Putting centre dominance under the microscope

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We make various short points on the phenomenon of centre dominance in SU(2). The Z(2) dominance seen in Wilson loops is related to the loop distribution and to half-odd-integer representations of the group. The distributions also make it clear that, in this picture, the requirement of vortices for confinement is essentially trivial. We confirm that the same effect appears in the positive plaquette model. The simple random vortex picture is shown to give a substantial fraction of the string tension.

1. INTRODUCTION

There is now considerable evidence that vortices of Z(N) flux are a useful way of looking at confinement in SU(N) gauge theories. The centre Z(N) naturally has a special role in the theory; the corresponding vortices can easily be shown to cause confinement in a simple model with minimal mathematical baggage.

The nature of real, physical vortices is less trivial, and their relationship to the other mechanisms proposed for confinement still only partly understood. The formation of vortices is due to the tradeoff between action and entropy; as argued by ’t Hooft some time ago \cite{1}, it is a dynamical question about the phase structure of the theory whether they are realised under the prevailing physical conditions. If the phase is the appropriate one, entropic arguments suggest that a vortex should form on some scale around that of the correlation length, while if it has a finite extension the contribution to the action can be small. The path-ordered product of fields around the outside gives \( \exp(2\pi i/N) \).

An important consequence of this is that vortices are related to the homotopy of the quotient group SU(N)/Z(N); it is too simplistic to consider the Z(N) part alone as being responsible for the behaviour. This is enough to invalidate the old claim that centre vortices cannot explain the string tension seen in the adjoint representation of SU(2), which has just the symmetry SU(2)/Z(2). A specific mechanism for understanding the behaviour in higher representations has been proposed \cite{2}.

Much of the evidence that vortices thought of in this way are genuine physical objects has come from ‘projection vortices’ in SU(2) \cite{3}, where the field is reduced to its Z(2) components. Here one finds the effect of confinement remains (though of course this claim is made of various other pictures). The vortices have excellent scaling properties, and present some hope for understanding the behaviour of SU(2) representations higher than \( j = 1/2 \).

It has been suggested \cite{4} that the sign of the Wilson loop in SU(2), or its projection onto Z(N) for general SU(N), is a counter for vortices: in the SU(2) case an even (odd) number of vortices pierce the loop if the trace of the loop is positive (negative). It was shown that this indeed reproduces the heavy quark potential — so well, in fact, that the dynamics here clearly contains not just confinement, but everything else too, so that despite the simplicity of the picture it is harder to investigate the physics of confinement directly; in other words, ‘vortices’ is intended here in a broader sense than just those objects causing a linear potential between quarks. However, it is another hopeful sign that Z(N) effects are the important ones, and therefore that the more specific way of looking at vortices described above will encapsulate the physics. Here, we shall investigate the causes of this result for the Wilson loop. This is a summary of results presented in ref. \cite{5}.
2. DISTRIBUTIONS AND REPRESENTATIONS

One (hitherto unregarded) way of looking at the properties of Wilson loops is via their distribution. We define \( \rho(W_0(A)) \) to be the normalised distribution of the trace of the Wilson loop \( W_0 \) of area \( A \) in the fundamental representation such that its integral over \(-1 < W_0 < 1\) is unity. This is shown in figure 1 for various small loops. For loops much larger than the correlation length, the value corresponds to a random walk in the gauge manifold, for which the distribution (shown as the lowest curve) is

\[
\rho(W_0) = \frac{2}{\pi}(1 - W_0^2)^{1/2}.
\]

The distribution allows us to make an important point simply: if Wilson loops count vortices, eliminating vortices trivially removes confinement, as we have only the right hand half of the distribution where no exponential decay to zero is possible.

