Localization properties and high-fidelity state transfer in electronic hopping models with correlated disorder

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Abstract

We investigate a tight-binding electronic chain featuring diagonal and off-diagonal disorder, these being modelled through the long-range-correlated fractional Brownian motion. Particularly, by employing exact diagonalization methods, we evaluate how the eigenstate spectrum of the system and its related single-particle dynamics respond to both competing sources of disorder. Moreover, we report the possibility of carrying out efficient end-to-end quantum-state transfer protocols even in the presence of such generalized disorder due to the appearance of extended states around the middle of the band in the limit of strong correlations.

Keywords: diffusive spreading, correlated disorder, localization, quantum-state transfer

1. Introduction

In the past few decades, there has been a growing interest in investigating quantum transport properties of low dimensional disordered lattices [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], most of them based on Anderson scaling theory. In general lines, it is well established that there are no extended eigenstates in low-dimensional systems for any amount of uncorrelated disorder. The breakdown of standard Anderson localization theory was put forward about thirty years ago by Flores and Dunpap [17, 18]. They pointed out that the presence of short-range correlations in the disorder distribution yielded the appearance of extended states in the spectrum of disordered chains. That could explain to a great extent some unusual transport properties of several types of polymers [17, 18]. Right after this discovery, a handful of works came along to investigate the role of

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disorder correlations, either short- or long-ranged, in wide variety of physical systems [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. Particularly, it was shown in Refs. [19, 21] that long-range correlated random potentials can actually allow for mobility edges in 1D disordered models. In Ref. [19], that specific kind of fluctuations was generated using the trace of a fractional Brownian motion whose intrinsic correlations decay following a power law. Through numerical renormalization methods, it was show that this model exhibits a phase of extended states around the center of the band [19]. Tackling the same problem, the authors in [21] applied a analytical perturbation technique and came up with a direct relationship between the localization length and the characteristics of the intrinsic correlations in the disorder distribution. A few years later, the above results were validated through experiments carried out in microwave guides featuring correlated scatters [41]. The authors demonstrated that intrinsic long-range correlations within the scatters distribution ultimately improve the wave transmission. On the theoretical side, the Anderson model with long-range correlated hopping fluctuations (off-diagonal disorder) was studied in Refs. [20, 32]. Likewise, it was found that strong correlations promote the appearance of a phase of extended states close to the center of the band.

In this work we provide further progress along those lines. In particular, we consider two sources of disorder acting simultaneously on the potentials as well as on the hopping strengths of the chain, both exhibiting long-range correlated fluctuations generated by the fractional Brownian motion. This model embodies a generalized disordered scenario which we aim to push on its capability of supporting extended states in the middle of the band thereby weakening Anderson localization. By looking at the participation ratio of eigenstates and also at the dynamics of the system through its mean square displacement for an delta-like initial state we find out the chain allows for propagating modes if substantial long-range correlations are taking place in both sources of disorder. Looking forward possible applications in the field of quantum-information processing, we also investigate whether such a model of generalized disorder would allow for realizing weak-coupling quantum-state transfer protocols [42, 43, 44, 45]. The point is that when designing chains for transmitting quantum states from one point to another – which is a crucial requirement in quantum networks [46] – one should take into account the possibility of undesired fluctuations taking place due to experimental errors [40, 47, 48, 49, 50, 51, 52, 53], that including correlated noise [40, 47, 48]. Our calculations reveal that an electron (or a properly encoded qubit) can
be almost fully transferred through the noisy bulk of the chain depending upon specific sets of parameters.

2. Model and Formalism

We consider a $N$-site linear chain described by the electronic tight-binding Hamiltonian ($\hbar = 1$)

$$H = \sum_{n=1}^{N} \epsilon_n |n\rangle \langle n| + \sum_{n=1}^{N-1} J_n (|n\rangle \langle n+1| + \text{h.c.}),$$

written in the Wannier basis set $\{|n\rangle\}$ accounting for the electron position, where $\epsilon_n$ is the on-site potential and $J_n$ is the hopping strength, those being the source of static disorder. Those parameters are here expressed in terms of energy unit $J \equiv 1$. Specifically, we assume that both quantities fluctuate such that their corresponding disorder distributions come with intrinsic long-range correlations modelled via the fractional Brownian motion \[19, 22, 24, 25\]

$$\epsilon_n, J_n = \frac{N}{2} \sum_{k=1}^{N/2} \frac{1}{k^{\gamma/2}} \cos \left( \frac{2\pi nk}{N} + \phi_k \right).$$

