Damage recognition based on Generalized Genetic Algorithm

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Abstract—A new optimization approach to solving problems concerning structural damage identification with finite element model is introduced. In this approach, the generalized genetic algorithm is applied to damage recognition with its advantage of directed evolution and two-phase transformation. To employ the method, objective function is constructed by the residual of modal frequency and incomplete modal mode. Meanwhile the recombination and mutation operators are also improved with sparsity constrain. The results show that modified generalized genetic algorithm performs better in convergence speed and accuracy than traditional optimization approach in damage identification.

1. INTRODUCTION
Structural systems are always subjected to variable damage and deterioration during their service life because of environmental and operational factors[1], resulting in unbearable losses of both personal and property. Thus, Structural health monitoring (SHM) becomes significantly important to ensure lifetime safety of these structures. Structural damage identification is the focal point of SHM, consisting of damage existence, localization and severity identification. As the vibration-based methods are endowed with the advantage of convenient data accessing and notable efficiency, they have become a dominant research topic in recent years[2].

The Finite Element Model(FEM) method is a standard tool for modelling structural behavior by using test data to modify the analytical model towards real structures[3]. Different from the traditional damage index, it can infer precise damage location and severity simultaneously[4]. When FEM is adopted, damages are iterated by a certain optimization algorithm, until the objective function reaches the minimum value. Thus, the identification problem transforms into a constrained optimization problem. Teughels successfully used the FEM on damage identification of the Z24 bridge[5], while Nizar compared different objective functions for FEM method on the IASC-ASCE benchmark[6].

The Genetic Algorithm(GA) is an evolutionary optimization method using different individuals called chromosomes to imitate structural parameters. Population evolves iteratively towards a better solution in a process inspired by a nature evolution. Tran-Ngoc applied GA in damage identification of NAMO bridge with FEM[7]. Adam integrated GA with Gaussian mixture models for the damage detection of Z-24 bridge[8]. However, SHM problems usually have characteristics incompatible with traditional GA, and mismatch may arise in directly application[9-10]. So it is important to consider features of SHM for the improvement of GA in SHM[11-12].

This paper presents a new method using Generalized Genetic Algorithm(GGA) combined with sparsity constrains. Its objective function is the residual of modal data between real damaged structure and finite model. According to the numerical experiment, GGA shows better accuracy with a faster convergence speed.
2. THEORY

2.1. Fundamental of the vibration system

The eigenvalue equation of a structure with n degrees of freedom can be written as

\[(K - \lambda_i M)\varphi_i = 0\] (1)

Where \(K\) and \(M\) are the stiffness and mass matrices of the whole structure, \(\lambda_i = \omega_i^2\), \(\omega_i\) stands for the \(i\)th modal frequency, while \(\varphi_i\) stands for the \(i\)th modal mode.

In most cases, the damage caused by degradation and cracks only results in losses of stiffness, while the mass matrices remained unchanged. So the real dynamic characteristic is represented by the following equation:

\[du = -\Delta K KK d\]

\[(K - \lambda_i M)\varphi_i = 0\] (2)

Where \(\Delta K\) is loss of stiffness matric, consisting of each element which can be further expressed as

\[\Delta K_k = K_k - Ke_k K_e\] (4)

Where \(Ke_k\) and \(Ke_e\) are the stiffness matrices of element \(k\) before and after damage. \(\alpha_k\) refers to the degree of stiffness loss, ranging from 0 to 1.

Using FEM analysis software like ANSYS, the \(i\)th modal parameter \(\lambda_i^*, \varphi_i^*\) can be obtained from any model input. During model updating, the following equation is used to determine the residual between real damaged structure and finite model:

\[g_i = MAC_i(1 - MAC_i) + \gamma_i \times ||\lambda_i - \lambda_i^*||\] (i = 1, 2, ..., n) (5)

Where \(n\) refers to the total modal order in use, and \(\beta_i\), \(\gamma_i\) are calculation parameters determined before iteration. \(MAC_i\) is modal assurance criterion of the \(i\)th modal mode, signifying the correlation of real damaged structure and finite model:

\[MAC_i = \left(\varphi_i^T \varphi_i\right)^{1/2} \left(\varphi_i^d \varphi_i^d\right)^{1/2}\] (6)

When the finite model literally equals to the damaged structure, equation (5) has a value of zero. It is apparent that the further FEM towards real structure, the larger value of equation (5). So this function can be useful in damage identification. While the full mode is not required in this function, extra error originated from order-extended is avoided compared to traditional damage index method. Considering the fact that we can only acquire a few modal orders from the vibration test on actual structures, solving of equation (5) becomes severely ill-posed and ill-conditioned. So it is necessary to transform it into an optimization problem and apply the objective function combined with sparsity restrains, which can be expressed as follow:

\[\min f(\alpha) = ||a||\]

s.t. \(g(\alpha) = 0\)

\[0 \leq \alpha_k \leq 1\] (i = 1, 2, ..., \(N_e\)) (7)

Where \(N_e\) is the number of potential damage elements. 1-norm is used to promote the imitation of sparsity of real damage condition[13].

