From Analogue Models to Gravitating Vacuum

G.E. Volovik
Low Temperature Laboratory, Aalto University, Finland
L.D. Landau Institute for Theoretical Physics, Moscow, Russia

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We discuss phenomenology of quantum vacuum. Phenomenology of
macroscopic systems has three sources: thermodynamics, topology and sym-
metry. Momentum space topology determines the universality classes of fermionic vacua. The vacuum in its massless state belongs to the Fermi-point universality class, which has topologically protected fermionic quasi-particles. At low energy they behave as relativistic massless Weyl fermions. Gauge fields and gravity emerge together with Weyl fermions at low energy. Thermodynamics of the self-sustained vacuum allows us to treat the problems related to the vacuum energy: the cosmological constant problems. The natural value of the energy density of the equilibrium the self-sustained vacuum is zero. Cosmology is the process of relaxation of vacuum towards the equilibrium state. The present value of the cosmological constant is very small compared to the Planck scale, because the present Universe is very old and thus is close to equilibrium.

0.1 Introduction. Phenomenology of quantum vacuum

0.1.1 Vacuum as macroscopic many-body system

The practical use of Analogue Gravity started 30 years ago with the remark “that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow” [1]. In this paper by Unruh, the space-time geometry of the gravitational field has been modeled by the effective flow of a liquid, which plays the role of aether. The other types of analogue aether are represented by elastic media [2, 3, 4, 5, 6, 7]; topological matter with Weyl fermions [8, 9, 10]; fermionic matter experiencing the four dimensional quantum Hall effect [11]; etc.

Now the aether is becoming far more than the analogy. The aether of the 21-st century is the quantum vacuum, which is a new form of matter. This is the real substance, which however has a very peculiar properties strikingly different from the other forms of matter (solids, liquids, gases, plasmas, Bose condensates, radiation, etc.) and from all the old aethers. The new aether is Lorentz invariant with great accuracy (except probably of the neutrino sector of the quantum vacuum [12], where in principle the Lorentz invariance can be spontaneously broken [13, 14]). It has equation of state $p = -\epsilon$; as follows from the cosmological observations its energy density is about $10^{-29}\text{g/cm}^3$ (i.e. the quantum aether by 29 orders magnitude lighter than water); and it is actually anti-gravitating.

Quantum vacuum can be viewed as a macroscopic many-body system.
Characteristic energy scale in our vacuum (analog of atomic scale in quantum liquids) is Planck energy $E_P = (\hbar c^5/G)^{1/2} \sim 10^{19}$ GeV $\sim 10^{32}$K. Our present Universe has extremely low energies and temperatures compared to the Planck scale: even the highest energy in the nowadays accelerators is extremely small compared to Planck energy: $E_{\text{max}} \sim 10$ TeV $\sim 10^{17}$K$\sim 10^{-15}E_P$. The temperature of cosmic background radiation is much smaller $T_{\text{CMBR}} \sim 1$ K $\sim 10^{-32}E_P$.

Cosmology belongs to ultra-low frequency physics. Expansion of Universe is extremely slow: the Hubble parameter compared to the characteristic Planck frequency $\omega_P = (c^5/G\hbar)^{1/2}$ is $H \sim 10^{-60}\omega_P$. This also means that at the moment our Universe is extremely close to equilibrium. This is natural for any many-body system: if there is no energy flux from environment the energy will be radiated away and the system will be approaching the equilibrium state with vanishing temperature and motion.

According to Landau, though the macroscopic many-body system can be very complicated, at low energy, temperature and frequency its description is highly simplified. Its behavior can be described in a fully phenomenological way, using the symmetry and thermodynamic consideration. Later it became clear that another factor also governs the low energy properties of a macroscopic system – topology. The quantum vacuum is probably a very complicated system. However, using these three sources – thermodynamics, symmetry and topology – one may try to construct the phenomenological theory of the quantum vacuum near its equilibrium state.

### 0.1.2 3 sources of phenomenology: thermodynamics, symmetry and topology

Following Landau, at low energy $E \ll E_P$ the macroscopic quantum system (our Universe is an example) contains two main components: vacuum (the ground state) and matter (fermionic and bosonic quasiparticles above the ground state). The physical laws which govern the matter component are more or less clear to us, because we are able to make experiments in the low-energy region and construct the theory. The quantum vacuum occupies the Planckian and trans-Planckian energy scales and it is governed by the microscopic (trans-Planckian) physics which is still unknown. However, using our experience with a similar condensed matter systems we can expect that the quantum vacuum component should also obey the thermodynamic laws, which emerge in any macroscopically large system, relativistic or non-relativistic. This approach allows us to treat the cosmological constant problems.
Cosmological constant was introduced by Einstein [15], and was interpreted as the energy density of the quantum vacuum [16, 17]. Astronomical observations [18, 19] confirmed the existence of cosmological constant which value corresponds to the energy density of order $\Lambda_{\text{obs}} \sim E_{\text{obs}}^4$ with the characteristic energy scale $E_{\text{obs}} \sim 10^{-3}$ eV. However, naive and intuitive theoretical estimation of the vacuum energy density as the zero-point energy of quantum fields suggests that vacuum energy must have the Planck energy scale: $\sim E_{P}^4 \sim 10^{120} E_{\text{obs}}$. The huge disagreement between the naive expectations and observations is naturally resolved using the thermodynamics of quantum vacuum discussed in this review. We shall see that the intuitive estimation for the vacuum energy density as $\sim E_{P}^4$ is not completely crazy, but this is valid for the Universe when it is very far from equilibrium. In the fully equilibrium vacuum the relevant vacuum energy, which enters Einstein equations as cosmological constant, is zero.

The second element of the Landau phenomenological approach to macroscopic systems is symmetry. It is in the basis of the modern theory of particle physics – the Standard Model, and its extension to higher energy – the Grand Unification (GUT). The vacuum of Standard Model and GUT obeys the fundamental symmetries which become spontaneously broken at low energy, and are restored when the Planck energy scale is approached from below. In the GUT scheme, general relativity is assumed to be as fundamental as quantum mechanics.

This approach contains another huge disagreement between the naive expectations and observations. It concerns masses of elementary particles. The naive and intuitive estimation tells us that these masses should be on the order of Planck energy scale: $M_{\text{theor}} \sim E_{P}$, while the masses of observed particles are many orders of magnitude smaller being below the electroweak energy scale $M_{\text{obs}} < E_{\text{ew}} \sim 1 \text{ TeV} \sim 10^{-10} E_{P}$. This is called the hierarchy problem. There should be a general principle, which could resolve this paradox. This is the principle of emergent physics based on the topology in momentum space, which demonstrates that our intuitive estimation of fermion masses of order $E_{P}$ is not completely crazy, but it can be valid only for such vacua where the massless fermions are not protected by topology.

### 0.1.3 Vacuum as topological medium

Topology operates in particular with integer numbers – topological charges – which do not change under small deformation of the system. The conservation of these topological charges protects the Fermi surface and another
object in momentum space – the Fermi point – from destruction. They survive when the interaction between the fermions is introduced and modified. When the momentum of a particle approaches the Fermi surface or the Fermi point its energy necessarily vanishes. Thus the topology is the main reason why there are gapless quasiparticles in topological condensed matter and (nearly) massless elementary particles in our Universe.

