Seesaw Mechanism and Its Implications

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Abstract

The seesaw mechanism is introduced and some of its different realizations and applications are discussed. It is pointed out how they can be used to understand the bi-large mixing patterns among neutrinos in combination with the assumptions about high scale physics such as grand unification or quasi-degeneracy of neutrino masses.

I. INTRODUCTION

The discovery of neutrino masses and mixings has been an important milestone in the history of particle physics and rightly qualifies as the first evidence for new physics beyond the standard model. The amount of new information on neutrinos already established from various neutrino oscillation searches has provided very strong clues to new symmetries of particles and new directions for unification. Enough puzzles have emerged making this field a hotbed for theory research with implications ranging all the way from supersymmetry and grand unification to cosmology and astrophysics.

A major cornerstone for the theory research in this field has been the seesaw mechanism introduced 25 years ago in four independently written papers [1] to understand why neutrino masses are so much smaller than the masses of other fermions of the standard model. Even though there was no solid evidence for neutrino masses then, there were very well motivated extensions of the standard models that led to nonzero masses for neutrinos. It was therefore incumbent on those models that they have a mechanism for understanding why upper limits on neutrino masses known at that time were so small and the seesaw mechanism was introduced in the context of specific such models in the year 1979 e.g. horizontal, left-right and SO(10) models to achieve this goal. A general operator description of small neutrino mass without any specific model was written down the same year [2]. A very minimal nonsupersymmetric SO(10) model was constructed soon after as an application [3]. It was clear from this early enthusiasm about the idea that if the experimental evidence for neutrino masses ever appeared then, seesaw mechanism would be a major tool in understanding its various ramifications. As we see below, this has indeed turned out to be the case.
II. SEESAW MECHANISM

To appreciate the simplicity and beauty of the seesaw mechanism, let us start with a discussion of neutrino mass in the standard model. It is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ group under which the quarks and leptons transform as follows: Quarks $Q^T = (u_L, d_L)(3, 1, \frac{2}{3})$; leptons $L^T = (\nu_L, e_L)(1, 2, -\frac{1}{2})$; Color Gauge Fields $G_a(8, 1, 0)$; Weak Gauge Fields $W^\pm, Z, \gamma(1, 3 + 1, 0)$.

The electroweak symmetry $SU(2)_L \times U(1)_Y$ is broken by the vacuum expectation of the Higgs doublet $< H_0 > = v_{wk} \approx 246$ GeV, which gives mass to the gauge bosons and the fermions, all fermions except the neutrino. The model had been a complete success in describing all known low energy phenomena, until the evidence for neutrino masses appeared.

Note that there is no right handed neutrino in the standard model and this directly leads to the fact that neutrinos are massless at the tree level. This result holds not only to all orders in perturbation theory but also when nonperturbative effects are taken into account due to the existence of an exact B-L symmetry of the standard model. It would therefore appear that nonzero neutrino mass ought to be connected to breaking of B-L symmetry.

A simple way to generate neutrino masses is to introduce right handed neutrinos $N_R$, one per family into the standard model. The standard model Lagrangian now allows for a new Yukawa coupling of the form $h_\nu \tilde{L}HN_R$ which after electroweak symmetry breaking leads to a neutrino mass $\sim h_\nu v_{wk}$. Since $h_\nu$ is expected to be of same order as the charged fermion couplings in the model, this mass is much too large to describe neutrino oscillations. Luckily, since the $N_R$’s are singlets under the standard model gauge group, they are allowed to have Majorana masses unlike the charged fermions. We denote them by $M_R N_R^T C^{-1} N_R$ (where $C$ is the Dirac charge conjugation matrix). The masses $M_R$ are not constrained by the gauge symmetry and can therefore be arbitrarily large (i.e. $M_R \gg h_\nu v_{wk}$). This together with mass induced by Yukawa couplings (called the Dirac mass) leads to a the mass matrix for the neutrinos (left and right handed neutrinos together) which has the form

$$M_\nu = \begin{pmatrix} M_D^0 & M_D^T \\ M_D & M_R \end{pmatrix}$$  \hspace{1cm} (1)

where $M_D$ and $M_R$ are $3 \times 3$ matrices. Diagonalizing this mass matrix, one gets the mass matrix for the light neutrino masses to be as follows:

$$M_\nu = -M_D^T M_R^{-1} M_D$$  \hspace{1cm} (2)

