The use of quantile methods in economic history

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**ABSTRACT**

Quantile regression and quantile treatment effect methods are powerful econometric tools for considering economic impacts of events or variables of interest beyond the mean. The use of quantile methods allows for an examination of impacts of some independent variable over the entire distribution of continuous dependent variables. Measurement in many quantitative settings in economic history have as a key input continuous outcome variables of interest. Among many other cases, human height and demographics, economic growth, earnings and wages, and crop production are generally recorded as continuous measures, and are collected and studied by economic historians. In this paper we describe and discuss the broad utility of quantile regression for use in research in economic history, review recent quantitative literature in the field, point to potential limits in its use, and provide an illustrative example of the use of these methods based on 20,000 records of human height measured across 50-plus years in the 19th and 20th centuries. We suggest that, despite limitations in certain settings, there is still considerably more room in the literature on economic history to convincingly and productively apply quantile regression methods.

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Quantile regression; quantile treatment effects; economic history; practitioners

1. Introduction

Correctly capturing key patterns and relationships in economic history often requires the measurement of entire distributions of variables. For example, consider human demographics. Stunting, or very low heights owing to impaired growth in childhood, is associated with lower cognitive ability in later life, poorer health during adulthood, and reduced labor market earnings (Jayachandran and Pande 2017; Fogel 2004; Floud et al. 2011). And while stature is known to be broadly associated with health and other measures of well-being in many populations and time-periods (Costa 2004; Vogl 2014; Deaton 2007), these relationships are not constant over the entire range of heights, nor even monotonic in all populations. Thus, if one wishes to evaluate some historical event as an input to height or other demographic measures, it is necessary to determine how the event has impacted heights across the entire distribution. Similarly, consider the distribution of wealth, discussed for example in Latzko (2020); Canbakal and Filiztekin (2021). Identical mean movements in wealth may underlie vastly different distributional patterns, from movements of many individuals at low points in the wealth distribution, to movements of very few people at the top of the wealth distribution. What’s more, truncation in measures at certain points of the distribution may imply challenges in even correctly measuring mean wealth (Conley and Galenson 1994). For both of these reasons, beyond a focus on the mean, one may wish to consider how certain historical events affect and shift the entire distribution of wealth. Phenomena such as this are encountered in many other applications in economic history, such as hedonic analyses of real estate prices, analyses in changes in wages and labor supply measured as hours worked, and analyses of how institutions shape later economic development to name but a few. In each case, the distributional effects of policies or events can be as, or even more, important than their mean effects in the population under study.

In this article we seek to motivate the importance of considering the distributional impacts of historical events, and specifically survey a broad series of
methods which are ideally designed for such analyses. In particular, we lay out a range of methods related to quantile analyses, such as quantile regression, and the estimation of quantile treatment effects. These methods have emerged in a long line of theoretical and computational advances in the econometric literature, first fully described in Koenker and Bassett (1978), and comprehensively discussed in a range of papers or books since (Koenker 2017; Koenker and Hallock 2001; Lamarche 2019; Koenker 2005). While four decades have passed since the publication of Koenker and Bassett (1978)’s seminal paper on quantile regression, this is still an area of active research, in particular with recent advances considering quantile treatment effects, and identification in broader circumstances. We note that while this is quite a broad scope, we do not extend to additionally consider more general non-parametric analyses and descriptive analyses in economic history (examples of such methods can be found in Deaton (1989); Wachter (1981)). While also illuminating, this article focuses on modeling and estimation using quantile regression and other related methods.

Despite being well-suited to applications in economic history where dependent variables of interest are often continuously distributed, these are arguably under-utilised in empirical applications. As well as providing an overview of these methods including recent extensions which are likely to be of particular interest to practitioners in economic history, we provide a discussion of both the benefits and the drawbacks of quantile methods in economic history.1 In a survey across all principal economic history journals we find around 50 papers that have used quantile regression in some way. We review the arguments for the use of quantile methods in these papers, which generally suggest one of three principal motivations: (a) quantile methods allow us to learn more in a given context; (b) quantile methods are more closely linked to particular economic models in certain settings; and (c) quantile methods allow for certain complications to be resolved which are present in other modeling settings. Of course, there are reasons why quantile methods may not be well suited to studies in economic history where measurement and data collection is challenging. Among other things such methods require continuously measured outcomes, and can be richer where larger databases are available, both of which may be relevant limitations in research where data collection is challenging and costly.

This article thus seeks to motivate the “use” of quantile analyses in economic history in two ways. The first is to describe how they can be productively used by practitioners in empirical studies to capture relationships of interest which may not be gleaned from simple mean or other average estimators. And the second is to document how they have been used (and arguably under-used) in the literature on economic history up to this point.

In what remains of this paper, we first provide a brief primer on the use of quantile methods in Section 2. We then provide a much deeper summary of key methods for distributional analyses in empirical methods in Section 3. In Section 4 we provide both an overview of papers based on quantile analysis methods in economic history, review arguments presented in these papers underlying the importance of estimation based on quantile methods, and lay out a number of potential drawbacks related to these models in research in economic history. Finally, in Section 5 we document a specific example based on microdata on human height covering around 20,000 individuals exposed to divergent rates of economic growth throughout their life. In closing, we make a number of points related to the computational implementation of these methods.

2. A Primer on regression and quantile regression

In regression analysis, a researcher seeks to understand how some variable or group of variables (known as the “independent variable(s)”) impacts some other variable (known as “the dependent variable”). The regression is a statistical technique by which one seeks to use data to estimate relationships between the dependent variable and each independent variable. These relationships are summarized from data as a numerical ‘parameter’ when a regression is estimated using data. Under a number of assumptions, most importantly that there are no unobserved variables which are correlated with both the independent and dependent variable, one can interpret regression parameters as causal relationships, and as capturing the expected change in the dependent variable if one increases a given independent variable by one unit, holding all else constant.

Frequently, the estimation of these regression parameters is conducted using a statistical technique known as ordinary least squares, or OLS. OLS is a technique which seeks to find the parameters which best explain the linear relationship between the dependent and independent variable by choosing the single value for each that minimizes the sum of the squared distance between the regression predicted value and the actual value for each dependent variable over all

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1. Lamarche (2019) places quantile methods in a broader context of quantile regression in economic history.
observations in the data. While OLS is perhaps the statistical technique most frequently used in regression analysis in economic history, and in quantitative modeling in economics and the social sciences more generally, it is not the only statistical technique which can be used to estimate regressions.

