Conservation-Based Modeling and Boundary Control of Congestion with an Application to Traffic Management in Center City Philadelphia

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Abstract—This paper develops a conservation-based approach to model traffic dynamics and alleviate traffic congestion in a network of interconnected roads (NOIR). We generate a NOIR by using the Simulation of Urban Mobility (SUMO) software based on the real street map of Philadelphia Center City. The NOIR is then represented by a directed graph with nodes identifying distinct streets in the Center City area. By classifying the streets as inlets, outlets, and interior nodes, the model predictive control (MPC) method is applied to alleviate the network traffic congestion by optimizing the traffic inflow and outflow across the boundary of the NOIR with consideration of the inner traffic dynamics as a stochastic process. The proposed boundary control problem is defined as a quadratic programming problem with constraints imposing the feasibility of traffic coordination, and a cost function defined based on the traffic density across the NOIR.

I. INTRODUCTION

In the process of urbanization and the rapid popularization of private vehicles, the problem of urban traffic congestion has become more and more prominent and produced numerous negative impacts on economy [1], [2] and environment [3], [4]. Traffic congestion can destroy the urban environment and ecology. Due to the low-speed driving conditions, the emission of greenhouse gas, noxious gas and noise will increase, and that will badly affect human health [5].

Over the past decades, a large number of scholars have developed prediction, control, and optimization methods to solve the challenges of traffic congestion in urban areas. Ref. [6] offers an integration of fuzzy rule-based systems and the genetic algorithms to model and predict the traffic coordination. Refs. [7] and [8] develop the traffic predictive approaches by relying on driver behavior and bus driving intervals. With the rapid development of V2X and autonomous driving technology, floating car data (FCD) technology has been widely used to estimate the traffic state [9], [10].

Researchers have also developed different model-based and model-free approaches to obtain dynamics of traffic coordination and control congestion. The model-based macroscopic fundamental diagram (MFD), whose applicability for urban traffic is experimentally verified in [11], is an efficient tool to obtain dynamics of an urban traffic network. Ref. [12] applies the MFD model to evaluate the traffic accumulation amount, and estimate the traffic state. Refs. [13], [14] integrate MFD with perimeter control to improve the mobility of a traffic network. Moreover, Refs. [15]–[19] apply the cell transmission model (CTM) method to enhance the efficiency and accuracy of the network modeling by partitioning the traffic network into homogeneous road elements. Recently, with the improvement of computing capacity and the development of AI technology, reinforcement learning (RL) method has attracted more and more attention. Ref. [20] presents an overview of the recently-developed RL algorithms in the area of adaptive traffic signal control. In Refs. [21]–[24], researchers integrate the model-free methods with RL approaches to optimally plan the functionalities of traffic signals. The model predictive control (MPC) is another commonly used tool for controlling the traffic dynamics in urban networks. Refs. [14], [13], [25], [26] and [27] apply the MPC approach to assign optimal boundary control variables. Ref. [28] integrates the MPC and mixed-integer linear programming (MILP) to manage the complexity of traffic coordination optimization.

This paper offers an integration of mass conservation law and MPC-based boundary control to obtain the traffic dynamics and alleviate the traffic congestion. We use the Simulation of Urban Mobility (SUMO) software to convert the real street map data into a directed graph representing a network of inter-connected roads (NOIR). While we previously modeled traffic inner dynamics as a time-invariant stochastic process in Refs. [25]–[27], this paper applies the mass conservation law to model the traffic inner dynamics as a non-stationary stochastic process and obtain the traffic feasibility conditions at the interior nodes. Compared to Refs. [25]–[27] that control the congestion only through the inlet

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boundary nodes, we apply the MPC to control the boundary inflow through the NOIR inlets, and the boundary outflow through the NOIR outlets. For the case study, we use the proposed model and control approach to evaluate congestion management in a certain area of Center City Philadelphia with the map shown in Fig. [I].

This paper is arranged in the following structure: Section II explains the preliminary notions of graph theory. The problem statement is presented in Section III and followed by obtaining the traffic network dynamics and providing the feasibility conditions in Section IV. Section V presents the problem statement is presented in Section III and followed by obtaining the traffic network dynamics and providing the feasibility conditions in Section IV. Section V presents the boundary control approach based on the MPC method to control the traffic congestion. Then, the simulation results are reported in Section VI and followed by the conclusion in Section VII.

