Spatial meshing theory of involute spiroid gear drive with line contact

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Received: 12 September 2019; Revised: 17 November 2019; Accepted: 29 January 2020

Abstract
In the paper, spatial meshing theory of involute spiroid gear drive with line contact (ISGDWLC), which is a new type of worm drive with large ratio, crossed axes and high load capacity, is presented completely for the further researches of load capacity, wear and efficiency. The methods of modeling and motion simulation are also brought forward on CATIA. Firstly, based on the spatial meshing theory, the theoretical model of ISGDWLC is proposed, including equations of flanks, meshing equation, nonundercutting conditions, conjugate zone, sliding angle and induced normal curvature. Secondly, a set of formulas for the design of ISGDWLC is provided, which has solved the problems such as undercutting, interference and redundant line of contact. In addition, in the light of simultaneous equations method the length of contact is calculated accurately, easily and rapidly. Finally, a numerical example is supplied and its sliding angle, induced normal curvature and length of contact are analyzed utilizing MATLAB. The calculations are done in a second and the analytic solutions are got. The results show that ISGDWLC not only has a simple calculation and stable instantaneous transmission ratio but also has a slight edge in local meshing performances compared to other spiroid gear drive. It has the potential to be widely used. At present, a prototype has been established according to the above work.

Keywords: Involute spiroid gear, Spatial meshing theory, Screw involute surface, Local meshing performances, Length of contact

1. Introduction

Since the spiroid gear drive invented in the 1950s (Saari, 1954), many countries, research institutes and scientists have made great contributions to its development. Theoretically, the tooth surface of classical spiroid gear drive is helical surface with variable lead, worm wheel needs to be machined by special hob and the tooth surface can’t be ground (Goldfarb, 2004, 2006a, 2006b) (Fu, 1990), which is one of the reasons why this type of transmission cannot be widely used. In order to solve these problems, the involute spiroid gear drive is developed, which is a kind of transmission with large ratio and crossed axes. It can be divided into two types: (1) One flank of worm is contact with matched worm wheel surface along points and the other flank along a line (Yang et al., 1995); (2) Both flanks of worm are contact with matched worm wheel surfaces along lines (Baidu Wenku, 2019a, 2019b) (Zhang and Nie, 2011a, 2011b, 2011c). In this paper, the latter is discussed as it possesses higher carrying capacity, and named involute spiroid gear drive with line contact (ISGDWLC). Figure 1 shows that ISGDWLC consists of a conical worm and a face type gear. It is not only similar to the spiroid gear in appearance, but also has the advantages that the spiroid gear owns, as well as the following characteristics:

1) High transmission accuracy. The flanks of the worm and worm wheel are screw involute surfaces, and their respective leads are fixed values, which are beneficial to improve the stability of the transmission and precision.
2) Convenient and various machining ways. The manufacturing technology of screw involute surface is relatively mature (Savas et al., 2015), and the conical face gear with an involute gear line can be machined with hob or with disc tool (Frackowiak and Wojtko, 2018). It is seen that employing ISGDWLC is able to greatly increase the flexibility of
 maching.

3) Grindable surface. The flanks can be ground for the screw involute surface is developable ruled surface (Litvin and Fuentes, 2004).

In previous work, researchers have done many works about theory, geometry and machine of ISGDWLC. But there are some problems that they don’t solve such as incomplete spatial meshing theory, interference, local meshing performances and so on. For the further research, in the paper spatial meshing theory is presented completely. In addition, the equations of worm wheel flanks are simplified and they proves again the flanks are screw involute surface.

Then a simple method, named simultaneous equations method, is used to calculate the transient length of contact. Moreover, a way to solve the problem of interference omitted in previous work is proposed on the basis of Three-dimensional model and motion simulation on CATIA V5 R21. At last, a virtual prototype is established and its meshing characteristics and length of contact are calculated.

The paper basic structure is as follows. Firstly, the spatial meshing theory of ISGDWLC are introduced. Secondly, geometric design formulas supplemented will be list in a table, and some parameters about length of contact and interference will be presented. Thirdly, a numerical example is supplied and the local meshing performances are analyzed. The results are compared with the research of other spiroid gear.

