Feasibility, strategy, methodology, and analysis of probe measurements in plasma under high gas pressure

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Abstract. This paper reviews existing theories for interpreting probe measurements of electron distribution functions (EDF) at high gas pressure when collisions of electrons with atoms and/or molecules near the probe are pervasive. An explanation of whether or not the measurements are realizable and reliable, an enumeration of the most common sources of measurement error, and an outline of proper probe-experiment design elements that inherently limit or avoid error is presented. Additionally, we describe recent expanded plasma-condition compatibility for EDF measurement, including in applications of large wall probe plasma diagnostics. This summary of the authors’ experiences gained over decades of practicing and developing probe diagnostics is intended to inform, guide, suggest, and detail the advantages and disadvantages of probe application in plasma research.

1. Introduction

It is known [1] that, in a low-pressure plasma when there are no collisions between atoms and electrons flowing to the electric probe, the probe electron current $I_e$ may be calculated using the Langmuir formula

$$I_e(V) = \frac{2\pi N_e S}{m^2} \int_{-V}^{0} f(\varepsilon)(\varepsilon - eV) d\varepsilon,$$

where $f(\varepsilon)$ is the electron distribution function (EDF), $N_e$ is the electron density, $e$ is the magnitude of electron charge, $S$ is the probe surface, $m$ is the mass of electron, $V$ is the (negative) probe potential and $\varepsilon$ is the electron energy. In equation (1), it is assumed that

$$4\sqrt{2}\pi m^{-3/2} \int_0^\infty \sqrt{\varepsilon} f(\varepsilon) d\varepsilon = 1.$$

At present, the most published probe measurements of the EDF [2-6] are conducted at low gas pressure and interpreted with the Druyvesteyn formula, which is simple to obtain from equation (1) by the double differentiation over the probe potential [7]

$$\frac{m^2}{2\pi N_e S e^3} \times \frac{d^2I_e}{dV^2}.$$

The method for finding the plasma parameters by applying equations (1) and (2) is limited by the important requirements that the mean-free-path of electrons $\lambda$ is much greater than both the Debye
shielding radius \( r_D \) and the probe radius \( a \). It is still generally accepted that, due to violation of those conditions, the probe method is not applicable (in noble gases) at pressures of greater than, typically, 10 Torr. For advancing the probe method into the region of high pressures in order to find the EDF, a number of theories have been proposed. In this paper, we briefly review the available theories and analysis of a number of experimental studies on the EDF measurements and electron densities in high (medium) pressure plasma.

2. The effect of sink of electrons to the probe

The effect of the electron sink to the probe is connected to reducing electron density near the probe due to electron absorption (recombination) by the probe surface. The distortion of the EDF occurs due to the finiteness of the rate of diffusion replenishment of the region of electrons depleted due to their flow to the probe. This reduces the probe current at any energy. In the sink theory, it is assumed that at distances from the probe surface starting at the infinity to the mean free path of electrons, \( \lambda \), electrons flow to the probe due to diffusion and from distance \( \lambda \) to the probe surface they go to the probe freely without collisions. It is also assumed that the mean-free-path of electrons and the probe radius are much greater than \( r_D \).

The first was Swift’s calculation \cite{8}, where for a spherical probe the variation of the EDF, \( f(\epsilon, \lambda) \), at the boundary of the collisionless region from the undisturbed EDF, \( f(\epsilon) \), was modeled. For the dependence of the electron current on the retarding potential, expression

\[
I_e(V) = \frac{2\pi N_e e \epsilon}{m^2} \int_0^\infty f(\epsilon)(e - eV)d\epsilon
\]

was obtained. Here,

\[
\delta(\epsilon) = \frac{3a^2}{4\lambda(\epsilon)(a + \lambda(\epsilon))}
\]

is called as the sink parameter (of electrons to the probe). The influence of this effect on the standard (that is, when equation (2) is valid for the EDF) probe measurements of space potential, plasma density, and the average energy (temperature) of the electrons has been investigated, for example, in Refs. \[8-10\].

At present, in the experimental practice of probe measurements, the cylindrical probes are used much more often than spherical ones, since they are the simplest in manufacturing and the influence of the probe holder on measurements is minimal. The calculations in Ref. \[9\], resembling the are calculations from Ref. \[8\] but appropriate for the current to a cylindrical probe having length \( l \) and radius \( a \), also give equation (3) but with the sink parameter

\[
\delta(\epsilon) = [3a/4\lambda(\epsilon)]\ln \left[ 1 + \frac{2l}{a + \lambda} \right].
\]

Note that, in the last case, the electron diffusion flux was taken into account not only on the side surface of the probe, but also, on one probe end. If we take into account the diffusion of electrons to both ends of the probe in equation (3), we obtain

\[
\delta(\epsilon) = \frac{3a}{4\lambda(\epsilon)} \ln \left[ 1 + \frac{l}{a + \lambda} \right].
\]

A generalized theory for the sink effect of electrons on a probe of arbitrary shape is also possible. Such a generalization has been conducted, for example, in Ref. \[11\], where the electron current was determined for a probe made in the form of an ellipsoid of revolution with semi-axes of \( l/2 \) and \( a \). Using the coordinates of an elongated or an oblate ellipsoid of revolution as a result of calculations similar to Ref. \[8-10\], the expression for the current to the probe also yields expression (3), where the sink parameters are:

a) for a prolate ellipsoid of revolution \((l/2 > a)\)
\[ \delta(\varepsilon) = \frac{3S}{32 \pi \lambda(\varepsilon)} \ln \left( \frac{\sigma_0 + 1}{\sigma_0 - 1} \right); \quad b = \sqrt{\left( \frac{l}{2} + \lambda \right)^2 - (\alpha + \lambda)^2}; \quad \sigma_0 = \frac{\lambda}{b}; \quad (6) \]

b) for an oblate ellipsoid of revolution \((l/2 < a)\)

\[ \delta(\varepsilon) = \frac{3S}{32 \pi \lambda(\varepsilon)} \left[ \pi - 2 \tan^{-1} \sigma_0 \right]; \quad b = \sqrt{(\alpha + \lambda)^2 - (l/2 + \lambda)^2}; \quad \sigma_0 = \frac{\lambda}{b}. \quad (7) \]

