Into the bulk: reconstructing spacetime from the $c=1$ matrix model

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Abstract

We write down exact solutions in the collective field theory of the $c=1$ matrix model and in dilaton-gravity coupled to a massless scalar. Using the known correspondence between these two theories at the null boundaries of spacetime, we make a connection between scalar fields in these two theories in the bulk of spacetime. In the process, we gain insight into how a theory containing gravity can be equivalent to one without gravity. We analyze a simple time-dependent background as an example.
1 Introduction

One of the hallmarks of gauge/gravity correspondence is the emergent nature of a non-compact spacetime dimension, with the gauge theory dual living on the boundary of the gravitational—or string theory—spacetime. The situation with c=1 string theory is somewhat similar. On the string/gravity side, we have Liouville string theory, which is a sub-critical string theory with a target spacetime of dimension two. The corresponding effective spacetime action is dilaton-gravity coupled to a massless scalar known as the ‘tachyon’. On the gauge side, we have a gauged Matrix Quantum Mechanics in a double scaling limit, a large N system living in one dimension: time. Using the collective field formalism, this MQM can be rewritten as a 1+1 dimensional field theory of a single scalar, corresponding to the density of matrix eigenvalues. It can be said that the spacial dimension in which Liouville strings propagate is emergent, as it is generated from the matrix eigenvalues. It is less clear how gravity emerges in this picture.

The purpose of this paper is to explore in some detail how gravity arises from the c=1 matrix model when the so called ‘leg-pole’ factors are taken into account, extending the results of [1]. We will think about this construction as a toy model for holography, as follows. The spacetime of Liouville string theory (see Figure 1(a)) is flat, and can be parametrized by two coordinates $x$ and $t$, or $x^\pm = t \pm x$. The string coupling varies with space like $g_s \sim \exp(2x)$, and the strong coupling region at large $x$ is shielded by the presence of a tachyon background, $T_0 \sim \exp(2x)$ which repels strings away from this region. In addition, the same quantum improvement which leads to the inhomogeneous string coupling (and which is necessary in a noncritical string theory) also makes the tachyon massless. Finally, in two target space dimensions, there are no transverse oscillators in the quantization of the string world-sheet, so the tachyon is the only propagating degree of freedom. From the infinite ladder of string states, only some special discrete states at discrete euclidean momenta remain. These lead to short distance bulk interactions between the tachyon quanta, described at the lowest order by dilaton-gravity. Tachyon pulses enter from $I^-$ to be scattered by the tachyon wall and return to $I^+$. In the bulk, these pulses can interact with each other via either tachyon three-point (and higher) vertices, or by exchanging gravitons and dilatons (and more massive string fields). We will reconstruct the gravitational interaction between these pulses and the resulting metrics from boundary data alone.

The matrix model, and the corresponding collective field theory, can be thought of as providing boundary data for tachyon scattering. In particular, together with the leg-pole transform, the matrix model allows us to calculate the exact shape of the outgoing tachyon pulse given the incoming pulse. It is in this sense that the matrix model provides us with a holographic description of dilaton-gravity. The two dimensional gravity background has two null boundaries, $\mathcal{I}^\pm$, and the one dimensional gauge theory supplies the scattering matrix between them.
At the same time, we have an equivalence between two different theories of a scalar field in 1+1 dimensions: one with gravity (dilaton-gravity interacting with the tachyon field) and one without (collective field theory for MQM). We will see in detail how it is possible for these two theories to be equivalent, shedding perhaps some light on how gravity can be an emergent theory.

Naively, the spacetime on which the collective field theory lives is fixed. As was first pointed out in [2], this is not the case. Sufficiently large fluctuations of the Fermi sea of eigenvalues can in fact make the spacetime on which the collective field lives time-dependent, changing its structure, for example, by introducing space-like infinities [3]. In such scenarios, it would be most interesting to be able to calculate the metric and dilaton of the equivalent string theory target space. We will see that our methods make this partially possible, and will calculate the metric and the dilaton for a particular time-dependent scenario.

We work in the convention where $\alpha' = 1$. In order to avoid complications arising from the tachyon background, we will be focusing on bulk processes which do not involve it. To separate these bulk processes from the interactions with the background, we will make the background parametrically small by taking the string coupling at the tachyon wall to be large.

The reminder of the paper is organized as follows: In Section 2 we will discuss exact solutions to the collective field equations of motion, in the chiral, or lightcone, formalism. In Section 3 we will solve the equations of motion of dilaton-gravity coupled to a scalar perturbatively to third order in the scalar field. In Section 4 we will
tie the results of two previous sections together, and extract some information about the relationship between the tachyon and the collective field of matrix eigenvalues. Finally, in Section 5, we will use our techniques to study a particular time-dependent background.

2 Solutions in chiral formalism

In this section, we will obtain explicit formulas linking the profile of outgoing fluctuations in the collective field to the profile of incoming fluctuations. We will start with a brief review of the salient facts and definitions in the matrix model. For more details, please see [4], [5], and [6] (chapter 5).

The c = 1 MQM has as its fundamental degrees of freedom non-interacting fermions in the upside down harmonic oscillator potential, with the Hamiltonian

$$ H = \frac{1}{2} p^2 - \frac{1}{2} x^2. \quad (1) $$

The curvature of the potential is fixed by taking $\alpha' = 1$. The effective (or bosonized) picture for this system is that of a Fermi fluid moving in phase space $(x, p)$. Due to the incompressibility of this fluid, in the classical limit it is sufficient to give the position of the Fermi surface, often presented as $p_{\pm}(x, t)$, the upper and lower branches of the Fermi surface as a function of $x$. The local density of fermions is then given by the distance between the two branches of $p$:

$$ \varphi(x, t) \equiv \frac{1}{2} (p_+(x, t) - p_-(x, t)) . \quad (2) $$

The static Fermi surfaces are constant energy hyperbolas given by $E = \frac{1}{2} p^2 - \frac{1}{2} x^2 = \mu$. For $\mu < 0$ the left and the right branches of the hyperbola do not interact: any small fluctuation around this static background evolves by moving along one arm from $x = \infty$ towards finite $x$ and back out to $x = \infty$ along the other arm of the same branch.

