Exclusive semileptonic decays of $B$ mesons to orbitally excited $D$ mesons in the relativistic quark model

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Abstract

Exclusive semileptonic $B$ meson decays to orbitally excited $D$ mesons are investigated in the infinitely heavy quark limit in the framework of the relativistic quark model based on the quasipotential approach. The $B \to D^{**} e \nu$ Isgur-Wise functions $\tau_{3/2}(w)$ and $\tau_{1/2}(w)$ are determined. It is found that the relativistic transformation of the meson wave functions (Wigner rotation of the light quark spin) contribute already at the leading order of the heavy quark expansion.

The investigation of semileptonic decays of $B$ mesons to excited $D$ meson states is a problem which is important both from a theoretical and experimental point of view. In particular, these decays can provide an additional source of information for the determination of the Cabibbo-Kobayashi-Maskawa matrix element $V_{cb}$ as well as on the relativistic quark dynamics inside heavy-light meson. The experimental data on these decays are becoming available now [1–3], and the $B$ factories will provide more accurate and comprehensive data. The presence of the heavy quark in the initial and final meson state in these decays considerably simplifies their theoretical description. A good starting point in this analysis is the infinitely heavy quark limit, $m_Q \to \infty$ [4]. In this limit the heavy quark symmetry arises, which strongly reduces the number of independent weak decay form factors [2]. The heavy quark mass and spin decouple then and all meson properties are determined by light degrees of freedom alone. As a result the heavy quark degeneracy of levels emerges. The spin $s_q$ of the light quark couples with its orbital momentum $l$ ($j = l \pm s_q$), resulting for $P$-wave mesons in two degenerate $j = 3/2$ states ($J^P = 1^+, 2^+$) and two degenerate $j = 1/2$ states ($0^+, 1^+$). The heavy quark symmetry also predicts that the weak decay form factors

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$^1$ Here $J = j \pm 1/2$ and $P$ are the total angular momentum and parity of the meson.
for $B \rightarrow D^{**}e\nu$ decays, where $D^{**}$ is a generic $P$-wave $D$ meson state, can be expressed in terms of two independent Isgur-Wise functions $\tau_{3/2}$ and $\tau_{1/2}$.

In preceding papers we have calculated the mass spectra of orbitally and radially excited states of heavy-light mesons as well as different weak decays of $B$ mesons to ground state heavy and light mesons in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. Let us now apply this model to the investigation of semileptonic $B$ decays to $P$-wave $D$ mesons in the heavy quark limit.

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation of the Schrödinger type

$$\left( \frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R} \right) \Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M) \Psi_M(q),$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_q^2 - m_Q^2)^2}{4M^3}.$$  

Here $m_q, m_Q$ are the masses of light and heavy quarks, and $p$ is their relative momentum. In the centre-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}.$$  

The kernel $V(p, q; M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz-structure of the confining quark-antiquark interaction in the meson. In constructing the quasipotential of quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by

$$V(p, q; M) = \bar{u}_q(p)\bar{u}_Q(-p) \left\{ \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_\mu^\mu \gamma_\nu^\nu + V_{\text{conf}}^V(k) \Gamma_{Q;\mu} + V_{\text{conf}}^S(k) \right\} u_q(q)u_Q(-q),$$

where $\alpha_s$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge and $k = p - q$; $\gamma_\mu$ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \left( \frac{1}{\epsilon(p) + m} \right) \chi^\lambda,$$

with $\epsilon(p) = \sqrt{p^2 + m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} k_\nu,$$
where $\kappa$ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

\[
V_{\text{conf}}^V(r) = (1 - \varepsilon)Ar, \\
V_{\text{conf}}^S(r) = \varepsilon Ar + B,
\]

reproducing

\[
V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B,
\]

where $\varepsilon$ is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in $v^2/c^2$, can be found in Refs. [11,12] and for heavy-light mesons in [3]. All the parameters of our model like quark masses, parameters of the linear confining potential, mixing coefficient $\varepsilon$ and anomalous chromomagnetic quark moment $\kappa$ were fixed from the analysis of heavy quarkonia masses [11] and radiative decays [13]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.30$ GeV have usual quark model values. The value of the vector-scalar mixing coefficient $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion [14] and meson radiative decays [13]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia $3P_J$- states [11]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1 + \kappa)$ and thus vanishes for the chosen value of $\kappa = -1$.

