Reasoning About Liquids via Closed-Loop Simulation

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Abstract—Simulators are a powerful tool for reasoning about a robot’s interactions with its environment. However, when simulations diverge from reality, that reasoning becomes less useful. In this paper, we evaluate closing the loop between simulation and reality by using observations of real liquids to correct errors when tracking the liquid’s state in a simulator. Our results show that closed-loop simulation is an effective way to prevent large divergence between the simulated and real liquid states, and additionally that this method can enable reasoning about liquids that would otherwise be infeasible.

I. INTRODUCTION

Liquids are a ubiquitous part of human environments, appearing in many common household tasks. Recent work in robotics has begun to investigate ways in which robots can reason about and manipulate liquids. Many have shown success in tasks involving liquids using even relatively weak models of the physics underlying liquid flow [52, 25, 22]. Other work has shown how using a physics-based model to reason about liquids can enable richer understanding of actions that involve liquids [11].

Physics-based models are powerful tools for enabling robots to reason about their environments. Work on rigid-body actions using physics-based models has enabled robots to perform a wide-variety of tasks [30, 20, 5]. However, without feedback from real sensory data, over time they will diverge from their corresponding real environments. For rigid-body models, there has been some work connecting the modeled state of the system to the state of the real environment [16, 27]. There has also been work showing it is possible to connect the two for some deformable objects [28]. For liquids, though, while there has been work in real environments enabling task-specific reasoning, and work in simulators using physics-based models to enable reasoning on a wide variety of tasks, there has not yet been any work connecting the physics simulation with real-time perception.

In this paper, we investigate using physics-based models of liquids to track liquids in real environments. Closing the loop between simulation and reality is important for transferring any type of robotic reasoning from the simulation to the real robot, but it is especially so for liquids. Unlike rigid bodies, modeling liquids is much higher dimensional and lacks the same kind of inherent structure, and thus small perturbations can quickly lead to large deviations. Figure 1 shows a comparison between real liquid (Figure 1b) and the result of performing a carefully tuned liquid simulation with the same setup (Figure 1c). It is clear that without any feedback, the liquid simulator and the real liquid have significant differences. With feedback from real sensory data, however, the simulator can track the real liquid with much more accuracy, as shown in Figure 1d. Here we look at ways to incorporate sensory feedback into physics-based liquid models and show how it can be used to solve diverse robotics tasks. For the purposes of this paper, we assume that our robot has access to 3D mesh models of the objects in its environment and their poses, a task solved in prior work [26, 7], and that it can differentiate between liquid and everything else in its raw observations, which has also been shown to be possible by prior work [24].

In this paper we first discuss other work in robotics related to ours, followed by a detailed description of the liquid simulator we use as the base for our closed-loop physics-based model. Next we describe two different methods for using the observations of real liquid to correct errors in the base liquid simulator. After that we describe three experiments we performed using this methodology and their results. We end the paper with a discussion of the implications of our method and future work. Our results here show that using closed-loop liquid simulations, robots can perform reasoning tasks about liquids that would otherwise be much more challenging.

II. RELATED WORK

Liquid simulation and fluid mechanics are well researched in the literature [11]. They are commonly used to model fluid flow in areas such as mechanical and aerospace engineering, and to model liquid surfaces in computer graphics [2, 6, 17]. While there hasn’t been any work yet, to the authors’ knowledge, to combine real world liquid observations with liquid simulations, there has been some work combining machine learning with simulation. Work by Ladický et al. [12] used regression forests to learn the update rules for particles in a particle-based liquid simulator. There has also been some work combining real world observations with deformable object simulation. Schulman et al. [28], by applying forces...
in the simulator in the direction of the gradient of the error between the observation and simulation, were able to model cloth based on real observations. Our warp field method, described in section IV-B applies a similar concept to liquids.

In robotics, there has been some work using simulators to reason about liquids. Kunze and Beetz [10, 11] employed a simulator to reason about a robot’s actions as it attempted to make pancakes, which involved reasoning about the liquid batter. Yamaguchi and Atkeson [34, 33] used a simulator to reason about pouring. However, in [10, 11, 34, 33] they do not simulate liquids; rather they use small, rigid spheres as a stand-in for liquid. Schenck and Fox [24] used a finite element method liquid simulator to train a deep network on the tasks of detecting and tracking liquids. In this paper, similar to [24], we directly simulate liquids.

Yamaguchi and Atkeson did, although, follow up their simulated work with pouring on a real robot [35]. Several others have also performed the pouring task using a real robot [25, 24, 22]. However, most of those simply dump the entire contents from the source container into the target, bypassing the need to reason in any detail about the liquid dynamics. Only [25, 22] actually attempted to pour specific amounts of liquid, requiring at least a partial understanding of liquids on the robot’s part.

