Generalized complex marginal deformation of pp-waves and giant gravitons

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Abstract

We present the Penrose limits of a complex marginal deformation of $\text{AdS}_5 \times S^5$, which incorporates the $SL(2,\mathbb{R})$ symmetry of type IIB theory, along the $(J,0,0)$ geodesic and along the $(J,J,J)$ geodesic. We discuss giant gravitons on the deformed $(J,0,0)$ pp-wave background.

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1 Introduction

The marginal deformation [1] introduces phases in the superpotential which breaks the $SO(6)_{R}$ R-symmetry group to its $U(1) \times U(1) \times U(1)_{R}$ Cartan subgroup. In the gravity side [2], the $U(1) \times U(1)$ non-R-symmetry maps to a two-torus. The dual geometry is obtained by applying an $SL(2, \mathbb{R})$ transformation which acts on the Kähler modulus of the corresponding two-torus or equivalently a TsT (T-duality, shift, T-duality) transformation. The phases in the gauge theory can be complexified. In the dual geometry, it corresponds to a specific $SL(3, \mathbb{R})$ transformation which consists of the $SL(2, \mathbb{R})$ transformation and an S-duality transformation $SL(2, \mathbb{R})_s$ or equivalently an STsTS (S-duality, T-duality, shift, T-duality, S-duality) transformation [2, 3]. The three-parameter generalization is proposed as a dual geometry to a non-supersymmetric marginal deformation of $\mathcal{N} = 4$ super Yang-Mills theory [3].

The charges of chiral superfields under the $U(1) \times U(1)$ symmetry in the gauge theory corresponds to the angular momenta along the two-torus in the dual geometry. In terms of the angle coordinates $(\phi_1, \phi_2, \phi_3)$ of $S^5$, there are four possible BPS geodesics, $(J_{\phi_1}, J_{\phi_2}, J_{\phi_3}) \sim (J, 0, 0), (0, J, 0), (0, 0, J)$ and $(J, J, J)$. The Penrose limit along the first three geodesics and the Penrose limit along the fourth geodesic are two distinct pp-waves. The pp-waves are discussed in [4, 2, 5, 14].

A point graviton which has an angular momentum about the sphere of $AdS_m \times S^n$ blows up into a spherical brane [6]. A giant graviton is a spherical $(n - 2)$-brane which wraps a part of $S^n$. A dual giant graviton is a spherical $(m - 2)$-brane which wraps a spatial part of $AdS_m$. Both are BPS objects, which have the same quantum numbers as the Kaluza-Klein mode of the point graviton [7, 8]. Giant gravitons in the Penrose limit of $AdS_5 \times S^5$ are studied in [9].

Giant gravitons on the three-parameter non-supersymmetric background [3] are discussed in [10, 11]. It is shown in [11] that the (dual) giant gravitons do not depend on the deformation parameters $\gamma_i$, $(i = 1, 2, 3)$. (Dual) giant gravitons in the supersymmetric deformation are obtained by setting $\gamma_i = \gamma$. D3-brane (dual) giant gravitons and D5-brane dual giant gravitons on $\gamma$-deformed $AdS_5 \times S^5$ are discussed in [12]. Giant gravitons in the Penrose limits of marginally deformed $AdS_5 \times S^5$ along the $(J, 0, 0)$ geodesic and along the $(J, J, J)$ geodesic are considered in [13, 14]. It is shown in [13] that the giant graviton on the deformed $(J, 0, 0)$ pp-wave is independent of the deformation parameter $\gamma$ and energetically degenerate with the Kaluza-Klein point graviton whereas the giant graviton on the deformed $(J, J, J)$ pp-wave does not retain its round three-sphere shape. In [14], the Penrose limits of the complex marginal deformation of $AdS_5 \times S^5$ along the $(J, 0, 0)$ geodesic and along the $(J, J, J)$ geodesic are studied. Giant gravitons and dual giant gravitons are discussed on the deformed $(J, 0, 0)$ pp-wave. It is shown that the giant gravitons are not energetically degenerate with the point graviton and exist only up to a critical value of $\sigma$. They are energetically unfavorable but nevertheless perturbatively stable.

