Reappearance of the Kondo effect in serially coupled symmetric triple quantum dots

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Abstract – We investigate the spectral properties of a serially coupled triple quantum dot (STQD) system by means of the hierarchical equations of motion (HEOM) approach. We find that with the increase of the interdot coupling $t$, the first Kondo screening is followed by another Kondo effect reappearing due to the transition from the respective Kondo singlet state of individual QD to the coherence bonding state generated among the three QDs. The reappearance of the Kondo effect results in the three-peak structure of the spectral functions of peripheral QD-1(3). By investigating the susceptibility $\chi$, we find that the local susceptibility of intermediate QD-2 is a positive value at weak interdot coupling, while it changes into a negative value at strong interdot coupling, at which the STQD system gives rise to the reappearance of the Kondo effect. We also find that the slopes of $1/\chi$ will deviate from a straight line behaviour at low temperature in the reappearing Kondo regime. In addition, the influence of temperature and dot-lead coupling strength on the reappearing Kondo effect as well as the Kondo-correlated transport properties are afterwards exploited in detail.

Introduction. – The triple quantum dot system as the simplest device provides an ideal platform for investigating the coded qubit, frustration and quantum teleportation [1]. The investigation of the TQD is just the first step to study the multiple-“impurity” configurations. More important applications of TQD are in the field of quantum computation and quantum information processing due to their extended freedom of coupling and geometric arrangement. Significantly, it leads to more interesting physics such as Fano resonances [2], Aharonov-Bohm oscillations [3], quantum phase transitions [4] and Kondo effect [5,6].

At low temperatures, the Kondo effect exhibits in the nanoscale Coulomb blockade systems with degenerate ground states. As a many-body phenomenon, the Kondo effect in the QDs system acquires great interest for its significant role in contributing to the quantum transport properties of multiple-“impurity” configurations.

Remarkably, experimental studies of varied configurations of the TQD structure [7–12] make it possible to investigate the characteristics of the Kondo effect in nanoscale setups. Recently, a vast amount of theoretical works have been carried out to study triangular [13,14] and mirror symmetry TQD [15,16], which are expected to exhibit novel Kondo physics. Mitchell et al. present the phase diagram for the triangular geometry of the TQD system and examine the interplay between frustration and the Kondo effect [17]. Ferromagnetic and antiferromagnetic Kondo physics phases [6,18] and two-channel Kondo physics [19,20] are shown in both the triangular geometry and mirror symmetry of the TQD system. Also the $SU(3)$ Kondo effect [21] and $SU(4)$ Kondo effect [22] are interesting phenomena in the triangular geometry of TQD due to the different dot occupancies.

On the other hand, the studies on the Kondo physics of serially coupled TQD (STQD) are not sufficient. One additional QD inserting the middle of the serially coupled double QD (SDQD) to form STQD seems lacking

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of surprises. However, some pioneer works have proved that the STQD is not the simple extension of SDQD but it has more complex physics than the latter, such as the non-Fermi-liquid behavior [23], two-channel Kondo effect [24], nonequilibrium Kondo transport [25] and the novel phase diagram [5]. This indicates that the low temperature Kondo effects in STQD deserve more detailed studies (with accurate approaches) to reveal the rich physics induced by the additional spin that couples to the others on both sides but decouples to the baths. In the present work, we deal with this issue by fully considering the electron-electron (e-e) interactions in each dot and adopting the hierarchical equations of motion (HEOM) approach [26,27]. The spectral functions and nonequilibrium Kondo transport properties are calculated with a high accurate level. The Kondo effects with some intriguing properties being not presented in SDQDs are explored. We find a reappearance of the Kondo effect phenomenon in the STQD system by analysing the spectral properties and susceptibilities. The thermodynamic properties and dynamical properties of the reappearing Kondo effect are also exploited.

