Convection and segregation in a flat rotating sandbox

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Abstract. A flat box, almost completely filled with a mixture of granulate, is rotated slowly about its horizontal central axis. In this experiment, a regular vortex flow of the granular material is observed in the cell plane. These vortex structures have a superficial analogy to convection rolls in dissipative structures of ordinary liquids. Whereas in the latter, the origin of the convection can often be attributed to gradients e.g. of densities or surface tensions, there is no trivial explanation at present for the convection of the granulate in the rotating container. Despite the simplicity of the experiment, the underlying mechanisms for convection and segregation are difficult to extract. Here, we present a comprehensive experimental study of the patterns under various experimental conditions and propose a mechanism for the convection.

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1. Introduction

Granular media are ubiquitous in everyday life, e.g. in the form of food and pharmaceutical products, in mining, agriculture or in the construction trade. In many industrial branches, products are packed, transported and distributed in granular form. Even geophysical processes are often related to the granular structure of soil. From an engineering point of view, packing properties and granular dynamics under external agitation pose a multitude of problems. Under shearing, vibration or rotation, granular matter in some respects may behave similarly to liquids, but often, unexpected and even counterintuitive effects are observed ([1–4] and references therein).

Moreover, even relatively simple equipment can produce a rich variety of novel and challenging experimental phenomena that are beyond our current understanding. That is what makes granular science sometimes quite exciting. For example, if a bidisperse mixture of differently sized grains is rotated horizontally in a cylindrical drum, a complex scenario of segregation processes is found. Radial segregation takes place within a few revolutions. The smaller particles form a core in the central drum area. In long tubes, an additional axial segregation can be observed at longer time scales. The segregated components form axial stripe patterns. Merging of these stripes (coarsening) can follow on ever longer timescales.
Axial demixing in a horizontal rotating container was first described by Óyama [5] in 1939. However, the effect was known to engineers even earlier (see the discussion in [3]). Nowadays the rotating drum represents a standard system to study granular segregation, and has become one of the classical experiments in granular physics. In addition to cylinders [3, 6–16], axial segregation was also observed in containers with square cross sections [6, 15] and in conical [17, 18] or spherically shaped tumblers [19, 20]. For a general review of pattern formation in rotating drums, see [21].

Our experiment differs from previous studies in two respects. Firstly, the three-dimensional (3D) geometry is replaced by a quasi 2D thin rectangular box. Secondly, the cell is packed with granulate at unconventionally large fill ratios, so that almost no fluidization of the grains is possible. Under these conditions, we observe effects that are absent in loosely filled containers. We find convection rolls that differ from other known patterns in granular media. This system was introduced recently [22]. In the present study, we describe various aspects of the experimental patterns. We propose a mechanism for the convection mechanism from these observations.

2. The setup and a general description of the experiment

The experiment is depicted schematically in figure 1: a flat transparent container is slowly rotated about its central horizontal axis, and the typical rotation rate is 20 rpm. Experiments have shown that the results are comparable for rotation rates at least between 1 and 50 rpm. Inertia influences are negligible (Froude number <0.025 at 20 rpm). The sample widths are between \( w = 6 \) cm and 50 cm, and heights range from \( h = 3 \) cm to 8 cm. The container depth was fixed at \( d = 5 \) mm, except for the experiments in section 3.3. The dimensions \( w \times h \times d = 50 \text{ cm} \times 8 \text{ cm} \times 5 \text{ mm} \) shall be assumed as default if no other size is explicitly given.

Pattern formation and dynamics are extracted from transmission images taken at regular intervals. Typically every 20 full turns, the rotation is gradually slowed down until the cell stops in an upright position. A camera takes a picture in transmitted light before the rotation continues. The front of the cell is defined here as the side whose upper edge moves towards the observer during the rotation. The rotation sense indicated in figure 1 will be considered as standard unless otherwise stated.

In most experiments, the cell is filled with a mixture of \( (350 \pm 50) \mu \text{m} \) and \( (900 \pm 100) \mu \text{m} \) diameter roughly spherical soda-lime glass beads (MGL type, Eisenwerk Würth) of density \( \rho = 2.490 \pm 0.001 \text{ g cm}^{-3} \) (Quantachrome Ultrapynometer 1000) in the 50%/50% volume ratio. In
some experiments, we use slightly different mixtures. The principal effects do not depend on the exact mixture composition. The fill level $C$ of the container plays a crucial role and is defined as the ratio of the net volume of glass beads, determined from their weight and density, to the entire volume of the box. Granular motion decreases with increasing $C$. A thin fluidized layer forms only when $C$ is smaller than $\approx 0.66$. For higher fill ratios, no global motion can be observed but individual particles are able to rearrange among their neighbors. The width of the fluidized layer is related to the depth $d$ of the container, and increases with the free volume above the granular bed.

Two distinct phenomena are observed in the flat cell. They differ completely in the resulting segregation patterns and in the motion of granulates. The transition between them is found at a critical fill height $C_C \approx 0.60$, i.e. at a relatively high fill ratio where fluidization is almost impossible.

At lower fill ratios $C < C_C$, chute flow occurs every time the cell plane passes a certain tilt angle (roughly the angle of repose) during rotation. This leads to an axial segregation just like in usual cylindrical drum experiments, connected with a local modulation of the fill height (figure 4(a), cf also [7, 8]). At long time scales, the stripe pattern coarsens. In contrast to cylindrical mixer geometries, the dynamics is limited to two avalanches during a full rotation (360°). If the fill ratio exceeds $C_C$, convection rolls associated with another type of segregation emerge (figure 2). The limited free space allows the formation of only two shallow fluidized zones on the top and bottom edges (marked in figure 2(b)). These fluidized zones sandwich the segregation pattern. When the fill ratio is increased further, the flow amplitude decays to zero at $C \approx 0.66$.

The number of rolls is roughly given by the container aspect ratio $w/h$. It can adopt even or odd values. It may vary slightly during an experiment when individual rolls or roll pairs are formed or extinguished. Even single rolls are possible when the aspect ratio is close to one. The maximum number of rolls is principally only limited by the aspect ratio of the container, and we observed up to 16 rolls in long cells.

