Could the Casimir Effect explain the Energetics of High-Temperature Superconductors?

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It is proposed that the Casimir energy of the electromagnetic field in between the layers of cuprate superconductors could significantly contribute to the energy balance in these materials. Since the Casimir energy is strongly dependent on the distance between the layers a corresponding dependence is predicted for the superconducting transition temperatures. Comparison with the experimental data on the transition temperatures of epitaxial superlattices of alternating layers of YBCO and PrBCO of varying thickness shows that these are well reproduced.

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The question has remained open how Cooper pairs can be stable at around 100K where some high temperature superconductors (HTSCs) are still superconducting. In particular, the phonon-mediated attractive electron-electron interaction of BCS theory is known to be too weak at these temperatures, see, e.g., [1, 2].

For a new approach to this problem, let us reconsider a feature that HTSCs have in common, namely parallel superconducting layers which are separated by layers of essentially insulating material. Since in between any two conducting surfaces there occurs a Casimir effect, see [3], the effect also occurs between the parallel superconducting layers in HTSCs, as was first pointed out in [4]. Before estimating its significance in HTSCs, let us briefly review the Casimir effect’s underlying mechanism.

We begin by recalling that the ground state energy of a quantum harmonic oscillator is not necessarily fixed. It can be lowered, for example, by coupling the oscillator to degrees of freedom which decrease the amplitude of its zero-point fluctuations. This happens, e.g., when these extra degrees of freedom effectively increase the oscillator’s mass. An example would be the vibrational oscillator of a diatomic molecule that captures a neutron. Similarly, the ground state energy of the electromagnetic field can be lowered by suppressing the zero point fluctuations of some of its electromagnetic field modes. As Casimir showed, this happens, for example, when the electromagnetic field couples to conducting charge carriers which are confined to two parallel plates, in which case certain modes in the c-direction become suppressed. In the case of HTSCs, as the temperature is lowered below \( T_c \), superconducting charge carriers form in parallel layers and they therefore lower the electromagnetic ground state energy. Our aim here is to estimate if this lowering of the energy might be sufficient to make the formation of Cooper pairs energetically favorable at temperatures as high as 100K.

The derivation of the Casimir effect for two conducting objects from first principles requires the calculation of the ground state energy of the quantum system that consists of both, the electromagnetic field and the charge carriers. This calculation is hard and it is instructive to consider first the simplified case where the two objects are what may be called ideal conductors, namely conductors whose conductivity and Meißner effect expel electromagnetic fields of all wavelengths with vanishing penetration depths. In this case, the charge degrees of freedom are effectively integrated out and one is left with the noninteracting quantum field theory of electromagnetism together with the boundary condition that the electromagnetic fields vanish at the objects’ surfaces. Only those electromagnetic modes which obey this boundary condition contribute their ground state energy. In this way, the total ground state energy of the system depends on the objects’ (shape and) distance, \( a \). The distance dependence of the energy implies a force, the Casimir force. These calculations require renormalization but are straightforward for simple geometries such as two parallel plates. The Casimir force between two ideally conducting, neutral and parallel plates of large area \( A \) and distance \( a \) is given by:

\[
F(a) = -\frac{\pi^2 \hbar c A}{240a^3}
\]  

(1)

 Corrections that take into account the finite conductivity of real metals have been calculated for geometries such as parallel plates and a plate and a sphere, along with corrections for finite surface roughness and finite temperature, see [5]. Recent experiments measured the Casimir force between a metallic plate and sphere down to distances around 100nm, confirming the theoretical predictions to a precision of 0.5% [6].

From Eq(1) the ground state energy of the quantum system consisting of the electromagnetic field and two ideally conducting, neutral and parallel plates reads:

\[
E(a) = -\frac{\pi^2 \hbar c A}{720a^3}
\]  

(2)
The integration constant is chosen such that the energy of the system vanishes at infinite plate separation, or equivalently, in the absence of conducting plates. We see that a system of two conducting parallel plates at a distance \(a\) is energetically lower than the same system of two parallel plates at distance \(a\) if the two plates are insulators. Indeed, if two parallel insulating plates possess a microscopic mechanism which allows them to create ideally conducting charge carriers at an energy expense of no more than \(E(a)\) (and correspondingly less at finite temperature), then the two plates are energetically driven to use this microscopic mechanism to create ideally conducting charge carriers.

