STRANGENESS PHOTOPRODUCTION FROM THE
DEUTERON AND HYPERON–NUCLEON INTERACTION

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Abstract

Pronounced effects due to final state hyperon–nucleon interaction are predicted in strangeness photoproduction reaction on the deuteron. Use is made of the covariant reaction formalism and the $P$–matrix approach to the hyperon–nucleon interaction.

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The first measurement of kaon photoproduction on the deuteron are anticipated this year [1]. These data may open a new window on the ΛN and ΣN forces since the final state YN interaction (FSI) plays an important role in the γd → K+YN reaction. This problem has been addressed by several authors starting from the pioneering paper by Renard and Renard [2-3]. Two points make the present work different from the previous studies: (i) the use of the covariant formalism both for the reaction mechanism and the deuteron wave function, and (ii) the P–matrix approach to the FSI which takes into account the subnuclear degrees of freedom and disentangle the dynamical singularities from kinematical threshold effects [4]. Our main result is a prediction of the spectacular effects in the reaction cross section due to the YN FSI.

The reaction γd → K+YN, Y = Λ, Σ0 is a 2 → 3 process. The corresponding double differential cross–section reads

\[ d^2\sigma = \frac{1}{2^{11/2} \pi^5 kM_dE_K} \frac{\lambda^{1/2}(s_2,m_Y^2,m_n^2)}{s_2} \int d\Omega_{Yn}^* |T|^2. \]  

(1)

Here k, p_K^2, E_K and Ω_K correspond to the deuteron rest system with z-axis defined by the incident photon beam direction k. The solid angle Ω_{Yn}^* is defined in the Yn center-of-momentum system. The quantity \( \lambda(x,y,z) \) is the standard kinematical function

\[ \lambda(x,y,z) = x^2 - 2(y+z)x + (y-z)^2. \]

We shall use the covariant relativistic approach to calculate the amplitude \( T \) of the process γd → K+YN. The amplitude will be approximated by the two leading diagrams, namely the tree (pole, or plane waves) graph and the triangle graph with FSI. It will be demonstrated that within the covariant approach one easily retrieves the usual nonrelativistic impulse approximation and the Migdal-Watson approach to FSI. We start with the tree diagram. To calculate it two blocks have to be specified: (i) the elementary photoproduction amplitude \( M_{\gamma K} \) on the proton, and (ii) the deuteron vertex \( \Gamma_d \). The elementary amplitude used in the present calculation was derived from the tree level effective Lagrangian [5]. Taken into account were resonances with the spin \( \leq 5/2 \) in the s–channel the spin–1/2 resonances in the u–channel, and K*(892) and K1(1270) resonances in the t–channel. This amplitude
has the following decomposition over invariant terms \([6]\)

\[
M^{\gamma K} = \pi_Y \sum_{j=1}^{6} A_j M_j (s', t', u') u_p ,
\]

(2)

where \(s' = (k + p_p)^2\), \(t' = (k - p_K)^2\), \(u' = (k - p_Y)^2\).

The decomposition of the deuteron vertex function \(\Gamma_d\) in independent Lorentz structures has the form \([7]\)

\[
\Gamma_d = \sqrt{m_N} \left[ (p_p + p_n)^2 - M_d^2 \right] \left[ \varphi_1(t_2) \frac{(p_p - p_n)_\mu}{2m_N^2} + \varphi_2(t_2) \frac{1}{m_N} \gamma_\mu \right] \mathcal{E}^{\mu}(p_d, \lambda). \tag{3}
\]

Here \(t_2 = (p_d - p_n)^2\), \(\mathcal{E}^{\mu}(p_d, \lambda)\) is the polarization 4-vector of the deuteron with momentum \(p_d\) and polarization \(\lambda\).

Now we can write the following expression for the tree diagram

\[
T^{(t)} = \pi_Y \left\{ \left( \sum_{j=1}^{6} A_j M_j (s', t', u') \right) S(p_p) \Gamma_d \right\} u_n^c ,
\]

(4)

where \(u_n^c\) is a charge conjugated neutron spinor.

