Directional Spatial Channel Estimation For Massive FD-MIMO in Next Generation 5G Networks

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Abstract—Full-dimensional (FD) channel state information at transmitter (CSIT) has always been a major limitation of the spectral efficiency of cellular multi-input multi-output (MIMO) networks. This letter proposes an FD-directional spatial channel estimation algorithm for frequency division duplex massive FD-MIMO systems. The proposed algorithm uses the statistical spatial correlation between the uplink (UL) and downlink (DL) channels of each user equipment. It spatially decomposes the UL channel into azimuthal and elevation dimensions to estimate the array principal receive responses. An FD spatial rotation matrix is constructed to estimate the corresponding transmit responses of the DL channel, in terms of the frequency band gap between the UL and DL channels. The proposed algorithm shows significantly promising performance, approaching the ideal perfect-CSIT case without UL feedback overhead.

Index Terms— Full-dimensional MIMO; spatial correlation; frequency division duplex; 5G; massive MIMO; CSI.

I. INTRODUCTION

Full dimensional massive multi-input multi-output (FD-mMIMO) is a key technology for boosting the spectral efficiency (SE) of 5G cellular networks [1]. Performance improvement of FD-mMIMO systems is achieved by using adaptive transmission at the base-station (BS) over the FD cell space. However, the assumption of perfect FD channel state information at the transmitter (FD-CSIT) is vital for achieving optimality, which is not feasible in practice [2, 3].

Hence, typical CSIT acquisition algorithms in time division duplex systems reasonably assume channel reciprocity, where the downlink (DL) channel can be approximated by the transpose of the uplink (UL) channel. In frequency division duplex (FDD) systems, channel reciprocity is not applicable due to the frequency band offset Ω. Consequently, channel quantization and limited-feedback algorithms have been widely considered [4]. Current LTE-A Pro standards [5] define double-codebooks for tracking the channels small- and large-scale variations. For FD-mMIMO systems, the double-codebooks are extended to scan the azimuthal and elevation dimensions, providing a Kronecker-product (KP) based beamforming algorithm [6].

However, channel quantization represents a major limitation of the network spatial degrees of freedom (DoFs), regardless of the number of transmit antennas [7]. Hence, the design of the beamformed CSI-reference-signals (CSI-RS) is widely studied in recent standards [8, 9], where the DL pilots are distributed across several FD beamforming directions. CSI-RS algorithms have shown scalability and performance improvement with limited feedback overhead; however, they may result in blind coverage spots if scanning precision is insufficient. Furthermore, an FDD mMIMO system based on DL spatial channel estimation (FMMSE) [7] has been recently proposed, to remove the limitation of the channel quantization; though, it suffers from sub-optimal performance in FD systems due to the missing elevation DoFs.

In this work, a directional spatial channel estimation (D-SCE) algorithm is proposed for FDD FD-mMIMO systems. The UL channel is spatially projected over the FD space of a pre-designed discrete Fourier transform (DFT) codebook. The FD spatial power spectrum of the UL channel is estimated to obtain the instantaneous receive response of the antenna array. The estimated array response is spatially rotated in terms of Ω to compensate for the spatial deviation of the principal DL channel clusters and thereby attain the corresponding transmit response. Finally, the UL channel is spatially beamformed towards the principal set of the estimated transmit responses. The minimum mean square error (MMSE) criterion is applied to refine the estimation accuracy. The proposed D-SCE algorithm shows promising SE improvement, without the requirement of channel quantization or feedback overhead.

This paper is organized as follows. In Section II, the spatial channel modeling is presented. Section III introduces the proposed D-SCE algorithm. Performance results are discussed in Section IV and conclusions are drawn in Section V.

Notations: (x)T , (x)H and (x)−1 denote the transpose, Hermitian, and inverse operations on x. x ⊗ y stands for the Kronecker product of x and y, while ℛ and |x| represent the mean and absolute value of x. x ∼ ℂℳ(0, σ2) denotes a complex Gaussian random variable with zero mean and variance σ2, while {x} indicates the set of possible x values. xκ, κ ∈ {ul, dl} denotes the link direction of x.

