Recent developments in nucleon spin structure
with focus on $h_{1L}^\perp$ and pretzelosity $h_{1T}^\perp$

Dedicated to 65 anniversary of Professor Klaus Goeke*

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Abstract. The leading twist transverse momentum dependent parton distribution functions (TMDs) $h_{1L}^\perp$ and $h_{1T}^\perp$, which is sometimes called “pretzelosity,” are studied. For $h_{1L}^\perp$ we consider a “Wandzura–Wilczek-type” approximation, which follows from QCD equations of motion upon the neglect of pure twist-3 terms and allows to express it in terms of transversity. On the basis of available data from HERMES we test the practical usefulness of this approximation and discuss how it can be further tested by future CLAS and COMPASS data. We review the theoretical properties of pretzelosity and observe an interesting relation valid in a large class of relativistic models: The difference between helicity and transversity distributions, which is often said to be a ‘measure of relativistic effects’ in the nucleon, is nothing but the transverse moment of pretzelosity. We discuss preliminary deuteron target data from COMPASS on the single spin asymmetry (SSA) in semi-inclusive deep inelastic scattering (SIDIS) related to pretzelosity, and make predictions for future experiments at JLab, COMPASS and HERMES.

1 Introduction

Processes like SIDIS, hadron production in $e^+e^-$ annihilations or the Drell–Yan process [1–16] factorize at leading twist [17–20] and allow to access information on transverse momentum dependent fragmentation functions and TMDs [21, 22]. The latter contain novel information on the nucleon structure. In order to be sensitive to “intrinsic” transverse parton momenta it is necessary to measure adequate transverse momenta in the final state, for example, in SIDIS the transverse momenta of produced hadrons with respect to the virtual photon.

The eight leading-twist TMDs [9], and further subleading-twist structures [23, 24] describe the structure of the nucleon in these reactions

\[ f_a^a, f_T^a, g_{1L}^a, g_{1T}^a, h_{1L}^a, h_{1T}^a, h_{1L}^{1,a}, h_{1T}^{1,a}, e^a, g_T^a, h_L^a, \ldots \]  \tag{1}

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which are functions of $x$ and $p_T^2$. (The dots denote thirteen further twist-3 TMDs. The renormalization scale dependence is not indicated for brevity.) Integrating over transverse momenta one is left with six independent “collinear” parton distribution functions (pdfs) [25, 26]

\[ f_1^a(x), \quad g_1^u(x), \quad h_1^a(x), \quad e^a(x), \quad g_1^d(x), \quad h_2^a(x), \]  

(2)

where we have $j(x) = \int d^2 p_T j(x, p_T^2)$ for $j = f_1^a, e^a, g_T, h_L$ while $g_1^u(x) = \int d^2 p_T g_{1T}^u(x, p_T^2)$ and $h_2^a(x) = \int d^2 p_T \{h_1^T(x, p_T^2) + 2(2M_N^2)h_{1T}^a(x, p_T^2)\}$.

The fragmentation of unpolarized hadrons is described in terms of two fragmentation functions, $D_1^a$ and $H_1^{a\perp}$, at leading-twist. In SIDIS (with polarized beams and/or targets, where necessary) it is possible to access information on the leading twist TMDs by measuring the angular distributions of produced hadrons. Some data on such processes are available [27–45]. The fragmentation functions and TMDs in SIDIS and other processes were subject to numerous studies in the literature [46–73]. This is true especially for the prominent transversity Boer–Mulders function $h_1^{a\perp}$ and the Collins fragmentation function $H_1^{a\perp}$. Among the so far less considered functions are $h_2^{a\perp}$ and the ‘pretzelosity’ distribution $H_1^{a\perp}$.

The purpose of this lecture (based on the works [67, 68]) is fourfold. First, we discuss whether some of the unknown TMDs could be approximated in terms of (possibly) better known ones. Second, we review what is known about $h_2^{a\perp}$. Third, we mention the models these TMDs were calculated. Fourth, we present estimates for SSAs in which these functions enter, and discuss the prospects to measure these SSAs in experiments at Jefferson Lab and COMPASS.

The process of SIDIS is sketched in Fig. 1.

