Stochastic String Topography and Multivacua

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Abstract

The suggestion that there exist causally disconnected universes or subuniverses to explain the values of physical parameters such as the cosmological constant is discussed. A statistical model of the string landscape/topography is formulated using a stochastic Langevin equation for string and supergravity potentials. A Focker-Planck equation for the probability density of superpotentials is derived and the possible non-supersymmetric multivacua describing string/M-theory topography are investigated. The stochastic fluctuations of the superpotentials and their associated vacuum states can possibly lead to a small positive cosmological constant.

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1 Introduction

There has for many active physicists been a disturbing increase in the attention paid to the notion that we need a large, and possibly infinite, number of universes that make up a multiverse, $\mathcal{M}$. These universes are causally disconnected and therefore no observations can ever be performed to detect them. Much of the argument evolving from the multiverse point of view stems from various variations of the anthropic principle [1, 2, 3].

There have been two developments in physics that have been triggers for well-respected physicists to take such a serious step. The first is the egregious disagreement between naïve quantum field theory calculations of the cosmological constant $\Lambda$ and the observations of the CMB and supernovae SNIa [4, 5]. The cosmological constant was introduced by Einstein in his theory of gravity to stabilize the universe through the repulsive force it causes in his field equations. This constant came and went in the development of cosmology, but has now made a triumphant return.
due to the remarkable discovery that the expansion of the universe appears to be accelerating.

The cause for this acceleration could be the existence of dark energy, transparent to electromagnetic radiation, which can be identified as a smooth background of vacuum energy density. This identification is supported by WMAP and type SNIa supernova data \[4, 5\], for in the equation of state for a perfect fluid \( p = w\rho \) the parameter \( w \) is measured to be \( w = -1.02 \pm 0.5 \) and for the cosmological constant \( w = -1 \). However, the consensus opinion in the physics community is that all attempts have failed to explain why the naive quantum field theory calculation of the cosmological constant \( \Lambda \) is 122 orders of magnitude larger than the “observed” value, \( \rho_{\text{vac}} \sim (10^{-3} \text{eV})^4 \), where \( \Lambda = \rho_{\text{vac}}/M_P^2 \), \( \rho_{\text{vac}} \) is the vacuum energy density and \( M_P^2 \) is the reduced Planck mass \[6\]. To make matters worse, there is the preposterous outcome of the observed acceleration of the universe that the dominance of the cosmological constant in the evolution of the universe should have happened within our present epoch. This “coincidence” has also defied any simple explanation.

The second motivation is the observation that string theory must have a very large number (“googleplex”) of vacuum states \[7\]. This has been suspected (known) for a long time but certain string aficionados have emphasized this circumstance in the recent literature \[7, 8, 9, 10\], and there is less attempt to ignore or deny that this phenomenon will play a fundamental role in the development of string theory in the future. Of course, string theory has always suffered from the lack of a unique way to compactify the 10/11-dimensional theory to the 4 known spacetime dimensions. There is an infinite number of ways to perform such a compactification. The problem lies in the moduli fields that are ubiquitous in string theory and that reliable calculations in string theory can only be performed when the theory is supersymmetric. Recently, it has been proposed that string theories can exist in dimensions higher than the critical 10/11 dimensions \[11\], in which case the lack of uniqueness of string theory clearly increases enormously.

The original motivations for string theory were twofold: 1) It would provide a consistent way to combine quantum theory with gravity, 2) It could lead to a unification of the known four forces of Nature. It is often stated by string theorists that string theory is “the only way to unify Einstein’s GR with quantum theory.” This supposition has never been proved and alternative ways of unifying the two pillars of modern physics have been pursued over several years, and others will no doubt make their appearance in future \[12\].