The formal interpretation of the coefficients in OLS regression models is in estimating impacts of changes in independent variables on the average (mean) of the dependent variable in the data. As an example, if one wishes to consider how the adoption of some technological innovation impacts agricultural productivity in a sample of farms where productivity is measured as output of crops in kilograms per year, and estimates a regression parameter of 5, this implies that on average the technology has increased output by 5 kg per year in the sample of farms. However, this average impact may hide considerable heterogeneity of impacts of the technology. While it may be true that all farms increased their productivity by 5 kg on average, it may also be true that only relatively more productive farms increased their productivity, or even that more productive farms increased productivity, while some less productive farms actually decreased productivity as a result of the technology. These distribitional impacts are potentially of more interest than mean impacts, as they allow for a richer understanding of how the technology works, whether it may affect the market structure, and at which points of the productivity distribution these impacts are observed.

This logic can be formalized through an alternative regression technique, known as Quantile Regression. In quantile regression, rather than estimating the impact of changes in independent variables on the mean of the dependent variable, one can estimate the impact of changes in independent variables on any quantile of the distribution of the dependent variable. A quantile is simply the point in the entire distribution of the variable below which a particular proportion of the population falls. For example, the 5th quantile over a variable is the point at which 5% of the population has a value below this point, while 95% of the population has a value above this point. The 50th quantile, also known as the median, is the value of the variable at which 50% of the population has a realization below this value, and 50% of the population has a realization above this value. One can consider any quantile between 0 and 100. By using quantile regression, questions can be answered such as how does a change in an independent variable impact individuals who are very low in the distribution of the dependent variable? Or how does a change in an independent variable impact individuals who are very high in the distribution of the dependent variable? Similarly, one could consider the impacts of changes in independent variables on the dependent variable at the mean of the dependent variable, or at any point across the entire range of the dependent variable. Note here then, that these quantile regressions can consider a particular type of heterogeneity in data: this is how returns to particular variables change across the entire distribution of the dependent variable.

The formal theory behind quantile regression was introduced by Koenker and Bassett (1978), and has since been extended to a range of alternative settings including the conception of quantile treatment effects, and models to correct for cases where relevant unobserved variables exist, where the dependent variable is measured with error, to panel data, and so forth. In the following section we provide considerable more mathematical formulation to the ideas laid out in this section.

3. Quantile regression

Generally, in empirical applications one wishes to consider the impact of some group of independent variables \( x_k \) for \( k \in \{1, \ldots, K\} \) on a specific dependent variable, denoted \( y \). Where observations \( i \) refer to units (such as individuals), this is often parameterized using a linear regression model:

\[
y_i = \beta_0 + \beta_1 x_i + \ldots + \beta_k x_k + u_i, \tag{1}
\]

where \( u_i \) is an unobserved error term. Frequently, and indeed, in a the majority of papers in economic history (see Section 4) estimation is implemented using the ordinary least squares (OLS) method. Mathematically, this procedure simply consists of finding the \( K \) parameter estimates of \( \beta \) which minimize the following problem:

\[
\hat{\beta} = \arg \min_{\beta \in \mathbb{R}^K} \sum_{i=1}^{N} (y_i - x_i \beta)^2,
\]

where \( x_i \) refers to the vector of values of each the independent variables \( x_k \) for unit \( i \). This optimization returns a vector of parameters capturing the mean impacts of each \( x_k \) on the outcome of interest \( y \). While mean impacts across the distribution of \( y \) may be a logical summary parameter in many cases, it is not the only point with meaningful empirical content. Often, other points of the distribution of \( y \)
may be as, or more important, than the mean, particularly in historical processes where extreme outcomes may be of particular interest, including inequality studies. This suggests the importance of modeling options allowing for additional heterogeneity. Below we describe a range of quantile estimation techniques which allow for focus across any points of the distribution of outcomes of interest.

3.1. The linear quantile regression model

Quantile regression considers how some independent variable of interest affects a dependent variable at particular (and generally varying) points of the full distribution of some dependent variable \( y \). Thus, at a minimum, \( y \) requires a distribution with considerable variation, and is inappropriate in cases where \( y \) is a binary or categorical measure. We will denote as \( \tau \) the quantiles of the distribution, such that (for example) \( \tau = 0.5 \) indicates the median of the distribution, and \( \tau = 0.1 \) indicates the 10th percentile, or the point of the distribution below which 10% of the observations of \( y \) are observed. As originally documented in Koenker and Bassett (1978), we can estimate the parameter vector \( \beta(\tau) \) capturing the impact of each \( x_i \) on the \( \tau \)th quantile of \( y \) based on the following minimization problem:

\[
\beta(\tau) = \arg \min_{\beta \in \mathbb{R}^p} \left[ \frac{1}{N} \sum_{i=1}^{N} \tau y_i x_i \beta + \frac{1}{N} \sum_{i=1}^{N} (1-\tau) |y_i - x_i \beta| \right].
\] (2)

Note that here, estimation is based on absolute deviations of \( x_i \beta \) from \( y_i \) rather than quadratic distances in OLS, and indeed, in the case that \( \tau = 0.5 \) (the median) this formula collapses to the Least Absolute Deviations estimator. In all cases except for the median, this minimization problem uses the quantity \( \tau \) to ‘tilt’ the estimates toward data which is lower or higher in the distribution of \( y \), as can be observed in the two terms within the parentheses in Equation (2): when \( \tau \) is between 0 and 0.5, more weight is given to the right-hand summation for units \( i \) whose \( y_i \) is less than the conditional mean \( x_i \beta \), whereas when \( \tau \) is between 0.5 and 1, more weight is assigned to the left-hand summation considering units whose \( y_i \) is greater than the conditional mean.

Equation (2) is the well-known linear quantile regression model which is implemented as standard in many computational languages. Frequently, rather than focusing on a particular quantile of interest, estimates \( \beta(\tau) \) are documented over a range of quantiles in a graphical manner (refer for example to the illustration in Section 5 of this paper). What’s more, with modern computational tools, standard errors can be calculated quite simply along the range of the distribution of \( y \), which permits formal hypothesis tests and other inferential procedures at each quantile considered. Formally, following notation from Lamarche (2019), the approximate distribution of the vector \( \beta(\tau) \) can be written as:

\[
\beta(\tau) \sim \mathcal{N} \left( \beta, \frac{1}{N} \tau (1-\tau) H^{-1} J H^{-1} \right),
\]

where \( \mathcal{N}(\cdot) \) refers to a Gaussian distribution and the matrices \( J = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i x_i' \), and \( H = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} f_i(x_i \beta(\tau)) x_i x_i' \), with \( f_i(\cdot) \) being the conditional density function of \( y \). Note here two key implications: firstly, that this inference holds asymptotically, i.e. as the sample size grows, and secondly that a key ‘ingredient’ is the estimate of the density \( f_i(x_i \beta(\tau)) \) at quantile \( \tau \). If one is willing to assume that error terms are identical and independently distributed (iid), the \( f_i \) term is identical among all units \( i \), and estimation is simplified, for example using the fitted value of the density at the quantile of interest. However, if heteroscedasticity robust estimates are desired, more complex ‘sandwich’ estimates are necessary. While these are also implemented as standard in computational routines in languages such as Stata or R, further background on these procedures can be found in Koenker (2005, chapter 3). Alternative options which avoid the estimation of these matrices consist of using bootstrap resampling methods for inference. A review of such resampling methods in quantile regression is provided by He (2018). Finally, note that solutions have also been proposed to resolve cases where clustered inference is desired, allowing for correlated shocks within groups, as well as heteroscedasticity (Hagemann 2017), once again based on bootstrap resampling procedures.