Since as already noted $M_R$ can be much larger than $M_D$ which is likely to be of order $= h_\nu v_{wk}$, one finds that $m_\nu \ll m_{e,u,d}$ very naturally. This is known as the seesaw mechanism [1] and it provides a natural explanation of why neutrino masses are small.

Seesaw mechanism of course raises its own questions:

- what is the scale of $M_R$ and what determines it ?
- Is there a natural reason for the existence of the right handed neutrinos ?
- Is the seesaw mechanism by itself enough to explain all aspects of neutrino masses and mixings.
Below, we try to answer some of these questions and discuss how far one can go towards explaining observations.

### III. WHY SEESAW MECHANISM IS SO APPEALING?

Even though the standard model has enjoyed incredible success in explaining all low energy observations, it has long been recognized that it cannot be a complete theory due to many issues it leaves unaddressed, e.g. the gauge hierarchy problem, strong CP problem as well as the fermion mass and mixing problem etc. Besides, it also has some aesthetic inadequacies that cry out for new physics. In the latter category are such questions as (i) an omission of the right handed neutrino which simple quark lepton symmetry observed in weak interaction would have demanded; (ii) a somewhat adhoc definition of the electric charge in terms of an U(1) charge called “weak hypercharge Y” i.e. $Q = I_{3L} + \frac{Y}{2}$ and (iii) the origin of parity violation. It turns out that inclusion of the three right handed neutrinos helps to remove the aesthetic cloud that hangs over the standard model while maintaining all its phenomenological successes.

The fact that the addition of one right handed neutrino per generation to the standard model restores quark lepton symmetry is obvious. But in 1973, Pati and Salam pointed out that there is a compelling symmetry reason for including the right handed neutrino if one considers leptons as a fourth color in Nature \[4\], the other three being associated with quarks. In fact in the presence of the $N_R$’s, the minimal anomaly free gauge group of weak interactions expands beyond the standard model and becomes the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ \[5\] which is a subgroup of the $SU(2)_L \times SU(2)_R \times SU(4)_c$ group introduced by Pati and Salam. We will see below that both these gauge groups lead to a picture of weak interaction which is fundamentally different from that envisaged in the standard model in that weak interactions like the strong and electromagnetic ones are parity conserving at very high energies and observed maximally parity violating V-A structure of low energy weak processes is a consequence of gauge symmetry breaking.

To see this explicitly, we consider the left-right symmetric theory of weak interactions under which fermions and Higgs bosons transform as follows: $Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} (2,1,\frac{1}{3})$; $Q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix} (2,1,-\frac{1}{3})$; $L_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (2,1,-1)$; $L_R \equiv \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} (1,2,-1)$. The Higgs fields transform as $\phi (2,2,0)$; $\Delta_L (3,1,2)$; $\Delta_R (1,3,2)$.

It is clear that this theory leads to a weak interaction Lagrangian of the form

$$L_{wk} = g \left( j^\mu_L \cdot \bar{W}_{L,\mu} + j^\mu_R \cdot \bar{W}_{R,\mu} \right)$$

which is parity conserving prior to symmetry breaking. Furthermore, in this theory, the electric charge formula is given by \[6\]:

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2},$$

where each term has a physical meaning unlike the case of the standard model. When only the gauge symmetry $SU(2)_R \times U(1)_{B-L}$ is broken down, one finds the relation $\Delta I_{3R} =$
This connects $B - L$ breaking i.e. $\Delta(B - L) \neq 0$ to the breakdown of parity symmetry i.e. $\Delta I_{3R} \neq 0$. It also reveals the true meaning of the standard model hypercharge as $\frac{Y}{2} = I_{3R} + \frac{B - L}{2}$.