The focus of this study is on highly filled cells where convection is found. Axial segregation and chute flow will be mentioned only in passing. An example of a convection roll is shown in figure 2. Four rolls are decorated by dark bands enriched with small beads. Fluidized regions are seen at the bottom and top edges.

In order to explore the granular packing in more detail, we have carried out x-ray transmission measurements of a fully developed convection roll in the cell. Figure 3 shows an optical image of a typical segregation structure (a) and an x-ray transmission image (b) of the same region. As one can see, the x-ray transmission image is very sensitive to the local composition of the granular mixture. In the x-ray image, the optically darker regions appear systematically brighter than the rest, i.e. the absorption is lower. In a control experiment, we have verified that small beads and large beads have the same absorption characteristics in monodisperse samples. This indicates that the regions enriched with small beads must have a lower packing density than the optically brighter regions containing a more balanced mix of small and large beads. It is well known that a bidisperse mixture of spherical grains will always have a higher packing density than a monodisperse ensemble of spheres (e.g. [23] and

\begin{footnote}{The definition of the fill level should not be confused with usual definitions of packing density where the occupied volume is related to the bulk of the particles. The values for $C$ and $C_C$ depend on the parameters of the cell and beads. The values given here correspond to a box with default dimensions and granular composition as given in the text.}

\end{footnote}
Figure 2. Example of convection rolls in a densely filled container after 18,000 rotations. We present the front view (top) and the corresponding rear view (bottom) of the same experiment. The bottom image has been mirrored horizontally to facilitate direct comparison of the patterns: only minor differences are seen between the front and the back. The horizontal dark bands, enriched with small particles, evidence slight differences of both container sides in regions where the flow is upwards/downwards. The container border is indicated by a black line. The small gap on top of the granulate corresponds to the free space in the container cell. In the bottom image, the separation between fluidized regions along the outer edges and the central convection region is marked by a dashed white line \((w \times h \times d = 160 \text{ mm} \times 40 \text{ mm} \times 5 \text{ mm}, C = 0.625)\); see the movie 1 (available from stacks.iop.org/NJP/14/015001/mmedia).

Figure 3. Convection roll seen in transmission in visible light (a) and contrast enhanced x-ray \((\lambda = 0.05 \text{ nm})\) (b). The sense of convection is clockwise. Brightness indicates the number of photons that hit the detector. In the optical image, regions enriched with small particles appear dark as light is scattered more intensely. In the x-ray image, regions enriched with small particles appear brighter, indicating lower absorption. Both pictures show roughly the same global pattern. The slight warp in the x-ray image is a device artefact.
references therein). Here, the packing of the regions enriched with the smaller beads is obviously closer to the lower packing density of monodisperse grains than the rest of the cell. There are two reasons: firstly, the packing density curves as a function of mixture composition are in general asymmetric, with the maximum shifted to a higher content of large particles. Secondly, the dark regions cover less than 25% of the cell area, whereas the brighter regions cover the rest. Thus, the relative excess of small particles in the dark regions is higher than the relative excess of large beads in the bright regions.

We are aware of three studies that deal with rotation experiments using highly filled long cylindrical drums [9–11]. Nakagawa et al [9] used magnetic resonance imaging to discover radial ring-like segregation and axial migration of particles inside drums with a high fill level. Kuo et al [10] ‘froze’ and cut drums that had been rotated at high fill levels to conclude indirectly on the existence of some kind of convective currents. Inagaki and Yoshikawa [11] conjectured a global convection roll in drums of this type. In simulations of a truly 2D cell, Awazu [24] reported global convection in a single, elongated roll. However, even though Awazu also considered a flat, almost full cell, these numerical results are too far from the situation in our experiment to compare them directly. In addition to these experiments at a high fill level, Losert et al [16] also speculate that slow convective currents occur in axial segregation bands in half-filled rotating drums.

In the following, we provide a thorough characterization of the convection phenomena in almost completely filled rotating containers. Of course, the definition of an ‘almost filled’ container is vague. In the context of this paper, we use this phrase to describe containers that are filled above the critical fill level for convection patterns, where chute flow effects are suppressed. We demonstrate how the convection depends on external parameters. Our aim is not to examine quantitative effects of parameter changes. Rather we are interested in general properties that may help us to elucidate the convection mechanism.

3. Characterization of the convection

3.1. Variation of fill level: transition from axial segregation to convection rolls

In our previous study we have demonstrated on the basis of time–space plots the qualitative differences between both dynamical regimes, chute flow and convection, at different fill ratios (figure 3 in [22]). Here we show in a simpler, direct way that a transition between both regimes can be triggered if one changes the container volume during an experiment. First, an axial segregation pattern develops at a low filling state (\(C \approx 0.52 < C_C\)) (figure 4(a)). The motion of the beads follows primarily the sense of the container rotation. This pattern is persistent for a period of several thousands of rotations. Then, the height of the container is decreased to obtain a higher filling state (\(C \approx 0.64 > C_C\)). After this reduction, the stripe pattern almost instantly transforms (figure 4(b)). The smaller beads (dark region) first migrate towards the outer zones and later they are drawn into the central region by convective motion (figure 4(c)). It is possible that the undulation of the top edge of the granulate supports the onset of the convection structure. However, we have not tested this idea further. Convection starts in uniformly filled containers as well (see below). When the container height is increased again to restore the original low fill level, the system returns to the chute flow regime and axial segregation. This demonstrates strikingly how transitions between chute flow and convection can be initiated.
Figure 4. (a) Axial segregation pattern after 10,000 rotations in a container with a moderate fill level \((C \approx 0.52)\). The pattern formed during the first 1000 rotations and remained stationary afterwards. (b) After a further 2000 rotations with a reduced container height, \(C \approx 0.64\), the material is redistributed. Small beads accumulate at the top and bottom sides and the stripe structure dissolves. (c) A convection structure is clearly established after 5000 more rotations. Regions enriched with small bead stripes decorate the rolls; see the movie 2 (available from stacks.iop.org/NJP/14/015001/mmedia).

