Vibration analysis of concrete bridges during a train pass-by using various models

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Abstract: The vibration of a bridge must be determined in order to predict the bridge noise during a train pass-by. It can be generally solved with different models either in the time domain or the frequency domain. The computation cost and accuracy of these models vary a lot in a wide frequency band. This study aims to compare the results obtained from various models for recommending the most suitable model in further noise prediction. First, train-track-bridge models in the time domain are developed by using the finite element method and mode superposition method. The rails are modeled by Timoshenko beam elements and the bridge is respectively modeled by shell elements and volume elements. Second, power flow models for the coupled system are established in the frequency domain. The rails are modelled by infinite Timoshenko beams and the bridge is respectively represented by three finite element models, an infinite Kirchhoff plate, and an infinite Mindlin plate model. The vibration at given locations of the bridge and the power input to the bridges through the rail fasteners are calculated using these models. The results show that the shear deformation of the bridge deck has significant influences on the bridge vibration at medium-to-high frequencies. The Mindlin plate model can be used to represent the U-shaped girder to obtain the power input to the bridge with high accuracy and efficiency.

Key Words: railway bridges; vibration and noise; train-track-bridge interaction; power flow

1. Introduction
More and more elevated concrete bridges have been constructed for the growing metro lines in the suburbs of large cities due to its lower construction costs compared with tunnels. However, the noise from the elevated systems under the moving trains has adverse effects on the residents’ well-being and health. Rolling noise and bridge noise are the two dominant noise of the elevated metro system when the train speeds are less than 120 km/h. The prediction and control of rolling noise has been extensively investigated in the past, and a comprehensive review on this area can be found in the book by Thompson [1]. Nevertheless, the structure-borne noise from railway concrete bridges [2-8] only gained lots of concerns recently.

The vibration analysis of a bridge under moving trains is the basis for the bridge noise prediction. Janssens and Thompson [9] proposed a statistical energy analysis (SEA)-based method to predict vibration and noise from railway steel bridges in a large frequency range. Zhang et al. [4] studied the low-frequency concrete bridge noise by employing the three-dimensional finite element method for the vibration analysis and boundary element method for the acoustical analysis. Li et al. [2, 5] adopted the modal superposition method to simulate the train-track-bridge vibration in the time domain, and
the three-dimensional and 2.5-dimensional modal acoustic transfer vector techniques in the frequency domain for the prediction of bridge noise below 200 Hz. To overcome the inefficiency of finite element method based vibration analysis and boundary element method based acoustical analysis, Li et al. [7] utilized the finite element and statistical energy analysis method to form a hybrid procedure for bridge noise prediction. Another attempt to tackle the problem is the force-method based power flow scheme proposed by Li and Wu [10] for the investigation of the train-track-bridge dynamic interaction in the frequency domain. Li et al. [8] then combined this power flow method with the 2D infinite/finite element method to predict the rail noise and bridge noise below 1000 Hz with good balance between accuracy and efficiency.

In our previous studies, different methods have been proposed for the simulation of bridge vibration during a train pass-by. It is then necessary to find which method can simulate the bridge vibration with both efficiency and accuracy. In this study, three finite element models of the bridge are first compared in the train-track-bridge interaction analysis by using both of the time domain and the frequency domain methods. Then two infinite plate models are adopted in the frequency domain vibration analysis to investigate the accuracy of the simple models for the representation of the complex real bridge.

2. Train-track-bridge interaction analysis in time domain

2.1. Calculation method

Train-track-bridge dynamic interaction analysis has been widely used to obtain the time-varying responses of the coupled system. Li et al. [11] have proposed a mode superposition method based scheme to deal with this problem with the aid of the developed computer program named as VBC2.0.

![Figure 1. The train-track-bridge interaction model in time domain.](image)

In VBC2.0 program, a coupled train-track-bridge system is divided into the vehicle subsystem, the track subsystem, and the bridge subsystem (see Figure 1). Each subsystem can be expressed by an equation of motion. For train-track-bridge system, the equation of motion could be expressed as [2, 11]:

\[
\begin{align*}
\ddot{q}_v &= \Phi_v^T f_v - 2\xi_v \omega_v q_v - \omega_v^2 q_v \\
\ddot{q}_t &= \Phi_t^T f_t - 2\xi_t \omega_t q_t - \omega_t^2 q_t \\
\ddot{q}_b &= \Phi_b^T f_b - 2\xi_b \omega_b q_b - \omega_b^2 q_b
\end{align*}
\]

