Finite Element Failure Analysis of GFRP Laminates in Plate-Cone Reticulated Shell

Xing Wang,1 Yu Jiang,1 Yonghui Huang,2 Yue Huang,2,3 and Fan Wang4

1School of Civil Engineering, Guangzhou University, Guangzhou 510006, China
2Guangzhou University-Tamkang University Joint Research Center for Engineering Structure Disaster Prevention and Control, Guangzhou University, Guangzhou 510006, China
3School of Civil Engineering, Qingdao University of Technology, Qingdao 266033, China
4Guangzhou University Library, Guangzhou University, Guangzhou 510006, China

Correspondence should be addressed to Yonghui Huang; huangyh@gzhu.edu.cn and Yue Huang; jeff.yue.huang@gmail.com

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Plate-cone reticulated shell is a new type of spatial structures with good mechanical behavior, technical economy, and architectural appearance. In this paper, using ANSYS software, the strength failure analysis model of composite laminates is established in cooperation with the Strength Criterion of Hoffman. The effects of layer number, laying direction, and thickness of laminates on the ultimate strength of laminates are studied by detailed parametric analysis, which provides a theoretical basis for the design of composite plate-cone reticulated shell and GFRP laminated plates. Some important conclusions are obtained and can be applied to engineering practice.

1. Introduction

Plate-cone reticulated shell is an emerging spatial structure in recent years, which is developed based on Kaiser aluminum stressed-skin dome. The structure is assembled with cone elements and truss members, which are connected at the joints using bolts [1] as shown in Figures 1 and 2. Plate-cone reticulated shell is a special type stress-skin structures of half continuity and half lattice since ventral members (also ventral members and bottom members) of common double-layer reticulated shell are replaced by cone elements. The cone plates can be made of conventional materials such as aluminum alloy, and steel or light composite materials such as carbon fiber reinforced polymer (CFRP) or glass fiber reinforced polymer (GFRP). Plate-cone reticulated shell is an effective structure that can make full use of the strength and stiffness of the plates. At the same time, it integrates load-bearing, enclosure and decoration into a whole. Because of the high strength-to-weight ratio, good technical and economic benefits, and distinctive architectural visual effect, the plate-cone reticulated shell has been widely used as a long-span spatial structure [2, 3]. The idea of plate-cone reticulated shell was originated from the Kaiser aluminum stressed-skin dome. The first Kaiser aluminum stressed-skin dome was built on the Hawaiian village of Honolulu, USA, in 1957 with a span of 44.2 m [4]. To date, there are many other aluminum stressed-skin domes which have been built in schools, banks, city centers, conference halls etc., such as three Temcor aluminum dome stadiums with 71 m span on Elmira College, New York, and the airlines dome with 60 m span on Schipol Airport of Amsterdam, Holland [1]. Since the 1990s, composite materials such as FRP have been increasingly used in civil engineering [5]. Because FRP is much lighter and stronger than ordinary steel and aluminum alloy, it is more preferred to be used as the prefabricated cone element in plate-cone reticulated shells.

Many studies have been carried out in relation to conventional single-layer or double-layer reticulated dome. Among them, Fan et al. [6] studied the elasto-plastic stability of seven types of commonly used single-layer reticulated shells. Xiong et al. [7] investigated the elasto-plastic stability of single-layer latticed shells with aluminum alloy gusset joints. Hiyama et al. [8] investigated the global buckling behaviors of an aluminum alloy double-layer spatial latticed...
structure with tubular pipes, ball connections, and joining bolts via experimentation and analysis. Xie and Li [9] studied the natural vibration characteristics of an aluminum alloy double-layer reticulated shell with various structural dimensions. Zhi et al. [10] examined the failure mechanisms of single-layer reticulated domes subjected to seismic loads. Zhai et al. [11] carried out the dynamic response analysis of the reticulated domes under blast loading using the finite element (FE) software ANSYS/LS-DYNA for studying damage model and damage assessment. Lin et al. [12] studied the failure modes of a reticulated dome in a small airplane.

