Enhanced subthreshold $e^+e^-$ production in short laser pulses

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The emission of $e^+e^-$ pairs off a probe photon propagating through a polarized short-pulsed electromagnetic (e.g. laser) wave field is analyzed. A significant increase of the total cross section of pair production in the subthreshold region is found for decreasing laser pulse duration even in case of moderate laser pulse intensities.

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The history of the study of $e^+e^-$ production in $\gamma'\gamma$ interaction starts with the pioneering work by Breit and Wheeler [1] published in 1934. About thirty years later, Reiss [2] and Narozhnyi, Nikishov and Ritus [3, 4] have analyzed the $e^+e^-$ emission off a photon $\gamma'$ propagating in the field of an intensive polarized monochromatic electromagnetic (e.m.) plane. The $e^+e^-$ production probabilities were found using the non-perturbative Volkov solutions for the electron and positron wave functions [5].

If one identifies the external e.m. field with a laser pulse then most of the early work considers long lasting pulses where the temporal shape can be neglected. We denote this approach as the infinite pulse approximation (IPA). In IPA, electrons $e^-$ and positrons $e^+$ become quasi-particles with effective quasi-momenta and effective (dressed) masses. Differential and total probabilities of the $e^+e^-$ pair emission depend on the reduced strength of the e.m. field $A^\mu$, $\xi^2 = -e^2(A^2)/(M^2_e) = s^2/s^2_{thr}$, where $M_e$ is the electron mass (we use $c = \hbar = 1$, $e^2/4\pi = \alpha = 1/137$). Furthermore, the dimensionless variable $\xi = s_{thr}$ is introduced, where $s$ is the square of the total energy in the center of mass system (c.m.s.) of the Breit-Wheeler process $\gamma' + \gamma \rightarrow e^+ + e^-$ and $s_{thr} = 4M_e^2$ is its threshold value. The Ritus variable is then defined by $\kappa = 2\xi/\xi$, [3]. The case of $\xi > 1$ corresponds essentially to multi-photon processes. Within IPA, the minimum number of photons $\gamma$ in the reaction $\gamma' + n\gamma \rightarrow e^+ + e^-$ is defined as $n_{min} = I(\xi) + 1$, where $I(\xi)$ is the integer part of $\xi$. First evidence of the multi-photon Breit-Wheeler process with $\xi = 3.83$ and $0.1 < \xi < 0.35$ was detected at SLAC in the E-144 experiment [6], where the application of IPA is justified since the used laser pulses contain around $10^6$ cycles in a shot.

The rapidly evolving laser technology [7] can provide the laser power up to $10^{24} - 10^{25}$ W/cm$^2$ in near future which is sufficient for the formation of positrons from cascade processes in the photon-electron-positron plasma [8–10] generated by photon-laser [11, 13], electron-laser [14, 15] or laser-laser interactions [16, 17] (see [18] for surveys). The next generation of optical laser beams are expected to be essentially short (femtosecond duration) with only a few oscillation of the e.m. field in the pulse to be expected at ELI [19] and CLF [20] facilities. This requires the generalization of the IPA multi-photon process $\gamma' + n\gamma \rightarrow e^+ + e^-$ to a finite pulse duration. Formally, this generalization may be done in a straightforward manner by substituting the expansion in Fourier series into Fourier integrals with taking into account the Volkov solution for the finite wave field. In practice, an evaluation of the total cross section requires the calculation of five-dimensional integrals with rapidly oscillating integrands which is rather demanding. Therefore, previous considerations are often restricted to the analysis of the three-dimensional differential cross sections, see for example [12] for finite beam size effects in $e^+e^-$ pair production (cf. also [21] and references therein).

The aim of the present Letter is to elaborate a method for the calculation of the total cross section in the subthreshold (multi-photon) region accounting for the effect of finite laser pulse duration in $e^+e^-$ pair production off a probe photon. We denote such a process with a finite pulse and plane wave fronts as finite pulse approximation (FPA). In this case, the in/out fermion states refer to the vacuum. Moreover, due to the modulation of the pulse envelope function, the power spectrum contains frequencies $> \omega$ (see below) which enhance the pair production in the subthreshold region even for moderately strong laser intensities.

