SPECTRUM OF THE $Y=2$ PENTAQUARKS

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ABSTRACT

By assuming a mass formula for the spectrum of the $Y = 2$ pentaquarks, where the chromo-magnetic interaction plays a main role, and identifying the lightest state with the $\Theta^+(1540)$, we predict a spectrum in good agreement with the few $I = 0$ and $I = 1$ candidates proposed in the past.

Keywords: Pentaquarks, Chromo-magnetic interaction

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1 Introduction

The quark model is a successful theory, which is widely used to classify hadrons and calculate their masses and decays. According to the simple quark model, mesons are formed by a quark and an antiquark of the opposite colour, and baryons by three quarks in a colour singlet. However, besides these states, QCD does not exclude the existence of multiquark (tetraquark, pentaquark,...) states, which may transform as higher SU(3)F representations.

In fact, recently, the discovery of a narrow ($Y = 2, I = 0$) KN resonance $\Theta^+$ at 1540 MeV [1], just the value predicted [2] in the Skyrme model [3], has motivated the search [4] [5] [6] [7] [8] [9] [10] [11] [12] to understand it as a $uudd\bar{s}$ state. A study of pentaquarks has been performed many years ago [13], at the time when bubble chambers, the best device to detect these particles, were working. To get narrow widths, one considered [13] P-wave states with the constituents separated by the orbital centrifugal barrier in two clusters such that the $\bar{q}$ could not form a colour singlet with a quark of its cluster, as $(qq)(\bar{q})^6$ or $(qq)(\bar{q})^8$. The contribution of the chromo-magnetic force, which accounts for the mass splittings within the SU(6)FS multiplets [14], was supposed to be the sum of the ones coming from the two clusters, since the centrifugal barrier suppresses the contact interaction.

In the following section we shall propose a mass formula for the positive parity pentaquarks built with 4$q$, $L = 1$ and a $\bar{q}$ in S-wave with respect to them or with 4$q$, $L = 0$ and a $\bar{q}$ in P-wave with respect to them. We shall extend our consideration to the negative parity states built with 4$q$ and a $\bar{q}$ in S-wave.

2 A mass formula for pentaquarks

In [5] the authors consider two 2$q$ clusters transforming as the $(\bar{3}, 1, 3)$ of $SU(3)_C \times SU(2)_S \times SU(3)_F$ in P-wave, which implies the most negative contribution from the chromomagnetic interaction for the 4$q$, $L = 1$ states transforming as a $(3, 1, 6)$ representation. In fact the contribution for the interaction of two quarks is given by:

$$\Delta m_{qq} = -C_{qq} \left[ C_6(2q) - \frac{1}{2}C_3(2q) - \frac{1}{3}C_2(2q) - 4 \right]$$

where $C_6(2q)$, $C_3(2q)$ and $C_2(2q)$ are the SU(6)$_{CS}$, SU(3)$_C$ and SU(2)$_S$ Casimir operators, respectively. The consequences of eq.(1) are reported in Table 1.

The contribution of the chromo-magnetic interaction of the 4$q$ with the $\bar{q}$, the $\bar{s}$ for the $Y = 2$ states, depends on their relative position. In the approach developed in [13] $(qq\bar{q})(qq)$ clusters where considered. Starting with 4$q$, $L = 1$ states, one has the same overlap of the $\bar{q}$ with the two $qq$ clusters, which implies:

$$\Delta m_{4q,\bar{q}} = C_{4q,\bar{q}} \left[ C_6(p) - \frac{1}{3}C_2(p) + \frac{1}{3}C_2(t) - \frac{4}{3} \right]$$

where $C_6(p)$ and $C_6(t)$ are the Casimir of SU(6)$_{CS}$, $p$ and $t$ are the representations for pentaquark and 4$q$ states, respectively, and $C_2(p)$ and $C_2(t)$ are the Casimir of SU(2)$_S$. 

