New bosonic hard string scattering amplitudes and extended Gross conjecture in superstring theory

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Abstract

We discover new hard string scattering amplitudes (HSSA) with polarizations orthogonal to the scattering plane in the NS sector of 10\textit{D} open superstring theory. The corresponding HSSA in the bosonic string theory are of subleading order in energy. In addition, all HSSA are found to be proportional to each other and the ratios among these HSSA are calculated to justify Gross conjecture in 1988 on high energy symmetry of string theory. Moreover, together with these new HSSA, the total number of HSSA calculated in the first massive level of NS sector are found to agree with those of hard polarized fermion string scattering amplitudes (PFSSA) in the R sector calculated recently.

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I. INTRODUCTION

In contrast to high energy behaviour of quantum field theories, e.g. the well-known asymptotic freedom of QCD discovered in early 70s [1], many important issues of high energy behaviour of string theory [2] remained to be studied. Historically, it was first conjectured in 1988 by Gross [3] in his pioneer works on this subject that there exist infinite linear relations [4] among hard string scattering amplitudes (HSSA) of different string states in the hard scattering limit. Moreover, these linear relations are so powerful that they can be used to solve all HSSA and express them in terms of one amplitude. This conjecture was later corrected and proved by Taiwan group using the decoupling of zero norm states [5, 6] in [7–12]. For more details, see the recent review [13, 14].

In addition to the bosonic string theory, there has been attempts to extend Gross conjecture (GC) to the NS sector of superstring theory [12, 15]. Recently, the present authors calculated a class of polarized fermion string scattering amplitudes (PFSSA) [16] at arbitrary mass levels which involve the leading Regge trajectory fermion string states of the R sector, and successfully extended GC to the fermionic sector. Since it is a nontrivial task to construct the general massive fermion string vertex operators [17, 22], the proof of extended
GC for the case of superstring theory remains an open question.

One important issue of extended GC for superstring is to compare the HSSA of NS and R sectors at each fixed mass level of the theory. Since so far the complete hard PFSSA is available only for the first massive level (no non-leading Regge trajectory fermion string states) in the R sector\cite{16}, as a first step, in this paper we will calculate the complete HSSA for the first massive level of the NS sector. As one consistent check of spacetime SUSY, we will calculate and count the number of HSSA for both NS and R sectors and check whether they match to each other or not.

To be more specific, the two HSSA we want to compare are

\begin{equation}
\langle V^{(-1)}\text{Tensor}^{(-1)}\phi^{(0)}\phi^{(0)} \rangle
\end{equation}

for the NS sector and

\begin{equation}
\langle \psi^{(-\frac{1}{2})}\chi^{(-\frac{1}{2})}\phi^{(-1)}\phi^{(0)} \rangle
\end{equation}

for the R sector. In Eq.(1.1) the first vertex $V^{(-1)}$ is chosen to be a massless vector, the second vertex $\text{Tensor}^{(-1)}$ is one of the two massive tensor states and the third and the fourth vertices are tachyons $\phi^{(0)}$. The total ghost charges sum up to $-2$. In Eq.(1.2) the first vertex $\psi^{(-\frac{1}{2})}$ is chosen to be a massless spinor, the second vertex $\chi^{(-\frac{1}{2})}$ is the $10D$ massive Majorana spinor and the third and the fourth vertices are tachyons $\phi^{(-1)}$ and $\phi^{(0)}$ in $(-1)$ and $(0)$ ghost pictures respectively. Again, the total ghost charges sum up to $-2$.

The number of HSSA in Eq.(1.2) was calculated to be 16 recently\cite{16}. On the other hand, The HSSA of Eq.(1.1) with polarizations on the scattering plane was calculated in\cite{12} and was listed in Eq.(3.31). We will use slightly different approach to obtain the result in this paper. Since the number of bosonic HSSA calculated in\cite{12} was less than 16, there exist HSSA with polarizations orthogonal to the scattering plane. These bosonic HSSA were mainly due to the ”worldsheet fermion exchange” and were first noted in\cite{12}. Motivated by the recent result of PFSSA\cite{16}, we will calculate in this paper the complete boconic HSSA including all HSSA with polarizations orthogonal to the scattering plane.

In the next section, we will first calculate all physical states which include zero norm states (ZNS) of the first mass level in the NS sector of old covariant quantization string spectrum. In particular, we will pay attention to ZNS and the enlarged gauge symmetries induced by them. Surprisingly, in constrast to the open bosonic string theory\cite{5}, there is NO ”inter-particle gauge transformation” for the two positive-norm states at this mass level.
Since at each fixed mass level, it is believed that string states of both NS sector and R sector form a large gauge multiplet, this result seems to tell us that for the case of superstring, one cannot ignore the effect of R sector when considering high energy symmetry of superstring theory.

II. ZNS AND ENLARGED GAUGE TRANSFORMATIONS IN THE NS SECTOR

In this section, we will first calculate all physical states for the first massive level in the NS sector (0 ghost picture). This will include all ZNS and the corresponding enlarged gauge transformations induced by them. Surprisingly, in constrast to the open bosonic string theory [5], there is NO inter-particle enlarged gauge transformation for the positive-norm anti-symmetric spin three state and symmetric spin two state at this mass level. The calculation is lengthy but elementary and we will mainly list the results.