We consider the e.m. four-potential $A \sim (\phi, A)$ in FPA, depending solely on the invariant phase $\phi = k \cdot x$,

$$A(\phi) = f(\phi)(a_1 \cos \phi + a_2 \sin \phi) ,$$

where $|a_1| = |a_2| = a$, $a_1a_2 = 0$ for circular polarization. We employ here the envelope function $f(\phi) = 1/\cosh(\phi/\Delta)$, where $\Delta = \pi \tau_0/\pi N$, and $N$ characterizes the number of cycles in a pulse; $\tau_0 = 2\pi/\omega$ is the time of one cycle for the laser frequency $\omega$. Thus, $\tau$ is the time scale of the pulse duration. The case of pulses obeying $\omega \tau \gg 1$ has been analyzed in [22].

Utilizing the e.m. potential [11] in the Volkov solutions leads to two significant modifications of the transition amplitude. Besides physical asymptotic momenta and masses, the finite time $\tau$ requires Fourier integrals in the integrand of invariant amplitudes, and the discrete harmonics become continuous. Thus, the $S$ matrix element
of the process $\gamma' \to e^+(\gamma) + e^-(\gamma)$, where $e^\pm(\gamma)$ refers to
Volkov states in the field \([11]\), is expressed as

$$S = \int_\xi^\infty dl \, M(l) \left( \frac{2(\pi)^4 l^4 \delta(k' + lk - p - p')}{\sqrt{2p_0^2 p'_0 2\omega^2}} \right).$$

(2)

where the transition matrix $M(l)$, similarly to the case of the non-linear Compton effect \([22, 29]\) as a crossed channel of the pair production, consists of four terms

$$M(l) = \sum_{m=0}^{3} M^{(m)} C^{(m)}(l),$$

(3)

where

$$C^{(m)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi \, \chi^{(m)}(\phi) \, e^{i\phi - \imath P(\phi)}.$$

(4)

Here, $\chi^{(m)} = (1, f^2, f, f(\phi) \sin \phi)$ with $m = 0, 1, 2, 3$ and

$$P(\phi) = zP_0(\phi, \phi_0) - \xi^2 \zeta u \int_0^{\phi} d\phi' \, f^2(\phi'),$$

(5)

$$P_0(\phi, \phi_0) = \int_{-\infty}^{\phi} d\phi' \cos(\phi' - \phi_0) f(\phi'),$$

(6)

where $u = (k \cdot k')^2 / (4k \cdot p(k \cdot p'))$, $z = 2\xi \sqrt{u(u_0 - u)/u_0}$, $u_0 = l/\zeta$. The angle $\phi_0$ is related to the azimuthal angle of the positron in the $e^+e^-$ rest frame by $\phi_0 = \phi_p + \pi$ and can be determined through invariants $\alpha_{1,2} = e ((a_{1,2} \cdot \rho)/(k \cdot p) - (a_{1,2} \cdot p')/(k' \cdot p'))$ as $\cos \phi_0 = \alpha_1 / z$, $\sin \phi_0 = \alpha_2 / z$. Here, $p_{e^-} \equiv p' \sim (p_0', p')$ and $p_{e^+} \equiv p \sim (p_0, p)$. The transition operators $M^{(2,3)}$ are the same as in IPA \([9]\), while the operators $M^{(0,1)} = \tilde{u}_{p'} M^{(0,1)} v_p$, read now

$$M^{(0)} = \tilde{f}', \quad M^{(1)} = \frac{e^2 A k \bar{k} k' \bar{k} A}{4(k \cdot p)(k \cdot p')}.$$  

(7)

where $u_{p'}$ and $v_p$ are the free-field Dirac spinors of the outgoing electron and positron, respectively; $\epsilon'$ is the polarization four-vector of the probe photon $\gamma'$ with four-momentum $k' \sim (\omega', k')$, and $k \sim (\omega, k)$ is the four-momentum of the e.m. (laser) field \([11]\). Feynman's slash notation is employed, e.g. $\bar{A} = A / \gamma$, as four-product with the Dirac $\gamma$ matrices. The integrand of the function $C^{(0)}$ does not contain the envelope function and needs a regularization, e.g. using a prescription given in Ref. \([22]\)

$$C^{(0)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi \, e^{i\phi - \imath P(\phi)} \times (z \cos(\phi - \phi_0) f(\phi) - \xi^2 \zeta u f^2(\phi)) .$$

(8)

