The Solar System’s Post-Main Sequence Escape Boundary

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ABSTRACT
The Sun will eventually lose about half of its current mass nonlinearly over several phases of post-main sequence evolution. This mass loss will cause any surviving orbiting body to increase its semimajor axis and perhaps vary its eccentricity. Here, we use a range of Solar models spanning plausible evolutionary sequences and assume isotropic mass loss to assess the possibility of escape from the Solar System. We find that the critical semimajor axis in the Solar System within which an orbiting body is guaranteed to remain bound to the dying Sun due to perturbations from stellar mass loss alone is \( \approx 10^3 \) AU - \( 10^4 \) AU. The fate of objects near or beyond this critical semimajor axis, such as the Oort Cloud, outer scattered disc and specific bodies such as Sedna, will significantly depend on their locations along their orbits when the Sun turns off of the main sequence. These results are applicable to any exoplanetary system containing a single star with a mass, metallicity and age which are approximately equal to the Sun’s, and suggest that few extrasolar Oort Clouds could survive post-main sequence evolution intact.

Key words: planet-star interactions, planets and satellites: dynamical evolution and stability, stars: evolution, stars: AGB and post-AGB, Oort Cloud, minor planets, asteroids: general

1 INTRODUCTION
The fate of our Solar System is of intrinsic human interest. From the present day, the Sun will continue to exist in a main sequence phase for at least 6 Gyr longer, and then experience post-main sequence phases before ending its life as a white dwarf. The post-main sequence phases will be violent: the Sun’s radius and luminosity will likely vary by several orders of magnitude while the Sun ejects approximately half of its present-day mass through intense winds. The effect on orbiting bodies, including planets, asteroids, comets and dust, may be catastrophic.

The future evolution of the Solar System between now and the end of the Sun’s main sequence will be largely unaffected by the Solar mass loss. The Sun’s current mass loss rate lies in the range \( \sim 10^{-14} M_\odot/\text{yr} \) - \( 10^{-13} M_\odot/\text{yr} \) (e.g. Cohen [2011]; Priti`ev & Priti`ev [2011]). As demonstrated by Veras et al. [2011], all orbiting objects would evolve adiabatically due to this small mass loss. Thus, assuming the mass loss rate remains constant by the end of the main sequence, their eccentricities will remain unchanged, and their semimajor axes would increase by at most \( \approx 0.055\% \). The Sun’s main sequence mass loss rate, has, however, varied over time. Wood et al. [2002] and Zendelas et al. [2010] suggest that this mass loss rate was orders of magnitude greater when the Sun began life on the main sequence, and is a monotonically decreasing function of time. Therefore, 0.055% represents an upper bound. Regardless, this potentially slight orbital expansion, uniformly applied to all orbiting objects, is not predicted to change the dynamics of the Solar System.

Instead, the primary driver of dynamical change in the Solar System during the Sun’s main sequence will arise from the mutual secular perturbations of the orbiting bodies. Kholshevnikov & Kuznetsov [2007] provide a comprehensive review of the investigations up until the year 2007 which have contributed to our understanding of this evolution. These studies describe orbital evolution beyond \( 10^4 \) yrs from now in a primarily qualitative manner because numerical integrations typically cannot retain the orbital phase information of the terrestrial planets over the Sun’s main sequence lifetime. Further, Mercury might suffer a close encounter with Venus (Laskar [2008]), and the orbital motion of the other inner planets is predicted to be chaotic. The outer planets, however,
are predicted to survive and harbor quasi-periodic motion. Since this review, additional studies have primarily focused on the complex evolution of the terrestrial planets; Laskar et al. (2011) helps affirm the intractability of terrestrial orbital phases over long times. Other studies Batygin & Laughlin 2008 and Laskar 2008 corroborate previous results about how the outer Solar System planets (Jupiter, Saturn, Uranus, and Neptune) should remain stable in near-circular (e < 0.1) orbits, even though the orbits themselves may be chaotic (Hayes 2007, 2008; Hayes et al. 2010).

This uncertainty of a planet’s orbital architecture at the beginning of the Sun’s post main sequence phases renders difficult the determination of the fate of individual presently-known bodies. Regardless, several studies have explored the effect of the Sun’s post-main-sequence evolution on the planets. The outcome for the Earth is controversial because investigators differ on whether the Earth’s expanding orbit will “outrun” the Sun’s expanding envelope, and on how to model the resulting tidal interactions (see Schröder & Connon Smith 2008 and references therein). Duncan & Lissauer (1998) consider the stability and orbits of Jupiter, Saturn, Uranus and Neptune with mass scalings and a post main-sequence mass loss prescription from Sackmann et al. (1993). They find that this prescription yields “no large growth in planetary eccentricities”, but mention the possibility of Oort Cloud comets being ejected during periods of brief, rapid mass loss. Complimentary studies of other planetary systems have explored in more detail how Oort Cloud comets’ analogues may become unbound (Alcock et al. 1986; Parriott & Alcock 1998).

Our study is devoted to exploring the prospects for dynamical ejection from the Solar System by using realistic multi-phasic nonlinear mass loss prescriptions from the SSE code (Hurley et al. 2000) and by treating isolated objects beyond the influence of any potentially surviving planets. We do so by using the analytical mass loss techniques in Veras et al. (2011), which yield a critical semimajor axis, a_{crit}, within which an orbiting object is guaranteed to remain bound. In Section 2 we present the solar evolutionary models used, highlighting those which are later applied to N-body simulations that help reaffirm the validity of a_{crit}. In Section 3, we define a_{crit}, plot it as a function of Solar evolution model, and explore the properties of some representative orbits. In Section 4, we explore the implications for the scattered disc and the Oort Cloud. We discuss the results in Section 5 and conclude in Section 6.

