Shortcut loading a Bose–Einstein condensate into an optical lattice

Xiaoji Zhou, Shengjie Jin and Jörg Schmiedmayer

1 School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, People’s Republic of China
2 Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, Stadionallee 2, A-1020 Vienna, Austria
3 Author to whom any correspondence should be addressed.

E-mail: xjzhou@pku.edu.cn and schmiedmayer@atomchip.org

Keywords: shortcut, optical lattice, fidelity, excited bands

Abstract

We present an effective and fast (few microseconds) procedure for transferring a Bose–Einstein condensate from the ground state in a harmonic trap into the desired bands of an optical lattice. Our shortcut method is a designed pulse sequence where the time duration and the interval in each step are fully optimized in order to maximize robustness and fidelity of the final state with respect to the target state. The atoms can be prepared in a single band with even or odd parity, and superposition states of different bands can be prepared and manipulated. Furthermore, we extend this idea to the case of two-dimensional or three-dimensional optical lattices where the energies of excited states are degenerate. We experimentally demonstrate various examples and show very good agreement with the theoretical model. Efficient shortcut methods will find applications in the preparation of quantum systems, in quantum information processing, in precise measurement and as a starting point to investigate dynamics in excited bands.

1. Introduction

Efficient preparation and manipulation of ultracold atomic gases in optical lattices (OL) have applications in many fields, including quantum simulation of many-body systems, the realization of quantum computation, quantum optics, and high-precision atomic clocks [1–6]. There is a common concern how to quickly transfer the Bose–Einstein condensates (BECs) from the initial harmonic trap into a desired band of an OL with high fidelity and robustness. For example, to load atoms into the ground band in an OL, one chooses to ramp up the lattice depth adiabatically, the time scale usually lasts up to tens of milliseconds. To shorten the time of transfer, different techniques were proposed, sharing the concept of ’shortcuts to adiabaticity’ [7]. They promise to reach the same target state as the adiabatic process but within a very short time. One kind of shortcut is the continuous action method, including counter-diabatic driving, fast-forward protocols and inverse engineering. They are developed and exploited extensively in rapid manipulations of cold atoms, such as expansion/compression, rotation, transport and loading, etc [8–15]. The other is optimal control [16, 17] or composite pulses like in nuclear magnetic resonance [18]. These techniques have been used for atomic clocks, atomic interferometry and quantum computing [19–21]. As for loading atoms into an OL, theoretical proposals such as adding a supplementary driving potential [12] are very attractive.

Recently ultracold gases in higher bands of OL attracted much attention. Many interesting many-body phenomena, e.g., supersolid quantum phases in cubic lattices [22], quantum stripe ordering in triangular lattices [23], orbital degeneracy [24] can appear with ultracold atoms in excited-band states. However, the most widely used adiabatic approaches cannot directly transfer atoms into the excited bands. Several experimental techniques have been developed including: (i) coherent manipulation of vibrational bands by stimulated Raman transitions [25], (ii) using a moving lattice to load a BEC into an excited-band [26], (iii) swapping population to selectively exciting the atoms into the P-band [27] or F-band [28] of a bipartite square OL, these approaches required to transfer atoms into the S-band firstly, or (iv) adiabatically loading the BEC into the excited band using a phase imprint in a superlattice [29]. Fast and high fidelity shortcut directly loading into the desired band is lacking.
In this paper, we demonstrate an effective method for transferring atoms from an harmonic trap into the desired band of an OL. This shortcut stems from nonholonomic coherent control [30, 31], and is composed by standing-wave pulse sequences which are imposed on the system before the lattice is switched on. The time duration and interval in each step are optimized in order to reach the target state with a high fidelity and robustness. This process can be completed within several tens of microseconds, reducing the loading time by up to three orders of magnitude as compared to adiabatic loading. It can be applied to load different excited bands and open up the possibility to study their dynamic behavior. Furthermore, we demonstrate the manipulation of the superposition of Bloch states and loading into two-dimensional (2D) and three-dimensional (3D) OL. Our experimental results are in good agreement with the theoretical model.

The structure of this manuscript is organized as follows. In section 2, we introduce our idea of shortcut loading and optimization of pulse sequences to the S-band with zero quasi-momentum. The demonstration of loading atoms into odd parity excited bands such as the D-band in OL are given in section 3. In section 4, the shortcut loading atoms into S-band with non-zero quasi-momentum and into superpositions of band states are implemented. The case of 2D or 3D OL with degenerate energies of the excited states are shown in section 5. Finally, the main results are summarized in section 6.

2. The shortcut loading method

2.1. The idea of shortcut

We consider the general situation for transferring atoms into an OL. Before the preparation, atoms are confined in a weak harmonic trap \( V_{\text{harm}} = \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2) \) with the initial wave function \( |\psi_i\rangle = |p = 0\rangle \), as shown in figure 1(a), where \( m \) is the atom mass, \( \omega \) is the trap frequency and \( p \) the atomic momentum. The loading process then transfers atoms into a target state of an OL. The lattice is constructed by a set of laser beams with electric field amplitude \( E_i \), whose potential can be written as

\[
V(\vec{r}) = \sum_{i,j} \vec{E}_i \cdot \vec{E}_j \cos((\vec{k}_i - \vec{k}_j) \cdot \vec{r} + (\alpha_i - \alpha_j)),
\]

where \( k_i = \lambda_i/(2\pi) \) is the wave number, \( \lambda_i \) is the wavelength and \( \alpha_i \) is the initial phase of laser beam \( i \). For a cubic lattice, we can assume \( k_i = -k_i \) and \( i = x, y, z \) as shown in figure 1(b). When neglecting atom–atom interactions because loading time is very short and for the lattice laser sufficiently far detuned, the single-atom Hamiltonian in OL is given by \( (\hbar = 1) \),

\[
\hat{H} = \frac{\vec{p}^2}{2m} + V(\vec{r}).
\]

According to the Bloch’s theorem, the eigenstates of the Hamiltonian \( \hat{H} \) can be expressed as \( |n, \vec{q}\rangle = u_{n,\vec{q}}(\vec{r}) e^{i\vec{q}\cdot\vec{r}} \), with the index of the energy band \( n = 1, 2, 3... \) and the quasi-momentum \( \vec{q} \).

