Black holes in many dimensions at the LHC: testing critical string theory

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We consider black hole production at the LHC in a generic scenario with many extra dimensions where the Standard Model fields are confined to a brane. With ∼ 20 dimensions the hierarchy problem is shown to be naturally solved without the need for large compactification radii. We find that in such a scenario the properties of black holes can be used to determine the number of extra dimensions, n. In particular, we demonstrate that measurements of the decay distributions of such black holes at the LHC can determine if n is significantly larger than 6 or 7 with high confidence, and thus can probe one of the critical properties of string theory compactifications.

One of the most difficult questions facing theoretical high-energy physics is how to consistently combine General Relativity with Quantum Mechanics, as naive quantization produces unrenormalizable divergences. This issue is exacerbated by the hierarchy problem, which asks why the electroweak scale, $M_{ew} \sim \text{TeV}$, is so small compared with the (reduced) Planck scale, $M_{Pl} \sim 10^{18}$ GeV, which is associated with the energy at which non-renormalizable Einstein gravity becomes strong. It appears that resolution of these puzzles may require a complete theory of quantum gravity.

In recent years it has been proposed that the fundamental scale of gravity might not be $M_{Pl}$, but rather $M_s \sim \text{TeV}$ [1, 2]. There is then no large hierarchy between the gravitational and electroweak scales. In this scenario, the observed weakness of gravity results from the presence of extra dimensions with large radii. In the simplest picture, gravity is able to propagate in all $D$ dimensions, but the Standard Model (SM) fields are restricted to a 3 + 1 dimensional “brane”. The strength of gravity at long distances is then diluted by the volume of the extra dimensions. Here, we examine a scenario where the number of extra dimensions is large. In this case, as we will see below, additional hierarchies do not arise between $M_s$ and the size of the additional dimensions. In particular, we examine the properties of black hole (BH) production and decay at the LHC with different numbers of extra dimensions and show that the number of additional dimensions $n$ can be determined at high confidence, in particular when $n$ is large. Our results hold in the generic case where the size of the BH is much less than the curvature of the additional dimensions, and where the SM is confined to a 3-brane.

As of now, the best candidate for a complete theory of quantum gravity is (critical) string theory (CST), which reduces to Einstein gravity at low energies and allows for the computation of finite $S$-matrix amplitudes. For CST to be a consistent theory there are three essential ingredients: (i) the fundamental objects of the theory are no longer point-like and must have a finite size of order $M_s$, the string scale; (ii) supersymmetry must be a good symmetry, at least at scales $\gtrsim M_s$; (iii) space-time must be ten or eleven dimensional, (i.e., $D = 4 + n = 10$, if the string coupling is perturbative, $D = 11$ if it is non-perturbative), with the additional dimensions being compactified at a radius $R_c \gtrsim 1/M_s$. Most research in string theory so far has focused on critical string theories, where the world-sheet anomalies are automatically canceled. It is precisely this anomaly cancelation that requires $D = 10$. However, there are consistent non-critical backgrounds of string theory in arbitrary numbers of dimensions. Here, the anomalies are canceled by solving the equations of motion taking into account the tree level moduli potential as well as contributions to the equations of motion from other sources such as fluxes, orientifolds, and branes [3]. In either case, the common expectation is that $M_s$ is slightly below or equal to $M_{Pl}$ which would imply that the predictions of CST are difficult to test directly. Currently there is no evidence for any of these basic assumptions. If indeed $M_s \sim M_{Pl}$ it may be that CST can never be directly tested in laboratory experiments. Furthermore, even if supersymmetry and/or extra dimensions were discovered in future experiments, this would be no guarantee that CST represents the correct theory of nature.

We will show in this paper that the number of compactified large dimensions can be determined from black hole production at the LHC. This would provide a probe of classes of CST models. Specifically, if $n > 6(7)$ is measured with high confidence then present CST compactifications would be tested. As a proof of principle for our proposal, we will show that there exists a region in the parameter space where we can experimentally exclude the case $n \leq 6(7)$ at 5σ significance.

