Multicomponent dipole-mode spatial solitons

Anton S. Desyatnikov and Yuri S. Kivshar

Nonlinear Physics Group, Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia

Kristian Motzcek and Friedemann Kaiser

Institute of Applied Physics, Darmstadt University of Technology, D-64289 Darmstadt, Germany

Carsten Weilnau and Cornelia Denz

Institute of Applied Physics, Westfälische Wilhelms-Universität Münster, D-48149 Münster, Germany

We study (2+1)-dimensional multicomponent spatial vector solitons with a nontrivial topological structure of their constituents, and demonstrate that these solitary waves exhibit a symmetry-breaking instability provided their total topological charge is nonzero. We describe a novel type of stable multicomponent dipole-mode solitons with intriguing swinging dynamics.

Recent progress in the study of spatial optical solitons and their interaction, as well as the extensive experimental demonstrations of stable self-focussing of light in different types of nonlinear bulk media, open the road for new concepts to control the diffraction of optical beams and to design new devices for optical switching and storage. Many novel fundamental concepts recently suggested in the physics of spatial optical solitons and storage [1] have complex structures and, in many cases, their total intensity profile exhibits multiple humps [2].

In a bulk medium, vector solitons exist in different forms and, as was recently shown for two-component self-trapped beams, many types of multipole vector solitons can be predicted and analyzed for an isotropic nonlinear bulk medium with saturable nonlinearity [3]. Recently, an important generalization of this concept to the case of N-component two-dimensional vector solitons was suggested for an example of a thresholding nonlinearity [4]. In particular, Musslimani et al. [4] predicted multihump N-component composite spatial solitons that carry different topological charges ('spins') and, therefore, can provide exciting possibilities for 'spin-dependent' interactions of self-trapped optical beams [5].

The purpose of this Letter is twofold. First, we study in more detail the dynamics of multicomponent spatial solitons carrying topological charges in different components and demonstrate that, in contrast to the conjecture of their stability made in Ref. [5], these vector solitons demonstrate a symmetry-breaking instability in all the cases where their total angular momentum is nonzero. Second, based on earlier studies of two-component vector solitons [6] and the conceptual approach developed in Ref. [6], we propose a novel type of stable multicomponent vector solitons consisting of two perpendicular dipole components trapped by the soliton-induced waveguide. These vector solitons are studied here for the case of N = 3 components, which are shown to be the building blocks for the solitons composed of N in-coherently coupled dipole-mode beams [7]. Additionally, we demonstrate numerically that these novel vector solitons are very robust for a broad range of their parameter space, and they demonstrate intriguing swinging dynamics outside the stability domain, resembling long-lived excitations and vibrations of molecules.

We consider the interaction of N mutually incoherent (2+1)-dimensional optical beams propagating in a bulk saturable medium, described by the normalized equations \( (j = 1, 2, \ldots, N) \),

\[
\frac{\partial E_j}{\partial z} + \Delta_\perp E_j - \frac{E_j}{1 + \sum |E_j|^2} = 0,
\]

where \( \Delta_\perp \) is the transverse Laplacian and \( z \) is the propagation coordinate. Equations (1) describe, in a rather simplified isotropic approximation, screening spatial solitons in photorefractive materials [8].

To describe multicomponent vector solitons in the framework of the model (1), first we look for stationary solutions in the form \( E_j(x, y, z) = u_j(x, y) \exp(-i\beta_jz) \), where \( \beta_j \) is the propagation constant and \( u_j(x, y) \) is the envelope of the \( j \)-th component. Then, introducing the dimensionless parameter \( \lambda_j = (1 - \beta_j)/(1 - \beta_1) \) and normalizing the field amplitudes, \( u_j \rightarrow \sqrt{\lambda_j} u_j, \) and the coordinates, \( (x, y) \rightarrow (x, y)/\sqrt{1 - \beta_1} \), we obtain

\[
\Delta_\perp u_j - \lambda_j u_j + F(I)u_j = 0,
\]

where \( I = \sum |u_j|^2 \) is the normalized total intensity, and \( F(I) = I(1 + s I)^{-1} \) with the effective saturation parameter \( s = 1 - \beta_1 \).

First of all, following Musslimani et al. [4], we seek multicomponent radially symmetric solutions of Eqs. (1) for which the main component \( u_1(x, y) = U_1(r) \) has no nodes, but each of the components \( u_k \) \( (k > 1) \) carries a different topological charge, \( u_k(x, y) = U_k(r) \exp(i m_k \theta) \). We denote such states as \( (0, \ldots, m_k, \ldots) \), and an example for \( N = 3 \) is presented in Fig. 1(a), where the same intensity distribution corresponds to two different states, \((0, +1, +1)\) and \((0, +1, -1)\).

