Neutrino masses, mixing, Majorana CP-violating phases and \((\beta\beta)_{0\nu}\) decay

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Abstract. Predictions of the effective Majorana mass \(|\langle m \rangle|\) in \((\beta\beta)_{0\nu}\) decay for 3-\(\nu\) mixing and massive Majorana neutrinos are reviewed in the present study. The physics potential of the experiments, searching for \((\beta\beta)_{0\nu}\) decay and having sensitivity to \(|\langle m \rangle| > 0.01\) eV for providing information on the type of the neutrino mass spectrum, the absolute scale of neutrino masses and on the Majorana CP-violation phases in the PMNS neutrino mixing matrix, is also discussed.

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1. Introduction

There has been remarkable progress in the study of neutrino oscillations over the last several years. Experiments with solar, atmospheric and reactor neutrinos [1]–[6] have provided compelling evidence for the existence of neutrino oscillations driven by nonzero neutrino masses and neutrino mixing. Evidence for oscillations of neutrinos was obtained also in the first long-baseline accelerator neutrino experiment K2K [7].

The hypothesis of solar neutrino oscillations which, in 1967, were first predicted to cause a solar neutrino deficit [8] and later, in one variety or another, were considered as the most natural explanation for the observed [1, 2] solar neutrino deficit (see e.g. [9]–[11]), has received a convincing confirmation from the measurement of the solar neutrino flux through the neutral current reaction on deuterium by the SNO experiment [3, 4]. Analysis of the solar neutrino data obtained by Homestake, SAGE, GALLEX/GNO, SK and SNO experiments showed that the data favour the large mixing angle (LMA) MSW solution for the solar neutrino problem. The first results of the KamLAND reactor experiment [6] have confirmed (under the very plausible assumption of CPT invariance) the LMA MSW solution, establishing it essentially as a unique solution for the solar neutrino problem. This remarkable result brought us, after more than 30 years of research, initiated by the pioneer works of Pontecorvo [8, 12] and the experiment of Davis et al [13], very close to a complete understanding of the true cause of the solar neutrino problem.

A combined two-neutrino oscillation analysis of the solar neutrino and KamLAND data, performed before the latest (salt-phase) SNO results were announced, identified two distinct solution subregions within the LMA solution region, LMA-I and II (see e.g. [14, 15]). The best-fit values of two-neutrino oscillation parameters, namely the solar neutrino mixing angle $\theta^\odot$ and the mass-squared difference $\Delta m^2_{\odot}$ in the two subregions, LMA-I and -II, read (see e.g. [14]) $\Delta m^2_{\odot}^{I} = 7.3 \times 10^{-5} \text{eV}^2$, $\Delta m^2_{\odot}^{II} = 1.5 \times 10^{-4} \text{eV}^2$ and $\tan^2\theta^I_\odot = \tan^2\theta^I_\odot = 0.46$. The LMA-I solution was preferred statistically by the data. At 90% CL, it was found in (see e.g. [14])

$$\Delta m^2_{\odot} \simeq (5.6 - 17) \times 10^{-5} \text{eV}^2, \quad \tan^2\theta^I_\odot \simeq (0.32 - 0.72). \quad (1)$$

In September 2003, the SNO collaboration published data from the salt phase of the experiment [4]. In particular, for the ratio of the CC and NC event rates, the collaboration found $R_{CC/NC} = 0.306 \pm 0.026 \pm 0.024$ and, correspondingly, $R_{CC/NC} \leq 0.41$ at high CL. As was shown in [16], an upper limit of $R_{CC/NC} < 0.5$ implies a significant upper limit on $\Delta m^2_{\odot}$ smaller than $2 \times 10^{-4} \text{eV}^2$: $\Delta m^2_{\odot} \lesssim 1.7 \times 10^{-4} \text{eV}^2$. Thus, the latest SNO data on $R_{CC/NC}$ imply stringent constraints on the LMA-II solution. A combined statistical analysis of data from the solar neutrino and KamLAND experiments, including the latest SNO results, showed [18] (see also e.g. [19]) that the LMA-II solution is allowed only at 99.13% CL. Furthermore, the data have substantially reduced the maximum allowed value of $\sin^2\theta^I_\odot$ (see [16, 17]), thereby excluding the possibility of maximal mixing at 5.4 standard deviations (S.D.) [4, 18]. The best-fit value of $\theta^I_\odot$ corresponds to $\cos 2\theta^I_\odot = 0.40$, whereas, at 90% (95%) CL, one has $\cos 2\theta^I_\odot \geq 0.24$ (0.22). This has very important implications, in particular for predictions of the effective Majorana mass in neutrinoless double-beta decay, $|\langle m \rangle|$ [21]–[24].

Future data from SNO on the day/night effect and the spectrum of $e^-$ from the CC reaction, and future high statistics data from KamLAND, may resolve the LMA-I–LMA-II solution ambiguity and can constrain further the solar neutrino mixing angle (see [16, 18, 20] and references therein).

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There is also strong evidence for oscillations of the atmospheric $\nu_\mu (\bar{\nu}_\mu)$ from the Zenith angle dependence of the sub-GeV and multi-GeV $\mu$-like events, which were observed in the Super-Kamiokande experiment [25]. The experimental results are best described in terms of dominant two-neutrino $\nu_\mu \rightarrow \nu_\tau (\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ oscillations with maximal mixing, $\sin^2 2\theta_{\text{atm}} \equiv 1$. A combined analysis [26] of atmospheric neutrino data and the data from the K2K long-baseline accelerator experiment [7] shows that, at 90% CL, the neutrino mass-squared difference responsible for the atmospheric neutrino oscillations lies in the interval

$$2.0 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m^2_{\text{atm}} \lesssim 3.2 \times 10^{-3} \text{ eV}^2. \quad (2)$$

The $\Delta m^2_{\text{atm}}$ best-fit value found in [26] reads $\Delta m^2_{\text{atm}}|_{\text{BF}} = 2.6 \times 10^{-3} \text{ eV}^2$. Preliminary results from an improved analysis of the SK atmospheric neutrino data, performed recently by the SK collaboration, gave [25]

$$1.3 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m^2_{\text{atm}}| \lesssim 3.1 \times 10^{-3} \text{ eV}^2, \quad 90\% \text{ CL}, \quad (3)$$

with best-fit value $|\Delta m^2_{\text{atm}}| = 2.0 \times 10^{-3} \text{ eV}^2$. Adding the K2K data [7], Fogli et al [27] find the same best-fit value and

$$1.55 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m^2_{\text{atm}}| \lesssim 2.60 \times 10^{-3} \text{ eV}^2, \quad 90\% \text{ CL}. \quad (4)$$

Recently, the SK collaboration presented the first evidence of an ‘oscillation dip’ in the $L/E$ dependence, where $L$ and $E$ are the distance travelled by neutrinos and the neutrino energy respectively, of a particular selected sample of $\mu$-like events [28]. Such a dip is predicted due to the oscillatory dependence of the $\nu_\mu \rightarrow \nu_\tau (\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ oscillation probability on $L/E$: the $\nu_\mu \rightarrow \nu_\tau (\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ transitions of atmospheric neutrinos are predominantly two-neutrino transitions governed by vacuum oscillation probability. The dip in the observed $L/E$ distribution corresponds to the first oscillation minimum of the $\nu_\mu (\bar{\nu}_\mu)$ survival probability, $P(\nu_\mu \rightarrow \nu_\mu)(P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu))$, as $L/E$ increases starting from values for which $\Delta m^2_{\text{atm}} L / (2E) \ll 1$ and $P(\nu_\mu \rightarrow \nu_\mu) \approx 1$. This beautiful result represents the first ever observation of a direct effect of the oscillatory dependence on $L$ and $E$ of the probability of neutrino oscillations in vacuum.

Interpretation of the solar and atmospheric neutrinos, and of the KamLAND data in terms of neutrino oscillations, requires the existence of three-neutrino mixing in the weak charged lepton current:

$$\nu_{\text{L}} = \sum_{j=1}^{3} U_{\ell j} \nu_{\text{L}}^j. \quad (5)$$

Here, $\nu_{\text{L}}$ (where $l = e, \mu, \tau$) are the three left-handed flavour neutrino fields, $\nu_{\text{L}}^j$ is the left-handed field of the neutrino $\nu_j$ having a mass $m_j$ and $U$ is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix [29]. Actually, all existing neutrino oscillation data, except the data of the LSND experiment [30], can be described if we assume the existence of three-neutrino mixing in vacuum, (equation (5)), and we will consider this possibility in what follows.5

3 These are $\mu$-like events for which the relative uncertainty in the experimental determination of the $L/E$ ratio does not exceed 70%.

4 In the LSND experiment, indications for oscillations $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ with $(\Delta m^2)_{\text{LSND}} \simeq 1 \text{ eV}^2$ were obtained. The LSND results are being tested in the MiniBooNE experiment at Fermilab [31].

