Supersymmetry confronts $B_s \to \mu^+\mu^-$: Present and future status

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Abstract

The purely leptonic rare decay $B_s \to \mu^+\mu^-$ is very sensitive to supersymmetric contributions which are free from the helicity suppression of its Standard Model diagrams. The recent observation of the decay by the LHCb experiment and the first determination of its branching fraction motivate a review of their impact on the viable parameter space of supersymmetry. In this paper we discuss the implications of the present and expected future accuracy on $\text{BR}(B_s \to \mu^+\mu^-)$ for constrained and unconstrained MSSM scenarios, in relation to the results from direct SUSY searches and the Higgs data at the LHC. While the constraints from $\text{BR}(B_s \to \mu^+\mu^-)$ can be very important in specific SUSY regions, we show that the current result, and even foreseen future improvements in its accuracy, will leave a major fraction of the SUSY parameter space, compatible with the results of direct searches, unconstrained. We also highlight the complementarity of the $B_s \to \mu^+\mu^-$ decay with direct SUSY searches.
1 Introduction

The rare decay $B_s \rightarrow \mu^+\mu^-$ has been recognised as a probe of new physics beyond the Standard Model (SM) and one of the high priority channels for study in the LHC $B$ physics program. Because its SM predicted rate is made very small by a helicity suppression, it may reveal the contributions of additional diagrams arising in extensions of the SM, which do not suffer from the same suppression. In particular, in supersymmetric extensions of the SM (SUSY), its decay amplitude receives an enhancement by a factor of order $\tan^3 \beta$ [1–3], where $\tan \beta$ is the ratio of vacuum expectation values of the two Higgs fields, and the branching fraction can be larger than in the SM by one order of magnitude, or more. The sensitivity of $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ has been discussed extensively in the literature in the past years, mostly in constrained versions of the Minimal Supersymmetric Standard Model (MSSM) [4–21] and, more recently, in the phenomenological MSSM (pMSSM) [22–24]. The recent observation of the decay with the determination of its branching fraction by the LHCb experiment to a value very close to the SM prediction [25] excludes very large deviations, motivating a review of its implications on the viability of SUSY. In this paper, we discuss these implications in the context of constrained and unconstrained MSSM models, with R-parity and CP conservation, with an eye also on the future experimental progress of this measurement at the LHC. We show that the constraining power of $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ is important in specific regions of the MSSM, but leaves substantial room for the SUSY parameters. We quantify this by studying the fraction of the MSSM model points, obtained in flat scans of the parameter space, which are compatible with the present and future $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ constraints in the framework of the Constrained MSSM (CMSSM) and the phenomenological MSSM (pMSSM) with 19 free parameters, and account for the results on the Higgs mass and direct SUSY searches at ATLAS and CMS.

The paper is organised as follows. In section 2 we discuss the SM prediction for $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, the SUSY contributions and the current experimental results. Section 3 discusses the experimental prospects at the LHC experiments. The constraints derived are described in section 4 for the CMSSM and the more general case of the pMSSM. Conclusions are provided in section 5.

2 Current status

2.1 SM prediction

In the SM, the flavour changing neutral current (FCNC) decay $B_s \rightarrow \mu^+\mu^-$ proceeds via $Z$ penguin and box diagrams and is helicity suppressed. The average branching fraction can be expressed as [26–30]:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = \frac{G_F^2 \alpha^2}{64\pi^3} \left| f_{B_s} m_{B_s}^3 V_{tb} V_{ts}^* \right|^2 \tau_{B_s} \sqrt{1 - \frac{4m_{\mu}^2}{m_{B_s}^2}} \left\{ \left(1 - \frac{4m_{\mu}^2}{m_{B_s}^2}\right)|C_{Q_1} - C'_{Q_1}|^2 + |(C_{Q_2} - C'_{Q_2}) + 2(C_{10} - C'_{10})\frac{m_{\mu}}{m_{B_s}}|^2 \right\} \right.,$$

where $G_F$ is the Fermi coupling constant, $\alpha$ is the fine-structure constant, $f_{B_s}$ is the decay constant of the $B_s$ meson, $m_{B_s}$ is its mass, $V_{tb} V_{ts}^*$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $\tau_{B_s}$ is the lifetime of the $B_s$ meson, and $C_{Q,i}$ and $C'_{Q,i}$ are the Wilson coefficients of the effective Lagrangian.
Figure 1: Dominant $B_s \to \mu^+\mu^-$ diagrams in the SM, 2HDM and MSSM.