II. SYSTEM MODEL

In this work, we consider a DL multi-user FD mMIMO system, as shown in Fig. 1. A BS is mounted with NT = Nb × Nh planar uniform rectangular array (URA), where NT and Nb are the elevation and azimuthal antenna elements, respectively. There are K FD uniformly distributed users with Mf antennas. Each UE accepts d independent DL spatial streams x ∼ ℂℳ(0, 1/Pl), where P is the user transmit power. The received DL signal at the kth user is expressed as
is the separation distance, with \( \mathcal{G} \) and \( \mathcal{S} \) spatially-correlated channel model [10], where the channel coefficient is given by

\[
\eta = \begin{bmatrix} a_{\kappa}^n_c(r_c) \end{bmatrix}
\]

for \( \kappa \in \{ul, dl\} \) and \( \mathcal{S} \) spatial channel of the BS and UE are the BS and UE spatial antenna patterns, respectively. The steering element \( \tau_{(m,n,c)_k} \) of the channel coefficient is given by

\[
\tau_{(m,n,c)_k} = \frac{1}{\sqrt{C}} \sum_{\ell=0}^{C-1} \sqrt{\alpha_k} G_{\ell,k} r_{(m,n,c)_k},
\]

where \( h_{\ell,k} \in \mathbb{C}^{M_x \times N_t} \), \( \forall k \in \{0,1, \ldots, K - 1\} \) is the DL spatial channel of the \( k^{th} \) user, \( V_k \in \mathbb{C}^{N_t \times 1} \) is the zero-forcing precoding matrix given as \( V_k = (H_{\ell,k})^H (H_{\ell,k} H_{\ell,k})^{-1} \), and \( n_k \) is the additive white Gaussian noise. We adopt the spatially-correlated channel model [10], where the channel is described by its major \( C \) scattering clusters, spatially distributed over the FD cell space as \( Z \) rays per cluster.

The exact spatially-correlated FD-MIMO channel model, introduced in Section II, can be rewritten only by its predominant spatial clusters, distributed over the FD cell space as

\[
\mathbf{H}_k = \frac{1}{\sqrt{C}} \sum_{\ell=0}^{C-1} g_{\kappa,c}(\phi_c),
\]

where \( h_{\kappa,c} \in \{ul, dl\} \) is the UL/DL spatial channel matrix of an arbitrary user, \( g_{\kappa,c} \) is the \( \ell^{th} \) cluster gain of the UL/DL channel, and \( a_{\kappa,c}(\phi_c) \) is the UL receive or DL transmit antenna FD response of the \( \ell^{th} \) cluster, with \( \phi_c \) as the FD spatial angle of the corresponding antenna response. The FD antenna response \( a_{\kappa,c}(\phi_c) \) is composed of the horizontal and vertical responses by the Kronecker product as \( a_{\kappa,c}(\phi_c) = a_{\kappa,c}(\theta_c) \otimes a_{\kappa,c}(\phi_c) \), where the horizontal \( a_{\kappa,c}(\theta_c) \) and vertical \( a_{\kappa,c}(\phi_c) \) antenna response, in the azimuthal direction \( \theta_c \) and elevation direction \( \phi_c \) of the \( \ell^{th} \) cluster, are given by

\[
a_{\kappa,c}(\theta_c) = \begin{bmatrix} 1, e^{-j2\pi\Delta_h \cos \phi_c}, \ldots, e^{-j2\pi\Delta_h (N_k - 1) \cos \phi_c} \end{bmatrix}^T,
\]

\[
a_{\kappa,c}(\phi_c) = \begin{bmatrix} 1, e^{-j2\pi\Delta_v \cos \phi_c}, \ldots, e^{-j2\pi\Delta_v (N_k - 1) \cos \phi_c} \end{bmatrix}^T,
\]

where \( \Delta_h \) and \( \Delta_v \) are the horizontal and vertical antenna physical spacing, respectively. The \( \ell^{th} \) FD cluster can be sampled in the DFT domain as

\[
\mathcal{H}_{\kappa,c}(b) = \sum_{n=0}^{N_t-1} g_{n,c} e^{-j2\pi\Delta_n \cos \phi_c} e^{-j2\pi b n / N_t}, b = 0, \ldots, N_t-1,
\]

where \( \Delta_n \) denotes the effective antenna spacing of the entire antenna array. The magnitude of \( |\mathcal{H}_{\kappa,c}(b)| \) is described by

\[
|\mathcal{H}_{\kappa,c}(b)| = |g_{n,c}| \times \left| \frac{\sin \left( \frac{\pi}{T} \left( -2\pi\Delta_n \sin (90 - \phi_c) + \frac{\pi}{2} b \right) \right)}{\sin \left( \frac{\pi}{T} \left( -2\pi\Delta_n \sin (90 - \phi_c) + \frac{\pi}{2} b \right) \right)} \right|.
\]