2 SOLAR EVOLUTION MODELS

We utilize the SSE code (Hurley et al. 2000), which yields a complete stellar evolutionary track of a star for a given initial stellar mass, metallicity and mass loss prescription. We denote the Sun’s mass as M_\odot(t), and define a Solar Mass \equiv 1 M_\odot to be the current value of the Sun’s mass. We assume that the Sun’s current metallicity is [Fe/H]_\odot \equiv 0.02. We also assume that the Sun has already been evolving on the main sequence for exactly 4.6 Gyr.

\[ \frac{dM_*}{dt} = \frac{4 \times 10^{-13}}{M_\odot} L_* R_* (t) M_\odot \text{ yr}^{-1} \]

where \( L_* \) represents the Solar luminosity and \( \eta \) is an im-
important dimensionless coefficient whose magnitude helps
determine the sequence of stellar evolutionary phases. 
Sackmann et al. (1993) compute evolutionary models for
the Sun, and state that $0.4 < \eta < 0.8$ represent “reason-
able mass-loss rates”. This conforms with the typically-
used value of $\eta = 0.5$ (e.g. Kudritzki & Reimers 1978; 
Hurley et al. 2000; Bonsor & Wyatt 2010). An updated
version of the Reimers Law (Schröder & Cuntz 2003) 
contains two new multiplicative factors:

$$\left( \frac{T_{\text{eff}}}{4000\text{K}} \right) \left[ 1 + \frac{g_\odot}{4300g_\star} \right]^{3.5} (4)$$

where $g_\star$ and $T_{\text{eff}}$ are the star’s surface gravity and effective
temperature, and $g_\odot$ is the current value of the Solar
surface gravity. The product of both these additional factors
is not far from unity. According to Schröder & Cuntz
(2003), who used the RGBs and Horizontal Branches of 2
globular clusters with very different metallicities for cal-
ibration, their effective value of $\eta$ in the original Reimers
law would be about 0.5.

All these considerations dissuade us from selecting
a single value of $\eta_\odot$ and instead lead us to consider
the entire range $0.2 \leq \eta_\odot \leq 0.8$; we sample $\eta_\odot$ values in this
range in increments of 0.01. Representative evolutionary
sequences in this range are displayed in the upper panel
of Fig. 4. In all cases, the Sun undergoes a $\gtrsim 0.5$ Gyr-
long transition towards the foot of the RGB after leaving
the main sequence. The Sun moves on to the RGB before end-
uring a period of core helium burning and subsequent
evolution on the early AGB, as defined by Hurley et al.
(2006). Then, for $\eta_\odot \leq 0.74$, the Sun will enter a ther-
mal pulsing AGB phase before becoming a white dwarf.
This intermediate phase is bypassed for $0.75 \leq \eta_\odot \leq 0.80$.

The different evolutionary sequences demonstrate
that the greatest amount of mass lost and the greatest
mass loss rate may occur during the RGB, Early AGB,
or Thermally Pulsing AGB phases. In all cases the total
percent of the Sun’s current mass which is lost is
$\approx 46.5\%-49.0\%$. The value of $\eta$ determines how much en-
velope mass is lost on the RGB, and hence, the amount
of envelope mass which remains at the start of the AGB.
For $\eta_\odot \lesssim 0.5$, most of this disposable mass is reserved
for the AGB, whereas for $\eta_\odot \gtrsim 0.5$, most of this mass is
lost during the RGB phase. When $\eta$ is large enough
($\eta_\odot \gtrsim 0.75$), then enough of the envelope mass is lost dur-
ing the RGB to cause the Sun to bypass the Thermally
Pulsing AGB phase. In the extreme case of $\eta_\odot \gtrsim 0.94$
(not modeled here), the Sun would bypass Helium burn-
ing and the AGB phases entirely, and would transition to
a pure Helium white dwarf directly from the RGB phase.

The lower panel zooms in and isolates the thermally
pulsing AGB phase for $\eta_\odot = 0.2 - 0.4$, where the mass
loss rate is greatest. Notice that the timescale for this
mass loss is $\approx 0.1 - 0.3$ Myr, orders of magnitude shorter
than the mass loss on the RGB.

3 THE CRITICAL SEMIMAJOR AXIS

3.1 Equations

A star which is expunging mass beyond an orbiting object
will cause the latter’s orbit to expand. If the mass loss is
large enough during one orbital period, then deformation
of the orbit can cause the object to escape the system. We
quantify these claims by defining a dimensionless “mass
loss index” (Veras et al. 2011):

$$\Psi \equiv \frac{\text{mass loss timescale}}{\text{orbital timescale}} = \frac{\alpha}{n\mu}$$

$$= \frac{1}{2\pi} \left( \frac{\alpha}{1M_\odot/\mu} \right) \left( \frac{a}{1\text{AU}} \right)^{\frac{7}{2}} \left( \frac{\mu}{1M_\odot} \right)^{-\frac{1}{2}}, (5)$$

where $\alpha$ represents the Solar mass loss rate, $a$ and $n$
represent the orbiting object’s semimajor axis and mean
motion, and $\mu = M_\star + M_\odot$, where $M_\star$ represents
the mass of the object. We can assume $\mu \approx M_\star$, because
every known object which orbits the Sun has a mass that is
less than 0.1% of $1M_\odot$. However, observations cannot
yet exclude the possibility of the existence of a Jovian
planet orbiting at a distance of at least several thou-
sand AU (Iorio 2011). Therefore, “undiscovered” plan-
ets beyond $\sim 10^3$ AU might be massive enough to make
a nonnegligible contribution to $\mu$. Even if such a planet
formed within the orbit of Neptune, gravitational scatter-
ing could have placed the planet in its present location
(Veras et al. 2009). Alternatively, the planet could have
previously been free-floating and subsequently captured
by the Solar System.

Dynamical evolution critically depends on $\Psi$. When
$\Psi \ll 1$, the orbiting object evolves “adiabatically” such
that $a$ increases but its eccentricity, $e$, remains constant.
Examples of objects with $\Psi \ll 1$ include all 8 of the
inner and outer planets, both currently (see Section 1)
and during any plausible post-main sequence Solar evol-
ution phase. Alternatively, when $\Psi \gg 1$, in the “run-
away” regime, $a$ continues to increase but now $e$ may
achieve any value from zero to unity. Veras et al. (2011)
demonstrate that the bifurcation point beyond which the
motion is no longer adiabatic is not sharp and occurs at
$\Psi_{\text{bif}} \sim 0.1 - 1.0$. In fact, if $\Psi$ is instead defined as
$\alpha T/\mu$, where $T$ is the orbital period of the orbiting
object, then this definition differs from Eq. (5) by a factor
of $1/(2\pi) \approx 0.16$. Therefore, both definitions effectively
bound the approximate location of the bifurcation point
when $\Psi = 1$, and we consider both possibilities in our
analysis.

The Sun loses mass nonlinearly, as showcased in
Fig. 4. One can apply the analytical linear results from
Veras et al. (2011) to a nonlinear mass loss profile by
partitioning the profile into $N$ smaller approximately lin-
ear stages, with mass loss rates of $\alpha_1, \alpha_2, ... \alpha_i, ... \alpha_N$. If all
these segments occur in the adiabatic regime, then we can
relate the semimajor axis and eccentricity at the start of
the first segment to these same elements at the end of
segment $i$ with:

$$a_i = a_0 \left( \frac{\mu_i}{\mu_0} \right)^{-1} \equiv a_0 \beta_i^{-1}, (6)$$
\[ e_i = e_0, \quad (7) \]

Because the orbiting object’s eccentricity does not change in this regime, the body will remain bound in the absence of additional perturbations.

