Electromagnetic Meson Form Factors in the Salpeter Model

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Abstract

We present a covariant scheme to calculate mesonic transitions in the framework of the Salpeter equation for $qar{q}$-states. The full Bethe Salpeter amplitudes are reconstructed from equal time amplitudes which were obtained in a previous paper [1] by solving the Salpeter equation for a confining plus an instanton induced interaction. This method is applied to calculate electromagnetic form factors and decay widths of low lying pseudoscalar and vector mesons including predictions for CEBAF experiments. We also describe the momentum transfer dependence for the processes $\pi^0, \eta, \eta' \rightarrow \gamma\gamma^*$. 
I. INTRODUCTION

In two previous papers [1,2] we presented a quark model for light mesons based on the Salpeter equation. We investigated a kernel that incorporates confinement and a residual instanton induced quark interaction [3,4,5], which in this framework leads to the correct masses and flavor mixing of the π and η mesons. In general we obtained a satisfactory description of the mass spectrum of the low lying pseudoscalar and vector mesons. We also calculated various decay observables such as the weak decay constants, the γγ-decay width of the pseudoscalars and the leptonic widths of vector mesons. A comparison with nonrelativistic results revealed the relevance of the relativistic treatment (including the correct normalization of the bound states) for the description of these observables, especially for the two photon width of the pion.

All these transitions involve a non-hadronic final state and therefore could be calculated in the rest frame of the bound state where also the amplitudes were determined. A relativistic quark model however should also be able to describe reactions with a mesonic final state. If we consider e.g. weak decays of heavy to light mesons or electromagnetic scattering with large momentum transfer, the outgoing meson will recoil with relativistic velocity. The calculation of such transitions between mesonic states therefore involves a boost of at least one of the meson amplitudes. A covariant formulation of the Salpeter equation [6] enables to treat this boost correctly and thus to investigate the region of large momentum transfer which will be available e.g. in CEBAF experiments in the near future [7,8,9].

A second important ingredient of any relativistic quark model is an adequate treatment of the off-shell properties of the quarks. Especially for mesonic states with large binding energy the negative energy Dirac-components become essential. If one considers form factors at high momentum transfer, the quarks are highly off shell. The Salpeter model presented here allows for a consistent inclusion of these effects.

In section II of this paper we briefly repeat the covariant formulation of the Salpeter equation and the calculation of the full Bethe-Salpeter (BS) amplitude as well as its transformation properties. In section III we sketch the calculation of the electromagnetic current in the Mandelstam formalism. Finally we present our results for the electromagnetic form factors for 0− → 0− and 1− → 0− transitions, the corresponding decay widths and the form factors for the processes π0, η, η′ → γγ∗.

II. THE SALPETER EQUATION AND THE RECONSTRUCTION OF THE FULL BETHE SALPETER AMPLITUDE

The Bethe-Salpeter equation for the amplitude

\[ [\chi_P(x)]_{\alpha\beta} = \langle 0 | T \Psi^1_\alpha(\eta_1 x) \Psi^2_\beta(-\eta_2 x) | P \rangle, \]

in momentum space reads [10,2]:

\[ \chi_P(p) = S^F_1(p_1) \int \frac{d^4p'}{(2\pi)^4} [ -i K(P,p,p') \chi_P(p')] S^F_2(-p_2) \]

in momentum space reads [10,2]:
where \( p_1 = \eta_1 P + p \), \( p_2 = \eta_2 P - p \) denote the momenta of the quark and antiquark respectively, \( P \) is the four momentum of the bound state and \( S^F \) and \( K \) are the Feynman propagators and the irreducible interaction kernel. Here \( \eta_1, \eta_2 \) are two arbitrary real numbers satisfying \( \eta_1 + \eta_2 = 1 \).

The Salpeter equation neglects retardation effects in the interaction kernel in the rest frame of the bound state. This can be written covariantly as \( K(P, p, p') = V(p_\perp, p'_\perp) \) where \( p_\perp = p - (P p/P^2) P \). Furthermore it is assumed that the propagators are given by their free form \( S^F_i(p) = i/(\not{p} - m_i + i\epsilon) \) with \( m_i \) an effective constituent quark mass. Introducing the Salpeter or equal time amplitude in the rest frame of the bound state

\[
\Phi(\not{p}) := \left. \int dp^0 \chi_P(p^0, \not{p}) \right|_{p=(M,\vec{0})}
\]

one arrives at the well-known Salpeter equation \([1]\), i.e.