Most of the aforementioned studies are focused on the structural behavior of reticulated domes of homogeneous materials. In contrast, the studies on the GFRP laminates in plate-cone reticulated shell are limited, among which Robak [13] studied the structural use of plastics pyramids in double-layer space grids, Wang and Wang [14] researched the preliminary application of GFRP on plate-cone reticulated shells and analyzed the plate-cone reticulated shell as a whole, material design and structure design have not been done. Composite materials possess some distinct mechanical properties, such as anisotropy, nonhomogeneity, low interlaminar shear modulus and low interlaminar shear tensile strength, geometric nonlinearity, and material nonlinearity. These mechanical characteristics make the problem more complicated and challenging than the conventional materials that is homogeneous, continuous, linear elastic, and isotropic. Hence, it is necessary to investigate the behavior and strength of the composite plate-cone reticulated shell with FRP laminates.

In this paper, based on the composite mechanics and the theory of plate and shell, the elastic stress in the principal direction of single-layer plate of plate-cone reticulated shell is calculated by using finite element analysis. Then, the ultimate strength of each layer under each load step is obtained by the Strength Criterion of Hoffman, and the failure load of the first failed layer of laminated plate (defined as the first layer strength) and the failure load of the last failed layer (defined as the last layer strength) are obtained and evaluated. The strength and main influencing factors of composite laminates are studied comprehensively and deeply which provides theoretical foundation for the design of composite plate-cone reticulated shells with laminated plates.

2. Failure Analysis of GFRP Composite Laminates

Generally, in composite laminates, the fiber orientation in each layer is different. In some cases, even the material properties and thickness at different layers are different. Hence, the resistance of each layer of fibers to external load is different. It is highly unlikely for all layers of fibers to reach the ultimate strength and fail at the same time under certain external loads. Instead, the failure of the laminates often initiates from the weakest layer and propagate layer by layer.

The different strength criteria of single-layer composite plate has been proposed for the different situations. Strength Criteria of Tsai–Hill did not consider the influence of difference with tensile strength and compressive strength on material failure. Hoffman considered the factor of difference
with tensile strength and compressive strength and supplied some linear terms shown as follows [15]:

\[
C_1(\sigma_2 - \sigma_3)^2 + C_2(\sigma_3 - \sigma_1)^2 + C_3(\sigma_1 - \sigma_2)^2 + C_4\sigma_1 + C_5\sigma_2 + C_6\sigma_3 + C_7\tau_{23} + C_8\tau_{31} + C_9\tau_{12}^2 < 1. \tag{1}
\]

Considering the orthotropy and plane stress state of the single-layer plate, the Strength Criteria of Robert [15] which is shown in equation (2) was derived from equation (1):

\[
F = \frac{\sigma_1^2}{X_1X_c} + \frac{\sigma_2^2}{X_2Y_c} + \frac{X_c - X_1}{X_1X_c}\sigma_1 + \frac{Y_c - Y_1}{Y_1Y_c}\sigma_2 + \frac{\tau_{12}^2}{S^2} < 1, \tag{2}
\]

where \(F\) is the failure value and \(F \geq 1.0\) means material failure occurs. \(X_1, X_2, Y_1, Y_2, \) and \(S\) are the basic strength of the single-layer FRP laminate which can be obtained from the material property test, \(\sigma_1\) and \(\sigma_2\) are the principal stresses along the principal directions, and \(\tau_{12}\) is the maximum shear stress.

To perform the failure analysis, the normal stress \(\{\sigma_x, \sigma_y, \tau_{xy}\}\) of each layer of laminates along the reference axis of element’s coordinate \(x, y, \) and \(z\) are calculated firstly. Because the layering angle of each layer is different, the calculated normal stress of each layer may not follow the principal direction of elasticity. In order to carry out strength failure analysis, the normal stress \(\{\sigma_x, \sigma_y, \tau_{xy}\}\) of laminates under the element’s coordinate system should be transformed into the principal stress \(\{\sigma_1, \sigma_2, \tau_{12}\}\) along the principal direction of elasticity of the layer by using equation (3). Then, the strength of each layer is calculated according to the Strength Criterion of Hoffman which is shown in equation (2).

