Aeroservoelastic Model Modification and Uncertainty Quantification with Bayesian Posteriori Estimation*

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A half model of a scaled aircraft is designed and tested in a wind tunnel. Based on the step-sinusoidal swept frequency test, the aeroservoelastic frequency response function is measured and compared with the predictions of a theoretical model. According to the discrepancies, the key uncertainty factors such as actuator model uncertainty, aileron aerodynamic uncertainty and structural damping uncertainty are investigated. Therefore, the original theoretical model is modified and updated with these model uncertainties considered, respectively. Motivated by the still existing discrepancies of experimental data and comprehensive updated theoretical outcomes, we take the above aeroservoelastic model uncertainty as an aleatory probabilistic one. Hence, the probability of each model uncertainty is quantified by Bayesian Posteriori Estimation theory. In addition, the uncertainty probabilities of aeroservoelastic response and aeroservoelastic stability boundary are predicted. The aeroservoelastic uncertainty quantification methodology with model uncertainty is validated by comparing the probabilistic predictions of the theoretical model set with experimental data. Taking model uncertainty into consideration, the present method is able to predict the probabilistic aeroservoelastic stability bounds and response as well. Results indicate that the updated comprehensive model is the best theoretical model with the highest posterior probability. In addition, the unsteady aerodynamics due to aileron deflection has the most significant effect on aeroservoelastic response.

Key Words: Aeroservoelasticity, Uncertainty Quantification, Wind Tunnel Test, Model Uncertainty, Bayesian Theory

1. Introduction

Aeroservoelasticity involves the multidisciplinary coupling of structural dynamics, aerodynamics and controls. Various uncertainties and nonlinearity exist inevitably within the aeroservoelastic models of airplanes and missiles that are rather complicated. Pettit summarized the application of uncertainty quantification in aeroelasticity. This paper discusses the studies on the stability, response and design problems of aeroelastic systems considering the influence of all kinds of uncertainty, such as data error, model error and numerical error. Notably, it has become an important academic branch of aeroelasticity and aeroservoelasticity research in recent years. Uncertainty sources of aeroservoelastic systems include uncertainty of input signals, model uncertainty among several models, parametric uncertainty of a specific model, uncertainty of output signals, etc. Among them, model uncertainty and parametric uncertainty are the most important for theoretical aeroservoelastic analysis. Abundant studies have concentrated on parametric uncertainty, while few are focused on the model uncertainty. It is because researchers usually select a “best” model in their minds, so they don’t care about other “average” models. The most commonly used quantification methods for aeroservoelastic parameter uncertainty include the interval analysis method, structured singular value theory, Monte Carlo Simulation (MCS), Polynomial Chaos Expansion (PCE), etc. However, few methods can be found to quantify the influence of model uncertainty to aeroservoelastic behavior. In general, without consideration of parametric uncertainty, model uncertainty in a model set refers to the distance between different models’ outcomes and experimental data. It is mainly brought by the introduction of modeling assumptions and simplification in the process of mathematical modeling and numerical algorithms. Until now, the major available method for model uncertainty quantification is based on Bayesian Posteriori Estimation. It is a good tool to quantify the probability distribution of model uncertainty in a model set, based on known experimental results. It can tackle with several theoretical models simultaneously while the mentioned MC and other methods cannot. Motivated by this advantage, the model uncertainty of the modified and updated models can be quantified in this current work by Bayesian probabilistic theory.

This current work focuses on reducing the gap of frequency response between wind tunnel testing and theoretical aeroservoelastic models. Therefore, the paper is organized as follows. First, the setup and results of wind tunnel testing are introduced. Due to the gap of theoretical results and experimental data, six different theoretical models are obtained by model modification and updating, based on an original model. Next, the sixth model is selected as the best theoretical model, to quantify model uncertainty probability for aeroservoelastic systems using Bayesian Maximum Posteriori Estimation. Uncertainty probability quantification for all six models is carried out. Finally, the method is validated by comparison between the theoretical results and those of experiments.
2. Setup and Results of Aeroservoelastic Wind Tunnel Testing