5 For a similar analysis taking into account the LSND evidences for neutrino oscillations, see [32].
The PMNS mixing matrix $U$ can be parametrized by three angles $\theta_{\text{atm}}$, $\theta_{\odot}$ and $\theta$, and, depending on whether the massive neutrinos $\nu_j$ are Dirac or Majorana particles, by one or three CP-violating phases [33]–[35]. In the standard parametrization of $U$ (see e.g. [36]), the three mixing angles are denoted as $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$:

$$
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} e^{i\alpha_{21}/2} & s_{13} e^{i\alpha_{31}/2} \\
    -s_{12}c_{13} - c_{12}s_{13}s_{23} e^{i\delta} & s_{12}s_{23} e^{i\delta} & 0 \\
    s_{12}s_{23} - c_{12}c_{23}s_{13} e^{i\delta} & (-c_{12}c_{23} - s_{12}s_{23}s_{13}) e^{i\delta} & c_{23} e^{i\delta} \sin\alpha_{31}/2
\end{pmatrix},
$$

where we have used the usual notations $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$; $\delta$ is the Dirac CP-violation phase and $\alpha_{21}$ and $\alpha_{31}$ are two Majorana CP-violation phases [33]–[35]. If we identify the two independent neutrino mass-squared differences in this case, $\Delta m_{21}^2$ and $\Delta m_{31}^2$, with the neutrino mass squared differences that induce the solar and atmospheric neutrino oscillations, $\Delta m_{\odot}^2 = \Delta m_{21}^2 > 0$, $\Delta m_A^2 = \Delta m_{31}^2$, we have $\theta_{12} = \theta_{\odot}$, $\theta_{23} = \theta_{\text{atm}}$ and $\theta_{13} = \theta$. The angle $\theta$ is limited by the data from the CHOOZ and Palo Verde experiments [37, 38]. The oscillations between flavour neutrinos are insensitive to the Majorana CP-violating phases $\alpha_{21}$ and $\alpha_{31}$ [33, 39]. Information about these phases can be obtained, in principle, from the $(\beta\beta)_{0v}$-decay experiments [22, 24], [40]–[43] (see also [44]–[46]). Majorana CP-violating phases might be at the origin of the baryon asymmetry of the Universe [47].

The precise limit on the angle $\theta$ from the CHOOZ and Palo Verde data is $\Delta m_{\text{atm}}^2$-dependent (see e.g. [48]). For the values of $\Delta m_{\text{atm}}^2$ found in [26] and equation (2), the upper bound on $\sin^2 \theta$ at 90% (99.73%) CL reads $\sin^2 \theta < 0.03$ (0.05). Using the latest SK preliminary result, one gets, at 90% (99.73%) CL, from a combined 3-$\nu$ oscillation analysis of the solar neutrino, CHOOZ and KamLAND data [18],

$$
\sin^2 \theta < 0.047 (0.074).
$$

Somewhat better limits on $\sin^2 \theta$ compared with the existing one can be obtained in the MINOS, OPERA and ICARUS experiments [49, 50]. Various options are currently being discussed (experiments with off-axis neutrino beams, more precise reactor antineutrino and long-baseline experiments, etc; see e.g. [51, 52]) on how to improve the sensitivity to $\sin^2 \theta$ by at least a factor of 5 or more, i.e. to values of $\sim 0.01$ or smaller.

The combined 3-$\nu$ oscillation analysis of the solar neutrino, CHOOZ and KamLAND data also showed [18] that the allowed ranges of the solar neutrino oscillation parameters do not differ substantially for $\sin^2 \theta < 0.05$ from those derived in the two-neutrino oscillation analyses. At 90% CL, for instance, one finds

$$
0.23 \lesssim \sin^2 \theta_{\odot} \lesssim 0.38 \quad \text{for} \quad \sin^2 \theta = 0.0,
$$

$$
0.25 \lesssim \sin^2 \theta_{\odot} \lesssim 0.36 \quad \text{for} \quad \sin^2 \theta = 0.04.
$$

The best-fit values in both cases read $\sin^2 \theta_{\odot} = 0.30$. The allowed values of $\Delta m_{\odot}^2$ in the LMA-I region also do not change considerably with respect to those obtained in the two-neutrino oscillation analyses and are given at 90% CL by [18]

$$
5.6 \times 10^{-3} \text{eV}^2 \lesssim \Delta m_{\odot}^2 \lesssim 9.2 \times 10^{-5} \text{eV}^2 \quad \text{for} \quad \sin^2 \theta = 0.0,
$$

$$
6.1 \times 10^{-5} \text{eV}^2 \lesssim \Delta m_{\odot}^2 \lesssim 8.5 \times 10^{-5} \text{eV}^2 \quad \text{for} \quad \sin^2 \theta = 0.04.
$$
The $\Delta m^2_{2\odot}$ best-fit value is practically the same for the two values of $\sin^2 \theta$: $\Delta m^2_{2\odot} \approx 7.2 \times 10^{-5}$ eV$^2$.

Note that the atmospheric neutrino and K2K data do not allow one to determine the sign of $\Delta m^2_{\text{atm}}$. This implies that if we identify $\Delta m^2_{\text{atm}}$ with $\Delta m^2_{31}$ for three-neutrino mixing, one can have $\Delta m^2_{31} > 0$ or $\Delta m^2_{31} < 0$. The two possibilities correspond to two different types of neutrino mass spectra: with normal hierarchy, $m_1 < m_2 < m_3$, and with inverted hierarchy, $m_3 < m_1 < m_2$ (see e.g. [40]). We will use the terms normal hierarchical (NH) and inverted hierarchical (IH) for the two types of spectra in the case of strong inequalities between the neutrino masses, i.e. if $m_1 \ll m_2 \ll m_3$ and $m_3 \ll m_1 < m_2$, respectively. The spectrum can also be of quasi-degenerate (QD) type: $m_1 \approx m_2 \approx m_3$ and $m^2_{1,2,3} \gg |\Delta m^2_{\text{atm}}|$.

The sign of $\Delta m^2_{\text{atm}}$ can be determined in very long baseline neutrino oscillation experiments at neutrino factories (see e.g. [54]), e.g. using combined data from long baseline oscillation experiments at the JHF facility and with off-axis neutrino beams [55]. Under some rather special conditions, it might be determined also in experiments with reactor $\bar{\nu}_e$ [53, 56].

As is well known, neutrino oscillation experiments allow one to determine differences of squares of neutrino masses, but not the absolute values of the masses. Information on the absolute values of neutrino masses of interest can be derived in the $^3\text{H} \beta$-decay experiments studying the electron spectrum [57]–[59] and from cosmological and astrophysical data (see e.g. [60]–[62]). The currently existing most stringent upper bounds on the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ were obtained in the Troitzk [58] and Mainz [59] $^3\text{H} \beta$-decay experiments and read

$$m_{\bar{\nu}_e} < 2.2 \text{ eV} \quad (95\% \text{ CL}).$$

We have $m_{\bar{\nu}_e} \approx m_{1,2,3}$ for the QD neutrino mass spectrum. The KATRIN $^3\text{H} \beta$-decay experiment [59] is expected to reach a sensitivity of $m_{\bar{\nu}_e} \sim (0.20–0.35) \text{ eV}$, i.e. to probe the region of QD neutrino mass spectrum. Data from the WMAP experiment on the cosmic microwave background radiation was used to obtain an upper limit on the sum of neutrino masses [61]:

$$\sum_j m_j < 0.70 \text{ eV} \quad (95\% \text{ CL}).$$

A conservative estimate of all the uncertainties related to the derivation of this result (see e.g. [63]) leads to a less stringent upper limit, at least by a factor of $\sim 1.5$ and possibly by a factor of $\sim 3$. The WMAP and future PLANCK experiments can be sensitive to $\sum_j m_j \approx 0.4 \text{ eV}$. Data on weak lensing of galaxies by large-scale structure, combined with data from the WMAP and PLANCK experiments may allow one to determine $(m_1 + m_2 + m_3)$ with an uncertainty of $[62] \delta \sim 0.04 \text{ eV}$.

After the spectacular experimental progress made during the last several years in the studies of neutrino oscillations, further understanding of the structure of the neutrino masses and mixing, of their origins and of the status of the CP symmetry in the lepton sector requires a large and challenging programme of research to be pursued in neutrino physics. The main goals of such a

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6 This definition of the IH spectrum corresponds to a convention we will call A (see e.g. [53]), in which the neutrino masses are not ordered in magnitude according to their index number. We can also always number the neutrinos with definite mass in such a way that $[24, 48] m_1 < m_2 < m_3$. In this convention called B [53], the IH spectrum corresponds to $m_1 \ll m_3 \equiv m_3$. We will use convention B in our further analysis.
research programme should include the following:

- High-precision determination of the neutrino mixing parameters which control the solar and the dominant atmospheric neutrino oscillations, $\Delta m^2_{\odot}$, $\theta_{\odot}$ and $\Delta m^2_{\text{Atm}}$, $\theta_{\text{Atm}}$.
- Measurement of, or improving by at least a factor of 5–10 the existing upper limit on, the value of the only small mixing angle $\theta (\approx \theta_{13})$ in the PMNS matrix $U$.
- Determination of the type of the neutrino mass spectrum (NH or IH or QD).
- Determining or obtaining significant constraints on the absolute scale of neutrino masses, or on the lightest neutrino mass.
- Determining the nature of massive neutrinos, which can be Dirac or Majorana particles.
- To establish whether the CP symmetry is violated in the lepton sector (i) due to the Dirac phase $\delta$ and/or (ii) due to the Majorana phases $\alpha_{21}$ and $\alpha_{31}$ if the massive neutrinos are Majorana particles.
- Searching with increased sensitivity for possible manifestations, other than flavour neutrino oscillations, of the nonconservation of individual lepton charges $L_l$, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$ and $\tau \rightarrow \mu + \gamma$ decays.
- Understanding at a fundamental level the mechanism giving rise to the neutrino masses and mixing and to the $L_\mu$ nonconservation, i.e. finding the theory of neutrino mixing. Progress in the theory of neutrino mixing might also lead, in particular, to a better understanding of the possible relation between CP violation in the lepton sector at low energies and generation of the baryon asymmetry of the Universe.

