Quantum ring with the Rashba spin-orbit interaction in the regime of strong light-matter coupling

V. K. Kozin\textsuperscript{1,2}, I. V. Iorsh\textsuperscript{1}, O. V. Kibis\textsuperscript{3,2} and I. A. Shelykh\textsuperscript{1,2}

\textsuperscript{1}ITMO University, Saint Petersburg 197101, Russia
\textsuperscript{2}Science Institute, University of Iceland, Dunhagi 3, IS-107, Reykjavik, Iceland and
\textsuperscript{3}Department of Applied and Theoretical Physics, Novosibirsk State Technical University,
Karl Marx Avenue 20, Novosibirsk 630073, Russia

We developed the theory of electronic properties of semiconductor quantum rings with the Rashba spin-orbit interaction irradiated by an off-resonant high-frequency electromagnetic field (dressing field). Within the Floquet theory of periodically driven quantum systems, it is demonstrated that the dressing field drastically modifies all electronic characteristics of the rings, including spin-orbit coupling, effective electron mass and optical response. Particularly, the present effect paves the way to controlling the spin polarization of electrons with light in prospective ring-shaped spintronic devices.

I. INTRODUCTION

Rapidly developing field of spintronics deals with spin related phenomena in mesoscopic transport.\textsuperscript{1-4} Generally, the spins of individual carriers can be controlled either by application of an external magnetic field or via change of the strength of the spin-orbit interaction (SOI) in the system. The second approach forms the basis for so-called non-magnetic spintronics which attracts enormous interest of the scientific community. Particularly, the two mechanisms of the SOI are relevant for semiconductor structures: The Dresselhaus SOI caused by the inversion asymmetry of the crystal lattice and the Rashba SOI originated from the inversion asymmetry of the structure as a whole. The latter mechanism is of specific interest for spintronic applications since it becomes dominant in conventionally used InAs/GaSb-, AlSb/InAs- and GaAs/GaAlAs-based nanostructures\textsuperscript{5-7} and can be easily tuned by an external gate voltage.\textsuperscript{8-10} Recently, the alternative way of tuning SOI by purely optical methods was developed.\textsuperscript{11-13} It is based on the regime of strong light-matter coupling when the system “electron + electromagnetic field” cannot be divided into weakly interacting optical and electronic subsystems. As a consequence, the hybrid electron-field object — so-called “electron dressed by electromagnetic field” (dressed electron) — appears as an elementary quasiparticle.\textsuperscript{14,15} The physical properties of dressed electrons can differ sufficiently from their “bare” counterparts as it has been demonstrated for wide variety of condensed-matter structures, including bulk semiconductors,\textsuperscript{16-18} quantum wells,\textsuperscript{19-23} quantum rings,\textsuperscript{24-29} graphene,\textsuperscript{30-38} topological insulators,\textsuperscript{39} etc. From viewpoint of spintronic applications, it is crucially important that the SOI strength can be modified by laser irradiation\textsuperscript{40} since this allows direct optical tuning of spin relaxation time in 2D electron gas\textsuperscript{41} and, therefore, paves the way to optically controlled spintronic devices.\textsuperscript{12}

Although the first ferromagnetic spintronic device (the Datta-Das spin transistor\textsuperscript{42}) has been realized experimentally, its technological production remains challenging due to the difficulties with the efficient spin injection from ferromagnetic contacts. Therefore, design of non-magnetic spintronic devices which do not need the presence of ferromagnetic elements is still an actual problem. As a possible way to solve the problem, it was proposed to use semiconductor quantum rings (QRs) with the Rashba SOI which induces the phase shift between the spin waves propagating in the clockwise and counterclockwise directions. In turn, this results in the large conductance modulation due to the interference of the spin waves.\textsuperscript{43} As a consequence, physical basis of various QR-based non-ferromagnetic spintronic devices — including spin transistors, spin filters and quantum splitters — appears.\textsuperscript{44-49} In the aforementioned previous studies on the subject, the spin properties of QRs were assumed to be controlled by gate voltage. As to the optical methods of the spin control of QRs, they are escaped attention before. The present theoretical research is aimed to fill partially this gap in the spintronics of QRs.