This description is unfortunately singular, at the place where the upper and the lower edge of the Fermi sea join. To avoid this singularity, we will use an equivalent description with $\mu > 0$ and allow the fluctuation to propagate from the left to the right along the upper branch of the hyperbola $p^2 - x^2 = 2\mu$.

To calculate the relationship between the incoming and the outgoing pulse, we will use the chiral (or lightcone) formalism [7]. The chiral formalism is based on the simple observation that if the Fermi surface is given at $t=0$ by $G(x, p) = 0$ for some function $G$, then the time evolution of this Fermi surface is given by

$$ G \left( \frac{1}{2} (p + x) e^{-t} - \frac{1}{2} (p - x) e^t, \frac{1}{2} (p + x) e^{-t} + \frac{1}{2} (p - x) e^t \right) = 0 . \quad (3) $$
Consider now the fluctuations of the upper branch of the hyperbola given by $p^2 - x^2 = 2\mu$, with $\mu > 0$. We define the fluctuation field $\eta$ with

$$p(x, t) = \sqrt{2\mu + x^2 + 2\sqrt{\pi} \partial_x \eta} . \quad (4)$$

It will turn out that $\eta$ is best thought of as a function of $\sigma$ such that $x = \sqrt{2\mu} \sinh \sigma$, so that $\partial_x \eta = (2\mu + x^2)^{-1/2} \partial_{\sigma} \eta(\sigma) \approx |1/x| \partial_\sigma \eta(\sigma)$. The fluctuations come in from $x \to -\infty$ and exit at $x \to \infty$. Let us see this in detail.

Consider the following exact time-dependent profile of type (3) for some function $f_{\text{in}}(\sigma) \ll \mu$, whose meaning will become clear in a moment:

$$p^2 - x^2 = 2\mu - 2f'_{\text{in}} \left( \ln \left( \frac{1}{2\mu} (p - x) e^t \right) \right) . \quad (5)$$

This can be rewritten as

$$p + x = \frac{2\mu}{p - x} - \frac{2}{p - x} f'_{\text{in}} \left( \ln \left( \frac{1}{2\mu} (p - x) \right) + t \right) . \quad (6)$$

Assuming that $f_{\text{in}}(y)$ has finite support on some interval near $y = 0$, for $t \to -\infty$, $f_{\text{in}}$ in nonzero only if $p - x$ is large. Then, the right hand side of our equation is small, and we must have $p \approx -x$. Therefore, $x$ is large and negative, and $p - x \approx -2x$. Substituting this in, we get

$$p = -x - \frac{\mu}{x} + \frac{1}{x} f'_{\text{in}} \left( \ln \left( -\frac{2}{\mu} x \right) + t \right) , \quad (7)$$

which for large negative $x$ can be rewritten as

$$p = \sqrt{x^2 + 2\mu} - \frac{1}{x} f'_{\text{in}} (t - \sigma) , \quad (8)$$

with $x = \sqrt{2\mu} \sinh \sigma \approx -\sqrt{\mu/2} \exp(-\sigma)$. We can now identify $f_{\text{in}}$ with the early time $\eta$ in equation (4), which is right-moving as expected. To be precise, for $t \to -\infty$, $\eta(\sigma, t) = (2\sqrt{\pi})^{-1} f_{\text{in}}(t - \sigma)$.

The same analysis applies to late time fluctuations at large positive $x$. Starting with a time-dependent profile given by

$$p^2 - x^2 = 2\mu - 2f'_{\text{out}} \left( -\ln \left( \frac{1}{2\mu} (p + x) e^{-t} \right) \right) , \quad (9)$$

we can identify $\eta(\sigma, t) = (2\sqrt{\pi})^{-1} f_{\text{out}}(t - \sigma)$ at late times, $t \to -\infty$, with $x \approx \sqrt{\mu/2} \exp(\sigma)$.

The crucial observation now is that the profiles in equations (5) and (9) are exact solutions and valid at all times. Therefore, if the incoming profile is $f_{\text{in}}$, the outgoing
profile can be obtained from setting the right hand sides of equations (5) and (9) equal:

\[ f'_{in} \left( \ln \left( \sqrt{\frac{1}{2\mu}} (p-x)e^{t} \right) \right) = f'_{out} \left( -\ln \left( \sqrt{\frac{1}{2\mu}} (p+x)e^{-t} \right) \right), \quad (10) \]

We now substitute the expression for \( x+p \) from equation (6), and define \( y = \ln((p-x)e^{t}/\sqrt{2\mu}) \) to get

\[ f'_{in}(y) = f'_{out} \left( y - \ln \left( 1 - \mu^{-1} f'_{in}(y) \right) \right), \quad (11) \]

or, defining \( y(z) \) by \( z = y - \ln \left( 1 - \mu^{-1} f'_{in}(y) \right) \), \( f'_{out}(z) = f'_{in}(y(z)) \), which is nothing more but the time delay equation in [8, 1]. More interestingly, we can find the profile at any time. Given the incoming profile \( f_{in} \), equation (5) can be solved for \( p \) as a function of \( x \) treating \( f_{in} \) as a small perturbation. \( \eta \) as a function of \( x \) (or \( \sigma \)) and \( t \) can then be read off.

To second order in \( f_{in} \), we get

\[ p = \sqrt{x^2 + 2\mu} - \frac{1}{\sqrt{x^2 + 2\mu}} f'_{in}(t - \sigma) \]
\[ + \frac{e^\sigma}{\sqrt{2\mu(2\mu + x^2)}} f''_{in}(t - \sigma)f''_{in}(t - \sigma) \]
\[ - \frac{1}{2(2\mu + x^2)^{3/2}} (f'_{in}(t - \sigma))^2 + o((f'_{in})^3), \quad (12) \]

or

\[ 2\sqrt{\pi} \partial_\sigma \eta(\sigma, t) = 2\eta_{in}(t - \sigma) - \frac{e^\sigma}{4\mu \cosh \sigma} (f'_{in}(t - \sigma))^2 + o((f'_{in})^3). \quad (13) \]

This can be integrated with respect to \( \sigma \)

\[ 2\sqrt{\pi} \eta(\sigma, t) = f_{in}(t - \sigma) - \frac{e^\sigma}{4\mu \cosh \sigma} (f'_{in}(t - \sigma))^2 + o((f'_{in})^3). \quad (14) \]

