On the limiting behaviour of hard processes in QCD at small $x$. *

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Abstract

At sufficiently small $x$ where pQCD methods become insufficient we rewrote sum of dominant Feynman diagrams for single-scale hard processes in the form of the effective field theory where a quasiparticle corresponds to pQCD ladder. We explain that pQCD series are divergent at small $x$ since pQCD produces tachyon in effective field theory and because of degeneracy of vacua. Within the WKB approximation we found the hierarchy among quasiparticle interactions due to smallness of the QCD coupling constant which justifies restriction by the triple quasiparticle interaction in the Lagrangean of effective field theory. Degeneracy of vacua is removed by accounting for the classical solutions -kinks which cannot be decomposed into series over powers of $\alpha_s$ but describe color inflation, Bose-Einstein condensation of quasiparticles. Quantum fluctuations around the WKB solution lead to the spontaneous violation of two-dimensional translation symmetry and the appearance of zero modes- "phonons" which are relevant for the black disc behaviour of small $x$ processes. Account of gluon exchanges between overlapping ladders neglected in the first approximation produces a color network occupying a "macroscopic" longitudinal volume. We discuss briefly possible role of new QCD phenomena in the two-scale hard processes.

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I. INTRODUCTION

One of the challenging properties of the perturbative QCD is the conflict between predicted within DGLAP approximation rapid increase of cross sections of hard processes with energy [1,2] and the probability conservation. The QCD evolution equation describes well the experimental data (see review [3] where $F_2(x,Q^2) \propto x^{-\mu(Q^2)}$ for $\mu(Q^2) \approx 0.2$ and $Q^2 \approx 10\text{GeV}^2$). The slope $\mu$ is increasing with $Q^2$. Cross sections of diffractive processes like diffractive electroproduction of vector mesons and photoproduction of hidden heavy flavor mesons increase even faster $\propto x^{-2\mu(Q^2)}$ [4,5]. This behaviour has been observed recently (for the review and references see ref. [3]). Predicted by DGLAP increase of parton distributions with energy is too fast to keep evident condition $\sigma_{\text{tot}} \geq \sigma_{\text{diff}}$ for cross sections of hard processes i.e. the probability conservation [6]. The pQCD factorization theorems contradict the Froissart type bound for the cross-section of scattering of spatially small colorless dipole that follows from the nonlinear relations between the Green functions in QCD and analytic properties of amplitudes in the plane of momentum transfer [7]:

$$\sigma(s) \sim \log^2(s/s_0)$$

where $s$ is the center of mass energy of colliding particles. The same puzzle exists in the other approximations such as LO and NLO BFKL approximation [9,10]. The closely connected property of pQCD is the fast increase of the correlators of local currents with distance [11,12]. The presence of long range interaction suggests the instability of the pQCD vacuum. The puzzles reveal itself in the hard phenomena where the virtualities of the interacting partons are large and therefore they are unrelated to the poorly understood physics of quark confinement and spontaneous broken chiral symmetry.

The pQCD calculation within the DGLAP approximation [14] of the scattering of the color-neutral two-gluon dipole of size $\approx 1/Q$ off a proton target violates the LT approximation at $x \leq x_{\text{cr}}(Q^2) \sim 10^{-5}-10^{-4}$, i.e. in the kinematics conveniently achievable at LHC. (The estimate is given for the scale of hard processes $Q^2 \sim 10 \text{ GeV}^2$ chosen to guarantee the smallness of the running invariant charge.) At these scales the higher twist contributions blow up. This scale of $x$ seems to be significantly larger than the values of $x$ where different pQCD approximations start to diverge in the predicted behavior of the amplitudes of the hard processes [15]. Thus the violation of the LT approximation, of DGLAP,LO+NLO BFKL approximations, and maximal value permitted by probability conservation for the amplitudes of hard processes are achieved at the values of $x$ where pQCD calculation of LT contributions is still more or less unambiguous. Thus the challenging problem is to develop technology of calculations and to predict existence and probability of new QCD hard phenomena at small $x$ where conventional pQCD approaches should fail.

