The $\phi$ meson with finite momentum in a dense medium

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The dispersion relation of the $\phi$ meson in nuclear matter is studied in a QCD sum rule approach. In a dense medium, longitudinal and transverse modes of vector particles can have independently modified dispersion relations due to broken Lorentz invariance. Employing the full set of independent operators and corresponding Wilson coefficients up to operator dimension 6, the $\phi$ meson QCD sum rules are analyzed with changing densities and momenta. The non-trivial momentum dependence of the $\phi$ meson mass is found to have opposite signs for the longitudinal and transverse modes. Specifically, the mass is reduced by 5 MeV for the longitudinal mode, while its increase amounts to 7 MeV for the transverse mode, both at a momentum scale of 1 GeV. In an experiment which does not distinguish between longitudinal and transverse polarizations, this could in principle be seen as two separated peaks at large momenta. Taking however broadening effects into account, the momentum dependence will most likely be seen as a small but positive effective mass shift and an increased effective width for non-zero momenta.

I. INTRODUCTION

The study of light vector mesons in a dense medium can provide important insights to our understanding of the origin of hadron masses, which is closely related to the breaking of chiral symmetry in vacuum. Vector mesons are especially well suited for experimental measurements of in-medium effects, as they can decay into dileptons, which do not feel the strong interaction and are therefore less distorted by the presence of the nuclear medium compared to hadronic decay products. Initially, the $\rho$ meson mass shift in nuclear matter was considered to be a suitable probe for the restoration of chiral symmetry at finite density. With the help of QCD sum rules, it was (within certain approximations) possible to relate this mass shift to the reduction of the chiral condensate $\langle \bar{q}q \rangle$ ($q$ here stands for $u$ or $d$ quarks) at finite density [1]. Such a mass shift was later reported in Ref. [2] from measurements of dilepton spectra in 12 GeV pA reactions at the E325 experiment at KEK. It was, however, also realized that QCD sum rules in the $\rho$ meson channel can be satisfied equally well by both mass shifted and broadened $\rho$ meson peaks [3, 4], the latter being obtained for instance in hadronic effective theory calculations [5, 6]. The experimental findings of Ref. [7] point to similar conclusions of a broadened $\rho$ meson without any mass shift.

With the difficulty of drawing any definite conclusions about the behavior of the $\rho$ meson at finite density and its relationship to chiral symmetry, attention has turned to mesons with smaller widths such as the $\omega$ and the $\phi$. About the $\omega$, a lot of experimental work has been done in recent years. See for instance Ref. [8] for a review. On the theoretical side, a suggestion was put forward by one of the present authors that the finite density behavior of the $\omega$ together with the axial-vector meson $f_1(1285)$ could serve as an indicator of the restoration of chiral symmetry in nuclear matter [9]. In this work, we will however focus on the $\phi$ meson and its modification at finite density. Specifically, we study the momentum dependence of the $\phi$ meson energy (e.g. its dispersion relation) at finite density. While the dispersion relation of any particle in vacuum is fixed by Lorentz symmetry, this is no longer the case in a medium, which serves as a specific frame of reference. This can hence lead to a modified dispersion relation at finite density. The $\phi$ meson is in this context presently of particular interest, as its dispersion relation will be studied at the J-PARC E16 experiment, which will start running in 2020 [10].

The main goal of this work is to determine the non-trivial dispersion relation of the $\phi$ meson and to make predictions for the E16 experiment at J-PARC. We furthermore study the longitudinal and transverse polarizations of the $\phi$, which are equal in vacuum, but can behave differently in nuclear matter. For this purpose, we make use of the QCD sum rule method, for which the effect of broken Lorentz invariance is encoded as expectation values of non-scalar QCD operators (see Refs. [11, 12]). These expectation values always vanish in vacuum, but become finite in a hot or dense medium. The most prominent ones are $\langle \bar{q}q^n q \rangle_\rho$, $\langle ST \bar{q} q^n iD^\mu q \rangle_\rho$ and $\langle ST G^{\mu
u}_{\rho} G_\rho^{\mu
u} \rangle_\rho$ (see Ref. [13] for a more complete list), of which the first one vanishes in the vector channel considered here. As will be shown in the later sections of this paper, the second and third play the dominant roles in determining the dispersion relations of the longitudinal and transverse modes of the $\phi$. After a separate QCD sum rule study of the longitudinal and transverse modes, we will discuss what effects the modified dispersion relations may have on future experimental dilepton measurements.

