Superconductor-ferromagnet-superconductor junctions in flux and phase qubits.

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Abstract. $\pi$-junction qubit based on superconducting quantum interference device ring has been already proposed. In this qubit, a quantum two-level system is spontaneously generated without any bias flux applied. Flux qubit with a single $\pi$-junction concerns the large inductance $L$ of the loop, which makes the qubit vulnerable to decoherence caused by magnetic fluctuations in the environment. To overcome this difficulty one can use three-junction $\pi$-qubit, comprising three $\pi$-junctions in a loop with negligible small inductance or unharmonic two-junction interferometer exploiting non-sinusoidal current-phase relations of the junctions. In the paper we theoretically investigate the macroscopic quantum dynamics of proposed qubits. The measurement process in two types of qubits has been analyzed in the frame of Lindblad equation formalism. Different mechanisms giving a non-sinusoidal CPR in SFS structures are analyzed as well. It is shown, that the largest second harmonic can be obtained for the SFS junction in the “clean” limit with a thin ferromagnet layer at low temperature.

1. Introduction.

Several types of Josephson $\pi$-junctions with spontaneous phase $\pi$-shift between superconducting electrodes have been demonstrated over the last few years [1]-[2]. These $\pi$-junctions are based on nonhomogeneous superconducting LOFF-type state (Larkin, Ovchinnikov, Fulde, Ferrell) in the superconductor/ferromagnet (S/F) heterostructures. The critical current through the Josephson junction with ferromagnet spacer oscillates as a function of the ferromagnet layer thickness [3], and this effect allows to construct Josephson junctions with negative critical current, that is equivalent to the phase $\pi$-shift across the junction.

At the same time several types of Josephson-junction quantum bits (qubits) have been suggested [3]-[5], and multi-qubit systems are now under development to come to the simplest quantum computations. Nevertheless, the novel qubit design remains topical as yet. In particular, disadvantage of the early proposed phase and flux qubits (with Josephson characteristic energy $E_J$ much higher than Coulomb characteristic energy $E_C$) is associated with application of half a magnetic flux quantum needed to set the required operating point. Some attempts have been made to resolve this problem with the help of phase $\pi$-shifter, based on Josephson $\pi$-junction. But both the five-junction loop suggested by Blatter et al. [4] and the $\pi$-ring suggested by Kawabata et al. [5] (figure 1a) require large inductance $L_{loop}$ to form the double-well potential. This implies a large size of the qubit loop and makes the qubit
vulnerable to decoherence caused by magnetic fluctuations in the environment. One way to overcome this difficulty is to use three \( \pi \)-junctions connected in series in a superconducting ring. In this case, magnetic energy of the loop can be much smaller than Josephson-junctions energy \( E_J \). In fact, the qubit corresponds to the well-known three-junction persistent-current qubit (figure 1b,c) [3] but this \( \pi \)-persistent-current qubit doesn’t need application of half a flux quantum. Moreover, the widely discussed \( \phi \)-junction with non-sinusoidal current-phase relation (CPR)
\[
I_j(\phi) = A_j \sin \phi_j - B_j \sin 2\phi_j + \ldots \quad (j = 1,2)
\]
could be realized on the base of SFS or S-FN-S [6] structures as well. Inserting couple of the \( \phi \)-junctions into a low-inductive dc SQUID (\( l \ll 1 \)) one can come to the so-called “silent” qubit which also doesn’t need application of half a flux quantum (figure 1d). The three-junction persistent-current \( \pi \)-qubit and the two-junction silent qubit are discussed here.

2. Quantum superposition

To compare these two types of qubits we started from the case of symmetric interferometers, where \( E_{11} = E_{12} = E_1 \neq E_{13} \) for three-junction qubit and \( E_{11} = E_{12} = E_j \) for two-junction qubit. In both systems the height of the barrier between the wells depends on similar parameters \( \alpha_\pi = E_{13}/E_3 \) for \( \pi \)-persistent-current qubit and \( \alpha_S = B_{1,2}/A_{1,2} \) for two-junction silent qubit. The principal requirement to the qubit formation is relatively large value of critical current of the third junction for three-junction qubit and large second harmonic in the current-phase relation for two-junction qubit: \( \alpha_\pi, \alpha_S > 0.5 \).

