An approach for multi-objective optimization of vehicle suspension system

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Abstract. In this paper, a half car model of with nonlinear suspension systems is selected in order to study the vertical vibrations and optimize its suspension system with respect to ride comfort and road holding. A road bump was used as road profile. At first, the optimization problem is solved with the use of Genetic Algorithms with respect to 6 optimization targets. Then the $k-\varepsilon$ optimization method was implemented to locate one optimum solution. Furthermore, an alternative approach is presented in this work: the previous optimization targets are separated in main and supplementary ones, depending on their importance in the analysis. The supplementary targets are not crucial to the optimization but they could enhance the main objectives. Thus, the problem was solved again using Genetic Algorithms with respect to the 3 main targets of the optimization. Having obtained the Pareto set of solutions, the $k-\varepsilon$ optimality method was implemented for the 3 main targets and the supplementary ones, evaluated by the simulation of the vehicle model. The results of both cases are presented and discussed in terms of convergence of the optimization and computational time. The optimum solutions acquired from both cases are compared based on performance metrics as well.

1. Introduction

The multibody dynamics have been used extensively by automotive industry so as to model and design the vehicle and its parts i.e. suspension system, chassis, tires. Depending on the complexity and the depth of each study, various models have been developed, so as to describe the dynamic behavior of the vehicle. These models vary from simple (Quarter Car Model [1]) to more complex and detailed ones (Half Car Model [2] or Full Car Model [3, 4]), simulating the tyre, the body and the suspension system of the vehicle. The latter is considered one of the most important subsystems of the vehicle, because not only it transmits the loads from the lower part of the vehicle to the upper, but also it ensures the comfort of the travel by absorbing any unwanted vibrations. Through the suspension system, various aspects of the dynamic behavior can be studied, with the main ones being the ride comfort of the passengers and the road holding of the vehicle [5]. The aforementioned characteristics depict one of the main conflicts in the automotive industry: the trade-off between the ride comfort and the handling of the vehicle. The suspension systems are simulated in multibody dynamics with springs and dampers. In literature, different systems can be found, depending on the needs of each study. The most common ones are the passive systems [6], but the need to adjust the characteristics of the vehicle in order to be able to cope with different road surfaces, has led to the development of the semi-active [7] and active suspension systems [4]. The main disadvantage of these suspension types...
is that they can be too complex and expensive at times. In order to achieve similar behaviour of the vehicle, certain nonlinearities can be introduced in the components of the passive suspension system. These nonlinearities were investigated in previous work [5], [8] and are discussed in this paper as well. Finally, in order to investigate how the passengers are affected by the driving experience various models have been introduced simulating also the body of the driver and the seats of the vehicle [9]. In this way, the vibrations that are transmitted to the driver can be studied in depth taking into consideration also ISO 2631 which illustrates methods to evaluate the human exposure to the vibrations induced by the road.

The rapid growth of modern optimization methods has been beneficial to the study of the suspension systems. Taking advantage of the versatility of the methods, as well as the advances in computer analysis, fast optimal solutions to the key parameters of the suspension systems are produced. Various optimization problems have been formed, involving either one optimization target (single – objective SOO) or multiple (multi – objective MOO). The MOO offers an insight of the conflicted targets that were mentioned before, whereas the SOO results in a much simpler and non-time consuming study. The most common multi – objective optimization method is the Pareto Front, where the different targets are separated throughout the optimization process, and the result of the method is a set of optimal solutions. The Pareto Front optimization method has been extensively used in the study of the dynamical behavior of the vehicle with the use of different targets including both the ride comfort of the passengers as well as the road holding of the vehicle [5]. Different characteristics of the performance of the vehicle have been used in order to achieve better optimization results such as the root mean square of the body acceleration, the variance of the tire forces [3] among others. The great variety in the possible objective functions often leads to an exaggeration in the selected number of targets, which increases the complexity of the problem in question.