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta \\
-\sin^2 \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}. \tag{3}
\]

In order to obtain the normal stress of each layer of laminates, nonlinear static analysis of the plate-cone reticulated shell under step-by-step loading was carried out using commercial finite element software ANSYS. Then, the principal stress of the single-layer plate is obtained according to equation (3). The ultimate strength of each layer at each load step is obtained by Strength Criterion of Hoffman, and the ultimate strength of the first failed layer (defined as the first layer strength) and the ultimate strength of the last failed layer (defined as the last layer strength) are analyzed and evaluated.

It should be pointed out that the strength of the first failed layer is generally considered as the ultimate strength of the composite laminates in engineering design, so the stiffness reduction of the laminate plate caused by the failure of a certain layer is generally not considered in the structural analysis. Nevertheless, the ultimate strength of the last failed layer of the laminate is also calculated in this paper in order to evaluate the interval between the ultimate strength of the first layer and that of the last layer. This is critical and essential to identify a reasonable design of the laminate composite, which is characterized by the full use of the potential of composite materials.

3. Finite Element Analysis

3.1. Geometry and Material Properties of the Model. A plate-cone cylindrical reticulated shell with quadrangular pyramids is studied using finite element analysis, as shown in Figure 3. The span of structure \(S = 30\) m, length \(L = 45\) m, vector height \(F = 10\) m, and thickness \(h = 1.5\) m. The top connecting truss members are steel pipes of \(\Phi 108 \times 5.0\) mm, and the triangle plates of cones are orthogonal symmetric GFRP laminated plates that consist of four layers of laminates with total thickness equal to 8 mm. The matrix of each layer is composed of glass/epoxy resin, and the laying mode of laminated plates is [0/90]°, i.e., the orientation of fibers are 0° in the first and the fourth layers and 90° in the second and the third layers. The material properties of the GFRP plates are listed in Table 1. The steel pipes are considered as ideal elastic plastic material with elastic modulus \(E = 206\) GPa, yield strength \(f_y = 235\) MPa, and Poisson ratio \(\mu = 0.3\).

3.2. Finite Element Model. For the composite plate-cone reticulated shell, the FEM analysis model only with the top member, without bottom members and middle members, is adopted in this paper. The model considers the joints of the top members as hinged and the bottom joints as rigid (as shown in Figure 2). In the ANSYS model, the spatial truss element Link8 with 2 nodes and 6 degrees of freedom is used to model the top truss members, and the finite strain shell element Shell181 with 4 nodes and 24 degrees of freedom is used to model the triangular plate elements. This element type was demonstrated to suit for analyzing layered composite structures where material properties were varied at different layers [16–18].

The translational displacements of bottom nodes along two longitudinal edges are restricted in three directions. Uniformly distributed vertical loads of 2 kN/m² is applied perpendicular to the shell surface. Self-weight of the structure is also considered. Figure 4 shows the finite element model of the plate-cone cylindrical reticulated shell, which contains 326 nodes, 600 shell elements, and 275 link elements.

3.3. Numerical Results. The displacements and internal forces of the plate-cone cylindrical reticulated shell are
obtained. Figure 5 shows the vertical displacement of the plate-cone reticulated shell. Figures 6 and 7 present the axial force of the top member and the laminates. Figures 8 and 9 show the stresses in each layer of laminates.

From Figure 5, it can be seen that the maximum displacement of the plate-cone reticulated shell is 6.18 mm. The ratio of the maximum displacement to the span of the structure is 1/4854, which reveals that the composite plate-cone reticulated shell has large stiffness similar to the conventional steel plate-cone reticulated shell.

