Weighted Bott-Chern and Dolbeault cohomology for LCK-manifolds with potential

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Abstract
A locally conformally Kähler (LCK) manifold is a complex manifold, with a Kähler structure on its covering \( \tilde{M} \), with the deck transform group acting on \( \tilde{M} \) by holomorphic homotheties. One could think of an LCK manifold as of a complex manifold with a Kähler form taking values in a local system \( L \), called the conformal weight bundle. The \( L \)-valued cohomology of \( M \) is called Morse-Novikov cohomology. It was conjectured that (just as it happens for Kähler manifolds) the Morse-Novikov complex satisfies the \( dd^c \)-lemma. If true, it would have far-reaching consequences for the geometry of LCK manifolds. Counterexamples to the Morse-Novikov \( dd^c \)-lemma on Vaisman manifolds were found by R. Goto. We prove that \( dd^c \)-lemma is true with coefficients in a sufficiently general power \( L^a \) of \( L \) on any LCK manifold with potential (this includes Vaisman manifolds). We also prove vanishing of Dolbeault and Bott-Chern cohomology with coefficients in \( L^a \). The same arguments are used to prove degeneration of the Dolbeault-Frölicher spectral sequence with coefficients in any power of \( L \).

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1 Introduction

1.1 LCK manifolds and $dd^c$-lemma

A locally conformally Kähler (LCK) manifold is a complex manifold which admits a Kähler metric on its covering $\tilde{M}$ such that the monodromy acts on $\tilde{M}$ by Kähler homotheties. For more details and the reference on this subject, please see Section 2.

The LCK property is equivalent to existence of a Hermitian form $\omega$ on $M$ satisfying $d\omega = \omega \wedge \theta$, where $\theta$ is a closed 1-form. This form is called the Lee form of an LCK-manifold.

One can consider the Kähler form on $\tilde{M}$ as a Kähler form on $M$ taking values in a 1-dimensional local system, or, equivalently, in a flat line bundle $L$. This bundle is called the weight bundle of $M$.

Cohomology of this local system is known as the Morse-Novikov cohomology of an LCK manifold. In locally conformally Kähler geometry, the Morse-Novikov cohomology share many properties of the Hodge decomposition with the usual cohomology of the complex manifolds. The locally conformally Kähler form represents a cohomology class (called the Morse-Novikov class) of an LCK manifold, encoding the topological properties of an LCK structure. However, the $dd^c$-lemma, which plays the crucial role for the Kähler geometry, is invalid in the Morse-Novikov setting. The main question of the locally conformally Kähler geometry is to find a replacement of the $dd^c$-lemma which would allow one to study the interaction between the complex geometry and the topology of a manifold.
The statement of the \( dd^c \)-lemma seems, on the first sight, to be technical. It says that on any compact Kähler manifold \((M, I)\), one has \( \text{im} \, d \cap \ker d^c = \text{im} \, dd^c \), where \( d^c = IdI^{-1} \) is the twisted de Rham differential. However, it is used as a crucial step in the proof of the degeneration of the Dolbeault-Frölicher spectral sequence, and in the proof of homotopy formality of Kähler manifolds.

For an LCK manifold, one replaces the de Rham differential by its Morse-Novikov counterpart \( d_\theta := d + \theta \), where \( \theta \) is the connection form of its weight bundle; the twisted de Rham differential is replaced by \( d^c_\theta = Id_\theta I^{-1} \). It was conjectured in [OV2] that the \( d_\theta d^c_\theta \)-lemma would hold on any LCK manifold, giving \( \text{im} \, d_\theta \cap \ker d^c_\theta = \text{im} \, d_\theta d^c_\theta \). The implication of the \( d_\theta d^c_\theta \)-lemma would include the topological classification of LCK structures on some manifolds (such as nilmanifolds) and a construction of automorphic Kähler potentials on LCK manifolds with vanishing Morse-Novikov class. However, this conjecture was false, as shown by R. Goto ([G]).

### 1.2 Weighted Bott-Chern cohomology

When the \( d_\theta d^c_\theta \)-lemma is false, one needs to study a more delicate cohomological invariant, called the weighted Bott-Chern cohomology of a manifold:

\[
H^{p,q}_{BC}(M, L) := \frac{\ker d_\theta \cap \ker d^c_\theta}{\text{im} \, d_\theta d^c_\theta}|_{\Lambda(p,q)(M)}.
\]

In [G], Goto has shown that the Bott-Chern cohomology group is responsible for the deformational properties of an LCK manifold, and computed it for certain \((p, q)\) and certain examples of LCK manifolds, called the Vaisman manifolds (see Subsection 2.2).