where $f_{B_s}$ is the $B_s$ decay constant, $m_{B_s}$ is the $B_s$ meson mass and $\tau_{B_s}$ is its mean lifetime. $C_{Q_1}$ and $C_{Q_2}$ are the Wilson coefficients of the semileptonic scalar and pseudo-scalar operators, while $C_{10}$ is the axial semileptonic Wilson coefficient. The $C'_i$ terms correspond to the chirality flipped coefficients. In the SM, only $C_{10}$ is non-vanishing and it gets its largest contributions from a $Z$ penguin top loop (75%) and a $W$ box diagram (24%) (see Fig. 1). The SM expected value is evaluated using $m_{B_s}^{\text{MS}}(m_b) = (5.36677 \pm 0.00024)$ GeV and $m_t^{\text{pole}} = (173.5 \pm 0.6 \pm 0.8)$ GeV, corresponding to $C_{10} = -4.16 \pm 0.04$, from which the following SM prediction for the branching fraction is derived:

$$\text{BR}(B_s \to \mu^+\mu^-) = (3.53 \pm 0.38) \times 10^{-9}, \quad (2.2)$$

where we used the numerical values of $m_{B_s} = (5.36677 \pm 0.00024)$ GeV, $|V_{tb}V_{ts}^*| = 0.0404 \pm 0.0011$, $\tau_{B_s} = (1.497 \pm 0.015)$ ps [31,32] and $f_{B_s} = (234 \pm 10)$ MeV. The value of $f_{B_s}$ is extracted from the average of the lattice results reported by the ETMC-11 [34,35] and HPQCD-12 [36] Collaborations and represents the dominant source of systematic uncertainty (8.7%) in the SM prediction. The top mass determination and the choice of the renormalisation scheme for its running have an important impact on the evaluation of the $B_s \to \mu^+\mu^-$ branching fraction, as discussed in [37]. The effect is illustrated in Fig. 2 where we show the SM central value for BR($B_s \to \mu^+\mu^-$) as a function of the top pole mass value. A change of $\pm 2$ GeV in the top mass corresponds to a $\pm 10^{-10}$ change in the branching fraction value. Other sources of uncertainty include the choice of scale for the calculation of the fine-structure constant and parametric uncertainties. Adding all these uncertainties in quadrature, a total theoretical uncertainty of 11% is estimated.

2.2 SUSY contributions

The $B_s \to \mu^+\mu^-$ decay may receive very large enhancements within specific extensions of the SM. In particular, in the MSSM the Higgs-mediated scalar FCNCs do not suffer from the same

1Note that $C_{Q_{1,2}} = m_b C_{S,P}$. 

\[\]
Figure 2: BR($B_s \rightarrow \mu^+\mu^-$) vs. the top quark pole mass. The black (lower) line corresponds to the CP-averaged branching ratio, while the red (upper) line shows the untagged value.

helicity suppression as the SM diagrams, thus leading to possible drastic enhancements at large values of $\tan \beta$ [1–3]. In this case, the $C_{Q1}$, $C_{Q2}$ coefficients give the dominant contributions. For positive values of $C_{Q2}$, the interference with the term proportional to $(C_{Q1}^2 + C_{Q2}^2)$ is destructive. The upper bound on BR($B_s \rightarrow \mu^+\mu^-$) is more easily evaded or, conversely, an appropriate pseudo-scalar contribution may lead to a suppression of this decay mode to rates below the SM expectation. In the MSSM, the largest contribution to $C_{Q1}$ and $C_{Q2}$, in the large $\tan \beta$ region, reads [3,20]:

$$C_{Q1} \approx -C_{Q2} \approx -\mu A_t \frac{\tan^2 \beta}{(1 + \epsilon_b \tan \beta)^2} \frac{m_t^2}{m_{\tilde{t}}^2} \frac{m_b m_{\mu}}{4 \sin^2 \theta_W M_W^2 M_A^2} f(x_{i\mu}) ,$$  