The paper is organized as follows. In Sec. II, we derived the effective Hamiltonian of the irradiated QR with the Rashba spin-orbit interaction within the Floquet theory of periodically driven quantum systems. In Sec. III, the elaborated theory is applied to analyze spin and optical
characteristics of the irradiated QR. As to Sec. IV, it contains conclusions and acknowledgements.

II. MODEL

To describe an irradiated QR (see Fig. 1), we have to start from the Hamiltonian describing an irradiated two-dimensional (2D) electron system with the Rashba spin-orbit interaction,

$$\hat{H}_{2D} = \frac{(\hat{p} - eA)^2}{2m} + \alpha [\sigma_x(\hat{p}_y - eA_y) - \sigma_y(\hat{p}_x - eA_x)],$$

where $\hat{p} = (\hat{p}_x, \hat{p}_y)$ is the operator of electron momentum, $m$ is the effective electron mass, $e$ is the electron charge, $\alpha$ is the Rashba spin-orbit coupling constant, $\sigma_x, \sigma_y$ are the Pauli matrices, $A = (A_x, A_y) = (E_0/\omega, 0)$ is the vector potential of a linearly polarized electromagnetic wave (dressing field) in the 2D plane, $E_0$ is the electric field amplitude of the wave, and $\omega$ is the wave frequency which is assumed to be far from resonant electron frequencies. Applying the standard approach to transform the 2D electron system into the one-dimensional (1D) ring-shaped one, we arrive from the 2D Hamiltonian at the Hamiltonian of irradiated QR,

$$\hat{H}_{QR} = \hat{H}' + \sum_{n=1}^{2} \hat{V}_n e^{in\omega t} + \text{H.c.,}$$

which consists of the time-independent Hamiltonian of unirradiated QR,

$$\hat{H}' = \frac{\hat{p}^2}{2mR^2} + \frac{\alpha}{R} [\sigma_y \hat{p}_z - i\hbar \frac{\sigma_x}{2}] + \frac{eE_0^2}{4m\omega^2},$$

and the periodical part with the two harmonics originating from the irradiation,

$$\hat{V}_1 = \frac{eE_0}{2mR\omega} \left( \sin \phi \hat{p}_z - i\hbar \frac{\cos \phi}{2} \right) + \frac{\alpha eE_0}{2\omega} \sigma_y,$$

$$\hat{V}_2 = \frac{eE_0^2}{8m\omega^2},$$

where $R$ is the QR radius, $\hat{p}_z = -i\hbar \partial/\partial \phi$ is the operator of angular momentum along the axis $z$, $\phi$ is the parameter describing the strength of electron-field coupling, and $\gamma_2 = mRa/\hbar$ is the dimensionless parameter describing the strength of Rashba spin-orbit coupling. As expected, the Hamiltonian exactly coincides with the Hamiltonian of unirradiated QR in the absence of the field ($E_0 = 0$).

III. RESULTS AND DISCUSSION

To consider the Schrödinger problem with the effective Hamiltonian, let us start from its part. Two exact eigenstates of the Hamiltonian can be written as

$$\Psi_1(\phi) = e^{i\gamma_1 \phi} \left( \cos(\xi/2) e^{i\phi/2} - \sin(\xi/2) e^{i\phi/2} \right)$$

and

$$\Psi_2(\phi) = e^{i\gamma_2 \phi} \left( \sin(\xi/2) e^{i\phi/2} \cos(\xi/2) e^{i\phi/2} \right),$$

where

$$\xi = \arctan \left[ \frac{2m^*Ra/\hbar}{2(m^*/m) (eE_0\alpha/\omega^2\hbar^2) + 1} \right]$$

