Fermion analysis of IR modified Hořava–Lifshitz gravity: tunneling and perturbation perspectives

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Abstract
In this paper, we investigate the fermion Hawking radiation and quasinormal (QN) modes in infrared (IR) modified Hořava–Lifshitz (HL) gravity under tunneling and perturbation perspectives. Firstly, through the fermion tunneling in IR modified HL gravity, we obtain the Hawking radiation emission rate, tunneling temperature and entropy for the Kehagias–Sfetsos black hole. It is found that the results of fermion tunneling are consistent with the thermodynamics results obtained by calculating surface gravity. Secondly, we numerically calculate the low-lying QN mode frequencies of fermion perturbations by using WKB formulas including the third-order and sixth-order approximations simultaneously. It turns out that the actual frequency of fermion perturbation is larger than that in the Schwarzschild case, and the damping rate is smaller than that for the pure Schwarzschild. The results of fermion perturbation suggest the QN modes could live longer in HL gravity.

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1. Introduction

Recently, Hořava presented a power counting renormalizable gravity theory at the Lifshitz point which is called Hořava–Lifshitz (HL) gravity \cite{1}. It exhibits a broken Lorentz symmetry at short distances and reduces to usual general relativity (GR) gravity at large distances, in particular, $\lambda = 1$ which controls the contribution of the extrinsic curvature trace. With the HL gravity theory putting forth, HL gravity is intensively investigated in many aspects involving basic formalism \cite{2}, cosmology \cite{3}, various black hole solutions and their thermodynamics \cite{4–7} and so on.

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In the subsequent developments of HL gravity, people are trying to find the influence of matter fields for HL gravity. Which analog of the matter energy–momentum tensor could be used to act as gravitational source at GR? The pioneering works involve geodesic analysis by various methods including the optical limit of a scalar field theory [8], super Hamiltonian formalism [9], foliation preserving diffeomorphisms [10], Lorentz-violating modified dispersion relations [11] and so on. The optical limit presented by Capasso and Polychronakos [8] could offer a deformed geodesic equation by the generalized Klein–Gordon action. It is shown that the particles do not move in a geodesic manner. Deviations from geodesic motion appear both in flat and Schwarzschild-like spacetimes. Similar result deviations from GR also are found in [9–11] with various above-mentioned approaches.

Considering a vanishing cosmological constant \( \Lambda = 0 \), Kehagias and Sfetsos [5] propose an asymptotically flat black hole solution by introducing the addition term proportional to the Ricci scalar of three geometry \( \mu^4 R^{(3)} \), which indicates the Minkowski vacuum and modified GR at infrared (IR) modification. Since then, many people have focused on its phenomenology involving strong field gravitational lensing [12, 13], scalar field quasinormal (QN) modes [12, 14], time-like geodesic motion [15], thin accretion disk [16] and observation constraints [17] as well as its thermodynamics analysis [18–20], which presents the black hole entropy \( S = A/4 + \pi/\alpha \ln(A/4) \) via the first law of thermodynamics where \( \alpha \) is the Ho\'rava parameter. This entropy could be treated as generalized uncertainty principle quantum correction entropy [19] or casting entropy [20]. If fermions are tunneling in this kind of HL gravity, might it keep this logarithm entropy? Motivated by this, we investigate fermion tunneling in section 3.

On the other hand, as we know that through some additional fields (e.g. scalar or fermion fields), the black hole suffers a damping oscillation phase which is named as ‘quasinormal models’ (QNMs) or ‘QN ringing’. As a result, the normal model oscillation is replaced by a complex frequency which encodes the black hole’s important information such as mass, charge, momentum and the dimensions of spacetime. The real part of complex frequency represents the actual frequency and the imaginary part of it represents the damping of the oscillation. It is believed that these QN frequencies could be detected by LIGO, VIRGO, TAMT, and GEO600 in future. So according to this observable’s QNMs, some people have used massless scalar field to obtain the important low-lying QN frequencies which could live longer and be detected easily [12, 14]. It is interesting that the QNMs of a massless scalar field are longer lived and have larger real oscillation frequency in HL gravity than in GR. Otherwise, as one kind of basic particles, the fermions could provide us with much important information. Motivated by the situations above, we will evaluate the QNMs for the massless fermion perturbations in section 4.

As we already know, due to the different kinetic terms, HL gravity with \( \lambda \neq 1 \) is significantly different from GR. Hence, the HL gravity with \( \lambda \neq 1 \) has been extensively studied in the literature. The relevant works are mainly concentrated on the basic problems such as how to find various exact black hole solutions, the application of cosmology, the constraints of various fundamental parameters and so on. Despite that more attention has been paid to the properties of HL black holes, there are few works referring to the fermion analysis, especially to the \( \lambda \neq 1 \) case. So far as we know, the works relevant to the fermion analysis focus mainly on the IR modified HL gravity including Dirac perturbations [21, 22] and fermion tunneling for \( z = 4 \) black holes [23] and so on. In Wang and Gui’s work [21], the QN frequencies of massless Dirac field perturbation are evaluated by the third-order WKB approximation. In Varghese and Kuriakose’s work [22], the evolution of Dirac perturbations is also investigated using time domain integration and third-order WKB methods. In Chen, Yang and Zu’s work [23], the fermion tunneling is investigated in the background of (3+1) dimensions and (4+1) dimensions black holes in the \( z = 4 \) HL gravity.
This paper is organized as follows. In section 2, we present the Kehagias–Sfetsos (KS) black hole solutions. In section 3, we calculate the fermion tunneling emission rate of Hawking radiation, tunneling temperature, and entropy. In section 4, we use the third- and sixth-order WKB formulas to numerically calculate frequencies simultaneously. In section 5, we present the conclusions. We adopt the signature $(-, +, +, +)$ and put $\hbar, c,$ and $G$ equal to unity.

