Voltage Stability Analysis of Synchronized Current Phasor Control Based on Bifurcation Theory

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Abstract. Synchronized Current Phasor Control (SCPC) is a novel inverter control method proposed recently. This paper investigates the voltage stability of a simple SCPC grid based on the bifurcation theory. Firstly, the expression of the bifurcation point of the simple SCPC grid is derived, and then the variation of voltage stability domain is analysed according to the trajectory of bifurcation point when the inverter droop parameters change. The theoretical analysis results show that the droop parameters have an obvious influence on the voltage stability, which provides guidance for parameter setting.

1. Introduction

In recent years, Renewable Energy Sources (RES) continue to develop rapidly worldwide, and its penetration rate in grid is increasing[1]. Especially with the rapid development of photovoltaic generation, HVDC transmission technology and energy storage technology, inverter has become one of the main generation forms of RES. Inverter has the characteristics of small capacity, large quantity and wide distribution[2]. Therefore, the increase of its penetration presents new challenges to the operation control of grid.

Many control methods have been proposed to manage inverter, which can be classified as: the centralized control[3], the master-slave control[4] and the peer-to-peer control[5]. Among them, the peer-to-peer control has the advantages of autonomous operation, coordinated control, no communication, etc. A typical peer-to-peer control method is P-f/Q-V droop control, which has been widely used in microgrids[6]. In this control method, the frequency is adjusted based on the changed active power output of the inverter and the voltage amplitude is adjusted based on the changed reactive power output of the inverter, which can achieve good active power distribution. But the frequency and voltage offset caused by P-f/Q-V droop control have a bad effect on power quality. Recently a novel Synchronized Current Phasor Control (SCPC) was proposed to solve these problems[7]. In SPPC, a Synchronized Rotating Frame (SRF) is created according to the clock signal provided by Global Positioning System (GPS) or BeiDou Navigation Satellite System (BDS), so the SRF at each renewable energy source is synchronized. Then SCPC fixes current phasor of all inverters on the d-axis of the SRF so that the frequency is fixed on 50Hz. Further, a I-V droop control is used to achieve the real and reactive power allocation according to the capacity of each inverter. Therefore, the problem of frequency stability is avoided, but the influence of SCPC on the voltage stability of grid is not clear, so it is necessary to investigate this problem.

The bifurcation analysis is a theory used widely to track the state of voltage stability, which takes the bifurcation point of inverter output power and load voltage (PV) curve as the critical point of
voltage collapse\cite{8}. Hence, this paper uses the bifurcation analysis to investigate the voltage stability of SCPC grid and analyse the influence of inverter droop parameters on voltage stability.

2. The principle of SCPC

As mentioned above, the SCPC has the characteristics of fixed frequency and same phase angle by fixing the phasor of the inverter output current to the d-axis of the SRF, then the amplitude is regulated through I-V droop curve to achieve the real power and reactive power allocation. In this section, the principles of SCPC to achieve the above effects will be explained in detail.

2.1. The principle of Synchronized Current Phasor

The key of SCPC is that the reference rotating frame of each inverter is synchronous. Thanks to the clock signal provided by GPS or BDS, a signal changing in the range of $0 \sim 2\pi$ in each period is formed, which is used as the real-time phase angle of dq rotating frame. So, the SRF with the fixed rotating frequency and the same phase angle is formed in the whole grid. Then the phasor of inverter output current is fixed on the d-axis of the SRF, thus the frequency and phase angle of all inverter in the whole network is fixed\cite{7}. The control diagram of the SCPC is shown in Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.pdf}
\caption{The control diagram of the SCPC.}
\end{figure}

Figure 1. The control diagram of the SCPC.

2.2. The principle of I-V droop control

In the two-inverter system shown in Figure 2, the current phase angle of the two inverters is 0. So, the amplitude of load voltage is:

$$V_{pcc} = (I_1 + I_2)|Z_{load}| \tag{1}$$

It can be seen from equation (1) that the load voltage is proportional to the algebraic sum of the inverter output current, so the load voltage can be adjusted in the required range by adjusting the output current amplitude of the inverter. Therefore, the I-V droop control is proposed to regulate the voltage by changing the inverter output current amplitude\cite{7}. The I-V droop curve is shown in Figure 3.