We first consider the 1D case for simplicity. The potential can be expressed as \( V(x) = \frac{V_0}{2}(1 + \cos 2kx) \), where \( V_0 \) is the lattice depth (here the harmonic trap is ignored during the preparation process because it is small compared with the OL potential). The Bloch states can be written as

\[
\psi_{n,\vec{q}}(x) = \sum_{\ell = -\infty}^{\infty} u_{n,\ell}(x) e^{i\vec{q}\cdot\vec{x}}.
\]
\[ |n, q\rangle = \sum_{\ell} c_{n,\ell} |2\ell k + q\rangle. \] (3)

This target state \(|\psi_0\rangle\) can be decomposed over a reduced basis of plane waves \(2\ell k + q\). In the quasi-momentum space, for a pure Bloch state at \(q = 0\), the parity is given by \(\Omega = \sum_{\ell} c_{n,\ell} - c_{n,-\ell}\) and \(\Omega = 0\) for a state with odd parity and \(\Omega = 1\) stands for a state with even parity. As shown in figure 1(c), the Bloch state with \(n = 1, 3, 5\ldots\) correspond to the S-, D-, G-bands with even parity, and \(n = 2, 4,\ldots\) to the P- and F-bands with odd parity. (\(V_r = 10E_r\), where \(E_r\) is one-photon recoil energy \(E_r = k^2/(2m)\), where the interaction-induced finite width of the momentum distribution is about \(\pm 0.03\ell\)). The corresponding wave functions for the different Bloch states (S, P, D, F, G) with \(q = 0\) are also shown in figure 1(d).

To achieve fast loading we will apply an \(m\)-step preloading sequence on the initial state \(|\psi_0\rangle\) before switching on the lattice with the optical depth \(V_0\). The state after the preloading sequence \(|\psi_f\rangle\) is given by:

\[ |\psi_f\rangle = \prod_{j=m}^{1} U_j |\psi_0\rangle, \]

where \(U_j = e^{-i\hat{H}_j/\hbar}\) is the evolution operator of the \(j\)th process. For the target state \(|\psi_0\rangle\), the parameters \(\hat{H}_j\) and \(t_j\) can be determined via maximizing the fidelity

\[ \zeta = |\langle \psi_0 | \psi_f \rangle|^2. \]

When \(\zeta = 1\) all the atoms would be prepared in the state \(|\psi_0\rangle\), \(1 - \zeta\) describes the difference between the achieved atomic state \(|\psi_f\rangle\) and the target state \(|\psi_0\rangle\). In the other word, the deviation rate \(N_\zeta\) is:

\[ N_\zeta = 1 - |\langle \psi_0 | \psi_f \rangle|^2. \]

Our goal is to properly choose \(\hat{H}_j\) and \(t_j\) so that \(N_\zeta\) is small enough to be neglected in the experiment. However, considering the Pauli exclusion principle of fermions and the wide momentum distribution of thermal atoms, this shortcut method cannot generalize to fermions or thermal atoms directly.

### 2.2. Calculating the time sequences

One obvious choice for each \(\hat{H}_j\) is to take the Hamiltonian corresponding to the interaction of atoms with a standing wave with the same periodicity as the OL. For this purpose, the power of the same laser as the one used for the final lattice loading is simply adjusted and each Hamiltonian \(\hat{H}_j\) is obtained after substitution of \(V_0\) by the new lattice depth \(V_j\). More precisely, as \(\hat{H}_j\) has spatial periodicity, we get its eigenstates by solving the equation

\[ \hat{H}_j |n, q, V_j\rangle = E_{n,q} |n, q, V_j\rangle, \] (32)

where \(E_{n,q}\) is the corresponding eigen-energy. We use the notation \(|n, q, V_j\rangle\) for denoting the Bloch states for a \(V_j\) lattice depth. Since only states with \(q = 0\) are initially populated, no other quasi momenta can be populated during the sequence of pulses. Then the state of the system can be written in the momentum eigenstates basis \(|2\ell k + q\rangle\), independent on the potential depth \(V_j\), and the evolution operator can be written as the following matrix:

\[ \hat{U}_j (V_j, t_j) = \hat{C}(V_j) \hat{E}(V_j, t_j) \hat{C}(V_j)^\dagger, \] (7)

where \(\hat{C}(V_j)\) is the unitary matrix of transition between the Bloch states basis and the momentum eigenstates basis with matrix elements

\[ \hat{C}(V_j)_{\ell \ell'} = \langle 2\ell' k + q | n, q, V_j \rangle, \]

and \(\hat{E}(V_j, t_j)\) is a diagonal matrix with elements

\[ \hat{E}(V_j, t_j)_{\ell m} = \exp(-iE_{n,q}(V_j)t_j). \] (9)

Because of the simple form of the potential, it is easy to obtain these matrices, from which the wave function’s evolution can be calculated optimally for a specific target state. We can obtain the values of \(V_j\) and \(t_j\) by optimizing for the specific target state using for example a gradient descent algorithm.

Let us start with four steps, for which depth \(V_j\) and duration time \(t_j (j = 1, 2, 3, 4)\) are independently adjusted as shown in figure 2(a2). Optimizing for maximum fidelity (minimal deviation rate \(N_\zeta\) and constraining \(t_j\) between 0 \(\mu\)s to 50 \(\mu\)s and \(V_j\) from \(0\) to \(30E_r\)) we find \(N_\zeta\) to be in the rage of \(10^{-3}\) or smaller for lattice depth of up to \(30E_r\) (figure 2(b)).

