Acquisition and Research of PSF of L-R image reconstruction

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Abstract: In order to make the quality of image reconstruction more accurate, this paper adopts the method of PSF to image reconstruction, analyzes and studies how to obtain the PSF. Firstly, the principle of reconstructed image is modeled, the principle of reconstructed image is analyzed. Then the parameters of the PSF are simulated and the L-R algorithm is analyzed to observe and estimate the reconstructed image. Finally, multiple iterations of L-R algorithm can obtain the appropriate PSF to achieve and the best image reconstruction quality. The analysis can find a suitable PSF.

1. Introduction

PSF can improve the quality of image restoration in image reconstruction, so it is necessary to find an appropriate PSF in image reconstruction. In this paper, L-R algorithm is used to study the acquisition of PSF for several iterations. Considering the influence of PSF itself on image reconstruction, the acquisition of appropriate PSF is analyzed and studied.

2. Model building

2.1 PSF Model

The process of image reconstruction, PSF itself will cause image degradation, so as long as the PSF function model is established, the influence of PSF on image reconstruction needs to be considered. Therefore, it is essential to modify the impact of the PSF model.

A model can be created between the observed value and the target value:

\[ f_1 = f_2 \otimes h \]  

Among them: \( f_1 \) -- observed value; \( f_2 \) -- Target value; \( \otimes \) - convolution factor; \( h \) -- Systematic quantification.

PSF can describe the system process of system imaging. Therefore, \( h \) here refers to the value of PSF. According to Equation (1), in an ideal state, the image reconstruction established by the PSF contains no noise, and the observed value is the convolution of the target value and the PSF. However, no matter what method is used for image reconstruction, generally the image will be disturbed by noise. When there is noise, the model can be establish:

\[ f_1 = f_2 \otimes h + n \]  

Among them: \( n \)-- Add noise.
2.2 Image Degradation Model

Image degradation is the image blur caused by signal distortion and noise caused by external environment in the process of obtaining image information. There are two common methods in image fuzzy processing. One is reducing exposure time, the other is image processing. This paper mainly aims at image processing, simulates the degradation process of the initial image blur, and selects the algorithm to restore the image, which is one of the best means to process the fuzzy image.

The degenerate expression of the image:

\[ b(x, y) = a(x, y) \times g(x, y) + c(x, y) \]  

(3)

As shown in Figure 1, \( a(x, y) \) is the original image; \( g(x, y) \) is the degenerate function; \( c(x, y) \) is the noise-adding function; \( b(x, y) \) is the fuzzy function.

Figure 1. Image degradation model

2.3 Gaussian Noise Model

The most common noise caused by external environment is Gaussian noise. In the whole image processing, it is which we need to get rid of, and denoise is also an essential link in image reconstruction.

Gaussian noise is a kind of normal noise and the most common noise in nature. The following is the expression for its probability density:

\[ P(z) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-u)^2}{2\sigma^2}} \]  

(4)

Among them: \( z \) -- random variable, represents gray value; \( u \) -- Expected value of noise; \( \sigma \) -- standard deviation of noise.

3. Simulation of PSF and Lucy-Richardson algorithm

3.1 Simulation Of PSF

In order to find out the appropriate PSF, this paper compares the parameters of the PSF obtained when the neutron point sources with different incident quantities are incident at the offset center of 1cm.[2]

By analyzing the characteristics of energy deposition data, it is found that its distribution is standard normal distribution, so the following expression can be used:

\[ h(r, x, y) = c(r)e^{-[a(r)(x-x_0)^2+b(r)(y-y_0)^2]} \]  

(5)

Among them: \( h(r, x, y) \)-- that's the PSF; \( r \) -- distance from the source point to the axis; \( x_0, y_0 \)-- the center of the function.

Table 1. The PSF parameters of different energies at the incidence of 1cm at the offset center.[1]

| The incident energy /MeV | 2    | 5    | 8    | 11   | 14   |
|-------------------------|------|------|------|------|------|
| peak \( (\times 10^3)/MeV \) | 2.653| 5.144| 6.357| 7.892| 8.774|
| \( a/cm^{-2} \)          | 0.281| 0.257| 0.204| 0.196| 0.187|
| \( b/cm^{-2} \)          | 0.265| 0.249| 0.193| 0.188| 0.182|

According to Equation (5), when a and b are larger, the normal standard distribution of the PSF is smaller, and the PSF of the imaging system is better. It can be seen from several simulations in Table 1 that with the increase of incident energy, the values of a and b will be smaller, so the larger the normal standard distribution of PSF is, the lower the image resolution will be. This is because the increase of incident energy will lead to the increase of secondary particles.
3.2 Lucy - Richardson Algorithm

Lucy-Richardson algorithm, also referred to as L-R algorithm, is a new iterative algorithm proposed by Lucy and Richardson et al. based on Bayesian iterative algorithm in the 1970s. The basic central idea of L-R is to make the output image be the maximum likelihood estimation of the original image through its own iterative process.\cite{4} The key technique of L-R algorithm is how to determine the point function. As long as the appropriate point function is found, the image can be restored clearly even with few iterations.

