Interference enhanced polarization entanglement and the concept of an Entangled-Photon Laser

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(Dated: March 31, 2022)

Inspired by laser operation, we address the question of whether stimulated emission into polarization entangled modes can be achieved. We describe a state produced by stimulated emission of the singlet Bell state and propose a setup for creating it. As a first important step towards an entangled-photon laser we demonstrate experimentally interference-enhanced polarization entanglement.

In the past two decades there has been a wealth of fascinating experiments on the production and applications of entangled photons. Whereas the earlier research mainly focused on fundamental tests of quantum mechanics [3, 4, 5, 18, 19, 20, 21], recent fundamental tests of quantum mechanics. So far, mainly pairs of entangled photons, produced by parametric down-conversion inside non-linear crystals [3, 4, 5, 18, 19, 20, 21] have been used in the experiments and only few experiments addressed three- and four-photon quantum correlations [5, 22, 23, 24]. At the other end of the optical energy range, where the number of photons is large and the optical fields can be described by in- and out-of-phase amplitudes with respect to a classical electro-magnetic field, quantum correlations may still play an important role. For example, squeezed light fields [22, 23] can be used to create entangled light beams (entangled in the noise-level of the quadrature components) and to implement quantum teleportation [24, 25, 26].

The nature of the quantum correlations at the few photon level and at the high-intensity level are very different. For entangled photons the quantum correlation is between degrees of freedom for each particle (e.g. their polarization degrees of freedom). On the other hand, the quantum correlation at the high-intensity level is between photon numbers in the entangled modes.

In this Letter, we introduce the notion of laser-like operation for polarization entangled photons. The quantum correlations will be *both* between the polarization degree of freedom for each particle and between the photon numbers in the entangled modes. Therefore, the proposed device will operate in the new and virtually unexplored regime of many-particle entanglement.

As the acronym LASER (Light Amplification by Stimulated Emission of Radiation) indicates, entangled laser operation would mean that a (spontaneously created) photon pair in two polarization-entangled modes stimulates a (spontaneously created) photon pair in the other polarization-entangled modes, inside a non-linear gain medium, the emission of additional pairs. For this process a non-linear gain medium is required for which we consider type-II parametric down conversion [28]. A simplified interaction Hamiltonian (24, 25) for the nonlinear interaction between a classical pump field and two polarization-entangled modes is given by

\[
\hat{H}_{\text{int}} = e^{i\phi} \kappa \hat{K}^\dagger + e^{-i\phi} \kappa \hat{K},
\]

where \(\hat{K}^\dagger \equiv (\hat{a}^\dagger \hat{b}^\dagger - \hat{a} \hat{b})\) and \(\hat{K} \equiv (\hat{a} \hat{b}^\dagger - \hat{a}^\dagger \hat{b})\), are the creation and annihilation operators of polarization entangled photon pairs in modes \(a\) and \(b\). Horizontal and vertical polarization are represented by \(h\) and \(v\), and \(\kappa\) is a real-valued coupling coefficient. When acting on the vacuum state the time evolution operator \(\hat{U} = \exp(i\hat{H}t/\hbar)\) yields

\[
\hat{U}(\tau)\ket{0} = e^{-\tau} \sum_{n=0}^{\infty} \tau^n \left( \sum_{m=0}^{n} (-1)^m |n-m, m,n-m\rangle \right),
\]

where \(\tau = \frac{\kappa t}{\hbar}\), \(r = \tanh \tau\) and \(q = 2 \ln(\cosh \tau)\). We used the shorthand notation \(|ij;k,l\rangle\) for \(|i\rangle_{ah}|j\rangle_{av}|k\rangle_{bh}|l\rangle_{bv}\) and \(\ket{0}\) represents \(|0,0,0,0\rangle\). The fact that for a given \(n\), all terms in Eq. (2) have equal absolute amplitudes is the result of stimulated emission. This is in contrast to the state obtained by the product of \(n\) entangled pairs in the \(\Psi^-\) state, which would yield a binomial distribution over the terms. An important consequence of the equally weighted terms for a given \(n\) is that they form a highly polarisation entangled state of \(2n\) photons. In addition the state (2) is entangled in photon numbers in the two modes, though only weakly since the terms with different \(n\)’s have different weights \(r^n (r < 1)\).

