Dissipative Effects in Photon Diagnostics of Quark Gluon Plasma

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Abstract

The effects of dissipation on the space time evolution of matter formed in ultra-relativistic heavy ion collision is discussed. The thermal photon spectra for RHIC and LHC energies with viscous flow is considered. The effects of viscosity in the thermal single photon spectra is seen to be important in QGP phase as compared to the hadronic phase. Recently available data from WA80 group at SPS energies for $S + Au$ collision are compared with theoretical calculations. The experimental data do not appear to be compatible with the formation of matter in the pure hadronic phase.

Keywords: Quark Gluon Plasma, Hydrodynamics, Dissipation, Photon Spectra.
I. Introduction

The primary motivation for studying heavy ion collisions at ultrarelativistic energies is that such reactions provide an unique opportunity to probe hadronic matter at very high temperature and/or density. Numerical simulations of the QCD equation of state on the lattice predict that at a sufficiently high temperature (~160 – 200) MeV, a phase transition should occur from the colour confined chirally asymmetric phase of QCD, the hadronic phase, to a (locally) colour deconfined, chirally symmetric phase, referred to as the Quark-Gluon Plasma (QGP) \[1\]. The QCD phase transition is of great relevance in cosmology also \[2\]; within the standard big bang model, the early universe should have experienced such a transition a few microseconds after the big bang. Although the order of this phase transition remains to be settled, a substantial volume of work has already been carried out, assuming it to be of first order both in the laboratory as well as in the early universe. In this work, we shall tacitly assume that the hadron to QGP phase transition is indeed of first order.

In most of the works aimed at devising signals of the above mentioned phase transition in the laboratory, the usual scenario is as follows: after a canonical (proper) time \(\sim 1 \text{ fm/c}\) (the strong interaction time scale), the excited matter within the reaction volume of the colliding heavy ions comes to a (local) thermal equilibrium with a temperature \(T_i\). If this temperature \(T_i\) is larger than the critical temperature \(T_c\) of the QCD phase transition, then the excited matter would exist in the form of QGP. The QGP then evolves in space and time, cooling down to the critical temperature \(T_c\) when the confinement phase transition starts; for a first order transition the system remains in a mixture of QGP and hadronic matter at a temperature \(T_c\) for some time the duration of which is determined by the relative number of the degrees of freedom in the QGP and hadronic phases. Such a configuration continues until all the latent heat which maintains the temperature at \(T_c\) by compensating the cooling due to expansion is fully exhausted. At this stage all of the QGP converts to hadronic phase and the system continues to cool further, its dynamics being governed by the hadronic equation of state, until the mean free paths of the constituent hadrons become too long to maintain a collective behaviour. At this point, usually referred to as freeze-out era, the momentum distributions of the particles become frozen at the characteristic values when the particles suffer the last collision, and the particles free stream thereafter, carrying the space time integrated informations to the detectors. For the sake of simplicity, most studies on QGP diagnostics assume that the system behaves as an ideal i.e. nondissipative, fluid throughout the entire evolution.

Given the tremendous importance of the issue, it is imperative to understand how far, if at all, such an idealisation is justified. Even though a rigorous treatment
of the dissipative effects in relativistic fluid dynamics is still beset with several technical as well as conceptual difficulties [3, 4], one must admit that in any realistic scenario, dissipative effects, in principle should have an important role. In this work we consider the relativistic fluid to be weakly dissipative, as has been argued to be the case by earlier authors [3] and explore the ensuing consequences on the estimation of the initial temperature and the photon spectra, data for which have become available of late. Since our motivation in the present work is to study the relative importance of the dissipative effect vis-a-vis an ideal fluid dynamic scenario that has been routinely used by a number of authors [3, 7, 8, 9] to compare the currently available data and/or to estimate experimentally measurable quantities at RHIC or LHC, we adopt the same evolution scenario (Bjorken scaling flow), together with the same formation time $\tau_i (\approx 1 \text{ fm/c})$. It must be mentioned at the outset that our present purpose is demonstrative rather than making firm phenomenological comparisons.

