Adaptive Exploration-Exploitation Tradeoff for Opportunistic Bandits

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Abstract

In this paper, we propose and study opportunistic bandits - a new variant of bandits where the regret of pulling a suboptimal arm varies under different environmental conditions, such as network load or produce price. When the load/price is low, so is the cost/regret of pulling a suboptimal arm (e.g., trying a suboptimal network configuration). Therefore, intuitively, we could explore more when the load is low and exploit more when the load is high. Inspired by this intuition, we propose an Adaptive Upper-Confidence-Bound (AdaUCB) algorithm to adaptively balance the exploration-exploitation tradeoff for opportunistic bandits. We prove that AdaUCB achieves $O(\log T)$ regret with a smaller coefficient than the traditional UCB algorithm. Furthermore, AdaUCB achieves $O(1)$ regret when the exploration cost is zero if the load level is below a certain threshold. Last, based on both synthetic data and real-world traces, experimental results show that AdaUCB significantly outperforms other bandit algorithms, such as UCB and TS (Thompson Sampling), under large load fluctuations.

1 Introduction

In this paper, we propose and study opportunistic bandits - a new variant of bandits where the regret of pulling a suboptimal arm varies under different environmental conditions, such as network load or produce price. When the load/price is low, so is the cost/regret of pulling a suboptimal arm (e.g., trying a suboptimal network configuration). Therefore, intuitively, we could explore more when the load is low and exploit more when the load is high. Inspired by this intuition, we propose an Adaptive Upper-Confidence-Bound (AdaUCB) algorithm to adaptively balance the exploration-exploitation tradeoff for opportunistic bandits. We prove that AdaUCB achieves $O(\log T)$ regret with a smaller coefficient than the traditional UCB algorithm. Furthermore, AdaUCB achieves $O(1)$ regret when the exploration cost is zero if the load level is below a certain threshold. Last, based on both synthetic data and real-world traces, experimental results show that AdaUCB significantly outperforms other bandit algorithms, such as UCB and TS (Thompson Sampling), under large load fluctuations.

Motivating scenario 1: price variation.

Motivating scenario 2: load variation.

Network configuration is widely used in wireless networks, data-center networks, and the Internet to control network topology, routing, load balancing, and thus the overall performance. For example, in a cellular network, a cell tower has a number of parameters to configure, including radio spectrum, transmission power, antenna angle and direction, etc. The configuration of such parameters can greatly impact the overall performance, e.g., coverage, throughput, and service quality. A network configuration can be considered as an arm, where its performance needs to be learned. Networks are typically designed and configured to handle the peak load, and thus we hope to learn the best configuration for the peak load.

Network traffic load fluctuates over time. When the network load is low, we can inject dummy traffic into the network so that the total load, the real load plus the dummy load, resembles the peak load. It allows us to learn the performance of the configuration under the peak load. At the same time, the regret of using a suboptimal configuration is low because the real load affected is low. Furthermore, in practice, we can set the priority of the dummy traffic to be lower than the real traffic. Because networks handle high priority traffic first, low priority traffic results in little or no impact on the high priority traffic. In this case, the regret on the actual load is further reduced, or even negligible (when the suboptimal configuration is sufficient to handle the real load).

Opportunistic bandits. Motivated by these application scenarios, we study opportunistic bandits in this paper. Specifically, we define opportunistic bandit as a bandit problem with the following characteristics: 1) The best arm does not change over time. 2) The exploration cost (regret) of a suboptimal arm varies depending on a time-varying external condition that we refer to as load (which is the price in the first scenario). 3) The load is revealed before an arm is pulled, so that one can decide which arm to pull depending on the load. As its name suggests, in opportunistic bandits, one can leverage the opportunities of load variation to achieve a lower regret. In addition to the previous two examples, opportunistic bandit algorithms can be applied to other scenarios that share the above characteristics.
We note that opportunistic bandits significantly differs from non-stationary bandits (Garivier and Moulines 2011; Besbes, Gur, and Zeevi 2014). In non-stationary bandits, the expected reward of each arm varies and the optimal arm may change over time, e.g., because of the shift of interests. In opportunistic bandits, the optimal arm does not change over time, but the regret of trying a suboptimal arm changes depending on the load. In other words, in non-stationary bandits, the dynamics of the optimal arm make finding the optimal arm more challenging. In contrast, in opportunistic bandits, the time-varying nature of the load provides opportunities to reduce the regret of finding the fixed optimal arm. Because of such fundamental differences, in non-stationary bandits, one can show polynomial regret (e.g., $\Omega(T^2/3)$) (Besbes, Gur, and Zeevi 2014) because one has to keep track of the optimal arm. In opportunistic bandits, we can show $O(\log T)$ (or even $O(1)$ in certain special case) regret because we can push more exploration to slots when the regret is lower.

We also note the connection and difference between opportunistic bandits and contextual bandits (Zhou 2015; Wu et al. 2015). Broadly speaking, opportunistic bandits can be considered as a special case of contextual bandits where we can consider the load as the context. However, applying existing general contextual bandits algorithms will not generate optimal regrets because they do not take advantages of the unique properties of opportunistic bandits, in particular, the optimal bandit remains the same, and regrets differ under different contexts (i.e., load).

**Contributions.** In this paper, we propose an Adaptive Upper-Confidence-Bound (AdaUCB) algorithm to dynamically balance the exploration-exploitation tradeoff in opportunistic bandits. The intuition is clear: we should explore more when the load is low and exploit more when the load is high. The design challenge is to quantify the right amount of exploration and exploitation depending on the load. The analysis challenge is due to the inherent coupling over time and thus over bandits under different conditions. In particular, due to the randomness nature of bandits, the empirical estimates of the expected rewards could deviate from the true values, which could lead to suboptimal actions when the load is high. We address these challenges by studying the lower bounds on the number of pulls of the suboptimal arms under low load. Because the exploration factor is smaller under higher load than that under lower load, it requires less information accuracy to make the optimal decision under higher load. Thus, with an appropriate lower bound on the number of pulls of the suboptimal arms under low load, we can show that the information obtained from the exploration under the lower load is sufficient for accurate decisions under the higher load. As a result, the explorations under high load are reduced and thus so does the overall regret. In summary, our contributions are as follows:

- To the best of our knowledge, this is the first work proposing and studying opportunistic bandits that aims to adaptively balance the exploration-exploitation tradeoff considering load-dependent regrets. The work applies to scenarios with time-varying load and load-dependent regret.
- We propose AdaUCB, an algorithm that adjusts the exploration-exploitation tradeoff according to the load level. Under AdaUCB, the system explores more when the load is low and exploits more when the load is high.
- We prove that AdaUCB achieves $O(\log T)$ regret with a smaller coefficient than the traditional UCB algorithm. Furthermore, AdaUCB achieves $O(1)$ regret when the exploration cost is zero if the load level is smaller than a certain threshold.
- We evaluate the proposed algorithm with both synthetic data and real-world traces. We show that AdaUCB significantly outperforms other bandit algorithms, such as UCB and TS (Thompson Sampling), under large load fluctuations.

2 System Model

We study an opportunistic bandit problem, where the exploration cost varies over time depending on an external condition, called load here. Specifically, consider a $K$-armed stochastic bandit system. At time $t$, each arm has a random nominal reward $X_{k,t}$, where $X_{k,t} \in [0, 1]$ are independent across arms, and i.i.d. over time, with mean value $\mu_k$. Let $\mu^* = \max_k \mu_k$ be the maximum expected reward and $k^* = \arg \max_k \mu_k$ be the best arm. The arm with the best nominal reward does not depend on the load and does not change over time.

Let $L_t \geq 0$ be the load at time $t$. For simplicity, we assume $L_t \in [0, 1]$. The agent observes the value of $L_t$ before making the decision; i.e., the agent pulls an arm $a_t$ based on both $L_t$ and the historical observations, i.e., $a_t = \Gamma(L_t, H_{t-1})$, where $H_{t-1} = (L_1, a_1, X_{a_1,1}, \ldots, L_{t-1}, a_{t-1}, X_{a_{t-1},t-1})$ represents the historical observations. The agent then receives an actual reward $L_t X_{a_t,t}$. While the underlying nominal reward $X_{a_t,t}$ is independent of $L_t$ conditioned on $a_t$, the actual reward depends on $L_t$. We also assume that the agent can observe the value of $X_{a_t,t}$ after pulling arm $a_t$ at time $t$.

