Global asymptotical stability of almost periodic solutions for a non-autonomous competing model with time-varying delays and feedback controls

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ABSTRACT

In this paper, we are concerned with a non-autonomous competing model with time delays and feedback controls. Applying the comparison theorem of differential equations and by constructing a suitable Lyapunov functional, some sufficient conditions which guarantee the existence of a unique globally asymptotically stable nonnegative almost periodic solution of the system are established. An example with its numerical simulations is given to illustrate the feasibility of our results.

1. Introduction

In recent years, the competitive prey-predator systems have been investigated by many scholars. The dynamical behavior such as permanence, global attractivity and global stability of continuous differential competitive prey-predator systems was extensively studied and many excellent results were reported. For example, Sarwardi et al. [27] focused on the local and the global stability and the bifurcations of a competitive prey-predator system with a prey refuge, Ko and Ahn [16] discussed the global attractor, persistence and the stability of all non-negative equilibria of a diffusive one-prey and two-competing-predator system with a ratio-dependent functional response, Pan et al. [25] considered Gause's principle in interspecific competition of the cyclic predator-prey system, Qun Liu et al. [23] addressed the global stability of a stochastic predator-prey system with infinite delays. For more related works on this topic, one can see [2,3,6,9–12,14,15,18,20,24,26,28,29,31,34,35,38]. In 2001, Zhang et al. [36] investigated the permanence of the following non-autonomous competing model

\[
\frac{dx_1(t)}{dt} = x_1(t) \left[ \frac{r_1(t)}{x_1(t) + k_1(t)} - a_1(t)x_1(t) - b_1(t)x_2(t) - c_1(t) \right],
\]
where \( x_1(t), x_2(t) \) represent the densities of two competing populations, respectively. \( r_i, a_i, b_i, c_i, k_i : (0, + \infty) \to + \infty, i = 1, 2 \), are continuous functions.

In real world, the situation of competitive populations is often distributed by unpredictable forces which can result in changes in parameters of systems such as the intrinsic growth rates, thus it is important to investigate models with control variables which are so-called disturbance functions \([13,21,22,30,37]\). Moreover, the unpredictable forces are seldom happened immediately, there is a time delay. Inspired by the above analysis, we modify system (1) as the following non-autonomous competing model with multiple delays and feedback controls

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= x_1(t) \left[ \frac{r_1(t)}{x_1(t) + k_1(t)} - a_1(t)x_1(t) - b_1(t)x_2(t) - c_1(t) - d_1(t)u_1(t - \tau_1(t)) \right], \\
\frac{dx_2(t)}{dt} &= x_2(t) \left[ \frac{r_2(t)}{x_2(t) + k_2(t)} - b_2(t)x_1(t) - a_2(t)x_2(t) - c_2(t) - d_2(t)u_2(t - \tau_2(t)) \right], \\
\frac{du_1(t)}{dt} &= -a_1(t)u_1(t) + \beta_1(t)x_1(t - \sigma_1(t)), \\
\frac{du_2(t)}{dt} &= -a_2(t)u_2(t) + \beta_2(t)x_2(t - \sigma_2(t)),
\end{align*}
\]

where \( u_1, u_2 \) are the control variables. Many scholars \([7,30]\) argue that periodic phenomenon and almost periodic phenomena are widespread in nature, and almost periodic phenomenon is more frequent than periodic one. Hence, they have been the object of intensive analysis by numerous authors. In particular, there have been extensive results on existence of almost periodic solutions of differential equations in the literatures (see \([1,8,17,19,33]\)). The main object of this paper is to investigate the almost periodic solutions of model (2). By the analysis on the almost periodic solutions of model (2), we can find the coexistence conditions of two competing populations, which can help human beings to control ecological balance. Also the analysis results enrich and develop the ecological theory. We believe that this investigation on the global stability of two competitive populations has important theoretical value and tremendous potential for application in administering process for both ecology and mathematical ecology.

Let \( R \) and \( R^+ \) denote the set of all real numbers and nonnegative real numbers, respectively. Let \( f \) be a continuous bounded function defined on \( R \) and we set \( f^u = \sup_{t \in R} f(t) \) and \( f^l = \inf_{t \in R} f(t) \). In order to obtain our main results, throughout this paper, we assume that

\[(H1) \quad r_i, k_i, a_i, b_i, c_i, d_i, \alpha_i, \beta_i \text{ are all positive almost periodic functions, which are bounded above by positive constants and } \inf \{(r_i/k_i)^l, a_i^l\} > 0, i = 1, 2.\]
\( \tau_i, \sigma_i \) are positive almost periodic functions, continuously differentiable and satisfy

\[
\max \left\{ \sup_{s \in \mathbb{R}} \tau_i(s), \sup_{s \in \mathbb{R}} \sigma_i(s) \right\} < 1
\]

for \( i = 1, 2 \), where \( \dot{g}(t) \) expresses the derivative of \( g \) with respect to time \( t \) for any differentiable \( g \).

Denote

\[
\tau = \max_{1 \leq i \leq 2} \left\{ \max_{s \in \mathbb{R}} \tau_i(s), \sup_{s \in \mathbb{R}} \sigma_i(s) \right\}.
\]

We consider (2) together with the following initial condition

\[
\begin{align*}
x_1(\theta) &= \varphi_1(\theta) \geq 0, \quad \theta \in [-\tau, 0], \\
x_2(\theta) &= \varphi_2(\theta) \geq 0, \quad \theta \in [-\tau, 0].
\end{align*}
\] (3)

The innovativeness of this article is listed as follows:

- A non-autonomous competing model has been generalized to a non-autonomous competing model with time-varying delays and feedback controls.
- Some new sufficient conditions which guarantee the existence of a unique globally asymptotically stable nonnegative almost periodic solution of the system are presented. The research shows that the delay and feedback control term have an important effect on global asymptotical stability of almost periodic solutions of considered system.
- Constructing an appropriate Lyapunov functional to handle the global asymptotical stability of almost periodic solutions is a challenging work. The analytic results of this article will enrich and develop the stability theory of delayed differential equations and also improve the earlier works.

The remainder of the paper is organized as follows: in Section 2, several useful lemmas are introduced. In Section 3, by applying the comparison theorem of the differential equation and constructing a suitable Lyapunov functional, a set of sufficient conditions which ensure the existence of a unique globally asymptotically stable nonnegative almost periodic solution for system (2) are established. In Section 4, a suitable example with its simulations is given to illustrate the feasibility of the main results. Brief conclusions are drawn in Section 5.

