Trade and the Location of Two Industries: A Two-Factor Model

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We study a two-country two-factor model with free entry and monopolistic competition. There are two industries employing immobile labor as fixed input and mobile capital as marginal input. Firms cannot move across countries, but only move across industries within a country. The two industries can differ in three aspects: factor intensities, transport costs and demand elasticities. The two countries are identical except for size. The production specialization and trade pattern are the results of the interaction of two effects: the market access effect and the wage differential effect.

KEYWORDS: market access effect, production specialization, trade pattern, wage differential effect

1. Introduction

The last few decades have witnessed a remarkably large reduction of trade costs and a closer economic integration. The European Union (EU) is unquestionably the most integrated regional trade agreement (RTA) in the world. Although member countries hope that deep integration will accelerate their economies, the impacts of trade integration on various industries are still controversial.

The effects of country size differences have been a focus of attention for many years in the literature of new trade theory (NTT) (e.g. Markusen, 1981; Markusen and Melvin, 1981; Krugman, 1980; Helpman and Krugman, 1985) and new economic geography (NEG) (e.g. Krugman, 1991; Krugman and Venables, 1995). The most famous result in this area is the home market effect (HME) of Krugman (1980) and Helpman and Krugman (1985). They study a world economy producing differentiated varieties subject to internal increasing returns to scale. When the differentiated products are costly to trade, firms concentrate in large countries in order to save transport costs.

Some subsequent studies have tried to assess the robustness of the home market effect and to analyze the influence of this effect on economic distribution and trade pattern by using alternative assumptions. For example, Davis (1998) investigates whether the result of home market effect depends on the assumption of zero transport costs for the homogeneous good. He shows that in a focal case in which differentiated and homogeneous goods have identical transport costs, the home market effect disappears. Moreover, several studies examine the economic location issue by incorporating a more realistic agricultural sector. For instance, keeping the usual immobile labor sector, Takatsuka and Zeng (2012a, 2012b) build a general equilibrium model with general agricultural transport costs. Takatsuka and Zeng (2012a) show the exact threshold value of the agricultural transport cost for the HME in terms of firm share to occur. In contrast, Takatsuka and Zeng (2012b) demonstrate that the HME always exists for any agricultural transport costs by including mobile capital as the second production factor.

Another notable study of Takahashi et al. (2013) successfully revisits the home market effect by removing the homogeneous good in the footloose capital model. They show that the larger country has a higher wage rate and the wage differential evolves in a bell shape in terms of transport costs.

This article aims to study how the size of a country matters in determining its production specialization and trade pattern with multiple industries. Specifically, we try to answer the following questions: how does a difference in size between two countries affect the equilibrium wages of these countries? What is the impact of transport costs on the wage differential? How does a continuous fall in transport costs affect the production specialization and trade pattern?

Some of these issues are firstly addressed in Amiti (1998). Amiti studies the relation between country size and inter-industry trade in a general equilibrium model with two countries, two imperfectly competitive industries. The two countries are identical except for size. Transport costs affect both industries. The production specialization and trade pattern are results of interplay of two effects: the “market access” effect and “production cost” effect. The latter attracts
firms to the smaller country due to lower wages. The prices in the smaller country are lower than in the larger country in each industries.

Another study of Laussel and Paul (2007) investigate the issues by a two country and one factor model. The only factor is labor which can freely move across industries but cannot move internationally. They show that if the two countries are sufficiently close in size and demand elasticities differ across industries, a continuous fall in transport costs is associated with a reversal in the pattern of trade at some intermediate level. On the other hand, if the two countries are very different in size and demand elasticities differ across industries, the larger country is always a net exporter of the less differentiated good.

Chen and Zeng (2014) demonstrate an evolution of spatial sorting of high- and low-quality firms in a general equilibrium model with segmented preferences and firm heterogeneity in demand elasticity. They show that in two regions with identical population sizes, high-quality firms agglomerate in the region that accommodates more high-skilled labor, whereas low-quality firms move to the region that hosts more low-skilled labor in response to trade liberalization.

Since Martin and Rogers (1995), most existing footloose capital models assume that capital is the fixed input and labor is the marginal input in the manufacturing production. A merit of this setup is that firms’ relocation can be described by the mobile capital. In a recent paper of Peng and Zeng (2014), their roles are exchanged to analyze the impact of globalization on an economy without industrial relocation. Their model turns out to be more tractable. We now apply their idea to investigate the location of two industries for three reasons. First, although the total number of firms in a country becomes constant because of the labor immobility across countries, the firms can relocate across industries. This model is expected to provide more detailed information of industrial relocation in the domestic market. Second, this approach can examine the robustness of the results based on the existing models. Third, in some manufacturing industries (e.g. the design and processing industry of platinum, gold, and silver jewelry), workers with intellectual property or know-how are employed as fixed input while capital taking the form of raw materials is used as marginal input.

As in Amiti (1998) and Laussel and Paul (2007), there are two kinds of driving forces in this article. The first one is “wage differential effect” first called in Laussel and Paul (2007), which is similar to the “production cost effect” of Amiti (1998). In our paper, this effect is quite different from theirs. The immobile labor is chosen as marginal input in both of these two papers, therefore, the prices are lower in the smaller country in each industry. By contrast, mobile capital is chosen as the only marginal input thus the prices equal across countries in each industry in this paper. The wage differential effect in this article arises only from the different fixed costs. We seek to contribute to the literature by showing that if prices equal across countries in each industry, some new results can be generated.

The wage differential evolves in a bell shape regarding transport costs both in Amiti (1998) and in this article. However, this property is totally tractable in our model when two industries only differ in factor intensities. While the results in Amiti (1998) are restricted in interior equilibrium, we extend it to the corner equilibrium. We show the home market effect in terms of wages not only in interior equilibrium but also in corner equilibrium. When two industries only differ in demand elasticity, Amiti (1998) shows that the smaller country is more specialized in the production of low-elasticity goods and hence a net exporter of low-elasticity goods at transport cost level close to autarky. In contrast, when transport cost level is close to autarky, the market access effect dominates in this article. Thus, the larger country is more specialized in the production of low-elasticity goods at transport cost level close to autarky. Our results of trade pattern show that the larger country is a net exporter of both industry goods when trade is close to autarky. The net export of high-elasticity goods increases and that of low-elasticity goods decreases when trade is close to free. Our results of trade pattern contrast with Laussel and Paul (2007), who employ a one-factor model. The result of trade pattern in Laussel and Paul (2007) depends on the differential of country size. Since the immobile labor is the only factor in Laussel and Paul (2007), if the two countries are very different in size, most of the firms are located in the larger country. Thus, the larger country is always a net exporter of the low-elasticity goods.

When two industries only differ in transport costs, Amiti (1998) shows that the smaller country is always more specialized in the production of low transport cost goods and hence a net exporter of low transport cost goods. In contrast, in this article our results show that when transport costs are close to autarky level, the larger country is more specialized in the production of low transport cost goods. The results of trade pattern are similar to the case that two industries differ only in terms of demand elasticity.

The rest of the paper is constructed as follows. Section 2 sets up the model. Section 3 studies the equilibrium of the model. Section 4 accesses the wage differential effect. Section 5 analyzes the pattern of trade and production specialization. Finally, Section 6 makes some concluding comments.

