A New Modified Conjugate Gradient Method and Its Global Convergence Theorem

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(Received March 24, 2013; Accepted June 25, 2013; Available online March 01, 2020)

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Abstract:

In this article, we try to proposed a new conjugate gradient method for solving unconstrained optimization problems, we focus on conjugate gradient methods applied to the non-linear unconstrained optimization problems, the positive step size is obtained by a line search and the new scalar to the new direction for the conjugate gradient method is derived from the quadratic function and Taylor series and by using quasi newton condition and Newton direction while deriving the new formulae. We also prove that the search direction of the new conjugate gradient method satisfies the sufficient descent and all assumptions of the global convergence property are considered and proved. In order to complete the benefit of our research we should take into account studied the numerical results which are written in FORTRAN language when the objective function is compared our new algorithm with HS and PRP methods on the similar set of unconstrained optimization test problems which is very efficient and encouragement numerical results.

Keywords: optimization, quadratic function, Taylor series, global convergence.
1. Introduction:

Conjugate Gradient methods (CG) contain a type of unconstrained optimization algorithms which are known by low memory requirements in strong and global convergence feathers.

This study focuses on conjugate gradient approaches for solving nonlinear unconstrained optimization problems [1].

Where \( f: \mathbb{R}^n \rightarrow \mathbb{R} \) is a continuously differentiable function, that is bounded from below, a nonlinear conjugate gradient method constructs a sequence using iteration \( x_k, k \geq 1 \) starting from an initial point \( x_0 \in \mathbb{R}^n \):

\[
x_{k+1} = x_k + s_k \quad \text{..........................(2)}
\]

\[
s_k = \alpha_k d_k \quad \text{..........................(3)}
\]

Where \( d_k \) is a search direction, and the positive step size \( \alpha_k \) is obtained by a line search such that \( \alpha_k > 0 \). Often the sharpest descent path is used in the first iteration, namely \( d_1 = -g_1 \), and the other search direction can be defined recursively [2]:

\[
d_{k+1} = -g_{k+1} + \beta_k d_k, \quad \text{..................(4)}
\]

Here \( \beta_k \) is the CG updates parameter and \( g_k = \nabla f(x_k)^T \) and different CG methods correspond to different scalar \( \beta_k \) choices. If we let \( \| \cdot \| \) represent the Euclidean norm and define \( y_k = g_{k+1} - g_k \), we may get a list of some options for the CG update parameter like thus:

\[
\beta_{HS}^k = \frac{g_{k+1}^T y_k}{d_k^T g_k} \quad \text{..........................Hestenes and Stiefel (1952) [3]}
\]

\[
\beta_{FR}^k = \frac{\| g_{k+1} \|^2}{\| g_k \|^2} \quad \text{..........................Fletcher and Reeves (1964)[4]}
\]

\[
\beta_{D}^k = \frac{g_{k+1}^T g_{k+1}}{d_k^T g_k} \quad \text{..........................Daniel (1967)[5]}
\]

\[
\beta_{PRP}^k = \frac{g_{k+1}^T y_k}{\| g_{k+1} \|^2} \quad \text{..........................Polak Ribiere and Polyak (1969)[6]}
\]

\[
\beta_{GD}^k = -\frac{g_{k+1}^T g_{k+1}}{d_k^T g_k} \quad \text{..........................Fletcher (1987)[7]}
\]
\[ \beta^L_k = \frac{g_{k+1}^T y_k}{-d_k^T g_k} \]  
Liu and Storey (1991) [8]

\[ \beta^B_k = \frac{||g_{k+1}||^2}{d_k^T y_k} \]  
Dai and Yuan (1999) [2]

\[ \beta^V_k = (y_k - 2d_k \frac{||y_k||^2}{d_k^T y_k}) g_k^T \frac{g_{k+1}}{d_k^T y_k} \]  
Hager and Zhang (2005) [9]

More detail see [10-12].

