The scientific field of traffic engineering encompasses a rich set of mathematical techniques, as well as researchers with entirely different backgrounds. This paper provides an overview of what is currently the state-of-the-art with respect to traffic flow theory. Starting with a brief history, we introduce the microscopic and macroscopic characteristics of vehicular traffic flows. Moving on, we review some performance indicators that allow us to assess the quality of traffic operations. A final part of this paper discusses some of the relations between traffic flow characteristics, i.e., the fundamental diagrams, and sheds some light on the different points of view adopted by the traffic engineering community.

PACS numbers: 02.50.-r, 45.70.Vn, 89.40.-a

Keywords: xxx

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The scientific field of traffic engineering encompasses a rich set of mathematical techniques, as well as researchers with entirely different backgrounds. This paper provides an overview of what is currently the state-of-the-art with respect to traffic flow theory. Starting with a brief history, we introduce the microscopic and macroscopic characteristics of vehicular traffic flows. Moving on, we review some performance indicators that allow us to assess the quality of traffic operations. A final part of this paper discusses some of the relations between traffic flow characteristics, i.e., the fundamental diagrams, and sheds some light on the different points of view adopted by the traffic engineering community.

Because of the large diversity of the scientific field (en-
I. A BRIEF HISTORY OF TRAFFIC FLOW THEORY

Historically, traffic engineering got its roots as a rather practical discipline, entailing most of the time a common sense of its practitioners to solve particular traffic problems. However, all this changed at the dawn of the 1950s, when the scientific field began to mature, attracting engineers from all sorts of trades. Most notably, John Glen Wardrop instigated the evolving discipline now known as traffic flow theory, by describing traffic flows using mathematical and statistical ideas \cite{115}.

During this highly active period, mathematics established itself as a solid basis for theoretical analyses, a phenomenon that was entirely new to the previous, more ‘rule-of-thumb’ oriented, line of reasoning. Two examples of the progress during this decade, include the fluid-dynamic model of Michael James Lighthill, Gerald Beresford Whitham, and Paul Richards (or the LWR model for short) for describing traffic flows \cite{72,102}, and the car-following experiments and theories of the club of people working at General Motors’ research laboratory \cite{17,40,41,50}. Simultaneous progress was also made on the front of economic theory applied to transportation, most notably by the publication of the ‘BMW trio’, Martin J. Beckmann, Charles Bartlett McGuire, and Christopher B. Winsten \cite{6}.

From the 1960s on, the field evolved even further with the advent of the early personal computers (although at that time, they could only be considered as mere computing units). More control-oriented methods were pursued by engineers, as a means for alleviating congestion at tunnels and intersections, by e.g., adaptively steering traffic signal timings. Nowadays, the field has been kindly embraced by the industry, resulting in what is called intelligent transportation systems (ITS), covering nearly all aspects of the transportation community.

In spite of the intense booming during the 1950s and 1960s, all progress seemingly came to sudden stop, as there were almost no significant results for the next two decades (although there are some exceptions, such as the significant work of Ilya Prigogine and Robert Herman’s, who developed a traffic flow model based on a gas-kinetic analogy \cite{101}). One of the main reasons for this, stems from the fact that many of the involved key players returned to their original scientific disciplines, after exhausting the application of their techniques to the transportation problem \cite{92}. Note that despite this calm period, the application of control theory to transportation started finding new ways to alleviate local congestion problems.

At the beginning of the 1990s, researchers found a revived interest in the field of traffic flow modelling. On the one hand, researchers’ interests got kindled again by the appealing simplicity of the LWR model, whereas on the other hand one of the main boosts came from the world of statistical physics. In this latter framework, physicists tried to model many particle systems using simple and elegant behavioural rules. As an example, the now famous particle hopping (cellular automata) model of Kai Nagel and Michael Schreckenberg \cite{92} still forms a widely-cited basis for current research papers on the subject.

In parallel with this kind of modelling approach, many of the old ‘beliefs’ (e.g., the fluid-dynamic approach to traffic flow modelling) started to get questioned. As a consequence, a plethora of models quickly found its way to the transportation community, whereby most of these models didn’t give a thought as to whether or not their associated phenomena corresponded to real-life traffic observations.

We note here that, whatever the modelling approach may be, researchers should always compare their results to the reality of the physical world. Ignoring this basic step, reduces the research in our opinion to nothing more than a mathematical exercise!

As the international research community began to spawn its traffic flow theories, Robert Herman aspired to bring them all together in december 1959. This led to the tri-annual organisation of the International Symposium on Transportation and Traffic Theory (ISTTT), by some heralded as ‘the Olympics of traffic theory’ because the symposium talks about the fundamentals underlying transportation and traffic phenomena. Another example of the evolution of recent developments with respect to the parallels between traffic flows and granular media, is the bi-annual organisation of the workshop on Traffic and Granular Flow (TGF), a platform for exchanging ideas by bringing together researchers from various scientific fields.

Nowadays, the research and application of traffic flow theory and intelligent transportation systems continues. The scientific field has been largely diversified, encompassing a broad range of aspects related to sociology, psychology, the environment, the economy, … The global avidity of the field can be witnessed by the exponentially growing publication output. Keeping our previous comment in mind, researchers from time to time just seem to ‘add to the noise’ (mainly due to the sheer diversity of the literature body), although there occasionally exist exceptions such as the late Newell, as subtly pointed out by Michael Cassidy in \cite{100}.

As a final word, we refer the reader to two personalised views on the history of traffic flow theory, namely the musings of the late Gordon Newell and Denos Gazis \cite{42,99}. We furthermore invite the reader to cast a glance at the ending pages of Wardrop’s paper \cite{115}, in which a rather colourful discussion on the introduction of mathematics to traffic flow theory has been written down.
II. MICROSCOPIC TRAFFIC FLOW CHARACTERISTICS

Road traffic flows are composed of drivers associated with individual vehicles, each of them having their own characteristics. These characteristics are called microscop ic when a traffic flow is considered as being composed of such a stream of vehicles. The dynamical aspects of these traffic flows are formed by the underlying interactions between the drivers of the vehicles. This is largely determined by the behaviour of each driver, as well as the physical characteristics of the vehicles.

Because the process of participating in a traffic flow is heavily based on the behavioural aspects associated with human drivers [39], it would seem important to include these human factors into the modelling equations. However, this leads to a severe increase in complexity, which is not always a desired artifact [76]. However, in the remainder of this section, we always consider a vehicle-driver combination as a single entity, taking only into account some vehicle related traffic flow characteristics.

Note that despite our previous remarks, we do not debate the necessity of a psychological treatment of traffic flow theory. As the research into driver behaviour is gaining momentum, a lot of attention is gained by promising studies aimed towards driver and pedestrian safety, average reaction times, the influence of stress levels, aural and visual perceptions, ageing, medical conditions, fatigue, . . .

A. Vehicle related variables

Considering individual vehicles, we can say that each vehicle $i$ in a lane of a traffic stream has the following informational variables:

- a length, denoted by $l_i$,
- a longitudinal position, denoted by $x_i$,
- a speed, denoted by $v_i = \frac{dx_i}{dt}$,
- and an acceleration, denoted by $a_i = \frac{dv_i}{dt} = \frac{d^2x_i}{dt^2}$.

Note that the position $x_i$ of a vehicle is typically taken to be the position of its rear bumper. In this first approach, a vehicle’s other spatial characteristics (i.e., its width, height, and lane number) are neglected. And in spite of our narrow focus on the vehicle itself, the above list of variables is also complemented with a driver’s reaction time, denoted by $\tau_i$.

With respect to the acceleration characteristics, it should be noted that these are in fact not only dependent on the vehicle’s engine, but also on e.g., the road’s inclination, being a non-negligible factor that plays an important role in the forming of congestion at bridges and tunnels. We do not use the derivative of the acceleration, called jerk, jolt, or surge (jerk is also used to represent the smoothness of the acceleration noise [52]).

Except in the acceleration capabilities of a vehicle, we ignore the physical forces that act on a vehicle, e.g., the earth’s gravitational pull, road and wind friction, centrifugal forces, . . . A more elaborate explanation of these forces can be found in [27].

B. Traffic flow characteristics

Referring to Fig. 1, we can consider two consecutive vehicles in the same lane in a traffic stream: a follower $i$ and its leader $i + 1$. From the figure, it can be seen that vehicle $i$ has a certain space headway $h_{s_i}$ to its predecessor (it is expressed in metres), composed of the distance (called the space gap) $g_{s_i}$ to this leader and its own length $l_i$:

$$h_{s_i} = g_{s_i} + l_i.$$  

By taking, as stated before, the rear bumper as a vehicle’s position, the space headway $h_{s_i} = x_{i+1} - x_i$. The space gap is thus measured from a vehicle’s front bumper to its leader’s rear bumper.

![Figure 1](image)

**FIG. 1:** Two consecutive vehicles (a follower $i$ at position $x_i$ and a leader $i + 1$ at position $x_{i+1}$) in the same lane in a traffic stream. The follower has a certain space headway $h_{s_i}$ to its leader, equal to the sum of the vehicle’s space gap $g_{s_i}$ and its length $l_i$.

Analogously to equation (1), each vehicle also has a time headway $h_{t_i}$, (expressed in seconds), consisting of a time gap $g_{t_i}$ and an occupancy time $\rho_i$:

$$h_{t_i} = g_{t_i} + \rho_i.$$  

Both space and time headways can be visualised in a time-space diagram, such as the one in Fig. 2. Here, we have shown the two vehicles $i$ and $i + 1$ as they are driving. Their positions $x_i$ and $x_{i+1}$ can be plotted with respect to time, tracing out two vehicle trajectories. As the time direction is horizontal and the space direction is vertical, the vehicles’ respective speeds can be derived by taking the tangents of the trajectories (for simplicity, we have assumed that both vehicles travel at the same constant speed, resulting in parallel linear trajectories).
Accelerating vehicles have steep inclining trajectories, whereas those of stopped vehicles are horizontal.

When the vehicle’s speed is constant, the time gap is the amount of time necessary to reach the current position of the leader when travelling at the current speed (i.e., it is the elapsed time an observer at a fixed location would measure between the passing of two consecutive vehicles). Similarly, the occupancy time can be interpreted as the time needed to traverse a distance equal to the vehicle’s own length at the current speed, i.e., $\rho_i = l_i / v_i$; this corresponds to the time the vehicle needs to pass the observer’s location. Both equations (1) and (2) are furthermore linked to the vehicle’s speed $v_i$ as follows [74]:

$$\frac{h_{s_i}}{h_{t_i}} = \frac{g_{s_i}}{g_{t_i}} = \frac{l_i}{\rho_i} = v_i. \quad (3)$$

As the above definitions deal with what is called single-lane traffic, we can easily extend them to multi-lane traffic. In this case, four extra space gaps — related to the vehicles in the neighbouring lanes — are introduced, namely $g_{s_i}^{l}$ at the left-front, $g_{s_i}^{r}$ at the left-back, $g_{s_i}^{r,b}$ at the right-back. The four corresponding space headways, $h_{s_i}^{l,f}$, $h_{s_i}^{l}$, $h_{s_i}^{r,f}$, and $h_{s_i}^{r,b}$, are introduced in a similar fashion. The extra time gaps and headways are derived in complete analogy, leading to the four time gaps $g_{t_i}^{l,f}$, $g_{t_i}^{l}$, $g_{t_i}^{r,f}$, and $g_{t_i}^{r,b}$, and the four corresponding time headways $h_{t_i}^{r,f}$, $h_{t_i}^{r}$, $h_{t_i}^{r,f}$, and $h_{t_i}^{r,b}$.

In single-lane traffic, vehicles always keep their relative order, a principle sometimes called first-in, first-out (FIFO) [24]. For multi-lane traffic however, this principle is no longer obeyed due to overtaking manoeuvres, resulting in vehicle trajectories that cross each other. If the same time-space diagram were to be drawn for only one lane (in multi-lane traffic), then some vehicles’ trajectories would suddenly appear or vanish at the point where a lane change occurred.

In some traffic flow literature, other nomenclature is used: space for the space gap, distance or clearance for the space gap, and headway for the time headway. Because this terminology is confusing, we propose to use the unambiguously defined terms as described in this section.

III. MACROSCOPIC TRAFFIC FLOW CHARACTERISTICS

When considering many vehicles simultaneously, the time-space diagram mentioned in section II.B can be used to faithfully represent all traffic. In Fig. 3 we show the evolution of the system, as we have traced the trajectories of all the individual vehicles’ movements. This time-space diagram therefore provides a complete picture of all traffic operations that are taking place (accelerations, decelerations, . . .).

![FIG. 2: A time-space diagram showing two vehicle trajectories $i$ and $i+1$, as well as the space and time headway $h_{s_i}$ and $h_{t_i}$ of vehicle $i$. Both headways are composed of the space gap $g_{s_i}$ and the vehicle length $l_i$, and the time gap $g_{t_i}$ and the occupancy time $\rho_i$, respectively. The time headway can be seen as the difference in time instants between the passing of both vehicles, respectively at $t_{i+1}$ and $t_i$ (diagram based on [74]).](image)

![FIG. 3: A time-space diagram showing several vehicle trajectories and three measurement regions $R_t$, $R_s$, and $R_{t,s}$. These rectangular regions are bounded in time and space by a measurement period $T_{mp}$ and a road section of length $K$. The black dots represent the individual measurements.](image)
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mean speed. We furthermore give a short discussion on
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some caution is advised: a too large measurement region
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Using these different methods of observation, we now
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discuss the measurement of four important macroscopic
traffic characteristics: density, flow, occupancy, and
mean speed. We furthermore give a short discussion on
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data.

A. Density

The macroscopic characteristic called density allows us
to get an idea of how crowded a certain section of a road
is. It is typically expressed in number of vehicles per
kilometre (or mile). Note that the concept of density to-
tally ignores the effects of traffic composition and vehicle
lengths, as it only considers the abstract quantity ‘number
of vehicles’.

Because density can only be measured in a certain spatial
region (e.g., $R_s$ in Fig. 3), it is computed for temporal
regions such as region $R_t$ in Fig. 3. When density can not be
exactly measured or computed, or when density mea-
surements are faulty, it has to be estimated. To this end,
several available techniques exist e.g., based on explicit
simulation using a traffic flow propagation model [24],
based on a traffic flow model [21], based on a complete traffic
state estimator using an extended Kalman filter [114], or based on a
non-linear adaptive observer [3]. . .