2. The Model

There are two manufacturing industries which employ labor, \( L \), and capital, \( K \). To rule out the H-O comparative advantage, each consumer is assumed to hold one unit of capital, so \( K = L \). Capital is perfectly mobile between the countries whereas labor can only move within a country. The two imperfectly competitive industries are labeled by
The population share in Home, \( \theta \), is assumed to be larger than 1/2. We denote variables pertaining to Foreign by an asterisk (*)

### 2.1 Consumers

Consumers in both countries have a Cobb-Douglas preference, given by \( U = C_1^{0.5} C_2^{0.5} \), where

\[
C_k = \left[ \sum_{i} n_i \frac{c_{ki}^{\sigma_k}}{\sigma_k} + \sum_{j} \left( \frac{m_{kj}}{\tau_k} \right)^{1-\sigma_k} \right]^{1/\sigma_k} 
\]

is the aggregate consumption of industry \( k \) goods produced in both countries. To focus on the impact of country size on production specialization and trade pattern, we keep other aspects of these two industries as symmetric as possible. Thus, consumers here have a preference with the same expenditure share on each industry goods. This assumption also helps to simplify mathematical expressions. In Eq. (2.1), \( \sigma_k > 1 \) is the elasticity of substitution between any pair of differentiated goods in industry \( k \), \( n_k \) is the number of industry \( k \) goods produced in the home country, \( c_{ki} \) is consumption in the home country of industry \( k \) good \( i \) produced in the home country, and \( m_{kj} \) is the amount of good \( j \) in industry \( k \) shipped from the foreign to the home country.

Let \( \phi = \phi_2 \equiv \tau_2^{1-\alpha_2} \) be the trade freeness of industry 2, and we assume \( \tau_1 = \tau_2^{\beta} \) with \( \beta \geq 1 \). The trade freeness of industry 1 then becomes \( \phi_1 = \phi^{\frac{\beta(n_2-1)}{n_2}} \). If \( \beta = 1 \), two industries have identical transport costs; if \( \beta > 1 \), transport costs of industry 1 goods are higher than that of industry 2 goods.

The price index is given as

\[
P_k = \left[ \sum_{i} n_i \left( P_{ki}^{1-\sigma_k} \right) + \sum_{j} \left( p_{kj}^* \right)^{1-\sigma_j} \phi_j \right]^{1/\sigma_k} ,
\]

where \( p_{ki} \) is the producer price set by firm \( i \) industry \( k \) in the home country and \( p_{kj}^* \) is the producer price set by firm \( j \) in industry \( k \) in the foreign country. Maximization of the utility function subject to the budget constraint allocates expenditure between the two industries as follows:

\[
P_1 C_1 = \frac{1}{2} Y, \quad P_2 C_2 = \frac{1}{2} Y. \tag{2.3}
\]

The consumer maximizes the sub-utility function subject to the budget constraint to derive demand functions for each industry 1 good produced in the home country and in the foreign country:

\[
c_{i} = \frac{1}{2} P_{ki}^{-\alpha_1} P_1^{\sigma_1 - 1} Y, \quad m_{ij} = \frac{1}{2} \tau_1^{1-\sigma_1} p_{kj}^* p_1^{\sigma_1 - 1} Y \tag{2.4}.
\]

The demand functions for industry 2 goods are derived in the same way.

### 2.2 Firms

Borrowing the technology from Peng and Zeng (2014), production for any variety of each firm requires \( \delta_k \) units of immobile labor as the fixed input and mobile capital as marginal input. We choose unit of products so that one unit of mobile capital is required to produce one unit of each variety as the marginal input. And it is clear that capital can be employed in two countries. Due to the free mobility of capital, the capital returns in two countries are equalized at equilibrium. We take this capital return as numeraire. As argued in Peng and Zeng (2014), this production technology describes the situation that labor is used to “design the production line” (Peng et al., 2006) that captures the diversity of differential products, while capital is used to produce each unit of the product, whose amount is dependent on the quantity of firms’ output. For example, in the design and processing industry of platinum, gold, and silver jewelery, workers with highly trained skills and know-how are employed as fixed input while capital takes the form of raw materials and is used as marginal input. We assume Samuelson’s iceberg international transportation costs: \( \tau (\geq 1) \) units of a manufactured good must be shipped for one unit of requirement in the other country. The production of each variety is split into domestic and foreign markets. A firm in industry \( k \) producing variety \( i \) in Home sets prices of goods to maximize its profit as

\[
\Pi_{ki} = p_{ki} X_{ki} - (\delta_k w + X_{ki}),\tag{2.5}
\]

where \( w \) is the nominal wage rate of each worker in Home.

Firms maximize profits with respect to quantity gives the optimal price as:
\[ p_{ki} = \frac{\sigma_k}{\sigma_k - 1}. \] (2.6)

Imposing the free entry and exit condition, by setting profits equal to zero, determines the quantity of output required to cover fixed cost:

\[ X_{ki} = (\sigma_k - 1)w\delta_k. \] (2.7)

The firms are assumed to be symmetric, so Eq. (2.7) does not depend on the firm name. Notations \( c_{ki}, m_{ki}, p_{ki}, \) and \( p^*_{ki} \) are simplified as \( c_k, m_k, p_k, \) and \( p^*_k, \) respectively.

3. Equilibrium of the Model

In this section, we solve the equilibrium of the model and obtain the equations of equilibrium distribution of firms which will be used later for studying the production specialization and trade pattern.

3.1 Equilibrium of the factor market

We turn to the factor market equilibrium conditions. First of all, the labor market equilibrium condition is

\[ n_1\delta_1 + n_2\delta_2 = L\theta, \quad n_1^*\delta_1 + n_2^*\delta_2 = L(1 - \theta). \] (3.1)

The left hand sides of the equations are total labor employment and the right hand sides are total labor supply for each country. On the other hand, the capital market equilibrium condition is

\[ (\sigma_1 - 1)\delta_1(n_1w + n_1^*w^*) + (\sigma_2 - 1)\delta_2(n_2w + n_2^*w^*) = L. \] (3.2)

3.2 The equilibrium distribution of firms

We need the product market equilibrium to close the model. If Home (resp. Foreign) accommodates some firms of industry \( k = 1, 2, \) then

\[ X_k = c_k + m_k^*, \quad (\text{resp. } X_k^* = c_k^* + m_k^*). \] (3.3)

holds. The price indexes are

\[ P_k = \frac{\sigma_k}{\sigma_k - 1}(n_k + n_k^*\phi_k)^{1/\sigma_k}, \quad P_k^* = \frac{\sigma_k}{\sigma_k - 1}(n_k^*\phi_k + n_k^*)^{1/\sigma_k}, \] (3.4)

where \( \phi_k = \tau_k^{1-\sigma_k}. \) Each country is endowed with equal capital to labor ratios, hence

\[ Y = L\theta(w + r), \quad Y^* = L(1 - \theta)(w^* + r*), \] (3.5)

where \( r \) and \( r^* \) are the capital prices in two countries, which are equal by the perfect mobility assumption.

Plugging Eqs. (2.2), (2.4), (2.7), (3.4), and (3.5) into (3.3), we obtain the following four equations

\[ \frac{L\theta(w + 1)}{n_k + n_k^*\phi_k} + \frac{L(1 - \theta)(w^* + 1)\phi_k}{\phi_k n_k + n_k^*} = 2\sigma_k w\delta_k, \quad k = 1, 2 \] (3.6)

\[ \frac{L\theta(w + 1)\phi_k}{n_k + n_k^*\phi_k} + \frac{L(1 - \theta)(w^* + 1)}{\phi_k n_k + n_k^*} = 2\sigma_k w^*\delta_k, \quad k = 1, 2 \] (3.7)

in an interior equilibrium. By multiplying (3.6) by \( n_k, \) (3.7) by \( n_k^* \) and adding them up, we obtain

\[ \sigma_1\delta_1(n_1w + n_1^*w^*) = \sigma_2\delta_2(n_2w + n_2^*w^*). \]

By use of (3.2), we thus have

\[ n_1w + n_1^*w^* = \frac{L\sigma_2}{\delta_1(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)}, \] (3.8)

\[ n_2w + n_2^*w^* = \frac{L\sigma_1}{\delta_2(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)}, \] (3.9)

\[ \theta w + (1 - \theta)w^* = \frac{\sigma_1 + \sigma_2}{2\sigma_1\sigma_2 - \sigma_1 - \sigma_2}. \] (3.10)

Equation (3.10) is the average labor income. Adding the capital returns, we obtain the total average income as

\[ 1 + \theta w + (1 - \theta)w^* = \frac{2\sigma_1\sigma_2}{2\sigma_1\sigma_2 - \sigma_1 - \sigma_2}. \]

Since labor is the fixed input of production, its share in the total production is
\[ \frac{\sigma_1 + \sigma_2}{2 \sigma_1 \sigma_2} = \frac{1}{2} \left( \frac{1}{\sigma_1} + \frac{1}{\sigma_2} \right). \]

It is well known that the share of the fixed input is 1/\( \sigma \) in the case of a single industry. Interestingly, we find the similar property in which the share becomes the average of two industries.