**Model and HEOM approach.** – A STQD system is the model we study (see inset of fig. 1(b)). The two peripheral quantum dots (QD-1 and QD-3) are directly coupled to the leads, while the intermediate QD (QD-2) is not directly coupled with the leads. The localized QDs constitute the open system of primary interest, and the surrounding reservoirs of itinerant electrons are treated as environment. The total Hamiltonian for the system is

\[ H = H_{\text{dots}} + H_{\text{leads}} + H_{\text{couplings}}, \]

where the interacting STQD is

\[ H_{\text{dots}} = \sum_{\sigma=1,2,3} \left[ \epsilon_{i\sigma} \sigma_{i\sigma}^\dagger \sigma_{i\sigma} + U_i n_{i\sigma} n_{i\sigma} \right] + \sum_{\sigma} \left( t_{12} \sigma_{1\sigma}^\dagger \sigma_{2\sigma} + t_{23} \sigma_{2\sigma}^\dagger \sigma_{3\sigma} + \text{H.c.} \right), \tag{1} \]

where \( \sigma_{i\sigma}^\dagger \) (\( \sigma_{i\sigma} \)) is the operator that creates (annihilates) a spin-\( \sigma \) electron with energy \( \epsilon_{i\sigma} \) (i = 1, 2, 3) in the dot i. \( n_{i\sigma} = \sigma_{i\sigma}^\dagger \sigma_{i\sigma} \) corresponds to the operator for the electron number of dot i. \( U_i \) is the on-dot Coulomb interaction between electrons with spin \( \sigma \) and \( \bar{\sigma} \) (opposite spin of \( \sigma \)), while \( t_{12} \) (\( t_{23} \)) is the interdot coupling strength between QD-1(3) and QD-2, determined by their overlapping integrals. For simplicity, we take \( t_{12} = t_{23} = t \) in our model.

In what follows, the symbol \( \mu \) is adopted to denote the electron orbital (including spin, space, etc.) in the system for brevity, i.e., \( \mu = \{ \sigma, i, \ldots \} \). The device leads are treated as noninteracting electron reservoirs and the Hamiltonian can be written as

\[ H_{\text{leads}} = \sum_{k\mu \rho \alpha=L,R} \epsilon_{\alpha k} d_{k\mu \alpha}^\dagger d_{k\mu \alpha}, \]

and the dot-lead coupling is

\[ H_{\text{couplings}} = \sum_{k\mu \alpha=L,R} t_{k\mu \alpha} \sigma_{k\mu \alpha}^\dagger \sigma_{\alpha k} + \text{H.c.}, \]

with \( \epsilon_{\alpha k} \) being the energy of an electron with wave vector \( k \) in the \( \alpha \) lead, and \( d_{k\mu \alpha}^\dagger \) (\( d_{k\mu \alpha} \)) the corresponding creation (annihilation) operator for an electron with the \( \alpha \)-reservoir state \( |k \rangle \) of energy \( \epsilon_{\alpha k} \).

To describe the stochastic nature of the transfer coupling, the term of dot-lead coupling can be defined in the reservoir \( H_{\text{leads}} \)-interaction picture as

\[ H_{\text{couplings}} = \sum_{n} \left[ f_n^\dagger (t) \sigma_{n \alpha} + \sigma_{n \alpha}^\dagger f_n (t) \right], \]

with \( f_n^\dagger = e^{i H_{\text{leads}} t} [\sigma_{n \alpha} t_{k\mu \alpha} d_{k\mu \alpha}^\dagger] e^{-i H_{\text{leads}} t} \) being the stochastic interaction operator and satisfying the Gauss statistics. Here, \( t_{k\mu \alpha} \) denotes the transfer coupling matrix element.

The influence of electron reservoirs on the dots is taken into account through the hybridization functions, which are assumed of Lorentzian form,

\[ \Delta_\alpha (\omega) = \pi \sum_\delta \Gamma_{\alpha k \delta} \delta (\omega - \epsilon_{\alpha k} - \Delta_\alpha) = \Delta W^2 / [2 (\omega - \epsilon_{\alpha k})^2 + W^2], \]

where \( \Delta \) is the effective quantum dot-lead coupling strength, \( W \) is the band width, and \( \mu_\alpha \) is the chemical potentials of the \( \alpha \) (\( \alpha = L, R \)) lead [27–30].

