Quantum key distribution based on quantum dimension and independent devices

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In this paper, we propose a quantum key distribution (QKD) protocol based on only a two-dimensional Hilbert space encoding a quantum system and independent devices between the equipment for state preparation and measurement. Our protocol is inspired by the fully device-independent quantum key distribution (FDI-QKD) protocol and the measurement-device-independent quantum key distribution (MDI-QKD) protocol. Our protocol only requires the state to be prepared in the two dimensional Hilbert space, which weakens the state preparation assumption in the original MDI-QKD protocol. More interestingly, our protocol can overcome the detection loophole problem in the FDI-QKD protocol, which greatly limits the application of FDI-QKD. Hence our protocol can be implemented with practical optical components.

Introduction - The unconditional security of a perfect quantum key distribution (QKD) protocol\textsuperscript{[1]} has been proved by applying entanglement distillation technology and an information theory approach\textsuperscript{[2,4]}. Unfortunately, in practice a practical QKD system is usually composed of imperfect devices. For example, a real source emits weak coherent pulses, which contain multiple photons and will leak the secret key information\textsuperscript{[5]}. A wavelength dependent beam splitter may leak the basis information\textsuperscript{[6]}, since it can be controlled by Eve to apply a man-in-the-middle attack. More generally, the imperfect device in a practical QKD system may be controlled by Eve, so that unconditional security of QKD can not be guaranteed\textsuperscript{[7]}. Since it is difficult to include all possible imperfections in a security analysis model, fully device-independent quantum key distribution (FDI-QKD) is a very hot topic since it can defend against all attacks introduced by imperfect quantum devices\textsuperscript{[8–12]}. The FDI-QKD protocol requires a violation of the Clauser-Horne-Shimony-Holt (CHSH)\textsuperscript{[13]} inequality between two remote parties Alice and Bob; then unconditional security of the secret key can be guaranteed by quantum mechanics and the no-signaling principle. However, in practical experimental realizations, the quantum channel is lossy and the detection efficiency is restricted. Thus, the FDI-QKD protocol is usually vulnerable to a no-fair-sampling attack, which can introduce the detection loophole problem\textsuperscript{[15]} in the Bell test. To avoid the detection loophole problem caused by the quantum channel loss, Lim et al.\textsuperscript{[16]} proposed a FDI-QKD protocol with a local Bell test, which requires Bell tests to be carried out only locally in Alice’s laboratory. But, it can not avoid the detection loophole problem introduced by the limited detection efficiency.

The most vulnerable device in a practical QKD system is the single-photon detector, which may be controlled by Eve if she can apply a light blinding attack.\textsuperscript{[17]} Recently, motivated by the time-reversed entanglement protocol\textsuperscript{[19,20]}, a measurement-device-independent quantum key distribution (MDI-QKD) protocol\textsuperscript{[17,18]} was proposed to avoid the detector side channel attack. In the MDI-QKD protocol, perfect Bennett-Brasard 1984 (BB84)\textsuperscript{[1]} quantum states are prepared on Alice’s and Bob’s sides respectively, and then the two photons will be transmitted to Eve to apply a Bell state projection measurement. By applying the time-reversed entanglement technique, the perfect entangled state can be assumed to be prepared on Eve’s side, and then Alice and Bob perform a perfect BB84 state measurement\textsuperscript{[1]}. Correspondingly, the final secret key can be generated after error correction and privacy amplification. The MDI-QKD protocol can also be realized in practice, where the actual weak coherent pulse laser will not weaken the unconditional security of the key when the decoy state method is applied.

In the spirit of the FDI-QKD protocol, we propose a QKD protocol to weaken the state preparation assumption in the MDI-QKD protocol. More precisely, we only require the state to be prepared in the two-dimensional Hilbert space, and Alice’s (Bob’s) encoding device is independent of Eve. In our protocol, the state measurement can be assumed to be a full black box, while the state preparation can be assumed to be a black box with a dimension restriction. Similarly to the security analysis method in the FDI-QKD protocol, we apply the maximal guessing probability to estimate Eve’s information. In the final security key rate formula, the upper bound of Eve’s information is estimated by the CHSH value violation, while Bob’s information can be calculated by the quantum bit error rate (QBER). Before proposing our protocol, we will give an introduction to the FDI-QKD and MDI-QKD protocols in the following.

