Optimization of transport support for forage harvesters

A M Valge¹, A I Sukhoparov¹ and A N Perekopsky¹

¹Federal State Budgetary Scientific Institution "Federal Scientific Agroengineering Center VIM", branch in 3, Filtrovskoje Shosse, p.o. Tiarlevo, RU196625 Saint Petersburg, Russia

E-mail: sukhoparov_ai@mail.ru

Abstract. Silage is the main type of feed in the diet of cattle. Violation of the agrotechnical terms of laying silos for more than 4 days contributes to a decrease in the quality of grass feed. Therefore, when harvesting silage, it is necessary to form a harvesting and transport complex of technical means that would ensure the timely filling of storage. The forage harvester performs the most energy-intensive operations in the technological process of harvesting feed from grasses. Therefore, the efficiency of the entire technological process depends on the optimal ratio of the number of combines and the vehicles that serve them. To optimize the system "technological machine-vehicles", it is important to apply the methods of the queuing theory, which takes into account the probabilistic downtime of forage harvesters. The use of this solution method in the formation of a harvesting and transport complex based on a statistical model allows us to ensure the flow of the technological process of delivering grass mass from the field in the storage during the acyclic movement of vehicles. Optimization according to the statistical model helps to reduce the cost of silage production by up to 5% due to a more rational amount of vehicle use.

1. Introduction

Most of the costs in the production of grass feed fall on herbs harvesting and storage. During this period, up to 90% of all work related to feed production is performed. In the structure of feed for cattle, the main part falls on feed from dried grasses – silage and haylage [1].

Silage harvesting is carried out by technological complexes consisting of forage harvesters and vehicles, the efficiency of which significantly depends on their correct ratio. The forage harvester is the most capital-intensive machine of all the technical means involved in the production of grass feed. Thus, the cost indicators of the feed obtained and the overall profitability of animal husbandry depend on the efficiency of forage harvester in the harvesting and transport complex [1, 2]. Depending on the productivity of the harvester, a logical chain of transporting feed from the fields to their storage location is built in order to ensure that the storages are filled within 4-5 days [3, 4, 5]. With the increase in the amount of energy resources in the production of feed, the cost of a unit of feed will also increase. In this regard, it becomes relevant to determine the number of vehicles required to service one forage harvester.

With the development of information technologies and the development of various mathematical programs that allow processing a large amount of data using a variety of mathematical methods, it became possible to solve problems to optimize the use of technical means [6, 7, 8, 9, 10]. The development of an approach to assess the efficiency of the harvesting and transport complex will allow it to be used in a formalized form directly in decision support tools. Operational decision-
making in the process of harvesting silage and haylage will contribute to the rational use of material and technical resources, taking into account the potential of fields, storage volumes and the distance between them [3, 11, 12, 13], which will affect the reduction of costs associated with the procurement of animal feed.

2. Materials and methods

As a rule, one forage harvester is serviced by several vehicles. After loading, the next vehicle transports the silage mass to the silo trench, unloads, and returns to the forage harvester for another load. With the right choice of the number of transport vehicles, their joint work takes place without machine downtime. Let's consider possible algorithms for determining the required number of vehicles needed to service a forage harvester.

Deterministic method

If several vehicles of the same type are assigned to each forage harvester, then the balance of the distribution of service time by their combine can be represented as the following identity:

\[ (N_r - 1) \cdot t_n = t_m + t_p \]  

(1)

where \( N_r \) – total number of vehicles, units;
\( t_n \) – loading time of one vehicle, hour;
\( t_m \) – time of movement of the vehicle from forage harvester to the storage and back, hour;
\( t_p \) – time of unloading the vehicle in the storage, hour.

Equation (1) is the main balance equation of the time of loading of the vehicle and transportation of the silage mass, in which the entire harvesting complex operates smoothly without downtime of the forage harvester and vehicles.

The time of movement of the vehicle to the storage and back is determined by the formula [14]:

\[ t_m = \frac{60 \cdot R}{V_1} + \frac{60 \cdot R}{V_2} \]  

(2)

where \( R \) – distance of transportation of silage mass from the field to the storage site, km;
\( V_1 \) and \( V_2 \) – vehicle speed with and without load, km/h.