In this paper, a hierarchical equations of motion approach (HEOM) developed in recent years is employed to study the STQD system [27–29]. The outstanding issue of characterizing both equilibrium and nonequilibrium properties of a general open quantum system are referred to in refs. [26,27,29]. The HEOM theory established based on the Feynman-Vernon path-integral formalism adopts a general form of the system Hamiltonian, in which all the system-bath correlations are taken into consideration. It is applicable to a wide range of system parameters without additional derivation and programming efforts and can characterize both static and transient electronic properties of a strongly correlated system [29].

The reduced density matrix of the quantum dots system \( \rho^{(0)} (t) = \text{tr}_{\text{res}} \rho_{\text{total}} (t) \) and a set of auxiliary density matrices \( \{ \rho_{\sigma_{j_1} \ldots j_n}^{(n)} (t); n = 1, \ldots, L \} \) are the basic variables in HEOM. Here \( L \) denotes the terminal or truncated tier level. The HEOM that governs the dynamics of open system assumes the form of [26,27]:

\[ \dot{\rho}_{\sigma_{j_1} \ldots j_n}^{(n)} = - (i \mathcal{L} + \sum_{r=1}^{n} \gamma_r) \rho_{\sigma_{j_1} \ldots j_n}^{(n)} - i \sum_j A_j^{(n+1)} \rho_{\sigma_{j_1} \ldots j_{n-1} j j_n}^{(n+1)} - i \sum_{r=1}^{n} (-)^{n-r} C_{j_r} \rho_{\sigma_{j_1} \ldots j_{r-1} j_r+1 \ldots j_n}^{(n-1)}, \tag{2} \]

for the n-th-order auxiliary density operator \( \rho^{(n)} \) can be defined via the auxiliary influence functional as

\[ \rho_{\sigma_{j_1} \ldots j_n}^{(n)} (t) \equiv \mathcal{U}_j^{(n)} (t, t_0) \rho (t_0), \]

with the reduced Liouville-space propagator \( \mathcal{U}_j^{(n)} (\psi, t; \psi_0, t_0) \) referred to in [26].

We adopt the index \( j = \{ j_1, \ldots, j_n \} \) and \( j_r = \{ j_1 \cdots \hat{j}_r \cdots j_{n+1} \} \). The action of superoperators, respectively, is

\[ A_j \rho_{\sigma_{j_1} \ldots j_n}^{(n+1)} = \sigma_{\sigma_{j_1} \ldots j_n}^{(n+1)} (t) \rho_{\sigma_{j_1} \ldots j_n}^{(n+1)} (t) \]

and

\[ C_{j_r} \rho_{\sigma_{j_1} \ldots j_r+1 \ldots j_n}^{(n-1)} = \sum_{\nu} \left\{ c^{(n)}_{\sigma_{j_1} \ldots j_n} \delta_{\sigma_{j_1} \ldots j_n}^{(n-1)} - (-1)^{n-1} c^{(n)}_{\sigma_{j_1} \ldots j_n} \delta_{\sigma_{j_1} \ldots j_n}^{(n-1)} \right\}. \tag{4} \]

where, the index \( j \equiv (\alpha \mu \nu) \) denotes the transfer of an electron to/from (\( o = +/ - \)) the impurity state \( | \sigma \rangle \), associated with the characteristic memory time \( \gamma^{-1}_m. \) The
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total number of distinct j indexes involved is determined by the preset level of accuracy for decomposing reservoir correlation functions by exponential functions. $a^j_\mu$ ($a^j_\mu$) corresponds to the creation (annihilation) operator for an electron with the $\mu$ electron orbital (including spin, space, etc.). The correlation function $C^\alpha_\mu(t)$ follows immediately the time-reversal symmetry and detailed-balance relations.

