Proton Radius from Muonic Hydrogen Spectroscopy and Effect of Atomic Nucleus Motion

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The proton radius has been measured in electron-proton scattering experiments and laser based spectroscopy of muonic hydrogen. The latter method is based on the precise calculations for the atomic energy levels in the approximation of static nucleus, and includes numerous corrective effects. The 4% discrepancy between two measuring methods is known as the proton radius puzzle. We suggest that this discrepancy may be caused by an additional electromagnetic interaction with the magnetic moment generated by the nucleus motion around the center of mass of the muonic hydrogen. Our estimation show that the effect of the atomic nucleus motion is high enough and may help to solve the proton radius puzzle.

I. INTRODUCTION

The proton charge radius is of fundamental importance for understanding the proton’s internal structure. From the elastic electron-proton scattering experiments [1–4] and corresponding analyses [5, 6], the average value for the proton radius yields roughly 0.878 ± 0.011 fm. However, according to a more precise spectroscopy of the muonic hydrogen [9, 10] the proton radius turns out to be of 0.841 ± 0.001 fm, a factor of 1.044 lower than the value from the electron-proton scattering experiments. This discrepancy is known as the “proton radius puzzle”.

A detailed analysis of the experimental and theoretical situation around the proton radius problem, including prospects for solutions to the proton radius puzzle, is given in the dedicated reviews [10–12].

II. MUONIC ATOM

According to Dirac equations with a Coulomb field, the atomic shells 2S and 2P of the hydrogen atom should have the same energy. The Lamb shift is the violation of this rule which is observed as electromagnetic transition between the energy levels of the aforementioned atomic shells.

As it was shown soon after its discovery, the Lamb shift is caused by the interaction of atomic electrons with vacuum fluctuations such as virtual electron-positron pairs.

The modern theory of light hydrogenic atoms based on the Dirac equation with a Coulomb [12] accounts for numerous corrections to the Lamb shift.

A theoretical description of the muonic atom is similar to that of the ordinary hydrogen, provided the electron mass \( m_e \) is replaced with the reduced mass of muon:

\[ m_\mu^* = \frac{m_\mu}{1 + \frac{m_e}{m_\mu}} = 95 \text{ MeV}/c^2, \]  

where \( m_\mu \) and \( m_e \) are the masses of the atomic nucleus (proton) and orbital muon, respectively.

The energy levels of the muonic atom are determined using the precise relativistic wave functions calculated in the Coulomb field of the static nucleus. However, there are specific effects for the muon hydrogen caused by that the orbital muon is \( m_\mu/m_e = 207 \) times closer to the nucleus.

In particular, due to a smaller muonic atom size, the wave-function of the orbital muon overlaps with the atomic nucleus \( (m_\mu/m_e)^3 \approx 10^7 \) times stronger than in the regular hydrogen that makes it more sensitive to the proton size. Also one may expect a significant sensitivity of the muonic hydrogen shells to the electromagnetic form factor of the proton, as well as to its polarization in the electric field of the muon.

As the size of virtual electron-positron pairs is compatible with the radius of the muonic hydrogen, the last is more sensitive to vacuum fluctuations, that makes it easier to measure the Lamb shift. In the ordinary hydrogen the degenerated \( 2S_{1/2} \) and \( 2P_{1/2} \) levels split due to the Lamb shift by 1058 MHz (4 \( \mu \)eV). Since in the muonic hydrogen two particles are very close to each other the corresponding Lamb shift is \( \approx 10^5 \) times higher (206 meV).

Consequently, the muonic atom shells are very sensitive to the spatial structure of the atomic nucleus and the Lamb shift experiment [9, 10] provides a significantly better precision for the proton radius measurement compared to the regular hydrogen spectroscopy and electron-proton scattering experiments.

How magnetic forces affects the muonic atom levels?

The atomic levels are classified by the orbital momentum \( \vec{l} \), the total momentum \( \vec{j} = \vec{l} + \vec{s} \), where \( \vec{s} \) is the orbital
particle spin, and the full moment of the atom $\vec{F} = \vec{j} + \vec{I}$, where $\vec{I}$ is the spin of the atomic nucleus.