The organisation of the present work is as follows. In the next section, we briefly recapitulate the space-time evolution [4] of the system with dissipation. In section III we discuss the effect of dissipation on the boundary or initial conditions governing the evolution equation. Section IV briefly recapitulates the formalism for photon diagnostics of QGP. Finally in section V, we present the results of our actual estimates at LHC, RHIC and SPS energies and compare them with the available data [10] at SPS. Section VI contains a brief summary and conclusions.

II. Viscous Hydrodynamics at Relativistic Energies

For a perfect fluid, the mean free path of the constituent particles is negligibly small compared to all the available length scales in the system. In a finite system like colliding nuclei, however large, such a criterion may not be fulfilled and as such, the changes in the collective quantities like pressure, energy density, number density, velocity etc. over a distance of a mean free path cannot always be ignored. This should lead to an inherent dissipative behaviour.

In the presence of dissipation, the kinetic energy of the fluid decays as heat energy. This requires a redefinition of the energy-momentum tensor and the particle current [11]. For the sake of brevity, we simply quote the result in the form of energy-momentum conservation in an imperfect fluid which reads [3]

$$\frac{d \epsilon}{d \tau} + \frac{(\epsilon + P)}{\tau} = (\frac{4}{3} \eta + \zeta)/\tau^2$$

(1)

Eq.(1) is the scaling Navier-Stokes(NS) equation where we have assumed the validity of the scaling solution [12], usually referred to as the Bjorken solution [13]. The rate of heat generation is $(4\eta/3 + \zeta)/\tau^2$, which vanishes in the case of an ideal fluid.

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\( \eta \to 0, \zeta \to 0 \). Obviously, the occurrence of the viscosity co-efficients \( \eta \) and \( \zeta \) in eq.(1) violates the invariance under \( \tau \to -\tau \), and consequently the reversibility \( \text{(4)}. \) As a result, entropy is generated during the temporal evolution of the system; thus, estimating the initial temperature from the final multiplicity in the usual manner \( \text{(3)} \) constitutes an overestimate (see Appendix-I).

We assume, without any loss of generality that the functional relation between the pressure and energy density in case of dissipative flow is same as that of ideal flow \( \text{(11).} \) We shall, as is the usual practice, assume that the baryon density is zero in the central region so that the thermal conductivity \( K = 0 \) \( \text{(3)} \) throughout.

IIa. Quark-Gluon Plasma Phase

The limits on the shear viscosity for the NS equation (eq. 1) in the QGP phase have been discussed in depth in \( \text{(5)} \) (see also \( \text{(15)} \)). The acceptable range of \( \eta \) is

\[
2T^3 \leq \eta \leq 3T^3(\tau T) \tag{2}
\]

Obviously, the range is quite small unless the product \( T\tau \) (very late times and/or large temperatures) is very large. As should be clear from what follows, such a situation is rather unlikely within the QGP phase.

The bulk viscosity \( \zeta \), in general can be as large as \( \eta \), or in certain circumstances, even larger \( \text{(16)} \). It was however shown by Weinberg \( \text{(16)} \) some time ago that if the trace of the energy momentum tensor be expressible as a function of \( \epsilon \) and the pressure is the same function of \( \epsilon \) as in the adiabatic case, then \( \zeta = 0 \). We thus obtain for the scaling NS equation in the QGP phase

\[
\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = \frac{4}{3} \frac{\eta}{\tau^2} \tag{3}
\]

Although eq. (3) is somewhat simpler than eq. (1), we work with the more general form for the time being. For \( \epsilon = 3aQT^4 \) and \( P = \epsilon/3 \), where \( a_Q = (\pi^2/90)g_{QGP} \), we can solve eq. (1) with the boundary condition \( T = T_i \) at \( \tau = \tau_i \):

\[
T = T_i(\tau_i/\tau)^{1/3} + \left[ \frac{1}{6} \eta Q_0 + \frac{1}{8} \zeta Q_0 \right] \frac{1}{a_Q \tau_i} \left[ (\tau_i/\tau)^{1/3} - \tau_i/\tau \right] \tag{4}
\]

where \( \eta Q_0 = \eta_Q/T^3 \) and \( \zeta Q_0 = \zeta_Q/T^3 \). The proper time \( \tau_Q \) taken by the system to cool down from the initial temperature \( T_i (> T_c) \) to the critical temperature \( T_c \) can be calculated by inverting eq. (4):

\[
\tau_Q = \tau_Q(\tau_i, T_i, T_c) \tag{5}
\]

