Making manifest the symmetry enhancement for coincident BPS branes

Sergei V. Ketov

Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany

Abstract. We consider \( g \) coincident M-5-branes on top of each other, in the KK monopole background \( Q \) of multiplicity \( N \). The worldvolume of each M-5-brane is supposed to be given by the local product of the four-dimensional spacetime \( R^{1,3} \) and an elliptic curve. In the coincidence limit, all these curves yield a single (Seiberg-Witten) hyperelliptic curve \( \Sigma_g \), while the gauge symmetry is enhanced to \( U(N) \). We make this gauge symmetry enhancement manifest by considering the hypermultiplet LEEA which is given by the spacetime \( N=2 \) non-linear sigma-model (NLSM) having \( Q \) as the target space. The hyper-Kähler manifold \( Q \) is given by the multicentre Taub-NUT space, which in the coincidence limit amounts to the multiple Eguchi-Hanson (ALE) space \( Q_{mEH} \). The NLSM is most naturally described in terms of the hyper-Kähler coset construction on \( SU(N,N)/U(N) \) in harmonic superspace, by using the auxiliary (in classical theory) \( N=2 \) vector superfields as Lagrange multipliers, with FI terms resolving the \( A_N \) singularity. The Maldacena limit, in which the hypermultiplet LEEA becomes extended to the \( N=4 \) SYM with the gauge group \( U(N) \), arises in quantum field theory due to a dynamical generation of the \( N=2 \) vector and hypermultiplet superfields, when sending the FI terms to zero.

1 Brane technology and KK monopoles

The (Seiberg-Witten-type) exact solution to the \( N=2 \) super-QCD can be identified with the LEEA of the effective (called \( N=2 \) MQCD) field theory defined in the worldvolume of the single M-5-brane, given by the local product of the uncompactified four-dimensional spacetime \( R^{1,3} \) and the hyperelliptic curve \( \Sigma_g \) of genus \( g = N - 1 \): Witten (1997) (see, e.g. Karch, Lüst and Smith (1998), Ketov (1997), Ketov (1998) for a review or an introduction). The hyperelliptic curve \( \Sigma_g \) is supposed to be holomorphically embedded into the hyper-Kähler four-dimensional multiple Taub-NUT space \( Q_{mTN} \) associated with the multiple KK-monopole. The identification of the low-energy effective actions (LEEA) in these two, apparently very different, field theories (the \( N=2 \) super-QCD in the Coulomb branch, on the one side, and the \( N=2 \) MQCD defined in the M-5-brane worldvolume, on the other side) is highly non-trivial, since the former is defined as the leading contribution to the quantum LEEA in a gauge field theory, whereas the latter is determined by the classical M-5-brane dynamics or the \( D = 11 \) supergravity equations of motion, whose BPS solutions preserving some part of supersymmetry are the M-theory branes under consideration.
1.1 Multiple KK monopole

The multiple KK monopole is a non-singular BPS solution to the eleven-dimensional supergravity equations of motion, given by the product of the seven-dimensional (flat) Minkowski spacetime $R^{1,6}$ and the four-dimensional Euclidean multicentre Taub-NUT space $Q_{mTN}$ (Townsend (1995)): 

$$ds_{[11]}^2 = dx^\mu dx^\nu \eta_{\mu\nu} + H(dy)^2 + H^{-1}(d\varrho + C \cdot dy)^2,$$

$$\nabla \times C = \nabla H,$$

$$F_{(4)} \equiv dA_{(3)} = 0,$$

where $\mu = 0, 1, 2, 3, 7, 8, 9$, $y = \{y_i\}$, $i = 4, 5, 6$, the eleventh coordinate $\varrho$ is supposed to be periodic (with the period $2\pi k$ – this is just necessary to avoid conical singularities of the metric), while the harmonic function $H(y)$ is given by

$$H(y) = 1 + \sum_{p=1}^{N} \frac{|k|}{2|y - y_p|}.$$  

(2)

As is clear from eq. (1), two adjacent KK monopoles are connected by a homology 2-sphere having poles at the positions of the two monopoles. Near a singularity of $H$, the KK circle $S^1$ contracts to a point. A holomorphic embedding of the Seiberg-Witten spectral curve $\Sigma$ into the hyper-Kähler manifold $Q_{mTN}$ is the consequence of the BPS condition (Mikhailov (1997))

$$Area_\Sigma = \left| \int_{\Sigma} \Omega_\Sigma \right|,$$

(4)

where $\Omega_\Sigma$ is the pullback of the Kähler $(1, 1)$ form $\Omega$ of $Q_{mTN}$ on $\Sigma$. Any four-dimensional hyper-Kähler manifold, in fact, possesses a holomorphic $(2, 0)$ form $\omega$, which is simply related to the Kähler form $\Omega$ as

$$\Omega^2 = \omega \wedge \bar{\omega}.$$  

(5)