In the old covariant first quantized (OCFQ) spectrum of 10D open superstring theory, the solutions of physical state conditions include positive-norm states and two types of ZNS. In the NS sector, the ZNS are [23]

Type I:

\[ G_{\frac{1}{2}} |\chi\rangle, \text{ where } G_{\frac{1}{2}} |\chi\rangle = G_{\frac{3}{2}} |\chi\rangle = L_0 |\chi\rangle = 0; \]  
(2.1)

Type II:

\[ \left( G_{\frac{3}{2}} + 2G_{\frac{1}{2}} L_{-1} \right) |\chi'\rangle, \text{ where } G_{\frac{1}{2}} |\chi'\rangle = G_{\frac{3}{2}} |\chi'\rangle = (L_0 + 1) |\chi'\rangle = 0. \]  
(2.2)

In this paper, we will use the notation of reference [23]. Note that while type I ZNS are zero-norm at any spacetime dimension, type II ZNS are zero-norm only at \( D = 10 \).

The most general positive-norm states for the first massive level in the NS sector can be calculated to be

\[ \left( \epsilon_{\mu\nu\rho} b_{\rho} b_{\mu} b_{\nu} + \epsilon_{\mu\nu} b_{\nu} b_{\mu} \alpha_{-1} + \epsilon_{\mu} b_{\mu} \right) |0, k\rangle \]  
(2.3)

with the on-shell conditions

\[ 3\epsilon_{\mu\nu\rho} k^\mu - \epsilon_{\mu\nu} = 0, \]  
(2.4)

\[ (\epsilon_{\mu\nu}) k^\mu + \epsilon_{\mu} = 0. \]  
(2.5)
The above on-shell conditions imply that we can divide Eq. (2.3) into two states, the anti-symmetric spin three state which is described by the doublet \( \{ \epsilon_{\mu\nu\rho}, \epsilon_{[\mu\nu]} \} \)

\[
\left( \epsilon_{\mu\nu\rho} b^\rho_{\mu\nu} - \epsilon_{[\mu\nu]} b^\rho_{\mu\nu} + \epsilon_{[\mu\nu]} b^\rho_{\mu\nu} \right) |0, k\rangle ; 3\epsilon_{\mu\nu\rho} k^\rho - \epsilon_{[\mu\nu]} = 0;
\]

and the symmetric spin two state which is described by the doublet \( \{ \epsilon_{(\mu\nu)}, \epsilon_{\mu} \} \)

\[
\left( \epsilon_{(\mu\nu)} b^\nu_{\mu\nu} + \epsilon_{\mu} b^\mu_{\mu\nu} \right) |0, k\rangle ; (\epsilon_{(\mu\nu)} k^\mu + \epsilon_{\mu}) = 0.
\]

We are now ready to calculate the ZNS. The most general form of type I ZNS turns out to be

\[
|ZNS\rangle = \left( k_{[\mu} \theta_{\nu]} b^\rho_{\mu\nu} b^\mu_{\mu\nu} b^\nu_{\mu\nu} - 2\theta_{[\mu\nu]} b^\rho_{\mu\nu} \alpha^\rho_{\mu\nu} + k_{[\mu(\theta_{\nu])} b^\rho_{\mu\nu} \alpha^\rho_{\mu\nu} + k_{(\mu(\theta_{\nu])} b^\rho_{\mu\nu} \alpha^\rho_{\mu\nu} + \theta_{\mu} b^\mu_{\mu\nu} \right) |0, k\rangle
\]

with the on-shell conditions

\[
2k^\mu \theta_{[\mu\nu]} + \theta_{\nu} = 0, \text{ (automatically } k \cdot \theta = 0).
\]

For the rest of this paper, we define \( e^P = \frac{1}{M^2}(E_2, k_2, 0) = \frac{k_2}{M^2} \) the momentum polarization, \( e^L = \frac{1}{M^2}(k_2, E_2, 0) \) the longitudinal polarization and \( e^T = (0, 0, 1) \) the transverse polarization on the scattering plane. In addition, we define \( e^{T_i} = \{ e^T, e^{T_3}, e^{T_3}, ..., e^{T_9} \} = \{ e^T, e^{T_i} \} \) to represent all transverse directions including directions \( e^{T_i} \) orthogonal to the scattering plane.

One obvious solution of Eq. (2.9) is to choose

\[
\theta^{(2)}_{[\mu\nu]} = \frac{1}{2} (e^I_{e^I - e^I_{e^I}}), e^I \neq e^P, e^J \neq e^P,
\]

\[
\theta^{(2)}_{\nu} = 0,
\]

and one obtains the anti-symmetric spin two ZNS

\[
|ZNS\rangle = \left( k_{[\mu} \theta_{\nu]} b^\rho_{\mu\nu} b^\mu_{\mu\nu} b^\nu_{\mu\nu} - 2\theta_{[\mu\nu]} b^\rho_{\mu\nu} \alpha^\rho_{\mu\nu} \right) |0, k\rangle, k^\mu \theta_{[\mu\nu]} = 0.
\]

It can be shown that the ZNS in Eq. (2.10) generates an enlarged on-shell gauge transformation

\[
\epsilon'_{[\mu\nu\rho]} = (\epsilon_{[\mu\nu\rho]} + k_{[\mu} \theta_{\nu]}), \epsilon'_{[\mu\nu]} = \epsilon_{[\mu\nu]} - \theta_{[\mu\nu]}
\]