is the angle between the local spin quantization axis and the $z$-axis (see Fig. 1). It follows from single-valuedness of the eigenstates, $\Psi_{1,2}(\phi) = \Psi_{1,2}(\phi + 2\pi)$, that the $z$-component of total angular momentum of electron, $j_z$, must satisfy the condition, $j_z = \lambda n + 1/2$, where
n = 0, 1, 2, ... is the orbital quantum number corresponding to the electron rotation in QR, and the sign \( \lambda = \pm \) describes the direction of the rotation (counterclockwise/straight). Omitting constant terms which only shift the zero energy, one can write the electron energy spectrum of the eigenstates \(|n, \lambda, s\rangle\) based on the three quantum numbers, \(|n, \lambda, s\rangle\), the eigenstates \(|n, \lambda, s\rangle\) can be written as

\[
|n, +, -1\rangle = e^{i\nu} \left( \begin{array}{c} \cos(\xi/2) \\
-\sin(\xi/2) e^{i\mu} \end{array} \right), \quad (15)
\]

\[
|n, +, +1\rangle = e^{i\nu} \left( \begin{array}{c} \sin(\xi/2) \\
\cos(\xi/2) e^{i\mu} \end{array} \right), \quad (16)
\]

\[
|n, -, +1\rangle = e^{-i\nu} \left( \begin{array}{c} \cos(\xi/2) \\
\sin(\xi/2) e^{i\mu} \end{array} \right), \quad (17)
\]

\[
|n, -, -1\rangle = e^{-i\nu} \left( \begin{array}{c} \sin(\xi/2) \\
\cos(\xi/2) e^{i\mu} \end{array} \right), \quad (18)
\]

for \( n = 1, 2, 3, ... \) and

\[
|0, +, -1\rangle = \left( \begin{array}{c} \cos(\xi/2) \\
-\sin(\xi/2) e^{i\mu} \end{array} \right), \quad (19)
\]

\[
|0, +, +1\rangle = \left( \begin{array}{c} \sin(\xi/2) \\
\cos(\xi/2) e^{i\mu} \end{array} \right) \quad (20)
\]

for \( n = 0 \). It follows from Eq. (14), particularly, that \( \varepsilon_{n=0} = \varepsilon_{n=1} \). This means that the states \(|n, -, s\rangle\) and \(|n-1, +, s\rangle\) are degenerated.

The eigenstates and eigenenergies \(|n, \lambda, s\rangle\) can be easily verified by direct substitution into the Schrödinger equation with the Hamiltonian \(|\hat{H}_0\rangle\). However, the total effective Hamiltonian \(|\hat{H}\rangle\) consists of the two parts, including both the discussed Hamiltonian \(|\hat{H}_0\rangle\) and the term \(|\hat{V}\rangle\). Therefore, we have to analyze the effect of the term \(|\hat{V}\rangle\) on the found solutions of the Schrödinger problem with the Hamiltonian \(|\hat{H}_0\rangle\). It follows from Eqs. (9) and (15)–(20) that \(|n', \lambda', s'|\hat{V}|n, \lambda, s\rangle \sim \delta_{n,\lambda} \lambda, s' \) for \( n, n' \geq 1 \). Thus, the term \(|\hat{V}\rangle does not split the degenerate states \(|n, -, s\rangle\) and \(|n-1, +, s\rangle\). Moreover, the conventional criterion of perturbation theory,

\[
\left| \frac{\langle n', \lambda', s'|\hat{V}|n, \lambda, s\rangle}{\varepsilon_{\lambda'} n' - \varepsilon_{\lambda n}} \right| \ll 1, \quad (21)
\]

can be satisfied for a broad range of QR parameters. This means that the term \(|\hat{V}\rangle\) can be treated as a weak perturbation which leads only to a relatively small displacement of the energy levels \(|\varepsilon_{\lambda n}\rangle\). Particularly, the criterion \(|\hat{V}\rangle\) is fulfilled for the first tens of energy levels \(|\varepsilon_{\lambda n}\rangle\) in the typical case of InGaAs-based QRs with the effective mass \( m = 0.045 m_e \), radius \( R \approx 200 \) nm and the Rashba coupling constant \( \alpha \approx 10^4 \) m/s. As a consequence, the effective Hamiltonian \(|\hat{H}\rangle\) can be reduced to the simplified Hamiltonian \(|\hat{H}_0\rangle\). Correspondingly, the found eigenstates and eigenenergies \(|\varepsilon_{\lambda n}\rangle\) can be applied to describe electronic properties of the irradiated QR.

