Virtual modelling of particles two-step feeding

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Abstract. The paper shows significant influence of mode and geometric parameters on the accuracy of particles feeding process. The model of the transformation of individual material batches into a continuous flow has been proposed based on the mathematical apparatus of random Markov processes with discrete space and time. The implementation and visualization of the mathematical model is carried out through the MAPLE universal mathematical package.

1. Introduction
Virtual modeling becomes an integral part of modern computer design, since it allows studying a large number of ways for compound machine designs and constructions. This approach makes it possible to simulate real conditions of their functioning on the computer and choose the best ones for developing more perfect products [1]. Besides, it reduces time and money consumption due to the substitution of many real processes with virtual ones. In [2] the properties of the adequacy of virtual models and the conditions associated with them for the appearance of the substitution effect are described. The authors developed an approach putting virtual computer models in an intermediate position between mathematical models and simulated physical objects. They considered virtual model as mathematical and physical one realized with the help of a physical device (i.e., computer). Moreover, a new fundamental property of virtual models, the ability to partially or completely substitute physical reality by computer models as adequacy and rudeness, can be added as key criteria of mathematical models. In this regard, the possibility of a superior substitution (i.e. ensuring the conditions not yet realized to fundamentally new solutions of real physical issues) seems to be of special interest. Among the others, processing bulk materials (loose, granular and, powder materials) comprising transportation, grinding, dosing, blending, classification, etc. are associated with particles movement. Meanwhile, the motion of a particle can be considered as the motion of a rigid body based on classical mechanics laws. Due to simultaneous movement of numerous particles it is almost impossible to count their collisions, even by advanced computers. That is why, various assumptions have been employed, where the bulk material was considered as a liquid or highly concentrated gas based on the laws of hydrodynamics or aerodynamics, respectively [3].

Continuous weighing of particles is the key operation of many technological processes. The main indicator of feeders is the feeding accuracy, which characterizes the uniformity of bulk material continuous flow at the outlet of a feeder and the deviation of actual productivity from the specified one.
The accuracy of agro-food particles feeding by vibrating feeders was considered in [4]. Feed-rate deviations caused by hopper refills widely used in the loss-in-weigh industrial technology was described in [5]. The general strategy feeding accuracy increase with this technology was presented by Coperion K-Tron [6]. Besides, feeding devices improvement was studied in [7-10]. To calculate the main parameters of the feeder, various mathematical models have been developed [11-13] and feeding accuracy has been increased.

The accuracy of nanoparticles feeding is of particular importance, since their content in the final product is tenth, and sometimes, even hundredths of a percent. For example, when modifying plastic lubricants with hybrid nanomaterials (graphene and carbon nanotubes), the content of these nanomaterials is less than 0.1%. Thus, even at capacities of tens of kilograms per hour for lubrication, the nanomaterials feeding productivity should be of the order of hundredths of a gram per second.

Weighing the material during its movement is the main reason limiting the increase in the continuous feeding accuracy for bulk material. Since dynamic loads on the weight sensor reduce its accuracy, a two-stage continuous feeding technology has been proposed [14]. The essence of the technology is that separate bulk material batches of weight $\Delta P$ are formed at the first stage of the process; at equal intervals of time $\Delta T$, they are supplied into a cylindrical tray vibrating torsionally [15]. The feeding capacity $Q$ can be evaluated as follows:

$$Q = \frac{\Delta P}{\Delta T}. \quad (1)$$

Considering the aforementioned, the aim of the present work is to develop a virtual model for a two-stage continuous feeding process, which would allow determining conditions and geometric parameters which have significant impact on the feeding accuracy.

### 2. Modeling of particles feeding process

We choose two-stage continuous weighing of bulk materials as modeling object and the device shown in Figure 1. A cylindrical tray (2) is installed on a base (1) which vibrates due to a vibrator (3). The frequency and amplitude of oscillations are controlled by unit No. 1 (5). Individual bulk material batches of weight $P$ are formed by a batcher (4) and are supplied into the tray (2) through a funnel (8) at regular intervals $T$. The batcher (4) is controlled by unit No. 2 (6) and a synchronizer (7). From the tray (2), the material is continuously supplied onto the inclined plate of a flow sensor (10), and then into a receiving container (9) or on a conveyor belt (not shown). The mathematical apparatus of random Markov processes is widely implemented for modeling bulk solid mixing [16], grinding and classification [17].

The process was considered to be discrete in space and time, so the tray and the conveyor belt along the length were divided into sections, and the transition of the system from one state to another occurs in a jump-like manner. The system was understood to be a bulk material distributed over the tray sections and the conveyor belt cells.