Analysis of expressions (6) and (7) shows that for \(l/2 = a\), that is for the case of a spherical probe, they give expression (4). If, however, an ellipsoid with semi-axes \(l/2\) and \(a\) is assumed to be an equivalent of a cylindrical probe for which \(l \gg a\), then for the value of \(\delta(\varepsilon)\) in equation (6) for the transition with limits \(\sigma_0 \to 1\) and \(\alpha \to \lambda\)

\[ \delta(\varepsilon) \approx \frac{3a}{4\lambda} \ln \left[ \frac{l}{a + \lambda} \right], \quad (8) \]

which practically coincides with equation (5).

So, as shown by the available calculations, the dependence of the electron current to the probe of arbitrary shape with the retarding potential (in the framework of the theory of the sink effect) can be described by unique equation (3), where the sink parameter is determined by the probe geometry and calculated either by equation (6) or equation (7), respectively, for a prolate or oblate probe. It is easy to see from equation (3) that, in the Langmuir mode of operation \((a/\lambda \ll 1)\), the current on the probe is determined by equation (1). For the opposite case \((a/\lambda \gg 1)\) the sink theory gives

\[ I_e(V) = \frac{2\pi N_e e^2}{m^2} \int_0^\infty f(e, e) de \delta(\varepsilon). \quad (9) \]

From equation (9) by the differentiation over probe potential [12] it is simple to obtain

\[ f(\varepsilon) = \frac{\delta(eV)m^2}{2\pi N_e e^3 V^3} \times \frac{dt}{dV}, \quad (10) \]

which is an analogue of the Druyvesteyn formula for the collisional plasma.

3. Kinetic probe theory

Within the framework of the sink theory, the influence of collisions at the distances less than \(\lambda\) from the probe surface on the electron current is neglected. But, strictly speaking, some of the electrons streaming toward the probe from a distance \(\lambda\) can be prevented from reaching its surface by collisional encounters with gas atoms. To develop more precise predictions, it is necessary to take into account those collisions and to choose realistic boundary conditions on the surface of the probe. For this purpose, a kinetic theory can be used.

3.1. Diffusion mode \((\lambda \ll a \ll \lambda_e)\)

The kinetic theory for this case in the diffusion mode, that is \(a/\lambda \gg 1\), has been studied in Ref. [12]. Since electrons undergo many collisions before they reach the probe, the EDF near the probe is weakly anisotropic and, hence, the two-term expansion is valid [5,6]. In a stationary weakly ionized plasma, the isotropic part of the EDF, neglecting inelastic collisions, is described by the kinetic equation

\[ \frac{1}{3} \delta v f_1 = -eE \frac{1}{3m} \partial \frac{\partial}{\partial v} \left( v^2 f_1^\ast \right) = \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v} (v^3 f_0 v e_n), \quad (11) \]

where \(e_n\) is the frequency of transport-inducing electron collisions with gas atoms, and \(f_1^\ast\) is the directed part of the EDF, which may be described by equation [13]

\[ v \nabla f_0 - \frac{eE}{m} \frac{\partial f_0}{\partial v} = -v e_n f_1^\ast. \quad (12) \]
If inequality $\bar{a} = a + d \ll \lambda_e$ holds, the EDF is nonlocal \cite{12} and it is possible to neglect the right-hand side in equation (11), which is responsible for energy losses in elastic collisions and is of the order of $(m/M)\bar{f}_0\nu_{en}$ in comparison with the left-hand side, which is of the order of $v^2\bar{f}_0/(\nu_{en}\bar{a})^2$. Here, $\bar{a}$ is the characteristic scale of the EDF variation near the probe and $d$ is the probe sheath/pre-sheath size. In this case, it is convenient \cite{12} to use as an independent variable $w$ instead of $\varepsilon$, where $w = \varepsilon + e\varphi(r)$ is the total energy of an electron with the kinetic energy $\varepsilon$ and $\varphi(r)$ is the plasma electric potential at distance $r$. After the calculation of the isotropic part of the EDF, we obtain

$$f_0(w,r) = C_0(w) + C_1(w) \int \frac{d\nu \nu_{en}(v)}{v^3 r^2},$$

(13)

where $C_0(w)$ and $C_1(w)$ are functions of the total electron energy, $\alpha = 1$ and $\alpha = 2$ are for cylindrical and spherical probes, respectively.

Function $C_0(w)$ may be found from the condition that at infinity the unknown distribution function is equal to the undistorted EDF. Assuming that potential $\varphi$ at the infinity satisfies the requirement $\varphi(\infty) = 0$, we find that $C_0(w) = f(w)$. The second boundary condition is $f(\varepsilon \geq eV, a) = 0$. Therefore, it is assumed that the density of "energetic" electrons having kinetic energy greater than $eV$ on the probe surface is zero. Then, it is found that the distribution function becomes

$$\begin{align*}
(f_0(w,r) &= f_0(w) & \text{for } w \leq eV, \\
(f_0(w,r) &= f_0(w) \left(1 - \int_0^\infty \frac{d\nu \nu_{en}(v)}{v^3 r^2} / \int_0^\infty \frac{d\nu \nu_{en}(v)}{v^3 r^2}\right) & \text{for } w \geq eV.
\end{align*}$$

(14)

For the case of the cylindrical probe in equations (14), the upper limit of integration $A$ may be taken as equal to the length of the probe $l$ and it is infinity for the spherical probe (some authors prefer to use $\pi l/4$ as the upper limit for the cylindrical probe which will change a bit some formulas below \cite{5,6}).