As a consistency check, we notice that in the large negative \( \sigma \) regime, \( \eta = \eta_{in} \), as expected, and that in the large positive \( \sigma \) regime, there are no left-moving terms (everything is a function of \( t - \sigma \)). In particular, if we take \( \sigma \to \infty \) in the above equation, then

\[ 2\sqrt{\pi} \eta(\sigma \to \infty) = f_{in}(t - \sigma) - \frac{1}{2\mu} (f'_{in}(t - \sigma))^2 + o((f'_{in})^3). \quad (15) \]

To third order, the calculation is a bit more messy. The result for \( \eta' \) is again a total derivative, and can be integrated to give

\[ 2\sqrt{\pi} \eta(\sigma, t) = f_{in}(t - \sigma) - \frac{e^\sigma}{4\mu \cosh \sigma} (f'_{in}(t - \sigma))^2 \]
\[ - \frac{(e^{2\sigma} + 3)e^\sigma}{48\mu^2 \cosh^3(\sigma)} (f'_{in}(t - \sigma))^3 \]
\[ + \frac{e^{2\sigma}}{8\mu^2 \cosh^2(\sigma)} (f'_{in}(t - \sigma))^2 f''_{in}(t - \sigma) + o((f'_{in})^4). \quad (16) \]
As a check, let us compute the large $\sigma$ limit of this expression from the time delay equation, $y = z + \ln (1 - \mu^{-1} f'_m(y))$. Expanding, we get that

\[
y(z) = z - \mu^{-1} f'_m(z) + \mu^{-2} f''_m(z) f'_m(z) - \frac{1}{2} \mu^{-2} (f'_m(z))^2 + o((f'_m)^3),
\]

and therefore

\[
f''_m(z) = f''_m(y(z)) = f''_m(z) - \mu^{-1} f'_m(z) f''_m(z) + \mu^{-2} (f'_m(z))^2 f''_m(z) + o(f'_m)^4),
\]

which can be integrated to give

\[
f''_m = f''_m(z) = f''_m(z) - \mu^{-1} f'_m(z) f''_m(z) + \mu^{-2} (f'_m(z))^2 f''_m(z) + o((f'_m)^4),
\]

implying that

\[
\eta''_m = \eta''_m(z) = \eta''_m(z) - \mu^{-1} \eta'_m(z) \eta''_m(z) + \mu^{-2} (\eta'_m(z))^3 \eta''_m(z) + o(\eta'_m)^4).
\]

This agrees with equation (16) when $\sigma \to \infty$.

Our procedure can clearly be extended to any order, and gives both the collective field profile at any time, and the outgoing profile for $t \to \infty$ in terms of the incoming field, as illustrated in Figure 1(b).

### 3 Dilaton-Gravity coupled to a massless scalar

Having studied the interior behaviour in the collective field theory, we now turn out attention to the interior behaviour of the dilaton-gravity theory.

As was described above, effective field theory for Liouville string theory is dilaton-gravity coupled to the (massless) tachyon scalar. Since the tachyon is a massless field and not actually tachyonic, the action for these three degrees of freedom is perfectly well defined. Denoting the dilaton field with $\Phi$ and the tachyon with $T$, we have [1]

\[
S = \frac{1}{2} \int dt dx \sqrt{-G} e^{-2\Phi} \left[ a_1 \left[ R + 4 (\nabla \Phi)^2 + 16 \right] - (\nabla T)^2 + 4 T^2 - 2V(T) \right],
\]

where we will take the tachyon potential to be

\[
V(T) = \frac{a_2 T^3}{3}.
\]

Here $a_1$ and $a_2$ are constants which were determined in [1] to be $a_1 = \frac{1}{2}$ and $a_2 = -2\sqrt{2}$. 

6
In conformal gauge, where the metric is $ds^2 = -e^{2\rho} dx^+ dx^-$, the equations of motion are [9]

\begin{align*}
2\partial^2_+ \Phi - 4 \partial_+ \rho \partial_+ \Phi &= a_1^{-1} \partial_+ T \partial_+ T \quad (23) \\
2\partial^2_\Phi - 4 \partial_\rho \partial_- \Phi &= a_1^{-1} \partial_- T \partial_- T \quad (24) \\
2\partial_+ \partial_- \Phi - 4 \partial_+ \Phi \partial_- \Phi - 4e^{2\rho} &= a_1^{-1} e^{2\rho} \left( T^2 - \frac{a_2}{6} T^3 \right) \quad (25) \\
4\partial_+ \partial_- \Phi - 4 \partial_+ \Phi \partial_- \Phi - 2 \partial_+ \partial_- \rho - 4e^{2\rho} &= a_1^{-1} \partial_+ T \partial_- T \quad (26) \\
&+ a_1^{-1} e^{2\rho} \left( T^2 - \frac{a_2}{6} T^3 \right) \\
e^{-2\rho} (\partial_+ \partial_- T - \partial_+ \Phi \partial_- T - \partial_- \Phi \partial_+ T) - T &= -\frac{a_2}{4} T^2 \quad (27)
\end{align*}

The first three equations are for the metric, the fourth is for the dilaton and the last is for the tachyon field. The last two equations can be combined to give a particularly simple relationship

\[ 2\partial_+ \partial_- (\rho - \Phi) + a_1^{-1} \partial_- T \partial_+ T = 0 . \quad (28) \]

In the absence of tachyon field, the above equation becomes $\partial_+ \partial_- (\rho - \Phi)$. Using up the left-over coordinate freedom $x_+ \to \tilde{x}_\pm (x_\pm)$, we could set $\Phi = \rho$, the Kruskal gauge. However, since we are dealing with a linear dilaton background, a more natural gauge would be the modified Kruskal gauge $\Phi = x^+ - x^- + \rho$. Either gauge choice is only possible in regions where the tachyon field is zero.

We will expand in powers of the tachyon field. To zeroth order, we have the linear dilaton background,

\[ \Phi_0 = 2x = x^+ - x^- , \quad \rho_0 = 0 . \quad (29) \]

The tachyon background is a solution to the linearized version of equation (27) in the linear dilaton background,

\[ \partial_+ \partial_- T - \partial_- T + \partial_+ T - T = 0 . \quad (30) \]

The most general static solution to this equation is

\[ T_0 = (b_1 x + b_2) e^{2x} . \quad (31) \]

We are working in the limit where the tachyon background can be neglected, $b_1, b_2 \to 0$, and will be expanding in powers of the incoming tachyon field: $T = T^{(1)} + T^{(2)} + T^{(3)} + ..., \text{ignoring } T_0$.

