Studying the fractal properties of Ceres

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Abstract. Currently, the asteroid Ceres belongs to small celestial bodies with the most well-known physical parameters. The study of the structural and real properties of Ceres is an urgent and modern task, the solution of which will make it possible to develop the evolutionary theory of a minor planet. In this work, the fractal properties of the dwarf planet Ceres were analyzed using data from the Dawn space mission. Using the expansion in a harmonic series in spherical functions the height parameters of the structural model of Ceres, a 3D model of Ceres was constructed. The analysis showed that the resulting system has a complex multiparameter fractal configuration. The study of such objects requires the use of harmonic multiparameter methods. Multivariate fractal analysis allows to represent systems similar to the Ceres model in the form of a spectrum of fractal dimensions. The advantage of fractal analysis is the ability to explore local areas of the physical surface. In this work, the Minkowski algorithm was used for this purpose. At the final stage, an overdetermined system was solved for various local areas of topocentric information in order to postulate a model that takes into account external measures. Fractal dimensions D are determined for local regions and the entire model of the planet. Fractal dimensions vary from 1.37 to 1.92 depending on the longitude and latitude of Ceres. The main results are as follows: 1) the structure of the Ceres surface varies more strongly in longitude; 2) the structure of Ceres is smoother in latitude; 3) the coefficient of self-similarity changes rather quickly in longitude, which indicates that different local regions of the minor planet were formed under the influence of various physical processes. It is necessary to emphasize that the resulting fractal dimensions are significantly scattered both in longitude and latitude of Ceres. This fact confirms the presence of a complex structure in the spatial model of a minor planet. This also applies to the actual physical surface of Ceres. The results of the work allow us to conclude that fractal modeling can give independent values of the fractal dimension both for the entire model of Ceres and for its local macrostructural regions.

1. Introduction
The study of small bodies in the solar system is an important and modern task [1]. This is based on solving the following problems. The first is the asteroid-cometary threat to the Earth [2]. The second is considering the fundamental foundations of the population of the Solar system by meteoroids and studying their dynamics and evolution [3]. An analysis of the physical characteristics of known asteroids indicates an insufficient amount of information about them [4]. Currently, there are many cases of the discovery of new small cosmic bodies, however, the information on their physical characteristics, unfortunately, does not change [4].
Based on the data of the NASA’s Dawn spacecraft, the elemental base of the geological structure of Ceres was determined and agreed. The results obtained in this work can be used to analyze planetophysical processes on Ceres [5]. It is necessary to emphasize, that in order to create a complete picture of both the current state and the evolution of Ceres, knowledge of the material composition must be integrated with the structural parameters of its surface [6, 7]. In this case, the Weierstrass-Mandelbrot fractal modeling method was used. According to this approach, the models are separate fractals and are described by multifractals - an interconnected system of fractals [8]. These composite fractals are recursive, since they are invariant for the entire model of a complex structure both in scale and in symmetry [8]. The model of the surface of Ceres was built by expanding the height functions into a regression harmonic series [9]. The order of expansion of the height function depends on the number of control points. As a result, using the harmonic expansion of the height functions into spherical functions [10], a three-dimensional Ceres model was constructed for its fractal analysis.

According to [11, 12, 13], we can conclude that the further development of the method of comparative fractal analysis will allow astronomers to study a larger number of local chemical and physical parameters and anomalies of Ceres.

2. Method of fractal analysis

In this work, we used the Hausdorff-Besicovitch dimension (HBD) method (HBD) [14]. Finding HBD is based on measuring the relative distances between points in the set. In this case, HBD is an equivalent value for the value of the fractal dimension D. An important task is to determine the large number of distances between individual points of the set [15].

Until now, when considering the magnitude of the set of points in space, a certain test function was chosen \( h(\delta) = \gamma(d)\delta^d \), which was used as a straight-line segment, square, circle, sphere or cube, and as a result, this measure covered the set under study. In this case, a measure was formed \( M_d = \sum h(\delta) \).

In the case of straight-line segments, squares, cubes, the geometric parameter \( \gamma(d) = \frac{\pi}{4} \) and \( \gamma(d) = \frac{\pi^d}{6^d} \) respectively, for circles and spheres. In the general case, with symptomatology \( \delta \rightarrow 0 \), the measure \( M_d \) becomes either equal to 0 or \( \infty \) (this depends on the value of the parameter \( d \))[16].

