Fat Branes, Orbifolds and Doublet-Triplet Splitting

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Abstract

A simple higher dimensional mechanism of the doublet-triplet splitting is presented in a five dimensional supersymmetric SU(5) GUT on $S^1/Z_2$. The splitting of multiplets is realized by a mass term of Higgs hypermultiplet which explicitly breaks SU(5) gauge symmetry. Depending on the sign of mass, zero mode Higgs doublets and triplets are localized on the either side of the fixed points. The mass splitting is realized due to the difference of magnitudes of the overlap with a brane localized or a bulk singlet field. No unnatural fine-tuning of parameters is needed. The proton stability is ensured by locality without symmetries. As well as a conventional mass splitting solution, it is shown that the weak scale Higgs triplet is consistent with the proton stability. This result might provide an alternative signature of GUT in future collider experiments.

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The doublet-triplet splitting problem is one of the notorious problems in grand unified theories (GUTs) [1]. Many proposals for this problem have been made in four dimensional models [2] or recently in higher dimensional models [3]. See also for string-derived models [4]. In our previous papers [5, 6], we have presented a doublet-triplet splitting mechanism in the context of the fat brane scenario [7]. In this approach, a conventional mass splitting, namely the weak-GUT scale splitting, can be realized by an overlap of zero mode wavefunctions between the doublet and the triplet Higgs fields without unnatural fine-tuning of parameters [5]. Interestingly, it has also been shown that an alternative mass splitting, namely the weak-TeV scale splitting, can be realized by the same mechanism [6]. The proton stability is guaranteed by the strong suppression of the coupling of matter fields to the triplet Higgs due to a small overlap of wavefunctions. This TeV scale triplet Higgs scenario deserves an attention as an alternative signature of GUT instead of the proton decay.

The setup itself has crucial problems although these scenarios are very attractive. In the fat brane scenario, extra dimensions are considered to be non-compact otherwise the effective four dimensional theory becomes vector-like. This non-compactness leads that the gravity and gauge fields cannot propagate in the bulk and have to be localized on a fat brane (domain wall) to obtain finite coupling constants. As is well known, it is highly nontrivial to realize the localization of the gravity and gauge fields on a domain wall in infinite extra dimensions. Even if the localization is realized by the “quasi-localization” mechanism [8, 9], zero mode wave functions have to be almost flat on a domain wall because of the charge universality constraints. It seems to be very difficult to obtain such flat wave functions.

To avoid this situation, we consider a theory on an orbifold.\(^2\) By construction, the bulk gravity and gauge fields propagate in a finite extra space and it is easy to obtain flat zero mode wave functions in flat extra dimensions. First explicit realization of the fat brane scenario on an orbifold has been given in Ref. [10]. By developing extra coordinate dependent vacuum expectation values (VEVs) of the parity odd scalar field, the kink solution (domain wall) is constructed. The chiral fermion zero modes which couples to the scalar field generating domain wall are localized on either fixed point depending on the sign of the coupling constant.

\(^2\)In the smooth compact dimensional case, \(S^1\) for instance, the anti-domain wall as well as the domain wall appears. This case cannot only yield chiral fermions but also cause the instability of the system.
In this letter, we apply this mechanism to the doublet-triplet splitting in a five-dimensional supersymmetric (SUSY) SU(5) GUT compactified on $S^1/Z_2$. There are fixed points at $y = 0, \pi R$, where $y$ denotes the fifth dimensional coordinate and $R$ is a compactification radius.

We shall focus on a Higgs sector. Two hypermultiplets are introduced as Higgs multiplets.

$$H_1 = (H_1(5), H_1^c(5^*)),$$  \hspace{1cm} $$H_2 = (H_2(5^*), H_2^c(5)),$$  \hspace{1cm} (1)

where the representations under SU(5) are specified. We assign their $Z_2$ parities as follows;

$$H_1(-y) = +H_1(y), \hspace{1cm} H_1^c(-y) = -H_1^c(y),$$  \hspace{1cm} (2)

$$H_2(-y) = +H_2(y), \hspace{1cm} H_2^c(-y) = -H_2^c(y).$$  \hspace{1cm} (3)

