Control of Magnetohydrodynamic Mixed Convection and Entropy Generation in a Porous Cavity by Using Double Rotating Cylinders and Curved Partition

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ABSTRACT: In this work, mixed convection and entropy generation analyses in a partitioned porous cavity with double inner rotating cylinders are explored under magnetic field effects. A curved partition shape is considered with identical rotating cylinders and an inclined magnetic field, while the right vertical wall moves with a constant speed in the y-direction. Numerical simulations are performed by considering various values of Rayleigh number, Hartman number, Darcy number, inclination of the magnetic field, size of the curved partitions, and rotational speeds of the inner cylinders and their vertical locations with the cavity. Complicated flow field with multicular structures are observed due to the complex interaction between the natural convection, moving wall, and rotational effects of inner cylinders. Improved heat-transfer performance is obtained with higher values of magnetic field inclination, higher values of permeability/porosity of the medium, and higher rotational speeds of the cylinders. Almost doubling of the average Nu number is obtained by decreasing the value of the Hartmann number from 25 to 0 or varying the magnetic field inclination from 90 to 0. When rotational effects of the cylinders are considered, average heat-transfer improvements by a factor of 5 and 5.9 are obtained for nondimensional rotational speeds of 5 and −5 in comparison with the case of motionless cylinders. An optimum length of the porous layer is achieved for which the best heat-transfer performance is achieved. As the curvature size of the partition is increased, better heat transfer of the hot wall is obtained and up to 138% enhancement is achieved. Significant increments of entropy generation are observed for left and right domains including the rotating cylinders. The magnetic field parameter also affects the entropy generation and contributions of different domains including the curved porous partition.