To simplify this difficult problem we restrict ourselves by the theoretical analysis of the one-scale hard processes like $\gamma^*(Q^2) + \gamma^*(Q^2) \rightarrow \text{hadrons}$, where $Q^2$ evolution is suppressed. Account of the running of the coupling constant and, to some extent, energy-momentum conservation suppress fast diffusion to large and small transverse distances [10]. In our consideration we will neglect diffusion to small distances since in any case it can not change properties of amplitudes near black body limit. However we will take into account V.Gribov diffusion to large impact parameters which is distinctive feature of ladder diagrams [16].
Moreover we restrict ourselves by the consideration of the kinematics of extremely small $x$ where diffractive production of large mass states is not hampered by energy-momentum conservation. In this kinematics assumption on the dominance of shadowing due to diffraction into small masses will be hardly consistent with the probability and energy-momentum conservation cf. [17]. So we assume dominance of processes of diffraction into large invariant masses.

The aim of this talk is to calculate limiting behaviour of small $x$ processes. Following V.Gribov Reggeon Calculus [18,19] we develop effective theory (EFT) describing the single scale hard processes and solve it within the WKB approximation. This approach helps to visualize the challenging problem: modern methods of pQCD produce tachyon in EFT and as in the theory of superconductivity [20] and in the string model in 26 dimensions [21] it is necessary to find the physical vacuum state of EFT and then to develop the perturbation theory. The ideas and methods of spontaneously broken continuous symmetries will be heavily used also. The new QCD phenomena we find may appear important for many-scale hard processes as well even if methods developed in the paper are not directly applicable.

We work in the kinematics where light-cone wave functions of the colliding $\gamma^*$ are dominated by configurations having large invariant mass and containing many constituents with large transverse momenta —the hard-QCD analogue of the triple-reggeon limit. Thus a necessary kinematical condition of applicability of our method is $x \leq x_{cr}(Q^2) \cdot 10^{-2}$, i.e. $x \sim 10^{-6} - 10^{-7}$. Here $x_{cr}(Q^2)$ can be determined from the condition that the contribution of one ladder in dipole–dipole scattering at central impact parameters reaches the black disk limit. The demand for the existence of the three-reggeon limit is the main kinematical limitation for the applicability of our approach.

To account for the coherence of high-energy processes and the rapid increase with energy of the amplitudes of hard small-$x$ processes, we construct an effective field theory (EFT) of interacting perturbative ladders, which are our quasiparticles and neglect initially by gluon interactions between ladders. We take account of the interaction between quasiparticles along the lines of Gribov’s Reggeon field theory [18]. (This approach is in the spirit of statistical models of critical phenomena which account for the interactions between major modes only. Specifics of the physics of large longitudinal distances are included in the concept of quasiparticle.) The interaction between quasiparticles when the amplitudes of hard processes are near the unitarity limit can be easily evaluated when the WKB approximation is combined with the smallness of the running coupling constant in pQCD. We showed that smallness of the running coupling constant helps to establish a hierarchy of multiladder interactions and the dominance of the triple-ladder vertex [13]. The smallness of the multiladder vertices in pQCD implies that basic phenomena characteristic of the BDL should be insensitive to the restriction by the triple-ladder vertex. This observation helps to fix the form of the Lagrangian of the EFT. Moreover smallness of effective triple ladder vertex justifies applicability of semiclassical approximation. Thus EFT is solvable within WKB approximation and leads to transition to BDL- ”black disk limit”. Derived in our papers Lagrangean of EFT is almost identical to that analysed in preQCD Reggeon Calculus [22–25] long ago. So in many situations we may translate results obtained in these papers to the language appropriate for the theoretical description of single scale hard QCD regime.

It follows from EFT that the transition to BDL in the one scale hard processes is a result of the existence of nonperturbative solutions of equations of EFT - kinks in rapidity-
impact parameter space. Transition due to the general EFT kink is suppressed by the factor $\sim \exp(-1/N_c\alpha_s)$. The critical kinks that correspond to actual transitions between vacua are the limiting cases of the families of these noncritical kinks.

The transition to BDL is of the inflationary type: in the case of collisions of two small dipoles the time scale $T_I \sim \exp(1/\mu(Q^2))/Q$ of the transition significantly smaller than the time needed for the formation of perturbative ladder $\propto 1/Q^{1-\bar{\mu}(Q^2)}$.

