Prediction of drag crisis on a circular cylinder using a new algebraic transition model coupled with SST DDES

A S Stabnikov and A V Garbaruk
Peter the Great St. Petersburg Polytechnic University (SPbPU)
Russia, Saint-Petersburg, 195251, Politechnicheskaya, 29
E-mail: an.stabnikov@gmail.com

Abstract. Flows with massive separation zones are frequent in industrial applications and pose a considerable challenge for contemporary turbulence solving techniques. Accurate prediction of these types of flows can be broken down into two distinct problems: prediction of separation points, which can be addressed by using laminar to turbulent transition models in RANS framework and description of the separated regions, best dealt with by using LES. The present work proposes a new algebraic transition model which is then coupled with the hybrid RANS-LES method DDES with the resulting approach tested on a flow around a circular cylinder on a wide range of Reynolds numbers ranging from sub- to super-critical. Results show that the proposed method is an improvement relative to both RANS and DDES methods and shows promise but requires further development.

1. Introduction
The aerodynamic drag crisis of bluff bodies is a phenomenon which involves the aerodynamic drag coefficient suddenly plunging with the rise of the Reynolds number. The cause of the crisis is shift of the position of laminar-turbulent transition in the flow. At low Reynolds numbers the transition takes place in the separated mixing layer, but with increasing Reynolds number the transition point moves upstream into the attached boundary layer. This shift results in the detachment point moving downstream, decreasing the low-pressure separation zone size behind the body and lowering the aerodynamic drag coefficient.

This phenomenon is very difficult to model because it requires precise description of the dependence of transition position on the Reynolds number and at the same time representation of the massive separation zone behind the body. The most common approach for describing the transition position is use of the Reynolds averaged Navier Stokes (RANS) equations coupled with special models for laminar – turbulent transition description. However, it is well known that global hybrid RANS-LES methods (HRLM), like DDES, perform better than RANS in describing massive separation flow simulations [1]. Coupling DDES with a transition model may, at least in theory, increase the precision of calculations for these types of flows.

Nowadays a wide range of different transition models is presented in literature. The most precise transition models are so called differential transition models, in which additional differential equations are solved for transitional variables such as intermittency ($\gamma$), critical Reynolds number ($Re_\theta$), “laminar kinetic energy” ($k_L$) or others. The most popular differential model is SST $\gamma$-$Re_\theta$ [2], which in general surpasses other models in accuracy. There have been a number of attempts of coupling differential models with HRLM, presented, for example in works [3-8], but all these attempts are not fully successful. The most significant problem is robustness of the resulting complex model: numerical and convergence problems are observed and these problems are to be expected, since they may arise when
using differential transitional models even in the “simpler” RANS framework. As a result, these approaches are not reliable enough to be used in engineering applications. Algebraic transition models, based on algebraic relationships for transition specific variables, could be considered as an alternative to differential models. Though they are inferior to differential models in accuracy, they are significantly more robust. So, the coupling of the algebraic transition models with HRLM looks like a promising approach. Recently, a new algebraic transition model KD [9] by Kubacki and Dick was proposed. However, it is based on the Wilcox k-ω model [10], which is inferior to the k-ω Menter SST model [11]. Taking all these facts into consideration, the goals of the current work are as follows:

- adaptation of the KD model [9] so as to be coupled with k-ω SST [11];
- testing of the new obtained model on transitional boundary layer flows;
- combining the model with SST DDES [1] approach and application to the drag crisis phenomenon prediction.

2. SST KD transition model formulation

2.1. RANS model formulation

In this model, intermittency is used in the production term in “k” equation of the k-ω SST (2003) model [11] which takes the form

\[ P = \gamma P_b + (1 - \gamma) P_{sep}; \] (1)

The model uses the following algebraic relation for the intermittency

\[ \gamma = \min\left(\max\left(\frac{k}{\nu A_\Omega}, 1.0\right), 0.0\right). \] (2)

Other model expressions are

\[ P_{sep} = C_{sep} F_{sep} \nu S^2; \] (3)

\[ F_{sep} = \min\left(\max\left(\frac{R_\nu}{2.2 A_\nu}, 1.0\right), 1.0\right); \quad R_\nu = \frac{\nu' S}{\nu}; \] (4)

\[ P_t = \nu_s S^2. \] (5)