This paper is organized as follows. In Section II, we give a brief description of the formalism of QCD sum rules, which is followed by a discussion of the used input parameters in Section III. Section IV is devoted to the detailed results obtained in this study and to a discussion of potential consequences for experimental measurements. The paper is summarized and concluded in Section V.
II. QCD SUM RULES WITH FINITE THREE-MOMENTUM

For the purpose of studying the φ meson in nuclear matter, let us first consider the correlation function of the vector current with strange quarks \( j_{\mu}(x) = \bar{s}(x)\gamma_\mu s(x) \).

\[
\Pi_{\mu\nu}(\omega, q^2) = i \int d^4x e^{iq\cdot x} \langle T\{j_{\mu}(x) j_{\nu}(0)\} \rangle, \tag{1}
\]

where \( \langle \cdot \rangle_{\rho} \) represents the expectation value taken with respect to the nuclear matter ground state with density \( \rho \). The nuclear medium is assumed to be at rest. In vacuum or in the \( |q^2| \rightarrow 0 \) limit there is only one invariant function in this correlation function, i.e. \( \Pi(\omega^2) = -\frac{1}{\omega^2}\Pi^\mu_\mu \). For finite \( q^2 \) in nuclear matter, however, longitudinal and transverse polarization states are distinguishable and their distinct components are expressed by

\[
\Pi_L(\omega^2, q^2) = \frac{1}{q^2}\Pi_{00}, \tag{2}
\]

\[
\Pi_T(\omega^2, q^2) = -\frac{1}{2}\left( -\frac{1}{q^2}\Pi^\mu_\mu + \Pi_L \right). \tag{3}
\]

In the deep space-like \( q^2 \) region (e.g. \( \omega \rightarrow i\infty \) with \( |q^2| \) held fixed [14]), these are calculable using the operator product expansion (OPE),

\[
\Pi^{\text{OPE}}(\omega^2, q^2) = \sum_n C_n(\omega^2, q^2)(O_n)_{\rho}. \tag{4}
\]

In the time-like \( q^2 \) region, the imaginary part of the correlation function is expressed by the spectral function (at zero temperature),

\[
\rho(\omega^2, q^2) = \frac{1}{\pi}\Im\Pi(\omega^2, q^2). \tag{5}
\]

For discussing the \( q^2 \) dependence, it is convenient to substitute \( \omega^2 \) by \( Q^2 = -q^2 \) and to re-express the OPE in terms of \( Q^2 \) and \( q^2 \), i.e. \( C_n(\omega^2, q^2) \rightarrow C_n(Q^2, q^2) \). After this substitution, it becomes more apparent that the \( q^2 \) dependence can be divided into two types: (i) trivial and (ii) non-trivial momentum dependence. The trivial dependence comes from the \( q^2 \) absorbed in \( Q^2 \), while the non-trivial one comes from terms in which \( q^2 \) cannot be absorbed in \( Q^2 \). Naturally, OPE terms which contain only the \( Q^2 \) dependence do not violate Lorentz symmetry and keep the ordinary form of the dispersion relation: \( \omega^2 = m_\phi^2 + q^2 \). This happens for scalar operators and their Wilson coefficients. On the other hand, a non-trivial \( q^2 \) dependence appears in Wilson coefficients of non-scalar operators and play an important role in medium. They cause the \( \phi \) meson to have not only a modified dispersion relation, \( \omega^2 = m_\phi^2 + q^2 \), but also to have different longitudinal and transverse polarization modes.

With the above change of variables, the remaining \( q^2 \) now represents only the non-trivial momentum dependence. The same substitution is also applicable to the spectral function side. In this work, we employ the following simple “pole + continuum” ansatz,

\[
\rho(s, q^2) \approx \frac{f(q^2)}{4\pi^2} \delta(s - m_\phi^2(q^2)) + \frac{1}{4\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \theta(s - m_0(q^2)). \tag{6}
\]

Here, all non-trivial momentum dependence is assumed to be encoded in the three spectral parameters, \( m_\phi(q^2) \), \( f(q^2) \), and \( m_0(q^2) \), which denote the mass of the \( \phi \), the coupling strength between \( \phi \) and \( j_{\mu}(0) \), and the threshold parameter of the continuum, respectively.

Using the variables \( Q^2 \) and \( q^2 \), the standard dispersion relation which connects the spectral function to the OPE side can be written as

\[
\Pi^{\text{OPE}}(Q^2, q^2) = \int_0^\infty ds \rho(s, q^2) \frac{Q^2}{s + Q^2}. \tag{7}
\]

which is analogous to the vacuum dispersion relation except for the additional \( q^2 \) dependence. Therefore, we can apply the same analysis method as we do in the vacuum case. In other words, we can employ QCD sum rules for studying the \( \phi \) meson in vacuum, at rest in medium, and in medium with finite 3-momentum within the same framework with changing density and 3-momentum. Note that this simplification is not applicable to baryons or charged mesons.

