Construction of an axonometric outline of a curved surface of a part

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Abstract. The article considers the geometric construction of the orthogonal isometry of a curved surface. To solve this problem, the authors propose to perform additional construction of the contour of the section of the surface in the orthogonal drawing and use the coordinates of the points of this section to construct an axonometric outline of the surface. The mismatch between the axonometric contour of the rotation surface and the corresponding contour of this surface in the projection drawing is illustrated. As a result, the proposed solution allows geometrically accurate construction of support and additional points of the axonometric contours of the revolution surfaces.

1. Introduction
The article discusses solutions to geometric problems associated with the construction of an axonometric drawing of curved surfaces. It is recommended to solve this problem in a certain sequence: first, on the orthogonal drawing to build a line of an axonometric surface contour, and already on its basis to build a outline of a curved surface in an axonometry. Such tasks are encountered in the process of developing architectural and design projects [1-8]. New approaches to their solution are relevant. Despite the introduction of modern computer technology, the a priori value of manual modeling of the architectural form based on axonometric projection as a process ahead of the emergence of a “conceivable concept” remains [4].

It is known that an axonometric drawing, in other words “axonometry”, is geometrically constructed according to a projection orthogonal drawing [9, 10]. The basis of the method for constructing axonometric images is that the object together with its coordinate system is projected parallel to the axonometry plane. The basic concepts of these geometric constructions are well known for a long time [7-9, 12]. The shape of the part can be different. It can be multifaceted, curved, or combine both surfaces.

2. Materials and methods
Any axonometric image of an object is performed by constructing its individual points [11]. Building polyhedral surfaces based on their specified geometric shape elements (vertices, faces, edges) does not cause difficulties. Axonometric projections are based on coordinates taken from an orthogonal drawing[9, 10, 14]. We can say that elements of an orthogonal drawing are transformed into elements of an axonometric drawing. In Figure 1, the projections of the pyramid vertices in axonometry are based on coordinates taken from the orthogonal drawing.
The construction of axonometry of curved surfaces is somewhat more complicated. Before proceeding to the solution of such problems, we first consider some features of the projection process associated with the concept of "surface outline". Obtaining an axonometric drawing is achieved in stages using two wrapping projecting surfaces of different projection directions [12].

It is believed that the projection drawing of the part itself is the result of the intersection of the wrapping, projecting surface with the projection plane. Recall that in orthogonal projection, the wrapping surface is a set of projecting lines that are tangent to the surface of the part and perpendicular to the plane of projections. The direction of projection is equal to the angle of 90 degrees. The line of contact of the wrapping projection surface with the surface of the part is called the visible contour line and its orthogonal projection on the projection plane is called the surface outline, or the visible contour projection. Note that this contour, which belongs to the orthogonal drawing, does not directly participate in the construction of the axonometric outline of the curved surface. The following arguments lead to this conclusion.

The axonometric projection of a curved surface is formed according to the same law as the orthogonal one. That is, the axonometric drawing of the part is the result of the intersection of the newly formed wrapping projection surface with the axonometric plane. To clarify, the newly formed enveloping projection surface is a set of projecting lines that are tangent to the surface of the part and parallel to the direction of axonometric projection. As is known, for orthogonal isometry, the projections of the axonometric projection direction are located at an angle of 45 degrees to the projections of the coordinate axes [9, 12].

It is easy to predict that the expected line of contact of the new wrapping projection surface with the surface of the part, the so-called visible contour line, is not present in the orthogonal drawing. This is due to the fact that there was no need to portray it. But now this complicates or makes it impossible to build its axonometric projection, i.e. the shape of the surface. In other words, there is no contour on an orthogonal drawing that can be used to draw an outline of a curved surface in axonometry. It follows that in order to solve the main problem, it is necessary to add a new visible contour line to the orthogonal drawing in accordance with the chosen direction of axonometric projection.

Consider a possible solution to this problem. As can be seen in Figure 2, the orthogonal drawing of the surface of revolution contains a base - this is a parallel, and the outline is the main meridian. The direction of the axonometric projection $S$ is selected in accordance with the direction of projection of the orthogonal isometry. We believe that the new “wrapping ray surfaces” touches a given surface of
revolution along a plane curve. This flat curve is the line of the visible contour. We construct it in the
drawing as a line of a section of given revolution surface. First we define the determinant of the
section plane. Obviously, it can be defined by three points A, B, C. Each of these points is the touch
point of the projecting lines of the wrapping surface with the specified rotation surface. As can be seen
on the horizontal projection of the orthogonal surface drawing, points A, B lie at the intersection of the
parallel and the meridian does not require additional construction. To set point C in the drawing, an
additional geometric solution is required. From Figure 2 we can see that the point of contact of the
projecting straight line with the given surface of revolution must lie in the axial horizontal projecting
plane. The meridian of the revolution surface and the tangent projection straight line belong to this
plane.

