Fluctuations of collective flows for event-by-event hydrodynamic evolution sources

Zhongqian Su\textsuperscript{1,2}, Yanchao Wang\textsuperscript{2}, Dayi Song\textsuperscript{2} and Weining Zhang\textsuperscript{1,3}

\textsuperscript{1}School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024

\textsuperscript{2}Liaoning Institute of Science and Technology, Benxi 117004

\textsuperscript{3}Email: weiningzhang1@163.com

Abstract. Through investigating the space-time evolution of the hydrodynamic particle-emitting sources with the fluctuating initial conditions generated by Heavy Ion Jet Interaction Generator (HIJING), we discussed the space-time and velocity evolution of quark gluon plasma (QGP). In order to detect the event-by-event inhomogeneity of the sources, we examine the distribution of the error-inverse-weighted fluctuations, between the two-pion Bose-Einstein correlation functions of single events. We find that the distribution becomes wide for the fluctuating initial sources with velocity \(v \neq 0\), which means the sources evolution faster than nonzero initial conditions. We also discussed the elliptic flow effect, velocity and temperature evolution in Heavy Ion Reactions, and get the same results.

1. Introduction

Relativistic hydrodynamics has been widely used in describing the evolution of Quark Gluon Plasma (QGP) [1-3]. It provides the link between the initial and final states of the systems produced in the collisions. In general, the initial quantities of relativistic heavy ion collisions systems are event-by-event fluctuations [4]. These initial fluctuations may affect the system evolution of space-time and lead to some changes of final particle observables relative to those associated with smoothed initial conditions [4-8]. Hanbury-Brown-Twiss (HBT) interferometry is a useful tool for probing the space-time structure of the particle-emitting sources in high energy heavy ion collisions [9-12]. Previous studies indicate that the single event HBT correlation functions of the final identical pions exhibit event-by-event fluctuations in the granular source model [13, 14]. In this case, we generate the event-by-event initial states by using the Heavy Ion Jet Interaction Generator (HIJING) [15] at the energies of the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). Then, we simulate the evolution of the sources with a (2+1) dimension relativistic hydrodynamics with the fluctuating initial conditions. Through investigating the effect of the fluctuating initial conditions on the space-time evolution of the particle-emitting sources on different velocity, we found that the particle-emitting sources with fluctuating initial conditions are inhomogeneous in space. The large width of the distribution of the error-inverse-weighted fluctuations between the correlation functions of single and mixed events with the fluctuating initial conditions.
2. Fluctuating initial conditions and equation of state

Once the equation of state (EOS) \( P = P(\varepsilon, n_b) \) is known, we can simulate the evolution of QGP by relativistic hydrodynamics. Here \( P \) denotes the local equilibrium pressure, \( n_b \) is the net charge density, \( \varepsilon \) is the energy density. In this work we consider the evolution with the Bjorken-cylinder geometry and zero net baryon density. Through solving the hydrodynamics equations, i.e., \( \partial_t T^{\mu\nu} = 0 \), we will obtain information about thermodynamic and the flow velocity of the QGP. Considering the Bjorken-cylinder geometry, after \( v_z = z/t \), the other quantities become independent of the space-time rapidity \( \eta = (1/2)\ln[(t+z)/(t-z)] \). Using this, we solve them numerically by applying the relativistic Harten-Lax-Leer-Einfeldt (RHLLE) algorithm.

The energy-momentum tensor of the fluid in relativistic hydrodynamics can be expressed as

\[
T_{\mu\nu} = \varepsilon u^\mu u^\nu - p \Delta_{\mu\nu}
\]

where \( T_{\mu\nu} \) is the energy-momentum stress tensor, \( p \) is local isotropic pressure density, \( \Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \) is a space projection in the local fluid rest system which is orthogonal to \( u_\mu \), i.e., \( \Delta_{\mu\nu} u^\nu = 0 \).

The EOS used in this paper is the parametric EOS (figure 2 [12]), which combines hadron resonance gas at low temperatures with lattice Quantum Chromodynamics (QCD) at high temperatures.