The second motivation is the desire of particle physicists to seek a unification of the known forces of nature. This reductionist approach could hopefully lead to an understanding of the many free parameters in the Standard Model of particle physics and our present (incomplete) model of cosmology \[1\]. So far, such attempts at unification have been unsuccessful. Grand Unified Theories failed to be falsified by observations (except for the simplest model: the \( SU(5) \) model of Georgi and

\[1\] The number of parameters in the minimum supersymmetric standard model (MSSM) is of order 100.
Glashow \[13\]). No one has yet succeeded in showing that at least one of the many possible vacuum states in string theory succinctly and correctly describes the Standard Model of particle physics with the gauge group $SU(3) \times SU(2) \times U(1)$ and 3 generations, although there are several published low-energy supergravity models which describe in $D = 4$ dimensions extensions of the standard model, e.g. $SO(32)$, $SO(10)$ and $E_8 \times E_8$. Due to the large number of possible vacuum states, there exist many gauge groups that could describe low-energy string theory. There is also no knowledge of the energy scale at which the necessary supersymmetry breaking of string theory should occur. Recently, it has been argued that the supersymmetry breaking scale should be large, perhaps of order 1000 TeV or even at the Plank mass scale $\sim 10^{19}$ GeV \[14\]. The reasons for this have to do with several difficulties associated with a low-energy supersymmetry breaking scale $\sim 1$ TeV, e.g. the failure to observe a light Higgsino and the fast proton decay associated with dimension-5 operators in MSSM. Even if supersymmetric particles are observed at the large hadron collider (LHC), it would be argued that the solution to the electroweak hierarchy problem and the cosmological constant problem are unavoidable fine-tuning problems associated with low-energy physics, and should be understood in the context of the anthropic principle. Without an observation of superpartners, the whole edifice of supersymmetry and superstrings is to be seriously questioned.

To suggest that we should deviate fundamentally from the way physics has been done over the past three-four centuries by constructing a “statistical theory of theories” \[8, 9\], due to the failure of string theory to provide a unique, falsifiable framework, is providing present day string theory with an aura of validity that has not yet been established. There is presently no known concrete prediction made by string theory that can be falsified, although more than 25 years have been devoted to discovering such a prediction \(^2\). Because of this lack of testability, it would seem inappropriate at present to use the elegance of string theory to justify its correctness \(^3\).

The need for a de Sitter or almost de Sitter universe to explain the acceleration of the expansion of the universe has also led to problems in string theory, and complicated potentials have been invented to produce possible meta-stable string solutions \(^2\). The prediction of the correct number of degrees of freedom according to the Bekenstein-Hawking entropy law for black holes only holds uniquely for extremal black holes, which are physically un-realizable objects \[15\]. This result does not constitute a laboratory experiment or astronomical observation and, therefore, does not comply with the standard requirements of well-tested physics theories.

\(^2\)In the 19th century, the existence and nature of atoms and molecules were a mystery. In order to understand atoms, Lord Kelvin (William Thompson) \[16\] invented an elegant mathematical solution to the problem based on knot theory and topology. He speculated that electromagnetic forces were propagated as linear and rotational strains in an elastic solid ("ether"), producing vortex atoms which generated the field. He proposed that these atoms consisted of tiny knotted strings, and the type of knot determined the type of atom. The correct theory of chemical bonding that was supported by experiment was much less elegant than Kelvin’s imaginative mathematical theory. In recent years, knot theory has become a subject of considerable interest to mathematicians.
that support “inflation” \[17\] and a positive cosmological constant \[18\]. In view of the special initial conditions required by standard inflationary cosmology, namely, a peculiar flat inflaton potential and sufficient initial homogeneity of spacetime, inflation theory is described by an “eternal” inflation scenario \[19, 20\]. This scenario requires a multiverse or anthropic principle basis for its development. In contrast, the alternative solution of initial value problems of early universe cosmology, provided by a bimetric gravity theory \[21\] with a variable speed of light, predicts a power spectrum in agreement with the data that does not require fine-tuned potentials or a large initial vacuum energy and homogeneous universe. It postulates a large speed of light or a small speed of gravitons in the early universe, depending upon which metric frame is used to solve the cosmology, and there is no need for a multiverse or the anthropic principle.