**Definition 1.1.** The local system \( L \) is a real, oriented line bundle with a flat connection. Trivializing this bundle, we can write its connection as \( \nabla_L = d - \theta \), where \( \theta \) is the Lee form of our LCK manifold. For arbitrary \( a \in \mathbb{C} \), the connection \( \nabla_{L^a} := d - a\theta \) is also flat. For \( a \in \mathbb{Z} \), the corresponding line bundle is identified with the \( a \)-th tensor power of \( L \), denoted as \( L^a \). One may think of the flat line bundle \( (L, \nabla_{L^a}) \) as of a real (or complex) power of \( L \). We denote this line bundle and its local system by \( L^a \), and call it \( a \)-th power of a weight bundle.

In this paper we compute the weighted Bott-Chern cohomology for \( L^a \), on LCK manifolds with potential, and show that it vanishes for all \( a \) outside of a countable subset of \( \mathbb{R} \) (Corollary 4.2). This implies \( dd^c \)-lemma for forms...
with coefficients in $L_a$, for these values of $a$. This result is based on a computation of Dolbeault cohomology with coefficients in $L_a$, which also vanish \textbf{(Theorem 3.2)}.

1.3 LCK manifolds with potential

\textbf{Definition 1.2.} A compact LCK manifold $(M, \omega, \theta)$ is called \textbf{LCK with potential} if $\omega = d_\theta d_\psi^c \psi$ for a proper, positive function $\psi$ which is called \textbf{LCK potential}. An equivalent definition will be given in Subsection 2.3.

LCK manifolds with potential are understood very well now. The following results were proven in \cite{OV3} and \cite{OV4}. Recall that a \textbf{linear Hopf manifold} is a quotient of $\mathbb{C}^n \setminus \{0\}$ by a $\mathbb{Z}$-action generated by a linear map with all eigenvalues $|\alpha_i| > 1$.

\textbf{Theorem 1.3.} Let $M$ be a compact complex manifold. Then $M$ admits an LCK metric with potential if and only if $M$ admits an embedding to a linear Hopf manifold. ■

\textbf{Theorem 1.4.} Let $M$ be an LCK manifold with potential. Then $M$ is a deformation of a Vaisman manifold \textbf{(Definition 2.2)}. In particular, $M$ is diffeomorphic to a principal $S^1 \times S^1$-bundle over a projective orbifold. ■

It would be nice to have a topological characterization of LCK manifolds with potential. Since \cite{OV2}, we were extending much effort trying to prove the following conjecture, which has many geometric consequences.

\textbf{Conjecture 1.5.} Let $(M, \omega, \theta)$ be a compact LCK manifold. Assume that $\omega$ is $d_\theta$-exact. Then $\omega$ is $d_\theta d_\psi^c$-exact, that is, $M$ is an LCK with potential.

This conjecture is still open. It would trivially follow if the $d_\theta d_\psi^c$-lemma were true, but it is known now to be false. However, a weaker conjecture still stands.

\textbf{Conjecture 1.6.} Let $(M, \omega, \theta)$ be a compact LCK manifold, $L$ its weight bundle, and $L_a$ the weight bundle to the power of $a \in \mathbb{R}$ \textbf{(Definition 1.1)}. Then, for all $a$ outside of a countable set, $d_\theta d_\psi^c$-lemma is true: for any $d_\theta$-exact $(1,1)$-form $\eta$, one has $\eta = d_\theta d_\psi^c \eta$. 
In this paper, we prove that Conjecture 1.6 is true for LCK manifolds with potential (Corollary 4.2). This is done by proving a vanishing result for weighted Dolbeault cohomology (Theorem 3.2).

2 Locally conformally Kähler geometry

In this section we give the necessary definitions and properties of locally conformally Kähler (LCK) manifolds.

2.1 LCK manifolds

Definition 2.1. A complex manifold \((M, I)\) is LCK if it admits a Kähler covering \((\tilde{M}, \tilde{\omega})\), such that the covering group acts by holomorphic homotheties.

Equivalently, there exists on \(M\) a closed 1-form \(\theta\), called the Lee form, such that \(\omega\) satisfies the integrability condition:

\[
d\omega = \theta \wedge \omega.
\]

Clearly, the metric \(g := \omega(\cdot, I\cdot)\) on \(M\) is locally conformal to some Kähler metrics and its lift to the Kähler cover in the definition is globally conformal to \(h\).

To an LCK manifold one associates the weight bundle \(L_\mathbb{R} \to M\). It is a real line bundle associated to the representation

\[
\text{GL}(2n, \mathbb{R}) \ni A \mapsto |\det A|^{\frac{1}{n}}.
\]

The Lee form induces a connection in \(L_\mathbb{R}\) by the formula \(\nabla = d - \theta\). \(\nabla\) is associated to the Weyl covariant derivative (also denoted \(\nabla\)) determined on \(M\) by the LCK metric and the Lee form. As \(d\theta = 0\), then \(\nabla^2 = d\theta = 0\), and hence \(L_\mathbb{R}\) is flat.