The life time of the QGP phase is \( \tau_Q - \tau_i \). It can be readily seen that in the presence of dissipation, the life time of the QGP phase becomes longer (due to viscous heating) relative to the ideal case, for the same values of \( T_i, \tau_i \) and \( T_c \). For an actual estimate we use \( \eta Q_0 = 2.5 \), as dictated by eq. (2) and \( \zeta Q_0 = 0 \) in the pure QGP phase.
IIb. The Mixed Phase

When the temperature of the system comes down to $T_c$, the system goes over to a mixed phase composed of QGP and hadronic matter. In this phase the temperature remains constant but the energy density changes due to the change in the statistical degeneracy on account of hadronization. We define $f_Q$ as the volume fraction of the QGP in the mixed phase, so that the energy density at any proper time $\tau$ during the life time of the mixed phase is given by

$$\epsilon(\tau) = f_Q(\tau)\epsilon_Q + (1 - f_Q(\tau))\epsilon_H$$

(6)

where $\epsilon_Q(\epsilon_H)$ is the energy density in the QGP(hadronic) phase at temperature $T_c$. Eq.(6) is obtained by a linear interpolation between the two points $(1, \epsilon_Q)$ and $(0, \epsilon_H)$ in the $(f_Q, \epsilon)$ plane. For the coefficients of viscosity, we can formally use a similar interpolation,

$$\eta(\tau) = f_Q(\tau)\eta_Q + (1 - f_Q(\tau))\eta_H$$

(7)

and

$$\zeta(\tau) = f_Q(\tau)\zeta_Q + (1 - f_Q(\tau))\zeta_H$$

(8)

where the subscript $Q(H)$ denotes the value in the QGP(Hadronic) phase at $T_c$.

The applicable limits on $\eta_Q$ and $\zeta_Q$ have already been discussed in the previous section. The corresponding situation in the hadronic phase is explained in the following section.

The evolution equation for $f_Q(\tau)$ in the mixed phase can be readily obtained from eqs. (1), (6), (7) and (8):

$$\frac{df_Q}{d\tau} + \left(1 - \frac{b}{\tau^2}\right)f_Q = \frac{c}{\tau^2} - \frac{a}{\tau}$$

(9)

where $a = (\epsilon_H + P_c)/\Delta \epsilon ; b = 4(\eta_Q - \eta_H)/3\Delta \epsilon ; c = (2\xi_0 + 4s\eta_H)/\Delta \epsilon ; \Delta \epsilon = \epsilon_Q - \epsilon_H$. The solution of this equation with the boundary condition $f_Q = 1$ at $\tau = \tau_Q$ is given by

$$f_Q = (ab - c) [Ei(b/\tau) - Ei(b/\tau_Q)] e^{-b/\tau} + (a + 1) \frac{\tau_Q}{\tau} e^{(b/\tau_Q - b/\tau)} - a$$

(10)

where $Ei(x)$ is the exponential function. For $b = c = 0(\eta = \xi = 0)$ we recover the solution for $f_Q$ in case of a perfect fluid. Note however that the bulk viscosity $\zeta$ in the mixed phase may be non-zero due to the change in sound velocity over a finite relaxation time. We use $\zeta = 2\eta/3$, following Hosoya and Kajantie. The time $\tau_H$, when the mixed phase ends, is obtained from eq. (10) by demanding $f_Q(\tau_H) = 0$. We thus have,

$$\tau_H = \tau_Q(\tau_Q)$$

(11)

for given equations of state in the QGP and the hadronic phases. The life time of the mixed phase is then $\tau_H - \tau_Q$. 