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1 $\gamma$ is used for the $SL(2, \mathbb{R})$ transformation and $\sigma$ is used for the $SL(2, \mathbb{R})_s$ transformation. Both are real parameters with unit period.
In this work, we study the Penrose limits of complex marginal deformation of $AdS_5 \times S^5$, which incorporates the $SL(2,\mathbb{R})$ symmetry of type IIB theory and observe giant gravitons on the deformed $(J,0,0)$ pp-wave background. In section 2 we review the generalized complex marginal deformation of $AdS_5 \times S^5$ [15, 16], and present the pp-wave geometries which are obtained by taking the Penrose limits along the $(J,0,0)$ geodesic and along the $(J,J,J)$ geodesic. In section 3 we study the giant graviton solution on the pp-wave background and check the stability by observing small fluctuations about the solution. In section 4 we summarize our results.

2 Generalized complex marginal deformation

The Lunin-Maldacena $SL(3,\mathbb{R})$ transformation, which generates the gravity dual of the complex marginal deformation [1, 2] is

$$\Lambda^T_{LM} = \begin{pmatrix} 1 & 0 & 0 \\ \gamma & 1 & \sigma \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.1)$$

The transformation can be generalized by an $SL(3,\mathbb{R})$ transformation

$$L = \begin{pmatrix} L_{11} & 0 & L_{13} \\ 0 & 1 & 0 \\ L_{31} & 0 & L_{33} \end{pmatrix}, \quad \det L = 1, \quad (2.2)$$

which corresponds to the $SL(2,\mathbb{R})$ symmetry of type IIB supergravity. The $SL(3,\mathbb{R})$ transformation $LA^T_{LM}$, therefore produces a generalized complex marginal deformation [2, 15]. We consider $AdS_5 \times S^5$ defined by

$$ds^2 = R^2 \left[ -dt^2 \cosh^2 \rho + \cos^2 \theta \sin^2 \theta \left( \sum_{i=1}^3 d\mu_i^2 + \sum_{i=1}^3 \mu_i^2 d\phi_i^2 \right) + \sinh^2 \rho d\Omega_3^2 + \sum_{i=1}^3 d\mu_i^2 + \sum_{i=1}^3 \mu_i^2 d\phi_i^2 \right],$$

$$\chi_0 = \tau_1, \quad e^{-\Phi_0} = \tau_2, \quad B_2 = 0, \quad C_2 = 0,$$
$$C_4 = 4R^4 e^{-\Phi_0} (\omega_4 + \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3),$$
$$F_5 = 4R^4 e^{\Phi_0} (\omega_{AdS_5} + \omega_{S^5}),$$
$$\omega_{AdS_5} = d\omega_4, \quad \omega_{S^5} = d\omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3,$$
$$d\omega_1 = -\cos \alpha \sin^3 \theta \cos \theta \sin \theta d\alpha \wedge d\theta,$$
$$\mu_1 = \cos \alpha, \quad \mu_2 = \sin \alpha \cos \theta, \quad \mu_3 = \sin \alpha \sin \theta. \quad (2.3)$$
where $R$ is the radius of $AdS_5$ and the radius of $S^5$. The complex marginal deformation of $AdS_5 \times S^5$ [10] is

$$ds^2 = R^2 H^{1/2} \left[ -dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_5^2 + \sum_{i=1}^3 (d\mu_i^2 + G\mu_i^2 d\phi_i^2) \right]$$

$$+ GP\mu_1^2 \mu_2^2 \mu_3^2 \left( \sum_{i=1}^3 d\phi_i \right)^2,$$

$$e^\Phi = \sqrt{GH} \tau_2^{-1},$$

$$\chi = H^{-1} (h + \tau_2^2 \hat{\gamma} \hat{g}),$$

$$B_2 = R^2 GQ_2 \omega_2 - 4R^2 \tau_2 \hat{\gamma} \omega_1 \wedge \sum_{i=1}^3 d\phi_i,$$

$$C_2 = R^2 GT \omega_2 - 4R^2 \tau_2 \hat{\gamma} \omega_1 \wedge \sum_{i=1}^3 d\phi_i,$$

$$C_4 = 4R^4 \tau_2 \omega_4 + 4R^4 \tau_2 G \left[ 1 - \hat{T} g_0 \right] \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3,$$

$$F_5 = 4R^4 \tau_2 (\omega_{AdS_5} + G\omega_{S^5}),$$

(2.4)

where

$$P = \hat{\gamma}^2 f - 2\hat{\gamma} \hat{\sigma} h + \hat{\sigma}^2 g,$$

$$Q = \hat{\gamma} f - \hat{\sigma} h,$$

$$T = \hat{\gamma} h - \hat{\sigma} g,$$

(2.5)

$$G^{-1} = 1 + P g_0,$$

$$H = f + \tau_2^2 \hat{\sigma}^2 g_0,$$

$$g_0 = \mu_1^2 \mu_2^2 + \mu_2^2 \mu_3^2 + \mu_3^2 \mu_1^2,$$

$$\omega_2 = \mu_1^2 \mu_2^2 d\phi_1 \wedge d\phi_2 + \mu_2^2 \mu_3^2 d\phi_2 \wedge d\phi_3 + \mu_3^2 \mu_1^2 d\phi_3 \wedge d\phi_1,$$