for the anti-symmetric spin three state in Eq. (2.6).
The second solution of Eq. (2.9) is to choose
\[ \theta^{(1)}_{\mu \nu} = \frac{1}{2M} \left( e^P e^K - e^K e^P \right), \quad e^K \neq e^P, \]  
(2.12)
\[ \theta^{(1)}_\nu = -2k^\mu \theta_{\mu \nu} = -\frac{1}{M} k^\mu \left( e^P e^K - e^K e^P \right) = e^K. \]  
(2.13)
For this choice, Eq. (2.8) got simplified and one obtains a vector ZNS
\[ |ZNS\rangle = \left( k_{(\mu \theta_{\nu})} b^{\frac{\mu}{2}} \alpha_{-1} + \theta_{\mu} b^{\frac{\nu}{2}} \right) |0, k\rangle \cdot k \cdot \theta = 0. \]  
(2.14)
It can be shown that the ZNS in Eq. (2.14) generates an enlarged on-shell gauge transformation
\[ \epsilon'_{(\mu \nu)} = \epsilon_{(\mu \nu)} + k_{(\mu \theta_{\nu})}, \quad \epsilon'_\mu = [\epsilon_\mu + \theta_{\mu}] \]  
(2.15)
for the symmetric spin two state in Eq. (2.7). To obtain the vector ZNS state in Eq. (2.14), one can alternatively propose
\[ |\chi\rangle = \left[ \frac{1}{2M^2} \left( k_{(\mu \theta_{\nu})} b^{\frac{\mu}{2}} b^{\frac{\nu}{2}} + \theta_{\mu} \alpha_{-1} \right) \right] |0, k\rangle \cdot k \cdot \theta = 0 \]  
(2.16)
and put it in Eq. (2.1).

The only type II ZNS is a scalar and can be calculated to be
\[ |ZNS\rangle = \left( 3k \cdot b + \alpha_{-1} \cdot b + 2 \left( k \cdot b + \alpha_{-1} \right) \right) \left( k \cdot \alpha_{-1} \right) |0, k\rangle. \]  
(2.17)
It can be shown that the ZNS in Eq. (2.17) generates an enlarged on-shell gauge transformation
\[ \epsilon'_{(\mu \nu)} = \epsilon_{(\mu \nu)} + \eta_{\mu \nu} + 2k_{(\mu} k_{\nu)}, \quad \epsilon'_\mu = \epsilon_\mu + 3k_{\mu} \]  
(2.18)
for the symmetric spin two state in Eq. (2.7).

We note that the number of ZNS in Eq. (2.10), Eq. (2.14) and Eq. (2.17) can be counted to be 36 + 9 + 1 = 46. On the other hand, the number of ”spurious states” in the −1 ghost picture were calculated \[18\] to be 36 + 9 + 1 = 46, which is consistent with the results calculated in the 0 ghost picture in this paper.

Surprisingly, in contrast to the open bosonic string theory \[5\], there is NO ”inter-particle gauge transformation” for the positive-norm anti-symmetric spin three state and symmetric spin two state at this mass level.

Here we briefly review the results of 26D open bosonic string theory at mass level \[M^2 = 4\] \[8\]. At this mass level, there are two positive-norm states, the symmetric spin three state and
the anti-symmetric spin two state. In addition there are three type I ZNS, the symmetric spin two, a vector and a scalar, and one type II vector ZNS. One can do a linear combination of the two degenerate vector ZNS to obtain the so-called $D_2$ vector ZNS $^{[5]}$

$$|D_2\rangle = \left(\frac{1}{2} k_\mu k_\nu \theta^2 \lambda \alpha^\mu - 2 \eta_{\mu\nu} \theta^2 \lambda \alpha^\mu \alpha^\nu - 9 k_\mu \theta^2 \lambda \alpha^\mu \alpha^\nu - 6 \theta^2 \lambda \alpha^\mu \alpha^\nu \right) |0, k\rangle, \quad k \cdot \theta^2 = 0 \quad (2.19)$$

which generates an "inter-particle gauge transformation" for the positive-norm symmetric spin three state and the anti-symmetric spin two state. Thus these two states form a large gauge multiplet. The other three ZNS generate gauge transformations for the symmetric spin three state. An important consequence of this inter-particle symmetry is the linear relation which relates hard string scattering amplitudes of the two positive-norm states. Indeed, all HSSA are proportional to each others with the ratios $^{[7, 8]}$

$$T_{TTT} : T_{LLT} : T_{LT} : T_{[LT]}$$

$$= \alpha^T_{\perp 1} \alpha^T_{\perp 2} : \alpha^L_{\perp 1} \alpha^L_{\perp 2} : \alpha^T_{\perp 1} \alpha^T_{\perp 2} = 8 : 1 : -1 : -1. \quad (2.20)$$

Although there is no "inter-particle gauge transformation" at mass level $M^2 = 2$ in the NS sector of the spectrum of the superstring, in the next section we will see that the 16 HSSA originated from the two positive-norm states are still related and are indeed again proportional to each others. Presumably, this is due to the spacetime SUSY and the spacetime massive fermion string scattering amplitudes of the fermionic sector of the theory $^{[16]}$.

III. HARD STRING SCATTERING AMPLITUDES IN THE NS SECTOR

In this section we will calculate all HSSA in Eq.(1.1) including some NEW ones which were not considered in $^{[12]}$. We will demonstrate the linear relations among the HSSA between the two positive-norm states, and thus extend and justify the original Gross conjecture to the case of superstring. Similar inter-particle symmetry was discovered $^{[7, 8]}$ in the 26D bosonic string theory.

For the case of the bosonic string, string scattering amplitudes with polarizations orthogonal to the scattering plane are of subleading order in energy in the hard scattering limit. However, it was noted that $^{[12]}$ for the HSSA of the NS sector of superstring, there existed leading order HSSA with polarizations orthogonal to the scattering plane. This was due to
the "worldsheet fermion exchange" in the correlation functions and was argued to be related to the massive spacetime fermion HSSA of the theory.

