"Pair" Fermi contour and repulsion-induced superconductivity in cuprates

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(Dated: November 9, 2018)

The pairing of charge carriers with large pair momentum is considered in connection with high-temperature superconductivity of cuprate compounds. The possibility of pairing arises due to some essential features of quasi-two-dimensional electronic structure of cuprates: (i) The Fermi contour with strong nesting features; (ii) The presence of extended saddle point near the Fermi level; (iii) The existence of some ordered state (for example, antiferromagnetic) close to the superconducting one as a reason for an appearing of "pair" Fermi contour resulting from carrier redistribution in momentum space. In an extended vicinity of the saddle point, momentum space has hyperbolic (pseudoeuclidean) metrics, therefore, the principal values of two-dimensional reciprocal reduced effective mass tensor have unlike signs. At small momenta of the relative motion of a pair with large pair momentum, the pairing is sensitive just to sign and value of the effective mass but not to only the value of the Fermi velocity as it is in the case of Cooper pairing. Nesting of the Fermi contour results in an increase of the statistical weight of the pair with large total momentum due to an extension of momentum space domain which corresponds to permissible values of the relative motion momentum. Rearrangement of holes in momentum space results in a rise of "pair" Fermi contour which may be defined as zero-energy line for relative motion of the pair. The superconducting gap arises just on this line. Pair Fermi contour formation inside the region of momentum space with hyperbolic metrics results in not only superconducting pairing but in a rise of quasi-stationary state in the relative motion of the pair. Such a state has rather small decay and may be related to the pseudogap regime of underdoped cuprates. Bounded states of the relative motion of the pair is studied both for attraction and repulsion between the components of the pair. It is concluded that the pairing in cuprates may be due to screened Coulomb repulsion. In this case, the superconducting energy gap in hole-doped cuprates exists in the region of hole concentration which is bounded both above and below. The superconducting state with positive condensation energy exists in more narrow range of doping level inside this region. Such hole concentration dependence correlates with typical phase diagram of cuprates. The pairing mechanism and the pair Fermi contour conception make possible a rise of the superconducting condensate and quasi-stationary states of pairs and may provide qualitative interpretation for the key experimental facts relating to cuprates, namely: (1) Observable values of the superconducting transition temperature; (2) The same symmetry and the same energy scale of the superconducting gap and the pseudogap; (3) Relatively small values of the coherence length; (4) Asymmetry of tunnel current-bias characteristic; (5) An "apparent" violation of the Ferrell-Glover-Tinkham low-frequency optical sum rule; (6) Quasi-particle peak and "dip-hump" structure of the angle-resolved photoemission and tunnel spectra; (7) Anomalous large (as compared with the prediction of the theories of the Bardeen-Cooper-Schrieffer type) values of $2\Delta/T_C$ ratio observed in extremely underdoped cuprates with hole doping; (8) The pair Fermi contour conception does not require any additional suggestion relating to a character of carrier or pair scattering such as the so-called "forward scattering" or "hot and cold spots" on the Fermi surface; (9) The pair Fermi contour conception is fairly consistent with a character of doping dependence of pseudogap state crossover temperature and also both superconducting transition temperature and superfluid density observed in HTSC cuprates.

PACS numbers:

I. INTRODUCTION

Recently, we proposed new mechanism of superconducting (SC) pairing in anisotropic quasi-two-dimensional (2D) electron system typical of high-temperature superconducting (HTSC) cuprate compounds. Pairs with large momentum $K$ ($K$-pairs) are considered; here, $K \approx 2k_F$, $k_F$ is the value of the Fermi momentum directed along $K$. It is well known that the Cooper channel of pairing becomes inefficient when the pair momentum exceeds a value of the order of $\Delta/v_F$; here, $\Delta$ is the SC gap at $K = 0$, $v_F$ is the Fermi velocity. The same relatively small value of the pair momentum corresponds to the wavelength of spatially inhomogeneous SC phase arising in weakly ferromagnetic electron system as well. The Cooper channel at $K \neq 0$ is suppressed due to Pauli’s exclusive principle which restricts the phase volume accessible for the electron states contributing to the $K$-pair state. This phase volume decreases rapidly with $K$ and vanishes at $K \sim \Delta/v_F$. Therefore, pairing
with large pair momentum may be possible if some rearrangement in the electron system provides finite (and sufficiently large) phase volume for the states forming K—pair. Any rearrangement of electrons in momentum space which transfers a part of them across the Fermi surface (FS) results in an increase of the energy of the electron subsystem of a crystal. If the electron subsystem interacts with some other one, for example, antiferromagnetic (AF) spin subsystem, and such an interaction results in a gain in total energy, new FS arises corresponding to new ground state of the system. All of undoped HTSC cuprates are AF insulators, therefore, the pairing mechanism \[1, 2, 3, 4, 5, 6, 7\] is not exotic, most likely, this mechanism is typical of doped HTSC cuprate compounds. Electronic structure and physical properties of such layered compounds as HTSC cuprates are studied in detail \[11, 12\]. Conducting \(\text{CuO}_2\) plane (one or more per unit cell) is the key structural unit of any HTSC cuprate compound. The neighboring planes are separated by reservoirs that is atomic layers which, under doping, inject carriers (holes, as usual) into these planes. The interplane coupling is very weak and this is just the reason one may consider HTSC cuprates as 2D electron system. In undoped cuprates, there is long-range AF order below the Neel temperature, \(T_N\). Doping leads to progressive destruction of long-range order and, as a final result, to a rise of strongly anisotropic metallic (M) state. Angle-resolved photoemission spectroscopy (ARPES) measurements \[14, 15, 16, 17\] result in unambiguous conclusion that, in the normal (N) state, any HTSC cuprate has large FS. Observed FS’s are in good agreement with band structure calculations \[19, 20, 21, 22, 23\] based on the density functional theory. In the case of hole-doped compounds, the Fermi contour (FC) that is cross-section of the FS which is parallel to conducting layers is a square with rounded corners. The FC of holes is centered at \((\pi, \pi)\) and exhibits strong nesting feature along \([100]\)-type directions. This is an evidence in behalf of the significant role of the interaction between next-nearest-neighbor atoms as far as taking account of nearest-neighbor interaction only results in the square FC with perfect nesting along \([110]\)-type directions exactly at half-filling \[11\]. It should be noticed that, in the case of electron-doped compounds, such as \(Nd_{2-x}Ce_xCuO_4\), any appreciable nesting of the FC is absent and the FC is closed to a circle. At approximately half-filling, long parts of the FC are situated close to the saddle points of hole dispersion \[11\]. Hole doping moves the Fermi level towards the saddle point whereas electron doping acts in reverse direction. Therefore, in hole-doped compound, nesting feature of hole FC appears in relatively wide concentration range. Weak dispersion along the nesting directions results in the fact that longitudinal (along the nested straight-line parts of the FC) component of the Fermi velocity sufficiently smaller than the transversal one \[24\]. This corresponds to an effective enhancement of 2D density of states in the vicinity of logarithmic van Hove singularity due to the saddle point \[29\]. Thus, there is an extended vicinity of the saddle point in which the principal values of 2D tensor of reversed reduced effective mass have unlike signs. One can say that, in such a vicinity, momentum space has hyperbolic (pseudoeuclidean) metrics. Due to nesting feature of the FC, the absolute values of the principal effective masses differ strongly from each other: positive longitudinal mass is essentially more than the absolute value of negative transversal mass. It should be pointed out once more that, in a case of any hole-doped cuprate compound, long straight-line parts of the FC are situated, in main, just in such “flat-band” or “extended van Hove singularity” vicinity \[11\].

In HTSC cuprates, the SC state appears in some doping interval, \(x_s < x < x^*\), bounded both above and below. Both superconducting transition temperature \(T_C\) anduperfluid density (or phase stiffness) \(\rho_s\) may demonstrate highly complicated dependence on doping in this interval \[12, 21\]. The absolute maximum of \(T_C\) corresponds to the optimal doping, \(x_{\text{opt}}\). Phase diagram typical of hole-doped HTSC cuprates is presented in Fig. 1.

In underdoped \(x < x_{\text{opt}}\) compounds, one-particle density of states is suppressed essentially at \(T < T^*\). Such a suppression may be interpreted as a rise of the so-called pseudogap in the excitation spectrum \[27\]. The temperature \(T^*\) corresponding to a crossover between the N state at \(T > T^*\) and the “pseudogap regime” at \(T < T^*\) decreases with doping increase and becomes approximately equal to \(T_C\) at \(x \approx x_{\text{opt}}\). The pseudogap \(\Delta^*\), just as the SC gap \(\Delta\), is strongly anisotropic, and also, the character of the anisotropy is the same both for \(\Delta^*\) and \(\Delta\) \[12\]. The maxima of their absolute values correspond to antinodal \([100]\)-type directions. The minimal values (which, possibly, are equal to zero) both of \(\Delta^*\) and \(\Delta\) correspond to nodal \([110]\)-type directions. Pseudogap evolution with temperature decrease from \(T^*\) to \(T_C\) was studied using ARPES technique \[23\] in underdoped single crystals \(Bi_2Sr_2CaCu_2O_{8+\delta}\). The pseudogap arises at \(T = T^*\) at four points of the FC corresponding to antinodal directions. Lowering of temperature from \(T^*\) leads to pseudogap extension in the directions of the corners of the FC. Thus, the FC turns out to be discontinuous and has the form of four arcs (rounded corners of the square FC) which are not connected with each other \[22\]. The arc length decreases gradually with temperature lowering. At \(T = T_C\), the FC disappears and, instead of it, the SC gap arises which has minimal (or equal to zero) values just in the points (corresponding to nodal directions) where the FC shrinks \[23\]. Knight shift measurements indicate that there is singlet pairing of carriers when the electron system of HTSC cuprates is in the SC state \[24, 30, 31\]. Therefore, observed momentum dependence of the SC gap may correspond to either anisotropic s-type or d-type of orbital symmetry \[22\]. The same orbital symmetry and the same energy scale of \(\Delta\) and \(\Delta^*\) enable one to suppose that the SC gap and the pseudogap are of the same origin. Thus, the pseudogap regime may be considered as an incoherent
state of paired charge carriers \[ \text{[14, 35].} \]

In the theory by Bardeen, Cooper and Schrieffer (BCS) which explains the conventional superconductivity successfully \[ \text{[2]}, \] attraction due to virtual phonon exchange is a driven force leading to pairing of carriers. In principle, phonon mechanism of Cooper pairing should not be excluded as a mechanism of HTSC \[ \text{[29]} \] although it is difficult to explain satisfactorily some essential features of HTSC state, for example, the symmetry of the SC gap. In view of the fact that the phase diagram of any HTSC cuprate has a region with long-range AF order, AF fluctuation exchange as a mechanism of pairing \[ \text{[37, 38, 39, 40]} \] seems as quite natural (neutron scattering experiments \[ \text{[11, 42, 43, 44]} \] exhibit broadened Bragg peaks up to the optimal doping). The other point of view is founded on the statement that ground state energy gain at the SC transition in HTSC cuprates is due to a lowering of the kinetic energy arising when two of like-charged carriers form a pair \[ \text{[45, 46, 47, 49, 51]} \]. In such a case, generally speaking, one needs no attraction between carriers and screened Coulomb repulsion remains as a natural essential interaction in the electron system.

AF fluctuations (short-range AF order) may lead to a specific quasi-one-dimensional (1D) self-organization in 2D electron system of HTSC cuprates. Elastic neutron scattering study in \( L a_{1-x}N d_{x}S r_{x}C u O_{4} \) compound enables one to assume that, at \( T < T_{c} \approx 80K \), holes doped into a crystal are situated in 1D antiphase boundaries (charge stripes) separating hole depleted domains with AF order \[ \text{[24, 25]} \]. A rise of such static stripe structure (in a general way, predicted in \[ \text{[54, 55, 56]} \] ) may be described as a transfer of excess holes from AF part of a stripe into antiphase boundary (M part of a stripe). In underdoped \( L a_{2-x}S r_{x}C u O_{4} \) which does not contain Nd atoms, dynamic (fluctuating) stripes were observed \[ \text{[57]} \]. Dynamic stripe magnitude, just as the magnitude of AF fluctuations, decreases with doping and, at \( x > x_{\text{opt}} \), neutron scattering technique does not make possible a resolution of strongly broadened stripe peaks of rather low intensity. A stripe structure may exist being independent of superconductivity but such a structure (just as AF fluctuations) and superconductivity are closely and in a nontrivial way connected with each other. As an indirect confirmation of this statement one may take into consideration the fact that, in \( L a_{1-x}N d_{x}S r_{x}C u O_{4} \) at \( x = 1/8 \) when static stripe magnitude is maximal, there is a local minimum on the doping dependence of the SC transition temperature \[ \text{[38]} \]. On the contrary, it is possible that dynamic stripes stimulate superconductivity \[ \text{[2]} \].

Experimental data available make possible to determine the main features and details of the electronic structure which are essential to understand the character of the SC state of HTSC cuprates and interpret their physical properties qualitatively. Firstly, all doped HTSC compounds have 2D electronic structure with strong nesting of the FC situated in an extended vicinity of the saddle point of the hole dispersion. Secondly, in all doped HTSC compounds, doping regions corresponding to AF and SC phases are close to each other and, in the SC region, there is short-range AF order resulting in stripe self-organization of spin and charge subsystems of the crystal. The theory here developed takes into account these principal features of the electronic structure and can qualitatively explain the key experimental facts relating both to N and SC state of HTSC cuprates.

The paper is organized as follows: The next section is dedicated to the formulation of the conditions under which the pairing with large pair momentum may be possible; in addition, we introduce the concept of the “pair” Fermi contour. In Sec. III we consider the problem of a single pair in momentum space with hyperbolic metrics and discuss the symmetry properties of the pair wave function. Sec. IV contains a discussion of the character of the two poles of the scattering amplitude corresponding to a quasi-stationary state of the pair and superconducting instability. Effective interaction between particles composing a pair with large momentum is considered in Sec. V. In Sec. VI we derive the equation defining the SC order parameter. The approximate solutions of this equation are presented in Sec. VII both in the case of attraction and repulsion between the components of the pair. In Sec. VIII, we discuss SC transition induced chemical potential shift, and then, in Sec. IX, doping dependent SC condensation energy is studied. In Sec. X, we consider some special case of SC state arising due to weak ferromagnetism, associated with stripe structure. Sec. XI is dedicated to a brief discussion of some key experimental results related to both N and SC state of HTSC cuprates; also, we discuss some other possible reasons of “opening” of the PFC and propose a qualitative interpretation of available experimental data in the scope of the theory developed here.