![Fig. 1. Kinematics of SIDIS, $lN \to l'hX$, and the definitions of azimuthal angles in the lab frame.](image)

We denote the momenta of the target, incoming and outgoing lepton by $P$, $l$ and $l'$ and introduce $s = (P + l)^2$, $q = l - l'$ with $Q^2 = -q^2$. Then $y = \frac{p_T}{P}$, $x = \frac{Q^2}{2p_T q}$, $z = \frac{P q}{P_T}$, and $\cos \theta_\gamma = 1 - \frac{2M_N^2}{Q^2 (1 - y)}$ where $\theta_\gamma$ denotes the angle between target polarization vector and momentum of the virtual photon $\gamma^*$. See Fig. 1, and $M_N$ is the nucleon mass. The component of the momentum of the produced hadron transverse with respect to $\gamma^*$ is denoted by $p_{h\perp}$ and $P_{h\perp} = |p_{h\perp}|$.

The cross section differential in the azimuthal angle $\phi$ of the produced hadron has schematically the following general decomposition [7, 74] (the dots indicate power suppressed terms):

\[
\frac{d\sigma}{d\phi} = F_{UU} + \cos(2\phi)F_{UU}^{\cos(2\phi)} + S_L \sin(2\phi)F_{UL}^{\sin(2\phi)} + \lambda \left[ S_L F_{LL} + S_T \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) F_{UT}^{\sin(\phi - \phi_S)} + \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + 3(\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right] + \ldots
\]

(3)

In $F_{XY}^{\text{weight}}$ the index $X = U(L)$ denotes the unpolarized (longitudinally polarized, helicity $\lambda$) beam. $Y = U(L, T)$ denotes the unpolarized target (longitudinally, transversely with respect to the virtual photon polarized target). The superscript reminds on the kind of angular distribution of the produced hadrons with no index indicating an isotropic $\phi$-distribution.

Each structure function arises from a different TMD. The chirally even $f$'s and $g$'s enter the observables in connection with the unpolarized fragmentation function $D_1^a$, the chirally odd $h$'s in connection with the chirally odd Collins fragmentation function $H_1^{a\perp}$

\[
F_{UU} \propto \sum_a c_a^2 f_1^a \otimes D_1^a, \quad F_{LT}^{\cos(\phi - \phi_S)} \propto \sum_a c_a^2 g_1^{1T} \otimes D_1^a, \quad F_{UT}^{\sin(\phi - \phi_S)} \propto \sum_a c_a^2 f_1^{1T} \otimes D_1^a,
\]

(4)

\[
F_{LL} \propto \sum_a c_a^2 g_1^a \otimes D_1^a,
\]

(5)
F^{\cos(2\phi)}_{UU} \propto \sum_a e_a^2 h_{1L}^a \otimes H_1^L, \quad F^{\sin(\phi+\phi_0)}_{UT} \propto \sum_a e_a^2 h_1^a \otimes H_1^a, \quad (6)

F^{\sin(2\phi)}_{UL} \propto \sum_a e_a^2 h_{1L}^a \otimes H_1^L, \quad F^{\sin(3\phi-\phi_0)}_{UT} \propto \sum_a e_a^2 h_{1T}^a \otimes H_1^a. \quad (7)

More precisely, TMDs and fragmentation functions enter the respective tree-level expressions in certain convolution integrals, indicated by \( \otimes \) in (4-7), which entangle transverse parton momenta from TMDs and fragmentation functions. (Going beyond tree-level description requires introduction of soft factors \([19, 20]\) from which we refrain here.) In general, such convolution integrals cannot be solved, unless one weighs the DIS counts with an adequate power of transverse hadron momentum \([9]\).

