**Kepler** photometry of RRc stars: peculiar double-mode pulsations and period doubling

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Accepted 2014 December 1. Received 2014 November 28; in original form 2014 October 11

**ABSTRACT**
We present the analysis of four first overtone RR Lyrae stars observed with the **Kepler** space telescope, based on data obtained over nearly 2.5 yr. All four stars are found to be multiperiodic. The strongest secondary mode with frequency \(f_2\) has an amplitude of a few mmag, 20–45 times lower than the main radial mode with frequency \(f_1\). The two oscillations have a period ratio of \(P_2/P_1 = 0.612 – 0.632\) that cannot be reproduced by any two radial modes. Thus, the secondary mode is nonradial. Modes yielding similar period ratios have also recently been discovered in other variables of the RRc and RRd types. These objects form a homogenous group and constitute a new class of multimode RR Lyrae pulsators, analogous to a similar class of multimode classical Cepheids in the Magellanic Clouds. Because a secondary mode with \(P_2/P_1 \sim 0.61\) is found in almost every RRc and RRd star observed from space, this form of multiperiodicity must be common. In all four **Kepler** RRc stars studied, we find subharmonics of \(f_2\) at \(\sim 1/2 f_2\) and at \(\sim 3/2 f_2\). This is a signature of period doubling of the secondary oscillation, and is the first detection of period doubling in RRc stars. The amplitudes and phases of \(f_2\) and its subharmonics are variable on a timescale of 10–200 d. The dominant radial mode also shows variations on the same timescale, but with much smaller amplitude. In three **Kepler** RRc stars we detect additional periodicities, with amplitudes below 1 mmag, that must correspond to nonradial g-modes. Such modes never before have been observed in RR Lyrae variables.

**Key words:** techniques: photometric – stars: horizontal branch – stars: oscillations – stars: variable: RR Lyrae – stars: individual: KIC 4064484; KIC 5520878; KIC 8832417; KIC 9453114

1 INTRODUCTION

RR Lyrae variables are evolved stars burning helium in their cores. In the Hertzsprung-Russell diagram they are located
at the intersection of the horizontal branch and the classical instability strip, in which the \( \kappa \)-mechanism operating in the H and He partial ionization zones drives the pulsation. They are classified according to their pulsation characteristics using a variant of the initial classification by Bailey (1902). The subclass of the RRab stars is by far the largest. These stars pulsate in the radial fundamental mode (\( F \)), with periods of 0.3 – 1.0 d, and peak-to-peak amplitudes in \( V \) ranging from few tenths of magnitude at long periods to more than 1 mag at short periods. Their light curves are asymmetric (steeper on the rising part). Their less numerous siblings are the RRc stars, which pulsate in the first overtone radial mode (\( 1O \)) with shorter periods in the range 0.2 – 0.5 d, and with more sinusoidal light curves with lower amplitudes of about 0.5 mag in \( V \), peak-to-peak. Even less numerous are the RRd stars, which pulsate simultaneously in the radial first overtone mode and the radial fundamental mode (\( F+1O \)).

Playing an important role in distance determination and in Galactic structure and evolution studies, RR Lyrae stars are among the best studied and most observed classes of variable stars. In recent years dozens of RR Lyrae stars have been observed with unprecedented precision from space by the MOST (Gruberbauer et al. 2007), CoRoT (e.g. Szabó et al. 2014) and Kepler (e.g., Kolenberg et al. 2010; Benkő et al. 2010) telescopes.

Nevertheless, many intriguing puzzles surround the RR Lyrae stars. The most stubborn problem is the Blazhko effect, a quasi-periodic modulation of pulsation amplitude and phase that has been known for more than 100 yr (Blazhko 1907). Dedicated ground-based campaigns (Jurcsik et al. 2009) and results of Kepler observations (Benkő et al. 2010) indicate that up to 50 per cent of the RRab stars show Blazhko modulation. For the RRc stars the incidence rate is probably lower. Ground-based observations show it is below 10 per cent (e.g. Mizerski 2003; Nagy & Kovács 2006). We lack high-precision space observations for these stars. Only recently the first Blazhko modulated RRc star was observed from space by Kepler (Molnár et al., in preparation). Despite many important discoveries, including detection of period doubling (Szabó et al. 2010) and of excitation of additional radial modes in Blazhko variables (Benkő et al. 2010, 2014; Molnár et al. 2012) our understanding of the Blazhko effect remains poor (for a review see Szabó 2014).

Another mystery is the mode selection process in RR Lyrae stars: we do not know why some stars pulsate in two radial modes simultaneously. The ability of current non-linear pulsation codes to model this form of pulsation is still a matter of debate; see, e.g., Kolláth et al. (2002) and Smolec & Moskalik (2008b) for opposing views. Recent discoveries of \( F+2O \) radial double-mode pulsations and the detection of non-radial modes in RR Lyrae stars (see Moskalik 2013, 2014 for reviews) make the mode-selection problem even more topical and puzzling. Particularly interesting is the excitation of non-radial modes, evidence of which is found in all subclasses of RR Lyrae variables. Their presence seems to be a frequent phenomenon in these stars.

This paper describes new properties of first overtone RR Lyrae stars, some of them revealed for the first time by the high-precision Kepler photometry. We present an in-depth study of four RRc stars in the Kepler field: KIC 4064484, KIC 5520878, KIC 8832417 and KIC 9453114.

Partial results of our analysis have been published in Moskalik et al. (2013). They have also been included in a review paper of Moskalik (2014). Here we present a full and comprehensive discussion of the results.

In Section 2 we describe the Kepler photometry and our methods of data reduction. Properties of the four RRc variables are summarized in Section 3. Our main findings are discussed in Sections 4 – 6, where we present the results of frequency analyses of the stars and describe amplitude and phase variability of the detected pulsation modes. In Section 7 we put the Kepler RRc stars in a broader context of other recently identified multimode RRc variables and discuss the group properties of this new type of multimode pulsators. Our conclusions are summarized in Section 8.

2 Kepler photometry

The Kepler space telescope was launched on 2009 March 6 and placed in a 372.5-d Earth trailing heliocentric orbit. The primary purpose of the mission was to detect transits of Earth-size planets orbiting Sun-like stars (Borucki et al. 2010). This goal was achieved by nearly continuous, ultrasensitive photometric monitoring of nearly 200 000 stars in the 115 deg\(^2\) field of view. A detailed description of the mission design and its in-flight performance is presented in Koch et al. (2010), Caldwell et al. (2010), Haas et al. (2010), Jenkins et al. (2010a,b) and Gilliland et al. (2010). Kepler’s primary mission ended after four years when the second reaction wheel failed in May 2013.

The Kepler magnitude system (\( K_p \)) corresponds to a broad spectral bandpass, from 423 nm to 897 nm. The time series photometric data delivered by the Kepler telescope come in two different formats: Long Cadence (LC) and Short Cadence (SC), with sampling rates of 29.43 min and 58.86 s,
respectively. The time of mid-point of each measurement is corrected to Barycentric Julian Date (BJD). Four times per orbital period the spacecraft was rotated by 90° to keep optimal illumination of its solar arrays. These rolls naturally organize the data into quarters, denominated Q1, Q2, etc., where each quarter lasts about 3 months (except the first and the last quarter, which are shorter). The data are almost continuous, with only small gaps due to regular data downlink periods and to infrequent technical problems (safe mode, loss of pointing accuracy). The typical duty cycle of the Keplertm light curve is 92 per cent.

2.1 Data reduction

The Kepler telescope is equipped with 42 science CCDs (Koch et al. 2010). A given star fell on four different CCDs with each quarterly roll of the spacecraft, then returned to the original CCD. As a result of different sensitivity levels, the measured flux jumps from quarter to quarter. For some stars slow trends also occur within each quarter, due to image motion, secular focus changes or varying sensitivity of the detector (Jenkins et al. 2010a). All these instrumental effects need to be corrected before starting the data analysis.

Our data reduction procedure is similar to that of Nemec et al. (2011). We use the ‘raw’ fluxes, properly called Simple Aperture Photometry fluxes (Jenkins et al. 2010a). The detrending is done separately for each quarter. The flux time series is first converted into a magnitude scale. Slow drifts in the magnitudes are then removed by subtracting a polynomial fit. Next, the data are fitted with the Fourier sum representing a complete frequency solution. The residuals of this fit are inspected for any additional low-level drifts, which are again fitted with a polynomial and subtracted from the original magnitudes (secondary detrending). The detrended data for each quarter are then shifted to the same average magnitude level. As the final step, all quarters are merged, forming a quasi-continuous $K_p$ magnitude light curve of a star. An example of a reduced light curve is displayed in Fig. 4.

3 RRC STARS IN THE KEPLER FIELD

More than 50 RR Lyrae variables are currently identified in the Kepler field (Kolenberg et al. 2014). At the time this study was initiated, only four of them were classified as RRc pulsators. Basic characteristics of these four stars stars are given in Table 1. The first four columns of the table list the star numbers, equatorial sky coordinates ($\alpha, \delta$) and mean $K_p$ magnitude, all taken from the Kepler Input Catalogue (KIC, Brown et al. 2011). Columns 5 and 6 contain the pulsation periods and the total (peak-to-peak) amplitudes. All four RRc stars turned out to be multiperiodic (Section 4), but they are all strongly dominated by a single radial mode. The periods and amplitudes given in the table correspond to this dominant mode of pulsation. They are determined from the Kepler light curves. Column 7 contains spectroscopic metal abundances, [Fe/H], from Nemec et al. (2013). Two stars in our RRc sample are metal-rich (KIC 5520878 and KIC 8832417), the other two are metal-poor. Column 8 indicates other identifications of the stars. KIC 9453114 is also known as ROTSE1 J190350.47+460144.8. None of the Kepler RRc variables has a GCVS name yet.

