Active Control of the Space-borne Antenna Reflector Considering Thermal Load

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Abstract. In order to improve the shape accuracy of the space-borne antenna reflector under the interference of uncertain load, the active control method of the reflector shape was studied. First, a finite element model of the thermal and mechanical coupling of the space-borne antenna reflector was established. Taking the uncertain load into account, the radial basis function (RBF) neural network was used to estimate the error, and the adaptive controller was designed according to the estimation error. Based on the Lyapunov stability theory, the stability of the controller was analyzed. Finally, the adaptive control of the space-borne antenna reflector was simulated numerically. The simulation results show that the method of RBF neural network adaptive control is effective by reducing the shape error of the reflector in a short time when considering the uncertain load.

1. Introduction
With the rapid development of communications, space science, and earth observation, the demand for large-aperture space-borne antenna reflectors is increasing. According to antenna theory, it is known that large-aperture antenna can transmit a large amount of high-resolution data [1,2], and high shape accuracy (small RMS error) can achieve high operating frequency band and wide frequency band. Many experiments have proved that thermal deformation is the main factor for the shape deviation of the reflector. In this case, it is necessary to actively control the shape of the reflector [3].

Active shape control of the reflector is divided into static shape control and dynamic shape control. The static shape control of the reflector assumes that the external load changes slowly. Song X S et al. [4] established an influence coefficient matrix according to the magnitude of the influence of each actuator on the deformation of the grid reflector. Furthermore, the least square method is used to solve the voltage value of each actuator, and the effectiveness of this method is verified by experiments. Wu et al. [5] used piezoelectric ceramic transducer (PZT) and large fiber composite (MFC) actuators to reduce the overall and local (high-order) surface errors of the reflector respectively. The antenna may also be subjected to thermal shocks when satellite in orbit. This rapid external load may cause vibrations on the reflector. In addition, static shape control only considers the final control force without considering the loading process. Unreasonable loading process will also lead to the vibration of the structure. Based on the above factors, the static shape control of the reflector has certain limitations.

In order to eliminate the vibration of the reflector structure, scholars have adopted a dynamic shape control method to control the reflector. Xun et al. [6] took a large cable net antenna as the research object, and used a fast model predictive control method to estimate the input voltage curve of the actuator. Luo et al. [7] proposed a hybrid control algorithm based on FLC, and designed a controller based on this algorithm to enhance the attenuation of free vibration of large truss structures.
In practical applications, due to the inhomogeneity of the structure, the assembly error and the external load, the reflector is essentially a nonlinear structure. In this case, the shape control of the reflector needs to adopt a nonlinear control method. Because the RBF neural network can approximate any nonlinear function with a compact set and arbitrary precision, this article adopts the RBF neural network adaptive control to control the shape of the reflector.

2. Thermal-mechanical coupling finite element modeling of the space-borne antenna reflector

The space-borne antenna reflector mainly consists of the following parts: reflecting surface, actuator parts, and driving device. The structure is shown in Figure 1.

\[ \varepsilon = [\varepsilon_{xx} \ \varepsilon_{yy} \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{xz}]^T \]

is the strain vector of the element, \( d = [u \ v \ w]^T \) is the displacement vector of the element. \( \nabla \) is the derivative operator. But the strain here should be the total strain, which is the sum of the force and thermal expansion.

\[ \{\varepsilon\} = \{\varepsilon\}_E + \{\varepsilon\}_T \]  

(2)

\( \{\varepsilon\}_E \) is the elastic strain and \( \{\varepsilon\}_T \) is the thermal strain. \( \Delta T \) represent the temperature increment, the anisotropic linear expansion coefficient is \( \alpha_1, \alpha_2, \alpha_3 \), and the thermal strain is supposed to be

\[ \{\varepsilon\}_T = [\alpha_1 \Delta T \ \alpha_2 \Delta T \ \alpha_3 \Delta T \ 0 \ 0 \ 0] \]

(3)

The relationship between stress and elastic strain is obtained as

\[ \{\sigma\} = [D]\{\varepsilon\}_E \]

\[ \{\varepsilon\}_E = \{\varepsilon\} - \{\varepsilon\}_T \]

