Students’ errors in solving combinatorics problems observed from the characteristics of RME modeling

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Abstract. This article was written based on the learning evaluation results of students’ errors in solving combinatorics problems observed from the characteristics of Realistic Mathematics Education (RME); that is modeling. Descriptive method was employed by involving 55 students from two international-based pilot state senior high schools in Banten. The findings of the study suggested that the students still committed errors in simplifying the problem as much 46%; errors in making mathematical model (horizontal mathematization) as much 60%; errors in finishing mathematical model (vertical mathematization) as much 65%; and errors in interpretation as well as validation as much 66%.

1. Introduction
Combinatorics is an initial material about probability consisting of basic rules of counting, permutation, and combinations. Probability is very useful concept in everyday life as its role is very important in decision making. At a higher level, permutation and combination materials are the basic materials in learning discrete mathematics [5, 11]. Combinatorics is a branch of contemporary mathematics in which it is widely used in business and industry [11]. Moreover, NCTM states that combinatorics is important as it is an integral part of the mathematics curriculum from the elementary level to the high school level [11].

Using the basic rules of calculation, permutations, and the right combination of problem solving skills are challenging. This is in line with Wraughton & Nolan [14] that understanding the rules of counting is challenging for students especially when they are struggling to determine when and how to apply rules of multiplication, permutation, and combinations in calculation.

According to Mashiach Eizenberg, M., & Zaslavsky, O. [10], combinatorics is one of the mathematical topics that is considered difficult to teach and learn. To overcome such challenges in learning combinatorics, one of the ways is to present the combinatorics problem from daily life (contextual). In studying permutations and combinations, students need real-world context that are relevant to stimulate learning and knowledge [3]. Along with this, Sukorianto et al. stated that permutation and combination materials require problems in the form of the real-world issues so that they can retain their knowledge [12].

In order for the real-world problems get mathematical solutions, it takes a simplification of the real situation into a mathematical form called modeling. This is in line with Gravemeijer [6] who stated that modeling is a bridge to convert contextual problems into formal forms. According to Heuvel-Panhuizen [13] and Maaß [9], a model is a form of mathematical representation of a problem. A mathematical model is a representation of a particular phenomenon created in the forms of images, graphs, schemes, equations, or algorithms.

Modeling is also concerned by Cheng [4] in which mathematical modeling is a process of representation of real-world problems in the form of mathematics in an attempt to find a solution of the problem. A model or modeling cannot be separated from the process of mathematization. Some people assume that mathematization is part of the modeling, while some others consider model as part of mathematization. Blum and Leiss [2] consider mathematization as part of the modeling. Mathematical modeling ability can be divided into five stages [1]: (1) simplifying the problem, (2) making
mathematical model (horizontal mathematization), (3) finishing mathematical model (vertical mathematization), (4) interpretation, and (5) validation. This study will identify errors in students’ answers in solving combinatorics problems observed from the mathematical modeling indicators [1].

Gravemeijer [6] asserts that the development model is formed through four stages, namely situational, referential, general, and formal. The situational stage is the most basic stage of modeling where knowledge and model are still developing in the context of problem situations selected. In the referential stage, the models and strategies developed refer to the context in which the student creates a model to describe the situation from the context. The model at this stage is called ‘model of’. In the general stage, it is developed by them has led to the search for solutions mathematically. Such model at this stage is called ‘model for’. Finally, at the formal stage, they have worked with symbols and mathematical representations.

2. Methodology
The research method was descriptive study by involving the students from State Senior High School (said ‘SMAN’) in Banten Province which are RSBI schools in 2015. They were SMAN Cahaya Madani Banten Boarding School (said ‘CMBBS’) and SMAN 4 Pandeglang. 55 XI science-major students were selected consisting of 30 students from SMAN CMBBS and 25 students from SMAN 4 Pandeglang. To seek the errors in the students’ answers in solving combinatorics problems, they had studied combinatorics materials with their respective teachers beforehand.

The instrument used was test. The items were in the form of short essay of 5 questions that considered modeling characteristics in RME whose indicators are: (1) simplifying the problem; (2) making mathematical model (horizontal mathematization); (3) finishing mathematical model (vertical mathematization); (4) interpretation; and (5) validation. The items for piloting study were tested before regarding its validity and reliability by involving 5 experts and 20 students. The test was conducted on May, 16th 2017 at SMAN 4 Pandeglang and May, 23rd 2017 at SMAN CMBBS.

Validity question item is conducted to determine validity of the instrument using content validity. It examined whether the material was really representative. Item 1 and item 5 represented the rules of multiplication and addition, item 2 represented permutation, and item 3 and item 4 represented combination.

3. Findings and Discussion
The percentage of students making errors in solving combinatorics problems observed from the mathematical characteristics of RME is presented in Table 1.

| Students’ errors                          | Percentage (%) of students of each question item | Average (%) |
|------------------------------------------|-------------------------------------------------|-------------|
| Errors in simplifying the problem        | 69 62 15 44 40                                  | 46          |
| Errors in making mathematical model      | 85 73 38 53 53                                  | 60          |
| Errors in finishing mathematical model   | 89 73 47 58 56                                  | 65          |
| Errors in interpretation                 | 89 73 47 58 64                                  | 66          |
| Validation                               | 89 73 47 58 64                                  | 66          |

From the table above, it can be seen that the most frequently committed errors by the students are those in interpretation and validation. It is because when they are not able to simplify the problem correctly, it will result in the incorrect creation of mathematical model, and their completion is wrong, so interpretation and validation are also wrong. Table 1 also shows that the error of each question begins
The findings of this study are that the students have difficulty in solving the combinatorics problem, especially in the step of making mathematical model (horizontal mathematization) and its completion. The results of this study are in line with Lockwood [7] in which students face difficulties when solving the problem of combinatorial calculation. In addition, students still make errors in the completion of permutation and combination [12].