The global convergence qualities of CG algorithms are the topic of this research. We use \( \beta_k^{hs} \), \( \beta_k^{pr} \) and compare it with our new \( \beta_k \) which will be derived later, the condition in (5) is used to avoid non-convergence in nonlinear functions that are utilized with inexact line search:

\[ ||g_{k+1}|| g_k > 0.2 \|g^T_{k+1}\| \]  
(5)

Which is called Powell restart condition because if the scalar \( \beta_k \) appears negative these strategies will restart the descent direction on the all iteration.

The search direction obtained by the new method at each iteration satisfies the sufficient descent condition as we prove. In order to grantee the global convergence of non-linear conjugate gradient methods for the CG line search we are often used Wolfe conditions, here we denote the Standard Wolfe line search which is mean that the step length \( \alpha_k \) in equation (3) is obtained such that [13-15]:

\[ f(x_k + \alpha_k d_k) - f(x_k) \leq \rho \alpha_k g_k^T d_k \]  
(6)

\[ ||g^T_{k+1}d_k|| < -\sigma g_k^T d_k \]  
(7)

Where \( d_k \) is a descent direction and \( 0 < \rho \leq \sigma < 1 \). The strong Wolfe conditions consists of (6) and by rewrite the equation (7), see [16]:

\[ g^T_{k+1}d_k \geq \sigma g_k^T d_k \]  
(8)

but we are attention on shows whether there is a conjugate gradient method that converges under Wolfe’s standard conditions or not. We develop a new formula \( \beta_k \) to show that this new conjugate gradient method is globally convergent if the classic Wolfe requirements (6) and (7) are met.

2: The Derivation of a new Scaled CG Method:

The derivation of most CG method are based, in some way, to the quadratic function and then generalized to non-quadratic functions by restart procedures. Hence, we may assume that our objective function \( f(x) \) is a convex function, then the new method depends on the quadratic form:

\[ f(x) = \frac{1}{2} x^T Gx + b^T x + a \]  
(10)

Where \( G \) is the Hessian matrix and \( b = g \) and \( a \) is a constant, so that \( y_k = g_{k+1} - g_k = G s_k \) and by subsisting the value of \( x_{k+1} = x_k + s_k \) in (10) we get [17]:

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\[ f(x_{k+1}) = \frac{1}{2}(x_k + s_k)^T G(x_k + s_k) + b^T (x_k + s_k) + a \]
\[ f(x_{k+1}) = \frac{1}{2}(x_k + s_k)^T (Gx_k + Gs_k) + (b^T x_k + b^T s_k) + a \]
\[ f(x_{k+1}) = \frac{1}{2}(x_k^T Gx_k + x_k^T Gs_k + s_k^T Gx_k + s_k^T Gs_k) + (b^T x_k + b^T s_k) + a \]

And if we assume that \( a = 1/2(s_k^T Gs_k + x_k^T Gs_k) \) then:

\[ f_{k+1} = f_k + \frac{1}{2} s_k^T Gs_k + b^T s_k \]

\[ \text{As we get:} \]

\[ \text{From (13) & (14) we have:} \]

\[ y_k = s_k \frac{[-2g_k^T s_k - 2(f_k - f_{k+1})]}{s_k^T s_k} \]

\[ G = \frac{-2g_k^T s_k - 2(f_k - f_{k+1})}{s_k^T s_k} \quad \text{I}_{n \times n} \]

\[ G^{-1} = \frac{s_k^T s_k}{-2g_k^T s_k - 2(f_k - f_{k+1})} \quad \text{I}_{n \times n} \]

\[ G^{-1} = \frac{s_k^T s_k}{2(g_k^T s_k + f_k - f_{k+1})} \quad \text{I}_{n \times n} \]

\[ d_{k+1} = \frac{s_k^T s_k}{2(g_k^T s_k + f_k - f_{k+1})} g_{k+1} \]

and since the direction is Newton direction then \( d_{k+1} = -G^{-1} g_{k+1} \) then

\[ y_k = s_k \frac{[-2g_k^T s_k - 2(f_k - f_{k+1})]}{s_k^T s_k} \]

\[ y_k = s_k \frac{[-2g_k^T s_k - 2(f_k - f_{k+1})]}{s_k^T s_k} y_k^T g_{k+1} \]

\[ \text{From (13) & (14) we have:} \]

\[ y_k^T d_{k+1} = -y_k^T g_{k+1} + \beta_k y_k^T d_k \]

\[ \text{and since the new direction satisfying (4) so we have:} \]

\[ y_k^T d_{k+1} = -y_k^T g_{k+1} + \beta_k y_k^T d_k \]

