Effects of Final-State Interaction and Screening on Strange- and Heavy-Quark Production

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Abstract. Final-state interaction and screening have a great influence on $q\bar{q}$ production cross sections, which are important quantities in many problems in quark-gluon plasma physics. They lead to an enhancement of the cross section for a $q\bar{q}$ color-singlet state and a suppression for a color-octet state. The effects are large near the production threshold. The presence of screening gives rise to resonances for $q\bar{q}$ production just above the threshold at specific plasma temperatures. These resonances, especially $c\bar{c}$ and $b\bar{b}$ resonances, may be utilized to search for the quark-gluon plasma by studying the temperature dependence of heavy-quark pair production just above the threshold.

1. Introduction

The cross sections for the production of $q\bar{q}$ pairs are important quantities in many problems in high-energy heavy-ion collisions. For example, the rate of approach to chemical equilibrium depends on the $s\bar{s}$ production cross section [1, 2], and the charm signal and background depend on the $c\bar{c}$ production cross section [3, 4].

In the lowest-order evaluation of $q\bar{q}$ production cross sections, the quark and the antiquark are described by plane waves, and their final-state interaction is not included. The quark and the antiquark however interact with each other through a color-Coulomb interaction, which has a great influence on the probability of $q\bar{q}$ production. The effects of final-state interactions can be approximately included in terms of a multiplicative $K$-factor [5]-[11]. While one can use the lowest-order [5] or the next-to-leading order results [11] for $q\bar{q}$ production, there are situations in which a perturbation expansion involving only the lowest two orders may not be sufficient, as in the case of large coupling constants and energies near the production threshold. A nonperturbative correction, based on the use of the distorted wave function in the presence of the color-Coulomb potential, can provide non-perturbative corrections to the cross sections [7]-[11]. Furthermore, in the quark-gluon plasma, the color-Coulomb interaction between a quark and an antiquark is screened to become a color-Yukawa interaction, and the screening needs

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to be taken into account when we study reactions in the quark-gluon plasma \([12]\).

2. The Importance of Final-State Interactions

The importance of final-state interaction can be assessed by studying the process \(e^+ e^- \rightarrow \) hadrons, which can be considered to go initially through the reaction \(e^+ e^- \rightarrow q \bar{q}\), where the produced \(q \bar{q}\) pair hadronizes subsequently to produce hadrons. The total cross section is

\[
\sigma(e^+ e^- \rightarrow \text{hadrons}) = N_c \sum_f \left( \frac{e_f}{e} \right)^2 \sigma_f(s),
\]

where \(\sigma_f\) in lowest-order QCD is

\[
\sigma_f(s) = \frac{4\pi}{s} \alpha^2 \sqrt{1 - \frac{4m_f^2}{s}} \left(1 + \frac{2m_f^2}{s}\right) \theta\left(1 - \frac{4m_f^2}{s}\right).
\]

Here, \(N_c = 3\) is the number of colors, \(e_f\) and \(m_f\) are the electric charge and rest mass of the quark \(q_f\) with flavor \(f\), and \(\alpha\) is the fine-structure constant. A convenient quantity to use to compare with experiment is the hadronic to muonic cross section ratio

\[
R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)}.
\]

Fig. 1. The ratio \(R = \sigma(e^+ e^- \rightarrow \text{hadrons})/\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)\) as a function of \(\sqrt{s}\). The dashed curve is the lowest-order result. The solid curve is obtained by using the \(K\)-factor of Eq. (14).