The last column of Table 1 lists Kepler observing runs analysed in this paper. Here we use only the Long Cadence photometry, collected in quarters Q1 to Q10. For KIC 8832417 and KIC 9453114 it is supplemented by 10 d of commissioning data (Q0). Due to the loss of CCD module no. 3, no photometry was obtained for KIC 4064484 in Q6 and Q10. The total time base of the data ranges from 774 d for KIC 4064484 to 880 d for KIC 8832417 and KIC 9453114.

Fig. 2 displays phased light curves of the dominant radial mode of Kepler RRc stars. This is low resolution version of the figure.

Table 2. Light curve shape parameters for the dominant radial mode of Kepler RRc stars. Values are determined from Q0+Q1 data.

| KIC   | $\log P_1$ [d] | $A_1$ [mmag] | $R_{21}$ | $\phi_{21}$ [rad] | $M - m$ |
|-------|----------------|--------------|----------|-------------------|--------|
| 8832417 | -0.605         | 138.40       | 0.1015   | 4.661             | 0.425  |
| 5520878 | -0.570         | 162.88       | 0.1078   | 4.704             | 0.427  |
| 4064484 | -0.472         | 190.50       | 0.1109   | 4.813             | 0.408  |
| 9453114 | -0.436         | 206.38       | 0.0982   | 4.928             | 0.430  |
other than the dominant one \((f_1 = 1/P_1)\) and its harmonics (Section 4). The light curves are typical for radial first overtone pulsators, with round tops, long rise times and low amplitudes. The change of slope which occurs shortly before the brightness maximum is also characteristic for the RRc variables (e.g. Lub 1977; Olech et al. 2001).

To describe light curve shape in a quantitative way we resort to a Fourier decomposition (Simon & Lee 1981). We fit the light curves of Fig. 2 with the Fourier sum of the form

\[
K_p(t) = A_0 + \sum_{k} A_k \cos(2\pi k f_1 t + \phi_k).
\]

and then compute the usual Fourier parameters: the amplitude ratio \(R_{21} = A_2/A_1\) and the phase difference \(\phi_{21} = \phi_2 - 2\phi_1\). They are listed in Table 2 together with the semi-amplitude of the dominant frequency, \(A_1 \approx A_{1\text{tot}}/2\). The last column of the table gives another light curve parameter: the interval from minimum to maximum, expressed as a fraction of the pulsation period. This quantity measures asymmetry of the light curve and is traditionally called \(M - m\) (Payne-Gaposchkin & Gaposchkin 1966) or a risetime parameter (e.g. Nemec et al. 2011). For all four variables in Table 2 this parameter is above 0.4, which agrees with their classification as RRc stars (Tsesevich 1975).

In Fig. 3 we compare the Fourier parameters and the peak-to-peak amplitudes \(A_{1\text{tot}}\) of the \textit{Kepler} RRc stars with those of RR Lyrae stars in the Galactic Bulge (Soszyński et al. 2011a). \textit{Kepler} non-Blazhko RRab stars (Nemec et al. 2011) are also displayed. All parameters are converted to a common photometric band, namely to Johnson \(V\). For the Bulge variables, \(R_{21}\) and \(\phi_{21}\) are transformed from \(I\) to \(V\) with the formula of Morgan, Simet & Bargenquast (1998).

For the amplitude, we use equation \(A_{1\text{tot}}(V) = 1.62 A_{1\text{tot}}(I)\), derived by us from photometry of RR Lyrae stars in M68 (Walker 1994). The amplitude transformation is the same for RRab and for RRc stars. In case of the \textit{Kepler} variables, \(\phi_{21}\) is transformed from \(K_p\) to \(V\) according to Eq. 2 of Nemec et al. (2011), but for \(A_{1\text{tot}}\) and \(R_{21}\) we used proportional scaling, which in our opinion is more appropriate. Using the same data as Nemec et al. (2011), we derive \(A_{1\text{tot}}(V) = 1.16 A_{1\text{tot}}(K_p)\) and \(R_{21}(V) = 0.975 R_{21}(K_p)\).

All \(K_p \rightarrow V\) transformation formulae are calibrated with RRab stars, thus applying them to the RRc stars can yield only approximate results. This approximation is still sufficiently accurate, particularly for \(A_{1\text{tot}}\) (where scaling is mode-independent) and for \(\phi_{21}\) (where colour-to-colour corrections are always small). We expect the approximation to be least accurate in the case of \(R_{21}\) for which colour-to-colour scaling is somewhat dependent on the pulsation mode (Morgan et al. 1998).

Fig. 3 shows that the Fourier parameters of variables listed in Table 1 (black diamonds) are very different from those of the \textit{Kepler} RRab stars. At the same time, they match typical values for the first overtone RR Lyrae stars well. The Figure proves that the variables of Table 1 belong to the population of RRc stars.

\section{Data Analysis and Results}

We analyzed the pulsations of \textit{Kepler} RRc stars with a standard Fourier transform (FT) combined with the multifrequency least-squares fits and consecutive prewhitening. We used well-tested software written by Z. Kołaczkowski (see Moskalik & Kołaczkowski 2009). The light curve was first fitted with a Fourier series representing variations with the dominant frequency, \(f_1\) (Eq. 1). After subtracting the fitted function (prewhitening), the residuals were searched for secondary periods. This was done with the FT, computed over the range from 0 to 24.5 d\(^{-1}\), which is the Nyquist frequency of the Long Cadence data. Newly identified frequencies were included into the Fourier series, which was fitted to the light curve again. The residuals of the fit were searched for more frequencies. The whole procedure was repeated until no new periodicities were found. In each step, all frequencies were
Figure 4. Prewhitening sequence for KIC 5520878. The upper panel displays the Fourier transform of the original Kp magnitude light curve. The middle and bottom panels show FTs after consecutive prewhitening steps (see text). Only Q1 data are analyzed here.

Table 3. Main frequency components identified in Q1 lightcurve of KIC 5520878.

| frequency | $f$ [d$^{-1}$] | $A$ [mmag] | frequency | $f$ [d$^{-1}$] | $A$ [mmag] | frequency | $f$ [d$^{-1}$] | $A$ [mmag] |
|-----------|---------------|-------------|-----------|---------------|-------------|-----------|---------------|-------------|
| $f_1$     | 3.715126      | 162.88      | $f_1-f_2$ | 2.16017       | 0.95        | $0.5f_2$  | 2.93694       | 5.74        |
| $2f_1$    | 7.430253      | 17.55       | $2f_1-f_2$| 1.55180       | 0.41        | $1.5f_2$  | 8.81428       | 1.65        |
| $3f_1$    | 11.145379     | 11.06       | $f_1+f_2$ | 9.59625       | 3.37        | $f_1-0.5f_2$ | 0.77949      | 0.95        |
| $4f_1$    | 14.860506     | 4.77        | $2f_1+f_2$| 13.31087      | 0.76        | $2f_1-0.5f_2$ | 4.48680      | 0.18        |
| $5f_1$    | 18.575632     | 2.61        | $3f_1+f_2$| 17.02511      | 0.16        | $f_1+0.5f_2$ | 6.64948      | 0.87        |
| $6f_1$    | 22.90759      | 1.39        | $4f_1+f_2$ | 20.73973      | 0.16        | $f_1-1.5f_2$ | 5.10593      | 0.26        |
| $7f_1$    | 26.005885     | 0.78        | $f_1-2f_2$| 8.03827       | 0.26        | $f_1+1.5f_2$ | 12.52531     | 1.13        |
| $8f_1$    | 29.721011     | 0.37        | $f_1+2f_2$| 15.46826      | 0.44        | $2f_1+1.5f_2$ | 16.24251     | 0.44        |
| $f_2$     | 5.87858       | 7.04        | $2f_1+2f_2$ | 19.18085      | 0.18        | $3f_1+1.5f_2$ | 19.95592     | 0.16        |
| $2f_2$    | 11.75877      | 0.65        | $f_1+3f_2$ | 21.35066      | 0.14        | $f_1+2.5f_2$ | 18.41040     | 0.14        |
| $3f_2$    | 17.62523      | 0.13        | $2f_1+2.5f_2$ | 22.12452     | 0.29        | $f_1+3.5f_2$ | 32.62050      | 0.12        |

optimized by the least-squares routine. The final fit yields frequencies, amplitudes and phases of all identified harmonic components.

All four Kepler RRc variables turned out to be multiperiodic. Very early in the analysis, we realized that the amplitudes and phases of the detected frequencies are not constant. Therefore, we first discuss only a short segment of available data, namely Q0+Q1. Our goal here is to establish the frequency content of the RRc light curves. We defer the analysis of the entire Q0–Q10 datasets and discussion of temporal behaviour of the modes to Section 5. In the following subsections, we present the results for each of the investigated RRc stars individually. We discuss the stars in order of decreasing amplitude stability, which nearly coincides with the order of increasing pulsation period.

4.1 KIC 5520878

Fig[4] shows the prewhitening sequence for KIC 5520878. The FT of its light curve (upper panel) is strongly dominated by the radial mode with frequency $f_1 = 3.715126$ d$^{-1}$ and amplitude $A_1 = 162.88$ mmag. After subtracting $f_1$ and its harmonics from the data, the FT of the residuals (middle panel) shows the strongest peak at $f_2 = 5.87858$ d$^{-1}$. This secondary mode has an amplitude of only $A_2 = 7.04$ mmag, 23 times lower than $A_1$. Its harmonic and several combinations with $f_1$ are also clearly visible. The period ratio of the two modes is $P_2/P_1 = 0.6320$. After prewhitening the light curve with both $f_1$, $f_2$ and their harmonics and combinations (bottom panel), the strongest remaining signal appears at $f_3 = 2.93694$ d$^{-1}$, i.e. at $\sim 1/2f_2$. Thus, $f_3$ is not an
Figure 5. Frequency spectra of the four Kepler RRc stars (Q0+Q1 data only) after pre-whitening by $f_1$ and its harmonics. FT of KIC 5520878 (upper panel) is the same as plotted in the middle panel of Fig. 4.

independent frequency, it is a subharmonic of $f_2$. A second subharmonic at $f_3 = 8.81428 \text{ d}^{-1} \sim 3/2f_2$ is also present. The remaining peaks in the FT correspond to linear combinations of $f_1$ with $f_3$ and $f_2$. In Table 3 we list all harmonics, subharmonics and combination frequencies identified in Q1 light curve of KIC 5520878.