This model captures the essence of opportunistic bandits and its assumptions are reasonable. For example, in the agriculture scenario, $X_{a_t,t}$ captures the effectiveness of a treatment, e.g., the survival rate or the yield of an antibiotic treatment. The value of $X_{a_t,t}$ can always be observed by the agent after applying treatment $a_t$ at time $t$. Conditioned on $a_t$, $X_{a_t,t}$ is also independent of $L_t$, the price of the commodity. Meanwhile, the actual reward, the monetary reward, is modulated by $L_t$ (the price) as $L_t X_{a_t,t}$. In the network configuration example, $X_{a_t,t}$ captures the impact of a configuration at the peak load, e.g., success rate, throughput, and service quality score. Because the total load (the real load plus the dummy load) resembles the peak load, $X_{a_t,t}$ is independent of the real load $L_t$ conditioned on $a_t$, and can always be observed. Further, because the real load is a portion of the total load and the network can identify real traffic from dummy traffic, the actual reward is thus a portion of the total reward, modulated by the real load as $L_t X_{a_t,t}$.

If system statistics are known a priori, then the agent will always pull the best arm and obtain the expected total reward.
Thus, the regret of a policy Γ is defined as
\[ R_\Gamma(T) = u^* \mathbb{E}[\sum_{t=1}^T L_t] - \sum_{t=1}^T \mathbb{E}[L_t X_{a_t,t}]. \] (1)

In particular, when \( L_t \) is i.i.d. over time with mean value \( \mathbb{E}[L_t] = \bar{L} \), the total expected reward for the oracle solution is \( u^* \bar{L} T \) and the regret is \( R_\Gamma(T) = u^* \bar{L} T - \sum_{t=1}^T \mathbb{E}[L_t X_{a_t,t}] \). Because the action \( a_t \) can depend on \( L_t \), it is likely that \( \mathbb{E}[L_t X_{a_t,t}] \neq \bar{L} \mathbb{E}[X_{a_t,t}] \).

### 3 Adaptive UCB

We first recall a general version of the classic UCB1 (Auer, Cesa-Bianchi, and Fischer 2002) algorithm, referred to as UCB(\( \alpha \)), which always selects the arm with the largest index defined in the following format:
\[ \hat{u}_k(t) = \bar{u}_k(t) + \sqrt{\frac{\alpha \log t}{C_k(t-1)}}, \quad 1 \leq k \leq K, \]

where \( \alpha \) is a constant, \( C_k(t-1) \) is the number of pulls for arm-\( k \) before \( t \), and \( \bar{u}_k(t) = \frac{1}{C_k(t-1)} \sum_{\tau=1}^{t-1} \mathbb{1}(a_\tau = k) X_{k,\tau} \). It has been shown that UCB(\( \alpha \)) achieves logarithmic regret in stochastic bandits when \( \alpha > 1/2 \) (Bubeck 2010). UCB1 in (Auer, Cesa-Bianchi, and Fischer 2002) is a special case with \( \alpha = 2 \).

**Algorithm 1** AdaUCB

1: **Init:** \( \alpha > 0.5, C_k(0) = 0, \bar{u}_k(0) = 1 \).
2: **for** \( t = 1 \) to \( K \) **do**
3: \hspace{1em} Pull each arm once and update \( C_k(t) \) and \( \bar{u}_k(t) \) accordingly;
4: **end for**
5: **for** \( t = K + 1 \) to \( T \) **do**
6: \hspace{1em} Observe \( L_t \).
7: \hspace{2em} Calculate UCB: for \( k = 1, 2, \ldots, K \),
   \[ \hat{u}_k(t) = \bar{u}_k(t) + \sqrt{\frac{\alpha(1-\tilde{L}_t) \log t}{C_k(t-1)}}, \] (2)
   where \( \tilde{L}_t \) is the normalized load defined in Eq. (4);
8: \hspace{1em} Pull the arm with the largest \( \hat{u}_k(t) \):
   \[ a_t = \arg \max_{1 \leq k \leq K} \hat{u}_k(t); \] (3)
9: Update \( \bar{u}_k(t) \) and \( C_k(t) \);
10: **end for**

In this work, we propose an AdaUCB algorithm for opportunistic bandits. In order to capture different ranges of \( L_t \), we first normalize \( L_t \) to be within \([0, 1]\):
\[ \tilde{L}_t = \frac{L_t}{l^{(+)} - l^{(-)}}, \] (4)

where \( l^{(-)} \) and \( l^{(+)} \) are the lower and upper thresholds for truncating the load level, and \([L_t]_t^{l^{(+)}} = \max\{l^{(-)}, \min(L_t, l^{(+)})\}\). To achieve good performance, the truncation thresholds should be appropriately chosen and can be learned online in practice, as discussed in Sec. 4.3. We note that \( \tilde{L}_t \) is only used in AdaUCB algorithm. The rewards and regrets are based on \( L_t \), not \( \tilde{L}_t \).

The AdaUCB algorithm adjusts the tradeoff between exploration and exploitation based on the load level \( L_t \). Specifically, as shown in Algorithm 1, AdaUCB makes decisions based on the sum of the empirical reward (the exploitation term) \( \bar{u}_k(t) \) and the confidence interval width (the exploration term). The latter term is proportional to \( \sqrt{1 - \tilde{L}_t} \). In other words, AdaUCB uses an exploration factor \( \alpha(1 - \bar{L}_t) \) that is linearly decreasing in \( \tilde{L}_t \). Thus, when the load level is high, the exploration term is relatively small and AdaUCB tends to emphasize exploitation, i.e., choosing the arms that perform well in the past. In contrast, when the load level is low, AdaUCB uses a larger exploration term and gives more opportunities to the arms with less explorations. Intuitively, with this load-awareness, AdaUCB explores more when the load is low and leverages the learned statistics to make better decisions when the load is high. Since the actual regret is scaled with the load level, AdaUCB can achieve an overall lower regret. Note that we have experimented a variety of load adaptation functions. The current one achieves superior empirical performance and is relatively easy to analyze, and thus adopted here.

### 4 Regret Analysis

Although the intuition behind AdaUCB is natural, the rigorous analysis of its regret is challenging. To analyze the decision in each slot, we require the statistics for the number of pulls of each arm. Unlike traditional regret analysis, we care about not only the upper bound, but also the lower bound for calculating the confidence level. However, even for fixed load levels, it is difficult to characterize the total number of pulls for suboptimal arms, i.e., obtaining the tight lower and upper bounds for the regret. The gap between the lower and upper bounds makes it even more difficult to evaluate the properties of UCB for general random load levels. Therefore, we start with the case of squared periodic wave load and Dirac rewards to illustrate the behavior of AdaUCB in Sec. 4.1. Then, we extend the results to the case with random binary-value load and random rewards in Sec. 4.2 and finally discuss the case with continuous load in Sec. 4.3.

Specifiically, we first consider the case with binary-valued load, i.e., \( L_t \in \{ \epsilon_0, 1 - \epsilon_1 \} \), where \( \epsilon_0, \epsilon_1 \in [0, 0.5) \). For this case, we let \( l^{(-)} = \epsilon_0 \) and \( l^{(+)} = 1 \). Then, \( \tilde{L}_t = 0 \) if \( L_t = \epsilon_0 \), and \( \tilde{L}_t = \frac{1 - \epsilon_0}{1 - \epsilon_1} \) if \( L_t = 1 - \epsilon_1 \). Therefore, the indices used by AdaUCB are given as follows:
\[ \hat{u}_k(t) = \begin{cases} \bar{u}_k(t) + \sqrt{\frac{\alpha \log t}{C_k(t-1)}}, & \text{if } L_t = \epsilon_0, \\ \bar{u}_k(t) + \sqrt{\frac{\alpha \epsilon_1 \log t}{(1 - \epsilon_0)C_k(t-1)}}, & \text{if } L_t = 1 - \epsilon_1. \end{cases} \] (5)

We focus on 2-armed bandits in analysis in the paper for easy illustration. The results can be extended to general multi-armed cases as discussed in Appendix C.1 and all experiments are run for general cases with \( K > 2 \) in Sec. 5.
4.1 AdaUCB under Periodic Square Wave Load and Dirac Rewards

We first study a simple case with periodic square wave load and Dirac rewards. In this scenario, the evolution of the system under AdaUCB is deterministic. The analysis of this deterministic system allows us to better understand AdaUCB and quantify the benefit of load-awareness.

Specifically, we assume the load is $L_t = \epsilon_0$ if $t$ is even, and $1 - \epsilon_1$ if $t$ is odd. Moreover, the rewards are fixed, i.e., $X_{k,t} = u_k$ for all $k$ and $t$, but unknown a priori. WLOG, we assume arm-1 has higher reward, i.e., $1 \geq u_1 > u_2 \geq 0$, and let $\Delta = u_1 - u_2$ be the reward difference.

