Preparation of a three-photon state in a nonlinear cavity–quantum dot system

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Abstract
We study theoretically the properties of a three-photon state prepared inside a semiconductor cavity, due to the interaction between a quantum dot and an electromagnetic field, and two consecutive spontaneous parametric downconversion (SPDC) processes. Thus, we consider a scheme involving three modes of the electromagnetic field, whose frequencies are given by the SPDC processes: \( \omega_0 \rightarrow \omega_1 + \omega_2 \) and \( \omega_2 \rightarrow \omega_1 + \omega_1 \). Furthermore, we study the low excitation regime in which a three-photon state is accessible within the system’s dynamics.

Keywords: three-photon state, nonlinear cavity, quantum dot, spontaneous parametric downconversion process, photonic crystal

(Some figures may appear in colour only in the online journal)

1. Introduction

During the last few years, several research groups have been studying the light–matter interaction in quantum dots (QDs) embedded in semiconductor microcavities, both experimentally [1–6] and theoretically [7–15]. Such investigations led to a new phenomenology which in turn has led to technological applications [16–21].

On the other hand, the generation of photon \( n \)-plets has been an interesting research branch, because it could allow researchers to prepare quantum states inside cavities which would be useful in quantum communication [22]. In particular, three-photon states can be obtained experimentally using spontaneous parametric downconversion (SPDC) [23–28], and third-order optical nonlinearities in assembled [29–31] systems. In this sense, although several groups have managed to prepare and control specific quantum states [32, 33], the preparation of arbitrary quantum states of light is still an experimental challenge.

Bearing in mind the cavity quantum electrodynamics (cQED) description of the light–matter interactions and the preparation of quantum states of light via SPDC processes, we consider that it is possible to set up an experimental design in which the initial state inside a cavity can be prepared, and finely controlled. Even though the cavity does not have to be microscopic, the experimental design is scalable from those of semiconductor microcavities. This means that the problem and the obtained results are not restricted to the optical region of the electromagnetic spectrum. Therefore, in order to prepare the quantum state inside a cavity, a mesoscopic nonlinear crystal can be included in the experimental design. On the other hand, it has recently been demonstrated that photonic crystal (PhC) cavities are capable of enhancing the harmonic generation produced by either a \( \chi^{(2)} \) or a \( \chi^{(3)} \) nonlinearity, which would finally yield an SPDC process [34–37]. Furthermore, it has been shown that the adequate geometry of the PhC [34], pump power [35] and whether the cavity is singly or doubly resonant [36] may lead to a 100% conversion.

In this sense, we consider a semiconductor cavity in which there are a QD and two nonlinear crystals. The former is coupled to a \( \omega_0 \) electromagnetic mode, so the cavity is filled with \( \omega_0 \) photons. Afterwards, these photons go through the nonlinear crystals and two SPDC processes take place: \( \omega_0 \rightarrow \omega_1 + \omega_2 \) and \( \omega_2 \rightarrow \omega_1 + \omega_1 \). Taking into account an exciton pumping and \( \omega_0 \) photon leakage from the cavity, which are both incoherent processes, a three-photon state is accessible in the \( \omega_1 \) electromagnetic mode.

The nonlinear cavity–QD system can be constructed using a GaAs substrate, over which several layers of Al\(_x\)Ga\(_{1-x}\)As, and a layer of Al\(_x\)In\(_{1-x}\)As in which the QDs are localized, are grown using the molecular beam epitaxy (MBE) method. In this particular construction, the optical properties are nonlinear [37, 38], and could therefore be used as a basis
we provide an electromagnetic mode inside a semiconductor cavity, and the following (\( g, n \)) yield the SPDC processes.

\[ H_{SPDC} = \zeta \left( \alpha_0^\dagger \alpha_0^\dagger a_2^\dagger a_2 + \alpha_0 \alpha_0^\dagger a_1^\dagger a_1 \right) + \xi \left( \alpha_1^2 a_2^\dagger a_2^\dagger + \alpha_1^\dagger a_2^2 \right), \]  

(2)

where \( \alpha_0^\dagger \) (\( a_0^\dagger \)) is the \( \omega_0 \) electromagnetic mode annihilation (creation) operator and \( \sigma^\dagger \) (\( \sigma \)) is the exciton annihilation (creation) operator. The \( \omega_0 \) electromagnetic mode and the \( \omega_2 \) exciton are coupled with an interaction strength \( g \) and their frequencies are close enough to resonance to allow for the rotating wave approximation, i.e. \( \Delta = \omega_0 - \omega_{qd} \ll \omega_0, \omega_{qd} \).

