Abstract

Recurrent neural networks (RNN) such as long-short-term memory (LSTM) networks are essential in a multitude of daily live tasks such as speech, language, video, and multimodal learning. The shift from cloud to edge computation intensifies the need to contain the growth of RNN parameters. Current research on RNN shows that despite the performance obtained on convolutional neural networks (CNN), keeping a good performance in compressed RNNs is still a challenge. Most of the literature on compression focuses on CNNs using matrix product (MPO) operator tensor trains. However, matrix product state (MPS) tensor trains have more attractive features than MPOs, in terms of storage reduction and computing time at inference. We show that MPS tensor trains should be at the forefront of LSTM network compression through a theoretical analysis and practical experiments on NLP tasks.

Since the beginning of the last decade, computational resources have taken a primary role in the advancing of artificial intelligence. With the advent of new architectures for neural networks, starting with AlexNet and convolutional neural networks, computational demand has accelerated to the point of doubling itself each hundred days [Amodei et al., 2019]. In fact from AlexNet to AlexZero, computing demand has increased 3000 times. This implies major challenges for resources such as memory. Neural network model size is a bottleneck. The use of memory and computer power for training neural nets is enormous. For example, AlexNet takes 1.5 weeks to train on ImageNet on one Nvidia TitanX GPU [Han et al., 2015]. Although this might seems like a long time, we have to consider that following the current trend in neural network modeling, a more sophisticated architecture, such as ResNet152, might take at least ten times longer to train. Model size is indeed a major challenge: larger model implies larger sizes, which implies higher energy consumption. In a recent tweet debate, Elliot Turner, the CEO and co-founder of Hologram AI, wrote that it costs $
we could embed the matrix into a tensor as follows. Suppose that we can write

\[ A \]

A tensor is usually denoted by calligraphic bold symbols, e.g.

\[ \mathbf{W} \]. For compression of weight matrices, we could embed the matrix into a tensor as follows. Suppose that we can write

\[ N = I_1 \times I_2 \times \cdots \times I_n \],

and Situnayake (2019). MIT Technology Review has identified it as one of the top 10 promising technologies of 2020 [Hao, 2020]. The present work is primarily about storage compression and compression for compute-constraint efficient inference on recurrent neural networks (RNN) and long-short-term memory networks (LSTM) based architectures. We show that LSTM architectures based on weight matrices given by tensor trains can achieve performances similar or better than classical LSTM models, even when the number of parameters is reduced by half or even 80% with respect a classical full LSTM model. Furthermore, we show theoretically and in practice through a series of experiments, that some tensor train networks can do inference in a fraction of time.

LSTM networks are essential in automatic speech recognition models, language modeling, machine translation, and handwritten recognition among many other difficult tasks. They have a simple but heavy architecture that requires the estimation of eight weight matrices during training, and the same number of matrix-vector multiplications during both inference and training. Therefore, the deployment of large LSTM based models on edge devices consumes substantial storage, memory and computational resources. Because of the dependencies of time steps, the operations in LSTM networks are not easy to parallelize. Although, specialized hardware such as NPU, and TPU (specifically for edge devices) support sigmoid and hyperbolic tangent operations, these hardware are not optimized for a large number of such operations.

Comprehension in the form of tensor decomposition or low-rank matrix approximations is about giving sound structure to the gate weights, so as to reduce the number of parameters [Pan et al., 2019; Tjandra et al., 2018; 2017; Kusupati et al., 2019; Winata et al., 2018; Indra Winata et al., 2019; Barone, 2016]. Alternatives to compression are quantization [Li et al., 2018; Alom et al., 2017; He et al., 2016; Hubara et al., 2016] and pruning [Ramakrishnan et al., 2019; Luo et al., 2017; Han et al., 2016; Hu et al., 2016]. Quantization is about reducing the computational time of multiplications of matrices and vectors using few precision-bits. Although this reduces memory use, it does not reduce the number of parameters. Pruning is about eliminating unnecessary weights (elements of the weight matrix) or even nodes in the network. However, pruning trains with full precision and hence it does not reduces memory during training, nor precision bits. Moreover, pruning patterns in the weight matrix might be chaotic, which implies non-standard ways to store information.

