Probing the degree of coherence through the full 1D-3D dimensional crossover

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We experimentally study a gas of quantum degenerate $^{87}$Rb atoms throughout the full dimensional crossover, from a one-dimensional (1D) system exhibiting phase fluctuations consistent with 1D theory to a three-dimensional (3D) phase-coherent system, thereby smoothly interpolating between these distinct, well-understood regimes. Using a hybrid trapping architecture combining an atom chip with a printed circuit board, we continuously adjust the system’s dimensionality and explore a wide range of chemical potentials $\mu$ and temperatures $T$ and study the phase fluctuations through the power spectrum of density ripples in time-of-flight expansion. We find that while $\mu$ controls the departure of the system from 3D, inside the crossover the fluctuations are dependent on both $\mu$ and $T$, with an increasing dependence on $T$ as the system becomes more 1D. Ultimately, the fluctuations are shown to be determined by the relative occupation of 1D axial collective excitations.

The dimensionality of a system can have dramatic effects on its properties, giving rise to a plethora of interesting behaviours. The nature of superfluid and superconducting phase transitions is well known to be radically different in systems of one, two, or three dimensions. The Mermin-Wagner-Hohenberg theorem [1, 2] dictates that at finite temperature more than two dimensions are required for true long range order. The transition in two dimensions (2D) is governed by a Kosterlitz-Thouless mechanism [3] of topological origin, and for three dimensions (3D) it is the paradigmatic example of symmetry breaking that is well described, at least qualitatively, by mean-field theories. In one dimension (1D) no such transition exists, but due to the enhanced role of both quantum and thermal fluctuations there is a richer set of physical regimes in 1D systems of interacting particles than in 2D or 3D [4, 5].

The stark contrast of transition phenomena makes the study of a system that lies in between two distinct dimensions — referred to as a dimensional crossover — of great fundamental interest, as well as offering the potential for practical applications. A typical example is provided by layered superconductors, either naturally occurring [6] or artificially controlled [7], presenting instances of the 2D–3D crossover. While the 2D–3D crossover in superconductors has been extensively studied, producing superconducting samples in the 1D–3D crossover is technologically more challenging, but remains a subject of intense research, with the ultimate goal to realise new high temperature superconductors [8–12]. Alternatively, the 1D–3D crossover could be partially accessed with superfluid $^4$He inside carbon nanotubes and nanopores, with the 1D regime being reached when the transverse size becomes on the order of a few angstroms [13, 14]—which is currently very difficult to obtain [15].

Conversely, ultracold atom experiments are naturally suited to study the 1D–3D crossover, where the external trapping geometry can be flexibly tuned to constrain atomic degrees of freedom, providing the means to effectively manipulate the dimensionality. Examples include single magnetically trapped systems [16], or arrays of systems with tuneable coupling in optical lattices [17]. In particular, purely 1D systems can now be routinely formed by employing the extremely tight traps generated by atom chips [18–20]. A major difference to 3D systems is the presence of both density [21, 22] and phase fluctuations, the latter having been studied by several experiments in various limited regimes [23–28]. However, a comprehensive experimental mapping of the phase fluctuations in the entire crossover remains elusive, since few experiments have access to the necessary tunability of trapping geometry, atom number, and temperature.

Here, we map out the full 1D–3D crossover by characterising the phase fluctuations in individual degenerate Bose gases over a wide parameter space from a chemically 1D system (i.e. with $\mu \lesssim \hbar \omega_{\perp}$ where $\mu$ is the zero temperature chemical potential and $\omega_{\perp}$ is the transverse trapping frequency), through to the 3D regime with $\mu \gg \hbar \omega_{\perp}$, where the fluctuations smoothly disappear. Using a combination of an atom chip and printed circuit board (PCB), we are able to control independently the axial and radial confinement, which allows measurement of the phase fluctuations across a wide range of external trap aspect ratios, in addition to studying a range of temperatures and atom numbers. The properties of the in-trap phase fluctuations are studied by observing the power spectrum of the density ripples which form as the atomic cloud expands during time of flight. For 1D clouds, when $\mu/\hbar \omega_{\perp}$ is sufficiently small, the experimental results agree well with 1D theoretical predictions, and for larger $\mu/\hbar \omega_{\perp}$ we observe an increasing divergence from the 1D theory (Fig. 1). A simple text book argument suggests $k_B T/\hbar \omega_{\perp}$ as the single parameter to determine the effective dimensionality of the system, regardless of interactions [30]. To investigate the validity of this, we explore the influence of all parameters on the fluctu-
Our experiment uses $^{87}$Rb atoms prepared in the $|F = 2, m_F = 2\rangle$ magnetic sub-state which are loaded into a Ioffe-Pritchard style wire trap generated by a combination of an atom chip and a PCB. A conventional cylindrically-symmetric H-trap configuration is used [19], in which transverse confinement (in the x-y plane) is realised by a single DC current-carrying wire on the atom chip together with an external bias field. Independent axial confinement (along z) is provided by two parallel wires on the PCB which is situated underneath the atom chip. We vary the current in the atom chip and PCB wires to create a number of traps with varying aspect ratio $\kappa = \omega_\perp/\omega_z$, with final trapping frequencies in the range $\omega_\perp/2\pi = (570 - 1380)$ Hz transversely, and $\omega_z/2\pi = (15 - 34)$ Hz axially.