Obviously, successful realization of the experimental part of this programme of research would be a formidable task and would require many years.

In the present paper, we will review the potential contribution of studies of neutrinoless double-beta ($\beta\beta_0\nu$) decay of certain even–even nuclei, $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$, to the programme of research outlined above. The $(\beta\beta)_0$ decay is allowed if the neutrinos with definite mass are Majorana particles (see e.g. [10, 64, 65] for reviews). Let us recall that the nature (Dirac or Majorana) of the massive neutrinos $\nu_j$ is related to the fundamental symmetries of particle interactions. The neutrinos $\nu_j$ will be Dirac fermions if the particle interactions conserve some lepton charge, e.g. the total lepton charge $L$. The neutrinos with definite mass can be Majorana particles if there does not exist any conserved lepton charge. As is well known, massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [66], which also provides a very attractive explanation for the smallness of the neutrino masses and, through the leptogenesis theory [47], for the observed baryon asymmetry of the Universe.

If the massive neutrinos $\nu_j$ are Majorana fermions, processes in which the total lepton charge $L$ is not conserved and changes by two units, such as $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$ and $\mu^+ + (A, Z) \rightarrow (A, Z + 2) + \mu^-$, should exist. The process most sensitive to the possible Majorana nature of the massive neutrinos $\nu_j$ is the $(\beta\beta)_0\nu$ decay (see e.g. [10]). If the $(\beta\beta)_0\nu$ decay is generated only by the $(V\!-\!A)$ charged current weak interaction via the exchange of the three Majorana neutrinos $\nu_j$ and the latter have masses not exceeding a few MeV, which will be assumed to hold throughout this paper, the dependence of the $(\beta\beta)_0\nu$-decay amplitude $A(\beta\beta)_0\nu$ on the neutrino mass and mixing parameters factorizes in the effective Majorana mass $\langle m \rangle$ (see e.g. [10, 65]):

$$A(\beta\beta)_0\nu \sim \langle m \rangle M,$$

(14)
where $M$ is the corresponding nuclear matrix element (NME) and

$$|⟨m⟩| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{iα_{21}} + m_3|U_{e3}|^2 e^{iα_{31}},$$ \hspace{1cm} (15)

$α_{21}$ and $α_{31}$ being the two Majorana CP-violating phases of the PMNS matrix\footnote{We assume that $m_j > 0$ and that the fields of the Majorana neutrinos $ν_j$ satisfy the Majorana condition $C(ν_j)^T = ν_j, \ j = 1, 2, 3,$ where $C$ is the charge conjugation matrix.} $[33, 35]$. Let us note that if CP invariance holds, one has $[67]$ $α_{21} = kπ, α_{31} = k′π$, where $k, k′ = 0, 1, 2, \ldots$. In this case,

$$η_{21} ≡ e^{iα_{21}} = ±1, \quad η_{31} ≡ e^{iα_{31}} = ±1$$ \hspace{1cm} (16)

represent the relative CP parities of the Majorana neutrinos $ν_1$ and $ν_2$ and $ν_1$ and $ν_3$, respectively. It follows from equation (15) that measurement of $|⟨m⟩|$ will provide information, in particular, on the neutrino masses. As equation (14) indicates, observation of the $(ββ)_{0ν}$ decay of a given nucleus and measurement of the corresponding half lifetime would allow one to determine $|⟨m⟩|$ only if the value of the relevant NME $M$ is known with a relatively low uncertainty.

The experimental searches for $(ββ)_{0ν}$ decay have a long history (see e.g. $[64, 65]$). Rather stringent upper bounds on $|⟨m⟩|$ have been obtained in the $^{76}$Ge experiments by the Heidelberg-Moscow collaboration $[68]$:

$$|⟨m⟩| < 0.35 \text{ eV}, \quad 90\% \text{ CL.}$$ \hspace{1cm} (17)

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant NME $[65]$, we get

$$|⟨m⟩| < (0.35–1.05) \text{ eV}, \quad 90\% \text{ CL.}$$ \hspace{1cm} (18)

The IGEX collaboration has obtained $[69]$:

$$|⟨m⟩| < (0.33–1.35) \text{ eV}, \quad 90\% \text{ CL.}$$ \hspace{1cm} (19)

Evidence for $(ββ)_{0ν}$ decay of $^{76}$Ge, taking place with a rate corresponding to $0.11 \text{ eV} \leqslant |⟨m⟩| \leqslant 0.56 \text{ eV}$ (95% CL), is claimed to have been obtained in $[70]$. The results presented in $[70]$ have been criticized in $[71]$. Even stronger evidence has reported recently in $[72]$, where the following value of $|⟨m⟩|$ has been given:

$$0.1 \text{ eV} \leqslant |⟨m⟩| \leqslant 0.9 \text{ eV}, \quad 99.73\% \text{ CL.}$$ \hspace{1cm} (20)

The results reported in $[72]$ will be checked in the currently running and future $(ββ)_{0ν}$-decay experiments (see below). However, it may take a very long time before a comprehensive check could be completed.

Higher sensitivity to $|⟨m⟩|$ is planned to be reached in several $(ββ)_{0ν}$-decay experiments of a new generation. The NEMO3 experiment $[73]$ with $^{100}$M and $^{82}$Se, which began to take data in July 2002, and the cryogenics detector CUORICINO $[75]$, which uses $^{130}$Te and is already operative, are expected to reach a sensitivity to values of $|⟨m⟩| \sim 0.2 \text{ eV}$. The first preliminary results from these two experiments were published recently $[73, 74]$ and they respectively read

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(90% CL): $\langle |m| \rangle < (1.2-2.7) \text{ eV}$ and $\langle |m| \rangle < (0.7-1.7) \text{ eV}$. Up to an order of magnitude, better sensitivity, i.e. to $\langle |m| \rangle \approx 2.7 \times 10^{-2}, 1.5 \times 10^{-2}, 5.0 \times 10^{-2}, 2.5 \times 10^{-2}$ and $3.6 \times 10^{-2} \text{ eV}$, is expected to be achieved in the CUORE, GENIUS, EXO, MAJORANA and MOON experiments \cite{75} with $^{136}\text{Xe}$, $^{76}\text{Ge}$, $^{136}\text{Xe}$, $^{76}\text{Ge}$ and $^{100}\text{Mo}$, respectively. Additional high-sensitivity experiments with $^{136}\text{Xe}$ (XMASS \cite{77}) and $^{48}\text{Ca}$ (CANDLES \cite{78}) are also being considered.

As we will discuss in what follows, the studies of $(\beta\beta)_{0v}$ decay and the measurement of a nonzero value of $\langle |m| \rangle \gtrsim 10^{-2} \text{ eV}$:

- Can establish the Majorana nature of massive neutrinos. The $(\beta\beta)_{0v}$ decay experiments are presently the only feasible experiments capable of doing that (see \cite{10}).
- Can give information on the type of the neutrino mass spectrum \cite{21, 23, 82} (see also \cite{40, 79, 81}). More specifically, a measured value of $\langle |m| \rangle \gtrsim 10^{-2} \text{ eV}$ can provide, in particular, unique constraints on, or even can allow one to determine the type of the neutrino mass spectrum if $m_{1,2,3}$ are Majorana particles \cite{21, 23}.
- Can provide also unique information on the absolute scale of neutrino masses, or on the lightest neutrino mass (see e.g. \cite{21, 22, 79}).
- With additional information from other sources ($^3\text{H} \beta$-decay experiments or cosmological and astrophysical data and considerations) on the absolute neutrino mass scale, the $(\beta\beta)_{0v}$-decay experiments can provide unique information on the Majorana CP-violation phases $\alpha_{21}$ and $\alpha_{31}$ \cite{22, 24, 40, 42, 43}.