Under these settings, we can easily obtain the bounds for the number of pulls for each arm by borrowing the idea from (Salomon, Audibert, and Alaoui 2011) [Salomon, Audibert, and Alaoui 2013]. The proofs of these results are included in Appendix A, which are similar to (Salomon, Audibert, and Alaoui 2011; Salomon, Audibert, and Alaoui 2013), except for the efforts of addressing the case of $L_t = 1 - \epsilon_1$.

First of all, we bound the total number of pulls for the suboptimal arm.

Lemma 1. In the opportunistic bandit with periodic square wave load and Dirac rewards, the number of pulls for arm-2 is uniformly bounded as: $C_2(t) \leq \frac{\alpha \log t}{\Delta^2} + 1$.

Next, we propose a lower bound on $C_2(t)$, which is the information used to make decisions in slot $2t + 1$ with $L_{2t+1} = 1 - \epsilon_1$, and show the bound in Lemma 1 indeed is tight.

Lemma 2. In the opportunistic bandit with periodic square wave load and Dirac rewards, the number of pulls for arm-2 is uniformly bounded as $C_2(t) \geq f(t) = f_{t}^{\epsilon} \cdot \min(h'(s), 1) ds - h(2)$, where $h(t) = \frac{\alpha \log t}{\Delta^2} \left(1 + \sqrt{\frac{2\alpha \log t}{(2t-1)\Delta^2}}\right)^{-2}$.

The lower bound in Lemma 2 indicates that we have sufficient explorations when $L_t = 1 - \epsilon_1$ for sufficiently large $t$, and AdaUCB will always pull the better arm when $L_t = 1 - \epsilon_1$ after a certain time, as shown in the following lemma.

Lemma 3. In the opportunistic bandit with periodic square wave load and Dirac rewards, there exists a constant $T_1$ independent of $T$ such that under AdaUCB, $a_t = 1$ when $L_t = 1 - \epsilon_1$ for $t \geq T_1$.

Combining Lemmas 1 and 3, we can easily bound the regret of AdaUCB:

Theorem 1. In the opportunistic bandit with periodic square wave load and Dirac rewards, the regret of AdaUCB is bounded as: $R_{\text{AdaUCB}}(T) \leq \frac{\alpha \log T}{\Delta^2} + O(1)$.

Remark 1: According to (Salomon, Audibert, and Alaoui 2011), the regret of UCB($\alpha$) is lower bounded by $\frac{\alpha \log t}{\Delta}$ for fixed load $L_t = 1$. Without load-awareness, we can expect that the explorations occur uniformly under different load levels. Thus, the regret of UCB($\alpha$) in this opportunistic bandit is roughly $\frac{\alpha \log t}{\Delta^2}$, and is much larger than the regret of AdaUCB for small $\epsilon_0$ and $\epsilon_1$. As an extreme case, when $\epsilon_0 = 0$, the regret of AdaUCB is $O(1)$, while that of UCB($\alpha$) is $O(\log T)$.

Remark 2: The above analysis provides us insights about the benefit of load-awareness in opportunistic bandits. With load-awareness, AdaUCB forces exploration to the slots with lower load and the information obtained there is sufficient to make good decisions in higher-load slots. Thus, the overall regret of AdaUCB is much smaller than traditional algorithms without load-awareness.

4.2 AdaUCB under Random Binary-Valued Load and Random Rewards

We now consider the more general case with random binary-valued load and random rewards. We assume that load $L_t \in \{\epsilon_0, 1 - \epsilon_1\}$ and $\mathbb{P}(L_t = \epsilon_0) = \rho \in (0, 1)$. We consider i.i.d random reward $X_{k,t} \in [0, 1]$ and $\mathbb{E}[X_{k,t}] = u_k$, where $1 \geq u_1 > u_2 \geq 0$ and $\Delta = u_1 - u_2$.

Compared with the deterministic case in Sec. 4.1, the analysis under random load and rewards is much more challenging. In particular, due to the reward randomness, there will be deviation for the empirical value $\hat{u}_k(t)$. Unlike Dirac reward, this deviation could result in suboptimal decisions even when $\epsilon_0$ and $\epsilon_1$ are small. Thus, we need to carefully lower bound the number of pulls for each arm so that the deviation is bounded with high probability. We only provide sketches for the proofs here due to the space limit and the detailed analyses can be found in Appendix B.

We consider a larger $\alpha$ ($\alpha \geq 2$ in general or larger when explicitly stated) for theoretical analysis purpose, similarly to earlier UCB papers such as (Auer, Cesa-Bianchi, and Fischer 2002). As we will see in the simulations, AdaUCB with $\alpha > 1/2$ works very well for general random load.

We first propose a loose but useful bound for the number of pulls for the optimal arm. Let $C_k^{(0)}(t)$ be the number of slots where arm-$k$ is pulled when $L_t = \epsilon_0$, i.e., $C_k^{(0)}(t) = \sum_{t=1}^{\tau} 1(L_{\tau} = \epsilon_0, a_{\tau} = k)$.

Lemma 4. In the opportunistic bandit with random binary-valued load and random rewards, for a constant $\eta \in (0, 1)$, there exists a constant $T_2$, such that under AdaUCB, for all $t \geq T_2$

$$\mathbb{P}(C_k^{(0)}(t) < \frac{(1-\eta)\rho t}{2}) \leq (t + 2)(t/2)^{-2\alpha} + 2e^{-\eta^2 t/2} + e^{-2\eta^2 t}.$$

Sketch of Proof: The intuition of proving this lemma is that, when $C_k^{(0)}(t)$ is too small, the optimal arm will be pulled with high probability. Specifically, we focus on the slot before $t$, referred to as $t'$, when the suboptimal arm is pulled under load $L_t = \epsilon_0$ for the last time. If $C_k^{(0)}(t) < \frac{(1-\eta)\rho t}{2}$, then $t' \geq t/2$ with high probability and $C_k^{(t')-1} \geq C_k^{(0)}(t') - 2 = \Theta(t)$. Using the fact that $\frac{\alpha \log t}{\Delta^2} \to 0$ as $t \to \infty$, we know there exists a constant $T_2$ such that for $t \geq T_2$, the confidence width $\sqrt{\frac{\alpha \log t}{C_k^{(0)}(t')}}$ will be sufficiently small compared with the gap $\Delta$. Moreover, the algorithm will pull the best arm when the UCB deviation
is sufficiently small. Then, we can bound the probability of the event \( C^0_1(t) < \frac{(1 - \eta) pt}{2} \) by using Chernoff-Hoeffding’s inequality for the deviation of UCBs.

Next we bound the total number of pulls of the suboptimal arm as follows:

**Lemma 5.** In the opportunistic bandit with random binary-valued load and random rewards, under AdaUCB, we have

\[
E[C_2(T)] \leq \frac{4\alpha \log T}{\Delta^2} + O(1).
\]

**Sketch of Proof:** To prove this lemma, we discuss the slots when the suboptimal arm is pulled under low and high load levels, respectively. When the load is low, i.e., \( L_t = \epsilon_0 \), AdaUCB becomes UCB(\( \alpha \)) and thus we can bound the probability of pulling the suboptimal arm similarly to (Auer, Cesa-Bianchi, and Fischer 2002). When the load is high, i.e., \( L_t = 1 - \epsilon_1 \), the index becomes \( \bar{u}_k(t) = \frac{\alpha \log t}{1 - \epsilon_0} + \frac{\sqrt{\alpha \log t}}{(1 - \epsilon_0) C^0_1(t - 1)} \). In this case, the high probability lower bound for the index of the optimal arm can be

\[
u_1 - \left(1 - \frac{\alpha}{1 - \epsilon_0}\right) \sqrt{\frac{\alpha \log t}{C^0_1(t - 1)}},
\]

which is smaller than \( u_1 \).

We can then bound this deviation with high probability using Lemma 4. With similar adjustment on the UCB index for the suboptimal arm, we can bound the probability of pulling the suboptimal arm under high load. The conclusion of the lemma then follows by combining the above two cases.

Now we further lower bound the pulls of the suboptimal arm with high probability.

**Lemma 6.** In the opportunistic bandit with random binary-valued load and random rewards, for a positive number \( \delta \in (0, 1) \), we have

\[
P \left\{ C_2(t) \leq \frac{\alpha \log t}{4(\Delta + \delta)^2} \right\} = O \left( t^{-2\alpha - 2} + e^{-\frac{\alpha^2(1 - \eta) pt}{2}} + t^{-(2\alpha(1 - \eta) pt/2)} \right).
\]

**Sketch of Proof:** Although the analysis is more difficult, the intuition of proving this lemma is similar to that of Lemma 4. If \( C_2(t) \) is too small at a certain slot, then we will pull the suboptimal arm instead of the optimal arm with high probability. To be more specific, we focus on the slot \( t' \) when the optimal arm is pulled for the last time before \( t \) under load \( L_t = \epsilon_0 \). According to Lemma 4, \( C_1(t') \geq C^0_1(t') \geq \frac{(1 - \eta) pt}{2} \) with high probability, indicating \( t' \geq (1 - \eta) pt/2 \) with high probability. Moreover, the index for the optimal arm \( u_1(t') \leq u_1 + \delta \) with high probability for a sufficiently large \( t' \), because \( \sqrt{\frac{\log t}{t}} \rightarrow 0 \) as \( t \rightarrow \infty \). Then we can bound the probability of the event that the index of the suboptimal arm \( u_2(t') \leq u_1 + \delta = u_2 + (\Delta + \delta) \) when \( C_2(t' - 1) < \frac{\alpha \log t}{4(\Delta + \delta)^2} \).