(2.3)

where $x_{i\mu} = m_{\tilde{t}}^2 / \mu^2$, with $m_{\tilde{t}}$ the geometric average of the two stop masses, and

$$f(x) = -x \frac{1}{1 - x} - x \frac{1}{(1 - x)^2} \ln x .$$  

(2.4)

The $\epsilon_b$ correction parametrises loop-induced non-holomorphic terms that receive their main contributions from higgsino and gluino exchange. Since $f(x) > 0$, the sign of $C_{Q1}$ is opposite to that of the $\mu A_t$ term. Here, Eq. (2.3) is given for purely illustrative purposes; in our numerical analysis we employ the result of a full calculation, which includes all relevant contributions. It must be pointed out that, whereas the MSSM may have a spectacular impact on the $B_s \rightarrow \mu^+\mu^-$ process, it is equally possible to effectively suppress the SUSY contributions by moving to regions of intermediate $\tan \beta$ values and/or large masses of the pseudo-scalar Higgs boson $A$. In such cases, the branching fraction does not deviate from its SM prediction, effectively preventing this decay from probing parts of the supersymmetric parameter space.
2.3 Experimental results

The $B_s \rightarrow \mu^+\mu^-$ decay has been the target of a dedicated effort at the Tevatron and the LHC. To date, the most constraining upper limit obtained by a single experiment comes from LHCb \cite{38}, $\text{BR}(B_s \rightarrow \mu^+\mu^-) < 4.5 \times 10^{-9}$ at 95\% C.L., based on 1.0 fb$^{-1}$ of data at 7 TeV. Searches leading to upper limits have been carried out also by CMS \cite{39} and ATLAS \cite{40}, while the CDF Collaboration reported an excess of events over the estimated background, corresponding to a value $\text{BR}(B_s \rightarrow \mu^+\mu^-) = (1.3^{+0.9}_{-0.7}) \times 10^{-8}$ \cite{41}. The combination of the LHCb, ATLAS and CMS results led to an upper bound of $4.2 \times 10^{-9}$ \cite{42} in Summer 2012.

More recently, the LHCb Collaboration has announced the first evidence for this decay and measured its branching fraction \cite{25} to be:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = (3.2^{+1.4}_{-1.2}(\text{stat})^{+0.5}_{-0.3}(\text{syst})) \times 10^{-9}. \quad (2.5)$$

This value is in excellent agreement with the SM prediction, leading to speculations on its implications on the viability of SUSY. However, it must be noted that the upper limit constraint derived from this result is somehow weaker compared to those from the earlier upper limits, while it is interesting to investigate the effect of the lower limit from (2.5).

Before discussing these implications, it is important to consider that the theoretical prediction of the branching fraction does not directly correspond to the quantity measured by the LHCb experiment. In fact, the theoretical predictions are CP-averaged quantities in which the effect of $B_s - \bar{B}_s$ oscillations is disregarded. On the contrary, the experimental measurement corresponds to an untagged branching fraction which is related to the CP-averaged value by the relation \cite{43,44}:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{untag}} = \left(1 + A_{\Delta \Gamma} y_s \right) \frac{1}{1 - y_s^2} \text{BR}(B_s \rightarrow \mu^+\mu^-), \quad (2.6)$$

where

$$y_s \equiv \frac{1}{2} \tau_{B_s} \Delta \Gamma_s = 0.088 \pm 0.014, \quad (2.7)$$

and

$$A_{\Delta \Gamma} = \frac{|P|^2 \cos(2\varphi_P) - |S|^2 \cos(2\varphi_S)}{|P|^2 + |S|^2}. \quad (2.8)$$