All orbiting bodies will be expanding their semimajor axis at the same rate. The mass loss index at the end of segment \( i \) will be:

\[ \Psi_i = \kappa \alpha_i \left( \frac{\alpha_0 \mu_0}{\mu_i^3} \right)^{\frac{\gamma}{2}} \quad (8) \]

where \( \kappa = 1 \) or \( 2\pi \), depending on whether \( \Psi \) is defined with respect to the mean motion (Eq. 5) or the orbital period. \( \kappa \) could also be treated generally as a tunable parameter which may better pinpoint the location of the bifurcation point in specific cases. The first instance at which the orbiting object may escape is when the evolution becomes non-adiabatic and hence the eccentricity can vary. This occurs when \( \Psi_i \approx 1 \). Therefore, the minimum percent of \( \mu_0 \) which must be retained in order to guarantee that the orbiting object will remain bound in a given segment \( i \) is:

\[ \beta_{\text{crit}, i} \equiv \left( \frac{\mu_0}{\mu_i} \right)_{\text{crit}} \approx \kappa^{\frac{1}{2}} \left( \frac{\alpha_0}{1 \text{AU}} \right)^{\frac{1}{2}} \left( \frac{\alpha_i}{1 \text{AU}} \right)^{\frac{1}{2}} \left( \frac{\mu_0}{1 \text{M}_\odot} \right)^{-\frac{1}{2}}. \quad (9) \]

Hence, the critical \( \alpha_0 \), termed \( a_{\text{crit}, i} \), at which an orbiting object will remain in the adiabatic regime, and thus bound to a 1 \( \text{M}_\odot \) star, is:

\[ a_{\text{crit}, i} \approx \kappa^{-\frac{1}{2}} \min \left[ \beta_i^2 \left( \frac{\alpha_i}{1 \text{M}_\odot/\text{yr}} \right)^{-\frac{1}{2}} \right], \quad (10) \]

where the minimum is taken over all segments \( i \).

### 3.2 Mass Loss Profile Partition

The accuracy of \( a_{\text{crit}} \) depends on how finely the Solar mass loss profile is partitioned into stages. Given the focused nature of our study, we simply adopt the most accurate option by treating each SSE timestep as a segment. These segments also include transitions between evolutionary phases, for which a mass loss and timescale is computed by SSE in every case. We choose output parameters which yield 3333 outputs total per simulation for the RGB, core helium burning, early AGB and thermally pulsing AGB phases, and at least 20 outputs for each of the other phases of evolution. Our choices are motivated by the observation that the segment which satisfies Eq. (10) always arises from the RGB or an AGB phase. The minimum SSE timesteps are \( \approx 10^3 \text{ yr} \) long, which is shorter than the orbital period of the objects we consider here. Equation (10) is valid for any relation between SSE timesteps and orbital period because, in the adiabatic regime, the semimajor axis and eccentricity evolution is independent of the location of the object along its orbit. We apply this output parametrization uniformly across the \( \eta_0 \) values sampled.

### 3.3 N-body Simulations

We supplement the computation of \( a_{\text{crit}} \) (from the stellar evolution profiles alone) with N-body simulations of orbiting objects. These simulations help affirm that objects with \( a < a_{\text{crit}} \) do remain bound, and provide a rough characterization of the motion of objects which are susceptible to escape. We select the 7 Solar evolutionary pathways corresponding to \( \eta_0 = [0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8] \) for these simulations, and use a second-order mixed-variable symplectic integrator from a modified version of the Mercury integration package. All orbiting objects are treated as test particles for computational ease and so that they could be integrated simultaneously. Because \( \mu \) is dominated by \( M_* \), an isolated giant planet would react to stellar mass loss nearly equivalently to an isolated test particle on the same orbit. After each Mercury timestep, we linearly interpolate the value of the Sun’s mass from the SSE output. This approximation is sufficient for numerical integrations which don’t feature close encounters.

Simulating orbital evolution throughout the remaining lifetime of the Solar System with numerical integrations and without utilizing secular approximations is computationally unfeasible and, for this project, unnecessary. Our focus is on the behavior of objects when they are most susceptible to escape. Therefore, having already identified when the greatest mass loss occurs, we simulate the entire thermally pulsating AGB phase for the \( \eta_0 = [0.2, 0.3, 0.4] \) simulations (see Fig. 1). The duration of these simulations is between \( 2.7 \times 10^4 \text{ yr} \) and \( 5.7 \times 10^5 \text{ yr} \). For \( \eta_0 = [0.5, 0.6, 0.7, 0.8] \), we simulate Solar mass evolution from when \( M_* = 0.98000 \text{M}_\odot \), which occurs during the RGB, until the beginning of the white dwarf phase (see Fig. 1). For these simulations, modeling the phases after the RGB may be important because despite their overall low mass loss rates, the AGB phases could feature short bursts of high mass loss. Also, \( \beta \) is lower for the phases beyond the RGB. The duration of these simulations is between \( 1.64 \times 10^5 \text{ yr} \) and \( 1.88 \times 10^7 \text{ yr} \).

We choose the initial conditions for the orbiting objects as follows. The orbital evolution of an orbiting object which is susceptible to escape is highly dependent on \( e \) and its true anomaly, \( f \) (see Veras et al. [2011]). For linear mass loss and \( \Psi \gg 1 \), the object’s eccentricity is likely to initially increase. However, the eccentricity must initially decrease if \( f_{\text{crit}} \leq f \leq (360^\circ - f_{\text{crit}}) \), where \( f_{\text{crit}} = 180^\circ - (1/2) \cos^{-1}(7/9) \approx 160^\circ \), and we wish to sufficiently sample this behavior as well. These considerations motivate us to choose \( f_0 \) in increments of \( 30^\circ \) from \( 0^\circ - 330^\circ \), supplemented by \( f_0 = [160^\circ, 170^\circ, 190^\circ, 200^\circ] \). In order to sample a wide range of eccentricities, we choose \( e_0 = [0.0, 0.2, 0.4, 0.6, 0.8, 0.95] \). Because application of Eq. (10) demonstrates that \( 10^3 \text{AU} \leq a_{\text{crit}} \leq 10^7 \text{AU} \), we choose \( a_0/\text{AU} = [10^2, 10^3, 3 \times 10^3, 5 \times 10^3, 10^4] \). The smallest of these values yields a period of 414 years for \( M_* = 0.5 \text{M}_\odot \). Therefore, we choose a timestep less than \( 1/50 \)th of this value, \( 1 \times 10^4 \text{ days} \), for all our numerical simulations. Because this timestep might not sample the pericenter sufficiently for highly eccentric orbits, we perform an identical set of simulations with a timestep

\[ \frac{\mu_0}{1 \text{AU}} \]

\[ 1 \text{AU} \]

\[ 1 \text{M}_\odot \]
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\[ a_0/(10^3 \text{ AU}) \]

\[ \eta_\odot \]