Equations (14) show that knowledge of the profile of the potential near the probe is necessary for calculating the EDF. However, if we assume that $\nu_{en}/\nu^3 = \text{const.}$, as it is valid in the approximation for the electron diffusion scattering cross-section in argon, the profile of the potential is not important for calculations. With somewhat less accuracy, the same is valid for other Ramsauer gases (krypton and xenon). Then, the current to the probe of an arbitrary shape is described by expression

$$I_w(V) = \frac{2\pi N_e s}{m^2} \int_{eV} \frac{f(\varepsilon) d\varepsilon}{\delta(\varepsilon)},$$

(15)

Equation (15) is also valid for any gas in the case of the thin near-probe sheath with thickness much smaller than the probe size (radius). This may be the case for the large electron density.

For the spherical geometry from equations (14) for the EDF we obtain $f_0(w,r) = f_0(w)(1 - a/r)$ and for cylindrical geometry we have $f_0(w,r) = f_0(w)\ln[r/a]/\ln[l/a]$. Since the directed part of the distribution function is $f_1 = \frac{v^3}{\nu_{en}(v)} f_0$ \cite{13}, for the dependence of the electron current on the cylindrical and spherical probes we have

$$I_{ec}(V) = \frac{8\pi^2 N_e d e}{3m \ln[l/a]} \int_{eV} \frac{f(w) v^3}{\nu_{en}} d\varepsilon, \quad I_{es}(V) = \frac{16\pi^2 N_e d e}{3m} \int_{eV} f(\varepsilon) \frac{v^3}{\nu_{en}} d\varepsilon,$$

(16)

respectively.

If we take into account that factor $\frac{v^3}{\nu_{en}}$ under the integral in equations (16) is equal to $2\lambda(\varepsilon)c/m$, then for currents on the spherical and cylindrical probes from equations (16) we obtain expressions

$$I_{ec}(V) = \frac{16\pi^2 N_e d e}{3m \ln[l/a]} \int_{eV} f(\varepsilon) \lambda(\varepsilon) c d\varepsilon, \quad I_{es}(V) = \frac{32\pi^2 N_e d e}{3m^2} \int_{eV} f(\varepsilon) \lambda(\varepsilon) c d\varepsilon,$$

(17)

respectively. It is clear that equations (17) can be obtained also in the sink theory from equation (9). Consequently, for the Ramsauer gases or thin near-probe sheath in the diffusion regime ($a >> \lambda$), the...
sink theory and the kinetic analysis give the same expressions (17) for the currents to probes. It is also possible to get expressions for the EDF similar to equation (10)

\[
 f(\varepsilon) = \frac{-3m^2 \ln[l/a]}{16\pi^2 N_e e^3 V_\lambda} \frac{dl_{ee}}{dV} \quad \text{and} \quad f(\varepsilon) = \frac{-3m^2}{32\pi^2 N_e a e^3 V_\lambda} \frac{dl_{ee}}{dV}
\]

(18)

for cylindrical and spherical probes, respectively.

3.2. Arbitrary case \((\lambda_e \gg a)\)

Reduction of the gas pressure leads to increasing \(\lambda_e\) and will make the condition \(\bar{a} \gg \lambda_e\) invalid. The kinetic approach may also provide a solution for this case. In this case the boundary condition \(f(\varepsilon \geq eV, a) = 0\) is not satisfactory [14-16]. It leads to the discontinuity of the distribution function at the point \(w = eV\). This in turn can lead to the following paradox [14]: under certain conditions, the current to the probe, calculated according to equation (3), can be greater (!), than the current to the probe in a collisionless-transport mode. This is due to the fact, that in some situations the value of \(\delta(\varepsilon)\) is not large enough compared to unity for a given type of the EDF (although the sink parameter can reach values of between 30 and 50). Therefore, it is necessary to be careful to use expressions (15) for the EDF which rapidly falling with energy, especially in the region of large retarding potentials. The use of the original expression (3) maybe more preferable.

If we take
\[
 f_0(w \geq eV, a) = \frac{4}{3} f_1(w \geq eV, a),
\]
as the second boundary condition, then the electron current to the probe will be determined by expression

\[
 I_e(V) = \frac{2\pi N_e eS}{m^2} \int_{eV}^{\infty} \frac{f(w)(w-eV)dw}{1+2\pi^2 a^2(w-eV)\lambda(w,a)}.
\]

(19)

where \(A = \infty\) for a spherical probe and \(A = l\) for a cylindrical probe. It is seen, that solution (19) for Ramsauer gases, when \(\lambda(e) \approx \text{const}\) and the potential profile is insignificant, practically coincides with equation (3) in the diffusion regime.

Once again, in the general case, knowledge of the potential profile is required to find the dependence of the electron current on the probe on the retarding potential. In this case, probe plasma diagnostics become very difficult. On the other hand, one can try to numerically estimate the value of the integral

\[
 I(w, a) = \int_r^{A+d} \frac{dr}{r^a \lambda(e)} + \frac{1}{\lambda(w)a} \int_r^{A+d} \frac{dr}{r^a},
\]

(20)

(outside the sheath/pre-sheath we assume that \(\varphi = 0\) and \(\varepsilon = w\)). Clearly valid, for the thin sheath (or small retarding potentials) in the diffusion-transport mode \((a \gg \lambda)\), is the integral \(I(w, a) \equiv ((\lambda(w)w)^{-1}) \times \ln[l/a]\). Likewise valid, for the current to the probe obtained above in the framework of the theory of the sink effect, are expression (3) and equation (19), obtained in the kinetic approach. Here we note that the choice of such an approximation for \(I(w, a)\) and for nonzero \(V\) turns out to be sufficiently accurate [4], as estimates show.

