Effective chiral theory of kaon-nucleon scattering

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Abstract

We apply the relativistic chiral $SU(3)$ Lagrangian density to kaon-nucleon scattering imposing constraints from the pion-nucleon $s$- and $p$-wave threshold parameters at chiral order $Q^2$. The $s$ and $u$ channel decouplet baryon exchange is included explicitly and is found to play a crucial role in understanding the empirical $s$- and $p$-wave nuclear kaon dynamics quantitatively.

1 Introduction

A good understanding of the $\bar{K}$-nucleon interaction is required for the description of $K^-$-atoms [1] and the subthreshold production of kaons in heavy ion reactions [2]. The ultimate goal is to relate the in-medium spectral function of kaons with the anticipated chiral symmetry restoration at high baryon density.

In early approaches the $\bar{K}$-nucleon scattering amplitudes are described in terms of a coupled channel $K$-matrix with the set of parameters adjusted to available elastic and inelastic cross sections at small laboratory momenta $p_{\text{lab}} \leq 200$ MeV. Due to the insufficient quality of the low energy elastic and inelastic $K^-p$ data the resulting scattering amplitudes remained ambiguous. The sign of the $K^-$-proton scattering length was only recently convincingly determined by a kaonic-hydrogen-atom measurement [3]. The sign of the $K^-$-neutron scattering length remains highly model dependent [4, 5]. This reflects the fact that there are no $K^-$-neutron (deuteron) scattering data available at low energies and therefore the isospin one scattering amplitude is constrained only indirectly for example by the $\Lambda \pi^0$ production data. As a consequence also the subthreshold $\bar{K}N$ scattering amplitudes, which determine the $\bar{K}$-spectral function in nuclear matter to leading orders in the density expansion, are poorly known. In the region of the $\Lambda^*(1405)$ resonance the isospin zero amplitudes of various analyses may differ by a factor of two [6, 7]. Here a profound theory is asked for. In particular it is desirable to utilize the chiral symmetry constraints of QCD. First attempts in this directions can be found in [6, 7]. The reliability of the extrapolated subthreshold scattering amplitudes can further be substantially improved by including both $s$- and $p$-waves in the
analysis of the empirical cross sections since for $p_{lab} > 200$ MeV the available data are much more precise than at $p_{lab} < 200$ MeV where one expects s-wave dominance.

In this work we apply the relativistic chiral $SU(3)$ Lagrangian including an explicit baryon decouplet field. As to our knowledge this is the first application of the chiral $SU(3)$ Lagrangian to the s- and p-wave nuclear kaon dynamics. It is argued that due to the rather large kaon mass the kaon-nucleon dynamics turns non-perturbative in contrast with the pion-nucleon system where chiral perturbation theory can be applied successfully [8]. This implies that a partial summation scheme is required in the strangeness sector [3, 4, 5]. We solve the Bethe-Salpeter equation for the scattering amplitude with the interaction kernel truncated at chiral order $Q^2$. This leads to s- and p-wave contributions in the scattering amplitude. As a crucial technical ingredient we propose to apply a subtraction scheme rather than a cutoff scheme as employed in [3, 4]. The subtraction point is identified with the hyperon mass as to protect the s-channel hyperon exchange term contributing in the $P_{11}$ and $P_{31}$ channels. The renormalization scheme is an important input of our chiral $SU(3)$-dynamics. In particular we avoid first, an uncontrolled breaking of the $SU(3)$-symmetry induced by channel dependent cutoff parameters as in [3] and second, a strong sensitivity of the $\Lambda^*(1405)$ resonance structure on the cutoff parameter implicit in [3].

We successfully adjust the set of parameters to describe the existing $K^-p$ elastic and inelastic cross section data including angular distributions to good accuracy. Moreover the measured spectral form of the $\Lambda^*(1405)$ and $\Sigma^*(1385)$ resonances are reproduced. As a result of our analysis we find a weakly repulsive s-wave $K^-\text{neutron}$ interaction and a subthreshold $K^-\text{nucleon}$ forward scattering amplitude with sizable contributions from p-waves.