From (13) & (14) we have:
\[
\frac{s_k^T s_k}{2(g_k^T s_k + f_k - f_{k+1})} y_k^T g_{k+1} = -y_k^T g_{k+1} + \beta_k y_k^T d_k
\] then:
\[
\beta_k = \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{s_k^T s_k}{2(g_k^T s_k + f_k - f_{k+1})} \right]
\]

\[\text{………………… (15)}\]

Then we have the new direction as \(d_{k+1} = -g_{k+1} + \beta_k d_k\), and \(\beta_k\) as we explain in equation (16) which is derived from the quadratic function.

3. Outlines Of The New Algorithm:

Step (1): Initialize select \(x_i \in \mathbb{R}^n\) and compute \(f(x_i), g(x_i)\), consider \(d_1 = -g_k\) and \(k = 1\).

Step (2): If \(\|g_k\| \leq \varepsilon\), stop. \(x_k\) is the optimal solution, else go to step (3).

Step (3): Compute \(\alpha_k\) satisfying the Wolfe conditions (6), (7) and update the variable \(x_{k+1} = x_k + \alpha_k d_k\), Compute: \(f_{k+1}, g_{k+1}, y_k, s_k\).

Step (4): Compute our new \(d_{k+1}\). If Powell restart satisfied then set \(d_{k+1} = -g_{k+1}\) else \(d_{k+1} = d_k\) and set new \(\alpha_k, k = k+1\) go to step (2).

4. Descent Property of The New Algorithm:

Assumptions (4.1):

i. set \(\xi = \{x \mid f(x) \leq f(x_i)\}\) is bounded, to be specific, there exists a factor \(B > 0\) such that \(\|x\| \leq B\) for all \(x \in \xi\).

ii. In some neighborhood \(N\) of \(\xi\) we assume that \(f(x)\) is continuously differentiable function \(f(x)\), and the gradient is globally lipschitz continuous, this means there exist a factor \(L > 0\) such that \(\|\nabla f(x) - \nabla f(y)\| < L\|x - y\|\), \(\forall x, y \in N\). [18]

Now we'll provide the following theorems, which guarantees the new algorithm's descent property:

**Theorem (4.2):**

Suppose \(\{x_k\}\) is a sequences generated from the suppose algorithm if there exist a constant \(\gamma > 0\) such that \(\|g_{k+1}\| < \gamma\) for all \(k\). then the search direction is descent direction for all \(k\).

Proof: using induction method for \(k=1\) we have \(d_1 = -g_k\) then \(d_1^T g_1 < 0\), then we suppose that \(d_k = -g_k + \beta_k d_k\) and \(d_k = -g_k + \beta_k d_k\) and

\[
\beta_k = \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{s_k^T s_k}{2(g_k^T s_k + f_k - f_{k+1})} \right]
\]