(1)

where \( q \), \( \Phi \), \( \omega \) and \( \xi \) are the modal coordinate vector, modal shape matrix, modal frequency matrix, and modal damping matrix of each subsystem respectively; \( f \) denotes the combination of the pseudo-forces within each subsystem and the external forces exerted by other subsystems connected to it; the subscript \( v \), \( t \) and \( b \) represent vehicle, track and bridge subsystems respectively; and the superscript \( T \) is the transpose operation of a matrix. Equation (1) can be solved using the fourth-order Runge-Kutta method. Roughness spectrum provided in ISO3095:2005 standard [12] is used for the excitation source of the train-track-bridge system by transforming it into samples of rail roughness in the space domain.
2.2. Finite element models

In this study, a 30-metre-long one-span U-shaped girder (see Figure 2) is used to investigate the effects of various models on the bridge vibration during a train pass-by. Three kind of finite element models of the bridge are respectively developed by using SEHLL181, SHELL63 and SOLID95 elements in the software ANSYS. According to the principle of at least six elements per wavelength, the element size should not exceed 0.20 m at the interest frequency range below 1500 Hz. The rails are simulated by BEAM188 elements with the lengths 40m longer than the one-span girder. The rails outside of the concerned span are connected to the rigid ground by rail fasteners. The parameters of the bridge, vehicle and rail are listed in Table 1.

![Figure 2. Cross-section of the bridge (unit: mm).](image)

Table 1. Main parameters of the models.

| Parameters        | Unit   | Value          |
|-------------------|--------|----------------|
| Bridge            | Young’s modulus | MPa   | 4×10^4       |
|                   | Density | kg/m^3 | 2600         |
|                   | Poission’s ratio | --   | 0.2          |
|                   | Young’s modulus | MPa   | 2.1×10^5     |
|                   | Density | kg/m^3 | 7800         |
|                   | Poission’s ratio | --   | 0.3          |
|                   | Vertical stiffness | MN/m | 60           |
|                   | Horizontal stiffness | MN/m | 20           |
| Rail              | Vertical damping | kN s/m | 80           |
| Rail fastener     | Horizontal damping | kN s/m | 60           |
| Trail             | Wheelset mass   | Kg    | 1150          |
| Wheel             | Contact stiffness | MN/m | 1.08×10^3   |
|                  | Wheelset mass   | Kg    | 1900          |
|                  | Contact stiffness | MN/m | 1.12×10^3   |

2.3. Effect of different models on the bridge vibration

Figure 3 shows the one-third octave-band acceleration level spectra for the bottom slab at the mid-span of the bridge on the pass-by time. It can be observed that various finite element models lead to similar results below 630 Hz. However, the relative deviation becomes prominent above 630 Hz. The bridge model using SHELL181 produces the computed vibration 7 dB larger than that using the SHELL63 model, because the former include the shear deformation and rotation inertia effects while the latter one neglects these effects. The Solid95 model gives larger bridge acceleration levels than the SHELL181 model mainly due to the local deformation under the concentrated forces of the fasteners.

Figure 4 illustrates the power transferred from the rails to the bridge using different models for the bridge. It can be seen from the figure that various models have slight influences on the bridge power for the frequencies above 630 Hz. The bridge models using SHELL181 elements and SOLID95 elements give consistent results about 4 dB larger than that using the SHELL63 model. The former two models both include shear deformation and rotation inertia effects while the latter one ignores these effects. It is noted that the solid element model has 16554 elements while the shell
model has only 1048 elements. It is more suitable using shell element to model the U-shaped girder, as it’s efficient and of high accuracy in terms of the computed power input to the bridge.

Figure 3. Vertical acceleration level spectra of bottom slab of bridge.

Figure 4. Power input to the U-shaped bridge by time domain method.

3. Train-track-bridge interaction analysis in frequency domain

3.1. Calculation method

Power flow analysis in the frequency domain could be used to investigate the vibration of a coupled system in the medium-to-high frequency range. Li et al [8, 10] proposed a force-method-based power flow approach to solve the train-track-bridge coupling system shown in Figure 5. Only the rigid wheel and the linearized contact spring are considered for the train modelling in the medium-to-high frequency range. The rail is modeled by infinite Timoshenko beam. The bridge can be represented by either a finite element model or an infinite plate model.

Figure 5. The force-method-based power flow model of coupled train-track-bridge system.