Using the above lemmas, we can further refine the upper bound on the regret of AdaUCB and show that AdaUCB achieves smaller regret than traditional UCB.

**Theorem 2.** Using AdaUCB in the opportunistic bandit with random binary-valued load and random rewards, if \( \alpha > 8 \) and \( \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} < \frac{1}{8} \), we have

\[
R_{\text{AdaUCB}}(T) \leq \frac{4\epsilon_0 \alpha \log T}{\Delta} + O(1).
\]

**Sketch of Proof:** The key idea of the proof is to find an appropriate \( \delta \in (0, \Delta) \) such that \( \alpha > 8(1 + \frac{\delta}{\Delta})^2 \) and \( \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} < \frac{1}{8(1 + \frac{\delta}{\Delta})^2} \). In fact, the existence of this \( \delta \) is guaranteed under the assumptions \( \alpha > 8 \) and \( \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} < \frac{1}{8} \). Using this \( \delta \), we can then use Lemma 6 to bound the probability of pulling the suboptimal arm when the load is high. This indicates most explorations occur when the load is low, i.e., \( L_t = \epsilon_0 \). The conclusion of this theorem then follows according to Lemma 5.

**Remark 3:** Although there is no tight lower bound for the regret of UCB(\( \alpha \)), we know that for traditional (load-oblivious) bandit algorithms, \( E[C_2(T)] \) is lower bounded by \( \frac{\log^T}{\pi L(u_2, u_1)} \) (Lai and Robbins 1985) for large \( T \), where \( KL(u_2, u_1) \) is the Kullback-Leibler divergence. Without load-awareness, the regret will be roughly lower bounded by \( \frac{\log T}{\pi L(u_2, u_1)} \). In contrast, with load-awareness, AdaUCB can achieve much lower regret than load-oblivious algorithms, when the load fluctuation is large, i.e., \( \epsilon_0 \) and \( \epsilon_1 \) are small.

Theorem 2 directly implies the following result.

**Corollary 1.** Using AdaUCB in the opportunistic bandit with random reward under i.i.d. random binary load where \( \epsilon_0 = 0 \), if \( \alpha > 8 \) and \( \epsilon_1 < \frac{\pi^2}{4} \), we have \( R_{\text{AdaUCB}}(T) = O(1) \).

**Remark 4:** We note that the above \( O(1) \) bound is in the sense of expected regret, which is different from the high probability \( O(1) \) regret bound (Abbasi-Yadkori, Pál, and Szepesvári 2011). Specifically, while the opportunistic bandits can model the whole spectrum of load-dependent regret, Corollary 1 highlights one end of the spectrum where there are “free” learning opportunities. In this case, we push most explorations to the “free” exploration slots and result in an \( O(1) \) expected regret. Note that even under “free” exploration, we assume here that the value of the arms can be observed as discussed in Sec. 2.

It is worth noting that there are realistic scenarios where the exploration cost of a suboptimal arm is zero or close to zero. Consider the network configuration case where we use throughput as the reward. In this case, \( X_{a_i, t} \) is the percentage of the peak load that configuration \( a_i \) can handle. Because of the dummy low-priority traffic injected into the network, we can learn the true value of \( X_{a_i, t} \) under the peak load. At the same time, the configuration \( a_i \), although suboptimal, may completely satisfy the actual load \( L_t \) because it is high priority and thus served first. Therefore, although a suboptimal arm, \( a_i \) sacrifices no throughput on the real load \( L_t \), and thus generates a real regret of zero. In other words, even if the system load is always positive, the chance of zero regret under a suboptimal arm is greater than zero, and in practice, can be non-negligible. To capture this effect, we can modify the regret defined in Eq. 1 by replacing \( L_t \) with \( 0 \) when \( L_t \) is smaller than a threshold.
Last, we note that, under the condition of Corollary 1, it is easy to design other heuristic algorithms that can perform well. For example, one can do round-robin exploration when the load is zero and chooses the best arm when the load is non-zero. However, such naive strategies are difficult to be extended to more general cases. In contrast, AdaUCB applies to a wide range of situations, with both theoretical performance guarantees and desirable empirical evaluations.

4.3 AdaUCB under Continuous Load

Inspired by the insights obtained from the binary-valued load case, we discuss AdaUCB in opportunistic bandits under continuous load in this section.

Selection of truncation thresholds. When the load is continuous, we need to choose appropriate \( l^-(\cdot) \) and \( l^+(\cdot) \) for AdaUCB. We first assume that the load distribution is \textit{a priori} known, and discuss how to choose the thresholds under unknown load distribution later. The analysis under binary-valued load indicates that, the explorations mainly occur in low load slots. To guarantee sufficient explorations for a logarithmic regret, we propose to select the thresholds such that:

- The lower threshold \( l^-(\cdot) \) satisfies \( \Pr\{L_t \leq l^-(\cdot)\} = \rho > 0 \);
- The upper threshold \( l^+(\cdot) \geq l^-(\cdot) \).

In the special case of \( l^+(\cdot) = l^-(\cdot) \), we redefine the normalized load \( \hat{L}_t \) in \( \hat{L}_t = 0 \) when \( L_t \leq l^-(\cdot) \) and \( \hat{L}_t = 1 \) when \( L_t > l^-(\cdot) \).

Regret analysis. Under continuous load, it is hard to obtain regret bound as that in Theorem 2 for general \( l^-(\cdot) \) and \( l^+(\cdot) \) chosen above. Instead, we will show logarithmic regret for general \( l^-(\cdot) \) and \( l^+(\cdot) \), and illustrate the advantages of AdaUCB for the special case with \( l^-(\cdot) = l^+(\cdot) \).

First, we show that AdaUCB with appropriate truncation thresholds achieve logarithmic regret as below. This lemma is similar to Lemma 5 and the detailed outline of proof can be found in Appendix C.2.

Lemma 7. In the opportunistic bandit with random continuous load and random rewards, under AdaUCB with \( \Pr\{L_t \leq l^-(\cdot)\} = \rho > 0 \) and \( l^+(\cdot) \geq l^-(\cdot) \), we have

\[
\mathbb{E}[C_2(T)] \leq \frac{4\alpha \log T}{\Delta^2} + O(1). \tag{8}
\]

Next, we illustrate the advantages of AdaUCB under continuous load by studying the regret bound for AdaUCB with special thresholds \( l^+(\cdot) = l^-(\cdot) \).

Theorem 3. In the opportunistic bandit with random continuous load and random rewards, under AdaUCB with \( \Pr\{L_t \leq l^-(\cdot)\} = \rho > 0 \) and \( l^+(\cdot) = l^-(\cdot) \), we have

\[
R_{\text{AdaUCB}}(T) \leq \frac{4\alpha \log T \mathbb{E}[L_t | L_t \leq l^-(\cdot)]}{\Delta} + O(1), \tag{9}
\]

where \( \mathbb{E}[L_t | L_t \leq l^-(\cdot)] \) is the expectation of \( L_t \) conditioned on \( L_t \leq l^-(\cdot) \).

Sketch of Proof: Recall that for this special case \( l^+(\cdot) = l^-(\cdot) \), we let \( \hat{L}_t = 0 \) for \( L_t \leq l^-(\cdot) \) and \( \hat{L}_t = 1 \) for \( L_t > l^-(\cdot) \).

Then we can prove the theorem analogically to the proof of Theorem 2 for the binary-valued case. Specifically, when \( L_t \leq l^-(\cdot) \), we have \( \hat{L}_t = 0 \) and it corresponds to the case of \( L_t = c_0 (\hat{L}_t = 0) \) in the binary-valued load case. Similarly, the case of \( L_t > l^+(\cdot) (\hat{L}_t = 1) \) corresponds to the case of \( L_t = 1 - c_1 \) under binary-valued load with \( c_1 = 0 \). Then, we can obtain results similar to Lemma 5 and show that the regret under load \( L_t > l^+(\cdot) \) is \( O(1) \). Furthermore, the number of pulls under load level \( L_t \leq l^-(\cdot) \) is bounded according to Lemma 7. The conclusion of the theorem then follows by using the fact that all load below \( l^-(\cdot) \) are treated the same by AdaUCB, i.e., \( \hat{L}_t = 0 \) for all \( L_t \leq l^-(\cdot) \).