Even though several studies have shown that compression and quantization do not hurt performance with feedforward neural networks nor convolutional neural networks (CNN), research with recurrent neural networks has shown that keeping a good level of precision in the computations is very challenging. However, we show that when adequate tensorization techniques are applied, the performance of compressed LSTM networks on natural language tasks is not affected. We shed light onto how two simple tensorization techniques are able to achieve even better performance than a full network model. These are the so-called matrix product operator (MPO) and the matrix product state (MPS) tensor trains. We start with a brief introduction of these tensor decompositions and its compression properties. In Section 4, we show how in theory MPS can be used efficiently for inference. Section 5 deals with training issues arising from tensor-train architectures. In Section 4, we shed light on how to improve the performance of tensor train architectures using knowledge distillation and regularization. Our theoretical results are corroborated in Section 5 through an extensive study on the performance of tensor train-based LSTM architectures on a NLP task.

1 Tensorization and Compression Techniques

A tensor is usually denoted by calligraphic bold symbols, e.g. \( \mathbf{W} \). For compression of weight matrices, we could embed the matrix into a tensor as follows. Suppose that we can write

\[ N = I_1 \times I_2 \times \cdots \times I_n \],
and $M = J_1 \times J_2 \times \cdots \times J_m$, for some positive integers $N, M, n, m$. Then we can think of the matrix $W$ as a tensor $W \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_n \times J_1 \times J_2 \times \cdots \times J_m}$. The element $w_{i_1\cdots i_n\cdots j_1\cdots j_m}$ of the matrix $W$ is mapped to the element $w_{i_1, i_2, \cdots, i_n, j_1, j_2, \cdots, j_m}$ of the tensor, so that, for example, following the big-endian colexicographic ordering, we map $i = i_n + (i_{n-1} - 1)I_n + (i_{n-2} - 1)I_{n-1} + \cdots + (i_1 - 1)I_2 \cdots I_n$, and $j = J_m + (J_{m-1} - 1)J_m + \cdots + (J_2 - 1)J_2 \cdots J_m$. For the sake of space, in what follows we briefly described two tensor train decompositions used recently for compressing neural networks. Other tensor decompositions also used in compression of CNNs are the Polyadic and Tucker decompositions [Kolda and Bader, 2009], and the tensor ring decomposition [Pan et al., 2019], also known as tensor-chain in the Physics literature [Cichocki et al., 2016].

**Tensor-train decomposition.** The tensor-train decomposition, also known as the matrix product state (MPS) in Physics, decomposed the tensor $W \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_n \times J_1 \times J_2 \times \cdots \times J_m}$ into a product of $n + m$ core tensors $W = \mathcal{A}^{(1)} \times^1 \mathcal{A}^{(2)} \times^1 \cdots \times^1 \mathcal{A}^{(n)} \times^1 \mathcal{B}^{(1)} \times^1 \cdots \times^1 \mathcal{B}^{(m)}$, so that $\mathcal{A}^{(k)} \in \mathbb{R}^{r_{k-1} \times I_k \times r_k}$, $\mathcal{B}^{(k)} \in \mathbb{R}^{r_{k-1} \times J_k \times r_k}$, $r_r = r_m = r_0$. We refer to the set of indices $\{r_k\}$, and $\{r_{rk}\}$ as the inner ranks of the core tensors. Here the notation $G^{(k)} \times^1 G^{(k+1)}$ stands for the mode-(3, 1) product between two tensors [Cichocki et al., 2016]. The leftmost panel of Figure 1 depicts an MPS tensor train with six cores.

![Figure 1](image.png)

Figure 1: A six-core MPS (left panel) and a three-core MPO (right panel). R represents the inner ranks of the cores.

**The matrix product operator (MPO) tensor.** A special case of tensor train corresponds to the case $W \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_n \times J_1 \times J_2 \times \cdots \times J_m}$, that is, when $m = n$. In this case the tensor may be rewritten as $W = \mathcal{A}^{(1)} \times^1 \mathcal{A}^{(2)} \times^1 \cdots \times^1 \mathcal{A}^{(n)}$ with $\mathcal{A}^{(k)} \in \mathbb{R}^{r_{k-1} \times I_k \times r_k}$, $k = 1, \ldots, n$, with $r_0 = r_n = 1$. The indexes are organized so that in the notation $W(h_1, h_2, \ldots, h_n)$, $h_k$ corresponds to $(i_k, j_k)$ in the form $i_k + (j_k - 1)I_k$. $i_k \in \{1, \ldots, I_k\}$, $j_k \in \{1, \ldots, J_k\}$, $k = 1, \ldots, n$. In an MPO the input and output entries are intertwined. The right panel of Figure 1 depicts an MPO tensor train with three cores. Note that a two-core MPO is a two-matrix decomposition often more efficient than a reduced matrix decomposition (e.g., singular value decomposition) of the weight matrix.