Topology provides the complementary anti-GUT approach in which the ‘fundamental’ symmetry and ‘fundamental’ fields of GUT gradually emerge together with ‘fundamental’ physical laws when the Planck energy scale is approached from above [8, 9, 10]. The emergence of the ‘fundamental’ laws of physics is provided by the general property of topology – robustness to details of the microscopic trans-Planckian physics. As a result, the physical laws which emerge at low energy together with the matter itself are generic. They do not depend much on the details of the trans-Planckian subsystem, being determined by the universality class, which the vacuum belongs to. Well below the Planck scale, the GUT scenario intervenes: the effective symmetries which emerged due to the topology reasons exhibit spontaneous breaking at low energy. This is accompanied by formation of composite objects, Higgs bosons, and gives tiny Dirac masses to quarks and leptons.

In the anti-GUT scheme, fermions are primary objects. Approaching the Planck energy scale from above, they are transformed to the Standard Model chiral fermions and give rise to the secondary objects: gauge fields $A_\mu$ and tetrad field $e_\mu^a$. The effective metric emerges as the composite object of tetrad field, $g^{\mu\nu} = \eta^{ab} e^\mu_a e^\nu_b$. In this approach, general relativity is the effective theory describing the dynamics of the effective metric experienced by the effective low-energy fermionic and bosonic fields. It is a side product of quantum field theory (or actually of the many-body quantum mechanics) in the vacuum with Fermi point. The emergence of the tetrad field before the metric field suggests that the effective theory for gravitational field must be of the Einstein-Cartan-Sciama-Kibble type [20], which incorporates the torsion field, rather than the original Einstein theory (see also Refs. 21 [22] for the other origin of torsion field from fermions).

Vacua with topologically protected gapless (massless) fermions are representatives of the broader class of topological media. In condensed matter it includes topological insulators (see review [25]), semimetals, topological superconductors and superfluids (see review [26]), states which experience quantum Hall effect, graphene, and other topologically nontrivial gapless and gapped phases of matter. Topological media have many peculiar properties: topological stability of gap nodes; topologically protected edge states including Majorana fermions; topological quantum phase transitions occur-
ring at $T = 0$; topological quantization of physical parameters including Hall and spin-Hall conductivity; chiral anomaly; topological Chern-Simons and Wess-Zumino actions; etc. [9] The modern aether – the quantum vacuum of Standard Model – is also the topologically nontrivial medium both in its massless and massive states, and thus must experience the peculiar properties.

In this review we mostly concentrate on the properties of quantum vacuum, which are relevant for the solution of main cosmological constant problem. This is determined by the more universal phenomenon – the thermodynamic behavior which emerges in any system in the limit of large number of elements. Topology in momentum space becomes important for the next steps: for dynamics of the cosmological constant.

0.2 Quantum vacuum as self-sustained medium

0.2.1 Vacuum energy and cosmological constant

There is a huge contribution to the vacuum energy density, which comes from the ultraviolet (Planckian) degrees of freedom and is of order $E_{Pl}^4 \approx (10^{28} \text{eV})^4$. The observed cosmological is smaller by many orders of magnitude and corresponds to the energy density of the vacuum $\rho_{\text{vac}} \sim (10^{-3} \text{eV})^4$. In general relativity, the cosmological constant is arbitrary constant, and thus its smallness requires fine-tuning. If gravitation would be a truly fundamental interaction, it would be hard to understand why the energies stored in the quantum vacuum would not gravitate at all [27]. If, however, gravitation would be only a low-energy effective interaction, it could be that the corresponding gravitons as quasiparticles do not feel all microscopic degrees of freedom (gravitons would be analogous to small-amplitude waves at the surface of the ocean) and that the gravitating effect of the vacuum energy density would be effectively tuned away and cosmological constant would be naturally small or zero [9, 28].

0.2.2 Variables for Lorentz invariant vacuum

A particular mechanism of nullification of the relevant vacuum energy works for such vacua which have the property of a self-sustained medium [21, 30, 31, 32, 33]. A self-sustained vacuum is a medium with a definite macroscopic volume even in the absence of an environment. A condensed matter example is a droplet of quantum liquid at zero temperature in empty space. The
observed near-zero value of the cosmological constant compared to Planck-scale values suggests that the quantum vacuum of our universe belongs to this class of systems. As any medium of this kind, the equilibrium vacuum would be homogeneous and extensive. The homogeneity assumption is indeed supported by the observed flatness and smoothness of our universe \cite{34,35,36}. The implication is that the energy of the equilibrium quantum vacuum would be proportional to the volume considered.

Usually, a self-sustained medium is characterized by an \textit{extensive conserved quantity} whose total value determines the actual volume of the system \cite{37,38}. In condensed matter, the quantum liquid at $T = 0$ is a self-sustained system because of the conservation law for the particle number $N$, and its state is characterized by the particle density $n$ which acquires a non-zero value $n = n_0$ in the equilibrium ground state. As distinct from condensed matter systems, the quantum vacuum of our Universe obeys the relativistic invariance with great accuracy. The Lorentz invariance of the vacuum imposes strong constraints on the possible form this variable can take: the particle density $n$ must be zero in the Lorentz invariant vacuum, since it represent the time component of the 4-vector. One must find the relativistic analog of the quantity $n$, which is invariant under Lorentz transformation. An example of a possible vacuum variable is a symmetric tensor $q^{\mu\nu}$, which in a homogeneous vacuum is proportional to the metric tensor

$$q^{\mu\nu} = q g^{\mu\nu}. \quad (1)$$

This variable satisfies the Lorentz invariance of the vacuum. Another example is the 4-tensor $q^{\mu\nu\alpha\beta}$, which in a homogeneous vacuum is proportional either to the fully antisymmetric Levi–Civita tensor:

$$q^{\mu\nu\alpha\beta} = q e^{\mu\nu\alpha\beta}, \quad (2)$$

or to the product of metric tensors such as:

$$q^{\mu\nu\alpha\beta} = q (g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}). \quad (3)$$

Scalar field is also the Lorentz invariant variable, but it does not satisfy another necessary condition of the self sustained system: the vacuum variable $q$ must obey some kind of the conservation law. Below we consider some examples satisfying the two conditions: Lorentz invariance of the perfect vacuum state and the conservation law.
0.2.3 Yang-Mills chiral condensate as example

Let us first consider as an example the chiral condensate of gauge fields. It can be the gluonic condensate in QCD \cite{39, 40}, or any other condensate of Yang-Mills fields, if it is Lorentz invariant. We assume that the Savvidy vacuum \cite{41} is absent, i.e. the vacuum expectation value of the color magnetic field is zero (we shall omit color indices):

\[
\langle F_{\alpha\beta} \rangle = 0 , \tag{4}
\]

while the vacuum expectation value of the quadratic form is nonzero:

\[
\langle F_{\alpha\beta} F_{\mu\nu} \rangle = \frac{q}{24} \sqrt{-g} g_{\alpha\beta\mu\nu} . \tag{5}
\]

Here \( q \) is the anomaly-driven topological condensate (see e.g. \cite{42}):

\[
q = \langle \tilde{F}^{\mu\nu} F_{\mu\nu} \rangle = \frac{1}{\sqrt{-g}} \epsilon^{\alpha\beta\mu\nu} \langle F_{\alpha\beta} F_{\mu\nu} \rangle , \tag{6}
\]

In the homogeneous static vacuum state, the \( q \)-condensate violates the \( P \) and \( T \) symmetries of the vacuum, but it conserves the combined symmetry \( PT \) symmetry.