4.2 KIC 8832417

Pulsations of this star are dominated by the radial mode with frequency $f_1 = 4.02339 \text{ d}^{-1}$. After prewhitening the data with $f_1$ and its harmonics (Fig. 5 second panel) we identify a secondary mode at $f_2 = 6.57203 \text{ d}^{-1}$. The resulting period ratio is $P_2/P_1 = 0.6122$. The subharmonic of the secondary frequency at $\sim 3/2f_2$ is clearly visible. The subharmonic at $\sim 1/2f_2$ is present as well, but its appearance is different. The peak is very broad and almost split into three close components. Such a pattern implies that we look either at a group of frequencies that are too close to be resolved, or at a single frequency with unstable amplitude and/or phase. The central component of this subharmonic power is located at $f_3 = 3.29848 \text{ d}^{-1} = 0.5019f_2$. The remaining peaks in the prewhitened FT correspond to linear combinations of $f_1$ with $f_2$ and $3/2f_2$.

4.3 KIC 4064484

Pulsations of KIC 4064484 are dominated by the radial mode with frequency $f_1 = 2.96734 \text{ d}^{-1}$. After prewhitening the data with $f_1$ and its harmonics (Fig. 5 third panel) we find a secondary mode at $f_2 = 4.82044 \text{ d}^{-1}$, yielding a period ratio of $P_2/P_1 = 0.6156$. The peak corresponding to $f_2$ is broadened and starts to split into several unresolved components. The same is true for the combination peaks at $f_2 - f_1$, $f_1 + f_2$, $2f_1 + f_2$, etc. As in the other two RRc stars, we find a subharmonic of the secondary frequency at $\sim 3/2f_2$. This peak is also broadened and split, and so are the corresponding combination peaks. Unlike the previous two RRc stars, in KIC 4064484 we do not detect a clear subharmonic at $\sim 1/2f_2$. There is an excess of power in the vicinity of this frequency, but it forms a dense forest of peaks, none of which stand out.
The dominant radial mode of KIC 9453114 has a frequency of \( f_1 = 2.73164 \text{ d}^{-1} \). After prewhitening the light curve with \( f_1 \) and its harmonics (Fig. 3, bottom panel) we find a secondary mode at \( f_2 = 4.44643 \text{ d}^{-1} \), yielding a period ratio of \( P_2/P_1 = 0.6143 \). A second peak of somewhat lower amplitude is present next to it, at \( f_2 = 4.55830 \text{ d}^{-1} \). Another much weaker peak can be identified at the same distance from \( f_2 \), but on its opposite side. Thus, the secondary frequency in KIC 9453114 is split into three resolved components, which form an approximately equidistant triplet. The same pattern is also seen at the combination frequencies \( f_2 - f_1, f_1 + f_2, 2f_1 + f_2 \), etc. The \( f_2 \) triplet can be interpreted in two physically different ways: either as a multiplet of nonradial modes (\( \ell \geq 1 \)) split by rotation or as a single mode undergoing a periodic or quasi-periodic modulation. A close inspection of the triplet shows that all three components are incoherent. Indeed, an attempt to prewhiten them with three sine waves of constant amplitudes and phases proves unsuccessful, leaving significant residual power. This casts doubt on the nonradial multiplet interpretation. We will return to this point in Section 5.1.2.

Although very weak, the subharmonics of \( f_2 \) are also present in KIC 9453114. The signal corresponding to \( \sim 3/2 f_2 \) is detected at \( f_{3/2} = 6.68487 \text{ d}^{-1} \). Its combination frequencies at \( \sim f_1 + f_{3/2} \) and \( \sim 2f_1 + f_{3/2} \) can be identified, too. All these peaks are split into resolved triplets. A second subharmonic \( (\sim 1/2f_2) \) is detected as well, as a marginally significant single peak at \( f_{3/2} = 2.27340 \text{ d}^{-1} \).

### 4.4 KIC 9453114

The results of the frequency analyses of Kepler RRc variables are summarized in Table 4. In Fig. 5 we display the Fourier transforms of their Q0+Q1 light curves, prewhitened of the dominant radial mode. For better comparison, the FTs are plotted vs. normalized frequency, \( f/f_1 \).

All four RRc variables are multiperiodic. Fig. 5 shows that their frequency spectra are remarkably similar and display peaks at the same places. In each star we detect a secondary mode, which appears at \( f_2/f_1 = 1.58 - 1.63 \) or \( P_2/P_1 = 0.612 - 0.632 \). This is always the highest secondary peak, yet its amplitude is only a few mmag and is 25 - 45 times lower than the amplitude of the radial mode. Without the benefit of the high-precision Kepler photometry, such a weak signal is very difficult to detect.

In each variable we identify at least one subharmonic of the secondary frequency. The signal at \( \sim 3/2f_2 \) is detected in all Kepler RRc stars. The second subharmonic at \( \sim 1/2f_2 \) is visible in three of the stars, although it is prominent only in KIC 5520878. The presence of subharmonics, i.e., frequencies of the form \((n+1/2)f_1\), is a characteristic signature of a period doubling (Bergé et al. 1986; see also Fig. 3 of Smoleč & Moskalik 2012). Thus, our finding constitutes the first detection of the period doubling phenomenon in the RRc variables (see also Moskalik et al. 2013; Moskalik 2014). These stars are thus the fourth class of pulsators in which period doubling has been discovered, following the RV Tauri stars (known for decades), and the Blazhko RRab stars (Kolenberg et al. 2010, Szabó et al. 2010) and the BL Herculis stars (Smoleč et al. 2012), only identified recently.

We note, that the subharmonics listed in Table 4 are never located at precisely \( 1/2f_2 \) and \( 3/2f_2 \). The deviations from the exact half-integer frequency ratios are very small, almost never exceeding 0.5 per cent, but they are statistically significant. We recall, that similar deviations are also observed for period doubling subharmonics in the Blazhko RRab stars (Szabó et al. 2010, 2014; Kolenberg et al. 2011; Guggenberger et al. 2012). This behaviour has been traced to the nonstationary character of the subharmonics, which causes their instantaneous frequencies to fluctuate around the expected values (Szabó et al. 2010). The same reasoning applies also to the RRc stars. As we will discuss in the next section, the subharmonics detected in these variables are nonstationary as well.

Finally, we note that a secondary mode with \( P_2/P_1 \sim 0.61 \) and its subharmonics are detected in every RRc star observed by Kepler. This suggests that excitation of this mode and the concomitant period doubling is not an exception, but is a common property of the RRc variables. We return to this point in Section 5.4.

### 4.5 Similarity of Kepler RRc stars

The results of the frequency analyses of Kepler RRc variables are summarized in Table 4. In Table 4, we display the Fourier transforms of their Q0+Q1 light curves, prewhitened of the dominant radial mode. For better comparison, the FTs are plotted vs. normalized frequency, \( f/f_1 \).

All four RRc variables are multiperiodic. Fig. 5 shows that their frequency spectra are remarkably similar and display peaks at the same places. In each star we detect a secondary mode, which appears at \( f_2/f_1 = 1.58 - 1.63 \) or \( P_2/P_1 = 0.612 - 0.632 \). This is always the highest secondary peak, yet its amplitude is only a few mmag and is 25 - 45 times lower than the amplitude of the radial mode. Without the benefit of the high-precision Kepler photometry, such a weak signal is very difficult to detect.

In each variable we identify at least one subharmonic of the secondary frequency. The signal at \( \sim 3/2f_2 \) is detected in all Kepler RRc stars. The second subharmonic at \( \sim 1/2f_2 \) is visible in three of the stars, although it is prominent only in KIC 5520878. The presence of subharmonics, i.e., frequencies of the form \((n+1/2)f_1\), is a characteristic signature of a period doubling (Bergé et al. 1986; see also Fig. 3 of Smoleč & Moskalik 2012). Thus, our finding constitutes the first detection of the period doubling phenomenon in the RRc variables (see also Moskalik et al. 2013; Moskalik 2014). These stars are thus the fourth class of pulsators in which period doubling has been discovered, following the RV Tauri stars (known for decades), and the Blazhko RRab stars (Kolenberg et al. 2010, Szabó et al. 2010) and the BL Herculis stars (Smoleč et al. 2012), only identified recently.

We note, that the subharmonics listed in Table 4 are never located at precisely \( 1/2f_2 \) and \( 3/2f_2 \). The deviations from the exact half-integer frequency ratios are very small, almost never exceeding 0.5 per cent, but they are statistically significant. We recall, that similar deviations are also observed for period doubling subharmonics in the Blazhko RRab stars (Szabó et al. 2010, 2014; Kolenberg et al. 2011; Guggenberger et al. 2012). This behaviour has been traced to the nonstationary character of the subharmonics, which causes their instantaneous frequencies to fluctuate around the expected values (Szabó et al. 2010). The same reasoning applies also to the RRc stars. As we will discuss in the next section, the subharmonics detected in these variables are nonstationary as well.

Finally, we note that a secondary mode with \( P_2/P_1 \sim 0.61 \) and its subharmonics are detected in every RRc star observed by Kepler. This suggests that excitation of this mode and the concomitant period doubling is not an exception, but is a common property of the RRc variables. We return to this point in Section 5.4.

### 5 VARIABILITY OF AMPLITUDES AND PHASES

#### 5.1 \( f_2 \) and its subharmonics

We start the discussion with the secondary mode, \( f_2 \), for which evidence of instability is most noticeable. Fig. 5 presents RRc frequency spectra computed for month-long subsets of available data. Already for this short timebase, the Fourier peaks corresponding to \( f_2 \) and its subharmonics are broadened or even split. This indicates that amplitudes and/or phases of these frequency components are variable. To examine this variability in detail, in this Section we analyze for each star the entire light curve, covering quarters Q0 – Q10.
5.1.1 Time domain

We examine the temporal behaviour of modes by applying a time-dependent Fourier analysis (Kovács, Buchler & Davis 1987). To this end, we subdivide the light curve into short overlapping segments with duration $\Delta t$, and then fit a Fourier series consisting of all significant frequency terms to each segment separately. All frequencies are kept fixed. The choice of $\Delta t$ is somewhat arbitrary and depends on how fast the amplitudes and phases change. With this procedure, we can follow the temporal evolution of all frequency components present in the data.