**Figure 2.** The Solar System’s critical semimajor axis, as a function of the Solar Reimers Mass Loss coefficient, \( \eta_\odot \). The curve is derived from Eq. (10). In the shaded pink region above the curve, orbiting bodies may escape the dying Sun on hyperbolic orbits. In the white region below the curve, bodies are guaranteed to remain bound on elliptical orbits. The colour of the line segments indicates the Solar phase in which orbiting bodies are most susceptible to escape: red = RGB, green = early AGB and blue = thermally pulsing AGB. Checkmarks and crosses provide information about the results of N-body integrations (see Section 3.3). Checkmarks indicate that every orbiting object in every N-body simulation remained bound. Each cross represents a different phase (red = RGB, green = early AGB, blue = thermally pulsing AGB) are represented on the curve, demonstrating the sensitivity of \( a_\text{crit} \) to the stellar model chosen. When the RGB evolution dominates the orbiting object’s motion, the critical semimajor axis is higher than in the AGB-dominated cases because RGB evolution takes place earlier in the Sun’s post-main sequence life, when the Sun has more mass. Any objects in the pink shaded region above the curve might escape; those in the white region below the curve are guaranteed to remain bound.

### 3.4 Visualizing the Critical Semimajor Axis

We plot \( a_\text{crit} \) from Eq. (10) assuming \( \kappa = 1 \) as a function of \( \eta_\odot \) in Fig. 2. The curve illustrates that for \( \eta_\odot \leq 0.5 \), \( a_\text{crit} \approx 10^3 \text{ AU} \), and for \( \eta_\odot = 0.7 \), \( a_\text{crit} \approx 10^4 \text{ AU} \). Other values of \( a_\text{crit} \) lie in-between these two extremes. The curve is coloured according to the stellar evolutionary phase at which an orbiting object is most susceptible to ejection, which depends on a combination of the mass loss rate and the mass remaining in the Sun. Three different phases (red = RGB, green = early AGB, blue = thermally pulsing AGB) are represented on the curve, demonstrating the sensitivity of \( a_\text{crit} \) to the stellar model chosen. When the RGB evolution dominates the orbiting object’s motion, the critical semimajor axis is higher than in the AGB-dominated cases because RGB evolution takes place earlier in the Sun’s post-main sequence life, when the Sun has more mass. Any objects in the pink shaded region above the curve might escape; those in the white region below the curve are guaranteed to remain bound.

The overlaid symbols indicate the results of the N-body simulations. Checkmarks indicate that every orbiting object from every \( \{ e_0, f_0 \} \) bin sampled remained bound (\( e < 1 \)) for the duration of the simulation. Crosses indicate that at least one orbiting object become unbound, either through the orbit becoming hyperbolic (\( e > 1 \)) or through the body achieving \( a \geq 10^6 \text{ AU} \). The number of crosses indicate the number of \( e_0 \) bins in which at least one orbiting object became unbound. The N-body simulations help affirm that all of the objects below the curve remained bound. This also holds true if \( \kappa = 2\pi \). In that case, the curve would keep the same form but be lowered by a factor of \((2\pi)^{2/3} \approx 3.4\). This value of \( \kappa \) would provide a more conservative estimate for the Solar System’s critical semimajor axis, and would still ensure that any known surviving planets and belts would remain bound. Objects with high values of \( e_0 \) are usually, but not always, prone to escape; additional cross symbols...
suggest that objects escaped at progressively lower values of \( \eta \). The number of crosses displayed do not show uniform patterns primarily because of the complex dependencies of the evolution in the adiabatic/runaway transitional regime on \( a, \epsilon \) and \( f \) (see Section 4.2). Also, the parameter space sampled is limited and the N-body simulations do not model the entire pre-white dwarf lifetime of the Sun.

Not shown in Fig. 2 are the results from the \( a_0 = 10^2 \) AU and \( a_0 = 10^5 \) AU simulations. In the former, every orbiting object remains bound (corresponding to a row of checkmarks). In the latter, at least one orbiting object escapes for each value of \( \eta_0 \) integrated (corresponding to a row of different numbers of crosses).

### 3.5 Properties of the Critical Stage

Here we take a closer look at the critical stage which yields \( a_{\text{crit}} \) through Figs. 3–6. Figures 3–4 demonstrate that the critical stage occurs nearly at the end of the Sun’s post-main sequence life, when the Sun will have already lost the majority of its disposable mass. The solid black curve in Fig. 3 is a poor proxy of the \( a_{\text{crit}} \) curve from Fig. 2. Both Figs. 3 and 4 suggest that \( \eta_0 = 0.7 \) represents a bifurcation point in Solar evolution, which is approximately where the thermally pulsing AGB stage becomes transitory. The critical time from the start of the Sun’s main sequence will be \( \approx 12.465 \pm 0.005 \) Gyr for \( \eta_0 \leq 0.7 \), and \( 12.326 \pm 0.0002 \) Gyr for \( \eta_0 \geq 0.7 \). These are remarkably well-constrained values relative to the curves in Fig. 2.

Now consider \( \alpha_i \) at the critical stage. Figure 5 shows that this mass loss rate varies between \( \approx 2 \times 10^{-7} M_\odot/\text{yr} \) and \( \approx 5 \times 10^{-6} M_\odot/\text{yr} \). Also, this rate closely mirrors the maximum rate achieved throughout the simulation, given by the dashed, brown curve. Therefore, by itself, \( \max(\alpha_i) \) is an excellent indicator of the stability prospects of an orbiting body. Also, this trend suggests that short violent outbursts from planet-hosting stars can jeopardize the survival of orbiting bodies. The Solar System is subject to the same danger even though the Sun is not expected to experience a violent outburst with \( \alpha > 5 \times 10^{-6} M_\odot/\text{yr} \) from these models. If we are underestimating the variability of the Sun after it turns off of the main sequence, then orbiting bodies can be in greater danger of escape than we currently expect.

If such variability is on a timescale which is shorter than the timesteps used in these simulations, then \( a_{\text{crit}} \) is being overestimated. Figure 6 illustrates the timestep from which the mass loss rate at the critical stage was computed. In all cases, this timestep is between approximately 980 yr and 5860 yr. The Sun’s current rotational period is less than one month; during a single rotation, prominences may erupt and sunspots may appear, among other phenomena. The Sun’s (unknown) post-main sequence variability will likely be more violent, but on a similar timescale. If so, then short bursts of intense mass loss could be important but are not modeled by the comparatively long SSE timesteps.

