Neutral current neutrino-nucleon interaction and the $\Delta(1232)$

A Mariano$^1$, C Barbero$^1$ and G López Castro$^2$

$^1$ Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, cc. 67, 1900 La Plata, Argentina
$^2$ Departamento Física, Centro de Investigación y de Estudios Avanzados del IPN, Apdo. Postal 14-740, 07000 México, D.F., México
E-mail: mariano@fisica.unlp.edu.ar

Abstract. Neutrino-induced one-pion production off nucleons and nuclei in the intermediate energy region, is a source of relevant data on hadronic structure. On the other hand, a proper understanding of these processes is very important in the analysis of neutrino oscillation experiments, where the weak production of pions is the main background. Then, it is important develop a consistent model to have under control the charged (CC) and neutral current (NC) pion production cross sections. Working within an isobar model previously developed to describe elastic and radiative pion-nucleon cross sections and pion photoproduction reactions, using the axial form factors (FF) already obtained from the study of the CC neutrino-nucleon currents, we analyze its predictive power in the NC current production case.

1. Introduction

The interest on inelastic weak pion production processes from $\nu N \rightarrow l\pi N'$ reactions is twofold. They are a powerful tool to study the hadron structure with high intensity neutrino beams and make possible to investigate the axial structure of the nucleon and baryon resonances. This information will enlarge our view of hadron structure beyond what is presently known about electromagnetic form factors (FF) from JLAB. For example FERMILAB has the NUMI beam and MINER$\nu$A(Main INjector ExpeRiment for $\nu$-A) is a neutrino detector designed to study $\nu$-nucleus interactions with unpreceding detail [1]. It will probe the four-momentum transfer squared ($Q^2$) dependence of the nucleon axial FF and also study neutrino induced $\pi N$ states dominated by resonances excitation. Pions are mainly produced by them and these reactions can be used to extract information on nucleon-to-resonance axial transition form factors through hadron effective models. Then, these determinations could be compared with those obtained from static quark models or lattice QCD calculations. In addition MINER$\nu$A has several nuclear targets, allowing a study of nuclear effects in neutrino interactions. Another fundamental question is the strangeness content of the nucleon spin which can be best unraveled with NC quasielastic $\nu$ scattering (NCQE) $\nu N \rightarrow \nu N$ (FINeSSE proposal) [2].

On the other hand they have become a very important element in the analysis of neutrino oscillation experiments which searching for the distorsion in the neutrino flux at a detector positioned far away from the source, and by comparing near and far neutrino energy spectra, one gains information about the oscillation probability.
In this moment new high quality data are becoming available from MiniBoone [3] and SciBoone [4], experiments full dedicated to measure cross sections, which will sum to the already obtained in the K2K [5] experiment. A proper understanding of these processes is very important in the analysis of neutrino oscillation experiments. For instance, the $\pi^0$ production in NC is the most important background to experiments that measure $\nu_\mu \rightarrow \nu_e$ oscillation in the energy range around 1 GeV. This is because the emerging $\pi^0$ can mimic the $\nu_e$ signal events when, for example one of the two photons in the $\pi^0 \rightarrow \gamma \gamma$ decay (used to detect the pion) is not detected. This can happen when the photon exits the detector before showering or does not have enough energy to initiate the shower. Similarly $\pi^+$ production by CC is an important source of background in $\nu_\mu \rightarrow \nu_x$ disappearance searches.

In the energy region relevant for MiniBooNE and SciBoone experiments that is between 0.5 and 2 GEV, the dominant process in addition to the QE scattering is the excitation of the $\Delta$ ($P_{33}(1232 \text{ MeV})$) resonance (figure 1(g),(h)). We also have in addition many non resonant contributions (figure 1(a)-(f)) to the CC1$\pi$ and NC1$\pi$ cross sections. It is then important to

Figure 1. Different amplitude contributions to $\nu N$ scattering
develop a consistent treatment from the point of view of the effective Lagrangian models for describing the $\Delta$ (spin 3/2 particle). These very actual experiments, require to include medium effects on the free nucleon cross section to make a comparison between theoretical results and the data. Nevertheless and before to analyze these effects, it is important that the model used to describe the free nucleon cross section incorporates consistently the resonances and also non resonant terms, since the interference between them could be important. This model should be an extension of one used satisfactorily in describing other reactions to get a confiable theoretical determination of the $\pi\pi$ background cross section, already at the free nucleon level, to be useful at the moment of the calibration of the experiments. Here we use the axial form factors already obtained from the study of the CC1$\pi$ production on free nucleons [6], within an isobar model we have developed to describe several other different processes. We will analyze its predictive power of the model on NC1$\pi$ production of free nucleons.

