The age problem in $\Lambda$CDM model

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ABSTRACT
The age problem in the $\Lambda$CDM model is re-examined. We define the elapsed time $T$ of an object is its age plus the age of the Universe when it was born. Therefore in any cosmology, $T$ must be smaller than the age of the Universe. For the old quasar APM 08279+5255 at $z = 3.91$, previous studies have determined the best-fit value of $T$, 1 $\sigma$ lower limit and the lowest limit to $T$ are 2.3, 2.0 and 1.7 Gyr, respectively. Constrained from SN1a+$R + A + d$, SN1a+$R + A + d + H(z)$, and WMAP5+2dF+SNLS+HST+BBN, the $\Lambda$CDM model can only accommodate $T(z = 3.91) = 1.7$ Gyr at 1 $\sigma$ deviation. Constrained from WMAP5 results only, the $\Lambda$CDM model can only accommodate $T(z = 3.91) = 1.7$ Gyr at 2 $\sigma$ deviation. In all these cases, we found that $\Lambda$CDM model accommodates the total age (14 Gyr for $z = 0$) of the Universe estimated from old globular clusters, but cannot accommodate statistically the 1 $\sigma$ lower limit to the best-fit age of APM 08279+5255 at $z = 3.91$. These results imply that the $\Lambda$CDM model may suffer from an age problem.

Key words: cosmological parameters-dark energy-cosmology: observations-cosmology: theory.

1 INTRODUCTION
Over the past decade, there are two most important reasons, one is the “age problem”, the other is the “dark energy problem”, to rule out with great confidence a large class of cold dark matter (CDM) cosmological models. A matter-dominated spatially flat Friedmann-Robertson-Walker (FRW) Universe (with age $T = 2/3H_0$), for example, is ruled out unless $h < 0.48$, compared with the 14 Gyr age of the Universe inferred from old globular clusters (Pont et al 1998). This is the “age problem”. If one considers the age of the Universe at high redshift, for instance, the 3.5 Gyr-old radio galaxy 53W091 at $z = 1.55$ and 4 Gyr-old radio galaxy 53W069 (Dunlop et al 1996; Spinrad et al 1997), this problem becomes even more acute.

The “dark energy problem” results from an increasing number of independent cosmological observations, such as measurements of intermediate and high redshift supernova Ia (SNIa), measurements of the Cosmic Microwave Background (CMB) anisotropy, and the current observations of the Large-Scale Structure (LSS) in the Universe. These cosmological observations have consistently indicated that the around 70% of the present Universe energy content, with a positive energy density but a negative pressure (called dark energy), is homogeneously distributed in the Universe and is causing the accelerated expansion of the Universe. The simplest and most theoretically appealing candidate of dark energy is the vacuum energy (or the cosmological constant $\Lambda$) with a constant equation of state (EoS) parameter $w = -1$. This scenario is in general agreement with the current astronomical observations, but has difficulties to reconcile the small observational value of dark energy density with estimates from quantum field theories (Peebles and Ratra 2003; Carroll 2001; Padmanabhan 2003; Sahni and Starobinsky 2000; Ishak 2007). The existence of such a “dark energy” not only explains the accelerated expansion of the Universe and the inflationary flatness prediction $\Omega_{\text{total}} \simeq 1$, but also reconciles the “age problem”. However, the discovery of an old quasar, the APM 08279+5255 at $z = 3.91$, has once again led to an “age problem”. The age of this quasar was initially estimated to be around 2-3 Gyr (Kossen and Hasinger 2003; Friaca et al. 2005) re-evaluated the age of APM 08279+5255 to be 2.1 Gyr by using an improved method (we will discuss the possible range of its age in more details later). For the currently accepted values of the matter density parameter $\Omega_m = 0.27 \pm 0.04$ (Spergel et al. 2003) and of the Hubble parameter $H_0 = 72 \pm 8$ km s$^{-1}$Mpc$^{-1}$ (Freedman et al. 2001), most of the existing dark energy scenarios cannot accommodate its age at such a high redshift if imposing a prior on $H_0$, such as $\Lambda$CDM model (Friaca et al. 2005; Alcaniz et al. 2003), parameterized vari-
able dark energy models (Dantas et al. 2007, Barboza and Alcaniz 2008), quintessence (Capozziello et al. 2007; Jesus et al. 2008), the $f(R) = \sqrt{R^2 - R_0^2}$ model (Movahed et al. 2007), braneworld models (Movahed and Ghassemi 2007; Pires et al. 2006; Movahed and Sheykhi 2008; Alam and Sahni 2006), holographic dark energy model (Wei and Zhang 2007), and other models (Sethi et al. 2005; Abreu et al. 2009; Santos et al. 2008).

