D-BRANES AND VECTOR BUNDLES ON CALABI-YAU MANIFOLDS: A VIEW FROM THE HELIX

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Abstract. We review some recent results on D-branes on Calabi-Yau (CY) manifolds. We show the existence of structures (helices and quivers) which enable one to make statements about large families of D-branes in various phases of the Gauged Linear Sigma Model (GLSM) associated with the CY manifold. A comparison of the quivers of two phases leads to the prediction that certain D-brane configurations will decay as one moves across phases. We discuss how boundary fermions can be used to realise various D-brane configurations associated with coherent sheaves in the GLSM with boundary. This is based on the talk presented by S.G. at Strings 2001, Mumbai.

1. Introduction

This talk is a summary of recent work done by us on D-branes in Calabi-Yau manifolds. Following the seminal work of Polchinski, there has been enormous progress in our understanding of both BPS and non-BPS branes in flat space. The situation for the case when spacetime is, say $M^4 \times X$, where $X$ is a Calabi-Yau three-fold is more complicated. Most of our understanding is limited to special regions in their moduli space such as the large-volume limit and Gepner points where one uses techniques specially adapted to the situation. Unlike the case of toroidal compactifications, these correspond to fewer unbroken supersymmetries and thus fewer constraints follow. For example, the BPS condition leaves open the possibility of walls of marginal stability in the moduli space, where a D-brane can decay. It is thus useful to understand the behaviour of D-branes away from the special points in regions where stringy corrections are important.

The moduli space of Calabi-Yau manifolds is the direct product of the Kähler moduli space and the complex moduli space. At special points in these moduli spaces, there are different descriptions of D-branes. For the quintic, two special points in the Kähler moduli space are the Gepner point and the large volume point. In the large volume limit where one has a nice geometric description of the Calabi-Yau manifold, D-branes which preserve A-type supersymmetry wrap special Lagrangian submanifolds while those that preserve B-type supersymmetry wrap holomorphic cycles of the Calabi-Yau space. More generally, including the gauge fields on the brane, we end up with B-branes being related to coherent sheaves. We will only consider B-branes in this talk.

At the Gepner point, one is in a non-geometric phase and the compactification is described by a suitable tensor product of $\mathcal{N} = 2$ minimal models. The quintic is given by tensoring five copies of the $k = 3$ minimal model. D-branes are described, for example, by constructing boundary states satisfying Cardy’s consistency condition. A large family of such states have been constructed using a
prescription due to Recknagel and Schomerus[7]. We will refer to these states as the RS boundary states.

**Example 1.1.** For the quintic, the B-type RS states are described by the following labels[8]

\[ |L_1, L_2, L_3, L_4, L_5; M \rangle \]

where \( L_i = 0,1 \) and \( M = 0,1,2,3,4 \). The \( M \) label reflects a quantum \( \mathbb{Z}_5 \) symmetry. The five boundary states given by \( \sum_i L_i = 0 \) are rigid i.e., they have no moduli associated with them.

The natural question to ask is what geometric objects in the large-volume limit correspond to the RS boundary states. The answer depends on the path taken from the Gepner point to the large volume point since B-branes can be transformed under monodromy around singular points[8, 9]. Further, not all B-branes at the Gepner point will be stable objects in the large-volume limit. This question was studied in the case of the quintic[8], where the large-volume charges of the RS boundary states were obtained. Subsequently, in [10], it was shown that the five \( \sum_i L_i = 0 \) states are given by the restriction of the bundles \( \Omega^p(p) \) on \( \mathbb{P}^4 \) to the quintic hypersurface[1].

It was also conjectured that all \( \sum_i L_i \neq 0 \) RS states are bound states of the five \( \sum_i L_i = 0 \). It is rather easy to verify this at the level of charges.

The method used in establishing the relationship between RS boundary states and the large-volume bundles makes use of mirror symmetry and is rather cumbersome. **Is there a more straightforward relationship which does not make use of mirror symmetry?** In this talk, we will see the existence of structures which enable us to make this correspondence in a simpler and more transparent manner.