In order to study stability of these composite solitons, we propagate them numerically and find that, provided...
the total angular momentum is nonzero, all these multicomponent solitons undergo a symmetry-breaking instability and fragment into a number of the fundamental solitons, as shown in Fig. 1(b) for the case \((0, +1, +1)\). This instability is similar to the instability of the vortex-mode solitons described earlier for the two-component model. The resulting incoherent superposition of two parallel dipole components \(u_2\) and \(u_3\) can be regarded as a generalization of a two-component dipole-mode soliton \(\{u_1, V\}\) to a three-component solution \(\{u_1, u_2, u_3\}\) at \(\lambda_2 = \lambda_3\) with the help of a transformation of the dipole components, \(V \rightarrow \{u_2, u_3\}\), where \(u_2 = V \cos \psi\) and \(u_3 = V \sin \psi\) (\(\psi\) is a transformation parameter). Such a straightforward generalization is indeed possible for the \(N\)-component system \(\{\}\).

\[
\text{FIG. 1. Evolution of the three-component soliton: (a) stationary solution at } z = 0, \text{ (b) the symmetry-breaking instability of the } (0, +1, +1) \text{ solution at } z = 80, \text{ (c) the long-lived quasi-stable propagation of the } (0, +1, -1) \text{ state at } z = 500.
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Total} & |u_1|^2 & |u_2|^2 & |u_3|^2 \\
\hline
\text{(a)} & & & \\
\text{(b)} & & & \\
\text{(c)} & & & \\
\hline
\end{array}
\]

\[
\text{FIG. 2. Families of the three-component dipole-mode solitons. (a) Soliton structure at } \lambda_2 = 0.5 \text{ and } \lambda_3 = 0.65, \text{ (b) the total and partial powers vs. } \lambda_3 \text{ at fixed } \lambda_2 = 0.5.
\]

The most important property of the \((0, +1, -1)\) solution is that its total angular momentum is zero, and this makes it stable. In our calculations, this vector soliton was observed to be unchanged for the distances of the order of \(10^3\) diffraction lengths. However, being launched with additional noise, this soliton displays slowly growing modulations, as shown in Fig. 1(c). The total intensity of the modulated rings in Fig. 1(c) preserves the initial ring profile, resembling an incoherent superposition of two perpendicular dipole components \(\{\}\). While the vector soliton, consisting of two crossed dipoles, has been shown to be unstable without the third main component \(\{\}\), we found that the three-component dipole-mode soliton is stable in our numerical simulations. Stabilization of the vector ring in the presence of the third component can be explained by the physics of the soliton-induced waveguides. Indeed, two crossed dipoles, \(u_2\) and \(u_3\), represent a vectorial guided mode of the induced waveguide. A nontrivial rotational transformation of such a solution (see Ref. \(\) for details) allows to find a whole family of possible superpositions of these modes, including, as a particular case, the vortex components shown in Fig. 1(a) and the \(N\)-component dipole-mode soliton.

In order to find the multicomponent solitary waves with a nontrivial geometry, we integrate the system \(\{\}\) numerically, by means of a relaxation technique, and find a novel class of the dipole-mode soliton that consists of perpendicularly oriented dipoles with different powers: the simplest possible solution of this type has \(N = 3\) components, and it is described by two independent parameters \((\lambda_2, \lambda_3)\), as shown in Fig. 2(a). The family of these solitons ranges from solutions where the fundamental mode dominates the entire structure to solutions where one of the dipoles dominates, as can be seen in Fig. 2(b), where for fixed \(\lambda_2 = 0.5\) the power of the components \(P_j = \int |u_j|^2 \, dr\) is shown as a function of \(\lambda_3\).

Numerical propagation of these solitons has shown that from the lower cutoff value for \(\lambda_3\), where the intensity of the \(u_3\)-component vanishes, up to a value of about \(\lambda_3 = 0.7\) these vector solitons are stable, whereas for higher \(\lambda_3\) they decompose to form different new structures. As can be seen from Fig. 3, an unstable soliton breaks the symmetry along both symmetry axes of the initial distribution. The products of this instability (see the last row in Fig. 3) are a fundamental vector soliton and a rotating dipole-mode soliton, recently introduced in Ref. \(\) as ‘a propeller soliton’. Those two simpler solitons fly away from each other after the break-up.

Near the instability threshold, for \(0.7 < \lambda_3 < 0.8\), we observe very interesting and intriguing dynamics, associated with weak oscillatory instabilities. Figure 4 shows a characteristic example of this dynamics, when the instability breaks the symmetry only along one of the symmetry axes (parallel to the orientation of the stronger dipole). The product of this instability is a structure consisting of a tripole, a dipole and a nodeless beam. This structure is remarkably long-lived and it has, as the snapshots show, a swinging behavior resembling a swinging mode of a three-atom molecule. We could observe almost three periods of such oscillations, until a strong energy-exchange between the two dipole beamlets sets in and destroys this structure. We expect that the vibrational degrees of freedom, which are likely associated
with long-lived soliton internal modes, should manifest themselves in the rich dynamics of soliton collisions, as it is known for the study of a two-component model [3].

which is more consistent with the experimentally studied photorefractive nonlinearities [10]. In order to verify this, we have used the $N$-component generalization of the Zozulya-Anderson model that takes into account the most important properties of photorefractive nonlinearities [11], and found similar classes of multicomponent localized solutions with perpendicularly oriented dipole components. Since the anisotropy allows stable stationary dipole modes oriented in two fixed directions only [10], these solutions are found to be stable even in anisotropic media with nonlocal nonlinear response. This allows us to expect the subsequent experimental observation of the novel type of vector solitons and swinging dynamics described above.

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