Short-Baseline Electron Neutrino Disappearance, Tritium Beta Decay and Neutrinoless Double-Beta Decay

Carlo Giunti
INFN, Sezione di Torino, Via F. Giuria 1, I–10125 Torino, Italy

Marco Laveder
Dipartimento di Fisica “G. Galilei”, Università di Padova, and INFN, Sezione di Padova, Via F. Marzolo 8, I–35131 Padova, Italy

(Dated: October 12, 2010)

We consider the interpretation of the MiniBooNE low-energy anomaly and the Gallium radioactive source experiments anomaly in terms of short-baseline neutrino oscillation mixing schemes. The separate fits of MiniBooNE and Gallium data are highly compatible, with close best-fit values of the effective oscillation parameters $\Delta m^2$ and $\sin^2 2\theta$. The combined fit gives $\Delta m^2 \gtrsim 0.1 \text{eV}^2$ and $0.11 \lesssim \sin^2 2\theta \lesssim 0.48$ at 2$\sigma$. We consider also the data of the Bugey and Chooz reactor antineutrino oscillation experiments and the limits on the effective electron antineutrino mass in $\beta$-decay obtained in the Mainz and Troitsk Tritium experiments. The fit of the data of these experiments limits the value of $\sin^2 2\theta$ below 0.10 at 2$\sigma$. Considering the tension between the neutrino MiniBooNE and Gallium data and the antineutrino reactor and Tritium data as a statistical fluctuation, we perform a combined fit which gives $\Delta m^2 \sim 2 \text{eV}$ and $0.01 \lesssim \sin^2 2\theta \lesssim 0.13$ at 2$\sigma$. Assuming a hierarchy of masses $m_1 , m_2 , m_3 \ll m_4$, the predicted contributions of $m_4$ to the effective neutrino masses in $\beta$-decay and neutrinoless double-$\beta$ decay are, respectively, between about 0.06 and 0.49 and between about 0.003 and 0.07 eV at 2$\sigma$. We also consider the possibility of reconciling the tension between the neutrino MiniBooNE and Gallium data and the antineutrino reactor and Tritium data with different mixings in the neutrino and antineutrino sectors. We find a 2.6$\sigma$ indication of a mixing angle asymmetry.

PACS numbers: 14.60.Pq, 14.60.Lm, 14.60.St

I. INTRODUCTION

Neutrino oscillations have been observed in solar, atmospheric and long-baseline reactor and accelerator experiments. The data of these experiments are well fitted in the framework of three-neutrino mixing, in which the three flavor neutrinos $\nu_e, \nu_\mu, \nu_\tau$ are unitary linear combinations of three massive neutrinos $\nu_1, \nu_2, \nu_3$ with the solar (SOL) and atmospheric (ATM) squared-mass differences

$$\Delta m^2_{21} = \Delta m^2_{\text{SOL}} \approx 8 \times 10^{-5} \text{eV}^2,$$

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| = \Delta m^2_{\text{ATM}} \approx 2 \times 10^{-3} \text{eV}^2,$$

where $\Delta m^2_{jk} = m_j^2 - m_k^2$ and $m_j$ is the mass of the neutrino $\nu_j$ (see Refs. [1, 8]).

Besides these well-established observations of neutrino oscillations, there are at least three anomalies which could be signals of short-baseline neutrino oscillations generated by a larger squared-mass difference: the LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal [9], the Gallium radioactive source experiments anomaly [10, 11], and the MiniBooNE low-energy anomaly [12]. In this paper we consider the MiniBooNE and Gallium anomalies, which can be explained by short-baseline electron neutrino disappearance [13, 14] in the effective framework of four-neutrino mixing, as explained in Sections III and IV. On the other hand, the LSND anomaly is disfavored by the results of the MiniBooNE $\nu_\mu \rightarrow \nu_e$ experiment [12, 16] and may require another explanation [17, 23].

In Refs. [13, 14] we proposed to explain the MiniBooNE low-energy anomaly through the disappearance of electron neutrinos due to very-short-baseline oscillations into sterile neutrinos generated by a squared-mass difference $\Delta m^2$ larger than about 20 eV$^2$. In that case, the analysis of the MiniBooNE data is simplified by the fact that the effective survival probability $P_{\nu_e \rightarrow \nu_e}$ is practically constant in the MiniBooNE energy range from 200 to 3000 MeV. In this paper we extend the analysis of MiniBooNE data to lower values of $\Delta m^2$, considering the resulting energy dependence of the effective short-baseline (SBL) electron neutrino and antineutrino survival probability

$$P_{\nu_e \rightarrow \nu_e}(L, E) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right),$$

where $L$ is the neutrino path length and $E$ is the neutrino energy (CPT invariance implies that the survival probabilities of neutrinos and antineutrinos are equal; see Ref. [3]).

The two-neutrino-like effective short-baseline survival probability in Eq. (3) is obtained in four-neutrino schemes (see Refs. [1, 14, 6, 4]), which are the simplest extension of three-neutrino mixing schemes which can ac-
commodate the two small solar and atmospheric squared-mass differences in Eqs. (1) and (2), and one larger squared-mass difference for short-baseline neutrino oscillations,

$$|\Delta m^2_{41}| = \Delta m^2 \gtrsim 0.1 \text{eV}^2. \quad (4)$$

The existence of a fourth massive neutrino corresponds, in the flavor basis, to the existence of a sterile neutrino $\nu_s$.

In this paper we consider 3+1 four-neutrino schemes, since 2+2 four-neutrino schemes are disfavored by the combined constraints on active-sterile transitions in solar and atmospheric neutrino experiments [3]. For simplicity, we consider only 3+1 four-neutrino schemes with

$$m_1, m_2, m_3 \ll m_4, \quad (5)$$

which give the $\Delta m^2_{41}$ in Eq. (4) and appear to be more natural than the other possible 3+1 four-neutrino schemes in which either three neutrinos or all four neutrinos are almost degenerate at a mass scale larger than $\sqrt{\Delta m^2}$ (see Refs. [1, 4, 6, 7]).