The state described in Eq. (2) would be a desirable output for what we call a polarization entangled-photon laser. It has similar features to the output of a conventional laser, in the sense that the photon number distribution broadens and shifts its peak as the average number of photons increases in marked contrast to the behaviour of a thermal or squeezing distribution. It does not however approach a stationary state, because no loss mechanisms are included in the description. In a conventional laser the intra-cavity photons can be seen as copies...
of one another (except for the one spontaneously emitted photon in the mode), which implies that a conventional laser can operate with continuous outcoupling. In contrast, the photons in a polarization entangled-photon laser form one complex entangled state. Therefore the entangled fields first have to build up inside a high finesse resonator and then they have to be switched out. This requires a pulsed laser operation.

At the heart of our design for an entangled photon laser is the bow-tie cavity, shown in Fig. 1(a). The reason for this design is two-fold. First, since we consider non-collinear down-conversion the modes diverge out of the gain region and have to be redirected into the non-linear interaction region in order to achieve efficient stimulated emission. Second, the two entangled modes are counter propagating modes in the same resonator and therefore experience exactly the same path length and mode structure. By including a $\lambda/2$ wave plate in the resonator, such that the vertical and horizontal field components will interchange every round trip, each photon will also experience exactly the same birefringence and spatial walk off as any other photon. Therefore, the bow-tie ring resonator together with the $\lambda/2$ wave plate guarantees the indistinguishability of all photons created throughout the amplification process which is the key to obtaining optimal entanglement. The resonator has to be interferometrically stable with respect to the phase of the pump laser pulses because the parametric down-conversion can be considered as a phase-dependent gain mechanism.

There are many experimental complications to be overcome before an entangled photon laser can be built, but as a first experimental feasibility study of the main operation mechanism we reduced the design to the double-pass configuration shown in Fig. 1(b). Photon pairs in the singlet $\Psi^{-}$ Bell-state can be created in the first and second pass of a noncollinear type-II parametric down-conversion crystal. Since the creation is rather inefficient, we restrict our attention to the observation of resonant enhancement of the spontaneous emission rate which is an adequate measure of how well the indistinguishability requirements can be met.

The time-evolution operator for the two-pass system is given by

$$\hat{U} = \hat{U}_2\hat{U}_1 = e^{\tau(\omega_2\hat{K}_2^\dagger - \omega_1\hat{K}_2^\dagger)} e^{\tau(\hat{K}_1^\dagger - \hat{K}_1)}$$

where $\theta = \phi + \pi/2$ and the subscripts on $\hat{K}$ denote on which pass they operate. Expanding to first order yields

$$\hat{U} = 1 + \tau \hat{K}_2^\dagger e^{i\theta} \tau \hat{K}_2$$

where the terms containing $\hat{K}$ are neglected, since they yield zero when they operate on the vacuum. If the entangled photon pairs created by $\hat{K}_1^\dagger$ and $\hat{K}_2^\dagger$ are indistinguishable, then

$$\hat{U} = 1 + \tau \hat{K}^\dagger(1 + e^{i\theta}).$$

Equation (3) is most easily understood as an interference pattern of two polarization-entangled photon-pair amplitudes. Generalizing for $n$ passes, the result is that the probability for obtaining a pair of entangled photons scales as $n^2$ for indistinguishable fields and as $n$ for distinguishable fields.

In addition to the general design of the counter-propagating entangled-mode configuration and the $\lambda/2$ wave-plate, two 1mm BBO crystals, rotated by 90° with respect to the main BBO crystal, and narrow-bandwidth filters f1 and f2 have been included in the detection stage in order to complete the birefringence and temporal walk-off compensation [15, 13].

Figure 2(a) shows the detected number of polarization-entangled photon pairs as function of the difference in path length between the entangled-photon feed-back loop and the delay of the reflected pump-pulse. Figure 2(b) shows that inside the region of interference, the coincidence rate oscillates with a period corresponding to half the wavelength of the pump field [14].