2. Predictions of the effective Majorana mass

The predicted value of $\langle |m| \rangle$ depends, for 3-ν mixing on \cite{83} (see also \cite{40, 79}): (i) $\Delta m^2_\odot$, (ii) $\theta_\odot$ and $\Delta m^2_{\text{sol}}$, (iii) the lightest neutrino mass and (iv) the mixing angle $\theta$. Using the convention (B), in which always $m_1 < m_2 < m_3$, one has $\Delta m^2_\odot \equiv \Delta m^2_{\text{sol}}$ and $m_3 = \sqrt{m_1^2 + \Delta m^2_{\text{atm}}}$, while either $\Delta m^2_\odot \equiv \Delta m^2_{\text{atm}}$ (normal mass hierarchy) or $\Delta m^2_\odot \equiv \Delta m^2_{32}$ (inverted mass hierarchy). In the first case, we have $m_2 = \sqrt{m_1^2 + \Delta m^2_{\odot}}$, $|U_{e1}|^2 = \cos^2 \theta_\odot (1 - |U_{e2}|^2)$, $|U_{e2}|^2 = \sin^2 \theta_\odot (1 - |U_{e3}|^2)$ and $|U_{e3}|^2 \equiv \sin^2 \theta$, whereas in the second case, $m_2 = \sqrt{m_1^2 + \Delta m^2_{\odot} - \Delta m^2_\text{atm}}$, $|U_{e1}|^2 = \cos^2 \theta_\odot (1 - |U_{e2}|^2)$, $|U_{e2}|^2 = \sin^2 \theta_\odot (1 - |U_{e1}|^2)$ and $|U_{e1}|^2 \equiv \sin^2 \theta$. The two possibilities for $\Delta m^2_\odot$ correspond also to the two different *hierarchical* types of neutrino mass spectrum—the NH, $m_1 \ll m_2 \ll m_3$, and the IH, $m_1 \ll m_2 \cong m_3$, respectively. Let us recall that, for the QD neutrino mass spectrum, we have $m_1 \cong m_2 \cong m_3$ and $m^2_{1,2,3} \gg \Delta m^2_\text{atm}$. For the allowed ranges of values of $\Delta m^2_\odot$ and $\Delta m^2_\text{atm}$ (see \cite{25}), the NH (IH) spectrum corresponds to $m_1 \lesssim 10^{-3} (2 \times 10^{-2}) \text{ eV}$, whereas one has a QD spectrum if $m_{1,2,3} \cong m_{\odot} > 0.20 \text{ eV}$. For $m_1$ lying in the interval between $\sim 10^{-3} (2 \times 10^{-2})$ and $0.20 \text{ eV}$, the neutrino mass spectrum is with partial normal (inverted) hierarchy (see e.g. \cite{40}).

Given $\Delta m^2_\odot$, $\Delta m^2_\text{atm}$, $\theta_\odot$ and $\sin^2 \theta$, the value of $\langle |m| \rangle$ depends strongly on the type of the neutrino mass spectrum as well as on the values of the two Majorana CP-violation phases of the PMNS matrix, $\alpha_{31}$ and $\alpha_{32}$ (see equation (15)). Note that for the QD spectrum, $m_1 \cong m_2 \cong m_3, m^2_{1,2,3} \gg \Delta m^2_\text{atm}, \Delta m^2_\odot$ and $\langle |m| \rangle$ is essentially independent of $\Delta m^2_\text{atm}$ and $\Delta m^2_\odot$.

\footnote{The quoted sensitivities correspond to values of the relevant NME from \cite{76}.}
Correspondingly, the two possibilities, $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$ and $\Delta m^2_{\odot} \equiv \Delta m^2_{32}$, lead effectively to the same predictions for $|\langle m \rangle|$.

### 2.1. NH neutrino mass spectrum

For the NH neutrino mass spectrum, one has

$$m_2 \cong \sqrt{\Delta m^2_{\odot}}, \quad m_3 \cong \sqrt{\Delta m^2_{\text{atm}}} \quad \text{and} \quad |U_{e3}|^2 \equiv \sin^2 \theta$$

and, correspondingly,

$$|\langle m \rangle| = \left| (m_1 \cos^2 \theta_{\odot} + e^{i\alpha_{21}} \sqrt{\Delta m^2_{\odot}} \sin^2 \theta_{\odot}) \cos^2 \theta + \sqrt{\Delta m^2_{A}} \sin^2 \theta e^{i\alpha_{31}} \right|$$

(21)

$$|\langle m \rangle| \lesssim \left| \sqrt{\Delta m^2_{\odot}} \sin^2 \theta_{\odot} \cos^2 \theta + \sqrt{\Delta m^2_{A}} \sin^2 \theta e^{i(\alpha_{31} - \alpha_{21})} \right|,$$

(22)

where we have neglected the term $\sim m_1$ in equation (22). In this case, one of the three massive Majorana neutrinos effectively ‘decouples’ and does not give a contribution to $|\langle m \rangle|$; however, the value of $|\langle m \rangle|$ still depends on the Majorana CP-violation phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$. This is a consequence of the fact that, in contrast with the case of massive Dirac neutrinos (or quarks), CP violation can take place in the mixing of only two massive Majorana neutrinos [33].

From equations (3) and (8)–(11), it follows that $\sqrt{\Delta m^2_{\odot}} \lesssim 10^{-2}$ eV, $\sin^2 \theta_{\odot} \lesssim 0.40$, $\sqrt{\Delta m^2_{A}} \lesssim 5.5 \times 10^{-2}$ eV (at 90% CL) and the largest neutrino mass enters into the expression for $|\langle m \rangle|$ with the factor $\sin^2 \theta < 0.05$. For these reasons, the predicted value of $|\langle m \rangle|$ is below $10^{-2}$ eV: for $\sin^2 \theta = 0.04$ (0.02), one finds $|\langle m \rangle| \lesssim 0.006$ (0.005) eV. Using the best-fit values of the indicated neutrino oscillation parameters, we get even smaller values for $|\langle m \rangle|$, $|\langle m \rangle| \lesssim 0.005$ (0.004) eV (see tables 1 and 2). Actually, from equation (21) and the allowed ranges of values of $\Delta m^2_{\odot}$, $\Delta m^2_{A}$, $\sin^2 \theta_{\odot}$, $\sin^2 \theta$ as well as of the lightest neutrino mass $m_1$ and the CP-violation phases $\alpha_{31}$ and $\alpha_{31}$, it follows that for the NH spectrum, there can be a complete cancellation between the contributions of the three terms in equation (21) and one can have $[22]|\langle m \rangle| = 0$.

### 2.2. IH spectrum

One has, for the IH neutrino mass spectrum (see e.g. [40]), $m_2 \cong m_3 \cong \sqrt{\Delta m^2_{A}}$, $|U_{e1}|^2 \equiv \sin^2 \theta$. Neglecting $m_1 \sin^2 \theta$ in equation (15), we find $[24, 40, 79]

$$|\langle m \rangle| \cong \sqrt{\Delta m^2_{A}} \cos^2 \theta \sqrt{1 - 2 \sin^2 \theta \sin^2 \left(\frac{1}{2} \alpha_{32}\right)}.$$ (23)

Even though one of the three massive Majorana neutrinos ‘decouples’, the value of $|\langle m \rangle|$ depends on the Majorana CP-violating phase $\alpha_{32} \equiv (\alpha_{31} - \alpha_{21})$. Obviously,

$$\sqrt{\Delta m^2_{A}} \cos^2 \theta |\cos 2\theta_{\odot}| \leq |\langle m \rangle| \leq \sqrt{\Delta m^2_{A}} \cos^2 \theta.$$

(24)

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9 This statement is valid, within the convention $m_1 < m_2 < m_3$ we are using, as long as there are no independent constraints on the CP-violating phases $\alpha_{21}$ and $\alpha_{31}$, which enter into the expression for $|\langle m \rangle|$. For the spectrum with normal hierarchy, $|\langle m \rangle|$ depends primarily on $\alpha_{21}$ ($|U_{e1}|^2 \ll 1$), whereas if the spectrum is with inverted hierarchy, $|\langle m \rangle|$ will depend essentially on $\alpha_{31} - \alpha_{21}$ ($|U_{e1}|^2 \ll 1$).
Table 1. The maximal values of $|\langle m \rangle|$ (in units of $10^{-3}$ eV) for the NH and IH spectra, and the minimal values of $|\langle m \rangle|$ (in units of $10^{-3}$ eV) for the IH and QD spectra, for the best-fit values of the oscillation parameters and $\sin^2 \theta = 0.0, 0.02$ and 0.04. The results for the NH and IH spectra are obtained for $\Delta m^2_{1}\text{atm} = 2.6 \times 10^{-3}$ eV$^2$ ($2.0 \times 10^{-3}$ eV$^2$ for values in parentheses) and $m_1 = 10^{-4}$ eV, whereas those for the QD spectrum correspond to $m_0 = 0.2$ eV (from [23]).

| $\sin^2 \theta$ | $|\langle m \rangle|_{\text{NH max}}$ | $|\langle m \rangle|_{\text{IH min}}$ | $|\langle m \rangle|_{\text{IH max}}$ | $|\langle m \rangle|_{\text{QD min}}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0             | 2.6 (2.6)       | 19.9 (17.3)     | 50.5 (44.2)     | 79.9            |
| 0.02            | 3.6 (3.5)       | 19.5 (17.0)     | 49.5 (43.3)     | 74.2            |
| 0.04            | 4.6 (4.3)       | 19.1 (16.6)     | 48.5 (42.4)     | 68.5            |

