Synchronized Vibration Transition of Three Exciters in Non-resonant Vibration System

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Abstract. The synchronized vibration transition has been proposed in the non-resonant vibration system with three exciters. Based on former man, the movement equations of self-synchronous vibration system with three rotors are replaced by differential equation of phase difference angle first, and the necessary conditions of synchronous movement for system are analyzed, stability and bifurcation of equilibrium points of vibration system are discussed. Firstly, dynamics model are established, differential equation of phase difference angle is deduced based on the dynamics equation of the vibration system. Then, the necessary conditions of synchronous movement are established, stability and bifurcation of equilibrium points of vibration system are discussed using Lyapunov theories. Finally, the effects of system parameters on synchronization stability about self-synchronous vibration system are investigated with numerical simulations.

1. Introduction

In the vibration system, if the excitation motor is properly installed, even if there is a certain initial phase difference between each of the two excitation motors, the rotation speeds of the two motors are the same and the phase difference angle is constant. In this way, self-synchronizing vibration is achieved. Reference 1-2 proposed the insight that the rotational speed of the two eccentric rotors fluctuates around the average rotational speed in the synchronized state. Reference 3 discussed the concept of self-synchronization and control synchronization. Reference 4 derives the coupling equation for the system to form a synchronous motion state. Reference 5 used the method of simulation analysis to express the formation and development of self-synchronous vibration in detail. Reference 6 derived the synchronization condition and stability criterion of the system in the near-synchronous state. Reference 7 discussed the equilibrium point stability and bifurcation characteristics of the system. Although the above research articles have studied various forms of self-synchronizing vibration, they are only for the two-rotor self-synchronizing vibration system. They have not conducted in-depth research on the synchronization stability of the three-rotor self-synchronizing vibration system. This article will expand research along this line of thought. Firstly, a system dynamics model is established. Then the fluctuation of the rotational speed of the three eccentric rotors around the average rotational speed is analyzed, and the differential equations of the three eccentric rotors is derived with respect to the phase difference angle of the eccentric rotor in the non-resonant vibration system with three exciters. Based on the differential equation of phase difference angle for the eccentric rotor, the synchronous motion necessity condition is derived and the stability point stability and bifurcation
characteristics of the system are discussed. Finally, this paper investigates the influence of various parameters on the investigated using simulation.

2. Vibration dynamics model

The mechanical model of the non-resonant vibration system with three exciters is shown. The Lagrange equation is used to establish the vibration equation, and the equation of motion in the three directions of the vibrating body and the equation of he non-resonant vibration system with the three exciters are obtained as follows.

\[
M \ddot{x} + c_x \dot{x} + k_x x = m_r (\dot{\phi}_1 \sin \phi_1 + \dot{\phi}_1^2 \cos \phi_1) - \sum_{i=2}^{3} m_r [\dot{\phi}_1 \sin \phi_1 + \dot{\phi}_1^2 \cos \phi_1] \\
M \ddot{y} + c_y \dot{y} + k_y y = m_r (\dot{\phi}_2 \sin \phi_2 + \dot{\phi}_2^2 \cos \phi_2) - \sum_{i=2}^{3} m_r [\dot{\phi}_2 \sin \phi_2 + \dot{\phi}_2^2 \cos \phi_2] \\
J_{\phi_1} \ddot{\phi}_1 + c_{\phi_1} \dot{\phi}_1 + k_{\phi_1} \phi_1 = c_x (\dot{\phi}_1 - \psi) - c_x (\phi_2 + \psi) - c_y (\phi_1 + \psi) + \sum_{i=2}^{3} m_r [\dot{\phi}_1 \cos(\phi_1 + \beta_1 + \psi) + \dot{\phi}_1^2 \sin(\phi_1 + \beta_1 + \psi)] \\
+ m_r [\dot{\phi}_2 \cos(\phi_2 - \beta_1 - \psi) + \dot{\phi}_2^2 \sin(\phi_2 - \beta_1 - \psi)] \\
J_{\phi_2} \ddot{\phi}_2 + c_{\phi_2} \dot{\phi}_2 + k_{\phi_2} \phi_2 = c_x (\phi_2 - \phi_1 - \psi) + m_r \sin(\phi_1 + \beta_1 + \psi) + m_r \cos(\phi_2 - \beta_1 - \psi) + \dot{\phi}_2^2 \sin(\phi_2 - \beta_1 - \psi)] = T_i, \\
J_{\psi} \ddot{\psi} + c_{\psi} \dot{\psi} + k_{\psi} \psi = c_x (\phi_2 + \psi) + m_r \sin(\phi_1 + \beta_1 + \psi) + \dot{\phi}_1^2 \sin(\phi_1 + \beta_1 + \psi) + \dot{\phi}_2^2 \sin(\phi_2 + \beta_1 + \psi)] = T_i, \quad i = 2, 3
\]