There has been some limited work on perceiving liquids from real data. In a separate paper, Yamaguchi and Atkeson [32] used optical flow combined with stereo vision to perceive liquid flows in 3D. Work by Griffith et al. [8] used liquids to assist a robot in understanding containers from sensory data. In both [24, 25], they use deep networks to both perceive liquids in color images and to reason about their behavior. In this paper, we use a thermal camera and heated water to allow our robot to perceive liquids and then reason about them.

III. OPEN-LOOP LIQUID SIMULATOR

The physics-based model used by our robot to reason about liquids is implemented from a liquid simulator. The simulator acts as the underlying model, while the observations are used to modify that model. The state of the simulator tracks the liquid over time, simulating it forward while the observations prevent it from deviating from the real liquid dynamics. In this section, we describe how the liquid simulator computes the dynamics of the liquid, and in the following section we describe how the observation modifies the liquid state.

To simulate the trajectory of liquid in a scene, the liquid is represented as a set of particles and the Navier-Stokes equations [11] are applied to compute the forces on each particle. The Navier-Stokes equations require certain physical properties of liquid (e.g., pressure, density) to be defined for all points in \( \mathbb{R}^3 \). This is implemented using Smoothed Particle Hydrodynamics (SPH) [31], which computes the value of a property at a specific location in space as the weighted average of the neighboring particles. This is in contrast to finite element liquid simulations [21], which divide the scene into a voxel grid and store the values of the given property at each location in the grid. One major disadvantage of the finite element simulations is that as the size of the environment grows, the requirements of the voxel grid in both memory and run time grows as \( O(n^3) \), making them inefficient for large environments with sparsely located liquids. This is the case for the simulations in this paper, and so we chose to use SPH, which is better suited to this type of task. The implementation used in this paper is based off the implementation from the particle simulation library Fluidix [15]. The rest of this section briefly describes that implementation.

Smoothed Particle Hydrodynamics is essentially a method for representing a continuous vector field of a physical property in space via a discrete set of particles. It is based around the following equation for evaluating that field at any arbitrary point in space, where \( A \) is the physical property in question:

\[
A(r) = \sum_j m_j A_j W(|r - r_j|, h)
\]

where \( m_j \) is the mass of particle \( j \), \( A_j \) is the value stored in particle \( j \), \( \rho_j \) is the density of particle \( j \), \( W \) is a kernel function that weights the contribution of each particle by its distance, and \( h \) is the cutoff distance for \( W \). In SPH, the mass \( m_j \) of each particle is constant, however the density \( \rho_j \) is not, and must be computed via the SPH equation above. That is, the physical value we want to compute \( A \) is set to be the density \( \rho \), which results in \( \rho \) appearing on the right side of the equation twice. The issue of recurrence (requiring the density to be known in order to compute the density) is handled by the density in the denominator canceling out:

\[
\rho(r) = \sum_j m_j \frac{\rho_j}{\rho_j} W(|r - r_j|, h) = \sum_j m_j W(|r - r_j|, h).
\]

To implement a liquid simulation using SPH, each particle must store 6 physical properties: 3D position (without orientation), velocity, force, mass, density, and pressure. As stated above, the mass for each particle is constant, however the density \( \rho \) appearing on the right side of the equation twice. The issue of recurrence (requiring the density to be known in order to compute the density) is handled by the density in the denominator canceling out:

\[
\rho(r) = \sum_j m_j \frac{\rho_j}{\rho_j} W(|r - r_j|, h) = \sum_j m_j W(|r - r_j|, h).
\]

The force due to viscosity is

\[
f_i^{\text{viscous}} = \sum_j -\mu \frac{m_j}{\rho_j} \left( \frac{v_i}{\rho_i^2} + \frac{v_j}{\rho_j^2} \right) \nabla^2 W(r_i - r_j),
\]

where \( \mu \) is the viscosity constant of the liquid (recall that \( v_i \) is the velocity of particle \( i \)) to compute the surface tension acting on each particle, we must first compute the normal of
each particle:

\[ n_i = \sum_j \frac{m_j}{\rho_j} \nabla W(r_i - r_j). \]

Intuitively, the normal \( n_i \) for any particle in the center away from the surface of the liquid will have approximately equal contributions from all directions, resulting in the magnitude of \( n_i \) being small. Conversely, for particles near the surface, \( n_i \) will have a large contribution from particles in the direction of the interior of the liquid and very little contribution in the direction of the surface, resulting in an \( n_i \) with a large magnitude in the direction away from the surface. The force due to surface tension is computed as

\[ f_{\text{tension}}^i = -\sigma \frac{n_i}{|n_i|} \sum_j \frac{m_j}{\rho_j} \nabla W(r_i - r_j) \]

where \( \sigma \) is the liquid’s tension constant. To prevent numerical instability when \( |n_i| \) is small, we only compute the tension force when the normal magnitude is greater than a threshold, i.e., the particle is near the surface.