A half-span aircraft model, assembled by fuselage, all-moving canard, wing, aileron, horizontal tail and elevator, is mounted vertically on the floor in a wind tunnel as shown in Fig. 1. This model has two degrees of rigid-freedom: plunge motion along the sliding rail and pitch motion about the mass center. During the whole test, a feedback control law is acting on the actuator of the elevator to stabilize the pitch and plunge motions. The experiment was carried out in the FD-09 wind tunnel of China Academy of Aerospace Aerodynamics. The dimension of the test section was 3 × 3 m. The wind tunnel tests include a step-sinusoidal swept frequency test, gust response test and time response test. The first and third experiments are related to the present study. The discretized frequency response function and the aeroservoelastic stability boundary were obtained in these two tests. The same aircraft model mounted with an engine nacelle was also tested in similar programs. More details of this model can be found in Wu et al.5) More details of this model can be found in Wu et al.5)

2.1. Wind tunnel testing setup

First, the step-sinusoidal swept frequency test was conducted at low velocities. In this test, the aileron, elevator and canard were driven by electronic actuators, respectively. A real-time step-sinusoidal function generated by computers was used to drive the aileron in the step sinusoidal swept test. The command signals and acceleration at the wing tip were measured. The command signals and responses were collected by NI PXI-4472B NI (National Instruments) data acquisition equipment, at the acquisition frequency of 1,000 Hz.

The step-sinusoidal swept test for this configuration is similar to that of Wu et al.5) It was conducted in the velocity range from 16 m/s to 24 m/s. The swept frequency range was 0.6 Hz to 7 Hz with increasing frequency steps of 0.2 Hz. Each fixed-frequency excitation was sustained for 4 s and dwell for 1 s. More details of the data acquisition can be found in Wu et al.5)

2.2. Methods of data processing

Since the frequencies of input command are given, the control surface is driven to vibrate harmonically when neglecting the flow disturbance. Hence, it’s reasonable to obtain the frequency response of acceleration by representing the response at certain frequencies using harmonic function approximation. For every time segment of swept command signal, \( u = u_0 \sin \omega t \), the acceleration response of the airfoil, \( a \), satisfies the harmonic function approximation. Considering the instability of the signal at the beginning and ending of the harmonic drive in every frequency, the first 0.5 s data segment and the last 0.5 s data should be omitted. The time response of the middle segment with 3 s is represented by the following sinusoidal function:

\[
a_k(t) = a_{0k} \sin(\omega_k t + \theta_k)
\]

Alternatively, the acceleration data in the time domain is transformed to the frequency domain by discrete Fast Fourier Transformation. In this method, it is also possible to calculate the magnitudes of frequency response function (FRF) in the frequency domain. The results of using the two methods are shown in Fig. 2.

From this figure, it’s easy to see that the results obtained using the two methods fit well with each other. The two methods can also be applied to processing response signals at other velocities. The function fitting method is adopted to obtain the FRF of wing tip acceleration in the following studies, without special illustration.

2.3. Results of step-sinusoidal swept frequency test

The FRF at several flow velocities is calculated, respectively. The magnitude of the FRF for wing tip acceleration is illustrated in Fig. 3. According to Fig. 3, the peak frequency of the 1st bending mode increases as velocity increases. The bending-mode frequency is 2.2 Hz at 16 m/s and 3 Hz at 24 m/s. Obviously, the magnitude of FRF changes slightly below the frequency of the 1st bending mode, but it increases sharply with the flow velocity when the frequency is over that. When the velocity is greater than 22 m/s, discretized FRF data ‘jumps’ a little due to the increases in flow turbulence.

2.4. Results of time response test of aeroservoelasticity

When the flow velocity is close to the flutter boundary, the model responses excited by the turbulent flow are large enough to measure. The response data of closed-loop aeroservoelasticity applying the control law of stabilization is
collected. At each velocity, the duration for data acquisition is 100 s. An averaged Fourier transformation is performed for each 10 seconds’ data segment. The magnitude of the acceleration in the frequency domain is shown in Fig. 4. The peak frequencies are 6.0 Hz, 6.2 Hz and 7.2 Hz at the velocity of 33 m/s, which is rather close to the flutter boundary. Since other tests of this model have to be conducted after the time response test, the state of flutter critical boundary is not reached for the sake of safety.