We emphasize that the sequence generated by the equation above exhibits a power-law spectrum $1/k^\gamma$ and $\phi_k$ represents a random phase uniformly distributed within the range $[0, 2\pi]$. For $\gamma = 0$, the sequence is fairly uncorrelated. On the other hand, $\gamma > 0$ brings about long-range correlations in the disorder sequence. Therefore, exponent $\gamma$ stand out as very important parameter in our work since it controls the degree of correlations within the disordered sequence. Hereafter, Eq. (2) will be used for generating disorder distributions for both $\epsilon_n$ and $J_n$ but with a few remarks: (i) for $\epsilon_n$ we attribute $\gamma \rightarrow \alpha$ and normalize the entire sequence so that $\langle \epsilon_n \rangle = 0$ and $\langle \epsilon_n^2 \rangle = 1$; (ii) for $J_n$ we set $\gamma \rightarrow \beta$ and redefine $J_n \rightarrow \tanh (J_n) + 2$ after normalization in order to rule out possible null hopping strengths. It is also important to note that each sequence for $\epsilon_n$ and $J_n$ is generated using distinct sets of phases, $\{\phi_k\}$. In summary, our model contains two independent parameters $\alpha$ and $\beta$ that account for the degree of correlations for both diagonal and off-diagonal sources of disorder.

Our quantities of interest are all obtained through exact diagonalization of Hamiltonian (1) which gives us the eigenvalues $\{E_j\}$ and its corresponding
eigenvectors $|\psi^j\rangle = f^j_n |n\rangle$. Our first task will be evaluating the participation ratio defined as \[24\]

$$\xi^j = \frac{1}{\sum_n |f^j_n|^4}. \quad (3)$$

This measure provides an estimate of the number of bare states a given eigenstate is spread on, i.e., it quantifies the degree of localization. In particular, the participation number becomes size-independent for localized wave-packets and diverges with $N$ for extended ones. In addition, we investigate the electronic time evolution through the chain. We initialize the initial wave-packet in $|\psi(0)\rangle = \sum_n c_n(0) |n\rangle$ where $c_n(0) = \delta_{n,n_0}$. The electronic state at time $t$ can thus be obtained from $|\psi(t)\rangle = \sum_n c_n(t) |n\rangle = e^{-iHt} |\psi(0)\rangle$, where

$$c_n(t) = \sum_j f^j_{n_0} f^j_n e^{-iE_j t}. \quad (4)$$

By using the relations above we can compute the width $\sigma$ of the electronic wave-packet through \[54\]

$$\sigma(t) = \sqrt{\sum_n (n - \langle n(t) \rangle)^2 |c_n(t)|^2}, \quad (5)$$

where $\langle n(t) \rangle = \sum_n n |c_n(t)|^2$ is the electronic average position. Note that $\sigma(t)$ goes from 0, for a wave function confined to a single site, to $O(N)$ for a wave extended over the whole system. Note that we can also compute the time-dependent participation number defined as $\xi(t) = 1/\sum_n |c_n(t)|^4$. Both quantities are distinct ways to obtain an estimate of the size of the wave-packet at time $t$ \[24\] \[54\].

3. Results

After having introduced our main tools in the previous section it is now time to investigate the actual role of diagonal and off-diagonal sources of disorder acting simultaneously in the chain.

3.1. Localization properties

We start our analysis showing results for the participation ratio of the entire eigenstates set. It should be emphasized that every quantity evaluated
in this work was properly averaged over many distinct realizations of disorder. The total number of eigenstates $N_E = NM$ was larger than $10^5$ for all calculations, $M$ being the number of samples. We averaged $\xi^j$ over a small window around energy $E$ and therefore we are looking towards the quantity $\xi(E) = (\sum_{E_j < E + \Delta E}^{E_j > E - \Delta E} \xi^j)/n(E)$, where $n(E)$ is the number of eigenvalues $\{E_j\}$ within the interval $[E - \Delta E, E + \Delta E]$. Herein we fix $\Delta E = 0.2$.

In Fig. 1 we plot the rescaled mean participation number $\xi/N$ versus
energy $E$ for many combinations of $\alpha$ and $\beta$. Calculations were done for $N = 1000$ up to 8000 sites. We observe in Figs. 1(a) and 1(b) that $\xi/N$ decreases as the system size $N$ is increased regardless of the $E$ value. This is a clear signature that all eigenstates become localized at the thermodynamic limit. On the other hand, Fig. 1(c) reveals a rather interesting behavior. Close to the band center, the rescaled participation number remains constant thus indicating the appearance of extended states at this region. For $|E| >> 0$ we observe that $\xi/N$ decreases with $N$ what indicates the presence of localized states far from the band center. Thereby, our calculations show that one-dimensional systems featuring both diagonal and off-diagonal disorder only display extended states whenever both sources of fluctuations are augmented with strong long-range correlations. If only either $\alpha$ or $\beta$ is greater than zero, the electron transport can be suppressed by the presence of uncorrelated randomness in the lattice. We can further observe this feature by analyzing Fig. 2 where we plot the mean participation number around the band center $\xi_0/N \equiv \xi(E \approx 0)/N$ versus $\alpha$ and $\beta$ for $N = 8000$. We note that only for $\alpha$ and $\beta$ larger than 2 we are to obtain the rescaled participation number $\xi_0/N \approx 0.58(2)$ which is very close to the corresponding value of extended states in ordered chains with open-boundary conditions, that is $2/3$. Our outcomes are also in agreement with the rescaled participation number for extended states in disordered systems found elsewhere [22, 24, 32].