For purposes of demonstration, we perform our calculations in the the large extra dimensions picture of Arkani-Hamed, Dimopoulos and Dvali (ADD) [2]. We emphasize that our results are general and we only use ADD as a calculational framework. Here, $M_s$ and $M_{Pl}$

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are related by $M^2_{pl} = V_n M^{n+2}_s$, where $V_n$ is the volume of the $n$ compactified large dimensions. For simplicity in what follows, we will assume that this $n$-dimensional space is compactified on a torus of equal radii so that $V_n = (2\pi R_c)^n$, where $R_c$ is the compactification radius. Given $M_{pl}$ and $M_s \sim$ a few TeV, $R_c$ becomes completely fixed by the relation above. Note that the case $n = 1$ is excluded while $n = 2$ with low $M_s$ is disfavored by current data. For the case of a torus, the graviton has Kaluza-Klein (KK) excitations $h^{(n)}_{\mu\nu}$, with masses given by $M^2_n = n^2/R_c^2$, where $n$ labels a set of occupation numbers. The KK graviton couplings to the Standard Model (SM) fields are described by the stress-energy tensor $T^\mu\nu$, given in $D$ dimensions by $\mathcal{L} = -\sum_n h^{(n)}_{\mu\nu} T^\mu\nu/M_{pl}^{4+n/2}$.

This scenario has three distinct experimental signatures which have been studied in some detail in the literature: (i) missing energy events associated with KK graviton emission in the collisions of SM fields; (ii) new contact interactions associated with spin-2 KK exchanges between SM fields; (iii) black hole production in particle collisions.

Is there any guide as to what values of $n > 6(7)$ we should consider? For $n \leq 6$ it is well known that the hierarchy problem is not truly solved. Although we have reduced $M_s$ to a few TeV, $M_s R_c \gg 1$, as seen in Fig. 1. By contrast, with $n$ large we could have $M_s R_c \lesssim 10$. Note that, if $M_s R_c < 1$ the theory would lose its predictive power since the compactification scale is above the cutoff. To obtain the interesting range of compactification radii, $1 \lesssim M_s R_c \lesssim 10$, requires $17 \lesssim n \lesssim 39$, hence we will focus on this set of values in what follows. If the compactification topology is a sphere, rather than a torus, this changes to $n \gtrsim 30$, as seen in Fig. 1. It is important to notice that this model does solve the hierarchy problem for large $n$, but this would lie outside the realm of CST. Note that some other modifications of the compactification geometry can obtain $R_c M_s \lesssim 10$ [11]. For such large values of $n$ the Kaluza-Klein masses are at the TeV scale. For example, in the ADD case, since each graviton KK state is coupled with 4 dimensional Planck strength, $M_{pl}$, to the SM fields, it is clear that this significantly weakens the KK contributions to the processes (i) and (ii) above. Thus, no meaningful collider constraints would be obtainable; this may also happen in the generic model we consider here. For example, in ADD with $n = 2$, precision measurements at the International Linear Collider at $\sqrt{s} = 1$ TeV will be sensitive to $M_s \lesssim 10$ TeV, while with $n = 21$, this drops to $M_s \lesssim 1$ TeV. Thus for reasonable values of $M_s$ the only signal for large $n$ in ADD is black hole production.

We now investigate BH production at the LHC in detail; for previous studies see [12]. When $\sqrt{s} \gtrsim M_s$, BHs are produced with a geometric (subprocess) cross section, $\hat{\sigma} \simeq \pi R_s^2$. We expect this to hold in all models which satisfy our assumptions. Here $R_s$ is the Schwarzschild radius corresponding to a BH of mass $M_{BH} \simeq \sqrt{\hat{s}}$. $R_s$ is given by

$$M_s R_s = \left[ \frac{\Gamma(n+2)}{(n+2)\pi^{(n+3)/2}} \frac{M_{BH}}{M_s} \right]^{1/(n+1)}.$$  (1)

Note that $\hat{\sigma} \sim n$ for large $n$. Numerical simulations and detailed arguments have shown that the geometric cross section estimate is good to within factors of a few [14]. The total number of BH events at the LHC with invariant mass above an arbitrary value $M_{BH,\text{min}}$ is shown in Fig. 2. The scale of the total inclusive BH cross-section, $\sim 100$ pb, is huge compared to that which is typical of new physics processes, $\sim 1$ pb. Thus, over much of the parameter space the LHC will be producing over a million BH events per year. This high rate means that there will be tremendous statistical power, and essentially all measurements will be systematics limited.