2. An asymptotically flat IR modified black hole solution in deformed HL gravity

In this section, we review briefly the KS black hole solutions under the limit of $\Lambda_{W} \to 0$ with running constant $\lambda = 1$ in the IR critical point $z = 1$. The space geometric is parameterized with Arnowitt–Deser–Misner (ADM) formalism,

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt).$$

The action for the fields of the HL theory is

$$S = \int dt d^3x \sqrt{gN}\left\{\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\alpha^2}C_{ij}C^{ij} + \frac{\kappa^2\mu}{2\alpha^2}e^{ij\ell}R^{(3)}_{\ell j}R^{(3)}_{\ell k} - \frac{\kappa^2\mu^2}{8}R^{(3)}_{ij}R^{(3)}_{ij} \right. $$

$$\left. + \frac{\kappa^2\mu^2}{8(1 - 3\lambda)}\left(1 - \frac{4\lambda}{4}(R^{(3)}_{ij})^2 + \Lambda_{W}R^{(3)} - 3\Lambda_{W}^2\right) + \mu^4R^{(3)}\right\},$$

where the second fundamental form, extrinsic curvature $K_{ij}$, and the Cotton tensor $C^{ij}$ are given as follows:

$$K_{ij} = \frac{1}{2N}\left(\frac{\partial}{\partial t}g_{ij} - \nabla_i N_j - \nabla_j N_i\right),$$

$$C^{ij} = \epsilon^{ijk}\nabla_k\left(R^{(3)}_{ij} - \frac{1}{4}R^{(3)}g_{ij}\right).$$

Here, $\kappa$, $\lambda$, $\alpha$, $\mu$ and $\Lambda_{W}$ are the constant parameters. The last term of metric equation (2) represents a soft violation of the detailed balance condition. Comparing the HL gravity action with that of GR gravity, we can obtain the speed of light $c$, Newton’s constant $G$ and the cosmological constant $\Lambda$:

$$c = \frac{\kappa^2\mu^2}{4}\sqrt{\frac{\Lambda_{W}}{1 - 3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda = \frac{3}{2}\Lambda_{W}.$$

In the limit of $\Lambda_{W} \to 0$, we can obtain a deformed action as follows:

$$S = \int dt d^3x (\mathcal{L}_0 + \mathcal{L}_1),$$

$$\mathcal{L}_0 = \sqrt{g}N\left\{\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2)\right\},$$

$$\mathcal{L}_1 = \sqrt{g}N\left\{\frac{\kappa^2\mu^2(1 - 4\lambda)}{32(1 - 3\lambda)}R^2 - \frac{\kappa^2}{2\alpha^2}\left(C_{ij} - \frac{\mu\alpha^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu\alpha^2}{2}R^{ij}\right) + \mu^4 R\right\}.$$
The lapse function is
\[ f(r) = 1 + \alpha r^2 - \sqrt{r(\alpha^2 r^3 + 4\alpha M)}, \]
where the parameter \( M \) is an integration constant related to the mass of black hole. Using the null hypersurface condition, one can find that there are two horizons, the inner \( r_- \) and outer event horizon \( r_+ \) in this space,
\[ r_{\pm} = M \left( 1 \pm \sqrt{1 - \frac{1}{2\alpha M^2}} \right). \]
Thermodynamic quantities including mass \( M_{KS} \), temperature \( T_{KS} \), and heat capacity \( C_{KS} \) and entropy \( S_{KS} \) presented in [18] are listed as
\[ M_{KS} = \frac{1 + 2\alpha r_\pm^2}{4\alpha r_\pm}, \]
\[ T_{KS} = \frac{2\alpha r_\pm^2 - 1}{8\pi r_+(\alpha r_+^2 + 1)}, \]
\[ C_{KS} = -\frac{2\pi}{\alpha} \left[ \frac{(\alpha r_+^2 + 1)^2 (2\alpha r_+^2 - 1)}{2\alpha^2 r_+^4 - 5\alpha r_+^2 - 1} \right], \]
\[ S_{KS} = \frac{A}{4} + \frac{\pi}{\alpha} \ln \left( \frac{A}{4} \right), \]
with the horizon area \( A = 4\pi r_+^2 \). Under the limit of \( \alpha \rightarrow +\infty \), the entropy reduces to the Bekenstein–Hawking entropy \( S_{BH} = A/4 \) for the Schwarzschild black hole.

