Modeling of gas flows in radial micro-nozzles

S P Kiselev¹,², V P Kiselev¹, V Yu Liapidevskii³,⁴ and V N Zaikovskii¹

¹Khrushchov Institute of Theoretical and Applied Mechanics, Russian Academy of Sciences, Siberian Branch, 630090 Novosibirsk, Russia
²Novosibirsk State Technical University, 630092 Novosibirsk, Russia
³Lavrent’ev Institute of Hydrodynamics, Russian Academy of Sciences, Siberian Branch, 630090 Novosibirsk, Russia
⁴Novosibirsk State University, 630090 Novosibirsk, Russia

E-mails: kiselev@itam.nsc.ru, kiselevvp@itam.nsc.ru, zaikovskii@itam.nsc.ru, liapid@hydro.nsc.ru

Abstract. In the present paper, results of an experimental and numerical study of gas flows in radial micro-nozzles are reported. The radial micro-nozzle consists of two disks spaced apart by less than one millimeter distance. To the inlet of the radial nozzle, a gas under high pressure is supplied; through the nozzle, the pressurized gas is ejected into ambient space. In the present study, we analyzed the flows in which the gas at the inlet to the micro-nozzle had a supersonic velocity. It was shown that, on the condition that the micro-nozzle width was smaller than some critical value, then, due to the wall friction force, a pseudo-shock arose in the micro-nozzle. In the pseudo-shock, the gas underwent deceleration to a subsonic velocity. A satisfactory agreement between the calculated and experimental data was achieved.

1. Introduction

Until recently, little attention was given to the study of supersonic gas flows in radial micro-nozzles. Exceptions here is the publication [1], in which supersonic gas flows in wider nozzles and at lower pre-chamber pressures in comparison with the present study were dealt with. However, by now novel technologies in which radial nozzles play an important role have emerged. S.P. Kiselev et al. [2] have reported experimental and numerical data proving the possibility of using radial micro-nozzles for microparticle acceleration and cold-spray application of coatings onto inner pipe surfaces. In [3], self-sustained oscillations that arose during the outflow of a supersonic jet into ambient space were studied. In the present paper, we report detailed data gained in experiments and numerical simulations in which the structure of supersonic gas flows in radial micro-nozzles was investigated.

2. Experimental procedure

The radial micro-nozzle was formed by two disks spaced apart by a distance smaller than one millimeter (this distance was the nozzle width). The experiments were carried out on a facility schematically shown in Figure 1. The experimental arrangement comprised an external disk 1 with radius \( r_e = 36 \) mm and an internal disk 2 of the same radius. The space between the disks was connected to a pre-chamber 3. The pre-chamber was a cylindrical tube with internal radius \( r_i = 5 \) mm and external radius \( r_o = 9 \) mm. The gas was supplied to the pre-chamber under pressure \( p_0 \) through...
twelve holes that were made on the external surface of the tube (in Figure 1, those holes are conventionally indicated with three arrows)
In equations (1), \( v \) is the radial component of gas velocity, \( p \) is the pressure, \( \rho \) is the gas density, \( T \) is the gas temperature, \( C_v \) and \( C_p \) are the specific heats of the gas at constant volume and pressure, \( R \) is the universal gas constant, and \( \tau_w \) is the wall friction force. Like in [1], the friction force \( \tau_w \) was calculated by the formulas

\[
\tau_w = C_f \frac{\nu v^2}{2}, \quad C_f = \frac{0.06}{(2 \text{Re})^{1/4}}, \quad \text{Re} = \frac{v h}{\nu},
\]

where \( \nu = \mu/\rho \) is the kinematic viscosity and \( \mu \) is the dynamic viscosity, which for the case of air was calculated by the Sutherland formula [4]. The dependence of drag coefficient \( C_f \) on the Reynolds number \( \text{Re} \) was chosen as the one in a turbulent flow of incompressible fluid. As it was shown in [1], the friction force \( \tau_w \) was calculated by the formulas (2) was adjusted so that a best agreement between the calculated and experimental data could be achieved in one of the experiments for a nozzle with \( h = 0.15 \text{ mm} \). Subsequent numerical calculations for the flows at other nozzle-width values and the comparisons of calculated with experimental data were carried out at the chosen value of the constant. Note that in the study of [1] a value of 0.08 was adopted for the empirical constant.

