Semi-empirical predictive equations for the initial amplitude of submarine landslide-generated waves: applications to 1994 Skagway and 1998 Papua New Guinea tsunamis

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Abstract
Accurate predictions of maximum initial wave amplitude are essential for coastal impact assessment of tsunami waves generated by submarine landslides. Here, we analyse the existing predictive equations for the maximum initial amplitude ($\eta_{\text{max}}$) of submarine landslide-generated waves and study their performance in reproducing real-world landslide incidents. Existing equations include various landslide parameters such as specific gravity ($\gamma_s$), initial submergence ($d$), slide length ($B$), width ($w$), thickness ($T$) and slope angle ($\theta$). To determine how landslide parameters affect wave amplitude, we conduct a systematic sensitivity analysis. Results indicate that the slide volume ($V = B \times w \times T$) and $d$ are among the most sensitive parameters. The data from the 1994 Skagway (observed $\eta_{\text{max}}$: 1.0–2.0 m) and 1998 Papua New Guinea (PNG) (observed $\eta_{\text{max}}$: 10–16 m) incidents provided valuable benchmarks for evaluating the performance of the existing equations. The predicted maximum initial amplitudes of 0.03–686.5 m and 3.7–6746.0 m were obtained for the 1994 and 1998 events, respectively, indicating a wide range for wave amplitudes. The predicted estimates for the smaller-sized event, i.e. the 1994 Skagway, appear to be more accurate than those made for the larger event, i.e. the 1998 PNG case. We develop a new predictive equation by fitting an equation to actual submarine landslide tsunamis: $\eta_{\text{max}} = 50.67 \left( \frac{V}{d} \right)^{0.34}$, where $V$ is the slide volume (km$^3$), $d$ is initial submergence depth (m), and $\eta_{\text{max}}$ is in metres. Our new equation gives wave amplitudes of 1.6 m and 7.8 m for the 1994 and 1998 landslide tsunamis, respectively, which are fairly consistent with real observations.

Keywords Landslide · Landslide-generated waves · Tsunami · Papua New Guinea
1 Introduction

Landslide-generated waves have been major threats to coastal areas and have led to destruction and casualties in several locations. In July 1998, a landslide tsunami in Sis-sano Lagoon, Papua New Guinea, generated a 15-m high wall of water (Fig. 1c) killing more than 2100 people (Tappin et al. 2008; Synolakis et al. 2002; Lynett et al. 2003; Heidarzadeh and Satake 2015). It is now well accepted that the source of the 1998 PNG tsunami was a combined earthquake–landslide source (Fig. 1a, b). The 1994 Skagway, Alaska (USA) landslide tsunami destroyed the railway dock; as a result, a construction worker was killed and the harbour was damaged (Kulikov et al. 1996; Rabinovich et al. 1999) (Fig. 1d–f). Another significant landslide tsunami was generated in Nice (France) airport on 16 October 1979 where a part of the new harbour extension, nearby to the Nice international airport, slumped into the Mediterranean Sea (Assier-Rzadkieicz et al. 2000; Dan

![Image](image_url)

Fig. 1  a The combined landslide (colour map) and earthquake (contour) source of the 1998 PNG tsunami based on Heidarzadeh and Satake (2015). b The 3D projection of the landslide source of the 1998 PNG tsunami. c The distribution of the surveyed tsunami run-up during the 1998 PNG event based on the data from Synolakis et al. (2002). d Location map showing the Skagway tide gauge position, which recorded the 1994 Skagway landslide tsunami. e Google-Earth image showing the Skagway harbour. f The tide gauge waveform of the 1994 Skagway tsunami digitized from Rabinovich et al. (1999). AST on the label of the horizontal axis is abbreviation for Atlantic Standard Time. Panels a–b are modified from Heidarzadeh and Satake (2015)
et al. 2007). The 2–3 m high tsunami swept away 11 people and left one death (Gennesseaux 1980; Sultan et al. 2004; Fine et al. 2005). Other major landslide events are 1946 Unimak (Alaska) (Okal et al. 2003), Storegga (approximately 8,100 years ago) (Harbitz 1992), 1741 Oshima–Oshima (Japan) (Satake and Kato 2001) and 1999 Izmit bay (Turkey) (Yalciner et al. 1999) tsunamis.

Due to the importance of landslide tsunamis for coastal safety, various studies have been conducted to characterize the waves generated by submarine failures, including experimental, numerical and analytical studies. These investigations over the past years have shed light on the processes of generation and propagation of landslide-generated waves. Wiegel (1955) was a pioneer in physical modelling of water waves generated by submarine landslides, who has been inspired by many researchers after his work. Fundamentals of landslide tsunami characteristics were discovered by Wiegel (1955) as he reported that the leading wave amplitude of landslide tsunamis increases by increasing block density; by decreasing initial block submergence; and by increasing incline angle. Harbitz and Pedersen (1992) presented analytical solutions for wave excitation by submarine landslides in order to study the influence of governing parameters. Harbitz and Pedersen (1992) found an expression for the relative importance of the effect of landslide volume versus the shear stress on the interface between fluid and sliding masses. In terms of numerical simulations, the research by Heinrich (1992) replicated the effect of water waves created by a submarine solid block descending a sloping beach in a wave flume by a standard finite difference technique. Likewise, Liu et al. (2005) presented a numerical model for landslide tsunami based on large eddy simulations technique validated by a set of large-scale experimental studies. Heidarzadeh et al. (2014) reviewed the existing numerical tools for modelling landslide-generated waves.

