Fast electrons interacting with chiral matter: mirror symmetry breaking of quantum decoherence and lateral momentum transfer

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(Dated: April 8, 2022)

Photons experience mirror asymmetry of macroscopic chiral media, as in circular dichroism and polarization rotation, since left and right handed circular polarizations differently couple with matter handedness. Conversely, free relativistic electrons with vanishing orbital angular momentum have no handedness so the question arises whether they could sense chirality of geometrically symmetric macroscopic samples. In this Letter, we show that matter chirality breaks mirror symmetry of the scattered electrons quantum decoherence, even when the incident electron wave function and the sample shape have a common reflection symmetry plane. This is physically possible since the wave function transverse smearing triggers electron sensitivity to the spatial asymmetry of the electromagnetic interaction with the sample, as results from our non-perturbative analysis of the scattered electron reduced density matrix, in the framework of macroscopic quantum electrodynamics. Furthermore, we prove that mirror asymmetry also shows up in the distribution of the electron lateral momentum, orthogonal to the geometric symmetry plane, whose non-vanishing mean value reveals that the electron experiences a lateral mechanical interaction entirely produced by matter chirality.

Molecular chirality is an important topic in science both for its implications in chemistry, biology, and medicine and for the physical effects produced by microscopic mirror symmetry breaking. The coupling of a chiral molecule with the electromagnetic field involves both its electric and magnetic dipoles [1] whose combined polar and axial characters trigger molecular sensitivity to radiation handedness, a property routinely used to detect chirality by means of circular dichroism and polarization rotation [2]. The large mismatch between molecular size and wavelength both makes such effects very small and establishes the dipolar interaction which forbids the molecule to sense the spatial variations of the field. However, macroscopic chiral samples with a large number of chiral molecules display macroscopic mirror symmetry breaking and consequently they support chiroptical effects driven by the symmetry of the spatial field profile, as the suppression of field mirror symmetry upon reflection by chiral molecular films (mirror optical activity) [3, 4], or the antenna emission of asymmetric fields in chiral metamaterials [5]. Tight subwavelength confinement of the field reduces the molecule-field spatial scale mismatch and accordingly it enhances the above chiroptical spatial effects, in analogy to other chiral nonanophotonic phenomena [6–10].

The ability of electron microscopes to focus the beam down to the subnanometer scale makes fast electrons an ideal continuum source of ultra-confined electromagnetic field [11] whose evanescent character can in principle be used to enhance spatially asymmetric chiroptical effects, this leading to indirect chiral sensing by detecting the diffraction radiation from a lateral nanostructure [3]. On the other hand, only few methods have been proposed to directly sense chirality in electron microscopes. Magnetically induced chirality can be probed by comparing electron energy loss spectra taken in opposite magnetic fields and under suitable electron scattering conditions (electron energy-loss magnetic chiral dichroism) [12, 13]. In the nonmagnetic case, the idea of a probing electron with an handedness able to sense matter chirality, has led to consider electron vortex beams [14, 15] carrying orbital angular momentum whose exchange with the sample has been shown to provide chiral information in both elastic scattering from crystals [16, 17] and inelastic scattering from plasmonic samples with chiral shape and single chiral molecules [18]. A different approach based on the photon-induced near-field electron microscopy technique [19–21] has been proposed [22] where the probing handedness is injected by means of circularly polarized radiation and the chirality of the plasmonic sample shape is retrieved from the spatial scan of the electron energy spectrum.

In this Letter, we show that a fast electron with initial mirror symmetric wave function, i.e. with no handedness, does experience microscopic chirality of a symmetrically shaped macroscopic sample (see Fig.1): the quantum decoherence of the scattered electron turns out to be mirror asymmetric due to a chiroptical spatial effect generalizing mirror optical activity. To this end, we evaluate the non-perturbative reduced density matrix of the scattered electron by using macroscopic quantum electrodynamics [23–26], an appropriate framework to deal with the electron-sample entanglement [27–29] which is ultimately responsible for electron decoherence [30–33]. Besides, by using the density matrix, we show that the distribution of the momentum component orthogonal to the reflection symmetry plane (see Fig.1) is not mirror symmetric thus showing that the scattered electron gains a net lateral momentum purely due to the sample chirality. We specialize our general discussion to an aloof electron probing a realistic chiral nanofilm where the above asymmetric effects are magnified by the relatively long effective interaction length.

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Macrosopic quantum electrodynamics provides the description of the quantized field coupled to the chiral sample via the spectral electric field \( \vec{E}_\omega (r) = \sum \lambda \int d^3r' \, G_{\omega \lambda}(r, r') \hat{f}_{\omega \lambda}(r') \) and Hamiltonian \( H_{em} = \int d\omega \, \hbar \omega \sum \lambda \int d^3r \, \hat{f}_{\omega \lambda}^\dagger(\omega) \cdot \hat{f}_{\omega \lambda}(\omega) \) is the annihilation operator of a polariton excitation of frequency \( \omega \) kind \( \lambda = e, m \) (electric and magnetic) position \( r \) and direction \( j \), whereas \( G_{\omega \lambda}(r, r') \) are two tensors related to the sample Green’s tensor \( G_{\omega \lambda}(r, r') \) [25] (see Sec.S1 of Supplemental Material) in turn satisfying the inhomogeneous Helmholtz equation

\[
\nabla \times \frac{1}{\mu} \nabla \times \frac{\omega^2}{c^2} (\varepsilon - \kappa^2) + \frac{\omega}{c} \left( \frac{\nabla \kappa}{\kappa} \right) x + \frac{2\omega\kappa}{c\mu} \nabla \cdot \kappa(r, r') = \delta(r-r') I
\]

where \( \varepsilon(r, \omega) \), \( \mu(r, \omega) \) and \( \kappa(r, \omega) \) are the medium permittivity, permeability and chiral parameter, respectively. The electron is assumed to be almost monoenergetic and highly directional with its momentum \( \vec{p} \) slightly differing from a reference momentum \( P_0 \) so that its paraxial Hamiltonian with no momentum recoil is \( H_{el} = V \cdot \vec{p} \) where \( V = eP_0/\sqrt{m^2c^2 + P_0^2} \) and \( \vec{p} \) are the electron velocity and momentum operator. In the same approximation, the chiral sample-electron interaction hamiltonian is \( H_{int} = e[V \cdot \hat{A}(\vec{R}) - \hat{\Phi}(\vec{R})] \) where \( -e < 0 \) and \( \hat{A} \) and \( \hat{\Phi} \) are the vector and scalar potential operators in the Coulomb gauge related to the transverse and longitudinal parts of the electric field operator. Time evolution produced by the Hamiltonian \( H_{em} + H_{el} + H_{int} \) can be analytically dealt with, in analogy to the achiral situation [34], and the rigorous scattering operator \( \hat{S} \) we have obtained is detailed in Sec.S2.2 of Supplemental Material. The incident electron wave function is \( \psi_i(\vec{R}) = \phi_i(\vec{R}_i) e^{i\vec{k}_R \cdot \vec{R}_i/\sqrt{\hbar}} \) where subscripts \( v \) and \( t \) henceforth label the parts of a vector parallel and orthogonal to \( V \) (see Fig.1), \( \phi_i \) is the initial transverse wave function profile, \( E_i \) is the initial electron energy and \( \ell \) is the longitudinal quantization length. Since no radiation is initially involved in the scattering setup we are considering, the initial electron-field state is \( |\Psi_i\rangle = |\phi_i\rangle \otimes |0\rangle \) where \( |0\rangle \) is the polariton vacuum state. The final state \( |\Psi\rangle = \hat{S} |\Psi_i\rangle \) displays electron-field entanglement, it resulting from processes of electron energy loss and multipolariton excitation of any order, so that measurement predictions about the electron (with the field left unmeasured) have to be performed by resorting to the reduced density matrix (RDM) \( \rho_{el}(\vec{R}, \vec{R}') = \psi_i(\vec{R}) \psi_i^\dagger(\vec{R}') \gamma(\vec{R}, \vec{R}') \) (see Sec.S2.3 of Supplemental Material) where

\[
\gamma(\vec{R}, \vec{R}') = e^{i(\Phi(\vec{R}_i) - \Phi(\vec{R}_i')) - \frac{1}{2} [\Delta(\vec{R}, \vec{R}_i) + \Delta(\vec{R}', \vec{R}_i')]_z} \Delta(\vec{R}, \vec{R}')
\]

is the decoherence factor (DF) of the scattered electron with

\[
\Phi(\vec{R}_i) = \frac{4\alpha}{c} \int_0^\infty q dq \int_{\infty}^{\infty} dq' \sin \left[ \frac{\omega}{V} (q-q') \right] J \cdot \text{Im} \left[ \int \vec{u}_v \cdot G_{\omega}(\vec{R}_i + q\vec{u}_v, \vec{R}_i + q'\vec{u}_v) \vec{u}_x \right],
\]

\[
\Delta(\vec{R}, \vec{R}') = \frac{4\alpha}{c} \int_{\infty}^{\infty} dq e^{-i\frac{\omega}{V}(\vec{R}_i - \vec{R}')} \int_{\infty}^{\infty} dq' e^{i\frac{\omega}{V}(q'-q)} J \cdot \text{Im} \left[ \int \vec{u}_v \cdot G_{\omega}(\vec{R}_i + q\vec{u}_v, \vec{R}_i' + q'\vec{u}_v) \vec{u}_x \right], \tag{3}
\]

where \( \alpha \) is the fine-structure constant and \( \vec{u}_v = \vec{V}/V \) is the velocity unit vector. The first exponential in the right hand side of Eq.(2) physically arises from elastic processes due to the zero-point fluctuations of the field whereas the second exponential containing \( \Delta(\vec{R}, \vec{R}') \) provides the inelastic contribution to electron decoherence, usual mechanisms in achiral environments [35, 36] that we have here generalized in the presence of chiral matter. From the above RDM it is possible to evaluate the energy distribution and, most importantly for our purposes, the transverse momentum distribution (TMD) (see Sec.S2.4 of Supplemental Material) which is

\[
\frac{d \rho}{d^2P_t} = \frac{1}{(2\pi\hbar)^2} \int d^2R_i \int d^2R'_i \frac{e^{-i\hat{p}_t \cdot (\vec{R}_i - \vec{R}_i')}}{\phi_i(\vec{R}_i) \phi_i^\dagger(\vec{R}_i') \gamma(\vec{R}_i, \vec{R}_i')} . \tag{4}
\]

As a prelude to the discussion of the asymmetric features gained by the electron upon scattering, it is essen-
FIG. 2. (a) Scattering of an aloof electron by a chiral nanofilm deposited on a substrate. (b) Film permittivity ε and chiral parameter κ. (c) Symmetric and antisymmetric parts of Δ for two electron velocities β = V/c (columns). For graphical clarity purpose, suitable level curves are plotted as dotted, dashed and solid black lines.

Consider the Green’s tensor behavior under spatial reflections. To avoid macroscopic geometrical distortion, possibly hiding microscopic chirality, we consider a chiral sample displaying a mirror symmetry plane π and the mirror reflection through it, \( \mathbf{r}_M = \mathbf{R} \mathbf{r} \), so that \( \varepsilon(\mathbf{R} \mathbf{r}) = \varepsilon(\mathbf{r}) \), \( \mu(\mathbf{R} \mathbf{r}) = \mu(\mathbf{r}) \) and \( \kappa(\mathbf{R} \mathbf{r}) = \kappa(\mathbf{r}) \) (see Fig.1). Now the mirror image of the Green’s tensor is \( \mathbf{G}(\mathbf{r};\mathbf{R} \mathbf{r}) = \mathbf{R} \mathbf{G}(\mathbf{r};\mathbf{R} \mathbf{r}) \mathbf{R} \) which is easily seen to satisfy Eq.(1) with the sign flip \( \kappa \rightarrow -\kappa \), so that \( \mathbf{G}(\mathbf{r};\mathbf{R} \mathbf{r}) \) is the Green’s tensor of the opposite enantiomeric sample, a restatement of reflection invariance of electrodynamics. As a remarkable consequence, \( \mathbf{G}(\mathbf{r};\mathbf{R} \mathbf{r}) \) cannot coincide with \( \mathbf{G}(\mathbf{r};\mathbf{R} \mathbf{r}) \) (see Sec.S2.5 of Supplemental Material), since otherwise identical sources in different from, the optical lateral force experienced by chiral nanoparticles [37, 38].

To discuss our general results in a realistic situation, we consider the setup sketched in Fig.2(a) where an aloof electron travels in vacuum (\( \varepsilon_1 = 1 \)) with its velocity \( \mathbf{V} = \mathbf{V} e_x \) parallel to a homogeneous chiral nanofilm of thickness \( d = 50 \text{ nm} \) and deposited onto a substrate (\( \varepsilon_2 = 1.48 \)), assuming infinite extension along the y axis and finite length \( L \) along the x axis. We assume an elliptical Gaussian transverse profile for the initial wave function, \( \phi_i(Y,Z) = \sqrt{\frac{2}{\pi \sigma_y \sigma_z}} e^{-\frac{Y^2}{2 \sigma_y^2} - \frac{(Z L)^2}{2 \sigma_z^2}} \), of widths \( \sigma_y \) and \( \sigma_z \) and impact parameter \( b \). In Fig.2(b) we plot the nanofilm permittivity \( \varepsilon \) and chiral parameter \( \kappa \) modeling typical chiral molecules with ultraviolet resonance at 3.54 eV embedded in a dielectric matrix, with negligible magnetic response (\( \mu = 1 \)) [39]. Such maximally symmetric setup has \( \pi = \frac{
abla}{2} \phi \) as its reflection symmetry plane so that, from the comparison of Fig.1 and Fig.2(a), we get \( \mathbf{R} = e_x e_x - e_y e_y + e_z e_z \).
\( \gamma(X, Y, Z, Z') \) and its mirror symmetry breaking for various interaction lengths \( L \) and two electron velocities, for \( X = 0 \) and \( Z' = -3 \) nm. The modulus of \( \gamma \) in Fig.3(a) turns out to be localized around the chosen reference \( Z' \) and its spatial spread, for a fixed interaction length, is smaller at lower electron velocities, as a consequence of the evanescent mechanism ruling \( \Delta \) discussed in Fig.2. In addition the spatial spot of \( \gamma \) remarkably shrinks at larger interaction lengths, for a fixed electron velocity, an effect resulting from the non-perturbative character of the DF in Eq.(2) and testifying that first-order perturbation theory is not adequate in the here considered setup for interaction lengths of the order of a ten of microns (see Sec.2.6 of Supplemental Material). In Fig.3(b) we plot the modulus of the DF mirror asymmetry degree, defined as \( \text{Asym} \left[ \gamma(R, R') \right] = 2 \left( \gamma(R, R') - \gamma(R, R')^\dagger \right) / (\gamma(R, R') + \gamma(R, R')^\dagger) \), which using Eq.(2) turns out to be

\[
\text{Asym} \left[ \gamma \right] = 2i \tan \Delta_A. \tag{7}
\]

DF mirror asymmetry is again more pronounced at lower electron velocities owing to the above discussed behavior of \( \Delta_A \) and it remarkably increases at larger interaction lengths.