From (8), the leakage of each channel cluster becomes range-limited with the number of the transmit antennas. Hence, with large antenna arrays, the channel dimension reduces to fewer and more predominant clusters. This leads to significant estimation precision if the directions of only the most predominant DL clusters are sufficiently approached. Furthermore, the \( \ell^{th} \) channel cluster gain is expressed as

\[
g_{\kappa,c} = A_{\kappa,c} \gamma_{\kappa,c} \Upsilon_{\kappa,c},
\]

where \( A_{\kappa,c} \) is a constant to represent the transmit power, and antenna gain. \( \gamma_{\kappa,c} \) and \( \Upsilon_{\kappa,c} \) are the large- and small-scale factors of the \( \ell^{th} \) channel cluster. In dense environments, where low mobility conditions are applicable, e.g., 30 km/h,
the surrounding scatterers and Doppler shift slowly vary between two successive UL and DL transmissions of the same user. Thus, it is reasonable to assume that the average gain of both channel clusters is constant [10], being given as 

\[ \text{Correlation Coefficient} = \rho \]

where \( \rho \) denotes the statistical expectation. The optimal DL transmit response to maximize the received power of the \( C \) channel clusters is given by 

\[ \tilde{a}_{\text{dl},c}(\phi_c) = \arg \max_{\phi} (\tilde{a}_{\text{dl},c}(\phi_c)^H a_{\text{dl},c}(\phi_c) a_{\text{dl},c}(\phi_c)^H \tilde{a}_{\text{dl},c}(\phi_c)) \]

(11)

where \( \tilde{a}_{\text{dl},c}(\phi_c) \) and \( a_{\text{dl},c}(\phi_c) \) are the estimated and actual DL transmit responses at the BS. Thus, the optimal transmit response \( \tilde{a}_{\text{dl},c}(\phi_c) \) should spatially align with the global set of the principal eigenvectors of the actual DL response \( a_{\text{dl},c}(\phi_c) \), where \( \phi_c = \phi_c \). However, in FDD networks, the BS only accesses the UL receive response \( a_{\text{dl},c}(\phi_c) \) since the transmit \( a_{\text{dl},c}(\phi_c) \) and receive \( a_{\text{dl},c}(\phi_c) \) antenna responses are not reciprocal, due to \( \Omega \). Furthermore, no closed-form relation between \( a_{\text{dl},c}(\phi_c) \) and \( a_{\text{dl},c}(\phi_c) \) exists in the literature, because it is a non-convex problem [5]. Hence, the knowledge of the actual DL transmit response \( a_{\text{dl},c}(\phi_c) \) is not available at the BS.

In this work, we transform the optimization problem in (11), with the pre-knowledge requirement of the DL antenna response \( a_{\text{dl},c}(\phi_c) \), into a search problem of the closest possible spatial directions, observed from the spatial power spectrum of the UL channel, as it will be discussed subsequently.

Assuming a standard antenna sector of 120°/90° coverage in both the azimuthal and elevation directions, the FD cell space is spatially divided into \( U \) and \( Q \) elevation and azimuthal subspaces, with an arbitrary scanning precision. Then, we define an FD-DFT beamforming codebook at the BS to project the UL channel clusters over the virtual beamforming subspaces. An approximate estimate of the spatial power spectra of the UL/DL channels, averaged over the \( C \) channel clusters within the entire FD space \( \phi \) is given by

\[ P_c(\phi) = \left[ a_{c}^H(\phi) (H_c^H H_c)^{-1} a_{c}(\phi) \right]^{-1}. \]

(12)