The two subsequent SPDC processes generate two more electromagnetic modes with frequencies \( \omega_1 \) and \( \omega_2 \). In the first process, one \( \omega_0 \) photon generates one \( \omega_1 \) and one \( \omega_2 \) photon (\( \omega_0 \rightarrow \omega_1 + \omega_2 \)), whereas in the second process one \( \omega_2 \) generates two \( \omega_1 \) photons (\( \omega_2 \rightarrow 2\omega_1 \)). Both processes may be described in an effective way with the following Hamiltonian [30]:

\[ H_{IC} + H_{SPDC}. \]  

(3)

The dynamical behavior and the incoherent pumping and loss of the dot–cavity system are included in the master equation, which in the Lindblad notation is written as

\[ \dot{\rho} = i [\rho, H] + \frac{P}{2} (2\sigma^\dagger \rho \sigma - \{ \sigma^\dagger, \rho \}) + \kappa \left( 2a_0^\dagger a_0 - \{ a_0^\dagger, a_0, \rho \} \right), \]  

(4)

where \( H \) is the Hamiltonian given in equation (3), \( \kappa \) is the rate at which \( \omega_0 \) photons escape from the cavity and \( P \) is the rate at which the excitation is pumped to the cavity and is linked to the rate at which electron–hole pairs relax into the dot.

Furthermore, the system’s energy levels and its connection via the master equation given in equation (4) are shown schematically in figure 2. Each energy level is
associated with a quantum state written as $|a, i, j, k\rangle$, where $a$ is the QD state (either ground or excited) and $i, j$ and $k$ are the photon numbers in the $\omega_0$, $\omega_1$ and $\omega_2$ modes of the electromagnetic field, respectively. The presented scheme shows the energy levels accessible for the $|g, n_0, n_1, n_2\rangle$ state by just one process.

3. Results

We solved the master equation given in (4) numerically using the following parameter values: the dipole-like interaction constant between the QD and the $\omega_0$ mode is set as $g = 5$ meV; the excitation energy of the QD is set as $\omega_{qd} = 500$ meV which in turn is tuned perfectly with the $\omega_0$ mode, i.e. $\omega_{qd} = \omega_0$. These values are usual for $\lambda$ cavities operating in the infrared region of the electromagnetic spectrum. Furthermore, restricting our results to the low-excitation regime, we set the incoherent pumping rate $P$ as 0.1 $\mu$eV, whereas the cavity loss for the fundamental mode $\omega_0$ is taken as $\kappa = 0.1$ meV. On the other hand, the SPDC rates are taken as $\xi = 1$ meV and $\zeta = 3$ meV, following the recommendations presented in [30].

We consider the QD in its excited state and the electromagnetic field to be in a vacuum state in all of its modes, as the system’s initial condition. With this setup, we observe that the state of the $\omega_1$ electromagnetic mode reaches a so-called three-photon state, which is a superposition of $3n$-photon Fock states, within the system’s dynamics. These results are shown in figure 3.

In this way, we have obtained results similar to those presented in [30], considering explicitly the interaction between a QD in a semiconductor cavity and an electromagnetic field. These results are very interesting, since we have shown that a three-photon state can be prepared inside a semiconductor cavity made of PhC capable of enhancing the harmonic generation produced by either a $\chi^{(2)}$ or $\chi^{(3)}$ nonlinearity.