In 3+1 four-neutrino schemes the effective mixing angle in the effective short-baseline electron neutrino survival probability in Eq. (3) is given by (see Refs. [1, 4, 6, 7])

$$\sin^2 2\theta = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2 \right). \quad (6)$$

In this paper we assume that the value of $|U_{e4}|^2$ is so small that the effective short-baseline muon neutrino survival probability is practically equal to unity and short-baseline $\nu_\mu \leftrightarrow \nu_e$ are negligible [4]. This assumption is justified by the lack of any indication of $\nu_\mu \rightarrow \nu_e$ transitions in the MiniBooNE experiment [12, 16] and the limits on short-baseline muon neutrino disappearance found in the CDHSW [24], CCFR [25] and MiniBooNE [26] experiments. We do not consider the MiniBooNE antineutrino data [27], which have no statistical uncertainties which are too large to constraint new physics [13].

The plan of the paper is as follows. In Section II we discuss the analysis of MiniBooNE data. In Section III we present an update of the analysis of Gallium data published in Ref. [1] and the combined analysis of MiniBooNE and Gallium data. In Section IV we discuss the implications of the measurements of the effective electron neutrino mass in Tritium $\beta$-decay experiments and their combination with reactor neutrino oscillation data. In Section V we present the results of the combined analysis of MiniBooNE, Gallium, reactor and Tritium data and in Section VI we present the corresponding predictions for the effective masses measured in $\beta$-decay and neutrinoless double-$\beta$-decay experiments. In Section VII we calculate the mixing angle asymmetry between the neutrino and antineutrino sectors which could explain the tension between the neutrino and antineutrino data under our short-baseline $\nu_e$-disappearance hypothesis. In Section VIII we draw the conclusions.

### II. MINIBOONE

The MiniBooNE experiment was made with the purpose of checking the indication of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations generated by a $\Delta m^2 \gtrsim 0.1 \text{eV}^2$ found in the LSND experiment [4]. The MiniBooNE collaboration did not find any indication of such oscillations in the $\nu_\mu \rightarrow \nu_e$ channel [12, 16]. On the other hand, the MiniBooNE collaboration found an anomalous excess of low-energy $\nu_e$-like events in the data on the search for $\nu_\mu \rightarrow \nu_e$ oscillations [12, 16], as shown in Fig. 1.

As in Refs. [13, 15], we consider an explanation of the low-energy MiniBooNE anomaly based on the possible short-baseline disappearance of electron neutrinos, taking into account a possible overall normalization factor $f_\nu$ of the calculated $\nu_e$-induced and misidentified $\nu_\mu$-induced events which contribute to the observed number of $\nu_e$-like events. The normalization factor $f_\nu$ could be due mainly to the uncertainty of the calculated neutrino flux (see Ref. [28]). Since the misidentified $\nu_\mu$-induced and $\nu_e$-induced events dominate, respectively, at low and high energies (see Fig. 2), the low-energy excess can be fitted with $f_\nu > 1$ and the high-energy data can be fitted compensating $f_\nu > 1$ with the disappearance of $\nu_\mu$'s.

In Refs. [13, 15] we considered only very-short-baseline $\nu_e$ disappearance due to a $\Delta m^2 \gtrsim 20 \text{eV}^2$, which generates a survival probability $P_{\nu_e \rightarrow \nu_e}$, which is constant in the MiniBooNE energy range, from 200 to 3000 MeV. In this paper we extend the analysis to lower values of $\Delta m^2$, considering the resulting energy dependence of the survival probability. In this case, the theoretical number of $\nu_e$-like events in the $j$th energy bin is given by

$$N_{\nu_e,j}^{\text{the}} = N_{\nu_e,j}^{\nu_e,\text{the}} + N_{\nu_e,j}^{\nu_\mu,\text{the}}, \quad (7)$$

where

$$N_{\nu_e,j}^{\nu_e,\text{the}} = f_\nu P_{\nu_e \rightarrow \nu_e} N_{\nu_e,j}^{\nu_e,\text{cal}} \quad (8)$$

is the number of $\nu_e$-induced events and

$$N_{\nu_e,j}^{\nu_\mu,\text{the}} = f_\nu N_{\nu_e,j}^{\nu_\mu,\text{cal}} \quad (9)$$

is the number of misidentified $\nu_\mu$-induced events. Here $N_{\nu_e,j}^{\nu_e,\text{cal}}$ and $N_{\nu_e,j}^{\nu_\mu,\text{cal}}$ are, respectively, the number of $\nu_e$-induced and misidentified $\nu_\mu$-induced events calculated by the MiniBooNE collaboration for the $j$th energy bin.
Construction of the MiniBooNE least-squares function allows a value of $N_{\nu_e}$ detected and predicted charged-current quasi-elastic events [31] allows a value of $f_\nu$.

Here we use directly the order to constrain its value in the least-squares analysis. Ref. [15] we used this estimate of the uncertainty of $f_\nu$.

The MiniBooNE measurement of a ratio 1.21 ± 0.24 of detected and predicted charged-current quasi-elastic $\nu_\mu$ events [31] allows a value of $f_\nu$ as large as about 15%. In Ref. [15] we used this estimate of the uncertainty of $f_\nu$ in order to constrain its value in the least-squares analysis.

Here we use directly the $\nu_\mu$ data given in Ref. [29] for the construction of the MiniBooNE least-squares function

$$\chi^2_{\text{MiniBooNE}} = \sum_{j=1}^{11} \left( \frac{N_{\nu_e,j}^{\text{the}} - N_{\nu_e,j}^{\exp}}{\sigma_{\nu_e,j}} \right)^2 + \sum_{j,k=1}^{8} \left( N_{\nu_{\mu},j}^{\text{the}} - N_{\nu_{\mu},j}^{\exp} \right) \left( (V_{\nu_{\mu}}^{-1})_{jk} \left( N_{\nu_{\mu},k}^{\text{the}} - N_{\nu_{\mu},k}^{\exp} \right) \right).$$

(10)