\[
\Phi(\not{p}) = \int \frac{d^3p'}{(2\pi)^3} \frac{\Lambda^+_{\perp}(\not{p}) \gamma^0 [(V(\not{p}, \not{p}') \Phi(\not{p}')) \gamma^0 \Lambda^+_{\perp}(-\not{p})]}{M + \omega_1 + \omega_2} - \int \frac{d^3p'}{(2\pi)^3} \frac{\Lambda^+_{\perp}(\not{p}) \gamma^0 [(V(\not{p}, \not{p}') \Phi(\not{p}')) \gamma^0 \Lambda^-_{\perp}(-\not{p})]}{M - \omega_1 - \omega_2}
\]

with the projectors \( \Lambda^\pm_{\perp} = (\omega_i \pm H_i)/(2\omega_i) \), the Dirac Hamiltonian \( H_i(\not{p}) = \gamma^0(\vec{\gamma}\not{p} + m_i) \) and \( \omega_i = (m_i^2 + \not{p}^2)^{1/2} \) as well as \( V(\not{p}, \not{p}') := \left. V(p_\perp, p'_\perp) \right|_{p=(M,\vec{0})} \).

The amplitudes \( \Phi \) have been calculated by solving the Salpeter equation for an interaction kernel including a confining plus a residual instanton induced interaction. The parameters have been fixed in order to reproduce the masses of the pseudoscalar and vector mesons, the weak decay constant of the pion and the leptonic width of the \( \rho \)-meson. The results have been presented in ref. \([1]\), we use model V1 therein for the following calculations. The confinement interaction has been described by a timelike vector spin structure

\[
\left[ V_C^V(\not{p}, \not{p}') \Phi(\not{p}') \right] = -V_C((\not{p} - \not{p}')^2) \gamma^0 \Phi(\not{p}') \gamma^0
\]

as a scalar confinement leads to an RPA-instability of the Salpeter equation \([12][13]\). The scalar function \( V_C \) in coordinate space is given by a linearly rising potential \( V_C(r) = a_c + b_c r \) in analogy to nonrelativistic quark models, see e.g. \([14][15]\).

In order to reproduce the spectrum of the pseudoscalar mesons, we used an additional instanton induced interaction given by 't Hooft \([16][17][18]\). It acts only on pseudoscalar and scalar mesons and has the form

\[
[V_T(\not{p}, \not{p}')] \Phi(\not{p}')] = 4 G \left[ 1 \text{ tr } (\Phi(\not{p}')) + \gamma^5 \text{ tr } (\Phi(\not{p}')) \gamma^5 \right]
\]

where \( G \) is a flavor dependent coupling constant. Here summation over flavor and a regularizing Gaussian function have been suppressed (see \([1]\) for more details).

The calculation of transition matrix elements between bound states involves the knowledge of the full BS amplitude \( \chi_P(p) \) which has to be reconstructed from the equal time amplitude \( \Phi(\not{p}) \). From the Bethe-Salpeter equation itself one finds that the amputated BS amplitude or vertex function \( \Gamma_P(p) := \left[ S^F_2(p_1) \right]^{-1} \chi_P(p) \left[ S^F_2(-p_2) \right]^{-1} \) may be computed in the rest frame from the equal time amplitude as
\[ \Gamma_P(p) \big|_{p=(M,\vec{0})} = \Gamma(p') = -i\int \frac{d^3p'}{(2\pi)^4} [V(\vec{p},\vec{p}')\Phi(p')] \]  

(7)

From the transformation law for the Dirac field operators \( U_\Lambda \Psi(x) U_\Lambda ^\dagger = S_\Lambda ^{-1} \Psi(\Lambda x) \) and the corresponding properties for the bound state \(| P \rangle\) with mass \( M \) one derives for the transformation property of the BS amplitudes under a Lorentz transformation \( \Lambda \):

\[ \chi_{\Lambda P}^{J M J} (p) = \sum_{M'_J} S_\Lambda \chi_{P}^{J M J'} (\Lambda^{-1}p) \ S^{-1}_\Lambda \ D_{M'_J M J}^{I*} (u(\Lambda, P)) , \]  

(8)

where \( u(\Lambda, P) := \Lambda^{-1}_P \Lambda \Lambda_P \) is the corresponding Wigner rotation and we defined the boost \( \Lambda_P \) by \( P = \Lambda_P (M, \vec{0}) \).