![Figure 1. Two-step particles feeding: 1- base; 2- cylindrical tray; 3- vibrator; 4- batcher; 5- control unit No. 1; 6- control unit No. 2; 7- synchronizer; 8- funnel; 9- container; 10- flow sensor.](image-url)
Figure 2 shows the Markov chain for the task under consideration. The first \( N_1 \) cells simulate the vibrating tray, whereas the following \( N_2 \) - the conveyor belt.

![Markov chain of the tray and the conveyor belt.](image)

**Figure 2.** Markov chain of the tray and the conveyor belt.

Two processes take place on the conveyor: cell redistribution of particles and movement of the particles along the vibrating tray. The intensity of the particle redistribution is determined by the probabilities \( p(i, i + 1) \), and particles movement along the tray – by the probabilities \( p_M(i, i + 1) \).

The state of the system is described by the \( S(i) \) vector:

\[
S(i) = \{s(1), s(2), s(3), ..., s(j), ..., s(N)\},
\]

where \( i \) is transition number, \( j \) – cell number, and \( N \) – the total amount of cells:

\[
N = N_1 + N_2,
\]

where \( N_1 \) is the number of tray sections or cells, \( N_2 \) is the number of conveyor belt sections (figure 2).

Thus, \( N_1 \) elements of the state vector characterize the distribution of particles along the tray sections, whereas \( N_2 \) elements – along the conveyor cells. Depending on the task, the numerical values of \( s(j) \) elements can be expressed in grams, cubic centimeters, or individual batch weight fractions.

The continuous feeding process is described by the following relationships:

\[
\begin{aligned}
S(1) &= S(0) \times P \times P_M \\
S(2) &= S(1) \times P \times P_M \\
&\vdots \\
S(i) &= S(i-1) \times P \times P_M \\
&\vdots \\
S(n) &= S(n-1) + SL \\
S(n) &= S(n) \times P \times P_M \\
&\vdots \\
S(k) &= S(k-1) \times P \times P_M
\end{aligned}
\]

where \( S(i) \) is the vector of the system state after transition \( i \), \( P \) is the matrix of transition probabilities, \( P_M \) is the movement matrix, \( n \) is the number of material batch loading steps (transitions), \( n=Lm \), \( m=1,2,3,... \) (integers), \( L \) is the number of batch loading steps; \( k=T_F/\Delta T \), where \( k \) is the number of transitions, after which the process ends, and \( T_F \) is the process ending time, \( \Delta T \) is the time of one transition. Matrices that remain unaltered at changing transition number are generally used. The matrix of transition probabilities is given as follows:
In matrix (5) first five elements simulate the vibrating tray, and the rest ones model the conveyor belt. The numerical values of the matrix elements simulating the vibrating tray depend on the mode parameters (tray slope angle, frequency and amplitude of tray torsional vibrations), as well as the friction coefficients of the material being weighed (external friction coefficient, internal friction coefficient, movement), and are determined when identifying the parameters of a mathematical model for a real process (equal values of 0.5 were chosen herein).

Since in the real implementation of the two-step feeding technology of bulk material batches are supplied into the tray periodically at time intervals $\Delta T$, the vector of the system state must periodically change in the mathematical model.

For a transition, the material can come from cell $i$ not only to neighboring cell $(i+1)$ but also to cell $(i+2)$. If during one transition the batch material is redistributed among several cells, the conversion of individual batches on the vibrating tray is carried out faster. At the same time, according to calculations, the material batch leaves the tray more rapidly. In all cases, transition time and of tray sections number, maybe easily calculated from an experiment.

The elements related to the conveyor are always constant (all the elements on the first diagonal located above the main diagonal of the matrix are equal to 1).

The movement matrix has the following form:

$$P_{m} =
\begin{bmatrix}
0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

### (5)
At each transition, a multiple of $\Delta T/\Delta t$, where $\Delta t$ is the time of one transition, the following procedure should be performed:

$$ S(0) = S(0) + S_L, $$

(7)

where $S_L$ is the vector, which elements characterize particles distribution shape along tray sections at loading.

In fact, the considered mathematical model allows simulating the fluctuation of batch shape for particles moving along the vibrating tray. Besides, it is possible to transform individual batches into a continuous flow with a periodic distribution change of next batch in initial tray sections.

In the virtual model, the batcher’s functioning, i.e. the process of forming individual material batches is proposed to be simulated using a random number generator and a filter providing the required deviations of the individual batch weight from a given value. The range of numerical deviation values depends on the particular batcher used in two-stage feeding. This process (i.e., individual batch feeding into the vibrating tray), as well as the periodic redistribution of the material in the initial sections is performed by employing specially developed software.