Furthermore, it is relevant to point out that, generally speaking, $\gamma$ is related to the so-called Hurst exponent $H$ through $H = (\gamma - 1)/2$ which describes the long-term memory of a given series. The set spanned by Eq. (2) is said to be nonstationary when $\gamma > 1$ and persistent (anti-persistent) when $\gamma > 2$ ($\gamma < 2$). When $\alpha = 2$ the series corresponds exactly to the trace of the Brownian motion. Moreover, as shown in [19] in the case of on-site disorder only, $\alpha = 2$ marks the transition point between Anderson-like insulator and metallic phases with sharp mobility edges.

### 3.2. Time dynamics and quantum-state transfer

The interplay between localized and delocalized states we have seen in the previous section allows for a rich variety of dynamical regimes [25]. Our goal now is explore how the competition between two independent sources of correlated disorder reflects upon the spreading profile of the initial state of a single electron. Right after that we will tackle a very appealing application of such platforms in the context of quantum information processing.
Figures 3 and 4 show a summary of our calculations for the time-dependent spread and participation number for an initial delta-like state prepared at the (N/2)th site, that is \( f_n(0) = \delta_{n,N/2} \). Those coefficients at a later time are evaluated through Eq. (4) for \( N = 1000 \) up to 8000 for various combinations of \( \alpha \) and \( \beta \). For comparison purposes, time and functions of interest were rescaled by the system size \( N \). We computed \( f_n(t) \) until a stationary state could be reached after multiple reflections of the wave packet on the lattice boundaries. Therefore, for \( \alpha \) and \( \beta \) larger than 2 [see Figs. 3(c) and 4(c)] we obtained a sharp curve collapse thus implying that the wave packet spreads ballistically before reaching the boundaries of the chain. For \( \alpha \) or \( \beta \) less than 2, on the other hand, panels (a) and (b) of Figs. 3 and 4 there is clearly no collapse, thus suggesting a much slower electronic dynamics along the chain [22].
In general lines, our results show that chains with correlated disorder in both diagonal and off-diagonal terms can only support the presence of extended states once both sources of disorder display strong enough correlations, that is $\alpha, \beta > 2$. Still, it is very impressive that two competing and independent sources of noise allow for coherent transmission of electronic excitations through the chain. That could, for instance, find many applications in quantum communication protocols [42, 55]. Now, we evaluate the robustness of a quantum-state transfer scheme [43, 44] against our generalized disorder model.

First, let us make further assumptions towards the configuration of the system. We now consider a chain made up by $N + 2$ sites [described by the very same Hamiltonian in Eq. (1) now with $N \rightarrow N + 2$], such that the first and last one will act as, respectively, sender and receiver parties. For those, particularly, we set $\epsilon_1 = \epsilon_{N+2} = 0$ and $J_1 = J_{N+1} = g$ that is, disorder is only present along the communication channel itself (sites 2 to $N + 1$). The transfer scheme is based on the weak-coupling model [43, 44] – usually worked out in the context of spin chains – where $g$ is set several orders of magnitude weaker than the energy scale of the channel. That forces both end sites to span their own subspace, with a couple of eigenstates taking the form $|\psi^\pm\rangle \approx (|1\rangle \pm |N + 2\rangle)/\sqrt{2}$, so that state transmission takes place via coherent dynamics between them. Naturally, nearly-perfect transmission shall be expected in ordered chains. If that is not the case, the presence of generalized disorder breaks down the mirror and particle-hole symmetries of the system thus damaging the effective two-body coupling between the ends of the chain [44].

We are now about to show that a high-fidelity quantum-state transfer protocol can actually be realized in the presence of correlated fluctuations, involving the whole channel. Let us outline the transfer protocol following the original proposal from Ref. [42]. Suppose that Alice wishes to send an arbitrary qubit $|\varphi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$ to Bob, where $|0\rangle_i$ ( $|1\rangle_i$) denotes the absence (presence) of an electron at site $i$. Then, she arranges for an initial state of the form $|\Psi(0)\rangle = |\varphi\rangle_1|0\rangle_2 \ldots |0\rangle_{N+2}$. By letting the system evolve following its natural Hamiltonian dynamics, she expects, in the best-case scenario, to have $|\Psi(\tau)\rangle = |0\rangle_1|0\rangle_2 \ldots |0\rangle_{N+1}|\varphi\rangle_{N+2}$ so Bob can properly retrieve the qubit. A measure for the figure of merit of the protocol can obtained by averaging the input fidelity over the whole Bloch sphere (for
Figure 5: Maximum fidelity versus $\alpha$ averaged over 500 independent realizations of disorder in a 50-site channel (52 sites total) for (a) $\beta = 0$ and (b) $\beta = 3$. Solid, dashed, dotted lines display results for $g/J = 0.2$, 0.1, 0.01, respectively. $F_{\text{max}} \equiv \max\{F(t)\}$ was evaluated over time window $tJ \in [0, 2 \times 10^5]$.