It follows from the Hamiltonian \(|\hat{H}_0\rangle\) that the irradiation of QR results in the two main effects: First, it renormalizes the electron effective mass \(|\varepsilon_{\lambda n}\rangle\) and, second, it leads to the unusual spin-orbit coupling \( \sim l_s \sigma \) described by the third term of the Hamiltonian \(|\hat{H}_0\rangle\). In turn, these effects lead to the dependencies of the spin angle \(|\varepsilon_{\lambda n}\rangle\) and the energy levels \(|\varepsilon_{\lambda n}\rangle\) on the irradiation intensity, which are plotted in Fig. 2. It follows from Fig. 2a that the irradiation very strongly effects on the spin angle \(|\varepsilon_{\lambda n}\rangle\).

Namely, the relatively weak irradiation can decrease the angle to tens percents of its initial value in the unirradiated QR, \( \xi_0 = \arctan(2a m R/h) \). Since the modulation of spin orientation by various external actions lies in the core of modern spintronics, the found strong dependence of the spin polarization on the irradiation can be used, particularly, in prospective ring-shaped spintronic devices operated by light. It follows from Fig. 2b that the irradiation also strongly效果s on the energy of the electron levels in QR and their spin splitting. Such a light-induced modification of the energy spectrum \(|\varepsilon_{\lambda n}\rangle\) can manifest itself, particularly, in optical measurements discussed below.

Let us consider a QR irradiated by a two-mode electromagnetic wave consisting of a strong dressing field (which renormalizes the energy spectrum of electrons according to the aforesaid) and a relatively weak probe field with the frequency \( \Omega \) (which can detect the discussed renormalization of the energy spectrum). The optical spectrum of absorption of the probe field can be obtained with using the conventional Kubo formalism \(|\varepsilon_{\lambda n}\rangle\). Within this approach, the longitudinal conductivity describing the response of the QR to the probe field polarized along the \( j = x, y \) axis reads

\[
\sigma_{jj} = \sum_{n, \lambda, s} \sum_{n', \lambda', s'} \frac{\langle f(\varepsilon_{\lambda n}^*) - f(\varepsilon_{\lambda n}') \rangle \langle \langle n', \lambda', s'|\hat{\sigma}_j|n, \lambda, s\rangle \rangle^2}{(\varepsilon_{\lambda n}' - \varepsilon_{\lambda n})(\varepsilon_{\lambda n}' - \varepsilon_{\lambda n} + \hbar \Omega + i\Gamma)} \times \langle \hat{\sigma}_j \rangle, \quad (22)
\]

where \( f(\varepsilon) \) is the Fermi-Dirac distribution function, \( \hat{\sigma}_j = \hat{p}_j / m \) is the velocity operator, and \( \Gamma = \hbar / \tau \) is the broadening of energy levels depending on the electron relaxation time, \( \tau \). Substituting Eqs. (14)–(20) into Eq. (22), one can calculate the sought absorption spectrum of the probe field (see Fig. 3), which is represented by the real part of the conductivity, \( \text{Re}(\sigma_{jj}) \). In the absence of the dressing field, the absorption spectrum of the QR plotted in Fig. 3a consists of the three peaks corresponding to the following electron transitions: \(|5, +, +1\rangle \rightarrow |4, +, +1\rangle, \)
FIG. 2: Electronic characteristics of InGaAs-based QR (the electron effective mass is \( m = 0.045m_0 \), the Rashba coupling constant is \( \alpha = 10^4 \) m/s and the QR radius is \( R = 200 \) nm) irradiated by a dressing field with the frequency \( \omega = 1.6 \cdot 10^{12} \) rad/s: (a) Dependence of the spin angle, \( \xi \), on the irradiation intensity, \( I \); (b) Dependence of the first nine electron energy levels, \( \varepsilon_{\lambda n} \), on the irradiation intensity, \( I \), for the counterclockwise electron rotation in the ring (\( \lambda = + \)), where the dashed and solid lines correspond to the different spin orientations (\( s = \pm 1 \)).