The ratios \(R\) calculated with Eqs. (1) and (2) are shown as the dashed curve in Fig. 1 and are compared with experimental data. The departure of the experimental data from the lowest-order results of (1) - (2) can be assessed by defining an empirical
multiplicative $K$-factor for $q\bar{q}$ production:

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = N_c \sum_f \left( \frac{e_f}{e} \right)^2 K_f(s) \sigma_f(s).$$  \hspace{1cm} (4)

A comparison of Eq. (4) and the experimental $R$ ratios indicates that the empirical values of $K_c(s)$ for $c\bar{c}$ production are near unity for $\sqrt{s} \gg 2m_c$, as predicted by perturbative QCD \cite{5}. However, $K_c(s)$ is much greater than unity near the threshold. The large deviation from unity arises predominantly from the final-state interaction between $q$ and $\bar{q}$. The magnitude of the deviation indicates that close to the threshold a perturbative treatment, including only the leading order and the next-to-leading order, is inadequate. Similar effects of initial- and final-state interactions are known in many other areas of physics \cite{13,14}. The lowest-order QCD results need to be corrected by a non-perturbative $K$-factor to bring the theoretical results into approximate agreement with the observed values.

3. $K$-factor for Color-Coulomb Interactions

To study the corrections to the lowest-order cross sections for $q\bar{q}$ production, we consider the case of a small relative velocity between $q$ and $\bar{q}$. The color-Coulomb interaction between $q$ and $\bar{q}$ is

$$V(r) = -\frac{\alpha_{\text{eff}}}{r},$$  \hspace{1cm} (5)

where $\alpha_{\text{eff}} = C_f \alpha_s$, $\alpha_s$ is the usual QCD running coupling constant, and $C_f$ is

$$C_f = \begin{cases} 4/3, & \text{for color-singlet;} \\ -1/6, & \text{for color-octet.} \end{cases}$$  \hspace{1cm} (6)

For outgoing states relevant to a produced $q\bar{q}$ pair, the wave function of the quark in the field of the antiquark is given by \cite{15}

$$\psi = Ne^{i\mathbf{p} \cdot \mathbf{r}} \left(1 - \frac{i}{2E} (\mathbf{\alpha} \cdot \nabla)\right) u_1 F_1(-i\xi, 1, -i(\mathbf{p} \cdot \mathbf{r})).$$  \hspace{1cm} (7)

Here, $u$ is a free quark spinor, $\mathbf{\alpha}$ is a Dirac matrix, $1 F_1$ is the confluent hypergeometrical function, $N$ is the normalization constant

$$|N|^2 = \frac{2\pi \xi}{1 - e^{-2\pi \xi}}$$  \hspace{1cm} (8)

where

$$\xi = \frac{\alpha_{\text{eff}}}{v},$$  \hspace{1cm} (9)

and $v$ is the asymptotic relative velocity of the quark and the antiquark,

$$v = \frac{\sqrt{s^2 - 4sm^2_q}}{s - 2m^2_q}.$$  \hspace{1cm} (10)
The square of the wavefunction at contact, which is a generalization of the familiar ‘Gamow factor’ \cite{13}, gives the corrective $K$-factor

$$K = |\psi(0)|^2 = \frac{2\pi \xi}{1 - e^{-2\pi \xi}} (1 + \alpha_{\text{eff}}^2),$$

(11)

where the spins of both quarks have been averaged over.

In order to obtain a generalized correction factor that is valid not only when $q$ and $\bar{q}$ have low relative velocities, but also have large relative velocities, we use the interpolation technique suggested by Schwinger \cite{16}. In the high energy limit, the next-to-leading order QCD correction for $e^+ + e^- \rightarrow q + \bar{q}$ is given by \cite{5}

$$K = 1 + \frac{\alpha_s}{\pi}.$$  

(12)

The transition from the low-velocity corrective factor to relativistic velocities can be accommodated with the introduction of a function $f(v)$

$$f(v) = \alpha_{\text{eff}} \left[ \frac{1}{v} + v \left( -1 + \frac{3}{4\pi^2} \right) \right].$$

(13)