Figure 1 shows the solutions of the transverse distributions of energy density at \( z = 0 \) and \( t = 2 \) and \( 5 \text{ fm}/c \) for the two single-event sources with the HIJING initial conditions. In figure 1(a1) and (a2), we show the evolution of energy with the velocity \( v = 0 \), while (b1) and (b2) mean the evolution with the velocity \( v \neq 0 \). The grid sizes for the HLLE are taken to be \( \Delta x = \Delta y = 0.1 \text{ fm} \), and the time step is taken to be \( \Delta t = 0.99 \cdot \Delta x \). From figure 1 it can be seen that the transverse distributions of the energy density have large event-by-event fluctuations in space. These fluctuations are reserved even at the late stage of the evolution at \( t = 9 \text{ fm}/c \). In figure 2, we show the evolutions of press, temperature, the velocity of sound and entropy with energy.

![Figure 1](image1.png)

**Figure 1.** Transverse distributions of energy density of the two single events with different impact parameter \( b \) and \( t \).
3. **Numerical result of fluctuating initial sources**

Using RHLLE numerical method, we compute the space-time evolution of the hydrodynamic particle-emitting sources with the fluctuating initial conditions in addition to temperature and radial velocity profiles. In figure 3 and figure 4, we sketch the temperature and velocity profiles at different times for two considering cases of an ideal fluid with initial velocity \( v = 0 \) and \( v \neq 0 \). The solid lines are for the evolution with initial velocity \( v \neq 0 \), and the dash lines are for \( v = 0 \). We can see the comparison of temperature and transverse velocity profiles in \( z = 0 \) plane for two different cases. From the figures, we find the initial velocity almost has no impact on the temperature evolution, even in the bigger \( r \). As for velocity evolution, the initial velocity will increase the hydrodynamics evolution, especially in smaller time (figure 4). And in the figures the lines with same color mean the curve at the same time. As for the different times for figure 4 from figure 3, it is only for clear distinction.

**Figure 2.** The parametric equation of state.

**Figure 3.** Temperature profiles at different times.

**Figure 4.** Velocity profiles at different times.
The HBT correlation function of identical pions is defined as the ratio of the two-pion momentum distribution \( P(\vec{p}_1, \vec{p}_2) \) to the product of the single-pion momentum distribution \( P(\vec{p}_1)P(\vec{p}_2) \). Assuming that final identical pions are emitted at the space-time configuration characterized by a freeze-out temperature \( T_f \), we can generate the pion momentum \( \vec{p}(i=1,2) \) according to Bose-Einstein distribution, and then construct the single-event and mixed-event two-pion correlation functions \([5,14]\). In order to observe the event-by-event fluctuations, we investigate the ratio of signal to noise of the fluctuations between the correlation functions of single and mixed events \([c_s(q_i) - c_m(q_i)]\):

\[
f(q_i) = \left| \frac{c_s(q_i) - c_m(q_i)}{\Delta(c_s(q_i) - c_m(q_i))} \right|
\]

where \( i \) is the index of the component of relative momentum \( q \), and \( \Delta(c_s(q_i) - c_m(q_i)) \) is the error of the fluctuation.

In figure 5, we show the distributions \( dN/df \) of the \( f \) calculated with the variables of transverse relative momentum \( q_{tr} \) and relative momentum \( q \) of the pion pairs obtained from the 80 events with impact parameter \( b = 4 \) and 8, respectively. And \( \sigma = 0.3, 0.6, 0.9 \) is size of the particle-emitting sources generated by Heavy Ion Jet Interaction Generator (HIJING). In calculations we take the width of the relative momentum \( q_{tr} \) bin to be 10\( \text{MeV}/c \) and use the bins in the region \( 20 \leq q_i \leq 200\text{MeV}/c \). The number of correlated pion pairs for each event is taken to be \( N_{\pi\pi} = 10^6 \). In figure 5, the solid lines are the results for the fluctuating initial conditions with initial velocity \( v \neq 0 \). For comparison, the corresponding distributions for the events with the initial velocity \( v = 0 \), which are obtained by averaging over 400 random HIJING events, are shown with the dashed lines. One can see that the \( f \) distributions for \( v \neq 0 \) are wider than the corresponding results for \( v = 0 \).