In the subsection A of this section, we will first calculate ratios among HSSA with polarizations on the scattering plane by using the hard Ward identities (or decoupling of ZNS in the hard scattering limit), and then by using explicit HSSA calculations. In subsections B and C, we will calculate two types of new HSSA with polarizations orthogonal to the scattering plane.

A. Hard SSA with polarizations on the scattering plane

We first do the hard Ward identity (HWI) calculation for polarizations on the scattering plane. In the hard scattering limit, one identifies $e^L = e^P$. In the following calculation, each scattering amplitude has been assigned a relative energy power. For each longitudinal $L$ component, the energy order is $E^2$ while for each transverse $T$ component, the energy order is $E$. This is due to the definitions of $e^L$ and $e^T$ in the paragraph below Eq.(2.9) where $e^L$ got one energy power more than that of $e^T$.

In the HWI calculation, we put the ZNS in the second vertex and suppress the indices for the other vertices. The ZNS in Eq.(2.10) gives the HWI

$$\left(\alpha^{-1}_L b^T_{\frac{1}{2}} - \alpha^{-1}_T b^L_{\frac{1}{2}}\right) |0, k\rangle = 0. \tag{3.21}$$

For the choice of $\theta^{(1)} = e^L$, the ZNS in Eq.(2.14) gives the HWI

$$\left[\left(\frac{2}{b^L_{\frac{1}{2}}} + \left(\frac{\alpha^L_{-1}}{M}\right) \left(b^L_{\frac{1}{2}}\right)\right) \right] |0, k\rangle. \tag{3.22}$$

For the choice of $\theta^{(1)} = e^T$, the ZNS in Eq.(2.14) gives the HWI

$$\frac{1}{b^T_{\frac{1}{2}}} + \frac{1}{2} \left[\left(\frac{\alpha^L_{-1}}{M}\right) \left(b^T_{\frac{1}{2}}\right) + \left(\frac{\alpha^T_{-1}}{M}\right) \left(b^L_{\frac{1}{2}}\right)\right] |0, k\rangle = 0. \tag{3.23}$$

The ZNS in Eq.(2.17) gives the HWI

$$\left[\frac{3}{M} b^L_{\frac{1}{2}} + \alpha^L_{-1} b^L_{-1} \eta_{uv} + \frac{2}{M^2} \left(b^L_{-1}\right) \left(\alpha^L_{-1}\right)\right] |0, k\rangle = 0 \tag{3.24}$$
where
\[ \eta_{\mu\nu} = -e_\mu^P e_\nu^P + e_\mu^L e_\nu^L + e_\mu^T e_\nu^T + \sum_{i=1}^{7} e_\mu^T e_\nu^T \alpha_{\frac{T}{T}}^i b_{\frac{T}{T}}^i \rightarrow \text{not contribute} \] (3.25)

In sum, ZNS in Eq.(2.17) gives the HWI
\[ \left[ (2) \sqrt{3} M b_{\frac{L}{2}}^L + (2) M^2 \alpha_{\frac{L}{2}}^T b_{\frac{T}{2}}^T + (4\rightarrow 2) 2 \alpha_{\frac{T}{2}}^L b_{\frac{T}{2}}^L \right] |0, k\rangle = 0. \] (3.26)

The naive energy order of $2\alpha_{\frac{L}{2}} b_{\frac{L}{2}}^L$ term in Eq.(3.26) is $E^4$, the HWI forces its energy order to drop to $E^2$. Similar calculations were performed in Eq.(3.21), Eq.(3.22) and Eq.(3.23).

We note that the energy order $E^3$ term in Eq.(3.23) gives
\[ \alpha_{\frac{L}{2}}^L b_{\frac{T}{2}}^T + \alpha_{\frac{T}{2}}^T b_{\frac{L}{2}}^L = 0, \] (3.27)

which together with Eq.(3.21) force the following two $E^3$ order terms vanish
\[ \alpha_{\frac{L}{2}}^L b_{\frac{T}{2}}^T = 0 = \alpha_{\frac{T}{2}}^T b_{\frac{L}{2}}^L. \] (3.28)

Thus the real leading order terms are those of energy order $E^2$ in Eq.(3.22) and Eq.(3.26)
\[ \alpha_{\frac{L}{2}}^L b_{\frac{T}{2}}^L + M b_{\frac{L}{2}}^L = 0, \] (3.29)
\[ 3M b_{\frac{L}{2}}^L + M^2 \alpha_{\frac{T}{2}}^T b_{\frac{T}{2}}^T + 2 \alpha_{\frac{T}{2}}^L b_{\frac{T}{2}}^L = 0, \] (3.30)

which can be solved to give the ratios
\[ b_{\frac{L}{2}}^L : \alpha_{\frac{L}{2}}^L b_{\frac{T}{2}}^T : \alpha_{\frac{T}{2}}^T b_{\frac{T}{2}}^L = -\frac{1}{M} : \frac{1}{M^2} : 1. \] (3.31)

The result in Eq.(3.31) is consistent with the calculation obtained in [12] by a slightly different approach.