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Table 1: Chromomagnetic splittings for 2q states.

| $SU(3)_C \times SU(2)_S$ | $\frac{\Delta m_{qq}}{c_{qq}}$ |
|--------------------------|--------------------------|
| (3, 1)                   | $-2$                     |
| (6, 3)                   | $-\frac{1}{3}$           |
| (3, 3)                   | $+\frac{2}{3}$           |
| (6, 1)                   | $+1$                     |

To account for the low value of the mass of $\Theta^+$, despite the expected contribution of the kinetic energy associated to the orbital angular momentum, we may assume that the overlap of the $\bar{s}$ with the quarks is the same as in $K^{(*)}$ particles; we take:

$$C_{4q, \bar{q}} = \frac{3}{16}(m_{K^*} - m_K) \quad (3)$$

To compute the r.h.s. of eq.(2), one needs to know the $SU(6)_{CS}$ transformation properties of each pair of two $q$ clusters in P-wave, which, if the two clusters have the same transformation properties, depends on the $SU(3)_F$ behaviour of the 4$q$’s. This information is described in Table 2 together with the contribution of the chromo-magnetic interaction to the mass splitting.

From eqs.(1,2) and Tables 1 and 2 we derive the contribution to the pentaquark masses coming from the chromo-magnetic interaction. By including the contribution of the quark masses, the kinetic energy associated to the angular momentum and the spin-orbit term, one gets:

$$m(p_1) = \Sigma_{i=1,4} m_{q_i} + m_{\bar{q}} + \Delta m_{qq}^1 + \Delta m_{qq}^2 +$$
$$C_{4q, \bar{q}} \left[ C_6(p) - C_6(t) - \frac{1}{3} C_2(p) + \frac{1}{3} C_2(t) - \frac{4}{3} \right] +$$
$$+ K_1 + a \bar{L} \cdot \bar{S}_q \quad (4)$$

where $\Delta m_{qq}^1 + \Delta m_{qq}^2$ is the contribution of the chromo-magnetic interaction coming from the two 2$q$ clusters according to eq.(1). A similar form holds for the positive parity states built with 4$q$ in S-wave and $\bar{q}$ in P-wave with respect to them:

$$m(p_2) = \Sigma_{i=1,4} m_{q_i} + m_{\bar{q}} +$$
$$C_{qq} \left[ C_6(t) - \frac{1}{3} C_2(t) - \frac{26}{3} \right] + K_2 + b \bar{L} \cdot \bar{S}_q + c \bar{L} \cdot \bar{S}_{\bar{q}} \quad (5)$$

Finally for the negative parity states built with 4$q$ and a $\bar{q}$ in S-wave one has the mass formula:

$$m(s) = \Sigma_{i=1,4} m_{q_i} + m_{\bar{q}} + C_{4q, \bar{q}} \left[ C_6(p) - C_6(t) - \frac{1}{3} C_2(p) + \frac{1}{3} C_2(t) - \frac{4}{3} \right] +$$
$$C_{qq} \left[ C_6(t) - \frac{1}{3} C_2(t) - \frac{26}{3} \right] \quad (6)$$
| $2q \times 2q$ | $SU(6)_{CS}$ | $SU(3)_F \times SU(2)_S$ | $\Delta m_{Cqq}$ |
|----------------|-------------|--------------------------|----------------|
| $[(3, 1) \times (3, 1)]_A$ | 210         | (6, 1)                   | -4             |
| $[(3, 1) \times (6, 3)]_A$ | 210         | (6, 3)                   | $-\frac{7}{3}$ |
| $[(3, 1) \times (6, 3)]_S$ | 105         | (3, 3)                   | $-\frac{7}{3}$ |
| $(3, 1) \times (6, 1)$ | $\frac{1}{2}210 + \frac{\sqrt{3}}{2}105'$ | (15 + 3, 1)      | -1             |
| $(3, 3) \times (6, 3)$ | $\frac{\sqrt{3}}{2}210 - \frac{1}{2}105'$ | (15 + 3, 1)      | $+\frac{1}{3}$ |
| $(\bar{3}, 1) \times (\bar{3}, 3)$ | $\frac{1}{\sqrt{2}}210 - \frac{1}{\sqrt{2}}105'$ | (15 + 3, 3)      | $-\frac{5}{3}$ |
| $(3, 3) \times (6, 3)$ | $\frac{1}{\sqrt{2}}210 + \frac{1}{\sqrt{2}}105'$ | (15 + 3, 3)      | $+\frac{1}{3}$ |
| $(\bar{3}, 3) \times (6, 3)$ | 105'        | (15 + 3, 5)              | $+\frac{1}{3}$ |
| $[(3, 3) \times (3, 3)]_A$ | 105'        | (15' + 6, 5 + 1)         | $+\frac{4}{3}$ |
| $[(3, 3) \times (6, 1)]_A$ | 105'        | (15' + 6, 3)             | $+\frac{5}{3}$ |
| $[(3, 3) \times (3, 3)]_S$ | $\sqrt{\frac{7}{3}}105 - \frac{1}{\sqrt{3}}15$ | (15, 3)          | $+\frac{3}{3}$ |
| $[(3, 3) \times (6, 1)]_S$ | $\frac{1}{\sqrt{3}}105 + \sqrt{\frac{7}{3}}15$ | (15, 3)          | $+\frac{3}{3}$ |

