A proposal for the measurement of Rashba and Dresselhaus spin-orbit interaction strengths in a single sample

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We establish an exact analytical treatment for the determination of the strengths of the Rashba and Dresselhaus spin-orbit interactions in a single sample by measuring persistent spin current. A hidden symmetry is exploited in the Hamiltonian to show that the spin current vanishes when the strength of the Dresselhaus interaction becomes equal to the strength of the Rashba term. The results are sustained even in the presence of disorder and thus an experiment in this regard will be challenging.

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Spintronics is a rapidly growing field that aims at the achievement of efficient quantum devices, such as the magnetic memory circuits and computers, in which one needs to manipulate the spin of the electron rather than its charge. Magnetic nano-structures and quasi-one dimensional semiconductor rings have been acknowledged as ideal candidates for testing the effects of quantum coherence in low dimensions, and have been extensively investigated to test their potential as new generation quantum devices mentioned above.

Spin-orbit interaction (SOI) in semiconductors is a central mechanism that determines the essential physics in the meso- and nano-scales, and is largely responsible for the prospect of semiconducting structures as potential quantum devices. Rashba spin-orbit interaction (RSOI) and the Dresselhaus spin-orbit interaction (DSOI) are the two typical spin-orbit interactions that one encounters in a conventional semiconductor.

The Rashba spin-orbit field in a solid is attributed to an electric field that originates from a structural inversion asymmetry whereas, the Dresselhaus interaction comes from bulk inversion asymmetry. Quantum rings formed at the interface of two semiconducting materials are ideal candidates to unravel the interplay of the two kinds of SOI. A quantum ring in a heterojunction, formed by trapping a two-dimensional gas of electrons in a quantum well, generates a band offset at the interface of two different semiconducting materials. This creates an electric field. This electric field is described by a potential gradient normal to the interface. The potential at the interface is asymmetric, leading to the presence of a RSOI. On the other hand, at such interfaces, the bulk inversion symmetry is naturally broken, and DSOI plays its part.

Needless to say, an accurate estimation of the SOI's is crucial in the field of spintronics. The RSOI can be controlled by a gate voltage placed in the vicinity of the sample. Thus, in principle, all possible values of the RSOI can be achieved. Measurement of RSOI has already been reported in the literature. Comparatively speaking, reports on the techniques of measurement of the DSOI are relatively few, and are mainly based on an optical monitoring of the spin precession of the electrons, measurement of electrical conductance of nano-wires, or photo-galvanic methods.

In the present communication we propose a method to measure the strengths of both the RSOI and DSOI by measuring the spin current flowing through a single sample. An important issue, while the measurement of a spin current in a mesoscopic system is concerned, is the non-conservation of spin caused by the ubiquitous presence of the spin-orbit coupling. A proper definition of spin current is therefore urgent, and its measurement in a sample is challenging. It has only recently been possible, using a novel version of the Doppler effect, to quantify the spin current in a ferromagnetic wire, and by using spin relaxation modulation induced by spin injection.

Inspired by the success in the measurement of spin current we provide a completely analytical scheme that immediately suggests an experiment to measure the strengths of either RSOI or the DSOI in the same sample. Our system is a mesoscopic ring grafted at a heterojunction, but in such a manner that the spin-orbit interaction is effective only in a fraction of the perimeter of the ring (see Fig. 1). The spin current should be measured in the SO interaction-free region where the spin flip scattering is absent. One thus need not worry about the non-conservation of spin and the spin current in any ‘bond’ in the non-shaded region in Fig. 1 is the same, and becomes a well defined quantity.
In this letter we prove that, in such a mesoscopic ring the spin current will be zero whenever the strengths of the RSOI and the DSOI are equal. As we have already mentioned above, the RSOI strength can in principle, be smoothly varied with applied gate voltage. This leads to the idea of estimating the strength of the DSOI if we happen to know a measured value of the RSOI. The idea is as follows. We devise a unitary transformation that acts on the full Hamiltonian of the system to extract a subtle symmetry that is finally exploited to achieve the result. Incidentally, similar kind of symmetry has previously been reported in literature\cite{Li, Zhang}. In this work we have focused on the effect of the symmetry on the spin current for a system in the presence of both the Rashba and the Dresselhaus interactions. Our result is analytically exact, and is true for any value of the spin-orbit interactions. It should therefore be observable in a suitably designed experiment. In addition, we numerically calculate the persistent spin current in the system, and determine its dependence on the length of the sample involving the SOI (determined by the angle $\gamma$ in Fig. 1). The numerical results corroborate our analytical work, and remain robust even in the presence of disorder.