3.2. Influence of initial preparation

Most techniques of filling the container with the premixed beads lead to an unavoidable initial segregation, often similar to stratification patterns in thin cells filled from one side [25]. A typical example can be seen in figure 8. These inhomogeneities of the initial state are, however, not essential for the formation of rolls. In order to demonstrate this, we have prepared a well-defined demixed initial state where the larger species is stratified below the smaller beads (figure 5(a)). We leave enough space for sufficient fluidization. After 70 rotations, the beads are satisfactorily well mixed (figure 5(b)). If the experiment were to continue in this geometry, the beads would start to segregate axially during further rotation. However, we decrease the volume as described previously, to enter the convection regime (figure 5(c)). Thus we can conclude that the initial state of mixing is not relevant for the onset of convection.

3.3. Influence of the cell thickness

In order to answer the question of whether the observed structures are unique to a particular cell thickness, experiments with variable thicknesses \(d\) have been performed. In thin cells, down to about 3 mm, the convection mechanism continues to exist. Even in a quasi-2D monolayer of beads in a flat container, we find flow and segregation structures; however, these are beyond the scope of the present study.

Convection was also found in cells of 10 mm thickness. With increasing cell thickness, the convection patterns are more difficult to observe with our transmission technique. In a thick sample cell \((d = 20 \text{ mm})\) the granulate structure looks different: the fluidized edges (section 2)
Figure 5. (a) Demixed initial state to avoid non-uniform filling. Small beads (dark region) are stacked above large beads (bright region). $C = 0.522$. (b) After 70 rotations the beads are well mixed. (c) After the cell volume was reduced, a convection pattern forms. The image shows the fully developed pattern after 25,000 rotations. $C = 0.630$; see movies 3 and 4 (available from stacks.iop.org/NJP/14/015001/mmedia).

grow with cell thickness, so that in the central part of a 50 mm high cell of 20 mm thickness, only a narrow core of jammed granular material remains. There is at least one reference that mentions currents in the core of the granular bed, in a rotating cylinder of a high fill ratio [10]. However, the structure of the currents in that geometry is completely different from that observed in the flat cells.

3.4. Role of the interstitial medium

In vibrated systems, it has been demonstrated that air and viscous drag may play a non-negligible role in the formation of granular convection structures [26, 27]. Even though the velocity of grain motion in our experiments is much lower, it is interesting to examine the influence of air on our experiment. So far, all the experiments described above have been performed under atmospheric air pressure. In order to qualify the role of the interstitial fluid, we have tested the effect of reduced air pressure. We had to modify the setup for the evacuation experiment. A container ($w \times h \times d = 140 \text{ mm} \times 35 \text{ mm} \times 5 \text{ mm}$; with small slits at the top and bottom sides) is enclosed in an outer vessel consisting of a large glass cylinder (inner diameter 20 cm, length 20 cm) and two aluminum side walls. The side walls contain orifices for the vacuum pump and a pressure gauge. The rotating cell inside is driven by a motor outside of the cylinder, with the driving axis led through a vacuum feedthrough. As in any granular experiment, we cannot measure the local pressure inside the granular bed. The actual pressure may be somewhat higher than the pressure given by the gauge because of continuous degassing of the large granular surface. We evacuated the setup until a sufficiently low inner pressure level was established. The minimum pressure obtained with our pump was 3 Pa. At this pressure, the granular convection showed no systematic differences from the dynamics at ambient pressure. The variation of convection velocities between low pressure and atmospheric...
Figure 6. Time–space plot of granulate in (a) vacuum and (b) water, derived from the vertical flow component in the horizontal cell midplane (cf the dashed line in figure 1). Blue and red colors stand for downward and upward motions, respectively. Container dimensions are 14 cm × 3.5 cm × 5 mm. The fill ratio was $C = 0.61$ in the vacuum experiment and $C = 0.60$ in the underwater experiment.

Pressure experiments was within the fluctuations observed for different runs of the experiment at ambient pressure.

In the following plots we characterize the granular dynamics quantitatively by the vertical flow component of the convection patterns. We construct space–time plots from the spatial profiles of this flow component, taken from a horizontal line in the middle of the front views along the rotational axis. The local flow velocity is determined with the following procedure: we evaluate correlations of grain patterns between consecutive images in 8 mm × 8 mm regions. As in figure 1, red and blue colors correspond to upward and downward flows, respectively. Similar plots for a uniform rotation sense of the cell can be found in [22] for different fill levels of the cell.

The convection dynamics are quantitatively characterized in figure 6(a). In the vacuum experiment with a container of aspect ratio 4 we find between 4 and 5 rolls. Such a long-term fluctuation of the number of rolls in the convection system is also common in the atmospheric pressure experiments (see next section).

The same setup was used to immerse the complete container and the granulate in water, with a small amount of detergent added. In order to avoid trapped air bubbles in the container that would inevitably disturb the grain motion, the following procedure was found useful: As in the vacuum experiment, the setup was evacuated before the start of the experiment, and simultaneously water was soaked into the cylinder enclosing the experiment. The presence of water did not influence the qualitative findings for the granular dynamics in the rotating flat cell (an exemplary structure has been used to construct figure 1). Moreover, in this test...
we can rule out the influence of electrostatic charges on the basic convection mechanism. The quantitative results are found in figure 6(b). Again, four to five rolls are formed in a container with aspect ratio 4. Convection flow amplitudes are of the same order of magnitude as in the vacuum experiment.

3.5. Modification of the rotation scheme

In the standard experiment, the sense of rotation is maintained. This means that the grains slide in a defined direction when one of the cell planes is on the top side. This broken symmetry is manifest in details of the segregation texture. Figure 2 shows circular assemblies of small beads on the front and rear sides. Moreover, one acknowledges narrow dark bands of small beads that are accumulated on the front side of the rotating cell in regions where convective flow is directed upward. At the rear side, these bands are located in regions where flow is downward. After each half rotation of the cell, the front and rear sides as well as up and down flows alternate, so that the structures on both sides of the cell are equivalent, but the above-mentioned asymmetry prevails.