String theory does not appear to be able to solve the “cosmological constant problem” using a “monovacuum”. We cannot, of course, exclude the possibility that some future form of string theory will solve this problem, but in the present form of the theory the possibility of doing so appears to face difficulties, even when mechanisms to break supersymmetry are invoked, e.g. employing flux tubes and D-branes. These mechanisms must use largely unknown non-perturbative physics to break the string supersymmetry. This supersymmetry breaking is essential to remove the massless moduli fields that disagree with experiment.

The very large number of vacuum states that could possibly support a small positive cosmological constant has led to the suggestion that there can exist a large or even infinite number of universes all with different values of \( \Lambda \), some of which could contain a vacuum density consistent with observations. After all, if you take a large enough sample of universes, then one can argue that some of them surely must have the correct vacuum state that supports a very small \( \Lambda \) and would “make life possible.” As has been argued recently \[22\], the fact that we observe galaxies in our universe and that the cosmological constant \( \Lambda \) has to be smaller than some given value to allow for the formation of galaxies \[23\], is simply an observational fact and any statistical arguments that claim to “explain” this fact are irrelevant. It has also be pointed out that we can conceive of universes in which both the cosmological constant and the matter density are much higher and yet galaxies can form \[24\]. Adherents of the anthropic principle disagree with this conclusion, arguing that the anthropic principle scenario and its consequences for the cosmological constant problem is serious science \[25\]. There are presently no compelling reasons to believe that the anthropic principle can lead to a falsification of the string landscape/topography scenario.

There have been times in the past when it seemed difficult and, indeed, maybe impossible to explain some experimental observation, e.g. the puzzle as to why the Sun could keep shining when its fuel should have been spent a long time ago. This led to speculations that if it did not continue to shine, then life on our planet could not exist and therefore some “anthropic” principle must be postulated to account for this dilemma. Another example is that it took almost 50 years from the discovery
of superconductivity by Kammerling-Onnes in 1911 to the understanding of this phenomenon by Bardeen, Cooper and Schrieffer [26]. The lack of a solution of the problem was due to the continuing attempt to use naive perturbation theory when, all along, a non-perturbative attack on the problem was required. Perhaps after many years of trying to solve the problem of superconductivity, some physicists began to feel that there was no solution, and that life as we know it could not exist, if the phenomenon of superconductivity was not explained and that only an “anthropic principle” argument could save the day. As far as we know, this need was not overtly publicized by any of the theorists who failed to understand the superconductivity phenomenon 4.

Given all of these arguments for and against the need for an anthropic principle or multiverse solution to physical theory, we shall nonetheless in the following pursue a statistical interpretation of the string theory multivacuum problem. We can conceive of many or even an infinite number of moduli super-potentials that describe the possible vacua of string theory as random fields. In Section 2, we shall adapt modified versions of well-known methods for analyzing stochastic processes to string theory to construct a Langevin equation for the string states, such that the supersymmetric string vacua undergo a deformation due to the stochastic fluctuations of the moduli potentials. In Section 3, we derive a Focker-Planck equation for the string probability distributions describing super-potentials 5, while in Section 4, these methods are applied to supergravity models for the moduli Kähler potentials. In Section 5, we use the stochastic fluctuations to attempt a solution of the cosmological constant problem in string theory, and in Section 6 we end with conclusions.

By following a statistical approach to string theory and its vacua, we abandon a complete deterministic approach to string theory as far as the solutions of the theory and its vacuum states are concerned. Perhaps, we can regard this purely statistical approach to the problem as an interim stage of the unification of forces and quantum gravity and hope that in the future a more unique and deterministic formulation of string theory will be discovered, allowing for calculations using a unique vacuum state.

2 Stochastic String Theory

In string/M-theory the space of solutions is controlled by the moduli space of supersymmetric vacua. The varying dynamical moduli fields determine the size and shape parameters of compact internal spaces that are required by 4-dimensional string theories.