Meanwhile, since the RHS of (3.10) is constant, this equation shows that \( w \) is negatively related to \( w^* \). Namely, \( w \) increases if and only if \( w^* \) decreases.

Equations (3.1), (3.2), (3.8), (3.9) and (3.10) provide simple expressions for variables \( n^*_1, n_2^*, n_2^* \), \( w^* \) in terms of \( n \) and \( w \):

\[ n^*_1 = \frac{L \sigma_2 - (2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2) n_1 w \delta_1}{\sigma_1 + \sigma_2 - (2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2) \theta \delta w \delta_1} \left(1 - \frac{1}{\delta} \right), \tag{3.11} \]

\[ n_2 = \frac{\theta L - \delta_1 n_1}{\delta_2}, \tag{3.12} \]

\[ n^*_2 = \frac{L(1 - \theta) - \delta_1 n_1^*_1}{\delta_2} = \frac{1 - \theta L}{\delta_2} - \frac{L \sigma_2 - (2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2) n_1 w \delta_1}{\sigma_1 + \sigma_2 - (2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2) \theta \delta_1} \left(1 - \frac{1}{\delta} \right), \tag{3.13} \]

\[ w^* = \frac{\sigma_1 + \sigma_2}{(2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2)(1 - \theta)} - \frac{\theta}{1 - \theta} w. \tag{3.14} \]

Note that \( n_1 \) and \( w \) can be obtained from Eqs. (3.6) and (3.7) for two different industries.

Equation (3.12) and the first equality of (3.13) show that the pair of \( n_1 \) and \( n_2 \) and the pair of \( n^*_1 \) and \( n^*_2 \) are negatively related.

Finally, in an interior equilibrium, taking the ratio of the equilibrium product market conditions for industry \( k \) in Foreign to Home derives

\[ \frac{\theta(w + 1) \left( \frac{n_k}{n^*_k} \phi_k + 1 \right) + \phi_k (1 - \theta)(w^* + 1) \left( \frac{n_k}{n^*_k} \phi_k + 1 \right)}{\phi_k \theta(w + 1) \left( \frac{n_k}{n^*_k} \phi_k + 1 \right) + (1 - \theta)(w^* + 1) \left( \frac{n_k}{n^*_k} \phi_k + 1 \right)} = \frac{w}{w^*}. \tag{3.15} \]

### 3.3 Autarky and free trade

In the case of autarky, we have \( \phi_k = 0 \). Equation (3.6) degenerates to \( L \theta \delta_1 w / n_k = 2 \sigma_1 \delta w \) for \( k = 1, 2 \). By use of (3.12), we can solve out \( n_1 \) and \( w \). Meanwhile, Eqs. (3.7), (3.11)–(3.14) can be used to solve all other variables. The solution is

\[ w = w^* = \frac{\sigma_1 + \sigma_2}{2 \sigma_1 \sigma_2 - \sigma_1 - \sigma_2}, \tag{3.16} \]

\[ n_1 = \frac{L \phi \sigma_2}{\delta_1 (\sigma_1 + \sigma_2)}, \quad n_2 = \frac{L \phi \sigma_1}{\delta_2 (\sigma_1 + \sigma_2)}, \quad n^*_1 = \frac{L(1 - \theta) \sigma_2}{\delta_1 (\sigma_1 + \sigma_2)}, \quad n^*_2 = \frac{L(1 - \theta) \sigma_1}{\delta_2 (\sigma_1 + \sigma_2)}. \tag{3.17} \]

The results of (3.17) reveal that \( n_1/n_2 = n^*_1/n^*_2 \). Therefore, the larger country is just a scale expansion of the smaller country in the case of autarky. In fact, since there is no trade, the demand and supply of each industry in each country are balanced. On the other hand, each firms in each industry produces the same amount of output in equilibrium.

To show how wages and firm location change at \( \phi = 0 \), we calculate the derivatives of these important variables as follows.

\[ \frac{\partial \phi}{\partial \phi} \bigg|_{\phi=0} = \frac{2(2\theta - 1)\sigma_1^2 \sigma_2}{(2\sigma_1 \sigma_2 - \sigma_1 - \sigma_2)^2} > 0, \tag{3.18} \]

\[ \frac{\partial \phi^*}{\partial \phi} \bigg|_{\phi=0} = -\frac{2(2\theta - 1)\sigma_1^2 \sigma_2}{(2\sigma_1 \sigma_2 - \sigma_1 - \sigma_2)^2} < 0, \tag{3.19} \]

\[ \frac{\partial n_1}{\partial \phi} \bigg|_{\phi=0} = -\frac{L(2\theta - 1)\sigma_1 \sigma_2}{\delta_1 (\sigma_1 + \sigma_2)^2} < 0, \quad \frac{\partial n_2}{\partial \phi} \bigg|_{\phi=0} = \frac{L(2\theta - 1)\sigma_1 \sigma_2}{\delta_2 (\sigma_1 + \sigma_2)^2} > 0, \tag{3.20} \]

\[ \frac{\partial n^*_1}{\partial \phi} \bigg|_{\phi=0} = \frac{L(2\theta - 1)\sigma_1 \sigma_2}{\delta_1 (\sigma_1 + \sigma_2)^2} > 0, \quad \frac{\partial n^*_2}{\partial \phi} \bigg|_{\phi=0} = -\frac{L(2\theta - 1)\sigma_1 \sigma_2}{\delta_2 (\sigma_1 + \sigma_2)^2} < 0. \tag{3.21} \]

Therefore, \( w, n_2 \) and \( n^*_1 \) increase while \( w^*, n_1 \) and \( n^*_2 \) decrease at \( \phi = 0 \).

While we have only an interior equilibrium at \( \phi = 0 \), the equilibrium at \( \phi_k = 1 \) can be either interior or corner. If \( \sigma_1 = \sigma_2 \), the equilibrium at \( \phi = 1 \) is interior and we have
\[ w = w^* = \frac{1}{\sigma - 1}, \quad n_1 = \frac{L\theta}{2\delta_1}, \quad n_2 = \frac{L\theta}{2\delta_2}, \quad n_1^* = \frac{L(1 - \theta)}{2\delta_1}, \quad n_2^* = \frac{L(1 - \theta)}{2\delta_2}. \]

On the other hand, if \( \sigma_1 > \sigma_2 \), there exists a threshold value of \( \phi^* \in (0, 1) \), such that a corner equilibrium with \( n_1^* = 0 \) occurs when \( \phi \in [\phi^*, 1] \). Such a corner equilibrium is solved in Appendix A and its uniqueness is examined in Appendix B. At \( \phi = 1 \), the wages are given by (3.16) again while the industrial location at the corner equilibrium is given by

\[ n_1 = \frac{L\sigma_2}{\delta_1(\sigma_1 + \sigma_2)}, \quad n_1^* = 0, \quad n_2 = \frac{L}{\delta_2} \left[ \frac{\sigma_1(\theta - \sigma_2(1 - \theta))}{\sigma_1 + \sigma_2} \right], \quad n_2^* = \frac{L(1 - \theta)}{2\delta_2}. \]