We can thus construct an approximate correction factor for the color-Coulomb interaction for all relative velocities as \cite{10}

$$K = \frac{2\pi f(v)}{1 - \exp[-2\pi f(v)]} (1 + \alpha_{\text{eff}}^2).$$

(14)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The $K$-factor for $q\bar{q}$ production as a function of $\sqrt{s}$ for the color-Coulomb $q\bar{q}$ interaction.}
\end{figure}
In Fig. 2 we show the $K$-factor calculated with Eq. (14) for the color-Coulomb interaction as a function of $\sqrt{s}$. It gives an enhancement for a color-singlet state and a suppression for a color-octet state. The correction factors deviate significantly from unity near the threshold. As a test of the reliability of these correction factors, we use the $K$-factor of Eq. (14) for the production of color-singlet $q\bar{q}$ pairs to calculate the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ [11]. The results of $R$ ratios are given as the solid curve in Fig. 1, which are in reasonable agreement with experiment. This agreement lends support to the application of the $K$-factor (14) to other processes.

4. K-factor for a Color-Yukawa Interaction

If one places a quark and an antiquark in a quark-gluon plasma, their interaction will be screened to become a color-Yukawa interaction:

$$V(r) = -\frac{\alpha_{\text{eff}} e^{-r/\lambda_D}}{r},$$

(15)

where the Debye screening length $\lambda_D$ is inversely proportional to the quark-gluon plasma temperature $T$ [17]. To obtain the corrective $K$-factor we evaluate the wave function for a quark and an antiquark in the color-Yukawa potential using the phase-amplitude method of Calogero [18].

![Graph](image)

**Fig. 3.** The $K$-factor for $q\bar{q}$ production as a function of $\xi$ and $\eta$ for the color-Yukawa $q\bar{q}$ interaction.

We introduce $a_B = 1/\mu|\alpha_{\text{eff}}|$, where $\mu$ is the reduced mass. The quantity $a_B$ is the Bohr radius for an attractive potential and the appropriate length scale for a repulsive potential. The Schrödinger equation in $x = r/a_B$ contains only the
Coulomb parameter $\xi = \alpha_{\text{eff}}/v$ and the dimensionless screening length parameter
\[ \eta = \frac{\lambda_D}{\alpha_H}. \] (16)

The $K$-factor is therefore only a function of $\xi$ and $\eta$. The quantity $K(\xi, \eta)$ has been evaluated in Ref. [12], and we summarize the main results here.

In Fig. 3 we display $K(\xi, \eta)$ for the color-Yukawa potential for $-1 \leq \xi \leq 1$ and $\eta = 0.1, 0.2, ..., 0.7$. It is greater than unity for positive $\xi$, corresponding to an attractive interaction. It is less than unity for negative $\xi$, corresponding to a repulsive interaction. In the limit $\eta \to \infty$, the corrective factor $K(\xi, \eta)$ approaches the Gamow factor, which is a function of $\xi$ only.

In Fig. 4a we show $K(\xi, \eta)$ as a function of $\eta$ over the range $0.4 \leq \eta \leq 2$. We find that $K(\xi, \eta)$ has a maximum at $\eta = 0.835$. The larger the values of $\xi$, the greater is the maximum of $K(\xi, \eta)$. The peak values of the $K$-factor are much greater than the corresponding Gamow factor, as indicated by the ratio $K(\xi, \eta)/(\text{Gamow factor})$ in Fig. 4b.

![Fig. 4](image-url)

(Fig. 4) (a) The $K$-factor for $q\bar{q}$ production as a function of $\xi$ and $\eta$ for the color-Yukawa $q\bar{q}$ interaction. (b) The ratio of $K$ to the Gamow factor as a function of $\xi$ and $\eta$.

The prominent peak of $K(\xi, \eta)$ at $\eta = 0.835$ is due to the emergence of the lowest bound state of the system into the continuum to become a $q\bar{q}$ resonance.
as the screening length decreases. Movement of bound states into the continuum is familiar in the context of a single-particle system in a finite-range potential, as in nuclear single-particle states \[21\] and in the scattering of an electron from an atom\[19\]. Numerical calculations show that \( \eta \approx 0.84 \) is the screening length parameter for which the lowest bound state in the screened potential becomes unbound \[20\]. This coincides with the location of the peaks of \( K(\xi, \eta) \) in \( \eta \). We have found that there is a similar peak of \( K(\xi, \eta) \) at \( \eta = 3.23 \), corresponding to the second s-wave bound state emerging into the continuum to become a \( q\bar{q} \) resonance.