4Recently, a possible resolution of the cosmological constant problem was proposed based on the idea that the vacuum is unstable in the presence of a gravitational field, causing fermion condensates to produce a non-perturbative vacuum energy gap that leads to an exponential suppression of the cosmological constant in the early universe [27].

5Stochastic probability methods have been applied to Einstein’s general relativity theory [28] and the problem of the cosmological constant [29-30].
ory. In a supersymmetric theory these moduli fields are massless scalar fields and cannot describe the real world. The continuum of supersymmetric moduli fields has an exact super-particle degeneracy and the associated cosmological constant \( \Lambda \) is zero. Therefore, we must seek non-supersymmetric moduli field solutions and the number of these solutions is so huge that we may as well say that it is infinite. The local minima of the potentials \( V(\phi) \) correspond to the possible values of the cosmological constant and describe the topographical values associated with valleys, while the peaks surrounding the extremal valleys are the mountain tops describing saddle points or maximum values of the moduli field solutions for the potentials. The maximum extremal values of the potentials are meta-stable points in the topography and if the local minima are absolute minima, then the “true” vacuum states are stable. The simplest solution of 10/11 string/M-theory is a flat Minkowski space manifested by the massless moduli fields. No such massless particles have been observed and we must modify the shape of this flat topography and fix the values of the moduli by minimizing the potential energy. If the minima are positive, then this indicates that the solutions are described by de Sitter or near de Sitter solutions. It is claimed that such solutions have been found by an intricate and complicated combination of string techniques, including D-brane/anti-brane configurations and fluxes of fields that are higher-dimensional generalizations of electromagnetic field sources [18].

Let us consider string stochastic processes. The potential energy \( V(\phi) \) for the moduli fields \( \phi \) satisfies the equation

\[
\partial_\phi V(\phi) - f(\phi) = 0,
\]

where \( \partial_\phi = \partial V(\phi) / \partial \phi \) and we adopt a real representation for the moduli fields \( \phi \). The \( \phi \) symbolically denotes the complete set of supermoduli fields \( \phi_i \) associated with a string model and \( f \) denotes the vector set \( f_i \). For a flat Minkowski vacuum solution \( \langle V(\phi) \rangle = \langle f(\phi) \rangle = 0 \), otherwise, for supersymmetric AdS and dS solutions \( \langle V(\phi) \rangle \) and \( \langle f(\phi) \rangle \) do not in general vanish. The infinite number of solutions for \( V(\phi) \) will be described by a stochastic process with random variables \( F(\phi) \). The Langevin [31] string potential equation then reads

\[
\partial_\phi f(\phi) = -\sigma f(\phi) + F(\phi),
\]

where \( \sigma \) is a constant. We assume that the stochastic fluctuations of the superpotentials \( V(\phi) \) are described by Gaussian fluctuations and the stochastic variables \( F(\phi) \) satisfy

\[
\langle F(\phi) \rangle = 0, \quad \langle F(\phi) F(\psi) \rangle = C \delta(\phi - \psi),
\]

where \( C \) is a constant.

Solving for \( f \) gives

\[
f = f_0 \exp(-\sigma \phi) + \int_0^\phi d\psi \exp(-\sigma(\phi - \psi)) F(\psi),
\]
where we have cast (2) into an integral form using either the Ito or Stratonovich integral calculus. By using (3) we obtain

\[ \langle f^2(\phi) \rangle = f_0^2 \exp(-2\sigma\phi) + \frac{C}{2\sigma}(1 - \exp(-2\sigma\phi)). \]  

(5)

For large \( \phi \) we shall make this equal \( 3E_s/m \), where \( E_s \) is the string energy scale and \( m \) is the mean moduli field mass. It follows that

\[ \langle F(\phi)F(\psi) \rangle = \frac{6E_s}{m}\sigma\delta(\phi - \psi). \]  

(6)

Integrating Eq. (4) we obtain

\[ V = V_0 + \frac{V_0}{\sigma}(1 - \exp(-\sigma\phi)) + \int_0^\phi d\psi \int_0^\psi d\chi \exp(-\sigma(\psi - \chi))F(\chi), \]

(7)

where \( V_0 = V(0) \). This yields the mean square displacement

\[ \langle (V(\phi) - V_0)^2 \rangle = \frac{V_0^2}{\sigma^2}(1 - \exp(-\sigma\phi))^2 \]

+ \( \frac{3E_s}{m\sigma^2}(2\sigma\phi - 3 + 4\exp(-\sigma\phi) - \exp(-2\sigma\phi)) \).  