The semiclassical treatment, used here and in all previous studies [14], may receive potentially large corrections from two sources: (i) distortions from the finite compactification scale as $R_s$ approaches $R_c$, and (ii) quantum gravity. Case (i) is easily controllable. We know that in 5 dimensions the critical point for instabilities due to finite compactification is $(R_s/R_c)^2 \approx 0.1$ [12]. For LHC energies we always have $(R_s/R_c)^2 << 0.1$, so these corrections are negligible. In more dimensions the ratio of the volume of a BH with fixed $R_s$ to the volume of the torus with fixed $R_c$ drops rapidly with $n$, so we expect the corrections to be even smaller. Case (ii) is more problematic; we estimate the quantum gravity effects by looking at the corrections from higher curvature terms in the action, e.g.

$$S = \frac{M^2_{pl} - 2}{2} \int d^D x \left( R + \frac{\alpha_1}{M^2_s} \mathcal{L}_2 + \frac{\alpha_2}{M^4_s} \mathcal{L}_3 + \ldots \right).$$  (2)

Here $R$ is the Ricci scalar, and $\mathcal{L}_i$ is the $i$th order Lovelock invariant, with $\mathcal{L}_2$ being the Gauss-Bonnet term [16]. This equation also defines our convention for the funda-
highly asymmetric states, with high angular momentum, and possibly a non-zero charge. However, they quickly shed their charge and angular momentum by emitting bulk graviton modes and soft brane modes, and relax to a simple Schwartzchild state; their decay then proceeds primarily by thermal emission of Hawking radiation \cite{12} until $M_{BH} \sim M_*$, where quantum gravity effects will mediate the final decay. The Hawking temperature is given by

\[ T_H = \frac{(n+1)M_*}{4\pi} \frac{\Gamma\left(\frac{2+n}{2}\right)}{(n+2)\pi^{(n+3)/2}} \frac{M_{BH}}{M_*}^{-1/(n+1)}. \] \( (3) \)

From this we can see that, at fixed $M_{BH}$, higher dimensional BHs are hotter. Since the average multiplicity goes inversely with the temperature, a low dimensional BH will emit many quanta before losing all of its energy. By contrast, the decay of a high dimension BH will have fewer final state particles, and each emitted quanta will carry a larger fraction of the BH energy. We will use this difference to obtain experimental resolution on $n$. It was seen in \cite{12} that for $n \leq 6$ an error of $\pm 0.75$ could be obtained. However, as $n$ gets large the BH properties at adjacent $n$ converge, so it is \textit{a-priori} unclear at what level $n$ can be determined, if at all, in this case.

The previous argument suggests we examine the final state multiplicity, or the individual particle $p_T$ distributions as a probe of $n$. The multiplicity is affected by two major sources of uncertainty: (a) contributions from initial and final state radiation that produce additional jets, and (b) the details of the final quantum gravity decay of the BH are unknown. In what follows we will assume that this \textit{remnant} decay is primarily 2-body. However, this is clearly model-dependent; we prefer observables that are independent of this assumption, disfavoring the multiplicity. By contrast, the $p_T$ spectra of individual particles, particularly at high-$p_T$, will be mostly sensitive to the \textit{initial} temperature of the BHs. There are many such distributions that one could consider. In particular, one would like to examine all possible distributions and see that the candidate BH states are coupling equally to each SM degree of freedom, verifying that these are gravitational phenomena \cite{14}. For illustration we will focus here on the $p_T$ and individual jet $p_T$ distributions for the BH final state.