The conventional QCD sum rule analysis, which will be followed here, is based on the Borel transform which is defined as,

\[
\Pi(M^2) = \lim_{Q^2 \rightarrow M^2} \Pi(Q^2) (\frac{Q^2}{s})^n \left( \frac{d^4}{dQ^2} \right)^n (\Pi(Q^2)). \tag{8}
\]

where \( M \) is called Borel mass. The analysis is performed through the following processes. For a given \( \rho \) and \( |q^2| \), we can express the two spectral parameters, \( m_\phi \) and \( f \), as functions of \( M^2 \) and \( s_0 \),

\[
m_\phi(M^2, s_0) = \sqrt{M^2 - \frac{\Pi(\Pi(M^2, s_0))}{\Pi(M^2, s_0)}}, \tag{9}
\]

\[
f(M^2, s_0) = 4\pi^2 M^2 \Pi(M^2, s_0)e^{m_\phi(M^2, s_0)/M^2}. \tag{10}
\]

Here, \( \Pi(M^2, s_0) \equiv \Pi^{\text{OPE}}(M^2, q^2) - \frac{1}{M^2} \int_0^{s_0} ds e^{-s/M^2} \rho(s, q^2) \) and \( \Pi(M^2, s_0) \equiv \partial\Pi(M^2, s_0)/\partial(1/M^2) \). These functions are relevant only inside a so-called Borel window \( (M_{\text{min}}, M_{\text{max}}) \), which is determined by the conditions that the pole contribution is larger than the continuum by 50% and that the dimension 6 contribution of the OPE series is smaller than 10% of the total series. Specifically, we have

\[
M_{\text{max}} : \int_0^{s_0} ds e^{-s/M^2} \rho(s, q^2) > 0.5, \tag{11}
\]

\[
M_{\text{min}} : \frac{\Pi(M^2, q^2)}{\Pi^{\text{OPE}}(M^2, q^2)} < 0.1. \tag{12}
\]
We furthermore define average values of $m_\phi$ and $f$ inside this window as follows:

\[ \overline{m}_\phi(s_0) = \int_{M_{min}}^{M_{max}} dM \frac{m_\phi(M^2, s_0)}{M_{max} - M_{min}}, \]  
\[ \overline{f}(s_0) = \int_{M_{min}}^{M_{max}} dM \frac{f(M^2, s_0)}{M_{max} - M_{min}}. \]  

The next step is to find a threshold value $\overline{s}_0$ that makes the curve $m_\phi(M^2, s_0)$ the flattest. This is achieved by finding the minimum of $\chi_2^2$, defined as

\[ \chi_2^2(s_0) = \int_{M_{min}}^{M_{max}} dM \frac{(m_\phi(M^2, s_0) - \overline{m}_\phi(s_0))^2}{M_{max} - M_{min}}. \]

Once $\overline{s}_0$ is determined, the other spectral parameters are obtained by their average values at this threshold value i.e. $\overline{m}_\phi(\overline{s}_0)$ and $\overline{f}(\overline{s}_0)$. The above process is repeated for all $\rho$ and $|\vec{q}|$ values to be investigated.

### III. INPUT PARAMETERS: NUMERICAL VALUES AND UNCERTAINTIES

The complete set of operators relevant to the OPE of the vector channel up to dimension 6, discussed in Ref. [15], is given by,

**Scalar Operators**

\[ \bar{s}s, \]  
\[ G_0 \equiv \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}, \]  
\[ j^2 \equiv g^2 (D_\mu G^a_{\alpha\mu}) (D_\nu G^a_{\beta\nu}), \]  
\[ j^2 \equiv g^2 \bar{s}t^a \gamma_\mu s \bar{s}t^a \gamma_\mu s, \]

**Non-Scalar(Twist) Operators**

\[ A_{\alpha\beta} \equiv gST \bar{s}(D_\mu G_{\alpha\mu}) \gamma_\beta s, \]  
\[ B_{\alpha\beta} \equiv gST \bar{s}(D_\mu \bar{G}_{\beta\mu}) \gamma_\alpha s, \]  
\[ C_{\alpha\beta} \equiv m_s ST \bar{s}D_\alpha D_\beta s, \]  
\[ F_{\alpha\beta} \equiv ST \bar{s}\gamma_\alpha D_\beta s, \]  
\[ H_{\alpha\beta} \equiv g^2 ST \bar{s}t^a \gamma_\alpha s \bar{s}t^a \gamma_\beta s, \]  
\[ K_{\alpha\beta\gamma\delta} \equiv ST \bar{s}\gamma_\alpha D_\beta \bar{s}\gamma_\delta D_\gamma s, \]  
\[ G_{2\alpha\beta} \equiv \frac{\alpha_s}{\pi} ST G^a_{\alpha\mu} G^a_{\beta\mu}, \]  
\[ X_{\alpha\beta} \equiv \frac{\alpha_s}{\pi} ST G^a_{\mu\nu} D_\beta D_\gamma G^a_{\mu\nu}, \]  
\[ Y_{\alpha\beta} \equiv \frac{\alpha_s}{\pi} ST G^a_{\alpha\mu} D_\mu D_\gamma G^a_{\beta\nu}, \]  
\[ Z_{\alpha\beta} \equiv \frac{\alpha_s}{\pi} ST G^a_{\alpha\mu} D_\mu D_\gamma G^a_{\beta\nu}, \]  
\[ G_{4\alpha\beta\gamma\delta} \equiv \frac{\alpha_s}{\pi} ST G^a_{\alpha\mu} D_\beta D_\gamma G^a_{\delta\mu}. \]

$ST$ here stands for the operation that makes the Lorentz indices symmetric and traceless. Non-scalar operators can also be categorized according to their twist (= dimension - spin).