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IIc. Hadronic Phase

The pure hadronic phase starts at $T = T_c$ at the time $\tau_H$ so that the solution of eq. (1) in the hadronic phase is again eq. (4) with the appropriate substitution of the $a_Q$ by $a_H$, $T_i$ by $T_c$ and $\tau_i$ by $\tau_H$:

$$T = T_c(\tau_H/\tau)^{1/3} + \left(\frac{1}{6} \eta_{H0} + \frac{1}{8} \zeta_{H0}\right)\frac{1}{a_H \tau_H} \left[\left(\frac{T_H}{\tau}\right)^{1/3} - \frac{T_H}{\tau}\right]$$

In the hadronic phase, the co-efficient of the shear viscosity can be obtained from the estimated transport cross-section according to the relation $\eta_H \approx T/\sigma_{tr} \sim (T/200MeV)(0.5 - 1)/\text{fm}^3[3]$. The expected range of $\sigma_{tr} \sim 10 - 20 \text{ mb}$ which corresponds to $\eta_{H0} \sim 0.75 - 1.5$ at $T_c$. In what follows we shall use the larger value of 1.5 for $\eta_{H0}$ as a conservative case. The value of the bulk viscosity can be non-zero in the hadronic phase due to lack of chemical equilibrium and its consequence on the sound velocity [18], we again take $\zeta_H = 2\eta_H/3$ [15].

As mentioned in Section I, the system freezes out at a time denoted by $\tau_f$, when it becomes too dilute to maintain a collective flow. There is as yet no universal criterion for determining the instant of freeze-out, for a detailed discussion, see [6]. In the present context, we describe freeze-out through a freeze-out temperature $T_f$ which we treat as a free parameter ($\leq 140 \text{ MeV}$). Then from eq.(12) $\tau_f$ is given by

$$\tau_f = \tau_f(\tau_H, T_c, T_f)$$

so that the life time of the hadronic phase is $\tau_f - \tau_H$.

III. Hydrodynamic Flow and Connections with Observables

In the case of an ideal fluid, the conservation of entropy implies that the rapidity density $dN/dy$ is a constant of motion for the isentropic flow [13]. In such circumstances, the experimentally observed multiplicity, $dN/dy$ may be related to a combination of the initial temperature $T_i$ and the initial time $\tau_i$ as $T_i^3 \tau_i$. Assuming an appropriate value of $\tau_i$ (taken to be $\sim 1 \text{ fm}/c$), one can estimate $T_i$.

For dissipative systems, such an estimate is obviously inapplicable (see appendix I). Generation of entropy during the evolution invalidates the role of $dN/dy$ as a handy constant of motion. Moreover, the irreversibility arising out of dissipative effects implies that estimation of the initial temperature from the final rapidity density is no longer a trivial task. We can, nevertheless, relate the experimental $dN/dy$ to the freeze-out temperature $T_f$ and the freeze-out time $\tau_f$ by the relation,

$$\frac{dN}{dy} = \pi R_A^2 a_H T_f^3 \tau_f/c$$

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where $R_A$ is the radius of the colliding nuclei (we consider $AA$ collision for simplicity) and $c$ is a constant $\sim 3.6$. The parameter $a_H$ occurring in eq. (14) is the so called statistical degeneracy ($a = (\pi^2/90)g$) for an effectively massless hadronic gas. In this work we assume the hadronic matter to consist of $\pi, \rho, \omega$ and $\eta$, which allows us to estimate $a_H$ through a parametrization introduced earlier [19]. It must also be mentioned that eq. (14) still constitutes a tremendous amount of idealization; for a discussion of all the caveats, see [6]. For our present purpose, we assume that eq. (14) does hold at the freeze-out era. We shall also neglect transverse expansion in this work, which would amount to an overestimation of $\tau_f$, for a given $T_f$ and $dN/dy$. We however do not consider this to be a major factor; this is to be discussed in some detail in Sec. V.

IV. Photon Spectra

In this section we will briefly discuss the photon spectra due to an expanding quark gluon plasma. In the QGP phase the main contribution comes from the annihilation $(q\bar{q} \rightarrow g\gamma)$ and Compton processes $(q\bar{q}g \rightarrow q\bar{q}\gamma)$. In the hadronic phase (composed of $\pi, \rho, \omega, \eta$), an array of reactions like $\pi\rho \rightarrow \pi\gamma$, $\pi\eta \rightarrow \pi\gamma$, $\pi\pi \rightarrow \eta\gamma$, $\pi\pi \rightarrow \rho\gamma$, $\pi\pi \rightarrow \gamma\gamma$ and the decays $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and $\omega \rightarrow \pi^0\gamma$ are considered [20].