Eddy viscosity is defined as a sum of “small-scale” and “large-scale” parts

\[ \nu_T = \nu_s + \nu_i; \] (6)

where

\[ \nu_s = \frac{a_s k_s}{\omega}, \text{ with } \omega = \max[a_s, \omega, \nu' S]; \] (7)

\[ \nu_i = \frac{a_i k_i}{\omega}, \text{ with } \omega = \max[a_i, \omega, \nu' S]; \] (8)

\[ k_s = f_{ss} k; \quad k_i = k - k_s; \quad f_{ss} = \exp\left(-\left(\frac{C_{ss} \nu \Omega}{k}\right)^2\right); \] (9)
\[ C_{SS} = C_s (1.0 + C_A \beta \omega^2); \quad \beta = 1 - \tanh \left( \frac{k}{C_W \nu \Omega} \right); \quad \psi = \tanh \left( \frac{-\Omega(S - \Omega)}{C_\nu (\beta' \omega^2)} \right). \] (10)

The model constants are as follows:
\[ A_y = 1.2, \quad C_s = 2.5, \quad C_A = 1.0, \quad C_\nu = 10.0, \quad C_W = 10.0, \quad C_{sep} = 2.0, \quad A_\nu = 550.0, \quad a_i = 0.45, \quad a_s = 0.3, \quad C_{lim} = 7.0 / 8.0 \] (11)

In the equations, \( k \) is the turbulence kinetic energy and \( \omega \) is the specific dissipation rate of the SST model, \( \Omega \) is the vorticity tensor magnitude, \( \nu \) is the kinematic viscosity and \( F_2 \) is the original function from Menter SST model [11].

The main difference between original KD model [9] and proposed model is the adoption of another form of the onset parameter \( Re_\Omega = \frac{k}{\nu \Omega} \) used in \( \gamma \) (2) and \( f_{SS} \) functions (9). The original model uses the form based on the distance to the wall \( y \): \( Re_y = \frac{\sqrt{k} \nu}{\nu} \), but in the proposed modification we change it to \( Re_\Omega = \frac{k}{\nu \Omega} \), used in the previous version of the model by Kubacki and Dick [12]. Additional changes are as follows. The computation of the small-scale (7) and large-scale (8) eddy viscosities are adapted to the SST model formulation. The power in the \( f_{SS} \) function (9) has changed from 2 to 4. The constants (11) were calibrated using the T3A transitional boundary layer case [13]. Two constants have changed values: \( A_\nu = 1.2 \) and \( C_s = 2.5 \). The model is coupled with 2003 Menter SST model [11] in the same way as the original model is coupled to \( k-\omega \) Wilcox model [10] in [9]. We will be referring to the new modified model as SST KD.

### 2.2. SST KD DDES approach

The new model, SST KD was applied as a base model for the DDES approach instead of the standard SST model [1]. In addition, an enhanced version of the DDES with length scale adapted to shear layers \( \Delta_{SLA} \) was used [14]. The model will be referred to as SST KD DDES \( \Delta_{SLA} \).

### 3. Testing on boundary layer flows

As was mentioned in previous section, the proposed SST KD model was calibrated using the T3 series transitional boundary layer cases, following the experimental work by Rolls Royce [13]. Only the cases without pressure gradient (T3A-, T3A and T3B) were considered. The cases have different inlet turbulence intensities and Reynolds numbers, namely T3A- with 0.87% freestream intensity, T3A with 3.3% and T3B with 6.2% with corresponding Reynolds numbers of \( 1.0 \times 10^6 \), \( 3.0 \times 10^6 \) and \( 3.3 \times 10^6 \). The length scale for the Reynolds numbers is \( 1m \). Even at the lowest intensity, the bypass transition scenario takes place.

The computational domain contains a freeflow region upstream of the plate’s leading edge to minimize the pressure gradient effect. The boundary conditions are as follows: uniform velocity profile was set at the inlet boundary, constant pressure at outlet, symmetry at upper boundary and the freeflow region upstream of the plate, no-slip wall at the plate. Turbulence characteristics at the start of the plate were chosen to coincide with experimental values. A computational grid of \( 568 \times 88 \) cells was used.

