Robust Imitation Learning from Corrupted Demonstrations

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Abstract

We consider offline Imitation Learning from corrupted demonstrations where a constant fraction of data can be noise or even arbitrary outliers. Classical approaches such as Behavior Cloning assumes that demonstrations are collected by an presumably optimal expert, hence may fail drastically when learning from corrupted demonstrations. We propose a novel robust algorithm by minimizing a Median-of-Means (MOM) objective which guarantees the accurate estimation of policy, even in the presence of constant fraction of outliers. Our theoretical analysis shows that our robust method in the corrupted setting enjoys nearly the same error scaling and sample complexity guarantees as the classical Behavior Cloning in the expert demonstration setting. Our experiments on continuous-control benchmarks validate that our method exhibits the predicted robustness and effectiveness, and achieves competitive results compared to existing imitation learning methods.

1 Introduction

Recent years have witnessed the success of using autonomous agent to learn and adapt to complex tasks and environments in a range of applications such as playing games [e.g. 43, 61, 70], autonomous driving [e.g. 4, 28], robotics [19], medical treatment [e.g. 76] and recommendation system and advertisement [e.g. 37, 67].

Previous success for sequential decision making often requires two key components: (1) a careful design reward function that can provide the supervision signal during learning and (2) an unlimited number of online interactions with the real-world environment (or a carefully designed simulator) to query new unseen region. However, in many scenarios, both components are not allowed. For example, it is hard to define the reward signal in uncountable many extreme situations in autonomous driving [30]; and it is dangerous and risky to directly deploy a learning policy on human to gather information in autonomous medical treatment [76]. Therefore an offline sequential decision making algorithm without reward signal is in demand.

Imitation Learning (IL) [48] offers an elegant way to train intelligent agents for complex task without the knowledge of reward functions. In order to guide intelligent agents to correct behaviors, it is crucial to have high quality expert demonstrations. The well-known imitation learning algorithms such as Behavior Cloning (BC, [49]) or Generative Adversarial Imitation Learning (GAIL, [21]) require that the demonstrations given for training are all presumably optimal and it aims to learn the optimal policy from expert demonstration data set. More specifically, BC only uses offline demonstration data without any interaction with the environment, whereas GAIL requires online interactions.
Figure 1: Reward vs. percentage of corruptions in Hopper environment from the PyBullet\cite{12} with corrupted demonstrations. We fix the sample size for the demonstration data set, and vary the fraction of corruptions $\epsilon$ up to 20%. Shaded region represents one standard deviation for 20 trials. Our algorithm Robust Behavior Cloning (RBC) on corrupted demonstrations has nearly the same performance as BC on expert demonstrations (this is the case when $\epsilon = 0$), which achieves expert level. And it barely changes when $\epsilon$ grows larger to 20%. By contrast, the performance of vanilla BC on corrupted demos fails drastically. The detailed experimental setup and comparisons with existing methods are included in Figure 2 and Figure 3.

However in real world scenario, since the demonstration is often collected from human, we cannot guarantee that all the demonstrations we collected have high quality. This has been addressed in a line of research \cite{6, 59, 65, 66, 71}. An human expert can make mistakes by accident or due to the hardness of a complicated scenario (e.g., medical diagnosis). Furthermore, even an expert demonstrates a successful behavior, the recorder or the recording system can have a chance to contaminate the data by accident or on purpose \cite{15, 46, 78}.

This leads to the central question of the paper:

Can the optimality assumption on expert demonstrations be weakened or even tolerate arbitrary outliers under offline imitation learning settings?

More concretely, we consider corrupted demonstrations setting where the majority of the demonstration data is collected by an expert policy (presumably optimal), and the remaining data can be even arbitrary outliers (the formal definition is presented in Definition 2.1).

Such definitions allowing arbitrary outliers for the corrupted samples have rich history in robust statistics \cite{23, 24}, yet have not been widely used in imitation learning. This has great significance in many applications, such as automated medical diagnosis for healthcare (\cite{76}) and autonomous driving \cite{42}, where the historical data (demonstration) is often complicated and noisy which requires robustness consideration.

However, the classical offline imitation learning approaches such as Behavior Cloning (BC) fails drastically under this corrupted demonstration settings. We illustrated this phenomenon in Figure 1. We use BC on Hopper environment (a continuous control environment from PyBullet \cite{12}), and the performance of the policy learned by BC drops drastically as the fraction of corruptions increases in the offline demonstration data set.

In this paper, we propose a novel robust imitation learning algorithm – Robust Behavior Cloning (RBC, Algorithm 1), which is resilient to corruptions in the offline demonstrations. Particularly, our RBC does not require potentially costly or risky interaction with the real world environment or any human annotations. In Figure 1, Our RBC on corrupted demonstrations has nearly the same performance as BC on expert demonstrations (this is the case when $\epsilon = 0$), which achieves expert...
level. And it barely changes when $\epsilon$ grows larger to 20%. The detailed experimental setup and comparisons with existing methods (e.g., [59]) are included in Section 5.

1.1 Main Contributions

• (Algorithm) We consider robustness in offline imitation learning where we have corrupted demonstrations. Our definition for corrupted demonstrations significantly weakens the presumably optimal assumption on demonstration data, and can tolerate a constant $\epsilon$-fraction of state-action pairs to be arbitrarily corrupted. We refer to Definition 2.1 for a more precise statement.

To deal with this issue, we propose a novel algorithm Robust Behavior Cloning (Algorithm 1) for robust imitation learning. Our algorithm works in the offline setting, without any further interaction with the environment or any human annotations. The core ingredient of our robust algorithm is using a novel median of means objective in policy estimation compared to classical Behavior Cloning. Hence, it’s simple to implement, and computationally efficient.

• (Theoretical guarantees) We analyze our Robust Behavior Cloning algorithm when there exists a constant fraction of outliers in the demonstrations under the offline setting. To the best of our knowledge, we provide the first theoretical guarantee robust to constant fraction of arbitrary outliers in offline imitation learning. We show that our RBC achieves nearly the same error scaling and sample complexity compared to vanilla BC with expert demonstrations. To this end, our algorithm guarantees robustness to corrupted demonstrations at no cost of statistical estimation error. This is the content of Section 4.