5. Strange and charm \( q\bar{q} \) production

Basic processes for \( q\bar{q} \) production in nucleon-nucleon collisions and in a quark-gluon plasma are \( gg \to q\bar{q} \) and \( q\bar{q} \to g^* \to q\bar{q} \). We can use the \( K \)-factor to correct the lowest-order result of Ref. \[22\] for \( q\bar{q} \) production in these processes as well. The corrected cross section for \( gg \to q\bar{q} \), (where \( q \) refers to \( s \) or \( c \) quarks), averaged over initial gluon types and polarizations and summed over final colors and spins, is

\[
\sigma_{gg}(M_{q\bar{q}}) = K_{gg} \frac{\pi \alpha_s^2}{3 M_{q\bar{q}}^2} \left\{ (1 + \eta_m + \frac{1}{16} \eta_m^2) \ln \left( \frac{1 + \sqrt{1 - \eta_m^2}}{1 - \sqrt{1 - \eta_m}} \right) \right. \\
- \left. \left( \frac{7}{4} + \frac{31}{16} \eta_m \right) \sqrt{1 - \eta_m} \right\},
\]

(17)

where \( \eta_m = 4 m_q^2 / M_{q\bar{q}}^2 \), \( m_q \) is the mass of the quark and \( M_{q\bar{q}} \) is the invariant mass of the produced \( q\bar{q} \) pair. For \( q\bar{q} \) production by gluon fusion, \( q\bar{q} \) pairs are produced with a relative color-octet/color-singlet ratio of \[19\]

\[
\frac{(\text{color - octet})}{(\text{color - singlet})} = \frac{(d_{abc}/\sqrt{2})^2}{(\delta_{ab}/\sqrt{3})^2} = \frac{5}{2}.
\]

(18)

Taking these weights into account, we can write the corrective factor for the gluon fusion mode as \( K_{gg} = [5K(\text{octet}) + 2K(\text{singlet})]/7 \).

Similarly, the corrected cross section for \( q\bar{q} \to g^* \to q\bar{q} \), (where \( q \) refers to \( s \) or \( c \) quarks), averaged over initial and summed over final colors and spins, can be written as

\[
\sigma_{q\bar{q}}(M_{q\bar{q}}) = K_{q\bar{q}} \frac{8 \pi \alpha_s^2}{27 M_{q\bar{q}}^2} \left( 1 + \frac{\eta_m}{2} \right) \sqrt{1 - \eta_m}.
\]

(19)

For \( q\bar{q} \) production by quark-antiquark annihilation through a virtual gluon in lowest order, the produced \( q\bar{q} \) pair is in a color-octet state. The corrective factor to be used is the color-octet corrective factor, \( K_{q\bar{q}} = K(\text{color octet}) \).

As illustrative examples we shall study two cases with different plasma screening lengths. For the quark-gluon plasma with \( N_f \) flavors and \( N_c = 3 \), lowest-order
perturbative QCD gives a Debye screening length of  

$$\lambda_{D}^{(\text{PQCD})} = \frac{1}{\sqrt{(\frac{N_c}{4} + \frac{N_f}{6})g^2T}}.$$  

(20)

For a coupling constant $\alpha_s = g^2/4\pi = 0.3$ and $N_f = 3$, the Debye screening length at a temperature of 200 MeV is $\lambda_D \approx 0.4$ fm. We shall examine the cases of $\lambda_D = 0.2$ and 0.4 fm, corresponding respectively to temperatures of 400 MeV and 200 MeV in this perturbative QCD estimate.

The final-state interaction enhances the production cross section for color-singlet states, and suppresses the production for color-octet states. The gluon fusion modes produce $q\bar{q}$ pairs in a superposition of color-singlet and color-octet states, and the larger magnitude of the color-singlet $K$-factor dominates the correction even though the singlet weight factor is lower. The net result is an enhancement of production by gluon-fusion. On the other hand, for $s\bar{s}$ and $c\bar{c}$ production by $q\bar{q}$ annihilation, the intermediate virtual gluon selects color-octet states only. The color interaction is repulsive, so the cross section is suppressed.