From Figures 6 and 7, it can be seen that the composite plate-cone reticulated shell has the same rules for static internal force distribution as the common steel plate-cone reticulated shell [19, 20]. The maximum axial compression force of top members is −32.732 kN, and the maximum axial compression force of plates is -51.055 kN. The strength of each layer of laminates was calculated according to equations (2) and (3). The results show that the strength of the first and fourth layer of composite laminates is 0.007029, and the strength of the second and third layer of composite laminates.

**Table 1: Material properties of the GFRP laminate.**

| Property | Value |
|----------|-------|
| $E_X$ (GPa) | 53.74 |
| $E_Y = E_Z$ (GPa) | 17.95 |
| $G_{XY} = G_{XZ}$ (GPa) | 8.63 |
| $G_{YZ}$ (GPa) | 5.98 |
| $PR_{XY} = PR_{XZ}$ | 0.25 |
| $X_t = X_c$ (GPa) | 0.49 |
| $Y_t$ (GPa) | 1.034 |
| $Y_c$ (GPa) | 0.027 |
| $S_t$ (GPa) | 0.138 |
| $S_c$ (GPa) | 0.041 |

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**Figure 3: Geometric parameters of the plate-cone cylindrical reticulated shell.**

**Figure 4: FE model of the plate-cone cylindrical reticulated shell.**
laminates is 0.071455. Therefore, the composite plate-cone reticulated shell has notable residual strength. However, the internal force distribution is not uniform among layers of laminates, so it is necessary to analyze the reasonable laying design for laminates of the composite plate-cone reticulated shell.

4. Parametric Analysis

4.1. Effect of the Number of Laminate Layers. In order to evaluate the effects of various design parameters on the behavior of the GFRP laminates, a parametric analysis is conducted using the FE model established above. Firstly, the effect of the number of laminate layers on the strength of each layer is studied. There are two groups of laying mode used in this plate-cone cylindrical reticulated shell. The first group is composed of laminates with fibers oriented in 90° and 0°: [90/0/90] (mode 1), [90/0] (mode 2), [90/0/90/0/90] (mode 3), [90/0/90] (mode 4), and the number of the laminate layers for modes 1 to 4 is 3 to 6, respectively. The second group is composed of laminates with fibers oriented in −45° and +45°: [−45/+45/−45] (mode 5), [−45/+45] (mode 6), [−45/+45/−45/+45/−45] (mode 7), and [+45/−45/+45] (mode 8), and the number of laminate layers for modes 5 to 8 is also 3 to 6, respectively. A schematic diagram showing the different modes is presented in Figure 10. The total thickness of the composite laminates is 8 mm in all cases, and the thickness of each layer is equal.

Figures 11 and 12 show the ultimate strength of the reticulated shell composed with the first group (modes 1∼4) and the second group (modes 5∼8) of laminates, respectively.

It can be seen from the figures that the number of laminate layers has insignificant effect on the failure load of the composite plate-cone reticulated shells with quadrangular cones for the same laying mode and total thickness. Taking the first group as an example, it is found that the first failed layer of laminates is the layer with fibers oriented in 90° occurring in the direction of 90° fibers, while the failure of the last layer occurs in the direction of 0° fibers. The first layer strength and the last layer strength are almost the same even though the number of layers is different. The first layer strength and the last layer strength for the mode with four layers of laminates (i.e., laying mode 2) are only 8.16% and 4.75% greater than that of the mode with three layers of laminates (i.e., laying mode 1), respectively. Therefore, it can be concluded that the influence of the number of layers on the ultimate strength and failure sequence of each layer of laminates can be ignored if the total thickness of the GFRP laminates is the same. Considering the material cost and time consumption, it is recommended that the layer number of laminates should not be too much, and the total layer number of laminates is suitable to be 4 layers.

4.2. Effect of Laying Direction. The laying direction of laminates is another critical parameter of laminates. Because the laying structures are different, that is to say, if the order of each layer in the laminate is different, the ultimate strength of the laminate may be totally different even for the same material system. Therefore, for the composite plate-cone reticulated shell, studying the influence of laying direction on the strength of the laminate is very necessary and of significant importance in practice.