In Fig. 6 we present results of such an analysis for KIC 5520878. For this star we adopted $\Delta t = 10$ d. The plot displays amplitudes and phases of the secondary mode, $f_2$, and of its two subharmonics, $1/2f_2$ and $3/2f_2$. The amplitude of the secondary mode varies in a rather irregular fashion, with a timescale of $\sim 200$ d. The range of these variations is extremely large: from almost zero to 9.6 mmag. In other words, the amplitude of $f_2$ fluctuates by nearly 100 per cent!

Both subharmonics of $f_2$ display large, irregular changes as well. Interestingly, although occurring with approximately the same timescale, they do not seem to be correlated with variations of the parent mode – the maximum (minimum) amplitudes of the subharmonics in some instances coincide with maximum (minimum) amplitude of $f_2$, but in other instances they do not. The amplitude variability of the secondary mode and its subharmonic is accompanied by irregular variability of their phases (Fig. 6, bottom panel).

In Fig. 7 we compare amplitude variability of all four Kepler RRc stars. Only 150 d of data is displayed. For the consecutive objects (top to bottom) we adopted $\Delta t = 10$ d, 5 d, 5 d and 3 d, respectively. Strong amplitude variations of secondary periodicities are found in all the stars. These variations are always irregular. As such, they are not compatible with beating of two or more stable modes. The timescale of amplitude changes is not the same in every RRc variable. It ranges from $\sim 200$ d (KIC 5520878) to $\sim 10$ d (KIC 9453114) and becomes progressively shorter as we go from shorter to longer period pulsators. This tendency explains why Fourier peaks in Fig. 6 become broader with the increasing pulsation period of the star.

As in the case of KIC 5520878, secondary periodicities in all RRc stars display significant phase variations (not shown). They occur on the same timescale as the variations of the respective amplitudes. We recall here that a phase change of a mode is equivalent to a change of its frequency. Indeed, the difference between the instantaneous and the mean frequency is

$$\Delta \omega = 2\pi \Delta f = \frac{d\phi}{dt}. \quad (2)$$

Thus, the frequencies of the secondary mode and its subharmonics are not constant in Kepler RRc stars, but fluctuate on a timescale of $10-200$ d. This is the reason why the values of $f_3/f_2$ and $f_2/f_2$ (see Table 4) deviate from the exact half-integer ratios. These deviations are larger in stars in which phase variations are faster.

5.1.2 Frequency domain

For each studied RRc star we computed the Fourier transform of the entire light curve (Q0–Q10), after prewhitening it with the dominant (radial) frequency and its harmonics. The resulting frequency patterns in the vicinity of the secondary mode, $f_2$, and its subharmonic, $3/2f_2$, are displayed in Fig. 8.

Because the secondary mode in RRc stars has variable amplitude and phase, it cannot be represented in the FT by a single sharp peak. Fig. 8 confirms this. In the case of KIC 5520878, $f_2$ is visibly broadened, but still does not split into resolved components. For the other three stars, the mode splits into a quintuplet of well-separated, equally-
and 

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multiplet component is very weak. Therefore, variations of

Prewhitened frequency spectra of

of a quasi-period is responsible for splitting

lation of a single mode. In the latter picture, the existence

this mode. For KIC 8832417 and KIC 4064484 we find

triplet can be used to estimate the timescale of variability o f

of the secondary mode, \( f_2 \). Right column: FT of its subharmonic at \( 3/2f_2 \).

Figure 8. Prewhitened frequency spectra of Kepler RRc stars, computed for the entire light curves (Q0 – Q10). Left column: FT of the secondary mode, \( f_2 \). Right column: FT of its subharmonic.

spaced peaks (see also Fig. 12). Such a pattern suggests that

\( f_2 \) might correspond to a multiplet of a \( \ell = 2 \) nonradial mode. However, this is not the only possible interpretation.

We notice that each component of the quintuplet is broadened and in fact forms a band of power. None of them can be attributed to a stable mode with a well-defined amplitude and frequency. We can interpret this in two different ways. The \( f_2 \) frequency pattern might be explained as a rotationally split multiplet of nonradial modes, of the same \( \ell \) and different \( m \), all of which are nonstationary. Alternatively, the observed pattern might result from a quasi-periodic modulation of a single mode. In the latter picture, the existence of a quasi-period is responsible for splitting \( f_2 \) into well-separated components (Benkő, Szabó & Paparó 2011) and the irregularity of the modulation causes these components to be broadened. In Section 7.2.3 we will present arguments in favour of this latter interpretation.

The separation between the components of the \( f_2 \) multiplet can be used to estimate the timescale of variability of this mode. For KIC 8832417 and KIC 4064484 we find \( \sim 75 \) d and \( \sim 29 \) d, respectively. In KIC 9453114 the separation implies timescale of \( \sim 19 \) d. However, in this object every second multiplet component is very weak. Therefore, variations of \( f_2 \) will be dominated by a timescale half as long, i.e. \( \sim 9.5 \) d. The derived numbers confirm our earlier assessment that the variability of the secondary mode is faster in stars of longer periods.

In the right column of Fig. 8 we display the Fourier transform of the subharmonic, \( 3/2f_2 \). This signal is visible only in the FT of the first three stars. In KIC 5520878 the peak is broadened, but not yet resolved into separate components. This is the same behaviour as displayed by its parent mode (\( f_2 \)). In KIC 8832417 the subharmonic is already split, but because components are much broader than in the case of \( f_2 \), the splitting appears incomplete. Finally, in KIC 4064484 the subharmonic is resolved into three separate bands of power. In both KIC 8832417 and KIC 4064484 the splitting pattern of the subharmonic looks different than for its parent mode. Nevertheless, the frequency separations of multiplet components for \( 3/2f_2 \) and for \( f_2 \) are roughly the same. This shows that both signals vary on approximately same timescale.

5.2 The radial mode

5.2.1 Time domain

In Fig. 8 we show the temporal behaviour of the dominant radial mode in Kepler RRc stars. In every object we find changes of both the amplitude (\( A_1 \)) and the phase (\( \phi_1 \)). This variability has two components: a long-term drift with a timescale of many months, and a much faster quasi-periodic modulation which is superimposed on this drift.

With the exception of KIC9453114, the long-term amplitude changes are rather small, less than \( \pm 4 \) per cent. We are not sure if these changes are real. They might be of instrumental origin, resulting, e.g., from a slow image motion, coupled with a contamination by a nearby faint star. Amplitude fluctuations of this size are found in many stars observed with the Kepler telescope. On the other hand, the amplitude variations found in KIC9453114 are much larger (\( \pm 8 \) per cent) and they are almost certainly intrinsic to the star. We note that these variations do not resemble the Blazhko modulation as we currently know it (Benkő et al. 2010, 2014).

In two of the studied stars the radial mode displays a large, long-term drift of the pulsation phase, amounting to almost \( 0.3 \) rad in KIC 4064484 and \( 2.9 \) rad in KIC9453114. These variations are orders of magnitude too fast to be explained by stellar evolution. We note that nonevolutionary phase changes are observed in many RRc and RRab stars (e.g. Jurcsik et al. 2001, 2012, Le Borgne et al. 2007), as well as in many overtone Cepheids (e.g. Berdnikov et al. 1997; Moskalik & Kółaczkowski 2009). They are also occasionally detected in \( \delta \) Sct stars (e.g. Bowman & Kurtz 2014). Currently, their nature remains unexplained (see however Derekas et al. 2004).

We now turn our attention to the short timescale variations. They are clearly visible in three of the Kepler RRc stars: KIC 8832417, KIC 4064484 and KIC9453114. These variations are very small: changes of \( A_1 \) are of the order of \( \pm 1.5 \) per cent, or equivalently of \( \pm 2.5 \) mmag, and changes of \( \phi_1 \) of the order of \( \pm 0.01 \) rad. Such a small effect could be detected only with ultraprecise photometry, such as delivered by Kepler. The rapid variability of the radial mode is faster in stars of longer periods. We recall that the same tendency was found previously for the secondary mode, \( f_2 \). In Fig. 10 we compare temporal behaviour of the two modes in KIC9453114. It is immediately obvious that the amplitude of the radial mode, \( A_1 \), and the amplitude of the secondary mode, \( A_2 \), vary with the same timescale. We also note that the two amplitudes seem to be anticorrelated and a maximum (minimum) of one of them in most cases coincides with a minimum (maximum) of the other. This behaviour is common to all Kepler RRc stars.
Figure 9. Amplitude and phase variations of the dominant radial mode in Kepler RRc stars. This is low resolution version of the figure.

Figure 10. Amplitude variations in KIC9453114. Upper panel: dominant radial mode, $f_1$. Bottom panel: secondary mode, $f_2$.

5.2.1.1 Period variations Phase variations of the dominant radial mode (Fig. 9) are equivalent to variations of its period, $P_1$. The latter can be computed with Eq. 2. Slow long-term phase drifts observed in KIC 4064484 and KIC 9453114 correspond to slow long-term changes of $P_1$, with a total range of $\sim 7$ s and $\sim 49$ s, respectively. The short timescale phase variations result in additional quasi-periodic modulation of $P_1$, imposed on the long-term trends. The maximum range of these rapid variations is $\pm 3.2$ s in KIC8832417, $\pm 5.0$ s in KIC4064484 and $\pm 16.5$ s in KIC9453114. Changes of the radial mode period in the last object are displayed in Fig. 11.