Fully device-independent quantum key distribution - The FDI-QKD protocol considers two remote parties Alice and Bob to share a secret key, where Alice (Bob) privately chooses a random input number $x \in \{0,1\}$ ($y \in \{0,1,2\}$) and collects an output $a \in \{0,1\}$ ($b \in \{0,1\}$). By considering all of the input and output random numbers, the data can be divided into two parts. The first part considers the input data $\{x \in \{0,1\}, y \in \{0,1\}\}$ and the corresponding output data $\{a, b\}$, which can be
used for estimating the CHSH value. While the second part considers the input data \( \{ x = 0, y = 2 \} \) and the corresponding output data \( \{ a, b \} \), which can be used for estimating the QBER and generating the final secret key. More precisely, Alice and Bob determine the conditional probabilities \( \{ p(a, b|x, y), a, b, x, y \in \{ 0, 1 \} \} \) to estimate the practical CHSH value \( g \) as the following equation

\[
g = \sum_{a, b, x, y} (-1)^{a+b+xy}p(a, b|x, y),
\]

where the local hidden variable (LHV) bound of \( g \) is 2, while the quantum non-local theory guarantees that \( g \leq 2\sqrt{2} \). From considerations of quantum mechanics, the conditional probability value can be given by

\[
p(a, b|x, y) = Tr(pA_{xa} \otimes B_{yb}) \quad x, y, a, b \in \{ 0, 1 \},
\]

where \( \rho \) is the state shared between Alice and Bob, \( A_{xa} \) and \( B_{yb} \) are measurement operators with the input parameters \( \{ x, y \} \) and the output parameters \( \{ a, b \} \) respectively. Note that, projective measurements can be assumed without loss of generality due to the fact that the quantum dimension has no restriction. The degree of unpredictability of Alice’s measurement outcome \( a \) can be quantified by the maximal guessing probability (Bob’s measurement outcome \( b \) can be analyzed similarly)

\[
p_{\text{guess}}(a) = \max_{a} p(a|x) = \max_{a} \sum_{b} p(a, b|x, y).
\]

where the second equation use the no-signaling principle. By applying the FDI-QKD security analysis result given by Masanes et al. [12], the min-entropy bound of Eve’s reduced state conditioned on Alice’s system can be given by the following equation

\[
H_{\text{min}}(a|x, E) = -\log_{2}p_{\text{guess}}(a) = \log_{2}(1 + \sqrt{2 - \frac{p}{4}}) \equiv f(g).
\]

From this equation, we can see that Eve cannot get any information \( (H_{\text{min}}(a|x, E) = 1) \) when the CHSH value reaches \( 2\sqrt{2} \). If the CHSH value can be obtained from the LHV theory, Eve can get all of the secret information \( (H_{\text{min}}(a|x, E) = 0) \). In the previous analysis, security of the FDI-QKD protocol can be proved without any other assumptions about the practical devices, thus the quantum system can be assumed to be prepared in a Hilbert space of arbitrary dimension. When the quantum state is prepared in two dimensional Hilbert space \( [21] \), the upper bound of the maximal guessing probability can be given by

\[
p_{\text{guess}}(a) \geq p_{\text{guess}}(a)_{\text{2dimensional}},
\]

this inequality can be explained by the fact that the dimension of Eve’s state preparation black box has been restricted to 2; thus she has restricted information with which to guess Alice’s measurement outcome \( a \) compared with the original protocol. Correspondingly, the lower bound of the min-entropy function can be estimated by

\[
H_{\text{min}}(a|x, E)_{\text{2dimensional}} \geq H_{\text{min}}(a|x, E).
\]