The action we consider is given by

$$S_5 = \int d^5 x \left\{ \left[ H_i^\dagger e^{-V} H_i + H_i^c e^V H_i^c \right] \theta_2 \bar{\theta}_2 + \left[ H_i^c (\partial_y - \frac{1}{\sqrt{2}} \Phi - \Phi^c H_i + \frac{1}{2} M(y) H_i H_i^c + \delta(y - \pi R) S_{MP} H_1 H_2 \right] \theta_2 \bar{\theta}_2 + h.c. \right\} \hspace{1cm} (i = 1, 2),$$  \hspace{1cm} (4)

where the action $S_5$ is written in terms of 4D, $N = 1$ superspace formalism [13], $S$ is a singlet chiral superfield, which is assume to be localized on the brane at $y = \pi R$. $M_5, M_P$ are Planck scales in five and four dimensions, which are related by $M_5^3 \pi R = M_P^3$. Here, the vector superfield $V$ and the chiral superfield $\Phi$ in the adjoint representation are explicitly given by

$$V = -\theta \sigma^\mu \bar{\theta} A_\mu + i \bar{\theta}^2 \theta \lambda_1 - i \theta^2 \bar{\theta} \bar{\lambda}_1 + \frac{1}{2} \theta^2 \bar{\theta}^2 D,$$  \hspace{1cm} (5)

$$\Phi = \frac{1}{\sqrt{2}} (\Sigma + i A_5) + \sqrt{2} \theta \lambda_2 + \theta^2 F,$$  \hspace{1cm} (6)

where $A_\mu (\mu = 0, 1, 2, 3)$ is a gauge field in four dimensions, $\lambda_{1,2}$ are gauginos, $F, D$ are auxiliary fields, $A_5$ is an extra component of the gauge field, $\Sigma$ is a real scalar field in the adjoint representation. Note that an extra dimensional coordinate dependent mass term is introduced in the second line of Eq. (4). A mass parameter $M(y)$ is given by

$$M(y) = M_5 \text{ diag}(2, 2, 2, -3, -3) \varepsilon(y),$$  \hspace{1cm} (7)

3For other applications, see [11, 12].

4The cases that is localized at $y = 0$ and constant in the bulk will be discussed later.
where \( \varepsilon(y) \) is a sign function with respect to \( y \). Thus, an SU(5) is explicitly broken to the standard model (SM) gauge group at the cutoff scale \( M_5 \). Throughout this paper, we consider the following SUSY vacuum at \( M_5^5 \)

\[
\langle \Sigma \rangle = \langle H_5 \rangle = \langle H_5^c \rangle = \langle S \rangle = 0. \tag{8}
\]

Let us concentrate on fermionic components of the action to study zero modes in the background (7) and (8),

\[
S_5 \supset \psi_i^c i\sigma^\mu \partial_\mu \bar{\psi}_i + \bar{\psi}_i i\sigma^\mu \partial_\mu \psi_i - \psi_i^c \partial_y \bar{\psi}_i + \bar{\psi}_i \partial_y \psi_i + \frac{1}{2} M(y) (\psi_i^c \psi_i + \bar{\psi}_i \bar{\psi}_i)
\]

\[ - \delta(y - \pi R) \left( \frac{S}{M_P} \bar{\psi}_1 \psi_2 + \frac{S^*}{M_P} \bar{\psi}_2 \psi_1 \right) (i = 1, 2), \tag{9}\]

where \( (\psi_i, \psi_i^c) \) denote obviously the fermionic components of \( (H_i, H_i^c) \). By expanding in modes as

\[
\psi_i(x, y) = \sum_n \psi_i^{(n)}(x) f_i^{(n)}(y), \quad \psi_i^c(x, y) = \sum_n \psi_i^{c(n)}(x) f_i^{c(n)}(y), \tag{10}\]

we obtain mode equations

\[
0 = m_n \bar{\psi}_1^{c(n)} f_1^{c(n)} + \psi_1^{c(n)} \partial_y \bar{f}_1^{c(n)} - \frac{1}{2} M(y) \bar{\psi}_1^{(n)} \bar{f}_1^{(n)}, \tag{11}\]

\[
0 = m_n \bar{\psi}_2^{c(n)} f_2^{c(n)} + \psi_2^{c(n)} \partial_y \bar{f}_2^{c(n)} - \frac{1}{2} M(y) \bar{\psi}_2^{(n)} \bar{f}_2^{(n)}, \tag{12}\]