The probability is normalized to some time unit. In IPA, one can use the time of one cycle, $\tau_0$. In FPA, a proper time unit is provided by the pulse width, which is $N$ times greater, $\tau = N \tau_0$, where $N$ is the number of the cycles in a pulse. Therefore, for a convenient comparison of IFA and FPA results, the latter one is scaled by $1/N$. Thus, the probability of the $e^+e^-$ pair emission reads

$$W = \frac{\alpha M^2}{4\omega' N} \int \frac{d\phi_p}{2\pi} \int_1^{\infty} dl \int_0^{\infty} dw \frac{w(l, \xi, u, \phi_0)}{w^2/2 - \Gamma^2} \left( \frac{\xi^2}{2u - 1} \right),$$

(9)

$$w(l, \xi, u, \phi_0) = (2u_0 + 1)[C^{(0)}(l)]^2 + [C^{(3)}(l)]^2 + \Re C^{(0)}(l) \left( \xi^2 C^{(1)}(l) - \frac{2}{\zeta} [\alpha_1 C^{(2)}(l) + \alpha_2 C^{(3)}(l)] \right) \epsilon,$$

(10)

where $\alpha_0 = 1/\Delta$, $\epsilon = \frac{1}{\xi^2} \frac{\sin(\phi - \phi_0) f(\phi)\bar{f}(\phi)'}{\epsilon}$ is a rather small contribution for a finite pulse duration $\Delta = \pi N$ with $N \geq 2$ because of (i) the factor $1/\Delta$ and (ii) the derivative $f'/(\phi)$ in the integrand has a maximum value at the boundaries of the pulse with $\phi \sim 0.9\Delta$, where this function is suppressed. In fact, the numerical evaluation shows that the contribution of $\mathcal{O}(\Delta)$ can be omitted (we find $|\mathcal{O}(\Delta)| < 0.1 (0.05)$ for $\Delta = 2\pi (5\pi)$). This approximation allows to express the basic functions $C^{(m)}(l)$ via new functions $Y_l$ and $X_l$

$$C^{(0)}(l) = \tilde{Y}_l(z) e^{i\phi_{\lambda}}, \quad C^{(1)}(l) = Y_l(z) e^{i\phi_{\xi}},$$

(11)

$$C^{(2)}(l) = \frac{1}{2} \left( Y_{l+1}(z) e^{i(l+1)\phi_{\lambda}} + Y_{l-1}(z) e^{i(l-1)\phi_{\xi}} \right),$$

(12)

with

$$\tilde{Y}_l(z) = \frac{\Delta}{2} \int_0^\infty d\psi f^1(\psi + \phi_{\lambda}) e^{i\psi - izf(\psi + \phi_{\xi}) \sin \psi},$$

$$Y_l(z) = 1 / 2 \int_{-\infty}^{\infty} d\psi f^1(\psi + \phi_{\lambda}) e^{i\psi - izf(\psi + \phi_{\xi}) \sin \psi},$$

$$X_l(z) = 1 / 2 \int_{-\infty}^{\infty} d\psi f^1(\psi + \phi_{\lambda}) e^{i\psi - izf(\psi + \phi_{\xi}) \sin \psi},$$

($$f^1(m) = \phi \epsilon \frac{\sin(\xi^2 \zeta u \tan \phi_{\Delta})}{\Delta}.$$)

(13)

The function $C^{(3)}$ emerges from $C^{(2)}$ by the substitutions $1/2 \rightarrow 1/2i$ and sign showed in the bracket to $\pi - \pi$. In the last line, $m = 1, 2$ is a label on the l.h.s., while on the r.h.s. it is the power of the envelope function, as follows from Eqs. \([11], [22]\): the exponential term results from an analytic evaluation of the last term in Eq. (5) for the chosen envelope function.

The partial probability $w(l)$ in Eq. \([10]\) reads

$$w(l, \xi, u, \phi_p) = 2\tilde{Y}_l(z) + \xi^2 (2u - 1) \times \left( Y_{l-1}^2(z) + Y_{l+1}^2(z) - 2\Re \tilde{Y}_l(z) X_l(z) \right).$$

(14)
which resembles the expression for the probabilities \( w_n \) in case of IPA (cf. Ref. 4) arising upon the substitutions \( \int dlw(l) = \sum \con w_n, \int Y^2 = J^2_{n\xi}, Y^2_{n\pm 1} = J^2_{n\pm 1}, \) Re\( Y(l)X^*_n(z) \to J^2_{n} \) with Bessel functions \( J_n \).