2. $\Delta(1232)$ amplitude
Effective Lagrangians used to built non resonant Feynman graphs are rough well known and the corresponding amplitudes can be obtained straightforward [6], then we concentrate in those including the resonance terms

$$\hat{\mathcal{L}}_{\Delta} = \hat{\mathcal{L}}_{\Delta\text{free}} + \hat{\mathcal{L}}_{\Delta\pi N} + \hat{\mathcal{L}}_{\Delta W N},$$

where $\hat{\mathcal{L}}_{\Delta\text{free}}$ is the free $\Delta$ contribution, $\hat{\mathcal{L}}_{\Delta\pi N}$ accounts for the strong interaction and $\hat{\mathcal{L}}_{\Delta W N}$ the weak one. $\hat{\mathcal{L}}_{\Delta}$ depends in each of its pieces on an arbitrary parameter $A$ through the matrix [7]

$$A_{\mu\nu}(A) = g_{\mu\nu} + \frac{1}{2}(1 + 3A)\gamma_{\mu}\gamma_{\nu},$$

being invariant under the contact transformation

$$\psi_{\Delta}^\mu \to \psi_{\Delta}^\mu + a\gamma^\mu\gamma_\alpha \psi_{\Delta}^\alpha, \quad A \to A' = \frac{A - 2a}{1 + 4a},$$

where $a$ ($a \neq -1/4$) is another arbitrary parameter. This field transformation assures that spurious spin-1/2 components ($\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$) are removed from the field describing a free $\Delta$ particle. Physical amplitudes should be independent of this parameter and in what follows we give the set Feynman rules to built the contributions shown in figures 1(g) and 1(h). The effective weak $\Delta$-excitation (V-A) vertex amplitude reads

$$\langle \Delta|V_{\mu}(0)|N \rangle = \sqrt{2}C_B^V\pi_{\Delta\nu} \left[ (G_M(Q^2) - G_E(Q^2))K_{\nu\mu}^C \right](T^\dagger \cdot B^*)u_N,$$

where $K_{\nu\mu}^M$, $K_{\nu\mu}^E$, and $K_{\nu\mu}^C$ are electromagnetic vertex functions [8], $B \equiv W, Z$ are isospin wavefunctions, $C_B^V = \sqrt{2}\cos\theta$ and $C_Z^V = (1 - 2\sin^2\theta_W)$, and

$$\langle \Delta|A_{\mu}(0)|N \rangle = iC_B^A\pi_{\Delta\nu} \left[ -D_1(Q^2)g_{\nu\mu} + \frac{D_2(Q^2)}{m_N^2}(p + p_{\Delta})^\alpha \times (g_{\nu\mu}q_\alpha - q_{\nu}g_{\alpha\mu}) - \frac{D_3(Q^2)}{m_N^2}p_{\nu}q_\mu \right](T^\dagger \cdot B^*)u_N,$$

where $C_B^A = \sqrt{2}\cos\theta$ and $C_Z^A = 1$, and where the $\Delta$ deformation has been not considered ($D_4 = 0$). The strong $\pi N\Delta$ formation vertex amplitude reads

$$\mathcal{V}(\pi N \to \Delta) = - \left( \frac{f_{\Delta\pi N}}{m_\pi} \right) q_{\pi\nu} u_N.$$
while the bare $\Delta$ propagator is

$$G_{\alpha\beta}^0(p) = \frac{\not{p} + m_\Delta}{p^2 - m_\Delta^2} \left\{ -g_{\alpha\beta} + \frac{1}{3} \gamma_\alpha \gamma_\beta + \frac{1}{3m_\Delta} \left( \gamma_\alpha P_\beta - \gamma_\beta P_\alpha \right) + \frac{2}{3m_\Delta^3} P_\alpha P_\beta \right\} - \frac{2(p^2 - m_\Delta^2)}{3m_\Delta^3} \left[ \gamma_\alpha P_\beta - \gamma_\beta P_\alpha - (\not{p} + m_\Delta^2)\gamma_\alpha \gamma_\beta \right].$$

