Research on Position Error of External Radiation Radar Sensor Based on 3D Positioning Algorithm

Liu Duo
School of Electronic Engineering, Naval University of Engineering Wuhan, 430030, 1197635141@qq.com China

Abstract. The paper establishes a measurement-launch station correlation model of a single-frequency off-line radar system, derives the principles of the three-dimensional TOA positioning method and the two-dimensional TOA positioning method to estimate the target height, and gives the analysis of the GDOP positioning accuracy of the corresponding algorithm. Through the simulation verification of a three-transmission single-receiving external radiation source positioning model, the effects of different algorithms on positioning accuracy and measurement-transmission station correlation are analyzed. The results show that using the two-dimensional TOA positioning method to estimate the target height can improve the measurement-transmitting station correlation probability of the single-frequency off-line radiation source radar.

Keywords: Passive radar with external radiation source, passive positioning, time difference of arrival, azimuth of arrival, three-dimensional positioning algorithm.

1. Introduction
The traditional radar is a transceiver, which receives the target echo of its own transmitted signal, analyses the characteristic parameters of the target echo signal, and extracts the relevant information parameters of the target to achieve the positioning and tracking of the target. Because it needs to emit electromagnetic waves by itself, it is easy to be discovered, which greatly reduces its own viability. The external radiation source radar itself does not need to emit an electromagnetic wave signal, and the target can be detected by measuring the direction angle or the time difference of arrival of the third-party radiation source signal and the echo signal reflected by the target. It is difficult to be discovered by reconnaissance and can avoid being attacked, so the external radiation source radar system has a strong survivability.

The single-frequency external radiation source radar refers to an external radiation source radar that uses a single-frequency network radiation source, and belongs to a type of multi-base radar (multi-transmission single/multi-reception radar). It not only has the advantages of single-frequency network and traditional external radiation source radar, but also has the following advantages compared to multi-frequency external radiation source radar: 1) The system uses multi-transmission single-receiving mode and each echo signal is processed at the same time. The frequency network adopts the mode of multi-single-receiving (time-sharing processing) or single-receiving and multi-receiving, which has the advantages of high system performance and low hardware cost; 2) There is only one
transmitting station, the equipment is centralized, and the system maintenance is simple and convenient; 3) The received data is centralized in one node, the processing is centralized and the software cost is low; 4) There is less communication between the nodes and higher security (strong anti-radiation missile capability), and the multi-frequency network data is scattered among the nodes, and the needs between the nodes are jointly processed. Frequent communication, low security performance. In recent years, digital broadcasting and television signals have gradually replaced analogy signals. The single frequency network technology has been widely used because of its many advantages such as frequency saving and high spectrum utilization. The detection of external radiation sources based on single-frequency network digital broadcast TV signals has also become a research hotspot in recent years. This paper first models the measurement-transmission station correlation problem, then from the perspective of target positioning algorithm selection, analyses the positioning accuracy of different positioning methods and the impact on the measurement-transmission station correlation problem. The external radiation source positioning model was simulated and analysed [1].

2. Positioning of external radiation source radar sensor

2.1. Basic principles of three-dimensional scaling technology

The positioning algorithm only needs to obtain the location coordinates of the unknown target in the network based on the network connectivity and other information, which will reduce the requirements of the target hardware and make it more suitable for large-scale WSN. The positioning performance of the positioning algorithm without ranging is hardly affected by the environment. Even if its positioning accuracy will be reduced, it can meet the needs of some applications [2].

The three-dimensional scaling technology TOA is a data analysis technology that converts similar information between entities into spatial geometric information. It is often used in exploratory data analysis or information visualization. It was originally used in data analysis of psychometrics and is now used as a general purpose. The data analysis technology is widely used in various fields. Suppose the dissimilarity between the research subjects \(i\) and \(j\) is denoted by \(p_{ij}\), and the dissimilarity \(p_{ij}\) between the research subjects constitutes the dissimilarity matrix \(p_{ij}\). The coordinate matrix for constructing points on the three-dimensional space is represented by \(X_{nm}\), where \(n\) is the number of coordinate points, \(m\) is the dimension of the coordinate points, the coordinates of the physical objects \(i\) and \(j\) are \(K\), respectively, and the distance between \(L\) and \(M\) on the three-dimensional space is denoted by \(d_{ij}\). The three-dimensional scaling technique is to use the dissimilarity between the entities to construct the relative coordinate map of the points on the three-dimensional space, so that \(p_{ij}\) and \(d_{ij}\) are as close as possible. In the three-dimensional scale, the closeness is measured by the strength coefficient (STRESS). Define the threat coefficient as:

\[
\text{Stress} = \sum \left[ f(p_{ij}) - d_{ij} \right]^2
\]

