Information theory in high energy physics
(extensive and nonextensive approach)

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Abstract: The application of information theory approach (both in its extensive and nonextensive versions) to high energy multiparticle production processes is discussed and confronted with experimental data on $e^+e^-$ annihilation processes, pp and $\bar{p}p$ scatterings and heavy ion collisions.

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1 Introduction

High energy multiparticle production processes are most naturally described by statistical models (see [1,2] for a historical background and [3,4] for the most recent developments). Their results are usually interpreted in the thermodynamical sense, with temperature $T$ and chemical potential $\mu$ entering with their usual meaning. However, it has been recognized that in this branch of physics one very frequently encounters ”thermal-like” form of distributions in some variable $x$, say $\propto \exp(-x/T)$, without system under consideration being in any kind of thermal equilibrium [5]. It is enough that out of the huge number of produced secondaries only some part is registered by detectors, and out of them only one or two are selected for final scrutiny. The averaging emerging this way is then equivalent to the action of some ”heat bath” characterized by parameter $T$. Once this is realized it is then obvious that there are situations in which such ”heat bath” is more complicated (for example nonextensive) and needs additional parameter(s) to be
described properly \[6,7,8\]. In this way the nonextensive Tsallis statistics characterised by the non-extensivity parameter \(q\) enters in a natural way \[9,10,11,12\]. On the other hand, such apparently ”thermal-like” behaviour of some distributions arises very naturally in many physical applications of information theory (MaxEnt) \[13\] (with Shannon form of the corresponding information entropy, its nonextensive form uses Tsallis entropy characterised by the same parameter \(q\) as mentioned above, such that for \(q \rightarrow 1\) one recovers the usual Shannon entropy).

2 Results

The usefulness of information theory is most obvious in situations when one has to ”guess” the most probable (and least biased) distribution \(p(x)\) of some quantity \(x\) using only limited amount of information given in terms of finite number \(n\) of some observables: \(R_{k=1,...,n} = \langle R_k(x) \rangle\) \((p(x)\) is normalized to unity and average \(\langle \ldots \rangle\) is with respect to \(p(x)\) in extensive and to \([p(x)]^q\) in nonextensive approaches, respectively \[13,9\]. In high energy multiparticle production processes it allows to find, in a maximally model independent way, the real information content of the experimental data considered \[14,15,16,17,18\]. From \[14\] we know therefore that: (i) - particles produced in high energy collisions are mostly located in one dimensional phase space, i.e., they have limited transverse (with respect to collision axis) momenta \(p_T\) \((\langle p_T \rangle\) is finite) and are fully characterized by their longitudinal momenta \(p_L = \mu_T \sinh y\) \(^\star\); (ii) - only fraction \(K \in (0,1)\) (called inelasticity) of the initially available energy \(W\) is converted into produced particles. As was shown recently by us \[18\] the fact that multiplicity distribution of produced secondaries is no longer of Poissonian type but follows much broader Negative Binomial (NB) form can be accounted for only by using nonextensive approach with \(q\) given by the parameter \(k\) of NB, \(q = 1 + 1/k\), which represents the amount of dynamical fluctuations in the number of produced secondaries \[18\].

We are usually interested in single particle rapidity distributions, which in the information theory approach are given by

\[
p(y) = \frac{1}{N} \frac{dN}{dy} = \frac{1}{Z_q} \exp_q \left(- \beta_q \cdot \mu_T \cosh y \right).
\]  

\(^\star\) Here \(\mu_T = \sqrt{\mu^2 + \langle p_T^2 \rangle}\) is the so called transverse mass of particle with mass \(\mu\) and transverse momentum \(p_T\) and \(y\) denotes rapidity of the particle, variable defined in such way that its energy is \(E = \mu_T \cosh y\).
This form is identical with that used in statistical models but now $Z_q$ and $\beta_q$ are no longer free parameters to be fitted when comparing with experimental data but instead are given by the normalization condition and energy conservation constraint,

$$
\int_{-Y_m}^{Y_m} dy p(y) = 1 \quad \text{and} \quad \int_{-Y_m}^{Y_m} dy \mu_T \cdot \cosh y \cdot [p(y)]^q = \frac{\kappa_q \cdot W}{N}
$$

(2)

