Time-domain calculation method for structural fatigue life under multi-axis correlation random excitation

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Abstract. Based on the typical aerospace frame structure, the fatigue life law of the structure under multi-axis correlation time-domain random excitation is studied. By constructing multi-axis time-domain random excitation with different correlation coefficients, multi-axial fatigue theory and structural vibration fatigue life time-domain method are used to calculate the fatigue life of the structure under different excitations; by constructing the structural finite element model in MSC. Pastran and nCode-DesignLife, and Co-simulation calculates the structural fatigue life. The results show that there is a negative correlation between the correlation coefficient of the applied multi-axis time-domain random excitation and the structural fatigue life, that is, the larger the correlation coefficient, the shorter the fatigue life.

1. Introduction
At present, the methods for calculating the structural vibration fatigue life mainly include the frequency-domain method and the time-domain method. The calculation method of structural frequency-domain fatigue life is to use the spectral parameters to describe the magnitude information of the response in the frequency-domain, and combine the fatigue life curve of the material with the fatigue cumulative damage theory for life calculation. The frequency-domain method has the characteristics of simple thinking and small calculation, and has been paid attention to by academic circles and engineering circles. The earliest method of calculating the frequency-domain fatigue life is the peak-distribution method [1], but the amplitude is the main influencing factor of structural fatigue damage. The fatigue life calculation error of the peak-distribution method is large, and it is common with the progress of fatigue research. It is believed that the calculation of fatigue life using rain-flow amplitude is most consistent with the fatigue damage mechanism[2], but the relationship between the probability density of rainfall amplitude and power spectral density is very complicated[3-5]. The distribution of rain-flow amplitude in the ideal narrow-band process is Rayleigh distribution [6]. The distribution of rain-flow amplitude in the general broadband stochastic process is an important research content of the frequency-domain method [7-12], and the existing rain-flow amplitude distribution models have limitations, and there are still many problems in the calculation of vibration fatigue life.

The time-domain vibration fatigue life calculation method firstly performs the time-domain simulation of the response spectral density of the structure, obtains the time history of the response, uses the cycle counting method to perform the cycle counting process, and then calculates the fatigue life based on the fatigue performance curve of the material and the fatigue cumulative damage theory.
process of calculating vibration fatigue life, the fatigue cumulative damage theory usually uses Miner linear cumulative damage theory, and the cycle counting method adopts the most commonly used and most suitable rain damage cycle counting method [13,14].

The time-domain method for calculating the vibration fatigue life is the closest to the general cycle fatigue life calculation method. However, since the calculation of the time-domain simulation of the stochastic process is very large, it is difficult to apply in engineering calculation, so the current calculation of vibration fatigue life on the domain method is few. Generally speaking, only the uniaxial time-domain method is theoretically and experimentally studied [15, 16]. There is still a lack of sufficient knowledge of the multi-axis time-domain method to calculate the structural fatigue life, and lacking experimental and simulation data to determine the influence of random excitation correlation coefficient on multi-axis vibration effect. In this paper, the typical aerospace frame structure is taken as an example to compare the vibration fatigue life of the structure under multi-axis random excitation with different correlation coefficients, so that the law is preliminarily recognized.

2. Construction of multi-axis correlation time-domain excitation

For a given acceleration excitation power spectral density curve, three sets of acceleration excitation time-domain sample signals $a_1$, $a_2$ and $a_3$ with the correlation coefficient of 0, the equal root mean square value and the equal maximum value are obtained by the random sine wave superposition method. Further, the acceleration in the X, Y, Z directions with different correlation coefficients is constructed to excite the time-domain sample signals $a_4$, $a_5$ and $a_6$.

\[ a_i = \sum_{k=1}^{M} A_k \sin(2\pi f_k t + \varphi_k) \]  
(1)

\[ \Delta f = (f_u - f_l) / M \]  
(2)

\[ a_k^2 = 2G(f)\Delta f \text{ for } k = 1, 2, ..., M \]  
(3)

\[ f_k = f_l + \left(k - \frac{1}{2}\right)\Delta f \]  
(4)