In Fig.4 we display the dependence of the overall phenomenology concerning lateral momentum transfer and energy exchange on the interaction length and the electron velocity, for two initial electron configurations: a tightly confined and distant electron with \((\sigma_y, \sigma_z, b) = (3, 3, 18) \) nm (first row) and a laterally wider and closer one with \((\sigma_y, \sigma_z, b) = (500, 3, 9) \) nm (second row). In Figs.4(a) and 4(b) we respectively plot mean value \((P_y)\) and root-mean-square \(\sqrt{\langle P_y^2 \rangle - \langle P_y \rangle^2}\) (see Sec.S3.4 of Supplemental Material) of the electron lateral momentum (both normalized with the reference momentum \( P_0 = mV/\sqrt{1 - (V/c)^2} \) showing that they increase both as the interaction length increases and as the electron velocity decreases, as a consequence of the properties of \( \gamma \) discussed in Fig.3. However, the laterally wider and closer electron (second row) displays larger lateral momentum gain, since mirror symmetry breaking is more pronounced close to the film (see Fig.3(b)), and it has smaller momentum uncertainty. To quantify the efficiency of the lateral momentum transfer, in Fig.4(c) we plot the peak factor \( \langle P_y \rangle / \sqrt{\langle P_y^2 \rangle - \langle P_y \rangle^2} \) showing that the laterally wider and closer electron (second row) experiences a lateral momentum transfer two order of magnitudes more efficient than the tight confined and distant electron (first row). This is in agreement with the above general observation that the lateral smearing of the electron wave function rules the mirror symmetry breaking of its RDM. Remarkably, the lateral momentum transfer efficiency is globally larger at larger electron velocities. For completeness, in Figs.4(d) and 4(e) we respectively plot the loss mean value \(\langle E \rangle - E_i\) and root-mean-square

\[
\sqrt{\langle E^2 \rangle - \langle E \rangle^2} \tag{see Sec.S3.5 of Supplemental Material}.
\]

In Fig.3 we display the scattered electron DF

\[ F_e = F_{x} e_z, \text{ and } F_e = F_{y} e_y; \] accordingly asymmetric effects show up along the \( y \)-axis which is the lateral direction. Planar geometry enables analytical evaluation of the relevant Green’s tensor component along the electron velocity \( G_{xxx}(x - x', y - y', z - z') \) and its reflected part in vacuum \((z < 0, z' < 0) \) turns out to be mirror asymmetric (see Secs.S3.1 and S3.2 of Supplemental Material). Moreover, the antisymmetric part of \( G_{xxx} \) is produced by the reflection coefficients \( R_{SP}, R_{PS} \) resulting from the \( S - P \) polarization coupling in turn triggered by matter chirality [3]. The first of Eqs.(3) provides the phase factor \( \Phi(Z) \), which is irrelevant in our symmetry analysis, whereas the second equation, after setting \( X = X - X', Y = Y - Y', Z = Z + Z' \), yields \( \Delta(R, R') = \Delta_S(X, Y, Z) + i \Delta_A(X, Y, Z) \) where the real \( \Delta_S \) and \( \Delta_A \) are proportional to the interaction length \( L \) and are respectively symmetric and antisymmetric under the lateral reflection \( Y \rightarrow -Y \), they stemming from the symmetric and antisymmetric parts of \( G_{xxx} \) (see Sec.S3.4 of Supplemental Material). All the electron mirror symmetry breaking effects are a consequence of \( \Delta_A \) which evidently vanishes in the achiral limit \( \kappa \rightarrow 0 \). In Fig.2(c) we plot \( \Delta_S \) and \( \Delta_A \) at \( X = 0 \) and \( Z < 0 \) (vacuum) for the interaction length \( L = 1 \) \( \mu \)m, for two electron velocities. Their relevant features are a consequence of the photon momentum \( k_x = -\omega/V \) selected by the electron velocity and entailing the fully evanescent character of the field and the corresponding enhancement of the film chiral response. At lower electron velocities such evanescent character is more pronounced, since the photon momentum parallel to the film is \( \sqrt{\omega/V^2 + k_z^2} \), and accordingly both \( \Delta_S \) and \( \Delta_A \) show tighter spatial confinement and larger amplitudes.
energy $E_0 = VP_0$. The energy loss absolute value evidently increases as both the interaction length and electron velocity increase as expected. On the other hand the energy root-mean-square has the same behavior of the lateral momentum root-mean-square. Figures 5(a), (b), (d) and (e) provide the overall information that for the here considered relatively large interaction lengths (up to twenty microns), both the lateral momentum $P_y$ and the longitudinal one $P_x = E/V$ very slightly depart from the reference momentum $P_0$ (for any electron velocity) and this self-consistently validates the above paraxial approximation without momentum recoil used to describe the electron dynamics.

In conclusion, we have shown that fast electrons with no handedness (i.e. carrying no orbital angular momentum) can experience the microscopic chirality of geometrically symmetric samples owing to the combination of electron lateral quantum smearing with the spatial asymmetry of the electromagnetic interaction. Matter chirality breaks mirror symmetry of electron quantum decoherence, an effect detectable through bi-prism electron holography [40, 41] or in a two-slits interference experiment [42, 43]. This result is in agreement with the fact that the spatial coherence of inelastically scattered electrons is highly correlated to the optical properties of the sample. As a consequence, we predict that the electron lateral momentum distribution is asymmetric as well thus yielding a net lateral momentum transfer which can be detected through momentum-resolved energy loss spectroscopy [44].

ACKNOWLEDGEMENTS

The author acknowledges PRIN 2017 PELM (grant number 20177PSCCKT).

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Fast electrons interacting with chiral matter: mirror symmetry breaking of quantum decoherence and lateral momentum transfer: Supplemental Material

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(Dated: April 8, 2022)

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arXiv:2204.03490v1  [quant-ph]  7 Apr 2022
Macroscopic quantum electrodynamics (MQED) is a powerful tool providing the quantum description of the electromagnetic field in absorbing linear media [1, 2] with any optical properties, encompassing spatial nonlocality and nonreciprocity [3]. We here review the MQED formalism in inhomogenous chiral media characterized (in the frequency domain $e^{-i\omega t}$) by the classical constitutive relations

$$
D_\omega = \varepsilon_0 \varepsilon_\omega E_\omega - \frac{i}{c} \kappa H_\omega,
$$

$$
B_\omega = \frac{i}{c} \kappa E_\omega + \mu_0 \mu H_\omega,
$$

(S1)

where $\varepsilon(\mathbf{r}, \omega)$ and $\mu(\mathbf{r}, \omega)$ are the relative permittivity and permeability and $\kappa(\mathbf{r}, \omega)$ is the Pasteur parameter controlling the coupling between electric and magnetic responses, i.e. the medium chirality; as required by causality, such parameters satisfy the Kramers-Kronig dispersion relations. The details of the general quantization scheme are reported in Ref.[3] and we here summarize the specific results pertaining chiral media.

### S1.1. Green’s tensor

The classical Green’s tensor is among the basic tools of MQED and hence we briefly discuss here some of its properties. The classical field produced by a current density distribution $J_\omega(\mathbf{r})$ in the presence of a chiral medium is

$$
E_\omega(\mathbf{r}) = i\omega \mu_0 \int d^3\mathbf{r}' G_\omega(\mathbf{r}, \mathbf{r}') J_\omega(\mathbf{r}')
$$

(S2)

where the Green’s tensor $G_\omega(\mathbf{r}, \mathbf{r}')$ is the solution of the inhomogenous Helmholtz equation

$$
\left\{ \nabla \times \frac{1}{\mu} \nabla \times -\frac{\omega^2}{c^2} \left( \varepsilon - \frac{\kappa^2}{\mu} \right) + \frac{\omega}{c} \left[ \left( \nabla \times \frac{\kappa}{\mu} \right) \times +2\frac{\kappa}{\mu} \nabla \right] \right\} G_\omega(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') I,
$$

(S3)

with the boundary conditions $G_\omega(\mathbf{r}, \mathbf{r}') \to 0$ for $\mathbf{r}, \mathbf{r}' \to \infty$. Here we have considered the general inhomogeneous situation where $\varepsilon, \mu$ and $\kappa$ are position dependent and it encompasses the case of piecewise homogeneous media. A relevant property of the Green’s tensor is that its analytic continuation $G_\Omega(\mathbf{r}, \mathbf{r}')$ to the $\Omega$ complex plane is analytic along the whole upper half plane $\text{Im } \Omega > 0$. In addition it satisfies the Schwartz reflection principle

$$
G^*_\Omega(\mathbf{r}, \mathbf{r}') = G_{-\Omega^*}(\mathbf{r}, \mathbf{r}')
$$

(S4)

and it displays the asymptotic behavior

$$
G_\Omega(\mathbf{r}, \mathbf{r}') \to -\frac{\Omega^2}{\omega^2} \delta(\mathbf{r} - \mathbf{r}')
$$

(S5)

for $\Omega \to \infty$. By using all these properties it is simple to show that the imaginary part of the Green’s tensor satisfies the important integral relation

$$
\int_0^{+\infty} d\omega \omega \text{Im} \left[ G_\omega(\mathbf{r}, \mathbf{r}') \right] = \frac{\pi}{2} c^2 \delta(\mathbf{r} - \mathbf{r}') I.
$$

(S6)

Since chiral media are reciprocal, the Green’s tensor satisfies the Onsager reciprocity relation

$$
G^T_\omega(\mathbf{r}, \mathbf{r}') = G_\omega(\mathbf{r}', \mathbf{r}).
$$

(S7)

### S1.2. Field quantization

Medium absorption forbids radiation to be regarded as an isolated system and hence, as suggested by the fluctuation-dissipation theorem, a noise current $j_N^\omega(\mathbf{r})$ is introduced to account for the medium degrees of freedom. Moreover, the noise current encodes the full radiation-medium state since its radiated field is provided by $E_\omega(\mathbf{r})$ =
\[ i \omega \mu_0 \int d^3 r' G_{\omega} (r, r') j_\omega (r') \] (see Eq.(S2)). Quantization is performed by requiring that the electric and magnetic induction field operators satisfy the standard vacuum QED commutation relations and this leads to a radiation-medium description in terms of boson polaritonic excitations. Due to magnetoelastic coupling, both electric \((e)\) and magnetic \((m)\) contributions of the noise currents are involved and hence two field operators \( \hat{f}_{\omega e} (r) = \hat{f}_{\omega e j} (r) e_j \) and \( \hat{f}_{\omega m} (r) = \hat{f}_{\omega m j} (r) e_j \) are introduced in the Schrödinger picture whose components satisfy the commutation relations
\[
\left[ \hat{f}_{\omega \lambda j} (r), \hat{f}_{\omega' \lambda' j'} (r') \right] = 0, \\
\left[ \hat{f}_{\omega \lambda j} (r), \hat{f}_{\omega' \lambda' j'}^\dagger (r') \right] = \delta (\omega - \omega') \delta_{\lambda \lambda'} \delta_{j j'}, \delta (r - r') \quad (S8)
\]
so that \( \hat{f}_{\omega \lambda j}^\dagger (r) \) is the creation operator of the \( j \) component of a polaritonic excitation of frequency \( \omega \) and kind \( \lambda = e, m \) at the position \( r \). The electromagnetic Hamiltonian is
\[
\hat{H}_{\omega m} = \int_0^\infty d \omega \ h \omega \sum_{\lambda} \int d^3 r \ \hat{f}_{\omega \lambda}^\dagger (r) \cdot \hat{f}_{\omega \lambda} (r) \quad (S9)
\]
since it predicts the correct time evolution of the annihilation operators
\[
e^{i \hat{H}_{\omega m} t} \hat{f}_{\omega \lambda} e^{-i \hat{H}_{\omega m} t} = \hat{f}_{\omega \lambda} e^{-i \omega t}. \quad (S10)
\]
The electric field operator \( \hat{E} = \int_0^{+\infty} d \omega \ (\hat{E}_{\omega e} + \hat{E}_{\omega m}^\dagger) \) has positive frequency part
\[
\hat{E}_{\omega e} (r) = \sum_{\lambda} \int d^3 r' G_{\omega \lambda} (r, r') \hat{f}_{\omega \lambda}^\dagger (r') \quad (S11)
\]
where the tensors \( G_{\omega e} \) and \( G_{\omega m} \) are obtained from the Green’s tensor \( G_{\omega} \) by means of
\[
\begin{pmatrix} G_{\omega e} \\ G_{\omega m} \end{pmatrix}_{(r, r')} = -i \omega \mu_0 \sqrt{\frac{\hbar}{\pi}} \begin{pmatrix} U_{ee} & U_{em} \\ U_{me} & U_{mm} \end{pmatrix}_{(r)} \begin{pmatrix} i \omega G_{\omega} \\ G_{\omega} \times \nabla \end{pmatrix}_{(r, r')} \quad (S12)
\]
with the matrix \( U_{\lambda \lambda'} \) satisfying the equation
\[
\begin{pmatrix} U_{ee} & U_{em} \\ U_{me} & U_{mm} \end{pmatrix}^T \begin{pmatrix} U_{ee} & U_{em} \\ U_{me} & U_{mm} \end{pmatrix} = \begin{pmatrix} \varepsilon_0 \text{Im} \left( \varepsilon - \frac{\kappa^2}{\mu} \right) & i \sqrt{\frac{\varepsilon \mu_0}{\varepsilon_0 \mu_0}} \text{Im} \left( \frac{\kappa}{\mu} \right) \\ -i \sqrt{\frac{\varepsilon_0}{\varepsilon_0 \mu_0}} \text{Im} \left( \frac{\kappa}{\mu} \right) & - \frac{\varepsilon_0}{\mu_0} \text{Im} \left( \frac{1}{\mu} \right) \end{pmatrix}. \quad (S13)
\]
It is worth noting that the tensors \( G_{\omega e} \) and \( G_{\omega m} \) satisfy the identity
\[
\sum_{\lambda} \int d^3 s \ G_{\omega \lambda} (r, s) G_{\omega \lambda}^\dagger (r', s) = \frac{\hbar \omega^2}{\varepsilon_0 e^2} \text{Im} \left[ G_{\omega} (r, r') \right] \quad (S14)
\]
which is very useful since, among other things, when combined with Eqs.(S8) and (S11), it yields the relation
\[
\left[ \mathbf{F} \cdot \hat{E}_{\omega e} (r), \mathbf{F}' \cdot \hat{E}_{\omega e}^\dagger (r') \right] = \delta (\omega - \omega') \frac{\hbar \omega^2}{\varepsilon_0 e^2} \mathbf{F} \cdot \text{Im} \left[ G_{\omega} (r, r') \right] \mathbf{F}' \quad (S15)
\]
where \( \mathbf{F} \) and \( \mathbf{F}' \) are two vectors. Equation (S15) provides the commutator of electric field components at different points and different frequencies and it highlights the role played by the imaginary part of the Green’s tensor.