### 2 Storage and Computing time

In addition to the compression properties of MPS and MPO tensor train decompositions, we are interested in the order of operations required at inference, that is, to obtain the products $WX$ between the weight matrices and the input or output vectors involved in the computations of the gates of a LSTM unit.

Let $R = \max\{r_0, \ldots, r_m, r_0, \ldots, r_m\}$, and let $I = \max\{I_1, \ldots, I_n\}$, and $J = \max\{J_1, \ldots, J_m\}$. Also, denote by $O_{\text{MPS}}$ and $O_{\text{MPO}}$, respectively, the order of the number of operations required at inference by an MPS and an MPO LSTM cell. The main result of this section is the following.

**Theorem 1** The storage required for an MPS tensor train is of the order $O((R(I + J)) + R^2((n - 1)I + (m - 1)J))$, while for an MPO tensor train is $O((I + J)R + (n - 2)R^2)$. Furthermore, the number of operations required to compute $WX$ for MPS is order $O_{\text{MPS}} = O(R(N + M) + R^2[(n - 1)N + (m - 1)M])$. For MPO, the weight matrix needs to be reconstructed, so the multiplication requires $O_{\text{MPO}} = O(NM[R + R^2(n - 2)])$.

**Proof:** For MPS, the dimensions of the core tensors are $(1 \times I_1 \times r_{r_1}, r_{r_1} \times I_2 \times r_{r_2}, \ldots, r_{r(n-1)} \times I_n \times r_{r_0}, r_{r_0} \times J_1 \times r_{r_1}, \ldots, r_{r(m-1)} \times J_m \times 1)$. The storage needed for this decomposition is $\sum_{i=1}^n r_{r(i-1)}I_i + \sum_{j=1}^m r_{r(j-1)}R_{rj}J_j$, where $r_0 = r_m = 1$. Thus, we have the upper bound $R(I + J) + R^2[(n - 1)I + (m - 1)J]$. For MPO, the storage needed is $\sum_{i=1}^n r_{r(i-1)}I_iJ_i$, where $r_0 = r_n = 1$, which is bounded from above by $2RIJ + R^2(n - 2)IJ$. 

3
Next, turning to the computation aspects of tensor train, let us denote by \( j = (j_1, \ldots, j_m) \), \( i = (i_1, \ldots, i_h) \), \( h = (k_1, \ldots, k_{h-1}, h_1) \), \( h = (h_1, \ldots, h_{m-1}) \). For MPS, by organizing the terms adequately, one can compute \( \mathbf{W}(\mathbf{i}) \) as

\[
\mathbf{W}(\mathbf{i}) = \sum_{j} \sum_{h_{m-1}} \mathcal{A}^{(1)}_{(i_1, k_1)} \cdots \mathcal{A}^{(n)}_{(i_{n-1}, k_{n-1}, h_1)} \mathcal{B}^{(1)}_{(h_1, j_1, h_1)} \cdots \mathcal{B}^{(m)}_{(h_{m-1}, j_m, h_{m-1})} \mathcal{X}(\mathbf{j})
\]

\[
= \sum_{j} \sum_{h_{m-1}} \left( \sum_{k}, \mathcal{A}^{(1)}_{(i_1, k_1)} \cdots \mathcal{A}^{(n)}_{(i_{n-1}, k_{n-1}, h_1)} \right) \sum_{j} \mathcal{B}^{(1)}_{(h_1, j_1, h_1)} \cdots \mathcal{B}^{(m)}_{(h_{m-1}, j_m, h_{m-1})} \mathcal{X}(\mathbf{j})
\]

\[
= \sum_{h_{m-1}} \mathcal{G}_{h_m}(\mathbf{i}) \mathcal{X}(\mathbf{j}) \sum_{h} \mathcal{B}^{(1)}_{(h_1, j_1, h_1)} \cdots \mathcal{B}^{(m)}_{(h_{m-1}, j_m, h_{m-1})} = \sum_{h_{m-1}} \mathcal{G}_{h_m}(\mathbf{i}) \mathcal{X}(\mathbf{j}) \mathcal{G}_{h_m}(\mathbf{j})
\]

