From zero resistance states to absolute negative conductivity in microwave irradiated 2D electron systems

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Recent experimental results regarding a 2D electron gas subjected to microwave radiation reveal that magnetoresistivity, apart from presenting oscillations and zero resistance states, can evolve to negative values at minima. In other words, the current can evolve from flowing with no dissipation, to flow in the opposite direction of the dc bias applied. Here we present a theoretical model in which the existence of radiation-induced absolute negative conductivity is analyzed. Our model explains the transition from zero resistance states to absolute negative conductivity in terms of multiphoton assisted electron scattering due to charged impurities and shows how this transition can be driven by tuning microwave frequency and intensity. This opens the possibility of controlling the magnetoconductivity in microwave driven nanodevices and understanding the novel optical and transport properties of such devices.

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The effect of an AC field on the electronic transport properties of nanostructures has been an active research topic in the last years. One reason for this is that the AC field can profoundly modify the electronic structure and the dynamical properties of electrons in the nanostructure. The application of an external AC potential also allows the electronic properties to be tuned in a controllable way\textsuperscript{22}. Ten years ago, transport experiments on AC-driven weakly coupled semiconductors superlattices revealed a fascinating, non-intuitive, behavior: for certain parameters of the AC potential and stationary electric field, the electronic current flowed uphill presenting absolute negative conductivity (ANC)\textsuperscript{23}. In the rapid developing field of nanoelectronics, it can be expected that nanodevices, will soon routinely incorporate two-dimensional electron systems. A deep knowledge of how this structures respond to external electromagnetic fields is thus of high importance.

Recently two experimental groups\textsuperscript{24,25} have announced the existence of oscillations and zero resistance states (ZRS) in the longitudinal magnetoconductivity ($\rho_{xx}$) of two dimensional electron systems (2DES) subjected to microwave (MW) radiation and a perpendicular magnetic field ($B$). One of the most controversial topics in this field, has been the existence of ANC and even the very presence of ZRS has been also questioned. Most experimental works\textsuperscript{15,16,17,18,19,20,21,22,23,24,25}, report clearly such vanishing dissipation states but not ANC. Only Willett et al\textsuperscript{12}, and very recently Zudov et al\textsuperscript{12}, have reported minima in which $\rho_{xx}$ is distinctly negative. On the other hand many theoretical contributions have been presented to explain $\rho_{xx}$ oscillations with $B$ and the possibility of ZRS and ANC\textsuperscript{24,15,16,17,18,19,20,21,22,23,24,25}.

Here we present a microscopical model which explains how the system evolves from ZRS to ANC. The proposed theory is based on how the Larmor orbits dynamics are driven by the MW field and the external DC bias ($E_{DC}$) in the transport direction ($x$-direction). It shows how this transition can be driven by tuning the MW parameters. We start from an exact expression for the electronic wave function dressed by photons. This many body wave function is expressed in terms of photosatellites and energy sidebands\textsuperscript{28,29}. At high MW-power and low frequency, multi-photon processes become relevant in assisting electron-charged impurity scattering, responsible for $\rho_{xx}$. This exact solution for the electronic wave function of a 2DES in a perpendicular $B$-field, a DC electric field and MW radiation, is given by\textsuperscript{23,28,29}.

$$\Psi(x,t) \propto \phi_n(x - X - x_d(t), t) \sum_{p=-\infty}^{\infty} J_p \left[ \propto \frac{eE_0x}{\hbar w} \right] e^{ipxt}(1)$$