Where $a_i$ is the acceleration excitation time-domain sample signal, that is, when $i$ is 1, 2, 3, respectively, the acceleration excitation time-domain sample signals $a_1$, $a_2$, $a_3$; $A_k$ is the amplitude; $f_k$ is the frequency; $t$ is the time; $\varphi_k$ obeys a uniform distribution within $0\sim2\pi$; $G(f)$ is a unilateral power spectral density function; $M$ is a positive integer and is set to be sufficiently large; $f_u$ and $f_l$ are respectively the upper and lower frequency of the power spectral density function.

\[ a_4 = a_i \sin \beta + a_3 \cos \beta \]  
(5)

\[ a_5 = a_i \sin \beta + a_3 \cos \beta \]  
(6)

\[ a_6 = a_2 \sin \beta + a_3 \cos \beta \]  
(7)

\[ \rho^{a_4\phi_0} = \frac{Cov(a_4, a_5)}{\sqrt{D(a_4)}\sqrt{D(a_5)}} \]  
(8)
\[
\rho_{aa} \equiv \frac{\text{Cov}(a_4, a_5)}{\sqrt{D(a_4)D(a_5)}} \tag{9}
\]

\[
\rho_{ab} \equiv \frac{\text{Cov}(a_4, a_6)}{\sqrt{D(a_4)D(a_6)}} \tag{10}
\]

Where, \(\text{Cov}(a_4, a_5), \text{Cov}(a_5, a_6), \text{Cov}(a_4, a_6)\) are the covariance of \(a_4, a_5, a_6\), respectively; \(D(a_4), D(a_5), D(a_6)\) are the variance of \(a_4, a_5, a_6\), respectively; \(\rho_{a_4a_5}, \rho_{a_5a_6}, \rho_{a_4a_6}\) are the correlation coefficients of \(a_4, a_5, a_6\), respectively; \(\beta\) is an angle parameter.

Through the above method, the multi-axis random excitation with different correlation coefficients can be constructed by changing the value of the parameter \(\beta\). As shown in Table 1, the relationship between the two.

| Parameter \(\beta\) | Correlation coefficient |
|---------------------|------------------------|
| \(\pi/6\)           | (0.25, 0.25, 0.25)     |
| \(\pi/4\)           | (0.50, 0.50, 0.50)     |

3. **Simulation Analysis of Multi-axial Fatigue Life of Structure**

3.1. **Structural finite element modelling**
Create a three-dimensional model of a typical aluminum alloy frame in Catia, mesh it in Hypermesh, divide the hexahedral mesh as much as possible, and finally import the mesh information of the model into MSC.Patran, as shown in Figure 1. Total: 38,643 nodes, 27,444 units, of which 128 6-node pentahedral units, 27,316 8-node hexahedral units. To facilitate the application of constraints and excitations, the MPC is established, and nodes are established at the \([0, 0, -0.01]\) position. As the master node of the MPC, the nodes on the bolt holes are selected as slave nodes.

![Figure 1. Finite element mode.](image)

3.2. **Multi-axis fatigue life simulation analysis**
Four sets of transient dynamics analysis were carried out respectively. The loading conditions were as shown in Table 2. The modal superposition method was used to calculate the stress-time data of the model nodes, and then the multi-axis fatigue analysis process was established in the fatigue analysis software. The fatigue life under each working condition is shown in Table 3.
Table 2. Loading conditions.

| Loading conditions | Correlation coefficient |
|--------------------|-------------------------|
| 1                  | (0, 0, 0)               |
| 2                  | (0.25, 0.25, 0.25)      |
| 3                  | (0.50, 0.50, 0.50)      |
| 4                  | (1.00, 1.00, 1.00)      |

Table 3. The most vulnerable node number and fatigue life.

| Node | Fatigue life of all working conditions /h |
|------|------------------------------------------|
|      | 1    | 2    | 3    | 4    |
| 1    | 5.69 | 5.34 | 4.24 | 1.87 |
| 2    | 10.31| 10.23| 8.25 | 3.29 |
| 3    | 53.08| 49.46| 43.64| 11.19|
| 4    | 50.82| 41.56| 30.81| 14.37|
| 5    | 62.66| 51.28| 37.45| 15.15|

According to the Table 3, when the structural member is subjected to multi-axis related loads, the correlation coefficient between the shafts is increased, the stress on the structural members is increased, the damage is also increased, and the fatigue life is shortened.

