A Study on Modeling of Effective Series Resistance for Lithium-ion Batteries under Life Cycle Consideration

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Abstract. This paper presents a modeling of effective series resistance for Lithium-ion batteries, which is focusing on the effect of life cycles in aging cells during operations. A computer-based sequential control system is developed to prepare aging cells and automatically characterize the information of testing batteries. Several aspects of testing parameters during the charge and discharge, such as characteristics of the effective series resistance, amplitudes of the pulse current, changes of the increasing resistance, state of charge, capacity and operating cycles, are considered and analyzed to implement in the effective series resistance model. A methodology based on the experiment of pulse tests is applied as sequential steps for modeling the effective series resistance with life cycle consideration. Comparison results between the proposed model and measured values over the life cycle of the battery show the satisfactory verification with the maximum error lower than 4%.

1. Introduction

In high power and high voltage applications with smart grid technologies, a battery-based energy storage system (BESS) is one of the important parts to maintain the grid stability and flexibility of the system. The large-scale BESSs require an accurate battery management system (BMS) for controlling huge batteries for safety. A Lithium-ion battery (LIB) is a promising energy storage device applying in these applications. Hence, the efficient battery modeling is the key to obtain the most significant performance from the battery. There are various methods to gain an equivalent electrical circuit model (EECM) of the battery [1-6]. However, those research methods only focus on battery modeling at the initial state of cell life without considering the aging conditions. A prediction of the lifetime and state of health (SOH) of the batteries can be implied from life cycles. In order to develop a precise BMS for the large-scale BESSs, the variety of algorithms to control the aging batteries and understanding of battery behaviors for life-cycles are especially needed. One essential part of the EECM is the value of ohmic effective series resistance (ESR) because this parameter can lead to battery lifetime determination.

In this research, we propose the modeling of effective series resistance (ESR) extracted by the developed battery test system based on the experiments of the pulse tests to investigate the cell at the beginning of life and the resistance rise after performing the cycle tests. The testing cell used in the research is a rechargeable Lithium manganese oxide battery (LMO), which has dominant features and widely used in electric vehicles and many energy storage applications. The aim of this study is to model the effective series resistance (ESR) characteristic considering with the life cycle changes to understand and estimate nonlinear behaviors of the LIB during long cycling operations and to prepare essential knowledge for engineers who develop the model-based algorithms for the BMS.
2. Li-ion Battery Equivalent Circuit Model and Effective Series Resistance Determination

ECCM has different structures depending on the viewpoint, scope of the study, accuracy of the model and various types of the battery chemistry [7, 8]. The EECM generally comprises of three main parts as follows: 1) the equivalent voltage source called open circuit voltage (OCV), which has a nonlinear potential characteristic depending on the property of electrode materials and the state of charge (SOC) of the battery; 2) a bulk ohmic equivalent resistance called an effective series resistance (ESR) representing the resistance of electrodes, electrolyte, separator and connecting wires inside the battery, which mainly affects the changes of ohmic resistance values in the battery; 3) a pair of equivalent RC-transient elements, which reflects the resistances during the process of charge transfer and charge diffusion (R1 and R2) and capacitances of the double-layer structure and diffusion process of the battery (C1 and C2). For the scope of this study, the model of the ESR will be investigated only in cases of life cycle considerations. However, the details of OCV and RC-transient elements was also introduced in the previous works [9, 10]. Figure 1 proposes a combination of ECCM for the LIB with effective series resistances and additional resistances due to the life cycle changes shown in a dot-line block. The ESR under the life cycle effect (ESRcy) comprises of effective series resistances at the beginning of life (ESR0_chg and ESR0_dis) and life cycle resistances (Rcy_chg and Rcy_dis) for the charge and discharge operations, respectively.

The ESR characteristic of the LIB can be derived by the experiments based on the galvanostatic intermittent titration technique (GITT), which was originally proposed to characterize the kinetic parameters of the chemistry batteries [11]. The ESR is determined by a ratio of voltage and current based on Ohm’s law during the immediate step changes of respective pulse tests without the effects of transient elements [12]. It is usually indicated in a function of the SOC between 0% and 100% separately for the charge and discharge. The ESR can be calculated as shown in equation (1), while the SOC is calculated based on the coulomb counting method as shown in equation (2).