(8)

For large \( \phi \) this gives

\[ \langle (V(\phi) - V_0)^2 \rangle = \frac{6E_s}{m\sigma}\phi. \]  

(9)

### 3 String Diffusion Focker-Planck Equation

Let us introduce the probability distribution function \( p(V(\phi), f(\phi)) \), which determines the probability \( P \) to have \( V(\phi) \) and \( f(\phi) \) for a given \( \phi \):

\[ P(V(\phi), f(\phi)) = p(V(\phi), f(\phi), \phi)dV(\phi)df(\phi). \]  

(10)

If the fluctuations are small during the shifts \( \Delta V(\phi) \) and \( \Delta f(\phi) \), then we can assume that \( V(\phi) \) and \( f(\phi) \) do not change significantly. We can then assume that the string state at \( V(\phi) + \Delta V(\phi) \) and \( f(\phi) + \Delta f(\phi) \) only depends on the string state at the values \( V(\phi) \) and \( f(\phi) \), whereby the string state is Markovian. Let us denote by the variable \( u \) the pair \( u(\phi) = (V(\phi), f(\phi)) \). We introduce the string transfer function \( \Psi(V, f, \Delta V, \Delta f, \Delta \phi) \) which describes the probability to go from string state 1 to string state 2. The evolution of the string Markov state depends on the transfer function and the initial string state. We have

\[ \int d\Delta \phi \Psi(u, \Delta u, \Delta \phi) = 1, \]  

(11)

yielding a unit probability.
The stochastic string Kolmogorov equation is [33]:

\[ p(u, \phi + \Delta \phi) = \int d\Delta u p(u - \Delta u, \phi) \Psi(u - \Delta u, \Delta u, \Delta \phi). \] (12)

For small values of \( \Delta \phi \) we can expand \( p(u) \):

\[ p(u, \phi + \Delta \phi) = p(u, \phi) + \partial_\phi p \Delta \phi + ... \] (13)

and obtain the Focker-Planck equation (FPE) [33]:

\[ \partial_\phi p(u, \phi) = -\partial_u \left[ f(u)p(u, \phi) \right] + \frac{1}{2} \partial_u^2 \left[ F^2(u)p(u, \phi) \right]. \] (14)

In general, the probability evolution of the string system in terms of the \( V(\phi), f(\phi) \) “phase” space is described by the moments \( \langle (F(\phi))^n \rangle \). However, for our string Focker-Planck equation, we solve the evolution only in terms of the first and second moments.

A useful solution of the FPEs can be obtained for stationary random \( u(\phi) \) processes. We assume that a string system subjected to \( V(\phi) \) and \( f(\phi) \) fluctuations can be described by a stationary behavior of the probability distribution density \( p \). This means that the string system can form a state for which the probability density \( p_S(u) \) has a shape that does not change as \( u \) develops. The sample paths \( u \) will in general not satisfy a steady state value \( u_S \), so that the \( V \) and \( f \) variables continue to fluctuate. However, these fluctuations are such that \( u \) and \( u + \Delta u \) have the same probability density, \( p_S(u) \).

The stationary solution \( p_S \) of the FPE satisfies

\[ \partial_\phi p(u, \phi) + \partial_u I(u, \phi) = 0, \] (15)

where

\[ I(u, \phi) = f(u)p(u, \phi) - \partial_u [F^2(u)p(u, \phi)]. \] (16)

The stationary FPE is then given by

\[ \partial_u I_S(u) = 0, \] (17)

which implies that \( I_S(u) = \text{constant} \).

