Prediction of vibration characteristics of blisks using similitude models

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ABSTRACT
This study investigates the determination approach of distorted scaling laws for predicting the dynamic characteristic of an aero engine’s blisk. Based on the dynamic scaling laws of typical thin-walled structures, an assumption of geometrically complete scaling laws is firstly proposed and numerically validated. For distorted models of disk thickness, in order to simplify the design procedure, a simplification condition is proposed and applied to the first 10 orders’ distorted scaling laws (blade-dominated vibrations) by combining sensitivity analysis. Next, the 11th–14th orders’ distorted scaling laws are determined for disk-dominated vibrations. Numerical validation demonstrates that distorted scaling laws possess a good accuracy. Finally, the applicability of these new scaling laws is validated by the experimental data. The results indicate that, by using the new scaling laws, the simple models can predict vibration characteristics of blisk by employing similitude models.

1. Introduction

The blisks are widely used in the mechanical engineering due to their great stiffness and light weight, such as steam turbine, gas turbine, and aero engine (Liao et al., 2007; Sichani et al., 2012). Moreover, the vibration characteristic of blisks is one of the most significant research interests (Holland and Epureanu, 2013). However, employing the prototype directly in vibration experiments is arduous to machine or test, and it is time-consuming. Fortunately, the similitude design can address these problems. In addition, for some special cases, the thickness of the scaled-down model is too small for a machine. For example, the blade thickness of a scaled-down model of blisks may be too small. Therefore, distorted similitude design is a powerful tool to predict the vibration characteristics due to the limitation of structural parameters.

A number of investigations have been studied in attempts to understand the vibration characteristics of blisks. Effects of small structural variations on the natural characteristics and responses of periodic structures were presented (Wei and Pierre, 1988a,b). In their studies, blisks can be simplified as the nearly periodic structures with cyclic symmetry and the perturbation approach is employed to analyze the mistuning effects. Lim et al. (2007) advocated a new modeling by dividing the system into a cyclic symmetry model with a tuned bladed disk component. A
reduced-order modeling approach of multi-stage mistuned bladed disks was developed (Laxalde and Pierre, 2011). The experimental modal analysis of an academic bladed disk was demonstrated, and the mistuning of each blade was identified by using a base excitation (Nyssen and Golinval, 2016).

In terms of the similitude design, the differential equation method was developed to determine the scaling laws. Morton (1988) developed the scaling laws of fiber isotropic composites and established them by a dimensional analysis method, and the impact load is considered. Rezaeepazhand et al. (1995) and Rezaeepazhand and Simites (1996) further investigated distorted scaling laws of laminated structures for predicting the buckling and vibration characteristics. In their studies, the scaling factors of distorted materials and geometrical parameters were established by considering the number of piles of the laminated plates and shells. Wu (2003, 2005, 2006) presented the scaling laws for predicting the dynamic responses of a full-size beam by using a complete similitude model, and scaling laws were validated by using free and forced vibration characteristics. Afterwards, both elastically supported conditions and subjected to a moving force were investigated for a beam. The scaling laws of symmetric laminated plates and cylindrical shells were presented by using the governing equation (Singhatanadgid and Unghbakorn, 2005; Unghbakorn and Wattanasakulpong, 2007). Both complete and distorted similitude scaling laws were derived for predicting the buckling problems. Yazdi and Rezaeepazhand (2011) and Yazdi (2014) derived the scaling law of the nonlinear vibration frequency between the prototype and experimental models, and the similitude design approach of vibration amplitudes were determined by considering the number of piles. The scaling laws of the forced response and energy response were established for the rectangular flexural plates by employing the energy distribution method (De Rosa et al., 2011; De Rosa and Franco, 2015; Meruane et al., 2016). Adams and Melz (2016) and Adams et al. (2017) proposed a scaling method of the vibration structures by using global sensitivity analysis.