### 2. Preliminaries

In order to obtain the main result of this paper, we shall first state several lemmas which will be useful in the proving the main result.

**Lemma 2.1 ([5]):** Let \( a > 0, b > 0 \).

(i) If \( \frac{dx(t)}{dt} \geq b - ax \), then \( \liminf_{t \to +\infty} x(t) \geq b/a \) for \( t \geq 0 \) and \( x(0) > 0 \);

(ii) If \( \frac{dx(t)}{dt} \leq b - ax \), then \( \limsup_{t \to +\infty} x(t) \leq b/a \) for \( t \geq 0 \) and \( x(0) > 0 \).
Lemma 2.2 ([5]): Let $a > 0, b > 0$.

(i) If $\frac{dx(t)}{dt} \geq x(b - ax)$, then $\lim \inf_{t \to +\infty} x(t) \geq \frac{b}{a}$ for $t \geq 0$ and $x(0) > 0$;

(ii) If $\frac{dx(t)}{dt} \leq x(b - ax)$, then $\lim \sup_{t \to +\infty} x(t) \leq \frac{b}{a}$ for $t \geq 0$ and $x(0) > 0$.

Definition 2.1 ([4]): A function $f(t, x)$, where $f$ is an $m$-vector, $t$ is a real scalar and $x$ is an $n$-vector, is said to be almost periodic in $t$ uniformly with respect to $x \in R^n$, if $f(t, x)$ is continuous in $t \in R$ and $x \in R$, and if for any $\varepsilon > 0$, it is possible to find a constant $l(\varepsilon) > 0$ such that in any interval of length $l(\varepsilon)$ there exists a $\sigma$ such that the inequality

$$\| f(t + \sigma, x) - f(t, x) \| = \sum_{i=1}^{m} |f_i(t + \sigma, x) - f_i(t, x)| < \varepsilon$$

is satisfied for all $t \in R$ and $x \in R^n$. The number $\sigma$ is called the $\varepsilon$-translation number or $\varepsilon$-almost period of $f(t, x)$.

Definition 2.2 ([32]): A function $f : R \to R$ is said to be asymptotically almost periodic function if there exists an almost-periodic function $h(t)$ and a continuous function $r(t)$ such that $f(t) = h(t) + r(t), r(t) \to 0$ as $t \to +\infty$.

Lemma 2.3: Any solution $(x_1(t), u_1(t), x_2(t), u_2(t))^T$ of system (2) satisfies

$$\lim \sup_{t \to +\infty} x_i(t) \leq x_i^* := \frac{\left( \frac{r_i}{k_i} \right)^u}{a_i}, \quad \lim \sup_{t \to +\infty} u_i(t) \leq u_i^* := \frac{\beta_i^u x_i^*}{\alpha_i}, \quad i = 1, 2.$$ 

Proof: Let $(x_1(t), u_1(t), x_2(t), u_2(t))^T$ be any positive solution of system (2). It follows from system (2) that

$$\frac{dx_i(t)}{dt} \leq x_i(t) \left( \left( \frac{r_i}{k_i} \right)^u - a_i^l x_i(t) \right), \quad i = 1, 2.$$ 

By Lemma 2.2, we get

$$\lim \sup_{t \to +\infty} x_i(t) \leq \frac{\left( \frac{r_i}{k_i} \right)^u}{a_i^l} := x_i^*, \quad i = 1, 2. \quad (4)$$

It follows from (2) that for any $\varepsilon > 0$, there exists a $T_1 > 0$ such that for all $t > T_1,$

$$\frac{du_i(t)}{dt} \leq -\alpha_i^l u_i(t) + \beta_i^u (x_i^* + \varepsilon),$$

Applying Lemma 2.1, we have

$$\lim \sup_{t \to +\infty} u_i(t) \leq \frac{\beta_i^u (x_i^* + \varepsilon)}{\alpha_i^l}, \quad i = 1, 2.$$ 

Letting $\varepsilon \to 0$ in the above inequality leads to

$$\lim \sup_{t \to +\infty} u_i(t) \leq \frac{\beta_i^u x_i^*}{\alpha_i^l} := u_i^*, \quad i = 1, 2. \quad (5)$$
Let
\[
\Omega = \{(x_1(t), u_1(t), x_2(t), u_2(t)) \mid 0 \leq x_1(t) \leq x_1^*,
\]
\[
0 \leq x_2(t) \leq x_2^*, 0 \leq u_1(t) \leq u_1^*, 0 \leq u_2(t) \leq u_2^*\}.
\]

**Lemma 2.4:** \(\Omega \neq \emptyset\).

**Proof:** By (H1) and (H2), there exists a sequence \(\{t_n\}\) with \(t_n \to +\infty\) as \(n \to +\infty\) such that

\[
\begin{align*}
& r_i(t + t_n) \to r_i(t), \quad a_i(t + t_n) \to a_i(t), \quad b_i(t + t_n) \to b_i(t), \\
& c_i(t + t_n) \to c_i(t), \quad d_i(t + t_n) \to d_i(t), \quad \alpha_i(t + t_n) \to \alpha_i(t), \\
& \beta_i(t + t_n) \to \beta_i(t), \quad \tau_i(t + t_n) \to \tau_i(t), \quad \sigma_i(t + t_n) \to \sigma_i(t), \quad k_i(t + t_n) \to k_i(t),
\end{align*}
\]
as \(n \to +\infty\), where \(i = 1, 2\). Let \(z(t) = (x_1(t), u_1(t), x_2(t), u_2(t))^T\) be a solution of system (2) satisfying \(0 \leq x_1(t) \leq x_1^*, 0 \leq x_2(t) \leq x_2^*, 0 \leq u_1(t) \leq u_1^*, 0 \leq u_2(t) \leq u_2^*\) for \(t > T\) for some \(T > 0\). Clearly, the sequence \(z(t + t_n)\) is uniformly bounded and equicontinuous on each bounded subset of \(R\). Therefore by Ascoli’s theorem we can conclude that there exists a subsequence \(z(t + t_k)\) which converges to a continuous function \(\tilde{z}(t) = (\tilde{x}_1(t), \tilde{u}_1(t), \tilde{x}_2(t), \tilde{u}_2(t))^T\) as \(k \to +\infty\) uniformly on each bounded subset of \(R\). Let \(T_0 \in \mathbb{R}\) be given. We may assume that \(t_k + T_0 \geq T\) for all \(k\). For \(t \geq 0\), we have

