Charmless two body hadronic decays of $\Lambda_b$ baryon

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Abstract

Using a theoretical framework based on the next-to-leading order QCD improved effective Hamiltonian, we have estimated the branching ratios and asymmetry parameters for the two body charmless nonleptonic decay modes of $\Lambda_b$ baryon i.e. $\Lambda_b \rightarrow p(\pi/\rho)$, $p(K/K^*)$ and $\Lambda(\pi/\rho)$, within the framework of generalized factorization. The nonfactorizable contributions are parametrized in terms of the effective number of colors, $N_{\text{eff}}^c$. So in addition to the naive factorization approach ($N_{\text{eff}}^c = 3$), here we have taken two more values for $N_{\text{eff}}^c$ i.e., $N_{\text{eff}}^c = 2$ and $\infty$. The baryonic form factors at maximum momentum transfer ($q^2_m$) are evaluated using the nonrelativistic quark model and the extrapolation of the form factors from $q^2_m$ to the required $q^2$ value is done by assuming the pole dominance. The obtained branching ratios for $\Lambda_b \rightarrow p\pi$, $pK$ processes lie within the present experimental upper limit.

1 Introduction

The principal interest in the study of weak decays of bottom hadrons in the context of Standard Model (SM) lies in the fact that they provide valuable information on the weak rotation matrix - the Cabibbo Kobayashi and Maskawa matrix. In fact $b$-decays determine five of its matrix elements: $V_{cb}$, $V_{ub}$, $V_{td}$, $V_{ts}$ and $V_{td}$. The dominant decay modes of bottom hadrons are those involving $b \rightarrow c$ transitions. There are also rare decay modes which proceed through the CKM suppressed $b \rightarrow u$ spectator tree diagram and/or $b \rightarrow s$ ($b \rightarrow d$) penguin amplitudes with, in general, both QCD and electroweak penguins participating. The study of exclusive charmless nonleptonic bottom decays is of great interest for several reasons. First of all, they proceed in general through the $W$-loop diagrams, the so called penguin diagrams without CKM suppression and through the CKM suppressed spectator diagrams. Thus the salient feature in charmless bottom decays is that the loop graphs are as important as
the tree graphs. In some cases the loop graphs may even be dominant over the tree graphs. Furthermore, as most of these decays proceed through more than one amplitudes with different CKM phases, there will in general be interference and so, there is an opportunity to observe direct CP violation. Hence the analysis and measurement of charmless hadronic $b$-decays will enable us to understand the QCD and electroweak penguin effects as well as the origin of CP violation in the Standard Model and provide a powerful tool of seeing physics beyond the SM.

Recently, there has been a remarkable progress in the study of exclusive charmless bottom meson decays both experimentally and theoretically. Experimentally, CLEO \cite{1} has discovered many new two body decay modes

\begin{equation}
B \to \eta'K^\pm, \eta'K^0, \pi^\pm K^\mp, \pi^0 K^\pm, \rho^0 \pi^\pm, \omega K^\pm
\end{equation}

and found a possible evidence for $B \to \phi K^*$. Moreover, CLEO has provided new improved upper limits for many other decay modes. With $B$ factories Babar and Belle starting to collect data, many exciting years in the arena of $B$ physics and CP violation are expected to come. Theoretically many significant improvements and developments have taken place over the past years. For example, a next-to-leading order effective Hamiltonian for current-current operators and QCD as well as electroweak penguin operators have become available. The renormalization scheme and scale problems with factorization approach for matrix elements can be circumvented by employing scale- and scheme-independent Wilson coefficients. Incorporating all these improved results, the exclusive two body charmless hadronic decays of $B$ mesons and their CP asymmetries have been extensively studied in Refs. \cite{2-6}.

It is also interesting to study the charmless nonleptonic decays of bottom baryon system. Recently some data on bottom baryon $\Lambda_b$ have appeared. For instance, OPAL has measured its lifetime and the production branching ratio for the inclusive semileptonic decay \cite{7}. Furthermore, measurements of the nonleptonic decay $\Lambda_b \to \Lambda J/\psi$ has also been reported \cite{8}. Certainly we expect more data in the bottom baryon sector in the near future.

In this paper we would like to study the charmless hadronic decays of $\Lambda_b$ baryon i.e. $\Lambda_b \to p(\pi/\rho), p(K/K^*)$ and $\Lambda_b \to \Lambda(\pi/\rho)$. Experimentally, only upper limits on the branching ratios for rare $\Lambda_b$ decay modes $\Lambda_b \to p\pi$ and $\Lambda_b \to pK$ have been observed \cite{9}. The standard theoretical framework to study the nonleptonic $\Lambda_b$ decays is based on the effective Hamiltonian approach, which allows us to separate the short- and long-distance contributions in these decays using the Wilson operator product expansion \cite{10}. QCD perturbation theory is then used in deriving the renormalization-group improved short distance contributions \cite{11,12}. This program has now been carried out up to and including next-to-leading order terms \cite{13,14}. But the long-distance part in the two body decays $B_i \to B_f M$ (where $B_i(B_f)$ are the initial(final) baryons and $M$ is the final pseudoscalar/vector meson) involves the transition matrix
element $\langle B_f M | O_i | B_i \rangle$, where $O_i$ is an operator in the effective Hamiltonian. Calculation of these matrix elements from the first principle is not yet possible and hence some approximation has to be adopted to deal with these matrix elements. The one we use here is based on the idea of factorization in which the final state interactions has to be absent and hadronic matrix elements in the $B_i \rightarrow B_f M$ transition, factorize into a product of two comparatively tractable matrix elements, one involving the form factors and the other, the decay constant. It is customarily argued that the final state interactions (FSI) are expected to play a minor role in charmless hadronic $b$-decays due to large energy release in these decay processes. Motivated by the phenomenological success of factorization in charmless nonleptonic $B$ decays [2-6], we would like to pursue this framework for charmless $\Lambda_b$ decays. The renormalization scheme and scale problems with factorization approach for matrix elements can be circumvented by employing the scale and scheme independent effective Wilson coefficients. The form factors at maximum recoil have been calculated using the nonrelativistic quark model [14] and the nearest pole dominance has been used to extrapolate them to the required $q^2$ point.

