Double Degenerate Stars

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Abstract. Regardless of the formation mechanism, an exotic object, Double Degenerate Star (DDS), is introduced and investigated, which is composed of baryonic matter and some unknown fermion dark matter. Different from the simple White Dwarfs (WDs), there are additional gravitational force provided by the unknown fermion component inside DDSs, which may strongly affect the structure and the stability of such kind of objects. Many possible and strange observational phenomena connecting with them are concisely discussed. Similar to the normal WD, this object can also experience thermonuclear explosion as type Ia supernova explosion when DDS’s mass exceeds the maximum mass that can be supported by electron degeneracy pressure. However, since the total mass of baryonic matter can be much lower than that of WD at Chandrasekhar mass limit, the peak luminosity should be much dimmer than what we expect before, which may throw a slight shadow on the standard candle of SN Ia in the research of cosmology.

PACS numbers: 04.40.Dg, 97.20.Rp, 97.60.Bw, 97.60.Jd

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It is widely believed that White Dwarfs (WDs) are the end stage of the low or the intermediate mass stars’ evolution and are very dense objects. Opposite to so many uncertainties under extreme densities inside Neutron Stars (NSs), the physical bases are much simple and well established. From observational side, there are abundant WDs in our Galaxy due to the high frequent birth rate for their progenitor stars and the relatively slowly cooling efficiency after they were born. This is why the WD’s structure is treated as one of the best understood areas of astrophysics now and an excellently educational stuff in so many astronomy textbooks, but they are still interesting objects for scientists and catch much attention at all times especially after Sloan Digital Sky Survey (SDSS) data release 4 (DR4) catalog of WDs, which provides quite well opportunity for a detailed comparison between theoretical models and observations.

From theoretical viewpoint, WDs and NSs are all special cases of Fermion stars (FSs), in which the inward self-gravity is balanced by the degeneracy pressure of fermions. In fact, one-component FSs except WDs and NSs are theoretically oversimplified and well studied celestial objects for a quite long time. Their maximum
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mass can span several orders of magnitude and which make them be one kind of potential replacers of black holes. Although the observational evidences for black holes exist have so far been quite abundant and becoming even more strong recently, the existence of them is still and will continue to be an hotly debated question in the following decades before people can test GR theory through observations of material falling into black holes as the next generation space-borne plans will do. Instead of putting a supermassive black hole in the center of galaxies, some peculiar objects such as FSs were constructed theoretically and some of them can also be confronted with many known observation constraints quite well. One famous model among them is the extended Neutrinos Star composed of degenerate heavy neutrinos,\cite{2} this object has neither an event horizon nor a singularity, with shallow potential and is benefit to explain the soft spectrum radiation of accreting baryonic matter. However, the 15.2-year orbit measurements of S2 surrounding a dark object around SgrA* near the Milky Way center seem exclude the possibility of the massive, degenerate FS in our galaxy center since the strongly constrain form the central density structure.\cite{3} Anyway, even one component FSs really exist, it would be very difficult to observe them directly, since the composition of them entirely is dark matter, which only participates in the gravitational and sometimes in weak interaction, does not emit or reflect light.

In this Letter, we concentrate on an idealized celestial objects from theoretical side, namely Double Degenerate Stars (DDSs), sometimes which look like WDs but in fact are assumed to be composed of the normal matter with uniform chemical composition and a sort of unknown fermions (perhaps some dark matter composition) with mass $m_f$. Actually, the DDSs we concerned here are some kind of fermion-fermion stars.\cite{4} It is necessary to emphasize that they have normal matter surfaces, on which thermal emission due to the cooling or some internal heating process can be observed. Of course, researchers can also identify the characteristic spectrums due to different chemical composition in their crust and atmosphere. If they exist and happen to be located in binary systems, we can directly observe them and distinguish them from the normal WDs by their peculiar behaviour.