Note that if we drop the term in the second line of the previous equation we get the so called Rarita-Schwinger (RS) propagator. Several approaches use the vertexes shown above together the RS propagator, being this procedure inconsistent [9]. Until this moment we have introduced bare vertexes and propagator for the $\Delta$, and they should be dressed by the $\pi N$ interaction to all orders [10]. Numerical calculation of the dressed vertexes and propagator is a difficult task because as we are in presence of a non-renormalizable effective theory one must included cutting FF to calculate loop integrals, which introduce model dependencies. Nevertheless it has been shown that for $W \sim m_\Delta$, where $W$ is the $\pi N$ invariant mass, one can assume that a dressed vertex can be replaced for the bare one but with an effective (strong, electromagnetic or weak) coupling constant [10]. The unstable character of the resonance is included in the unperturbed vertex by the replacement $G = G^0(m_\Delta \rightarrow m_\Delta - i\Gamma_\Delta/2)$, where $m_\Delta$ now is the dressed mass and $\Gamma_\Delta$ the resonance width, this independent-energy self-energy approximation is called the complex mass scheme [7].

In terms of the center of mass (CM) variables the total cross section for the $\nu N \rightarrow l\pi N'$ process reads ($E_{\nu,CM}^{EM} = \frac{m_N E_{\nu,Lab}}{\sqrt{2E_{\nu,CM}^2 + m_N^2}}$) reads [6]

$$\sigma(E_{\nu,CM}) = \frac{m_N^2 N_i}{(2\pi)^3 E_{\nu,CM}} \sqrt{s} \int_{-E_{\nu,CM}}^{E_{\nu,CM}} dE_\pi \int_{E_\pi}^{E_{\nu,CM}} dE_\pi' \int_{-1}^{+1} d\cos\theta \int_0^{2\pi} d\eta \frac{1}{16} \sum_{\text{spin}} |\mathcal{M}|^2,$$

where $\sqrt{s} = E_{\nu,CM}^2 + E_{\pi,CM}^2$, and where $\mathcal{M} = M_B + M_R$ is separated in a background and resonant contribution. In the background amplitudes the vector FF are fixed through the CVC hypothesis, the axial ones using the PCAC hypotesis, and the strange FF obtained from parity violating electron scattering. Nevertheless the resonance parameters should be fixed by fitting cross sections calculated within our consistent isobar model (CIM), to different experimental data. For example the fitting to the elastic $\pi^+ p \rightarrow \pi^+ p$ cross section data, leads to $f_2^{2\Delta N\pi}/4\pi = 0.317 \pm 0.003$, $m_\Delta = 1211.7 \pm 0.4 MeV$ and $\Gamma_\Delta = 92.2 \pm 0.4 MeV$ [11]. We will assume the CVC hypothesis for the vector sector of the resonance FF. Then we can take them from a fit to the experimental data analysis for the $M_{1+}^{3/2}$, $E_{1+}^{3/2}$ amplitude multipoles, extracted from the $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ processes. We got the dressed values $G_M \equiv G_M(0) = 2.97 \pm 0.08$ and $G_E \equiv G_E(0) = 0.055 \pm 0.010$, and the bare values $G_M^0 = 1.69 \pm 0.02$ and $G_E^0 = 0.028 \pm 0.008$ through an evaluation of the loop integrals [10]. Finally the axial FF at $Q^2 = 0$, $D_i(0)$, $i = 1, 3$, are obtained by comparing the non-relativistic limit of the previous matrix element in the $\Delta$ rest frame with the non-relativistic quark model, and all can be expressed in terms of $D_1(0)$ [12]. Making a fit to the flux averaged differential cross section $\frac{d\sigma}{dQ^2}$ in the CC1$\pi \nu p \rightarrow \mu^- p \pi^+$ reaction we get $D_1(0) = 2.35 \pm 0.05$, or as in the usual nomenclature $C_2^A(0) = 1.35 \pm 0.03$ [6].