Next we turn to a simpler control: keeping the lattice strength \(V_j = V_{0j}\) for \(j = 1, 3\) (fixed to the final lattice potential \(V_{0j}\), and \(V_j = 0\) for \(j = 2, 4\), the times \(t_j\) being free parameters. This makes the sequence very easy to implement experimentally. A series of on and off pulses can be combined to a pulse sequence of length \(m\), where the \(j\)th component is composed of a duration \(t_{j1}\) where the OL is on and an interval \(t_{j2}\) where the OL is off (figure 2(a4)). To obtain an optimized shortcut scheme we have to find the proper time sequences so that the fidelity \(\zeta = |\langle \psi_f | \psi_0 \rangle|^2 \to 1\). From the green points in figure 2(b), we can see the deviation rate is still lower than 0.1% for all lattice depths. If we only use one optimized pulse, as shown in figure 2(a3), the fidelity is lower than 99% for most of the considered OL depths, but still much better than just switching on the OL (figure 2(a1)). On
the other side using more pulses, extending the sequence figure 2(a4) there is still a small improvement. In addition, the typical times of the loading process under a two-level model approximation are given in figure 2(c) for one pulse and two pulses. The improvement of fidelity comes at the expense of the loading time.

In the rest of our study we choose the simple scheme illustrated in figure 2(a4).

2.3. Experimental methods to probe the final state
All our measurements are done in absorption imaging after 31 ms of time of flight (TOF). The image thereby reflects the momentum of the atoms after the release from the optical lattice.

If we switch off the lattice abruptly (non-adiabatic switch off (NASI)), we project the wave function of the atoms in the lattice onto its momentum states. If there is coherence in the trapped wave function, one observes diffraction peaks after TOF.

If we switch off the lattice adiabatically, then we map the wave function in different bands to different momentum components. This so called band mapping (BM) [25, 33–35] allows to investigate in which bands the atoms reside. In addition the distribution of the atoms inside the mapped Brillouin zone allow to measure the distribution of quasi momenta. In our experiments the 'adiabatic switch off' is accomplished by exponentially ramping down the OL lattice potential in the form $e^{-t/\tau}$ where a characteristic decay time $\tau = 100 \mu s$ for a total length of 500 $\mu s$.

2.4. Experimental measurement for loading atoms into S-band
To demonstrate our shortcut approach, we prepare a nearly pure BEC of about $1.5 \times 10^5$ $^{87}$Rb atoms in a hybrid trap which is formed by overlapping a single-beam optical dipole trap with wavelength 1064 nm and a quadrupole magnetic trap. The resulting potential has harmonic trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (28, 55, 65)$ Hz, respectively, and a temperature of about 60 nK. The lattice is implemented by a standing wave created by two counter-propagating laser beams along the $x$-direction, with the lattice constant being $\lambda/2 = 426$ nm, the recoil energy $E_r$ being 3.16 kHz.

We start with sequence figure 3(a1), switching the OL with $V_0 = 10E_r$ on abruptly and hold the atoms in the lattice for some time $t$, and here we choose $t = 2$ ms. The absorption image after BM (figure 3(a2)) shows a significant fraction of atoms at momentum $\pm 2k$, which means that they are in excited bands (here the D-band). We integrated this image along the direction perpendicular to the $x$-axis and fit the experimental data points by three bi-modal functions, as shown in figures 3(a3) and (a4), respectively. The bi-modal function contains a Gaussian form that represents thermal atoms and an inverted parabolic function that denotes the condensate in S- and D-band. The blue and green area size equal to the atom numbers for S- and D-band, respectively. The measured fidelity from figures 3(a3) and (a4) is $\zeta = 72.6\%$.

We then realized an optimized 2 pulse shortcut sequence $(t_{11}, t_{12}, t_{21}, t_{22}) = (5.5, 21.0, 13.0, 6.1) \mu s$ with the fixed depth $V_0 = 10E_r$ (figure 3(b)). Employing BM we verify that the atoms are distributed in the first Brillouin zone that means atoms occupy $|S\rangle$ band. Similar to above, we can measure the fidelity of our final state prepared by the shortcut method, and obtain $\zeta = 99.2\%$. Here we should point that the total condensate atom number is almost conserved after the pulse sequence, which indicates the atoms in D-band are not killed but transferred into S-band by the designed pulse sequence.

![Figure 2](image-url)

Figure 2. (a) Four different time schemes for non-adiabatic loading into the ground (S) band. The OL turns on abruptly (a1); a four-step preloading sequence with both the potential depth and the duration of each step set as free parameters (a2); a one pulse (a3) and a two pulses (a4) preloading sequence with fixed potential depth and variable duration in each step. (b) Deviation rate $N_f$ for different lattice depths, where the blue diamonds, red circles, black stars and green points corresponding to (a1)–(a4), respectively. A logarithmic scale for the vertical axis is used. (c) Typical loading time for one pulse (a3)(solid line) and two pulse(a4) (dash line).
To further show that the coherence is not destroyed in the transfer process, we first hold the atoms in the OL for 2 ms and then use two additional inverted pulses to transfer the atoms back to the original state \( |\psi_0\rangle \), as shown in figure 3(c). We study the state of the atoms by a non-adiabatic switching off (NAS). The images obtained with the initial condensate, the atoms loaded in the OL, and after two additional inverted pulses are shown in figures 3(c2)–(c4). In figure 3(c3), we can see the interference peaks, similar to the familiar pattern observed in adiabatic loading experiments, which indicates a successful loading without significant excitation and heating. Comparing figures 3(c2) and (c4), we know there is little heating or disturbing effect on our BEC, which proves the effectiveness of our ‘preparing’ process of the ground state of OL [36].

2.5. Robustness analysis

In order to analyze the robustness of the pulse sequences, we use a two-level model approximation to draw the trajectory of the evolution on the Bloch sphere. Considering the parity of the bands, a two-level model \( |S\rangle \) and \( |D\rangle \) with corresponding eigenvalues \( E_S \) and \( E_D \) is sufficient when the OL depth is low. As shown in figure 4(a), we choose the aimed state as the \( S \)-band with zero quasi-momentum. The polar axis \( \hat{z} \) represents the Bloch state \( |S\rangle \) \( (|D\rangle) \) in the positive (negative) direction. Consequently, the initial plane wave represented by axis \( \hat{n} \) can be set as \( |\psi_0\rangle = \left( \cos \frac{\beta}{2}, \sin \frac{\beta}{2} \right)^T \), where \( \beta \) is the angle between axis \( \hat{z} \) and \( \hat{n} \). Obviously, when the pulse is imposed, the action can be seen as a counterclockwise rotation of \( \varphi \) around axis \( \hat{z} \) (the state \( |S\rangle \)) for a vector in the Bloch sphere. Likewise, during the time interval with the pulse being off, it is equivalent to an counterclockwise rotation of \( \theta \) around the axis \( \hat{n} \) (the plane wave with zero momentum) for a vector in the Bloch sphere.