Two transition models were compared on the three boundary layer flows: suggested SST KD model and its original form, the \( k-\omega \) KD model [9]. As can be seen in figure 1, while not being perfect, the proposed model leads to better agreement with experimental results in all three cases.
Figure 1. Comparison of skin friction coefficient distributions for the T3A-, T3A and T3A cases, obtained by the new SST KD model, original $k-\omega$ KD model and experimental results

4. Drag crisis problem

4.1. Problem formulation

The computations of a flow around a circular cylinder were carried out using 3D URANS and DDES approaches. Models used in the URANS framework: SST [11], SST $\gamma$-$Re_{\theta}$ transitional model [2], $k-\omega$ KD [12] transitional model, and the proposed transitional model SST KD. SST DDES $\Delta_{SLA}$ and SST KD DDES $\Delta_{SLA}$ [14] models were also used for the computations. The computational results were compared with results from the experimental study by Schewe [15]. The range of Reynolds numbers considered is from $5.0 \times 10^4$ to $1.3 \times 10^6$ fully capturing the drag crisis for a circular cylinder.

Figure 2. Flow around a circular cylinder computational domain and grid

The computational domain has a cylindrical form with the centre at $(x, y) = (0, 0)$ and diameter 80D. The spanwise size of the domain is $5D$. The boundary conditions are as follows: constant velocity at the inlet part of the boundary, along with low turbulence conditions for turbulent characteristics (corresponding to $\nu_r/\nu = 0.01$ and $Tu = 0.1\%$). No-slip conditions were used at the cylinder wall and constant pressure at the outlet part of the external boundary. Periodic conditions were applied at the spanwise boundaries.

The computational grid consists of 3 structured blocks (figure 2). Different grids were used for different Reynolds numbers and formulations. The $\Delta y^+$ for the wall cells is always less than 1 and the expansion rate less than 1.2. In all cases the solution is grid independent. The average amount of cells is around 13 million.
Incompressible numerical computations were carried out in an academic code NTS (Numerical Turbulence Simulation). Third order upwind scheme was used for convective flux approximations in frame of URANS whereas hybrid third order upwind-fourth order central difference scheme was employed for DDES. For all transport equations, including the equations for turbulent variables, the second order discretization scheme for the convective terms was selected. For each test case all simulations were initialized using a steady state RANS solution.

Two-stage strategy was used for unsteady simulations. The transient simulations were carried out for about 200 non-dimensional times and then about 100 times were used for averaging. The time step was chosen to ensure Courant number $C_u < 0.5$ in the separated region.

Note, that no problems with convergence of iterations were observed neither for URANS nor for DDES.

4.2. Results

Figure 3 (left) shows the comparison of average drag coefficient $C_d(Re)$ dependencies for RANS models with experimental results. Firstly, one can see that all three transition models predict drag crisis phenomenon better than fully turbulent SST, however none of them predict shape and critical Reynolds number well. The proposed SST KD model significantly overperforms its original form $k-\omega$ KD model in predicting the drag crisis. Overall, the differential SST $\gamma-Re$ model and the SST KD model are of comparable quality on the computations of this flow. The differential model predicts drag crisis position more precisely, while SST KD is better at predicting drag coefficient values at lower and higher Reynolds numbers.

By looking at SST KD DDES $\Delta_{SLA}$ (figure 3, right) one can notice the overall improvement in the predictions when compared to the baseline SST DDES $\Delta_{SLA}$. However, in the current state, 3D URANS SST KD does a better job at predicting drag coefficient values than the SST KD DDES $\Delta_{SLA}$. Perhaps, further calibration for the DDES coupling is necessary.

![Figure 3.](image.png)

**Figure 3.** Comparison of computational 3D URANS drag coefficient with experimental results (left) and obtained from DDES models and SST KD model with experimental results (right)

5. Conclusion

The proposed SST KD model is a valid algebraic transition model which is superior to its baseline model, the $k-\omega$ KD in both boundary layer attached flow and the complex flow around a cylinder at wide range of Reynolds numbers.

By introducing an algebraic transition model into DDES framework we were able to improve drag crisis prediction without significant increasing model complexity and computational requirements. The DDES approach based on the SST KD performs better than baseline DDES in predicting the drag crisis phenomena, but probably requires further tweaking and calibration.
Acknowledgments
The reported study was funded by RFBR, project number 19-31-27001. The results have been obtained with use of computational resources of Supercomputer Center at Peter The Great Saint-Petersburg Polytechnical University (www.spbstu.ru) and supported by the Academic Excellence Project 5-100 proposed by Peter the Great St. Petersburg Polytechnic University.

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