3.2. Quantile treatment effects

While this quantile regression can be used for \( x_i \) of arbitrary forms and dimensions, a particular case of interest is that of treatment effect models, with some binary ‘treatment’ of interest, which we denote \( D_i \). Note that while quantile regression requires continuous distributions for the outcome variable of interest, there is no limit on the nature of independent variables \( x_i \), including binary and categorical measures. The standard average treatment effect aims to identify the mean impact of receiving treatment (versus not
receiving treatment) on dependent variable $y$: $\Delta = E(y \mid D=1) - E(y \mid D=0)$. This assumes independence of $D$ from unobservables. Extending this average treatment effect to its quantile treatment effect (QTE) analogue gives:

$$\Delta(\tau) = Q_y(\tau \mid D=1) - Q_y(\tau \mid D=0), \quad (3)$$

where $Q_y(\tau)$ refers to the value of $y$ at a particular quantile $\tau$ of the distribution of $y$. Such an estimate, beyond the mean, is likely to be relevant for a range of historical policies, for example examining whether exposure to a specific type of economic or political system, exposure to a historical environmental shock, or exposure to a historical policy has divergent impacts across the distribution of individual outcomes capturing well-being. All such examples refer to binary ‘treatment’ statuses, and hence are potentially appropriate for QTE methods and their extensions discussed in the following sub-section, provided that outcomes of interest $y$ are continuous measures. In practice, these estimates can be generated using regression following the procedure laid out in the previous section. We return to discuss computational implementations in the final section of this paper.

This setting can be extended considerably, for example to include covariates, and/or to explicitly consider selection into treatment. This can be done in a number of ways, such as by attempting to correct for selection non-parametrically (Bitler, Gelbach, and Hoynes 2006; Firpo 2007), or extending to panel data settings and using recent methods such as difference-in-differences (Callaway and Li 2019). A particular case where QTE methods are extended to deal with endogeneity is laid out in the following sub-section when discussing instrumental variables and Local QTEs. While the inclusion of covariates can complicate the QTE setup (Koenker 2017), suggestions on how to deal with this have been proposed in Firpo (2007); Fröölich and Melly (2010); Callaway and Collins (2018) based on propensity score methods and re-weighting techniques. For applied work, a particularly useful reference discussing a range of practical estimation methods for quantile treatment effects is the paper of Fröölich and Melly (2010).

### 3.3. Quantile regression extensions

While standard quantile regression and QTE implementations can provide illustrative results which allow for an understanding of effects beyond the mean, recent theoretical and computation advances in these methods offer considerable additional benefits, including the possibility to address a number of issues in standard quantile regression models, and more explicitly address questions of causality. We discuss the broad scope of these recent advances here, providing key references. Full modeling considerations are available in the papers discussed below, however particularly useful overviews of these methods at more length can also be found in the survey articles or handbook chapters of Koenker (2017) (discussing all below points), or Wei (2018); Lamarche (2019); Melly and Wüthrich (2018); Chernozhukov, Hansen, and Wüthrich (2018) with focuses on particular points.

#### Models to correct for measurement error.

In the case of variables collected over a considerable time-frame from diverse data sources as is often the case in studies in economic history, measurement error or issues of sample selection are valid concerns. Fortunately, a number of methods suggest ways that measurement error can be accounted for within the framework of quantile regression (under assumptions about the nature of these errors). For example, Wei and Carroll (2009) document potential estimation methods when in the presence of measurement error in independent variables, and alternative methods are proposed by Wang, Stefanski, and Zhu (2012). The crux of these models is that if rather than observing a true variable $x$ we observe some noisy proxy, we must consider the conditional expectation of $x$ based on the noisy proxy, rather than simply using the proxy in the regression. Additionally, even if there is limited error in the measurement of variables, samples may be selected in certain ways, for example if only certain types of records survive following collection of historical data, or if data are based on surviving individuals. Arellano and Bonhomme (2017) propose methods for quantile regression under selection based on an exclusion restriction for estimation, while Blanco, Flores, and Flores-Lagunes (2013) (see also Lamarche (2019)) propose partial identification methods as way ahead in such circumstances. Computational implementations of such selection models are available, for example via the programs of Biewen and Erhardt (2020); Siravegna (2020). Interested readers are pointed to the handbook chapter of Wei (2018) which provides a deeper discussion of these issues.

#### Endogeneity, instrumental variables and causality.

In many settings of interest in economic history, independent variables of interest will be endogenous—
correlated with unobserved factors which are themselves correlated with outcomes of interest. As is the case in standard econometric models, quantile regressions or QTE models do not allow for causal inference when in the presence of endogeneity. However, there is a rich stream of work which seeks to extend standard linear instrumental variable (IV) models into a quantile framework. Here in particular we discuss the Local Quantile Treatment Effect (LQTE) approach, which, under a number of key assumptions provides consistent (causal) estimates for certain sub-groups of the population across the entire distribution of the outcome variable.

The LQTE framework extends the well-known Local Average Treatment Effect (LATE) framework laid out in Imbens and Angrist (1994). Here, causal estimation using an instrumental variable is based on a monotonocity assumption that requires that the instrumental variable induces exogenous variation in the dependent variable of interest which shifts all individuals weakly in the same direction, for example acting as an as-good-as-randomly assigned incentive or disincentive to uptake an endogenous variable of interest. Here we summarize this and the other identifying assumptions, as well as how an LQTE is estimated, with a binary instrumental variable denoted \(Z\). A chapter length discussion of these methods, as well as extensions to other circumstances is provided by Melly and Wüthrich (2018). Following the notation of Section 3.2, consider an outcome variable of interest \(y\), a binary treatment variable of interest \(D\), as well as the instrumental variable \(Z\). Under 5 assumptions, instrumental independence (or validity), the exclusion restriction, instrumental relevance, monotonicity and the stable unit treatment value assumption (SUTVA) limiting interaction between treatment statuses, Imbens and Angrist (1994) prove that the following Wald estimator gives the LATE:

\[
E(y_i | Z_i = 1) - E(y_i | Z_i = 0)
\]

\[
E(y_i | D_i = 1) - E(y_i | D_i = 0)
\]

Note that this LATE is a consistent estimate of the impact of treatment (\(D_i\)) on outcomes (\(y_i\)) for those individuals whose treatment status would be changed by the instrument (the ‘compliers’).