**S1.3. Vector and scalar potentials in the Coulomb gauge**

In order to deal with the electron-radiation coupling, the vector and scalar potentials, \( \hat{A} = \int_0^{+\infty} d \omega (\hat{A}_{\omega e} + \hat{A}_{\omega m}^\dagger) \) and \( \hat{\phi} = \int_0^{+\infty} d \omega (\hat{\phi}_{\omega e} + \hat{\phi}_{\omega m}^\dagger) \), are required. Their positive frequency parts are defined by the relation \( \hat{E}_{\omega e} = i \omega \hat{A}_{\omega e} - \nabla \hat{\phi}_{\omega e} \) so
that, after choosing the Coulomb gauge $\nabla \cdot \hat{A}_\omega = 0$, the terms $i\omega \hat{A}_\omega$ and $-\nabla \hat{\phi}_\omega$ can be identified with the transverse and longitudinal parts of the field $\hat{E}_\omega$ so that

$$
\hat{A}_\omega (r) = \frac{1}{i\omega} \int d^3r' \left[ \frac{1}{(2\pi)^3} \int d^3k \, e^{ik(r-r')} \left( I - \frac{kk}{k^2} \right) \right] \hat{E}_\omega (r'),
$$

$$
\hat{\phi}_\omega (r) = \int d^3r' \left[ \frac{1}{(2\pi)^3} \int d^3k \, e^{ik(r-r')} \frac{kk}{k^2} \right] \cdot \hat{E}_\omega (r'). \quad (S16)
$$

We highlight that the choice of the Coulomb gauge is not incompatible with a possible spatial inhomogeneity of the medium (responsible for $\nabla \cdot \hat{E}_\omega \neq 0$) since we are describing the electromagnetic field by means of both the vector and scalar potentials. This is made manifest by rewriting Eqs.(S16) as

$$
\hat{A}_\omega (r) = \frac{1}{i\omega} \left[ \hat{E}_\omega (r) + \nabla \int d^3r' \frac{\nabla' \cdot \hat{E}_\omega (r')}{4\pi |r-r'|} \right],
$$

$$
\hat{\phi}_\omega (r) = \int d^3r' \frac{\nabla' \cdot \hat{E}_\omega (r')}{4\pi |r-r'|}, \quad (S17)
$$

where we have used the Fourier integral representation of the Coulomb potential

$$
\frac{1}{4\pi r} = \frac{1}{(2\pi)^3} \int d^3k e^{ikr} k^2. \quad (S18)
$$

## S2. Interaction of a Fast Electron with a Chiral Medium

### S2.1. Interaction Hamiltonian

The minimal coupling Hamiltonian describing a relativistic electron interacting with a chiral medium is

$$
\hat{H} = c\sqrt{m^2c^2 + [\hat{P} + e\hat{A}(\hat{R})]^2} - e\phi(\hat{R}) + \hat{H}_{em} \quad (S19)
$$

where $m$ and $-e < 0$ are the electron rest mass and charge, $\hat{R}$ and $\hat{P}$ are the electron position and momentum operators ($[\hat{R}, \hat{P}] = i\hbar$), $\hat{A}$ and $\hat{\phi}$ are the above defined vector and scalar potentials and $\hat{H}_{em}$ is the electromagnetic Hamiltonian defined of Eq.(S9). Since $\hat{A} \cdot \hat{P} - \hat{P} \cdot \hat{A} = i\hbar \nabla \cdot \hat{A}$, the trasversality of the vector potential in the Coulomb gauge implies that $(\hat{P} + e\hat{A})^2 = \hat{P}^2 + 2e\hat{A} \cdot \hat{P} + e^2\hat{A}^2$ and hence, by exploiting the weakness of the electron-radiation coupling, the term $e^2\hat{A}^2$ can be neglected while the square root can be expanded up to the first order in the term $2e\hat{A} \cdot \hat{P}$ thus obtaining

$$
\hat{H} = c\sqrt{m^2c^2 + \hat{P}^2} + e \frac{c}{\sqrt{m^2c^2 + \hat{P}^2}} \hat{A} \cdot \hat{P} - e\phi(\hat{R}) + \hat{H}_{em}. \quad (S20)
$$

To further simplify the Hamiltonian, we now perform the standard paraxial approximation, which is adequate to deal with highly collimated electron beams routinely used in TEM microscopes. Accordingly we consider electron states whose momentum distribution is strongly localized around a central momentum $\hat{P}_0$ ($|\hat{P} - \hat{P}_0| \ll \hat{P}_0$) so that, by linearizing the first term of Eq.(S20) around $\hat{P}_0$ and evaluating the second term at $\hat{P}_0$, after neglecting an irrelevant constant contribution, we get

$$
\hat{H} = \sqrt{V \cdot \hat{P} + \hat{H}_{em}} + e \left[ V \cdot \hat{A}(\hat{R}) - \hat{\phi}(\hat{R}) \right] \left[ H_0 \right] \quad (S21)
$$

where $V = c\hat{P}_0/\sqrt{m^2c^2 + \hat{P}_0^2}$. Due to the relation

$$
e^{\hat{P} \cdot (V \cdot \hat{P})t} \hat{R} e^{-\hat{P} \cdot (V \cdot \hat{P})t} = \hat{R} + Vt, \quad (S22)$$

we get

$$
\hat{H} = \sqrt{V \cdot \hat{P} + \hat{H}_{em}} + e \left[ V \cdot \hat{A}(\hat{R}) - \hat{\phi}(\hat{R}) \right] \left[ H_0 \right] \quad (S21)
$$

where $V = c\hat{P}_0/\sqrt{m^2c^2 + \hat{P}_0^2}$. Due to the relation

$$
e^{\hat{P} \cdot (V \cdot \hat{P})t} \hat{R} e^{-\hat{P} \cdot (V \cdot \hat{P})t} = \hat{R} + Vt, \quad (S22)$$

we get

$$
\hat{H} = \sqrt{V \cdot \hat{P} + \hat{H}_{em}} + e \left[ V \cdot \hat{A}(\hat{R}) - \hat{\phi}(\hat{R}) \right] \left[ H_0 \right] \quad (S21)
$$
the vector $\mathbf{V}$ can be interpreted as the free electron velocity which is independent on momentum due to paraxial approximation. As highlighted by the curly brackets, the Hamiltonian has a main term $\hat{H}_0$ accounting for the bare electron-radiation system and a term $\hat{H}_{\text{int}}$ describing their coupling. By using Eq.(S9) and Eqs.(S17) we get

$$\hat{H}_0 = \mathbf{V} \cdot \hat{\mathbf{P}} + \int_0^\infty d\omega \hbar \omega \sum_\lambda \int d^3r \hat{f}_{\omega \lambda}^\dagger (\mathbf{r}) \cdot \hat{f}_{\omega \lambda} (\mathbf{r}),$$

$$\hat{H}_{\text{int}} = \int_0^{+\infty} d\omega \frac{e^{i\omega t}}{i\omega} \int d^3r \left[ \mathbf{Q}_\omega (\hat{\mathbf{R}}, \mathbf{r}) \cdot \hat{\mathbf{E}}_\omega (\mathbf{r}) - \mathbf{Q}_\omega^* (\hat{\mathbf{R}}, \mathbf{r}) \cdot \hat{\mathbf{E}}_\omega^\dagger (\mathbf{r}) \right]$$

(S23)

where

$$\mathbf{Q}_\omega (\mathbf{R}, \mathbf{r}) = \delta (\mathbf{R} - \mathbf{r}) \mathbf{V} + (\mathbf{V} \cdot \nabla \mathbf{r} - i\omega) \frac{1}{4\pi |\mathbf{R} - \mathbf{r}|} \nabla \mathbf{r}.$$  

(S24)

The delta function contribution of $\mathbf{Q}_\omega$ provides the direct interaction between the electron and the radiation electric field whereas the seconds accounts for the coupling of the electron with the Coulomb field produced by the medium inhomogeneity.

### S2.2. $S$-operator

Let us now consider a typical scattering situation where a free electron in the distant past is made to interact with the chiral medium and it is subsequently collected by a detector in the far future. The time evolution of an initial electron-radiation state $|\Psi_i\rangle = |\Psi (-\infty)\rangle$, in the interaction picture, is provided by $|\Psi (t)\rangle = \hat{U} (t) |\Psi_i\rangle$ where the time evolution operator $\hat{U} (t)$ is the solution of the equation

$$i\hbar \frac{d\hat{U}}{dt} = \left( e^{i\hat{H}_0 t} \hat{H}_{\text{int}} e^{-i\hat{H}_0 t} \right) \hat{U}$$  

(S25)

with initial value $\hat{U} (-\infty) = 1$. Dynamics of fast electrons interacting with non magnetoelectric media has been investigated to some extent [4–6] and we here extend those results to chiral media. We start by using Eqs.(S10) and Eqs.(S22), to cast Eq.(S25) as

$$\frac{d\hat{U}}{dt} = \left\{ \int_0^{+\infty} d\omega \int d^3r \frac{e^{i\omega t}}{\hbar \omega} \left[ -e^{-i\omega t} \mathbf{Q}_\omega (\hat{\mathbf{R}} + \mathbf{V} t, \mathbf{r}) \cdot \hat{\mathbf{E}}_\omega (\mathbf{r}) + e^{i\omega t} \mathbf{Q}_\omega^* (\hat{\mathbf{R}} + \mathbf{V} t, \mathbf{r}) \cdot \hat{\mathbf{E}}_\omega^\dagger (\mathbf{r}) \right] \right\} \hat{U}$$  

(S26)

which we solve by means of the trial solution

$$\hat{U} = e^{i\hat{W}}$$  

(S27)

where

$$\hat{W} = \Xi (\hat{\mathbf{R}}, t) + \int_0^{+\infty} d\omega \int d^3r \left[ \mathbf{N}_\omega (\hat{\mathbf{R}}, \mathbf{r}, t) \cdot \hat{\mathbf{E}}_\omega (\mathbf{r}) + \mathbf{N}_\omega^* (\hat{\mathbf{R}}, \mathbf{r}, t) \cdot \hat{\mathbf{E}}_\omega^\dagger (\mathbf{r}) \right]$$  

(S28)

and the unknown function $\Xi$ and vector $\mathbf{N}_\omega$ are such that $\Xi (\hat{\mathbf{R}}, -\infty) = 0$, $\mathbf{N}_\omega (\hat{\mathbf{R}}, -\infty) = 0$, as required by the initial value of $\hat{U}$. In order to substitute Eq.(S27) into Eq.(S26), we use the Sneddon’s formula for the derivative of the exponential of an operator [7]

$$\frac{d}{dt} e^{i\hat{W}} = i \int_0^1 d\varphi \ e^{i\varphi \hat{W}} \frac{d\hat{W}}{dt} e^{i(1-\varphi)\hat{W}}.$$  

(S29)

whose evaluation requires the analysis of the commutator of $\hat{W}$ and $\frac{d\hat{W}}{dt}$. From Eq.(S15), we easily get

$$\left[ \hat{W}, \frac{d\hat{W}}{dt} \right] = \frac{2i\hbar}{\pi \varepsilon_0 c^2} \int_0^{+\infty} d\omega \omega^2 \int d^3r \int d^3r' \text{Im} \left\{ \mathbf{N}_\omega (\hat{\mathbf{R}}, \mathbf{r}, t) \cdot \text{Im} [\mathbf{G}_\omega (\mathbf{r}, \mathbf{r'})] \frac{\partial \mathbf{N}_\omega^* (\hat{\mathbf{R}}, \mathbf{r}', t)}{\partial t} \right\}$$  

(S30)
whose independence on the polaritonic operators $\hat{F}_{\omega \lambda}, \hat{F}^\dagger_{\omega \lambda}$ implies that it commutes with both $\hat{W}$ and $\frac{d\hat{W}}{dt}$. As a consequence, the relation $[e^{i\hat{W}}, \frac{d\hat{W}}{dt}] = \left[\hat{W}, \frac{d\hat{W}}{dt}\right]$ allows Eq.(S29) to be written as

$$
\frac{d}{dt} e^{i\hat{W}} = \left(-\frac{1}{2} \left[\hat{W}, \frac{d\hat{W}}{dt}\right] + i \frac{d\hat{W}}{dt}\right) e^{i\hat{W}}
$$

(S31)

which, once compared with Eq.(S26), states that $e^{i\hat{W}}$ is the evolution operator if

$$
\frac{\partial N_\omega}{\partial t} = \frac{i e}{\hbar \omega} e^{-i\omega t} Q_\omega(\hat{R} + \mathbf{V} t, \mathbf{r}),
$$

$$
\frac{\partial \Xi}{\partial t} = \frac{1}{2i} \left[\hat{W}, \frac{d\hat{W}}{dt}\right].
$$

(S32)

After noting that Eq.(S24) yields

$$
e^{-i\omega t} Q_\omega(\hat{R} + \mathbf{V} t, \mathbf{r}) = e^{-i\omega t} \delta(\hat{R} - \mathbf{r} + \mathbf{V} t) \mathbf{V} + \frac{d}{dt} \left[\frac{e^{-i\omega t}}{4\pi|\hat{R} + \mathbf{V} t - \mathbf{r}|} \nabla \mathbf{r}\right],
$$

(S33)

the integration of the first of Eqs.(S32) is straightforward and we get

$$
N_\omega(\hat{R}, \mathbf{r}, t) = \frac{ie}{\hbar \omega} \left[e^{i\varphi(\hat{R}_v - \mathbf{r_v})} \delta(\hat{R}_v - \mathbf{r_v} + \mathbf{V} t) u_v + \frac{e^{-i\omega t}}{4\pi|\hat{R} + \mathbf{V} t - \mathbf{r}|} \nabla \mathbf{r}\right]
$$

(S34)

where $\theta(\xi)$ is the unit step function, $u_v = \mathbf{V}/V$ is the electron velocity unit vector and we have introduced the decomposition $\mathbf{F} = \mathbf{F}_t + \mathbf{F}_v$ of a vector $\mathbf{F}$ into its parts transverse $(t)$ and parallel $(v)$ to $\mathbf{V}$, i.e.

$$
\mathbf{F}_t = \mathbf{F} - (u_v \cdot \mathbf{F}) u_v,
$$

$$
\mathbf{F}_v = (u_v \cdot \mathbf{F}) u_v = F_v u_v.
$$

(S35)

The second of Eqs.(S32) can also be integrated by using Eq.(S30) and Eq.(S34) and, after some tedious but straightforward algebra, we get

$$
\Xi(\hat{R}, t) = \frac{4\alpha}{c} \text{Im} \int_0^+ d\omega \int_{-\infty}^{\hat{R}_v + \mathbf{V} t} d\mathbf{r}_v \int_{-\infty}^{\hat{R}_v + \mathbf{V} t} d\mathbf{r}_v' e^{i\varphi(\mathbf{r}_v - \mathbf{r}_v')} \text{Im} \left[u_v \cdot G_\omega(\mathbf{R}_t + \mathbf{r}_v, \mathbf{R}_t) + \frac{e^{-i\omega t}}{4\pi|\hat{R} + \mathbf{V} t - \mathbf{r}|} \nabla \mathbf{r}\right] u_v +
$$

$$
+ \frac{\alpha}{\pi c} \int_0^+ d\omega \int_0^{\mathbf{V} t} d^3 \mathbf{r} \int_{-\infty}^{\hat{R}_v + \mathbf{V} t} d\mathbf{r}_v' e^{i\varphi(\mathbf{r}_v + \mathbf{V} t - \mathbf{r}_v')} \nabla \mathbf{r}_v \cdot \text{Im} \left[G_\omega(\mathbf{r}, \mathbf{R}_t + \mathbf{r}_v u_v]\mathbf{u}_v +
$$

$$
+ \frac{\alpha}{4\pi^2 c} \int_0^{+\infty} d\omega \int_0^{\mathbf{V} t} d^3 \mathbf{r} \int_{-\infty}^{\mathbf{V} t} d^3 \mathbf{r}_t \int_{-\infty}^{t} d\tau \int_0^\infty d\mathbf{r} \frac{\nabla \mathbf{r} \cdot \text{Im} [G_\omega(\mathbf{r}, \mathbf{r}_t)] \nabla \mathbf{r}_t}{|\mathbf{R} + \mathbf{V} \tau - \mathbf{r}| |\hat{R} + \mathbf{V} \tau - \mathbf{r}|},
$$

(S36)

where

$$
\alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \approx \frac{1}{137}
$$

(S37)

is the fine structure constant. By using the integral relation of Eq.(S6) it is simple proving that the last contribution in Eq.(S36) can be written as

$$
\frac{\alpha}{4\pi^2 c} \int_0^{+\infty} d\omega \int_0^{\mathbf{V} t} d^3 \mathbf{r} \int_{-\infty}^{t} d\tau \frac{\nabla \mathbf{r} \cdot \text{Im} [G_\omega(\mathbf{r}, \mathbf{r}_t)] \nabla \mathbf{r}_t}{|\mathbf{R} + \mathbf{V} \tau - \mathbf{r}| |\hat{R} + \mathbf{V} \tau - \mathbf{r}|} = \frac{\alpha c}{2} \int_0^t d\tau \int_0^\infty d\mathbf{r} \frac{1}{r^2}
$$

(S38)

which, although infinite, will be omitted since it provides an unobservable phase factor (being independent on $\hat{R}$).

