New type of stochastic resonance in an active bath

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We report on the observation of bona fide stochastic resonance (SR) in a nonGaussian active bath without any periodic forcing. Particles hopping in a nanoscale double-well potential under the influence of correlated Poisson noise display a series of equally-spaced peaks in the residence time distribution. Maximal peaks are measured when the mean residence time matches a double condition on the interval and correlation timescales of the noise, demonstrating a new type of SR. The experimental findings agree with a simple model that explains the emergence of SR without periodicity. Correlated nonGaussian noise is common in living systems, suggesting that this type of SR is widespread in this regime.

Stochastic resonance (SR) occurs when a weak periodic signal is enhanced in the presence of noise, and the enhancement shows resonant behavior as the noise is tuned [1]. A prototypical setting of SR is a Brownian particle hopping in a bistable potential under the influence of thermal noise [2]. When periodic forcing is added, it may synchronize with the thermally-induced hopping process, as manifested by peaks in the residence time distribution (RTD) corresponding to the external force period. SR is a universal phenomenon that has been explored in diverse fields ranging from climatology to biology [1,3-7]. The validity of SR as a bona fide resonance that attains maximal synchronization has been extensively discussed [8-12].

Can random signals also induce bona fide resonance? – Gammaitoni et al. [1,13] demonstrated bona fide resonance under a periodic signal with random amplitudes. However, whether fully random fluctuations, with random amplitude and random period, lead to bona fide resonance remains an open question. The present study answers this question affirmatively by demonstrating SR of a Brownian particle in a double-well potential under nonGaussian active fluctuations without any periodic forcing.

Such active baths with nonGaussian statistics have become a timely topic, as mounting evidence suggests they are prevalent in living systems. In the active baths around swimming bacteria [14-21] or in the cellular milieu [22-25], diffusion is governed by the coaction of uncorrelated thermal fluctuations of the solvent and correlated fluctuations induced by active components. While a common model of active baths has been the Active Ornstein–Uhlenbeck (AOU) noise [14,26], theoretical studies showed that this Gaussian process suppresses SR as correlation increases [27]. This result is considered counter-intuitive, as active noise generally enhances transport and diffusion, calling for a more realistic active bath model. Indeed, recent experiments provided evidence for nonGaussian noise processes in biological and artificial systems of active swimmers [16,21,28-30], as well as in living cells [31,32] which the Gaussian AOU process cannot capture. On the other hand, a recent theoretical study proposed that the non-Gaussian diffusion observed in a bacterial suspension can be explained in terms of a colored Poisson process [33]. This motivates us to explore the nonGaussian nature of active baths using Exponentially Correlated Poisson (ECP) noise.

Here, we investigate the stochastic dynamics of a Brownian particle in an optically-generated symmetric double-well potential under the influence of an ECP noise. The ECP process generates finite-amplitude active bursts, decaying exponentially with correlation time \( \tau_c \) and separated by random intervals with an average time \( \tau_p \). We find that the particle position distribution exhibits two symmetric peaks that generally follow a Gaussian distribution. However, when \( \tau_c < \tau_p \) and the active noise is stronger than the thermal noise, the peaks develop exponential side-tails, a signature of a nonGaussian active bath [16,29,34]. In this nonGaussian regime, the RTD of the particle exhibits a series of exponentially decaying peaks at integral multiples of \( \tau_p \). The strength of the first peak can be maximized by changing either \( \tau_c \) or \( \tau_p \), according to the resonance condition \( 4\tau_c \approx \tau_p \approx \bar{\tau}_d/2 \), where \( \bar{\tau}_d \) is the average residence time. This observation establishes the existence of bona fide stochastic resonance without periodic forcing, as the observed dynamics is driven solely by the nonGaussian ECP process.

We further examined how ECP affects the generic form of SR, i.e., under periodic forcing. We observe that the suppression of SR in the regime of Gaussian fluctuation, a first demonstration of the predicted degradation of SR in the presence of colored noise [27]. Strikingly, we find that the SR significantly recovers in the nonGaussian regime of ECP noise, \( \tau_p > \tau_c \), even for large \( \tau_c \). Overall, our results propose the correlated-Poisson process as a strong generator of SR, with direct implications on stochastic processes in living systems.