Furthermore, to show the dependence of those variable when \( \phi \) approaches 1, we have the following results of derivatives.

\[
\begin{align*}
\frac{\partial w}{\partial \phi} &\bigg|_{\phi = 1} = -\frac{L^2(1 - \theta)(2\theta - 1)\sigma_2^2}{\delta^2(\sigma_1 + \sigma_2)(2\sigma_1\sigma_2 - \sigma_1)n_1^2} < 0, \\
\frac{\partial w^*}{\partial \phi} &\bigg|_{\phi = 1} = \frac{L^2(2\theta - 1)\sigma_2^2}{\delta^2(\sigma_1 + \sigma_2)(2\sigma_1\sigma_2 - \sigma_1)n_1^2} > 0, \\
\frac{\partial n_1}{\partial \phi} &\bigg|_{\phi = 1} = \frac{L(1 - \theta)(2\theta - 1)\sigma_2}{\delta(\sigma_1 + \sigma_2)} > 0, \\
\frac{\partial n_2}{\partial \phi} &\bigg|_{\phi = 1} = -\frac{L(1 - \theta)(2\theta - 1)\sigma_2}{\delta(\sigma_1 + \sigma_2)} < 0.
\end{align*}
\]

Therefore, the wages in Home decrease and the wages in Foreign increase to the same level of (3.16). Regarding the firm location, industry 1 agglomerates in Home and expands while industry 2 shrinks and moves to Foreign for \( \phi \in [\phi^*, 1] \).

### 4. The Wage Differential

There are two kinds of driving forces in this model: the market access effect and the wage differential effect. The interaction between these two effects determines the production specialization and the direction of trade. In this section, we examine the wage differential.

**Proposition 4.1.** For both interior equilibrium and corner equilibrium, we have \( w > w^* \) for \( 0 < \phi < 1 \).

**Proof:** Firstly, we show that \( w/w^* \neq 1 \) for \( 0 < \phi < 1 \) at the interior equilibrium. To the contrary, assume that \( w = w^* \) for some \( \phi \in (0, 1) \). Equation (3.15) becomes:

\[ \frac{\theta(\frac{\sigma_1}{\sigma_2} \phi_k + 1) + \phi_k(1 - \theta)(\frac{\sigma_1}{\sigma_2} + \phi_k)}{\phi_k(\frac{\sigma_1}{\sigma_2} \phi_k + 1) + (1 - \theta)(\frac{\sigma_1}{\sigma_2} + \phi_k)} = 1, \]

implying

\[ n_k/n_k^* = \frac{\theta - \phi_k(1 - \theta)}{1 - \theta(1 + \phi_k)} > \frac{\theta}{1 - \theta}, \quad k = 1, 2. \]

The above inequalities \( n_1/n_1^* > \theta/(1 - \theta) \) and \( n_2/n_2^* > \theta/(1 - \theta) \) violate (3.1).

Next, by use of (3.18) and (3.19) we have

\[ \frac{\partial w}{\partial \phi} w^* \bigg|_{\phi = 0} = \frac{2(2\theta - 1)\sigma_2^2}{(1 - \theta)(\sigma_1 + \sigma_2)(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)} > 0. \]

Given the same wage level of (3.16) at \( \phi = 0 \), we know that \( w > w^* \) for \( \phi \in (0, 1) \) as long as the equilibrium is interior.

Finally, Appendix A shows that \( w > w^* \) also holds at the only possible corner equilibrium with \( n_1^* = 0 \). Therefore, the positive wage differential of \( w > w^* \) occurs at any \( \phi \in (0, 1) \), no matter whether the equilibrium is interior or corner.

The above result is called the home market effect in terms of wages (Takahashi et al. 2013), which is firstly obtained by Krugman (1980) for the case of a single industry, saying that “the larger country, other things being equal, will have the higher wage.” Amiti (1998) and Laussel and Paul (2007) also have the home market effect in terms of wages. While this effect in Amiti (1998) are restricted in interior equilibrium, we extend it to the corner equilibrium. On the other hand, unlike Amiti (1998) and this article, Laussel and Paul (2007) employ a one-factor model. The relation between wage ratio \( w/w^* \) and transport costs are monotonic. In contrast, the numerical results presented in Amiti (1998) and this article suggest that the relation between \( w/w^* \) and \( \phi \) is bell-shaped. In our model, this bell-shaped pattern is analytically shown (see Lemma 5.1) when two industries differ only in terms of factor intensity.
5. The Industrial Location and Trade Pattern

In this section, we focus on three differences between two industries: the relative factor intensity, the transport costs and the demand elasticity. We study how the differences affect the firm location and trade pattern when transport costs fall from a prohibitive level to zero.

As noted earlier, there are two opposite effects in our model: the market access effect and the wage differential effect. Reducing transport costs from the autarky level to some finite level makes the large market more attractive. Because the firms in the smaller country find that the gains in exports do not offset the sales lose in the domestic market so the amount of output is insufficient to cover fixed costs and this leads to the exit of some firms. On the other hand, the demand for factors in the larger country would increase. An increase in the demand for capital in the larger country results in capital flowing from the smaller country to the larger country. In contrast, an increase in the demand for labor in the larger country pushes up wages in the larger country since labor is not mobile. A lower wage rate in the smaller country offsets the locational advantage of the larger country.

5.1 Different factor intensity

Our first case is that two industries only differ in the factor intensity of production. Namely, we have $\sigma_1 = \sigma_2 = \sigma$, $\beta = 1$ while $\delta_1 \neq \delta_2$. The model is completely solved in this case.

**Lemma 5.1.** If two industries only differ in factor intensity, the wages and the firm numbers are given by

$$w = \frac{(1 - \theta)(\sigma - 1) + \theta \phi + (1 - \theta)\phi + \sigma(1 - \theta)\phi^2}{(1 - \theta)(\sigma - 1) + \theta \phi + (1 - \theta)\phi + (1 - \theta)(\sigma - 1)\phi^2},$$

$$w^* = \frac{(1 - \theta)(\sigma - 1) + \theta \phi + (1 - \theta)\phi + \sigma(1 - \theta)\phi^2}{(1 - \theta)(\sigma - 1) + \theta \phi + (1 - \theta)\phi + (1 - \theta)(\sigma - 1)\phi^2},$$

$$n_1 = \frac{L_0}{2\delta_1}, \quad n_2 = \frac{L_0}{2\delta_2}, \quad n^*_1 = \frac{L(1 - \theta)}{2\delta_1}, \quad n^*_2 = \frac{L(1 - \theta)}{2\delta_2}.$$  

When $\phi$ increases from 0 to 1, wage rate $w$ exhibits a bell shape while $w^*$ has a U shape.

**Proof:** By use of $\sigma_1 = \sigma_2 = \sigma$ and $\phi_1 = \phi_2$, we have $n_1/n_2 = n^*_1/n^*_2$ from Eq. (3.15). Then Eqs. (3.11), (3.12), and (3.13) immediately imply (5.3). Furthermore, (5.1) and (5.2) can be easily derived from (3.14) and one of (3.6). Now we treat $w$ of (5.1) as a function of $\phi$. Evidently, for any $w^*$, equation $w(\phi) = w^*$ has at most two solutions of $\phi$. Therefore, in the $\phi$-$w$ plane, the curve $w(\phi)$ crosses any horizontal line $w = w^*$ at most twice. Given inequality (3.18) at $\phi = 0$ and equality (3.16) at both $\phi = 0$ and $\phi = 1$, $w(\phi)$ has a bell shape. Since $w$ and $w^*$ are negatively related according to (3.10), curve $w^*(\phi)$ has a U shape. 

