Jeans instability of a monatomic gas in the presence of thermal radiation

A Kumar\textsuperscript{1,a}, D L Sutar\textsuperscript{2,b} and R K Pensia\textsuperscript{3}

\textsuperscript{1}Research Scholar, Faculty of Science, Pacific Academy of Higher Education and Research University, Udaipur 313003 India
\textsuperscript{2}Research Scholar, Mewar University, Gangrar, Chittorgarh 312901, India
\textsuperscript{3}Department of Physics, Govt. Girls College, Neemuch 458441, India

E-mail: \textsuperscript{a)akchoudhary23456@gmail.com, b)devilalsutar833@gmail.com}

Abstract. The Jeans instability in magneto-radiative gas dynamics [MRGD] is discussed in the framework of magneto-gas dynamics [MGD] incorporating rotation and viscosity of the medium. The system of the present problem is assumed to adopt the Landau and Lifshitz model. The dispersion relation is obtained with the help of linearized perturbation equations. We find that the Jeans condition remains valid but the expression of the critical Jeans wave number is modified. Owing to the inclusion of radiative effect the sound speed changes which means that the critical wavelength depends on the amount of rate of radiation. We hope that our discussion on the present problem will help to improve our knowledge of interstellar medium [ISM].

1. Introduction

During few decade or so vigorous research activities have been launched to understand the various astrophysical problems. The simplest theory that describes the aggregation of masses in space the self-gravitational instability. The system comprising of particles that can aggregate together depends on the relative magnitude of the gravitational force to pressure force. Whenever the internal pressure of a gas is too weak to balance the self-gravitational force of a mass density perturbation, a collapse occurs. Such mechanism was first discovered by Jeans’s [1]. It was shown that the disturbances would grow if their wavelength excelled a certain minimum wavelength $\lambda_j$, given by $\lambda_j = \left( \frac{\pi c^2}{G \rho} \right)^{1/2}$, where $\rho$ and $c$ denote the density and sound velocity respectively and $G$ is the gravitational constant. The Jean’s criteria of instability are of central importance in understanding the process of formation of stars, planets, comets, asteroids, dark molecular clouds, commentary tails solar prominence and other astrophysical objects. A detailed contribution of the self-gravitational instability with different assumptions on the magnetic field and rotation has been given by Chandrasekhar [2]. In this connection, many investigators have discussed the gravitational instability of a homogeneous plasma considering the effects of various parameters [3-8]. On the other hand, the Coriolis force has been a subject of great interest for the last several decades. This interest was motivated by numerous mechanical applications in various disciplines, such as to understand the relation between angular momentum and rotational kinetic energy and how an inertial force can have
a significant effect on the moment of a body and still without doing any work. The magnetic force may be viewed as a kind of Coriolis force due to Thomas rotations, induced by successions of non-collinear Lorentz boosts. Maheshwari and Bhatia [9] have discussed the frictional effects with neutrals, in the presence of Hall currents, when the magnetic field is uniform and horizontal. Sankhla and Bhatia [10] have studied the combined influence of Hall current and Coriolis forces on the stability of a self-gravitating plasma. The characteristics of the magnetohydrodynamic wave in self-gravitating rotating plasma have been discussed by Hau and Chou [11]. Prajapati et al. [12] have investigated the problem of self-gravitational instability of rotating anisotropic heat conducting plasma. Recently, interest has been developed in understanding the role of thermal radiations in the star formation and molecular cloud condensation process in connection with thermal instability. Gomez-Pelayo and Moreno-Insertis [13] have investigated the problem of the thermal instability in a cooling and expanding medium including self-gravity and conduction in the neutral fluid dynamics. Shaikh et al. [14] have investigated the self-gravitational instability of thermally conducting partially ionized Hall plasma in the presence of a variable magnetic field. Recently, Prajapati et al. [15] have discussed the problem of self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss functions and electron inertia.