Active Bath Model– We consider the one-dimensional motion of a Brownian particle (a 2 \( \mu \)m diameter polystyrene bead) in a symmetric double-well potential, \( V_{pot}(x) = E_0[-2(x/l_m)^2 + (x/l_m)^4] \), where \( x \) is the particle
position, $\pm x_m$ are the potential minima, and $E_b$ is the barrier height in an active bath of temperature $T = 296 \pm 1 \text{K}$.

The motion of the particle is described by the overdamped Langevin equation:

$$\gamma \frac{dx}{dt} = -\frac{\partial V_{\text{ext}}(x)}{\partial x} + \xi_h + \xi_{\text{act}}.$$  (1)

The thermal noise $\xi_h$ is Gaussian distribution with zero mean and no memory, $\langle \xi_h(t) \xi_h(t') \rangle = 2\gamma^2 D \delta(t-t')$, where $\gamma$ is the dissipation coefficient in the solvent and $D = k_B T / \gamma$ is the thermal diffusivity of the particle. Without active noise, the particle is in thermal equilibrium, and the average barrier crossing time is the Kramers time, $\tau_K = \tau_e \exp(E_b / k_B T)$, where $\tau_e = 2\pi \sqrt{\gamma \tau_b}$ is the relaxation time associated with the potential minima at $\pm x_m$ and $\tau_b = \gamma x_m^2 / 4 E_b$ is the relaxation time associated with the negative curvature of the potential at the barrier maximum.

FIG. 1. (a) PDF of the particle position in the double-well potential, with $E_b / k_B T = 3$ and $x_m = 50 \text{nm}$, in the presence of ECP noise of fixed strength $f_{\text{act}} \approx 0.5 \text{pN}$ and Poisson interval $\tau_p \approx 28 \text{ms}$, for correlation times $\tau_c \approx 21 \text{ms}$ (olive), 7 ms (black), 0.28 ms (blue, numerical result). The gray curve is the theoretical PDF, $P(x) \sim \exp(-U(x) / k_B T)$. (b) The particle’s residence time distribution (RTD) for $f_{\text{act}} \approx 0.5 \text{pN}$, $\tau_c \approx 0.7 \text{ms}$, and $\tau_p \approx 0$. Here, the average residence time $\tilde{\tau}_p \approx 12.6 \pm 0.1 \text{ms}$ is much lesser than the Kramers’ time, $\tau_K \approx 79 \text{ms}$ (dashed vertical line). (c) The particle’s RTD, where the residence time is normalized by Poisson interval $\tau_p / \tau_p$, for the same $f_{\text{act}}$ and $\tau_p$ as in (a) with $\tau_c \approx 21 \text{ms}$ (olive), 7 ms (black), and 3.5 ms (purple). Inset: (Numerical result) Plot of the strength of the first peak as a function of $\tau_c / \tau_p$. The dashed vertical line denotes $\tau_c / \tau_p = 0.25$. (d) (Numerical result) the strength of the first peak in the RTD for the particle as a function of $\tau_p / \tilde{\tau}_p$ for fixed $\sqrt{c} \approx 20 \text{pN}$ and $\tau_c \approx 1.25 \text{ms}$ (orange), 7.5 ms (olive), and 20 ms (violet). The gray open circle is the plot of the second peak strength for $\tau_c \approx 7.5 \text{ms}$. The vertical dashed line corresponds to $\tau_p / \tilde{\tau}_p = 0.5$. (e) Normalized average residence time as a function of $\tau_p$ for the like-colored data in (d). (f) Effective shape of the double-well potential in the presence of ECP noise. The positive active burst $\xi_{\text{act}}(t)$ (blue curve) supplies energy to the particle during $\tau_c$, effectively lowering the barrier height by lifting the left well. Subsequently, the strength of the active burst is significantly reduced during time $\tau_p - \tau_c$, the left potential well is lowered back to its original position, and the thermal condition is recovered.
The active noise $\xi_{\text{act}}$ is an Exponentially Correlated Poisson (ECP) process with zero mean $\langle \xi_{\text{act}} \rangle = 0$, and correlation
\[
\langle \xi_{\text{act}}(t)\xi_{\text{act}}(t') \rangle = f_{\text{act}}^2 \exp(-|t-t'|/\tau_c) .
\] (2)

