Transfer Matrix Mathematical Method for Evaluation the DBR Mirror for Light Emitting Diode and Laser

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Abstract: Applied mathematics is one of the most widely sciences used to serve other sciences, especially physical mathematics, which is one of the most important applied mathematics topics used in physics. In this work, we used the Transfer matrix mathematical method for evaluation the Distributed Bragg Reflectors mirrors for vertical cavity and the wavelength And The method of light extraction from the martials pay attention from the researchers since the LED and Laser become the most popular devices in different applications. This study focused on the development of simulation method to design and optimization for the DBR(Distributed Bragg Reflectors) structures, which are commonly used in designing DBR mirrors for vertical cavity surface emitting GaN based LEDs and lasers. From the multi-layer simulation, the simulation results are given for the investigation of several properties. First, the wavelength(nm) vs. reflectance(%) can be calculated in the DBR structures that TiO$_2$ and SiO$_2$ thin films are stacked alternately. As a results, it is suggested that highly efficient DBR structures can be designed and manufactured using simulation methodology.

1. Introduction
The Vertical Cavity surface-emitting laser (VCSEL) and LEDs is a modern solid-state light device, with a very promising future and application [1-3]. In VCSEL laser structure produces a much lesser round trip, thus requiring a higher reflectivity to sustain oscillation and achieved the laser condition. This issue can be solved by using several multilayer mirrors to improve the reflectivity and reach the gain more than the losses. For the design of a resonator for VCSEL, it is of great importance to understand the theory and calculation of reflectivity and transmissivity using this mathematical model by MATLAB software.

Distributed Bragg Reflectors (DBRs) are multi-layer reflectors consisting of two materials with different refractive indices of layer cycles. The most general method of calculating the reflectance R of a multilayer is based on a matrix formulation of the boundary conditions at the film surfaces, derived from Maxwell’s equations [4]. Fresnel reflections occur at each interface due to the difference in refractive index. The refractive index difference between the two materials is small, so that the Fresnel reflection at one interface is very small. However, many DBR structures are composed of many interfaces and the thickness of both materials is chosen so that all reflected light can make constructive interference.

It can be seen that this condition is satisfied when the thickness of two materials is 1/4 wavelength of light for vertical incidence. In case of vertical incidence, equation (1) is obtained.
\[ T_{lh} = \frac{\lambda_{lh}}{4} = \frac{\lambda_o}{4n_{lh}} \]  

(1)

Where \( l \) and \( h \) are the vacuum Bragg wavelengths of light, \( T_l \) and \( h \) are the thickness of the low and high-refractive index material, and \( n_l \) and \( n_h \) are the refractive indices of the low- and high-materials. In order to obtain a high reflectance in a multi-layer thin film structure in which a low refractive index and a high refractive index are periodically stacked, the thickness of each layer becomes 1/4 of a wavelength and alternately use \( n_h \) and \( n_l \) having a large and small refractive index [5-6].

2. Simulation method

The function principle of nonconductive multilayers comes from the various interference effects that appear when they are reflected by multiple insulator substrates. The mathematical principles of nonconductive multi-layer films can be seen to be due to the various interference effects that appear when rays are reflected by multiple insulator substrates. A representative feature is shown in Figure 1, which is a comparison between a single thin film and multiple thin films. The incident light is reflected between the reflected light on both surfaces of the material with different refractive indices.

![Figure 1. The reflectance and transmittance in single and multi-layer films.](image)

Reflected light from a single film is the result of interference between the two layers of light reflected from the air layer and the surface of the base layer, and reflected light from multiple films is the result of interference from the various lights reflected from each of the different surfaces. In order to describe the interference of light reflected and transmitted by other contact surfaces of multiple film layers, the structure shown in Figure 2.
Figure 2. Reflectivity and transmittance calculations in multilayer thin film structure with refractive index $n_i$ and thickness $d_i$.

The characteristic matrix $M_i$ of the dielectric material having the refractive index $n_i$, and $n_o = 1$ which is the refractive index of air and the thickness $d_i$ is expressed by equation (2).

$$M_i = \begin{bmatrix}
\cos \frac{2\pi n_i}{\lambda} d_i & -i \frac{\sin 2\pi n_i}{\lambda} d_i \\
-i n_i \sin \frac{2\pi n_i}{\lambda} d_i & \cos \frac{2\pi n_i}{\lambda} d_i
\end{bmatrix}$$

(2)

The characteristic matrix of the multilayer thin film having $N$ dielectric thin films is shown in equation (3).

$$M = M_1 M_2 M_3 \ldots \ldots M_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

(3)

The reflectance $R$ and the transmittance $T$ can be obtained from Eqs. (4) and (5) [7-10].

$$R = \frac{2n_0 + BnTn_o}{A_n_0 + BnTn_0} \frac{-C-Dn_T}{A + C + Dn_T}$$

(4)

$$T = \frac{2n_0}{A_n_0 + BnTn_0} + C + Dn_T$$

(5)

3. Simulation and Characterization for Distributed Bragg Reflector Optimization Design

Based on low refractive index ($n_l$) and high refractive index ($n_h$) thin films, type and number of each layer pair, and thickness of each layer, the reflectance ($R$) value and transmittance ($T$) value can be calculated with respect to the wavelength of the incident light. In this simulation, when the reflectance is calculated through simulation, $R + T = 100\%$. The transmittance can be automatically obtained. The wavelength of the incident light is simulated at each wavelength band at intervals of 1 nm at $\lambda = 500$ to 600 nm. Reflectance (%) spectrum vs. wavelength (nm) results were obtained.
In the DBR structure, the outer surrounding has a value of \( n_o = 1 \) which is the refractive index of air. In order to obtain a high reflectance in a multi-layer dielectric thin film structure in which a low refractive index and a high refractive index are periodically piled up, the DBR structure was simulated by alternately depositing a high refractive index (\( n_h \)) and a low refractive index (\( n_l \)) dielectric with a relatively large and small refractive index. The thickness of the two materials satisfying the condition of 1/4 wavelength is selected.