\[
0 = m_n \bar{\psi}_1^{(n)} f_1^{(n)} - \psi_1^{(n)} \partial_y \bar{f}_1^{(n)} - \frac{1}{2} M(y) \bar{\psi}_1^{c(n)} \bar{f}_1^{c(n)} + \delta(y - \pi R) \frac{S^*}{M_P} \bar{\psi}_2^{(n)} \bar{f}_2^{(n)}, \tag{13}\]

\[
0 = m_n \bar{\psi}_2^{(n)} f_2^{(n)} - \psi_2^{(n)} \partial_y \bar{f}_2^{(n)} - \frac{1}{2} M(y) \bar{\psi}_2^{c(n)} \bar{f}_2^{c(n)} + \delta(y - \pi R) \frac{S^*}{M_P} \bar{\psi}_1^{c(n)} \bar{f}_1^{c(n)}, \tag{14}\]

where a mass in four dimensions is defined as

\[
-i \sigma^\mu \partial_\mu \bar{\psi}_1^{(n)} = m_n \psi_1^{(n)}, \quad -i \sigma^\mu \partial_\mu \psi_1^{c(n)} = m_n \bar{\psi}_1^{(n)}. \tag{15}\]

It is easy to find solutions of above equations (11)-(14) in the bulk,

\[
f_{1,2}^{(n)}(y) = N_n \exp \left[ \frac{1}{2} \int_0^y dx_5 M(x_5) \right] \cos(m_n y), \tag{16}\]

\[
f_{1,2}^{c(n)}(y) = N_n^c \exp \left[ -\frac{1}{2} \int_0^y dx_5 M(x_5) \right] \sin(m_n y), \tag{17}\]

where \( N_n^{(c)} \) are normalization constants.

\(^5\)Later, we consider the VEV of \( S \). We assume that \( S \) will take VEV below the energy scale of \( M_5 \) by, for example, an inverted hierarchy scenario.
On the other hand, the following boundary conditions at $y = \pi R$ should be satisfied from (13) and (14),

\[
\tan(m_n \pi R) = -\frac{S}{2 M_P} \frac{\psi_2}{\psi_1'} \exp \left[ \int_0^{\pi R} dy M(y) \right], \quad (18)
\]
\[
\tan(m_n \pi R) = -\frac{S}{2 M_P} \frac{\psi_1}{\psi_2'} \exp \left[ \int_0^{\pi R} dy M(y) \right]. \quad (19)
\]

Mass eigenvalues can be obtained by eliminating $\psi_1, \psi_2$ in (18) and (19) as

\[
m_n = \frac{1}{R} \left( n + \frac{1}{\pi} \arctan \left[ \frac{S}{2 M_P} \exp \left( \int_0^{\pi R} dy M(y) \right) \right] \right) (n = 0, 1, 2...). \quad (20)
\]

For $m_0 = 0$, zero mode wave functions take the form

\[
f_{1,2}^{(0)}(y) \simeq \exp \left[ \frac{1}{2} \int_0^y dx_5 M(x_5) \right], \quad (21)
\]
\[
= \left\{ \begin{array}{ll}
\sqrt{\frac{2 M_5}{e^{2 M_5 \pi R} - 1}} \exp[M_5 y] & \text{(for triplets)} \\
\sqrt{\frac{3 M_5}{1 - e^{-3 M_5 \pi R}}} \exp[-\frac{3}{2} M_5 y] & \text{(for doublets)}
\end{array} \right. \quad (22)
\]

It turns out that the triplet Higgs zero mode is peaked at $y = \pi R$, while the doublet Higgs zero mode at $y = 0$.

The doublet-triplet splitting is realized by the coupling of Higgs doublets to $S$

\[
\delta(y - \pi R) \left[ \frac{S}{M_P} H_1 H_2 \right]_{g^2} \Rightarrow \left\{ \begin{array}{l}
m_3 = \frac{(S)}{M_P} e^{2 M_5 \pi R} e^{2 M_5 \pi R} \simeq \frac{(S)}{M_P} 2 M_5 \\
m_2 = \frac{(S)}{M_P} 3 M_5 e^{-3 M_5 \pi R} \simeq \frac{(S)}{M_P} 3 M_5 e^{-3 M_5 \pi R} \simeq m_W
\end{array} \right. \quad (23)
\]

where the weak scale $m_W$ is $m_W \simeq 100$ GeV, $m_{2,3}$ are the doublet, triplet Higgs masses respectively. Remarkably, the triplet mass is unsuppressed since an overlap with a singlet is large, while the doublet mass is exponentially suppressed since an overlap with a singlet is very small.