### 2. The Gauged Linear Sigma Model

The Gauged Linear Sigma Model (GLSM) introduced by Witten provides a field-theoretic description of the moduli spaces associated with Calabi-Yau manifolds[11]. In this description, the Gepner point and the large volume point are seen to be suitable limit points in *phases* of the GLSM. The field content of the GLSM are: \( \Phi_i \) are \((2,2)\) chiral supermultiplets with charge \( Q_i \) with components \((\phi_i, \psi_{\pm i}, F_i)\); \( P \) is a chiral supermultiplet with charge \( Q_p = -\sum_i Q_i \) and components \((p, \psi_{\pm p}, F_p)\) and an abelian vector multiplet \( V \) whose field strength is part of a twisted chiral superfield \( \Sigma \) with components \((\sigma, \lambda_{\pm}, D, v_{01})\). The Fayet-Iliopoulos D-term and

\[ \Omega \] is the cotangent bundle to \( \mathbb{P}^4 \) and \( \Omega^p(p) = \wedge^p \Omega \otimes \mathcal{O}(p){
theta term are described by a single complex parameter $t = \frac{\theta}{2\pi} + ir$. We will consider two situations:

(i) Let us assume that there is no superpotential for the chiral superfields. When $r \gg 0$, the low energy theory is the the non-linear sigma model associated with the total space of a line-bundle $O(Q_p)$ over the weighted projective space $X = \mathbb{P}^{Q_1,\cdots,Q_n}$. This is a non-compact Calabi-Yau manifold. In the limit when $r \ll 0$, one is in the orbifold phase. The low-energy theory is described by the orbifold $\mathbb{C}^n/\mathbb{Z}_{|Q_p|}$.

(ii) Suppose we have a superpotential $W = PG(\Phi)$ where $G$ is a polynomial of degree $|Q_p|$. Now in the limit when $r \gg 0$, the low-energy theory is given by the non-linear sigma model associated with the Calabi-Yau manifold $M$ given by the hypersurface $G = 0$ in $X = \mathbb{P}^{Q_1,\cdots,Q_n}$. The field $p$ has zero vev and its fluctuations are massive. This is the Calabi-Yau phase. In the limit $r \ll 0$, one obtains an Landau-Ginzburg (LG) orbifold as the low-energy theory. This the LG phase. The infrared fixed points of this ultraviolet theory are supposed to be the large volume and Gepner points respectively.

In the following, we shall use $X$ to represent the ambient weighted projective space in which the Calabi-Yau manifold $M$ is an hypersurface.

Since the RS boundary states occur in the Gepner point, it is of interest to see how they are realised in the LG orbifold. It was argued in [12] that the only possible boundary conditions on individual LG fields are Dirichlet and take the form $\phi_i = 0$. This implies that the B-type RS states are branes localised at the orbifold singularity! In the case, when there is no superpotential, such branes are called fractional branes(see [9] and references therein). This is indeed the first clue that RS states must be related in some form to D-branes associated with the orbifold $\mathbb{C}^n/\mathbb{Z}_{|Q_p|}$. We now proceed to discuss D-branes associated with orbifolds.

3. D-branes on orbifolds and Quivers

Building on the work of Douglas and Moore[13], it was soon realised that the various D-branes associated with the orbifold $\mathbb{C}^n/\Gamma$, where $\Gamma$ is a discrete group are associated with representations of a quiver, the McKay Quiver[14]. The vertices of the quiver are in one-to-one correspondence with the fractional branes discussed in the previous section. The field content of the worldvolume gauge theory of the D-brane associated with the quiver is encoded as follows: The positive integers $n_i$ lead to a gauge group $\prod_i U(n_i)$ and each arrow corresponds to bi-fundamental scalar fields (as well as their supersymmetric partners).

**Example 3.1.** Consider $\mathbb{C}^5/\mathbb{Z}_5$, which is the orbifold associated with the quintic example. The quiver (see figure 2) has five vertices associated with the five fractional branes. It is in agreement with the fact that there are five $\sum_i L_i = 0$ RS boundary states.