Cosmological term

Let us choose the vacuum action in the form

\[
S_q = \int d^4 x \sqrt{-g} \epsilon(q) , \tag{7}
\]

with \( q \) given by \( \text{(6)} \). The energy-momentum tensor of the vacuum field \( q \) is obtained by variation of the action over \( g^{\mu\nu} \):

\[
T^q_{\mu\nu} = - \frac{2}{\sqrt{-g}} \frac{\delta S_q}{\delta g^{\mu\nu}} = \epsilon(q) g_{\mu\nu} - 2 \frac{\partial \epsilon}{\partial q} \frac{\partial q}{\partial g^{\mu\nu}} . \tag{8}
\]

Using \( \text{(5)} \) and \( \text{(6)} \) one obtains

\[
\frac{\partial q}{\partial g^{\mu\nu}} = \frac{1}{2} q g^{\mu\nu} . \tag{9}
\]

and thus

\[
T^q_{\mu\nu} = g_{\mu\nu} \rho_{\text{vac}}(q) , \quad \rho_{\text{vac}}(q) = \epsilon(q) - q \frac{\partial \epsilon}{\partial q} . \tag{10}
\]
In Einstein equations this energy momentum tensor plays the role of the cosmological term:

\[ T_{\mu\nu}^q = \Lambda g_{\mu\nu} , \quad \Lambda = \rho_{\text{vac}}(q) = \epsilon(q) - q \frac{\partial \epsilon}{\partial q} . \]  

(11)

It is important that the cosmological constant is given not by the vacuum energy as is usually assumed, but by the equivalent of the grand potential in condensed matter systems – the thermodynamic potential \( \rho_{\text{vac}} = \epsilon(q) - \mu q \), where \( \mu \) is thermodynamically conjugate to \( q \) variable, \( \mu = d\epsilon/dq \). Below, when we consider dynamics, we shall see that this fact reflects the conservation of the variable \( q \).

The crucial difference between the vacuum energy \( \epsilon(q) \) and thermodynamic potential \( \rho_{\text{vac}} = \epsilon(q) - \mu q \) is revealed when we consider the corresponding quantities in the ground state of quantum liquids, the energy density \( \epsilon(n) \) and the grand potential \( \epsilon(n) - \mu n \). The first one, \( \epsilon(n) \), has the value dictated by atomic physics, which is equivalent to \( E_\text{P}^4 \) in quantum vacuum. On the contrary, the second one equals minus pressure, \( \epsilon(n) - \mu n = -P \), according to the Gibbs-Duhem thermodynamic relation at \( T = 0 \). Thus its value is dictated not by the microscopic physics, but by external conditions.

In the absence of environment, the external pressure is zero, and the value of \( \epsilon(n) - \mu n \) in a fully equilibrium ground state of the liquid is zero. This is valid for any macroscopic system, and thus should be applicable to the self-sustained quantum vacuum, which suggests the natural solution of the main cosmological constant problem.

**Conservation law for \( q \)**

Equation for \( q \) in flat space can be obtained from Maxwell equation, which in turn is obtained by variation of the action over the gauge field \( A_\mu \):

\[ \nabla_\mu \left( \frac{\partial \epsilon}{\partial q} \tilde{F}_{\mu\nu} \right) = 0 . \]  

(12)

Since \( \nabla_\mu \tilde{F}_{\mu\nu} = 0 \), equation (30) is reduced to

\[ \nabla_\mu \left( \frac{\partial \epsilon}{\partial q} \right) = 0 . \]  

(13)

The solution of this equation is

\[ \frac{\partial \epsilon}{\partial q} = \mu , \quad (14) \]
where $\mu$ is integration constant. In thermodynamics, this $\mu$ will play the role of the chemical potential, which is thermodynamically conjugate to $q$. This demonstrates that $q$ obeys the conservation law and thus can be the proper variable for description the self-sustained vacuum.

### 0.2.4 4-form field as example

Another example of the vacuum variable appropriate for the self-sustained vacuum is given by the four-form field strength $[43, 44, 45, 46, 47, 48, 49, 50, 51]$, which is expressed in terms of $q$ in the following way:

$$F_{\alpha\beta\gamma\delta} \equiv q e_{\alpha\beta\gamma\delta} \sqrt{-\det g} = \nabla_{[\alpha} A_{\beta\gamma\delta]} , \quad (15a)$$

$$q^2 = -\frac{1}{24} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} , \quad (15b)$$

where $e_{\alpha\beta\gamma\delta}$ the Levi–Civita tensor density; $\nabla_{\alpha}$ the covariant derivative; and the square bracket around spacetime indices complete anti-symmetrization.

Using vacuum action (7) with the above $q$ one obtains the Maxwell equation

$$\nabla_{\alpha} \left( \sqrt{-\det g} F_{\alpha\beta\gamma\delta} \frac{\partial \epsilon(q)}{\partial q} \right) = 0 . \quad (16)$$

This equation reproduces the known equation $[43, 44]$ for the special case $\epsilon(q) = \frac{1}{2} q^2$. Using (15a) the Maxwell equation is reduced to

$$\nabla_{\alpha} \left( \frac{\partial \epsilon(q)}{\partial q} \right) = 0 . \quad (17)$$

The first integral of (17) with integration constant $\mu$ gives again Eq.(14), which reflects the conservation law for $q$.

Variation of the action over $q^{\mu\nu}$ gives again the cosmological constant (11) with $\Lambda = \rho_{\text{vac}} = \epsilon(q) - \mu q$. This demonstrates the universality of the description of the self-sustained vacuum: description of the quantum vacuum in terms of the field $q$ does not depend on the microscopic origin of this field.

### 0.2.5 Aether field as example

Another example of the vacuum variable $q$ may be through a four-vector field $u^\mu(x)$. This vector field could be the four-dimensional analog of the concept of shift in the deformation theory of crystals. (Deformation theory can be described in terms of a metric field, with the role of torsion and curvature
fields played by dislocations and disclinations, respectively. A realization of $u^\mu$ could be also a 4-velocity field entering the description of the structure of spacetime. It is the 4-velocity of “aether”.