To discuss the implications of these observations, note that in stage I, the gauge symmetry is broken by the Higgs multiplets $\Delta_L(3,1,2) \oplus \Delta_R(1,3,2)$ to the standard model and in stage II by the bidoublet $\phi(2,2,0)$ as in the standard model. In the first stage, the right handed neutrino picks up a mass of order $f \lesssim \Delta^0_R \equiv \bar{f}v_R$. Denoting the left and right handed neutrino by $(\nu, N)$ (in a two component notation), the mass matrix for neutrinos at this stage looks like

$$M^0_{\nu} = \begin{pmatrix} 0 & 0 \\ 0 & f v_R \end{pmatrix}$$

(5)

At this stage, familiar standard model particles are all massless. As soon as the standard model symmetry is broken by the bidoublet $\phi$ i.e. $\langle \phi \rangle \equiv \text{diag}(\kappa, \kappa')$, the W and Z boson as well as the fermions pick up mass. I will generically denote $\kappa, \kappa'$ by a common symbol $v_{wk}$. The contribution to neutrino mass at this stage look like

$$M^0_{\nu} = \begin{pmatrix} f v_L & h v_{wk} \\ h v_{wk} & f v_R \end{pmatrix}$$

(6)

where $v_L = \frac{\nu_{vL}}{\nu_{vR}}$. The appearance of the $f v_L$ term is a reflection of parity invariance of the model. Note that except for the $\nu\nu$ entry, the neutrino mass matrix in Eq.(6) is exactly in the same form as in Eq. (1). Diagonalizing this matrix, we get a modified seesaw formula for the light neutrino mass matrix

$$M_\nu = f v_L - h^{-1}_v f_R^{-1} h_v \left( \frac{v_{wk}^2}{v_R} \right)$$

(7)

The important point to note is that $v_L$ is suppressed by the same factor as the second term in Eq. (7) so that despite the new contribution to neutrino masses, seesaw suppression remains [7]. This is called the type II seesaw whereas the formula in Eq. (1) is called type I seesaw formula.$^1$

An important physical meaning of the seesaw formula is brought out when it is viewed in the context of left-right models. Note that $m_\nu \to 0$ when $v_R$ goes to infinity. In the same limit the weak interactions become pure V-A type. Therefore, left-right model derivation of the seesaw formula smoothly connects smallness of neutrino mass with suppression of V+A part of the weak interactions providing an important clarification of a major puzzle of the standard model i.e. why are weak interactions are near maximally parity violating ? The answer is that they are near maximally parity violating because the neutrino mass happens to be small. This point was emphasized in the fourth paper in Ref. [1].

$^1$The triplet contribution to neutrino masses without the seesaw suppression was considered in [8] and triplet contribution by itself with seesaw suppression outside the framework of parity symmetric models have been considered in [9]. In the limit of large right handed neutrino masses, type II seesaw formula for neutrino masses reduces to the triplet seesaw formula.
In a subsequent section, we will discuss the connection of the seesaw mass scale with the scale of grand unification, which is suggested by the value of the atmospheric $\Delta m^2_A$. SO(10) is the simplest gauge group that contains the right handed neutrino needed to implement the seesaw mechanism and also it is important to note that the left-right symmetric gauge group is a subgroup of the SO(10) group, which therefore provides an attractive overall grand unified framework for the discussion of neutrino masses. The extra bonus one may expect is that since bigger symmetries tend to relate different parameters of a theory, one may be able to predict neutrino masses and mixings. We will present a model where indeed this happens.

We also note that since type I seesaw involves the Dirac mass of the neutrino, which is likely to scale with generation the same way as the charged fermions of the standard model, unless there is extreme hierarchy among the right handed neutrinos, one would expect the $\nu$ spectrum to be hierarchical. On the other hand, it has been realized for a long time [11] that if neutrino masses are quasi-degenerate, it is a tell-tale sign of type II seesaw with the triplet vev term being the dominant one. However, a normal hierarchy can also arise with type II seesaw as we discuss in the example below.