In order to constrain parameters in our model further, let us study the proton decay. We assume that the matter fields are localized on the brane at $y = 0$. Then, the coupling of matter fields to the triplet Higgs $H_3$ is given by

\[
\delta(y) \left[ \frac{Y}{\sqrt{M_P}} H_3 QQ \right]_{g^2}, \quad (24)
\]

where $Y$ is Yukawa coupling, $Q$ mean the SM matter superfields. Integrating out with respect to the fifth coordinate $y$ and substituting the value of zero mode wave functions
of Higgs triplets at $y = 0$ yield a 4D effective yukawa coupling $Y_{\text{eff}}$, which is determined by a normalization constant,

$$Y_{\text{eff}} \simeq \sqrt{\frac{2M_5}{M_P}} e^{-M_5 \pi R}.$$  \hspace{1cm} (25)

Let us first discuss proton decay constraints from dimension six operator. There are two sources for dimension six operator, one is $X$, $Y$ gauge boson exchange, the other is the triplet Higgs scalar exchange at tree level. In our scenario, the amplitude of $X$, $Y$ gauge boson exchange is $1/m^2_{X,Y} \simeq 1/M_5^2$, where $m_{X,Y}$ are $X$, $Y$ gauge boson masses of order 5D Planck scale $M_5$ since GUT symmetry is broken by VEV of a hypermultiplet in the adjoint representation $\langle A^c A \rangle$. Namely, the constraint is simply $M_5 > 10^{16}$ GeV. On the other hand, the amplitude of the proton decay process from the triplet Higgs boson exchange is

$$\frac{(Y_{\text{eff}})^2}{m^2_3} \simeq \left( \frac{M_P}{2M_5 \langle S \rangle} \right)^2 \left( \frac{2M_5}{M_P} e^{-2M_5 \pi R} \right) = \frac{M_P}{2M_5 \langle S \rangle^2} e^{-2M_5 \pi R}.$$  \hspace{1cm} (26)

Therefore, the triplet Higgs boson exchange constraints to be satisfied leads to

$$\sqrt{\frac{2M_5}{M_P} \langle S \rangle} e^{M_5 \pi R} > 10^{16} \text{ GeV}. \hspace{1cm} (27)$$

Using the second relation in (23), the upper bound on the compactification scale is obtained as

$$R^{-1} < 5.5 \times 10^{17} \text{ GeV}. \hspace{1cm} (28)$$

Next we turn to the constraints from dimension five operators. The proton decay amplitude coming from the dimension five operator is evaluated

$$\frac{g^2 Y_{\text{eff}}^2}{16\pi^2 m_3 M_\lambda} \simeq \frac{g^2 y_1 y_2 (2M_5/M_P e^{-2M_5 \pi R})}{16\pi^2 m_3 M_\lambda}, \hspace{1cm} (29)$$

where $g$ is an SU(2) gauge coupling, $y_{1,2}$ are Yukawa coupling of the first and the second generations in five dimensions, and $M_\lambda$ is a gaugino mass. Comparing 4D GUT case, the following condition can be obtained,

$$\frac{1}{\langle S \rangle} \times 10^{16} e^{-2M_5 \pi R} < 1 \Leftrightarrow \frac{3M_5}{M_P} \times 10^{14} e^{-5M_5 \pi R} < 1,$$  \hspace{1cm} (30)

the second expression can be obtained by using the second relation in (23). Summarizing the proton decay constraints, $X$, $Y$ gauge boson exchange amplitudes proportional to $M_5^{-2}$ gives a lower bound for $M_5$, namely a upper bound for the compactification scale.
On the other hand, the amplitudes from Higgs triplet scalar exchange and the dimension five operator are roughly proportional to $M_5/M_P$ for $M_5/M_P \ll 1$, therefore a upper bound for $M_5$ is obtained, namely a lower bound for the compactification scale is obtained. Searching for allowed parameter region satisfying the constraints of X, Y gauge boson exchange, (28) and (30) with $M_5^2 \pi R = M_P^2$, the following results are obtained