$S$ and $P$ are related to the Wilson coefficients by:

$$S = \sqrt{1 - 4 \frac{m_{\mu}^2}{M_{B_s}^2} \frac{m_{b}^2}{M_{B_s}^2} \frac{1}{2m_{\mu} m_{b} + m_{s}} \frac{C_{Q_1} - C'_{Q_1}}{C_{10}^{SM}}}, \quad (2.9)$$

$$P = \frac{C_{10}}{C_{10}^{SM}} + \frac{M_{B_s}^2}{2m_{\mu} m_{b} + m_{s}} \frac{1}{C_{10}^{SM}} \frac{C_{Q_2} - C'_{Q_1}}{C_{10}^{SM}}, \quad (2.10)$$

and

$$\varphi_S = \arg(S), \quad \varphi_P = \arg(P). \quad (2.11)$$
The resulting untagged branching fraction can be directly compared to the experimental measurement. The SM expectation for this corrected branching fraction is:

$$BR(B_s \to \mu^+\mu^-)_{\text{untag}} = (3.87 \pm 0.46) \times 10^{-9}.$$  \hfill (2.12)

In the MSSM, the difference between the CP-averaged and the untagged values of the branching fraction depends on the specific SUSY parameters which enter $A_{\Delta R}$ but the shift is typically within $\pm 10\%$. The distribution of the branching fraction values, from our CMSSM scan discussed below, is shown in Fig. 3.

Figure 3: Distribution of $BR(B_s \to \mu^+\mu^-)_{\text{untag}}$ for CMSSM points. The general shape with entries at values below and above the SM expected value of $3.87 \times 10^{-9}$ persists when restricting to the points compatible with the results of LHC SUSY searches.

### 3 Experimental Prospects

The LHC experiments, in particular LHCb, will keep improving the precision in the determination of the $B_s \to \mu^+\mu^-$ branching fraction. The latest LHCb measurement offers valid guidance for estimating the evolution of the measurement accuracy for increasing statistics. By symmetrising the statistical uncertainty of the result to $\approx 1.3$ and using Gaussian statistics, we study the statistical accuracy as a function of the integrated luminosity. At 14 TeV centre of mass energy, the $B_s$ production cross section is approximately a factor of two larger compared to 7 TeV. The systematic uncertainties are expected to become important once the statistic uncertainties drop. These factors are taken into account in the following estimate:

$$\sigma(BR(B_s \to \mu^+\mu^-))(L) \approx \sqrt{\frac{1.3^2}{L} + \sigma_{\text{syst}}^2}$$  \hfill (3.13)

for 7 and 8 TeV operations, and

$$\sigma(BR(B_s \to \mu^+\mu^-))(L) \approx \sqrt{\frac{1.3^2}{2L - L_0} + \sigma_{\text{syst}}^2}$$  \hfill (3.14)
Figure 4: Expected uncertainty in the branching fraction of $B_s \to \mu^+\mu^-$ vs. the integrated luminosity recorded by LHCb (solid lines). The red (upper) line refers to an ultimate systematic uncertainty of 5% and the green (lower) to an ultimate systematic uncertainty of 1%. The dashed lines show the precision of LHC combinations, assuming comparable sensitivity for the LHCb and CMS experiments in the same time period.

for the 14 TeV data, where $L$ is the integrated luminosity, $L_0$ the total integrated luminosity taken at 7 and 8 TeV and $\sigma_{syst}$ the expected systematic uncertainty. With the improvements in the computing power for lattice calculations the uncertainty on $f_{B_s}$, the dominant source of theory uncertainty in the SM prediction, is likely to decrease to $\sim 1\%$ [45]. Fig. 4 shows the expected precision in $\text{BR}(B_s \to \mu^+\mu^-)$ as a function of the integrated luminosity for LHCb, assuming $\approx 3.5 \text{ fb}^{-1}$ at 7 and 8 TeV, and two different scenarios for the systematic uncertainties: 5% and, optimistically, 1%. This shows the importance of improvements in the systematic errors over a long period of time. The systematic uncertainty will largely depend on the accuracy available for the determination of the fragmentation function ratio $f_d/f_s$. We do not consider here improvements to the analysis and the detector performance, which are difficult to quantify at present but may lead to an additional reduction of the statistical uncertainties. The upgraded LHCb experiment plans to collect 50 fb$^{-1}$ of data after ten years of running [46], providing an ultimate uncertainty of $\lesssim 2 \times 10^{-10}$. In addition, the general purpose experiments can provide useful results and the CMS experiment has demonstrated a sensitivity quite close to that of LHCb. If this performance can be extrapolated to the future data sets, taking into account the larger event pile-up, and the higher energies, the LHC combinations will show improvements of $\gtrsim \sqrt{2}$ on the statistical error compared to the results of LHCb alone. Since the systematic uncertainty on $f_d/f_s$ is common to all the experiments, it is assumed to be fully correlated in this study.