The nonperturbative transition produces ladders which strongly overlap in the impact parameter space. Due to exchange by constituents between overlapping ladders system of ladders becomes color network. [13] However to evaluate properties of this color network, it is necessary to describe the BDL transitions (the kinks) directly in terms of the QCD language, which is yet not done.

To summarise, we were able to show that QCD can be formulated as solvable in quasiclassical approximation effective EFT (analogue of V.Ginzburg- L.Landau theory of superconductivity), and this EFT has a transition to the BDL limit, which in the QCD language is a color network.

For two scale processes like small dipole scattering off hadron(nucleus) target related effects should reveal itself at extremely large energies where QCD evolution is restricted by dipole fragmentation region. The challenging question is to find quasiparticles which dominated at lesser energies.

II. EFFECTIVE FIELD THEORY IN QCD

We assume, as it was mentioned above, that the dominant degrees of freedom—the quasiparticles of EFT, are pQCD color-singlet ladders -"Pomerons" [13]. In the derivation of the EFT from Feynman diagrams we follow Gribov’s Reggeon calculus [19,18] and take advantage of the simplifications due to the smallness of the running coupling constant.

The equations of the EFT can be derived as Lagrange equations of motion from an effective Lagrangian that has a form

$$L = p\partial_y q - q\partial_y p - \alpha' p\Delta q - \mu pq - \kappa pq(p + q) - c_{\text{dipole}}\int \exp(-BQ/2)q(y, b - B)d^2B\delta(y)$$

$$- c_{\text{dipole}}\int \exp(-BQ/2)p(y, b - B)d^2B\delta(y - Y)$$

(2.1)

Here $c_{\text{dipole}}$ accounts for the normalization of the virtual photon wave function. The first three terms have a straightforward interpretation in the case of noninteracting quasiparticles. They follow from the Mellin transformation of the Green function of the free quasiparticle, $G = [j - 1 - \mu(Q^2) - \alpha'k^2]^{-1}$, in the plane of complex angular momentum $j$ in the crossed channel. Since $\mu \geq 0$ Green function has pole in the unphysical region forbidden by probability conservation and analytic properties of amplitudes in the plane of momentum transfer i.e. pQCD "Pomerons" is tachyon of EFT. The fields $p(y, b) = \psi^+$ and $q(y, b) = \psi$ are the quasiparticle fields, analogous to Gribov’s Pomeron fields. We denote $\partial_y = \partial_{\log(x_0/x)}$ where $y$
is rapidity and \( x = Q^2/(2pq) \). The quantity \( x_0 \approx 0.1 - 10^{-2} \) denotes the length of the fragmentation region where there are no \( \log(x_0/x) \) factors. \( F_{2p}(x, Q^2) \), \( xG_p(x, Q^2) \sim (x_0/x)^{\mu(Q^2)} \) with \( \mu > 0; \mu(Q^2) \sim \alpha_s N_c/\pi + ... \) has been calculated in pQCD (for a review and references see Ref. [15]).

The third term describes the dependence on the collision energy of the essential impact parameters. We assume that it has the form \( \alpha'_p p \Delta b q \) natural for ladder diagrams [18]. Here \( \vec{b} \) is a two-dimensional impact parameter, and \( \alpha'_p \) is the 'Pomeron' slope. \( \alpha'_p \) is small within pQCD- \( \alpha'_p \propto N_c \alpha_s(Q^2)/Q^2 \). At the same time, near the BDL the effective \( \alpha'_p \) cannot be small [14]. Exact form of this term is unknown since it is sensitive near unitarity limit to unknown nonleading order terms in running coupling constant. So we choose this term to account for V.Gribov diffusion within pQCD ladder.

The evaluation within the WKB approximation of multi-Pomeron vertices near the BDL [13] shows that, the relative contribution of the fourth and higher multi-Pomeron vertices is suppressed by powers of \( \alpha_s \) compared to the triple-ladder term. Thus for the description of hard QCD phenomena it should be sufficient to restrict ourselves to the triple-ladder interaction. In the lowest order in the coupling constant, the triple-Pomeron vertex,

\[
L_4 = \kappa pq(p + q), \quad \kappa \propto i \frac{N_c^2 \alpha_s^2}{\lambda}
\]

is due to the interaction of ladders via one gluon loop. This estimate, accounts QCD evolution, a running coupling constant, and Sudakov form factors which suppress nonperturbative contribution discussed in Ref. [27]. The existence (but not the properties) of new QCD phenomena is not sensitive to the actual value of \( \lambda \approx Q \) which is the characteristic transverse momentum of the constituents of the pQCD ladder where it splits into two new ladders.