Nomenclature

\[ S_i \] : Coordinate system \((i = 0, P, 1, 2)\)
\[ \omega^{(i)} \] : Angular velocity \((i = 1, 2)\)
\[ T \] : Time
\[ \phi_{\omega}^{(i)} \] : Angular velocity vector \((i = 1, 2)\)
\[ \phi_t \] : Rotation angle \((i = 1, 2)\)
\[ \alpha, \beta \] : Worm tooth flanks
\[ \alpha^*, \beta^* \] : Worm wheel tooth flanks matched with \(\alpha\)
\[ r^i_j \] : Position vector of an arbitrary point on \(i\) in coordinate system \(S_j\) \((i = \alpha, \beta, \alpha^*, \beta^*, j = 0, P, 1, 2)\)
\[ r^i_j \] : Radius of base cylinders of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ u_i \] : Surface parameter of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ \theta_i \] : Surface parameter of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ l_i \] : Offset of starting points of helix on base cylinders of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ \lambda^i_{\rho_b} \] : Lead angle on base cylinders of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ p_{\rho} \] : Screw parameter on base cylinders of \(i\) \((i = \alpha, \beta)\)
\[ P_{\rho} \] : Screw lead of \(i\) \((i = \alpha, \beta, \alpha^*, \beta^*)\)
\[ n^i_{\rho} \] : Unit normal to \(i\) in \(S_i\) \((i = \alpha, \beta)\)
\[ V^i_{12} \] : Relative velocity vector at any point on \(i\) in \(S_i\) between the worm and the worm wheel \((i = \alpha, \beta)\)
\[ A \] : Center distance
\[ i_{12} \] : Transmission ratio
\[ \phi_{\omega}^i \] : Meshing equation of \(i\) \((i = \alpha, \beta)\)
\[ M_{\rho} \] : Matrix \(4 \times 4\) for transformation from \(S_j\) to \(S_i\)
\[ \theta_{\omega} \] : Sliding angle at arbitrary meshing point on first line of contact on \(i\) \((i = \alpha, \beta)\)
\[ K_{\rho}^i \] : Induced normal curvature of ISGDWLC at any meshing points on \(i\) \((i = \alpha, \beta)\)
\[ z_i \] : Number of worm \((i = 1)\) and worm gear teeth \((i = 2)\)
\[ \delta_i \] : Half cone angle of worm \((i = 1)\) and worm wheel top-based cone \((i = 2)\)
\[ m_{si} \] : Modulus along generatrix of worm top-based cone
\[ m_i \] : Axial modulus of \(i\) \((i = \alpha, \beta)\)
\[ t_{si} \] : Tooth pitch along generatrix of worm top-based cone
\[ t_{si} \] : Distance between worm top-based cone and worm root-based cone
\[ h_a \] : Addendum of worm
\[ h_a^* \] : Addendum coefficient
\[ h^*_f \] : Dedendum coefficient
\[ h_f \] : Dedendum of worm
\[ h \] : Tooth depth
\[ d_{1ca} \] : Small end diameter of worm top cone
\[ d_{1cm} \] : Small end diameter of worm middle cone
\[ d_{1cf} \] : Small end diameter of worm root cone
\[ d_{1cp} \] : Small end diameter of worm design root cone
\[ \varepsilon \] : Contact ratio
\[ L \] : Worm working length
\[ E \] : Offset of small end face of worm
\[ W_{1a} \] : Tooth thickness of worm in middle cone
\[ W_{1ca} \] : Addendum thickness of worm
\[ W_{1cf} \] : Dedendum thickness of worm
\[ \Delta_{1ca} \] : Offset of worm \((i = 1)\) or worm wheel \((i = 2)\) top cone
\[ \Delta_{1cp} \] : Offset of design root cone of worm
\[ R_{a1}, R_2 \] : Outer and inner ring diameter of worm wheel
\( Q_{ij} \): Crossing point between first line of contact on \( i \) \((i = \alpha, \beta)\) and top cones of worm \((j = 1)\) or worm wheel \((j = 2)\)

\( u_{ij} \): Generatrix length of worm at \( Q_{ij} \)(\(i = \alpha, \beta, j = 1\))

\( \Delta u_i \): Length of first line of contact on \( i \) \((i = \alpha, \beta)\)

\( a_{ij}, b_{ij}, c_{ij} \): Coefficients for calculation \( u_{ij} \) \((i = \alpha, \beta, j = 1, 2)\)

\( Q_{jei} \): Crossing point between first line of contact on \( i \) \((i = \alpha, \beta)\) and small end \((j = s)\) or big end face \((j = b)\) of worm

\( u_{jei} \): Generatrix length of worm at \( Q_{jei} \)(\(i = \alpha, \beta, j = s, b\))

\( C_{gi} \): Parameter defined for calculating \( l_{\beta} \) \((i = 1, 2, 3)\)

2. Spatial meshing theory

In this section, the spatial meshing theory of ISGDWLC is completed, including equations of flanks, meshing equation, conditions of nonundercutting, envelopes to family of contact lines, sliding angle and induced normal curvature.

As shown in Fig. 2 and Fig. 3, a fixed coordinate system \( S_{0}\{O_{0}, X_{0}, Y_{0}, Z_{0}\} \) and an auxiliary coordinate system \( S_{p}\{O_{p}, X_{p}, Y_{p}, Z_{p}\} \) are employed in the gear house. Coordinate systems \( S_{i}\{O_{i}, X_{i}, Y_{i}, Z_{i}\} \) and \( S_{j}\{O_{j}, X_{j}, Y_{j}, Z_{j}\} \) are associated with worm and worm wheel respectively. The axis \( Z_{i} \) and \( Z_{j} \) are coincident with worm and worm wheel axial line severally. The worm revolves around the axis \( Z_{i} \) with \( \omega_{1} \) and the worm wheel is rotated about \( Z_{j} \) with \( \omega_{2} \). \( \varphi_{1} \) and \( \varphi_{2} \) are the rotation angles of the worm and worm wheel in several after time \( T \).