In the case of small field intensity, \( \xi \ll 1 \), implying \( z \ll 1 \), and denoting \( l = n + \epsilon \), where \( n \) is the integer part of \( l \), one can use the following decomposition

\[
Y_1 \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} dl \psi e^{i\psi - izf(\psi + \phi_0)} \sin \psi f(\psi + \phi_0) \\
- \frac{1}{2\pi} \int_{-\infty}^{\infty} dl \sum \con k \frac{(iz)^k}{k!} \sin \psi e^{i(n+\epsilon)\psi} f^{k+1}(\psi + \phi_0)
\]

and analog for the function \( X_1(z) \) with the substitution \( f^{k+1}(\psi + \phi_0) \) for \( f^k(\psi) \).

The dominant contribution to the integral with a rapidly oscillating integrand stems from the term with \( k = n \), which results in

\[
Y_{k+\epsilon}(z) \approx \frac{z^n}{2^n k!} e^{-i\epsilon\phi_0} f_F^{(k+1)}(\epsilon), \\
X_{k+\epsilon}(z) \approx \frac{z^n}{2^n k!} e^{-i\epsilon\phi_0} f_F^{(k+1)}(\epsilon),
\]

where the function \( f_F^{(k)}(\epsilon) \) is the Fourier transform of the function \( f^k(\psi) \). For the above envelope function it can be calculated analytically using the theory of residues. Results of the leading orders \( n = 1 \) are

\[
Y_{0+\epsilon}(z) = \frac{\Delta e^{-\pi|\epsilon|\Delta/2}}{1 + e^{-\pi|\epsilon|\Delta}} e^{-i\epsilon\phi_0}, \\
Y_{1+\epsilon}(z) = \frac{\Delta^2|\epsilon| e^{-\pi|\epsilon|\Delta/2}}{2} e^{-i\epsilon\phi_0}, \\
X_{1+\epsilon}(z) = \frac{\Delta(\Delta^2\epsilon^2 + 1) e^{-\pi|\epsilon|\Delta/2}}{4} e^{-i\epsilon\phi_0}.
\]

The representation of Eq. (16) evidences (i) a fast decrease of \( Y_{n+\epsilon} \) with increasing \( |\epsilon| \) and (ii) the \( \phi_0 \) dependence disappears in \( Y_{n+\epsilon} \) and \( X_{n+\epsilon} \). This allows to express the integral over \( dl \) in (9) in a form useful for a qualitative analysis:

\[
\left( \frac{\omega N}{\alpha M^2} \right) W^{FPA} = \int_{-\infty}^{\infty} d\epsilon \int_0^{n_0} du \frac{w(n = n_0, \epsilon, \xi, u)}{\sqrt{u - 1}} \\
+ \sum_{n=n_0+1} \int_{-\infty}^{\infty} d\epsilon \int_0^{n_0} du \frac{w(n, \epsilon, \xi, u)}{\sqrt{u - 1}}
\]

with \( u_0 = (n + \epsilon)/\xi; n_0 = 1 \) for \( \xi \leq 1 \), and \( n_0 = I(\xi) \) for \( \xi > 1 \); The lower limit in integral over \( \epsilon \) in the second term reads \( \nu = -n \) for \( \epsilon > 1 \) and \( n = n_0 + 1 \), and \( \nu = -1 \) in other cases. This equation shows that, contrary to IPA where at given \( \xi > 1 \) (i.e. below threshold, \( s < s_{thr} = 4M_2^2 \)) only harmonics with \( n > I(\xi + 1) \) contribute, in FPA the harmonic with \( n = I(\xi) \) also contributes.

Consider, as a check of the normalization, the pair production above threshold with \( \xi = 1 - \delta s/2 < 1 \), where \( \delta s_0 \) is the energy excess \( \delta s_0 = s - s_{thr} \). Utilizing the explicit expressions (17) for the leading contribution \( Y_1 \) one can get a relation between emission probabilities in IPA (cf. 4) and FPA:

\[
W^{FPA} = W^{IPA}(n = 1, \xi, \bar{u}_1) I(\Delta, \xi), \\
I(\Delta, \xi) = \frac{\Delta^2}{N} \int_{-1}^{1} \frac{e^{-\pi|\epsilon|\Delta}}{(1 + e^{-\pi|\epsilon|\Delta})^2},
\]

where \( \bar{u} \) is an effective value of \( u \) in nth term of Eq. (13). The dependence of \( W \) on \( \bar{u} \) is rather week compared to the dependence on \( \xi \) and can be disregarded. Thus, in the limit \( \pi\Delta\delta_{s_0}/s \gg 1 \), IPA and FPA practically coincide since \( I(\Delta, \xi) \approx \Delta^2 |\xi| (1 + e^{-\pi\Delta\delta_{s_0}/s})^{-1} \approx \frac{\Delta^2}{N} = 1 \).

