Analysis of Linear Precoding Techniques for Massive MIMO-OFDM Systems under various scenarios

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Abstract: Emerging 5G technology is largely supported by the base station signal processing techniques such as precoding. Precoding combines the input signals in a predefined way and deliver them in a right proportion to the multiple antenna elements. Such precoding algorithms are crucial for the design of the emerging 5G technology; massive MIMO system. In this paper, the performance of various linear precoding algorithms such as Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Conjugate Gradient (CG) based precoding are analyzed in terms of Bit Error Rate (BER) and achievable sum-rate. The results are analyzed under various channel conditions such as rural, sub urban and urban. The above algorithms are tested for MIMO-OFDM system. OFDM system simulated here uses 52 subcarriers and a Base Station equipped with more than 100 antennas serving multiple users simultaneously. The channel models are stochastic channels simulated using WINNER II modeling.

Keywords-Massive MIMO, Linear precoding, Zero-Forcing(ZF), Conjugate gradient, Minimum Mean Square Error (MMSE).

INTRODUCTION

Transmit and receive diversity of Multiple Input Multiple Output (MIMO) helps to overcome the interference due to fading effect of the wireless channels by providing multiple links between the antennas over which the different versions of the same signal is being transmitted. This helps in reducing the loss due to fading by duplicate transmissions at the BS tower and maximal ratio combining at the receiving Mobile Terminal (MT). Throughput can also be increased by the virtual channels across the multiple transmitting and receiving antennas. This traditional MIMO is the older one which is also termed technically as Single User MIMO (SU-MIMO) since it serves only one user at a time.

MU-MIMO shown in Figure 1 is the extension of SU-MIMO in combination with Space Division Multiple Access (SDMA) [1]. But, the major challenge of this technology is the interference between users as multiple users are being served by the single BS. Solving this issue is being handled by transmit side signal processing operations such as beamforming (precoding). Thus with the thorough knowledge of channel, MU-MIMO can improve the gain and throughput.
Figure 2 makes a breakthrough in recent wireless systems by accommodating antennas in the order of hundreds. Massive MIMO systems offer coverage to much smaller $K$ number of users per cell by means of a BS with considerable $M$ number of antennas, $M << K$ [2]. But, in comparison with MU-MIMO, massive MIMO can provide coverage to more number of users.

Precoding is the signal processing operation that combines the input signals in a predefined way and deliver them in a right proportion to the multiple antenna elements. Such precoding algorithms are crucial for the design of the MU-MIMO and massive MIMO systems. For MU-MIMO systems with limited number of antennas linear precoding techniques fail to eliminate Multi User Interference (MUI). At the transmit side Base Station a simple technique of precoding is enough to deal with the MUI by using an algorithm called as Zero Forcing (ZF) precoding [3]. Hence in this paper we make a survey on the performance analysis of various precoding techniques under various scenarios like urban, and rural. The simulation results are given in terms probability of error for a massive MIMO-OFDM system which is the technology used in 4 G LTE standard.

1. LINEAR PRECODING ALGORITHMS
MU-MIMO and massive MIMO with the help of precoding (digital beamforming) and precoding allows the transmission of different signals over each antenna element with appropriate gain and phase tuning. Thus, the digital precoders are designed to generate multiple main beams so as to focus towards multiple users at the same time as shown in Figure 3.

![Figure 3. Digital Precoding](image)

Precoding in massive MIMO facilitates multi-stream data transmission intended for different users by weighting the signal emerging from every antenna element appropriately. The objective of the precoder design is to compute the precoding weight vector so as to maximize the link throughput at the receiver. Keke Zu (2013) [4] has worked out on many precoding algorithms for Multi User-MIMO (MU-MIMO) systems that open a way for its extension to large scale MU-MIMO systems. But, the complexity of the precoder design increases proportionally with the dimension of the antenna array. Hence, the precoding algorithms selected for massive MIMO systems have to make a trade-off between complexity and performance.

1.1 Minimum Mean Square Error (MMSE) Algorithm

Bahrami et al. (2008) [5] shown that linear precoding algorithms such as Maximal Ratio Transmission (MRT), ZF and Minimum Mean Square Error (MMSE) are simpler and of low complexity but, with the penalty on performance degradation.

The performance of various algorithms like ZF and MMSE (Regularised ZF) are compared for Rayleigh fading channel and channel correlation in Gao et al. (2011) as a function of the number of BS antennas. Under realistic propagation scenario the benefits of very large, but limited, antenna elements were studied in this literature. It shows that these algorithms reach 98% of the optimal capacity limits of DPC precoding with limited number of 20 BS antennas itself. The performance of massive MIMO system using various linear precoding algorithms is discussed in Hoydis et al. (2012) [6].