The expectation values of most of the above operators are not well known. Therefore, we often have to rely on assumptions and approximations which can give no more than order of magnitude estimates. In this section, we will succinctly discuss these techniques used to evaluate the various condensates appearing in this work. The final values and uncertainties of all the input parameters are summarized in Table I.

We will throughout this work make use of the linear density approximation, which can give a qualitatively good description to the condensates at the level of the normal nuclear matter density $\rho_0$. This approximation can be expressed as,

\[ \langle O \rangle_\rho \approx \langle O \rangle_0 + \rho \langle O \rangle_N, \]

where $\langle O \rangle_0 = \langle 0|O|0 \rangle$ and $\langle O \rangle_N = \langle N(0)|O|N(0) \rangle$, respectively. Here, $|N(q')\rangle$ is a one-nucleon state normalized as $\langle N(q')|N(q')\rangle = (2\pi)^3 \delta^{(3)}(q' - q')$. Lorentz indices of the nucleon matrix elements will be projected onto the nucleon four-momentum $p_\mu = (M_N, 0, 0, 0)$. Therefore, throughout this paper, non-scalar condensates whose Lorentz indices are omitted, should be understood to be defined as $\langle O_{\mu_1...\mu_2} \rangle_N \equiv O \cdot ST (p_{\mu_1} ... p_{\mu_2})$.

Under the above basic assumption, the condensates of the relevant operators are estimated as discussed below.

1. Scalar matrix elements

The matrix elements of scalar operators have been frequently discussed in the literature (see for instance Ref. [13] and the references cited therein). We thus here only give the final expressions, which read

\[ \langle \bar{s}s \rangle_\rho = 0.8 \langle \bar{q}q \rangle_0 + \frac{\sigma_{\pi N}}{2m_s}, \]
\[ \langle G_0 \rangle_\rho = \langle G_0 \rangle_0 + \frac{8}{9} \rho \sigma_{\pi N} + \frac{\sigma_{\pi N} - M_N}{2m_s}, \]

where $\sigma_{\pi N} = 2m_s\langle \bar{q}q \rangle_N$ and $\sigma_{\pi N} = m_s \langle \bar{s}s \rangle_N$. The ratio $\langle \bar{s}s \rangle_0 / \langle \bar{q}q \rangle_0 \approx 0.8$ is taken from an old QCDSR analysis [16]. For the scalar four-quark condensates, we have

\[ \langle j^2 \rangle_\rho = (4\pi\alpha_s)^2 \sum \langle \bar{q}_\gamma q_a \rangle_\rho \sim 0, \]
\[ \langle j^2 \rangle_\rho = \frac{16}{36} \langle \bar{s}s \rangle^2_\rho, \]
\[ \langle \bar{s}s \rangle_\rho = -\frac{16}{36} 4\pi\alpha_s \langle \bar{s}s \rangle^2_\rho. \]

The condensate on the first line is ignored here because it is proportional to $\alpha_s^2$ by use of the equation of motion. The others are estimated using the vacuum saturation approximation, which is assumed to hold at finite density in the same way as in vacuum.
2. Twist-2 matrix elements

Matrix elements of twist-2 operators can be related to the moments of parton distribution functions measured in DIS experiments [17],

\[ F = \frac{1}{2M_N} A_{2}^{s}, \]

\[ K = \frac{1}{2M_N} A_{4}^{s}, \]

\[ G_{2} = -\alpha_{s} \frac{A_{2}^{g}}{\pi M_{N}}, \]

\[ G_{4} = \alpha_{s} \frac{A_{4}^{g}}{\pi M_{N}}, \]

where \( A_{n}^{q/g} \) is the \( n \)-th moment of the strange quark/gluon distribution function of the nucleon. The values of \( A_{2}^{s}, A_{4}^{s}, A_{2}^{g}, \) and \( A_{4}^{g} \) are listed in Table I. They are extracted from the parton distributions provided in Ref. [18] (see Ref. [13] for more details).

3. Quark twist-4 matrix elements

The \( A, B, C, \) and \( H \) condensates were recently estimated in Ref. [19], making use of the general assumption \( \langle s \Gamma O \rangle_{N} \approx \langle \bar{q} \Gamma q \rangle_{N} \frac{A_{4}^{s}}{A_{2}^{s}} \). Here, we just show the final expressions, and refer the interested reader to Ref. [19].

\[ A = \frac{1}{2M_N} K_{a}^{s} \frac{A_{2}^{s}}{A_{2}^{s}}, \]

\[ B = \frac{1}{2M_N} K_{b}^{s} \frac{A_{2}^{s}}{A_{2}^{s}}, \]

\[ C = -m_s c_s^2 \frac{A_{2}^{s}}{A_{2}^{s}}, \]

\[ H = \frac{1}{2M_N} (K_{u}^{s} - \frac{1}{2} K_{ud}^{s}) \left( \frac{A_{2}^{s}}{A_{2}^{s}} \right)^{2}. \]

\( e_s^2 \) is the second moment of the twist-3 strange quark distribution function and \( K_{a}^{s}, K_{b}^{s}, K_{u}^{s}, \) and \( K_{ud}^{s} \) denote matrix elements of up or down quark twist-4 operators. Six different sets of \( K_{a}^{s}, K_{b}^{s}, \) and \( K_{u}^{s} \) are given in Ref. [20]. For their numerical values, we simply take the average over all the six sets for each parameter. Their uncertainty is estimated as half of the difference between maximum and minimum of all sets.