A number of researchers have proposed empirical equations for the prediction of maximum initial landslide tsunami amplitude. However, it is essential to examine how accurately these equations can reproduce actual landslide events. Here, we study the performance of the existing empirical and semi-empirical equations for the prediction of landslide tsunami amplitudes. Empirical equations are beneficial for predicting the amplitudes of landslide-generated waves in a timely way and can be readily employed for preliminary hazard analysis. The maximum initial amplitude of landslide waves around the source region is considered as the key performance indicator in this research. This parameter is difficult to measure directly; but it is available through experimental and numerical studies for some of the landslide events, as discussed later in this article. We study the existing equations and examine their performance by reproducing two actual landslide tsunami events for which real measurements are available: the 1994 Skagway and the 1998 PNG events. Finally, we propose a new empirical equation for prediction of maximum initial amplitude of landslide tsunamis based on data from actual events.

2 Methodology and data

The initial landslide-generated wave features are strongly influenced by landslide kinematics. Hence, determining the slide law of motion is necessary. Most landslide tsunami studies have employed the kinematic equations developed by Watts et al. (2005) for slide motion; here, we benefit from the same equations. The characteristic length (s0) and characteristic time (t0) for slide kinematics are defined as follows:
where \( u_t \) is terminal velocity and \( a_0 \) is the initial acceleration of the sliding mass.

We study the existing predictive equations for 2D (in section) characteristic tsunami amplitude generated by submarine landslides. The term characteristic amplitude refers to the maximum initial tsunami amplitude (\( \eta_{\text{max}} \) in Fig. 2). To find the effect of each landslide parameter (i.e. length, width, thickness, slope angle, initial slide submergence depth and slide specific gravity; Fig. 2) on the maximum initial wave amplitude, sensitivity analyses were conducted. We plotted the predicted wave amplitudes against single parameters in 2D plots (i.e. the wave amplitude versus each landslide parameter) and a combination of two parameters in 3D plots (e.g. the wave amplitude versus length of slide and slope angle). The mathematical toolbox of MATLAB (version: R2018a; Mathworks 2019) was used for our 2D and 3D analyses.

In the next stage, we evaluate the performance of the predictive equations by using them to reproduce the maximum initial wave amplitudes of two case studies, the 1994 Skagway and 1998 PNG, and compare the predicted values with the real available measurements for both events. The measured data from two tsunamis provide a valuable benchmark for evaluating the performance of different predictive equations. However, considering the different values reported for the initial landslide parameters in the literature, we used a sensible range for each landslide parameter.

We develop a new predictive equation for estimating the maximum initial wave amplitude by fitting an equation to the actual data from landslide tsunami incidents. The linear regression methodology is applied for deriving the new equation, which is the most widely used statistical technique for estimating cause–effect relationships (Iquebal and Himadri 2012). Our linear regression was performed through the stochastic optimization technique of genetic algorithm (GA) (Mathworks 2019). The GA Toolbox employs a cost function

\[
s_0 = \frac{u_t^2}{a_0} \tag{1}
\]

\[
t_0 = \frac{u_t}{a_0} \tag{2}
\]
to build a set of versatile routines for implementing a wide range of genetic algorithm methods.

Table 1 provides the data for the landslide parameters of each event based on the existing literature including slide specific gravity ($\gamma_s$), initial slide submergence ($d$), slide length ($B$), width ($w$), thickness ($T$) and slope angle ($\theta$). A representative value for each landslide parameter was defined, which is the most likely value for each parameter (Table 1); these representative values were used to reproduce a corresponding wave amplitude by the predictive equations rather than a range of amplitude given by the parameter range. To reproduce the highest and the lowest possible wave amplitudes using the proposed equations, we considered all combinations of the ranges of the initial landslide parameters. For this, the ‘ndgrid’ function in MATLAB (Mathworks 2019) was employed to transform the specified domain for six landslide parameters into arrays, which is ultimately used as inputs for the existing predictive equations.