\[
\begin{align*}
x_1(t + t_k + T_0) &= x_1(t_k + T_0) + \int_{T_0}^{t+T_0} \left[ x_1(s + t_k) \left( \frac{r_1(s + t_k)}{x_1(s + t_k) + k_1(s + t_k)} \right. \\
& \quad - a_1(s + t_k)x_1(s + t_k) - b_1(s + t_k)x_2(s + t_k) - c_1(s + t_k) \\
& \quad \left. - d_1(s + t_k)u_1(s + t_k - \tau_1(s + t_k)) \right) \right] ds \\
u_1(t + t_k + T_0) &= u_1(t_k + T_0) - \int_{T_0}^{t+T_0} \left[ \alpha_1(s + t_k)u_1(s + t_k) \\
& \quad - \beta_1(s + t_k)x_1(s + t_k - \sigma_1(s + t_k)) \right] ds,
\end{align*}
\]
\[
\begin{align*}
x_2(t + t_k + T_0) &= x_2(t_k + T_0) + \int_{T_0}^{t+T_0} \left[ x_2(s + t_k) \left( \frac{r_2(s + t_k)}{x_2(s + t_k) + k_2(s + t_k)} \right. \\
& \quad - b_2(s + t_k)x_1(s + t_k) - a_2(s + t_k)x_2(s + t_k) - c_2(s + t_k) \\
& \quad \left. - d_2(s + t_k)u_2(s + t_k - \tau_2(s + t_k)) \right) \right] ds \\
u_2(t + t_k + T_0) &= u_2(t_k + T_0) - \int_{T_0}^{t+T_0} \left[ \alpha_2(s + t_k)u_2(s + t_k) \\
& \quad - \beta_2(s + t_k)x_2(s + t_k - \sigma_2(s + t_k)) \right] ds.
\end{align*}
\]

(6)
According to Lebesgue’ dominated convergence theorem, and letting \( k \to +\infty \) in (6), we have

\[
\tilde{x}_1(t + T_0) = \tilde{x}_1(T_0) + \int_{T_0}^{t+T_0} \left[ \tilde{x}_1(s) \left( \frac{r_1(s)}{\tilde{x}_1(s) + k_1(s)} - a_1(s)\tilde{x}_1(s) - b_1(s)\tilde{x}_2(s) - c_1(s) - d_1(s)\tilde{u}_1(s - \tau_1(s)) \right) \right] \, ds,
\]

\[
\tilde{u}_1(t + T_0) = \tilde{u}_1(t_k + T_0) - \int_{T_0}^{t+T_0} \left[ \alpha_1(s)\tilde{u}_1(s) - \beta_1(s)x_1(s - \sigma_1(s)) \right] \, ds,
\]

\[
\tilde{x}_2(t + T_0) = \tilde{x}_2(T_0) + \int_{T_0}^{t+T_0} \left[ \tilde{x}_2(s) \left( \frac{r_2(s)}{\tilde{x}_2(s) + k_2(s)} - b_2(s)\tilde{x}_1(s) - a_2(s)\tilde{x}_2(s) - c_2(s) - d_2(s)\tilde{u}_2(s - \tau_2(s)) \right) \right] \, ds,
\]

\[
\tilde{u}_2(t + T_0) = \tilde{u}_2(T_0) - \int_{T_0}^{t+T_0} \left[ \alpha_2(s)\tilde{u}_2(s) - \beta_2(s)\tilde{x}_2(s - \sigma_2(s)) \right] \, ds. \tag{7}
\]

Since \( T_0 \in \mathbb{R} \) is arbitrarily given, \( \tilde{z}(t) = (\tilde{x}_1(t), \tilde{u}_1(t), \tilde{x}_2(t), \tilde{u}_2(t))^T \) is a solution of system (2) on \( \mathbb{R} \). Then \( 0 \leq \tilde{x}_1(t) \leq x_1^*, 0 \leq \tilde{x}_2(t) \leq x_2^*, 0 \leq \tilde{u}_1(t) \leq u_1^*, 0 \leq \tilde{u}_2(t) \leq u_2^* \) for \( t \in \mathbb{R} \). Thus \( z(t) \in \Omega \). This completes the proof. \( \blacksquare \)

3. Stability of almost periodic solution

In this section, we will consider the stability of the almost periodic solution of system (2).

**Theorem 3.1:** In addition to (H1) and (H2), assume that (H3) \( \delta > 0 \), where \( \delta = \min\{\delta_1, \delta_2, \delta_3, \delta_4\} \) and

\[
\delta_1 = \min \left\{ a_1^l - \frac{r_1^u}{(k_1^u)^2} - b_2^u - \frac{\beta_1^u}{1 - \sigma_1(\xi_1^{-1}(t))} \right\},
\]

\[
\delta_2 = \min \left\{ \alpha_1^l - \frac{d_1^u}{1 - \tau_1(\eta_1^{-1}(t))} \right\},
\]

\[
\delta_3 = \min \left\{ a_2^l - \frac{r_2^u}{(k_2^u)^2} - b_1^u - \frac{\beta_2^u}{1 - \sigma_2(\xi_2^{-1}(t))} \right\},
\]

\[
\delta_4 = \min \left\{ \alpha_2^l - \frac{d_2^u}{1 - \tau_2(\eta_2^{-1}(t))} \right\},
\]

where \( \xi_i^{-1}, \eta_i^{-1} \) are the inverse functions of \( \xi_i(t) = t - \sigma_i(t), \eta_i(t) = t - \tau_i(t) \), respectively, \( i = 1, 2 \). Then for any two positive solutions \( z(t) = (x_1(t), u_1(t), x_2(t), u_2(t))^T \) and \( \tilde{z}(t) = (\tilde{x}_1(t), \tilde{u}_1(t), \tilde{x}_2(t), \tilde{u}_2(t))^T \) of system (2), we have

\[
\lim_{t \to +\infty} |x_1(t) - \tilde{x}_1| = 0, \quad \lim_{t \to +\infty} |x_2(t) - \tilde{x}_1| = 0, \quad \lim_{t \to +\infty} |u_1(t) - \tilde{u}_1| = 0, \quad \lim_{t \to +\infty} |u_2(t) - \tilde{u}_2| = 0, \quad i = 1, 2.
\]
Proof: Let \( x_i(t) = e^{p_i(t)} \), \( i = 1, 2 \). Then system (2) can be transformed into

\[
\frac{dp_1(t)}{dt} = \frac{r_1(t)}{e^{p_1(t)} + k_1(t)} - a_1(t) e^{p_1(t)} - b_1(t) e^{p_2(t)} - c_1(t) - d_1(t) u_1(t - \tau_1(t)),
\]
\[
\frac{du_1(t)}{dt} = -\alpha_1(t) u_1(t) + \beta_1(t) e^{p_1(t-\sigma_1(t))},
\]
\[
\frac{dp_2(t)}{dt} = \frac{r_2(t)}{e^{p_2(t)} + k_2(t)} - b_2(t) e^{p_1(t)} - a_2(t) e^{p_2(t)} - c_2(t) - d_2(t) u_2(t - \tau_2(t)),
\]
\[
\frac{du_2(t)}{dt} = -\alpha_2(t) u_2(t) + \beta_2(t) e^{p_2(t-\sigma_2(t))},
\]