More recent studies were developed by Luo et al. (2015), and the dynamic similitude design method was investigated to obtain approximate and accurate distorted scaling laws of typical thin walled structures. The determination principles and structural size intervals of scaling laws were proposed to predict natural characteristics and dynamic response of the prototype. Luo et al. (2016a,b) and Ye et al. (2018) investigated the design of similitude models of rotor systems, and the balancing method of similitude models was discussed.

Up to now, there have been a plenty of theoretical studies about similitude design of typical thin-walled plates and shells (Coutinho et al., 2016; De Rosa and Franco, 2015; Meruane et al., 2016), but the ability to predict vibration characteristics of blisks is still missing by using similitude experimental models. In this study, a simplification condition is proposed in order to simplify the determining procedure of scaling laws. Moreover, a different method of determining scaling laws is presented for the blisks based on the sensitivity analysis.

The structure of the article is as follows. Geometrically complete scaling law of simple blisks is firstly assumed and numerically validated in Section 2. In order to simplify the determining procedure of the first 10 orders’ scaling laws, a simplification condition is proposed and distorted scaling laws are obtained based on sensitivity analysis and numerically validated in Sections 3 and 4. Moreover, experimental validation of distorted similitude models is presented in Section 5. Finally, in Section 6, sensitivity comparisons and similitude design procedure are given out, which guide to the design of experimental models for blisks.

2. Geometrically complete scaling law

2.1. Natural characteristics

The academic blisk is considered as a cyclic symmetrical structure with 18 sectors made of aluminium. The structure is clamped at its inner ring, and the boundary conditions respect the cyclic
symmetry. The finite element (FE) model is constructed and the natural characteristics of the cyclic structure are analyzed. The cyclic symmetry is applied to the finite model, and the 20-node SOLID186 elements having 6DOF at each node are selected to build the prototype. The FE model is meshed by the sweeping method, and the model consists of 13,340 elements. Geometrical characteristics of the academic structure and the clamped FE model of the blisk are depicted in Fig. 1. Young’s modulus $E$ is 70 GPa, Poisson ratio $\nu$ is 0.3, and density $\rho$ is 2770 kg m$^{-3}$.

The natural frequencies and vibration mode of the academic structure are shown in Table 1. Combining Fig. 2 and Table 1, the vibration mode of the academic blisk can be divided into two types according to the blade’s contribution to total blisk strain energy: blade-dominated and disk-dominated (Klauke et al., 2009). The first 10 orders’ natural frequencies are approximately close in the interval [598.49 Hz, 644.64 Hz] and the vibration mode are blade-dominated, which can be distinguished by the number of nodal diameter (ND) and circle (NC) lines. For the blade-dominated cyclic symmetry mode (BDCSM), the maximum value of nodal diameter depends on blade number $N$. Furthermore, the 11th~14th orders’ natural characteristics are disk-dominated cyclic symmetry mode (DDCSM).

2.2. Geometrically complete scaling laws

Due to the complexity of governing equation of an academic blisk, the governing equation of the blisk is arduous to derive and obtain. Furthermore, the classical similitude design method, the differential equation method is not suitable for the prediction of vibration characteristics of the blisk. According to the blisk structure, the blisk consists of thin walled annular plates and thin walled rectangular plates.

According to the governing equation, geometrically complete scaling law of natural frequencies can be denoted as $\lambda_{\omega} = \frac{1}{\lambda} \sqrt{\frac{\lambda_E}{\lambda_\rho}}$ for typical thin-walled structures, such as thin-walled annular or rectangular plates.

Consequently, geometrically complete scaling law of the blisk can be supposed as:

$$\lambda_{\omega} = \frac{1}{\lambda} \sqrt{\frac{\lambda_E}{\lambda_\rho}} \tag{1}$$

where, $\lambda$ is the scaling factor of geometrical parameters; $\lambda_{\omega}$ is the scaling factor of natural frequencies; $\lambda_E$ is the scaling factor of Young’s modulus; and $\lambda_\rho$ is the scaling factor of density.