Using the spatial region $R_s$, the density $k$ for single-lane
traffic is defined as:

$$ k = \frac{N}{K}, \quad (4) $$

with $N$ the number of vehicles present on the road seg-
ment. If we consider multi-lane traffic, we have to sum
the partial densities $k_l$ of each of the $L$ lanes as follows:

$$ k = \sum_{i=1}^{L} k_l = \frac{1}{K} \sum_{i=1}^{L} N_i, \quad (5) $$

in which $N_i$ now denotes the number of vehicles present
in lane $l$ (equation 5 is not the same as averaging over
the partial densities of each lane $\frac{N}{K}$).

In general, density can be defined as the total time spent
by all the vehicles in the measurement region, divided by
the area of this region $\frac{R}{s}$. This generalisation allows us to
compute the density at a point using the mea-
surement region $R_t$:

$$ k = \frac{\sum_{i=1}^{N} T_i}{T_{mp}} = \frac{1}{T_{mp}} \sum_{i=1}^{N} \frac{d\xi}{v_i} = \frac{1}{T_{mp}} \sum_{i=1}^{N} \frac{1}{v_i}, \quad (6) $$

with $T_i$ the travel time and $v_i$ the speed of the $i^{th}$ vehicle.

Extending the previous derivation to multi-lane traffic is
done straightforward using equation 5:

$$ k = \frac{1}{T_{mp}} \sum_{i=1}^{L} N_i \frac{1}{v_i}, \quad (7) $$

with now $v_i$ denoting the speed of the $i^{th}$ vehicle in lane
$l$.

As we now can obtain the density in both spatial and tem-
poral regions, $R_s$ and $R_t$ respectively, it would seem a
logical extension to find the density in the region $R_{t,s}$.
In order to do this, however, we need to know the travel
times $T_i$ of the individual vehicles, as can be seen in equa-
tion 6. Because this information is not always available,
and in most cases rather difficult to measure, we use a dif-
ferent approach, corresponding to the temporal average
of the density. Assuming that at each time step $t$, during
a certain time period $T_{mp}$, the density $k(t)$ is known in
consecutive regions $R_s$, the generalised definition leads to
the following formulation:

$$ k = \begin{cases} \frac{1}{T_{mp}} \int_{t=0}^{T_{mp}} k(t) \, dt & \text{(continuous),} \\ \frac{1}{T_{mp}} \sum_{t=1}^{T_{mp}} k(t) & \text{(discrete).} \end{cases} \quad (8) $$
For multi-lane traffic, combining equations \ref{eq:5} and \ref{eq:8} results in the following formula for computing the density in region $R_{t,s}$ using measurements in discrete time:

$$k = \frac{1}{T_{mp}} \frac{1}{K} \sum_{i=1}^{L} \sum_{l=1}^{S} N_i(t), \quad (9)$$

where $N_i(t)$ denotes the number of vehicles present in lane $l$ at time $t$.

There exists a relation between the macroscopic traffic flow characteristics and those microscopic characteristics defined in section \ref{sec:B}. For the density $k$, this relation is based on the average space headway $\bar{h}_s$ \cite{27,115}:

$$k = \frac{N}{K} = \frac{N}{\sum_{i=1}^{S} h_{si}} \frac{1}{\frac{1}{N} \sum_{i=1}^{S} h_{si}} = \frac{1}{\bar{h}_s}, \quad (10)$$

with $\bar{h}_s^{-1}$ the reciprocal of the average space headway.

2. Passenger car units

When considering heterogeneous traffic flows (i.e., traffic streams composed of different types of vehicles), operating agencies usually don’t express the macroscopic traffic flow characteristics using the raw number of vehicles, but rather employ the notion of passenger car units (PCU). These PCUs, sometimes also called passenger car equivalents (PCE), try to take into account the spatial differences between vehicle types. For example, by denoting one average passenger car as 1 PCU, a truck in the same traffic stream can be considered as 2 PCUs (or even higher and fractional values for trailer trucks).

Let us finally note that, because density is essentially defined as a spatial measurement, it is one of the most difficult quantities to obtain. It is interesting to notice that at this moment, it is theoretically possible for video cameras to measure density over a short spatial region. However, to our knowledge there currently exists no commercial implementation.

B. Flow

Whereas density typically is a spatial measurement, flow can be considered as a temporal measurement (i.e., region $R_t$). Flow, which we use as a shorthand for rate of flow, is typically expressed as an hourly rate, i.e., in number of vehicles per hour. Note that sometimes other synonyms such as intensity, flux, throughput, current, or volume\cite{123} are used, typically depending on a person’s scientific background (e.g., engineering, physics, …).

Measuring the flow $q$ in region $R_t$ for single-lane traffic, is done using the following equation, which is based on raw vehicle counts:

$$q = \frac{N}{T_{mp}}, \quad (11)$$

with $N$ the number of vehicles that has passed the detector’s site. For multi-lane traffic, we sum the partial flows of each of the $L$ lanes:

$$q = \sum_{l=1}^{L} q_l = \frac{1}{T_{mp}} \sum_{l=1}^{L} N_l, \quad (12)$$

with now $N_l$ denoting the number of vehicles that passed the detector’s site in lane $l$. Note that we assume that each lane has its own detector, otherwise we would be dealing with an average flow across all the lanes.

Generally speaking, flow can defined as the total distance travelled by all the vehicles in the measurement region, divided by the area of this region \cite{27,56}. In analogy with equation \ref{eq:6}, this generalisation allows us to compute the flow using the spatial measurement region $R_s$:

$$q = \sum_{i=1}^{N} X_i = \frac{1}{K} \frac{1}{\int dt} \sum_{i=1}^{N} v_i \Delta t = \frac{1}{K} \sum_{i=1}^{N} v_i, \quad (13)$$

with now $X_i$ the distance travelled by the $i$th vehicle during the infinitesimal time interval $dt$. The extension to multi-lane traffic is straightforward:

$$q = \frac{1}{K} \sum_{l=1}^{L} \sum_{i=1}^{N_l} v_{i,l}. \quad (14)$$

Considering consecutive flow measurements in region $R_{t,s}$, we can derive a formulation corresponding to the temporal average of the flow, similar to that of equation \ref{eq:3}. Assuming that at each time step $t$, during a certain time period $T_{mp}$, the flow $q(t)$ is known in consecutive regions $R_s$, the generalised definition leads to the following equations:

$$q = \begin{cases} \frac{1}{T_{mp}} \int_{t=0}^{T_{mp}} q(t) \, dt \quad \text{(continuous)}, \\ \frac{1}{T_{mp}} \sum_{t=1}^{T_{mp}} q(t) \quad \text{(discrete)}, \end{cases} \quad (15)$$
For multi-lane traffic, combining equations (14) and (15) results in the following formula for computing the flow in region $R_{t,s}$ using measurements in discrete time:

$$q = \frac{1}{T_{mp}} \frac{T_{mv}}{K} \sum_{i=1}^{L} \sum_{l=1}^{L} N_{i,l}(t),$$  

(16)

where $v_{i,l}(t)$ denotes the speed of the $i^{th}$ vehicle in lane $l$ at time $t$.

In analogy with equation (10), there exists a relation between the flow $q$, and the average time headway $\bar{h}_t$ with $t$:

$$q = \frac{N}{T_{mp}} = \frac{N}{\sum_{i=1}^{N} h_{i,t}} = \frac{1}{\bar{h}_t},$$  

(17)

with $\bar{h}_t^{-1}$ the reciprocal of the average time headway.

2. Oblique cumulative plots

As stated before, flows are always expressed as a rate. In contrast to this, we can also consider the raw vehicle counts at a certain location (i.e., measurement region $R_t$). If we plot the cumulative number of passing vehicles (denoted by $N$) with respect to time for different regions (e.g., inductive loop detectors), we get a set of curves such as the one in the left part of Fig. 4. These curves are called cumulative plots (or $(t,N)$ diagrams), and although their origins date back as far as 1954 with the work of Karl Moskowitz [25], it was Gordon Newell who applied them later on to their full potential (initially in the context of queueing theory) [95, 96, 97, 98] (a similar method was applied by John Luke, in the field of continuum mechanics [23, 75]).

The key benefit of these cumulative plots, comes when comparing observations stemming from multiple detector stations at a closed section of the road that conserves the number of vehicles (i.e., no on- or off-ramps), in which case we also speak of input-output diagrams. If there are two detector stations, then the upstream and downstream stations measure the input, respectively output, of the section. Similarly like in queueing theory, the upstream curve is sometimes called the arrival function, whereas the downstream one is called the departure function [95].

As the method is based on counting the number of individual vehicles at each observation location (whereby each vehicle is numbered with respect to a single reference vehicle), this results in a monotonically increasing function $N(t)$ (sometimes called the Moskowitz function, after its ‘inventor’), which increases each time a vehicles passes by. At each time instant $t$, the cumulative count is defined as:

$$N(t) = \sum_{t'=0}^{t} q(t') = N(t-1) + q(t).$$  

(18)

The time needed to travel from one location to another can easily be measured as the horizontal distance between the respective cumulative curves. Similarly, the vertical distance between these curves allows us to derive the accumulation of vehicles on the road section, which
gives an excellent indication of growing and dissipating queues (i.e., congestion). Furthermore, if we compute the slope of this function at each time instant \( t \), we obtain the flow \( q(t) = [N(t + \Delta t) - N(t)]/\Delta t \). Finally, because \( N(t) \) essentially is a step function, we can define a smooth approximation \( \tilde{N}(t) \). This results in an everywhere differentiable function, allowing us to compute instantaneous flows and local densities as \( q = \partial \tilde{N}(t, x)/\partial t \) and \( k = -\partial \tilde{N}(t, x)/\partial x \), respectively [27].

The main disadvantage of the method is the fact that these cumulative functions increase very rapidly, thereby masking the subtle differences between different curves. Cassidy and Windover therefore proposed to subtract a background flow \( q_b \), from these curves, resulting in functions \( N(t) - t q_b \) [13]. Based on this; Muñoz and Daganzo furthermore introduced enhanced clarity by overlaying this cumulative plot with a set of oblique lines with slope \(-q_b\) [88]. Choosing an appropriate value for \( q_b \), allows us to nicely enhance the characteristic undulations that are expressed by the different oblique curves.

An example of an oblique plot can be seen in the right part of Fig. 4; the cumulative count at each time instant can be read from an axis that is perpendicular to the oblique (slanted) overlayed dashed lines (e.g., we can see a count of some 30000 vehicles at 14:00). Note that the accumulation can still be measured by the vertical distance between two curves (i.e., at a specific time instant), but the travel time should now be measured along one of the overlayed oblique lines. Such a pair of cumulative curves can be thought of as a flexible plastic garden hose: whenever there is an obstruction on the road, the outflow of the section will be blocked, resulting in a local thickening of this ‘hose’ (i.e., the accumulation of vehicles on the section).

Using these oblique cumulative plots, we can now inspect the traffic dynamics in much more detail than was previously possible. For example, looking again at the right part of Fig. 4, we can see how the specific traffic stream characteristics propagate from one detector station to another. Even more visible, is a queue that starts to grow at approximately 11:00 (i.e., the time of the appearance of a “bulge”), dissipating at approximately 12:30. As data curves from upstream detectors lie above data curves from downstream detectors, we see a decrease in the road section’s output. Careful investigation of the traffic data revealed that the detector stations recorded a rather low flow (approximately 2500 vehicles per hour as opposed to a nominal flow of 4500 vehicles per hour), whereby all vehicles drove at a low speed (between 20 and 60 km/h as opposed to 110 km/h). This gives sufficient evidence to conclude that an incident probably occurred shortly after 11:00, consequently obstructing a part of the road and leading to a build up of vehicles in the section.

Let us finally note that although oblique cumulative plots currently are not a mainstream technique used by the traffic community, we predict their rising popularity: they are one of the most simple, yet powerful, techniques for studying local traffic phenomena, giving traffic engineers practical insight into the formation of bottlenecks. Some recent examples include the work of Muñoz and Daganzo [85, 86, 87, 89], Cassidy and Bertini [7, 15], Cassidy and Mauch [16], and Bertini et al. [8].

### C. Occupancy

Notwithstanding the importance of measuring traffic density, most of the existing detector stations on the road are only capable of temporal measurements (i.e., region \( R_i \)). If individual vehicle speeds can be measured, by double inductive loop detectors (DLD) for example, then density should be computed using equation (6).

However, in many cases these vehicle speeds are not readily available, e.g., when using single inductive loop detectors. The detector’s logic therefore resorts to a temporal measurement called the occupancy \( \rho \), which corresponds to the fraction of time the measurement location was occupied by a vehicle:

$$\rho = \frac{1}{T_{mp}} \sum_{i=1}^{N} \theta_{ti},$$  \hspace{1cm} (19)

In the previous equation, \( \theta_{ti} \) denotes the \( i^{th} \) vehicle’s on-time, i.e., the time period during which it is present above the detector (it corresponds to the shaded area swept by a vehicle at a certain location \( x_j \) in Fig. 2). Note that this on-time actually corresponds to the effective vehicle length as seen by the detector, divided by the vehicle’s speed [20]:

$$\theta_{ti} = \frac{l_i + K_i}{v_i},$$  \hspace{1cm} (20)
with \( l_i \) the vehicle’s true length and \( K_{ld} > dx \) the finite, non-infinitesimal length of the detection zone. If we define \( \bar{\tau}_t \) as the average on-time (based on the vehicles that have passed the detector during the observation period), then we can establish a relation between the occupancy and the flow [27] using equations (11) and (19):

\[
\rho = \left( \frac{N}{T_{\text{rep}}} \right) \left( \frac{1}{N} \sum_{i=1}^{N} \bar{\tau}_t \right) = q \bar{\tau}_t. \tag{21}
\]

Furthermore, it is as before possible to define the occupancy for generalised measurement regions, using the total space consumed by the shaded areas of vehicles in a time-space diagram (e.g., Fig. 4), divided by the area of the measurement region [14, 27]. Continuing our discussion, assume that individual vehicle lengths and speeds are uncorrelated; it can then be shown that [27]:

\[
\rho = \bar{L} k \Rightarrow k = \frac{\rho}{\bar{L}}, \tag{22}
\]

with \( \bar{L} \) the average vehicle length (note that this can correspond to the concept of passenger car units defined in section III A). Multiplying equation (22) by 100, allows us to express the occupancy as a percentage. For multi-lane traffic, the occupancy is derived in analogy to equation (2):

\[
\rho = \sum_{i=1}^{L} \rho_i = \frac{1}{T_{\text{rep}}} \sum_{i=1}^{L} \sum_{t=1}^{N_i} \bar{\tau}_{t,i}, \tag{23}
\]

with now \( \bar{\tau}_{t,i} \) the on-time of the \( i^{th} \) vehicle in lane \( l \). Note that the total occupancy derived in this way, can exceed 1 (but is bounded by \( \bar{L} \)); if desired, it can be normalised through a division by \( \bar{L} \) to obtain the average occupancy.

