GPS Rapid Static and Kinematic Positioning Based on GPS Active Network

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1 Introduction

The traditional GPS kinematic positioning is based on single reference station. It is mainly used for short baselines (i.e., < 20 km) when the GPS errors are spatially correlated. It is difficult to achieve high accuracy and reliability when baselines become longer or there is a strong ionospheric activity, as the propagation errors cannot be cancelled out by the difference method. In recent years, using a GPS reference network to improve the precise static and kinematic positioning efficiency has been paid much attention to [1-4]. Most of the researches concentrate on the medium size network (with baselines over 100 km) in order to reduce the number of reference stations. The basic principle of the network approach is to determine the spatial distribution of main GPS errors (i.e., propagation errors), then GPS errors at a user location can be predicted as the network corrections. In general, there are two approaches to generate the network corrections. The first approach tries to separate the total error into different components, such as orbital, ionospheric and tropospheric errors (similar to WADGPS) [1, 3], while another approach directly apply a total correction to the user's observation [4], which is similar to DGPS. There are two types of positioning methods with GPS reference networks. One is so called the 'virtual reference station' method which simulates an observation file at a location close to the user that consider all error effects on the basis of reference network estimation [1, 3]. As the virtual reference station is close to the user station, the data is just like a super-short baseline and easy can be processed by any conventional GPS software. Another method is to predict the total error effects at the user location and then to remove them from observation at the user station [2].
Hong Kong Lands Department is implementing a project to extend the scope of the existing permanent GPS reference stations to an active control system\(^1\). There are totally 13 stations which cover the whole territory and the station spacing is about 10 to 15 km. Currently 6 reference stations (Fig. 1) have been established and dual frequency GPS data are collected continuously everyday. The objective of the active network is to provide a fundamental geodetic infrastructure for engineering surveying (RTK and rapid static surveying), DGPS, and scientific research (deformation monitoring, atmosphere monitoring). The reason for the design of such a dense network is to ensure the users to achieve cm-level positioning accuracy within a short period even using only one low cost single frequency receiver.

However, as Hong Kong is located at low latitude (22°N) and may experience strong ionospheric activities. Even with short baselines (< 10 km), the integer ambiguity is still very difficult to resolve. In this study, we try to use the reference network to estimate the errors in GPS data, then to apply the corrections to the user’s data. As the reference station is shortly spaced in Hong Kong GPS active network, an epoch-by-epoch and satellite-by-satellite network correction algorithm is adopted. In Section 2 of this paper, the algorithms for ambiguity resolution between the reference stations, the error correction calculation and network correction generation will be described.

2 GPS network correction generation

GPS network correction is referred to the error correction at a user location calculated by the GPS reference network observation and the precise coordinates of the reference stations. The correction can be generated in real time, near real time or post-process according to the methods adopted and the baseline length between the reference stations. No matter what approach is adopted, positioning with a reference network can be summarized in the following steps. The first step tries to fix the integer ambiguities between reference stations using the precise coordinates of the stations. The second step determines the errors in GPS data of the reference network. Then the errors are interpolated at a user location. Finally the user applies the corrections to its data and uses conventional GPS processing techniques.

2.1 Ambiguity determination between the reference stations

In order to obtain the network correction, first the ambiguities between the reference stations have to be resolved. For high accuracy positioning, the double difference (DD) observations are commonly used. The double difference observation for \( L_1 \) and \( L_2 \) can be expressed as \(^6\)

\[
\lambda_1 \Delta D (\phi_1) = \Delta D (\rho) + \lambda_1 N_1 - \Delta Ion_1 + \Delta Trop + \epsilon_1
\]

\[
\lambda_2 \Delta D (\phi_2) = \Delta D (\rho) + \lambda_2 N_2 - \Delta Ion_2 + \Delta Trop + \epsilon_2
\]

where the subscript for station and satellite are omitted, and \( \Delta D (\cdot) \) is the double difference for (\(\cdot\) ); \( \phi \) is the carrier phase observable (cycle); \( \rho \) is the geocentric distance between the station and the satellite(m); \( N \) is the DD carrier phase ambiguity (cycle); \( \lambda \) is the wavelength of the carrier phase (m); \( \Delta Ion \) is the ionospheric refraction (m); \( \Delta Trop \) is the tropospheric delay (m); \( \epsilon \) is the

Fig. 1 The layout of the existing Hong Kong GPS active network

Data analysis and results using one-day observa-

tion from the active network are presented in Sec-

 tion 3. The conclusions and suggestions are given in Section 4 of this paper.
measurement noise.