• (Empirical support) We validate the predicted robustness and show the effectiveness of our algorithm on a number of different high-dimensional continuous control benchmarks. The vanilla BC is fragile indeed with corrupted demonstrations, yet our Robust Behavior Cloning is computationally efficient, and achieves nearly the same performance compared to vanilla BC with expert demonstrations. Section 5 also shows that our algorithm achieves competitive results compared to existing imitation learning methods.

Notation. Throughout this paper, we use $\{c_i\}_{i=1,2,3}$ to denote the universal positive constant. We utilize the big-$O$ notation $f(n) = O(g(n))$ to denote that there exists a positive constant $c_1$ and a natural number $n_0$ such that, for all $n \geq n_0$, we have $f(n) \leq c_1 g(n)$.

Outline. The rest of this paper is organized as follows. In Section 2, we formally define the setup and the corrupted demonstrations. In Section 3, we introduce our RBC and the computationally efficient algorithm (Algorithm 1). We provide the theoretical analysis in Section 4, and experimental results in Section 5. We leave the detailed discussion and related works in Section 6. All proofs and experimental details are collected in the Appendix.

2 Problem Setup

2.1 Reinforcement Learning and Imitation Learning

Markov Decision Process and Reinforcement Learning. We start the problem setup by introducing the Markov decision process (MDP). An MDP $M = \langle S, A, r, P, \mu_0, \gamma \rangle$ consists of a state space $S$, an action space $A$, an unknown reward function $r : S \times A \rightarrow [0, R_{\text{max}}]$, an unknown transition kernel $P : \Delta(S) \rightarrow \Delta(S)$, an initial state distribution $\mu_0 \in \Delta(S)$, and a discounted factor $\gamma \in (0, 1)$. We use $\Delta$ to denote the probability distributions on the simplex.
An agent acts in a MDP following a policy $\pi(\cdot|s)$, which prescribes a distribution over the action space $A$ given each state $s \in S$. Running the policy starting from the initial distribution $s_1 \sim \mu_0$ yields a stochastic trajectory $T := \{(s_i, a_i, r_i)\}_{i=1}^{\infty}$, where $s_i, a_i, r_i$ represent the state, action, reward at time $t$ respectively, with $a_i \sim \pi(\cdot|s_i)$ and the next state $s_{i+1}$ follows the unknown transition kernel $s_{i+1} \sim P(\cdot|s_i, a_i)$. We denote $\rho_{\pi, t} \in \Delta(S \times A)$ as the marginal joint stationary distribution for state, action at time step $t$, and we define $\rho_{\pi} = (1 - \gamma) \sum_{i=1}^{\infty} \gamma^t \rho_{\pi, t}$ as visitation distribution for policy $\pi$. For simplicity, we reuse the notation $\rho_{\pi}(s) = \int_{a \in A} \rho_{\pi}(s, a) da$ to denote the marginal distribution over state.

The goal of reinforcement learning (RL) is to find the best policy $\pi$ to maximize the expected cumulative return $J_\pi = \mathbb{E}_{T \sim \pi}[\sum_{t=1}^{\infty} \gamma^t r_t]$. Common RL algorithms (e.g., please refer to [62]) requires online interaction and exploration with the environments. However, this is prohibited in the offline setting.

**Imitation Learning.** Imitation learning (IL) aims to obtain a policy to mimic expert’s behavior with demonstration data set $D = \{(s_i, a_i)\}_{i=1}^{N}$ where $N$ is the sample size of $D$. Note that we do not need any reward signal. Tradition imitation learning assumes perfect (or near-optimal) expert demonstration – for simplification we assume that each state-action pair $(s_i, a_i)$ is drawn from the joint stationary distribution of an expert policy $\pi_E$:

$$(s_i, a_i) \sim \rho_{\pi_E} \tag{1}$$

Learning from demonstrations with or without online interactions has a long history (e.g., [21, 49]). The goal of offline IL is to learn a policy $\hat{\pi}^{\text{IL}} = \mathbb{A}(D)$ through an IL algorithm $\mathbb{A}$, given the demonstration data set $D$, without further interaction with the unknown true transition dynamic $P$.

**Behavior Cloning.** The Behavior Cloning (BC) is the well known algorithm [49] for IL which only uses offline demonstration data without any interaction with the environment. More specifically, BC solves the following Maximum Likelihood Estimation (MLE) problem, which minimizes the average Negative Log-Likelihood (NLL) for all samples in offline demonstrations $D$:

$$\hat{\pi}^{\text{BC}} = \arg \min_{\pi \in \Pi} \frac{1}{N} \sum_{(s, a) \in D} -\log(\pi(a|s)). \tag{2}$$

Recent works [1, 52, 73, 74] have shown that BC is optimal under the offline setting, and can only be improved with the knowledge of transition dynamic $P$ in the worst case. Also, another line of research considers improving BC with further online interaction of the environment [5] or actively querying an expert [55, 56].

### 2.2 Learning from corrupted demonstrations

However, it is sometimes unrealistic to assume that the demonstration data set is collected through a presumably optimal expert policy. In this paper, we propose **Definition 2.1** for the corrupted demonstrations, which tolerates gross corruption or model mismatch in offline data set.

**Definition 2.1 (Corrupted Demonstrations).** Let the state-action pair $(s_i, a_i)_{i=1}^{N}$ drawn from the joint stationary distribution of a presumably optimal expert policy $\pi_E$. The corrupted demonstration data $D$ are generated by the following process: an adversary can choose an arbitrary $\epsilon$-fraction ($\epsilon < 0.5$) of the samples in $[N]$ and modifies them with arbitrary values. We note that $\epsilon$ is a constant independent of the dimensions of the problem. After the corruption, we use $\mathcal{D}$ to denote the corrupted demonstration data set.
This corruption process can represent gross corruptions or model mismatch in the demonstration data set. To the best of our knowledge, Definition 2.1 is the first definition for corrupted demonstrations in imitation learning which tolerates arbitrary corruptions.

In the supervised learning, the well-known Huber’s contamination model ([23, 24]) considers $(x, y) \sim (1 - \epsilon)P + \epsilon Z$, where $x \in \mathbb{R}^d$ is the explanatory variable (feature) and $y \in \mathbb{R}$ is the response variable. Here, $P$ denotes the authentic statistical distribution such as Normal mean estimation or linear regression model, and $Z$ denotes the outliers.