We shall begin our discussion with the internal structure of DDSs. To make a simple model, we have assumed that the constituent matter is “cold” (fully degenerate) gas, except the quantum pressure of its electrons or its fermions without any interactions among them, such as the neutronization and pyconuclear reactions at sufficiently high density, the Coulomb corrections at low density, the Thomas-Fermi correction and so on. Now the Equation of State (EOS) can be expressed by some very simple functions of the dimensionless Fermi momentum $x_i = p_F/m_ic$,

\begin{align}
\rho_i(x_i) &= \frac{\pi}{3} \frac{g_i m_i^4 c^3}{h^3} \left[ 3 x_i \sqrt{x_i^2 + 1} \left( 2x_i^2 + 1 \right) - 3 \sinh^{-1}(x_i) \right], \\
P_i(x_i) &= \frac{\pi}{3} \frac{g_i m_i^4 c^5}{h^3} \left[ x_i \sqrt{x_i^2 + 1} \left( 2x_i^2 - 3 \right) + 3 \sinh^{-1}(x_i) \right],
\end{align}

where $\rho_i$, $P_i$, $m_i$ and $g_i$ are energy density, degenerate pressure, rest mass and degeneracy factor of the $i$—th component. Latin character $i$ ($i = e$, $f$) denotes the electron or
certain fermion. The only one difference between WDs and so-called FSs we should keep in mind is that the gravitation of WDs comes mainly form nucleons since the charge neutral condition and the electron’s mass is about 1800 times smaller than that of a proton, whereas the gravitation of FSs is completely come from component fermions themselves.

If we further assume that these objects are spherically symmetric, non-rotating, non-magnetic and in hydrostatic equilibrium, then the problem is simple enough to deal with. The gravitational field inside these object can be expressed by the internal metric

\[ ds^2 = -c^2 B(r) dt^2 + A(r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]  

Combining the above EOS with the continuity equation and the hydrostatic equilibrium equations for these objects.

\[ A(r) = \left[ 1 - \frac{2Gm(r)}{r c^2} \right]^{-1}, \]  

\[ \frac{d \ln B(r)}{dr} = \frac{2Gm(r)}{r^2 c^2} \left[ 1 + \frac{4\pi r^3 \sum_i \rho_i(r)}{m(r) c^2} \right] A(r), \]  

\[ \frac{dm(r)}{dr} = 4\pi r^2 [\rho_p(r) + \rho_e(r) + \rho_f(r)], \]  

\[ \frac{dP_f(r)}{dr} = -\frac{c^2}{2} \frac{d \ln B(r)}{dr} \left[ \rho_f(r) + \frac{P_f(r)}{c^2} \right], \]  

\[ \frac{dP_e(r)}{dr} = -\frac{c^2}{2} \frac{d \ln B(r)}{dr} \left[ \rho_p(r) + \rho_e(r) + \frac{P_e(r)}{c^2} \right], \]

where \( A(r) \) and \( B(r) \) are the metric coefficients, \( m(r) \) denotes the “gravitational mass” inside radius \( r \), which is the mass a distant observer would measure by its gravitational effects, for example, on orbiting movement or on gravitational lensing. Here \( \rho_p = m_p \mu_e \frac{8n_e}{3} \left( \frac{h}{m_e c} \right)^3 x_e^3 \) is the mass density of proton, \( m_p \) is the mass of proton, \( \mu_e \) is the mean atomic mass of electron. In addition, the particle number confined within a sphere of radius \( r \) satisfies

\[ \frac{dN_i(r)}{dr} = 4\pi r^2 \frac{8\pi g_i}{3} \left( \frac{h}{m_i c} \right)^3 x_i^3 \left[ 1 - \frac{2Gm(r)}{r c^2} \right]^{-1/2}. \]  