II. ELECTRON AND HOLE PAIRS. PAIR FERMI CONTOUR

Let us consider two electrons or two holes with total momentum \( \mathbf{K} = \mathbf{k}_{+} + \mathbf{k}_{-} \) where \( \mathbf{k}_{+} \) and \( \mathbf{k}_{-} \) are momenta of the particles composing a pair. This is a pair of noninteracting particles. Thus, now and below in this Section, we do not fall outside the limits of usual one-particle approximation and the pair here introduced may be named as a slave pair. Further, taking account of the screened Coulomb interaction between particles composing the pairs, we use such pairs to construct a SC state. Filling of the states inside the FC results in the fact that permissible values of a momentum of the relative motion of the \( K \)-pair, \( \mathbf{k} = (k_{+} - k_{-})/2 \), belong to a certain domain of momentum space. Such a domain, which we denote as \( \Xi_{K} \), has a form dependent on \( \mathbf{K} \) and on a shape of the FC \( \Xi \). Typical of any hole-doped HTSC cuprate FC is a square with rounded corners as it is represented schematically in Fig. 2. If the pair momentum is directed along \( [100] \) and \( K < 2k_{F} \) the corresponding domain has a
form shown in the same Figure. It is clear from Fig. 2 how one can define such a domain at any given $K$. It should be noted that the area (labeled by the same symbol $\Xi_K$) of the domain $\Xi_K$ tends to zero when $K \to 2k_F$. Thus, the statistical weight of the states which compose pairs with $K = 2k_F$ is equal to zero even in the case of perfect nesting [23, 68]. It should be noted that the model used in [23, 68] does not take into account the existence of an extended saddle point and hyperbolic metrics of momentum space, therefore, a pairing with $K = 2k_F$ turns out to be impossible. For arbitrary direction of $K$, there are, generally speaking, eight domains corresponding to the pair momenta which are equivalent to given vector $K$ (for special, antinodal or nodal, direction there are four equivalent vectors). However, at preassigned direction of $K$ from $2k_F$, the statistical weight (the area $\Xi_K$) depends on the $K$ direction decreasing from a maximal value for antinodal to a minimal one for nodal directions. It is quite obvious that the binding energy of $K$–pair has to increase with $\Xi_K$, therefore, one may expect that a rise of the SC condensate should be due to pairs with momenta corresponding to antinodal directions.

There is an experimental evidence in behalf of the consideration of hole pairs with large total momentum. As an example, one may consider the so-called “commensurate” neutron resonance (41 meV) peak below $T_C$ which is usually associated with a rise of resonance collective triplet $\pi$–mode [61] corresponding to the saddle point. Recently, in [12, 13] was observed “incommensurate” magnetic fluctuations in HTSC cuprates. As it is pointed out in Ref. [11], the incommensurate mode transforms continuously into the commensurate one demonstrating, in addition, a negative (downward away from the commensurate momentum) dispersion [14]. Such a tendency of softening of this triplet $\pi$–mode can be interpreted as an indirect evidence of the phase transition possibility associated with softening of a certain singlet mode corresponding to large and “incommensurate” pair momentum.

Almost straight-line parts of the FC belong to the region of momentum space with hyperbolic metrics. Therefore, the energy of the relative motion of a pair inside $\Xi_K$, 

$$\varepsilon_r(K, k) = \varepsilon \left( \frac{K}{2} + k \right) + \varepsilon \left( \frac{K}{2} - k \right) - 2\varepsilon \left( \frac{K}{2} \right), \tag{1}$$

at relatively small $k$ may be approximately represented as

$$\varepsilon_r(K, k) \approx \frac{\hbar^2}{2m}(\nu k_1^2 - k_2^2), \tag{2}$$

where $\varepsilon(k)$ is hole dispersion and, as it follows from a symmetry consideration, the coordinate axes are directed parallel (the $k_1$–axis) and perpendicular (the $k_2$–axis) to the FC (Fig. 2). These coordinate axis directions correspond to the principal directions of 2D tensor of reduced reciprocal effective mass (\nu/m and $-1/m$ are dependent on $K$ principal values of this tensor). Due to strong nesting of the FC, the absolute values of the effective masses differ considerably from each other, namely, a dimensionless parameter $\nu << 1$.

The domain $\Xi_K$ consists of two parts, $\Xi_K(–)$ and $\Xi_K(+)$, in which the energy of the relative motion of $K$–pair is negative and positive, respectively. The domain $\Xi_K′$, also shown in Fig. 2, corresponds to a pair with total momentum $K′$ ($K′$–pair) outside of the FC; thus $K′ > 2k_F$. This domain consists of two parts, $\Xi_K′(–)$ and $\Xi_K′(+)$, corresponding to negative and positive energy of the relative motion of $K′$–pair as well. Excitations composing $K$–pair inside the FC are electrons whereas holes are the excitations which compose $K′$–pair outside of the FC. Both $\Xi_K$ and $\Xi_K′$ belong to the region of momentum space inside 2D Brillouin zone which has hyperbolic metrics.

In contrast to Cooper pairs with $K = 0$, the energy of the relative motion of $K$–pairs at $K \approx 2k_F$ is sensitive not only to the value of the Fermi velocity but to signs and absolute values of the effective masses. In the case of hyperbolic metrics, $K$–pair density of states exhibits a logarithmic van Hove singularity corresponding to zero energy of the relative motion as it is shown schematically in Fig. 3. Weak dispersion along one of the directions in 2D momentum space (the $k_1$–axis in Fig. 2) leads to the fact that $K$–pair density of states has almost 1D character [23].

By definition of the ground state of the electron system, all pair states inside $\Xi_K$ are occupied whereas the states inside $\Xi_K′$ are vacant. Such a filling of the states in momentum space corresponds to spatially homogeneous state of the electron system. At given $K$, states of the relative motion of $K$–pair are characterized by the relative motion density of states, $g_K(\varepsilon)$. Upper edge of pair density of states in the domain $\Xi_K$ corresponds to the Fermi level (Fig. 3). There is a finite energy gap $\delta_{KK′}$ between the upper edge of $g_K(\varepsilon)$ and the lower one relating to pair density of states, $g_{K′}(\varepsilon)$, in the domain $\Xi_K′$ as it is seen from Fig. 3. Therefore, any transfer of a pair from $\Xi_K$ into $\Xi_K′$ is necessarily connected with an energy increase due to an increase of center-of-mass energy. However, it should be noted that the pairs having positive energy of the relative motion leave the domain $\Xi_K′$ whereas the pairs with negative energy arrive at the domain $\Xi_K′$.

Such transfers of pairs from the domain $\Xi_K$ into a region of momentum space outside of the FC may bear a relation to well-known spatially inhomogeneous (stripe) structure in which there is an alternation of hole enriched and depleted 1D regions [12]. The region of momentum space into which $K$–pairs may transfer is either that part, $\Xi_K′$, of the domain $\Xi_K$ which is situated outside of the FC or the domain $\xi_K′$ corresponding to total pair momentum $K′$ (Fig. 2). As far as relative motion density of states which corresponds to $K′$–pair belonging to the subdomain $\xi_K′$, is considerably greater than density of states corresponding to the subdomain $\xi_K$ we
may restrict ourselves to a consideration of the transfers \( \Xi^{(+)}_{K} \rightarrow \Xi^{(-)}_{K'} \) only. Suppose that a number of pairs, \( \delta N \), passes from \( \Xi^{(+)}_{K} \) into \( \Xi^{(-)}_{K'} \), so that in the subdomain \( \Xi^{(+)}_{K} \) (in the zero-temperature limit and in the absence of an interaction between holes), vacant pair states arise in a certain (small in comparison with \( \delta \varepsilon_{K,K'} \)) energy interval near \( 2E_F \). The same number of pairs occupies a small energy interval (which may be determined using pair number conservation condition) near the lower edge of the band corresponding to the subdomain \( \Xi^{(-)}_{K'} \). Thus, the energy increase due to pair transfers from \( \Xi^{(+)}_{K} \) into \( \Xi^{(-)}_{K'} \) may be estimated as \( \delta N \cdot \delta \varepsilon_{K,K'} \).

Transfers of pairs, \( \Xi^{(+)}_{K} \Rightarrow \Xi^{(-)}_{K'} \), in momentum space may be related to transfers of holes from AF parts of stripes into M parts in real space (Fig. 4). An enhancement of AF correlations due to such transfers results in some reducing of the energy which might compensate the energy increase due to the excitation of hole pairs leading to transfers \( \Xi^{(+)}_{K} \Rightarrow \Xi^{(-)}_{K'} \). An energy gain due to a removal of \( K \)--pairs from \( \Xi^{(+)}_{K} \), that is from AF parts of stripes, may be estimated phenomenologically if one introduces depending on doping parameter \( I = I(x) \) which may be treated as nearest-neighbor spin correlation function being a measure of AF short-range order. Let us assume that each hole pair transferring from the subdomain \( \Xi^{(+)}_{K} \) into the subdomain \( \Xi^{(-)}_{K'} \) gives an energy gain equal to \( I \). Then, total decrease in the energy of holes due to such transfers of \( \delta N \) hole pairs may be estimated as \( -\delta N \cdot I \). Thus, a rise of the stripe structure lowers the ground state energy provided that

\[
I > \delta \varepsilon_{K,K'}.
\] (3)

The existence in the domain \( \Xi_{K} = \Xi^{(-)}_{K} + \Xi^{(+)}_{K} \) of hole filled part (for which we use above introduced notation \( \Xi^{(-)}_{K} \)) and vacant part \( \Xi^{(+)}_{K} \) makes possible pairing of carriers in the vicinity of the lines separating filled and vacant subdomains. The energy of the relative motion of \( K \)--pair with respect to the value of the chemical potential is negative inside \( \Xi^{(-)}_{K} \) and positive inside \( \Xi^{(+)}_{K} \), therefore, the lines separating these subdomains (the lines of zero relative-motion energy) play role of a peculiar “pair” Fermi contour (PFC) on which the SC gap may arise. Such a conclusion is related both to the domains \( \Xi_{K} \) and \( \Xi_{K'} \), therefore, PFC is situated both inside and outside of the parent FC. If the value of the vector \( K - K' \) which may be considered as a reciprocal spatial scale of the stripe structure appreciably exceeds a character scale, \( \delta k_c \sim \Delta \), of non-zero SC order parameter in momentum space, one may consider the pairing problems in \( \Xi_{K} \) and \( \Xi_{K'} \) independently from each other. In the following, we consider just the case when \( |K - K'| >> \delta k_c \).

III. PROBLEM OF A SINGLE PAIR

First of all, let us consider a hole \( K \)--pair taking into account two-particle potential interaction between the particles composing the pair. As stated above, the momenta of interacting particles are confined inside the domain \( \Xi_{K} \). We suppose that this domain belongs to a region of momentum space with hyperbolic metrics. A wave function of \( K \)--pair may be written as

\[
\Psi_{K}(r_{+}, r_{-}) = \frac{1}{\sqrt{S}} \varphi_{K}(r) e^{iK \cdot R}.
\] (4)

Here, \( r_{+} \) and \( r_{-} \) are radius vectors of the particles, \( R = (r_{+} + r_{-})/2, r = r_{+} - r_{-} \), \( \varphi_{K}(r) \) is a wave function of the relative motion, \( S \) is a normalizing area.

On account of the crystal symmetry, all wave functions \( \varphi_{gK} \) corresponding to the momenta \( g \cdot K \) turn out to be equivalent; here, \( g \) is a crystal symmetry group transformation. Therefore, the \( K \)--pair wave function taking into account the crystal symmetry should be represented as a linear combination of the form

\[
\Psi_{K}^{(\Gamma)}(r) = \sum_{[gK]} c_{gK}^{(\Gamma)} \varphi_{gK}.
\] (5)

A choice of the coefficients \( c_{gK}^{(\Gamma)} \) is determined by the irreducible representation \( \Gamma \) of the crystal symmetry group according to which the wave function Eq. (5) transforms under the action of crystal symmetry operators \( \hat{g} \). It should be noted especially that the wave function Eq. (5) corresponds to current-less state in view of the fact that \( \sum \hat{g} K = 0 \).

Taking account of the fact that the domains \( \Xi_{gK} \) corresponding to equivalent momenta, \( g K, \) either do not overlap at all or overlap in a small way (Fig. 2) and, also, a scattering of \( K \)--pair from any such a domain into an equivalent one corresponds to rather large change in the total momentum of the pair, one can, in the first approximation, neglect any inter-domain scattering. Then, with regard for Eqs. (1), (2), the equivalent Hamiltonian of the relative motion of \( K \)--pair may be presented in the form

\[
\hat{H}_{K} = -\frac{\hbar^2}{2m} \left( \nu \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) + U_{K}^{*}(r),
\] (6)

where \( r = \sqrt{x_1^2 + x_2^2}, U_{K}^{*}(r) \) is an effective potential energy of the particles composing \( K \)--pair. This energy, which will be defined and discussed in detail in Sec. V, depends on the domain \( \Xi_{K} \) in which scattering due to interaction is permitted. When the area \( \Xi_{K} \) is large enough one can suppose that \( U_{K}^{*}(r) \sim \Xi_{K} \).

Generally speaking, all of the eigen-functions of the operator Eq. (6) belong to continuous spectrum. Therefore, it is quite natural to represent such a function in the form of a sum of an incident wave with the momentum \( q \) and scattered (expanding) wave,