Note that if we apply equation (22) to measurement region \( R_s \) based on the density in equation (1), then the occupancy \( \rho \) can be written as:

\[
\rho = \left( \frac{1}{N} \sum_{i=1}^{N} l_i \right) \frac{\sum_{i=1}^{N} l_i}{K} = \frac{1}{K} \sum_{i=1}^{N} l_i. \tag{24}
\]

So the occupancy now represents the ‘real density’ of the road, i.e., the physical space that all vehicles occupy.

In the past, density was sometimes referred to as concentration. Nowadays however, concentration is used in a more broad context, encompassing both density and occupancy whereby the former is meant to be a spatial measurement, as opposed to the latter which is considered to be a temporal measurement [39].

D. Mean speed

The final macroscopic characteristic to be considered, is the mean speed of a traffic stream; it is expressed in kilometres (or miles) per hour (the inverse of a vehicle’s speed is called its pace). Note that speed is not to be confused with velocity; the latter is actually a vector, implying a direction, whereas the former could be regarded as the norm of this vector.

I. Mathematical formulation

If we base our approach on direct measurements of the individual vehicles’ speeds, we can generally obtain the mean speed as the total distance travelled by all the vehicles in the measurement region, divided by the total time spent in this region [27, 36]. This gives the following derivations for the spatial and temporal regions, \( R_s \) and \( R_t \) respectively:

\[
\bar{v}_s = \frac{\sum_{i=1}^{N} X_i}{N} = \left\{ \begin{array}{l}
\frac{\sum_{i=1}^{N} v_i dx_i}{N dx_i} = \frac{1}{N} \sum_{i=1}^{N} v_i \quad \text{(region } R_s), \\
\frac{\sum_{i=1}^{N} dx_i}{N v_i} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{v_i} \quad \text{(region } R_t) \end{array} \right. \tag{25}
\]

with now \( X_i \) and \( T_i \) the distance, respectively time, travelled by the \( i^{th} \) vehicle and \( N \) the number of vehicles present during the measurement. The mean speed computed by the previous equations, is called the average travel speed (the computation also includes stopped vehicles), which is more commonly known as the space-mean speed (SMS); we denote it with \( \bar{v}_s \) (note that in some engineering disciplines, the sole letter \( u \) is used to denote a mean speed, however, this is ambiguous in our opinion).

It is interesting to see that the spatial measurement is based on an arithmetic average of the vehicles’ instantaneous speeds, whereas the temporal measurement is based on the harmonic average of the vehicles’ spot speeds. If we instead were to take the arithmetic average of the vehicles’ spot speeds in the temporal measurement region \( R_t \), this would lead to what is called the time-mean speed (TMS); we denote it by \( \bar{v}_t \):

\[
\bar{v}_t = \frac{1}{N} \sum_{i=1}^{N} v_i \quad \text{(region } R_t). \tag{26}
\]

Similarly, we can compute the time-mean speed for measurement region \( R_s \), by taking the harmonic average of the vehicles’ instantaneous speeds. With respect to both
space- and time-mean speeds. Wardrop has shown that the following relation holds [113]:

$$\bar{v}_t = \bar{v}_s + \sigma_s^2, \tag{27}$$

with $\sigma_s^2$ the statistical sample variance defined as follows:

$$\sigma_s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (v_i - \bar{v}_s)^2, \tag{28}$$

in which $v_i$ denotes the $i$th vehicle’s instantaneous speed. One of the main consequences of equation (27), is that the time-mean speed always exceeds the space-mean speed (except when all the vehicles’ speeds are the same, in which case the sample variance is zero and, as a consequence, the time- and space-mean speeds are equal). So a stationary observer will most likely see more faster than slower vehicles passing by, as opposed to e.g., an aerial photograph in which more slower than faster vehicles will be seen [27]. Despite this mathematical quirk, the practical difference between SMS and TMS is often negligible for free-flow traffic (i.e., light traffic conditions); however, under congested traffic conditions both mean speeds will behave substantially differently (i.e., around 10%).

Using equation (27), we can also estimate the space-mean speed, based on the time-mean speed and approximating the variance of the SMS with that of the TMS [9]:

$$\bar{v}_s = \bar{v}_t - \frac{\sigma_s^2}{\bar{v}_s},$$

$$\bar{v}_s \approx \bar{v}_t - \frac{\sigma_t^2}{\bar{v}_s},$$

$$\bar{v}_s - \bar{v}_t \approx -\frac{\sigma_t^2}{\bar{v}_s},$$

$$\bar{v}_s^2 - \bar{v}_t \bar{v}_s \approx -\sigma_t^2,$$

$$\bar{v}_s^2 - 2 \bar{v}_s \bar{v}_t + \frac{\bar{v}_t^2}{4} \approx \frac{\bar{v}_t^2}{4} - \sigma_t^2,$$

$$\left(\bar{v}_s - \bar{v}_t\right)^2 \approx \frac{\bar{v}_t^2}{4} - \sigma_t^2,$$

$$\bar{v}_s \approx \frac{\bar{v}_t}{2} + \sqrt{\frac{\bar{v}_t^2}{4} - \sigma_t^2} \quad \forall \bar{v}_t \geq 2 \bar{v}_s \tag{29}$$

In general, using the space-mean speed is preferred to the time-mean speed. However, in most cases only this latter traffic flow characteristic is available, so care should be taken when interpreting the results of a study (unless of course when SMS and TMS are negligibly different).

The extension of equation (26) to multi-lane is straightforward; for example, the space-mean speed is computed as follows:

$$\bar{v}_s = \frac{\sum l \sum i \bar{v}_{i,l} \cdot N_l}{\sum l N_l} \tag{30}$$

with now $v_{i,l}$ the instantaneous (or spot) speed of the $i$th vehicle in lane $l$.

2. Fundamental relation of traffic flow theory

There exists a unique relation between three of the previously discussed macroscopic traffic flow characteristics: density $k$, flow $q$, and space-mean speed $\bar{v}_s$ [113]:

$$q = k \bar{v}_s. \tag{31}$$

This relation is also called the fundamental relation of traffic flow theory, as it provides a close bond between the three quantities: knowing two of them allows us to calculate the third one (note that the time-mean speed in equation (26) does not obey this relation). In general however, there are two restrictions, i.e., the relation is only valid for (1) continuous variables [124], or smooth approximations of them, and (2) traffic composed of substreams (e.g., slow and fast vehicles) which comply to the following two assumptions:

**Homogeneous traffic**

There is a homogeneous composition of the traffic substream (i.e., the same type of vehicles).

**Stationary traffic**

When observing the traffic substream at different times and locations, it ‘looks the same’. Putting it a bit more quantitatively, all the vehicles’ trajectories should be parallel and equidistant [27]. A stationary time period can be seen in a cumulative plot (e.g., Fig. 4) where the curve corresponds to a linear function.

The latter of the above two conditions, is also referred to as traffic operating in a steady state or at equilibrium. Based on equations (5) and (12) using partial densities and flows for different substreams (e.g., vehicle classes with distinct travel speeds, macroscopic characteristics of different lanes, . . . ), we can now calculate the space-mean speed, using relation (31), in the following equivalent ways:
\[ \bar{v}_s = q / k, \]
\[ = \frac{\sum_{c=1}^{C} q_c}{\sum_{c=1}^{C} k_c}, \]
\[ = \frac{\sum_{c=1}^{C} q_c}{\sum_{c=1}^{C} \frac{q_c}{v_{s_c}}}, \]
\[ = \frac{\sum_{c=1}^{C} k_c \bar{v}_{s_c}}{\sum_{c=1}^{C} k_c}, \]

in which \( C \) denotes the number of substreams, \( q_c, k_c, \bar{v}_{s_c}, \) and \( \bar{v}_{s_c} \) the flow, density, space, and time-mean speed respectively of the \( c \)-th substream. In the above derivations, equation (32) should be used when both the flows and densities are known, equation (33) should be used when both the flows and space-mean speeds are known, and equation (34) should be used when both the densities and space-mean speeds are known.

As can be seen in equation (34), the space-mean speed is calculated by averaging the substreams’ space-mean speeds using their densities as weighting factors. Similarly, the time-mean speed can be derived by using the flows as weighting factors for the substreams’ time-mean speeds:

\[ \bar{v}_t = \frac{\sum_{c=1}^{C} q_c \bar{v}_{t_c}}{\sum_{c=1}^{C} q_c}, \]

Because density can not always be easily measured, we can compute it using the fundamental relation (31). Density can then be directly derived from flow and space-mean speed measurements, or if the latter are not available, they can be estimated from occupancy measurements; in [20, 21, 22]. Coifman provides a nice set of techniques for dealing with these difficulties.

### E. Moving observer method and floating car data

When measuring and/or computing the macroscopic traffic flow characteristics in the previous sections, we always assumed a fixed measurement region. There exists however yet another method, based on what is called a moving observer [112]. The idea behind the technique is to have a vehicle drive in both directions of a traffic flow, each time recording the number of oncoming vehicles and the net number of vehicles it gets overtaken by, as well as the times necessary to complete the two trips. Note that the assumption of stationary traffic still has to hold, i.e., the round trip should be completed before traffic conditions change significantly.

Using this method, it is then possible to derive the flow and density of the traffic stream in the direction of interest [27, 59]. However, the main disadvantage of this method is that, in order to obtain an acceptable level of accuracy on a road with a low flow, a very large number of trips are required [39, 84, 110].

One of the techniques that has entered the picture during the last decade, is the use of so-called floating cars or probe vehicles. They can be compared to the moving observer method, but in this case, the vehicles are equipped with GPS and GSM(C)/GPRS devices that determine their locations based on the USA’s NAVSTAR-GPS (or Europe’s planned GNSS Galileo), and transmit this information to some operator. Initially, this allows an agency, e.g., a parcel delivery service, to track its vehicles throughout a network, based on their locations. Nowadays, the technique has evolved, resulting in several completed field tests of which the main goal was to estimate the traffic conditions based on a small number of probe vehicles. During field measurements, floating cars can mimic several types of behaviour, most notably by traveling at the traffic flows’ mean speed, or by trying to travel at the road’s speed limit, or even by chasing another randomly selected vehicle from the traffic stream.

Some examples of studies and experiments with floating car data (FCD) are given in the following. Firstly, Fastenrath gives an overview of a telematic field trial (Vehicle Relayed Dynamic Information, VERDI) that addresses issues such as economical, political, and technical constraints [38]. Secondly, Westerman provides an overview of available techniques for obtaining real-time road traffic information, with the goal of controlling the traffic flows through telematics [113], and Wermuth et al. describe a ‘TeleTravel System’ used for surveying individual travel behaviour [117]. Then, Taale et al. compare travel times from floating car data with measured travel times (using a fleet of sixty equipped vehicles driving around in Rotterdam, The Netherlands), concluding that they correspond reasonably well [107]. Next, Michler derives the minimum percentage of vehicles necessary, in order to estimate traffic stream characteristics for certain traffic patterns (e.g., free-flow and congested traffic) based on rigid statistical grounds [81], and Linauer and Lehrs measure the travel time between points in a road network, based on a high number of users that submit a low number of GSM hand-over messages [73]. In addition, Demir et al. accurately reconstruct link travel times during periods of traffic congestion, using only a very limited number of FCD-messages with a small number of users [33]. A final, more regional, example is the founding of the government-supported Belgian ‘Telematics Cluster’ [125], a platform for encouraging the use of telematics solutions for ITS. The initiative already includes some 57 members, stimulating the cooperation between users, telecommunication companies, and the automotive industry.

In conclusion, we can state that the use of probe vehicles provides an effective way to gather accurate current travel times in a road network, thereby allowing good up-to-date estimations of traffic conditions. The technique will continue to grow and evolve, already by introducing personalised traffic information to drivers, based on their location and the surrounding traffic conditions. This development is furthermore stimulated by the fact that GSM market penetration still rises above 70% [73], and
Despite the obvious major advantage of obtaining accurate information on the traffic conditions, the technique suffers from a jurisprudential battle, in that there are many delicately privacy concerns involved with respect to the mobile operator that wants to track individual people’s units (not to mention the monetary cost associated with the numerous induced communications).

IV. PERFORMANCE INDICATORS

After considering the previously mentioned macroscopic traffic flow characteristics, we now take a look at some popular performance indicators used by traffic engineers when assessing the quality of traffic operations. We concisely discuss the peak hour factor, the reliability of travel times, the levels of service, and a measure of efficiency of a road. For a more complete overview, we refer the reader to [105].

A. Peak hour factor

During high flow periods in the peak hour, a possible indicator for traffic flow fluctuations is the so-called peak hour factor (PHF). It is calculated for one day as the average flow during the hour with the maximum flow, divided by the peak flow rate during one quarter hour within this hour [79]:

\[
PHF = \frac{\bar{q}_{60}}{\bar{q}_{15}}
\]

(36)

For example, suppose we measure flows on a main unidirectional road with three lanes, during a morning peak: from 07:00 to 08:00 we measure consecutively 3500, 6600, 6200, and 4500 vehicles/hour during each quarter. The total average flow \( \bar{q}_{60} \) is 5200 vehicles/hour, with a peak 15 minute flow rate \( \bar{q}_{15} \) = 6600 vehicles/hour. The PHF is therefore equal to 5200/6600 = 0.78.

Note that some manuals express the peak 15 minute flow rate as the number of vehicles during that quarter hour, necessitating an extra multiplication by 4 in the denominator of equation [36] to convert the flow rate to an hourly rate.

We can immediately see that the PHF is constrained to the interval [0.25,1.00]; the higher the PHF, the flatter the peak period (i.e., a longer sustained state of high flow). Typically, the PHF has values around 0.7 – 0.98. Note that two of the obvious problems with the PHF are, on the one hand, the question of when to pick the correct 15 minute interval, and on the other hand the fact that some peak periods may last longer than one hour.

