Cluster formation in two-component Fermi gases

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Two-component fermions are known to behave like a gas of molecules in the limit of Bose-Einstein condensation of diatomic pairs tightly bound with zero-range interactions. We discover that the formation of cluster states occurs when the effective range of two-body interaction exceeds roughly 0.46 times the scattering length, regardless of the details of the short-range interaction. Using explicitly correlated Gaussian basis set expansion approach, we calculate the binding energy of cluster states in trapped few-body systems and show the difference of structural properties between cluster states and gas-like states. We identify the condition for cluster formation and discuss potential observation of cluster states in experiments.

Introduction: A fermion is a particle that follows Fermi-Dirac statistics, which gives rise to Pauli exclusion principle [1]. The Pauli exclusion principle states that two or more identical fermions cannot occupy the same quantum state within a quantum system. The result is the emergence of Fermi pressure that prevents white dwarfs and neutron stars from gravitational collapse [2].

Fermi pressure is also responsible for stabilizing dilute two-component Fermi gases [3–6]. As the strength of two-body interaction changes, the two-component Fermi gases experience a crossover from weakly correlated Bardeen-Cooper-Schrieffer (BCS) pairing to a Bose-Einstein condensate (BEC) of tightly bound pairs [7, 8]. In the BEC limit of the BCS-BEC crossover, two unlike fermions form a molecule, which is a composite boson [3, 4]. The system is then governed by dimer-dimer interactions between molecules. Structureless bosons with two-body interactions are known to form cluster states, such as three-body Efimov states [9]. However, such cluster states haven’t yet been found in two-component Fermi gases in the BEC limit. Previous numerical calculations in few-body systems [10, 11] and larger systems [4, 12] focused on the zero-range limit and no cluster states were found in this regime. The reason is that the Fermi pressure between identical fermions prevents such cluster states to form. However, such cluster states could in principle exist with finite-range interactions.

In this Letter, we show that the formation of cluster states occurs when the effective range of the two-body interaction exceeds roughly 0.46 times the scattering length, regardless of the details of the short-range interaction.

The identification of such cluster states are crucial in three ways. First, the formation of clusters in small two-component Fermi gases in three dimensions (3D) corresponds to a phase transition from a droplet-like phase to a gas-like phase in many-body systems. This is reminiscent of the Luttinger liquid and gas of molecule transition in one dimension [13]. Second, the condition for cluster formation is important for preparing such systems in the lab. The magnitude of effective range can approach that of the scattering length in the vicinity of a Feshbach resonance [14]. Current technology allows preparing Fermi gases interacting through large effective range [15, 16]. The stability of the system and the atom loss rate will be strongly affected by the cluster formation. Third, two component Fermi gases share similarities with the low-density region of a neutron star interior [17]. The scaled interaction strength varies in different regions and the effective range is, in general, not negligible [17]. Therefore, the parameter regime in certain parts of the neutron star can overlap with that studied in this Letter. Our results can provide insights in stability of local regions inside a neutron star.

Previous studies have discussed the instability of trapped fermionic gases with attractive interactions within the mean-field framework and the range of interaction is identified as an important factor [5]. Here, we pursue an ab initio calculation using explicitly correlated Gaussian (ECG) basis set expansion approach to study small systems consisting up to 6 particles. The microscopic approach has been proven to be successful in understanding the physics in larger systems [6, 18].

System Hamiltonian: We consider equal mass two-component Fermi gases in 3D consisting of \( N \) spin-up and \( N \) spin-down atoms \((N_{\uparrow} = N_{\downarrow} = N)\) under external spherically symmetric harmonic confinement with angular trapping frequency \( \omega \). The system Hamiltonian \( H \) reads

\[
H = \sum_{i=1}^{N} \left( -\frac{\hbar^2}{2m} \nabla_{i}^2 + \frac{1}{2} m \omega^2 r_{i}^2 \right) + \sum_{i=1}^{N} \left( -\frac{\hbar^2}{2m} \nabla_{i}^2 \right) + \frac{1}{2} m \omega^2 r_{i}^2 \right) + \sum_{i=1}^{N} \sum_{i=1}^{N} V_{2b}(r_{i,i'}), \tag{1}
\]

where \( m \) denotes the mass of a single atom, \( \vec{r}_{i} \) and \( \vec{r}_{i}' \) denote the position vector of the \( i \)th spin-up and down atom with respect to the trap center, respectively. We define the harmonic oscillator length \( a_{ho} = \sqrt{\hbar/(m \omega)} \) and harmonic oscillator energy \( E_{2b} = \hbar \omega \). \( V_{2b} \) is the interspecies two-body interaction potential that depends on the interparticle distance \( r_{i,i'} \).