It will be convenient to absorb a factor of the dilaton background into $T$ by defining a new field $S = e^{-\Phi_0} T = e^{-2x} T = S^{(1)} + S^{(2)} + S^{(3)} + ...$. To lowest order the equation of motion is simply

\[ \partial_+ \partial_- S^{(1)} = 0 . \quad (32) \]
The rescaled tachyon field $S$ is a massless scalar; above equation has solutions of the form $S^{(1)} = S^{(1)}_{-}(x_{-}) + S^{(1)}_{+}(x_{+})$. Since the region $x \to +\infty$ is the strong coupling region, protected by the tachyon condensate, $S$ cannot have left-moving incoming excitations, and we are left with $S^{(1)} = S^{(1)}_{-}(x_{-})$.

To second order, we can linearize equations (23-26) in gravity and dilaton fluctuations about the background, $\Phi = \Phi_{0} + \delta$, to obtain

$$\partial_{+}^{2} \delta - 2 \partial_{+} \rho = \frac{1}{2a_{1}} (\partial_{+} T^{(1)})^{2}$$

(33)
$$\partial_{-}^{2} \delta + 2 \partial_{-} \rho = \frac{1}{2a_{1}} (\partial_{-} T^{(1)})^{2}$$

(34)
$$\partial_{+} \partial_{-} \delta - 4 \rho + 2 \partial_{+} \delta - 2 \partial_{-} \delta = \frac{1}{2a_{1}} (T^{(1)})^{2}$$

(35)
$$2 \partial_{+} \partial_{-} \delta + 2 \partial_{+} \delta - 2 \partial_{-} \delta - \partial_{+} \partial_{-} \rho - 4 \rho = \frac{1}{2a_{1}} \left( \partial_{+} T^{(1)} \partial_{-} T^{(1)} + (T^{(1)})^{2} \right) .$$

(36)

The tachyon equation of motion at this level is

$$\partial_{+} \partial_{-} T^{(2)} - \partial_{-} T^{(2)} + \partial_{+} T^{(2)} - T^{(2)} = - \frac{a_{2}}{4} (T^{(1)})^{2}$$

(37)

or

$$\partial_{-} \partial_{+} S^{(2)} = - \frac{a_{2}}{4} e^{2x} (S^{(1)})^{2} .$$

(38)

This last equations is easy to solve for $S^{(2)}(x_{-})$

$$S^{(2)} = - \frac{a_{2}}{4} e^{x^{+}} \int^{x^{+}}_{x^{-}} dx^{-} e^{-x^{-}} (S^{(1)}(x^{-}))^{2} .$$

(39)

Defining $\Omega = 2(\partial_{-} - \partial_{+}) \delta + 4 \rho$, we can combine equations (33-35) into

$$(\partial_{+} - 2) \Omega = \frac{1}{a_{1}} \left( (T^{(1)})^{2} - (\partial_{+} T^{(1)})^{2} \right)$$

(40)

$$(\partial_{-} + 2) \Omega = \frac{1}{a_{1}} \left( -(T^{(1)})^{2} + (\partial_{-} T^{(1)})^{2} \right)$$

(41)

$$\partial_{+} \partial_{-} \delta = \Omega + \frac{1}{2a_{1}} (T^{(1)})^{2}$$

(42)

while equations (35) and (36) give

$$\partial_{+} \partial_{-} (\delta - \rho) = \frac{1}{2a_{1}} \partial_{+} T^{(1)} \partial_{-} T^{(1)} .$$

(43)

These four equations can be integrated explicitly to give $\rho$ and $\delta$. They are more equations (four) than unknown functions (two), consistency requires that $T$ satisfy the 1st order equation (30), whose most general solution is

$$T^{(1)} = \sqrt{a_{1}} e^{x^{+} - x^{-}} \left( f_{+}(x^{+}) + f_{-}(x^{-}) \right) .$$

(44)
It is easy to show that in that case, the first two equations give
\[ \Omega = -e^{2x^+-2x^-} \left( f_+^2 + 2f_-f_+ + f_-^2 - \int_{x^+} dx^+ \left( f'_+ \right)^2 - \int_{x^-} dx^- \left( f'_- \right)^2 + 4A \right), \] (45)
and the third gives
\[ \delta = \frac{1}{4} \int_{x^-} dx^- e^{2x^+-2x^-} \left[ (f'_+)^2 - f_-^2 \right] + \frac{1}{4} \int_{x^+} dx^+ e^{2x^+-2x^-} \left[ (f'_+)^2 - f_-^2 \right] - \frac{1}{4} e^{2x^+-2x^-} \int_{x^+} dx^+ (f'_+)^2 - \frac{1}{4} e^{2x^+-2x^-} \int_{x^-} dx^- (f'_-)^2 + \int_{x^-} dx^- e^{-2x^-} f_- \int_{x^+} dx^+ e^{2x^+} f_+ - Ae^{2x^+-2x^-} + \alpha_+(x^+) - \alpha_-(x^-). \] (46)
From the definition of \( \Omega, \) \( \rho \) is then
\[ \rho = \frac{1}{4} \int_{x^-} dx^- e^{2x^+-2x^-} \left[ (f'_+)^2 - f_-^2 \right] + \frac{1}{4} \int_{x^+} dx^+ e^{2x^+-2x^-} \left[ (f'_+)^2 - f_-^2 \right] - \frac{1}{8} e^{2x^+-2x^-} \left( f_+^2 + 4f_-f_+ + f_-^2 + 2 \int_{x^+} dx^+ (f'_+)^2 + 2 \int_{x^-} dx^- (f'_-)^2 \right) - \frac{1}{2} e^{2x^+} f_+ \int_{x^-} dx^- e^{-2x^-} f_- - \frac{1}{2} e^{-2x^-} f_- \int_{x^+} dx^+ e^{2x^+} f_+ - Ae^{2x^+-2x^-} + \frac{1}{2} \partial_+ \alpha_+(x^+) + \frac{1}{2} \partial_- \alpha_-(x^-). \] (47)
and the fourth equation is satisfied automatically (it is in fact implied by the other three combined with (30)).