To simulate the flow of liquid in a scene during an interaction, we assume the simulator is given 3D models of the objects that interact with the liquid as well as their 6D poses over the course of the interaction (obtained for example from an object tracking system such as [31]). We initialize the liquid particles in the scene (details on this in section IV) and simulate the particles forward at each timestep as the simulator tracks the objects' poses.

Our liquid simulator is implemented using the particle simulation library Fluidix [15], which efficiently computes particle interactions on the GPU. We performed a best-first grid search over the space of parameters (e.g., the viscosity constant) to find the set of values that best match the real liquid dynamics. For each set of parameters in the grid, we used the evaluation criteria described in section IV to score them with respect to the data we collected (described in section V-B), and selected the parameters that best fit the real data. In doing so, we attempted to make our open-loop simulation as close as possible to the real liquid dynamics. For all the simulations in this paper, we use between 2,000 and 8,000 particles.\(^1\)

For a detailed derivation of smoothed particle hydrodynamics, please refer to [31].

### IV. Closed-Loop Liquid Simulators

While liquid simulators model fluid dynamics based on physical properties, they often don’t model every possible force that could affect the liquid; and even the best simulators still have some error relative to real liquids. Over time, even small errors can lead to a large divergence between real and simulated liquid behavior. While this may not be a problem in some cases (e.g., in 3D animation it may only be necessary for a liquid to appear realistic but have no need to model actual liquid dynamics), in robotics if we wish to use liquid simulation as the robot’s internal model of its environment, it must match the real liquid dynamics as closely as possible.

One potential method for alleviating this issue is to improve the fidelity of the simulator. However, this method has many pitfalls. It requires knowledge of every possible force that could affect the trajectory of the liquid, not only the standard forces such as pressure and viscosity, but also forces for example due to vacuum suction (as in the case of a plunger), which may require modeling additional elements of the environment. It can also be very brittle, as every property of every object in the environment must be known ahead of time (e.g., the friction constants over the entire surface of every object). Finally, and most importantly, even if the simulator is as accurate as possible, it will still have some small amount of error, and a purely open-loop system has no way to correct that error, and thus it will accumulate into a large error over time.

We propose two methods for dealing with accumulated error when tracking real liquid dynamics using a simulator. Both methods involve closing the loop, that is, utilizing observations of real liquid dynamics in order to better approximate them in the simulation. The first, inspired by standard Bayes filters in robotics, is a MAP filter, which uses the observation to “correct” simulation errors relative to the observation. The second, based on modeling physical forces in the simulator, applies a warp field based on the observation which pulls particles toward observed liquid. We describe these two methods in the following sections.

#### A. MAP Filter Simulator

We use a maximum a posteriori (MAP) filter as one of our closed-loop simulation methods. We model each particle as its own filter, with its own set of hypotheses, and use the MAP hypothesis at each timestep to compute the dynamics.

Let \( P_t \) be a set of liquid particles at time \( t \), \( O_t \) be the objects and their corresponding 6D poses, and \( I_t \) be the observation. We define \( S(P_{t-1}, O_t) = P_t \) to be the function as described in section III that computes the state of the liquid particles at timestep \( t \) given the previous state of the liquid particles. Each particle \( P_t^i \in P_t \) has an associated filter \( X_t^i \) with hypotheses \( x_t^i, x_t^{i,2}, ..., x_t^{i,N} \), where each represent the full state of a liquid particle.

At the beginning of each timestep \( t \), \( N = 1 \), and so the state of the single hypothesis \( x_t^{i,1} \) is copied into the liquid particle \( P_{t-1}^i \). Next, all the liquid particles are propagated forward in time by one step via \( S \) using the objects and their poses \( O_t \). Since \( S \) is not probabilistic, the dynamics sampling must be done separately. Given a liquid particle \( P_t^i \), we sample one hypothesis \( x_t^{i,N} \) for each location in a grid centered at that liquid particle’s position. The grid has dimension \( 3 \times 3 \times 3 \) and the size of each grid cell was set at a small, fixed constant (we use 5mm in this paper). This results in 27 hypotheses sampled for each liquid particle.

