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Phys. Rev. Lett. 111, 053003 — Published 31 July 2013
DOI: 10.1103/PhysRevLett.111.053003
Quantum quench in an atomic one-dimensional Ising chain

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We study non-equilibrium dynamics for an ensemble of tilted one-dimensional atomic Bose-Hubbard chains after a sudden quench to the vicinity of the transition point of the Ising paramagnetic to anti-ferromagnetic quantum phase transition. The quench results in coherent oscillations for the orientation of effective Ising spins, detected via oscillations in the number of doubly-occupied lattice sites. We characterize the quench by varying the system parameters. We report significant modification of the tunneling rate induced by interactions and show clear evidence for collective effects in the oscillatory response.

PACS numbers: 37.10.Jk, 67.85.Hj, 03.75.Lm, 05.30.Rt

Ultracold atomic ensembles confined in optical lattice potentials have proven to offer unique access to the study of strongly correlated quantum phases of matter [1, 2]. Unprecedented control over system parameters as well as exceptionally good isolation from the environment allow for implementation and quantitative simulation of lattice Hamiltonians [3, 4], not only bridging the fields of atomic and condensed matter physics in the study of ground-state phases, but also opening fundamentally new opportunities to explore out-of-equilibrium physics in essentially closed quantum systems [5, 6]. For example, the rapid time-dependent control available over system parameters makes it possible to observe dynamics arising from a quantum quench, where a parameter such as the lattice depth is changed suddenly in time [7–9]. Recently it was demonstrated that 1D chains of bosonic atoms with a superimposed linear gradient potential exhibit a quantum phase transition to a density-wave-ordered state, in which empty sites alternate with doubly-occupied sites (“doublons”). Beginning in a Mott-insulator phase of a Bose-Hubbard (BH) system [10, 11], where the on-site interactions dominate over tunneling and the atoms are exponentially localized on individual lattice sites, a gradient potential is added until the potential difference between adjacent sites matches the on-site interaction energy, and atoms can again resonantly tunnel. This was monitored for individual 1D chains with a length of about 10 sites with the quantum gas microscopy technique [12], and effectively maps onto a 1D Ising model [13], making it possible to simulate the transition from 1D paramagnetic (PM) spin chains to anti-ferromagnetic (AFM) spin chains in the context of ultracold atoms.

In this letter, we explore the dynamics of a quantum quench for bosonic atoms in such a tilted optical lattice [14–16]. Specifically, we quench the strength of the tilt to be near the phase transition point between PM and AFM regimes, and hence take the system far out of equilibrium, inducing strong oscillations in the number of doublons, which we detect through molecule formation. We find clear indications for the collective character of the ensuing dynamics.

We consider a 1D atomic ensemble in a tilted optical lattice potential near zero temperature. For sufficiently weak interaction energy, much smaller than the band gap, the system is described by the 1D single-band BH Hamiltonian [10] augmented by a tilt [13, 16]

\[ \hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) + E \sum_i \hat{n}_i + \sum_i \varepsilon_i \hat{n}_i. \] (1)

As usual, \( \hat{a}_i^\dagger \) (\( \hat{a}_i \)) are the bosonic creation (annihilation) operators at the \( i \)th lattice site, \( \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i \) are the number operators, \( J \) is the tunnel matrix element, and \( U \) is the on-site interaction energy. The linear energy shift from site to site is denoted by \( E \), and \( \varepsilon_i \) accounts for a weak external confinement. For sufficiently small tilt (\( E \ll U \)) and sufficiently strong interactions (\( U \gg J \)) the ground state of the system for one-atom commensurate filling is a Mott insulator with exactly one atom per site. When the tilt is ramped from \( E < U \) to \( E > U \) across \( E \approx U \) at finite \( J \), the system establishes a regular periodic pattern of dipole states, for which one finds double occupancy at every second site with empty sites in between [13, 14], see insets to Fig. 1. The formation of such density-wave ordering is due to an effective nearest-neighbor constraint, which results in correlated tunneling of all particles in the 1D chain. When, in

![Graph](image_url)