The BPS states in M-theory, whose zero modes appear in the effective field theory defined in the M-5-brane worldvolume, correspond to the minimal area (BPS) M-2-branes ending on the M-5-brane. The spacial topology of an M-2-brane determines the type of the corresponding supermultiplet in the spacetime: a cylinder ($Y$) leads to an $N=2$ vector multiplet, whereas a disc ($D$) gives rise to
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Since the pullback \( \omega_Y \) on \( Y \) is closed (Hennigson and Yi (1998)), there exists a meromorphic differential \( \lambda_{SW} \) satisfying the relations

\[
\omega_Y = d\lambda_{SW} \quad \text{and} \quad Z = \int_Y \omega_Y = \oint_{\partial Y} \lambda_{SW},
\]

where \( Z \) is the central charge and \( \partial Y \in \Sigma \). Hence, \( \lambda_{SW} \) can be identified with the Seiberg-Witten differential, which determines the spacetime \(\mathbb{N}=2\) gauge LEEA (see also Fayyazuddin and Spalinski (1997)).

### 1.2 \(\mathbb{N}=2\) QCD LEEA in Coulomb branch from brane dynamics

The geometrical origin and the physical interpretation of the hyperelliptic curve \( \Sigma_g \), parameterizing the exact Seiberg-Witten solution to the LEEA of \(\mathbb{N}=2\) supersymmetric QCD in the Coulomb branch, becomes transparent when using brane technology after M-theory resolution of UV singularities (Witten (1997)), where \( \Sigma_g \) appears to be the part of the M-5-brane worldvolume in eleven dimensions. The Nambu-Goto (NG) term (proportional to the M-5-brane worldvolume) of the effective M-5-brane action in the low-energy approximation gives rise to the scalar non-linear sigma model (NLSM) having the special geometry after the KK compactification of the six-dimensional NG action on the Seiberg-Witten curve \( \Sigma_g \) down to four spacetime dimensions. This is enough to unambiguously restore the full \(\mathbb{N}=2\) supersymmetric Seiberg-Witten LEEA, by the use of \(\mathbb{N}=2\) supersymmetrization of the special bosonic NLSM, when considering its complex scalars as the leading components of abelian \(\mathbb{N}=2\) vector multiplets in four spacetime dimensions and then deducing the Seiberg-Witten holomorphic potential out of the already derived special Kähler NLSM potential (see Ketov (1998) for a review).

Being applied to a derivation of the hypermultiplet LEEA of \(\mathbb{N}=2\) super-QCD in the Coulomb branch, brane technology suggests to dimensionally reduce the effective action of a D-6-brane (to be described in M-theory by a KK-monopole) down to four spacetime dimensions (Ketov (1998)). In a static gauge for the D-6-brane, the induced metric in the brane worldvolume is given by

\[
g_{\mu\nu} = \eta_{\mu\nu} + G_{mn} \partial_\mu y^m \partial_\nu y^n,
\]

where \(\mu, \nu = 0, 1, 2, 3, 7, 8, 9\), \( m, n = 4, 5, 6, 10 \), and \( G_{mn} \) is the multicentre ETN metric. After expanding the NG-part of the D-6-brane effective action

\[
S_{NG} = \int d^7 \xi \sqrt{-\det(g_{\mu\nu})}
\]

up to the second-order in the spacetime derivatives, and performing plain dimensional reduction down to four dimensions, one arrives at the hyper-Kähler NLSM

\[
S[y] = -\frac{1}{2} \int d^4 x G_{mn}(y) \partial_\mu y^m \partial_\nu y^n, \quad \mu = 0, 1, 2, 3,
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a hypermultiplet (Mikhailov (1997)). Since the pullback \( \omega_Y \) on \( Y \) is closed (Hennigson and Yi (1998)), there exists a meromorphic differential \( \lambda_{SW} \) satisfying the relations

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\]
whose $N=2$ supersymmetrization yields the full hypermultiplet LEEA, in precise agreement with the $N=2$ supersymmetric quantum field theory calculations in harmonic superspace (Ivanov, Ketov and Župnik (1997)).