\[ r_{i,i'} = |\vec{r}_{i} - \vec{r}_{i'}|. \]
We considered three different short-range potentials. (i) An attractive Gaussian potential with a repulsive core, \( V_{2b}^{(i)}(r) = V_0 \exp(-r^2/(4a_s^2)) - 2V_0 \exp(-r^2/(2a_s^2)) \); (ii) An attractive Gaussian potential, \( V_{2b}^{(ii)}(r) = V_0 \exp(-r^2/(2a_s^2)) \); and (iii) A modified attractive Gaussian potential, \( V_{2b}^{(iii)}(r) = rV_0 \exp(-r^2/(2a_s^2)) \). The scattering phase shift of two particles at low energy can be expanded as \(-k \cot(\delta(k)) = 1/a_s - r_{\text{eff}} k^2/2\), where \( k \) is the wave vector, \( a_s \) the s-wave scattering length, and \( r_{\text{eff}} \) the effective range. For a fixed \( r_0 \), we adjust \( V_0 \) such that \( V_{2b}(r) \) has a certain \( a_s \). For all three types considered in this work, shallow attractive potentials do not support two-body s-wave bound state in free-space. As the depth increases, the potential supports successively more two-body bound states [6]. In this work, we only consider attractive potentials that support one s-wave bound state in free space. We then calculate \( r_{\text{eff}} \) for such potential. Solid, dashed, and dotted lines in Fig. 1 show potentials (i), (ii), and (iii) that produce the same \( a_s = 0.2a_{ho} \) and \( r_{\text{eff}} = 0.09a_{ho} \), respectively. We’ll show later that the condition for cluster formation does not strongly depend on the type of potential. This indicates that the physics is well described by the two parameters \( a_s \) and \( r_{\text{eff}} \) alone. Although all three types of the potentials are short-range, type (i) simulates the repulsive core in realistic atom-atom interactions. Furthermore, for a fixed \( a_s \), potential type (i) can produce a larger range of \( r_{\text{eff}} \), allowing us to study a wider parameter regime.

![FIG. 1](image.png)

We solve the time-independent Schrödinger equation for the Hamiltonian given in Eq. (1) using ECG basis set expansion approach [19]. After separating off the center-of-mass degrees of freedom, we expand the eigenstates of the relative Hamiltonian in terms of ECG basis functions, which depend on a number of nonlinear variational parameters that are optimized through energy minimization [10, 11, 19, 20].

**Bound state energy**: Trapped (1,1) system, consisting one spin-up and one spin-down particle has been solved analytically. The exact energy spectrum for zero-range interaction is given in Ref. [21] while the contribution from the effective range is discussed in Ref. [22]. When \( a_s \) is positive and much smaller than \( a_{ho} \), the binding energy of the two-particle pair with short-range interaction can be approximated by \( E_s = -\frac{\hbar^2}{(ma_s^2)} \). For \( a_s = 0.2a_{ho} \), the binding energy is approximately \(-25E_{ho}\). The range dependence of the binding energy is negative, i.e., the binding energy decreases for increasing \( r_{\text{eff}} \).

For trapped two-component Fermi gases with more particles, i.e., \((N,N)\) systems, two particles with opposite spin form a molecule just like in (1,1) system. Such molecules were often treated as a single composite boson and an effective model with effective dimer-dimer interactions are shown to be very accurate in the zero-range limit [3, 10, 11]. In this effective model, the dimer-dimer interaction has a scattering length \( a_{dd} = 0.608a_s \), a small positive value compared to \( a_{ho} \), and does not support any bound state of two dimers [3]. This indicates that a dilute two-component Fermi gas behaves like a Bose gas with hard-core repulsive interaction in the BEC and zero-range interaction limit. However, dimer-dimer interaction isn’t \textit{a priori} repulsive and cluster states can in principle exist. An important question is what is the condition for cluster to form. In the following, we’ll show that the effective range \( r_{\text{eff}} \) plays an important role.