Table 2. Same as table 1, except for the 90% CL allowed values of $\Delta m^2_{1}\text{⊙}$ and $\theta_{\text{⊙}}$ obtained in [18], and of $\Delta m^2_{\text{atm}}$ given in equation (6) (equation (8) for results in parentheses) (from [23]).

| $\sin^2 \theta$ | $|\langle m \rangle|_{\text{NH max}}$ | $|\langle m \rangle|_{\text{IH min}}$ | $|\langle m \rangle|_{\text{IH max}}$ | $|\langle m \rangle|_{\text{QD min}}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0             | 3.7 (3.7)       | 10.1 (8.7)      | 56.3 (50.6)     | 47.9            |
| 0.02            | 4.7 (4.6)       | 9.9 (8.6)       | 55.1 (49.6)     | 42.8            |
| 0.04            | 5.5 (5.3)       | 11.4 (9.9)      | 54.0 (48.6)     | 45.4            |

The upper and lower limits correspond respectively to the CP-conserving cases $\alpha_{32} = 0$, or $\alpha_{21} = \alpha_{31} = 0, \pm \pi$, and $\alpha_{32} = \pm \pi$, or $\alpha_{21} = \alpha_{31} + \pi = 0, \pm \pi$. Most remarkably, since $\cos 2\theta_{\text{⊙}} \sim 0.40$ according to the solar neutrino and KamLAND data, we get a significant lower limit on $|\langle m \rangle|$, typically exceeding $10^{-2}$ eV, in this case [21, 22] (tables 1 and 2). Using, for example, the best-fit values of $\Delta m^2_{1}$ and $\tan^2 \theta_{\text{⊙}}$, one finds: $|\langle m \rangle| \gtrsim 0.017$ eV. The maximal value of $|\langle m \rangle|$ is determined by $\Delta m^2_{1}$ and can reach, as it follows from equations (3) and (4), $|\langle m \rangle| \sim 0.050–0.055$ eV. The indicated values of $|\langle m \rangle|$ are within the range of sensitivity of the next generation of $\beta\beta$-decay experiments.

The expression for $|\langle m \rangle|$, equation (23), permits us to relate the value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ to the experimentally measured quantities [24, 40] $|\langle m \rangle|$, $\Delta m^2_{\text{atm}}$ and $\sin^2 2\theta_{\text{⊙}}$:

$$\sin^2 \frac{\alpha_{31} - \alpha_{21}}{2} \approx \left( 1 - \frac{|\langle m \rangle|^2}{\Delta m^2_{1} \cos^4 \theta} \right) \frac{1}{\sin^2 2\theta_{\text{⊙}}} .$$

(25)

A more precise determination of $\Delta m^2_{1}$ and $\theta_{\text{⊙}}$ and a sufficiently accurate measurement of $|\langle m \rangle|$ could allow one to get information about the value of $(\alpha_{31} - \alpha_{21})$, provided the neutrino mass spectrum is of the IH type.

2.3. Three QD neutrinos

In this case, it is convenient to introduce $m_0 \equiv m_1 \equiv m_2 \equiv m_3$, $m_0^2 \gg \Delta m^2_{1}$, $m_0 \gtrsim 0.20$ eV. The mass $m_0$ effectively coincides with the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ measured in the $^3\text{H}$ $\beta$-decay experiments: $m_0 = m_{\bar{\nu}_e}$. Thus, $m_0 < 2.2$ eV, or if we use a conservative cosmological...
upper limit [63], $m_0 < 0.7$ eV. The QD neutrino mass spectrum is realized for values of $m_0$, which can be measured in the $^3$H $\beta$-decay experiment KATRIN.

The effective Majorana mass $|\langle m \rangle|$ is given by

$$|\langle m \rangle| \approx m_0 |\cos^2 \theta_\odot + \sin^2 \theta_\odot e^{i\alpha_{21}} \cos^2 \theta + e^{i\alpha_{31}} \sin^2 \theta|$$

(26)

$$|\langle m \rangle| \approx m_0 |\cos^2 \theta_\odot + \sin^2 \theta_\odot e^{i\alpha_{21}}| = m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \left(\frac{\alpha_{21}}{2}\right)}.$$  

(27)

Similar to the case of the IH spectrum, one has

$$m_0 |\cos 2\theta_\odot| \lesssim |\langle m \rangle| \lesssim m_0.$$  

(28)

For $\cos 2\theta_\odot \sim 0.40$, favoured by the solar neutrino and the KamLAND data, one finds a non-trivial lower limit on $|\langle m \rangle|$, $|\langle m \rangle| \gtrsim 0.07$ eV. For the 90% CL allowed ranges of values of the parameters, one has $|\langle m \rangle| \gtrsim 0.043$ eV (tables 1 and 2). Using the conservative cosmological upper bound on the sum of neutrino masses, we get $|\langle m \rangle| \lesssim 0.70$ eV. Also, in this case, one can obtain, in principle, direct information about one CP-violation phase from the measurement of $|\langle m \rangle|$, $m_0$ and $\sin^2 2\theta_\odot$:

$$\sin^2 \frac{\alpha_{21}}{2} \approx \left(1 - \frac{|\langle m \rangle|^2}{m_0^2}\right) \frac{1}{\sin^2 2\theta_\odot}.$$  

(29)

The specific features of the predictions of $|\langle m \rangle|$ for the three types of neutrino mass spectra discussed above are evident from figures 1 and 2, where the dependence of $|\langle m \rangle|$ on $m_1$ for the LMA solution is shown. For instance, if $\Delta m^2_\odot = \Delta m^2_{21}$, which corresponds to a spectrum with normal hierarchy, $|\langle m \rangle|$ can lie anywhere between 0 and the currently existing upper limits, given by equations (19) and (20). This conclusion does not change even under the most favourable conditions for the determination of $|\langle m \rangle|$, namely when $\Delta m^2_{\text{atm}}$, $\Delta m^2_\odot$, $\theta_\odot$ and $\theta$ are known with negligible uncertainty, as seen in figures 1 and 2.

3. Constraining the lightest neutrino mass

By observing the $(\beta\beta)_{0\nu}$ decay of a given nucleus, it would be possible to determine the value of $|\langle m \rangle|$ from the measurement of the associated lifetime of the decay. This would require knowledge of the NME of the process. At present, there exists large uncertainty in the calculation of $(\beta\beta)_{0\nu}$-decay NMEs (see e.g. [65, 84]). This is reflected, in particular, in the factor of $\sim 3$ uncertainty in the upper limit on $|\langle m \rangle|$, which is extracted from the experimental lower limits on the $(\beta\beta)_{0\nu}$-decay half lifetime of $^{76}$Ge. Recently, encouraging results on the problem of calculating the NMEs have been obtained in [85]. The observation of a $(\beta\beta)_{0\nu}$ decay of one nucleus would probably lead to searches and, eventually, to the observation of decay of other nuclei. One can expect that such a progress, in particular, will help to solve the problem of sufficiently precise calculation of the NMEs for the $(\beta\beta)_{0\nu}$ decay.

In this section, we consider briefly the information that future $(\beta\beta)_{0\nu}$ decay and/or $^3$H $\beta$-decay experiments can provide on the lightest neutrino mass $m_1$, without taking into account the possible effects of currently existing uncertainties in the evaluation of $(\beta\beta)_{0\nu}$-decay NMEs.