In the case of negligible normalized inductances \( l \) the total flux \( \Phi \) through the loop approaches the applied magnetic flux \( \Phi_e \). The Hamiltonian for three-junction persistent-current qubit in terms of effective phases \( \theta = (\phi_1 + \phi_2)/2 \) and \( \varphi = \phi_1 - \phi_2 \) can be written as follows:
\[
\hat{H}_s = -\frac{1}{2} E_\theta \left( 1 + \alpha_\pi \right) \frac{\partial^2}{\partial \theta^2} - 2E_\pi \cos(\theta) + \alpha_\pi E_j \cos(\varphi - 2\theta),
\]
where \( \phi_e = 2\pi \Phi_e/\Phi_0 \); the phase of the third Josephson junction defined as \( \varphi_3 = \phi_e - \pi - 2\theta \). Hamiltonian for two-junction qubit seems very similar:
\[
\hat{H}_s = -\frac{1}{2} E_\theta \frac{\partial^2}{\partial \theta^2} - 2E_\pi \cos(\theta) + \alpha_S E_j \cos(2\theta).
\]

In the Hamiltonian (2) for two-junction qubit the effective mass of the system doesn’t depend on the barrier shape parameter \( \alpha_S \) and the flux dependence of the qubit potential is implicit. The energy gap \( \Delta \) between ground and first excited level should always be much less than forbidden gap \( \Delta^* \) between the

Figure 1. Phase (a), flux (b, c) qubits with \( \pi \)-junctions and silent qubit (d). The slight flux \( \Phi_{e1} \) is meant for logic and readout operations, flux \( \Phi_{e2} \) is to change critical current of the third Josephson element (\( \pi \)-interferometer or S-FNF-S junction).

Figure 2. The ground energy level splitting \( \Delta \) and the forbidden gap \( \Delta^* \) between the ground level and the next one versus ratio \( s \) of the characteristic Josephson energy \( E_J \) to characteristic Coulomb energy \( E_Q \).
first excited level and the next one. Figure 2 shows that the requirement can be met if the ratio $s$ of the characteristic Josephson energy $E_J$ to characteristic Coulomb energy $E_Q$ exceeds $10^2$.

One can describe the impact of magnetic flux on three-junction and two-junction qubit with the following perturbation operators:

$$\tilde{V}_s = \alpha_s E_J (\phi_\alpha \sin(2\theta) - \frac{1}{2} \phi_\beta \cos(2\theta)),$$

$$\tilde{V}_s = E_J \frac{\phi_\beta}{4} \left[ \frac{1}{2} \cos(\theta) - \alpha_s \cos(2\theta) \right].$$

The Hamiltonians (1) and (2) with flux-dependent terms (3) have been analyzed numerically and in the frame of perturbation theory, exploiting generalized Mathieu functions as the functions of zero-order approximation. Calculated energy spectrum dependence on the ratio of the characteristic Josephson energy to Coulomb energy $s$ is similar for both types of qubits that leads to similar conditions for decoupling of the basis from the higher energy levels. At the same time, in two-junction qubit with identical junctions the eigenstates don’t depend on the external flux in the first order due to specific form of perturbation operator (3b), and even in the case of controlled junction asymmetry ($\eta = A_1/A_2 \neq 1$), that is necessary for measurement process, the basis gap is a parabolic function of applied flux $\phi_e$. It means that at operating point the two-junction qubit is strongly decoupled from the fluctuation in the controlling circuits and the eigenstates of the silent qubit remains the superposition of flux basis states $|L\rangle$ and $|R\rangle$ for relatively large values of external flux $\Phi_e$ (see figure 3a).

Readout and logic operations with three-junction and two-junction qubits have been considered in the frame of non-selective approach to continuous quantum measurements exploiting the so-called Lindblad-type equation for the density operator $\rho$:

$$\dot{\rho} = -\frac{i}{\hbar} [\hat{H}, \rho] - \frac{k}{2} [\hat{C}, [\hat{C}, \rho]].$$

Here the measurable value $C$ corresponds to interferometer phase $\theta = (\phi_1 + \phi_2) / 2$ (and the circulating current $I_{circ}$) and $k$ is the measurement accuracy. For small values of double-well potential asymmetry the Lindblad equation [7] was resolved and the rate of decoherence processes in three-junction and two-junction qubits have been calculated. Figure 3b presents decay of coherent superposition of eigenstates (entanglement varies from 0 for the pure state to 1) for different values of effective measurement accuracy $K = kx^2 / 2\pi \Delta$. The specificity of qubit type is hidden in the parameters $\Delta$ (gap) and $x$, which is the matrix element of the interferometer phase $\theta$ in the basis of calculated qubit eigenstates.

Figure 3. (a) Areas of coherent superposition $a |L\rangle + b |R\rangle$ existence for $\pi$-persistent-current and silent qubits. The eigenstates for external fluxes $\Phi_{e1} = 0$ and $\Phi_{e1} = 0.1$ are presented in the inset. (b) Von Neumann entropy as a measure of entanglement for two types of qubit at different values of measurement accuracy $K = kx^2 / 2\pi \Delta$. 