3. Fermion tunneling of IR modified HL gravity

In this section, we investigate the Hawking radiation of KS black hole in IR modified HL gravity with fermion tunneling. The tunneling probability, temperature and entropy are expected to be obtained. The Dirac equation in the KS black hole spacetime can be written as
\[ [\gamma^a e^a_{\mu} \partial_\mu + m F_{\mu}] \psi = 0, \]
where \( m \) is the mass of fermions. \( e^a_{\mu} \) is the inverse of the tetrad \( e_\mu^a \) defined by the black hole metric \( g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu \) with the Minkowski metric \( \eta_{ab} = \text{diag}(-1, 1, 1, 1) \). \( \gamma^a \) is the Dirac matrix and \( \Gamma_{\mu} \) is the spin connection given by
\[ \Gamma_{\mu} = \frac{1}{4}[\gamma^a, \gamma^b] e^a_{\mu} e^b_{\nu}, \]
where the covariant derivative of \( e_{\nu} \) is given by Christoffel symbols \( \Gamma^a_{\mu\nu} \) as
\[ e_{\nu} = \partial_\mu e_{\nu} - \Gamma^a_{\mu\nu} e_{ba}. \]
We choose following \( \gamma \) matrix:
\[ \gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -i \sigma^3 & 0 \\ i \sigma^3 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & -i \sigma^1 & 0 \\ i \sigma^1 & 0 & 0 \end{pmatrix}, \]
where \( \sigma^i \) is the Pauli sigma matrix,
\[ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]
In the presentation of $\sigma^i$, the spin-up wavefunction is written as
\[
\psi^\uparrow(t, r, \theta, \phi) = \left(\begin{array}{c} A(t, r, \theta, \phi) \xi^\uparrow \\ B(t, r, \theta, \phi) \xi^\downarrow \end{array}\right) \exp\left[\frac{i}{\hbar} I^\uparrow(t, r, \theta, \phi)\right],
\]
where $\xi^\uparrow$ denotes the eigenvector of a spin-up state with the eigenvalue $+1$, and $\xi^\downarrow$ denotes the eigenvector of a spin-down state with the eigenvalue $-1$. $I^\uparrow$ is the action of radiation particles with spin up. According to $\epsilon^\mu a = \delta^\mu$, we have
\[
e^\mu = \text{diag}\left(\sqrt{f}, \frac{1}{r}, \frac{1}{r \sin \theta}\right).
\]
Submitting equation (22) into Dirac equation (16), the frame $e^\mu_a$ should satisfy following relation:
\[
\left(\begin{array}{c} y^0 e_0^\mu \partial_t + y^1 e_1^\mu \partial_r + y^2 e_2^\mu \partial_\theta + y^3 e_3^\mu \partial_\phi + y^a e^\mu_a \Gamma^a_{\mu\nu} + \frac{m}{\hbar}\end{array}\right) \Psi^\uparrow = 0.
\]
Simplifying above equation (23), we can obtain
\[
\left(\begin{array}{c} y^0 \sqrt{f} \partial_t + \sqrt{f} y^1 \partial_r + \frac{y^2}{r} \partial_\theta + \frac{y^3}{r \sin \theta} \partial_\phi + y^a e^\mu_a \Gamma^a_{\mu\nu} + \frac{m}{\hbar}\end{array}\right) \Psi^\uparrow = 0.
\]
If we neglect the small quantity $\Gamma^a_{\mu\nu}$, equation (24) could be simplified as
\[
\left(\begin{array}{c} y^0 \sqrt{f} \partial_t + \sqrt{f} y^1 \partial_r + \frac{y^2}{r \sin \theta} \partial_\phi + \frac{m}{\hbar}\end{array}\right) \Psi^\uparrow = 0.
\]
On the benefit of $\gamma$ metrics (equation (20)), we have
\[
\gamma^0 \partial_t \Psi^\uparrow = \left(\begin{array}{c} A \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ 0 \\ -B \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ 0 \end{array}\right), \quad \gamma^1 \partial_r \Psi^\uparrow = \left(\begin{array}{c} B \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ 0 \\ -A \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ 0 \end{array}\right).
\]
\[
\gamma^2 \partial_\theta \Psi^\uparrow = \left(\begin{array}{c} 0 \\ iB \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ 0 \\ -iA \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \end{array}\right), \quad \gamma^3 \partial_\phi \Psi^\uparrow = \left(\begin{array}{c} 0 \\ 0 \\ B \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \\ -A \frac{1}{\hbar} \exp\left(-\frac{i}{\hbar} I^\uparrow\right) \end{array}\right).
\]
Substituting equations (26) and (27) into the Dirac equation (25), we can obtain
\[
\frac{1}{\sqrt{f}} \left(\begin{array}{cc} A \partial_t I^\uparrow & 0 \\ -B \partial_t I^\uparrow & 0 \end{array}\right) + \sqrt{f} \left(\begin{array}{cc} 0 & 0 \\ -A \partial_\theta I^\uparrow & 0 \end{array}\right) + \frac{1}{r} \left(\begin{array}{cc} iB \partial_\phi I^\uparrow & 0 \\ 0 & -iA \partial_\phi I^\uparrow \end{array}\right) + \frac{1}{r \sin \theta} \left(\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}\right) + \frac{m}{\hbar} \left(\begin{array}{cc} A \partial_\phi I^\uparrow & 0 \\ 0 & A \partial_\phi I^\uparrow \end{array}\right) = \left(\begin{array}{c} 0 \\ B \partial_\phi I^\uparrow \end{array}\right).
\]
This equation could be reduced to four components of \((t, r, \theta, \phi)\) as

\[
\frac{A}{\sqrt{f(r)}} \partial_t I_t + B \sqrt{f(r)} \partial_r I_t + mA = 0, \quad (29)
\]

\[
-\frac{B}{\sqrt{f(r)}} \partial_t I_t - A \sqrt{f(r)} \partial_r I_t + mB = 0, \quad (30)
\]

\[
\frac{B}{r} \left( i \partial_\theta I_\theta + \frac{1}{\sin \theta} \partial_\phi I_\phi \right) = 0, \quad (31)
\]

\[
\frac{A}{r} \left( i \partial_\theta I_\theta + \frac{1}{\sin \theta} \partial_\phi I_\phi \right) = 0. \quad (32)
\]

