Slow cross-symmetry phase relaxation in complex collisions

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We discuss the effect of slow phase relaxation and the spin off-diagonal $S$-matrix correlations on the cross section energy oscillations and the time evolution of the highly excited intermediate systems formed in complex collisions. Such deformed intermediate complexes with strongly overlapping resonances can be formed in heavy ion collisions, bimolecular chemical reactions and atomic cluster collisions. The effects of quasiperiodic energy dependence of the cross sections, coherent rotation of the hyperdeformed $\simeq (3 : 1)$ intermediate complex, Schrödinger cat states and quantum-classical transition are studied for $^{24}$Mg$^{+}^{28}$Si heavy ion scattering.

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I. INTRODUCTION

The dominating idea in the modern theory of highly excited strongly interacting systems is that phase randomization time is the shortest time scale of the problem\textsuperscript{[1]}. Applica-
tion of this idea to the theory of quantum chaotic scattering for colliding systems with rotationally invariant Hamiltonians implies the absence of correlations between the reaction amplitudes carrying different total spins [1]. However, though seemingly plausible, the assumption of a very fast phase relaxation is not consistent with many data sets on complex quantum collisions. In particular, the anomalously long-lived spin off-diagonal $S$-matrix correlations have been identified from the data on forward peaking of evaporating protons in nucleon induced [2, 3] and photonuclear [4] reactions. Such long-lived correlations reflect an anomalously slow phase relaxation, which is many orders of magnitude longer than the energy relaxation. This provides a manifestation of a new form of matter: thermalized non-equilibrated matter introduced by one of us in Refs. [5, 6]. The effect is of primary importance for many-qubit quantum computation since anomalously long “phase memory” can extend the time for quantum computing far beyond the quantum chaos border [2, 4].

An effect of a very slow phase relaxation has also been strongly supported by numerical calculations for $H+D_2$ [7], $F+HD$ [8] and $He+H_2^+$ [9] state-to-state chemical reactions. In these calculations, a slow phase relaxation manifests itself in stable rotating wave packets of the intermediate complexes [10]. Interestingly, this same effect of stable coherent rotation was originally revealed for heavy ion collisions, e.g., for $^{19}F+^{89}Y$ [11], $^{28}Si+^{64}Ni$ [12], $^{12}C+^{24}Mg$ [13, 14], $^{24}Mg+^{24}Mg$ and $^{28}Si+^{28}Si$ [15], $^{58}Ni+^{46}Ti$ and $^{58}Ni+^{62}Ni$ [16] collisions.

In this paper we reveal the effect of slow phase relaxation for yet another heavy ion scattering system, $^{24}Mg+^{28}Si$. This effect is studied in relation to quasiperiodic energy dependence of the cross sections, coherent rotation of the hyperdeformed $\simeq (3 : 1)$ intermediate complex, Schrödinger cat states and quantum-classical transition for $^{24}Mg+^{28}Si$ heavy ion scattering.

II. CROSS SECTION ENERGY AUTOCORRELATION FUNCTION

We consider first the scattering of spinless collision partners with spinless collision fragments in the exit channel. Using the semiclassical asymptotics of Legendre polynomials for $J \gg 1$, where $J$ is the total spin of the system, we represent the cross section in the form

$$d\sigma(E, \theta)/d\theta \equiv \sigma(E, \theta) = \sigma_d(\theta) + \delta\sigma(E, \theta), \quad (1)$$
with $\delta\sigma(E, \theta) = \delta\sigma^{(+)}(E, \theta) + \delta\sigma^{(-)}(E, \theta)$, $\delta\sigma^{(\pm)}(E, \theta) = |\delta f^{(\pm)}(E, \theta)|^2$ and

$$
\delta f^{(\pm)}(E, \theta) = \sum_J (2J + 1)W(J)^{1/2}\delta\tilde{S}^J(E)\exp[iJ(\Phi \pm \theta)].
$$

(2)

In these expressions, $\sigma_d(\theta) = |F_d(\theta)|^2$ is the energy independent potential scattering cross section, $\Phi$ is the average deflection angle obtained from a linear approximation for the $J$-dependence of the potential phase shifts in the entrance and exit channels, $W(J)$ is the average partial reaction probability, and $\delta\tilde{S}^J(E)$ are normalized, $\langle |\delta\tilde{S}^J(E)|^2 \rangle = 1$, energy fluctuating around zero $S$-matrix elements corresponding to time-delayed collision processes. The brackets $\langle ... \rangle$ stand for the energy $E$ averaging. In the expression for $\sigma(E, \theta)$ we have dropped (i) the highly oscillating angle interference term between the $\delta f^{(+)}(E, \theta)$ and $\delta f^{(-)}(E, \theta)$ amplitudes, and (ii) the interference terms between the energy smooth potential scattering amplitude $F_d(\theta)$ and energy fluctuating amplitudes $\delta f^{(\pm)}(E, \theta)$. This is because the excitation functions data for $^{24}\text{Mg}+^{28}\text{Si}$ scattering [17] were obtained by averaging over a wide $\Delta\theta_{c.m.} \simeq 77^\circ - 98^\circ$ angular range.