The questions are contextual, but they prefer them in the form of numbers because they find it more difficult to model it. It is suggested that permutation and combination problems expressed in the form of context require more mathematical thought, but students will find it more difficult [7].

Question number 1 has got the highest percentage of errors in simplifying the problem compared to other items as much 69%. The text of question number 1 is “In an entry selection test at a favorite high school in Banten, each participant gets a test number consisting of four different digits from 0, 2, 3, 4, 5, 7, and 8. How many test numbers can the admission committee make if the number of the test forms a number in-thousand as the quotient by 5?”

There are three kinds of students’ answers. First, some of them did not know the characteristic of the numbers which are divisible by 5. Second, some others knew the characteristic of the numbers divisible by 5, but misplace 0 in the position of thousands. Third, some others knew the characteristic of the numbers divisible by 5 and knew that 0 cannot be placed in the form of thousand, but forgot the context of the question in which ‘the mixed numbers are formed by different numbers’. Figure 1 is an example of a student's answer regarding the second stage that is the completion stage of this model or an error in the vertical mathematical stage. It is the most frequently done by the students compared to stage one and stage three. This error resulted in the incorrect stage of interpretation of the solution.

![Figure 1. Students’ answers on question number 1](image)

For question number 2, this is a case of cyclical permutation. Table 1 shows that there are 62% of the students who committed errors in simplifying the problem (horizontal mathematization) so the errors in the next stage become greater. The text for question number 2 was “A family consisting of father, mother, brother, and sister are having dinner together. They sit around a table. If the father and mother always sit close together, then determine the number of different ways of seating from the family!”

One of the students' answers in Figure 2. below is one of the representatives of students whose answers are toward the expected solution. The students made a model of the given problem, then made formal mathematics (vertical mathematization) that led to the final solution. Here, the students had not finished the model correctly seen in the drawings made by them where AB (representing father and mother) were drawn close together. However, when at the completion stage, they separated A and B by calculating the cyclical permutation of 4, also the factorial number 2 as the solution from the adjacent A and B. So, they committed errors in completing the model as well as its interpretation.
Figure 2. Students’ answers on question number 2

For question number 3, Table 1 shows that the students’ errors seen from the modeling characteristics were the smallest among other questions. More than half of them had completed each step of the modeling characteristics stage. The question text was “To paint the mosque walls, Mr. Hamid will create mixed colors that are formed from three basic colors. There are six different basic colors. How many mixed colors can Pak Hamid make?”

The incorrect examples of student’s answers are shown in Figure 3. There are three kinds of answers that are displayed. Two answers included using a horizontal mathematical process by creating a simplified model of the problem, but the students had not been able to make all the possible answers requested. While, one other answer used formal mathematics. It can be seen that they have not been able to distinguish the use of formal mathematical formulas between permutation and combination.

Figure 3. Students’ answers on question item 3

Another combination-related case question is on item 4, that is “In a formative test of mathematics, the teacher gives seven multiple choice questions and three short-essay questions. Each student may work on a problem that is considered easier first. Not all questions should be answered. They may choose five multiple choice questions and two short-essay questions. If question number 4 and 7 on multiple choice must be done, how many ways are available to choose the rest of the questions?”

In contrast to the number 3 above, although it is also the case of combination, many students committed errors. It is because the level of difficulty of number 3 which is different from number 4. Their understanding was tested in operating the problem of two combinations in one case. One of the students’ answers is presented in Figure 4.
Figure 4 shows that the students commit errors in the vertical mathematical stage in which there is information left in the calculation of the remaining multiple choice questions that must be selected. Multiplication operation between two combinations is correct, but because there is an error in the data input, the final answer is also wrong. Such errors often occur where they understand the formal mathematical formulas without understanding the meaning of the symbols they write.

Meanwhile, for question number 5, it is a case on multiplication and addition whose text is "A motor vehicle serial number consists of four-digit numbers. Determine the number of serial numbers that can be created if number 5 appears twice. Draw the position of number 5 in various sequence variations. (Hint: The numbers that can be used are 0, 1, ..., 9)"

The answers given by the students vary greatly in which 53% of them committed errors in the horizontal mathematical stage in creating mathematical model. Figure 5 shows that they understand the occurrence of number five is twice, but do not understand the calculation, so that the numbers in the grid box are not clear in meaning.

It suggests that they have not been able to interpret the symbols used in the process of problem solving, so that they had difficulty in the final process of calculation. It is also seen that they were not accustomed to solving challenging contextual problems and training their way of thinking in simplifying (modeling) the given problem.

4. Conclusions
Based on the finding of the study, students’ errors in solving the combinatorics problems observed from the modeling characteristics suggest that their ability in solving the combinatorics problems is still low. It also suggests that the still-low ability is especially regarding horizontal and vertical mathematization because these two stages are the core of modeling-problem completion process under real-world context. Combining the combinatorics problems under real-world contexts that has its own challenges must be
continuously trained to the students, so that they are accustomed to finding solutions and being able to create mathematical model to find solutions to the problems they face. For a more effective learning, teachers should make preparation in designing instructional materials in accordance with real-life contexts that train students’ thinking skills. Among them is teaching materials that help bring up the modeling characteristics built by students so that they are trained to constrict their own thoughts, and the teachers are guiding them to lead to formal mathematics.

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