Suppose that \( z(t) = (x_1(t), u_1(t), x_2(t), u_2(t))^T \) and \( \tilde{z}(t) = (\tilde{x}_1(t), \tilde{u}_1(t), \tilde{x}_2(t), \tilde{u}_2(t))^T \) are any two positive solutions of system (2). Consider the product system of system (8)

\[
\frac{dp_1(t)}{dt} = \frac{r_1(t)}{e^{p_1(t)} + k_1(t)} - a_1(t) e^{p_1(t)} - b_1(t) e^{p_2(t)} - c_1(t) - d_1(t) u_1(t - \tau_1(t)),
\]
\[
\frac{du_1(t)}{dt} = -\alpha_1(t) u_1(t) + \beta_1(t) e^{p_1(t-\sigma_1(t))},
\]
\[
\frac{dp_2(t)}{dt} = \frac{r_2(t)}{e^{p_2(t)} + k_2(t)} - b_2(t) e^{p_1(t)} - a_2(t) e^{p_2(t)} - c_2(t) - d_2(t) u_2(t - \tau_2(t)),
\]
\[
\frac{du_2(t)}{dt} = -\alpha_2(t) u_2(t) + \beta_2(t) e^{p_2(t-\sigma_2(t))},
\]
\[
\frac{dp_1(t)}{dt} = \frac{r_1(t)}{e^{p_1(t)} + k_1(t)} - a_1(t) e^{p_1(t)} - b_1(t) e^{p_2(t)} - c_1(t) - d_1(t) \tilde{u}_1(t - \tau_1(t)),
\]
\[
\frac{d\tilde{u}_1(t)}{dt} = -\alpha_1(t) \tilde{u}_1(t) + \beta_1(t) e^{\tilde{p}_1(t-\sigma_1(t))},
\]
\[
\frac{dp_2(t)}{dt} = \frac{r_2(t)}{e^{p_2(t)} + k_2(t)} - b_2(t) e^{p_1(t)} - a_2(t) e^{p_2(t)} - c_2(t) - d_2(t) \tilde{u}_2(t - \tau_2(t)),
\]
\[
\frac{d\tilde{u}_2(t)}{dt} = -\alpha_2(t) \tilde{u}_2(t) + \beta_2(t) e^{\tilde{p}_2(t-\sigma_2(t))}.
\]

Define

\[
V(t) = V_1(t) + V_2(t),
\]
where

\[ V_1(t) = |p_1(t) - \tilde{p}_1(t)| + |u_1(t) - \tilde{u}_1(t)| + |p_2(t) - \tilde{p}_2(t)| + |u_2(t) - \tilde{u}_2(t)|, \]

\[ V_2(t) = \int_{t-\tau_1(t)}^{t} \frac{\beta_1^u}{1 - \tilde{\sigma}_1(\xi_1^{-1}(s))} |p_1(s) - \tilde{p}_1(s)| \, ds \]
\[ + \int_{t-\tau_1(t)}^{t} \frac{d_1^u}{1 - \tilde{\tau}_1(\eta_1^{-1}(s))} |u_1(s) - \tilde{u}_1(s)| \, ds \]
\[ + \int_{t-\tau_2(t)}^{t} \frac{d_2^u}{1 - \tilde{\tau}_2(\eta_2^{-1}(s))} |u_2(s) - \tilde{u}_2(s)| \, ds \]
\[ + \int_{t-\sigma_2(t)}^{t} \frac{\beta_2^u}{1 - \tilde{\sigma}_2(\xi_2^{-1}(s))} |p_2(s) - \tilde{p}_2(s)| \, ds. \]

Calculating the right derivative \( D^+ V_1(t) \) of \( V_1(t) \) along the solutions of system (2), we have

\[ D^+ V_1(t) = \text{sgn}(p_1(t) - \tilde{p}_1(t))|\dot{p}_1(t) - \dot{\tilde{p}}_1(t)| + \text{sgn}(u_1(t) - \tilde{u}_1(t))|\dot{u}_1(t) - \dot{\tilde{u}}_1(t)| \]
\[ + \text{sgn}(p_2(t) - \tilde{p}_2(t))|\dot{p}_2(t) - \dot{\tilde{p}}_2(t)| + \text{sgn}(u_2(t) - \tilde{u}_2(t))|\dot{u}_2(t) - \dot{\tilde{u}}_2(t)| \]
\[ = \text{sgn}(p_1(t) - \tilde{p}_1(t))|r_1(t)\left[ \frac{1}{e^{p_1(t)} + k_1(t)} - \frac{1}{e^{\tilde{p}_1(t)} + k_1(t)} \right] \]
\[ - a_1(t)\left[ e^{p_1(t)} - e^{\tilde{p}_1(t)} \right] - b_1(t)\left[ e^{p_2(t)} - e^{\tilde{p}_2(t)} \right] \]
\[ - d_1(t)[u_1(t - \tau_1(t)) - u_1(t - \tau_1(t))] + \text{sgn}(u_1(t) - \tilde{u}_1(t)) \]
\[ \times \left| - \alpha_1(t)[u_1(t) - \tilde{u}_1(t)] + \beta_1(t)\left[ e^{p_1(t-\sigma_1(t))} - e^{\tilde{p}_1(t-\sigma_1(t))} \right] \right| \]
\[ + \text{sgn}(p_2(t) - \tilde{p}_2(t))|r_2(t)\left[ \frac{1}{e^{p_2(t)} + k_2(t)} - \frac{1}{e^{\tilde{p}_2(t)} + k_2(t)} \right] \]
\[ - b_2(t)\left[ e^{p_1(t)} - e^{\tilde{p}_1(t)} \right] - a_2(t)\left[ e^{p_2(t)} - e^{\tilde{p}_2(t)} \right] \]
\[ - d_2(t)[u_2(t - \tau_2(t)) - u_2(t - \tau_2(t))] + \text{sgn}(u_2(t) - \tilde{u}_2(t)) \]
\[ \times \left| - \alpha_2(t)[u_2(t) - \tilde{u}_2(t)] + \beta_2(t)\left[ e^{p_2(t-\sigma_2(t))} - e^{\tilde{p}_2(t-\sigma_2(t))} \right] \right| \]
\[ \leq -a_1^l|p_1(t) - \tilde{p}_1(t)| - a_1^l|u_1(t) - \tilde{u}_1(t)| \]
\[ - a_2^l|p_2(t) - \tilde{p}_2(t)| - a_2^l|u_2(t) - \tilde{u}_2(t)| \]
\[ + \frac{\beta_1^u}{(k_1^u)^2} |p_1(t) - \tilde{p}_1(t)| + b_1^l|p_2(t) - \tilde{p}_2(t)| \]
\[ + d_1^l|u_1(t - \tau_1(t)) - \tilde{u}_1(t - \tau_1(t))|. \]
\[ + \frac{r_2^u}{(k_2^u)^2} |p_2(t) - \tilde{p}_2(t)| + b_2^u|p_1(t) - \tilde{p}_1(t)| \\
+ d_2^u|u_2(t - \tau_2(t)) - \bar{u}_2(t - \tau_2(t))| \\
+ \beta_1^u|p_1(t - \sigma_1(t)) - \tilde{p}_1(t - \sigma_1(t))| + \beta_2^u|p_2(t - \sigma_2(t)) - \tilde{p}_2(t - \sigma_2(t))| \]