![Fig. 1 Transmission principle of ISGDWLC: a) sketch of structure; b) meshing diagram of \( \beta \); c) meshing diagram of \( \alpha \).](image)

![Fig. 2 Position vector of arbitrary point: a) on \( \alpha \); b) on \( \beta \). \( \mathbf{n}_{\alpha}^{x} \) and \( \mathbf{n}_{\beta}^{x} \) are on the planes tangent to base cylinders of \( \alpha \) and \( \beta \), respectively.](image)

The worm tooth flanks \( \alpha \) and \( \beta \) are screw involute surfaces as shown in Fig. 2. The screw involute surface is generated by a straight line that performs a screw motion being tangent to the helix on the base cylinder (Litvin and Fuentes, 2004). The position vectors \( r_{\alpha}^{x} \) and \( r_{\beta}^{x} \) can be deduced as:

\[
r_{\alpha}^{x}(u_{\alpha}, \theta_{\alpha}) = \left[ r_{\alpha}^{0}\cos \theta_{\alpha} - u_{\alpha} \cos \lambda_{\alpha}^{x} \sin \theta_{\alpha}, r_{\alpha}^{0}\sin \theta_{\alpha} + u_{\alpha} \cos \lambda_{\alpha}^{x} \cos \theta_{\alpha}, u_{\alpha} \sin \lambda_{\alpha}^{x} + p_{\alpha} \theta_{\alpha} - l_{\alpha} \right]^T
\]

\[
r_{\beta}^{x}(u_{\beta}, \theta_{\beta}) = \left[ r_{\beta}^{0}\cos \theta_{\beta} + u_{\beta} \cos \lambda_{\beta}^{x} \sin \theta_{\beta}, r_{\beta}^{0}\sin \theta_{\beta} - u_{\beta} \cos \lambda_{\beta}^{x} \cos \theta_{\beta}, -u_{\beta} \sin \lambda_{\beta}^{x} + p_{\beta} \theta_{\beta} - l_{\beta} \right]^T
\]

where \( p_{\alpha} = r_{\alpha}^{0}\tan \lambda_{\alpha}^{x} \) and \( p_{\beta} = r_{\beta}^{0}\tan \lambda_{\beta}^{x} \). For an observer located on the positive axis \( Z_{i} \), angle is measured anticlockwise. The unit normals to flanks \( \alpha \) are represented in \( S_{i} \) as

\[
\mathbf{n}_{\alpha}^{x} = \left[ -\sin \lambda_{\alpha}^{x} \sin \theta_{\alpha}, \sin \lambda_{\alpha}^{x} \cos \theta_{\alpha}, -\cos \lambda_{\alpha}^{x} \right]^T
\]

\[
\mathbf{n}_{\beta}^{x} = \left[ -\sin \lambda_{\beta}^{x} \sin \theta_{\beta}, \sin \lambda_{\beta}^{x} \cos \theta_{\beta}, -\cos \lambda_{\beta}^{x} \right]^T
\]
Fig. 3 Coordinate systems and geometric dimensioning of ISGDWLC. Coordinate systems $S_1\{O_1, X_1, Y_1, Z_1\}$ and $S_2\{O_2, X_2, Y_2, Z_2\}$ are fixed in the gear house. $S_1\{O_1, X_1, Y_1, Z_1\}$ and $S_2\{O_2, X_2, Y_2, Z_2\}$ are employed with worm and worm wheel respectively. The worm revolves around the axis $Z_1$ with $\omega^{(1)}$ and the worm wheel is rotated about $Z_2$ with $\omega^{(2)}$.

Fig. 4 Schematic diagram of geometric calculation of worm in plan $X_1 = 0$. First lines of contact on $\alpha$ and $\beta$ at $\phi = 0$ are projected on the plan. Other solid lines excluding the axes are the shape of worm intersecting with plan $X_1 = 0$ and trimmed suitably to fit first lines of contact.

According to the theory of relative motion (Litvin, 1989), the relative velocity vectors $V^{\alpha(12)}_1$ and $V^{\beta(12)}_1$ can be figured out in $S_1$ as

$$ V^{\alpha(12)}_1 = \begin{bmatrix} \omega^{(2)}(u_\alpha \sin \lambda_\alpha^\alpha + p_\theta \theta_\alpha - l_\alpha) \cos \phi_\alpha - \omega^{(1)}(r_\alpha^\alpha \sin \theta_\alpha + u_\alpha \cos \lambda_\alpha^\alpha \cos \theta_\alpha) \\ -\omega^{(2)}(u_\alpha \sin \lambda_\alpha^\alpha + p_\theta \theta_\alpha - l_\alpha) \sin \phi_\alpha + \omega^{(1)}(r_\alpha^\alpha \cos \theta_\alpha - u_\alpha \cos \lambda_\alpha^\alpha \sin \theta_\alpha) \\ -\omega^{(2)}[r_\alpha^\alpha \cos(\theta_\alpha + \phi_\alpha) - u_\alpha \cos \lambda_\alpha^\alpha \sin(\theta_\alpha + \phi_\alpha) - A] \end{bmatrix} $$

(5)

$$ V^{\beta(12)}_1 = \begin{bmatrix} \omega^{(2)}(-u_\beta \sin \lambda_\beta^\beta + p_\theta \theta_\beta - l_\beta) \cos \phi_\beta - \omega^{(1)}(r_\beta^\beta \sin \theta_\beta - u_\beta \cos \lambda_\beta^\beta \cos \theta_\beta) \\ -\omega^{(2)}(-u_\beta \sin \lambda_\beta^\beta + p_\theta \theta_\beta - l_\beta) \sin \phi_\beta + \omega^{(1)}(r_\beta^\beta \cos \theta_\beta + u_\beta \cos \lambda_\beta^\beta \sin \theta_\beta) \\ -\omega^{(2)}[r_\beta^\beta \cos(\theta_\beta + \phi_\beta) + u_\beta \cos \lambda_\beta^\beta \sin(\theta_\beta + \phi_\beta) - A] \end{bmatrix} $$