**IV. SEESAW AND LARGE NEUTRINO MIXINGS**

While seesaw mechanism provides a simple framework for understanding the smallness of neutrino masses, it does not throw any light on the question of why neutrino mixings are large. The point is that mixings are a consequence of the structure of the light neutrino mass matrix and the seesaw mechanism is only statement about the scale of new physics. This can also be understood by doing a simple parameter counting. If we work in a basis where the right handed neutrino masses are diagonal, there are 18 parameters describing the seesaw formula for neutrino masses - three RH neutrino masses and 15 parameters in the Dirac mass matrix. On the other hand, there are only nine observables (three masses, three mixing angle and three phases) describing low energy neutrino sector. Thus there are twice as many parameters as observables. As a result, the neutrino mass matrix needs inputs beyond the simple seesaw mechanism to fix the neutrino mass matrix.

In order to understand large mixings, one has to go beyond the simple seesaw mechanism to particular models. This is any way necessary to limit the scale of the right handed neutrino far below the Planck scale as seems to be the case. Many such models have been considered that use horizontal symmetry, grand unification, discrete symmetries, assumption of single right handed neutrino dominance etc. [10] to derive large mixings. In the following section, I will focus on a recently discussed minimal SO(10) model, where without any assumption other than SO(10) grand unification, one can indeed predict all but one neutrino parameters. I will then consider a case where assumption of quasi-degeneracy in the neutrino spectrum at high scale leads in a natural way via radiative corrections to large mixings at low energies.

To understand the fundamental physics behind neutrino mixings, we first write down the neutrino mass matrix that leads to maximal solar and atmospheric mixing. We consider the case of normal hierarchy where we have
\[
M_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix}
ce & b\epsilon & d\epsilon \\
b\epsilon & 1 + a\epsilon & -1 \\
d\epsilon & -1 & 1 + \epsilon 
\end{pmatrix}
\]  

(8)

where \(\epsilon \simeq \sqrt{\frac{\Delta m_{10}^2}{\Delta m_A^2}}\) and parameters \(a, b, c, d\) are of order one. Any theory of neutrino which attempts to explain the observed mixing pattern for the case of normal hierarchy must strive to get a mass matrix of this form. In the next section, we give a simple example of a minimal SO(10) grand unified theory that gives this mass matrix without any extra assumptions.

V. A PREDICTIVE MINIMAL SO(10) THEORY FOR NEUTRINOS

The main reason for considering SO(10) for neutrino masses is that its 16 dimensional spinor representation consists of all fifteen standard model fermions plus the right handed neutrino arranged according to the it \(SU(2)_L \times SU(2)_R \times SU(4)_c\) subgroup [4] as follows:

\[
\Psi = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\
d_1 & d_2 & d_3 & e \end{pmatrix}
\]  

(9)

There are three such spinors for three fermion families.

In order to implement the seesaw mechanism in the SO(10) model, one must break the B-L symmetry, since the right handed neutrino mass breaks this symmetry. One implication of this is that the seesaw scale is at or below the GUT scale. Secondly in the context of supersymmetric SO(10) models, the way B-L breaks has profound consequences for low energy physics. For instance, if B-L is broken by a Higgs field belonging to the 16 dimensional Higgs field (to be denoted by \(\Psi_H\)), then the field that acquires a nonzero vev has the quantum numbers of the \(\nu_R\) field i.e. B-L breaks by one unit. In this case higher dimensional operators of the form \(\Psi \Psi \Psi \Psi_H\) will lead to R-parity violating operators in the effective low energy MSSM theory such as \(QLd^c, u^c d^c \) etc which can lead to large breaking of lepton and baryon number symmetry and hence unacceptable rates for proton decay. This theory also has no dark matter candidate without making additional assumptions.