\[
\varphi_{K}(r) \Rightarrow \varphi_{Kq}(r) = e^{iqr} + \chi_{Kq}(r).
\] (7)
Fourier transform of the scattered wave,
\[ \tilde{\chi}_{Kq}(k) = \int \chi_{Kq}(r)e^{-ikr}d^2r, \] (8)
is a solution of the integral equation
\[ [\omega - \omega(k)]\tilde{\chi}_{Kq}(k) = \frac{1}{1 + u_0 B_K(\omega)} = u(k - q) + \int u(k - k')\tilde{\chi}_{Kq}(k')\frac{d^2k'}{(2\pi)^2}. \] (9)
Here, \( \omega(k) = \nu k_x^2 - k_y^2, \) \( h^2\omega/2m \) is an energy of the incident wave, \( u(k) = 2m\tilde{U}_K(k)/h^2, \) \( \tilde{U}_K(k) \) is the Fourier transform of the effective interaction energy introduced in Eq. (6). One has to integrate in Eq. (6) over the domain \( \Xi_K \) which is the domain of definition of momenta \( k \) and \( k' \). As far as this domain is small in comparison with 2D Brillouin zone one can approximately take \( u(k - k') \approx u(0) \equiv u_0 \). This approximation leads directly to the solution of the integral Eq. (9) in the form [3],
\[ \tilde{\chi}_{Kq}(k) = -\frac{u_0}{1 + u_0 B_K(\omega)}(\omega - \omega - i0 \cdot \text{sgn} \omega), \] (10)
where signum function provides a necessary condition in order that Eq. (10) were an expanding wave. The function \( B_K(\omega) \) is defined as
\[ B_K(\omega) = \int_{(\Xi_K)} \frac{1}{\omega(k) - \omega - i0 \cdot \text{sgn} \omega(2\pi)^2} = B_{K1}(\omega) + iB_{K2}(\omega). \] (11)
At real argument, the functions \( B_{K1}(\omega) \) and \( B_{K2}(\omega) \) are written in the form
\[ B_{K1}(\omega) = \int_{(\Xi_K)} \frac{1}{\omega(k) - \omega(2\pi)^2} \delta(\omega-k_0 - \omega)\frac{d^2k}{(2\pi)^2}, \]
\[ B_{K2}(\omega) = \pi \cdot \text{sgn} \omega \int_{(\Xi_K)} \delta(\omega(k) - \omega)\frac{d^2k}{(2\pi)^2}, \] (12)
where the integral defining \( B_{K1}(\omega) \) has meaning of Cauchy principal value.
A denominator of the scattering amplitude,
\[ f_K(\omega) = \frac{1}{1 + u_0 B_K(\omega)}, \] (13)
generally speaking, is not equal to zero at any real value of the argument \( \omega \). The case when the function \( B_{K2}(\omega) \) is equal to zero identically inside some interval of \( \omega \) may be considered as an exception. In such a case, scattering amplitude poles, which are the solutions of the equation,
\[ 1 + u_0 B_{K1}(\omega) = 0, \] (14)
correspond to bounded states.
When some complex value, \( \omega = \omega_K^{(0)} - i\Gamma_K \), is a solution of Eq. (14) and, in addition, \( B_{K2}(\omega_K^{(0)}) \neq 0, \omega_K^{(0)} \) makes sense of the energy of quasi-stationary state (QSS) provided that \( 0 < \Gamma_K << \omega_K^{(0)} \). At \( |\omega - \omega_K^{(0)}| << \omega_K^{(0)} \), the function \( B_{K1}(\omega) \) may be approximately written as
\[ B_{K1}(\omega) \approx B_{K1}(\omega_K^{(0)}) + B'_{K1}(\omega_K^{(0)}) \cdot (\omega - \omega_K^{(0)}). \] (15)
Here, the prime denotes differentiation with respect to \( \omega \). The scattering amplitude is represented in the form
\[ f_K(\omega) \approx \frac{1}{B'_{K1}(\omega_K^{(0)})} \cdot \frac{1}{\omega - \omega_K^{(0)} + i\Gamma_K}, \] (16)
where QSS decay is written as
\[ \Gamma_K \approx B_{K2}(\omega_K^{(0)})/B'_{K1}(\omega_K^{(0)}). \] (17)
In the case of tetragonal crystal (2D symmetry group \( C_{4v} \)), one can separate all of the equivalent vectors \( \hat{g}K \) into two subsets. One of them, which contains the vector \( K \) itself, also contains all of the vectors \( \hat{g}K \) related to each other by reflections with respect to the coordinate axes, \( k_x \) and \( k_y \). The other subset is generated in a similar way by the vector resulting from the reflection of \( K \) with respect to a diagonal of the square Brillouin zone. The coefficients \( c_{ijk}^{(A)} \) corresponding to the trivial irreducible representation \( A_{1g} \) are equal to each other. In the case of the irreducible representation \( B_{1g} \), the coefficients \( c_{ijk}^{(B)} \) have one and the same absolute value and differ in sign for the two subsets of the full set \( \hat{g}K \). Taking into account the explicit form, Eq. (10), of the functions \( \tilde{\chi}_{gKq}(k) \) one may easily conclude that, in the case of an appropriate choice of coordinate axis directions \( \omega(k) = \nu k_x^2 - k_y^2 \), for any \( gK \) belonging to the first subset of the full set of the vectors, \( \hat{g}K \), whereas \( \omega(gK) = \nu k_y^2 - k_x^2 \) when \( \hat{g}K \) belong to the second one. Thus, the wave function Eq. (5) corresponding to the irreducible representation \( A_{1g} \) has the form
\[ \Psi_{Kq}^{(A_{1g})} \sim \frac{2\omega}{(\omega + k_x^2) \cdot (\omega + k_y^2)}, \] (18)
provided that \( \nu << 1 \). Full symmetry of this function with respect to the crystal group enables one to relate it to \( s \)-type orbital symmetry. Under the same condition, \( \nu << 1 \), the wave function corresponding to the irreducible representation \( B_{1g} \) may be written as
\[ \Psi_{Kq}^{(B_{1g})} \sim \frac{k_x^2 - k_y^2}{(\omega + k_x^2) \cdot (\omega + k_y^2)}. \] (19)
This function may be conditionally related to \( d \)-type orbital symmetry.
In the coordinate representation, the wave function of the relative motion of \( K \)-pair corresponding to an expanding wave, at \( \omega > 0 \), has the form
\[ \chi_{Kq}(r) \sim \frac{f_K(\omega)}{4\sqrt{\nu}} \left\{ \begin{array}{ll} H_0^{(2)}(\sqrt{\nu \sqrt{\frac{x^2}{\nu} + \frac{y^2}{\nu}}}) \frac{2i}{\pi} \nabla \cdot \frac{K_0(\sqrt{\nu \sqrt{\frac{y^2}{\nu} - \frac{x^2}{\nu}}})}{K_0(\sqrt{\nu \sqrt{\frac{x^2}{\nu} + \frac{y^2}{\nu}}})} \end{array} \right\}. \] (20)
Here, $H_0^{(2)}(z)$ and $K_0(z)$ are Hankel and modified Bessel functions, respectively. The upper (low) row corresponds to $0 < |y| < |x|/\sqrt{\nu}$ ($|x|/\sqrt{\nu} < |y| < \infty$). Thus, the plane $x,y$ is separated into four sectors. Two sectors, namely, $0 < |y| < |x|/\sqrt{\nu}$, correspond to expanding waves whereas in the another two, $|x|/\sqrt{\nu} < |y| < \infty$, the wave function, Eq. (20), decays with the distance from the center. The straight lines, $y = \pm x/\sqrt{\nu}$, separating these sectors are caustics on which the function, Eq. (20), has a logarithmic singularity. At $\omega < 0$, we have

$$
\chi_{Kq}(r) = -\frac{f_K(\omega)}{4\sqrt{\nu}} \left\{ \frac{H_0^{(2)}(\sqrt{-\omega} y^2 - x^2/\nu)}{2i/\pi K_0(\sqrt{-\omega} y^2 - x^2/\nu)} \right\}. 
$$

(21)

Here, the upper (low) row corresponds to $0 < |x|/\sqrt{\nu} < |y|$ ($|y| < |x|/\sqrt{\nu} < \infty$). A character of coordinate dependence of this function, that is a disposition of the sectors corresponding to expanding and damping waves, is clear from the definition, Eq. (21). It should be noted that Eqs. (20), (21) are related to the case when the energy of the relative motion of $K$-pair is measured from the center–of–mass energy. One can easily see that a character spatial scale for the pairs considered here (which plays role of a coherence length) is determined by the value of the order of $|\omega|^{-1/2}$ and may be estimated as a few interatomic distances $^{[2]}$.

IV. QUASI-STATIONARY STATES

One can calculate the function Eq. (12) which defines the scattering amplitude. For the sake of simplicity, we suppose that the domain $\Xi_K$ being the domain of integration in Eq. (12) is a long and narrow rectangular strip which is roughly similar to real domain $\Xi_K$ in the case of anisotndal direction. We denote a length and a width of the strip as $\Delta k_1$ and $\Delta k_2$, respectively. Coordinate $k_1$-axis is directed along one of the principal directions of 2D reciprocal effective mass tensor which corresponds to positive effective mass, $m_1 = m/\nu$. Another axis, $k_2$, is directed along the principal direction corresponding to negative effective mass, $m_2 = -m$. Using such an approximation, one can represent the function $B_{K2}(\omega)$ at $\omega > 0$ in the explicit form,

$$
B_{K2}(\omega) = \frac{1}{2\pi \sqrt{\nu}} \left\{ \ln \frac{\sqrt{\omega - 1} + \sqrt{\omega - 1} + \omega}{\sqrt{\omega}}, \ln \frac{\sqrt{\omega + 1} + \sqrt{\omega + 1} - \omega}{\sqrt{\omega}} \right\}, 
$$

(22)

where $\omega - 1 = (\Delta k_2/2)^2$, $\omega + 1 = \nu(\Delta k_1/2)^2$. The upper (low) row corresponds to $0 \leq \omega - 1 \leq \omega_1 - \omega$ ($0 \leq \omega_1 - \omega \leq \omega - 1$). The function Eq. (22) is defined inside the interval $0 \leq \omega \leq \omega_1$ and depends on both $\omega - 1$ and $\omega_1$. Taking into account that $\nu << 1$ and $\Delta k_1 >> \Delta k_2$ one may assume, for the sake of simplicity, that $\omega - 1 = \omega_1 \equiv \omega_1$. This assumption using later on is sufficient to study the main features of the scattering amplitude. Thus, one can see that the upper solution in Eq. (22) has not a domain of definition whereas the lower one is defined inside the whole of the interval $0 \leq \omega \leq \omega_1$. Similar consideration provided that $\omega < 0$ leads to the explicit expression for $B_{K2}(\omega)$ at any $\omega$,

$$
B_{K2}(\omega) = \frac{\text{sgn} \omega}{2\pi \sqrt{\nu}} \ln \left\{ \sqrt{\omega_1 - |\omega| + \sqrt{\omega^2}} \right\}. 
$$

(23)

Here, $0 \leq |\omega| \leq \omega_1$. It should be noted that, at $|\omega| > \omega_1$, we have $B_{K2}(\omega) = 0$.

The function $B_{K2}(\omega)$ is connected with the density of states of the relative motion of $K$-pair inside the domain $\Xi_K$,

$$
B_{K2}(\varepsilon) = \pi \varepsilon \cdot g_K(\varepsilon) \cdot \text{sgn} \varepsilon, 
$$

(24)

where $\varepsilon = (h^2 \omega_1/m)$ is an energy width of the domain $\Xi_K$, $a^2 = S/N$, $N$ is a number of unit cells in conducting plane, $\varepsilon = (h^2 \omega/m)$. Average (inside the domain $\Xi_K$) density of states per unit cell is defined as

$$
g_K = \frac{1}{\varepsilon_0} \int_{-\varepsilon_0/2}^{\varepsilon_0/2} g_K(\varepsilon), d\varepsilon. 
$$

(25)

One can rewrite Eq. (25) in the form

$$
g_K = \frac{1}{\varepsilon_0} \cdot \Xi_K a^2 = \frac{m a^2}{\pi^2 h^2} \cdot \frac{1}{\sqrt{\nu}}. 
$$

(26)

Due to the condition that $\nu << 1$, average density of pair states inside the domain $\Xi_K$ may be considerably more in comparison with total average (inside 2D Brillouin zone) density of states which is equal to $ma^2/\pi^2 h^2$. This is a consequence of peculiar features of saddle point vicinity in HTSC cuprates (known as extended saddle point) associated with hyperbolic metrics and strong effective mass anisotropy.

The function defined by Eq. (23) has a logarithmic singularity at $|\omega| \to 0$,

$$
B_{K2}(\omega) \sim \frac{\text{sgn} \omega}{4\pi \sqrt{\nu}} \ln \left| \frac{4\omega_1}{\omega} \right|, 
$$

(27)

that is the well-known, typical of any 2D system, logarithmic van Hove singularity in density of states due to saddle points. Near the edges of the energy band, $-\omega_1 \leq \omega \leq \omega_1$, in which the function $B_{K2}(\omega)$ is defined, this function behaves as follows:

$$
B_{K2}(\omega) \sim \frac{\text{sgn} \omega}{2\pi \sqrt{\nu}} \sqrt{1 - |\omega|/\omega_1}, \quad |\omega_1 - |\omega|| << \omega_1. 
$$

(28)

The function $B_{K2}(\omega)$, Eq. (23), is plotted in Fig. 5.

Now, let us consider the function $B_{K1}(\omega)$ defined in Eq. (12). Taking into account Eq. (24) we have

$$
B_{K1}(\omega) = \int_{-\omega_1}^{\omega_1} \frac{1}{\nu k_1^2 - k_2^2 - \omega (2\pi)^2} d^2 k = \frac{1}{\pi} \int_{-\omega_1}^{\omega_1} B_{K2}(\omega') \cdot \text{sgn} \omega' \cdot \frac{d\omega'}{\omega' - \omega}. 
$$

(29)
First of all, let us estimate $B_{K1}(\omega)$ using the average value, Eq. (26), of pair density of states. We obtain

$$B_{K1}(\omega) \approx \frac{1}{2 \pi^2 \sqrt{\nu}} \int_{-\omega_1}^{\omega_1} \frac{d\omega'}{\omega' - \omega} = \frac{1}{2 \pi^2 \sqrt{\nu}} \ln \frac{\omega_1 - \omega}{\omega_1 + \omega}.$$  

(30)

The function Eq. (30) is presented in Fig. 6 (dashed line). It should be noted that just the function of the form Eq. (30) was used [62] to analyze the ARPES experiment. More rigorous treatment of such self-energy structure was done in [58] from the analysis of energy and momentum distribution curves. The explicit expression of $B_{K1}(\omega)$,

$$B_{K1}(\omega) = \frac{\omega}{\pi^2 \sqrt{\nu}} \int_{0}^{\infty} \ln \left[ \frac{\sqrt{\omega^2 - \omega^2} + \sqrt{\omega^2}}{\sqrt{\omega^2}} \right] \frac{d\omega'}{\omega'^2 - \omega^2},$$  

(31)

in the form of a combination of elementary or special functions is unknown. It is obvious that, at $|\omega| \rightarrow \infty$

$$B_{K1}(\omega) \sim -\frac{\Xi_K}{(2\pi)^2} \cdot \frac{1}{\omega}, \quad |\omega| \rightarrow \infty. \quad (32)$$

At $\omega \rightarrow \pm 0$, we have from the definition, Eq. (29),

$$B_{K1}(\pm 0) = \pm \frac{1}{2 \pi^2 \sqrt{\nu}} \int_{0}^{1} \ln \frac{1 - x}{1 + x} dx = \pm \frac{1}{8 \sqrt{\nu}}.$$  

(33)

Thus, a logarithmic singularity of density of states appears in $B_{K1}(\omega)$ as a finite discontinuity at $\omega \rightarrow \pm 0$. At $\omega = \pm \omega_1$, the function $B_{K1}(\omega)$ has the finite values,

$$B_{K1}(\pm \omega_1) = \mp \frac{1}{\pi^2 \sqrt{\nu}} \int_{0}^{1} \ln \left| \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x^2}} \right| dx \approx \mp 0.164 \frac{\nu}{\sqrt{\nu}},$$  

(34)

with $|B_{K1}(\pm \omega_1)| > |B_{K1}(\pm 0)|$. The function $B_{K1}(\omega)$, Eq. (31), is plotted in Fig. 6 (solid line).

The obtained function, $B_{K1}(\omega)$, allows us to analyze qualitatively the solutions of the Eq. (14) which determines the poles of the scattering amplitude. In the case of repulsion between particles composing $K$–pair ($u_0 > 0$) the solutions corresponding to positive energy ($\omega > 0$) exist provided that $-u_0 B_{K1}(\omega_1) < 1$. One of the solutions, $\omega_{QSS}^+$, corresponding to greater energy, exists at as much as desired value of the coupling constant, $u_0$. The second solution, $\omega_{SC}^+$, exists in a bounded interval of coupling constant values: $|B_{K1}^+(0)| < u_0 < |B_{K1}^-(\omega_1)|$. The first solution takes place when $\omega_1 < \omega_{QSS}^+ < \infty$ and the second one exists in an energy interval bounded both above and below, $0 < \omega_{SC}^+ < \omega_1$. At $\omega > 0$, as it is shown in Fig. 5, $B_{K2}(\omega) > 0$, therefore, the sign of a decay which corresponds to both poles of the scattering amplitude is determined by the behavior of the function $B_{K1}(\omega)$. In the interval $\omega_1 < \omega < \infty$, this function increases with $\omega$, therefore, positive decay, $\Gamma_{QSS}^+ > 0$, corresponds to the pole $\omega_{QSS}^+$. Hence, this pole may be really associated with a quasi-stationary state. But in fact, as it follows from the definition, Eq. (12), and Fig. 5, $\Gamma_{QSS}^- = +0$, therefore, the approximation used here leads to the pole $\omega_{QSS}^-$ being a real stationary state. Indeed, one can see that, due to hyperbolic metrics of momentum space, at $\omega > \omega_1$, any decomposition of $K$–pair becomes impossible because of the restrictions connected with momentum and energy conservation.

