Research on the State fragility Assessment under the Influence of Environmental Change

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Abstract. In the past few decades, the impact of environmental change on the polity, economy, safety, and community of a country has also been taken seriously in many countries. To measure the comprehensive impact of environmental change on the country, this article first uses the principal component analysis method to construct a set of comprehensive indexes. Then, OLS method has been applied to analyse the relationship between environmental factors and other indexes (polity, economy, community, and safety), to further describe the impact of environmental change on other factors. Additionally, this paper employs the projection pursuit method based on the simulated annealing algorithm to construct the national fragility index, which is regarded as a comprehensive impact index reflecting the influence of environmental change on a state. Thus, the overall effect of environmental change on the country (national fragility) is manifested by the above OLS-Projection Pursuit two-step conduction model. In order to assess the degree of the environmental change influence, this paper considers the national fragility indexes of 80 countries and adopts the fuzzy C-means clustering method to classify the countries into three categories: fragile states, vulnerable states and stable states. By using our model to empirically analyse the data of India and Yemen, it is found that India is a vulnerable country and Yemen is a fragile country, which is consistent with expectations and reflects the rationality of the model.

Keywords: environmental change; state fragility; principal component analysis; ols-projection pursuit model; fuzzy C-means clustering method.

1. Introduction
The impact of environmental change is already being realized and vary from region to region, including increased droughts, shrinking glaciers, and sea level rise. Many of these effects may have the potential to cause the weakening and breakdown of social and governmental structures, which means the adverse environmental change may increase the fragility of a state.
A fragile state means a country where the government cannot or do not want to supply people with basic necessities. There are various factors and indicators that can affect the fragility of a state, such as environmental change effects, social and economic influences. We are facing increasingly weather-related disasters, range from monsoons to drought in the world (Peter S.et al., 2003). Environmental change has an immense impact on state security, society and economy (Carment D.et al., 2009). It is necessary to set up models to measure the impact of environmental change and how it influences on fragility of a country.

When it comes to the influence of environmental change, increased droughts, changing animal and plant ranges, shrinking glaciers and the rise of sea level are usually mentioned (Peter S.et al.,2003). The paper chooses some factors to represent the impact of environmental change and use four impact indexes to reveal how does environmental change effect the fragility of a country.

The index of environmental change is constituted by carbon emissions, the Palmer drought index (Loukas.et al., 2009), the days of extreme weather and species richness. These factors can represent the degree of environmental change. If governments disregard these factors, that is, allow the trend of environmental degradation, which will threaten the fragility of a country. Crossing thresholds in locally environmental conditions, however subject farmers to significant production, consumption and income losses, and unimaginable pressure which coerces some into committing suicide (Nicholls et al. 2006; Guiney 2012), defaulting on loan repayment (Shiferaw et al. 2014) or implementing coping strategies that weaken their ability to appropriately adjust to future shocks (Nelson et al. 2007). All these are threats to agricultural production, food availability, and stability in such countries (Selvaraju et al. 2011).

As regards the influence of environmental change on fragility, the paper analyse it from four aspects, including economy, safety, politics and community.

In terms of environmental change and impact of economy, Zhang et al. Found that carbon emission reduction will ease energy supply pressure, accelerate the development of non-fossil energies, improve industry structure, and downsize energy-intensive industries. Temple found that the power sector of Bangladesh might be severely affected sector due to environmental change and the power outages result in a loss of industrial output worth $1 billion a year. It reduces the GDP growth by about half a percentage point in Bangladesh (Temple 2002). Alkhathlan et al. adopted both ARDL and Johansen cointegration approaches to investigate the relationships between energy consumption, CO 2 emissions, and economic growth for Saudi Arabia during the period from 1980 to 2008. (Alkhathlan et al.,2012) They found that energy consumption and carbon dioxide emissions do not necessarily result in economic output.

Apart from economy, society, safety and community is also affected by environmental change. Be them droughts, floods, heat wave, or in any other form, extreme events manifest either in isolation or in combination, impact negatively on crops, livestock, human health (Yiran and Stringer 2016), and the primary impacts of environmental change on society result from extreme events in Bangladesh (Rodrigo 2002). The studies also show that changing environment will have severe implications on the agriculture (Karim et al. 1999), water resources (Shahid 2011a), and public health (Shahid 2010a).