Hence, the correlation coefficient of the UL and DL clusters is calculated as

\[ \rho = \frac{\int (P_{ul}(\phi) - \bar{P}_{ul}(\phi))(P_{dl}(\phi) - \bar{P}_{dl}(\phi)) d\phi}{\sqrt{(P_{ul}(\phi) - \bar{P}_{ul}(\phi))^2 \sqrt{(P_{dl}(\phi) - \bar{P}_{dl}(\phi))^2}}. \]

(13)

The empirical cumulative density function (ECDF) of the correlation coefficient is shown in Fig. 2, for different \( \Omega \) values. As can be noticed, the UL and DL spatial spectra, and hence, the receive and transmit responses are highly correlated in the spatial domain, due to the small channel spatial variance over the closely-spaced antenna elements, e.g., for 50% of the channel samples, a correlation coefficient \( \rho = 0.8588 \) is observed for \( \Omega = 200 \text{ MHz} \).

Fig. 3 depicts the decomposable spatial spectra of the UL channel across the entire azimuthal space of each elevation subspace, with \( U = 4 \) and \( Q = 120 \). It is evident that the fourth elevation subspace \( (u = 3) \) is the best-match-subspace for maximizing the received signal-to-interference-noise ratio (SINR), capturing the additional UL spatial elevation DoFs. It is worth mentioning that \( Q \gg N_t \) should be satisfied in order to fully utilize the spatial DoFs of the antenna array. Fig. 4 exemplifies the impact of \( \Omega \) on the spatial deviation between the observed UL AoAs and actual DL AoDs across the principal elevation space. It is worth noting that a spatial shift between the most dominant clusters of the UL and DL channels appears, which is a function of \( \Omega \).
B. **FD Directional Spatial Channel Estimation**

The proposed D-SCE algorithm decomposes the observed UL channel into 2D projections over the azimuthal and elevation dimensions, using a pre-designed FD beamforming codebook. The principal sub-array receive responses in both dimensions are estimated to satisfy the maximization of the spatial SINR. Then, the receive responses are spatially rotated, in terms of $\Omega$, to estimate the BS transmit responses. The observed UL channel is spatially beamformed towards the directions of the estimated transmit responses. Finally, the MMSE criterion is applied to enhance the estimation precision.

First, an arbitrary FD beamforming codeword $W_n(\theta, \varphi) \in \mathbb{C}^{N_t \times 1}$ is composed as

$$W_n(\theta, \varphi) = \frac{1}{\sqrt{N_t}} \left( a_n^h(\theta) \otimes a_n^v(\varphi) \right).$$  

Accordingly, an FD beamforming spatial codebook is constructed as $\{W_n(\theta_q, \varphi_u)\}$, to cover the FD antenna sector by the discrete direction set $\{\theta_q, \varphi_u\}, \forall q = 0, 1, \ldots, Q - 1, u = 0, 1, \ldots, U - 1$. Next, the UL FD spatial spectrum is estimated according to (12), where the array responses are substituted by the FD codewords from the codebook as $a_n(\phi) = W_n(\theta_q, \varphi_u)$. The principal elevation subspace $\varphi_u$ of the UL spatial spectrum is obtained based on the maximization of the average received power over the corresponding $Q$-codeword azimuthal discrete space $\{\theta_q\}$ as

$$\varphi_u = \arg \max_{\{\varphi_u\}} \left( \mathbb{E} \left( \| P_{ul}(\theta_q, \varphi_u) \|^2 \right) \right).$$  

The strongest $N_i$ azimuthal directions are estimated as

$$\hat{\theta}_n = \arg \max_{\{\theta_q\}} \left( \| P_{ul}(\theta_q, \varphi_u) \|^2 \right), \forall n = 0, 1, \ldots, N_i - 1. \tag{16}$$

The FD array principal receive response matrix $A_R(\theta, \varphi_u) \in \mathbb{C}^{N_t \times N_i}$ can be given by

$$A_R(\theta, \varphi_u) = \left[ \left( a_n^h(\hat{\theta}_0) \otimes a_n^v(\varphi_u) \right)^T \ldots \left( a_n^h(\hat{\theta}_{N_i-1}) \otimes a_n^v(\varphi_u) \right)^T \right], \tag{17}$$

where $\theta = \{\hat{\theta}_n\}_{n=0}^{N_i-1}$ is the discrete set of the estimated principal azimuthal subspaces. A spatial rotation matrix $\Gamma \in \mathbb{C}^{N_t \times N_t}$ is constructed to compensate for the frequency band gap between the UL and DL channels, with each rotation column vector given by