The solution of the stationary FPE [17] reads

\[
\begin{align*}
p_S(u) &= \frac{C}{F^2(u)} \exp\left(2 \int_q dq \frac{f(q)}{F^2(q)}\right) \\
&- \frac{1}{F^2(u)}I \int_u dy \exp\left(2 \int_y dq \frac{f(q)}{F^2(q)}\right),
\end{align*}
\] (18)

where \( C \) denotes a normalization constant. When there is no flow of probability out of the string state, then for suitable boundary conditions, \( I = 0 \). We then obtain

\[
p_S(u) = \frac{C}{F^2(u)} \exp\left(2 \int_u dq \frac{f(q)}{F^2(q)}\right). \] (19)
Here, we have defined the stochastic integrals by using the Ito prescription \[32\] which leads to a consistent calculus for random Gaussian fluctuations. In this case, the normalization constant $C$ is given by

$$C^{-1} = \int_{u_1}^{u_2} du \frac{1}{F^2(u)} \exp \left( 2 \int_{u_1}^{u} dq \frac{f(q)}{F^2(q)} \right) < \infty. \quad (20)$$

4 Low-energy Supergravity and Stochastic Kähler Potential Fluctuations

Kachru et al. [18] (KKLT) have shown how geometric moduli can be stabilized in IIB orientifold or F-theory compactifications with $\mathcal{N} = 1$ supersymmetry in $D = 4$ dimensions. They begin with type IIB string compactification with NS and RR 3-form fluxes, $H$ and $F$, respectively, through the three-cycles of a Calabi-Yau manifold $M$. This yields a classical superpotential \[34\]:

$$\tilde{W} = \int_M \Omega \wedge G, \quad (21)$$

where $G = F - \tau H$ and $\tau$ is the axion-dilaton. In supergravity theory the effective 4-dimensional scalar potential is given by \[35\]

$$V = \exp(K)(G_{ij}D_i W D_j \bar{W} - 3|W|^2), \quad (22)$$

where we have chosen units for which $M_P^2 = 1$ and

$$D_i = \partial_i W + (\partial_j K) W. \quad (23)$$

Moreover,

$$G_{ij} = \partial_i \bar{\partial}_j K \quad (24)$$

is the Kähler metric obtained from the classical Kähler potential

$$K = - \log(\int_M \Omega \wedge \bar{\Omega}) - \log(\tau - \bar{\tau}) - 2 \log(\int_M J^3). \quad (25)$$

Here, $J$ is the form and $\Omega$ is the holomorphic 3-form on $M$. The indices $i, j$ run over all the scalar fields, including the complex structure moduli, $z_j$, the dilaton $\tau$, and the complex Kähler moduli, $\rho_k = b_k + i\sigma_k$. The $b_k$ denote an axion charge arising from the RR 4-form and $\sigma_k$ is the 4-cycle volume. In terms of the $t^i$ which measure areas of 2-cycles the classical volume is

$$V = \int_M J^3 = \frac{1}{6}\kappa_{ijk} t^i t^j t^k, \quad (26)$$

and

$$\sigma_k = \partial_{t_k} V = \frac{1}{2}\kappa_{ijk} t^i t^j. \quad (27)$$
The origin of the supergravity action can be either type IIB string theory or the heterotic string theory. Type IIB string theory is compactified on a Calabi-Yau manifold with Kähler moduli. In general the number of moduli is large, but in the literature usually only one or two moduli are analyzed and the dilaton and the classical complex structure moduli have been integrated out \[8, 36, 37, 38, 39\].