INTRODUCTION

Heat-transfer enhancement techniques are increasingly gaining significant attention from scientists and engineers since they have a direct impact on the energy savings and the improvement of system performances in a broader sense. Particularly, many innovative techniques for heat-transfer improvement or control of inside cavities and enclosures have been developed widely. Among passive techniques we can cite the use of fins, baffles, or dimples, addition of nanoparticles to the fluid, insertion of a porous medium, and the use of discrete heating sources in different configurations. However, passive techniques mainly require power inputs such as moving bodies, especially rotating cylinders, magnetohydrodynamics (MHDs), and electrohydrodynamics. In fact, recent studies are investigating the combination of one or more passive and active techniques. For instance, a numerical study with an active inner rotating cylinder in a square enclosure was performed. The size, rotational speed, and thermal conductivity of the cylinder were investigated, and it was noted that rotating a cylinder slowly with a moderate size is effective for heat-transfer enhancement. Recently, Sasmal et al. investigated the influence of Grashof number, Prandtl number, power-law index, and rotational velocity on the steady laminar natural convection heat transfer in a power-law fluid from an isothermal rotating cylinder positioned at the center of a lengthy enclosure. They found that for a rotating cylinder, at low Gr and Ra numbers, the heat transfer is enhanced irrespective of the nature of the fluid. Also, they presented a simple expression for the estimate of the average Nusselt number as a function of Gr, Pr, power-law index, and rotational velocity for any further application. Several studies on the entropy generation (EG) analysis during natural convection in various enclosures and processes involving different practical applications have been elaborated during the last decade. A numerical study of the influence of an inclined magnetic field (MF) on mixed...
convection and EG of non-Newtonian power-law fluid inside a partially heated cavity with an adiabatic rotating cylinder under the influence of an inclined MF was performed by Selimefendigil and Öztürk.16 Alsabery et al.18 focused on mixed convection and transient EG caused by a rotating heated inner cylinder within a square cavity composed of a flexible side wall. It was observed that both the flow and the heat-transfer rate are highly affected by using a very flexible wall. It was found that using a heated rotating cylinder with a flexible wall causes several deformation paths presenting various effects on the EG and convection heat transfer. Similar to these findings, Selimefendigil and Öztürk19 gave additional confirmation that the interaction between a fluid and a flexible wall affects both the flow structure and the heat-transfer rate. Another effective passive technique is the use of a porous medium, and many researchers have evaluated its impact on natural and forced convection.20–23 Misirlıoğlu24 investigated the steady heat transfer in the presence of a rotating cylinder in a square cavity and in a fluid-saturated porous medium. Results showed that at a low angular speed, the direction of the cylinder rotation and the presence of a clear fluid (no porous medium) have significant effects on both natural and forced regimes. However, the insertion of a porous medium highly attenuates these effects that will disappear at high rotation speeds. The enhancement for the heat transfer is reached when the forced convection starts to dominate and when the spin velocity rises. Recently, Raizah and Aly25 simulated the suspension of a nanoencapsulated phase change material during the double-diffusive convection flow inside a porous enclosure. They investigated the effects of several parameters such as melting temperature, buoyancy ratio, and size of the rotating cylinder. One of the main findings was that the increase of the cylinder radius significantly enhances the double-diffusive convection and it diminishes the intensity of a phase change zone. Moreover, for low porosity corresponding to low Darcy numbers, the heat transfer is intensified and the concentration distributions are well developed. However, the streamlines are highly decreased due to the resistance of the porous medium reflected by the low permeability. Moreover, the nanofluids have been widely used to enhance the thermal properties of fluids and hence the heat-transfer rate within enclosures. The rotating cylinder as one of the most effective ways to boost the heat transfer in cavities was used with a nanofluid by Roslan et al.26 They concluded that the heat-transfer rate increases by increasing the nanoparticle concentration for a rotating cylinder placed in the center of the cavity. The effect of using a nanofluid with rotating cylinders in cavities has been investigated over the last decade; see the extensive bibliography in refs 27 and 28. Similarly, hydro-magnetic mixed convection inside porous enclosures has been widely investigated. Chatterjee et al.29 evaluated numerically the hydro-magnetic mixed convection transport within an enclosure filled with a high-electrical conducting fluid exposed to a MF and in the presence of a thermally conductive rotating cylinder. There are several recent studies investigating the effects of parameters related to the combination of passive and active techniques.30–32 However, the combination of passive and active techniques still requires a lot of attention, mainly to obtain the optimal configuration in terms of technical and economic performances. In this context, this study will present a detailed numerical investigation of the control of MHD natural convection and EG inside a porous cavity with two rotating cylinders separated by a curved partition. Based on the above-mentioned literature survey, this contribution is a combination of two passive (porous medium and curved partition) and two active (double rotating cylinders and MHD) techniques. Considering the complexity of the problem, the effect of MF strength, inclination, permeability, rotational speed, vertical position of cylinders, and porosity of the medium on the heat transfer and flow patterns will be carefully investigated.

### MATHEMATICAL MODELING

#### Physical Problem

A schematic representation of the studied domain is shown in Figure 1. The computational model consists of a differentially heated square cavity of length H divided into three compartments A, B, and C. The left vertical plane is subjected to high temperature $T_h$, while the right one is maintained at a constant cold temperature $T_c$ and a vertical velocity $v_y$. The other horizontal walls are adiabatic. The two layers A and C are separated by a curved porous partition B of thickness $L_p$ and curvature radius $r_p$. Two identical circular cylinders of radius $r = 0.05 H$ having a rotational speed $\omega$ are placed in layers A and C. Here, $(x_{c1}, y_{c1})$ and $(x_{c2}, y_{c2})$ denote the coordinates of the cylinder centers. The whole computational domain is subjected to an external MF $\mathbf{B}_0$ inclined by an angle $\gamma$ with the horizontal.