The complexification of the weight bundle will be denoted by \(L\). The Weyl connection extends naturally to \(L\) and its \((0,1)\)-part endows \(L\) with a holomorphic structure.

\[\text{In conformal geometry, the weight bundle usually corresponds to } |\det A|^{\frac{1}{n}}. \text{ For LCK-geometry, } |\det A|^{\frac{1}{n}} \text{ is much more convenient.}\]
2.2 Vaisman manifolds

**Definition 2.2.** A Vaisman manifold is an LCK manifold with $\nabla^g$-parallel Lee form, where $\nabla^g$ is the Levi-Civita connection.

The following definition is implicit in the work of Boyer & Galicki, see [BG]:

**Definition 2.3.** A Sasakian manifold is an odd-dimensional contact manifold $S$ such that its symplectic cone $CS$ is equipped with a Kähler structure, compatible with its symplectic structure, and the standard symplectic homothety map $\rho_t : CS \rightarrow CS$ is holomorphic.

Compact Vaisman manifolds can be described in terms of Sasakian geometry as follows.

**Theorem 2.4.** [OV1] Let $(M, I, g)$ be a compact Vaisman manifold. Then:

1) The monodromy of $L_{\mathbb{R}}$ is $\mathbb{Z}$.
2) $(M, g)$ is a suspension over $S^1$ with fibre a compact Sasakian manifold $(W, g_W)$. Moreover, $M$ admits a conic Kähler covering $(W \times \mathbb{R}_+, t^2 g_W + dt^2)$ such that the covering group is an infinite cyclic group, generated by the transformation $(w, t) \mapsto (\varphi(w), qt)$ for some Sasakian automorphism $\varphi$ and $q \in \mathbb{Z}$.

The typical example of a compact Vaisman manifold is the diagonal Hopf manifold $H_A := \mathbb{C}^n/\langle A \rangle$ with $A = \text{diag}(\alpha_i)$, with $|\alpha_i| < 1$. An explicit construction of the Vaisman metric on $H_A$ is given in [OV6]. Other Vaisman metrics appear on compact complex surfaces, [Be].

Among the LCK manifolds which do not admit Vaisman metrics are some of the Inoue surfaces (cf. [It], [Be]) and their generalizations to higher dimensions ([OT]). The rank 0 Hopf surfaces are also non-Vaisman ([GO]).

2.3 LCK manifolds with potential

**Definition 2.5.** ([OV3]) A compact complex manifold $(M, I)$ is LCK with potential if it admits a Kähler cover $(\tilde{M}, \Omega)$ with global potential $\varphi : \tilde{M} \rightarrow \mathbb{R}_+$, such that $\varphi$ is proper (preimages of compacts are compact) and the monodromy map $\tau$ acts on $\varphi$ by multiplication with a constant: $\tau(\varphi) = \text{const} \cdot \varphi$. 
Remark 2.6. In [OV3, Proposition 2.5] it was proven that $f$ is proper if and only if the monodromy of the weight bundle is discrete in $\mathbb{R}_+$, that is, isomorphic to $\mathbb{Z}$.

Vaisman manifolds are LCK with potential (the potential is equal to the squared norm of the Lee field), which can be easily seen from the Sasakian description given above ([Ve1]). LCK metrics with potential are in one to one correspondence with strongly pseudoconvex shells in affine cones, as shown in [OV6].

We summarize the main properties of compact LCK manifolds with potential:

Theorem 2.7.

(i) ([OV3]) The class of compact LCK manifolds with potential is stable to small deformations.

(ii) ([OV4, Theorem 2.1]) Any LCK manifold with potential can be deformed to a Vaisman manifold. Moreover, the set of points which correspond to Vaisman manifolds is dense in the moduli of compact LCK manifolds with potential

(iii) ([OV3]) Any compact LCK manifold with potential can be holomorphically embedded into a Hopf manifold. Moreover, a compact Vaisman manifold can be holomorphically embedded in a diagonal Hopf manifold.

2.4 Morse–Novikov complex and cohomology of local systems

Let $M$ be a smooth manifold, and $\theta$ a closed 1-form on $M$. Denote by $d_\theta : \Lambda^i(M) \to \Lambda^{i+1}(M)$ the map $d - \theta$. Since $d\theta = 0$, $d_\theta^2 = 0$.