Dimension \( D \) for a fractal set is critical. The measure \( M_d \), when \( D \) is adopted, changes from 0 to \( \infty \):

\[
M_d = \gamma(d)\delta^d = \gamma(d)N(\delta)\delta^d \quad \text{for} \quad \delta^d \rightarrow 0, \quad \delta \rightarrow 0^+.
\]

The measure of a fractal set \( d \) is denoted as \( M_d \). In the case of \( d = D \), \( M_d \) can be an integer value, but it can also be 0 or \( \infty \). In this case, an important task is to determine the value of \( d \) at which the change in \( M_d \) occurs in an abrupt manner [17]. It is necessary to emphasize that in the presented definition, the Hausdorff-Besicovitch dimension acts as a local property, that is, it describes the characteristics of sets of points in the limit for an infinitely small size \( \delta \) of a test function covering the set. Hence, the fractal dimension \( D \) can serve as a local characteristic of the set. However, there are several details to consider. As an example, the definition of the Hausdorff-Besicovitch dimension does not prohibit covering a certain set with spheres of unequal radii, provided that the latter are less than \( \delta \), in this case the \( d \)-measure acts as a "lower bound", that is, it is the minimum value that is obtained when all possible coverings [18].

Equation (5) for determining the fractal dimension is applied in practice. If you cover a complex nonlinear line with many squares with sides \( \delta \), you can calculate the number of squares required to cover a given line. Let's designate this quantity as \( N(\delta) \). Further, in accordance with equation (5), it is necessary either to find \( M_d(\delta) \), or to continue the calculations and determine \( N(\delta) \) by reducing the sizes of the sides of the squares covering this line.

Since it follows from definition (5) that in the limiting case for small \( \delta \)

\[
L = N(\delta)\delta^3 \delta \rightarrow 0^+ \rightarrow S_0\delta^{-1},
\]
the D for the nonlinear curve model can be computed graphically by measuring the tangent of the slope of the line representing the versus $ln N(\delta)$ from $ln \delta$ [19].

3. Analysis of the results of studying the fractal properties of Ceres

When analyzing the elevation data, the entire surface was divided into pixels and the average value of the surface height was measured at each pixel. The obtained parameters are used to construct a 3D model of the asteroid's surface.

As a result, a three-dimensional model of Ceres was built. The model is a three-dimensional image according to the latitudes and longitudes of Ceres (Figure 1).

Let's take a look at the locations of the major geographic features. The largest mountain on Ceres, Mount Akhuna, is a cryovolcano 5 kilometers high. Its apex corresponds to a point with coordinates – 10° lat., + 315° long.

Large craters with corresponding coordinate centers are also distinguished: Urvara (–45° lat., 250° long.), Rongo (5° lat., 348° long.), Danto (7° lat., 135° long.), Okator (9° lat., 239° long.), Ikapati (34° lat., 45° long.), Khaulani (10° lat., 18° long.), Yalode (–40° lat., 288° long.).

![Figure 1. 3D-model of Ceres](image1)

![Figure 2. 2D-model of Ceres](image2)
Figure 2 shows a 2D model of Ceres. Highlands and lowlands are visible on it according to the given scale.

Figure 3 shows a map of the height isohypses of the Ceres surface. Isohypses show equal levels of heights and depths and serve for clarity of the analysis. It can be said that the set of isohypsum represents the altitude model of the minor planet.

It should be emphasized that the constructed model has approximated values of heights, since during the calculations they were averaged over the pixels of the surface images. However, as Figure 3 shows, smooth surfaces are not present on this asteroid. If we also take into account the fact that Ceres rotates on its axis, we can conclude that landing on this planet will be a rather difficult task.

Figure 3. Isohypses of the Ceres surface

4. Summary and conclusions

As a result, it was found that the minor planet is a silicate body filled with water. In the process of evolution, Ceres was heated and structurally altered, and geological activity is observed on the planet and at the present time [20].

In the present study, a multifractal approach was used to analyze the structure of a minor planet. As a result, it was found that the surface of Ceres contains many artifacts and formations that confirm the impact on the planet of a large number of meteoroids. Ceres models, built using satellite data, are structured by pixels, within the boundaries of which the average values of the surface heights were determined. Based on these results, the 3D model of the Ceres structure was built.

It is necessary to emphasize, that the constructed model is smoothed, since the average values of heights were used in the calculations [21, 22]. The pixel parameters have been entered into a digital database that will be used for further research [23, 24]. In the future, the analysis of Ceres can be continued using fractal geometry and determining the similarity coefficient [25].

The results of the work can be used in research focused on the study of the meteoroid component of the solar system and space threats [26, 27, 28]. It is planned to use the method developed by us to analyze the nonlinear terms of the physical libration of the Moon too. [29, 30].

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4
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