So the relationship between stress and total strain is

\[ \{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon\}_T) \]

Where \([D]\) is the elastic coefficient matrix. Using the same element and the same shape function to interpolate the temperature increment and displacement inside the element with the nodal temperature rise and nodal displacement, the expression of the temperature increment and displacement are given as

\[ \Delta T = [N]_T(\Delta T)^e \left[ \begin{array}{c} u \\ v \\ w \end{array} \right] = [N]\{\delta\}^e \]

(4)

\( (\Delta T)^e \) is the element node temperature increment matrix, \( \{\delta\}^e \) is the element node displacement matrix. The elastic energy is only caused by elastic strain \( \{\varepsilon\}_E \), so the elastic energy of element e is
\begin{equation}
U^e = \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV
\end{equation}

\begin{equation}
= \frac{1}{2} \int_{V_e} (\{\varepsilon\} - \{\varepsilon\}_T)^T [D] (\{\varepsilon\} - \{\varepsilon\}_T) dV
\end{equation}

\begin{equation}
= \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV + \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV - \frac{1}{2} \int_{V_e} ((\{\varepsilon\}_T^T [D] \{\varepsilon\}_T + \{\varepsilon\}_T^T [D] \{\varepsilon\}_T)) dV
\end{equation}

Because of

\begin{equation}
\{\varepsilon\}_T^T [D] \{\varepsilon\}_T = \{\varepsilon\}_T^T [D] \{\varepsilon\}_T
\end{equation}

The element elastic strain energy is given as

\begin{equation}
U^e = \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV - \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV + \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV
\end{equation}

\begin{equation}
\{\varepsilon\} = [B] \{\delta\}^e
\end{equation}

Substituting the above formula into the strain energy of the element, the following equation can be obtained

\begin{equation}
U^e = \frac{1}{2} \int_{V_e} \{\delta\}^e^T [k]^e \{\delta\}^e dV - \int_{V_e} \{\delta\}^e^T [k]^e \{\delta\}^e dV + \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV
\end{equation}

\begin{equation}
\{\delta\} = \int_{V_e} [B]^T [D] \{\varepsilon\}_T dV
\end{equation}

Where \([k]^e\) is the element stiffness matrix.

\begin{equation}
\{Q\}_T = \int_{V_e} [B]^T [D] \{\varepsilon\}_T dV
\end{equation}

Where \([Q\]_T is the thermal load.

\begin{equation}
C = \frac{1}{2} \int_{V_e} \{\varepsilon\}_T^T [D] \{\varepsilon\}_T dV
\end{equation}

Its extreme condition is given as

\begin{equation}
\frac{\partial U}{\partial \{\delta\}} = 0
\end{equation}

The following equation can be obtained as

\begin{equation}
[K]\{\delta\} = \{Q\}_T
\end{equation}

Where \(K\) is the total stiffness matrix of the structure, and the displacement of each node of the reflector thermal deformation can be obtained by solving equation (6).

3 Controller design and stability analysis

3.1 Controller design
Considering the damping of the system, the system dynamics equation is given as

\begin{equation}
M \ddot{\delta} + C \dot{\delta} + K \delta = F(t) + \tau
\end{equation}

Where \(M\) is the total mass matrix of the structure, \(C\) is the total damping matrix, \(K\) is the total stiffness matrix, \(F(t)\) is the force of the actuator, and \(\tau\) is the external force of the structure (thermal shock or external random interference load, etc.), \(\ddot{\delta}\) is the nodal acceleration, \(\dot{\delta}\) is the nodal velocity, \(\delta\) is the nodal displacement.