It is therefore interesting to study the influence of a reversal of the rotation sense. First, we have checked how the flow changes at long terms when we sporadically reverse the sense of rotation of the cell (figures 7(a) and (b)). Reversal of the rotational sense in most cases only slightly disturbs the lateral positions of the convection currents. With the exception of the first reversal in figure 7(a), the global roll structure and convection velocities are largely preserved after the reversal. The rolls may subsequently drift in the cell, but there appears to be continuity of the convection when the rotation of the cell is reversed sporadically. Locally, details of the flow profiles may fluctuate and the flow amplitude as well as the number of rolls may change, but this is also the case during unidirectional rotation (see figure 7(a) at about 100 000 rotations).

In addition, we have tested periodical changes of the directional sense (figures 7(c) and (d)). The plot in figure 7(c) in the region labeled $\pm 360^\circ$ was obtained while the sense of rotation of the cell was reversed after each full turn. It is obvious that the convective motion is still present, although the amplitude is low. In the following period, we have reversed the rotation every half turn (labeled $\pm 180^\circ$), and in the last part of figure 7(c), the cell has been switched to uniform rotation again. The convection pattern remains uninfluenced, but the velocity of flow increases noticeably.

Figure 7(d) demonstrates that a cradle-like $\pm 180^\circ$ back-and-forth rotation not only preserves an already developed convection scenario as in figure 7(c), but also creates convection in a freshly filled cell. We note that in this $\pm 180^\circ$ experiment, the two cell planes are different. There is only one chute plane and no exchange between the two planes.

The aspect ratio of the cell is 10 in figures 7(a) and (b) and the number of rolls is on average 8. In figures 7(c) and (d), the aspect ratio is 6.25 and the number of rolls is 6 on average.

3.6. Influence of side walls

In most experiments, convection starts from one of the lateral cell ends (figure 7(a)) or sets in rather uniformly (figure 7(b)). The upper edge of the granular bed often has a slightly tilted height profile after filling, i.e. at least on one side the local fill level is somewhat lower than in the central part. Then the convective flow starts at the lower filled sides of the cell.

We can manipulate the initial local fill level by spinning the box moderately around its central vertical axis before the experiment, so that the material is centrifuged to the lateral...
Figure 7. Time-space plots derived from the vertical flow component in the horizontal cell midplane (cf the dashed line in figure 1). Blue and red colors stand for downward and upward motions, respectively. (a, b) Sporadic reversal of the sense of rotation (at the positions of the arrows). In most cases, this reversal has only a minor influence on the convection. In both experiments, the cell height was 5 cm and the fill level $C = 0.646$. (c, d) Periodical reversal of the rotation sense. $\pm 180^\circ$ indicates a reversal every half rotation and $\pm 360^\circ$ indicates a reversal every full rotation of the cell. The label continuous marks the time period where we rotate the cell unidirectionally again. Note the different color scales and different flow magnitudes. The magnitudes of the individual experiments cannot be directly compared because of different cell heights and different cell preparations (see section 3.6). Fill levels were $C = 0.655$ (c) and $C = 0.645$ (d).
cell edges. Then, one obtains a height profile where at the center of the cell there is more free volume on top of the granulate than on the left- and right-hand sides (figure 8, top). When we subsequently start our rotation experiment with such an inhomogeneous initial grain distribution, the convection tends to start in the central region of the container. Such a preparation was used for the experiment in figure 7(d). For better visualization, we show the graph of the vertical convection velocity component in the cell midplane in figure 8. It is evident that the convection flow amplitude is initially highest in the central region (after 1500 and 4000 rotations, respectively) and equalizes later (10000 rotations) together with the homogeneous redistribution of the granulate. The conclusion is that the side walls may play a certain role in the convection pattern (they fix the wavelength), but are not necessary for the formation of convection structures. The intrinsic convection pattern would also form in the absence of lateral confinement, in an infinitely extended cell. This is a noticeable aspect when the convection rolls are compared to structures observed in shaken containers (section 4).

3.7. Control and locking of the convection structures

It is evident from the previous section that the locations and dynamics of the rolls can be influenced when the free space above the granulate in the container is modified. A related phenomenon is observed when the container edge itself is modulated. One can control the wave length of the emerging convection pattern within a certain range when a well-defined wave length is imprinted as a variable height pattern at one or both of the horizontal edges. Such
Figure 9. The cell is modified by different patterns of ledges. (a) Steps on one of the horizontal confining borders, (b) steps on both borders keeping the height constant and (c) assembly of pins on one side. The obstacles’ sizes are exaggerated in the images. (d) Three steps modulate the cell edge on one side. Steps support the formation of convection streams and lock them at their positions. Arrows indicate the flow direction. Only the central part of the cell is shown after 1300 rotations. (e) In the same experiment after 14 300 rotations, also the neighboring regions have developed visible convection. The natural wavelength is slightly larger than the step distances (cell dimensions 500 mm × 31.5 mm × 5 mm, step width 10 mm, step height 1 mm and step distance 30 mm), see the movie 6 (available from stacks.iop.org/NJP/14/015001/mmedia). (f) The cell is modified by alternating steps on both sides, keeping the height of the cell constant. Note that convection in this experiment is weak at the center because of the rather high fill level. Image taken after 5000 rotations (step width and distance 36 mm; height 1 mm), see the movie 7.

A modulation of container edges locks the position of the convection streams (examples are schematically shown in figures 9(a)–(c)).

If the cell edge contains ledges or dimples (for example, as in figure 9) the convection pattern will start at these obstacles. Figures 9(a)–(d) show a container with three ledges at the upper container edge located near the center of the cell. These ledges protrude 1 mm into the cell and span the complete cell depth \( d = 5 \) mm. Convection sets in first at the ledge positions. This is visible by the dark arches of small particles flowing downwards. At later stages, the convection pattern penetrates the rest of the cell (figure 9(e)). A minor detail is the slightly different segregation pattern in the central structured cell region versus the unstructured lateral regions. At the positions of ledges or dimples, the free space above the granulate adjusts to almost the same level as in other regions of the upper edge. The upper surface of the granular bed roughly follows the modulation of the cell boundary. However, the compensation is incomplete: it seems that the free space at the position of dimples is on average slightly larger, at the level of resolution.
of the images, than that at the ledge positions. Figures 9(b) and (f) demonstrate that not the modulation of the local cell height but the modulation of the boundary shapes is essential. Here, the ledges of the top and bottom edges are staggered such that the local cell height is uninfluenced. As a consequence, the convection structure again follows the modulation of the container boundaries. Small-sized particles accumulate below the ledges and the convection current tends to point in the directions away from ledges and toward dimples. Even though we have no simple explanation for this observation, it may help to evaluate the validity of models for the convection mechanism.