On the other hand, one may break B-L by a 126 dimensional Higgs field. The member of this multiplet that acquires vev has \(B-L = 2\) and leaves R-parity as an automatic symmetry of the low energy Lagrangian. There is a naturally stable dark matter in this case. It has recently been shown that this class of models lead to a very predictive scenario for neutrino mixings [12–15]. We summarize this model below.

As already noted earlier, any theory with asymptotic parity symmetry leads to type II seesaw formula. It turns out that if the B-L symmetry is broken by 16 Higgs fields, the first term in the type II seesaw (effective triplet vev induced term) becomes very small compared to the type I term. On the other hand, if B-L is broken by a 126 field, then the first term in the type II seesaw formula is not necessarily small and can in principle dominate in the seesaw formula. We will discuss a model of this type below.

The basic ingredients of this model are that one considers only two Higgs multiplets that contribute to fermion masses i.e. one 10 and one 126. A unique property of the 126 multiplet is that it not only breaks the B-L symmetry and therefore contributes to right handed neutrino masses, but it also contributes to charged fermion masses by virtue of
the fact that it contains MSSM doublets which mix with those from the 10 dimensional multiplets and survive down to the MSSM scale. This leads to a tremendous reduction of the number of arbitrary parameters, as we will see below.

There are only two Yukawa coupling matrices in this model: (i) $h$ for the 10 Higgs and (ii) $f$ for the 126 Higgs. SO(10) has the property that the Yukawa couplings involving the 10 and 126 Higgs representations are symmetric. Therefore if we assume that CP violation arises from other sectors of the theory (e.g. squark masses) and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then it will have only nine parameters. Noting the fact that the (2,2,15) submultiplet of 126 has a pair of standard model doublets that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows [12]:

\[
M_u = h\kappa_u + f v_u \\
M_d = h\kappa_d + f v_d \\
M_\ell = h\kappa_d - 3 f v_d \\
M_{\nu D} = h\kappa_u - 3 f v_u
\]  

where $\kappa_{u,d}$ are the vev’s of the up and down standard model type Higgs fields in the 10 multiplet and $v_{u,d}$ are the corresponding vevs for the same doublets in 126. Note that there are 13 parameters in the above equations and there are 13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). Thus all parameters of the model that go into fermion masses are determined.

To determine the light neutrino masses, we use the seesaw formula in Eq. (7), i.e.

\[
M_\nu = f v_L - h_{\nu T} f_R^{-1} h_\nu \left( \frac{v_{ub}^2}{v_R} \right).
\]

The coupling matrix $f$ is nothing but the 126 Yukawa coupling that appears in Eq. (10). Thus all parameters that give neutrino mixings except an overall scale are determined. These models were extensively discussed in the last decade [13] using type I seesaw formula. Their predictions for neutrino masses and mixings are either ruled out or are at best marginal.

There has been a revival of these models due to an observation for a two generation version of it [14]. It was pointed out in Ref.[14] that if the direct triplet term in type II seesaw dominates, then it provides a very natural understanding of the large atmospheric mixing angle for the case of two generations without invoking any symmetries. Subsequently it was shown [15] that the same $b-\tau$ mass convergence also provides an explanation of large solar mixing as well as small $\theta_{13}$ making the model realistic and experimentally interesting.

A simple way to see how large mixings arise in this model is to note that when the triplet term dominates the seesaw formula, we have the neutrino mass matrix $M_\nu \propto f$, where $f$ matrix is the 126 coupling to fermions discussed earlier. Using the above equations, one can derive the following sumrule (sumrule was already noted in the second reference of [13]):

\[
M_\nu = c (M_d - M_\ell)
\]

where numerically $c \approx 10^{-9}$ GeV. To see how this leads to large atmospheric and solar mixing, let us work in the basis where the down quark mass matrix is diagonal. All the
quark mixing effects are then in the up quark mass matrix i.e. \( M_u = U^T_{CKM} M^d_{u} U_{CKM} \). Note further that the minimality of the Higgs content leads to the following sumrule among the mass matrices:

\[
k \tilde{M}_\ell = r \tilde{M}_d + \tilde{M}_u
\]