## 2 Symmetry enhancement: two coincident D-6-branes

As is well known, the isolated singularities of the harmonic function (2) are just the coordinate singularities of the **eleven-dimensional** metric (1), though they are truly singular with respect to the (dimensionally reduced) **ten-dimensional** metric to be associated with the D-6-branes in the type-IIA picture. The physical significance of these ten-dimensional singularities can therefore be understood due to the illegitimate neglect of the KK modes related to the compactification circle $S^1$ in ten dimensions, since the KK particles (also called D-0-branes) become massless near the D-6-brane core (Townsend (1995)). Their inclusion is equivalent to accounting the instanton corrections in the four-dimensional $N=2$ supersymmetric gauge field theory.

When some parallel and similarly oriented D-branes coincide, it is usually accompanied by **symmetry enhancement** (Hull and Townsend (1995), Witten (1996)). Since the brane singularities become non-isolated in the coincidence limit, first, they have to be resolved by considering the branes to be separated by some distance $r$. Then one takes the limit $r \to 0$. In the simplest non-trivial case of two D-6-branes, one can consider the harmonic function

$$H(y) = \lambda + \frac{1}{2} \left\{ \frac{1}{|y - \xi e|} + \frac{1}{|y + \xi e|} \right\}, \quad r = 2\xi ,$$

(10)
describing the double-centered Taub-NUT metric in (1) with a non-vanishing constant potential at infinity, whose both centers are on the line $e$ in sixth direction, $e^2 = 1$. After being substituted into eq. (1), eq. (10) describes two parallel and similarly oriented KK-monopoles in M-theory, which dimensionally reduce to the double D-6-brane configuration in ten dimensions. The homology 2-sphere connecting the KK monopoles contracts to a point in this limit, which gives rise to a curvature singularity of the dimensionally reduced metric in ten dimensions. From the eleven-dimensional viewpoint, M-2-branes can wrap about the 2-sphere connecting the KK monopoles, while the energy of the wrapped M-2-brane is proportional to the area of the sphere (Hull and Townsend (1995)). When the sphere collapses, its area vanishes and, hence, an additional massless vector state appears due to the zero mode of the wrapped M-2-brane. One thus expects the gauge symmetry enhancement from $U(1) \times U(1)$ to $U(2)$ associated with the $A_1$-type singularity (Ooguri and Vafa (1996)). From the ten-dimensional viewpoint, the wrapped M-2-branes are just the (6-6) superstrings stretched between the D-6-branes, so that it is the massless zero modes of these 6-6 superstrings that become massless in the coincidence limit.

In order to make this symmetry enhancement manifest, let’s start from the hypermultiplet low-energy effective action, which is obtained by spacetime $N=2$
supersymmetrization of the bosonic NLSM (9) and whose hyper-Kähler (double Taub-NUT) metric is determined by the harmonic function (10). In terms of this NLSM metric, the symmetry enhancement amounts to the appearance of gauged isometries in the NLSM target space, while the latter can be made manifest in the N=2 harmonic superspace, as we are now going to demonstrate. First, let’s note that the N=2 supersymmetric double Taub-NUT NLSM is known to be equivalent to the one with the mixed (=Eguchi-Hanson-Taub-NUT) metric (Gibbons, Olivier, Ruback and Valent (1988)). The mixed NLSM is described by the following N=2 harmonic superfield action over the analytic subspace:

\[
S_{\text{mixed}}[q,V] = \int_{\text{analytic}} \left\{ q^{A+} D_+^{++} q^+_A + V_3^{++} \left( \frac{1}{2} \xi^{AB} q_{A}^+ q_{B}^+ + \xi^{++} \right) + \frac{1}{4} \lambda (q^{A+} q_{A}^+)^2 \right\},
\]