### 2 WW-type approximation for \( h_{1L}^a(x) \)

It is clear that there cannot be any exact relations among TMDs, because all TMDs are independent functions \([23]\). One may, however, find approximate relations as follows. From QCD equations of motion (eom), one obtains among others the following exact relations \([8]\)

\[
g_{TT}^{(1)a}(x) = x g_{T}^{a}(x) - x \tilde{g}_{T}^{a}(x), \quad -2 h_{1L}^{(1)a}(x) = x h_{L}^{a}(x) - x \tilde{h}_{L}^{a}(x), \quad (8)
\]

with \( h_{1L}^{(1)a}(x) = \int d^{2}p_{T} \frac{p_{T}^{a}}{2M_{S}} h_{1L}^{(1)a}(x, p_{T}^{2}) \), \( g_{TT}^{(1)a}(x) \) analog, and \( \tilde{g}_{T}^{a}(x), \tilde{h}_{L}^{a}(x) \) denoting pure twist-3 terms due to quark-gluon-quark correlations (and current quark mass terms). In the next step, we recall the relations among the collinear pdfs (2) \([26, 75, 76]\)

\[
g_{T}^{a}(x) = \int_{x}^{1} \frac{dy}{y} g_{T}^{a}(y) + \tilde{g}_{T}^{a}(x), \quad h_{L}^{a}(x) = 2x \int_{x}^{1} \frac{dy}{y^2} h_{T}^{a}(y) + \tilde{h}_{L}^{a}(x), \quad (9)
\]

where \( \tilde{g}_{T}^{a}(x), \tilde{h}_{L}^{a}(x) \) denote pure twist-3 (and mass) terms \([77, 78]\), though different ones than in (8). Eqs. (9) isolate “pure twist-3 terms” in the “twist-3” pdfs \( g_{T}^{a}(x), h_{L}^{a}(x) \), because in (2) the underlying “working definition” of twist \([79]\) (a pdf is “twist \( t \)” if its contribution to the cross section is suppressed, in addition to kinematic factors, by \( 1/Q^{t-2} \) with \( Q \) the hard scale) differs from the strict definition of twist (mass dimension of the operator minus its spin).

The remarkable observation is that \( \tilde{g}_{T}^{a}(x) \) is consistent with zero within error bars \([80–82]\) and to a good accuracy

\[
g_{T}^{a}(x) \approx \frac{\int_{x}^{1} \frac{dy}{y} g_{T}^{a}(y)}{} \quad \text{(exp. observation)} \quad (10)
\]

which is the “Wandzura–Wilczek (WW) approximation”, whose experimental status is demonstrated on Fig. 2.

Lattice QCD \([83, 84]\) and the instanton model of the QCD vacuum \([85]\) support this observation. Interestingly the latter predicts also \( \tilde{h}_{L}^{a}(x) \) to be small \([86]\), such that

\[
h_{L}^{a}(x) \approx 2x \int_{x}^{1} \frac{dy}{y^2} h_{T}^{a}(y) \quad \text{(prediction)} \quad (11)
\]

On the basis of this positive experimental and theoretical experience one may suspect that the analog terms in the relations (8) could also be negligible. If true one would have

\[
g_{TT}^{(1)a}(x) \approx x \int_{x}^{1} \frac{dy}{y} g_{T}^{a}(y), \quad (12)
\]

\[
h_{1L}^{(1)a}(x) \approx -x^2 \int_{x}^{1} \frac{dy}{y^2} h_{1}^{a}(y), \quad (13)
\]

that could be satisfied with an accuracy similar to (10). This remains to be tested in experiment.
An immediate application (or test) for the relations (12, 13) is provided by the following single/double spin asymmetries (SSA/DSA) in SIDIS

$$A_{UL}^{\sin 2\phi} \propto \sum_a e_a^2 h_{1L}^{(1)a} H_{1L}^{(a)},$$

$$A_{UT}^{\cos(\phi - \phi_S)} \propto \sum_a e_a^2 g_{1T}^{(1)a} D_{1T}^{(a)},$$

where the Collins fragmentation function $H_{1L}^{(a)}$ [4–6] in (14) was determined very recently from SIDIS data [32–34, 36] on the SSA

$$A_{UT}^{\sin(\phi + \phi_S)} \propto \sum_a e_a^2 h_{1L}^{(a)} H_{1L}^{(a)},$$

together with $e^+e^-$ annihilation data [40, 41] giving rise to a first but already consistent picture [60–62]. The $D_{1T}^{(a)}$ in (15) is the unpolarized fragmentation function which enters, of course, also the respective denominators in (14-16) that are proportional to $\sum_a e_a^2 f_{1T}^{(a)} D_{1T}^{(a)}$.

