Nonlinear vibration of a hemispherical dome under external water pressure.

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Abstract

The aim of this study was to analyse the behaviour of a hemi-spherical dome when vibrated under external water pressure, using the commercial computer package ANSYS 11.0. In order to achieve this aim, the dome was modelled and vibrated in air and then in water, before finally being vibrated under external water pressure. The results collected during each of the analyses were compared to the previous studies, and this demonstrated that ANSYS was a suitable program and produced accurate results for this type of analysis, together with excellent graphical displays. The analysis under external water pressure, clearly demonstrated that as external water pressure was increased, the resonant frequencies decreased and a type of dynamic buckling became likely; because the static buckling eigenmode was similar to the vibration eigenmode. ANSYS compared favourably with the in-house software, but had the advantage that it produced graphical displays. This also led to the identification of previously undetected meridional modes of vibration; which were not detected with the in-house software.

1. Introduction

Structures designed to withstand external water pressure usually appear in the form of thin-walled curved shells constructed from metals. These thin walled curved shell designs are used, as it is more efficient to withstand an external pressure in a membrane manner rather than through bending [1, 2]. To this end there has been extensive research and development regarding the collapse of pressure vessels under uniform external pressure. However, there are many factors that must be considered in the design of a pressure hull and one of the lesser investigated features, is that of vibration and its effects. Vibration in thin-walled pressure vessels can be the result of numerous cyclic forces, due to machinery or hydrodynamic flow. If these vibrations occur at resonant frequencies it is possible for the structure to fail at a lower pressure than predicted by static evaluation. Previous studies [1, 3, 4] encompassed the comparison of theoretical analyses, provided by in-house finite element computer programs (VIBCONE, VIBDOME, VIBVVMC and SUBPRESS) developed by Ross [5 to 9], together with the experimental analyses for a number of hemi-ellipsoidal prolate and oblate domes with varying aspect ratios. The present investigation has been developed, using the earlier results for the hemispherical dome with aspect ratio of 1.0. In the present study, the ANSYS 11.0 finite element analysis software was used to carry out a number of theoretical analyses.

Using the commercial computer package ANSYS, this paper investigated the behaviour of the hemispherical dome when vibrated under external water pressure. The dome was modelled and vibrated in air and then in water, before finally being vibrated under external water pressure. The results of the
analyses clearly demonstrated that as external water pressure was increased, the resonant frequencies decreased. The manuscript contains some new results that should be of interest to the researchers and engineers in the relevant field. The analysis also allowed for dynamic action between the water and the shell. The nonlinearity comes from the geometrical nonlinearity caused by the compression of the shell, due to the increasing the external hydrostatic pressure.

2. Theory of Dome Failure and Vibration

Almost all pressure vessel designs incorporate some form of dome; often as caps to cylinders. These domes fall into three main groups, PROLATE (aspect ratio of greater than 1.0), HEMI-SPHERICAL (aspect ratio equal to 1.0) and OBLATE (aspect ratio of less than 1.0). Previous static theoretical and experimental research [1, 2] has demonstrated that under uniform external water pressure hemi-ellipsoidal prolate domes and hemi-spherical domes tend to buckle in a lobar manner whilst hemi-ellipsoidal oblate domes tend to buckle axisymmetrically, as shown in Figure 1.

![Figure 1. Buckled prolate, hemi-spherical and oblate domes (Left to right).](image1)

Figures 2 & 4 show the lobar modes of vibration that occur with prolate and hemispherical domes and Figure 3 shows the modes of vibration that occur with oblate domes.

![Figure 2. Lobar eigenmodes for prolate & hemispherical domes.](image2)

![Figure 3. Axisymmetrical eigenmodes for oblate domes.](image3)
This suggests that the result of a dome vibrating at a resonant frequency under external water pressure may trigger a mode of dynamic buckling [1] at a lower pressure than static buckling. Research [1, 2] has shown that as the external water pressure increases so too does the compressive stresses the dome experiences [1]. This results in a decrease of structure stiffness and magnitude of resonant frequencies such that as the static limit is approached the resonant frequencies approach zero.