The paper is organized as follows. The kinematics of hyperon decays is presented in section II. In section III we discuss the effective Hamiltonian together with the quark level matrix elements and the numerical values of the Wilson coefficients. Using the factorization ansatz we evaluate the matrix elements in the nonrelativistic quark model in section IV. Section V contains our results and discussions.

2 Kinematics of Hyperon decays

In this section we have presented the kinematics of nonleptonic hyperon decays. The most general Lorentz-invariant amplitude for the decay $\Lambda_b \rightarrow B_f P$ (where $P$ is a pseudoscalar meson) can be written as

$$\mathcal{M}(\Lambda_b \rightarrow B_f P) = i \bar{u}_f(p_f) (A + B \gamma_5) u_{\Lambda_b}(p_i)$$

(2)

where $u_f$ and $u_{\Lambda_b}$ are the Dirac spinors for $B_f$ and $\Lambda_b$ baryons; $A$ and $B$ are parity violating S-wave and parity conserving P-wave amplitudes respectively. The corresponding decay rate ($\Gamma$) and up-down asymmetry parameter are given as [13, 16]

$$\Gamma = \frac{p_c}{8\pi} \left\{ \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right\}$$

$$\alpha = -2\kappa \text{ Re} \left( \frac{A^* B}{|A|^2 + \kappa^2 |B|^2} \right)$$

(3)

where $m_i$, $m_f$ and $m_P$ are the masses of the initial, final baryons and pseudoscalar meson respectively, $p_c$ is the c.m. momentum and $\kappa = p_c/(E_f + m_f) = \sqrt{(E_f - m_f)/(E_f + m_f)}$. 

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For the $\Lambda_b \to B f V$ (where $V$ is the vector meson) decay mode, the general form for the amplitude is given as

$$M(\Lambda_b \to B f V) = \bar{u}_f (p_f) \epsilon^\mu [A_1 \gamma_\mu \gamma_5 + A_2 (p_f)_\mu \gamma_5 + B_1 \gamma_\mu + B_2 (p_f)_\mu] u_{\Lambda_b} (p_i) \tag{4}$$

where $\epsilon^\mu$ is the polarization vector of the emitted vector meson. The corresponding decay rate and asymmetry parameter are given as \[16\]

$$\Gamma = \frac{p_c E_f + m_f}{8\pi m_i} \left\{ 2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2} (|S + D|^2 + |P_1|^2) \right\}$$

$$\alpha = \frac{4m_V^2 \text{Re}(S^* P_2) + 2E_V^2 \text{Re}(S + D)^* P_1}{2m_V^2 (|S|^2 + |P_2|^2) + E_V^2 (|S + D|^2 + |P_1|^2)} \tag{5}$$

with

$$S = -A_1$$
$$D = -\frac{p_c^2}{E_V (E_f + m_f)} (A_1 - m_i A_2)$$
$$P_1 = -\frac{p_c}{E_V} \left( \frac{m_i + m_f}{E_f + m_f} B_1 + m_i B_2 \right)$$
$$P_2 = \frac{p_c}{E_f + m_f} B_1 \tag{6}$$

### 3 Effective Hamiltonian

The effective Hamiltonian $\mathcal{H}_{\text{eff}}$ for the hadronic charmless $\Lambda_b$ decays is given as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{uq} V_{ub}^* \left[ c_1(\mu) O_1^u(\mu) + c_2(\mu) O_2^u(\mu) \right] - V_{tq} V_{tb}^* \sum_{i=3}^{10} c_i(\mu) O_i(\mu) \right\} + \text{h.c.} , \tag{7}$$

where $q = d, s$ and $c_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale $\mu$. The operators $O_{1-10}$ are given as

$$O_1^u = (\bar{u} b)_{V-A} (\bar{q} u)_{V-A} , \quad O_2^u = (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A} ,$$

$$O_{3(5)} = (\bar{q} b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)} ,$$

$$O_{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)} ,$$

$$O_{7(9)} = \frac{3}{2} (\bar{q} b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A(V-A)} ,$$

$$O_{8(10)} = \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)} \tag{8}$$
where $O_{1,2}$ are the tree level current-current operators, $O_{3-6}$ are the QCD and $O_{7-10}$ are the electroweak penguin operators. \langle \bar{q}_1 q_2 \rangle_{(V \pm A)}$ denote the usual $(V \pm A)$ currents. The sum over $q'$ runs over the quark fields that are active at the scale $\mu = O(m_b)$ i.e. ($q' \in u, d, s, c, b$). The Wilson coefficients depend (in general ) in the renormalization scheme and the scale $\mu$ at which they are evaluated. In the next to leading order their values obtained in the naive dimensional regularization (NDR) scheme at $\mu = m_b(m_b)$ as [17]

\[
\begin{align*}
  c_1 &= 1.082, \\
  c_2 &= -0.185, \\
  c_3 &= 0.014, \\
  c_4 &= -0.035, \\
  c_5 &= 0.009, \\
  c_6 &= -0.041, \\
  c_7 &= -0.002 \alpha, \\
  c_8 &= 0.054 \alpha, \\
  c_9 &= -1.292 \alpha, \\
  c_{10} &= 0.263 \alpha .
\end{align*}
\]