(2.6)

and

$$f = (L_{33} + L_{13} \tau_1)^2 + L_{13}^2 \tau_2^2,$$

$$g = (L_{31} + L_{11} \tau_1)^2 + L_{11}^2 \tau_2^2,$$

$$h = (L_{33} + L_{13} \tau_1) (L_{31} + L_{11} \tau_1) + L_{11} L_{13} \tau_2^2.$$

(2.7)

The $SL(2, \mathbb{R})$ transformation (2.2) can be identified with torus parameters from an eleven dimensional viewpoint. The parametrization considered in [15] [16] is

$$L_{11} = 1, \quad L_{13} = \frac{r_2}{R_1} \cos \xi, \quad L_{31} = 0, \quad L_{33} = 1,$$

(2.8)
with a constraint

\[ r_3 = \frac{R_3}{\sin \xi}, \]  

(2.9)

\( R_i, (i = 1, 3) \) are the torus radii before the torus deformation and \( r_3 \) is the torus radius of the third direction after the deformation. \( \xi \) is the intersection angle between the direction along the first direction and the direction along the third direction. The geometry can be simplified by identifying the axion-dilaton coupling with the torus modulus of the rectangular torus before the torus deformation as

\[ \tau = \tau_1 + i\tau_2 = il, \quad l := \frac{R_1}{R_3}. \]  

(2.10)

The deformed AdS\(_5\) \( \times S^5 \) is [15]

\[
ds^2 = R^2 \tilde{H}^{1/2} \left[ -dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + \sum_{i=1}^{3} \left( d\mu_i^2 + \tilde{G} \mu_i^2 d\phi_i^2 \right) + 9 \tilde{G} \tilde{P} \mu_1^2 \mu_2^2 \mu_3^2 d\psi^2 \right],
\]

\[
e^\Phi = \sqrt{\tilde{G} \tilde{H} l^{-1}},
\]

\[
\chi = \tilde{H}^{-1} (l \cot \xi + \hat{\gamma} \hat{\sigma} l^2 g_0),
\]

\[
B_2 = R^2 \tilde{G} \tilde{Q} \omega_2 - 4R^2 \hat{l} l \omega_1 \wedge \sum_{i=1}^{3} d\phi_i,
\]

\[
C_2 = R^2 \tilde{G} \tilde{T} \omega_2 - 4R^2 \hat{l} l \omega_1 \wedge \sum_{i=1}^{3} d\phi_i,
\]

\[
C_4 = 4R^4 l \omega_1 + 4R^4 \tilde{G}(1 - \hat{\sigma} \hat{T} g_0) \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3,
\]

\[
F_5 = 4R^4 l (\omega_{AdS_5} + \tilde{G} \omega_{S^5}),
\]

(2.11)

where

\[
\tilde{G}^{-1} = 1 + \tilde{P} g_0,
\]

\[
\tilde{H} = \csc^2 \xi + \hat{\sigma}^2 l^2 g_0,
\]

\[
\tilde{P} = \hat{\gamma}^2 \csc^2 \xi - 2 \hat{\gamma} \hat{\sigma} l \cot \xi + \hat{\sigma}^2 l^2,
\]

\[
\tilde{Q} = \hat{\gamma} \csc^2 \xi - \hat{\sigma} l \cot \xi,
\]

\[
\tilde{T} = \hat{\gamma} l \cot \xi - \hat{\sigma} l^2.
\]

(2.12)

We study the Penrose limits of (2.4) along the \((J,0,0)\) geodesic and along the \((J,J,J)\) geodesic.
The parametrization to take the Penrose limit along the \((J, 0, 0)\) geodesic is
\[
\Xi_1 := f, \quad \rho = \frac{y}{\Xi_1^{1/4} R}, \quad \alpha = \frac{r}{\Xi_1^{1/4} R}, \\
t = x^+ + \frac{x^-}{2\Xi_1^{1/2} R^2}, \quad \phi_1 = x^+ - \frac{x^-}{2\Xi_1^{1/2} R^2}, \\
r^2 = \sum_{i=1}^4 (x^i)^2, \quad y^2 = \sum_{a=5}^8 (x^a)^2.
\] (2.13)