For a pure boost \( \Lambda_P \) we can thus calculate the full BS amplitude in any reference frame as

\[ \chi_P (p) = S_{\Lambda_P} \chi_{(M,\vec{0})} (\Lambda_P^{-1} p) \ S^{-1}_{\Lambda_P} , \]  

(9)

Because of the covariant ansatz of the interaction kernel this kinematical boost gives the solution of the equation for any momentum \( \vec{P} \) of the bound state.

III. TRANSITION AMPLITUDES IN THE SALPETER FORMALISM

The general prescription for the calculation of any current matrix element between bound states has been given by Mandelstam [15], see e.g. [16] for a textbook treatment. Consider for example the electromagnetic current operator: it may be calculated from the BS amplitudes and a kernel \( K^{(\gamma)} \) as shown in Fig.1. \( K^{(\gamma)} \) denotes a kernel irreducible with respect to the incoming and outgoing quark antiquark pair, i.e. it includes all diagrams that may not be divided by just cutting the quark and the antiquark line.

In lowest order the kernel shown in Fig.2 reads explicitly

\[ K^{(\gamma)}_\mu (P, q, p, p') = \]  

\[ -e_1 \gamma^{(1)}_\mu S_2^{F^{-1}} (-P/2 + p) \delta(p' - p + q/2) - e_2 \gamma^{(2)}_\mu S_1^{F^{-1}} (P/2 + p) \delta(p' - p - q/2) \]  

(10)

where \( p \) and \( p' \) denote the relative momenta of the incoming and outgoing \( q\bar{q} \) pair, \( e_1 \) and \( -e_2 \) are the charge of the quark and antiquark, \( q = P - P' \) is the momentum transfer of the photon and we use without loss of generality \( \eta_1 = \eta_2 = 1/2 \), as the result is independent of this choice.

The Dirac coupling to pointlike particles is consistent with the use of free quark propagators. As one of the tasks of this work is to investigate whether the various properties of the low lying mesons may be described in terms of constituent quarks, we neglect their internal structure in the present treatment. The Ward identity for the free two particle propagator is thus trivially fulfilled.

For equal mesons in the initial and final state and zero photon momentum this is consistent with the general normalization condition for bound states as given by Cutkosky [17], which we already used in [12]. In this way the form factor is properly normalized.
For the electromagnetic current coupling e.g. to the first quark we have explicitly:

$$
\langle P' | j^{(1)}_\mu(0) | P \rangle = -e_1 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr} \left\{ \Gamma_{P'}(p - q/2) S^F_1(P/2 + p - q) \gamma_\mu S^F_1(P/2 + p) \Gamma_P(p) S^F_2(-P/2 + p) \right\}
$$

With the transformation properties given in eq.(8) one can show that the current matrix elements transform covariantly – the evaluation thus may be performed in any reference frame. The actual calculation of the current is done in the rest frame of the incoming meson. According to eq.(7) the vertex function of the initial meson then does depend only on the spacelike three momentum $\vec{p}$ and not on $p^0$.

For $Q^2 = 0$ and elastic transitions we have $P' = P = (M, \vec{0})$, so that also the outgoing vertex function $\bar{\Gamma} = -\gamma_0 \Gamma^+ \gamma_0$ does depend only on $\vec{p}$. The only $p^0$ dependence is thus contained in the one particle propagators so that the integral may be calculated analytically by contour integration according to the Feynman prescription. The remaining integration in $|\vec{p}|$ is done numerically.

For space-like momentum transfer $Q^2 > 0$ or nonequal mesons the outgoing vertex function has to be boosted according to eq.(9):

$$
\Gamma_{P'}(p - q/2) = S_{\Lambda_{P'}} \Gamma_{(M', \vec{0})}(\vec{p}_{\text{out}}) S^{-1}_{\Lambda_{P'}}.
$$

with $p_{\text{out}} = \Lambda_{P'}(p - q/2)$. This vertex function thus also depends on the zero component $p^0$ of the relative momentum of the incoming $q\bar{q}$ pair, although it has no singularities on the real axis. This means that we cannot close the contour in the $p^0$-plane, as we don’t know the analytic structure of $\Gamma$. We therefore perform the $p^0$ integration by principal value technique. The real part of the form factor is given by sum of the residues of the six poles of the one particle propagators via

$$
i \int_a^b dp^0 \frac{f(p^0)}{p^0 - p_k^0 \pm i\epsilon} = \pm \pi f(p_k^0) + i \int_a^b dp^0 \frac{f(p^0)}{p^0 - p_k^0}$$

for an isolated pole and

$$
i \int_a^b dp^0 \frac{f(p^0)}{(p^0 - p_k^0 \pm i\epsilon)^2} = \pm \pi f'(p_k^0) + i \int_a^b dp^0 \frac{f(p^0) - f(p_k^0)}{(p^0 - p_k^0)^2}$$

for a double pole and $a < p_k^0 < b$. Here $f(p^0)$ is a real function and the phase $i$ comes from the product of the one particle propagators. This summation over all the one-particle poles means that we take into account the positive and negative energy component of the amplitude, which is important for reactions involving relativistically bound states or large momentum transfer. The imaginary part vanishes due to the hermiticity of the current and time reversal invariance.

