Modeling of capillary discharge plasmas for wakefield acceleration and beam transport

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Abstract. Next generation accelerators demand sophisticated beam sources to produce ultra-low emittances with large gradients. The subsequent beamline optics are equally critical to transporting these beams between accelerating stages or to interaction points. Capillary discharge plasmas may address each of these challenges. Capillaries have been demonstrated as sources capable of increasing the peak energy and beam quality of laser wakefield accelerators, and as active plasma lenses featuring orders-of-magnitude increases in peak magnetic field. These systems are sensitive to energy deposition, heat transfer, ionization dynamics, and magnetic field penetration; therefore, improved modeling will enable advances in capillary design. We present simulations of capillary discharge waveguides and active plasma lenses in using FLASH, a publicly-available multi-physics code in development at the University of Chicago. We report on the implementation of a 2D, cylindrically symmetric capillary model for capturing plasma density and temperature evolution with realistic conductivities and magnetic fields. We then illustrate the use of laser energy deposition to model low density channel formation for the matching and guiding of intense laser pulses. Lastly, we discuss simulations of active capillary plasmas with different fill species, which show agreement with experimental observations of nonlinearities in the current density profile and magnetic field.

1. Introduction

High energy, high brightness electron beams are essential to meeting the demands of future accelerators. Improved electron sources and optics are required to achieve next generation collider designs. Capillary discharge plasmas promise tailored plasma profiles for a range of advanced accelerator applications. These devices have been successfully used as waveguides to couple intense laser pulses, increasing the acceleration length and peak energy during laser wakefield acceleration [2, 3, 4]. Owing to their capacity to generate high azimuthal magnetic fields, capillaries have also been employed as compact, efficient lenses for beam transport [5, 6] and for multi-stage coupling of laser plasma accelerators [7]. Recently, these devices have also been used as a tunable dechirper for electron beams [8]. The breadth of prospective applications merits continued investigation into capillary dynamics.

The plasma dynamics within a capillary discharge are governed by Ohmic heating of the plasma balanced by conductive dissipation at the capillary wall. The discharge current drives a strong, time-dependent magnetic field, which necessitates self-consistent magnetohydrodynamics (MHD) modeling. Thermal conductivity, and magnetic field transport require additional coupled models, and a detailed equation of state must be implemented to compute ionization and augment the hydrodynamic equations. The capillary discharge system may be contrasted with that of a Z-pinch system. While the Z-pinch produces very high magnetic pressure, the relatively low magnetic pressure in the capillary cannot...
overcome the plasma pressure, thereby resulting in the formation of a low density channel. Previously published works studying these systems have demonstrated the applicability of an MHD approach to capillary plasmas for laser wakefield sources [11, 12, 13, 14]. However, there remains a dearth of publicly available codes for modeling these systems.

In this paper, we demonstrate a two dimensional model for capillary discharge plasma simulation using the publicly available multiphysics code FLASH [10]. We apply this model to the simulation of cylindrical capillary geometries, and compare it against previously published analytical and computational benchmarks [11, 12, 13, 14]. We then demonstrate the application of this code to active plasma lenses, and illustrate the formation of nonlinearities in the magnetic field contributing to focusing aberrations. Finally, we demonstrate the simulation of a laser heater in FLASH for sub-channel formation within a capillary, which has been recently used to improve laser guiding and peak energy gain [4].

2. FLASH Model

We have implemented a model for capillary discharge plasmas using the FLASH code [10]. FLASH advances fluid quantities on a block-structured mesh by solving the Euler equations for inviscid fluid flow, including diffusive and dissipative effects such as convection, emissivity, heat exchange, and energy deposition (e.g. laser deposition via $Q_{las}$). The resulting equations describe the evolution of the system.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P_{\text{tot}} = 0 \quad (2)$$

$$\frac{\partial (\rho E_{\text{tot}})}{\partial t} + \nabla \cdot [(\rho E_{\text{tot}} + P_{\text{tot}}) \mathbf{v}] = Q_{\text{las}} - \nabla \cdot \mathbf{q} \quad (3)$$

FLASH employs a 3T model, which separately accounts for the internal energies of electron, ion, and radiation components of a given fluid. Therefore, the above fluid equations must be solved for each species, with the relevant diffusive or source terms included for each additional process. Equations of state, along with transport coefficients, are either pre-tabulated for the relevant parameter space, or computed dynamically using internal models.