### 3.6 Orbital Properties

#### 3.6.1 Overview

Here we analyze some representative orbits from the N-body simulations. Because the timescale for mass loss from the bottom panel of Fig. 1 is orders of magnitude shorter than the top panel, the difference in orbital properties is pronounced. Specifically, for \( a_0 > 10^3 \) AU and \( \eta_0 < 0.5 \), all eccentricity variations will often occur during a single orbit. Alternatively, for \( \eta_0 \geq 0.5 \), these variations occur over many orbits. We consider one case from the former category and two cases from the latter in detail. Additionally, we consider the eccentricity evolution for one case in the runaway regime, with \( a_0 = 10^5 \) AU and \( \eta_0 = 0.2 \), and demonstrate here that objects near their apocenters do remain bound in this case, as predicted by the linear theory.
loss ceased at this point, then the body would maintain this nearly circular orbit.

Mass loss has a dramatic effect on the evolution of $f$. A system no longer is adiabatic when $df/dt = 0$ \citep{Veras2011}. At this point, $f$ stops circulating and begins to librate. The lower left plot illustrates how the mass loss slows $df/dt$ for each orbiting object. Although none of the orbiting objects achieve $df/dt = 0$, the $f_0 = 90^\circ$ object (black curves) comes closest. This is also the object whose $a$ and $e$ values are increased by the greatest amount. Alternatively, the highest ending value of $df/dt$ is associated with the $f_0 = 240^\circ$ object, whose eccentricity decreases the most. In the linear mass loss approximation, for $\Psi \gg 1$, bodies closest to pericenter are most susceptible to escape. This plot helps demonstrate how no such correlation exists if the evolving regime is not runaway.

Although the bottom-right plot might suggest that some planets appear to be leaving this system, all bodies will remain bound, orbiting the Solar white dwarf with their values of $a$ and $e$ attained at $t = 12461.9$ Myr, assuming no additional perturbations.

3.6.2 Example Bounded Orbits for $\eta_\odot < 0.5$

We consider here short orbits ($a_0 = 3 \times 10^3$ AU) which are significantly disrupted by mass loss that takes place entirely within an orbital period ($\eta_\odot = 0.3$, see Fig. 4) and remain ellipses. Figure 7 illustrates the orbital evolution of 16 objects, all with different $f_0$ values, which all begin the N-body simulation with small-eccentricity ($e_0 = 0.2$) orbits. These objects may be treated as low-eccentricity, inner Oort Cloud bodies. Despite the small $e_0$, the semimajor axes all increase by a factor of $\approx 1.5 - 2.2$, and the final values of $e$ range from $\approx 0.1 - 0.6$. If this mass was lost adiabatically, then the semimajor axis would increase by a factor of $\approx 1.5$, the lower bound of the actual increase in $a$. Further, the $f_0 = 240^\circ$ object (brown curves) becomes nearly circularized at $t \approx 12461.67$ Myr; if mass

3.6.3 Example Escape Orbits for $\eta_\odot \geq 0.5$

Here we consider longer orbits ($a_0 = 1 \times 10^4$ AU) which are significantly disrupted by mass loss that takes place over several orbital periods ($\eta_\odot = 0.5, 0.6$, see Fig. 4) and which can cause an orbiting object to escape. We consider highly eccentric orbits ($e_0 = 0.8$) and compare two types of escape in Fig. 5: the left panels showcase the $f_0 = 0^\circ$ (red) object achieving an orbit with $e = 0.99872$ and $a > 10^6$ AU (two orders of magnitude higher than $a_0$), while the right panels showcase the $f_0 = 160^\circ$ (pink) object achieving a hyperbolic orbit. The only initial parameter changed in the two plot columns was the value of $\eta_\odot$ (0.5 on the left, and 0.6 on the right).

As evidenced by the plots in the upper two rows, the first strong burst of mass loss at $t \approx 12.22$ Gyr provides a stronger orbital kick for $\eta_\odot = 0.6$ Solar evolution. However, as indicated by Fig. 4, the second, stronger burst of mass loss at $t \approx 12.36$ Gyr yields a lower value of $a_{crit}$ for $\eta_\odot = 0.5$. Regardless, this second strong burst of mass loss for both $\eta_\odot$ values are roughly comparable, as the final range of $a$ and $e$ values are similar.

The difference is enough, however, to cause escape in two different ways, one with an object which was at pericenter at the beginning of the simulation, and one with an object which was near-apocenter. This figure provides further confirmation of the difficulty with correlating $f_0$ with $f$ for objects evolving in the transitional regime between the adiabatic and runaway regimes. Both $f$ vs $t$ plots demonstrate the expected change of behavior during the second strong mass loss event. However, they fail to distinguish between both types of escape, and neither technically reach $df/dt = 0$. However, for the left and right panels, respectively, $df/dt = 0.068^\circ$/Myr and $df/dt = 0.078^\circ$/Myr at $t = 12461.9$ Myr.

The bottom panels show the spatial orbits of the objects; the left panel illustrates all 16, and the right panel features only 4 (for added clarity), including the escap-
ing object. Due to the number of simulation outputs, lines are not connected between the outputted points. These plots explicitly illustrate how two bursts of mass loss from the Sun cause two distinct orbit expansions. Additionally, the apparent “noise” about the equilibrium orbits attained before the second mass loss event demonstrate the gradual expansion of the orbit due to minor mass loss events. Comparing the colours and positions of both plots shows almost no correlation with the identical initial conditions. The escaping pink body on the right panel never reaches a distance to the Sun which is closer than its initial pericenter value (2000 AU); the two dots for which \( x > 1.5 \times 10^4 \) AU indicate the path along which the body escapes.

3.6.4 Example Orbits for Rampant Escape

We now consider objects in considerable danger of escape. These can reside at typical Oort Cloud distances, \( a_0 \sim 10^5 \) AU. The strong mass loss from the \( \eta_\odot = 0.2 \) evolutionary track (with maximum \( \alpha \approx 7.5 \times 10^{-9} M_\odot/\text{yr} \); see Fig. 5) yields \( \Psi \gtrsim 30 \gg 1 \) and places these objects in the runaway regime. Here we determine if, unlike in the adiabatic regime, we can link the fate of an object with \( f_0 \) amidst nonlinear mass loss.