Consider the Morse–Novikov complex, (see e.g. [P], [R], [M])

\[
\Lambda^0(M) \xrightarrow{d_\theta} \Lambda^1(M) \xrightarrow{d_\theta} \Lambda^2(M) \xrightarrow{d_\theta} \cdots
\]

Its cohomology is the Morse–Novikov cohomology of $(M, \theta)$.

In Jacobi and locally conformal symplectic geometry, this object is called Lichnerowicz-Jacobi, or Lichnerowicz cohomology, motivated by Lichnerowicz’s work [L] on Jacobi manifolds (see e.g. [LLMP] and [B]).
Obviously, the flat line bundle $L$ can be viewed as a local system associated with the character $\chi : \pi_1(M) \to \mathbb{R}^{>0}$ given by the exponential $e^\theta \in H^1(M, \mathbb{R}^{>0})$, considered as an element of $\mathbb{R}^{>0}$-valued cohomology. Then we have:

**Proposition 2.8.** (see e.g. [N]) The cohomology of the local system $L$ is naturally identified with the cohomology of the Morse–Novikov complex $(\Lambda^*(M), d_\theta)$.

The following result was proven in [LLMP] and, with a different method, in [OV2]:

**Theorem 2.9.** The Morse–Novikov cohomology of a compact Vaisman manifold vanishes identically.

On the other hand on one of the Inoue surfaces (which is LCK but non-Vaisman) the Morse–Novikov class of $\omega$ is non-zero, see [B, Theorem 1].

### 3 Weighted Dolbeault cohomology for LCK manifolds with potential

Let $M$ be an LCK manifold with potential, and $\tilde{M}$ its $\mathbb{Z}$-covering equipped with the automorphic Kähler metric. In [OV3] it was shown that the metric completion $\tilde{M}_c$ of $M$ is a Stein variety with at most one isolated singularity. Moreover, $\tilde{M}_c$ is obtained from $\tilde{M}$ by adding one point, called “the origin”. Denote this point by $c$.

**Remark 3.1.** If $M$ is Vaisman, then $\tilde{M}$ is a true (Riemannian) cone and the fibres are Sasakian. In the general case, nothing more precise can be said neither on the metric of $\tilde{M}$ nor on the contact metric structure of the fibres.

Since $\tilde{M}_c$, a singular variety, to control what happens in the neighbourhood of $c$ we need some technique borrowed from algebraic geometry which we briefly explain below. Note that we could arrive at the same results by using $L^2$-estimates, but the computations and technicalities would have been much more involved.
3.1 The generic vanishing theorem

Let $M$ be an LCK manifold with potential, $\theta$ its Lee form, $\widetilde{M}$ its $\mathbb{Z}$–cover and denote by $t : \widetilde{M} \to \tilde{M}$ the monodromy action. Let also $\tilde{M}_c$ be its Stein completion with the point $c$.

The map $t$ extends to a map $\tilde{t}$ on $\widetilde{M}_c$ by setting $\tilde{t}(c) = c$. For simplicity we denote by $R$ be the local ring of $c$, i.e. $R = \mathcal{O}_{\tilde{M}_c,c}$ and by $m$ its maximal ideal. The map $\tilde{t}$ induces an automorphism of local $C^\infty$–algebras of $R$, hence $\tilde{t}$ induces a linear automorphism $L(\tilde{t})$ of the Zariski tangent space at $c$, $m/m^2$.

We denote the eigenvalues of this linear map by $\alpha_1, \ldots, \alpha_N$; the multiplicative monoid generated by them $\Delta_{\tilde{t}} = \{ \alpha_1^{k_1}, \ldots, \alpha_N^{k_N} | k_1, \ldots, k_N \in \mathbb{N} \}$ will also be called the spectrum of $\tilde{t}$.

Let now $\alpha \in \mathbb{C}$ be arbitrary and let $L_\alpha$ be the flat line bundle on $M$ corresponding to $\alpha \cdot \theta$.

The main result of this section is:

**Theorem 3.2.** In the above assumptions, $H^q( M, \Omega^p_M \otimes L_\alpha ) = 0$, for any $q$ and any $\alpha \notin \Delta_{\tilde{t}}$.

We describe the main steps of the proof and give the details in the next section.