and

\[
D^+ V_2(t) = \frac{\beta_1^u}{1 - \dot{\sigma}_1(\xi_1^{-1}(t))} |p_1(t) - \tilde{p}_1(t)| - \beta_1^u|p_1(t - \sigma_1(t)) - \tilde{p}_1(t - \sigma_1(t))| \\
+ \frac{d_1^u}{1 - \dot{\tau}_1(\eta_1^{-1}(t))} |u_1(t) - \bar{u}_1(t)| - d_1^u|u_1(t - \tau_1(t)) - \bar{u}_1(t - \tau_1(t))| \\
+ \frac{d_2^u}{1 - \dot{\tau}_2(\eta_2^{-1}(t))} |u_2(t) - \bar{u}_2(t)| - d_2^u|u_2(t - \tau_2(t)) - \bar{u}_2(t - \tau_2(t))| \\
+ \frac{\beta_2^u}{1 - \dot{\sigma}_2(\xi_2^{-1}(t))} |p_2(t) - \tilde{p}_2(t)| - \beta_2^u|p_2(t - \sigma_2(t)) - \tilde{p}_2(t - \sigma_2(t))|.
\]

Then it follows from (10)–(12) that

\[
D^+ V(t) = D^+ V_1(t) + D^+ V_2(t) \\
\leq \left[-a_1^l + \frac{r_1^u}{(k_1^u)^2} + b_2^u + \frac{\beta_1^u}{1 - \dot{\sigma}_1(\xi_1^{-1}(t))}\right] |p_1(t) - \tilde{p}_1(t)| \\
+ \left[-a_1^l + \frac{d_1^u}{1 - \dot{\tau}_1(\eta_1^{-1}(t))}\right] |u_1(t) - \bar{u}_1(t)| \\
+ \left[-a_2^l + \frac{r_2^u}{(k_2^u)^2} + b_1^u + \frac{\beta_2^u}{1 - \dot{\sigma}_2(\xi_2^{-1}(t))}\right] |p_2(t) - \tilde{p}_2(t)| \\
+ \left[-a_2^l + \frac{d_2^u}{1 - \dot{\tau}_2(\eta_2^{-1}(t))}\right] |u_2(t) - \bar{u}_2(t)| \\
\leq - \min \left\{a_1^l - \frac{r_1^u}{(k_1^u)^2} - b_2^u - \frac{\beta_1^u}{1 - \dot{\sigma}_1(\xi_1^{-1}(t))}\right\} |p_1(t) - \tilde{p}_1(t)| \\
- \min \left\{a_1^l - \frac{d_1^u}{1 - \dot{\tau}_1(\eta_1^{-1}(t))}\right\} |u_1(t) - \bar{u}_1(t)| \\
- \min \left\{a_2^l - \frac{r_2^u}{(k_2^u)^2} - b_1^u - \frac{\beta_2^u}{1 - \dot{\sigma}_2(\xi_2^{-1}(t))}\right\} |p_2(t) - \tilde{p}_2(t)| \\
- \min \left\{a_2^l - \frac{d_2^u}{1 - \dot{\tau}_2(\eta_2^{-1}(t))}\right\} |u_2(t) - \bar{u}_2(t)|
\]
\[\begin{align*}
&\leq -\delta [p_1(t) - \tilde{p}_1(t) + \left| u_1(t) - \tilde{u}_1(t) \right| + |p_2(t) - \tilde{p}_2(t)| + |u_2(t) - \tilde{u}_2(t)|] \\
&= -\delta V_1(t).
\end{align*}\]

Integrating the above inequality over internal \([T_0, t]\), it follows that for \(t \geq T_0\),

\[V(t) + \delta \int_{T_0}^{t} V_1(s) \, ds \leq V(T_0) < +\infty.\]

Then

\[\lim_{t \to +\infty} \sup_{t \geq T_0} \int_{T_0}^{t} V_1(s) \, ds \leq \frac{V(T_0)}{\delta} < +\infty.\]

Thus we have

\[\lim_{t \to +\infty} V_1(t) = 0.\]

By the definition of \(V_1(t)\), we can conclude that

\[\lim_{t \to +\infty} |x_i(t) - \bar{x}_i| = 0, \quad \lim_{t \to +\infty} |u_i(t) - \bar{u}_i| = 0, \quad i = 1, 2.\]

The proof of Theorem 3.1 is complete. \hfill \blacksquare

**Theorem 3.2:** If (H1)–(H3) hold, then there exists a unique globally asymptotically stable nonnegative almost periodic solution of systems (2).