(13)

where the tilde denotes the fact that we have made the mass matrices dimensionless by dividing them by the heaviest mass of the species i.e. up quark mass matrix by \( m_t \), down quark mass matrix by \( m_b \) etc. \( k, r \) are functions of the symmetry breaking parameters of the model. Using the Wolfenstein parameterization for quark mixings, we can conclude that that we have

\[
M_{d,\ell} \approx m_{b,\tau} \left( \begin{array}{ccc} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right)
\]

(14)

where \( \lambda \sim 0.22 \) and the matrix elements are supposed to give only the approximate order of magnitude.

An important consequence of the relation between the charged lepton and the quark mass matrices in Eq. (13) is that the charged lepton contribution to the neutrino mixing matrix i.e. \( U_\ell \approx 1 + O(\lambda) \) or close to identity matrix. As a result the neutrino mixing matrix is given by \( U_{PMNS} = U_\ell^T U_\nu \approx U_\nu \), since in \( U_\ell \), all mixing angles are small. Thus the dominant contribution to large mixings will come from \( U_\nu \), which in turn will be dictated by the sum rule in Eq. (12). Let us now see how this comes about.

As we extrapolate the quark masses to the GUT scale, due to the fact that \( m_b - m_\tau \approx m_\tau \lambda^2 \) for a wide range of values of \( \tan \beta \), the neutrino mass matrix \( M_\nu = c(M_d - M_\ell) \) takes roughly the form

\[
M_\nu = c(M_d - M_\ell) \approx m_0 \left( \begin{array}{ccc} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{array} \right)
\]

(15)

This mass matrix is in the form discussed in Eq. (8) and it is easy to see that both the \( \theta_{12} \) (solar angle) and \( \theta_{23} \) (the atmospheric angle) are now large. The detailed magnitudes of these angles of course depend on the details of the quark masses at the GUT scale. Using the extrapolated values of the quark masses and mixing angles to the GUT scale, the predictions of this model for various oscillation parameters are given in Ref.[15]. The predictions for the solar and atmospheric mixing angles fall within 3 \( \sigma \) range of the present central values. Specifically the prediction for \( U_{e3} \) (see Fig. 1) can be tested in MINOS as well as other planned Long Base Line neutrino experiments such as Numi-Off-Axis, JPARC etc.

There is a simple explanation of why the \( U_{e3} \) comes out to be large. This can also be seen from the mass sumrule in Eq.12. Roughly, for a matrix with hierarchical eigen values as is the case here, the mixing angle \( \tan 2\theta_{13} \sim \frac{M_{\nu,13}}{M_{\nu,33}} \sim \frac{\lambda^3 m_\tau}{m_\nu(M_\nu) - m_\tau(M_\nu)} \). Since to get large mixings, we need \( m_\nu(M_\nu) - m_\tau(M_\nu) \approx m_\tau \lambda^2 \), we see that \( U_{e3} \approx \lambda \) upto a factor of order one. Indeed the detailed calculations lead to 0.16 which is not far from this value.
FIG. 1. The figure shows the predictions of the minimal SO(10) model for $\sin^2 2\theta_A$ and $U_{e3}$ for the allowed range of parameters in the model. Note that $U_{e3}$ is very close to the upper limit allowed by the existing reactor experiments.

VI. CP VIOLATION IN THE MINIMAL SO(10) MODEL

In the discussion given above, it was assumed that CP violation is non-CKM type and resides in the soft SUSY breaking terms of the Lagrangian. The overwhelming evidence from experiments seem to be that CP violation is perhaps is of CKM type. It has recently been pointed out that with slight modification, one can include CKM CP violation in the model [16]. The basic idea is to include all higher dimensional operators of type $h'\Psi\bar{\Delta}\Sigma/M$ where $\bar{\Delta}$ and $\Sigma$ denote respectively the 126 and the 210 dimensional representation. It is then clear that those operators transforming as 10 and 126 representations will simply redefine the $h, f$ coupling matrices and add no new physics. On the other hand the higher dimensional operator that transforms like an effective 120 representation will add a new piece to all fermion masses. Now suppose we introduce a parity symmetry into the theory which transforms $\Psi$ to $\Psi^*$, then it turns out that the couplings $h$ and $f$ become real and symmetric matrices whereas the 120 coupling (denoted by $h'$) becomes imaginary and antisymmetric. This process introduces three new parameters into the theory and the charged fermion masses are related to the fundamental couplings in the theory as follows:

$$
M_u = h\kappa_u + fv_u + h'v_u \\
M_d = h\kappa_d + fv_d + h'v_d \\
M_l = h\kappa_d - 3fv_d - 3h'v_d \\
M_{\nu_D} = h\kappa_u - 3fv_u - 3h'v_u
$$

Note that the extra contribution compared to Eq. (10) is antisymmetric which therefore does not interfere with the mechanism that lead to $M_{\nu,33}$ becoming small as a result of $b - \tau$ convergence. Hence the natural way that $\theta_A$ became large in the CP conserving case remains.

Let us discuss if the new model is still predictive in the neutrino sector. Of the three new parameters, one is determined by the CP violating quark phase. the two others are determined by the solar mixing angle and the solar mass difference squared. Therefore we
lose the prediction for these parameters. However, we can predict in addition to \( \theta_A \) (see above), \( \theta_{13} \) and the Dirac phase for the neutrinos.

VII. RADIATIVE GENERATION OF LARGE MIXINGS: ANOTHER APPLICATION OF TYPE II SEESAW

As alluded before, type II seesaw liberates the neutrinos from obeying normal generational hierarchy and instead could easily be quasi-degenerate in mass. This provides a new mechanism for understanding the large mixings. The basic idea is that at the seesaw scale, all mixings angles are small. Since the observed neutrino mixings are the weak scale observables, one must extrapolate \([17]\) the seesaw scale mass matrices to the weak scale and recalculate the mixing angles. The extrapolation formula is 

\[
M_\nu(M_Z) = I M_\nu(v_R) I
\]

where 

\[
I_{\alpha\alpha} = \left(1 - \frac{h_\alpha^2}{16\pi^2}\right).
\]

Note that since 

\[
h_\alpha = \sqrt{2} m_\alpha/v_{wK} \quad (\alpha \text{ being the charged lepton index}),
\]

in the extrapolation only the \( \tau \)-lepton makes a difference. In the MSSM, this increases the \( M_{\tau\tau} \) entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown \([18]\) that if the muon and the tau neutrinos are nearly degenerate but not degenerate enough in mass at the seesaw scale, the radiative corrections can become large enough so that at the weak scale the two diagonal elements of \( M_\nu \) become much more degenerate. This leads to an enhancement of the mixing angles to become almost maximal value. This can also be seen from the renormalization group equations when they are written in the mass basis \([19]\). Denoting the mixing angles as \( \theta_{ij} \) where \( i, j \) stand for generations, the equations are:

\[
\frac{ds_{23}}{dt} = -F_\tau c_{23}^2 (s_{12} U_{\tau 1} D_{31} + c_{12} U_{\tau 2} D_{32}), \quad (18)
\]

\[
\frac{ds_{13}}{dt} = -F_\tau c_{23} c_{13}^2 (c_{12} U_{\tau 1} D_{31} + s_{12} U_{\tau 2} D_{32}), \quad (19)
\]

\[
\frac{ds_{12}}{dt} = -F_\tau c_{12} (c_{23}s_{13}s_{12} U_{\tau 1} D_{31} - c_{23}s_{13}c_{12} U_{\tau 2} D_{32} + U_{\tau 1} U_{\tau 2} D_{21}). \quad (20)
\]