2.3 Numerical validation

In order to effectively validate the geometrically complete scaling law of natural frequency for blisks, and different materials’ similitude models are as following: model M1 is made of titanium
alloy (Young modulus $E$ is 114 GPa and density $\rho$ is 4430 kg $\cdot$ m$^{-3}$) and geometrically complete scaling factor $\lambda_1$ is 4; model M2 is made of stainless steel (Young modulus $E$ is 210 GPa and density $\rho$ is 7850 kg $\cdot$ m$^{-3}$) and geometrically complete scaling factor $\lambda_2$ is 0.5. The longitudinal wave speeds of titanium alloy and stainless steel are 5072 and 5172 m/s, respectively. We assume that these two longitudinal wave speeds are the same in the following analysis.

By using modal analysis in ANSYS, similitude models are established and analyzed by employing 20-node SOLID186 element. Errors of natural frequencies between the prototype and similitude models can be calculated by:

$$
\eta = \frac{|\omega_p - \omega_{pr}|}{\omega_p} \times 100\%
$$

(2)

where, subscript $p$ denotes prototype; subscript $pr$ is the predicted results of similitude model.

Errors between the prototype and similitude models can be calculated by Eq. (2), and the natural characteristics of similitude models are shown in Table 2.

**Table 1.** The first 14 orders' natural characteristics.

| Order $i$ | 1   | 2    | 3    | 4    | 5    | 6    | 7    |
|-----------|-----|------|------|------|------|------|------|
| Natural   | 598.49 | 611.67 | 612.21 | 636.36 | 641.69 | 643.37 | 644.09 |
| Vibration mode | ![Vibration mode](image1) | ![Vibration mode](image2) | ![Vibration mode](image3) | ![Vibration mode](image4) | ![Vibration mode](image5) | ![Vibration mode](image6) | ![Vibration mode](image7) |

| Order $i$ | 8   | 9    | 10   | 11   | 12   | 13   | 14   |
|-----------|-----|------|------|------|------|------|------|
| Natural   | 644.43 | 644.59 | 644.64 | 1053.3 | 1159.2 | 1245.5 | 2290.2 |
| Vibration mode | ![Vibration mode](image8) | ![Vibration mode](image9) | ![Vibration mode](image10) | ![Vibration mode](image11) | ![Vibration mode](image12) | ![Vibration mode](image13) | ![Vibration mode](image14) |

Figure 2. The first 10 orders' natural frequencies for disk thickness' distorted model.
From Table 2, errors of the first 14 orders’ natural frequencies are within 0.4%. Moreover, the vibration mode of similitude models is same as the prototype. Therefore, similitude models can accurately predict the dynamic characteristics of the prototype by using Eq. (1).
3. Distorted models for disk thickness

In order to simplify the design method of distorted similitude models, the geometrically complete similitude models are firstly introduced as transitional models. The transitional model is defined as: the scaling factors of geometrical parameters are completely coincident between the prototype and transitional model. The geometrically complete scaling law between the prototype and transitional model can be denoted as:

\[ k_p/C_0 t = k E/C_0 q \]

where, subscript t represents transitional model; \( k_p/C_0 t \) is the scaling factor of geometrical parameter between the prototype and transitional model; \( k E/C_0 q \) is the scaling factor of structural parameters between the prototype and transitional model.

Sensitivity of the structural vibration behavior is defined as the variation rate of vibration characteristic parameters with respect to structural parameters \( j \) (mass, stiffness, and geometrical parameters) (Lee and Jung, 1997a,b). Furthermore, the sensitivity analysis is employed to obtain distorted similitude models based on the transitional models. A distorted model can be defined as follows: ensure one structural parameter of the transitional model changes in a small range, and other parameters keep unchanged. Material parameters need to be same between the distorted model and transitional model.

3.1. Distorted scaling laws

The characteristics of the transitional model are described as: geometrically complete scaling factor \( \lambda_p/t \) is 0.8, Young modulus \( E \) is 210 GPa, and density \( \rho \) is 7850 kg \cdot m^{-3}.

In order to establish the distorted similitude model for the disk thickness, the sensitivity analysis of natural frequency’s scaling factor with respect to disk thickness’ scaling factor is firstly analyzed. Because values of the first 10 orders’ natural frequencies are close and around 640 Hz, the 11th~14th orders’ distorted scaling laws are firstly determined.