In order to extend this framework to a quantile framework, distributional results are provided by Imbens and Rubin (1997); Abadie (2002). These give cumulative density functions specifically for the compliers, in this case which we denote \(F_{1c}\) and \(F_{0c}\). Here superscript \(c\) refers to the population of compliers, and \(F\) refers to the cumulative density of outcomes for cases where treatment is received (\(F_{1c}\)) or not received (\(F_{0c}\)). Given these definitions based on an IV and compliers, it is very easy to return to the notation from Section 3.2, which gives the LQTE estimator. As a clear parallel to Equation (3), this is defined as:

\[
\Delta(\tau) = Q_{1c}(\tau) - Q_{0c}(\tau),
\]

where as previously, \(\tau\) refers to quantiles of the relevant conditional density function above.\(^6\)

It is worth noting that these methods, while more demanding than standard QTE models given the required conditions of instrumental validity, are potentially well suited to applications in economic history where concerns exist relating to endogeneity. For example, Aaronson et al. (2021) estimate the impacts of the number of children born to women – a clearly endogenous variable – on maternal labor supply, using data over more than two centuries, by leveraging the birth of twins as an IV. In this case monotonicity assumptions are very likely met given that twin births should have weakly positive impacts on completed fertility.\(^7\) Very recent work by Valencia Caicedo (2021) provides considerable other examples of the use of IV in economic history (though not discussing quantile methods), describing among other examples IVs based on map borders or other geographical features such as rivers, or slopes of terrain, geographic suitability indexes, or Bartik-type instruments. All such instruments in economic history could be productively introduced into a quantile framework if distributional outcomes are considered.

Finally, prior to turning to applied examples of the use of quantile regression, we note that this particular implementation of IV and endogeneity corrections via LQTE methods is not the only way forward. Regression discontinuity designs can be similarly cast in this “Local” framework, while the instrumental variable quantile regression models (IVQR) of Chernozhukov and Hansen (2005) provides estimation methods which return average Quantile Treatment Effects (rather than LQTEs), but requiring alternative assumptions.\(^8\) These methods are further discussed in Melly and Wüthrich (2018) and Chernozhukov, Hansen, and Wüthrich (2018) respectively, and are likely to lead to more productive ways forward when non-binary endogenous variables are considered—in which case LQTE models are less appropriate—or in cases when broader non-local QTEs are desired, at the cost of alternative assumptions. A valuable review of these methods is provided in Chernozhukov and Hansen (2013).
Other extensions and methods. A range of other contexts which are potentially of use in quantitative studies of economic history can be productively studied in quantile settings. This includes settings such as longitudinal or panel data and difference-in-difference models, regression discontinuity designs, and non-parametric analyses. We briefly discuss these settings in turn below, pointing interested readers to relevant references.

Extensions of quantile regressions to panel data following individuals over time have been proposed in Koenker (2004) where challenges arise given desires to estimate movements across distributions within individuals (or panel units) with potentially few repeated individual level data-points. Koenker (2004) proposes using a penalized estimator to control for relevant individual-level effects, and much additional work has been conducted to take forward these techniques (a complete discussion is provided in Lamarche (2019, pp. 11-14)). Recent work of Gu and Volgushev (2019) has suggested using a grouped fixed effect approach, also based on penalized estimators to group similar individuals in a panel setting. Alternative lines of work, such as Arellano and Bonhomme (2016), suggest viewing (potential individual-level) variation as a problem of unobserved heterogeneity and estimating using an iterative process, super-imposing simulation based estimation procedures on top of standard quantile regression procedures. A particular setting of interest in cases of longitudinal data consists of the estimation of difference-in-differences models where exposure to some treatment varies within a panel over time such that baseline differences between exposed and unexposed individuals can be captured using pretreatment periods. These models have been extended to a quantile setting; see for example Callaway, Li, and Oka (2018) for a setting with two time periods and Callaway and Li (2019b) for a broader panel setting. Computational software is also available to implement these methods in Callaway (2019).

The regression discontinuity design provides credible identification in cases where some dependent variable of interest is moved discontinuously by some arbitrary cutoff. The use of these methods in economic history has been surveyed by Valencia Caicedo (2021), where examples are often based on distance to geographical features or map boundaries. The regression discontinuity design has been extended to a quantile framework in a very flexible way by Frandsen, Frölich, and Melly (2012), which allows for the estimation of quantile treatment effects 'local' to a particular discontinuity, and accompanying computational routines are available to implement this method.

Standard non-parametric regression implementations can capture fully flexible relationships between the mean of some variable $y$ and some independent variable $x$. Rather than parametric assumptions and linear functional forms (as imposed in Equation 1), the relationship is allowed to vary freely along the support of $x$, allowing for $x$ to have a non-linear impact on the mean of $y$. This logic can be extended to a quantile regression, if rather than considering a non-parametric relationship between the mean of $y$ and $x$, a non-parametric relationship between specific quantiles $\tau$ of $y$ and $x$ are estimated. This is a particularly flexible way to model heterogeneity which may be well-suited to historical outcomes over which there are few prior assumptions related to the nature of the relationship under study. A review of the methods available in Koenker (2017, Section 3), while computational resources for the implementation of such routines are available in, among others, Koenker (2021); Lipsitz et al. (2017).

4. Applications in economic history

4.1. Quantile regression in economic history research

To have some idea about the extensiveness of the use of quantile regression in economic history, we begin by running a search within the main economic history journals, covering the period from 2000 until the present.9 Namely, we searched within the Journal of Economic History, Explorations in Economic History, Economic History Review, Cliometrica, and the European Review of Economic History. But we also complemented these searches with searches of other economic history journals as discussed below, estimating that around 50-55 journal articles have made use of quantile regression as an analytical tool during the last two decades, although only a handful of these are dated pre-2005.