Covariant equations (1) and (4) can be easily reduced to the standard impulse approximation. Neglecting the spin summation in the matrix element (4) (factorization conjecture) and introducing the deuteron wave function as a product \([8]\) \(\Psi_d = [2(2\pi)^3 M_d]^{1/2} S(P_p) \Gamma_d\), one retrieves the nonrelativistic impulse approximation

\[
\frac{d^2 \sigma}{d|p_K|d\Omega_K} = \frac{p_K^2|p_Y^*|}{64\pi^2 k E_K \sqrt{s_2}} \int d\Omega_Y S(p_n) |M^{\gamma K}|^2 |\psi_d|^2 , \tag{5}
\]

where \(p_Y^*\) corresponds to the \(YN\) center–of–momentum system. The main physical difference between the covariant deuteron vertex used in the present calculation and the nonrelativistic wave function entering into (5) is that the former contains singlet and triplet \(p\)–wave components absent in the later \([8]\).

Next consider the loop (triangle) diagram with \(YN\) FSI. The corresponding amplitude is given by

\[
T^{(l)} = \int \frac{d^4 p_n}{(2\pi)^4} \pi_Y(p_Y^*) \left\{ \left( \sum_{j=1}^{6} A_j M_j \right) S(p_p) \Gamma_d C S(p_n) T_{YN} S(p_Y) \right\} \pi(p_n)^c . \tag{6}
\]
Here \( C = \gamma_2\gamma_0 \) is the charge-conjugation matrix, \( T_{Yn} \) is a four-fermion hyperon-nucleon vertex, this vertex being “dressed” by corresponding spinors constitutes the hyperon-nucleon amplitude \( F_{Yn} \).

The comprehensive treatment of the loop diagram will be presented in the forthcoming detailed publication while here we resort to a simple approximation with the aim to expose the effects of the FSI. Namely, only positive frequency components are kept in all three propagators \( S(p_j) \), \( j = p, n, Y \), then the integration over the time component \( dp^0_n \) is performed and the deuteron wave function is introduced in the same way as it was done in arriving to equation (5). Thus we get the following expression for \( T^{(l)} \):

\[
T^{(l)} = -\sqrt{(2\pi)^3 2M_d} \int \frac{dp^*}{(2\pi)^3} \frac{M^\gamma K}{p^*^2 - p^{'*}^2 - i0} F_{Yn}(E^*_{Yn}) ,
\]

where \( p^*^2 \) and \( p^{'*}^2 \) are the \( Yn \) momenta in the center–of–mass \( Yn \) system before and after the rescattering, \( F_{Yn}(E^*_{Yn}) \) is the half-off-shell \( Yn \) scattering amplitude at the energy \( E^*_{Yn} = p^{'*}^2 / m_{YN} \). The use of the nonrelativistic propagator in (7) is legitimate since FSI is essential at low \( YN \) relative momenta. In the kinematical region where FSI is important the amplitude \( M^\gamma K \) and the deuteron wave function \( \psi_d \) are smoother functions of \( p^* \) compared to the scattering amplitude \( F_{Yn} \). Therefore one can set \( p^* = p^{'*} \) in their arguments and take them off the integral. Next recall that as it was shown above the tree (plane waves) amplitude \( T^{(l)} \) allows the representation

\[
T^{(l)} \approx \sqrt{(2\pi)^3 2M_d} M^\gamma K \psi_d .
\]