We can obtain many different sequences for the same \( |\psi_0\rangle \) with \( \zeta \rightarrow 1 \). Figures 4(a1) and (a2) show the trajectories for one pulse and two pulses, respectively. The path (from point A to C) and the path (from point A to B, E to M) for \( \hat{U}_{11} \) caused the change in the phase between Bloch bands. On the other hand, the path (from point C to S) and the path (from point B to E, M to S) for \( \hat{U}_{12} \) mainly caused the change in the proportion of Bloch bands. From the analysis of the trajectories on the Bloch sphere, we find that if the track is symmetric, it is the most robust. This means, if one pulses for

\[ \varphi_1 = \theta_1; \]

or two pulses for

\[ \varphi_1 = \theta_2 \quad \text{and} \quad \theta_1 = \varphi_2; \]

the \( d^2\zeta/d\gamma^2 \) is the minimum, where \( \gamma \) are the parameters in experiment such as \( t_0 \) and \( V_0 \), which reminds us at a phase matching condition. This matching conditions have mirror symmetry about the center of the whole loading path. Therefore, we should choose proper rotation angles to make the fidelity maximum and get the
highest robustness. Considering the influence of higher bands, the sequence with phase matching in the two-level approximation will have to be corrected in order to satisfy the multi-level condition.

The variation of the fidelity $\zeta$ with respect to the pulse amplitude (mismatch with the lattice depth) is shown in figure 4(b). The experimental results are in good agreement with the theoretical predictions. The diamond points, which represents the shortcut time sequence considering the phase matching and multi-level correction, is the most robust. There is less than 0.2% variation for $VE_{0.5r0}$. On the contrary, there is 2% variation when we do not consider the phase matching as shown in the dotted points. Furthermore, although both one pulse and two pulses sequences satisfy the phase matching conditions, figure 4(c) indicates that the robustness and fidelity of one pulse sequence is a lower than two-pulse sequence with respect to variation of the time.

As for the sensitivity of the sequences against the lattice depth, from the experiment results shown in figure 4(d), we know that the higher the lattice potential, the smaller the robustness. Figure 4(e) gives the theoretical calculation for the robustness, where $R$ is the reciprocal of the total differential of $N$ in equation (6), and $R_0$ is the value of robustness for $VE_{5E_r}$. Theory curves are shown as the dashed-dotted (one pulse), dotted (two-pulse sequences process without phase matching) and solid (two-pulse sequences process with phase matching) curves, while the corresponding experimental data are shown as square, dotted and diamond points, respectively. (d) The variation of the fidelities with $\delta t_{12}$ for different depth. (e) Theoretical comparison between one pulse and two-pulse sequences for the robustness varying with depth.

3. Loading atoms into higher bands

The above shortcut method can be adapted to load atoms into excited bands in an OL. Since the initial state is of even parity and the parity of wave function remains unchanged during the pulse sequence, we can easily load atoms into higher bands of even parity such as $D$- and $G$-band. By adding a shift of the lattice phase we can change the parity to transfer atoms into odd parity bands such as the $P$- or $F$-bands.

3.1. Loading atoms into $D$-band

Similar to the loading method into the $S$-band, we can numerically maximize the fidelity $\zeta$ to obtain the time sequence to load atoms into the $D$-band. The time sequence is $t_{11}, t_{12}, t_{13}, t_{22} = (24.5, 28.8, 8.1, 2.2) \mu s$ for
\( V_0 = 10E_r \), and the experimental image by BM is shown in figure 5(a). However, we could not get the fidelity from this image because both \( D \)- and \( P \)-band atoms are distributed at \( \pm 2k \), as illustrated in figure 5(b).

We can measure the loading fidelity by the oscillations of the relative population in different momenta from the images by NAS. After the preparation process, atoms are in state

\[
|\psi_f\rangle = \frac{1}{\sqrt{N}} \sum_{n} f_n |n, 0\rangle.
\]

(12)

At holding time \( t \) in the OL, the atomic state becomes \( |\psi_f\rangle = \sum_n f_n e^{-i\omega_c t} |n, 0\rangle \), where \( E_{n,0} \) and \( |n, 0\rangle \) are the eigen-energy and eigenstate of \( H_{\text{FF}} \), \( H_{\text{FF}} |n, 0\rangle = E_{n,0} |n, 0\rangle \). Therefore, the number of atoms \( N_f(t) \) in the state \( |\psi_f\rangle = |0, 0\rangle \) at time \( t \) is given by \( N_f(t) = N |\langle 0, 0 |\psi_f\rangle|^2 \), and satisfies

\[
W_f(t) = \frac{N_f(t)}{N} = \left| \sum_n f_n c_{n,0} e^{-i\omega_c t} \right|^2,
\]

(13)

where \( N \) is the total atom number and \( c_{n,0} \) is defined in section 2.1. Equation (13) shows that the atom number \( N_f(t) \) oscillates with time \( t \).

Figure 5(c) shows the momentum distribution along \( \ell \) direction extracted from experimental images obtained by NAS. By a bi-modal fit we obtain \( N_f \) and \( W_f \) for different momentum states \( |\ell\rangle = \pm 2k \). The experimental values \( W_f(t) \) oscillate with time as shown in figure 5(d) and are well described by the theoretical model equation (13). From a fit to the experimental data, blue solid line for \( W_0(t) \) and red dash line for \( W_{\pm 1}(t) \) (we set \( W_1(t) = W_{-1}(t) \) in the theory calculations) we extract the fidelity of the preparation process: 98.2% for \( 10E_r \) and 97.3% for \( 20E_r \) using the loading time sequence (17.2, 25, 12.5, 1.1) \( \mu s \). After loading, the lifetime of atoms in \( D \)-band can be measured [37].

