STAR CLUSTERS*

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Abstract
This review concentrates almost entirely on globular star clusters. It emphasises the increasing realisation that few of the traditional problems of star cluster astronomy can be studied in isolation: the influence of the Galaxy affects dynamical evolution deep in the core, and the spectrum of stellar masses; in turn the evolution of the core determines the highest stellar densities, and the rate of encounters. In this way external tidal effects indirectly influence the formation and evolution of blue stragglers, binary pulsars, X-ray sources, etc. More controversially, the stellar density appears to influence the relative distribution of normal stars. In the opposite sense, the evolution of individual stars governs much of the early dynamics of a globular cluster, and the existence of large numbers of primordial binary stars has changed important details of our picture of the dynamical evolution. New computational tools which will become available in the next few years will help dynamical theorists to address these questions.

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1. Introduction

The study of star clusters is pursued as vigorously today as it has ever been. Their importance for astrophysics generally stems from their role as relics of the formation of the Galaxy (including the problem of their ages). In addition they are an important test bed for the theories of stellar evolution and stellar dynamics. (No-one would claim that understanding the dynamics of star clusters is one of the most pressing problems of astronomy, but the processes which occur in the star clusters, which are relatively easily observed, presumably also occur in galactic nuclei.)

Another reason for the continuing high level of cluster studies is the development of observational technique, which has extended studies from the optical into the UV, IR, X-ray and radio wave bands. Especially in the latter two, unexpected new discoveries have opened up fresh theoretical problems, while at the same time shedding light on areas that were already under investigation for other reasons.

This review looks at globular star clusters from the dynamical point of view. This bias is less serious than might be thought, because it is gradually being realised that the main problems of star clusters cannot be understood in isolation: for example, the dynamics of the clusters depends, in part, on the rate at which stars lose mass by internal evolution; while the dynamics controls the rate at which stars interact, and hence (in all probability) the formation of the cluster X-ray sources, millisecond pulsars, blue stragglers, etc. The idea that such aspects of cluster evolution are all interrelated is not entirely new, but for many years most research proceeded as though they could be understood independently of each other. Now, each specialty in cluster studies is finding that further progress depends on viewing each star cluster as a kind of ecosystem of interrelated species. Therefore, when we look a little deeply at the clusters from the dynamical point of view, we see the same objects as those studied by the X-ray astronomer or the observer interested in colour-magnitude diagrams, and many of the same problems.

2. External influences

For many years dynamicists thought that the study of globular clusters was fundamentally the same as that of isolated stellar systems. It was known that clusters were tidally limited, in a way that was successfully described by King models (King 1966), but the influence of the tide on the behaviour deep in the core was ignored until the work of Spitzer & Chevalier (1973), who showed that the rate of core collapse was greatly affected by tidal effects. This prediction has since been amply confirmed by surveys of the surface density profiles of observed clusters, which show that clusters with collapsed cores are prevalent at small galactocentric distances (Chernoff & Djorgovski 1989), where the tidal effects are strongest.

(Incidentally, the identification of post-collapse clusters, starting with M15, is one of the significant triumphs of the last decade or two. It brings to a conclusion a story started with the classical theoretical study of Hénon (1961). Even though the core of M15 now turns out to be resolvable (Lauer et al 1991; but cf. Yanny et al 1993), it is still best understood as a post-collapse cluster (Grabhorn et al 1992).)

While the Galactic tide controls the extent and influences the core dynamics of the existing clusters, it is also one of the main destructive processes. At present clusters are
destroyed by this and other processes at a rate of about 5 per Gyr (Hut & Djorgovski 1992). Consideration of the joint action of tidal processes and internal evolution helps to explain the observed relatively narrow distribution of cluster masses at the present (Fall & Rees 1977). On the other hand studies of the median cluster mass in other (extragalactic) cluster systems suggests that this is almost independent of the strength of tidal effects (van den Bergh 1993a), and so the role of tides may not be decisive.

Where the tide continues to influence the properties of existing clusters is in the spectrum of stellar masses. Internal processes drive the less massive stars to larger radii within the cluster, where they are relatively more easily stripped by the tide (cf. Chernoff & Weinberg 1990). Now that star counts in a number of clusters extend to sufficiently faint magnitudes, it can be seen that the present-day mass function is flattest (most depleted in low masses) in clusters which are most subject to tidal disruption (Capaccioli et al 1993).