(6)

By right of the meshing theory for gear drives, the meshing equations of $\alpha$ and $\beta$ can be obtained:

$$ \phi^{\alpha}(u_\alpha, \theta_\alpha, \phi_\alpha) = n^\alpha_\alpha \cdot V^{\alpha(12)}_1 = -\omega^{(2)} \sin(\theta_\alpha + \phi_\alpha) [u_\alpha + (p_\theta \theta_\alpha - l_\alpha) \sin \lambda_\alpha^\alpha] + \omega^{(2)} r_\alpha^\alpha \cos \lambda_\alpha^\alpha \sin(\theta_\alpha + \phi_\alpha) + 1 $$

(7)

$$ \phi^{\beta}(u_\beta, \theta_\beta, \phi_\beta) = n^\beta_\beta \cdot V^{\beta(12)}_1 = -\omega^{(2)} \sin(\theta_\beta + \phi_\beta) [-u_\beta + (p_\theta \theta_\beta - l_\beta) \sin \lambda_\beta^\beta] + \omega^{(2)} r_\beta^\beta \cos \lambda_\beta^\beta \sin(\theta_\beta + \phi_\beta) - 1 $$

(8)

There are two solutions to $\phi^{\alpha}(u_\alpha, \theta_\alpha, \phi_\alpha) = 0$. So two lines of contact exist on $\alpha$. The equations of them are ciphered as

$$ r_\alpha^\alpha = M_\alpha u_\alpha \theta_\alpha $$

$$ \theta_\alpha + \phi_\alpha = -\pi $$

$$ u_\alpha \geq 0 $$

(9)

$$ r_\beta^\beta = M_\beta u_\beta \theta_\beta $$

$$ u_\beta = \frac{r_\beta^\beta \cos \lambda_\beta^\beta \sin(\theta_\beta + \phi_\beta) + 1}{\sin(\theta_\beta + \phi_\beta)} - (p_\theta \theta_\beta - l_\beta) \sin \lambda_\beta^\beta $$

$$ u_\beta \geq 0 $$

(10)
In the paper, lines expressed by Eqs. (9) and (10) are named first line of contact and second line of contact respectively. Second line of contact on $\alpha$ should not appear on solid model resulting from the small siding angle. Similarly, the first line and second line of contact on $\beta$ are expressed respectively in $S_0$ as

\[ r^\alpha_0 = M_\alpha r^\alpha_0 (u_\alpha, \theta_\alpha) \]
\[ \theta_\alpha + \phi_1 = 0 \]
\[ u_\alpha \geq 0 \]

\[ r^\beta_0 = M_\beta r^\beta_0 (u_\beta, \theta_\beta) \]
\[ u_\beta = -\frac{r^\beta_0 \cos \lambda^\beta_0 [\cos(\theta_\beta + \phi_1) - 1]}{\sin(\theta_\beta + \phi_1)} + \tau(p_\beta \theta_\beta - l_\beta) \sin \lambda^\beta_0 \]
\[ u_\beta \geq 0 \]

Without exception, first line of contact is the generatrix of $\beta$. The position of second contact line on $\beta$ contradicts that of first lines of contact on $\alpha$. As a result, second contact line on flank $\beta$ is not considered in the design. The equations of flanks $\alpha'$ and $\beta'$ of worm wheel can be deduced in $S_2$. Utilizing the relationship between surface parameters of worm and those of worm wheel, equations of flanks $\alpha'$ and $\beta'$ can be simplified to position vectors $r^\alpha_2$ and $r^\beta_2$ at any points on worm wheel flanks as

\[ r^\alpha_2 (u_\alpha, \theta_\alpha) = \left[ -r_{2\alpha} \cos \theta_\alpha + u_\alpha \cos \lambda_{2\alpha} \sin \theta_\alpha, -r_{2\alpha} \sin \theta_\alpha - u_\alpha \cos \lambda_{2\alpha} \cos \theta_\alpha, -u_\alpha \sin \lambda_{2\alpha} + l_\alpha - r_{2\alpha} \theta_\alpha \tan \lambda_{2\alpha} \right] \] (13)

\[ r^\beta_2 (u_\beta, \theta_\beta) = \left[ -r_{2\beta} \cos \theta_\beta + u_\beta \cos \lambda_{2\beta} \sin \theta_\beta, -r_{2\beta} \sin \theta_\beta - u_\beta \cos \lambda_{2\beta} \cos \theta_\beta, -u_\beta \sin \lambda_{2\beta} - l_\beta + r_{2\beta} \theta_\beta \tan \lambda_{2\beta} \right] \] (14)

where

\[
\begin{align*}
    u_\alpha &= p_\alpha \frac{\pi + \phi_2}{\sin \lambda_{2\alpha}} \sin \lambda_{2\alpha} - l_\alpha - u_\alpha \\
    \theta_\alpha &= -\phi_2 \\
    r^\alpha_2 &= A + r^\alpha_\alpha \\
    l_\alpha &= (p_\alpha \pi + l_\alpha) \cot \lambda_{2\alpha} \\
    \lambda^\alpha_{2\alpha} &= \frac{1}{2} \pi - \lambda_{2\alpha} \\
    u_\beta &= u_\alpha + \frac{l_\beta + p_\beta \phi_2}{\sin \lambda_{2\beta}} \\
    \theta_\beta &= -\phi_2 \\
    r^\beta_2 &= A - r^\beta_\beta \\
    l_\beta &= l_\beta \cot \lambda_{2\beta} \\
    \lambda^\beta_{2\beta} &= \frac{1}{2} \pi - \lambda_{2\beta}
\end{align*}
\] (15)