whose analytic hypermultiplet superfields \( q_{A}^+ \), \( A = 1, 2 \), in the pseudo-real notation \( q_{a}^+ = (\tilde{q}^+, q^+) \), \( a = 1, 2 \), belong to a linear representation 2 of \( SU(1,1) \) whose \( U(1) \) subgroup is gauged by the use of the auxiliary N=2 vector gauge analytic superfield (Lagrange multiplier) \( V_3^{++} \). The parameters \( \lambda \) in eqs. (10) and (11) can be identified, whereas the parameter \( \xi^{++} = \xi_{ij} u_i^+ u_j^+ \) in eq. (11) is simply related to the constant \( \xi \) appearing in eq. (10) as \( |\xi|^2 = -\frac{1}{8}(\xi^{ij})^2 \). The hyper-Kähler NLSM metric, which is deduced out of the superspace action (11) after eliminating all the auxiliary fields in components, yields the double Taub-NUT metric, as can be verified by explicit calculation (Gibbons, Olivier, Ruback and Valent (1988)). This is, in fact, ensured by the manifest \( U(1)_A \times U(1)_{PG} \) symmetry of the superspace action (11), where the first \( U(1)_A \) factor is the unbroken part of the \( SU(2)_A \) automorphisms of the \( N = 2 \) supersymmetry algebra rotating two supercharges, whereas the second \( U(1)_{PG} \) symmetry only acts on the pseudo-real indices \( a = 1, 2 \) and thus implies an abelian isometry of the NLSM metric. In fact, any four-dimensional hyper-Kähler metric having the \( U(1)_{PG} \) isometry is a multicentre Taub-NUT metric (see Gibbons, Olivier, Ruback and Valent (1988) and references therein). The mixed four-dimensional hyper-Kähler metric of the N=2 supersymmetric NLSM (11) clearly interpolates between the Eguchi-Hanson (EH) metric (\( \lambda = 0 \)) and the Taub-NUT (\( \xi = 0 \)), both having the maximal isometry group \( U(2) \). The action of the \( U(2) \) isometry is linear in both limiting cases, while it is even holomorphic in the second case. In the harmonic superspace approach, this symmetry enhancement can be simply understood either as the restoration of the \( SU(2)_A \) automorphism symmetry in the Taub-NUT limit, or as the restoration of the \( SU(2)_{PG} \) symmetry in the Eguchi-Hanson limit.

When using the component results collected in the Appendix B of (Ketov (1998)), it is not difficult to verify that the spacetime vector gauge field belonging to the \( V^{++} \) supermultiplet becomes \textit{dynamical} due to quantum fluctuations of the hypermultiplets \( q_{A}^+ \) (see, e.g. sect. 8.3 of Polyakov (1987) for a similar phenomenon in two spacetime dimensions). After taking into account the \( U(1) \)
gauge symmetry and N=2 supersymmetry, this implies the dynamical generation of the extra physical N = 2 vector gauge superfield, in full agreement with the predictions of brane technology.

3 Symmetry enhancement: N coincident D-6-branes

The D = 11 supergravity approximation to M-theory is only valid for well-separated KK monopoles. When the KK monopoles coincide, their low-energy dynamics is to be approximated by weakly coupled superstrings propagating in the multi-Eguchi-Hanson (ALE) background (Sen (1997)). This background naturally originates from the multi-Taub-NUT space. Indeed, if all the D-6-branes coincide, they can be described in M-theory by sending all the moduli y_p in the harmonic function (2) to zero, so that the additive constant (asymptotic potential), which is equal to one in eq. (2), can be ignored near the core of the D-6-branes on top of each other. The multi-Eguchi-Hanson space thus possesses A_{N−1} simple singularity which implies the enhanced non-abelian gauge symmetry SU(N) in the effective supersymmetric field theory defined in the common worldvolume of the coincident D-6-branes.

The effective gauge field theory is supposed to be defined in the limit where the gravity decouples. The D = 11 supergravity has a 3-form A^{[11]}(3) which is decomposed in the full spacetime given by the product of the D-6-brane worldvolume R^{1,6} and the multi-Taub-NUT space Q_{mTN} as

\[
A^{[11]}(3) = \sum_{p=1}^{N} A^{[7]}_{p(1)} \wedge \omega^{[4]}_{p(2)},
\]