We calculate the relative ground state energies of (1,1), (2,2), and (3,3) systems, \( E_{11}, E_{22}, \) and \( E_{33} \), respectively, for fixed \( a_s = 0.2a_{ho} \) and a wide range of \( r_{\text{eff}} \). The center-of-mass energy is simply \( \hbar \omega \) for all systems and is not included in our results. To identify the cluster formation between dimers, we follow Ref. [10] and subtract the energy of molecules from the four- and six-body systems, i.e., we plot \( \Delta E_{22} = E_{22} - 2E_{11} \) and \( \Delta E_{33} = E_{33} - 3E_{11} \) as a function of \( r_{\text{eff}} \) in Fig. 2. Circles, squares, and diamonds are for potential type (i), (ii), and (iii), respectively. For small \( r_{\text{eff}} \), our results agree with previous studies: \( \Delta E_{22} \) and \( \Delta E_{33} \) are positive, and agree with the zero-range ground state energy for two and three weakly repulsive bosons with dimer-dimer scattering length \( a_{dd} = 0.608a_s \), which are marked as dotted lines in the insets. This indicates that the ground state is indeed a gas-like state. \( \Delta E_{22} \) and \( \Delta E_{33} \) remain largely unchanged as \( r_{\text{eff}} \) gradually increases. As \( r_{\text{eff}} \) approaches 0.9\( a_{ho} \), \( \Delta E_{22} \) and \( \Delta E_{33} \) decrease suddenly and turn negative for all three types of interactions, signalling the formation of bound states between dimers. Note that the magnitude of \( \Delta E_{33} \) is several times larger than \( \Delta E_{22} \), indicating that the bound state in (3,3) system is indeed a three-dimer bound state, instead of a two-dimer bound state plus a single dimer. Our calculations show that \( r_{\text{eff}} \) at which cluster starts to form is largely independent of the details of the two-body interactions and is roughly the same for (2,2) and (3,3) systems. This shows that \( r_{\text{eff}} \) is the deciding factor for cluster formation in two-component Fermi gases.
Our physics picture is as follows. The existence of cluster states in two-component Fermi gases is determined by the counterbalance between the Fermi pressure between like-particles and the dimer-dimer interaction. The Fermi pressure is a short-range effect, preventing like fermions from getting close to each other through the requirement of anti-symmetrization. On the other hand, cluster formation requires all particles, like or unlike, to be at distances of the order of $r_{\text{eff}}$ [3]. When $r_{\text{eff}}$ is small, the anti-symmetrization of two like fermions cannot happen within such small distance. Therefore, such interaction cannot support bound states between dimers. When $r_{\text{eff}}$ becomes larger, the dimer-dimer interaction can potentially overcome the Fermi pressure, allowing anti-symmetrization of two like fermions to happen within the range of $r_{\text{eff}}$.

Structural properties: In order to take a peak at the wave function, we calculate several structural properties for both the gas-like state and the cluster state interacting through potential type (i). First, we consider the spherically symmetric radial density $P_1(r)$, which tells the likelihood of finding a particle at distance $r$ from the trap center, with normalization $4\pi \int_0^\infty P_1(r) r^2 dr = 1$. Figure 3(a) and (d) show the radial density $P_1(r)$ for gas-like and cluster state, respectively. For both gas-like and cluster state, $P_1(r)$ peaks at the trap center and decays towards the edge. The overall extent of a single particle can be measured by expectation value $\langle r \rangle = 4\pi \int_0^\infty P_1(r) r^2 dr$. For both (2, 2) and (3, 3) systems, the expectation values for the cluster states are about 30% lower than gas-like states (see more details in Supplemental Material [23]). We confirm that the expectation values do not differ much for different $r_{\text{eff}}$ within the gas-like state and cluster state. The drop in $\langle r \rangle$ represents a sudden change from the gas-like state to the cluster state.