4. Gluon twist-4 matrix elements

The \( X, Y, \) and \( Z \) condensates are generally not well known. To have a rough idea on their numerical values and systematic uncertainties, we average two independent estimation methods and define the uncertainty as half of the difference between them. Specifically, we have

\[ (X_1, Y_1, Z_1) = \left( \frac{1}{2M_N}, \frac{0.32}{8.1}, \frac{3}{2M_N} \right), \]

\[ (X_2, Y_2, Z_2) = \left( -\frac{(G_0)_N}{4}, \frac{3G_2 + (G_0)_N}{48}, 3Y_2 \right), \]

where \( M^0_N \) denotes nucleon mass in the chiral limit [21]. The first estimate, \((X_1, Y_1, Z_1)\), is taken from Ref. [21]. The second one, \((X_2, Y_2, Z_2)\), is based on the method proposed in Ref. [11],

\[ \langle D_{\mu} G_{\rho \sigma} \ldots \rangle_N \approx -i P_{\mu}^{g} \langle G_{\rho \sigma} \ldots \rangle_N \approx -i \frac{1}{2} p_{\mu} \langle G_{\rho \sigma} \ldots \rangle_N. \]

Here, \( P_{\mu}^{g} \) denotes the average momentum of the gluon in the nucleon, which is assumed to carry half of the nucleon momentum \( p_{\mu} \). In case of \( Z \), however, the two estimates give quite similar values, so we just pick the first estimate because its uncertainty (which is related to \( K_{u}^{s} \)) is about 20 times larger than that of the second.

### Table I: Input parameters, given at a renormalization scale of 1 GeV.

| input parameter | value (uncertainty) | reference |
|-----------------|---------------------|-----------|
| \( \alpha_s \) | 0.472(0.024) \(^a\) | [22] |
| \( m_s \) | 0.124(0.011) GeV \(^b\) | [23] |
| \( \rho_0 \) | 0.17 fm \(^c\) | |
| \( M_N \) | (0.93827 + 0.93957)/2 GeV | [22] |
| \( M^0_N \) | 0.75 GeV | [21] |
| \( \langle \bar{q} q \rangle_0 \) | -(0.272)/1.35 GeV | [24] |
| \( \langle G_0 \rangle_0 \) | 0.012(0.004) GeV | [25, 26] |
| \( \sigma_{\pi N} \) | 0.045(0.0036) GeV | [27] |
| \( \sigma_{s N} \) | 0.06(0.007) GeV | [23] |
| \( A_2^g \) | 0.784(0.017) | |
| \( A_2^s \) | 0.053(0.013) | |
| \( A_2^g \) | 0.367(0.023) | [13] |
| \( A_2^s \) | 0.00121(0.00044) | |
| \( A_4^g \) | 0.0208(0.0023) | |
| \( e_2^s \) | 0.00155(0.00318) | |
| \( K_{a}^{s} \) | 0(1.73) GeV \(^d\) | |
| \( K_{b}^{s} \) | -0.057(0.26) GeV \(^d\) | |
| \( K_{u}^{s} \) | -0.411(1.73) GeV \(^d\) | |
| \( K_{ud}^{s} \) | -0.083 GeV \(^d\) | |
| \( X \) | 0.103(0.0814) GeV | |
| \( Y \) | -0.0094(0.0094) GeV | |

\(^a\) Eq. (9.4) in [22] with \( \frac{A_{(3)}^{G}}{2M_N} = \frac{A_{(2)}^{G}}{2M_N} = \frac{332 \pm 17}{2M_N} \) MeV is used at the renormalization point \( \mu = 1 \) GeV.

\(^b\) Multiplied by 1.35 to rescale to \( \mu = 1 \) GeV [22].

\(^c\) Divided by 1.35 because \( m_s \langle \bar{q} q \rangle \) is an RG invariant.

### IV. RESULTS

#### A. Spectral parameters \( (m_\phi, f, s_0) \)

The main result of this work is the computed momentum dependence of the three spectral parameters \( [m_\phi(q^2), f(q^2), s_0(q^2)] \) at normal nuclear matter density. In Fig. 1 we plot these parameters up to \(|q^2| = 2.0 \)
GeV together with their vacuum values. The parameters exhibit a common behaviour at finite density. First, all of them are shifted negatively at zero momentum. This is caused primarily by the modification of the dimension-4 scalar condensates, shown in Eqs. (33) and (34), the magnitude of the shift being especially sensitive to the value of the strange sigma term $\sigma_{sN}$ [28]. The transverse mode parameters then increase, while the longitudinal ones decrease with growing $|\vec{q}|$.