Our simulation instantiates a gas filling the capillary at uniform density, with an initial temperature of $\sim0.5\sim1$ eV and corresponding ionization computed assuming local thermal equilibrium (LTE). As noted in previous studies, the initial breakdown of the discharge lasts $<10$ ns, and does not substantially affect the system steady state [11]. The capillary wall is modeled as a boundary region with electron thermal conductivity $K_e$ computed by FLASH using a temperature dependent Spitzer-based model [15]. At each step, the temperature at the boundary is computed, averaged between the adjacent cell and first boundary block, and the corresponding conductivity is computed by FLASH before advancing thermal diffusion at the boundary. For the simulations in question, electron conductivity dominates ion (lattice) conductivity, and the heat transfer is computed according to

$$\frac{\partial e_e}{\partial t} = \nabla \cdot K_e \nabla T_e \quad (4)$$

We use a tabulated equation of state generated from the IONMIX code [16], which provides equation of state and radiative properties, including opacity, for LTE and non-LTE plasmas. The modest initial temperature of the fill species enables more accurate computation of state variables using the IONMIC model. Because the gas breakdown is not modeled, this assumption is reasonable, as the plasma temperature rises to several eV within 10s of ns, which is short compared to the 200 ns current rise times.

To model the discharge, we specify the azimuthal magnetic field $B_\theta$ at the boundary. FLASH updates the magnetic field explicitly on an auxiliary mesh using a second order integrator. This magnetic field
| Quantity | Units | 2D Benchmark | Lens | Heater |
|----------|-------|--------------|------|--------|
| \(n_0\)  | \(\text{cm}^{-3}\) | \(1.467 \times 10^5\) | \(1\) | \(1\) |
| \(T_{e,0}\) | eV | \(1\) | H | H/Ar |
| \(R\) | \(\mu\text{m}\) | \(250\) | 250 | 400 |

Table 1: Basic physical parameters for 2D benchmark (Sec. 3), lens (Sec. 4), and laser heater (Sec. 5) simulations. Each simulation uses a linearly rising discharge with peak current \(I_0 = 400\) A and period 200 ns rise time. Heater laser parameters are specified in Sec. 5.

is directly related to the current running through the gas, via Ampere’s Law. As the capillary walls are insulators, the entire discharge current runs within the open volume of the capillary, therefore the field at the surface of the capillary is uniquely specified. The energy deposited in the plasma is computed using the Spitzer resistivity [15], which is implemented in FLASH using expressions of the form found in the NRL Plasma Formulary [17]. According to this model, the resistivity varies as a function of the temperature and the Coulomb logarithm. Furthermore, the Coulomb logarithm varies as a function of the ionization state, which is updated self-consistently using the tabulated equation of state model. For these reasons, the distribution of current, and subsequently the magnetic field, can vary strongly within the capillary while still satisfying the magnetic field condition on the boundary.

The 3T model used by FLASH is necessary to account for electron-ion relaxation, which is relevant in cases of sharply varying ionization, high lattice conductivity, and/or high radiation flux. For the systems discussed in this paper, the relaxation time is short relative to the characteristic discharge duration \(\tau_{ei} \ll T_I\), and equilibrium is achieved within nanoseconds during benchmark simulations.

3. Two Dimensional Cylindrical Benchmarks
We have applied our model to capillary simulations using a 2D R-Z geometry. This geometry was chosen to exploit symmetries in the cylindrical capillary as well as additional laser propagation symmetries, and to significantly reduce the runtime to reasonable values for parametric scans. The simulations demonstrate the expected archetypal behavior from a capillary discharge, consisting of a three phase progression to a steady state system, as depicted in fig. 1.