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Figure 1. The dependence of $|\langle m \rangle|$ on $m_1$ for the LMA-I solution, for $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\Delta m^2_{\odot} = \Delta m^2_{32}$, and for the best-fit values of the solar neutrino oscillation parameters found in [18] and of $\Delta m^2_{\text{atm}}$ in [27], and for three fixed values of $\sin^2 \theta = 0.0; 0.02; 0.04$ (top, middle and bottom panels). For CP-conservation, the allowed values of $|\langle m \rangle|$ are constrained to lie: for (i) $\Delta m^2_{\odot} = \Delta m^2_{21}$ and the middle and bottom panels (top panel)—(a) on the lower thick solid lines (on the lower thick solid line) if $\eta_{21} = \eta_{31} = 1$, (b) on the long-dashed lines (on the lower thick solid line) if $\eta_{21} = -\eta_{31} = 1$, (c) on the dash-dotted lines (on the dash-dotted lines) if $\eta_{21} = \eta_{31} = -1$; and for (ii) $\Delta m^2_{\odot} = \Delta m^2_{32}$ (all panels)—(a) on the upper thick solid line if $\eta_{21} = \eta_{31} = \pm 1$, (b) on the dotted lines if $\eta_{21} = -\eta_{31} = \pm 1$. Values of $|\langle m \rangle|$ in the black regions signal CP-violation (from [23]).
Figure 2. The dependence of $|\langle m \rangle|$ on $m_1$ for the LMA-I solution, for $\Delta m^2_\odot = \Delta m^2_{31}$ and $\Delta m^2_\odot = \Delta m^2_{32}$, and for the 90% CL allowed values of $\Delta m^2_\odot$ and $\sin^2 \theta_\odot$ found in [18] and of $\Delta m^2_{3\text{mix}}$ in [27], and for three values of $\sin^2 \theta = 0.0; 0.02; 0.04$ (top, middle and bottom panels). For CP conservation, the allowed values of $|\langle m \rangle|$ are constrained to lie: for (i) $\Delta m^2_\odot = \Delta m^2_{21}$ and the middle and bottom panels (top panel), in the medium-gray and light-gray regions (a) between the two lower thick solid lines if $\eta_{21} = \eta_{31} = 1$, (b) between the two long-dashed lines (between the two lower thick solid lines) if $\eta_{21} = -\eta_{31} = 1$, (c) between the three thick dash-dotted lines and the axes (between the dash-dotted lines and the axes) if $\eta_{21} = -\eta_{31} = -1$, (d) between the three thick short-dashed lines and the axes (between the dash-dotted lines and the axes) if $\eta_{21} = \eta_{31} = -1$; and for (ii) $\Delta m^2_\odot = \Delta m^2_{32}$ and the middle and bottom panels (top), in the light-grey regions (a) between the two upper thick solid lines if $\eta_{21} = \eta_{31} = \pm 1$, (b) between the dotted and the thin dash-dotted (the dotted and the thick short-dashed) lines if $\eta_{21} = -\eta_{31} = 1$, (c) between the dotted and the upper thick short-dashed (the dotted and the thick short-dashed) lines if $\eta_{21} = -\eta_{31} = -1$. For CP violation, the allowed regions for $|\langle m \rangle|$ cover all grey + black regions. Values of $|\langle m \rangle|$ in the black regions signal CP violation (from [23]).
An experimental upper limit on $|\langle m \rangle|$, $|\langle m \rangle| < |\langle m \rangle|_{\text{exp}}$, will determine a maximal value of $m_1$, $m_1 < (m_1)_{\text{max}}$ for the normal mass hierarchy, $\Delta m^2_\odot \equiv \Delta m^2_{21}$ (figures 1 and 2). For the QD spectrum, for instance, we have $m_1 \gg \Delta m^2_\odot$, $\Delta m^2_{21}$, and up to corrections $\sim \Delta m^2_\odot \sin^2 \theta_\odot/(2m^2_1)$ and $\sim \Delta m^2_\Lambda \sin^2 \theta/(2m^2_1)$, one finds \[ (m_1)_{\text{max}} \approx \frac{|\langle m \rangle|_{\text{exp}}}{|\cos 2\theta_\odot \cos^2 \theta - \sin^2 \theta|}. \] (30) We get similar results for inverted mass hierarchy, $\Delta m^2_\odot \equiv \Delta m^2_{23}$, provided the experimental upper limit $|\langle m \rangle|_{\text{exp}}$ is larger than the minimal value of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{min}}$ (figures 1 and 2), predicted by taking into account all uncertainties in the values of the relevant input parameters ($\Delta m^2_\Lambda$, $\Delta m^2_\odot$, $\theta_\odot$, etc.). If $|\langle m \rangle|_{\text{exp}} < |\langle m \rangle|_{\text{min}}$, then either (i) the neutrino mass spectrum is not of the inverted hierarchy type or (ii) there exist contributions to the $(\beta\beta)_{0v}$-decay rate other than due to the light Majorana neutrino exchange (see e.g. [87]) that partially cancel the contribution from the Majorana neutrino exchange. The indicated result might also suggest that the massive neutrinos are Dirac particles.

Measurement of $|\langle m \rangle| = (|\langle m \rangle|)_{\exp} \gtrsim 0.02 \text{ eV}$ if $\Delta m^2_\odot \equiv \Delta m^2_{21}$ and measurement of $|\langle m \rangle| = (|\langle m \rangle|)_{\exp} \gtrsim \sqrt{\Delta m^2_\Lambda} \cos^2 \theta$ if $\Delta m^2_\odot \equiv \Delta m^2_{32}$ would imply that $m_1 \gtrsim 0.02 \text{ eV}$ and 0.04 eV respectively and, thus, a neutrino mass spectrum with partial hierarchy or of the QD type [40] (figures 1 and 2). The lightest neutrino mass will be constrained to lie in a rather narrow interval, $(m_1)_{\text{min}} \lesssim m_1 \lesssim (m_1)_{\text{max}}$.\(^\dagger\) The limiting values of $m_1$ correspond to the case of CP conservation. For $\Delta m^2_\odot \ll m^2_1$, (i.e. for $\Delta m^2_\odot \lesssim 10^{-4} \text{ eV}^2$), as can be shown [22], we have $(m_1)_{\text{min}} \approx (|\langle m \rangle|)_{\exp}$ for $\Delta m^2_\odot \approx \Delta m^2_{21}$ and $\sqrt{(m_1)_{\text{min}}^2 + \Delta m^2_\Lambda} \approx (|\langle m \rangle|)_{\exp}$ for $\Delta m^2_\odot \approx \Delta m^2_{32}$.

A measured value of $|\langle m \rangle|$, satisfying $(|\langle m \rangle|)_{\exp} < (|\langle m \rangle|)_{\text{max}}$, where, e.g. for a QD spectrum, $(|\langle m \rangle|)_{\text{max}} \approx m_1 \approx m_{\tilde{\nu}}$, would imply that at least one of the two CP-violating phases is different from zero: $\alpha_{21} \neq 0$ or $\alpha_{31} \neq 0$.\(^\ddagger\)

If the measured value of $|\langle m \rangle|$ lies between the minimal and maximal values of $|\langle m \rangle|$, which are predicted for the IH spectrum,

$|\langle m \rangle|_{\pm} = \left| \sqrt{\Delta m^2_\Lambda - \Delta m^2_\odot} \cos^2 \theta_\odot \pm \sqrt{\Delta m^2_\Lambda \sin^2 \theta_\odot} \right| \cos^2 \theta$, \hspace{1cm} (31)

$m_1$ again would be found from above, but we would have $(m_1)_{\text{min}} = 0$ (figures 1 and 2).

A measured value of $m_{\tilde{\nu}}$, $(m_{\tilde{\nu}})_{\exp} \gtrsim 0.20 \text{ eV}$, satisfying $(m_{\tilde{\nu}})_{\exp} > (m_1)_{\text{max}}$, where $(m_1)_{\text{max}}$ is determined from the upper limit on $|\langle m \rangle|$ if the $(\beta\beta)_{0v}$ decay is not observed, might imply that the massive neutrinos are Dirac particles. If $(\beta\beta)_{0v}$ decay has been observed and $|\langle m \rangle|$ measured, the inequality $(m_{\tilde{\nu}})_{\exp} > (m_1)_{\text{max}}$, with $(m_1)_{\text{max}}$ determined from the measured value of $|\langle m \rangle|$, would lead to the conclusion that there exist contribution(s) to the $(\beta\beta)_{0v}$-decay rate other than from the light Majorana neutrino exchange (see e.g. [87] and references therein) that partially cancels the contribution from the Majorana neutrino exchange.

\(^\dagger\) Analytic expressions for $(m_1)_{\text{min}}$ and $(m_1)_{\text{max}}$ are given in [22].

\(^\ddagger\) Note that, in general, the knowledge of the value of $|\langle m \rangle|$ alone will not allow to distinguish the case of CP conservation from that of CP violation.
These can be derived from equations (21), (24) and (26) and correspond to CP-conserving \(0^2\) experiment [49].

The results for the NH and IH spectra are obtained for \(m_1 = 10^{-4}\) eV, whereas those for the QD spectrum correspond to \(m_0 = 0.2\) eV (from [23]).

### Table 3

| \(\sin^2 \theta\) | \(|\langle m \rangle|_{\text{NH}} \) | \(|\langle m \rangle|_{\text{IH}} \) | \(|\langle m \rangle|_{\text{QD}} \) |
|-------------------|---------------------|---------------------|---------------------|
| 0.0               | 2.8 (3.1) [2.8 (3.1)] | 17.8 (13.8) [15.5 (12.0)] | 53.0 (57.8) [46.4 (50.6)] |
| 0.02              | 3.8 (4.2) [3.7 (4.1)] | 17.4 (13.5) [15.2 (11.7)] | 52.0 (56.6) [45.5 (49.6)] |
| 0.04              | 4.8 (5.3) [4.6 (5.0)] | 17.0 (13.3) [14.8 (11.5)] | 50.9 (55.5) [44.5 (48.6)] |

### 4. Determining the type of the neutrino mass spectrum

The possibility of distinguishing between the three different types of neutrino mass spectrum, NH, IH and QD, depends on the allowed ranges of values of \(|\langle m \rangle|\) for the three spectra. More specifically, it is determined by the maximal values of \(|\langle m \rangle|\) for the NH and IH spectra, \(|\langle m \rangle|_{\text{NH}}\) and \(|\langle m \rangle|_{\text{IH}}\), and by the minimal values of \(|\langle m \rangle|\) for the IH and QD spectra, \(|\langle m \rangle|_{\text{IH}}\) and \(|\langle m \rangle|_{\text{QD}}\). These can be derived from equations (21), (24) and (26) and correspond to CP-conserving values of the Majorana phases [82] \(\alpha_{21}\) and \(\alpha_{31}\). The minimal value, \(|\langle m \rangle|_{\text{QD}}\), scales to a good approximation with \(m_0\) and, thus, is reached for \(m_0 = 0.2\) eV.