Table 2: $SU(6)_{CS}$ and $SU(3)_F \times SU(2)_S$ transformation properties and chromo-magnetic contribution to the mass splitting for the two pair of diquarks in the $4q, L = 1$ states
For the $Y = 2$ states we are interested the $q$’s are $u$ or $d$ quarks and $\bar{q}$ is $\bar{s}$ and the sum
\[ \Sigma_{i=1,4} m_{q_i} + m_{\bar{q}} = m_N + m_K + 2 C_{qq} + 4 C_{4q,\bar{q}} \]  
(7)

By relating $C_{qq}$ and $C_{4q,\bar{q}}$ to the splittings in the $SU(6)_{FS}$ multiplets, one has:
\[ C_{qq} = \frac{m_{\Delta} - m_N}{4} = 72.5 \text{MeV} \]  
(8)
\[ C_{4q,\bar{q}} = \frac{3}{16} (m_{K^*} - m_K) \simeq C_{qq} \]  
(9)

so that the r.h.s. of eq.(7) is
\[ 940 + 500 + 435 = 1875 \text{MeV} \]  
(10)

$K_1$ will be fixed by identifying the lightest positive parity $Y=2$, $I=0$ state with the $\Theta^+(1540)$, while the ratio $\frac{K_2}{K_1}$ is found by considering that the kinetic energy term associated to the orbital motion is inversely proportional to the reduced mass of the states in P-wave and so we get:
\[ \frac{K_2}{K_1} = \frac{4 m_{u,d} + m_{s}}{4 m_{s}} = \frac{7}{8} \]  
(11)

The last equality follows from taking the ratio $\frac{m_{u,d}}{m_s} = \frac{5}{8}$. We take $a = 40 \text{MeV}$, as in $[9]$, $b = 20 \text{MeV}$ and $c = 50 \text{MeV}$ since the giro-magnetic factors are inversely proportional to the masses.

With these values one finds for the $I = 0, 1, 2$ states the spectrum described in Fig.1-3 (4) for the positive (negative) parity states. In Fig 4 also the isospin is reported above each line.

The comparison with experiment will be performed in the next section, after deducing the consequences of an approximate selection rule, one can reasonably assume for pentaquark decays.

3 General properties of pentaquark decays

The dynamics for pentaquark decays may be different from the case of $\Delta$ and $\rho$ decays, where one has to create a $q\bar{q}$ pair, with the $\bar{q}$ forming a meson with one of the initial quarks or with the initial quark.