It is noteworthy that the above lemma also shows that wage differential $w - w^*$ and $w/w^*$ evolve as a bell-shaped curve when trade costs fall.

The reason that two countries share the same production pattern for any transport costs is not mysterious. The fact of equal wages at autarky means that firms in each industry have the same output. The total output share in industry $k$ across countries should be equal to the consumption share (or labor share, since preferences are identical). Therefore, the relative firm numbers in industry $k$ across countries are equal to the consumption share: $n_1/n_2 = n^*_1/n^*_2 = \theta/(1 - \theta)$. On the other hand, the consumption share between industry 1 and industry 2 goods in each country should be equal to the production share $n_1X_1/(n_1X_1 + n_2X_2)$. It explains the result of $n_1/n_2 = \delta_2/\delta_1$ at autarky. To explain the same ratio holds for all trade costs, it is helpful to notice an important difference between Amiti (1998) and this article. The immobile labor is the only marginal input. Therefore, even when the transport costs fall, two countries keep the same production pattern.

Although the share of firm location is constant, the trade pattern varies. The larger country is a net exporter of good $k$ if its total export of good $k$ is larger than its total import of good $k$, both being measured in numeraire units. Since there are only two countries share the same production pattern, the larger country is a net exporter of good $k$ if and only if the smaller country is a net importer of the good. Let $E_k$ be the net export of good $k$ of the larger country. It is calculated as

$$E_k(\phi) = p_1n_km_\ell - p_\ell n_km_\ell = \frac{L\phi_k}{2} \left[ \frac{n_k(1 + w^*)(1 - \theta) - n_k\phi}{n_k + n_k\phi} - \frac{n_k(1 + w)\theta}{n_k + n_k\phi} \right]$$

$$= \frac{L\sigma(1 - \theta)(2\theta - 1)(1 - \phi)\phi}{2(1 - \theta)(\sigma - 1) + 2[1 - 2\theta(1 - \theta)]\sigma\phi + 2\theta(1 - \theta)(1 + \sigma)\phi^2},$$

where the last equality is from (2.4), (5.1), (5.2), (5.3), and $\phi_1 = \phi_2 = \phi$. 


Curve $E_k(\phi)$ has only one stationary point

$$
\phi_1 = \frac{\sqrt{(1-\theta)\theta(\sigma-1)}}{\sqrt{\sigma} + \sqrt{(1-\theta)\theta(\sigma-1)}} \in [0, 1].
$$

Furthermore, we have

$$
E'_k(0) = \frac{(2\theta - 1)\sigma}{2(\sigma - 1)} > 0, \quad E'_k(1) = -\frac{\theta(1-\theta)(2\theta - 1)\sigma}{2(\sigma - 1)} < 0.
$$

Therefore, $E_k(\phi)$ increases in $\phi$ in $[0, \phi_1)$ and decreases in $\phi$ in $(\phi_1, 1]$.

Proposition 5.2. (i) If two industries differ only in factor intensity of production, the firm number of each industry in each country is independent of trade costs. (ii) The wage rate in the larger country is higher and the wage differential evolves in a bell shape. (iii) The larger country is a net exporter of both industry goods. The volume of net export also has a bell shape.

Note that the capital endowment of each resident in either country is the same. The capital is mobile. Since the larger country is a net exporter in both industries, the trade balance condition implies that capital flows from the smaller country to the larger one.

Corollary 5.3. For $\phi \in (0, 1)$, the larger country is a net importer of capital.

5.2 Different demand elasticity

In our second case, two industries are identical in all respects except the demand elasticity. Namely, $\delta_1 = \delta_2 = \delta$, $\beta = 1$, $\sigma_1 > \sigma_2$. We first provides a result for the industry location.

Lemma 5.4. When two industries differ only in demand elasticity, there exists a $\tilde{\phi}_2 \in (0, 1)$ at which $n_1/n_2 = n^*_1/n^*_2$. If trade is close to autarky, we have inequality $n_1/n_2 < n^*_1/n^*_2$; if trade is close to free, we have inequality $n_1/n_2 > n^*_1/n^*_2$.

Proof: Firm numbers $n_1$ and $n^*_1$ are non-negative. According to Appendix A, there exists a threshold value of $\phi^*$, such that $n^*_1 = 0$ for all $\phi \geq \phi^*$. The detail of this corner equilibrium is solved in Appendix A, and the uniqueness is considered in Appendix B. Note that $n_1/n_2 - n^*_1/n^*_2 = 0$ holds at $\phi = 0$. By use of Eqs. (3.17), (3.20) and (3.21), the differentiation of $n_1/n_2 - n^*_1/n^*_2$ with respect to $\phi$ at $\phi = 0$ is

$$
\frac{\partial}{\partial \phi} \left( \frac{n_1}{n_2} - \frac{n^*_1}{n^*_2} \right)_{\phi=0} = -\frac{(2\theta - 1)\sigma_2}{(1-\theta)\sigma_1} < 0.
$$

Therefore, $n_1/n_2 - n^*_1/n^*_2 < 0$ holds when $\phi$ is close to zero. Since $n_1/n_2 > 0 = n^*_1/n^*_2$ at $\phi^*$, there exists a $\tilde{\phi}_2 \in (0, \phi^*) \subset (0, 1)$ at which $n_1/n_2 - n^*_1/n^*_2 = 0$. \hfill \Box

Figure 1 provides a simulation result, in which parameters are given as $\theta = 0.6$, $\sigma_1 = 3$, $\sigma_2 = 1.5$, $\delta = 1$. In the right panel, we can see that $n_1/n_2 < n^*_1/n^*_2$ holds for $\phi \in (0, \tilde{\phi}_2)$ and $n_1/n_2 > n^*_1/n^*_2$ holds for $\phi \in (\tilde{\phi}_2, 1)$. Namely, the larger country is relatively more specialized in the production of low-elasticity goods when transport costs are high and is relatively more specialized in the production of the high-elasticity goods when trade costs are low.

Now, we analyze the trade pattern. Unfortunately, we have no explicit form for the wages and firm numbers in this case. However, the derivatives of $E_k(\phi)$ at $\phi = 0$ and $\phi = 1$ can be calculated as

$$
E'_k(\phi)_{\phi=0} = \frac{L\sigma_1\sigma_2\phi^{2/(\sigma_1+\sigma_2)}}{(\sigma_1 + \sigma_2 - 2\sigma_1\sigma_2)^2} \left[ \frac{(2\theta - 1)(\sigma_1 - 1)(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)}{\sigma_2 - 1} + o(\phi) \right] > 0,
$$

$$
E'_k(1) = \frac{L\theta(1-\theta)(2\theta - 1)(\sigma_1 + \sigma_2)}{2(2\sigma_1\sigma_2 - \sigma_2 - \sigma_1)} > 0,
$$

$$
E'_k(0) = \frac{L\sigma_1\sigma_2}{2\sigma_1\sigma_2 - \sigma_2 - \sigma_1} > 0,
$$

$$
E'_k(1) = -\frac{L\theta(1-\theta)(2\theta - 1)(\sigma_1 + \sigma_2)(2\sigma_2 - 1)}{2(2\sigma_1\sigma_2 - \sigma_2 - \sigma_1)} < 0,
$$