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And to check the descent for above direction for \( k + 1 \), multiplying (4) by \( g_{k+1}^T \) then:
\[
g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{\alpha_k^2 \|d_k\|^2}{2(\alpha_k g_{k+1}^T d_k - \rho \alpha_k g_{k+1}^T d_k)} \right] g_{k+1}^T d_k
\]
Using Wolfe condition (6) and since \( s_k = \alpha_k d_k \) then we have:
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{\alpha_k^2 \|d_k\|^2}{2\alpha_k g_{k+1}^T d_k (1 - \rho)} \right] g_{k+1}^T d_k
\]
and since \( g_k^T = -d_k \) then
\[
g_k^T d_k = -\|g_k\|^2 \quad [19], \text{ then we have:}
\]
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{\alpha_k^2 \|d_k\|^2}{-2\alpha_k \|d_k\|^2 (1 - \rho)} \right] g_{k+1}^T d_k
\]
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{y_k^T g_{k+1}}{y_k^T d_k} \left[ 1 + \frac{\alpha}{2(\rho-1)} \right] g_{k+1}^T d_k
\]
Assume \( \rho \in (0, 1/2) \), then \( \frac{\alpha}{2(\rho-1)} \) (very small and negative) and we assume that
\( \tau = \frac{\alpha}{2(\rho-1)} \) so we have:
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + (\tau + 1) \frac{y_k^T g_{k+1}}{y_k^T d_k} g_{k+1}^T d_k
\]
and since \( g_{k+1}^T d_k = g_{k+1}^T d_k - g_k^T d_k + g_k^T d_k \) therefore
\[
g_{k+1}^T d_k = y_k^T d_k + g_k^T d_k \quad \text{and since} \quad g_k^T d_k < 0 \text{ then:} \quad g_{k+1}^T d_k < y_k^T d_k
\]
by substations this relation on the following equation we get:
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + (\tau + 1) \frac{\|g_{k+1}\|^2}{g_{k+1}^T d_k} (g_{k+1}^T d_k)
\]
\[
g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + (\tau + 1) \|g_{k+1}\|^2
\]
\[
g_{k+1}^T d_{k+1} \leq (\tau + 1) \|g_{k+1}\|^2 \quad \Rightarrow \quad g_{k+1}^T d_{k+1} \leq \tau \|g_{k+1}\|^2
\]
assume \( \tau = \xi \) and \( \tau \) is negative as we prove, then \( g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 \)
which is descent direction for all \( k \). Hence, the proof is completed by induction.

\[\text{(4.3) Global Convergence Theorem:}\]
We have the following lemma (4.4) and (4.5) for any conjugate Gradient method with the strong Wolfe line search, which were first found by zoutendijk [20] and Wolfe.
Lemma (4.4): Assume that Assumptions i and ii, and the descent condition are true, and that $\alpha_k$ is obtained using the strong Wolfe line search [6]. Then there's

$$\sum_{k=1}^{\infty} -\alpha_k g^T_k d_k < \infty$$

............... (17)

Lemma (4.5): Consider any conjugate gradient method in the form (2) is gained by the strong Wolfe line search. Assume assumptions i ii, and the descent condition are true. Then the so-called Zoutendijk condition is true:

$$\sum_{k=1}^{\infty} \frac{(g^T_k d_k)^2}{\|d_k\|^2} < +\infty$$

...............(18)

Proof:

Since $d_{k+1} = -g_{k+1} + \beta_k d_k$

or $d_{k+1} + g_{k+1} = \beta_k d_k$

(Dai and Yuan, 1999). Taking the square of each said and noting:

$$\beta_k^2 = \left( \frac{y^T_k g_{k+1}}{y^T_k d_k} \right)^2 \left( 1 + \frac{s^T_k s_k}{g^T_k s_k + f_k - f_{k+1}} \right)^2$$

$$\leq \left( \frac{y^T_k g_{k+1}}{y^T_k d_k} \right)^2 \left( 1 + \frac{\alpha^2 \|d_k\|^2}{\alpha_k g^T_k d_k - \rho \alpha_k g^T_k d_k} \right)^2$$

$$\leq \left( \frac{y^T_k g_{k+1}}{y^T_k d_k} \right)^2 \left( 1 + \frac{\alpha^2}{1 - \rho} \right)^2 \left( 1 + \frac{\alpha}{\sigma - 1} g^T_k g_{k+1} \right)^2$$

let $\gamma^2 g_{k+1} < \|g_{k+1}\| < \omega^2$ and let:

$$\gamma^2 = \left( 1 + \frac{\alpha^2}{1 - \rho} \right)^2 \frac{\omega^2}{\sigma - 1}$$

$$\beta_k \leq \left( \frac{\omega^2}{g^T_k d_k} \right)^2 \frac{\gamma^2}{\sigma - 1}$$

$$\|d_{k+1}\|^2 = \frac{\|d_k\|^2}{\left( g^T_k d_k \right)^2} - 2d^T_{k+1} g_{k+1} - \|g_{k+1}\|^2$$