where $e$ is the electron charge, $\phi_n$ is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator, $w$ the MW frequency, $E_0$ the intensity for the MW field, $X$ is the center of the orbit for the electron motion, $x_d = A \cos wt$ where $A \propto \frac{eE_0}{m^2w^2}$ and $J_p$ are Bessel functions. The wave function includes the energy sidebands $\epsilon_n, \epsilon_n \pm \hbar w, \epsilon_n \pm 2\hbar w, ..., \epsilon_n \pm phw$ and shows that, due to the MW radiation, the centers of electronic orbits are not fixed, but instead oscillate back and forth harmonically with $w$ and amplitude $A$. Electrons suffer scattering due to charged impurities that are randomly distributed in the sample. To proceed we calculate the electron-charged impurity multi-photon assisted transition rate $W_{n,m}$, from an initial state $\Psi_n(x,t)$, to a final state $\Psi_m(x,t)$\textsuperscript{20}. $W_{n,m} = W_{n,m}(0)[B_0 + B_1 + B_2]$ where,
\( W_{n,m}(0) \) is the transition rate when \( J_0 \simeq 1 \), and

\[
B_0 = \left[ J_0^2(A_m)J_0^2(A_n) + \sum_{s} 2J_s^2(A_m)J_s^2(A_n) \right] \times \frac{\Gamma}{[\hbar w_c(n - m)]^2 + \Gamma^2} \tag{2}
\]

\[
B_1 = \left[ \sum_{s} J_s^2(A_m)J_{s+1}^2(A_n) + J_{s+1}^2(A_m)J_s^2(A_n) \right] \times \frac{\Gamma}{[\hbar w_c(n - m) + \hbar w]^2 + \Gamma^2} \tag{3}
\]

\[
B_2 = \left[ J_0^2(A_m)J_0^2(A_n) + \sum_{s} J_s^2(A_m)J_{s+2}^2(A_n) + J_{s+2}^2(A_m)J_s^2(A_n) \right] \times \frac{\Gamma}{[\hbar w_c(n - m) + 2\hbar w]^2 + \Gamma^2} \tag{4}
\]

Here \( A_s \propto \frac{E_s}{w} \) and \( \Gamma \) is the state (Landau level) broadening due to different scattering mechanisms. Multiphoton processes have been considered up to 2 photons.

The average effective distance advanced by the electron in every scattering jump is given by: \( \Delta X_{MW}^{\pm} = \Delta X_0 + A \cos \omega \tau \), where \( \Delta X_0 \) is the effective distance advanced when there is no MW field present and \( 1/\tau = W_{n,m} \) (\( \tau \) being the impurity scattering time). From here, the longitudinal conductivity \( \sigma_{xx} \) can be calculated through the relationship \( \sigma_{xx} \propto \int \rho(E_n) \Delta X_{MW}^{\pm}(f_i - f_f) dE_n \), where \( f_i \) and \( f_f \) are the distribution functions for the initial and final Landau states respectively:

\[
\sigma_{xx} = \sigma_{xx}(0)[B_0[f(E_n) - f(E_m)] + B_1[f(E_n) - f(E_m + \hbar w)] + B_2[f(E_n) - f(E_m + 2\hbar w)]]
\]

(5)

and \( \sigma_{xx}(0) \) is the conductivity when \( J_0 \simeq 1 \). To obtain \( \rho_{xx} \) we use the relation \( \rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xy}} \simeq \frac{\sigma_{xx}}{\sigma_{xy}} \), where \( \sigma_{xy} \simeq \frac{\hbar c}{2\pi} \) and \( \sigma_{xx} \ll \sigma_{xy} \).