3. Theoretical Model Updating and Modification

The theoretical model of an aeroservoelastic system is composed of several subsystems, including elastic structure, aerodynamics, actuator system, sensor system and stabilization control system. The aeroservoelastic system itself is constructed by connecting the individual part according to their physical relationships using the SYSIC command in Matlab software.

For every part in the aeroservoelastic system, the dynamic equations are established, respectively. All of the driven actuators are electronic motors in a Maxon Remax series. Based on the cut-off frequency of the actuators, a three-order transfer function for their dynamics is expressed as

\[
G_{ac}(s) = \frac{2\pi \cdot 60}{s + 2\pi \cdot 60} \cdot \frac{(2\pi \cdot 9)^2}{s^2 + 2 \cdot 0.62 \cdot (2\pi \cdot 9)s + (2\pi \cdot 9)^2}
\]  

(2)

The motion equation of the elastic structures under aerodynamic forces can be written as

\[
M\ddot{q} + C\dot{q} + Kq + M_\delta \delta = F_i + F_d
\]  

(3)

where, \( M, \ C, \ K \) and \( M_\delta \) denote generalized mass matrix, generalized damping matrix and generalized stiffness matrix, respectively.\( n \in R \) is the number of modes and \( \delta \) is the deflection angle of the control surface. \( F_i, F_d \in R^n \) denote generalized aerodynamics caused by elastic mode motion and the deflection of control surfaces. The Double-Lattice Method is adopted in the calculation of unsteady aerodynamics \( F_i \) and \( F_d \) in the frequency domain. They are then represented in the form of state space equations in the time domain by rational function approximation. Two aerodynamic lag roots, 0.1 and 0.3, are selected here using the Minimum State Method.\( ^{12} \) The finite element model and modal analysis are employed to characterize the structural dynamics. The finite element model is verified according to an impact ground vibration test. In the ground vibration test, the first 10 lower normal frequencies are identified by the POLYMAX algorithm,\( ^{13} \) embedded in the modal vibration test software.

Since the TCP accelerometer mounted at the wing tip of the aircraft model has excellent dynamic performance, its transfer function is denoted as unity. As mentioned before, a control law is acting on the elevator during the whole test to stabilize the plunge and pitch rigid motion. The control law is expressed as

\[
y_{\text{elevator}} = 0.4u_{\omega} + 2.3u_{\theta} + 40.0u_{H}
\]  

(4)

where \( u_{\omega}, u_{\theta} \) and \( u_{H} \) are the input signals of control system. \( u_{\omega} \) and \( u_{\theta} \) denote the angular velocity and angular displacement of pitch motion, respectively. \( u_{H} \) represents the displacement of plunge motion. \( y_{\text{elevator}} \) denotes the output signal of the control system. It drives elevator movement using the actuator.

Up to now, all of the individual models of the aeroservoelastic system have been constructed. Next they are connected to a whole aeroservoelastic state-space model. Its root locus is depicted in Fig. 5. It can be seen clearly that instability occurs at the velocity of 33.4 m/s, where the 2nd bending mode cuts through the imaginary axis. From the experimental aspect, in the wind tunnel time response test, there is no obvious phenomenon of instability at \( V = 33 \text{ m/s} \) mentioned in the above section. Hence, this theoretical model may be reliable since it doesn’t conflict with experimental stability results. This theoretical model is called Model 1 for convenience, in order to distinguish it from the following modified models.

3.1. Uncertainty sources in theoretical models

The wing tip acceleration-to-command signal in the frequency domain is denoted as the FRF. The magnitude between the prediction of Model 1 and the experimental outcome is compared in Fig. 6. The phase difference is slight
and therefore it is not considered in this research. From the comparison, the difference in the FRF magnitude between them is relatively significant, though the aeroservoelastic stability boundary agrees fairly well. Hence, from the aspect of FRF, the accuracy of Model 1 needs much improvement. It was then necessary to investigate the uncertainty source and to modify Model 1 in order to reduce the difference of FRF between the predictions of the theoretical model and experimental results.