The ratios in Eq.(3.31) can be rederived by calculating a set of sample HSSA. We first calculate
\[ \langle V^{(-1)}(\text{Tensor}^{(-1)}\phi^{(0)}\phi^{(0)}) \rangle \]
\[ = \int dz_1 dz_2 dz_3 dz_4 \left| \frac{z_{13} z_{14} z_{34}}{d z_1 dz_3 dz_4} \right| \left( \frac{\psi_{1}^H}{1} e^{-\phi_{1}^i e^{ik_1} X_1} \right) \left( \frac{\psi_{2}^H}{1} e^{-\phi_{2}^i e^{ik_2} X_2} \right) \times \left( \frac{k_{3}^H}{4} \psi_{3}^H e^{ik_3 X_3} \right) \left( \frac{k_{4}^H}{4} \psi_{4}^H e^{ik_4 X_4} \right) \right) \] (3.32)
where the first vertex $V^{(-1)}$ is chosen to be a vector, the second vertex is the $(\alpha^{T}_{-1}) (b^{T}_{\frac{1}{2}}) |0;k)$ tensor state and the third and the fourth vertices are tachyons $\phi^{(0)}$. The total ghost charges sum up to $-2$. We first put $\epsilon = e^{T}$ to get

$$\langle V^{(-1)}Tensor^{(-1)}_0 \phi^{(0)} \phi^{(0)} \rangle$$

$$= \epsilon_{\mu_{1}} \epsilon_{\mu_{2}} \epsilon_{\mu_{3}} \epsilon_{\mu_{4}} \int dz_{2} \left[ \eta_{\mu_{1} \mu_{2}} \eta_{\mu_{3} \mu_{4}} \frac{z_{23}}{z_{12} z_{24}} - \frac{\eta_{\mu_{1} \mu_{3}} \eta_{\mu_{2} \mu_{4}}}{z_{14} z_{24}} + \frac{\eta_{\mu_{1} \mu_{4}} \eta_{\mu_{2} \mu_{3}}}{z_{12} z_{24}} \right]$$

$$\times \left| z_{12} \right|^{k_{1} \cdot k_{2} - 1} \left| z_{13} \right|^{k_{1} \cdot k_{3} + 1} \left| z_{14} \right|^{k_{1} \cdot k_{4} + 1} \left| z_{23} \right|^{k_{2} \cdot k_{3}} \left| z_{24} \right|^{k_{2} \cdot k_{4}} \left| z_{34} \right|^{k_{3} \cdot k_{4} + 1} \left( \frac{k_{3}^{T}}{z_{23}} + \frac{k_{4}^{T}}{z_{24}} \right).$$

(3.33)

The next step is to fix the gauge $z_{1} = 0, z_{2} = z_{2}, z_{3} = 1, z_{4} \to \infty \ (0 < z_{2} < 1)$ and use $k_{1} \cdot k_{2} = -\frac{z}{2} + 1, k_{2} \cdot k_{3} = -\frac{k}{2} + \frac{1}{2}$ to perform the integration to obtain

$$\langle V^{(-1)}Tensor^{(-1)}_0 \phi^{(0)} \phi^{(0)} \rangle$$

$$= (-k_{3}^{T}) \left[ (\epsilon_{1} \cdot \epsilon_{2}) (k_{3} \cdot k_{4}) - (k_{3} \cdot \epsilon_{1}) (k_{4} \cdot \epsilon_{2}) \frac{s}{u + 1} + (k_{3} \cdot \epsilon_{2}) (k_{4} \cdot \epsilon_{1}) \frac{s}{t + 1} \right]$$

$$\times \frac{\Gamma(-\frac{s}{2}) \Gamma(-\frac{t}{2} + \frac{1}{2})}{\Gamma(\frac{u}{2} + \frac{1}{2})}.$$

(3.35)

Finally in the hard scattering limit we have

$$k_{1} = (E, -E, 0), k_{2} = (E, E, 0), k_{3} = (-E, -E \cos \theta, -E \sin \theta), k_{4} = (-E, E \cos \theta, E \sin \theta)$$

(3.37)

and

$$\epsilon_{1} = \epsilon_{2} = e^{T} = (0, 0, 1), s = 4E^{2}, \frac{s}{u} \to \frac{-1}{\cos^{2} \frac{\theta}{2}}, \frac{s}{t} \to \frac{-1}{\sin^{2} \frac{\theta}{2}}, k_{3} \cdot k_{4} = \frac{-s}{2} - 1.$$

(3.38)

After a bit calculation, one ends up with

$$\langle V^{(-1)}Tensor^{(-1)}_0 \phi^{(0)} \phi^{(0)} \rangle$$

$$= (-k_{3}^{T}) \left[ -2E^{2} + \frac{4E^{2} \sin^{2} \frac{\theta}{2} \cos^{2} \frac{\theta}{2}}{\cos^{2} \frac{\theta}{2}} + \frac{4E^{2} \sin^{2} \frac{\theta}{2} \cos^{2} \frac{\theta}{2}}{\sin^{2} \frac{\theta}{2}} \right] B$$

(3.39)

$$= -2k_{3}^{T} E^{2} B$$

(3.40)

where $B$ is the hard scattering limit of Eq. (3.36).

Similar calculations can be performed if one replaces the second vertex $\alpha^{T}_{-1} b^{T}_{\frac{1}{2}}$ by
\( \alpha L_{1} b_{1}^{L} \) and \( b_{T}^{2} |0; k \rangle \) respectively. The results are

\[
\langle V^{(-1)}_{\text{Tensor}}(\alpha L_{1} b_{1}^{L}) \phi^{(0)}(0) \rangle = \frac{-2k_{3}^{T}E^{2}B}{M^{2}}, \tag{3.41}
\]

\[
\langle V^{(-1)}_{\text{Tensor}}(b_{T}^{2}) \phi^{(0)}(0) \rangle = \frac{2k_{3}^{T}E^{2}B}{M}. \tag{3.42}
\]

These results justify the ratios calculated in Eq. (3.31). They also agree with the results obtained by the saddle-point method [12].

