37] S. S. AbdusSalam, B. C. Allanach, F. Quevedo, F. Feroz, and M. Hobson, Fitting the Phenomenological MSSM, arXiv:0904.2548.

[38] M. Bridges, K. Cranmer, F. Feroz, M. Hobson, R. de Austri, et. al., A Coverage Study of the CMSSM Based on ATLAS Sensitivity Using Fast Neural Networks Techniques, JHEP 1103 (2011) 012, [arXiv:1011.4306].

[39] F. James and M. Roos, Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations, Comput.Phys.Commun. 10 (1975) 343–367.

[40] F. Feroz, K. Cranmer, M. Hobson, R. de Austri, and R. Trotta, Challenges of Profile Likelihood Evaluation in Multi-Dimensional SUSY Scans, arXiv:1101.3296.

[41] S. Ambrosanio, G. D. Kribs, and S. P. Martin, Signals for gauge mediated supersymmetry breaking models at the CERN LEP-2 collider, Phys.Rev. D56 (1997) 1761–1777, [hep-ph/9703211].

[42] Particle Data Group Collaboration, K. Nakamura et. al., Review of particle physics, J. Phys. G37 (2010) 075021.
[43] S. Abel, M. J. Dolan, J. Jaeckel, and V. V. Khoze, *Phenomenology of Pure General Gauge Mediation*, JHEP 12 (2009) 001, [arXiv:0910.02674].

[44] T. Gherghetta, G. F. Giudice, and J. D. Wells, *Phenomenological consequences of supersymmetry with anomaly-induced masses*, Nucl. Phys. B559 (1999) 27–47, [hep-ph/9904378].

[45] K. Desch, J. Kalinowski, G. A. Moortgat-Pick, M. Nojiri, and G. Polesello, *SUSY parameter determination in combined analyses at LHC / LC*, JHEP 0402 (2004) 035, [hep-ph/0312069].

[46] M. Muhlleitner, A. Djouadi, and Y. Mambrini, *SDECAY: A Fortran code for the decays of the supersymmetric particles in the MSSM*, Comput. Phys. Commun. 168 (2005) 46–70, [hep-ph/0311167].

[47] W. Beenakker, R. Hopker, M. Spira, and P. M. Zerwas, *Squark and gluino production at hadron colliders*, Nucl. Phys. B492 (1997) 51–103, [hep-ph/9610490].

[48] Atlas Collaboration Collaboration, G. Aad et. al., *Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in sqrt(s) = 7 TeV proton-proton collisions*, ATLAS-CONF-2011-086.

[49] G. Bertone et. al., *Global fits of the cMSSM including the first LHC and XENON100 data*, arXiv:1107.1715.