3 Existing predictive equations

Several equations have been proposed for the prediction of the maximum initial wave amplitude ($\eta_{\text{max}}$; Fig. 2) generated by submarine landslides. Table 2 lists seven such predictive equations. The equations by Watts et al. (1998, 2003, 2005) and Jilani and Ataie-Ashtiani (2008) benefit from the following fundamental relationships for terminal velocity ($u_t$) and initial acceleration ($a_0$) of landslides:

$$u_t = \sqrt{gB\frac{\pi(\gamma_s - 1)\sin \theta}{2C_d}}$$  \hspace{1cm} (3)

$$a_0 = g\frac{\gamma_s - 1}{\gamma_s + C_m}\sin \theta$$  \hspace{1cm} (4)

where $B$ is slide length, $C_d$ is the drag coefficient, $\theta$ is the angle of the slope, $\gamma_s$ is the slide specific gravity, $g$ is gravitational acceleration, and $C_m$ is the added mass coefficient. In this study, we assumed no deformation for the sliding mass; consequently, the shape-related

| Parameter | 1994 Skagway; range (representative value) | 1998 PNG; range (representative value) |
|-----------|-------------------------------------------|---------------------------------------|
| $T$ (m)   | 10–20 (15)                                 | 500–900 (760)                         |
| $B$ (m)   | 100–600 (350)                              | 3000–7000 (4500)                     |
| $w$ (m)   | 330–390 (360)                              | 3000–7000 (5000)                     |
| $d$ (m)   | 26–155 (88)                                | 1000–2000 (1500)                     |
| $\gamma_s$ | 1.75–1.85 (1.8)                           | 1.9–2.2 (2.15)                       |
| $\theta$ (°) | 9–26 (17.5)                             | 5–15 (8)                            |

Reference Campbell (1995), Kulikov et al. (1996), Watts et al. (2003), Campbell and Nottingham (1999) Heinrich et al. (2001), Synolakis et al. (2002), Heidarzadeh and Satake (2015)

* $T$, slide thickness; $B$, slide length; $w$, slide width; $d$, initial submergence depth; $\gamma_s$ is the specific gravity of slide material; $\theta$ slope angle
Table 2 Existing predictive equations for estimating the maximum initial wave amplitude of submarine landslides

| Authora | Equationb | Approach | Range of validity |
|---------|-----------|----------|------------------|
| HRB-92  | \( \eta_{\text{max}} = \frac{\tau_B}{2\mu C_0} \) | Analytical–numerical | \( X_f < x - B - C_0 \) |
| HRB-92  | \( \eta_{\text{max}} = \frac{\tau_B}{2\mu C_0(1 - F_s)} \) | Analytical–numerical | \( X_f > B \) |
| WTS-98  | \( \eta_{\text{max}} = \left( \frac{0.338B}{(\sqrt{\frac{gd}{\mu}})} \right) \) | Experimental | \( 3 < \frac{\eta_{\text{max}}}{\eta_{i, \text{tr}}} < 4.5 \) |
| GRL-02c | \( \eta_{\text{max}} = 0.217 + \left( 3.83 \frac{B}{w} \right) - 0.632 \left( \frac{w}{B} \right)^2 \) | Numerical | N/A |
| MRT-03  | \( \eta_{\text{max}} = 0.3945V \) | Regression to real events data | N/A |
| WTS-03  | \( \eta_{\text{max}} = 0.2193T \left( 1 - 0.754 \sin \theta + 0.1704 \sin^2 \theta \right)(\frac{\theta \sin \theta}{d}) \) | Experimental | N/A |
| WTS-05  | \( \eta_{\text{max}} = S_0 \left( 0.05741 - 0.0431 \sin \theta \right) + \left( \frac{T}{B} \right) \left( \frac{B \sin \theta}{d} \right)^{1.25} \) | Experimental | \( \theta \in [5, 30]^\circ \) |
| JIL-07  | \( \eta_{\text{max}} = S_0 f_1 \left( \frac{T}{B}, \theta \right) \left( \frac{d}{r} \right)^{f_2} \) | Experimental | \( d/d_{\text{ref}} \in [0.06, 3] \) |

\( \eta_{\text{max}} \) is wave zero-to-trough (or zero-to-crest) amplitude, \( B \) and \( w \) are the slide length and width, \( d \) is the initial slide submergence, \( V \) is the volume of slide (for MRT-03 equation, the input value for \( V \) must be in \( 10^6 \text{ m}^3 \)); \( \tau \) is the shear stress; \( \theta \) is the angle of the slope; \( \gamma_\mu \) is the specific gravity of water; \( \gamma_\mu \) is a nondimensional parameter that depends on the initial submergence depth \( d \) and the slope angle \( \theta \). \( C_H \) is a function of the shape of the sliding mass and the slope angle \( \theta \). We note that the type of the sliding mass, either a solid block or a deforming material, affects the characteristics of the generated waves (Watts et al. 2005). Based on the research by Grilli and Watts (2005), the maximum initial wave is larger for deformable slides in comparison to solid block failures.