This scenario may apply to HTSCs. They possess parallel layers which are initially insulating but can become superconducting at low enough temperature, i.e., there exists some microscopic mechanism that allows these layers to create superconducting charge carriers. The formation of the Cooper pairs leads not to ideal conductivity but to superconductivity, which in turn leads to a partial suppression of the fluctuations of certain electromagnetic modes. This leads, therefore, to some lowering of the vacuum energy of the electromagnetic field. If this lowering of the electromagnetic zero-point energy at the onset of superconductivity is large enough then the Cu-O layers’ initially non-superconducting charge carriers are indeed energetically driven to utilize the available microscopic mechanism in order to become Cooper pairs. We notice here the importance of the non-superconducting state being an insulating state. This is because the size of the change of the electromagnetic ground state energy is crucial and it depends on the difference in conductivity in the two phases.

In this scenario, it would be the very effects of superconductivity which enable and stabilize superconductivity. Cooper pairs would derive their stability collectively, across layers. Namely, Cooper pairs would be stable because if sufficiently many of the Cooper pairs on opposing layers were to break up then the suppression of some electromagnetic modes’ zero-point fluctuations would cease and their electromagnetic ground state energy would have to go back up to its unsuppressed level.

While a Casimir effect must occur in HTSCs it is, \textit{a priori}, not clear if it is indeed strong enough to lead to this scenario, especially because the superconducting Cu-O layers are less efficient at suppressing electromagnetic modes than ideal conductors would be. Let us, therefore, consider the well-studied material \(YBa_2Cu_3O_{7-x}\) (YBCO) which becomes superconducting at around 92K. The crystallographic unit cell contains two copper oxide layers at a distance of \(a_b \approx 0.39\,\text{nm}\) and neighboring such bi-layers are separated by a layer of essentially nonconducting material of width \(a_i \approx 1.17\,\text{nm}\). The area density of superconducting charge carriers on each individual Cu-O layer may reach on the order of \(n_s \approx 10^{14}/\text{cm}^2\).

For our purposes, the case of YBCO is of particular interest because of the availability of experimental data, $\text{[3, 4]}$, on epitaxial superlattices in which slabs of YBCO alternate with slabs of insulating material, namely \(PrBa_2Cu_3O_{7-x}\) (PrBCO). For example, in the experiments reported in $\text{[3]}$, the authors varied the thickness of the YBCO slabs from \(M = 1\) to \(M = 8\) unit cells and the thickness, \(a_m\), of the PrBCO slabs from \(N = 1\) to \(N = 16\) unit cells, i.e., in the range \(a_m \approx 2\,\text{nm}\) to \(20\,\text{nm}\). The superconducting transition temperature was measured as a function, \(T_c(N, a_m)\), of \(N\) and \(a_m\). The results are summarized in Fig.3 of $\text{[3]}$. Since \(T_c\) is known to be a function of the layer separation and since the Casimir effect is sensitive to varying layer separations those data provide a good testing ground for the present ansatz.

Let us initially consider the simplified case in which the Cu-O layers in their superconducting state are taken to be ideal conductors in the above sense, which means that they cause electromagnetic fields of all wavelengths to vanish on their surface. We also assume that the Cu-O layers are of negligible thickness and separated by vacuum. In this case, the charge degrees of freedom are effectively integrated out and we are left with the free quantum field theory of electromagnetic fields with the boundary condition that these fields vanish on each of the superconducting Cu-O layers. The ground state energy of the electromagnetic field between any two ideally conducting layers of distance \(a\) is then lowered by the amount given in Eq.\text{[2]} and we can apply this result to all the inter-layer distances that occur in the superlattice.

Concretely, each period of the superlattice contains a slab of \(M\) unit cells of YBCO (each cell with one bi-layer of Cu-O), followed by a slab of insulating PrBCO of thickness \(a_m\). Therefore, in each period of the superlattice, the case that two neighboring superconducting Cu-O layers are separated by the distance \(a_b\) occurs \(M\) times. The case that two neighboring superconducting Cu-O layers are separated by the distance \(a_i\) occurs \((M - 1)\) times and finally the case that two neighboring superconducting Cu-O layers are separated by the distance \(a_i + a_m\) occurs once per period of the superlattice. The reduction of the ground state energy of the combined electromagnetic and charge carrier system, within the volume given by one period of the superlattice times an area \(A\) in the \(ab\) plane, is therefore given by:

\[
E^{\text{(period)}} = -\frac{\pi^2\hbar c A}{720} \left( \frac{M}{a_b^3} + \frac{M - 1}{a_i^3} + \frac{1}{a_m^3} \right)
\]