When travelling around, people like to know how long a specific journey will take (e.g., by public transport, car, bicycle, …). This notion of an expected travel time is one of the most tangible aspects of journeying as perceived by the travellers. When people are travelling to their work, they are required to arrive on time at their destinations. Based on this premise, we can naturally state that people reason with a built-in safety margin: they consider the average time it takes to reach a destination, and use this to decide about their departure time.

Aside from the above obvious human rationale, there is also an increased interest in obtaining precise information with respect to travel times in the context of advanced traveller information systems (ATIS). Here, an essential ingredient is the accurate prediction of future travel times. Coupled with incident detection for example, drivers can obtain correct travel time information, thereby staying informed of the actual traffic conditions and possibly changing their journey. The requested information can reach the driver by means of a cell-phone (e.g., as a feature offered by the mobile service provider), it can be broadcasted over radio (e.g., the Traffic Message Channel – TMC), or it can be displayed using variable message signs (VMS) above certain road sections (e.g., dynamic route information panels – DRIPs), …

B. Travel times and their reliability

The travel time of a driver completing a journey, can be defined as ‘the time necessary to traverse a route between any two points of interest’ [111]. In this context, the experienced dynamic travel time, starting at a certain time \( t_0 \) over a road section of length \( K \) is defined as follows [9]:

\[
T(t_0) = \int_0^K \frac{1}{v(t,x)} dx \quad \forall \ t \geq t_0,
\]

(37)

for which it is assumed that all local instantaneous vehicle speeds \( v(t,x) \) are known at all points along the route, and at all time instants (hence the term dynamic travel time). In most cases however, we do not know all the \( v(t,x) \), but only a finite subset of them, defined by the locations of the detector stations (demarcating section boundaries). The travel time can then be approximated using the recorded speeds at the beginning and end of a section (there is an underlying assumption here, namely that vehicles travel at a more or less constant speed between detector locations). As stated earlier, the experienced travel time requires the knowledge of local vehicle speeds at all time instants after \( T_0 \). Because this is not always possible, a simplification can be used, resulting in the so-called experienced instantaneous travel time:

\[
\overline{T}(t_0) = \int_0^K \frac{1}{v(t_0,x)} dx,
\]

(38)
In general, we can derive the travel time using equation (35), i.e., the total distance travelled by all the vehicles, divided by their space-mean speed:

\[ T(t_0) = \frac{K}{\bar{v}_s(t_0)} \]  

(39)

in which an accurate estimation of the space-mean speed \( \bar{v}_s(t_0) \) at time \( t_0 \) is necessary (e.g., by taking the harmonic average of the recorded spot speeds).

2. Queueing delays

Traffic congestion nearly always leads to the build up of queues, introducing an increase (i.e., the delay) in the experienced travel time. The congestion itself can have originated due to traffic demand exceeding the capacity, or because an incident occurred (e.g., road works, a traffic accident, ...). This can create incidental (non-recurrent) or structural (recurrent) congestion. Congestion can thus be seen as a loss in travel time with respect to some base line reference. Two such commonly used references are the travel time under free-flow conditions, and the travel time under maximum (i.e., capacity) flow. The delay is typically expressed in vehicle hours. As stated earlier, there are several ways to inform a driver of the current and predicted travel time. Using DRIPs it is possible to advertise the extra travel time (the delay) than a spatial estimation (i.e., the travel time increases to some 22 minutes (e.g., due to an occurring incident)).

Another method for measuring the travel time, is based on historical data, namely cumulative plots (introduced in section III B 2). As mentioned earlier, the travel time can then be measured as the distance along the horizontal (or oblique) time axis; any excess due to delays can then easily be spotted on a set of oblique cumulative plots.

Based on cumulative plots of consecutive detector stations, we can calculate the travel time between the upstream and downstream end of a road section. To illustrate this, let us reconsider the cumulative curves shown in Fig. 4 of section III B 2. The evolution of the travel time during the day for these curves, is depicted in the top part of Fig. 5. The derived histogram (indicative of the underlying travel time probability density function), in the bottom part of the figure, shows that the mean travel time during the day is approximately 4 minutes.

```
FIG. 5: Top: The evolution of the travel time during one day, based on the cumulative plots from section III B 2. As can be seen, an incident likely occurred at 11:00, increasing the travel time from 4 to 7 minutes. Furthermore, at approximately 18:45 in the evening, all traffic seemed to simultaneously slow down for a period of some 10 minutes. Bottom: Based on the calculated travel times during the day, we can derive a histogram that is an approximation of the underlying travel time probability density function. The mean is located around 4 minutes.
```

We already mentioned the likely occurrence of an incident at 11:00, resulting in the formation of a queue. During this period, the travel time shot up, reaching first 5, then 7 minutes. Looking at the top part of Fig. 5 we furthermore notice a slight increase in the travel time at approximately 18:45, for a short period of some 10 minutes. Investigation of the detector data, revealed that the flow remained constant at about 4500 vehicles per hour, but the speed dropped to some 90 km/h (as opposed to 110 km/h); we can conclude that all vehicles were probably simultaneously slowing down during this period (perhaps a rubbernecking effect). Another possibility is a platoon of slower moving vehicles, but then it would seem to have dissipated rather quickly after 10 minutes.

Using ample historical data, we can analyse the travel time over a period of many weeks, months, or even years. This would allow us to make intuitive statements such as:

“The typical travel time over this section of the road during a working Monday, lies approximately between 4 and 6 minutes. There is however an 8% probability that the travel time increases to some 22 minutes (e.g., due to an occurring incident).”

Finally note that, besides the two previously mentioned techniques for estimating travel times, an extensive overview can be found in the Travel Time Data Collection Handbook [111]. Another concise but more theoretically-oriented overview is provided by Bovy and Thijs [9].
As mentioned in the introduction of this section, people reason about their expected travel times based on a built-in safety margin. Central to this is the concept of the average travel time. The reliability of such a travel time is then characterised by its standard deviation. Drivers typically accept (and sometimes expect) a small delay in their expected travel time. A traveller knows the expected travel time because of the familiarity with the associated trip. To the traveller, this is personal historical information, for instance obtained by learning the trip’s details (e.g., the traffic conditions during a typical morning rush hour) [5].

Directly linked to the reliability of a certain expected travel time, is its variability. They are said to be unreliable when both expected and experienced travel times differ sufficiently. A typical characterisation of reliability involves the mean and standard deviation (i.e., the variance, which is a measure of variability) of a travel time distribution [19]. An example of such a travel time distribution for one day is shown in the histogram in the bottom part of Fig. 5.

Both first- and second-order measures of a distribution are by themselves insufficient to capture the complete picture. In order to grasp the notion of the previously mentioned safety margin, another typical statistical measure is considered, namely the 90th percentile. The rationale behind the use of this percentile is that travellers adopt a certain ‘safe’ threshold with respect to their expected journey times. Considering the 90th percentile, this means that only one out of ten times the experienced travel time will differ significantly from the expected travel time. Travel time reliability can thus be viewed upon as a measure of service quality (similar to the concept of ‘quality of service’ (QoS) in telecommunications).

There has been some research into the analytic form of travel time distributions (e.g., the work of Arroyo and Kornhauser, concluding that a lognormal distribution seems the most appropriate [3]). There exist however significant differences between travel time distributions: in general, a smaller standard deviation indicates a better service quality and reliability. In contrast to this, a large standard deviation is indicative of chaotic behaviour of the traffic flow, the latter being totally unstable. Furthermore, travel time distributions can have a long tail; this signifies seldom events (e.g., incidents), that can have significant repercussions on the quality of traffic operations.

Historically, one of the main performance indicators to assess the quality of traffic operations was the level of service (LOS), introduced in the 1960s. It is represented as a grading system using one of six letters (A – F), whereby LOS A denotes the best operating conditions and LOS F the worst. These LOS measures are based on road characteristics such as speed, travel time, occupancy and drivers’ perceptions of comfort, convenience, … [11]. As is customary among traffic engineers, these representative statistics of these characteristics are collectively called measures of effectiveness (MOE).

Levels A through D are representative for free-flow conditions whereby LOS A corresponds to free flow, LOS B to reasonable free flow, LOS C to stable traffic operations, and LOS D to bordering unstable traffic operations. LOS E is reminiscent of near-capacity flow conditions that are extremely unstable, whereas LOS F corresponds to congested flow conditions (caused by either structural or incidental congestion) [79].

As an example, we provide an overview of the different levels of service in Table I (based on [79], in similar form originally published in the Highway Capacity Manual (HCM) of 1985 as the Transportation Research Board’s (TRB) special report #209).

| LOS | Density (veh/km) | Occupancy (%) | Speed (km/h) |
|-----|-----------------|---------------|--------------|
| A   | 0 → 7           | 0 → 5         | ≥ 97         |
| B   | 7 → 12          | 5 → 8         | ≥ 92         |
| C   | 12 → 19         | 8 → 12        | ≥ 87         |
| D   | 19 → 26         | 12 → 17       | ≥ 74         |
| E   | 26 → 42         | 17 → 28       | ≥ 48         |
| F   | 42 → 62         | 28 → 42       | < 48         |

TABLE I: Level of service (LOS) indicators for a motorway (adapted from [79], in similar form originally published in the 1985 HCM).

Calculating levels of service can be done using a multitude of methods; some examples include using the density (at motorways), using the space-mean speed (at arterial streets), using the delay (at signalised and unsignalised intersections), … [11].

The distinction
between different LOS is primarily based on the measured average speed, and secondly on the density (or occupancy). Furthermore, as traditional analyses only focus on a select number of hours, a new trend is to conduct whole year analyses (WYA) based on aggregated measurements such as e.g., the monthly average daily traffic (MADT) and the annual average daily traffic (AADT)\cite{10}. The MADT is calculated as the average amount of traffic recorded during each day of the week, averaged over all days within a month. Averaging the resulting twelve MADTs gives the AADT.

Regarding the use of the LOS, we note that it is a rather old-fashioned method for evaluating the quality of traffic operations. In general, it is difficult to calculate, mainly because the defined standards at which the different levels are set, always depend on the specific type of traffic situation that is studied (e.g., type of road, . . . ). This makes the LOS more of an engineering tool, used when assessing and planning operational analyses. Instead of using the LOS, we therefore propose to adopt the more suited approach based on oblique cumulative plots (we refer the reader to section\textsuperscript{11112}). These allow for example to assess the differences between travel times under free-flow and congested conditions, thereby giving a more meaningful and intuitive indication of the quality of traffic operations to the drivers.

**D. Efficiency**

In\cite{18}, Chen et al. state that the main reason for congestion is not demand exceeding capacity (i.e., the number of travellers who want to use a certain part of the transportation network, exceeds the available infrastructure’s capacity), but is in fact the inefficient operation of motorways during periods of high demand. In order to quantify this efficiency, they first look at what the prevailing speed is when a motorway is operating at its maximum efficiency, i.e., the highest flow (corresponding to the effective capacity, which is different from the HCM’s capacity which is calculated from the road’s physical characteristics). Based on the distribution of 5-minute data samples from some 3300 detectors, they investigate the speed during periods of very high flows. This leads them to a so-called sustained speed $v_{sust} = 60$ miles per hour (which corresponds to 60 mi/h $\times$ 1.609 $\approx$ 97 km/h).

The performance indicator they propose, is called the efficiency $\eta$ and it is based on the ratio of the total vehicle miles travelled (VMT), divided by the total vehicle hours travelled (VHT). Note that as the units of VMT and $v_{sust}$ should correspond to each other, we propose to use the terminology of total vehicle distance travelled (VDT) instead of the VMT, in order to eliminate possible confusion. Both VDT and VHT are defined as follows:

$$VDT = q K,$$

$$VHT = \frac{VDT}{\bar{v}_s},$$

with, as before, $q$ the flow, $K$ the length of the road section, and $\bar{v}_s$ the space-mean speed. Using the above definitions, we can write the efficiency of a road section as:

$$\eta = \frac{VDT}{VHT} = \frac{q K}{\frac{VDT}{\bar{v}_s}},$$

The efficiency is expressed as a percentage, and it can rise above 100% when the recorded average speeds surpass the sustained speed $v_{sust}$. In general, the discussed efficiency can also easily be calculated for a complete road network and an arbitrary time period. It can furthermore be seen as the ratio of the actual productivity of a road section (the output produced by this section during one hour), to its maximum possible production (the input to the section) under high flow conditions.

Note that as a solution to their original claim (“congestion arises due to inefficient operation”), Chen et al. propose to increase the operational efficiency, mainly through the technique of suitable ramp metering (using an idealised ramp metering control practice that maintains the occupancy downstream of an on ramp to its critical level). But in our opinion, they neglect to take into account the entire situation, i.e., they fail to consider the extra effects induced by holding vehicles back at some on ramps (e.g., the total time travelled by all the vehicles, including delays), thus rendering their statement practically worthless by giving a feeble argument. Careful examination of their reasoning, reveals that these extra effects are dealt with by shifting demand during the peak periods... but this just confirms our hypothesis that congestion occurs when demand exceeds capacity, even when this capacity is for example controlled through ramp metering!

In contrast to the work of Chen et al., Brilon proposes another definition for the efficiency (now denoted as $E$): it is expressed as the number of vehicle kilometres that are produced by a motorway section per unit of time\cite{10}:

$$E = q \bar{v}_s T_{mp},$$

with now $q$ the total flow recorded during the time interval $T_{mp}$. Brilon concludes that in order for motorways to operate at maximum efficiency, their hourly flows typically have to remain below the capacity flow (e.g., at 90% of $q_{cap}$). Brilon also proposes to use this point of maximum efficiency as the threshold when going from LOS D to LOS E.
V. FUNDAMENTAL DIAGRAMS

Whereas the previous sections dealt with individual traffic flow characteristics, this section discusses some of the relations between them. We first give some characterisations of different traffic flow conditions and the rudimentary transitions between them, followed by a discussion of the relations (which are expressed as fundamental diagrams) between the traffic flow characteristics, giving special attention to the different points of view adopted by traffic engineers.

A. Traffic flow regimes

Considering a stream of traffic flow, we can distinguish different types of operational characteristics, called regimes (two other commonly used terms are traffic flow phases and states). As each of these regimes is characterised by a certain set of unique properties, classification of them is sometimes based on occupancy measurements (see for example the discussion about levels of service in section IV C), or it is based on combinations of different macroscopic traffic flow characteristics (e.g., the work of Kerner [55]).