Consider the symmetry of the spacetime, we adopt the action below as

\[
I_\uparrow = -\omega t + W(r) + \Theta(\theta, \phi). \quad (33)
\]

Substituting equation (33) into equations (29), (30), (31), (32), we can obtain

\[
-\frac{A}{\sqrt{f(r)}} \omega + B \sqrt{f(r)} \partial_r W + mA = 0, \quad (34)
\]

\[
\frac{B}{\sqrt{f(r)}} \omega - A \sqrt{f(r)} \partial_r W + mB = 0, \quad (35)
\]

\[
B \left( i \partial_\theta \Theta + \frac{1}{\sin \theta} \partial_\phi \Theta \right) = 0, \quad (36)
\]

\[
A \left( i \partial_\theta \Theta + \frac{1}{\sin \theta} \partial_\phi \Theta \right) = 0. \quad (37)
\]

Because the contribution of \(\Theta\) on an outgoing particle is equal to that of \(\Theta\) on an incoming particle, equations (36) and (37) do absolutely nothing that are useful to the calculation of the tunneling probability. We only need to consider the action of the radial direction, i.e. equations (34) and (35), whose solvability condition is that the determinant of the coefficients \(A\) and \(B\) is zero. Namely,

\[
\begin{vmatrix}
-\omega + \frac{1}{\sqrt{f}} + m \\
\frac{1}{\sqrt{f}} \partial_r W
\end{vmatrix} = 0. \quad (38)
\]

By direct integration of determinant (38), \(W\) could be obtained as

\[
W_\pm(r) = \pm \int \sqrt{\omega^2 - m^2 f(r)} \frac{dr}{f(r)}, \quad (39)
\]

Using the condition \(f(r) \to 0\) near horizon \(r_+\), the numerator of an integrated fraction in equation (39) is reduced to \(\sqrt{\omega^2 - m^2 f(r)} \to \omega\). Hence, the terms containing mass (\(\sim m^2 f\)) has nothing to do with the tunneling probability. So, \(W_\pm(r)\) is applicable to the whole fermions, no matter massive or massless particles. Adopting the contour integration, we can obtain

\[
W_\pm(r) = \pm i \pi \frac{\omega}{f(r_+)} \quad (40)
\]
where ‘+’ denotes outgoing fermions, ‘−’ denotes incoming ones, and ‘′ means the first-order derivation of \( f(r) \) with respect to \( r \):

\[
f'(r_+) = \frac{df(r)}{dr} = 2\alpha r - \frac{2\alpha^2 r^3 + 2\alpha M}{\sqrt{\alpha^2 r^4 + 4\alpha Mr}}.
\]

(41)

It is well known that the tunneling probability could be related to the imaginary part of the action. Thus, the tunneling probability of the emission fermion is written as follows:

\[
\Gamma = \frac{P(\text{emission})}{P(\text{absorption})} = \frac{\exp(-2\text{Im} I^+)}{\exp(-2\text{Im} I^-)} = \frac{\exp(-2\text{Im} W^+)}{\exp(-2\text{Im} W^-)}.
\]

(42)

Substituting \( W^\pm (r) \) into equation (42), we can obtain

\[
\Gamma = \exp\left[-\frac{4\pi \omega}{f'(r_+)}\right] = \exp\left[-\frac{2\pi r_+\alpha r^3 + 4M + (\alpha r^3 + M\alpha)\sqrt{1 + 4M/\alpha^2 r^2}}{4\alpha Mr_+^2 - 2\alpha^2 r_+^2 M - \alpha^2 M^2}\right].
\]

(43)

According to the usual relation between the inverse temperature \( \beta \) and tunneling probability, \( \Gamma = \exp(-\beta \omega) \), we can obtain the fermion tunneling temperature

\[
T_{\text{fermion}} = \frac{f'(r_+)}{4\pi} = \frac{1}{2\pi} \left[ \alpha r_+ - \frac{\alpha^2 r_+^3 + \alpha M}{\sqrt{\alpha^2 r_+^4 + 4\alpha Mr_+}} \right].
\]

(44)

If we choose the mass function defined by equation (12) in [18], we can obtain

\[
T_{\text{fermion}} = \frac{2\alpha r_+^2 - 1}{8\pi r_+(1 + \alpha r_+^2)}.
\]

(45)

which is just the Hawking temperature of the black hole in IR deformed HL gravity [18]. As a thermodynamical system, the first law \( dM = TdS \) gives black hole entropy,

\[
S = \int T^{-1} dM + S_0 = \int dr_+ \left( \frac{1}{T} \frac{dM}{dr_+} \right) + S_0
\]

(46)

\[
= \pi \left( r_+^2 + \frac{1}{\alpha} \ln r_+^2 \right) + S_0,
\]

