Phenomenological Estimation of Parameters in minimal Supergravity Model

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Abstract

Several parameters of the minimal supergravity model are estimated by the method of the renormalization group. In this model, five arbitrary parameters \( (m_0, m_{1/2}, \tan \beta, A_0, \text{sig}(\mu)) \) are contained. \( \tan \beta \) is evaluated as 7.5 by the likelihood analysis of the gauge and Yukawa coupling constants. Further, the trilinear coupling constant \( A_0 \) is fixed by the equation of \( B_0 = (A_0 - 1)m_0 \) at the GUT scale \( M_X \approx 2 \times 10^{16} \text{(GeV)} \) and by the Higgs potential at the Z-boson mass scale \( M_Z = 91 \text{(GeV)} \). As the results, (1) allowed \( m_0 - m_{1/2} \) regions, (2) the sparticle mass spectra, and (3) the lightest supersymmetric particle mass are shown in this paper.

1 Introduction

In the minimal supergravity model (mSUGRA), five arbitrary parameters \( (m_0, m_{1/2}, \tan \beta, A_0, \text{sig}(\mu)) \) are included. If these parameters are specified, all the mass of the particles are derived by the analysis of renormalization group flow. Recently, the mass of Standard model (SM) Higgs boson has been evaluated by LEP. The reported Higgs mass is slight smaller than expected. This fact encourages the mSUGRA, because such light value of the Higgs mass is supported as one of the features of this model. If the mass of the lightest neutral Higgs boson in mSUGRA is close to the mentioned above the mass of Higgs boson in the SM, a constraint condition among \( m_0, m_{1/2} \) and \( \tan \beta \) is obtained. In order to determine their values completely, some additional conditions are introduced as follows:

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i) Assuming the SU(5) GUT scenario, the gauge and the Yukawa coupling constants of D-type quarks and leptons are unified at GUT scale \( M_X \), respectively [4].

ii) As one of the conditions of mSUGRA, \( B_0 \) is equal to \((A_0 - 1)m_0 \) at \( M_X \) [5]. At \( M_Z \) scale, two different \( B(M_Z) \) values are obtained; (a) by RGE development from \( M_X \) scale, and (b) by the Higgs potential at \( M_Z \) scale directly. \( A_0 \) is tuned numerically to make both \( B(M_Z) \) values match from each other.

iii) The condition \( \sigma(\mu) > 0 \) is derived from the analysis of the g-2 experiment [5].

As the results, \( m_0 - m_{1/2} \) regions, mass spectra, and the lightest supersymmetric particle (LSP) is shown in the following sections by making use of the above conditions in this paper.

2 Analysis of Parameters

2.1 On the \( \tan \beta \)

On the optimization of \( \tan \beta \) value is discussed by the two-loop level renormalization group equations (RGEs) [4, 5] of the gauge and Yukawa coupling constants in this section. Boundary conditions of the RGEs are given at Z-boson mass scale \( M_Z \). Their values at \( M_Z \) scale are referred from the latest Particle Data Group [6].

1) gauge coupling constants

The average value \( \alpha_X \) among \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) is introduced around GUT scale \( M_X \). \( \chi^2 \) fitting function is made of the squared sum of each differences between \( \alpha_X \) and \( \alpha_i (i = 1 \sim 3) \) as follows:

\[
\chi^2 = \sum_{i=1}^{3} \left( \frac{\alpha_i - \alpha_X}{\sigma_i} \right)^2 
\]

where, \( \sigma_i \) is the standard deviation of \( \alpha_i \). The energy scale where the \( \chi^2 \) becomes minimal is defined as \( M_X \) value. The value of \( \chi^2 \) is shown as a function of \( \tan \beta \) in fig.1. By the way, another mass scale parameter \( M_{SUSY} \simeq 1(\text{TeV}) \) is used to be introduced as a turning-on point of the sparticles’ effect, however, since \( a_3 \) is estimated 0.118 by the latest P.D.G., such \( M_{SUSY} \) becomes less than the scale \( M_Z [5] \). Therefore, the \( M_{SUSY} \) is neglected in this paper.