The rate of emission of photons per unit volume from QGP phase is given by [20],

$$E \frac{dR}{d^3p} = \frac{5}{9} \frac{\alpha\alpha_s T^2 e^{-E/T}}{(2\pi^2)^2} \ln \left( \frac{2.912 E}{g^2 T} + 1 \right)$$

(15)

For our exploratory calculations we will consider the rates of photon emission from the QGP and the hadronic matter to be equal as in [20, 21]. It is a very useful first approximation for identifying the $p_T$ window where we concentrate on the signature of QGP. The $p_T$ distribution of photons is obtained by convoluting the basic rates with the space time history,

$$\frac{dN}{d^2p_T dy} = \pi R_A^2 \int \left[ \left( E \frac{dR}{d^3p} \right)_{QGP} \Theta(\epsilon - \epsilon_Q) + \left\{ \left( E \frac{dR}{d^3p} \right)_{QGP} f_Q + \left( E \frac{dR}{d^3p} \right)_{H} (1 - f_Q) \right\} \Theta(M) \right. + \left. \left( E \frac{dR}{d^3p} \right)_{H} \Theta(\epsilon_H - \epsilon) \right] \tau d\tau d\eta$$

(16)

where $\Theta(M) = \Theta(\epsilon_Q - \epsilon)\Theta(\epsilon - \epsilon_H)$. The above equation along with eqs. (4), (10) and (12) is used to evaluate the photon spectra.
V. Results

As we have mentioned in section III, estimation of the initial energy density /temperature from the final state rapidity density is no longer a trivial task in the presence of dissipation. The principle however is quite straightforward. In our algorithm, we tacitly assume that the initial formation time $\tau_i$ of the thermalised system, be it in the QGP phase or the hadronic phase, has the canonical value of 1 fm/c. That the initial formation time (and consequently the initial temperature $T_i$) is a poorly known quantity is beyond dispute. Indeed, it is now widely believed that at RHIC or LHC, the gluons may equilibrate considerably earlier than 1 fm/c. However, even at these energies, the u/d quarks appear to equilibrate only $\sim$ 1 fm/c. Moreover, all the works aimed at comparing the experimental data with theoretical predictions take, $\tau_i$ to be, as a rule, equal to or greater than 1 fm. Most of these works also use the Bjorken scaling hydrodynamics to emulate the space-time evolution of the system. For a meaningful comparison with these works, we should also therefore adopt the same scenario.

We treat $T_i$ as a parameter; for each $T_i$, we let the system evolve forward in time under the condition of dissipative fluid dynamics (eq. (1)) till a given freeze-out temperature $T_f$ is reached. Thus $\tau_f$ is determined. We then compute $dN/dy$ at this instant of time from eq. (14) and compare it with the experimental $dN/dy$. The value of $T_i$ for which the calculated $dN/dy$ matches the experimental number is taken to be the approximate initial temperature. Schematically the calculation for $T_i$ proceeds (forward in time) as follows.

$$\tau_Q(T_i) \rightarrow \tau_H(\tau_Q(T_i)) \rightarrow \tau_f(\tau_H(\tau_Q(T_i)))$$

i.e. $\tau_H$ depends on $T_i$ through $\tau_Q$ and finally $\tau_f$ depends on $T_i$ through $\tau_H$ (cf eqs. (5),(11) and (13)). We then solve the non-linear eq. (14) to estimate $T_i$ for given values of $dN/dy$ and $T_f$. We adopt a truly conservative viewpoint in the sense that we start with values of $T_i$ less than $T_c$ so that the system is formed in the hadronic phase, and allow values of $T_i \geq T_c$ only if the discrepancy between the calculated and experimental (or estimated) values of $dN/dy$ necessarily demands it. In these circumstances, we naturally let the system evolve first in the quark gluon plasma phase, then the mixed phase and the hadronic phase.

The result of these calculations are summarized in table I. For the sake of comparison, we also list the corresponding values as calculated in the ideal case (superscript Bj). For RHIC and LHC, the experimental $dN/dy$ values are the usual

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1 The assumption of 1 fm/c as the initial formation time need not be taken seriously. For our present purpose, one can always adopt the viewpoint that we compare the temperature at a proper time 1 fm/c, a sufficiently early instant of time in the evolution history, in the dissipative vis-a-vis ideal fluid scenarios.
estimates obtained from extrapolating the pp data [26]. For SPS, the \( dN/dy \) is known from actual measurement [27].