Table 2 reveals that these equations have been developed employing different approaches such as experimental, analytical and numerical methods. Some equations require two/three initial landslide parameters for predicting the maximum initial wave amplitude such as those by Murty (2003) and Grilli et al. (2002), whereas other equations use five/six initial landslide parameters; e.g. those by Watts et al. (2005) and Jilani and Ataie-Ashtiani (2008). Except for the equations by Murty (2003) and Grilli et al. (2002), the other five equations directly or indirectly include six landslide parameters: the slide specific gravity \( (\gamma_s) \), the initial slide submergence \( (d) \), length of the slide \( (B) \), width of the slide \( (w) \), thickness of the slide \( (T) \) and angle of the slope \( (\theta) \). According to Table 2, the slide length \( (B) \) and thickness \( (T) \) appear in most equations. We note that slide width \( (w) \) is taken as a unit since many of

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Footnotes:

1. Abbreviations are: Harbitz and Pedersen (1992), HRB-92; Watts (1998), WTS-98; Grilli et al. (2002), GRL-02; Murty (2003), MRT-03; Watts et al. (2003), WTS-03; Watts et al. (2005), WTS-05; Jilani and Ataie-Ashtiani (2008), JIL-07.  
2. HRB-92, WTS-98, GRL-02c, MRT-03, WTS-03, WTS-05, JIL-07.  
3. In this dimensionless coefficient of the failing mass, i.e. the drag coefficient \( (C_D) \) and the added mass coefficient \( (C_m) \), are constant and assumed to have the value of 1. \( C_m \) is a nondimensional parameter that depends on the initial submergence depth \( (d) \) and the slope angle \( (\theta) \). \( C_H \) is a function of the shape of the sliding mass and the slope angle \( (\theta) \). We note that the type of the sliding mass, either a solid block or a deforming material, affects the characteristics of the generated waves (Watts et al. 2005). Based on the research by Grilli and Watts (2005), the maximum initial wave is larger for deformable slides in comparison to solid block failures.
these equations are developed for 2D-in-section cases; thus, slide width is hidden in the equations. This indicates the importance of slide volume \( V = B \times w \times T \) for the prediction of the maximum initial tsunami amplitudes generated by submarine landslides. It is noted that equations with a small number of initial landslide parameters (e.g., Murty 2003) could be helpful when only minimum information is available for a particular event.

As the wavelengths of landslide-generated waves are shorter compared to earthquake-generated waves, they show greater dispersive effects (Watts et al. 2005; Heidarzadeh et al. 2014, 2020; Yalciner et al. 2014). While the phase velocity \( c \) of long oceanic waves (representing earthquake tsunamis) takes the form of \( c = \sqrt{gh} \), in which \( h \) is water depth, it becomes \( c = \sqrt{gL} \) for short waves (representing most landslide tsunamis) where \( L \) is the wavelength indicating that \( c \) is a function of \( L \). Therefore, longer waves travel faster than shorter waves for landslide tsunamis, which causes wave dispersion. The three equations by WTS-98, -03 and -05 (see Table 2) account for the effects of dispersion through introduction of the Ursell parameter \( U \) in their formulations:

\[
U = \frac{HL^2}{h^3}
\]  

where \( H \) is wave height and \( h \) is water depth. The dispersion effect is also considered by HRB-92 (Table 2) by introducing a dispersion relation (see Eq. 25 in Harbitz and Pedersen 1992) in their equations.

### 4 Sensitivity analysis

In order to determine how different values of landslide parameters affect wave amplitude, we conducted a series of sensitivity analyses. Figures 3 and 4 present the results of the sensitivity analyses for single landslide parameters. Regarding the slide length \( B \) (Fig. 3a), it can be seen that all equations show a direct linear relationship between slide length and tsunami amplitude. For slide length in the range of 400–900 m, wave amplitudes predicted by most equations are up to 20 m, whereas the equation by Watts (1998), WTS-98, predicts approximately three times larger tsunami amplitudes than MRT-03 and HRB-92. The sensitivity analysis for the initial slide submergence \( d \) (Fig. 3b) shows that a rise in submergence depth causes a reduction in wave amplitude in an exponential manner. The slide width \( w \) and wave amplitude are directly related to each other (Fig. 3c). However, the amplitude predictions by Harbitz and Pedersen (1992), HRB-92, are constantly larger than those given by other equations. According to Fig. 3d, there is a direct linear relationship between slide specific gravity \( \gamma_s \) and wave amplitude. Although the equations by WTS-05 and JIL-07 predict similar values, the equations by WTS-98 and HRB-92 deliver five and 10 times higher amplitudes, respectively.