The nonzero value of the 4-vector in the vacuum violates the Lorentz invariance of the vacuum. To restore this invariance one may assume that $u^\mu(x)$ is not an observable variable, instead the observables are its covariant derivatives $\nabla_\nu u^\mu \equiv u^\mu_\nu$. This means that the action does not depend on $u^\mu$ explicitly but only depends on $u^\mu_\nu$:

$$S = \int_{\mathbb{R}^4} d^4x \, \epsilon(u^\mu_\nu),$$

with an energy density containing even powers of $u^\mu_\nu$:

$$\epsilon(u^\mu_\nu) = K + K^{\alpha\beta}_{\mu\nu} u^\alpha_\mu u^\beta_\nu + K^{\alpha\beta\gamma\delta}_{\mu\nu\rho\sigma} u^\alpha_\mu u^\beta_\nu u^\gamma_\rho u^\delta_\sigma + \ldots.$$  (19)

According to the imposed conditions, the tensors $K^{\alpha\beta}_{\mu\nu}$ and $K^{\alpha\beta\gamma\delta}_{\mu\nu\rho\sigma}$ depend only on $g_{\mu\nu}$ or $g^{\mu\nu}$ and the same holds for the other $K$–like tensors in the ellipsis of (19). In particular, the tensor $K^{\alpha\beta}_{\mu\nu}$ of the quadratic term in (19) has the following form in the notation of Ref. [52]:

$$K^{\alpha\beta}_{\mu\nu} = c_1 \, g^{\alpha\beta} g_{\mu\nu} + c_2 \, \delta^\alpha_\mu \delta^\beta_\nu + c_3 \, \delta^\alpha_\nu \delta^\beta_\mu,$$  (20)

for real constants $c_n$. Distinct from the original aether theory in Ref. [52], the tensor (20) does not contain a term $c_4 \, u^\alpha u^\beta g_{\mu\nu}$, as such a term would depend explicitly on $u^\mu$ and contradict the Lorentz invariance of the quantum vacuum.

The equation of motion for $u^\mu$ in flat space,

$$\nabla_\nu \frac{\partial \epsilon}{\partial u^\mu_\nu} = 0,$$  (21)

has the Lorentz invariant solution expected for a vacuum-variable $q$–type field:

$$u^\mu_\nu = q \, g_{\mu\nu}, \quad q = \text{constant}.$$  (22)

With this solution, the energy density in the action (18) is simply $\epsilon(q)$ in terms of contracted coefficients $K$, $K^{\mu\nu}$, and $K^{\mu\nu\rho\sigma}$ from (19). However, just as for previous examples, the energy-momentum tensor of the vacuum field obtained by variation over $g^{\mu\nu}$ and evaluated for solution (22) is expressed again in terms of the thermodynamic potential:

$$T^q_{\mu\nu} = \frac{2}{\sqrt{-g}} \left( \frac{\delta S}{\delta g^{\mu\nu}} \right) = g_{\mu\nu} \left( \epsilon(q) - q \frac{\partial \epsilon(q)}{\partial q} \right) = \rho_{\text{vac}}(q) g_{\mu\nu},$$  (23)

which corresponds to cosmological constant in Einstein’s gravitational field equations.
0.3 Thermodynamics of quantum vacuum

0.3.1 Liquid-like quantum vacuum

The zeroth order term \( K \) in (19) corresponds to a “bare” cosmological constant which can be considered as cosmological constant in the “empty” vacuum – vacuum with \( q = 0 \):

\[
\Lambda_{\text{bare}} = \epsilon(q = 0) . \tag{24}
\]

The nonzero value \( q = q_0 \) in the self-sustained vacuum does not violate Lorentz symmetry but leads to compensation of the bare cosmological constant \( \Lambda_{\text{bare}} \) in the equilibrium vacuum. This illustrates the important difference between the two states of vacua. The quantum vacuum with \( q = 0 \) can exist only with external pressure \( P = -\Lambda_{\text{bare}} \). By analogy with condensed-matter physics, this kind of quantum vacuum may be called “gas-like” (Fig. 1). The quantum vacuum with nonzero \( q \) is self-sustained: it can be stable at \( P = 0 \), provided that a stable nonzero solution of equation \( \epsilon(q) - q \frac{d\epsilon}{dq} = 0 \) exists. This kind of quantum vacuum may then be called “liquid-like”.

The universal behavior of the self-sustained vacuum in equilibrium suggests that it obeys the same thermodynamic laws as any other self-sustained macroscopic system described by the conserved quantity \( q \), such as quantum liquid. In other words, vacuum can be considered as a special quantum medium which is Lorentz invariant in its ground state. This medium is characterized by the Lorentz invariant “charge” density \( q - \) an analog of particle density \( n \) in non-relativistic quantum liquids.

Let us consider a large portion of such vacuum medium under external pressure \( P \) \[29\]. The volume \( V \) of quantum vacuum is variable, but its total “charge” \( Q(t) \equiv \int d^3r \ q(r,t) \) must be conserved, \( dQ/dt = 0 \). The energy of this portion of quantum vacuum at fixed total “charge” \( Q = qV \) is then given by the thermodynamic potential

\[
W = E + PV = \int d^3r \ \epsilon(Q/V) + PV , \tag{25}
\]

where \( \epsilon(q) \) is the energy density in terms of charge density \( q \). As the volume of the system is a free parameter, the equilibrium state of the system is obtained by variation over the volume \( V \):

\[
\frac{dW}{dV} = 0 . \tag{26}
\]
Figure 1: Vacuum as a medium obeying macroscopic thermodynamic laws. Relativistic vacuum possesses energy density, pressure and compressibility but has no momentum. In equilibrium, the vacuum pressure $P_{\text{vac}}$ equals the external pressure $P$ acting from the environment. The “gas-like” vacuum may exist only under external pressure. The “liquid-like” vacuum is self-sustained: it can be stable in the absence of external pressure. The thermodynamic energy density of the vacuum $\rho_{\text{vac}}$ which enters the vacuum equation of state $\rho_{\text{vac}} = -P_{\text{vac}}$ does not coincide with the microscopic vacuum energy $\epsilon$. While the natural value of $\epsilon$ is determined by the Planck scale, $\epsilon \sim E_P^4$, the natural value of the macroscopic quantity $\rho_{\text{vac}}$ is zero for the self-sustained vacuum which may exist in the absence of environment, i.e. at $P = 0$. This may explains why the present cosmological constant $\Lambda = \rho_{\text{vac}}$ is small.
This gives an integrated form of the Gibbs–Duhem equation for the vacuum pressure:

\[ P_{\text{vac}} = -\epsilon(q) + q \frac{d\epsilon(q)}{dq} = -\rho_{\text{vac}}(q) , \]  

(27)

whose solution determines the equilibrium value \( q = q(P) \) and the corresponding volume \( V(P,Q) = Q/q(P) \).