3.2. Loading atoms into G-band

For even higher bands such as the \( G \)-band, theoretical calculations suggest a fidelity \( \zeta = 99.2\% \) for a 5\( E_r \) deep lattice using an eight pulses sequence as: \( 32, 39, 40, 14, 14, 13, 13, 14, 13, 14, 11, 43, 11, 12 \) \( \mu s \). The experimental image by BM, and its momentum distribution is given in figure 6(b). We could not get the fidelity from the image by BM because both \( G \)- and \( F \)-band atoms are distributed at \( \pm 4k \).

Applying this shortcut method to load atoms into \( G \)-band at \( q = 0 \), a dynamical oscillation is clearly visible in the images with NAS, as shown in figure 6(c). This is best understood when looking at the corresponding
The extended band structure as drawn in figure 6(d), where the energy gaps between different bands are marked with $A_s (s = 1, 2, 3, 4, 5, 6)$: after loading into G-band, the atoms fall down into the F-band due to the small gap between G- and F-bands [38]. During the G-band preparation process, we ignored the effect of the harmonic trap because the time of the shortcut is very short. However, when we observe the atoms in OL for a long time, the weak harmonic trap will affect the dynamics. Once the BEC is in the F-band, it continues to lose momentum while gaining potential energy from the harmonic confinement. This corresponds to the BEC traversing dynamically along the F-band from $A_1, A_6$ to $A_2, A_5$ in figure 6(d). Once arriving at $A_2$ or $A_5$, the atoms face different dynamics depending on the lattice strength. If the lattice strength is small and the Bragg reflections at $A_2$ and $A_5$ are weak, the BEC will continue into the D-band by a Landau–Zener transition. After evoloving along the entire D-band, the BEC comes to the band gap between D- and P-bands at $A_3$ and $A_4$. Due to the large gap between D- and P-bands all the atoms at $A_3(A_4)$ will be Bragg reflected to $A_4(A_3)$, without tunneling into the P-band. Afterwards, the BEC will reverse its dynamics by moving up in momentum from $A_4, A_5$ to $A_3, A_2$. It eventually arrives at $A_6, A_1$, completing half of an oscillating cycle. As illustrated in figure 6(c), the oscillation period is about 24 ms.

The above atomic oscillation depends on the optical depth. When the OL is strong such as $V_0 = 15E_r$, the oscillation only exists within the F-band with a period of 17 ms. When the lattice strength is intermediate, for example $V_0 = 7.5E_r$, there is a superposition of two kinds of oscillations: across both the F- and D-bands, and within the F-band [39].

3.3. Loading atoms into P-band

The parity of quantum states in the P-band with $q = 0$ is odd, $\psi(-x) = -\psi(x)$ [26, 40, 41–43], one has to change the parity in order to load into the P-band. This can be done by a spatial shift of the OL. Our preparation process therefore consists of two series of pulses, as shown in figures 7(a) and (b). In the first series of pulses from $t_0$ to $t_1$, the atom experiences a spatial potential $V_{\text{even}}(x) = V_0 \cos^2 (kx)$. In the second series of pulses from $t_1$ to $t_2$, the atom experiences a potential $V_{\text{odd}}(x) = V_0 \cos^2 (kx + 3\pi /4)$. The coefficients $c_\ell$ ($c_\ell'$) defined in equation (3) and the distribution in the Bloch bands at the same times are shown in figures 7(c) and (d), respectively. At time $t_1$, all the components $c_\ell$ in the parity $\Omega$ would satisfy $c_{\ell} - c_{-\ell} = 0$, as shown in figure 7(c1) and the energy bands S, D and G… shown in figure 7(d1). However, from the view of the second series of pulses, by the lattice shift, the coefficient $c_{\ell}'$ should be multiplied by a phase according to $l$, i.e. $c_{\ell}'(E) = c_{\ell'} e^{i\ell (3\pi /4)}E$, and the relation between coefficients becomes $c_{\ell}' - (-c_{\ell}) = 0$. In our loading process, the first series of pulses ensure that coefficients $c_{\ell}'$ with even $\ell$ are zero. At the beginning of the second series of pulses, the parity of states can be completely changed as shown in figure 7(c2) and the corresponding energy bands P and F… are shown in figure 7(d2). From $t_1$ to $t_2$, the parity is unchanged. So only P-band state is populated at time $t_2$, as shown in figures 7(c3) and (d3).
In the experiment, we use two acousto-optic modulators to form our designed pulse sequence with the frequency difference $\delta \omega = 182.5 \text{ MHz}$ which corresponds to a phase shift between two pulses series by $\frac{3\pi}{4}$. Four special pulses are used to transfer atoms into $P$-band for $V_{\text{E}}(x) = V_0 \cos^2(kx)$, as shown in figure 8(a). The momentum distributions of absorption image along $\hat{x}$ direction after NAS are shown in figure 8(b). The momentum distribution nearly equals to zero at $k_0$ and has significant peaks at $k_\pm$. The small asymmetric at $k_\pm$ is caused by the population in other bands. To obtain the loading fidelity of $P$-band, we can measure the oscillations of $W(t)\ell = 0, -2$ as shown in figure 8(c), which is similar to $D$-band. By comparing the experimental data with the peripheral contour of the beating signal, we find the initial quantum state is $|\psi(t = 0)\rangle = \sqrt{0.9}|P, q = 0\rangle + \sqrt{0.05}|D, q = 0\rangle + \sqrt{0.05}|S, q = 0\rangle$ [39]. The corresponding fidelity is about 90% in $P$-band.

After loading atoms into $P$-band and holding for a longer time we can observe the quantum equilibration in dilute Bose gases [44]. In a similar way, we can also transfer atoms into the $F$-band with two sets of standing wave pulses $V_0 \cos^2(kx)$ and $V_0 \cos^2(kx + 3\pi/4)$ [38].