We compare the regret of AdaUCB and conventional bandit algorithms by an example, where the load level \( L_t \) is uniformly distributed in \( [0, 1] \). In this simple example, the regret of AdaUCB with thresholds \( l^-(\cdot) = l^+(\cdot) \) is bounded by \( R_{\text{AdaUCB}}(T) \leq \frac{4\alpha \log T}{\Delta^2} \cdot \frac{\rho}{2} + O(1) \), since \( \mathbb{E}[L_t | L_t \leq l^-(\cdot)] = \rho/2 \). However, for any load-oblivious bandit algorithm such as UCB(\( \alpha \)), the regret is lower bounded by \( \frac{\Delta \log T}{\alpha} \cdot \frac{\rho}{2} + O(1) \). Thus, AdaUCB achieves much smaller regret for a positive number \( \rho \) close to 0.

Remark 5: From the above analysis, we can see that the selection of \( l^+(\cdot) \) does not affect the order of the regret \( (\alpha \log T) \). However, for a fixed \( l^-(\cdot) \), we can further adjust \( l^+(\cdot) \) to control the explorations for the load in the range of \( (l^-(\cdot), l^+(\cdot)) \). Specifically, with a larger \( l^+(\cdot) \), more explorations happen under the load between \( l^-(\cdot) \) and \( l^+(\cdot) \). These explorations accelerate the learning speed but may increase the long term regret because we allow more explorations under load \( L_t > l^-(\cdot) \). The behavior is opposite if we use a smaller \( l^+(\cdot) \). In addition, appropriately chosen thresholds also handle the case when the load has little or no fluctuation, i.e., \( L_t \approx c \). For example, if we set \( l^-(\cdot) = c \) and \( l^+(\cdot) = 2c \), AdaUCB degenerates to UCB(\( \alpha \)).

E-AdaUCB. In practice, the load distribution may be unknown \textit{a priori} and may change over time. To address this issue, we propose a variant, named Empirical-AdaUCB (E-AdaUCB), which adjusts the thresholds \( l^-(\cdot) \) and \( l^+(\cdot) \) based on the empirical load distribution. Specifically, the algorithm maintains the histogram for the load levels (or its moving average version for non-stationary cases), and then select \( l^-(\cdot) \) and \( l^+(\cdot) \) accordingly. For example, we can select \( l^-(\cdot) \) and \( l^+(\cdot) \) such that the empirical probability \( \Pr\{L_t \leq l^-(\cdot)\} = \Pr\{L_t \geq l^+(\cdot)\} = 0.05 \). We can see that, in most simulations, E-AdaUCB performs closely to AdaUCB with thresholds chosen offline.

5 Experiments

In this section, we evaluate the performance of AdaUCB using both synthetic data and real-world traces. We use the classic UCB(\( \alpha \)) and TS (Thompson Sampling) algorithms as comparison baselines. In both AdaUCB and UCB(\( \alpha \)), we set \( \alpha \) as \( \alpha = 0.51 \), which is close to 1/2 and performs better than a larger \( \alpha \). We note that the gap between AdaUCB and the classic UCB(\( \alpha \)) clearly demonstrates the impact of opportunistic learning. On the other hand, TS is one of the
most popular and robust bandit algorithms applied to a wide range of application scenarios. So we apply it here as a reference. However, because AdaUCB and TS (or other bandit algorithms) improve UCB on different fronts, so their comparison does not clearly show the impact of opportunistic bandit.

**AdaUCB under synthetic scenarios.** We consider a 5-armed bandit with Bernoulli rewards, where the expected reward vector is \([0.05, 0.1, 0.15, 0.2, 0.25]\). Fig. 1(a) shows the regrets for different algorithms under random binary-value load with \(\epsilon_0 = \epsilon_1 = 0\) and \(\rho = 0.5\). AdaUCB significantly reduces the regret in opportunistic bandits. Specifically, the exploration cost in this case can be zero and AdaUCB achieves \(O(1)\) regret. For continuous load, Fig. 1(b) shows the regrets for different algorithms with beta distributed load. AdaUCB still outperforms the UCB(\(\alpha\)) or TS algorithms. Here, we define \(l'_p^-\) as the lower threshold such that \(\mathbb{P}\{L_t \leq l'_p^-\} = \rho\), and \(l'_p^+\) as the upper threshold such that \(\mathbb{P}\{L_t \geq l'_p^+\} = \rho\). More simulation results can be found in Appendix D.2 where we study the impact of environment and algorithm parameters such as load fluctuation and the thresholds for load truncation. These simulation results demonstrate that, with appropriately chosen parameters, the proposed AdaUCB and E-AdaUCB algorithms achieve good performance by leveraging the load fluctuation in opportunistic bandits. As a special case, with a single threshold \(l'^+ = l'^- = l'^{0.05}_p\), AdaUCB still outperforms UCB(\(\alpha\)) and TS, although it may have higher regret at the beginning.

**AdaUCB applied in MVNO systems.** We now evaluate the proposed algorithms using real-world traces. In an MVNO (Mobile Virtual Network Operator) system, a virtual operator, such as Google Fi [Project Fi, https://fi.google.com](https://fi.google.com), provides services to users by leasing network resources from real mobile operators. In such a system, the virtual operator would like to provide its users high quality service by accessing the network resources of the real operator with the best network performance. Therefore, we view each real mobile operator as an arm, and the quality of user experienced on that operator network as the reward. We use experiment data from Speedometer [Speedometer, https://storage.cloud.google.com/speedometer](https://storage.cloud.google.com/speedometer) and another anonymous operator to conduct the evaluation. More details about the MVNO system can be found in Appendix D.2. Here, using insights obtained from simulations based on the synthetic data, we choose \(l'^-\) and \(l'^+\) such that \(\mathbb{P}\{L_t \leq l'^-\} = \mathbb{P}\{L_t \geq l'^+\} = 0.05\). As shown in Fig. 2 the regret of AdaUCB is only about 1/3 of UCB(\(\alpha\)), and the performance of E-AdaUCB is indistinguishable from that of AdaUCB. This experiment demonstrates the effectiveness of AdaUCB and E-AdaUCB in practical situations, where the load and the reward are continuous and are possibly non-stationary. It also demonstrates the practicality of E-AdaUCB without a priori load distribution information.

### 6 Conclusions and Future Work

In this paper we study opportunistic bandits where the regret of pulling a suboptimal arm depends on external conditions such as traffic load or produce price. We propose AdaUCB that opportunistically chooses between exploration and exploitation based on the load level, i.e., taking the slots with low load level as opportunities for more explorations. We analyze the regret of AdaUCB, and show that AdaUCB can achieve provably lower regret than the traditional UCB algorithm, and even \(O(1)\) regret, under certain conditions. Experimental results based on both synthetic and real data demonstrate the significant benefits of opportunistic exploration under large load fluctuations.

This work is a first attempt to study opportunistic bandits, and several open questions remain. First, although AdaUCB achieves promising experimental performance under general settings, rigorous analysis with tighter performance bound remains challenging. Furthermore, opportunistic TS-type algorithms are also interesting because TS-type algorithms often performs better than UCB-type algorithms in practice. Last, we hope to investigate more general relations between the load and actual reward.

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Appendices

A. AdaUCB under Dirac Rewards

A.1 Proof of Lemma 1

We can verify that the conclusion holds for \( t = 1 \) and 2. For \( t \geq 3 \), we show the result by contradiction. If the conclusion is false, then there exists a \( \tau \geq 3 \) such that \( C_2(\tau - 1) \leq \frac{\alpha \log(\tau - 1)}{\Delta} + 1 \) but \( C_2(\tau) > \frac{\alpha \log \tau + 1}{\Delta} \). Because \( \log \tau > \log(\tau - 1) \), we know that arm-2 is pulled in slot \( \tau \). Thus, if \( L_\tau = \epsilon_0 \),

\[
u_1 + \frac{\alpha \log \tau}{C_1(\tau - 1)} \leq \nu_2 + \frac{\alpha \log \tau}{C_2(\tau - 1)}, \tag{10}
\]

indicating that \( \Delta = \nu_1 - \nu_2 < \frac{\alpha \log \tau}{\epsilon_0 (\tau - 1)} \), and thus \( C_2(\tau - 1) < \frac{\alpha \log \tau}{\Delta} \). Then \( C_2(\tau) \leq C_2(\tau - 1) + 1 < \frac{\alpha \log \tau + 1}{\Delta} \). Similarly, if \( L_\tau = 1 - \epsilon_1 \), we have \( C_2(\tau - 1) < \frac{\alpha_1 \log \tau}{\Delta} \times \frac{\alpha \log \tau + 1}{\Delta} \) and \( C_2(\tau) \leq C_2(\tau - 1) + 1 < \frac{\alpha \log \tau + 1}{\Delta} \). This contradicts the definition of \( \tau \) and completes the proof of this lemma.