where $o(\phi)$ is the Landau symbol “small” o. Therefore, $E_k(\phi)$ is positive when $\phi$ is close to autarky level. On the other hand, $E_k(\phi)$ increases while $E'_k(\phi)$ decreases when $\phi$ is close to free trade level. We perform simulations to show some properties of the net export of the larger country. In Fig. 2 the solid line depicts the net export of high-elasticity goods while the dotted line depicts the net export of low-elasticity goods. Figure 2 illustrates that the larger country is always a net exporter of the high-elasticity goods. On the other hand, when transport costs are close to free trade level, the large
country is a net importer of the low-elasticity goods. This result is different from that of Laussel and Paul (2007), which employ a one-factor model. Since the immobile labor is the only factor, the result of trade pattern in Laussel and Paul (2007) depends on the differential of country size. If the two countries are very different in size, most firms are located in the larger country. Thus, the larger country is always a net exporter of the low-elasticity goods. If the two countries are sufficiently similar in size, a continuous fall in transport costs from a prohibitive level to zero is associated with a reversal in the pattern of trade at some intermediate level. Finally, we examine the capital flow. Let $h$ denote the capital share employed in the Home country. The ratio of home to foreign demands for capital is

$$
\frac{h}{1-h} = \frac{n_1(\sigma_1 - 1)w^\delta_1 + n_2(\sigma_2 - 1)w^\delta_2}{n_1^*(\sigma_1 - 1)w^\delta_1 + n_2^*(\sigma_2 - 1)w^\delta_2}
\left(\frac{\sigma_2 - 1}{\sigma_2 - 1}wL\delta + \frac{n_1(\sigma_1 - \sigma_2)w^\delta_1}{(\sigma_2 - 1)wL(1-\theta) + n_1^*(\sigma_1 - \sigma_2)w^\delta_1} \right),
$$

(5.5)

where the second equality is from Eq. (3.12) and the first equality of (3.13).

According to Eq. (3.12) and the first equality of (3.13) again, we know that $n_1/n_2 = n_1^*/n_2^*$ holds only if they are equal to $\theta/(1-\theta)$. Therefore, $n_1/n_2 > n_1^*/n_2^*$ implies $n_1/n_2 > \theta/(1-\theta) > n_1^*/n_2^*$ so that $h/(1-h) > \theta/(1-\theta)$ holds.

Thus capital employment in the larger country is more than its endowment and capital flows from the smaller country to the larger country. Similarly, capital flows from the larger country to the smaller country when $n_1/n_2 < n_1^*/n_2^*$.

The evolution result of industrial location when trade costs fall is summarized as follows.

**Proposition 5.5.** When trade is close to autarky, the larger country is relatively more specialized in the production of the low-elasticity goods. When trade is close to free, the larger country is relatively more specialized in the production of the high-elasticity goods.

Intuitively, the lower is $\sigma$, the higher is the mark-up over marginal cost. Suppose a decrease of price, firms in the industry with lower demand elasticity increase sales more. Firms in both countries have incentives to enter the low-elasticity industry (a decrease of $n_1/n_2$ or $n_1^*/n_2^*$). Again, there is an trade-off between the market access effect and the wage differential effect. When transport costs are relatively high, the market access effect dominates. Firms in the larger country succeed in entering the low elasticity industry. Thus, the specialization degree of the low elasticity industry in the larger country increased (a decrease of $n_1/n_2$). As transport costs continue to fall, the magnitude of market access effect decreases. On the other hand, the lower wages give the firms in the smaller country an advantage of entering the low elasticity industry. Now, it’s the wage differential effect dominates. Firms in the smaller country then succeed in increasing the specialization degree of industry 2 (a decrease of $n_1^*/n_2^*$). Figure 1 illustrates that $n_1^*/n_2^*$ continues to fall as transport costs approach the free trade level.

This result is contrastive to Amiti (1998), which shows that the smaller country is relatively more specialized in the
production of low-elasticity goods when transport costs are close to autarky level. The reason is that the wage differential effect in this article and "production cost effect" in Amiti (1998) are quite different. A Leontief composite of capital and labor is chosen as both fixed input and marginal input in Amiti (1998). In other words, labor wages enter both the fixed cost and marginal cost in Amiti (1998). By contrast, the wage differential effect arises only from the fixed costs, and prices are equal across countries in this article. On the other hand, Chen and Zeng (2014) assume labor as fixed cost and marginal as fixed input. It implies that the wage differential effect in this article or Chen and Zeng (2014) is weaker than in Amiti (1998).

5.3 Different levels of transport costs

In our third case, two industries are identical except their trade costs: \( \delta_1 = \delta_2 = \delta \), \( \sigma_1 = \sigma_2 = \sigma \) but \( \tau_1 > \tau_2 \) (i.e., \( \beta > 1 \)). Namely, industry 1 goods are bulkier to transport. Similar to Lemma 5.4, we have the following conclusion on industrial location.

**Lemma 5.6.** There exists a \( \phi_3 \in (0, 1) \) at which \( n_1/n_2 = n_1^*/n_2^* \) holds. We have inequality \( n_1/n_2 < n_1^*/n_2^* \) when \( \phi \) is close to 0, and inequality \( n_1/n_2 > n_1^*/n_2^* \) when \( \phi \) is close to 1.

**Proof:** It can be proved in the same way as in Lemma 5.4. The only difference is to replace (5.4) by

\[
\frac{\partial}{\partial \phi} \left( \frac{n_1}{n_2} - \frac{n_1^*}{n_2^*} \right) \bigg|_{\phi=0} = -\frac{(2\theta - 1)}{(1 - \theta)\theta} < 0.
\]

Figure 3 gives a simulation result in which parameters are given as \( \theta = 0.6, \beta = 2, \sigma = 3, \delta = 1 \). We can visually see the threshold value \( \phi_3 \), above which the larger country is relatively more specialized in the production of low-elasticity goods.
Therefore, $E_1(\phi)$ is positive when $\phi$ is close to autarky level. On the other hand, $E_2(\phi)$ increases while $E_3(\phi)$ decreases when $\phi$ is close to free trade level. We perform some simulations to reveal the trade pattern in the larger country. Figure 4 illustrates that the larger country is always a net exporter of the high transport cost goods. On the other hand, when trade is close to free, the larger country is a net importer of the low transport cost goods.

Finally, we check the capital movement by use of the capital employment ratio Eq. (5.5). It’s easy to find that

$$\frac{h}{1-h} = \frac{w\theta}{w^*(1-\theta)} > \frac{\theta}{1-\theta}.$$  

The capital employment in the larger country is more than its endowment. Capital flows from smaller country to larger country at all transport cost levels.

We summarize the result of industrial location and trade pattern as follows.

**Proposition 5.7.** When trade is close to autarky, the larger country is relatively more specialized in the production of low transport cost goods and is a net exporter of the low transport cost goods. When trade is close to free, the larger country is relatively more specialized in the production of high transport cost goods and is a net export of the high transport cost goods. Capital flows from the smaller country to the larger one.

The intuition behind the results is similar to Case 2. While Amiti (1998) assumes a Leontief composite of labor and capital as their fixed and marginal inputs, we assume the immobile labor as fixed input in this model. Firms can only move across industries within a country. As transport costs fall from autarky to a finite level, firms in both countries have incentives to enter the low transport cost industry (a decrease of $n_1/n_2$, $n_1^*/n_2^*$), since transport costs of industry 2 goods are lower. The interaction between the market access effect and the wage differential effect determines the results. Specifically, when transport costs are high, the market access effect is strong. Firms in the larger country succeed in entering the low transport cost industry (a decrease of $n_1/n_2$). As transport costs are close to free trade, the magnitude of market access effect decreases. On the other hand, the lower wage rate gives the firms in the smaller country an advantage of entering the low transport cost industry (a decrease of $n_1^*/n_2^*$). The wage differential effect dominates and Fig. 3 illustrates that $n_1^*/n_2^*$ continues to fall as transport costs are closing to free trade level.