However the physical matrix elements [B_M | \mathcal{H}_{\text{eff}} | \Lambda_b] are obviously independent of both scheme and the scale. Hence the dependence in the Wilson coefficients must be cancelled by the corresponding scheme and scale dependence of both scheme and the scale. Hence the dependence on the Wilson coefficients, $c_i^{\text{eff}}$, which are scheme and scale independent i.e.,

\[\langle \bar{q} \gamma \mu \mathcal{H}_{\text{eff}} | b \rangle = \sum_{i,j} c_i^{\text{eff}}(\mu) \langle \bar{q} \gamma \mu | O_j | b \rangle^{\text{tree}}.\]

The effective Wilson coefficients $c_i^{\text{eff}}(\mu)$ may be expressed as [2-6]

\[
\begin{align*}
  c_1^{\text{eff}} |_{\mu = m_b} &= c_1(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu), \\
  c_2^{\text{eff}} |_{\mu = m_b} &= c_2(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu), \\
  c_3^{\text{eff}} |_{\mu = m_b} &= c_3(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu) - \frac{\alpha_s}{24\pi} (C_t + C_p + C_g), \\
  c_4^{\text{eff}} |_{\mu = m_b} &= c_4(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu) + \frac{\alpha_s}{8\pi} (C_t + C_p + C_g), \\
  c_5^{\text{eff}} |_{\mu = m_b} &= c_5(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu) - \frac{\alpha_s}{24\pi} (C_t + C_p + C_g), \\
  c_6^{\text{eff}} |_{\mu = m_b} &= c_6(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu) + \frac{\alpha_s}{8\pi} (C_t + C_p + C_g), \\
  c_7^{\text{eff}} |_{\mu = m_b} &= c_7(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu) + \frac{\alpha_s}{8\pi} C_e, \\
  c_8^{\text{eff}} |_{\mu = m_b} &= c_8(\mu) + \frac{\alpha_s}{4\pi} \left( \gamma^{(0)T} \ln \frac{m_b}{\mu} + \tilde{r}^T \right) c_i(\mu), \\
\end{align*}
\]
where $\hat{r}^T$ and $\gamma^{(0)T}$ are the transpose of the matrices $\hat{r}$ and $\gamma^{(0)}$ arise from the vertex corrections to the operators $O_1 - O_{10}$ derived in [13], which are explicitly given in Ref. [6].

The quantities $C_t$, $C_p$ and $C_g$ are arising from the penguin type diagrams of the operators $O_{1,2}$, the penguin type diagrams of the operators $O_3 - O_6$ and the tree level diagrams of the dipole operator $O_g$ respectively which are given in the NDR scheme (after $\overline{\text{MS}}$ renormalization) by

\begin{align}
C_t &= -\left(\frac{\lambda_u}{\lambda_t} \tilde{G}(m_u) + \frac{\lambda_c}{\lambda_t} \tilde{G}(m_c)\right) c_1 \\
C_p &= \tilde{G}(m_q) + \tilde{G}(m_b) + \sum_{i=u,d,s,c,b} \tilde{G}(m_i)(c_4 + c_6) \\
C_g &= -\frac{2m_b}{\sqrt{\langle k^2 \rangle}} c_g^{\text{eff}}, \quad c_g^{\text{eff}} = -1.043 \\
\tilde{G}(m_q) &= \frac{2}{3} - G(m_q, k, \mu) \\
G(m, k, \mu) &= -4 \int_0^1 dx \, x(1-x) \ln \left( \frac{m^2 - k^2 x(1-x)}{\mu^2} \right), \quad (13)
\end{align}

(12)

It should be noted that the quantities $C_t$, $C_p$ and $C_g$ depend on the CKM matrix elements, the quark masses, the scale $\mu$ and $k^2$, the momentum transferred by the virtual particles appearing in the penguin diagrams. In the factorization approximation there is no model independent way to keep track of the $k^2$ dependence; the actual value of $k^2$ is model dependent. From simple kinematics of charmless nonleptonic $B$ decays [18] one expects $k^2$ to be typically in the range

\begin{equation}
\frac{m_b^2}{4} \leq k^2 \leq \frac{m_b^2}{2}. \quad (14)
\end{equation}