It can be readily seen from table I that in all cases, the initial temperatures for the dissipative case are lower than in the ideal case. Lifetimes for the dissipative case are also comparatively longer. One also finds that the lower the freeze-out temperature, the lower the initial temperature although the dependence is rather weak.

It is evident from Table I that both in RHIC and LHC, the initial temperature is substantially higher than \( T_c \) even for dissipative dynamics. (We use the canonical value of 160 MeV for \( T_c \) in this work.)

The situation at SPS demands special discussion. Please therefore refer to Table 2a. We find that an initial QGP phase may be formed only if we use \( T_c \) lower than 160 MeV. For example, \( T_c \) of 150 MeV allows a QGP phase with an initial temperature of 154 MeV only, and that too for a freeze-out temperature of 100 MeV. Thus an unambiguous conclusion about the formation of the QGP on the basis of presently available SPS data appears to be doubtful. We therefore explore the situation in somewhat greater detail in Table 2b.

In Table 2b we study, on the one hand, the possibility that the system is formed entirely in the hadronic phase with dissipative dynamics. We look at the most conservative scenario of very low freeze-out temperature, consistent with the argument of Hama and Navarra [28] for the case of \( S + Au \) system. We find that to obtain a \( dN/dy \) of 225, an initial temperature of 262 MeV is required which is obviously much larger than the usual \( T_c \) of 160 MeV (or even 200 MeV). It is thus fair to conclude that for a reasonable formation time of 1 fm/c, the present SPS data are not compatible with a purely hadronic phase even if dissipative effects are taken into account. This, together with the results detailed in Table 2a makes the situation most interesting indeed. We thus explore the remaining possibility that the initial system is formed in the mixed phase, i.e., \( T_i = T_c \).

Table 2b shows that this case fits the data reasonably well. Depending on the choice of \( T_f \), the initial system could be a mixture of 80 – 90% quark matter and the rest hadronic matter, co-existing in thermal equilibrium at a temperature \( T_c \). It thus appears that there is a strong hint of deconfinement phase transition of first order in the currently available data but for a clear indication of the formation of the QGP phase, one must go to higher energies at RHIC or LHC. This situation is also similar to that advocated by Werner [29] of late, where he suggests the formation of quark blobs in hadronic matter at SPS energies.

Effects of these considerations on QGP diagnostics are of direct relevance in the present scenario. One of the promising signals of QGP formation is direct photons [6, 9]. We thus show in figs. 1 and 2 the calculated photon spectra for ideal as well as dissipative fluid dynamics at RHIC and LHC energies respectively. The
immediate feature that stands out from these figures is that the effect of dissipation is stronger in the QGP phase than in the hadronic phase. This is to be expected as the temperature in the QGP phase is higher, corresponding to the larger values of the co-efficients of viscosity. It should also be noted that even if the estimate of $T_i$ is lower than the ideal case by only 10% or less, the change in the photon spectra at high $p_T$ is quite sizeable.

In fig. 3, we compare the photon spectra corresponding to the configuration of Table 2b (initial mixed phase at a temperature of 160 MeV) with the experimental data of WA80\cite{10, 30}. For the sake of comparison, we also show the photon spectra from purely hadronic phase with the initial configuration of Table 2b. Obviously, the agreement with the initial mixed phase is better, considering the fact that the experimental data gives the upper limit for the photon spectra (fig.3). Before closing this section, we feel that a few comments on our neglect of transverse expansion are in order. As we already mentioned, this amounts to an overestimate of $\tau_f$, especially for RHIC and LHC energies. It is well known that the transverse flow effects at SPS energies are not at all large \cite{21}. Nevertheless, an inspection of Table 1, reveals that the change due to dissipative effects in total lifetime or $T_i$ relative to the ideal case is less than 10% at both RHIC and LHC energies. Since the same effect is seen both in $\tau_f$ and $\tau_H$, it is reasonable to expect that even in the presence of transverse flow, the same qualitative features would obtain. This however ought to be verified in a full (3+1) dimensional calculation which we plan to perform in the near future.