For the case of the collisionless-transport mode \((a \ll \lambda)\), when condition \(\bar{a} \ll \lambda_e\) is automatically satisfied, equation (19) implies equation (2) obtained by Druyvesteyn under the condition of absence of collisions between atoms and electrons flowing to the electric probe. The measurements of the EDF is also possible when collisions in the sheath/pre-sheath are taken into account and instead of condition \(a \ll \lambda\), it is sufficient that a much weaker requirement \(\bar{a} \ll \lambda_e\) is satisfied. This circumstance, first noted in Ref. [14], has a simple physical meaning: electrons, which come to the probe from the unperturbed plasma, lose an insignificant fraction of their energy even in collisions in the sheath/pre-sheath.
So, if we ignore the influence of the near-probe sheath, the theory of the sink effect and the kinetic theory give virtually identical descriptions of the current to the probe in the diffusion regime. Obviously, for the zero-retarding potential, the saturation currents completely coincide if, in the theory [8], we assume that the motion of electrons to the probe is diffusive up to its surface. The importance of the kinetic approach [12] lies in the fact that it allowed us to drop the condition \( a \ll \lambda \) and estimate the influence of collisions in the sheath. The main point is that the kinetic approach helped to indicate the limit \( (a \ll \lambda) \) of the parameters in the plasma, for which measurements of the EDF are possible.

We note that expression (19), for the case of a spherical probe, was first obtained in Ref. [17]. In Ref. [17], also for the first time, to the best of our knowledge, the Monte Carlo method was used for simulating the collection of electrons by such a probe for a thin sheath of space charge. As the simulation for \( \lambda = \text{const} \) shows, while \( a/\lambda \) increments across the range from 3 to 10, the electron current can be described within the limits of error bar as

\[
I(V) = \frac{2\pi NeS}{m^2} \int_{eV}^{\infty} f(w)(w-eV)dw_{1.3+\delta(1-\frac{eV}{kT})}, \quad \delta = \frac{3a^2}{4\lambda(a+\lambda)}. \tag{21}
\]

An attempt was made to refine the calculation of currents for cylindrical and spherical probes in Refs. [15,16] with a boundary condition that is slightly different from that used in Refs. [17]. The fact is that, in these works, the boundary condition \( f_0(W \geq eV, a) = \frac{4}{\gamma} f_1(W \geq eV, a) \) was used, which, strictly speaking, is valid only for completely isotropic EDF. Near the surface of the probe, the EDF associated with the case of small \( V \) is noticeably anisotropic and, in Ref. [15,16], the so-called effective boundary condition \( f_0(W \geq eV, a) = \gamma(\frac{a}{\lambda})^2 f_1(W \geq eV, a) \) [4,18] was accounted for in the EDF calculation.

The results for the dependence of the currents on the retarding potential for such a choice can be presented in the form

\[
I(V) = \frac{2\pi NeS}{m^2} \int_{eV}^{\infty} f(w)(w-eV)dw_{\frac{4}{\gamma}+\frac{5}{4}a^2(w-eV)I(w,a)}. \tag{22}
\]

The coefficient \( \gamma = \gamma(\frac{a}{\lambda}) \) in equation (22) varies slightly from 4/3 to 0.71 as \( \frac{a}{\lambda} \) changes from zero to infinity. In the case of a spherical probe, for which \( x = \frac{a}{\lambda} \geq 1 \), we have \( \gamma = 0.71 \pm 0.4/x \).

So, to date, a number of slightly different descriptions of the current to the probe have been proposed, using different boundary conditions. Obviously, in the diffusion limit and for a thin sheath, interpreting the current-voltage characteristic using different boundary conditions should give practically identical results. Further, it should be expected that, at zero retarding potential, effective boundary conditions will yield a closer result to the simulation results (if they are accurate enough). Assuming that the model current (17) is found to an accuracy of 1-2%, we estimate the accuracy of the use of equations (3), (19), (22) to be comparable.

Below, the electron current (3) obtained by Swift [8] is denoted by \( I_s(V) \); current (19) with the unity in puncture the integrand as \( I_{10} \) (index 0 should remind us that the sheath is infinitely thin), similarly the current (22) is like \( I_{\gamma (0)} \) (\( \gamma (0) \)) simulated by the Monte Carlo current method (21) as \( I_{\gamma 0} \). The relationship of the currents (3), (19), (22) to the simulated one is shown in figure 1(a, b, c) for the values \( a/\lambda = 3 \) and \( a/\lambda = 10 \), and \( a/\lambda = 50 \). As can be seen, the effective boundary conditions give more accurate values only for the saturation currents. However, as the retarding potential \( V \) (given in figure 1 in units of electron temperature) increases, the discrepancy between the currents \( I_{\gamma 0} \) and \( I_{\lambda 0} \) increases noticeably. For example, for \( V = 6 \), the discrepancy reaches 40% even at \( a/\lambda = 10 \) while, for \( a/\lambda = 3 \), the discrepancy exceeds 50%. On the other hand, for \( a/\lambda = 10 \), the currents \( I_s(0) \) and \( I_{10}(0) \) differ by only about 4% from \( I_{\lambda 0}(0) \). Consequently, the accuracy, for example, \( I_{10} \) for large \( V \) is not worse than 8% (interestingly, this is only 7% greater than the accuracy of \( I_s(V) \)). For large \( a/\lambda = 50 \), the differences in the currents (3), (19), (22) do not exceed 10-15%, which we interpret as the precision of the prediction.
Thus, current (19) is better than current (22) in terms of corresponding to the model current. This is a consequence of the fact that the effective boundary conditions are valid only for neutral particles. Simplifying (without taking into account the influence of the sheath) the description of the current within the framework of the sink theory (3) also gives a very good approximation. It is easy to see that if we discard the influence of the sheath and assume the motion of the electrons to the probe diffusively down to its surface, for the sink parameter, we have

$$\delta(e) = \frac{3a}{4\lambda(e)}$$

which exactly coincides with the sink parameter in the thin sheath limit.