At $0 < \omega < \omega_1$ the function $B_{K1}(\omega)$, on the contrary, decreases with $\omega$, therefore, finite and negative decay, $\Gamma_{SC}^+ < 0$, corresponds to the pole. This fact may be considered as an evidence of an instability with respect to a rise of $K$–pairs, and imaginary part of the pole, $\Gamma_{SC}^+$, may be directly connected with SC gap in one-particle excitation spectrum. However, the presence of positive real part of the pole, $\omega_{SC}^+ > 0$, indicate that a rise of the SC state becomes possible only if an energy increase connected with the finite value of $\omega_{SC}^+$ were compensated by sufficient energy decrease produced by corresponding rearrangement of the electron system which does not bear a direct relation to $K$–pair formation. As an example of such a rearrangement in HTSC cuprates, one may consider above mentioned rise of spatially inhomogeneous spin and charge structure because of partial restoration of AF order. Thus, because of the positive sign of the real part of the scattering amplitude, the SC pole without any renormalization of the ground state may be considered as corresponding to a metastable state. The QSS state has to be related not to a minimum but a maximum of total energy.

In the case when attraction between particles composing $K$–pair dominates ($u_0 < 0$), at $0 < |u_0| < B_{K1}(\omega)$, as it is seen from Fig. 6, there is a solution, $\omega_{SC}^-$, of Eq. (14) which exists in infinite energy region, $-\infty < \omega_{SC}^- < -\omega_1$, and, in the approximation used here, has an infinitesimal decay, $\Gamma_{SC}^- = -0$. On the contrary, another pole, $\omega_{QSS}^-$, existing inside the energy interval, $-\omega_1 < \omega_{QSS}^- < 0$, bounded both above and below, corresponds to real QSS with finite and rather large decay as one can see from Fig. 5. Therefore, in spite of the fact that, due to assumption that $\omega_1 = \omega_{+1}$, there is obvious symmetry of the function $B_{K1}(\omega)$ with respect to a change of the sign of the argument, namely, $B_{K1}(\omega) = -B_{K1}(\omega)$, there is essential asymmetry in a character of solutions of Eq. (14) with respect to the sign of the coupling constant.

V. INTERACTION BETWEEN PARTICLES COMPOSING $K$–PAIR

The point of view [64, 65, 66] that there are incoherent electron or hole pairs in the pseudogap regime leads to definite conclusion concerning the sign of the interaction energy which governs the pairing in HTSC cuprates. Namely, one may propose that the only essential interac-
tion between electrons is screened Coulomb repulsion,

\[ U(r) = \frac{e^2}{r} \cdot \exp\left(-\frac{r}{r_0}\right), \tag{35} \]

where \( r_0 = \left[4\pi e^2 N(E_F)\right]^{-1/2} \) is a screening length, \( N(E_F) \) is one-particle density of states on the Fermi level, \( E_F \). The potential \((5.1)\) may be expanded into Fourier integral or discrete Fourier series inside 2D Brillouin zone, \( Z_B \):

\[ U(r) = \int_{Z_B} \tilde{U}(k) e^{ikr} \frac{d^2k}{(2\pi)^2} = \frac{1}{S} \sum_{k \in Z_B} \tilde{U}(k) e^{ikr} \tag{36} \]

Here, \( \tilde{U}(k) \) is the Fourier transform of the potential \( Eq. \,(35) \), \( S \) is a normalizing area. A momentum \( k \) of the relative motion of a pair with total momentum \( K \) belongs to some domain \( \Xi_K \) inside the Brillouin zone. Therefore, to describe the effective interaction, \( U^*_K(r) \), of the particles composing a pair one can use the expression \( Eq. \,(36) \) in which, however, the only essential momenta of the particles composing a pair essentially. Thus, we have approximately,

\[ \text{where} \quad N \quad \text{where} \quad \text{posing a pair with total momentum} \quad \text{that} \quad \text{effective interaction between the particles composing a pair with total momentum} \quad \text{may be approximately considered as a contact interaction of the form} \]

\[ U^*_K(r) = \int_{\Xi_K} \tilde{U}(k) e^{ikr} \frac{d^2k}{(2\pi)^2} = \frac{1}{S} \sum_{k \in \Xi_K} \tilde{U}(k) e^{ikr}. \tag{37} \]

If the domain \( \Xi_K \) includes many quantum states and, on the other hand, this domain is small enough in comparison with the Brillouin zone area,

\[ \frac{(2\pi)^2}{S} \ll \Xi_K \ll \frac{(2\pi)^2}{a^2}, \tag{38} \]

one can easily estimate the expression \( Eq. \,(37) \) using the theorem about the mean value. Let us suppose that \( \tilde{U}(k) \approx \tilde{U}(0) \) inside the whole of the domain \( \Xi_K \). Then, at \( r \neq 0 \), oscillating terms in the sum, \( Eq. \,(37) \), suppress each other essentially. Thus, we have approximately,

\[ U^*_K(r) \approx U^*_K(0) \cdot \delta(r), \tag{39} \]

where

\[ U^*_K(0) = \frac{1}{S} \sum_{k \in \Xi_K} \tilde{U}(k) = \tilde{U}(0) \frac{\Xi_K}{(2\pi)^2}. \tag{40} \]

Taking account that

\[ \delta(r_0) = \sum_{k \in Z_B} e^{ikr} \Rightarrow a^2 \delta(r), \tag{41} \]

where \( N \) is a number of unit cells corresponding to the area \( S \), \( a \) is an interatomic distance, one can conclude that the effective interaction between the particles composing a pair with total momentum \( K \) may be approximately considered as a contact interaction of the form

\[ U^*_K(0) \approx \tilde{U}(0) \frac{\Xi_K a^2}{(2\pi)^2} \delta(r). \tag{42} \]

The Fourier transform of the effective interaction \( Eq. \,(42) \) has the form

\[ \tilde{U}^*_K(k) = \tilde{U}(0) \frac{\Xi_K a^2}{(2\pi)^2} = \frac{2\pi r_0 \Xi_K a^2}{2\pi}. \tag{43} \]

Here we take into account the explicit form of the Fourier transform of the screened Coulomb potential \( Eq. \,(35) \), \( \tilde{U}(k) = (2\pi e^2 r_0) / (1 + k^2 r_0^2) \). Now, the coupling constant may be written in the form

\[ u_0 = \frac{r_0}{\pi a^*} \Xi_K a^2 \tag{44} \]

where \( a^* = \hbar^2 / me^2 \) is an effective Bohr radius. An increase of carrier concentration due to doping leads to a decrease of the screening length and, as a result, to a decrease of the coupling constant. In Fig. 7, we represent a qualitative comparison of typical of HTSC cuprates phase diagram (Fig.1) and Fig. 6 which we consider here as a dependence of \( K \)-pair energy \( \omega \) on, increasing with doping, inverse value of the coupling constant, \( u_0^{-1} \). One can see from Fig. 7 that a crossover line, \( T^*(x) \), separating normal and pseudogap states in phase diagram is in accordance with the line which determines the energy of QSS with positive infinitesimal decay, \( \omega_{QSS}^+ \). Besides, there is an obvious accordance between SC phase region bounded by the line \( T_C(x) \) in phase diagram and the line which determines the solution leading to SC instability, \( \omega_{SC}^2 \). Indeed, as it is clearly seen from Fig. 7, both functions of doping, \( T_C(x) \) and \( \omega_{SC}^2(x) \), have finite domains of definition bounded above and below.

VI. SUPERCONDUCTING PAIRING

The presence of negative-decay poles in the scattering amplitude corresponding to the relative motion of electron or hole pair with large (of about \( 2k_F \)) total momentum bears evidence to a possibility of SC pairing both at attraction and repulsion between the particles composing the pair. A consequence of a rise of spatially inhomogeneous electron structure such as stripe structure is that a number of real \( K \)-pairs belonging to the domain \( \Xi_K \) must leave this domain and form new real pairs with a momentum \( K' \) (\( K' \)-pairs) in a domain \( \Xi_{K'} \) outside of the FC. The states inside \( \Xi_K \) and \( \Xi_{K'} \) having become vacant and remaining filled are separated from each other by a line which is, by our definition, the PFC. The area \( \Xi_K^{(+)} \) corresponding to vacant states inside \( \Xi_K \) is, generally speaking, not equal to the area \( \Xi_K^{(-)} \) of the filled part of the domain \( \Xi_{K'} \), if one takes into account the fact that, in a general case, the areas of AF and M parts of a stripe not equal to each other. The values of each of the areas, \( \Xi_K^{(+)} \) and \( \Xi_K^{(-)} \) are dependent on the AF energy which determines the position of the chemical potential \( 2\mu \) of pairs with respect to the edges of the energy bands corresponding to the domains \( \Xi_K \) and \( \Xi_{K'} \). These energy bands and relevant densities of
states, $g_K(\varepsilon)$ and $g_{K'}(\varepsilon)$, are represented schematically in Fig. 3. One can see that one part of the PFC, which is situated in the domain $\Xi_K$ (the boundary between $\Xi^{(-)}_K$ and $\Xi^{(+)}_K$), may be related to AF part of a stripe whereas another part, separating $\Xi^{(-)}_{K'}$ and $\Xi^{(+)}_{K'}$, belongs to M part of a stripe. An excitation of carriers (a rise of holes above and electrons below the chemical potential level) leads to a possibility of their pairing. Formally, one can consider (1) a scattering of pairs in AF part of a stripe (in the domain $\Xi_K$), (2) a scattering of pairs in M part of a stripe (in the domain $\Xi_{K'}$), and also, (3) a scattering which includes transfers of pairs between AF part of a stripe (the domain $\Xi_K$) and M part of a stripe (the domain $\Xi_{K'}$) as it is in the case discussed below in Sec. X. In such a case, (3), pairs are spatially separated and the interaction leading to their scattering is reduced. As it is already mentioned, the condition $|K' - K| >> \delta k_c$ allows us, in the first approximation, to consider pairing in the domains $\Xi_K$ and $\Xi_{K'}$ independently of each other thus restricting ourselves to one of the cases (1) or (2). It should be noted that the equation determining the SC order parameter has a set of solutions and, among them, there are solutions which, in the cases (1) and (2), may turn out to be trivial (corresponding order parameter becomes equal to zero). However, in such a case, a nontrivial solution may arise just due to the third, (3), possibility (for example, in the case of weak ferromagnetic ordering, $\text{[5]}$). In this Section, we restrict ourselves to the case (1) and consider SC pairing near the part of the PFC belonging to AF part of a stripe. Thus, we suppose that rather thin M parts of stripes do not affect the PFC (belonging to AF part of a stripe). Therefore, we write the equivalent Hamiltonian corresponding to relative motion of $K$–pairs in the form

$$
\hat{H}_K = \sum_{k} \left[ (\varepsilon_{k^+} - \mu) \hat{a}_{k^+} \hat{a}_{k^+} + (\varepsilon_{k^-} - \mu) \hat{a}_{k^-} \hat{a}_{k^-} \right] + \frac{1}{2} \sum_{k, k'} \hat{U}'_K(k - k') \hat{a}_{k^+} \hat{a}_{k^-} \hat{a}_{k'^+} \hat{a}_{k'^-},
$$

where $\varepsilon_{k^\pm} \equiv \varepsilon(k^\pm)$ is hole dispersion, $\hat{U}'_K(k - k')$ is Fourier transform of the effective interaction energy, Eq. (37), $k' \pm = K/2 \pm k'$, $\hat{a}_{k^\pm \sigma} (\hat{a}_{k^\pm \sigma}^\dagger)$ creates (annihilates) a hole with a momentum $k^\pm$ and spin quantum number $\sigma$. The symbol $\uparrow (\downarrow)$ is referred to $\sigma = 1/2 (\sigma = -1/2)$, $\mu$ is a hole chemical potential. The summation in Eq. (45) is taken over all range of values of momenta of the relative motion of $K$–pair. Note that the summation in the Hamiltonian Eq. (45) should be taken over only two (instead of three in general case) variables, $k$ and $k'$, just as in the case of BCS Hamiltonian, thus taking into account the only interaction between particles composing $K$–pairs. Also, it should be noted that the summation in Eq. (45) should be restricted by the corresponding domain $\Xi_K$ as it is discussed in Section V.

As usual, to diagonalize the Hamiltonian Eq. (45) approximately one can introduce creation and annihilation operators of new quasiparticles using the well-known Bogoliubov-Valatin transformation [$\text{3}3$, $\text{7}$]:

$$
\begin{align}
\hat{a}_{k^\pm \uparrow} &= u_{KK'} \hat{b}_{k^\pm +1} + v_{KK'} \hat{b}_{k^\pm -1}, \\
\hat{a}_{k^\pm \uparrow}^\dagger &= u_{KK'} \hat{b}_{k^\pm +1}^\dagger + v_{KK'} \hat{b}_{k^\pm -1}^\dagger, \\
\hat{a}_{k^\pm \downarrow} &= u_{KK'} \hat{b}_{k^\pm -1} - v_{KK'} \hat{b}_{k^\pm +1}, \\
\hat{a}_{k^\pm \downarrow}^\dagger &= u_{KK'} \hat{b}_{k^\pm -1}^\dagger - v_{KK'} \hat{b}_{k^\pm +1}^\dagger.
\end{align}
$$

The Hamiltonian, up to the terms of the order of $\hat{b}^2$, is written as

$$
\hat{H}_K = \hat{H}_{K0} + \hat{H}^{(0)}_K + \hat{H}^{(1)}_K.
$$

The ground state energy has the form

$$
E_{K0} = 2 \sum_k \xi_{KK'K} v^2_{K K'} + \sum_k \Delta_{KK} u_{K K} v_{K K},
$$

It is obvious that stripe structure periodicity in a conducting plane and the difference $|K' - K|$ have to be correlated. For simple 1D stripe structure, $K$ ($K'$)–pair state arises due to a mixing of only two $K$–states corresponding to either $[100]$ ($k_1$–axis) or $[010]$ ($k_2$–axis) direction. Thus, one may expect a rise of an array of alternating $CuO_2$ planes with 1D stripes which are perpendicular to each other in the neighboring planes. The pair states formed by the quadruples of $K$–pairs correspond to more complicated periodic 2D stripe structure. Real (nonperiodic) AF short-range-order fluctuations may be described as linear combination of $K$ ($K'$)–pair as well.