This result contrasts with Amiti (1998), which shows that the larger country is always more specialized in production of the high transport cost goods. This difference in the results between Amiti (1998) and our model can be intuitively explained as follows. First of all, mobile capital is chosen as one of the fixed input in Amiti (1998). The larger country is more attractive for firms in the high transport cost industry because they can save on transport costs there. The market access effect dominates, firms in the high transport cost industry relocate to the large market. In contrast, in our model, immobile labor is chosen as the fixed input. Firms can only move across industries within a country. Therefore, firms in the high transport cost industry only try to move to the low transport cost industry domestically.
Second, the wage differential effect in this article and “production cost effect” in Amiti (1998) are quite different. As discussed in Section 5.2, wages enter both the fixed cost and marginal cost in Amiti (1998). However, the wage differential effect arises only from the fixed costs and prices are equal across countries in this article. In other words, the wage differential effect is weaker in this article than in Amiti (1998). Therefore, when trade is close to autarky (market access effect is more stronger), the market access effect dominates and the larger country has a higher degree of production specialization in the low transport cost industry.

6. Concluding Remarks

In this article, we studied how different industries locate and what are the trade patterns when countries have different size. Both industries have technologies of increasing returns to scale. They may differ in factor intensity, demand elasticity, and trade costs. We have shown how the equilibrium results from the interplay of two forces, the market access effect and the wage differential effect. We show the home market effect in terms of wages not only in interior equilibrium but also in corner equilibrium. Unlike the existing literature, we choose mobile capital as marginal market access effect and the wage differential effect. We show the home market effect in terms of wages not only in interior equilibrium but also in corner equilibrium. Unlike the existing literature, we choose mobile capital as marginal input, thus the prices are equal between countries in this article. We seek to contribute to the literature by showing that if prices are equal across countries in each industry, some new results can be generated.

We obtain the following conclusions. First, if two industries differ only in factor intensity, two countries share the same production pattern \( n_1/n_2 = n_1^*/n_2^* \) for any trade costs. The wage rate in the larger country is higher and the wage differential evolves in a bell shape when trade costs fall. The larger country is a net exporter of both industry goods. The volume of the net export also evolves in a bell shape.

Second, if two industries differ only in demand elasticity, the larger country is relatively more specialized in the production of low-elasticity goods when trade is close to autarky; the larger country is relatively more specialized in the production of high-elasticity goods when trade is close to free. Our results suggest that the larger country is a net exporter of both industry goods when trade is close to autarky.

Third, if two industries differ only in transport costs, the larger country is relatively more specialized in the production of low transport cost goods and hence a net exporter of these goods when trade is close to autarky; the larger country is relatively more specialized in the production of high transport cost goods and hence a net exporter of these goods when trade is close to free.

Appendix A: The Corner Equilibrium (\( \phi_2 \equiv \phi \))

We examine a possible corner solution \( n_1^* = 0 \). By use of the labor market equilibrium equations, we can obtain \( n_2^*, n_2, w \) and \( w^* \) in terms of \( n_1 \) as follows.

\[
\begin{align*}
   n_2^* &= \frac{L(1 - \theta)}{\delta_2}, \\
   n_2 &= \frac{L\theta}{\delta_2} - n_1 \frac{\delta_1}{\delta_2}, \\
   w &= \frac{L\sigma_2}{\delta_1 n_1(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)}, \\
   w^* &= \frac{\delta_1 n_1(\sigma_1 + \sigma_2) - L\theta\sigma_2}{(1 - \theta)\delta_1 n_1(2\sigma_1\sigma_2 - \sigma_1 - \sigma_2)}.
\end{align*}
\]

Plugging the equations above into the following market-clearing condition of industry 2

\[
\frac{L\theta(1 + w)}{n_2 + n_2^*\phi} + \frac{L(1 - \theta)(1 + w^*)\phi}{\delta n_2 + n_2^*} - 2\sigma_2w\delta_2 = 0,
\]

we obtain \( An_1^2 - Bn_1 + C = 0 \), where

\[
\begin{align*}
   A &= 2\delta_1^2\sigma_2(\sigma_1 + \sigma_2)\phi > 0, \\
   B &= \delta_1 [(1 - \theta)[\theta(\sigma_1 + \sigma_2) + 2(1 - \theta)\sigma_1\sigma_2 + 2\sigma_2^2]\phi^2 \\
   &+ 2\sigma_2(\sigma_1 + 2\sigma_2)\phi + (1 - \theta)[\theta\sigma_2(2\sigma_2 - 1) + \sigma_2(2\sigma_2 - \theta)]] > 0, \\
   C &= L^2\sigma_2^2[2\sigma_2(1 - \theta + \theta\phi)(\theta - \theta\phi + \phi) - (1 - \theta)(1 - \phi^2)\theta] > 0.
\end{align*}
\]

This quadratic function of \( n_1 \) has two roots

\[
n_1 = n_{1a} = \frac{B - \sqrt{B^2 - 4AC}}{2A} > 0 \quad \text{and} \quad n_1 = n_{1b} = \frac{B + \sqrt{B^2 - 4AC}}{2A} > 0.
\]

They correspond to two values of \( n_2 \)

\[
n_{2a} = \frac{2A L\theta - B\delta_1 + \delta_1 \sqrt{B^2 - 4AC}}{2A\delta_2}, \quad n_{2b} = \frac{2A L\theta - B\delta_1 - \delta_1 \sqrt{B^2 - 4AC}}{2A\delta_2},
\]

from (A-1). Multiply the numerator of \( n_{2a} \) and \( n_{2b} \), we have
Appendix B: The Uniqueness of Corner Equilibrium

The following simulations in Fig. 5 show that a threshold value of \( \sigma_2 \) in Appendix A, we know that \( n_1 = n_{1\alpha} \) and \( n_2 = n_{2\alpha} \) in this corner equilibrium. Consequently,

\[
\frac{w - w^*}{\sigma_1 n_{1\alpha} - \sigma_2 n_{2\alpha}} = \frac{\sqrt{B^2 - 4AC} + 4L_2^2 \delta_1 \Phi - B}{4\delta_2^2 \sigma_2 \Phi n_{1\alpha} (2\sigma_1 - \sigma_1 - 2\sigma_2)(1 - \theta)}. 
\]

If \( 4L_2^2 \delta_1 \Phi - B \geq 0 \), we obtain \( w - w^* \geq 0 \) immediately. When \( 4L_2^2 \delta_1 \Phi - B < 0 \), the numerator is still positive since

\[
B^2 - 4AC - (4L_2^2 \delta_1 \Phi - B)^2 = 16L_2^4 \delta_1^2 \sigma_2^2 (1 - \theta)(1 - \phi)(2\theta - 1) - \Phi \sigma_2 (1 - \theta)(1 - \phi) \sigma_2 \geq 0.
\]

Therefore, we have \( w - w^* \geq 0 \) again.

In the case of \( \sigma_1 < \sigma_2 \), the corner equilibrium with \( n_1^* = 0 \) does not exist. This is because there is no \( \phi_1 \in (0, 1) \) satisfying the market-clearing condition of industry 1 in the smaller country:

\[
N_1 \text{st}(\phi) = \frac{L(\sigma_1 + 1)(1 - \phi) + L(1 - \theta)(w^* + 1)}{n_1 + n_1^* \phi} = \frac{2\sigma_1 w^* \delta_1}{n_1 + n_1^*} \leq 0.
\]

The following simulations in Fig. 5 show that a threshold value of \( \phi_1 \in (0, 1) \) meeting \( N_1 \text{st}(\phi) = 0 \) only exists when \( \sigma_1 > \sigma_2 \).

Appendix B: The Uniqueness of Corner Equilibrium

We first arrange all possible corner equilibria in the table below, and then provide some explanations.