In tables 1 and 2 (taken from [23]), we show the values of (i) \(|\langle m \rangle|_{\text{NH}}\), (ii) \(|\langle m \rangle|_{\text{IH}}\) and (iii) \(|\langle m \rangle|_{\text{QD}}\) \((m_0 = 0.2\) eV), calculated for the best-fit and the 90% CL allowed ranges of values of \(\tan^2 \theta_{\odot}\) and \(\Delta m_{\odot}^2\) in the LMA solution region. In table 3 (from [23]), we give the same quantities, \(|\langle m \rangle|_{\text{max}}\), \(|\langle m \rangle|_{\text{min}}\) and \(|\langle m \rangle|_{\text{QD}}\), calculated using the best-fit values of the neutrino oscillation parameters, including 1 S.D. (3 S.D.) ‘prospective’ uncertainties of 5% (15%) on \(\tan^2 \theta_{\odot}\) and \(\Delta m_{\odot}^2\) and of 10% (30%) on \(\Delta m_{\odot}^2\). As evident from tables 1–3, the possibility of determining the type of the neutrino mass spectrum if \(|\langle m \rangle|\) is found to be nonzero in the \((\beta\beta)_{0v}\)-decay experiments of the next generation depends crucially on the precision with which \(\Delta m_{\odot}^2\), \(\theta_{\odot}\), \(\Delta m_{\odot}^2\), \(\sin^2 \theta\) and \(|\langle m \rangle|\) will be measured. It depends crucially also on the values of \(\theta_{\odot}\) and \(|\langle m \rangle|\). The precision itself in the measurement of \(|\langle m \rangle|\) in the next generation of \((\beta\beta)_{0v}\)-decay experiments, given the latter sensitivity limits of \((1.5–5.0) \times 10^{-2}\) eV, depends on the value of \(|\langle m \rangle|\). Precision in the measurements of \(\tan^2 \theta_{\odot}\) and \(\Delta m_{\odot}^2\), used to derive the numbers in table 3 can be achieved, e.g. in the solar neutrino experiments and/or in the experiments with reactor \(\nu\) [53, 88]. If \(\Delta m_{\odot}^2\) lies in the interval \((2.0–5.0) \times 10^{-3}\) eV, as is suggested by the current data [25, 28], its value will be determined with a \(\sim 10\%\) error (1 S.D.) by the MINOS experiment [49].

High-precision measurements of \(\Delta m_{\odot}^2\), \(\tan^2 \theta_{\odot}\) and \(\Delta m_{\odot}^2\) are expected to take place in the next \(\sim 6–7\) years. We will assume in what follows that the problem of measuring or tightly constraining \(\sin^2 \theta\) will also be resolved in the indicated period. Under these conditions, the highest uncertainty in the comparison of the theoretically predicted value of \(|\langle m \rangle|\) with

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\(^{12}\) See [43, 82] for further details concerning the calculation of the uncertainty in \(|\langle m \rangle|\).
that determined in the \((\beta\beta)_{0\nu}\)-decay experiments would be associated with the corresponding \((\beta\beta)_{0\nu}\)-decay NMEs. We will also assume in what follows that, by the time one or more \((\beta\beta)_{0\nu}\)-decay experiments of the next generation will be operative (2009–2010), at least the physical range of variation of the values of the relevant \((\beta\beta)_{0\nu}\)-decay NMEs will be unambiguously determined.

Following [43, 82], we will parametrize the uncertainty in \(|\langle m \rangle|\) resulting from imprecise knowledge of the relevant NMEs—we will use the term ‘theoretical uncertainty’ for the latter—through a parameter \(\zeta, \zeta \geq 1\), defined as

\[
|\langle m \rangle| = \zeta(|\langle m \rangle|_{\text{exp}})_{\min} \pm \Delta,
\]

where \(|\langle m \rangle|_{\text{exp}})_{\min}\) is the value of \(|\langle m \rangle|\) obtained from the measured \((\beta\beta)_{0\nu}\)-decay half lifetime of a given nucleus using the largest NME and \(\Delta\) is the experimental error. An experiment measuring a \((\beta\beta)_{0\nu}\)-decay half lifetime will thus determine a range of \(|\langle m \rangle|\) corresponding to \(\zeta(|\langle m \rangle|_{\text{exp}})_{\min} - \Delta \leq |\langle m \rangle| \leq \zeta(|\langle m \rangle|_{\text{exp}})_{\min} + \Delta\).

The currently estimated range of \(\zeta^2\) for experimentally interesting nuclei varies from 3.5 for \(^{48}\text{Ca}\) to 38.7 for \(^{130}\text{Te}\) (see e.g. table 2 in [65, 84]). For \(^{76}\text{Ge}\) and \(^{82}\text{Se}\), it is \(\sim 10\) [65]. The actual uncertainties can be smaller [85].

To be possible to distinguish between the NH and IH spectra, between the NH and QD spectra and between IH and QD spectra, the following inequalities must hold, respectively:

\[
\zeta |\langle m \rangle|_{\text{max}}^{\text{NH}} < |\langle m \rangle|_{\text{min}}^{\text{IH}},
\]

\[
\zeta |\langle m \rangle|_{\text{max}}^{\text{NH}} < |\langle m \rangle|_{\text{min}}^{\text{QD}},
\]

\[
\zeta |\langle m \rangle|_{\text{max}}^{\text{IH}} < |\langle m \rangle|_{\text{min}}^{\text{QD}}, \quad \zeta \geq 1.
\]

These conditions imply, as can be demonstrated [82], upper limits on \(\tan^2 \theta_\odot\) that are a function of neutrino oscillation parameters and of \(\zeta\).

In figure 3 (taken from [82]), the upper bounds on \(\tan^2 \theta_\odot\), for which one can distinguish the NH spectrum from the IH spectrum and from that of the QD type, are shown as a function of \(\Delta m_\odot^2\) for \(m^2_A = 3 \times 10^{-3} \text{eV}^2\), \(\sin^2 \theta = 0.05\) and 0.0 and different values of \(\zeta\). For the NH versus IH spectrum, results for \(\sin^2 \theta = 0.01\) are also shown. For the QD spectrum, values of \(m_0 = 0.2; 1.0 \text{eV}\) are used.

The dependence of the maximal value of \(\tan^2 \theta_\odot\) of interest on \(m_0\) and \(\sin^2 \theta\) in the NH versus QD case is rather weak as demonstrated in figure 3. This is not so in what concerns the dependence on \(\sin^2 \theta\) in the NH versus IH case: the maximal value of \(\tan^2 \theta_\odot\) under discussion can increase noticeably (e.g. by a factor of \(\sim 1.2-1.5\)) when \(\sin^2 \theta\) decreases from 0.05 to 0. As it follows from figure 3, it would be possible to distinguish between the NH and QD spectra for the values of \(\tan^2 \theta_\odot\) favoured by the data for values of \(\zeta \simeq 3\) or even somewhat higher. In contrast, the possibility of distinguishing between the NH and IH spectra for \(\zeta \simeq 3\) depends critically on the value of \(\sin^2 \theta\): as figure 3 indicates, this would be possible for the current best-fit value of \(\tan^2 \theta_\odot\) and, for instance, \(\Delta m_\odot^2 = (5.0-15) \times 10^{-5} \text{eV}^2\), provided \(\sin^2 \theta \lesssim 0.01\).

In figure 4 (taken from [82]), we show the maximal value of \(\tan^2 \theta_\odot\) permitting one to distinguish between the IH and QD spectra as a function of \(\Delta m_\odot^2\), for \(\sin^2 \theta = 0.05\) and 0.0,
Figure 3. The upper bound on $\tan^2 \theta_\odot$, for which one can distinguish the NH spectrum from the IH spectrum and from that of QD type, as a function of $\Delta m^2_\odot$ for $\Delta m^2_\odot = 3 \times 10^{-3}$ eV$^2$ and different values of $\zeta$. The lower (upper) line corresponds to $\sin^2 \theta = 0.05 \ (0)$. For NH versus IH, there is a third (middle) line corresponding to $\sin^2 \theta = 0.01$ (from [82]).

$\Delta m^2_\odot = 7.0 \times 10^{-5}$ eV$^2$, $m_0 = 0.2, 0.5, 1.0$ eV and $\zeta = 1.0, 1.5, 2.0, 3.0$. The upper bound on $\tan^2 \theta_\odot$ of interest depends strongly on the value of $m_0$. It decreases with increase in $\Delta m^2_\odot$. As follows from figure 4, for the values of $\Delta m^2_\odot$ favoured by the data and for $\zeta \gtrsim 2$, distinguishing between the IH and QD spectra for $m_0 \approx 0.20$ eV requires too small, from the point of view of the existing data, values of $\tan^2 \theta_\odot$. For $m_0 \gtrsim 0.40$ eV, the values of $\tan^2 \theta_\odot$ of interest fall in the ranges favoured by the solar neutrino and KamLAND data even for $\zeta = 3$. 