Measurement-device-independent quantum key distribution - In contrast to the previous FDI-QKD protocol, the MDI-QKD protocol can remove all detector side channel attacks, and has no restrictions of limited detection efficiency and practical quantum channel losses. But, the security of the original protocol relies on the assumption that the legitimate users can perfectly characterize the encoding systems. The basic idea of the MDI-QKD protocol is to consider that Alice and Bob have characterized states, and then the signals interfere at a 50:50 beam splitter on Eve’s side. This is followed by a polarizing beam splitter, and the signals are projected into either the horizontal or vertical polarization state. An appropriate measurement can guarantee the projection into the two Bell states \( |\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle) \) and \( |\psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle) \). Following the time-reversed entanglement idea, the original MDI-QKD protocol can be assumed to be the entanglement-based BB84 protocol, where the prepared state can be guaranteed to be the maximal entangled state, which will be transmitted to Alice and Bob. Then Alice and Bob apply a perfectly characterized BB84 state measurement, and the final secret key can be established after error correction and privacy amplification. In the entanglement based BB84 protocol, Eve’s information about the final secret key can be estimated from the phase error rate introduced in the quantum. Bob’s uncertainty about Alice’s measurement outcome can be directly calculated from the conditional Shannon entropy function.

Unlike the FDI-QKD protocol, the MDI-QKD protocol has no detection loophole restriction, which requires Alice and Bob to have almost perfect state preparation. However, the state preparation may have some imperfections, which cannot be discovered by the legal parties; thus the imperfection maybe utilized by Eve to apply an attack. Note that the security of MDI-QKD has a quantum dimension restriction, and it can be easily verified that Eve can get all of the secret key information if a high-dimensional state has been prepared by Alice(Bob). Thus it is a natural question to ask if MDI-QKD protocol can generate unconditional security key based only on dimension restriction\([22]\). Fortunately, the answer is positive if we consider that Alice (Bob) and Eve share independent devices.

MDI-QKD with independent devices - We assume that the state preparation box on Alice’s side has random classical input number \( x \) and hidden variable \( \lambda_A \), which can be used to decide Alice’s state preparation \( \rho_{x, \lambda_A} \). Similarly, the state preparation \( \sigma_{y, \lambda_B} \) on Bob’s side can be controlled by the hidden variable \( \lambda_B \) and input random number \( y \). Unlike in the original MDI-QKD protocol, we
do not need to perfectly characterize the state preparation process, and we also require that Alice (Bob) and Eve share independent devices, which can be illustrated by the following equations

\[
P(\lambda_A|B) = P(\lambda_A|E) = P(\lambda_A),
\]
\[
P(\lambda_B|A) = P(\lambda_B|E) = P(\lambda_B),
\]
\[
P(\lambda_E|A) = P(\lambda_E|B) = P(\lambda_E),
\]

(7)

where \(\lambda_E\) is the hidden variable controlled by Eve’s device. In the independent-devices model [23], we can easily prove that Alice’s (Bob’s) state should be prepared using the input number \(x (y)\) and hidden variable \(\lambda_A (\lambda_B)\): neither of them can be controlled or known by Eve’s device. Because of the independent devices, Eve cannot control or know the state preparation on Alice and Bob’s sides through the hidden variable \(\lambda_E\), and it can easily be verified that the MDI-QKD protocol has no security if the hidden variable \(\lambda_A (\lambda_B)\) in the state preparation black box is controlled by Eve.

We first consider that pure states have been prepared by Alice and Bob in the two dimensional Hilbert space with given input numbers \(x, y \in \{00, 01, 10, 11\}\).

\[
\begin{align*}
\text{Alice} & : \{ |\phi_{00}\rangle_{\lambda_A}, |\phi_{01}\rangle_{\lambda_A}, |\phi_{10}\rangle_{\lambda_A}, |\phi_{11}\rangle_{\lambda_A} \}, \\
\text{Bob} & : \{ |\phi_{00}\rangle_{\lambda_B}, |\phi_{01}\rangle_{\lambda_B}, |\phi_{10}\rangle_{\lambda_B}, |\phi_{11}\rangle_{\lambda_B} \},
\end{align*}
\]

(8)

where the state preparation can be assumed to be controlled only by the input numbers.

More generally, when mixed states have been prepared by Alice and Bob in the two dimensional Hilbert space with input numbers \(x, y \in \{00, 01, 10, 11\}\), the detailed state preparation sets on Alice’s and Bob’s sides can be given by