Since the branching ratios depend crucially on the parameter $k^2$, here we would like to take a specific value for it from the above mentioned range. Here we will use for two-body penguin induced decays $\Lambda_b \to B_f \, M$ as done for the charmless $B \to PP$ decays [19]. Assuming that in the rest frame of the $\Lambda_b$ baryon, the spectator diquarks both in the initial and final baryon have negligible momentum and the momentum shared equally between the
two quarks of the emitted meson, the average momentum transfer for \( b \to qu\bar{u} \) transitions \((q=d\) for \( \Lambda_b \to p(\pi/\rho) \) and \( q=s\) for \( \Lambda_b \to p(K/K^*) \) and \( \Lambda(\pi/\rho) \) transitions) is given as
\[
\langle k^2 \rangle = m_b^2 + m_q^2 - 2m_b E_q ,
\]
(15)
The energy \( E_q \) of the \( q\)-quark in the final meson is determinable from
\[
E_q + \sqrt{E_q^2 - m_q^2 + m_u^2 + \sqrt{4(E_q^2 - m_q^2) + m_u^2}} = m_b ,
\]
(16)
where \( m_b \), \( m_q \) and \( m_u \) denote the masses of the decaying \( b\)-quark, daughter \( q\)-quark and the \( u\)-quark created as \( u\bar{u}\) pair from the virtual gluon, photon or \( Z \) particle in the penguin loop.

For numerical calculation we have taken the CKM matrix elements expressed in terms of the Wolfenstein parameters with values \( A = 0.815 \), \( \lambda = \sin \theta_c = 0.2205 \), \( \rho = 0.175 \) and \( \eta = 0.37 \) [6]. Using the mass renormalization equations with three loop \( \beta \) function, the values of the current quark masses are evaluated at various energy scales in Ref. [20]. Since the energy released in the decay mode \( \Lambda_b \to p\pi^- \) is of the order of \( m_b \), we take the current quark mass values at scale \( \mu \sim m_b \) from [20] as: \( m_u(m_b) = 3.2 \) MeV, \( m_d(m_b) = 6.4 \) MeV, \( m_s(m_b) = 90 \) MeV, \( m_c(m_b) = 0.95 \) GeV and \( m_b(m_b) = 4.34 \) GeV. Thus we obtain \( k^2/m_b^2 = 0.5 \) for \( b \to du\bar{u} \) transitions and \( k^2/m_b^2 = 0.499 \) for \( b \to su\bar{u} \) transitions. Using these values of \( k^2 \) the estimated values of the effective renormalization scheme and scale independent Wilson coefficients for \( b \to d \) and \( b \to s \) transitions are given in Table-1.

4 Evaluation of the matrix elements

After obtaining the effective Wilson coefficients now we want to calculate the matrix element \( \langle B_f M|O_i|\Lambda_b \rangle \) where \( O_i \) are the four quark current operators listed in eqn. (8), using the factorization approximation. In this approximation, the hadronic matrix elements of the four quark operators \( (\bar{u}b)(V-A)(\bar{q}u)(V-A) \) split into the product of two matrix elements, \( \langle M|(\bar{q}u)(V-A)|0 \rangle \) and \( \langle B_f|(\bar{u}b)(V-A)|\Lambda_b \rangle \) where Fierz transformation has been used so that flavor quantum numbers of the currents match with those of the hadrons. Since Fierzing yield operators which are in the color singlet-singlet and octet-octet forms, this procedure results in general the matrix elements which have the right flavor quantum numbers but involve both singlet-singlet and octet-octet current operators. However, there is no experimental information available for the octet-octet part. So in the factorization approximation, one discards the color octet-octet piece and compensates this by treating \( N_c \), the number of colors as a free parameter, and its value is extracted from the data of two body nonleptonic decays.
The matrix elements of the \((V - A)(V + A)\) operators i.e. \((O_6 \& O_8)\) can be calculated as follows. After Fierz ordering and factorization they contribute as \([21]\)

\[
\langle B_f | M | O_6 | A_b \rangle = -2 \sum_{q'} \langle M | \bar{q}(1 + \gamma_5)q'|0 \rangle \langle B_f | \bar{q}'(1 - \gamma_5)b|A_b \rangle
\]  

(17)

Using the Dirac equation the matrix element can be rewritten as

\[
\langle B_f | M | O_6 | A_b \rangle = - \left[ R_1 \langle B_f | V_\mu | A_b \rangle - R_2 \langle B_f | A_\mu | A_b \rangle \right] \langle M | (V - A)_\mu | 0 \rangle ,
\]  

(18)

with

\[
R_1 = \frac{2m_M^2}{(m_b - m_u)(m_q + m_u)} , \quad R_2 = \frac{2m_M^2}{(m_b + m_u)(m_q + m_u)} ,
\]  

(19)

where the quark masses are the current quark masses. The same relation works for \(O_8\).

Thus under the factorization approximation the baryon decay amplitude is governed by a decay constant and baryonic transition form factors. The general expression for the baryon transition is given as

\[
\langle B_f(\bar{p}_f)|V_\mu - A_\mu|A_b(\bar{p}_i)\rangle = \bar{u}_{B_f}(\bar{p}_f) \left\{ f_1(q^2)\gamma_\mu + if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu \right\} u_{A_b}(\bar{p}_i)
\]  