In addition to this magnetic fields can provide pressure support and inhibit the contraction and fragmentation of interstellar clouds. The magnetic field interacts directly only with the ions, electrons and charged grains in the gaseous medium. The collisions of the ions with the predominantly neutral gas in the clouds are responsible for the indirect coupling of the magnetic field to the bulk of the gaseous medium. The degree to which the magnetic pressure is important depends on the fields strength and the fractional abundance. The effects of simultaneous inclusion of viscosity, rotation, electron inertia and magnetic field on Jean’s instability of a monatomic gaseous medium is not investigated yet.

In view of the importance of thermal radiation effects in astrophysical contexts, we have incorporated thermal radiation effects in the investigation of Jean’s instability of an infinite homogeneous, viscous, rotating and magnetized monatomic gaseous medium in the presence of a finite electron inertia.

2. Linearized perturbation equations

We consider an infinite homogeneous rotating hot gaseous plasma in the presence of magnetic field \( \mathbf{B} (B_x, B_y, B_z) \) and rotation assuming as \( \Omega(0, \Omega_y, \Omega_z) \). The linearized perturbation equations of the problem are written as

\[
\begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \rho - \left(\frac{p}{T} + K_B \alpha T^3\right) \nabla \delta T - \rho \nabla \delta U - \frac{1}{4 \pi \mu} (\nabla \times \mathbf{B}) \times \mathbf{B} - \rho \theta \nabla^2 \mathbf{v} - 2 \rho (\mathbf{v} \times \mathbf{\Omega}) = 0
\end{align*}
\]

(1)

\[
\frac{\partial \delta \rho}{\partial t} + \rho \mathbf{v} \cdot \nabla \delta \rho = 0
\]

(2)

\[
\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \frac{c^2 \beta}{\omega_{pe} \alpha} \nabla^2 \mathbf{b}
\]

(3)

\[
4\pi G \rho \delta U = 0
\]

(4)

\[
\mathbf{\nabla} \cdot \mathbf{b} = 0
\]

(5)

\[
3K_B \left( \frac{\rho}{m} + \alpha T^3 \right) \frac{\partial \delta T}{\partial t} - \frac{p}{\rho} \nabla \delta \rho - K_B \left( \frac{1}{m} + \frac{\alpha T^3}{\rho_0} \right) \frac{\partial \delta \rho}{\partial t} = 0
\]

(6)

The parameters \( \rho, p, T, B, K_B, \Omega \) denote the density, pressure, temperature, magnetic field, Boltzmann constant, viscosity and t the time, \( \alpha = \frac{4\pi \gamma k_\perp^3}{45 \hbar c^3} \), is his Planck’s constant \( c \) is the velocity of light, \( \mu \) is the permeability of magnetic field, \( G \), gravitational constant, \( m \) is the mass of particle, \( \omega_{pe} \) electron plasma frequency. All perturbed quantity are given as \( q = q + \delta q \), where \( v, \delta \rho, \delta T, \delta U, b \), are the perturbed velocity, fluid density, magnetic field, temperature, gravitational potential, and magnetic field respectively.

3. Dispersion relation and discussions

We assume that all the perturbed quantity vary as \( \exp(i \omega t + ik_x) \)
\( \omega \) is the frequency of harmonic disturbances and \( k \) is the wave number in the \( z \)-direction. Using equations (1) – (7) we obtain the following dispersion relation.

\[
a_0 \sigma^2 + a_2 \sigma^4 + a_4 \sigma^2 + a_6 = 0
\]

We have assumed the following substitutions.

\[
i \omega = \sigma, \quad \Omega_j^2 = k^2 c_R^2 - 4 \pi \rho \Omega, \quad c_R^2 = \frac{5 K_B T}{3 m} \left[ 1 + \frac{2 a_2 (\delta T^2)}{5 [n + 2 a(\delta T^3)]} \right]
\]

\( n = \frac{\rho}{m} \) is number density of the particles in the unperturbed state.