Here, $f_{\text{act}} = \sqrt{C/(1+\lambda)}$ characterizes the strength of active noise, where $C$ is the variance of the active burst, and $\lambda$ is the Poisson parameter that determines the average number of waiting events between successive kicks. The average waiting time is then $\tau_p = \lambda \Delta t$, where $\Delta t$ is the noise input interval ($\Delta t = 0.07$ ms for this setup). The significance of our noise generation approach is that all three parameters, $C$, $\tau_c$, and $\tau_p$, can be independently varied. In particular, $\xi_{\text{act}}$ becomes exponentially-correlated Gaussian noise (AOU) in the limit $\tau_p = 0$, and white Gaussian noise when both $\tau_c$ and $\tau_p$ vanish. On the other hand, for $\tau_c = 0$ and $\tau_p > 0$, $\xi_{\text{act}}$ is white Poisson noise [35,36] (see Fig. S1(a) in the SI).

Double-well potential generation.— The double-well potential was generated experimentally using the optical feedback trap technique [37-42]. To this end, a colloidal particle suspended in deionized water was trapped in a harmonic potential, $V_{\text{op}}(x,t) = (k/2)(x-x_0)^2$, generated using optical tweezers, where $x_0$ is the center of the trap and $k$ is its stiffness. The particle position $x$ with respect to the trap center $x_0$ was measured with high precision ($\sim$1 nm). Then, the feedback force, $f_{\text{BW}} = -\partial_x V_{\text{BW}}(x)$, required for generating the double-well potential was computed. To impose active noise, we added to the feedback force $f_{\text{BW}}$ numerically-generated ECP noise, $\xi_{\text{act}}(t) = -k y(t)$, with normally-distributed random amplitude $y$ of mean zero, $\langle y \rangle = 0$, and variance $\sigma^2_y$. Each $y(t)$ is randomly drawn from a Poisson process with an average interval $\tau_p$, and decays exponentially with correlation time $\tau_c$ (see S1(b) in the SI). The resultant force $f_{\text{BW}} + \xi_{\text{act}}$ was applied to the particle as a feedback force, by shifting the trap center to $x(t) = (1+4E_k/\lambda^2 k)x(t)-(4E_k/\lambda^4 k)x^3(t)+y^2(t)$. In this study, we set $E_k = 3k_BT$ and $x_0 = 50$ nm. In addition, the trap stiffness of the optical trap $k \approx 10$ pN/μm was obtained experimentally from the equipartition theorem [43].

Generation of nonGaussian PDFs by ECP noise— Figure 1(a) shows the PDFs of the particle position in the symmetric double-well potential, in the presence of ECP noise of fixed strength $f_{\text{act}} \approx 0.5$ pN and Poisson time $\tau_p \approx 28$ ms ($\gg \tau_c \approx 4$ ms). For $\tau_c < \tau_p$, the PDF exhibits two symmetric peaks, centered around $\pm x_0$. The central region of the PDF is described by a Boltzmann distribution $P(x) \sim \exp(-U(x)/k_BT)$, where $U(x)$ is the effective double-well potential (see Fig. S2 in the SI), albeit with a reduced effective barrier height. However, for $\tau_c > \tau_p$, both the effective barrier height and well separation are larger than their values in a thermalized system. Remarkably, each peak is Gaussian near the center, develops nonGaussian outer tails. The PDFs become nonGaussian only when $\tau_c \leq \tau_p$ and $f_{\text{act}} \gtrsim f_{\text{th}}$, where $f_{\text{th}} = (8k_BTc/\lambda^4)^{1/2} \approx 0.4$ pN is the thermal force strength at the potential wells. This condition for obtaining a nonGaussian PDF is similar for diffusion in a simple harmonic potential (Fig. S3 in the SI).

Bona fide resonance without periodic forcing.— From the measurement of the particle RTD, we estimated the probability that the particle remains within the potential well for time $\tau_d$, which is determined based on the particle trajectories (as in [1,8,13]; see Fig. S4 in the SI for typical trajectories). In the absence of active noise, the RTD decays exponentially, with a mean residence time $\bar{\tau}_d$ equal to the Kramers time $\tau_k$ [1,2,44]. In the presence of active noise, in the regime $\tau_p < \tau_c$, the RTD remains exponential (Fig 1(b)), with $\bar{\tau}_d < \tau_k$.