From the above figure, the absolute gap of FRF magnitudes between the theoretical model and experiment becomes significant over the frequency of 3 Hz. This infers that the dynamics of the theoretical model become less accurate in the high-frequency range. Since the input signal comes from the computer command, we assumed there is no uncertainty in the inputs. Except for the measurement error of accelerators, the remaining uncertainty source comes from model uncertainty and parameter variation. From the modeling process for the aeroservoelastic system, we can investigate the uncertainty in the subsystems. This means the major factors that may affect the FRF include: (1) uncertainty from the structural dynamic model; (2) uncertainty from the actuator model; and (3) uncertainties from the calculation of unsteady aerodynamics in the frequency domain and that from the rational functions approximation process. It was also assumed there is no uncertainty in the accelerometer dynamics due to the fact that the TCP accelerometer has small additional mass and a high cutoff frequency. The above three major factors in the calculations are analyzed and considered for model modification in the following sections.

3.2. Theoretical model modification

3.2.1. Actuator model modification—Theoretical Model 2

The theoretical transfer function for the electronic actuator is expressed in Eq. (2). A swept frequency test for actuators on the ground was done to verify the transfer function. In this test, an aileron is connected to the actuator. The computer-generated sinusoidal swept command acts on the actuator, and it drives aileron motion. The deflection angle of aileron is measured to calculate the frequency response function. Therefore, the input-output characteristics of the actuator are measured in the test. The results of the swept frequency test for actuators, as well as the fitted transfer function with three orders, are shown in Fig. 7.

From the figure, when the amplitude of the sinusoidal command increases from 5 deg to 10 deg, the amplitude-frequency and phase-frequency relationships change significantly. When the command magnitude reaches 10 deg, the performance of the actuator declines significantly over the frequency of 7 Hz. This infers that the actuator’s transfer function is related with command amplitude. As we know, the Maxon electronic actuators have current-loop feedback, speed feedback and location feedback control laws. Hence, the actuator system itself has excellent command-following performance without external loading. The decaying phenomenon may be caused, to a great extent, by the external loading from the attached aileron. According to this, we modified the transfer function of actuators considering the inertial loading from the aileron.

Investigating the mechanism of actuators, the rotation limitation of the actuator shaft attracted our attention. To protect the actuator from burning up due to large current, the limit
angular velocity was set to $|\omega| < 6000$ rpm, and the maximum angular acceleration was set as $\dot{\theta} < 1500$ rps$^2$ in the driving software. With these two limitations, it was easy to ensure the acceleration limit was reached first. Since the actuator output declines over the frequency of 7 Hz, we assumed the angular acceleration reaches its limit value frequencies over 7 Hz when the command amplitude reaches 10 deg. The maximum angular acceleration of the actuator itself is relatively small, that is $\dot{\theta}_0 = 10 \times (2\pi \times 7^2) / 360 = 53.7$ rps$^2$. The total angular acceleration of the actuator with inertial loading of the aileron is expressed as follows:

$$\ddot{\theta} = \ddot{\theta}_0 + \frac{M(\theta_1)}{J_a} = \ddot{\theta}_0 + \frac{J_1 \dot{\theta}_1}{J_a} \tag{5}$$

where $J_a$ and $J_1$ denote inertia moments of the actuator shaft and the aileron, respectively. Since $\dot{\theta}_0$ is much smaller than the limit of angular acceleration, it can be neglected without loss of accuracy in approximation. Hence, in the ground swept frequency test at a limit state, we have

$$|\ddot{\theta}|_{\text{max}} = \frac{J_1 \dot{\theta}_1}{J_a} = \theta_{0\text{max}} \omega^2 \frac{J_1}{J_a} \times a \tag{6}$$

where $|\ddot{\theta}|_{\text{max}}$ represents the acceleration limit, and $a$ denotes the decaying coefficient of the response to angular command after 7 Hz. From the ground swept frequency test after 7 Hz, $a$ can be measured for every discretized frequency. Consequently, $J_1 / J_a$ for different angle amplitudes can be obtained using Eq. (6). The results are shown in Table 1.

From Table 1, the obtained $J_1 / J_a$ of different command amplitudes and frequencies are very close to each other. This agrees well with common sense that $J_1 / J_a$ should be a constant for a specified actuator and aileron. Therefore, $J_1 / J_a$ is calculated and verified by several experimental data. The averaged value is 29.28.