**Step 1: reduction to the local cohomology.** One has the following exact sequence, (see Corollary 3.6) which follows from Theorem 3.5:

$$
\begin{align*}
0 \to & \quad H^0( M, \Omega^i_M \otimes L_\alpha ) \to H^0( \tilde{M}, \Omega^i_{\widetilde{M}_c} ) \overset{1-\alpha}{\to} \\
& \quad \overset{\tilde{t} - \alpha}{\to} H^0( \tilde{M}, \Omega^i_{\widetilde{M}_c} ) \to H^1( M, \Omega^i_M \otimes L_\alpha ) \to \cdots \quad (3.1)
\end{align*}
$$

We are thus reduced to the study of the maps

$$
H^j \left( \tilde{M}, \Omega^i_{\widetilde{M}_c} \right) \overset{1-\alpha}{\to} H^j \left( \tilde{M}, \Omega^i_{\widetilde{M}_c} \right).
$$

Denote by $\Omega^i_{\tilde{M}_c}$ be the exterior $j$–power of the sheaf of Kahler differentials on $\tilde{M}_c$ and by $S$ its stalk at $c$. Using cohomology with supports, we have an exact sequence

$$
\begin{align*}
0 \to & \quad H^0_m( S ) \to H^0 \left( \tilde{M}_c, \Omega^i_{\tilde{M}_c} \right) \to H^0 \left( \tilde{M}, \Omega^i_{\widetilde{M}_c} \right) \to \\
& \quad \to H^1_m( S ) \to H^1 \left( \tilde{M}_c, \Omega^i_{\tilde{M}_c} \right) \to \cdots
\end{align*}
$$
Since $\tilde{M}_c$ is Stein, $H^i\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right) = 0$ for all $i \geq 1$, we obtain isomorphisms

$$H^i\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \simeq H^i_{m+1}(S),$$

and an exact sequence

$$0 \rightarrow H^0_{m}(S) \rightarrow H^0\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right) \xrightarrow{\alpha} H^0\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \rightarrow H^1_{m}(S) \rightarrow 0$$

These induce the commutative diagrams

$$\begin{array}{ccc}
H^i\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) & \xrightarrow{\alpha} & H^i_{m+1}(S) \\
\downarrow & & \downarrow \\
H^i\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) & \xrightarrow{\alpha} & H^i_{m+1}(S)
\end{array} \quad (3.2)$$

and a commutative diagram with exact rows

$$\begin{array}{ccc}
0 & \rightarrow & H^0_{m}(S) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^0\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^0\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^1_{m}(S) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^1\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^1\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^2_{m}(S) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^2\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & H^2\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \\
\downarrow & \searrow & \downarrow \\
0 & \rightarrow & 0
\end{array} \quad (3.3)$$

Eventually, notice that $H^i_{m+1}(S)$ and $H^0\left(\tilde{M}_c, \Omega^i_{\tilde{M}_c}\right)$ are $R$–modules.

**Step 2: algebraic proof of generic vanishing.** At this step we use the following fact, which will be proven in section 3.3.

**Theorem 3.3.** For any local noetherian $\mathbb{C}$–algebra $R$ endowed with a $\mathbb{Z}$–action given by an automorphism of local $\mathbb{C}$–algebras $\tilde{t}$ and any $R$–module $N$ endowed with the induced action $t_N : N \otimes_R R \xrightarrow{id_N \otimes \tilde{t}} N \otimes_R R$, i.e.

$$t_N(rm) = \tilde{t}(r)t_N(m), \quad \text{for all } r \in R, \ m \in N,$$

the map $t_M - \alpha$ is a $\mathbb{C}$–linear isomorphism for all $\alpha \in \mathbb{C} \setminus \Delta_{\tilde{t}}$.

**Step 3.** Using the above commutative diagrams (3.2), (3.3), we conclude that for each $\alpha \in \mathbb{C} \setminus \Delta_{\tilde{t}}$ and any $i, j \geq 0$ the map

$$t - \alpha : H^i\left(\tilde{M}, \Omega^i_{\tilde{M}}\right) \xrightarrow{} H^i\left(\tilde{M}, \Omega^i_{\tilde{M}}\right)$$

is an isomorphism. The conclusion follows from the exact sequence (3.1).
3.2 Proof of Step 1: reduction to the local cohomology

Definition 3.4. Let $F$ be a sheaf over a topological vector space. Denote by $F_x$ the stalk of $F$ in $x \in M$, and let $\text{God}(F)$ be a sheaf defined by $\text{God}(F)(U) := \prod_{x \in U} F_x$. The natural sheaf embedding $F \hookrightarrow \text{God}(F)$ is apparent. The sheaves $\text{God}_i(F)$ are defined inductively: $\text{God}_0(F) := F$, $\text{God}_1(F) := \text{God}(F)$ and $\text{God}_{i+1}(F) := \text{God}(\text{God}_i(F)/\text{God}_{i-1}(F))$. This gives an exact sequence

$$0 \rightarrow F \rightarrow \text{God}_1(F) \rightarrow \text{God}_2(F) \rightarrow \cdots$$

called the Godement resolution of $F$.