The values of $K$ in Figure 3 can be determined by setting the maximum output voltage $U_{\text{max}}$ when the output current is 0 and the minimum output voltage $U_{\text{min}}$ when the output current is maximum. The complete I-V droop control equation is:

$$I = \begin{cases} I_{\text{max}} & U \leq U_{\text{min}} \\ K(U - U_{\text{max}}) & U > U_{\text{min}} \end{cases} \tag{2}$$

It can be seen from equation (2) that the I-V droop control only plays a role when the voltage is greater than $U_{\text{min}}$. When the voltage is lower than $U_{\text{min}}$, the grid is in the state of overload and the inverter faces the danger of overcurrent. Therefore, the output current of the inverter needs to be fixed to the $I_{\text{max}}$ through the limiter.

For different inverters, $U_{\text{min}}$ and $U_{\text{max}}$ have the same value, so the slope of droop curve is determined by $I_{\text{max}}$. Hence the load current is shared according to the proportion of inverter
capacity, which is reflected in $I_{\text{max}}$. The synchronized phasor of inverter output current determines the inverter power factor is equal to the load power factor, so there is no need to decouple the active power and reactive power. What’s more, compared to the P-E/Q-V droop control, the I-V droop control eliminates the inherent delay of P and Q measurement by simplifying the power flow control to current flow control.

To sum up, many advantages of SCPC make it have great developing potential in such grid dominated by inverters.

3. Voltage stability analysis of SCPC based on bifurcation theory

In this section, the bifurcation analysis is used to investigate the voltage stability of the SCPC grid and the influence of inverter droop parameters on voltage stability is analysed.

3.1. Simplified model of grid

In order to study the voltage stability of a SCPC grid, the complex grid model is firstly simplified into a simple one as shown in Figure 4:

![Figure 3. I-V droop characteristic.](image)

![Figure 4. Simplified model of grid.](image)

The model consists of an inverter, a transmission line, and a constant impedance load. And the model is simplified as follows:

1) By ignoring the transient process of the inverter, the output current of the inverter shows the characteristic of SCPC.
2) Ignore the resistance of the transmission line.
3) Due to the extensive existence of reactive power compensation in grid, it is considered that the reactive load is completely compensated locally when investigating the voltage stability under active power disturbance.

Under the assumptions above, the active power flowing of the inverter can be expressed as:

$$P - U_1 I \cos \alpha = 0$$  \hspace{1cm} (3)

Since the resistance of the transmission line and the reactive load are ignored, it can be considered that the active power of inverter is equal to the active power of load $GU_2^2$, and the reactive power of the inverter is the reactive loss of the transmission line. By substituting the partial relations of I-V droop expression $I = K(U_1 - U_{\text{max}})$ into equations (3), where the expression for $K$ is $I_{\text{max}}/(U_{\text{min}} - U_{\text{max}})$, the following equation can be obtained:

$$GU_2^2 - KU_1 (U_1 - U_{\text{max}}) \cos \alpha = 0$$  \hspace{1cm} (4)

The relationship between $U_1$ and $U_2$ is $U_1 \cos \alpha = U_2$, so equation (4) can be changed to:
\[ G = \frac{K}{\cos \alpha} + \frac{K U_{\text{max}}}{U_2} = 0 \]  

(5)

Since the current phase angle is 0, the voltage phase angle \( \alpha \) is expressed as: \( \cos \alpha = 1/(1+G^2X^2)^{1/2} \). Therefore, the relationship between load voltage and load conductance is:

\[ U_2 = \frac{K U_{\text{max}}}{K \sqrt{1+G^2X^2} - G} \]  

(6)

The output active power of the inverter is:

\[ P = U_2^2 G = \left( \frac{K U_{\text{max}}}{K \sqrt{1+G^2X^2} - G} \right)^2 G \]  

(7)

According to equation (6) and equation (7), the load voltage \( U_2 \) and the active power \( P \) of the inverter corresponding to the load conductance \( G \) can be calculated. Suppose the parameters are shown in Table 1:

| Parameter | Value |
|-----------|-------|
| \( X \)   | 0.3 p.u. |
| \( I_{\text{max}} \) | 1.2 p.u. |
| \( U_{\text{max}} \) | 1.1 p.u. |
| \( U_{\min} \) | 1.0 p.u. |

Table 1. System parameters.