Next we must compute \( P(x_t^{i,N}|I_t, P_t) \), the probability of each hypothesis given the observation and the set of liquid particles. This can be challenging, as, unlike in simulation, it is usually not possible to fully observe the state of a real environment. Additionally, often the environment can only be observed in 2D via a camera image. This is especially true for many liquids, which do not appear on modern depth cameras (we use water for the experiments in this paper, which is not visible to the depth sensors on depth cameras). Thus for this paper, the observation \( I_t \) is an image of a real environment, where each pixel is labeled as liquid or not-liquid.

\(^1\)This range was selected for efficiency reasons.
To compute the observation probability for each hypothesis, we first apply Bayes rule
\[
P(x_t^{i,n} | I_t, P_t) \propto P(I_t | x_t^{i,n}, P_t) P(x_t^{i,n} | P_t).
\]
The prior \( P(x_t^{i,n} | P_t) \) is taken into account by the dynamics function \( S \), thus since we uniformly sample around the result of \( S \), we assume a uniform prior, with one exception. Not all sampled filter particles are physically feasible (e.g., a particle may have been sampled inside a 3D object mesh), so we discard those which are infeasible and set the distribution to be uniform over the remaining hypotheses. Thus, for all feasible hypotheses,
\[
P(x_t^{i,n} | I_t, P_t) \propto P(I_t | x_t^{i,n}, P_t).
\]

When computing \( P(I_t | x_t^{i,n}, P_t) \), what we really want to know is which \( x_t^{i,n} \in X_t^i \) maximizes this probability. However, the interaction between \( I_t \), \( x_t^{i,n} \), and \( P_t \) is highly complex and difficult to compute analytically. Instead, we approximate this value with an activation function \( \Psi \), which monotonically increases with \( P(I_t | x_t^{i,n}, P_t) \) and thus preserves the argmax property over \( X_t^i \). We define \( \Psi \) to be
\[
\Psi(I_t, x_t^{i,n}, P_t) = \sum_{j \in \text{liquid}(I_t)} \frac{W(|x_t^{i,n} - j_t|, h)}{\Phi(j_t, P_t) + 1}
\]
where \( \text{liquid}(I_t) \) is the set of all liquid pixels in \( I_t \), \( W \) is a kernel function, \( x_t^{i,n} \) is \( x_t^{i,n} \) projected onto the image plane, \( h \) is the limiting radius for \( W \), and \( \Phi \) returns the coverage of \( j_t \) by \( P_t \). Intuitively, this function sums the number of liquid pixels around \( x_t^{i,n} \), weighted by their distance to \( x_t^{i,n} \) divided by their coverage, i.e., how well explained that pixel is by \( P_t \). Thus, the more liquid pixels around a hypotheses, the higher its \( \Psi \) value, and the less the pixels are covered by the liquid particles, the higher the \( \Psi \) value.

For \( W \) we use a squared exponential kernel with a length scale of \( \frac{1}{10} \), and we set the limiting radius to 100. Intuitively, this means that the unit length under this kernel is 33 pixels with a limiting radius of 100 pixels. We compute \( \Phi(j_t, P_t) \) for a pixel \( p_t \) by projecting the liquid particles onto the image plane. This is done by first placing a small, fixed radius sphere at each liquid particle, then projecting that particle onto the image, taking into account occlusions due to objects. We then sum the number of spheres that projected onto \( j_t \) and set that value as \( \Phi(j_t, P_t) \).

Once this is computed for all hypotheses in \( X_t^i \), we select the hypothesis with the maximum value and discard the rest. On the next timestep, we replace the corresponding liquid particle with the state from this hypothesis. We also adjust the velocity to match the change in position from \( P_{t-1} \) so as to preserve the correct momentum.

### B. Warp Field Simulator

The second method we use for closing the loop in the simulator is a warp field. Here the observation applies a force in the simulator that attempts to make the liquid particles better match the observed liquid. Each observation point is essentially a magnet in the scene, pulling nearby particles towards it. However, if all observation points pulled with the same amount of force, then particles would tend to clump around a subset of the observation points, leaving other observation points with no nearby particles as the forces from the former cancel out those from the latter. Thus the amount of force an observation point applies to nearby particles must vary with the number of nearby particles. When taken together, all the observation points create a field of forces that warp the particles to better match the real liquid observations.

Once again let \( P_t \) be a set of liquid particles in a scene at time \( t \), \( O_t \) be the objects and their corresponding 6D poses, \( I_t \) be the observation, and \( S \) be the function that computes the dynamics of the particles for a single timestep. Like in the previous method, the observation \( I_t \) is in the form of an image, with each pixel labeled as liquid or not-liquid.