$$ESR = \frac{\Delta V}{\Delta I}$$  \hspace{1cm} (1)

$$SOC(t) = SOC_0 - \frac{TF}{T_0}(I_b(t)/3600Q_B)dt$$  \hspace{1cm} (2)

where ESR is the effective series resistance of the cell (mΩ), \(\Delta I\) represents the difference of immediate step change of the pulse current measured before and after applying the pulse tests, \(\Delta V\) represents the difference of immediate measured voltage change at the immediate step change of the pulse currents, SOC is the state of charge at the immediate step change of the pulse test (%), SOC0 is the initial SOC before the tests, \(I_b\) is the battery testing current (mA) that indicates a plus sign for discharge and a minus sign for charge, T0 and TF are the initial time and final time of testing that are limited within the operating voltage range of the cell and Q_B is a battery capacity (Ah).

![Figure 1. Li-ion battery model with ESR under the life cycle consideration.](image)

3. Effective Series Resistance Characterization with the Developed Test System

The test system is developed with a computer based control in a LabVIEW environment, which is used to prepare aging cells with a safety and then to characterize the ESR at desired life cycle operations. The
desired cycles will be set at 0 cycle, 400 cycles, 1,200 cycles and 2,400 cycles to represent the information of a new cell and the cell at equivalent ages of 1 year, 3 years and 6 years, respectively. The testing procedure and algorithms of charge/discharge sequential controls for the ESR characterization are proposed in figure 2.

A 3,500 mAh pouch-cell LMO with a nominal voltage of 3.8 V has been tested and connected in parallel with a programmable charge system, which is used to control the constant current between 0 and 3,500 mA (1C-rate) and to control the constant voltage between 3.0V and 4.2VDC for the state of charge (SOC) of the cell between 0 and 100%. A programmable discharge system is also connected in parallel with the cell to control the constant discharge current between 0 and 17,500 mA (5C-rates). The testing information are recorded with the sampling rate of 1 sampling per second that is enough to observe a slow response of electrochemical changes. The cell is tested in a temperature chamber at constant 20°C, which is only investigated the effect of life cycles without the effects of temperature changes. The developed algorithm is designed to support the transferring of operations between the continuous and pulse operations. For each pulse test, the step-size of pulse current is applied for 1% SOC and resting for 60 min until the cell voltage has reached to its limitation. The characterization results will be discussed in the next section.

**Figure 2.** A proposed test system and testing procedure for ESR characterization.
4. Modeling of Effective Series Resistance
This section proposes a methodology for modeling the ESR under life cycle changes. The ESR characteristics of the cell at the difference of testing conditions will be investigated in order to determine the accurate ESR mathematical model.

The life cycles that affected the effective series resistance (ESR) can be established with the life cycle resistance model ($R_{cy}$) using the experimental relationship of the resistance changes by comparing the cell at the beginning of life with the desired life cycles as 400, 1,200 and 2,400 cycles. In order to study only the effect of life cycles, the effect of temperatures will be ignored. The $R_{cy}$ model will be examined in the function of operating cycle and SOC under the constant temperature (20°C) as shown in equation (3).

$$R_{cy} = f(N, SOC)$$  \hspace{1cm} (3)

where $N$ is an operating cycle of the charge and discharge tests (cycles) and SOC is the state of charge during the tests.

The mathematical model of life cycle resistance can be extracted from the experimental relationship between the increasing resistances of the ESR and the increasing cycles of the tested cell. The SOCs extracted from the test system, as described in Section 3, are separated into 2%, 5%, 10%, 30%, 50%, 70%, 90% and 99% to easily observe the increasing resistances due to the cycle changes. The ESR characteristics for the cycles at different SOCs during the charge and discharge are shown in figure 3(a) and 3(b), respectively. The figures show that the trend of resistances increases with the increasing cycle. These trends have slightly different and mostly exhibit a linear relationship for each SOC.