As it was mentioned above, the result of the Pareto Front method is a set of optimal solution. Each one of these solutions is optimal in respect to the targets involved in the study. The selection of the optimum solution lies upon the designer, who is responsible of evaluating the solutions based on requested demands. This process can be tiresome and time – consuming, due to the potentially large number of optimal solutions and the complexity of the problem in question. In order to overcome this problem, various methods of solutions sorting have been developed. The aim of these methods is to sort the solutions, according to various indexes, aiming to produce one optimum solution or to decrease the number of optimal solutions significantly. Das (1999) introduced the k – optimality method which is based on the partial dominance between the solutions [10], and later Gobi et al. (2006, 2013) expanded this theory, launching the k – ε optimality method, which shows not only the dominance between different solutions but also the measure of their dominance [11].

The object of this paper is to optimize the parameters of the nonlinear suspension system of a heavy vehicle using the Pareto Front optimization method and extract one optimal solution based on the k – ε algorithm. In order to save computational time, a new approach is proposed, which involves the separation of the optimization targets to primary and supplementary ones, and it is described in the sections below. This paper is organized as follows: in Chapter 2 a brief description of the models used is provided, in Chapter 3 the road excitation applied is illustrated, in Chapter 4 the optimization methods are described in detail, while in Chapter 5 the optimization procedure and the results are highlighted and finally in Chapter 6 conclusions are presented.

2. Simulation Models
In the current study, the dynamic behavior of a heavy vehicle is investigated, as far as the vertical vibrations induced by the road surface are concerned. Thus, a half car model was chosen, including the front and rear axle of the vehicle allowing the pitch phenomena to be observed.

As shown in Figure 1, the vehicle model consists of three basic subsystems: the tires, the suspension systems and the body of the vehicle. The latter is considered as a rigid body of mass m, equal to the half of the total mass of the vehicle. The distance of the front and rear unsprung masses (m_{F and} m_{R}) from the center of mass is equal to a_{F and} a_{R} respectively. The front and rear tires are
modelled as linear springs, which receive as input the irregularities of the road profile. The governing equations of the model were described in detail in previous work [5].

Ideally, a suspension system should adjust its characteristics in order to function properly under different road conditions, ensuring the comfort of the travel and the safety of the vehicle. This adjustment is achieved through either the use of controllers (active suspensions) or through added nonlinearities in the components of the suspension (springs and dampers). These nonlinearities are achieved by adding a nonlinear term in the suspension spring force which is described mathematically by equation (1) and represents a Duffing oscillator which could simulate with higher accuracy spring like variable coil spring or variable coil pitch.

\[ F_{spring} = K_l \cdot x + K_{nl} \cdot x^3 \]  

where \( x \) is the suspension travel. In Tables 1 and 2 the parameters of the model are explained and their values are given.

Additionally, the seat and the driver of a vehicle are modelled in order to investigate in depth the ride comfort of the passenger, as depicted in Figure 2. Different parts of the human body such as the pelvis, the diaphragm, the thorax etc. are described via several \( m \)–\( c \)–\( k \) subsystems. This model was based in the work of Abbas et al. (2010) [13], the equations of the model as well as the parameters of each body part are taken from this research paper.

Important quantities or metrics indicating the dynamical behavior of the vehicle model are the suspension travel and the tire deflection. The suspension travel of the suspension is the term \( (z_s - z_F - a_F \cdot \theta) \) and \( (z_s - z_R - a_R \cdot \theta) \) for the front and rear suspension respectively and the tire deflection is \( (z_F - z_{Road_F}) \) and \( (z_R - z_{Road_R}) \) for the front and rear tire respectively. Both quantities

### Table 1. Nomenclature of vehicle parameters.

| Parameters          | Subscripts |
|---------------------|------------|
| \( z \)             | S          |
| \( \theta \)        | F          |
| \( z_{\text{road}} \) | R          |
| \( m \)             | I          |
| \( C \)             | \( n \)    |
| \( K \)             | T          |
| \( a \)             |            |

- Vertical motion coordinate
- Pitch motion coordinate
- Road excitation
- Mass
- Damper’s coefficient
- Spring’s stiffness coefficient
- Distance of the centre of the mass of the vehicle

### Table 2. Parameters of the Half Car Model.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( m_s \) [kg] | 2200   | \( a_F \) [m] | 1.61   |
| \( I_s \) [kg·m²] | 1142   | \( a_R \) [m] | 1.67   |
| \( m_F \) [kg] | 50.00  | \( K_{TF} \) [N/m] | 4.00·10⁵ |
| \( m_R \) [m] | 100.00 | \( K_{TR} \) [N/m] | 8.00·10⁵ |

Figure 1. Half Car Model.  
Figure 2. Seat – Driver Model.
are key parameters in the optimization study that follows in the next chapters. Other important metrics of the dynamical behavior of the vehicle are the tire forces which are described in equation (2) and the ratio of the tire force to the static force between the tire and the ground [12]. If the value of this ratio exceeds 1, then loss of contact between the wheel and the ground is indicated.