Figure 11. Period variations of the dominant radial mode in KIC9453114. Fifth order polynomial fit (red thick line) represents the slow period drift.
5.2.2 Frequency domain

Because of quasi-periodic modulation, we expect the dominant radial mode to appear in the FT as a multiplet structured around the frequency $f_1$. Since the variations are very small ($\sim$1.5 per cent in amplitude), the modulation side-peaks should be much lower than the central peak of the multiplet. Therefore, in order to extract them from the FT, we first need to remove the central peak.

This is not a straightforward task. The amplitude $A_1$ and phase $\phi_1$ of the radial mode undergo slow long-term changes (Fig.4). As a result, the central component of the multiplet is not coherent. The standard prewhitening method will fail in such a case, leaving unremoved residual power. To remedy this situation, we applied the time-dependent prewhitening, a new technique described in Appendix A. In this procedure, we subtract from the light curve a sine wave with varying amplitude and phase. The functional form of $A_1(t)$ and $\phi_1(t)$ is determined by the time-dependent Fourier analysis of the data. With the proper choice of the length of the light curve segment, $\Delta t$, we can remove the central peak of the multiplet, leaving the side-peaks unaffected. The appropriate value of $\Delta t$ has to be longer than the quasi-period of the modulation, but short enough to capture the long-term drift of $A_1$ and $\phi_1$.

We applied this technique to all Kepler RRc stars except KIC 5520878, for which the radial mode does not show any short timescale variations. In Fig.10 (bottom panels) we display the prewhitened FT of the radial mode in KIC 8832417, KIC 4064484 and KIC 9453114. For removing the central peak of the multiplet we adopted $\Delta t = 100 \, \text{d}, 30 \, \text{d}$ and $20 \, \text{d}$, respectively. In the upper panels of the Figure we plot for comparison the frequency pattern of the secondary mode, $f_2$. In each of the three stars, the dominant radial mode, $f_1$, splits into a quintuplet of equally spaced peaks. This is particularly well visible for KIC 4064484, where the structure is nicely resolved and very clean. In KIC 9453114 the quintuplet pattern is also clear, but additional peaks appearing in the vicinity of $\Delta f = -0.10 \, \text{d}^{-1}$ and $\Delta f = +0.07 \, \text{d}^{-1}$ slightly confuse the picture. Compared to $f_2$, the modulation side peaks of the radial mode are significantly narrower (i.e. more coherent). They are also much lower, never exceeding 0.9 mmag. This latter property bears witness to the extremely low amplitude of the radial mode modulation. Apart from these differences, the quintuplet splitting patterns of $f_1$ and $f_2$ are very similar. In particular, while separation between the quintuplet components differs from star to star, it is always the same for both modes. This means that the secondary mode, $f_2$, and the dominant radial mode, $f_1$, are both modulated with a common timescale.

6 ADDITIONAL FREQUENCIES

A Fourier transform of the entire light curve (Q0–Q10) offers not only better frequency resolution, but also a much lower noise level than the FT computed from a single quarter of data (Figs.4, 5). This advantage can be used to search for additional, low amplitude periodicities hidden in the light curve.

6.1 KIC 5520878

In order to lower the noise in the FT as much as possible, we first subtracted from the light curve the dominant frequency $f_1$ and its harmonics, as well as eight other frequency components higher than 0.4 mmag ($f_2$, $f_2-f_1$, $f_1+f_2$, $2f_1+f_2$, $1/2f_2$, $3/2f_2$, $f_1-1/2f_2$, $f_1+3/2f_2$). This step was performed with the time-dependent prewhitening method, adopting $\Delta t = 20 \, \text{d}$. The prewhitened FT yields a rich harvest of additional low-amplitude periodicities. We list them in Table 5. Four segments of the prewhitened FT are displayed in Fig.13.

In total, we detected 46 new, significant frequencies. All have amplitudes below 0.4 mmag. We checked that none of them can be explained as a linear combination of $f_1$, $f_2$ or $1/2f_2$. Only 15 of the new frequencies are independent,
Figure 13. Additional low amplitude frequencies in KIC 5520878. Frequency spectrum was computed for the entire light curve (Q1–Q10), prewhitened of the dominant radial mode, its harmonics and all peaks higher than 0.4 mmag. Dashed lines indicate subtracted frequencies $1/2f_2$ and $3f_1$.

i.e., correspond to pulsation modes. The remaining frequencies are linear combinations of the independent ones and $f_1$. In fact, except of $f_4$, $f_7$ and $f_8$, all other independent frequencies generate at least one combination peak. This is a very important observation. It proves that these periodicities originate in KIC 5520878 itself, and are not introduced by blending with another star.

The strongest of the low-amplitude modes appear at $f_{3a} = 1.12535 \, \text{d}^{-1}$ and $f_{3b} = 1.13577 \, \text{d}^{-1}$. These two modes form a well-resolved doublet. The same doublet structure is apparent in their linear combinations with $f_1$. Another well-resolved doublet with slightly smaller separation is formed by $f_{12a} = 10.72126 \, \text{d}^{-1}$ and $f_{12b} = 10.72867 \, \text{d}^{-1}$. Two equidistant triplets are also found: $f_6$ with separation of $\delta f_6 = 0.00405 \, \text{d}^{-1}$ and $f_{10}$ with separation of $\delta f_{10} = 0.00245 \, \text{d}^{-1}$. With the timebase of 869 d, we consider the $f_{10}$ triplet to be only marginally resolved. We note that $\delta f_{10}$ is not very different from $1/372.5 \, \text{d} = 0.00268 \, \text{d}^{-1}$. Therefore, splitting of $f_{10}$ might be an artefact, caused by the instrumental amplitude variation with the orbital period of the telescope. We think this is unlikely. If such a modulation is present in the data, it should affect all the frequencies. Clearly, this is not the case. Judging from behaviour of the dominant radial mode (Fig. 9), any instrumental amplitude variation in the KIC 5520878 dataset cannot be larger than $\pm 4$ per cent. Such a small modulation cannot explain relatively large sidepeaks of the $f_{10}$ triplet.

Two modes attract special attention. For $f_9$ and $f_{10}$ we find period ratios of $P_9/P_9 = 0.7427$ and $P_9/P_{10} = 0.7480$, respectively. These values are close to canonical period ra-
Table 5. Low-amplitude periodicities in KIC 5520878

| frequency | $f$ [d$^{-1}$] | $A$ [mmag] |
|-----------|---------------|------------|
| $f_4$     | 1.03272       | 0.11       |
| $f_{5a}$  | 1.12535       | 0.35       |
| $f_{5b}$  | 1.13577       | 0.33       |
| $f_{5a} - f_1$ | 2.59012   | 0.07       |
| $f_{5b} - f_1$ | 2.5793       | 0.04       |
| $f_{5a} - 2f_1$ | 6.3049       | 0.04       |
| $f_{5b} - 2f_1$ | 6.2941       | 0.03       |
| $f_{5a} - 3f_1$ | 10.0201      | 0.04       |
| $f_{5b} - 3f_1$ | 10.0094      | 0.03       |
| $f_{5a} - 4f_1$ | 13.7353      | 0.03       |
| $f_{5b} - 4f_1$ | 13.7245      | 0.03       |
| $f_{5a} + f_1$ | 4.84070      | 0.20       |
| $f_{5b} + f_1$ | 4.85087      | 0.21       |
| $f_{5a} + 2f_1$ | 8.5558       | 0.04       |
| $f_{5b} + 2f_1$ | 8.56611      | 0.06       |
| $f_{5a} + 3f_1$ | 12.2709      | 0.03       |
| $f_{5b} + 3f_1$ | 12.2813      | 0.03       |
| $f_{5a} + 4f_1$ | 15.9589      | 0.03       |
| $f_6$     | 1.49877       | 0.14       |
| $f_7$     | 1.50317       | 0.19       |
| $f_8$     | 1.50686       | 0.11       |
| $f_{10a}$ | 5.21814       | 0.13       |
| $f_{10b}$ | 8.9332        | 0.05       |
| $f_{10c}$ | 12.6483       | 0.03       |
| $f_7$     | 1.80060       | 0.06       |
| $f_8$     | 2.63891       | 0.08       |
| $f_9$     | 2.75926       | 0.10       |
| $f_9 + f_1$ | 6.4744       | 0.04       |
| $f_{10a}$ | 2.77651       | 0.19       |
| $f_{10b}$ | 2.77900       | 0.19       |
| $f_{10c}$ | 2.78141       | 0.22       |
| $f_{10a} + f_1$ | 6.49165      | 0.08       |
| $f_{10b} + f_1$ | 6.49419      | 0.08       |
| $f_{10c} + f_1$ | 6.49645      | 0.08       |
| $f_{10a} + 2f_1$ | 10.2067      | 0.03       |
| $f_{11}$  | 4.69968       | 0.18       |
| $f_{11} - f_1$ | 0.98052      | 0.14       |
| $f_{11} - 2f_1$ | 2.72874      | 0.06       |
| $f_{11} + f_1$ | 8.41479      | 0.05       |
| $f_{11} + 2f_1$ | 12.1298      | 0.04       |
| $f_{12a}$ | 10.72126      | 0.09       |
| $f_{12b}$ | 10.72867      | 0.07       |
| $f_{12a} + f_1$ | 7.0061       | 0.05       |
| $f_{12a} + f_2$ | 14.4363      | 0.04       |
| $f_{12a} + 2f_1$ | 18.1516      | 0.03       |
| $f_{12b} + 2f_1$ | 18.1586      | 0.02       |

Table 6. Low-amplitude periodicities in KIC 8832417

| frequency | $f$ [d$^{-1}$] | $A$ [mmag] |
|-----------|---------------|------------|
| $f_4$     | 1.18653       | 0.25       |
| $f_4 + f_1$ | 5.20990      | 0.12       |
| $f_{5a}$  | 3.08589       | 0.07       |
| $f_{5b}$  | 3.09347       | 0.08       |
| $f_6$     | 6.65658       | 0.32       |
| $f_6 - f_1$ | 2.63315      | 0.10       |
| $f_6 + f_1$ | 10.67988     | 0.11       |
| $f_7$     | 9.51594       | 0.08       |