In calculating the cross section energy autocorrelation function,

$$
C(\varepsilon) = \langle \sigma(E + \varepsilon, \theta)\sigma(E, \theta) \rangle / \langle \sigma(E, \theta) \rangle^2 - 1,
$$

(3)

we take into account the $S$-matrix spin off-diagonal correlation [18]

$$
\langle \delta\tilde{S}^J(E + \varepsilon)\delta\tilde{S}^{J'}(E)^* \rangle = \Gamma / (\Gamma + \beta|J - J'| + i\hbar\omega(J - J') - i\varepsilon).
$$

(4)

Here, $\omega$ is the angular velocity of the coherent rotation of the intermediate complex, $\beta$ is the spin phase relaxation width and $\Gamma$ is the total decay width of the intermediate complex.

We take $W(J)$ in the $J$-window form, $W(J) = W(|J - I(E)|/g)$, where the average spin $I(E)$, for a given c.m. energy of the collision partners, is close to the grazing orbital momentum. The $J$-window width $g$ relates to the effective number of partial waves, $g + 1$, contributing to $\delta\sigma(E, \theta)$. For the analyzed $^{24}\text{Mg}+^{28}\text{Si}$ scattering we estimate $g \simeq 1 - 5$, which is revealed by the shape of the measured elastic scattering angular distributions [17] corresponding to the maxima of the excitation functions. Although these angular distributions show regular oscillations with a well-defined period, they clearly deviate from the square of a single Legendre polynomial. We estimate the energy dependence of $I(E)$ in the linear approximation and obtain $I(E) = \bar{I} + \bar{I}(E - \bar{E})/\Delta E$. Here, $\bar{I} = I(\bar{E})$, $\bar{E}$ is the energy corresponding to the center of the energy interval over which the cross section is measured,
\[ \Delta E = 2(E - B)/I \] and \( B \) is the Coulomb barrier for the collision partners in the entrance channel for a configuration of the two touching spherical nuclei \(^{24}\text{Mg}\) and \(^{28}\text{Si}\).

We calculate \( C(\varepsilon) \) under the conditions \( g \geq 1, \beta \leq \Gamma \) and \( \beta \ll \hbar \omega \). We take \( W(J) \) in the Gaussian form, \( W(J) \propto \exp\left[-(J - I(\varepsilon))^2/g^2\right] \). Generalizing the calculations in [19] to the case of finite \( \beta \) and arbitrary \( \Delta E \) for the normalized \((C(\varepsilon = 0) = 1)\) cross-section energy autocorrelation function, we obtain

\[
C(\varepsilon) = \frac{\exp[-\varepsilon^2/(2\hbar\omega)^2d^2]}{\Re[1/(1 - \exp[-\pi\Gamma/(\hbar\omega - i\beta)])]} \Re\left[\frac{\exp[i\pi|\varepsilon|/(\hbar\omega - i\beta)]}{1 - \exp[i\pi(|\varepsilon| + i\Gamma)/(\hbar\omega - i\beta)]}\right],
\]  

(5)

where \( d^2 = g^2/(1 - \hbar\omega/\Delta E)^2 \). The above expression for \( C(\varepsilon) \) has been obtained for \( d \geq 1 \).

One can see that \( \beta/\hbar \) has the physical meaning of the imaginary part of the angular velocity signifying the space-time delocalization of the nuclear molecule and the damping of the coherent rotation of the intermediate complex [18]. For \( \beta = 0 \), \( C(\varepsilon) \) is an oscillating periodic function with period \( \hbar\omega \). For finite \( \beta \), the amplitude of the oscillations in \( C(\varepsilon) \) decreases with increasing \( |\varepsilon| \): the larger \( \beta \) the stronger the damping of the oscillations. For \( \beta = 0 \) and \( \hbar\omega = \Delta E \), Eq. (5) transforms to the result in Ref. [19].