However, for $\tau_p \gtrsim \tau_c$ and $f_{\text{act}} \gtrsim f_{\text{th}}$, the RTD displays a series of consecutive peaks, each centered at an integral multiple of $\tau_p$, i.e., $(\tau_d)_n = n\tau_p$ (Fig. 1(c)). The heights of these peaks decrease exponentially with their order $n$. In addition, the height of the first peak increases with the correlation time $\tau_c$, and assumes a maximum at a finite value of $\tau_c$. For further quantification, we measured the strength of the first peak (area under the peak) [8,45] (Fig. 1(c) inset), and found that it attains a maximum at $\tau_c \approx \tau_p/4$.

Furthermore, we measured the RTD of the particle as a function of $\tau_p$ while maintaining the active burst strength $\sqrt{C}$ and the correlation time $\tau_c$ constant. Similar to the above, a series of exponentially-decaying peaks centered at $n\tau_p$ are evident for $\tau_p \gtrsim \tau_c$ (Fig. S5 in the SI). For a given $\tau_c$, the strength of the first peak is maximal when $\tau_p \approx \tau_c/2$ (Fig. 1(d)). Note that for a fixed $\sqrt{C}$, a change in the Poisson interval changes the strength of active noise $f_{\text{act}} = \sqrt{C/(1+\lambda)}$. Thus, we find that maximal synchronization can be achieved between the particle residence time and active noise arrival interval by appropriately selecting the active noise parameters ($\tau_c$, $\tau_p$, or $f_{\text{act}}$). The pronounced maximum of the first peak strength demonstrates bona fide SR [1,13,46]. Importantly, the SR observed here is generated solely by the ECP noise, without any periodic modulation of the double-well potential. For $\tau_p \gg \tau_k$, the RTD again shows monotonic exponentially-decaying behavior because the
kicking events are rare, and the mean residence time $\bar{\tau}_d$ saturates to the Kramers time $\tau_k$, as shown in Fig. 1(e).

The observed barrier crossing enhancement and particle synchronization in the presence of ECP noise can be intuitively explained: The active noise randomly injects energy into the system, with a mean interval $\tau_p$, and each pulse decays with a correlation time $\tau_c$. For $\tau_p < \tau_c$, several bursts kick the particle during its thermal relaxation time, thus increasing its effective temperature. As a result, active noise drives the system faster than the thermal relaxation, which enhances the barrier crossing rate. For $\tau_p > \tau_c$ and $\tau_c$, each active burst drives the particle up to its decay time $\tau_c$, and ceases to act on the particle during time interval $\tau_p - \tau_c$, allowing to recover the thermal condition (Fig. 1(f)). Thus, non-Gaussian noise randomly modulates the barrier height and potential-well separation, with an average modulation period $\tau_p$. The best timing for the particle to cross the barrier is when the effective barrier height is the lowest. This optimality be achieved by varying either $\tau_p$, which controls the noise strength $f_{aw}$, or $\tau_c$, which controls the energy input duration $\tau_r$ and relaxation duration $\tau_p - \tau_r$. Thus, we see that the resonance condition, $4\tau_c \approx \tau_p \approx \bar{\tau}_d/2$, can be achieved by varying the noise parameters.

Recovery of generic SR by ECP noise.— To gain further insight, we studied the generic form of SR with asymmetric modulation of the double-well potential in the presence of ECP noise. To this end, the double-well potential was periodically tilted, $V(x,t) = V_{DW}(x) - Ax\sin(2\pi t/\tau_{mod})$, with an amplitude $A$ and period $\tau_{mod}$. In the absence of active noise and for modulation time lesser than the Kramers time, $\tau_{mod} < \tau_k$, the RTD shows a series of peaks centered at odd multiples of $\tau_{mod}/2$ (black curve in Fig. 2(a)). On increasing the modulation time toward the resonant condition $\tau_{mod} \approx 2\tau_k$, the barrier crossing rate of the particle becomes synchronized with the modulation period, and a single peak centered at $\tau_{mod}/2$ is observed (blue curve in Fig. 2(a)). The SR phenomenon under periodic forcing can also be identified through the power spectrum density (PSD) of the particle fluctuations [1]. A sharp peak is observed at the modulation frequency and a weak peak at the third harmonic (Fig. 2(a) inset). Thus, our optical feedback trap method can precisely measure bona fide SR in a thermal bath under periodic forcing. Compared to the previous experimental works [44,46], which studied the SR of Brownian particles in double-well potentials with inter-well separation greater than 1 $\mu$m, we demonstrated here SR in a nanoscale double-well potential well separated by $2\chi_m = 100$ nm.