After loading pre-cooled atoms ($\sim 10\mu$K) into the H-trap, condensates of approximately $10^5$ atoms are produced after 1.5 s of radio-frequency (RF) evaporation, and are then held in an RF shield [31] with energy corresponding to 12 kHz above the trap bottom for a further 150 ms to ensure thermal equilibrium (corresponding to several tens to hundreds of collisions per particle for all traps considered [32]). An additional adjustable hold time of up to 700 ms is applied to allow for controllable losses through collisions with the background gases in the vacuum chamber, and thus varying the final atom number $N = (0.5 - 10) \times 10^4$. By adjusting the final RF evaporative cooling frequency the temperature of the sample can be set in the range $T = (70 - 540)$ nK. Optical density (OD) images are acquired via standard absorption imaging techniques [33] with a probe beam along the x-direction after a time of flight $t_{\text{tof}} = 34$ ms. In-trap phase fluctuations initially present in the gas are transformed into density ripples during the time-of-flight expansion. To suppress undesirable diffraction fringes in the OD images, we ensure optimal focusing of the imaging objective following the technique described in [34] (see Supplementary Material [35]). The insets in Fig. 1 show a set of typical OD images exhibiting the density ripples of varying strength dependent on dimensionality.

To quantitatively analyse the spatial frequency content of the images, we calculate the power spectrum of the density ripples using the following steps. First, several hundred OD images of clouds under a chosen set of experimental conditions are acquired and post-selected such that the standard deviation in atom number and temperature is approximately 5% of the respective value of the set. The thermal component of the gas is then fitted to each image to obtain the temperature [36–38], and is then removed before further analysis. Next, each column density image is integrated along the remaining transverse direction of the cloud (i.e. along y) to obtain the axial 1D line density $n_1(z)$, which is then subtracted from the mean of the set $\langle n(z) \rangle$, leaving only the residual line density $\delta n_1(z) = n_1(z) - \langle n(z) \rangle$ for each individual shot $i$. We then take the Fourier transform of the density residuals

$$\delta \tilde{n}_i(q) = \int \delta n_1(z) e^{-iqz} \, dz,$$

(1)

where $q$ is the angular spatial frequency, and use $\delta \tilde{n}_i(q)$ to calculate a dimensionless power spectrum for each individual realisation, normalised by atom number,

$$|\rho_i(q)|^2 = \frac{1}{N_i^2} |\delta \tilde{n}_i(q)|^2.$$

(2)
Finally, we compute the mean power spectrum for the ensemble $\langle |\rho(q)|^2 \rangle$, providing a single spectrum for each set of experimental conditions. This quantity is equivalent to the structure factor $S(q)$ used in studies of density correlations such as in superfluid $^4$He [39]. Here however, measuring the structure factor after time of flight provides information about the phase correlations in-trap.