(47)

where we adopt \( M_{\text{KS}} \) equation (12). If we adopt \( S_0 = \pi \ln \pi /\alpha \), the final logarithmic entropy obtained through fermion tunneling is

\[
S = \frac{A}{4} + \frac{\pi}{\alpha} \ln \frac{A}{4}.
\]

(48)

Based on the surface gravity defined by

\[
\kappa_+ = \frac{1}{2} \frac{df}{dr_{\text{r}_+}} = \frac{2\alpha r_+^2 - 1}{4\alpha r_+(1 + \alpha r_+^2)},
\]

(49)

the thermodynamic temperature (equation (13)) and the thermodynamic entropy (equation (15)) are obtained in previous research [18]. It is interesting that the results equations (13) and (15)) based on surface gravity are in agreement with fermion tunneling results (equations (45) and (48)).
4. Fermion perturbations of IR modified HL gravity

In this section, we evaluate the QN modes of fermion perturbation by using the third-order and sixth-order WKB formulas, simultaneously. In order to obtain the QN frequencies, we should proceed from the Dirac equation (16). According to the relation $e_{b\mu} = \eta_{ab} e_{\mu}^b$, we can have $e_{b\mu} = \text{diag}(-\sqrt{f}, 1/\sqrt{f}, r, r \sin \theta)$. Then, based on $e_{b\nu;\mu} = \partial_{\mu} e_{\nu}^b - \Gamma_{\mu}^{\alpha} e_{\alpha}^b$, the non-zero covariant derivative could be listed as

$$
\begin{align*}
  e_{0;1}^2 &= \frac{\sqrt{f}}{2} f', \\
  e_{10}^2 &= -\frac{f'}{2\sqrt{f}}, \\
  e_{21}^2 &= -1, \\
  e_{12}^2 &= \sqrt{f}, \\
  e_{23} &= r \sin \theta \cos \theta, \\
  e_{13} &= \sqrt{r} \sin^2 \theta, \\
  e_{31} &= -\sin \theta, \\
  e_{32} &= -r \cos \theta.
\end{align*}
$$

(50)

Substituting equations (50) into equation (17), we can obtain the spin connections as

$$
\begin{align*}
  \Gamma_t &= -\frac{1}{4} f' \gamma^0 \gamma^1, \\
  \Gamma_r &= -\frac{\sqrt{f}}{2} \gamma^1 \gamma^2, \\
  \Gamma_\theta &= -\frac{1}{2} (\sin \theta \sqrt{f} \gamma^1 \gamma^3 + \cos \theta \gamma^2 \gamma^3), \\
  \Gamma_\phi &= -\frac{1}{2} (\sin \theta \sqrt{f} \gamma^1 \gamma^3 + \cos \theta \gamma^2 \gamma^3).
\end{align*}
$$

(51)

Considering the symmetry of frame equation (22), the Dirac equation (16) could be rewritten as

$$
[\gamma^0 e_0^0 (\partial_t + \Gamma_t) + \gamma^1 e_1^r (\partial_r + \Gamma_r) + \gamma^2 e_2^\theta (\partial_\theta + \Gamma_\theta) + \gamma^3 e_3^\phi (\partial_\phi + \Gamma_\phi)] \Phi = 0,
$$

(52)

where we adopt the massless Dirac field to simplify the perturbation problem.

Based on the spin connections (equation (51)) and the anticommutation relation of $\gamma$ metrics, equation (52) could be reduced to a simple form as follows:

$$
\frac{\gamma^0}{\sqrt{f}} \frac{\partial \Phi}{\partial t} + \sqrt{f} \gamma^1 \left( \frac{\partial}{\partial r} + \frac{1}{r} \right) \Phi + \gamma^2 \left( \frac{\partial}{\partial \theta} + \frac{1}{2 \cot \theta} \right) \Phi + \gamma^3 \left( \frac{\partial}{\partial \phi} \right) \Phi = 0,
$$

(53)

where $\Phi(t, r, \theta, \phi) = f^{-1/4}(r)\Psi(t, r, \theta, \phi)$. Then, we could adopt an ansatz as follows:

$$
\Phi = \frac{\Omega(\theta, \phi)}{r \sqrt{\sin \theta}} e^{-i\omega t} \begin{pmatrix}
  F(r) \\
  F(r) \\
  iG(r) \\
  iG(r)
\end{pmatrix}.
$$

(54)

Substituting the ansatz (54) into equation (53), we can obtain three equations: one equation of $\Omega$ refers to variables $(\theta, \phi)$ and two equations of $G(r)$ and $F(r)$ refer to variable $r$, which are listed as

$$
\begin{align*}
  -i \gamma^1 \gamma^0 \left( \gamma^2 \partial_\theta + \frac{\gamma^3}{\sin \theta} \right) \Omega(\theta, \phi) &= k \Omega(\theta, \phi), \\
  -\omega \frac{dG(r)}{dr} + \frac{k \sqrt{f}(r)}{r} G(r) &= 0, \\
  \omega \frac{dF(r)}{dr} + \frac{k \sqrt{f}(r)}{r} F(r) &= 0,
\end{align*}
$$