The results shown in Fig. 1 have rather large uncertainties, and the modification of $\sigma_{sN}$ (see Ref. [28] for a detailed discussion). A fit to the dispersion relation curves then gives $b_T = 0.0067 \pm 0.0034 \, \text{GeV}^{-2}$ and $b_L = -0.0048 \pm 0.0008 \, \text{GeV}^{-2}$.

FIG. 1: Plot of $m_\phi$, $f_\phi$, and $s_\phi$ as a function of $|\vec{q}|$. The black dots denote the vacuum results. Blue (red) lines show the results for the transverse (longitudinal) mode.

Furthermore, we investigated which condensates primarily determine the 3-momentum dependence of $\Delta m_\phi(q^2)$ for both polarization modes. For this purpose, we simply set each condensate to zero and compute how $\Delta m_\phi(q^2)$ changes. As a result, we found that only the two dimension-4 twist-2 condensates, i.e. $F$ and $G_2$, significantly change the momentum dependence as illustrated in Fig. 2. It can be seen in this figure that, for the longitudinal mode, the $F$ term has practically no effect while the negative mass shift is almost completely caused by the $G_2$ term. The behavior of the transverse mode, on the other hand, results from a large positive contribution from the $F$ term, which is reduced somewhat by the $G_2$ term. All other condensates only have minor effects below $|\vec{q}| = 2 \, \text{GeV}$.

FIG. 2: The effects of of the non-scalar condensates $F$ and $G_2$ on $\Delta m_\phi(q^2)$. The upper (lower) plot shows the behavior of the longitudinal (transverse) mode.

in the same way and the results are shown as bands in Fig. 1. As can be observed there, the tendency of the momentum dependence is maintained even when taking into account the full error ranges of the various input parameters.

Both longitudinal and transverse modes exhibit an essentially quadratic behavior with respect to momentum $q$. Therefore, it is worthwhile to recast and parametrize the $m_{\phi,L/T}(q)$-plot of Fig. 1 into the following simple formula:

$$
\frac{m_{\phi,L/T}(q^2)}{m_{\phi,vac}^L/q^2} = 1 + \left( a + \frac{b_L/T}{|q|^2} \right) \frac{p}{\rho_0}.
$$

From our result at zero momentum, we have $m_{\phi, vac} = 1.020 \, \text{GeV}$ and $a = -0.0087$, which strongly depends on the chosen value of $\sigma_{sN}$ (see Ref. [28] for a detailed discussion). A fit to the dispersion relation curves then gives $b_T = 0.0067 \pm 0.0034 \, \text{GeV}^{-2}$ and $b_L = -0.0048 \pm 0.0008 \, \text{GeV}^{-2}$.

The total uncertainty of $\Delta m_\phi(q^2)$ is estimated as $\sqrt{\sum_i \left( \Delta \delta m_\phi \right)^2}$, for completeness, uncertainties of the other spectral parameters are computed in the same way and the results are shown as bands in Fig. 1. As can be observed there, the tendency of the momentum dependence is maintained even when taking into account the full error ranges of the various input parameters.

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$$

From our result at zero momentum, we have $m_{\phi, vac} = 1.020 \, \text{GeV}$ and $a = -0.0087$, which strongly depends on the chosen value of $\sigma_{sN}$ (see Ref. [28] for a detailed discussion). A fit to the dispersion relation curves then gives $b_T = 0.0067 \pm 0.0034 \, \text{GeV}^{-2}$ and $b_L = -0.0048 \pm 0.0008 \, \text{GeV}^{-2}$.

The results shown in Fig. 1 have rather large uncertainties coming from the errors of $m_{\tilde{s}s}, \alpha_s, \langle \tilde{s}s \rangle_0$ and $\langle G_0 \rangle_0$, which determine the vacuum spectral parameters, and the modification of $\langle \tilde{s}s \rangle_\rho$ and $\langle G_0 \rangle_\rho$ at finite $\rho$. These uncertainties, however, only lead to an overall shift at zero momentum. Because our main interest is the momentum dependence of $m_\phi$ which is governed by the non-scalar condensates, we investigated the total uncertainty of $\Delta m_{\phi}(q^2) = m_{\phi}(q^2) - m_{\phi}(0)$ for which such an effect is reduced. The error of all contributing parameters (here generally denoted as $\delta a_i$) is given in Table 1. The total uncertainty is estimated as $\sqrt{\sum_i \left( \Delta \delta m_\phi \right)^2}$. For completeness, uncertainties of the other spectral parameters are computed in the same way and the results are shown as bands in Fig. 1. As can be observed there, the tendency of the momentum dependence is maintained even when taking into account the full error ranges of the various input parameters.