A.2 Proof of Lemma 2

The conclusion is true when \( t = 1 \) and \( t = 2 \). For \( t \geq 3 \), we prove the conclusion by contradiction. If the conclusion is false, then there exists \( \tau \geq 3 \), such that \( C_2(2(\tau - 1)) \geq f(\tau - 1) \) but \( C_2(2\tau) < f(\tau) \). Noticing that for \( s > 2 \), we have \( f(s) \in [0, 1] \), and thus \( f(\tau) \leq f(\tau - 1) + 1 \). Therefore, \( C_2(2\tau) < C_2(2(\tau - 1)) + 1 \), indicating that \( C_2(2\tau) = C_2(2(\tau - 1)) = C_2(2(\tau - 1)) \). Hence, arm-1 is pulled at \( 2\tau - 1 \) and \( 2\tau \). In particular, at time \( 2\tau \), we have

\[
u_1 + \sqrt{2\tau - 1 - C_2(2\tau - 1)} \geq \nu_2 + \sqrt{2\tau - 1 - C_2(2\tau - 1)}. \tag{11}
\]

This implies that

\[
\frac{\Delta}{\sqrt{\alpha \log \tau}} \geq \frac{\Delta}{\sqrt{\alpha \log(2\tau)}} \geq \frac{1}{\sqrt{C_2(2\tau - 1)}} \frac{1}{\sqrt{2\tau - 1 - C_2(2\tau - 1)}},
\]

On the other hand, one can easily show that for a sufficiently large \( \tau \), we have \( C_2(2\tau - 1) \leq (2\tau - 1)/2 \) and thus \( \frac{1}{\sqrt{2\tau - 1 - C_2(2\tau - 1)}} \leq \frac{\sqrt{2}}{\sqrt{2\tau - 1}} \). Consequently,

\[
\frac{\Delta}{\sqrt{\alpha \log \tau}} \geq \frac{1}{\sqrt{C_2(2\tau - 1)}} \frac{\sqrt{2}}{\sqrt{2\tau - 1}}. \tag{12}
\]

Thus, \( C_2(2\tau) = C_2(2\tau - 1) \geq h(\tau) \geq f(\tau) \). This contradicts the definition of \( \tau \) and completes the proof.

A.3 Proof of Lemma 3

According to the load we considered here, \( \mu_t = 1 - \epsilon_1 \) when \( t \) is an odd number, and thus \( t \) can be represented as \( t = 2\tau + 1 \), where \( \tau = (t - 1)/2 \). With the setting of

\[
0 \leq \epsilon_0, \epsilon_1 < 0.5, \text{ we have } \frac{1 - \epsilon_0}{\epsilon_1} \to 1. \text{ On the other hand, noting that } \frac{\log(2\tau + 1)}{\log \tau} \to 1 \text{ and } \frac{\log \tau}{2\tau + 1} \to 0 \text{ as } \tau \to \infty, \text{ we know that there exists a number } T_1 \text{ such that for } t = 2\tau + 1 \geq T_1,
\]

\[
\frac{\log(2\tau + 1)}{\log(2\tau - 1 - \Delta)} - 1 < \frac{1 - \epsilon_0}{\epsilon_1}. \tag{13}
\]

Thus, according to Lemma 2, we have \( C_2(2\tau) \geq f(\tau) \), and thus for \( t \geq T_1 \),

\[
\hat{u}_2(t) = u_2 + \sqrt{\frac{\alpha_1 \log(2\tau + 1)}{(1 - \epsilon_0)C_2(2\tau)}} < u_1 < \hat{u}_1(t). \tag{14}
\]

This implies the conclusion of this lemma, i.e., \( a_t = 1 \) when \( L_t = 1 - \epsilon_1 \) for \( t \geq T_1 \).

B. AdaUCB under Random Binary-Valued Load and Random Rewards

B.1 Proof of Lemma 4

Let \( t' < t \) be the last slot before \( t \) when the suboptimal arm (arm-2) is pulled under \( L_t = \epsilon_0 \). Then, we can separate the probability \( \Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}\} \) according to the value of \( t' \) as follows:

\[
\Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}\} = \Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}, t' < t/2\} + \Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}, t' \geq t/2\}. \tag{15}
\]

Note that if \( t' < t/2 \), then the optimal arm will be pulled in all remaining slots \( \tau > t' \) with \( L_\tau = \epsilon_0 \).

For the first term, if \( t' < t/2 \), then \( t - t' \geq t/2 \), and with high probability, there will be at least \( \frac{(1 - \eta)pt}{2} \) low-load slots according to Chernoff-Hoeffding’s inequality. Specifically, for a given \( t' < t/2 \),

\[
\Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}, t' < t/2\} \leq e^{-2\eta^2 t'}, \quad t' < t/2.
\]

Summing over all possible \( t' < t/2 \), we have

\[
\Pr\{C_1^{(0)}(t) < \frac{(1 - \eta)pt}{2}, t' < t/2\} \leq te^{-\eta^2 t'/2}. \tag{16}
\]

For the second term, we consider the distribution for the total number of low-load slots, i.e., \( \sum_{t=1}^{\tau} I(L_t = \epsilon_0) = C_1^{(0)}(t) + C_2^{(0)}(t) \). We know that

\[
\Pr\{C_1^{(0)}(t) + C_2^{(0)}(t) < (1 - \eta)pt\} \leq e^{-2\eta^2 t'}. \tag{17}
\]
Therefore,
\[
\mathbb{P}\{C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}, t' \geq t/2\} \leq \mathbb{P}\{C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}, t' \geq t/2, C_1^{(0)}(t) + C_2^{(0)}(t) \geq (1-\eta)\rho t\} + \mathbb{P}\{C_1^{(0)}(t) + C_2^{(0)}(t) < (1-\eta)\rho t\} \leq \mathbb{P}\{C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}, t' \geq t/2, C_1^{(0)}(t) + C_2^{(0)}(t) \geq (1-\eta)\rho t\} + e^{-2\eta^2 t}.
\] (18)

According to the definition of \(t'\), we know that if \(C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}\) and \(C_1^{(0)}(t) + C_2^{(0)}(t) \geq (1-\eta)\rho t\), then \(C_2(t' - 1) \geq (1-\eta)\rho t - \frac{(1-\eta)\rho t}{2} - 1 = \frac{(1-\eta^2)\rho t}{2} - 1\). Thus,
\[
\mathbb{P}\{C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}, t' \geq t/2, C_1^{(0)}(t) + C_2^{(0)}(t) \geq (1-\eta)\rho t\} \leq \mathbb{P}\{C_2(t' - 1) \geq \frac{(1-\eta^2)\rho t}{2} - 1, t' \geq t/2\}. \] (19)

Moreover, since arm-2 is pulled at time \(t'\), this requires that
\[
\hat{u}_2(t') = \hat{u}_2(t') + \sqrt{\frac{\alpha \log(t')}{C_2(t' - 1)}} \geq \hat{u}_1(t') = \hat{u}_1(t') + \sqrt{\frac{\alpha \log(t')}{C_1(t' - 1)}}. \] (20)

This requires that at least one of the events \(\hat{u}_1(t') < u_1\) and \(\hat{u}_2(t') > u_1\) should be true. However, for a fixed \(t' \geq t/2\), similar to (Auer, Cesa-Bianchi, and Fischer 2002), we have
\[
\mathbb{P}\{\hat{u}_1(t') < u_1\} \leq e^{-2\alpha \log t' - (t')^{-2}}. \] (21)

Moreover, because \(\frac{\log(t')}{(1-\eta)\rho t'/2 - 1} \to 0\) as \(t' \to \infty\), there exists a \(T_2\) such that for \(t \geq T_2\), \(\frac{\alpha \log(t')}{(1-\eta)\rho t'/2 - 1} \leq \Delta/2\). Thus,
\[
\mathbb{P}\{\hat{u}_2(t') \geq u_1\} \leq \mathbb{P}\{\hat{u}_2(t') \geq u_2 + \frac{\alpha \log(t')}{C_2(t' - 1)}\} \leq (t')^{-2\alpha}. \] (22)

Considering all possible values of \(t'\), we have
\[
\mathbb{P}\{C_1^{(0)}(t) < \frac{(1-\eta)\rho t}{2}, t' \geq t/2, C_1^{(0)}(t) + C_2^{(0)}(t) \geq (1-\eta)\rho t\} \leq \sum_{t'=t/2}^t 2(t')^{-2\alpha} \leq (t + 2)(t/2)^{-2\alpha}. \] (23)

The conclusion of the lemma then follows by combining Eqs (16), (18), and (23).