As our model includes confinement, we also have mesons with mass $M$ larger than the sum $m_1 + m_2$ of the constituents. The pinching singularities, that appear in general for such states for values of the relative momentum $\vec{p}$ where both particle and antiparticle are on mass shell, are canceled by the zeros of the corresponding trace of the spin part, so that
the integral remains well defined. This is due to the fact that the projection of the vertex function on positive energies \( \Gamma_{pos}(\vec{p}) := \Lambda_1^+ (\vec{p}) \Gamma (\vec{p}) \Lambda_2^- (\vec{p}) \) vanishes if both the quark and the antiquark are on shell. This means that the decay amplitude of the meson bound states into a free quark and antiquark vanishes so that confinement in this channel is guaranteed. The remaining integration in \(|\vec{p}|\) and \(\cos \Theta_p\) is done numerically.

The spin part of the current is evaluated by a standard trace technique appropriate for the particle antiparticle formalism.

The radial part of the vertex functions as well as the Salpeter amplitudes have been expanded in a basis of eleven Laguerre functions. The results are found to be stable within a large range of the scale parameter of the basis.

IV. RESULTS AND DISCUSSION

As already mentioned in sec.II, the parameters of the model were adjusted in a previous work \[1\] (model V1) to reproduce the mass spectrum and the decay observables with non-hadronic final states of the pseudoscalar and vector mesons (we refrained from readjusting them in order not to loose predictive power). The electromagnetic transitions calculated below thus have no free parameters, and give a further test of the Salpeter model for mesons as well as predictions for future experiments.

A. The Form Factors \(M \rightarrow M' \gamma^*\)

The electromagnetic form factor \(f(Q^2)\) of pseudoscalar mesons is defined by

\[
\langle P' | j_\mu(0) | P \rangle = e f(Q^2) (P' + P)_\mu
\]  

with \(e\) the total electric charge of the meson. Consider first the pion which in a constituent quark model is the most deeply bound state. In Fig.3 we compare our results (full line) for \(Q^2 \cdot f(Q^2)\) with experimental data up to 10 GeV\(^2\) \[18\]. The agreement is rather good even for high momentum transfer and shows that \(f(Q^2)\) behaves as \(1/Q^2\) for large \(Q^2\) (the error bars for some of the data points are still very large, so that new CEBAF experiments \[7\] for this process are interesting). It supports the hypothesis of Isgur and Llewellyn Smith \[19\] stating that the form factor in this region should be explained by nonperturbative effects. We also would like to mention similar calculations in the quasi-potential formalism by Tiemeijer and Tjon \[20\] and in a separable ansatz including chiral symmetry breaking by Ito, Buck and Gross \[21\]. Their calculations show a stronger fall off for higher momentum transfer. To analyze this effect we performed a calculation where we neglected the \(p^0\) dependence of the outgoing vertex function (dashed line). Obviously the inclusion of this fourth component is essential for the correct treatment at high momentum transfer. A covariant treatment of the four relative coordinates of the BS amplitude thus is mandatory for such processes.

However for small momentum transfer of the order of the quark mass we find that the form factor of the pion is not strictly monotonic and depends sensitively on the form of the interaction kernel, so that the determination of the charge radius becomes ambiguous. This strong dependence is a result of the large binding energy of the pion and shows the limits
of the Salpeter formalism for such states, at least for models that include a confinement interaction.

In the case of the kaon we obtain a very good description of the form factor at small momentum transfer, see Fig. 4. We find an electromagnetic charge radius for the charged kaon of \( r_{K^+}^{\text{calc}} \approx 0.60 \text{ fm} \) as compared to \( r_{K^+}^{\text{exp}} = 0.58 \pm 0.04 \text{ fm} \) \(^{22}\) or \( r_{K^0}^{\text{exp}} = 0.53 \pm 0.05 \text{ fm} \) \(^{23}\) from electron scattering data. For the neutral kaon we obtain \( r_{K^0}^{\text{calc}} = -0.070 \text{ fm} \) as compared to \( r_{K^0}^{\text{exp}} = -(0.054 \pm 0.026) \text{ fm}^2 \) \(^{24}\). In view of these agreements it is interesting to test the charged form factor in the region of high momentum transfer, which will be accessible in a CEBAF experiment \(^{8}\). Our prediction is plotted in Fig. 5 and compared to a vector dominance model (VDM). A deviation from a simple \( \rho \)-monopole ansatz \( F(x) = 1/(1 + Q^2/M_\rho^2) \) is predicted to appear for \( Q^2 > 1 \text{ GeV}^2 \).