By inserting Eq.(S34) and (S36) into Eq.(S28), we get the full time evolution operator $\hat{U} = e^{i\hat{W}}$ as

$$
\hat{U} = \exp \left[i\Xi(\hat{R}, t)\right] \exp \left\{i \int_0^{+\infty} d\omega \int d^3 \mathbf{r} \left[N_\omega(\hat{R}, \mathbf{r}, t) \cdot \hat{E}_\omega(\mathbf{r}) + N^*_\omega(\hat{R}, \mathbf{r}, t) \cdot \hat{E}^\dagger_\omega(\mathbf{r})\right]\right\}
$$

(S39)
and two observations are in order. First, if the medium inhomogeneity is weak, all the terms containing the gradient operator \( \nabla \) can be neglected since it operates on the electric field (or the Green’s tensor) whose divergence is vanishingly small together with the volume charge density. In this circumstance the obtained expression of \( \hat{U} \) coincides with the one deduced in literature for non magnetoelectric media (apart from the expression of the electric field in Eq. (S11) which is different due to the presence of both electric and magnetic polaritonic excitations). Second, in the limiting case \( t \to +\infty \) we are mainly concerned here, the second contributions to Eq. (S34) and Eq. (S36) vanish since they explicitly contain the Coulomb potential evaluated at the infinitely far position of the electron, and accordingly the S-operator \( \hat{S} = \hat{U}(+\infty) \) turns out to be

\[
\hat{S} = e^{i\Phi(\hat{R})} \hat{a}^{\dagger}(\hat{R}) \hat{a}(\hat{R})
\]  

(S40)

where, after relabelling \( r_e \to q \) and \( r'_e \to q' \), we have introduced the phase factor \( \Phi \) and the operator \( \hat{a} \) which, for a vector \( \mathbf{F} \), are given by

\[
\Phi(\mathbf{F}) = \frac{4\alpha}{c} \int_0^{+\infty} d\omega \int_0^{+\infty} dq \int dq' \sin \left( \frac{\omega}{\mathbf{V}} (q - q') \right) \text{Im} \left[ \mathbf{u}_e \cdot \mathbf{G}_\omega (\mathbf{F}_t + q \mathbf{u}_v, \mathbf{F}_t + q' \mathbf{u}_v) \mathbf{u}_v \right],
\]

(S41)

\[
\hat{a}(\mathbf{F}) = \int_0^{+\infty} d\omega e^{i\frac{\omega}{\mathbf{h} c}} \int dq e^{i\frac{\omega}{\mathbf{h} c} (\mathbf{F}_e - q)} \mathbf{u}_v \cdot \hat{\mathbf{E}}_\omega (\mathbf{F}_t + q \mathbf{u}_v).
\]

(S42)

Note that the phase factor \( \Phi(\mathbf{F}) = \Phi(\mathbf{F}_e) \) only depends on the transverse part \( \mathbf{F}_t \) of the vector \( \mathbf{F} \). We stress that the obtained S-operator in Eq. (S40) coincides with the one discussed in literature for non magnetoelectric media, and we have here extended its validity to chiral media in full generality.

As for the second contributions to Eq. (S34) and Eq. (S36) vanish since we are mainly concerned here, the second contributions to Eq. (S34) and Eq. (S36) vanish since

\[
\int dq e^{i\frac{\omega}{\mathbf{h} c} (\mathbf{F}_e - q)} \mathbf{u}_v \cdot \hat{\mathbf{E}}_\omega (\mathbf{F}_t + q \mathbf{u}_v).
\]

(S43)

The commutator \( \left[ \hat{a}(\hat{R}), \hat{a}^{\dagger}(\hat{R}) \right] = \Delta(\hat{R}, \hat{R}) \)

(S42)

where the have introduced the function of two vectors \( \mathbf{F} \) and \( \mathbf{F}' \)

\[
\Delta(\mathbf{F}, \mathbf{F}') = \frac{4\alpha \mathbf{e}}{c} \int_0^{+\infty} d\omega e^{-i\frac{\omega}{\mathbf{h} c} (\mathbf{F}_e - \mathbf{F}'_e)} \int dq dq' e^{i\frac{\omega}{\mathbf{h} c} (q - q')} \text{Im} \left[ \mathbf{u}_v \cdot \mathbf{G}_\omega (\mathbf{F}_t + q \mathbf{u}_v, \mathbf{F}_t' + q' \mathbf{u}_v) \mathbf{u}_v \right]
\]

which plays a fundamental role in our analysis (see below). Note that the Onsager reciprocity of the Green’s tensor (see Eq. (S7)) implies that

\[
\Delta(\mathbf{F}', \mathbf{F}) = \Delta^*(\mathbf{F}, \mathbf{F}')
\]

(S44)

and consequently \( \Delta(\mathbf{F}, \mathbf{F}) \) is a real number and \( \Delta(\hat{R}, \hat{R}) \) is an hermitian operator in agreement with the fact that, from Eq. (S42), it is the commutator of two hermitian conjugated operators. Note also that the relation

\[
\Delta(\mathbf{F}, \mathbf{F}') = \Delta(\mathbf{F}_t, (\mathbf{F}_e - \mathbf{F}'_e) \mathbf{u}_v, \mathbf{F}'_t)
\]

(S45)

holds, so that \( \Delta(\mathbf{F}, \mathbf{F}) = \Delta(\mathbf{F}_t, \mathbf{F}_t') \) only depends on \( \mathbf{F}_t \). Since both \( \hat{a}(\hat{R}) \) and \( \hat{a}^{\dagger}(\hat{R}) \) evidently commute with their commutator, we can resort to the Baker-Campbell-Hausdorff identity \( e^{\mathbf{X} + \mathbf{Y}} = e^{-\frac{1}{2}[\mathbf{X}, \mathbf{Y}]} e^{\mathbf{X}} e^{\mathbf{Y}} \) to cast Eq. (S40) in the form

\[
\hat{S} = e^{i\Phi(\hat{R}_e)} e^{-\frac{1}{2} \Delta(\hat{R}_e, \hat{R}_e)} \hat{a}^{\dagger}(\hat{R}) e^{-\hat{a}(\hat{R})}
\]

(S46)

where we have made explicit the dependence of \( \Phi \) and \( \Delta \) (with equal arguments) on the transverse component \( \hat{R}_e \) of the position operator.

\subsection*{S2.3. Electron reduced density matrix}

The S-operator obtained in the above section provides the full time evolution of the initial electron-radiation state \( |\Psi_i\rangle \) toward the final state \( |\Psi\rangle = \hat{S} |\Psi_i\rangle \). Since radiation is not excited at first, the initial state we choose is

\[
|\Psi_i\rangle = |\psi_i, 0\rangle \equiv |\psi_i \rangle \otimes |0\rangle
\]

(S47)
where $|\psi_i\rangle$ is a free electron state of energy $E_i$ [8], i.e.
\[ \mathbf{V} \cdot \hat{\mathbf{P}} |\psi_i\rangle = E_i |\psi_i\rangle \tag{S48} \]
and $|0\rangle$ is the vacuum radiation state defined by $\hat{f}_{\omega \lambda j}(\mathbf{r}) |0\rangle = 0$. Accordingly $|\Psi_i\rangle$ is an eigenstate of $\hat{H}_0$ with eigenvalue $E_i$. After applying the $S$-operator of Eq.(S46) to such initial state and using the second of Eqs.(S41) and Eq.(S11), we get
\[ |\Psi\rangle = e^{i\Phi(\mathbf{R}_i)} e^{-\frac{i}{\hbar} \Delta(\mathbf{R}_i, \mathbf{R}_i)} \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \int d\xi \ T(\mathbf{R}, \xi) \hat{f}^\dagger (\xi) \right]^n |\psi_i, 0\rangle \tag{S49} \]
where the exponential of $\hat{a}^\dagger(\mathbf{R})$ has been expanded, we have introduced the compact index $\xi = (\omega, \lambda, j, \mathbf{r})$ to label the elementary polaritonic excitation and, for notational convenience, we have set
\[ \hat{f}(\xi) = f_{\omega \lambda j}(\mathbf{r}), \quad \int d\xi = \int d\omega \sum_j \int d^3\mathbf{r}, \quad T(\mathbf{R}, \xi) = \frac{e^{i\Phi(\mathbf{R}_i)}}{\hbar \omega} \int dq \ e^{-i\omega(q) r} \mathbf{u}_v \cdot \mathbf{g}^*_{\omega \lambda j}(\mathbf{R}_i + q \mathbf{u}_v, \mathbf{r}) \mathbf{e}_j. \tag{S50} \]

In order to analyze the final state $|\Psi\rangle$, it is convenient to use the representation provided by the states
\[ |\mathbf{R}, \xi_1 \ldots \xi_n\rangle = |\mathbf{R}\rangle \otimes \left[ \frac{1}{\sqrt{n!}} \hat{f}^\dagger (\xi_1) \ldots \hat{f}^\dagger (\xi_n) |0\rangle \right] \tag{S51} \]
where $|\mathbf{R}\rangle$ is the eigenvector of the electron position operator $\hat{\mathbf{R}}$ (i.e. $\hat{R} |\mathbf{R}\rangle = \mathbf{R} |\mathbf{R}\rangle$). The state $|\mathbf{R}, \xi_1 \ldots \xi_n\rangle$ corresponds to the electron at the position $\mathbf{R}$ and the radiation with $n$ quanta distributed on the $n$ polaritonic excitations $\xi_1 \ldots \xi_n$. These states are an orthonormal basis of the tensor product of the electron Hilbert space and the radiation Fock space due to the relations
\[ \langle \mathbf{R}, \xi_1 \ldots \xi_n | \mathbf{R}', \xi'_1 \ldots \xi'_{n'} \rangle = \delta (\mathbf{R} - \mathbf{R}') \frac{\delta_{n,n'}}{n!} \sum_{\pi \in S_n} \left[ \delta \left( \xi_1 - \xi'_{\pi(1)} \right) \ldots \delta \left( \xi_n - \xi'_{\pi(n)} \right) \right], \tag{S52} \]
\[ \int d^3\mathbf{R} \sum_{n=0}^{\infty} \int d\xi_1 \ldots d\xi_n |\mathbf{R}, \xi_1 \ldots \xi_n\rangle \langle \mathbf{R}, \xi_1 \ldots \xi_n | = I, \tag{S53} \]
where the sum in the first equation spans the $n!$ permutation $\pi$ of the symmetric group $S_n$ and
\[ \delta (\xi - \xi') = \delta (\omega - \omega') \delta_{\lambda \lambda'} \delta_{jj'} \delta (\mathbf{r} - \mathbf{r'}). \tag{S53} \]

Accordingly the state $|\Psi\rangle$ of Eq.(S49) can be written as
\[ |\Psi\rangle = \int d^3\mathbf{R} \sum_{n=0}^{\infty} \int d\xi_1 \ldots d\xi_n |\mathbf{R}, \xi_1 \ldots \xi_n\rangle \langle \mathbf{R}, \xi_1 \ldots \xi_n | \Psi \rangle \tag{S54} \]
where
\[ \langle \mathbf{R}, \xi_1 \ldots \xi_n | \Psi \rangle = \psi_i(\mathbf{R}) e^{i\Phi(\mathbf{R}_i)} e^{-\frac{i}{\hbar} \Delta(\mathbf{R}_i, \mathbf{R}_i)} \frac{1}{\sqrt{n!}} T(\mathbf{R}, \xi_1) \ldots T(\mathbf{R}, \xi_n) \tag{S55} \]
and $\psi_i(\mathbf{R}) = \langle \mathbf{R} | \psi_i \rangle$ is the initial electron wavefunction. Equation (S54) clearly reveals the entanglement between electron and radiation created by their interaction produced by the chiral medium. In the present work we are mainly interested in spatial effects produced by the chiral medium on the electron state, regardless of the radiation state. Accordingly, due to the entanglement, it is necessary to describe the electron in terms of its reduced density operator $\hat{\rho}_e$ obtained by taking the partial trace of the density operator $\hat{\rho} = |\Psi\rangle \langle \Psi |$ over the radiation states. In the position representation which is here the most convenient one, we have
\[ \hat{\rho}_e = \int d^3\mathbf{R} \int d^3\mathbf{R}' \rho_e(\mathbf{R}, \mathbf{R}') |\mathbf{R}\rangle \langle \mathbf{R}'| \tag{S56} \]
where the reduced density matrix is
\[ \rho_e(R, R') = \sum_{n=0}^{\infty} \int d\xi_1 \ldots d\xi_n \langle R, \xi_1 \ldots \xi_n | \Psi \rangle \langle \Psi | R', \xi_1 \ldots \xi_n \rangle. \] (S57)