As long as for pentaquarks, all the elementary fermions in the final state are present in the initial one, which makes possible the hypothesis that the decay is a consequence of the separation of its constituents. Also, at difference with what happens for the decay of the previously mentioned ordinary hadrons, with the orbital angular momentum not conserved (changing from 0 to 1) as well as the spin, for the decay of pentaquarks with $L = 1$, to the initial orbital momentum of the $4q$’s in the initial state corresponds the relative angular momentum of the emitted meson with respect to the final baryon. So $L$ and $S$ may be both conserved. One may even assume that, as in the hypothesis that the amplitude is proportional to the scalar product of the initial and final wave-functions, also $SU(6)_{CS}$ and (or) $SU(6)_{FS}$ $[18]$ are conserved in pentaquark decays.
Figure 1: The spectrum of the pentaquark with $I = 0^+$

Figure 2: The spectrum of the pentaquark with $I = 1^+$

Figure 3: The spectrum of the pentaquark with $I = 2^+$
Figure 4: The spectrum of the pentaquark with negative parity

[19]. The consequences of $SU(6)_{CS}$ conservation for the decay of a pentaquark into a meson baryon final states are very restrictive [19], since the pseudoscalar mesons are $SU(6)_{CS}$ singlets. According to the $SU(6)_{CS}$ transformation properties of the baryon of the 56 of $SU(6)_{FS}$, as a 70 the $(8,1/2)^+$ and as a 20 the $(10,3/2)^+$, only the pentaquarks transforming as a 70 (20) of $SU(6)_{CS}$ may decay into a final state containing a pseudoscalar meson and a $(8,1/2)^+$ ($(10,3/2)^+$) baryon. Let us therefore consider the $SU(6)_{CS}$ products:

\[
210 \times \bar{6} = 1134 + 56 + 70 \quad (12)
\]

\[
105 \times \bar{6} = 560 + 70 \quad (13)
\]

\[
105' \times \bar{6} = 540 + 70 + 20 \quad (14)
\]

\[
\bar{15} \times \bar{6} = \bar{70} + 20 \quad (15)
\]

The combinations allowed to decay into a pseudoscalar meson and a $(8,1/2)^+$ (or a $(10,3/2)^+$) baryon, which transform as a 70 (20) of $SU(6)_{CS}$, are [9] for the $4q, L = 1$ states:

\[
|70, (1, S = 1/2), S_z = \frac{1}{2} > = \]

\[
\frac{1}{\sqrt{3}} \left\{ \frac{1}{\sqrt{2}} (210, (3, S = 1), S_z = 1 >_a |\bar{6}, (\bar{3}, S = 1/2), S_z = -1/2 >^a
\right.
\]

\[
-\frac{1}{2} (210, (3, S = 1), S_z = 0 >_a |\bar{6}, (\bar{3}, S = 1/2), S_z = 1/2 >^a
\]

\[
+\frac{1}{2} (210, (3, S = 0) >_a |\bar{6}, (\bar{3}, S = 1/2), S_z = 1/2 >^a}\}
\]

(16)
and the $S = 1/2$ (3/2) state constructed by composing the $S = 1$ of the 105 (15) with the $S = 1/2$ of the 6; $a = 1, 2, 3$ is a colour index to be saturated to get a colour singlet. This motivates our attention to the $SU(6)_{CS}$ transformation properties of the pentaquarks. In fact the states, which transform as a 70(20), are allowed to decay into a $8, 1/2^+(10, 3/2^+)$ and a pseudoscalar meson, while the ones transforming as the 70, 540, 560, 1134 representations are forbidden by the $SU(6)_{CS}$ selection rule to decay into those channels; we expect for them a three-body decay induced by the $SU(6)_{CS}$ violating creation of a $q\bar{q}$ pair. In general the pentaquarks, which behave as a 70 or a 20 of $SU(6)_{CS}$, are lighter, since these representations have lower $SU(6)_{CS}$ Casimirs.

In Table 3 we write for the positive parity states built with $4q, L = 1$ and a $\bar{q}$ in S-wave the mass (in MeV) of the $I = 0$ states, transforming as a 70, of the $I = 1$, transforming as a 70 or a 20 and of the $I = 2$ as a 20, which are the only ones allowed by $SU(2)_I$ and $SU(6)_{CS}$ to decay into a meson and a baryon of the 56 of $SU(6)_{FS}$. Small mixing of states with the same $I, J$ quantum numbers are present.

As shown in [20] also the states, which are allowed to have two-bodies decays by the $SU(6)_{CS}$ selection rule, may have a narrow width for a small overlap of the wave functions of the initial and the final states.