We studied the effects of the relativistic treatment by calculating only those contributions, where the quarks have positive energy. Apart from the noncovariance of the calculation this would correspond to the use of a reduced Salpeter equation. We find that the contribution of the negative energy states to the charged kaon form factor (and therefore to the normalization) is 25% for zero momentum transfer. It gives a radius of \( r_{K^0}^{\text{exp}} > 0.67 \text{ fm} \), which is off the experimental result. Relativistic effects thus play an important role for light mesons even at small \( Q^2 \).

The form factor \( f_{\rho\pi}(Q^2) \) for the process \( \rho \to \pi \gamma^* \) (or \( \omega \to \pi \gamma^* \)) may be defined according to

\[
\langle \pi(P') \mid j_\mu(0) \mid \rho(P, \lambda) \rangle = e \frac{f_{\rho\pi}(Q^2)}{M_\rho} \epsilon_{\mu\nu\sigma\tau} \epsilon_\nu(P, \lambda) P^{\sigma} P^{\tau}
\]

where \( \epsilon(P, \lambda) \) denotes the polarization vector of the \( \rho \) (or \( \omega \))-meson with spin projection \( \lambda \).

The transition \( \omega \to \pi \gamma^* \) has been measured in the timelike region via the decay \( \omega \to \pi^0 \mu^+ \mu^- \) \(^{25}\), where the normalized quantity \( F_{\omega\pi}(Q^2) := f_{\omega\pi}(Q^2)/f_{\omega\pi}(0) \) is fitted by a simple pole ansatz \( F_{\omega\pi}(Q^2) = 1/(1 + Q^2/\Lambda_{\omega\pi}^2) \) with \( \Lambda_{\omega\pi}^{\text{exp}} = (0.65 \pm 0.03) \text{ GeV} \). We compare the experimental results and fit to our calculation in the space like region \(^{9}\) in Fig. 6. Our curve would correspond on this scale to \( \Lambda_{\omega\pi}^{\text{calc}} = 0.63 \text{ GeV} \). The extrapolation of the data to the space like region and our prediction thus agree excellently, especially if we compare to a simple \( \rho \)-pole motivated by vector dominance, i.e. \( \Lambda_{\omega\pi}^{VDM} = 0.77 \text{ GeV} \), which is far off the experimental data. Thus we have found a process, where a relativistic quark model is superior to the phenomenological vector dominance model even at small momentum transfer.

The \( \rho \to \pi \gamma^* \) form factor, which in our model is degenerate with \( \omega \to \pi \gamma^* \), is particularly interesting for the calculation of meson exchange currents of the deuteron form factor. As there is also experimental interest in this quantity \(^{26}\), we plot our prediction for large momentum transfer in Fig. 6 and compare it to a simple \( \rho \)-pole and to the pole fit of \(^{23}\) for

\[\text{a calculation of the quantity} \quad F_{\omega\pi}(Q^2) \quad \text{becomes meaningless in the timelike region, as our model does not guarantee confinement in the} \ \rho \to \pi q\bar{q} \text{ channel, so that the graph diverges for} \quad Q^2 \to -(m_q + m_{\bar{q}})^2.\]
\[ \omega \to \pi^0 \gamma^* \] discussed above\(^2\). At momentum transfer larger than 1 GeV\(^2\), which is particularly important for relativistic calculations of the deuteron, we find a deviation even from the latter one. The calculation by Ito et al. \cite{21} obtained similar results in this context for \[ \rho \to \pi \gamma^* \]. These authors also discussed contributions beyond the impulse approximation including an interaction current \cite{27}, however neglecting the confinement problem. Our absolute value of \( f_{\rho\pi}(0) \) is less accurate and will be discussed in the context of the electromagnetic decay widths.