Figure 1 shows the Mass-Radius \((M-R)\) relations for fully degenerate WDs, pure FSs with different fermion mass and some DDSs with fixed fermion number. The maximum of these curves corresponds to the Oppenheimer–Volkoff limits for degenerate stars.\(^\text{[7]}\) For WDs, the limiting value is \( R_{Ch} = 2.64 \times 10^{-2} m_e^{-1} m_p^{-1} \mu_e^{-1} \left( 3h^3/\pi c G \right)^{1/2} \approx 1.02 \times 10^3 \text{ km} \left( \mu_e/2 \right)^{-1} \), \( M_{Ch} = 0.195 m_p^{-2} \mu_e^{-2} \left( 3c^3 h^3/\pi G^3 \right)^{1/2} \approx 1.39 M_{\odot} \left( \mu_e/2 \right)^{-2} \), which is the Chandrasekhar mass with general relativity corrections. However, for FSs, the limiting values is \( R_{OV} = 0.218 \sqrt{2/g_f m_f^{-2} \left( 3h^3/\pi c G \right)^{1/2}} \approx 8.09 \text{ km} \sqrt{2/g_f \left( m_f c^2/\text{GeV} \right)^2} \).
$M_{OV} = 0.025 \sqrt{2/g_f m_f^{-2}} (3c^3 h^3 / \pi G^3)^{1/2} \approx 0.627 M_\odot \sqrt{2/g_f (m_f c^2/\text{GeV})^{-2}}$, which strongly depend on the fermion mass. The curves to the right of the maximum are stable branch, where the radius decrease with increasing mass as we known well in degenerate stars, while those left from the maximum represent unstable configurations, will suffer gravitational collapse by some unstable modes and will finally spiral into certain points on $M-R$ plot as the central particle number density tends to infinity. In Newton’s theory of gravity, the upper mass limit of WDs, i.e. Chandrasekhar mass limit, is $M_{Ch} \approx 1.457 \cdot (2/\mu_e)^2 M_\odot$, as the center number density tends to infinity and the radius tends to zero. Instead, the critical radius $R_{Ch}$ for stability can be reasonably settled in the framework of general relativity (GR).

Figure 1 also shows that the largest dimensionless surface potential $(2GM/Rc^2)$ of equilibrium FSs ($\sim 0.23$, does not depend on the details characters of the fermion) can be much higher than that of WDs ($\sim 4.0 \times 10^{-3} (\mu_e/2)^{-1}$), which implies that general relativity is more important in determining the structure of FSs as their central density is high enough.

Since the mean distance between stars in typical galaxy should be $\sim 1$ pc and the compact objects people observed in the X-ray binary systems are merely of a few solar masses (typical value for stellar mass black hole candidates $\sim 10 M_\odot$), the DDS we constructed in stellar level should subject to these constraints. Furthermore, because there is no dissipation of energy due to friction and no effectively viscous processes to transport angular momentum for those unknown particles, the significant mass growth of DDS itself in relatively short time duration seems impossible to realize by accretion of dark matter. Thus, we can simply but appropriately assume that the DDS may satisfy the condition with conserved fermion number. Considering such number conserved pattern and further assuming that they are composed by 0.1 GeV fermions (the low limit for the mass range of Weakly-Interacting Massive Particle, which is selected somewhat arbitrarily from almost completely uncertain region at present), we can obtain the $M-R$ relations for DDSs for fixed fermion mass but with different fermion number, as shown in Figure 1.

The calculated results from the numerical solution of the structure equations show that DDSs with smaller electron number are composed of a double component core and a pure fermion envelope. As normal matter increases, the core size will increase (the dotted lines with circles or triangles in Fig. 1), larger gravity offered by normal matter inside the core may act on the fermions in the outer envelope and need more pressure to balance its structure and causes the invisible fermion surface of DDS shrink progressively (the solid lines with circles, triangles or asterisks in Fig. 1). After that, we will confront with two kinds of situations. Firstly, if there are a sufficient number of fermions inside DDSs, the objects may always have pure fermion envelope and their structure are always dominated by fermion component (such as $N_f = 1.25 \times 10^{57}$ and $1.12 \times 10^{58}$ curves in Fig.1). Secondly, if fermions are not too many, as baryonic matter infuse in, the visible normal matter surface will gradually grow up and eventually exceed the invisible fermion surface (the turn off point C in Fig. 1). After that, a seemed unstable $M$-
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The relation branch appears in Fig. 1, which corresponds to the transition process from fermion dominated (C) to electron dominated (D). Then the DDSs’ structure are mainly determined by the normal baryonic matter and looked more like WDs.