Under the condition of ensuring other geometrical and material parameters unchanged between the transitional model and distorted model, disk thickness is altered. Taking the 11th order’s distorted scaling laws as an example, the 11th order’s natural frequencies are fitted and can be expressed as:

\[ \lambda_{o,t} = 1.001 \lambda_{o,t-m}^{0.7784} \]

where, subscript \( H \) denotes the disk thickness of distorted models.

Efficiency of the fitting curve is determined by adopting adjusted square \( \bar{R}^2 \). The formula of the adjusted square \( \bar{R}^2 \) can be denoted as

\[
\bar{R}^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}
\]

where, \( y_i \) is the observed value, \( \hat{y}_i \) is the predicted value, and \( \bar{y} \) is the mean of observed values.

| Order | Fitting equation | Adjusted square \( \bar{R}^2 \) | Sensitivity | Distorted scaling law |
|-------|------------------|-----------------------------|-------------|-----------------------|
| 12    | \( \lambda_{o,t} \) = 1.001 \( \lambda_{o,t-m}^{0.7784} \) | 0.9996 | 0.7792 | \( \lambda_{o,p} = \frac{1.001 \lambda_{o,t}^{0.7784}}{\lambda_{o,t}} \) |
| 13    | \( \lambda_{o,t} \) = 1.001 \( \lambda_{o,t-m}^{0.856} \) | 0.9996 | 0.8569 | \( \lambda_{o,p} = \frac{1.001 \lambda_{o,t}^{0.856}}{\lambda_{o,t}} \) |
| 14    | \( \lambda_{o,t} \) = 0.9982 \( \lambda_{o,t-m}^{2.5701} \) | 0.9985 | 0.5691 | \( \lambda_{o,p} = \frac{0.9982 \lambda_{o,t}^{2.5701}}{\lambda_{o,t}} \) |
\[
\bar{R}^2 = 1 - \left[ \frac{n' - 1}{n' - (k + 1)} \right] \left( 1 - \frac{\sum (Y_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2} \right) (i = 1, 2, \cdots, n')
\]  
(5)

where, \( n' \) is fitted point numbers; \( k \) is the order of fitted polynomial; \( \bar{Y} \) is the fitted value; \( \bar{Y} \) is the average value; \( Y \) is the true value.

If \( \bar{R}^2 > 0.99 \) is satisfied, the fitting curve is thought to be effective and acceptable by comparing and analyzing in this research.

Sensitivity of the 11th order’s natural frequencies with respect to disk thickness can be denoted as:

\[
\Phi_{H11} = \frac{d\dot{\lambda}_{o11}}{d\dot{H}_{1-m}} \bigg|_{\dot{H}_{1-m}=1} = 0.846 \dot{\lambda}_{H}^{-0.1557} = 0.846
\]  
(6)

Therefore, for the disk thickness’s similitude models, the 11th order’s distorted scaling laws can be described as:

\[
\dot{\lambda}_{o} = \dot{\lambda}_{o,p-t} \cdot \dot{\lambda}_{o,1-m} = \frac{1}{\lambda_{p-t}} \sqrt{\lambda_E / \lambda_{o}} \cdot 1.002^{0.8443} = \frac{1.002^{0.8443}}{\lambda_{p-t}} \sqrt{\lambda_E / \lambda_{o}}
\]  
(7)

Similarly, the 12th~14th orders’ sensitivities and distorted scaling laws can be derived and shown in Table 3.

In addition, the first 10 orders’ natural frequencies of distorted models can be obtained by using ANSYS and depicted in Fig. 2.