Here $N_{\nu_{\mu},j}^{\exp}$ are the numbers of measured $\nu_{\mu}$-like events in 11 reconstructed neutrino energy bins and $N_{\nu_{\mu},j}^{\exp}$ are the numbers of measured $\nu_\mu$ charged-current quasi-elastic events in 8 reconstructed neutrino energy bins. The theoretical number of $\nu_\mu$ events in the $j$th energy bin is given by

$$N_{\nu_{\mu},j}^{\text{the}} = f_\nu N_{\nu_{\mu},j}^{\text{cal}},$$

(11)

where $N_{\nu_{\mu},j}^{\text{cal}}$ is the number of $\nu_\mu$ events calculated by the MiniBooNE collaboration [29]. In order to take into account the correct statistical uncertainty corresponding to the rescaling of the number of $\nu_\mu$ events due to $f_\nu$ in Eq. (11), we used the covariance matrix $V_{\nu_{\mu}}$ given by

$$(V_{\nu_{\mu}})_{jk} = (V_{\nu_{\mu}}^{\text{cal}})_{jk} + (f_\nu - 1) N_{\nu_{\mu},j}^{\text{cal}} \delta_{jk},$$

(12)

where $V_{\nu_{\mu}}^{\text{cal}}$ is the 8 × 8 covariance matrix of $\nu_\mu$ events presented by the MiniBooNE collaboration in Ref. [29]. We did not use the complete 19 × 19 covariance matrix of $\nu_e$ and $\nu_\mu$ events given in Ref. [29] because the correlations involving $\nu_e$ events have been obtained without taking into account the energy-dependent disappearance of electron neutrinos that we want to test. Assuming the correlations given in that 19 × 19 covariance matrix would suppress the energy dependence of $P_{\nu_{\mu} \rightarrow \nu_e}^{(j)}$. Therefore, for the uncertainties $\sigma_{\nu_{\mu},j}$ in Eq. (10) we used only the diagonal elements of the $\nu_e$ covariance matrix $V_{\nu_e}^{\text{cal}}$ given in Ref. [29], corrected by the change of statistical
uncertainty corresponding to the variation of expected events due to $f_\nu$ and $P_{\nu_e \rightarrow \nu_\mu}$ in Eqs. (7)–(9):

$$
\sigma^2_{\nu_e,j} = (V^{\text{cal}}_{\nu_e,j})_{jj} + N^\text{the}_{\nu_e,j} - N^\text{cal}_{\nu_e,j},
$$

with $N^\text{cal}_{\nu_e,j} = N^\nu_{e j} + N^\nu_{\mu j}$. The result of the minimization of $\chi^2_{\text{MB}e}$ is shown in Fig. 1b, in which the solid histogram corresponds to the best-fit values of $f_\nu$, $\sin^2 2\vartheta$ and $\Delta m^2$ in the first column of Tab. I. From Fig. 1b, one can see that the fit is acceptable for all the $\nu_e$ energy bins, including the first three bins which are out-of-fit in Fig. 1a. In Tab. I we give separately the contribution to $\chi^2_{\text{MB}e}$ of the first three low-energy $\nu_e$ bins and the sum of the contributions of the other $\nu_e$ energy bins and all the $\nu_\mu$ energy bins. In this way one can see that with $f_\nu = 1$ and $P_{\nu_e \rightarrow \nu_\mu} = 1$ (Null Hypothesis), although the global value $\chi^2 = 19.7$ is compatible with the number of degrees of freedom, NDF = 19, almost all the $\chi^2$ is due to the anomalous contribution 14.3 of the first three low-energy $\nu_e$ bins, whereas the other 16 $\nu_e$ and $\nu_\mu$ energy bins are overfitted, with the excessively small $\chi^2$ contribution of 5.4. This overfitting, which is probably due to an overestimate of the uncertainties, remains in the fit of the data with our hypothesis of $f_\nu > 1$ and $\nu_e$ disappearance. On the other hand, our hypothesis clearly explains the low-energy anomaly reducing the $\chi^2$ contribution of the first three low-energy $\nu_e$ bins to the acceptable best-fit value of 2.0.

Figure 2 shows the allowed regions in the $\sin^2 2\vartheta - \Delta m^2$ plane and the marginal $\Delta \chi^2$’s for $\sin^2 2\vartheta$ and $\Delta m^2$, from which one can infer the corresponding uncorrelated allowed intervals. One can see that the indication in favor of neutrino oscillations is not strong, being at the level of about 79% C.L. (1.2$\sigma$). It is interesting to notice that the best-fit value of $\Delta m^2$ is about 2 eV$^2$, which is approximately the same best-fit value obtained in Ref. [14] from the fit of the neutrino data of Gallium radioactive source experiments and the antineutrino data of the Bugey and Chooz reactor experiments under the hypothesis of $\nu_e$ and $\bar{\nu}_e$ disappearance. The results of the combined analysis of MiniBooNE neutrino data and the data of these other experiments is discussed in the following Sections.

III. GALLIUM RADIOACTIVE SOURCE EXPERIMENTS

The GALLEX [11, 33, 34] and SAGE [11, 35–37] collaborations tested the respective Gallium solar neutrino detectors in so-called ”Gallium radioactive source experiments” which consist in the detection of electron neutrinos produced by intense artificial $^{51}$Cr and $^{37}$Ar radioactive sources placed inside the detectors. Taking into account the uncertainty of the cross section of the detection process $\nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^-$ estimated in Ref. [38], the ratio $R$ of measured and predicted $^{73}\text{Ge}$ event rates are

$$
R_{\text{Cr1}} = 0.99_{-0.12}^{+0.11},
$$

$$
R_{\text{Cr2}} = 0.81_{-0.11}^{+0.10},
$$

$$
R_{\text{SAGE}} = 0.99_{-0.12}^{+0.12},
$$

$$
R_{\text{Ar}} = 0.79_{-0.10}^{+0.09},
$$

and the average ratio is

$$
R_{\text{Ga}} = 0.80_{-0.05}^{+0.05}.
$$

Thus, the number of measured events is about 2.7$\sigma$ smaller than the prediction.