![Figure 3. The ESR characteristics due to the cycle changes at different SOCs.](image)

From the previous study [13], the experimental voltage profile of the Li-ion battery shows that the flat plateau voltage occurs around the midpoint as 50% of SOC and illustrates a linear voltage trend. Therefore, the SOC at 50% (As shown with the triangle symbol) can be referred to be the reference SOC to observe the variations of resistance due to the cycle changes. In figure 3, the differences of ESR at the beginning of life and at the end of tested cycles for 50% SOC are 18.38% and 15.08% for the charge and discharge, respectively. The relationship between the initial ESR and the resistance increases due to the cycle changes can be illustrated in equation (4) and equation (5), respectively.

$$ESR_{cy} = ESR_0 + R_{cy}$$ \hspace{1cm} (4)

$$R_{cy} = k_{cy}N^Z$$ \hspace{1cm} (5)
where \( \text{ESR}_c \) is an effective series resistance of the LIB, which is considering the cycle effect (mΩ), \( \text{ESR}_0 \) is the effective series resistance at the beginning of life of the testing cell (mΩ), \( R_{cy} \) is a life cycle resistance (mΩ), \( N \) is an operating cycle of the charge and discharge tests (cycles), \( Z \) is an exponent of cycle (\( Z \) equals to 1 for the ESR linear relationship) and \( k_{cy} \) is the rate of change of increasing resistance due to the life cycle changes (\( \mu \Omega \text{.cycle}^{-1} \)).

The rate of change of increasing resistance (\( k_{cy} \)) for each SOC can be obtained in term of the slope of ESR function. The experimental linear relationship of the ESR can be directly expressed by fitting in MATLAB to determine the rate of change of the resistance due to the life cycles as shown in table 1.

Table 1. Rate of change of increasing resistance due to life cycles for SOCs during the charge and discharge tests.

| SOC (%) | Charge | | | | Discharge | | | |
|--------|--------|---|---|---|--------|---|---|---|
|        | ESR\(_0\) (mΩ) | \( k_{cy} \) (\( \mu \Omega \text{.cycle}^{-1} \)) | SSR (mΩ\(^2\)) | ESR\(_0\) (mΩ) | \( k_{cy} \) (\( \mu \Omega \text{.cycle}^{-1} \)) | SSR (mΩ\(^2\)) |
| 2      | 23.14  | 2.349 | 0.209 | 23.00 | 2.429 | 0.196 |
| 5      | 21.71  | 2.185 | 0.184 | 21.90 | 2.123 | 0.031 |
| 10     | 20.64  | 1.744 | 0.452 | 20.76 | 1.779 | 0.119 |
| 30     | 20.41  | 1.669 | 0.339 | 20.49 | 1.371 | 0.129 |
| 50     | 20.38  | 1.663 | 0.179 | 20.48 | 1.387 | 0.151 |
| 70     | 20.34  | 1.671 | 0.124 | 20.32 | 1.458 | 0.010 |
| 90     | 20.85  | 1.393 | 0.053 | 20.41 | 1.431 | 0.035 |
| 99     | 20.99  | 1.488 | 0.114 | 20.93 | 1.461 | 0.040 |

Table 1 shows the rate of change of increasing resistance for each SOC, the effective series resistance at the beginning of life (ESR\(_0\)) and the sum of square of residual errors (SSR) during the charge and discharge tests. At low SOC, the rate of change of increasing resistance is high for both of the charge and discharge. Then, the rate has been reduced and slightly changed along the SOC corresponding to the trend of the ESR characteristic for each cycling interval.

From Table 1, the model equation of the rate of increasing resistance, in a function of SOC for the cycles, is proposed in (6) and the extracted parameters for the charge and discharge can be obtained by a non-linear least square estimation technique in MATLAB as shown in table 2.

\[
k_{cy} = \frac{1}{(k_1 \cdot \ln(SOC) + k_2)}
\]

where \( k_{cy} \) is the rate of change of increasing resistance due to the life cycle changes (\( \mu \Omega \text{.cycle}^{-1} \)), \( k_1 \) is a pre-logarithm factor of the cycle effect (cycle.mΩ\(^{-1}.%^{-1} \)), \( k_2 \) is a constant factor due to the cycle effect (cycle.mΩ\(^{-1} \)) and SOC is a state of charge of the cell (%).

A comparison between the rate of change of increasing resistance due to the life cycles fitting from the experiments and the proposed key model with the extracted parameters in Table 2 is shown in figure 4. The comparison shows the concordance along the SOCs between the experiment and model with the sum of square of residual errors (SSR) for the charge and discharge only of 7.1380x10\(^{-2}\) mΩ\(^2\).cycle and 7.0575x10\(^{-2}\) mΩ\(^2\).cycle, respectively. From equation (5), the life cycle resistance model (\( R_{cy} \)) in a function of the SOC can be completed by substituting equation (6) into equation (5). Therefore, the effective series resistance model considering with the cycle effect (ESR\(_{cy} \)) in equation (4) can be represented by the extended equation as illustrated in equation (7).
Table 2. Extracted parameters for the equation of the rate of change of increasing resistance due to life cycles