Consider now the case of subthreshold pair production with \( \xi = 1 + \delta s/s > 1 \), where \( \delta s = s_{thr} - s \) is the "lack of energy". The probability has the following form

\[
W^{FPA} = I_1 W^{IPA}(n = 1) + CW^{IPA}(n = 2) + ..., \tag{21}
\]

with \( I_1(\Delta, \delta s/s) \approx e^{-\pi\Delta\delta s/s}(1 + e^{-\pi\Delta\delta s/s}) \) and \( C = (1/\pi^2) \int_{-\Delta}^{\pi\Delta} a^2 \exp(-x)(1 - \exp(-x))^{-2} dx \approx 2/3 \) for \( \delta s/s \lesssim 1 - 0.6\Delta/N \). The terms in r.h.s. of (21) are meant to have the same functional dependence on \( \xi \) and \( \bar{u}_{1,2} \) as in IPA. One can expect a significant enhancement of pair production for the short pulse because the probability of single photon events (\( n = 1 \)) is much greater than the probability of the two-photon events (\( n = 2 \):

\[
W^{IPA}(n = 1)/W^{IPA}(n = 2) \sim \xi^{-2} \gg 1. \tag{21}
\]

When the length of the pulse increases the contribution of the first term in Eq. (21) decreases exponentially due to \( I_1 \), and the prediction of FPA approaches to the IPA one.

The probability and the cross section are related to each other (22) as \( d\sigma = 2(2|\Delta|^2 M^2 \Delta/\alpha) d\sigma \). The total cross section of \( e^+e^- \) production is calculated using Eqs. (9), (14) and (16). The cross sections are exhibited in Fig. 1 as a function of \( \sqrt{s} \) in the threshold region for finite pulses with \( \Delta = \pi N \). The left and right panels correspond to \( \xi = 0.01 \) and 0.1, respectively. The dashed and thick solid curves are for \( N = 2 \) and 5, respectively. The thin solid curve is the IPA result. The thin dashed curve, labelled by "B-W", corresponds to the Breit-Wheeler process (1) practically coinciding with the lowest harmonic (\( n = 1 \)). One can see that in the subthreshold region, \( \sqrt{s} = 0.85 - 1.02 \) MeV, the cross section for short pulses is significantly greater than in IPA and the difference may reach one or two orders of magnitude.
for \( \xi = 0.1 \) and \( \xi = 0.01 \), respectively. When \( \xi \) and/or \( \zeta \) increase, the contribution of higher terms with \( n \geq 1 \) becomes finite that brings an additional (increasing) dependence on \( \Delta \) (cf. Eq. (10)).

The total cross section in a wider region of \( \sqrt{s} \) is exhibited in Fig. 2 left panel. At \( \sqrt{s} \simeq 0.55 \text{ MeV} \) the multiphoton events with \( l \geq 4 \) become important. In general, the total cross section in FPA has also the step-like structures similar to IPA. However, a decrease of the pulse duration leads to a smoothing. One can also see some enhancement of the cross section for a short pulse with \( N = 2 \) compared to the case of a longer pulse with \( N = 5 \). The total cross sections of the \( e^+e^- \) pair production as a function of \( \xi^2 \) at three values of \( \xi_{\text{min}} = \xi = 0.5, 1.1 \) and 3.8. Notations as in Fig. 1.

In summary, we have considered the total cross section of \( e^+e^- \) production off a probe photon interacting with a semi-intensive short laser pulse in the subthreshold region defined by multi-photon interactions. We find a nontrivial dependence of the cross section (production probability) on the pulse duration. Just below the threshold of the weak-field Breit-Wheeler process, the short laser pulses increase the cross section up to two orders of magnitude relative to a monochromatic plane wave. This effect must be taken into account in the evaluation of \( e^+e^- \) pair production in cascade processes produced by high-power laser fields.

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