System (1) was solved numerically by the explicit Lax–Wendroff difference scheme [5]. In calculations of flows with shock waves by this scheme, shock-wave smearing by scheme viscosity occurs. In the region \( r < r < r_e \), the problem about the decay of a pressure discontinuity specified at the initial time at some section \( r = r_e \) of the nozzle was solved:

\[
p = p_0, \quad T = T_0 = T_{\infty}, \quad v = 0, \quad r < r < r_e; \quad p = p_\infty, \quad T = T_{\infty}, \quad v = 0, \quad r_e < r < r_e, \quad (3)
\]

where \( p_0 \) is the pressure in the pre-chamber, \( p_0 > p_\infty \). After the decay of the discontinuity, a rarefaction wave propagates to the left; behind this wave, the gas undergoes acceleration as it moves toward the right-hand boundary of the nozzle. Initially, at the left boundary of the calculation domain, \( r = r_e \), a condition of symmetry is posed. This condition holds till the time \( t = t_* \), unless at this boundary a critical condition \( M_1 = 1 \) is achieved. Afterwards, all the three characteristics penetrate into the nozzle and, therefore, nozzle choking conditions, \( p = p_\infty, T = T_{\infty}, v = v_c = c \), are to be posed at the left boundary (here, \( M = \nu/c \) is the Mach number, \( c = \sqrt{\gamma RT} \) is the speed of sound, and \( \gamma = 1.4 \)). Marked with the asterisks are the values of flow quantities at the critical point \( r = r_e \); these values can be expressed in terms of the corresponding parameter values in pre-chamber using the well-known isentropic formulas [4]:

\[
p_\infty = \beta^{\gamma-1} p_0, \quad T_\infty = \beta T_0, \quad v_c = \beta^{\gamma/2} c_0, \quad \beta = 2/\gamma + 1.
\]

At the right boundary of the calculation domain, \( r = r_e \), at a subsonic speed \( M_e = v_e/c_e < 1 \) the pressure in ambient volume, \( p = p_\infty \), was posed, whereas at a supersonic speed \( M_e > 1 \), conditions of symmetry were used. The change of boundary conditions was due to the fact that at subsonic velocity only one, left characteristic penetrated into the calculation domain, whereas at supersonic velocity all the three characteristics were coming out from this domain.

Consider a problem about the decay of a discontinuity in a nozzle with \( h = 0.15 \text{ mm} \) which occurs at the point \( r_0 = 23 \text{ mm} \). Initially, a pressure drop in a gas at rest with temperature \( T_0 = 300 \text{ K} \) is specified. At the left of the point \( r = 23 \text{ mm} \), a uniform pressure \( p_0 = 1.43 \text{ MPa} \) is assumed, and at the right of this point, we have \( p_\infty = 0.1 \text{ MPa} \). After the decay of the discontinuity, in the nozzle there
arises an unsteady gas flow that reaches a steady state in a time \( t_i = 3 \mu s \). The establishment of the steady nozzle flow is illustrated in Figure 2.

![Figure 2](image1.png)

**Figure 2.** The calculated distributions of flow quantities in the nozzle at times \( t_i \) (0; 0.6; 1.2; 1.8; 2.4; 3.0; \( \mu s \)): (a) – \( p(r, t_i) \); (b) – \( M(r, t_i) \); (c) – \( T(r, t_i) \)