\[
\begin{align*}
\text{Alice} & : \{ \sum_{\lambda_A} \sqrt{p_{\lambda_A}} |\phi_{00}\rangle_{\lambda_A} |\lambda_A\rangle, \\
& \quad \sum_{\lambda_A} \sqrt{p_{\lambda_A}} |\phi_{01}\rangle_{\lambda_A} |\lambda_A\rangle, \\
& \quad \sum_{\lambda_A} \sqrt{p_{\lambda_A}} |\phi_{10}\rangle_{\lambda_A} |\lambda_A\rangle, \\
& \quad \sum_{\lambda_A} \sqrt{p_{\lambda_A}} |\phi_{11}\rangle_{\lambda_A} |\lambda_A\rangle \}, \\
\text{Bob} & : \{ \sum_{\lambda_B} \sqrt{p_{\lambda_B}} |\phi_{00}\rangle_{\lambda_B} |\lambda_B\rangle, \\
& \quad \sum_{\lambda_B} \sqrt{p_{\lambda_B}} |\phi_{01}\rangle_{\lambda_B} |\lambda_B\rangle, \\
& \quad \sum_{\lambda_B} \sqrt{p_{\lambda_B}} |\phi_{10}\rangle_{\lambda_B} |\lambda_B\rangle, \\
& \quad \sum_{\lambda_B} \sqrt{p_{\lambda_B}} |\phi_{11}\rangle_{\lambda_B} |\lambda_B\rangle \},
\end{align*}
\]

(9)

where \(\sum_{\lambda_A} p_{\lambda_A} = \sum_{\lambda_B} p_{\lambda_B} = 1\). The mixed state is prepared by considering different hidden variables \(\lambda_A\) and \(\lambda_B\). For example, if Alice receives the input number 00, the mixed state \(\sum_{\lambda_A} p_{\lambda_A} |\phi_{00}\rangle_{\lambda_A} |\lambda_A\rangle\) will be transmitted to Eve correspondingly.

To relax the state preparation limitation in the MDI-QKD protocol, we propose our protocol as shown in Fig. 1. Note that we assume the state preparation and measurement boxes are independent, which is reasonable in practical experimental realization. We must take this assumption since Eve cannot distinguish any pure state \(|\phi\rangle_{\lambda_A} (|\phi\rangle_{\lambda_B})\) from Alice’s (Bob’s) encoding ensembles via the LHV \(\lambda_A (\lambda_B)\); thus any pure state \(|\phi\rangle_{\lambda_A} (|\phi\rangle_{\lambda_B})\) will be treated equally by Eve.

Our protocol is illustrated in Fig.1. The detailed steps are described as follows:

**Step 1. State preparation:** Alice prepares two sets of quantum states \(\{\rho_{00}, \rho_{01}\}\) and \(\{\rho_{10}, \rho_{11}\}\) in the two dimensional Hilbert space, then she randomly chooses one of the quantum states, which will be sent to Eve in the middle of the quantum channel. Similarly, Bob prepares three sets of quantum states \(\{\sigma_{00}, \sigma_{01}\}, \{\sigma_{10}, \sigma_{11}\}\) and \(\{\sigma_{20}, \sigma_{21}\}\), then he randomly chooses one of the quantum states, which will be sent to Eve in the middle of the quantum channel.

Without loss of generality, in practical experimental realization, we can assume that \(\{\rho_{00}, \rho_{01}\}\) are eigenstates of the Pauli operator matrix \(Z\), and \(\{\rho_{00}, \rho_{01}\}\) are eigenstates of the Pauli operator matrix \(X\). \(\{\sigma_{00}, \sigma_{01}\}\) are eigenstates of the operator matrix \(Z^2 - X^2\), \(\{\sigma_{10}, \sigma_{11}\}\) are eigenstates of the operator matrix \(Z^2 - X^2\), and \(\{\sigma_{20}, \sigma_{21}\}\) are eigenstates of the pauli operator matrix \(Z\).

**Step 2. State measurement:** By considering all of Alice’s state preparation sets and Bob’s first two sets of state preparation, Alice and Bob save the classical data when Eve gets the measurement corresponding to the projection into the Bell state \(|\psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)\). The measurement results will be noted as the set \(S_1\).

Similarly, by considering Alice’s first state preparation set and Bob’s third state preparation set, Alice and Bob save the classical data when Eve gets the measurement corresponding to the projection into the Bell states \(|\psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)\) and \(|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)\). The measurement result will be noted as set \(S_2\). Note that Bob should flip his bit value, so that the classical bit 0 (1) will be changed to bit 1 (0).