![Graph showing the relationship between relative probe current and retarding probe voltage](image)

**Figure 1.** The relationship between the electron currents of a spherical probe for a thin sheath of space charge: (top/left) $a/\lambda = 3$, (top/right) $a/\lambda = 10$ and (bottom) $a/\lambda = 50$. $I_s/I_m$ (solid), $I_{10}/I_m$ (dashed) and $I_{p0}/I_m$ (dotted).

### 4. Experimental measurements at high gas pressures

Starting with the analysis, we first note that experimental measurements of the EDF are practically absent: only a very few papers are known to the authors. Let's start with the experiment [12], where electro-kinetic characteristics were measured in a discharge plasma in argon at pressures up to 100 Torr. In that work, it was concluded that it is possible to measure the EDF and the electron density according to equations (17). As shown in [11], this conclusion, generally speaking, is not confirmed by the experimental data given in the same work [12].

We recall that the measurements were made in a positive column of the argon discharge using a cylindrical probe of radius $a = 0.1$ mm and a length $l = 0.3$ cm located on the axis of the discharge tube. The probe current-voltage characteristics provided in Ref. [12] show that, for example, at a
pressure $p = 62$ Torr and for the discharge current of 35 mA, the electron saturation current is approximately 0.45 mA. Let us estimate the value of the sink parameter for the experimental conditions of Ref. [12] and compare it with the calculated value. For the discharge current, $I_d$ through the tube and for the electron saturation current to the probe, $I_e^{\text{sat}}$, we have, respectively,

\[ I_d = 0.43 N_e(0) e S_d v_d \quad \text{and} \quad I_e^{\text{sat}} = \frac{N_e(0) e S_d}{4(1+\delta)}. \tag{23} \]

where $N_e(0)$ is the electron density at the center of the discharge tube, $S_d$ is the area of its cross section, and $v_d$ is the drift velocity of the electrons. Thus, in order to estimate the value of the electron discharge parameter for the probe, we obtain the relation

\[ \delta \approx \frac{I_d S_d}{I_e^{\text{sat}} S_d 1.7 v_d} = 1. \]

Under the experimental conditions, the average $E/N$ value was $3.5 \cdot 10^{-17}$ Vcm$^2$ and, according to [19,20], the drift velocity is $4.5 \cdot 10^5$ cm/s. The characteristic electron energy is from 5 to 7 eV, so we assume the value of the average velocity to be $\bar{v} = 10^6$ cm/s. Thus, for an experimentally determined value of the sink parameter and taking into account $S/S_d = 6.6 \cdot 10^{-4}$, we obtain $\delta_{\text{exp}} = 5$. At the same time, the calculated value of the sink parameter in the case under consideration is 50! That is, an order of magnitude difference is observed. This, of course, is a rough estimate since, in the experiment [12], the EDF is far from Maxwellian and has the form $f(\varepsilon) \sim \exp\left(-\frac{\varepsilon}{\varepsilon_*}\right)$ with a characteristic energy from 6 to 7 eV. More accurate calculations, when the saturation current is calculated according to equations (16) with the experimentally found EDF (with the electron density estimated by the current of the tube), show the discrepancy between theory and experiment to be a factor of approximately from 3 to 3.5.

The following experiments [11] were carried out in helium gas discharge plasma using the current-voltage characteristics of the "thick" ($a_1 = 0.6$ mm) and "thin" ($a_2 = 0.045$ mm) cylindrical probes at pressures from 0.9 to 3.3 Torr. In the chosen pressure range, the thicker probe sink parameter reached values of $\delta \approx 10$, while the thin probe parameter $\delta$ is less than 1. Since $N_e \sim I_e^{\text{sat}}(0)(1 + \delta)$, the ratio of the saturation currents of the two probes, under the same experimental conditions, will be determined by

\[ \frac{I_e^{\text{sat}}(0)}{I_e^{\text{sat}}(0)} = \frac{S_1 1 + \delta_2}{S_2 1 + \delta_1}, \]

where $I_e^{\text{sat}}(0), I_e^{\text{sat}}(0), S_1, S_2, \delta_1$ and $\delta_2$ are saturation currents, areas and sink parameters for probe 1 and probe 2 respectively. Thus, a comparison of the experimentally obtained and calculated ratios of saturation currents makes it possible to judge the applicability of the sink theory for finding the plasma parameters. Such a comparison of the ratios shows that, at the maximum pressure (3.3 Torr), there is a noticeable (approximately threefold) increase in the experimental ratios of the saturation currents over the calculated ones, that is similar to obtained in [12].