This presents an interesting challenge. The observation points (liquid pixels in \( I \)) are defined in the 2D image plane, however the liquid particles in the simulator are in 3D space. In order to compute the warp field, we first project the liquid particles into the image plane, compute the forces applied by the field, then project those forces back into full 3D space. While the conversion from 3D to 2D and back may introduce some inaccuracies in some cases, such as when the camera is at an extreme angle relative to the motion of the liquid, for our purposes this is an accurate enough approximation.

Let the notation \( P_t^i \) denote the particle \( P_t^i \) projected into the image plane. The force due to the observation warp field is computed as
\[
\vec{f}^{i,\text{obs}}_t = \sum_{j \in \text{liquid}(I_t)} \lambda \frac{u^{ij}_t}{\Phi(j_t, P_t) + 1} W(|P_t^i - j_t|, h)
\]
where \( \lambda \) is the warp constant, \( u^{ij}_t \) is a unit vector pointing from particle \( P_t^i \) to liquid pixel \( j_t \), \( \Phi(j_t, P_t) \) is the coverage of pixel \( j_t \), and \( W \) is a kernel function (with \( h \) as the range limit). We use a squared exponential function for \( W \) with a length scale of \( \frac{1}{10} \) and a range limit of 100 pixels in all our experiments.

The coverage of a pixel \( \Phi(j_t, P_t) \) is a measure of how many liquid particles “cover” it, that is, how many liquid particles are nearby. The force applied to each particle by each liquid pixel is divided by that pixel’s coverage, thus as more particles cover an observed liquid pixels, it pulls particles to it with less force. Conversely, pixels that have lower coverage pull particles to them with more force. To compute the coverage \( \Phi(j_t, P_t) \) for a given pixel \( j_t \), we place a sphere of fixed radius at each particle location, then project those spheres onto the image plane, taking into account occlusions due to objects in the environment. We define the coverage as the number of spheres that projected onto pixel \( j_t \).

The force \( \vec{f}^{i,\text{obs}}_t \) is then projected back into 3D space. Finally, we apply the SPH equation to smooth the forces across the particles
\[
\vec{f}^{i,\text{obs}}_t = \sum_{j \in \text{liquid}(I_t)} \frac{m_j \vec{f}^{i,\text{obs}}_t}{r_{ij}^3} W(|r_i - r_j|, h).
\]
The resulting force \( \vec{f}^{i,\text{obs}}_t \) is then added to the other forces described in section III.

### V. Experimental Setup

#### A. Robotics & Sensors

The robot used in the experiments in this paper was an upper-torso robot with two 7-DOF arms, each with an electric...
parallel gripper. A table was fixed in front of the robot. To sense its environment, the robot used its Asus Xtion Pro RGBD camera, which recorded both color and depth images at 640 × 480 resolution at 30 Hz during each interaction, and its Infrared Cameras Inc. 8640P Thermal Imaging camera, which recorded thermographic images at 640 × 512 resolution at 30 Hz during each interaction. The thermal camera was used in combination with heated water to acquire the ground truth pixel labelings. The cameras were locked in fixed relative positions and placed just below the robot’s head at approximately chest height.

B. Data Collection

1) Pouring: We collected 16 pouring interactions. We varied the source container (cup, Figure 2a, or bottle, Figure 2b) and its initial fill amount (empty, 30%, 60%, or 90% full). Before each pouring interaction, a bowl (the pan, Figure 2d) was placed on the table in front of the robot. Next, the robot held the ends of the pipe junction (Figure 2c) with its grippers over the bowls and recorded from its RGBD and thermal cameras while 1 liter of water was poured in the top opening. Each leg of the pipe junction could be fully blocked or partially blocked, i.e., the flow going to that leg could be partially restricted or entirely stopped. A diagram of the pipe junction and how the blockages affected flow is shown in Figure 3. The blockage can be placed in either leg, for a total of 5 possible configurations.

2) Pipe Junction: We collected 5 pipe junctions interactions. Before each of the pipe junction interactions, two bowls were placed in the robot’s gripper, filled with water, and the gripper moved over the bowl. The robot then proceeded to rotate its wrist along a fixed trajectory such that the opening of the container tilted down towards the bowl and water poured out. During each pouring interaction, the robot recorded from its RGBD and thermal cameras as well as its joint poses. We collected two trials for each combination of source container and fill amount.

C. Data Processing

Before we can use our simulators to track the flow of liquid in the interactions described in the previous section, we must first perform some post-processing on the data. First, both the open-loop and closed-loop simulators require the object poses to be known over the course of the interaction. We utilize an object tracking method based on point cloud data to do this. Second, both closed-loop simulators require an image with pixels labels for the liquid. We use a thermal camera to acquire this labeling. In this paper we perform these steps offline, however both are capable of operating in real-time in online situations.