To calculate these distributions, we have simulated BH events using a modified version of CHARYBDIS \cite{21}, linked to PYTHIA \cite{22}. First, a large sample of BHs with masses above a critical value $M_{min} = M_*$ is generated. From these we select events by cutting on the reconstructed invariant mass, $M_{inv}$ of the event, defined by summing over all visible final state particles or jets with rapidity $|\eta| < 3$, and with $p_T \geq 50$ GeV. We would like to select events where the BH mass is large enough that the event is in the geometrical regime, and quantum gravity corrections are small. To do this, one would need to extract from the data an estimate of the size of $M_*$.\(^{1}\)

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1. We note that this is related to the other definitions in the literature by $M_* = (8\pi)^{-n/2}M_{DL,N} = 2(2\pi)^{n-1} \pi^{2+n}M_{CTR} \equiv (2\pi)^{-n/(2+n)}M_D$, as used in Gubin \textit{et al.} \cite{8},.
While we have no fundamental model for the quantum gravity effects near threshold, we can assume that there will be a turn-on for BH production near $M_{\ast}$, and the cross-section will then asymptote to the geometric value. While this will not lead to a precision determination of $M_{\ast}$, it can clearly be used to set an optimum cut on $M_{\text{inv}}$. In the context of a particular model of the threshold based on the action [2], we find that $M_{\text{inv}} \geq 2M_{\ast}$ is a reasonable cut [19]. We include initial-state radiation in the simulations, since that can lead to a contamination of lower $\sqrt{s}$ events in our sample. In the case of jets, for simplicity we turn off hadronization, and simply look at the parton-level characteristics.

To be specific, we generate a “data” set of $\sim 300k$ events with $n = 21$ and $M_{\ast} = 1$ TeV. We use this size sample as a conservative lower estimate of BH production. If the cross section is within an order of magnitude of that in Fig. 2, the LHC will collect many millions of events, giving an increase in statistical power over that presented here. Alternatively, if we employed a stiffer cut on the lower value of $M_{\text{inv}}$, this would yield a lower statistical sample, similar to the size of $300k$ events considered here, and we would expect our results to then qualitatively hold in this case as well. These “data” events are then compared to a number of template sets of events. We then ask at what confidence the template can be excluded by performing a $\chi^2$ test using only the resulting $p_T$ distribution (shown in Fig. 2). We examine the range $2 \leq n \leq 21$, and $0.75 \leq M_{\ast} \leq 5$ TeV. The lower bound on $M_{\ast}$ comes from non-observation at the Tevatron and cosmic rays [24], while the upper bound is set by demanding that the LHC be able to collect at least 50k events given the cross-section uncertainties. We then determine whether the CST region can be probed at high confidence within this scenario. For this test case, we find at least a $5\sigma$ exclusion for the entire CST region using the $p_T$ distribution alone, or $\sim 40\sigma$ using the jet-$p_T$ spectrum. Though the statistical power in jets is much higher, it suffers from more systematic uncertainties. Fig. 2 shows the 3, 5, and $10\sigma$ exclusion contours in the $(n, M_{\ast})$ plane obtained using the $p_T$ distribution for this test case. If the LHC collects a few million events rather than the 300k sample used here, simple scaling tells us that the $5\sigma$ curve excludes $n \leq 20$, and the $10\sigma$ curve excludes $n \leq 11$.

We have shown that the CST region can be excluded within this scenario if $n = 21$. What about other values of $n$? On changing the number of dimensions used in generating the “data”, we find that for any $n \geq 15$ the CST region can be excluded by at least $5\sigma$, with 300k events. We would, of course, like to know in what region of the parameter space this type of definitive test can be performed. A more detailed study of the parameter space is in progress [10].

In conclusion, we have shown that if there exist many TeV sized extra dimensions with the SM fields confined to a 3-brane, then there exists an observable that can probe classes of critical string theory models.

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FIG. 3: Exclusion curves in the $(n, M_{\ast})$ plane, assuming the data lies at the point (21, 1 TeV). Points outside the curves are excluded at 3, 5, or $10\sigma$.

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