But if one takes values of $\Omega_m$ and $H_0$ with 1σ deviation below $\Omega_m = 0.27 \pm 0.04$ (Spergel et al. 2003) and $H_0 = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ (Freedman et al. 2001), the age problem in $\Lambda$CDM model (Friaca et al. 2005) or in holographic dark energy model can be alleviated (Wei and Zhang 2007). In other words, the “age problem”, to a certain extent, is dependent on the values of the matter density parameter and the Hubble constant one takes. Because the estimations of Hubble constant may have somewhat large systematic errors at present, there are still debates on the value of $H_0$ in literatures. To consider the “age problem” in a consistent way, unlike done previously by taking a set of cosmological parameters a priori (Friaca et al. 2005; Wei and Zhang 2007), we do not take any special value of $\Omega_m$ or $H_0$ with prejudice. We instead obtain directly observational constraints on $H_0$ and $\Omega_m$ from SNIa, CMB, baryon acoustic oscillation (BAO), and $H(z)$ data points in the framework of the $\Lambda$CDM model, then investigate the “age problem” in the parameter space allowed by these observations. We will also discuss the “age problem” in $\Lambda$CDM model with the parameters from the five-year WMAP (WMAP5) data and other observations.

The structure of this paper is as follows. In section II, we consider constraints on the parameters of the $\Lambda$CDM model from SNIa, CMB, BAO, and $H(z)$ observations, and present the parameters from the five-year WMAP data. Using these best-fit values, the “age problem” is discussed, and the possible range of the age of the quasar APM 08279+5255 is also addressed in section III. Conclusions and discussions are given in section IV.

2 OBSERVATIONAL CONSTRAINTS ON $\Lambda$CDM

In this section, we will consider observational constraints on $\Lambda$CDM with SNIa, parameters measured from CMB and BAO, and $H(z)$ data. We will also list the parameters constrained from WMAP5 data and other observations in the framework of the $\Lambda$CDM model.

2.1 Observational constrains on $\Lambda$CDM from SNIa, $R$, $A$, $d$, and $H(z)$

To consider observational bounds on $\Lambda$CDM model for a flat Universe, we use the recently published 182 gold SNIa data with 23 SNIa at $z \geq 1$ obtained by imposing constraints $A_c < 0.5$ (excluding high extinction) (Riess et al. 2007). Each data point at redshift $z_i$ includes the Hubble-parameter free distance modulus $\mu_{\text{obs}}(z_i) = m_{\text{obs}} - M$ (where $M$ is the absolute magnitude) and the corresponding error $\sigma^{\prime}(z_i)$. The resulting theoretical distance modulus $\mu_{\text{th}}(z)$ is defined as

$$\mu_{\text{th}}(z) = 5 \log_{10}d_L(z) + 25, \quad (1)$$

where the luminosity distance in units of Mpc is expressed as

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')}, \quad (2)$$

where $H = H_0 E$ with $E = [\Omega_m(1 + z)^3 + (1 - \Omega_m)]^{1/2}$, here $\Omega_m$ includes baryon and cold dark matter. We treat $H_0$ as a parameter and do not marginalize it over.

In order to break the degeneracies among the parameters, we consider three $H_0$-independent parameters. One is the shift parameter $R$ measured from CMB observation, defined as (Bond et al. 1997; Melchiorri and Griffiths 2001)

$$R \equiv \Omega_m^{1/2} \int_0^{z_i} \frac{dz}{E(z)}, \quad (3)$$

where $z_i = 1089$ is the redshift of recombination. The shift parameter $R$ was found to be $R = 1.70 \pm 0.03$ (Wang 2006) from WMAP three-year data recently. The other two $H_0$-independent parameters are $A$ parameter and the distance ratio $d$, which are closely related to the measurements of the BAO peak in the distribution of Sloan Digital Sky Survey (SDSS) luminous red galaxies (LRG), and are defined as respectively

$$A \equiv \Omega_m^{1/2} (z_i E(z_i))^{-1/3} \left( \int_0^{z_i} \frac{dz}{E(z)} \right)^{2/3}, \quad (4)$$

$$d \equiv \left( \frac{z_i}{E(z_i)} \right)^{1/3} \left[ \frac{\int_0^{z_i} \frac{dz}{E(z)}}{f_0^{\sigma}} \right]^{2/3}, \quad (5)$$

where $z_i = 0.35$ is the effective redshift of the LRG sample. Measured from the SDSS BAO, the $A$ parameter and the distance ratio $d$ were found to be $A = 0.469 \pm 0.017$ and $d = 0.0979 \pm 0.0036$ (Eisenstein et al. 2005).