3. Stellar Influences on Cluster Dynamics

3.1 Mass Loss

We have seen that external influences have a large effect on the size, stellar population and internal dynamics of star clusters. Now we turn to the ways in which the stellar population itself affects the dynamics. One of the most important is the mass lost in the evolution of the more massive stars. Obvious though this effect is, it was almost entirely ignored in dynamical studies for many years, with the notable exception of the work of Angeletti & Giannone (1977). Recent work on this mechanism stems from a paper by Applegate (1986). Mass loss through stellar evolution leads to an increase in the radius of a cluster. Indeed the mean core radii of the older clusters in the LMC exceed those of the younger objects, but the magnitude of the effect is not yet in satisfactory quantitative agreement with dynamical models (Elson 1992).

Mass loss through stellar evolution is a serious disruptive process especially when the initial mass function is relatively flat (Chernoff & Weinberg 1990). To some extent this may be an artefact of the dynamical method they used (Fokker-Planck models), as N-body models indicate that clusters may survive a period of severe mass loss after a period of dynamical readjustment (Fig.1), even though the Fokker-Planck model suggests that clusters with identical parameters should disrupt. The fact that a cluster may well survive even with a relatively shallow mass spectrum helps to understand how the clusters may still possess enough neutron stars (born from the evolution of very high-mass stars) to explain the observed numbers of X-ray sources and pulsars.

One should add that the initial parameters of the models shown in Fig.1 and discussed in Chernoff & Weinberg (1990) are quite restrictive: in addition to the mass spectrum, the survival of a cluster depends on its initial size relative to the tidal radius, a parameter that has been relatively little explored (but see Phinney 1992). This initial condition probably also leaves an imprint on the anisotropy of the stellar velocity distribution. Anisotropy is manifest in proper motion studies of clusters such as M13 (Cudworth et al 1985), whereas N-body models (Giersz & Heggie 1993) show little anisotropy if the initial radius of the system is too large.
3.2 Binaries

The second main influence of the stellar content of a cluster on its dynamics is the presence of binary stars. As with the role of stellar mass loss, their effects were almost completely disregarded until recently, with a few exceptions (Hills 1975, Spitzer & Mathieu 1980). What has transformed the picture is the observational discovery of binaries of various types, using an increasing variety of techniques (see Hut et al 1992 for a review and many further references). Their effects on the dynamics are two-fold. First, they limit the depth of core collapse, since encounters between binaries generate kinetic energy more efficiently than other processes (Fig. 2). Second, their interactions can power much of the post-collapse evolution of a cluster. As they interact they are destroyed, like a non-renewable source of energy, but in tidally limited models the reserves may be sufficient to sustain a cluster for its entire lifetime.

The discovery of primordial binaries has given us a picture of dynamical evolution which is in some ways simpler than those previously available. One of the central questions was the dominant mechanism for generating energy. The most promising answer was the evolution of binaries in tidal two-body encounters (Statler et al 1985, Stodólkiewicz 1985), but this was complicated by their small semi-major axis: in any three-body interaction at least two of the stars are bound to collide (Hut & Inagaki 1985, Cleary & Monaghan 1990). Dynamically the most effective primordial binaries are those with much larger semi-major axes, for which this problem is only now being explored (McMillan 1993). As we shall see, an understanding of stellar collisions is still of great importance, but for different reasons.

4. Dynamical influences on the Stellar Content

In discussing the role of tidal processes on the stellar mass function we have already mentioned one way in which dynamics influences the kinds of stars found in globular clusters. Another, which has been known for a long time and has been the impetus for some observational studies (e.g. Richer & Fahlman 1989), is mass segregation.

Mass segregation might be expected to give rise to a colour gradient, but the question of colour gradients in globular star clusters has an unhappy history, because the centre of a cluster may be assigned to what is nothing more than a chance concentration of giants, and will therefore appear red. In recent years, however, it has been found (Djorgovski et al 1991; see also Cederbloom et al 1992) that most clusters exhibit flat colour profiles. Most interestingly, the exceptions are mainly those with post-core collapse profiles, which tend to be bluer at the centre. This is one of the clearest and most puzzling observational indications that dynamics influences the stellar content of globular star clusters. Interestingly, it was foreshadowed by early IUE observations (Dupree et al 1979) which indicated that the core radius was smaller at shorter wavelengths (cf. also Djorgovski & Piotto 1992).