- **Analytical treatment for the determination of SOI strengths:** Let us refer to Fig. 1. Only the bold portion of the arc contains the spin-orbit interactions. Within a tight-binding framework the Hamiltonian for such a ring reads,

$$H = H_0 - i\alpha H_{RSO} + i\beta H_{DSO}$$

(1)

where,

$$H_0 = \sum_n c_n^\dagger \epsilon_n c_n + \sum_n \left( c_{n+1}^\dagger t c_n + \text{h.c.} \right).$$

(2)

The Rashba and the Dresselhaus spin-orbit parts of the Hamiltonian, viz, $H_{RSO}$ and $H_{DSO}$, are given by,

$$H_{RSO} = \sum_n \left( c_n^\dagger \sigma_x \cos \varphi_n c_{n+1} + c_n^\dagger \sigma_y \sin \varphi_n c_{n+1} + \text{h.c.} \right)$$

(3)

$$H_{DSO} = \sum_n \left( c_n^\dagger \sigma_y \cos \varphi_n c_{n+1} + c_n^\dagger \sigma_z \sin \varphi_n c_{n+1} + \text{h.c.} \right)$$

(3)

where, $n = 1, 2, \ldots, N$ is the site index along the azimuthal direction $\varphi$ of the ring. The other factors in Eq. 3 are as follows.

$$c_n = \begin{pmatrix} c_{n+1}^\dagger \\ c_n \end{pmatrix}; \quad \epsilon_n = \begin{pmatrix} \epsilon_n^\dagger & 0 \\ 0 & \epsilon_n \end{pmatrix}; \quad t = t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here $\epsilon_{\sigma}$ is the site energy of an electron at the $n$-th site of the ring with spin $\sigma (\uparrow, \downarrow)$. $t$ is the nearest-neighbor hopping integral. $\alpha$ and $\beta$ are the isotropic nearest-neighbor transfer integrals which measure the strengths of Rashba and Dresselhaus SOI, respectively, and $\varphi_n, \varphi_{n+1} = (\varphi_{n+1} + \varphi_{n+1}) / 2$, where $\varphi_n = 2\pi(n-1)/N$. $\sigma_x$, $\sigma_y$ and $\sigma_z$ are the Pauli spin matrices. $c_{n+1}^\dagger$ ($c_n$) is the creation (annihilation) operator of an electron at the site $n$ with spin $\sigma (\uparrow, \downarrow)$. The form of the Hamiltonian written in the case of a ring is discussed elsewhere\cite{Wang}.

We now define a transformation,

$$U = \sigma_z \left( \frac{\sigma_x + \sigma_y}{\sqrt{2}} \right)$$

(4)

which, by definition is unitary. It is simple to verify that,

$$U \sigma_z U^\dagger = -\sigma_y; \quad U \sigma_y U^\dagger = -\sigma_x; \quad U \sigma_x U^\dagger = -\sigma_z.$$  

(5)

Using the operator defined in Eq. 4 we now make a change of basis and describe the full Hamiltonian in terms of the operators $\tilde{c}_n = U c_n$, and $\tilde{c}_n^\dagger = c_n^\dagger U^\dagger$. The Hamiltonian in the new basis reads,

$$\tilde{H} = \tilde{H}_0 - i\beta \tilde{H}_{RSO} + i\alpha \tilde{H}_{DSO}$$

(6)

where,

$$\tilde{H}_0 = \sum_n \tilde{c}_n^\dagger \epsilon_n \tilde{c}_n + \sum_n \left( \tilde{c}_{n+1}^\dagger t \tilde{c}_n + \text{h.c.} \right)$$

(7)

and, the spin-orbit parts of the Hamiltonian in this new basis are given by,

$$\tilde{H}_{RSO} = \sum_n \left( \tilde{c}_n^\dagger \sigma_x \cos \varphi_n \tilde{c}_{n+1} + \tilde{c}_n^\dagger \sigma_y \sin \varphi_n \tilde{c}_{n+1} + \text{h.c.} \right)$$

(8)

$$\tilde{H}_{DSO} = \sum_n \left( \tilde{c}_n^\dagger \sigma_y \cos \varphi_n \tilde{c}_{n+1} + \tilde{c}_n^\dagger \sigma_z \sin \varphi_n \tilde{c}_{n+1} + \text{h.c.} \right).$$