Rusek et al. (2013) [7] elaborates on the computation of precoding matrix $W$ using MMSE or Regularized Zero Forcing (RZF) algorithm. The precoding matrix $W$ using MMSE algorithm is given in Equation (1) as

$$ W = H^H (HH^H + \delta I)^{-1} $$

(1)
where $H$ is the knowledge of channel obtained using uplink pilot transmission assuming channel reciprocity, $\delta$ is the regularization factor and $I$ is the identity matrix. As $\delta \to 0$, the second term $\delta I$ becomes negligible and hence MMSE precoder reaches ZF precoder as given in Equation (2)

$$W = H^H (HH^H)^{-1}$$  \hspace{1cm} (2)

In the above equation $HH^H$ is the Gram matrix $G$ that has to be inverted for the computation of $W$ as given in Figure 4.

**Figure 4.** Computation of $W$ of ZF precoder

In case of matched filter precoding, as $\delta \to \infty$ the weight vector calculation is as follows

$$W = H^H (\delta I)^{-1} = H^H \delta^{-1}$$ \hspace{1cm} (3)

The above algorithm becomes much simpler since it avoids the computation of large size Gram matrix inversion $G^{-1}$. But the algorithm provides much inferior performance in most of the scenarios (results provided in section IV). With the calculated weight vector $W$, the data vector to be transmitted over “T” number of antennas is

$$X = H^H (HH^H + \delta I)^{-1} S$$ \hspace{1cm} (4)

The above transmitted data sequence is sent over the channel where it is added with Additive White Gaussian Noise (AWGN) of zero mean and unit variance. Hence the data vector received by the receiver is given by

$$Y = HX + N$$ \hspace{1cm} (5)

Here $H$ is the channel gain obtained through the channel state information (CSI) obtained through the uplink pilot information.

2. **LOW COMPLEXITY REGULARIZED ZERO-FORCING ALGORITHMS**

The above algorithms show good performance while producing complications in finding inversions. The algorithm shown in the next section provides simplicity in implementing the inverse modules of the algorithms ZF or MMSE algorithms.

2.1 **CG based precoding algorithm**
In Bei Yin et al. (2014) [8,10], Conjugate Gradient (CG) solves problems of the form

$$\hat{x} = \arg \min_x |b - Gx|$$

(10)

where $G$ is a positive definite matrix and $b$ is column vector containing the transmitted symbol vector. With $A=G=HH^H+\delta I$ by using the above mentioned algorithm $G^{-1}$ is approximated with the solution obtained in “N” iterations where Gram matrix $G$ is of size $N \times N$. The result obtained is $G^{-1}b$ at the end of final iteration which is then multiplied with $H^H$ then compute the transmitted symbol vector.

Algorithm: CG algorithm for Gram matrix inversion

// System to be solved: $\hat{x} = A^{-1}b$

// Input:

\[ A = G = HH^H + \delta I, \quad b = s \]

x_0 = 0; r_0 = b; d_0 = r_0

// Iteration starts: for k=1,2,…..N

$$\alpha_k = \frac{r_k^T r_{k-1}}{d_{k-1}^T Ad_{k-1}}$$

Approximate solution:

$$x_k = x_{k-1} + \alpha_k d_{k-1}$$

Residual correction

$$r_k = r_{k-1} - \alpha_k Ad_{k-1}$$

Improvement parameters

$$\beta_k = \frac{r_k^T r_k}{r_{k-1}^T r_{k-1}}$$

//Search directional vector

3. SIMULATION RESULTS

The simulation is done for the massive MIMO-OFDM system with 52 sub carriers and 64 QAM modulated symbol under various propagation scenarios. The QAM modulated symbols intended for “N” number of users are transmitted over parallel OFDM modulators with 52 subcarriers per OFDM symbol. The propagation scenarios considered are rural, urban and suburban scenarios and the comparison of performances are shown in Figure 5, 6 and 7 respectively. All the above scenarios are simulated in WINNER-II channel model [9]. The number of transmitting antennas are 100 at the base station and number of users are 4.
From the above simulation results it is clear that Matched filter precoding performance is poor compared with the other two algorithms irrespective of the scenario.
4. CONCLUSION

The comparison of all the above mentioned precoding algorithms is done in rural, urban and sub urban scenarios for MIMO-OFDM system with 64 QAM modulation. The MMSE or Regularized Zero-Forcing (RZF) outperforms Matched Filtering (MF) precoding algorithm in terms of probability of bit error in all scenarios. The complexity of MMSE can be greatly reduced by approximating the matrix inversions by suitable CG algorithm. The performance of CG precoding is also compared in the above scenario and it is proved that it shows better performance in rural macro cell scenario than the urban scenarios. Hence we conclude that this algorithm is better implemented for rural environments. In future the algorithm will be simulated for higher order of users and the effects will be observed.

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