Figure 1 shows examples of typical experimental data, together with the corresponding density ripples spectra, calculated according to the procedure outlined above. Cloud parameters $(N, T, \omega_\perp, \omega_z)$ are varied to move from a 3D condensate with no visible density ripples to a deeply quasi-condensate regime with strong density ripples. The theoretical predictions are generated using a stochastic model for the in-trap phase distribution that reproduces Bogoliubov results [29], and has been applied successfully in the 1D regime [40, 41], 2D regime [42], and elongated 3D regime (if the phase varies only axially) [43]. Multiple such realisations of a one-dimensional phase $\phi(z)$ are generated and then imprinted onto the zero-temperature ground state wave function $\psi(r) = \sqrt{n(r)}e^{i\phi(z)}$, constituting an ensemble of initial states (see Supplementary Material [35]). To obtain the density profiles after time of flight in the absence of interactions during expansion (pure ballistic expansion), we numerically propagate the initial states using the free Schrödinger equation. The density ripple power spectra are then extracted in the same manner as for the experimental data. In order to account for modifications due to the effects of mean-field interactions, we additionally propagate the same initial states using the full 3D Gross-Pitaevskii equation (GPE), with the corresponding spectra also shown in Fig. 1 for comparison. Such simulations are computationally demanding, and so an analytic hydrodynamic scaling approach has been previously used for clouds with little axial expansion (although with a discrepancy between the simulated and measured size of the fluctuations) [24]. Alternatively, experiments can be restricted to the simpler case in which it is valid to neglect interactions during expansion, limiting studies to the case of high aspect ratios of typically $\gtrsim 100$ [26, 28]. Using the combination of a graphics processing unit (GPU) together with the algorithm developed in Ref. [44] we are able to directly simulate the full TOF expansion for several thousands of realisations [35].

At the 1D side of the crossover ($\hat{\mu} \lesssim 1$) the quasi-condensates display large phase fluctuations and both theoretical models (with/without interactions) show excellent agreement with each other and with the experimental data in Fig. 1, justifying a non-interacting expansion in this regime. On the opposite side, the fluctuations at the 3D end of the crossover are suppressed [25, 45], as expected for a true 3D BEC in which there is long-range order, with full phase coherence across the ensemble even at finite temperature [46–49]. In between these two well-understood distinct regimes, we measure significant fluctuations even when $\hat{\mu}, \hat{T} \ll 1$ is not satisfied, consistent with previous results [23, 24] where it was found that such systems can acquire some 1D characteristics (typical of a quasi-condensate) [43] when the phase coherence length is smaller than the extent of the sample [50]. Neither theoretical model fully accounts for the observed drop in amplitude — however the inclusion of interactions captures well the shift in peak position to lower spatial frequency. The experimentally measured power spectra smoothly interpolate between the expectations for the 1D and 3D limits, with progressively closer agreement with the 1D stochastic model as the 1D regime is approached.

In our analysis two quantities are extracted - the position of the first maximum $q_{pk}$ and its average peak amplitude $\langle |\rho(q_{pk})|^2 \rangle$. These are obtained by fitting to the experimental spectra with the analytical result provided in [51] modified with a local density approximation to account for the inhomogeneous density profile.
of the condensate [28]. The relevance of the peak position is discussed in the supplementary material [35]. The average peak amplitude (|⟨ρ(qpk)⟩|^2) is a pertinent quantity to monitor in the crossover regime as it directly relates to the strength of the fluctuations—and thus, indirectly, the dimensionality of the system—while being only affected by interactions during time-of-flight expansion towards the 3D regime [52]. To identify the relevant parameters and understand their role in governing dimensionality of the physical system, we studied the dependency of (|⟨ρ(qpk)⟩|^2) on: the ratio of cloud length to thermal phase coherence length $\tilde{L} = L/\lambda_T$; aspect ratio of the trapping frequencies $\kappa = \omega_\perp/\omega_z$; reduced chemical potential $\tilde{\mu}$; and reduced temperature $\tilde{T}$. Here the chemical potential is determined from a local density approximation which gives $\mu = \hbar \omega_\perp \left(\sqrt{1 + 4an(0)} - 1\right)$, and the thermal phase coherence length is defined as $\lambda_T = 2\hbar^2 n(0)/mk_BT$.

The results are shown in Fig. 2. The aspect ratio $\kappa$ is often seen as a measure of dimensionality [23, 24, 45, 53], where a high (low) aspect ratio is a feature of a 1D (3D) system. Our data, however, indicate that this is not a critical parameter, and that even at high aspect ratios it remains possible to form coherent 3D gases as long as $\tilde{\mu}, \tilde{T} \gg 1$. As can be seen in Fig. 2a, at a fixed aspect ratio the size of the fluctuations can vary dramatically, i.e. the dimensionality is not solely driven by $\kappa$. In a 1D gas at finite temperature, the two-point phase correlation drops exponentially with the distance between two points. The characteristic length scale of this decay is $\lambda_T$. In practice, experiments (including the ones reported here) explore systems with finite length $L$, so that the ratio $L/\lambda_T$ becomes a quantity that determines whether or not phase fluctuations are actually observed [43, 50, 54, 55]. For $L \ll a$ a 1D system can have the appearance of a 3D system, but this can be interpreted as a finite size effect rather than a consequence of changed dimensionality. In an actual 3D gas, the quantity $\lambda_T$ loses its physical significance and phase fluctuations vanish as long as $T$ is below the critical temperature for condensation. Our data, displayed in Fig. 2b, show an expected reduction of (|⟨ρ(qpk)⟩|^2) as $L$ decreases. At higher values of $L$, a spread of values for (|⟨qpk⟩|^2) occurs, supporting the notion that $L$ on its own is not indicative of the dimensionality of the system.