**Step 3. CHSH and QBER value estimation:** Alice and Bob apply the set \(S_1\) to estimate the CHSH value \(g\), and the QBER value \(e\) can also be estimated by applying the
Step 4. Error correction and privacy amplification: By applying an error correction code, Alice and Bob can establish an identical classical binary number to eliminate the bit error. Since Eve can get secret key information from the error correction step and the non-maximally violated CHSH value, Alice and Bob construct the final secret key by applying a privacy amplification protocol.

Security analysis model and final secret key rate - To analyze our protocol, we can assume it to be realized in the following way (Bob’s state preparation can be analyzed similarly), Alice first prepares a pair of systems in the singlet state. If she wishes to prepare state $|\psi\rangle$, she will measure one particle in the basis $\{|\psi\rangle, |\psi^+\rangle\}$, and the other particle will also collapse to one of these states. Based on the measurement outcomes, the second quantum state will be sent to Eve if Alice gets the measurement outcome $|\psi\rangle$, while the singlet state will be discarded if Alice gets the measurement outcome $|\psi^+\rangle$. These two cases are demarcated by fragments (1) and (2) in Fig. 2. The quantum state prepared with the two different protocols cannot be distinguished, thus security of the two protocols is equivalent.

Without loss of generality, we can assume that the singlet states are prepared by Eve in fragment (3), then she will send one of these states to Alice. By applying the corresponding two-dimensional state measurement, Alice informs Eve that she should save the measurement outcomes if she gets the quantum state $|\psi\rangle$. Note that Eve is more powerful than in the previous protocol, thus security of the original protocol is not weakened in the present protocol. Next we transform the protocol in fragment (3) to the one in fragment (4). By considering the state measurement equipment as a black box, we have the protocol shown in fragment (4). Note that Eve’s ability will be enhanced in fragment (4). Obviously, this protocol in fragment (4) can be assumed to be a DI-QKD protocol, in which we can apply the CHSH value to estimate Eve’s information.

The main difficulty in this work is to obtain the final secret key rate. We first calculate the final key rate with pure state preparation on Alice’s and Bob’s sides respectively. Combining the CHSH value $g$ with the QBER value $e$, we can get the final secret key rate $R$ from the following formula

$$R \geq H_{\text{min}}(a|x, \lambda_A, \lambda_B, E)_{2\text{dimension}} - H(a|\bar{b}, \lambda_A, \lambda_B)_{S_2}$$

$$\geq H_{\text{min}}(a|x, \lambda_A, \lambda_B, E) - h(e)$$

$$= H_{\text{min}}(a|x) - h(e)$$

$$\geq 1 - \log_2(1 + \sqrt{2 - \frac{4}{e^2}}) - h(e),$$

where $h(x) = -x \log_2(x) - (1 - x) \log_2(1 - x)$ is the binary entropy function, $\bar{b}$ is the bit flip value of $b$, the second inequality can be obtained by utilizing the formula (6).

Since the state preparation can be controlled only by the input random number, the third equation can be proved simply by considering the formula (8).

As in the pure state preparation case, the final secret key rate with mixed state preparation can be given by

$$R \geq \sum p_{\lambda_A} p_{\lambda_B} \{H_{\text{min}}(a|x, \lambda_A, \lambda_B, E)_{2\text{dimension}} - H(a|\bar{b}, \lambda_A, \lambda_B)_{S_2}\}$$

$$\geq \sum p_{\lambda_A} p_{\lambda_B} \{-\log_2 p_{\text{guess}}(a|\lambda_A, \lambda_B) - h(e_{\lambda_A, \lambda_B})\}$$

$$\geq \int \{p_{\lambda_A} p_{\lambda_B} f(g_{\lambda_A, \lambda_B}) - h(\int p_{\lambda_A} p_{\lambda_B} e_{\lambda_A, \lambda_B})\}$$

$$= f(g) - h(e)$$

$$= 1 - \log_2(1 + \sqrt{2 - \frac{4}{e^2}}) - h(e),$$

where $g_{\lambda_A, \lambda_B}$ and $e_{\lambda_A, \lambda_B}$ are the CHSH and QBER values with given hidden variables $\lambda_A$ and $\lambda_B$ respectively. In practical experimental realizations, the observed CHSH value is $g = \sum p_{\lambda_A} p_{\lambda_B} g_{\lambda_A, \lambda_B}$, and the observed QBER
value is \( e = \sum p_{\lambda A} p_{\lambda B} e_{\lambda A, \lambda B} \). The fourth inequality is based on the concave function \( h(e) \), and the fifth inequality is based on the convex function \( f(g) \).