**Proof:** By (3) and Lemma 2.4, there exists a bounded positive solution \(z(t)\) of system (2), \(t \geq 0\). Suppose that \(z(t) = (x_1(t), u_1(t), x_2(t), u_2(t))^T\) is any solution of system (2), then there exists a sequence \(\{t'_{k}\}, t'_{k} \to +\infty \text{ as } k \to +\infty\), such that \((x_1(t + t'_{k}), u_1(t + t'_{k}), x_2(t + t'_{k}), u_2(t + t'_{k}))^T\) is a solution of the following system:

\[
\begin{align*}
\frac{dx_{1}(t)}{dt} &= x_{1}(t)\left[\frac{r_{1}(t + t'_{k})}{x_{1}(t) + k_{1}(t + t'_{k})} - a_{1}(t + t'_{k})x_{1}(t) - b_{1}(t + t'_{k})x_{2}(t)
\right. \\
&\quad \left. - c_{1}(t + t'_{k}) - d_{1}(t + t'_{k})u_{1}(t - \tau_{1}(t + t'_{k}))\right], \\
\frac{du_{1}(t)}{dt} &= -\alpha_{1}(t + t'_{k})u_{1}(t) + \beta_{1}(t + t'_{k})x_{1}(t - \sigma_{1}(t + t'_{k})), \\
\frac{dx_{2}(t)}{dt} &= x_{2}(t)\left[\frac{r_{2}(t + t'_{k})}{x_{2}(t) + k_{2}(t + t'_{k})} - b_{2}(t + t'_{k})x_{1}(t) - a_{2}(t + t'_{k})x_{2}(t)
\right. \\
&\quad \left. - c_{2}(t + t'_{k}) - d_{2}(t + t'_{k})u_{2}(t - \tau_{2}(t + t'_{k}))\right], \\
\frac{du_{2}(t)}{dt} &= -\alpha_{2}(t + t'_{k})u_{2}(t) + \beta_{2}(t + t'_{k})x_{2}(t - \sigma_{2}(t + t'_{k})).
\end{align*}\]

From above discussion and Lemma 2.3, we have that \((x_1(t + t'_{k}), u_1(t + t'_{k}), x_2(t + t'_{k}), u_2(t + t'_{k}))^T\) and \((x'_{1}(t + t'_{k}), u'_{1}(t + t'_{k}), x'_{2}(t + t'_{k}), u'_{2}(t + t'_{k}))^T\) are uniformly bounded. Thus \((x_1(t + t'_{k}), u_1(t + t'_{k}), x_2(t + t'_{k}), u_2(t + t'_{k}))^T\) are uniformly bounded.
and equi-continuous. In view of Ascoli’s theorem, there exists a uniformly convergent subsequence \( \{(x_1(t + t_k), u_1(t + t_k), x_2(t + t_k), u_2(t + t_k))^T \} \subseteq \{(x_1(t + t'_k), u_1(t + t'_k), x_2(t + t'_k), u_2(t + t'_k))^T \} \) such that for any \( \varepsilon > 0 \), there exists a \( k(\varepsilon) > 0 \) with the property that if \( m, k \geq k(\varepsilon) \), then

\[
|x_i(t + t_m) - x_i(t + t_k)| < \varepsilon, \quad |u_i(t + t_m) - u_i(t + t_k)| < \varepsilon, \quad i = 1, 2,
\]

which implies that \( (x_1(t + t_k), u_1(t + t_k), x_2(t + t_k), u_2(t + t_k))^T \) is asymptotically almost periodic function, then \( z(t + t_k) = (x_1(t + t_k), u_1(t + t_k), x_2(t + t_k), u_2(t + t_k))^T \) is the sum of an almost periodic function \( \Upsilon_1(t + t_k) = (\mu_1(t + t_k), v_1(t + t_k), \kappa_1(t + t_k), \nu_1(t + t_k))^T \) and a continuous function \( \Upsilon_2(t + t_k) = (\mu_2(t + t_k), v_2(t + t_k), \kappa_2(t + t_k), \nu_2(t + t_k))^T \) defined on \( R \) such that

\[
\Upsilon(t + t_k) = \Upsilon_1(t + t_k) + \Upsilon_2(t + t_k), \quad t \in R,
\]

where \( \Upsilon_1(t + t_k) \) and \( \Upsilon_2(t + t_k) \) satisfy

\[
\lim_{k \to +\infty} \Upsilon_1(t + t_k) = \Upsilon_1(t), \quad \lim_{k \to +\infty} \Upsilon_2(t + t_k) = 0,
\]

where \( \Upsilon_1(t) \) is an almost periodic function.

Now we will prove that \( \Upsilon_1(t + t_k) = (\mu_1(t + t_k), v_1(t + t_k), \kappa_1(t + t_k), \nu_1(t + t_k))^T \) is an almost periodic solution of system (2). In view of (H1) and (H2), there exists a sequence \( \{t_n\} \) with \( t_n \to +\infty \) as \( n \to +\infty \) such that

\[
\begin{align*}
& r_i(t + t_n) \to r_i(t), \quad a_i(t + t_n) \to a_i(t), \quad b_i(t + t_n) \to b_i(t), \\
& c_i(t + t_n) \to c_i(t), \quad d_i(t + t_n) \to d_i(t), \quad \alpha_i(t + t_n) \to \alpha_i(t), \\
& \beta_i(t + t_n) \to \beta_i(t), \quad \tau_i(t + t_n) \to \tau_i(t), \quad \sigma_i(t + t_n) \to \sigma_i(t), \quad k_i(t + t_n) \to k_i(t),
\end{align*}
\]

as \( n \to +\infty \), where \( i = 1, 2 \). Obviously, \( z(t + t_n) \to \Upsilon_1(t) \) as \( n \to +\infty \), then

\[
\frac{d\mu_1(t)}{dt} = \lim_{n \to +\infty} \frac{dx_1(t + t_n)}{dt} = \left. \lim_{n \to +\infty} x_1(t + t_n) \right| \frac{r_1(t + t_n)}{x_1(t + t_n) + k_1(t + t_n)} - a_1(t + t_n)x_1(t + t_n) - b_1(t + t_n)x_2(t + t_n) - c_1(t + t_n) - d_1(t + t_n)u_1(t + t_n - \tau_1(t + t_n)) = \mu_1(t) \left[ \frac{r_1(t)}{\mu_1(t) + k_1(t)} - a_1(t)\mu_1(t) - b_1(t)\kappa_1(t) - c_1(t) - d_1(t)v_1(t - \tau_1(t)) \right].
\]
Applying a similar argument as that in (15), we get
\[ \frac{dv_1(t)}{dt} = -\alpha_1(t)v_1(t) + \beta_1(t)\mu_1(t) + \sigma_1(t), \]
\[ \frac{dk_1(t)}{dt} = k_1(t) \left[ \frac{r_2(t)}{k_1(t) + k_2(t)} - b_2(t)\mu_1(t) - a_2(t)k_1(t) - c_2(t) - d_2(t)v_1(t - \tau_2(t)) \right], \]
\[ \frac{dv_1(t)}{dt} = -\alpha_2(t)v_2(t) + \beta_2(t)k_1(t - \sigma_2(t)). \]

This proves that \( z(t) \) satisfies system (2) and \( z(t) \) is a nonnegative almost periodic solution, by Theorem 3.1, it follows that there exists a unique globally asymptotically stable nonnegative almost periodic solution of system (2). The proof of Theorem 3.2 is complete.