II. PRELIMINARY NOTIONS OF GRAPH THEORY

In this paper, an NOIR is represented by graph \( G(V,E) \) where \( V \) and \( E \subset V \times V \) define nodes and edges of graph \( G \), respectively. We use \( i \in V \) to represent a road element in the NOIR. Note that all of the road elements partitioned and generated by SUMO in the NOIR are unidirectional. The bidirectional road is presented by two parallel one-way road elements (See Fig[I]). Also, since we are more interested in the boundary traffic dynamics, the traffic light controller is not in consideration in this paper. Edge \((i,j)\in E \) represents a directed connection from road element \(i\in V\) to road element \(j\in V\).

Set \( V \) can be partitioned as \( V=V_{in}\cup V_{out}\cup V_{f} \), where subsets \( V_{in} = \{1,\ldots,N_{in}\} \), \( V_{out} = \{N_{in}+1,\ldots,N_{out}\} \), and \( V_{f} = \{N_{out}+1,\ldots,N\} \) define the index numbers of inlets, outlets, and interior road elements respectively. For every road element \(i\in V\), sets

\[
I_{i} = \{ j : (j, i) \in E \}, \quad (1a)
\]

\[
O_{i} = \{ j : (i, j) \in E \} \quad (1b)
\]

define in-neighbors and out-neighbors. Traffic is directed from in-neighbor \(j\in I_{i}\) towards \(i\in V\setminus V_{in}\), or it is directed from \(i\in V\setminus V_{out}\) towards out-neighbor \(j\in O_{i}\).

III. PROBLEM STATEMENT

We implement the mass-conservation law to obtain traffic dynamics at every road element \(i\in V\). Let \(s_{i}[k]\) denote the external flow, and \(\rho_{i}[k]\), \(y_{i}[k]\), and \(z_{i}[k]\) denote traffic density, network inflow, and network outflow of road element \(i\in V\), respectively. Then, traffic dynamics at road element \(i\in V\) can be defined by

\[
\rho_{i}[k+1] = \rho_{i}[k] + y_{i}[k] - z_{i}[k] + s_{i}[k] \quad (2)
\]

where \(k=0,1,2,\cdots\) denotes the discrete sampling time.

External flow \(s_{i}[k]\) quantifies the traffic inflow entering the NOIR through inlet road element \(i\in V_{in}\), or the traffic outflow leaving the NOIR through outlet road element \(i\in V_{out}\) within time interval \([t_{k},t_{k+1}]\). We define \(s_{i}[k]\) as follows:

\[
s_{i}[k] = \begin{cases} u_{i}[k] \geq 0 & i \in V_{in} \\ -v_{i}[k] \leq 0 & i \in V_{out} \\ 0 & i \in V_{f} \end{cases} \quad (3)
\]

Network inflow \(y_{i}[k]\) and network outflow \(z_{i}[k]\) are given by

\[
y_{i}[k] = \begin{cases} 0 & i \in V_{in} \\ v_{i}[k] & i \in V_{out} \\ \sum_{j \in O_{i}} q_{i,j}[k] z_{j}[k] & i \in V_{f} \end{cases} \quad (4a)
\]

\[
z_{i}[k] = \begin{cases} 0 & i \in V_{out} \\ u_{i}[k] & i \in V_{in} \\ p_{i}[k] \rho_{i}[k] & i \in V_{f} \end{cases} \quad (4b)
\]

where

\[
p_{i}[k] = \begin{cases} 1 & \text{If } i \in V_{in} \cup V_{out} \\ \frac{z_{i}[k]}{\rho_{i}[k]} & \text{If } i \in V_{f} \text{ and } \rho_{i} = 0 \\ \frac{z_{i}[k]}{\rho_{i}[k]} & \text{If } i \in V_{f} \text{ and } \rho_{i} \neq 0 \end{cases} \quad (5)
\]

is the outflow probability of road element \(i\in V_{f}\) at discrete time \(k\), \(q_{i,j}[k] \in [0,1]\) is the fraction of outflow traffic directed from \(j \in V \setminus V_{out}\) to \(i \in O_{j}\) at every discrete time \(k\), and

\[
\sum_{i \in O_{j}} q_{i,j}[k] = 1 \quad (6)
\]

at every interior road \(i \in V_{f}\).

