A PROOF OF CONCEPT EXPERIMENT TO CONSTRAIN THE FOREGROUND SPECTRUM FOR
GLOBAL 21 CM COSMOLOGY THROUGH PROJECTION-INDUCED POLARIMETRY

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Abstract

Detecting the cosmological global (sky-averaged) 21 cm signal as a function of observed frequency will provide a powerful tool to study the ionization and thermal history of the intergalactic medium (IGM) in the early Universe (~400 million years after the Big Bang). The greatest challenge in conventional total-power global 21 cm experiments is the removal of the foreground synchrotron emission (~10^3-10^4 K) to uncover the weak cosmological signal (tens to hundreds of mK) since the intrinsic smoothness of the foreground spectrum is corrupted instrumental effects. Although the EDGES team has recently reported an absorption profile at 78 MHz in the sky-averaged spectrum, it is necessary to confirm this detection with an independent approach. In this paper, we present a new polarimetry-based, approach which relies on the dynamic characteristics of the foreground emission at the circumpolar region to track and remove the foreground spectrum directly, without relying on generic foreground models as in conventional approaches. Due to asymmetry and the Earth’s rotation, the projection of anisotropic foreground sources onto a wide-view antenna pointing at the North Celestial Pole (NCP) induces a net polarization with distinctive temporal variations. Different from the total-power approach, the Cosmic Twilight Polarimeter (CTP) is designed to measure and separate the varying foreground from the isotropic cosmological background simultaneously. By combining preliminary results of the prototype instrument with numerical simulations, we evaluate the practicality and feasibility for implementing this technique to obtain an independent global 21 cm measurement in the near future using an upgraded CTP.

Subject headings: dark ages, reionization, first stars - techniques: polarimetric - methods: observational

1. INTRODUCTION

Measuring the redshifted hyperfine transition 21 cm line of neutral hydrogen (HI) has been considered the primary means to probe the thermal and ionization history of the intergalactic medium (IGM) in the high-redshift Universe, during the three main phases known as: the Dark Ages (1,100 ≥ z ≥ 30), Cosmic Dawn (30 ≥ z ≥ 15), and the Epoch of Reionization (EoR, 15 ≥ z ≥ 6) [Furlanetto et al. 2006; Furlanetto 2006; Pritchard & Loeb 2010, 2012; Liu et al. 2013; Barkana 2016]. Development of large interferometric arrays has been focusing on measuring the power spectrum of spatial fluctuations of the 21 cm brightness temperature at the end of the EoR (e.g., Parsons et al. 2010; Tingay et al. 2013; Bowman et al. 2013; van Haarlem et al. 2013; Paciga et al. 2013; Meléndez et al. 2013; DeBoer et al. 2017).

For the last decade or so, there are also other efforts in searching for the sky-averaged monopole component of the redshifted 21 cm signal, where one employs a single dipole antenna or a compact array consisting of a small number of antenna elements to measure the cosmological signal averaged across all sky directions over a large frequency range (~40 ≤ ν ≤ 200 MHz). Theoretical studies have shown that global measurement of the 21 cm signal can complement the power spectrum measurements by providing meaningful physical parameters for the evolution of the IGM (e.g., Furlanetto 2006; Pritchard & Loeb 2012; Liu et al. 2013; Mirocha et al. 2013, 2015, 2017).

Conventional total-power global 21 cm experiments point their antennas at the zenith and integrate the sky signal into a single averaged spectrum containing both the cosmological 21 cm background along with the foreground synchrotron emission from both Galactic and extragalactic sources. Some of the current and past global 21 cm experiments include: the Experiment to Detect the Global EoR Signature (EDGES I & II, Bowman et al. 2008; Bowman & Rogers 2010; Mensa et al. 2017), the Shaped Antenna Measurement of the Background Radio Spectrum (SARAS 1 & 2, Patra et al. 2013; Singh et al. 2017), the Broadband Instrument for Global Hydrogen Reionization Signal (BIGHORN, Sokolowski et al. 2015a), the Large-Aperture Experiment to Detect the Global the Dark Ages (LEDa, Greenhill & LEDa Collaboration 2015), the Sonda Cosmológica de las Islas para la Detección de Hidrógeno Neutro (SCI-HI, Voytek et al. 2014), and the Probing Radio Intensity at high-z from Marion (PRiZM, Philip et al. 2018).