VI. Summary

In this work, we have explored the effects of dissipation in the space time evolution of matter formed in very energetic collisions of heavy ions. We find that even with the assumption of scaling hydrodynamic flow and weak dissipation, the deviation from the ideal case is not negligible. The presence of dissipation reduces the rate of cooling, resulting in a longer total lifetime. For a given total multiplicity in the final state, the estimated initial temperature for the dissipative case is smaller than in the ideal situation. This effect is most pronounced at SPS energies. It appears that the presence of dissipation at RHIC or LHC energies may not change the estimated $T_i$ to such an extent as to qualitatively invalidate the conclusions derived from the assumption of ideal hydrodynamics. At the level of the present analysis, we find that the measured $dN/dy$ of 225 cannot be easily reproduced without a first order phase transition (Table 2b). The most likely situation seems to be an initial configuration of a mixed state of hadronic and quark matter.

In the thermal single photon spectra, we observe that the effect of dissipation should show up at the high $p_T$ sector. This is quite natural, given the fact that dissipation results in a lower initial temperature. In all cases, the effect of viscosity is
important in the QGP phase as compared to the hadronic phase. Recently available data from the WA80 group at SPS energies for S + Au collisions have been compared with theoretical calculations without any additional free parameters. We find that the parameters extracted from the $dN/dy$ analysis (Table 2b) yield a reasonably good fit to the data, if the initial configuration is indeed a mixed phase. The data are not compatible with formation of matter in a pure hadronic phase. These conclusions are not expected to be materially altered due to transverse flow which we have not taken into account for the present.
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Table Captions

- Table 1: Comparison of time scales for the different phases and initial temperatures between Bjorken hydrodynamics (superscript Bj) and viscous hydrodynamics at RHIC and LHC energies. $dN/dy$ is the multiplicity, $T_i$, $T_c$ and $T_f$ are the initial, critical and freeze-out temperatures respectively. $\tau_i$, $\tau_Q$, $\tau_H$ and $\tau_f$ are the initial thermalisation time, time when phase transition starts, time when mixed phase ends and freeze-out time respectively.

- Table 2a: Same as Table 1 at SPS energies. The value of $dN/dy$ is taken from experiment.

- Table 2b: Same as Table 2a when matter is formed in pure hadronic or mixed phase.

Figure Captions

- Figure 1: Photon spectrum at RHIC energies for ideal and viscous flow. The inputs are taken from table 1, with freeze-out temperature $T_f = 100$ MeV. QM(HM) denotes the contributions from pure QGP(hadronic) and QGP(hadronic) part of the mixed phase.

- Figure 2: Photon spectra at LHC energies with and without viscous flow.

- Figure 3: Photon Spectra at SPS energies with viscous flow and comparison with WA80 data. The dashed curve denotes the photon spectrum with inputs from no phase transition scenario of table 2b. The solid curve denotes the photon spectrum with inputs from table 2b. The freeze-out temperature is taken as 65 MeV as calculated by applying the method of ref.[28]. The arrow denotes the experimental data from WA80[30, 10].
APPENDIX-I

In case of ideal fluid, the conservation of entropy implies that the rapidity density $dN/dy$ is a constant of motion. In such cases the experimentally measured value of the final rapidity density, which is a quantity depends on the integration over the space time history, can be connected to the initial temperature. In these calculations one assumes the validity of the extrapolation from the freeze point (with temperature $T_f$ and time $\tau_f$) to the initial point specified by temperature $T_i$ and proper time $\tau_i$ backward in time. But in the presence of dissipative effects the process is irreversible, and one should take care of this fact. In this appendix we will show that if we solve the eq.(1) backward in time then we always overestimate the initial temperature for a given value of $dN/dy$ and $\tau_i$.