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After considering the magnetic flux impact on three-junction and two-junction qubit one can conclude, that

- Both qubit types don’t demand persistent magnetic biasing and large loop inductances.
- Persistent-current $\pi$-junction qubit isn’t so exacting for the technological spread in the F-layer thickness and allows simple realization of two independent single-qubit logic gates.
- Two-junction silent qubit potential is protected from external sources of decoherence due to special form of Hamiltonian and, hence, specific behaviour of qubit basis.

3. SFS $\varphi$-junctions

Non-sinusoidal CPR required for two-junction qubit existence is not so exotic for the Josephson junctions with a weak link containing ferromagnet [8]. But until now there was no complete answer for a very simple question: in what structure one can obtain the largest value of higher harmonics amplitudes in CPR? In this paper we present the analysis of the higher harmonics amplitudes for SFS, SIFS and SIFIS junctions in the “clean” and the “dirty” limit.

Recently experimental evidences of non-sinusoidal CPR existence have been obtained [9], [10]. This CPR is peculiar to SFS junctions in the “clean” limit [11], [12]. Different types of a scattering also influence the CPR in the “dirty” limit, for example, a parallel spin-flip scattering [13] and the scattering of $s$-electrons into $d$-band [14] in the F-layer. CPR is sensitive to introducing of a thin dielectric spacer into the junction as well as to the S/F boundary resistance [15], [16]. Small S/F boundary resistance, which provides a significant proximity effect, gives a positive second harmonic, while dielectric spacers into SIFS and SIFIS junctions could make the second harmonic $B$ negative [15]. At the same time, different theoretical models [8, 11-17] give similar results: the second harmonic oscillates and decreases with the ferromagnet thickness $d$ twice faster than the first one. Near 0-\(\pi\) transition, where the first harmonic is small in comparison with the second one, the last is always positive, that excludes the $\varphi$ ground state for uniform junctions.

Critical current of a junction containing a ferromagnetic layer with complex coherence length $\xi_F$ ($\xi_F^{-1} = \xi_1^{-1} + \imath \xi_2^{-1}$) falls down as $\exp(-d/\xi_2)$ and oscillates with a period $2\pi\xi_2$ [8] in the “dirty” limit, when the electron mean free path $l << \xi_1, \xi_2$ depends on $l$ as well as on the length of a pair-breaking scattering in the ferromagnet (the spin-flip scattering [13] or the scattering into d-band [17], both influence on CPR). The pair-breaking scattering makes $\xi_1 < \xi_2$ [1], [14] and the Josephson current decays very fast with the ferromagnet thickness $d$ in the “dirty” limit. It was shown in different models, that the second harmonic $B(d)$ falls down and oscillates with $d$ twice faster than $A(d)$. Therefore the second harmonic can be detected only near the first 0-\(\pi\) transition where $A \to 0$ and it is not so easy. In the experiments [9], [18] it was detected only when the technology was improved and

\[ Figure 4. \text{The first three harmonics of the CPR as a function of the temperature for an SFS junction in the “clean” limit at } d = 0.3\xi_2 \text{ (a) (far from the 0-\(\pi\) transition) and } d = 0.55\xi_2 \text{ (b) (near the 0-\(\pi\) transition).} \]
the first 0-π transition moved to smaller $d$.
In the “clean” limit the Josephson current decays much slower. The cleaner is the ferromagnet, the larger is $l$, and the slower is the decay. In the limit $l \gg d$, $\xi_{1,2}$ the current decreases as $1/d$ [11].
Moreover, the “clean” SFS junction has non-sinusoidal CPR. Usually, $\xi_2 = v_F/2h$, where $v_F$ is the Fermi velocity, $h$ is the exchange magnetic energy in the ferromagnet [12][17][19]. It was established experimentally, that for pure ferromagnetic metals Ni has the smallest exchange field [20-22].
It was established on the solution of the Eilenberger equations [11]. Higher harmonics decay faster with the temperature [12],[23]. We have also checked this statement for models described in [11], [12]. The result is presented in figure 4. It is clear, that near the critical temperature $T_C$ superconducting gap $\Delta \rightarrow 0$ and equations become linear and their simple exponential solutions yield only sinusoidal CPR. Hence, one can conclude, that the largest second harmonic can be obtained for the SFS junction in the “clean” limit with a thin ferromagnetic layer at low temperature ($\approx 0.1T_C$). But its experimental observation at contemporary technology creates some difficulties, that lie in very fast oscillations of first harmonic amplitude $A(d)$ and creation of $d$ with high enough precision to achieve the 0-π transition.
New ways to create Josephson junctions with non-sinusoidal CPR, namely with a significant negative second harmonic, open by fabrication of non-uniform “long” Josephson junctions with a variation of the F-layer thickness along the junction [24],[25], or “clean” SFS junctions with a barrier of a dilute ferromagnetic alloys like Pd-Fe, for example.

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