Among systematic errors in these experiments, the...
In this study, we extend that framework to a working prototype instrument, the Cosmic Twilight Polarimeter (CTP), along with more in-depth numerical simulation to further evaluate the proposed technique. In conjunction, we also explore the use of sophisticated pattern recognition algorithms like the singular-value decomposition (SVD) in separating complex instrument systematics from the signal of interest.

In Section 2, we review the general approach of the PIPE and how it can be used to constrain the foreground spectrum. In Section 3 details on the instrumentation and calibration procedures for the CTP are provided. This is followed by the evaluation on the preliminary observation with the PIPE simulation including some of the major instrumental effects in Section 4. Some of the related implementation aspects and mitigation strategies are also discussed in Section 5. Implications of the PIPE on extracting the global 21 cm signal using the SVD are examined in Section 6. Finally, we conclude this work in Section 7 with lessons learned and the near-future plans on upgrading the CTP to better suit for searching the global 21 cm signal. This paper has been adapted from the Ph.D. dissertation by Nhan (2018).

2. PROJECTION-INDUCED POLARIZATION EFFECT

As described in NB17, by observing the sky with a broad-beam antenna, projection of the asymmetric foreground emission (as shown in the Haslam map, Haslam et al. [1982]) onto the antenna plane can give rise to a net induced polarization. This induced polarization can be quantified with the composite Stokes vector as a function of time and frequency, \( \mathbf{S}_{\text{net}}(t, \nu) = \{ I_{\text{net}}(t, \nu), Q_{\text{net}}(t, \nu), U_{\text{net}}(t, \nu), V_{\text{net}}(t, \nu) \} \). We simulated the PIPE by convolving the the antenna beams to the Haslam map scaled to the CTP’s range (60-80 MHz) using a power-law function with a spectral index of \( \beta = 2.47 \). The net induced polarization is subsequently computed as a vector sum of the Stokes parameters calculated from all directions.

We simulate the foreground to revolve about the stationary antenna centered at the North Celestial Pole (NCP) at a rate of \( \omega_{\text{sky}} \), the net induced polarization

![FIG. 1.— The majority of the diffuse emission is concentrated on the Galactic disk, with sparsely distributed extragalactic sources above and below the disk, as shown (here in equatorial projection) in the Haslam full-sky map at 408 MHz, Haslam et al. [1982].](image)
is maximum (minimum) when it is parallel (orthogonal) to one of the dipoles. As a result, the Stokes parameters $Q_{\text{net}}(t, \nu)$ and $U_{\text{net}}(t, \nu)$, which are measures of the net linear polarization at $(0^\circ, 90^\circ)$ and $(45^\circ, -45^\circ)$ respectively, peak twice per sidereal day. In the special case with an idealized Gaussian beam, $Q_{\text{net}}(t, \nu)$ and $U_{\text{net}}(t, \nu)$ are sinusoidal functions with an angular frequency equaling to twice the sky revolving rate ($2\omega_{\text{sky}}$).

By Fourier decomposing the waveforms of Stokes parameters at each observed frequency, the magnitude of the power spectral density (PSD) of Stokes waveforms are computed for each harmonic mode $n$ to construct the Stokes spectra as (Heinzel et al. 2002)

$$S_{n,\nu}^w = \frac{(\Delta t)^2}{s_1^2} \left| \sum_{t=1}^{M} w_{\text{BH4}}(t)s_i(t)e^{-i2\pi t/(nM\Delta t)} \right|^2,$$

where $s_i$ is one of the four Stokes parameters, $w_{\text{BH4}}(t)$ is the four-term Blackman-Harris window function to prevent spectral leakage, $s_1 = \sum_{t=0}^{M} w_{\text{BH4}}(t)$ is the normalization for discrete data length of $M$ with the averaging interval $\Delta t$.

In NB17, with the absence of antenna beam’s spectral dependence, we showed that the foreground spectrum $T_b(\nu)$ can be constrained by either $S_{Q,2}^w$ or $S_{U,2}^w$. The input global 21 cm background model $\delta T_{b,21\text{cm}}(\nu)$ can then be extracted by iteratively scaling and subtracting the second-harmonic Stokes spectrum from the total sky spectrum $S_0^w$, i.e., $\delta T_{b,21\text{cm}}(\nu) = T_{\text{sky}}(\nu) - T_b(\nu) = S_0^w - AS_{Q,2}^w$, where $A$ is some best-fitted scaling constant.