with \( \mathcal{G}_{h_m}(\mathbf{i}) = \sum_{k} \mathcal{A}^{(1)}_{(i_1, k_1)} \cdots \mathcal{A}^{(n)}_{(i_{n-1}, h, h_1)} \) and \( \mathcal{G}_{h_m}(\mathbf{j}) = \sum_{h} \mathcal{B}^{(1)}_{(h_1, j_1, h_1)} \cdots \mathcal{B}^{(m)}_{(h_{m-1}, j_m, h_{m-1})} \). The tensors \( \mathcal{F}_{h_m}(\mathbf{i}) \) and \( \mathcal{G}_{h_m}(\mathbf{j}) \) can be pre-computed. The number of operations required is of order \( \Theta(r_m)[N + M] \). The sums for all \( h_{m-2} \), \( \mathcal{D}_{h_m}(j_{m-1}, j_m) = \sum_{h_{m-2}} \mathcal{B}^{(m-1)}_{(h_{m-2}, j_{m-1}, h_{m-1})} \mathcal{B}^{(m)}_{(h_{m-1}, j_m, h_{m-1})} \) can be computed in \( \Theta(r_{m-2}r_{m-1}(n-1)) \). Hence, the sums for all \( h_{m-3} \), \( \mathcal{D}_{h_{m-2}}(j_{m-2}, j_{m-1}, j_m) = \sum_{h_{m-3}} \mathcal{D}^{(m-2)}_{(h_{m-3}, j_{m-2}, j_{m-1}, h_{m-2})} \mathcal{D}^{(m)}_{(h_{m-2}, j_{m-1}, h_{m-1})} \) can be computed in \( \Theta(r_{m-3}r_{m-2}r_{m-1}J) \). Continuing with this reasoning, we conclude that the sums, for all \( h_{m} \), \( \mathcal{G}_{h_m}(\mathbf{j}) \) can be computed in \( \Theta((n-1)r_{c_{(m-1)}}r_{c_{(m-2)}}r_{c_{(m-1)}}) \). Similarly, the sums \( \mathcal{F}_{h_m}(\mathbf{j}) \) can be computed in order \( \Theta((n-1)r_{c_{(m-1)}}r_{c_{(m-2)}}r_{c_{(m-1)}}) \). Thus, we have the upper bound \( \Theta(R(N + M) + R^2[(n-1)N + (m-1)M]) \).

The result for MPO is found in a similar way using the fact that now we have only one set of indices \((h_1, \ldots, h_n) = (i_1 + (j_1 - 1)I_1, \ldots, i_h + (j_h - 1)I_1) \) of dimension \( NM \).

An important corollary of the above result is the comparison at inference time, that is, in the computation of the forward pass, between MPS and MPO, and the full model multiplication \( \mathbf{W}x \).

**Corollary 1** Let \( n^{-1} > 1 \) be a given compression rate. Suppose that \( n \) is the factorization of the dimensions of \( \mathbf{W} \). For large \( N \) and/or \( M \), \( O_{\text{avar}}/NM = \kappa([N+M]/(1+J)) \), and \( O_{\text{avar}}/NM = \kappa([NM]/IJ) \). In particular:

(a) inference with MPS is more efficient than reconstructing the weight matrix if \( \kappa < (1+J)/(N+M) \);

(b) for \( n > 2 \), the efficiency gain of MPS over MPO at inference \( O_{\text{avar}}/O_{\text{marg}} = NM(I+J)/[IJ(N+M)]; \) for \( n = 2 \), \( O_{\text{avar}}/O_{\text{marg}} = NM(I+J)/[2IJ(N+M)]; \)

**Proof:** We suppose that \( n = m \). From Theorem 1 for a given compression rate \( \kappa \) we need to have \( \kappa MN = (I+J)[R_a + (n-1)R_a^2] = IJ[2R_a + (n-2)R_a^2] \), where \( R_a, R_o \) are the corresponding inner ranks of MPS and MPO. This yields \( R_o = [2(n-1)]^{-1}(\sqrt{n} + (4\kappa(n-1)NM)/(IJ) - 1) \), and \( R_a = [(n-2)]^{-1}(\sqrt{n} + (\kappa n(n-2)NM)/(1+J) - 1) \) for \( n > 2 \), and \( R_o = \kappa NM/(2IJ) \), for \( n = 2 \). This implies \( O_{\text{marg}} = \kappa NM/(N+M)/(1+J) \). For large \( N \) and/or \( M \), this also implies \( O_{\text{marg}} = \kappa(N^2M^2/IJ) \), for \( n > 2 \); for \( n = 2 \), \( O_{\text{marg}} = (\sqrt{2}/2)(N^2M^2/IJ) \).

In particular, inference with MPS is more efficient that reconstructing the weight matrix when \( \kappa < (1+J)/(N+M) \).