What feature of the stellar distribution is responsible for the colour gradient? Recent work (Djorgovski et al 1991) has confirmed for several clusters an older finding for M15 (Auriere & Cordoni 1981) that red giants are relatively depleted in the central regions of post-collapse clusters. By contrast the distribution of stars on and near the horizontal branch is quite strongly enhanced in dense cores.

These issues have important repercussions in the study of colour-magnitude diagrams. The tip of the red giant branch is truncated in denser stellar environments (Djorgovski &
Piotto 1993). Even more dramatic are the effects on the horizontal branch (e.g. Bailyn et al 1992), and this has rather wide implications in a variety of cluster studies. It is well known that the primary parameter which determines the morphology of the horizontal branch is the metal abundance (cf. Lee 1991 for a recent discussion), but it has been known for many years that at intermediate abundances the morphology may be very different in two clusters which have the same abundance; in other words some “second parameter” influences its morphology. To many experts the best candidate is the age of the cluster, but it now appears that the stellar density is another plausible candidate (Buonanno 1993). At any rate, it is no longer very meaningful to refer to the colour-magnitude diagram of a globular cluster: for some clusters, important details vary from one part of the cluster to another.

Mass segregation appears to work in the wrong direction to account for these observations, and it seems likely that an explanation will involve an understanding of physical stellar encounters. Fortunately, this difficult problem has been under study now for some time (e.g. Davies et al 1991, Benz & Hills 1992, Rasio 1993), motivated partly by attempts to understand the population of blue stragglers. Now that high-resolution studies even of relatively dense cluster cores are becoming possible (Yanny et al 1993, Ferraro 1993) these are being found in increasing numbers in several clusters. They may well have more than one origin (e.g. Bailyn 1992), but mergers resulting from stellar collisions is likely to dominate in some clusters.

Finally we turn to the high-energy astrophysicist’s view of globular clusters (cf. Bailyn 1991 for a review). The variable X-ray sources discovered in the 1970s are binary stars in which one component is a neutron star, and the more recent discovery of millisecond pulsars, some of which are in binaries, has further opened a new window into dynamical processes occurring in star clusters. Two considerable uncertainties in explaining the origin of these objects are, first, the retention of neutron stars in the shallow potential well of a typical globular cluster, and, second, the probability that the birth of a neutron star within a binary may well disrupt the binary. Hills (1977) suggested that the solution to the latter problem lay with exchange reactions, in which a single neutron star is likely to displace one component of a binary in an encounter, because of its relatively high mass. A recent encounter between a binary pulsar and a single star in M15 is the most plausible explanation of both its eccentricity and its large distance from the core of the cluster (Phinney & Sigurdsson 1991).

5. Some Highlights of Cluster Research

The foregoing sections of this review span the links between the environment, the internal dynamics and the stellar content of globular star clusters. They also show how observations at various wavelengths are used to help provide an integrated picture of these aspects; for example, radio observations of millisecond pulsars, optical photometry for colour-magnitude diagrams, ultra-violet surface brightness profiles, X-ray studies, and optical observations from space. On the other hand the previous sections fail to touch on some of the central problems which have engaged much attention in recent years. The aim of the present section is to redress the balance a little by underscoring some important recent advances and problems.
Foremost among these must be the realisation that even the Galactic system of globular clusters is relatively differentiated by kinematics, Galactic distribution and composition (see Zinn 1990, 1991 for reviews). At the very least the system can be divided into two components, and there is even an extreme view that there is no real distinction between the youngest globular clusters and the oldest open clusters.

The implications of these findings for theories of cluster formation and galactic evolution are less clear, partly because the relative dating of the two main classes of Galactic globular clusters has not yet been determined. It is even possible that the diversity in several cluster properties reflects successive mergers rather than changing circumstances of formation within the Galaxy (e.g. van den Bergh 1993b).

Motivated largely by the cosmological problem of the age of the universe, substantial effort has been applied to the determination of cluster ages. While it is now clear that there is a considerable spread in ages, the main thing that has been learned about absolute ages is the existence of several factors which make this question difficult to answer.

Much research has been devoted to the investigation of elemental abundances within individual clusters (see Norris 1988 for a review). While some variations (within an individual cluster) are associated with stellar evolution, in all clusters except two (ωCen and M22) the variations in primordial abundances are small, and in some cases the variations in the abundance of iron are less than 0.15dex.