In the presence of ECP noise at $\tau_p \approx 0$, corresponding to the Gaussian regime of the active bath we observe suppression of the SR: If the barrier height is modulated sinusoidally with a period $\tau_{mod} \approx 2\tau_k$, the intensities of the resonant peaks in the RTD as well as PSD decrease as the correlation time $\tau_c$ increases (Fig. 2(b) and inset). The peaks disappear completely when $\tau_c \gg \tau_p$ and the active noise is stronger than the thermal noise. Our experimental observation agrees with the theoretical prediction in [27]. However, in the non-Gaussian regime of the active bath, the

![Figure 2](image-url)
resonant peak reappears at finite $\tau_p$, as shown in Fig. 2(c). Likewise, a sharp peak at the modulation frequency is observed in the PSD (Fig 2(c) inset). The resonant peaks increase with $\tau_p$ and recover back to the purely thermal level when $\tau_p \gg \tau_K$, even active noise stronger than the thermal noise, $f_{aw} \geq f_a$ (Fig. S6 in the SI). Thus, we recovered bona fide resonance in the active bath under periodic forcing.

SR recovery in the presence of ECP noise at finite nonzero intervals, $\tau_p > 0$, can be explained as follows: For $\tau_p << \tau_c$, the ECP noise supplies energy to the particle continuously and in a random direction for an average time $\tau_c$. This noise might therefore counteract the barrier crossing process, with its typical timescale $\tau_K$, and the tilting with its period $\tau_{mod}$, thereby repressing the resonant behavior for long correlations $\tau_p > \tau_p \gg \tau_c$. However, for $\tau_p > \tau_c \gg \tau_p$, the noise strength decreases as $f_{aw} \sim (1+\lambda^2)^{-1/2}$; furthermore, each active pulse completely decays before the arrival of another. During this waiting period, $\tau_p - \tau_c$, the particle is free of active noise (as in Fig. 1(f)) and equilibrate by dissipating energy into the thermal bath. Thus, the thermal SR condition is recovered. This is evident from our observation that the central region of the effective potential—for which the PSD shows a sharp resonant peak (Fig. S6 in the SI)—fits well to a thermally-activated potential (Fig. S6 inset in the SI).

To sum, we studied the dynamics of a colloidal particle in a symmetric double-well potential in the presence of exponentially-correlated Poisson (ECP) noise. The RTD exhibited a series of peaks at integral multiples of the noise arrival interval. The strength of the first peak was maximized by either changing the correlation time or the Poisson interval, demonstrating bona fide SR in the Brownianian system without symmetry breaking. The generic form of SR with asymmetric modulation of the double-well potential, which deteriorates with Gaussian correlated noise, was recovered with nonGaussian correlated noise having a large Poisson interval. The findings of this study indicate that nonGaussian active fluctuations may lead to the synchronization of various biomolecular processes, such as protein folding, enzymatic reactions, and signal transduction inside living cells.

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Figures

FIG. S3. (a) Traces of the Poisson white noise of average interval $\tau_p = 50$ ms and variance $\sigma^2/(1 + \lambda) = (16 \text{ nm})^2$, where $\lambda = \tau_p/\Delta t$, generated from Gaussian white noise of variance $\sigma^2 = (500 \text{ nm})^2$, using Poisson counting process (Eq. S2). (b) The exponentially correlated Poisson noise of variance $\sigma^2/(1 + \lambda) = (16 \text{ nm})^2$ with $\tau_p = 50$ ms and $\tau_c = 10$ ms generated from white Poisson noise in panel (a) using Eq. (S3). (c) Probability distribution function (PDF) of the noise in panel (b). The solid red curve is the Gaussian fitting. (d) Autocorrelation function (ACF) of the noise in (b). The solid curve is fit to the exponential decay with a decay time of $9.9 \pm 0.7$ ms.
FIG. S4. Effective double-well potentials $U(x)/k_B T \sim -\ln P(x)$ for the like colors data in Fig. 1(a) in the main text. The dotted curves represent the experimental results. The solid curves were obtained from the numerical simulation of Eq. (2) in the main text. The gray curve is the theoretical plot of the symmetric double-well potential with $E_u/k_B T = 3$ and $x_u = 50$ nm.