Figure 3 shows the simulation result of increasing the number of layers pairs by stacking TiO\(_2\) 54 nm on the top and 103.4 nm SiO\(_2\) on the bottom so that the reflectance peak at \( \lambda = 600 \) nm wavelength band. As shown in Figure 4, it can be seen that the reflectance value increases as the number of pairs increases. It can be used as a manufacturing condition of the DBR structure with high efficiency while minimizing the number of thin film layers.

![Wavelength vs Reflectivity](image)

**Figure 3.** The reflectance spectrum by the number of pair(1pair~5pair) of SiO\(_2\)(103.4)/TiO\(_2\)(54nm).

Figure 4 shows that the reflectance is peaked at \( \lambda = 500 \) nm and the thickness of the TiO\(_2\) layer is 43.4 nm and the SiO\(_2\) layer is 85.6 nm thick. The number of pairs is increased from 1 pair (total 2 layers) to 5 pairs (total 10 layers) This is the simulation result for the structure.

1 pair (total 2 layers) laminate shows a low reflectance at a wavelength of \( \lambda = 500 \) nm. However, as the number of dielectric thin film layers increases, the reflectance gradually increases and reflectance approaches 100% at 5 pairs.
It can be seen that the width of the reflectance spectrum having the peak value of the reflectance at the short wavelength band is narrow and the width of the reflectance spectrum having the reflectance value at the long wavelength band is widened.

Based on the simulation results, it was found that the combination of TiO$_2$ and SiO$_2$ combination with the peak value of reflectance at $\lambda = 500$ and 600 nm with reflectance peak at design wavelength reflectance. In the visible wavelength range of $\lambda = 500$ nm and 600 nm.

A DBR structure with 30 layers, 5 pairs each with a flat reflectance value, was constructed through simulation (Figure 6).
As shown in Figure 6, as the number of thin film pairs increases, the absolute value of reflectance also increases, and the wavelength range of $\lambda = 400$ to $600$ nm is enlarged and widened. As a result, when the reflectance value is close to 100% and has a cut-off characteristic.

When 15 pairs (total 30 layers) were composite using TiO$_2$ / SiO$_2$ 1 pair ~ 5 pair, the reflectance value close to 100% was obtained in the visible region wavelength band.

It was confirmed that the best combination is to simulate the DBR structure for various combinations and to arrange the thickness of the dielectric thin film sequentially from below. By using the simulation method, we can calculate the wavelength vs. reflectance spectrum according to the DBR structure fabrication conditions and the layer structure, and it can be used to design the actual structure and to present the optimal experimental conditions.

4. Conclusion
In this study, optical characteristics simulation method for DBR structure, which is a key component of LED chip and LCD inspection equipment, is widely used in optical film manufacturing and we propose an optimal DBR structure through optical simulation. The following results were obtained from the simulation results. The refractive index and transmittance of incident light were calculated and compared using the refractive index values of TiO$_2$ and SiO$_2$ in multilayer thin film structures in which TiO$_2$ and SiO$_2$ thin films were alternately laminated. As a result, optimization of highly efficient distributed Bragg reflector design through simulation and can be used to fabricate DBR structures.

References
[1] Oster A, Zorn M, Vogel K, Fricke J, Juergen S, Wilfried J, Weyers M and Traenkle G 2001 SPIE OPTO. (San Jose, CA) 4286
[2] Jewell J L, Harbison J P, Scherer A, Lee Y H and Florez L T 1994 Chaos, Solitons Fract. 4 1575-96
[3] Dave Wiston Retrieved from http://www-ocs.colorado.edu/SimWindows/simwin.html
[4] Macleod H A 1986 Thin-Film Optical Filters (England: Adam Hilger Ltd. Bristol)
[5] Dobrowolski J A 1997 Optics and Photonics News 8 24-33
[6] Jambunathan R and Singh J 1997 IEEE J. Quantum Electron. 33 1180-9
[7] Lin H Y, Chen K J, Wang S W, Lin C C, Wang K J, Li J R, Lee P T, Shih M H, Li X, Chen H M and Kuo H C 2015 Opt. Exp. 23 A27-A33

[8] Wang D X, Ferguson I T, Buck J A and Nicol D 2006 Proc. SPIE, Nanoengineering: Fabrication, Properties, Optics, and Devices III p 63270Q

[9] Chen A, Yuan Q and Zhu K 2015 Appl. Surf. Sci. 360 693-7

[10] Kischkat J, Peters S, Gruska B, Semtsiv M, Chashnikova M, Klinkmüller M, Fedosenko O, Machulik S, Aleksandrova A, Monastyrskyi G, Flores Y and Masselink WT 2012 Appl. Opt. 51 6789-98