The first phase consists of uniform Ohmic heating from the discharge current, which is supported by the presence of a linear magnetic field from the central axis out to the radial wall of the capillary. This heating drives additional ionization of the background gas. Once the ionization fraction nears 100%, the heat entering the system is no longer immediately absorbed internally. As a result, the plasma begins to redistribute itself, and thermal conduction at the wall of the capillary becomes the primary heat sink. This behavior is characteristic of the second phase of the capillary system. Finally, the system reaches a steady state wherein Ohmic heating along the capillary is balanced by cooling at the wall. This balance produces a peak in temperature on axis, and a corresponding minimum in density, leading to channel formation. Figure 1 depicts channel formation via temperature and density profiles alongside analytical predictions given in ref. [14]. We note that the analytical formula is only valid for values of the radius \(r \ll R\), but the behavior at small \(r\) is consistent with theory.

4. Plasma Lens Considerations
We next considered use-cases for active plasma lenses, which leverage the strong azimuthal magnetic fields of the capillary to provide axisymmetric focusing for beams. Figure 2 depicts the radial magnetic field profile for two capillary simulations with equivalent geometries and discharge currents, but one containing Argon and the other Hydrogen. For high discharge currents, significant aberrations have been observed in lenses filled with Hydrogen and Helium, resulting from the redistribution of current along the capillary due to the temperature-dependent conductivity of the plasma [18]. Our simulations reproduce
Figure 1: Temporal evolution of radial (a) density and (b) temperature distributions are plotted above for each of the three phases of capillary evolution, alongside comparisons to the steady-state analytic predictions, with the form given in [14].

A nonlinear distribution consistent with the J-T model described in ref [18], with a gradient enhancement of 1.16 and scaled temperature of $u_0 = 0.09$, which are reasonable in comparison to values presented in recent studies with a Helium lens [6].

These aberrations have been observed to be significantly reduced by using heavier fill species such as Argon [6]. One explanation for this discrepancy is that the higher mass of Argon slows the thermal equilibration within the channel. These simulations corroborate the persistence of aberration-free magnetic fields in an Argon-filled capillary over modest discharge times.

5. Laser Deposition
Laser heating is a promising technique for shaping the radial profile of the electron-density profile in capillary discharges [12]. Depositing energy along the central axis of the capillary reduces the on-axis plasma density, resulting in a sharper radial profile [19, 20]. Laser heating has been used to produce narrow plasma channels, permitting laser guiding over long distances, resulting in the acceleration of an electron beam to 8 GeV in a single stage [4]. More generally, the creation of a near-hollow channel plasma could permit the independent control of focusing fields and accelerating fields, thus minimizing the emittance growth of a matched witness beam [9].

FLASH includes a laser energy deposition unit, which models collisional absorption (e.g. inverse Bremsstrahlung) using collision rates computed from electron temperature and density in the plasma. For cold, underdense plasmas, such as those produced by the discharge, this process is the dominant absorption mechanism and adequately describes the heating of the plasma from lasers with powers comparable to those used in experiment [21, 22, 4]. The FLASH implementation has been well benchmarked against other community codes, such as HYDRA [23].

We performed simulations with a 1.053 $\mu$m-wavelength heater laser for a range of laser energies, spot sizes, and pulse lengths. Fig. 3 shows the time evolution of the on-axis electron density from a simulation with a 1 ns, 1 J, 50 $\mu$m-radius heater pulse. After about 150 ns, the density reaches a steady-state parabolic profile near the central axis, with a spot size $W_0 = 75.8 \mu$m. The laser pulse locally heats electron, reducing the on-axis density by 58% compared to the pre-pulse value. The on-axis density then
undergoes strongly damped oscillations with a period of about 12 ns, corresponding to the round-trip
time defined by the ion acoustic velocity of 50 $\mu$m/ns, commensurate with peak temperatures during
laser heating.