The experimental data [$\text{29}$, $\text{30}$] bear evidence to singlet pairing in HTSC cuprates. Therefore, we write the equivalent Hamiltonian corresponding to relative motion of $K$–pairs in the form
where, related to the value of the chemical potential, an energy of the relative motion of $K$-pair is defined as

$$2\xi_{Kk} = \varepsilon(k_+) + \varepsilon(k_-) - 2\mu. \quad (49)$$

Diagonal, with respect to quasiparticle operators, part of the Hamiltonian is given by

$$\hat{H}_k^{(0)} = \sum_{k,\beta=\pm 1} \eta_{K\beta}(k) \hat{b}_{k,\beta}^\dagger \hat{b}_{k,\beta}, \quad (50)$$

where the energies corresponding to two branches ($\beta = \pm 1$) of one-particle excitation spectrum are equal to each other,

$$\eta_{K\beta}(k) = \sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2}. \quad (51)$$

Nondiagonal, with respect to quasiparticule operators, part of the Hamiltonian has the form

$$\hat{H}_k^{(1)} = \sum_k \left[2\xi_{Kk} u_{Kk} v_{Kk} - (v_{Kk}^2 - u_{Kk}^2)\Delta_{Kk}\right] \times \left(\hat{b}_{k,+1}^\dagger \hat{b}_{k,-1}^\dagger + \hat{b}_{k,-1} \hat{b}_{k,+1}\right). \quad (52)$$

Here we define the order parameter as

$$\Delta_{Kk} = \frac{1}{S} \sum_{k'} \tilde{U}_k^* (k-k') u_{Kk'} v_{Kk'} \cdot f_{k'}, \quad (53)$$

where $f_{k'} = (1 - n_{k',+1} - n_{k',-1})$,

$$n_{k,\beta} \equiv \langle \hat{b}_{k,\beta}^\dagger \hat{b}_{k,\beta}\rangle = \frac{1}{\exp(\eta_{K\beta}(k)/T) + 1} \quad (54)$$

are quasiparticle occupation numbers.

A choice of the amplitudes in Bogoliubov-Valatin transformation Eq. (46) is determined by the conditions that, in the zero-temperature limit, the subdomain $\Xi_{K}^{(-)}$, in which kinetic energy of the relative motion of $K$-pair is negative, $2\xi_{Kk} < 0$, must be filled and the nondiagonal part, Eq. (52), of the Hamiltonian vanishes. In addition, the condition $v_{Kk}^2 + u_{Kk}^2 = 1$ preserving Fermi’s commutation relations for quasiparticle operators must be fulfilled. These conditions yield

$$v_{Kk}^2 = \frac{1}{2} \left(1 - \frac{\xi_{Kk}}{\sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2}}\right), \quad \text{and} \quad u_{Kk} v_{Kk} = -\frac{1}{2} \frac{\Delta_{Kk}}{\sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2}}. \quad (55)$$

Now, one can obtain the equation determining the order parameter,

$$\Delta_{Kk} = -\frac{1}{25} \sum_{k'} \tilde{U}_k^* (k-k') \Delta_{Kk'} \cdot f_{k'}, \quad (56)$$

where summation is taken over the whole of the domain $\Xi_{K}$. It should be noted that kinetic energy of the relative motion of $K$-pair inside $\Xi_{K}^{(+)}$ is defined within the limits $0 \leq 2\xi_{Kk} \leq 2\varepsilon_{K+}$, where $\varepsilon_{K+}$ is an energy width of the subdomain $\Xi_{K}^{(+)}$; inside $\Xi_{K}^{(-)}$ we have $-2\varepsilon_{K-} \leq 2\xi_{Kk} \leq 0$ where $\varepsilon_{K-}$ is an energy width of the subdomain $\Xi_{K}^{(-)}$ (Fig. 3). Inside the whole of the domain $\Xi_{K}$, the excitation energy Eq. (51) is positive by definition, $\eta_{K\beta}(k) = \sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2} > 0$, and hence, in the zero-temperature limit, the factor $f_k = 1$ at any $k$ belonging to $\Xi_{K}$. Therefore, in the zero-temperature limit, the equation Eq. (56) transforms into the form

$$\Delta_{Kk} = -\frac{1}{25} \sum_{k'} \tilde{U}_k^* (k-k') \Delta_{Kk'} \cdot f_{k'}. \quad (57)$$

It is obvious that, in the case of repulsion between particles composing $K$-pair, BCS – like solution, independent of $k$, is absent.

VII. SUPERCONDUCTING GAP

The solutions of Eq. (57) for SC energy gap in the cases of attraction ($\tilde{U}_k^* (k-k') < 0$) and repulsion ($\tilde{U}_k^* (k-k') > 0$) between particles composing $K$-pair differ from each other essentially. First of all, we consider the case of attraction and restrict ourselves to the simplest approximation Eq. (43), namely, $\tilde{U}_k^* (k-k') \equiv U_k = \text{const}$. Such an approximation, just as BCS one, enables one to get an explicit expression for SC energy gap. A magnitude of the coupling constant $U_k$ depends on the pairing mechanism which is not under discussion here. The only circumstance we have to take into account is that one may neglect a predominance of repulsion as compared to attraction in comparatively narrow energy region corresponding to a vicinity of zero-energy line of electron or hole dispersion that is in the vicinity of the PFC. Let $2\xi$ be characteristic energy width of such a region and suppose, for the sake of simplicity, that $\xi$ is more less than any characteristic scale relating to each of the subdomains $\Xi_{K}^{(-)}$ and $\Xi_{K}^{(+)}$; that is $\xi << \varepsilon_{K-}$, $\xi << \varepsilon_{K+}$. The approximation $\tilde{U}_k^* (k-k') = \text{const}$ results in that there is a solution of the Eq. (57) independent of the momentum of the relative motion of $K$-pair that is $\Delta_{Kk} \Rightarrow \Delta_{K}$. We restrict ourselves to a consideration of such a solution only. Reducing, as usual, the sum in Eq. (57) into an integral over $\xi_{Kk}$ and introducing an average density of states, $g_K$, related to unit area, one can obtain the order parameter in the form

$$\Delta_{K} \approx \xi \cdot \exp \left(-\frac{1}{g_K U_k}\right). \quad (58)$$

that is a solution which formally coincides with BCS solution. It should be noted that $g_K$ is more less as compared with the total density of states on the Fermi level. Therefore, in the case of typical of phononic pairing mechanism coupling constant value, one obtains the energy gap, Eq. (58), which should be certainly more less in comparison with the gap were arisen due to conventional Cooper pairing on the full FC.
Now, let us consider the case when a repulsion between particles composing $\mathbf{K}$–pair dominates. In this case, one has not to take into account the existence of any bosonic degree of freedom (phononic, electronic, magnetic or some else) as a necessary condition of a rise of a bound state of $\mathbf{K}$–pair. Screened Coulomb repulsion Eq. (35) becomes the only essential interaction with the effective coupling constant, Eq. (43).

As one can see from Eq. (57) there is no solution of constant signs inside the domain $\Xi_K$ provided that $U_K > 0$. Therefore, to obtain an approximate solution of Eq. (57) we suppose that the order parameter dependence on the momentum of the relative motion of $\mathbf{K}$–pair is given by a discontinuous function changing its sign on the PFC. Restrict ourselves to a consideration of the simplest case when the order parameter has independent of $\mathbf{k}$ and different from each other values inside the subdomains $\Xi_K^(-)$ and $\Xi_K^+$. Namely, omitting the label $K$ in the definition of the order parameter, we assume that $\Delta_K \equiv \Delta_- > 0$ inside $\Xi_K^-$ and $\Delta_K \equiv -\Delta_+ < 0$ inside $\Xi_K^+$. One can take note of the fact that a singularity (a discontinuous character of the order parameter in contrast to the solution of constant signs as it follows from Eq. (58) in the case of attraction) on the line separating $\Xi_K^-$ and $\Xi_K^+$ may be considered as one more manifestation of hyperbolic metrics of momentum space in the same way as in the case of QSS when the wave functions, Eqs. (20), (21), of the relative motion of $\mathbf{K}$–pair, written in the special case of the PFC position coinciding with the pair center-of-mass energy, $2\varepsilon(K/2)$, change their behavior on the PFC as well.

The assumption relating to a character of the solution of Eq. (57) allows us to rewrite this equation in the form of the system of two equations for $\Delta_-$ and $\Delta_+$. In this connection, one should take into account the above mentioned remark that effective interaction matrix element $U_K \sim \Xi_K^-$ when both $\mathbf{k}$ and $\mathbf{k}'$ belong to the subdomain $\Xi_K^-$, that is a scattering due to the interaction is restricted to this subdomain. However, if $\mathbf{k}$ belongs to $\Xi_K^-$ and $\mathbf{k}'$ belongs to $\Xi_K^+$ and hence the scattering is possible in the whole of the domain $\Xi_K$, we have $U_K \sim \Xi_K$. Taking account of that $\Xi_K << (2\pi)^2/\alpha^2$, that is, in any case, the scattering region is more less as compared with 2D Brillouin zone, one may neglect a dependence of $U_K(k-k')$ on the momenta $\mathbf{k}$ and $\mathbf{k}'$ except for that which is already taken into account by the factor $\Xi_K$ in the definition of $U_K(k-k')$. Assuming that $U_K(k-k') \sim \Xi_K\alpha^2$ where both $\mathbf{k}$ and $\mathbf{k}'$ belong to $\Xi_K$ and the coupling constant is defined by Eq. (43), one can rewrite Eq. (57) in the form

$$(1-\alpha)\Delta_- + \Delta_+ = \frac{U_K\Xi_K a^2 h_\alpha}{2S}\Delta_+ \sum_{k \in \Xi_K^-} \frac{f_k}{\sqrt{\xi_k^2 + \Delta_+^2}},$$

$$\Delta_- + \alpha \Delta_+ = \frac{U_K\Xi_K a^2 h_\alpha}{2S}\Delta_- \sum_{k \in \Xi_K^+} \frac{f_k}{\sqrt{\xi_k^2 + \Delta_-^2}}. \tag{59}$$

We denote here $h_\alpha \equiv (1-\alpha + \alpha^2)$ and $\alpha \equiv \Xi_K^(-)/\Xi_K$. The factor $1-\alpha + \alpha^2 > 0$ when $\alpha$ varies within the range $0 \leq \alpha \leq 1$. It arises as a difference between squared “nondiagonal” and a product of “diagonal” interaction matrix elements with respect to the labels ($\pm$) and depends on a filling of the states inside the PFC. Thus, it takes into account statistical correlations in the system of fermions composing $\mathbf{K}$–pairs.

Dependence of AF energy on the doping level allows us to hunt down the evolution of the PFC due to a variation of doping. If $I(x) < \delta_0(KK) \equiv I_m$, the PFC is absent and $\varepsilon_{K-} = \varepsilon_0$ whereas $\varepsilon_{K+} = 0$ where $\varepsilon_0$ is an energy width of the domain $\Xi_K$. At $I(x_2) = I_m$, there is an “opening” of the PFC at two points, $a$ and $a'$, which are situated on the $k_1$–axis as it is shown in Fig. 2. A decrease of doping, $x < x_2$, leads, first of all, to a rise, and then, to an extension of the subdomain $\Xi_K^+$ which is accompanied with the corresponding decrease of the subdomain $\Xi_K^-$. Thus, the PFC length increases and then, after being up the maximal length value corresponding to a certain doping level, becomes to decrease, shrinking lastly at two points, $b$ and $b'$, on the $k_2$–axis (Fig. 2). Such a shrinking corresponds to a value of doping level, relating to AF energy $I(x_1) = I_M$. If one assume that the pair condensate density is directly connected to the PFC length, one can qualitatively explain both rather small values of the condensation density and a peculiar doping dependence of $T_C$ observed in cuprates. It should be noted that, generally speaking, the choice of the domains $\Xi_K$ and $\Xi_K'$ themselves depends on doping as well since, in the last analysis, one has to determine the antinodal vectors $\mathbf{K}$ and $\mathbf{K}'$ minimizing the total electron energy. Later on, for the sake of simplicity, we take into account such a dependence assuming that it is explicitly included into AF energy $I(x)$ and screening length $\rho_0(x)$ doping dependences. Such an evolution of the PFC length with a variation of doping has to determine, in main, doping dependence of both superconducting transition temperature and superfluid density.

For the sake of simplicity, we suppose that $\varepsilon_{K-}$ and $\varepsilon_{K+}$ are linear functions of doping:

$$\varepsilon_{K-}(x) = \varepsilon_0 \cdot \frac{x - x_1}{x_2 - x_1}, \quad \varepsilon_{K+}(x) = \varepsilon_0 \cdot \frac{x_2 - x}{x_2 - x_1}. \tag{60}$$

Here, $x_1 \leq x \leq x_2$. Let us introduce a “reduced” doping level,

$$y = \frac{x - x_1}{x_2 - x_1}, \tag{61}$$

varying within the limits of interval $0 \leq y \leq 1$. Then we have $\varepsilon_{K-}(y) = \varepsilon_0 \cdot y$, $\varepsilon_{K+}(y) = \varepsilon_0 \cdot (1 - y)$. As another one simplification, we assume that the density of states is constant inside the whole of the domain $\Xi_K$, $g_K = \Xi_K/(2\pi)^2\varepsilon_0$. From this assumption, it follows immediately that $\Xi_K^-/\Xi_K = \varepsilon_{K-}/\varepsilon_0$ that is $\alpha \equiv y$.

Reducing the summation over momenta in Eq. (59) to an integration over the energy of the relative motion of
one can rewrite, in the zero-temperature limit, the system of equations, Eq. (59), in the form

\[
(1 - y)\delta_- + \delta_+ = w_K(y) \cdot h_y \cdot \delta_- \cdot \ln \left( \frac{1 - y}{\sqrt{(1 - y)^2 + \delta_+^2}} \right),
\]

\[
\delta_- + y\delta_+ = w_K(y) \cdot h_y \cdot \delta_+ \cdot \ln \left( \frac{1 - y}{\sqrt{(1 - y)^2 + \delta_-^2}} \right)
\]

where \( h_y \equiv (1 - y + y^2), \delta_+ \equiv \Delta_+ / \epsilon_0 \) and

\[
w_K(y) = \frac{\pi^2 y_0^2(y)}{\epsilon_0 a^2} \left[ \frac{\Xi_K^2}{2\pi^2} \right].
\]

The screening length \( r_0 = r_0(y) \) is, generally speaking, a decreasing function of doping level. For the sake of simplicity, we use a linear approximation for the coupling parameter, Eq. (64),

\[
w_K(y) = w_K \cdot \left( 1 - \frac{y}{y_b} \right),
\]

where \( w_K \) is determined by Eq. (64) at \( y = 0 \), and \( y_b > 1 \).

It should be noted that the chemical potential \( \mu \) as a point of reference of kinetic energy of the relative motion of \( K^- \) pair changes due to a rise of SC order as compared with its value in the normal (nonsuperconducting) state. However, the corresponding shift of the chemical potential is quite small (of the order of the SC gap). Thus, calculating the values of the parameters \( \delta_- \) and \( \delta_+ \) one need not consider the chemical potential shift arising due to SC condensation of \( K^- \) pairs. So, we can adopt approximately that \( \mu \) is determined by the only parameter \( I \) and equals to the value which corresponds to the PFC at given \( I \) in normal state. However, in contrast to BCS theory, in our case, just the chemical potential shift determines, in the SC condensation energy and thus a doping dependence of the superconducting transition temperature.