\[
|7,+,1\rangle \rightarrow |6,+,1\rangle \text{ and } |7,+,1\rangle \rightarrow |6,+,1\rangle \text{ (peak 1); } |6,+,1\rangle \rightarrow |5,+,1\rangle , |8,+,1\rangle \rightarrow |7,+,1\rangle \text{ and } |8,+,1\rangle \rightarrow |7,+,1\rangle \text{ (peak 2); } |7,+,1\rangle \rightarrow |6,+,1\rangle \text{ and } |9,+,1\rangle \rightarrow |8,+,1\rangle \text{ (peak 3).}
\]

The evolution of this spectrum under influence of the dressing field is presented in Figs. 3b–3d. In the absence of the dressing field, the highest peak 3 originates from the transitions \( |6,+,1\rangle \rightarrow |5,+,1\rangle \) and \( |8,+,1\rangle \rightarrow |7,+,1\rangle \) since the chosen Fermi energy, \( \mu = 1 \) meV, lies in the middle between the corresponding levels (see Fig. 2b). Since the dressing field increases the distance between the energy levels, it shifts the peaks to the right and deforms them (see Figs. 3b–3d). It should be noted that the shape of the spectrum at the irradiation intensity \( I = 1000 \) W/cm\(^2\) is very similar to case of unirradiated QR (compare Figs. 3a and 3d). Physically, the similarity appears since the Fermi energy, \( \mu = 1 \) meV, lies at this intensity again in the middle between the corresponding levels (see Fig. 2b). However, the highest peak in this case arises from the peak 1 in Fig. 3a and, therefore, corresponds to the transitions \( |5,1,+1\rangle \rightarrow |4,1,+1\rangle , |7,1,+1\rangle \rightarrow |6,1,+1\rangle \text{ and } |7,1,-1\rangle \rightarrow |6,1,-1\rangle \).

FIG. 3: Absorption spectra of the probe field with the frequency \( \Omega \) for the InGaAs-based QR (the electron effective mass is \( m = 0.045m_0 \), the Rashba coupling constant is \( \alpha = 10^4 \) m/s, the electron relaxation time is \( \tau = 70 \) ps, the temperature is \( T = 5 \) K, the Fermi energy is \( \mu = 1 \) meV and the QR radius is \( R = 200 \) nm) irradiated by a dressing field with the frequency \( \omega = 1.6 \cdot 10^{12} \) rad/s and different irradiation intensities, \( I \).

IV. CONCLUSIONS AND ACKNOWLEDGEMENTS

In conclusion, we demonstrated that the key electronic characteristics of QRs with the Rashba spin-orbit interaction — the structure of electron energy levels and the spin polarization of electrons — strongly depend on an off-resonant irradiation. Particularly, the modification of both electron effective mass and the spin-orbit coupling appear. It is shown that the irradiation-induced renormalization of electron energy spectrum can be observed in state-of-the-art optical experiments, whereas the light sensitivity of the spin orientation can be exploited in prospective spintronic devices operated by light.