(where $\pm Y_m$ are maximal rapidities available in rest frame of hadronizing source, see [15,18] for details). There are therefore two parameters: nonextensivity $q$ responsible for dynamical fluctuations and $q$-inelasticity $\kappa_q$ to be deduced directly from data essentially in a model independent way **. This method works perfectly well for $pp$ and $\bar{p}p$ collisions [18], cf. Fig. 1a (where it provides us with the first model independent estimations of energy dependence of the mean inelasticity parameter and with its distribution). Here we show that it works also quite well for similar data on rapidity distributions obtained in $Au+Au$ collisions, cf. Fig. 1b. The $Au+Au$ data are for the most central events (covering collisions proceeding

** Here $\exp_q(x/\Lambda) = [1 + (1 - q)x/\Lambda]^{1/(1-q)} \overset{q \rightarrow 1}{\longrightarrow} \exp(x/\Lambda)$. Possible question concerning physical meaning of $\kappa_q$ for $q \not= 1$ case is solved by noticing [18] that the inelasticity parameter in this case, which has physical meaning, is $K_q = \kappa_q/(3 - 2q)$.

Fig. 1. Examples of applying eq. (1) to: (a) rapidity spectra for charged pions produced in $pp$ and $\bar{p}p$ collisions at different energies [19,20,21,22]; (b) similar data obtained for the most central $Au+Au$ collisions [23]; (c) rapidity spectra measured in $e^+e^-$ annihilations at 91.2 GeV [24] (dotted line is for $K_q = 1$ and $q = 1$ whereas full line is our fit with $K_q = 1$ and $q = 0.6$).
with impact parameter range 0 − 6%). They can be fitted choosing $K_q = 1$ and then $q = 1.29, 1.26$ and 1.27 for energies 19.6, 130 and 200 GeV, respectively (the $q$-inelasticity was therefore equal to $\kappa_q = 0.42, 0.48$ and 0.46)***. The most interesting are, however, results for $e^+e^-$ annihilations (cf. Fig. 1b) for which, by definition, $K_q = 1$ (i.e., always all energy of initial leptons is available for the production of secondaries) and which can be fitted only with $q < 1$ (in our case $q = 0.6$).

This point deserves closer scrutiny. One could argue that because fit in Fig. 1b is not perfect (there are some discrepancies for small rapidities and there is a tail at large values of $y$) there is nothing to be said before they are not addressed. But results for $q = 1$ clearly show that these discrepancies are not connected with the particular value of $q$ but rather with some additional mechanisms operating here action of which would, however, change our results only slightly (for example, a possibility of two rather than one source or $y$-dependent $\langle p_T \rangle$, as mentioned already in [25]).

With the above reservations let us then take a closer look at the possible origin of $q < 1$. We have already encountered similar situation when in [17] we have fitted single particle distributions assuming implicitly that $K_q = 1$ and discovering then that one could get fairly good agreement with data for $q < 1$ only. That was because in this case only $q < 1$ leads effectively to cutting-off a part of the phase space (once it is taken too big) mimicking therefore action of the inelasticity parameter (cf. [18]). On the other hand, we know that $q \neq 1$ signals presence of fluctuations in the system [10,11,12] and is given by normalized variations of these fluctuations, in our case they would be fluctuations of temperature $T = 1/\beta$ parameter. So far it was widely discussed only for the $q > 1$ case [11,12] but formally it covers the $q < 1$ case as well, cf. [11]. However, in this case temperature $T$ does not reach an equilibrium state because in this case

$$T = T_0 - (1 - q)E$$

(3)

instead remaining constant, $T = T_0$, as is the case for $q > 1$. In this case we have a kind of dissipative transfer of energy from the region where (due to fluctuations) the temperature $T$ is higher (for example, in our case from the quark ($q$) and antiquark ($\bar{q}$) jets formed in the first $e^+e^- \rightarrow q + \bar{q}$ to gluons and $q\bar{q}$ pairs and later

*** Actually data in Fig. 1b. are presented for the so called pseudorapidity $\eta$ defined not by energy $E$ and longitudinal momentum $p_L$ as is the case of rapidity $y$ but by total momentum $p$ and longitudinal momentum $p_L$ instead.
on to finally observed hadrons). It means therefore that $q < 1$ signals that in the reaction considered, where $K_q = 1$ and we have to account for the whole energy exactly, conservation laws start to be important and it is not possible for stationary state with constant final temperature to develop but temperature $T$ depends on the energy and for large energies tends to zero (notice that from eq. (3) one has limitation on the allowed energy of the secondaries: $E \leq T_0/(1 - q)$).  