By definition, both unknown quantities \( \delta_- \) and \( \delta_+ \) in Eq. (63) are nonnegative: \( \delta_- \geq 0, \delta_+ \geq 0 \). As one can easily see from Eq. (63), the system of equations, Eq. (63), leads to the trivial solution \( \delta_- = \delta_+ = 0 \) both at \( y = 0 \) and \( y = 1 \). As it also follows from Eq. (63), nontrivial solutions (if they exist) \( \delta_- \) and \( \delta_+ \) coincide, \( \delta_- = \delta_+ \) at \( y = 0.5 \). Nontrivial solutions turn out to be possible under the condition that the coupling parameter, Eq. (64), is large enough. Dependence of this parameter on doping, Eq. (65), leads to an asymmetry of the functions \( \delta_-(y) \) and \( \delta_+(y) \) that is, in general case (except as some special values of doping level), \( \delta_-(y) \neq \delta_+(y) \). Doping dependence of \( \delta_-(y) \) and \( \delta_+(y) \) is represented schematically in Fig. 8.

VIII. CHEMICAL POTENTIAL SHIFT

In spatially homogeneous system, the value \( 2E_F \) of the chemical potential of pairs indicates that the whole of the domain \( \Xi_K \) is filled whereas all of the states inside any domain \( \Xi_{K'} \) are vacant. A rise of spatially inhomogeneous stripe structure leads to a hole redistribution between \( \Xi_K \) and \( \Xi_{K'} \) with the result that the PFC arises. Thus, the full PFC may be treated, in the zero-temperature limit, as a line separating filled and vacant pair states in momentum space. The possibility of pairing itself resulting in an opening of the SC gap on the PFC arises just as a result of such a redistribution which may be due to above discussed partial restoration of AF order. The number of vacant states inside \( \Xi_K \) and, on the other hand, filled states inside \( \Xi_{K'} \) is governed by the value \( I \) of AF energy which determines the position \( 2\mu \) of the chemical potential of pairs with respect to the edges of the energy bands corresponding to the domains \( \Xi_K \) and \( \Xi_{K'} \). The densities of states, \( g_K(\varepsilon) \) and \( g_K'(\varepsilon) \), corresponding to these domains are represented in Fig. 3. It is generally assumed [36] that the interactions between carriers which are not included into the equivalent Hamiltonian of pairs such as Eq. (45) do not affect essentially the difference between the free energy values in the N and SC states. Therefore, to calculate the chemical potential shift due to the SC transition it is necessary to take into account the Hamiltonian Eq. (45) only. It should be noted that, in conventional superconductors, the chemical potential shift is equal to zero exactly because of the exact electron-hole symmetry of the excitation spectrum [71].

In HTSC cuprates however, there is no sufficient reason for such a statement [19] just because of essential electron-hole asymmetry [14]. To evaluate the chemical potential shift \( \mu' \) due to a condensation of \( K^- \) pairs belonging to the domain \( \Xi_K \) one has to take into account that a formal definition of an average number of particles inside \( \Xi_K \),

\[
\langle N_K \rangle = 2 \sum_{k \in \Xi_K} v_{Kk}^2 + \sum_{k \in \Xi_K} (u_{Kk}^2 - v_{Kk}^2)(n_{k,+1} + n_{k,-1}),
\]

takes into consideration the particles which may pass from \( \Xi_K \) into \( \Xi_{K'} \). In thermal equilibrium, such a passage is compensated by the particles passing from \( \Xi_{K'} \) into \( \Xi_K \). Therefore, the conserving quantity is a sum \( \langle N_K \rangle + \langle N_{K'} \rangle \) where the second term is an average number of particles inside \( \Xi_{K'} \). However, the condensation may be considered in each of the domains \( \Xi_K \) and \( \Xi_{K'} \) independently if, as it is accepted above, \( |K' - K| >> \delta_K \).
In this case, one has to take into account the only passages of particles across the PFC ($\Xi_K$+, which if one consider a condensation of $K$-pairs only) bearing in mind that the position of the PFC in the normal state is determined by the AF energy $I$ which is considered here as an external parameter. As far as the chemical potential shift due to SC condensation of pairs is also small together with $\Delta_{KK}$, such an approximation only slightly affects the introduced below coefficients $\lambda$ and $\tau$ and does not lead to any qualitatively new results. Strictly speaking, the pairing interaction, when $\Delta_{KK} \neq 0$, leads to, tending to zero with $\Delta_{KK}$, a small change in the fluxes of particles between the domains $\Xi_K$ and $\Xi_K'$. Thus, one obtains Eq. (67) in the form

$$\sum_{k \in \Xi_K} \frac{1}{\sqrt{\xi_{kk}^{(2)} + \gamma_{kk}^{(2)}}}.$$  

First of all, let us consider the case of repulsion between particles composing $K$-pair. Reducing the summation over momentum to integral over $\xi_{kk}$, after integration, one obtains Eq. (67) in the form

$$\left[\sqrt{(\varepsilon_{KK} - \mu')^2 - \varepsilon_{KK}} - \sqrt{(\varepsilon_{KK} + \mu')^2 - \varepsilon_{KK}}\right] + \frac{\Delta_\mu^2 + \Delta_\mu^2 - \mu^2 + \Delta_\mu^2}{\mu^2 + \Delta_\mu^2} = 0.$$  

As far as $\Delta_- << \varepsilon_{KK}$, $\Delta_+ << \varepsilon_{KK}$, in any case, and the chemical potential shift $\mu'$ measured from the PFC position at $\Delta \to 0$ is small together with $\Delta$, one can approximately rewrite Eq. (68) as

$$\sqrt{\mu^2 + \Delta_\mu^2} - \sqrt{\mu^2 + \Delta_\mu^2} \approx 2\mu' + \frac{\Delta_\mu^2}{2\varepsilon_{KK}} - \frac{\Delta_\mu^2}{2\varepsilon_{KK}}.$$  

(69)

Assuming that $\mu' = \mu'_1 + \mu'_2$ where $\mu'_1$ ($\mu'_2$) is a quantity of the first (second) order with respect to $\Delta$, the equation Eq. (69) may be solved with the use of the method of successive approximations. Thus, finally we have

$$\mu' = \frac{\varepsilon_0}{2\varepsilon_{KK}} + \frac{\Delta_\mu^2 - \Delta_\mu^2}{2\sqrt{\mu^2 + \Delta_\mu^2}} + \frac{\varepsilon_0}{16} \frac{(3\delta_1^2 + \delta_2^2)(\delta_2^2 + 3\delta_1^2)}{(\delta_2^2 + \delta_1^2)} \left(\frac{\delta_2^2}{1 - y} - \frac{\delta_1^2}{y}\right).$$  

(70)

Two values of the order parameter, $\delta_-$ and $\delta_+$, are not independent. They are connected with each other as it follows from the equations Eq. (63). For example, the first of these equations allows us to express $\delta_+$ as a function of $\delta_-$,

$$\delta_+ = \delta_- \cdot \gamma(\delta_-(y); y), \quad \gamma(\delta_-(y); y) \equiv \exp\left[w_\mu(y)\ln\left|\frac{y + \sqrt{y^2 + \delta_-^2}}{\delta_-^2 - (1 - y)}\right|\right].$$  

(71)

The expression Eq. (70) can be rewritten as

$$\mu' = \varepsilon_0 \cdot \delta \cdot \gamma(\delta_-(\lambda + \tau\delta)),$$  

(72)

where we denote $\delta_- \equiv \delta$, and

$$\lambda \equiv \frac{1}{2\sqrt{\varepsilon_{KK}}} \frac{1 - \gamma^2}{\sqrt{1 + \gamma^2}};$$

$$\tau \equiv \frac{1}{16} \frac{(3 + \gamma^2)(1 + 3\gamma^2)}{(1 + \gamma^2)^2} \left(\frac{\gamma^2}{1 - y} - \frac{1}{y}\right).$$  

(73)

In the case of attraction between particles composing $K$-pair, we have the only value of the order parameter, Eq. (58), which is independent of the momentum of the relative motion. Therefore, to obtain the chemical potential shift due to SC condensation one should formally write $\delta_+ = -\delta_-$. Then we have

$$\mu' = -\frac{\varepsilon_0}{4\gamma(1 - y)} \cdot \delta' \cdot \mu',$$  

(74)

Thus, the approximation we use here leads to the absence of the term which is linear in $\delta$ and the chemical potential shift turns out to be proportional to $\delta'$. In the symmetrical case, $y = 0.5$, when the PBC bisects the domain $\Xi_K$, $\mu' = 0$: as it is seen from Eq. (74), $\mu' < 0$ ($\mu' > 0$) at $0 < y < 0.5$ ($0.5 < y < 1$).

It should be noted that a necessity of $k$-dependence of the SC gap and corresponding displacement of the chemical potential from its value in the normal state was established phenomenologically by Hirsch [12] in his theory of hole superconductivity. It is clear that “the gap slope” introduced by Hirsch is directly related to our simple discontinuous solution of the gap equation (6.13) whereas the linear term in the chemical potential shift, arising just in the case when $\Delta_+ \neq \Delta_+$, corresponds to Hirsch’s “electron-hole symmetry-breaking term” being the difference, $\mu'$, between the chemical potential values in the superconducting and the normal state. One can see quite easily that, in the case of electron-hole asymmetry observed in tunnel current-bias characteristics, such a chemical potential shift is a direct consequence of the particle conservation law. Indeed, if one considers a redistribution of particles due to SC condensation inside the domain $\Xi_K = \Xi_K^+ \Xi_K^-$ only it becomes obvious that in the case when $\Delta_+ \neq \Delta_+$, the value 1/2 of the coherence factor $v_{KK}$, Eq. (55), cannot correspond to the position of the chemical potential relating to the normal state (see Fig. 9). So, some chemical potential shift is
needed to satisfy the condition that the number of transfers of holes from $\Xi_K^{-}$ must be equal to the number of transfers into $\Xi_K^{+}$. The sign of such a shift is determined by the sign of the difference $\Delta_{-} - \Delta_{+}$. Thus, Hirsch’s statement that $\mu' > 0$ is valid, generally speaking, in the case when $\Delta_{-} > \Delta_{+}$. One can also note that the so-called superconducting “Fermi surface” introduced in \cite{72} as “the locus in $k$-space of quasiparticle states of minimum energy”, in a sense, plays the role which, indeed, plays the PFC in the analysis of ARPES spectra and some other phenomena typical of HTSC cuprates.

IX. CONDENSATION ENERGY

The existence of the solution of Eq. (57) for SC order parameter in doping interval $x_1 < x < x_2$ does not mean yet that superfluid SC state arises in the whole or, at least in some part, of this interval. A phase transition from non-superfluid N state into superfluid SC state occurs under a necessary condition: namely, the condensation energy defined as a difference between N state and SC state values of the ground state energy must be positive.

In the zero-temperature limit, a contribution into the condensation energy which is associated with a condensation of $K$–pairs inside the domain $\Xi_K$ only, in accordance with Eq. (48), may be written as

$$E_{OS} = \sum_{k \in \Xi_K} \xi_{Kk} - \left( \sum_{k \in \Xi_K} \left[ \frac{\xi_{Kk}^2}{\sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2}} + \frac{1}{2} \frac{\Delta_{Kk}^2}{\sqrt{\xi_{Kk}^2 + \Delta_{Kk}^2}} \right] \right). \tag{75}$$

Reducing the summation over momentum in Eq. (75) to the integration over $\xi_{Kk}$, one has to take into account that the energy $\xi_{Kk}$, Eq. (49), measured from the chemical potential of N phase varies within the interval $-\xi_{Kk} \leq \xi_{Kk} \leq \xi_{Kk}^{+}$. Therefore, above discussed chemical potential shift due to SC condensation, $\mu'$, leads to

$$\sum_{k \in \Xi_K} 1 \Rightarrow S g_K \int_{-\mu'}^{\xi_{Kk}^{-}} d\xi + S g_K \int_{-\mu}^{\xi_{Kk}^{-} + \mu'} d\xi. \tag{76}$$

Here we take into account the fact that SC order parameter has a discontinuity on the PFC. Thus, correct to the terms of the order of $\Delta^2$, Eq. (75) may be represented in the form

$$E_{OS} = E_{0N} - S \frac{\gamma}{\epsilon_0} \xi_0 \cdot \delta \cdot (\lambda + c \delta), \tag{77}$$

where $\delta \equiv \Delta_{-}/\epsilon_0$ is a dimensionless order parameter and

$$E_{0N} = -S \frac{g_K}{\epsilon_0} \xi_k y^2 \tag{78}$$

is the corresponding contribution into the ground state energy of the N phase. The parameters $\lambda$ and $c$ are connected with $\lambda$ and $\tau$ in Eq. (73) in accordance with the relations

$$\lambda = 2y\lambda, \quad c = 2y\tau + \frac{1 + \gamma^2}{4}. \tag{79}$$

The second term in the second expression in Eq. (79) which is not connected with the chemical potential shift due to SC condensation may be formally related to a direct contribution of the pairing interaction in the Hamiltonian Eq. (45) into the condensation energy whereas the contributions associated with the coefficients $\lambda$ and $\tau$ may be related to a renormalization of kinetic energy of the relative motion of $K$–pairs being a result of SC condensation. Although such a separation \cite{13, 19, 23} of the condensation energy into kinetic and potential energy contributions has an arbitrary character (it is clear that both contributions vanish when the coupling constant in Eq. (45) tends to zero), it enables one to imagine more clear how hyperbolic metrics of momentum space affects the ground state of the electron system.

As it follows from Eq. (77), an energy gain due to the condensation of $K$–pairs is possible when

$$\lambda + c \delta > 0. \tag{80}$$

It is seen from Eq. (77) that this gain is, in main, due to a renormalization of the kinetic energy of the relative motion of $K$–pair. Indeed, the chemical potential shift due to a rise of a condensate of $K$–pairs results in the corresponding shift of the position of the PFC. Provided that the condition Eq. (80) is satisfied the PFC is shifted in a way that there is an extension of the part, $\Xi_K^{-}$, of the domain $\Xi_K$ in which the energy of the relative motion of $K$–pair is negative. The ground state energy decreases just due to a filling of the states which arise as a result of such PFC shift.

In this connection, one relevant optical experiment \cite{23} consistent with the conception elaborating here should be noted. An estimation of the superfluid density $\rho_s$ which is directly connected with IR reflection, indicate that, in several HTSC cuprates, $\rho_s$ significantly exceeds the value obtained from optical conductivity by means of Kramers–Kronig relations under the condition that one takes into account an energy interval comparable to the SC gap \cite{23}. This contradiction may be eliminated if one considerably extends the interval of integration. In conventional superconductors, as it follows from the BCS theory, each Cooper pair leads to an energy gain of the order of $\Delta$. The energy width of the condensation region in the vicinity of the FC is of the same order, $\sim \Delta$. Therefore, the condensation energy turns out to be of the order of $\Delta^2$. This explains the fact that, using the Kramers–Kronig relations, we can restrict ourselves to a finite interval of integration having a character energy width of about $\Delta$. The presence of the linear term ($\sim \Delta$) in the condensation energy in Eq. (77) clearly indicate that each $K$–pair also leads to an energy gain of the order of $\Delta$ but this gain must be associated with the condensation region in momentum space, which is connected.
not with the FC but with the PFC, having a character energy width of about a character kinetic energy of the relative motion inside the subdomain $\Xi_K$, namely, $\varepsilon_{K}$, which is usually more large as compared to $\Delta$.

Let us define the condensation energy per unit area as

$$
\varepsilon_c \equiv (E_{0N} - E_{OS})/4g_K\varepsilon^2S = \delta \cdot (\lambda + \epsilon g) \quad (81)
$$

and study qualitatively its dependence on doping level. In Fig.10, we represent a plot of the function $\varepsilon_c$ calculated numerically for some values of the coupling parameter, Eq. (64). It is obvious that there exists a certain minimal value of this parameter which corresponds to a start of SC condensation. This conclusion is in agreement with a finite value of the scattering amplitude at $\omega = \omega_1$ obtained in Sec. IV.