![Figure 5](image-url)

**Figure 5.** (Color online) Distributions \( dN/df \) for 80 FIC events with impact parameters \( b = 4 \text{fm} \) (the left figure for \( v_2 \) and the middle figure for \( v_3 \)) and \( b = 8 \text{fm} \) (the right figure for \( v_2 \)).

The widths of the distributions for \( v \neq 0 \) increase with impact parameter. But the widths of the distributions for \( v = 0 \) are small and almost invariant with the increase of impact parameter. This is because that the fluctuations of the single-event correlation functions for \( v = 0 \) are always small. And from figure 5, we also can find the triangulation (\( v_3 \) in figure 5) wider than elliptic flow (\( v_2 \) in figure 5).

In experiments the number of correlated pion pairs in one event, \( N_{\pi\pi} \), is limited. For the central collisions at RHIC energy, the event multiplicity of identical pion, \( M_x \), is about several hundreds and \( N_{\pi\pi} \) is about \( 10^5 \sim (M_x^2/2) \). The signatures of \( dN/df \) for the inhomogeneous particle-emitting sources are hopefully to be detected in the heavy ion collisions at RHIC and LHC.
4. Conclusions
Using the hydrodynamic model with HIJING event-by-event fluctuating initial conditions, we investigate the space-time evolution of the particle-emitting sources in relativistic heavy ion collisions for different initial velocities. The results indicate that the fluctuating initial conditions may lead to event-by-event inhomogeneous particle-emitting sources. In order to observe the fluctuations of the correlation functions, we investigate the distributions of $f$, the fluctuations between the two-pion correlation functions of single and mixed events with their error-inverse weights. We find that the widths of the distributions $dN/df$ for $v \neq 0$ are much wider than those for $v = 0$. Correspondingly, the values of the triangulation for $v \neq 0$ are larger. For $v \neq 0$, $f$ increases with the impact parameter of collisions, because the smaller number of the hot spots in the system with larger impact parameter may lead to a larger source granularity. Our results indicate that these signatures are hopefully to be detected in the heavy ion collisions at RHIC and LHC.

Acknowledgements
This research was financially supported by the Youth Fund of Liaoning Institute of Science and Technology (Qn201601, Qn201603), the Liaoning Provincial Education Department Foundation (L2017lkyqn-01, L2017lkyqn-07).

References
[1] Rischke D H 1998 Proceedings of the 11th Chris Engelbrecht Summer School in Theoretical Physics Cape Town Februa4-13 arXiv:nucl-th/9809044
[2] Kolb P F and Heinz U arXiv:nucl-th/0305084
[3] Huovinen P, Kolb P F, Heinz U W, Ruuskanen P V and Voloshin S A 2001 Phys. Lett. B 503 58
[4] Adare A, Luzum M and Petersen H arXiv: 1212.5388 [nucl-th]
[5] Zhang W N, Ren Y Y and Wong C Y 2006 Phys. Rev. C 74 024908
[6] Heinz U W, Song H and Chaudhuri A K arXiv:nucl-th/0510014
[7] Zhang W N, Yang Z T and Ren Y Y 2009 Phys. Rev. C 80 044908
[8] Rischke D H and Gyulassy M 1996 Nucl. Phys. A 608 479
[9] Wong C Y 1994 Introduction to High-Energy Heavy-Ion Collisions (chap 17) Singapore: World Scientific
[10] Wiedemann U A and Heinz U 1999 Phys. Rep. 319 145-230
[11] Weiner R M Phys. Rep. 327 249-346
[12] Shen C, Heinz U W, Huovinen P and Song H 2010 Phys. Rev. C 82 054904
[13] Wong C Y and Zhang W N 2004 Phys. Rev. C 70 064904
[14] Zhang W N, Li S X, Wong C Y, et al. 2005 Phys. Rev. C 71 064908
[15] Wang X N 1997 Phys. Rep. 280 287-371