### B. New hard SSA I

In addition to the three HSSA calculated in Eq. (3.40), Eq. (3.41) and Eq. (3.42), it was noted that [12] there existed HSSA with polarizations orthogonal to the scattering plane. In particular, one can replace \( b_{T}^{2} \) operator in the vertex by \( b_{T}^{i} \) where \( T_{i} \) represents directions orthogonal to the scattering plane. Note that the corresponding string scattering amplitudes are of subleading order in energy for the case of bosonic string theory. We first propose the following HSSA (we use the obvious brief notation)

\[
\langle b_{T}^{i}, \alpha^{T} b_{T}^{j}, \phi_{3}, \phi_{4} \rangle. \tag{3.43}
\]

The calculation of the above amplitude is similar to Eq. (3.35) of the calculation of the amplitude

\[
\langle b_{T}^{i}, \alpha^{T} b_{T}^{j}, \phi_{3}, \phi_{4} \rangle. \tag{3.44}
\]

Instead of Eq. (3.35), one obtains

\[
\langle b_{T}^{i}, \alpha^{T} b_{T}^{j}, \phi_{3}, \phi_{4} \rangle = \left( -k_{3}^{T} \right) \left[ (e^{T_{i}} \cdot e^{T_{j}})(k_{3} \cdot k_{4}) - (k_{3} \cdot e^{T_{i}})(k_{4} \cdot e^{T_{j}}) \frac{s}{u+1} \right] \tag{3.45}
\]

\[
\times \frac{\Gamma \left( \frac{s}{2} \right) \Gamma \left( \frac{s}{2} + \frac{1}{2} \right)}{\Gamma \left( \frac{s}{2} + \frac{1}{2} \right)}.	ag{3.46}
\]

Finally in the hard scattering limit, after a bit calculation, one ends up with

\[
\langle b_{T}^{i}, \alpha^{T} b_{T}^{j}, \phi_{3}, \phi_{4} \rangle = \left( -k_{3}^{T} \right) \delta_{ij} (k_{3} \cdot k_{4}) B = \delta_{ij} 2k_{3}^{T}E^{2}B. \tag{3.47}
\]

Eq. (3.47) contributes 7 more HSSA in the NS sector at this mass level.
C. New hard SSA II

So far all HSSA calculated in this section are from the positive-norm symmetric spin two state. One expects HSSA contributed from the anti-symmetric spin three state. In this subsection we propose the following HSSA

$$\langle b^{T_K}_{T_1}, b^{T_L}_{T_1} b^{T_I}_{T_1}, \phi_3, \phi_4 \rangle,$$

which was not considered in [12]. The calculation of this HSSA is lengthy but straightforward. We begin with the standard calculation of string scattering amplitude

$$\left\langle V^{-1} T_{\text{Tensor}}^{(-1)} \phi^{(0)} \phi^{(0)} \right\rangle$$

$$= \int d\Omega_1 d\Omega_2 \frac{|z_{13} z_{14} z_{34}|}{d\Omega_1 d\Omega_2 d\Omega_3} \left\langle \left( \epsilon^{T_K}_{\mu_1} \psi^{\mu_1}_1 e^{-\phi_1} e^{ik_1 x_1} \right) \left( \epsilon^{T_L}_{\mu_2} e^{T_I}_{\mu_3} \psi^{\mu_3}_2 \psi^{\mu_4}_2 \psi^{\mu_5}_2 e^{-\phi_1} e^{ik_2 x_2} \right) \right\rangle$$

$$\times \left( k^3_{\mu_5} \psi^{\mu_5}_3 e^{ik_3 x_3} \right) \left( k^4_{\mu_6} \psi^{\mu_6}_4 e^{ik_4 x_4} \right)$$

$$= \epsilon^{T_K}_{\mu_1} \epsilon^{T_l}_{\mu_2} e^{T_I}_{\mu_3} k^3_{\mu_5} k^4_{\mu_6} \int d\Omega_2 |z_{13} z_{14} z_{34}| \left\langle \left( \psi^{\mu_1}_1 \psi^{\mu_2}_2 \psi^{\mu_3}_2 \psi^{\mu_4}_2 \psi^{\mu_5}_2 \psi^{\mu_6}_4 \right) \right\rangle$$

$$= \int d\Omega_2 \left| z_{13} z_{14} z_{34} \right| e^{-\ln |z_{12}|} \left| z_{13}^{k_1-k_2} z_{14}^{k_1-k_3} \right| \left| z_{13}^{k_1-k_3} z_{14}^{k_1-k_4} \right| \left| z_{13}^{k_2-k_3} z_{24}^{k_2-k_4} z_{34}^{k_3-k_4} \right|$$

$$= \int d\Omega_2 \left| z_{12}^{k_1-k_2-1} z_{13}^{k_1-k_3+1} z_{14}^{k_1-k_4+1} \right| \left| z_{23}^{k_2-k_3} z_{24}^{k_2-k_4} z_{34}^{k_3-k_4+1} \right|$$

and

$$\left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \psi^{\mu_3}_2 \psi^{\mu_4}_2 \psi^{\mu_5}_2 \psi^{\mu_6}_4 \right\rangle$$

$$= - \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_2 \psi^{\mu_4}_2 \right\rangle \left\langle \psi^{\mu_5}_2 \psi^{\mu_6}_4 \right\rangle + \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_2 \psi^{\mu_5}_2 \right\rangle \left\langle \psi^{\mu_4}_2 \psi^{\mu_6}_4 \right\rangle$$

$$+ \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_3 \psi^{\mu_4}_3 \right\rangle \left\langle \psi^{\mu_5}_3 \psi^{\mu_6}_4 \right\rangle - \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_4 \psi^{\mu_5}_4 \right\rangle \left\langle \psi^{\mu_4}_2 \psi^{\mu_6}_4 \right\rangle$$

$$- \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_4 \psi^{\mu_5}_4 \right\rangle \left\langle \psi^{\mu_4}_2 \psi^{\mu_6}_4 \right\rangle + \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \right\rangle \left\langle \psi^{\mu_3}_4 \psi^{\mu_5}_4 \right\rangle \left\langle \psi^{\mu_4}_4 \psi^{\mu_6}_4 \right\rangle$$

$$= \sum_{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5 \mu_6} \left\langle \psi^{\mu_1}_1 \psi^{\mu_2}_2 \psi^{\mu_3}_2 \psi^{\mu_4}_2 \psi^{\mu_5}_2 \psi^{\mu_6}_4 \right\rangle$$