In order to better describe above indicators, we have studied a large number of methods, such as principal components analysis, which has been used to show the effect of atmospheric circulation over 10-year period (Jones et al. 1995). Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The paper finally chooses it to establish indicators. Moreover, to measure the impact of indicators on fragility, we need construct a set of evaluation methods. The paper use projection pursuit method, which does not need to subjectively weight each indicator. The evaluation model based on this method can be more objective and reliable. This idea originated in Kruskal, (1969) and Friedman and Tukey, (1974), using the term “projection pursuit” describing a technique for exploratory analysis of multivariate data. It is useful for an initial data analysis, especially when data are in a high-dimensional space.
The remainder of the paper is divided as follows. The second section describes the construction and measurement of index system. The third section use empirical analysis to reveal the Impact of environmental change index on other indicators. The fourth and fifth sections provides the results of fragility calculated by projection pursuit method and classified by fuzzy clustering method. An empirical analysis has been done in sixth section. The last section is the conclusion.

2. The Construction and Measurement of the Index System

2.1. The construction of index system
the paper constructs an index of environmental change to manifest the impact of environmental change on a country’s security, politics, economy and community. The impact of these four aspects will affect on national fragility further. In this case, the paper builds up four indexes, namely the index of security impact, the index of political impact, the index of economic impact, and the index of social impact, to evaluate the fragility of a country. The transmission mechanism in the paper is that environmental change affects four aspects of security, politics, economy, and community, and then affects the country's fragility.

2.1.1. Index of environmental change. the paper selected the following subordinate indicators to build up the index of environmental change: Palmer Drought index, which is used to measure the extent of drought in some area; species richness, which indicates the change in the number of species; the days of extreme weather, measuring extreme climate change; carbon emissions, used to measure sea level changes and iceberg reductions.

2.2. Index of safety impact.
The change in the environment may lead to the reduction in food production. The possible famines will threaten the stability of the country in all aspects. Therefore, per capita food production is selected as one of the constituent elements of the safety impact index; environmental change such as extreme weather, storms, etc. may be important to the national energy facilities’ safety. In United States, many military facilities along the coast of the have been affected by storms, including more than 20 nuclear facilities and countless processing plants in the past few years. Thus, the number of nuclear facilities damaged by the environmental change is regarded as one of the important subordinate factors. Crime rate is the third subordinate factors chosen for the index of safety impact.

2.2.1. Index of political impact. The adverse consequences of the environmental change may affect the stability of the political situation. In terms of the composition of the index of political impact, the paper chooses the number of organized conflicts to reflect the degree of political turmoil; political duration, which can reflect the stability of the political situation; the quality of public administration, reflecting the quality of national administrative affairs and representing whether a government has the ability to maintain the state’s political stability.

2.2.2. Index of economic impact. The change in the environment may lead to the reduction in grain production, which might reduce the income of related industries and may ultimately affect the country’s macroeconomic development. Therefore, the paper selects GDP, inflation rate, and fixed asset investment to reflect the development and stability of macroeconomics to determine whether a country has sufficient economic strength to maintain the stability of the country. In addition, the paper also selects Gini coefficient to show the fairness of income distribution. If the adverse effect of the environmental change on the economy causes the gap in the income distribution of a country to expand, it will become a hidden danger of national stability.

2.2.3. Index of community impact. The change in the environment might have an impact on society and further affect the country’s fragility. For example, the adverse effects of the environmental change
will lead to severe poverty, unemployment and environmental refugees which are the condition for the
development of terrorism. Therefore, it is necessary to pay attention to the social impact of
environmental change. Due to the adverse effects of poor food production and even the outbreak of
war, immigration may occur. The paper selects the net outflow of population to represent the number
in loss of migrants in a country. The net inflow of population, reflecting the inflow of environmental
refugees in countries receiving immigration; the per capita medical expenditure, reflecting the costs
that people need to pay for medicine; the unemployment rate, reflecting the degree of unemployment.
If the unemployed population is too high, it will threaten the stability of the country.