The fine (spin-orbit) interaction of magnetic moments shifts $j = 1/2$ level with respect to $j = 3/2$ state. Interaction of the nucleus magnetic moment with the atomic magnetic fields (hyper-fine interaction) further splits the energy levels of $S$ and $P$ shells on two different levels depending on the orientation of the nucleus spin $\vec{I}$. Hence the energy level depends on the total angular moment $\vec{F}$ of the atom. Thus, both the regular hydrogen and the muonic hydrogen atoms manifest six separated levels shown in Fig. 1. Since in the muonic atom particles are 207 times closer than in regular hydrogen the separation of energy levels is obviously higher for the muonic atom.

In the rigorous theory [13] (p. 72, Eq. (2)) the part of the Hamiltonian responsible for the hyper-fine interaction is proportional to the magnetic moment of the nucleus. For example, the hyper-fine splitting in the muonic hydrogen in the $S$ state, including nucleus size effects, is proportional to the so called Fermi energy $E_F$:

$$E_F = \mu_p \frac{8\alpha^4}{3\pi^3} \frac{m_p^2 m_n^2}{(m_p + m_n)^3} \propto \mu_p m_n^2,$$

where $m_{\mu,p}$ are the atomic particle masses defined in Eq. (1), $\mu_p$ is the nucleus (proton) magnetic moment, and $n$ - principal quantum number. With this relation we emphasize that in the rigorous approach [13, 14] the energy of the hyper-fine interaction is proportional to the magnetic moment of the static atomic nucleus and second power of a lepton mass.

In this article, we focus on the hyper-fine interaction of the atomic muon with the atomic nucleus and consider the validity of the static nucleus approximation to the interpretation of the very precise results of the Lamb Shift Experiment.

### III. LAMB SHIFT EXPERIMENT

The Lamb shift experiment is focused on the transition between $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ states (Fig. 1) of muonic hydrogen [15], and aims to improve the precision of the proton radius measurement by a factor of 20. The method of this experiment is as follows:

Low-energy negative muons enters the pure hydrogen-filled vessel, where they come to a rest. The target vessel is exposed to 5 T magnetic field used to focus the low-momentum beam.

When negative muons stop, the excited muonic hydrogen atoms are formed, a majority of which de-excite within 100 ns to the ground state. However, about 1% of muonic atoms populate a meta stable $2S_{1/2}^{F=1}$ state. At the atmospheric pressure, this shell de-populates quickly due to a very frequent collisions of muonic atom with gas molecules.

In order to reduce the de-population rate of the meta stable state the pressure in the Lamb shift experiment is as low as $10^{-3}$ bar and, correspondingly, the $2S_{1/2}^{F=1}$ state lifetime is of 1.3 $\mu$s. This relatively short period is long enough to generate a trigger for the state of art laser system.

![Fig. 1. Hyperfine terms of hydrogen-like atoms with the principal quantum number $n = 2$. (a) Regular hydrogen. The 4 $\mu$eV interval is the “classical” Lamb shift with $2S_{1/2}$ state higher than $2P_{1/2}$. (b) Muonic hydrogen. The vacuum polarization pulls the $2S_{1/2}$ states below the $2P_{1/2}$ states. The $2P_{3/2}^{F=1}-2P_{3/2}^{F=2}$ splitting is 3.4 meV [11].](image)

#### A. Laser System

The function of the tunable laser system is to stimulate a resonant transition between the aforementioned $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ muonic hydrogen levels. The proton radius is determined from the observed resonance frequency. The laser system is a cascade of several laser based devices with the wavelength of the final device tunable around 50 THz. On the muon stop trigger, with a rate of $10^3$ s$^{-1}$, the laser system generates a 5 ns light pulse with the delay of 1.5 $\mu$s only, that is compatible with the lifetime of the initial $2S_{1/2}^{F=1}$ state. A powerful light pulse (0.25 $mJ$) illuminates the mirrored inner space of the target volume that contains hydrogen with recently formed muonic atoms. If the tunable laser is on resonance with the $2S_{1/2}^{F=1}-2P_{3/2}^{F=2}$ transition frequency, then such transition occurs in about 30% of muonic atoms. Within a very short lifetime of 8.5 ps, the muonic hydrogen $2P_{3/2}^{F=2}$ state de-excite to $1S_{1/2}$ ground state emitting 1.9 keV photons. These photons are detected by the array of avalanche photo-diodes covering a large area around the muon stop volume.