Given an orbifold singularity, one might resolve the singularity by blowing up. It is not known in general, whether there exists crepant resolutions (i.e., the resolved space is a Calabi-Yau manifold). of orbifold singularities except for two cases: $\mathbb{C}^2/\Gamma$ (where $\Gamma$ is a discrete sub-group of $SU(2)$) and $\mathbb{C}^3/\Gamma$ (where $\Gamma$ is an abelian sub-group of $SU(3)$). In these cases, one can see (based on the work of Ito and Nakajima[15]) that the fractional branes - $\{S^0\}$ - provide a basis for $K_c(X)$. $K_c(X)$ represents the K-theory classes for all D-branes/vector bundles with compact support i.e., those
that live on the exceptional divisors corresponding to the resolution of the singular space \( X \). Further, these are dual to tautological bundles on the orbifold - \( \{ R_a \} \) - which form a basis for \( K(X) \), the K-theory classes associated with the unresolved space. This is one version of the McKay correspondence. The duality is

\[
\langle R_a, S^b \rangle \equiv \chi(R_a, S^b) = \int_X \text{ch}(R_a) \text{ch}(S^b) \text{Td}(X) = \delta_a^b .
\]

This is reminiscent of what we saw in the LG and CY phases. In the LG phase, all branes had support only on singularity. Hence, when the singularity is blown up – these branes necessarily have support on the associated exceptional divisors. The \( \{ S_a \} \) above have the same property. Further, the fractional branes form a basis for the branes near the orbifold point and hence it is natural to identify them with the \( \{ S_a \} \). Using these observations, Douglas and Diaconescu proposed the existence of tautological bundles \( \{ R_a \} \) in more general situations\[16\]. Further, they gave an inverse toric algorithm to construct the tautological bundles. This approach has two problems: it is tedious and one does not know how to handle situations where the ambient space has singularities which the CY hypersurface does not inherit. In next couple of sections, we will see that the \( \{ R_a \} \) are the foundation of a helix and can be obtained by studying the large-volume monodromy of the simplest brane of all – the six-brane. This is associated with the line bundle \( O \). The special role played by the line bundle \( O \) is very much in line with recent K-theoretic considerations based on Sen’s study of non-BPS branes\[17\]. Just as the D9-brane (and its antibrane) in IIB string theory generate all other branes in flat spacetime, it is natural to expect the six-brane (wrapping all of \( M \)) to generate all lower branes and hence charges.

4. Helices and Mutations

**Definition 4.1.** A coherent sheaf \( E \) on a variety \( X \) (of dimension \( n \)) is called **exceptional** if

\[
\text{Ext}^i(E, E) = 0 , \quad i \geq 1 \\
\text{Ext}^0(E, E) = \mathbb{C} ,
\]

where \( \text{Ext}^i(E, F) \) is the sheaf-theoretic generalisation of the cohomology groups \( H^i(X, E^* \otimes F) \) for vector bundles \( E \) and \( F \).
Definition 4.2. An ordered collection of exceptional sheaves $\mathcal{E} = (E_1, \ldots, E_k)$ is called a strongly exceptional collection if for all $a < b$, one has

$$\text{Ext}^i(E_b, E_a) = 0, \quad i \geq 0$$

$$\text{Ext}^i(E_a, E_b) = 0, \quad i \neq i_0,$$

for some $i_0$ (which is typically zero).

This implies that there exist Ramond ground states only in the sector with charge $i_0$. Further, $\chi(E_a, E_b)$ is an upper-triangular matrix with ones on the diagonal.

New exceptional collections can be generated from old ones by a process called mutation.

Definition 4.3. A left mutation of an exceptional pair $(E_a, E_{a+1})$ in an exceptional collection is defined by

$$L_a(E_a, E_{a+1}) = (L_{E_a}(E_{a+1}), E_a)$$

where we have introduced a new sheaf $L_{E_a}(E_{a+1})$ which is defined through exact sequences (see [21] for details). For example, when $\text{Ext}^0(E_a, E_{a+1}) \neq 0$ and the (evaluation) map $\text{Ext}^0(E_a, E_{a+1}) \otimes E_a \to E_{a+1}$ is injective, then $L_{E_a}(E_{a+1})$ is defined by

$$0 \to \text{Ext}^0(E_a, E_{a+1}) \otimes E_a \to E_{a+1} \to L_{E_a}(E_{a+1}) \to 0$$

The Chern characters of the new sheaves are given by

$$\text{ch}(L_{E_a}(E_{a+1})) = \text{ch}(E_{a+1}) - \chi(E_a, E_{a+1})\text{ch}(E_a)$$

Further, the collection is assumed to be strongly exceptional (with $i_0 = 0$) and hence $\chi(E_a, E_{a+1}) = \dim \text{Ext}^0(E_a, E_{a+1})$.