Transform the modal coordinates of Eq. (7), the following equation is obtained as

\begin{equation}
\delta = \sum_{i=1}^{n} \phi_i q_i
\end{equation}

In fact, the lower-order modes play a greater role, so the mode truncation method is used to reduce
the order of the model. The number of truncated modes is $k$, then Eq. (8) can be reduced to

$$\delta = [\phi_c \phi_{uc}]^{T} q_c$$

(9)

Where $\phi_c = [\phi_1 \phi_2 \cdots \phi_k]$ is the first $k$-order mode shape matrix, $\phi_{uc}$ is the remaining mode shape matrix, $q_c$ is the first $k$-order mode coordinates, and $q_{uc}$ is the remaining-order mode coordinates. After truncating the mode, $\delta = \phi_c q_c$, multiply both sides of the Eq. (7) by $\phi_c^T$,

$$\phi_c^T M \phi_c = I$$

$$\phi_c^T K \phi_c = diag(\omega_1^2, \omega_2^2, \cdots \omega_k^2)$$

Eq. (7) can be simplified to

$$M_4 \ddot{q}_c + C_4 \dot{q}_c + K_4 q_c = T + F$$

(10)

Where $M_4 = I, K_4 = diag(\omega_1^2, \omega_2^2, \cdots \omega_k^2)$, $C_4 = \beta_4 M_1 + \beta_2 K_1$, $T = \phi_c^T F(t), F = \phi_c^T \tau$.

Due to various uncertainties, the mass matrix, damping matrix and stiffness matrix will always be different from the theoretical values.

$$\Delta M = M_0 - M_1, \Delta C = C_0 - C_1, \Delta K = K_0 - K_1$$

(11)

Where $M_0, C_0, K_0$ are the theoretical values of the mass matrix, damping matrix, and stiffness matrix, and $M_1, C_1, K_1$ are their actual values. By Substituting Eq. (11) into Eq. (10), the following equation is given as

$$M_0 \ddot{q}_c + C_0 \dot{q}_c + K_0 q_c = T + f(q_c)$$

(12)

Where $f(q_c) = f(q_c, \dot{q}_c, \ddot{q}_c) = \Delta M \ddot{q}_c + \Delta C \dot{q}_c + \Delta K q_c + F$.

If $f(V)$ is known, the adaptive law can be designed according to the calculated control torque as

$$T = M_0 (\ddot{q}_c - k_v \dot{e} - k_p e) + C_0 \dot{q}_c + K_0 q_c - \hat{f}(V)$$

(13)

$$\dot{e} = k_v \dot{e} - k_p e = M_0^{-1} f(V) - \hat{f}(V)$$

(14)

Where $k_v = 2 \alpha l_{k \times k}$, $k_p = \alpha^2 l_{k \times k}$.

Introducing Eq. (14) and after some manipulations, the resulting state-space expression in the modal space is

$$\dot{X} = AX + B \left( f(V) - \hat{f}(V) \right)$$

(15)

Where $A = \begin{bmatrix} O & I \\ -k_v & -k_p \end{bmatrix}, B = \begin{bmatrix} O \\ M_0^{-1} \end{bmatrix}, X = [e \ \dot{e}]^T, f(V) - \hat{f}(V) = f(V) - f^*(V) + f^*(V) - \hat{f}(V)$

(16)

Definition $\eta = f(V) - f^*(V)$, $f^*(V) = w^*^T h(V)$, $\hat{f}(V) = \hat{w}^T h(V), w^*^T, \hat{w}^T$ are the optimal weights and estimated weights. For the RBF neural network used in this article, the weights are mainly optimized and approximated without considering the center coordinates of the radial basis function and The width is optimized, so the above definition is reasonable.

Eq. (15) can be transformed into

$$\dot{X} = AX + B (\eta - \hat{w}^T h)$$

(17)

Where $\hat{w}^T = \hat{w}^T - w^*^T$. 
3.2 Stability analysis

For the closed-loop system of the above formula, the following Lyapunov function is designed for stability analysis.

\[ V = \frac{1}{2} X^T P X + \frac{1}{2\lambda} \| \tilde{w} \|^2 \]  \tag{18} 

Where P is symmetric positive definite and satisfies.

\[ PA + A^T P = -Q \]  \tag{19} 

Where \( Q \gg 0 \), and the derivative of Eq. (18) is

\[ \dot{V} = \frac{1}{2} \left( (X^T P \dot{X} + \dot{X}^T P X) \right) + \frac{1}{\lambda} \text{tr}(\tilde{w}^T \tilde{w}) \]

\[ = \frac{1}{2} \left( (X^T P (AX + B(\eta - \tilde{w}^T h)) + (X^T A^T + (\eta - \tilde{w}^T h)B^T) P X) \right) + \frac{1}{\lambda} \text{tr}(\tilde{w}^T \tilde{w}) \]

\[ = -\frac{1}{2} X^T Q X + \eta^T B^T P X - \text{tr}(\tilde{w}^T \tilde{w}) \]

due to

\[ h^T \tilde{w} B P X = \text{tr}(B^T P X h^T \tilde{w}) \]