2 Relativistic chiral $SU(3)$ interaction terms

We recall the relevant interaction terms of the relativistic chiral $SU(3)$ Lagrangian density. The systematic construction principle can be found in [10]. A systematic regrouping of interaction terms accompanied by an appropriate subtraction scheme leads to manifest chiral power counting rules [11]. At chiral order $Q^2$ we collect the relevant interaction terms:

$$\mathcal{L} = \text{tr } B (i \not \partial - m_B) B + \frac{1}{4} \text{tr } (\partial^\mu \Phi) (\partial_\mu \Phi) + \frac{i}{8 f_\pi^2} \text{tr } B \gamma^\mu \left[ \Phi, (\partial_\mu \Phi) \right]_-, B \right)_- + \frac{F}{2 f_\pi} \text{tr } B \gamma_5 \gamma^\mu \left[ (\partial_\mu \Phi) , B \right]_+ + \frac{D}{2 f_\pi} \text{tr } B \gamma_5 \gamma^\mu \left[ (\partial_\mu \Phi) , B \right]_+$$
the nuclear kaon dynamics. Kinematical structures which we find crucial for a quantitative description of heavy baryon formalism presented for example in [13]. However, we refer the reader to [14] and p-wave dynamics. If the heavy baryon expansion is applied to (1) the free parameters of the local 4-point interaction terms describe s-wave range and can be absorbed into the local 4-point interaction terms. The remaining 11 pions. Similarly the parameters (2) sector its effect in (1) can be reliably determined only in an analysis of meson-baryon scattering since in the SU(2) sector its effect can be absorbed into the local 4-point interaction terms. The remaining 11 free parameters of the local 4-point interaction terms describe s-wave range and p-wave dynamics. If the heavy baryon expansion is applied to (1) the local 4-point interaction terms can be mapped onto corresponding terms of the heavy baryon formalism presented for example in [13]. However, we prefer the manifestly covariant form (1). The interaction terms of (1) predict well defined kinematical structures which we find crucial for a quantitative description of the nuclear kaon dynamics.

\[
\begin{align*}
&+ \text{tr} \Delta_{\mu} \cdot \left( (i \partial - m_{\Delta}) g^{\mu \nu} - i (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) + i \gamma^\mu \gamma^\nu \right) \Delta^\nu \\
&+ \frac{C}{2 f_{\pi}} \text{tr} \left\{ \left( \Delta_{\mu} \cdot (\partial_{\nu} \Phi) \right) \left( g^{\mu \nu} - \frac{1}{2} Z \gamma^\mu \gamma^\nu \right) B + \text{h.c.} \right\} \\
&+ \frac{1}{2} g_0^{(s)} \text{tr} \bar{B} B \text{tr} (\partial_{\mu} \Phi) (\partial^{\mu} \Phi) + \frac{1}{2} g_1^{(s)} \text{tr} \bar{B} (\partial_{\mu} \Phi) \text{tr} (\partial^{\mu} \Phi) B \\
&+ \frac{1}{4} g_F^{(s)} \text{tr} \bar{B} \left[ \left[ (\partial_{\mu} \Phi), (\partial^{\mu} \Phi) \right]_{+}, B \right]_{-} + \frac{1}{4} g_D^{(s)} \text{tr} \bar{B} \left[ \left[ (\partial_{\mu} \Phi), (\partial^{\mu} \Phi) \right]_{+}, B \right]_{+} \\
&+ \frac{1}{4} g_0^{(V)} \left( \text{tr} \bar{B} i \gamma^\mu (\partial^{\nu} B) \text{tr} (\partial_{\nu} \Phi) (\partial_{\eta} \Phi) + \text{h.c.} \right) \\
&+ \frac{1}{8} g_1^{(V)} \left( \text{tr} \bar{B} (\partial_{\mu} \Phi) i \gamma^\mu \text{tr} (\partial_{\nu} \Phi) (\partial^{\nu} B) + \text{h.c.} \right) \\
&+ \frac{1}{8} g_2^{(V)} \left( \text{tr} \bar{B} (\partial_{\nu} \Phi) i \gamma^\mu \text{tr} (\partial_{\mu} \Phi) (\partial_{\eta} B) + \text{h.c.} \right) \\
&+ \frac{1}{8} g_F^{(V)} \left( \text{tr} \bar{B} i \gamma^\mu \left[ \left[ (\partial_{\nu} \Phi), (\partial_{\eta} \Phi) \right]_{+}, (\partial^{\nu} B) \right]_{-} + \text{h.c.} \right) \\
&+ \frac{1}{8} g_D^{(V)} \left( \text{tr} \bar{B} i \gamma^\mu \left[ \left[ (\partial_{\nu} \Phi), (\partial_{\eta} \Phi) \right]_{+}, (\partial^{\nu} B) \right]_{+} + \text{h.c.} \right), \\
&+ \frac{1}{2} g_1^{(T)} \text{tr} \bar{B} (\partial_{\mu} \Phi) i \sigma^{\mu \nu} \text{tr} (\partial_{\nu} \Phi) B \\
&+ \frac{1}{4} g_D^{(T)} \text{tr} \bar{B} i \sigma^{\mu \nu} \left[ \left[ (\partial_{\nu} \Phi), (\partial_{\eta} \Phi) \right]_{-}, B \right]_{+} \\
&+ \frac{1}{4} g_F^{(T)} \text{tr} \bar{B} i \sigma^{\mu \nu} \left[ \left[ (\partial_{\nu} \Phi), (\partial_{\eta} \Phi) \right]_{+}, B \right]_{-}
\end{align*}
\]