(20)

where \(q = p_i - p_f\). In order to evaluate the form factors at maximum momentum transfer, we have employed nonrelativistic quark model \([14]\), where they are given as:

\[
f_1(q_m^2)/N_{fi} = 1 - \frac{\Delta m}{2m_i} + \frac{\Delta m}{4m_i m_q} \left(1 - \frac{\Lambda_b}{2m_f}\right) (m_i + m_f - \eta \Delta m)
\]

\[- \frac{\Delta m}{8m_i m_f m_Q} \bar{\Lambda} (m_i + m_f - \eta \Delta m)\]

\[
f_3(q_m^2)/N_{fi} = \frac{1}{2m_i} - \frac{1}{4m_i m_f} (m_i + m_f - \eta \Delta m) - \frac{\bar{\Lambda}}{8m_i m_f m_Q} [(m_i + m_f) \eta + \Delta m]
\]

\[
g_1(q_m^2)/N_{fi} = \eta + \frac{\Delta m \bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q}\right) \eta
\]

\[
g_3(q_m^2)/N_{fi} = - \frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q}\right) \eta
\]  

(21)

with \(\bar{\Lambda} = m_f - m_q\), \(\Delta m = m_i - m_f\), \(q_m^2 = \Delta m^2\), \(\eta = 1\), \(m_Q\) and \(m_q\) are the constituent quark masses of the interacting quarks of initial and final baryons.
with values $m_u=338$ MeV, $m_d=322$ MeV, $m_s=510$ MeV and $m_b=5$ GeV. $N_{fi}$ is the flavor factor:

$$N_{fi} = \text{flavor spin } \langle p | b_i^* b_b | \Lambda_b \rangle \text{flavor spin},$$

which is equal to $1/\sqrt{2}$ for $\Lambda_b \to p$ and $1/\sqrt{3}$ for $\Lambda_b \to \Lambda$ transitions. Since the calculation of $q^2$ dependence of form factors is beyond the scope of the nonrelativistic quark model we will follow the conventional practice to assume a pole dominance for the form factor $q^2$ behaviour as

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_V^2)^2}, \quad g(q^2) = \frac{g(0)}{(1 - q^2/m_A^2)^2}$$

where $m_V(m_A)$ is the pole mass of the vector (axial vector) meson with the same quantum number as the current under consideration. The pole masses are taken as $m_V = 5.32(5.42)$ GeV and $m_A = 5.71(5.86)$ GeV for $b \to d(b \to s)$ transitions. Assuming a dipole $q^2$ behaviour for form factors, and taking the masses of the particles from Ref. [9] the obtained values of the form factors at zero momentum transfer are given in Table-2.

The matrix element $\langle M | (V - A)_\mu | 0 \rangle$ is related to the decay constants of the charged pseudoscalar and vector mesons $f_P$ and $f_V$ as

$$\langle 0 | A_\mu | P(q) \rangle = i f_P q_\mu, \quad \langle 0 | A_\mu | V(\epsilon, q) \rangle = f_V m_V \epsilon_\mu$$

The decay constants for the neutral mesons (i.e. $\pi^0$ and $\rho^0$) are taken to be $1/\sqrt{2}$ times that of the corresponding charged mesons. Thus we obtain the transition amplitudes for various $\Lambda_b \to B_f P$ decay modes as given below.

### 4.1 $\Lambda_b \to B_f P$ transitions

1. $\Lambda_b \to p\pi^-$

Since in this case the final state has isospin $I_f = 3/2, 1/2$ we have $\Delta I = 3/2$ and $1/2$. From the flavor-flow topologies for $b \to d u \bar{u}$ transitions, we find that the isospin decomposition of the effective Hamiltonian as follows: the tree diagrams have $\Delta I = 3/2, 1/2$, the QCD penguins $\Delta I = 1/2$ and the electroweak penguins $\Delta I = 3/2, 1/2$ components. Hence both tree and penguins (QCD as well as the electroweak penguins) contribute to this channel and hence the amplitude is given as

$$M(\Lambda_b \to p\pi^-) = i \frac{G_F}{\sqrt{2}} f_\pi \bar{u}_p(p_f) \left\{ V_{ub} V_{ud}^* a_1 - V_{ib} V_{td}^* \left( a_4 + a_{10} + (a_6 + a_8) R_1 \right) \right\}$$

$$\times \left[ f_1(m_\pi^2)(m_i - m_f) + f_3(m_\pi^2)m_\pi^2 \right]$$

$$+ \left\{ V_{ub} V_{ud}^* a_1 - V_{ib} V_{td}^* \left( a_4 + a_{10} + (a_6 + a_8) R_2 \right) \right\}$$

$$\times \left[ g_1(m_\pi^2)(m_i + m_f) - g_3(m_\pi^2)m_\pi^2 \gamma_5 \right] u_\Lambda_b(p_i).$$ 

(25)
2. **Λ̅b → pK⁻**

It can be seen from the flavor-flow topologies for \( b \to su \bar{u} \) transitions that the effective Hamiltonian has isospin components as: the tree diagram with \( \Delta I = 1, 0 \), the QCD penguins with \( \Delta I = 0 \) and the electroweak penguins with \( \Delta I = 1, 0 \) components. Since the final state (pK) has isospin 1 and 0, we have \( \Delta I = 1, 0 \) for this process. Thus we find that tree, QCD as well as the electroweak penguins will contribute to this channel and obtain the amplitude as

\[
M(\Lambda̅b \to pK⁻) = iG_F \sqrt{2} f_K \bar{u}_p(p_f) \left\{ V_{ub} V_{us}^* \left( a_1 - V_{tb} V_{ts}^* (a_4 + a_{10} + (a_6 + a_8) R_1) \right) \right. \\
\times \left. \left( f_1(m_K^2)(m_i - m_f) + f_3(m_K^2)m_K \right) \right. \\
+ \left. \left. \left( V_{ub} V_{us}^* \left( a_1 - V_{tb} V_{ts}^* (a_4 + a_{10} + (a_6 + a_8) R_2) \right) \right) \right. \\
\times \left. \left. \left( g_1(m_K^2)(m_i + m_f) - g_3(m_K^2)m_K \right) \gamma_5 \right) \right. u_{\Lambda^b}(p_i) .
\] (26)