By using the projections of Eq.(S55) we straightforwardly get
\[ \rho_e(R, R') = \psi_i(R) \psi_i^*(R') e^{i[\Phi(R_i) - \Phi(R'_i)]} e^{-\frac{1}{2} [\Delta(R_i, R_i) + \Delta(R'_i, R'_i)]} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \int d\xi T(R, \xi) T^*(R', \xi) \right)^n \] (S58)
which can be casted as
\[ \rho_e(R, R') = \psi_i(R) \psi_i^*(R') e^{i[\Phi(R_i) - \Phi(R'_i)]} e^{-\frac{1}{2} [\Delta(R_i, R_i) + \Delta(R'_i, R'_i)]} e^{\Delta(R, R')} \] (S59)
where we have summed the power series and we have used the integral
\[ \int d\xi T(R, \xi) T^*(R', \xi) = \Delta(R, R') \] (S60)
which is a simple consequence of Eqs.(S50) and (S15). Since \( \psi_i(R) \psi_i^*(R') \) is the density matrix of the fully coherent initial electron state, it is convenient writing Eq.(S59) as
\[ \rho_e(R, R') = \psi_i(R) \psi_i^*(R') \gamma(R, R') \] (S61)
where
\[ \gamma(R, R') = e^{i[\Phi(R_i) - \Phi(R'_i)]} e^{-\frac{1}{2} [\Delta(R_i, R_i) + \Delta(R'_i, R'_i)]} e^{\Delta(R_i, R'_i) u_i, R'_i) \} \] (S62)
quantifies the decoherence of the electron produced by its interaction with the chiral medium. Note that we have used Eq.(S45) to make explicit the dependence of \( \Delta(R, R') \) on the difference of \( R_i \) and \( R'_i \) which is also true for \( \gamma_i \), i.e.
\[ \gamma(R, R') = \gamma(R_i + (R_i - R'_i) u_i, R'_i). \] (S63)
Note that \( \gamma(R, R) = 1 \) so that \( \rho_e(R, R) = |\psi_i(R)|^2 \), in agreement with the normalization condition
\[ \text{Tr}(\hat{\rho}_e) = 1. \] (S64)

**S2.4. Electron transverse momentum and energy distributions**

The reduced density matrix derived in the above section fully describes the state of the electron after its interaction with the chiral film. We here extract from it the electron energy and transverse momentum distribution which is routinely measured in standard electron microscopes (momentum resolved energy-loss spectroscopy). We start by considering the electron states \( |P_t, E \rangle \) of definite transverse momentum \( P_t \) and energy \( E \), whose wavefunctions are
\[ \langle R | P_t, E \rangle = \frac{1}{2\pi \hbar \ell} e^{\frac{i}{\pi \hbar} P_t R_t} e^{i \frac{P_t}{2\pi \hbar} R_t} \] (S65)
where \( \ell \) is the longitudinal quantization length and \( E = \frac{hV}{\ell} - 2\pi m \), with \( m \) integer. These states are an orthonormal basis since
\[ \int d^2P_t \sum_E |P_t, E \rangle \langle P_t, E| = I, \]
\[ \langle P_t, E | P'_t, E' \rangle = \delta(P_t - P'_t) \delta_{E,E'}, \] (S66)
and the continuum limit is obtained at the end of the calculations by letting \( \ell \to +\infty \) and by using the relation
\[ \sum_E \to \frac{\ell}{2\pi \hbar V} \int dE. \] (S67)
By inserting the completeness relation (first on Eqs.(S66)) into the reduced density matrix normalization condition (Eq.(S64)) we get

\[ 1 = \int d^2 P_t \sum_{E} \frac{1}{(2\pi\hbar)^2} \int d^3 R' \int d^3 R e^{-\frac{i}{\hbar} P_t \cdot (R_t - R')} e^{-i \frac{E-E_t}{\hbar} (R_t - R')} \rho_e (R, R'). \]  

(S68)

Since the impinging electron has energy \( E_i \), we set for its initial wavefunction

\[ \psi_i (R) = \frac{1}{\sqrt{\ell}} \phi_i (R_t) e^{i \frac{E}{\hbar} R_t}, \]

(S69)

where \( \phi_i (R_t) \) is an arbitrary transverse profile normalized as \( \int d^2 R_t |\phi_i (R_t)|^2 = 1 \) and accordingly, by using Eq.(S61) and Eq.(S69), Eq.(S68) can be written as

\[ 1 = \int d^2 P_t \sum_{E} \frac{1}{(2\pi\hbar)^2} \int d^2 R_t \int d^2 R'_t e^{-\frac{i}{\hbar} P_t \cdot (R_t - R'_t)} \phi_i (R_t) \phi_i^* (R'_t) \times \]

\[ \times \sum_{E} \frac{1}{(2\pi\hbar)^3} \int dR_v \int dR_v' e^{-i \frac{E-E_t}{\hbar} (R_t - R_v)} \gamma (R_t + (R_v - R'_v) u_v, R'_t). \]  

(S70)

After introducing the variable \( Q = R_v - R_v' \) and noting that the relation

\[ \int_{-\ell/2}^{\ell/2} dR_v \int_{-\ell/2}^{\ell/2} dR_v' F (R_v - R_v') = \int_{-\ell}^{\ell} dQ (|Q|) F (Q), \]

(S71)

holds for any function \( F (R_v - R_v') \), the continuum limit of Eq.(S70) can easily be evaluated with the help of Eq.(S67) and it is

\[ 1 = \int d^2 P_t \int dE \frac{d\mathcal{P}}{d^2 P_t dE} \]  

(S72)

where

\[ \frac{d\mathcal{P}}{d^2 P_t dE} = \frac{1}{(2\pi\hbar)^3 \ell^2} \int d^2 R_t \int d^2 R'_t \int dQ e^{-\frac{i}{\hbar} P_t \cdot (R_t - R'_t)} e^{-i \frac{E-E_t}{\hbar} Q} \phi_i (R_t) \phi_i^* (R'_t) \gamma (R_t + Q u_v, R'_t) \]  

(S73)

is the transverse momentum and energy probability distribution of the electron after the interaction with the chiral film. By integrating this expression over \( E \) and over \( P_t \), we respectively get the transverse momentum and energy probability distributions

\[ \frac{d\mathcal{P}}{d^2 P_t} = \frac{1}{(2\pi\hbar)^2} \int d^2 R_t \int d^2 R'_t e^{-\frac{i}{\hbar} P_t \cdot (R_t - R'_t)} \phi_i (R_t) \phi_i^* (R'_t) \gamma (R_t, R'_t), \]

\[ \frac{d\mathcal{P}}{dE} = \frac{1}{2\pi\hbar \ell} \int d^2 R_t \int dQ e^{-i \frac{E-E_t}{\hbar} Q} |\phi_i (R_t)|^2 \gamma (R_t + Q u_v, R_t). \]  

(S74)

It is crucial here evaluating the mean value and variance of both transverse momentum and energy. We start by decomposing transverse position and momentum into their orthogonal components, i.e. \( R_t = R_1 u_1 + R_2 u_2 \) and \( P_t = P_1 u_1 + P_2 u_2 \), where \( u_1 \) and \( u_2 \) are orthogonal unit vectors spanning the transverse plane and we consider the \( n \)-th order moments

\[ \langle P^n_{j} \rangle = \int d^2 P_t \frac{d\mathcal{P}}{d^2 P_t} P^n_{j}, \]

\[ \langle E^n \rangle = \int dE \frac{d\mathcal{P}}{dE} E^n. \]

(S75)

By using the distributions of Eqs.(S74) we get

\[ \langle P^n_{j} \rangle = \int d^2 R_t \int d^2 R'_t \left[ \left( -\frac{\hbar}{i} \frac{\partial}{\partial R_j} \right)^n \delta (R_t - R'_t) \right] \phi_i (R_t) \phi_i^* (R'_t) \gamma (R_t, R'_t), \]

\[ \langle E^n \rangle = \int d^2 R_t \int dQ \left[ \left( -V \frac{\hbar}{i} \frac{\partial}{\partial Q} \right)^n \delta (Q) \right] e^{i \frac{E}{\hbar} Q} |\phi_i (R_t)|^2 \gamma (R_t + Q u_v, R_t). \]

(S76)
which can be written, after integrating by parts $n$-times and performing the integration over $R'_t$ and $Q$ with the help of the delta functions, as

$$\langle P^n_j \rangle = \int d^2 R_t |\phi_t|^2 \{ \hat{P}^n_j [\phi_t (R_t) \gamma (R_t, R'_t)] \}_{R'_t = R_t},$$

$$\langle E^n \rangle = \int d^2 R_t |\phi_t|^2 \{ \hat{E}^n \left[ e^{\frac{i}{\hbar} Q_t Q} (R_t + Q u, R_t) \right] \}_{Q=0},$$

where we have introduced the transverse momentum $\hat{P}_j = \frac{\hbar}{i} \frac{\partial}{\partial R'_t}$ and energy $\hat{E} = V \frac{\hbar}{i} \frac{\partial}{\partial Q}$ operators. For $n = 1$ these relations readily provide the transverse momentum and energy mean values

$$\langle P_j \rangle = \int d^2 R_t |\phi_t|^2 \hat{P}_j \phi_t + \int d^2 R_t |\phi_t|^2 \left( \hat{P}_j \Theta \right)_{Q=0 R'_t = R_t},$$

$$\langle E \rangle = E_i + \int d^2 R_t |\phi_t|^2 \left( \hat{E} \Theta \right)_{Q=0 R'_t = R_t},$$

where

$$\Theta (R_t + Q u, R'_t) = \log \gamma (R_t + Q u, R'_t).$$

Since $\int d^2 R_t |\phi_t|^2 \hat{P}_j \phi_t$ is the transverse momentum mean value of the initial electron state, Eqs. (S78) remarkably show that both expectation values are the sum of the initial contribution plus a term resulting from the interaction with the chiral medium which is the average on the initial electron state of suitable derivatives of $\Phi$ and $\Delta$. Analogously, Eqs.(S77) for $n = 2$ enable the evaluation of the variances which, after some algebra, turn out to be

$$\langle P^2_j \rangle - \langle P_j \rangle^2 = \left[ \int d^2 R_t |\phi_t|^2 \hat{P}_j^2 \phi_t - \left( \int d^2 R_t |\phi_t|^2 \hat{P}_j \phi_t \right)^2 \right] +$$

$$+ 2 \left\{ \int d^2 R_t |\phi_t|^2 \left( \hat{P}_j \phi_t \right) \left( \hat{P}_j \Theta \right)_{Q=0 R'_t = R_t} - \left( \int d^2 R_t |\phi_t|^2 \hat{P}_j \phi_t \right) \left[ \int d^2 R_t |\phi_t|^2 \left( \hat{P}_j \Theta \right)_{Q=0 R'_t = R_t} \right] \right\} +$$

$$+ \left\{ \int d^2 R_t |\phi_t|^2 \left[ \left( \hat{P}_j \Theta \right)^2 + \hat{P}_j^2 \Theta \right]_{Q=0 R'_t = R_t} - \left[ \int d^2 R_t |\phi_t|^2 \left( \hat{P}_j \Theta \right)_{Q=0 R'_t = R_t} \right]^2 \right\},$$

$$\langle E^2 \rangle - \langle E \rangle^2 = \int d^2 R_t |\phi_t|^2 \left[ \left( \hat{E} \Theta \right)^2 + \hat{E}^2 \Theta \right]_{Q=0 R'_t = R_t} - \left[ \int d^2 R_t |\phi_t|^2 \left( \hat{E} \Theta \right)_{Q=0 R'_t = R_t} \right]^2 .$$

From the first of Eqs.(S80), the transverse momentum variance is the sum of the transverse momentum variance of the initial electron state (first term in square brackets) plus two contributions produced by the interaction of the electron with the chiral medium (second and third term in curly brackets). On the other hand, from the second of Eqs.(S80), the energy variance has no initial electron contribution as expected since the impinging electron has definite energy $E_i$.

**S2.5. Mirror asymmetry of electron decoherence and lateral momentum transfer**

The obtained expressions of the reduced density matrix, transverse momentum and energy distributions are rather general with regard to their dependence on the Green’s tensor, they formally holding also for achiral media. On the other hand, medium chirality has impact on the Green’s tensor behavior under spatial reflections, a topic we here discuss as a prelude to the investigation of the mirror symmetry breaking of the electron quantum properties. To this end we consider the geometric spatial reflection $r^M = R r$ through an arbitrary mirror plane $\pi$ where $R = I - 2 n n$ is the reflection tensor and $n$ is the unit vector normal to $\pi$. In order to avoid geometrical mirror asymmetries, we hereafter focus on a geometrically mirror symmetric medium for which

$$\varepsilon (R r, \omega) = \varepsilon (r, \omega),$$

$$\mu (R r, \omega) = \mu (r, \omega),$$

$$\kappa (R r, \omega) = \kappa (r, \omega).$$
Since the electric field and the current densities are polar vectors, their mirror images are $E^M_\omega (r) = \mathcal{R}E_\omega (\mathcal{R}r)$ and $j^M_\omega (r) = \mathcal{R}j_\omega (\mathcal{R}r)$ so that Eq.(S2) implies that the mirror image of the Green’s tensor is

$$G^M_\omega (r, r') = \mathcal{R}G_\omega (\mathcal{R}r, \mathcal{R}r').$$

(S82)

Now it is simple showing that $G^M_\omega (r, r')$ satisfies the equation

$$\left\{ \nabla \times \frac{1}{\mu} \nabla \times - \frac{\omega^2}{c^2} \left( \varepsilon - \frac{\kappa^2}{\mu} \right) + \frac{\omega}{c} \left[ \left( \nabla - \frac{\kappa}{\mu} \right) \times + 2 \frac{\kappa}{\mu} \nabla \times \right] \right\} G^M_\omega (r, r') = \delta (r - r') I$$

(S83)

which is identical to Eq.(S3) except for the sign switch of the chirality parameter $\kappa$. In other words, $G^M_\omega (r, r')$ is the Green’s tensor of the opposite enantiomeric medium which is the mirror image of the original one (since microscopically each chiral molecule is reflected into its opposite enantiomer). This observation is a mere restatement of reflection invariance (or mirror symmetry) of classical electrodynamics which states that if a complete experiment (i.e. field, sources and matter) is subjected to mirror reflection, the resulting experiment should, in principle, be realizable.