The one-dimensional regime is reached when the transverse fluctuations are chemically and thermally frozen out, i.e. $\tilde{\mu}, \tilde{T} \ll 1$ [20]. Figure 2c shows the amplitude of the phase fluctuations over the $\tilde{\mu}, \tilde{T}$ parameter space explored, and shows that $\tilde{T}$ plays little role in determining the size of the fluctuations when the system is chemically 3D. For example, when $\tilde{\mu} \gtrsim 4.5$ the fluctuations are always small regardless of the large variation in $\tilde{T}$. However, when $\tilde{\mu}$ decreases the importance of $\tilde{T}$ on the fluctuations in the system increases, as can be seen in the inset of Fig. 2c. These observations indicate that in the crossover regime the role of $\tilde{\mu}$ dominates that of $\tilde{T}$, quantitatively supporting previous qualitative statements in the literature [16, 56].

To further understand the driving parameters for the phase fluctuations, we alternatively examine the data in terms of the occupation of excitations present in the system. Excitations of the quasi-condensate can be split into two categories - high-energy ($\epsilon_j > \hbar \omega_\perp$) free particle-like, and low-energy ($\epsilon_j < \hbar \omega_\perp$) axial phonon-like excitations. The latter exhibit a 1D character since their wavelengths are larger than the radial size of the cloud but smaller than its axial size [43]. Thus, only the low-energy excitations contribute to fluctuations of the phase. The axial spectrum of low-energy excitations for an elongated 3D condensate is $\epsilon_j = \hbar \omega_z \sqrt{j(j+3)}/4$ [57], and if $k_BT \gg \hbar \omega_z$ the occupation of each mode $j$ can be approximated to $N_j = k_BT/\epsilon_j$. We propose that the relevant quantity is the relative total population of 1D excitations present in the system, $N_{1D}/N$, where

$$N_{1D} = \sum_j \frac{\epsilon_j < \hbar \omega_\perp}{k_BT/\epsilon_j}. \quad (3)$$

In other words, this quantity compares the number of quasi-particles contributing to the phase fluctuations with the total number of atoms in the quasi-condensate. Experimentally we expect this to be proportional to the contrast of the observed density ripples, and thus lead to a linear relationship between (|⟨ρ(qpk)⟩|^2) and $N_{1D}/N$. The

FIG. 3. Dependency of the amplitude of the density ripple power spectrum (|⟨ρ(qpk)⟩|^2) on the relative number of 1D excitations $N_{1D}/N$. A fit to the power law function (|⟨ρ(qpk)⟩|^2) = $A(N_{1D}/N)^\alpha + C$ (red dashed line) is applied to the data (blue circles), which gives $\alpha = 0.997 \pm 0.001$, and the shaded area gives the 95% prediction band of the fit. Error bars for the data points represent the statistical uncertainty as two standard deviations from the mean, and are obtained by bootstrapping.
two quantities are plotted in Fig. 3, where we have fitted a power law function to the data, with power $\alpha$. We find $\alpha = 1$ within errors, thus supporting a linear dependency. In contrast to when the data is plotted against aspect ratio or relative thermal phase coherence length (Fig. 2), it is striking that our data, which cover a broad experimental parameter space, now appear to collapse onto a single, approximately linear curve. This is a strong indication that $N_{1D}/N$ is the more relevant quantity to predict the strength of the phase fluctuations.