(55-57)

where $k = -l$ or $l + 1$ and the coordinate transformation $dr = f(r) \, ds$ is adopted. Eliminating $F(r)$ (or $G(r)$) in equations (56) and (57), we can obtain two second-order differential equations of $G(r)$ (or $F(r)$),

$$
\frac{d^2 F}{dr^2} + (\omega^2 - V_1) F = 0,
$$

(58)
Figure 1. The potentials $V_1(r, k)$ (solid line) versus radial coordinate $r$ with $k = 1, 2, 3$. Meanwhile, we also draw the Schwarzschild case with the dotted line for comparison.

\[
\frac{d^2 G}{dr^2} + (\omega^2 - V_2)G = 0, 
\]  

(59)

where $V_1$ and $V_2$ are supersymmetric partners with same spectra,

\[
V_1 = \frac{\sqrt{f(r)}}{r^2} \left( |k| \sqrt{f(r)} + r \frac{d f(r)}{dr} + f(r) \right) \ k = l + 1, 
\]  

(60)

\[
V_2 = \frac{\sqrt{f(r)}}{r^2} \left( |k| \sqrt{f(r)} - r \frac{d f(r)}{dr} + f(r) \right) \ k = -l. 
\]  

(61)

In the following, we use equation (58) containing potential $V_1$ to evaluate the QN mode frequencies of the massless Dirac field by the third- and sixth-order WKB approximations. Here, $V_1(r, k)$ is plotted in figure 1, which illustrates clearly that with bigger $|k|$, $V_1$ is higher than that of the Schwarzschild case (dotted lines). Moreover, the gap between KS and Schwarzschild increases greatly with increasing $k$, in particular near the maximum points. So we can expect that the QNMs of fermion perturbations could be live longer and the actual frequencies increase because there is more lower potential for IR modified HL gravity.

According to the potential $V_1(r, k)$ (equation (60)), the massless Dirac QN modes in the KS black hole spacetime satisfies the boundary conditions,

\[
\Phi(x) \sim \exp(\pm i\omega), \quad x \to \pm \infty. 
\]  

(62)

where $\omega = \text{Re}(\omega) + i\text{Im}(\omega)$. The real part $\text{Re}(\omega)$ determines its actual oscillation frequency and the absolute value of the imaginary part $|\text{Im}(\omega)|$ determines the damping rate.

In the various methods to obtain the frequencies of QNMs, the WKB numerical formulas are convenient to give accurate frequency values for the longer lived QNMs. This method is originally shown by Schutz et al [24] and is later developed to the third order by Iyer et al [25, 26]. At a later time, the WKB approximation of QNMs is expanded to the sixth order by Konoplya [27]. Then after that, this method is extensively used in various spacetimes [28]. In
this paper, we numerically calculate the low-lying mode frequencies through the sixth-order WKB formula which has the form [27] as follows:
\[
\frac{iQ_0}{\sqrt{2Q_0}} - \Lambda_2 - \Lambda_3 - \Lambda_4 - \Lambda_5 - \Lambda_6 = n + 1/2,
\]  
where \(Q\) is a ‘reverse potential’ given by \(Q = \sqrt{\omega^2 - V}\). \(Q_i^0\) denotes the \(i\)th derivative of \(Q\) at its maximum point with respect to the ‘tortoise coordinate’ \(r_*\). The results of the third orders could also be obtained by equation (63) without \(\Lambda_4, \Lambda_5\) and \(\Lambda_6\). Considering that the WKB approximation fails to calculate the higher order modes, we evaluate only low-lying QNM modes, i.e. \((k < n)\) by various overtones \(n\). The correctional terms of \(\Lambda_2\) and \(\Lambda_3\) are given in [25, 26]. The correctional terms of \(\Lambda_4, \Lambda_5\) and \(\Lambda_6\) are given in [27]. It turns out that WKB series shows well convergence in all sixth orders for the Dirac field, which is similar to the scalar field case [12, 14]. In this paper, we analyze the effect of parameter HL gravities on QNM modes through two kinds of \(\alpha\): one is fixed and the other is changed.

For the fixed \(\alpha\), we consider only the first five low-lying modes \(k = 1, 2, 3, 4, 5\) with \(0 \leq n < k\). The results are listed in table 1. The QN mode frequencies for positive \(k\) are plotted in figure 2, which illustrates that the real part Re \(\omega\) decreases with increasing mode number \(n\) for the given angular momentum number \(k\). Else, the absolute value of the imaginary part \(|\text{Im} \omega|\) increases as \(n\) increases, which indicates that higher modes decay faster than the low-lying ones. Compared with Schwarzschild results (solid quare points), the Re \(\omega\) of \(\alpha\) is larger than the Schwarzschild limit, while the damping rate \(|\text{Im} \omega|\) is smaller than the pure Schwarzschild case.

For the changed \(\alpha\), we treat 1/2\(\alpha\) changed in [0, 1] as a whole. Three important kinds of low-lying modes, i.e. \((k = 1, n = 0), (k = 2, n = 0)\) and \((k = 3, n = 0)\), are listed in table 2–4, respectively. According to these three tables, we plot the real part Re \(\omega\) and the imaginary part Im \(\omega\) of the third- and sixth-order results in figure 3. Here, it should be noted that the horizontal abscissa denotes the value of 1/2\(\alpha\). We impose the interpretations on these data and draw conclusions from them.

