Building a wear resistance model of drilling operation using locally adaptive regression models

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Abstract. The paper deals with building mathematical models of metal cutting processes in the context of the optimum performance problem. A new class of wear resistant models called «locally adaptive models» is proposed. A practical case solution is given.

1. Introduction
Determining the optimum performance is one of the main problems of the metal cutting theory. To solve this problem, it is necessary to use models describing wear resistance of a tool. A technique of optimum performance determining depends on the model type. The paper highlights an algorithm of building a basic wear resistance model of a cutting tool in the case of drilling operation. For this building locally adaptive regression modeling is used.

2. Problem statement
Wear resistance of a drill can be described by the total length of the bores [1-2], have been made until the drill is not blunted \(L\), mm or by operating time until resharpening is required \(T\), min. Wear resistance depends on FPR rate \(S\), mm per revolution) and revolution frequency \(n\), revolutions per minute) or revolution rate \(V\), meters per minute).

The mentioned parameters are interrelated through the following formulas:

\[L = S \cdot T,\quad S_m = S \cdot n,\quad V = \frac{\pi d n}{1000},\]

where \(d\) is the size of the processed bore, mm.

Thus, the general wear resistance model can be described as follows:

\[L = f(S, n)\quad \text{or}\quad T = f(S, n).\]

Estimating the model parameters (2) is based on wear resistance experiment data. After that, the obtained model is used for optimum performance defining. For example, it can be done using the cost reduction criterion [1-2]

\[Q(n, S) = \frac{c}{L(n, S)} + \frac{D}{S n} + E,\]

where \(C, D, E\) are economic parameters.

3. Theory
In prediction modeling when building dependencies the following observing equations are considered:

\[y = f^T(x)\theta + e = \sum_{l=1}^{m} f_l(x)\theta_l + e,\]
where \( y \) is the response; \( f^T(x) = (f_1(x), f_2(x), \ldots, f_m(x)) \) is the vectorial function of the independent variable \( x = (x_1, \ldots, x_k)^T \) that varies in the domain \( X \); \( \theta = (\theta_1, \ldots, \theta_m)^T \) is the vector of unknown parameters that need to be determined on the basis of experimental results; \( e \) is the error. The structure of the vector \( f(x) \) cannot be always uniquely determined. There are often the cases when in different parts of the definition domain the response dependence is not the same. One of the most efficient techniques of building dependencies is the adaptive regression models technique derived from the fuzzy system concept [3-5].

The main idea of building locally adaptive regression models (LAR) is to use not only the predictors defined on all the definition domain of some factor, but also use the predictors defined on the local subdomains of factors. Normally, in order to obtain necessary smoothness of the required response dependency on the chosen factors, such subdomains are defined with intersection. Let the definition domains of the factors \( x_1, \ldots, x_k \) be divided into the local subdomains \( s_1, \ldots, s_k \). For the factor \( x_j \) we will call the local subdomains “fuzzy partitions” and will define them as \( R_{j1}, R_{j2}, \ldots, R_{js_j} \), \( j = 1, \ldots, k \). Whether the factor is contained in some fuzzy partition or not can be determined using the value of the indicator function \( \mu_j(x_j) \in [0, 1], i = 1, \ldots, s_j \), \( \mu_j(x_j) \in [0, 1], j = 1, \ldots, s_2, \ldots, \mu_j(x_k) \in [0, 1], i = 1, \ldots, s_k \). The introduced indicator function is almost the same as the membership function of linguistic variables [5-6]. An indicator function type can be different, for example the function can be triangle, trapezoidal or non-linear. In order to exclude the presence of some artifacts of the required dependency in the local subdomains it is necessary to normalize the indicator functions that means that the following conditions must be satisfied

\[
\sum_{i=1}^{s_j} \mu_i(x_j) = 1, \sum_{i=1}^{s_j} \mu_j(x_j) = 1, \ldots, \sum_{i=1}^{s_j} \mu_k(x_j) = 1. \tag{5}
\]

For clarity, let’s consider the special case when we have two input factors. Let’s divide the action areas of the cardinal variables \( x_1, x_2 \) into fuzzy partitions. Assume that in particular partitions the response properties are linear. In this case, decision tree for two factors is the following:

\[
\Pi_{ij} : \text{If} \quad (x_1 \in R_{i1} \land x_2 \in R_{j2}) \text{ then } \quad y_{ij} = \theta_0 + \theta_{i01} + \theta_{i02} + (\theta_1 + \theta_{i11} + \theta_{i12}) x_1 + (\theta_2 + \theta_{i21} + \theta_{i22}) x_2. \tag{6}
\]