The journal which has most frequently published articles employing this technique is the Journal of Economic History, which published 18 articles using quantile regressions or non-parametric analyses for a wide range of topics from 2000, including: the market for paintings in Florence and Italy between 1285 and 1550 (Etro 2018); the interaction between inequality and financial development in the US during the late nineteenth century (Jaremski and Fishback 2018); fluctuations in technology during the Great Depression in the US (Watanabe 2016); the credibility of fixed
exchange rates during the classical gold standard era (Mitchener and Weidenmier 2015); and inequality of wealth in the Ottoman Empire (Coşgel and Ergene 2012); just to mention those published after 2010. These few examples make clear the broad applicability of quantile methods to questions of interest in economic history, covering issues in micro, macro and financial economics.

Explorations in Economic History has also published articles using quantile regression based methods with some frequency. A scoping review identified six such articles. Namely, Walker (2000), which explores the degree of economic opportunities in San Francisco compared to other regions around mid-nineteenth century; Dupont (2007), which tests for contagion in bank runs in Kansas during the panic of 1893; Canaday (2008), which deals with the relationship between wealth and wealth accumulation by both blacks and whites in South Carolina between 1910 and 1919, and its determinants; Drelichman and González-Agudo (2014), which reconstructs housing costs for various social groups and traces the effect of exogenous shocks on the rental market for Toledo, Spain, between 1489 and 1600; Alvarez and Ramos-Palencia (2018), which deals with the relationship between human capital and male labor earnings in eighteenth-century Spain; and Callaway and Collins (2018), which measures the union wage premium for several US-cities circa 1950, using unconditional quantile methods.

The Economic History Review, in turn, has published five articles where quantile regressions were used: Temin and Voth (2008) analyses the cost and availability of private bank credit between 1702 and 1724; Gazeley and Newell (2011), in turn, estimates urban poverty among working families in the British Isles circa 1904; Brown and Guinnane (2018) deals with the causes of fluctuations in infant mortality rates in Bavaria during the 1820s-1910s; Artunc (2019) examines the composition of firm ownership and entrepreneurship in Egypt between 1910 and 1949; while Karagedikli and Tunçer (2021) estimates real hedonic house prices and urban wealth inequality for the housing market between 1720 and 1814 in the Ottoman Empire. Note that as above, these articles show both a broad scope of themes, as well as a broad scope of geographic and temporal settings which have been productively analyzed with these models.

The European Review of Economic History has also published five articles making use of quantile regression: Koepke and Baten (2005), which provides the first anthropometric estimates of the biological standard of living in Europe during the first millennium AD; Dincecco (2009), which performs a statistical analysis of political regimes and sovereign credit risk in Europe from 1750 to 1913; Dribe (2009), who analyses the importance of demand and supply factors in the Swedish fertility transition between 1880 and 1930; Kholodilin (2016), that analyses the housing rental dynamics of Berlin during World War I; and more recently, Jorge-Sotelo (2019) focused on the impact of currency depreciation on international capital flows in Spain between 1928 and 1931 crisis.

Clomiometrica has published six articles using quantile regressions, the first of these less than a decade ago: Carson (2012), compared body mass index values of late 19th- and early 20th-century amongst African-American groups (i.e. blacks versus mulatto); Ogasawara and Kobayashi (2015), which deals with the impact of social workers on reducing infant mortality rates in inter-war Tokyo; Du Plessis, Janssen, and Von Fintel (2015), who for the period 1700-1725, estimated hedonic slave price indices and the value of their marginal productivity; González-Val, Tirado-Fabregat, and Viladecans-Marsal (2017), who analyzed the impact of market potential on the structure and growth of some Spanish cities during 1860–1960; Ogasawara and Matsushita (2019), which deals with the treatment effects of piped water on diseases in industrializing Japan (1920s-1930s); and finally, Keywood and Baten (2021), who tests the relationship between elite numeracy and elite violence in Europe from 500 to 1900.

We also run searches in other economic history journals, but where quantile regression was observed to be somewhat less frequently used. For example, in this journal, Historical Methods, Carson (2018) examines correlates of weights of Mexican residents of the United States in the 19th and 20th centuries and the previously mentioned paper of Conley and Galenson (1994) has been influential. In the main Spanish economic history journal, Revista de Historia Económica – Journal of Iberian and Latin American Economic History, quantile regressions were used in only four papers; in the Australian Economic History Review in only one article; in the Economic History of Developing Regions, similarly in only one article; while in the Scandinavian Economic History Review it has appeared twice. Finally, in the three main business history journals (Business History, Business History Review and Enterprise & Society), it was used in three articles, twice at Business History and once at Enterprise & Society, although this is rather unsurprising since this sub-discipline cultivates a less quantitative approach to history.
4.2. The potential for quantile regression methods in economic history

To consider more carefully the potential for quantile regression in research in economic history, in this section we lay out both the arguments for and the arguments against the use of quantile regression. In particular, when focusing on the arguments for implementing quantile regression or quantile analyses in economic history, we focus on arguments presented in papers, and the justification used in this literature when discussing these methods. We also note some general drawbacks which may be of relevance when considering the potential of these methods, particularly in settings which are common in economic history where data collection and samples sizes may be a considerable challenge.

4.2.1. Arguments for the use of quantile methods in economic history

Throughout the extant literature, papers applying quantile methods present three broad arguments for its justification as an improvement over standard analyses, often explicitly contrasting with OLS as a ‘baseline’ alternative. These can be described as (1) Reasons relating to what we can learn, and richer findings from these models; (2) Theoretical justifications based on explicit modeling arguments; and (3) Technical reasons why quantile methods may resolve certain issues present in non-quantile settings.

A first argument, which appears to be the most common in papers applying quantile regression in economic history is simply that we learn more when applying quantile regression than when simply estimating mean response function. This is relevant in settings when one cares specifically about distributional impacts of some policy. For example, if we are not indifferent about how an independent variable affects the shape of some outcome distribution, quantile regression may be more useful than OLS. A natural counterpoint to this, discussed in the following subsection, is that if we are completely indifferent to where a dependent variable shifts a distribution, only caring about how average outcomes are changed in a population, quantile regression will not be a necessary tool. In understanding the relevance of the argument that quantile regression allows us to learn more, it is useful to consider particular arguments in this line. For an illustration of this, consider the argument laid out by Walker (2000), in his study of demographic factors and wealth. He states, in arguing for the importance of quantile regression:

“The standard hedonic regression approach assumes that the goods or services transacted are of homogenous quality and purpose, only differentiated by their observable (hedonic) characteristics. Real estate does not exactly fit these assumptions. In particular, unobserved quality will almost certainly increase with a tenant’s income. Towards the upper tail of the income distribution, a house may acquire additional purposes besides providing shelter; for example, it could be used as an object of conspicuous consumption, broadcasting the wealth or political clout of its resident. ... The assumptions of the standard approach, however, could reasonably hold in the neighborhood of a particular point of the income distribution of tenants. This suggests a quantile regression approach”

Álvarez and Ramos-Palencia (2018, p. 39).