In the above solution, \( A \) is an arbitrary integration constant, and \( \alpha_\pm \) are arbitrary integration functions. \( \alpha_\pm \) can be removed from the solution by a coordinate transformation which respects conformal gauge, namely (to linear order) \( x^\pm \to x^\pm + \alpha_\pm(x^\pm). \) In the interest of simplicity, we will adopt a coordinate system where \( \alpha_\pm = 0 \) for now, and return to the issue of coordinate ambiguity later.

In contrast with \( \alpha_\pm, \) the constant \( A \) cannot be set to zero by a coordinate change. Its presence, however, is contrary to our implicit boundary conditions, since \( Ae^{2x^+-2x^-} \) is large for \( x^- \to -\infty. \) If we imagine that the incoming tachyon pulse is localized (as shown in Figure 1(a)), the metric before the pulse arrives should be flat. As we will see in a moment, inclusion of a nonzero \( A \) corresponds to a black hole background. We will therefore set \( A = 0 \) as well. Similar arguments apply to the region of spacetime where \( x^+ \to \infty. \)

The general solution in equations (45,46,47) is more than what we require. Because our theory has only one asymptotic weakly coupled region, and because we have

\footnote{Note on notation: anytime a limit is not shown for an integral, it is \( +\infty \) for an upper limit, and \( -\infty \) for a lower limit. Integrals with no limits at all should be interpreted as being over the entire real line.}
ignored the presence of the background which can reflect back a scalar pulse, $S^{(1)}$ has only one component, and not two: $S^{(1)} = S^{(1)}(x) = \sqrt{a} f_-(x)$. Therefore (dropping the (1) subscript on $S$ for brevity),

$$\Omega = - \frac{1}{a_1} e^{2x^+-2x^-} \left( S^2 - \int_{x^-}^{x^+} dx^- (S')^2 \right),$$

$$\delta = \frac{1}{4a_1} \int_{x^-}^{x^+} dx^- e^{2x^+-2x^-} [(S')^2 - S_2^2] - \frac{1}{4a_1} e^{2x^+-2x^-} \int_{x^-}^{x^+} dx^- (S')^2 \\text{(48)}$$

and

$$\rho = \frac{1}{4a_1} \int_{x^-}^{x^+} dx^- e^{2x^+-2x^-} [(S')^2 - S_2^2] - \frac{1}{8a_1} e^{2x^+-2x^-} \left( S_2^2 + 2 \int_{x^-}^{x^+} dx^- (S')^2 \right) \\text{(49)}$$

If the incoming pulse is localized around some $x^-$, and we look at larger values of $x^-$, the metric and the dilaton outside the pulse simplify to

$$\delta = \rho = \frac{1}{4a_1} e^{2x^+-2x^-} \int_{x^-}^{x^+} dx^- e^{2x^+-2x^-} [(S')^2 - S_2^2] - \frac{1}{4a_1} e^{2x^+-2x^-} \int_{x^-}^{x^+} dx^- (S')^2 \ \text{(50)}$$

which imply

$$\Omega = \frac{1}{a_1} e^{2x^+-2x^-} \int_{x^-}^{x^+} dx^- (S')^2. \ \text{(52)}$$

Notice that this is nothing else but the standard 2D black hole, which in Kruskal gauge $\rho = \Phi$ is given by

$$e^{-2\rho} = e^{-2\Phi} = \frac{m}{2} - 4(y^+ - y_0^+)(y^- - y_0^-). \ \text{(53)}$$

Changing variables $y^\pm - y_0^\pm = \mp e^{2\tilde{y}^\pm}$ and linearizing, we get

$$e^{2\rho} = 1 - \frac{m}{8} e^{2\tilde{y}^+ - 2\tilde{y}^-} \ \text{and} \quad e^{2\Phi} = \frac{1}{4} e^{2\tilde{y}^+ - 2\tilde{y}^-} \left( 1 - \frac{m}{8} e^{2\tilde{y}^+ - 2\tilde{y}^-} \right) \ \text{(54)}$$

or

$$\rho = - \frac{m}{16} e^{2\tilde{y}^+ - 2\tilde{y}^-} \ \text{and} \quad \Phi = \text{const.} + (\tilde{y}^+ - \tilde{y}^-) - \frac{m}{16} e^{2\tilde{y}^+ - 2\tilde{y}^-}. \ \text{(55)}$$

To compare with equation (51), let $\tilde{y}^+ = x^+ + C e^{2x^+}$ and $\tilde{y}^- = x^-$. Then, for large negative $x^+$, the metric and the dilaton in (51) and (55) agree, with

$$m = \frac{4}{a_1} \int dx^- (S')^2 \quad \text{and} \quad C = \frac{1}{4a_1} \int dx^- e^{-2x^-} [(S'_-)^2 - S_2^2]. \ \text{(56)}$$

Notice that the mass is simply the integral over the stress energy of the incoming pulse, as expected, and that, had we included the integration constant $A$, it would have contributed to the mass, signaling the presence of an undesirable black hole background unrelated to the tachyon pulse.
To third order, we only need the tachyon equation, which now includes interactions with the metric and the dilaton, or

\[
\partial_- \partial_+ S^{(3)} = -\frac{a^2}{2} e^{2x}(S^{(1)})(S^{(2)}) + \frac{1}{2} \Omega S^{(1)} + \partial_+ \delta \partial_- S^{(1)} + \partial_- \delta \partial_+ S^{(1)}.
\]  

(57)

With the explicit forms of \(\Omega\) and \(\delta\) above, this equation can be integrated as well. Let us treat a special case, where we will imagine that the incoming field is made up of two well separated pulses with finite support, \(S^{(1)}_-(x^-) = S^{(1A)}_-(x^-) + S^{(1B)}_-(x^-)\), with the A pulse centered around \(x^A_0\) and the B pulse centered around \(x^B_0 = x^A_0 + T\), with \(T\) large. We will think of \(S^{(1A)}_-(x^-)\) as a source for the second order fields (tachyon, dilaton and metric) and examine scattering of the second pulse, B, from this background. For \(x_- \to +\infty\), the outgoing third order tachyon field is

\[
S^{(3)} = -\frac{a^2}{2} \int_{x^+}^{x^-} dx^+ \int_{x^-}^{x^+} e^{x^+ - x^-} (S^{(2)}_-)^A - \frac{1}{2} \Omega - \partial_+ \partial_- \delta \right) S^{(1B)}_-. \n\]