By taking \(R \to \infty\), we obtain the pp-wave geometry
\[
ds^2 = -2dx^+dx^- - \left[y^2 + (1 + P)r^2\right] (dx^+)^2 + dr^2 + r^2 d\Omega_3^2 + dy^2 + y^2 d\Omega_3^2, \\
e^\Phi = \Xi_1 \tau_2^{-1}, \\
B_2 = \frac{r^2}{\Xi_1^{1/2}} Q(\cos^2 \theta dx^+ \wedge d\phi_2 - \sin^2 \theta dx^+ \wedge d\phi_3), \\
C_2 = \frac{r^2}{\Xi_1^{1/2}} T(\cos^2 \theta dx^+ \wedge d\phi_2 - \sin^2 \theta dx^+ \wedge d\phi_3), \\
C_4 = -\tau_2 \Xi_1 (y^4 dx^+ \wedge d\Omega_3 + r^4 dx^+ \wedge d\tilde{\Omega}_3),
\] (2.14)

where
\[
d\tilde{\Omega}_3^2 = d\theta^2 + \cos^2 \theta d\phi_2^2 + \sin^2 \theta d\phi_3^2, \\
d\Omega_3 = \cos \theta \sin \theta d\theta \wedge d\phi_2 \wedge d\phi_3.
\] (2.15)

The parametrization to take the Penrose limit along the \((J, J, J)\) geodesic [16] is
\[
\Xi_2 := f + \frac{1}{3} \hat{\sigma}^2 \tau_2^2, \quad \theta_0 = \frac{\pi}{4}, \quad \alpha_0 = \arccos \left(\frac{1}{\sqrt{3}}\right), \\
\alpha = \alpha_0 - \frac{x^2}{\Xi_2^{1/4} R}, \quad \theta = \theta_0 + \sqrt{\frac{3}{2}} \frac{x^1}{\Xi_2^{1/4} R}, \quad \rho = \frac{y}{\Xi_2^{1/4} R}, \\
\varphi^1 = \sqrt{\frac{3 + P}{2}} \frac{1}{\Xi_2^{1/4} R} \left(x^3 - \frac{1}{\sqrt{3}} x^4\right), \quad \varphi^2 = \sqrt{\frac{2(3 + P)}{3}} \frac{x^4}{\Xi_2^{1/4} R}, \\
t = x^+ + \frac{1}{2\Xi_2^{1/2} R^2} x^-, \quad \psi = x^+ - \frac{1}{2\Xi_2^{1/2} R^2} x^-, 
\] (2.16)

where the spherical coordinates and the torus coordinates are related by
\[
\phi_1 = \psi - \varphi_2, \quad \phi_2 = \psi + \varphi_1 + \varphi_2, \quad \phi_3 = \psi - \varphi_1.
\] (2.17)
By taking $R \to \infty$ and shifting the coordinate $x^-$ as $x^- \to x^- - \frac{\sqrt{3}}{\sqrt{3} + p}(x^1 x^3 + x^2 x^4)$, we obtain the pp-wave geometry in the homogeneous plane wave form \[ \text{[17]} \]

$$ds^2 = -2dx^+ dx^- + \frac{2\sqrt{3}}{\sqrt{3} + p}(x^3 dx^1 + x^4 dx^2 - x^1 dx^3 - x^2 dx^4) dx^+ + \sum_{i=1}^{8} (dx^i)^2$$

$$e^\Phi = \sqrt{\frac{3}{\sqrt{3} + p}} \zeta_2^{-1},$$

$$B_2 = \frac{Q}{\sqrt{3}} \zeta_2^{-1/2} dx^3 \wedge dx^4 + \frac{2Q}{\sqrt{3} + p} \zeta_2^{-1/2} dx^+ \wedge (x^2 dx^3 - x^1 dx^4)$$

$$- \frac{2\sqrt{3}}{3} \hat{\sigma} \zeta_2^{-1/2} dx^+ \wedge (x^2 dx^1 - x^1 dx^2),$$

$$C_2 = \frac{T}{\sqrt{3}} \zeta_2^{-1/2} dx^3 \wedge dx^4 + \frac{2T}{\sqrt{3} + p} \zeta_2^{-1/2} dx^+ \wedge (x^2 dx^3 - x^1 dx^4)$$

$$- \frac{2\sqrt{3}}{3} \hat{\tau} \zeta_2^{-1/2} dx^+ \wedge (x^2 dx^1 - x^1 dx^2),$$

$$C_4 = 4R^4 \tau_2 \omega_4$$

$$+ 2\tau_2 \zeta^{-1} \left(1 - \frac{1}{3} T \hat{\sigma}\right) dx^+ \wedge (x^2 dx^1 \wedge dx^2 \wedge dx^4 - x^1 dx^2 \wedge dx^3 \wedge dx^4). \quad (2.18)$$