In the following sections, we discuss the regimes known as free-flow traffic, capacity-flow traffic, congested, stop-and-go, and jammed traffic. Our discussion of these regimes is in fact based on the commonly adopted way of looking at traffic flows, as opposed to for example Kerner’s three-phase traffic theory that includes a regime known as synchronised traffic (we refer the reader to section IV D for more details). We conclude the section with a note on the transitions that occur from one regime to another.

1. Free-flow traffic

Under light traffic conditions, vehicles are able to freely travel at their desired speed. As they are largely unimpeded by other vehicles, drivers strive to attain their own comfortable travelling speed (we assume that in case a vehicle encounters a slower moving vehicle ahead, it can easily change lanes in order to overtake the slower vehicle). Notwithstanding this ability for unconstrained travelling, drivers have to take into account the maximum allowed speed (denoted by $v_{\text{max}}$), as well as road-, engine-, and other vehicle characteristics. Note that in some cases, depending on the country under scrutiny, drivers perform speeding.

In essence, the previous description of free-flow traffic considers a traffic flow to be unrestricted, i.e., no significant delays are introduced due to possible overtaking manoeuvres. As a consequence, the free-flow speed (by some called the nominal speed) is the mean speed of all vehicles, travelling at their own pace (e.g., 100 km/h); it is denoted by $v_f$.

Free-flow traffic occurs exclusively at low densities, implying large average space headways according to equation (10). As a result, small local disturbances in the temporal and spatial patterns of the traffic stream have no significant effects, hence traffic flow is stable in the free-flow regime.

2. Capacity-flow traffic

When the traffic density increases, vehicles are driving closer to each other. Considering the number of vehicles that pass a certain location alongside the road, an observer will notice an increase in the flow. At a certain moment, the flow will reach a maximum value (which is determined by the mean speed of the traffic stream and the current density). This maximum flow is called the capacity flow, denoted by $q_c$, $q_{\text{cap}}$, or even $q_{\text{max}}$. A typical value for the capacity flow on a three-lane Belgian motorway with $v_{\text{max}}$ equal to 120 km/h, can reach a maximum of some 7000 vehicles [113]. According to equation (17), the average time headway is minimal at capacity-flow traffic, indicating the (local) formation of tightly packed clusters of vehicles (i.e., platoons), which are moving at a certain capacity-flow speed $v_c$ (or $v_{\text{cap}}$) which is normally a bit lower than the free-flow speed. Note that some of these fast platoons are very unstable when they are composed of tail-gating vehicles: whenever in such a string a vehicle slows down a little, it can have a cascading effect, leading to exaggerate braking of following vehicles. Hence, these latter manoeuvres can destroy the local state of capacity-flow, and can in the worst case lead to multiple rear-end collisions. At this point, traffic becomes unstable.

The calculation of the capacity flow is a daunting task, holding traffic engineers occupied for the last six decades. The fact of the matter is that there exists no rigorous definition for the concept of ‘capacity’. As a result, after many years of research, this culminated in the publication of the fourth edition of the already previously mentioned Highway Capacity Manual. It contains an impressive overview, spanning methodologies for assessing the capacity at specific types of road infrastructures (motorway facilities, weaving sections, on- and off-ramps, signalised and unsignalised urban intersections, ... ) [1].

3. Congested, stop-and-go, and jammed traffic

Considering the regime of capacity-flow traffic, it is reasonable to assume that drivers are more mentally aware and alert in this regime, as they have to adapt their driving style to the smaller space and time headways under high speeds. However, when more vehicles are present, the density is increased even further, allowing a sufficiently large disturbance to take place. For example, a driver with too small space and time headways, will have to brake in order to avoid a collision with the leader directly
in front; this can lead to a local chain of reactions that disrupts the traffic stream and triggers a breakdown of the flow. The resulting state of saturated traffic conditions, is called congested traffic.

The moderately high density at which this breakdown occurs, is called the critical density, and is denoted by \( k_{\text{crit}} \) (for a typical motorway, its value lies around 25 vehicles (PCUs) per kilometre per lane, \([113]\)). From this knowledge, we can derive the optimal driving speed for single-lane traffic flows as \( v = q_{\text{cap}} / k_{\text{crit}} = 2000 \div 25 = \approx 85 \text{ km/h} \).

Higher values for the density indicate almost always a worsening of the traffic conditions; congested traffic can result in stop-and-go traffic, whereby vehicles encounter so-called stop-and-go waves. These waves require them to slow down severely, or even stop completely. When traffic becomes motionless, the space headway reaches a value around 140 vehicles (PCUs) per kilometre. As already stated in the introduction of section 3A.2, density ignores the effects of traffic composition and vehicle lengths. For a typical value of some 140 vehicles/km/lane for the jam density, this means that we express the density by using passenger car units (see section 3A.2 for more details). Suppose now for example that an average trailer truck equals 4.5 PCUs, then the jam density would decrease to some 140 \( \div 4.5 \approx 31 \) trucks for this class of vehicles. As a consequence, the value of the jam density is different for each vehicle class.

Note that the jam density is typically expressed in vehicles per kilometre. As already stated in the introduction of section 3A, density ignores the effects of traffic composition and vehicle lengths. For a typical value of some 140 vehicles/km/lane for the jam density, this means that we express the density by using passenger car units (see section 3A.2 for more details). Suppose now for example that an average trailer truck equals 4.5 PCUs, then the jam density would decrease to some 140 \( \div 4.5 \approx 31 \) trucks for this class of vehicles. As a consequence, the value of the jam density is different for each vehicle class.

4. A note on the transitions between different regimes

Streams of traffic flows can be regarded as many-particle systems (e.g., gasses, magnetic spin systems, . . . ); as they have a large number of degrees of freedom, it is often intractable when it comes to solving them exactly. However, from a physical point of view, these systems can be described in the framework of statistical physics, whereby the collective behaviour of their constituents is approximately treated using statistical techniques.

Within this context, the changeover from one traffic regime to another, can be looked upon as a phase transition. Within thermodynamics and statistical physics, an order parameter is often used to describe the phase transition: when the system shifts from one phase to another (e.g., at a critical point for liquid-gas transitions), the order parameter expresses a different qualitative behaviour. Two examples of such an order parameter that is applicable to traffic flows, can be found in Schadschneider et al. who considered nearest neighbour correlations \([104]\), and in Jost and Nagel who devised a measure of inhomogeneity \([53]\) (we refer the reader to our work in \([78]\) for an example in which they are used and compared when tracking phase transitions).

There exists a difference in which a phase transition can express itself. This difference is designated by the order of the transition; generally speaking, the two most common phase transitions are first-order and second-order transitions. According to Ehrenfest’s classification, first-order transitions have an abrupt, discontinuous change in the order parameter that characterises the transition. In contrast to this, the changeover to the new phase occurs smoothly for second-order transitions \([71, 119]\). Note that higher-order phase transitions also exist, e.g., in superconducting materials \([23]\).

With respect to the description of regimes in traffic flows, it is commonly agreed that there exists a first-order phase transition when going from the capacity-flow to the congested regime. The point at which this transition occurs, is the critical density. Studying the phase transitions encountered in fluid dynamics, there exists a transition from the laminar flow (i.e., a fluid flowing in layers, each moving at a different velocity) to the turbulent flow (i.e., the disturbed random and unorganised state in which vortices form). However, the transition here is triggered by an increase in the velocity of the fluid, as opposed to the transition in traffic flows where a change in the density can lead to a cascading instability. In this respect, the analogy for traffic flows holds better when comparing them to gas-liquid transitions. Here free-flow traffic corresponds to a gaseous phase, in which particles are evenly spread out in the system. At the point of the phase transition, liquid droplets will form, coagulating together into bigger droplets. This leads to a state where both gaseous and liquid phases coexist, typically in the form of a big liquid droplet surrounded by gas particles. For even higher densities, particles are so close to each other, and the only remaining state is the liquid phase \([17, 52, 53, 54, 63, 93]\).

In conclusion, we refer the reader to the work of Tampère, where an excellent overview is given, detailing the different traffic flow regimes, their transitions, and mechanisms with respect to jamming behaviour \([108]\).

B. Correlations between traffic flow characteristics

Whereas the previous sections all treated the macroscopic traffic flow characteristics on an individual basis, this section considers some of the relations between them. We start our discussion with a look at the historic origin of fundamental diagrams, after which we shed some light on the different classical approaches. The section concludes with some considerations with respect to empirical measurements.

1. The historic origin of the fundamental diagram

As in many scientific disciplines, the resulting statements and theories are often preceded by an investigation of obtained experimental data, which serves as empirical ev-
As can be seen from Greenshields’ relation, when increasing the density from zero to the jam density $k_j$, the mean speed will monotonically decrease from the free-flow speed $\bar{v}_f$ to zero (note that we dropped the ‘s’ or ‘t’ subscript from the mean speed, as it is not sure whether or not Greenshields used space- or time-mean speed, respectively). The relation can be understood intuitively, by assuming that drivers will tend to slow down in crowded traffic, because this naturally gives them more time to react to changes (e.g., sudden braking of the lead vehicle). As it is reasonable to assume that the mean speed remains unaffected for very low densities, Greenshields furthermore flattened the upper-left part of the regression line (corresponding to the free-flow speed), although this effect is not incorporated in equation (44).

$$\bar{v} = \bar{v}_f \left(1 - \frac{k}{k_j}\right).$$  

(44)

Although Greenshields’ derivation of the linear relation between density and space-mean speed appears elegant and simple, it should nevertheless be taken with a grain of salt. The fact of the matter is that his hypothesis is, as can be seen in Fig. 3 based on only seven measurement points, which comprise aerial observations taken on September 3rd (Monday, Labor Day), 1934 [79]. One of the problems is that these observations are not independent. An even more serious problem is that six of these observations were obtained for free-flow conditions, whereas the one single point that indicates congested conditions, was obtained at an entirely different road, on a different day [35]!

Some twenty years later, Lighthill and Whitham developed a theory that describes the traffic flows on long crowded roads using a first-order fluid-dynamic model [72]. As one of the main ingredients in their theory, they postulated the following fundamental hypothesis: “at any point of the road, the flow $q$ is a function of the density $k$”. They called this function the flow-concentration curve (recall from section III C that density in the past got sometimes referred to as concentration).

Continuing their reasoning, Lighthill and Whitham then referred to Greenshields’ earlier work, relating the space-mean speed to the density, and, by means of equation (44), thus relating the flow to the density. The existence of the concept of the flow-concentration curve mentioned above, was justified on the grounds that it describes traffic operating under steady-state conditions, i.e., homogeneous and stationary traffic as explained in section III D 2. In this context, the flow-concentration curve therefore describes the average characteristics of a traffic flow. So Greenshields first fitted a regression line to scarce data, after which his functional form seemed to be taken for granted for the following seventy years. The key aspect in Lighthill and Whitham’s (and also Richards’ [102]) approach, lay in the fact that they broadened the flow-concentration curve’s validity, including also conditions of non-stationary traffic. They also stated that, because of e.g., changes in the traffic composition, the curve can vary from day to day, or even within a day (e.g., rush hours, ...). The same statement holds also true when considering the flow-concentration curves of different vehicle classes (e.g., cars and trucks).

The term fundamental diagram itself, is historically based on Lighthill and Whitham’s fundamental hypothesis of the existence of such a one-dimensional flow-concentration curve. As traffic engineers grew accustomed to the graphical representation of this curve, they started talking about the diagram that represents it, i.e., the ‘fundamental diagram’ [45].

In its original form, the fundamental diagram represents an equilibrium relation between flow and density, denoted by $q_e(k)$. But note that, because of the fundamental relation of traffic flow theory (see section III D 2), it is equally justified to talk about the $\bar{v}_s(k)$ or the $\bar{v}_s(q)$ fundamental diagrams. Due to this equilibrium property, the traffic states (i.e., the density, flow, and space-mean speed) can be thought of as ‘moving’ over the fundamental diagrams’ curves.

2. The general shape of a fundamental diagram

We now give an overview of some of the qualitative features of the different possible fundamental diagrams, representing the equilibrium relations between density,
space-mean speed, and average space headway, and flow. Note that in each example, we consider a possible fundamental diagram, as they can take on many (functional) shapes.

**Space-mean speed versus density**

We start our discussion based on the equilibrium relation between space-mean speed and density, i.e., the $\tau_s(k)$ fundamental diagram. The main reason for starting here, is the fact that this diagram is the easiest to understand intuitively. Complementary to the example of Greenshields in Fig. 6, we give a small overview of its most prominent features:

- the density is restricted between 0 and the maximum density, i.e., the jam density $k_j$,
- the space-mean speed is restricted between 0 and the maximum average speed, i.e., the free-flow speed $\tau_{ff}$.
- as density increases, the space-mean speed monotonically decreases,
- there exists a small range of low densities, in which the space-mean speed remains unaffected and corresponds more or less to the free-flow speed,
- and finally, the flow (equal to density times space-mean speed), can be derived as the area demarcated by a rectangle who’s lower-left and upper-right corners are the origin and a point on the fundamental diagram, respectively.

**Space-mean speed versus average space headway**

Microscopic and macroscopic traffic flow characteristics are related to each other by means of equations (10) and (17). According to the former, density $k$ is inversely proportional to the average space headway $h_s$. We can therefore derive a fundamental diagram, similar to the previous one, by substituting the density with the average space headway. As as result, the abscissa gets ‘inverted’, resulting in the fundamental diagram as shown in Fig. 7.

**Flow versus density**

Probably the most encountered form of a fundamental diagram, is that of flow versus density. Its origins date back to the seminal work of Lighthill and Whitham who, as described earlier, referred to it as the flow-concentration curve. An example of the $q_e(k)$ fundamental diagram is depicted in Fig. 8.

The interesting features of this type of fundamental diagram, can be summed as follows:

- for moderately low densities (i.e., below the critical density $k_c$), the flow increases more or less linearly (this is called the free-flow branch of the fundamental diagram),
• near the critical density $k_c$, the fundamental diagram can bend slightly, due to faster vehicles being obstructed by slower vehicles, thereby lowering the free-flow speed \cite{97}.
• at the critical density $k_c$, the flow reaches a maximum, called the capacity flow \cite{128} $q_{\text{cap}}$.
• in the congested regime (i.e., for densities higher than the critical density), the flow starts to degrade with increasing density, until the jam density $k_j$ is reached and traffic comes to a stand still, resulting in a zero flow (this is called the congested branch of the fundamental diagram).
• the space-mean speed $\bar{v}_x$ for any point on the $q_c(k)$ fundamental diagram, can be found as the slope of the line through that point and the origin.