The equation
\[
D_i W \equiv \partial_i W + (\partial_i K)W = 0
\] (28)
determines the supersymmetric vacua. The cosmological constant becomes
\[
\Lambda_{\text{susy}} = -3 \exp(K)|W|^2,
\] (29)
corresponding to AdS or Minkowski spacetimes, and the latter hold for \(D_i W = W = 0\). Choices of fluxes \(G\) determine the \(z_j\) and the \(\tau\). Moreover, when the superpotential is independent of \(\rho_k\), then the \(3|W|^2\) cancels yielding a no-scale potential \(V\). This means that the stabilized supersymmetric potential is independent of the Kähler moduli. KKLT suggested two mechanism for stabilizing the Kähler moduli \[18\]: (1) brane instantons and (2) gaugino condensates. Such contributions are included as non-perturbative \(W_{\text{np}}\) effects in the superpotential. Recently, Brustein and de Alwis \[37\] and Balasubramanian and Berglund \[38\] showed that stringy corrections to the Kähler potential can give rise to non-supersymmetric vacua, including metastable de Sitter spacetimes. They also claim that the cosmological constant problem can be solved by a 1-loop correction to the cosmological constant such that the scale of supersymmetry breaking takes a realistic value. However, they are still faced with the discretuum of values of possible \(W\) and suggest that the anthropic principle should be used as a selective principle or, failing this, that the cosmological constant problem is an inevitable problem of fine-tuning.

We shall treat the “lifting” of the Kähler moduli to de Sitter spacetime by our stochastic method. We postulate the SDE:
\[
\partial_i W = f_i(K, W) + h_i(K, W),
\] (30)
where
\[
f_i(K, W) = -\partial_i K W,
\] (31)
and \(h_i\) is a stochastic variable which satisfies
\[
\langle h_i(K, W) \rangle = 0. \tag{32}
\]
We can cast this into an integral form
\[
W = W^0 + g^{ij} \int dY_i f_j(Y) + g^{ij} \int dY_i h_j(Y), \tag{33}
\]
where \(g^{ij}\) is a moduli field metric.
Let us denote by $w$ the absolute value of the superpotential, $w = |W|$, and by $\Phi$ the absolute value of the moduli variables, $\Phi = |\phi|$. For homogeneous Markov processes, the FPE becomes

$$
\partial_t p(w, \Phi_i) = -\partial_w [D_i(w)p(w, \Phi_i)] + \frac{1}{2} \partial^2_w [(H^2(w))_i p(w, \Phi_i)],
$$

(34)

where now $\partial_i = \partial_{\Phi_i}$, and the real drift $D_i(w)$ and the real diffusion $H_i(w)$ are independent of $\Phi_i$ and $H^2 = g^{kl} H_k H_l$.

The real stationary probability density solution, $p_S$, is given by

$$
\partial_\Phi p_S(w, \Phi) + \partial_w I(w, \Phi) = 0,
$$

(35)

where $\partial_\Phi p = \partial p/\partial \Phi$ and we have for convenience of notation suppressed the indices $i, j$ and

$$
I(w) = D(w)p(w) - \partial_w [H^2(w)p(w)].
$$

(36)

The stationary solution to the equation

$$
\partial_w I(w) = 0
$$

(37)

is given by

$$
p_S(w) = \frac{C}{H^2(w)} \exp \left( 2 \int^w dX \frac{D(X)}{H^2(X)} \right)
- \frac{1}{H^2(w)} I(w) \int^w dY \exp \left( 2 \int^Y dX \frac{D(X)}{H^2(X)} \right).
$$

(38)

For no flow of probability, $I(w) = 0$, we obtain

$$
p_S(w) = \frac{C}{H^2(w)} \exp \left( 2 \int^w dX \frac{D(X)}{H^2(X)} \right),
$$

(39)

where

$$
C^{-1} = \int^{w_2}_{w_1} dw \frac{1}{H^2(w)} \exp \left( 2 \int^w dX \frac{D(X)}{H^2(X)} \right).
$$

(40)

A generic feature of the solution for the stationary probability density (39) is that for a vanishing stochastic average $\langle |H(w)|^2 \rangle \to 0$, the probability density will have a maximum value at, say, $w_{\text{max}}$. Thus, the minimum values of the potentials for non-supersymmetric vacua will occur in the neighborhood of the superpotential value $w_{\text{max}}$ for small values of $H(w)$.