Fig. 4 depicts the radial electron-density profiles at 0.2 ns intervals following the laser. The density
profiles have been fit to a parabola, and resulting values of the spot size, $W_0$, as well as the coefficient
standard deviations, are shown in the figure. For parabolic density profiles and with non-relativistic laser
intensities ($a_0 < 1$), the matched laser spot size can be estimated by setting the right-hand-side of Eq. (1)
in Ref. [24] to zero:

$$W_0 = \left( \frac{r_0^2}{\pi r_e \Delta n} \right)^{1/4}, \quad (5)$$

where $r_0$ and $\Delta n$ are the radius and depth of the secondary channel, respectively, and $r_e$ is the classical
electron radius. Eq. (5) is used to calculate the spot sizes for the parabolic secondary channels shown in
6. Conclusions
We have developed 2D simulations of cylindrical capillary discharge plasmas using the FLASH code. Custom boundary conditions were used to capture realistic conductivities and magnetic field evolution, and verified the use of tabulated equations of state for interoperability with different gas species. Our simulations show good agreement with published simulations and analytical models. We then performed simulations of active plasmas lenses with different fill species, which show agreement with observations of nonlinearities in the resultant current density profile and magnetic field. We then demonstrated the use of FLASH’s laser deposition module for modeling a laser heater. Our simulations confirm the formation of a low density channel with a reduced spot size for a Gaussian laser pulse.

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References
[1] Geddes C G R, Toth Cs, van Tilborg J, Esarey E, Schroeder C B, Bruhwiler D, Nieter C, Cary J and Leemans W P 2004 Nature 431 538-541
[2] Kaganovich D, Ting A, Moore C I, Zigler A, Burris HR, Ehrlich Y, Hubbard R and Sprangle P 1999 Phys. Rev. E 59 R4769

Figure 4: Electron-density profiles beginning from the end of the heater laser pulse. Dotted lines represent the parabolic fits to the central density, and spot size $W_0$ is given and alongside standard deviations of the fitting coefficients (small values indicating a better fit).

Fig. 4. Future investigations will consider the effects of varying capillary length and diffraction in the heater laser generating a matched spot which depends on the longitudinal coordinate and time-of-arrival of the drive laser beam.
[3] Leemans W P et al. 2014 Phys. Rev. Lett. 113 245002
[4] Gonsalves A J et al. 2019 Phys. Rev. Lett. 122 084801
[5] van Tilborg J et al. 2015 Phys. Rev. Lett. 115 184802
[6] Lindstrom C A et al. 2018 Phys. Rev. Lett. 121 194801
[7] Steinke S et al. 2016 Nature 530 190-193
[8] D’Arcy R et al. 2019 Phys. Rev. Lett. 122 034801
[9] Schroder C B, Esarey E, Benedetti C and Leemans W P 2013 Phys. Plasmas 20 080701
[10] Fryxell B et al. 2000 Astrophys. J., Suppl. Ser. 131 273
[11] Bobrova N A, Esaulov A A, Sakai J-I, Sasorov P V, Spence D J, Butler A, Hooker S M and Bulanov S V 2001 Phys. Rev. E 65 016407
[12] Bobrova N A, Sasorov P V, Benedetti C, Bulanov S S, Geddes C G R, Schroeder C B, Esarey E, and Leemans W P 2013 Phys. Plasmas 20 020703
[13] Bagdasarov G A et al. 2017 Phys. Plasmas 24 083109
[14] Bagdasarov G A et al. 2017 Phys. Plasmas 24 053111
[15] Atzeni S and Meyer-ten-Vehn J 2004 The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter (Oxford: Clarendon Press)
[16] Macfarlane J J 1989 Comp. Phys. Comm. 56 259-278
[17] Huba, J D 2006 NRL Plasma Formulary NAVAL RESEARCH LAB WASHINGTON DC PLASMA PHYSICS DIV
[18] van Tilborg J et al. 2017 Phys. Rev. Accel. Beams 20 032803
[19] Durfee C G and Milchberg H M 1993 Phys. Rev. Lett. 71 2409
[20] Vollbeyn P, Esarey E, and Leemans W P 1999 Phys. Plasmas 6 2269
[21] Gibbon P and Forster E 1996 Plasma Phys. Control. Fusion 38 769
[22] Krue W 2018 The physics of laser plasma interactions (Boca Raton: CRC Press)
[23] Orban C, Fatenejad M, Chawla S, Wilks S C, and Lamb D Q 2013 A Radiation-Hydrodynamics Code Comparison for Laser-Produced Plasmas: FLASH versus HYDRA and the Results of Validation Experiments Preprint arXiv:1306.1584
[24] Benedetti C, Schroeder C B, Esarey E, and Leemans W P 2012 Phys. Plasmas 19 053101