What is remarkable here is that reflection invariance additionally entails the mirror asymmetry of the Green’s tensor of a chiral medium, i.e. $G^M_\omega (r, r') \neq G_\omega (r, r')$. To prove this we note that the assumption $G^M_\omega (r, r') = G_\omega (r, r')$ provides the equations

$$\nabla \times \left[ \frac{1}{\mu} \nabla \times G_\omega (r, r') \right] = \frac{\omega^2}{c^2} \left( \varepsilon - \frac{\kappa^2}{\mu} \right) G_\omega (r, r') + \delta (r - r') I,$$

$$\nabla \times \left[ \frac{1}{\mu} \nabla \times \mathcal{R}G_\omega (\mathcal{R}r, \mathcal{R}r') \right] = \nabla \times \left[ \left( \frac{1}{\mu} \nabla \log \sqrt{\frac{\mu}{\kappa}} \right) \times G_\omega (r, r') \right].$$

(S84)

where the first of Eqs.(S84) is obtained by taking the sum of Eqs.(S3) and (S83) and the second is deduced by taking the difference of Eqs.(S3) and (S83), solving the resultant equation for $\nabla \times G_\omega (r, r')$ and subsequently applying to it the operator $\nabla \times (\mu^{-1} \bullet)$. Now Eqs.(S84) are manifestly incompatible since the right hand side of the second one, as opposed to the first one, explicitly depends on $\nabla \mu$, $\nabla \kappa$ and the first order spatial derivatives of $G_\omega (r, r')$ and in addition it does not contain a delta function. We conclude that $G_\omega (r, r')$ and $G^M_\omega (r, r')$ can not coincide, i.e.

$$\mathcal{R}G_\omega (\mathcal{R}r, \mathcal{R}r') \neq G_\omega (r, r')$$

(S85)

which states the mirror asymmetry of the Green’s tensor of a chiral sample. Evidently in an achiral medium where $\kappa = 0$, Eqs.(S3) and (S83) coincide so that, by uniqueness of solutions, $G^M_\omega (r, r') = G_\omega (r, r')$, i.e. the Green’s tensor is mirror symmetric.
Mirror symmetry breaking of the electron quantum properties is a consequence of the Green’s tensor mirror asymmetry. In order to discuss such effects, with reference to Fig.(S1), we here consider a chiral medium displaying a reflection symmetry plane $\pi$, the reflection $R$ through $\pi$ (so that Eqs.(S81) are satisfied) and an electron whose velocity $V$ lies on $\pi$ and whose initial wave function is left invariant by $R$, i.e.

$$\phi_i (R R_i) = \phi_i (R_i) .$$  \hspace{1cm} (S86)

The considered reflection $R$, when acting on a vector $F$, does not affect its part parallel to the electron velocity, i.e. $R F_v = F_v$. In addition, since the transverse part $F_t$ lies in the plane orthogonal to $\pi$, it can be decomposed as $F_t = F_x + F_n$ where

$$F_x = \frac{1}{2} (F_t + R F_t) ,$$

$$F_n = \frac{1}{2} (F_t - R F_t) = \frac{1}{2} (F - R F) ,$$  \hspace{1cm} (S87)

are its parts parallel ($\pi$) and orthogonal ($n$) to the reflection plane $\pi$ (see Fig.(S1)). We refer to $F_n$ as the lateral part of the vector $F$. The chosen configuration is geometrically mirror symmetric and the occurrence of any mirror symmetry breaking is accordingly of pure physical origin and due to the medium chirality. Since $R u_v = u_v$, Eq.(S85) readily yields

$$u_v \cdot G_\omega (R r, R r') u_v \neq u_v \cdot G_\omega (r, r') u_v ,$$  \hspace{1cm} (S88)

which combined with the first of Eqs.(S41) and Eq.(S43) directly implies that

$$\Phi (R R_t) \neq \Phi (R_t) ,$$

$$\Delta (R R_t, R R') \neq \Delta (R_t, R') ,$$  \hspace{1cm} (S89)

so that, from Eq.(S62), we eventually get

$$\gamma (R R_t, R R') \neq \gamma (R_t, R') .$$  \hspace{1cm} (S90)

This proves that the mirror symmetry of the electron quantum decoherence is broken by medium chirality. Besides, from the first of Eqs.(S74) we get

$$\frac{d \mathcal{P}}{d^2 P_t} \bigg|_{R P_t} - \frac{d \mathcal{P}}{d^2 P_t} \bigg|_{P_t} = \frac{1}{(2 \pi \hbar)^2} \int d^2 R_t \int d^2 R'_t e^{-i P_t \cdot (R_t - R'_t)} \phi_i (R_t) \phi^*_i (R'_t) \left[ \gamma (R R_t, R R'_t) - \gamma (R_t, R'_t) \right]$$  \hspace{1cm} (S91)

which, due to the mirror asymmetry of the electron quantum decoherence (see Eq.(S90)), remarkably reveals that the transverse momentum distribution is not left invariant by the reflection $P_t \rightarrow R P_t$, i.e. it is mirror asymmetric.

One of the most relevant consequences of this fact is that $\langle P_n \rangle_i$, the mean value of the lateral momentum, can not be vanishing in spite of the fact that the impinging electron has not such a momentum contribution since

$$\langle P_n \rangle_i = \int d^2 R_t \phi_i^* \left( \frac{\hbar}{i} \nabla_{R_t} \right) \phi_i + \int d^2 R_t |\phi_i|^2 \left( \frac{\hbar}{i} \nabla_{R_t} \right) \log \left[ \frac{\gamma (R_t, R'_t)}{\gamma (R R_t, R R'_t)} \right] \bigg|_{R'_t = R_t} ,$$

$$\langle P_n \rangle = \int d^2 R_t |\phi_i|^2 \left( \frac{\hbar}{i} \nabla_{R_t} \right) \log \left[ \frac{\gamma (R_t, R'_t)}{\gamma (R R_t, R R'_t)} \right] \bigg|_{R'_t = R_t} .$$  \hspace{1cm} (S92)

The first of these equations states that the contribution to $\langle P_n \rangle$ that the electron gets as a result of its interaction with the medium (the second term in the right hand side) is generally non-vanishing and it is independent on the medium chirality since it survives in the achiral case where $\gamma (R R_t, R R'_t) = \gamma (R_t, R'_t)$. On the other hand, the second of Eqs.(S92) dramatically states that $\langle P_n \rangle$ is here solely provided by the electron-medium interaction and that it does not vanish only if the quantum electron decoherence is mirror asymmetric $\gamma (R R_t, R R'_t) \neq \gamma (R_t, R'_t)$, that is to say if the medium is chiral.

The non-vanishing lateral momentum $\langle P_n \rangle$ transferred to the electron is related to a mechanical interaction that the chiral medium is able to exert on the electron along the direction normal to the reflection symmetry plane $\pi$. In
spite of this lateral interaction, in our description the mean value of the electron transverse position \( \langle R_t \rangle = \text{Tr}(\hat{\rho}_e \hat{R}_t) \) does not change upon scattering since, from Eq.(S61), \( \rho_e \langle R, R \rangle = |\psi_i(R)|^2 \) and hence

\[
\langle R_t \rangle = \int d^3R |\phi_i(R)|^2 R_t
\]  

(S93)

which is the mean value of the transverse position of the initial electron wavefunction. This is due to the paraxial approximation adopted to describe the electron where the electron Hamiltonian is \( \mathbf{V} \cdot \hat{\mathbf{P}} = \mathbf{V}\hat{P}_t \) (see Eq.(S21)) which evidently commutes with the transverse position operator \( \hat{R}_t \). Accordingly \( \hat{R}_t \) commutes with the full Hamiltonian \( \hat{H} \) of Eq.(S21) and its mean value can not vary due the Ehrenfest theorem

\[
\frac{d}{dt} \langle R_t \rangle = \frac{1}{i\hbar} \langle \Psi(t)|[\hat{R}_t, \hat{H}]|\Psi(t)\rangle.
\]  

(S94)

### S2.6. Weak coupling regime

The reduced density matrix in Eq.(S61) and the transverse momentum distribution in Eq.(S74) have been rigorously evaluated by using S-operator of Eq.(S46) which in turn has been obtained by solving the time evolution equation in Eq.(S25) with no approximation. As a consequence our treatment is non-perturbative and it enables the investigation of the electron-chiral medium interaction beyond the weak coupling regime. For example, this can occur by letting the electron to fly near a large chiral sample providing a long interaction length (see below). This possibility is very important for our purposes since it could enhance the mirror symmetry breaking effects discussed in Sec.S2.5. In fact such asymmetry effects are a physical consequence of medium chirality which is generally very weak (the Pasteur parameter in Eq.(S1) is typically \( |\kappa| \sim 10^{-4} \)) and accordingly the asymmetry is expected to be very tiny in the weak coupling regime.

To discuss the weak coupling regime, we start by examining the full electron-radiation state of Eq.(S49) which turns out to be the superposition of an infinite number of states, the nth of which having precisely \( n \) elementary polaritonic excitations and being an eigenstate of eigenvalue \( E_i \) of the free electron-radiation Hamiltonian \( \hat{H}_0 \) due to the relation

\[
\left[ \hat{H}_0, \int d\xi T(\hat{R}, \xi) \hat{f}^\dagger(\xi) \right] = 0,
\]  

(S95)

which can easily be obtained from the first of Eqs.(S23) and the third of Eqs.(S50) with the help of the basic electron position-momentum commutation relations and the polaritonic commutation relations of Eqs.(S8). As a consequence the term with \( n = 0 \) is basically a renormalization of the bare electron initial state whereas the nth term describes a process where the electron loses energy due to the generation of \( n \) polaritonic excitations. The weak coupling regime occurs when the only relevant process is the generation of a single polaritonic excitation and accordingly all the terms with \( n > 1 \) can be neglected, a situation described by the standard first order perturbation theory. Now Eq.(S58) reveals that the reduced density matrix gets a contribution from each of the \( n \)-polaritonic excitation process and that higher order terms can be neglected if

\[
|\Delta \langle R, R' \rangle| \ll 1.
\]  

(S96)

which accordingly is the condition characterizing the weak coupling regime. This condition is also accompanied by the fact that \( |\Phi(R)| \ll 1 \) and hence the first order expansion of the reduced density matrix reads

\[
\rho_e \langle R, R' \rangle \simeq \psi_i(R) \psi_i^* (R') [1 + \Delta \langle R, R' \rangle] =
\]

\[
= \psi_i(R) \psi_i^* (R') \left\{ 1 + \frac{4\alpha}{c} \int_0^\infty dq \int dq' e^{-i\Phi(R_q - R_{q'})} \int d\xi \int d\xi' e^{i\Phi(\xi - \xi')} \text{Im} [u_v \cdot G_{\omega} (R_q + q u_v, R_{q'} + q' u_v)] \right\},
\]  

(S97)

which formally coincides with the results of Ref.[9] where the authors consider the interaction of fast electrons with non magnetoelectric media. Analogously, the first order expansion of the electron energy distribution in the second of Eqs.(S74) is

\[
\frac{d\mathcal{P}}{dE} \simeq \delta(E - E_i) + \Gamma \left( \frac{E_i - E}{\hbar} \right)
\]  

(S98)
where
\[
\Gamma(\omega) = \frac{4\alpha}{\hbar c} \int d^2R_t |\phi_i(R_t)|^2 \int_0^\infty dq \int_0^\infty dq' e^{i\hat{\omega}(q-q')} \text{Im} [u_y \cdot G_{\omega}(R_t + qu_y, R_t + qu'_y)u_y]
\]  
(S99)
is the well-known expression of the electron energy loss probability pertaining non magnetoelectric media [8].

S3. ALOOF ELECTRON TRAVELING PARALLEL TO A CHIRAL FILM

In order to specialize the above general results we hereafter focus on an aloof electron traveling parallel to a chiral film. With reference to left side of Fig.S2, the homogenous chiral film of thickness \( d \), permittivity \( \varepsilon \), permeability \( \mu \approx 1 \) and Pasteur parameter \( \kappa \) is in vacuum \( (\varepsilon_1 = 1) \) and it lies onto the substrate of permittivity \( \varepsilon_2 \). The medium (substrate + chiral film + vacuum) has the mirror symmetry plane \( \pi = \{ z \} \) and the reflection through such plane is provided by the tensor
\[
R = e_x e_x - e_y e_y + e_z e_z.
\]  
(S100)
The electron has initial mirror symmetric wavefunction \( \psi_i(\mathbf{R}) = \psi_i(\mathbf{R}) \) and its velocity is along the \( x \) axis, so that \( u_y = e_x \). Such configuration is of the kind used in Sec.2.5. to discuss the mirror symmetry breaking effects and the decomposition of a vector \( \mathbf{F} \) described in Fig.(S1) here becomes
\[
\mathbf{F}_v = F_x e_x,
\]
\[
\mathbf{F}_\pi = F_z e_z,
\]
\[
\mathbf{F}_n = F_y e_y,
\]  
(S101)
so that asymmetric effects show up along the \( y \)-axis which is the lateral direction. In the chosen aloof configuration the initial electron wavefunction \( \phi_i(\mathbf{R}) \) is non-vanishing only in vacuum \( (Z < 0) \) so that, from Eq.(S61), the electron reduced density matrix does not vanish only in vacuum as well. Accordingly we will evaluate the Green’s tensor component \( e_x \cdot G_x(\mathbf{R}, \mathbf{R}')e_x \) and the fundamental quantities \( \Phi(\mathbf{R}) \) and \( \Delta(\mathbf{R}, \mathbf{R}') \) only for \( z < 0, \ z' < 0, \ Z < 0 \) and \( Z' < 0 \). For the sake of clarity, in the right side of Fig.S2 we have summarized the quantities used in the below analysis.

S3.1. Green’s tensor component along the electron velocity

In order to evaluate the Green’s tensor component \( e_x \cdot G_x(\mathbf{R}, \mathbf{R}')e_x \) in the vacuum half-space we start noting that an elementary current density \( J_\omega(\mathbf{r}) = J_0 \delta (\mathbf{r} - \mathbf{r}')e_x \) located in vacuum \( (Z' < 0) \) produces in \( z < 0 \) the field
\[
\mathbf{E}_\omega(\mathbf{r}) = (i\omega \mu_0 J_\omega) G_x(\mathbf{r}, \mathbf{r'}) e_x = \mathbf{E}_\omega^{(0)}(\mathbf{r}) + \mathbf{E}_\omega^{(r)}(\mathbf{r})
\]  
(S102)
where \( \mathbf{E}_\omega^{(0)} \) is the free-space field produced by the elementary current
\[
\mathbf{E}_\omega^{(0)} = (i\omega \mu_0 J_\omega) \left( 1 + \frac{\nabla \nabla \cdot}{k_\omega^2 \varepsilon_1} \frac{e^{ik_\omega \sqrt{\varepsilon_1} |\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} e_x \right)
\]  
(S103)
and \( \mathbf{E}_\omega^{(r)} \) is the field reflected by the chiral film interface (see left side of Fig.S2). To evaluate the reflected field it is convenient to cast the free-space field as
\[
\mathbf{E}_\omega^{(0)} = \left( i\omega \mu_0 J_\omega \right) \frac{i}{8\pi^2} \int d^2k|| e^{ik|| (r_1-r'_1)} \frac{e^{ik_1 z - z'}}{k_{1z}} \left\{ - \frac{k_y}{k_{1z}} \mathbf{u}_S + \left( \frac{k_{1z}^2 k_x}{k_y || k_{1z}^2 ||} \right) \left[ \mathbf{u}_p - \frac{k_y}{k_{1z}} \text{sgn} (z-z') e_z \right] \right\}
\]  
(S104)
where we have used the Weyl representation of the spherical wave
\[
\frac{e^{ik_\omega \sqrt{\varepsilon_1} |\mathbf{r} - \mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} = \frac{i}{2\pi} \int d^2k|| e^{ik|| (r_2-r'_2)} \frac{e^{ik_2 z - z'}}{k_{2z}}.
\]  
(S105)
Now in Ref.[10] it has been shown that an incident field
$$\varepsilon_1 = 1$$

$$\phi_i(Y, Z)$$

vacuum

$$E_0^{(i)}(r) = (i\omega_0 \varepsilon_{0}) E_0^{(i)}(r, r') e_z$$

$$E_0^{(f)}(r) = (i\omega_0 \varepsilon_{0}) E_0^{(f)}(r, r') e_z$$

$$\begin{align*}
\varepsilon, \kappa & \quad d \\
& \quad \text{chiral film}
\end{align*}$$

substrate

$$\begin{align*}
\varepsilon_2 & \\
& \quad \text{chiral film}
\end{align*}$$

FIG. S2. **Left:** Geometry of the interaction between the aloof electron traveling in vacuum parallel to the x axis nearby the chiral film deposited onto the substrate. The red spot displays the initial electron wavefunction transverse profile $$\phi_i(Y, Z)$$ whereas the green arrows sketch the vacuum and reflected fields, $$E_0^{(i)}$$ and $$E_0^{(f)}$$, used to evaluate the Green’s tensor component along the electron velocity. **Right:** List of quantities used in the evaluation.