1) Object Tracking: We use the software program DART [26] (Dense Articulated Real-Time Tracking) to track the objects in each interaction. DART uses depth images to track objects over time. We initialize the pose of the bowls by using the Point Cloud Library’s built-in tabletop segmentation algorithm to find the point cluster on the table, and then set their initial pose to the centroid. We initialize the containers by computing the robot’s forward kinematics to find the gripper pose. Once initialized, DART returns a pose for each container at each point in time over the interaction.

2) Liquid Labeling: For each pouring and pipe junction interaction, the water was heated to a temperature significantly above the surrounding environment but below its boiling point. The interactions were recorded with a thermal camera, and the thermal image was simply thresholded to locate the liquid
pixels. Figure 4b shows an example thermal image recorded during a pipe junction interaction, and Figure 4c shows its corresponding thresholded values.

In addition to generating labels from the thermal image, it must also be calibrated to the depth image (the object poses generated by DART, and thus the entire simulator, operate in the depth camera frame of reference). That is, for each pixel in the thermal camera, we must determine which pixel in the depth camera it corresponds to. This is not as simple as it may appear. Water is not visible in the depth image as the projected infrared light does not reflect properly off the surface. However, our depth camera also collects color images and calibrates it to the depth frame automatically. We can use the color image then to calibrate the thermal camera.

While there exist methods for doing a full registration between color and thermal images [19], these tend to be noisy and unreliable. In this paper, because the water remains at a fixed distance from the camera, we use a simpler solution. First we take a checkerboard pattern printed on a wooden board and place it under a high-intensity halogen lamp. The light and dark pattern on the board absorbs light from the lamp at different rates, causing the dark squares to heat faster than the light squares. We then hold this board in front of both the thermal and color cameras at the same distance as the water. The differential heating causes the checkerboard pattern to be visible in both cameras, allowing us to find correspondence points between the two images. We then use these points to compute an affine transformation between the images, and use it to transform the thermal image onto the color image. Figures 4a and 4b show an example color image and its corresponding thermal image transformed onto the color space (the thermal camera has a narrower field of view than the color camera, which is why there are no thermal values around the edge of Figure 4b). Figure 4c shows the thresholded thermal image overlaid onto the color image.

D. Evaluation Criteria

We use two criteria for evaluating our methodology. The first is intersection over union (IOU). In this case, the state of the liquid simulation is projected into the camera by placing small spheres at each particle location and projecting those into the camera, taking into account occlusions by objects. We then compare the set of pixels labeled as liquid by this projection to the set of pixels labeled as liquid by the thermal image. The IOU is simply the intersection of these two sets divided by the union.

When comparing the probability of multiple simulations for the purposes of estimating hidden state, we use $P(\hat{I}_\pi | I_\pi)$ where $I_\pi$ is a set of predicted images for interaction $\pi$, and $\hat{I}_\pi$ is the set of ground truth images. To compute this, we first apply Bayes rule

$$P(\hat{I}_\pi | I_\pi) \propto P(I_\pi | \hat{I}_\pi) P(\hat{I}_\pi).$$

For our experiments, we assume the prior $P(\hat{I}_\pi)$ is uniform. To compute $P(I_\pi | \hat{I}_\pi)$, we assume each pixel is independent and simply multiply their individual probabilities together

$$P(I_\pi | \hat{I}_\pi) = \prod_{t=1}^{T} \prod_j P(j | \hat{j}).$$

where we set $P(j | \hat{j})$ equal to $\delta$ if $j$ and $\hat{j}$ are equal (both liquid or both not-liquid), and to $1-\delta$ if they are not. Due to the large number of pixels across all images and timesteps, we set $\delta = 0.50001$ to prevent underflow. After computing the probabilities, we then normalize them so they sum to 1.

VI. Experiments & Results

We ran three experiments to evaluate our simulators at tracking the state of real-world liquids. The first utilized the pouring interactions and focused on quantitatively evaluating the open and closed loop simulators. The second and third experiments test our simulation methods at estimating the state of an unknown variable in the environment. This is an important ability for a robot, as often liquids are occluded by containers or other objects, forcing robots to reason about the hidden state of the liquids based on outcomes during an interaction, something that is not always necessary during rigid object interactions. Our second two experiments examine two different cases of hidden state estimation using liquids.

A. Comparing Open and Closed Loop Simulation Methods

To compare each of the three simulation methods (open loop, MAP filter, and warp field), we simulated them on the data collected for each pouring interaction. At the start of each interaction, we fill the 3D model of the container with the same amount of liquid as was filled in the real container. We then simulate the liquid forward in time, updating the object poses based on the tracked poses acquired using DART. We evaluate each method by comparing their IOUs, computed as described in section V.D.