What is the reason for the observed discrepancy between theory and experiment? Here we can note a number of factors. The main factor is, apparently, the presence in the gas-discharge plasma of the electric field, which causes the transfer of electrons from the unperturbed plasma to the depleted region of the probe region. Since in the theory it is assumed that the current to the probe is determined only by the gradient drift, for its applicability it is necessary to have the ratio

\[ \xi = \frac{eE}{mp} \frac{\partial f}{\partial \varepsilon} \approx \frac{eE\bar{\varepsilon}}{\bar{\varepsilon}} \ll 1, \tag{24} \]

where $\bar{\varepsilon}$ is the characteristic energy of the electron distribution.
Requirement (24) for the case of spherical geometry can be obtained [21] from the following consideration. Solving the equation $\Delta N_e(\rho) = 0$ for the density profile of $N_e(r)$ under the boundary condition $N_e(a) = 0$ in the diffusion mode yields $N_e(r) = N_e(1 - a/r)$. Hence, for the electron current we have

$$I_e^{sat} = \int D_e \nabla N_e dS = \frac{eSN_eD_e}{a}, \quad (25)$$

(here $D_e = \lambda\bar{v}/3$) which actually coincides with the value of the saturation current

$$I_e^{sat} = \frac{N_e eS\bar{v}}{4(1 + \delta_c)},$$

where $\delta_c = 3a/(4\lambda)$, given by the theory of flow in the diffusion regime.

As shown by (25), the value of the electron density gradient is $\sim N_e/a$, that is, the spherical probe perturbs the plasma at a distance of $\sim a$. It is obvious that the electron current transported by an externally applied (created not by a probe) electric field $E$ in a section equal to $\pi(\rho + \rho)^2$, should be much less than the diffusive-motion current per probe, that is,

$$\frac{eSN_eD_e}{a} \gg e\mu E N_e 4\pi a^2, \quad (26)$$

or, taking into account the relation $\frac{\mu_e}{D_e} = e / (kT_e)$, we find the condition $\frac{eEa}{kT_e} \ll 1$.

Calculation [20] for a spherical probe shows that the saturation current for a Maxwellian EDF with allowance for the external electric field increases approximately in proportion to the growth of the parameter $\lambda_e \equiv eEa / (2kT_e)$. Unfortunately, such calculations for a cylindrical probe are not known to us.

When speaking about estimates of the applicability limits (while satisfying the inequalities $a + d \ll \lambda_e$ and (28)) of the theory for a cylindrical probe, we note that there are significantly different definitions of the quantity $\bar{a}$. For example, in Ref. [12], we can say, the upper, or conservative, limit is $\bar{a} = l/2$. Probably a more accurate (mid-conservative) estimate can be obtained by setting the unperturbed EDF at a distance where $N_e(\bar{a}) = 0.5N_e$. Then, for a cylindrical probe, we have $\bar{a} \approx a\sqrt{l/\lambda}$ (with an analogous choice for a spherical probe $\bar{a} = 2a$). Finally, the lower estimate can be obtained by analogy with a spherical probe. It is $\bar{a} = a\ln[l/a]$ [22]. With this choice of $\bar{a}$, by analogy with a spherical probe (that is, analogously to equation (26)) for a probe perpendicular to the field (for large electric fields it is advisable to place a probe perpendicular to electric field $E$), one can obtain

$$\frac{eSN_eD_e}{a\ln[l/a]} \gg e\mu E N_e 2a(1 + \ln[l/a]), \quad (27)$$

whence we find the requirement

$$\xi = \frac{eEa\ln[l/a]1+\ln[l/a]}{kT_e\pi} = \frac{eE(1+\ln[l/a])}{kT_e\pi} \ll 1. \quad (28)$$

Further, the usual estimate $\bar{a} \ll \lambda_e$ was given in Ref. [12] within an order of magnitude. In fact, this condition looks more exactly like $(\lambda_e / \bar{a})^2 \gg 1$ [21]. Then, when choosing the lower limit, it is likely that the condition $\lambda_e > 3\bar{a} = 3a\ln[l/a]$ will be sufficient for the feasibility of the theory of the sink effect, as in this case $(\lambda_e / \bar{a})^2 \sim 10$.

As shown by the experimental data [11,12], the conditions (24) (with the choice of $\bar{a} = l/2$) or (28) (where $\bar{a} = a\ln[l/a]$) are certainly not satisfied. For example, when choosing $\bar{a} = l/2$ and the maximum test pressure in [11], the value $\xi$ was of the order of unity and, in experiments in Ref. [12], it was from 2 to 2.5. Even when choosing the lower estimate for $\bar{a} = a\ln[l/a]$, we have $\xi \approx 1$ in Ref. [12] and $\xi \approx 0.35$ in Ref. [11].
Let us now briefly discuss the possible influence of ionization processes in the probe sheath on the probe currents. Expression for the sink parameter \( \delta = \frac{3a}{4\lambda} \ln[l/a] \) shows that when the probe length \( l \) tends to infinity, the current density vanishes. At the same time, obviously, when recombination and ionization of particles in the near-probe sheath in the plasma are taken into account, the current density \( j_e \) on such a probe must be finite [23]. It was shown in Ref. [24] that for an infinite probe at the center of a discharge tube of radius \( R \), the electron density

\[ j_e \approx \left[ 1 + \frac{3a}{4\lambda} \ln[R/(2.4a)] \right]^{-1}, \quad (29) \]

i.e., instead of expression \( \delta = \frac{3a}{4\lambda} \ln[l/a] \), when ionization in the plasma volume is taken into account, we have \( \delta = \frac{3a}{4\lambda} \ln[R/(2.4a)] \). It is clear that in the case when the ionization length is

\[ L_i = \sqrt{\frac{D_e}{Z_i}} = \frac{R}{2.4} \leq l, \]

the known expression for the sink parameter, generally speaking, can no longer be used. Interestingly, in most studies of a gas-discharge plasma, tubes of a radius from 1 to 2 cm are used, the characteristic probe length is from 0.5 to 1 cm, i.e., \( l \) and \( L_i \) are practically equal.