### 0.3.2 Macroscopic energy of quantum vacuum

Since the vacuum energy density is the vacuum pressure with minus sign, equation (27) suggests that the relevant vacuum energy, which is revealed in thermodynamics and dynamics of the low-energy Universe, is the equivalent of the grand potential:

\[ \rho_{\text{vac}}(q) = \epsilon(q) - q \frac{d\epsilon(q)}{dq} . \]  

(28)

This is confirmed by Eqs. (11) and (23) for energy-momentum tensor of the self-sustained vacuum, which demonstrates that it is the grand potential \( \rho_{\text{vac}}(q) \) rather than the energy density \( \epsilon(q) \), which enters the equation of state for the vacuum and thus corresponds to the cosmological constant:

\[ \Lambda = \rho_{\text{vac}} = -P_{\text{vac}} . \]  

(29)

While the energy of microscopic quantity \( q \) is determined by the Planck scale, \( \epsilon(q_0) \sim E_\text{P}^4 \), the relevant vacuum energy which sources the effective gravity is determined by a macroscopic quantity – the external pressure. In the absence of an environment, i.e. at zero external pressure, \( P = 0 \), one obtains that the pressure of pure and equilibrium vacuum is exactly zero:

\[ \Lambda = -P_{\text{vac}} = -P = 0 . \]  

(30)

Equation \( \rho_{\text{vac}}(q) = 0 \) determines the equilibrium value \( q_0 \) of the equilibrium self-sustained vacuum. Thus from the thermodynamic arguments it follows that for any effective theory of gravity the natural value of \( \Lambda \) is zero in equilibrium vacuum.

This result does not depend on the microscopic structure of the vacuum from which gravity emerges, and is actually the final result of the renormalization dictated by macroscopic physics. Note that we have two types of cancellation. The Planck scale quantities \( \epsilon(q) \) and \( q d\epsilon/dq \) cancel each other in equilibrium. The same occurs with the Planck scale contribution of zero-point energy of the bosonic and fermionic matter fields to the vacuum energy.
Figure 2: Contribution of different energy scales into the macroscopic energy of the self-sustained system at $T = 0$. Zero point energy $\rho_{zp}$ of the effective bosonic and fermionic quantum fields gives rise to the diverging contribution to the energy of the system. In quantum vacuum it is of order of $E_P^4$. In equilibrium this contribution is compensated without fine-tuning by microscopic degrees of freedom of the system (by trans-Planckian degrees of quantum vacuum correspondingly).
\( \rho_{\text{vac}} \), it is naturally compensated by microscopic degrees of freedom of the self sustained quantum vacuum. The vacuum variable \( q \) is adjusted automatically to nullify the relevant vacuum energy, \( \rho_{\text{vac}}(q_0) = \rho_{\text{zp}} + \rho_{\text{micro}} = 0 \). The actual spectrum of the vacuum energy density (meaning the different contributions to \( \rho_{\text{vac}} \) from different energy scales) is not important for the cancellation mechanism, because it is dictated by thermodynamics. The particular example of the spectrum of the vacuum energy density is shown in Fig. 2, where the positive energy of the quantum vacuum, which comes from the zero-point energy of bosonic fields, is compensated by negative contribution from trans-Planckian degrees of freedom [56].

Using the quantum-liquid counterpart of the self-sustained quantum vacuum as example, one may predict the behavior of the vacuum after cosmological phase transition, when \( \Lambda \) is kicked from its zero value. The vacuum will readjust itself to a new equilibrium state with new \( q_0 \) so that \( \Lambda \) will again approach its equilibrium zero value [29]. This process depends on details of dynamics of the vacuum variable \( q \), and later on we shall consider some examples of dynamical relaxation of \( \Lambda \).

### 0.3.3 Compressibility of the vacuum

Using the standard definition of the inverse of the isothermal compressibility, \( \chi^{-1} \equiv -V \frac{dP}{dV} \) (Fig. 4), one obtains the compressibility of the vacuum by varying Eq. (27) at fixed \( Q \equiv qV \) [29]:

\[
\chi_{\text{vac}}^{-1} = -V \frac{dP_{\text{vac}}}{dV} = \left[ q^2 \frac{d^2 \epsilon(q)}{dq^2} \right]_{q=q_0} > 0.
\]  

(31)

A positive value of the vacuum compressibility is a necessary condition for the stability of the vacuum. It is, in fact, the stability of the vacuum, which is at the origin of the nullification of the cosmological constant in the absence of an external environment.

From the low-energy point of view, the compressibility of the vacuum \( \chi_{\text{vac}} \) is as fundamental physical constant as the Newton constant \( G_N = G(q_0) \). It enters equations describing the response of the quantum vacuum to different perturbations \( \chi_{\text{vac}} \). While the natural value of the macroscopic quantity \( P_{\text{vac}} \) (and \( \rho_{\text{vac}} \)) is zero, the natural values of the parameters \( G(q_0) \) and \( \chi_{\text{vac}}(q_0) \) are determined by the Planck physics and are expected to be of order \( 1/E_P^2 \) and \( 1/E_P^4 \) correspondingly.
0.3.4 Thermal fluctuations of $\Lambda$ and the volume of Universe

The compressibility of the vacuum $\chi_{\text{vac}}$, though not measurable at the moment, can be used for estimation of the lower limit for the volume $V$ of the Universe. This estimation follows from the upper limit for thermal fluctuations of cosmological constant [57]. The mean square of thermal fluctuations of $\Lambda$ equals the mean square of thermal fluctuations of the vacuum pressure, which in turn is determined by thermodynamic equation [37]:

$$
\left\langle (\Delta \Lambda)^2 \right\rangle = \left\langle (\Delta P)^2 \right\rangle = \frac{T}{V_{\chi_{\text{vac}}}}.
$$

(32)

Typical fluctuations of the cosmological constant $\Lambda$ should not exceed the observed value:

$$
\left\langle (\Delta \Lambda)^2 \right\rangle < \Lambda_{\text{obs}}^2.
$$

Let us assume, for example, that the temperature of the Universe is determined by the temperature $T_{\text{CMB}}$ of the cosmic microwave background radiation. Then, using our estimate for vacuum compressibility $\chi_{\text{vac}}^{-1} \sim E_{\text{P}}^4$, one obtains that the volume $V$ of our Universe highly exceeds the Hubble volume $V_H = R_H^3$ -- the volume of visible Universe inside the present cosmological horizon:

$$
V > \frac{T_{\text{CMB}}}{\chi_{\text{vac}} \Lambda_{\text{obs}}^2} \sim 10^{28} V_H.
$$

(33)

This demonstrates that the real volume of the Universe is certainly not limited by the present cosmological horizon.