Figure 9 confirms that we can. The results agree well with the linear theory. At the highest eccentricities sampled (\( e_0 = 0.95 \)), the objects initially at apocenter (orange; \( f_0 = 180^\circ \)) remain bound to the dying Sun, and, further, suffer the greatest eccentricity decrease. Additionally, 4 other objects nearest to the apocenter of their orbits also remain bound. As the initial eccentricity of the objects is decreased (from 0.95 in the top panel to 0.6 in the bottom panel), more bodies which are increasingly further away from their initial apocenter are allowed to survive. Also, as the initial eccentricity is decreased, more bodies which are progressively further away from their initial apocenter suffer a net eccentricity decrease. Note finally that the results are not symmetric about \( f_0 = 180^\circ \); objects approaching apocenter are slightly more protected than objects leaving apocenter.

4 APPLICATION TO THE SCATTERED DISC, OORT CLOUD AND OTHER TRANS-NEPTUNIAN OBJECTS

4.1 Overview

The Solar System architecture beyond the Kuiper Belt is often divided into two distinct regions: the scattered disc and the Oort Cloud. The former is possibly an ancient, eroding remnant of Solar System formation, whereas the latter is thought to dynamically interact with the local Galactic environment and is continuously depleted and replenished. Both regions may supply each other with mass throughout the lifetime of the Solar System.

However, the definitions of these regions are not standardized. The boundary of the scattered disc is empirical and refers to objects with perihelia beyond Neptune \((q > 30 \text{ AU})\) and semimajor axes beyond the 2:1 resonance with Neptune \((a > 50 \text{ AU})\) (Luu et al. 1997; Gomes et al. 2005). Other papers have set a variety of numerical bounds based on their empirical readings of the semimajor axis - eccentricity phase space for objects beyond Neptune. In an early study, Duncan & Levison (1997) distinguished the Kuiper Belt from a scattered disc of objects with \( q \approx 32 – 48 \text{ AU} \), \( e \lesssim 0.8 \) and \( i \lesssim 50^\circ \). Alternatively, sometimes the scattered disc is considered to be a subclass of the Kuiper Belt (e.g. Gladman & Chirikov 2000) and contains objects which satisfy the simpler constraint \( q \gtrsim 33 \text{ AU} \) (Tiscareno & Malhotra 2003; Volk & Malhotra 2008) and \( e \gtrsim 0.34 \) (Volk & Malhotra 2008), whereas Kaib & Quinn (2008) give \( q \gtrsim 40 \text{ AU} \) and Sheppard (2010) claim \( q \sim 25 – 35 \text{ AU} \) as the criterion for inclusion. Further, Levison et al. (2004) demonstrate how the scattered disc population has changed with time. For example, at an age of \( 10^6 \) yrs, most scattered disc objects had \( q < 10 \text{ AU} \). More generally, Levison et al. (2008) define the currently-observed scattered disc to be composed of objects with perihelia close enough to Neptune’s orbit to become unstable over the (presumably main-sequence) age of the Solar System. However, some scattered disc objects may not be primordial, and instead represent a small transient population supplied by the Kuiper Belt. Also, because scattered disc objects interact with Neptune, some are depleted and diffuse into the Oort cloud.

If the scattered disc is insufficiently replenished from the Oort Cloud and Kuiper Belt, then the scattered disc may be depleted relative to its current level by the end of the Sun’s main sequence. Whatever disc objects which might remain will continue to interact with Neptune as their orbits move outward. If the orbits expand adiabatically, the scattered disc object will continue to interact with Neptune as during the main sequence. If the orbits expand non-adiabatically, then the scattered disc objects’ eccentricity will change, and may no longer interact with Neptune. However, the semimajor axis of a scattered disc object which is high enough to cause non-adiabatic evolution is generally too high (\( a > 10^5 \text{ AU} \)) to remain in that population. Therefore, the scattered disc population will be subject to adiabatic mass loss evolution, and continue to interact by diffusing chaotically in semimajor axis (Yabushita 1980), likely helping to repopulate the Oort Cloud.

Beyond the scattered disc resides the Oort Cloud, which was originally postulated to contain \( \sim 10^{13} \) comets of observable size with \( 5 \times 10^4 \text{ AU} < a < 1.5 \times 10^5 \text{ AU} \), \( 0 < e < 1 \) and isotropically-distributed inclinations (Oort 1950). Some of these comets occasionally enter the inner Solar System and can achieve perihelia of a few AU. These intruders are thought to have been jostled inward by forces external to the Solar System. Many subsequent investigations since this seminal work have refined this basic picture for the outermost region of the Solar System.

The external perturbations may arise from passing stars and/or the local Galactic tide. Heisler & Tremaine (1986) demonstrated that tides likely dominate injection events into the inner Solar System, while passing stars continuously randomize the Oort Cloud comet orbital parameter distribution (Dybczynski 2002). Morris & Malter (1986) demonstrate how Galactic forces vary the orbital angular momentum of an Oort Cloud
comet during a single orbit, which, hence, may be neither an ellipse nor closed. Therefore, if a typical comet’s bound orbit significantly varies from Keplerian motion due to external perturbations, the analytics in Veras et al. (2011) cannot be applied to these objects without incorporating models of those perturbations.

However, if we do treat bound Oort Cloud cometary orbits as approximately elliptical, then we can estimate the effect of stellar mass loss on the cloud. One major refinement of Oort’s original model is the division of the Oort Cloud into an “inner” and “outer” population at $a = 2 \times 10^4$ AU (Hills 1981). This bifurcation point, which determines if a comet is observable only when it is part of a shower, is still widely utilized in modern simulations and computations. Figure 2 indicates that this bifurcation value is a factor of 2–20 times as high as the Solar System’s post main-sequence escape boundary. Hence, comets from both populations will be susceptible to escape due to post main-sequence mass loss. Moreover, in a seminal study, Duncan et al. (1987) approximated the inner edge of the Oort Cloud to be $\approx 3000$ AU, whereas Brass (2008) use this same value to divide the Oort Cloud further into a third “innermost” region. High-eccentricity bodies with semimajor axes approximately equal to this potential second bifurcation point in Oort Cloud structure may be susceptible to escape during the Sun’s post-main-sequence evolution if $\eta \lesssim 0.6$ (see Fig. 2).