FIG. S5. PDF of the particle position in harmonic potential $V_{op}(x) = (k/2)(x - x_c)^2$ in the presence of ECP noise. Here, the active noise is injected to the particle in the form of feedback force $f_{act}(t) = -k y(t)$ that corresponds to the shift of the potential center by $x_c(t) = x(t) - y(t)$. (a) (experimental result) PDF of the particle position in the harmonic potential of stiffness $k = 9.1 \text{ pN} \mu\text{m}^{-1}$ in the thermal bath (gray circles), in the presence of non-Gaussian noise of strength $\sqrt{C} = 4.6 \text{ pN}$ and correlation time $\tau_c = 17.5 \text{ ms}$ with Poisson interval $\tau_p = 14 \text{ ms}$ (wine circles), and 35 ms (dark cyan bars). The solid curves are the Gaussian fittings. (b) (numerical result) PDF of the particle position in the harmonic potential in the thermal bath (white circles), in the presence of non-Gaussian noise of fixed strength $f_{act} = 0.5 \text{ pN}$ and correlation time $\tau_c = 25 \text{ ms}$ with $\tau_p = 5 \text{ ms}$ (blue circles), 50 ms (pink bars), and 250 ms (black bars). The solid curves are the Gaussian fittings. The PDFs are non-Gaussian only when $\tau_c \leq \tau_p$ and $f_{act} \gg f_{th}$, where $f_{th} = \sqrt{k_B T/\mu} \approx 0.2 \text{ pN}$ is the thermal strength.
FIG. S6. Trajectories of the particle in a double-well potential, $E_b/k_B T = 3$ and $x_m = 50 \text{ nm}$, in the presence of ECP noise of fixed strength $f_{\text{act}} \approx 0.5 \text{ pN}$ and Poisson interval $\tau_p \approx 28 \text{ ms}$ with correlation time (a) $\tau_c \approx 0.28 \text{ ms}$ (blue, numerical result), (b) 7 ms (black), and (c) 21 ms (green). The PDFs in Fig.1(a) of the main text are from these trajectories.

FIG. S7. (numerical result) Normalized residence time distributions $\tau_d/\tau_p$ of the particle in double-well potential with $E_b/k_B T = 3$, $x_m = 50 \text{ nm}$ for fixed $\sqrt{C} \approx 20 \text{ pN}$ with $\tau_c = \tau_p \approx 7.5 \text{ ms}$ (yellow), with $\tau_c \approx 2 \text{ ms}$ and $\tau_p \approx 8 \text{ ms}$ (black bars), and 15 ms (pink).
FIG. S8. (numerical result) Power spectral density in the presence of the ECP noise of strength $f_{\text{act}} \approx 1\, \text{pN}$ (greater than thermal strength $f_{\text{th}} \approx 0.4\, \text{pN}$) with $\tau_c \approx 25\, \text{ms}$ and $\tau_p \approx 1000\, \text{ms}$, under the same resonant condition of Fig. 2(c) in the main text. Inset: the effective double-well potential (blue circles) for the same data in the main panel. The cyan solid curve fits well with the symmetric double-well potential $V_{\text{DW}}(x)$ of $E_n/k_B T = 3$ and $x_m = 50\, \text{nm}$ with fitting parameter $E_n/k_B T = 3.09 \pm 0.06$ and $x_m = 54.9 \pm 0.2\, \text{nm}$.

Noise generation procedure
The active noise in the main text is generated as follows: Let $q_n$ be the sequence of identically distributed random numbers that follow a Gaussian distribution,

$$P(q) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{q^2}{2\sigma^2}\right). \quad (1)$$

From these random numbers, we can generate Poisson white noise as [2,3]

$$q_p(t) = \sum_{i=1}^{N(t)} q_i \delta(t-t_i). \quad (2)$$

where $N(t)$ represents a Poisson counting process with mean arrival time $\tau_p$ (see Fig. S1 (a)). The exponentially correlated Poisson numbers $y_n$ of correlation time $\tau_c$ can be generated recursively using the following relation [1],

$$y_n = q_0 \exp(-n/\tau_c) + \sqrt{1 - \exp(-2/\tau_c)} \sum_{i=1}^{n} q_i \exp(-i/\tau_c). \quad (3)$$

The time traces of the ECP are shown in Fig. S1(b). The time autocorrelation of the ECP noise is shown in Fig. S1(d).

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