The mutation of a strongly exceptional collection may not continue to be strongly exceptional. If the mutated collection is also strongly exceptional, the mutation is called admissible. The mutations that we consider in this paper (in order to generate $S_i$) are assumed to be admissible though we do not always verify this explicitly.

Definition 4.4. An exceptional collection $(E_i, i \in \mathbb{Z})$ is called a helix of period $p$ if for all $s$ the following condition is satisfied:

All pairs $(E_{s-1}, E_s)$, $(E_{s-2}, L^1(E_s))$, $\ldots$, $(E_{s-p+1}, L^{p-2}(E_s))$ admit left mutations and $L^{p-1}(E_s) = E_{s-p}$.

Thus, a sequence of $(p-1)$ left mutations of a helix brings one back to an element of the helix modulo a shift of $p$. Each collection $(E_i, E_{i+1}, \ldots, E_{i+p})$ is called a foundation of the helix $\{E_i\}$. Any helix is determined uniquely by any of its foundations. One can also define the helix using right mutations. We shall henceforth use the term helix for the foundation of a helix since we will have no need to distinguish them.

Example 4.5. The collection of line bundles $\mathcal{R} = \{\mathcal{O}, \mathcal{O}(1), \ldots, \mathcal{O}(n)\}$ is the foundation of a helix of period $n$ on the complex projective space $\mathbb{P}^n$.\footnote{This is somewhat different from the definition of Rudakov and is the one used by Bondal [18]. An exceptional collection is one for which the $\text{Ext}^i(E_a, E_b) = 0$ for $a > b$.}

\footnote{The process of mutation has been related to brane creation on the mirror in \cite{19}. See also \cite{20}.}
4.1. Two conjectures and their consequences.

**Conjecture 4.6.** The large-volume monodromy \((t \to t + 1)\) on \(\mathcal{O}\) (i.e., the bundle which wraps all of \(X\)) produces an exceptional collection which is a helix with foundation \(\mathcal{R} \equiv \{ R_1 = \mathcal{O}, R_2, \ldots, R_p \} \).

**Remarks**
1. The period \(p\) of the helix reflects a quantum \(\mathbb{Z}_p\) symmetry associated with \(X\).
2. The procedure seems to work for cases when the space \(X\) is not fully resolved\(^2\).
3. In cases where there are many Kähler moduli, the monodromy action \(t_i \to t_i + 1\) \((i = 1, \ldots, d)\) generates a \(d\)-dimensional lattice of which the helix is a one-dimensional lattice\(^22\).
4. This conjecture has been verified in a variety of examples\(^2, 22, 23\).

**Conjecture 4.7.** All exceptional bundles on \(X\) are generated by mutations.

**Remarks**
1. There exists a mutated helix with foundation \(\{ S^p, \ldots, S^1 = \mathcal{O} \}\) with \(S^a = L_{-1}(R_a)\) with the property:
   \[
   \chi(R_a, S^b) = \delta^b_a,
   \]
   i.e., the \(S^a\) are dual to the \(R_a\).
2. The restriction of the \(S^a\) to the Calabi-Yau hypersurface gives the \(\sum_i L_i = 0\) RS states in all examples that we have considered.
3. For \(X = \mathbb{P}^n\), it is known that \(\mathcal{R}\) form a basis for all sheaves on \(\mathbb{P}^n\). This follows from Beilinson’s theorem\(^24\). This suggests that there exists a generalisation for weighted projective spaces as well by using methods followed in \(^25, 26\) for vector bundles on \(\mathbb{P}^n\). Thus, one anticipates that all coherent sheaves on \(X\) will be given as the cohomology of complexes of associated with the \(R_a\) (or \(S_a\)).

5. Quivers from Helices

Given a helix \(\mathcal{R} \equiv \{ R_1, \ldots, R_p \}\), one can construct a quiver as follows\(^20\):
Consider a quiver with \(p\) vertices with the \(i\)-th vertex associated with \(R_i\). Draw \(\dim[\text{Hom}(R_i, R_j)]\) arrows beginning at vertex \(i\) and ending at vertex \(j\). The relations of the quiver are the obvious ones. The following example illustrates the general situation.