where \( D_{ij} = (m_i + m_j) / (m_i - m_j) \) and \( U_{\tau 1,2,3} \) are functions of the neutrino mixings angles. The presence of \( (m_i - m_j) \) in the denominator makes it clear that as \( m_i \simeq m_j \), that particular coefficient becomes large and as we extrapolate from the GUT scale to the weak scale, small mixing angles at GUT scale become large at the weak scale. It has been shown recently that indeed such a mechanism for understanding large mixings can work for three generations \([20]\). It was shown that if we identify the seesaw scale neutrino mixing angles with the corresponding quark mixings and assume quasi-degenerate neutrinos, the weak scale solar and atmospheric angles get magnified to the desired level while due to the extreme smallness of \( V_{ub} \), the magnified value of \( U_{e3} \) remains within its present upper limit. Such a situation can naturally arise in a parity symmetric model with quark-lepton unification. In figure 2, we show the evolution of the mixing angles to the weak scale. A requirement for this scenario to work is that the common mass of neutrinos must be larger than 0.1 eV, a result that can be tested in neutrinoless double beta experiments.
FIG. 2. Radiative magnification of small quark-like neutrino mixings at the see-saw scale to bilarge values at low energies. The solid, dashed and dotted lines represent $\sin \theta_{23}$, $\sin \theta_{13}$, and $\sin \theta_{12}$, respectively.

VIII. OTHER REALIZATIONS OF SEESAW

As we saw from the previous discussion, the conventional seesaw mechanism requires rather high scale for the B-L symmetry breaking and the corresponding right handed neutrino mass (of order $10^{15} \text{ GeV}$). There is however no way at present to know what the scale of B-L symmetry breaking is. There are for example models bases on string compactification [22] where the $B - L$ scale is quite possibly in the TeV range. In this case small neutrino mass can be implemented by a double seesaw mechanism suggested in Ref.[21]. The idea is to take a right handed neutrino $N$ whose Majorana mass is forbidden by some symmetry and a singlet neutrino $S$ which has extra quantum numbers which prevent it from coupling to the left handed neutrino but which is allowed to couple to the right handed neutrino. One can then write a three by three neutrino mass matrix in the basis $(\nu, N, S)$ of the form:

$$M = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{pmatrix}$$  \hspace{1cm} (21)

For the case $\mu \ll M \approx M_{B-L}$, (where $M_{B-L}$ is the $B - L$ breaking scale), this matrix has one light and two heavy states. The lightest eigenvalue is given by $m_\nu \sim m_d M^{-1} \mu M^{-1} m_D$. There is a double suppression by the heavy mass compared to the usual seesaw mechanism and hence the name double seesaw. A generalization of this mechanism to the case of three generations is straightforward. One important point here is that to keep $\mu \sim m_D$, one also needs some additional gauge symmetries, which often are a part of the string models. In fact this mechanism is sometimes invoked in string models with TeV scale $Z'$ to understand neutrino masses [22].

It has recently been noted [23] that if there is parity symmetry in these models, the 13 and 31 entries of the above neutrino mass matrix get filled by a small seesaw suppressed entry. This has interesting applications in neutrino model building.

There are also other alternatives to seesaw, [24] discussed in literature, that we do not discuss.
IX. CONCLUSION

In summary, the seesaw mechanism is by far the simplest and most appealing way to understand neutrino masses. It not only improves the aesthetic appeal of the standard model by restoring quark-lepton symmetry but it also makes weak interactions asymptotically parity conserving. Further more it connects neutrino masses with the hypothesis of grand unification. In this talk I have discussed three different realizations of this mechanism: type I, type II and double seesaw. I have also noted how using the type II seesaw mechanism, one can have simple understanding of large neutrino mixings among neutrinos. A particularly interesting framework is provided by the minimal SO(10) model with 126 Higgs fields, which provides a very predictive framework for neutrinos.

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Note added: After the Seesaw25 conference in Paris, it came to the attention of the author as well as others in the community that there was an early 1977 paper by P. Minkowski (Phys. Lett. B 67, 421 (1977)) where the seesaw (type I) formula was also discussed.
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