4. Test verification

4.1. Multi-axis vibration strain test analysis

In order to verify the construction method of multi-axis correlation time-domain excitation and the accuracy of fatigue life simulation calculation, a three-axis vibration test was carried out on the typical frame structure, and the measurement points were placed at the six points most vulnerable to fatigue simulation. Strain measurement was performed.

There are 6 measuring points in the X direction and 4 measuring points in the Y direction. A total of 10 measuring points are arranged. The arrangement of the measuring points is shown in Figure 2. The unidirectional strain conditions were tested and three sets of tests were performed, as shown in Table 4.

Figure 2. Measuring point layout.  
Figure 3. Test system layout.

Table 4. Test condition.

| Loading conditions | Correlation coefficient |
|--------------------|-------------------------|
| 1                  | (1.00, 1.00, 1.00)      |
| 2                  | (0.50, 0.50, 0.50)      |
| 3                  | (0, 0, 0)               |
4.2. Multi-axis vibration strain simulation calculation
The simulation results of the strains under the corresponding working conditions were carried out for the six measuring points tested in the strain test, and the simulation results as shown in Figure 6 and Figure 7 were obtained.

4.3. Comparative analysis of experiment and simulation
As shown in Table 5 and Table 6, the results of the comparison between the strain test values of the X-direction and the Y-direction and the measured values of the measured point strain, respectively, are the minimum error of 0.21% and the maximum error of 26.75%, considering the test accuracy. And the accuracy of the simulation, the error is within the acceptable range.

Furthermore, it can be concluded that the correlation coefficient between the random excitations in different directions has a greater influence on the strain of the structural members, and the strain of the structural members increases with the increase of the correlation coefficient, indicating the signal correlation of the three vibration directions. The higher the degree, the stronger the vibration of the structural member, and the correspondingly the fatigue damage. Therefore, the simulation method and
the experimental results are compared to verify the accuracy of the multi-axis correlation time-domain excitation construction method and fatigue life simulation calculation.

### Table 5. X-direction strain comparison.

| Node | Loading condition 1 RMS/με | Simulation value | Error | Loading condition 2 RMS/με | Simulation value | Error | Loading condition 3 RMS/με | Simulation value | Error |
|------|---------------------------|------------------|-------|---------------------------|------------------|-------|---------------------------|------------------|-------|
| 1    | 90.25                     | 77.81            | 13.78%| 75.57                     | 71.68            | 5.15% | 72.34                     | 70.10            | 3.10% |
| 2    | 100.20                    | 108.02           | 7.80% | 81.87                     | 94.82            | 15.82%| 64.03                     | 74.43            | 16.24%|
| 3    | 68.77                     | 71.67            | 4.22% | 62.21                     | 61.22            | 1.59% | 54.52                     | 52.61            | 3.50% |
| 4    | 51.49                     | 57.10            | 10.90%| 52.20                     | 55.30            | 5.94% | 40.30                     | 49.41            | 22.61%|
| 5    | 45.15                     | 39.65            | 12.18%| 25.12                     | 32.02            | 27.47%| 26.51                     | 30.11            | 13.58%|
| 6    | 40.55                     | 49.31            | 21.60%| 30.04                     | 29.30            | 2.46% | 28.87                     | 31.77            | 10.05%|

### Table 6. Y-direction strain comparison.

| Node | Loading condition 1 RMS/με | Simulation value | Error | Loading condition 2 RMS/με | Simulation value | Error | Loading condition 3 RMS/με | Simulation value | Error |
|------|---------------------------|------------------|-------|---------------------------|------------------|-------|---------------------------|------------------|-------|
| 1    | 73.28                     | 77.91            | 6.32% | 66.20                     | 71.35            | 7.78% | 57.39                     | 69.72            | 21.49%|
| 2    | 78.07                     | 96.10            | 23.10%| 72.15                     | 84.82            | 17.56%| 59.71                     | 75.68            | 26.75%|
| 3    | 108.80                    | 110.40           | 1.47% | 89.24                     | 89.43            | 0.21% | 83.33                     | 84.80            | 1.76% |
| 4    | 100.50                    | 106.50           | 5.97% | 97.51                     | 92.20            | 5.45% | 76.29                     | 84.89            | 11.27%|