In order to calculate the exclusive semileptonic decay rate of the $B$ meson it is necessary to determine the corresponding matrix element of the weak current between meson states. The matrix element of the weak current $J^W = \bar{c}\gamma_\mu(1 - \gamma^5)b$ between $B$ meson and orbitally excited $D^{**}$ meson in the quasipotential method has the form [15]

\[
\langle D^{**} | J^W(0) | B \rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{D^{**}}(p) \Gamma_\mu(p,q) \Psi_B(q),
\]

where $\Gamma_\mu(p,q)$ is the two-particle vertex function and $\Psi_{B,D^{**}}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to $\Gamma$ come from Figs. 1 and 2. In the heavy quark limit $m_{b,c} \to \infty$ only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes at $1/m_Q$ order. As we limit our analysis here to the leading order of the heavy quark expansion, only the vertex function $\Gamma^{(1)}$ is necessary. It looks like

\[
\Gamma^{(1)}(p,q) = \bar{u}_c(p_c)\gamma_\mu(1 - \gamma^5)u_b(q_b)(2\pi)^3\delta(p_q - q_q),
\]

where [15]

\[2\] The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz-structure of the $q\bar{q}$-interaction.
\[ p_{c,q} = \epsilon_{c,q}(p) \frac{p_{D^{**}}}{M_{D^{**}}} \pm \sum_{i=1}^{3} n^{(i)}(p_{D^{**}}) p_{i}^{i}, \]
\[ q_{b,q} = \epsilon_{b,q}(p) \frac{p_{B}}{M_{B}} \pm \sum_{i=1}^{3} n^{(i)}(p_{B}) q_{i}, \]

and
\[ n^{(i)}(p) = \left\{ \frac{p_{i}^{i}}{M}, \delta_{ij} + \frac{p_{i}^{j} p_{j}^{i}}{M(E + M)} \right\}. \]

The wave function of a \( P \)-wave \( D^{**} \) meson at rest is given by
\[ \Psi_{D^{**}}(p) \equiv \Psi_{D^{**}}^{JM}(p) = \mathcal{Y}_{j}^{JM} \psi_{D^{**}}(j)(p), \quad (11) \]

where \( J \) and \( M \) are the total meson angular momentum and its projection, while \( j \) is the light quark angular momentum. \( \psi_{D^{**}}(j)(p) \) is the radial part of the wave function, which has been determined by the numerical solution of eq. (1) in [6]. The spin-angular momentum part \( \mathcal{Y}_{j}^{JM} \) has the following form
\[ \mathcal{Y}_{j}^{JM} = \sum_{\sigma_{Q} \sigma_{q}} \langle j M - \sigma_{Q}, \frac{1}{2} \sigma_{Q} | J M \rangle \langle 1 M - \sigma_{Q} - \sigma_{q}, \frac{1}{2} \sigma_{q} | j M - \sigma_{Q} \rangle Y_{1}^{M - \sigma_{Q} - \sigma_{q}} \chi_{Q}(\sigma_{Q}) \chi_{q}(\sigma_{q}). \quad (12) \]

Here \( \langle j_{1} m_{1}, j_{2} m_{2} | J M \rangle \) are Clebsch-Gordan coefficients, \( Y_{j}^{m} \) are spherical harmonics, and \( \chi(\sigma) \) (where \( \sigma = \pm 1/2 \)) are spin wave functions,
\[ \chi(1/2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi(-1/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]