**FIG. 1:** (color online) Adiabatic tilt through the phase transition point. The number of doublons \( N_d \) (circles) as detected through Feshbach molecule formation and the number of singly occupied sites \( N_s \) (triangles) are plotted versus the tilt \( E_f \) at the end of the adiabatic ramp for \( V_i = 10 E_R \) and \( \alpha_i = 225(5) \alpha_R \), giving \( U = 0.963(20) \) kHz for \( V_i = 20 E_R \). The full ramp takes 40 ms. The solid lines are fits to guide the eye using the error function. The dashed line indicates \( E_f = 1.010(20) \) kHz. The insets show a schematic of the Ising phase transition for a 1D Mott-insulator chain from the PM (left) to the AFM (right) ground state [12]. All error bars in this figure and in the following figures denote the one-sigma standard error.
FIG. 2: (color online) (a) Reversibility of the phase transition. The number of doublons $N_d$ is plotted as the system is driven from the PM into the AFM ground state ($t \leq 40$ ms) and back ($t > 40$ ms) for $V_z = 10E_R$. The dashed line is a fit using two concatenated error functions to guide the eye. For this measurement $V_{c,y} = 25E_R$. (b) Phase transition for various values of the lattice depth $V_z = 8$ (circles), 10 (triangles), 12 (squares), and $14E_R$ (diamonds). The ramp rate is $E = (17.3, 17.3, 8.6, 5.8)$ Hz/ms for $V_z = (8, 10, 12, 14)E_R$. The solid lines are fits using the error function to determine the half width $w$. For this measurement $V_{c,y} = 30E_R$. We determine $U$ to $(1.233, 1.324, 1.401, 1.467)$ kHz and $E_c$ to $(1.309, 1.371, 1.431, 1.487)$ kHz for $V_z = (8, 10, 12, 14)E_R$ with typical error of 20 Hz for $U$ and $E_c$. The positions of $E_c$ are indicated by the dashed lines. (c) Half width $w$ of the phase transition obtained from the fit as a function of $V_z$. The dashed line shows the calculated $4J$. (d) Same data as in (b), but normalized and plotted as a function of $(E_f - U)/J$. The dashed line indicates the quantum critical point.

called the course of the ramp, the tilt compensates for the interaction energy ($E \approx U$), the particles are allowed to resonantly tunnel onto the neighboring sites, however, only when the neighboring particle has not yet tunnelled itself. This system, when $|U - E|/J \ll U$, can be mapped onto an effective Ising spin model [12, 13], where the two distinct ground states (PM and AFM order, respectively) are connected via a quantum phase transition with the quantum critical point $E_c = U + 1.85J$ [17].

Starting with a Cs Bose-Einstein condensate [18, 19] we prepare an ensemble of 1D Bose chains by first creating a 3D Mott insulator in a cubic optical lattice with unity filling [20]. Initially, the lattice depth $V_y = 20E_R$ ($q = x, y, z$), where $E_R = 1.325$ kHz [21] is the photon recoil energy. The residual harmonic confinement along the vertical $z$-direction is $V_z = 11.9(0.2)$ Hz. A tilt $E$ of up to $1.7$ kHz along $z$ is controlled by a magnetic force $\nabla B$. A Feshbach resonance allows us to control $U$ independent of $J$ [22, 23].

We first drive the transition adiabatically from the PM to the AFM state. We do this in parallel for an ensemble of about 2000 1D systems ("tubes" or "Ising chains") with an average length of 40 sites [20]. We start with a pre-tilt of $E \approx 0.7U$ and then lower $V_z$ to 10$E_R$ ($J = 25.4$ Hz [10]) within 10 ms to allow for particle motion along the vertical direction. Effectively, our 3D one-atom-per-site Mott insulator has now turned into an ensemble of 1D tubes with near unity occupation as horizontal tunnel coupling between the tubes can be neglected on the timescale of the experiment. We ramp through the PM-to-AFM transition with a ramp speed of $E = 17.3$ Hz/ms up to a final value $E_f$ and project the ensemble of 1D many-body systems onto number states by quickly ramping $V_z$ up to its initial value and instantly reversing the tilt. We detect double occupancy through Feshbach molecule formation and detection [20, 24, 25] with an overall doublon detection efficiency of 80(3)%.

Fig. 1 plots the number of doublons $N_d$ as a function of $E_f$. The transition from an ensemble of singly occupied sites to an ensemble with predominant double occupation can be seen clearly. The transition point is in very good agreement with the expectation from the BH model, with $U = 0.963(20)$ kHz and $E_c = 1.010(20)$ kHz. When ramping fully through the transition we create $N_d = 3 \times 10^4$ doublons, corresponding to ensembles with 79(2)% double occupancy. We believe that defects in the initial one-atom Mott shell due to finite temperature and the superfluid outer ring are the main limitations to a higher conversion efficiency. We estimate the average length of AFM chains to $\approx 13$ lattice sites [20]. Note that this state with exceptionally high double occupancy is ideal for creating ultracold samples of ground-state molecules [25].