Through the results of the passenger model, the ride comfort could be measured via the Vibration Dose Value (VDV) and the Crest Factor (CF) which are calculated through equations (3) and (4), respectively. These characteristics are proposed by the ISO 2631 standard which evaluates the human exposure to whole-body vibration.

$$VDV_{head} = \left( \int_0^T (a_{wh}(t))^4 \, dt \right)^{\frac{1}{2}}$$  \hspace{1cm} (3)

$$CF_{head} = \frac{\text{max}(\text{acc}_{head})}{\text{rms}(\text{acc}_{head})}$$ \hspace{1cm} (4)

3. Road excitation

One of the important aspects in the analysis of the dynamic behavior of a vehicle is the road profile that is used as an input function. Road profile generation is a strong asset for the researchers, as it enables them to test the vehicle under different road conditions. In this research, a road bump was generated as it is shown in Figure 3, using a sinusoidal function [7].

![Figure 3. Road Bump](image)

The geometrical aspects of the bump were chosen as follows: its height was set to 0.05m and its length was set to 2m. The velocity of the vehicle was considered constant, at 10 m/s. Finally, a time lag between the front and rear axles of the vehicle was applied. Specifically, the two wheels follow the same trajectory with a time delay due to the distance $a_F + a_R$ between them ($t_{\text{distance}} = \frac{a_F + a_R}{v}$), as it is shown in Figure 3.

4. Optimization

In this work, a multi-objective optimization approach was followed. The optimization problem is expressed as a problem of minimization of the objective functions, with the selected design variables being subjected to a number of inequality constraints [5].

In multi-objective optimization the objective function is not a scalar number, as in SOO problems, but a vector. Thus, in order to rank the different solutions, the concept of partial ordering is adopted. The Pareto dominance, which is used in this paper, is based on this concept. The solutions of the Pareto set are non-dominated by any other solution and are considered as equally optimal solutions [5]. When dealing with complex problems, the size of the Pareto set might be quite large, increasing significantly the workload so as to select one optimum solution based on this set. In order to bypass this problem, many algorithms have been developed. Their goal is to reduce the size of the Pareto set or even extract one optimum solution to the problem. Das (1998) introduced the k-optimality method, which is based on the partial dominance of the Pareto solutions, locating solutions which achieve a better trade-off between the optimization targets and eventually “offer something more” than the other optimal solutions. The solutions are sorted using the k index, which is a measure.
of their optimality and can only acquire integer values [10]. Based on the aforementioned method, Levi et al (2005) introduced the \( k - \sum \) optimality method, which is implemented in this paper, which not only vets the as \( k \)-method, but also measures the entity of this variation. In \( k-e \), the \( k \) index can receive not only integer values on contrary with \( k \) method, but also real values. In this way, a continuous degree of optimality is achieved. These two methods were described in detail in previous work [5].

5. Optimization procedure.

The objective of this report is the optimization of the dynamic behavior of a heavy vehicle under specific road conditions. In the multi – objective approach that was adopted, the design variables were chosen as the key parameters of the suspension systems, and are shown in equation (5).

\[
\begin{align*}
\text{constant} & \sum \text{deviation} < Z_k: B_k: J_{k_1}:B_{k_i}:J_{n_k}:J_{n_k_0} \\
\end{align*}
\]