We neglect eikonal-type inelastic rescatterings since a bare particle may have one inelastic collision and any number of elastic collisions only, [28,18]. For the interactions that rapidly increase with energy, requirements of causality, positivity of probability for physical processes, and energy-momentum conservation can be hardly satisfied within such a set of diagrams [17]. In contrast, the contribution of rescatterings due to an inelastic diffraction into mass \( M^2 \), where \( \beta = Q^2/(Q^2 + M^2) \), is not too small, dominates in two-scale hard small-\( x \) phenomena at \( x \approx x_{cr} \). We include this contribution in the scale factor of the source.

EFT is formally different from the approaches suggested in Refs. [29] [30], [31]. EFT accounts for the increase with energy of essential impact parameters, neglect elastic eikonal rescatterings etc. However major difference is in the account of kinks and quantum fluctuations around kinks, of the phenomenon of spontaneously broken continuous symmetry. Moreover within EFT the contribution of the quantum fluctuations around the pQCD vacuum is negligible in one-scale hard processes.

Within the approximations made in this paper the coupling of the pQCD ladder to a hadron can be treated as the interaction with a source. The actual form of these term is unimportant for most of the results obtained in this paper, so we refer a reader to ref. [13] for details and here just include this interaction into Lagrangian 2.1.
III. CRITICAL PHENOMENA IN HARD QCD NEAR UNITARITY LIMIT

The form of Lagrangian (2.1) relevant for hard QCD phenomena coincides with the particular preQCD model for the Lagrangian of V.Gribov Reggeon Field Theory analysed in refs. [22–24]. Thus in the further analysis we use WKB solution of Lagrangian equations of motion and quantum fluctuations around them found in refs. [22–26] but interpret them in terms of hard QCD phenomena.

Let us formulate here the main properties of the WKB solution.

Distinctive property of the Lagrangian (2.1) is the existence, apart of the usual perturbative vacua

\begin{equation}
\phi = 0, \phi_0 = 0,
\end{equation}

of the two new vacua:

\begin{equation}
\phi = \mu/\kappa, \phi_0 = 0 \quad \text{and} \quad \phi = 0, \phi_0 = \mu/\kappa.
\end{equation}

The detailed analytical analysis of the model is possible in 1+1 dimensions only, where the equations of motion are reduced to ordinary differential equation. In refs. [22–24] the family of kinks, characterized by a 2d velocity parameter \( v \) has been found. These kinks interpolate between 3 vacua eqs. (3.1,3.2). The action of the kink is finite

\begin{equation}
S \sim (\mu/\kappa)^2 (2 - v) \phi_0^2,
\end{equation}

where \( \phi_0 \) is the field value at \( b = vY \) and \( v \) is the kink velocity. It is proportional to \( 1/(N_c^2 \alpha_s^2) \), where we used the dependence of \( \mu \) and \( \kappa \) on \( N_c \) discussed in section 2. For the value of parameter \( v = 2 \) we obtain critical kinks with zero action. The existence of these critical kinks is crucial for the quantisation of the theory. The classical contributions of these kinks into wave function are not exponentially suppressed. Quantum fluctuations around these kinks are described by a positive quadratic form, cf. [24].

The characteristic property of kinks is their step function form. One of the functions \( p \) or \( q \) behaves approximately like a step function

\begin{equation}
p(q) \sim \theta(v \sqrt{\alpha' \mu} (y - y_0) - |\vec{b} - \vec{b}_0|)
\end{equation}

where \( v \) is the kink velocity (a free parameter, \( v=2 \) for critical kink). The solution contains arbitrary parameter \( y_0 \) that helps to understand why the physics related to the fragmentation can be hidden into the properties of the source. The arbitrary solution depends also on \( b - \vec{b}_0 \). The value of \( b_0 \) is not fixed by equations. This is a zero mode relevant for the appearance of ”phonons” in quantum fluctuations.

In physical 2+1 dimensions there is no analytical expression for the solution giving critical kinks with zero action as well as a full classification of kinks. So in 2+1 dimensions we rely on the results of the numerical simulations made in [22,23], which found the same properties of kinks for the 2+1 dimensional theory as for 1+1 dimensional one.