3. Results for NC
We have four processes of interest in NC production of $\pi$
our results within the CIM model to the $\nu_\mu d \rightarrow \nu_\mu p\pi^- p_s$ process measured in the Argonne (ANL) 12 foot deuterium bubble chamber [13, 14] exposed to a broad neutrino beam from the Zero Gradient Synchrotron (ZGS) with $0.3 \leq E_\nu \leq 1.5 \text{ GeV}$. The cut in the $E_\nu$ energy is to avoid double-pion production and $p_s$ denotes the spectator proton in the reaction, and it is expected that due to the small binding energy of $d$ ($\sim 2 \text{ MeV}$) one can compare the data with the theoretical calculations for $\nu_\mu n \rightarrow \nu_\mu p\pi^-$. This experiment only gives results for the flux averaged $\nu_\mu p \rightarrow \nu_\mu n\pi^+$ and $\nu_\mu p \rightarrow \nu_\mu p\pi^0$ NC cross sections.

In addition we have to our disposal data on NC reactions coming from simulations obtained with the NUANCE software package, which includes Fermi motion, Pauli blocking and final state interactions (FSI), to extract single-pion production on free nucleons from a nuclear target. A reanalysis was done in 2002 by E. Hawker [15] (NUANCE 2002) on the Gargamelle bubble chamber NC1π data. This detector is filled with propane-freon ($C_2H_8 + CF_3Br$) at CERN [16]. The NUANCE simulation code to get free cross section from the propane-freon target was used, since in this experiments only the integrated flux averaged cross section an the R-ratios between NC and CC cross sections were reported. No cuts in the neutrino energy was done and the flux runs on an interval $1\text{GeV} \leq E_\nu \leq 8\text{GeV}$.

In the figure 2 we compare theoretical results using eq. (1) within the CIM model for the $\nu_\mu n \rightarrow \nu_\mu p\pi^-$ reaction with the data of [13]. We make the cut $W \leq 1.45\text{GeV}$ in order to keep us within the first resonance region and to keep valid the complex mass scheme. Also we report the values for the R-ratios measured in the ANL experiment and our calculated values in the table 1. The $\sigma(\nu n \rightarrow \nu p\pi^-)$ prediction with the CIM is consistent with the experimental cross section in spite of the large errobars. The replacement of the full propagator by the RS one, leads to appreciable changes and the B-R interference is important. $R_{\pm}$ fall into the limits of the experiment and also the data are consistent with the fact that in our model both $\nu n \rightarrow \nu p\pi^-$ and $\nu p \rightarrow \nu n\pi^+$ amplitudes have the same isospin coefficients. Our calculated value for $R_0$ is out of the experiment limits, nevertheless it must be taken into account that being $\nu p \rightarrow \nu n\pi^+$ and $\nu p \rightarrow \nu p\pi^0$ measured as different channels of the same experiment, while $\nu n\pi^+$ events have a 30% of background events, $\nu p\pi^0$ have a 65% [14].

Now we show in the figure 3 results for the $\nu_\mu p \rightarrow \nu_\mu p\pi^0$ reaction not measured in ANL but reported in the CERN experiment, being its cross section extracted with the NUANCE code, and compare it with our prediction with the CIM model. CIM results are consistent with the

**Table 1.** Comparison of R-ratios for the reactions measured in [13, 14] with results obtained with the CIM model.

| R_− | $\frac{\sigma(\nu n \rightarrow \nu p\pi^-)}{\sigma(\nu p \rightarrow \mu^- p\pi^+)}$ | 0.11 ± 0.022(Derrick80) | 0.104(CIM) |
| R_+ | $\frac{\sigma(\nu p \rightarrow \mu^- p\pi^+)}{\sigma(\nu p \rightarrow \mu^- p\pi^-)}$ | 0.12 ± 0.04(Derrick81) | 0.104(CIM)(isos. sym.) |
| R_0 | $\frac{\sigma(\nu p \rightarrow \mu^- p\pi^-)}{\sigma(\nu p \rightarrow \mu^- p\pi^0)}$ | 0.09 ± 0.05(Derrick81) | 0.207(CIM) |
Figure 2. Theoretical calculations for the $\nu_\mu n \rightarrow \nu_\mu p\pi^-$ reaction are compared with the data of [13]. With B we indicate non resonant contributions plus the 1(g) one of the figure 1, and while with R only the pole contribution of the graph 1(h). We also show the full amplitude obtained with the RS propagator.