The work was partially supported by RISE Program (project CoExAN), FP7 ITN Program (project NOT- EDEV), Russian Foundation for Basic Research (project 17-02-00053), Rannis project 163082-051, and Ministry of Education and Science of Russian Federation (projects 3.4573.2017/6.7, 3.2614.2017/4.6, 14.Y26.31.0015).
A. S. Sheremet, O. V. Kibis, A. V. Kavokin, and I. A. She-
D. Yudin and I. A. Shelykh, Two-dimensional electron gas
13
J. P. Heida, B. J. van Wees, J. J. Kuipers, T. M. Klapwijk,
S. P. Goreslavskii and V. F. Elesin, Electric properties of
16
M. O. Scully and M. S. Zubairy, Quantum Optics (Cam-
15
Q. T. Vu, H. Haug, O. D. Mucke, T. Tritoschler, M. We-
gen, G. Khitrova, and H. M. Gibbs, Light-Induced Gaps in
Semiconductor Band-to-Band Transitions, Phys. Rev. Lett.
19
Q. T. Vu and H. Haug, Detection of light-induced band
gaps by ultrafast femtosecond pump and probe spec-
troscopy, Phys. Rev. B 71, 035305 (2005).
A. Mysyrowicz, D. Hulin, A. Antonetti, A. Migus, W.
T. Masselink, and H. Morkoc, “Dressed Excitons” in a
Multiple-Quantum-Well Structure: Evidence for an Optical
Stark Effect with Femtosecond Response Time, Phys. Rev.
Lett. 56, 2748 (1986).
M. Wagner, H. Schneider, D. Stehr, S. Winnerl, A.
M. Andrews, S. Schortner, G. Strasser, and M. Helm,
Observation of the Intraexciton Butler-Townes Effect in
GaAs/AlGaAs Semiconductor Quantum Wells, Phys. Rev.
Lett. 105, 167401 (2010).
M. Teich, M. Wagner, H. Schneider, and M. Helm, Semi-
 conductor quantum well excitons in strong, narrowband
derahertz fields, New J. Phys. 15, 065007 (2013).
O. V. Kibis, How to suppress the backscattering of con-
duction electrons?, Europhys. Lett. 107, 57003 (2014).
S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. She-
ykh, Transport properties of a two-dimensional electron
gas dressed by light, Phys. Rev. B 91, 155312 (2015).
O. V. Kibis, Dissipationless Electron Transport in Photon-
Dressed Nanostructures, Phys. Rev. Lett. 107, 106802
(2011).
F. K. Joibari, Y. M. Blanter, and G. E. W. Bauer, Light-
induced spin polarizations in quantum rings, Phys. Rev. B 
90, 155301 (2014).
H. Sigurdsson, O. V. Kibis, and I. A. Shelykh, Optically
induced Aharonov-Bohm effect in mesoscopic rings, Phys.
Rev. B 90, 235413 (2014).
O. V. Kibis, H. Sigurdsson, I. A. Shelykh, Aharonov-Bohm
effect for excitons in a semiconductor quantum ring dressed
by circularly polarized light, Phys. Rev. B 91, 235308
(2015).
M. Hasan, I. V. Iorsh, O. V. Kibis, I. A. Shelykh, Optically
controlled periodical chain of quantum rings, Phys. Rev.
B 93, 125401 (2016).
V. K. Kozin, I. V. Iorsh, O. V. Kibis, and I. A. Shelykh,
Periodic array of quantum rings strongly coupled to circu-
larly polarized light as a topological insulator, Phys. Rev.
B 97, 035416 (2018).
F. J. Lopez-Rodriguez and G. G. Naumis, Analytic solu-
tion for electrons and holes in graphene under electromag-
netic waves: Gap appearance and nonlinear effects, Phys.
Rev. B 78, 201406(R) (2008).
T. Oka and H. Aoki, Photovoltaic Hall effect in graphene,
Phys. Rev. B 79, 081406(R) (2009).
O. V. Kibis, Metal-insulator transition in graphene
induced by circularly polarized photons, Phys. Rev. B 81,
165433 (2010).
T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Dem-
ler, Transport properties of nonequilibrium systems under
the application of light: Photoinduced quantum Hall insu-
lators without Landau levels, Phys. Rev. B 84, 235108
(2011).
G. Usaj, P. M. Perez-Piskunow, L. E. F. Foa Torres, and
C. A. Balseiro, Irradiated graphene as a tunable Floquet
topological insulator, Phys. Rev. B 90, 115423 (2014).
K. Kristinsson, O. V. Kibis, S. Morina, and I. A. Shelykh,
Control of electronic transport in graphene by electromag-
netic dressing, Sci. Rep. 6, 20082 (2016).
O. V. Kibis, S. Morina, K. Dini, and I. A. Shelykh, Mag-
netoelectronic properties of graphene dressed by a high-