Dealing with corrupted $x$ and $y$ in high dimensions has a long history in the robust statistics community [e.g. 10, 11, 57, 75]. However, it’s only until recently that robust statistical methods can handle constant $\epsilon$-fraction (independent of dimensionality $\mathbb{R}^d$) of outliers in $x$ and $y$ [14, 25, 31, 35, 38, 39, 41, 50, 60]. We note that in Imitation Learning, the data collecting process for the demonstrations does not obey i.i.d. assumption in traditional supervised learning due to the temporal dependency.

3 Our Algorithm

Equation (2) directly minimizes the empirical mean of Negative Log-Likelihood, and it is widely known that the mean operator is fragile to corruptions [23, 24]. Indeed, our experiment in Figure 1 demonstrates that in the presence of outliers, vanilla BC fails drastically. Hence, we consider using a robust estimator to replace the empirical average of NLL in eq. (2) – we first introduce the classical Median-of-Means (MOM) estimator for the mean estimation, and then adapt it to dealing with loss functions in robust imitation learning problems.

The vanilla MOM estimator for one-dimensional mean estimation works like following: (1) randomly partition $N$ samples into $M$ batches; (2) calculates the mean for each batch; (3) outputs the median of these batch mean.

The MOM mean estimator achieves sub-Gaussian concentration bound for one-dimensional mean estimation even though the underlying distribution only has second moment bound (heavy tailed distribution) (interested readers are referred to textbooks such as [3, 26, 47]). Very recently, MOM estimators are used for high dimensional robust regression [7, 22, 25, 35, 41] by applying MOM estimator on the loss function of empirical risk minimization process.

3.1 Robust Behavior Cloning

Inspired by the MOM estimator, a natural robust version of eq. (2) can randomly partition $N$ samples into $M$ batches with the batch size $b$, and calculate

$$\hat{\pi} = \arg\min_{\pi \in \Pi} \text{median}_{1 \leq j \leq M} (\ell_j(\pi)),$$

where the loss function $\ell_j(\pi)$ is the average Negative Log-Likelihood in the batch $B_j, j \in [M]$:

$$\ell_j(\pi) = \frac{1}{b} \sum_{(s,a) \in B_j} -\log(\pi(a|s)).$$

Our idea eq. (3) minimizes the MOM of NLL, which extends the MOM mean estimator to the loss function for robust imitation learning. Although eq. (3) can also achieve robust empirically result, we propose Definition 3.1 for theoretical convenience, which optimizes the min-max version (MOM tournament [25, 34, 35, 41]) to handle arbitrary outliers in demonstration data set $(s, a) \in D$. 

5
Definition 3.1 (Robust Behavior Cloning). We split the corrupted demonstrations $D$ into $M$ batches randomly $\{B_j\}_{j=1}^M$, with the batch size $b \leq \frac{1}{M}$. The Robust Behavior Cloning solves the following optimization

$$
\hat{\pi}^{RBC} = \arg \min_{\pi \in \Pi} \max_{\pi' \in \Pi} \text{median}_{1 \leq j \leq M} (\ell_j(\pi) - \ell_j(\pi')).
$$

The workhorse of Definition 3.1 is eq. (5), which uses a novel variant of MOM tournament procedure for imitation learning problems.

In eq. (4), we calculate the average Negative Log-Likelihood (NLL) for a single batch of state-action pair $(s, a)$, and $\hat{\pi}^{RBC}$ is the solution of a min-max formulation based on the batch loss $\ell_j(\pi)$ and $\ell_j(\pi')$. Though our algorithm minimizes the robust version of NLL, we do not utilize the traditional iid assumption in the supervised learning.

The key results in our theoretical analysis show that the min-max solution to the median batch of the loss function is robust to a constant fraction of arbitrary outliers in the demonstrations. The intuition behind solving this min-max formulation is that the inner variable $\pi'$ needs to get close to $\pi_E$ to maximize the term median $\text{median}_{1 \leq j \leq M} (\ell_j(\pi) - \ell_j(\pi'))$; and the outer variable $\pi$ also need to get close to $\pi_E$ to minimize the term. In Section 4, we show that under corrupted demonstrations, $\hat{\pi}^{RBC}$ will be close to $\pi_E$. In particular, $\hat{\pi}^{RBC}$ in Definition 3.1 has the same error scaling and sample complexity compared to $\hat{\pi}^{BC}$ in the expert demonstrations setting.

Algorithm design. In Section 4, we provide rigorous statistical guarantees for Definition 3.1. However, the objective function eq. (5) is not convex (in general), hence we use Algorithm 1 as a computational heuristic to solve it.

In each iteration of Algorithm 1, we randomly partition the demonstration data set $D$ into $M$ batches, and calculate the loss $\ell_j(\pi) - \ell_j(\pi')$ by eq. (4). We then pick the batch $B_{\text{Med}}$ with the median loss, and evaluate the gradient on that batch. We use gradient descent on $\pi$ for the arg min part and gradient ascent on $\pi'$ for the arg max part.

In Section 5, we empirically show that this gradient-based heuristic Algorithm 1 is able to minimize this objective and has good convergence properties. As for the time complexity, when using back-propagation on one batch of samples, our RBC incurs overhead costs compared to vanilla BC, in order to evaluate the loss function for all samples via forward propagation. Empirical studies in Appendix B show that the time complexity of RBC is comparable to vanilla BC.

4 Theoretical Analysis

In this section, we provide theoretical guarantees for our RBC algorithm. Since our method (Definition 3.1) directly estimates the conditional probability $\pi(a|s)$ over the offline demonstrations, our theoretical analysis provides guarantees on $\mathbb{E}_{s \sim \rho_{\pi_E}} \left\| \hat{\pi}^{RBC}(\cdot|s) - \pi_E(\cdot|s) \right\|_{TV}^2$, which upper bounds the total variation norm compared to $\pi_E$ under the expectation of $s \sim \rho_{\pi_E}$. The ultimate goal of the learned policy is to maximize the expected cumulative return, thus we then provide an upper bound for the sub-optimality $J_{\pi_E} - J_{\hat{\pi}^{RBC}}$.