![Figure 2. Construction of a line of touch of the enveloping projection surface with the surface of rotation](image)

Next, we show how to find the desired point of tangency C. Note that all constructions associated
with finding this point must be solved in the level plane. Therefore, we turn the meridian with the
expected point C around the axis of the surface of revolution to the position of the main meridian. On
the horizontal projection (Picture 2), point 1 should be turned to point 2 in parallel. Then (this can be
seen in the frontal projection of the drawing) in the plane of the main meridian we draw a straight
tangent to the meridian and parallel to the direction of the axonometric projection. The angle of this
straight line is 35 degrees. The tangent point (C) is constructed as the intersection point of two
mutually perpendicular lines. One of which is a direct tangent to the meridian, and the second passes
through the center of the arc of the meridian. Now it remains to turn the meridian back to its original
position, and with it, the tangent point C should be rotated to the original position of the meridian.

Thus, in the drawing, three points A, B, C are defined. They belong to the secant plane. These
points are points of visibility, since they belong to the outline of the surface of revolution during its
axonometric projection. That is, these three constructed reference points define the line of touch, or the line of the visible contour in the drawing. Next, you need to determine the intermediate points of this line. Since the construction of axonometry requires the natural value of the coordinates of the points, it is therefore rational to solve this problem by applying methods of transforming the drawing. We use the method of replacing projection planes. An additional vertical projection plane is located perpendicular to the secant plane (A, B, C) and parallel to the axial, horizontally projecting plane (the meridian of the surface of revolution, the tangent projection line and the tangent point C belong to this plane). We use the method of replacing projection planes. An additional vertical projection plane is located perpendicular to the secant plane (A, B, C) and parallel to the axial horizontally projecting plane (the meridian of the revolution surface, the tangent projection line and the tangent point C belong to this plane). To solve the main problem, the meridian is projected in full size on the additional projection plane, and the secant plane is projected by a degenerate projection. A further solution is easy to see in the drawing. The desired line of the visible contour is defined on the frontal and horizontal projection planes by three reference points (A, B, C).

Next, you need to determine the intermediate points of this line. Therefore, on the additional projection plane, we construct a series of lines parallel to the horizontal projection plane. These are degenerate projections of parallels. The next step is to find their intersection points with a degenerate secant plane.

These points (3, 4), like points (A, B, C) belong to the parallels of a given surface of revolution. We complete the missing projections of these points and construct their smooth contour on the frontal and horizontal projection planes. Thus, the resulted drawing is defined by the initial contours of the surface of revolution under consideration and supplemented by a smooth contour of the points (A, B, C, 3, 4). These points are points of visibility and belong to the outline of the surface of revolution during its axonometric projection. This can be seen on the additional projection plane in the direction of axonometric projection.

For greater clarity, the parallels meridians are plotted on an axonometric drawing and a cut is made. Additionally, an isometry of the meridian is constructed and it passes through the top of the surface of revolution and points A, B.

3. Results
Now, according to this drawing, we construct the orthogonal isometry of the surface of revolution. Figure 3 shows an isometry of the base of the revolution surface in the form of an ellipse and the outline of the surface of revolution. The construction of an isometric surface shape is performed according to the coordinates of the smooth contour of the points (A, B, C, 3, 4).

![Figure 3. Construction of an axonometric outline of the revolution surface](image-url)
4. Conclusion
Analysis of the image of these two contours on an isometry allows us to conclude that the use of an additional circuit specially constructed on an orthogonal drawing makes it possible to most accurately convey the outline shape of a curved surface in axonometry.

References
[1] Bolshakova M S 2018 Axonometric constructions in graphic design Visual Culture: Design, Advertising.Inf. Technologies: Materials of the XVII Vseros. Scientific Practical conf. [Omsk: Omsk State Technical University Publishing House] p 15
[2] Eliseev N A 2013 Development of the axonometry in works of scientists of petersburg XXX Symposium of the Intern. Committee for the History of Technology [M.-SPb.: ICONTEC] p 9
[3] Efimov D I 2014 E-learning in continuing education [Ulyanovsk: Publishing house: Ulyanovsk State Technical University Number] pp 263-264
[4] Malakhov S, Boranov S 2017 Axonometry role as a model motivating the archofunction J.Innovative Project B 2 3 (7) [Samara: State Technical University] p 67
[5] Matveeva M, Kalyuzhnova N and Violin S 2019 IOP Conf. Series: Materials Science and Engineering. 667 doi:10.1088/1757-899X/667/1/012036 6.
[6] Peshkov V V 2019 DOI: 04010 (https://doi.org/10.1051/matecconf/201821204010)
[7] Puras I Y 2016 Modern trends in graphic design Business and Design Review B 1 (2) p 8 3
[8] Rauschenbakh B V 2001 Geometry of a Picture and Visual Perception (SPb.: ABC-Classic) p 320
[9] Chetverukhin N F 1963 Descriptive Geometry (M.: Higher School) pp 255 – 372
[10] Glazunov EA, Chetverukhin N.F. 1953 Axonometry (Moscow: State Publishing House of Technical and Theoretical Literature) p. 292
[11] Gordon V O, Sementsov-Ogievsky M A 2009 Course Descriptive Geometry (Moscow: High School) p 272 p.
[12] Klimukhin A G 1963 Descriptive Geometry (Moscow: Stroyizdat) p 368
[13] Klimukhin A G 2007 Descriptive Geometry (M: Architecture) p 333
[14] Koroev Y I 2015 Descriptive Geometry (Moscow: KNORUS) p 423
[15] Kravcova L I, Kostrubova I I and Smolkova E F 2014 Technical drawing (Irkutsk: ISTU) p 88