**Example 5.1.** Consider \(X = \mathbb{P}^4\). Here \(\mathcal{R} = \{ \mathcal{O}, \mathcal{O}(1), \ldots, \mathcal{O}(4) \}\).
- \(\text{Hom}(\mathcal{O}(i), \mathcal{O}(i + 1)) = \text{multiplication by } \phi_i\) \((i = 1, \ldots, 5)\) where \(\phi_i\) are homogeneous coordinates on \(\mathbb{P}^n\).
- Relations: Consider \(\text{Hom}(\mathcal{O}, \mathcal{O}(2))\). One has the relation \(\phi_1 \phi_2 = \phi_2 \phi_1\).
- The associated quiver called the *Beilinson quiver* (see figure 3).
- There is a one-to-one correspondence between representations of this quiver and coherent sheaves (and hence D-branes) on \(\mathbb{P}^4\).

It is useful to study the two quivers associated with the two phases of the GLSM associated with the quintic. If we ignore the superpotential, we obtain the orbifold \(\mathbb{C}^5/\mathbb{Z}_5\) in the LG phase and the total space of the line bundle \(\mathcal{O}(-5)\) on \(\mathbb{P}^4\) in the CY phase. As we have seen the D-branes in the LG phase are associated with representations of the McKay quiver while D-branes in the CY phase are associated
with representations of the Beilinson quiver. Comparing the two quivers (see figures 2 and 3), we see that the McKay quiver has extra arrows connecting vertex 5 to vertex 1. If these arrows were present in the CY phase, we would require maps of negative degree. One of the characteristics of the LG phase is that the $p$ field has non-zero vacuum expectation value (vev). This suggests that in the LG phase, these Hom's are given by maps of the form $p\phi_i$, there are five in all which accounts for the extra five arrows in the McKay quiver.

**Conjecture 5.2.** All D-brane configurations that make use of the links that disappear when one goes from the LG to the CY phase should decay.\[10\].

In more general examples one has several phases. The above picture suggests that to each phase (more specifically, a limit point) has its own quiver whose representations give rise to D-brane configurations. This will enable one to handle complicated situations where hybrid phases appear and when there is no orbifold/LG phase. The disappearance of links between phases gives us classes of D-branes that are expected to decay.

**Example 5.3.** Another example which illustrates the scenario is that of the local model for a flop given by the total space of line bundles $O(-1) \oplus O(-1)$ over $\mathbb{P}^1$. The GLSM has four fields $(\phi_1, \ldots, \phi_4)$ with charges $(1, 1, -1, -1)$. There are two phases: $r \gg 0$ where $\phi_3, \phi_4$ have vanishing vevs and $r \ll 0$ where $\phi_1, \phi_2$ have vanishing vevs. The flop-transition is given in figure 4. This example also illustrates

\[\text{Figure 3. The Beilinson quiver for } \mathbb{P}^4\]

\[\text{Figure 4. The quivers for the flop transition}\]

the fact that flop-transition corresponds to a *change of t-structure* in the derived category naturally associated with representations of the quiver.\[27\].
6. Coherent Sheaves in the GLSM with Boundary

It is of interest to see how D-brane configurations associated with coherent sheaves can be realised in the GLSM with boundary. We will for simplicity consider the case when a coherent sheaf $E$ is given as the cohomology of a monad. The more general case is briefly discussed at the end of this section. The basic idea is to consider the following complex (monad) of holomorphic vector bundles $A$, $B$, and $C$

$$0 \to A \xrightarrow{a} B \xrightarrow{b} C \to 0,$$

which is exact at $A$ and $C$. The holomorphic vector bundle

$$E = \ker b/\text{Im } a$$

is the cohomology of the monad.

In a field-theoretic realisation, this is implemented as follows. Consider fermions $\pi_a$ ($a = 1, \ldots, \text{rk } B$). The map $a$ is realised as the gauge invariance

$$\pi_a \sim \pi_a + E^i_a(\phi)\kappa_i,$$

where $\kappa_i$ are sections of $A$ ($i = 1, \ldots, \text{rk } A$). This gauge-invariance is fixed by the condition(s) $E^i_a \pi_a = 0$. The map $b$ is implemented by the holomorphic constraint

$$J^m_a(\phi)\pi_a = 0 \ (m = 1, \ldots, \text{rk } C) .$$

In the GLSM, we will be interested in the case where the boundary preserves the linear combinations of the bulk $(2, 2)$ supersymmetry associated with B-type boundary conditions. In this regard, it is useful to see how the bulk fields decompose. We obtain: (i) A $(2, 2)$ chiral multiplet $\Phi$ decomposes into a scalar and Fermi chiral multiplet $(\Phi', \Xi)$ respectively. (ii) A twisted chiral multiplet $\Sigma$ becomes an unconstrained complex multiplet. (iii) The combination $\tilde{v}_0 = v_0 + \eta^2 \sqrt{2}$ behaves as the boundary gauge field.