This shows part (a) of the statement. For part (b), we just need to compute the efficiency gain of MPS over MPO which by definition, and given the computations above is easily seen to be \( NM(I+J)/[[N+M]/IJ, \) for \( n > 2 \). This ratio is halved when \( n = 2 \).

**Remarks:** The theorem shows that MPS and MPO tensor train decompositions are very effective at compressing a weight matrix. However, the MPO bound is larger than the case where the dimensions are not intertwined (MPS). The key to keep this bound small is to combined the row dimensions \((J_{R(a)}, I_{R(a)})\) and a permutation of the column dimensions \((J_{R(o)}, I_{R(o)})\) so that \( J_{R(o)} \) and \( I_{R(a)} \) be kept as small as possible.

Assuming that \( M = N \) and \( J = I \), the efficiency gain of MPS in the multiplication is of order \( \Theta(N/I) \). Assuming that \( I = N^{1/\alpha} \), we get an efficiency of order \( N^{(n-1)/\alpha} \). So there are great gains at inference time when MPS are used.

### 3 Initialization and layer normalization for tensor weights

The usual way to initialize the weights is by random assignment (e.g., by Glorot initialization [Glorot and Bengio 2010]). This is usually done with a Uniform\((-\sqrt{\pi}/2, \sqrt{\pi}/2\)) or a Normal\((0, \sqrt{\pi}/2\)) [Ijandra et al.]}
When the weights are given by product of tensors, what need to be initialize are the tensor components. The typical weight is given by the form \( w = \sum_{k_1,k_2,\ldots,k_n=1} \alpha^{(1)}_{k_1,k_2,\ldots,k_n} \alpha^{(2)}_{k_1,k_2,\ldots,k_n} \cdots \alpha^{(n)}_{k_1,k_2,\ldots,k_n} \). So we need to know the distributions of products and sums of products. This is not straightforward as sum of products of uniform or normal variables does not result in a uniform or normal variable. But since usually \( n \) is large, and assuming identically distributed terms, by the central limit theorem, the sum should be closely distributed as a normal variable. So we just need to know its mean and variance. Assuming that all individual tensor terms are centered at zero yields zero-mean terms. For the variance, assuming that \( \text{Var}(\alpha^{(j)}) = \sigma^2 \), gives \( \text{Var}(w) = \sigma^2 n \prod_{k=0}^{n-1} r_k \). To obtain a normally distributed weight with variance \( 1/\beta^2 \) we set \( \sigma^2 = \left[ \prod_{k=0}^{n-1} r_k \right]^{-1} \beta^2 - 1/2n \). Alternatively, if we would like to have a more uniform distribution, say \( \text{Uniform}(0, \beta) \) for some \( \beta > 0 \) (e.g., \( \beta = 1/\sqrt{n} \)), we ask for the resulting normal distribution to be flat in the region \( (-\beta, \beta) \). That is, we ask \( \text{Prob}(|W| \leq \beta) = 1 - \alpha \) for some small \( \alpha > 0 \). This implies \( \sigma^2 = \beta^2 \Phi^{-1}(1 - \alpha/2)^2/n \prod_{k=0}^{n-1} r_k \). The individual term distributions are, respectively, \( \text{Uniform}(-b, b) \) for some \( b = \sqrt{n\beta}/ \Phi^{-1}(1 - \alpha/2)^{1/n} \prod_{k=0}^{n-1} r_k \).

**Normalization.** Without any constraints on the core tensors, the tensor decomposition is not necessarily unique [Kolda and Bader 2009]. Also, because the weights are computed as products of core tensors, backpropagation computations for a given tensor weight involves the weights of all the other tensors. These two facts make training of tensor-compressed RNN harder than training of non-constrained RNN. In our experiments, layer normalization [Ba et al. 2016] stabilized the tensor weights. So we have adopted this technique as standard when training tensor-compressed RNN.

### 4 Knowledge distillation

It is harder for a compressed network to achieve the same level of accuracy than a larger network. Instead of training a compressed network from scratch, one can use the knowledge gained by a larger network to help training the compressed network. Knowledge distillation [Hinton et al. 2014; Howard et al. 2017; Polino et al. 2018; Phuong and Lampert 2019] is a technique widely used to mitigate the problem. The classification scores from the larger network are incorporated in the loss function associated with the training of the smallest network. This is usually done through a cross-entropy loss. For a linear output network (that is, with no sigmoid transformation of the output), the “cross-entropy” is replaced by squared errors on the activations.