Simply according to the depiction of equilibrium configuration of degenerate stars in the textbook, you might easily come to the “common sense” conclusions that DDSs will lose their stability from C to D since the radius increase as the mass increase, whose behaviour seems more like those self-bound strange stars. As we know, the self-contained stability analysis especially with larger nonlinear perturbations acting on the equilibrium solution is very complicated, highly model dependent without general procedure can be devised and goes far beyond the capability of our recent work. For simplicity, our discussion shall be restricted to linear stability analysis. We find that DDSs even in the transition state from C to D are in the minimum-energy configuration with \( \frac{dE}{dn_{c,e}} = 0 \) and \( \frac{d^2E}{dn_{c,e}^2} > 0 \), where \( E \) is the total energy of the system, which means that DDSs can maintain stable before they arrive at the maximum mass (as shown in Table 1) just form the view of dynamic. However, there is one more complication that needs to be kept in mind, it is about the applicability of the EOS we used. When the central density for baryonic matter is high enough, the Fermi energy of the electrons may exceed the threshold for the inverse \( \beta \)-decay reactions, then the \( \beta \) equilibrium conditions between the chemical potential of nucleons and electrons, \( \mu_n = \mu_p + \mu_e \), even the more practical nuclear EOS should be considered. To some extent, the EOS we adopted here is too simplified.

Table 1 lists some physical parameters for the maximum mass DDSs under some selected fermion number \( N_f \) and mass \( m_f \). Because of the enormous mass range spanned for fermion and the completely free input value for fermion number, their reasonable analysis requires a deep physical understanding of the nature of Dark Matter and the structure formation for this kind of objects over a vast range of parameter space. We just give some demonstrations and reveal some important properties of these objects here. For fixed fermion mass DDSs, such as \( m_f = 0.1 \text{ GeV} \), the first to fourth rows in Table 1: As the fermion number increases, you can see that the maximum and permissible baryonic number inside these objects will decrease, i.e. the total rest mass for normal matter will reduce, besides the normal matter component will be buried more deeply inside stars. The letter f and e appearing in fourth column represent whether fermion or electron component only exist at core. In addition, DDSs with the total rest mass of fermion being \( 1M_\odot \) are taken as examples from fourth to sixth rows in Table 1. It is clear that the allowable maximum baryonic number will become smaller and the normal component will be compressed more deeply in the centre as the composed fermion becomes more heavier. As a demonstration, the metric coefficients of two maximum mass DDSs are plotted in Fig. 2, which can be used to construct the Newtonian gravitational scalar potential (\( \sim \ln B(r)/2 \)) in weak field limit.

Strictly speaking, there is no relationship between the evolution and the \( M-R \) relation for a given fermion number DDS, the following descriptions just help us to understand. We just imagine that DDSs originate from innate fermion star seeds
formed at very early universe and evolve just by accreting baryonic matter. At the beginning, any gas or dust near or bumped into the innate seed tends to be pulled into them. Friction within the accreting material causes it to lose mechanical energy, spiral and sink into the deep center. As matter sinks in, concentrates and compresses together continuously, the core density increases remarkably and the core temperature also increases simultaneously due to the release of gravitational potential energy. If the total mass for the sunked gas is large enough and if there is no suitable and efficient cooling process, the central temperature may keep on increasing and eventually reach the critical value at which hydrogen burning can ignite. The normal matter core may arrive at some evolutionary stage, whose behaviour will be quite similar to the main sequence of normal stars and is supported against gravitational contraction by the outward thermal pressure provided by the nuclear reactions. We plan to study such a kind of objects, give more detail and quantitative descriptions in our future work. In this study, we just concentrate on DDSs, completely ignore the temperature contribution and merely treat the sunk normal matter core as zero-temperature degenerate gas. In addition, to our knowledge, the influence of rapid rotation, strong magnetic field, finite temperature, the coulomb corrections at low density and the neutronization and pyconuclear reactions at high density can remarkably affect the internal structure of the DDSs, we should gradually include them in our continuous work.