Final HERMES [28–30] and preliminary CLAS [37] data on (14) and preliminary COMPASS data [44] on (15) are available, such that first tests of the WW-type approximations (12, 13) are now or soon possible. A test of the approximation (12) was suggested in [66] along the lines of the study of the SSA (15) discussed previously also in [46].

In this lecture we present a test of the approximation (13). Under the assumption that this approximation works, we shall see that it yields results for the SSA (14) compatible with data [28–30]. From another point of view our work provides a first independent cross check from SIDIS for the emerging picture of $H_{1L}^{(a)}$ [60–62].

Among the eight structure functions in SIDIS described in terms of twist-2 TMDs and fragmentation functions the SSAs (14, 15) are the only ones, for which WW-type approximations could be of use. Exact com-relations exist for all eight twist-2 pdfs in (1). But the relations (8) are special in that they connect the respective TMD pdfs, namely $g_{1T}$ and $h_{1L}$, to “collinear” twist-3 pdfs, namely $g_T$ and $h_1$. Those in turn are related to twist-2 pdfs, $g_1$ and $h_1$, by means of (experimentally established or theoretically predicted) WW-approximations (10, 11). Experiments may or may not confirm that the WW-type approximations (12, 13) work.

What would it mean if (12, 13) were found to be satisfied to within a very good accuracy? First, that would be of practical use for understanding and interpreting the first data [28–44]. Second, it would call for theoretical explanations why pure twist-3 terms should be small. (Only for the smallness of the “collinear” pure twist-3 terms in (10, 11) lattice QCD [83, 84] and/or instanton vacuum [85, 86] provide explanations.)

What would it mean if (12, 13) were found to work poorly? This scenario would be equally interesting. In fact, all TMDs in (1) are independent structures, and any of them contains different type of information on the internal structure of the nucleon. The measurement of the complete set of all eighteen structure functions available in SIDIS [7] is therefore indispensable for our aim to learn more about the nucleon structure.

One type of information accessible in this way concerns effects related to the orbital motion of quarks, and in particular correlations of spin and transverse momentum of quarks which are dominated by valence quarks and hence play a more important role at large $x$. E.g. it was shown that spin-orbit correlations may lead to significant contributions to partonic momentum and helicity distributions [70] in the large-$x$ limit. Spin-orbit correlations are presumably of similar importance for transversity, and crucial for $h_{1L}^{(a)}$, which is a measure for the correlation of the transverse spin and the transverse momentum of quarks in a longitudinally polarized nucleon.
3 Properties and experimental check of WW-type approximation for $h_{1L}^{\perp(1)u}(x)$

In order to model $h_{1L}^{\perp(1)u}(x)$ by means of the WW-type approximation (13) one inevitably has to use, in addition, models for the transversity pdf. Fig. 3a shows four different models: saturation of the Soffer bound [87] at the low initial scale of the leading order parameterizations [89] (choosing $h_{1}^u > 0$ and $h_{1}^d < 0$), the chiral quark soliton model ($\chi$QSM) [88], the non-relativistic model assumption $h_{1}^u(x) = g_{1}^u(x)$ at the low scale of the parameterization [89], and the hypercentral model [90]. All curves in Fig. 3 are leading-order evolved to 2.5 GeV$^2$ which is a relevant scale in experiment, see below.

These (and many other [91,92]) models agree on that $h_{1}^u(x) > 0$ and $h_{1}^d(x) < 0$ with $|h_{1}^u(x)| < |h_{1}^d(x)|$, though the predictions differ concerning the magnitudes, see Fig. 3a. Models in which antiquark distribution functions can be computed, e.g. [88], predict that the transversity antiquark pdfs are far smaller than the quark ones.

Let us establish first a robust feature of the relation (13), namely the ratio $R = h_{1L}^{\perp(1)u}(x)/h_{1}^u(x)$ exhibits little dependence on the transversity model, see Fig. 3b. A “universal” behaviour of this ratio at large $x$ is not surprising. By inspecting (13) one can prove:

- for $x \to 1$, it behaves like $R \approx (1 - x)$,
- for $x \to 0$, it also vanishes: $R \to 0$,
- a common feature: $|R| \lesssim 0.1$.