Second, we consider the scaled distribution function between unlike pair $4\pi P_{12}(r)r^2$, which tells the likelihood of finding two unlike particles at distance $r$ from each other, with normalization $4\pi \int_0^\infty P_{12}(r) r^2 dr = 1$. Since we consider the BEC limit, it is natural to expect a sharp peak at a short distance that is of the order of $a_s$ as two unlike particles form a molecule. Indeed, for both the gas-like and cluster state, a sharp peak at short distance is identified. A difference is that the distribution function for cluster state vanishes at the order of trap length $a_{\text{ho}}$, while exhibits a lower and wider second peak for the gas-like state. This lower and wider peak corresponds to the distribution between two unlike particles that belongs to different dimers.
Third, we consider the scaled distribution function between like pair $4\pi P_{11}(r)r^2$, which tells the likelihood of finding two like particles at distance $r$ from each other. With normalization $4\pi \int_0^\infty P_{11}(r)r^2\,dr = 1$. The scaled distribution function between like pair is a good gauge of the overall extent of the system. For gas-like state, a broad distribution centered around the trap length $a_{ho}$ is observed, confirming that the system is indeed extended to the confinement of the trap. For cluster state, we find that a peak appears at a similar distance to the unlike pair, which is an order of magnitude smaller than gas-like state. This difference clearly distinguishes the two states. Further details on the expectation values of distances between unlike and like pairs are discussed in Supplemental Material [23].

**Condition for cluster formation:** For a certain $a_s$, both the ground state energies and the structural properties exhibit a clear transition between the gas-like and cluster state. Although our calculations are performed with short-range interactions due to the counterbalance between the Fermi pressure and the material limit, i.e., with infinitely large scattering length. This could explain the observation in Ref. [24] where no cluster states were observed, confirming that the system is indeed extended.

To observe cluster formation, it is important to know the relation between $a_s$ and $r_{eff}$ in realistic interactions. Neutral atom interactions are best modelled by van der Waals potential $V_{2h,vdW}(r_{ij}) = -C_6/r_{ij}^6$. van der Waals length and so called mean scattering length are defined as $R_{vdW} = (mC_6/\hbar^2)^{1/2}/2$ and $\bar{a} = [4\pi/\Gamma(1/4)^2] R_{vdW}$, where $\Gamma(x)$ is the Gamma function. Reference [28] obtained the relation between $r_{eff}$ and $a_s$ for van der Waals potential:

$$r_{eff} = \Gamma(1/4)^4/(6\pi^2)\bar{a} \left[ 1 - 2\bar{a}/a_s + 2 (\bar{a}/a_s)^2 \right].$$

This formula has been shown to work reasonably well near broad Feshbach resonances [29]. In the case of $^{40}$K, $R_{vdW}$ was calculated to be 64.9$a_0$, where $a_0$ denotes the Bohr radius [30]. The scattering length is tunable in the vicinity of a Feshbach resonance between $|f,m_f| = [9/2, -9/2]$ and $[9/2, -5/2]$, which is centred at $B_{ok} = 224.21 G$ [25]. Together with Eq. (2), we determine that the system is in gas-like state for magnetic field 200.3G $< B < 233.9G$ (see more details in Supplemental Material [23]). Within this range, the two component Fermi gas went through the BEC-BCS crossover as observed in the experiments [25]. We expect cluster to form outside this range. This is consistent with the observations that two component Fermi gases are generally stable near Feshbach resonances [3].

Cluster formation can potentially be observed through rf spectroscopy in a magnetically trapped two-component Fermi gas with thousands of atoms [25, 26]. Previous experimental measurements [25] on $|f,m_f| = [9/2, -9/2]$ and $[9/2, -5/2]$ of $^{40}$K are performed with $215G < B < 230G$, which lies entirely within the gas-like state according to our prediction. Our results suggest that cluster formation may be observed further away from the Feshbach resonance. A more direct reproduction of the system that we studied, i.e., a trapped system with only few atoms, can potentially be realized in optical tweezer systems [31, 32] and optical microtraps [33].