There is one more piece of information revealed by the $q_c(k)$ fundamental diagram: When taking the slope of the tangent in any point of the diagram, we obtain what is called the kinetic wave speed. These speeds $w$ correspond to shock waves encountered in traffic flows (e.g., the stop-and-go waves). As can be seen from the figure, the shock waves travel forwards, i.e., downstream, in free-flow traffic ($w > 0$), but backwards, i.e., upstream, in congested traffic ($w < 0$).

The above shape of the $q_c(k)$ fundamental diagram is just one possibility. There exist many different flavours, originally derived by traffic engineers seeking a better fit of these curves to empirical data. After the work of Greenshields, another functional form — based on a logarithm — was proposed by Greenberg \cite{43}. Another possible form was introduced by Underwood \cite{112}. All of the previous diagrams are called single-regime models, because they formulate only one relation between the macroscopic traffic flow characteristics for the entire range of densities (i.e., traffic flow regimes) \cite{78}. In contrast to this, Edie started developing multi-regime models, allowing for discontinuities and a better fit to empirical data coming from different traffic flow regimes \cite{35}.

We refer the reader to the work of Drake et al. \cite{34} and the book of May \cite{79} for an extensive comparison and overview of these different modelling approaches (note that Drake et al. used time-mean speed).

During the last two decades, other, sometimes more sophisticated, functional relationships between density and flow have been proposed. Examples are the work of Smulders who created a non-differentiable point at the critical density in a two-regime fundamental diagram \cite{106}, the METANET model of Messmer and Papageorgiou who’s single-regime fundamental diagram contains an inflection point near the jam density \cite{80}, the work of De Romph who generalised Smulders’ functional description of his two-regime fundamental diagram \cite{103}.

The typical triangular shape of the fundamental diagram introduced by Newell, resulting in only two possible values for the kinematic wave speed $w$ \cite{97}. … As can be seen, these fundamental diagrams sometimes take on non-convex forms, depending on the existence of inflection points in the functional relation between flow and density. In general, they can be convex, concave, (dis)continuous, piecewise-linear, everywhere differentiable, have inflection points, … Variations in shape will continue to be proposed, as it is for certain that there is no general consensus among traffic engineers regarding the correct shape of this fundamental diagram. To illustrate this, a more exotic approach is based on catastrophe theory, which is, in a sense, a three-dimensional model that jointly treats density, flow, and space-mean speed.

Acha-Daza and Hall applied the technique, resulting in a satisfactory fit with empirical data \cite{2}.

The most extreme argument with respect to the shape of the fundamental diagram, came from Kerner who questioned its validity, and consequently rejected it altogether by replacing it with his fundamental hypothesis of three-phase traffic flow theory (refer to section \ref{V.D} for more details) \cite{55}.

### Space-mean speed versus flow

An often spotted shape is that of the $\bar{v}_x(q)$ fundamental diagram, depicted in Fig. 9. As opposed to the earlier discussed $q_c(k)$ fundamental diagram, the space-mean speed versus flow curve no longer embodies a function in the strict mathematical sense: for each value of the flow, there exists two different mean speeds, namely one in the free-flow regime (upper branch) and one in the congested regime (lower branch).

![Fig. 9: A fundamental diagram relating the flow $q$ to the space-mean speed $\bar{v}_x$. The capacity flow $q_{\text{cap}}$ is located at the right edge of the diagram, i.e., it is defined as the maximum average flow. Note that there are two possible speeds associated with each value of the flow.](image)

Some people, e.g., economists who use the flow to represent traffic demand, find this kind of fundamental diagram easy to cope with. But in our opinion, we are convinced however, that this diagram is rather difficult to understand at first sight. We believe the $\bar{v}_x(k)$ fundamental diagram is a much better candidate, because density can intuitively be understood as a measure for how crowded traffic is, as opposed to some flow giving rise to two different values for the space-mean speed.

As a final comment, we would like to point out that the previously discussed bivariate functional relationships between the traffic flow characteristics (e.g., density and flow), are based on observations. More importantly, this
means that there is no direct causal relation assumed between any two variables. Fundamental diagrams sketch only possible correlations, implying that the nature of the transitions between different traffic regimes thus remains to be explored (see section V A 3 for a discussion).

3. Empirical measurements

As mentioned earlier, the fundamental diagrams discussed in the previous section represent equilibrium relations between the macroscopic traffic flow characteristics of section [11]. In sharp contrast to this, real empirical measurements from detector stations do not describe such nice one-dimensional curves corresponding to the functional relationships.

As an illustrative example, we provide some scatter plots in Fig. 10. The shown data comprises detector measurements (the sampling interval was one minute) during the entire year 2003; they were obtained by means of a video camera [113] located at the E17 three-lane motorway near Linkeroever [129], Belgium. Because of the nature of this data, we only obtained flows, occupancies, and time-mean speeds. After calculating the average vehicle length, the occupancies were converted into densities using equation [22]. Using these recorded time series, we then constructed scatter plots of the density, time-mean speed, flow, and average space headway. Note that no substantial changes are introduced in these plots due to e.g., our using of densities calculated from occupancies, instead of using real measured densities.

The occurrence of all this scatter in the data, leads some traffic engineers to question the validity of the fundamental diagram. More specifically, the behaviour in congested traffic seems ill-defined to some. As stated earlier, Kerner is the most intense opponent in this debate, as he outright rejects Lighthill and Whitham’s hypothesis that remained popular over the last fifty years. Despite this criticism, the fundamental diagram remains, to the majority of the community, a fairly accurate description of the average behaviour of a traffic stream. Cassidy even provided quantifiable evidence of the existence of well-defined bivariate relations between traffic flow characteristics. The key here was to separate stationary periods from non-stationary ones in the detector data (i.e., stratifying it) [14, 27]. Prior work of Del Castillo and Benitez resulted in a more mathematically justified method, for fitting empirical curves in data regions of stationary traffic, after construction of a rigid set of properties that all fundamental diagrams should satisfy [31, 32].

As a final note, we remark that the distribution of the cloud-like data points of the diagrams in Fig. 10 is a result of various kinds of phenomena. First and foremost, there is the heterogeneity in the traffic composition (fast passenger cars, slow trailer trucks, …). Secondly, as already mentioned, the non-stationary behaviour of traffic introduces a significant amount of scatter in the congested regime. Thirdly, each scatter plot is dependent on the type of road, and the time of day at which the measurements were collected. In this respect, the influence of (changing) weather conditions is not to be underestimated (e.g., rain fall results in different diagrams). In conclusion, it is clear that if we want these scatter plots to better fit the fundamental diagrams, all data points should be collected under similar conditions. Even more so, the relative location on the road at which the data points were recorded plays a significant role: e.g., a jam that propagates upstream, passing an on-ramp will show different effects.

FIG. 10: Illustrative scatter plots of the relations between traffic flow characteristics as measured by video camera CLO3 located at the E17 three-lane motorway near Linkeroever, Belgium. The measured occupancies were converted into densities, the time-mean speed remained unchanged. Shown are scatter plots of a \((k,\bar{v})\) diagram (top-left), a \((h,\bar{v})\) diagram (top-right), a \((k,q)\) diagram (bottom-left), and a \((q,\bar{v})\) diagram (bottom-right).

As the dimension of time is removed in these scatter plots, Daganzo calls them time-independent models [27]. It is important to understand that these scatter plots are not fundamental diagrams, because the latter represent one-dimensional equilibrium curves. According to Helbing, a better designation would be regression models [47]. In this dissertation, we introduce a terminology based on phase spaces (or equivalently state spaces), resulting in e.g., the \((k,q)\) diagram (note that we dropped the adjective ‘fundamental’).
depending on where the observations were gathered (upstream, right at, or downstream of the on-ramp) and on whether or not the particular bottleneck was active \[79\].

C. Capacity drop and the hysteresis phenomenon

In the early sixties, traffic engineers frequently observed a discontinuity in the measurements near the capacity flow. To this end, Edie proposed a two-regime model that included such a discontinuity at the critical density \[q_{cap}\]. Nowadays, this typical form of the \(q_{c}(k)\) fundamental diagram is known as a reversed lambda shape (the name was originally suggested by Koshi et al. \[77\]).

An example of such a reversed \(\lambda\) fundamental diagram, is shown in the left part of Fig. 11. Note however, that the depicted discontinuity apparently leads to overlapping branches of the free-flow and congested regimes, resulting in a multi-valued fundamental diagram.

![Diagram of fundamental diagram](image)

FIG. 11: Left: the typical inverted \(\lambda\) shape of the \((k,q)\) fundamental diagram, showing a capacity drop from \(q_{cap}\) to below \(q_{out} \ll q_{cap}\) (i.e., the queue discharge flow). The hysteresis effect occurs when going from the congested to the free-flow branch, as indicated by the three arrows (1) – (3). Right: a \((k,q)\) diagram based on empirical data of one day, obtained by video camera CLO3, at the E17 three-lane motorway near Linkeroever, Belgium. The black dots denote minute measurements, whereas the thick solid line represents the time-traced evolution of traffic conditions. The observed hysteresis loop was based on consecutive 5-minute intervals covering a period that encompasses the morning rush hour between 06:30 and 09:30.

Considering the left part of Fig. 11, it appears the flow can take on two different values (hence the name ‘two-regime, two-capacity’ model) depending on the traffic conditions, i.e., whether traffic is moving from the free-flow to the congested regime on the equilibrium curve or vice versa. In order to comprehensively understand this hysteretic behaviour, we consider the following intuitive sequence of events:

1. In the free-flow regime, the flow steadily rises with increasing density, small perturbations in the traffic flow have no significant effects (see section V.A.1).
2. At the critical density \(k_{c}\), traffic is said to be metastable: for small disturbances, traffic is stable, but when these disturbances are sufficiently large, they can lead to a cascading effect (see section V.A.2), resulting in a breakdown of traffic and kicking it onto the congested branch. The state of capacity flow at \(q_{cap}\) is destroyed, due to a sudden decrease of the flow, called the capacity drop.
3. In order to recover from the congested to the free-flow regime, the traffic density has to be reduced substantially (in comparison with the reverse transition), i.e., well below the critical density \(k_{c}\). After this recovery, the flow will not be equal to \(q_{cap}\), but to \(q_{out} < q_{cap}\), which is called the outflow from a jam or the queue discharge capacity.

The above sequence signifies a hysteresis loop in the flow versus density fundamental diagram: going from the free-flow to the congested regime occurs via the capacity flow, but the reverse transition proceeds via another way. The phenomenon was first observed by Treiterer and Meyers, who used aerial photography to calculate densities and space-mean speeds, extracted from a platoon of moving vehicles \[11\]. Hall et al. later observed a similar phenomenon \[46\].

The right part of Fig. 11 shows a \((k,q)\) diagram, obtained with empirical data collected at Monday September 10th, 2001. The data was recorded by video camera CLO3, at the E17 three-lane motorway near Linkeroever, Belgium. The small dots represent minute-based measurements, whereas the thick solid line represents the time-traced evolution of traffic conditions. The observed hysteresis loop was based on consecutive 5-minute intervals covering a period that encompasses the morning rush hour between 06:30 and 09:30.

Zhang is among the few who try to give a possible rigorous mathematical explanation for the occurrence of this hysteresis phenomenon \[120\]. His exposition is based on the behaviour of individual drivers during car-following: central to his interpretation is the existence of an asymmetry between accelerating and decelerating vehicles (a related notion was already explored by Newell back in 1963 \[94\]). The former are associated with larger space headways, whereas the latter typically have smaller space headways. Both observations can be understood when considering the characteristic ‘harmonica’ effect of a string of consecutive vehicles: when the next stop-and-go wave is encountered, a driver is more alert as he typically has to brake rather hard in order to avoid a collision. But once this wave has passed, a driver gets more relaxed, resulting in a larger response time when applying the gas pedal. The deceleration reaction leads to a sudden decrease of the space headway, whereas the acceleration reaction leads to a gradually developing larger space headway. To this end, Zhang introduces three distinct traffic phases, respectively called the acceleration phase, the deceleration phase, and a strong equilibrium (indicating a constant speed). Because the space headway is thus treated differently under these qualitatively different circumstances, the result is that there are now different functional relations for the \(\tau_{k_{c}}(\bar{h}_{s})\) fundamental diagram. As a consequence, a hysteresis loop can appear in the (density,flow) state space. Note that Zhang’s work describes a continuous loop in state space, whereas in most cases hysteresis is assumed to follow a discontinuous fundamental diagram. Furthermore, as there are three
different ways for vehicles to reside in a traffic stream (i.e., Zhang’s traffic phases), there are now three different capacities related to these conditions; it is the capacity under a stationary equilibrium flow that should be considered as the ideal capacity of a roadway [121].

Note that depending on the location where the traffic stream measurements were performed, the transition from the free-flow to the congested regime and vice versa does not always have to pass via the capacity flow. Instead, observations can indicate that the traffic state can jump abruptly from one branch to another in the diagram [39]. A possible explanation is that upstream of a jam, vehicles arrive with high speeds, resulting in strong decelerations; a detector station located at this point would observe traffic jumping from the uncongested branch immediately to the congested branch, without necessarily having to pass via the capacity [55]. This has led Hall et al. to believe the reversed lambda shape is more correctly replaced by a continuous but non-differentiable inverted V shape [46].

Continuing this latter train of thought, Daganzo believes that many of these ‘extravagant’ phenomena (e.g., a multi-valued fundamental diagram) are uncalled for. Applying the stratification methodology of Cassidy [14], the scatter in the empirical data may vanish, restoring a smooth continuous equilibrium relation between density and flow. One way of explaining the high tip of the lambda, is to assume that it is caused by statistical fluctuations that comprise platoons of densely packed vehicles [27].

During the last seventy years, there has been a continuing quest to find the ‘correct’ form of the fundamental diagrams. In this respect, we like to stress the fact that ‘only looking at the measurements’ is not sufficient: traffic engineers wanting to mine the gigabytes of empirical data, should always look at the global picture. This means that the typical driving patterns, as well as the local geometry/infrastructure, should also be taken into account, so that the local measurements can be interpreted with respect to the traffic flow dynamics. If this is neglected, the danger exists that traffic is only sampled at discrete locations, giving a sort of ‘truncated’ view of the occurring dynamical processes.