We introduce boundary Fermi multiplets satisfying

$$\overline{\mathcal{D}}\Pi_a = \sqrt{2}\Sigma' E_a(\Phi')$$

where $\Sigma'$ is a boundary chiral multiplet. Similarly, the holomorphic constraints require the addition of a boundary chiral multiplet.

So far the story seems similar to $(0, 2)$ constructions for vector bundles in the GLSM. One new ingredient is that large volume monodromy of the vector bundles must be implemented correctly. For example, for the quintic, under $t \to t + 1$, $E \to E \otimes \mathcal{O}(1)$. In the monad construction, it is easy to see that this corresponds to a simple shift in the $U(1)$ charges of the boundary fermions.

In the GLSM, this is done by the addition of boundary contact terms – part of which can be fixed in the Non-Linear Sigma Model (NLSM) limit of the GLSM. Further, the boundary conditions in the GLSM must have a proper NLSM limit and requires careful treatment of the fields in the vector multiplet [12, 28]. See [3] for more details.

One consequence of Beilinson’s theorem, for say, $\mathbb{P}^n$, is that all coherent sheaves arise from complexes of length less than or equal to $n$ ($n = 2$ is a monad). Thus, in order to describe D-branes associated with complexes of arbitrary length, one needs to extend the above discussion to this situation. This requires ones to deal with nested gauge invariances and more fields. Further, in the GLSM with boundary it is more natural to consider direct sums of line-bundles rather than vector bundles.
This can also be implemented in the GLSM with boundary provided one uses first-order actions for the bosonic boundary superfields. This also seems to make the implementation of the large volume monodromy much simpler[3].

We illustrate it with an example in $\mathbb{P}^4$. Consider ten fermi superfields $\Pi_{ij}$ subject to the constraint $(i,j,k = 1,2,\ldots,5$ and $E^k_{ij} = \frac{1}{2}(\phi_i \delta^k_j - \phi_j \delta^k_i))$

$$\mathcal{D}_{ij} \Pi_{ij} = \sqrt{2} \Sigma_k E^k_{ij}(\Phi')$$

where $\Sigma_k$ are five bosonic superfields subject to the constraint ($E_k = \phi_k$)

$$\mathcal{D}_{ij} \Sigma_k = \sqrt{2} N E^k_{ij}(\Phi')$$

Here we have introduced a chiral fermi superfield $N$. The consistency condition between the two constraints is $\sum_k E^k_{ij} E_k = 0$. This is the statement that the composition of two consecutive maps in the following complex vanish.

$$0 \rightarrow \mathcal{O}(-2) \xrightarrow{E_k} \mathcal{O}(-1)^{\oplus 5} \xrightarrow{E^k_{ij}} \mathcal{O}^{\oplus 10} \rightarrow \mathcal{T}^2(-2) \rightarrow 0$$

One can verify that the number of massless fermions are as expected and are sections of $\mathcal{T}^2(-2)$. ($\mathcal{T}$ is the tangent bundle to $\mathbb{P}^4$.)

7. Conclusion

In this talk we have seen a correspondence between D-branes at special limit points in various phases of CY manifolds and representations of quivers. Using this, we have seen how one can predict that certain D-branes will decay as one moves from one phase to another. However, the precise points at which they decay needs more detail and is discussed in the talk by Douglas at this conference. One may wonder if all D-branes on the Calabi-Yau manifold arise from the quivers that we considered. This clearly cannot be the case since not all vector bundles on a Calabi-Yau manifold arise as restrictions of bundles from the ambient variety. This is also related to the fact that the RS states are only a subset of the all boundary states satisfying Cardy’s consistency condition. There are other arguments based on completeness in an axiomatic formulation of string field theory[29] which also lead to similar conclusions.

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