We begin the theoretical analysis by Assumption 4.1, which simplifies our analysis and is common in literature [1, 2]. By assuming that the policy class $\Pi$ is discrete, our upper bounds depend on the quantity $\log(|\Pi|)/N$, which matches the error rates and sample complexity for using BC with expert demonstrations [1, 2].

Assumption 4.1. We assume that the policy class $\Pi$ is discrete, and realizable, i.e., $\pi_E \in \Pi$.

1Without loss of generality, we assume that $M$ exactly divides the sample size $N$, and $b = \frac{N}{M}$ is the batch size.
Algorithm 1 Robust Behavior Cloning.

1: **Input:** Corrupted demonstrations $D$
2: **Output:** Robust policy $\hat{\pi}_{RBC}$

3: Randomly initialize $\pi$ and $\pi'$ respectively.
4: for $t = 0$ to $T - 1$, do
5: Randomly partition $D$ to $M$ batches with the batch size $b \leq \frac{1}{3\epsilon}$.
6: For each batch $j \in [M]$, calculate the loss $\ell_j(\pi) - \ell_j(\pi')$ by eq. (4).
7: Pick the batch with median loss within $M$ batches
8: end for
9: **Return:** Robust policy $\hat{\pi}_{RBC} = \pi$.

4.1 The upper bound for the policy distance

We first present Theorem 4.1, which shows that minimizing the MOM objective via eq. (5) guarantees the closeness of robust policy to optimal policy in total variation distance.

**Theorem 4.1.** Suppose we have corrupted demonstration data set $D$ with sample size $N$ from Definition 2.1, and there exists a constant corruption ratio $\epsilon < 0.5$. Under Assumption 4.1, let $\tau$ to be the output objective value with $\hat{\pi}_{RBC}$ in the optimization eq. (5) with the batch size $b \leq \frac{1}{3\epsilon}$, then with probability at least $1 - c_1\delta$, we have

$$
E_{s \sim \rho_{\pi}} \|\hat{\pi}_{RBC} - \pi\|_{TV}^2 = O \left( \frac{\log(|\Pi|/\delta)}{N} + \tau \right).
$$

The proof is collected in Appendix A. We note that the data collection process does not follow the iid assumption, hence we use martingale analysis similar to [1, 2]. The first part of eq. (6) is the statistical error $\frac{\log(|\Pi|/\delta)}{N}$, which matches the error rates of vanilla BC for expert demonstrations [1, 2]. The second part is the final objective value in the optimization eq. (5) $\tau$ which includes two parts – the first part scales with $O(1/\epsilon)$, which is equivalent to the fraction of corruption $O(\epsilon)$. The second part is the sub-optimality gap due to the solving the non-convex optimization. Our main theorem – Theorem 4.1 – guarantees that a small value of the final objective implies an accurate estimation of policy and hence we can certify estimation quality using the obtained final value of the objective.

4.2 The upper bound for the sub-optimality

Next, we present Theorem 4.2, which guarantees the reward performance of the learned robust policy $\hat{\pi}_{RBC}$.

**Theorem 4.2.** Under the same setting as Theorem 4.1, we have

$$
J_{\pi_E} - J_{\hat{\pi}_{RBC}} \leq O \left( \frac{1}{(1 - \gamma)^2} \sqrt{\frac{\log(|\Pi|/\delta)}{N} + \tau} \right),
$$

with probability at least $1 - c_1\delta$. 


The proof is collected in Appendix A. We note that the error scaling and sample complexity of the statistical error log(|I|/δ)/N in Theorem 4.2 match the vanilla BC with expert demonstrations [1, 2].

Remark 4.1. The quadratic dependency on the effective horizon (\(\frac{1}{(1-\gamma)^2}\) in the discounted setting or \(H^2\) in the episodic setting) is widely known as the compounding error or distribution shift in literature, which is due to the essential limitation of offline imitation learning setting. Recent work [52, 73] shows that this quadratic dependency cannot be improved without any further interaction with the environment or the knowledge of transition dynamic \(P\). Hence BC is actually optimal under no-interaction setting. Also, a line of research considers improving BC by further online interaction with the environment or even active query of the experts [5, 55, 56]. Since our work, as a robust counterpart of BC, focuses on the robustness to the corruptions in the offline demonstrations setting, it can be naturally used in the online setting such as DAGGER [56] and [5].

5 Experiments

In this section, we study the empirical performance of our Robust Behavior Cloning. We evaluate the robustness of Robust Behavior Cloning on several continuous control benchmarks simulated by PyBullet [12] simulator: HopperBulletEnv-v0, Walker2DBulletEnv-v0, HalfCheetahBulletEnv-v0 and AntBulletEnv-v0. Actually, these tasks have true reward function already in the simulator. We will use only state observation and action for the imitation algorithm, and we then use the reward to evaluate the obtained policy when running in the simulator.

5.1 Experimental setup

For each task, we collect the presumably optimal expert trajectories using pre-trained agents from Standard Baselines\(^2\). In the experiment, we use Soft Actor-Critic [20] in the Standard Baselines3 pre-trained agents, and we consider it to be an expert. We provide the hyperparameters setting in Appendix B.

For the continuous control environments, the action space are bounded between -1 and 1. We note that Definition 2.1 allows for arbitrary corruptions, and we choose these outliers’ action such that it has the maximum effect, and cannot be easily detected. We generate corrupted demonstration data set \(D\) as follows: we first randomly choose ϵ fraction of samples, and corrupt the actions. Then, for the option (1), we set the actions of outliers to the boundary (−1 or +1). For the option (2), the actions of outliers are drawn from a uniform distribution between −1 and +1.

We compare our RBC algorithm (Algorithm 1) to a number of natural baselines: the first baseline is directly using BC on the corrupted demonstration \(D\) without any robustness consideration. The second one is using BC on the expert demonstrations (which is equivalent to ϵ = 0 in our corrupted demonstrations) with the same sample size.