(58)

(59)

Combining all our previous results,

\[
-\frac{a^2}{2} e^{x^+ - x^-} (S^{(2)}_-)^A + \frac{1}{2} \Omega - \partial_+ \partial_- \delta = \frac{a^2}{8} e^{2x^+ - x^-} \int_{x^-}^{x^+} e^{-x_-} (S^{(1A)}(x^-))^2 - \frac{1}{2a_1} e^{2x^+ - 2x^-} \int_{x^-}^{x^+} \left( \partial_- S^{(1A)}_-(x^-) \right)^2 ,
\]

(60)

and therefore

\[
S^{(3)} = \frac{a^2}{16} e^{2x^+} \int_{x^-}^{x^+} e^{-x_-} (S^{(1B)}_-(x^-)) \int_{x^-}^{x^+} e^{-x_-} (S^{(1A)}(x^-))^2 - \frac{1}{4a_1} e^{2x^+} \int_{x^-}^{x^+} e^{-2x_-} S^{(1A)}_-(x^-) \int_{x^-}^{x^+} \left( \partial_- S^{(1A)}_-(x^-) \right)^2 .
\]

(61)

The first term is due to a Feynman diagram shown in Figure 2(a) and the second due to that in Figure 2(b). In the latter case, it is the total stress energy of the pulse which determines the result, in other words, pulse B scatters from the dilaton-gravity background created by the first pulse.

4 Relationship between spacetime and matrix model

In this section, we will confirm that the results of the two preceding sections are linked by the leg-pole transform on the boundary, and discuss a strategy towards extending the correspondence into the bulk.

The leg-pole transform connects the incoming tachyon field profile to the incoming collective field via [1]

\[
S_{in}(x^-) = -\int dv \ K(v - x^-) \eta_{in}(v)
\]

(62)

\[
\eta_{in}(\sigma^-) = -\int dv \ K(\sigma^- - v) \ S_{in}(v)
\]

(63)
and the outgoing fields via

\[ S_{\text{out}}(x^+) = \int dv \ K(x^+ - v + \ln(\mu/2)) \eta_{\text{out}}(v) \]  
\[ \eta_{\text{out}}(\sigma^-) = \int dv \ K(v - \sigma^- + \ln(\mu/2)) S_{\text{out}}(v). \]  

(64)  

(65)

\( x^\pm = t \pm x \) and \( \sigma^\pm = t \pm \sigma \) are lightcone coordinates in spacetime and in the collective field theory, as shown in Figure 1. The kernel \( K \) of the leg-pole transform is given by

\[ K(v) = -\frac{w}{2} J_1(w), \quad \text{where} \quad w = 2 \left(\frac{2}{\pi}\right)^{1/8} e^{\nu/2}. \]

(66)

It derives its name from the poles in its Fourier transform,

\[ K(\omega) = \int dv \ e^{-i\omega v} \ K(v) = \left(\frac{2}{\pi}\right)^{i\omega/4} \frac{\Gamma(-i\omega)}{\Gamma(i\omega)}. \]

(67)

The frequency space expression was originally derived by comparing the S-matrix of the matrix model with world-sheet results in Liouville string theory [10, 11].

The Appendix collects some useful formulas about the leg-pole kernel, which are easily derived in frequency space.

The shifts of \( \ln(\mu/2) \) in the outgoing formulas (64) and (65) are related to the position of the tachyon wall. We will be taking \( \mu \to 0 \), which takes the tachyon wall deeply into the strong coupling region and allows us to neglect, for the most part, scattering from the tachyon background.
Minus signs in equations (62) and (63) are a result of using the positive energy hyperbola in Section 2.

We combine the scattering formula (20) with the leg-pole transforms to compute the scattering of the field $S$. We will do this order by order, and compare with the results of the previous section. We will assume that the incoming pulse is well localized (with Gaussian fall-off, for example) around $x^- = 0$.

To first order in $S$ (and $\eta$) we have

$$S^{(1)}_{out}(x^+) = -\int du \left( \int dv \ K(x^+ + \ln(\mu/2) - v) K(v - u) \right) S_{in}(u). \quad (68)$$

The kernel in the bracket can be, at first approximation, thought to be local, and centered around $u = x^+ + \ln(\mu/2)$. Therefore, for an incoming pulse centered around $x^-$, the bulk of the outgoing pulse is centered around $x^+ = x^- - \ln(\mu/2)$, indicating that the scattering takes place at string coupling $e^{2x} = e^{x^+ - x^-} = 1/\mu = g_{st}$, as expected (up to a coupling independent shift). The shape of the tachyon background can be deduced from the detailed shape of the scattered pulse, but we are not interested in it.

To second order, we have

$$S^{(2)}_{out}(x^+) = \int dv \ K(x^+ + \ln(\mu/2) - v) \left( -\frac{\sqrt{\pi}}{\mu} \eta'_{in}(v)^2 \right) \left( \int dv \ K(v - u_1) K'(v - u_2) \right) \times S_{in}(u_1) S_{in}(u_2) \quad (69)$$

We are interested in the region where $x^+ + \ln \mu$ is large and negative, so we can use formula (85) in the Appendix to obtain

$$S^{(2)}_{out}(x^+) = \frac{\sqrt{2}}{\mu} \int du \ e^{x^+ + \ln(\mu/2) - u} (S_{in}(u))^2 = \frac{1}{\sqrt{2}} \int du \ e^{x^- - u} (S_{in}(u))^2. \quad (70)$$

Notice that the answer is independent of $\mu$: this is bulk scattering, and does not depend on the position of the tachyon wall. The answer is in agreement with equation (39), with $a_2 = -2\sqrt{2}$.