3.8. Plumes and oscillations of the flow amplitude

In all cases where periodic modulation of the cell boundaries is imposed, there is competition between the natural and geometrically imprinted wave lengths. If the difference between these wavelengths is too large, the rolls do not lock to the geometrical constraints.

In particular, this was evident when we performed experiments with very narrow ledges in the top edge. The original idea was to create arbitrary edge shapes by placing small screws equidistantly along the cell edge and to vary the local cell height by adjusting the screw lengths inside. If the natural wave length of the convection is much larger than the screw distance, one might expect that the granulate experiences some local average of the container boundary. However, this concept of an ‘effective’ container wall did not work. Instead, the very character of the convection changed. The rolls did not flow continuously as in the previous experiments, but exhibited bursts in the convective flow and in the segregation patterns like plumes in a high-Rayleigh-number thermal convection experiment (cf e.g. [28]). One typical case is depicted in figure 10(a). Groups of three pins are screwed 3 mm into the container, whereas the remaining pins end the same level with the container border (see figure 9(c)). The result of this modification is quite a complex interaction of flow processes. The convection is characterized by an intermittent ascent of ‘plumes’ of small particles in the granulate. In general, plumes rise at horizontal positions that are between two groups of pins. The downward motion is less peculiar as it proceeds as a continuous flow at the position of the pin groups. Narrow stripes of small particles (dark) form on the top side, apparently influenced by the inward piercing pins. Time–space plots of horizontal cross sections show the segregation (figure 10(b)) and vertical flow field (figure 10(c)). The cross sections were taken along the rotation axis from the transmission profiles of the cell (figure 10(b)) and the calculated vertical flow velocity components (figure 10(c)). Plumes are reflected as short horizontal dark and red stripes in figures 10(b) and (c), respectively. Two exemplary plumes are encircled by dashes in figure 10(a). Their positions in the space–time diagrams are indicated by arrows. The ‘natural’ number of rolls in this geometry (cell aspect ratio) would be approximately 6. A frustrated region forms at the center of the cell because of the incompatibility of the wave length imprinted by the pin settings. This is seen especially in the flow profile of figure 10(c).

3.9. Further observations

3.9.1. Tilted rolls. The following experiment shows that the phenomenon of convection in containers with a high fill ratio is not restricted to the glass bead ensembles but that mixtures of different materials (even of different densities) produce similar patterns. We have filled a cell with a mixture of glass spheres (650 ± 100 µm) and poppy seeds (ρ ≈ 0.6 g cm⁻³; size
Figure 10. (a) Segregation pattern after modification of the cell by pins as sketched in figure 9(c), after 20,000 rotations. Plumes move upwards; two examples are marked by dashed ellipses. Another representation of the same experiment is given as time–space plots of the cell cross sections along the central horizontal axis of (a) the segregation pattern and (b) the vertical velocity. Plumes are represented in a pronounced modulation of the segregation pattern and the flow velocity in time. The positions of the pins are indicated above the time–space plots (cell width 48 cm; pins are screws of metric size 5 mm). The transparent upper cell edge is not visible in the image; it is at the same height as the screw ends of the upper row of screws; see the movie 8 (available from stacks.iop.org/NJP/14/015001/mmedia).

≈ 1.0 mm × 0.7 mm × 0.5 mm). Irrespective of the composition that consists of materials with varying size, density and shape, convection sets in. An interesting aspect of this system is that after approximately 10,000 rotations, the rolls become tilted in the central region.

Figure 11 shows two representative snapshots of the segregation patterns found in this system. The local direction of flow is sketched by arrows. The dark areas are enriched with poppy seeds. The fill level is clearly above $C_C$, but has not been determined exactly in this experiment. The image on the rear side of the cell is the same as that on the front side.

3.9.2. Traveling rolls and stripes. Traveling waves in a rotating drum have been described in the literature for special parameter sets or after special initial preparations of the mixers [7, 11–13]. Other interesting objects are the so-called ‘fountain’ bands that emit traveling waves...
waves propagating in opposite directions [13]. In dilute rotating suspensions of granulates, hydrodynamic effects may produce traveling waves of regions of accumulated grains [29–31].

Similar structures can also be observed in the flat cells. At a subcritical fill level, within the chute flow regime, traveling bands of segregated small beads can be observed. They can travel in either direction along the cell axis and they are reflected at the cell boundaries. Their motion may be very regular and periodic. At supercritical cell fill levels, in the convection regime, we find traveling convection rolls under certain conditions, particularly at long time scales. These structures will not be considered here.

4. Comparison with shaken systems

Convective motion of granular material has also been reported in shaken systems, in avalanches [32], subsurface layers [33] and in a few shear studies [34]. In the literature, a long list of papers deals with granulate in shaken containers. It is well known that in these vibrated systems many types of convection structures can form (e.g. [35]). An extensive overview of convection rolls in shaken containers is beyond the scope of this paper, but we compare some general features of shaken systems with our rotating container experiment. Afterwards, we report some shaking experiments with the very sample cells used in the rotation experiment.

4.1. Energy input

In shaken systems, the granulate has to overcome the Earth’s gravitational acceleration $g$. Typical thresholds reported in the literature are $\approx 1.2 \, g$ and above [1, 36]. The selection of patterns depends on the shaking strength [35, 37]. In contrast, in the rotating cell experiment the effective gravitational acceleration is less than $g$. During the phase when the granulate moves,
the cell plane forms an angle with the horizontal that is comparable to the angle of repose, and the effective (in-plane) acceleration is $\approx 0.8\,\text{g}$ when the grains slide. Inertia effects are negligible and the observed cell dynamics (measured in units of cell rotations) is independent of the driving angular velocity of the cell in a large parameter range.