In Fig. 1 the calculated \( \rho_{xx} \) as a function of \( B \) for different MW power at fixed frequency \( \nu = w/2\pi = 700\text{GHz} \) is shown. Our results follow the qualitative experimental behavior, and show that ANC occurs at the principal minimum, as the incident radiation power is increased. The inset shows the evolution from ZRS to negative conductivity. The physical explanation is as follows. In Fig. 2 we represent schematic diagrams to describe \( \rho_{xx} \) evolution at minima. In Fig. 2a orbits are moving forwards and on average the electron advances a shorter distance than in the no MW case, \( \Delta X_{MW} < \Delta X_0 \). This corresponds to a decrease in the conductivity but \( \rho_{xx} > 0 \) still. If we raise the MW power we will eventually reach the situation depicted in Fig. 2b, where orbits are moving forwards but their amplitude \( A \) is larger than the electronic jump. In this case the jump is blocked by the Pauli exclusion principle because the final state is occupied. **This is the physical origin of the ZRS**. At fixed photon frequency, small values for MW-power correspond to transitions where no photon absorption or emission are involved, i.e., the arguments of Bessel functions are so small that only \( J_0 \) terms need to be taken into account. This implies that only direct, \( J_0 \rightarrow J_0 \), transitions are relevant. This corresponds to Fig. 2a and 2b. If we further increase the MW-power, while keeping the frequency constant, the argument of the Bessel functions will become larger, and higher order sidebands must be considered: multi-photon transitions become relevant. Transitions such \( J_1 \rightarrow J_0 \) and \( J_2 \rightarrow J_0 \), which correspond to one photon and two photons processes respectively, can then play and effective role in the current. At minima and for \( A \) larger than the electron scattering jump (large MW-power), we find a situation where \( \Delta X_{MW} < 0 \). However for multi-photon transitions \( J_1 \rightarrow J_0 \) or \( J_2 \rightarrow J_0 \) etc, the difference of distribution functions \( (f_i - f_f) > 0 \). These processes \( \Delta X_{MW} < 0 \) and \( (f_i - f_f) > 0 \) produce negative contributions to the current and are the physical origin of ANC.

One surprising effect in the experimental results is the positive peak in the middle of \( \rho_{xx} \) negative minimum. This is also observed in calculated results. (see inset of Fig. 1). The explanation for this can be readily obtained. In a regime with \( \Delta X_{MW} < 0 \) and finite temperature, direct \( (J_0 \rightarrow J_0) \) transitions, can correspond to negative values for the difference of electronic distribution functions, \( (f_i - f_f) \). This is due to the fact that in such a regime, the final state is always deeper in energy than the initial state, with respect to the Fermi energy. Considering the smearing of the distribution
function at finite temperature, the final result is that 

\[ f_i < f_f \Rightarrow (f_i - f_f) < 0. \]

When \( \Delta X^{MW} < 0 \) and \((f_i - f_f) < 0\), an effective positive net current will be produced giving rise to a positive \( \rho_{xx} \). In the inset of Fig. 1 it can also be observed that an increase in MW-power produces an increase in the positive and negative part of the minimum in a similar way. The explanation has to do with the corresponding increase in the amplitude \( A \) and, as a consequence, in \( \Delta X^{MW} \). This has a similar impact on both the one-photon, \( J_1 \rightarrow J_0 \), transitions (negative contributions) and on the direct \( J_0 \rightarrow J_0 \) one, (positive contributions). Positive net values for \( \rho_{xx} \) in the middle of the main minima, have been experimentally obtained by other groups \(^7_{12}\), notably in ref \(^7\), where this effect was termed ”breakdown of ZRS”.

Fig. 3 shows \( \rho_{xx} \) versus \( B \) for different MW-frequencies at fixed MW-power. ANC is achieved for all the cases presented and the growth of the central peak as frequency is increased is clearly visible (see inset). The explanation is as follows. At constant \( E_0 \), lower \( w \) corresponds to high values for the Bessel function arguments, i.e., \( J_0 \)'s are decreasing and \( J_{n \neq 0} \)'s are increasing. In this regime, direct transitions (positive contributions) become less important than multi-photon, \( J_n \rightarrow J_0 \) transitions (negative contributions). The first ones are represented by the \( B_0 \) term in the scattering rate \( W_{n,m} \) and \( \sigma_{xx} \), and the second ones by \( B_1 \) and \( B_2 \), (see Eqs. 2-5). Eventually the positive contributions are totally compensated by the negative ones and the central peak at minimum vanishes. As we increase the frequency, the reverse, occurs giving rise to a distinct positive peak in the middle of the negative minimum.