$$E_\omega^{(i)} = \int d^2k || e^{ik_1 \cdot r_1} e^{ik_2 z} \left[ U_S^{(i)} u_S + U_P^{(i)} \left( u_P - \frac{k_{\parallel}}{k_{1z}} e_z \right) \right]$$

(S106)

Impinging onto the chiral film/substrate composite produces in $$z < 0$$ the reflected field

$$E_\omega^{(r)} = \int d^2k || e^{ik_1 \cdot r_1} e^{-ik_2 z} \left[ \left( R_{SS} U_S^{(i)} + n R_{SP} U_P^{(i)} \right) u_S + \left( n R_{PS} U_S^{(i)} + R_{PP} U_P^{(i)} \right) \left( u_P + \frac{k_{\parallel}}{k_{1z}} e_z \right) \right]$$

(S107)

in which the reflection matrix is given by

$$\begin{pmatrix}
R_{SS} & n R_{SP} \\
n R_{PS} & R_{PP}
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \left( M^{(1)} - M^{(2)} \right) \left( M^{(1)} + M^{(2)} \right)^{-1}$$

(S108)

where

$$M^{(1)} = \begin{pmatrix}
C_+ + \frac{k_\omega}{k_{\parallel}} S_+^{(1)} & n \left( \frac{k_\omega}{k_{\parallel}} \varepsilon_{k_{\parallel} z} C_+ - S_+^{(1)} \right) \\
n \frac{k_{\parallel} k_{1z}}{k_\omega} \varepsilon_{k_{\parallel} z} C_+ & C_+ + \frac{k_\omega}{k_{\parallel}} S_+^{(1)}
\end{pmatrix} \left( C_+ + \frac{k_\omega}{k_{\parallel}} S_+^{(1)} \right),$$

$$M^{(2)} = \begin{pmatrix}
\frac{k_\omega}{k_{1z}} \varepsilon_{k_{\parallel} z} C_+ + S_+^{(2)} & n \frac{k_{\parallel} k_{1z}}{k_\omega} \varepsilon_{k_{\parallel} z} C_+ + S_+^{(2)} \\
n \frac{1}{\varepsilon_{k_{\parallel} z}} \varepsilon_{k_{\parallel} z} C_+ - S_+^{(2)} & \left( C_+ + \frac{k_\omega}{k_{\parallel}} S_+^{(2)} \right)
\end{pmatrix} \left( C_+ + \frac{k_\omega}{k_{\parallel}} S_+^{(2)} \right),$$

(S109)

and

$$C_\pm = \frac{1}{2} \left[ \cos (K_{z+} d) \pm \cos (K_{z-} d) \right],$$

$$S^{(1)}_\pm = \frac{1}{2} \frac{k_\omega (n + \kappa)}{K_{z \pm} n} \sin (K_{z+} d) \pm \frac{k_\omega (n - \kappa)}{K_{z -} n} \sin (K_{z-} d),$$

$$S^{(2)}_\pm = \frac{1}{2} \left[ \frac{K_{z \pm} n}{k_\omega (n + \kappa)} \sin (K_{z+} d) \pm \frac{K_{z -} n}{k_\omega (n - \kappa)} \sin (K_{z-} d) \right].$$

(S110)

It is worth noting that the reflection coefficients $$R_{13}$$ defined in Eq. (S108) are left invariant by chirality reversal $$\kappa \rightarrow -\kappa$$ as can easily be proved with help of the matrices $$M^{(i)}$$ of Eqs. (S109). The only term affected by chirality reversal is the chiral refractive index $$n = \sqrt{\varepsilon \kappa^2} / \kappa$$ whose sign is switched by the reversal.
Since along the stripe $z' < z < 0$ the free-space field in Eq.(S104) has the same structure of the field in Eq.(S106) with S and P amplitudes

$$ U_S^{(i)} = (i\omega\mu_0 J_\omega) \frac{i}{8\pi^2} e^{-ik'_i r'_i} e^{-ik_{1z}z'} \left( -\frac{k_y}{k_\parallel} \right), $$
$$ U_P^{(i)} = (i\omega\mu_0 J_\omega) \frac{i}{8\pi^2} e^{-ik'_i r'_i} e^{-ik_{1z}z'} \left( \frac{k_1^2 k_x}{k_\parallel k_{1z}^2 \varepsilon_1} \right), $$

the reflected field is readily given by Eq.(S107) and it is

$$ E^{(r)} = (i\omega\mu_0 J_\omega) \frac{i}{8\pi^2} \int d^2 k_i e^{ik_i (r_i - r'_i)} e^{-ik_{1z} (z + z')} \left[ \left( -R_{SS} \frac{k_y}{k_\parallel} + n R_{SP} \frac{k_1^2 k_x}{k_\parallel k_{1z}^2 \varepsilon_1} \right) u_S + \right. $$
$$ + \left. \left( -n R_{PS} \frac{k_y}{k_\parallel} + R_{PP} \frac{k_1^2 k_x}{k_\parallel k_{1z}^2 \varepsilon_1} \right) \left( u_P + \frac{k_\parallel}{k_{1z}} e_z \right) \right]. $$

(S112)

Projecting the field in Eq.(S102) onto the $x$ axis and using Eqs.(S104) and (S112) we get the $xx$ component $G_{ωxx}(r, r') = G_{ωxx}^{(0)}(r, r') + G_{ωxx}^{(r)}(r, r')$

(S113)

where

$$ G_{ωxx}^{(0)}(r, r') = \frac{i}{8\pi^2} \int d^2 k_i e^{ik_i (r_i - r'_i)} \frac{e^{ik_{1z} (z + z')}}{k_{1z}} \left( 1 - \frac{k_1^2}{k_\parallel k_{1z}^2 \varepsilon_1} \right), $$

$$ G_{ωxx}^{(r)}(r, r') = \frac{i}{8\pi^2} \int d^2 k_i e^{ik_i (r_i - r'_i)} \frac{e^{ik_{1z} (z + z')}}{k_{1z}} \left[ \left( -R_{SS} \frac{k_1^2}{k_\parallel^2} + R_{PP} \frac{k_1^2 k_x^2}{k_\parallel k_{1z}^2 \varepsilon_1} \right) - n \left( -R_{SP} \frac{k_1^2}{k_\parallel^2 \varepsilon_1} + R_{PS} \frac{k_x k_y}{k_{1z}^2} \right) \right]. $$

(S114)

are the $xx$ components of the free-space and reflected parts of the Green’s tensor, $G_{ωω}^{(0)}(r, r')$ and $G_{ωω}^{(r)}(r, r')$.

### S3.2. Mirror asymmetry of the Green’s tensor as due to mirror optical activity

Green’s tensor mirror asymmetry of a geometrically mirror symmetric chiral medium (see Eq.(S85)) is a very general property which is independent on the medium shape or inhomogeneity. In the specific configuration we are examining here (left side of Fig.S2) such mirror asymmetry can be related to mirror optical activity (MOA), a chiroptical phenomenon investigated in Refs.[10, 11] where the field reflected by a homogeneous chiral film $E_ω^{(i)}(r)$ has always indefinite parity (asymmetry) even if the incident field $E_ω^{(i)}(r)$ is mirror symmetric. To show such connection and gain further insight about Green’s tensor asymmetry, we start by briefly summarizing MOA description. We say that a field $E_ω(r)$ is a mirror symmetric field (MSF) if it coincides with its mirror image $E_ω^M(r) = R E_ω(\bar{r})$, i.e. $E_ω(r) = E_ω^M(r)$. For the reflection through the $xz$ we are considering (see Eq.(S100)) the S and P unit vectors (see right side of Fig.S2) evidently satisfy the momentum space symmetry relations

$$ R u_S (R k_\parallel) = -u_S (k_\parallel), $$
$$ R u_P (R k_\parallel) = u_P (k_\parallel), $$

(S115)

so that it is straightforward deducing from Eq.(S106) that $E_ω^{(i)}$ is a MSF if and only if it has antisymmetric and symmetric S and P amplitudes respectively

$$ U_S^{(i)} (R k_\parallel) = -U_S^{(i)} (k_\parallel), $$
$$ U_P^{(i)} (R k_\parallel) = U_P^{(i)} (k_\parallel). $$

(S116)

Such momentum space characterization of the incident MSF enables to prove that the reflected field of Eq.(S107) is such that

$$ E_ω^{(r)} - E_ω^{(r)M} = 2n \int d^2 k_i e^{ik_i r_i} e^{-ik_{1z}z} \left[ R_{SP} U_P^{(i)} u_S + R_{PS} U_S^{(i)} \left( u_P + \frac{k_\parallel}{k_{1z}} e_z \right) \right]. $$

(S117)
where use has been made of the fact that the reflection coefficients \( R_{1i}(k_0^2) \) in Eq.(S108) are rotationally invariant around the z-axis. Now the right hand side of Eq.(S117) does not vanish due to the mixing reflection coefficients \( R_{SP} \) and \( R_{PS} \) which result from the S-P coupling mediated by chirality and thus vanishing in a achiral medium (see Ref.[10] for further details). Therefore, the reflected field \( \mathbf{E}_m^{(r)} \) is not a MSF and MOA is described by the \( R_{SP} \) and \( R_{PS} \) reflection coefficients.

To show the impact of MOA on the Green’s tensor \( xx \) component evaluated in the above section, it is sufficient to observe that the free-space field \( \mathbf{E}_m^{(0)} \) of Eq.(S103) is produced by an elementary current parallel to the x axis and located at \( \mathbf{r}' \) so that it is a MSF with respect the reflection through the plane orthogonal to the y axis and containing \( \mathbf{r}' \), namely

\[
(r || - r ||') M = R(r || - r ||').
\]  
(S118)

Accordingly Eq.(S104) shows that \( \mathbf{E}_m^{(0)} \) depends on \( (r || - r ||') \) and that its S and P amplitudes are antisymmetric and symmetric, respectively; this explaining the symmetry of the free-space part of the Green’s tensor \( xx \) component

\[
G_{\omega xx}^{(0)} (\mathbf{r}, \mathbf{r}') = G_{\omega xx}^{(r)} (\mathbf{r}, \mathbf{r}')
\]  
(S119)

which is evident from the first of Eqs.(S114). Due to MOA the reflected field \( \mathbf{E}_m^{(r)} \) of Eq.(S112) is not a MSF and this entails the asymmetry of the reflected part of the Green’s tensor \( xx \) component. As a matter of fact from the second of Eqs.(S114) we get

\[
G_{\omega xx}^{(r)} (\mathbf{r}, \mathbf{r}') - G_{\omega xx}^{(r)} (\mathbf{r}, \mathbf{r}') = 2i \int d^2k_{||} e^{i k_{||} (r || - r ||')} \frac{-e^{ik_{1z}(z+z')}}{k_{1z}} \left( R_{SP} \frac{k_{1z}^2}{k_{2z}^2 e_1} + R_{PS} \right) \frac{k_x k_y}{k_{||}^2}
\]  
(S120)

clearly revealing that the general mirror asymmetry of the Green’s tensor stated in Eq.(S85) is here due to MOA since its right hand side only contains the mixing reflection coefficients \( R_{SP} \) and \( R_{PS} \).

**S3.3. Evaluation of the electron reduced density matrix**

We evaluate here the phase factor \( \Phi (\mathbf{R}) \) and the fundamental quantity \( \Delta (\mathbf{R}, \mathbf{R}') \) for \( Z < 0 \) and \( Z' < 0 \). Since the electron velocity is parallel to the x axis, the t and v parts of \( \mathbf{R} \) and \( \mathbf{R}' \) are (see Eq.(S35))

\[
R_t = Ye_y + Ze_z, \quad R_v = X,
\]
\[
R'_t = Y'e_y + Z'e_z, \quad R'_v = X',
\]  
(S121)

and accordingly, from the first of Eqs.(S41) and Eq.(S43) we have

\[
\Phi (\mathbf{R}_t) = \frac{4\alpha}{c} \int_0^\infty dq \int_{-\infty}^{+\infty} dq' e^{i \mathbf{q} \cdot (\mathbf{r}_t - \mathbf{r}_t')} \text{Im} \left[ G_{\omega xx} (\mathbf{r}, \mathbf{r}') \right]_{\mathbf{r}_v = qe_x + Ye_y + Ze_z, \quad \mathbf{r}_v' = q'e_x + Ye_y + Ze_z}
\]
\[
\Delta (\mathbf{R}, \mathbf{R}') = \frac{4\alpha}{c} \int_0^{\infty} dq e^{-i \mathbf{q} \cdot (\mathbf{R} - \mathbf{R}')} \int_{-\infty}^{+\infty} dq' e^{i \mathbf{q}' \cdot (\mathbf{r} - \mathbf{r}')} \text{Im} \left[ G_{\omega xx} (\mathbf{r}, \mathbf{r}') \right]_{\mathbf{r}_v = qe_x + Ye_y + Ze_z, \quad \mathbf{r}_v' = q'e_x + Ye_y + Ze_z}
\]  
(S122)

By exploiting the rotational invariance of the vacuum longitudinal wavenumber \( k_{1z}(k_0^2) \) and of the reflection coefficients \( R_{1i}(k_0^2) \), from Eqs.(S113) and (S114) we get the imaginary part of the \( xx \) component of the Green’s tensor

\[
\text{Im} \left[ G_{\omega xx} (\mathbf{r}, \mathbf{r}') \right] = \frac{1}{8\pi^2} \int d^2k_{||} e^{ik_{||} (r || - r ||')} \text{Re} \left[ e^{ik_{1z}(z+z')} \left( 1 - \frac{k_x^2}{k_{1z}^2 e_1} \right) + e^{-ik_{1z}(z+z')} \frac{k_x k_y}{k_{||}^2} \right]
\]  
(S123)

where

\[
\Upsilon (\omega, k_{||}) = \left( R_{SS} \frac{k_x^2}{k_{||}^2} + R_{PP} \frac{k_z^2}{k_{2z}^2 e_1} \right) - \left( R_{SP} \frac{k_z^2}{k_{2z}^2 e_1} + R_{PS} \right) \frac{k_x k_y}{k_{||}^2}
\]  
(S124)
which inserted into Eqs.(S122) yields
\[
\Phi (R_t) = \frac{\alpha}{2\pi^2 c} \int_0^{+\infty} d\omega \int d^2k || \left[ \text{Im} \int dq \int dq' e^{i(k_x + \bar{\nu})(q-q')} \right] \text{Re} \left[ \frac{1}{k_{1z}} \left( 1 - \frac{k_x^2}{k_0^2 \varepsilon_1} \right) + \frac{e^{-ik_{1z}Z}}{k_{1z} Y} \right],
\]
\[
\Delta (R, R') = \frac{\alpha}{2\pi^2 c} \int_0^{+\infty} d\omega e^{-i\bar{\nu}(X-X')} \int d^2k || e^{ik_y(Y-Y')} \left[ \int dq \int dq' e^{i(k_x + \bar{\nu})(q-q')} \right] \times
\]
\[
\times \text{Re} \left[ \frac{\alpha}{2\pi^2 c} \int_0^{+\infty} d\omega \int d^2k || \left( 1 - \frac{k_x^2}{k_0^2 \varepsilon_1} \right) + \frac{e^{-ik_{1z}Z}}{k_{1z} Y} \right].
\] (S125)