Hardly, however, this circumstance can strongly affect the value of the saturation current, because one can expect that the "effective" sink parameter is described by an expression of the type of

\[ \delta_{eff} \approx \frac{3a}{4\lambda} \ln \left[ \frac{L_i}{(l + L_i)a} \right], \]

i.e. it is determined by the geometric mean of \( L_i \) and \( l \). If we assume that \( \delta_{eff} \) is determined by this relation, then under the experimental conditions [12], a decrease of \( \delta_{eff} \) in comparison with \( \delta \) is only 5%. Consequently, one does not expect such a significant divergence between theory and experiment.

At the same time in Ref. [12], a decrease of \( \delta_{eff} \) in comparison with \( \delta \) can be about 40%. On the other hand, the role of ionization waves (traveling striations) is completely unclear, when, according to [25], the densities of charged particles at the maximum and minimum of the wave can differ by several times.

In conclusion, let us briefly consider the possible error in finding the sink parameter. As noted in Ref. [11], the sink theory for the diffusion regime for \( \lambda = const \) and the Maxwellian EDF gives the same values of the saturation current as the Bohm theory [2]. For example, for a cylindrical probe represented by an ellipsoid of revolution, we have

\[ I_{e sat}^{3at} = \frac{N_e S}{4(1+\delta_E)}, \quad \text{where} \quad \delta_E = \frac{\delta S}{16\pi C_s D_e}, \quad (30) \]

where \( C_s \) and \( D_e \) are the capacity of the body in the form of an ellipsoid with semi-axes \( a, l/2 \) and the diffusion coefficient of electrons \( D_e = \lambda \beta^3/3 \). The capacity \( C_s \) is then calculated according to

\[ C_{se} = \frac{l/1-(2a/l)}{2 \tan^{-1} \sqrt{1-(2a/l)^2}}, \quad (31) \]

where subscript “se” is for an elliptical body.

At the same time, there are calculations (see, for example, [26]), which for a cylindrical segment (subscript “sc”) of length \( l \) and radius \( a \) give (here \( \Lambda = \ln(l/a) \))

\[ C_{sc} = \frac{l}{2 \ln(l/a)} \left( 1 + \frac{1-\ln z}{\Lambda} + \frac{1+(1-\ln 2)^2-\pi^2/12}{\Lambda^2} + O(\Lambda^{-3}) \right). \quad (32) \]
The ratio $\psi(x) = C_{sc}/C_{se}$, as a function of $x = l/a$, is shown in figure 2. As can be seen, the capacity of the cylinder is always greater than the capacity of the ellipsoid. For a prolate probe with values $l/a$ in the range from 50 to 250, generally speaking, the sink parameter should be reduced by an average of approximately 5-8%.

It is interesting that for $l/a \sim 10$ such a change can already be from 20% to 25%. In this connection, we note that the decrease in the experimental $\delta_{exp}$ in comparison with the calculated $\delta$ for a thick probe observed in the experiment [12] is apparently partly due to this factor. For Ref. [12], this decrease is about 12%.

![Figure 2](image_url)

**Figure 2.** Function $\psi$ depending on parameter $x = l/a$.

5. Large wall probes
The methods of probe measurements in plasmas at high pressure, described above, are based on the assumptions of the EDF nonlocality in a plasma and that the probe disturbs the plasma properties within a small fraction of electron energy relaxation length $\lambda_e$ only. In Ref. [27], an application of a small wall probe for the EDF measurements, in the case when the plasma dimension is smaller than $\lambda_e$, has been discussed. Typically, a wall probe is an electrically isolated segment of the plasma volume wall which collects the current from the plasma for different probe potentials. An area of a small probe is much smaller that plasma wall area. The wall probe does not require a probe holder and thus may reduce the disturbance of plasma. The distortion of the EDF measurements by the ion current may be significantly reduced due to the much greater probe radius of curvature than for a cylindrical or spherical probe. It is probably possible to modify the above probe theories to conduct absolute measurements with small wall probe [28]. However, small wall probes typically cannot measure the EDF for energies below from 3 to 5 average electron energies in the central part of the plasma which has lowest electric potential. Those electrons are imprisoned in the volume and cannot reach plasma walls.
There are a number of publications about the EDF measurements with such probes (see Ref. [29] and references therein). However, they are inconclusive about real possibility and reliability of measurements with small wall probe. At present the small wall probes cannot be recommended for reliable practical use and require additional confirmation and validation.

A method for electron energy spectra studies using a large wall probe has been developed in Ref. [30]. The area of the large wall probe may be significantly larger than for a typical spherical or cylindrical Langmuir and small wall probe resulting in a drastic increase in the probe sensitivity. That is especially important for the measurements of energetic electrons in the tail of the EDF as their density is much smaller than the density of thermal electrons. It was demonstrated that the large wall probe allows us to conduct measurements for gas pressures from a fraction of one Torr to atmospheric pressures [6,30,31] and maybe higher. The theory of the large wall probe has not been developed yet. However, it was experimentally demonstrated in different gases at pressures of about one Torr, that, in the used experimental device, the EDF is proportional to the second derivative of electron probe current $I_e$ with respect to the probe potential $V$ [30].

The sketch of the experimental device used in Ref. [30] is shown in figure 3. The discharge takes place between a plane disc-shaped molybdenum cathode (C) and anode (A), schematically shown in figure 3. The plasma volume is bounded by a cylindrical stainless-steel wall (W). The cathode and anode are 2.5 cm in diameter. The distance between the cathode and anode is 1.2 cm. The wall W was used as a large wall probe.