### Table 1. Various low-lying overtones, QN frequencies for a fixed \(\alpha = 0.5\).

| \(|k|\) | \(n\) | Third | Sixth | Schwarzschild (sixth) |
|---|---|---|---|---|
| 1 | 0 | \(0.200\, 527 - 0.071\, 134i\) | \(0.199\, 480 - 0.065\, 511i\) | \(0.182\, 642 - 0.094\, 937i\) |
| 2 | 0 | \(0.419\, 782 - 0.070\, 205i\) | \(0.420\, 578 - 0.069\, 468i\) | \(0.380\, 069 - 0.096\, 366i\) |
| 1 | 0 | \(0.393\, 067 - 0.213\, 644i\) | \(0.396\, 805 - 0.208\, 570i\) | \(0.355\, 860 - 0.297\, 269i\) |
| 3 | 0 | \(0.635\, 027 - 0.070\, 293i\) | \(0.635\, 293 - 0.070\, 174i\) | \(0.574\, 094 - 0.096\, 307i\) |
| 1 | 0 | \(0.617\, 529 - 0.211\, 964i\) | \(0.618\, 963 - 0.211\, 069i\) | \(0.557\, 016 - 0.292\, 717i\) |
| 2 | 0.583\, 072 - 0.356\, 887i | \(0.586\, 030 - 0.353\, 696i\) | \(0.526\, 534 - 0.499\, 713i\) |
| 4 | 0 | \(0.849\, 148 - 0.070\, 367i\) | \(0.849\, 265 - 0.070\, 339i\) | \(0.767\, 354 - 0.096\, 270i\) |
| 3 | 0.836\, 227 - 0.211\, 639i | \(0.836\, 870 - 0.211\, 407i\) | \(0.754\, 300 - 0.290\, 969i\) |
| 4 | 0 | \(0.810\, 528 - 0.354\, 547i\) | \(0.811\, 997 - 0.353\, 679i\) | \(0.729\, 754 - 0.491\, 909i\) |
| 3 | 0.772\, 412 - 0.500\, 217i | \(0.774\, 470 - 0.498\, 069i\) | \(0.696\, 728 - 0.702\, 337i\) |
| 5 | 0.106\, 830 - 0.070\, 410i | \(0.106\, 890 - 0.070\, 401i\) | \(0.960\, 293 - 0.096\, 254i\) |
| 1 | 1.052\, 590 - 0.211\, 548i | \(1.052\, 930 - 0.211\, 469i\) | \(0.949\, 759 - 0.290\, 149i\) |
| 2 | 1.032\, 170 - 0.353\, 663i | \(1.032\, 970 - 0.353\, 352i\) | \(0.929\, 491 - 0.488\, 114i\) |
| 3 | 1.001\, 690 - 0.497\, 438i | \(1.002\, 930 - 0.496\, 649i\) | \(0.901\, 073 - 0.692\, 514i\) |
| 4 | 0.961\, 423 - 0.643\, 574i | \(0.962\, 684 - 0.642\, 041i\) | \(0.866\, 730 - 0.905\, 116i\) |
Figure 2. Massless Dirac QN mode frequencies. For the convenience of comparison, double results including the KS solution (solid triangle) and the Schwarzschild solution (solid square) are given simultaneously.

Table 2. The QN frequencies of null overtone modes ($n = 0, k = 1$).

| $1/2\alpha$ | Third       | Sixth       |
|-------------|-------------|-------------|
| 0           | 0.176 452 $– 0.100 109i$ | 0.182 642 $– 0.094 937i$ |
| 0.1         | 0.179 189 $– 0.098 175i$ | 0.184 899 $– 0.092 152i$ |
| 0.2         | 0.181 871 $– 0.096 140i$ | 0.187 179 $– 0.089 352i$ |
| 0.4         | 0.187 091 $– 0.091 667i$ | 0.191 448 $– 0.083 907i$ |
| 0.5         | 0.189 632 $– 0.089 156i$ | 0.193 342 $– 0.081 265i$ |
| 0.6         | 0.192 118 $– 0.086 395i$ | 0.195 055 $– 0.078 615i$ |
| 0.8         | 0.196 809 $– 0.079 810i$ | 0.197 915 $– 0.072 858i$ |
| 1           | 0.200 527 $– 0.071 134i$ | 0.199 480 $– 0.065 511i$ |

(1) The real part $\text{Re} \omega$ increases with increasing $1/2\alpha$ and the absolute value of the imaginary part $|\text{Im} \omega|$ decreases with increasing $1/2\alpha$, which indicates that these QNMs could live longer.

(2) The gap between the third- and sixth-order results is visibly displayed in the imaginary part. Their real parts basically have the same values, except for $k = 1$ modes. In general, the average relative magnitudes of the gaps are approximately given as

$$\frac{|6^{th} \text{Im} (\omega) – 3^{rd} \text{Im} (\omega)|}{3^{rd} \text{Im} (\omega)} \approx 10\%, \quad (64)$$

$$\frac{|6^{th} \text{Re} (\omega) – 3^{rd} \text{Re} (\omega)|}{3^{rd} \text{Re} (\omega)} \approx 0. \quad (65)$$

Hence, the orders of WKB approximations have the tremendous bearing on the damping rate, more than on the actual frequency.
Figure 3. The real parts $\text{Re}\, \omega$ and the imaginary parts $\text{Im}\, \omega$ versus radial $r$ under the third-order (solid triangle) and the sixth-order (solid pentagram) WKB approximations.