L-R model establishment:

$$f(x, y)^{k+1} = f(x, y)^{k} \left[\frac{g(x,y)}{h(r,x,y) \otimes f(x,y)^{k}} \oplus h(r,x,y)\right] \quad (6)$$

Among them: $f(x, y)^{k}$ -- iterate over the original image $k$ times; $\otimes$ -- correlation operation.

4. Image restoration

4.1 Image Restoration

Analysis and research are carried out in two forms: L-R iterative restoration of images containing PSF and L-R iterative restoration of images without PSF.

The experiment of the first group is shown in Figure 2. Figure (a) is an original image. Figure (b) is an image containing only the PSF; Figure (c) is the image of the actual PSF function after noise is added; Figure (d), Figure (e), and Figure (f) Figure (g) is the image of 20, 220, 420, and 620 iterations of Figure (c).

By comparing figure (a) and figure (b), it can be seen that the PSF image alone has almost no effect on the quality of the original image, and only the image edge will blur, which is almost the same as the original image.

By comparing figure (a) and Figure (c), it can be seen that when noise is added to the PSF image, the image becomes blurred obviously, which is far from the original image.

The image restoration of fuzzy image (c) was carried out by using L-R algorithm for multiple iterations, and the image (d), (e), (f) and (g) were compared. With the increase of iteration times, the image clarity became higher and higher.

Figure 2: L-R Iterative restoration image of the image with PSF that has noise or not

It can be seen from the above that, when the image created by the PSF is only affected by the external environment, when there is only noise, multiple L-R iterations are carried out on the image. As the number of iterations increases, the image will be restored better and clearer.

The second group of experiments is shown in Figure3. Figure (a) is an original image. Figure (b) is a fuzzy image; Figure (c) is the image after noise is added to the fuzzy image; Figure (d), Figure (e), and Figure (f) Figure (g) is the image of 20, 220, 420, and 620 iterations of Figure (c).

The comparison between figure (a) and Figure (b) shows that motion blur has a great impact on the
details of the original image.

By comparing figure (a) and Figure (c), it can be seen that when noise is added to the blurred image, the image becomes more blurred, which is far from the original image, and the outline of the original image can hardly be seen.

The image of fuzzy and noisy image c) is restored, and the L-R algorithm is used to carry out multiple iterations, and the image (d), (e) and (f) were compared. With the increase of iterations, the clarity of the image became higher and higher.

It can be seen from the above that, when the image reconstruction is affected by the external environment, the image distortion is relatively serious, and repeated iteration of the image can also achieve the image restoration. The higher the number of iterations of the image, the better the restoration.

4.2 To Obtain PSF
For the above two groups of experiments, by comparing the image s in Figure 1 and Figure 2 with the same number of iterations respectively, it is obvious that the image restoration effect is better than the image processed by the PSF. Therefore, it is the most important to find the appropriate PSF in image processing. The following is the analysis from the Angle of, which is constant between neutron coding hole and the source space.

Research point 1: The obstruction of neutron coding holes of different thickness to neutron incident energy affects the acquisition of PSF.

As shown in Figure 4, with the increase of incident energy, the neutron penetration capability is continuously enhanced, so the PSF obtained on the detector is also larger and larger. This conclusion has been verified in Table 1 above.

Research point 2: obtaining the PSF of the same thickness neutron hole from different angles.

As shown in Figure 5, when the change of the source point is within the controllable range, the imaging effect of the PSF obtained on the detector almost changes little, so when the PSF is in the
working environment of space invariance, it is suitable to study the reconstruction image of the PSF.

![Image](image.png)

Figure 5 1 - source region, 2 - $\theta_1$, 3- $\theta_2$, 4-offset P2, 5-center P1, 6-neutron encoding hole, 7-center P1 image, 8-offset P2 image, 9-neutron detector

The PSF obtained by different angles for the same thickness of the neutron hole

It can be seen from the above that, using the space invariance of the PSF, it is found that with the increase of different incident energy, the normal distribution of the PSF is also larger, while with the increase of the thickness of the neutron coding hole, the standard normal distribution of the PSF is decreasing, finds the appropriate thickness, and changes the neutron incident energy. [6] Combining equation (5), a suitable PSF can be obtained Image reconstruction has a good effect on image restoration.

5. Conclusion

From the above experimental results, it can be concluded that when using the PSF to restore the image, the effect of image restoration is much better than using the algorithm directly. This paper analyzes and studies the acquisition of appropriate PSF, and compares the first group and the second group of experiments. It is found that under the same number of iterations of L-R iterative algorithm, according to figure 1 and figure 2, it is obvious that the image restoration effect with PSF is much better. Therefore, in the case of image processing, it is necessary to analyze and study the appropriate PSF. In the first research point, it is found that with the increase of incident energy, the incident energy is proportional to the neutron secondary particle, which is equivalent to the decrease of the thickness of the neutron coding hole, and the standard normal distribution of the PSF obtained on the neutron detector is increasing, which makes the ability of the PSF to restore the image worse, so the incident energy is larger in the environment, a better PSF can be obtained by properly increasing the thickness of the coding hole; In the second study point, as long as the other conditions of the point spread function are unchanged, to change the incident angle, only the position of the incident point in the source area needs to be changed. According to figure 4, when $\theta_1 = \theta_2$, the influence of the PSF on the image is almost the same, means to say, when the point function has space invariant property, the PSF can restore the image better.

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