We also consider 9 $H(z)$ data points in the range $0 < z < 1.8$ as shown in Table II (Jimenez et al. 2003; Simon et al. 2005; Abraham et al. 2004; Treu et al. 1999; Dunlop et al. 1996; Spinrad et al. 1997; Nolan et al. 2003; Samushia and Ratra 2006). These 9 $H(z)$ data points have been used to test dark energy models recently.

These three $H_0$-independent parameters are extensively used to constrain dark energy models (e.g., Liddle et al. 2006; Nesseris and Perivolaropoulos 2004; Yang et al. 2008). Some points regarding the use of these parameters have been raised and discussed. This is an issue that deserves additional clarification. However, many authors have shown that these parameters are effective to break the degeneracies among the parameters (Liddle et al. 2006; Nesseris and Perivolaropoulos 2004; Yang et al. 2008).

Since the SNIa, CMB, BAO, and 9 $H(z)$ data points are effectively independent measurements, we can simply minimize their total $\chi^2$ value given by

$$\chi^2(\Omega_m, H_0) = \chi^2_R + \chi^2_A + \chi^2 + \chi^2_{\text{SNIa}} + \chi^2_H, \quad (6)$$

to find the best-fit values of the parameters of the $\Lambda$CDM
we quote the results obtained by Dunkley et al. (2008). They model, with

\[ H(z) = \frac{d - 302.2}{1.2}, \]

\[ \chi^2_A = \frac{R - 1.70}{0.03}, \]

\[ \chi^2_H = \frac{A - 0.0979}{0.0036}, \]

\[ \chi^2_{\Lambda} = \frac{\sum_i (H_{\text{obs}}(z_i) - H_{\text{th}}(z_i))^2}{\sigma^2_{H_i}}, \]

and

\[ \chi^2_{\text{SNIa}} = \frac{\sum_i (\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i))^2}{\sigma^2_{\mu}}, \]

Fitting SNIa, CMB, and BAO, we find the best-fit values of the parameters at 68.3% confidence: \( \Omega_m = 0.288 \pm 0.008 \) and \( H_0 = 63.7 \pm 3 \text{ km s}^{-1}\text{Mpc}^{-1} \) with \( \chi^2_{\text{min}} = 163.39 \) \( (\chi^2_{\text{min}}/\text{dof}=0.89, p(\chi^2 > \chi^2_{\text{min}}) = 0.87) \), as shown in Table 1.

If the 9 \( H(z) \) data points are also included in fitting, we find the best-fit values of the parameters at 68.3% confidence: \( \Omega_m = 0.302 \pm 0.009 \) and \( H_0 = 63.6 \pm 3 \text{ km s}^{-1}\text{Mpc}^{-1} \) with \( \chi^2_{\text{min}} = 181.57 \) \( (\chi^2_{\text{min}}/\text{dof}=0.95, p(\chi^2 > \chi^2_{\text{min}}) = 0.73) \), as shown in Table 2.

All these results are consistent with \( \Omega_m = 0.27 \pm 0.04 \) (Spergel et al. 2003) measured from WMAP in the \( \Lambda \)CDM model and \( H_0 = 62.3 \pm 1.3 \) (random )\pm5.0 (systematic) \text{ km s}^{-1}\text{Mpc}^{-1} from HST Cepheid-calibrated luminosity of Type 1a SNIa observations (Sandage et al. 2006).