where the 2-forms ω_p in Q_{mTN} have been introduced in subsect. 1.1, whereas A_p are N massless vectors (1-forms) in R^{1,6}. In addition, there are 3N scalar fields associated with the translational zero modes (or moduli) y_p. All these vectors and scalars are the bosonic components of N massless vector supermultiplets in 1 + 6 dimensions, each having 8_B + 8_F on-shell components. The gauge group of the effective field theory (in the case of separated KK monopoles) in the Coulomb branch is therefore given by U(1)^N. Since the intersection matrix of 2-cycles in Q_{mTN} is just given by the Cartan matrix of A_{N−1}, the abelian gauge symmetry U(1)^N is to be enhanced to U(N) (thus defining the non-abelian ‘Coulomb branch’) in the coincidence limit (Sen (1997)). The area of the 2-cycles vanishes in this limit, so that the M-2-branes wrapped around these 2-cycles give rise to the additional massless vectors which are the M-2-brane zero modes. In the type-IIA picture, the 6-6 strings stretched between separated D-6-branes do not contribute to the effective LEEA in the Coulomb branch. However, since the zero modes of 6-6 strings become massless when the brane separation vanishes, they do contribute to the LEEA in the non-abelian Coulomb branch. After plain dimensional reduction from R^{1,6} to R^{1,3}, the effective N=1 super-Yang-Mills theory in 1 + 6 dimensions yields the N=4 super-Yang-Mills theory in 1 + 3 dimensions, which has the same number of on-shell components.
We arrive at the same conclusions on the quantum field theory side of the story in the four-dimensional spacetime, when we consider the corresponding hypermultiplet LEEA given by the gauged N=2 supersymmetric NLSM associated with the coset SU(N, N)/U(N) in harmonic superspace,

\[ S_{\text{non-abelian}}[q, V] = \int_{\text{analytic}} \left\{ \text{tr}_F (\frac{\tilde{q}}{\mathbf{T}} + D^{++} q^+) + \text{tr}_C (V^{++} \cdot \xi^+) \right\}, \quad (13) \]

where \( D^{++} = D^{++}_Z + iV^{++} \), the covariant derivative \( D^{++}_Z \) has central charge \( Z \), and the hypermultiplets \( q \) are supposed to belong to the fundamental (F) representation of \( SU(N, N) \) whose \( U(N) \) subgroup is gauged by the use of the N=2 vector gauge superfield \( V^{++} \) valued in the Lie algebra of \( U(N) \). The Fayet-Iliopoulos (FI) terms are now valued in the Cartan (C) subalgebra of \( U(N) \). These FI terms are apparently necessary in the action, in order to resolve the \( A_N \) singularity which would appear in their absence.

The N=2 hypermultiplet superpropagator in the harmonic superspace \( Z = \{ x^\mu, \theta_i^\alpha, \bar{\theta}_i^\alpha \} \) reads (see, e.g., Ivanov, Ketov and Zupnik (1997))

\[ i \langle q^+ (1) \bar{q}^+ (2) \rangle = -\frac{1}{\Box^2} (D^{++}_Z)^4 (D^{++}_Z)^4 \left\{ \delta^{12} (Z_1 - Z_2) \frac{e_{vz}^{(2)} - e_{vz}^{(1)}}{(u_1^+ u_2^+)^3} \right\}, \quad (14) \]

where we have introduced the so-called ‘bridge’ \( e_{vz} \) associated with the abelian N=2 vector gauge superfield whose N=2 chiral field strength is constant and equal to the central charge \( Z \). It is now straightforward to evaluate the local part of the one-loop effective action \( i \text{Tr} \log D^{++} \) in the LEEA approximation, with the latter being defined by the condition \( |k|^2 \ll |\xi|^2 \) for all external momenta \( k^\mu \). This should yield

\[ S_{N=2 \text{ SYM}}[V] = g_{\text{YM}}^2 \int_{\text{full}} \text{tr} \sum_{n=2}^\infty \frac{(-i)^n}{n} \int du_1 du_2 \cdots du_n \times \frac{V^{++}(Z, u_1)V^{++}(Z, u_2) \cdots V^{++}(Z, u_n)}{(u_1^+ u_2^+ u_3^+ \cdots u_n^+ u_1^+)} \times \frac{V^{++}(Z, u_1)V^{++}(Z, u_2) \cdots V^{++}(Z, u_n)}{(u_1^+ u_2^+ u_3^+ \cdots u_n^+ u_1^+)} \], \quad (15) \]

with the induced gauge coupling \( g_{\text{YM}}^2 \sim N \log \left( 1 + \frac{|\xi|^2}{|Z|^2} \right) \). The action (15) is known as the N=2 supersymmetric Yang-Mills action in harmonic superspace (Zupnik (1987)).

I conclude this section with a few comments.

If the \((1 + 6)\)-dimensional \( N = 1 \) supersymmetric effective field theory were compactified on the circle \( S^1 \), this would give rise to the gauge field theory in \((1 + 5)\) dimensions, whose T-dual is an \((2, 0)\) supersymmetric gauge field theory with \( N \) tensor multiplets. Therefore, in the type-IIB case, we do not get an enhanced gauge symmetry but \( N \) tensor multiplets instead.