All the $I = 2$ states are degenerate with $I = 0$ states; as stressed by Rossi and Veneziano [21], the $I_3 = 0$ mass eigenstates have not definite isospin: in our case the diagonal states are the ones with $(wu)(dd)$ or $(ud)(ud)$ clusters.

By considering $(4q, L = 0)$, we can build positive (negative) parity states by combining them with a $q$ with $L = 1(0)$ with respect to them. Within the approximation first suggested in [13] of requiring that the $\bar{q}$ should form a meson only with a $q$ in the same cluster, one expects a narrow width for the $J = 1/2^+$ state built with $(uudd, I = 0, S = 1, L = 0)$ and a $\bar{s}$ in P-wave respect to them.
Table 3: Positive parity states allowed to decay into a pseudoscalar meson and a baryon of the 56 of $SU(6)_{FS}$.

In Table 4 we write the $SU(6)_{CS}$ transformation properties and the masses of the negative parity states.

Table 4: Negative parity states.

We expect that the states with $S = 1/2$, which transform as the 70 of $SU(6)_{CS}$ and are allowed to decay into the final state consisting of a pseudoscalar meson and a $1/2^+$ octet state, have a very large width as the $SU(6)_{CS}$ singlet [22] $f_0(680)$ meson, which makes very difficult to identify them. Instead the D-wave resonance, for which the orbital angular momentum changes by two units (from 0 to 2), are expected to have smaller widths, which made possible their identification. In fact the only negative parity $Y = 2$ states, for which evidence has been found, are a $D_03$ and a $D_{15}$, both with a mass near to our prediction. The lightest state has $I = 0$ and $J^P = \frac{1}{2}^+$ and its identification with $\Theta^+(1540)$ implies a value $K_1 = 308$ which is a rather reasonable value. This state is almost the same considered by previous authors [6] [11], where a variational approach has been developed to reach a higher precision for the determination of the spectrum. Apart the fact of predicting a $I = 0$ state as the lowest one with $Y = 2$ the spectrum resulting from eq.(5-7) and Table 2 is in good agreement with the evidence found for the $Y = 2$ exotic states. In fact the two $I = 1$ positive parity states, $P_{11}(1720)$ and $P_{13}(1780)$ found in [15] are predicted properly. As long as negative parity states, one predicts a $D_{03}(1899)$ near to the $D_{03}(1865)$ found in [16] and a $D_{15}(2117)$ to be compared with the evidence for a $D_{05}(2074)$ [17].
and (2150) [16].

To every $Y = 2$ state, independently of its isospin, will correspond a $Y = -1, I = \frac{3}{2}$ state. By neglecting the $SU(3)_F$ violation in the chromo-magnetic interaction, the spin-orbit term and the kinetic energy, one expects a spectrum for these states given by the sum of the spectra of the $Y = 2, I = 0, 1, 2$ states translated by the difference of the constituent masses of the strange and light quarks, 175 MeV.

So the 1862 MeV $\Xi^\pm$ state seen by NA49 [23] could be identified with a member of the 27 of $SU(3)_F$ representation, which contains the $P^{11}_{1720}$.

We may conclude that the chromo-magnetic interaction, successful in reproducing the mass splitting within the ordinary $SU(6)_{FS}$ hadron multiplets [14], seems rather promising to describe the spectrum of pentaquark states.

The symmetry with respect to $SU(6)_{FS}$ would also have important consequences. In fact by eqs.(12,15) and the tensor products:

$$56 \times 35 = 1134 + 700 + 70 + 56$$

we reach the conclusion that the pentaquarks transforming as the exotic $SU(3)_F$ representations cannot decay into the final state consisting of a pseudoscalar or a vector meson and a baryon of the octet $1/2^+$ or of the decuplet $3/2^+$, if their $4q$’s transform as the $105 + 105'$ of $SU(6)_{FS}$ [4], have their couplings to these states proportional with ratios dictated by $SU(6)_{FS}$ symmetry if their $4q$’s transform as the 126 or 210.