As one can see from Fig. 10, calculated condensation energy has negative sign inside some region of doping level. This fact is, mainly, due to negative sign of the chemical potential shift $\mu$ level. This fact is, mainly, due to negative sign of the energy has negative sign inside some region of doping energy width of about a character kinetic energy of the relative motion.

In Fig.10, we represent a plot of the function $\varepsilon_c$ and study qualitatively its dependence on doping level. In this connection, it should be noted that such a choice of the parameters $\Delta$ and $\Delta_+$ has to correspond to more symmetric tunnel current-bias characteristics in extremely underdoped regime in comparison with the optimal one.

**X. SUPERCONDUCTIVITY AT WEAK FERROMAGNETIC ORDERING**

It is known \[22, 23\] that, due to doping, charge-density wave (CDW) may arise in AF phase resulting from spin-density wave (SDW). A coexistence of SDW and CDW may lead to a rise of weak ferromagnetic state as it is, for example, in a boride family \[7\]. Charge ordering is also known in doped superconducting cuprates containing bismuth \[22\]. 1D stripe structure of such cuprates leads naturally to a modulation of spin and charge density and, as a result, makes possible a rise of weak ferromagnetism. In this Section, using the conception of PFC, we briefly discuss a possibility of superconductivity in cuprates with weak ferromagnetic ordering in AF parts of stripes \[2\].

If one suppose that a value of corresponding spontaneous magnetization is large enough, $\mu_B H_c >> \Delta$ (here, $\mu_B$ is the Bohr magneton, $H_c$ is ferromagnetic Weiss field, $\Delta$ is SC gap in the absence of the ferromagnetic order), the only trivial solution for SC order parameter may be obtained when one considers the domains $\Xi_K$ and $\Xi_K'$ independently from each other. Indeed, when Weiss field is large enough, the subbands corresponding to spin quantum numbers of opposite sign become removed with respect to each other so that the domain of definition of a momentum of $K$–pair relative motion turns out to be empty. Such a conclusion is consistent with that which follows from the problem of Cooper pairing in a weak ferromagnetic studied in \[2, 10\]. A similarity between Cooper pairing and $K$–pairing becomes clear if one takes into account the fact that, in the case of Cooper pairing, the momenta of particles composing a pair play role of the momenta of the relative motion as well. Thus, to obtain a nontrivial solution for SC order parameter one needs in a consideration of above mentioned electron and hole transfers across the parts of the PFC, one of which is related to AF part of a stripe and belongs to the domain $\Xi_K$ and another one corresponds to M part of a stripe associated with the domain $\Xi_K'$. Therefore, in the following we take into account the transfers $\Xi_K \leftrightarrow \Xi_K'$ only. Note that in \[23\] the model with strong tendency to spatial modulation due to correlations between $M$ and AF parts of a stripe structure is considered in connection with the problem of high-temperature superconductivity in cuprates.

To obtain the Hamiltonian of hole pairs in the case discussed here, one needs in taking account of the fact that each hole passing from $\Xi_K$ into $\Xi_K'$ in momentum space passes from $M$ part into AF part of a stripe in real space. As a result, this hole arises in a region with weak ferromagnetic ordering due to a coexistence of SDW and CDW. It seems quite natural to propose that an average magnetization in this region is proportional to the following we take into account the transfers $\Xi_K \leftrightarrow \Xi_K'$ only. A dimensionless parameter $\chi$ has to be considered as a small quantity, however, as it is noted above, we suppose that $\Delta/\chi << \chi << 1$. One can write a hole energy as

$$
\varepsilon_{\sigma k_{\pm}} \equiv \varepsilon_{\sigma}(k_{\pm}) = \varepsilon(k_{\pm}) + \chi I\sigma \cdot \Theta_k, \quad (82)
$$

Here, $\Theta_k$ is a characteristic function such that $\Theta_k = 1$ if a hole momentum belongs to $\Xi_K$, and $\Theta_k = 0$ in any other case. Thus, the Hamiltonian of pairs corresponding to Eq. (45) has the form

$$
\hat{H}_K = \sum_k \left[ (\varepsilon_{\uparrow k_{\pm}} - \mu)\hat{a}_{k_{\pm}}^{\dagger} \hat{a}_{k_{\pm}} + (\varepsilon_{\downarrow k_{\pm}} - \mu)\hat{a}_{k_{\pm}}^{\dagger} \hat{a}_{k_{\pm}} - \mu S \sum_{k,k'} \hat{U}_K(k-k')\hat{a}_{k_{\pm}}^{\dagger} \hat{a}_{k_{\pm}} \hat{a}_{k'-\downarrow} \hat{a}_{k'-\downarrow} \right] \quad (83)
$$

The summation over $k$ and $k'$ is taken over all range of momenta of the relative motion of $K$ and $K'$–pairs. As it is well known \[23\], spin-dependent kinetic energy in the Hamiltonian of pairs...
results in a redefinition of quasi-particle energies: instead of Eq. (51) we have

\[ \eta_{K\beta}(k) = \sqrt{\varepsilon_{K}^{2} + \Delta_{K}^{2} + \frac{\beta}{2} \chi I \cdot \Theta_{k}}, \tag{84} \]

where \( \beta = \pm 1 \). One can see that \( \eta_{\pm}(k) \geq 0 \) at any \( k \) inside the domain \( \Xi_{K} \), and therefore the corresponding factor \( f_{k} = 1 \) in the zero-temperature limit. On the other hand, inside the domain \( \Xi_{K} \), the condition that \( \eta_{\pm}(k) \geq 0 \) may be satisfied if only

\[ \chi I \sqrt{1 - (2\Delta_{K}/\chi I)^{2}} \leq 2\xi_{K} \leq 2\varepsilon_{K+}, \tag{85} \]

where \( \varepsilon_{K+} \) is an energy width of the subdomain \( \Xi_{K}^{(+)} \). Under the condition Eq. (85), the factor \( f_{k} = 1 \) whereas in the opposite case when

\[ 0 \leq 2\xi_{K} \leq \chi I \sqrt{1 - (2\Delta_{K}/\chi I)^{2}}, \tag{86} \]

this factor is equal to zero because of the equalities \( \eta_{+1}(k) = 0, \eta_{-1}(k) = 1 \) in the zero-temperature limit. Thus, the factor \( f_{k} \) excludes some part of the domain \( \Xi_{K} \) from the sum in the equations Eqs. (56), (57) which determine SC order parameter. It should be noted that the condition \( \Delta \ll \chi I \) allows us to simplify Eqs. (85), (86) as far as \( \sqrt{1 - (2\Delta_{K}/\chi I)^{2}} \approx 1 \).

Let us consider the simplest solution of Eq. (57) when \( \Delta_{K} = \Delta_{-} > 0 \) inside the subdomain \( \Xi_{K}^{(-)} \) and \( \Delta_{K} = -\Delta_{+} < 0 \) inside the subdomain \( \Xi_{K}^{(+)} \). Then, one can reduce the summation over momenta in Eq. (59) to an integration over the energy of the relative motion of pairs according to

\[ \sum_{k \in \Xi_{K}^{(-)}} 1 \Rightarrow S g_{K} \int_{-\varepsilon_{K}^{(-)}}^{0} d\xi, \quad \sum_{k \in \Xi_{K}^{(+)}} 1 \Rightarrow S g_{K} \int_{\chi I/2}^{\varepsilon_{K}^{+}} d\xi \tag{87} \]

where \( \varepsilon_{K}^{(-)} \) is an energy width of the subdomain \( \Xi_{K}^{(+)} \).

Let us assume, for the sake of simplicity, that \( \varepsilon_{K}^{(-)} = \varepsilon_{K}^{(+)} \); then average densities of states, \( g_{K} \) and \( g_{K}^{*} \), are equal to each other, \( g_{K} = g_{K}^{*} \). Also, we assume that \( \Xi_{K}^{(-)} = \Xi_{K}^{(+)} \), then we have simply \( a = 0.5 \). With the help of Eq. (87), the equations Eq. (59) may be rewritten as

\[ \frac{\delta_{+}}{2} + \delta_{+} = \bar{w}_{K} \cdot \left( 1 - \frac{y}{y_{b}} \right) \left( 1 - y \right) \delta_{-} \cdot \ln \frac{2(1 - y)}{\bar{w}_{K}}, \]
\[ \delta_{-} + \frac{\delta_{+}}{2} = \bar{w}_{K} \cdot \left( 1 - \frac{y}{y_{b}} \right) \left( 1 - y \right) \delta_{+} \cdot \ln \frac{2\varepsilon_{0} y_{b} (1 - y)}{\chi I}, \tag{88} \]

where \( \bar{w}_{K} = 3w_{K}/4 \). The second of the equations Eq. (88) is linear, therefore,

\[ \delta_{+} = \gamma \delta_{-}, \tag{89} \]

where, as before, \( \delta_{\pm} = \Delta_{\pm}/\varepsilon_{0}, \varepsilon_{K+} = \varepsilon_{0} \cdot (1 - y) \),

\[ \gamma = \frac{1}{\bar{w}_{K} \cdot \left( 1 - \frac{y}{y_{b}} \right) \left( 1 - y \right) \ln \frac{2\varepsilon_{0} y_{b} (1 - y)}{\chi I} - \frac{1}{2}}. \tag{90} \]

As far as \( \gamma \leq 0 \) by definition and on account of Eq. (3), the solution of the equations Eq. (88) exists within a bounded range of AF energy values,

\[ \delta_{\Xi_{K}^{(-)}} \leq 2 \Delta_{-} \leq 2 \varepsilon_{K+}, \tag{89} \]

Thus, the inequalities Eq. (91) determine that range of doping in which the SC order parameter may be unequal to zero.

The first of the equations Eq. (88) leads to dependent on doping absolute value of the SC order parameter,

\[ \delta_{-} = 2(1 - y) \cdot \exp \left[ -\frac{1}{2 \bar{w}_{K} \left( 1 - y \right) \left( y_{b} - y \right)} \right]. \tag{92} \]

In spite of the fact that the expression Eq. (92) is formally similar to BCS gap, Eq. (92) depends on the coupling constant \( w_{K} \) in essentially complicated way because the parameter \( \gamma_{K} \), Eq. (90), depends on \( w_{K} \) itself. The pre-exponential factor in Eq. (92) is limited by a character value of kinetic energy of the relative motion of the SC pair inside the subdomain \( \Xi_{K}^{(+)} \). This energy itself depends on that to what extent the domain \( \Xi_{K} \) is filled by carriers or, in other words, to what extent the PFC is opened due to a rise of a stripe structure.

Considering hole number conservation inside the domain \( \Xi_{K}^{(+)} \), one can obtain the chemical potential shift in the form of Eqs. (72), (73) where the parameter \( \gamma \) is defined by Eq. (90). In Fig. 11, doping dependence of the condensation energy, Eq. (81), and the gap parameter, Eq. (92), are represented for the value of the coupling constant \( w_{K} = 1 \). It should be noted that, in the case discussed here, this constant has to be considered as a reduced value of the parameter defined by Eq. (43) because of a spatial separation of carriers composing a pair.

As one can see, the SC energy gap takes finite values at certain doping levels corresponding to zero condensation energy. As far as the SC transition temperature \( T_{C} \) is directly connected with the condensation energy, it is obvious that, in contrast to well-known consequence of the BCS theory, namely, \( 2\Delta_{HF}/T_{C} \approx 3.5 \), any universal, independent on doping, relation between \( \Delta \) and \( T_{C} \) is absent. Therefore, as it follows from the consideration of doping dependencies of SC gap and condensation energy presented in Sections IX and X, one must not consider as striking the large values of the ratio \( 2\Delta_{HF}/T_{C} \) observable in underdoped HTSC cuprates [80].

**XI. SUMMARY AND CONCLUSIONS**

Specific quasi-two-dimensional electron structure of HTSC cuprate compounds results from their layered
crystal structure and chemical composition. Namely, if carrier concentration is close to the half-filling, the considerable part of the hole Fermi contour with strong nesting features turns out to be belonging to an extended region of momentum space with hyperbolic metrics. Such an electron structure leads to a possibility of a rise of electron and hole pairs with large, of the order of doubled Fermi momentum, total momentum and comparatively small momentum of the relative motion of the pair \[ \hbar k_{F} \].

Due to hyperbolic metrics of momentum space bounded states may exist both in the case of attraction and repulsion between particles composing the pair. As it follows from the qualitative analysis of the energy dependence of the scattering amplitude, in both cases (repulsion and attraction), the scattering amplitude has two poles. In the case of repulsion, one of the poles, with larger positive energy and infinitesimal positive decay, may be associated with a quasi-stationary state of the relative motion of the pair, whereas the second pole, with smaller positive energy and finite negative decay, may lead to a development of superconducting instability and the imaginary part of this pole may be related to the SC gap. An evaluation of Coulomb repulsion in the electron system with and without QSS’s indicates that, provided that QSS concentration exceeds a certain value, an energy gain is possible and thus incoherent QSS’s may exist resulting in some suppression of one-particle density of states.

Therefore, a rise of such QSS’s may be directly related to the pseudogap state being one of the remarkable features of underdoped HTSC cuprates. It is obvious that positive real part of any of the two poles results in an energy increase when the pairs arise and this increase might hardly be compensated only due to a rise of the SC condensate or sufficiently large QSS concentration. One can assume that such an energy increase is associated with the fact that, to give rise to a creation of the pairs, at least a part of the domain of definition of a momentum of the relative motion of the pair were free of carriers. To set this part free one must annihilate the particles composing the pair inside the FC and then create them in a new pair state outside of the FC. Such a transfer of a number of pairs in momentum space may result in a redistribution of carriers in real space. Total energy increase which is due to this redistribution may be compensated when, as it appears to be just the case being related to hole-doped HTSC cuprates, there is an energy decrease due to partial restoration of AF ordering in hole-depleted regions arisen. An alternation of hole-depleted and enriched regions in real space forms charge and spin spatial structure (irregular and dynamic, generally speaking) of the electron system. Under definite conditions (in particular, in underdoped regime), this structure, associated closely with short-range AF order fluctuations, becomes apparent as quasi-regular static or dynamic 1D stripe structure.

Such phase separation accompanied with a change of a filling of hole states in momentum space may correspond to the minimum of total energy of the electron system. Short-range AF ordering stabilizing stripes due to redistribution of hole pairs is intrinsic but not unique possible attribute of such a self-organization. For example, long-range orbital magnetic ordering \[ \xi_{K} \], known as a flux phase state, may play the same role as well. Although there is not any direct evidence of existence of flux phase state in cuprates, this phase perhaps is associated with the so-called hidden-order-parameter region in the phase diagram \[ \xi_{K} \].

Such a scenario of total energy gain due to carrier redistribution is not the only one. Another one possibility, analogous to that which may be in the case of the superconductivity \[ \xi_{K} \], may be related to the problem here discussed. Namely, one may to consider an increase of AF transition temperature resulting from a change of an excitation energy distribution due to transitions from \( \xi_{K}^{(+)} \) into \( \xi_{K}^{(−)} \).