$$F = \left[ 1, e^{-j2\pi (f_{ul} \Gamma)} , \ldots, e^{-j2\pi (N_i-1)(f_{ul} \Gamma)} \right]^T, \tag{18}$$

where $f_{ul}$ and $f_{dl}$ are the operating center frequencies of the DL and UL channels, respectively. The corresponding FD transmit response matrix $A_T(\Phi, \Psi) \in \mathbb{C}^{N_t \times N_t}$ is estimated by spatially rotating the obtained receive responses through $\Gamma$ by

$$A_T(\Phi, \Psi) = A_R(\theta, \varphi_u) \Gamma^T, \tag{19}$$

where $\Phi$ and $\Psi$ denote a rotated set of the azimuthal subspaces $\theta$ and the elevation $\varphi_u$ subspace. A rough estimate of the DL channel is calculated by beamforming the observed UL channel over the estimated transmit responses as given by

$$H_{dl}^{(1)} = H_{ul}^{(1)} A_T(\Phi, \Psi). \tag{20}$$

Finally, the estimated transmit response matrix is refined by applying the MMSE criterion. The MMSE approach seeks a matrix $G$ to minimize the corresponding MSE as

$$\text{MSE} = \mathbb{E} \left\{ (GH_{ul}^{(1)} A_T(\Phi, \Psi) - H_{dl}) (GH_{ul}^{(1)} A_T(\Phi, \Psi) - H_{dl})^H \right\}. \tag{21}$$

The $G$ matrix is expressed as: $G = (H_{dl}^{(1)} A_T(\Phi, \Psi) + \sigma^2 I)^{-1} (H_{ul}^{(1)} A_T(\Phi, \Psi))$. Then, the final DL channel estimate is expressed as

$$H_{dl}^{(2)} = H_{ul}^{(1)} GA_T(\Phi, \Psi). \tag{22}$$

IV. **Numerical Results**

We adopt an $8 \times 8$ URA transmit antenna setup at the BS and $2 \times 1$ receive antennas at the user side. The 3GPP FD spatial channel, as described by (2)-(3), defines $C = 12$ for line of sight (LoS) and $C = 20$ for non-LoS, with $Z = 20$. The detailed simulation parameters are listed in Table I. The performance of the proposed D-SCE is compared with the state-of-the-art CSIT harvesting standards for FD-mMIMO channels as follows:

**Beamformed CSI-RS**

The original 3GPP proposal [8] has shown significant CSIT acquisition gain. The enhanced-CSI-RS (E-CSI-RS) algorithm [9] is an extension of the CSI-RS standard, proposed from Intel Research. E-CSI-RS algorithm adopts dual FD-DFT codebooks. The first $L$-codeword codebook $J$ defines the
actual CSI-RS span, physically beamformed across the FD cell space. Thus, the beamformed DL channel $H_{dl}^{bf}$ at the user side is de-beamformed to obtain an approximate estimate of the actual channel $H_{dl}^{approx}$ span given as

$$H_{dl}^{approx} = H_{dl}^{bf} J J^H.$$ (23)

At the user side, the estimate of the DL full-span channel $H_{dl}^{approx}$ is spatially projected over the second $N$—codeword codebook $\Sigma$. Finally, each UE feeds-back its serving BS with an index $\kappa$ of $B = \log_2 (N)$ bits, to select the closest-match codeword to its estimated DL channel as

$$\kappa = \arg \max_\Sigma |H_{dl}^{approx} \Sigma|^2.$$ (24)

Hence, the actual CSIT gain is $L$; however, the effectively harvested gain is $N$, where $N \gg L$.