3. **Λb → Λπ⁰**

For \( \Lambda_b \to \Lambda\pi^0 \) we have only \( \Delta I = 1 \) and from the flavor-flow diagrams for \( b \to su \bar{u} \) processes, we find that only the tree and electroweak penguins will contribute to this channel.

\[
M(\Lambda_b \to \Lambda\pi^0) = iG_F f_\pi \bar{u}_\Lambda(p_f) \left\{ V_{ub} V_{us}^* \left( a_2 - V_{tb} V_{ts}^* \left( \frac{3}{2}(a_9 - a_7) \right) \right) \right. \\
\times \left. \left[ \left( f_1(m_\rho^2)(m_i - m_f) + f_3(m_\pi^2)m_\pi \right) \right. \\
\left. \left. + \left( g_1(m_\rho^2)(m_i + m_f) - g_3(m_\rho^2)m_\pi \right) \gamma_5 \right) \right. u_{\Lambda_b}(p_i) .
\] (27)

4.2 **Λb → BfV transitions**

Here we obtain the transition amplitudes for \( \Lambda_b \to B_f V \) decay channels. As seen from the flavor-flow diagrams for \( \Lambda_b \to B_f P \) processes, in this case also the \( \Lambda_b \to p\rho \) and \( pK^* \) receive contributions from tree as well as QCD and electroweak penguins where as \( \Lambda_b \to \Lambda \rho \) has only tree and electroweak penguin contributions. Thus we obtain the corresponding transition amplitudes as follows.

1. **Λb → pρ⁻**

\[
M(\Lambda_b \to p\rho⁻) = G_F \sqrt{2} f_\rho m_\rho e^{i\mu} \bar{u}_p(p_f) \left\{ V_{ub} V_{ud}^* \left( a_1 - V_{tb} V_{td}^* (a_4 + a_{10}) \right) \right. \\
\times \left. \left( f_1(m_\rho^2) - f_2(m_\rho^2)(m_i + m_f) \right) \gamma_\mu + 2 f_2(m_\rho^2)(p_f)_\mu \right) \] (28)
the others depend strongly on it. Therefore for charmless 

\[ N \]

amplitudes depend dominantly on 

\[ a \]

coefficients given as 

\[ b \]

can be seen that the coefficients 

\[ c \]

and 

\[ d \]

for the effective number of colors.

\[ \varepsilon \]

decays of \( \Lambda \)

\[ e \]

implies 

\[ f \]

can be reliably predicted within factorization approach even in the absence of 

\[ g \]

current-current amplitudes, 

\[ h \]

a recent analysis of \( B \rightarrow D \pi \) data gives \( N_{eff} \sim 2 \) [24]. On the other hand Mannel et al [24] have used \( N_{eff} = \infty \) to study the nonleptonic 

\[ i \]

decays of \( \Lambda_b \) baryon. So here we have taken three sets of values i.e., 2, 3 and \( j \) for the effective number of colors.

\[ k \]

It should be noted from Table-1 that the dominant coefficients are \( a_1, a_2 \) for current-current amplitudes, \( a_4 \) and \( a_6 \) for QCD penguin induced amplitudes and \( a_9 \) for electroweak penguin induced amplitudes. Furthermore, it can also be seen that the coefficients \( a_1, a_4, a_6 \) and \( a_9 \) are in general \( N_{eff} \) stable, whereas the others depend strongly on it. Therefore for charmless \( b \) decays whose amplitudes depend dominantly on \( N_{eff} \) stable coefficients, their decay rates can be reliably predicted within factorization approach even in the absence of information on non-factorizable effects.

\[ l \]

- \( \left( g_1(m^2) + g_2(m^2)(m_i - m_f) \right) \gamma_{\mu} + 2g_2(m^2)(p_f)\gamma_5 \right] u_{\Lambda_b}(p_i) \cdot \]

\[ m \]

2. \( \Lambda_b \rightarrow pK^{*-} \)

\[ n \]

(29)

\[ o \]

(30)

\[ p \]

The coefficients \( a_1, a_2 \cdots a_{10} \) are combinations of the effective Wilson coefficients given as 

\[ q \]

(31)

where \( N_{eff} \) is the effective number of colors treated as free parameter in order to model the nonfactorizable contributions to the matrix elements and its value can be extracted from the two body nonleptonic \( B \) decays. Naive factorization implies \( N_c = 3 \). A recent analysis of \( B \rightarrow D \pi \) data gives \( N_{eff} \sim 2 \) [24]. On the other hand Mannel et al [24] have used \( N_{eff} = \infty \) to study the nonleptonic 

\[ r \]

decays of \( \Lambda_b \) baryon. So here we have taken three sets of values i.e., 2, 3 and \( s \) for the effective number of colors.

\[ t \]