We shall analyse this distribution via a Fourier analysis,

\[
\rho(\sin \phi^2) = \sum_{n=1}^{\infty} (a_n \cos(n - \frac{1}{2})\phi + b_n \sin n\phi),
\]

where the units \( W_0 \equiv \sin \phi^2 \) have been chosen so that \(-\pi < \phi < \pi\). This form is such that

\[
\rho(\pm \pi) = 0.
\]

The expectation value of \( W_0 \) is easily found to be

\[
\langle W_0 \rangle = 3b_0/\pi.
\]

We can similarly find the expectation value in the centre dominance picture by assuming a value \(-1\) where \( \rho < 0 \) and \( 1 \) where \( \rho > 0 \). In this case,

\[
\langle W_0^{Z(2)} \rangle = \sum \frac{nb_n}{n^2 - 1/4}.
\]

The formulae differ in two ways: firstly by the presence of all odd terms for \( n > 1 \) in (5), and secondly by an overall factor of \( 3\pi/16 \) which disappears in ratios and is therefore unimportant. The first is investigated in figure 2, where we plot \(|b_2/b_1|\) and \(|b_3/b_1|\). Shown in the same graph are the corresponding values from a simple approximation, where we have selected loops at random from the plaquette distribution and multiplied them as if uncorrelated: this corresponds to an area law \( \langle \text{Tr}_j(A) \rangle = \text{Tr}_j(\text{plaquette})^A \) for all representations \( j \).

2.1. Representations

The above clearly demonstrates centre dominance in this sense, but we can go further. It turns out that the Fourier series \( \sum a_n \cos n\phi \) corresponds term by term with the representations of the gauge group, namely,

\[
\langle \text{Tr}_{n-1/2}(W_0) \rangle = \frac{(-1)^{n-1/2}\pi b_n}{4n}
\]

for half-odd-integer representations, and

\[
\langle \text{Tr}_{n-1}(W_0) \rangle = \frac{(-1)^{n-1}\pi a_n}{4(n-1/2)}
\]
for the rest. Note that the $a_0$ term is the trivial identity representation, corresponding to the distribution $[1]$. So centre dominance results simply from the fact that the higher representations for $n - \frac{1}{2}, n > 1$ decay more quickly than the rest. In particular, if the scaling is Casimir-like, so that the string tension is proportional to $j(j + 1)$ for representation $j$, the $3/2$ contribution would be expected to decay exponentially 5 times faster than the $1/2$. The central issue of the character expansion was noted in ref. [6].

3. RANDOM VORTICES

As has been known for a long time, arguably the simplest way of generating confinement in SU(2) is to fill the vacuum randomly with vortices which flip the sign of a Wilson loop. This can be shown without reference to the lattice: take an area $A$ embedded in a much larger area $A'$, and throw vortices into $A'$ up to the required area density $p_A$. For non-interacting vortices where perimeter effects are ignored, the distribution of such objects within the area $A$ is a simple binomial and gives an area law for the Wilson loop with string tension $K = 2p_A$ as $A'$ is allowed to tend to infinity. The value of $p_A$ found for projection vortices (in an indirect projection, and therefore possibly not optimal) in [1] was $1.9 \pm 0.2$ fm$^{-2}$ for $K = 440$ MeV, or around 3/4 of the required value. This density is found to have excellent scaling behaviour [6,8].

4. POSITIVE PLACETTE MODEL

The positive plaquette model [9] is an alternative regularisation of SU(2) in which negative action is forbidden. It has all the physical features of SU(2) [10]. Its coupling is renormalised so that $\beta = 1.9$ is slightly weaker than $\beta = 2.5$ in ordinary SU(2). We can use it to show that the centre dominance effect in the Wilson loop does not require negative plaquettes. Figure 3 shows the heavy quark potential in this picture compared with that where only the sign of the loops is taken. It is clear that the centre dominance effect is present here too. Another talk at this conference presented similar results [11].

5. CONCLUSIONS

More work needs to be done connecting vortices with other pictures of confinement; some results exist in the case of monopoles [12]. Also, the case of SU(3) needs to be explored further.

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