In order to discuss the influence of laying direction on the strength of the structure, the mechanical properties of eight kinds of laying directions (total thickness is 8 mm, and the thickness of each layer is uniform) are calculated using finite element analysis. Combining with the Strength Criterion of Hoffman, the ultimate strength and the failure sequence of layers of laminates are obtained. The detailed results are shown in Table 2.

It can be found from Table 2 that the laying direction of the laminate has significant influence on the strength of the structure. It can also be observed that

(1) The ultimate strength of the plate-cone reticulated shell with different laying modes are different. The first layer strength of laminates with laying mode of [0/90] and [90/0] are 26.6 kN/m², while the last layer strength is about 130 kN/m². However, for laminates with laying modes of [−45/+45] and [+45/−45], all of four layers almost fail concurrently, and the failure load is about 44.5 kN/m².
(2) The failure sequence of layers is also different. The first failed layers of laminates with laying modes of [90/0]°S, [+45/0]°S, and [90/+45]°S are the first and fourth layers, while the last failed layers are the second and third layers. However, for laminates with laying modes of [0/90]°S, [0/-45]°S, and [+45/0]°S, the failure sequence of layers are just the opposite, that is, the first failed layers are the second and third layers, and the last failed layers are the first and fourth layers. For laminates with laying modes of [−45/+45]°S and [+45/−45]°S, all four layers failed simultaneously.

(3) The strength interval between the first layer strength and the last layer strength is also different. For laminates with laying modes of [0/90]°S and [90/0]°S, there is the largest strength interval between the first

| Layer | Strength | | | | |
|-------|----------|---|---|---|---|
| 1     | 10.321   | 2.0683 | -13.774 | -27.449 | -32.732 |
| 2     | 10.835   | 3.0259 | -11.829 | -24.730 | -29.725 |
| 3     | 10.580   | 3.2284 | -11.065 | -23.468 | -28.272 |
| 4     | 10.184   | 3.0243 | -10.913 | -23.009 | -27.692 |
| 5     | 9.8917   | 2.7595 | -10.994 | -22.902 | -27.511 |
| 6     | 9.7321   | 2.5733 | -11.105 | -22.915 | -27.480 |
| 7     | 9.6628   | 2.4795 | -11.178 | -22.946 | -27.491 |
| 8     | 9.6447   | 2.4525 | -11.201 | -22.959 | -27.498 |

Figure 6: Axial force of top members of the plate-cone reticulated shell (kN).
| \(-5105.5\) | 10200 | 1690.7 |
| \(-1858.6\) | 14846 | 721.73 |
| \(-4273.80\) | 5523 | 56.963 |
| \(-45046\) | 8621.8 | 1510.5 |
| \(-4389\) | 20952 | 382.81 |
| \(-41243\) | 7413.9 | 2540.7 |
| \(-30878\) | 8814.4 | 2856.7 |
| \(-4154\) | 19331 | 202.8 |
| \(-27094\) | 3330.5 | 3249.4 |
| \(-3034\) | 29135 | 292.9 |
| \(-16425\) | 3399.5 | 2433.4 |
| \(-2320.3\) | 5554.2 | 355.05 |
| \(-1598.5\) | 127.78 | 320.94 |

| \(-47257\) | 9731.6 | 1699 |
| \(-1667.7\) | 7732.0 | 119.8 |
| \(-13320\) | 705.88 | 6.42 |
| \(-44829\) | 9645.1 | 77.367 |
| \(-4665.8\) | 1435.1 | 112.43 |
| \(-43752\) | 9094 | 1136.6 |
| \(-1609.8\) | 12559.0 | 577.41 |
| \(-12289\) | 9115.4 | 234.12 |
| \(-399.54\) | 8968.6 | 384.14 |
| \(-43330\) | 9094.6 | 917.59 |
| \(-1613.7\) | 12904.0 | 240.19 |
| \(-43213\) | 9615.8 | 501.33 |
| \(-43200\) | 9016.5 | 785.24 |
| \(-1621.4\) | 12951.0 | 126.88 |
| \(-43180\) | 8919 | 178.79 |
| \(-43170\) | 8992 | 712.6 |
| \(-1624.5\) | 12974.0 | 52.833 |
| \(-43170\) | 8941.6 | 630.45 |
| \(-43167\) | 8963.3 | 688.31 |
| \(-43167\) | 8963.3 | 688.31 |