With the ground swept frequency test for actuators, we verified the fact that the actuator model is related to external loading. Afterwards, we modified the real actuator model in the wind tunnel test. In real wind tunnel test, not only the inertial forces act on the actuator, but also the aerodynamic force of control surfaces. Therefore, we consider the case of the maximum angular acceleration of the actuator shaft with the effect of aerodynamics. Since static aerodynamic loading may take great effect on the actuator shaft, a quasi-steady aerodynamic model was employed to estimate aerodynamic loading on the actuator shaft, and the aerodynamic moment about the aileron shaft can be expressed as

$$M_{LE} = \eta \left( \frac{\pi}{4} \rho V^2 b^2 \dot{\theta} + \frac{\pi}{4} \rho V b^2 \dot{\theta} \right) = \eta (A_{a0} \dot{\theta} + A_{a1} \dot{\theta}) \tag{7}$$

where $\eta$ is the introduced aerodynamic efficiency coefficient of the aileron. $M_{LE} (\eta = 1)$ means the moment of plate quasi-steady aerodynamics without the effect of the wing. The parameters of the tested aileron are: inertia moment is $J_1 = 3.564 \times 10^{-4}$ kgm$^2$, chord length is $b = 0.085$ m, and span length is $l = 0.41$ m.

| Frequency | 7 Hz | 8 Hz | 9 Hz | 10 Hz | 11 Hz | 12 Hz |
|-----------|------|------|------|-------|-------|-------|
| 5 deg     | 29.29| 28.50| 28.25| —     | —     | —     |
| 10 deg    | 29.29| 28.50| 28.25| —     | —     | —     |

Fig. 8. The bode diagram for an actuator with the angle amplitude of 6 deg at $V = 24$ m/s.

The maximum angular acceleration generated by the aerodynamic and inertial loading in the wind tunnel is expressed as

$$|\ddot{\theta}|_{\text{max}} = \frac{J_1 \dot{\theta}_1 + M_{LE}}{J_a} = \theta_{0\text{max}} \left( \omega^2 J_1 + \eta(\omega A_{a0} + i\omega A_{a1}) \right) \times a_i \tag{8}$$

where $a_i$ denotes the decaying coefficient of the response to angular command when angular acceleration reaches the limit with the effect of aerodynamics.

The frequency response curve of the actuator with an aerodynamic load at the velocity of 24 m/s is shown in Fig. 8. In this case, the third-order transfer function is fitted as

$$G_{a2}(s) = \frac{2\pi \cdot 60}{s + 2\pi \cdot 60} \frac{(2\pi \cdot 6.5)^2}{s^2 + 2 \cdot 0.65 \cdot (2\pi \cdot 6.5)s + (2\pi \cdot 6.5)^2} \tag{9}$$

By modifying the actuator model from Eq. (2) to Eq. (9), a new theoretical aeroservoelastic system, noted as Model 2, is obtained. The amplitude of the bode diagram for Model 2 is shown in Fig. 9. From this figure, we can find that the amplitude of the frequency response decreases significantly at frequencies over 4 Hz, due to the limitation in actuator angular acceleration.

3.2.2. Structural damping modification—Theoretical Model 3

A ground vibration test for the aircraft model was carried out to verify the finite element model before the wind tunnel test. An impact test, together with POLYMAX identification in LMS software, was used for the key normal modes. The difference between the frequency of the first seven key
modes in the theoretical model and that of experimental modes was less than 5%. However, the differences in structural damping were rather significant. Generally, there is no physical damping element in the finite element model. Therefore, in order to represent the real damping properties, generalized damping for each elastic mode was specified for the theoretical aeroservoelastic model. The effect of structural damping on aeroservoelastic response was studied by increasing the structural damping of Model 1 from 1% to 10%.

With the actuator model described in Eq. (2), the magnitude of frequency response function is shown in Fig. 10. According to the figure, generalized damping mainly decreases the magnitude of the frequency response at all frequencies. As can be seen, when generalized damping increased, the frequency of maximum magnitude also increased, which is not consistent with the 1st bending peak frequency in the experimental data. Hence, it is not proper to set a large value for the generalized structural damping.