The theoretical prediction of the rate is based on the calculation of the detection cross section presented in Ref. [38]. It is possible that a part of the observed deficit is due to an overestimation of this cross section [10, 37, 39], because only the cross section of the transition from the ground state of $^{71}$Ga to the ground state of $^{71}$Ge is known with precision from the measured rate of electron capture decay of $^{71}$Ge to $^{71}$Ga. Electron neutrinos produced by $^{51}$Cr and $^{37}$Ar radioactive sources can be absorbed also through transitions from the ground state of $^{71}$Ga to two excited states of $^{71}$Ge, with cross sections which are inferred using a nuclear model from $p + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + n$ measurements [40]. This calculation has large uncertainties [41, 42]. However, since the contribution of the transitions to the two excited states
null hypothesis (null Hyp.). The following six lines correspond to the case of \( f_\nu = 1 \) and no oscillations (null Hyp.). The following six lines correspond to the case of \( f_\nu > 1 \) and \( \nu_e \) disappearance (our Hyp.). The last three lines give the parameter goodness-of-fit (PG) \([32]\). In the MB\( \nu \) column, the value of \( \chi^2 \) in the null Hyp., the number of degrees of freedom in the null Hyp., (which is equal to the number of energy bins), and the value of \( \chi^2_{\text{min}} \) in our Hyp. are shown as the sum of the contributions of the first three low-energy \( \nu_e \) bins and the other \( \nu_e \) and \( \nu_\mu \) energy bins. In the MB\( \nu \)+Ga and \((\text{MB}\nu\text{+Ga})+\text{(Re}^3\text{H})\) columns the value of \( \chi^2_{\text{min}} \) in our Hyp. is shown as the sum of the contribution of the first three MiniBooNE low-energy \( \nu_e \) bins and the other contributions.

| Hyp.          | \( \chi^2 \) | NDF  | GoF   | \( \chi^2_{\text{min}} \) | NDF  | GoF   | \( \chi^2_{\text{min}} \) | NDF  | GoF   | \( \chi^2_{\text{min}} \) | NDF  | GoF   | \( \chi^2_{\text{min}} \) | NDF  | GoF   |
|---------------|---------------|------|-------|---------------------------|------|-------|---------------------------|------|-------|---------------------------|------|-------|---------------------------|------|-------|
| Null Hyp.     | 14.3          | 5.4  | 9.4   | 51.5                      |      | 0.41  | 0.051                      |      | 0.71  |                            |      |       |                            |      |       |
| NDF           | 3             | 16   | 4     | 58                        |      |       |                            |      |       |                            |      |       |
| GoF           | 0.41          | 0.051| 0.71  |                           |      |       |                            |      |       |                            |      |       |
| Our Hyp.      | 2.0           | 7.6  | 1.8   | 22.2                      | 9.2  | 49.1  | 4.1                        | 63.4 |       |                            |      |       |
| NDF           | 16            | 2    | 20    | 56                        |      | 78    |                           |      |       |                            |      |       |
| GoF           | 0.89          | 0.40 | 0.93  | 0.73                      | 0.08 |       |                            |      |       |                            |      |       |
| \( \sin^2 2\theta \) | 0.32          | 0.27 | 0.28  | 0.042                     | 0.062|       |                            |      |       |                            |      |       |
| \( \Delta m^2_{\text{at}} \) | 1.84          | 2.09 | 1.92  | 1.85                      | 1.85 |       |                            |      |       |                            |      |       |
| \( f_\nu \)   | 1.26          |      | 1.25  | 1.17                      |      |       |                            |      |       |                            |      |       |
| PG \( \Delta \chi^2 \) | 0.098         | 0.01 | 6.97  |                           |      |       |                            |      |       |                            |      |       |
| NDF           | 2             | 2    | 2     |                            |      |       |                            |      |       |                            |      |       |
| GoF           | 0.95          | 0.99 | 0.03  |                            |      |       |                            |      |       |                            |      |       |

**TABLE I.** Values of \( \chi^2 \), number of degrees of freedom (NDF) and goodness-of-fit (GoF) for the fit of different combinations of MiniBooNE (MB\( \nu \)), Gallium (Ga), and reactor (Re) data. The first three lines correspond to the case of \( f_\nu = 1 \) and no oscillations (null Hyp.). The following six lines correspond to the case \( f_\nu > 1 \) and \( \nu_e \) disappearance (our Hyp.). The last three lines give the parameter goodness-of-fit (PG) \([32]\). In the MB\( \nu \) column, the value of \( \chi^2 \) in the null Hyp., the number of degrees of freedom in the null Hyp., (which is equal to the number of energy bins), and the value of \( \chi^2_{\text{min}} \) in our Hyp. are shown as the sum of the contributions of the first three low-energy \( \nu_e \) bins and the other \( \nu_e \) and \( \nu_\mu \) energy bins. In the MB\( \nu \)+Ga and \((\text{MB}\nu\text{+Ga})+\text{(Re}^3\text{H})\) columns the value of \( \chi^2_{\text{min}} \) in our Hyp. is shown as the sum of the contribution of the first three MiniBooNE low-energy \( \nu_e \) bins and the other contributions.

is only 5% \([38]\), even the complete absence of such transitions would reduce \( R^{\text{Ga}} \) to about \( 0.90 \pm 0.05 \), leaving an anomaly of about 1.8\( \sigma \).

Here we consider the electron neutrino disappearance explanation of the Gallium radioactive source experiments anomaly \([13–15, 43–45]\), (another interesting explanation through quantum decoherence in neutrino oscillations has been proposed in Ref. \([21]\)).

In Ref. \([14]\) we have analyzed the data of the Gallium radioactive source experiments in terms of the effective survival probability in Eq. \([3]\). Here we update that analysis taking into account the revised value of \( R^{\text{GALLEX}} \) in Eq. \([14]\) published recently in Ref. \([11]\) and taking into account the asymmetric uncertainties of \( R^{\text{GALLEX}} \), \( R^{\text{CRI37}} \) and \( R^{\text{SAGE}} \) (which have been symmetrized for simplicity in the analysis presented in Ref. \([14]\)). Following the method described in Ref. \([14]\), we obtained the best-fit values of \( \sin^2 2\theta \) and \( \Delta m^2 \) in the second column of Tab. \[1\] and the allowed regions in the \( \sin^2 2\theta \)–\( \Delta m^2 \) plane shown in Fig. \[3\]. The indication in favor of neutrino oscillations is at the level of about 98% C.L. (2.3\( \sigma \)).