![Figure 3](image-url)

**Figure 3.** Schematic diagram of experimental device: cathode C, grounded anode A, and cylindrical wall W. Typical structures of the discharge include negative glow (NG), anode glow (AG) and Faraday dark space FDS. The cathode sheath boundary (located in the NG) is indicated by the dashed line.

![Figure 4](image-url)

**Figure 4.** High-energy portion of $d^2I/dV^2$ (absolute value) in neon (3 Torr), argon (0.5 Torr) and oxygen (20%)/argon (80%) (0.5 Torr) dc discharge. The discharge currents were 10 mA, 2mA and 3 mA, respectively. The maxima at 16 eV and 11.5 eV are due to collisions of neon and argon metastable atoms with slow electrons. The maximum at $\approx 4$ eV is due to electron detachment from oxygen, i.e. $O^- + O \rightarrow O_2 + e$ [30].

The plasma of the device typically contains two group of electrons. Low-energy groups of thermal electrons with Maxwellian distributions have electron temperatures at the order of 0.1 eV. The distribution of energetic electron groups is far from equilibrium and exhibits several peaks in the EDF tail. The density of these electrons may be from $10^2$ to $10^5$ times less than that of the low-energy
electrons. The origination of the high-energy electrons is collisions of metastable excited atoms between themselves and with electrons in the reactions

\[ A^+ + A^+ \rightarrow A^- + A + e, \]
\[ A^+ + e \rightarrow A^- + A + e, \]
\[ A^+ + e \rightarrow A + e. \]  \hspace{1cm} (33)

Here, \( A^+ \) denotes an exited atom, and \( A^- \) and \( A^- \) denote atomic and molecular ions. The first two reactions in (33) represent the Penning ionization process, whereas the third describes the super-elastic collision of an exited atom with a low-energy thermal electron.

Figure 4 shows the maxima in \( d^2I/dV^2 \) measured with the wall probe in the discharge plasma in neon, argon and oxygen (20%)/argon (80%) mixture. The peaks for pure neon and argon at 16.6 eV and 11.5 eV are the result of collisions of slow thermal electrons with the excited atoms (the third reaction in (33). The small peak in the oxygen argon mixture near 4 eV is due to electron detachment from negative oxygen ions. The energetic electrons from the detachment have energy of 3.6 eV.

Figure 5 shows \( d^2I/dV^2 \) in the afterglow of a helium/argon mixture dc pulsed discharge. During the discharge, the plasma contains energetic primary electrons accelerated in the near-cathode sheath in the direction of the anode. These electrons ionize the gas thus producing the plasma. They also create metastable atoms, which in turn generate groups of energetic electrons. Upper curve is 85 \( \mu s \) after the pulse and lower curve is 450 \( \mu s \). The maximum at about 4 eV is connected to Penning ionization of argon atoms by helium metastable. The maxima at about 14.5 eV and about 20 eV are deactivation of helium metastable atoms by slow electrons.

**Figure 5.** \( d^2I/dV^2 \) in the afterglow of a helium/argon mixture (4 Torr, argon is 0.002%) dc pulsed discharge. Upper (blue) curve is 85 \( \mu s \) after the pulse and lower (red) curve is 450 \( \mu s \). The maximum at 4 eV is connected to Penning ionization of argon atoms by helium metastable. The maxima at 14.5 eV and 20 eV are deactivation of helium metastable atoms by slow electrons.
The area under a peak in the energy spectrum is proportional to the density of energetic electrons resulting from a specific reaction. The ratio of different constituents of a gas mixture can be obtained from the areas under the peaks corresponding to those constituent reactions. For example, the ratio of the peak areas at 4, 15 and 20 eV in figure 5 yields the ratio between argon density atoms, helium metastable atoms and slow electrons. Furthermore, the area of the maximum at 4 eV declines with time in afterglow due to reduction of helium metastable atom density as argon atom density is unchanged during afterglow. As a calibrated gas mixture has been used with 0.002% of argon (density of argon atoms $N_{Ar}$ is $2.8 \times 10^{12}$ cm$^{-3}$), density of metastable helium atoms and electrons can be measured. It was found for the blue curve that density of metastable helium atom $N_{He}$ is $2.2 \times 10^{11}$ cm$^{-3}$ and density of electrons $N_{e}$ is $3.6 \times 10^{11}$ cm$^{-3}$. For red curve $N_{He} = 6.2 \times 10^{10}$ cm$^{-3}$ and $N_{e} = 1.6 \times 10^{11}$ cm$^{-3}$.

6. Conclusions

Thus, at present a number of theories exist, which allow probe measurements of the EDF under high gas pressures (up to an atmospheric pressure and higher) for which the simple Druyvesteyn formula cannot be used. So far, those theories have been used in a very limited number of works, which does not allow a reliable check of their accuracy. Those theories (especially for the arbitrary case) probably cannot provide accurate interpretations of measurements for high pressure comparable with the Druyvesteyn formula (for low pressure), as they are more complicated and take into account many additional and sometime unknown factors (like plasma potential behavior near the probe). Analysis of a few experimental data shows that, due to a number of reasons, the use of high pressure theory for the processing of probe IV traces may lead to significant measurement errors. For high pressure, influence of a probe-holder may be greater than for lower pressure. Electron reflection from dirty probe surface may also be more important for higher pressure [32]. Other effects, which can distort the probe measurements enumerated, for example, in reviews [4,5], may also become significant for higher pressure EDF measurements. However, for a definitive answer to the question of how these factors affect the results of the experiment, of course, additional studies are required. A small wall probe can be a useful for some measurements but, so far, no reliable experimental confirmation exists. A large wall probe can provide higher sensitivity, but results may be spoiled by variation of plasma properties under changed probe potential [33,34]. Nevertheless, the large wall probe may be useful, for example, in analysis of energetic electrons in plasmas.

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