Figure 3. Theoretical calculations for the $\nu_\mu n \rightarrow \nu_\mu p\pi^0$ reaction are compared with the simulation of [15]. Lines convention is the same as in figure 2.

NUANCE simulation for $\sigma(\nu_\mu n \rightarrow \nu_\mu p\pi^0)$ cross section in spite of the $W$ cut in our calculations, which could be understood examining the $d\sigma/dW$ distribution for the CERN experiment, which shows a not appreciable number of events in the $\nu_\mu p\pi^0$ channel for $W > 1.45\text{GeV}$. Finally we mention that in CERN experience they report

$$R_0 = \frac{< \sigma(\nu p \rightarrow \nu p\pi^0) > + < \sigma(\nu n \rightarrow \nu n\pi^0) >}{2 < \sigma(\nu n \rightarrow \mu^-\pi^0) >} = 0.45 \pm 0.08,$$

in comparison with out theoretical value $R_0 = 0.766$. The lower value of $R_0$ regards our calculation could be due to our $W$ cut, but as is shown in the NUANCE simulation the measured value for $\sigma(\nu n \rightarrow \nu n\pi^0)$ (equal to $\sigma(\nu p \rightarrow \nu p\pi^0)$ within the CIM) is critically low, which could be due to the challenging final state to detect.

4. Conclusions

In summary, we have proved the predictive power of our consistent dynamical model also for weak NC1$\pi$ pion production. The model was previously used to describe pion-nucleon elastic and radiative scattering, pion photoproduction and CC1$\pi$ in a satisfactory way. We show, as in our previous calculations, the importance of using the full $\Delta$ propagator, which is consistent
with the $N \rightarrow \Delta$ weak vertex. Also we again mention the importance of the interference between the resonant and background contributions in the weak amplitude.

Actual Miniboone results [3] correspond to an energy region $0.4 < E_\nu < 2.4 \text{GeV}$ and no invariant mass cut is used, events occurring in the region $W < 1.6 \text{GeV}$, which is above the $\Delta$ one. Then, it is imperative to add consistently to the resonant amplitude the second resonance region ($N(1440)P_{11}, N(1520)D_{13}, N(1535)S_{11}$, etc.) contributions. Also we need to incorporate energy dependent contributions to the $\Delta$ self-energy but keeping invariance on contact transformations of the Lagrangian. Finally medium effects should be incorporated in a way this invariance and also electromagnetic gauge invariance are not violated.

Acknowledgments
A. M. and C. B. fellow to CONICET (Argentina), and this work was supported by the UNLP under the grant for visits 2009. G.L.C. acknowledges partial support from UNLP, under grant for visits 2008.

References
[1] Gran R 2007 Minerva Collaboration Preprint hep-ex/07113029
[2] Raaf J L 2005 Nuc. Phys. Proc. Suppl. B 139 47
[3] Aguilar-Arevalo A A et al 2009 Miniboone Collaboration Preprint hep-ex/0904.3159
[4] Hiraide K 2009 SciBoone Collaboration Preprint hep-ex/0909.5127
[5] Hasegawa M et al 2008 K2K Collaboration Phys. Rev. Lett. 95 252301
[6] Barbero C, López Castro G and Mariano A 2008 Phys. Lett. B 664 70
[7] El-Amiri M, López Castro G and Pestieau J 1992 Nucl. Phys. A 543 673
[8] Jones H F and Scadron M D 1973 Ann. Phys. 81 1
[9] Hernandez E, Nieves J and Valverde M 2007 Phys. Rev. D 76 033005
[10] Mariano A 2007 J. Phys. G: Nucl. Part. Phys. 34 1627
[11] López Castro G and Mariano A 2001 Phys. Lett. B 517 339
[12] Sato T, Uno D and Lee T S H 2002 Phys. Rev. C 67 065201
[13] Derrick M et al 1980 Phys. Lett. B 92 363
[14] Derrick M et al 1981 Phys. Rev. D 23 569
[15] Hawker E A 2002 Proceedings of the Second International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region http://www.ps.uci.edu/~nuint/proceedings/hawker.pdf
[16] Krenz W et al 1978 Nuc. Phys. B 135 45