![Fig. 5. Cross sections for $s\bar{s}$ production (a) by gluon fusion, and (b) by $q\bar{q}$ annihilation.](image)

In Figures 5 we present the cross sections for $s\bar{s}$ production in a quark-gluon plasma with screening lengths of 0.2 and 0.4 fm (solid curves). The cross sections
for the color-Coulomb interaction without screening are shown as dashed curves. The lowest-order cross sections are shown as dashed-dot curves for comparison.

For an $s\bar{s}$ pair, $a_B$ is about 3 fm for a color-singlet state and about 24 fm for a color-octet state. Therefore, the relevant screening length parameter $\eta$ is small for the cases of $\lambda = 0.2$ and 0.4 fm. This implies that screening reduces the final-state interaction significantly, and $K \approx 1$ for $s\bar{s}$ production in a quark-gluon plasma. For a plasma with a screening length of 0.2 to 0.4 fm, the effect of screening reduces the final-state interaction so that the cross sections are close to the lowest-order cross sections. The situation is quite different from the case without screening. The final-state color-Coulomb interaction leads to a $K$-factor which is significantly different from unity. As shown in Fig. 5, in the case with final-state color-Coulomb interaction, the $s\bar{s}$ production cross section through gluon fusion is greatly enhanced, while the production through $q\bar{q}$ annihilation is suppressed from the lowest-order cross sections.

Fig. 6. Cross sections for $c\bar{c}$ production (a) by gluon fusion, and (b) by $q\bar{q}$ annihilation.

The effect of screening manifests itself for $c\bar{c}$ production in a different way, as shown in Fig. 6. The quantity $a_B$ for a $c\bar{c}$ pair is about 0.5 fm for a color-singlet state, and about 4 fm for a color-octet state. Thus, for a color-singlet $c\bar{c}$ system, the screening length parameter for $\lambda = 0.4$ fm is $\eta \approx 0.8$, close to the value $\eta = 0.834$.
for the occurrence of the production resonance. Thus, when the screening length is near 0.4 fm, screening greatly enhances the production cross section for color-singlet $c\bar{c}$ near threshold. The theoretical $c\bar{c}$ production cross sections through the gluon fusion mode therefore exhibit a marked rise at small velocities (Fig. 6a).

For $c\bar{c}$ production by $q\bar{q}$ annihilation, the intermediate virtual gluon selects color-octet states only. The color interaction is therefore repulsive, and suppresses the cross section. With typical screening lengths of 0.2 and 0.4 fm, the corrected cross sections are near the color-Coulomb values for $c\bar{c}$, while for $s\bar{s}$ production they are close to the lowest-order tree-level values, in accordance with the different values of $\eta$ for the two flavors.

6. Conclusions and Discussions

In reactions involving $q\bar{q}$ production, the quark and the antiquark are subject to final-state interactions due to their mutual interaction. The lowest-order cross sections for these processes can be corrected by using an approximate corrective $K$-factor to take into account the $q-\bar{q}$ interaction. We have obtained the $K$-factor for final-state color-Coulomb and color-Yukawa interactions. The corrective $K$-factor for the color-Yukawa interaction depends on two dimensionless parameters: the usual Coulomb parameter $\xi = \alpha_{\text{eff}}/v$, and the screening length parameter $\eta = \lambda_D/a_p$. For attractive Yukawa potentials we observe prominent peaks of the $K$-factor as a function of the screening length parameter $\eta$. The peaks are located at $\eta = 0.835$ and $\eta = 3.23$, corresponding to two lowest $s$-wave $q\bar{q}$ bound states emerging into the continuum to become $q\bar{q}$ resonances as the screening length decreases. We have calculated the cross sections for two typical choices of the Debye screening length (0.2 fm and 0.4 fm), corresponding to plasma temperatures of approximately 400 and 200 MeV respectively. While the corrections to the cross section in the color-Coulomb limit are of similar magnitude for $s\bar{s}$ and $c\bar{c}$ pairs in the same velocity range, they are considerably different for the two systems in the presence of screening with a color-Yukawa interaction. This arises because the quantity $a_B$ is much smaller for the $c\bar{c}$ system than for the $s\bar{s}$ system. For the color-singlet case, a screening length of 0.4 fm corresponds to $\eta \approx 0.8$, which is near a zero-energy $c\bar{c}$ resonance. This is in contrast to $s\bar{s}$ production for $\lambda_D = 0.2 - 0.4$ fm, for which the corresponding screening length parameters of $\eta = 0.04 - 0.08$ are very small, and are far from $\eta = 0.835$ for a $q\bar{q}$ resonance.