To third order, we will assume that the incoming tachyon profile is made up of two pulses, just like we did in Section 2. Then,

$$S^{(3)}_{out}(x^+) = \int dv \ K(x^+ + \ln(\mu/2) - v) \left( \frac{2\pi}{3\mu^2} (\partial_v - 1)(\eta'_{in}(v))^3 \right) \quad (71)$$

$$= \frac{2\pi}{3\mu^2} \int du_1 du_2 du_3 \left( 3 \left( S_{in}^B(u_1) S_{in}^A(u_2) S_{in}^A(u_3) \right) \times \left( \int dv \ (1 - \partial) K(x^+ + \ln(\mu/2) - v) K'(v - u_1) K'(v - u_2) K'(v - u_3) \right) \right). \quad (71)$$

13
Using equation (86) in the Appendix, this becomes,

\[ \frac{1}{2} \int du_1 du_2 du_3 \left( e^{2x^+ - u_1 - u_2} \delta(u_2 - u_3) + e^{2x^+ - 2u_1} \delta''(u_2 - u_3) \right) \frac{S_{in}^B(u_1) S_{in}^A(u_2) S_{in}^A(u_3)}{1} = \]

\[ \frac{1}{2} \int du_1 e^{2x^+ - u_1} S_{in}^B(u_1) \int du_2 e^{-u_2} \left( S_{in}^A(u_2) \right)^2 - \frac{1}{2} \int du_1 e^{2x^+ - 2u_1} S_{in}^B(u_1) \int du_2 (\partial S_{in}^A(u_2))^2 \]

which agrees with equation (61) for \( a_1 = \frac{1}{2} \) and \( a_2 = -2\sqrt{2} \), as before.

Now that we know the boundary profiles match up, we can explore the relationship between bulk fields. From equation (39), using our established value for \( a_2 \), we have, to second order

\[ S(x^-, x^+) = S_{in}(x^-) + \frac{1}{\sqrt{2}} e^{x^+} \int_{x^-}^{x^+} dx^- e^{-x^-} \left( S_{in}(x^-) \right)^2. \]  

(73)

We can try to obtain the relationship between \( S \) and \( \eta \) at the same time \( t \) up to second order by solving for \( S_{in} \) in terms of \( \eta(\sigma_+, \sigma_-) \) at time \( t \). Concretely, equation (14) implies that, to second order in \( \eta \),

\[ \eta_{in}(\sigma-t) = \eta(\sigma, t) + \frac{e^\sigma}{4\mu \cosh \sigma} (\partial_\sigma - \eta(\sigma, t))^2 + o(\eta^3). \]  

(74)

While on the surface it would appear that the right hand side depends on both \( \sigma^- \) and \( \sigma^- \), we know that this is not the case. We will now apply the equal-time leg-pole transform to obtain \( S_{in} \)

\[ S_{in}(t - x) = \int dv K(-v + x) \eta_{in}(\sigma^- = t - v) \]

\[ = \int dv K(-v + x) \left( \eta(v, t) + \frac{e^v}{4\mu \cosh v} (\partial_\sigma - \eta(v, t))^2 \right) \]

(75)

which we can then plug into equation (73).

This procedure can be extended to higher orders in perturbation theory, and would allow us to relate the metric and the dilaton, as well higher order corrections to the tachyon field to the collective field, via equations (50,49,61). Therefore, at least in principle, we can write the explicit field redefinition linking dilaton-gravity coupled to a massless scalar to a theory with only a single scalar field. Notice though that going beyond the third order would require the inclusion of effects of heavy string states into the gravity action. The map is nonlocal, which should come as no surprise, since it can be interpreted as a result of integrating out dilaton-gravity.

Due to diffeomorphism invariance in the dilaton-gravity theory, this field redefinition cannot be unique. We have fixed the coordinate invariance by asking the fields to be related at equal times, hence picking a particular coordinate system in the gravitating theory. However, for localized pulses, the ambiguity results in at most exponentially small corrections at large \( x \).
The argument for this last fact rests on form of the detailed agreement between the collective field and the tachyon on the boundary. Let us assume a well localized incoming tachyon pulse. Under the transform (63), for \( \sigma^- \) large and negative, the incoming collective field has a form \( \eta_{in} = A_1 e^{\sigma^-} + A_2 e^{2\sigma^-} + \ldots \), while for \( \sigma^- \) large and positive, the fall-off is much more rapid. The outgoing collective field has the same form. Therefore, the outgoing tachyon field for large negative \( x^+ \) must also be sum of terms of the form \( e^{kx^-} \) with \( k \) a positive integer. Now, consider the effect of a change of coordinates \( x^+ \to \tilde{x}^+ \) on equations (39) and (61). For these equations to only contribute terms in the form \( e^{k\sigma^-} \), the change of coordinates much be limited to \( x^+ \to x^+ + \sum_k B_k e^{kx^-} \). Therefore, on the boundary the coordinates can be fixed up to exponentially small ambiguity. A similar argument holds for the incoming boundary, and the coordinate \( x^- \).

In the bulk, the coordinate changes are limited to those which maintain conformal gauge. This is because at the lowest order, both the collective field and the rescaled tachyon field \( S \) are massless scalars, with an equation of motion \( \partial_+ \partial_{-} S = \partial_+ \partial_{-} \eta = 0 \). To maintain conformal gauge, the bulk coordinate changes must be of the form \( x^\pm \to X^\pm(x^\pm) \), where the functions \( X^\pm \) must be of the form discussed in the previous paragraph, and are fixed up to exponentially small corrections.

While explicit, the procedure for connecting bulk fields in the two theories described in this section is not straightforward. In the next section, we will simply use our results from Section 3 to discuss some interesting examples beyond localized pulses.

5 Example: time-dependent background

In this section we will employ our results to make a connection between the matrix model and spacetime physics in a time-dependent scenario. For convenience, especially when comparing our results with previous work on this background [2, 12-15], in this section we take our matrix model background to be the left branch of \( x^2 - p^2 = 2\mu \), and define the fluctuation field \( \eta \) in the standard way [16], \( (p_+ - p_-)/2 = \sqrt{x^2 - 2\mu} + \sqrt{\pi} \partial_\eta \), which is compatible with our definition in equation (4). On the left branch of the hyperbola, we have \( x = -\sqrt{2\mu} \cosh \sigma \) and we will take \( \sigma \) to be negative, so that for large \( x \), \( x \approx -\sqrt{\mu/2} \exp(-\sigma) \).