3. Finite Element Method for Vibration Analysis Under Water

3.1 Analysis using ANSYS 11.0

The hemi-spherical dome was modelled using quadrilateral shell elements of two types, Shell 93 and Shell 63; whose geometry is defined by nodes. These elements are similar in that they are both quadrilateral, but Shell93 has six degrees of freedom per node, translation is allowed in all coordinate directions and rotation around the coordinate axes is possible. Shell 63 has, however, only 3 translation degrees of freedom per node.

Shell 93 is an 8 node element particularly well suited to modelling curved shells and has plasticity, stress stiffening, large deflection and large strain capabilities. This element was suitable for vibration in air. The Shell 63 element can only represent a linear surface and displacement between nodes with accuracy; it is suitable for vibration in water.

Fluid 30 3-D acoustic elements were used to model the water around the dome to the geometry of the pressure tank held by the department. The element is ideal for modelling the fluid medium and the interface in fluid/structure interaction problems in particular submerged structural dynamics. The translational degrees of freedom are only valid on nodes that are next to the interface with structural element. The interaction of the fluid and structure at a mesh interface causes the acoustic pressure to exert a force applied to the structure and the structural motions produce an effective fluid load. It is necessary to define whether the fluid elements are in contact with a structure or not.

3.2 Vibration in Air.

The hemi-spherical dome was modelled in ANSYS using the 8 node Shell 93 elements which were the most accurate for this type of curved structure, as shown in Figure 5. The dome had a radius of 100 mm, an aspect ratio (AR) of 1.0 and a thickness of 2mm.

**Figure 4.** Lobar pattern for higher frequency complex Eigen modes.

**Figure 5.** Modelled hemi-spherical dome.
3.2.1 *Results of the Vibration in Air.* The results of the modal analysis were carefully scrutinised and the exact modes of vibration along with their resonant frequencies identified through the animation function offered by ANSYS; modes of vibration are shown in Figure 6 & Table 1.

![Figure 6](image1.png)

**Table 1.** Lobar vibration results for Domes 1, 2 and 3 in air.

| Mode of Lobar Vibration (n=?, m=2) | Frequency (Hz) | Difference between Dome 1 and Dome 3 results (Hz) |
|----------------------------------|----------------|-----------------------------------------------|
|                                  | Dome 1         | Dome 2 | Dome 3 |                                      |
| n=1                              | 1439.6         | 1439.7 | 1439.8 | 0.2                                  |
| n=2                              | 2240.7         | 2240.3 | 2240.2 | 0.5                                  |
| n=3                              | 2384.8         | 2383.5 | 2383.3 | 1.5                                  |
| n=4                              | 2459.4         | 2456.4 | 2455.8 | 3.6                                  |
| n=5                              | 2530.4         | 2524.0 | 2522.8 | 7.6                                  |

Table 1 display's the results of the lobar eigenmodes for the three domes; the mesh density of Dome 3 being higher than Dome 2 and the mesh density of Dome 2 being higher than Dome 1. From Table 1, it can be seen that the resonant frequencies converge with increasing mesh density. The ANSYS results compared with the experimental and theoretical results from previous work [1, 10, 11], as shown in Figure 7.

![Figure 7](image2.png)

**Figure 7.** Comparison of previous experimental (Exp.), theoretical (CON-conical element) and current ANSYS results.
3.3 Vibration Analysis in Water

There had been significant research into the vibration of thin-walled pressure vessels carried out at the University of Portsmouth by Ross [1], Ross and Mackney [4] and Ross et al [5 to 9]. This included the vibration of various domes in air and then in water. It encompassed both the experimental and theoretical using in-house finite element programs. This work had shown that the introduction of water around the structures had significantly reduced the resonant frequencies and had demonstrated the accuracy of Ross’ programs for this type of analysis.

The geometry of the water around the dome was based on the pressure tank that was used for the experimental analysis in water and would be used in future work. This tank has an internal diameter of 284 mm and a depth of 449 mm. The fluid-structure mesh is shown in Figure 8.