#### B. Resonance Frequency and Proton Radius

The resonance frequency is determined by observing the yield of the 1.9 keV photons in a function of the varying wavelength. Such dependency manifests a peak
at \( f_R = 49.885 \text{ THz} \) corresponding to the \( 2S_{1/2}^{-1}2P_{3/2}^{F=2} \) transition.

The value for the proton radius yields from thus measured resonant frequency \( f_R \) using the relation [10] (p. 14, Eq. (2.18)) obtained for the Lamb shift with the precise relativistic wave functions:

\[
f_R/\text{THz} = 50.7700 - 1.2634 r_p^2 + 0.008394 r_p^3,
\]

where the proton radius is given in femtometers (\( fm \)) and is defined as the RMS radius of the spherically symmetric distribution of the proton charge density. In terms of energy, the Lamb shift is parameterized [15] (p. 1, Eq. (1.1)) as:

\[
\Delta E_R/\text{meV} = 209.9685 - 5.2248 r_p^2 + 0.0347 r_p^3.
\]

Here, the major contribution (\( \approx 209 \text{ meV} \)) is given by the effects of vacuum polarization. This "pedestal" term also accounts for more than 20 corrections that influence the energy levels of \( 2S_{1/2}^{-1} \) or \( 2P_{3/2}^{F=2} \) state [12, 15].

The second and third terms of Eq. (4) are sensitive to the proton radius. The most sensitive to the proton radius second term is given by the relation [12] (p. 110, Eq. (6.3)) or [15] (p. 136, Eq. (D.5)):

\[
\Delta E_2 = -\frac{2}{3} \alpha n^{-3} m_p^3 r_p^2 \rho_{l0} = -5.2248 r_p^2 = -4.03 \text{ meV},
\]

where \( n = 2 \) is the atomic principle quantum number of only \( S \)-state, since \( P \)-state energy level is insensitive to the nucleus size, \( m_p \) - the reduced mass of the muon, \( \alpha \) - Sommerfeld’s constant, and \( r_p = 0.878 \text{ fm} \) is used for the numerical estimate.

\[ \text{IV. ADVANTAGES AND QUESTIONS} \]

Although according to Eq. (5), the effect of proton size in the muonic \( 2S_{1/2}^{-1}2P_{3/2}^{F=2} \) Lamb shift is quite small, it is about hundred times higher than in the ordinary hydrogen. That’s what makes the muonic hydrogen spectroscopy a very precise and sensitive measuring instrument.

However, the basic relationships for the proton radius Eqs. (3) and (4) include numerous contributions, and there is no guarantee that there are no other effects that can alter the interpretation of the measured frequencies and the corresponding transition energies.

\[ \text{A. Nucleus Spin} \]

The proton radius yields from the Eq. (4) which is based on the precise relativistic wave functions calculated in a static Coulomb field. It is important to notice that the Dirac equation with a static Coulomb field accounts for the spin of electrons and muons, but ignores the spin of the atomic nucleus. Therefore, the interaction of the atomic magnetic moment with the magnetic moment of proton is accounted separately [15]. In particular, according to the precise calculations performed for the \( 2P_{3/2}^{F=2} \) state, the interaction with nucleus spin contributes as much as 1.3 meV [15] to the \( 2S_{1/2}^{-1}2P_{3/2}^{F=2} \) energy interval.

\[ \text{B. Static Nucleus Approximation} \]

From the relevant reviews [16, 17], we know that the energy levels of muonic hydrogen are calculated using the wave functions obtained in the approximation of an infinitely heavy stationary nucleus.

However, the recoil corrections are taken into account and summarized in Table 2.2 [15]. From this Table the total recoil effect may be estimated as 0.0705 meV, or 1.7% of the proton radius effect (Eq. (5)).

Although the influence of the finite mass of the nucleus is taken into account through the reduced muon mass and the nucleus recoil momentum [15, 18], the static nucleus approximation may not correspond to reality.

The reason is that both particles are actually "orbiting" the common center of mass. Hence the atomic nucleus can generate an additional magnetic moment which is referred below as the "induced magnetic moment". As a result, the Coulomb potential which has been used for precise calculations of atomic \( S \)- and \( P \)-levels, has to be modified accordingly.

\[ \text{V. EFFECT OF ORBITING NUCLEUS} \]

As it is known from the atomic physics the magnetic moment of the orbital electron may be presented as

\[
\mu_e = \frac{e}{2m_e c} \sqrt{l(l+1)} = \mu_B \sqrt{l(l+1)},
\]

where \( l \) is the orbital moment of electron, \( \mu_B \) stands for the Bohr magneton. In the muonic atom the muon mass should be used on place of the electron mass.