It is important to note that the wave functions entering the weak current matrix element \( (1) \) are not at rest in general. E.g., in the \( B \) meson rest frame the \( D^{**} \) meson is moving with the recoil momentum \( \Delta \). The wave function of the moving \( D^{**} \) meson \( \Psi_{D^{**} \Delta} \) is connected with the \( D^{**} \) wave function at rest \( \Psi_{D^{**}0} = \Psi_{D^{**}}^{(j)} \) by the transformation [13]
\[ \Psi_{D^{**} \Delta}(p) = D_{c,q}^{1/2}(R_{L \Delta}^{W}) D_{q}^{1/2}(R_{L \Delta}^{W}) \Psi_{D^{**}0}(p), \quad (13) \]

where \( R^{W} \) is the Wigner rotation and the rotation matrix \( D^{1/2}(R) \) in spinor representation is given by
\[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{c,q}^{1/2}(R_{L \Delta}^{W}) = S^{-1}(p_{c,q}) S(\Delta) S(p), \quad (14) \]

where
\[ S(p) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left( 1 + \frac{\alpha p}{\epsilon(p) + m} \right) \]
is the usual Lorentz transformation matrix of the four-spinor. For electro-weak \( B \) meson decays to \( S \)-wave final mesons such transformation contributes at first order of the \( 1/m_{Q} \).
expansion, while for the decays to excited final mesons it gives a contribution already to the leading term, due to the orthogonality of the initial and final meson wave functions.

In the infinitely heavy quark limit \((m_{b,c} \rightarrow \infty)\) all form factors of the semileptonic \(B \rightarrow D^{**}e\nu\) decays are related to two independent Isgur-Wise form factors \(\tau_{1/2}\) and \(\tau_{3/2}\) by

\[
\begin{align*}
\langle D_0(1/2)(v')|\bar{c}\gamma_\mu(1-\gamma^5)b|B(v)\rangle &= 2\sqrt{M_BM_{D_0(1/2)}} \tau_{1/2}(w)(v'_\mu - v_\mu), \\
\langle D_1(1/2)(v')|\bar{c}\gamma_\mu(1-\gamma^5)b|B(v)\rangle &= 2\sqrt{M_BM_{D_1(1/2)}} \tau_{1/2}(w)\{i\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v^\beta v'\gamma \nonumber \\
&\quad + (1-w)e^*_\mu + (\epsilon^* \cdot v)v'_\mu\}, \\
\langle D_1(3/2)(v')|\bar{c}\gamma_\mu(1-\gamma^5)b|B(v)\rangle &= \sqrt{M_BM_{D_1(3/2)}} \tau_{3/2}(w)\{i(w+1)\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{\*\alpha}v^\beta v'\gamma \\
&\quad + (1-w^2)e^*_\mu - (\epsilon^* \cdot v)[3v_\mu - (w-2)v'_\mu]\}, \\
\langle D_2(3/2)(v')|\bar{c}\gamma_\mu(1-\gamma^5)b|B(v)\rangle &= \sqrt{3M_BM_{D_2(3/2)}} \tau_{3/2}(w)\{-i\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{\*\alpha}v_\eta v^\beta v'\gamma \\
&\quad + [(w+1)e^*_\mu v^\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\mu]\},
\end{align*}
\tag{15}
\]

where \(v(v')\) is the four-velocity of the initial (final) meson, \(w = v \cdot v'\), and \(\epsilon^\mu, \epsilon^{\mu\nu}\) are polarization vector and tensor of \(D_1\) and \(D_2\) mesons, respectively.

To calculate the corresponding matrix elements, we substitute the vertex function \(\Gamma^{(1)}\) (14) in the matrix element of the weak current between meson states (1) and take into account the wave function properties (11)–(13). Then, in the limit \(m_{b,c} \rightarrow \infty\) we find that the heavy quark symmetry relations (13) are exactly satisfied in our model. The resulting expressions for the Isgur-Wise functions \(\tau_{3/2}\) and \(\tau_{1/2}\) are

\[
\begin{align*}
\tau_{3/2}(w) &= \frac{\sqrt{2}}{3}\frac{1}{(w+1)^{3/2}} \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}_{D(3/2)}(p) + \frac{2\epsilon_q}{M_{D(3/2)}(w+1)} \Delta \\
&\quad \times \left[ -2\epsilon_q \frac{\partial}{\partial p} + \frac{p}{\epsilon_q + m_q} \right] \psi_B(p),
\end{align*}
\tag{16}
\]