### Table 1. The observational \( H(z) \) (km s\(^{-1}\)Mpc\(^{-1}\)) data with 1 \( \sigma \) uncertainty (Jimenez et al. 2003; Simon et al. 2005; Samushia and Ratra 2006)

| \( z \) | \( H(z) \) |
|---|---|
| 0.09 | 69 |
| 0.17 | 83 |
| 0.27 | 70 |
| 0.40 | 87 |
| 0.88 | 117 |
| 1.30 | 168 |
| 1.43 | 177 |
| 1.53 | 140 |
| 1.75 | 202 |

2.2 Observational constraints on \( \Lambda \)CDM from WMAP5 data and other observations

WMAP data are analyzed in the framework of the \( \Lambda \)CDM model, \( H_0 \) (or \( h \)) and \( \Omega_m h^2 \) can be constrained directly. Here we quote the results obtained by Dunkley et al. (2008). They constrained baryon and cold dark matter density parameters \( \Omega_b h^2 \) and \( \Omega_c h^2 \), dimensionless Hubble parameter \( h \), cosmological constant density parameter \( \Omega_{\Lambda} \), the scalar spectral index \( n_s \), the optical depth to reionization \( \tau \), and the linear theory amplitude of matter fluctuations on \( 8h^{-1}\text{Mpc} \) scales \( (\sigma_8) \), with five-year WMAP (WMAP5) data. They found the best-fit value of \( \Omega_b h^2 = \Omega_c h^2 = 0.02273 \pm 0.00062, h = 0.7193^{+0.027}_{-0.026} \).

Recently, the case of coupling neutrino mass was considered to constrain cosmological parameters. For example, Vacca et al. (2009) constrained \( \Omega_\nu h^2, \Omega_\nu h^2, \tau, n_s, \) the ratio of the sound horizon to the angular diameter distance at recombination \( \theta_0 \), the amplitude of the scalar fluctuations at a scale of \( k = 0.002 \text{ Mpc}^{-1} \) \( (A_s) \), the sum of neutrino mass \( (M_\nu) \), the energy scale in dark energy \( (\Lambda) \), and the coupling parameter between CDM and dark energy \( (\beta) \), with observations of WMAP5, 2dF galaxy redshift survey \( (2dF) \), Supernova Legacy Survey \( (SNLS) \), Hubble Space Telescope \( (HST) \), and Big Bang Nucleosynthesis \( (BBN) \). They found the best-fit values of \( \Omega_b h^2, \Omega_\nu h^2, H \) as \( 10^{11} \Omega_b h^2 = 2.258 \pm 0.061, \Omega_\nu h^2 = 0.1098 \pm 0.0040, H_0 = 70.1 \pm 2.1 \text{ km s}^{-1}\text{Mpc}^{-1} \). We will use these results to discuss the “age problem” in \( \Lambda \)CDM in the next section.

3 AGE PROBLEM IN \( \Lambda \)CDM

Old high-redshift objects are usually used to test dark energy model or constrain parameters (see e.g. Lima et al. 2009). Recently, the quasar APM 08279+5255 at \( z = 3.91 \) have been used to test many dark energy models, such as \( \Lambda \)CDM model (Friaca et al. 2005; Alcaniz et al. 2003), \( \Lambda(t) \) model (Cunha and Santos 2004), parametrized variable Dark Energy Models (Barboza and Alcaniz 2008; Dantas 2007), quintessence (Capozziello et al. 2007; Jesus et al. 2008), the \( f(R) = \sqrt{R^2 - R_0^2} \) model (Movahed et al. 2007), braneworld modes (Movahed and Ghassemi 2007; Pires et al. 2006; Movahed and Sheykhi 2008; Alam and Sahni 2006), holographic dark energy model (Wei and Zhang 2007, Granda et al. 2009), and other models (Sethi et al. 2005; Abreu et al. 2009; Santos et al. 2008). It was shown that the quasar APM 08279+5255 \( z = 3.91 \) cannot accommodated most dark energy models. In order to understand the problem better, we first discuss the possible range of the age of APM 08279+5255.

3.1 The age of APM 08279+5255

APM 08279+5255 is an exceptionally luminous broad absorption line \( (\text{BAL}) \) quasar at redshift \( z = 3.91 \). From \textit{XMM-Newton} observations of APM 08279+5255, Hasinger et al. (2002) have derived an iron overabundance of \( \text{Fe/O} \) of \( 3.3 \pm 0.9 \) (here the abundance ratio has been normalized to the solar value) for the BAL system. Using an \( \text{Fe/O}=3 \) abundance ratio, derived from X-ray observations, Komossa and Hasinger (2003) estimated the age of the quasar APM 08279+5255 to lie within the interval \( 2-3 \text{ Gyr} \). An age of 3 \text{ Gyr} is inferred from the temporal evolution of \( \text{Fe/O} \) ratio in the giant elliptical model \( \text{(M4a)} \) of Hamann and Ferland (1993, hereafter HF93). For the ‘extreme model’ M6a of HF93, the \( \text{Fe/O} \) evolution would be faster, and \( \text{Fe/O}=3 \) is already reached after 2 \text{ Gyr}.