From Tab. \[2\] and the comparison of Figs. \[2\] and \[3\] one can see that the fits of MiniBooNE and Gallium data lead to remarkably similar results: the best-fit values of the oscillation parameters are very close and the allowed regions in the \( \sin^2 2\theta \)–\( \Delta m^2 \) plane are highly compatible. This is certainly an impressive success of our hypothesis of electron neutrino disappearance.

The results of the combined fit of MiniBooNE and Gallium data are shown in the third column of Tab. \[1\] and in Fig. \[4\]. The separate data sets are well fitted by the electron neutrino disappearance hypothesis: the \( \chi^2 \) contribution of the first three MiniBooNE low-energy \( \nu_e \) bins is 2.2, that of the other 16 MiniBooNE \( \nu_e \) and \( \nu_\mu \) energy bins is 7.4, and that of the 4 Gallium data is 1.9. The consistency of the combined fit is also supported by the excellent value of the parameter goodness-of-fit. Combining the two data sets improves the indication in favor of neutrino oscillations to the level of about 99.5% C.L.
tium β-electron energy spectrum in the process and mixing of neutrinos through the measurement of the neutrino mass (see Refs. [2, 8]).

The most accurate measurements of the effective electron neutrino disappearance that have been performed in the Mainz [46] and Troitsk [47]:

\[ m_\beta = -0.6 \pm 2.2 \pm 2.1 \text{eV}^2 \quad \text{(Mainz)}, \]

\[ m_\beta = -2.3 \pm 2.5 \pm 2.0 \text{eV}^2 \quad \text{(Troitsk)}. \]

These measurements can be interpreted and combined in order to derive upper bounds for the effective mass \( m_\beta \) through a \( \Delta \chi^2 \) analysis in the physical region \( m_\beta \geq 0 \). In Fig. 5 we plotted the corresponding \( \Delta \chi^2 \)'s as a function of \( m_\beta \). One can see that

\[ m_\beta \leq 2.3 \text{eV} \quad \text{(Mainz, 95\% C.L.)}, \]

\[ m_\beta \leq 2.0 \text{eV} \quad \text{(Troitsk, 95\% C.L.)}, \]

in approximate agreement with the corresponding values in Refs. [46, 47]. The combined upper bound is

\[ m_\beta \leq 1.8 \text{eV} \quad \text{(Mainz+Troitsk, 95\% C.L.)}. \]

In 3+1 four-neutrino schemes, taking into account that the mass splittings among \( m_1, m_2, m_3 \) in Eqs. (11) and (12) are negligible for a measurement of \( m_\beta \) at the scale of 0.1 - 1 eV, the effective mass is given by

\[ m_\beta^2 \simeq \left(1 - |U_{e4}|^2 \right) m_1^2 + |U_{e4}|^2 m_2^2 = m_1^2 + |U_{e4}|^2 \Delta m_{41}^2. \]

In 3+1 schemes of the type in Eq. (13), we have

\[ m_\beta \geq |U_{e4}| \sqrt{\Delta m^2}. \]
The effective mixing angle in short-baseline electron neutrino disappearance experiments is related to $|U_{e4}|$ by Eq. (28). Inverting this relation and taking into account that the value of $|U_{e4}|$ must be small in order to fit the data of solar neutrino experiments with neutrino oscillations, we have

$$|U_{e4}|^2 = \frac{1}{2} \left( 1 - \sqrt{1 - \sin^2 2\theta} \right).$$

In Fig. 6 we show the limits in the $\sin^2 2\theta - \Delta m^2$ plane obtained from Eqs. (27) and (28) and the results in Eqs. (21) and (22) of the Mainz and Troitsk Tritium $\beta$-decay experiments. Notice that for small values of $\sin^2 2\theta$ the bounds are practically linear in the log-log plot in Fig. 6 because in this case $|U_{e4}|^2 \approx \sin^2 2\theta/4$ and the inequality in Eq. (27) leads to

$$\log \Delta m^2 \lesssim 2 \log 2 + 2 \log m_{\beta}^{ub} - \log \sin^2 2\theta,$$

where $m_{\beta}^{ub}$ is the upper bound for $m_\beta$.

In the fourth column of Tab. I and in Fig. 6 we report the results of the analysis of the data of the Bugey and Chooz reactor experiments presented in Ref. [14], with the addition in the analysis of the results of the Mainz and Troitsk Tritium $\beta$-decay experiments, which affect the high-$\Delta m^2$ region. As already commented in Ref. [14], the reactor data are compatible with both the Null Hypothesis of absence of electron antineutrino disappearance and Our Hypothesis of electron antineutrino disappearance, with a hint in favor of electron antineutrino oscillations due to a $\Delta m^2$ of about 2 eV.

V. COMBINED ANALYSIS

The results of the combined analysis of MiniBooNE, Gallium, reactor and Tritium data are presented in the last column of Tab. I and in Fig. 7. One can see that the goodness-of-fit is high. The separate data sets are fitted fairly well by the electron neutrino disappearance hypothesis: the $\chi^2$ contribution of the first three MiniBooNE low-energy $\nu_e$ bins is 4.1, that of the other 16 MiniBooNE $\nu_e$ and $\nu_\mu$ energy bins is 7.5, that of the 4 Gallium data is 6.3, that of the 56 reactor degrees of freedom is 49.0 and that of the 2 Tritium degrees of freedom is 0.57. On the other hand, the 3% parameter goodness-of-fit of the combined analysis of neutrino MiniBooNE and Gallium data and antineutrino reactor and Tritium data is rather low.

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2 As erratum, let us notice that in the fourth column of Table III in Ref. [14] there is a small mistake in the evaluation of the parameter goodness-of-fit. The correct values are $\Delta \chi^2_{\min} = 0.52$ and GoF = 0.47. We also notice that in the version of Ref. [14] published in Phys. Rev. D the value of $\sin^2 2\beta_{\text{Ga}}$ for the Ga+Bu+Ch analysis (last column of Table III in Ref. [14]) is different from the correct one, which is 0.054 (see the arXiv version of Ref. [14]).