4.2. Symmetry

Under vertical shaking in the Earth’s gravitational field, the up–down symmetry is broken. The forces of the container onto the grains can be transmitted only upward. Downward, gravity accelerates the grains. The energy distribution along the container height is different. It can be higher in the bottom region or near the surface. A resulting density inversion can lead to convection [38]. Under horizontal vibration, similar convection rolls are found as under vertical shaking, but gravity influences the particle dynamics as it acts perpendicularly to the container forces [39]. In our rotating cell, the symmetry of the top and bottom sides of the cell is conserved, because the rotation permanently exchanges the top and bottom edges of the container.

4.3. Role of side walls

Side walls may drive convection patterns in vertically shaken containers [40] or can confine the rolls [38]. If convection patterns are driven by friction at the side walls, they always appear pairwise either downwards or upwards at the sides [37, 40]. In contrast, we have shown in section 3.6 that side walls are not necessary for convection in the rotation experiment reported here. Any number of rolls (even or odd) can be observed in the cell. The container walls confine the convection structure laterally, but there is not necessarily the same flow direction at opposite side walls in a symmetric cell; up and down flows are equivalent.

4.4. Role of air

Owing to the high particle velocities in shaken containers, air can play an important role in the convection structures observed. However, there is no clear trend, as in most experiments air increases the effect [26, 27], but the opposite has also been found [41]. In shaken shallow layers, convection is found also in the absence of air [42]. In section 3.4 we have shown that in the rotating cell, interstitial air is irrelevant. Convection still persists at pressures two orders of magnitude lower than the threshold where the mean free path of the air molecules becomes comparable to the particle size. At that pressure (Knudsen regime [26, 27, 43]), friction and viscosity of the air have no major influence.

4.5. Shaking of our flat container

The most straightforward comparison between the dynamics of grains in shaken and rotated cells can be made when the same containers that were used in the rotation experiment are vibrated vertically, at comparable fill levels. In the following, we give a qualitative but not exhaustive description of this comparison.

A cell as depicted in figure 2 was used to test whether vertical shaking instead of slow rotations of the almost filled cell produces comparable results. The energy input during shaking is always considerably larger than in the rotation experiment. We drive the cell with an amplitude
Figure 12. Schematic sketch of the flow in the vertically vibrating cell for low (a, c) and high (b, d) filling. A heap forms either with two rolls (a, b) under symmetric driving or with a single roll (c, d) under slightly asymmetric driving. The lateral expansion of the convection structures depends on the fill level.

of up to 6 g. At 1 g, no collective motion is observable. The strongest convection is found at 3 g and 30 Hz frequency. Irrespective of the fill level of the container, the observed flow structures are qualitatively different from those described in the rotation experiment.

For lower fill heights ($C < C_C$), a single heap develops in the cell (sketched in figure 12(a)) and a single roll pair emerges, with the direction of flow indicated by arrows in the picture. At high filling level ($C > C_C$), the heaping effect leads to the loss of free volume above the granulate in the center of the cell. Thus, there is practically no collective transport in the cell center. However, the free volume at the sides of the cell (sketched in figure 12(b)) allows the formation of two rolls acting only at the side walls of the container. At the center, no motion is detectable within the experimental time frame. In both situations, the flow is downward at the cell side walls.

Thus, both structures differ qualitatively from the rotating cell experiment. If the vibration is not exactly orthogonal to the cell orientation, any slight asymmetry moves the peak of the granular bed to the one of the lateral sides. In that case, only a single elongated roll forms in the whole cell at lower filling (figure 12(c)), whereas at the high fill level (figure 12(d)), a single localized convection roll forms at the side that contains the free volume. At the opposite side of the cell, there is no measurable transport on the time scale of the experiment. Stable segregation patterns were not observed except in the situation of figure 12(a), where a part of the small beads accumulated in the upwards-flowing region at the center.

These phenomena are well known from other vibration experiments [36, 44, 45]. The beads tend to be driven downwards at the side walls and always move upwards in the inner region towards the peak of the granular surface. It is obvious that the driving mechanism for convection in vibration experiments must be essentially different from that in the rotation experiments and that the characteristic flow and segregation patterns described in the previous sections cannot be reproduced by vibration techniques. Furthermore, much higher accelerations are needed to drive convection in the vibrating cells. The convection occurs on completely different time scales compared to the rotating cell structures.

5. Characterization of the fluidized zones

Figure 13 sketches the direction of particle circulation observed in a cross section of the cell perpendicular to the rotation axis. Figure 13(a) defines the frame of reference with respect to the cell rotation. At subcritical fill levels (figure 13(b)), there is a global circulation of the
Figure 13. General circulation schemes in the rotating flat cell, seen from the side. (a) Cross-section of the flat rotating container. The reference system is given by the sense of rotation and the definition of the front view. (b) At the lower fill level the whole granular bulk circulates. (c) At the higher fill level (\(C > C_c\)) and continuous cell rotation, only shallow fluidized regions rotate. In the central parts, one has essentially up and down net flows in the cell plane within the convection rolls. This is symbolized by two cross-sections in the respective upward and downward convection roll segments. (d) At periodical reversal after every half rotation, the two fluidized regions rotate in opposite directions. (e) At periodic reversal after every full cell rotation, the observable motion is ‘outward’ on both cell planes. Dark and light shaded regions in (b)–(e) mark the fluidized zones and convection regions, respectively. White areas indicate the free space above the granulate, see movie 10 (available from stacks.iop.org/NJP/14/015001/mmedia).

granular material with the rotation of the cell rotation. This is the consequence of the sliding during the chute flow being somewhat faster at the current top side of the cell than at the bottom side [46]. The velocity of this motion has been found to be of the order of several hundreds of micrometers per cell rotation, so that the number of cell rotations necessary for a full orbit of the granulate will be of the order of 1000. The situation changes at supercritical fill levels where this global convection breaks down. Two fluidized regions remain at the upper and lower edges of the granular bed. Figure 13(c) sketches a global circulation stream in each of these fluidized layers. The velocity of motion is roughly of the same order of magnitude as in the global circulation at subcritical cell filling. We observe tracers at both cell edges to track the sense of direction in the fluidized zones. In the standard experiment with uniform sense of cell rotation, the two fluidized edges develop circulation with the same sense as the cell rotation. Interestingly, the situation changes when the cell is rotated \(\pm 180^\circ\) back and forth. We always start the \(\pm 180^\circ\) rocking motion with the rotation sense depicted in figure 13(a) and then reverse it. In that experiment, the sense of circulation of one of the two fluidized zones reverses. Top and bottom sides of the cell can now be clearly distinguished because one of the cell planes is always above the other during the half-circle. The granular dynamics in the fluidized zones reorganizes such that at the ‘top’ side, flow in these zones is toward the center of the cell, while at the ‘bottom’ side, the fluidized material flows away from the center (see figure 13(d)).