1 Electronic address: Oleg.Kibis(c)nstu.ru
2 D. D. Awschalom, D. Loss, N. Samarth, Semiconductor
Spintronics and Quantum Computation (Springer-Verlag,
Berlin, 2002).
3 I. Zutic, J. Fabian and S. D. Sarma, Spintronics: Funda-
mentals and applications, Rev. Mod. Phys. 76, 323 (2004).
4 D. D. Awschalom and M. E. Flatté, Challenges for semi-
conductor spintronics, Nat. Phys. 3, 153 (2007).
5 M. Cahay, Spin transistors: Closer to an all-electric device, 
Nat. Nanotechnol. 10, 21 (2015).
6 B. Miller, D. M. Zumbuhl, C. M. Marcus, Y. B. Lyanda-
Geller, D. Goldhaber-Gordon, K. Campman and A.C. Gos- 
sard, Gate-Controlled Spin-Orbit Quantum Interference
Effects in Lateral Transport, Phys. Rev. Lett. 90, 076807
(2003).
7 A. Studenikin, P. T. Coleridge, N. Ahmed, P. Poole and
A. Sachrajda, Experimental study of weak antilocalization
effects in a high-mobility In0.53Ga0.47As/In0.52Al0.48As quantum well, 
Phys. Rev. B 68, 035317 (2003).
8 Ghosh, C. J. B. Ford, M. Pepper, H. E. Beere and D. A.
Ritchie, Possible Evidence of a Spontaneous Spin Polarization
in Mesoscopic Two-Dimensional Electron Systems, 
Phys. Rev. Lett. 92, 116601 (2004).
9 J. Nitta, T. Akazaki, H. Takayanagi, T. Enoki, Gate
Control of Spin-Orbit Interaction in an Inverted
In0.53Ga0.47As/In0.52Al0.48As Heterostructure, Phys. Rev.
Lett. 78, 1335 (1997).
10 G. Engels, J. Lange, Th. Schäpers, H. Lüth, Experimental
approach to spin splitting in modulation-
doped InxGa1-xAs/InP quantum wells for B−0, 
Phys. Rev. B 55, R1958 (1997).
11 J. P. Heida, B. J. van Wees, J. J. Kuipers, T. M. Klapwijk,
G. Borghs, Spin-orbit interaction in a two-dimensional
electron gas in an InAs/AlSb quantum well with gate-
controlled electron density, Phys. Rev. B 57, 11911 (1998).
12 A. A. Pervishko, O. V. Kibis, S. Morina, and I. A. Shelykh, 
Control of spin dynamics in a two-dimensional electron
gas by electromagnetic dressing, Phys. Rev. B 92, 205403
(2015).
13 A. S. Sheremet, O. V. Kibis, A. V. Kavokin, and I. A. She-
ykh, Datta-and-Das spin transistor controlled by a high-
frequency electromagnetic field, Phys. Rev. B 93, 165307
(2016).
14 D. Yudin and I. A. Shelykh, Two-dimensional electron gas
in the regime of strong light-matter coupling: Dynamical
conductivity and all-optical measurements of Rashba and
Dresselhaus coupling, Phys. Rev. B 94, 161404(R) (2016).
15 C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg,
Atom-Photon Interactions: Basic Processes and Application-
s (Wiley, Chichester, 1998).
16 M. O. Scully and M. S. Zubairy, Quantum Optics (Cam-
bridge University Press, Cambridge, 2001).
17 S. P. Goreslavskii and V. F. Elesin, Electric properties of
a semiconductor in the field of a strong electromagnetic
wave, JETP Lett. 10, 316 (1969).
18 Q. T. Vu, H. Haug, O. D. Mucke, T. Tritoschler, M. We-
gen, G. Khitrova, and H. M. Gibbs, Light-Induced Gaps in
Semiconductor Band-to-Band Transitions, Phys. Rev.