We also investigate the empirical performance of the baseline which achieves the state-of-the-art performance: Behavior Cloning from Noisy Observation (Noisy BC). Noisy BC is a recent offline imitation learning algorithm proposed by [59], which achieves superior performance compared to [5, 6, 71]. Similar to our RBC, Noisy BC does not require any environment interactions during training or any screening/annotation processes to discard the non-optimal behaviors in the demonstration data set.

The Noisy BC works in an iterative fashion: in each iteration, it reuses the old policy iterate

\(^2\)The pre-trained agents were cloned from the following repositories: https://github.com/DLR-RM/stable-baselines3, https://github.com/DLR-RM/rl-baselines3-zoo.
\( \pi^{\text{old}} \) to re-weight the state-action samples via the weighted Negative Log-Likelihood

\[
\pi^{\text{new}} = \arg \min_{\pi \in \Pi} \frac{1}{N} \sum_{(s,a) \in D} - \log(\pi(a|s)) \ast \pi^{\text{old}}(a|s).
\]

Intuitively, if the likelihood \( \pi^{\text{old}}(a|s) \) is small in previous iteration, the weight for the state-action sample \((s,a)\) will be small in the current iteration. Noisy BC outputs \( \hat{\pi} \) after multiple iterations.

### 5.2 Convergence of our algorithm

We first illustrate the convergence and the performance of our algorithm by tracking the metric of different algorithms vs. epoch number in the whole training process. More specifically, we evaluate current policy in the simulator for 20 trials, and obtain the mean and standard deviation of cumulative reward for every 5 epochs. This metric corresponds to theoretical bounds in Theorem 4.2.

We focus on four continuous control environments, where the observation space has dimensions around 30, and the action space has boundary \([-1, 1]\). In this experiment, we adopt option (1), which set the actions of outliers to the boundary \((-1 \text{ or } +1)\). We fix the corruption ratio as 10% and 20%, and present the Reward vs. Epochs. Due to the space limitation, we leave the experiments for all the environments to Figure 4 in Appendix B.

As illustrated in Figure 4, Vanilla BC on corrupted demonstrations fails to converge to expert policy. Using our robust counterpart Algorithm 1 on corrupted demonstrations has good convergence properties. Surprisingly, our RBC on corrupted demonstrations has nearly the same reward performance vs. epochs of directly using BC on expert demonstrations.

**Computational consideration.** Another important aspect of our algorithm is the computational efficiency. To directly compare the time complexity, we report the reward vs. wall clock time performance of our RBC and “Oracle BC”, which optimizes on the expert demonstrations. The experiments are conducted on 1/2 core of NVIDIA T4 GPU, and we leave the results to Table 2 in Appendix B due to space limitations. When using back-propagation on batches of samples, our RBC incurs overhead costs compared to vanilla BC, in order to evaluate the loss function for all samples via forward propagation. Table 2 shows that the actual running time time of RBC is comparable to vanilla BC.

### 5.3 Experiments under different setups

In this subsection, we compare the performance of our algorithm and existing methods under different setups.

**Different fraction of corruption.** We fix the sample size as 60,000, and vary the corruption fraction \(\epsilon\) from 0% to 20%. Figure 1 has shown that our RBC is resilient to outliers in the demonstrations ranging from 0 to 20% for the Hopper environment. In Figure 2, the full experiments validate our theory that our Robust Behavior Cloning nearly matches the expert performance for different environments and corruption ratio. By contrast, the performance of vanilla BC on corrupted demos fails drastically.

**Different sample size in \(D\).** In this experiment, we fix the fraction of the corruption \(\epsilon = 15\%\), set the actions of outliers to the boundary \((-1 \text{ or } +1)\), and vary the sample size of the demonstration data set. It is expected in Theorem 4.2 that larger sample size of \(D\) leads to smaller suboptimality gap in value function. Figure 3 validates our theory: Our RBC on corrupted demonstrations has nearly the same reward as Oracle BC (BC on expert demonstrations), and the sub-optimality gap gets smaller as sample size \(N\) grows larger. However, directly using BC on corrupted demonstrations cannot improve as sample size grows larger.

The Oracle BC on expert demonstrations is a strong baseline which achieves reward performance at expert level with very few transition samples in demonstrations. This suggests that compounding
error or distribution shift may be less of a problem in these environments. This is consistent with
the findings in [5].

In Figure 2 and Figure 3, our algorithm achieves superior results compared to the state-of-the-art
robust imitation learning method [59] under different setups. The key difference between our method
and the re-weighting idea [59] is that we can guarantee removing the outliers in the objective
function eq. (5), whereas the outliers may mislead the re-weighting process during the iterations. If
the outliers have large weight in the previous iteration, the re-weighting process will exacerbate
the outliers’ impacts. If an authentic and informative sample has large training loss, it will be
down-weighted incorrectly, hence losing sample efficiency. As in Figure 3, our RBC benefits from
the tight theoretical bound Theorem 4.2, and achieves superior performance even the sample size
(the number of trajectories) is small.

6 Related Work

Imitation Learning. Behavior Cloning (BC) is the most widely-used imitation learning algorithm
[48, 49] due to its simplicity, effectiveness and scalability, and has been widely used in practice.
From a theoretical viewpoint, it has been showed that BC achieves informational optimality in
the offline setting [52] where we do not have further online interactions or the knowledge of the
transition dynamic $P$. With online interaction, there’s a line of research focusing on improving BC
in different scenarios – for example, [56] proposed DAgger (Data Aggregation) by querying the
expert policy in the online setting. [5] proposed using an ensemble of BC as uncertainty measure
and interacts with the environment to improve BC by taking the uncertainty into account, without
the need to query the expert. Very recently, [51, 73, 74] leveraged the knowledge of the transition
dynamic $P$ to eliminate compounding error/distribution shift issue in BC.

Besides BC, there are other imitation learning algorithms: [21] used generative adversarial
networks for distribution matching to learn a reward function; [54] provided a RL framework to deal
with IL by artificially setting the reward; [18] unified several existing imitation learning algorithm
as minimizing distribution divergence between learned policy and expert demonstration, just to
name a few.

Offline RL. RL leverages the signal from reward function to train the policy. Different from
IL, offline RL often does not require the demonstration to be expert demonstration [e.g. 16, 17, 33]
(interested readers are referred to [36]), and even expects the offline data with higher coverage for
different sub-optimal policies [9, 27, 53]. Behavior-agnostic setting [44, 45] even does not require
the collected data from a single policy.