In the present work, the cell distribution of the batch after its loading into the cylindrical tray was calculated using the T-Flex CAD software. The MAPLE universal mathematical package (Waterloo Maple Corporation, Waterloo, ON, Canada) was chosen herein as a modelling tool for developing the model. This software can be used for numerical and symbolic calculations; it enables simulation of technical systems and visualization of a wide variety of calculations and physical processes. Besides, it provides tools for preparing technical documentation. Moreover, the software contains a unit for estimating root mean square deviations of the material weight in the sampler cell from the required values based on the given capacity. This unit makes it possible to use the virtual program to determine the accuracy of continuous weighing depending on the mode and geometric parameters of the device, on which the two-stage feeding technology is implemented.

3. Results and discussion

Figure 3 shows the distribution of the material along the sections: cells 1 to 19 simulate the vibrating tray, and cells 20 to 50 – the conveyor belt. Figures 3a-3c presents options where the next material batch is loaded every 30, 15 and 10 steps, respectively. In all the histograms, the material is distributed directly after loading the next batch. Initially, this material is distributed only along cells 1 and 2, and then, under the action of vibration, it is distributed along the rest tray cells. The ratio of the weight of the material located on cell $i$ to the weight of the individual batch represents specific gravity. Using a relative weight enables scaling any performance, without changing the vector of the system state $S(0)$ and the matrix of transition probabilities $P$.

As can be seen from the histograms, the distribution of the material among the sections when supplied at the 10th (Figure 3c) or the 15th (Figure 3b) steps is more uniform than when the material is supplied at the 30th step (Figure 3a). It should be noted that the actual weight of the individual material batch shown in Figure 3a is twice larger compared to Figure 3b, or three times more compared to Figure 3c, since the feeding process with the same capacity is simulated.

At first glance, it may seem that the easiest way to increase the uniformity of the continuous flow of the bulk material at the outlet of the dispenser is to supply the individual batches into the vibrating tray more often (i.e., to reduce the $T$ value). However, in this case, the weight of one batch will decrease, and consequently, weighing errors will increase. We assume that the batcher weighs the individual batches with an accuracy of 0.001 g. If the batch weight is 1 g, then the error will be 0.1 %, but if it is 0.5 g, then the error will be 0.2 %. Naturally, increasing the error of weighing the individual batch will increase the error of the continuous feeding.
Figure 3. Histograms of the distribution of bulk material along the sections of the tray and conveyor: the material is supplied at the 30 (a), 15 (b), and 10 (c) steps.

Figure 4 shows the distribution of the material immediately after loading the third batch. It can be seen that the third portion was distributed on the first two sections of the vibrating tray. The forms of the previous (first and second) batches changed, and they merged into a continuous flow.

The calculation of the movement of the individual material batch along the inclined vibrating tray using the model described above yields the similar results.

Figure 4. Distribution of the material immediately after loading the third batch into the vibrating tray.

The next batch is loaded prior to complete discharge of the current batch from the tray (here next batch is loaded at 30 steps). Since the "stretching" batch is faster than its discharging from the tray, the adjacent batches are interconnected. Therefore, the individual material batches are converted into a continuous flow. In a real feeder, the variable parameters are as follows: tray slope angle, amplitude of torsional vibrations, and weight of the individual batch weight.

To identify the parameters of the proposed mathematical model for a specific bulk material, it is sufficient to carry out nine simple experiments with three different amplitudes of torsional vibrations and three different tray slope angles. In these experiments, it is necessary to determine the batch
distribution immediately after loading into the tray, time when the particles are discharged from the tray, time during which all particles are discharged from the tray, and (4) conveyor belt cell distribution of the particles.

The adequacy of the model to the real process is achieved by changing the elements of the matrix of transition probabilities. The optimization objective is the minimum deviation from a given performance at any specified time.

Conclusion
The virtual model of the two-stage continuous weight feeding of bulk materials presented herein allows using the minimum of experimental data to estimate mode and geometric parameters of the vibrating feeder, which provides the specified performance and accuracy. Besides, the model enables tracing the physical evolution of the feeding process, monitoring the planned sequence of bulk material batch conversion into a continuous flow, and visualizing the geometric details of each device component.

Thus, the proposed model can be used in the design of new devices for continuous weight feeding of bulk materials with high accuracy, even at low performance (less than 0.1 g s⁻¹). Moreover, this model can be applied for educational purposes, since it allows elucidation of the type and operation method for equipment involved in the two-step feeding of bulk materials.

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