Finally, we like to agree with Zhang’s comments: the root cause of most of the differences in the construction of fundamental diagrams, is the erroneous treatment of data (e.g., mixing data stemming from different traffic flow regimes) [120]. Because fundamental diagrams imply the notion of an equilibrium, care should be taken when using the data, i.e., only considering stationary periods after removing the transients.

D. Kerner’s three-phase theory

In the mid-nineties, Kerner and other fellow researchers, studied various traffic flow measurements stemming from detector stations along German motorways. Initially, they agreed with the classic notion of Lighthill and Whitham’s fundamental hypothesis of the existence of one-dimensional equilibrium relation between the macroscopic traffic flow characteristics (see section V.B.1 for more details). However, upon discovery of a rich and complex set of empirical tempo-spatial patterns in congested traffic flow, Kerner decided to abolish this hypothesis, as it could not adequately capture all of these observed patterns. As a consequence, Kerner rejects all traffic flow theories and models that are based on this one-dimensional equilibrium relation [55].

In the search for a more correct theory that could accurately describe empirical traffic flow observations, Kerner developed what is known as the three-phase theory of traffic flow.

1. Free flow, synchronised flow, and wide-moving jam

In section V.A we elaborated on a classic approach to traffic flow, general assuming two qualitatively different regimes, namely free-flow and congested traffic. Based on empirical findings, Kerner and Reborn in 1996 proposed three different regimes, separating the congested regime into two other regimes. This led them to the introduction of the following regimes [61]:

- free flow,
- synchronised flow,
- and wide-moving jam.

The main difference between synchronised flow and the wide-moving jam, is that in the former low speeds but high flows (comparable to free-flow traffic) can be observed, whereas in the latter both low speeds and low flows are observed. The description by the term ‘synchronised’ was based on the discovery that the time series of flows, densities, and mean speeds exhibited large degrees of correlation among neighbouring lanes. And although synchronised flow is treated as a form of congestion, it nevertheless is characterised by a high continuous flow. Furthermore, a typical tempo-spatial region of synchronised flow has a fixed downstream front (that could be located at a bottleneck’s position), whereas both the upstream and downstream fronts of a wide-moving jam can propagate undisturbed in the upstream direction of a traffic stream [61].

Kerner distinguishes several congestion patterns with respect to traffic flows. A first typical pattern is a synchronised-flow pattern (SP), which can be further classified as a moving SP (MSP), a widening SP (WSP), and a localised SP (LSP). An SP can only contain synchronised flow; as we will shortly mention in section
2. Fundamental hypothesis of three-phase traffic theory

Central to Kerner’s theory, is the fundamental hypothesis of three-phase traffic theory, which basically states that hypothetical steady states of synchronised flow, cover a two-dimensional region in a flow versus density diagram (as opposed to the classic notion of a one-dimensional equilibrium relation). An example of such a diagram can be seen in Fig. 12.

The curve of free flow (denoted by $F$) is reminiscent of observations in the classic free-flow regime. It levels of a bit towards the capacity flow and wide-moving jams. Just as with the SP, there exist different types of GP. These are a dissolving GP (DGP), a GP under weak congestion, and a GP under strong congestion. A final often encountered pattern occurs when two bottlenecks are spatially close to each other, resulting in what is called an expanded congested pattern (EP).

Taking the above considerations into account, the discovery and distinction between both types of congested traffic patterns should be made on the basis of tempo-spatial plots of the speed, rather than the flow (because the flow in synchronised traffic is difficult to differentiate from that of free-flow traffic) [55]. To this end, Kerner et al. developed two applications that are capable of accurately estimating, automatically tracking, and reliably predicting the above mentioned congested traffic patterns. Their models are the Forecasting of Traffic Objects (FOTO) and Automatische StauDynamikAnalyse (ASDA) [63].

$$q(k) = \frac{1}{T} \left( 1 - \frac{k}{k_{jam}} \right), \quad (45)$$

with $T$ the time gap in congested traffic flows; it is used to tune the outflow from a jam. Because wide-moving jams travel undisturbed, their outflow — caused by vehicles that leave the downstream front — can be either free flow or synchronised flow. Typical values for this outflow range from 1500 to 2000 vehicles/hour/lane [55]. The average flow rate within such a wide-moving jam can be almost zero, meaning that vehicles continuously encounter stop-and-go waves.

Related to the wild scatter in the $(k,q)$ diagram of three-phase traffic theory, is the microscopic behaviour of individual vehicles. The explanation given by Kerner and Klenov, is that vehicles in synchronised flow do not assume a fixed preferred distance to their direct frontal leader, but rather accept a certain range of distances. Within this range, drivers have both the tendency to over-accelerate when they think there is the ability to overtake, and the tendency for drivers to adjust their speed to that of their leader, when this overtaking can not be fulfilled [55, 58].

3. Transitions towards a wide-moving jam

The breakdown of traffic from the free-flow to the wide-moving jam state, is nearly always characterised by two successive $F \rightarrow S$ and $S \rightarrow J$ transitions, between free flow and synchronised flow, and synchronised flow and wide-moving jam respectively. In the first stage, a state of free flow changes to synchronised flow by the $F \rightarrow S$ transition. Central to the idea of this phase transition, is the fact that there is no explicit need for an external disturbance for its occurrence. A sufficiently large (i.e., supercritical) internal disturbance inside the traffic stream

![FIG. 12: The flow versus density relation according to Kerner’s three-phase traffic theory. The curve of free flow (denoted by $F$) is intersected by the line $J$, denoting the steady propagation of wide-moving jams. The line $J$ also intersects the curve of free flow in the outflow from a jam $q_{out} \ll q_{cap}$ at the associated density $k_{out}$.](image-url)
(e.g., a lane change) causes a nucleation effect that instigates the $F \rightarrow S$ transition. Once it has set in, the onset of congestion is accompanied by a sharp drop in the mean vehicle speed. During the second stage, a set of narrow-moving jams can grow inside the tempo-spatial region of synchronised flow. A narrow-moving jam is different from a wide-moving jam, in that vehicles typically do not on average come to a full stop inside the jam. But, due to a compression of synchronised flow (an effect termed the pinch effect), these narrow-moving jams can coalesce into a wide-moving jam, thereby completing the cascade of the $F \rightarrow S \rightarrow J$ transition, resulting in stop-and-go traffic [57].

With respect to the flow versus density diagram in Fig. 12, it can be seen that the line $J$ actually divides the region of synchronised flow in two parts. Points that lie underneath this line, characterise stable traffic states where no $S \rightarrow J$ transition can occur. Points above the line $J$ however, characterise metastable traffic states, meaning that sufficiently large disturbances can trigger a $S \rightarrow J$ transition [55, 66].

Note that the direct $F \rightarrow J$ transition between free flow and wide-moving jam can also occur, but it has a very small probability, i.e., the critical perturbation needed, is much higher than that of the frequently occurring $F \rightarrow S$ transition between free flow and synchronised flow. So in general, wide-moving jams do not emerge spontaneously in free flow, but a situation where such a transition may occur, is when an off-ramp gets filled with slow-moving vehicles. This results in a local obstruction at the motorway’s lane directly adjacent to the off-ramp, which can cause a local breakdown of the upstream traffic, resulting in a wide-moving jam. Finally, it is important to distinguish the nature of this transition from that of the $F \rightarrow S$ transition: the former is a transition induced by an external disturbance of the local traffic flow, whereas the latter is considered as a spontaneous transition due to an internal disturbance within the local traffic flow (e.g., a lane change) [55].

4. From descriptions to simulations

As Kerner himself describes his three-phase theory, it is a qualitative theory. In essence, it gives no explanation of why certain transitions occur, as it only describes them [55]. However, some exemplary microscopic traffic flow models have already been developed (i.e., treating all vehicles and their interactions individually). These models can reproduce the different empirical tempo-spatial patterns described by Kerner’s theory. As examples, we mention two models based on cellular automata: a first attempt was made by Knospe et al., who developed a model that takes into account a driver’s reaction to the brake-lights of his direct frontal leader [65]. Kerner et al. refined this approach by extending it; their work resulted in a family of models based on the notion of a synchronisation distance for individual vehicles; they are commonly called the KKW-models (from its three authors, Kerner, Klenov, and Wolf) [54].

The theory can describe most of the encountered tempo-spatial features of congested traffic. And at the moment, successful microscopic models have been developed, but the work is not yet over: an important challenge that remains for theoreticians, is the mathematical derivation of a consistent macroscopic theory (i.e., one that treats traffic at a more aggregate level as a continuum) [55]. In pursuit of such a model, Kim incorporated Kerner’s traffic regimes into a broader framework, encompassing six different possible states: the transitions between these states are tracked with a modified macroscopic model that uses concepts from fuzzy logic theory [64].

E. Theories of traffic breakdown

A central question that is often asked in the field of traffic flow theory, is the following: “What causes congestion?” Clearly, the answer to this question should be a bit more detailed than the obvious “Because there too many vehicles on the road!” With respect to the phase transitions that signal a breakdown of the traffic flow, various — seemingly contradicting — theories exist. Are they merely a matter of belief, or can they be rigourously ‘proven’? Opinions are divided, but nowadays, two qualitatively different mainstream theories exist, attributed to different schools of thought [12, 77, 108].

The European (German) school

In the early seventies, Treiterer and Meyers performed some aerial observations of a platoon of vehicles. As they constructed individual vehicle trajectories, they could observe a growing instability in the stream of vehicles, leading to an apparently emerging phantom jam (i.e., a jam ‘out of nothing’) [111].

Some twenty years later, in the mid-nineties, Kerner and Konhäuser made detailed studies of traffic flow measurements, obtained at various detector stations along German motorways. Their findings indicated that phantom jams seemed to emerge in regions of unstable traffic flow [59]. This stimulated Kerner and Rehborn to further research efforts directed towards the behaviour of propagating jams [61, 62]. They proposed a different set of traffic flow regimes, culminating in what is now called three-phase traffic theory (see section V.2 for more details) [55, 57, 62]. The main idea supported by followers of this school of researchers, is that traffic jams can spontaneously emerge, without necessarily having an infrastructural reason (e.g., on-ramps, incidents, …) [63]. In dense enough traffic, phase transitions from the free-flow to the synchronised-flow regime can occur, after which a local instability such as e.g., a lane change can grow (the so-called pinch effect), triggering a stable jam leading to stop-and-go behaviour [57]. Kerner’s three-phase theory stands out as an archetypical example of these modern views. But although his theory has, in our opinion, been worked out well enough, he more than frequently encounters harsh criticisms when conveying it to most audiences (perhaps the main cause for this human behaviour is the fact that Kerner always mentions the same view, i.e., “all existing traffic flow theories are wrong”).
Inspired by Kerner’s work, Helbing et al. gave in 1999 an extended treatise on the different types of congestion patterns that can be observed in the vicinity of spatial inhomogeneities (e.g., on-ramps). Their work resulted in a universal phase diagram, containing a whole plethora of patterns of congested traffic states (called homogeneously congested traffic – HCT, oscillatory congested traffic – OCT, triggered stop-and-go traffic – TSG, pinned localised cluster – PLC, and moving localised cluster – MLC), each one having unique characteristics. In that same year, Lee et al. studied the patterns that emerge at on-ramps, thereby agreeing with the findings of Helbing et al. As the previous research into congestion patterns was largely based on the use of analytical traffic flow models and computer simulations, the need for validation with empirical data grew. In 2000, the work of Treiber et al. among others, proved the existence of the previously mentioned congestion patterns.

At this point, it is noteworthy to mention the seminal work of Nagel and Schreckenberg, who in 1992 developed a model that describes traffic flows in which local jams can form spontaneously. As many variations on this model have been proposed, later work also focussed on the stability of traffic flows in these models, e.g., the work of Jost and Nagel.

The Berkeley school
Including names such as the late Newell, Daganzo, Bertini, Cassidy, Muñoz, …, the ‘Berkeley school’ (University of California) supports the theory that all congestion is strictly induced by bottlenecks. The hypothesis holds for both recurrent and, in the case of an incident, non-recurrent congestion.

The main starting point states that there is always a ‘geometrical’ explanation for the breakdown. This explanation is based on the presence of road inhomogeneities such as on- and off-ramps, tunnels, weaving areas, lane drops, sharp bends, elevations, … Once a jam occurs due to such a (temporary) bottleneck, it does not dissipate immediately; as a result, drivers can wonder why they enter and exit a congestion wave, without there being an apparent reason for its presence (since it happened earlier and the cause e.g., an incident, already got cleared). Daganzo uses this line of reasoning as an explanation for the dismissal of phantom jams.

The school uses a specific terminology with respect to bottlenecks (being road inhomogeneities). Two qualitatively different regimes exist: the free-flow regime and the queued regime. The latter occurs when a bottleneck becomes active, which will result in a queue growing upstream of the bottleneck while a free-flow regime exists downstream. The bottleneck capacity is then defined as the maximum sustainable flow downstream (which is different from the maximum flow that can be observed prior to the bottleneck’s activation).

The location of these bottlenecks has some peculiarities involved: one of them is the concept of a capacity funnel. It assumes that drivers are at times more alert, e.g., when they are driving on a motorway and nearing an on-ramp in rather dense traffic conditions. This impels them to accept shorter headways, so they are driving closely behind each other at a relatively high speed. Once they have passed the on-ramp’s location, they tend to relax, resulting in larger headways. The effect is that the bottleneck’s actual position is located more downstream.

Shortly after the publication of Kerner and Rehborn’s findings about the peculiar phase transitions that seemed to occur on German motorways, Daganzo et al. provided a swift response where they stated that the occurring phase transitions could also be caused by bottlenecks in a predictable way. They implied that no spontaneously emerging traffic jams are suggested, and that the observed traffic data from both German and North American motorways did not contradict their own statements about the cause of the phase transitions.

In short, the subtle difference between their work and that of Kerner and Rehborn, is that instabilities in the traffic stream are the result and not the cause of the queues that emerge at active bottlenecks. With respect to a spontaneous breakdown of traffic flow at on- and off-ramps (i.e., bottlenecks), Daganzo also states that this can be explained using a simple traffic flow model operating under the assumption of a too high inflow from the on-ramp or a caused by blocking of the off-ramp.