(3.52)
We can now use \( k_1 \cdot k_2 = -\frac{i}{2} + 1, k_2 \cdot k_3 = -\frac{i}{2} + \frac{1}{2} \) and perform the integration to get

\[
\left\langle V^{(-1)} T_{\text{Tensor}}^{(1)} \phi^{(0)} \phi^{(0)} \right\rangle
= \frac{\mu_1^L T_{i} T_{j} k_3^3 k_4^4}{\epsilon_{\mu_1} \epsilon_{\mu_2} \epsilon_{\mu_3} \epsilon_{\mu_4}} \left( \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} \right)
\times \int dz_2 \left( \frac{1 - z_2}{z_2 - 1} \right)^{k_1 \cdot k_2 - 1} (1 - z_2)^{k_2 \cdot k_3} \]

We can now use \( k_1 \cdot k_2 = -\frac{i}{2} + 1, k_2 \cdot k_3 = -\frac{i}{2} + \frac{1}{2} \) and perform the integration to get

\[
\left\langle V^{(-1)} T_{\text{Tensor}}^{(1)} \phi^{(0)} \phi^{(0)} \right\rangle
= \frac{\mu_1^L T_{i} T_{j} k_3^3 k_4^4}{\epsilon_{\mu_1} \epsilon_{\mu_2} \epsilon_{\mu_3} \epsilon_{\mu_4}} \left( \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} \right)
\times \int dz_2 \left( \frac{1 - z_2}{z_2 - 1} \right)^{k_1 \cdot k_2 - 1} (1 - z_2)^{k_2 \cdot k_3} \]

The next step is to fix the gauge \( z_1 = 0, z_2 = z_3 = 1, z_4 \to \infty (0 < z_2 < 1) \) to get

\[
\left\langle V^{(-1)} T_{\text{Tensor}}^{(1)} \phi^{(0)} \phi^{(0)} \right\rangle
= \frac{\mu_1^L T_{i} T_{j} k_3^3 k_4^4}{\epsilon_{\mu_1} \epsilon_{\mu_2} \epsilon_{\mu_3} \epsilon_{\mu_4}} \left( \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} + \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} - \eta^{\mu_1 \mu_2 \eta^{\mu_3 \mu_5} \eta^{\mu_4 \mu_6}} \right)
\times \int dz_2 \left( \frac{1 - z_2}{z_2 - 1} \right)^{k_1 \cdot k_2 - 1} (1 - z_2)^{k_2 \cdot k_3} \]

In the hard scattering limit, the HSSA reduces to

\[
\left\langle b^{T_{i}} b^{T_{j}} \right\rangle
= \left[ -\delta_{IK} \frac{1}{M} \left( \frac{-t}{2} \right) k_4^{T_{i}} + \delta_{JK} \frac{1}{M} \left( \frac{-u}{2} \right) k_3^{T_{i}} \right] B.
\]

There are three cases to further reduce the result:

**Case one:** for \( e_{T}^{T} = e_{T}^{T}, e_{T}^{T} = e_{T}^{T}, e_{T}^{T} = e_{T}^{T} \),

\[
\left[ -\frac{1}{M} \left( \frac{-t}{2} \right) k_4^{T_{j}} + 0 + \frac{1}{M} \left( \frac{-u}{2} \right) k_3^{T_{j}} \right] = \frac{1}{M} \left( \frac{t}{2} \right) \times 0 + \frac{1}{M} \left( \frac{-u}{2} \right) \times 0 = 0.
\]
Case two: for $\epsilon^{Tk} = e^{T}, \epsilon^{Ti} = e^{T}, \epsilon^{Tj} = e^{Tj}$,

$$
\left[ -0 \times \frac{1}{M} \left( -\frac{t}{2} \right) k^T_{4j} + 0 \times \frac{1}{M} \left( -\frac{t}{2} \right) k^T_{4i} + 0 \times \frac{1}{M} \left( -\frac{u}{2} \right) k^T_{3j} - 0 \times \frac{1}{M} \left( -\frac{u}{2} \right) k^T_{3i} \right] = 0.
$$

(3.63)

Case three: for $\epsilon^{Tk} = e^{Tk}, \epsilon^{Ti} = e^{T}, \epsilon^{Tj} = e^{Tj}$,

$$
\left[ -0 \times \frac{1}{M} \left( -\frac{t}{2} \right) k^T_{4j} + \delta_{jk} \frac{1}{M} \left( -\frac{t}{2} \right) k^T_{4k} + 0 \times \frac{1}{M} \left( -\frac{u}{2} \right) k^T_{3j} - \delta_{jk} \frac{1}{M} \left( -\frac{u}{2} \right) k^T_{3k} \right]
= \left[ \delta_{jk} \frac{1}{2M} (t + u) k^T_{3k} = \frac{-2E^2 \delta_{jk} k^T_{3k}}{M}. \right. \left. (j, k = 3, 4, 5, ..., 9). \right)
$$