High-energy heavy-ion collisions have become the focus of intense research because of the possibility of producing a quark-gluon plasma during such collisions. The suppression of $J/\psi$ production has been suggested as a probe of the screening between a charm quark and a charm antiquark in the plasma, because $J/\psi$ production is suppressed in a quark-gluon plasma above a temperature $T_{c\bar{c}}$, such that $c$ and $\bar{c}$ cannot form a bound state [23, 24]. The presence of the quark-gluon plasma is signalled by a substantial decrease of the probability for $J/\psi$ production above $T_{c\bar{c}}$. 
From our results, the occurrence of $q\bar{q}$ resonances may provide a complementary signal to search for the quark-gluon plasma. These $q\bar{q}$ resonances give rise to prominent peaks of the $K$-factor as a function of the screening length parameter, which is the ratio of the screening length $\lambda_D$ to $a_B$. The screening length is inversely proportional to the temperature \(^{[17]}\). Thus, $q\bar{q}$ resonances lead to prominent peaks of the $K$-factor at specific plasma temperatures. We have seen in Fig. 6 that large values of the $K$-factor near the threshold give rise to a narrow peak in the heavy-quark production cross section just above the threshold. The occurrence of a $q\bar{q}$ resonance will be accompanied by a much enhanced $q\bar{q}$ production cross section just above the threshold. The enhancement will be a function of the temperature. Here, the quark-gluon plasma is signalled by the presence of a $c\bar{c}$ resonance just above the threshold at $T_{c\bar{c}}$.

The search for $q\bar{q}$ screening resonances in the quark-gluon plasma can make use of the peaks of the $K$-factor at $\eta = 0.835$ and $\eta = 3.23$. The resonance at $\eta = 3.23$ may not lead to realizable enhancements because it corresponds to temperatures much below the estimated quark-gluon plasma transition temperature (of approximately $150 – 200$ MeV). Using the perturbative QCD estimates, the screening length parameter $\eta = 0.835$ corresponds to a $c\bar{c}$ resonance at $T_{c\bar{c}} \approx 165$ MeV and a $b\bar{b}$ resonance at $T_{b\bar{b}} \approx 393$ MeV. These $T_{c\bar{c}}$ and $T_{b\bar{b}}$ estimates from PQCD are approximate and uncertain, as lattice gauge theory gives Debye screening lengths of about half the PQCD estimates \(^{[24]}\). The Debye screening length needs to be determined by experimental searches for these $c\bar{c}$ and $b\bar{b}$ resonances using the peaks in the $K$-factors. Temperature dependence of this type arises from the nature of screening between the interacting heavy quark and its antiquark partner, which is an important property to identify the deconfined quark-gluon plasma. A search for the quark-gluon plasma using heavy-quark resonances will require the measurement of the production yield of heavy-quark pairs near the threshold, and a method to estimate the temperature of the environment in which the production takes place. The enhancement will occur either for the production of heavy-quark pairs by the collision of the constituents of the thermalized quark-gluon plasma, or by the collision of the partons in nucleon-nucleon collisions in a quark-gluon plasma environment.

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