The compatibility equation of the coupled system can be obtained by considering the internal forces of the spring-dashpot pairs within the system as unknowns [8, 10]

$$\delta(\omega) F(\omega) + \Delta_p(\omega) = -A(\omega) F(\omega)$$

where $\omega$ is the angular frequency; $F(\omega)$ is the unknown force vector of the spring-dashpot pairs; $\Delta_p(\omega)$ is the relative compression displacement vector of the spring-dashpot pairs caused by the unit harmonic excitation $P(\omega)$ exerted on the released rail through the active wheel; $\delta(\omega)$ is the dynamic flexibility matrix of the released structure; and $A(\omega)$ is the dynamic flexibility matrix corresponding to the stiffness and damping of the spring-dashpot pairs.

The unknown spring-force $F(\omega)$ can be obtained from equation (2). Then the vibration velocities and power of the rail and bridge could be calculated based on the released structures under known forces. The wheel-rail contact force $F_c(\omega)$ could be obtained through the compatibility condition at the wheel-rail interface [1].

The moving roughness method [1] is used in the frequency domain analysis but the wheel position changes when the trains are moving on the bridge. In this study, two typical load cases were chosen to
include the effect of the wheel positions. In case A, the front and rear bogies of two adjacent vehicles locate at the middle of the bridge symmetrically, and in case B the two bogies of one vehicle load on the ends of the bridge symmetrically. In both load cases, each wheel is respectively chosen as the active wheel in turn. And the vibration of the bridge is calculated by averaging the results obtained from various load cases and active wheel scenarios.

3.2. Finite element models and infinite plate models of the bridge

Finite elements of the bridge can be used to calculate the dynamic flexibility matrix of the released bridge in Equation (2) by using mode superposition method. Thus the three bridge models used in the time domain analysis are also adopted in the frequency domain analysis. In addition, the flexibility of the bottom slab of the U-shaped girder can be represented by an infinite plate [13]. As there are rail support blocks on the slab, the thickness of infinite plate could be set to 1.4 times of the bottom slab thickness according to parameter analysis. The infinite Kirchhoff plate (thin plate theory) and Mindlin plate (thick plate theory) [13] are used to model the bridge separately in the power flow analysis.

3.3. Effect of different bridge models on the bridge vibration

Figure 6 shows the acceleration levels of the bottom slab beneath the rail by using different bridge models. It can be seen from Figure 6.(a) that three finite element models of the bridge give similar results in the frequency range of 40 Hz to 400 Hz. For the frequency above 630 Hz, the difference becomes distinct, which matches the trend obtained through the time domain analysis. Figure 6.(b) also indicates the effects of shear deformation and rotation inertia do have large influence on the bridge vibration above 630 Hz.

![Figure 6](image1)

**Figure 6.** Vertical acceleration level spectra of bottom slab of bridge: (a) finite element model; and (b) infinite plate model.

![Figure 7](image2)

**Figure 7.** Power flow to the U-shaped bridge: (a) finite element model; and (b) infinite plate model.
Figure 7 shows the bridge power obtained with different finite element models and infinite plate models. It can be observed that the bridge power will be underestimated by up to 10 dB in the frequency range from 315 Hz to 1250 Hz if the shear effect is not included.

4. Comparison and justification of the two methods
In this study, bridge vibration under a train pass-by was computed both in the time domain and frequency domain. The two methods provide similar results in terms of the effect of shear deformation on the bridge vibration, though they have some difference in the vibration amplitudes, as can be seen in Figure 8. Overall, both the time domain and frequency domain method can be used to compute the bridge vibration under a train pass-by with fairly good accuracy. And the Mindlin plate model which includes shear deformation and rotation inertia effects can be used in the power flow analysis in the medium-to-high frequency range above 200 Hz accurately and efficiently.

Figure 8. Power flow to the bridge obtained from different methods and models.

5. Conclusions
In this study, three finite element models have been used to calculate the bridge vibration during a train pass-by in both the time domain and frequency domain. Two theoretical plate models have also been adopted in the frequency domain analysis. The conclusions obtained from the calculated results can be summarized as follows:

(1) Both of the time domain and frequency domain analysis method could provide reasonable vibration results of bridge in the coupled train-track-bridge system. It is better to use the time domain method in the low frequency range (below 200 Hz) for better accuracy and the frequency domain method in medium-to-high frequency range for higher efficiency.

(2) The Mindlin plate model can give good results compared with the solid finite element model but the former is more efficient in the modelling and computing procedures.

The simulated bridge power by the train-track-bridge interaction analysis can be used to predict the bridge noise by combing it with the noise radiation models. The equivalent thickness of the infinite plate for a real bridge should be further investigated in the future.

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