Regarding the slide thickness \( T \), although all proposed equations show a direct relationship between wave amplitude and slide thickness, JIL-07 provides larger amplitudes with an exponential curve. For example, for the slide thickness of 50 m, the predicted amplitude varies in the range of 4.5–21.5 m (Fig. 4a). With regard to slope angle, Fig. 4b reveals contradictory behaviours in wave amplitude as slope angle changes: HRB-92 and WTS-98 indicate a direct linear relationship between slope angle and wave amplitude with gradually rising rates, whereas WTS-05 shows an increase in wave amplitude for \( \theta = 15–35^\circ \) in an exponential manner followed by amplitude decrease for \( \theta = 35–60^\circ \). As compared to WTS-05, JIL-07’s equation behaves in an opposite manner.
Figures 5, 6 and 7 give the results of sensitivity analyses for two landslide parameters simultaneously. Based on Fig. 5, all the existing equations show that an increase in the slide length (B) combined with a decrease in the initial slide submergence (d) results in an increase in wave amplitude. However, these equations (i.e. WTS-98, WTS-03, WTS-05, and JIL-07) do not yield similar results. Given same ranges for initial landslide parameters, JIL-07 reaches a value of approximately 60 m for the maximum wave amplitude, while WTS-03 gives a value of approximately 12 m. Figure 6 examines the effects of the slide length (B) and the slope angle (θ) on wave amplitude. The results indicate that the equations by WTS-05, WTS-03 and HRB-92 produce an increase in wave amplitude by simultaneous increase in B and θ. For WTS-98, there are three limitations for its range of validity; one of the limitations implies that the slope angle must be 45 degrees; consequently, the effect of slope angle on wave amplitude is not given by WTS-98. A direct linear relationship between slide length and wave amplitude is seen for WTS-98. In contrast with other equations, JIL-07’s equation behaves differently where an increase in slope angle results in a decrease in wave amplitude. The effects of specific gravity (γ_s) and slide length (B) on wave amplitude are illustrated in Fig. 7. An increase in the length and specific gravity
of the slide (i.e. $\gamma_s$ and $B$, respectively) is followed by an increase in the maximum wave amplitude (Fig. 7). Although three equations (i.e. WTS-98, WTS-05, and JIL-07) demonstrated similar behaviour, the maximum wave amplitudes are significantly different. The WTS-05 and JIL-07 equations predict maximum wave amplitudes of 15–30 m, whereas WTS-98 predicted a maximum wave amplitude of approximately 120 m.

Overall, sensitivity analyses indicate that the slide volume ($V$) and the initial slide submergence ($d$) are among the most sensitive parameters on the maximum initial landslide wave amplitudes: higher slide lengths and lower initial slide submergence depths lead to higher wave amplitudes. Our study shows that the predictions made by different equations for the maximum initial landslide wave amplitudes vary in a wide range and are sometimes divided by one order of magnitude. This clearly highlights the need for further experimental and numerical studies on the subject.

5 Performance to the 1994 Skagway and 1998 Papa New Guinea tsunamis

The 1994 Skagway, Alaska landslide tsunami event, which destroyed 300 m of the railway and claimed the life of one construction worker, was applied as one of the benchmark events here. This event occurred at low tide, and no earthquake was reported at that time. A tide gauge belonging to the National Oceanic and Atmospheric Administration of the USA, located in Skagway harbour (see Fig. 1d, e), recorded approximately 1 m zero-to-crest wave amplitude with a wave period of ~3 min (Fig. 1f) (Kulikov et al. 1996). Watts et al. (2005) reported 2.1 m as the maximum initial wave amplitude. Therefore, we considered a range from 1 to 2.1 m as the observed initial wave amplitude for this event (Table 3, first row; Fig. 8a, pink lines). We employ the existing predictive equations (Table 2) to estimate the maximum amplitude of the initial landslide tsunami wave using the initial landslide parameters given in Table 1. The outcome is shown in Table 3 and Fig. 8a. In Table 3, the representative wave amplitudes correspond to the representative values for initial landslide
The predicted values range from 0.03 to 686.5 m giving five orders of magnitude difference between the minimum and maximum predictions. In terms of representative wave amplitudes, the predictions are in the range of 0.8–19.4 m (Table 3, last column; Fig. 8a, yellow circles), whereas observed tsunami amplitude was 1.0–2.1 m. Most existing equations, except for WTS-98, fairly reproduce the observation (Fig. 8a).