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Figure 4. The upper bound on \( \tan^2 \theta_\odot \) allowing one to discriminate between the IH and QD neutrino mass spectra, as a function of \( \Delta m^2_\alpha \) for different values of \( \zeta \). The lower (upper) line corresponds to \( \sin^2 \theta = 0.05 \) (0) (from [82]).

These quantitative analyses show that if \(|\langle m \rangle|\) is found to be nonzero in future (\(\beta\beta\))\(_{0v}\)-decay experiments, it would be easier, in general, to distinguish between the spectrum with NH and those with IH or of QD type using the data on \(|\langle m \rangle|\) \(\neq 0\) than to distinguish between the IH and QD spectra. Discriminating between the latter would be less demanding if \(m_0\) is sufficiently large.
5. Constraining the Majorana CP-violation phases

The problem of detection of CP violation in the lepton sector is one of the most formidable and challenging problems in the study of neutrino mixing. As was noticed in [22], the measurement of $|\langle m \rangle|$ alone could exclude the possibility that the two Majorana CP-violation phases $\alpha_{21}$ and $\alpha_{31}$, present in the PMNS matrix, are equal to zero. However, such a measurement cannot rule out, without additional input, the case of the two phases taking different CP-conserving values. The additional input needed for establishing CP violation could be, for example, measurement of neutrino mass $m_{\bar{\nu}e}$ in the $3^H\beta$-decay experiment KATRIN [59] or cosmological determination of the sum of the three neutrino masses [62], $\Sigma = m_1 + m_2 + m_3$ or derivation of a sufficiently stringent upper limit on $\Sigma$. At present, no viable alternative to the measurement of $|\langle m \rangle|$ for obtaining information on the Majorana CP-violating phases $\alpha_{21}$ and $\alpha_{31}$ exists or can be foreseen to exist in the next ~8 years.

The possibility of obtaining information on the CP violation due to the Majorana phases $\alpha_{21}$ and $\alpha_{31}$ by measuring $|\langle m \rangle|$ was studied by a number of authors [24, 40, 41, 42, 44], and more recently, it was studied in [43, 45]. Barger et al [45] considered in their analysis, in particular, the effect of uncertainty in the knowledge of NMEs on the measured value of $|\langle m \rangle|$. After making a certain number of assumptions about the experimental and theoretical developments in the field of interest that may occur by the year 2020, they claim to have shown ‘once and for all that it is impossible to detect CP violation from $(\beta\beta)_{0\nu}$ decay in the foreseeable future’. A different approach to the problem was used in [43], where an attempt was made to determine the conditions under which CP violation might be detected from a measurement of $|\langle m \rangle|$ and $m_{\bar{\nu}e}$ or $\Sigma$, or of $|\langle m \rangle|$ and a sufficiently stringent upper limit $\Sigma$. We will summarize below results of the latter study.

The analysis in [43] is based on prospective input data on $|\langle m \rangle|$, $m_{\bar{\nu}e}$, $\Sigma$, $\tan^2 \theta_\odot$, etc. The effect of the NME uncertainty was included in the analysis. For example, for the IH spectrum ($m_1 \ll m_2 \simeq m_3$, $m_1 < 0.02$ eV), a ‘just-CP-violating’ region [40]—a value of $|\langle m \rangle|$ in this region would signal unambiguously CP violation in the lepton sector due to Majorana CP-violating phases—would be present if

$$|\langle m \rangle|_{\text{exp}}^{\text{max}} < \sqrt{\left(\Delta m_{\text{atm}}^2\right)_{\text{min}}},$$

$$|\langle m \rangle|_{\text{exp}}^{\text{min}} > \sqrt{\left(\Delta m_{\text{atm}}^2\right)_{\text{max}} (\cos 2\theta_\odot)_{\text{max}}},$$

where $|\langle m \rangle|_{\text{exp}}^{\text{max}}$ is the largest (smallest) experimentally allowed value of $|\langle m \rangle|$, taking into account both the experimental error on the measured $(\beta\beta)_{0\nu}$ decay half lifetime and the uncertainty due to the evaluation of the NMEs. Condition (38) depends crucially on the value of $(\cos 2\theta_\odot)_{\text{max}}$ and it is less stringent for smaller values of $(\cos 2\theta_\odot)_{\text{max}}$ [22].

Using the parametrization given in equation (32), the necessary condition permitting us to establish, in principle, that the CP symmetry is violated due to the Majorana CP-violating phases reads

$$1 \leq \zeta < \frac{\sqrt{\left(\Delta m_{\text{atm}}^2\right)_{\text{min}}}}{\sqrt{\left(\Delta m_{\text{atm}}^2\right)_{\text{max}} (\cos 2\theta_\odot)_{\text{max}} + 2\Delta}}.$$
Obviously, the smaller the \((\cos 2\theta)_{\text{max}}\) and \(\Delta\) values, the larger the ‘theoretical uncertainty’, which might allow one to make conclusions concerning the CP-violation of interest.

A similar analysis was performed for QD neutrinos mass spectrum. The results can be summarized as follows. The possibility of establishing that the Majorana phases \(\alpha_{21}\) and \(\alpha_{31}\) have CP-nonconserving values requires quite accurate measurements of \(|\langle m\rangle|\) and, e.g. of \(m_{\bar{\nu}_e}\) or \(\Sigma\), and holds only for a limited range of values of the relevant parameters. More specifically, to prove that CP violation associated with Majorana neutrinos takes place requires, in particular, a relative experimental error on the measured value of \(|\langle m\rangle|\) not higher than 15–20\%, a ‘theoretical uncertainty’ in the value of \(|\langle m\rangle|\) due to imprecise knowledge of the corresponding NMEs smaller than a factor of 2, a value of \(\tan^2 \theta_{\odot} \gtrsim 0.55\), and values of the relevant Majorana CP-violating phases \((\alpha_{21}, \alpha_{32})\) typically within the ranges of \(\sim (\pi/2–3\pi/4)\) and \(\sim (5\pi/4–3\pi/2)\).

6. Conclusions

Future \((\beta\beta)_{0\nu}\)-decay experiments have a remarkable physics potential. They can establish the Majorana nature of the neutrinos with a definite mass \(\nu_j\). If the latter are Majorana particles, the \((\beta\beta)_{0\nu}\)-decay experiments can provide unique information on the type of the neutrino mass spectrum and on the absolute scale of neutrino masses. They can also provide unique information on the Majorana CP-violation phases present in the PMNS neutrino mixing matrix. Knowledge of values of the relevant \((\beta\beta)_{0\nu}\)-decay NMEs with a sufficiently low uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of \((\beta\beta)_{0\nu}\)-decay half lifetime.

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Notes added. After the submission of this article for publication, the KamLAND experiment reported new data [89] on the spectrum of \(e^+\) produced by reactor \(\tilde{\nu}_e: \tilde{\nu}_e + p \rightarrow e^+ + n\). The data correspond to a statistics of \(766.3\) Ty and clearly show a distortion of the \(e^+\) spectrum, compatible with that expected for \(\tilde{\nu}_e\) oscillations, driven by the neutrino mass-squared difference responsible for the solar neutrino transitions, \(\Delta m_{\odot}^2\), and lying in the LMA-I subregion of the LMA solution region. A combined analysis of the global solar neutrino and KamLAND 766 Ty data in terms of neutrino oscillations reveals [91] that the LMA-II solution is excluded at \(4\sigma\). A three-flavour neutrinos oscillation analysis of the data shows [91] that \(\sin^2 \theta < 0.05\) at 99.73\% CL and that, for \(\sin^2 \theta = 0.0 (0.02)\), for instance, one has, at 90\% CL, \(0.23 \lesssim \sin^2 \theta_{\odot} \lesssim 0.34\) and \(7.5 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m_{\odot}^2 \lesssim 9.2 (9.0) \times 10^{-5} \text{ eV}^2\). The corresponding best-fit values read: \(\sin^2 \theta_{\odot} \approx 0.28\) and \(\Delta m_{\odot}^2 \approx 8.4 \times 10^{-5} \text{ eV}^2\). We see that the maximal allowed values of \(\sin^2 \theta_{\odot}\) are smaller than those in equations (8) and (9), whereas the best-fit and the minimal values of \(\Delta m_{\odot}^2\) are larger than those used (equations (10) and (11)). The former implies that the minimal
values of $|\langle m \rangle|$ for IH and QD neutrino mass spectra are somewhat larger than those shown in figures 1 and 2; the latter means that the predictions for $|\langle m \rangle|$ in the case of the NH neutrino mass spectrum, illustrated in figures 1 and 2, will be somewhat modified. However, taking into account implications of the new KamLAND data will lead, in general, to minor modifications of the results on the effective Majorana mass $|\langle m \rangle|$ and the physics potential of the $(\beta\beta)_{0\nu}$-decay experiments, presented in this paper.

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