In the following we will use the $\chi$QSM [93,94], see Fig. 3c, which has several advantages: it describes the twist-2 pdfs $f_2^u(x)$ and $g_2^u(x)$ within (10–30)% accuracy [95], it is derived from the instanton vacuum model [96,97] which predicts the “collinear WW-type approximation” (11) to work well [86], and below we will use information on the Collins effect from the analysis [61] where this model was used. This helps to minimize the model-dependence in our study. But we shall see that our conclusions do not depend on the choice of model.

The SSA as defined in the HERMES experiment is given by

$$A_{UL}^{\sin 2\phi} = \frac{\sum_i \sin(2\phi_i)(N_i^- - N_i^+)}{\sum_i \frac{1}{2}(N_i^- + N_i^+)} = \frac{\int \frac{d\gamma}{(1-y)^2}}{\int \frac{d\gamma}{(1-y+y^2)^2}} \frac{2^{\sin 2\phi}}{F_{UU}^{\sin 2\phi}}$$

where $N_i^- (N_i^+)$ denotes the number of events $i$ with target polarization antiparallel (parallel) to the beam. Had the events in the numerator of (17) been weighted by $P_{UL}(M_N m_h)$ in addition to sin$(2\phi)$, the convolution integral could be solved in a model independent way with the result given in terms of the transverse moment of $h_{1L}^u$ and an analog moment for $H_{1L}^u$ [9]. Including such an additional weight makes data analysis more difficult due to acceptance effects. Omitting it, however, forces one to resort to models.

Here we shall assume the distributions of transverse parton momenta to be Gaussian. Such an Ansatz satisfactorily describes data on many hard reactions [55], provided the transverse
momenta are much smaller than the hard scale of the process, i.e. \( \langle P_{h\perp} \rangle \ll \langle Q \rangle \) which is the case at HERMES. In fact, the \( z \)-dependence of \( \langle P_{h\perp} \rangle \) at HERMES [30] is well described in the Gauss Ansatz [58]. Of course, one has to keep in mind that it is a crude approximation, and it is not clear whether it works also for polarized distribution and fragmentation functions. Moreover, since also unintegrated versions of (8) hold, this Ansatz cannot be equally valid for all TMDs.

What is convenient for our purposes is that the Gauss Ansatz allows to solve the convolution integral. Using the WW-type approximation (13) with \( h_1^\perp(x) \) from the \( \chi \) QSM [88], and information on the Collins effect from [60–62], we obtain the results shown in Fig. 4. The error bands of the theoretical curves reflect the present uncertainties in the quantitative understanding of the Collins effect, see [67] for explicit expressions and details.

Our results shown in Fig. 4 for pion production from proton and deuteron targets are consistent with the HERMES data [28–30], and do not exclude that (13) is a useful approximation.

Further insights are expected from the CLAS experiment at Jefferson Lab, which promises higher statistics due to two orders of magnitude higher luminosity, and provides access to much larger \( x \) and larger \( z \) than HERMES and COMPASS. Large \( \langle z \rangle \) may also enhance the SSA due to Collins function \( H_1^{-(1/2)a}(z) \propto z D_1^a(z) \), as observed in [61]. This makes CLAS an ideal experiment for studies of this SSA in particular and spin-orbit correlations in general. Comparison of the various data sets will also allow to draw valuable conclusions on the energy dependence of the process, possible power-corrections, etc.

The preliminary data from CLAS [37] have shown non-zero SSAs for charged pions, and a compatible with zero within error bars result for \( \pi^0 \). Within our approach it is possible to understand the results for \( \pi^+ \) and \( \pi^0 \), however, we obtain for \( \pi^- \) an opposite sign compared to the data. In view of this observation, it is worth to look again on Fig. 4b which shows HERMES data on the \( \pi^- \) SSA. Does Fig. 4b hint at an incompatibility? Charged pions and in particular the \( \pi^- \) may have significant higher twist contributions, in particular from exclusive vector mesons and semi-exclusive pion production at large \( z \).