4. Preparation and manipulation of superposition of Bloch states

Above, we presented a shortcut loading method to transfer atoms into one band at zero quasi-momentum. We now extend our scheme to load atoms into $S$-band with non-zero quasi-momentum and superpositions of band states. Furthermore, it can be used to construct $\pi/2$ pulse or $\pi$ pulse between $S$- and $D$-bands, and implement a Ramsey interferometer (RI) with motional states [45].
4.1. Preparation of atoms in the S-band with non-zero quasi-momentum

If we want to load atoms into the S-band with non-zero quasi-momentum \( q_0 \), the BEC, initially with \( p = 0 \) (figure 9(a)), should be accelerated to obtain a momentum \( p_0 \). We can use a magnetic field gradient provided by coils, to accelerate a BEC to momentum \( p = -0.8k \) within 2 ms. Immediately afterwards the designed pulse sequence is used to transfer atoms into the S-band of the OL at quasi-momentum \( q_0 \). The corresponding experimental absorption images by NAS after step 1 and step 2 are shown in figure 9(b). After step 3, the final state \( \psi_{q_0} = |\psi_{-0.8k}\rangle \) is shown in figure 9(c). The momentum distribution as measured by NAS (left image) has significant peaks at 0.8k and -1.2k. The right image as obtained by BM shows a significant peak at 0.8k, which verifies the effectiveness of the loading.

4.2. Preparation of atoms in superposition of Bloch states

We can also choose the target state as a superposition states, such as \( \psi_{q_0} = |S\rangle + |D\rangle \), as illustrated in figure 10 [37]. Using a shortcut time sequence as \( (30, 6.4, 8.7, 4.5) \mu s \) we extract a measured fidelity of \( \zeta = 0.995 \) from the fits to the measured \( W(t) (\ell = 0, \pm 1) \) as a function of the holding time \( t \).

4.3. Manipulation of Bloch states

Our shortcut method can also be employed to manipulate Bloch states [46–49], for instance to design \( \pi/2 \) or \( \pi \) pulses between S- and D-bands constituting a pseudo-spin system. Unlike conventional RI where selection rules can be used to prepare population in two states, the lattice band transition, similar to transition between vibration states in molecules [50], have no selection rules. For an arbitrary initial superposition between S and D-band states, \( |\psi_i\rangle = \sin \frac{\theta}{2} |S\rangle + e^{i\phi} \cos \frac{\theta}{2} |D\rangle \), a \( \pi/2 \) pulse \( \hat{U}_{\pi/2} \) should transfer the initial states to the final states by
5. Loading atoms into 2D and 3D optical lattice

The above discussion has been focused on 1D OL. We now show how these shortcut pulses can be extended to 2D or 3D OL, such as the square, triangular, hexagonal OLs [51–54]. Among these OLs, the square lattice is the simplest, where the total potential energy is the sum of potential energies in \( \hat{k}_x \) and \( \hat{k}_y \) directions, while this is not true for the triangular OL.

5.1. 2D square optical lattice

The potential energy for 2D square OL can be divided into potential energies in the \( \hat{x} \) and \( \hat{y} \) directions if \( ||\hat{k}_x|| = 0 \) or the electric field \( \hat{E}_x \perp \hat{E}_y \) in equation (1), in this case, the Hamiltonian of OL is given by

\[
\hat{H} = -\frac{1}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \left[ V(x) \cos(2K_x x) + V(y) \cos(2K_y y) \right],
\]

and the wave functions can be separated in the form of \( \psi(\vec{r}) = \psi_x(x) \psi_y(y) \).

In figure 11 we draw the schematic of Bloch bands of the square lattice for \( V_x = 11E_r \) and \( V_y = 9E_r \). There is a difference between the lattice depths in the \( \hat{x} \) and \( \hat{y} \) directions in order to avoid energy degeneracy. The S-band is the ground band, and there are two P-bands, \( P_x \) and \( P_y \), and three D-bands \( D_{x}, D_y \) and \( D_{xy} \). Figure 11(d) shows the energy bands along the \( \hat{k}_x \) direction at \( q_y = 0 \). For Bloch states \( \psi_{2d} \) in each band, we can arrange their eigenstates according to their eigenenergies, which are shown in the second column of figure 12(a). The first state is S-band and the second and third states are \( |P_x^{\uparrow}\rangle \) and \( |P_x^{\downarrow}\rangle \) respectively. The 4th, 5th and 6th states are \( |D_{x}^{\uparrow}\rangle \), \( |D_{y}^{\downarrow}\rangle \) and \( |D_{xy}^{\downarrow}\rangle \) respectively. At the same time, the first column in figure 12(a) displays the product forms of the corresponding two one-dimensionalBloch states.

For loading atoms into 2D square lattice, the evolution operator can be separated in the \( \hat{x} \) and \( \hat{y} \) directions \( \hat{U} \psi = \hat{U}_x \psi_x \hat{U}_y \psi_y \). If the target state in the square lattice \( |\psi_{\theta}\rangle = \sum_{n_x} \alpha_{n_x} |n_x = i, n_y = j\rangle \) can be written in the form of the product of two 1D states \( |\psi_{\theta}\rangle = \sum_{n_x} \alpha_{n_x} |n_x = i\rangle \otimes \sum_{n_y} \beta_{n_y} |n_y = j\rangle \). Then coefficients of these two forms should satisfy

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m
\end{bmatrix}
\begin{bmatrix}
\beta_1 & \beta_2 & \cdots & \beta_m
\end{bmatrix}
= 
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} \\
\gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mm}
\end{bmatrix}.
\]

5. Loading atoms into superposition states

(a) Schematic illustration of the superposition states \( |S\rangle + |D\rangle / \sqrt{2} \) in OL with \( q = 0 \) for \( V = 10E_r \). (b) The relative population as a function of holding time \( t \) (similar to figure 8(b)).
We could use two separate 1D pulse sequences in the $x$ and $y$ directions to obtain target state with close to 100% fidelity. If equation (17) does not hold, we can use numerical optimization to obtain fidelities as high as possible.

In figure 12(b), we demonstrate this loading sequences for $6d_2y=\bar{n}$, where the pulse sequences in two directions are independent but end at the same time, and the red part of the pulse sequence represents the laser with a phase shift that breaks the parity conservation. Figure 12(c) displays the calculated time sequences for three different target states and their corresponding calculated fidelities for $\psi_{\text{sd}}=|1\rangle$, $|6\rangle$ and $\frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|4\rangle$.