### 3 Giant graviton on the deformed pp-wave

We study giant gravitons on the deformed pp-wave \[ \text{[2.14]} \]. A static gauge for a brane which wraps the $(\theta, \phi_2, \phi_3)$ directions is

$$\sigma^0 = \tau, \quad \sigma^1 = \theta, \quad \sigma^2 = \phi_2, \quad \sigma^3 = \phi_3, \quad (3.1)$$

and

$$X^+ = \lambda \tau, \quad X^- = \mu \tau. \quad (3.2)$$

The fields on the three-sphere \[ \text{[2.15]} \] can be parameterized as

$$X^1 = r \cos \theta \cos \phi_2, \quad X^2 = r \sin \theta \cos \phi_3,$$

$$X^3 = r \cos \theta \sin \phi_2, \quad X^4 = r \sin \theta \sin \phi_3. \quad (3.3)$$

\( \omega_1 \) in \[ \text{[2.8]} \] is solved as \( \omega_1 = \frac{1}{4\pi} \zeta_2^{-1/2} \left(\left(\frac{1}{2} \chi - \zeta\right) x^1 dx^2 - \zeta x^2 dx^1\right) + O(R^{-3}). \) \( \zeta = \frac{\chi^2}{x^0} \) is chosen, which is consistent with \[ \text{[13]} \].
We turn off the fields on $AdS_5$

$$X^a = 0, \ (a = 5, 6, 7, 8). \quad (3.4)$$

A D3-brane is described by the Dirac-Born-Infeld action and the Wess-Zumino term:

$$S = S_{DBI} + S_{WZ}$$

$$= -T_3 \int d^4 \sigma e^{-\Phi} \sqrt{-\det P(g - B_2)} - T_3 \int P[C_q \wedge e^{-B_2}]. \quad (3.5)$$

$P$ denotes the pullback of the spacetime field to the brane worldvolume. The D3-brane action in the deformed geometry (2.14) is

$$S = -T_3 \int d^4 \sigma \tau^2 \sqrt{2 \lambda \mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau^2 \Xi^{-1}) - \lambda r^4}. \quad (3.6)$$

The action does not depend on $\gamma$ while it depends on $\sigma$ as well as $\tau_1$ and $\tau_2$.

The lightcone momentum of the D3-brane is

$$P^+ = -\frac{\delta L}{\delta \mu} = \frac{M \lambda r^3}{\Xi_1 \sqrt{2 \lambda \mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau^2 \Xi^{-1})}}, \quad (3.7)$$

and the lightcone Hamiltonian is

$$P^- = H_{lc} = -\frac{\delta L}{\delta \lambda} = \frac{Mr^3}{\Xi_1} \left[ \frac{\mu + \lambda r^2 (1 + \hat{\sigma}^2 \tau^2 \Xi^{-1})}{\sqrt{2 \lambda \mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau^2 \Xi^{-1})}} - r \right]. \quad (3.8)$$

where $M := 2\pi^2 \tau_2 T_3$. The lightcone Hamiltonian can be written as

$$H_{lc} \sim \frac{M^2 r^6}{2 \Xi_1^2 P^+} + \frac{P^+(1 + \hat{\sigma}^2 \tau^2 \Xi^{-1}) r^2}{2} - \frac{Mr^4}{\Xi_1}. \quad (3.9)$$

For $0 \leq \hat{\sigma} < \frac{\sqrt{\Xi_1}}{\sqrt{3} \tau_2}$, the Hamiltonian is extremized at

$$r_0 = 0, \ r_{\pm} = \frac{P^+ \Xi_1}{3M} \left( 2 \pm \sqrt{1 - 3\hat{\sigma}^2 \tau^2 \Xi^{-1}} \right). \quad (3.10)$$

---

\footnote{We choose the minus sign for the Wess-Zumino term as it is done in [12] since it is consistent with the conventions of [2, 3].}