In the formula, \(x, y, \psi\) respectively represent the displacements in the horizontal direction, the vertical direction and the torsional vibration direction. \(\phi_i (i=1, 2, 3)\) are the corners of the eccentric rotor. \(m_i (i=1, 2, 3)\) indicates the mass of the eccentric mass. \(r_1 (i=1, 2, 3)\) is the radius of the eccentric block. \(M\) is the total mass of the system (including the vibrating body and the motor). \(I_i\) is the distance between \(O_i\) and \(O\). \(J_{\phi_i}\) is the moment of inertia of the body around the \(O_i\) point. \(J_0 (i=1, 2, 3)\) is the moment of inertia of the eccentric block \(i\) around the \(O_i\) point. \(c_{x}, c_y c_{\psi}\) are the damping of the spring in \(x, y\) and \(\psi\) direction respectively; \(c_i (i=1, 2, 3)\) respectively represent the damping of the motor \(i\) shaft; \(k_x, k_y, k_{\psi}\) are the stiffness of the spring in \(x, y\) and \(\psi\) respectively. \(T_i (i=1, 2, 3)\) is the input torque of the motor \(i\). The non-resonant vibration system with three exciters can be expressed as

\[
x = a_1 \cos \phi_1 - a_2 \cos \phi_2 - a_3 \cos \phi_3 \\
y = b_1 \sin \phi_1 + b_2 \sin \phi_2 + b_3 \sin \phi_3 \\
\psi = c_1 \sin(\phi_1 + \beta_1) + c_2 \sin(\phi_2 + \beta_1) + c_3 \sin(\phi_3 + \beta_1)
\]

3. Synchronization necessity conditions

In the near-synchronous state, the phase angle of the two eccentric rotors can be expressed as \(\phi = \tau + \alpha\).

In the formula, \(\tau = \omega t\), \(\omega\) is the average rotational speed, \(\tau\) is the average phase angle, and \(\alpha_i (i=1, 2, 3)\) is the disturbed phase angle of the eccentric rotor \(i\). Among them, \(\tau \geq \alpha_i \cdot |\alpha_i| < 1\). By transforming the differential of time \(t\) into the differential of the new variable \(\tau\) in \([0, 2\pi]\), so \(\dot{\phi}_i = \dot{\phi}_i (0 + \alpha_i)\) and \(\dot{\phi}_i = \omega_i \alpha_i\). the motion state of the three eccentric rotors is analyzed to establish the relationship between the input torque of the three motors in equation (1) and the rotational speed of the motor rotor. The power formula of the motor provides us with an extremely convenient transition. It not only reduces the number of variables in mathematical form, but also provides a theoretical basis for us to explain the law of energy transfer in the synchronous process. The input torque of the three-motor to the vibrating machine is expressed as \(T_i = F_i (1 - \alpha_i) / \omega_i\). Neglecting the high-order small term of \(\psi\) in equation (1) to obtain three eccentric rotors, \(|\alpha_i| < 1\), so \(k_i / M = \omega_i^2 >> \omega_i^2 (1 + \alpha_i)^2\), and \(k_i / M = \omega_i^2 >> \omega_i^2 (1 + \alpha_i)^2\). The case where the radius and mass of the three eccentric rotors are equal is analyzed. When \(m = m_i, r_i = r\), \(c = c_i (i=1, 2, 3)\), then \(J_0 = J, \chi_1 = \chi_2 = \chi_3 = \chi\). Set up, \(\Delta P_i = P_i - P_i > 0\), \(\Delta F_i = F_i + F_i\). Let \(\Delta \alpha_1 = x_1, \Delta \alpha_2 = x_2, \Delta \alpha_3 = y_1, \Delta \alpha_3 = y_1\). From the nature of the equilibrium point in the multidimensional plane, \(x_1 = y_1 = 0, x_2 = y_2 = 0\). The resulting \(x_1\) and \(y_1\) are the equilibrium points of the system. That is, when the system is in stable operation, the difference angle between every two axes has a stable value. Since
\[ \Delta \alpha_1 \text{ and } \Delta \alpha_2 \text{ are independent of each other. It is a mathematical condition to analyze the region where the solution exists, which is a necessary condition for the self-synchronization of the system.} \]