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**References**

[1] Photon Nucleus
   LEPS Collaboration, T. Nakano et al., *Phys. Rev. Lett.* **91** (2003) 012002;
   Kaon Nucleus
   DIANA Collaboration, V.V. Barmin et al., *Yad. Fiz.* **66** (2003) 1763 (*Phys. Atom. Nucl.* **66** (2003) 1715);
   Photon Deuteron
   CLAS Collaboration, S. Stepanyan et al., *Phys. Rev. Lett.* **91** (2003) 252001;
   Photon Proton
   SAPHIR Collaboration, J. Barth et al., *Phys. Lett.* **B572** (2003) 127;
   CLAS Collaboration, V. Kurakovsky and S. Stepanyan, 8th International Conference on Interactions of Particle and Nuclear Physics, New York (2003), AIP *Conf. Proc.* **698** (2004) 543;
   Neutrino ($D + Ne$)
A.E. Asratyan, A.G. Dolgenko and M.A. Kubantsev, *Phys. Atom. Nucl.* **67** (2004) 704 and *Yad. Phys.* **67** (2004) 543; HERMES Collaboration, A. Airapetian et al., *Phys. Lett.* **B585** (2004) 13.

[2] D. Diakonov, V. Petrov and M. Polyakov, *Z. Phys.* **A359** (1997) 305; H. Weigel, *Eur. Phys. J.* **A2** (1998) 391.

[3] T.H.R. Skyrme, *Nucl. Phys.* **31** (1962) 556; E. Witten, *Nucl. Phys.* **B223** (1983) 433; G. S. Adkins, C. R. Nappi and E. Witten, *Nucl. Phys.* **B238** (1983) 552; E. Guadagnini, *Nucl. Phys.* **B236** (1984) 35; M. Chemtob, *Nucl. Phys.* **B256** (1985) 600.

[4] F. Stancu and D. O. Riska, *Phys. Lett.* **B575** (2003) 242.

[5] R. L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91** (2003) 232003.

[6] B. K. Jennings and K. Maltman, *Phys. Rev. D69* (2004) 094020.

[7] R. Bijker, M.M. Giannini and E. Santopinto, *Eur. Phys. J.* **A22** (2004) 319.

[8] C. E. Carlson, D. Carone, J. Kwee and V. Nazaryan, *Phys. Lett.* **B573** (2003) 101 and **B579** (2004) 52.

[9] F. Buccella and P. Sorba, *Mod. Phys. Lett.* **A19** (2004) 1547.

[10] F. Close and J.J. Dudek, *Phys. Lett.* **B583** (2004) 278.

[11] J.J. Dudek, hep-ph/0403235.

[12] H. Högaasen and P. Sorba, *Mod. Phys. Lett.* **A19** (2004) 2403.

[13] H. Högaasen and P. Sorba, *Nucl. Phys.* **B145** (1978) 119; M. De Crombrugghe, H. Högaasen and P. Sorba, *Nucl. Phys.* **B156** (1979) 347.

[14] A. De Rújula, H. Georgi and S.L. Glashow, *Phys. Rev. D12* (1975) 147.

[15] R. A. Arndt and L. D. Roper *Phys. Rev. D31* (1985) 2230.

[16] The Particle Data Group *Phys. Lett.* **B111** (1982) 1.

[17] J. S. Hyslop, R. A. Arndt, L. D. Roper and R. L. Workman *Phys. Rev. D46* (1992) 961.

[18] F. Gursey and L.A. Radicati, *Phys. Rev. Lett.* **13** (1964) 173.

[19] F. Buccella, Selection rules for multiquark decays, submitted to *Mod. Phys. Lett. A*, hep-ph/0411244.

[20] C. E. Carlson, D. Carone, J. Kwee and V. Nazaryan, *Phys. Rev. D70* (2004) 037501.

[21] G.C. Rossi and G.Veneziano, *Phys. Lett.* **B70** (1977) 255 and **B597** (2004) 338.
[22] R. L. Jaffe, *Phys. Rev.* **D17** (1978) 1444.

[23] NA49 Collaboration, C. Alt et al., *Phys. Rev. Lett.* **92** (2004) 042003