The corresponding form factors in the strange sector \( K^{*\pm} \to K^{\pm} \gamma^* \) and \( K^{*0} \to K^0 \gamma^* \) are extremely interesting quantities, as the currents coupling to the quark and the antiquark differ because of their unequal masses. Our results in Fig.\(8\) show for the neutral process at least at small momentum transfer a nearly VDM-like behavior. However in the charged case the picture is totally different: the negative interference between the two currents leads to a zero in the form factor at \( Q^2_0 = 2.7 \text{ GeV}^2 \), a region which is already highly relativistic. The effect may be understood qualitatively in a VDM type model, where the coupling to the quark and antiquark is assumed to be proportional to their respective magnetic moments and to a propagator of the corresponding vector meson \( \rho \) or \( \Phi \). The result however depends sensitively on the ratio of the mass of the strange (\( m_s \)) and nonstrange (\( m_n \)) constituent quark. We varied these masses keeping the sum of them as well as the other parameters fixed and obtained a dependence on the ratio \( m_s/m_n \) that is listed in Tab.\(\|\). Our original fit in \cite{1} used \( m_n = 170 \text{ MeV} \) and \( m_s = 390 \text{ MeV} \), i.e. corresponds to a ratio \( m_s/m_n = 2.3 \).

Because of the accurate results for the corresponding decay widths (see next section) we consider these calculations even more reliable than in the \( \rho \pi \gamma \) case. In view of the sensitivity of the zero in the form factor we therefore would encourage an experimental investigation of this interesting phenomenon e.g. at the CEBAF facility, thus providing empirical information on the ratio of the constituent quark masses.

**B. The decay width \( M \to M' \gamma \) for the ground state mesons**

There exist several measurements of decay processes of an excited meson to a state with lower mass by emission of a single real photon \cite{28}. They provide a suitable test of the BS amplitudes especially for resonances where no detailed study of form factors is available. The results for the transitions between vector and pseudoscalar mesons are summarized in Tab.\(\|\). If the mass difference is large, the final meson is emitted with relativistic velocity, so that a covariant framework is essential.

In the semirelativistic ansatz of Godfrey and Isgur \cite{14} the nonrelativistic decay formulae have been modified by terms \( (m/E)^n \) with \( m \) the quark mass and \( E \) its energy, which however forces to use new ad hoc parameters \( n \). The relativistic framework presented here includes these effects automatically.

The widths for decays with a pion in the final state are generally underestimated by a

\(^2\)assuming SU(2) flavor symmetry for \( F_{\omega\pi} \) and \( F_{\rho\pi} \) which of course must not be true experimentally
factor of two. Our results are consistent with a calculation of Tiemeijer \cite{28} in a similar equal

time formalism. This again indicates that the Salpeter formalism is not fully satisfactory in
the case of such deeply bound states as the pion.

The transitions of strange mesons $K^* \to K\gamma$ are in excellent agreement with measure-
ments for both the neutral and the charged channel, consistent with the good results of the
kaon form factor. The kaon therefore seems to be well understood in the Salpeter model.
Again the width for the charged decay does sensitively depend on the ratio of the constituent
quark masses, see Tab.\textcolor{red}{I}, giving a ratio of $m_s/m_n = 1.5 - 1.9$, whereas the neutral decay is
almost stable and does not restrict this quantity.

The electromagnetic decays involving a $\eta$ ($\eta'$) meson in the final (initial) state are a
sensitive test of the $n\bar{n}$- and $s\bar{s}$-component of their BS amplitude and therefore of the
interaction that induces the flavor mixing. The decays $\rho, \omega \to \eta\gamma$ involve the $n\bar{n}$-component,
$\Phi \to \eta\gamma$ the $s\bar{s}$-component. The agreement is excellent for all the three values re-
confirming the good description of the mixing coefficients for the $\eta$ given in \cite{1}. For the $\eta'$ decays into
$\rho$ and $\omega$ we find an overestimation of a factor of around two consistent with the fact that
the $n\bar{n}$-component of this meson is too large in our model compared to the semi-empirical
value extracted from the $J/\Psi$-decay \cite{1}.

Our prediction for the decay width $\Phi \to \eta'\gamma$ includes the estimated error from the
inaccuracy in the calculated meson masses which enters the transition matrix elements. As
we underestimate the $s\bar{s}$-component of the $\eta'$ only by a small amount \cite{1}, we expect our
result for this experimental value to be quite accurate.