What may be the magnetic moment of an orbiting nucleus? Let’s first estimate it in classical approximation, where one may consider a hydrogen-like atom as two charged particles rotating around the common center of mass with the same angular frequency \( \omega \). The absolute values of the orbital angular moments \( l_{\mu,p} \) and magnetic moments \( \mu_{\mu,p} \) for each particle are:

\[
\begin{align*}
l_{\mu,p} &= m_{\mu,p} \omega r_{\mu,p}^2, \\
\mu_{\mu} &= -\frac{e}{2m_\mu} l_{\mu} = -\frac{e}{2} \omega r_{\mu}^2, \\
\mu_p &= \frac{e}{2m_p} l_p = \frac{e}{2} \omega r_p^2,
\end{align*}
\]
where $\omega$ is the angular frequency, $m_{\mu,p}$ are the masses of muon and proton, respectively, and $r_{\mu,p}$ are corresponding particle orbit radii. Since both particles "orbit" the common center of mass, the rotation periods are identical and the obvious relation holds: $m_{\mu}r_{\mu} = m_{p}r_{p}$. For convenience, using Eq. (7), we express the orbital magnetic moment of the proton $\mu_p$ in terms of the magnetic moment of the orbiting partner, i.e., muon:

$$\mu_p = \mu_{\mu}\left(\frac{m_p}{m_{\mu}}\right)^2. \quad (8)$$

It is interesting to notice that although the total orbital moment $l_{\mu} + l_{p}$ coincides with the orbital moment calculated for the muon reduced mass orbiting the motionless nucleus, the effective magnetic moment of such atom does not follow this rule, and in the limit of equal masses the effective magnetic moment zeroes.

In accordance to Eq. (6) the smallest nonzero value for $\mu_\mu$ is $\sqrt{2}\mu_{\mu}(p)$ where $\mu(p) = 4.5 \times 10^{-26} \text{ JT}^{-1}$ is the nominal magnetic moment of muon.

A. Resonance Shift

Fortunately, the estimation of the induced magnetic moment effect in the $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ resonant transition may be done without going into detailed theoretical calculations. With such goal in mind, we focus on the $2P_{3/2}^{F=2}$ and $2P_{3/2}^{F=1}$ states of a muonic atom, which have opposite orientations of the nucleus spin (Fig. 4).

Precision calculations for static nucleus show ([III], p. 27, Eq. (32) and p. 60, Fig. 4) that interaction of the nucleus magnetic moment with the atomic magnetic field results in the following hyper fine splitting of $2P_{3/2}^{F=2}$ and $2P_{3/2}^{F=1}$ states:

$$\Delta E = (3.3926 - 0.1446) \text{ meV} \approx 3.25 \text{ meV}, \quad (9)$$

which is obviously proportional to the doubled value of the nominal magnetic moment $\mu_N$ of the nucleus (proton) via Fermi energy, as prescribed by Eq. (2) for all hyper fine effects.

Since the "radius" of the proton orbit is very small compared to the atomic radius, we reasonably assume that the orbital proton interacts with the same magnetic field as in the center of the atom. Hence, the effective magnetic moment may be considered as a vector sum of the nucleus nominal magnetic moment and its orbital magnetic moment.

Correspondingly the effect of the induced magnetic moment may be considered as a small linear perturbation of the value $\Delta E$ from Eq. (9). In order to estimate the effect of the nucleus orbital motion we scale a halve of this precise value by the ratio of the induced magnetic moment of the orbital proton (Eq. (8)) to its nominal magnetic moment $\mu(p) = 1.41 \times 10^{-26} \text{ JT}^{-1}$ used in Eq. (9):

$$\Delta W \approx \frac{\mu_p}{2\mu(p)} \Delta E = \frac{\mu_{\mu}(p)}{2\mu(p)\left(\frac{m_p}{m_{\mu}}\right)^2} \Delta E \approx 0.1 \text{ meV}. \quad (10)$$

Note that the magnetic moment of the orbital proton is opposite to that of negative muon, and similar to the static nucleus, the induced magnetic moment shifts the $2F_{3/2}$ state to higher energies, as well as the transition energy, leading to a higher proton radius, provided the measured resonant transition energy $\Delta E_R$ in Eq. (4) remains unchanged. The estimate Eq. (10) constitutes 2.5% of the energy specified in Eq. (5), that correspondingly translates to the $\approx 1\%$ effect in terms of proton radius.