In summary, we consider two intervals at 95% C.L. for the branching fraction values:

$$1.1 \times 10^{-9} < \text{BR}(B_s \to \mu^+\mu^-) < 6.4 \times 10^{-9}$$

(3.15)
corresponding to the current LHCb result of Eq. (2.5) and

\[ 3.1 \times 10^{-9} < \text{BR}(B_s \to \mu^+\mu^-) < 4.6 \times 10^{-9} \]  

which represents a realistic estimate of the LHC ultimate relative accuracy of \( \sim 5\% \), when including an estimated improved theory uncertainty of \( \sim 8\% \) in the determination of the rate of this process, if the central value meets the SM prediction.

### 4 Constraints in MSSM models

We study the effect of imposing the constraints of Eqs. (3.15) and (3.16) on the CMSSM and pMSSM by performing broad scans of the model parameters and studying the fraction of points compatible with those \( B_s \to \mu^+\mu^- \) rates. The parameters are varied in flat scans within their ranges given below. The SUSY mass spectra are obtained with SOFTSUSY 3.3.4 \[47\] and the value of \( \text{BR}(B_s \to \mu^+\mu^-) \) with SuperIso v3.4 \[30, 48\]. We select points where the lightest SUSY particle is the \( \tilde{\chi}_0^1 \) neutralino and which are consistent with the LEP and LEP-2 limits on SUSY particles. These points are referred to as “accepted” points in the following. Then, we test each point for compatibility with the results of the LHC SUSY and Higgs searches.

#### 4.1 CMSSM

First, we consider the effect of \( \text{BR}(B_s \to \mu^+\mu^-) \) in the CMSSM parameter space, where we perform flat scans varying the CMSSM parameters in the ranges:

\[ m_0, m_{1/2} \in [50, 3000] \text{ GeV}; \quad \tan\beta \in [1, 60]; \quad A_0 \in [-10, 10] \text{ TeV}; \quad \text{sign}(\mu) > 0 . \]  

Since the effects on \( \text{BR}(B_s \to \mu^+\mu^-) \) are small for negative values of the \( \mu \) parameter, we choose \( \text{sign}(\mu) > 0 \) in the scans, which is also in better agreement with the muon \((g - 2)\) constraint.

Results are given in graphical form in Fig. 5 where we show the \( \text{BR}(B_s \to \mu^+\mu^-) \) values as functions of the four CMSSM parameters, comparing the totality of the generated points to those consistent with the lightest Higgs boson \( h \) mass range, \( 123 < M_h < 129 \text{ GeV} \) \[49, 50\]. The \( \text{BR}(B_s \to \mu^+\mu^-) \) admits a lower value of about \( 1.5 \times 10^{-9} \), which is still larger than the present experimental lower bound derived from the LHCb measurement, so that the experimental lower limit does not yet imply the exclusion of portions of the CMSSM parameter space. Branching fraction values below \( \sim 3 \times 10^{-9} \) can be reached for \( m_{1/2} \lesssim 1 \text{ TeV}, \quad 0 \lesssim A_0 \lesssim 6 \text{ TeV} \) and \( \tan\beta \gtrsim 20 \). However, once the Higgs mass limits are imposed, the vast majority of the allowed points have the \( \text{BR}(B_s \to \mu^+\mu^-) \) at values which are equal to, or larger than, the SM prediction, with the exception of a few points located in a region at large \( m_0 \), very small \( m_{1/2} \sim 50 - 100 \text{ GeV} \) and \( A_0 \sim 5 \text{ TeV} \). These points are all excluded by \textit{e.g.} the LEP or Tevatron direct SUSY search limits as they lead to too light gluinos and neutralinos.