Friaca et al. (2005) re-evaluated the age of APM 08279+5255 by using a chemodynamical model for the evolution of spheroids. An age of 2.1 \text{ Gyr} is set by the condition that \( \text{Fe/O} \) abundance ratio of the model
reaches 3.3, which is the best-fitting value obtained in Hasinger et al. (2002). An age of 1.8 Gyr is set when the Fe/O abundance ratio reaches 2.4, 1 σ deviation from the best-fitting value 3.3.

An age of 1.5 Gyr is set when the Fe/O abundance ratio reaches 2. But there is a correlation between Fe/O values and the neutral hydrogen column density $N_H$, in the sense that lower Fe/O values are obtained for higher values of $N_H$. Because of this, a value of Fe/O as low as two is highly improbable, as it would require $N_H$ in excess of $1.2 \times 10^{22}$ cm$^{-2}$ (see from Fig. 3 of Hasinger et al. 2002), which seems to be ruled out from the determinations of $N_H$ by other Chandra and XMM observations. Even considering only the XMM2 data set, the lowest value of Fe/O is 2.4 at 1.28 $\times 10^{22}$ cm$^{-2}$ for 1 σ deviation from the best-fit 3.3 (see Fig. 3 of Hasinger et al. 2002).

Based on the above discussions, we obtain the following estimates of the age of APM 08279+5255 since the initial star formation and stellar evolution in the galaxy: (1) the best estimated value is 2.1 Gyr; (2) 1 σ lower limit is 1.8 Gyr; (3) the lowest limit is 1.5 Gyr.

When using the best available WMAP5 polarization results (Dunkley et al. 2008), the polarization optical depth $\tau$ values imply a peak epoch of reionizing photons at $z = 10.8 \pm 1.4$. However considering that the much smaller and more preliminary WMAP first year data set implied $z = 17 \pm 10$ for the peak epoch of reionization, we take the peak reionization redshift as between $z = 8 - 14$. Suppose that the Population III stars are mostly responsible for the reionization, then we can further estimate that these star formation processes start as early as $z = 15 - 17$ in high density peaks. This also agrees with recent results based on the new Hubble WFC3/IR imaging in the Ultradeep Field, which suggest that HST has now in fact seen the tail-end of this reionizing population at $z = 8 - 10$ (Yan et al. astro-ph/0910.0077). We therefore conclude that the quasar APM 08279+5255 started its initial formation process at least 0.2-0.3 Gyr since the beginning of the Universe.

Therefore, we define the elapsed time $T$ of an object is its age plus the age of the universe when it was born. Since the spirit of testing cosmological models with ages of astrophysical objects is to examine if such objects can have sufficient time to be formed given the age of the Universe at that specific redshift, and we need to calculate the elapsed time $T$ of the objects since the beginning of the Universe, and compare this time ($T$ in all following figures) with the elapsed time (age) of the Universe at that specific redshift since the beginning of the Universe. Based upon the above discussions, we obtain the following estimates of the elapsed time $T$ of APM 08279+5255 since the beginning of the Universe: (1) the best estimated value is 2.3 Gyr; (2) 1 σ lower limit is 2.0 Gyr; (3) the lowest limit is 1.7 Gyr.

| Observations | $\Omega_m$ | $H_0$ | $\chi_{\text{min}}^2$/dof | $p$ |
|--------------|------------|-------|----------------|-----|
| SNIa+R + A + d | 0.288 ± 0.008 | 63.7 ± 3 | 0.89 | 0.85 |
| SNIa+R + A + d + $H(z)$ | 0.302 ± 0.009 | 63.6 ± 3 | 0.95 | 0.73 |

Table 2. The best values of the parameters ($\Omega_m$, $H_0$) of ΛCDM model with the corresponding $\chi^2_{\text{min}}$/dof and $p(\chi^2 > \chi^2_{\text{min}})$ fitting from SNIa+R + A + d and SNIa+R + A + d + $H(z)$ observations with 1 σ confidence level, here $H_0$ with dimension km s$^{-1}$Mpc$^{-1}$.

In this subsection, we use APM 08279+5255 to discuss the age problem in the ΛCDM model in the parameter space ($\Omega_m - H_0$ plane) allowed by SNIa, CMB, BAO, and H(z) observations, obtained in the previous section.