#### Governing Equations

The generalized Darcy–Brinkman–Forchheimer extended porous model equation with MF effects is considered. Impacts of various effects such as thermal radiation and viscous dissipation are ignored. The Eckert number is much less than 1 in this work, and many studies have been conducted that ignored the viscous dissipation in the energy equation for nanofluid convection with MF effects.33–35 The magnetic Re number is considered to be small when the induced MF impacts are small and displacement currents effects are ignored. Important applications, significance, and impacts of displacement currents have been mentioned in a recent work in ref 36. Neglecting those effects for MHD convection of
nanofluid have been considered in many studies.\textsuperscript{37–39} 2D, steady, laminar flow equations with Boussinesq approximation are given as
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)
\[
\frac{1}{\nu} \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{1}{\rho_u} \frac{\partial p}{\partial x} + \frac{\nu}{\partial x} (\nabla^2 u) - \nu \frac{\partial u}{\partial K} - \frac{F_c}{\sqrt{u}} u \sqrt{u^2 + v^2} + \frac{\sigma_{eff} B_0^2}{\rho_u} (\nu \sin(\gamma) \cos(\gamma) - u \sin^2(\gamma))
\]  
(2)
\[
\frac{1}{\nu} \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{1}{\rho_u} \frac{\partial p}{\partial y} + \frac{\nu}{\partial y} (\nabla^2 v) - \nu \frac{\partial v}{\partial K} - \frac{F_c}{\sqrt{u}} v \sqrt{u^2 + v^2} + \frac{\sigma_{eff} B_0^2}{\rho_u} (\nu \sin(\gamma) \cos(\gamma) - \nu \cos^2(\gamma))
\]  
(3)
\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{eff}}{\nu^2} T
\]  
(4)

For nondimensionalization, the following parameters are used
\[
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{v_0}, \quad V = \frac{v}{v_0}, \quad P = \frac{P}{\rho_0 v_0^2}, \quad \theta = \frac{T - T_s}{T_h - T_c}
\]  
(5)

Nondimensional equations are stated as
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  
(6)
\[
\frac{1}{\nu} \left( \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + d_1 \frac{1}{\epsilon Re} (\nabla^2 U) - d_1 \frac{U}{Da Re} - \frac{F_c}{\sqrt{Da}} U \sqrt{U^2 + V^2} + d_2 \frac{Ha^2}{Re} (V \sin(\gamma) \cos(\gamma) - \nu \sin^2(\gamma))
\]  
(7)
\[
\frac{1}{\nu} \left( \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + d_1 \frac{1}{\epsilon Re} (\nabla^2 V) - d_1 \frac{V}{Da Re} - \frac{F_c}{\sqrt{Da}} V \sqrt{U^2 + V^2} + d_2 \frac{Gr}{Re^2} \theta + d_2 \frac{Ha^2}{Re} (U \sin(\gamma) \cos(\gamma) - \nu \cos^2(\gamma))
\]  
(8)
\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = d_4 \frac{1}{Re Pr} \nabla^2 \theta
\]  
(9)
with \(d_1 = \nu_{eff}/\nu, d_2 = \beta_{eff}/\beta, d_3 = \beta_{eff}/\beta, \) and \(d_4 = \alpha_{eff}/\alpha.\) In the above given equations, \(Re, Da, Ha, Pr,\) and \(Gr\) are the Reynolds number, Darcy number, Hartmann number, Prandtl number, and Grashof number, respectively. The Forchheimer coefficient is represented by \(F_c\) and \(Ra\) is the Rayleigh number. They are defined as
\[
Re = \frac{\nu H}{\nu}, \quad Da = \frac{K}{H^2}, \quad Ha = B_0 H \sqrt{\frac{\sigma}{\rho \nu}}, \quad Pr = \frac{\nu}{\alpha}, \quad Gr = \frac{g(T - T_s)H^3}{\nu^2}, \quad F_c = \frac{1.75}{\sqrt{150 \nu^2}}, \quad Ra = Gr Pr
\]  
(10)

The domains (A) and (C) have the same porosity of \(\varepsilon_s\) and permeability of \(\kappa_t\), while for domain (B), the porosity and permeability are \(\varepsilon_s\) and \(\kappa_p\).