Obviously, Equations (13) and (14) are the position vectors at arbitrary points of screw involute surfaces. Parameters in Eq. (15) decide the sizes, positions and shapes of surfaces. Based on the theory of gearing meshing (Fu, 1999) (Dong, 1989), conditions of nonundercutting of $\alpha$ can be expressed in $S_1$ as

\[ u_\alpha = -(p_\alpha \theta_\alpha - l_\alpha) \sin \lambda_{2\alpha} \]
\[ r^\alpha_1 = r^\alpha_2 (u_\alpha, \theta_\alpha) \]
\[ u_\alpha = \frac{p_\alpha \theta_\alpha - l_\alpha}{\sin \lambda_{2\alpha}} \]
\[ r^\alpha_1 = r^\alpha_2 (u_\alpha, \theta_\alpha) \] (17)
Limiting flank $\alpha$ with Eq. (16), the undercutting will be avoided theoretically. In the same way, two limiting lines on $\beta$ are represented in $S_1$ as

$$
\begin{align*}
\lambda = \frac{\theta + \cos \beta}{\alpha} & \sin \beta \\
\lambda = \frac{\theta + \cos \beta}{\alpha} & \cos \beta
\end{align*}
$$

(18)

In general, $u_\beta < 0$ is induced in Eqs. (18) and (19) for $\theta_\beta < 0$ largely in the spiroid worm in practice. Therefore singular points will not appear on $\beta$. The envelopes to family of contact lines on $\alpha$ and $\beta$ can be expressed as

$$
\begin{align*}
u = \frac{\theta + \cos \beta}{\alpha} & \sin \beta \\
u = \frac{\theta + \cos \beta}{\alpha} & \cos \beta
\end{align*}
$$

(19)

Equation (20) coincides exactly with the limiting line Eq. (16). And obviously the envelope on $\beta$ doesn’t exist. By definition (Wu, 2009), the equations of conjugate zone of $\alpha$ and $\beta$ are ciphered out as

$$
\begin{align*}
u_\alpha + (\theta + \cos \beta) & \sin \beta \\
u_\beta + (\theta + \cos \beta) & \cos \beta
\end{align*}
$$

(20)

Equation (20) coincides exactly with the limiting line Eq. (16). And obviously the envelope on $\beta$ doesn’t exist. By definition (Wu, 2009), the equations of conjugate zone of $\alpha$ and $\beta$ are ciphered out as

$$
\begin{align*}
u_\alpha + (\theta + \cos \beta) & \sin \beta \\
u_\beta + (\theta + \cos \beta) & \cos \beta
\end{align*}
$$

(21)

Any values of $u_\alpha$, $u_\beta$, $\theta_\alpha$ and $\theta_\beta$ satisfy the inequalities, meaning that the conjugate zone of $\alpha$ is limited only by condition of undercutting and helix on base cylinder. The solid model of $\beta$ is only limited by the base cylinder. The sliding angles at arbitrary meshing points on first lines of contact on $\alpha$ and $\beta$ can be calculated as

$$
\begin{align*}
\theta_\alpha = \arcsin\left(\frac{|(u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)| - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)}{(u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) + (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) + (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)}\right)

\theta_\beta = -\pi - \phi
\end{align*}
$$

(24)

$$
\begin{align*}
\theta_\alpha = \arcsin\left(\frac{|(u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)| - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)}{(u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) + (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) - (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha) + (u_\alpha \sin \lambda_\alpha + \theta_\alpha - l_\alpha)}\right)

\theta_\beta = -\pi - \phi
\end{align*}
$$

(25)

Sliding angle is a value to preliminarily estimate the transmission efficiency. In order to get higher transmission efficiency, the sliding angle should be as close as possible to $90^\circ$.

Induced normal curvature is usually used to estimate local contact stress level on the basis of Hertz theory. The smaller the value of induced normal curvature, the smaller local contact stress may be. Induced normal curvatures of ISGDWLC at any meshing points can be attained as
3. Design of ISGDWLC

In the light of previous research, some conditions of limitation wasn’t applied on the geometry design, and a phenomenon of interference wasn’t mentioned. The following will modify and supplement the previous geometric calculation methods appropriately, summarize a set of formulas for the design of ISGDWLC, solve the interference problem by combining three-dimensional model and calculate the length of first line of contact.

3.1 Design of worm and worm wheel

In the paper, with reference to the early related researches the main formulas are listed in Table 1. Center distance \( A \), transmission ratio \( i_{12} \), contact ratio \( \varepsilon \), number of worm threads \( z_1 \) and worm wheel teeth \( z_2 \), lead angles \( \lambda_\alpha^* \) and \( \lambda_\beta^* \) should be determined in advance. Main geometric parameters of ISGDWLC are indicated in Fig. 3 and Fig. 4. Calculation formulas of worm and worm wheel are list in Table 1. The definitions of parameters in previous works (Baidu Wenku, 2019a, 2019b) (Zhang and Nie, 2011a, 2011b, 2011c) are used in the paper. In addition, two key points need to be highlighted, namely:

1) Parameter \( E \) and \( \delta_1 < \lambda_\alpha^* \) are deduced to avoid undercutting and second contact line on solid model in meshing, which didn’t consider or considered inadequately in previous research. The point with \( \theta_\alpha = -0.5\pi \) of second contact line on flank \( \alpha \) is on plane \( Z_1 = -E \) and worm top cone at the same time.