The closest relation between offline RL and IL is the learning of stationary visitation distribution,
where learning such visitation distribution does not involve with reward signal, similar to IL. A
line of recent research especially for off-policy evaluation tries to learn the stationary visitation
distribution of a given target policy [e.g. 13, 40, 44, 45, 64]. Especially [32] leverages the off-policy
evaluation idea to IL area.

Robustness in IL and RL. There are several recent papers consider corruption-robust in
either RL or IL. In RL, [79] considers that the adversarial corruption may corrupt the whole episode
in the online RL setting while a more recent one [78] considers offline RL where $\epsilon$-fraction of
the whole data set can be replaced by the outliers. However, the $\epsilon$ dependency scales with the
dimension in [78], yet $\epsilon$ can be a constant in this paper for robust offline IL. Many other papers
consider perturbations, heavy tails, or corruptions in either reward function [8] or in transition
dynamic [58, 63, 72].

The most related papers follow a similar setting of robust IL are [6, 59, 65, 66, 71], where
they consider imperfect or noisy observations in imitation learning. However, they do not provide
theoretical guarantees to handle arbitrary outliers in the demonstrations. And to the best of our
knowledge, we provide the first theoretical guarantee robust to constant fraction of arbitrary outliers
in offline imitation learning. Furthermore, [65, 66, 71] require additional *online interactions* with the environment, and [6, 71] require annotations for each demonstration, which costs a significant human effort. Our algorithm achieves robustness guarantee from purely *offline* demonstration, without the potentially costly or risky interaction with the real world environment or human annotations.

### 6.1 Summary and Future Works

In this paper, we considered the corrupted demonstrations issues in imitation learning, and proposed a novel robust algorithm, Robust Behavior Cloning, to deal with the corruptions in offline demonstration data set. The core technique is replacing the vanilla Maximum Likelihood Estimation with a Median-of-Means (MOM) objective which guarantees the policy estimation and reward performance in the presence of constant fraction of outliers. Our algorithm has strong robustness guarantees and has competitive practical performance compared to existing methods.

There are several avenues for future work: since our work focuses on the corruption in offline data, any improvement in *online IL* which utilizes BC would benefit from the corruption-robustness guarantees and practical effectiveness by our *offline* RBC. Also, it would also be of interest to apply our algorithm for real-world environment, such as automated medical diagnosis and autonomous driving.
Figure 2: Reward vs. Percentage of Corruption with demonstration data of size 60,000. We vary the corruption ration $\epsilon$ from 0% to 20%. In (a), the outliers are set randomly at the boundary -1 or 1. In (b), the outliers are randomly drawn from a uniform distribution between $[-1, 1]$. For different algorithms, we report their reward performance after training with 200 epochs. The shaded region represents the standard deviation for 20 trials. Vanilla BC (purple line) on corrupted demonstrations fails drastically. Surprisingly, our RBC (red line) on corrupted demonstrations has nearly the same performance as the expert (grey dashed line), and achieves competitive results compared to the state-of-the-art robust imitation learning method (blue line) [59].
Figure 3: Reward vs. Sample Size with fixed $\epsilon = 0.15$ in the demonstrations. We set the outliers randomly at the boundary -1 or 1, and vary the sample size from 5,000 to 80,000. For different algorithms, we report reward performance after training with 200 epochs. The shaded region represents the standard deviation for 20 trials. Vanilla BC (purple line) on corrupted demonstrations fails drastically, and cannot converge to expert level with large sample size. Oracle BC (green line) denotes BC on expert demonstrations with the same sample size. Our RBC (red line) achieves nearly the same reward performance as Oracle BC even the sample size of the demonstration is small, and achieves superior results compared to the state-of-the-art robust imitation learning method (blue line) [59].
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A Proofs

The analysis of maximum likelihood estimation is standard in i.i.d. setting for the supervised learning setting [69]. In our proofs of the robust offline imitation learning algorithm, the analysis for the sequential decision making leverages the martingale analysis technique from [2, 77].

Our Robust Behavior Cloning (Definition 3.1) solves the following optimization

$$\hat{\pi}^{RBC} = \arg \min_{\pi \in \Pi} \max_{\pi' \in \Pi} \text{median}_{1 \leq j \leq M} (\ell_j(\pi) - \ell_j(\pi')) ,$$  

(8)

where the loss function $\ell_j(\pi)$ is the average Negative Log-Likelihood in the batch $B_j$:

$$\ell_j(\pi) = \frac{1}{b} \sum_{(s,a) \in B_j} -\log(\pi(a|s)).$$  

(9)

This can be understood as a robust counterpart for the maximum likelihood estimation in sequential decision process.

With a slight abuse of notation, we use $x_i$ and $y_i$ to denote the observation and action, and the underlying unknown expert distribution is $y_i \sim p(\cdot|x_i)$ and $p(y|x) = f^*(x,y)$. Following Assumption 4.1, we have the realizable $f^* \in \mathcal{F}$, and the discrete function class satisfies $|\mathcal{F}| < \infty$.

Let $\mathcal{D}$ denote the data set and let $\mathcal{D}'$ denote a tangent sequence $\{x'_i, y'_i\}_{i=1}^{\lvert \mathcal{D} \rvert}$. The tangent sequence is defined as $x'_i \sim \mathcal{D}_i(x_{1:i-1}, y_{1:i-1})$ and $y'_i \sim p(\cdot|x'_i)$. Note here that $x'_i$ follows from the distribution $\mathcal{D}_i$, and depends on the original sequence, hence the tangent sequence is independent conditional on $\mathcal{D}$.

For this martingale process, we first introduce a decoupling Lemma from [2].