We calculate the final secret key rate with different QBER and CHSH inequality values in Fig. 3.

From the calculation result, we can see that the maximal tolerable QBER can reach nearly 0.5 if the CHSH value reaches \( 2\sqrt{2} \), while the maximal tolerable QBER is 0 if the CHSH value can be explained by the LHV theory. The calculation result shows that this protocol is much more robust than the previous MDI-QKD protocol when \( g \) approaches a high value.

In our security analysis, we assume that the state prepared on Alice’s (Bob’s) side has no correlation with the previous or the following state. Where the prepared quantum states can be assumed to be uncorrelated with each other, the eavesdropper and hidden variables have no memory in Alice and Bob’s devices, and thus the quantum de Finetti theorem \( [24] \) can be directly applied to make our protocol secure against the most general attack. Other applications of this protocol and a more general security analysis are interesting open questions for future research.

Conclusion - We propose a QKD protocol, the security of which is based only on quantum states prepared in the two dimensional Hilbert space and the independent of devices between Alice (Bob) and Eve. Our protocol can also be practically realized with current experimental technology.

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[1] C. H. Bennett and G. Brassard, Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India. New York: IEEE, 1984. 175C179
[2] H.-K. Lo and H. F. Chau, Science 283, 2050 (1999).
[3] P.W. Shor and J. Preskill, Phys. Rev. Lett. 85, 441 (2000).
[4] R. Renner, PhD thesis, Diss. ETH No 16242, quant-ph/0512258 (2005).
[5] G. Brassard, N. Lutkenhaus, T. Mor, and B. C. Sanders, Physical Review Letters, 85(6):1330 (2000).
[6] H. W. Li, S. Wang, J. Z. Huang, et al., Phys. Rev. A 84(6):062308 (2011).
[7] V. Scarani, et al., Reviews of modern physics. 81(3): p. 1301 (2009).
[8] S. Pironio, A. Acn, N. Brunner, N. Gisin, S. Massar, and V. Scarani, New J. Phys. 11, 045021 (2009).
[9] M. McKague, New J. Phys. 11, 103037 (2009).
[10] E. Haggi, Ph.D. thesis, ETH Zurich, 2010, arXiv:1012.3878
[11] E. Haggi and R. Renner, arXiv:1009.1833
[12] L. Masanes, S. Pironio, and A. Acn, Nat. Commun. 2, 238 (2011).
[13] J. F. Clauser, M.A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[14] Anupam Garg, N.D. Mermin, Phys. Rev. D 25 (12): 3831C5 (1987).
[15] Charles Ci Wen Lim, Christopher Portmann, Marco Tomamiche, Renato Renner, and Nicolas Gisin, Phys. Rev. X 3, 031006 (2013)
[16] Lydersen L, Wiechers C, Wittmann C, et al. Hacking commercial quantum cryptography systems by tailored bright illumination. Nat Photonics, 2010, 4: 686C689
[17] S. L. Braunstein and S. Pirandola, Phys. Rev. Lett. 108, 130502 (2012).
[18] H.-K. Lo, M. Curty, and B. Qi, Phys. Rev. Lett. 108, 130503 (2012).
[19] E. Biham, B. Huttner, and T. Mor, Phys. Rev. A 54, 2651 (1996).
[20] H. Inamori, Algorithmica 34, 340 (2002).
[21] H-W Li, P. Mironowicz, M. Pawlowski, Z-Q Yin, Y-C Wu, S. Wang, W. Chen, H-G Hu, G-C Guo, and Z-F Han, Phys. Rev. A 87, 020302(R), (2013)
[22] Z-Q. Yin, C-H. F. Fung, X-F. Ma, C-M. Zhang, H-W. Li, W. Chen, S. Wang, G-C, Guo, Z-F Han, Phys. Rev. A 88, 062322 (2013).
[23] J. Bowles, M. T. Quintino, N. Brunner, e-print arXiv:1311.1525 (2013).
[24] M. Christandl, R. Konig, and R. Renner, Phys. Rev. Lett. 102, 020504 (2009).