\[ D = \frac{M}{\Delta T_1} (k_y - k_y') \geq 1 \]

Among them, \( \Delta T_1 = T_1 - T_2 \) \( \Delta T_2 = T_2 - T_3 \). The necessary condition for self-synchronization of the non-resonant vibration system with three exciters is \( D \geq 1 \). If \( D < 1 \), then there is no solution in equation (2), that is, the "self-synchronizing" vibration system can only operate without synchronization. Therefore, increasing of \( D \), the stable synchronous motion can be achieved in the non-resonant vibration system with three exciters.

**4. Synchronization stability condition**

There are various methods for determining the balance position and the stability of the motion state. In this paper, the Lyapunov one-time approximation stability method is used. The Lyapunov primary approximation stability discriminant method is defined as: if the real part of all the eigenvalues of the approximation equation is negative, the zero solution of the original equation is asymptotically stable; if the real part of at least one eigenvalue of the approximation equation is positive, the zero solution of the original equation is unstable; if the first approximation equation has zero real eigenvalues and the real part of the other roots is negative, the zero solution stability of the original equation cannot be judged, and its stability is related to the nonlinear term. In the system, when \( \Delta T_1 = \Delta T_2, k_y > k_y' \), then \( \Delta \bar{E}_1 \approx \Delta \bar{E}_1, \mu > 0 \). In the system itself, \( \eta > 0 \), the Routh-Hurwitz criterion is used to determine that the real part of the characteristic equation (8) is negative, and the stability of the system equilibrium point is obtained.

\[ \mu \cos x_1 + \mu \cos y_1 - \mu_{12} \cos (x_1 + y_1) > 0, \mu \cos x_1 \cos y_1 - \mu_{12} \cos (x_1 + y_1) > 0 \]

**5. Synchronized Vibration Transition**

The main structural parameters of the universal synchronous test rig are determined as follows. \( M=148 \text{kg} \), \( m_1= m_2= m_3=3.5 \text{kg} \), \( J=17 \text{kg}\cdot\text{m}^2 \), \( J_{01} = J_{02} = J_{03}=0.01 \text{ kg}\cdot\text{m}^2 \), \( r_1 = r_2 = r_3=0.08 \text{m} \), \( k_y=77600 \text{N/m} \), \( k_x=30000 \text{ N/m} \), \( k_y'=30000 \text{ N/m} \), \( c_x = c_y = 1000 \text{ N/s/m} \), \( c_y = 1000 \text{ N/s/rad} \), \( l_1 = l_3=0.4 \text{m} \), \( l_2=0.2 \text{m} \), \( c_1 = c_2 = c_3 = 0.01 \text{N/s/rad} \). The response curves of the three-rotor self-synchronized vibration are obtained the equation (1) and the electromagnetic torque equations and the rotor motion equations about the rotors. The response of the self-synchronizing system is shown in Figure 1 when \( \Delta T_{12}=\Delta T_{31} \) and the eccentric mass distances are equal. It can be seen from Figure 1 that the phase difference of the eccentric block changes continuously, and finally tends to be stable. The phase difference of the two exciters is each the phase difference between the exciter 1 and the exciter 2 and the phase difference between the exciter 2 and the exciter 3. They all tend to be consistent after a transitional process, and are each stable at around 1 rad or -3 rad. Because of the positional arrangement of the motor, the speed and phase difference of the exciter will not be exactly the same. Therefore, the magnitude and direction of the exciting force are constantly changing. Therefore, the amplitude of each vibration direction at the center of the vibrating machine changed, accompanied by the occurrence of torsional vibration (but the amplitude of the torsional oscillation of the steady state is small). Therefore, the system gradually tends to stabilize the synchronous vibration, that is, the system realizes the self-synchronization stable state.