0.4 Dynamics of quantum vacuum

0.4.1 Action

In section 0.2 a special quantity, the vacuum “charge” $q$, was introduced to describe the statics and thermodynamics of the self-sustained quantum vacuum. Now we can extend this approach to the dynamics of the vacuum charge. We expect to find some universal features of the vacuum dynamics, using several realizations of this vacuum variable. We start with the 4-form field strength [43, 44, 45, 46, 47, 48, 49, 50, 51] expressed in terms of $q$. The low-energy effective action takes the following general form:

$$
S = -\int_{\mathbb{R}^4} d^4 x \sqrt{|g|} \left( \frac{R}{16\pi G(q)} + \epsilon(q) + \mathcal{L}_M(q, \psi) \right),
$$

(34a)

$$
q^2 \equiv -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}, \quad F_{\kappa\lambda\mu\nu} \equiv \nabla_{[\kappa} A_{\lambda\mu\nu]},
$$

(34b)

$$
F_{\kappa\lambda\mu\nu} = q \sqrt{|g|} \epsilon_{\kappa\lambda\mu\nu}, \quad F^{\kappa\lambda\mu\nu} = q e^{\kappa\lambda\mu\nu} / \sqrt{|g|}.
$$

(34c)
where $R$ denotes the Ricci curvature scalar; and $\mathcal{L}^M$ is matter action. The vacuum energy density $\epsilon$ in (34a) depends on the vacuum variable $q$ which in turn is expressed via the 3-form field $A_{\lambda\mu\nu}$ and metric field $g_{\mu\nu}$ in (34b). The field $\psi$ combines all the matter fields of the Standard Model. All possible constant terms in matter action (which includes the zero-point energies from the Standard Model fields) are absorbed in the vacuum energy $\epsilon(q)$.

Since $q$ describes the state of the vacuum, the parameters of the effective action – the Newton constant $G$ and parameters which enter the matter action – must depend on $q$. This dependence results in particular in the interaction between the matter fields and the vacuum. There are different sources of this interaction. For example, in the gauge field sector of Standard Model, the running coupling contains the ultraviolet cut-off and thus depends on $q$: 

$$\mathcal{L}^{G,q} = \gamma(q) F_{\mu\nu} F_{\mu\nu},$$ (35)

where $F_{\mu\nu}$ is the field strength of the particular gauge field (we omitted the color indices). In the fermionic sector, $q$ should enter parameters of the Yukawa interaction and fermion masses.

### 0.4.2 Vacuum dynamics

The variation of the action (34a) over the three-form gauge field $A$ gives the generalized Maxwell equations for $F$-field,

$$\nabla_\nu \left( \sqrt{|g|} \frac{F^{\kappa\lambda\mu\nu}}{q} \left( \frac{d\epsilon(q)}{dq} + \frac{R}{16\pi} \frac{dG^{-1}(q)}{dq} + \frac{d\mathcal{L}^M(q)}{dq} \right) \right) = 0.$$ (36)

Using (34c) for $F^{\kappa\lambda\mu\nu}$, we find that the solutions of Maxwell equations (36) are still determined by the integration constant $\mu$

$$\frac{d\epsilon(q)}{dq} + \frac{R}{16\pi} \frac{dG^{-1}(q)}{dq} + \frac{d\mathcal{L}^M(q)}{dq} = \mu.$$ (37)

### 0.4.3 Generalized Einstein equations

The variation over the metric $g^{\mu\nu}$ gives the generalized Einstein equations,

$$\frac{1}{8\pi G(q)} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{1}{16\pi} \frac{dG^{-1}(q)}{dq} \nabla_\mu \nabla_\nu G^{-1}(q) - g_{\mu\nu} \nabla_\rho G^{-1}(q) - \left( \epsilon(q) - q \frac{d\epsilon(q)}{dq} \right) g_{\mu\nu}$$

$$+ q \frac{\partial \mathcal{L}^M}{\partial q} g_{\mu\nu} + T^M_{\mu\nu} = 0,$$ (38)
where $\Box$ is the invariant d’Alembertian; and $T^M_{\mu\nu}$ is the energy-momentum tensor of the matter fields, obtained by variation over $g^{\mu\nu}$ at constant $q$, i.e. without variation over $g^{\mu\nu}$, which enters $q$.

Eliminating $dG^{-1}/dq$ and $\partial L^M/\partial q$ from (38) by use of (37), the generalized Einstein equations become

$$\frac{1}{8\pi G(q)} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{1}{8\pi} \left( \nabla_\mu \nabla_\nu G^{-1}(q) - g_{\mu\nu} \Box G^{-1}(q) \right) - \rho_{\text{vac}} g_{\mu\nu} + T^M_{\mu\nu} = 0,$$

(39)

where

$$\rho_{\text{vac}} = \epsilon(q) - \mu q.$$  

(40)

For the special case when the dependence of the Newton constant and matter action on $q$ is ignored, (39) reduces to the standard Einstein equation of general relativity with the constant cosmological constant $\Lambda = \rho_{\text{vac}}$.

### 0.4.4 Minkowski-type solution and Weinberg problem

Among different solutions of equations (36) and (38) there is the solution corresponding to perfect equilibrium Minkowski vacuum without matter. It is characterized by the constant in space and time values $q = q_0$ and $\mu = \mu_0$ obeying the following two conditions:

$$\left[ \frac{d\epsilon(q)}{dq} - \mu \right]_{\mu = \mu_0, \ q = q_0} = 0,$$

(41a)

$$\left[ \epsilon(q) - \mu q \right]_{\mu = \mu_0, \ q = q_0} = 0.$$  

(41b)

The two conditions (41a)–(41b) can be combined into a single equilibrium condition for $q_0$:

$$\Lambda_0 \equiv \left[ \epsilon(q) - q \frac{d\epsilon(q)}{dq} \right]_{q = q_0} = 0,$$

(42)

with the derived quantity

$$\mu_0 = \left[ \frac{d\epsilon(q)}{dq} \right]_{q = q_0}.$$  

(43)

In order for the Minkowski vacuum to be stable, there is the further condition: $\chi(q_0) > 0$ where $\chi$ corresponds to the isothermal vacuum compressibility [31] [29]. In this equilibrium vacuum the gravitational constant $G(q_0)$ can be identified with Newton’s constant $G_N$. 
Let us compare the conditions for the equilibrium self-sustained vacuum, (42) and (43), with the two conditions suggested by Weinberg, who used the fundamental scalar field $\phi$ for the description of the vacuum. In this description there are two constant-field equilibrium conditions for Minkowski vacuum, $\partial \mathcal{L}/\partial g_{\alpha\beta} = 0$ and $\partial \mathcal{L}/\partial \phi = 0$, see Eqs. (6.2) and (6.3) in [58]. These two conditions turn out to be inconsistent, unless the potential term in $\mathcal{L}(\phi)$ is fine-tuned (see also Sec. 2 of Ref. [59]). In other words, the Minkowski vacuum solution may exist only for the fine-tuned action. This is the Weinberg formulation of the cosmological constant problem.

The self-sustained vacuum naturally bypasses this problem [33]. Equation $\partial \mathcal{L}/\partial g_{\alpha\beta} = 0$ corresponds to the equation (42). However, the equation $\partial \mathcal{L}/\partial \phi = 0$ is relaxed in the $q$-theory of self-sustained vacuum. Instead of the condition $\partial \mathcal{L}/\partial q = 0$, the conditions are $\nabla_{\alpha}(\partial \mathcal{L}/\partial q) = 0$, which allow for having $\partial \mathcal{L}/\partial q = \mu$ with an arbitrary constant $\mu$. This is the crucial difference between a fundamental scalar field $\phi$ and the variable $q$ describing the self-sustained vacuum. As a result, the equilibrium conditions for $g_{\alpha\beta}$ and $q$ can be consistent without fine-tuning of the original action. For Minkowski vacuum to exist only one condition (42) must be satisfied. In other words, the Minkowski vacuum solution exists for arbitrary action provided that solution of equation (42) exists.