The EG equations are stated as in the following\textsuperscript{40,41}
\[
S = \frac{\kappa_{eff}}{T_0} \left( \frac{\partial T}{\partial X} \right)^2 + \left( \frac{\partial T}{\partial Y} \right)^2 + \frac{\mu_{eff}}{T_0} \left( \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} \right)^2 + \frac{\mu_{eff}}{T_0} \left( \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} \right)^2 \left( u \sin(\gamma) - \nu \cos(\gamma) \right)^2
\]  
(11)

In the above representation, contribution of different parts to the total EG are shown with the average temperature of \(T_a = (T_h + T_c)/2.\) The representations due to the heat transfer, viscous dissipation, and MF effects are shown by the first, second, and third terms on the right-hand side of the above given equations.

Boundary conditions in a nondimensional form are stated as follows:
- At the left vertical wall, \(U = V = 0, \theta = 1\)
- At the right vertical wall, \(U = 0, V = 1, \theta = 0\)
- At the horizontal walls, \(U = V = 0, \frac{\partial \theta}{\partial n} = 0\)
- At the interface between the domains, \(U_i = U_p, V_i = V_p, \frac{\partial \theta}{\partial n} = \frac{\partial \theta}{\partial n}\)
- At the left rotating cylinder surfaces, \(U = -\Omega(Y - Y_s), V = \Omega(X - X_s), \frac{\partial \theta}{\partial n} = 0\)
- At the right rotating cylinder surfaces, \(U = -\Omega(Y - Y_s), V = \Omega(X - X_s), \frac{\partial \theta}{\partial n} = 0\)

where the nondimensional rotational speed is denoted by \(\Omega = \omega r/\nu_0.\) Local and average Nu numbers are calculated as
\[
Nu_y = -\frac{k_{eff}}{k_f} \frac{\partial \theta}{\partial n}, \quad Nu_m = \frac{1}{H} \int_0^H Nu_y \, dy
\]  
(12)

Cu–water nanofluid was used as the heat-transfer fluid. A solid volume fraction of 0.02 Cu nanoparticles are used in the heat-transfer fluid, and effective thermophysical property relations for the thermal conductivity and electrical conductivity are based on the Maxwell–Garnett\textsuperscript{42} and Maxwell model.\textsuperscript{43} The viscosity of the nanofluid is based on the Brinkmann\textsuperscript{44} model. Our aim in this study is not to explore the combined impacts of nano fluid behavior with the rotating cylinder and curvature effects of the partitioned zone but to take
the advantages of using nanosized particles in the base fluid under the MF as it has been shown in previous studies.45–47

Solution Method and Code Validation. As the solver, Galerkin-weighted residual finite element method (FEM) is considered. The theory and applications of FEM have been presented in refs 48–50. The FEM has been successfully applied for analyzing the convective heat transfer of the nanofluid under MF effects.51–54 In the method, any field variable of interest (ff) is approximated as

\[ \mathbf{f}_f = \sum_{i=1}^{N_F} \Phi_i \mathbf{F}_f \]  

(13)

The shape function is represented by \( \Phi_i \), while FF denotes the elemental nodal value. Different ordered Lagrange FEM is used. When the approximated variables are used in the governing equations, residuals will be obtained. It will be set to zero in an average sense as

\[ \int_V W_r \, dV = 0 \]  

(14)

where \( W_r \) is the weight function. A convergence criterion of \( 10^{-6} \) is selected. Grid independence tests were performed to assure mesh independence of the solution. Figure 2a shows the

Figure 2. Grid independence test results (a) and grid distribution (b) \( (Ra = 10^5, Ha = 25, \gamma = 45^\circ, Da_2 = 5 \times 10^{-3}, \epsilon_1 = 0.5, \Omega = -1, y_{c1} = 0.25H, y_{c2} = 0.75H, r_p = 0.15H, and L_p = 0.15H) \).

comparison results of the average Nu number for different grid sizes, and grid G5 with 39 118 number of elements is selected. A grid distribution near the rotating cylinder and curved partition is shown in Figure 2b. Refinements near the interfaces and wall are performed.