Both longitudinal and transverse modes exhibit an essentially quadratic behavior with respect to momentum $q$. Therefore, it is worthwhile to recast and parametrize the $m_{\phi,L/T}(q)$-plot of Fig. 1 into the following simple formula:

$$
\frac{m_{\phi,L/T}(q^2)}{m_{\phi,vac}^L/q^2} = 1 + \left( a + \frac{b_L/T}{|q|^2} \right) \frac{p}{\rho_0}.
$$

From our result at zero momentum, we have $m_{\phi, vac} = 1.020 \, \text{GeV}$ and $a = -0.0087$, which strongly depends on the chosen value of $\sigma_{sN}$ (see Ref. [28] for a detailed discussion). A fit to the dispersion relation curves then gives $b_T = 0.0067 \pm 0.0034 \, \text{GeV}^{-2}$ and $b_L = -0.0048 \pm 0.0008 \, \text{GeV}^{-2}$.

Furthermore, we investigated which condensates primarily determine the 3-momentum dependence of $\Delta m_{\phi}(q^2)$ for both polarization modes. For this purpose, we simply set each condensate to zero and compute how $\Delta m_{\phi}(q^2)$ changes. As a result, we found that only the two dimension-4 twist-2 condensates, i.e. $F$ and $G_2$, significantly change the momentum dependence as illustrated in Fig. 2. It can be seen in this figure that, for the longitudinal mode, the $F$ term has practically no effect while the negative mass shift is almost completely caused by the $G_2$ term. The behavior of the transverse mode, on the other hand, results from a large positive contribution from the $F$ term, which is reduced somewhat by the $G_2$ term. All other condensates only have minor effects below $|\vec{q}| = 2 \, \text{GeV}$.

FIG. 1: Plot of $m_\phi$, $f_\phi$, and $s_\phi$ as a function of $|\vec{q}|$. The black dots denote the vacuum results. Blue (red) lines show the results for the transverse (longitudinal) mode.

FIG. 2: The effects of of the non-scalar condensates $F$ and $G_2$ on $\Delta m_\phi(q^2)$. The upper (lower) plot shows the behavior of the longitudinal (transverse) mode.
B. Polarization-averaged spectra with finite widths

In most experiments, dilepton measurements have so far not distinguished the two polarization modes. Thus, in general, the sum of both modes will be measured together with the vacuum peak in polarization-averaged spectra\(^1\). Moreover, even though we have assumed the \(\phi\) meson width to be zero in our analysis, this is not the case in reality (see for instance Refs. [30, 31]). Thus, to have a more realistic idea about the behavior of the dilepton spectrum at normal nuclear matter density and non-zero momentum, we artificially introduce a width using the relativistic Breit-Wigner form,

\[
\rho(s) = \frac{f(q^2)}{4\pi^2} \frac{\sqrt{s}\Gamma/\pi}{(s - m^2_\phi(q^2))^2 + s\Gamma^2},
\]

where the values of \(m_\phi(q^2)\) and \(f(q^2)\) are taken from our results. The in-medium width is set to \(\Gamma = 15.3\) MeV, the value reported in the E325 experiment [29]. We then fit the polarization-averaged peak, \(\frac{1}{2}(\rho_L(\omega) + 2\rho_T(\omega))\), using a single peak which has the same Breit-Wigner form. These are shown in Fig. 3 together with the vacuum peak \((\Gamma_{\text{vac}}=4.26\) MeV) for selected momenta.

The effective mass and width extracted from the fit are plotted as a function of \(|q^2|\) in Fig. 4. As can be observed there, the mass shifts of the two modes partially cancel each other. Because there are two transverse and only one longitudinal mode, the average mass however tends to the transverse side and is thus positive. Furthermore, as the two components move away from each other, this leads to an increasing width with increasing momentum.

![FIG. 3: Polarization-averaged spectral function (pink, dashed line), its single peak fit (green line) at normal nuclear matter density. The vacuum \(\phi\) meson peak is shown as a black dotted line for comparison.](image)

![FIG. 4: Effective mass (upper plot) and width (lower plot) of the single peak fit, shown as a function of \(|q^2|\). In the upper plot, the central values of the transverse (longitudinal) masses are shown as blue (red) dashed lines for comparison.](image)

C. Discussion

Let us discuss potential implications of the obtained results for future dilepton measurements in the \(\phi\) meson mass region. As can be seen in the lower right plot of Fig. 3, due to the opposite behavior of the longitudinal and transverse modes, the (angle averaged) dilepton spectrum develops a double peak for momenta above about 1 GeV. Care is however needed when applying this result to experimental dilepton spectra, as \(\phi\) mesons with large momenta likely travel outside of the nucleus before they decay into dileptons, which are therefore not strongly affected by the dense medium. Indeed, in the E325 experiment a finite density effect was only observed for dileptons with \(\beta\gamma = \frac{|q^2|}{m}\) values smaller than 1.25 [29]. We therefore expect the average increase of the mass (green curve of the upper plot in Fig. 4) and the increase of the width (lower plot in Fig. 4) to be the most easily and likely detectable finite momentum effects.