According to the model and calculated results presented above, high values for the ratio \( \frac{E_0}{w} \) is a first and important condition to obtain photo-satellites and the consequent ANC. However this is not sufficient, and an additional condition has to be fulfilled concerning the external DC bias applied in the \( x \) direction, as we explain below.

In Fig. 4a we represent calculated \( \rho_{xx} \) versus \( B \) at fixed \( w \) and \( E_0 \) and at increasing external DC bias \( E_{DC} \). We can observe at the two main minima how the \( \rho_{xx} \) negative values are shifted to positive as \( E_{DC} \) is raised. At the same time the central peak gets larger. The explanation can be seen from the schematic diagrams in Fig. 4b and 4c. In both cases we are in a regime where \( \Delta X^{MW} < 0 \). At lower bias (\( E_{DC1} \), Fig 4.b), one-photon transitions giving negative current (\( J_1 \rightarrow J_0 \)) dominate and, except for...
FIG. 4: (a) Calculated $\rho_{xx}$ versus $B$ at two minima for fixed MW-frequency and intensity and different external DC bias ($E_{DC}$). For increasing $E_{DC}$, $\rho_{xx}$ minimum becomes positive. (b) Schematic diagrams explanatory for the lower bias case ($E_{DC1}$). In this situation the Fermi energy slope is smaller making the $J_1 \rightarrow J_0$ transitions predominant which provides negative contributions to the current. (c) Schematic diagram for higher bias ($E_{DC2}$). In this case the former $J_1 \rightarrow J_0$ negative contributions become positive because the final Landau state is now below the Fermi energy and therefore is not empty. As a result, most contributions are positive shifting the minimum of $\rho_{xx}$ upwards and causing the central peak to increase. Temperature 500 mK.

in the central peak, they can compensate the positive direct ($J_0 \rightarrow J_0$) contributions. We observe that the final state for $J_1 \rightarrow J_0$ is well above the Fermi energy and the corresponding difference ($f_i - f_f$) > 0, is important making the negative contributions relevant. At higher bias $E_{DC2}$, we reach the situation depicted in Fig. 4c. The corresponding slope for the Fermi energy is larger than in the $E_{DC1}$ case. Now the final state is below $E_F$, i.e., it is not empty making the corresponding difference ($f_i - f_f$) < 0 and the contribution to the current evolves from negative to positive. The final result is that the $\rho_{xx}$ negative values get smaller for increasing $E_{DC}$ becoming eventually zero or positive. Another important results is that the central $\rho_{xx}$ peak becomes much larger which can be explained with similar arguments.

Therefore we can state that high values for $\frac{E_{DC}}{E_F}$ and a regime of lower values for $E_{DC}$, constitute the two main conditions which have to be fulfilled in order to obtain ANC. Thus according to our theoretical model and taking into account of the experimental difficulties to measure negative resistance, we believe that by tuning the appropriate MW parameters and external DC bias, it would be possible to experimentally observe the evolution from ZRS to ANC.

In summary, we have presented here a theoretical model to explain the physics behind the existence of radiation-induced ANC in 2D electron systems. We are able to explain the transition from ZRS to ANC in terms of multi-photon assisted processes. It has also been shown how this transition can be tuned by the parameters of the microwave field. In addition, the external DC bias has been found to be of decisive importance in achieving ANC. We think that this work sheds some light on the controversy whether ANC actually occurs in these structures and which are the main parameters that govern the evolution from ZRS to ANC. The basic understanding of this process will be of importance to the future implementation of workable nanodevices.