The age-redshift relation for a spatially flat, homogeneous, and isotropic universe with the vacuum energy reads

$$T(z) = \int_z^\infty \frac{dz'}{H_0(1+z')\sqrt{\Omega_m(1+z')^3 + (1-\Omega_m)}}. \quad (12)$$

With this equation, one can calculate the age of the Universe at any redshift in the framework of the ΛCDM model.

Taking $\Omega_m = 0.288$ and $H_0 = 63.7$ km s$^{-1}$Mpc$^{-1}$ obtained from fitting SNIa, CMB, and BAO observations, we find the present age of the Universe is $T = 15.0$ Gyr, larger than 14 Gyr estimated from old globular clusters (Pont et al. 1998), but ΛCDM just accommodates the lowest limit to the elapsed time ($T = 1.7$ Gyr) of APM 08279+5255 at the 1 σ deviation, as shown in figure 1. There is no way with these parameters ΛCDM can accommodate statistically even the 1 σ lower limit to the elapsed time: $T = 2.0$ Gyr.

Similarly, taking $\Omega_m = 0.302$ and $H_0 = 63.6$ km s$^{-1}$Mpc$^{-1}$ obtained from fitting SNIa, CMB, BAO, and H(z) observations, we find the present age of the Universe is $T = 14.8$ Gyr, again larger than 14 Gyr estimated from old globular cluster (Pont et al. 1998), but ΛCDM just ac-
commodates the lowest limit to the elapsed time \((T = 1.7 \text{ Gyr})\) of \(08279+5255\) at the 1 \(\sigma\) deviation, as shown in figure 2. There is also no way with these parameters \(\Lambda\)CDM can accommodate statistically even the 1 \(\sigma\) lower limit to the elapsed time: \(T = 2.0 \text{ Gyr}\).

As shown in figures 1 and 2, the only way to reconcile the elapsed time \(T\) of \(08279+5255\) with the age of the Universe at \(z = 3.91\) in the \(\Lambda\)CDM model, is to take smaller values of \(H_0\) and \(\Omega_m\), which certainly contradict many other independent observations. We therefore conclude that the \(\Lambda\)CDM model suffers possibly from an age problem. These discussions are summarized in Table 3.

### 3.3 Parameters constrained from WMAP5+2dF+SNLS+HST+BBN

Here we use \(08279+5255\) to discuss the age problem in the \(\Lambda\)CDM model with the parameters constrained from WMAP5 data and other observations. Taking the best-fit values of \(\Omega_m h^2 = 0.1326\) and \(h = 0.719\), obtained from fitting WMAP5 data (Dunkley et al. 2008), we find the present age of the Universe is \(T = 13.7 \text{ Gyr}\), which is less than 14 Gyr estimated from old globular clusters (Pont et al. 1998). However with the 1 \(\sigma\) deviations of \(\Omega_m h^2\) and \(h\), \(\Lambda\)CDM can accommodate the age of old globular clusters; for instance, taking \(\Omega_m h^2 = 0.12643\) and \(h = 0.692\), we obtain the present age of the Universe is \(T = 14.1 \text{ Gyr}\). This situation is depicted in upper right corner of Fig. 3. However, with such 1 \(\sigma\) deviations, \(\Lambda\)CDM still cannot accommodate even the lowest limit to the age of \(08279+5255\) at redshift \(z = 3.91\); only with 2 \(\sigma\) deviations, \(\Lambda\)CDM can accommodate the lowest limit: \(T = 1.7 \text{ Gyr}\). Even with 2 \(\sigma\) deviations of \(\Omega_m h^2\) and \(h\), \(\Lambda\)CDM cannot accommodate the 1 \(\sigma\) lower limit to the best-fit age: \(T = 2.0 \text{ Gyr}\); statistically there is no feasibility to be consistent with the best-fit value: \(T = 2.3 \text{ Gyr}\). Therefore the age of \(08279+5255\) conflicts with WMAP5 results severely within the \(\Lambda\)CDM model.