Table 7. Low-amplitude periodicities in KIC 9453114

| frequency | $f$ [d$^{-1}$] | $A$ [mmag] |
|-----------|---------------|------------|
| $f_4$     | 0.91199       | 0.23       |
| $f_4 + f_1$ | 3.64315      | 0.10       |
| $f_5$     | 1.87588       | 0.09       |
| $f_5 - f_1$ | 0.85617      | 0.10       |
| $f_5 - 2f_1$ | 3.58750      | 0.16       |

6.2 KIC 4064484, KIC 8832417 and KIC 9453114

In the other three Kepler RRc stars only very few additional low-amplitude modes can be found. In KIC 8832417 and KIC 9453114 we detected 5 and 2 such modes, respectively. Their frequencies are listed in Tables 5 and 6. About
Table 8. Known RR Lyrae variables with $P_2/P_1 \sim 0.61$

| Star             | Type | $P_1$ [d] | $P_2/P_1$ | $A_2$ [mmag] | subharmonics of $f_2$ | ref. |
|------------------|------|-----------|-----------|--------------|-----------------------|-----|
| ω Cen V10        | RRc  | 0.3750    | 0.6137    | 6.4          |                       | 2   |
| ω Cen V19        | RRc  | 0.3000    | 0.6119    | 7.1          |                       | 2   |
| ω Cen V81        | RRc  | 0.3894    | 0.6138    | 7.2          |                       | 2   |
| ω Cen V87        | RRc  | 0.3965    | 0.6219    | 6.3          |                       | 2   |
| ω Cen V105       | RRc  | 0.3353    | 0.6138    | 12.5         |                       | 2   |
| ω Cen V350       | RRc  | 0.3791    | 0.6084    | 6.2          |                       | 2   |
| OGLE-LMC-RRLYR-11983 | RRc  | 0.3402    | 0.6026    | 51.5:        |                       | 3   |
| OGLE-LMC-RRLYR-14178 | RRc  | 0.3634    | 0.6103    | 27.8:        |                       | 3   |
| SDSS Stripe 82-1528004 | RRc  | 0.3276    | 0.6068    | ?            |                       | 4   |
| SDSS Stripe 82-3252839 | RRc  | 0.3112    | 0.6238    | ?            |                       | 4   |
| CoRoT 0105036241  | RRc  | 0.3729    | 0.6125    | 3.3          |                       | 6   |
| CoRoT 0105735652  | RRc  | 0.2792    | 0.6150    | 2.2          |                       | 6   |
| KIC 4064484      | RRc  | 0.3370    | 0.6150    | 4.6          | yes                   | 7   |
| KIC 5520878      | RRc  | 0.2692    | 0.6320    | 7.0          | yes                   | 7   |
| KIC 8832417      | RRc  | 0.2485    | 0.6122    | 7.1          | yes                   | 7   |
| KIC 9453114      | RRc  | 0.3661    | 0.6144    | 4.6          | yes                   | 7   |
| EPIC 60018224    | RRc  | 0.3063    | 0.6145    | 11.4         | yes                   | 8   |
| EPIC 60018238    | RRc  | 0.2748    | 0.6030    | 2.0          |                       | 8   |
| EPIC 60018678    | RRc  | 0.4325    | 0.6200    | 3.8          | (yes)                 | 8   |
| AQ Leo           | RRd  | 0.4101    | 0.6211    | 2.5          | yes                   | 1   |
| CoRoT 0101368812 | RRd  | 0.3636    | 0.6141    | 5.5          | yes                   | 5   |
| EPIC 60018653    | RRd  | 0.4923    | 0.6163    | 8.5          | yes                   | 8   |
| EPIC 60018662    | RRd  | 0.4175    | 0.6170    | 6.9          | yes                   | 8   |

REFERENCES: 1 - Gruberbauer et al. (2007); 2 - Olech & Moskalik (2009); 3 - Soszyński et al. (2009); 4 - Süveges et al. (2012); 5 - Chadid (2012); 6 - Szabó et al. (2014); 7 - this paper; 8 - Molnár et al. in preparation.

7 DISCUSSION

7.1 A new period ratio for RR Lyrae stars

It is striking that all four RRc stars prominent in the Kepler field that have been analyzed in this paper show an additional frequency with a period ratio of $\sim 0.61$ to the main radial mode. This is not the first time such a period ratio has been seen in RR Lyrae stars. It was first found in a double-mode variable AQ Leo (Gruberbauer et al. 2007). Since then, low amplitude modes yielding similar period ratios have been detected in 18 other RR Lyrae variables, observed both from the ground and from space. We list these objects in Table 8 where we also include, for completeness, the four Kepler RRc stars. For each variable we provide its pulsation type, a period of the dominant radial mode, $P_1$, a period ratio of the secondary mode and the dominant mode, $P_2/P_1$, and an amplitude of the secondary mode, $A_2$. In the case of double-mode variables in which two radial modes are present, $P_1$ refers to the first radial overtone. The amplitudes $A_2$ are given in different photometric systems by different authors and, consequently, are not directly comparable to each other. We quote them here only to provide a rough estimate of the strength of the secondary mode.

Comments on individual stars

ω Cen variables V10, V81, V87 and V350: Detection of the secondary mode in these stars is unambiguous, but identification of its true period is hindered by daily aliases. Depending on the choice of an alias we find period ratio of either $P_2/P_1 \sim 0.80$ or $0.61$. Only in the case of V10 is the former alias slightly higher, which led Olech & Moskalik (2009) to designate this variable a candidate double overtone pulsator. In V81, V87 and V350 the latter alias is higher. So far, no unambiguous 1O+2O double-mode RR Lyrae stars have been found in any stellar system (see, however, Pigulski 2014). On the other hand, the existence of RR Lyrae stars with $P_2/P_1 \sim 0.61$ is well established. On these grounds, we consider the period ratio of $\sim 0.61$ to be more likely in these four variables.

ω Cen variables V19 and V105: In these stars the identification of the correct alias and consequently of the true period of the secondary mode is unambiguous.
OGLE-LMC-RRLYR-11983: In the original paper of Soszyński et al. (2009) the star was classified as a fundamental mode pulsator (RRab star). This was based on the measured value of the Fourier phase $\phi$. However, the harmonic of the primary mode in this variable is weak and its amplitude is determined with a large error of almost 30 per cent. Consequently, $\phi$ is not measured accurately enough to distinguish between an RRc and an RRab light curve. In fact, $\phi = 3.358$, derived by Soszyński et al. (2009), does not fit to either type. We have reclassified this star using the empirical period luminosity relation for the Wesenheit index, $W_I = I - 1.55(V - I)$. With $W_I = 18.376$ mag, OGLE-LMC-RRLYR-11983 is placed firmly among the RRc stars of the LMC.

OGLE-LMC-RRLYR-14178: The star was classified by Soszyński et al. (2009) as an overtone pulsator (RRc star). The value of the Wesenheit index $W_I = 17.795$ mag supports this classification.

SDSS Stripe 82 variables 1528004 and 3252839: The secondary frequency in these stars was detected with a principal component analysis, applied to multiband photometric data (Süveges et al. 2012). The authors did not identify the dominant radial mode. On the basis of the empirical period-amplitude diagram (their Fig. 9) we classify variable 3252839 as an RRc star. In the case of 1528004, the period of the radial mode points towards overtone pulsations, but its amplitude is too high for an RRc type. We classify this variable as a probable RRc star. Amplitudes of the secondary mode were not published.

CoRoT 0105036241 and CoRoT 0105735652: The secondary mode in these stars was detected with photometry collected by the CoRoT space telescope (Szabó et al. 2014). No subharmonics were found.

EPIC 60018224 $\equiv$ EV Psc: The secondary mode was discovered during 9-d long engineering test run of Kepler K2 mission (Molnár et al. in preparation). Subharmonics of the secondary mode at $\sim 1/2 f_2$ and $\sim 3/2 f_2$ are also clearly visible.

EPIC 60018238: No subharmonics of the secondary mode were detected in this star.

Figure 14. Petersen diagram for RR Lyrae stars of Table 5. RRc stars are plotted with black asterisks and RRd stars with red filled circles.

EPIC 60018678: In addition to the secondary mode, marginally significant subharmonics at $\sim 1/2 f_2$ and $\sim 3/2 f_2$ are also present in this star (Molnár et al. in preparation).

AQ Leo: The secondary mode in this double-mode star was discovered with photometry collected by the MOST space telescope (Gruberbauer et al. 2007). A subharmonic of a secondary mode at $\sim 1/2 f_2$ was also found, although it was not recognized as such at the time.

CoRoT 0101368812: The secondary mode with a period ratio of $\sim 0.61$ to the first radial overtone was discovered by Chadid (2012). In addition, its subharmonic at $\sim 3/2 f_2$ was also detected, but not recognized as such by the author. It is denoted in the original paper as $f_4 - f_1$.

EPIC 60018653: In addition to the secondary mode, subharmonics at $\sim 1/2 f_2$ and $\sim 3/2 f_2$ are also clearly visible in this star (Molnár et al. in preparation).

EPIC 60018662: In addition to the secondary mode, a weak subharmonic at $\sim 1/2 f_2$ was also detected in this star (Molnár et al. in preparation).

Objects listed in Table 5 form a sample of 23 RR Lyrae variables in which a period ratio of $\sim 0.61$ has been found. In most of these stars the main mode is the first radial overtone (RRc type). Four variables belong to the group of double-mode pulsators (RRd type), where two radial modes are excited. Even in the latter case, the first radial overtone strongly dominates, having an amplitude 1.8 - 3.0 times higher than the fundamental mode (Gruberbauer et al. 2007; Chadid 2012; Molnár et al. in preparation.). In Fig. 14 we plot all 23 stars on the Petersen diagram. They form a tight, almost horizontal progression, with values of $P_2/P_1$ restricted to a narrow range of 0.602 - 0.632. Both RRc and RRd variables follow the same trend. Clearly, the RR Lyrae star with $P_2/P_1 \sim 0.61$ form a highly homogenous group, constituting a new, well defined class of multimode pulsators.