Finally we give some observable predictions for such a kind of objects.

(1) Since there is additional gravitational force provided by the unknown fermion component inside DDSs, more electron degenerate pressure are needed to maintain the structure. Therefore, one distinguishing characteristic of DDSs is that they must have a smaller visible radius compared with corresponding WDs. Thus, we can provide another model instead of strange dwarfs to explain some strange WDs’ combined observations, which appear to have significantly smaller radii than that expected for a standard electron degenerate WD EOS.

(2) As we know, the leading model for type-Ia supernova (SN Ia) is still degenerate thermonuclear explosion of a accreting carbon-oxygen WD in a closed binary system as WD’s mass grows to the Chandrasekhar Mass. DDS we introduced here can also experience such a kind of explosion when its mass exceeds the maximum mass permissible by electron degeneracy pressure. Since its maximum mass of baryonic matter should be smaller than Chandrasekhar mass limit of WD, moreover the normal matter now is situated in a more deep potential well, the corresponding binding energy of DDS is much larger than that of WD with the same mass, the production of radioactive nuclide $^{56}$Ni (determines the peak luminosity) and the total kinetic energy after the explosion (determines the expansion time scale) should be much different to that of standard SN Ia, which may throw a slight shadow on the standard candle of SN Ia in the research of cosmology. Thus, it is worth rechecking and simulating the Phillips Relation (light curve width-luminosity relationship for SN Ia) for such a kind of objects in a near future.

(3) Despite the mass for baryonic matter should be smaller than Chandrasekhar
mass, considering the invisible fermion component, the gravitational mass of DDS can be much larger than the upper mass limit for a WD even for an NS ($\sim 3.1M_\odot$). If DDSs really exist, we hope to find them in some binary systems.

Acknowledgments: This research was supported by the National Natural Science Foundation of China under Grants No 10221001. LUO Xin-lian would like thank the hospitality of Center for Gravitational Wave Astronomy at UTB, where some of the work was performed. The authors wish to thank an anonymous referee for his valuable suggestions to our work.

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Figures and Table Caption

Fig1. The Mass-Radius relations for fully degenerate White Dwarfs, pure Fermi Stars with different fermion mass and some Double Degenerate Stars with fixed fermion number. The Straight lines with slope 1 at top left is the black hole limit.

Fig2. Metric Coefficients inside DDSs.

Table1. The physical parameters for some maximum mass DDSs.
Table 1.

| $m_f$(GeV) | $N_f$     | $N_e(10^{56})$ | Core (km) | Radius (km) | Mass ($M_\odot$) |
|------------|-----------|----------------|-----------|-------------|------------------|
| 0.1        | $1.25 \times 10^{54}$ | 8.02           | 90.9      | 537         | 1.35             |
| 0.1        | $3.94 \times 10^{55}$ | 7.38           | 111       | 209         | 1.25             |
| 0.1        | $1.25 \times 10^{57}$ | 7.06           | 142       | 627         | 1.30             |
| 0.1        | $1.12 \times 10^{58}$ | 7.03           | 135       | 1850        | 2.18             |
| 0.01       | $1.12 \times 10^{59}$ | 8.28           | 934       | $7.78 \times 10^5$ | 2.39             |
| 0.3        | $3.72 \times 10^{57}$ | 4.46           | 34.5      | 115         | 1.74             |