**Kronecker-based Beamforming**

The KP-based CSIT approach [6] is an extension of the current 2D-MIMO CSIT standards. KP-based CSIT algorithms adopt the current double-codebooks in LTE-Pro standards [5] for the azimuthal scanning, where the azimuthal $t^{th}$ codeword, for $N_h = 8$ horizontal antenna setup, is given by

$$(T)_t = \left( \frac{1}{\sqrt{8}} \right) \left[ \varpi, e^{j \frac{\pi t}{4}}, e^{j \frac{3 \pi t}{4}}, e^{j \frac{5 \pi t}{4}}, e^{j \frac{7 \pi t}{4}}, e^{j \frac{9 \pi t}{4}}, e^{j \frac{11 \pi t}{4}}, e^{j \frac{13 \pi t}{4}} \right]^T,$$ (25)

where $\varpi = \{1, e^{j \frac{\pi t}{4}}, e^{j \frac{3 \pi t}{4}}, e^{j \frac{5 \pi t}{4}}, e^{j \frac{7 \pi t}{4}}, e^{j \frac{9 \pi t}{4}}, e^{j \frac{11 \pi t}{4}}, e^{j \frac{13 \pi t}{4}} \}$. For the elevation scanning, a DFT-based codebook $a_k^{\varphi}(\varphi)$ is used, and the final FD-KP-based codebook is generated by $W(\theta, \varphi) = T \otimes a_k^{\varphi}(\varphi)$.

Fig. 5 depicts the average SE performance in bits/sec/Hz of the perfect CSIT-based and the proposed FD D-SCE algorithms, under different antenna setups, $d = 2$. B = 209.28 per user per channel coherence time, which overwhelms the UL control channel.

Fig. 6 shows the SE performance comparison of the perfect CSIT-based, proposed FD D-SCE, E-CSI-RS, KP-based, 2D FMMSCE and 2D RVQ algorithms. The proposed D-SCE with $B = 0$ feedback overhead bits outperforms the E-CSI-RS with $B = 7$ bits per user per channel coherence time, especially in the high SINR region, approaching the perfect CSIT case. The E-CSI-RS suffers from performance degradation due to the channel approximation, which contributes to the residual inter-user interference. The proposed D-SCE provides significant outperformance than the KP-based CSIT with $B = 10$ bits, e.g., 4-elevation and 256-azimuthal codewords, respectively, due to the insufficient CSIT precision of the adopted azimuthal LTE-Pro dual-codebook. FMMSCE and conventional RVQ clearly suffer from performance degradation due to the missing elevation beamforming, with $B = 0$ and 9 bits, respectively.

Moreover, the proposed D-SCE algorithm shows robustness to the UL estimation error as: $H_{ul} = H_{al} + Y$, where $Y \in \mathbb{C}^{N_t \times M_r}$ is the Gaussian estimation error of the UL channel.
with variance $\sigma^2$. As shown in Fig. 7, the D-SCE algorithm shows significant robustness against the UL channel estimation error, when sufficient scanning precision is available, e.g., maximum SE loss of 15 bps/Hz when $\sigma^2 = 0.3$, with $Q = 120$. This is due to the high density of the precise beamforming directions with small spatial offsets around the optimal DL transmit responses.

Furthermore, the precision of the azimuthal $Q$ and elevation $U$ DoFs is shown to influence the performance of the proposed D-SCE algorithm. Fig. 8 and 9 present the mean loss of the overall SE with different $Q$ and $U$ values. With low azimuthal scanning precision $Q$, referenced to the $Q = 120$ case, the D-SCE algorithm exhibits consistent performance degradation because the FD DoFs of the antenna array can not be fully utilized, as shown in Fig. 8. Due to the small Doppler shift in the elevation direction, and hence, the small spatial channel elevation variance, selecting the elevation DoFs $U$ size does not provide fast performance improvement. Hence, as depicted in Fig. 9, referenced to the $U = 4$ case used in this work, increasing the elevation scanning precision $U \gg 4$ does not offer significant spatial DoFs. The performance gain of the proposed D-SCE algorithm is due to i) the proper reduction of the spatial channel span, and ii) the on-the-go sufficient estimation of the principal transmit responses.

V. CONCLUSION

A novel FD directional spatial channel estimation (D-SCE) algorithm has been proposed. It blindly utilizes the statistical spatial correlation between the UL and DL channels, to attain higher CSIT harvesting gain. Compared to the state-of-the-art standard CSIT estimation algorithms, the proposed D-SCE algorithm shows significant performance improvement without FD CSIT overhead. With simple implementation complexity, zero CSIT overhead, and scalability with the size of the transmit array, the proposed D-SCE algorithm is a strong candidate for FD-mMIMO FDD systems.

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