Similarly, taking the best-fit values of \(\Omega_m h^2 = 0.11208\) and \(H_0 = 70.1 \text{ km s}^{-1}\text{Mpc}^{-1}\), obtained from fitting WMAP5+2dF+SNLS+HST+BBN (Vacca et al. 2009), we find the present age of the Universe is \(T = 14.5 \text{ Gyr}\), larger than 14 Gyr estimated from old globular clusters (Pont et al. 1998). With 1 \(\sigma\) deviations of \(\Omega_m h^2\) and \(h\), \(\Lambda\)CDM can accommodate the lowest limit: \(T = 1.7 \text{ Gyr}\), as shown in the upper right corner of Fig. 4. For \(T = 2.0\) and 2.3 Gyr, the situation is qualitatively similar to that shown in Fig. 3, with however slightly less severe conflicts with the \(\Lambda\)CDM model. These discussions are summarized in Table 4.

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**Figure 2.** The 68.3%, 95.4% and 99.7% confidence regions in the \(\Omega_m-H_0\) (km s\(^{-1}\)Mpc\(^{-1}\)) plane fitting from SNIa, CMB, BAO, and \(H(z)\) observations, compared with the same lines as in Fig 1.

**Figure 3.** The upper right corner shows the 1 \(\sigma\) allowed parameter space with \(\Omega_m h^2 \geq 0.1264\) and \(h \geq 0.692\) from WMAP5, which can accommodate \(T = 14.0 \text{ Gyr}\) at \(z = 0\). However \(T \geq 1.7 \text{ Gyr}\) at \(z = 3.91\) for \(08279+5255\) cannot be accommodated statistically.

**Figure 4.** The upper right corner shows the 1 \(\sigma\) allowed parameter space with \(\Omega_m h^2 \geq 0.1081\) and \(H_0 \geq 68 \text{ (km s}^{-1}\text{Mpc}^{-1})\) from WMAP5+2dF+SNLS+HST+BBN, which can accommodate both \(T = 14.0 \text{ Gyr}\) at \(z = 0\) and \(T = 1.7 \text{ Gyr}\) at \(z = 3.91\) for \(08279+5255\). However \(T \geq 2.0 \text{ Gyr}\) at \(z = 3.91\) cannot be accommodated statistically.
of observational data sets. Here obtained the best-fit values of the parameter at 68.3% confidence are: Ωm = 0.288 ± 0.008, H0 = 63.7 ± 3 Mpc s⁻¹ km⁻¹ with χ² min = 163.39 (p(χ² > χ² min) = 0.87). In the Ωm−H0 parameter space allowed by these observations, the ΛCDM model accommodates the total age (14 Gyr for z = 0) of the Universe estimated from old globular clusters (Pont et al. 1998), but just accommodate the lowest limit to the elapsed time (T = 1.7 Gyr) of APM 08279+5255 at 1 σ deviation. There is no way, with these parameters, ΛCDM can accommodate statistically even the 1 σ lower limit to the elapsed time: T = 2.0 Gyr.

If H(z) observations are also included, the best-fit values of the parameter at 68.3% confidence are: Ωm = 0.302 ± 0.009 and H0 = 63.6 ± 3 km s⁻¹ Mpc⁻¹ with χ² min = 181.57 (p(χ² > χ² min) = 0.73). In this case the, the ΛCDM model is also consistent with the total age of the Universe estimated from old globular clusters (Pont et al. 1998), but just accommodate the lowest limit to the elapsed time (T = 1.7 Gyr) of APM 08279+5255 at 1 σ deviation. Constrained from WMAP5 only, the ΛCDM model can accommodate the total age of the Universe estimated from old globular clusters but cannot accommodates the lowest limit to the age (T = 1.7 Gyr) of APM 08279+5255 at redshift z = 3.91 at 1 σ deviation; only 2 σ deviations can accommodate even the absolute lower limit of T = 1.7 Gyr. Even 2 σ deviations of Ωm h² and h cannot accommodate the 1 σ lower limit of T = 2.0 Gyr.

Constrained from WMAP5+2dF+SNLS+HST+BBN, the ΛCDM model can accommodate the total age of the Universe estimated from old globular clusters (Pont et al. 1998), and can accommodate the lowest limit to the elapsed time (T = 1.7 Gyr) of APM 08279+5255 at redshift z = 3.91 at 1 σ deviation. The situation is qualitatively similar to the case constrained from WMAP5 only, with however slightly less severe conflicts with the ΛCDM model.