If \( W^* \) is a large network that we wish to compress and \( W \) is a smaller network, then knowledge distillation on activations corresponds to adding to the loss function the sum of squared errors \( \sum_k \| W^* x_k - W x_k \|_2^2 \), where \( \| \cdot \|_2 \) denotes the Euclidean distance. This is the same as \( \text{Trace}(W^* - W)(W^* - W)^{\dagger} \), where \( S = \sum_k x_k x_k^\dagger \) is the covariance matrix of the data (assuming that the input vectors are centered), and the superscript \( \dagger \) indicates matrix transposition. The importance of the structure in \( W^* \) is what we refer to as knowledge distillation on weights since the added loss \( \text{Trace}((W^* - W)(W^* - W)^{\dagger}) = \| W^* - W \|_2^2 \) becomes the squared of the Frobenius norm of the difference between the weights. One can also see this last setup as a regularization or prior knowledge on the tensor decomposition. In our experiments we have tried these two linear distillation techniques. The original loss function \( L(W) \) is replaced by \( \lambda L(W, \lambda) = L(W) + \lambda \text{Trace}(W^* - W)(W^* - W)^{\dagger} \), for some appropriate value of the hyper-parameter \( \lambda \). Knowledge distillation is done by first training a full non-compressed network, and then using the last value of \( W^* \) to start training a tensor-compressed network.

### 5 Experiments

For our experiments, we set up a language model task. This consists of estimating probabilistic sentence models \( p(w_t | w_0, w_1, \ldots, w_{t-1}) \), for each string of words \( w_0, w_1, \ldots, w_T \). Each LSTM cell in the network models one of these conditional probabilities. Our architecture may be seen in Figure 2. The task was run on two largely different size corpuses, both publicly available: The Penn Treebank (PTB) dataset [Taylor et al. 2003], and the Google One Billion Words (Google corpus) dataset [Chelba et al. 2013]. Both corpuses are widely used in machine learning and NLP research [Devlin et al. 2018; Gulraian et al. 2017]. We recall that a treebank is a parsed text corpus with annotated semantic and/or syntactic structure.

PTB, which was built in the period 1989-1996, is one of the first large such corpus. It comprises millions of words from annotated text from diverse sources such as the Wall Street journal. For our experiments, we used
Figure 2: The LSTM network architecture for the Penn Treebank and the Google One-billion dataset

the built-in Pytorch Penn Treebank corpus [Petrochuk, 2020]. The training and test sets comprise, respectively, 929,580 and 82,420 sentences. The vocabulary size is 10,000 words.

The Google corpus is a much larger corpus and a harder task than PTB. It was built from the 2011 Workshop on Statistical Machine Learning 1 by only selecting the English corpora. It has a vocabulary size of 793,471 words. For our experiments, we selected the first 10 million sentences of the corpus, and gather the 100,000 most frequent words to form our vocabulary. The training set consists of 7 millions sentences. The test set contains 2 millions sentences. We measure the performance of the compressed models with perplexity. This is a goodness-of-fit measure commonly used in NLP tasks. Its logarithm is proportional to the log-likelihood. The comparisons of perplexity are made on the testing subsets of the corpuses.

5.1 Perplexity versus compression

Our first experiment consists of measuring the performance of MPS-based and MPO-based tensorized LSTM architectures under different compression rates ranging from 1.59 to more than 53. For this task we choose the PTB corpus [Taylor et al., 2003].

We consider three different decompositions for the weight matrices, yielding two, three or four cores for MPO, and four, six and eight cores for MPS. The compression rates correspond to different values of the inner rank parameters. For this experiment we fix the inner rank parameters to the same value $R$. For each type of decomposition MPS and MPO we try different values of $R$ ranging from 20 to 365, so as to obtain similar compression rates for both models. In the results below, the rates has been binned to ease their comparison. The MPS and MPO architectures compared are described in Table 1. In our experiments, we set the sentence length to $T = 35$ words. The embedding size or dimension of the word vector was set to 650. The hidden dimension of the LSTM cell was also fixed at 650. We also used a batch of size 20. There are four weight matrices associated with the input vector $x$, and four weight matrices associated with the LSTM hidden output vector $h$. By stacking the weight matrices together, we form two matrices $W_x$ and $W_h$ of sizes $(4 \times 650) \times 650$ associated with $x$ and $h$, respectively. Each one of these matrices was represented as a tensor train, either MPS or MPO, from the start of the training of the models. That is, in this task, the models considered are fully tensorized from the beginning. The exception is the full model, which is the model without any tensor decomposition on the weight matrices. The compression rates are computed as number of weight parameters of the full model divided by the number of weight parameters of the tensorized model given in Theorem 1.