Lett. 92, 217403 (2004).
19 Q. T. Vu and H. Haug, Detection of light-induced band
 gaps by ultrafast femtosecond pump and probe spec-
frequency field, Phys. Rev. B 93, 115420 (2016).
37 O. V. Kibis, K. Dini, I. V. Iorsh, I. A. Shelykh, All-optical band engineering of gapped Dirac materials, Phys. Rev. B 95, 125401 (2017).
38 I. V. Iorsh, K. Dini, O. V. Kibis, I. A. Shelykh, Optically induced Lifshitz transition in bilayer graphene, Phys. Rev. B 96, 155432 (2017).
39 N. H. Lindner, G. Refael and V. Galitski, Floquet topological insulator in semiconductor quantum wells, Nat. Phys. 7, 490 (2011).
40 H. C. Koo, J. H. Kwon, J. Eom, J. Chang, S. H. Han, and M. Johnson, Control of spin precession in a spin-injected field effect transistor, Science 325, 1515 (2009).
41 T. Bergsten, T. Kobayashi, Y. Sekine, and J. Nitta, Experimental Demonstration of the Time Reversal Aharonov-Casher Effect, Phys. Rev. Lett. 97, 196803 (2006).
42 J. Nitta, F. E. Meijer and H. Takayanagi, Spin-interference device, Appl. Phys. Lett. 75, 695 (1999).
43 J. Nitta and T. Koga, Rashba Spin-Orbit Interaction and Its Applications to Spin-Interference Effect and Spin-Filter Device, J. Supercond. 16, 689 (2003).
44 D. Frustaglia and K. Richter, Spin interference effects in ring conductors subject to Rashba coupling, Phys. Rev. B 69, 235310 (2004).
45 I. A. Shelykh, N. T. Bagraev, N. G. Galkin, L. E. Klyachkin, Interplay of h/e and h/2e oscillations in gate-controlled Aharonov-Bohm rings, Phys. Rev. B 71, 113311 (2005).
46 A. G. Aronov and Y. B. Lyanda-Geller, Spin-orbit Berry phase in conducting rings, Phys. Rev. Lett. 70, 343 (1993).
47 M. Popp, D. Frustaglia and K. Richter, Spin filter effects in mesoscopic ring structures, Nanotechnology 14, 347 (2003).
48 A. A. Kiselev and K. W. Kim, J., T-shaped spin filter with a ring resonator, Appl. Phys. 94, 4001 (2003).
49 I. A. Shelykh, N. G. Galkin, and N. T. Bagraev, Quantum splitter controlled by Rasha spin-orbit coupling, Phys. Rev. B 72, 235316 (2005).
50 F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, One-dimensional ring in the presence of Rashba spin-orbit interaction: Derivation of the correct Hamiltonian, Phys. Rev. B 66, 033107 (2002).
51 F. Casas, J. A. Oteo, and J. Ros, Floquet theory: exponential perturbative treatment, J. Phys. A 34, 16 (2001).
52 N. Goldman and J. Dalibard, Periodically Driven Quantum Systems: Effective Hamiltonians and Engineered Gauge Fields, Phys. Rev. X 4, 031027 (2014).
53 A. Eckardt and E. Anisimovas, High-frequency approximation for periodically driven quantum systems from a Floquet-space perspective, New J. Phys. 17, 093039 (2015).
54 F. Nagasawa, D. Frustaglia, H. Saarikoski, K. Richter and J. Nitta, Control of the spin geometric phase in a semiconductor quantum ring, Nat. Commun. 4, 2526 (2013).
55 H. Brueck and K. Flensberg, Many-Body Quantum Theory in Condensed Matter Physics (University Press, Oxford, 2001).