It should be noted from Table-1 that the dominant coefficients are \( a_1, a_2 \) for current-current amplitudes, \( a_4 \) and \( a_6 \) for QCD penguin induced amplitudes and \( a_9 \) for electroweak penguin induced amplitudes. Furthermore, it can also be seen that the coefficients \( a_1, a_4, a_6 \) and \( a_9 \) are in general \( N_{eff} \) stable, whereas the others depend strongly on it. Therefore for charmless \( b \) decays whose amplitudes depend dominantly on \( N_{eff} \) stable coefficients, their decay rates can be reliably predicted within factorization approach even in the absence of information on non-factorizable effects.

\[ u \]

\[ v \]

\[ w \]

\[ x \]

\[ y \]

\[ z \]
4.3 Classification of the factorized amplitudes

Applying the effective Hamiltonian (7), the factorizable decay amplitudes for \( \Lambda_b \to B_f M \) decay processes obtained within the generalized factorization approach are given in eqns (25-30). In general the two body charmless \( B \) meson decays are classified into six classes:

- Class-I decay modes dominated by the external W-emission characterized by the parameter \( a_1 \).
- Class-II decay modes dominated by the color suppressed internal W-emission characterized by the parameter \( a_2 \).
- Class-III decays involving both external and internal W-emissions described by \( a_1 + r a_2 \).
- Class-IV decays are dominated by the QCD penguin parameter \( a_4 + Ra_6 \).
- Class-V modes are those whose amplitudes are governed by the effective coefficients \( a_3, a_5, a_7 \) and \( a_9 \).
- Class-VI modes involving the interference of \( a_{even} \) and \( a_{odd} \).

Assuming the same classification for charmless \( \Lambda_b \) decays we now find the classes for the decay processes we are interested in:

a. \( \Lambda_b \to p(\pi/\rho) \)

These decays proceed at the tree level through \( b \to u \bar{u} d \) and at the loop level via \( b \to d \) penguins. Since in terms of the Wolfenstein parametrization,

\[
V_{ub}V_{ud}^* \simeq A\lambda^3(\rho - i\eta), \quad V_{tb}V_{td}^* \simeq A\lambda^3(1 - \rho + i\eta)
\]

are of the same order of magnitude, it is clear that these decays are tree dominated as the penguin contributions are suppressed by the smallness of penguin coefficients. Hence these decay modes belong to Class-I category.

b. \( \Lambda_b \to p(K/K^*) \)

These decays proceed at the tree level through \( b \to u \bar{u} s \) and via \( b \to s \) penguins. In this case

\[
V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta), \quad V_{tb}V_{ts}^* = -A\lambda^2,
\]

the magnitude of \( V_{tb}V_{ts}^* \) is approximately \((10^2)\) times larger than that of \( V_{ub}V_{us}^* \). Hence these processes are dominated by the QCD penguin coefficients and belong to Class-IV.

c. \( \Lambda_b \to \Lambda(\pi/\rho) \)

These decays proceed at the tree level through internal W emission \( b \to u \bar{u} s \) and via \( b \to s \) electroweak penguins. Since the magnitude of \( V_{tb}V_{ts}^* \) is larger than \( V_{ub}V_{us}^* \), these decays are dominated by the electroweak penguins and belong to Class-V. Since the electroweak coefficients are smaller than those of tree and QCD penguin coefficients, the branching ratios for these type transitions are in general smaller than the other decay modes that we have considered.
5 Results and Discussions

After obtaining the transition amplitudes for various decay processes we now proceed to estimate their branching ratios and asymmetry parameters. Comparing the evaluated transition amplitudes for $\Lambda_b \rightarrow B_f M$ processes (eqns. (25-30)) with the corresponding generalized amplitudes given in eqns. (2,4) one can easily determine the coefficients $A$, $B$, $A_1$, $A_2$, $B_1$ and $B_2$. Hence the branching ratios and asymmetry parameters are estimated with eqns. (3,5,6). Using the various pseudoscalar and vector meson decay constants (in MeV) as $f_\pi = 130.7$, $f_K = 159.8$, $f_K^* = 221$ and $f_\rho = 216$, the estimated branching ratios and asymmetry parameters are presented in Tables-3 and 4 respectively for three different sets of effective number of colors. It is seen that the branching ratios are maximum for $N_{c_{eff}} = \infty$, however $\alpha$ is stable for all three sets. The estimated branching ratios for $\Lambda_b \rightarrow p\pi$ and $pK$ for all three sets of $N_{c_{eff}}$ lies below the present experimental upper limit $BR(\Lambda_b \rightarrow p\pi/pK) < 5 \times 10^{-5}$ [9]. It should also be noted that the decay modes $\Lambda_b \rightarrow \Lambda(\pi/\rho)$ have the smallest branching ratios in comparison to the others. This is so because these decay modes receive contributions from CKM as well as color suppressed tree and electroweak penguin diagrams and moreover they are dominated by the later. It is naively believed that in charmless $b$-decays the contributions from the electroweak penguin diagrams are negligible compared to QCD penguins because of smallness of electroweak Wilson coefficients. Thus the estimated branching ratio for $\Lambda_b \rightarrow \Lambda(\pi/\rho)$ are one order smaller than the $\Lambda_b \rightarrow p(\pi/\rho), p(K/K^*)$ transitions.