The spectral function $A(\omega)$ which exhibits prominent Kondo signatures at low temperatures can be evaluated in two ways: either with a time-domain scheme or by calculations in the frequency domain. The time-domain scheme starts with the evaluation of system correlation functions $\tilde{C}_{\mu}(t)$ and $\tilde{C}_{\mu,\alpha}(t)$ via the time evolution of the HEOM propagator $\tilde{G}_{\mu}(t)$ at $t > 0$. The spectral function obtained straightforwardly by a half Fourier transform is

$$A_\mu(\omega) = \frac{1}{\pi} \text{Re} \left\{ \int_0^\infty dt \{ \tilde{C}_{\mu}(t) + [\tilde{C}_{\mu,\alpha}(t)]^* \} e^{i\omega t} \right\}. \quad (5)$$

The frequency domain scheme involves the half Fourier transform of HEOM to evaluate $C_{AB}(\omega) = \int_0^\infty dt C_{AB}(t)e^{i\omega t}$. The spectral function is then

$$A_\mu(\omega) = \frac{1}{\pi} \text{Re}\{ \tilde{C}_\mu(\omega) + \tilde{C}_\mu,\alpha(-\omega) \}. \quad (6)$$

The total system spectral function is $A(\omega) = \Sigma_{\mu=1}^{N}\alpha A_\mu(\omega)$. The details of the HEOM formalism and the derivation of the spectral function of the system are supplied in refs. [26–28]. Consequently, the varied physical quantities such as the local magnetic susceptibility and current can be acquired via the HEOM-space linear response theory.

Results and discussion. – We present the numerical solution of the STQD model in fig. 1(b) using the HEOM method. For simplicity, the three QDs in this model are assumed equivalent and possess the electron-hole symmetry. So we will take the same parameters for the three QDs in calculations as follows: $\epsilon_i$ ($i = 1, 2, 3$) = $-1.0$ meV and $U_i$ ($i = 1, 2, 3$) = 2.0 meV. In fig. 1 we show the single-dot spectral function of the STQD system $A(\omega)$, as a function of frequency $\omega$ for different times of interdot coupling strengths $t$. The quantum dot-lead coupling strength is $\Delta = 0.2$ meV, the band width is $W = 2.0$ meV and temperature is $K_B T = 0.03$ meV. We note that for a weak interdot coupling strength, the spectral function both for QD-1(3) and QD-2 exhibits a single Kondo peak centered at $\omega = 0$. A continuous transition is observed from the Kondo state exhibiting a single-peak Kondo resonance to another exhibiting a double peak by increasing the interdot coupling strength $t$. The most interesting issue in the STQD system is that the Kondo effect reappears at the strong interdot coupling strength for the QD-1(3) (fig. 1(a)), accompanied by a three-peak structure of the spectral function. The width of the central Kondo peak ($\omega = 0$) broadens and the height increases with $t$. Markedly, the substantially larger interdot couplings revives the reappearing single Kondo peak in the density of state of STQD (fig. 1(b)). On the contrary, the Kondo peak of the spectral function for QD-2 disappears (inset of fig. 1(a)) with the increase of the interdot coupling strength.

The physical picture is thus described as follows. At small interdot coupling strength, such as $t = 0.05$ meV, the quantum dot-lead coupling strength is much larger ($\Delta = 0.2$ meV), QD-1(3) prefers to form its respective Kondo singlet with the delocalized electrons of leads (upper part of the inset fig. 1(b)), with the result that the QD-1(3) moment screened by the left (right) lead is dominant and the spectral function shows a similar behavior to the single QD system. Just as the Kondo state of single QD, a single peak Kondo resonance emerges on the QD-1(3). Meanwhile, the left (right) lead and QD-1(3) constitute an effective surrounding reservoir to QD-2. A lower single Kondo peak is observed on QD-2 due to the weak interdot coupling strength between QD-2 and QD-1(3).