**Final remarks:** We studied the formation of cluster state in two-component Fermi gases by numerically calculating the ground state energy of a trapped few-body system interacting through various types of short-range interactions. Our findings corroborate the picture that the counterbalance between the Fermi pressure and the short-range interactions determines the boundary between the gas-like and cluster state. Although our calculations are performed with short-range interactions due to numerical limitation, the existing range of data strongly suggests that cluster states are absent for the zero-range interaction. This confirms the conclusion in Ref. [4, 10–12]. Third, as we increase $a_s$, the region of cluster state shrinks. This indicates that it could be more difficult to find cluster states as the system moves towards the unitarity limit, i.e., with infinitely large scattering length. This could explain the observation in Ref. [24] where no
to the numerical restrictions, we expect such result to apply to realistic interactions because of the short-range nature of the Fermi pressure. We also provide an estimate for the parameter regime where such cluster states could potentially be identified in the experiments.

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[1] E. Fermi, Nuovo Cimento 11, 157 (1934).
[2] D. Koester and G. Chanmugam, Reports on Progress in Physics 53, 837 (1990).
[3] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Phys. Rev. Lett. 93, 090404 (2004).
[4] G. E. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. 93, 200404 (2004).
[5] B. M. Fregoso and G. Baym, Phys. Rev. A 73, 043616 (2006).
[6] D. Blume, Reports on Progress in Physics 75, 046401 (2012).
[7] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92, 040403 (2004).
[8] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Phys. Rev. Lett. 92, 120403 (2004).
[9] C. H. Greene, P. Giannakeas, and J. Pérez-Ríos, Rev. Mod. Phys. 89, 035006 (2017).
[10] D. Blume and K. M. Daily, Phys. Rev. A 80, 053626 (2009).
[11] D. Blume and K. M. Daily, Comptes Rendus Physique 12, 86 (2011).
[12] D. Blume, J. von Stecher, and C. H. Greene, Phys. Rev. Lett. 99, 233201 (2007).
[13] K. T. Law and D. E. Feldman, Phys. Rev. Lett. 101, 096401 (2008).
[14] P. Dyke, S. E. Pollack, and R. G. Hulet, Phys. Rev. A 88, 023625 (2013).
[15] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).
[16] E. L. Hazlett, Y. Zhang, R. W. Stites, and K. M. O’Hara, Phys. Rev. Lett. 108, 045304 (2012).
[17] P. van Wyk, H. Tajima, D. Inotani, A. Ohnishi, and Y. Ohashi, Phys. Rev. A 97, 013601 (2018).
[18] J. Levinsen, P. Massignan, S. Endo, and M. M. Parish, Journal of Physics B: Atomic, Molecular and Optical Physics 50, 072001 (2017).
[19] J. Mitroy, S. Bubin, W. Horiiuchi, L. Adamowicz, W. Cencek, K. Szalewicz, J. Komasa, D. Blume, and K. Varga, Rev. Mod. Phys. 85, 693 (2013).
[20] X. Y. Yin and D. Blume, Phys. Rev. A 92, 013608 (2015).
[21] T. Busch, B.-G. Englert, K. Rzaewski, and M. Wilkens, Found. of Phys. 28, 549 (1998).
[22] F. Werner and Y. Castin, Phys. Rev. A 86, 013626 (2012).
[23] See Supplemental Material at inset link for more details on expectation values related to structural properties and the Feshbach resonance in \(^{40}\)K.
[24] M. M. Forbes, S. Gandolfi, and A. Gezerlis, Phys. Rev. A 86, 053603 (2012).
[25] C. A. Regal and D. S. Jin, Phys. Rev. Lett. 90, 230404 (2003).
[26] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92, 083201 (2004).
[27] M. Bartenstein, A. Altmeyer, S. Riedl, R. Geursen, S. Jochim, C. Chin, J. H. Denschlag, R. Grimm, A. Simonis, E. Tiesinga, C. J. Williams, and P. S. Julienne, Phys. Rev. Lett. 94, 103201 (2005).
[28] B. Gao, Phys. Rev. A 58, 1728 (1998).
[29] C. L. Blackley, P. S. Julienne, and J. M. Hutson, Phys. Rev. A 89, 042701 (2014).
[30] A. Derevianko, W. R. Johnson, M. S. Safronova, and J. F. Babb, Phys. Rev. Lett. 82, 3589 (1999).
[31] A. M. Kaufman, B. J. Lester, and C. A. Regal, Phys. Rev. X 2, 041014 (2012).
[32] L. R. Liu, J. D. Hood, Y. Yu, J. T. Zhang, N. R. Hutzler, T. Rosenband, and K.-K. Ni, Science 360, 900 (2018).
[33] F. Serwane, G. Zürn, T. Lompe, T. B. Ottenstein, A. N. Wenz, and S. Jochim, Science 332, 336 (2011).
Supplemental Material for “Cluster formation in two-component Fermi gases”