Table 3. The QN frequencies of null overtone modes ($n = 0, k = 2$).

| 1/2$\alpha$ | Third                        | Sixth                        |
|-------------|------------------------------|------------------------------|
| 0           | $0.378627 - 0.096542\, 4i$   | $0.380069 - 0.096366\, i$   |
| 0.1         | $0.381664 - 0.094904\, 6i$   | $0.383040 - 0.094794\, i$   |
| 0.2         | $0.384885 - 0.093172\, 8i$   | $0.386218 - 0.093061\, i$   |
| 0.4         | $0.391946 - 0.089317\, 9i$   | $0.393230 - 0.089069\, i$   |
| 0.5         | $0.395827 - 0.087121\, 2i$   | $0.397087 - 0.086766\, i$   |
| 0.6         | $0.399797 - 0.084675\, 9i$   | $0.401200 - 0.084209\, i$   |
| 0.8         | $0.409203 - 0.078696\, 4i$   | $0.410286 - 0.078038\, i$   |
| 1           | $0.419782 - 0.070205\, 1i$   | $0.420578 - 0.069468\, i$   |

Moreover, when the horizontal abscissa of figure 3 approaches the Schwarzschild case, namely $1/2\alpha \rightarrow 0$ (or $\alpha \rightarrow +\infty$), the real frequencies $\text{Re}\, \omega$ decrease and the damping rate $|\text{Im}\, \omega|$ increases. In other words, the HL gravities have the longer lived and bigger actual frequency than that of the usual Schwarzschild case. This specific phenomenon is also observed in the massless scalar field perturbation [12, 14].
Table 4. The QN frequencies of null overtone modes ($n = 0$, $k = 3$).

| $1/2\alpha$ | Third       | Sixth       |
|-------------|-------------|-------------|
| 0           | 0.573 685  − 0.096 324i | 0.574 094  − 0.096 307i |
| 0.1         | 0.578 089  − 0.094 773i | 0.578 503  − 0.094 783i |
| 0.2         | 0.582 754  − 0.093 108i | 0.583 169  − 0.093 135i |
| 0.4         | 0.592 999  − 0.089 343i | 0.593 402  − 0.089 371i |
| 0.5         | 0.598 659  − 0.087 176i | 0.599 051  − 0.087 189i |
| 0.6         | 0.604 747  − 0.084 754i | 0.605 126  − 0.084 744i |
| 0.8         | 0.618 500  − 0.078 810i | 0.618 839  − 0.078 742i |
| 1           | 0.635 027  − 0.070 293i | 0.635 293  − 0.070 173i |

5. Conclusion

In this paper, we have investigated fermion tunneling and perturbation in the IR modified Hořava–Lifshitz (HL) gravity. We summarize what has been achieved.

(1) For the fermion Hawking radiation, we consider the symmetrical characteristic of spacetime and adopt the action with the form of equation (33). Through the decomposition of Dirac equation (16), we can obtain the imaginary part of the fermion action which could help us obtain the tunneling probability according to equation (42). Then, on the benefit of $\Gamma = \exp(-\beta\omega)$, the tunneling temperature could be given as equation (44). So, according to the first law $dM = T\,dS$, we have a tunneling entropy (equation (48)) naturally. It is interesting that the tunneling Hawking temperature and the tunneling entropy are in agreement with that obtained by calculating surface gravity.

(2) For the fermion perturbation, we obtain low-lying damped quasinormal (QN) modes by using the sixth-order WKB approximations, as well as the third-order formulas. In order to obtain a detailed analysis on these obtained QN frequencies, we adopt two kinds of methods: one is to fix the Hořava parameter $\alpha$ (fixed $\alpha$) and the other is to change $\alpha$ in the range $[0, +\infty]$ (varied $\alpha$).

For the fixed $\alpha$ case, the results show quite an impact on the HL gravity on QN frequencies. This is profoundly manifested in the following ways: the actual frequencies become bigger and the damping rate becomes slower, which indicates these low-lying modes could be longer lived than that of usual Schwarzschild. This fact could also be explained by the perturbation potential $V_1$ (equation (60)) illustrated in figure 2, which shows that the potential contained HL gravity (solid lines) is lower than that of Schwarzschild (dotted lines).

For the varied $\alpha$ case, we have numerically calculated three kinds of important low-lying modes ($k = 1$, $n = 0$), ($k = 2$, $n = 0$) and ($k = 3$, $n = 1$) by the third- and sixth-order WKB approximations. The results listed in tables 2, 3 and 4 show three facts as follows. (i) With increasing $1/2\alpha$, the real part of frequencies increases and the absolute value of the imaginary part decreases, which is illustrated in figure 3. In other words, if the parameter $1/2\alpha$ becomes larger, the actual frequency of QNMs will be larger with a longer damping rate. (ii) The real part is not sensitive to the third- or sixth-order WKB approximates. This fact is also illustrated in the left curves in figure 3, except for some small $1/2\alpha$ modes of $k = 1$. (iii) Against the real part, the imaginary part is sensitive to our WKB approximates methods. The gap of the third- and sixth-order results is basically unchanged. This fact is also illustrated in the right curves in figure 3. In other words, if this specific information could be tested by LIGO, VIRGO, TAMT, GEO600, it will support HL gravity forcefully and energetically.
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