With the order increasing, influences of disk thickness with respect to natural frequency are becoming smaller as shown in Fig. 2, which means that sensitivities of the disk thickness are smaller. Because the first 10 order’s vibration mode are coincident, a simplification condition is proposed in order to simplify the design process of distorted models. The simplification condition can be written as:

\[
\zeta = \left| \frac{\Phi_{\min}}{\Phi_{\max}} \times 100\% \right| \leq 5\%
\]  
(8)

where, \( \Phi_{\min} \) is the minimum value of the first 10 orders’ sensitivities; \( \Phi_{\max} \) is the maximum value of the first 10 orders’ sensitivities. If the ratio of sensitivity value with respect to the maximum value is less than 5%, the natural frequency is almost affected by the disk thickness. For the first 10 orders’ natural frequencies, the distorted scaling laws of natural frequency can be determined as \( \dot{\lambda}_{o,1-m} = 1 \) if sensitivity is satisfied as Eq. (8).

The first order’ sensitivity is obtained as 0.2507, and the maximum sensitivity \( \Phi_{\max} \) is 0.8569 by combining Fig. 2 and Table 3. According to the simplification condition, for the first 10 orders’ distorted scaling laws, the first 3 orders’ distorted scaling laws are indispensable between the transitional model and distorted model. Then, the 4th~7th orders’ scaling laws are unnecessary to calculate and can be denoted as \( \dot{\lambda}_{o,1-m} = 1 \). Similarly, the first 3 orders’ distorted scaling laws can be obtained as:

\[
\dot{\lambda}_{o1} = \frac{-0.0836\dot{\lambda}_{H1-m}^{-2.999}+1.084}{\lambda_{p-t}} \sqrt{\lambda_E / \lambda_{o}}
\]  
(9a)

\[
\dot{\lambda}_{o2} = \frac{-0.061\dot{\lambda}_{H1-m}^{-2.314}+1.061}{\lambda_{p-t}} \sqrt{\lambda_E / \lambda_{o}}
\]  
(9b)

\[
\dot{\lambda}_{o3} = \frac{-0.05724\dot{\lambda}_{H1-m}^{-2.918}+1.057}{\lambda_{p-t}} \sqrt{\lambda_E / \lambda_{o}}
\]  
(9c)
Considering the 4th~10th orders’ sensitivities is satisfied as the simplification condition Eq. (8), the 4th~10th orders’ distorted scaling laws can be denoted as:

\[ \lambda_{io} = \frac{1}{\lambda_{p-t}} \sqrt{\frac{\lambda_E}{\lambda_p}} \]  

(10)

### 3.2. Numerical validation

In order to effectively validate the accuracy of disk thickness’ distorted scaling laws, different disk thickness’ distorted models are selected. Distorted models’ materials are keeping the same as the transitional model, and disk thickness’ scaling factors are \( \lambda_{H_1} = 0.85 \) and \( \lambda_{H_2} = 1.2 \), respectively. Natural characteristics are calculated and prediction results of distorted model M1 are depicted in Fig. 3. Furthermore, the results of distorted model M2 are similar.

Error ratios of the first 14 orders’ natural frequencies are within 2.5% and the vibration modes are identical between the prototype and distorted models. Therefore, the distorted scaling laws can accurately predict the first 14 orders’ natural characteristics between the prototype and distorted models.

### 4. Distorted models for blade length

#### 4.1. Distorted scaling laws

The design method of distorted models for blade length is investigated based on sensitivity analysis, and the numerical validation is presented in this section. The transitional model is firstly selected, and the geometrical scaling factor is defined as \( \lambda_{p-t} = 1.2 \). Material of the transitional model is stainless steel, and Young’s modulus \( E \) is 210 Gpa and density \( \rho \) is 7850 kg \( \cdot \) m\(^{-3} \).

For the 11th~14th orders’ distorted scaling laws, the design method of distorted similitude models is similar with the design of disk thickness’ distorted models.

Refer to the Fig. 3, natural frequencies of different distorted models are calculated by using ANSYS and the 11th~14th orders’ natural frequencies are firstly fitted. Moreover, the 11th~14th orders’ sensitivities and distorted scaling laws are determined and listed in Table 4 in the same way.

The first 10 orders’ natural frequencies of distorted models are depicted in Fig. 4.