2) Yukawa coupling constants

When the SU(5) scenario is presumed [4], Yukawa coupling constants of D-type
quark and lepton is supposed to coincide at $M_X$. Their $\chi^2$ fitting function is defined as the following equations:

$$\chi^2 = \sum_{i=1,2,3}^{q=D,L} \left( \frac{\alpha_q - \alpha_{X_i}}{\alpha_{X_i}} \right)^2 \quad (2)$$

$$\alpha_{X_i} = (\alpha_{D_i} + \alpha_{L_i}) / 2 \quad (3)$$

where, the suffix $i$ denotes the generation, and $q$ implies D-type quarks or Leptons. Moreover, with taking the CKM matrix into consideration, Yukawa coupling constants of RGEs are diagonalized. The $\tan \beta$ dependence on $\chi^2$ at $M_X$ scale is shown in fig. 2. In the minimal SO (10) scenario, once the mass spectra of the particles in the lower energy scale region are fixed to their observed values, required unification of Yukawa coupling constants is failed. With likelihood analysis of $\tan \beta$, it is failed to determine the optimized value of $\tan \beta$ to unify the Yukawa coupling constants.

According to the above mentioned 1) and 2), the value of $\tan \beta$ is implied as much as 7.5. As the result, in order to keep the unification among gauge coupling constants, it is impossible to unify the Yukawa coupling constants at $M_X$ as shown in fig.3 and fig.4.
Figure 2:

Figure 3:
2.2 On the $A_0$

As mentioned in avant, one $B(M_Z)$ value is obtained by 1-loop level RGEs [6, 9] development with boundary constraint condition $B_0 = (A_0 - 1)m_0$ at $M_X$ scale. Another $B(M_Z)$ is calculated at $M_Z$ as

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2 \beta}{\tan^2 \beta}$$

(4)

$$B(M_Z) = -\frac{1}{2\mu} \left( m_{h_d}^2 - m_{h_u}^2 + 2\mu^2 \right) \sin 2\beta$$

(5)

where, $m_{h_d}$ and $m_{h_u}$ are soft breaking mass parameters of two Higgs doublet at $M_Z$. $A_0$ is determined by the numerical tuning to coincide both $B(M_Z)$ values. It is shown that $A_0$ at $m_{1/2}/m_0 = 8/3$, $1$, $3/8$ as functions of $\tan \beta$ in fig.5.

2.3 On the sig($\mu$)

sig($\mu$) > 0 is assumed by the result of the g-2 experiment [5].

2.4 On the experiment result of LEP.

As the experimental results of LEP [10], the lightest neutral Higgs mass in the SUSY models exists in the region from 90(GeV) to 130(GeV). However, the lightest neutral
Higgs mass is supposed to be beyond 115(GeV) like SM Higgs in this paper, because such small mass as 90(GeV) makes difficulties to evaluate $m_0$ and $m_{1/2}$.

3 Numerical results

i) $m_0 - m_{1/2}$ plane

Higgs mass is calculated by the 1-loop level RGE analysis as follows [11]:

$$m_h = m_{h,(tree)} + \Delta m_{h,(1-loop)}$$

(6)

Corresponding $m_0 - m_{1/2}$ regions are shown with several Higgs mass settings (115, 120 and 130 (GeV)) with tan $\beta$ = 3, 8, and 20 in fig.5, fig.6, and fig.7, respectively.