The entanglement was assessed by looking at the visibility of the coincidence rate variation as function of the difference in angle between the polarizers in front of detectors D1 and D2 (see Fig. 3). In particular we fix one of the polarizers at 45° and thus avoid the experimentally favoured basis $H, V$. This visibility was measured both in the maximum overlap and in the no overlap region with results of 0.92 and 0.83 respectively. The higher quality obtained when the two passes are indistinguishable can be explained as a by-product of the exchange in polarization in the feedback loop. This has the additional effect of eliminating the spectral distinguishability [33] between the down-converted photons. In conclusion, the high interference contrast indicates that the experimental setup operates under the important indistinguishability requirements between pairs created at different times.

Related experiments on interference enhanced emission and stimulated emission of photon pairs that are not entangled in polarization have been reported and analysed in Refs. [24, 34, 36]. Experiments and theory involving single-photon injection into type-II crystals (rather than pair-photon injection as presented in this letter) have been reported in Ref. [25, 27].

Having demonstrated the feasibility of one of the main mechanisms of an entangled photon laser we now briefly discuss what the prospects are for the construction and possible applications of an operational device. First of all, just like squeezing experiments, the device will be very sensitive to loss, both in the generation of the entangled output modes as well as during the propagation and detection of the output state. With current technology it is possible to obtain a round-trip cavity loss of 2-3% (including switching elements) and a pair-photon creation efficiency of 0.5 per pump pulse per pass of the nonlinear material. Under these conditions polarization-entangled output modes with 10 to 1000 photons per pulse are con-
Acknowledgements

We thank Piero Varisco and William Irvine for their experimental progress, we will always have to face the situation of losing photons in the process of creating, transporting and analyzing the desired state (2). Since the desired state is one large complex entangled state (entangled in polarization and photon numbers) one might think that loss destroys all the interesting properties of the state. However, on the contrary, the complex entanglement has the remarkable feature that the loss or measurement of one (or more) particles does not eliminate all the entanglement between the remaining particles. To illustrate this important property let us focus on the (un-normalised) four-particle terms in Eq (2):

\[ |2,0;0,2⟩ − |1,1;1,1⟩ + |0,2;2,0⟩. \]

Consider the case that a polarisation measurement in the H, V basis is made on one particle, say in mode \( a \), and the measurement result is \( h \). The state of the remaining three particles will be

\[ \sqrt{2}|1,0;0,2⟩ − |0,1;1,1⟩ \]

which contains (non-maximally) three-particle entanglement. This feature of robustness seems to indicate a connection with high entanglement persistency states.

In this letter we introduced the concept of an entangled photon laser and studied crucial experimental aspects of its construction. Currently we are exploring the multi-particle correlations of the entangled photon laser for novel secure quantum communication, quantum cloning [40], and entanglement purification [41].

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FIG. 1: The basic design of a polarization entangled-photon laser is shown in (a). A linear pulsed pump resonator overlaps inside a nonlinear crystal with two counter-propagating polarization entangled modes. After the intra-cavity fields have built up during several roundtrips the total field is coupled out by two optical switches. Reducing the design to a double pass of the crystal leads to setup (b). A frequency-doubled mode-locked Ti:Sapp laser (80 MHz rep. rate, $\lambda_0 = 780$ nm) pumps a 2 mm BBO crystal. Pinholes perform spatial selection of the entangled modes. The pump is reflected onto itself by mirror M3 which is mounted on a computer-controlled translation stage. Photon pairs are observed by coincidence-detection between avalanche photodiodes D1 and D2 after passing through polarizers, Pol1 and Pol2, and 5nm bandwidth filters f1 and f2.

FIG. 2: Graph (a) shows the number of detected photon pairs around the region of overlap between the reflected pump and the photon-pair wavepacket from the first pass. Graph (b) is a fine scan of the zero delay region. An average over many oscillations gives us a period of $200 \pm 10$nm which corresponds with the expected value of half the wavelength of the pump field $\lambda_p/2 = 195$nm.

FIG. 3: (a), (b), (c) and (d) show the interference fringes for different relative orientations of the polarizers in front of the two detectors. In particular, one polarizer was fixed at $45^\circ$ while the other was varied from $45^\circ$ (a), through $75^\circ$ (b) and $105^\circ$ (c) to $135^\circ$ (d). Graph (e) shows the average of the maxima of the interference as a function of the relative angle. The visibility of this curve is $0.92 \pm 0.02$ which is a direct measurement of the entanglement in the overlap region.