From Fig. 2, the variation trend of the first 10 orders’ natural frequencies is different. Observed from Fig. 4, the 2nd order’s sensitivities are same as the 3rd order’s sensitivities. Similarly, the 4th~10th orders’ sensitivities are coincident. The distorted scaling laws can be determined as follows:

\[ \lambda_{io,2-3} = \frac{-0.417 + 1.417 \lambda_{L_{1-m}}^{-1.271}}{\lambda_{p-t}} \sqrt{\frac{\lambda_E}{\lambda_p}} \]  

(11a)
Similarly, the first 10 orders’ distorted scaling laws can be obtained and shown in Table 5.

By comparing the first 14 orders’ sensitivities, sensitivity ratios are not satisfied as Eq. (8). Therefore, the first 10 orders’ distorted scaling laws cannot be ignored between the transitional model and distorted model.

### 4.2. Numerical validation

Different blade length’s distorted model are selected based on the transitional model, and blade length’s scaling factors are defined as $\lambda_{Lt-m} = 0.85$ and $\lambda_{Lt-m} = 1.2$, respectively. Material parameters of distorted models are same as the transitional model.

By modal analysis, the first 14 orders’ natural characteristics are obtained. Based on distorted scaling laws, prediction results and error ratios can be calculated and shown in Fig. 5, respectively. Furthermore, the results of distorted model M2 are similar.

Error ratios between the prototype and distorted models are calculated within 1% and the vibration mode are identical. Therefore, distorted scaling laws can accurately predict natural characteristics of the prototype for blade length’s distorted models.

### 5. Experimental validation

#### 5.1. Test preparation

Experimental results are presented for distorted models in order to validate the distorted scaling laws, and error analysis is presented in this section.

For experimental structures, the scaling factors of distorted model M1 and M2 are taken as 1.2 and 0.8, respectively. Blade length and disk thickness are distorted parameters of model M1 and M2, respectively. In the experiment, the test setup is depicted in Fig. 6 for experimental prototype and distorted models.

A simple test procedure is presented for natural characteristics of experimental blisks:

1. The structures are installed in the experimental system and the acceleration sensor is fixed on the top of a blade.
2. Sensitivity and other parameters are set by hammering the test system.
3. Test models of the prototype and distorted models are established, and the point 1h is set up as response point of the acceleration sensor.
4. Move the modal hammer and measure the frequency responses of all points for test models.
Analyze and identify the stabilization diagram of experimental results, and vibration mode of experimental prototype and model are identical and depicted in Table 6. The experimental results are conformed to experimental characteristics of thin-walled annular plates. Observed from Table 6, the forced response of individual blades is increased by structural mistuning. The structural mistuning is known to be caused by small derivations from blade to blade, and unavoidable geometric and material imperfection leads to the mistuning.

Moreover, the first 13 orders’ natural frequencies of experimental models and predicted results are shown in Fig. 7. Figure 7 indicates that the error ratio of natural frequencies between the prototype and distorted model are within 10% (Simitses and Rezaee Pazhand, 1992), and the vibration modes are identical. Consequently, the disk thickness’ distorted model can predict dynamic characteristics of the prototype by employing distorted scaling laws.

5.3. Results validation for blade length distorted model

Similarly, for blade length distorted model, experimental results and error ratios of natural frequencies are depicted in Fig. 8.
The reasons of exceptive error are as follows: the machining of the experimental blisks, such as the dimensional error, deviation of material parameters. For example, precision of blade thickness is formidable to ensure and the blade thickness will make great effects on the test results. According to Table 6, localized vibrations are caused by the machining of blade thickness.

Experimental error ratios are higher than numerical error ratios, and other reasons can be classified as follows:

1. Test setup and fixture are important influence factors.
2. Measurement accuracy of a test system, for example, the precision of the acceleration sensor.
3. Random errors of the test procedure.

Figure 8 refers that error ratios between the prototype and distorted model are within 10% except the 11th order natural frequency, and the vibration mode is identical. Accordingly, the blade length’ distorted model can predict dynamic characteristics of the prototype by distorted scaling laws within an acceptable error interval.