A simultaneous look at the Eqs. 8 and 9 reveal that in the new basis, the strengths of the Rashba and Dresselhaus spin-orbit interactions viz, $\alpha$ and $\beta$ have been interchanged. As a consequence, the polarized spin current operator in the quantized direction (+Z), for the free region, defined as $J_{\sigma}^z = \frac{1}{2} (\sigma_{\sigma} \hat{x} + \hat{x} \sigma_{\sigma})$ flips its sense of circulation, and becomes $-J_{\sigma}^z$. Here, $\sigma = \sum_n n \sigma_n c_n$. This immediately leads to the most important observation that, as soon as $\alpha = \beta$, $H$ and $\tilde{H}$ become identical. That is, the full Hamiltonian remains invariant under the unitary transformation defined by Eq. 4 whenever the strengths of the Rashba and the Dresselhaus spin-orbit interactions become equal. The energy eigenvalues obviously remain unchanged in this case.

At the same time, from the last equation in Eq. 5 we see that $U \sigma_z U^\dagger = -\sigma_z$, which means that, the spin current $J_{\sigma}^z$ calculated in the original basis becomes exactly equal to the spin current $-J_{\sigma}^z$ calculated in the new basis. This can happen only when $J_{\sigma}^z = 0$ as $\alpha = \beta$. The RSOI can be controlled by a gate voltage, and hence its strength is determined. So, an experiment on the measurement of spin current in a mesoscopic ring, where the RSOI is continuously varied, will show a complete disappearance of the spin current as soon as the strength of the DSOI becomes equal to that of the RSOI. We thus have a means.
of determining DSOI from a measurement of the spin current. The spin current, however, as stated earlier, is to be measured in that region of the ring which is free from the spin-orbit coupling.

Needless to say, the strength of the Rashba interaction can be obtained by the same method, if we know the Dresselhaus interaction beforehand.

• **Calculation of persistent spin current:** In the second quantized form the spin current operator $J^z_s$ for the free region can be written as,

$$J^z_s = 2i\pi \sum_n \left( c_{n+1}\sigma_z c_n - c_n^\dagger \sigma_z c_{n+1}^\dagger \right). \tag{9}$$

Therefore, for a particular eigenstate $|\psi_k\rangle$ the persistent spin current becomes, $J^z_s|\psi_k\rangle = \langle \psi_k | J^z_s | \psi_k \rangle$, where $|\psi_k\rangle = \sum_p a_{p,\uparrow}^k | p \uparrow \rangle + a_{p,\downarrow}^k | p \downarrow \rangle$. Here $| p \uparrow \rangle$'s and $| p \downarrow \rangle$'s are the Wannier states and $a_{p,\uparrow}^k$'s and $a_{p,\downarrow}^k$'s are the corresponding coefficients. In terms of these coefficients, the final expression of persistent spin current for k-th eigenstate reads,

$$J^z_{s,k} = 2\pi it \sum_n \left\{ a_{n,\uparrow}^{k,s} a_{n+1,\uparrow} a_{n,\uparrow}^{k,s} - a_{n+1,\uparrow} a_{n,\uparrow}^{k,s} \right\}$$

$$- 2\pi it \sum_n \left\{ a_{n,\downarrow}^{k,s} a_{n+1,\downarrow} a_{n,\downarrow}^{k,s} - a_{n+1,\downarrow} a_{n,\downarrow}^{k,s} \right\}. \tag{10}$$

Let us rename the polarized spin current $J^z_{s,k}$ as $J^k_s$ for the sake of simplicity. At absolute zero temperature ($T = 0$ k), net persistent spin current in a mesoscopic ring for a particular filling can be obtained by taking the sum of individual contributions from the energy levels with energies less than or equal to Fermi energy $E_F$. Therefore, for $N_e$ electron system total spin current becomes $J_s = \sum_{k=1}^{N_e} J^k_s$.

We measure spin current in the region which is free from the SOI, and, since the spin currents between any two neighboring sites in this interacting free region are identical to each other we compute the current only in a single bond.

In the present work we compute all the essential features of persistent spin current and related issues at absolute zero temperature and choose the units where $e = h = 1$. Throughout our numerical work we fix $t = 1$ and measure the energy scale in unit of $t$.

• **Numerical results:** In Fig. 2 we present the variation of the spin current as a function of the RSOI and the DSOI for a 40-site ordered half-filled ring. The angle $\gamma = \pi$. Figs. 2(a) and (b) refer to the cases where only the RSOI and the DSOI are present respectively. The RSOI and the DSOI drive the spin current in opposite directions (with equal magnitudes when $\alpha = \beta$), as is evident from the figure.