\begin{align}
1.8 \times 10^{17} \text{ GeV} &< R^{-1} < 5.5 \times 10^{17} \text{ GeV}, \\
2.6 \times 10^9 \text{ GeV} &< \langle S \rangle < 6.7 \times 10^{17} \text{ GeV}, \\
2.1 \times 10^9 \text{ GeV} &< m_3 < 3.9 \times 10^{17} \text{ GeV},
\end{align}

where the upper (lower) bounds of (31) ((32) and (33)) come from the dimension six proton decay constraints, while the other bounds come from the cutoff scale $\langle S \rangle$ ($\langle S \rangle < M_P$).

Note that the upper bound for $\langle S \rangle$ becomes more stringent since $\langle S \rangle$ depends on $M_5$ or $R^{-1}$ through the doublet Higgs mass in (23). A large $\langle S \rangle$ corresponds to a small $M_5$, namely a small $R^{-1}$ and vice versa. It is very interesting in that an intermediate scale triplet Higgs can be consistent with the proton stability. It should be stressed that the proton stability can be ensured not by symmetries but by the locality only, namely by strong suppression of coupling constants coming from overlap of wave functions between the triplet Higgs and the SM matter.

So far, we have discussed the doublet-triplet splitting and the proton stability in the case with localized singlet at $y = \pi R$. It is also interesting other situations. What’s happen when $S$ is localized at $y = 0$? In this case, the splitting is realized by

\begin{equation}
\delta(y) \left[ \frac{S}{M_P} H_1^H_2 \right] g^2 \Rightarrow \begin{cases} 
2 M_5 \frac{\langle S \rangle}{M_P} e^{-2M_5 \pi R} \\
M_5 \frac{3M_5}{M_P} 1 - e^{-3M_5 \pi R} \end{cases},
\end{equation}

then the ratio between the doublet and the triplet Higgs masses is modified as

\begin{equation}
\frac{m_3}{m_2} \simeq \frac{2}{3} e^{-2M_5 \pi R} \ll 1.
\end{equation}

The triplet Higgs mass is extremely smaller than the doublet Higgs mass since an overlap between the triplet Higgs localized on $y = \pi R$ and $S$ is very small. The following argument leads that this case is inconsisitent with the proton stability. Consider the dimension six operator constraints from the triplet Higgs boson exchange (27). The result is

\begin{equation}
\sqrt{\frac{2M_5}{M_P}} \langle S \rangle e^{-M_5 \pi R} > 10^{16} \text{ GeV}.
\end{equation}
One can easily check that (36) has no solution. Thus, this case is excluded.

Finally, we consider the case that $S$ is a bulk field and has a constant profile. In this case, the mass splitting is realized by both (23) and (34), which means

$$m_3 \simeq 2M_5 \frac{\langle S \rangle}{M_P}, \quad m_2 \simeq 3M_5 \frac{\langle S \rangle}{M_P} \simeq m_W.$$  \hfill (37)

Remarkably, the triplet Higgs and the doublet Higgs masses are comparable since exponentially suppressed contributions are negligible. Furthermore, the triplet Higgs mass is predicted\(^6\),

$$m_3 \simeq \frac{2}{3}m_2 \sim 67 \text{ GeV}.$$  \hfill (38)

The proton decay constraints in this case are

$$R^{-1} < 6.8 \times 10^{17} \text{ GeV (dim 5)},$$ \hfill (39)
$$R^{-1} < 3.5 \times 10^{17} \text{ GeV (triplet Higgs exchange)},$$ \hfill (40)
$$R^{-1} > 5.5 \times 10^{11} \text{ GeV (X, Y gauge boson exchange)}.$$ \hfill (41)

The allowed parameters are obtained as

$$5.5 \times 10^{11} \text{ GeV} < R^{-1} < 3.5 \times 10^{17} \text{ GeV},$$  \hfill (42)
$$160 \text{ GeV} < \langle S \rangle < 8.0 \text{ TeV},$$  \hfill (43)

where as in the case with $S$ localized at $y = \pi R$, a large $\langle S \rangle$ coresponds to a small $M_5$, namely a small $R^{-1}$ and vice versa. Thus, the bulk constant $S$ case is very interesting in that the extremely light Higgs triplets is consistent with the proton decay constraints and might provide an alternative signature of GUT in future collider experiments, as discussed in [6]. The detailed analysis of collider signatures of light Higgs triplets is recently presented in Ref. [14]. Before summarizing this paper, we briefly comment on the gauge coupling unification. As discussed in [6], we can consider the possibility that extra fields $5 + 5^*$ are introduced to recover the gauge coupling unification. However, in the present case, since SU(5) is broken at the cutoff scale $M_5$, we do not consider the gauge coupling unification.