where we introduce the meson octet field $\Phi = \sum_i \Phi_i \lambda_i$, the baryon octet field $B = \sum_i B_i \lambda_i / \sqrt{2}$ and the completely symmetric baryon decouplet field $\Delta$. The parameter $f_{\pi} \simeq 93$ MeV is determined from the weak decay width of charged pions. Similarly the parameters $F \simeq 0.45$ and $D \simeq 0.80$ are constrained by the weak decay widths of the baryon octet states and $C \simeq 1.5 - 1.7$ is constrained by the hadronic decay width of the baryon decouplet states. We point out that the parameter $Z$ in (1) can be reliably determined only in an $SU(3)$ analysis of meson-baryon scattering since in the $SU(2)$ sector its effect can be absorbed into the local 4-point interaction terms. The remaining 11 free parameters of the local 4-point interaction terms describe s-wave range and p-wave dynamics. If the heavy baryon expansion is applied to (1) the local 4-point interaction terms can be mapped onto corresponding terms of the heavy baryon formalism presented for example in [13]. However, we prefer the manifestly covariant form (1). The interaction terms of (1) predict well defined kinematical structures which we find crucial for a quantitative description of the nuclear kaon dynamics.
Further interaction terms are induced by the explicit chiral symmetry breaking of QCD:

\[ \mathcal{L}_{\chi-SB} = b_D \text{tr} \bar{B} [\chi, B]_+ + b_F \text{tr} \bar{B} [\chi, B]_- + b_0 \text{tr} \bar{B} B \text{tr} \chi + d_D \text{tr} (\bar{\Delta}_\mu \cdot \Delta^\mu) \chi + d_0 \text{tr} (\bar{\Delta}_\mu \cdot \Delta^\mu) \text{tr} \chi, \]
\[ \chi = 2 \chi_0 - \frac{1}{4 f_\pi^2} [\Phi, [\Phi, \chi_0]]_+, \]
\[ \chi_0 = \frac{1}{3} \left( m_\pi^2 + 2 m_K^2 \right) 1 + \frac{2}{\sqrt{3}} \left( m_\pi^2 - m_K^2 \right) \lambda_8. \] (2)