In conclusion, we have experimentally studied the onset of phase fluctuations in degenerate Bose gases in equilibrium through the 1D–3D dimensional crossover, by characterising the density ripples power spectrum in free expansion. We have observed that the previously developed stochastic model based on exponential decay of the phase correlations correctly describes the data in 1D, but shows a gradual departure as the dimensionality is tuned towards 3D. On the 3D side, an almost complete suppression of fluctuations is measured, as expected for a 3D BEC with uniform phase. Several parameters which can conceivably affect the dimensionality were investigated, with $\tilde{\mu}$ found to determine whether the system is able to exhibit 1D character, rather than trap aspect ratio, relative thermal coherence length, or $T$. Indeed, for $\tilde{\mu} \lesssim 4.5$ the gas exhibits phase fluctuations with a strength that is clearly dependent on $T$, whereas for larger values of $\tilde{\mu}$ the system appears effectively 3D, regardless of $T$—as expected, since in 3D long-range order is possible even at finite temperature. The temperature dependence in the low-$\tilde{\mu}$ regime is understood in terms of the number of low-energy axial modes that can be populated. Finally, we have shown that a better predictor for the strength of the phase fluctuations is indeed the relative population of these modes. An important extension to this work will be to realise a similar experiment in a non-equilibrium setting with rapid changes from 1D to 3D and vice versa. It would also be of great interest to experimentally explore all possible crossovers involving any dimension, from 0D (where excitations are frozen along all directions) to 3D. Such a set-up can be realised with cold atom systems by combining established techniques, including atom chips and optical-dipole traps.

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INTERACTIONS DURING TIME-OF-FLIGHT

In one-dimensional (1D) systems the in-trap interaction energy is small and is quenched quickly during time-of-flight (TOF) \( t_{\text{tof}} \), and it has been shown that in this case a purely ballistic expansion model is sufficient to describe the density ripples [1, 2]. However, for clouds in the 1D-3D crossover the influence of the mean-field interaction during TOF is not negligible, and the expansion should described by the Gross-Pitaevskii equation. In the early stages of expansion (\( t_{\text{tof}} \lesssim 1/\omega_\perp \)) the relative amount of interaction energy present is significant, and therefore the cloud undergoes hydrodynamic expansion. At longer times (\( t_{\text{tof}} \gg 1/\omega_\perp \)) the density has reduced significantly and the interaction energy is effectively gone, and the cloud then evolves according to the linear Schrodinger equation with the expansion being said to be ballistic.

In the case of ballistic expansion, the transverse and axial dimensions are separable, and the main contribution to the axial velocity distribution comes from the axial gradient of the in-trap phase \( \phi(z, t = 0) \), given by

\[
v(z, t) = \frac{\hbar}{m} \nabla \phi(z, t),
\]

where \( m \) is the atomic mass. After time-of-flight this gives rise to interference and the formation of density ripples. The phenomenon is the matter-wave analog of the temporal Talbot effect [3–9]. Since in trap the phase effectively fluctuates at all spatial frequencies \( q \), each particular \( t_{\text{tof}} \) will be equal to the Talbot time \( t_{\text{Talbot}} \) for some specific value of \( q \), effectively amplifying the power spectrum \( \langle |\rho(q)|^2 \rangle \) at that \( q \). In fact, fractional Talbot times also exist, i.e. \( t_{\text{Talbot}}/n \) for \( n = \{1, 2, 3, \ldots \} \), and as such the power spectrum displays multiple discretely spaced peaks with decreasing amplitude - an example is shown in Fig. 1. The \( n^{\text{th}} \) peak position \( q_n \) is approximately given by [1],

\[
q_n = \sqrt{\frac{\pi m (2n-1)}{h t_{\text{tof}}}},
\]

and becomes more accurate for large \( n \) (see Fig. 1). Accounting for finite optical imaging resolution suppresses all but the first peak \( q_1 \) (to avoid confusion in the main text we refer to this peak \( q_1 \) as \( q_{\text{pk}} \)).

To account for the hydrodynamic expansion, we perform a numerical GPE calculation as follows. For a given atom number and trapping geometry we first numerically find the zero-temperature ground state of the system using the split-step Fourier method [10–13] in imaginary time [14, 15]. To model the effect of finite temperature we then imprint on this ground state wave function a phase that fluctuates axially but is invariant along the transverse direction. The phase is generated by a stochastic method which reproduces the correlation statistics from the Bogoliubov theory

\[
\langle (\phi(z) - \phi(0))^2 \rangle = \frac{z}{\lambda_T},
\]

where \( \lambda_T = 2\hbar^2 \rho(0)/mk_B T \) is the phase coherence length, and works equally well in both the 1D [16] and elongated 3D regimes [17].