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EXPECTATION VALUES RELATED TO STRUCTURAL PROPERTIES

We follow the notation in the main text. To gain more insights from the structural properties, we consider three expectation values calculated from the spherically symmetric radial density \( P_1(r) \), the scaled distribution function between unlike pair \( 4\pi P_{12}(r)r^2 \), and the scaled distribution function between like pair \( 4\pi P_{11}(r)r^2 \). The expectation value \( \langle r \rangle = 4\pi \int_0^\infty P_1(r)r^3dr \) measures the overall extent of a single particle, while \( \langle r_{12} \rangle = 4\pi \int_0^\infty P_{12}(r)r^3dr \) and \( \langle r_{11} \rangle = 4\pi \int_0^\infty P_{11}(r)r^3dr \) measure the expected distance between unlike and like particles, respectively.

\[
\langle r_{12} \rangle = 4\pi \int_0^\infty P_{12}(r)r^3dr \\
\langle r_{11} \rangle = 4\pi \int_0^\infty P_{11}(r)r^3dr
\]

FIG. S1. (Color online) Panel (a), (b), and (c) show the expectation values \( \langle r \rangle \), \( \langle r_{12} \rangle \), and \( \langle r_{11} \rangle \) as a function of effective range \( r_{\text{eff}} \) for \( a_s = 0.2a_{ho} \). Blue circles and red squares are for (2, 2) and (3, 3) systems, respectively.

Circles and squares in Fig. S1 show the three expectation values for (2, 2) and (3, 3) systems, respectively. In both systems, a sudden decrease of all three expectation values occurs at around \( r_{\text{eff}} = 0.9a_{ho} \), where the transition from gas-like state to cluster state occurs. Notably, \( \langle r_{11} \rangle \) decreases by an order of magnitude as cluster forms.

ADDITIONAL DETAILS ON THE FESHBACH RESONANCE IN \( ^{40}K \)

We consider the Feshbach resonance between \( |f, m_f\rangle = |9/2, -9/2\rangle \) and \( |9/2, -5/2\rangle \) in \( ^{40}K \) [1]. We plot the relation between \( a_s \) and the magnetic field \( B \) in Fig. S2 according to the background scattering length \( a_{bg} = 174a_0 \), resonance peak \( B_{pk} = 224.21 \pm 0.05G \) and width \( w = 9.7 \pm 0.6G \), measured with high precision in Ref. [1]. Together with Eq. (2) in the main text, we can determine the boundary between the cluster state and the gas-like state for this Feshbach resonance.

\[
B_{pk} = 224.21 \pm 0.05G \\
w = 9.7 \pm 0.6G
\]

FIG. S2. (Color online) Dashed line shows \( a_s \) as a function of magnetic field \( B \) near the Feshbach resonance between \( |f, m_f\rangle = |9/2, -9/2\rangle \) and \( |9/2, -5/2\rangle \) of \( ^{40}K \). Dotted vertical line marks the position of Feshbach resonance. The regime of cluster state is determined according to condition \( 0 < r_{\text{eff}} < 0.46a_{s} \).

[1] C. A. Regal and D. S. Jin, Phys. Rev. Lett. 90, 230404 (2003).