The code is validated by using different available studies in the literature. In the first work, effects of MF in a square cavity under free convection are analyzed numerically, and the results are compared with the ones available in ref 55. Figure 3 shows the comparison results of the average Nu number at \( Gr = 2 \times 10^5 \) considering various MF strengths. The highest deviation below 5% is achieved. Impacts of using rotational effects of the inner cylinder on convection are considered in Figure 4 from the available numerical results of Roslan et al.26 At two different sizes and rotational speeds, comparisons of average Nu number are presented in Figure 2b, and the highest deviation below 5% is obtained between different cases.

RESULTS AND DISCUSSION

In this section, several numerical results are presented and discussed. A parametric study is performed by considering the Rayleigh number, Hartman number, Darcy number, and inclination of the MF. The results concern mainly the streamlines profile, the isotherms profile, and the Nusselt number variation. During the simulations, the Re number based on the moving wall velocity is taken as 200, while the rotational speed of the cylinders is varied. Figure 5 shows the impact of the Rayleigh number variation on the flow and temperature profiles. For \( Ra = 10^5 \), the left side of the cavity (compartment A) is the seat of a multicellular flow due essentially to the competition between the upward thermal buoyancy forces and the downward movement created by the cylinder rotation. By increasing the Rayleigh number, the buoyancy forces become dominant and eliminate the different instabilities. These forces generate a strong flow toward the top of the cavity. For the right side of the cavity (compartment C), the ascending movement of the cold wall generates a large counterclockwise rotating cell. By increasing the Rayleigh number, a strong diffusion of the hot fluid at the top of the cavity is observed. This diffusion allows the rotating cylinder to push the cold fluid toward the porous
medium (compartment B) and also the formation of a small cell in its basal area. Regarding the temperature profile, we note that for Ra = 10^4 and 10^5, the lower cylinder creates an almost isothermal zone near the lower part of the heated wall. An unstable reverse thermal stratification (hot fluid is under the cold fluid) located under the lower cylinder is observed. In this zone, forced convection dominates and the rotating lower cylinder causes enthalpic advection. When Ra reaches 10^6, two phenomena can be recorded: first the disappearance of the reverse stratification and second the transition from a horizontal to vertical stratification at the top of compartment B of the porous medium.

Figure 6a confirms the isothermal zone observed at the lower corner of the cavity for Ra = 10^4 and 10^5. Indeed, the local Nusselt number is 0 at y = 0. Whatever the value of the Rayleigh number, the profile of the local Nusselt number is similar, showing a peak observed at y = y_2 = 0.25H in the vicinity of cylinder 1; this peak is due to the forced convection generated by the rotation of this cylinder. It is also noted that there is a degradation of the local Nusselt number at both extremities of the hot wall. As expected, the average Nusselt number (Figure 6b) grows as the Rayleigh number increases, and 318% heat transfer improvement is reached for Ra = 10^6 compared to the case at Ra = 10^3. Figure 7a–c shows a strong effect of the MF intensity on the streamlines in the entire enclosure. By increasing the Hartman number, we can highlight the birth of a multilayer cell in compartment A, the transition from a vertical flow to an oblique flow in the center of the B compartment, and finally the creation of a small cell under cylinder 2 in addition to the acceleration and displacement of the vortex of the main cell to the middle of the cold wall for compartment C. However, the effect of the change in the MF angle has less influence on the structure of the flow in all areas (Figure 7d,e), except under cylinder 1 where secondary cells appear for a large Ha, and in the porous medium area (B), the distortions of the streamlines become more pronounced. The increase of the medium permeability (Figure 7g–i) mainly affects the position and intensity of the cell vortex created by the movement of the cold wall in the C zone of the enclosure.