**Figure 7:** Axial force of laminates of the plate-cone reticulated shell (N).

strength and the last strength. The ratio between the last layer strength and the first layer strength for laminates with laying modes of \([0/90]_S\) and \([90/0]_S\), \([+45/90]_S\) and \([+90/+45]_S\), and \([+45/0]_S\) and \([+0/+45]_S\) are 4.89, 2.95, and 1.27, respectively. And for laminates with laying modes of \([-45/+45]_S\) and
| x       | y       | z       | x       | y       | z       | x       | y       | z       |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| -2.13   | -0.21   | -0.23   | -0.96   | -1.08   | -0.11   | -0.22   | -0.18   | -0.21   |
| 0.0904  | -0.016  | 0.0016  | 0.9738  | -0.018  | 0.0028  | 0.9616  | -0.018  | 0.0028  |
| -0.37   | 0.20    | 0.32    | -0.009  | 0.018   | 0.036   | -0.009  | 0.018   | 0.036   |
| -0.093  | -0.028  | -0.029  | -0.093  | -0.028  | -0.029  | -0.093  | -0.028  | -0.029  |
| 0.102   | -0.053  | -0.055  | 0.104   | -0.053  | -0.055  | 0.104   | -0.053  | -0.055  |
| 0.0721  | -0.096  | -0.118  | 0.0721  | -0.096  | -0.118  | 0.0721  | -0.096  | -0.118  |
| 0.0102  | -0.057  | -0.060  | 0.0102  | -0.057  | -0.060  | 0.0102  | -0.057  | -0.060  |
| -0.20   | -0.020  | -0.023  | -0.20   | -0.020  | -0.023  | -0.20   | -0.020  | -0.023  |
| -0.346  | -0.111  | -0.096  | -0.346  | -0.111  | -0.096  | -0.346  | -0.111  | -0.096  |
| 0.0489  | -0.055  | -0.057  | 0.0489  | -0.055  | -0.057  | 0.0489  | -0.055  | -0.057  |
| 0.137   | 0.214   | 0.298   | 0.137   | 0.214   | 0.298   | 0.137   | 0.214   | 0.298   |
| -0.019  | -0.008  | -0.009  | -0.019  | -0.008  | -0.009  | -0.019  | -0.008  | -0.009  |
| 0.0205  | 0.05    | 0.049   | 0.0205  | 0.05    | 0.049   | 0.0205  | 0.05    | 0.049   |
| 0.186   | 0.028   | 0.031   | 0.186   | 0.028   | 0.031   | 0.186   | 0.028   | 0.031   |
| -0.030  | -0.028  | -0.029  | -0.030  | -0.028  | -0.029  | -0.030  | -0.028  | -0.029  |
| -0.33   | -0.091  | -0.091  | -0.33   | -0.091  | -0.091  | -0.33   | -0.091  | -0.091  |
| 0.0252  | -0.145  | -0.145  | 0.0252  | -0.145  | -0.145  | 0.0252  | -0.145  | -0.145  |
| -0.030  | 0.0152  | 0.0152  | -0.030  | 0.0152  | 0.0152  | -0.030  | 0.0152  | 0.0152  |
| 0.0269  | 0.0141  | 0.0141  | 0.0269  | 0.0141  | 0.0141  | 0.0269  | 0.0141  | 0.0141  |
| -0.010  | -0.091  | -0.091  | -0.010  | -0.091  | -0.091  | -0.010  | -0.091  | -0.091  |
| 0.0248  | -0.305  | -0.305  | 0.0248  | -0.305  | -0.305  | 0.0248  | -0.305  | -0.305  |
| 0.0299  | -0.028  | -0.028  | 0.0299  | -0.028  | -0.028  | 0.0299  | -0.028  | -0.028  |
| 0.0001  | 0.0152  | 0.0152  | 0.0001  | 0.0152  | 0.0152  | 0.0001  | 0.0152  | 0.0152  |
| 0.0034  | 0.040   | 0.040   | 0.0034  | 0.040   | 0.040   | 0.0034  | 0.040   | 0.040   |
| -0.018  | 0.0013  | 0.0013  | -0.018  | 0.0013  | 0.0013  | -0.018  | 0.0013  | 0.0013  |
| 0.0001  | 0.0152  | 0.0152  | 0.0001  | 0.0152  | 0.0152  | 0.0001  | 0.0152  | 0.0152  |
| -0.0081 | 0.036   | 0.036   | -0.0081 | 0.036   | 0.036   | -0.0081 | 0.036   | 0.036   |
| -0.0001 | 0.0152  | 0.0152  | -0.0001 | 0.0152  | 0.0152  | -0.0001 | 0.0152  | 0.0152  |