**Remark 3.1:** Generally, it is difficult to construct a suitable Lyapunov functional to obtain the results we need. In this paper, we skillfully construct a suitable Lyapunov functional which does not appear in previous papers to achieve our goal. In this sense, the paper has novelty of techniques.

4. **An example and its computer simulations**

In this section, we present an example with its numerical simulations to demonstrate the analytical predictions obtained in the previous section.

**Example 4.1:** Consider the following system
\[ \frac{dx_1(t)}{dt} = x_1(t) \left[ \frac{r_1(t)}{x_1(t) + k_1(t)} - a_1(t)x_1(t) - b_1(t)x_2(t) - c_1(t) - d_1(t)u_1(t - \tau_1(t)) \right], \]
\[ \frac{du_1(t)}{dt} = -\alpha_1(t)u_1(t) + \beta_1(t)x_1(t - \sigma_1(t)), \]
\[ \frac{dx_2(t)}{dt} = x_2(t) \left[ \frac{r_2(t)}{x_2(t) + k_2(t)} - b_2(t)x_1(t) - a_2(t)x_2(t) - c_2(t) - d_2(t)u_2(t - \tau_2(t)) \right], \]
\[ \frac{du_2(t)}{dt} = -\alpha_2(t)u_2(t) + \beta_2(t)x_2(t - \sigma_2(t)), \]

where \( r_1(t) = 0.4 + 0.3 \sin \sqrt{2}t, r_2(t) = 0.3 + 0.1 \cos \sqrt{2}t, \]
\( k_1(t) = 0.04 + 0.03 \cos \sqrt{2}t, \]
\( k_2(t) = 0.02 + 0.01 \sin \sqrt{2}t, b_1(t) = 0.3 + 0.2 \cos \sqrt{5}t, b_2(t) = 0.1 + 0.3 \sin \sqrt{2}t, c_1(t) = 0.1 + 0.2 \sin \sqrt{2}t, c_2(t) = 0.2 + 0.1 \sin \sqrt{5}t, d_1(t) = 0.3 + 0.2 \sin \sqrt{5}t, d_2(t) = 0.4 + 0.1 \)
Figure 1. Dynamical behavior of system (16): times series of $x_1, u_1, x_2, u_2$.

$\sin \sqrt{2}t, a_1(t) = 4 + 0.2 \sin \sqrt{2}t, a_2(t) = 3 + 0.2 \cos \sqrt{2}t, \alpha_1(t) = 4 + 0.3 \cos \sqrt{2}t, \alpha_2(t) = 3 + 0.4 \sin \sqrt{5}t, \beta_1(t) = 0.4 + 0.2 \sin \sqrt{5}t, \beta_2(t) = 0.3 + 0.1 \cos \sqrt{5}t, \tau_i(t) = \sigma_i(t) = 0.023 \sin t, i = 1, 2$. Then we get $(r_1/k_1)^u = 70, (r_2/k_2)^u = 40, a_1^l = 3.8, a_2^l = 2.8, x_1^* = 18.4, x_2^* = 15.7, \beta_1^u = 0.6, \beta_2^u = 0.4, \alpha_1^l = 3.7, \alpha_2^l = 2.6, u_1^* = 2.9, u_2^* = 2.5, b_1^u = 0.5, b_2^u = 0.4, d_1^l = 0.5, d_2^l = 0.5, \delta_1 = 1.21, \delta_2 = 1.12, \delta_3 = 1.09, \delta_4 = 1.24, \delta = 1.09$. We can easily check that all the conditions (H1)–(H3) in Theorem 3.2 are fulfilled. Then we can draw the conclusion that system (16) has a unique globally asymptotically stable nonnegative almost periodic solution which is shown in Figure 1.

5. Conclusions

In the present paper, a non-autonomous competing model with time-varying delays and feedback controls has been investigated. With the aid of the comparison theorem of differential equations and constructing a suitable Lyapunov functional, we establish some sufficient conditions which guarantee the existence of a unique globally asymptotically stable nonnegative almost periodic solution of the model. We find that under some suitable conditions, the two competitive species can come to a dynamical balance. The time delays and feedback control effect play an important role in affecting the fate of two competitive species. Our results are new and complement of the existed results in [36]. However, in real complex situation, considering that the discrete competing models are more appropriate to describe the dynamics relationship between two competitive species, thus the investigation on the discrete competitive species models has some theoretical and practical meanings and value to a certain extent. We will leave this topic for our future work.