Figure 8 presents the variation of the local Nusselt number along the hot wall as a function of the MF intensity and the porosity and permeability of the porous medium. Regardless of the varied parameter, the profile of the local Nusselt number curve is almost the same. Nu represents usually a maximum in the vicinity of the rotating cylinder 1 and a decay at the upper and lower wall extremities. The best conditions to improve the heat transfer are the decrease of the Hartman number (MF intensity) and the increase of the porosity and the Darcy number (permeability).

Figure 9 illustrates the evolution of the average Nusselt number on the hot wall. Decreasing the Hartman number from 25 to 0 or increasing the MF inclination from 90 to 0 allows to almost double the value of the Nusselt number. The reduction of natural convection in cavities with higher MF strength in the presence of nanoparticle loading has been shown in various studies.6–58 Permeability and porosity have a very dramatic effect on the heat transfer. Indeed, an increase in the Darcy number from 10^{-4} to 10^{-3} or porosity from 0.25 to 0.75 allows to multiply the Nusselt number by a factor of 22 and 42.

Figure 10 displays the impact of changing angular rotation speed of the cylinder (Ω) on the streamlines and isotherms. For stationary cylinders (Ω = 0), Figure 10b,e reveals a flow driven by a trade-off between the buoyancy forces on the left side of the enclosure and the movement of the lid creating a large recirculation zone all along the C compartment. Indeed, the upward flow due to the natural convection on the A side of the enclosure is countered by the forced flow due to the cold wall motion. This competition generates a strong thermal gradient in

**Figure 5.** Rayleigh number impacts of flow pattern (FP) (a–c) and thermal pattern (TP) variations (d–f) (Ha = 15, γ = 45°, Da1 = 5 × 10^{-3}, ε1 = 0.5, Da2 = 5 × 10^{-3}, Ω = −1, y_1 = 0.25H, y_2 = 0.75H, r_v = 0.15H, and L_p = 0.15H).

**Figure 6.** Local (a) and average Nu (b) variations of the hot wall (x = 0) for different Ra numbers (Ha = 15, γ = 45°, Da1 = 5 × 10^{-3}, ε1 = 0.5, Da2 = 5 × 10^{-3}, Ω = −1, y_1 = 0.25H, y_2 = 0.75H, r_v = 0.15H, and L_p = 0.15H).
the contact zone between the hot and cold flows in the upper part of compartment C. A negative value of the cylinder rotation ($\Omega = -5, \text{Figure 10a,b}$) creates a recirculation zone and amplifies the distortion just above cylinder 1 in compartment A. The rotation of cylinder 2 extends the clustering of the isotherms not only to the top of the C compartment but also to the whole porous medium (zone B).

By reversing the rotation direction of the cylinders ($\Omega = 5$), the recirculation zone near cylinder 1 disappears, while another recirculation zone is formed near cylinder 2. It should be noted that cylinder 2 rotation promotes the penetration of the hot fluid in the center of the porous medium (compartment B). As seen in Figure 11 the effects of vertical displacement of the first and second cylinders on the flow are totally independent, that is, each cylinder influences only its own compartment where it is located. For a low position of cylinder 1 ($y_{c1} = 0.25H$), only one recirculation zone is formed above this rotating cylinder. By placing cylinder 1 in the middle or upper position, a biconvex flow takes place. The created cells are located on both sides of the cylinder for $y_{c1} = 0.75H$, while they are situated in the upper left corner for $y_{c1} = 0.5H$. The effect of displacement of the second cylinder is practically negligible; in fact, the displacement of the lid dominates the movement in compartment C by creating a large rotating cell. The only thing that can be recorded is that for $y_{c2} = 0.75H$, there is an increase in the rotational intensity of this vortex.

Figure 7. Effects of MF strength (a–c), inclination (d–f), and permeability of the medium (g–i) on the flow pattern distributions ($Ra = 10^5, Da_2 = 5 \times 10^{-3}, \varepsilon_1 = 0.5, \Omega = -1, y_{c1} = 0.25H, y_{c2} = 0.75H, r_p = 0.15H, \text{and } L_p = 0.15H$).