\[
\begin{align*}
\tau_{1/2}(w) &= \frac{1}{3\sqrt{2}}\frac{1}{(w+1)^{1/2}} \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}_{D(1/2)}(p) + \frac{2\epsilon_q}{M_{D(1/2)}(w+1)} \Delta \\
&\quad \times \left[ -2\epsilon_q \frac{\partial}{\partial p} - \frac{2p}{\epsilon_q + m_q} \right] \psi_B(p),
\end{align*}
\tag{17}
\]

where the arrow over \(\partial/\partial p\) indicates that the derivative acts on the wave function of the \(D^{**}\) meson. The last terms in the square brackets of these expressions result from the wave function transformation (13) associated with the relativistic rotation of the light quark spin (Wigner rotation) in passing to the moving reference frame. These terms are numerically important and lead to the suppression of the \(\tau_{1/2}\) form factor compared to \(\tau_{3/2}\). Note that if we had applied a simplified nonrelativistic quark model [14,16] these important contributions would be missing. Neglecting further the small difference between the wave functions \(\psi_{D(1/2)}\) and \(\psi_{D(3/2)}\), the following relation between \(\tau_{3/2}\) and \(\tau_{1/2}\) would have been obtained [17]

\[
\tau_{1/2}(w) = \frac{w+1}{2} \tau_{3/2}(w).
\tag{18}
\]
However, we see that this relation is violated if relativistic transformation properties of wave function are taken into account. At the point $w = 1$, where the initial $B$ meson and final $D^{*\ast}$ are at rest, we find instead the relation

$$
\tau_{3/2}(1) - \tau_{1/2}(1) \approx \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \bar{\psi}_{D^{*\ast}}(p) \frac{p}{\epsilon_q + m_q} \psi_B(p),
$$

(19)

obtained by assuming $\psi_{D(3/2)} \approx \psi_{D(1/2)} \approx \psi_{D^{*\ast}}$. This relation (19) coincides with the one found in Ref. [13] where the Wigner rotation was also taken into account.

In Table I we present our numerical results for $\tau_j(1)$ and its slope $\rho_j^2 = -\frac{1}{\tau_j} \frac{\partial}{\partial w} \tau_j|_{w=1}$ in comparison with other model predictions [17–23]. Moreover, we plot $\tau_j(w)$ for $B \to D^{*\ast}e\nu$ and $B_s \to D^{*\ast}s e\nu$ decays as function of $w$ in Figs. 3, 4. The corresponding decay rates and branching ratios are given in Tables II and III. We see that most of the above approaches predict close values for the function $\tau_{3/2}(1)$ and its slope $\rho_{3/2}^2$, while the results for $\tau_{1/2}(1)$ significantly differ from each other. This difference is a consequence of a different treatment of the relativistic quark dynamics. Nonrelativistic approaches predict $\tau_{3/2}(1) \approx \tau_{1/2}(1)$ (see (18)), while the relativistic treatment leads to $\tau_{3/2}(1) > \tau_{1/2}(1)$ (see (19)). Our results for the branching ratios of $B \to D_{1,2}(3/2)e\nu$ decays are consistent with available experimental data [22], which at present require to use some assumptions about the branching fractions of the $D_J$ mesons.

Finally, let us test the fulfilment of the Bjorken sum rule [24] in our model. This sum rule states

$$
\rho^2 = \frac{1}{4} + \sum_m |\tau_{1/2}^{(m)}(1)|^2 + 2 \sum_m |\tau_{3/2}^{(m)}(1)|^2 + \cdots,
$$

(20)

where $\rho^2$ is the slope of the $B \to D^{(s)}e\nu$ Isgur-Wise function, $\tau_{1/2}^{(m)}$ and $\tau_{3/2}^{(m)}$ are the form factors describing the orbitally excited states discussed here and their radial excitations, and ellipses denote contributions from non-resonant channels. We see that the contribution of the lowest lying $P$-wave states ($m = 0$) implies the bound

$$
\rho^2 > \frac{1}{4} + |\tau_{1/2}(1)|^2 + 2|\tau_{3/2}(1)|^2 = 0.80,
$$

(21)

which is in agreement with the slope $\rho^2 = 1.02$ in our model [14].