Taking into account the ratio (2), the total number of vehicles required for the maintenance of one forage harvester and the smooth operation of the harvesting and transport complex will be:

\[ N_r = \frac{\frac{60 \cdot R}{V_1} + \frac{60 \cdot R}{V_2} + t_p}{t_n} + 1 \]  

(3)

Statistical method

Methods of queuing theory are widely used in the literature to study systems of the "technological machine - transport vehicles" type [15].

In our case, it is advisable to use a mathematical model in the form of a closed system of queuing theory. The model is presented in the form of \( N \) service channels – forage harvesters (leading machine). Vehicles in this model have the status of serviced vehicles (requirements requests). It is assumed that the time of service (loading of the vehicle) is distributed according to the exponential law with the parameter \( \mu \) (the intensity of service of the next requirement). The flow of requirements (vehicles) entering the service is taken as a stationary Poisson flow with an intensity of \( \lambda \). Requests that find all channels busy are queued and waiting for them to be released. In such a system, there is always, with some probability, downtime of both the main and transport machines. Therefore, there is
a compromise task of completing the harvesting complex with such a number of vehicles that would ensure a minimum of downtime costs for both types of machines.

When working, there may be several harvesting machines. They are served by a group of transport vehicles. The number of leading and of servicing machines remains constant for the review period. After loading, of serviced machines are sent to the feed storage and after unloading, they return again to service the main machines in the field (closed system). The of service machines form an input stream of requests, which obeys the Poisson distribution:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t},$$

where $n$ – the number of events during time $t$;

$\lambda$ – the intensity of receipt of requests on service (1/hour).

Each application is served by the leading machine for some time, obeying the exponential distribution law:

$$P(t) = e^{-\mu t},$$

where $\mu$ – the average service time (a parameter of the distribution law).

In general, in a closed queuing system, the number of machines remains constant. If there is a free the servicing machine, it immediately starts servicing, if all the leading machines are occupied, then the application is queued. The total queue length cannot be greater than the number of serving machines.

To create a mathematical model of a closed system in accordance with the procedure [15], the Kolmogorov graph is used in Figure 1.

![Kolmogorov graph for a closed queuing system](image)

Where the figure is characterized by the following notation:

$N$ – number of driving machines (forage harvesters);

$m$ – number of the servicing machines (transport vehicles);

$\mu$ – the intensity of application servicing by the leading machines, $\mu = \text{const}$;

$P_0$ – probability of downtime of all leading machines;

$P_1$ – the probability of downtime of the 1st leading machine.

For the graph on Figure 1 in the steady-state mode, the system of equations describing the relationship between the various probabilities of the states of the system has the form (7):
for $n < N$

$$- m \lambda P_0 + \mu P_1 = 0,$$

$$m \lambda P_0 - (m - 1) \lambda P_1 - \mu P_1 + 2 \mu P_2 = 0,$$

$$(m - 1) \lambda P_1 - (m - n) \lambda P_n - 2 \mu P_2 + 3 \mu P_3 = 0.$$  \hspace{1cm} (6)

for $n \geq N$

$$ (m - n + 1) \lambda P_{n-1} - (m - n) \lambda P_n - N \mu P_n + N \mu P_{n-1} = 0$$

......................................................................................................................... (7)

$$\lambda P_{m-1} - N \mu P_m = 0$$

By converting equations (6) and (7), the following basic relations for determining the system parameters are obtained [15]:

- the probability of downtime of all leading machines due to the absence of the servicing machines

$$P_0 = \left[ 1 + \sum_{n=1}^{N-1} \frac{m! \Psi^n}{n!(n-m)!} + \sum_{n=m}^{\infty} \frac{m! \Psi^n}{n!(m-n)! N! N^{n-N}} \right]^{-1} \hspace{1cm} (8)$$

where

$$\Psi = \frac{\lambda}{\mu};$$

- the average number of the servicing machines in the queue:

$$N_r = \sum_{n=1}^{N-1} \frac{(N-n)m! \Psi^n}{n!(m-n)!}; \hspace{1cm} (9)$$

- the average number of driving machines idle while waiting for the servicing machines:

$$N_K = \sum_{n=m}^{\infty} \frac{(n-N)m! \Psi^n}{n!(n-N)(m-n)!N!}; \hspace{1cm} (10)$$

The system "technological machine - transport vehicles" is a stochastic system in which, with any combination of their number, downtime and queues of leading and servicing machines are possible.