C. The Form Factors $M \to \gamma\gamma^*$

The structure of the BS amplitude for neutral pseudoscalar mesons may be tested by
the production via a virtual and an (almost) real photon as done at $\gamma\gamma$ facilities of $e^+e^-$
colliders \cite{30,31}. In lowest order the process is given by the graphs in Fig.\textcolor{red}{9}. If one of the
photons is on shell, i.e. $q_1^2 = 0$, the amplitude may be parameterized as

$$T_{\mu\nu}(q_1, q_2) = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta f_{M\gamma}(Q^2_2)$$\hspace{1cm}(17)$$

Referring to the parameterization of the experimental data given in \cite{30,31} we define a width

$$\Gamma(Q^2) = \frac{M^3}{64\pi} f_{M\gamma}^2(Q^2)$$\hspace{1cm}(18)$$

which for $Q^2 = 0$ gives the decay width for a pseudoscalar into two real photons.

In Fig.\textcolor{red}{14} we have plotted the results for $\Gamma(Q^2)$ for the $\pi^0, \eta$ and $\eta'$ including the ex-
perimental results \cite{30,31}. The decay widths have already been published and are in good
agreement with experimental data \cite{1}.

\textsuperscript{3}from SU(2)-isospin symmetry the decay width $\rho^0 \to \pi^0\gamma$ should be the same as for $\rho^\pm \to \pi^\pm\gamma$
and a factor nine smaller than the width for $\omega \to \pi^0\gamma$ and therefore this experimental value has to
be considered with care
The width for the process $\pi^0 \to \gamma\gamma^*$ depends sensitively on the quark mass, a result that has also been found by Ito et al. [21]. Their optimal value of the nonstrange quark mass $m_n = 250\,\text{MeV}$ is in rough agreement with our result of $m_n = 170\,\text{MeV}$, which had been adjusted in order to obtain the correct pion decay constant. However as in the case of the charged pion form factor we find an unnatural structure for $\Gamma_{\pi^0 \to \gamma\gamma}(Q^2)$ for low momentum transfer due to the strong binding of the pion. Also in this process the width for larger momentum transfer is somewhat underestimated.

The structure of the $\eta$ form factor can be described almost quantitatively up to 4 GeV$^2$, which underlines the good description of this meson for the observables discussed above. Although the $\eta'$ width is too small by about 30%, the dependence on $Q^2$ is well reproduced up to 8 GeV$^2$.

V. SUMMARY AND CONCLUSION

Starting from a relativistic quark model based on the Salpeter equation that includes confinement and an instanton induced flavor mixing interaction, we investigated the electromagnetic properties of the light pseudoscalar and vector mesons including the isoscalar states. In order not to loose predictive power we used the BS amplitudes of a former work [1], all the present results thus were calculated with no additional free parameter.

Especially for the $\eta$ and K meson, but also for the lowest vector mesons we find an excellent description of all available observables. The flavor mixing for the $\eta$, which can be measured in the decays $\rho, \omega, \Phi \to \eta\gamma$, is correctly reproduced. The kaon form factor for small momentum transfer as well as the $K^* \to K\gamma$-widths are in almost quantitative agreement. The phenomenological extrapolation of the $\omega \to \pi\gamma^*$ form factor from the data in the time like region agrees with our quark model prediction in the space like region, but differs significantly from the standard vector dominance model. We present predictions for the processes $\rho \to \pi\gamma^*$ and the kaon form factor in the large $Q^2$ regime which will be measured at CEBAF in near future. We found that the transition form factor $K^{*\pm} \to K^{\pm}\gamma^*$ represents an interesting observable, as its form depends strongly on the ratio of strange and nonstrange quark mass. Because of the negative interference of the current coupling to quark and antiquark one obtains a zero of the amplitude, which we predict at $Q_0^2 \approx 2.7\text{GeV}^2$. From the decay width into a real photon we find $m_s/m_n \approx 1.9 - 1.5$.

The description of the $1/Q^2$-behaviour of the charged pion form factor in the region of high momentum transfer is possible only if the BS amplitude is boosted correctly, i.e. if the full dependence of the relative four momentum is taken into account. However we reach the limits of the Salpeter ansatz in the case of the pion due to its strong binding. We find that the charged and neutral form factor on the scale of the quark mass become extremely sensitive to the interaction – in our model there are structures not apparent in experimental data. We could not find a kernel that includes confinement and is able to describe the form factors in this region.

\footnote{A larger quark mass would give a smaller width at zero momentum transfer, but also a smaller slope}
The results show that a relativistic treatment of constituent quarks in the framework of the Salpeter model for mesons including a relativistic normalization and covariant boosting of the amplitudes is able to describe almost quantitatively the various properties of the ground state pseudoscalar and vector mesons. In view of this success we will apply the formalism to a detailed study of the complete meson spectrum including one-gluon-exchange for heavy systems, which will be done in a future work.