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References
[1] J.O. Alzabut, G.T. Stamovb, and E. Sermutlu, Positive almost periodic solutions for a delay logarithmic population model, Math. Comput. Model. 53(1–2) (2011), pp. 161–167.
[2] S. Baek, W. Ko, and I. Ahn, Coexistence of a one-prey two-predators model with ratio-dependent functional responses, Appl. Math. Comput. 219(4) (2012), pp. 1897–1908.
[3] L.M. Cai, J.Y. Yu, and G.T. Zhu, A stage-structured predator-prey model with Beddington-DeAngelis functional response, J. Appl. Math. Comput. 26(1–2) (2008), pp. 85–103.
[4] F.D. Chen, Almost periodic solution of the non-autonomous two-species competitive model with stage structure, Appl. Math. Comput. 181(1) (2006), pp. 685–693.
[5] F.D. Chen, Z. Li, and Y.J. Huang, Note on the permanence of a competitive system with infinite delay and feedback controls, Nonlinear Anal. Real World Appl. 8(2) (2007), pp. 680–687.
[6] Z.J. Du and Z.S. Feng, Periodic solutions of a neutral impulsive predator-prey model with Beddington-DeAngelis functional response with delays, J. Comput. Appl. Math. 258 (2014), pp. 87–98.
[7] A.M. Fink, Almost Periodic Differential Equations, Lecture Notes in Mathematics Vol. 377, Springer, Berlin, 1974.
[8] J.B. Geng and Y.H. Xia, Almost periodic solutions of a nonlinear ecological model, Commun. Nonlinear Sci. Numer. Simul. 16(6) (2011), pp. 2575–2597.
[9] L.N. Guin, Existence of spatial patterns in a predator-prey model with self- and cross-diffusion, Appl. Math. Comput. 226 (2014), pp. 320–335.
[10] Y.X. Guo, Exponential stability analysis of traveling waves solutions for nonlinear delayed cellular neural networks, Dyn. Syst. Int. J. 32(4) (2018), pp. 490–503.
[11] Y.X. Guo, Globally robust stability analysis for stochastic Cohen-Grossberg neural networks with impulse control and time-varying delays, Ukrainian Math. J. 69(8) (2018), pp. 1220–1233.
[12] M.A. Han, X.Y. Hou, L.J. Sheng, and C.Y. Wang, Theory of rotated equations and applications to a population model, Discrete Contin. Dyn. Syst. A 38(4) (2018), pp. 2171–2185.
[13] H.F. Hao and W.T. Li, Positive periodic solutions of a class of delay differential system with feedback control, Appl. Math. Comput. 148(1) (2004), pp. 35–46.
[14] P. Huang, X. Li, and B. Liu, Almost periodic solutions for an asymmetric oscillation, J. Differ. Equ. 263(12) (2017), pp. 8916–8946.
[15] T.K. Kar and B. Ghosh, Sustainability and optimal control of an exploited prey predator system through provision of alternative food to predator, Biosystems 109(2) (2012), pp. 220–232.
[16] W. Ko and I. Ahn, A diffusive one-prey and two-competing-predator system with a ratio-dependent functional response: I, long time behavior and stability of equilibria, J. Math. Anal. Appl. 397(1) (2013), pp. 9–28.
[17] Y.K. Li and X.L. Fan, Existence and globally exponential stability of almost periodic solution for Cohen-Grossberg BAM neural networks with variable coefficients, Appl. Math. Model. 33(4) (2009), pp. 2114–2120.
[18] M.M. Li and J.R. Wang, Exploring delayed Mittag-Leffler type matrix functions to study finite time stability of fractional delay differential equations, Appl. Math. Comput. 324 (2018), pp. 254–265.
[19] Y.K. Li, T.W. Zhang, and Z.W. Xing, The existence of nonzero almost periodic solution for Cohen-Grossberg neural networks with continuously distributed delays and impulses, Neurocomputing 73(16–18) (2010), pp. 3105–3113.
[20] Y.N. Li, Y.G. Sun, and F.W. Meng, New criteria for exponential stability of switched time varying systems with delays and nonlinear disturbances, Nonlinear Anal.: Hybrid Syst. 26 (2017), pp. 284–291.

[21] X. Liao, Z. Ouyang, and S. Zhou, Permanence of species in nonautonomous discrete Lotka-Volterra competitive system with delays and feedback controls, J. Comput. Appl. Math. 211(1) (2008), pp. 1–10.

[22] P. Liu and Y.K. Li, Multiple positive periodic solutions of nonlinear functional differential system with feedback control, J. Math. Anal. Appl. 288(2) (2003), pp. 819–832.

[23] Q. Liu, Y.L. Liu, and X. Pan, Global stability of a stochastic predator-prey system with infinite delays, Appl. Math. Comput. 235 (2014), pp. 1–7.

[24] M.A. Menouer, A. Moussaoui, and E.H. Ait Dads, Existence and global asymptotic stability of positive almost periodic solution for a predator-prey system in an artificial lake, Chaos Solitons Fractals 103 (2017), pp. 271–278.

[25] Q.H. Pan, H.Y. Wang, L.Y. Chen, Z. Huang, and M.F. He, Gause’s principle in interspecific competition of the cyclic predator-prey system, Phys. A Statist. Mech. Appl. 396 (2014), pp. 108–113.

[26] B. Sahoo and S. Poria, Effects of supplying alternative food in a predator-prey model with harvesting, Appl. Math. Comput. 234 (2014), pp. 150–166.

[27] S. Sarwardi, P.K. Mandal, and S. Ray, Analysis of a competitive prey-predator system with a prey refuge, Biosystems 110(3) (2012), pp. 133–148.

[28] V.Yu. Slyusarchuk, Almost periodic solutions of nonlinear discrete systems that can be not almost periodic in Bochner’s sense, J. Math. Sci. 212(3) (2016), pp. 335–348.

[29] M. Vasilova, Asymptotic behavior of a stochastic Gilpin–Ayala predator-prey system with time-dependent delay, Math. Comput. Model. 57(3–4) (2013), pp. 764–781.

[30] C. Wang, Almost periodic solutions of impulsive BAM neural networks with variable delays on time scales, Commun. Nonlinear Sci. Numer. Simul. 19(8) (2014), pp. 2828–2842.

[31] W.T. Wang, Positive pseudo almost periodic solutions for a class of differential iterative equations with biological background, Appl. Math. Lett. 46 (2015), pp. 106–110.

[32] Q. Wang and B.X. Dai, Almost periodic solution for n-species Lotka-Volterra competitive system with delay and feedback controls, Appl. Math. Comput. 200(1) (2008), pp. 133–146.

[33] K. Wang and Y.L. Zhu, Stability of almost periodic solution for a generalized neutral-type neural networks with delays, Neurocomputing 73(16–18) (2010), pp. 3300–3307.

[34] Y.H. Xia, New results on the global asymptotic stability of a Lotka-Volterra system, J. Appl. Math. Comput. 36(1–2) (2011), pp. 117–128.

[35] J.B. Xu, Z.D. Teng, and H.J. Jiang, Permanence and global attractivity for discrete nonautonomous two-species Lotka-Volterra competitive system with delays and feedback controls, Period. Math. Hungar. 63(1) (2011), pp. 19–45.

[36] L.J. Zhang, H.F. Huo, and J.F. Chen, Asymptotic behavior of the nonautonomous competing system with feedback controls, J. Biomath. 16(4) (2001), pp. 405–410.

[37] T.W. Zhang, Y.K. Li, and Y. Ye, Persistence and almost periodic solutions for a discrete fishing model with feedback control, Commun. Nonlinear Sci. Numer. Simul. 16(3) (2011), pp. 1564–1573.

[38] Z. Zhao, Z.X. Li, and L.S. Chen, Existence and global stability of periodic solution for impulsive predator-prey model with diffusion and distributed delay, J. Appl. Math. Comput. 33(1–2) (2010), pp. 389–410.