details, see [42]):

$$F(t) = \frac{1}{2} + \frac{|c_{N+2}(t)|}{3} + \frac{|c_{N+2}(t)|^2}{6}$$

which is basically a monotonic function of the transition amplitude between sender and receiver sites, $c_{N+2}(t) \equiv \sum_j f_1^j f_{N+2}^j e^{-iE_j t}$, [cf. Eq. (4)].

Here we are concerned with the maximum fidelity $F_{\text{max}} = \max\{F(t)\}$ achieved during a given interval since the dynamics time scale of the system varies considerably sample by sample. In particular, we evaluated $F_{\text{max}}$ over $tJ \in [0, 2 \times 10^5]$ for about 500 independent realizations of disorder and averaged them out for every system configuration as shown in Fig. 5. There, it is clear that an efficient transfer protocol can be performed through our noisy channel once supported by prominent intrinsic correlations in both sources of disorder [see Fig. 5(b)]. We observe that $F_{\text{max}}$ tends to saturate after $\alpha > 2$, thus pointing out the crucial role of extended states in the process. We also highlight in Fig. 5(b) that we are able to achieve nearly perfect fidelities provided $g$ is weak enough, in order to avoid mixing between the channel and sender/receiver subspaces.

What is most impressive in the results shown above is that even though the noisy channel must be augmented with strong long-range correlations in order to establish successful quantum-state transfer rounds we must point out the fact that considerable amounts of disorder are still present in the
That ultimately destroys the mirror and particle-hole symmetries of the spectrum [50] and so, intuitively, it should not allow for an effective resonant interaction between the outer ends of the chain. Fortunately, it actually does. A very useful picture of this can be put forward by writing down the sender/receiver decoupled Hamiltonian with renormalized parameters obtained through second-order perturbation theory in $g$ [(for details, see Ref. [44]), $H_{\text{eff}} = h_1|1\rangle\langle 1| + h_{N+2}|N+2\rangle\langle N+2| - J'|1\rangle\langle N+2| + \text{h.c.}$, where

$$h_1 = -g^2 \sum_k |f_2^k|^2 / E_k,$$  

$$h_{N+2} = -g^2 \sum_k |f_{N+1}^k|^2 / E_k,$$  

$$J' = g^2 \sum_k f_2^k f_{N+1}^{k*} / E_k,$$  

with the sum in $k$ running over the normal modes of the channel only. Recalling that sites 1 (sender) and $N+2$ (receiver) are tuned to the middle of the band, $\epsilon_1 = \epsilon_{N+2} = 0$, the existence of delocalized states at this region of the spectrum provided the degree of correlations $\alpha$ and $\beta$ are high enough (that is, greater than 2) is such that it masks the overall asymmetric nature of the chain yielding rather balanced distributions of amplitudes $f_2^k$ and $f_{N+1}^k$. Hence, $h_1 \approx h_{N+2}$ what triggers an effective two-site dynamics with negligible local impurities. Moreover, since the renormalized parameters [Eqs. (7) through (9)] scales with $E_k^{-1}$, the outskirts of the band, filled by localized-like states (thus more spatially asymmetric), have a much weaker influence on them.

4. Conclusions

In this work we considered an electronic tight-binding chain with correlated disorder in both diagonal and off-diagonal terms of the Hamiltonian. The fractional Brownian motion was used to generate each corresponding disorder distributions. We analyzed the localization properties of the system, accounted by the participation ratio of its entire spectrum, and also evaluated the electronic dynamics profile along the chain. We showed the model supports extended states only if both sources of disorder contain strong intrinsic long-range correlations, for at least $\alpha, \beta > 2$. We also investigated a possible application for this class of chains in the context of quantum-state
transfer protocols. By perturbatively coupling both communicating parties to the noisy chain, it is possible to transmit an excitation from one end of the chain to another with very high fidelities as long as a proper set of delocalized states is available in the spectrum in order to overcome the spatial asymmetry induced by disorder.

By tackling the properties of a standard electronic hopping model augmented with twofold long-range-correlated disorder, we set the ground for further studies along that direction involving other classes of many-body interacting models. Moreover, we also highlight the importance of investigating special types of disorder that might occur in solid-state devices for quantum information processing tasks [40].

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