\footnote{The conjugate momenta are defined by $P_{\pm} = \frac{\delta L}{\delta (\sigma, X^7)}$. The upper index and the lower index are related by $P^\pm = -P_{\mp}$.}
The lightcone Hamiltonian has local minima at \( r = r_0 \) and \( r = r_+ \), and a local maximum at \( r = r_- \). The radii do not depend on \( \gamma \) while they depend on \( \sigma \) as well as the axion-dilaton parameters \( \tau_1 \) and \( \tau_2 \). The corresponding lightcone energies are

\[
E_0 = 0,
E_{\pm} = \frac{(P^+)^2 \Xi_1}{27M} \left[ 1 + 9\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1} \mp (1 - 3\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})^{3/2} \right].
\] (3.11)

For \( \hat{\sigma} = 0 \), we have \( E_+ = E_0 \), i.e., the giant graviton is degenerate with the point graviton. For \( 0 < \hat{\sigma} < \frac{\sqrt{3}}{\sqrt{3} \tau_2} \), we have \( E_+ > E_0 \), i.e., the degeneracy is lifted. The energy of the point graviton is less than the energy of the giant graviton. Therefore the giant graviton becomes energetically unfavorable. For \( \hat{\sigma} = \frac{\sqrt{3} \tau_1}{\sqrt{3} \tau_2} \), we have \( r_+ = r_- \) and \( E_+ = E_- \), i.e., there is a saddle point at \( r = r_{\pm} \). The lightcone Hamiltonian has one minimum at \( r = r_0 \). For \( \hat{\sigma} > \frac{\sqrt{3} \tau_1}{\sqrt{3} \tau_2} \), the giant graviton disappears. The result is consistent with the results of \[13\] \[14\].

The lightcone Hamiltonian in the Penrose limit of (2.11) is obtained by substituting \( \Xi_1 = \csc^2 \xi \) and \( \tau = \tau_1 + i\tau_2 = il \). The lightcone Hamiltonian for \( \xi = \frac{\pi}{3} \) in the units of \( M = 1 \) and \( P^+ = 1 \) is plotted in Figure 1. It is qualitatively the same as the one plotted in \[14\]. The lightcone Hamiltonian for \( \hat{\sigma}l = \frac{1}{3} \) in the units of \( M = 1 \) and \( P^+ = 1 \) is plotted in Figure 2. As the intersection angle \( \xi \) decreases, the minimum value of the lightcone Hamiltonian increases.

![Figure 1: Lightcone Hamiltonian with \( \xi = \frac{\pi}{3} \) as a function of \( r \). \( \hat{\sigma}l = 0 \) (solid), \( \hat{\sigma}l = \frac{1}{3} \) (dot-dashed), \( \hat{\sigma}l = \frac{2}{3} \) (dashed), \( \hat{\sigma}l = \frac{4}{3} \) (dotted), \( \Xi_1 = \csc^2 \xi \), \( \tau_2 = l \), \( M = 1 \) and \( P^+ = 1 \).](image)

We examine the spectrum of small fluctuations about the giant graviton solution following the method of \[13\] \[14\] \[18\]. We fix the lightcone coordinates as

\[
X^+ = \tau, \quad X^- = \nu \tau + \epsilon \delta x^-.
\] (3.12)
Figure 2: Lightcone Hamiltonian with $\hat{l} = \frac{1}{3}$ as a function of $r$. $\xi = \frac{2}{7}$ (solid), $\xi = \frac{3}{7}$ (dashed), $\xi = \frac{4}{7}$ (dotted), $\Xi_1 = \csc^2 \xi$, $\tau_2 = l$, $M = 1$ and $P^+ = 1$.

The ansatz for the perturbed configuration is

$$r = r_0 + \epsilon \delta r,$$

$$X^1 = r \cos \theta \cos \phi_2, \quad X^2 = r \sin \theta \cos \phi_3,$$

$$X^3 = r \cos \theta \sin \phi_2, \quad X^4 = r \sin \theta \sin \phi_3,$$

$$X_a = \epsilon \delta x^a, \quad (a = 5, \ldots, 8).$$

The components of the pullback $D_{\mu\nu} = P [g - B_2]_{\mu\nu}$ up to the second order of $\epsilon$ are

$$D_{\tau\tau} = -2 \nu - (1 + P) r_0^2 + \epsilon [ -2 \partial_\tau \delta x^- - 2(1 + P) r_0 \delta r ]$$

$$+ \epsilon^2 [ - \sum_a (\delta x^a)^2 - (1 + P) \delta r^2 + \sum_{I=1+a} (\partial_\tau \delta x^I)^2 ],$$