In summary, we have presented a simple higher dimensional mechaism of the doublet-triplet splitting in a five dimensional SUSY SU(5) GUT on $S^1/Z_2$. The splitting of

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\(^6\)Taking into account SUSY breaking, triplet Higgsinos obtain an additional SUSY breaking mass.
multiplets is realized by a mass term of Higgs hypermultiplet which explicitly breaks SU(5) gauge symmetry. Depending on the sign of mass, zero mode Higgs doublet and triplet are localized on the opposite side of the fixed points. The mass splitting is realized due to the difference of magnitudes of the overlap with a brane localized singlet field not due to the boundary conditions. An unnatural fine-tuning of parameters is not necessary. As well as a conventional doublet-triplet splitting solution, it has been shown that the weak scale Higgs triplet is consistent with the proton stability. This result might provide an alternative signature of GUT in future collider experiments.

We would like to stress that the proton stability is ensured by locality without symmetries! In a recent orbifold GUT literature [15], dimension-5 baryon- and lepton-number violating operators are forbidden by a $U(1)_R$ symmetry. However, dimension-5 and -6 baryon- and lepton-number violating operators in our model are strongly suppressed by a small overlap of wave functions in spite of an orbifold setup is adopted.

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References

[1] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[2] E. Witten, Phys. Lett. B105, 267 (1981); A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B115, 380 (1982); B. Grinstein, Nucl. Phys. B206, 387 (1982); S. Dimopoulos and F. Wilczek, Santa Barbara preprint, NSF-ITP-82-07 (1982); K. Inoue, A. Kakuto and H. Takano, Prog. Theor. Phys. 75, 664 (1986); E. Witten, arXiv:hep-ph/0201018;
[3] Y. Kawamura, Prog. Theor. Phys. 105 999 (2001); Prog. Theor. Phys. 105 691 (2001); A. Hebecker and J. March-Russell, Nucl. Phys. B613, 3 (2001) [arXiv:hep-ph/0106166]; M. Kakizaki and M. Yamaguchi, Prog. Theor. Phys. 107, 433 (2002) [arXiv:hep-ph/0104103]; see also Refs. [5, 6].

[4] A. E. Faraggi, Nucl. Phys. B428 111 (1994) [hep-ph/9403312]; Phys. Lett. B520 337 (2001) [hep-ph/0107094].

[5] N. Maru, Phys. Lett. B522, 117 (2001) [arXiv:hep-ph/0108002].

[6] N. Haba and N. Maru, Phys. Lett. B532, 93 (2002) [arXiv:hep-ph/0201216].

[7] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D61, 033005 (2000) [arXiv:hep-ph/9903417].

[8] G. R. Dvali, G. Gabadadze and M. A. Shifman, Phys. Lett. B497, 271 (2001) [arXiv:hep-th/0010071].

[9] M. Kolanovic, arXiv:hep-th/0301116.

[10] H. Georgi, A. K. Grant and G. Hailu, Phys. Rev. D63, 064027 (2001) [arXiv:hep-ph/0007350].

[11] D. E. Kaplan and T. M. Tait, JHEP 0111, 051 (2001) [arXiv:hep-ph/0110126].

[12] Y. Grossman and G. Perez, Phys. Rev. D67, 015011 (2003) [arXiv:hep-ph/0210053].

[13] N. Arkani-Hamed, T. Gregoire and J. Wacker, JHEP 0203, 055 (2002) [arXiv:hep-th/0101233].

[14] K. Cheung and G. C. Cho, Phys. Rev. D67, 075003 (2003) [arXiv:hep-ph/0212063].

[15] See, for example, L. J. Hall and Y. Nomura, Phys. Rev. D64, 055003 (2001) [arXiv:hep-ph/0103125].