Note that the free-space contribution to \( \Phi \) is independent on \( R_t \) and hence we hereafter neglect it since it provides an unobservable phase factor in the reduced density matrix. The integrals over \( q' \) can be analytically carried out
\[
\text{Im} \int dq' e^{i(k_x + \bar{\nu})(q-q')} = \text{P.V.} \frac{1}{k_x + \bar{\nu}},
\]
\[
\int dq' e^{i(k_x + \bar{\nu})(q-q')} = 2\pi\delta \left( k_x + \frac{\omega}{V} \right),
\] (S126)
where \( \text{P.V.} \) stands for the Cauchy principal value, whereas the remaining integral over \( q \) can be replaced by the effective electron path length which can be approximated by \( L \), the longitudinal length of the chiral film (see Fig.1 of the main text). By inserting these integrals into Eqs.(S125) we get
\[
\Phi (R_t) = \frac{L\alpha}{2\pi^2 c} \text{P.V.} \int_0^{+\infty} d\omega \int d^2k || \frac{1}{k_x + \bar{\nu}} \text{Re} \left( \frac{e^{-ik_{1z}Z}}{k_{1z} Y} \right),
\]
\[
\Delta (R, R') = \frac{L\alpha}{\pi c} \int_0^{+\infty} d\omega \int d^2k || e^{-i\bar{\nu}(X-X')} e^{ik_y(Y-Y')} e^{\frac{k_y^2}{k_0^2 \varepsilon_1} + k_y^2} \frac{e^{\frac{k_x^2}{k_0^2 \varepsilon_1} + k_x^2} \text{Im} (\bar{\nu})_{k_x=-\bar{\nu}}}{\sqrt{k_y^2 \left( \frac{1}{\beta_x} - \varepsilon_1 \right) + k_y^2}}.
\] (S127)

Note that the free-space part of the Green’s tensor does not affect \( \Delta (R, R') \) since its contribution in the curly bracket in the second of Eqs.(S125) is purely imaginary because \( k_{1z} = i\sqrt{k_x^2 \left( \frac{1}{\beta_x} - \varepsilon_1 \right) + k_y^2} \) for \( k_x = -\omega/V \). In addition, due to the planar geometry of the chiral film, the phase factor \( \Phi \) is a function only of \( Z \) whereas \( \Delta (R, R') \) only depends on \( X - X', Y - Y' \) and \( Z + Z' \). Equations (S127) enable the evaluation of the reduced density matrix of Eq.(S61).

### S3.4. Mirror symmetry breaking effects produced by mirror optical activity

The Green’s tensor component \( xx \) is not symmetric under mirror reflections owing to MOA, as discussed above, and this entails that the aloof electron-chiral film interaction has broken mirror symmetry. In order to discuss this crucial point, from the second Eqs.(S127) it is convenient noting that the decomposition
\[
\Delta (R, R') = \Delta_S (X - X', Y - Y', Z + Z') + i\Delta_A (X - X', Y - Y', Z + Z')
\] (S128)
holds, in which
\[
\Delta_S (\bar{X}, \bar{Y}, \bar{Z}) = \frac{L\alpha}{\pi c} \int_0^{+\infty} d\omega e^{-i\bar{\nu} \bar{X}} \int dky \cos(k_y \bar{Y}) e^{\frac{k_y^2}{k_0^2 \varepsilon_1} + k_y^2} \text{Im} \left( R_{SS} \frac{k_y^2}{k_x^2} + R_{PP} \frac{k_y^2}{k_0^2 \varepsilon_1} \frac{k_y^2}{k_x^2} \right)_{k_x=-\bar{\nu}},
\]
\[
\Delta_A (\bar{X}, \bar{Y}, \bar{Z}) = \frac{L\alpha}{\pi c} \int_0^{+\infty} d\omega e^{-i\bar{\nu} \bar{X}} \int dky \sin(k_y \bar{Y}) e^{\frac{k_y^2}{k_0^2 \varepsilon_1} + k_y^2} \text{Im} \left[ -n \left( R_{SP} \frac{k_y^2}{k_0^2 \varepsilon_1} + R_{PS} \frac{k_y^2}{k_0^2 \varepsilon_1} \right) \frac{k_y}{k_x^2} \frac{k_x}{k_y} \right]_{k_x=-\bar{\nu}},
\] (S129)
are symmetric and antisymmetric under the reflection through the \(xz\) plane, respectively, since
\[
\Delta_S(\tilde{X}, -\tilde{Y}, \tilde{Z}) = \Delta_S(\tilde{X}, \tilde{Y}, \tilde{Z}),
\]
\[
\Delta_A(\tilde{X}, -\tilde{Y}, \tilde{Z}) = -\Delta_A(\tilde{X}, \tilde{Y}, \tilde{Z}).
\]

Note that \(\Delta_A\) is entirely due to MOA since it only contains the mixing reflection coefficients \(R_{SP}\) and \(R_{PS}\) and accordingly its sign is switched by chirality reversal \(\kappa \rightarrow -\kappa\) since it is proportional to the chiral refractive index \(n\).

Equation (S128) shows that the mirror symmetry of \(\Delta\) is broken by MOA and it enables the decoherence factor of Eq.(S62) to be written as
\[
\gamma(\mathbf{R}, \mathbf{R}') = e^{i[\Phi(\tilde{Z}) - \Phi(\tilde{Z}')]} e^{\Delta_S(\tilde{X}-\tilde{X}', \tilde{Y}-\tilde{Y}', \tilde{Z}+\tilde{Z}') - \frac{1}{2} \left[ \Delta_S(0,0,2\tilde{Z}) + \Delta_S(0,0,2\tilde{Z}')\right]} e^{i\Delta_A(\tilde{X}-\tilde{X}', \tilde{Y}-\tilde{Y}', \tilde{Z}+\tilde{Z}')}
\]
(S131)

which, in agreement with the general case of Eq.(S90), evidently has broken mirror symmetry here due the last exponential factor containing \(\Delta_A\). In order to quantitatively its absolute mirror asymmetry we consider the dimensionless quantity
\[
\text{Asym} \left[ \gamma(\mathbf{R}, \mathbf{R}') \right] = \frac{2\gamma(\mathbf{R}, \mathbf{R}') - \gamma(\mathbf{R}, \mathbf{R}')}{\gamma(\mathbf{R}, \mathbf{R}') + \gamma(\mathbf{R}, \mathbf{R}')}\]
(S132)

which compares \(\gamma(\mathbf{R}, \mathbf{R}')\) and its mirror image \(\gamma(\mathbf{R}, \mathbf{R}')\) through the ratio between their difference and their arithmetic average. Inserting Eq.(S131) into Eq.(S132) we obtain
\[
\text{Asym} \left[ \gamma(\mathbf{R}, \mathbf{R}') \right] = 2i \tan \left[ \Delta_A(\tilde{X}-\tilde{X}', \tilde{Y}-\tilde{Y}', \tilde{Z}+\tilde{Z}') \right]
\]
(S133)

clearly elucidating that MOA breaks the mirror symmetry of the electron decoherence. Since \(\Delta_A\) is proportional to \(L\), the absolute asymmetry of the decoherence factor can be enhanced in the presence of long effective interaction lengths. For an initial mirror symmetric electron wave function (see Eq.(S86)), Eq.(S133) also provides the absolute mirror asymmetry of the electron reduced density matrix (see Eq.(S61)).

As discussed in Sec.2.5, the electron transverse momentum distribution lacks mirror symmetry along the lateral direction. In order to relate such general effect to MOA in the configuration we are examining, we here focus on the distribution of the lateral moment component \(P_y\) which is given by \(\frac{d\varphi}{dP_y} = \int dP_z \frac{d\varphi}{dP_z}\). From the first of Eqs.(S74), we straightforwardly get
\[
\frac{d\varphi}{dP_y} = \frac{1}{2\pi\hbar} \int dY \int dY' \int dZ e^{-iP_y(\tilde{Y}-\tilde{Y}')} \phi_i(\tilde{Y}, \tilde{Z}) \phi_i^*(\tilde{Y}', \tilde{Z}) \left[ \gamma(\mathbf{R}, \mathbf{R}) \right]_{Z'=Z}
\]
(S134)

which, after using Eq.(S131) and performing the change of integration variables \(Y' = Y - \tilde{Y}, Z = \tilde{Z}/2\), becomes
\[
\frac{d\varphi}{dP_y} = \frac{1}{4\pi\hbar} \int d\tilde{Y} \int d\tilde{Z} e^{-iP_y\tilde{Y}} \tilde{\phi}_i(\tilde{Y}, \tilde{Z}) e^{i\Delta_S(0,0,\tilde{Z}) - \Delta_S(0,0,\tilde{Z})} e^{i\Delta_A(0,0,\tilde{Z})}
\]
(S135)

where
\[
\tilde{\phi}_i(\tilde{Y}, \tilde{Z}) = \int dY \phi_i(Y, \tilde{Z}/2) \phi_i^*(Y - \tilde{Y}, \tilde{Z}/2)
\]
(S136)

is the autoconvolution of the initial electron wavefunction. For the initial mirror symmetric electron wave function \(\phi_i(-Y, Z) = \phi_i(Y, Z)\), the autoconvolution \(\tilde{\phi}_i(\tilde{Y}, \tilde{Z})\) is even in \(\tilde{Y}\). Now the real quantities \(\Delta_S(0, \tilde{Y}, \tilde{Z}) - \Delta_S(0,0,\tilde{Z})\) and \(\Delta_A(0, \tilde{Y}, \tilde{Z})\) are even and odd in \(\tilde{Y}\), respectively, and hence we conclude that \(\frac{d\varphi}{dP_y}\) of Eq.(S135) can not be either even or odd in \(P_y\) since it is the Fourier transform of a function whose parity is indefinite. To explicitly highlight its asymmetry we write Eq.(S135) as
\[
\frac{d\varphi}{dP_y} = \frac{1}{4\pi\hbar} \int d\tilde{Y} \int d\tilde{Z} \tilde{\phi}_i(\tilde{Y}, \tilde{Z}) e^{i\Delta_S(0,0,\tilde{Z})} \times \left\{ \cos \left( \frac{1}{\hbar} P_y \tilde{Y} \right) \cos \left[ \Delta_A(0, \tilde{Y}, \tilde{Z}) \right] + \sin \left( \frac{1}{\hbar} P_y \tilde{Y} \right) \sin \left[ \Delta_A(0, \tilde{Y}, \tilde{Z}) \right] \right\}
\]
(S137)

where the even and odd contributions to the transverse momentum distribution are separated. The odd part is entirely due to \(\Delta_A\) so that the mirror symmetry of the electron lateral momentum distribution is broken by MOA.
As discussed in Sec.S2.5, the asymmetry of the transverse momentum distribution has the remarkable consequence that the electron gains a net lateral momentum upon scattering. To explicitly discuss such phenomenon for the aloof electron we are considering, we evaluate the mean value and variance of $P_y$. After substituting Eq.(S128) into Eq.(S79), and using the fact that the impinging electron with mirror symmetric wavefunction has no transverse momentum, $\int dY \int dZ \phi_i^* \left( \frac{\hbar}{i} \frac{\partial}{\partial Y} \right) \phi_i = 0$, the firsts of Eqs.(S78) and Eqs.(S80) with $j = y$, after some algebra, yield

$$\langle P_y \rangle = \int dY \int dZ |\phi_i|^2 A, \quad \langle P_y^2 \rangle - \langle P_y \rangle^2 = \int dY \int dZ \phi_i^* \left( \frac{\hbar}{i} \frac{\partial}{\partial Y} \right)^2 \phi_i + \int dY \int dZ |\phi_i|^2 S + \left\{ \int dY \int dZ |\phi_i|^2 A^2 - \left\{ \int dY \int dZ |\phi_i|^2 A \right\}^2 \right\},$$

(S138)

where

$$A(Z) = \left[ \frac{\hbar}{i} \frac{\partial \Delta_A(0, \tilde{Y}, 2Z)}{\partial \tilde{Y}} \right]_{\tilde{Y}=0}, \quad S(Z) = \left[ -\frac{\hbar^2}{2} \frac{\partial^2 \Delta_S(0, \tilde{Y}, 2Z)}{\partial \tilde{Y}^2} \right]_{\tilde{Y}=0}. \quad (S139)$$

The first of Eqs.(S138) makes explicit that the mean value of $P_y$ is fully due to MOA, as expected and it is one of the main results of this work. The second of Eqs.(S138) reveals that the variance of $P_y$ has two contributions that add up to the initial variance $\int dY \int dZ \phi_i^* \left( \frac{\hbar}{i} \frac{\partial}{\partial Y} \right)^2 \phi_i$. The one containing $\Delta_S$ (through $S$) is not vanishing even in the achiral limit $\Delta_A = 0$ and it accounts for the natural momentum distribution reshaping produced by the interaction of the electron with the medium. On the other hand, the contribution containing $\Delta_A$ (through $A$) is entirely produced by MOA and it is expected to be very small as compared to the previous one.

### S3.5. Electron energy mean value and variance

As discussed in section S2.5, the interaction of the electron with the chiral medium results from the combined effect of an infinite number of processes in each of which the energy lost by the electron is transferred to a finite number of polaritonic excitations. As a consequence, the energy of the electron after the interaction is not definite and it has the probability distribution discussed in section S2.4 which, for the aloof electron we are considering, can be obtained by substituting the decoherence factor of Eq.(S131) into the second of Eqs.(S74), thus getting

$$\frac{d\mathcal{P}}{dE} = \frac{1}{2\pi \hbar V} \int dY \int dZ \int dQ \phi_i^* (\frac{E}{|\phi_i|})^2 e^{\Delta_S(Q, 0, 2Z) - \Delta_S(0, 0, 2Z)}, \quad (S140)$$

This expression does not contain $\Delta_A$ and hence we conclude that the electron-medium energy exchange is not affected by MOA, as expected since the electron energy is invariant under mirror reflection. In analogy with the above discussed transverse momentum, after substituting Eq.(S128) into Eq.(S79), the seconds of Eqs.(S78) and Eqs.(S80), after some algebra, yield the energy mean value and variance

$$\langle E \rangle = E_i + \int dY \int dZ |\phi_i|^2 \sigma^{(1)}, \quad \langle E^2 \rangle - \langle E \rangle^2 = \int dY \int dZ |\phi_i|^2 \sigma^{(2)} + \left\{ \int dY dZ |\phi_i|^2 \left( \sigma^{(1)} \right)^2 - \left\{ \int dY \int dZ |\phi_i|^2 \sigma^{(1)} \right\}^2 \right\}, \quad (S141)$$

where

$$\sigma^{(n)}(Z) = \left[ \left( V \frac{\hbar}{i} \frac{\partial}{\partial \bar{X}} \right)^n \Delta_S(\bar{X}, 0, 2Z) \right]_{\bar{X}=0}. \quad (S142)$$

Using the first of Eqs.(S129) it is simple proving that $\sigma^{(1)}(Z) < 0$ so that the first of Eqs.(S141) shows that the energy mean value is smaller than the initial energy $E_i$, as expected since part of the initial electron energy is delivered to
the field.

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