B. Orbital Proton and Bohr Magneton

The estimate Eq. (10) is based on Eq. (5), which suggests a direct proportionality of the induced magnetic moment to the second power of the proton orbit radius. This relation is based on the assumption of classical behaviour of atomic components, i.e., that there is a strict correlation between "coordinates" of two atomic particles.

In quantum case one may attribute the "orbital" nucleus with the magnetic moment defined by Eq. (6) where the mass of electron is replaced with the proton mass and magnetic moment of the orbital particle is inversely proportional to the first power of its mass.

Hence, following the mass dependence of the orbital magnetic moment in quantum case, one may estimate the orbital magnetic moment of proton in the muonic hydrogen as:

$$\mu_p = \mu(p)\left(\frac{m_p}{m_{\mu}}\right). \quad (11)$$

As we have an additional magnetic moment, one may expect that atomic levels with given $F$ are split in accordance to the mutual orientation of the nucleus spin and its orbital moment. For the projection of the resulting magnetic moment $\mu_\mu$ to the direction of the magnetic field in the center of the atom one may write

$$\mu_\mu = \mu(p) \pm \mu_\mu = \mu(p)\left(1 \mp \frac{\mu(p)}{\mu_\mu}\left(\frac{m_p}{m_{\mu}}\right)\right), \text{ or } \mu(p). \quad (12)$$

Following the arguments from the classical consideration above one may expect that the resulting projection $\mu_\mu$ will be higher than the nominal magnetic moment $\mu(p)$ and, correspondingly, the $2P_{3/2}$ energy level may be higher by the value

$$\Delta W \approx \frac{\mu_p}{2\mu(p)} \Delta E = \frac{\mu_{\mu}(p)}{2\mu(p)\left(\frac{m_p}{m_{\mu}}\right)^2} \Delta E \approx 0.5 \text{ meV}. \quad (13)$$

Note that this value has to be added to the right side of Eq. (4) and compared to 4.03 meV from the relation...
Eq. (5). Comparing the estimation Eq. (13) with the value from Eq. (5) one may conclude that the additional shift 0.5 meV translates to 6% higher proton radius \( r_p = 0.89 \) fm, provided that experimental value \( f_R \) in the left side of Eq. (4) remains as is. The last value for the proton radius almost fits the experimental value interval \((0.867, 0.889)\) fm, that was estimated from various scattering experiments.

What would be the effect of the nucleus motion in \( P \)-state of the ordinary hydrogen? In this relation it is interesting to notice that from the measurement of the 1S – 3S transition in regular hydrogen \([19]\) the proton radius yields as high as 0.877 fm, while from the 2S – 4P transition frequency, measured in Garching \([20]\) and 2S–2P \([21]\), it yields a lower values - 0.835 and 0.833 fm, respectively.

VI. CONCLUSION

We have considered the effect of the nucleus orbital motion around the atom’s center of mass as a perturbation to the accurately calculated data on the muonic hydrogen hyper-fine structure with static nucleus. We have shown that nucleus motion is a possible source for the atomic levels distortion, leading to the underestimation of the proton radius. Hence, the interaction of the orbital atomic nucleus with the atomic magnetic field needs to be strictly accounted when calculating the transition energies.

The effect of nucleus motion may significantly change the estimate of proton radius from the Lamb’s shift experiment to higher values, thereby reducing, or even eliminating, the discrepancy that makes up the proton radius puzzle.