2.3. Index Measure Based on Principal Component Analysis
For the above five indicators, this paper uses the principal component analysis method to build.

2.3.1. Index of Environmental Change $\theta$.

2.3.2. Step One: standardize the raw data. There are four indicators for the principal component
analysis, namely $X_1, X_2, X_3, X_4$. There are 80 countries chosen as the evaluation objects. Set the value
of the $i^{th}$ indicator of the $j^{th}$ evaluation object as $a_{ij}$. Convert each indicator value $a_{ij}$ to a standardized
indicator value:

$$ a_i^* = \frac{a_i - \mu_j}{s_j}, \quad i = 1, 2, \ldots, 80; \quad j = 1, 2, 3, 4 $$

Where, $\mu_j = \frac{1}{n} \sum_{i=1}^{n} a_{ij}$; $s_j = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (a_{ij} - \mu_j)}$, $j = 1, 2, 3, 4$ , Which respectively are the sample
mean and the sample standard deviation of the $j^{th}$ index.

Correspondingly, the standardized indicator variable is:

$$ x_i^* = \frac{s_j - \mu_j}{s_j}, \quad j = 1, 2, 3, 4 $$

Step Two: calculate the correlation coefficient matrix $R$. The correlation coefficient matrix
$R = \left( r_{ij} \right)_{4 \times 4}$

$$ r_{ij} = \frac{\sum_{k=1}^{80} a_{ik} a_{kj}}{80-1} \cdot \; i, j = 1, 2, 3, 4 $$

Step Three: calculate the eigenvalue and eigenvector. Calculate the eigenvalue of the correlation
coefficient matrix $R \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0$, and their corresponding eigenvectors $u_1, u_2, u_3, u_4$, where
$u_j = [u_{1j}, u_{2j}, u_{3j}, u_{4j}]^T$, we can get the four principal component expressions composed of
eigenvectors:

$$ y_1 = 0.1484x_1^* + 0.5627x_2^* + 0.8013x_3^* + 0.1385x_4^* $$
$$ y_2 = -0.1409x_1^* - 0.7513x_2^* + 0.4791x_3^* + 0.4315x_4^* $$
$$ y_3 = 0.7184x_1^* + 0.0520x_2^* - 0.2793x_3^* + 0.6350x_4^* $$
$$ y_4 = -0.6649x_1^* + 0.3409x_2^* - 0.2244x_3^* + 0.6256x_4^* $$

Where, $y_k$ is the $k^{th}$ principal component.

Step Four: Select $p (p \leq m)$ main components to get the expression of Index of environmental
impact. We can get information contribution rate and cumulative contribution rate by calculating
eigenvalues. The information contribution rate of Principal component $y_k$ is:

$$ b_j = \frac{\lambda_j}{\sum_{j=1}^{4} \lambda_j}, \quad j = 1, 2, 3, 4 $$
The cumulative contribution rate of Principal components \(y_1,\ldots,y_p\) is:

\[
\alpha_p = \frac{\sum_{k=1}^{p} \lambda_k}{\sum_{k=1}^{p} \lambda_k}
\]

Take \(\alpha_p = 0.85\) here, the top \(P\) variables \(y_1,\ldots,y_p\) are selected as \(P\) principal components, replace the original four variables, so that \(P\) principal components can be comprehensively analyzed.

**Table 1. Principal component analysis results of the index of environmental change**

| Number | Principal components | Characteristic root | Information contribution rate | Cumulative contribution rate |
|--------|----------------------|---------------------|-------------------------------|-----------------------------|
| 1      | \(y_1\)              | 1.5639              | 0.3910                        | 0.3910                      |
| 2      | \(y_2\)              | 1.1033              | 0.2758                        | 0.6668                      |
| 3      | \(y_3\)              | 0.7879              | 0.1970                        | 0.8638                      |

It can be seen that the cumulative contribution rate of the first three eigenvalues has already reached 85\%, which means the principal component analysis has a good effect. The first three principal components can be selected and the fourth principal component can be omitted.