New data expected from CLAS with \( E_{\text{beam}} = 6 \text{ GeV} \) [98], will increase the existing statistics by about an order of magnitude and more importantly provide comparable to \( \pi^+ \) sample of \( \pi^0 \) events. Neutral pion sample is not expected to have any significant contribution from exclusive vector mesons, neither it is expected to have significant higher twist corrections due to semi-exclusive production of pions with large \( z \) [99], where the separation between target and current fragmentation is more pronounced. The JLab upgrade to 12 GeV [100] will allow to run at an order of magnitude higher luminosity than current CLAS, providing a comprehensive set of single and double spin asymmetries covering a wide range in \( x \) and \( z \). That will allow detailed studies of kinematic dependence of target SSA and clarify the situation.

COMPASS has taken data with a longitudinally polarized deuteron target which are being analyzed, and soon also a proton target will be used. The 160 GeV muon beam allows to extend the measurements of SSAs to small \( x \). With the cut \( Q^2 > 1 \text{ GeV}^2 \) the average \( \langle Q^2 \rangle \) at
COMPASS is comparable to that at HERMES. Therefore Fig. 4 shows roughly our predictions for COMPASS. One may expect $A_{UL}^{sin2\theta}$ to be substantially smaller than $A_{UL}^{sin(\phi+\phi_s)}$ especially at small $x$. It will be interesting to see whether these predictions will be confirmed by COMPASS.

4 What do we know about $h_{1T}^\perp$?

This TMD is defined as the coefficient of the correlation
\[ \frac{1}{2\pi^2}[(S_T p_T)(p_T s_T) - \frac{1}{2} p_T^2 (S_T s_T)], \]
where $S_T$ and $s_T$ denote the spin vectors of proton and quark, which are transverse with respect to the virtual photon momentum. Let us summarize briefly what we know about pretzelosity.

(1) It is chirally odd and so there is no gluon analog of pretzelosity.

(2) It has a probabilistic interpretation [8].

(3) At large-$x$ it is predicted to be suppressed by $(1-x)^2$ compared to $f_1^a(x)$ [72].

(4) It is expected to be suppressed also at small $x$ compared to $f_1^a$.

(5) It must satisfy the positivity condition [73]:
\[ |h_{1T}^{\perp}(x, p_T^2)| \leq \frac{1}{2} \left( f_1^a(x, p_T^2) - g_1^q(x, p_T^2) \right) \quad \text{or} \quad |h_{1T}^{\perp}(x)| \leq \frac{1}{2} \left( f_1^a(x) - g_1^q(x) \right). \]  

With the Soffer inequality $|h_1^q(x)| \leq \frac{1}{2} (f_1^q + g_1^q)(x)$ [87], we obtain the remarkable bound:
\[ |h_{1T}^{\perp}(x)| + |h_1^q(x)| \leq f_1^a(x). \]  

(6) In the limit of a large number of colors $N_c$, it was shown that for $x N_c \approx O(1)$ and $p_T \approx O(1)$ (the same pattern holds also for antiquarks) [103]:
\[ \frac{h_{1n}^\perp + h_{1d}^\perp}{h_{1n}^\perp - h_{1d}^\perp} \approx O(1/N_c) \]  

(7) It requires the presence of nucleon wave-function components with two units orbital momentum difference, e.g. $s-d$ interference or quadratic in $p$-wave component [72].

(8) In some sense it ‘measures’ the deviation of the ‘nucleon shape’ from a sphere [104] (that is why it was called pretzelosity).

(9) In simple (spectator-type) models, it has been related to chirally odd generalized parton distributions [101].

(10) It was observed in the bag model [68] that
\[ h_{1T}^{\perp}(x) = g_1^q(x) - h_1^q(x) \]  

and since the difference of $g_1^q(x)$ and $h_1^q(x)$ is a measure for relativistic effects in nucleon’ [26] one may view the transverse moment of pretzelosity as a measure for relativistic effects.

An important question is: How general the relation (21)? It is clear that this relation cannot be strictly valid in QCD, where all TMDs are independent. However, could it nevertheless allow to gain a reasonable estimate for $h_{1T}^{\perp}(x)$ in terms of transversity and helicity? Until clarified by experiment, we can address this question only in models.