We, however, stress that for independently measured longitudinal and transverse spectra, the mass shifts are larger. It would hence be interesting to measure not only the angle averaged dileptons but also their angular distri-

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\(^1\) In an actual experiment such as that reported in Ref. [29], \(\phi\) mesons are produced in nuclei with a finite size. A large fraction of them decay into dileptons only after they have moved outside of the dense nucleon region. Such dileptons will only contribute as vacuum peaks to the observed spectrum.
V. SUMMARY AND CONCLUSIONS

In this paper, we have analyzed the strange quark vector channel at finite density and momentum in a QCD sum rule approach. This was done with the goal of studying the dispersion relation of the longitudinal and transverse modes of the $\phi$ meson in nuclear medium. A linear combination of them will be measured at the E16 experiment at J-PARC. We found that both longitudinal and transverse dispersion relations are modified significantly compared with their vacuum form. At a momentum scale of 1 GeV, the longitudinal $\phi$ meson mass is reduced by 5 MeV, while its transverse counterpart is increased by 7 MeV. With a simultaneously increasing width, it is however unlikely that the two modes will be seen as separate peaks. Instead, their effect might only be detectable as an increasing effective mass with width increasing momentum. Assuming the width to be fixed at 15.3 MeV in nuclear matter, we estimate the mass shift of the combined (one longitudinal and two transverse modes) $\phi$ meson peak at a momentum of 1 GeV to be about 4 MeV, while the width can be expected to increase by about 7 MeV at the same momentum. It remains to be seen whether e.g. the E16 experiment will have a sufficient resolution to detect these effects.

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APPENDIX

In this appendix, we give the principle formulas needed for the numerical analyses of this paper. They can be obtained by substituting the OPE results given in Ref. [15] into Eqs.(2) and (3). For non-scalar operators, $\hat{O}$ denotes $\mathcal{O} \times (M_N)^s$ where $s$ is spin of the operator.

\begin{equation}
\Pi_L(Q^2, q^2) = -\frac{3m_s^2}{2\pi^2Q^2} - \frac{1 + \alpha_s/\pi}{4\pi^2} \ln \frac{Q^2}{\mu^2} + \left(2 - \frac{8m_s^2}{3Q^2} \right) \frac{m_s\langle ss \rangle}{Q^4} + \left(\frac{1}{12} + \frac{m_s^2}{9Q^2} \right) \frac{\langle G^2 \rangle}{Q^4} + \left(\frac{1}{81} + \frac{2\ln \frac{Q^2}{\mu^2}}{27} \right) \frac{\langle j^2 \rangle}{Q^6} - \frac{2(\hat{\sigma}^2)}{Q^6} \\
+ \frac{4(\hat{s}j\bar{s})}{9Q^6} + \left(2 - \frac{3m_s^2}{Q^2} \right) \frac{\hat{F}}{Q^4} - \frac{10\hat{K}}{3Q^6} + \left(\frac{3}{4} - \frac{11m_s^2}{12Q^2} \right) \frac{\hat{A}}{Q^4} - \left(\frac{1}{3} + \frac{4m_s^2}{3Q^2} \right) \ln \frac{Q^2}{\mu^2} \frac{\hat{G}_2}{Q^4} + \left(\frac{205}{216} - \frac{11}{36} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{G}_4}{Q^6} + \frac{\hat{A}}{2Q^6} - \frac{5\hat{B}}{2Q^6} + \frac{7\hat{C}}{Q^6} + \frac{2\hat{H}}{Q^6} + \left(\frac{1}{6} + \frac{5}{24} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{X}}{Q^6} - \left(\frac{49}{144} + \frac{1}{8} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{Y}}{Q^6} - \left(\frac{11}{16} + \frac{5}{8} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{Z}}{Q^6},
\end{equation}

\begin{equation}
\Pi_T(Q^2, q^2) = \Pi_L(Q^2, 0) + \frac{\hat{q}^2}{Q^2} \left\{ - \left(4 - \frac{9m_s^2}{Q^2} \right) \frac{\hat{F}}{Q^4} + \frac{18\hat{K}}{Q^6} - \left(\frac{7}{6} - \frac{11m_s^2}{Q^4} \right) \frac{\hat{A}}{Q^4} - \left(\frac{2}{3} + 3m_s^2 \right) \ln \frac{Q^2}{\mu^2} \right\} \frac{\hat{G}_2}{Q^4} - \left(\frac{559}{120} - \frac{33}{20} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{G}_4}{Q^6} - \frac{3\hat{A}}{2Q^6} + \frac{7\hat{B}}{2Q^6} - \frac{5\hat{C}}{Q^6} - \frac{4\hat{H}}{Q^6} - \left(\frac{1}{5} + \frac{7}{24} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{X}}{Q^6} + \left(\frac{149}{240} + \frac{1}{24} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{Y}}{Q^6} - \left(\frac{239}{240} + \frac{7}{8} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{Z}}{Q^6} \right\} + \frac{\hat{q}^4}{Q^4} \left\{ \left(\frac{181}{45} - \frac{22}{15} \ln \frac{Q^2}{\mu^2} \right) \frac{\hat{G}_4}{Q^6} - \frac{16\hat{K}}{Q^6} \right\}.
\end{equation}

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