4. Summary and conclusions

In this paper, we have studied theoretically the preparation of a three-photon state inside a semiconductor cavity made of PhC capable of enhancing the harmonic generation produced by either a $\chi^{(2)}$ or $\chi^{(3)}$ nonlinearity. The three-photon state is the result of the interaction between a QD embedded in the cavity and a $\omega_0$ mode of the electromagnetic field, and two SPDC processes yielding two more modes of the electromagnetic field: $\omega_0 \rightarrow \omega_1 + \omega_2$ and $\omega_2 \rightarrow \omega_1 + \omega_1$. To study the system’s dynamics we have solved a Lindblad master equation numerically considering both an incoherent excitation pump rate and $\omega_0$-photon leakage from the cavity. In this way, we observed that the three-photon state is accessible within the dynamics in the $\omega_1$ mode in a low-excitation regime.

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Appendix. Dynamics of the density operator’s matrix elements

On the basis \{\{g, n_0, n_1, n_2\}; \{e, n_0, n_1, n_2\}\} of product states between the QD and the electromagnetic modes, the matrix elements of the density operator are

\[
\rho_{a, i, j, k; l, m, n} = \langle a, i, j, k | \rho | b, l, m, n \rangle,
\]

where \(a\) and \(b\) are either \(g\) or \(e\).

In this notation, the density operator’s matrix elements satisfy the following differential equations:

\[
\partial_t \rho_{g, i, j, k; l, m, n} = \left[ i \omega_0 \left( l - i + \frac{m - j}{3} + \frac{2 n - k}{3} \right) - \frac{l + i}{2} \right] \rho_{g, i, j, k; l, m, n} + i g \left( \sqrt{l + 1} \rho_{g, i, j, k; l, m, n} - \sqrt{l} \rho_{e, i, j, k; l, m, n} \right) + \kappa \sqrt{(l + 1)(l + 1)} \rho_{g, i, j, k; l, m, n} + i \zeta \left( \sqrt{l} \rho_{g, i, j, k; l, m, n} - \sqrt{l - 1} \rho_{e, i, j, k; l, m, n} \right) + i \zeta \left( \sqrt{l} \rho_{e, i, j, k; l, m, n} - \sqrt{l - 1} \rho_{g, i, j, k; l, m, n} \right) + \sqrt{(l - 1)n} \rho_{g, i, j, k; l, m, n} - \sqrt{l} \rho_{g, i, j, k; l, m, n} + \sqrt{(l + 1)n} \rho_{g, i, j, k; l, m, n} + \sqrt{l} \rho_{e, i, j, k; l, m, n} + \sqrt{l + 1} \rho_{e, i, j, k; l, m, n} + \sqrt{l - 1} \rho_{e, i, j, k; l, m, n} + \sqrt{l} \rho_{e, i, j, k; l, m, n} + \sqrt{l + 1} \rho_{e, i, j, k; l, m, n}
\]

(A.1)

plus the Hermitian conjugate of (A.4).

Once the system of linear equations is solved, we obtain the density operator of the cavity–QD system as a function of time: \(\rho(t)\). This operator has four quantum numbers associated, one to the QD and one to each of the modes of the electromagnetic field, and its matrix elements are thus given by

\[
\rho_{a, i, j, k; l, m, n}(t) = \langle a, i, j, k | \rho(t) | b, l, m, n \rangle.
\]

(A.5)

Nevertheless, in this particular case we are only interested in the degree of freedom associated with the \(\omega_1\) mode, so it is convenient to consider the reduced (to the \(\omega_1\) subsystem) density operator instead of the complete operator. The reduced operator is denoted as \(\rho^{(3)}(t)\), and is obtained from the complete operator by performing partial trace over all the remaining degrees of freedom:

\[
\rho^{(3)}_{j}(t) = \langle i | \rho^{(3)}(t) | j \rangle = \sum_{a, n, i, m} \langle a, n, i, m | \rho(t) | a, n, j, m \rangle.
\]

(A.6)

Finally, once we have obtained the reduced density operator for the \(\omega_1\) mode, we compute its Wigner function as in e.g. [44].

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