Because the precise coordinates of the reference stations are known, the main concerns for ambiguity determination between low reference stations are baseline length or other error effects. The tropospheric delay can be modeled with a standard model (such as Hopfield Model) with the meteorological measurements at sites. The ambiguity $N$ of $L_1$ and $L_2$ can then be calculated if the other errors are ignored:

$$N = \frac{DD(\phi_1) - DD(\phi_2) + N_w}{2}$$

For short baselines, the approximate $N$ will normally be close to an integer (the fraction part within 0, 3 cycles), as the double difference procedure cancels most of GPS errors. For the observation data collected in March, 2001 in Hong Kong GPS active network, however, we found that the integer ambiguities for independent $L_1$ and $L_2$ observation can not be obtained with the above method in most of the time in a day. The estimated ambiguities using Eq. 3 vary greatly, sometimes may reach a few cycles [5].

In this study, the following algorithm is applied for the ambiguity resolution among reference stations. First we try to determine the widelane ambiguity based on Eq. (4).

$$N_w = N_1 - N_2 = \frac{DD(\phi_1) - DD(\phi_2) - DD(\rho)}{\lambda_w}$$

where $\lambda_w$ is the wavelength of the widelane combination.

Due to the long wavelength (0.86 m) and low ionospheric effect, the widelane ambiguity is much easier to determine than $L_1$ or $L_2$. In data processing, we always select the shorter independent baselines in the network to minimize the errors in double difference observations. Meanwhile, to further reduce noises in the measurements, the estimated widelane ambiguity using Eq. (4) can also be smoothed by averaging over a period of time. After the widelane ambiguity is determined, we make use of the ionosphere-free combination and widelane ambiguity to determine $L_1$ and $L_2$ ambiguities, as expressed in Eq. (5):

$$\begin{cases} N_1 = \frac{f_1}{f_{w}} DD(\phi_1) - \frac{f_2}{f_{w}} (DD(\phi_2) + N_w) - \frac{DD(\rho)}{c} (f_1 + f_2) \\ N_2 = N_1 - N_w \end{cases}$$

where $f_w$ is the frequency of the widelane combination and $c$ is the speed of light.

With Eq. (5), the ionospheric delay has been eliminated. However, other errors such as the tropospheric residual and measurement noises remain in the data. More reliable result may be obtained by averaging over a period of time. As the observation is collected continuously all the time, the cycle slip detection and recovery are not a serious problem for reference stations.

Because the ambiguity determination is performed at epoch-by-epoch and satellite-by-satellite base, after $L_1$ and $L_2$ integer ambiguities are fixed, a reliability test is needed to check the correctness of the ambiguity fixing. Normally, the real ambiguities calculated by Eq. (5) are within a few cycles of the true values, then we examine all combinations of ambiguities for all satellites as conventional ambiguity searching methods. By fixing the ambiguities, we solve for the coordinates of reference stations again using ionosphere-free combination. The estimated coordinates with the true ambiguity combination should be closed to ‘true’ coordinates of the reference stations. The Fisher ratio test is also applied to the residuals for the validation.

2.2 Network correction

After the ambiguity for $L_1$ and $L_2$ have been determined, the network correction can be calculated. Firstly the corrections based on baselines among the reference network are obtained by simply calculating the residuals by fixing the ambiguities to their integers. Then the corrections generated by the network are interpolated to the user location. As we use the double difference (DD) observation for positioning, the correction is also based on double difference observables.

1) DD residual correction

With the precise coordinates of the reference station and $L_1$ and $L_2$ ambiguities, the DD residuals
can be expressed as
\[
\text{Res}_1 = \lambda_1 DD(\phi_1) - (DD(\rho) + \lambda_1N_1 + \Delta Trop)
\]
(6)
\[
\text{Res}_2 = \lambda_2 DD(\phi_2) - (DD(\rho) + \lambda_2N_2 + \Delta Trop)
\]
(7)
The DD residuals consist of all errors, such as ionospheric errors, tropospheric residual, orbit error, clock error and measurement errors. It is the sum of total error effects. Depending on the baseline length, the main parts of the correction are ionospheric error, tropospheric errors and orbit errors.