With the increase of the interdot coupling strength, each spin of adjacent QDs is asymptotically antiferromagnetic correlated (|↓, ↑, ↓⟩ or |↑, ↓, ↓⟩) (lower part of inset fig. 1(b))), three onsite local moments bind at $J_{\text{eff}} = 4t^2/U$ into a rigid antiferromagnetic spin chain. A flipping of QD-1 spin creates a state that has a finite projection on the excited high-energy state of the STQD system. The electron with the opposite spin hoping from the lead into the QD will break the antiferromagnetic spin chain and increase the energy of the STQD system by $J_{\text{eff}} = 4t^2/U$. Leading to two splitting peaks appears at $\omega \sim \pm J_{\text{eff}}$. If the interdot coupling strength is further increased, an additional single-peak Kondo resonance emerges at $\omega = 0$. The mechanism can be understood as follows. As an antiferromagnetically ordered spin chain, the coherence bonding state of the STQD system persists in a local moment phase. Due to the odd spins, there is a residual spin $\frac{1}{2}$ which can also be screened by the conduction electrons of leads. Finally, as a result of collective spin screening, the additional single-peak Kondo resonance emerges at $\omega = 0$. The coexistence of the Kondo singlet state of QD-1(3) and a residual spin $\frac{1}{2}$ local moment are the reasons of the three-peak structure of the spectral function observed.

Fig. 1: (Color online) (a) The spectral function $A(\omega)$ of the QD-1(3) in the STQD system with different interdot couplings $t$. (b) The density of state (DOS) of the STQD system with substantially larger interdot couplings $t$. The inset of (b) shows the schematic description of the STQD device. The parameters adopted are $\epsilon_i$ ($i = 1, 2, 3$) = $-1.0$ meV and $U_i$ ($i = 1, 2, 3$) = 2.0 meV, $W = 2.0$ meV, $\Delta = 0.2$ meV, $K_B T = 0.03$ meV.
in STQD. For substantially larger interdot couplings, the electrons belong to all the three QDs and are actually itinerant. Only the complete screening of the entire STQD associated with the reappearing single Kondo peak formation will be preservative. The two peaks at $\omega \sim \pm J_{\text{eff}}$ indicating the partial screening results will be quenching.

For strong interdot couplings, the spectral density of QD-2 develops a dip and vanishes at the Fermi level (inset of fig. 1(a)). The dip is associated with the underscreening of the magnetic moment of QD-2 and leads to a complete suppression of the spectral density at the Fermi level. The width of the dip increases with increasing $t$ and suppresses the Kondo resonance. The variation of this dip can be interpreted in terms of a Kondo hole formed as a consequence of the strong coupling between QD-2 and QD-1(3).

Significantly, we note that the transition of the spectral function for QD-1(3) is smooth (crossover) and there is no abrupt phase transition. The behavior of this reappearing Kondo effect of the STQD system possesses some intriguing properties which are not presented in single or double QDs systems.

To further elucidate the physical mechanism of the reappearing Kondo effect, we then study the characters of the susceptibility. The susceptibility of the STQD system vs. interdot coupling strength $t$ is studied in fig. 2(a). By comparison, the parameters adopted are the same as fig. 1. Firstly, the susceptibility of the STQD system decreases monotonically with interdot coupling strength $t$. At very strong interdot coupling, there is no local moment of the STQD system. The QDs spins are completely screened by conduction electrons with the result of a finite value of susceptibility. Similarly, the local susceptibilities both for QD-1(3) (curve (I)) and QD-2 (curve (II)) showed in the inset of fig. 2(a) share the same transition behaviour to the total susceptibility of the STQD system. Importantly, the local susceptibility of QD-2 is extremely sensitive to $t$ than QD-1(3). One interesting issue is the transition of the local susceptibility of QD-2 induced by $t$, from the positive value at weak interdot coupling to the negative value at strong interdot coupling. This behavior also demonstrates the Kondo physics of STQD depicted in the inset of fig. 1(b): at weak interdot coupling strength, the Kondo singlet state of the QD-1(3) forming with the itinerant electron of the left (right) lead is predominant, and the QD-2 approaches an isolated particle corresponding to a free spin, as sketched in the upper part of inset fig. 1(b). It leads to the positive response to the external magnetic field both for three QDs. But with increasing interdot coupling strength, the three QDs are tending to generate a coherence bonding state and the spin of the adjacent QDs are beginning to form an antiparallel arrangement, as sketched in the lower part of inset fig. 1(b). This degenerate ground states are on the lowest energy scale and removed by external magnetic field, leading to the existence of the only one arrangement ($|↓, ↑, ↓\rangle$ or $|↑, ↓, ↑\rangle$). Therefore, the response to the external magnetic field of QD-2 is different from QD-1(3). When the spins of QD-1 and QD-3 are parallel with the external magnetic field, the spin of QD-2 must become antiparallel with the external magnetic field. As a consequence, the QD-2 exhibits a negative susceptibility for strong interdot couplings. The negative value is approximately associated with the formation of a spin-$\frac{1}{2}$ magnetic moment on STQD, which can also be screened asymptotically by the conduction electrons of leads, leading to the reappearance of the Kondo effect. It is needed to pay more attention that only the result of susceptibility of the STQD system can be measured experimentally, but the local susceptibility of each QD cannot be acquired directly by experiment observation. Here, we show the variation of the local susceptibility on each QD only for the analysis of such Kondo physics.