Interestingly, the relation (21) does not only hold in the bag model [68], but is found [68] to be satisfied also in the spectator model [105]. In fact, it was conjectured [68] that (21) could hold in a large class of relativistic quark models, which was subsequently confirmed in the constituent quark model [106] and the relativistic model of the proton [107] (but not in a

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1 Actually, in the decomposition of the gluon analog of the above correlator structure appears in Ref. [101] has been called $h_{1T}^{\perp}$ [68]. In Ref. [102] it was given a different name.) This gluon TMD, however, has different properties compared to our quark pretzelosity. For example, the $h_{1T}^{\perp}$ of [101] is ‘odd’ under time-reversal while $h_{1T}^{\perp}$ with $a = q$, $\bar{q}$ is ‘even’.
variant of the spectator model [108]). These are all quark models. The limitations of (21) are nicely illustrated in the ‘quark target model’ where in addition to quarks there are also gluons, and (21) is not satisfied [101]. Thus, the explicit inclusion of gluon degrees of freedom spoils this relation, i.e. it can be valid in ‘no-gluon-models’ only.

It would be interesting and instructive to know the necessary and sufficient conditions for the relation (21) to hold in a (quark) model.

5 Preliminary COMPASS data & prospects at JLab

In SIDIS pretzelosity gives rise (in combination with the Collins fragmentation function [5,4]) to the SSA

$$A_{UT}^{\sin(3\phi - \phi_S)} = C_{\text{Gauss}} \frac{\sum_n e_n^2 x H_{UT}^{(1)}(x) H_{1}^{(1/2)}(z)}{\sum_n e_n x f_T^n(x) D^n_T(z)}$$

(22)

where we assumed the Gauss Ansatz, whose parameters are contained in the factor $C_{\text{Gauss}}$. This factor is a slowly varying function of the Gauss model parameters and can be well approximated for practical purposes by its maximum $C_{\text{Gauss}} \leq C_{\text{max}} = 3/(2\sqrt{2})$, see for further details [68].

At COMPASS the $\sin(3\phi - \phi_S)$ and other SSAs were measured on a deuteron target [44]. By saturating the positivity bound for the transverse moment of pretzelosity (point (5) in Sec. 4) we estimated [68] the maximum effect for the SSA. We used the parameterizations [89,109], and for $H_{1}$ the information from [61,62].

Fig. 5. The transverse target SSA $A_{UT}^{\sin(3\phi - \phi_S)}$ for deuteron estimated on the basis of the positivity bound vs. preliminary COMPASS data [44] for positive (a) and for negative (b) hadrons. (c) The same for the $\pi^+$ production from proton in the kinematics of CLAS 12 as function of $x$ (error projections from [100]). Solid curve presents the prediction of relativistic covariant model [107]. The shaded area is the region allowed by positivity (18).

The results are shown in Fig. 5a,b and compared to the preliminary data [44]. At small $x$ the preliminary data favor that pretzelosity does not reach the bound. Whether due to the expected suppression at small $x$ or opposite signs of $u$ and $d$-flavors, see Sec. 3, cannot be concluded.

The important observation is that preliminary COMPASS data [44] do not exclude a sizeable effect in the region $x > 0.1$, see Fig. 5a,b, where JLab can measure with precision. This is demonstrated in Fig. 5c showing the $\pi^+$ SSA from a proton target in the kinematics of CLAS with 12 GeV beam upgrade (with error projections for 2000 hours run time [100]).

It could even more promising to look at SSAs due to Collins effect, like $A_{UT}^{\sin(3\phi - \phi_S)}$, in kaon production. The statistics for kaon production is lower than for pion production, but the SSA might be larger as it is suggested by a model [110] of the Collins function. With a RICH detector at CLAS the kaon SSAs could be measured in the valence-$x$ region [100].

6 Conclusions

The longitudinal SSAs in SIDIS [28–31] were subject to intensive, early studies [47–51] that were based on assumptions concerning the flavour dependence of $H_{1}$ [49,52,54] that turned out
not to be supported by data on the Collins effect from SIDIS with transverse target polarization [32–34, 36] and $e^+e^-$-annihilations [40, 41]. These data give rise to a new, consistent picture of $H_1^+ [60–62]$ which invites reanalyses of longitudinal SSAs.