The parameters \( b_D, b_F, \) and \( d_D \) are determined to leading order by the baryon octet and decouplet mass splitting. The empirical estimates are \( b_D \simeq (0.064 \pm 0.004) \text{ GeV}^{-1}, b_F \simeq - (0.209 \pm 0.004) \text{ GeV}^{-1}, \) and \( d_D \simeq (0.87 \pm 0.04) \text{ GeV}^{-1}. \) They induce well defined meson-octet baryon-octet interaction vertices. The parameter \( b_0 \) is related to the pion-nucleon sigma-term \( \sigma_{\pi N} = -2 m_\pi^2 (b_D + b_F + 2 b_0) \) with the empirical value \( \sigma_{\pi N} = (45 \pm 10) \text{ MeV}. \)

The chiral Lagrangian is a powerful tool once it is combined with appropriate power counting rules leading to a systematic approximation strategy. In the \( \pi N \) sector the \( SU(2) \) chiral Lagrangian was successfully applied demonstrating good convergence properties of the perturbative chiral expansion. In the \( SU(3) \) sector the situation is more involved due in part to the rather large kaon mass \( m_K \simeq m_N/2. \) Here the perturbative evaluation of the chiral Lagrangian cannot be justified and one must change the expansion strategy. Rather than expanding directly the scattering amplitude one may expand the interaction kernel according to chiral power counting rules. This is in analogy to the treatment of the \( e^+ e^- \) bound state problem of QED where a perturbative evaluation of the interaction kernel can be justified. For details on the formalism how to extract the interaction kernel from the relativistic chiral Lagrangian and how to then solve for the Bethe-Salpeter scattering equation with a physical renormalization scheme we refer to \[17\].

3 Results

We first discuss the effect of the leading interaction term of the chiral \( SU(3) \) Lagrangian density suggested long ago by Tomozawa and Weinberg. If taken as input for the multi-channel Bethe-Salpeter equation, properly furnished with a renormalization scheme leading to a subtraction point close to the baryon octet mass, a rich structure of the scattering amplitude arises. In Fig. 1 we show the solution of the multi-channel Bethe-Salpeter as a function of the kaon...
mass. For physical kaon masses the isospin zero scattering amplitude shows a resonance structure at energies where one would expect the $\Lambda^*(1405)$ resonance. We point out that the resonance structure disappears as the kaon mass is decreased. Already at a hypothetical kaon mass of 300 MeV the $\Lambda^*(1405)$ resonance is not formed anymore. Fig. 1 nicely demonstrates that the chiral $SU(3)$ Lagrangian is necessarily non-perturbative in the strangeness sector. This confirms the findings of [6, 7]. However, note that in previous works the $\Lambda^*(1405)$ resonance is a result of a fine tuned cutoff parameter which gives rise to a different kaon mass dependence of the scattering amplitude [8]. In our scheme the choice of subtraction point close to the baryon octet mass follows necessarily form the compliance of the relativistic chiral Lagrangian with chiral counting rules. Moreover, the identification of the subtraction point with the $\Lambda(1116)$ mass in the isospin zero channel protects the hyperon exchange s-channel pole contribution.

Figure 1: Real (l.h.s.) and imaginary (r.h.s.) part of the isospin zero s-wave $K^-$-nucleon scattering amplitude as it follows from the $SU(3)$ Weinberg-Tomozawa interaction term in a coupled channel calculation. We use $f_\pi = 93$ MeV and identify the subtraction point with the $\Lambda(1116)$ mass.
Figure 2: $K^-$-proton elastic and inelastic cross sections. The data are taken from [18]. The solid line includes effects of s- and p-waves. The dotted line shows the s-wave contribution only.