3.2.3. Aerodynamic efficiency modification of the control surface—Theoretical Model 4

When the Double-Lattice Method (DLM) is employed to calculate unsteady aerodynamics due to aileron motion, the aileron is assumed to be an idealized flat plate without any disturbance. The aerodynamic pressure at $k = 0.2$ is shown in Fig. 11. In this figure, the influence of static aerodynamic loading and disturbance by gear rod is neglected in the theoretical aerodynamic model, and the aerodynamic pressure on the aileron front is very steep. However, due to the obstacle of the main wing and connecting rod, the aerodynamic pressure may be less than the ideal one. Considering these factors, it is assumed that the aerodynamic efficiency of the aileron is 80% of its nominal value using DLM. By updating the aerodynamic model in the frequency domain, the theoretical aeroservoelastic model is denoted as Model 4. The consequent magnitude of frequency response function is illustrated in Fig. 12. From the figure, though it’s assumed that the decreased amplitude of the pressure coefficient for aileron motion at all reduced frequencies is the same, it is evident to suppress the response amplitude at high frequencies.

3.2.4. Rational function modification of aerodynamics —Theoretical Model 5

As mentioned above, state space equations are adopted for the aeroservoelastic theoretical models. Hence, it’s necessary to fit the unsteady aerodynamics in the frequency domain using rational function approximation. The Minimum State algorithm is used with two aerodynamic lag roots, 0.1 and 0.3,
chosen in Model 1. In Model 5, we change two alternative lag roots to 0.3 and 0.5 to construct an updating theoretical model. The magnitude of the frequency response function is shown in Fig. 13. The frequency response function without rational function approximation is also depicted for comparison. According to this figure, the predictions of Model 5 are closer to the predictions of the frequency-domain models. Consequently, it’s more reasonable to select 0.3 and 0.5 as the lag roots.

3.2.5 Comprehensive modification model—Theoretical Model 6

The essential criterion for an available theoretical model is that its prediction agrees with the experimental data. Although factors such as sensor noise, velocity fluctuations in the wind and nonlinearity have a certain effect on the frequency response of the system, they are not as important as the factors mentioned above. Therefore, these factors are not considered one by one in the study. All the main factors discussed in the previous sections are analyzed comprehensively here. Take $\gamma = 0.5$, the actuator model is shown in Eq. (9). The aeroservoelastic model is comprehensively modified, in which the generalized damping for each elastic mode is 0.01, the aerodynamic efficiency of the aileron is 0.8, and the aerodynamic lag roots are 0.3 and 0.5. This updated model is denoted as Model 6. The magnitude of its FRF is shown in Fig. 14, where it is compared with the results of Model 1.

According to the comparisons between the six models and the experiment, the results of Model 6 are the closest to the experiment data while there is a rather large gap. Therefore, it is necessary to introduce model uncertainty and to quantify the effects on aeroservoelastic stability and response. The Bayesian Theory is adopted for model uncertainty quantification in the following section.

4. Bayesian Posteriori Estimation Theory

Assuming there are $N$ theoretical models $M_k$, $k = 1, \cdots, N$, the post-conditional probability of Model $M_k$ with experimental data $y = [y_1, \cdots, y_L]$ can be expressed as

$$\Pr(M_k|y) = \frac{\Pr(y|M_k)\Pr(M_k)}{\sum_{k=1}^{N} \Pr(y|M_k)\Pr(M_k)}$$

(10)

where $\Pr(y|M_k)$ denotes the conditional probability of experimental data $y$ under the condition of Model $M_k$. $\Pr(M_k)$ is the prior probability of Model $M_k$. The theoretical models are assumed to satisfy a uniform distribution without any prior information about the assessment of the model. Hence, the prior probability of Model $M_k$ is $\Pr(M_k) = 1/N$. However, the probability can be given based on experts’ experience, if there is any prior information about the prediction precision of the model.

Assuming the $j$th response of Model $M_k$ is $\hat{y}_{kj}$, the experimental output data is $y_j$, and measurement error can be expressed as

$$y_j = \hat{y}_{kj} + \epsilon_{kj}$$

(11)

where $\epsilon_{kj}$ denotes the uncertainty of the theoretical model. $\epsilon_{kj}$ is generally assumed to satisfy identical independent Gaussian distribution with the mean value of 0 and the variance of $\sigma^2_k$. According to the length of $L$ experimental data and the output of the models, the expression for variance using the Maximum Likelihood Theory can be written as

$$\sigma^2_k = \left( \frac{1}{L} \sum_{i=1}^{L} \epsilon^2_{ki} \right) / L$$

(12)