The IOU for the three simulation methods is shown in Table 1. The table shows that all of the methods tend to perform better when more liquid is involved. We notice that the bottle, while taller, meaning if they are not. Due to the large quantity of pixels.

TABLE I: IOU of Open and Closed Loop Methods

|        | Open-Loop | MAP Filter | Warp Field |
|--------|-----------|------------|------------|
| Cup    | 60.17%    | 75.58%     | 79.94%     |
| Bottle | 67.25%    | 77.12%     | 79.41%     |
| 30%    | 35.56%    | 65.22%     | 67.01%     |
| 60%    | 77.62%    | 79.85%     | 82.80%     |
| 90%    | 77.94%    | 80.69%     | 83.22%     |
| Overall| 65.66%    | 76.03%     | 78.41%     |

Even in log-space, values would still periodically underflow with higher values for $\delta$ due to the large quantity of pixels.

2The 4 pouring interactions where the container was left empty were not included in this analysis because the union part of the IOU would be 0, resulting in a division by 0.
the pouring interactions, aggregated by the ground truth fill V-D.

We compute the probability of each simulation by evaluating the probability of their predicted images as described in section V-D. Instead, the task of the robot was to estimate the amount of liquid in the container was not given to the robot. More specifically, the robot needs to run multiple simulations for each interaction, one for each possible fill amount, and compare the predictions of each simulation to the observation.

For each interaction, the robot ran 5 simulations: one for both legs unblocked, one for the right leg partially blocked, one for the right leg fully blocked, one for the left leg partially blocked, and one for the left leg fully blocked. The probability of each simulation is computed using the method described in section V-D.

Figure 6 shows the probability for each of the simulated blockages over time for one of the interactions using the best closed-loop method (warp field). The robot ran one simulation for each blockage type, and the diagrams across the top of the figure indicate where the blockage in that simulation was placed. The color bordering each diagram corresponds to the color of the line indicating that simulations probability over time. After only a short time window, the robot is able to correctly place the highest probability on the correct simulation in every case.

C. Solving the Pipe Junction Task

The final experiment we performed was the pipe junction task. Here the task is for the robot to find the blockage in a pair of connected pipes simply by observing the liquid as it exits the pipes, a situation the robot may find itself in if, say, trying to diagnose a broken sink. We assume that the robot knows a priori the default, unblocked flow rate of liquid through the pipes, and thus must use the change in flow to find the blockage. To test this, a pipe T-junction was held inverted over two bowls such that the legs of the T emptied into different bowls, both visible to the robot. However, the task is to find the blockage based only on the output of the pipes, so the T-junction was held high enough so that the robot could only see the openings on the bottom and not the top opening. To simulate a constant flow into the pipes, a container with exactly 1 liter of water was tilted at a constant angular velocity so that the liquid flowed into the top opening of the junction. The type of blockage used (if any), unblocked, partially blocked, or blocked, was placed inside the pipe, not visible to the robot. We used the data collected during the pipe junction interactions to evaluate the robot on this task.

To solve this task, like in the previous experiment, the robot needs to run multiple simulations with different values for the hidden state (the pipe blockages) and compare their outcomes. For each interaction, the robot ran 5 simulations: one for both legs unblocked, one for the right leg partially blocked, one for the right leg fully blocked, one for the left leg partially blocked, and one for the left leg fully blocked. The probability of each simulation is computed using the method described in section V-D.

Figure 5 shows the probability distribution over the initial fill amounts computed by the robot. They are aggregated by the true fill amounts. From top to bottom they are empty, 30% full, 60% full, and 90% full (as indicated by the *). The blue bars show the distribution for the open loop method, the cyan for the MAP filter, and the red for the warp field method. The robot ran one simulation for each blockage type, and the diagrams across the top of the figure indicate where the blockage in that simulation was placed. The color bordering each diagram corresponds to the color of the line indicating that simulations probability over time. After only a short time window, the robot is able to correctly estimate the initial amount of liquid, placing over 70% probability on the correct simulation in every case.

B. Estimating the Initial Amount of Liquid

We evaluated all three simulation methods on the same hidden state task. For each pouring interaction, the initial amount of liquid in the container was not given to the robot. Instead, the task of the robot was to estimate this amount based on the observations and its own liquid simulations. To do this, the robot needs to run multiple simulations for each interaction, one for each possible fill amount, and compare the predictions of each simulation to the observation.