For PTB, the full model perplexity is 79.91. It contains $3.38 \times 10^6$ parameters. The perplexity results associated with five compression rate bins for the different tensorized LSTM models are shown in Table 2 and also in the leftmost panels of Figure 3. Together with the main statistics associated with the compression rate bins, we also show the minimum perplexity achieved in each compression bin.

5.2 Performance with knowledge distillation

As expected, the best results are linked to lower compression rates. In fact, perplexity increases nearly quadratically with compression, specially for MPO models (see panel A of Figure 3). Our second experiment

1https://www.statmt.org/wmt11/translation-task.html
Table 2: MPS and MPO tensor train perplexity results on the Penn Treebank corpus. SD stands for standard deviation; “at rate” is the compression rate associated with the minimum perplexity at the corresponding compression bin. Numbers in boldface are the minimum perplexities achieved.

| Compression Rate | MPS Mean (SD) | Minimum [at rate] | MPO Mean (SD) | Minimum [at rate] |
|------------------|---------------|-------------------|---------------|-------------------|
| 1 - 1.8          | 83.11 (0.47)  | 82.78 [1.51]      | 83.13 (0.00)  | 83.13 [1.51]      |
| 1.8 - 2.6        | 84.73 (3.11)  | 82.03 [1.79]      | 83.39 (0.79)  | 82.50 [1.79]      |
| 2.6 - 3.4        | 85.88 (3.43)  | 82.94 [2.68]      | 83.76 (1.04)  | 82.07 [2.68]      |
| 3.4 - 6          | 86.89 (3.22)  | 83.13 [3.38]      | 85.04 (1.04)  | 83.46 [3.82]      |
| 6 - 53           | 87.68 (3.34)  | 83.91 [6.00]      | 89.06 (3.16)  | 85.30 [6.49]      |

Figure 3: Perplexity of different tensor train models on the Penn Treebank corpus. KDW stands for knowledge distillation on weights, KDA for knowledge distillation on activations, and A for alone, that is, no knowledge distillation.

is to study the improvement, if any, associated with using knowledge distillation in the training of tensorized models. For this task, we only look at the perplexity results of the lower compression models two-core MPO and four-core MPS. The results with knowledge distillation on activations (KDA) and on weights (KDW) are displayed in the middle and rightmost panels of Figure 3. The statistics are also displayed in Table 3. The perplexity means and standard deviations are computed from different models with compression rates in the corresponding bins, as well as, from different values of the hyper-parameter $\lambda \in \{0.5, 1, 5, 10, 50, 100, 500, 5000\} \times 10^{-6}$ (see Section 4). The results are similar for MPS and MPO, achieving perplexities very close to the full model for the lowest compression rates near 1.8. We stress that this compression rate corresponds to a reduction of 44% in the number of weight parameters.

Table 3: Knowledge distillation on MPS and MPO tensor trains. Perplexity results on the Penn Treebank corpus. Numbers in boldface are the minimum perplexities achieved.

| Compression Rate | MPS Mean (SD) | Minimum [at rate] | MPO Mean (SD) | Minimum [at rate] |
|------------------|---------------|-------------------|---------------|-------------------|
| 1.8 - 2.6        | 84.17 (3.73)  | 80.20 [1.80]      | 86.07 (4.74)  | 83.74 [1.80]      |
| 2.6 - 3.4        | 87.83 (4.96)  | 81.79 [2.68]      | 84.72 (4.08)  | 80.42 [2.68]      |
| 3.4 - 6          | 88.50 (0.71)  | 88.00 [3.74]      | 84.25 (1.47)  | 81.23 [3.82]      |
| 6 - 53           | 89.86 (5.94)  | 83.59 [6.99]      | 89.29 (4.33)  | 86.03 [6.00]      |
| 1.8 - 2.6        | 84.34 (3.02)  | 80.74 [1.80]      | 81.40 (0.89)  | 80.82 [1.80]      |
| 2.6 - 3.4        | 90.47 (5.52)  | 85.41 [2.60]      | 81.79 (0.76)  | 81.23 [2.68]      |
| 3.4 - 6          | -             | -                 | 85.58 (2.64)  | 82.72 [3.82]      |
5.3 Performance of tensorized LSTM cells with knowledge distillation for the Google corpus

Our third experiment consists of trying tensor train compression on a large dataset. Due to time and computer memory constraints, we only run our experiments with four-core MPS (MPS2) and two-core MPO (MPO2) with knowledge distillation. This choice is based on the good performance of this setup for the smaller Penn Treebank corpus. The tensor-train setup is summarized in Table 1. For this experiment, we set the sentence length to $T = 20$ words. The embedding size or dimension of the word vector was set to 256. The size of the LSTM cell was fixed to 2048. The full model contains $18.8 \times 10^6$ parameters.