Organizing the above equation, the following equation is given as

\[ \dot{V} = -\frac{1}{2} X^T Q X + \frac{1}{\lambda} \text{tr}(-\lambda B^T P X h^T \tilde{w} + \tilde{w}^T \tilde{w}) + \eta^T B^T P X \]  \tag{20} 

Design the following weight adaptive algorithm

\[ \dot{\tilde{w}}^T = \lambda B^T P X h^T + k_1 \lambda \| X \| \tilde{w}^T \]

\[ \dot{\tilde{w}} = \lambda h X^T P B + k_1 \lambda ||X|| \tilde{w} \]  \tag{21} 

It can be proved that the above adaptive algorithm can ensure the stability of the system by designing appropriate control parameters.

Substituting equation (21) into equation (20)

\[ \dot{V} = -\frac{1}{2} X^T Q X + \frac{1}{\lambda} \text{tr}(k_1 \lambda \| X \| \tilde{w}^T \tilde{w}) + \eta^T B^T P X \]

\[ = -\frac{1}{2} X^T Q X + k_1 \| X \| \| \tilde{w}^T \tilde{w} \| \| \tilde{w} \| \] 

According to the nature of the norm

\[ \text{tr}(\tilde{w}^T \tilde{w}) = \text{tr}(\tilde{w}^T (\tilde{w}^* + \tilde{w})) < \| \tilde{w} \|_F \| \tilde{w}^* \|_F - \| \tilde{w} \|_F^2 \]

Therefore

\[ \dot{V} < -\frac{1}{2} X^T Q X + k_1 \| X \| \| \tilde{w} \|_F \| \tilde{w}^* \|_F - \| \tilde{w} \|_F^2 \| \tilde{w} \|_F^2 + \eta^T B^T P X \]

\[ < -\frac{1}{2} \lambda_{\min}(Q) \| X \|^2 + k_1 \| X \| \| \tilde{w} \|_F \| \tilde{w}^* \|_F - k_1 \| X \| \| \tilde{w} \|_F + \| \eta \| \lambda_{\max}(P) \| X \| \]

\[ < -\| X \| \left( \frac{1}{2} \lambda_{\min}(Q) \| X \| + k_1 \left( \| \tilde{w} \|_F - \frac{w_{\max}}{2} \right)^2 - \frac{k_1}{4} w_{\max}^2 - \| \eta \| \lambda_{\max}(P) \right) \]

According to \( \dot{V} < 0 \), the following inequalities can be derived as

\[ \frac{1}{2} \lambda_{\min}(Q) \| X \| > \frac{k_1}{4} w_{\max}^2 + \| \eta \| \lambda_{\max}(P) \]

Therefore, \[ \| X \| >> \frac{2}{\lambda_{\min}(Q)} \left( \frac{k_1}{4} w_{\max}^2 + \| \eta \| \lambda_{\max}(P) \right) \]  \tag{22} 

It can be obtained from equation (22) that increasing the eigenvalue of Q and reducing the eigenvalue of P can ensure that \( \dot{V} < 0 \) can also reduce the convergence value of X and improve the convergence effect of the system.
4 Control simulation
Establish the finite element model of the reflector in the finite element analysis software, and derive the mass matrix and stiffness matrix of the reflector model, combining the RBF neural network and the above-mentioned adaptive weight algorithm to realize adaptive control of the space-borne antenna reflector model.