**Figure 8**: Normal and shear stress ($\sigma_x$, $\sigma_y$, $\tau_{xy}$) of the first and fourth layers of laminates (MPa).
|          | \( \sigma_x \) (MPa) | \( \sigma_y \) (MPa) | \( \tau_{xy} \) (MPa) |
|----------|-----------------|-----------------|-----------------|
| 2.35     | 0.217           | 0.0076          |
| -3.06    | -0.271          | -0.247          |
| -2.029   | -0.0683         | -0.0093         |
| 0.17     | 0.284           | 0.0097          |
| -2.49    | 0.318           |                |
| 1.98     | -1.74           | -0.0964         |
| -3.61    | -0.346          | -0.025          |
| -0.892   | -0.164          | 0.407           |
| 1.34     | -1.86           | -0.357          |
| -3.42    | -0.136          | -0.0030         |
| -2.41    | -0.309          | -0.0350         |
| 0.766    | -0.984          | -0.304          |
| -1.02    | -0.188          | -0.0439         |
| -0.696   | -0.0064         | -0.0101         |
| -0.263   | -0.0597         |                |

Figure 9: Normal and shear stress \( (\sigma_x, \sigma_y, \tau_{xy}) \) of the second and third layers of laminates (MPa).
Based on the above analysis, it can be concluded that the laying direction of laminate is a key factor affecting the ultimate strength of GFRP laminate of the composite plate-cone reticulated shell.

4.3. Effect of Thickness of Laminate. In order to study the influence of the thickness of laminates on the ultimate strength of laminates, the parametric analysis for laminates with different total thickness and different thickness of individual layer is conducted. The triangular plates of cone elements are adopted with different laying design, i.e., the laying directions are all $[90/0]_S$, but the total thickness of the laminates and the thickness of each layer are different. In this paper, the mechanical properties of six kinds of laminates with different laying modes (seen in Table 3) are calculated, and the ultimate strength of the structure under uniformly distributed loads are obtained by combining the Strength Criterion of Hoffman; more detailed results are shown in Table 3.

From Table 3, it can be seen that the thickness of composite laminates has a significant effect on the ultimate strength of laminates in plate-cone reticulated shells. With
the change of the thickness of each layer, the first layer strength and last layer strength of the structure vary remarkably. Generally, the first layer and last layer strengths will increase with the increase of the total thickness and the thickness of the second and fourth layers. It can also be found that with the change of total thickness, the strength interval between the first layer strength and last layer strength of the laminates changes slightly. The ratios between the last layer ultimate strength $Q_2$ and the first layer ultimate strength $Q_1$ are about 5.