0.5 Cosmology as approach to equilibrium

0.5.1 Energy exchange between vacuum and gravity+matter

In the curved Universe and/or in the presence of matter, $q$ becomes space-time dependent due to interaction with gravity and matter (see [37]). As a result the vacuum energy can be transferred to the energy of gravitational field and/or to the energy of matter fields. This also means that the energy of matter is not conserved. The energy-momentum tensor of matter $T_{\mu\nu}^M$, which enters the generalized Einstein equations (39), is determined by variation over $g^{\mu\nu}$ at constant $q$. That is why it is not conserved:

$$\nabla_\nu T^{M\mu\nu} = -\frac{\partial \mathcal{L}_M}{\partial q} \nabla_\mu q. \quad (44)$$

The matter energy can be transferred to the vacuum energy due to interaction with $q$-field. Using (37) and the equation (40) for cosmological constant one obtains that the vacuum energy is transferred both to gravity and matter.
with the rate:

$$\nabla \mu \Lambda \equiv \nabla_{\mu} \rho_{\text{vac}} = \left( \frac{d\epsilon(q)}{dq} - \mu \right) \nabla_{\mu} q = -\frac{R}{16\pi} \frac{dG^{-1}(q)}{dq} \nabla_{\mu} q + \nabla_{\nu} T^{\mu\nu}. \quad (45)$$

The energy exchange between the vacuum and gravity+matter allows for the relaxation of the vacuum energy and cosmological “constant”.

### 0.5.2 Dynamic relaxation of vacuum energy

Let us assume that we can make a sharp kick of the system from its equilibrium state. For quantum liquids (or any other quantum condensed matter) we know the result of the kick: the liquid or superconductor starts to relax back to the equilibrium state, and with or without oscillations it finally approaches the equilibrium \([60, 61, 62, 63, 64]\). The same should happen with the quantum vacuum. Let us consider this behavior using the realization of the vacuum \(q\) field in terms of the 4-form field, when \(\mu\) serves as the overall integration constant. We start with the fully equilibrium vacuum state, which is characterized by the values \(q = q_0\) and \(\mu = \mu_0\) in (41). The kick moves the variable \(q\) away from its equilibrium value, while \(\mu\) still remains the same being the overall integration constant, \(\mu = \mu_0\). In the non-equilibrium state which arises immediately after the kick, the vacuum energy is non-zero and big. If the kick is very sharp, with the time scale of order \(t_P = 1/E_P = \sqrt{G_N}\), the energy density of the vacuum can reach the Planck-scale value, \(\rho_{\text{vac}} \sim E_P^4\).

For simplicity we ignore the interaction between the vacuum and matter. Then from the solution of dynamic equations (37) and (39) with \(\mu = \mu_0\) one finds that after the kick \(q\) does return to its equilibrium value \(q_0\) in the Minkowski vacuum. At late time the relaxation has the following asymptotic behavior: \([30]\)

$$q(t) - q_0 \sim q_0 \frac{\sin \omega t}{\omega t}, \quad \omega t \gg 1,$$

where oscillation frequency \(\omega\) is of the order of the Planck-energy scale \(E_P\). The gravitational constant \(G\) approaches its Newton value \(G_N\) also with the power-law modulation:

$$G(t) - G_N \sim G_N \frac{\sin \omega t}{\omega t}, \quad \omega t \gg 1.$$  \quad (47)

The vacuum energy relaxes to zero in the following way (see Fig. 3):

$$\rho_{\text{vac}}(t) \propto \frac{\omega^2}{t^2} \sin^2 \omega t, \quad \omega t \gg 1,$$

(48a)
Figure 3: Sketch of the oscillating decay of the cosmological constant after sharp kick. Frequency of oscillations $\omega \sim E_p$. If the Universe starts expansion from the state with vacuum energy $\rho_{\text{vac}} \sim E_p^4$, in the process of relaxation the vacuum energy will reach the observed value of cosmological constant at present time: $\Lambda(t_{\text{present}}) \sim E_p^2/t_{\text{present}}^2 \sim (10^{-3} \text{ eV})^4$.

For the Planck scale kick, the vacuum energy density after the kick, i.e. at $t \sim 1/E_p$, has a Planck-scale value, $\rho_{\text{vac}} \sim E_p^4$. According to (48a), at present time it must reach the value

$$\rho_{\text{vac}}(t_{\text{present}}) \propto \frac{E_p^2}{t_{\text{present}}^2} \sim E_p^2 H^2, \quad (48b)$$

where $H$ is the Hubble parameter. This value approximately corresponds to the measured value of the cosmological constant.

This, however, can be considered as an illustration of the dynamical reduction of the large value of the cosmological constant, rather than the real scenario of the evolution of the Universe. We did not take into account quantum dissipative effects and the energy exchange between vacuum and matter. Indeed, matter field radiation (matter quanta emission) by the oscillations of the vacuum can be expected to lead to faster relaxation of the initial vacuum energy [65],

$$\rho_{\text{vac}}(t) \propto \Gamma^4 \exp(-\Gamma t), \quad (48c)$$

with a decay rate $\Gamma \sim \omega \sim E_p$.

Nevertheless, the cancellation mechanism and example of relaxation provide the following lesson. The Minkowski-type solution appears without
fine-tuning of the parameters of the action, precisely because the vacuum is characterized by a constant derivative of the vacuum field rather than by a constant vacuum field itself. As a result, the parameter $\mu_0$ emerges in (41a) as an integration constant, i.e., as a parameter of the solution rather than a parameter of the action. Since after the kick the integration constant remains intact, the Universe will return to its equilibrium Minkowski state with $\rho_{\text{vac}} = 0$, even if in the non-equilibrium state after the kick the vacuum energy could reach $\rho_{\text{vac}} \sim E_\nu^4$. The idea that the constant derivative of a field may be important for the cosmological constant problem has been suggested earlier by Dolgov [66, 67] and Polyakov [68, 69], where the latter explored the analogy with the Larkin–Pikin effect [70] in solid-state physics.

0.5.3 Minkowski vacuum as attractor

The example of relaxation of the vacuum energy in Sec. 0.5.2 has the principle drawback. Instead of the fine-tuning of the action, which is bypassed in the self-sustained vacuum, we have the fine-tuning of the integration constant. We assumed that originally the Universe was in its Minkowski ground state, and thus the specific value of the integration constant $\mu = \mu_0$ has been chosen, that fixes the value $q = q_0$ of the original Minkowski equilibrium vacuum. In the 4-form realization of the vacuum field, any other choice of the integration constant ($\mu \neq \mu_0$) leads to a de-Sitter-type solution [30]. Though it is not excluded that the Big-Bang started after the kick from the equilibrium Minkowski vacuum, it is instructive to consider the scenarios which avoid this fine-tuning and obtain the natural relaxation of $\mu$ to $\mu_0$ from any initial state. Such relaxation as we know occurs in quantum liquids. So we must relax the condition on $\mu$: it should not serve as an overall integration constant, while remaining the conjugate variable in thermodynamics. Then using the condensed matter experience one may expect that the Minkowski equilibrium vacuum becomes an attractor and the de-Sitter solution with $\mu \neq \mu_0$ will inevitably relax to Minkowski vacuum with $\mu = \mu_0$.