The specific implementation is divided into four steps: (1) construct the dissimilarity matrix \(p_{ij}\); (2) multiply each element in matrix \(p_{ij}\) to obtain \(P^{(2)}\); (3) double-centralize \(P^{(2)}\), that is, both sides of \(E\) are multiplied by the centre at the same time. See (2) for the calculation formula of matrix \(J\) and \(J\).

\[
J = E - n^{-1}J
\]

Where \(E\) is the identity matrix of order \(n\), \(J\) is the full matrix of order, and matrix \(B\) after double centralization is shown in equation (3).
Perform singular value decomposition on $B$ to find the largest $w$ positive eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_w$ and corresponding $w$ eigenvectors $e_1, e_2, \ldots, e_w$. The $w$ vectors form a $w$-dimensional diagonal matrix $\Lambda$, and the $w$ eigenvectors form an $n \times w$-dimensional matrix $V$. The relative coordinates of the transmitting station are 

$$X = V\Lambda^{-1}$$

2.2. TOA positioning algorithm

The specific positioning process of the TOA algorithm is divided into the following stages: (1) Each transmitting station first uses the classic TOA algorithm to construct the initial local coordinate system. (2) Use the incremental greedy method to convert the initial local coordinates of all targets into global relative coordinates. (3) According to the coordinate information of the beacon transmitting station in the external radiation source radar sensor, convert the global relative coordinates into the initialized global absolute coordinate system. (4) According to the local attributes of the network, let the unknown target select its true 1-hop neighbour target to participate in the positioning solution. (5) Use the Gaussian kernel weighting mechanism to calculate the weight:

$$w_{ij} = \exp \left( \frac{\left(\hat{d}_{ij} - h_i^2\right)^2}{h_i^2} \right)$$

Where $\hat{d}_{ij}$ represents the measured distance (m) between the transmitting station $I$ and the target $j$; $h_i$ represents the maximum value $h_i = \max_j(\hat{d}_{ij})$ of the measured distance between the transmitting station $I$.

(6) Iteratively optimize the initial global absolute coordinates as the initial value of iteration to obtain the final global absolute coordinates.

The specific process of calculating the relative error of target positioning:

(1) Before each iteration, first calculate the average positioning error of the target according to equation (6)

$$AverageError(i) = \sum_{j=1}^{M} \left| d_{ij} - \hat{d}_{ij} \right| / M$$

In the formula: $M$ represents the number of neighbouring transmitting stations when participating in the positioning of the transmitting station $I$; $d_{ij}$ represents the Euclidean distance between the transmitting station $I$ and the neighbouring target $j$; $\hat{d}_{ij}$ represents the ranging or shortest path between the transmitting station $I$ and the neighbouring target $j$ distance.

(2) Use equation (7) to normalize the positioning error.

$$Norm(i) = \left\{ \sum_{j=1}^{M} \left| d_{ij} - \hat{d}_{ij} \right| \right\} / \left\{ \sum_{j=1}^{M} \hat{d}_{ij} \right\}$$
(3) Considering the location error of neighbor $j$ of unknown transmitting station $i$ and the error introduced by ranging $\hat{\delta}_{ij}$ between $j$ and $i$, the relative error of target $j$ relative to transmitting station $i$ can be calculated according to equation (7).

$$Res_{Error}(j) = AverageError(j) + Norm(i) \times \hat{\delta}_{ij}$$

(8)

With reference to the classic three-dimensional scaling technique, positioning is to locate unknown transmitters by minimizing the threat coefficient. In this paper, the threat coefficient is converted into a target cost function as shown in equation (9), and the target cost function is minimized by solving Conditions to locate unknown targets [3].

$$S = 2 \sum_{i=1}^{n} \sum_{j \in \text{transmitting stations}} w_{ij} \left( \hat{\delta}_{ij} - \|z_i - z_j\| \right)^2$$

(9)