To summarize, using the next-to-leading order QCD corrected effective Hamiltonian, we have obtained the branching ratios and asymmetry parameters for the charmless hadronic $\Lambda_b$ decays, within the framework of generalized factorization. The nonfactorizable contributions are parametrized in terms of the effective number of colors $N_{c_{eff}}$. So in addition to the naive factorization approach ($N_{c_{eff}} = 3$), here we have taken two more values for $N_{c_{eff}}$ i.e., $N_{c_{eff}} = 2$ and $\infty$. The baryonic form factors at maximum momentum transfer ($q_m^2$) are evaluated using the nonrelativistic quark model and the extrapolation of the form factors from $q_m^2$ to the required $q^2$ value is done by assuming the pole dominance. The obtained branching ratios for $\Lambda_b \rightarrow p\pi, pK$ processes lie within the present experimental upper limit. Though the branching ratios for these modes are small, they could be accessible at future hadron colliders with large $b$-production. Furthermore, with large data on $\Lambda_b$ baryon expected in the near future, these decay channels will serve as a testing ground to look for CP violation in and beyond Standard Model.

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Table 1: Numerical values of the effective Wilson coefficients $c_{eff}^i$ for $b \rightarrow s$ and $b \rightarrow d$ transitions evaluated at $k^2/m_b^2 = 0.499$ for $b \rightarrow s$ and at $k^2/m_b^2 = 0.5$ for the $b \rightarrow d$ processes.

|     |  $b \rightarrow s$ |  $b \rightarrow d$ |
|-----|-------------------|-------------------|
| $c_1^{eff}$ | 1.168 | 1.168 |
| $c_2^{eff}$ | -0.365 | -0.365 |
| $c_3^{eff}$ | 0.0225+0.0043i | 0.0224+0.0038i |
| $c_4^{eff}$ | -(0.0467 + 0.0129)i | -(0.0454 + 0.0115)i |
| $c_5^{eff}$ | 0.0133+i0.0043 | 0.0131+i0.0038 |
| $c_6^{eff}$ | -(0.0481 + i0.0129) | -(0.0475 + i0.0115) |
| $c_7^{eff}/\alpha$ | -(0.0299 + i0.0356) | -(0.0294 + i0.0329) |
| $c_8^{eff}/\alpha$ | 0.055 | 0.055 |
| $c_9^{eff}/\alpha$ | -(1.4268 + i0.0356) | -(1.426 + i0.0329) |
| $c_{10}^{eff}/\alpha$ | 0.48 | 0.48 |

Table 2: Values of the form factors at zero momentum transfer evaluated using the nonrelativistic quark model

$$
\begin{array}{ccccccc}
\text{Decay process} & f_1(0) & m_i f_2(0) & m_i f_3(0) & g_1(0) & m_i g_2(0) & m_i g_3(0) \\
\Lambda_b \rightarrow p & 0.043 & -0.022 & -0.009 & 0.092 & -0.02 & -0.047 \\
\Lambda_b \rightarrow \Lambda & 0.061 & -0.025 & -0.008 & 0.107 & -0.014 & -0.043 \\
\end{array}
$$

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Table 3: Branching ratios for various charmless $\Lambda_b \rightarrow B_f M$ decay modes.

| Decay processes | $N_{cJ} = 2$ | $N_{cJ} = 3$ | $N_{cJ} = \infty$ | Expt. [9] |
|-----------------|--------------|--------------|-------------------|-----------|
| $\Lambda_b \rightarrow p\pi^-$ | $8.52 \times 10^{-7}$ | $9.29 \times 10^{-7}$ | $11.57 \times 10^{-7}$ | $< 5 \times 10^{-5}$ |
| $\Lambda_b \rightarrow pK^-$ | $1.38 \times 10^{-6}$ | $1.54 \times 10^{-6}$ | $1.87 \times 10^{-6}$ | $< 5 \times 10^{-5}$ |
| $\Lambda_b \rightarrow \Lambda\pi^0$ | $1.2 \times 10^{-8}$ | $1.58 \times 10^{-8}$ | $3.22 \times 10^{-8}$ | - |
| $\Lambda_b \rightarrow p\rho^-$ | $1.22 \times 10^{-6}$ | $1.38 \times 10^{-6}$ | $1.55 \times 10^{-6}$ | - |
| $\Lambda_b \rightarrow pK^{*-}$ | $2.99 \times 10^{-7}$ | $2.71 \times 10^{-7}$ | $4.075 \times 10^{-7}$ | - |
| $\Lambda_b \rightarrow \Lambda\rho^0$ | $1.93 \times 10^{-8}$ | $2.52 \times 10^{-8}$ | $5.1 \times 10^{-8}$ | - |

Table 4: Asymmetry parameter ($\alpha$) for various charmless $\Lambda_b \rightarrow B_f M$ decay modes.

| Decay processes | $N_{cJ} = 2$ | $N_{cJ} = 3$ | $N_{cJ} = \infty$ |
|-----------------|--------------|--------------|------------------|
| $\Lambda_b \rightarrow p\pi^-$ | -0.77 | -0.77 | -0.77 |
| $\Lambda_b \rightarrow pK^-$ | -0.77 | -0.77 | -0.77 |
| $\Lambda_b \rightarrow \Lambda\pi^0$ | -0.89 | -0.89 | -0.89 |
| $\Lambda_b \rightarrow p\rho^-$ | -0.71 | -0.71 | -0.71 |
| $\Lambda_b \rightarrow pK^{*-}$ | -0.68 | -0.68 | -0.68 |
| $\Lambda_b \rightarrow \Lambda\rho^0$ | -0.78 | -0.78 | -0.78 |