Lemma A.1. [Lemma 24 in [2]] Let $\mathcal{D}$ be a dataset, and let $\mathcal{D}'$ be a tangent sequence. Let $\Gamma(f, \mathcal{D}) = \sum_{(x,y) \in \mathcal{D}} \phi(f, (x,y))$ be any function which can be decomposed additively across samples in $\mathcal{D}$. Here, $\phi$ is any function of $f$ and sample $(x,y)$. Let $\hat{f} = \hat{f}(\mathcal{D})$ be any estimator taking the dataset $\mathcal{D}$ as input and with range $\mathcal{F}$. Then we have

$$\mathbb{E}_\mathcal{D} \left[ \exp \left( \Gamma(\hat{f}, \mathcal{D}) - \log \mathbb{E}_{\mathcal{D}'} \exp(\Gamma(\hat{f}, \mathcal{D}')) - \log |\mathcal{F}| \right) \right] \leq 1.$$

Then we present a Lemma which upper bounds the TV distance via a loss function closely related to KL divergence. Such bounds for probabilistic distributions are discussed extensively in literature such as [68].

Lemma A.2. [Lemma 25 in [2]] For any two conditional probability densities $f_1, f_2$ and any state distribution $\mathcal{D} \in \Delta(\mathcal{X})$ we have

$$\mathbb{E}_{x \sim \mathcal{D}} \|f_1(x, \cdot) - f_2(x, \cdot)\|_{TV}^2 \leq -2 \log \mathbb{E}_{x \sim \mathcal{D}, y \sim f_2(\cdot|x)} \exp \left( -\frac{1}{2} \log \frac{f_2(x,y)}{f_1(x,y)} \right).$$
A.1 Proof of Theorem 4.1

With these Lemmas in hand, we are now equipped to prove our main theorem (Theorem 4.1), which guarantees the solution \( \hat{\pi}_{\text{RBC}} \) of eq. (5) is close to the optimal policy \( \pi_E \) in TV distance.

**Theorem A.1 (Theorem 4.1).** Suppose we have corrupted demonstration data set \( \mathcal{D} \) with sample size \( N \) from Definition 2.1, and there exists a constant corruption ratio \( \epsilon < 0.5 \). Under Assumption 4.1, let \( \tau \) to be the output objective value with \( \hat{\pi}_{\text{RBC}} \) in the optimization eq. (5) with the batch size \( b \leq \frac{1}{3\epsilon} \), then with probability at least \( 1 - c_1\delta \), we have

\[
E_{s \sim \rho_E} \| \hat{\pi}_{\text{RBC}}(\cdot|s) - \pi_E(\cdot|s) \|_{\text{TV}}^2 = O \left( \frac{\log(|\Pi|/\delta)}{N} + \tau \right).
\]

**Proof of Theorem 4.1.** En route to the proof of Theorem 4.1, we keep using the notations in Lemma A.1 and Lemma A.2, where the state observation is \( x \), the action is \( y \), and the discrete function class is \( \mathcal{F} \).

Similar to [2], we first note that Lemma A.1 can be combined with a simple Chernoff bound to obtain an exponential tail bound. With probability at least \( 1 - c_1\delta \), we have

\[
-\log E_{D'} \exp(\Gamma(\hat{f}, D')) \leq -\Gamma(\hat{f}, D) + \log |\mathcal{F}| + \log(1/\delta). \tag{10}
\]

Our proof technique relies on lower bounding the LHS of eq. (10), and upper bounding the RHS eq. (10).

Let the batch size \( b \leq \frac{1}{3\epsilon} \), which is a constant in Definition 3.1, then the number of batches \( M \geq 3\epsilon N \) such that there exists at least 66% batches without corruptions.

In the definition of RBC (Definition 3.1), we solve

\[
\hat{\pi}_{\text{RBC}} = \arg \min_{\pi \in \Pi} \max_{\pi' \in \Pi} \text{median}_{1 \leq j \leq M} (\ell_j(\pi) - \ell_j(\pi')). \tag{11}
\]

Notice that since \( \pi_E \) is one feasible solution of the inner maximization step eq. (11), we can choose \( \pi' = \pi_E \). Now we consider the objective function which is the difference of Negative Log-Likelihood between \( f \) and \( f^* \), i.e., \( \ell_j(f) - \ell_j(f^*) \), defined in eq. (4) where

\[
\ell_j(\pi) = \frac{1}{b} \sum_{(s,a) \in B_j} -\log(\pi(a|s)).
\]

Hence, we choose \( \Gamma(f, D) \) in Lemma A.1 as

\[
\Gamma_j(f, D) = \frac{N}{b} \sum_{i \in B_j} -\frac{1}{2} \log \frac{f^*(x_i, y_i)}{f(x_i, y_i)} = \frac{N}{2b} \sum_{i \in B_j} (\log f(x_i, y_i) - \log f^*(x_i, y_i)).
\]
which is the difference of Negative Log-Likelihood $N(\ell_j(f^*) - \ell_j(f))/2$ evaluated on a single batch $B_j, j \in [M]$. This is actually the objective function on a single batch appeared in eq. (5).

Lower bound for the LHS of eq. (10). We apply the concentration bound eq. (10) for such uncorrupted batches, hence the majority of all batches satisfies eq. (10). For those batches, the LHS of eq. (10) can be lower bounded by the TV distance according to Lemma A.2.

\[
\begin{align*}
- \log \mathbb{E}_{\mathcal{D}} \left[ \exp \left( \frac{N}{b} \sum_{i \in B_j} -\frac{1}{2} \log \left( \frac{f^*(x'_i, y'_i)}{\hat{f}(x'_i, y'_i)} \right) \right) \right] |\mathcal{D} | \\
&= - \frac{N}{b} \sum_{i \in B_j} \log \mathbb{E}_{x, y \sim \mathcal{G}_i} \left[ \exp \left( -\frac{1}{2} \log \frac{f^*(x, y)}{\hat{f}(x, y)} \right) \right] \\
&\geq \frac{N}{2b} \sum_{i \in B_j} \mathbb{E}_{x \sim \mathcal{G}_i} \left\| \hat{f}(x, \cdot) - f^*(x, \cdot) \right\|_{TV}^2,
\end{align*}
\]

where (i) follows from the independence between $\hat{f}$ and $\mathcal{D}'$ due to the decoupling technique, and (ii) follows from Lemma A.2, which is an upper bound of the Total Variation distance.