Here some terms (to be more specific, \( \theta_0 + \theta_1 x_1 + \theta_2 x_2 \)) are included into each branch of the tree. Such terms determine the linear response dependency on the input factors in the entire definition domain without taking into account its dividing into partitions. By virtue of the fact that in (6) multiple expressions can be “not false” at the same time, it will be better to describe the decision tree with the observation model

\[
y_{ij} = \theta_0 + \sum_{i=1}^{s_j} \mu_i(x_1)\theta_{0i1} + \sum_{j=1}^{s_k} \mu_j(x_2)\theta_{02j} + (\theta_1 + \sum_{i=1}^{s_j} \mu_i(x_1)\theta_{1i1} + \sum_{j=1}^{s_k} \mu_j(x_2)\theta_{12j}) x_1 + (\theta_2 + \sum_{i=1}^{s_j} \mu_i(x_1)\theta_{2i1} + \sum_{j=1}^{s_k} \mu_j(x_2)\theta_{22j}) x_2 + \varepsilon_{ij}. \tag{7}
\]

After estimating the parameters \( \theta \) the decision tree will be described as the following convolution
When solving the particular problem of the regression model. In the considered problem, the realized experiment layout makes some constraints for using more complex local models. In the considered problem, the realized experiment layout corresponds to a five-level full-factor experiment for two factors. It includes 25 different points, not counting duplicate observations. At the best case, this allows using as a local model a quadratic polynomial of two factors when the definition domain is subdivided into two partitions. As for the shape of the indicator function curve, we will use trapezoidal and sigmoid functions for the highlighted problem. The category named «the partition shape» should also include the coordinates of the neighbor fuzzy partitions’ intersection points and their intersection areas. Because the definition domains of the factors are symmetrical, the fuzzy partitions can also be located symmetrically with respect to the zero value. Width of partitions’ intersection areas directly impacts on the smoothness of transition of a regression dependency from one local model to another. Let’s denote the coordinate of the partitions’ intersection point as \( \bar{x}_\mu \). Also let’s denote the half of the intersection area width for the trapezoidal indicator functions as \( \Delta \mu \). When solving the particular problem of the regression estimation, the parameters \( \bar{x}_\mu \) and \( \Delta \mu \) can be adjusted. The sigmoid indicator function will be define as follows

\[
\mu(x) = \frac{1}{1 + e^{-d(x-\bar{x}_\mu)}},
\]

where \( \bar{x}_\mu \) is the symmetric point of the indicator function on the \( x \)-axis, \( \mu(\bar{x}_\mu) = 0.5; d \) is the flatness coefficient defining the slope of the indicator function.

4. Experimental results
The proposed technique has been applied to determining optimum performance \( (n^*, S^*) \) of drilling the stainless steel IXI8H9T with a drill, which diameter is \( \varnothing 4.2 \) mm, using liquid refrigerating agent. The steel is hard-to-treat.

To carry out the experiment, the bench-type drilling machine S-25 has been designed and modified in order to achieve smooth regulation of \( n \) and \( S \) in a wide range. The experiments have been made according to the layout of the five-level full-factor experiment. The factor \( S \) has been varied on the levels \{0.0280; 0.0450; 0.0621; 0.0790; 0.0962\}. The factor \( n \) has been varied on the levels \{750; 1098; 1447; 1795; 2145\}.

Earlier, in the works [1-2], the logarithmic quadratic model was used as the basic model describing the data of wear resistance experiments. The logarithms of the response values \( L \) were used in the aforementioned model. In this case, the logarithms were taken in order to reduce the response scale.
range. Using non-linear response transformation requires much more accurate response approximation provided by some regression model. Ranges of response scale also show that an experiment has been carried out for rather wide ranges of its input factors. Wide ranges can lead to so-called model creep, when the response properties are different for different parts of the definition domain.

For building a relevant model of the investigated process, it is highly important to choose the model that would have the optimal complexity and wouldn’t have the overfit effect. When we deal with the overfit effect, a model is strongly trained for describing training data, and its prediction properties are bad. Choosing a model, one should be focused on the so-called external quality criteria. The model, for which the value of the external criterion is minimal, should be chosen. In our work we use two such criteria. Let’s assume that the observation dataset is divided into two parts: \( A \) and \( B \).

For estimating prediction properties of the used regression models we will use the regularity property criterion:

\[
\Delta^2(B) = \Delta^2(B / A) = \left\| y_B - X_B \hat{\theta}_A \right\|^2,
\]

where \( X_B \) is the observation matrix for the \( B \)-part; \( \hat{\theta}_A \) is the parameters’ estimates obtained for the \( A \)-part.

In addition to the regularity property criterion, the well-known cross-validation criterion can be used:

\[
\Delta^2_{cv} = \sum (y_i - f^T(x_i) \hat{\theta}_{(i)})^2,
\]

where \( \hat{\theta}_{(i)} \) is the parameters’ estimate for the full dataset excluding the \( i \)-th observation.