We will focus on the following exact time-dependent profile in eigenvalue phase space

\[
(x + p + \lambda e^t)(x - p) = 2\mu ,
\]

which at large \( x \) and large negative \( t \) corresponds to \( \eta \approx -\lambda/2\sqrt{\pi} e^t x \approx \lambda/2 \sqrt{\mu/2\pi} e^{t-\sigma} \). This is the incoming \( \eta \) profile.

As has been shown in [12, 15], the exact effective action for the fluctuation \( \eta \) in the background with \( \lambda \neq 0 \) is the same as the effective action in the static background.
Figure 3: Spacetime of the time-dependent solution in different coordinate systems. (a) $\sigma^\pm$ or $x^\pm$ (b) $\tilde{\sigma}^\pm$ (c) $\tilde{x}^\pm$. Dashed arrows indicate coordinate change relating the regions in (b) and (c) to the region in (a).

$(\lambda = 0)$ under a change of coordinates from $\sigma$ to $\tilde{\sigma}$ given by $\sqrt{2\tilde{\mu}} \cosh \sigma = \sqrt{2\mu} \cosh \tilde{\sigma} + (\lambda/2)e^t$. For $\sigma$ and $\tilde{\sigma}$ large and negative, the change of coordinates is

$$e^{-\sigma^+} = e^{-\tilde{\sigma}^+} + \tilde{\lambda}$$

with $\tilde{\lambda} = \lambda/\sqrt{2\mu}$, $\sigma^+ = t + \sigma$ and $\tilde{\sigma}^+ = t + \tilde{\sigma}$.

To analyze the spacetime behaviour in this background, we first notice that under the leg-pole transform, the incoming profile $\eta_{in} \sim e^{t-\sigma}$ changes only by an infinite normalization constant. We have, therefore, an incoming field given by $S_{in} \sim e^{x^-}$.

Consider, therefore, a tachyon background $T_\lambda = \tilde{\mu}(e^{x^+-x^-} + \tilde{\lambda} e^{x^+})$ where $\tilde{\lambda}$ is a renormalized constant, and we have added back the standard stationary background term to regularize our problem. In this background, the form of small fluctuations must satisfy a linear version of equation (27),

$$\partial_+ \partial_- S = -\frac{a_2}{2} T_\lambda S$$

This equation can be transformed into one where the background is simply $T_0 = \tilde{\mu} e^{x^+-x^-}$ by the following change of variables

$$e^{-\tilde{x}^-} = e^{-x^-} - \tilde{\lambda} x^-$$

Since in the matrix model the time-dependent background is equivalent to one which is static, we would expect the same to be true in dilaton-gravity. The combined change of coordinates (77) and (79) relate these static backgrounds to each other, at least at large $x$ (or $\sigma$). The behaviour near the potential barrier is more complicated,
and hard to study in the spacetime picture since the exact form of the tachyon potential is not well defined.

The physical picture is illustrated in Figure 3. Figure 3(a) shows the time-dependent spacetime generated by the decaying Fermi sea [12] whose $I^+$ is incomplete. Figure 3(b) shows how this incomplete spacetime is related to the static spacetime obtained from the collective theory in the new coordinates. Figure 3(c) shows this relationship for the dilaton-gravity theory. The metric and the dilaton are trivial in the $\tilde{x}^\pm$ coordinates to this order.

To next order, we can calculate the second order tachyon field, as well as the metric and the dilaton:

\begin{align}
T^{(2)} &= -\frac{a_2}{4} \tilde{\mu} \hat{\lambda} e^{2x^+} \\
\delta &= -\frac{1}{8a_1} \tilde{\mu} \hat{\lambda} e^{2x^+} \\
\rho &= -\frac{1}{4a_1} \tilde{\mu} \hat{\lambda} e^{2x^+}
\end{align}

Since the effect of the second order field is small in the region where the coordinate changes (77) and (79) are nontrivial, they will not have a large effect on the spacetime analysis we have presented already.

6 Conclusion and further directions

The results of Section 3 can be used to rewrite the theory of a scalar coupled to dilaton-gravity without the need for the dilaton and gravity fields, at least to lowest order in those fields. Simply take the expressions (46) and (47) and substitute them back to the tachyon equation of motion, (27). The resulting equation of motion is of course nonlocal, as is expected when trying to integrate out gravitational interaction. Diffeomorphism invariance of the original theory manifest itself in the presence of the integration functions $\alpha^\pm(x^\pm)$.

In Section 4 we outlined a procedure for relating the solution of this nonlocal action to the simpler solutions of the collective field theory, using the known boundary correspondence. This is a toy model for the much more complicated problem of reconstructing the spacetime dynamics in AdS/CFT. Our simple example in Section 5 demonstrates how our results can be used to study time-dependent scenarios in Liouville string theory. It would be very interesting to see how these results can be used in more complicated scenarios, such as those involving space-like future boundaries [3, 17].
Acknowledgments

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Appendix: Integrals involving the leg-pole kernel $K$

Using the Fourier transform form of $K$, it is easy to show that the following integrals are true.

$$
\int dy \ K(y-x_1)K(y-x_2) = \delta(x_1-x_2) ,
$$

(83)

for any $x_1$ and $x_2$;

$$
\int dy \ K(x-y)K(y-x_1) = \sqrt{\frac{2}{\pi}} \left( (x-x_1) + 4\gamma - 2 + \ln \sqrt{\frac{2}{\pi}} \right) e^{x-x_1} ,
$$

(84)

for $x-x_1$ large and negative;

$$
\int dy \ K(x-y)\partial K(y-x_1)\partial K(y-x_2) = -\sqrt{\frac{2}{\pi}} \ e^{x-x_1} \delta(x_1-x_2)
$$

(85)

for $x-x_i$, $i = 1,2$ large and negative; and finally, for $x-x_i$ large and negative, and with $x_1-x_2 \gg |x_2-x_3|$,  

$$
\int dy \ (1-\partial)K(x-y)\partial K(y-x_1)\partial K(y-x_2)\partial K(y-x_3) = \frac{1}{\pi} e^{2x-x_1-x_2} \delta(x_2-x_3) + \frac{1}{\pi} e^{2x-2x_1} \delta''(x_2-x_3) .
$$

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