Upper bound for the RHS of eq. (10). Note that the objective is the median of means of each batches and $f^*$ is one feasible solution of the inner maximization step eq. (11). Since $\tau$ is the output objective value with $\hat{\pi}_{RBC}$ in the optimization eq. (5), this implies that $\ell_{Med}(\pi) - \ell_{Med}(\pi') \leq \tau$ for the median batch $B_{Med}$, which is equivalent to $-\Gamma_{Med}(f, \mathcal{D}) \leq N\tau/2$.

Hence for the median batch $B_{Med}$, the RHS of eq. (10) can be upper bounded by

\[
-\Gamma_{Med}(\hat{f}, \mathcal{D}) + \log |\mathcal{F}| + \log(1/\delta) \leq \log |\mathcal{F}| + \log(1/\delta) + N\tau/2.
\]

Putting together the pieces eq. (12) and eq. (13) for $B_{Med}$, we have

\[
\mathbb{E}_{s \sim \rho_E} \left\| \hat{\pi}^{RBC}(\cdot|s) - \pi_E(\cdot|s) \right\|_{TV}^2 = O\left( \frac{\log(|\mathcal{F}|/\delta)}{N} + \tau \right),
\]

with probability at least $1 - c_1\delta$.

\[\square\]

A.2 Proof of Theorem 4.2

With the supervised learning guarantees Theorem 4.1 in hand, which provides an upper bound for $\mathbb{E}_{s \sim \rho_E} \left\| \hat{\pi}^{RBC}(\cdot|s) - \pi_E(\cdot|s) \right\|_{TV}^2$, we are now able to present the suboptimality guarantee of the reward for $\hat{\pi}^{RBC}$. This bound directly corresponds to the reward performance of a policy.
Theorem A.2 (Theorem 4.2). Under the same setting as Theorem 4.1, we have

\[
J_{\pi_E} - J_{\hat{\pi}_{RBC}} \leq O \left( \frac{1}{(1 - \gamma)^2} \sqrt{\frac{\log(|F|/\delta)}{N} + \tau} \right),
\]

with probability at least \(1 - c_1 \delta\).

Proof of Theorem 4.2. This part is similar to [1], and we have

\[
(1 - \gamma)(J_{\pi_E} - J_{\hat{\pi}_{RBC}}) = E_{s \sim \rho_{\pi_E}} E_{a \sim \pi_E(s)} A^\pi_{RBC}(s, a)
\leq \frac{1}{1 - \gamma} \sqrt{E_{s \sim \rho_{\pi_E}} \|\hat{\pi}_{RBC}(\cdot | s) - \pi_E(\cdot | s)\|_1^2}
= \frac{2}{1 - \gamma} \sqrt{E_{s \sim \rho_{\pi_E}} \|\hat{\pi}_{RBC}(\cdot | s) - \pi_E(\cdot | s)\|_{TV}^2},
\]

where we use the fact that \(\text{sup}_{s,a,\pi} |A^\pi(s,a)| \leq \frac{1}{1 - \gamma} \) for the advantage function and the reward is always bounded between 0 and 1.

Combining Theorem 4.1, we have

\[
J_{\pi_E} - J_{\hat{\pi}_{RBC}} \leq O \left( \frac{1}{(1 - \gamma)^2} \sqrt{\frac{\log(|F|/\delta)}{N} + \tau} \right),
\]

with probability at least \(1 - c_1 \delta\). \(\Box\)

B Experimental Details

In this section, we provide the details of our algorithm RBC in different setups. All Behavior Cloning models were trained to minimize the mean-squared error regression loss on the demonstration data for 200 epochs using Adam [29]. In all setting, we fix the policy network as 3 hidden layer feed-forward Neural Network of size \{500, 500, 500\} with ReLU activation. More hyperparameters are provided in Table 1.

Table 1: Hyperparameters

| Hyperparameter       | Value          |
|----------------------|----------------|
| Parallel Environments| 20             |
| \(\ell_2\) regularization | 0             |
| Entropy coefficient  | 0.01           |
| Gradient clipping    | 0.1            |
| Learning rate        | \(7.5 \times 10^{-4}\) |

**Reward vs. Epochs.** We illustrate the convergence of our algorithm by tracking the reward performance for different continuous control environments simulated by PyBullet [12] simulator:
HopperBulletEnv-v0, Walker2DBulletEnv-v0, HalfCheetahBulletEnv-v0 and AntBulletEnv-v0. More specifically, we evaluate current policy in the simulator for 20 trials, and obtain the mean and standard deviation of cumulative reward for every 5 epochs. In this experiment, we adopt option (1) for the outliers, which set the actions of outliers to the boundary (−1 or +1). In Figure 4, we fix the corruption ratio as 10% and 20% with fixed demonstration data of size 60000, and present the Reward vs. Epochs. We note that the difference of purple curves in $\epsilon = 0.1$ and $\epsilon = 0.2$ is due to different random seed.

**Convergence time.** Another important aspect of our algorithm is the practical time complexity efficiency. To speed up our RBC, we pick multiple batches around the median batch in line 7 of Algorithm 1, and evaluate the gradient using back-propagation on these batches. The experimental setup is consistent with Figure 4. To directly compare the time complexity, we report the reward vs. wall clock time performance of our RBC and “Oracle BC”, which optimizes on the expert demonstrations.

We measure the convergence time by counting the elapsed time from zero to first achieving 95% of expert level. The experiments are conducted on 1/2 core of NVIDIA T4 GPU, and presented in Table 2, which shows that the actual running time time of RBC is comparable to vanilla BC.

|       | Hopper | HalfCheetah | Ant | Walker2D |
|-------|--------|-------------|-----|----------|
| Oracle BC | 88     | 174         | 49  | 159      |
| RBC   | 368    | 597         | 134 | 385      |
Figure 4: Reward vs. Epochs for offline Imitation Learning on four different continuous control tasks from PyBullet [12] with fixed demonstration data of size 60000. We choose the corruption ratio $\epsilon = 10\%, 20\%$. For every 5 epochs, we evaluate the current policy in the environment for 20 trials, and the shaded region represents the standard deviation. We note that the difference of purple curves between left and right is due to different random seed. Vanilla BC on corrupted demonstrations fails to converge to expert policy. Using the robust counterpart Algorithm 1 on corrupted demonstrations has good convergence properties. Surprisingly, our RBC on corrupted demonstrations has nearly the same reward performance of using BC on expert demonstrations.