3 We thank the anonymous referee of Phys. Rev. D for suggesting this interesting figure.
between the neutrino and antineutrino data sets is entirely due to the different requirements on the value of \( \sin^2 2\theta \), whereas they nicely agree on a best-fit value of \( \Delta m^2 \) at about 2 eV^2.

Since the 3\% parameter goodness-of-fit of the combined analysis shows a tension between the neutrino and antineutrino data (under our \( \nu_e \)-disappearance hypothesis) but is not sufficiently small to reject with confidence the compatibility of the neutrino and antineutrino data set, in the following part of this Section and in Section VII we consider the results and implications of the combined analysis. In Section VII we consider a possible difference between the effective mixing angles in the neutrino and antineutrino sectors.

Although the combined analysis of neutrino and antineutrino data favors smaller values of \( \sin^2 2\theta \) than those obtained from the analysis of MiniBooNE and Gallium data alone, the fit of the MiniBooNE and Gallium data remains better than in the case of no oscillations and \( f_\nu = 1 \).

Figure 8 shows the fit of MiniBooNE \( \nu_e \) data corresponding to the best-fit result of the combined analysis. One can see that the fit of the first three low-energy bins of the Gallium source experiments, the best-fit value of \( \Delta m^2 \) is not as good as that in Fig. 1b, but it is nevertheless acceptable and much better than that in Fig. 1a.

For the Gallium source experiments, the best-fit values of the oscillation parameters give \( R^{\text{GALLEX}} = 0.97 \), \( R^{\text{SAGE}} = 0.96 \) and \( R^{\text{SAGE}} = 0.96 \). Therefore, the experimental values of \( R^{\text{GALLEX}} \) and \( R^{\text{SAGE}} \) in Eqs. (14) and (16) are fitted very well and the Gallium \( \chi^2 \) contribution of 6.3 is almost equally due to the loose fits of \( R^{\text{Cr2}} \) and \( R^{\text{Ar}} \) in Eqs. (15) and (17).

Considering the combined fit of the results of MiniBooNE, Gallium, reactor and Tritium data as a fair indication in favor of a possible short-baseline electron neutrino disappearance generated by the effective mixing parameters \( \Delta m^2 \simeq 2 \text{ eV} \) and \( 0.01 \leq \sin^2 2\theta \leq 0.13 \), in the next Section we present the corresponding predictions for the effective neutrino masses in \( \beta \)-decay and neutrinoless double-\( \beta \)-decay experiments which could be measured in future experiments.
VI. PREDICTIONS FOR BETA-DECAY AND NEUTRINOLESS DOUBLE-BETA-DECAY EXPERIMENTS

In this Section we present predictions for the effective neutrino masses in \(\beta\)-decay and neutrinoless double-\(\beta\)-decay experiments obtained as a consequence of the combined fit of MiniBooNE, Gallium, reactor and Tritium data discussed in the previous Section.

Figure 10 shows the residual \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) as a function of the contribution \(|U_{e4}|\sqrt{\Delta m^2}\) to the effective mass \(m_\beta\) in \(\beta\)-decay experiments (see Eq. (27)). Since from the last column of Tab. I we have \(\sin^2 2\theta_{\text{bf}} \ll 1\), we obtain

\[
|U_{e4}|^2 \simeq \frac{\sin^2 2\theta_{\text{bf}}}{4} = 0.016, \quad (30)
\]

and the best-fit value of \(|U_{e4}|\sqrt{\Delta m^2}\) is

\[
(U_{e4}|\sqrt{\Delta m^2})_{\text{bf}} = 0.17 \text{ eV}, \quad (31)
\]

and

\[
0.06 \leq |U_{e4}|\sqrt{\Delta m^2} \leq 0.49 \text{ eV at } 2\sigma. \quad (32)
\]

This prediction is relevant for the KATRIN experiment [19], which is under construction and scheduled to start in 2012. The expected sensitivity of about 0.2 eV at 90% C.L. may be sufficient to observe a positive effect if \(|U_{e4}|\sqrt{\Delta m^2}\) is sufficiently large, as allowed by \(\Delta \chi^2\) in Fig. 10.

If massive neutrinos are Majorana particles, neutrinoless double-\(\beta\) decay is possible, with a decay rate proportional to the effective Majorana mass (see Refs. [2, 8, 50–52])

\[
m_{2\beta} = \sum_k U^2_{ek} m_k. \quad (33)
\]

The results of the combined fit of MiniBooNE, Gallium, reactor and Tritium data discussed in Section IV allow us to estimate the contribution of the heaviest massive neutrino \(\nu_4\) to \(m_{2\beta}\), which is approximately given by \(|U_{e4}|^2\sqrt{\Delta m^2}\), taking into account the mass hierarchy in Eq. (3).

Figure 11 shows \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) as a function of the contribution \(|U_{e4}|^2\sqrt{\Delta m^2}\) to \(m_{2\beta}\) in four-neutrino schemes. The best-fit value is:

\[
(U_{e4}|^2\sqrt{\Delta m^2})_{\text{bf}} = 0.02 \text{ eV}, \quad (34)
\]

and

\[
0.003 \leq |U_{e4}|^2\sqrt{\Delta m^2} \leq 0.07 \text{ eV at } 2\sigma. \quad (35)
\]

This range must be confronted with the expected contributions to \(m_{2\beta}\) coming from the three light massive...
neutrinos $\nu_1$, $\nu_2$, $\nu_3$. Assuming a hierarchy of masses,
\[ m_1 \ll m_2 \ll m_3 \ll m_4, \tag{36} \]
which is the most natural case compatible with the hierarchy in Eq. (5), we have
\[ m_{2\beta} \simeq |U_{e2}|^2 \sqrt{\Delta m_{\text{SOL}}^2} + |U_{e3}|^2 \sqrt{\Delta m_{\text{ATM}}^2} + |U_{e4}|^2 \sqrt{\Delta m_{\nu}^2}, \tag{37} \]
where we have neglected the contribution of the lightest massive neutrino $\nu_1$. From the 3$\sigma$ upper limits of the three-neutrino mixing parameters given in Ref. [5], we obtain
\[ 2 \times 10^{-3} \lesssim |U_{e2}|^2 \sqrt{\Delta m_{\text{SOL}}^2} \lesssim 4 \times 10^{-3} \text{eV}, \tag{38} \]
\[ |U_{e3}|^2 \sqrt{\Delta m_{\text{ATM}}^2} \lesssim 3 \times 10^{-3} \text{eV}. \tag{39} \]