Here it is clear that quantile regression has a clear theoretical basis, and is the natural method in this particular setting. Somewhat related arguments are at times laid out when considering the nature of independent variables. For example, Ogasawara and Kobayashi (2015) consider a historical policy which...
increased the supply of social workers at a district level, and their impact on rates of infant mortality. The authors suggest that given the nature of the policy, ex ante expectations are that impacts will be heterogeneous, with particular points of distributional interest:

“In particular, new districts were introduced into low-income areas where the infant mortality rate was very high ..., suggesting that the influence of social workers was stronger at the higher quantiles of the infant mortality rate than at the lower quantiles.”

Ogasawara and Kobayashi (2015, p. 105).

This argument relates the use of quantile methods to contextual factors, implying that these models are simply more empirically justified than OLS.

A final series of arguments focus on technical points which imply that quantile regression may offer improvements over competing methods. For example, Kholodilin (2016) notes that a benefit of quantile regression is robust to outliers in a way which OLS is not, and Conley and Galenson (1998); Walker (2000) note that censored outcomes will not affect estimates of quantile regression when estimating impacts at uncensored quantiles of the outcome distribution. Many papers make a mix of these more technical points with the first argument that quantile regression allows us to learn more. For example González-Val, Tirado-Fabregat, and Viladecans-Marsal (2017) notes both the benefit that quantile regression provides a “richer characterization of the data” (a point on what we can learn), and also notes that these methods allow for unobserved heterogeneity, are more robust to outliers, and are robust to heteroskedasticity (a technical point). Similarly, Carson (2012) notes two advantages in discussing determinants of BMI in the early 20th century: “more robust estimation in the face of an unknown truncation point” (a technical point), and “greater description of covariate effects across the BMI distribution” (a learning point).

All told, these arguments suggest that there are a number of benefits which quantile methods have offered in research in economic history, from greater richness of analyses, to greater fit with economic or contextual details, to more theoretical robustness.

4.2.2. Arguments against the use of quantile methods in economic history

Given that there appear to be 3 arguments frequently made for the use of quantile regression or other quantile methods in economic history, it is worth asking if there are arguments against a more widespread adoption of quantile regression in research in economic history. We suggest that there are at least two main reasons why one may not wish to estimate quantile regression, instead using a model such as OLS. The first is that heterogeneity may simply not be of interest, with average results being the useful summary statistic of interest. The second is that there may be data-based justifications which imply that quantile regression is not relevant, and not desired. In practice, this second argument is likely considerably more relevant in research on economic history.

If adopting quantile regression, researchers must present more information (effects at different quantiles), and adopt different tools than in the case of OLS. These costs, while arguably minor, may not be desired if they do not bring about benefits. One reason why these costs may not be justified is that researchers are actually interested in the mean of the outcome variable, rather than impacts across the distribution. Indeed, for this to be a true argument against quantile regression, it must be that researchers are truly interested in relationships between independent variables and the mean of the outcome variable, rather than the median of the outcome variable. If, instead, the median is a measure of interest, a quantile regression could be estimated simply focusing on quantile 50. However, there are valid cases where one may wish to focus on the mean rather than the median. The mean is more sensitive to outliers and highly skewed data than the median, and at times this may actually be a feature of the data which is relevant to consider. For example, if one does indeed want extreme outcomes to exert more weight on the summary statistic considered, then OLS regression may be a more relevant tool than quantile regression. For example, in the case of economic productivity as an outcome (e.g. agricultural productivity), one may be interested in the fact that there are highly productive units in an estimation sample, as there may be some expectation of long term productivity spillovers, and in these cases, the mean may be a more relevant statistic than the median. In this context, more technical argument against quantile regression is if some measure of central tendency is desired and data is normally distributed, then the mean is equal to the median, and in these cases OLS will be a more efficient estimator than quantile regression.

Apart from cases where one simply is not interested in distributional outcomes, a much more relevant case in which quantile regression is not likely to be an estimator of interest are those settings in which data do not allow for estimation of quantile regression. The data requirements for quantile regressions are not
trivial, specifically in cases where costly data collection is frequent, such as in research in economic history. In particular, a first requirement is that data needs to be approximately continuous for these methods to be feasible. This immediately rules out any measures where a dependent variable of interest is binary, or categorical without a very large number of categories. For example, quantile regression will necessarily be silent on what one can say if considering an outcome such as the determinants of particular political systems, such as democracy or autocracy, if these measures are binary. Similarly, many outcomes of interest such as years of achieved education have relatively few values, and as such, similarly the logic of quantiles does not exist in a fine grained way. Thus, a clear limit to quantile regression is imposed where variables are not collected in a continuous way. An additional restriction is that often in the case of economic history, data is hard won, and relatively small samples are available. In these cases, even if a variable is (theoretically) continuous, the estimated sample will not allow for the consideration of many quantiles. This point is raised by Buchinsky (1998), who notes that when few data points are available, few distinct quantiles will be available.

For both of these reasons – distributional indifference and data limitations, quantile regression should not be considered to be a catch-all technique. In Online Appendix A we provide an analysis of the body of papers published in a single economic history journal over the last 30 years to see in theory what sort of coverage of continuous variables are available, though note that this should be considered as a descriptive activity, not implying that in all cases where continuous outcome variables are available, quantile methods will necessarily be of interest.

5. An illustrative example based on 19th century demographics and economic growth

As a brief illustration of the use of quantile regression, and how it can shed light on patterns across the distribution of outcomes which are hidden by standard analyses, we consider the empirical setting described in Llorca-Jaña et al. (2019, 2021). As laid out there, we gathered information on all the 36,371 records of Military Personnel born in the 20th century and 3,283 record of Military Personnel born in the 19th century in Chile. These records relate mainly to soldiers or low-ranking officers (the Army’s Historical Archive). Full information on this process and these data are available in Llorca-Jaña et al. (2019, 2021). Of those individuals, we generate a final database of the Chilean-born individuals aged between 17-55, which here we cross with rates of economic growth in the province in which the individual was born. These rates of economic growth are calculated from historical evidence collected by Badia-Miró (2008), which is the best available sub-national evidence of economic conditions in Chile, covering the periods of 1890-1950. From these data sources, we are able to combine a large micro-level sample of human height as well as measures of economic growth by decade and province of birth. Finally, we have a database of 17,293 individuals. The reduction of the database is explained given the data availability of growth rates (1890-1940). Descriptive plots of these data on height and exposure to economic growth are provided in Figure 1, suggesting considerable variation in observed heights, and also in changes in economic conditions within provinces over time.