![Fluid-Structure Mesh](image)

**Figure 8.** Fluid-Structure Mesh.

To define the fluid it was necessary to create a second material model for water and define the density (1000 kg/m³) and the sonic speed (1490 m/s). It was also possible to introduce sound absorption, however, it was assumed this did not apply to this model.

It was important to define two separate types of the Fluid 30 elements, Type 1 - structure present and Type 2 – structure absent which is the KEYOPT (2) setting in the program (KEYOPT(2)=0 is structure present and KEYOPT(2)=1 is structure absent). The structure absent option removes the transitional degrees of freedom on the element nodes and was used for all elements not in immediate contact with the structure whilst Type 1 were only used for that layer in contact. The structural elements and the fluid elements in contact were considered to have the same displacement hence they used the same nodes at the interface, Due to the fact the fluid elements had four nodes on each side (four corner nodes) the structure could only be modelled with the Shell 63 elements.

3.3.1 Results of the Vibration under water. Once again the results obtained from ANSYS showed close correlation to the results from the in-house programmes and the experimental data as can be seen from Table 2 and Figure 9. As the frequencies increase the discrepancy between the ANSYS
set of results and the others tends to increase, however, this remained below 10% between experimental and ANSYS; indicating ANSYS results remained suitable for this study. This close relationship of the results also validated the method applied to modelling the fluid elements, the geometry used and the defining of the fluid/surface interface, a vital achievement in order to progress to applying pressure. The introduction of the fluid around the dome resulted in a significant drop in the resonant frequencies for the different modes for example n=2 reduced from 2240.7 Hz in air to 767 Hz in water, as shown in Table 2 & Figure 9.

Table 2. Results of vibration in water for in-house programs.

| Mode | Exp. Frequency (Hz) | VMC | CMC | CON | ANSYS |
|------|---------------------|-----|-----|-----|-------|
| n=1  | 414                 | 509 | 511 | 509 | 458.55|
| n=2  | 767                 | 720 | 722 | 727 | 751.7 |
| n=3  | 887                 | 821 | 822 | 836 | 908.54|
| n=4  | 964                 | 905 | 906 | 929 | 1015.9|
| n=5  | 1005                | 986 | 987 | 1020| 1097.3|

3.4 Vibration Analysis Under External Water Pressure

As previously described, the expectation was that as the pressure acting on the dome increased there would be a decrease in flexural stiffness. This would lead to a decrease in resonant frequencies and a potential failure mode that is a combination of instability and vibration referred to as dynamic buckling [1].

The hemi-spherical dome model was subjected to 0.25, 0.5, 0.75 and 0.95 times its critical static pressure ($P_{cr}$) and a modal analysis performed for each of these pressures. The critical static was taken from Ross’s experimental results [1] as 1.28 MPa.
In order to apply the pressure and analyse vibration, ANSYS required a two stage process. Firstly the hydrostatic pressure was applied to the structural elements as a PRESTRESS and a static analysis carried out. The result of this was a reduced stiffness matrix that considered the compression effects. This reduced stiffness matrix was then used in a modal analysis with the option of ‘consider PRESTRESS effects’ selected to calculate the new resonant frequencies.

3.4.1 Results of Vibration Under External Water Pressure. Figure 10, was supplied by Ross [1] and shows the results obtained from the SUBPRESS program for the vibration of the hemi-spherical dome under external water pressure, at various pressures, where $P_{cr}$ = the experimental static buckling pressure.
When the graph of the SUPRESS results (Figure 10) is compared to that of the ANSYS results, of Table 3 & Figure 11, it can be seen that the two display frequency values are in the same region. At the lower modes the SUBPRESS frequencies appear to be slightly higher than ANSYS with the opposite occurring as the lobar modes increase. However, as stated, the exact values from the two programmes cannot be compared with much accuracy but it appears that a good set of results had been obtained from ANSYS.