Step Five: the contribution rates of the top three principal components are regarded as weights, and the principal component expression of the index of environmental change is constructed:

\[
\theta = 0.391y_1 + 0.2758y_2 + 0.197y_3
\]

Substitute the top three principal component expressions to obtain the relationship between the index of environmental change \(\theta\) and the four impact indicators as follows:

\[
\theta = 0.1607x_1 - 0.0026x_2 + 0.3904x_3 + 0.2983x_4
\]

Where the index of environmental change \(\theta \in (0,1)\). Palmer Drought index \(x_1\), carbon emission \(x_4\), the days of extreme weather \(x_3\) and \(\theta\) are positive relations. Species richness \(x_2\) and \(\theta\) are negative relations. The larger \(\theta\) indicates that the climate is worse and the higher the country's fragility.

2.3.3. **Index of safety impact** \(I_s\). The principal component analysis method is also used here for analysis. It can be expressed as:

\[
I_s = 0.1283I_{s1} + 0.6796I_{s2} - 0.5694I_{s3}
\]

2.3.4. **Index of political impact** \(I_p\). Principal component analysis is used to analyse the three ordinate indicators. It can be expressed as:

\[
I_p = -0.5859I_{p1} + 0.0187I_{p2} + 0.5908I_{p3}
\]

2.3.5. **Index of economic impact** \(I_e\). Principal component analysis is used to analyse the four ordinate indicators. It can be expressed as:

\[
I_e = -0.4416I_{e1} - 0.4008I_{e2} + 0.3788I_{e3} + 0.5004I_{e4}
\]

2.3.6. **Index of social impact** \(I_c\). The principal component analysis method is also used here for analysis. It can be expressed as:

\[
I_c = 0.0433I_{c1} + 0.4818I_{c2} - 0.3805I_{c3} + 0.4911I_{c4}
\]

Through principal component analysis, we have obtained indicators of the index of environmental change, the index of safety impact, the index of political influence, the index economic impact and the index of social impact, which provides a complete set of indexes for doing the following empirical research.
3. Empirical research on the effect of environmental change on the four impact indexes

To manifest the relationship between the index of environmental change $\theta$ and the index of safety impact $I_s$, index of political impact $I_p$, index of economic impact $I_e$, index of social impact $I_c$, we use their scatter plots and curve fitting them separately. The results are as follows:

(a) The relation between $I_s$ and $\theta$
(b) The relation between $I_p$ and $\theta$
(c) The relation between $I_e$ and $\theta$
(d) The relation between $I_c$ and $\theta$

Fig. 1. The relations of indexes

Then we do a correlation test as following:

|       | $I_s$ | $I_p$ | $I_e$ | $I_c$ | $\ln \theta$ |
|-------|-------|-------|-------|-------|--------------|
| $I_s$ | 1.000 |       |       |       |              |
| $I_p$ | 0.792 | 1.000 |       |       |              |
| $I_e$ | 0.687 | 0.824 | 1.000 |       |              |
| $I_c$ | 0.724 | 0.661 | 0.718 | 1.000 |              |
| $\ln \theta$ | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 |

From the above table, $p<0.01$, it can be considered that there has a significantly negative correlation between $\ln \theta$ and $I_s$, $I_p$, $I_e$, $I_c$, which could also be seen from the figure.

From figure 5, it can be seen that the relationship between $\theta$, $I_s$, $I_p$, $I_e$, $I_c$ cannot be fully characterized by linear functions, so we have logarithmically processed it to make it conform to the linear function. The following results are obtained by the least square regression of $\ln \theta$ and each index:
Table 3. Regression analysis results

| Variable | Is   | Ip   | Ie   | Ic   |
|----------|------|------|------|------|
| ln9      | -0.2204  | -0.1407  | -0.1881  | -0.1852  |
| _cons    | 0.2803  | 0.5194  | 0.1234  | 0.0387  |

legend: * p<.1; ** p<.05; *** p<.01

Since p<0.01, we can see that the results are quite significant. We have obtained the specific effects of $\theta$ on the four impact indices of $I_s$, $I_p$, $I_e$, $I_c$, $\theta$ and $I_s$, $I_p$, $I_e$, $I_c$ are both negatively correlated, and the specific relationship between $\theta$ and $I_s$, $I_p$, $I_e$, $I_c$ is obtained by OLS model:

$\begin{align*}
I_s &= -0.2204 \ln \theta + 0.2803, \theta \in (0,1) \\
I_p &= -0.1407 \ln \theta + 0.5194, \theta \in (0,1) \\
I_e &= -0.1881 \ln \theta + 0.1234, \theta \in (0,1) \\
I_c &= -0.1852 \ln \theta + 0.0387, \theta \in (0,1)
\end{align*}$

4. Fragility index score based on projection optimization

4.1. Evaluation of fragility

Based on the foregoing, we have already demonstrated that the environmental change index will affect other indicators. Next, we use the previously constructed indicator system to further adopt a projection pursuit method to establish a scientifically sound scoring system.

The projection pursuit method can project the high-dimensional spatially dispersed data onto a low-dimensional space. Then, through calculation the projection index function is optimized to find the best projection value that reflects the characteristics of the data structure. Then the sample is evaluated according to the low-dimensional projection value, and the evaluation value is obtained. Moreover, the projection pursuit method has no problem of solving the weight and can avoid the problems caused by general evaluation methods. Therefore, it is scientific and reasonable to establish an evaluation model to measure the state’s fragility using the projection pursuit method in this paper.

First, we define the fragility index $\gamma_f \in [0,1]$, which represents the fragility of a state.

The closer $\gamma_f$ is to 1, the higher the state’s fragility, and the closer $\gamma_f$ is to zero, the lower the state’s fragility.

The evaluation indexes we selected are divided into Index of safety impact $I_s$, Index of political impact $I_p$, Index of economic impact $I_e$, Index of social impact $I_c$.

For this reason, we use the projection pursuit method to obtain the evaluation value of the fragility index $\gamma_f$. Considering that it may fall into a local optimum in the process of its use, we have added a simulated annealing algorithm to the traditional projection pursuit method. The results are optimized to avoid falling into a local optimum and improve the accuracy and objectivity of the evaluation value.

Step One: projecting to low dimension

Observe the index of 80 countries from a variety of perspectives, which can fully find and reflect the best projection.

We set $a_j$ as the direction vector of projection, $v_{ij}(i=1,2,...,80; j=1,2,3,4)$ indicates the $j^{th}$ index of the $i^{th}$ national sample. These four indicators are $I_s$, $I_p$, $I_e$, $I_c$ and get the linear projection of the sample $i$ in one-dimensional space is:

$Z_i = \sum_{j=1}^{80} a_j v_{ij}$
Step Two: constructing the projected index function

We set $Z_j$ value of projection, constructs a projection indicator function $Q(a)$ the accordance of determining the vector of projection.

$$\max Q(a) = S_j \cdot D_j,$$

$$\sum_{j=1}^{a_j} a_j^j = 1$$
$$a_j > 0$$

$S_j$ means the standard deviation of the projected value, means the local density of the projected value. They can be expressed as:

$$S_j = \sqrt{\frac{\sum (Z_j - \bar{Z})^2}{m-1}}$$

$$D_j = \sum \sum (R - r_i) \times u(R - r_i) , i = 1,2,3,4$$

Step Three: using simulated annealing to optimize the projection direction

The objective function is:

$$\max Q(a) = S_j \cdot D_j,$$

$$\sum_{j=1}^{a_j} a_j^j = 1$$
$$a_j > 0$$

The basic principle of simulated annealing is that the high-temperature particles slowly and naturally cool down, eventually achieving thermal equilibrium at a specific temperature and achieving the lowest energy state, which should follow the rules below:

1) If $E(i) \geq E(j)$, then accept that the state is transformed by next state;
2) If $E(i) \leq E(j)$, then the state has a certain probability to be accepted. The probability is:

$$\mu = \frac{E(i) - E(j)}{K T}$$

Where $K$ is the Boltzmann constant and $T$ is the temperature of the particle.

When the temperature drops to 0, the probability distribution of equilibrium $x_i$ is:

$$P_i^* = \frac{1}{\sum_{i} \sum_{x_{i,w}} P_i \cdot s}$$

Where $s$ represents the number of status,

When the temperature drops slowly, each temperature has experienced state transitions many times, and then reach to a thermal equilibrium at each temperature. At this time, the global optimum solution can be found with a probability of 1. Therefore, simulated annealing algorithm can be used to obtain the optimal solution in the projection pursuit method.