The knowledge of the family of kink solutions permits semiclassical quantization of the theory and to calculate S-matrix.

Remarkable property of the quantum fluctuations around critical kinks is the existence of zero modes-”phonons” in EFT, which are characterized by the linear dispersion:

\begin{equation}
E = i2 \sqrt{\alpha' \mu} k = 2 \sqrt{\alpha' \mu} P_{cl}
\end{equation}
where \( P_{cl} = \int d^2bp \frac{d\phi}{db} \) is the total classical momentum derived from the EFT action. (Energy and momentum of kinks are defined as components of \( \int d^2bT^{00} \) and \( \int d^2bT^{0i} \), where the energy momentum tensor of the EFT is found via the Nether theorem).

In addition, there exists a band of low lying states with a dispersion relation \( E \sim k^2 \), and a band of states with a gap \( \sim \mu \). It can be argued that the first set of states corresponds to unshifted quasiparticle fluctuations around perturbative vacuum (3.1) in the presence of a kink and viewed from the reference frame moving with a critical speed \( v_0 \). The higher modes can be interpreted as collective fluctuations of a condensate of ladders, i.e. quasiparticles interacting with kinks. It is easy to prove that these modes do not influence the expanding disk solution asymptotically, although can be important outside the disk, and near the transition to a black disk regime. Only solutions with linear spectrum are relevant for the asymptotic behaviour of high energy processes.

These results are valid in 1+1 dimensional model that was solved analytically and are confirmed by the numerical analysis of spectrum of quantum fluctuations around kinks that has been performed in physical 2+1 dimensional case in refs. [22,23]. Moreover, it was proved on the discrete lattice that this model can be continuously connected with the Ising model in transverse magnetic field.

To summarize, the quasiclassical solution of EFT has following distinctive features (see details in ref. [13]):

A) EFT has three degenerated "vacua" (3.1), (3.2), (The word "vacua" is in the brackets, since EFT is the theory with nonhermitian Hamiltonian). The true wave function of the physical basic state is a linear combination of these three vacua, and the Hamiltonian is diagonalized by "critical" kinks with zero action. If the initial state is a perturbative one interacting with a source, it evolves in rapidity and becomes a condensate of quasiparticles=ladders.

B) The S-matrix is given by the functional integral \( S(B,Y) = \int dp \int dq \exp(-L) \) where the corresponding action is calculated via the classical kink solution described above, but includes now the source terms-the coupling with virtual photons. The main contribution to S-matrix comes from quantum fluctuations around critical kinks with zero action. Spectrum of excitations begins from massless excitations- "phonons". The contribution of the kinks into the two- quasiparticle Green functions that determines the partial waves is:

\[
D(B,Y) = (\mu/\kappa)^2 \theta(B^2 - \alpha' \mu Y^2). \tag{3.5}
\]

\( (\kappa/\mu)^2 D \) as given by eq. (3.5) is 1 inside the black disk and suppressed exponentially as \( \mu/\kappa \sim 1/\alpha_s \), outside.

C) The obtained state is described by the asymptotic wave function

\[
\Psi(y) = \exp(-\frac{\mu}{\kappa} \int d^2bq(b,y) \theta(b^2 - \alpha' \mu Y^2)) |\phi_0\rangle \tag{3.6}
\]

Here \( |\phi_0\rangle \) is a perturbative vacuum. To the extent that correlations may be neglected the asymptotic vector (3.6) is a coherent state [24] and the S-matrix is given by

\[
S(B,Y) = \exp(-\frac{\mu}{\kappa} \theta(2\sqrt{\alpha' \mu Y} - B)), \tag{3.7}
\]
where we explore that the target is localized near the impact parameter \( b \sim B \). This is just the Froissart (BDL) behaviour. We want to draw attention that \( \Psi(y) \) can not be obtained by decomposition over powers of \( \alpha_s \).

D) In other words in the limit of infinite energies \( Y \to \infty \) produced state corresponds to a Bose-Einstein condensate of ladders in the entire space. However, for finite energies the solution is the black disk of radius \( R^2 \sim R_0^2 + \alpha' \mu Y^2 \). The equation (3.6) gives the exact form of the wave function of the Bose-Einstein condensate of ladders as a function of rapidity \( Y \).