2) The interference which wasn’t reported in previous works may occur at the position near the base cylinder of \( \alpha' \). The malfunction indicates that the aforementioned limitations can’t prevent from global interference thoroughly. As shown in Fig. 5, an interference between \( \alpha \) and \( \alpha' \) occurs near the base cylinder of worm wheel. We find that increasing inner ring diameter \( R_i \) and combining with motion simulation of the gear drive are able to avoid the interference simply and efficiently. The formula of \( R_i \) in Eq. (28) is proposed preliminarily considering the compactness of the gear drive and its value is allowed to adjust appropriately.

\[
\sqrt{E^2 + (r_{1E}^*)^2} \leq R_i \leq \sqrt{E^2 + (r_{2E}^*)^2}
\]  

(28)

3.2 Length of first line of contact

Load transmitting, friction, lubrication and wear will occur at the position of contact. Therefore lengths of first lines of contact are important to calculate the efficiency, carrying capacity, wear and so on. Section 2 has proved that first lines of contact on \( \alpha \) and \( \beta \) are straight lines which causes it is very easy to deduce the formulas of lengths changing over time and position. On the basis of geometrical parameters in Table 1 and simultaneous equations method, the functions of first lines of contact and top cones are deduced firstly, and then generatrix lengths \( u_{\alpha1}, u_{\alpha2}, u_{\beta1} \) and \( u_{\beta2} \) can be obtained respectively by detecting the crossing points \( Q_{\alpha1}, Q_{\alpha2}, Q_{\beta1} \) and \( Q_{\beta2} \) between first lines of contact and top cones of worm or worm wheel as shown in Fig. 6. The problem is turned into solving unitary quadratic equation. Based on the above principle, generatrix length at \( Q_{\alpha1} \) are presented as

\[
u_{\alpha1} = \frac{-b - \sqrt{b^2 - 4a_1c_1}}{2a_1}
\]  

(29)
### Table 1. Formulas of main parameters.

| Parameter | Formula | Value |
|-----------|---------|-------|
| $A$ [mm] | $A = 100$ | $6.337$ |
| $i_t$ | $i_t = 70$ | $6.337$ |
| $d_{ias}$ [mm] | $d_{ias} = d_{ias} - \frac{2h_t}{\cos \delta_i}$ | $38.216$ |
| $z_1$, $z_2$ | $i_{l2} = \frac{z_2}{z_1}$ | $31.129$ |
| $d_{ifa}$ [mm] | $d_{ifa} = d_{ifa} - \frac{2h_i}{\cos \delta_f}$ | $32.547$ |
| $\epsilon$ | $\epsilon = 7$ | $2.817$ |
| $\lambda_1^\alpha$, $\lambda_1^\beta$ [°] | $m_{sl} = \frac{2A \sin \lambda_1^\alpha}{(z_1 \tan \lambda_1^\alpha - z_i) [(\cos \lambda_1^\alpha - \delta)]}$ | $2.817$ |
| $h_1$, $h_1^\prime$ [mm] | $h_1 = h_1^\prime m_{sl} (h_1^\prime = 1)$ | $2.817$ |
| $t_{sl}$ [mm] | $t_{sl} = \frac{t_{sl}}{t_{si}}$ | $5.321$ |
| $P_{a1}$ [mm-1] | $P_{a1} = \frac{2 \pi r_1^a \tan \lambda_2^a}{\pi z_1}$ | $1390.018$ |
| $P_{b1}$ [mm-1] | $P_{b1} = \frac{2 \pi r_1^b \tan \lambda_2^b}{\pi z_1}$ | $2226.225$ |
| $L$ [mm] | $L = \pi c m_{sl}$ | $64.817$ |
| $W_{z1}$ | $W_{z1} = 0.5 \pi m_{sl}$ | $4.424$ |
| $m_a$, $m_b$ [mm] | $m_a = \frac{P_a}{\pi z_1}$ | $9.260$ |
| $m_b$ [mm] | $m_b = \frac{P_b}{\pi z_1}$ | $8.522$ |
| $r_1^a$, $r_1^b$ [mm] | $r_1^a = \frac{P_a}{2 \pi \tan \lambda_2^a}$ | $3.160$ |
| $r_1^b$ [mm] | $r_1^b = \frac{P_b}{2 \pi \tan \lambda_2^b}$ | $5.062$ |
| $R_1$ [mm] | $R_1 = \sqrt{(E + L)^2 + (r_1^a)^2}$ | $156.857$ |
| $l_a$ [mm] | $l_a = E + (E - \frac{r_1^a \cos \lambda_2^a}{\sin \lambda_2^a})\tan^2 \frac{\lambda_2^a}{2} \tan \lambda_2^a$ | $61.154$ |
| $l_{b1}$ [mm] | $l_{b1} = l_a + \pi r_1^a \tan \lambda_2^a - [C_{b1} - \frac{d_{ias}}{2} \tan \lambda_2^a + \tan \lambda_2^b]$ | $47.212$ |
| $\delta_2$ [°] | $\delta_2 = \arccos \left(1 + \frac{A^2}{[E + \frac{2}{3} L + (E + \frac{2}{3} L + \Delta l_{fa}) \tan^2 \delta_f]^2 \sin \delta_f} \right)$ | $80.707$ |
| $\Delta l_{fa}$ [mm] | $\Delta l_{fa} = [E + \frac{2}{3} L + (E + \frac{2}{3} L + \Delta l_{fa}) \tan^2 \delta_f] \frac{\sin \delta_f}{\sin \delta_f + \sin \delta_f} - \frac{E + \frac{2}{3} L}{\sin \delta_f \sin \delta_f}$ | $-1.386$ |
where