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[1] P. Lehmann, R. E. Taylor, and R. Wilson, “Electron-proton scattering at low momentum transfers,” Phys. Rev. 126, 1183 (1962).
[2] L. N. Hand, D. G. Miller, and R. Wilson, “Electric and magnetic formfactor of the nucleon,” Rev. Mod. Phys. 35, 335 (1963).
[3] J. J. Murphy, Y. M. Shin, and D. M. Skopik, “Proton form factor from 0.15 to 0.79 fm−2,” Phys. Rev. C 9, 2125 (1974) [erratum: Phys. Rev. C 10, 2111 (1974)].
[4] G. G. Simon, C. Schmitt, F. Borkowski, and V. H. Walther, “Absolute electron proton cross-sections at low momentum transfer measured with a high pressure gas target system,” Nucl. Phys. A 333, 381 (1980).
[5] I. Sick, “On the RMS radius of the proton,” Phys. Lett. B 576, 62 (2003).
[6] P. G. Blunden and I. Sick, “Proton radii and two-photon exchange,” Phys. Rev. C 72, 057601 (2005).
[7] P. J. Mohr, B. N. Taylor, and D. B. Newell, “CODATA Recommended Values of the Fundamental Physical Constants: 2006,” Rev. Mod. Phys. 80, 633 (2008).
[8] R. Pohl, A. Antognini, F. Nez, F. D. Amaro, F. Biraben, J. M. R. Cardoso, D. S. Covita, A. Dax, S. Dhawan, L. M. P. Fernandes et al. “The size of the proton,” Nature 466, 213 (2010).
[9] A. Antognini, F. Nez, K. Schuhmann, F. D. Amaro, FrancoisBiraben, J. M. R. Cardoso, D. S. Covita, A. Dax, S. Dhawan, M. Diepold et al. “Proton Structure from the Measurement of 2S – 2P Transition Frequencies of Muonic Hydrogen,” Science 339, 417 (2013).
[10] A. Antognini, F. Kottmann, F. Biraben, P. Indelicato, F. Nez and R. Pohl, “Theory of the 2S-2P Lamb shift and 2S hyperfine splitting in muonic hydrogen,” Annals Phys. 331, 127 (2013).
[11] R. Pohl, R. Gilman, G. A. Miller, and K. Pachucki, “Muonic hydrogen and the proton radius puzzle,” Ann. Rev. Nucl. Part. Sci. 63, 175 (2013).
[12] M. I. Eides, H. Grotch, and V. A. Shelyuto, “Theory of Light Hydrogenic Bound States,” Springer Tracts Mod. Phys. 222, pp. 1-262 (2007).
[13] A. Adamczak, D. Bakalov, L. Stoychev, and A. Vacchi, “Hyperfine spectroscopy of muonic hydrogen and the PSI Lamb shift experiment,” Nucl. Instrum. Meth. B 281, 72 (2012).
[14] R. N. Faustov, E. V. Cherednikova, and A. P. Martyinenko, “Proton polarizability contribution to the hyperfine splitting in muonic hydrogen,” Nucl. Phys. A 703, 365 (2002).
[15] A. S. Antognini, “The Lamb shift experiment in muonic hydrogen,” 186 pp, Dissertation, Ludwig Maximilians University, Munich, Germany (2005); https://core.ac.uk/download/pdf/11028066.pdf.
[16] C. E. Carlson, “The Proton Radius Puzzle,” Prog. Part. Nucl. Phys. 82, 59 (2015).
[17] A. Antognini, “Muonic atoms and the nuclear structure,” [arXiv:1512.01765 [physics.atom-ph]].
[18] S. G. Karshenboim, V. G. Ivanov, and E. Y. Korzinin, “Relativistic recoil corrections to the electron-vacuum-polarization contribution in light muonic atoms,” Phys. Rev. A 85, 032509 (2012).
[19] H. Fleuraey, S. Galtier, S. Thomas, M. Bonnard, L. Julien, F. Biraben, F. Nez, M. Abgrall, and J. Guéna, “New measurement of the 1S – 3S transition frequency of hydrogen: contribution to the proton charge radius puzzle,” Phys. Rev. Lett. 120, no.18, 183001 (2018).
[20] A. Beyer, L. Maisenbacher, A. Matveev, R. Pohl, K. Khabarova, A. Grinin, T. Lamour, D. C. Yost, T. W. Hänisch, N. Kolachevsky et al. “The Rydberg con-
stant and proton size from atomic hydrogen,” Science 358, no.6359, 79 (2017).
[21] N. Bezginov, T. Valdez, M. Horbatsch, A. Marsman, A. C. Vutha, and E. A. Hessels, “A measurement of the atomic hydrogen Lamb shift and the proton charge radius,” Science 365, no.6457, 1007 (2019).