(3.66)

In summary, we get the following ratios in the NS sector

$$
\langle b^T_{11}, \alpha^-_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^L_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^-_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^-_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^L_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^L_{11} b^T_{31} \rangle : \langle b^T_{11}, \alpha^L_{11} b^T_{31} \rangle
\right)
$$

(3.67)

$$
= -2k^T_3 E^2 B : \frac{-2k^T_3 E^2 B}{M^2} : \frac{2k^T_3 E^2 B}{M} : \delta_{ij} 2k^T_3 E^2 B : \delta_{ik} \frac{-2k^T_3 E^2 B}{M} \right)
$$

(3.68)

$$
= 1 : \frac{1}{M^2} : -\frac{1}{M^2} : -\delta_{ij} : \frac{\delta_{il}}{M} \left( i, j, k, l = 3, 4, 5, ..., 9 \right)
$$

(3.69)

where we have, for simplicity, omitted the last two tachyon vertices in the notation of each HSSA in Eq.(3.67). The ratios in Eq.(3.67) to Eq.(3.69) are the superstring version of the bosonic string ratios in Eq.(2.20). However, there is no inter-particle gauge transformation for the NS sector of the superstring case in the analysis of section II.

At this stage, one might think that there are $1 + 1 + 1 + 7 + 7 = 17$ HSSA in Eq.(3.67) which is inconsistent with the $2^4 = 16$ hard PFSSA calculated in [16]. To resolve the inconsistency, one notes that $b^L_{33}$ in the third amplitude in Eq.(3.67) cannot be alone a (high energy) physical state. So one needs to do a linear combination of the first three amplitudes in Eq.(3.67) to transform the HSSA from the operator basis to the physical state basis. The first physical state amplitude can be obtained by choosing

$$
\epsilon^{(\mu \nu)} = \epsilon^{T \mu, \epsilon^{T \nu}}, \epsilon^{(\mu \nu)} = -\epsilon^{(\mu \nu)} k^\mu = 0
$$

in Eq.(2.7) to obtain

$$
\left( \epsilon^{(\mu \nu)} b^\mu_{11} \alpha^-_{11} + \epsilon^{(\mu \nu)} b^\mu_{33} \right) |0; k\rangle
\right)
$$

(3.71)

$$
\rightarrow \left( b^T_{21} \alpha^-_{11} \right) |0; k\rangle \rightarrow 1.
$$

(3.72)
The second physical state amplitude can be obtained by choosing
\[ \epsilon_{(\mu\nu)} = \frac{1}{2} (e^P_{\mu} e^L_{\nu} + e^P_{\nu} e^L_{\mu}) , \epsilon_{\nu} = -\epsilon_{(\mu\nu)} k^\mu = \frac{1}{2} (e^P_{\mu} e^L_{\nu} + e^P_{\nu} e^L_{\mu}) k^\mu = \frac{1}{2} e^P_{\mu} k^\mu e^L_{\nu} = -\frac{M}{2} e^L_{\nu} \] (3.73)
in Eq. (2.7) to obtain
\[ \left( \epsilon_{(\mu\nu)} b^\mu_\perp \alpha^\nu_\perp + \epsilon_{\mu} b^\mu_\perp \right) |0; k\rangle \]
\[ \rightarrow \left( b^P_\perp \frac{\alpha^L_\perp}{2} + b^L_\perp \frac{\alpha^P_\perp}{2} - M b^L_\perp \right) |0; k\rangle = \left( 2 b^L_\perp \frac{\alpha^L_\perp}{2} - M b^L_\perp \right) |0; k\rangle \rightarrow \frac{2}{M^2} + 1. \] (3.75)

So the ratios of the hard SSA in the NS sector at this mass level are
\[ \left\langle b^T_\perp, \alpha^T_\perp, b^T_\perp \right\rangle : \left\langle b^T_\perp, \left( 2 b^L_\perp \frac{\alpha^L_\perp}{2} - M b^L_\perp \right) \right\rangle : \left\langle b^T_\perp, \frac{\alpha^T_\perp}{2}, \frac{b^T_\perp}{2} \right\rangle : \left\langle b^T_\perp, \frac{b^L_\perp}{2}, \frac{b^L_\perp}{2} \right\rangle \]
\[ = -2 k_3^T E B : -2 (\frac{2}{M^2} + 1) k_3^T E B : \delta_{ij} 2 k_3^T E B : \delta_{lk} \frac{-2 k_3^T E B}{M} \] (3.76)
\[ = 1 : 2 : -\delta_{ij} \frac{\delta_{lk}}{\sqrt{2}} \quad (i, j, k, l = 3, 4, 5, \ldots, 9) \] (3.77)

In sum, in the NS sector one gets
\[ 1 + 1 + 7 + 7 = 16 \] (3.78)

HSSA in Eq. (3.76). This result agrees with those of hard massive PFSSA in the R sector calculated recently [16].

IV. DISCUSSION

One main result of this work is the derivation of the superstring version of Eq. (2.20), namely, the ratios in Eq. (3.67) to Eq. (3.69). The important motivation to do this derivation is of course due to the recent calculation of the hard PFSSA in [16].

Both the effects of ”worldsheet fermion exchange” [12] and NO ”inter-particle gauge transformation” of the NS sector shown in this paper are related to the existence of R sector for the case of superstring. This means that it is important to include R sector [16] when considering high energy symmetry of superstring theory. Unfortunately, the construction of the general massive fermion string vertex operators is difficult at this stage. However, the results of this paper seem to indicate that spacetime SUSY may help to provide useful information.
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