In this work we did this for $A_{UL}^{\sin2}\propto \sum_a e_a^2 h_{1L}^{(1)a} H_{1a}^+$ from the particular point of view of the question whether there are useful, approximate relations among different TMDs. In fact, QCD equations of motion relate the TMDs entering this SSA to $h_{1L}^+$ and certain pure twist-3 (and quark mass) terms. Neglecting such terms yields an approximation for $h_{1L}^{(1)a}$ similar in spirit to the WW-approximation for $g_2^+(x)$ that is supported by data.

Our study reveals that data do not exclude the possibility that such WW-type approximations work. As a byproduct we observe that data on the two SSAs, $A_{UL}^{\sin2}$ and $A_{UL}^{\sin(\phi+\phi_S)}$, are compatible. This is important because both observables are due to (the same!) Collins effect.

In Ref. [66] predictions for $A_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2 g_{1T}^{(1)a} D_1^a$ were made assuming the validity of a WW-type approximation for the relevant pdf. Comparing these predictions to preliminary COMPASS data [44] one arrives at the same conclusion. Also here data do not exclude the possibility that the WW-type approximation works.

In order to make more definite statements precise measurements of these SSAs are necessary, preferably in the region around $x \approx 0.3$ where the SSAs are largest. An order of magnitude more data on target SSA expected from the CLAS upcoming run [98] will certainly improve our current understanding of this and other SSAs and shed light on spin-orbit correlations.

The value of precise $A_{UL}^{\sin2}$ data should not be underestimated. This SSA is in any case an independent source of information on the Collins effect. An experimental confirmation of the utility of the WW-type approximation (13), however, would mean that it is possible to extract information on transversity, via (13), from a longitudinally polarized target.

Another subject of this lecture were the properties of the pretzelosity distribution function $h_{LT}^+$, and the presentation of a study of this TMD in the bag model. In the bag and in some other quark models we observed an interesting relation, which can be summarized for illustrative purposes by the following assertion:

$$\text{helicity} - \text{transversity} = \text{transverse moment of pretzelosity}. \quad (23)$$

That the difference between the helicity and transversity distributions is ‘a measure of relativistic effects’ is known since long ago [26] (and was also recognized in a bag model calculation). However, now we are in a position to make this statement more precise. This difference is just $h_{LT}^+$ which thus ‘measures’ relativistic effects in the nucleon, and vanishes in the non-relativistic limit where helicity and transversity distributions become equal.

This relation is not supported in models with explicit gluon degrees of freedom [101], and, of course, cannot be true in QCD where all TMDs are linearly independent. Nevertheless, the relation (23), see Eq. (21) for its precise formulation, could turn out to be a useful approximation. In view of the numerous novel functions involved, any well-motivated approximation is welcome and valuable [69].

Besides being useful for extending our intuition on relativistic spin-orbit effects in the nucleon [104,72], the relation (23) has also an important consequence on transversity. In all quark models where (23) holds, $h_{LT}^+$ is negative. Since $g_1^+(x)$ is positive, this implies that $h_1^+(x) > g_1^+(x)$. For the $d$-flavor signs are reversed, but in any case $|h_1^+(x)| > |g_1^+(x)|$ which is confirmed in models, e.g., [92,88].

In the bag model, the negative sign of $h_{LT}^+$ arises because it is proportional to minus the square of the $p$-wave component of the nucleon wave function. Thus, in models with no higher orbital momentum ($d$-wave, etc.) components, $h_{LT}^+$ is manifestly negative ($h_{LT}^+$ has opposite sign dictated by SU(6) symmetry, and predicted in large $N_c$ [103]).

This prediction can be tested at JLab. Since the production of positive pions from a proton target is dominated by the $u$-flavor, one expects a negative $\sin(3\phi-\phi_S)$ SSA, see Fig. 5.

Forthcoming analyzes and experiments at COMPASS, HERMES and JLab [98,100] will provide valuable information on the pretzelosity distribution function, and will deepen our understanding of the nucleon structure.
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