Therefore, strong cancellations between the contributions of $\nu_2$ and $\nu_3$ are possible (albeit not likely [53]), whereas the range in Eq. (36) disfavors strong cancellations between the contributions of $\nu_2$ and $\nu_3$ and the contribution of $\nu_4$. In this case, $m_{2\beta} \simeq |U_{e4}|^2 \sqrt{\Delta m_{\nu}^2}$ leading to a possible observation of neutrinoless double-$\beta$ decay in future experiments which will be sensitive to values of $m_{2\beta}$ smaller than $10^{-1}$ eV (e.g. CUORE [44], EXO [55], SuperNEMO [56]; see the review in Ref. [52]).

On the other hand, if neutrinoless double-$\beta$ decay experiments which are sensitive to values of $m_{2\beta}$ of the order of $10^{-1}$ eV (e.g. CUORICINO [57], GERDA [58], Majorana [59]; see the review in Ref. [52]) will see a positive signal, maybe compatible with the signal asserted in Ref. [60], the mass hierarchy in Eq. (36) will become unlikely and the favorite 3+1 four-neutrino schemes will be those in which the three light neutrinos $\nu_1$, $\nu_2$ and $\nu_3$ are almost degenerate at the mass scale of $m_{2\beta}$.

VII. MIXING ANGLE ASYMMETRY?

The tension between neutrino and antineutrino data discussed in Section V could be due to a difference of the effective mixing angles in the neutrino and antineutrino sectors. Such a difference could be due to a violation of the fundamental CPT symmetry or to another unknown mechanism. Phenomenological analyses of different masses and mixings for neutrinos and antineutrinos have been presented in several publications [61–74].

In this section we consider the possibility that neutrinos and antineutrinos have different effective masses and mixings in short-baseline $\nu_e$ and $\bar{\nu}_e$ disappearance experiments. We fit the neutrino and antineutrino data with the survival probabilities
\[ P_{\nu_e \rightarrow \nu_e}(L, E) = 1 - \sin^2 2\theta_\nu \sin^2 \left( \frac{\Delta m_{\nu}^2 L}{4E} \right), \tag{40} \]
\[ P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(L, E) = 1 - \sin^2 2\theta_\bar{\nu} \sin^2 \left( \frac{\Delta m_{\nu}^2 L}{4E} \right), \tag{41} \]

...
the best fit is

\[ (|U_{e4}|\sqrt{\Delta m^2})_{\nu_e}^{bf} = 0.14 \text{ eV}, \] (45)

and

\[ (|U_{e4}|\sqrt{\Delta m^2})_{\bar{\nu}_e} \leq 1.29 \text{ eV \ at \ } 2\sigma. \] (46)

For the contribution \((|U_{e4}|\sqrt{\Delta m^2})_{\nu_e}\) to the effective electron neutrino mass in \(\beta^+\) decays we must consider the dashed line in Fig. 10, which has been obtained from the analysis of MiniBooNE and Gallium neutrino data: the best fit is

\[ (|U_{e4}|\sqrt{\Delta m^2})_{\nu_e}^{bf} = 0.38 \text{ eV}, \] (47)

and

\[ (|U_{e4}|\sqrt{\Delta m^2})_{\bar{\nu}_e} \geq 0.21 \text{ eV \ at \ } 2\sigma. \] (48)

Unfortunately the existing and foreseen experiments are \(\beta^-\) -decay experiments for which the contribution \((|U_{e4}|\sqrt{\Delta m^2})_{\nu_e}\) to the effective electron antineutrino mass is expected to be small. The future \(\beta^-\) -decay experiment will use either Tritium (KATRIN \[49\]) or \(^{187}\)Re (MARE \[75\]). If the mixing difference between the neutrino and antineutrino sectors will be confirmed with highest confidence by future neutrino oscillation data it will be interesting to study the possibility of making \(\beta^+\) -decay experiments for the search of the effective electron antineutrino mass, for which the dashed line in Fig. 10 and Eq. (48) give a reachable lower limit.

We do not consider here neutrinoless double-\(\beta\) decay in the case of a neutrino-antineutrino mixing difference, since the Majorana nature of neutrinos requires a treatment which goes well beyond the purposes of this paper (see Ref. \[76\]).

VIII. CONCLUSIONS

In this paper we have discussed a neutrino oscillation interpretation of the MiniBooNE low-energy anomaly and the Gallium radioactive source experiments anomaly in the framework of 3+1 four-neutrino mixing schemes. We have shown that the combined fit of MiniBooNE and Gallium data indicate a possible short-baseline electron neutrino disappearance generated by effective oscillation parameters \(\Delta m^2 \geq 0.1 \text{ eV}^2\) and \(0.11 \leq \sin^2 2\theta \leq 0.48\) at 2\(\sigma\), with best fit at \(\Delta m^2 \simeq 2 \text{ eV}^2\) and \(\sin^2 2\theta \simeq 0.3\) (see Fig. 3).

We have also considered the data of the Bugey and Chooz reactor neutrino oscillation experiments and the results of the Mainz and Troitsk Tritium \(\beta\)-decay experiments, which imply an upper bound on the effective electron neutrino mass of about 2 eV (see Fig. 5 and the combined upper bound in Eq. (25)). As already discussed in Ref. \[14\], the Bugey data give a faint indication of a possible short-baseline electron neutrino disappearance generated by effective oscillation parameters \(\Delta m^2 \simeq 2 \text{ eV}^2\) and \(\sin^2 2\theta \simeq 0.04\), which is compatible with Chooz and Tritium data (see Fig. 6).