\[
\begin{align*}
    a_{x1} &= \tan^2 \delta_1 \sin^2 \lambda_1^c - \cos^2 \lambda_1^c \\
    b_{x1} &= 2 \tan^2 \delta_1 \sin \lambda_1^c \{r_1^c \tan \lambda_1^c [-2(n+1)\pi - \varphi_1] - l_x - \Delta l_{isc}\} \\
    c_{x1} &= \{r_1^c \tan \lambda_1^c [-2(n+1)\pi - \varphi_1] - l_x - \Delta l_{isc}\}^2 \tan^2 \delta_1 -(r_1^c)^2
\end{align*}
\]

\[\text{(30)}\]

**Fig. 5** Interference between worm and worm wheel. A redundant intersecting line between worm and worm wheel appears in the inner ring of worm wheel. It means that interference between \(a\) and \(a'\) occurs.

**Fig. 6** Schematic diagram of calculating length of first line of contact: a) on plan \(X_0 = -r_1^c\); b) on plan \(X_0 = r_1^c\). The generatrices intersects with the small end of worm at \(Q_{sen}, Q_{sre}\) with the big end of worm at \(Q_{ben}, Q_{bref}\), with worm top cone \(Q_{bic}, Q_{bic}\) and with worm root cone \(Q_{bic}, Q_{bic}\). The functions of boundary lines can be presented easily. The length of generatrix at intersecting points can be obtained by solving the simultaneous equations of intersecting lines and the length of contact is determined by subtraction.

The others can be calculated in the same way. Moreover, arbitrary meshing points can be located easily, rapidly and accurately at any moment through simultaneous equations method. Furthermore the crossing points \(Q_{sen}, Q_{senf}, Q_{sref}\) and \(Q_{bref}\) should be taken into account and their positions can be determined in the same method. According to the position of crossing points, the length of first lines of contact need to be discussed in several cases as Table 2. The use of simultaneous equations method fully reflect the computational superiority of ISGDWLC.

| Case | \(\Delta u_x\) [mm] | Case | \(\Delta u_x\) [mm] |
|------|-----------------|------|-----------------|
| \(u_{x1} \leq u_{sre}\) and \(u_{x2} \geq u_{sre}\) | \(\Delta u_x = u_{x1} - u_{x2}\) | \(u_{x1} \leq u_{sre}\) and \(u_{x2} \geq u_{sre}\) | \(\Delta u_x = u_{x1} - u_{x2}\) |
| \(u_{x2} < u_{sre} < u_{x1}\) | \(\Delta u_x = u_{x1} - u_{x2}\) | \(u_{x1} < u_{sre} < u_{x2}\) | \(\Delta u_x = u_{x1} - u_{x2}\) |
| \(u_{x2} < u_{sre} < u_{x1}\) | \(\Delta u_x = u_{x1} - u_{x2}\) | \(u_{x1} < u_{sre} < u_{x2}\) | \(\Delta u_x = u_{x1} - u_{x2}\) |
| \(u_{x1} \leq u_{sre}\) or \(u_{x2} \geq u_{sre}\) | \(\Delta u_x = 0\) | \(u_{x1} < u_{sre}\) or \(u_{x2} \geq u_{sre}\) | \(\Delta u_x = 0\) |

4. Numerical example

A numerical example is implemented as follow based on MATLAB. The basic parameter values of the drive are shown in Table 1. Three-dimensional models of worm and worm wheel are established easily and accurately based on Eqs. (1), (2), (13) and (14). Firstly the starting point is determined by the function of worm wheel flank. Secondly the
generatrix and spiral line can be drawn. Thirdly generatrix sweeps along spiral line and the surface forms. Finally the top cone and root cone are used to restrict the surface and a model of worm wheel flank is completed. The rest of flanks can be established using the same method. The motion simulation is provided following the procedure as shown in Fig. 7 to certify that the interference is avoided and the drive can operate smoothly with constant transmission ratio.

![Flow chart of motion simulation. It is also the procedure to solve the interference between worm and worm wheel.](image)

**4.1 Lengths of first lines of contact**

The lengths of generatrix and first lines of contact related to $\varphi_1$ are deduced for the analysis of meshing performances according to Eqs. (29) and (30) and Table 2. In Fig. 8a and Fig. 8b, the lengths have obviously positive correlation with $\varphi_1$. When first lines of contact move to a position approaching end faces of worm, the lengths tend to equal and finally reached to the same value. In Fig. 8c and Fig. 8d, it can be seen that the length of first lines of contact of both flanks increase first, and then decrease with the increase of $\varphi_1$. The influences of the limitations of worm end faces are expressed as the plunges at the both ends of curves.

There are variances in the lengths of generatrices and lengths of first lines of contact for both flanks. Comparing the curves for two flanks in Fig. 8c and Fig. 8d, it is found that lengths of first line of contact and generatrix on $\alpha$ are less than those of $\beta$. The ineluctable variances are caused synthetically by differences of radius of base cylinders, lead angles and $d_{1\text{oa}}$. The above results can be proved by Three-dimensional model conveniently.