4.1 Simulation parameters
The damping parameters in the reflector model are set as: damping coefficient $\beta_1 = 2 \times 10^{-3}$, $\beta_2 = 10^{-4}$, the system parameters and the parameters in the controller are shown in Table 1 and Table 2.

| Table 1. The parameters of system |
| parameter | value |
|----------|------|
| Number of truncated modes | 19 |
| Mass matrix error | 0.01 $M_0$ |
| Stiffness matrix error | 0.01 $K_0$ |

| Table 2. The parameters of controller |
| parameter | value |
|----------|------|
| $\alpha$ | 10 |
| $Q$ | $50I_{38 \times 38}$ |
| $\gamma$ | 20 |
| $k_1$ | 0.01 |

The evaluation standard of the control effect is the root mean square value of the surface error of the reflector, and the root mean square value (RMS) is defined as

$$RMS = \sqrt{\frac{\sum_{i=1}^{k} (\delta_i - \delta_i^q)^2}{k}}$$

(23)

$\delta_i$ is the displacement of the i-th node, $\delta_i^q$ is the ideal displacement of the i-th node, and k is the number of nodes.

4.2 RBF neural network
The neural network topology diagram is shown in Figure 2. It can be seen from the figure that the RBF neural network is composed of three layers: input layer, middle layer and output layer.

![Figure 2. RBF neural network topology](image)

The first layer is the input layer, which represents the node displacement and velocity error of the reflecting surface. The second layer is the middle layer. The activation function of each middle layer node uses the radial basis function. The third layer is the output layer, which represents the estimated value of $f(V)$. The output layer result is related to the radial basis function and weight coefficient. The equation of the neural network model based on the radial basis function is given as
\[ f(V) = \sum \omega_j \varphi(\|V - V_j\|) \]

This paper uses the standard Gaussian function as the activation function, which can be expressed as
\[ \varphi(V) = e^{-\frac{\|V - c_j\|^2}{2\sigma_j^2}} \]
\(c_j\) is the base function center of the j-th central layer unit, and \(\sigma_j\) represents the variance. There are three parameters to be learned by the RBF neural network: \(c_j\), \(\sigma_j\), and the weight between the intermediate layer and the output layer.

The structure of the RBF neural network is 38-50-1, the input of the neural network is \(V = [q_c, \dot{q}_c]\), the parameters of the Gaussian function are set to \(c_j = \text{linspace}(-0.1:0.1:50)\), \(\sigma_j = 1\), the initial weight coefficients are set to 0.1.

### 4.3 Simulation results

Figure 3 is the RMS error curve of the RBF neural network adaptive control reflector. It can be seen from the figure that the initial profile error root mean square value is about 700 μm. When the control is stable, the root mean square value reaches approximately 0.8 μm. The simulation results can be concluded that the RBF neural network adaptive control can reduce the shape error of the reflector to a certain extent.

![RMS error curve of RBF adaptive control](image)

**Figure 3.** RMS error curve of RBF adaptive control.

Fig. 4 shows the acting force of actuator 1, actuator 2, actuator 3 and actuator 4 under RBF neural network adaptive control. Based on the RBF neural network adaptive control, the acting force of the actuator does not fluctuate sharply.

![Acting force curve of actuators](image)

**Figure 4.** Acting force curve of actuator 1,2,3,4 under RBF neural network adaptive control.

Fig. 5 is the surface error of the reflector at four different times during the control process. The initial
error is a random error distributed all over the reflector, as shown in Fig. 5(a), and at the end of the control, the displacement error of every free node is reduced to below 2.5 μm, which is represented in Fig. 5(d). Fig. 5 visually shows that the adaptive control of RBF neural network method is valid to solve the reflector shape control problem.

![Figure 5](image)

**Figure 5** The surface error under RBF neural network adaptive control: (a) control time t=0s; (b) control time t=2s; (c) control time t=4s; (d) control time t =6s.

5. Conclusion
This paper takes the space-borne antenna reflector as the research object, and adopts RBF neural network adaptive controller to control the surface error after thermal deformation. Taking into account the interference of uncertainty load, the active control of the reflector system is realized. The follow conclusion can be obtained.

1. The thermal-mechanical coupling finite element model of the space-borne antenna reflector is theoretically established. Based on the established finite element model, an adaptive control law of surface accuracy considering the uncertainty is proposed.

2. When there is a large load disturbance from the outside, the RBF neural network adaptive control has relatively small fluctuations and good robustness.

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