5. Conclusions

The strength of each layer in FRP laminate is one of the main controlling factors in the design of composite plate-cone reticulated shell. In this paper, using ANSYS software, a strength model of composite laminate is established using the Strength Criterion of Hoffman without considering the stiffness degradation of laminates. The effects of number of layers, laying direction, and thickness of laminates on the ultimate strength of laminates are studied in the parametric analysis. Based on the results of the numerical analysis, the following conclusions can be drawn:

(1) In the composite plate-cone reticulated shell, the influence of the number of laminate layers on the ultimate strength and failure sequence of each layer of laminated plates is negligible. Taking into account other factors, such as material cost and time consumption, the total layer number of laminate is recommended as four layers.

### Table 2: Uniformly distributed load when layer-by-layer failure of laminated plates.

| Laying mode   | First layer strength (kN/m²) | Layers failed at first | Last layer strength (kN/m²) | Layers failed at last |
|---------------|-----------------------------|------------------------|-----------------------------|-----------------------|
| [0/90]° S     | 26.6                        | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers | 130.0                       | 1\textsuperscript{st} and 4\textsuperscript{th} layers |
| [90/0]° S     | 26.5                        | 1\textsuperscript{st} and 4\textsuperscript{th} layers | 132.3                       | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers |
| [-45/+45]° S  | 44.5                        | All of four layers     | 44.5                        | All of four layers    |
| [+45/-45]° S  | 44.4                        | All of four layers     | 44.5                        | All of four layers    |
| [+45/0]° S    | 65.4                        | 1\textsuperscript{st} and 4\textsuperscript{th} layers | 83.0                        | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers |
| [0/+45]° S    | 65.5                        | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers | 83.0                        | 1\textsuperscript{st} and 4\textsuperscript{th} layers |
| [+45/90]° S   | 14.8                        | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers | 43.6                        | 1\textsuperscript{st} and 4\textsuperscript{th} layers |
| [90/+45]° S   | 14.8                        | 1\textsuperscript{st} and 4\textsuperscript{th} layers | 43.8                        | 2\textsuperscript{nd} and 3\textsuperscript{rd} layers |

Note: [3mm/1mm] S indicates that the thickness of the first and fourth layers is 3 mm, and that of the second and third layers is 1 mm.

### Table 3: Comparison of ultimate strength of laminated plates with different thicknesses.

| Thickness of each layer | Total thickness (mm) | First layer strength $Q_1$ (kN/m²) | Last layer strength $Q_2$ (kN/m²) | $Q_2/Q_1$ |
|-------------------------|----------------------|------------------------------------|-----------------------------------|-----------|
| [2 mm/2 mm] S           | 8                    | 26.6                               | 132.3                             | 4.99      |
| [1 mm/1 mm] S           | 4                    | 14.0                               | 76.8                              | 5.48      |
| [3 mm/1 mm] S           | 8                    | 23.2                               | 121.3                             | 5.23      |
| [1 mm/3 mm] S           | 8                    | 28.8                               | 134.0                             | 4.66      |
| [2 mm/1 mm] S           | 6                    | 18.5                               | 100.0                             | 5.40      |
| [1 mm/2 mm] S           | 6                    | 21.7                               | 108.5                             | 5.01      |

Note: [3mm/1mm] S indicates that the thickness of the first and fourth layers is 3 mm, and that of the second and third layers is 1 mm.
(2) The laying direction is one of the key factors affecting the ultimate strength of the composite laminate in the plate-cone reticulated shell. The influence of laying direction on the ultimate strength and strength interval is significant. It should be fully used in design of practical engineering to achieve reasonable strength interval of laminate, for making full use of potential and advantages of composite material.

(3) The influence of the thickness of composite laminate on the ultimate strength of laminate is significant. The first layer strength and last layer strength will increase with the total thickness increase, and the thickness of the second and fourth layers increase. But the influence of the total thickness on the strength interval is negligible.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare there are no conflicts of interest.

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