Therefore, the conditional probability of experimental data $y$ on the condition of model $M_k$ is

$$\Pr(y|M_k) = \prod_{i=1}^{L} \Pr(y_i|\hat{y}_{ki}, \sigma^2_k) = \left( \frac{1}{2\pi\sigma^2_k} \right)^{L/2} \exp\left(-\frac{1}{2\sigma^2_k} \sum_{i=1}^{L} \epsilon^2_{ki} \right)$$

(13)

The posterior probability of Model $M_k$ on the condition of $y$ can be easily obtained by substituting Eq. (13) into Eq. (10).

Based on the posterior probability of each model, the outcome of the aeroservoelastic stability or response can be predicted, considering the probability quantification of all the theoretical models. The prediction of model $M_k$ is denoted as $\gamma^*_k$. Using the additive adjustment factor algorithm, the prediction of aeroservoelastic response $\gamma_m$ can be written as

$$\gamma_m = \gamma^*_k \times \text{additive adjustment factor}$$

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\[ y_m = y_k^m + E_a^m \]  
(14)

where \( E_a^m \) is the additive adjustment factor to represent the model uncertainties in the model set composed of several theoretical models. From the aspect of probability, \( E_a^m \) is assumed to satisfy a normal distributed probability function. Hence, its mean value and variance can be calculated based on the posterior probability of each theoretical model uncertainty. The expression is written as follows:

\[ E(E_a^m) = \sum_{k=1}^{N} P(M_k)(y_m - y_k^m), \quad E(y_m) = y_k^m + E(E_a^m) \]  

(15)

\[ \text{Var}(E_a^m) = \sum_{k=1}^{N} P(M_k)(y_m - E(y_k))^2, \quad \text{Var}(y_m) = \text{Var}(E_a^m) \]  

(16)

5. Aeroservoelastic Uncertainty Quantification

Although we have made several modifications in theoretical models, discrepancies still exist between the experimental data and predictions from the comprehensive theoretical model, as illustrated in Section 3. These discrepancies can be depicted by model uncertainty embedded in each theoretical model. Therefore, the Bayesian Theory is introduced to quantify the probability of each model uncertainty. The criterion is how close the prediction of the theoretical model is to the experimental data. Meanwhile, after the probability quantification of model uncertainty, the influence on response and the stability of its aeroservoelastic system is evaluated from the aspect of probability description.

5.1. Quantification of model uncertainty

As mentioned above, our criterion is to evaluate the discrepancy between the theoretical model and experimental data, so model uncertainty and measurement uncertainty are not distinguished here. They are both depicted by the gap between theoretical outcome and experimental data. The gap is denoted in Eq. (11). Using the Maximum Likelihood Theory and Bayesian Maximum A-Posteriori (MAP) Estimation, the posterior probability of FRF by every theoretical model is calculated on the basis of the experimental FRF at \( V = 24 \text{ m/s} \). First, we have to suppose a-priori probability for each model. According to Section 3, the a-priori probability of modified theoretical Model 6 is considered to be the highest since its frequency response is closest to the experimental data. The uncertainty of other models is regarded to satisfy an identical uniform distribution. In order to investigate the influence of different a-priori probability distribution, three different a-priori probabilities are assumed for Model 6, shown in Table 2. In Case 1, Model 6 has a 25% a-priori probability, and in Case 2 it has a higher a-priori probability. By comparison, it is also assumed an identical a-priori probability of each model in Case 3. Second, the posterior conditional probabilities of every theoretical model are calculated in Eq. (13), respectively. The obtained posterior probabilities are shown in Table 2.

| Model | Case 1 | Case 2 | Case 3 |
|-------|--------|--------|--------|
| Prior | 0.15   | 0.10   | 0.17   |
| Post  | 0.099  | 0.055  | 0.053  |

According to the comparison of a-priori probability and post-test probability in the three cases, different a-priori probability distribution may lead to variance in the post-test conditional probability distribution. Generally speaking, higher a-priori probability corresponds to higher post-test probability in model uncertainty. Moreover, the post probability of Model 6 is higher than its a-priori probability in all three cases. From special Case 3, although the same a-priori probability for each model, that is 0.1667, the post-test probability of Model 6 is much higher than that of other models. It is obvious that the predictive response of Model 6 is closest to the experimental data. Besides, the post-test probabilities of Model 2 and Model 4 are higher than those of the other three models, suggesting that actuator uncertainty and aileron aerodynamic efficiency has significant influence on the FRF data. In addition, Model 1 and Model 5 have an almost the same post-test probability. This indicates that root uncertainty may have little effect on FRF.