At subleading order the chiral $SU(3)$ Lagrangian leads to basically 11 free parameters to be adjusted to empirical data. Chiral symmetry is found predictive in the $SU(3)$ sector since it reduces the number of free parameters. Static $SU(3)$ symmetry alone would predict 18 independent terms rather than the 11 chiral terms since $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$. The set of parameters is well determined by elastic and inelastic $K^-p$ cross section data together with empirical pion-nucleon threshold parameters. In Fig. 2 we present the result of our fit for the elastic and inelastic $K^-p$ cross sections. The data set is nicely reproduced including the rather precise data points for laboratory momenta $200 \text{ MeV} < p_{\text{lab}} < 300 \text{ MeV}$. In Fig. 2 the s-wave contribution to the total cross section is shown with a dashed line. Sizeable p-wave contributions are found only in the $\Lambda \pi_0$ production cross section. Note that the $\Lambda \pi_0$ channel carries isospin one and therefore provides a valuable constraint on the poorly known $K^-$-neutron interaction.
We worked out two crucial ingredients of a successful description of the s-wave kaon-nucleon dynamics. First it is found that the explicit $\Sigma^*(1385)$ contributions to s- and p-waves are important for a good fit. We emphasize that the $\Sigma^*(1385)$ induces s-wave range parameters which are $SU(3)$-independent from the chiral range parameters. The situation is different from the pion-nucleon system where the isobar induced s-wave range terms can be absorbed into the chiral range parameters. Second, we find that it is crucial to employ the relativistic chiral Lagrangian. It gives rise to well defined kinematical structures in the local 4-point interaction kernel of (1) which leads to a mixing of s-wave and p-wave parameters. Only in the heavy-baryon mass limit the parameters decouple into the s-wave and p-wave sector.

In Fig. 3 we show the $\Lambda^*(1405)$ and $\Sigma^*(1385)$ spectral functions measured in the reactions $K^-p \rightarrow \Lambda\pi^+\pi^-$ [20] and $K^-p \rightarrow \Sigma^+\pi^-\pi^+\pi^-$ [21] respectively. We point out that even though the spectral form of the $\Lambda^*(1405)$ provides an important constraint for the parameter set it does not by itself lead to a stringent determination of the isospin zero subthreshold scattering amplitude.
in the vicinity of the $\Lambda^*(1405)$ resonance. This is reflected in the fact that different analyses consistent with the low energy data set and the spectral form of the resonance may give rise to rather different subthreshold amplitudes [6, 7].

In Fig. 4 we confront our result with available differential cross sections from $K^-p$ proton scattering. The angular distribution pattern is consistent with a weak p-wave contribution. The linear slope in $\cos \theta$ reflects the interference of the p-wave contribution with a strong s-wave. The angular distributions of further inelastic $K^-p$ reactions (not shown) are reproduced with similar quality. We also reproduce the empirical $K^-p$ threshold branching ratios [22] to good accuracy. Here the isospin breaking effects are important. In order to unambiguously find a solution we constrain the free parameters to also reproduce the empirical s-wave scattering lengths and p-wave scattering volumes of the pion-nucleon sector. The pion-nucleon threshold parameters are evaluated perturbatively to subleading orders in the chiral expansion in accordance with
effective field theory calculations in the presence of an explicit isobar field. Here the small pion mass justifies the perturbative treatment. An accurate description of the pion-nucleon threshold parameters is obtained.

Finally we present our result for the $\bar{K}$-nucleon forward scattering amplitudes. In our approach it receives contributions from $s$- and $p$-wave channels:

$$f_{KN,\text{forward}}^{(l)}(\sqrt{s}) = f_{KN,s\text{-wave}}^{(l)}(\sqrt{s}) + p_{KN}^2 f_{KN,p\text{-wave}}^{(l)}(\sqrt{s})$$

where \(\sqrt{s} = \sqrt{m_N^2 + p_{KN}^2 + m_K^2 + p_{KN}^2}\). As can be seen from Fig. 5 we find sizeable $p$-wave contributions in the subthreshold amplitudes. In particular the $\Sigma(1385)$-resonance dominates the isospin one scattering amplitude. Note that $p$-wave channels contribute with a positive imaginary part for energies larger than the kaon-nucleon threshold but with a negative imaginary part for subthreshold energies. In particular recall that a negative imaginary part of a
subthreshold amplitude is not forbidden by the optical theorem which relates the imaginary part of the forward scattering amplitudes to the total cross section only for energies above threshold. We emphasize that our subthreshold amplitudes differ strongly from the ones of A.D. Martin [4]. Note that an important and highly model dependent input of Martin’s dispersion analysis of the \( K^- p \) forward scattering amplitude was the subthreshold amplitude which is not directly constrained by data. Clearly, one should investigate constraints from dispersion relations in a refined chiral \( SU(3) \) analysis.