5. Conclusion

Through simulation and experimental analysis, the vibration fatigue life of typical frame structures under multi-axis correlation time-domain random excitation is studied. The following conclusions can be drawn:

When the relevant time-domain random excitations of X, Y and Z directions are applied to the structural members, the correlation coefficient has a great influence on the structural stress and strain, and the stress and strain of the structural members will become larger as the correlation coefficient increases. It shows that the larger the correlation coefficient of the three vibration directions, the stronger the vibration of the structural member. Furthermore, combined with the fatigue damage theory, the fatigue life is shortened as the correlation coefficient increases, that is, the larger the inter-axis correlation coefficient, the shorter the fatigue life of the structural member.

References

[1] Lutes L D, Sarkani S. Stochastic Analysis of Structural and Mechanical Vibrations[M]. 1997.
[2] Wu Yisheng. A Formula For Fatigue Damage Calculation Under Random Loading[J]. The Ocean Engineering, 1994(1):94-103.
[3] Li Chang. A Approach Based on Power Spectral Density for Fatigue Life Estimation[J]. Machine Design & Research, 1994(1):94-103.
[4] Wang Lin, Ni Qiao, Zhang Qiang, etc. Estimation of Fatigue Life of Pressure Piping System under Random Excitations[J]. Journal of Huazhong University of Science and Technology (Natural Science Edition), 2003, 31(21):100-102.
[5] Wu Qihe, Ye Duyi, Yang Ying. A New Method for Prediction The Stochastic Fatigue Life of Components[J]. Engineering Mechanics, 1995, 12(2):87-94.

[6] Luo Hongyun, Chen Zhiyang, Zhang Weibo, etc. Life Prediction of Components under Narrow Band Random Loading[J]. Journal of Jilin Institute of Technology, 2000(4):32-34.

[7] Braccesi C, Cianetti F, Tomassini L. Random fatigue. A new frequency domain criterion for the damage evaluation of mechanical components[J]. International Journal of Fatigue, 2015, 70(70):417-427.

[8] Benasciutti D. Some analytical expressions to measure the accuracy of the “equivalent von Mises stress” in vibration multiaxial fatigue[J]. Journal of Sound & Vibration, 2014, 333(18):4326-4340.

[9] Carpinteri A, Spagnoli A, Vantadori S. Reformulation in the frequency domain of a critical plane-based multiaxial fatigue criterion[J]. International Journal of Fatigue, 2014, 67(67):55-61.

[10] Cristofori A, Benasciutti D, Tovo R. A stress invariant based spectral method to estimate fatigue life under multiaxial random loading[J]. International Journal of Fatigue, 2011, 33(7):887-899.

[11] Yan Tingfei, Zhang Jungang, Fang Guiqian, Shen Zhiqiang. A comparison between vibro-acoustic environment test and single environment test for a spacecraft antenna[J]. Spacecraft Environment Engineering, 2014, 31(2):154-157.

[12] Liu Mo, Feng Yaoqi, He Ling. The determination of conditions of multi-axis random vibration tests for satellite products[J]. Spacecraft Environment Engineering, 2013, 30(2):155-159.

[13] Anthes R J. Modified rainflow counting keeping the load sequence [J]. International Journal of Fatigue, 1997, 19(7):529-535.

[14] Repetto M P. Cycle counting methods for bi-modal stationary Gaussian processes[J]. Probabilistic Engineering Mechanics, 2005, 20(3):229-238.

[15] An Gang, Gong Xinmao. Fatigue Failure Analysis of Structures under Random Vibration Environment[J]. Mechanical Science And Technology, 2000(s1):40-42.

[16] Xu Fei, Xiao Shouting. The Power Spectral Density Method for The Estimation of The sonic Fatigue Life [J]. Journal of Mechanical Strength, 1996(4):38-42.