As a consequence, in the CMSSM, it is not possible to have \( \text{BR}(B_s \to \mu^+\mu^-) \) smaller than the SM prediction and at the same time be in agreement with the SUSY and Higgs search results. Therefore, if in the future the central measured value of \( \text{BR}(B_s \to \mu^+\mu^-) \) remains
Figure 5: Untagged $\text{BR}(B_s \to \mu^+\mu^-)$ vs. the CMSSM parameters $m_0$ (upper left), $m_{1/2}$ (upper right), $A_0$ (lower left), $\tan \beta$ (lower right). The solid line corresponds to the central value of the $\text{BR}(B_s \to \mu^+\mu^-)$ measurement, and the dashed lines to the $2\sigma$ experimental deviations. The green points are those in agreement with the Higgs mass constraint.

close to the SM prediction, the lower bound is unlikely to have any effect on constraining the CMSSM parameter space.

Figure 6 shows the fraction of CMSSM points compatible with the current LHCb measurement and the expected ultimate precision in the $(m_{1/2}, m_0)$ plane. They are compared to the region excluded at 95% C.L. by the ATLAS SUSY searches in channels with missing transverse energy (MET) obtained on 5.8 fb$^{-1}$ of data at 8 TeV [51] and the expected reach of 300 fb$^{-1}$ at 14 TeV [52], which shows that the sensitivity through the $B_s \to \mu^+\mu^-$ decay improves approximately as the reach of direct searches. However, while searches in the jets + MET channels are directly sensitive to the $m_{1/2}$ and $m_0$ parameters, the $B_s \to \mu^+\mu^-$ decay probes a complementary region of the CMSSM parameter space, accessible to direct searches only through the $H/A \to \tau\tau$ channel.

We quantify the fraction of the CMSSM points in agreement with the $\text{BR}(B_s \to \mu^+\mu^-)$
Figure 6: Fraction of CMSSM points compatible with the current (left) and ultimate (right) 95\% C.L. constraints on BR($B_s \to \mu^+\mu^-$) in the ($m_{1/2}, m_0$) parameter plane. The continuous line shows the parameter region excluded by the ATLAS SUSY searches at 8 TeV with 5.8 fb$^{-1}$ of data (from [51]) and the dotted line the reach estimated by CMS for searches at 14 TeV with 300 fb$^{-1}$ (from [52]).

Figure 7: Fraction of CMSSM points obtained through a 4-parameter flat scan passing the LHC SUSY constraints and in agreement with the present BR($B_s \to \mu^+\mu^-$) measurement of Eq. (3.15) (continuous line), and with the prospective range of Eq. (3.16) (dotted line), as a function of tan $\beta$.

As expected the BR($B_s \to \mu^+\mu^-$) provides us with a powerful constraint for CMSSM points having large values of tan $\beta$. The fractions of our generated CMSSM points, for which we also enforce the requirements to have masses of the sfermions below 3.5 TeV, of the gauginos below 3 TeV and of the CP-odd Higgs boson below 2 TeV to make the results
The dependence of the $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ values calculated at each pMSSM point with the most relevant pMSSM parameters is given in Fig. 8 for all the valid points and those having $123 < M_h < 129$ GeV. Contrary to the case of the CMSSM, here even after imposing the Higgs mass constraints a sizeable number of points with a value of $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ below the SM prediction (down to $0.5 \times 10^{-9}$) is obtained. These low values are reached for $\tan \beta \gtrsim 10$ and $m_{\tilde{t}_1} \gtrsim 300$ GeV. This observation is important for the prospect of improving the lower experimental bound on the decay rate.$^2$