An unambiguous characteristic of a phase transition is its reversibility. Fig. 2(a) shows the result when the linear ramp (from 0 to 40 ms) is reversed (from 40 to 80 ms). We find that a large fraction of the doublons, more than 85%, is returned to the one-atom-per-site Mott phase.

Adiabatic driving of the quantum phase transition from the PM into the AFM state is permitted by the finite tunneling rate $J$ between the sites through quantum fluctuations. In Fig. 2(b) we drive the phase transition at various values for the lattice depth $V_z$ in order to highlight the role of tunneling. To assure adiabaticity, we increase the ramp duration for larger $V_z$. Evidently, a reduced tunnel coupling for larger values of $V_z$ gives rise to a narrowing of the transition. Note that the transition point is shifted to a larger tilt as $V_z$ is increased due to the increase in $U$. We fit each data set with an error function of the form $\propto \exp((E - E_0)/w)$ and plot the extracted half-width $w$ as a function of $V_z$ in Fig. 2(c). We clearly find the width of the phase transition to scale with the tunnel matrix element. Interestingly, $w$ fits very well to the value $4J$. In Fig. 2(d) we plot the data shown in Fig. 2(b) as a function of $(E_f - U)/J$. The data collapses onto a single curve, confirming the role of $J$ as the natural energy scale [13].

Slow ramping through the critical point allows the system...
The shaded areas (red and blue) indicate the one-sigma statistical maximum as a function of offset. (d) Extracted number of doublons as a function of $\delta$.

The quadratic fit to the data for $f_\delta$ is justified, in the simplest approximation, by a two-state Rabi model for the case of a single double-well potential [14]. At the point $E \approx U$, the two Fock states [11] and [20], describing one atom in each site and two atoms in the left site, respectively, are resonantly coupled by $J$, leading to Rabi oscillations between the two states with frequency $\sqrt{2}J$, where the factor $\sqrt{2}$ ensues from bosonic enhancement. The probability to find the system in the dipole state [20] thus oscillates in the case of finite detuning from resonance with the generalized Rabi frequency $f_\delta = \sqrt{8J^2 + \delta^2}$ [14], which is quadratic for sufficiently small $\delta$.

Before we continue this discussion we make use of our capability to tune $U$ by means of a Feshbach resonance. We repeat the measurements shown in Fig. 3 for different values of $a_0$, which we tune after producing the 3D one-atom Mott-insulator state and before initializing the quench. We determine the resonance position $E_{\text{res}}$ and the on-resonance oscillation frequency $f_0 = f_\delta|_{\delta = 0}$. The result is shown in Fig. 4(a) and (b). For repulsive interactions we find good agreement for $E_{\text{res}}$ with $U$ calculated from lowest-band Wannier functions [10]. For attractive interactions there is a significant shift to lower absolute values of $E_{\text{res}}$ as corrections taking into account higher bands and the properly regularized pseudopotential would have to be included [23]. We attribute the considerable dependence of the data for $f_0$ on $a_0$ to an effectively altered tunnel barrier as $U$ and hence $E_{\text{res}}$ are changed [27, 28].

We now quantitatively compare our findings for $f_0$ with the result from the simplified double-well system. Firstly, to remove the effects of the modified tunneling rate due to the tilt and interactions, we interpolate our data to find $f_0$ for $a_0 = 0$ with the linear fit as shown in Fig. 4(b). The resulting values are shown in Fig. 4(c) for data sets taken at three different values of $V_c$, and are compared to the prediction for the double well on resonance, $f_0 = 2\sqrt{2}J$ (dotted line). Evidently, the tunneling in the 1D chains is significantly faster when compared to the double-well system. This arises due to the nearest-neighbor constraint present in the 1D chains, which prohibits having two adjacent dipoles and which leads to an increase of the oscillation frequency even for three sites by a further factor of $\sqrt{2}$, to $4J$ (dashed line) [14]. Through numerical simulations with up to 60 lattice sites we have verified that the oscillation frequency shows only a slight further increase for longer chains beyond 3 sites [20]. The weak harmonic trapping potential causes an additional small increase of the computed oscillation frequency. However, both effects are presently within our experimental uncertainty.