Therefore for the sum of the two diagrams we can write

\[
T^{(l)} + T^{(1)} = T^{(l)} \int \frac{dp^*}{(2\pi)^3} \Psi^{(-)}_{p^{'*}}(p^*) \equiv T^{(l)} / D,
\]

where

\[
\Psi^{(-)}_{p^{'*}}(p^*) = (2\pi)^3 \delta(p^* - p^{'*}) - \frac{F_{Yn}(E^*_{Yn})}{p^*^2 - p^{'*}^2 + i0} .
\]

and \( 1/D \) denotes the enhancement factor which will be calculated in the \( P \)-matrix approach. The \( P \)-matrix description of the \( YN \) interaction including threshold phenomena and the
resonance at 2.13 GeV close to the \( \Sigma N \) threshold was presented in [9]. According to [9] the 2.13 GeV structure is not a genuine six–quark state but the \( P \)-matrix partner of the deuteron (see also [10]). Using the connection between \( S \)- and \( P \)-matrices [11], one arrives at the following expression for the enhancement factor

\[
D^{-1} = \frac{e^{-ip'^*b}}{2ip'^*b} \left\{ \frac{e^{-ip'^*b}R(-ip'^*) - e^{ip'^*b}R(+ip'^*)}{R(+ip'^*)} \right\},
\]

(11)

where

\[
R(ip'^*) = (P^0_1 - ip'^*)(E - E_n + \frac{\Lambda^2_1}{P^0_1 - ip'^*} + \frac{\Lambda^2_2}{P^0_2 - ip'^*}),
\]

(12)

and where the elements of the \( P \)-matrix are given by [11]

\[
P_{ij} = P^0_i \delta_{ij} + \frac{\lambda_i \lambda_j}{E - E_n},
\]

(13)

and indices 1 and 2 correspond to \( \Lambda n \) and \( \Sigma^0 n \) channels, \( p'^*_2 \) is the momentum in the \( \Sigma^0 n \) channel. The numerical values of the \( P \)-matrix parameters entering into (11)-(13) may be found in [9].

Finally we present the results of the calculations obtained using the equations (1),(4),(9) and (11). Use has been made of the elementary photoproduction amplitude from [5] and the deuteron vertex function taken from the relativistic Gross model [8]. Calculations of the plane–waves diagram (4) with this input were performed in [12]. In Fig.1 the double differential cross section (1) is shown as a function of the photon energy in the \( \Lambda n \) invariant mass region close to the \( \Lambda n \) threshold (2.05\( GeV \leq \sqrt{s_{\Lambda n}} \leq 2.09\( GeV \)). The pronounced peak typical for the FSI is seen at the \( \Lambda n \) threshold.
FIG. 1. The double differential cross section as a function of the photon energy for $P_K = 1.4 GeV$ and $\theta_{\gamma K} = 1^0$.

In Fig. 2 the same cross section is plotted in a different kinematical conditions covering a wider region of the $NY$ invariant mass $(2.05 GeV \leq \sqrt{s_{\Lambda n}} \leq 2.17 GeV)$ including the $\Sigma N$ threshold. Apart from the structure at the $\Lambda n$ threshold a spectacular peak due to the $2.13 GeV$ resonance lying in the immediate vicinity of the $\Sigma^0 n$ threshold is seen. Also shown are the results obtained with the Verma–Sural potential of the $\Lambda N$ interaction [13]. The difference is quite distinct.
FIG. 2. The double differential cross section as a function of photon energy for $P_K = 0.426 GeV$ and $\theta_{\gamma K} = 15^0$. Full – line: $P$– matrix result, dashed line: Verma–Sural potential.

The main conclusion is that FSI effects in the $\gamma d \rightarrow K^+ \Lambda n$ reaction are measurable and distinctly reflect the underlying $NY$ interaction dynamics. Therefore detailed calculations along the lines outlined in this paper are highly desirable. The authors would like to thank V.A.Karmanov for fruitful discussions and suggestions. Valuable remarks by T.Mizutani and A.E.Kudryavtsev are gratefully acknowledged. Special thanks are to C.Fayard, G.H.Lamot, F.Rouvier and B.Saghai for stimulating contacts and hospitality at all stages of the present work. Financial support from the University Claude Bernard, DAPNIA (Saclay) and RFFI grant 970216406 is gratefully acknowledged.
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