4 Discussion and outlook

The result for the \( K^- \)-nucleon scattering amplitudes has interesting consequences for kaon propagation in dense nuclear matter. An attractive subthreshold scattering amplitude permits a kaon to propagate in nuclear matter with an energy smaller than its free space mass. To leading order in the nuclear density the kaon self energy \( \Pi_K(\omega, \vec{q}) \) is determined by the s- and p-wave scattering amplitudes implicit in Fig. 5. For isospin symmetric nuclear matter the low density theorem \([23, 4]\) leads to

\[
\Pi_K(\omega, \vec{q}) = -4 \pi \frac{\sqrt{s}}{m_N} \left( f_{K_N}^{(s\text{-wave})}(\sqrt{s}) \right) + \left( \frac{3 \omega^2}{5 m_N^2} k_F^2 + \vec{q}^2 \right) f_{K_N}^{(p\text{-wave})}(\sqrt{s}) \rho + \cdots \tag{4}
\]

where we imply isospin averaged scattering amplitudes and identify \( s \simeq (m_N + \omega)^2 - \vec{q}^2 \). In \([4]\) we assume \( \omega, |\vec{q}|, k_F \ll m_N \) and treat Fermi motion effects in a simplistic fashion. The dots in \([4]\) represent further terms of the density expansion. Note that according to \([4]\) at small momenta \( \vec{q} \) the self energy is not determined by the forward scattering amplitudes in contrast with the expectation at large momenta. Rather, an evaluation of the self energy requires the decomposition of the forward scattering amplitudes into its partial wave contributions.

The spectral function of the kaon is expected to show a complicated structure due to the formation of the \( \Lambda^*(1405)N^{-1} \) and \( \Sigma^*(1385)N^{-1} \) states. This is an immediate consequence of the resonance structure seen in the kaon-nucleon scattering amplitudes. At small energies \( \omega \simeq m_{\Lambda, \Sigma} - m_N \) the spectral function may couple also to the hyperon nucleon-hole states \( \Lambda N^{-1} \) and \( \Sigma N^{-1} \) though the spectral weight is expected to be rather small \([24]\).

As demonstrated in \([23]\) at intermediate nuclear densities \( \sim \rho_0 \) the low density theorem \([4]\) provides a good approximation to the self energy for energies
ω and momenta \( \vec{q} \) leading to a \( s = (m_N + \omega)^2 - \vec{q}^2 \) sufficiently below or above the resonance structures in the scattering amplitudes. On the other hand at zero momentum \( \vec{q} = 0 \) and energies \( \omega \) close to the kaon mass, a kinematical region of up most importance for the microscopic understanding of kaonic atoms, the low density expression is useless already at rather small densities \( \rho < 0.1 \rho_0 \). This is due to the presence of an active small scale determined by the binding energy of the \( \Lambda^*(1405) \) resonance [25]. Since already the vacuum scattering amplitude is a rather sensitive function of the kaon mass (see Fig. 1) one arrives at a self-consistent scheme where the feedback effect of an attractive kaon spectral function on the in-medium kaon-nucleon scattering process must be taken into account [25]. Note that though the isospin one amplitude is smaller than the isospin zero amplitude its contribution to the kaon self energy in isospin symmetric nuclear matter is enhanced by a factor three from isospin. We therefore expect that both the \( \Lambda^*(1405) \) and the \( \Sigma^*(1385) \) resonances need to be included in a self-consistent treatment of kaon propagation in nuclear matter. Work in this direction is in progress [26].

Rather than a result of perturbative chiral s-wave dynamics suggested in [27, 13] we arrive at an intriguing non-perturbative and much richer picture of the chiral nuclear kaon dynamics.

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