When the load conductance changes from 0.5 p.u. to 5 p.u., the curve describing the relationship between the active power and the load voltage is shown in Figure 5:

![Figure 5. The PV curve of the simplified model.](chart.png)

It can be seen from Figure 5 that with the voltage dropping, the active power of the inverter increases at first and then decreases. The turning point is the bifurcation point, which represents voltage instability. The bifurcation theory is used to analyse the motion trajectory of the bifurcation point when the inverter droop parameters change, so as to judge the influence of inverter droop parameters on the voltage stability. In this paper, the bifurcation point is obtained by differentiating the PV expression. The expressions of the bifurcation point and the inverter droop parameters are obtained as follows:
\[(X^2 - K^2 X^4) G^4 + (2K^2 X^2 + 1) G^2 - K^2 = 0 \]  

(8)

After the G is determined according to equation (8), the voltage and power of the bifurcation point can be obtained according to equation (6) and (7), which determine the bifurcation point.

3.2. Influence of inverter droop parameters

In this subsection, the trajectory of the bifurcation point when inverter droop parameters changes is analysed, as shown in Figure 6 to Figure 8. The black arrows in these figures indicate the inverter droop parameter increases in that direction. The red line arrows indicate the direction in which the load conductance G increases.

It can be seen from Figure 6 to Figure 8 that the system is easy to cross the bifurcation point under overload, resulting in voltage collapse. So, the larger load conductance of the bifurcation point corresponding to the change of parameters is, the larger the stability domain is. Data of the load conductance G corresponding to bifurcation points and the inverter droop parameters is shown in Table 2 to Table 4. It can be seen from Table 2 to Table 4 that decreasing \(U_{\text{max}}\) or increasing \(U_{\text{min}}\) and \(I_{\text{max}}\) can help to increase the voltage stability domain. Among them, when adjusting \(U_{\text{max}}\) and \(U_{\text{min}}\), the change of G corresponding to bifurcation points is roughly the same, while when adjusting \(I_{\text{max}}\), the change of G corresponding to bifurcation points is much smaller than that of \(U_{\text{max}}\) and \(U_{\text{min}}\). Therefore, special attention should be paid to the voltage stability of the system when setting \(U_{\text{max}}\) and \(U_{\text{min}}\).

| \(U_{\text{max}}\) | 1.05 | 1.06 | 1.07 | 1.08 | 1.09 | 1.10 |
|-------------------|------|------|------|------|------|------|
| \(G\)             | 3.035| 2.981| 2.930| 2.880| 2.831| 2.784|

| \(U_{\text{min}}\) | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 1.00 |
|-------------------|------|------|------|------|------|------|
| \(G\)             | 2.570| 2.610| 2.652| 2.695| 2.734| 2.784|

| \(I_{\text{max}}\) | 1.15 | 1.16 | 1.17 | 1.18 | 1.19 | 1.20 |
|-------------------|------|------|------|------|------|------|
| \(G\)             | 2.764| 2.768| 2.772| 2.776| 2.780| 2.784|
4. Conclusion
In this paper, the voltage stability of SCPC grid is investigated by bifurcation analysis. The theoretical analysis results show that the increase of $U_{\text{max}}$ will cause the bifurcation point to move to the left, and the load conductance corresponding to the bifurcation point will decrease, which has a negative impact on the voltage stability. Increasing $U_{\text{min}}$ or $I_{\text{max}}$ will cause the bifurcation point to move to the right, and the load conductance corresponding to the bifurcation point will increase, which improves the voltage stability, and the effect of adjusting $U_{\text{min}}$ and $U_{\text{max}}$ is more obvious than that of adjusting $I_{\text{max}}$.

To sum up, the theoretical analysis results in this paper can provide guidance for the reasonable setting of inverter droop parameters, thus improving the voltage stability.

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