5.2. 2D triangular optical lattice

For other 2D configurations, such as triangular OL constructed by three traveling-wave lasers with $|\bar{k}_i| = |\bar{k}_j| = |\bar{k}_\ell|$ and $\arg(\bar{k}_i, \bar{k}_j) = \pi/3 (i \neq j)$ (figure 13(a)) we cannot separate the variables in the $\hat{x}$ and $\hat{y}$ directions, and the Bloch states are written as

$$|n, \bar{q}\rangle = \sum_{\bar{c}_i, \bar{c}_\ell} \epsilon_{\bar{c}_i, \bar{c}_\ell} |\bar{c}_i \rangle \bar{b}_1 + |\bar{c}_\ell \rangle \bar{b}_2 + |\bar{q}\rangle,$$

where $\bar{b}_1 = \sqrt{3}k \hat{x}$ and $\bar{b}_2 = \sqrt{3}k\left(-\frac{1}{2} \hat{x} - \frac{\sqrt{3}}{2} \hat{y}\right)$. For the target state $|\psi_n\rangle = \sum_{\bar{c}} \gamma_{\bar{c}} |n, \bar{q}\rangle = |1, 0\rangle$, we can impose the same time sequence on the three traveling beams as shown in figure 13(b). The theoretical momentum distribution is shown in figure 13(c) for $V_0 = 10E_r$. Using the time sequence $(t_{13}, t_{12}, t_{21}, t_{23}) = (6, 22, 7, 10)$ s we can reach the theoretical fidelity $\zeta = 0.991$ for $V_0 = 10E_r$. The corresponding experimental image with NAS shown in figure 13(d) is in agreement with the theoretical result.

The corresponding theoretical population distributions in momentum space for higher bands in a 2D triangular lattice are shown in figure 14(a), where $|n = i\rangle (i = 2, 3 \ldots)$ represents the $i$th eigenstate with zero
quasi-momentum, and \(|n = 3 + 4\) represents the superposition of degenerate states, \(|n = 3\) and \(|n = 4\). For instance, if we choose the target state \(|\psi_5\rangle = |7, 0\rangle\), we can get \(\zeta = 0.92\) using time sequence \((t_{15}, t_{12}, t_{23}, t_{22}) = (22.1, 37.9, 79.9, 35.6)\) \(\mu s\) for the lattice depth \(V_0 = 10E_r\). The experimental results are shown in figure 14(b). To load atoms into other excited states, more complicated pulse sequences with different phases are required.

5.3. 3D optical lattice
For the simplest 3D cubic OL, the wave functions can be separated by variables in the \(\hat{x}, \hat{y}\) and \(\hat{z}\) directions, and we can also load BEC to arbitrary target states. A more complicated 3D lattice composed of a 2D triangular lattice in the \(\hat{x}–\hat{y}\) plane with \(\lambda = 1064\) nm and a 1D lattice in the \(\hat{z}\) direction with \(\lambda = 852\) nm is shown in figure 15(a). For this OL we can combine the 1D sequence and 2D sequence. Figure 15(b) shows the different time sequences on the \(\hat{x}–\hat{y}\) plane and the \(\hat{z}\) direction. The atoms can be transferred from the harmonic trap into the S-band of the OL in the \(\hat{x}–\hat{y}\) plane and \(\hat{z}\) direction, as shown in figure 15(c), or S-band in the \(\hat{x}–\hat{y}\) plane and D-band in the \(\hat{z}\) direction as shown in figure 15(d). The time sequences in figure 15(c) are \((6, 22, 7, 10)\) \(\mu s\) in the \(\hat{x}–\hat{y}\) plane and...
(24.5, 28.8, 8.1, 2.2) μs in the الز endpoint. The sequences in figure 15(d) are (6, 22, 7, 10) μs in the ز−ط plane and (5.5, 21.0, 13.0, 6.1) μs in the ز endpoint. The experimental results are in agreement with the theoretical calculations.

6. Conclusions

In summary, we present a method for effective preparation of a BEC in different bands of an optical lattice within a few tens of microseconds. This shortcut stems from nonholonomic coherent control, composed by pulse sequences which are imposed on the system before the OL switches on and fully optimized for high fidelity and robustness. With our approach, the BEC can be prepared in either pure Bloch states or superposition of states of different sequences which are imposed on the system before the OL switches on and fully optimized for high fidelity and robustness. Furthermore, this shortcut can also be successfully applied for 2D and 3D OLs. The experimental results are well described by the theoretical calculations. Because the duration of pulses is short enough, the atom–atom interaction can be neglected during the design of pulse sequences, and the numerical results show that the interaction leads to a change of fidelity less than 1% in our designed time sequence. This efficient shortcut not only provides applications in controllable quantum systems and quantum information processing, but also is helpful for the study of orbital OL, simulation of systems in condensed matter physics, and the precise measurements.

Acknowledgments

We thank J. Yin, P Zhang, B Wu, G J Dong, H W Xiong, B G Yang, Z H Zhao and X Chen for useful discussions. This work is supported by the National Key Research and Development Program of China (Grant No. 2016YFA0301501), and the NSFC (Grants No.11334001, No. 61475007 and No. 61727819). JS acknowledges support by the European Research Council, ERC-AdG, Quantum Relax.