The dashed line in Figure 2 shows the initial distribution of flow quantities. The thin solid line and the dash-and-dot lines show the flow quantities in the unsteady flow, whereas the thick solid line shows the flow quantities in the established flow. Evidently, the choking condition \( M = 1 \) is realized in the established flow at the nozzle inlet. Behind the inlet, the flow accelerates to a supersonic velocity \( M = 1.4 \) (\( r \approx 13.5 \text{ mm} \)) and, afterwards, the flow undergoes deceleration. The supersonic-flow deceleration region ends in a shock wave (\( r \approx 22 \text{ mm} \)), in which the supersonic flow with \( M > 1 \) passes over into a subsonic flow with \( M < 1 \). Unlike in pipe flows [4], here the gas flow behind the shock wave first undergoes deceleration and, then, acceleration to the sonic velocity, \( M = 1 \). The gas temperature grows in value behind the shock wave to attain a maximum at the point at which the gas deceleration gives way to gas acceleration. At the nozzle outlet, there establishes a sonic flow with \( M = 1 \), like it also occurs in uniform-section pipes [4].

The variation of pressure with radius in three radial nozzles of width \( h = 0.075; 0.15; 0.5 \text{ mm} \) as revealed in numerical calculations and experiments (respectively, lines and symbols) are shown in Figure 3. The data calculated by model 1 are shown with thick lines, and the data calculated by model 2 (this model is described below in Sec. 3.2), with thin lines.

![Figure 3](image2.png)

**Figure 3.** Normalized pressure versus radius for various micro-nozzles; the triangles and dashed lines refer to \( h = 0.075 \text{ mm} \) and \( p_0 = 1.4 \text{ MPa} \); the circles and solid lines, to \( h = 0.15 \text{ mm} \) and \( p_0 = 1.4 \text{ MPa} \); and the squares and dash-and-dot lines, to \( h = 0.5 \text{ mm} \) and \( p_0 = 1.15 \text{ MPa} \).
As it is evident from Figure 3, the calculated data well agree with the experimental data. In the nozzle with $h = 0.5 \text{ mm}$, a near-isentropic flow in which the pressure monotonically decreases with radius is realized. In the nozzle with $h = 0.15 \text{ mm}$, we have a flow with a shock wave located in the section $r \approx 22 \text{ mm}$. On decreasing the nozzle width to $h = 0.075 \text{ mm}$, a displacement of the shock wave to the inlet section is observed. Behind the shock wave, there establishes a constant gradient of pressure, which compensates the action of wall friction force on the gas flow in the nozzle.

### 3.2. Mathematical model 2

A mathematical unsteady pseudo-shock model for a barotropic gas flow in a long plane channel capable of providing a description to the transition from supersonic to subsonic flow was developed in [6]. The model is based on a two-layer flow scheme with mass transfer that includes a potential supersonic flow core and a turbulent boundary-layer flow. As the boundary layers join together, the model transforms into a single-layer model for barotropic vortex gas flow. In microchannels analyzed in the present study, the two-layer flow region of a length not exceeding ten channel widths can be neglected. In the latter case, the pseudo-shock can be represented in long-wave approximation with a weak shock wave and with a subsequent region of intense vortex flow.

In radial nozzles, the equations of swirling barotropic gas flow assume the form (cp. with equations (5) in [6]):

\[
\begin{align*}
\frac{\partial \rho v_r}{\partial t} + \frac{\partial \rho v_r v_r}{\partial r} &= 0, \\
\frac{\partial \rho v_r}{\partial r} + \frac{\partial (\rho (v_r^2 + q_r^2) + \rho p)}{\partial r} &= p - \frac{2r \tau_{w}}{h}, \\
\frac{\partial}{\partial t} (\rho \left(\frac{v_r^2}{2} + q_r^2 + \varepsilon\right)) + \frac{\partial}{\partial r} (\rho v_r \left(\frac{v_r^2}{2} + q_r^2 \right) + (\varepsilon + \frac{p}{\rho})v_r) &= -\kappa \rho q^2 - \frac{2r \tau_{w}}{h}, \\
p &= p_0 \left(\frac{\rho}{\rho_0}\right)^\gamma, \\
\varepsilon &= \frac{p}{(\gamma - 1)\rho}.
\end{align*}
\]

Here, the quantity $\omega = q / h$ describes the mean vorticity of the flow, and the constant quantity $\kappa$ defines the rate of energy dissipation. Note that the sub-class of continuous solutions (4) with $q \equiv 0$ coincides with the class of continuous solutions of the classical equations of barotropic gas flow. However, in the discontinuous solutions of equations (4), behind the shock-wave front there forms a vorticity leading to a smooth rise of pressure in the pseudo-shock.