To document these methods, we consider a particular model seeking to determine the effect of economic growth during an individual’s formative years on their adult height. Relationships between height and economic development have been discussed in the past, including over long periods, such as Peracchi’s (2008) study of Italian height from the 1730s-1980s. Here, we are interested in a model of the following type:

$$\text{Height}_{it} = \beta_0 + \beta_1 \text{Growth}_{pt}^9 + \beta_2 \text{Growth}_{pt}^{10}$$ (5)

where the height of an individual $i$ born in province $p$ and year $t$ is regressed on the growth rate in that province when the individual is born ($\text{Growth}_{pt}^9$), when they are aged 10 years ($\text{Growth}_{pt}^{10}$) and when they are aged 20 years ($\text{Growth}_{pt}^{20}$). As we lay out below, these decennial age ranges are chosen to ensure that each indicator will effectively refer to a different growth rate. In Online Appendix B we provide models which allow us to consider age more finely, and arguably in more logical age bins, but which are potentially subject to certain measurement issues. We now turn to discuss these measurement challenges more concretely.

**Modeling challenges in this setting**

In this paper we suggest that quantile methods can potentially be well suited to analyses in economic history. It is illustrative nevertheless to see that in any given setting challenges are likely to arise for reasons related to data measurement, and modeling more
Firstly, it is important to note that given challenges in collecting data on economic patterns around 200 years in the past, these measures of growth are the best available estimates, but should be considered as a noisy measure of the economic conditions during an individual's growing years. This model includes province and decade fixed effects (\( \mu_p \) and \( \lambda_t \) respectively), capturing idiosyncratic regional or temporal factors, such that measures of growth are not simply proxying regional or time-specific factors that correlated with height.

Secondly, we note that given challenges in capturing rates of economic growth, our measures simply refer to average growth over a 10-year period. Given this, we need to associate each individual with a growth rate at a given age. To ensure that growth rates map 1:1 to ages, we thus lay out a main specification where \( \text{Growth}_{0} \) refers to rates of growth in the decade when an individual was aged 0. However, the true exposure to this varies, and if an individual was born in the last year of the decade, they would only be exposed to this growth rate when aged 0-1, while for an individual born in the first year of the decade, they would be exposed to this growth rate when aged 0-9. Similarly, in the case of \( \text{Growth}_{10} \), this captures the growth rate in the decade in which the individual turned 10, and so could cover periods as extreme as those where an individual was aged 1-10, or 10-19. The variable thus guarantees that an individual was exposed to this growth rate at age 10, but other exposure ages may vary in this measure. Similar arguments exist for the variable \( \text{Growth}_{20} \). As such, all coefficients must be simply interpreted as impacts of growth on height during the decade in which an individual turned a particular age. In Appendix B we lay out an alternative model whereby more fine-grained measures of age exposure are used, but note that these may not uniquely capture different growth rates at each age. In this particular setting, this is a modeling challenge inherent in the frequency of measures of economic growth.

Empirical results

To consider what can be gained from estimating quantile regression in this particular setting of interest in economic history, and with these modeling challenges, we present estimates from Equation (5) by both standard OLS, and using quantile regression following Equation (2) at quantiles 5, 10, ..., 95. These results are displayed in Figure 2, where we plot estimates on each of the parameters \( \beta_1, \beta_2 \) and \( \beta_3 \) and the intercept, \( \beta_0 \) – on average (OLS, indicated by the solid horizontal black line), and across the distribution (quantile regression, indicated by the solid gray line). 95% confidence intervals are indicated by dashed lines (OLS) or shaded intervals (quantile regression).

The top-left panel documents a null effect of exposure to growth in the decade of birth on adult height when considering average effects, although patterns point to marginally negative effects lower in the distribution of adult height, and marginally positive effects higher in the distribution. Where results are more striking is in the top right-hand panel. While exposure to growth in the decade in which a child is aged 10 years is not observed to significantly
increase adult height in this specification, this effect is heterogeneous, with significant impacts at the lower end of the distribution of height, and null impacts among taller individuals. Here the value of considering quantile regression becomes more clear, even in this setting with measurement challenges in exposure to growth. These results point to a particular value of economic growth in human condition, which – at least in this early life period where children are growing significantly – is most relevant among those who are shorter. This is significant, as adult height is a well known marker of health, suggesting that economic growth during this period of childhood in this context has done the most to pick up individuals who have the worst health stocks. While some similar patterns may even persist when children are older (bottom left panel), these effects are observed to be most notable in the early years of life when children are particularly sensitive to their conditions. In particular, the pattern for growth in the decade aged 20 has broadly similar patterns to that when considering growth in the decade when aged 10. However, in the case of Growth when aged 20, results are largely insignificant when considering 95% CIs. As we laid out above, this measure (growth when aged 20) will cover certain individuals when they are aged from 11 to 20, and as such, is still a period in which humans are growing. Finally, the bottom right panel simply documents the intercept, which by definition will be lower at lower quantiles. In this particular setting, it allows us to visualize the variation in human height in centimeters conditional on growth, province, and year fixed effects, suggesting that moving from around the 5th to the 95th quantile, height will increase by nearly 20 centimeters.

While these results are descriptive, they are nonetheless able to point to certain distributional factors which are of relevance in understanding human demographics and sensitive periods of human capital accumulation, as well as the value of historical periods of growth – beyond just population averages.

6. Conclusions and ways ahead

This paper seeks to provide a preview of a range of empirical methods which are relevant to consider impacts of some independent variable(s) of interest, across the entire distribution of a continuous

![Figure 2. Quantile Regression.](image)
dependent variable of interest. We specifically seek to describe these methods, and motivate their adoption more widely in literature in economic history where considerable work is often spent to collect rich (continuous) outcome measures of interest. These methods can thus contribute to fully taking advantage of such data collection processes or existing data repositories which have been collected based on considerable efforts in collating, systematizing, or digitizing historical records.

We discuss both standard quantile regression methods originally laid out by Koenker and Bassett (1978) and also document how these have been fruitfully applied in a more recent “treatment effects” literature, with the application of Quantile Treatment Effects. We discuss a number of other extensions which may be of particular interest for researchers in economic history such as quantile regression with measurement error, additional ways to loosen parametric assumptions, and potential solutions to endogeneity in these models. This is a very large and ever-growing literature in econometrics, and so here, while aiming to provide a broad overview of the field, we do not claim to comprehensively survey the entire field nor the full depth of all models. Fortunately, there are a number of full textbook or handbook references such as work of Koenker (2004); Koenker et al. (2017), to which we point interested readers in cases where a more comprehensive econometric base of these models is desired.