This expectation is confirmed in the aether type realization of the vacuum variable in terms of a vector field as discussed in Sec. 0.2.5. The constant vacuum field $q$ there appears as the derivative of a vector field not for all vector fields $u_\beta$, but only for the specific solution $u_\beta^q$ corresponding to the equilibrium vacuum, $q g_{\alpha\beta} \equiv \nabla_\alpha u_\beta^q = u_{\alpha\beta}^q$. In this realization, the effective chemical potential $\mu \equiv d\epsilon(q)/dq$ appears only for the equilibrium states (i.e., for their thermodynamical properties), but $\mu$ does not appear as an integration constant for the dynamics. Hence, the fine-tuning problem of the integration constant is overcome, simply because there is no integration
Figure 4: Aether-field $q$ evolution and Minkowski attractor in a spatially flat Friedmann–Robertson–Walker universe in Dolgov model [67] (see [33] for details). The bare cosmological constant is $\Lambda_{\text{bare}} \sim E_p^4$. Four numerical solutions correspond to different boundary conditions, but all approach the Minkowski-spacetime solution (49). The Minkowski vacuum is an attractor because the vacuum compressibility (31) is positive, $\chi(q_0) > 0$.

constant.

The instability of the de-Sitter solution towards the Minkowski one has been already demonstrated by Dolgov [67], who considered the simplest quadratic choices of the Lagrange density of $u_\beta(x)$. But his result also holds for the generalized Lagrangian with a generic function $\epsilon(u_{\alpha\beta})$ in Sec. 0.2.5 [33]. In the Dolgov scenario the initial de-Sitter-type expansion evolves towards the Minkowski attractor by the following $t \to \infty$ asymptotic solution for the aether-type field $u_\beta = (u_0(t), 0)$ and Hubble parameter:

$$u_0(t) \to q_0 t, \quad H(t) \to 1/t.$$  (49)

At large cosmic times $t$, the curvature terms decay as $R \sim H^2 \sim 1/t^2$ and the Einstein equations lead to the nullification of the energy-momentum tensor of the $u_\beta$ field: $T_{\alpha\beta}[u] = 0$. Since (49) with $du_0/dt = H u_0$ satisfies the $q$–theory Ansatz $u_{\alpha\beta} = q g_{\alpha\beta}$, the energy-momentum tensor is completely expressed by the single constant $q$: $T_{\alpha\beta}(q) = [\epsilon(q) - q \delta(q)/dq] g_{\alpha\beta}$. As a result, the equation $T_{\alpha\beta}(q) = 0$ leads to the equilibrium condition (42) for the Minkowski vacuum and to the equilibrium value $q = q_0$ in (49).

Figure 4 shows explicitly the attractor behavior for the simplest case of Dolgov action, with the equilibrium value $q_0$ in (49) appearing dynamically. This simple version of Dolgov scenario does not appear to give a realistic
description of the present Universe [71] and requires an appropriate modification [72], which demonstrates that the compensation of a large initial vacuum energy density can occur dynamically and that Minkowski spacetime can emerge spontaneously, without setting a chemical potential. In other words, an “existence proof” has been given for the conjecture that the appropriate Minkowski value $q_0$ can result from an attractor-type solution of the field equations. The only condition for the Minkowski vacuum to be an attractor is a positive vacuum compressibility [31].

0.5.4 Remnant cosmological constant

Figure 5 demonstrates the possible more realistic scenario with a step-wise relaxation of the vacuum energy density [73]. The vacuum energy density moves from plateau to plateau responding to the possible phase transitions or crossovers in the Standard Model vacuum and follows, on average, the steadily decreasing matter energy density. The origin of the current plateau with a small positive value of the vacuum energy density $\Lambda_{\text{present}} = \rho_{\text{vac}} \sim (10^{-3} \text{eV})^4$ is still not clear. It may result from the phenomena, which occur in the infrared. It may come for example from anomalies in the neutrino...
sector of the quantum vacuum, such as non-equilibrium contribution of the
light massive neutrinos to the quantum vacuum [73]; reentrant violation
of Lorentz invariance [74] and Fermi point splitting in the neutrino sector
[13, 14]. The other possible sources include the QCD anomaly [75, 32, 76, 77,
78]; torsion [79]; relaxation effects during the electroweak crossover [31]; etc.
Most of these scenarios are determined by the momentum space topology of
the quantum vacuum.

0.6 Discussion

To study the problems related to quantum vacuum one must search for
the proper extension of the current theory of elementary particle physics –
the Standard Model – based on the topology in momentum space. However,
the gravitational properties of the quantum vacuum can be understood even
without that: by extending of our experience with self-sustained macroscopic
systems to the quantum vacuum. A simple picture of quantum vacuum is
based on three assumptions: (i) The quantum vacuum is a self-sustained
medium – the system which is stable at zero external pressure, like quantum
liquids. (ii) The quantum vacuum is characterized by a conserved charge $q$,
which is analog of the particle density $n$ in quantum liquids and which is
non-zero in the ground state of the system, $q = q_0 \neq 0$. (iii) The quantum
vacuum with $q = q_0$ is with a great precision a Lorentz-invariant state. The
latter is the only property which distinguishes the quantum vacuum from
the quantum condensed-matter systems (here we consider the properties of
the deep vacuum, on the Planck energy scale, and do not discuss subtle
effects of spontaneous violation of Lorentz symmetry which in principle may
occur in the infrared [12, 13, 14]).

These assumptions naturally solve the main cosmological constant prob-
lem without fine-tuning. In any self-sustained system, relativistic or non-
relativistic, in thermodynamic equilibrium at $T = 0$ the zero-point energy of
quantum fields is fully compensated by the microscopic degrees of freedom,
so that the relevant energy density is zero in the ground state. This con-
sequence of thermodynamics is automatically fulfilled in any system, which
may exist without external environment. This leads to the trivial result for
gravity: the cosmological constant in any equilibrium vacuum state is zero.
The zero-point energy of the Standard Model fields is automatically compen-
sated by the $q$–field that describes the degrees of freedom of the deep
quantum vacuum.

These assumptions allow us to suggest that cosmology is the process
of equilibration. From the condensed matter experience we know that the ground state of the system serves as an attractor: starting far away from equilibrium, the quantum liquid finally reaches its ground state. The same should occur in our Universe: starting far away from equilibrium in a very early phase of universe, the vacuum is moving towards the Minkowski attractor. We are now close to this attractor, simply because our Universe is old leaving a small remnant cosmological constant measured in present time. The $q$–theory transforms the standard cosmological constant problem into the search for the proper decay mechanism of the vacuum energy density and for the proper mechanism of formation of small remnant cosmological constant. For that we need the theory of dynamics of quantum vacuum, whose details depend on the topology of the quantum vacuum in momentum space.
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