If we reverse the sense of rotation every full turn (360°), the flow in the two fluidized regions is not as pronounced as in the other experiments. However, a continuous flow of the grains at the cell plates toward the outer cell edges is observable from tracer motion.
(figure 13(e)). Because of mass conservation, we assume that there must be an oppositely directed flow in the central cell plane (marked by ‘?’ in figure 13(e)), forming a roll pair that cannot be monitored with our optical observation technique.

In any case, it is obvious that the sense of rotation in the fluidized regions plays no role in the general appearance of the convection patterns in the cell plane. The in-plane convection patterns in the central, jammed region of the granulate cell develop irrespective of the circulation dynamics of the outer fluidized zones.

The fluidized regions develop some kind of axial segregation bands. A close inspection reveals that regions where the convective flow is directed towards the fluidized zone are enriched with smaller beads. There is a clear relation between segregation and the direction of the convection. The segregation of the upper fluidized zone is the same after half a cell rotation when the lower edge reaches the top, but the pattern is shifted axially by one convection roll width. Because of symmetry, bands of small and large beads oppose each other in the cell. This creates a synchronization effect of the two axial segregation bands. The cause of the axial segregation in the fluidized zone is a dynamic effect: when mixed material enters the fluidized layer and is transported axially by the drift, the smaller species is partially ‘sieved’ out (Brazil nut effect) and enters the central core again, whereas the larger species remains in the fluidized zone. This provides a natural explanation of why the fluidized zone is somewhat richer with smaller beads in the regions where the convection roll moves into it. The ring-shaped segregation pattern in the convection roll is a consequence of the above-described sieving. Since the convective flow is found not only in bidisperse mixtures but also in samples with a narrow bead size distribution (see figure 3(f) in [22]), it seems that this segregation is not the primary source for the dynamic pattern formation.

Systematic height differences of the upper edge of the granular bed, if any, are much smaller than in the case of pure axial segregation at lower fill ratios (cf figure 4(a) and [7, 8]). When one analyzes the averaged top edge from a series of images, height modulations with the wave length of the convection pattern are below experimental resolution.

6. Proposed convection mechanism

Based on the experimental observations presented above, we propose the following mechanism that drives the convection: figure 14 sketches the basic dynamic processes that we have to consider. The material moves exclusively during a short slide phase in each half turn of the cell. The essential motion is normal to the rotation axis, in the cell plane (vertical arrows in figures 14(a) and (b)). This sliding motion is modulated along the horizontal axis, as indicated in the figure schematically by the difference of the arrow lengths. After two opposite slides during a full turn of the cell, only a small residual net displacement of the beads at the center of the container remains, cf figure 14(c).

The first and probably most important experimental observation is that if the cell is halted at any moment during the rotation, the convection resumes unchanged after such an interruption. The flow and segregation patterns are the same as in a continuously rotating cell. The crucial point here is that when the cell is at rest, all information on the state of the convection roll structure must be coded in the static distribution of the grains, i.e. a ‘memory’ of the effective flow field pattern exists in the packing structure. When the cell rotation continues, for example after a stop in the upright orientation, the local vortex motion is resumed. This is independent of whether the container rotation is in the same sense as before the stop or in the opposite direction.
Figure 14. Downsizing of grains in the cell in two half-cycles of the container rotation (a, b) and the net convection as the effective particle motion averaged over a complete cell rotation cycle (c). Local packing density inhomogeneities in the fluidized zones are sketched. For clarity, the inhomogeneities are unrealistically exaggerated.

The second important feature of the convection pattern is that the direction of cell rotation is of minor importance. Continuous rotation with uniform sense produces similar flow patterns as a periodic change of the sense of cell rotation after each full turn or each half turn. This has been demonstrated in section 3.5. We can thus conclude that the differences between the front and the rear sides of the cell (as seen in figure 2) are not relevant for the convection mechanism.

Finally, we emphasize that in the search for the physical origin of the pattern, we have to look for a subtle mechanism: the convection velocity is orders of magnitude slower than the excitation dynamics (container rotation), and even moderate chute flow in the container at lower filling ratios will eliminate the convection mechanism.

Key for the elucidation of the mechanism is the fact that the convection pattern structure must be encoded in the packing of the grains. The vertical transport during the slide phase must be slightly modulated in the axial direction, and this modulation must change its sign after each half turn of the cell. There are two options to explain this: the first one would be to assume that the local packing can memorize a direction sense, i.e. a hypothetical ‘directed’ random packing of spheres exists. Even though in principle this may not be inconceivable, we discard this hypothesis as unlikely. The second option is to assume that the vortex direction is encoded in packing density gradients inside the container. This is sketched in figure 14. Density fluctuations are also the cause of the Rayleigh–Bénard convection in ordinary fluids, but this coincidence is only superficial.