Divide each term in above equation by $(d^T_{k+1} g_{k+1})^2$ and also using Wolfe condition (6) & (7) we get:

$$\frac{\|d_{k+1}\|^2}{(g^T_{k+1} d_{k+1})^2} \leq \frac{\|d_k\|^2}{(g^T_k d_k)^2} - \frac{2}{(g^T_{k+1} g_{k+1})^2} - \frac{\|g_{k+1}\|^2}{(d^T_{k+1} g_{k+1})^2}$$

$$= \left( \frac{g^T_k d_k}{g^T_{k+1} d_{k+1}} \right)^2 \left( 1 - \frac{\|g_{k+1}\|^2}{\sigma - 1} \right) + \frac{1}{(g^T_{k+1} g_{k+1})^2} + \frac{1}{\|g_{k+1}\|^2}$$

$$\leq \frac{\|d_k\|^2}{(g^T_k d_k)^2} + \frac{1}{\|g_{k+1}\|^2}$$

..........................(19)
Because \( \left\langle g^*_k, d_k \right\rangle \leq \frac{1}{\|g_k\|^2} \), then (19) shows that
\[
\frac{\|d_k\|^2}{(g^*_k d_k)^2} \leq \sum_{i=1}^k \frac{1}{\|g_i\|^2} \quad \forall \quad k
\]

now if the theorem is not true, there exist a constant \( c > 0 \) s.t.
\[
\|g_k\| \geq c \quad \forall \quad k
\]

Therefore it follow from (20),(21) that
\[
\frac{\|d_k\|^2}{(g^*_k d_k)^2} \leq \frac{k}{c^2}
\]

which implies that:
\[
\sum_{k=0}^\infty \frac{(g^*_k d_k)^2}{\|d_k\|^2} = \infty
\]

Which is conflict with Zoutendijk theorem hence \( \|g_k\|=0 \) therefore the algorithm is globally convergent.

6. Numerical experiments:

We now provide numerical tests comparing our novel approach to the HS and PRP algorithms on the same set of unconstrained optimization test functions, with the goal of determining which method is the most reliable and efficient for addressing any unconstrained optimization problem.

We investigated numerical trials with the same number of dimensions for each test function [21] for n=100, 300, 1000, 5000, 6000, and 10000.

All of the algorithms use the same line search and the same parameters. The number of iterations(No.I) and the number of function evaluations (No.F) are used to do the comparison. Our algorithms has stopped at once \( \|g_k\| \leq 10^{-5} \).

Table (6.1) Comparison algorithms for n=100, n=1000

| problems       | New n=100 No.I(No.F) | PR N=100 No.I(No.F) | HS n=100 No.I(No.F) | New n=1000 No.I(No.F) | PR N=1000 No.I(No.F) | HS n=1000 No.I(No.F) |
|----------------|----------------------|----------------------|---------------------|------------------------|----------------------|----------------------|
| Startit        | 8(18)                | 6(14)                | 6(14)               | 8(18)                  | 6(14)                | 6(14)                |
| Wolfe          | 44(89)               | 44(89)               | 205(411)            | 53(107)                | 64(129)              | 210(421)             |
| Rosenbrok      | 26(68)               | 26(65)               | 26(68)              | 27(70)                 | 27(67)               | 27(70)               |
| Powell         | 41(112)              | 46(116)              | 55(130)             | 43(116)                | 57(150)              | 64(169)              |
| Wood           | 78(162)              | 127(261)             | 98(202)             | 84(174)                | 146(299)             | 98(202)              |
| Sum            | 12(63)               | 12(63)               | 12(63)              | 18(80)                 | 22(104)              | 18(80)               |
| central        | 37(261)              | 31(205)              | 43(278)             | 40(302)                | 41(332)              | 48(345)              |
| Miele          | 67(208)              | 99(307)              | 52(159)             | 78(253)                | 101(322)             | 52(159)              |
| Fred           | 9(24)                | 9(25)                | 10(27)              | 9(24)                  | 10(27)               | 10(27)               |
| Nondiagonal    | 27(66)               | 29(77)               | 27(66)              | 27(65)                 | 30(78)               | 27(65)               |
| Shallo         | 11(27)               | 11(28)               | 11(27)              | 11(27)                 | 11(28)               | 11(27)               |
| Cubic          | 16(44)               | 16(44)               | 16(44)              | 16(44)                 | 16(44)               | 16(44)               |
| Beal           | 13(31)               | 10(23)               | 11(27)              | 13(31)                 | 11(27)               | 11(27)               |
| Osp            | 48(165)              | 49(164)              | 48(165)             | 199(554)               | 149(568)             | 203(586)             |
| Total          | 437(1338)            | 515(1481)            | 620(1681)           | 626(1865)              | 691(2189)            | 801(2263)            |
### Table (6.2) Comparison algorithms for n=5000,n=10000