In Section VII we have discussed the tension between the neutrino MiniBooNE and Gallium data and the antineutrino reactor and Tritium data. Considering such tension as a statistical fluctuation, we have presented the results of the combined analysis of MiniBooNE, Gallium, reactor and Tritium data: \(\Delta m^2 \simeq 2 \text{ eV}^2\) and \(0.01 \leq \sin^2 2\theta \leq 0.13\) at 2\(\sigma\), with best fit at \(\Delta m^2 \simeq 2 \text{ eV}^2\) and \(\sin^2 2\theta \simeq 0.06\) (see Fig. 4).

In Section VIII we have presented predictions for the effective neutrino masses in \(\beta\)-decay and neutrinoless double-\(\beta\)-decay experiments obtained as a consequence of the combined analysis of MiniBooNE, Gallium, reactor and Tritium data, assuming the hierarchy of masses in Eq. (5). The predicted interval for the contribution of \(m_4\) to the effective neutrino mass in \(\beta\)-decay is between about 0.06 and 0.49 eV at 2\(\sigma\). The upper part of this interval may be reached by the KATRIN experiment \[49\]. For neutrinoless double-\(\beta\)-decay we obtained a prediction for the contribution of \(m_4\) to the effective neutrino mass between about 0.003 and 0.07 eV at 2\(\sigma\), which may be reached in future experiments (see Ref. \[52\]).

We also considered, in Section VII the possibility of reconciling the tension between the neutrino MiniBooNE and Gallium data and the antineutrino reactor and Tri-
tium data discussed in Section IV with different mixings in the neutrino and antineutrino sectors. We found a 2.6σ indication of a mixing angle asymmetry (99.14% C.L.). We pointed out the possibility of checking the mixing difference between the neutrino and antineutrino sectors by measuring different effective electron antineutrino and neutrino masses in β− and β+ decay experiments.

The indication in favor of short-baseline disappearance of electron neutrinos imply the possible existence of a light sterile neutrino which could have important consequences in physics and cosmology. As far as the effective number of neutrino species in cosmology, $N_{\text{eff}}$, is concerned, the analysis of 7-years WMAP data has provided the following result: $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$ (68% C.L.) [99]. In 2011 the Planck experiment will measure $N_{\text{eff}}$ with a factor of 4 improvement in accuracy with respect to present data [100, 101]. In other words, the possibility of existence of a fourth light sterile neutrino could be pursued with 5σ significance.

Finally, we would like to encourage all experiments which can investigate the hypothesis of short-baseline electron neutrino disappearance.

Starting from 2010, at the same $L/E$ of MiniBoone, the magnetic off-axis near detector at 280 m of the T2K experiment [102] will count $\nu_e$ events with expected higher statistics and similar $\nu_\mu$ background contamination. A test of short-baseline oscillations may be done, although the accuracy suffers from the scarce knowledge of the neutrino flux and of the neutrino cross section at 1 GeV energies [12, 103].

A better measurement will be possible with the new CERN-PS neutrino beam [104, 105], thanks to the presence of 2 detectors at 140 m (NEAR) and 885 m (FAR). At $\Delta m^2 \approx 2 eV^2$, the oscillation length is about 1 km for 1 GeV neutrino energies. Therefore, one can reduce the systematic error of the Monte Carlo predictions by normalizing the high energy part of the $\nu_e$ spectrum at the NEAR location. In addition, a better $\nu_\mu$ background rejection will be possible using the liquid Argon technology. The interesting possibility of a $\nu_e$ tagging in the CERN-PS beam was also studied in this context [106].

New measurements with a radioactive source could be made in the SAGE experiment [10], with the Borexino detector and with the future LENS detector [79]. At $\Delta m^2 \approx 2 eV^2$, the oscillation length is about 1 m for 1 MeV neutrino energies. Therefore Borexino could measure the oscillation pattern over a distance of 4 m (the Borexino radius) using the well known $\nu_e$-$\mu$ scattering process and with a vertex resolution that at the moment is about 15 cm [107].

At the Gran Sasso laboratories a very interesting measurement could be realized by using the ICARUS 600 ton detector and new low-cost and high-power proton cyclotrons under development for commercial uses [108]. These provide electron neutrino beams with energy up to 52 MeV from muon decay-at-rest. A low-energy $\nu_e$ disappearance experiment (as well as $\nu_\mu$ disappearance and $\nu_\mu \rightarrow \nu_e$ measurements) can be performed with such devices due to the full efficiency of the ICARUS detector at 20 MeV energies. The expected event rate is about 400 charged-current electron neutrino events per year per ton with the ICARUS detector located at 50 m from the source [109]. The number of events is calculated assuming $10^{15}$ $\nu_e$’s per year and a fully efficient detector. Since at $\Delta m^2 \approx 2 eV^2$ the oscillation length is about 20 m for a 20 MeV neutrino energy, it is possible to measure the full oscillation pattern along the beam direction inside the ICARUS volume.

The disappearance of electron neutrinos can be investigated with high accuracy in future near-detector beta-beam [110] and neutrino factory [73, 111] experiments in which the neutrino fluxes will be known with high precision.

Furthermore, the MiniBooNE low-energy anomaly may be clarified by the ArgoNeuT, MicroBooNE [112] and BooNE experiments [113], and the magnetic off-axis near detector at 280 m of the T2K experiment has the unique opportunity to measure the charge of the events of the low-energy anomaly [114].

NOTE ADDED

After the completion of this work, two important experimental results have been presented at the Neutrino 2010 conference:

1. The MiniBooNE collaboration presented updated results on the search for short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations which are compatible with the LSND signal [115, 116]. The inclusion of these data in our framework will require a separate analysis in which the assumption of negligible $|U_{\mu 3}|^2$ is relaxed [117].

2. The MINOS collaboration presented an indication of a possible difference between the effective mixings of neutrinos and antineutrinos in long-baseline $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance [118]. This indication is analogous to that discussed in Section VII.

ACKNOWLEDGMENTS

We would like to thank E. Bellotti, C. Giganti, A. Longhin, F. Pietropaolo and A. Rubbia for interesting discussions and suggestions.

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