5 The Cosmological Constant and Stochastic Fluctuations

We shall assume that the stochastic supersymmetric breaking fluctuations contribute to the “lifting” of the supersymmetric value of the cosmological constant given by (29), so that the effective cosmological constant is given by

$$
\Lambda^2 = \Lambda^2_{\text{susy}} + \Lambda^2_{\text{fluct}} + \Lambda^2_{\text{np}},
$$

(41)
where

$$\Lambda_{\text{fluct}}^2 = \langle V^2(\phi) \rangle,$$

(42)

and $\Lambda_{\text{np}}$ denotes the contributions to the effective cosmological constant due to stringy non-perturbative effects.

Quantum “stringy” corrections to $\Lambda_{\text{susy}}$ will break supersymmetry and “hide” superpartners from observation. The scale of the non-perturbative supersymmetry breaking will occur at energies $E_{\text{sb}} > 1$ TeV. We assume that the stochastic fluctuations that break supersymmetry dominate the probabilistic determination of $\Lambda$, and in particular that this is true at much lower energies, $E_{\text{fluct}} \sim 10^{-3}$ eV, for small values of the stochastic fluctuations. Moreover, the magnitude of the value of $\Lambda_{\text{susy}}$ must be small, corresponding to small values of $K$ and $W$ in (29). Then, it is possible to obtain a small positive value for the effective cosmological constant $\Lambda$, corresponding to a vacuum energy density $\rho_{\text{vac}} \sim (10^{-3} \text{eV})^4$, at the peak value of $\langle V^2(\phi) \rangle$.

The actual supersymmetric breaking energy scale $E_{\text{sb}}$ could be at a large energy, $E_{\text{sb}} \gg 1$ Tev, but the stringy corrections would not significantly affect the “lifting” of $\Lambda_{\text{susy}}$, whereas the stochastic fluctuations could become significant already at the low-energy scale $E_{\text{fluct}} \sim 10^{-3}$ eV where the vacuum energy density $\rho_{\text{vac}}$ is found to give a positive de Sitter value of $\Lambda$. Since the value of $\Lambda$ fluctuates as the universe expands, it is possible to explain the “coincidence” of the onset of the acceleration of the expansion of the universe in the present epoch.

6 Conclusions

We have adopted the point of view that if string theory and its supergravity low-energy limit do correctly describe nature, then the string topography should be described by a stochastic probabilistic theory. We could interpret the different possible values of the cosmological constant as being associated with different universes, and the most likely value determined by our stochastic probability theory is the one describing our universe. However, we do not have to postulate a multiverse picture and its accompanying anthropic principle. The many possible extremal values of the superpotential in string theory are described by a random variable and a non-vanishing correlation function, formed from the stochastic random variable describing the superpotentials as functions of the moduli fields $\phi$. The most likely value of the superpotential is determined by a Focker-Planck equation for the superpotential probability distribution. Thus, a probabilistic depiction of string theory is interpreted as a statistical solution of the string topography within our universe and, as in quantum mechanics, a parallel universes interpretation is a possible philosophical interpretation of the world that can be avoided as a non-falsifiable paradigm.

Our approach is different from the one described by the anthropic principle, in that we postulate an ensemble of superpotentials and classically solve for the most likely potential by means of stochastic probability theory, because we do not
have enough information at present to solve string theory and obtain a unique one vacuum solution. Thus, we say that we live in one universe and we adopt a statistical approach to solving the superpotentials and finding the most likely value for the vacuum state in our universe. For the anthropic principle, on the other hand, each of the string superpotentials and vacuum states that make up the ensemble of universes represents a real result of an observation in one of the universes, and only by saying that one or another of the possible choices can sustain life can we decide which is associated with our universe.

It is possible that we shall never discover a way to describe string theory vacua by a monovacuum, leading to a conventional quantum field theory solution to the cosmological constant problem. Perhaps, only a statistical probability description of the string topography will lead to a string description of quantum gravity and particle physics.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada. I thank Martin Green for helpful discussions.

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