| Problems   | New n=5000 | PR N=5000 | HS N=5000 | New n=10000 | PR N=10000 | HS N=10000 |
|------------|------------|-----------|-----------|-------------|------------|------------|
|            | No.I(No.F) | No.I(No.F) | No.I(No.F) | No.I(No.F)  | No.I(No.F) | No.I(No.F) |
| Wolfe      | 314(630)   | 255(512)  | 150(304)  | 261(525)    | 215(432)   | 305(613)   |
| Rosenbrock | 27(70)     | 27(67)    | 27(70)    | 27(70)      | 27(67)     | 27(70)     |
| Powell     | 43(116)    | 57(150)   | 65(171)   | 43(116)     | 63(179)    | 65(171)    |
| Central    | 111(321)   | 47(148)   | 51(390)   | 40(302)     | 50(466)    | 51(390)    |
| Miele      | 81(273)    | 109(360)  | 54(174)   | 78(253)     | 109(360)   | 54(147)    |
| Nondiagonal| 27(65)     | 30(78)    | 27(65)    | 27(65)      | 30(78)     | 27(65)     |
| Shallo     | 11(27)     | 11(28)    | 11(27)    | 11(27)      | 11(28)     | 11(27)     |
| Beal       | 11(27)     | 11(27)    | 11(27)    | 11(27)      | 11(27)     | 11(27)     |
| Sum        | 30(136)    | 19(86)    | 30(136)   | 47(181)     | 24(97)     | 47(181)    |
| Osp        | 435(1353)  | 611(1859) | 435(1353) | 612(1979)   | 834(2546)  | 612(1979)  |
| Cubic      | 16(44)     | 16(44)    | 16(44)    | 16(44)      | 16(44)     | 16(44)     |
| Total      | 1106(3062) | 1193(3629)| 877(2761) | 1173(3589)  | 1390(2590) | 1226(3714) |

### Table (6.3) Comparison algorithms for n=300,n=6000

| Problems   | New n=300 | PR N=300 | HS n=300 | New n=6000 | PR N=6000 | HS n=6000 |
|------------|-----------|----------|----------|------------|-----------|-----------|
|            | No.I(No.F) | No.I(No.F) | No.I(No.F) | No.I(No.F)  | No.I(No.F) | No.I(No.F) |
| Gpowell-3  | 18(39)    | 23(49)   | 30(62)   | 19(42)     | 23(49)     | 31(64)    |
| Wolfe      | 46(93)    | 53(107)  | 43(87)   | 204(414)   | 294(590)   | 275(554)  |
| Cubic      | 16(44)    | 16(44)   | 16(44)   | 16(44)     | 16(44)     | 16(44)    |
| Shallo     | 11(27)    | 11(28)   | 11(27)   | 11(27)     | 11(28)     | 11(27)    |
| Beal       | 11(27)    | 11(27)   | 11(27)   | 11(27)     | 11(27)     | 11(27)    |
| Edger      | 6(16)     | 7(18)    | 7(18)    | 6(16)      | 7(18)      | 7(18)     |
| Total      | 108(246)  | 121(272) | 118(265) | 267(570)   | 362(756)   | 351(734)  |

7. Conclusion:

We searched in this research for a new modification conjugate gradient method procedure which depends on deriving the quadratic function. We have decided under our experiment that the global convergence for the suggested idea is stated also the numerical experiment explained in Tables (6.1),(6.2) and (6.3) are the efficient of the proposed algorithm with respect to regular HS and PRP methods on average and according to the numbers of results.

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