Table 3. Resonant frequencies of the dome, under external water pressure from the ANSYS analysis.

| Mode of lobar vibration (n=?) | Amount of critical pressure (Pcr) applied (Pcr=1.28 MPa) |
|-----------------------------|---------------------------------------------------------|
|                             | 0           | 0.25        | 0.5         | 0.75        | 0.95        |
| 1                           | 478.55      | 473.03      | 466.92      | 40.98       |             |
| 2                           | 751.7       | 740.91      | 729.29      | 715.77      |             |
| 3                           | 908.54      | 887.12      | 864.24      | 838.33      | 810.43      |
| 4                           | 1035.9      | 998.04      | 957.39      |             | 872.58      |
| 5                           | 1157.5      | 1094.9      | 1026.1      | 961.45      |             |
| 6                           | 1285.1      | 1187.8      | 1076.6      | 942.45      | 794.44      |
| 7                           | 1430        | 1286.6      | 1115.4      | 961.45      |             |
| 8                           | 1603.7      | 1405.7      | 1155.5      |             |             |
| 9                           | 1816.2      | 1562.6      | 1221.5      | 1074.5      | 717.73      |
| 10                          | 2071.1      | 1773.6      | 1540.7      | 1116        | 501.87      |
| 11                          | 2296.5      | 2043.7      | 1654.2      | 1124.9      | 520.76      |

Figure 11. Resonant frequency distributions of a hemi-spherical dome under external water pressure; results from ANSYS.
It is clear from the two graphs that the results from ANSYS followed the same trend as those from SUBPRESS. When $P_{cr}$ is 0 the values of the resonant frequencies increase as $n$ increases toward the maximum when $n$ approaches infinity. As pressure is applied the value of the frequencies at each mode decreases but the trend remains the same. However at $0.95P_{cr}$ a saddle point was identified at $n=4$ and a minimum value of resonant frequency was found at $n=10$ which Ross (2001) had identified as the static buckling eigen mode. This suggests that, as expected, a form of dynamic buckling may occur at a pressure below the domes static critical pressure due to vibrations at a certain resonant frequency.

4 Conclusions

This was a successful study, in that the objectives were systematically completed in order to achieve the aims. At each of the 3 stages the results were carefully considered and compared with known data from Ross. This allowed the modelling and methodology to be confirmed and ensured no errors were present at any junction. The comparison also allowed for comments to be made at the key stages.

As expected, ANSYS’s graphical ability proved a vital resource throughout the project and was clearly an advantage over the previous in-house programmes. It allowed for the identification of previously undetected meridional modes of vibration between lobar modes and for these to be visualised. The analysis of these modes in detail would be a natural progression to the strain of work.

The final analysis of the hemi-spherical dome under external water pressures generated data that supported results from the in-house program SUBPRESS. This data showed a marked decrease in frequency for the lobar modes of vibration as more pressure is applied. It also demonstrated that for lower percentages of the known critical pressure the value of $n$ increased in conjunction with an increase in resonant frequency. However as the critical static pressure is approached, this ceases to be the case and the dome achieved the first resonance at $n=10$; which is its eigenmode during static failure, hence a mode of dynamic buckling was likely at this pressure and above. This is a significant occurrence as the frequency involved (501.87 Hz) could easily be generated in a practical environment.

The procedure described in the report for the modelling of the dome in ANSYS can be applied to many other domes and allows for further work in this general field including the modelling and analysis of domes with different aspect ratios, the use of different materials and different fluid densities for example the comparison or fresh and salt water.

It is well known that any material and in particular, one with geometrical defects in a shell would affect its buckling load considerably. This is also the case for these spherical shells too. Some doublet frequencies would split into two distinct ones. Such defects will occur as the static buckling pressure is approached. This is because the increasing external pressure causes the shell to be in increasing compression, so that its flexural stiffness decreases and this causes the vibration frequencies to decrease and the vibration eigenmodes to become of similar form to the buckling eigenmodes. In contrast to this, if the internal pressure in the shell were increased, the flexural stiffness would increase and so to would the magnitudes of its resonant frequencies. These phenomena can easily be demonstrated with a violin string. If the violin string is tightened, so that its axial tension increases, its resonant frequencies will increase, the converse is true with submarine pressure hulls under external pressure; but in their case they can suffer dynamic buckling at an external pressure much less than that to cause static buckling.

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