Since one period of the superlattice contains \(2M\) layers of Cu-O, this energy reduction is shared by \(2Ma_n\) superconducting charge carriers, yielding the gap energy:

\[
2\Delta = \frac{E^{\text{(period)}}}{2MA n_s}
\]
In HTSCs, the value of the variable $\eta$, 

$$\eta = \frac{2|\Delta|}{k_B T_c}, \quad (5)$$

is thought to be around or somewhat larger than the BCS value of $\eta \approx 3.5$. We obtain for the temperature $T_c$ of the superconducting transition: 

$$T_c(M, a_m) = \frac{\pi^2 \hbar c}{1440 M n_s \eta k_B} \left( \frac{M}{a_b^3} + \frac{M - 1}{a_i^3} + \frac{1}{a_m^3} \right) \quad (6)$$

As Fig.1 shows, the so-predicted transition temperature

**FIG. 1:** $T_c$ for the ideal Casimir effect and $\eta = 3.5$, $n_s = 5 \times 10^{13}/\text{cm}^2$. The curves are, from bottom to top, for YBCO layer thicknesses $M = 1, 2, 3, 4, 8, \infty$ (the latter is the case of pure YBCO). Notice the temperature scale of kilo Kelvin.

curves are qualitatively of the right shape when compared with Fig.3 of [8]. However, the transition temperatures are four orders of magnitude too high! The onset of ideal conductivity on these Cu-O planes would provide far more Casimir energy than is needed to explain the Cooper pair binding energy. But the predicted Casimir effect for real Cu-O layers is of course weaker.

The expression Eq(2) for the Casimir energy function must be corrected, for example, to take into account that the space between the Cu-O layers is dielectric rather than a vacuum. It is also necessary to correct for the non-flatness of the Cu-O layers and for finite temperature effects. Most importantly, however, real superconducting Cu-O layers are not ideal conductors in the sense above. Real Cu-O layers merely suppress the penetration of electromagnetic fields and they do so less and less efficiently the shorter the wavelength. In particular, modes whose wavelengths in the $c$ direction are shorter than a certain length scale contribute to the ground state energy essentially the same amount whether the Cu-O planes are in their insulating or in their superconducting state. These modes do not contribute to the Casimir effect, i.e., to the lowering of the ground state energy at the onset of superconductivity in Cu-O layers.

Thus, while the Casimir energy, $E(a)$, of the idealized case given in Eq(2) diverges as $E(a) \propto a^{-3}$ for $a \to 0$, the corrected Casimir energy function $E_{corr}(a)$ must flatten at some critical length, say $a_c$. For $a \to 0$, it is to be expected that $E_{corr}(a) \to 0$ because the volume in which the lowering of the density of the electromagnetic ground state energy occurs, namely the volume between the plates, goes to zero as $a \to 0$.

The critical length, $a_c$, should be roughly in the range $10^0...10^2 \text{nm}$. Namely, while $a_c$ cannot be smaller than the coherence length in the $c$ direction, $a_c$ also should not be much larger than about $100nm$. The latter bound is suggested by the fact that measurements of the Casimir effect between non-superconducting metal objects have shown that the Casimir force persists even there, without changing sign, at least down to values of $a \approx 100nm$, see [6]. Let us write the corrected expression $E_{corr}(a)$ as:

$$E_{corr}(a) = -\frac{\pi^2 \hbar c A}{720 a^3} f(a) \quad (7)$$

Here, $f$ is a cutoff function of which we know that it obeys $f(a) \to 0$ probably at least as fast as $a^3$ for $a \ll a_c$, and we make the simplifying assumption that $f(a) \approx z$ for $a \gg a_c$ with $z = 1$. It is important to calculate $f$ for concrete materials but this is hard as it involves the renormalization of relativistic quantum electrodynamics in a highly nontrivial background. For now, our aim is to establish only roughly the potential role of the Casimir effect in HTSCs and for this limited purpose the precise behavior of $f$ should not matter. Also, while the value of $z$ is likely smaller than 1, any value of $z$ is straightforward to accommodate in our subsequent calculations. Below, we will therefore consider the simple example $f_1(a) = e^{-a_c/a}$, and for comparison also the example $f_2(a) = e^{-\sqrt{a_c/a}}$ which describes a slightly softer cutoff. In a further simplification, we treat the Casimir effects of neighboring pairs of layers as independent so that we can then use $f$ to calculate $E_{corr}$ as before. We obtain for the critical temperature:

$$T_c(M, a_m) = \frac{\pi^2 \hbar c (f(a_b) + (1 - 1/M)f(a_i) + f(a_m)/m)}{1440 n_s \eta k_B} \quad (8)$$