Given the above problem setting, the main purpose of this paper is to alleviate the traffic congestion by assigning the optimal external flow \(s_{i}[k]\). Assuming \(p_{i}[k]\) and \(q_{i,j}[k]\) are known at every interior road \(i \in V_{f}\), the external flow is determined by solving a quadratic programming problem with cost function

\[
C = \frac{1}{2} \sum_{i \in V_{in}} \sum_{l=0}^{N_{r}-1} u_{i}[k+l]^{2} + \sum_{j \in V_{out}} v_{j}[k+l]^{2} \quad (7)
\]

and the following inequality and equality constraints:

\[
\bigwedge_{i \in V_{in}} \bigwedge_{l=0}^{N_{r}-1} (u_{i}[k+l] \geq 0), \quad (8a)
\]

\[
\bigwedge_{j \in V_{out}} \bigwedge_{l=0}^{N_{r}-1} (v_{j}[k+l] \geq 0), \quad (8b)
\]

\[
\bigwedge_{i \in V_{f}} \bigwedge_{l=0}^{N_{r}-1} (\rho_{i}[k+l] \geq 0), \quad (8c)
\]

\[
\bigwedge_{i \in V_{f}} \bigwedge_{l=0}^{N_{r}-1} (\rho_{i}[k+l] \leq \rho_{i,max}), \quad (8d)
\]

\[
\sum_{i \in V_{in}} u_{i}[k+l] + \sum_{j \in V_{out}} v_{j}[k+l] = d_{0} \quad (8e)
\]

Note that \(N_{r} < \infty\) is the time horizon length and \(\rho_{i,max}\) is the maximum number of vehicles that can be accommodated at road element \(i \in V_{f}\). Constraint Eqs. (8a) and (8b) ensure that the traffic back-flow is avoided at every inlet or outlet.
Theorem 1. Assume graph $G(V, E)$ holds the following properties:

1) Traffic inflow directs from every inlet boundary road element towards an interior road element.
2) There is at least one directed path from every interior node to an outlet node.
3) Graph $G$ contains no isolated node.
4) No inlet boundary road element is directly connected to an outlet boundary road element.

Then, the traffic network dynamics (13) is BIBO stable.

Proof: If assumptions of Theorem 1 are satisfied, matrix $A[k]$ holds the following properties at every discrete time $k$:

1) All entries in matrix $A[k]$ are non-negative.
2) Column $i$ of matrix $A[k]$ sums up to 1, if $O_{i+k+1} \cap V_{out} = \emptyset$.
3) Elements of column $i$ of matrix $A[k]$ sums up to a positive number in interval $(0, 1)$, if $O_{i+k+1} \cap V_{out} \neq \emptyset$.

Thus, eigenvalues of matrix $A[k]$ are all less than one at every discrete time $k$.

Per traffic dynamics (13), we can define

$$
x[k+1] = \Theta_k \begin{bmatrix} x[1] \\ \vdots \\ B[k]s[k] \end{bmatrix},
$$

where

$$
\Theta_k = \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_1 & \Gamma_0 \end{bmatrix},
$$

$$
\Gamma_h = \prod_{j=k-h+1}^{k} A[j],
$$

for $h = 1, \cdots, k$, and $\Gamma_0 = I_{N_{out}} \in \mathbb{R}^{(N_{out}) \times (N_{out})}$ is an identity matrix. Because $x[1] < \infty$, and $s[k]$ is bounded at every discrete time $k$, we can write

$$
x[1] \leq \max(1)_{N_{out} \times 1},
$$

$$
B[k]s[k] \leq \max(1)_{N_{out} \times 1},
$$

where $\max(1)$ is bounded. If assumptions of Theorem 1 are satisfied, spectral radius $r$ of matrix $\Gamma_k$ is less than 1 at every discrete time $k$. Therefore, we can write

$$
x^T[k+1]x[k+1] \leq \max(1)_{N_{out} \times 1} \left( \sum_{t=0}^{k} \sum_{l=0}^{t} \Gamma_l^T \Gamma_h \right) \max(1)_{N_{out} \times 1}
$$

$$
\leq \frac{2r^t}{(1-r)} \frac{\max(1)_{N_{out} \times 1}}{1-r}
$$

which implies that $x^T[k+1]x[k+1]$ is bounded at every discrete time $k$, and thus the BIBO stability of traffic dynamics (13) is proven.

V. TRAFFIC CONGESTION CONTROL

This paper applies the model predictive control (MPC) approach to control congestion through optimizing the boundary inflow and outflow. For the proposed MPC control, we
use the linear time-varying dynamics (13) to predict traffic evolution within a future finite time horizon, and determine the optimal boundary external flow as a solution of the quadratic function subject to the inequality and equality constraints.