Theorem 3.5. Let $\widetilde{M} \xrightarrow{\pi} M$ be a manifold equipped with a free action of $\mathbb{Z}$, $M := \widetilde{M}/\mathbb{Z}$ its quotient, and let $F$ be a $\mathbb{Z}$-equivariant sheaf on $\widetilde{M}$. For any character $\alpha : \mathbb{Z} \rightarrow \mathbb{R}$, denote by $F^\alpha \subset \pi^* F$ the sheaf of automorphic sections of $\pi^* F$, associated with the character $\alpha$, considered as a sheaf on $M$.

Then one has the exact sequence

$$0 \rightarrow H^0(M, F^\alpha) \rightarrow H^0(\widetilde{M}, F) \xrightarrow{t-\alpha} H^1(\widetilde{M}, F) \rightarrow H^1(M, F^\alpha) \rightarrow \cdots$$

(3.4)

where $t$ is the associated action by the generator of $\mathbb{Z}$ acting on $\widetilde{M}$, and $\alpha$ is the multiplication by the number $\alpha(t)$.

Proof: Consider the Godement resolution $0 \rightarrow F \rightarrow F^1 \rightarrow F^2 \rightarrow \cdots$. Here $F^i = \text{God}(F^{i-1}/\text{im}(d)) = \text{God}(\text{coker}(d_{i-1}))$, and $d_i : F^{i-1} \rightarrow F^i$. Then

$$0 \rightarrow F^\alpha \rightarrow \pi_* F^\alpha \xrightarrow{t-\alpha} \pi_* F^\alpha \rightarrow 0$$

(3.5)

is an exact sequence of complexes of flabby sheaves over $M$.

Indeed, $F^\alpha = \ker(t-\alpha)$ and we only have to show that $t-\alpha$ is surjective. It is enough to make the proof at the level of sections of $F^k$. The argument is combinatorial. We look at $\widetilde{M}$ as $\bigcup_{i \in \mathbb{Z}} \widetilde{M}_i$ where $M_0$ is the fundamental domain of the $\mathbb{Z}$ action and $\widetilde{M}_i = t^i(M_0)$.

Then, given $f \in F^k(U)$, $U \subset \widetilde{M}$, it is enough to solve the equation $(t-\alpha)g = f$ for each $f_i = f|_{U_i}$, $U_i = U \cap \widetilde{M}_i$; this will give as solution the section $g_{i-1} \in F(U_{i-1})$, $i \in \mathbb{Z}$. The equation is

$$g_i - \alpha g_i - 1 = f_{i-1},$$

which can be solved recursively once we have chosen arbitrarily $g_0 \in F(U_0)$. 

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The long exact sequence associated to \((3.5)\) is precisely \((3.4)\). \(\blacksquare\)

Let now \(M\) be a locally conformally Kähler manifold with Kähler covering \(\widetilde{M}\) and monodromy \(\Gamma \cong \mathbb{Z}\). Consider the weight bundle \(L\) on \(M\), and let \(L_\alpha\) be its power associated with the character \(\alpha \in \text{Hom}(\Gamma, \mathbb{R})\). Since the automorphic forms on \(\widetilde{M}\) can be identified with forms on \(M\) with values in \(L\), from the above result we directly obtain:

**Corollary 3.6.** For a compact LCK manifold with monodromy \(\mathbb{Z}\) one has the exact sequence for the Dolbeault cohomology of \(M\) with values in \(L_\alpha\):

\[
0 \rightarrow H^0(M, \Omega^i_M \otimes L_\alpha) \rightarrow H^0\left(\widetilde{M}, \Omega^i_{\widetilde{M}}\right) \xrightarrow{t-\alpha} H^1(M, \Omega^i_M \otimes L_\alpha) \rightarrow \cdots
\]

### 3.3 Proof of Step 2: algebraic proof of generic vanishing.

**Proof of Theorem 3.3:** We perform several reductions. First, since completion is faithfully flat, we can assume the ring \(R\) complete (in its \(m\)-adic topology).

Next, we reduce to the case when \(R\) is regular. To do this, we choose a minimal system of generators for \(m_R\), say \(r_1, \ldots, r_n\) and define a map

\[
\pi : \mathbb{C}[[X_1, \ldots, X_n]] \rightarrow R,
\]

by \(X_i \mapsto r_i, i = 1, n\).

The action \(t\) on \(R\) lifts to an action \(\overline{t}\) on \(\mathbb{C}[[X_1, \ldots, X_n]]\) as follows. For each \(i\), choose a power series \(S_i\) which lifts \(r(t_i)\) and define \(\overline{t}\) on \(\mathbb{C}[[X_1, \ldots, X_n]]\) by sending \(X_i\) to \(S_i\). Note that since the map \(\pi\) induces an isomorphism at \(m/m^2\) level, the eigenvalues of \(L(t)\) and \(L(\overline{t})\) are the same.