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TABLES

TABLE I. Dependence of the $K$- and $K^*$-mass, weak decay constant $f_K$, charged kaon radius, decay widths and the zero of the form factor for $K^{*\pm} \to K^{\pm}\gamma$ on the ratio of the strange and nonstrange constituent quark mass $m_s/m_n$ (masses and decay constant given in MeV)

| $m_s/m_n$ | $M_K$ | $M_{K^*}$ | $f_K$ | $<r_{K^\pm}^2>$ [fm] | $\Gamma_{K^{*\pm} \to K^{\pm}\gamma}$ [keV] | $\Gamma_{K^{*0} \to K^{0}\gamma}$ [keV] | $Q^2_0$ [GeV$^2$] |
|----------|-------|-----------|-------|----------------|-----------------|-----------------|-----------------|
| 3.0      | 535   | 895       | 179   | 0.61           | 78              | 114             | 2.1             |
| 2.3      | 510   | 880       | 183   | 0.60           | 64              | 112             | 2.7             |
| 1.8      | 485   | 865       | 185   | 0.59           | 52              | 111             | 4.8             |
| 1.5      | 475   | 860       | 187   | 0.59           | 45              | 110             | >10             |
| 1.0      | 465   | 855       | 189   | 0.57           | 27              | 109             | –               |
| Experiment | 495   | 892       | 164   | 0.58±0.04      | 50±5            | 117±10          | –               |

TABLE II. Comparison of experimental and calculated electromagnetic meson decay widths

| Mesonic decay width [keV] | experimental | calculated |
|---------------------------|--------------|------------|
| $\Gamma(\rho^{\pm} \to \pi^{\pm}\gamma)$ | 68 ± 7 | 38 |
| $\Gamma(\rho^0 \to \pi^0\gamma)$ | 121 ± 31 | 38 |
| $\Gamma(\omega \to \pi\gamma)$ | 717 ± 43 | 335 |
| $\Gamma(K^{*\pm} \to K^{\pm}\gamma)$ | 50 ± 5 | 64 |
| $\Gamma(K^{*0} \to K^{0}\gamma)$ | 117 ± 10 | 112 |
| $\Gamma(\rho \to \eta\gamma)$ | 58 ± 10 | 50 |
| $\Gamma(\omega \to \eta\gamma)$ | 4.0 ± 1.7 | 5.6 |
| $\Gamma(\phi \to \eta\gamma)$ | 56.9 ± 2.9 | 60 |
| $\Gamma(\eta' \to \omega\gamma)$ | 5.9 ± 0.9 | 12.7 |
| $\Gamma(\eta' \to \rho\gamma)$ | 59 ± 6 | 122 |
| $\Gamma(\phi \to \eta'\gamma)$ | < 1.8 | 0.18±0.02 |
FIG. 1. The electromagnetic current in the Mandelstam formalism calculated from the BS amplitudes $\chi_P, \bar{\chi}_{P'}$ and the kernel $K^{(\gamma)}$ and the definition of the relevant momenta.

\[ \gamma^* q = K^{(\gamma)} \mu, q \]

FIG. 2. Perturbative expansion of the kernel $K^{(\gamma)}$ in lowest order of the strong interaction; the full circle denotes an inverse quark propagator.
FIG. 3. The pion form factor for large momentum transfer compared to results from pion photoproduction, see [18] and the references given therein. The solid curve represents the calculation with the correct boost of the vertex function, the dashed line is obtained, if the zero component of its relative momentum is neglected.
FIG. 4. The charged kaon form factor for small momentum transfer data from electron scattering [22,23].
FIG. 5. The charged kaon form factor for large momentum transfer (solid line) and comparison to a $\rho$-pole motivated by VDM (dashed line)
FIG. 6. Comparison of the normalized $\omega\pi\gamma^*$ form factor (solid line) in the space-like region with an extrapolation of experimental data in the time-like region \(^{25}\) (dotted line) and with a $\rho$-pole ansatz motivated by vector dominance (dashed line)
FIG. 7. The normalized \( \rho\pi\gamma^* (\omega\pi\gamma^*) \) form factor for large momentum transfer
FIG. 8. The normalized neutral (dotted line) and charged (solid line) $K^*K\gamma^*$ form factors compared to VDM (dashed line)
FIG. 9. Feynman graph for the decay into two photons

\[ \gamma q_1 \mu^+ q_2 \nu^+ = \chi + \chi \]

FIG. 10. The \( \pi^0, \eta \) and \( \eta' \to \gamma \gamma^* \) form factors compared to experimental data [30,31]
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