At this point, both $\eta$ and $n_s$ are still free parameters, but we notice that only their product enters in Eq(8). The question is, therefore, if, given a cutoff function, all six experimental curves reported in Fig.3 of [8] can be reproduced at once when appropriately choosing the two free parameters $a_c$ and $\eta \cdot n_s$. That these six curves can in fact be reproduced surprisingly well is shown in Figs2,3. The comparatively large magnitude of the ideal Casimir effect (i.e. the case $a_c = 0$), see Fig1, shows that, in principle, there is in this scenario no energetic barrier to raising $T_c$ to significantly higher temperatures. The challenge is to lower $a_c$, i.e., to find layered materials, such as cuprates, for which the Casimir effect is as large as possible.
For YBCO slab thicknesses of \(M\) there is a cutoff function \(T_c\) predicted for corrected Casimir effect with the cutoff function \(f_2(a) = e^{-\frac{a_i}{a}}\) and setting \(a_c = 2.7\)nm, \(\eta = 9.5\), \(n_\delta = 9 \times 10^{14}/\text{cm}^2\). The curves are, from bottom to top, for YBCO slab thicknesses of \(M = 1, 2, 3, 4, 8, \infty\).

**FIG. 2:** \(T_c\) predicted for corrected Casimir effect with the cutoff function \(f_2(a) = e^{-\frac{a_i}{a}}\) and setting \(a_c = 2.7\)nm, \(\eta = 9.5\), \(n_\delta = 9 \times 10^{14}/\text{cm}^2\). The curves are, from bottom to top, for YBCO slab thicknesses of \(M = 1, 2, 3, 4, 8, \infty\).

In conclusion, we have shown that the Casimir effect concerns the energy stored in electromagnetic fields, and on the assumptions made about the cutoff function \(f\) and on the assumption that the Casimir effect of neighboring pairs of layers can be treated as independent, i.e., that these Casimir energies are additive. These points will need to be investigated from first principles.

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[1] M. Tinkham, *Introduction to Superconductivity*, Dover Publications, Mineola, N.Y. (1996), P.W. Anderson, *The theory of superconductivity in the High-\(T_c\) Cuprates,
Princeton University Press, Princeton, N.J. (1997), Y. Yanase et al, Phys. Rept., 387, 1 (2003)

[2] J.R. Schrieffer, Theory of Superconductivity, Benjamin Inc., New York, N.Y. (1964)

[3] H.B.G. Casimir, D. Polder, Phys. Rev. 73, 360 (1948)

[4] A. Kempf, talk given at the 10th Marcel Grossmann Meeting, Rio de Janeiro, Brazil, 20-26.7.2003, gr-qc/0403112

[5] G. Barton, N.S.J. Fawsett, Phys. Rept. 170, 1 (1988), V.B. Bezerra, G.L. Klimchitskaya, V.M. Mostepanenko, Phys. Rev. A62, 014102 (2000), M. Bordag, U. Mohideen, V.M. Mostepanenko, Phys. Rept. 353, 1 (2001), Y. Aghababaie, C.P. Burgess, Phys. Rev. D70, 085003 (2004), S.K. Lamoreaux, Rep. Prog. Phys., 68, 201 (2005)

[6] R.S. Decca et al., Annals Phys. 318 37 (2005), quant-ph/0503105, U. Mohideen, A. Roy, Phys. Rev. Lett., 81, 4549 (1998)

[7] D.H. Lowndes, D.P. Norton, J.D. Budai, Phys. Rev. Lett. 65, 1160 (1990)

[8] Q. Li, X.X. Xi, X.D. Wu, A. Inam, S. Vadlamannati, W.L. McLean, Phys. Rev. Lett. 64, 3086 (1990)

[9] E.V.L. de Mello et al., Phys. Rev. B 66, 092504 (2002), C. Acha et al., Phys. Rev. B 57, R5630-R5633 (1998)

[10] S. Chakravarty, H.-Y. Kee, K. Völler, Nature, 428, 53 (2004)

[11] Z.K. Tang et. al., Science, 292 2462 (2001), M. Kociak et. al., Phys. Rev. Lett. 86 2416 (2001)

[12] I. Takesue, et. al., Phys. Rev. Lett. 96, 057001 (2006)