While we argue that these models are well suited to research in economic history, we suggest that there is scope for considerably more work in this line. Nevertheless, we do note that the application of quantile methods in economic history may also be limited in certain settings. Most clearly, this is the case where available data, or variables of interest, are binary, or based on few categorical outcomes. Similarly, historical analyses may often be based on relatively few observations, in which case the ability of quantile analysis to distinguish between quantiles will be limited. In an empirical setting laid out in this paper we also note that the frequency of measurements in certain settings may be limited, though this is a more general modeling concern, not limited to quantile methods.

Fortunately, where suitable and of interest, these methods are accompanied by a range of computational tools which mean that their application can be viewed as part of a quite standard toolbox for interested practitioners. In closing, it is worth pointing to the functionality of these packages, which are significant and generally open source contributions of the methods discussed in this paper, allowing for these methods to be adopted at relatively low cost.

Computational languages widely used in economics and economic history such as R, Stata, Julia, Python, MATLAB and so forth generally all have a standard implementation of quantile regression allowing for simple implementation of these models. For example, (Koenker 2021) provides the R quantreg package which contains (among many other things) a standard quantile regression interpretation as rq, while Stata's qreg provides a stable option for both estimation and various inference procedures. However, many extensions to these commands’ ‘standard’ procedures are available, including packages to extend analyses to quantile treatment effects such as ivqte in Stata (Fröölich and Melly 2010) or the QTE package in R (Callaway 2019). Each of these QTE libraries extends in numerous ways to cases where treatment assignment is endogenous, and, in the case of QTE to difference-in-differences models (Callaway 2019). Within the ‘universe of quantreg in R there are many other extensions, including a wide range of inference procedures, non-linear models, LASSO models, and plotting functions. Given the highly applied nature of these methods, and the considerable recent extensions in the field, many new papers are also accompanied by computational code – most frequently in R or Stata – including the examples discussed in the paper such as recent advances in selection models (Siravegna 2020; Biewen and Erhardt 2020), and quantile analyses in regression discontinuity designs (Frandsen, Fröölich, and Melly 2012). All in all, these models present an extremely flexible, accessible and extendable series of analytical routes to researchers in economic history, and should be viewed as a key component of analyses, allowing for a much richer consideration of the distributional effects of historical phenomena in economic processes.

Notes

1. In an Online Appendix, we also survey the usage of these methods in economic history, an analysis of their use, and potential for their use in papers published over the past 30 years in the field of economic history.

2. Heterogeneity can be observed for a number of reasons in empirical studies. In this paper we are interested in heterogeneity in the impacts of some independent variable(s) over the full distribution of a dependent variable of interest. A useful alternative example of the interest of heterogeneity in economic history is provided by Bisin and Moro (2020) who discuss
heterogeneity in the context of differential take-up of some treatment, and resulting sample-based heterogeneity due to the estimation of Local Average Treatment Effects (LATEs).

3. There are also other reasonable approaches generally included as standard in statistical software, such as using a kernel density estimator or residual quantile function to estimate the density at quantile \( \tau \). These are all available in programs such as Stata with the qreg program, or R with the quantreg library.

4. Formally, this is \( Q_\tau(y) = \inf \{ y : y \geq \tau \} \), where \( F() \) refers to the cumulative density function of \( y \).

5. In extensive form, these can be represented (Abadie 2002) as: 
\[
F'_\gamma(y) = \frac{E[1_Y \leq y | D = 1] - E[1_Y \leq y | D = 0]}{E[D = 1] - E[D = 0]}, \quad \text{and} \\
F'_\gamma(y) = \frac{E[1_Y \leq y (1-D) | Z = 0] - E[1_Y \leq y (1-D) | Z = 0]}{E[1-Y \leq y (1-D) | Z = 0] - E[1-Y \leq y (1-D) | Z = 0]}
\]

6. In the interests of completeness, this is: 
\[
Q_\tau(y) = \inf \{ y : F(y) \geq \tau \} \quad Q_\tau(y) = \inf \{ y : F(y) \geq \tau \} \text{ where all notation follows that of Section 3.2.}
\]

7. Note however that in general, the assumptions necessary for identification with instruments are not trivial. Bhalotra and Clarke (2019) discuss a number of considerations related to these instruments in a standard linear model. An entirely different take on IV style models which precluded the LQTE approach described here and allows IVs to generate variation locally in endogenous variables is the work of Checher (2005); Ma and Koenker (2006). This suggests productive ways forward in a quantile framework even if instruments generate shifts at only specific points of endogenous variables of interest.

8. In particular, these methods require a rank preservation assumption, restricting the ranks which individuals can take in terms of the ordering of the outcome variable to be the same across differing potential IV assignments.

9. As we describe in Online Appendix A which quantifies use of these methods over a longer period in a single journal, there is even less use of quantile regression prior to 2000. Two notable exceptions are the studies of Conley and Galenson (1998, 1994) which provided early illustrations of the power of quantile regression in historical analyses.

10. Additional papers discuss descriptive quantile procedures for correcting for partially observed data (Wachter 1981; Komlos 2004).

11. There are many historical precedents to the study of height in economic history, although less work extending to quantile analyses. Among many other references Wachter and Trussell (1982) discuss historical measurement of height, though work goes back much further, for example Quetelet’s discussion of classifying populations.

12. We note that the sample reduction from the initial 36,371 digitized records owes to the availability of historical measures of growth. We calculate growth records in each decade as: 
\[
\frac{GDP_{p,t} - GDP_{p,t-1}}{GDP_{p,t-1}}, \quad \text{where} \\
p \text{ indexes provinces and} \ t \text{ indicates decades, and as such this will not be defined in the decade of 1950, given that comparable records are not available for 1960. We thus limit the final estimation sample to all individuals born in the 60 years between the 1890s and 1940s.}
\]

13. We acknowledge an anonymous referee for raising this point which we lay out in full in Online Appendix B.

14. We also note that we consistently use measures related to an individual’s province of birth, given that from military data we know where they are born. However, in the case of individuals moving between provinces in the country, these measures are noisy proxies of exposure to local economic conditions.

15. We note that even in cases where quantile regression suggests that parameters are constant across the distribution, one learns something beyond what is documented in OLS, as this is a result in itself, suggesting that impacts of the variable are indeed constant, and OLS is not masking considerable parameter heterogeneity across the distribution of the outcome variable.

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Data availability statement

The data described in this paper is available at the Harvard Dataverse: https://doi.org/10.7910/DVN/E0NT7E.

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