Figure 8. Impacts of MF strength (a), permeability (b), and porosity of the medium (c) on the local $Nu$ variations of the hot wall ($x = 0$) ($Ra = 10^5, Da_2 = 5 \times 10^{-3}, \Omega = -1, y_{c1} = 0.25H, y_{c2} = 0.75H, r_p = 0.15H, \text{and } L_p = 0.15H$).

The results showed in Figure 11 are confirmed in Figure 12c,d which represents the variation of the local Nusselt number in the position of the rotating cylinders. As previously observed, a modification of $y_{c2}$ has partially no significant incidence on the flow and therefore on the heat transfer (Figure 12d), whereas the variation of $y_{c1}$ demonstrates that it is possible to control and improve the transfer on a section of the hot wall. Indeed, the peak of the Nusselt number is always attached to the position of cylinder 1. From Figure 12a–d, it can be concluded that the rotation of the cylinders considerably improves the heat transfer at any point of the hot plate. Similar trends in the Nusselt number versus varying angular rotational speeds for rotating cylinder effects on convection in cavities have been shown in several studies.7 However, a clockwise rotation ($\Omega < 0$) has even more impact on the enhancement of the heat transfer. As a matter of fact, an increase by a factor of 5 (for $\Omega = 5$) and 5.9 (for $\Omega = -5$) was recorded in comparison with the case of a stationary cylinder. The effect of the curvature of the...
porous medium is described in Figure 13a. Whatever the value of curvature $r_p$, no change in the flow topography has been reported in the C compartment. However, the A compartment is strongly affected by the $r_p$ variation. An increase in the curvature value is coupled with a reduction in the zone A size. As $r_p$ increases from 0 to 0.1$H$, an intensification of the flow is observed in the gap between the cylinders.

Figure 9. Effects of MF strength (a), inclination (b), permeability (c), and porosity of the medium (d) on the average Nu variations of the hot wall ($x=0$) ($Ha=15$, $\gamma=45^\circ$, $Da_1=5 \times 10^{-3}$, $e_1=0.5$, $Da_2=5 \times 10^{-3}$, $\Omega=-1$, $y_{c1}=0.25H$, $y_{c2}=0.75H$, $r_p=0.15H$, and $L_p=0.15H$).

Figure 10. FP (a–c) and TP (d–f) variations with different rotational speeds of the cylinders ($Ra=10^7$, $Ha=15$, $Da_1=5 \times 10^{-3}$, $e_1=0.5$, $Da_2=5 \times 10^{-3}$, $y_{c1}=0.25H$, $y_{c2}=0.75H$, $r_p=0.15H$, and $L_p=0.15H$).

Figure 11. FP variations with different vertical positions of cylinder centers ($Ra=10^7$, $Ha=15$, $Da_1=5 \times 10^{-3}$, $e_1=0.5$, $Da_2=5 \times 10^{-3}$, $\Omega=-1$, $r_p=0.15H$, and $L_p=0.15H$).
between the rotating cylinder and the hot plate. By further increasing $r_p$ to 0.2H, instabilities characterized by the formation of two cells below and above cylinder 1 are noted. As shown in Figure 13d, exactly the same finding is noticed by the variation of the parameter $L_p$ from 0.1H to 0.3H.

The local Nusselt number as a function of the layer size is plotted in Figure 14a. As shown in the figure, there is an optimal size (here 0.2H) allowing the best heat transfer. It should be noted that for $y/H$ less than 0.2, the size of the porous zone has no significant impact on this transfer. Figure 14b allows to estimate the gain expected by adjusting and optimizing the size of the B compartment (porous zone). An improvement on the average Nusselt number of about 27 and 60% is noted for $L_p = 0.2H$ in comparison to $L_p = 0.1H$ and $L_p = 0.3H$, respectively.