Finally, we compare the on-resonance oscillatory dynamics to follow adiabatically the many-body ground state. Non-equilibrium dynamics, however, arises when the initial Mott state is quenched by a rapidly applied tilt to the vicinity of the transition [14, 16]. In our experiment we initiate non-equilibrium dynamics by first tilting the lattice and then quickly lowering $V_c$ to $10E_R$. We let the system evolve for a hold time $t_h$ and finally record the number of doublons $N_d$ as before. For the subsequent measurements $U = 1.019(20)$ kHz. Fig. 3(a) shows the result for different values of $\delta = E - U$ close to the transition. The ensemble of 1D chains exhibits large amplitude oscillations for $N_d$. The amplitude is maximal for $\delta = 0$ and significantly lower for $|\delta| > 0$. For $\delta \approx 0$, which we call the resonant case, the first maximum is reached at $t_h \approx 3$ ms. The oscillations damp out to an equilibrium value over a time scale of about 30 ms. We fit the data using exponentially damped sinusoids. From these we obtain the frequency $f_\delta$, the peak value of the first oscillation $N_d^{\text{max}}$, and the decay rate $\gamma$. Away from resonance, $N_d^{\text{max}}$ is lower, however $f_\delta$ is higher and, interestingly, $\gamma$ is lower. This can be seen from Fig. 3(b), (c), and (d), where we plot $f_\delta$, $\gamma$, and $N_d^{\text{max}}$ as a function of $\delta$. We fit $f_\delta$ by a quadratic function, as will be justified below, and obtain the position of the minimum $E_{\text{res}}$. The data for $N_d^{\text{max}}$ and $\gamma$ are fitted by Lorentzians. The damping is consistent with collapse of oscillations in coherent dynamics as observed in our simulations of the constrained Ising model [20], see also Refs. [13, 16]. We find that the positions of the maxima for $N_d^{\text{max}}$ and $\gamma$ at $E = 1035(2)$ and $E = 1030(3)$ Hz, respectively, agree with the minimum for $f_\delta$ at $E_{\text{res}} = 1028(1)$ Hz. Within our error bars [26], all values are compatible with the value for $U$. We note that $N_d^{\text{max}}$ is only about 5% lower than the value for $N_d$ we obtained from the adiabatic sweep as shown in Fig. 1.
of the ensemble of 1D Ising chains with that of an ensemble of nearly isolated effective double-well systems to check the role of the constraint. The double wells are engineered by highly non-adiabatic loading of the 3D lattice, thereby creating a comparatively dilute sample of doublons. We clean out the singly occupied lattice sites by temporarily hiding the doublons in a molecular state and removing singular atoms by resonant light. We subsequently initiate the quench dynamics as before and detect the number of singly occupied sites \( [20] \) as a function of \( t_h \) as shown in Fig. 4(d). The oscillation frequencies in the two systems, determined to \( f_0 = 128(2) \text{ Hz} \) and \( f_0 = 93(2) \text{ Hz} \), respectively, differ by a factor 1.38(3), in good agreement with the collective effect discussed above.

In conclusion, we have investigated quench dynamics by tuning a tilt suddenly onto the transition from PM to AFM Ising order in a large ensemble of 1D atomic Mott-insulator chains. We observe significant shifts of the tunneling parameter arising from interactions, and collective effects in the oscillations arising from the effective constraint in the model. This study of quench dynamics opens the opportunity to explore many aspects of the dynamics in these systems that up to now were only addressed theoretically. This includes scaling relations for smaller quenches across the phase transition \([15, 29, 30]\), as well as possibly quenches at other resonance points (e.g., near \( E = U/2 \)).

We are indebted to R. Grimm for generous support, and thank Johannes Schachenmayer for discussions and contributions to numerical code development. We gratefully acknowledge funding by the European Research Council (ERC) under Project No. 278417, and support in Pittsburgh from NSF Grant PHY-1148957.

![FIG. 4: (color online) Dependence of the resonance frequency \( f_0 \) and of the on-resonance oscillation frequency \( f_0' \) on the on-resonance oscillation frequency \( f_0 \) as shown in Fig. 4(d). The oscillation frequencies in the two systems, determined to \( f_0 = 128(2) \text{ Hz} \) and \( f_0 = 93(2) \text{ Hz} \), respectively, differ by a factor 1.38(3), in good agreement with the collective effect discussed above.](image-url)
extrema for $N_d^{\text{max}}$, $\gamma$, and $f_{\delta}$ is about 10 Hz and hence considerably larger than the statistical error from the fits.

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