The second case study is the tsunami that struck Papua New Guinea (PNG) in 1998, where a moderate earthquake ($M_w = 7$) triggered a destructive landslide tsunami (Fig. 1a). The landslide-generated waves caused the destruction of three villages and the loss of more than 2100 people (Tappin et al. 2008; Heinrich et al. 2001; Synolakis et al. 2002). The tsunami was recorded on a few tide gauges in the far-field including stations of Lombrum (PNG), Rabaul (PNG), Malakal Island (Palau) and Yap Island (State of Yap) located 600–1450 km from the source, but no near-field tide gauge record is available (Heidarzadeh and Satake 2015). The tsunami recorded a maximum trough-to-crest wave height of 3–9 cm at the far-field tide gauges (Heidarzadeh and Satake 2015). In the near field, tsunami run-up was up to 15 m (Synolakis et al. 2002) (Fig. 1c). The 1998 PNG tsunami was a turning point in tsunami research since it showed that submarine......
mass failures could cause deadly tsunamis (Tappin et al. 2008). Based on the existing knowledge, the maximum initial wave amplitudes were reported as 11 m and 16 m by Heinrich et al. (2001) and Synolakis et al. (2002), respectively. Therefore, we considered a value in the range of 11–16 m as the observed initial wave amplitude for the PNG event (Table 4, first row; Fig. 8b, pink lines). The initial slide parameters are based on those shown in Table 1. The results of the predictions are given in Table 4 and Fig. 8b.
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In summary, the two benchmark tests reveal that the estimates of initial landslide waves made by different equations are divided by a few orders of magnitude. The predictions for the smaller-size event (i.e. 1994 Skagway) appear to be more accurate than those made for the larger event (i.e. 1998 PNG). The reason that many existing equations fail in the accurate prediction of larger-scale events could be due to the lack of enough physical studies in this field, in particular, the absence of enough large-scale landslide investigations. The equation by WTS-03 shows better performance in predicting the maximum initial wave amplitudes for both case studies. However, WTS-03’s equation requires several initial landslide parameters which may make it difficult to use this equation for events with limited information. Some models (e.g. MRT-03) show a good performance in predicting the 1994 event (Fig. 8a) but fail for the 1998 case (Fig. 8b); this could be attributed to the rather complex physics of submarine landslide failures and the limitations of the existing equations. Our results may reveal the importance of large-scale or field-scale laboratory experiments on landslide tsunamis; such tests are costly, but it appears they are necessary to advance the knowledge of landslide tsunamis.

Fig. 7 Effects of slide specific gravity ($\gamma_s$) and slide length ($B$) on the maximum initial landslide wave amplitude resulting from equations of Watts et al. (2005) (WTS-05), Watts (1998) (WTS-98) and Jilani and Ataie-Ashtiani (2008) (JIL-98). In each panel, while two slide parameters are varied, the other parameters are kept constant. The values of the constant slide parameters are shown at the bottom-right corner of the figure for each panel.
Table 3 Maximum and minimum observed (first row) and predicted (later rows) initial wave amplitudes for the 1994 Skagway landslide tsunami

| Predictive equations<sup>a</sup> | Highest possible value of $\eta_{max} (m)$ | Lowest possible value of $\eta_{max} (m)$ | Range of parameters | Representative value (m) |
|--------------------------------|--------------------------------|--------------------------------|-------------------|-------------------------|
| Real observed value<sup>b</sup> | N/A | 2.1 | 1.0 | N/A | N/A |
| Predicted by equations | | | | | |
| HRB-92 | 6.4 | 1.4 | $B = 100–600$ m $w = 330–390$ m $\gamma_s = 1.75–1.85$ $T = 10–20$ m $\theta = 9–26 \, (°)$ | 3.5 |
| WTS-98<sup>c</sup> | 686.5 | 0.9 | $B = 100–600$ m $d = 26–155$ m $\gamma_s = 1.75–1.85$ $T = 10–20$ m $\theta = 9–26 \, (°)$ | 19.4 |
| GRL-02 | 6.0 | 2.1 | $B = 100–600$ m $w = 330–390$ m | 3.1 |
| MRT-03 | 1.8 | 0.1 | $B = 100–600$ m $w = 330–390$ m $T = 10–20$ m | 0.8 |
| WTS-03 | 13.7 | 0.03 | $B = 100–600$ m $d = 26–155$ m $T = 10–20$ m $\theta = 9–26 \, (°)$ | 1.6 |
| WTS-05 | 54.6 | 0.1 | $B = 100–600$ m $d = 26–155$ m $\gamma_s = 1.75–1.85$ $T = 10–20$ m $\theta = 9–26 \, (°)$ | 1.3 |
| JIL-07 | 121.7 | 0.4 | $B = 100–600$ m $d = 26–155$ m $\gamma_s = 1.75–1.85$ $T = 10–20$ m $\theta = 9–26 \, (°)$ | 2.9 |

<sup>a</sup>Abbreviations are: Harbitz and Pedersen (1992), HRB-92; Watts (1998), WTS-98; Grilli et al. (2002), GRL-02; Murty (2003), MRT-03; Watts et al. (2003), WTS-03; Watts et al. (2005), WTS-05; Jilani and Ataie-Ashtiani (2008), JIL-07

<sup>b</sup>The real observed values are based on Kulikov et al. (1996) and Watts et al. (2005)

<sup>c</sup>It is noted that the equation for Watts (1998) (WTS-98) is valid for a single value of $\theta = 45°$, whereas the slope angle for the 1994 Skagway failure was $9° – 26°$ (Campbell 1995; Kulikov et al. 1996)
A new empirical equation for the maximum initial wave amplitude of submarine landslide tsunamis