Under the transformation $\tau \to -\tau$, eq.(1) becomes,

$$\frac{d\epsilon}{d\tau} + \frac{(\epsilon + P)}{\tau} = -\left(\frac{4}{3} \eta + \zeta\right) / \tau^2$$

(A.1)

For the equation of state of a massless gas, eq.(15) can be written as

$$\frac{d}{d\tau}(T \tau^{1/3}) = -\frac{b}{12 a_k \tau^{5/3}}$$

(A.2)

where $\eta = \eta_0 T^3$, $\zeta = \zeta_0 T^3$, $b = \frac{4}{3} \eta_0 + \zeta_0$, $a_k = (\pi^2/90) g_k$, $g_k$ is the statistical degeneracy. The solution of eq.(A.2) with the boundary condition $T = T_f$ at $\tau = \tau_f$ can be written as,

$$T \tau_f^{1/3} = K (dN/dy)^{1/3} - \frac{b}{8 a_k} \left(\tau_f^{-2/3} - \tau_i^{-2/3}\right)$$

(A.3)

where we have used the relation $T_f \tau_f^{1/3} = K(dN/dy)^{1/3}$. The initial temperature estimated from eq.(A.3) is

$$T_i \tau_i^{-1/3} = K (dN/dy)^{1/3} - \frac{b}{8 a_k} \left(\tau_f^{-2/3} - \tau_i^{-2/3}\right)$$

(A.4)

$T_i'$ denotes the initial temperature when we solve the hydrodynamic equation backward in time. Similarly we can solve the hydrodynamic eq.(1) with boundary condition $T = T_i$ at $\tau = \tau_i$, the solution is given by,

$$T \tau_i^{1/3} = T_i \tau_i^{-1/3} + \frac{b}{8 a_k} \left(\tau_i^{-2/3} - \tau_f^{-2/3}\right)$$

(A.5)

At the freeze-out time eq.(A.5) can be written in terms of $dN/dy$,

$$T_i \tau_i^{-1/3} = K (dN/dy)^{1/3} + \frac{b}{8 a_k} \left(\tau_f^{-2/3} - \tau_i^{-2/3}\right)$$

(A.6)

In case of ideal hydrodynamics,($b = 0$) $T_i' = T_i$. But in case of viscous flow for a given $dN/dy$ and $\tau_i$,

$$T_i' > T_i$$

(A.7)
### Table 1

|       | $T_f$ (MeV) | $T_c$ (MeV) | $\tau_i/\tau_i^{Bj}$ (fm/c) | $\tau_Q/\tau_Q^{Bj}$ (fm/c) | $\tau_H/\tau_H^{Bj}$ (fm/c) | $\tau_f/\tau_f^{Bj}$ (fm/c) | $T_i/T_i^{Bj}$ (MeV) |
|-------|-------------|-------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|---------------------|
| RHIC  | 100         | 160         | 1/1                         | 3.5/4.2                     | 35/34                       | 150/138.4                  | 225/258             |
| $(dN_{dy} = 1735)$ | 140         | 160         | 1/1                         | 3.6/4.2                     | 36/34                       | 54.7/50.5                 | 228/258             |
| LHC   | 100         | 160         | 1/1                         | 13.7/13.6                   | 117.2/109.8                 | 486.8/449.8               | 358/382             |
| $(dN_{dy} = 5624)$ | 140         | 160         | 1/1                         | 13.8/13.6                   | 118.2/109.8                 | 177.4/163.9              | 359/382             |
### Table 2a

| $T_f$ (MeV) | $T_c$ (MeV) | $\tau_i/\tau_i^{Bj}$ (fm/c) | $\tau_Q/\tau_Q^{Bj}$ (fm/c) | $\tau_H/\tau_H^{Bj}$ (fm/c) | $\tau_f/\tau_f^{Bj}$ (fm/c) | $T_i/T_i^{Bj}$ (MeV) |
|-------------|-------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|----------------------|
| SPS         | 100         | 120                         | 1/1                         | 3.5/1.7                     | 37.9/13.6                   | 67.5/23.5            |
|             | 150         | 1/1                         | 1.1/2.5                     | 18/20                       | 67.5/67.5                   | 154/203              |

($\frac{dN}{dy} = 225$)

### Table 2b

| $\frac{dN}{dy}$ | $T_i$ (MeV) | $\tau_i$ (fm/c) | $T_f$ (MeV) | $\tau_f$ (fm/c) |
|-----------------|-------------|-----------------|-------------|-----------------|
| 225             | 140         | 160             | 16          | 25              |
| 225             | 100         | 160             | 15          | 68              |
| 225             | 65          | 160             | 14          | 247             |

No Phase Transition

Formation in Mixed Phase
- - - no phase transition

--- formed in mixed phase

WA80 data (\(^{32}\text{S+Au}\))

Viscous flow

SPS