Where \( K_F, K_{nlF}, K_R \) and \( K_{nlR} \) are the linear and nonlinear coefficients of the front and rear springs and \( C_F \) and \( C_R \) are the linear coefficients of the front and rear dampers. The upper and lower bounds of these design variables, were chosen based on previous work conducted by the authors and are shown in Table 3 [5, 8]. As far as the constraints of the problem are concerned, they were selected in regard to the ride comfort of the passengers and the nonlinear part of the suspension systems, demanding the rms of the sprung mass acceleration to be less than 0.8 m/s\(^2\) and a 10 – 30% usage of the nonlinear part of the suspension (\( \frac{k_{nlF}x_{nlF}}{k_{F}+K_{nlF}x_{nlF}} \)), based on equation 1) through all the simulation. [5].

| Design Variables | Lower Bound | Upper Bound |
|------------------|-------------|-------------|
| \( J_{k_1} \) | 3.2 \times 10^4 | 1.5 \times 10^5 |
| \( C_F, C_R \) | 2.0 \times 10^3 | 1.0 \times 10^4 |
| \( J_{n_k} \) | 5 \times 10^5 | 3 \times 10^8 |

The selection of the vector of the objective function is of vital importance to the optimization procedure. As it was mentioned above six optimization targets were selected (equations (6) – (11)): the variance of the acceleration of the body of the vehicle \( (f_1) \), the mean of the variances of the front and rear suspension system \( (f_2) \), the mean of the variance of the front and rear tire deflections \( (f_3) \), the vibration dose value of the head \( (f_4) \), the crest factor of the head \( (f_5) \) and the variance of the pitch angle \( (f_6) \). The targets \( f_1, f_2, f_3 \) are linked to the ride comfort of the passengers whereas the remaining three are involved with the road holding of the vehicle. In the first segment of the analysis, a multi – optimization problem was solved with all the aforementioned targets involved. In the latter part of the analysis, the targets were split into two groups: the main and the supplementary ones. In the first group, the variance of the body acceleration \( (f_1) \) and the mean variances of the suspension travels \( (f_2) \) and the tire deflections \( (f_3) \) were included. The remaining three targets were used as supplementary ones: the targets \( f_4 \) and \( f_5 \) enhance the ride comfort of the passenger whereas target \( f_6 \) is linked with the road holding of the vehicle. The Pareto optimization procedure was applied in the half – car model.

\[
\begin{align*}
\text{in}\text{accsprung mass} & \text{mean}\text{var} & \text{var}\text{of}\text{travel}\text{of}\text{spring} & \text{var}\text{of}\text{travel}\text{of}\text{tire} \\
f_1 & = & \text{var}(\text{accsprung mass}) & \\
f_2 & = & 0.5 \cdot \text{[var}(\text{spring travel}) + \text{var}(\text{tire travel})]\text{]} & \\
f_3 & = & 0.5 \cdot \text{[var}(\text{tire}\text{deflection}) + \text{var}(\text{tire}\text{deflection})]\text{]} & \\
f_4 & = & VDV_{\text{head}} & \\
f_5 & = & CF_{\text{head}} & \\
f_6 & = & 10^4 \cdot \text{var}(\theta) & \\
\end{align*}
\]

6. Results

In this section the two resulting optimal set of solutions are presented. In both cases a threshold of 13% of the maximum value of every objective function was selected, based on previous work [5]. In Figure 4 the \( k - \sum \) levels of optimal solutions are depicted for both cases. For the first case, which is
presented in Figure 4 (a), the $k - \Sigma$ algorithm has detected one optimum solution of level 4, whereas for the second case (Figure 4 (b)) the algorithm has detected 3 solutions of level 4. As far as the second case is concerned, the solution with the maximum $k$ value was selected in order to compare the optimum solutions of the two problems.

Figure 4. Optimal solutions $f_2 = f_2(f_1, k)$, $k - \varepsilon$ levels (a) 6 targets case, (b) 3+3 targets case

![Figure 4](image)

Figure 5. Optimal solutions for Pareto Set (a) $k$-$\varepsilon$ levels (b-f) Pareto fronts of the Objectives