3 Summary

We have demonstrated that information theory (know also as MaxEnt method) can play very important role in high energy physics providing simple, highly model independent, estimations of single particle distributions and allowing to reliably estimate the amount of information provided by experimental data. Its nonextensive version extends applicability range of MaxEnt by including also some intrinsic fluctuations in the hadronizing system visible as broadening of the multiplicity distributions. As was shown here, single particle distribution data on all kinds of collisions, starting form very elementary $e^+e^-$ annihilations, via $pp$ and $\bar{p}p$ collisions, and ending with the most complicated heavy ion scatterings, can be described by formula (1) depending on very limited number of parameters: inelasticity $K$ and nonextensivity parameter $q$. Fairly good fits were obtained in all cases. To proceed further one should concentrate now on these parts of the phase space where discrepancies occur and then introduce one-by-one some additional hypothesis and check whether they lead to the better agreement with data (see, for example [25]). Such approach allows to avoid assumptions, which whereas looking promising, are not justified.

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*Actually, when fluctuations depend on it in the same way the relative variance $\omega$ remains constant and $q = 1 - \omega$, cf. [11].

**Notice that analysis of $p_T$ distribution in the same process using a kind of $q$-version of Hagedorn model [26] reports $q > 1$ instead. This is, however, what we call $q_T$ in [25] and this is different from $q$ considered here (called $q_L$ in [25], where we compared both $q$'s for $p\bar{p}$ collisions). The reason for such different behaviour is that $p_T$'s considered there are essentially not influenced by conservation laws but, in our language here, reflect instead a kind of stationary state with $q > 1$ and energy independent $T$.**
References

[1] L.D.Landau and S.Z.Bilenkij, Nuovo Cim. Suppl. 3 (1956) 15.
[2] R.Hagedorn, Riv. Nuovo Cim. 6 (1983) 1983.
[3] W.Broniowski, A.Baran and W.Florkowski, Acta Phys. Polon. B33 (2002) 4235.
[4] F.Becattini, Nucl. Phys. A702 (2002) 336.
[5] L.Van Hove, Z. Phys. C21 (1985) 93.
[6] A.R.Plastino and A.Plastino, Phys. Lett. A193 (1994) 140.
[7] M.Baranger, Physica A305 (2002) 27.
[8] M.P.Almeida, Physica A325 (2003) 426.
[9] C.Tsallis, in Nonextensive Statistical Mechanics and its Applications, S.Abe and Y.Okamoto (Eds.), Lecture Notes in Physics LPN560, Springer (2000). See also at http://tsallis.cat.cbpf.br/biblio.htm
[10] G.Wilk and Z.Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770.
[11] G.Wilk and Z.Wlodarczyk, Chaos, Solitons and Fractals 13/3 (2001) 581.
[12] C.Beck and E.G.D.Cohen, Physica A322 (2003) 267.
[13] P.Harremoës and F.Topsøe, Entropy 3 (2001) 191 and references therein.
[14] Y.-A.Chao, Nucl. Phys. B40 (1972) 475.
[15] G.Wilk and Z.Wlodarczyk, Phys. Rev. D50 (1994) 2318.
[16] T.Osada, M.Maruyama and F.Takagi, Phys. Rev. D59 (1999) 014024.
[17] F.S.Navarra, O.V.Utyuzh, G.Wilk and Z.Wlodarczyk, Nuovo Cim. 24C (2001) 725.
[18] F.S.Navarra, O.V.Utyuzh, G.Wilk and Z.Wlodarczyk, Phys. Rev. D67 (2003) 114002.
[19] C.De Marzo et al., Phys. Rev. D26 (1982) 1019.
[20] C.De Marzo et al., Phys. Rev. D29 (1984) 2476.
[21] R.Baltrusaitis et al., Phys. Rev. Lett. 52(1993) 1380.
[22] F.Abe et al., Phys. Rev. D41 (1990) 2330.
[23] B.B.Beck et al., Phys. Rev. Lett. 91 (2003) 052303.
[24] R.Barate et al. (ALEPH Collab.), Phys. Rep. 294 (1998) 1.
[25] F.S.Navarra, O.V.Utyuzh, G.Wilk and Z.Wlodarczyk, Information theory approach (extensive and nonextensive) to high energy multiparticle production processes, hep-ph/0312052 to be published in Physica A (2004).
[26] I.Bediaga, E.M.F.Curado and J.M.de Miranda, Physica A286 (2000) 156.