We also analyze the variation of the temperature-dependent susceptibility with different interdot coupling strengths $t$ (fig. 2(b)). For high temperature, the susceptibility of the STQD system is fitted well by a Curie-Weiss law $\chi = C/(T + \theta)$, where $C$ is a Curie constant, $T$ is the temperature and $\theta$ is a constant with a value in the thermal energy range ($0 < \theta < 300\,\text{K}$) [31]. We find that the slopes of $1/\chi K_B T$ curves at various interdot coupling strengths are almost the same value for high temperature. For weak interdot coupling strength, the susceptibility of the STQD system always shows a $T^{-1}$ temperature dependence. It is because that at weak coupling (especially if the temperature is much larger than the coupling energy), the response to the external magnetic field is positive both for the three QDs. More importantly, we focus on the low-temperatures behavior of the susceptibility. The slopes of $1/\chi$ deviates from the straight line at low temperature for the strong interdot coupling strength $t$, where the local susceptibility $\chi$ of QD-2 changes progressively from a positive value into a negative one (not shown here), under which, the STQD system gets into the reappearing Kondo regime. This provides another framework to study the reappearance of the Kondo effect of the STQD system.

The susceptibility of the STQD system for very low temperature ($T \sim 0$) is not presented in our work. It is because the HEOM method only studies the case of finite temperature but cannot deal with the zero-temperature case at present. The difficulty lies in the computational

Fig. 2: (Color online) (a) The susceptibility of the STQD system vs. interdot coupling strength $t$. The inset shows the local susceptibility of the QD-1(3) (I) and of the QD-2 (II). (b) The susceptibility of the STQD system $1/\chi$ vs. temperatures $K_B T$ with different interdot coupling strengths $t$. The parameters adopted are $\epsilon_i (i = 1, 2, 3) = -1.0\,\text{meV}$ and $U_i (i = 1, 2, 3) = 2.0\,\text{meV}$, $W = 2.0\,\text{meV}$, $K_B T = 0.03\,\text{meV}$. 

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The parameters adopted are $T = T_0$, here $T_0$ is the Kondo temperature of the STQD system. We predict that the reappearing Kondo effect of the STQD system, we find that the height of the reappearing Kondo peak rises rapidly with the increasing dot-lead coupling strength $\Delta$. We then examine the dot-lead coupling strength $\Delta$-dependent reappearing Kondo effect. The behavior of the reappearing Kondo effect at different temperatures is studied in fig. 3(a), where we plot the results of the spectral functions $A(\omega)$ of QD-1(3) with different temperatures $K_BT$. To probe distinctly the reappearing Kondo effect of the STQD system, we adopt a large dot-lead coupling $\Delta = 0.3$ meV and a strong interdot coupling $t = 0.25$ meV, the other parameters are the same as those in fig. 1. The STQD system depicts a different varying behavior of the Kondo effect from the single QD problem. Firstly, all the three splitting Kondo resonance peaks on QD-1(3) are robust at low temperatures. The reappearing Kondo effect also enhances with the low temperatures, leading to the increasing height of Kondo peaks for QD-1(3) with decreasing temperature. It deserves special attention that the three peaks transfer to a broad packet with the increasing of the temperature ($K_BT = 0.15$ meV). Finally, all the Kondo peaks disappear at the temperature $T > T_K$, here $T_K$ is the Kondo temperature of the STQD system. We predict that the Kondo temperature $T_K$ of the STQD system is higher than in the single QD system, due to the fact that higher temperature is needed to quench the three splitting Kondo peaks of the STQD system’s coherence bonding state.