Given traffic dynamics (13) at discrete time, the following predictive model can be used to model traffic coordination within the next \( N_r \) time steps:

\[
X[k] = G[k]x[k] + H[k]U[k]
\]

where

\[
X[k] = [x^T[k+1] \ldots x^T[k+N_r]]^T \in \mathbb{R}^{(N_r,N) \times 1},
\]

\[
G[k] = \begin{bmatrix}
A[k] \\
B[k] & B[k] & B[k] & \cdots \\
A^2[k] & A[k] & B[k] & B[k] & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
A^{N_r-1}[k] & \ldots & & & B[k] & B[k] & B[k] \\
\end{bmatrix} \in \mathbb{R}^{(N_r,N,N) \times N},
\]

\[
H[k] = \begin{bmatrix}
B[k] & B[k] & \cdots \\
A[k] & B[k] & B[k] & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
A^{N_r-1}[k] & \ldots & & & B[k] & B[k] & B[k] \\
\end{bmatrix}
\]

\[
x[k] = [\rho_1[k] \ldots \rho_N[k]]^T \in \mathbb{R}^{N \times 1},
\]

\[
U[k] = [s^T[k] \ldots s^T[k+N_r-1]]^T \in \mathbb{R}^{(N_r,N_r \times N_r)}.
\]

Now, we can rewrite the cost function (7) as

\[
C = \frac{1}{2} \langle U[k]^T U[k] \rangle.
\]

By using the predictive traffic coordination model (9), we can also rewrite the constraint equations (8) as follows:

\[
U[k] \geq 0_{(N_r,N_r \times N_r)\times 1},
\]

\[
G[k]x[k] + H[k]U[k] \leq 0_{N_r \times 1} \otimes x_{\max},
\]

\[
G[k]x[k] + H[k]U[k] \geq 0_{(N,N) \times 1},
\]

\[
(I_{N_r,N_r} \otimes 1_{N_r \times N_r}) U[k] = d_0_{N_r \times N_r}.
\]

where \( I_{N_r,N_r} \) is the identity matrix. Eq. (22a) integrates feasibility conditions (8a) and (8b). Constraint equations (22b), (22c), and (22d) are identical to (8c), (8d), and (8e) respectively.

**Theorem 2.** If \( w_i[k] \geq 0 \) at every \( i \in \mathcal{V}_{in} \), \( v_j[k] \geq 0 \) at every \( j \in \mathcal{V}_{out} \), \( \rho_i[k] \) is updated by dynamics (2), and \( \rho_i[0] \geq 0 \) at every node \( i \in \mathcal{V}_t \), then \( \rho_i[k] \geq 0 \) at every interior road element \( i \in \mathcal{V}_t \) and every discrete time \( k \).

**Proof:** By applying the mass conservation law in (2), traffic network dynamics can be obtained via Eq. (9b) at every interior road \( i \in \mathcal{V}_t \). Per Assumption (1), \( \rho_i[k] \in [0,1] \) at every \( i \in \mathcal{V}_t \) and all discrete times \( k \), which indicate that \((1 - \rho_i[k]) \geq 0 \) on the right-hand side of Eq. (9b). Also, \( g_{i,j}[k] \) is defined as a quantity in interval \([0,1]\) at every discrete sampling time \( k \). If \( \rho_i[0] \geq 0 \), \( u_i[k] \geq 0 \) at every \( i \in \mathcal{V}_t \), and \( v_j[k] \geq 0 \) at every \( j \in \mathcal{V}_{out} \), then the right-hand side of Eq. (9b) must be a non-negative quantity at every discrete sampling time \( k \). This implies that \( \rho_i[k] \geq 0 \) at every node \( i \in \mathcal{V}_t \) and discrete time \( k \).

Per Theorem 2, \( \rho_i[k] \geq 0 \) at every road \( i \in \mathcal{V}_t \) and discrete time \( k \). Therefore, condition (22a) is redundant, and conditions (22b), (22c), and (22d) are sufficient to determine the feasible optimal boundary input \( U^*[k] \) by solving the following quadratic programming problem:

\[
\min \frac{1}{2} \langle U[k]^T U[k] \rangle
\]

subject to equality constraint (22a) and inequality constraint

\[
-I_{N_r,N_r} U[k] \leq \begin{bmatrix} 0_{(N_r,N_r)\times 1} \end{bmatrix}
\]

Note that

\[
s^*[k] = \begin{bmatrix} I_{N_r} & 0_{N_r \times (N_r-1)N_r} \end{bmatrix} U^*[k]
\]

is the optimal boundary control at discrete time \( k \).