### B.2 Proof of Lemma 5

We can prove this lemma by borrowing ideas from the analysis for the traditional UCB policies (Auer, Cesa-Bianchi, and Fischer 2002; Bubeck 2010), except that we need more conditions to bound the UCB for the optimal arm when the load level is \(L_t = 1 - \epsilon_t\). Specifically, we analyze the probabilities for load levels \(\epsilon_0\) and \(1 - \epsilon_1\), respectively.

**Case 1:** \(L_t = \epsilon_0\)

When the load is \(L_t = \epsilon_0\), AdaUCB makes decisions according to the indices \(\hat{u}_k(t) = \bar{u}_k(t) + \sqrt{\frac{\alpha \log t}{C_k(t-1)}}\) for all \(k\), which is the same as traditional UCB. Thus, the suboptimal arm can be pulled when at least one of the following events is true:

\[
\hat{u}_1(t) \leq u_1, \] (24)

\[
\hat{u}_2(t) > u_2 + 2\sqrt{\frac{\alpha \log t}{C_2(t-1)}}, \] (25)

and

\[
C_2(t - 1) < \frac{4\alpha \log T}{\Delta^2}. \] (26)

Then we can bound the probabilities for the events \(\{24\}\) and \(\{25\}\) according to (Auer, Cesa-Bianchi, and Fischer 2002).

**Case 2:** \(L_t = 1 - \epsilon_1\)

When the load is \(L_t = 1 - \epsilon_1\), AdaUCB chooses the arm according to the indices \(\hat{u}_k(t) = \bar{u}_k(t) + \sqrt{\frac{\alpha \log t}{(1-\epsilon_0)C_k(t-1)}}\) for all \(k\). Thus, pulling the suboptimal arm requires one of the following events is true:

\[
\hat{u}_1(t) \leq u_1 - (1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}})\Delta/2, \] (27)

\[
\hat{u}_2(t) > u_2 + (1 + \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}})\sqrt{\frac{\alpha \log t}{C_2(t-1)}}, \] (28)

and

\[
C_2(t - 1) < \frac{4\alpha \log T}{\Delta^2}. \] (29)

In fact, if all the above events are false, we have

\[
\hat{u}_1(t) > u_1 - (1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}})\Delta/2 = u_2 + (1 + \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}})\sqrt{\frac{\alpha \log t}{C_2(t-1)}} \geq \hat{u}_2(t),
\]
indicating that arm-1 will be pulled. Now we only need to bound the probability of events \(\{27\}\) and \(\{28\}\).

For event \(\{27\}\), since we use a smaller index than the traditional UCB, we require a sufficiently large \(C_1(t-1)\) to guarantee that \(\hat{u}_1(t)\) is close enough to \(u_1\). Specifically, because \(\frac{\log t}{t} \to 0\) as \(t \to \infty\), there exists a constant number \(T_3\) such that \(\sqrt{\frac{\alpha \log t}{(1-\eta)\rho t'/2}} \leq \frac{\Delta}{2}\) for \(t \geq T_3\). Thus, for \(t \geq T_3\),

we have

\[
\mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\Delta}{2} \right\} 
\leq \mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\Delta}{2} \right\} + \mathbb{P}\left\{ C_1(t - 1) \leq \left(1 - \eta \right) pt \right\}
\]

\[
\leq \mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\alpha \log t}{C_1(t - 1)} \right\} + \mathbb{P}\left\{ C_1(t - 1) < \left(1 - \eta \right) pt \right\}
\]

\[
\leq \mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\alpha \log t}{C_1(t - 1)} \right\} + \mathbb{P}\left\{ C_1(t - 1) < \left(1 - \eta \right) pt \right\}
\]

Using the same argument as in (Auer, Cesa-Bianchi, and Fischer 2002), we have

\[
\mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\alpha \log t}{C_1(t - 1)} \right\} \leq t^{-2\alpha}.
\]

Moreover, we can bound \( \mathbb{P}\{ C_1(t - 1) < \left(1 - \eta \right) pt \} \) according to Lemma 3. Thus

\[
\mathbb{P}\left\{ \tilde{u}_1(t) \leq u_1 - \left(1 - \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\alpha \log t}{C_1(t - 1)} \right\} \leq t^{-2\alpha} + \frac{\alpha \log t}{\sqrt{C_2(t - 1)}}
\]

For event (28), it is equivalent to \( \{ \tilde{w}_1(t) > u_2 + \frac{\alpha \log t}{\sqrt{C_2(t - 1)}} \} \). Thus,

\[
\mathbb{P}\left\{ \tilde{w}_1(t) > u_2 + \left(1 + \sqrt{\frac{\epsilon_1}{1 - \epsilon_0}} \right) \frac{\alpha \log t}{C_2(t - 1)} \right\} \leq t^{-2\alpha}.
\]

Combining the above two cases together over all \( t \), and using the bound \( \sum_{t=1}^{T} \left[ t^{-2\alpha} + \frac{\alpha \log t}{\sqrt{C_2(t - 1)}} \right] = O(1) \), we obtain the conclusion of this lemma.

### B.3 Proof of Lemma 6

Let \( t' \) be the last slot before \( t \) when the optimal arm (arm-1) is pulled under load level \( L_t = \epsilon_0 \). According to Lemma 4, we know that \( C_1(t') \) is greater than \( (1 - \eta) pt/2 \) with high probability. Thus,

\[
\mathbb{P}\{ t' < (1 - \eta) pt/2 \} \leq (t + 2)(t/2)^{-2\alpha} + te^{-\eta t^2/2} + e^{-2\eta t^2}.
\]

Now we focus on the case where \( t' \geq (1 - \eta) pt/2 \). Again according to Lemma 4, we know that \( C_1(t') \geq C_1(t') \geq (1 - \eta) pt/2 \) with high probability. Because \( \frac{\log t'}{(1 - \eta) pt/2 - t} \to 0 \), there exists a constant \( T_4 \) independent of \( T \), such that for any \( t' \geq T_4 \), we have \( \sqrt{\frac{\alpha \log t'}{C_1(t' - 1)}} \leq \delta/2 \) if \( C_1(t' - 1) \geq (1 - \eta) pt/2 - 1 \). Thus, for \( t' \geq T_4 \),

\[
\mathbb{P}\{ \hat{u}_1(t') > u_1 + \delta \} \leq \mathbb{P}\{ \hat{u}_1(t') \leq u_1 + 2 + \sqrt{\frac{\alpha \log t'}{C_1(t' - 1)}} \}
\]

\[
C_1(t' - 1) \geq (1 - \eta) pt/2 - 1 \}
\]

\[
\leq \mathbb{P}\{ C_1(t' - 1) < (1 - \eta) pt/2 - 1 \}
\]

\[
\leq \frac{(1 - \eta) pt/2 - 1}{(1 - \eta) pt/2 - 1}
\]

\[
\leq \frac{\alpha \log t'}{4(\Delta + \delta)^2}.
\]

Note that \( C_2(t) < \frac{\alpha \log t}{4(\Delta + \delta)^2} \) indicates that \( C_2(t') < \frac{\alpha \log t}{4(\Delta + \delta)^2} \). Thus,

\[
\mathbb{P}\{ C_2(t) < \frac{\alpha \log t}{4(\Delta + \delta)^2} \}
\]

\[
\leq \sum_{n; n < \frac{\alpha \log t}{4(\Delta + \delta)^2}} \mathbb{P}\{ \hat{u}_1(t') \leq u_1 + \delta, C_2(t' - 1) = n \}
\]

\[
\mathbb{P}\{ \hat{u}_1(t') > u_1 + \delta \}
\]

\[
+ \sum_{n; n < \frac{\alpha \log t}{4(\Delta + \delta)^2}} \mathbb{P}\{ \hat{u}_2(t') \leq u_1 + \delta, C_2(t' - 1) = n \}.
\]