The region below the horizontal line is excluded by the chargino mass bound $m_{\chi^\pm} > 91(GeV)$. Moreover, the region in the left side of the ordinate line is excluded by the fact that it is not charged particle(tau slepton), the LSP must be neutral particle(neutralino).

ii) mass spectra

$m_0$ is assumed to coincide with the result of g-2 experiment. The predictions of the mass spectra with several different tan $\beta$ are listed on Table 1.
Figure 6:

Figure 7:
$m_h = 115\text{GeV}$

$m_h = 120\text{GeV}$

$m_h = 130\text{GeV}$

$\tan \beta = 20$

$m_{1/2} = 200\text{GeV}$

Figure 8:

$m_0 = 500\text{GeV}$

$m_0 = 800\text{GeV}$

$m_0 = 200\text{GeV}$

$LSP (\text{GeV})$

Figure 9:
Table 1: Mass Spectra of mSUGRA (unit is GeV).

| $\tan \beta$ (non-dimensional) | 3  | 8  | 20 |
|-------------------------------|----|----|----|
| $m_0$                         | 300| 300| 600|
| $m_h$                         | 120| 120| 120|
| $m_0$                         | 531| 316| 316|
| $\mu$                         | 1031| 626| 803|
| B                             | -416| -99 | -34 |
| $A_0$ (non-dimensional)       | 0.473| 1.70| 2.37|
| heavier neutral Higgs         | 1197| 709| 744|
| CP-odd Higgs                  | 1198| 713| 748|
| Charged Higgs                 | 1197| 709| 744|
| chargino1                     | 461| 271| 276|
| chargino2                     | 1041| 640| 813|
| neutralino1                   | 237| 140| 141|
| neutralino2                   | 461| 271| 276|
| neutralino3                   | -1032| -630| -807|
| neutralino4                   | 1042| 640| 812|
| $m_{\tilde{u}_1}$             | 1391| 861| 1006|
| $m_{\tilde{c}_1}$             | 1391| 861| 1006|
| $m_{\tilde{u}_2}$             | 1433| 884| 1026|
| $m_{\tilde{c}_2}$             | 1433| 884| 1026|
| $m_{\tilde{t}_1}$             | 1042| 580| 495|
| $m_{\tilde{t}_2}$             | 1341| 840| 820|
| $m_{\tilde{d}_1}$             | 1387| 859| 1005|
| $m_{\tilde{d}_2}$             | 1387| 859| 1005|
| $m_{\tilde{d}_3}$             | 1387| 859| 1005|
| $m_{\tilde{d}_4}$             | 1387| 859| 1005|
| $m_{\tilde{b}_1}$             | 1291| 770| 725|
| $m_{\tilde{b}_2}$             | 1385| 850| 863|
| $m_{\tilde{e}_1}$             | 366| 327| 614|
| $m_{\tilde{\mu}_1}$          | 366| 327| 614|
| $m_{\tilde{e}_2}$             | 481| 369| 637|
| $m_{\tilde{\mu}_2}$          | 481| 369| 637|
| $m_{\tilde{\tau}_1}$         | 365| 321| 515|
| $m_{\tilde{\tau}_2}$         | 481| 369| 637|
| $m_{\tilde{\nu}_e}$          | 482| 371| 638|
| $m_{\tilde{\nu}_\mu}$        | 482| 371| 638|
| $m_{\tilde{\nu}_\tau}$       | 482| 371| 638|
iii) lightest supersymmetric particle (LSP) The mass of the LSP as a function of \( \tan \beta \) is shown at \( m_0 = 200, 500 \) and \( 800 \) (GeV) in fig. 8. Note that the neutralino is most plausible candidate of the LSP.

4 Summary and Conclusion

The parameters of mSUGRA are estimated by several conditions phenomenologically. \( \tan \beta \) is estimated about 7.5 by using RGEs of the gauge and Yukawa coupling constants. \( A_0 \) is fixed by the constraint at \( M_X \) and the conditions at \( M_Z \). The allowed \( m_0 - m_{1/2} \) regions are shown with several values of the Higgs boson’s mass. The mass spectra and the mass of the LSP are calculated. Needless-to-say, it is important to find at least one supersymmetric particle experimentally for the reality of mSUGRA or other supersymmetric models. With the results shown in this paper, the supersymmetric particles are not so much heavy, and possible to be observed by colliders in the near future.

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