Although Eq. (5) is obtained for spinless reaction fragments it also holds for reaction products having intrinsic spins. This can be shown using the helicity representation for the scattering amplitude [18].

In Fig. 1 we present the energy autocorrelation functions for the \(^{24}\text{Mg} + ^{28}\text{Si}\) elastic and inelastic scattering [20], constructed from the data on the excitation functions measured on the \( E_{\text{c.m.}} = 49 - 57 \text{ MeV} \) energy interval [17]. The experimental excitation functions were averaged on the \( \theta_{\text{c.m.}} = 77^\circ - 98^\circ \) angle interval. The experimental \( C(\varepsilon) \)'s are not Lorentzian but oscillate with a period \( \simeq 0.75 \text{ MeV} \). The fit of the experimental \( C(\varepsilon) \)'s for all the elastic and inelastic channels is obtained with \( \Gamma = 0.15 \text{ MeV}, \beta = 0.1 \text{ MeV}, \hbar\omega = 0.75 \text{ MeV} \) and \( d = 5 \). The calculated \( C(\varepsilon) \)'s are normalized to the experimental data at \( \varepsilon = 0 \).

The extracted value of \( \hbar\omega \) suggests an anomalously strong deformation of the intermediate complex. Indeed, for \( J \simeq 34 - 38 \) [17], using the moment of inertia of a \( \simeq (2 : 1) \) superdeformed intermediate complex, corresponding to the two touched spherical colliding nuclei \(^{24}\text{Mg}\) and \(^{28}\text{Si}\), we have \( \hbar\omega \simeq 1.9 \text{ MeV} \). This value is bigger by a factor of about 2.5 than the period of oscillations in the experimental \( C(\varepsilon) \)'s. This reveals the excitation of \( \simeq (3 : 1) \) hyperdeformed coherent rotational states of the intermediate complex.

It should be noted that the intrinsic excitation energy of the intermediate complex, which
we obtain by subtracting deformation and rotation energy for the total energy, is about 15 MeV or more. This corresponds to the average level spacing of $D \approx 10^{-6}$ MeV or less. Therefore, the intermediate complex is in the regime of strongly overlapping resonances, $\Gamma/D \geq 10^5$. In this regime, the theory of quantum chaotic scattering and random matrix theory are conventionally assumed to apply [1]. In accordance with these approaches, which in particular reconfirmed the Ericson theory of the compound nucleus cross-section fluctuations [21], the spin off-diagonal $S$-matrix correlations vanish yielding $C(\varepsilon) = 1/[1 + (\varepsilon/\Gamma)^2]$. The Lorenzian curves presented in Fig. 1 with $\Gamma = 0.85$ MeV to fit the experimental data at $\leq 0.1$ MeV are clearly in contrast with the oscillations in all the experimental $C(\varepsilon)$'s. Within our approach, the limit of vanishing spin off-diagonal $S$-matrix correlations corresponds to $\beta \gg \Gamma$, where $\hbar/\beta$ is the characteristic spin phase relaxation time. Therefore, the persistence of the oscillations in $C(\varepsilon)$ indicates an anomalously long spin off-diagonal “phase memory”.

In Fig. 1 we present another possible fit of the experimental $C(\varepsilon)$'s with the same $\Gamma = 0.15$ MeV and $\hbar\omega = 0.75$ MeV, but with the different values $\beta = 0.03$ MeV and $d = 1$. One can see that both the fits are qualitatively and quantitatively indistinguishable. The question arises if the quantities of the interest, in particular the phase relaxation width $\beta$, can reliably be determined from the data.

III. TIME POWER SPECTRUM OF THE COLLISION

Consider the time $(t)$ power spectrum of the collision for the spinless reaction partners in the entrance and exit channels. Unlike the cross section energy autocorrelation function in the previous Section, the time power spectrum will be studied for a fine angular resolution. The time power spectrum is given by the Fourier component of the amplitude energy autocorrelation function [14, 21]. For $t \ll \hbar/D$, i.e. for $\Gamma/D \gg 1$, the continuous spectrum approximation is valid and we have [10, 18, 22]

$$P(t, \theta) \propto H(t) \exp(-\Gamma t/\hbar) \sum_{J,J'}[W(J)W(J')]^{1/2} \exp[i(\Phi - \omega t)(J-J') - \beta|J-J'|t/\hbar]P_J(\theta)P_{J'}(\theta).$$

(6)

Here, $P_J(\theta)$ are Legendre polynomials, and the Heaviside step function $H(t)$ signifies that the intermediate complex cannot decay before it is formed at $t = 0$.