We consider a vortex with clockwise sense of the local convection, as shown in figure 14(c), and we presume that packing is more loose in the bottom right corner (3) than in the bottom left corner (4). Figure 14(a) sketches the situation immediately before the critical angle for sliding is reached. As a consequence, the grains slide down more efficiently on the right (edge 2–3). In the fluidized top zone of the granulate, a sideward flux from (1) towards (2) balances this difference by transporting beads towards the right-hand side, diluting the corner (1). Very small density modulations are sufficient to explain our experimental observations. During the next half turn, edge 1–2 becomes the bottom edge and the material slides back as indicated in figure 14(b). Since corner (1) is packed less densely than corner (2), grains slide down earlier and faster on the left-hand side (edge 1–4). Corners (1) and (3) change their roles. This is balanced by a horizontal transport from (3) to (4) in the upper fluidized zone. This mechanism maintains the packing density modulation. The observed convection (figure 14(c)) results from the small net...
difference between two large displacements, i.e. the slide from (1) to (4) in one half cycle (a) and the slide from (4) to (1) in the second half cycle (b).

In order to get a realistic impression of these processes and the orders of magnitude of the involved parameters, one has to recollect that during each half turn the beads shift by roughly the height of the free volume above the granulate. This height ranges from a few hundreds of micrometers at high \( C \) to a few millimeters near the critical \( C_C \). The average convection velocity, i.e. the difference between the downward flows shown in figures 14(a) and (b), is of the order of 10, . . . , 50 \( \mu \)m per cell rotation, i.e. two orders of magnitude smaller. In order to achieve such a net difference between the two slides, the packing densities of opposite corners (e.g. (1) and (2)) need to differ by about 1%. The net axial transport in the fluidized zones is two orders of magnitude smaller than the downward slide. It is also much slower than the dynamics of individual beads in the fluidized zones (see section 5).

As the experiments show, the material in the central zone of the cell is characterized by slow collective dynamics. Thus, it is reasonable to assume that the relevant packing density modulations are those in the fluidized edges. If these ideas are correct, then there is some indication that the local composition of the mixture and local packing densities are interrelated: in the regions where the rolls transport grains into the fluidized zone (the region with lower packing density), we have an accumulation of smaller beads, whereas in the regions where the rolls transport material out of the fluidized zone (higher packing fraction), this zone contains a more balanced mixture of grains (cf section 5).

This model explains a number of features of the observed convection pattern, such as the independence of convection of the cell rotation sense. It is also in accordance with the observation that a modification of the cell edge geometry imposes a defined flow pattern: below ledges, the granular surface is slightly depressed, and a small net drift in the fluidized zone transports material there, increasing the packing density marginally. The observations that the same patterns are observed in vacuum, in air and under water are compatible with the proposed mechanism as well. Also, the robustness of the convection rolls with respect to the composition of the granulate is explainable.

Small initial packing density fluctuations will always be present in a broad wavelength range. From this spectrum, the wavelength of the convection pattern is selected in a similar way as in hydrodynamic systems where the container geometry plays the dominant role. Too broad convection rolls are damped because the lateral transport efficiency is too large, and for short wavelengths, the gradient in vertical transport is too large. Thus, the most efficient coupling of flow and packing structures is found for rolls of nearly circular geometry.

The question is how this proposed model could be tested. Since the differences in packing are probably very small, their confirmation requires highest precision packing fraction measurements. An attempt has been made to characterize the average local packing by means of x-ray transmission measurements (figure 3 in section 2). The findings of the x-ray experiment support our suggested driving mechanism at least indirectly, by demonstrating the connection between sample composition and packing in our cells. However, the differences inside the fluidized zone were too small to be detected in our x-ray images, so a direct test could not be achieved.

7. Summary

We have demonstrated that convective flow can be found in almost completely filled thin cells under various parameter conditions. Its mechanism has still not been completely resolved, but
it is obvious that a different explanation than for vibrated granular mixtures has to be found. Some features of the convection structures have been clearly identified. They appear in a certain cell thickness range and for a limited range of fill levels of the cell, but they are otherwise not very sensitive to the composition of the granular material. The patterns form under atmospheric pressure in air, but they are also present under low pressure at least down to a few Pascals, and they are equally present when the complete experiment is submerged in water.

The role of the initial preparation of the mixture in the formation of the convection rolls is not relevant. As we have shown, both a uniformly mixed initial preparation (figure 5(b)), as well as a partially segregated initial state due to an efficient filling technique (figure 8, top), and a completely segregated regular initial state (figure 5(a)) lead to qualitatively similar convection patterns.

There are several opportunities for manipulating the pattern wavelength and appearance. For example, a special structuring of one or both of the cell edges can be used to imprint a certain pattern wavelength. When the initial fill height of the cell is locally modulated, the convection pattern can be forced to start in certain well defined regions of the cell.

Another interesting property of the rolls is that a reversal of the sense of rotation of the container does not reverse the convective flows, but rather leaves the convection structure unchanged. We have also established that the observed pattern is the same when the cell is continuously rotated by several thousands of turns and when the cell is repeatedly stopped during the experiment to take photographs intermittently. When the cell is rotated back and forth repeatedly in 180° sweeps, the resulting convection pattern is qualitatively the same as for continuous rotation. This means that the local flow structure of the convection pattern (e.g. up or down) must be encoded in the particular arrangement of the grains in different parts of the cell, irrespective of the direction of cell rotation.

We have suggested a mechanism for the convection mechanism and for the segregation patterns observed in the vortices. It is assumed that small packing density fluctuations couple to the global vortex flow. Even though the experimental results of our study certainly support our concept, they provide no direct access to the fundamental parameter, namely the local packing density differences in the fluidized zones. Measurements of the local granulate content with available tomographic techniques are too insensitive to test this. One has to look for a technique that can distinguish local packing densities differing by one or a few per cent.

One of the predictions of our model is that segregation of the mixtures could play a noticeable role in the formation of local packing density differences. This means that in strictly monodisperse material, the convection mechanism would break down, but this is very difficult to test. So far, we have performed experiments only with the bidisperse samples and with rather broad distributions of grain sizes [22], where convection was also observed. In a monodisperse material, crystallization prevents the formation of convective flow and a direct search for convection in such samples is impossible.

Summarizing, even though the presented mechanism explains many of the observed phenomena and is compatible with all observations in the flat rotating container, many details are not yet fully understood. The role of the fluidized zones and the partial segregation of mixtures during the convection process should be studied in detail. Experiments with different mixture compositions will be helpful in obtaining a clearer picture of the mechanisms involved and the influence of segregation on the efficiency of the convection.
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