We thus assume that \(R\) is a power series ring, \(R = \mathbb{C}[[X_1, \ldots, X_n]]\) and show that it is enough to prove the statement of the theorem for \(N\) being a free module. To this end, recall that by Kaplansky’s theorem, \(N\) is free if and only if is projective, i.e. its projective dimension \(\text{pdim}(N)\) vanishes.

Let then \(N\) be an arbitrary regular \(R\)-module. Since \(R\) is regular, its global dimension is finite by Serre’s theorem, so \(\text{pdim}(N) < \infty\). We proceed by induction on \(\text{pdim}(N)\).

Let \(F\) be a free \(R\)-module surjecting onto \(N\), so there exists the exact sequence

\[
0 \rightarrow K \rightarrow F \rightarrow N \rightarrow 0.
\]
As pdim$(K) = \text{pdim}(N) - 1$, the induction applies to $K$. Letting $t_F, t_K, t_N$ be the induced actions on $F, K, N$ we see that for any $\lambda \in \mathbb{C}$ the following commutative diagram exists:
\[
\begin{array}{cccccc}
0 & \longrightarrow & K & \longrightarrow & F & \longrightarrow & N & \longrightarrow & 0 \\
& & t_K - \alpha & \downarrow & t_F - \alpha & \downarrow & t_N - \alpha & & \\
0 & \longrightarrow & K & \longrightarrow & F & \longrightarrow & N & \longrightarrow & 0
\end{array}
\]
which induces the long exact sequence
\[
0 \longrightarrow \ker(t_K - \alpha) \longrightarrow \ker(t_F - \alpha) \longrightarrow \ker(t_N - \alpha) \longrightarrow \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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\right).
\]
After conjugation by a linear automorphism, we may assume that $L(t)$ is in its Jordan normal form.

Let $\lambda \in \mathbb{C}$ be arbitrary. We claim that if $\lambda \not\in \Delta_t$ then the equation $t(x) - \lambda x = y$ has exactly one solution for each $y \in \mathbb{C}[[X_1, \ldots, X_n]]$. Indeed, to find $x$ it is enough to specify the values of $\partial^I x(0)$ for each multiindex $I = (k_1, \ldots, k_n)$. But $t(x) - \lambda x = y$ yields
\[
(\alpha_1^{k_1} \cdots \alpha_n^{k_n} - \lambda) \partial^I x(0) = \partial^I y(0)
\]
and thus, unless $\alpha_1^{k_1} \cdots \alpha_n^{k_n} - \lambda = 0$, the value $\partial^I x(0)$ is uniquely determined.
3.4 Degeneration of the Dolbeault-Frölicher spectral sequence with coefficients in a local system

The next result, interesting in itself, proves that on compact LCK manifolds with potential, the Dolbeault-Frölicher spectral sequence with coefficients in a local system

\[ E_1^{p,q}(M, \Omega^p_M \otimes L_\alpha) \Rightarrow H^{p+q}(M, L_\alpha(C)), \]

degenerates at \( E_2 \) level (where \( L_\alpha(C) \) denotes the local system associated to \( L_\alpha \)). This parallels the degeneration of this spectral sequence at \( E_1 \) level for compact Kähler manifolds (where \( L_\alpha \) is taken to be trivial). In particular, this gives a new proof to [Theorem 2.9](#) and produces new examples of compact complex manifolds that do not carry LCK metrics with potential. One of the approaches to finding such manifolds is due to S. Rollenske ([Rol](#)), who has shown that on a nilmanifold, the Dolbeault-Frölicher spectral sequence does not necessarily degenerate, and gave examples when the \( n \)-th differential \( d_n \) is non-zero, for arbitrarily high \( n \).

**Proposition 3.7.** Let \( M \) be a compact LCK manifold with potential, \( \alpha \in \text{Hom}(\Gamma, \mathbb{R}_+) \) a positive character, and \( L_\alpha \) the corresponding line bundle. For any \( p, q \), consider the map \( \partial_{p,q} : H^p(M, \Omega^q_M \otimes L_\alpha) \to H^p(M, \Omega^{q+1}_M \otimes L_\alpha) \).

Then \( \ker \partial_{q,p+1} = \text{im} \partial_{q,p} \), for all \( p, q \).