To ensure their optimal composition, we use the optimization criterion in the form of [16]:

$$y = \frac{C_B N - C_{BS} N + \mu C_K 2r}{G \mu} + \frac{C_{BS} N + m C_{KS}}{G \mu (1 - P_0)}, \hspace{1cm} (11)$$

where $C_B$ - cost of operating costs of the leading machine;

$C_{BS}$ - cost of downtime of the leading machine;

$C_K$ - the cost of operating of the servicing machine;

$C_{KS}$ - cost of downtime of the servicing machine;

$r$ - distance of transportation of silage mass from the field to the storage site;

$G$ - the amount of cargo transported for 1 flight.

After the conversion, we get the ratio for optimization. For a given number of leading machines, the optimal number of servicing machines is determined by the inequality [11]:

...
where

\[ C = \frac{C_{\text{RS}}}{C_{\text{RS}}} \]

Equation (12) is universal, since it includes only normalized variables and is suitable for determining the optimal parameters of any closed queuing system. For calculations based on formulas (8)-(12), a computer program [11] has been developed that allows us to obtain normalized optimal indicators for any multi-channel queuing system at known values, both for the forage harvester and for vehicles and the cost of their downtime. The results of the calculations for the program are presented in the table 1 and the table 2.

**Table 1.** Probabilities of downtime of forage harvesters at different parameters of the service system

| \( \lambda \) \( \mu \) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|----------------|-----|-----|-----|-----|-----|-----|
| 0.1            | 0.26 | 0.20 | 0.15 | 0.12 | 0.12 | 0.10 |
| 0.2            | 0.20 | 0.17 | 0.13 | 0.09 | 0.09 | 0.09 |
| 0.3            | 0.18 | 0.16 | 0.12 | 0.09 | 0.09 | 0.06 |
| 0.4            | 0.17 | 0.13 | 0.11 | 0.06 | 0.06 | 0.06 |
| 0.5            | 0.16 | 0.13 | 0.11 | 0.06 | 0.06 | 0.06 |
| 0.6            | 0.14 | 0.11 | 0.08 | 0.04 | 0.04 | 0.04 |
| 0.7            | 0.14 | 0.08 | 0.07 | 0.05 | 0.05 | 0.05 |
| 0.8            | 0.13 | 0.07 | 0.06 | 0.04 | 0.04 | 0.04 |
| 0.9            | 0.13 | 0.06 | 0.05 | 0.03 | 0.03 | 0.03 |

**Table 2.** Optimal number of vehicles to service two forage harvesters

| \( \lambda \) \( \mu \) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|----------------|-----|-----|-----|-----|-----|-----|
| 0.1            | 13  | 15  | 17  | 18  | 18  | 19  |
| 0.2            | 8   | 9   | 10  | 10  | 11  | 11  |
| 0.3            | 6   | 7   | 7   | 8   | 8   | 9   |
| 0.4            | 5   | 6   | 6   | 7   | 7   | 7   |
| 0.5            | 4   | 5   | 5   | 6   | 6   | 6   |
| 0.6            | 4   | 5   | 5   | 5   | 6   | 6   |
| 0.7            | 3   | 4   | 4   | 5   | 5   | 5   |
| 0.8            | 3   | 4   | 4   | 5   | 5   | 5   |
| 0.9            | 3   | 4   | 4   | 4   | 5   | 5   |

3. Results and Discussion

The comparison of the deterministic and statistical methods is considered by the example of determining the optimal number of transport vehicles required for the maintenance of a two forage harvesters.

Deterministic model

The work of two Maral-125M forage harvesters in a field for harvesting grass in a yield of 12.0 t/ha is simulated. Feed harvesters are serviced by GAZ-SAZ-3507 vehicles with a load capacity of 4 tons, the loading time of which is 10 min. The average distance of grass mass transportation is 5.0 km, the
average speed of vehicles with a load is 30.0 km/h, without a load – 40.0 km/h, the average time of unloading into a silo trench is 2.0 min.

With such initial data, using the formula (3), we obtain:

\[ N_T = \frac{60 \cdot 5 + 60 \cdot 5 + 2}{10} + 1 = 2.95 \approx 3 \text{ vehicles.} \]

For the maintenance of one forage harvester, three vehicles are required, and for the maintenance of two, respectively, 6 vehicles.