$^2$Note that the lower reachable value we obtain is smaller than the one obtained in the recent study of Ref. [21]. This is because we use the full MSSM expressions with no assumption on $C_{10}$, and use non universal masses for the SUSY particles.
Figure 8: Untagged BR($B_s \to \mu^+\mu^-$) vs. the parameters $\mu$ (upper left), $M_3$ (upper right), $A_t$ (middle left), $\tan\beta$ (middle right), $M_A$ (lower left) and $m_{\tilde{t}_1}$ (lower right). The solid line corresponds to the central value of the BR($B_s \to \mu^+\mu^-$) measurement, and the dashed lines to the $2\sigma$ experimental deviations. The green points are those in agreement with the Higgs mass constraint.
Figure 9: Variation of the untagged BR($B_s \to \mu^+\mu^-$) in the plane ($C_{10}, C_{Q_1}$). The dotted vertical lines delimit the range of $C_{10}$ in the CMSSM, and dashed lines the range in the pMSSM.

Figure 10: Constraints from BR($B_s \to \mu^+\mu^-$) in the ($M_A, \tan \beta$) and ($M_A, m_{\tilde{t}_1}$) parameter planes. The black points corresponds to all the valid pMSSM points and those in grey to the points for which $123 < M_h < 129$ GeV. The dark green points in addition are in agreement with the latest BR($B_s \to \mu^+\mu^-$) range given in Eq. (3.15), while the light green points are in agreement with the prospective LHCb BR($B_s \to \mu^+\mu^-$) range given in Eq. (3.16). The red line indicates the region excluded at 95% C.L. by the CMS $A/H \to \tau^+\tau^-$ searches (from [54]).

The BR($B_s \to \mu^+\mu^-$) dependence on the $C_{10}$ and $C_{Q_1} = -C_{Q_2}$ Wilson coefficients in the minimal flavour violation (MFV) framework [55, 56] is shown in Fig. 9. It is instructive to observe that the values of BR($B_s \to \mu^+\mu^-$) can decrease down to 0 for $C_{10} = C_{Q_1} = 0$. However, in the pMSSM, the variation of $C_{10}$ is limited to the interval [-5.0,-2.6], even when applying constraints from $B \to K^*\mu^+\mu^-$ observables, so that the lowest value which can be
reached is around $0.5 \times 10^{-9}$. The impact of the present and future determinations of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ on the parameters most sensitive to its rate: $(M_A, \tan \beta)$ and $(M_A, m_{\tilde{t}_1})$ is shown in Fig. 10, where we give all the valid pMSSM points from our scan, those with $123 < M_h < 129$ GeV and, highlighted in green, those in agreement with the present $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ range (3.15) and the ultimate constraint (3.16) at 95% C.L. As already discussed in Ref. [22], the constraints from $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ affect the same pMSSM region, at large values of $\tan \beta$ and small values of $M_A$, also probed by the dark matter direct detection constraints and, more importantly, the $H/A \rightarrow \tau^+ \tau^-$ direct Higgs searches at the LHC [54, 58]. The search for the $H/A \rightarrow \tau^+ \tau^-$ decay has already excluded a significant portion of the parameter space where large effects on $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ are expected. We also note that the stop sector is further constrained by direct searches in $b$-jets + MET channels, which disfavour small values of $m_{\tilde{t}_1}$. The figure shows that it is difficult for $M_A$ and $m_{\tilde{t}_1}$ to be simultaneously light.

In more quantitative terms, we compute the fractions of all the accepted pMSSM points, of those not excluded by the jets + MET and $H/A \rightarrow \tau^+ \tau^-$ searches by ATLAS [58] and CMS [54], and those also compatible at 90% C.L. with the ATLAS and CMS Higgs data using the analysis of Ref. [59], which are compatible with the (3.15) and (3.16) constraints at 95% C.L. on $B_s \rightarrow \mu^+ \mu^-$. Results are summarised in Fig. 11 and Table 2. The current LHCb result rules out just below 3% of the pMSSM points compatible with the LHC direct SUSY searches and the Higgs results. The projected bound, assuming the central value coincides with the SM expectation, will increase the reach by an order of magnitude to 30% of the points and severely constrain solutions with very large values of $\tan \beta$. By then, the direct searches for SUSY in channels with jets + MET and $H/A \rightarrow \tau^+ \tau^-$ will also have extended their sensitivity to a much larger part of the pMSSM parameter space. Extrapolating the current bounds to 300 fb$^{-1}$, the fraction of our scan points not excluded by the direct searches but excluded by the projected