In Fig. 2 we present $P(t, \theta)$ for three moments of time and for the two different sets of
the parameters for which the \(C(\varepsilon)\)'s were calculated in the previous Section (Fig. 1). The first set is: \(\Gamma = 0.15\ \text{MeV}, \hbar\omega = 0.75\ \text{MeV}, I = 36, \Phi = 0, \beta = 0.03\ \text{MeV}, d = 1\). For the second set we have different values of \(\beta = 0.1\ \text{MeV}, d = 5\) while the rest of the parameters is unchanged. For the reason discussed in [10] the time power spectra in Fig. 2 are scaled with the \(P_{\text{diag}}(t, \theta)\), which is given by Eq. (6), where only the spin diagonal terms \(J = J'\) are taken into account. Such a spin diagonal approximation corresponds to the limit of quantum chaotic scattering and random matrix theory [1]. Accordingly, deviation of the scaled time power spectra in Fig. 2 from a constant unity is a quantitative measure of the deviation of the collision process from the universal limit of the quantum chaotic scattering theory [1].

Fig. 2 illustrates a rotation of the two wave packets towards each other. As the wave packets rotate they also spread - the bigger \(\beta\) the faster the spreading. One observes that, for \(\beta = 0.03\ \text{MeV}\) and \(d = 1\), the contrast of the interference fringes, due to the interference between the near-side and far-side amplitudes [10], is very strong. These interference fringes is a manifestation of Schrödinger cat states in highly excited quantum many-body systems [22]. On the contrary, for \(\beta = 0.1\ \text{MeV}\) and \(d = 5\), the contrast of the interference fringes is greatly reduced indicating a quantum-classical transition in the collision process [10].

Our approach shows that the complicated many-body collision problem can be accurately represented by the simple picture of a weakly damped \((\beta << \hbar\omega)\) quantum rotator. This picture was obtained without introducing any collective degrees of freedom of the intermediate complex, such as its deformation and spatial orientation. The introduction of those degrees of freedom is known to be a successful approximation [23] for very low, closed to Yrast line, intrinsic excitations of the intermediate complex. Yet, in our case of high intrinsic excitations \((\geq 15\ \text{MeV})\), the collective degrees of freedom acquire large spreading widths [24], \(\Gamma_{\text{spr}} \gg \beta, \Gamma\), and by consequence they decay much faster than the average life-time of the intermediate complex.

Notice that \(P(t, \theta)\) can be obtained from the data for excitation function fluctuations for binary collisions, for fine energy and angular resolutions, provided the relative contribution of direct processes is significant \((\geq 70\%)\) [25]. The latter is usually the case for heavy-ion elastic and inelastic scattering. Experimentally, fine energy [25] and angular [26] resolutions required for the determination of \(P(t, \theta)\) are routinely achievable for heavy ion collisions [27]. Therefore, a reliable determination of the phase relaxation width \(\beta\), which is an important
new energy scale in quantum many-body systems \cite{2,4,10}, is experimentally possible.

**IV. CONCLUSION**

We have discussed the effects of slow phase relaxation, spin off-diagonal $S$-matrix correlations on the cross section energy oscillations and the time evolution of the highly excited intermediate system formed in the $^{24}\text{Mg}^{+}^{28}\text{Si}$ collision. Such quasiperiodic energy oscillations were observed experimentally. The effects of coherent rotation of the hyperdeformed $\simeq (3:1)$ intermediate complex, Schrödinger cat states and quantum-classical transition have been revealed for the $^{24}\text{Mg}^{+}^{28}\text{Si}$ heavy ion scattering.

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FIG. 1: Experimental (dots) and calculated $C(\varepsilon)$'s for $^{24}\text{Mg}+^{28}\text{Si}$ elastic and inelastic scattering. Dashed lines are obtained with $d = 1$ and $\beta = 0.03$ MeV, and dashed-dotted lines with $d = 5$ and $\beta = 0.1$ MeV (see text). Dotted lines are Lorentzians with $\Gamma = 0.085$ MeV.
FIG. 2: The time power spectra for the $^{24}\text{Mg} + ^{28}\text{Si}$ scattering obtained for the three moments of time with $T$ being a period of one complete revolution of the intermediate complex. Solid thin lines are obtained with $d = 1$ and $\beta = 0.03$ MeV, and solid thick lines with $d = 5$ and $\beta = 0.1$ MeV (see text).