2) DD ionospheric correction

As the spacing of Hong Kong GPS network is short and the ionospheric delay is the main problem for GPS data, alternatively we can also calculate double difference ionospheric correction separately:
\[
\Delta \text{Ion}_1 = \frac{f_1^2}{f_1^2 - f_2^2}[\{(\lambda_1 DD(\phi_1) - \lambda_2 DD(\phi_2)) - (\lambda_1N_1 - \lambda_2N_2)\}]
\]
(8)
\[
\Delta \text{Ion}_2 = \frac{f_1^2}{f_2^2}\Delta \text{Ion}_1
\]
(9)
With this method, we combine the observations to eliminate the range between receiver and satellite and only observation noises remains. Therefore, Eqs. (8) and (9) can be also used to study the ionospheric activity in the region of active network.

3) Correction interpolation

After all the corrections in the network have been calculated, the correction for the user station within the network has to be obtained in order to remove all or part of errors in the baseline observation. Considering the spatial distribution of the errors, the DD residual or DD ionospheric refraction measured from reference stations need to be interpolated at the user station. As Hong Kong network spacing is relatively short and up to now there are only 6 reference stations, a simple plane linear method is used:
\[
f(x, y) = a_1dx + a_2dy
\]
(10)
where \((x, y)\) is the horizontal coordinate of the reference station; \((dx, dy)\) is the relative plane coordinate from reference station to fiducial reference station; \(f(x, y)\) is the measurement (DD ionospheric refraction or DD residual); \(a_1, a_2\) are the coefficients to be estimated.

With three reference stations (including one master station) the coefficients can be obtained, which can then be used to generate network correction when the approximate coordinate of the user station is known, i.e. from pseudorange solution.

3 GPS rapid static and kinematic positioning with GPS active network

The following is an example to demonstrate the efficiency of the algorithms discussed in this paper. The 24-hour GPS data (with a 5 seconds interval) used for the test were obtained from the existing Hong Kong GPS network on 4 March 2001. Station HKFN is selected as the fiducial reference station, HKKT is used as a user station and baseline HKFN-HKKT (9.2 km) is used as the example.

Using Eq. (4) and Eq. (5), the widelane and \(L_1\) ambiguities for all satellites observed can be estimated for baseline HKFN-HKKT (Fig. 2 and Fig. 3 respectively). For simplicity, only the fraction parts of ambiguity estimations are shown in Fig. 2 and Fig. 3. It can be seen in Fig. 2 that almost all the \(L_w\) ambiguity variation are within 0.5 cycles while in the period of 0-3 UTC and 16-24 UTC hour (local time 0-11 am, and normally in this period the ionosphere is quiet or inactive) the variation is more smooth than in other period. However during 4-16 UTC hour (local time 12-24 pm, and in this period ionosphere activity is normally strong), strong variations can be seen and sometimes the ambiguities may reach 0.5 cycles. In this case the epoch-by-epoch \(L_w\) ambiguity determination may not provide a reliable answer especially when the baseline becomes longer. Test shows, in all cases, that the correct widelane ambiguities can be determined after averaging data for a few minutes.

Compared with the widelane ambiguities, the real-value \(L_1\) ambiguity variations in Fig. 3 is more smooth all the time but much noisier. Although most of them are within 0.5 cycles, there are still many epochs while the fraction of ambiguities exceed 0.5 cycles. By averaging over a period of time,
both $L_1$ and $L_2$ ambiguities can be obtained. DD residuals agree within 0.15 cycles, with a few peaks less than 0.5 cycles, which may be caused by the irregular error change for a satellite (as shown in Fig. 4(c)).

After the ambiguities for all reference stations have been resolved, the $L_1$ DD corrections for all baselines in the network related to the base reference station can be calculated using either Eq. (6) or Eq. (8). In Eq. (6), all error sources are considered, while in Eq. (8) only ionospheric error is considered. Then the network corrections at the user location can be interpolated using Eq. (10).