In this paper we have applied the relativistic quark model to the consideration of semileptonic decays of $B$ mesons to orbitally excited charmed mesons in the leading order of the heavy quark expansion. In particular, it has been found that the Lorentz properties and transformations of meson wave functions play an important role in the theoretical description of these decays. Thus, the Wigner rotation of the light quark spin gives a significant contribution already at the leading order of the heavy quark expansion. In conclusion let us mention that the corrections in inverse powers of the heavy quark mass $1/m_{c,b}$ to the decay rates might turn out to be non-negligible, especially for spin zero and spin one $D^{*\ast}$ mesons [17]. The investigation of such corrections in the framework of our model is an important task that will be considered elsewhere.

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TABLES

TABLE I. The comparison of our model results for the values of the functions $\tau_j$ at zero recoil of final $D^{**}$ meson and their slopes $\rho_j^2$ with other predictions.

|       | our | [17] | [19] | [20] | [21] | [18], [22] | [18], [23] |
|-------|-----|------|------|------|------|-------------|-------------|
| $\tau_{3/2}(1)$ | 0.49 | 0.41 | 0.56 | 0.66 | 0.54 | 0.52        |
| $\rho_{3/2}^2$    | 1.53 | 1.5  | 2.3  | 1.9  | 1.5  | 1.45        |
| $\tau_{1/2}(1)$  | 0.28 | 0.41 | 0.09 | 0.41 | 0.35 ± 0.08 | 0.22 | 0.06       |
| $\rho_{1/2}^2$   | 1.04 | 1.0  | 1.1  | 1.4  | 2.5 ± 1.0 | 0.83 | 0.73       |

TABLE II. Decay rates $\Gamma$ (in units $|V_{cb}/0.04|^210^{-15}$ GeV) and branching ratios (%) for $B \to D^{**}e\nu$ decays.

| Decay       | $\Gamma$ | Br | Br (CLEO) [1] | Br (ALEPH) [2] |
|-------------|----------|----|---------------|----------------|
| $B \to D_1(3/2)e\nu$ | 1.4      | 0.33 | 0.56 ± 0.13 ± 0.08 ± 0.04 | 0.74 ± 0.16   |
| $B \to D_2(3/2)e\nu$ | 2.1      | 0.52 | < 0.8         | < 0.2         |
| $B \to D_1(1/2)e\nu$ | 0.30     | 0.074 |               |               |
| $B \to D_0(1/2)e\nu$ | 0.25     | 0.062 |               |               |

TABLE III. Decay rates $\Gamma$ (in units $|V_{cb}/0.04|^210^{-15}$ GeV) and branching ratios (%) for $B_s \to D^{**}e\nu$ decays.

| Decay       | $\Gamma$ | Br |
|-------------|----------|----|
| $B_s \to D_{s1}(3/2)e\nu$ | 1.6      | 0.39 |
| $B_s \to D_{s2}(3/2)e\nu$ | 2.5      | 0.59 |
| $B_s \to D_{s1}(1/2)e\nu$ | 0.54     | 0.13 |
| $B_s \to D_{s0}(1/2)e\nu$ | 0.45     | 0.11 |
FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (9).

FIG. 2. Vertex function $\Gamma^{(2)}$ with the account of the quark interaction. Dashed lines correspond to the effective potential \[4\]. Bold lines denote the negative-energy part of the quark propagator.
FIG. 3. Isgur-Wise functions $\tau_{3/2}(w)$ (upper curve) and $\tau_{1/2}(w)$ (lower curve) for $B \rightarrow D^{**}e\nu$ decay.

FIG. 4. Isgur-Wise functions $\tau_{3/2}(w)$ (upper curve) and $\tau_{1/2}(w)$ (lower curve) for $B_s \rightarrow D_s^{**}e\nu$ decay.