B.1 \( n_2^* = 0 \)

In our model, two countries are identical except for size. It is easy to understand that the analysis of corner equilibrium \( n_2^* = 0 \) with \( \sigma_1 > \sigma_2 \) is symmetric to the case of \( n_1^* = 0 \) with \( \sigma_1 < \sigma_2 \). On the other hand, the corner equilibrium \( n_2^* = 0 \) with \( \sigma_1 < \sigma_2 \) is symmetric to the corner equilibrium \( n_1^* = 0 \) with \( \sigma_1 > \sigma_2 \). According to the results in Appendix A, we know that \( n_1^* = 0 \) with \( \sigma_1 < \sigma_2 \) is a possible corner equilibrium. In corner equilibrium \( n_2^* = 0 \) with \( \sigma_1 < \sigma_2 \), the analysis is symmetric to the case \( n_1^* = 0 \) with \( \sigma_1 > \sigma_2 \), therefore, the inequality \( w - w^* > 0 \) still holds.

B.2 \( n_1 = 0 \)

Corner equilibrium \( n_1 = 0 \) deserves more exploration, since the sizes of two countries are different. By use of the labor market equilibrium equations, we can obtain \( n_2^* \), \( n_2 \), \( w \) and \( w^* \) in terms of \( n_1^* \) as follows.
Table 2. Types of corner equilibria. Possible: ∅, Impossible: ×.

| Demand elasticity | Corner equilibrium | $n_1^*$ = 0 | $n_2^*$ = 0 | $n_1 = 0$ | $n_2 = 0$ |
|-------------------|-------------------|--------------|--------------|------------|------------|
| $\sigma_1 > \sigma_2$ | $n_1^* = 0$ | $n_2^* = 0$ | × | × | × |
| $\sigma_1 < \sigma_2$ | × | $n_1^* = 0$ | $n_2^* = 0$ | × | × |

$$n_2 = \frac{L\theta}{\delta_2}, \quad n_2^* = \frac{L(1-\theta)}{\delta_2} - n_1^* \frac{\delta_1}{\delta_2}.$$  \hspace{1cm} (B.1)

$$w^* = \frac{L\sigma_2}{\delta_1 n_1^*(2\sigma_1 \sigma_2 - \sigma_1 - \sigma_2)}, \quad w = \frac{\delta_1 n_1^*(\sigma_1 + \sigma_2) - L(1-\theta)\sigma_2}{\theta \delta_1 n_1^*(2\sigma_1 \sigma_2 - \sigma_1 - \sigma_2)}.$$  

Plugging the equations above into the following market-clearing condition for industry 2

$$N_2 (\phi) = \frac{L(1+w)\phi}{n_2 + n_2^*} + \frac{L(1-\theta)(1+w^*)}{\phi n_2 + n_2^*} - 2\sigma_2w^*\delta_2 = 0,$$

obtains  $\Delta n_1^* = B_1 n_1^* + C_1 = 0$, where

$$B_1 \equiv L \delta_1 \left[ \theta \left( (1-\theta)(\sigma_1 + \sigma_2) + 2\theta \sigma_1 \sigma_2 + 2\sigma_2^2 \right) \phi^2 
+ 2(1-\theta)\sigma_2 \left( \sigma_1 + \sigma_2 \right) \phi \theta(1-\theta)\sigma_1 (2\sigma_2 - 1) + \sigma_2 (2\sigma_2 - 1 + \theta) \right] > 0,$$

$$C_1 \equiv L^2 \sigma_2 \left( 2\sigma_2 (\theta + \phi - \theta\phi)(1-\theta) + (1-\theta)(1-\phi^2) \theta \right) > 0.$$  

This quadratic function of $n_1^*$ has two roots

$$n_1^* = n_{1a}^* = \frac{B_1 - \sqrt{B_1^2 - 4AC_1}}{2A} > 0 \quad \text{and} \quad n_1^* = n_{1b}^* = \frac{B_1 + \sqrt{B_1^2 - 4AC_1}}{2A} > 0.$$  

In the case of $n_{1b}^*$, we have

$$n_2^* = n_{2b}^* = \frac{2AL(1-\theta) - B_1 \delta_1 - \delta_1 \sqrt{B_1^2 - 4AC_1}}{2A \delta_2},$$

from full employment condition. From the numerator we have a quadratic function in terms of $\theta$:

$$\Phi_1 (\theta) \equiv \frac{2AL(1-\theta) - B_1 \delta_1}{L \delta_1^2} = (2\sigma_1 \sigma_1 - \sigma_1 - \sigma_2)(1-\phi^2) \phi^2 - \left[ (\sigma_1 + \sigma_2)(2\sigma_2 - 1) + 2\sigma_1 \sigma_2 \phi + (\sigma_1 + \sigma_2 + 2\sigma_2^2) \phi \theta \right] + 2\sigma_1 \sigma_2 \phi.$$  

For $\theta \in (1/2, 1)$, we have $\Phi_1 (\theta) = -2\sigma_2 [\sigma_1 + (\sigma_1 + \sigma_2) \phi \theta] < 0$. On the other hand, we have

$$\Phi_1 \left( \frac{1}{2} \right) = -\frac{1}{4} \left( \sigma_1 + \sigma_2 + 2\sigma_1 \sigma_2 + 4\sigma_2^2 \right) \phi^2 + \sigma_1 \sigma_2 \phi + 1 \left( \sigma_1 + \sigma_2 - 2\sigma_1 \sigma_2 - 4\sigma_2^2 \right)$$

$$\leq \frac{1}{4} \left( \sigma_1 + \sigma_2 - 16\sigma_2^2 \right) < 0,$$  

where the second inequality is due $\sigma_1 \leq \sigma_2$. Since $(2\sigma_1 \sigma_1 - \sigma_1 - \sigma_2)(1-\phi^2) > 0$, we know $\Phi_1 (\theta) < 0$ for $\theta \in (1/2, 1)$ by the properties of quadratic function. Accordingly, inequality $n_{2b}^* < 0$ holds for all $\phi \in (0, 1)$, so we only have $n_2^* = n_{2a}^*$ in this corner equilibrium. However, we show that $n_2^* = n_{2w}^*$ at $0$ for $\phi \in (\phi^2, 1)$. Note that $\phi^2$ is defined as the threshold value of trade freeeness that satisfies market-clearing condition of the industry 1 in the larger country:

$$N_1 (\phi) \equiv \frac{L(1+w) + L(1-\theta)(w^* + \phi \frac{\mu}{\phi + \mu - 1})}{n_1 + n_1^* \phi \frac{\mu}{\phi + \mu - 1}} - 2\sigma_1 w \delta_1 = 0.$$  

Although we have no closed-form solution of $\phi^2$, Fig. 6 shows that $n_{2a}^*$ is negative when $\phi \in (\phi^2, 1)$. It implies that when $\sigma_1 \leq \sigma_2$, $n_1 = 0$ is not a plausible corner equilibrium.

In the case of $\sigma_1 > \sigma_2$, $n_1 = 0$ is not a possible corner equilibrium. Because the threshold value of $\phi^2 \in (0, 1)$ which satisfies the market-clearing condition $N_1 (\phi) = 0$ does not exist. Figure 7 shows the result of our simulation. We can not observe such a $\phi^2 \in (0, 1)$ when $\sigma_1 > \sigma_2$.

B.3 $n_2 = 0$

By use of the symmetry of this model setting, we know that the analysis of corner equilibrium $n_2 = 0$ with $\sigma_1 > \sigma_2$ is symmetric to the case of $n_1 = 0$ with $\sigma_1 < \sigma_2$. On the other hand, the analysis of corner equilibrium $n_2 = 0$ with
is symmetric to the case of $n_1 = 0$ with $\sigma_1 > \sigma_2$. According to the results in the above subsection, we know that $n_2 = 0$ is not a possible corner equilibrium.

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