| Operation test | \( k_1 \) \( \text{(cycle.mΩ}^{-1}.\%^{-1}) \) | \( k_2 \) \( \text{(cycle.mΩ}^{-1}) \) | SSR \( \text{(mΩ}^2\cdot\text{cycle}) \)
|----------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Charge         | 6.3045x10^{-2}                              | 3.8016x10^{-1}                               | 7.1380x10^{-2}                               |
| Discharge      | 8.4867x10^{-2}                              | 3.5265x10^{-1}                               | 7.0575x10^{-2}                               |

\[
ESR_{cy} = ESR_0 + N^Z \cdot (k_1 \cdot \ln(SOC) + k_2)
\]  

where \( ESR_{cy} \) is an effective series resistance of the LIB, which is considering the cycle effect (mΩ), \( ESR_0 \) is the effective series resistance at the beginning of life of the testing cell (mΩ), \( N \) is an operating cycle of the charge and discharge tests (cycles), \( Z \) is an exponent of cycle (\( Z \) equals to 1 for an ESR linear relationship), \( k_1 \) is a pre-logarithm factor of the cycle effect (cycle.mΩ^{-1}.%^{-1}), \( k_2 \) is a constant factor due to the cycle effect (cycle.mΩ^{-1}) and SOC is a state of charge of the cell (%).

5. Model Verification and Discussion

This section shows a verification of the proposed ESR model under the life cycle consideration. The model in equation (7) is used for plotting the ESR characteristics that only prepare the testing information of a new cell to observe the cycle effects of the aging battery. The model and measure of ESRs for the charge and discharge operations along the SOCs at different operating cycles are compared in figure 5. The result of modeled ESRs show well-fittings with the measured values similar to both charge and discharge operations.

Figure 6 shows combination plots of the estimated ESR from the model against the measured ESR values for all cycles of the charge and discharge. A black diagonal axis represents a reference trend where the estimated ESR equals to the measured value. In the combine plot, each swarming of the resistances is classified by different colors for the different aging cycles. This plot is easy to observe the resistance increment. The ohmic resistance of the cell is grown up with increasing cycles.

(a) For charge operation
(b) For discharge operation

Figure 4. Comparison between the rate of change of increasing resistance fitting from the experiments and the proposed model.
The explanation of these characteristic changes in the LIB can be referred to the processes of intercalation and de-intercalation, which are the insertion and extraction of lithium at the spinel electrode. For long cycling operations, a spinel crystal structure of electrode has been distorted in a mechanism of the phase transitions. This distortion can cause stress and electromechanical grinding on spinel particles. Moreover, the loss of cycleable lithium is occurred during the charge and discharge by growing of the coated solid electrolyte interface (SEI) against the surface of the negative electrode affecting to the reducing of ability for the lithium intercalation. These phenomena result in the increase of the ohmic resistance between the electrodes and the fade of capacity in LIBs in the case of long cycling operation [14,15]. The ESR combined plots show well-fitting results of the model. The maximum and minimum estimation errors of the model for all cycles are only 3.863% and -2.456%, respectively.

6. Conclusion
In this paper, a model of effective series resistance (ESR) is investigated after performing the cycle tests. The experimental ESR characteristics extracted from the proposed battery test system are used to create the ESR model that is only used the characteristic information of a new cell to investigate the effect of life cycle operations. From the study, it is found that the increment of cycle-life causes to increase the value of ohmic effective series resistance of the cell. At high cycle-life as 2,400 cycles, the ESRs at the reference point of 50% SOC have increased by 18.38% and 15.08% when compared to the cell at the beginning of life during the charge and discharge, respectively. The model of ESR for life cycles is extracted and validated by comparing to the measured value. At the same time, combine plots of the...
ESR display the concordance between the measurement and model for the different cycles. The verified model show that the estimated ESRs are swarmed along the reference diagonal line with the error ranges of ±4% for all cycles. The developed algorithms and proposed procedures for the ESR modeling can be applied for analysis and study the characteristic changes of other types of Lithium-ion batteries. This study let us understand the changing of characteristics of a bulk resistance value as effective series resistance that affects the lithium-ion battery. However, the developed model should be modified and implemented with the BMS that are our next steps in the future. The accurate model can be further used as one of the indexes in the BMSs to estimate the state of health and cycling degradation of the battery.

7. References

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