**Fig. 1 System response when \( \Delta T_{12}=\Delta T_{31} \) and eccentric mass distance are equal**
The eccentricity is expressed as the product of the eccentric rotor mass and radius. When the mass \( m_3 \) of the eccentric rotor 3 is changed to 4 kg, the response of the eccentric mass distance of the self-synchronizing system is as shown in Fig. 2. It can be observed from the figure that when the eccentricity is not equal, the vibration shape is not significantly changed compared with the vibration shape of the eccentricity. Due to the eccentricity, the exciting force generated by the excitation motor is constantly changing. Therefore, the phase difference of the eccentric rotor is also constantly changing, and the vertical and horizontal displacements and the torsion angle of the vibrating body (at the center of mass) are constantly changing. As the rotational speed of the motor and the phase difference angle of the eccentric rotor gradually become uniform, the displacement of the system in all directions achieves a stable synchronization state. The analysis shows that under the condition of satisfying certain synchronization stability, the system can also restore the self-synchronization stable state by changing the eccentric mass distance of the rotor of the system.

After the system reaches the synchronous steady state, the voltage of one motor is cut off after 5s, which means \( \Delta T_{12} \neq \Delta T_{31} \). After 5s, the response of the system is obtained as shown in Fig. 3. The thin line in the graph of the speed in the figure is shown as the speed of the power-off motor (the operating frequency of the test system is 16 Hz). After the motor power is cut off, the rotation speed of the rotor undergoes an oscillation process. The vibrating machine undergoes a transition process with "shooting" characteristics in the horizontal, vertical, and torsional directions, and finally exhibits regular periodic vibration. At the same time, the phase difference angles of the three eccentric blocks also undergo a large oscillation process, reaching the stable value again, and the phase difference angle is stable at around 0, -\( \pi \) rad respectively. Therefore, the system is in a stable state.

6. Conclusion
This paper is the first to derive the differential equations of the eccentric rotor phase difference angle of the three-rotor self-synchronous vibration system on the basis of predecessors. It then analyzes the necessary conditions for the synchronous motion of the system for this system of equations. Finally, the Lyapunov one-time approximation stability method is applied to discuss the stability and bifurcation characteristics of the system. The effects of system parameters on the self-synchronization stability of the system are investigated by mathematical simulation and experiment. The system can achieve synchronous stability when it must meet certain self-synchronization conditions.

Experiments and simulations show that within a certain range, if the eccentricity of two co-rotating rotors is increased, the synchronization stability of the system can be good. For an eccentric rotor that is different from the other two rotors, the synchronization stability is more sensitive. If the eccentricity of the system is properly reduced, the synchronization stability can be better. Under the condition of satisfying certain synchronization stability, the three-motor reverse rotation system with different
eccentricity has better synchronization stability than the system with equal eccentricity.

For the three-rotor to the slewing system, after the system reaches the synchronous steady state, if the motor power supply that is different from the rotation direction of the other rotors is cut off, the system loses the synchronous stable state. Conversely, when one of the two rotors with the same direction of rotation is disconnected, the system remains in a self-synchronizing state. This shows that the system meets the synchronization stability condition at the time of power failure. Through experiments and simulation comparisons, it has been shown that the test and simulation results are consistent.

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