Figure 13. Effects of curvature (a−c) and size of porous layer (d−f) on FP distributions ($Ra = 10^5$, $Ha = 10$, $\gamma = 45^\circ$, $Da_1 = 5 \times 10^{-3}$, $\varepsilon_1 = 0.5$, $Da_2 = 5 \times 10^{-3}$, $\Omega = -1$, $y_{c1} = 0.25H$ and $y_{c2} = 0.75H$).

Figure 12. Local (a,c) and average Nu (b,d) variations of the hot wall ($x = 0$) with various rotational speeds (a,b) and different vertical positions of cylinder centers (c,d) ($Ra = 10^5$, $Ha = 15$, $Da_1 = 5 \times 10^{-3}$, $\varepsilon_1 = 0.5$, $Da_2 = 5 \times 10^{-3}$, $r_p = 0.15H$, and $L_p = 0.15H$).

EG analysis is also performed for different variations of the pertinent parameters. Impacts of MF parameters [strength (a) and inclination (b)] on the variation of normalized EG are shown in Figure 15. The contributions of individual parts A, B, and C on the EG are shown, while the normalization is performed by using the EG value at the first parameter of interest (the lowest one). The value of EG reduces for domain A with higher $Ha$ and $\gamma$, while its value rises for domain C. Highest temperature gradients are obtained for the left vertical wall of the heated section when the strength of the MF is intensified, which resulted in higher heat transfer irreversibility along with the higher contribution of the entropy due to the MF. When all of the domains (A, B, and C) are considered, the EG reduces up to the MF strength of $Ha = 10$ and then rises thereafter (Figure 16). However, the EG reduces with higher inclination of MF which is due to the lower heat-transfer irreversibility with higher $\gamma$ values. As the rotational speeds of the cylinders rise, EG rises significantly for domains A and B which is attributed to the higher intensification of temperature gradients near the vertical
walls. The impacts of rotation on domain B are very slight, and it has a very low contribution to the overall EG at the highest value of $\Omega$ (Figure 17). When the vertical locations of the rotating cylinder change, the EG is also affected. When the location of the first cylinder is changed in the $+y$ direction, the EG value is slightly reduced for part A, while it slightly increases for part C (Figure 18).

## Conclusions

Numerical work for the mixed convection with combined effects of inner double rotating cylinders and moving wall in a partitioned porous cavity is performed along with the EG. A curvature form of the partition is considered, while a uniform inclined MF is imposed throughout the whole computational domain. The cylinders are identical and rotate with the same speed. The following conclusions can be drawn:

- Complex flow field and multicellular flow structures are observed due to the interaction between the natural convection, the rotational effects of the double cylinders, and the moving wall.

- The flow field structures are less influenced by variation of MF inclination as compared to varying its strength. The best conditions for improving the $Nu$ number are...
obtained when decreasing the MF intensity and increasing the porosity and permeability of the medium.

The heat-transfer rate at any point of the hot plate is improved significantly by the rotation of the cylinders, while a clockwise rotation has even more impact on the Nu number enhancement.

An optimal size of the curved porous layer is obtained which allows the best heat transfer.

Better heat-transfer performance for the hot plate is achieved with higher curvature size.

The value of EG reduces up to the case with the Hartmann number of 10 and then increases for higher MF strength values, while for higher MF inclinations, the value of EG reduces.

Profound increments of the EG values are obtained for domains A and B with varying rotational speeds of the cylinders. When varying the vertical locations of the rotating cylinders, the EG value is influenced slightly.

The present study can be extended to include different thermal boundary conditions and nonuniform MF effects and considering nonidentical cylinders with different rotational speeds, different forms of the curvature of the partition, and transient flow effects which will increase the applicability of the study.

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**NOMENCLATURE**

- $B_0$: magnetic field strength
- $D_a$: Darcy number
- $Gr$: Grashof number
- $Ha$: Hartmann number
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