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1. F.Grossmann, T.Dittrich, P.Jung and P. Hanggi, Phys. Rev. Lett. 67, 516, (1991).
2. R. Lopez, R.Aguado, G.Platero and C.Tejedor, Phys. Rev. B, 64, 075319, (2001).
3. B.J. Keay et al., Phys. Rev. Lett. 75, 4098, (1995).
4. R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson, V. Umansky, Nature 420, 646 (2002).
5. M.A. Zudov, R.R. Du, N. Pfeiffer, K.W. West, Phys. Rev. Lett. 90, 046807 (2003).
6. R.G. Mani, V. Narayanamurti, K. von Klitzing, J.H. Smet, W.B. Johnson, V. Umansky, Phys. Rev. B 69, 161306(R) (2004).
7. R.G. Mani, Appl. Phys. Lett. 85, 4962, (2004); R.G. Mani, Physica E, 22, 1, (2004); R.G. Mani, Physica E, 25, 189, (2004).
8. S.A. Studenikin, M. Potemski, A. Sachrajda, M. Hilke, L.N. Pfeiffer, K.W. West, Phys. Rev. B, 71, 245313, (2005); S.A. Studenikin, M. Potemski, P.T. Coleridge, A. Sachrajda, Z.R. Wasilewski, Solid State Comm 129, 341 (2004).
9. C.L. Yang, M.A. Zudov, T.A. Knuuttila, R.R. Du, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 91, 096803, (2003); M.A. Zudov, Phys. Rev. B, 69, 041304, (2003).
10. R.R. Du, M.A. Zudov, C.L. Yang, L.N. Pfeiffer and K.W. West, Physica E, 22, 7 (2004); R.R. Du, M.A. Zudov, C.L. Yang, Z.Q. Yuan, L.N. Pfeiffer and K.W. West, J. Mod. Phys. B, 18, 3465, (2004).
11. J.H.Smiet, B. Gorschunov, C.Jiang, L.Pfeiffer, K.West, V. Umansky, M. Dressel, R. Dressel, R. Meisels, F.Kuchar, and K.von Klitzing, Phys. Rev. Lett. 95, 116804 (2005).
12 R.L. Willett, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 93 026804 (2004).
13 M.A. Zudov, R.R. Du, L.N. Pfeiffer and K.W. West, Phys. Rev. B, 73, 041303 (2006).
14 A.C. Durst, S. Sachdev, N. Read, S.M. Girvin, Phys. Rev. Lett. 91 086803 (2003)
15 C.Joas, J.Dietel and F. von Oppen, Phys. Rev. B 72, 165323, (2005); J.Dietel, L.J. Glazman, F.W.J. Hekking and F. von Oppen Phys. Rev. B 71 045329 (2005).
16 X.L. Lei, S.Y. Liu, Phys. Rev. Lett. 91, 226805 (2003);
17 X.L. Lei, S.Y. Liu, Phys. Rev. B 72, 075345 (2005);
18 V. Ryzhii and V. Vyurkov, Phys. Rev. B 68 165406 (2003); V. Ryzhii, Phys. Rev. B 68 193402 (2003); V.Ryzhii and R. Suris, J. Phys: Cond. Mat. 15, 6855, (2003) ; Ryzhii et al, Sov. Phys. Semicond. 20, 1299, (1986).
19 P.H. Rivera and P.A. Schulz, Phys. Rev. B 70 075314 (2004)
20 Junren Shi and X.C. Xie, Phys. Rev. Lett. 91, 086801 (2003).
21 Kang-Hun Ahn, J. Korean Phys. Soc., 47 (4), 666-672, (2005).
22 A.V. Andreev, I.L. Aleiner and A.J. Millis, Phys. Rev. Lett. 91, 056803 (2003)
23 J. Iñarrea and G. Platero, Phys. Rev. Lett. 94 016806, (2005)
24 T-K Ng and Lixin Dai, Phys. Rev. B, 72, 235333 (2005).
25 J. Iñarrea and G. Platero, Phys. Rev. B 72 193414 (2005)
26 J.Iñarrea, G. Platero and C. Tejedor, Phys Rev. B. 50 4581, (1994).
27 J.Iñarrea and G. Platero Phys Rev. B 51 5244, (1995).
28 E.H. Kerner, Can. J. Phys. 36, (3) 371-377 (1958) .
29 K. Park, Phys. Rev. B 69 201301(R) (2004).
30 B.K. Ridley. Quantum Processes in Semiconductors, 4th ed. Oxford University Press, (1993).