Next to calculate the time-of-flight expansion the wave function with fluctuating phase is evolved in real time using the GPE with the trapping potential removed.
until the interaction energy becomes negligible (typically 1 – 6 ms, depending on the transverse trapping frequency). We then expand the wave function ballistically (ignoring interactions) using the free Schrödinger equation out to the final time-of-flight of 34 ms [18]. The process is repeated with 200 realisations of random phases, and the resulting simulated images are then analysed in the same way as the experimental data to obtain the power spectrum of the density ripples \( |\rho(q)|^2 \). Examples of power spectra for clouds in the 1D and 3D regime, with \( \tilde{\mu} = 0.8 \) and 7.4 respectively, are shown in Fig. 2a–b. In this figure, we also compare the full GPE calculation with the pure ballistic expansion model to reveal the effect of interactions. As is apparent in the 3D case the interactions not only suppress the peak amplitude of the power spectrum but also shift it to a lower spatial frequency. In contrast, the influence of the interactions in the 1D regime is almost negligible.

The effect can be understood as follows, during the period of hydrodynamic expansion repulsive interactions between atoms induce an acceleration, broadening the velocity distribution, and cause the density ripples to spread out further than they would if the expansion was ballistic. This interaction induced spreading of the density ripples effectively suppresses the power spectrum not only reducing the amplitude but also shifting the spectrum to lower spatial frequencies, as can be seen in Fig. 2b. To observe this effect we extract the position of the first maximum \( q_1 \) from all of the experimental data, the results are shown in Fig. 2c. We observe a clear correlation in peak position with the reduced chemical potential \( \tilde{\mu} \) through the 1D-3D crossover and find clear agreement with our GPE simulation. Thus, the significance of interactions increases as the dimensionality gradually changes from 1D to 3D and should be accounted for when modelling systems within the 1D-3D crossover. However, they can be safely ignored close to or inside the 1D regime. The same shifting of the spectral peak positions was predicted and observed in 2D degenerate Bose gases [19, 20], however there was a small disparity between the prediction and observation.

**IMAGING**

To focus our optical imaging system on the plane of the atomic cloud we have used the technique described in [21]. This method utilises the sensitivity of the power spectrum \( |\rho(q)|^2 \) to defocusing effects which can be detrimental to measurements of phase fluctuations. As the imaging system is moved out-of-focus by a distance \( d \) from the focal plane the density ripples are blurred, and additional fringes appear since it is now rather the near-field diffraction pattern which will be imaged onto the camera. In the power spectrum this results in an attenuation of the amplitude and the creation of additional higher frequency maxima. The modified power spectrum \( |\rho(q, d)|^2 \) is related to the in-focus power spectrum \( |\rho(q, d = 0)|^2 \) by

\[
|\rho(q, d)|^2 = |\rho(q, 0)|^2 \cos^2 \left( \frac{q^2 d}{2k_0} \right), \tag{4}
\]

where \( k_0 \) is wave number of the probe light. A measurement of \( |\rho(q, d)|^2 \) is shown in Fig. 3, where the second and third order peaks (arising from the diffraction fringes) can be seen clearly as the system is moved out of focus. The optimal focal position \( d = 0 \) is determined by fitting the higher order peaks with Eq. 4.

Finite optical resolution can be accounted for in a simple approximation convolving the simulated density ripples with the point spread function (PSF) of the imaging system. The PSF of a diffraction-limited imaging system has the functional form of an Airy disk, which we approx-
FIG. 3. Power spectrum of the density ripples $\langle |\rho(q)|^2 \rangle$ across the focus. At each position $d$ the power spectrum $\langle |\rho(q)|^2 \rangle$ is calculated from approximately 30 images. Green (Red) circles (squares) are the measured second (third) peak positions, and the green (red) solid (dash-dotted) line is a fit using Eq. 4. The optimal focus position is indicated by $d = 0$ mm with an error of $\pm 20 \, \mu m$.

imate by fitting a Gaussian to the central lobe. Convolving the density ripples with the Gaussian-approximated PSF modifies the experimentally detected power spectrum in $q$-space, which is then given by

$$\langle |\rho(q)|^2 \rangle_{\text{exp}} = \langle |\rho(q)|^2 \rangle \exp \left( -\sigma_{\text{psf}}^2 q^2 \right),$$

where $\sigma_{\text{psf}}$ is the RMS width of the Gaussian. To obtain the value of $\sigma_{\text{psf}}$ we fitted measured power spectrum of clouds in the 1D limit $\tilde{\mu} \lesssim 1$ where the effect of interactions during expansion are negligible, yielding a value of $\sigma_{\text{psf}} \sim 4 \, \mu m$.

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