![Figure 5](image)
Whereas, in Figure 5 the Pareto fronts for all targets in respect to the first objective are depicted. Both sets of optimal solutions are compared: with blue dots stand the solutions of the 3+3 optimization targets case whereas the red ones represent the solutions of the 6 optimization targets case. Furthermore, the optimum solutions of each set are depicted with different colour (green / yellow), based on the results of the k –ε algorithm, which was applied in both cases. The results of both cases confirm the supplementary character of the last target: Figure 5(a) and Figure 5(e) follow the same pattern where the same applies for the other three Figures (5(b) – 5(d)). Moreover, the conflicting relationship between the different optimization targets is highlighted: the targets $f_3$ and $f_6$ are in prone of the suspension travel of the vehicle, so if their values are decreased then the value of the $f_1$ target, which indicates ride comfort, increases (Figure 5(a) and 5(e)). On the other hand, targets $f_2$ and $f_5$ represent the ride comfort, so their values are proportional to the ones of the first objective target (Figure 5(c) and 5(d)). Additionally, the effect of $f_3$ to the ride comfort and the road holding is illustrated in Figure 5c, a subject which was investigated in depth in a previous work [8].

The optimum solutions of each set are compared in Table 4. In Table 3 the values of the optimization targets are presented. In both tables Solution 1 stands for the optimum solution of the 6 optimization targets case, whereas Solution 2 stands for the optimum solution of the 3+3 optimization targets case. The two solutions are similar, but the solution of the 3+3 optimization problem dominates in 4 out of 6 optimization targets.

| Design Variables | Objective | Values |
|------------------|-----------|--------|
| $K_{d1}$ (N/m)   | $f_1$     | 0.450  |
| $C_l$ (Ns/m)     | $f_2(10^3)$ | 7.487 |
| $K_{d2}$ (N/m)   | $f_3(10^3)$ | 4.156 |
| $C_r$ (Ns/m)     | $f_4$     | 7.680  |
| $K_{d3}$ (N/m^3) | $f_5$     | 1.820  |
| $K_{d4}$ (N/m^3) | $f_6$     | 0.482  |

An important aspect of this analysis concerns the dynamic behavior of the vehicle. In Table 4, the resulting values of the design variables are depicted. Comparing the values it is shown that, the solution of the 3+3 targets case has produced softer suspension systems compared to the 6 targets case, without neglecting the road holding. This is depicted also in the fact that the root mean square of the body acceleration is in the same range for both solutions (~0.67 m/s^2) without risking the road holding of the vehicle in any of the cases, and this is not only a success of the alternative approach in the optimization procedure but also the advantage of the non-linear suspension system, discussed in detail in previous work [5, 8]. Discussing the dynamical behavior of the vehicle, the solution of the 3+3 targets case produced almost equal values of the maximum tire force only in the front suspension system (3578N), while in the rear suspension system the value of the tire force is higher (3908 for solution 1 and 4805 for solution 2), due to the lower suspension travel of the rear suspension system. Moreover, the maximum absolute value of the $f_3$ index are also in the same range for both solutions (~0.161 for the front suspension system, while for the rear the values are ranging from 0.171 – 0.212), way far from the limit of 1 which indicates the wheel lift off.

7. Conclusions
To sum up, this paper focuses on the comparison of two approaches for the solution of the same optimization problem, this of the optimum suspension design of a heavy vehicle, which involve different handling of the optimization targets. In the first approach, all the desired targets were introduced in the optimization process and then the resulting solutions were implemented in the k –ε algorithm. In the second one, only the main targets of the study were introduced in the optimization algorithm and the rest of them were calculated later. Finally, the resulting set of solutions was implemented again in the k –ε algorithm. The optimum solutions were compared in terms of the values
of each target through figures, the design variables, and important metrics describing the dynamic behaviour of the vehicle.

As it was illustrated in previous chapters, the solutions and the Pareto front produced by the 3+3 targets case dominate the ones of the 6 targets case in terms of the values of the optimization targets as well as the dynamic behaviour of the vehicle is concerned. It is important to mention that by increasing the complexity of an optimization problem does not necessarily mean that a better solution will be produced. As it was pointed out in this work, better results might be produced if the number of the main optimization targets is selected appropriately, and more focus is given in the selection of one optimum solution after the optimization process is finished by using as factors of the selection any other supplementary optimization targets. By adding more targets in the selection of the optimum solution, proved to be a more efficient process than adding them in the main optimization process. Apart from delivering a “better optimal” solution, is also less time consuming, due to the decreased complexity of the optimization process. Further work is in progress in order to extend the conclusions of this study and present them in more details.

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