**VI. SIMULATION RESULTS**

In this section, we present the simulation results of modeling and control in the example NOIR shown in Fig. 1. This particular NOIR consists of 259 road elements of Center City Philadelphia, where the index numbers of the road elements are shown in Fig. 1. We process the map data generated by SUMO and obtain the corresponding graph \( (V,E) \). Node set \( V = \{1, \ldots, 259\} \) can be expressed as \( \mathcal{V} = \mathcal{V}_{in} \cup \mathcal{V}_{out} \cup \mathcal{V}_t \) and \( \mathcal{V}_{in} = \{1, \ldots, 20\}, \mathcal{V}_{out} = \{21, \ldots, 42\}, \mathcal{V}_t = \{43, \ldots, 259\} \).

We set the whole simulation time as 3000s and the sampling interval as 20s, which indicates that the traffic coordination is simulated for 150 time steps. At every discrete time \( k \in \{1, \ldots, 150\} \) the outflow probability matrix \( P[k] \) and the fraction probability matrix \( Q[k] \) are randomly generated to simulate the uncertainty of human driving intent roughly. For simulation, we choose \( w_0 = 100 \), which implies 100 cars are permitted to cross the boundary of the NOIR shown in Fig. 1 at every discrete time \( k \). For every road \( i \in \mathcal{V}_t \),

\[
\rho_{i,max} = \frac{n_{i,lane} \cdot i_l}{l_{veh}}
\]

assigns an upper bound for the number of cars at road element \( i \in \mathcal{V}_t \), where \( l_{veh} = 4.5m \) is considered the same for all road elements, \( i_l \) is the length of road element \( i \in \mathcal{V}_t \) in the Center City area, and \( n_{i,lane} \) refers to the number of lanes at road element \( i \in \mathcal{V}_t \). Meanwhile, the initial traffic density \( \rho_i[0] \) is assigned randomly for every road element \( i \in \mathcal{V}_t \).

We plot simulation results for two inlet boundary road elements 8, 17 in \( \mathcal{V}_{in} \), two outlet boundary road elements 27, 35 in \( \mathcal{V}_{out} \), and two interior road elements 68, 119 in \( \mathcal{V}_t \). The locations of these six road elements are presented in Table 1.

Fig. 2 plots the density variations versus discrete sampling time \( k \) at the example interior road elements. It can be observed that the traffic density of road elements 68 and 119.
| Type     | Road Index | Name and Location               |
|----------|------------|---------------------------------|
| Inlet    | 8          | Cherry St. (N20th-N19th)        |
| Inlet    | 17         | S11th St. (Locust-Walnut)       |
| Outlet   | 27         | N16th St. (Race-Vine)           |
| Outlet   | 35         | Sansom St. (N20th-S19th)        |
| Interior | 68         | Filbert St. (N20th-N15th)       |
| Interior | 119        | JFK Blvd (N16th-N17th)          |

Table I: Example road elements in NOIR

*Fig. 2: Variation trends of traffic densities at example interior road elements*

reaches the steady-state condition when the discrete sampling time \( k > 20 \).

*Fig. 3* illustrates the variation of the external traffic flow at inlet road elements \( 8, 17 \in \mathcal{V}_{in} \) and outlet road elements \( 27, 35 \in \mathcal{V}_{out} \). The variation trends at inlet and outlet road elements are similar: After a period of variation, the external traffic inflows and the external traffic outflows reach the steady state condition. *Fig. 4* presents the net inflow and outflow of the NOIR versus discrete sampling time \( k \). It could be observed that, after starting the simulation, the amount of traffic inflow decreases while the amount of traffic outflow increases symmetrically and after about 30 sampling times they converge to a stable state. We can formulate this variation trend as

\[
\sum_{i \in \mathcal{V}_{in}} u_i[k] = \sum_{i \in \mathcal{V}_{out}} v_i[k] \equiv 50, \quad (27)
\]

when \( k > 30 \).

**VII. Conclusion**

This paper introduces a conservation-based modeling method to learn the traffic network dynamics and alleviate the traffic congestion. We apply the mass conservation law to model traffic coordination by a time-varying stochastic process where the real map data is used to define the traffic network. We offer an MPC control to manage traffic congestion by controlling the inflow and outflow at the boundary of the NOIR. The simulation results demonstrate that our proposed modeling and control approach can manage the traffic congestion effectively through optimizing the traffic inflow and outflow across the boundary of the NOIR. In our future work, we plan to obtain the traffic dynamics based on real traffic data and control congestion through the boundary ramp meters and traffic signals, situated at road intersections.

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