For each pouring interaction, the robot ran 4 simulations: one where the container was left empty, one where the container was filled to 30% full, one where the container was filled to 60% full, and one where it was filled to 90% full. For each simulation, the liquid particles are simulated forward in time as the object poses are updated via their tracked poses. We compute the probability of each simulation by evaluating the probability of their predicted images as described in section V-D.

Figure 5 shows the results of performing this for each of the pouring interactions, aggregated by the ground truth fill amount (indicated by the * in the x-axis of each graph). The blue bars show the probability distributions for the open-loop method, the cyan bars show the distribution for the MAP filter method, and the red bars show the distribution for the warp field method. All methods are easily able to correctly place the highest probability on the empty simulation when there is in fact no liquid in the interaction, which follows intuition as there are no observed liquid particles. Additionally, even though there is slightly more confusion, all of the methods place the highest probability on the 90% simulation when the containers start out 90% full. Again, this aligns with intuition as it is easy to distinguish “a lot” of liquid from “almost no” liquid. The most confusion occurs when trying to distinguish “a little” (30%) from “some” (60%). The open loop method is almost completely unable to distinguish between the two, both distributions being very similar. The MAP filter method is slightly better, but still gets confused when the true amount in the container is 60%. Only the warp field method is able to correctly estimate the initial amount of liquid, placing over 70% probability on the correct simulation in every case.

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Reasoning about Liquids Reasoning about liquids in the robotic literature on real robots has been limited to restricted tasks, such as pouring [25, 22, 3]. With our physics-based model, reasoning about liquids can be done on a much wider variety of tasks. The last two experiments in this paper both involve completely different tasks, one reasoning about pouring, the other about blockages in pipes, yet the same algorithm is able to solve both tasks, without any special knowledge aside from generic 3D models. This is the kind of reasoning that is only enabled through full physics-based models. Reasoning about concepts like persistence and conservation are straightforward under our model. For example, a robot using this model could observe a pouring interaction, and it would be immediately obvious that the new liquid in the target container originated in the source container, and that the overall liquid is the same at the end of the pour as it was at the beginning. This is the kind of reasoning that is difficult under non-physics based models.

Generalizing to Other Liquids Another advantage of our physics-based model is that it can generalize to different types of liquid. Yamaguchi and Atkeson [32] developed a model-based detector that could determine the location of liquids in a scene, and they showed that it could generalize to a wide array of liquid types. This is unlike learning-based models, which cannot generalize to liquids too different from their training set. With the alteration of a few physical parameters, a physics-based model can easily generalize to liquids as diverse as water, oil, honey, and even dough. It is currently an open challenge as to how to infer these parameters efficiently from observation.

Predicting Liquid Behavior While others have used physics-based models for liquids [11], none have yet combined them with real perception. As a result, due to the quick divergence of open-loop models with reality, there has been little prior work exploring the possible action spaces around liquids. Closed-loop liquid simulations enable robots to interact with liquids in a much wider variety of tasks. This can allow robots, using the same model, to perform tasks as diverse as carrying a container across a room without spilling its contained liquid, scooping liquid with a spoon, and ejecting liquid from a syringe in a controlled manner. Without closed-loop liquid simulations, each of these tasks would require developing a separate model. Using an algorithm such as model predictive control [4], the robot could plan for a short time horizon in the future using the open-loop simulation, but track the current state using the closed-loop simulation, thus preventing a fatal divergence from reality.

VIII. Conclusion

In this paper, we proposed two methods for tracking the state of liquid with a closed-loop simulator. The first, inspired by Bayes filter techniques in robotics, used a MAP filter to correct errors in the simulator. The second, inspired by the physical forces underlying the simulator, applied a warp field to the particles to correct the error. The results clearly show that both our closed-loop methods are better at tracking the liquid than the open-loop method. We also showed how these closed-loop simulations can be used to reason about and infer the hidden variables of an interaction involving liquids. To our knowledge, this is the first time real liquid observations have ever been combined with liquid simulations.

In the immediate future, we plan to continue this work along multiple avenues of investigation. Firstly, we plan to utilize deep learning methods like the ones in [24, 14] to perceive liquids, bypassing the need for a thermal camera. Deep learning can also be applied to perform system identification, i.e., to learn the correct physics models and update them in real-time based on perception. One of the drawbacks to our method is that, due to efficiency concerns, we only used on the order of thousands of particles, which sometimes resulted in discontinuous liquid in predicted images. By using deep learning, it may be possible to increase the efficiency of the simulation, enabling higher resolution liquid modeling. Additionally, we also plan to apply our methodology to solving closed-loop controls tasks with real liquids, something which was difficult or impossible before. Finally, we plan to make our data from this paper publicly available to other researchers.

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