In all stars of this class the amplitude of the additional mode is extremely low, in the mmag range. This amplitude is typically 20 - 60 times lower than the amplitude of the dominant radial mode. Detection of such a weak signal from the...
ground is difficult. The situation is very different for stars observed from space. With the sole exception of the Blazhko star CSS J235742.1–015022 (Molnár et al. in preparation), the secondary mode yielding period ratio of $P_2/P_1 \sim 0.61$ has been detected in every RRc and RRd pulsator for which high-precision space photometry is available. This is 13 objects out of total 14 observed from space. Such statistics strongly suggest that excitation of this additional mode is not an exception. It must be a common property of RRc and RRd variables. We expect that it should be found in many more stars, provided that good enough data become available.

### 7.1.1 Comparison with Cepheids

Detection of low amplitude secondary modes with a period ratio of $\sim 0.61$ to the main radial mode is not limited to the RR Lyrae stars. Similar modes are also found in Classical Cepheids of the Magellanic Clouds. So far, 173 such variables have been identified (Moskalik & Koślawski 2008, 2009; Szoszyński et al. 2008, 2010). In Fig. 16 we plot them on the Petersen diagram, together with their RR Lyrae counterparts. Both groups have very similar properties. In the case of Cepheids, just like in the RR Lyrae stars, the phenomenon occurs only in the first overtone variables or in the double-mode variables pulsating simultaneously in the fundamental mode and the first overtone (only one star). Apparently, excitation of the first radial overtone is a necessary condition. The amplitudes of the secondary modes are as low as in the RR Lyrae stars, with amplitude ratios of $A_2/A_1 < 0.055$ (Moskalik & Koślawski 2009). The measured period ratios are also almost the same in both types of pulsators, although in Cepheids the range of $P_2/P_1$ is somewhat broader ($0.599 – 0.647$). The only difference between the two groups is that Cepheids split in the Petersen diagram into three well-detached parallel sequences, whereas no such structure is evident in the case of RR Lyrae stars. This difference needs to be explained by future theoretical work.

### 7.1.2 Comparison with theoretical calculations

In the case of Classical Cepheids it has been shown that the period ratio of $\sim 0.61$ cannot be reproduced by two radial modes (Dziembowski & Smolec 2009; Dziembowski 2012). Since the main mode is radial, this implies that the secondary frequency, $f_2$, must correspond to a nonradial mode of oscillation. Theoretical analysis indicates that an f-mode of high spherical degree ($\ell = 42 – 50$) is the most likely candidate (Dziembowski 2012).

Figure 16. Linear period ratios $P_{30}/P_{10}$ and $P_{40}/P_{10}$. Model masses and luminosities are in the range of 0.55 – 0.75 $M_\odot$ and 40 – 70 $L_\odot$, respectively. Upper panel: models for metallicity of $Z = 0.001$. Bottom panel: models for metallicity of $Z = 0.01$. The RR Lyrae stars of Table A are plotted with filled circles for comparison.

$Z$ does not lower the computed values any further. We note in passing, that weak sensitivity to $Z$ is typical of period ratios of two overtones. This is different from behaviour of $P_{10}/P_{1}$, sensitivity of which to a metal abundance is much stronger (see e.g. Popielski et al. 2000). The models are compared with the RRc and RRd variables listed in Table A. The observed period ratios generally fall between theoretically predicted values of $P_{30}/P_{10}$ and $P_{40}/P_{10}$. Only in a handful of long period variables can the secondary period be matched with the third overtone, but for vast majority of the sample it cannot be matched with a radial mode. This result does not depend on the choice of $Z$. Therefore, as in the case of Cepheids, we conclude that also in the RR Lyrae stars the secondary mode with a period ratio of $\sim 0.61$ to the first radial overtone must be nonradial.

Model calculations show that two different types of nonradial modes are linearly unstable in the RR Lyrae stars (Van Hoolst et al. 1998; Dziembowski & Cassisi 1999). Low-degree modes of $\ell = 1 – 3$ are excited rather weakly and preferentially in the vicinity of radial modes. Their frequency spectrum is very dense, which makes it difficult to destabilize only a single isolated mode. The other group are strongly trapped unstable modes (STU modes) of high spherical degree ($\ell \geq 6$). Oscillations of this type are excited strongly,
with growth rates as large as in the case of radial modes. Most importantly, such oscillations can be excited at frequencies far apart from those of the radial modes. The strong trapping in the envelope, which makes their destabilization possible, usually selects from the dense spectrum only a single mode. We believe that excitation of STU modes is the most likely explanation for the puzzling period ratio of \( \sim 0.61 \) in the RRc and RRd variables. At this point, this is only a working hypothesis. Its verification requires detailed linear nonadiabatic pulsation calculations, similar to those performed by Dziembowski (2012) for the Classical Cepheids.

7.2 Period doubling of the secondary mode

In all four Kepler RRc stars not only do we see the additional frequency with a period ratio of \( \sim 0.61 \) to the main mode, but we also see its subharmonics at \( \sim 1/2f_2 \) and \( \sim 3/2f_2 \). Subharmonic frequencies are also detected in several other variables of Table 8. Noticeably, they are found only in those stars for which high precision space photometry is available. This is hardly surprising, considering that amplitudes of the subharmonics are even lower than in the case of \( f_2 \) and almost never exceed 3.0 mmag. In total, the additional mode \( f_2 \) is accompanied by subharmonics in 10 out of 13 RRc and RRd stars studied from space. Clearly, this is a common property of these variables.

The presence of subharmonics in the frequency spectra of RRc and RRd stars is very significant, as it is a characteristic signature of a period doubling behaviour. Period doubling is a phenomenon which is well known in many dynamical systems (Bergé et al. 1986). In the context of stellar pulsations, it results in an alternating light curve in which even and odd pulsation cycles have different shapes. Such a light curve repeats itself after two pulsation periods, not one. The appearance of subharmonics in the Fourier spectrum is a direct consequence of this property (e.g. Smolec & Moskalik 2012, their Fig. 3).

Light curves with alternating deep and shallow minima have been known for decades in RV Tauri stars. Only in the 1980s were they interpreted as resulting from period doubling (Buchler & Kovács 1987; Kovács & Buchler 1988). More recently, the period doubling effect has been discovered in two other types of pulsating variables – in Blazhko-modulated RRab stars (Kolenberg et al. 2010; Szabó et al. 2010) and in BL Herculis stars (Smolec et al. 2012). In the former case, its strength varies significantly over the Blazhko cycle, and does not repeat from one Blazhko cycle to the next. The picture is different in the BL Herculis stars, where the alternating light curve is strictly periodic (repetitive).

As was first recognized by Moskalik & Buchler (1990), period doubling in pulsating variables can be caused by a half-integer resonance between the modes. Through dynamical modelling, its origin in RV Tauri stars was traced back to the 5:2 resonance with the second overtone (Moskalik & Buchler 1990), in RRab stars to the 9:2 resonance with the 9th overtone (Kolláth et al. 2011) and in BL Herculis stars to the 3:2 resonance with the first overtone (Buchler & Moskalik 1992; Smolec et al. 2012). In these three classes of pulsators the period doubling affects the dominant mode. In the case of the Kepler RRc stars and the other RRc and RRd stars of Table 8, however, we do not observe this phenomenon in the main radial mode(s). Instead, it is the puzzling secondary nonradial mode with a period ratio of \( \sim 0.61 \) to the first overtone, that shows a period doubling behaviour. Therefore, as we do not know the identity of this mode, it is unclear which resonance (if any) is responsible for the period doubling in the RRc and RRd variables.

7.3 Variability of the secondary mode

In all four RRc stars observed with Kepler, the secondary nonradial mode, \( f_2 \), and its subharmonics at \( \sim 1/2f_2 \) and \( \sim 3/2f_2 \) display very strong variations of amplitudes and phases. Variability of the \( f_2 \) mode has also been found in other RRc stars observed from space (Szabó et al. 2014; Molnár et al. in preparation), indicating that this is yet another common property of all RRc pulsators.

Thanks to the superb quality of the Kepler data, we have been able to study this phenomenon with unprecedented detail. We have shown that variability of the secondary mode is quasiperiodic and occurs on a timescale of 10–200 d, depending on the star. In the frequency domain, it causes the mode to split into a quintuplet of equally spaced peaks. Each of these peaks is broadened, which reflects an irregular character of the modulation. The main radial mode, \( f_1 \), also varies in a quasiperiodic way, but this modulation is extremely small. In the frequency domain, the mode splits into a quintuplet as well. In every star the separation between the quintuplet components of \( f_1 \) and of \( f_2 \) is the same, which proves that both modes are variable with the same timescale.

The finding that the secondary mode and the main radial mode are both modulated on a common timescale has implications for understanding of the nature of the \( f_2 \) multiplet. Because \( f_1 \) corresponds to a radial mode (\( \ell = 0 \)), its quintuplet structure cannot be caused by rotational splitting. It can be interpreted only in one way – as resulting from a true, physical modulation of the mode’s amplitude and phase. The same frequency splitting and thus the same variability timescale of \( f_1 \) and \( f_2 \) indicates that modulations of the two modes are not independent. This in turn implies that quintuplet pattern of \( f_2 \) cannot correspond to a multiplet of nonradial modes, either. Accepting such a picture would make it difficult to understand why beating of rotationally split multiplet (\( f_2 \)) should occur on the same timescale as the true modulation of another mode (\( f_1 \)). We believe that such a coincidence is highly unlikely. This leads us to the conclusion that the observed modulation of the secondary mode, \( f_2 \), is due not to beating, but just as in the case of \( f_1 \), to a true physical modulation of a single pulsation mode.