A truly different gauge symmetry enhancement pattern appears when \( N \) of D-6-branes come on top of an orientifold six-plane, which leads to the \( SO(2N) \) gauge symmetry (Ooguri and Vafa (1996)). In M-theory, the orientifold six-plane
can be represented by the Atiyah-Hitchin space (Atiyah and Hitchin (1988)) instead of a KK monopole (Sen (1997)). Indeed, far from the origin the Atiyah-Hitchin space has the topology $R^3 \times S^1 / T_4$, i.e. it looks like $Q_{mTN}$ whose points are now supposed to be identified under the action of the discrete symmetry $T_4$ reversing signs of all four coordinates of $Q_{mTN}$. This matches the definition of the orientifold six-plane according to Sen (1997).

4 Large $N$ limit

In order to reproduce the Seiberg-Witten-type solution to $N = 2$ super-QCD from M-theory, merely a single and smooth M-5-brane in a KK-monopole background is needed. The M-5-brane worldvolume is to be compactified on the SW curve $\Sigma_g$ down to $(1+3)$ dimensions i.e. to the worldvolume of a D-3-brane. Taking $N$ M-5-branes (whose worldvolume is given by the local product $R^{1,3} \times \Sigma_1$) and allowing them to coincide in the background of $N$ KK monopoles yields (at a generic point in the moduli space) an $N = 2$ supersymmetric gauge field theory with the non-abelian gauge group $U(N)$ as the LEEA in the common (macroscopically $(1+3)$-dimensional) brane worldvolume. The KK monopoles merge in this process too, which also implies extra massless hypermultiplets in the LEEA, thus increasing their number to $N^2$, i.e. to the adjoint representation of the gauge group.

The coincidence limit corresponds to sending $|\xi| \to 0$ in the spacetime LEEA which is now given by a sum of the gauge-invariant action of a massless hypermultiplet in the adjoint of $U(N)$ and the $N=2$ super-Yang-Mills action (15). This sum,

$$\int_{\text{analytic}} \text{tr}_{\text{ad}} \left( q \, D^{++} q^+ \right) + S_{N=2 \text{ SYM}} \equiv S_{N=4 \text{ SYM}},$$

(16)

is, however, nothing but the N=4 supersymmetric Yang-Mills action in the N=2 harmonic superspace! The induced super-Yang-Mills gauge coupling constant in the limit $|\xi| \to 0$ is given by

$$N g^{2}_{\text{YM}} \sim \frac{|Z|^2}{|\xi|},$$

(17)

so that the non-vanishing central charge is really necessary for a dynamical generation of the N=2 supersymmetric Yang-Mills fields.

Our result is closely related to the recent conjecture of Maldacena (1997). He discussed ‘simple’ M-5-branes, having no Riemann surface in their worldvolumes, in the particular limit (down the ‘throat’) given by the product $AdS_7 \times S^4$ whose both radii are proportional to $N^{1/3}$. At large $N$, the Maldacena LEEA is given by a $(2,0)$ superconformally invariant gauge field theory in six dimensions, which is supposed to be dual to M-theory compactified on $AdS_7 \times S^4$ (Maldacena 1997). Down to four spacetime dimensions, Maldacena (1997) considered $N$ D-3-branes at large $N$ instead, and he argued that the N=4 super-Yang-Mills theory in
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the t’Hooft limit has to be dual to the IIB superstring theory compactified on $AdS_5 \times S^5$. The four-dimensional N=4 super-Yang-Mills theory is supposed to live on the boundary of the $AdS_5$-space, with the correspondence

$$Ng_{YM}^2 \sim (\alpha')^{-2} R_{AdS}^4, \quad g_{YM}^2 \sim g_{string}^2.$$

Note that the t’Hooft limit of large $Ng_{YM}^2$ is equivalent to $|\xi| \rightarrow 0$ in our approach because of eq. (17).

To conclude, the conformal (t’Hooft-Maldacena) LEA limit described by the N=4 super-Yang-Mills theory can be deduced from the hypermultiplet LEA near the singularity, after taking into account the dynamical generation of the massless hypermultiplets in the adjoint representation of the gauge group, and of the non-abelian N=2 vector gauge multiplets at a non-vanishing N=2 central charge. Our approach can be generalized to any simply-laced gauge group.

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