Many modern Oort Cloud investigations focus on formation and/or the orbital properties of observed comets instead of topics more relevant to this study, such as the eccentricity distribution of the vast majority of the remaining (currently unobservable) comets and the future evolution of the Oort Cloud over the next several Gyr. However, some authors (Emel’yanenko et al. 2007, Brass et al. 2011) present plots which illustrate that the eccentricity distribution of Oort Cloud comets do span from 0 to 1, with the minimum eccentricity typically increasing as a decreases. Therefore, the orbits studied in Sections 3.6.2 and 3.6.3 represent plausible families of
Figure 8. The orbital evolution of objects at $a_0 = 1 \times 10^4$ AU with $e_0 = 0.8$ for $\eta_0 = 0.5$ (left panels) and $\eta_0 = 0.6$ (right panels) from 12.2 Gyr. In the left panels, the $f_0 = 0^\circ$ (red) body escapes when, after achieving $e = 0.99872$, its semimajor axis exceeds $10^6$ AU. Alternatively, in the right panels, the $f_0 = 160^\circ$ (pink) body escapes by achieving a hyperbolic orbit ($e > 1$, $a < 0$). The $f$ vs $t$ panels demonstrate that the true anomaly cannot distinguish between these two types of escape in this case, and that bodies may escape just before achieving $df(t)/dt = 0$. The pericentric vs. near-apocentric $f_0$ values of the escaping bodies, as well as a comparison of the orbit locations in the bottom two plots, indicate that the phase information is easily lost on the approach to $df(t)/dt = 0$. The bottom plots also demonstrate how the orbital architecture changes over several orbital periods, in contrast to Fig. 7. Note that the ejected planet in the bottom-right plot never comes within its initial pericenter value (2000 AU) of the Sun.
"inner" Oort Cloud orbits, and the orbits presented in Section 3.6.4 represent a plausible family of "outer" Oort Cloud orbits.

Multiple subpopulations of the Oort Cloud help categorize recent discoveries. Sedna (Brown et al. 2004), the most distant Solar System object yet observed (at 90.32 AU), highlights the failure of the traditional scattered disc and Oort Cloud to partition the Solar System beyond the Kuiper Belt. With \( q \approx 76 \text{ AU and } a \approx 480 \text{ AU} \), Sedna does not fit inside either population, and demonstrates that the orbital parameter phase space of trans-Neptunian objects is larger than previously thought. If Sedna maintains its orbit until the Sun turns off of the main sequence, then the object is guaranteed to remain bound (Fig. 4) during post-main sequence mass loss, regardless of its position along its orbit. However, other objects a few hundred AU more distant might not survive. Two other objects which defy easy classification are 2006 SQ272 (Kaib et al. 2009), with \( q \approx 24 \text{ AU and } a \approx 796 \text{ AU} \), and 2000 OO57 (Veillet et al. 2000; Millis et al. 2002), with \( q \approx 21 \text{ AU and } a \approx 552 \text{ AU} \). The perihelions and aphelions of both objects imply that they can be classified as either scattered disc objects or Oort Cloud comets. Regardless, the short stability timescale (\( \sim 200 \text{ Myr} \)) predicted for 2006 SQ272 implies that it will not survive the Sun’s main sequence phase. 2000 OO57 will remain bound (Fig. 4) to the post-main sequence Sun if the object survives the Sun’s main sequence.

### 4.2 Depletion Characteristics

Although determining the fraction of the Oort Cloud which is depleted likely requires detailed modeling and extensive N-body simulations, we can provide some estimates here. We perform an additional 3 sets of simulations, for \( \eta_\odot = 0.2, 0.5 \) and 0.8, and focus on the region beyond \( 10^4 \text{ AU} \), where most Oort Cloud objects are thought to reside. These values of \( \eta_\odot \) represent, in a sense, one nominal and two extreme Solar evolutionary tracks. For each \( \eta_\odot \), we integrate with exactly the same conditions as described in Section 3.3, but now sample 72 uniformly-spaced values of \( f_0 \) (for 0.5\(^\circ\) resolution), 9 uniformly-spaced values of \( e_0 \) in the range [0.1–0.9], and the following 6 values of \( a_0/\text{AU} = 1.0 \times 10^4, 2.5 \times 10^4, 5.0 \times 10^4, 7.5 \times 10^4, 1.0 \times 10^5, 1.25 \times 10^5 \).

The results of these simulations are presented in Figs. 10-11. Fig. 10 is a cartoon which illustrates the relation between instability and initial conditions, and Fig. 11 plots the fraction of the Oort Cloud lost depending on the Solar evolutionary model adopted. The cartoon indicates unstable systems with red crosses, as a function of \( a_0, e_0 \) and \( f_0 \). The connection of these variables with instability helps describe the character of the mass loss. For example, the initial conditions which lead to instability for the \( \eta_\odot = 0.2 \) simulations (upper panel of Fig. 10) are patterned and focused towards the pericenter for higher \( a_0 \) and \( e_0 \) values, mimicking runaway regime behaviour and implying large mass loss rates (see Fig. 5). In contrast, the initial conditions which lead to instability for the \( \eta_\odot = 0.5 \) simulations (middle panel of Fig. 10) exhibit little order, implying that the mass loss rates are not large enough to reside fully in the runaway regime, but are large enough to allow for escape. The \( \eta_\odot = 0.8 \) simulations (lower panel of Fig. 10) provide a third alternative, showing a greater sensitivity to \( a_0 \) than to either \( e_0 \) or \( f_0 \). The bottom two panels indicate the difficulty in predicting the prospects for escape amidst nonlinear mass loss without appealing to N-body simulations.
There are other features of Fig. [10] worth noting. Not shown are objects with $e_0 \leq 0.2$, because all remain stable. This implies that in no case does the Sun lose enough mass for a long enough period of time to eject distant circular bodies. In the upper panel, at least one object for every other $(a_0, e_0)$ pair becomes unstable. Also, in this panel, asymmetric signatures about the pericenter indicate that of two bodies at equal angular separations from pericenter, the body approaching pericenter is more likely to escape. This asymmetry does not arise in the analytic linear theory [Veras et al. 2011], but may be guessed based on physical grounds, as a planet initially approaching pericenter is more likely to be ejected than a planet initially approaching apocenter. This tendency is also suggested in Fig. [9] with the top two curves in each panel. In the bottom-rightmost ellipse in the upper panel of Fig. [10] we superimposed two brown diamonds at the locations of $f_{\text{crit}}$ and $360^\circ - f_{\text{crit}}$ in order to determine how closely the escape boundaries mimic the analytic boundaries which ensure that a body’s eccentricity must initially decrease.

The middle and bottom panels of Fig. [10] indicate that the fraction of bodies which escape does not necessarily scale with $a_0$; the bottom panel displays a dramatic double-peaked red cross-dominated structure at $a_0 = 5 \times 10^4$ AU and $a_0 = 1 \times 10^5$ AU. One can glean understanding of those panels by noting three points: First, the orbital periods of objects in both panels is roughly equal to the mass loss timescales achieved for those Solar evolutionary tracks, placing the objects in a transitional regime. Second, the rate of change of eccentricity, which is proportional to $(e + \cos f)$ (see [Veras et al. 2011]), can assume either positive or negative values and increase or decrease depending on the value of $a_0$ in this transitional regime. This result is not exclusive to nonlinear mass loss prescriptions; one can show that even for a linear mass loss rate, the inflection point of the eccentricity with respect to time contains multiple extrema in the transitional regime. Third, the moment when the Sun becomes a white dwarf and stops losing mass freezes an object’s orbit, even if the orbit was approaching $e \to 1$. Therefore, if the $\eta_c = 0.8$ post-main-sequence mass loss lasted longer, the distribution of unstable systems in the bottom panel would no longer be bimodal.