Figure 3 shows the variation of the spin current as a function of Rashba SO interaction strength for a 40-site ordered ring in the half-filled case considering different values of $\beta$, when $\gamma$ is fixed at $\pi$.

![Fig. 2](image1.png)

**Fig. 2:** (Color online). Persistent spin current as a function of SO coupling strength for an ordered 40-site half-filled ring, when $\gamma$ is set at $\pi$. (a) $\beta = 0$ and (b) $\alpha = 0$.

![Fig. 3](image2.png)

**Fig. 3:** (Color online). Persistent spin current as a function of Rashba SO interaction strength for a 40-site ordered ring in the half-filled case considering different values of $\beta$, when $\gamma$ is fixed at $\pi$.

![Fig. 4](image3.png)

**Fig. 4:** (Color online). Persistent spin current as a function of $\gamma$ for a 40-site ordered ring in the half-filled case considering different values of $\alpha$, when $\beta$ is fixed at 0.
any strength of the RSOI, provided it equals the DSOI strength. Hence, it is expected to be observed in experiments as well. The region of the mesoscopic ring in which the SOI is ‘on’, plays an important role in determining the strength of the spin current in the system. To get an idea, we systematically study the variation of $J_s$ as a function of $\gamma$ with the DSOI $\beta = 0$. For a given value of the angle $\gamma$, the current increases with increasing values of the RSOI strength $\alpha$ (see Fig. 4).

Finally, in view of a possible experiment, we test the robustness of our results by considering a 40-site disordered ring. Disorder is introduced via a random distribution (width $W = 1$) of the values of the on-site potentials (diagonal disorder), and results averaged over 40 disorder configurations have been presented (see Fig. 5). Since $H_0$ remains invariant under the unitary transformation defined by Eq. 4, all the qualitative results should remain unchanged even in the presence of disorder. However, the disorder will reduce the amplitude of the spin current. This is precisely what we find in our numerical analysis. The current still becomes zero as soon as $\alpha = \beta$. This observation strengthens our claim that a suitably designed experiment on the measurement of the spin current will lead to an exciting method of measurement of the Dresselhaus spin-orbit interaction from a knowledge of the gate controlled Rashba spin-orbit interaction in a mesoscopic ring.

A relevant question in view of the above discussion is the possibilities of the experimental detection of persistent spin current. The spin current, as obtained from the theoretical calculations already existing in the literature as well as from the present calculation, is a robust phenomenon undisturbed by the presence of disorder. Determination of spin current is also very much possible, and already a few experiments have been carried out in this direction\textsuperscript{27–30}. For example, by studying the Kerr effect\textsuperscript{25–29} associated with spin accumulations induced by the spin current or by investigating the reciprocal spin Hall effect\textsuperscript{30} persistent spin current can be detected. It is also well known that a spin current may produce a spin torque which can be measured experimentally\textsuperscript{31–33}. This definitely provides a way of estimating the spin current. Probably a more convenient way of detecting persistent spin current is the measurement of the electric field and electric potential induced by it\textsuperscript{34–38}. This is quite analogous to the detection of persistent charge current in a mesoscopic ring by measuring its induced magnetic field\textsuperscript{39,40}. We can also measure the strengths of spin-orbit interactions by attaching two electrodes in a mesoscopic ring. In that case also the persistent spin current as well as the transport spin current have well defined expressions and simply by measuring the transport spin current we can have an estimate of the SO interaction strengths. These latter observations are the major issues of our forthcoming paper.

In conclusion, we present an exact analytical method which shows that the strengths of the Rashba or the Dresselhaus spin-orbit interactions can be determined in a single mesoscopic ring by noting the vanishing of the spin current in the sample. A unitary transformation is prescribed, which when applied to the spin-orbit Hamiltonian brings out a hidden symmetry. The symmetry is exploited to prove that, by making the strengths of the two interactions equal, one achieves a zero spin current in the system. We provide numerical results which support all our analytical findings, and show that the vanishing of the spin current is a robust effect even in the presence of disorder. This last observation gives us confidence to propose an experiment in this line.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5.png}
\caption{(Color online). Persistent spin current as a function of Rashba SO interaction strength for a 40-site disordered ($W = 1$) ring in the half-filled case considering different values of $\beta$, when $\gamma$ is fixed at $\pi$.}
\end{figure}

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