For the subset of the Google corpus used in this experiment, the full model perplexity is 67.48. Hence, the task appears simpler than for the PTB corpus, which had a larger perplexity. But in reality, the task is harder due to the vocabulary size. The size of the model and the amount of training data might explain the difference in perplexities. A summary of the results is displayed in Table 4. The perplexity means and standard deviations are computed as explained in Section 5.2. The MPO tensorized LSTM architecture reaches perplexities better than the full model. The MPS architecture yield perplexities close to but slightly larger than the full model.

Table 4: Knowledge distillation on MPS and MPO tensor trains. Perplexity results on the Google corpus. SD stands for standard deviation. Numbers in boldface are the minimum perplexities achieved.

| Tensor Train | Compression Rate | Knowledge Distillation |
|--------------|------------------|------------------------|
| MPO          | 2.5              | 70.59 (2.19)           |
|              | 6.7              | 70.31 (1.50)           |
| MPS          | 2.5              | 71.60 (2.59)           |
|              | 5.6              | 73.56 (2.41)           |
|              | 7.7              | 73.67 (2.14)           |

5.4 Benchmarking computing time at inference

As stressed in the introduction and in Section 2, MPS LSTM cells are much more efficient in computing the product $W_x$. In this experiment we compare the times needed to do this computation for MPS and MPO tensorized networks. We benchmarked the inference times (i.e., forward pass times) for two, three and four-core MPO, as well as for four, six and eight-core MPS tensorized LSTMs on CPU, at a fixed compression rate of about 1.8. The time was inferred using 1327 sentences of the Penn Treebank corpus. We run the inference task twelve times. The first two measures were discarded so as to get rid of hardware-related issues at the start of the experiment (i.e., loading in the cache, etc.). Table 5 shows the results in real seconds. The experiments were run on a 20-core Intel Xeon Processor E5 2698v4 CPU with 256 GB of memory RAM. Although, the multiplication after reconstruction has been optimized for MPO, MPS is still faster.

Table 5: Mean inference times for MPS and MPO tensor trains on the Penn Treebank corpus. The corresponding standard deviations are shown in parentheses.

| 4-core MPS | 2-core MPO | Tensor Train Decomposition |
|------------|------------|---------------------------|
| 1.39 (0.08)| 3.64 (0.12)| 1.90 (0.05)               |
| 4.25 (0.13)| 2.40 (0.07)| 6.77 (0.23)               |

6 Conclusions

Our goal is to find small models that perform well, as opposed to models that are just compressed networks for inference. Tensor decompositions greatly reduce the amount of storage both during training and inference. They also have the potential of accelerating computations at inference time in small storage and compute constraint devices. Our experimental results show that LSTM networks can be effectively compressed via tensor decompositions with some gain or very little lost in performance. Short MPS and MPO tensor trains, i.e., tensor decompositions with very few large inner rank tensors, appear to perform better. We show that up to 6x compression rates may be achieved without hurting by much perplexity. However, compression rates of about 2x seem optimal. Our results applied to both small and very large datasets. In our experiments, the compressed LSTM networks trained with knowledge distillation performed better on the large dataset. We believe that the reasons for this are the following: (a) the training set is very large, thus facilitating parameter estimation, and
more importantly generalization; (b) the covariance matrix of the data \( S \) must have several small eigenvalues which allows a more effective knowledge distillation on tensor train activations.

As mentioned earlier, compressing with tensors may also introduce computational gains at inference time if inference relies on CPUs as opposed to GPUs. This is the current situation on small constraint devices. At similar compression rates, tensor decompositions based on MPO appear to perform slightly better than tensor decompositions based on MPS. However, gains at inference time could be obtained only if one does not need to reconstruct the weight matrices of the LSTM cell. Following our arguments of Section 2, MPS tensor trains present such a property. Computation as well as compression goals should be balanced in computationally constrained devices. Our experiments show that MPS tensor train decompositions are the right answer to balance these two goals.

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