Where $N$ represents the number of transmitting stations in the external radiation source radar sensor; $n$ represents the number of unknown targets; $\hat{\delta}_{ij}$ represents the measured distance between the transmitting station $i$ and the target $j$; $w_{ij}$ represents the weighted value calculated in the previous step (when the transmitting station $i$ and target $j$ are not neighbours, $w_{ij} = 0$ or does not exist); $\|z_i - z_j\|$ represents the calculation formula of the distance between the transmission station $i$ and the estimated position of the target $j$ as follows:

$$\|z_i - z_j\| = \sqrt{(x_i - x_j)^2 - (y_i - y_j)^2}$$

(10)

After a simple operation, the cost function can become equation (11).

$$S = \sum_{i=1}^{n} S_i + c$$

(11)

Where $c$ is a constant independent of the coordinates of the transmitting station, $S_i$ is the local cost function of each unknown target, and its expression is shown in equation (12).

$$S_i = \sum_{j=1, j \neq i}^{n} w_{ij} \left( \hat{\delta}_{ij} - \|z_i - z_j\| \right)^2 + 2 \sum_{j=1}^{N} w_{ij} \left( \hat{\delta}_{ij} - \|z_i - z_j\| \right)^2$$

(12)

When there is no beacon transmitting station, that is, $N=n$, the relationship between $S$ and $S_i$ is (13).

$$\frac{\partial S}{\partial z_i} = 2 \frac{\partial S_i}{\partial z_i}$$

(13)
It can be seen from the above relationship that when each \( S_i \) takes the minimum value, \( S \) will obtain the minimum value, and then the position coordinates of the corresponding unknown target can also be obtained by minimizing the local cost function of each unknown target [4].

Next, the simplest and fastest method among the unconstrained optimization methods is used to represent the steepest descent method to optimize and solve its local cost function.

In numerical analysis, the basic idea of the iterative optimization method is: given the initial point \( z^{(0)} \), a series of points \( \{z^{(k)}\} \) are obtained. When \( k \to \infty \), \( z^{(k)} \) is the optimal solution when Eq. (12) takes the minimum value. The iterative formula is shown in Eq. (14).

\[
z^{(k+1)} = z^{(k)} + \alpha d_k
\]  

(14)

Where \( k \) is the number of iterations of the \( k \) cycle, \( z^{(k)} \) is the result of the \( k \) iteration, \( \alpha_k \) is the step size of the \( k+1 \) iteration, and \( d_k \) is the direction of the search. Let \( D^{(k)} = (z_1^{(k)} - z_1^{(k)}, z_2^{(k)} - z_2^{(k)}, \ldots, z_N^{(k)} - z_N^{(k)}) \) and \( B^{(k)} = (b_1^{(k)}, b_2^{(k)}, \ldots, b_N^{(k)}) \), then the coordinate \( z_i \) of the transmitting station \( i \) according to equation (14) satisfies equation (15). The method of choosing the iteration step size \( \alpha^{(k)} \) is to make the objective function \( S_i \) the smallest. The calculation formula is shown in equation (16).

\[
z_i^{(k+1)} = z_i^{(k)} - \alpha^{(k)} \times D^{(k)} \times B^{(k)}
\]  

(15)

\[
S_i \left( z_i^{(k)} - \alpha^{(k)} \times D^{(k)} \times B^{(k)} \right) = \min_{\alpha \geq 0} S_i \left( z_i^{(k)} - \alpha^{(k)} \times D^{(k)} \times B^{(k)} \right)
\]  

(16)

\[
\phi(\alpha) = z_i^{(k)} - \alpha \times D^{(k)} \times B^{(k)}
\]  

(17)

In this paper, the linear search is performed by quadratic interpolation, and the function values of \( \phi(\alpha) \) at \( \alpha_i, \alpha_{i-1}, \alpha_{i-2} \) are respectively \( \phi(\alpha_i), \phi(\alpha_{i-1}), \phi(\alpha_{i-2}) \) and Equation (18) can be obtained by using quadratic interpolation.

\[
\begin{align*}
\alpha_{i+1} &= \frac{1}{2} \left( \alpha_i + \alpha_{i-2} - \frac{c_i}{c_2} \right) \\
c_1 &= \frac{\phi_{i-2} - \phi_i}{\alpha_{i-2} - \alpha_i} \\
\phi_{i-1} &= \phi_i - \alpha_i \\
c_2 &= \frac{\alpha_{i-1} - \alpha_{i-2}}{\alpha_{i-1} - \alpha_{i-2}}
\end{align*}
\]  

(18)
Calculate $\phi_{i+1} = \phi(\alpha_{i+1})$, and then remove the point with the largest value of the corresponding function from $\alpha_i$, $\alpha_{i-1}$, $\alpha_{i-2}$ and substitute the new point $\alpha_{i+1}$ to get the new three points, and iteratively execute the above calculation process until the required accuracy is met \[5\].