We have already bounded the first term by (31). To bound the second term, we note that when \( C_2(t' - 1) = n < \frac{\alpha \log t}{4(\Delta + \delta)^2} \), we have \( \sqrt{\frac{\alpha \log t'}{C_2(t' - 1)}} \geq 2(\Delta + \delta) \sqrt{\frac{\log t'}{\log t}} \geq (2 - \delta)(\Delta + \delta) \) for a sufficiently large \( t \) (independent of \( T \)). Thus, for \( n < \frac{\alpha \log t}{4(\Delta + \delta)^2} \),

\[
\mathbb{P}\{ \hat{u}_2(t') \leq u_1 + \delta, C_2(t' - 1) = n \}
\]

\[
\leq \mathbb{P}\{ \hat{u}_2(t') + \frac{\alpha \log t}{C_2(t' - 1)} \leq u_2 + \Delta + \delta, C_2(t' - 1) = n \}
\]

\[
\leq \frac{\alpha \log t'}{C_2(t' - 1)} \leq \frac{\alpha \log t}{\sqrt{C_2(t' - 1)}}
\]

\[
\leq e^{-2(\frac{\delta}{\Delta + \delta})^2}\frac{\alpha \log t'}{2(\Delta + \delta)^2}.
\]

Combining Eqs (30), (31), and (32), and summing over all possible values of \( t' \), we have

\[
\mathbb{P}\{ C_2(t) < \frac{\alpha \log t}{4(\Delta + \delta)^2} \}
\]

\[
\leq (t + 2)(t/2)^{-2\alpha} + te^{-\eta t^2/2} + e^{-2\eta t^2} + t\left[ \frac{(1 - \eta) pt/2 - 2\alpha}{2} + \frac{(1 - \eta) pt/2 - 2\alpha(\frac{\delta}{\Delta + \delta})^2}{2} + \frac{(t + 2)(1 - \eta) pt/4}{4} + \frac{t e^{-\frac{\eta t^2}{2} - 2\alpha}}{2} + e^{-\eta t^2(1 - \eta) pt/2} \right].
\]
Theorem 2. To show this theorem, we only need to show that the number of pulls of the suboptimal arm under the higher load \(\tilde{L}_t = 1 - \epsilon_t\), is bounded by \(O(1)\).

To show this result, we analyze the probability of pulling the suboptimal arm under higher load, i.e., \(\tilde{L}_t = 1 - \epsilon_t\). We need to show that the number of pulls of the suboptimal arm by Lemma 5. To show this theorem, we only use the range of \(\alpha\).

To bound the probability of the event about \(\hat{u}_1(t)\), we note that \(C_1(t - 1) > (1 - \eta)\rho t/2\) with high probability and for a sufficient large \(t\), we have \(1 - \sqrt{\frac{\epsilon_l}{1 - \epsilon_0}} \sqrt{\frac{\alpha \log t}{C_1(t - 1)}} \leq \Delta/4\). Then, using a similar argument as in Lemma 5, we can bound the first event as:

\[
P\{\hat{u}_1(t) \leq u_2 + \frac{3\Delta}{4}\} \leq t^{-2\alpha} + (t + 2)(t/2)^{-2\alpha} + te^{-\eta t^2/2} + e^{-2\eta t^2},
\]

To bound the probability of the event about \(\hat{u}_2(t)\), we let \(\delta \in (0, \Delta)\) be a number satisfying \(\alpha > 8(1 + \frac{\delta}{\Delta})^2\) and \(\sqrt{\frac{\epsilon_l}{1 - \epsilon_0}} < \frac{\Delta}{8(\Delta + \delta)}\) (the existence of \(\delta\) is guaranteed by the range of \(\alpha\) and \(\sqrt{\frac{\epsilon_l}{1 - \epsilon_0}}\)). Note that when \(C_2(t - 1) \geq \frac{\alpha \log(t - 1)}{4(\Delta + \delta)^2}\), which occurs with high probability according to Lemma 6, we have \(\sqrt{\frac{\alpha \log t}{C_2(t - 1)}} \leq \Delta/4\). This leads to the following bound:

\[
P\{\hat{u}_2(t) > u_2 + \frac{3\Delta}{4}\} \leq P\left\{\hat{u}_2(t) > u_2 + \frac{3\Delta}{4}, C_2(t - 1) \geq \frac{\alpha \log(t - 1)}{4(\Delta + \delta)^2}\right\} + P\{C_2(t - 1) < \frac{\alpha \log(t - 1)}{4(\Delta + \delta)^2}\}.
\]

The second term is bounded according to Lemma 6. For the first term, we have

\[
P\{\hat{u}_2(t) > u_2 + \frac{3\Delta}{4}, C_2(t - 1) \geq \frac{\alpha \log(t - 1)}{4(\Delta + \delta)^2}\} \leq P\left\{\hat{u}_2(t) > u_2 + \sqrt{\frac{\alpha \log t}{C_2(t - 1)}}\right\} \leq e^{-2\alpha \log t/\zeta} = t^{-2\alpha / \zeta}.
\]

Considering the summation over all \(t\) and noting that \(2\alpha / \zeta > 1\) and \(2\alpha (\frac{1 - \delta}{\Delta + \delta})^2 > 2\), we know that the summation is bounded by \(O(1)\). This completes the proof of this theorem.

\[\begin{align*}
\hat{u}_1(t) &\leq u_1 - (1 - \epsilon_t) \frac{\Delta}{2}, \\
\hat{u}_2(t) &> u_2 + (1 + \epsilon_t) \sqrt{\frac{\alpha \log t}{C_2(t - 1)}}, \\
\end{align*}\]
Then we can bound the regret using the same argument for Case 2 in the proof of Lemma 5. The total regret are then bound by summing over the above two cases.

D Additional Simulations

D.1 Simulations under Synthetic Scenarios

We use the periodic square wave load to study the impact of load fluctuation. We use the square wave load defined in [4,1] and let \( l^- = \epsilon_0 \) and \( l^+ = 1 \) as in Sec. 4. In this case, the load fluctuation is larger when \( \epsilon_0 \) and \( \epsilon_1 \) are smaller. As we can see from Fig. 3 with adaptive exploration based on load level, AdaUCB significantly reduces the regret in opportunistic bandits, especially compared with UCB(\( \alpha \)). Comparing the results for different \( \epsilon_0 \) and \( \epsilon_1 \) values, we can see that the improvement of AdaUCB is more significant under loads with larger fluctuations. In particular, when \( \epsilon_0 = 0 \), i.e., the exploration cost is zero if the load is under certain threshold, the proposed AdaUCB achieves \( O(1) \) regret, as shown in Fig. 3(a).

We also test the performance of the algorithms under binary-valued load, which is the load in Sec. 4.2. The results are shown in Fig. 4. Here, let \( \epsilon_0 = \epsilon_1 = 0 \). It can be seen that the AdaUCB achieves \( O(1) \) regret which is consistent with the analytical result. Also, \( \rho \) is time proportion of low load. Thus, a larger \( \rho \) leads to a smaller regret for all algorithms.

We next investigate the performance of the algorithms under continuous load. Here, we assume the load is i.i.d. over time, following the beta distribution \( \text{Beta}(2,2) \). We study the impact of different truncation thresholds for AdaUCB.

We define \( l_p^- \) as the lower threshold such that \( P\{L_t \leq l_p^- \} = \rho \), and \( l_p^+ \) as the upper threshold such that \( P\{L_t \geq l_p^+ \} = \rho \). From Fig. 5 we can see that the selection of \( l^- \) and \( l^+ \) affects the performance of AdaUCB. Typically, a small \( l^- \) is sufficient to provide good performance, because there are also exploration opportunities when the load is between \( l^- \) and \( l^+ \). In addition, it can be seen that, with a lower \( l^- \), although the regret at very beginning grows a little bit faster, ultimately the performance of algorithm is better.

D.2 Simulations Setting in MVNO Systems

Here, we provide more details about the MVNO system mentioned in Sec. 5. In an MVNO system, a virtual operator, such as Google Fi (Project Fi, https://fi.google.com), provides services to users by leasing network resources from real mobile operators such as T-Mobile and Sprint. In such a system, the virtual operator would like to provide its users high quality service by accessing the network resources of the real operator with the best network performance. Therefore, we view each real mobile operator as an arm, and the quality of user experienced on that operator network as the reward. We run experiments based on traces collected from real cellular networks, provided by Speedometer (Speedometer, https://storage.cloud.google.com/speedometer). Speedometer is a custom Android mobile network measurement app developed by Google, running on thousands of volunteer phones. The data consists of ping, traceroute, DNS lookup, HTTP fetches, and UDP packet-loss measurements for two years. We use round-trip-time (RTT) as a performance indicator for the quality of user experience, and use the inverse of RTT (normalized to \([0,1]\)) as the reward. We consider a three-armed case, where we consider the three operators, Verizon, T-Mobile, and Sprint, as three arms, using data from Speedometer dataset. We use the load trace of another anonymous operator as the load of the virtual network. The load trace is illustrated in Fig. 6 which shows a clear semi-periodic nature.
Figure 3: Regret under periodic square wave load.

Figure 4: Regret under binary-valued load.

Figure 5: Regret under beta distributed load.
Figure 6: Normalized traffic load in a cellular network (see Sec. 5).