We then examine the dot-lead coupling strength $\Delta$-dependent reappearing Kondo effect. Figure 3(b) shows the spectral functions $A(\omega)$ of the QD-1(3) for the interdot coupling strength $t = 0.25$ meV case. We find that the height of the reappearing Kondo peak rises rapidly with the increasing dot-lead coupling strength $\Delta$. We can attribute this transformation to the aforementioned increase of the effective Kondo temperature as a function of the dot-lead coupling. The mechanism can be understood via the Kondo physics of the single QD system. According to the analytical expression for the Kondo temperature $T_K = \sqrt{\frac{\Delta}{2}} e^{-\pi\Delta/\Delta_0^2}$ [31] ($\Delta = 2\Delta$ as two leads in our system), the Kondo temperature $T_K$ increases with the dot-lead coupling strength $\Delta$ by augmenting the height of the Kondo peak. Since the temperature of the STQD system is fixed ($K_BT = 0.03$ meV), this leads to the Kondo effect enhancing with the increase of $\Delta$, accompanied with the rising height of the Kondo peak. Importantly, for the weak interdot coupling strength $t$, the strong dot-lead coupling strength $\Delta$ only heightens the single Kondo peak, but cannot develop the three-peak structure of the spectral function as shown in fig. 3(b). This happens because the reappearing Kondo effect originates from the coherence bonding state in the STQD system, which forms at a stronger interdot coupling strength $t$.

Finally, we study the Kondo-correlated transport properties through the STQD system. The current through the STQD device for the appropriate values of voltages and interdot couplings, with the same parameters as in fig. 1, are plotted in fig. 4. When the bias voltage is applied to the leads, the current flowing through the STQD system engenders and rapidly increases. Moreover, the current increases strongly with the interdot coupling strength $t$. The increasing current behavior can be explained according to the Kondo effects picture of the STQD. The Kondo peak in the density of state leads to the resonant transmission through the STQD system. For large bias voltage, more Kondo peaks in the density of state fall into the bias window. So the striking enhancement of transport is the summational results of the first appearance and reappearing Kondo resonances in the STQD device. As a matter of fact, the observation of such a variation will provide a remarkable phenomenon of quantum coherence transport between the Kondo many-body states.

**Conclusions.** - In summary, we have investigated the Kondo effect and spectral properties of the STQD system based on the HEOM method. A clear picture of the reappearance of the Kondo effect induced by the interdot coupling strength $t$ is described. These reappearing Kondo effects will develop a prominent transport behavior of the STQD system.
For weak interdot coupling, the conduction electrons of the lead screen the adjacent QD to lead the Kondo effect analogously to a single QD system. With the increase of the interdot coupling strength $t$, the STQD system asymptotically transforms from the Kondo singlet state of an individual QD to the coherence bonding state generated among the three QDs. So a crossover translation from the local Kondo screening of the constituents to the reappearing Kondo effect accompanied with a three-peak structure of the spectral function of QD-1(3). The properties of the susceptibility according to the interdot coupling strength $t$ displays a different temperature-dependent behaviour from the single QD system. The stronger dot-lead coupling strength $\Delta$ can also inhibit a different temperature-dependent behaviour from the local Kondo screening of the constituents to the reappearing Kondo effect. As a summational result of the first appearance and reappearing Kondo effects, the transport current through STQD increases monotonically with increasing $t$. The characteristic universal signatures of the STQD system in physical quantities may be observed in experiments and is a prerequisite for the understanding and design of more complex structures, such as Kondo lattices. We hope our work will inspire and encourage experimental investigations of Kondo physics in coupled three QDs and related systems.

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