Statistical model.

Consider the same model with a random change in service time and load time. Let's take the following averages:

- loading time of the vehicle \( t_n = 10 \text{ min.}; \)
- vehicle turnaround time (travel time from the field to the storage area and back, and time to unload) \( t_{n+\mu} = (10 + 7.5) + 2 = 19.5 \text{ min.} \)

We get the following queuing system parameters:

\[ \mu = \frac{60}{10} = 6; \quad \lambda = \frac{60}{19.5} \approx 3; \quad \frac{\lambda}{\mu} = 0.5 \]

Depending on the ratio of the cost of downtime of the forage harvester and the vehicle, the probability of downtime of the forage harvester and the required number of vehicles required to transport the silage mass from the field to the silo storage change. This is clearly shown in Figure 2 on the example of servicing two Maral-125M forage harvesters by GAZ-SAZ-3507 vehicles.

![Figure 2. The optimal number of vehicles for servicing the two forage harvester](image)

At \( \frac{\lambda}{\mu} = 0.5 \), depending on the ratio of the cost of downtime of the forage harvester and the vehicle, changes the probability of downtime of the forage harvester, Table 1, and the required number of vehicles, Table 2. If the ratio of the cost of downtime of the forage harvester and the vehicle is less than 2, for the maintenance of two Maral-125M combines, in this case, it is necessary from 4 to 5 GAZ-SAZ-3507 cars. At the same time, the probability of idle harvesters is 0.16-0.11. When the ratio of the costs downtime forage harvesters and vehicles is equal to two or more, six of vehicle with a load capacity of 4 tons are needed to service the mentioned combines. At the same time, the probability of
downtime of combine harvesters will be 0.06. To service one forage harvester, three vehicles are required, respectively.

In the given example, calculated on the basis of a statistical model, the number of cars is almost equal to the number obtained by the deterministic model (3) (the deviation was 1.7%), i.e. the solution of the problem of finding the optimal number of vehicles for servicing the harvesting machine is feasible using both methods. However, the solution based on the statistical model of the closed system of the queuing theory will be more preferable, since it takes into account the acyclic nature of the movement of vehicles, which is a more realistic condition for the operation of the harvesting and transport complex when harvesting silage.

Table 3 shows the results of time-lapse observations of the technological process of transporting feed from grasses in the production of silage. They were carried out in the farm "Kalozhtsy" of the Leningrad region, Russia. Observations were carried out on the work of the harvesting and transport complex consisting of a single forage harvester Maral-125M, and the vehicles were GAZ-SAZ-5307 vehicles. Two options were considered:

- 4 cars serve the harvesting machine, which is used in the farm (basic option);
- 3 cars serve the forage harvester (optimization option).

| Option     | Machine | Nom. | Downtime due to technological reasons, min. | Total downtime, h. | Costs for idle time, RUB. | Operating costs, RUB. |
|------------|---------|------|---------------------------------------------|-------------------|--------------------------|----------------------|
|            |         |      | average | min | max | lot |                  |                    |                        |                      |
| Basic      | Maral-125M | 1    | 1.2     | 0.2 | 2.8 | 5   | 0.1               | 1412               | 141                   | 4102                 | 65055                |
|            | GAZ-3507 | 4    | 4.7     | 0.4 | 8.5 | 95  | 7.46              | 531                | 3961                  |                      |                      |
| Optimization | Maral-125M | 1    | 1.0     | 0.4 | 5.2 | 42  | 0.72              | 1412               | 1017                  | 1033                 | 61986                |
|            | GAZ-3507 | 3    | 2.0     | 0.5 | 1.5 | 2   | 0.03              | 531                | 16                    |                      |                      |

More effective was the second option, in which the optimal number of vehicles was determined using the statistical model of a closed system of queuing theory. The formation of the harvesting and transport complex in the studied farm according to the calculated variant in comparison with the basic one contributed to a decrease in operating costs by 4.7%.

4. Conclusions
1. The system "harvesting machines - transport machines" is a stochastic system in which, with any combination of their number, downtime and queues of leading (harvester) and servicing machines (vehicles) are possible. Taking this into account, the closed system method of queuing theory was used to optimize the transport support of forage harvesters, and a statistical model was obtained.