Of importance for the modelling and dynamics of star clusters (see, for example, Meylan & Pryor 1993) is the accurate and increasingly extensive measurement of individual stellar radial velocities. Complementary information has come from the ever improving study of proper motions, with the result that for several clusters we now have three-dimensional information on both space and internal motions (e.g. Cudworth 1993).

Studies of globular clusters have gradually extended to include extragalactic cluster systems. Confined at first to the study of integrated properties, technical advances now make it possible to study colour-magnitude diagrams and structural properties of clusters in nearby galaxies (e.g. Christian & Heasley 1991, Cacciari et al 1993 and references therein). Finally, this brief review does no justice at all to the nearest cluster system – the open clusters of the Galaxy, despite their importance for such central problems as the distance scale, Galactic structure, and star formation.

6. The Next Few Years

In this concluding section we revert largely to the theoretical aspects which were discussed in §§2-4. We shall not even discuss the scientific problems specifically, but consider instead some of the tools which are used by theorists, and how they may be expected to develop in the foreseeable future.

The tool which many theorists would like to apply to these problems is \(N\)-body simulation, but it is exceptionally time-consuming. Great advances have been made in the last 30 years, partly because of improvements in general-purpose hardware, and partly because of the development of a small number of highly tuned and efficient codes which have been specially devised with these problems in mind (Aarseth 1985). These tools are sufficiently well developed to provide numerical simulations of systems with as many as 10000 particles (Aarseth & Spurzem 1993; this paper, Fig.3), though not yet routinely.
This is more than enough, however, for very detailed and realistic models of open star clusters, and yet little has been done on this problem with these methods since the work of Terlevich (1987).

At the present time the capabilities of $N$-body methods are undergoing rapid transformation, as special purpose hardware is currently under development (Makino et al 1992). Within a year or two it should be possible for the first time to model a cluster containing, say, 50000 stars. This is still small relative to a typical globular cluster, but it goes a long way to closing the number-gap which at present must be bridged with more or less simplified theories. What makes these developments still more relevant to the kinds of problems discussed in §§2-4 is that they should be able to include fairly realistic treatments of close stellar interactions, by means of smooth particle hydrodynamics, for example (cf. Rasio 1993). In this way one might hope for a better theoretical understanding of such questions as the expected numbers of blue stragglers, etc.

Continuing developments should eventually enable us to provide realistic models of individual star clusters. Indeed the most serious foreseeable difficulty is in correctly modelling the astrophysical aspects of such $N$-body models, e.g. the internal stellar evolution of binary stars. But even when such models become possible, some time must elapse before they become routine. Until this happens it is likely that much can be learned by rather more simplified techniques. One which deserves to be revived is the Monte Carlo method originally developed by Hénon (1973) and brought to an amazing level of realism by Stodólkiewicz (1985). It was somewhat eclipsed by the finite-difference method of Cohn (1979), but the latter becomes relatively rather expensive when it is made more realistic, unlike the Monte Carlo method. Until direct $N$-body simulation routinely achieves the required capability, the Monte Carlo method may well be the most effective tool for bridging the gap between the largest $N$-body simulations and the globular clusters.

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Figure Captions

Fig.1 Evolution of core radius in a large ensemble of 500-body systems (after Giersz & Heggie 1993). The initial conditions were King models with the stated scaled central potential $W_0$, and with a mass spectrum $f(m) \propto m^{-x}$. Tidal parameters and total mass (see title) have been chosen appropriate to the LMC. According to Fokker-Planck models of Weinberg (cf. Elson 1992) a cluster with $x = 0.5$ and $W_0 = 7$ should disrupt after about $3 \times 10^7$ yr.

Fig.2 Evolution of core radius in $N$-body models with and without primordial binaries. The initial conditions are described fully in Heggie & Aarseth (1992), where they are referred to as Models S and I. With primordial binaries the depth of core collapse is much shallower, and the effect would be greater in larger systems than in these 2500-body systems.

Fig.3 Advances in collisional $N$-body simulations. Only large models extending a long way into core collapse, or beyond, are included. For the most part the date is that of publication, except for the last points, where dates are provisional. Sources can be traced from the name and date, except for HARP (Makino et al 1992) for which data are conservative estimates only.
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