Earlier, in the work \[7\] some model classes were highlighted. The results is provided in the Table I, where \( \varepsilon^2 \) corresponds to the residual sum of squares for the considered models. For building the LAR models the trapezoidal indicator functions has been used. The optimal structures of the models in their class has been chosen on the basis of the regular property criterion \( \Delta^2(B) \) minimum. In addition, the values of two other criteria (\( \Delta^2_{cv} \) and \( \varepsilon^2 \)) have been computed.

After analyzing the Table I, it can be said, that using the ordinary quadratic and cubic polynomials doesn’t give the sufficient increase of the observation data approximation quality.

| Model                              | Number of parameters | \( \Delta^2(B) \) | \( \Delta^2_{cv} \) | \( \varepsilon^2 \) |
|------------------------------------|----------------------|------------------|------------------|------------------|
| Linear                             | 3                    | 47.0             | 2.07             | 89.4             |
| Linear with interactions           | 3                    | 47.0             | 2.07             | 89.4             |
| Quadratic                          | 4                    | 16.3             | 0.78             | 31.3             |
| Cubical                            | 10                   | 6.43             | 0.304            | 9.07             |
| Linear LAR, \( \bar{\mu} = 0, \Delta \mu = 0.5 \) | 9                    | 15.3             | 0.79             | 25.8             |
| Linear LAR with interactions       | 10                   | 10.9             | 0.53             | 17.4             |
| Quadratic LAR, \( \bar{\mu} = 0, \Delta \mu = 0.5 \) | 12                   | 5.45             | 0.229            | 7.15             |
| Quadratic LAR, 2 partitions on \( S \) , \( \bar{\mu} = 0, \Delta \mu = 0.5 \) | 10                   | 5.34             | 0.242            | 7.05             |

For the models presented in the Table I, checking their adequacy can be done by computing the \( F \)-statistics \( F = \hat{\sigma}^2_{LF} / \hat{\sigma}^2_e \), where \( \hat{\sigma}^2_{LF} = \varepsilon^2 / (N - s) \) is the estimate of the observation variance obtained using the model; \( \hat{\sigma}^2_e \) is the estimate of the observation variance obtained for the duplicate
observations. In our case, we have $\hat{\sigma}^2 = 0.0612$. It can also be noted, that in the Table 1 there are no models that can be regarded as adequate with the respect of the F-criterion. To make them adequate in term of this criterion, the approximation should be accurate with the variance $\hat{\sigma}^2_{LP} \approx 0.1$ for the model. That can be done if we tune the indicator functions’ parameters $\bar{x}$ and $\Delta x$. The results presented in the Table 1 allow assuming that the model has the optimal complexity when it includes from 10 to 12 parameters. As a result, the LAR model with the trapezoidal indicator function (the function’s parameters are $\bar{x} = -0.22, \Delta x = 0.73$) has been obtained. The optimal structure of the model consists of 12 parameters and provides the following criteria values: $\Delta^2(B) = 3.66, \Delta^2_{ck} = 0.1427, \varepsilon^2 = 3.94$. The Figure 1 shows the difference between properties of the estimated regressions for the quadratic and fuzzy-quadratic models. The label «LAR model Type 1» denotes the influence curve for the LAR model with the trapezoidal integral function, and the label «quadratic model» denotes the influence curve for the ordinary quadratic model. The label «y» denotes the observed response values. The FPR rate $S$ varies in the range $[-1, +1]$ for normalized units, that corresponds to the range $[0.0280; 0.0962]$.

![Figure 1](image.png)

**Figure 1.** Sectional view of the LAR model with the trapezoidal indicator functions and of the quadratic model for $n = 2145$.

5. **Discussion of results**

The Figure 1 shows that the influence curve for the LAR model has some salient points. They appear due to the fact that the indicator functions are piecewise-linear. This drawback can be overcome by using non-linear functions, for example sigmoid ones. Such approach has been considered. The corresponding LAR model has been obtained. Its optimal structure includes 12 parameters and provides the following criteria values: $\Delta^2(B) = 3.36, \Delta^2_{ck} = 0.134, \varepsilon^2 = 3.63$. It can be noted, that the considered LAR model has the better characteristics than the previously obtained model with trapezoidal indicator functions. The corresponding sectional views for the mentioned LAR and quadratic models are shown in the Figure 2. The label «LAR model Type 2» denotes the influence curve for the LAR model with sigmoid indicator function. There are no salient points on this curve.
Figure 2. Sectional view of the LAR model with the sigmoid indicator functions and of the quadratic model for \( n = 2145 \).

6. Conclusion
The comparative analysis of different wear resistance models of drilling operation has been made. The locally adaptive model with the quadratic basic part has been proposed as relevant for practical use. In order to take into account local response properties when building the model, the definition domains of the factors have been divided into two fuzzy partitions using the sigmoid indicator functions.

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