$$D_{\tau\theta} = D_{\theta\tau} = -\epsilon \delta \phi \delta x^- + \epsilon^2 \sum_{I=i+a} (\partial_\tau \delta x^I)(\partial_\theta \delta x^I),$$

$$D_{\tau\phi_2/\phi_2\tau} = \mp \Xi_1^{-1/2} Q r_0 \cos^2 \theta + \epsilon [ - \partial_{\phi_2} \delta x^- \mp 2 \Xi_1^{-1/2} Q r_0 \cos^2 \theta \delta r ]$$

$$+ \epsilon^2 [ \sum_{I=i+a} \partial_\tau \delta x^I \partial_{\phi_2} \delta x^I \mp \Xi_1^{-1/2} Q \cos^2 \theta \delta r^2 ],$$

$$D_{\tau\phi_3/\phi_3\tau} = \pm \Xi_1^{-1/2} Q r_0 \sin^2 \theta + \epsilon [ - \partial_{\phi_3} \delta x^- \pm 2 \Xi_1^{-1/2} Q r_0 \sin^2 \theta \delta r ]$$

$$+ \epsilon^2 [ \sum_{I=i+a} \partial_\tau \delta x^I \partial_{\phi_3} \delta x^I \pm \Xi_1^{-1/2} Q \sin^2 \theta \delta r^2 ],$$

(3.13)
\[ D_{ij} = r_0^2 g_{ij} + 2r_0 g_{ij} \partial r + \epsilon^2 \left[ g_{ij} \partial r^2 + \sum_{I=1}^{8} (\partial_I \partial x^I)(\partial_J \partial x^J) \right], \quad (i, j = \theta, \phi_2, \phi_3), \]

where the metric \( g_{ij} \) is defined as

\[
g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos^2 \theta & 0 \\ 0 & 0 & \sin^2 \theta \end{pmatrix}.
\] (3.15)

The D3-brane action is

\[
S = S_{\text{DBI}} + S_{\text{WZ}}
\]

\[
= -T_3 \int d^4 \sigma e^{-\Phi} \sqrt{-\text{det} P[G - B]} - T_3 \int P[C_4 - C_2 \wedge B_2]
\]

\[
= -T_3 \Xi^{-1} \tau_2 \int d\tau d^3 \sigma \sqrt{|g|} r_0^3 \left\{ 2\epsilon + r_0^2 u - r_0 \right\}
\]

\[
- \epsilon T_3 \Xi^{-1} \tau_2 \int d\tau d^3 \sigma \sqrt{|g|} \frac{r_0^3}{\sqrt{2\nu + r_0^2 u}} \left\{ 3\nu + 2r_0^2 u - 2r_0 \sqrt{2\nu + r_0^2 u} \right\}
\]

\[
- \epsilon^2 T_3 \Xi^{-1} \tau_2 \int d\tau d^3 \sigma \frac{r_0^3}{\sqrt{2\nu + r_0^2 u}} \left\{ \sqrt{|g|} \left[ 30\nu + 28r_0^2 u - (6\nu + 4r_0^2 u)^2 \right] 
\]

\[
- 12r_0 \sqrt{2\nu + r_0^2 u} \delta r^2 + \sqrt{|g|} r_0^2 \sum_{a} (\delta x^a)^2 - \sum_{I} \delta x^I \left[ (2\nu + r_0^2 u) \partial_I (\sqrt{|g|} g^{ij} \partial_J) 
\]

\[
+ (\partial_{\phi_2} - \partial_{\phi_3})(\sqrt{|g|} \Xi^{-1} Q^2 r_0^2)(\partial_{\phi_2} - \partial_{\phi_3}) - r_0^2 \partial_r \left( \sqrt{|g|} \partial_r \right) \delta x^R 
\]

\[
+ \sqrt{|g|} \frac{4(3\nu + r_0^2 u)}{2\nu + r_0^2 u} r_0 \delta r \delta x^R - \delta x^R \partial_r (\sqrt{|g|} g^{ij} \partial_J) \delta x^R 
\]

\[
+ \frac{r_0^2}{2\nu + r_0^2 u} \delta x^R \partial_r (\sqrt{|g|} \partial_r) \delta x^R \right\},
\] (3.16)

where

\[
u = 1 + \sigma^2 \tau_2 \Xi^{-1}.
\] (3.17)

In the first order in \( \epsilon \), \( \partial_r \delta x^- = 0 \) as the endpoints in \( \tau \) are fixed. From the second term, which is proportional to \( \delta r \) we get a constraint\footnote{The constraint is also obtained from (3.7) and (3.10) with \( \lambda := 1 \) and \( \mu := \nu \).}

\[
\nu_{\pm} = \frac{2r_0^2}{9} \left[ -1 - 3\sigma^2 \tau_2 \Xi^{-1} \pm \sqrt{1 - 3\sigma^2 \tau_2 \Xi^{-1}} \right].
\] (3.18)
\( \nu_+ \) minimizes the action.