To solve constrained optimization problems like equation (7), method of Lagrange Multiplier is applied to change the original problem into equation (8).
\[ \text{min. } F = \|\alpha\|_p + \sum_{i=1}^{N_e} \lambda_i g_i(\alpha) \]

s.t. \( 0 \leq \alpha_k \leq 1 \) \( (k = 1, 2, \cdots, N_e) \) \[ (8) \]

And the fitness function can be constructed as follow:

\[ \Psi = \frac{1}{1 + F} = \frac{1}{1 + \|\alpha\|_p + \sum_{i=1}^{N_e} \lambda_i g_i(\alpha)} \]

\[ (9) \]

2.2. Generalized Genetic Algorithm

To overcome the weakness in GA, Dong adapted GGA from GA to deal better with structural damage identification and optimization design, and he successfully applied it to the optimal placement of sensors on the Hong Kong Tsing Ma Bridge[14]. GGA get rid of the traversal search used in traditional stochastic optimization method including GA, and turns to directional evolution instead. Combined with population isolation mechanism, two-phase evolution, 2/4 competition between parents with children, and self-adopted operator, GGA outperforms GA in global optimization of Structural damage identification.

Like the real evolution in nature, GGA divides the evolution program into gradually-change phase and suddenly-change phase, which are quite different in operators. In gradually-change phase, population moves towards local optimal solution thorough global domain, mainly by recombination and selection. In suddenly-change phase, population reaches the local peak rapidly in each local optimal area. Throughout the program, the gradually-change phase dominates the evolution while the suddenly-change phase takes the rest. The transformation between phases is triggered once the local convergence occurs or the cluster head is renewed.

2.3. GGA combined with sparsity constrains

In engineering practice, the majority of damage occurs only in several structural elements, which is relatively quite a small part of the whole building. Hence, it is essential to consider the sparsity restrains in FEM to imitate the structural behaviour better. Operators used in adopted GGA are introduced as follow.

a) Selection Operator. For the purpose of fulfilling directional evolution, GGA conducts selection after every recombination or mutation. Both children and parents are sorted by their objective function and only the top two enter into the next step, as is called 2/4 competition. Before recombination or mutation, random sampling without replacement is employed to make full use of every chromosome in population.

b) Recombination Operator. Recombination operator differs in the two phases of evolution. In gradually-change phase, one-point crossover is conducted while multiple-point crossover is executed between two parents in suddenly-change phase. The top two with lowest value of objective function win the 2/4 competition and the others are abandoned.

With the recombination going on, the emerging children become random in wide distribution, in which most of them are far beyond the feasible region. For the characteristic of GGA, only the well-behaved individuals survive after the 2/4 competition, so it is essential to improve the quality of children after recombination. To quantify the quality, a new criterion is introduced by sparsity constrains to judge whether the chromosome is too far from initial status.

\[ F_{\text{iter}} \leq 2F_0 \] \[ (10) \]

Where \( F_{\text{iter}} \) is the value of equation (8) for children during recombination, and \( F_0 \) represents the initial gap between finite model and damage structure. If the children do not satisfy equation (10), we will repeat the operator with chromosome that stands for no damage, until the constrain is fitted.
3. IMPLEMENTATION RULES
The implementation rules of GGA with sparsity constrains are as follows:

a) Initialization. Input scale of each subpopulation \( N \), number of subpopulations \( m \), convergence parameter \( p \) and the other parameters in need. Set the generation \( k = 0 \). Divide the whole domain into several sub-domains with the initialized population. Then conduct the following evolutionary procedure in each subpopulation.

b) Evolution of subpopulations. Use different tactic according to the phase of subpopulation. The following steps are examples from the gradually-change phase:

- Select the parents \( V_1, V_2 \) from the subpopulation \( V_i(k) \).
- Apply recombination operator on \( V_1, V_2 \) to get two children. If they do not satisfy the condition of equation (10), extra-recombination is performed until the constrain is fitted. Then set them as \( S_1', S_2' \).
- Do 2/4 competition with \( V_1, V_2, S_1', S_2' \), and the survivals are \( S_1, S_2 \).
- Apply mutation operator on \( S_1', S_2' \), get the children \( S_1, S_2 \).
- Do 2/4 competition with \( S_1', S_2', S_1, S_2 \) and the survivals enter the parent population.
- For the suddenly-change phase, steps vary according to the information above.

c) Transformation. Examine every subpopulation. Change the phase into suddenly-change once local convergence occurs, and change back if the cluster head varies.

d) Adjust the calculation parameters. When the generation reaches the given value, set limit on range of parameters, and initiate the modification of the gene mutation probability.

e) Exchange between subpopulation. If \( k \) reached \( 10q(q = 1, 2, \ldots) \), move the best individual into each subpopulation to multiply excellent chromosome and accelerate convergence speed.

f) Termination examination. Check the global optimization constancy for convergence. If the best chromosome remains unchanged after continuous \( p \) generation, then the algorithm is stopped. Otherwise, \( k = k + 1 \), return to step (b) and continue with evolution process.

4. NUMERICAL EXPERIMENT

4.1. Two-dimension truss model
To verify the new algorithm in this paper, a two-dimension truss model composed of 25 elements is used for numerical experiment. The specific number of each node and element are showed in figure 1.

Casted in steel, the model density of all the elements is \( \rho = 7800 \text{ kg/m}^3 \), with elastic modulus of \( E = 206 \text{ GPa} \). Different kind of sections are displayed in table 1. It is assumed that when damage occurs, only the stiffness of elements decay while the mass matrices remain unchanged. The specific damage scenarios are presented in table 2.

We set three different types of damage, distinguished from each other as single element damage, little element damage, and multiple element damage. Under single element damage situation, only the element 5 suffers degradation, while the element 2, 9, 18 suffer degradation in little element damage
and six different elements damaged in multiple case. The top eight order of modes are used in calculation.

4.2. Identification Results

![Figure 1](https://via.placeholder.com/150)

**Figure 1.** Example of a figure caption.

| Element | Cross-section Area (m²) |
|---------|-------------------------|
| 1~6     | 0.0018                  |
| 7~12    | 0.0015                  |
| 13~17   | 0.0010                  |
| 18~25   | 0.0012                  |

**Table 1. Cross sections of the truss structure**

| Damage Cases | Damage Element | Specific Damage Situation |
|--------------|----------------|---------------------------|
| 1            | 5              | 5: 25%                    |
| 2            | 2,9,18         | 2,9: 25%; 18: 35%         |
| 3            | 2,4,24         | 2,4,24: 25%; 9,15,19: 35% |

**Table 2. Damage cases of the truss structure**

To compare GGA method with traditional GA in structural damage identification, these methods are first applied to damage case 1 with no noise.

| Algorithm          | Average Generations to Convergence | Accuracy |
|--------------------|-----------------------------------|----------|
| GA                 | 91                                | 60%      |
| GA with elitist preservation | 66                                | 100%     |
| GGA                | 31                                | 100%     |

**Table 3. Identification results for the 1st damage case with different algorithm**
Table 3 indicates that if there is no destabilization and complete modal mode is used, all the algorithms achieve a perfect recognition of the damage case 1. The only difference is the generation and the time towards convergence. However, in real life we can never get modal data without any noises with the complete mode shape, so it is essential to test our optimization method in complex engineering conditions.

After the noise added to the modal mode of the first eight orders, we execute the identification of the three cases with data in two thirds of the total freedoms. The results are shown in figure 3-5. It is obvious that when no noise is added, all the three damage scenarios can be perfectly identified without wrong detection, while the situation varies after noises are added to modal mode applied. For case one and case two, the noises do not affect the results much, but for scenario 3 where there are many
elements decayed, the recognition of element 4 and 17 becomes inaccurate in severity. The results show that GGA can be useful in damage identification with not too much noise, especially in scenarios with fewer damage elements.

5. NUMERICAL EXPERIMENT

This paper puts forward a new algorithm based on GGA combined with sparsity restrains for the optimization problems organized by FEM in structural damage recognition. The conclusions are follows.

a) GGA performs better than traditional GA method in accurate and convergence speed for damage identification problems because of its directivity and sparsity constrains.

b) When there is no noise, GGA has the ability to achieve unmistakable recognition, with limited ability in low-noise situation.

c) This method may fail to find all the potential damage elements when there are too many damage elements with severe noise.

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