Fig. 4 gives a comparison between measured DD residuals (Eq. (6)) and the predicted DD residuals based on the reference network. Fig. 4(a) shows the measured $L_1$ DD residuals of all satellites pair for baseline HKFN-HKKT, while Fig. 4(b) gives the interpolated $L_1$ DD residuals based on the data from other reference stations in the reference network. The difference between them is given in Fig. 4(c). It can be seen in Fig. 4(a) that during the day there is significant variation of $L_1$ DD residuals, some times over 1 cycle. Without proper corrections, it would be very difficult to resolve the integer ambiguity. The interpolated and measured $L_1$ DD residuals agree within 0.15 cycles, with a few peaks less than 0.5 cycles, which may be caused by the irregular error change for a satellite (as shown in Fig. 4(c)).

Similar to $L_1$ DD residual correction, the $L_1$ ionospheric error are also calculated and interpolated according to Eq. (8). Fig. 5 compares the accuracy of the interpolated DD ionospheric delay with each other for all satellites. Compared with Fig. 4, it is very similar in both the shape and the size. Thus this clearly indicates that the major error in the Hong Kong active network is caused by the ionospheric activity.
From the above analysis, it is clearly shown that the network correction can considerably reduce the observation errors at the user station of the network. Here we just choose \( L_1 \) data for positioning test to show the improvement in ambiguity resolution and positioning accuracy. Both the rapid static positioning and kinematic positioning tests are performed.

Using the GPS rapid static positioning software developed at LSGI, baseline HKFN-HKKT is processed with the conventional GPS processing method and with the method discussed in this paper based on the network correction. The 24-hour data set for baseline HKFN-HKKT is divided into every 15 minutes. We try to solve the baseline with every 15-minute data. The hourly successful rates of ambiguity resolution with 15-minute GPS observation data in a day by both methods are shown in Fig. 6(a) and Fig. 6(b), respectively. It can be seen in Fig. 6(a) that only about 46% of 15-minute sessions while the integer ambiguities can be resolved with conventional method, mainly at the time when the ionosphere is quiet. After applying the network corrections developed in this study, the success rate for ambiguity resolution for the same baseline reaches 100% (Fig. 6(b)).

The improvements of positioning accuracy by different network corrections are given in Table 1. Compared with the conventional positioning method, the positioning accuracy is improved from 2 cm to 5 mm horizontally and from 4 cm to 3 cm vertically. As discussed in this paper the ionospheric error is the main errors source in the DD observation, either DD residuals or DD ionospheric delay may be used to generate network correction for the Hong Kong active network. Table 1 shows that there are no significant differences between the two methods.
Table 1  The 15-minute rapid static positioning improvement with different methods

| Method               | Success rate (\%) | Rms N/cm | Rms E/cm | Rms H/cm |
|----------------------|-------------------|----------|----------|----------|
| Conventional method  | 46                | 2.0      | 2.0      | 3.8      |
| Iono. correction     | 100               | 0.4      | 0.5      | 2.9      |
| L_i DD res. correction | 100             | 0.4      | 0.5      | 2.9      |

Because the network corrections are generated epoch-by-epoch, they can also be used for kinematic positioning. This time, we apply the network correction to the baseline HKFN-HKKT and then estimate the positions at every epoch to simulate a kinematic situation. Fig. 7 shows the positioning errors of every epoch. The overall rms in north, east and height are 1 cm, 1 cm and 3 cm, respectively.

Fig. 7  The kinematic positioning result with network corrections

4 Conclusions and suggestions

In this paper a data processing strategy for GPS rapid static and real time kinematic positioning using GPS reference network has been presented. The method contains three steps. Firstly the integer ambiguities among baselines within the reference network are determined. Then the DD residuals (or ionospheric residuals in the case of Hong Kong as ionospheric errors are the main problem for GPS data processing) can be calculated and interpolated at the user location. After applying the network correction to the user observation, conventional GPS processing methods can be used to determine the user position, in either rapid static or kinematic modes. The experiment shows that this method is very useful in the regions where the ionospheric activity is active. After applying these corrections to a user station observation, both the positioning accuracy and the success rate for ambiguity resolution can be significantly improved. In the example of the rapid static positioning of 15-minute observation, the success rate for ambiguity fixing has been improved from 46\% to 100\%, while the positioning accuracy is improved from 2 cm to 4mm-5mm horizontally and from 3.8 cm to 2.9 cm vertically.

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