3. Experimental simulation

The simulation scene is set as follows: There are 10 external radiation sources, 1 observation station, and 8 targets to be estimated in the scene. The positions of the external radiation sources are randomly distributed on the 30km×30km plane. The geometric distribution is shown in Figure 1. The distance and system error are set to [3, 1, 5, 3, 4, 4, 1, 2, 3, 4], the unit is km; the azimuth system error is set to [0.04, 0.04, 0.02, 0.02, 0.03, 0.03, 0.01, 0.01], the unit is rad. The measurement errors of the angle of arrival and the distance sum follow the Gaussian distribution with zero mean. The standard deviation of the angle of arrival measurement error is set to 0.015 rad to 0.040 rad, and the standard deviation of distance and measurement error $\sigma_t$ is set to 0.06 km to 0.30 km.

Receiving station coordinates (30, 40, 0.1) km, transmitting station 1 coordinates (-40, 30, 0.1) km, transmitting station 2 coordinates (0, 0, 0.2) km, transmitting station 3 coordinates (50, 0, 0.1) km, distance and measurement error 30m, site error 10m, flight height 7 to 11km randomly selected. The simulation analyses the performance of the two positioning methods from the two-dimensional positioning error and the estimated distance and error of the target. Among them, the two-dimensional positioning error is the difference between the target estimated value and the real value obtained by the positioning method, which is $\|\hat{x}_t - x_t\|$, and the distance and error are the difference between the estimated distance and the target distance and the measured value, which is $\|\hat{R} - R\|$, where $\hat{R}$ represents positioning. Then the target distance and information are solved by the estimated target position. Distance and error characterize the measurement-transmitting station correlation performance of the single-frequency off-line radiation source radar. The smaller the error, the better the performance. Figure 2 shows the simulation results of the positioning errors of the two positioning methods \[6\].

![Figure 1. Spatial position distribution of external radiation source-target-observation station](image-url)
4. Conclusions
In this paper, a single-frequency external radiation source radar measurement-transmission station correlation model is established, and the impact of measurement-transmission station correlation problems on subsequent data processing is analysed. Then, from the perspective of target positioning algorithm selection, the positioning accuracy of different positioning methods and the impact on the measurement-transmission station correlation problem are analysed, and simulation verification is carried out through a three-transmission single-receiving single-frequency external radiation source radar positioning model. The comprehensive two-dimensional positioning accuracy, target distance and estimation error of the target can be found. The two-dimensional positioning method for estimating the height of the target is more suitable for the positioning and measurement of the single-frequency off-line source radar-deblurring of the transmitting station.

References
[1] Nurge, M. A., Youngquist, R. C., & Starr, S. O. A satellite formation flying approach providing both positioning and tracking. Acta Astronautica, 122(5) (2016) 1-9.
[2] Roberts, G. W., Tang, X., & Brown, C. A review of satellite positioning systems for civil engineering. Proceedings of the Institution of Civil Engineers - Civil Engineering, 168(4) (2015) 185-192.
[3] Kjerstad, N. Satellite performance and positioning challenges at high latitudes. European journal of navigation, 13(2) (2015) 13-17.
[4] Sadeghi, M., Behnia, F., & Haghshenas, H. Positioning of geostationary satellite by radio interferometry. IEEE Transactions on Aerospace and Electronic Systems, 55(2) (2019) 903-917.
[5] Causa, F., Renga, A., & Grassi, M. Robust filter setting in gps-based relative positioning of small-satellite leo formations. Advances in space research, 62(12) (2018) 3369-3382.
[6] Kondratiuk, V., Kovalevskiy, E., & Ilnytska, S. Integrated positioning system with restricted access to navigation satellite signals. Nephron Clinical Practice, 5(1) (2017) 60-66.