The only way to reconcile the elapsed time T of APM 08279+5255 with the age of the Universe at z = 3.91 in the ΛCDM model, is to take very small values of H0 and Ωm, which certainly contradict many other independent observations. We therefore conclude that the ΛCDM model suffers from a problem with the estimated age of APM 08279+5255 at redshift z = 3.91, based on the currently best available data for the Hubble constant H0 and the matter density Ωm (recently Riess et al. 2009) obtained H0 = 74.2 ± 3.6 km s⁻¹ Mpc⁻¹, which may lead to more serious age problem in ΛCDM model). These results can be tested with future cosmological observations. Of course, new and more reliable determination of the age of APM 08279+5255 are also needed. We mention in passing that when the Dunlop et al. (1996) paper first came out, the 3.5 Gyr age of 53W091 at z = 1.5 was a significant problem for a non-Λ cosmology; however soon thereafter papers came out that did spectral energy distribution (SED) fitting of its Keck spectrum that allowed new age estimates of around 1.8 Gyr (Yi et al. 2000, Bruzual and Magris 1999).

Table 3. Summary on the constraints to the ages of the Universe at z = 0 and z = 3.91 in the ΛCDM model, using different combinations of observational data sets. Here H0 takes the dimension of km s⁻¹ Mpc⁻¹.

| Data                     | Cosmological parameters | Age of old globular clusters 14 Gyr | Elapsed time of APM 08279+5255 (Gyr) | Best value | 1σ lower limit | the lowest limit |
|--------------------------|-------------------------|------------------------------------|-------------------------------------|------------|----------------|-----------------|
| SNIa+R + A + d           | Ωm = 0.288 ± 0.008      | 1 σ                                | yes                                 | no         | no             | yes             |
|                          |                         | 2 σ                                | yes                                 | no         | no             | yes             |
|                          |                         | 3 σ                                | yes                                 | no         | no             | yes             |
| SNIa+R + A + d + H(z)    | Ωm = 0.302 ± 0.009      | 1 σ                                | yes                                 | no         | no             | yes             |
|                          |                         | 2 σ                                | yes                                 | no         | no             | yes             |
|                          |                         | 3 σ                                | yes                                 | no         | no             | yes             |
| WMAP5                    | Ωm h² = 0.02273 ± 0.00062 | 1 σ                             | yes                                 | no         | no             | no              |
|                          | Ωm h² = 0.1099 ± 0.0062 | 2 σ                             | yes                                 | no         | no             | yes             |
|                          | h = 0.719 ± 0.027       | 3 σ                             | yes                                 | no         | no             | yes             |
| WMAP5+SNLS+ HST+BBN+2dF | Ωm h² = 0.258 ± 0.061   | 1 σ                             | yes                                 | no         | no             | yes             |
|                          | Ωm h² = 0.1098 ± 0.0040 | 2 σ                             | yes                                 | no         | no             | yes             |
|                          | Ωm = 70.1 ± 2.1         | 3 σ                             | yes                                 | no         | no             | yes             |

4 CONCLUSIONS AND DISCUSSIONS
As previous many works have shown, the age problem in dark energy models is dependent on the values of Hubble constant and matter density one takes, at least to a certain degree. Because the estimations of Hubble parameter may have somewhat large systematic errors currently, there are still debates on the value of H0 in literatures. In the paper, we re-examine the age problem in the ΛCDM model in a consistent way, i.e., without requiring a priori values of the parameters. We define the elapsed time T of an object is its age plus the age of the universe when it was born. Therefore in any cosmology, T must be smaller than the age of the Universe. For the old quasar APM 08279+5255 at z = 3.91, previous studies have determined the best-fit value, 1 σ lower limit and the lowest limit to T are 2.3, 2.0 and 1.7 Gyr, respectively. In Table 1 we summarize the constraints to the ages of the Universe at z = 0 and z = 3.91, using different combination of observational data sets.

Fitting SNIa, CMB, and BAO observations, we have obtained the best-fit values of the parameter at 68.3% confidence: Ωm = 0.288 ± 0.008 and H0 = 63.7 ± 3 km s⁻¹ Mpc⁻¹ with χ² min = 163.39 (p(χ² > χ² min) = 0.87). In the Ωm−H0 parameter space allowed by these observations, the ΛCDM model accommodates the total age (14 Gyr for z = 0) of the Universe estimated from old globular clusters (Pont et al. 1998), but just accommodate the lowest limit to the elapsed time (T = 1.7 Gyr) of APM 08279+5255 at 1 σ deviation. There is no way, with these parameters, ΛCDM can accommodate statistically even the 1 σ lower limit to the elapsed time: T = 2.0 Gyr.

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