The same modulation timescale of \( f_1 \) and \( f_2 \) implies that both modes must be part of the same dynamical system, in other words that they must interact with each other. We can only speculate what the nature of this interaction might be. Cross-saturation is the most obvious possibility that comes to mind. In physical terms, the two modes compete for the same driving (\( \kappa \)-mechanism in the He\(^+\) partial ionization zone) and when the amplitude of one mode decreases, the amplitude of the other mode can increase. This kind of coupling has to occur always when the two modes use the same driving source. It also predicts that the ampli-
tude variations of the two modes should be anticorrelated. This is what we observe.

8 SUMMARY AND CONCLUSIONS

In this paper we present an in-depth analysis of four first overtone RR Lyrae stars (RRc stars) observed with the Kepler space telescope: KIC 4064484, KIC 5520878, KIC 8832417 and KIC 9453114. For our study we used the Long Cadence data (30 min sampling) gathered between Q0 and Q10, with a total timebase of 774 to 880 d, depending on the star. Our most important findings can be summarized as follows:

- None of the studied Kepler RRc stars displays a classical Blazhko effect, with a nearly coherent periodic (or multi-periodic) modulation of the amplitude and phase of the dominant Blazhko effect, with a nearly coherent periodic (or multi-periodic) modulation of the amplitude and phase of the dominant radial mode.
- In every Kepler RRc star we detect a secondary mode, \( f_2 \), with a period ratio of \( \sim 0.61 \) to the first radial overtone. The mode has a very low amplitude, more than 20 times below that of the dominant radial mode.
- Secondary modes with similar period ratios are also present in 19 other RR Lyrae variables. These stars are either of RRc or of RRd type, but never of RRab type. Apparently, the excitation of the secondary mode is somehow connected with the excitation of the first radial overtone. The observed period ratios are in a very narrow range of \( 0.602 \pm 0.032 \), defining a new class of multimode pulsators. Including the four RRc stars studied in this paper, this class has currently 23 members.
- The period ratio of \( \sim 0.61 \) is also observed in Classical Cepheids of the Magellanic Clouds. The stars are either single mode first overtone pulsators or \( F+1O \) double-mode pulsators, but never single mode fundamental pulsators. This property is the same in the case of Cepheids and in the case of RR Lyrae stars.
- In neither RR Lyrae stars nor Cepheids can the period ratio of \( \sim 0.61 \) be reproduced by two radial modes. In both types of variables the secondary mode must be nonradial.
- In every Kepler RRc star we detect at least one subharmonic of the secondary mode, at \( \sim 1/2f_2 \) or at \( \sim 3/2f_2 \). Similar subharmonics have also been found recently in several other RRc and RRd variables observed from space (Molnár et al. in preparation). They have also been retrospectively identified in two more RRd stars observed from space. Detection of subharmonics of \( f_2 \) is a signature of period doubling of this mode. After RV Tauri, Blazhko RRab and BL Herculis stars, the RRc and RRd stars are now the fourth group of pulsators in which period doubling has been found. Contrary to the former three types of variables, in the RRc and RRd stars the period doubling affects not the primary, but the secondary mode of pulsation.
- Judging from the results of space photometric observations, the excitation of the secondary mode with the period ratio of \( \sim 0.61 \) to the first overtone and concomitant period doubling of this mode must be a common phenomenon in RRc and RRd variables.
- In every Kepler RRc star the amplitude and phase of the secondary mode and its subharmonics are strongly variable, with timescales of \( 10 \) to \( 200 \) d. The main radial mode varies on the same timescale, but with an extremely low amplitude. Its variability can be detected only with high-precision photometry collected from space.
- In three Kepler RRc stars even more periodicities are detected, all with amplitudes well below 1 mmag. One of the low amplitude modes discovered in KIC 5520878 can be identified with the radial second overtone, but all others must be nonradial. Many of these modes appear at frequencies that are below that of the radial fundamental mode. As such, they cannot be acoustic oscillations (\( p \)-modes), but must be classified as gravity modes (\( g \)-modes). This is the first detection of such modes in the RR Lyrae stars. The result is very surprising. From the theoretical point of view, the \( g \)-modes are not expected to be excited in the RR Lyrae variables, because they are all strongly damped in the radiative interior of the star.

At the time of writing, several additional RR Lyrae variables have been found in the Kepler field, thanks to the efforts of many dedicated individuals, and often as a by-product of studies devoted to other objects. For example, short period eclipsing binaries can have periods and amplitudes in the range of RR Lyrae stars (e.g., Rappaport et al. 2013; Szabó et al. in preparation). Also, the citizen science project PlanetHunters (www.planethunters.org) has yielded some additional RR Lyrae stars in the Kepler field. Among the new finds are several RRc stars.

The sample of space-observed RRc and RRd variables will be further expanded by the Kepler K2 mission, which is already underway (Howell et al. 2014). It is observing in the plane of the ecliptic, and will switch the field of view every 3 months. With this strategy, a variety of stellar populations will be sampled, including not only the Galactic halo and the thick disc but also several globular clusters and the Galactic Bulge. The K2 mission can observe hundreds of RR Lyrae variables, among them tens of RRc and of RRd type (Molnár, Plachy & Szabó 2014). The mission will increase greatly the number such objects studied from space. It will be interesting to see whether they show characteristics similar to the four RRc variables discussed in this paper.

When writing of this manuscript was almost completed, a new paper on RRc stars has been submitted by Netzel et al. (2014). The authors have analyzed the OGLE-III photometry of the Galactic Bulge. In \( \sim 3 \) per cent of the studied variables, they have detected a low-amplitude secondary mode with the period ratio of \( \sim 0.61 \) to the first radial overtone. This result confirms our conclusion that excitation of this puzzling mode is a common phenomenon in RRc stars.

ACKNOWLEDGEMENTS

Funding for this Discovery mission was provided by NASA’s Science Mission Directorate. The authors gratefully acknowledge the entire Kepler team, whose outstanding efforts have made these results possible. This research has been supported by the Polish NCN through grant no. DEC-2012/05/B/ST9/03932. It has also been supported by the “Lendület-2009 Young Researchers” Program of the Hungarian Academy of Sciences, the Hungarian OTKA grant K83790, the National Science Foundation grant no. NSF PHY05-51164, the European Community’s Seventh Framework Programme (FP7/2007-2013).
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APPENDIX A: TIME-DEPENDENT PREWHITENING

In the standard prewhitening procedure a periodic signal is removed from the light curve by subtracting a sine function with constant frequency \((f_1)\), amplitude \((A_1)\) and phase \((\phi_1)\). The values of \(f_1\), \(A_1\) and \(\phi_1\) are determined from the data with a least-squares fit. The method works very well when the signal’s frequency, amplitude and phase are indeed non-variable. However, when this condition is violated, as is the case for the Kepler RRc stars, the standard procedure fails and leaves residual power in the frequency spectrum of the prewhitened data. This is illustrated in Fig. A1, where we display the prewhitening sequence for the dominant mode of KIC 5520878. The residuals in the FT (middle panel) have an amplitude of 2.4 mmag. This is only \(\approx 1.5\) per cent of the original peak’s amplitude, but in context of Kepler photometry this is huge.

To remedy this situation, we have developed a novel method that we call the time-dependent prewhitening. In this procedure, we subtract from the light curve a sine function not with constant, but with varying amplitude and phase. To model this variability, we adopt \(A_1(t)\) and \(\phi_1(t)\) determined from the data (Fig. 9) by the time-dependent Fourier analysis (Kovács et al. 1987). The frequency of the mode, \(f_1\), is kept fixed. In the bottom panel of Fig. A1 we display the results of the time-dependent prewhitening for KIC 5520878. The dominant frequency is now removed entirely, down to the noise level of the FT (55 \(\mu\)mag).

Time-dependent Fourier analysis and, consequently, also time-dependent prewhitening, have one free parameter: the length of the light curve segment, \(\Delta t\). The choice of this parameter determines the time resolution of the method. Variations occurring on timescales shorter than \(\Delta t\) will not be captured by the time-dependent Fourier analysis and consequently, they will not be subtracted by the prewhitening procedure.

We illustrate this property in Fig. A2 where we present prewhitening of the dominant frequency of KIC 4064484. In the upper panel we plot the results for \(\Delta t = 5\) d. The time-dependent Fourier analysis (left column) captures both the long-term trend and the rapid quasi-periodic modulation of the amplitude. Using this \(A_1(t)\) and concomitant \(\phi_1(t)\) (not shown), the time-dependent prewhitening removes from the FT all power associated with the mode (right column). The results for \(\Delta t = 30\) d are quite different. The time-dependent Fourier analysis now captures only the long-term trend, but the rapid modulation is averaged out. After the time-dependent prewhitening, we find four residual peaks in the FT. They are placed symmetrically around the (removed) central frequency, \(f_1\), forming an equidistant frequency multiplet. These peaks are the Fourier representation of the quasi-periodic modulation of the mode, which

\(\text{Figure A1. Prewhitening of the main frequency in KIC 5520878. Upper panel: Fourier transform of the original } K_p \text{ magnitude light curve (quarters Q1 – Q10). Middle panel: FT after standard prewhitening. Bottom panel: FT after time-dependent prewhitening (} \Delta t = 10 \text{ d).} \)
Figure A2. Time-dependent prewhitening of the dominant mode of KIC 4064484 for the light curve segment length of $\Delta t = 5$ d and $\Delta t = 30$ d. Left column: variations of the amplitude of the mode, $A_1(t)$, determined with the time-dependent Fourier analysis. Right column: FT of the prewhitend light curve. Frequency of the removed dominant mode, $f_1$, is indicated by the dashed line.

for $\Delta t = 30$ d is not captured by the method and not removed during prewhitening.

In the language of the Fourier analysis, $\Delta t = 5$ d corresponds to a bandwidth of $0.2 \text{ d}^{-1}$ ($1/\Delta t$). This is broader than the frequency multiplet in Fig. A2. As a result, the signal reconstructed with the time-dependent Fourier analysis captures all the Fourier power associated with the mode and all of it can be removed. For $\Delta t = 30$ d, the bandwidth is only $0.0333 \text{ d}^{-1}$. This is narrow enough to isolate only the central peak of the multiplet. Consequently, only this peak is subtracted in the prewhitening process.