To find the spectrum we decompose the solution as
\[
\delta x^I = \delta \tilde{x}^I e^{-i\omega^I Y_{l,\alpha}}. 
\] (3.19)

\( Y_{l,\alpha} \) are four-dimensional spherical harmonics which satisfy
\[
\frac{1}{\sqrt{|g|}} \partial_i (\sqrt{|g|} g^{ij} \partial_j) Y_{l,\alpha} = -q_l Y_{l,\alpha}, \quad q_l = l(l + 2). \] (3.20)

Due to the term \( \sim Q^2 \left[ (\partial_{\phi_2} - \partial_{\phi_3}) \delta x^I \right]^2 \) in the fifth line of the action (3.16) the degeneracy of the spherical harmonics is lifted. The spherical harmonics are diagonalized as
\[
\left( \frac{\partial}{\partial \phi_2} - \frac{\partial}{\partial \phi_3} \right)^2 Y_{l,\alpha} = -\alpha^2 Y_{l,\alpha}. \] (3.21)

The spectrum in the \( X^a \), \( (a = 5, \cdots, 8) \), directions is
\[
\omega_a^2 = 1 + \Xi^{-1} Q^2 \alpha^2 + \frac{1}{g} \left( 2 + \sqrt{1 - 3 \Xi^{-1} \sigma^2 \tau_2^2} \right)^2 q_l. \] (3.22)

The radial direction and the null direction \( X^- \) are coupled. The equations of motion are
\[
s := 1 - 3 \Xi^{-1} \sigma^2 \tau_2^2, \\
\left[ \frac{8}{3} (\sqrt{s} - s) + \frac{1}{9} (2 + \sqrt{s})^2 q_l + \Xi^{-1} Q^2 \alpha^2 - \omega^2 \right] \delta \tilde{r} - i \frac{\omega}{\rho_0} \left( \frac{6 \sqrt{s}}{2 + \sqrt{s}} \right) \delta \tilde{x}^-- = 0, \\
i \frac{\omega}{\rho_0} \left( \frac{6 \sqrt{s}}{2 + \sqrt{s}} \right) \delta \tilde{x}^- + \frac{1}{\rho_0^2} \left[ q_l - \frac{9}{(2 + \sqrt{s})^2} \omega^2 \right] \delta \tilde{x}^- = 0. \] (3.23)

The spectrum is
\[
t := \frac{4 \sqrt{s} (2 + \sqrt{s})}{3} + \Xi^{-1} Q^2 \alpha^2, \\
\omega_\pm^2 = \frac{t}{2} + \left( \frac{2 + \sqrt{s}}{3} \right)^2 q_l \pm 2 \sqrt{\frac{t^2}{16} + \frac{(2 + \sqrt{s})^2}{3}} q_l. \] (3.24)

\( \omega_\pm^2 \)'s are positive definite while \( \omega_\mp^2 \)'s are positive semidefinite. A zero mode occurs when \( l = 0 \). The spectrum is independent of the size \( \rho_0 \). The spectrum depends on the marginal deformation parameters \( \gamma \) and \( \sigma \) as well as the axion-dilaton parameters \( \tau_1 \) and \( \tau_2 \). There is no complex frequency. When \( \sigma \neq 0 \), giant gravitons are not energetically favorable but the spectrum of small fluctuations shows that the giant gravitons are perturbatively stable. The result is consistent with the results of [13, 14].
4 Discussion

We have studied the Penrose limits of the complex marginal deformation of $AdS_5 \times S^5$ which incorporates the $SL(2,\mathbb{R})$ symmetry of type IIB theory and have presented the pp-wave geometries along the $(J,0,0)$ geodesic and along the $(J,J,J)$ geodesic. We have shown that giant gravitons on the $(J,0,0)$ pp-wave depend the parameter $\sigma$ as well as the axion-dilaton parameters. Giant gravitons exist up to a critical value of $\sigma$, which depends on the axion-dilaton parameters. The spectrum of small fluctuations about the giant graviton solution is obtained. The giant gravitons are energetically unfavorable but perturbatively stable.

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