![Lengths of generatrix at crossing point and first line of contact related with $\varphi_1$: a) generatrix of $\alpha$; b) generatrix of $\beta$; c) first line of contact on $\alpha$; d) first line of contact on $\beta$. Length of generatrix at crossing point between generatrix and boundary of worm or wheel is obtained by solving the simultaneous equations of intersecting lines. $u_i$, $u_2$, $u_{\alpha}$ and $u_{\beta}$ are the lengths of generatrices at the crossing points between generatrix of flank $i$ ($i = \alpha, \beta$) with worm top cone, top cone of worm wheel, small end of worm and big end of worm, respectively. Length of first line of contact is calculated by lengths of generatrices subtraction considering the position of crossing points.](image)

**4.2 Meshing characteristics**

In the light of Eqs. (24) and (25), the data graphs regarding correlation among $|\theta_{\alpha1}|$, $\varphi_1$, $u_\alpha$ and among $|\theta_{\beta1}|$, $\varphi_1$, $u_\beta$ are given in Fig. 9. It is clear that ration angle $\varphi_1$ has little effect on $|\theta_{\alpha1}|$ and $|\theta_{\beta1}|$. With the increase of $u_\alpha$ and $u_\beta$, $|\theta_{\alpha1}|$ and $|\theta_{\beta1}|$ gradually close to $90.0^\circ$ and the rates of increases slow down. On the whole, the lubricating performance of the spiroid gearing is relatively favorable. According to the value range of $u_\alpha$ and $u_\beta$, the corresponding value range of $|\theta_{\alpha1}|$ is $[78.3^\circ, 83.0^\circ]$ approximately. Similarity, the value range of $|\theta_{\beta1}|$ is about $[71.6^\circ, 79.5^\circ]$. The data indicates that lubricating characteristic on $\alpha$ is better and the relative unsubstantial position is at the point where $u_\beta$ is the smallest, namely at the crossing point between tooth root of $\beta$ and small end face of worm. It is worth mentioning that the lubricating character can be improved by the increases of offset $E$ and the size of the gearing, but the processing cost will rise and the gear drive may take up more space.

As shown in Fig. 10, induced normal curvatures on both flanks of worm have monotonicity with the length of generatrix. The values vary considerably firstly and then steady at around 0. With the increase of $\varphi_1$, $K^\alpha_{\psi}$ increases while $K^\beta_{\psi}$ decreases. According to section 4.1, the corresponding range of $K^\alpha_{\psi}$ is about $[0.002\text{mm}^2, 0.010\text{mm}^2]$.
and that of $K_N^\alpha$ is $[0.017\text{mm}^{-1}, 0.034\text{mm}^{-1}]$. If the end faces of worm are ignored, the smaller the length of generatrix, the higher the absolute value of induced normal curvature on both flanks of worm. The sign of induced normal curvature depends on the direction of normal vector in Eqs. (3) and (4). $K_N^\alpha < 0$ and $K_N^\beta > 0$ in the range of lengths of generatrices prove once again that undercutting does not happen.

To validate our calculation, the comparison between the above results in this paper and the local meshing performances calculated (Zhao, 2018), whose research subject is conical surface enveloping spiroid drive, is carried. Both drives have the same center distance. Roughly, the local meshing performances of ISGDWLC has a slight edge, which proves the above formulas have some credibility. But it must be noted that ISGDWLCl has advantages for structure, calculation and transmission accuracy. Now a 3D model is built and a prototype has been established by Jilin Yu Hing Machinery Manufacturing Co., LTD. as shown in Fig. 11 and Fig. 12.

5. Conclusion

In the paper, the meshing theory, modeling and motion simulation of ISGDWLC are established, and a numerical example is implemented. The results of the performed research allow the following conclusion to be drawn:

1) Spatial meshing theory of ISGDWLC is presented completely. The equations for worm wheel flanks, which are the equations of screw involute surfaces, are deduced. According to the theory that the screw involute surface is generated by a straight line that performs a screw motion being tangent to the helix on the base cylinder, the model of worm wheel is established by CATIA successfully and simply. This property can be used to process worm wheel and the ways will be more flexible. Although in section 2 second lines of contact on $\alpha$ and $\beta$ are excluded, they may get good local performances and better use in same unknown methods.

2) The formulas for geometrical parameters of the gearing are mentioned completely. Sharping in the addendum and dedendum of worm is avoided. The problem of interference, which has not been described in previous literatures, is discovered and solved utilizing motion simulation on CATIA. Simultaneous equations method is used to calculate the length of generatrix, and the accurate solution can be obtained.

3) The numerical example investigation is completed. The attained outcome makes clear that, local meshing performances of ISGDWLC are relatively favorable, but there is difference between local meshing
performances of $\alpha$ and $\beta$ reflected by the length of first line of contact, sliding angle and induced normal curvature, which will affect the life of worm and restrict the application. Thus improving the asymmetry will be one of research direction in the future.

Overall, compared to other spiroid gear drive, ISGDWLC has some merits, such as accuracy, easy computation, relative good local meshing performances and high load capacity. Therefore, the gear drive is a possible potential for future application.

Acknowledgment

This work is supported by grants from Jilin province science and technology development plan (Product transformation of new type of spiroid gear drive, No. 20160307026G.X), China.

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