The sensitivity analysis of landslide parameters revealed that slide volume \( V \) and initial submergence depth \( d \) are among the most sensitive parameters for predicting the initial wave amplitude generated by a submarine landslide (Figs. 3, 4). Based on our sensitivity analysis, the maximum initial wave amplitude \( \eta_{\text{max}} \) is directly related to slide volume (i.e. \( V = B \times w \times T \)) but is inversely correlated with initial slide submergence (Fig. 3). Existing data available in the literature on tsunami events induced by submarine landslides (Table 5; Fig. 9) were used as the observational data for obtaining a new predictive equation. The genetic algorithm, in MATLAB (Mathworks 2019), was employed for deriving the following regression equation:

\[
\eta_{\text{max}} = 50.67 \left( \frac{V}{d} \right)^{0.34}
\]

where the slide volume \( V \) is in \( \text{Km}^3 \), the initial slide submergence \( d \) is in metres, and \( \eta_{\text{max}} \) is the maximum initial amplitude in metres. The two sides of Eq. (6) do not dimensionally agree because our purpose is to fit an equation to the existing field data, as is commonly done in the literature (e.g. equations by MRT-03 and GRL-02; Table 2). We note that the equation was fitted on some of the data listed in Table 5 (i.e. events of 1979 France; 1975 Canada; 1999 Turkey; 1994 PNG and 1994 USA). The new equation (Eq. 6)
Table 4. Maximum and minimum observed (first row) and predicted (later rows) initial wave amplitudes for the 1998 PNG landslide tsunami

|                  | Predictive equations | Highest possible value of $\eta_{\text{max}}$ (m) | Lowest possible value of $\eta_{\text{max}}$ (m) | Range of parameters | Representative value (m) |
|------------------|----------------------|-------------------------------------------------|-----------------------------------------------|--------------------|--------------------------|
| Real observed value$^b$ | N/A                  | 16.0                                            | 11.0                                         |                    | N/A                     |
| Predicted by equations |                      |                                                 |                                               |                    |                          |
| HRB-92           |                      | 110.0                                           | 21.9                                         | $B = 3000–7000$ m  | 42.7                    |
| WTS-98$^c$      |                      | 1553.8                                          | 39.2                                         | $B = 3000–7000$ m  | 222.5                   |
| GRL-02           |                      | 5.7                                             | 1.7                                          | $B = 3000–7000$ m  | 3.7                      |
| MRT-03           |                      | 17397.5                                         | 1775.3                                       | $B = 3000–7000$ m  | 6746.0                   |
| WTS-03           |                      | 75.5                                            | 1.2                                          | $B = 3000–7000$ m  | 8.7                      |
| WTS-05           |                      | 408.4                                           | 8.3                                          | $B = 3000–7000$ m  | 59.7                     |
| JIL-07           |                      | 2081.5                                          | 103.8                                        | $B = 3000–7000$ m  | 567.8                    |

$^a$ Abbreviations are: Harbitz and Pedersen (1992), HRB-92; Watts (1998), WTS-98; Grilli et al. (2002), GRL-02; Murty (2003), MRT-03; Watts et al. (2003), WTS-03; Watts et al. (2005), WTS-05; Jilani and Ataie-Ashtiani (2008), JIL-07.

$^b$ The real observed values are based on Synolakis et al. (2002) and Heinrich et al. (2001).

$^c$ It is noted that the equation for Watts (1998) (WTS-98) is valid for a single value of $\theta = 45^\circ$, whereas the slope angle for the 1998 PNG failure was $5–15^\circ$ (Heinrich et al. 2001 Synolakis et al. 2002; Heidarzadeh and Satake 2015).
Table 5  The observation data of real tsunami landslide events. Some of these data were employed as the input data for obtaining the new equation (Eq. 6). Here, $V$ and $d$ are slide volume and submergence depth, respectively. The parameter $\eta_{\text{max, cal}}$ represents prediction made by our equation (Eq. 6), while $\eta_{\text{max, obs}}$ is the actual observed value.