Step four: Calculating projection vulnerability index

Through the above optimization of the simulated annealing algorithm, we can get the final fragility index $\gamma_j$:

$$\gamma_j = \sum_{j=1}^{a_j} a_j \cdot \gamma_j$$

By this means, we get results and normalize it. We obtained the fragile states index $\gamma_j$ of 80 countries, and it fluctuate in the interval between 0 and 1. When this index become closer to 0, it means the state is more stable, vice versa.

The results of 80 countries are as follows:
Table 4. The result of 80 countries

| Country | \( Y_f \) | Country | \( Y_f \) | Country | \( Y_f \) | Country | \( Y_f \) |
|---------|----------|---------|----------|---------|----------|---------|----------|
| AFG     | 0.7323   | COM     | 0.7534   | GAB     | 0.5533   | ARG     | 0.3637   |
| IRQ     | 0.967    | LAO     | 0.6763   | SRB     | 0.6625   | SVK     | 0.2853   |
| NGA     | 0.8569   | COL     | 0.7072   | NAM     | 0.5154   | ITA     | 0.277    |
| KEN     | 0.8434   | MDG     | 0.6868   | MEX     | 0.5544   | ESP     | 0.279    |
| NER     | 0.8222   | GEO     | 0.7526   | CUB     | 0.5783   | POL     | 0.2578   |
| LBR     | 0.8238   | TZA     | 0.6      | KAZ     | 0.6381   | CHL     | 0.2835   |
| CMR     | 0.8367   | GTM     | 0.6445   | CYP     | 0.5689   | KOR     | 0.2695   |
| MRT     | 0.8422   | CHN     | 0.6749   | ZAF     | 0.5426   | JPN     | 0.1892   |
| EGY     | 0.8191   | DZA     | 0.7037   | MYS     | 0.5139   | USA     | 0.2293   |
| NPL     | 0.7857   | NIC     | 0.6177   | JAM     | 0.4401   | FRA     | 0.1801   |
| RCA     | 0.7793   | TKM     | 0.7333   | BRN     | 0.5045   | GBR     | 0.1906   |
| MLI     | 0.7227   | HND     | 0.6031   | BRA     | 0.4284   | DEU     | 0.1433   |
| KHM     | 0.729    | TUN     | 0.7113   | ATG     | 0.4611   | AUT     | 0.1415   |
| LBY     | 0.8249   | THA     | 0.6662   | MNG     | 0.4522   | CAN     | 0.1278   |
| IRN     | 0.7992   | IND     | 0.6327   | PAN     | 0.3713   | AUS     | 0.1565   |
| LBN     | 0.8264   | RUS     | 0.6546   | BHS     | 0.3588   | ISL     | 0.1531   |
| SLB     | 0.7423   | MDA     | 0.6668   | HRV     | 0.3875   | NOR     | 0.1125   |
| ZMB     | 0.7399   | TUR     | 0.6917   | GRC     | 0.3567   | NZL     | 0.0633   |
| SWZ     | 0.7248   | SAU     | 0.6     | HUN     | 0.3477   | SWE     | 0.0339   |
| PHL     | 0.7092   | VIE     | 0.6132   | ARE     | 0.3591   | FIN     | 0        |

Classification of fragility based on fuzzy clustering

In order to identify whether a country is fragile country, vulnerable country or stable country, we need to categorize these countries and get a classification criterion. We use fuzzy C-means clustering algorithm to classify them.

Unlike traditional C-means algorithm, the fuzzy c-means clustering algorithm has the concept of membership degree.