E) The transition occurs for given impact parameter \( b \) at rapidities \( b^2 \sim \mu Y(1/\lambda^2 + 4\alpha' \mu Y) \).

IV. KINKS AND QCD.

Some properties of classical solutions of EFT can be understood directly in QCD. A kink produces action proportional to \( (\mu / \kappa) \sim (1/N_c \alpha_s) \) in some power. The dependence of the S-matrix (3.7), of the critical kink action (understood as the limit of a family of kinks with nonzero action) on the coupling constant \( \alpha_s \), the spontaneously broken translational invariance, and existence of ”phonon” show that this is a novel nonperturbative QCD phenomenon.

The characteristic form of a kink is the step-function in \( Y \) space, with the width of the order \( \delta Y \approx \log(\delta E/Q) = 1/\mu \). Thus coherent length relevant for the evolution of kink is enhanced due to large Lorentz slowing down interaction factor as \( T_L \sim \delta E/Q^2 \approx \exp(1/\mu)/Q \approx 10^2/Q \). In other words, for sufficiently low \( x \) \( T_L \ll T_c \), where \( T_c \) is coherence length \( T_c \sim 1/(Qx^{1-\mu}) \). (This formulae differs from more familiar LT formulae \( T_c \sim 1/(2mN_cx) \). It follows from the violation of LT approximation in the kinematics near BDL.) This rapid transition to BDL can be called the ”color inflation”: one ladder due to the tunneling transition blows up during time \( T_L \) and creates an entire region of space filled with gluon ladders. During time \( T_L \sim (\mu^2/\kappa^2)R(Y)^2 \) ladders are created, where \( R(Y) \) is a black disk radius for a given rapidity \( Y \).

It is easy to evaluate density of ladders in coordinate space by solving discussed above diffusion equations cf. similar analysis in [24]. One obtains

\[
d_2^2 \sim N_c \alpha_s / \lambda^2
\]

Thus \( l_t^2 / d_2^2 \approx \alpha_s N_c \ll 1 \), where \( l_t \sim \kappa/\mu \sim \alpha_s N_c / \lambda \) is the characteristic scale of a ladder in a transverse parameter space. It follows from above estimate that pQCD ladders overlap significantly. But overlapping ladders can exchange by quarks and gluons because pQCD does not produce barriers between ladders. This distinctive feature of color network macroscopical in the longitudinal direction resembles quark-gluon plasma.

Understanding the actual longitudinal structure of the system can not be resolved within EFT. Indeed, even after the phase transition, the ladders continue to grow till color network achieves longitudinal length \( 1/(Qx^{1-\mu}) \).
V. OBSERVABLE PHENOMENA.

Investigation of one hard scale processes will be subject of research at the next generation of $e\bar{e}$ colliders. Two scale processes like structure functions of hadron (nuclear) target are dominated by $Q^2$ evolution at significant region of small $x$. At extremly small $x \ll x_{cr}10^{-2}$ $Q^2$ evolution will be restricted by the virtual photon fragmentation region. In this kinematics which can be achieved in cosmic ray physics QCD phenomena found in the paper may reveal itself. This contribution is additional to the contribution of large and moderate masses $M^2$ in the photon w.f. where $\beta = Q^2/(Q^2 + M^2)$ is not too small. Color inflation may reveal itself as threshold like increase of multiplicity of hadrons and softening of hadron distribution in longitudinal direction. Hard QCD phenomena discussed in the paper may reveal itself in the central pp,pA and AA collisions in the regime where hard collisions are near the BDL.

Promising way to identify the onset of the new QCD regime would be to measure Mueller-Navelet process [32]: $p + p \rightarrow jet + X + jet$ where the distance in the rapidity between high $p_t$ jets is large. Expected behaviour is: initial fast increase of cross section with $y$ predicted by pQCD should change to the fast decrease at larger $y$ because of the color inflation, i.e. disappearance of the long range correlations in the rapidity (coordinate) space near the BDL.

In the case of central heavy ion collisions one may put $\alpha_s' = 0$ because of large radius of heavy nuclei. So color inflation will reveal itself as the threshold behaviour in the hadron production, leading to the formation of the color network at sufficiently large energies.

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