| Event                      | $V$ (km$^3$) | $d$ (m) | Run-up (m) | $\eta_{\text{max, obs}}$ (m) | $\eta_{\text{max, cal}}$ (m) | References for observed values                          |
|---------------------------|--------------|---------|------------|-------------------------------|-------------------------------|---------------------------------------------------------|
| 1888 Ritter Island (PNG)  | 4–5          | 900     | 15         | 2.3                           | 8.4                           | Day et al. (2015), Johnson (1987)                        |
| 1741 Oshima–Oshima (Japan)| 2.4          | 1100    | 34         | 6.3                           | Ioki et al. (2019), Satake (2007) |
| 1946 Unimak (USA)         | 240          | 120     | 42         | 64.1                          | Watts et al. (2003), Okal et al. (2003) |
| 1975 Kitimat (Canada)     | 0.023        | 210     | N/A        | 4.2                           | 2.3                           | Murty (1979), Kirby et al. (2015)                        |
| 1979 Nice (France)        | 0.0022       | 40      | 2–3        | 1.4                           | 3.9                           | Murty (2003), Dan et al. (2007), Ioualalen et al. (2010) |
| 1994 Skagway (USA)        | 0.0032       | 88      | N/A        | 1.0–2.1                       | 1.6                           | Watts et al. (2003), Kulikov et al. (1996)              |
| 1998 PNG                  | 6            | 1500    | 15         | 11–16                         | 7.8                           | Heinrich et al. (2001), Synolakis et al. (2002)         |
| 1999 Izmit (Turkey)       | 0.0052       | 45–60   | N/A        | 1–2                           | 2.2                           | Tinti et al. (2006)                                     |
provides a preliminary prediction of the initial wave amplitude for potential submarine landslides for cases with minimum available landslide information. The linear regression for the new proposed equation yields an acceptable correlation coefficient of $R^2 = 0.92$. Table 5 and Fig. 10 compare the observed wave amplitudes ($\eta_{\text{max,obs}}$) with those predicted by Eq. (6) ($\eta_{\text{max,cal}}$). By applying our proposed equation to the two benchmark tests of the 1994 Skagway and 1998 PNG tsunamis, we obtained initial wave amplitudes of 1.6 m
and 7.8 m for these events, respectively, which are fairly close to the observed amplitudes (Table 5, column 6).

Here, we apply our Eq. (6) to two cases of submarine landslides to predict maximum initial wave amplitudes. Seismic and bathymetric surveys of submarine features provide valuable information for locating potential submarine landslides (e.g. Fu et al. 2017). As examples of previous geophysical investigations of the seafloor, Schwab et al. (2014) and Lindhorst et al. (2014) located potential submarine landslides offshore western Thailand (Andaman Sea) and inside Lake Ohrid (Macedonia/Albania), respectively. We benefit from landslide scenarios A and B of Schwab et al. (2014) for the Andaman Sea with respective volumes of 11.1 and 0.07 km³ at water depths of 880 and 1100 m, respectively (Table 6). For the lake Ohrid, Lindhorst et al. (2014) reported a slide with a volume of 0.063 km³ at a depth of 120 m. Our new proposed equation (Eq. 6) is applied to estimate the maximum initial wave amplitudes which resulted in predicted initial wave amplitudes of 11.4 and 1.9 m for the two landslides scenarios A and B of the Andaman Sea and 3.9 m for the Lake Ohrid (Table 6).

7 Conclusion

Existing equations for the prediction of maximum initial submarine landslide-generated waves were examined through sensitivity analyses and benchmark tests. The main findings are as follows:

- Among all involved parameters in the existing predictive equations, slide volume ($V$) appears in most equations, thus emphasising its importance in estimating the maximum initial wave amplitude generated by submarine landslides.
- The predictions of wave amplitude through existing equations are divided by a few orders of magnitude. In particular, the values range from 0.03 to 686.5 m for the prediction of the maximum initial amplitude of the 1994 Skagway and from 3.7 to 6746.0 m for the 1998 Papua New Guinea (PNG) tsunamis. The observed amplitudes for the two aforesaid events are in the ranges of 1.0–2.1 m and 11–16 m, respectively.
- By applying the existing predictive equations to the two cases of 1994 Skagway and 1998 PNG landslide tsunamis, we witnessed a better agreement between the observed and predicted maximum initial wave amplitudes for the small-scale landslide tsunami.

| Location of potential submarine landslide | $V$ (km³) | $d$ (m) | $\eta_{\text{max,cal}}$ (m) |
|------------------------------------------|-----------|--------|-----------------------------|
| Andaman Sea, Western Thailand            | 11.1*     | 880*   | 11.4                        |
|                                           | 0.07**    | 1100** | 1.9                         |
| Lake Ohrid, Macedonia/Albania***         | 0.063     | 120    | 3.9                         |

*Landslide data based on scenario A in Schwab et al. (2014)
**Landslide data based on scenario B in Schwab et al. (2014)
***Landslide data from Lindhorst et al. (2014)
(i.e. 1994 Skagway). For the large-scale tsunami (i.e. 1998 PNG), the predictions are scattered in a wider range. This may emphasize the essentiality of conducting large-scale or field-scale laboratory experiments on landslide-induced tsunamis.

- We put forward a new equation by benefiting from the existing information from previous real cases of submarine landslide tsunamis: 
\[ \eta_{\text{max}} = 50.67 \left( \frac{V}{d} \right)^{0.34}, \]
where \( V \) is slide volume (km\(^3\)), \( d \) is initial submergence depth (m), and \( \eta_{\text{max}} \) is in metres. This new equation resulted in wave amplitudes of 1.6 m and 7.8 m for the 1994 Skagway and 1998 PNG tsunamis, respectively, indicating fair agreement with real observed values for these tsunamis.

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**Compliance with ethical standards**

**Conflict of interest** The authors declare that they have no competing interests. The data and material used in this research are available in the body of the article.

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