The dividing space of traditional C-means clustering is:

\[
M_{hc} = \left\{ U = R^{cn} | \mu_{ik} \in [0,1], \forall i,k; \sum_{i=1}^{n} \mu_{ik} = 1, \forall k; 0 < \sum_{k=1}^{n} \mu_{ik} < n, \forall i \right\}
\]

According to the theory of fuzzy sets, fuzzy C-means clustering makes \( \{0,1\} \) extend from two-valued to the space, thus extending the hard C partition to the fuzzy C partition:

\[
M_{uc} = \left\{ U = R^{cn} | \mu_{ik} \in [0,1], \forall i,k; \sum_{i=1}^{n} \mu_{ik} = 1, \forall k; 0 < \sum_{k=1}^{n} \mu_{ik} < n, \forall i \right\}
\]

In order to make the clustering standard determined in the best class, the clustering loss function based on membership function is minimized, which can be expressed as:

\[
m \in \text{argmin}_J(U,v) = \sum_{i=1}^{n} \sum_{k=1}^{c} \left( \mu_{ik} \left( x_i \right) \right)^{2} \left| x_i - m_k \right|^{b}
\]

Where \( X = \{x_1, x_2, ..., x_n\} \) is data set; \( m_j \) is cluster center; \( U = \) fuzzy -partition of \( Y \); \( v = \{v_1, v_2, ..., v_c\} \) vectors of centers, \( \mu_{ik} \left( x_i \right) \) is the corresponding membership function for sample \( i \); \( b \) is the weighted index.

The optimal fuzzy clustering of \( Y \) is defined as the minimum value of \( J_f \). Seeking partial derivative, the minimum necessary to obtain the formula:

\[
m_j = \frac{\sum_{i=1}^{n} \left( \mu_{ik} \left( x_i \right) \right) v_i}{\sum_{i=1}^{n} \left( \mu_{ik} \left( x_i \right) \right)}
\]

\[
\mu_{ik} \left( x_i \right) = \frac{\left| x_i - m_j \right|^{-1/b}}{\sum_{i=1}^{n} \left| x_i - m_j \right|^{-1/b}}, \quad 1 \leq i \leq k
\]

The steps of fuzzy C-means algorithm are as follows:
Set the cluster class number \( m=3 \); set the weighted exponent \( B \) to 2; set the iterative convergence condition.

Initializing each cluster center \( v = \{v_1, v_2, \ldots, v_c\}, c = 3 \)

Current cluster center calculates the membership function according to formula 6. The center of each cluster is recalculated with the current membership function based on Formula 5.

Repeat the operation until the membership value of each sample is stable. When the algorithm converges, all kinds of clustering centers and membership degree values are obtained, and the classification of fuzzy clustering is completed.

Set the class number \( M \) as 3, we get the final 3 classifications as follows:

| Fragility  | Fragile states index \( \gamma_f \) |
|------------|-----------------------------------|
| Stable     | \( 0 \leq \gamma_f \leq 0.36 \)   |
| Vulnerable | \( 0.36 < \gamma_f \leq 0.70 \)   |
| Fragile    | \( 0.70 < \gamma_f \leq 1 \)     |

After getting rid of the extreme indicators, our results show that the critical point between fragile states and vulnerable states is 0.70 while the critical point between vulnerable states and stable states is 0.36.

5. Empirical analysis
This paper also analyzes the environmental fragility of Yemen and India and finds that Yemen's fragility value is \( \gamma_f = 0.9425 \in [0.70, 1] \), so Yemen is a fragile country; India's fragility value is \( \gamma_f = 0.6749 \in (0.36, 0.70] \), so it is judged that India is a vulnerable country.

6. Conclusion
In order to measure the impact of environmental change on the country, the paper uses principal component analysis to construct five impact indexes, including Index of environmental impact, Index of safety impact, Index of political impact, Index of economic impact and Index of social impact. Through the use of OLS and projection pursuit model to explore the comprehensive impact of environmental change on the national fragility, and using fuzzy C-means clustering method to classify the fragility of countries, the following results are obtained:

When the state fragility index \( \gamma_f \in [0.36, 0.7] \), the country is stable to environmental impacts; When \( \gamma_f \in [0.7, 1] \), the country is vulnerable to environmental impacts; when \( \gamma_f \in (0.36, 0.7] \), the country is fragile to environmental impacts. This paper then do empirical analysis to Yemen and India. The results of this model are in line with the actual conditions in these two countries. Therefore, the model has a certain degree of reliability and promotion value for assessing the impact of the environment on a country's fragility.

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