\[
\alpha_0 = 16 \pi^2 \mu^2 \rho^2
\]

\[
\alpha_2 = 16 \pi^2 \mu^2 \rho^2 \theta_k \Omega_j^2 + 4 \pi \mu \rho [k^2 \left( \frac{B^2 + B_z^2}{A_1^2} \right) + 16 \pi \mu \rho \theta_k \Omega^2]
\]

\[
\alpha_4 = 8 \pi \mu \rho \left( \frac{k^2 B_Z^2}{A_1^2} + 8 \pi \mu \rho \theta_k \Omega_2 \right) \Omega_j^2 + k^2 \left[ \frac{k^2 B^2 B_2^2}{A_1^2} + 16 \pi \mu \rho \left( B_y \Omega_2 - B_z \Omega_y \right)^2}{A_1} + 16 \pi \mu \rho \theta_k B_2^2 \Omega^2 \right]
\]

\[
\alpha_6 = k^4 \theta_k^2 B_2^2 \Omega_j^2
\]

where, \( A_1 = (af) \), and \( f = \left( 1 + \frac{c^2 k^2}{\omega^2_{pe}} \right) \)

Equation (8) gives the combined influence of rotation, viscosity, electron inertia, magnetic field and thermal radiation on Jean’s instability of the homogeneous gaseous plasma. Equation (8) does not allow a positive real or a complex root whose real part is positive and the system is stable because \( B_2 \neq 0 \) it follows that when \( \Omega_j^2 < 0 \) then one of the root of equation (8) is positive, that means instability occurs with

\[
k_j = \left( \frac{4 \pi \rho \Omega}{c_R^2} \right)^{1/2}
\]

where, \( k_j \) is modified critical Jeans wave number and corresponding critical wavelength is given by

\[
\lambda_j = \left( \frac{\pi c_R^2}{\rho \Omega} \right)^{1/2}
\]

The gaseous molecular cloud is unstable for all wavelengths of perturbation which are greater than Jeans wavelength and for all wave number which is less than Jeans wave number \( (k < k_j) \).

If we take \( B_2 = 0 \) then \( \alpha_6 = 0, \alpha_4 = 16 \pi \mu \rho \Omega_2^2 \left( 4 \pi \mu \rho \theta_k \Omega_j^2 + \frac{k^2 B^2}{A_1^2} \right) \) and \( V_2 = \frac{B^2}{4 \pi \mu \rho} \) is the Alfvén velocity.

Since \( \Omega_2 \neq 0 \) then from equation (8) we get modified critical Jean’s wavelength as.

\[
\lambda_{j1} = \left( \pi \left( c_R^2 + \frac{V_2^2}{f} \right) (G \rho)^{-1} \right)^{1/2}
\]

Comparing equations (10) and (11) we see that in the transverse direction the magnetic field increase the critical wavelength, therefore is evident that magnetic field has a stabilizing influence on the gaseous molecular cloud whereas the electron inertia has a destabilizing influence on the system. If we take \( \Omega_2 = 0 \) then modified critical Jeans wavelength is given as

\[
\lambda_{j2} = \left( \pi \left( c_R^2 + \frac{V_2^2}{f} \right) (G \rho - \frac{\Omega^2}{\pi})^{-1} \right)^{1/2}
\]

Comparing equations (11) and (12) we see that the rotation increase the critical Jean’s wavelength. Thus the rotation has a stabilizing influence on the gaseous plasma. Thus the stabilizing effects of magnetic field and rotation have been removed by finite electron inertia on Jean’s criterion of rotating gaseous plasma in the presence of thermal radiation.
4. Conclusions
In the present paper, we have investigated the problem of Jean’s instability of viscous rotating homogeneous monatomic gaseous plasma in the presence of thermal radiation, a finite electron inertia, and magnetic field. The equations of the problem are stated and the general dispersion relation is obtained, which is modified due to the presence of these parameters. The effect of the magnetic field is found to stabilize the system whereas the effect of finite electron inertia is found to destabilize the system. The rotation parameter increases the stabilizing effect of magnetic field.

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