References

[1] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. 80 885
[2] Derevianko A and Katori H 2011 Rev. Mod. Phys. 83 331
[3] Cronin A D, Schmiedmayer J and Pritchard D E 2009 Rev. Mod. Phys. 81 1051
[4] Cohen-Tannoudji C and Guéry-Odelin D 2011 Advances in Atomic Physics: An Overview (Singapore: World Scientific)
[5] H A C, Liu W M, Song J L and Zhou F 2008 Phys. Rev. Lett. 101 010402
[6] Liu W M, Fan W B, Zheng W M, Liang Q and Chui S T 2002 Phys. Rev. Lett. 88 170408
[7] Torrontegui E, Ibáñez S, Martínez-Garaot S, Modugno M, del Campo A, Guéry-Odelin D, Ruschhaupt A, Chen X and Muga J G 2013 Adv. At. Mol. Opt. Phys. 62 117
[8] Chen X, Ruschhaupt A, Schmidt S, del Campo A, Guéry-Odelin D and Muga J G 2010 Phys. Rev. Lett. 104 063002
[9] Schaff F, Song X L, Capuzzi P, Vignolo P and Labeyrie G 2011 Europhys. Lett. 93 23001
[10] Bason M G, Viteau M, Malossi N, Huillery P, Arimondo E, Ciampini D, Faoro R, Giovannetti V, Mannella R and Morsch O 2012 Nat. Phys. 8 147
[11] Martínez-Garaot S, Torrontegui E, Chen X, Modugno M, Guéry-Odelin D, Tseng Y S and Muga J G 2013 Phys. Rev. Lett. 111 213001
[12] Masuda S, Nakamura K and del Campo A 2012 Phys. Rev. Lett. 113 060303
[13] Rohringer W, Fischer D, Steiner F, Mazets I E, Schmiedmayer J and Trupke M 2015 Sci. Rep. 5 9820
[14] Masuda S and Rice S A 2015 J. Phys. Chem. B 119 11088
[15] Papoular D and Stringari S 2015 Phys. Rev. Lett. 115 025302
[16] Bulatov A, Vugmeister B, Burin A and Rabitz H 1999 Phys. Rev. A 60 4875
[17] Bucker B, Berrada T, van Frank S, Schaff J F, Schumm T, Schmiedmayer J, Jäger G, Grond J and Hohenester U 2013 J. Phys. B: At. Mol. Opt. Phys. 46 104012
[18] Levitt M H 1986 Prog. Nucl. Magn. Reson. Spectrosc. 18 61
[19] Schleider-Smith M H, Leroux I D and Vuletić V 2010 Phys. Rev. Lett. 104 073004
[20] Butts D L, Kotru K, Kinaat J M, Radiojevic A M, Timmons B P and Stoner R E 2013 J. Opt. Soc. Am. B 30 922
[21] Lee J H, Montano E, Deutsch I H and Jessen P S 2013 Nat. Commun. 4 2027
[22] Scarola V W and Das Sarma S 2005 Phys. Rev. Lett. 95 036003
[23] Wu C, Liu W V, Moore J and Das Sarma S 2008 Phys. Rev. Lett. 97 190406
[24] Lewenstein M and Liu W V 2011 Nat. Phys. 7 101
[25] Müller T, Polling S, Widera A and Bloch I 2007 Phys. Rev. Lett. 99 200405
[26] Browaeys A, Höffner H, McKenzie C, Rolston S L, Helmersen K and Phillips W D 2005 Phys. Rev. A 72 053605
[27] Wirth G, Ölschlager M and Hemmerich A 2011 Nat. Phys. 7 147
[28] Ölschlager M, Wirth G and Hemmerich A 2011 Phys. Rev. Lett. 106 015302
[29] Taie S, Ozawa H, Ichinoz E, Nishio T, Nakajima S and Takahashi Y 2015 Sci. Adv. 1 e1500854
[30] Lloyd S 1995 Phys. Rev. Lett. 75 346
[31] Harel G and Akulin V M 1999 Phys. Rev. Lett. 82 1
[32] Denschlag J H, Simsarian J E, Höffner H, McKenzie C, Browaeys A, Cho D, Helmersen K, Rolston S L and Phillips W D 2002 J. Phys. B: At. Mol. Opt. Phys. 35 3095
[33] Kohl M, Moritz H, Stöferle T, Günter K and Esslinger T 2005 Phys. Rev. Lett. 94 080403
[34] Kastberg A, Phillips W D, Rolston S L, Spreeuw R J C and Jessen P S 1995 Phys. Rev. Lett. 74 1542
[35] Greiner M, Bloch I, Mandel O, Hänsch T W and Esslinger T 2001 Phys. Rev. Lett. 87 160405
[36] Liu X X, Zhou X J, Xiong W, Vogt T and Chen X Z 2011 Phys. Rev. A 83 063402
[37] Zhai Y Y, Yue X G, Wu Y J, Chen X Z, Zhang P and Zhou X J 2013 Phys. Rev. A 87 063638
[38] Wang Z K, Yang B G, Hu D, Chen X Z, Xiong H W, Wu B and Zhou X J 2016 Phys. Rev. A 94 033624
[39] Hu D, Niu L X, Yang B G, Chen X Z, Wu B, Xiong H W and Zhou X J 2015 Phys. Rev. A 92 043614
[40] Müller T, Folling S, Widera A and Bloch I 2007 Phys. Rev. Lett. 99 200405
[41] Panahi P S, Lühmann D S, Struck J, Windpassinger P and Sengstock K 2012 Nat. Phys. 8 71
[42] Parker C V, Ha L and Chin C 2013 Nat. Phys. 9 769
[43] Wirth G, Ölschläger M and Hemmerich A 2011 Nat. Phys. 7 147
[44] Niu L X, Tang P J, Yang B G, Chen X Z, Wu B and Zhou X J 2016 Phys. Rev. A 94 063603
[45] Hu D, Niu L X, Jin S J, Chen X Z, Dong G J, Schmiedmayer J and Zhou X J 2017 arXiv:1712.07520
[46] Deng L, Hagley E W, Denschlag J, Simsarian J E, Edwards M, Clark C W, Helmerson K, Rolston S L and Phillips W D 1999 Phys. Rev. Lett. 83 5407
[47] Xiong W, Yue X G, Wang Z K, Zhou X J and Chen X Z 2011 Phys. Rev. A 84 043616
[48] Yue X G, Zhai Y Y, Wang Z K, Xiong H W, Chen X Z and Zhou X J 2013 Phys. Rev. A 88 013603
[49] Yang B G, Jin S J, Dong X Y, Liu Z, Yin L and Zhou X J 2016 Phys. Rev. A 94 043607
[50] Kneipp K, Wang Y, Kneipp H, Itzkan I, Dasari R R and Feld M S 1996 Phys. Rev. Lett. 76 2444
[51] Petsas K I, Coates A B and Grynberg G 1994 Phys. Rev. A 50 5173
[52] Becker C, Soltan-Panahi P, Kronjäger J, Dörscher S, Bongs K and Sengstock K 2010 New J. Phys. 12 065025
[53] Struck J, Ölschläger C, Le Targat R, Soltan-Panahi P, Eckardt A, Lewenstein M and Sengstock K 2011 Science 333 996
[54] Jo G B, Guzman J, Thomas C K, Hosur P, Vishwanath A and Stamper-Kurn D M 2012 Phys. Rev. Lett. 108 045305