**Table 6. Vibration mode of experimental models.**

| Order $i$ | 1 | 2 | 3 | 4 | 5 |
|-----------|---|---|---|---|---|
| Vibration mode | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) |

| Order $i$ | 6 | 7 | 8 | 9 | 10 |
|-----------|---|---|---|---|----|
| Vibration mode | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) |

| Order $i$ | 11 | 12 | 13 |
|-----------|----|----|----|
| Vibration mode | ![Image](image_url) | ![Image](image_url) | ![Image](image_url) |
6. Sensitivity comparison

In order to guide the design of distorted models, the sensitivity comparison is presented for significant structural parameters (Zhou, 2010). Under the condition of ensuring other geometrical and material parameters unchanged between the transitional model and distorted model, the blade thickness is variable value and natural characteristics of distorted models are calculated by using ANSYS.

Natural frequencies of blade thickness’ distorted models are fitted by Matlab, and sensitivity results of natural frequency with respect to blade thickness can be calculated. In addition, sensitivity results of natural frequency with respect to geometrical parameters (disk thickness, blade length and thickness) are depicted in Fig. 9.

Therefore, for the first 10 orders’ sensitivities, sensitivity of natural frequencies with respect to blade length is maximum value and disk thickness is a minimum influence factor. Furthermore, for the 11th~14th orders’ sensitivities, sensitivity of natural frequencies with respect to disk thickness is the maximum value and blade thickness is the minimum influence factor. In order to
investigate different orders’ dynamic characteristics of blisks, the sensitivity analysis can provide
guidance for the design and manufacture of blisks.

According to the engineering requirements, geometrically complete similitude model is firstly
designed due to complete scaling factor possess a good accuracy in a wider interval. Since the
geometrical parameters take great effect on the vibration mode of blisks, distorted scaling factor
is altered in a smaller interval and determined combining the sensitivity analysis.

7. Conclusions

In this paper, geometrically complete scaling law of blisks is established, and numerical validation
indicates that the geometrically complete scaling law can predict the dynamic characteristics. A
simplification condition is proposed and applied to the first 10 orders’ distorted scaling laws, and
distorted scaling laws are developed based on sensitivity analysis. Moreover, the effects of geomet-
crical parameters are analyzed, which can provide the guidance for the machine of blisks. Detailed
conclusions obtained from this study are listed as follows:

1. For the design procedure of distorted models for blisks, a simplification condition Eq. (8) is
   proposed and applied to the first 10 orders’ distorted scaling laws based on the sensitivity
   analysis.
2. Sensitivity analysis presents the effects of geometrical parameters with natural frequencies,
   and the blade length is a significant factor for the blade-dominated vibration mode.
3. Experimental validation suggests that the first 14 orders’ distorted scaling laws can predict
   the dynamic characteristics of the prototype within an acceptable error interval. Furthermore,
   the similitude design procedure of the blisks is presented.

Nomenclature

\[ E \] Young’s modulus
\[ \mu \] Poisson ratio
\[ \rho \] Density
\[ H \] Disk thickness
\[ L \] Blade length
\[ W \] Blade width
\[ T \] Blade thickness
\[ N \] Number of blades
\[ \lambda \] Scaling factor
\[ \text{subscript p} \] Prototype
\[ \text{subscript t} \] Transitional model
\[ \text{subscript m} \] Distorted model
\[ \Phi \] Sensitivity value
\[ \eta \] Error
\[ \xi \] Sensitivity ratio
\[ R^2 \] Adjusted square
\[ \text{FE} \] Finite element
\[ \text{ND} \] Nodal diameter
\[ \text{NC} \] Nodal circle
\[ \text{BDCSM} \] Blade-dominated cyclic symmetry mode
\[ \text{DDCSM} \] Disk-dominated cyclic symmetry mode
Funding

This work was supported by the National Science Foundation of China [grant numbers 11572082]; the Fundamental Research Funds for the Central Universities of China [grant numbers N160312001, N150304004]; and the Excellent Talents Support Program in Institutions of Higher Learning in Liaoning Province of China [grant numbers LJQ2015038].

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