In any of the cases considered here, a redistribution of carriers in momentum space may result in a rise of new zero-excitation-energy line separating occupied and vacant states in 2D Brillouin zone. We believe that, first of all, a rise of vacant states inside and occupied states outside of the FC must lead to a formation of pairs with total momenta corresponding to the largest areas \( \Xi_{K}^{−} \) and \( \Xi_{K′}^{−} \) with \( K \) and \( K′ \) along the antinodal directions. Such pairs have the largest binding energies and exist up to the temperature, \( T^* \), of the beginning of the pseudogap regime. Then, the lowering of the temperature from \( T^* \) to \( T_{C} \) results in a gradual rise of electron and hole pairs with total momenta having different values and directions and corresponding to some set of domains \( \Xi_{K} \) and \( \Xi_{K′} \). At last, at \( T = T_{C} \), there is a beginning of SC condensation of pairs into the state with the largest binding energy. SC condensation gives a start to the growth of the unoccupied part, \( \Xi_{K}^{(+)} \), of the “antinodal” domain \( \Xi_{K} \) at the expense of a redistribution of carriers both inside and outside of the FC. Thus, as a final result, zero-energy line for pair excitations arises inside the domain \( \Xi_{K} \) (similarly, such a line arises inside the corresponding domain \( \Xi_{K′} \) outside of the FC as well). Just this line may be treated as “pair” Fermi contour (PFC).

The conception of PFC and hyperbolic pairing enables one to explain qualitatively some general features of phase diagram and many surprising experimental data relating to high-\( T_{C} \) cuprates. In particular, a rise of both SC and pseudogap state may be considered as a manifestation of hyperbolic metrics of momentum space and screened Coulomb repulsion between holes. Therefore, both SC gap and pseudogap must have one and the same energy scale and their \( d \)-type “orbital” symmetry, in fact, is determined by the crystal symmetry. A character spatial scale of a pair both in QSS and SC state (the coherence length) is of the order of a few interatomic distances. As an evidence in favor of the PFC conception, one may consider an interpretation of two interesting experiments, relating to examination of electronic spectrum of several of HTSC cuprates with the help of ARPES technique. In the case, when an energy of excited electron energy and finite negative decay, may lead to a development of superconducting instability and the imaginary part of this pole may be related to the SC gap. An evaluation of Coulomb repulsion in the electron system with and without QSS’s indicates that, provided that QSS concentration exceeds a certain value, an energy gain is possible and thus incoherent QSS’s may exist resulting in some suppression of one-particle density of states. Therefore, a rise of such QSS’s may be directly related to the pseudogap state being one of the remarkable features of underdoped HTSC cuprates. It is obvious that positive real part of any of the two poles results in an energy increase when the pairs arise and this increase might hardly be compensated only due to a rise of the SC condensate or sufficiently large QSS concentration. One can assume that such an energy increase is associated with the fact that, to give rise to a creation of the pairs, at least a part of the domain of definition of a momentum of the relative motion of the pair were free of carriers. To set this part free one must annihilate the particles composing the pair inside the FC and then create them in a new pair state outside of the FC. Such a transfer of a number of pairs in momentum space may result in a redistribution of carriers in real space. Total energy increase which is due to this redistribution may be compensated when, as it appears to be just the case being related to hole-doped HTSC cuprates, there is an energy decrease due to partial restoration of AF ordering in hole-depleted regions arisen. An alternation of hole-depleted and enriched regions in real space forms charge and spin spatial structure (irregular and dynamic, generally speaking) of the electron system. Under definite conditions (in particular, in underdoped regime), this structure, associated closely with short-range AF order fluctuations, becomes apparent as quasi-regular static or dynamic 1D stripe structure. Such phase separation accompanied with a change of a filling of hole states in momentum space may correspond to the minimum of total energy of the electron system. Short-range AF ordering stabilizing stripes due to redistribution of hole pairs is intrinsic but not unique possible attribute of such a self-organization. For example, long-range orbital magnetic ordering \( \xi_{K} \), known as a flux phase state, may play the same role as well. Although there is not any direct evidence of existence of flux phase state in cuprates, this phase perhaps is associated with the so-called hidden-order-parameter region in the phase diagram \( \xi_{K} \).

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is near the Fermi level $E_F$ (less than about 100 meV), ARPES data [88] indicate unequivocally that electronic structure has 2D character and FC remains in the well-known form of a square with rounded corners [1]. If an electron is excited far from $E_F$ (about 500 meV) the electronic structure becomes rather 1D than 2D. It should be noted that, truly, a simple cross-shaped form of the FC, in fact, derived not directly from experimental data but offered as a result of motivated speculations based on the simplest 1D stripe model. Thus, the ARPES data [88] evidence the simultaneous existence of both FC and PFC.

Such fairly surprising conclusion is entirely consistent with the concept of the PFC introduced here. In fact, to detect the PFC using ARPES technique one needs an electron excitation with an energy which is essentially less than $E_F$: if excited state is situated near the PFC, the excited electron may easily find a partner to form a pair.

One can believe that the so-called “dip-hump structure” in the ARPES spectra [88] is one more evidence in favour of the concept of PFC: a hump, arising (just in the case of antinodal direction corresponding to the maximal value of $\Xi_K$) at energies which are essentially more than the energy related to the quasi-particle peak, may be connected with the excitations in the form of pairs near the PFC.

Another one unusual feature of HTSC cuprates with $d$-type symmetry of the SC gap can be qualitatively interpreted in the framework of the PFC conception. It is believed that impurity scattering has to lead to essential reducing of $T_C$ because the scattering of a pair into regions of momentum space corresponding to nodal directions means, in fact, a break of the pair. In this sense, nonmagnetic impurities play role similar to that which play magnetic impurities in conventional superconductors. However, the experimental fact is that the HTSC cuprates are weakly sensitive to impurity content.

Thus, there are no experimental data which were confirmed such a “destructive” influence of impurities on the SC state. In this connection, it should be noted that, to reconcile such a statement with the experiment, one has to suppose that any interaction resulting in a scattering of Cooper pair in $d$-type superconductor, including the interaction which leads to a binding in the pair itself, must possess the peculiar feature, namely, the scattering into “nodal regions” has to be more weak in comparison with the scattering into “antinodal regions” (the so-called “forward scattering”) [90, 91, 12]. Using the PFC conception, it is not necessary to consider any peculiar feature of a scattering as far as the scattering inside the domain $\Xi_K$, that is a variation of the direction of $K$–pair relative motion momentum, is, in fact, almost isotropic as in the case of $s$–scattering of Cooper pairs in conventional superconductors.

A problem connected with a strong anisotropy of reversed relaxation times, that is an existence of the so-called “hot” and “cold” spots on the Fermi surface can be qualitatively solved in the framework of the PFC conception too. Indeed, a rise of $K$–pairs results in their free in-plane motion without a change of charge density whereas a character of the interaction of paired carriers may be changed essentially: this interaction, being inside antinodal regions, turns out to be more weak as compared to the interaction of unpaired carriers inside nodal regions [3].

The idea we use here is based, in main, on the fact that the PFC should be “opened” that is, due to a rise of stripes and hyperbolic metrics of momentum space, some piece of the FC turns out to be the same as a line of zero kinetic energy of the relative motion of a hole pair with large momentum. If such a line corresponding to a certain $K$ is close enough to rather large piece of the real FC (such a case may occur, for example, just at $\nu << 1$ and $|2k_F - K| << k_F$) the pairing mechanism here discussed may be possible as well even without any hole redistribution both in momentum space (between the domains $\Xi_K$ and $\Xi_{K'}$) and in real space (that is without a rise of a stripe structure). In this case, the value of $\varepsilon_{K+}$ plays role of a cut-off parameter since it must appear in the arguments of the logarithmic functions in Eq. (63) together with the SC gap parameters $\Delta_-$ and $\Delta_+$. In a sense, the pairing problem becomes analogous to that which arises in the case of Cooper pairing in weak ferromagnets [1].

This statement is consistent with the results presented in Sections III and IV. Namely, at $\nu \to 0$ and $\omega \to 0$ (it is obvious that the parameter $\omega \to 0$ plays role of an energy distance between the FC and the line of zero energy of pair relative motion), the imaginary part of the SC pole of the scattering amplitude (which is proportional to the SC gap parameter) exceeds necessarily the real part of the pole due to a logarithmic singularity of $B_{K2}(\omega)$ at $\omega \to 0$, as one can see from Fig. 5. Thus, one can assume a possibility of the existence of crystals with such a form of the FC which is optimally conforming with the form and energy position of a line of zero kinetic energy of the relative motion of a hole pair with large total momentum (one can consider hyperbolic lines used here as a certain limiting case). As a result, in such crystals, the pairing mechanism discussed here may dominate (possibly, even without AF state in the neighborhood of the SC one and thus without a rise of stripes as it maybe occurs in cuprate compounds with more than one $CuO_2$ plane in the unit cell [13]).

It should be noted that a superconducting state with large $(K \approx 2k_F)$ total pair momentum was previously studied [14] in the framework of the microscopic model of the coexistence of superconductivity and antiferromagnetism or charge-density wave (a structural phase transition: in such a case, the momentum $K \approx 2k_F$ turns into a new vector of the reciprocal lattice). In this model, the state with large pair momentum arises as a result of coexistence of AF ordering and Cooper pairs with zero total momentum. Phenomenologically, such transitions as a break of corresponding symmetry is usually considered in the framework of, for example, SO(5) or SU(4) models.
Zhang et al. have assembled AF and d-wave SC order parameters into a five-dimensional vector and have postulated the symmetry of unified in such a way order parameter under rotations of an SO(5) group. However, to obtain the closed Lie algebra of fermion pairing and particle-hole operators describing antiferromagnetism and superconductivity, one has to consider a more general, than SO(5) symmetry, for example, the SU(4) symmetry. Such more general approach leads directly to a rise of the components of the unified order parameter corresponding to pairs with large, of the order of the AF vector, total momentum. Thus, one may conclude that \( K \) and \( K' \)– pairs introduced in our paper in a microscopic way are fully consistent with rather general symmetry constraints. Note that if the vector \( K \) coincides with the AF vector exactly the SC order parameter due to a rise of \( K \)– pairs and the AF (triplet) order parameter turn out to be connected to another one SC order parameter corresponding to pairs with zero total momentum. A small difference between \( K \) and the AF vector leads to a small total momentum of these pairs. Such pairs may be in singlet spin state (conventional Cooper pairs) or in triplet spin state. The case we discuss in this paper corresponds just to the latter of the two possibilities, namely, triplet AF order coexists with singlet SC order due to \( K \)– pairs and triplet SC order due to the pairs with small total momentum.

The phenomenological approach used here to take account of the influence of AF fluctuations on carrier pairing enables one to interpret qualitatively the key experimental data relating to HTSC cuprates. We believe that the principal conception of hyperbolic pairing and a rise of pair Fermi contour is an inherent feature of cuprate electron system which has to become apparent both in band scheme and in appropriate models of strongly-correlated systems, such as \( t - J \) model with regard for next-nearest-neighbor interactions (the so-called \( t - t' - J \) model), description of the electronic structure.

Acknowledgments

We are deeply grateful to A. F. Andreev, A. M. Dykhne, V. L. Ginzburg, Yu. Kagan, L. V. Keldysh and Yu. E. Lozovik for fruitful discussions. The work was supported, in part, the Russian scientific-educational program “Integration” (projects AO133 and AO155).
FIG. 1: Phase diagram (temperature vs doping level) typical of hole-doped HTSC cuprates.

FIG. 2: Typical of hole-doped HTSC cuprates, hole Fermi contour (FC) as a square with rounded corners (labeled by the Fermi energy, $E_F$) centered at $(\pi, \pi)$ . The domain of definition of momenta of the relative motion of $K$ and $K'$ pairs are denoted as $\Xi_K$ and $\Xi_{K'}$, respectively. Each such a domain consists of two parts, $\Xi_K^{(-)}$, $\Xi_K^{(+)}$ and $\Xi_{K'}^{(-)}$, $\Xi_{K'}^{(+)}$, respectively. Inside the subdomains $\Xi_K^{(-)}$, $\Xi_K^{(+)}$, $\Xi_{K'}^{(-)}$, $\Xi_{K'}^{(+)}$, the energy of the relative motion of corresponding pair measured from the pair chemical potential value $2\mu$ is negative (positive). Total pair momentum is directed along an antinodal direction. The lines separating the subdomains of negative and positive relative motion energy form the pair Fermi contour (PFC). Doping decrease results in an opening of the PFC at two points, $a$ and $a'$, on $k_1$–axis, corresponding to a doping level $x_2$. Then, there is a rise and an extension of the subdomains $\Xi_K^{(-)}$ and $\Xi_{K'}^{(-)}$ accompanied with the corresponding decrease of the subdomains $\Xi_K^{(+)}$ and $\Xi_{K'}^{(+)}$. The PFC shrinks at two points, $b$ and $b'$, on $k_2$–axis, corresponding to a doping level $x_1 < x_2$.  

FIG. 3: Top panel: a sketch of the domains $\Xi_K$, $\Xi_{K'}$ and hole distribution in the cases corresponding to a homogeneous state of the electron system (left top panel) and a stripe state (AF part of a stripe, middle top panel; M part of a stripe, right top panel). Occupied and unoccupied pair states are separated by the PFC. Occupied states inside the domains are shadowed. Bottom panel: relative-motion band diagram for homogeneous state (left bottom panel), AF part of a stripe (middle bottom panel) and M part of a stripe (right bottom panel).
FIG. 4: A sketch of a stripe ordering in momentum space (top panel) and real space (bottom panel). Arrows are hole transitions between AF and M parts of stripes.

FIG. 5: A plot of the function $B_{K2}(\omega)$, Eq. (4.2), schematically.

FIG. 6: Plots of the function $B_{K1}(\omega)$: solid line - Eq. (23), dashed line - Eq. (30). An illustration of the graphic solution of the equation Eq. (14), schematically.

FIG. 7: A comparison of typical HTSC cuprates phase diagram and the graphic solution of the equation Eq. (14), determining the poles of the scattering amplitude (schematically).

FIG. 8: Solutions, $\delta_-$ and $\delta_+$, of the system of equations, Eq. (63), plotted schematically as functions of reduced doping level. Solid (dashed) line corresponds to the value of the effective coupling constant $w_K = 2$ ($w_K = 4/3$).

FIG. 9: Solid line: energy dependence of the coherence factor $v_{kK}^2$ in the case when $\Delta_+ > \Delta_-$ (schematically). Dashed line corresponds to $v_{kK}^2$ without ane chemical potential shift $\mu'$. One can see that such a shift is a direct consequence of the particle number conservation inside the domain $\Xi_K$: this number in normal state (in this Figure: the rectangular area corresponding to the subdomain $\Xi_K^{(-)}$) has to be equal to the area bounded by the solid line.

FIG. 10: Condensation energy, Eq. (81), plotted schematically as a function of reduced doping level. Solid (dashed) line corresponds to the value of the effective coupling constant $w_K = 2$ ($w_K = 4/3$).

FIG. 11: Condensation energy $\varepsilon_c$ (solid line), Eq. (81), calculated in the case of weak ferromagnetic ordering, SC energy gap parameter $\delta_-$ (dashed line), Eq. (92), plotted schematically as functions of reduced doping level. The effective coupling constant $w_K = 1$. 

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