**Proof:** The monodromy map \( \tilde{t} \) on \( \tilde{M} \) is the exponential of a holomorphic vector field \( X \). This is proven in [OV5, Theorem 2.3](#) using the embedding of \( M \) in a Hopf manifold \( \mathbb{C}^N \setminus \{0\} / \langle A \rangle \) where \( A \) is linear, with all eigenvalues smaller than 1. The holomorphic vector field is then \( X = \log A \). In particular:

\[ \tilde{t}^*(\eta) = \text{Lie}_X \eta. \]

Let now \( [\eta] \in H^p(M, \Omega^{q+1}_M \otimes L_\alpha) \). A representative \( \eta \) can be seen as a \((q+1,p)\)-form on \( \tilde{M} \) which is \( \partial\)-closed and automorphic of weight \( \alpha \).

Suppose \( \eta \) is also \( \partial \)-closed. Then, since \( \tilde{t}^*(\eta) = \alpha \cdot \eta \), we obtain

\[ \alpha \cdot \eta = \text{Lie}_X(\eta) = di_X \eta + i_X d\eta, \]

by Cartan’s formula.

But \( \overline{\partial}(\eta) = \partial \eta = 0 \) by assumption, thus \( i_X d\eta = 0 \), and we are left with:

\[ \alpha \cdot \eta = \partial(i_X \eta) + \overline{\partial}(i_X \eta). \]
As $X$ is holomorphic, $i_X \eta$ is of type $(q,p)$, and hence $\overline{\partial}(i_X \eta)$ is of type $(q,p+1)$. On the other hand both $\partial i_X(\eta)$ and $\alpha \cdot \eta$ are of type $(q+1,p)$, implying $\overline{\partial}(i_X \eta) = 0$ and $\alpha \cdot \eta = \partial(i_X \eta)$.

This yields $\eta = \partial i_X (\frac{1}{\alpha} \eta)$, and hence $\eta \in \text{im}(\partial_{q,p})$. $lacksquare$

4 Weighted Bott-Chern cohomology for LCK manifolds with potential

We now generalize [OV2, Theorem 4.7]. We have:

**Proposition 4.1.** Let $(M,I,g)$ be a compact LCK manifold. Then the following sequence is exact:

$$H^{p-1}_\partial (\Omega^q_M \otimes L_\alpha) \oplus H^{q-1}_\partial (\Omega^p_M \otimes L_\alpha) \overset{\partial_\theta + \overline{\partial}_\theta}{\longrightarrow} H^{p,q}_{BC}(M,L_\alpha) \overset{\nu}{\longrightarrow} H^{p+q}(M,L_\alpha(\mathbb{C}))$$

(4.1)

where $\nu$ is a tautological map, $\partial_\theta = \partial - \vartheta^{1,0}$ and $\overline{\partial}_\theta = \overline{\partial} - \vartheta^{0,1}$.

**Proof:** We prove that $\text{im}(\partial_\theta + \overline{\partial}_\theta) = \ker \nu$. Let $\eta$ be a $(p,q)$-form with values in $L_\alpha$ whose class vanishes in the cohomology of the local system $L_\alpha(\mathbb{C})$. Then $\eta = d_\theta \beta$. Then $\eta$ decomposes as $\eta = \overline{\partial}_\theta \beta^{p,q-1} + \partial_\theta \beta^{p-1,q}$. On the other hand, as $\eta$ is of bidegree $(p,q)$, we have $\partial_\theta \beta^{p,q-1} = 0$ and $\overline{\partial}_\theta \beta^{p-1,q} = 0$, and hence $\beta^{p,q-1}$ and $\beta^{p-1,q}$ produce the cohomology classes in $[\beta^{p,q-1}] \in H^{p-1}_\partial (\Omega^q_M \otimes L_\alpha)$ and $[\beta^{p-1,q}] \in H^{q-1}_\partial (\Omega^p_M \otimes L_\alpha)$. Then $[\eta]_{BC} = \partial_\theta [\beta^{p,q-1}] + \overline{\partial}_\theta [\beta^{p-1,q}]$. $lacksquare$

By [Theorem 2.7 (ii)], the compact LCK manifolds with potential are topologically equivalent with Vaisman manifolds. Therefore, [Theorem 2.9] implies that their cohomology of the local system $L_\alpha(\mathbb{C})$ vanishes identically. Together with our main result [Theorem 3.2] this proves the following generic vanishing of Bott-Chern cohomology (we keep the notations in Section 3):

**Corollary 4.2.** Let $M$ be an LCK manifold with potential, $\alpha \in \mathbb{C}$ and $L_\alpha$ the flat line bundle corresponding to $\alpha \cdot \theta$. Then $H^{p,q}_{BC}(M,L_\alpha) = 0$ for all $\alpha \notin \Delta$.

**Remark 4.3.** Note that $H^{p,q}_{BC}(M,L_\alpha) = 0$ is equivalent with the $dd^c$-lemma at the level $(p,q)$, and hence our result says that, generically, a compact LCK manifold with potential satisfies the $dd^c$-lemma for all $(p,q)$. 
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