Equations (4) are hyperbolic equations that have one contact characteristic and two sonic characteristics:

\[
\frac{dr}{dt} = v, \quad \frac{dv}{dt} = v \pm \sqrt{c^2 + 3q^2}, \quad c^2 = \frac{Z P}{\rho}.
\]

Thus, equations (4) are equations of the gas-dynamic type, and the numerical solutions of the problem about the structure of the transonic flow in a radial nozzle can therefore be obtained by standard methods. In the present study, for performing a comparison between the solutions of equations (4), on the one hand, and the experimental data and numerical solutions obtained above within model 1, on the other hand, we used a method developed by S.K. Godunov. As the initial data, the exact supersonic solution of equations (4) with $q \equiv 0$ and $C_f = 0$ was used. The boundary conditions were the same as those for model 1. At the left boundary of the calculation domain, the flow was assumed irrotational. The value of $\kappa$ in our calculations weakly affected the solutions, and it therefore was put equal to zero. In determination of the dissipative term of (2), the air kinematic viscosity was assumed constant ($\nu = 1.2 \cdot 10^{-5} \text{ m} / \text{s}^2$). The calculated distributions of pressure for three designs of radial nozzle are shown in Figure 3 with thin lines together with their respective experimental data. It is seen that the
data calculated by model 2 are close to the data calculated by model 1, and both calculated datasets satisfactorily agree with the experiment.

4. Conclusion

In sufficiently wide plane and radial nozzles, the pseudo-shock region occupies a substantial portion of the channel. Under such conditions, during the transition from supersonic to subsonic flow, a predominant fraction of energy dissipation (up to 90%) is spent for generation of the mechanical energy of vortex flow in the turbulent boundary layer. In the microchannels, the pseudo-shock width decreases markedly, allowing the use of quasi-one-dimensional gas flow models for pseudo-shock modeling. For making a comparison between the dissipative processes related with the growth of entropy and the generation of vorticity in microchannels, in the present study we have considered two alternative approaches, the classical one-dimensional gas-dynamics equations and the vortex-flow equations for barotropic gas. The performed numerical calculations of gas flows in microchannels have shown that both models yield close results. This circumstance is related with the fact that, in transonic gas flows in microchannels, the boundary-layer perturbation or separation regions behind pseudo-shocks extend to a distance of order ten microchannel widths. The ratio between the length of the perturbed boundary layer behind pseudo-shock and the microchannel radius is low, and both regions contribute little to the deceleration of the gas flow. The results of numerical calculations by both models fairly well agree with experimental data.

Acknowledgements. This work was partially supported by the Russian Foundation for Basic Research under Grant No. 16-01-00156-a.

References

[1] Moller P 1966 J. Basic Eng. Series D 88 153-4
[2] Kiselev S P, Kiselev V P and Zaikovskii V N 2016 J. of Appl. Mech. and Tech. Phys. 57 237-46
[3] Kiselev S P, Kiselev V P, Klinkov S V, Kosarev V F and Zaikovskii V N 2017 Surface & Coating Technology 313 24-30
[4] Abramovich G N 1976 Applied Gas Dynamics (Moscow: Nauka) (in Russian)
[5] Anderson D A, Tannehill J C and Pletcher R H 1984 Computational Fluid Mechanics and Heat Transfer (Hemisphere Publishing Corporation)
[6] Lipatov I I, Liapidevskii V Yu and Chesnokov A A 2016 Doklady Academii Nauk 466 545-9 (in Russian)