Fig. [11] simply counts the unstable systems from Fig. [10] and both figures can be compared directly side-by-side. In the top panel of Fig. [11] over 80% of the highest eccentricity objects escape, and over half of all objects with $e_0 = 0.5$ escape. The escape fractions become nearly flat for $a \geq 5 \times 10^4$ AU. In contrast, in the middle panel, all $e_0$ curves except one maintain escape fractions which are $< 20\%$ for all values of $a_0$ that were sampled. In the bottom panel, the escape fractions spike at both $a_0 = 5 \times 10^4$ AU and $a_0 = 1 \times 10^5$ AU for the five highest eccentricity curves. The results indicate how sensitive Oort Cloud depletion is to both the Oort Cloud model adopted and the Solar evolution model adopted.

5 DISCUSSION

5.1 Planetary Perturbations

Perturbations from surviving planets which reside within a few tens of AU of the Sun will have no effect on the orbits described here, unless an external force is evoked, or one or more of the planets is ejected from the system and achieves a close encounter on its way out. As shown in [Veras et al. 2011], the pericenter of an orbiting isolated body experiencing isotropic mass loss cannot decrease. This holds true in both the adiabatic and runaway regimes, for any mass loss prescription. Therefore, mass loss cannot lower the pericenter of an Oort Cloud comet which was originally beyond Neptune’s orbit.

How the known surviving planets evolve themselves under their mutual gravitational attraction amidst post-main-sequence mass loss is less clear. With no mass loss, the giant planets are stable until the Sun turns off of the main sequence [Laskar 2008; Kholshevnikov & Kuznetsov 2011] indicate that Jupiter and Saturn need to be about 20 times more massive than their current values in order to suffer close encounters. However, [Duncan & Lissauer 1998] demonstrate that with post-main-sequence mass loss, giant planet evolution may not be quiescent. In one linear mass-loss approximation, they find that Uranus and Neptune’s orbits will eventually cross. More generally, if the planets remain in orbits under $\sim 100$ AU and beyond the Sun’s expanding envelope, they will evolve in the adiabatic regime, expanding their near-circular orbits all at the same rate. They will remain bound according to Figure [2]. However, as multiple planets expand their orbits and maintain their relative separations, the critical separations at which the planets will remain stable will vary [Debes & Sigurdsson 2002]. Also, as alluded to in Section 1, Jupiter and Saturn are predicted to largely maintain their current orbits until the Sun turns off of the main sequence. If this is true, then they will still be close to the 5:2 mean motion commensurability when significant stellar mass loss commences. How near-resonance behavior is linked to a potentially varying critical separation is not yet clear, and suggests that much is yet to be discovered about post-main-sequence planetary systems with multiple planets.

5.2 Beyond the Solar White Dwarf

After the Sun has become a white dwarf, surviving object orbits will retain the orbital parameters achieved at the moment the Sun stopped losing mass. These objects will forever remain on these orbits unless subjected to additional perturbations. For the surviving planets, the primary perturbations will come from one another, which could lead to instabilities on Gyr, or longer, timescales (see, e.g., Fig. 29 of [Chatterjee et al. 2008]). For objects further afield, the dominant perturbations will be external, and come from, for example, Galactic tides (e.g. Tremaine 1993) and passing stars (e.g. Veras et al.)

1 which scales as $M_* f(t)^{-2/3}$ [Tremaine 1993] and might harbor an entirely different functional form from the present-day prescription due to the collision of the Milky Way and
Figure 10. Initial condition stability snapshot for $\eta_\odot = 0.2$ (upper panel), $\eta_\odot = 0.5$ (middle panel) and $\eta_\odot = 0.8$ (lower panel). Each scaled ellipse is for a given triplet ($\eta_\odot$, $a_0$, $e_0$) and graphically represent the initial $f_0$ of unstable systems (large red crosses). The orange dot is a representation of the Sun, and the brown diamonds (at $\eta_\odot = 0.2$ and the highest values of $a_0$ and $e_0$) represent the values $f_{\text{crit}} = 180^\circ - (1/2) \cos^{-1} (7/9) \approx 160^\circ$ and $360^\circ - f_{\text{crit}} \approx 200^\circ$, which, for runaway mass loss, bound the region in which the eccentricity must initially decrease. For $\eta_\odot = 0.2$, strong “runaway” mass loss causes objects which are closest to pericenter to be lost more easily; the effect is enhanced for higher $a_0$ and $e_0$. For $\eta_\odot = 0.5$, the transitional regime between adiabatic and runaway dominates, and objects appear to go unstable at a variety of nonconsecutive values of $f_0$. For $\eta_\odot = 0.8$, the instability appears to be highly sensitive to $a_0$.

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5.3 Exoplanet Analogue

Although stellar evolutionary tracks are highly dependent on the zero-age main sequence mass and metallicity of a star, generally the maximum value of $a$ increases as the stellar mass increases. Therefore, for stars of approximately Solar metallicity which are more massive than the Sun, $a_{\text{crit}}$ is expected to be lower than for the Solar System, and orbiting exobodies would be more prone to escape. The methodology in this work can be applied to any extrasolar system for which a stellar evolutionary track can be estimated. Given recent detections of objects which may be orbiting parent stars at $a > 10^3$ AU, this type of analysis may become increasingly relevant.

6 CONCLUSION

The Solar System’s critical semimajor axis within which bodies are guaranteed to remain bound to the dying Sun during isotropic post-main sequence mass loss and in the absence of additional perturbations is $a_{\text{crit}}$. A more precise value can be obtained for a given Solar evolution model; this range encompasses many realistic evolutionary tracks. The most important Solar evolutionary phase for dynamical ejection may be the RGB ($\eta \gtrsim 0.7$), the thermally pulsing AGB ($\eta \lesssim 0.5$) or the early AGB ($0.5 \lesssim \eta \lesssim 0.7$). Objects with $a > a_{\text{crit}}$, such as Oort Cloud comets, and those with $a \gtrsim a_{\text{crit}}$, such as some scattered disc objects, may escape depending on their orbital architectures and position along their orbits. Quantifying the fraction of the population of these objects which escape would require detailed modeling of their (unknown) orbital parameters at the start of the Sun’s post-main sequence lifetime.

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