\footnote{In general non-SUSY MFV scenarios, $C_{10}$ can admit larger ranges, leading to constraints also coming from the lower bound of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ as shown in [57].}
Table 2: Fraction of pMSSM points, obtained through a 19-parameter flat scan, compatible with the BR($B_s \to \mu^+\mu^-$) constraint.

| Fraction of points                          | Current bounds | Projected bounds |
|--------------------------------------------|----------------|------------------|
| All pMSSM points                           | 95.3%          | 67.8%            |
| Accepted pMSSM points                      | 97.7%          | 78.1%            |
| Points not excluded by LHC searches        | 95.1%          | 63.3%            |
| Points compatible at 90% C.L. with Higgs results | 97.2%          | 70.0%            |

bounds on $B_s \to \mu^+\mu^-$ will decrease from 30% to $\sim 20\%$.

If SUSY is indeed realised in nature and a signal from the direct searches at ATLAS and CMS is observed by then, it would be interesting to perform a quantitative test of consistency between the mass of the SUSY states being observed and the branching fraction of this, until not long ago, elusive decay. In particular, if the pseudoscalar Higgs and the scalar top masses are determined by ATLAS and CMS, the precise value of BR($B_s \to \mu^+\mu^-$) can be used to severely constrain the combination of $(\mu A_t, \tan \beta)$ in the MSSM. Moreover, by constructing alternative observables such as double ratios of leptonic decays formed from the decays $B_s \to \mu^+\mu^-$, $B_u \to \tau \nu$, $D \to \mu \nu$ and $D_s \to \mu \nu/\tau \nu$, it could be possible to enhance the sensitivity of the individual decays, through the cancellation of hadronic uncertainties, and stronger constraints can be obtained [13,16].

5 Conclusions

The observation of the rare decay $B_s \to \mu^+\mu^-$ and the first determination of its branching fraction by the LHCb experiment represent a major milestone of the probe of physics beyond the SM through rare decays of $b$ hadrons. The excellent agreement of the measured value with the SM prediction has raised the question of its implications on the viability of SUSY. In this paper, we have reviewed the predictions for the branching fraction of this decay in the SM and the MSSM and discussed the impact of the new LHCb result and the expected final LHC accuracy on the SUSY parameter space in two models: the CMSSM and the pMSSM. We observe that, despite the significant differences between the two models, the sensitivity of the $B_s \to \mu^+\mu^-$ rate is significant in specific regions of the parameter space of these models, mostly at large values of $\tan \beta$, regions which are also probed by direct SUSY particle searches at ATLAS and CMS. As a result, the constraint derived from the current LHCb result removes $\sim 10\%$ of the scan points in the CMSSM and a few % in the pMSSM, which are not already excluded when the bounds from direct SUSY searches and the Higgs data are applied. This is a consequence of the suppression of the SUSY contributions for intermediate $\tan \beta$ values and/or large masses of the pseudo-scalar Higgs boson $A$, where the branching fraction in the MSSM does not deviate from its SM prediction. The situation in other constrained MSSM scenarios, such as AMSB and GMSB, is similar to that in the CMSSM, with high sensitivity at large $\tan \beta$ [16]. The improved accuracy on the branching fraction measurement expected from the
14 TeV runs together with the expected improvements in the theory uncertainties, will boost the sensitivity, in particular for the region $\tan \beta > 50$ which could be almost completely constrained, and underline the complementarity of direct and indirect searches for supersymmetry through the possibility of consistency checks, if the heavy Higgs bosons can be observed in the direct searches conducted by ATLAS and CMS.

**Acknowledgements**

We are grateful to Monica Pepe Altarelli for reviewing the text, to Guido Martinelli and Vittorio Lubicz for discussion on the expected accuracy of lattice calculations.

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