5.2. Aeroservoelastic uncertainty quantification of frequency response function

After the uncertainty probability of the six models, quantifying outcomes of the theoretical models, can be predicted with comprehensive consideration of the six models. An additive uncertainty is introduced in the best model, Model 6. Since Model 6 is the best model here, its a-priori probability is higher than those of the other five models. Therefore, Case 1 and Case 2 are used for response and stability computation, without consideration of Case 3 in Table 2. With the consideration of post-test probability and outcomes for the other five models, the mean value, variance, and upper and lower bounds with 95% confidence level of the aeroservoelastic FRF are calculated as shown in Fig. 15. From comparing the two figures, the response bound of the second a-priori probability assumption agrees better with the experimental data because Model 6 has a larger probability in the second case. From the comparison in this figure, the upper and lower bounds of 95% confidence level cover some of the experimental data. Hence, to some extent, the uncertainty probability quantification describes the effect of model uncertainty on the aeroservoelastic system based on the best theoretical model, and taking the influence of other models on the aeroservoelastic response into account. In addition, the bounds of Case 2 cover more experimental data. This indicates that modification of the original Model 1 is effective since Model 6 is more accurate than the other five models. However, the upper and lower bounds of FRF data at the 95% confidence level still does not cover experimental data at some fre-
5.3. Uncertainty quantification of the aeroservoelastic stability

The critical velocities of aeroservoelastic stability are calculated by introducing additive uncertainty under the uncertainty quantification of each model. First, the critical stability velocities of the six models calculated using the root locus method are 31.72 m/s, 31.71 m/s, 35.00 m/s, 31.72 m/s, 33.38 m/s, and 33.37 m/s. Then, we also select Model 6 was selected as the best model and uncertainty was introduced to consider the effects on the other five models. The mean value, variance, and upper and lower bounds of the 95% confidence level of critical stability velocity were also quantified, as illustrated in Table 3.

From the table, variance and upper and lower bounds of the 95% confidence level in Case 1 are rather low, which is related to the fact that the probability of Model 6 is given less than the second probability set. Since the critical stability velocity of the second post-test probability is greater than 31.5 m/s, it satisfies the experimental phenomenon that instability didn’t occur at \( V = 33 \) m/s.

6. Conclusion

The FRF and stability of an aircraft model were calculated from a theoretical aeroservoelastic model, and compared to experimental results from a step-sinusoidal swept test and time response test excited by turbulence. In order to depict the gap between the theoretical outcome and experimental result, model uncertainty was introduced and quantified using the Bayesian Posterior Estimation Theory. Consequently, their influence on aeroservoelastic stability and response was quantified by adding uncertainty. Using the wind tunnel test and aeroservoelastic model uncertainty quantification, the following conclusions were made:

1. The step-sine swept excitation technique is advantageous in a wind tunnel testing due to the high signal-to-noise ratio. The model uncertainties and their effect on aeroservoelasticity can be taken into consideration using the Bayesian Estimation theory. The probabilistic response and stability outcome more closely agree with experimental results.

2. An updated comprehensive model, denoted as Model 6, was chosen as the best model since it has the highest probability after model uncertainty quantification. The predictive results of the theoretical model were influenced significantly by the aerodynamic efficiency of aileron deflection. This suggests that in the future wind tunnel testing, dynamic pressure distribution measurement can be taken into consideration to evaluate the accuracy of aerodynamic modeling.

3. Though the theoretical model was modified and model uncertainty was introduced to predict aeroservoelastic FRF, the upper and lower bounds of FRF data at the 95% confidence level still does not agree with experimental data at some frequency points. The fact is that the aeroservoelastic FRF is not only related to the theoretical model itself, but also to input and output measurement.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No. 11302011, 11172025) and the Research Fund for the Doctoral Program of Higher Education of China (No. 20131102120051).

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