2. The problem of determining the optimal number of vehicles with a load capacity of 4 tons for servicing two Maral-125M forage harvesters, depending on the ratio of the cost of downtime of the forage harvesters and the vehicle, has been solved. When solving this problem, it was revealed that the probability of forage harvesters downtime will be 0.16-0.11 when using from 4 to 5 vehicles, and to ensure the probability of combine harvesters downtime below 0.06, 6 vehicles are needed.

3. The formation of the harvesting and transport complex according to the statistical model is more preferable than the deterministic one, since the probabilistic acyclic nature of the movement of vehicles from the field to the storage site of the silage mass is taken into account. The difference in the calculated number of transport vehicles required for the maintenance of harvesting machines according to the developed models is 1.7%.

4. For farms in the North-West of Russia, optimization of the number of vehicles serving the forage harvesters according to the statistical model helps to reduce the cost of silage production by up to 5%.
References
[1] Basnet C B, Foulds L R and Wilson J M 2006 Scheduling contractors’ farm-to-farm crop harvesting operations (International Transactions in Operational Research vol 13 (1)) pp 1–15
[2] Vindis P, Stajnko D, Lakota M and Mursec B 2012 Comparison of efficiency of single – row and self – propelled harvester on small farms (Actual Tasks on Agricultural Engineering-Zagreb vol 40) pp 311–320
[3] Valge A, Sukhoparov A and Papushin E 2020 Grass forage transportation process modeling (Engineering for Rural Development) pp 1201–1207
[4] Amiama C, Pereira J M, Castro A and Bueno J 2015 Modelling corn silage harvest logistics for a cost optimization approach (Computers and Electronics in Agriculture vol 118) pp 56–65
[5] Busato P, Sopegno A, Pampuro N, Sartori L and Berruto R 2019 Optimisation tool for logistics operations in silage production (Biosystems Engineering vol 180) pp 146–160
[6] Edwards G, Sørensen C G, Bochtis D D and Munkholm L J 2015 Optimised schedules for sequential agricultural operations using a Tabu Search method (Computers and Electronics in Agriculture vol 117) pp 102–113
[7] Valge A M 2013 The use of Excel and Mathcad systems in conducting research on the mechanization of agricultural production (Methodological guide) (Saint Petersburg GNU SZNINESKH Rosselhozakademii Publ) p 200
[8] Yezekyan T, Marinello F, Armentano G, Trestini S and Sartori L 2020 Modelling of Harvesting Machines’ Technical Parameters and Prices (Agriculture-Basel vol 10 iss 6) art number 194
[9] Brownell D K, Liu J, Hilton J W, Richard T L, Cauffman G R and Macafee B R 2012 Evaluation of two forage harvesting systems for herbaceous biomass harvesting (Transactions of the ASABE vol 55) pp 1651–1658
[10] Dudenhoeffer N E, Luck B D, Digman M F and Drewry J L 2018 Simulation of the forage harvest cycle for asset allocation (Applied Engineering in Agriculture vol 73) pp 327–334
[11] Bueno J, Amiama C and Pereira J M 2014 Discrete event simulation model for the harvest cycle of silage corn (VII CongresoIberico De Agroingeneria y CienciasHorticolas: Innovar y Producir Para El Futuro) pp 1064–1068
[12] Harmon J D, Luck B D, Shinners K J, Anex R P and Drewry J L 2018 Time-motion analysis of forage harvest: a case study (Transactions of the ASABE vol 61 no 2) pp 483–491
[13] Valge A, Sukhoparov A, Papushin E and Dobrinov A 2021 Evaluation effectiveness of forage harvesters in silage preparation (IOP Conf Series: Earth and Environmental Science vol 699) p 012050
[14] Popov V D and Valge A M 2004 Modeling and optimization of processes and technologies of grass forage harvesting in the conditions of the North-West of Russia (Saint Petersburg GNU SZNINESKH Rosselhozakademii Publ) p 176
[15] Wentzel E S 1972 Operations research (Moscow Soviet radio) p 552
[16] Kudryavtsev E M 1989 Complex mechanization, automation, and mechanical strength of construction (Moscow: Stroyizdat) p 246