The studies undertaken by this school, are heavily based on the researchers’ use of cumulative plots and elegantly simple traffic flow models, as opposed to the classic methodology that investigates time series of recorded counts and speeds. As stated earlier (see section), some recent examples include the work of Muñoz and Daganzo, Cassidy and Bertini, and Cassidy and Bertini.

Recently, Tampère argued that both theories, as enunciated by the two schools, are not entirely contradictory. His statement is based on the fact that the mechanisms behind the bottleneck-induced breakdown and spontaneous breakdown are approximately the same, only differing in the probability of such a breakdown (which is related to the instability of a traffic flow).

In our view, both theories are sufficiently different, but compatible, in that the first school elaborately describes traffic flow breakdown more or less as having an inherently probabilistic nature, whereas the second school treats breakdown a strict deterministic process. The former introduces a complex variety of congestion patterns, while the latter primarily focusses on an elegantly simple description of traffic flow breakdown. Even more characteristically, is the observation that most adept of the European school, inherently need stochasticity in the models in order to produce their sought phantom traffic jams (note that notwithstanding the fact that stochastic models are in a strict sense also deterministic, we nevertheless adopt in this dissertation, the convention that deterministic means ‘non-stochastic’). Our argument is in a way also supported by Nagel and Nelson, who state that the purpose of the traffic flow model (e.g., the effect of moving bottlenecks versus predicting mean traffic behaviour) decide whether or not stochasticity in the model is required.
Furthermore, there might be some room for stochasticity in the Berkeley models after all, with the work of Laval which suggests that (disruptive) lane changes form the main cause for instabilities in a traffic stream [69]. Deciding which school is right, is therefore in our opinion a matter of personal taste, but in the end, we agree with Daganzo when he states that research into bottleneck behaviour is the most important in the context of traffic flow theory [30].

VI. CONCLUSIONS

In this paper, an extensive account was given, detailing several aspects related to the description of traffic flows. Most importantly, we have introduced a nomenclature convention, built upon a consistent set of notations. Our discussion of traffic flow characteristics centred around the space and time headways as microscopic characteristics, with densities and flows as their macroscopic counterparts. Several noteworthy highlights are the technique of oblique cumulative plots and the derivation of travel times based on these plots. A finally large part of this paper reviewed some of the relations between traffic flow characteristics, i.e., the fundamental diagrams, and clarified some of the different points of view adopted by the traffic engineering community.

APPENDIX A: GLOSSARY OF TERMS

1. Acronyms and abbreviations

| Acronym | Description |
|---------|-------------|
| 4SM     | four step model |
| AADT    | annual average daily traffic |
| ABM     | activity-based modelling |
| ACC     | adaptive cruise control |
| ACF     | average cost function |
| ADAS    | advanced driver assistance systems |
| AIMSUN2 | Advanced Interactive Microscopic Simulator for Urban and Non-Urban Networks |
| AMICI   | Advanced Multi-agent Information and Control for Integrated multi-class traffic networks |
| AON     | all-or-nothing |
| ASDA    | Automatische StauDynamikAnalyse |
| ASEPI   | asymmetric simple exclusion process |
| ATIS    | advanced traveller information systems |
| ATMS    | advanced traffic management systems |
| BCA     | Burgers cellular automaton |
| BJH     | Benjamin, Johnon, and Hui |
| BJH-TCA | Benjamin-Johnson-Hui traffic cellular automaton |
| BL-TCA  | brake-light traffic cellular automaton |
| BML     | Biham, Middleton, and Levine |
| BML-TCA | Biham-Middleton-Levine traffic cellular automaton |
| BMW     | Beckmann, McGuire, and Winsten |
| BPR     | Bureau of Public Roads |
| CA      | cellular automaton |
| CA-184  | Wolfram’s cellular automaton rule 184 |
| CAD     | computer aided design |
| CBD     | central business district |
| CFD     | computational fluid dynamics |
| CFL     | Courant-Friedrichs-Lewy |
| ChSch-TCA | Chowdhury-Schadschneider traffic cellular automaton |
| CLO     | camera Linkeroever |
| CML     | coupled map lattice |
| CONTRAM | CONTinuous TRaffic Assignment Model |
| COMF    | car-oriented mean-field theory |
| CPM     | computational process models |
| CTM     | cell transmission model |
| DDE     | delayed differential equation |
| DFI-TCA | deterministic Fukui-Ishibashi traffic cellular automaton |
| DGP     | dissolving general pattern |
| DLC     | discretionary lane change |
| DLD     | double inductive loop detector |
| DNL     | dynamic network loading |
| DRIP    | dynamic route information panel |
| DTA     | dynamic traffic assignment |
| DTC     | dynamic traffic control |
| DTM     | dynamic traffic management |
| DUE     | deterministic user equilibrium |
| DynaMIT | Dynamic network assignment for the Management of Information to Travellers |
| DYNASMART | DYnamic Network Assignment-Simulation Model for Advanced Roadway Telematics |
| ECA     | elementary cellular automaton |
| EP      | expanded congested pattern |
| ER-TCA  | Emmerich-Rank traffic cellular automaton |
| FCD     | floating car data |
| FDE     | finite difference equation |
| FIFO    | first-in, first-out |
| FOTO    | Forecasting of Traffic Objects |
| GETRAM  | Generic Environment for TRaffic Analysis and Modeling |
| GHR     | Gazis-Herman-Rothery |
| GIS     | geographical information systems |
| GNSS    | Global Navigation Satellite System (e.g., Europe’s Galileo) |
| GoE     | Garden of Eden state |
| GP      | general pattern |
| GPRS    | General Packet Radio Service |
| GPS     | Global Positioning System (e.g., USA’s NAVSTAR) |
| GRP     | generalised Riemann problem |
| GSM     | Groupe Spéciale Mobile |
| GSMC    | Global System for Mobile Communications |
| HAP     | household activity pattern problem |
| HCM     | Highway Capacity Manual |
| HCT     | homogeneously congested traffic |
| HDM     | human driver model |
| HKM     | human-kinetic model |
| HRB     | Highway Research Board |
2. List of symbols

- $a_i$: the acceleration of vehicle $i$
- $C$: the number of substreams in a traffic flow
- $dx$: a single infinitesimal location in space
- $dt$: a single infinitesimal instant in time
- $\eta$: the efficiency of a road section
  
  (according to Chen et al., [18])
\( E \) the efficiency of a road section
(according to Brilon, [10])
\( F \) the free-flow curve in three-phase traffic theory
\( g_{s,i} \) the space gap of vehicle \( i \)
\( g_{s,i}^{L,B} \) the space gap at the left-back of vehicle \( i \)
\( g_{s,i}^{L,F} \) the space gap at the left-front of vehicle \( i \)
\( g_{s,i}^{R,B} \) the space gap at the right-back of vehicle \( i \)
\( g_{s,i}^{R,F} \) the space gap at the right-front of vehicle \( i \)
\( g_{t,i} \) the time gap of vehicle \( i \)
\( g_{t,i}^{L,B} \) the time gap at the left-back of vehicle \( i \)
\( g_{t,i}^{L,F} \) the time gap at the left-front of vehicle \( i \)
\( g_{t,i}^{R,B} \) the time gap at the right-back of vehicle \( i \)
\( g_{t,i}^{R,F} \) the time gap at the right-front of vehicle \( i \)
\( h_{s,i} \) the average space headway
\( h_{t,i} \) the space headway of vehicle \( i \)
\( h_{t,i}^{L,B} \) the space headway at the left-back of vehicle \( i \)
\( h_{t,i}^{L,F} \) the space headway at the left-front of vehicle \( i \)
\( h_{t,i}^{R,B} \) the space headway at the right-back of vehicle \( i \)
\( h_{t,i}^{R,F} \) the space headway at the right-front of vehicle \( i \)
\( \bar{h}_s \) the average time headway
\( \bar{h}_t \) the time headway of vehicle \( i \)
\( \bar{h}_{t,i}^{L,B} \) the time headway at the left-back of vehicle \( i \)
\( \bar{h}_{t,i}^{L,F} \) the time headway at the left-front of vehicle \( i \)
\( \bar{h}_{t,i}^{R,B} \) the time headway at the right-back of vehicle \( i \)
\( \bar{h}_{t,i}^{R,F} \) the time headway at the right-front of vehicle \( i \)
\( J \) the wide moving jam line \( J \) in three-phase traffic theory
\( k \) the density
\( k_c \) the density of the \( c \)-th substream in a traffic flow
\( k_c \) the critical density
\( k_{crit} \) the critical density
\( k_{jam} \) the jam density
\( k_{max} \) the jam density
\( i \) the density in lane \( l \)
\( k_{out} \) the density associated with the queue discharge capacity
\( k(t) \) the density at time \( t \)
\( K \) the length of a measurement region
(i.e., a certain road section)
\( K_{ld} \) the length of a detection zone
\( \bar{L} \) the average length of a vehicle
\( l_i \) the length of vehicle \( i \)
\( L \) the number of lanes on a road
\( N \) the number of vehicles in a measurement region
\( N_l \) the number of vehicles in the measurement region in lane \( l \)
\( \bar{N}_l(t) \) the number of vehicles in the measurement region in lane \( l \) at time \( t \)
\( N(t) \) a cumulative count function
\( \bar{N}(t) \) a smooth approximation of \( N(t) \)
\( \sigma_t \) the average on-time of a set of vehicles
\( \alpha_{i,l} \) the on-time of vehicle \( i \) in lane \( l \)
\( q \) the flow
\( \overline{q}_{15} \) the peak flow rate during one quarter hour within an hour
\( \overline{q}_{60} \) the average flow during the hour with the maximum flow in one day
\( q_0 \) a background flow
\( q_c \) the flow of the \( c \)-th substream in a traffic flow
\( q_{cap} \) the capacity flow
\( q_c(k) \) an equilibrium relation between the flow and the density
\( q_l \) the flow in lane \( l \)
\( q_{max} \) the capacity flow
\( q_{out} \) the outflow from a (wide moving) jam, the queue discharge capacity
\( q(t) \) the flow at time \( t \)
\( \rho \) the occupancy
\( \rho_{i,l} \) the occupancy of vehicle \( i \)
\( \rho_l \) the occupancy in lane \( l \)
\( R_s \) a spatial measurement region at a fixed time instant
\( R_t \) a temporal measurement region at a fixed location
\( \bar{R}_{l,s} \) a general measurement region
\( \sigma_s^2 \) the statistical sample variance of the space-mean speed
\( \sigma_t^2 \) the statistical sample variance of the time-mean speed
\( S \) the synchronised-flow region in three-phase traffic theory
\( \tau_i \) the reaction time of vehicle \( i \)'s driver
\( t \) a time instant
\( T_i \) the travel time of vehicle \( i \)
\( T_{mp} \) the duration of a measurement period
\( T(t_0) \) the experienced dynamic travel time, starting at time instant \( t_0 \)
\( \bar{T}(t_0) \) the experienced instantaneous travel time, starting at time instant \( t_0 \)
\( \bar{\tau}_c \) the capacity-flow speed
\( \bar{\tau}_{cap} \) the capacity-flow speed
\( \bar{\tau}_{ff} \) the free-flow speed
\( v_i \) the speed of vehicle \( i \)
\( v_{i,l} \) the speed of vehicle \( i \) in lane \( l \)
\( v_{i,l}(t) \) the speed of vehicle \( i \) in lane \( l \) at time \( t \)
\( v_{max} \) the maximum allowed speed (e.g., by an imposed speed limit)
\( \overline{v}_s \) the space-mean speed
\( \overline{v}_{x_c} \) the space-mean speed of the \( c \)-th substream
\( \overline{v}_{x_c}(t) \) an equilibrium relation between the SMS and the average space headway
\( \overline{v}_{x_c}(k) \) an equilibrium relation between the SMS and the density
\( \overline{v}_{x_c}(q) \) an equilibrium relation between the SMS and the flow
\( \overline{v}_{\text{sust}} \) the sustained speed during a period of high flow
\( \overline{v}_t \) the space-mean speed
\( \overline{v}_{t_c} \) the time-mean speed of the \( c \)-th substream
\( v(t, x) \) the local instantaneous vehicle speed at time instant \( t \) and location \( x \)
\( w \) the characteristic/kinematic wave speed (of a wide moving jam)
\( x_i \) the longitudinal position of vehicle \( i \)
\( X_i \) the distance travelled by vehicle \( i \)

**ACKNOWLEDGEMENTS**

Dr. Bart De Moor is a full professor at the Katholieke Universiteit Leuven, Belgium. Our research is supported by: Research Council KUL: GOA AMBioRICS, several PhD/postdoc & fellow grants, Flemish Government: FWO: PhD/postdoc grants, projects, G.0407.02 (support vector machines), G.0197.02 (power islands), G.0141.03 (identification and cryptography), G.0491.03 (control for intensive care glycemia), G.0120.03 (QIT), G.0452.04 (new quantum algorithms), G.0499.04 (statistics), G.0211.05 (Nonlinear), research communities (IC-CoS, ANMMM, MLDM), IWT: PhD Grants, GBOU (McKnow), Belgian Federal Science Policy Office: IUAP P5/22 (‘Dynamical Systems and Control: Computation, Identification and Modelling’), 2002-2006, PODO-II (CP/40: TMS and Sustainability), EU: FP5-Quprodis, ERNSI, Contract Research/agreements: ISMC/IPCOS, Data4s,TML, Elia, LMS, Mastercard.

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[122] Note that when calculating the total density using equation 3, the partial densities can also correspond without loss of generality to different vehicle classes instead of just different lanes 27,115.

[123] In most cases, volume denotes the number of vehicles counted during a certain time period, as opposed to flow which is just the equivalent hourly rate.

[124] Note that the hypothesis also assumes that the variables are spatially measured, e.g., space-mean speed.

[125] http://www.telematicscluster.be

[126] Note that in a broader sense, queueing delays also encompass delays at signalised and unsignalised intersections.

[127] The TRB was formerly known as the Highway Research Board (HRB).

[128] Note that this capacity flow is not an extreme value, i.e., it can be different from the maximum observed flow. The reason is that, with respect to the nature of the fundamental diagram, the capacity flow is taken to be an average value 44,72.

[129] The detector station is called CLO3, which is an acronym for ‘Camera Linkeroever’.

[130] But note that bottleneck-induced traffic flow breakdowns are not excluded by the theory of Kerner et al.

[131] In addition, they also provided a link with Kerner’s three-phase theory, whereby synchronised flow can correspond to HCT, OCT, or PLC, and moving jams can correspond to TSG or MLC states 49.