3. A PROOF-OF-CONCEPT EXPERIMENT

In this section, we provide a brief description of the CTP, a prototype instrument used to evaluate the PIPE technique.

This net polarization establishes a direct means to measure only the foreground spectrum but not that of the cosmological 21 cm background. The cosmological 21 cm signal is spatially isotropic since its brightness fluctuations at small angular scale ($< 2^\circ$, Bittner & Loeb 2011) are not resolvable compared to the foreground anisotropy at larger angular scale in the sky-averaged measurement. As illustrated in the three simple scenarios in Figure 2:

(a) When four point sources with identical brightness, at some arbitrary level, are placed at equal distance from the center of the FOV (left panel), no net polarization is produced due to symmetry (right panel). A similar argument can be applied to the isotropic 21 cm background, which is not expected to produce a net induced polarization in the absence other instrumental effects.

(b) With four point sources at equal distance, but one source stronger than the others, projection of the source onto the antenna plane induces a net polarization whose waveform is periodic related to the rate of the sources revolving about the center.

(c) Similarly, asymmetry in projection of the Haslam map revolving about the NCP also produces a net polarization, as a sinusoidal waveform with a twice diurnal (two cycles per sidereal day) period.

![Figure 2](image_url)

Fig. 2.— Illustrations of three simple scenarios for the PIPE: (a) Four identical point sources at equal distance from the center (left panel) produces zero net polarization (right panel). (b) In a similar situation with four point sources at equal distance, a net polarization is produced when one of the point sources is stronger than the remaining three. (c) As an example at 60 MHz, the polarization is produced due to symmetry (right panel). A similar argument can be applied to the asymmetric distribution of foreground emission from the Haslam map centered at the NCP produces a net polarization which can be used to track the foreground, with the signature twice-diurnal periodicity.

![Figure 3](image_url)

Fig. 3.— Front view of the titled sleeved dipole antenna pointing at the NCP. The antenna consists of dual-polarized dipoles between two circular disks. The antenna was mounted on top of the temperature-controlled enclosure for the FE electronics. The external conductive mesh skirt helps to ensure a symmetrically circular beam pattern.
3.1. Instrumentation and Data Acquisition

The CTP uses a broadband, sleeved dipole with a pair of orthogonal dual-polarized antenna. The antenna is surrounded by a conductive mesh skirt in attempt to enhance the beam symmetry between two polarizations. Besides the dual-polarization antenna, the system consisted of a thermally stabilized front-end (FE) stage along with a back-end (BE) instrument rack stored in a weatherproof and thermally regulated enclosure. The prototype was deployed during the Fall of 2017 at the Equinox Farm, LLC, in Troy, VA (38.0°N, 78.3°W). The general layout of the CTP is illustrated in Figure 3.

![Block diagram for the CTP’s data acquisition (DAQ) pipeline.](image)

Fig. 4.— Block diagram for the CTP’s data acquisition (DAQ) pipeline. The instrument FE (blue shaded) consists of a thermal controlled stage for the main RF chain and the calibrator tone. The FE instrument rack consist of the signal digitizer, and frequency counters for monitoring the power level of the calibrator tones with the General Purpose Interface Bus (GPIB). The sampled signal are channelized into spectra with the FFTW3 program before the signal gain and noise temperature are corrected. In the end, Stokes parameters are calculated and monitored as a function of time before applying the harmonic decomposition on them for further analysis.

The signal from each polarization was amplified and filtered through a radio-frequency (RF) module in the FE. Although the sleeved antenna was designed to operate between 60-120 MHz, a 20-MHz bandpass filter (BPF) centered at 75 MHz was used to reject radio frequency interference (RFI) from local digital TV stations and the FM band (88-108 MHz).

The output voltages from both polarizations of the antenna were digitized by a FPGA-based analog-to-digital converter (ADC), Signatec PX14400A, before being channelized with fast Fourier transform (FFT), using the FFTW8 software library, into complex voltages $V_X(t, \nu)$ and $V_Y(t, \nu)$ at a resolution bandwidth (RBW) of $\Delta \nu \sim 57.00$ kHz. A block diagram for the data acquisition (DAQ) pipeline is provided in Figure 4.

Subsequently, their autocorrelation ($\langle \tilde{V}_X \tilde{V}_X^* \rangle$, $\langle \tilde{V}_Y \tilde{V}_Y^* \rangle$) and cross-correlation ($\langle \tilde{V}_X \tilde{V}_Y^* \rangle$, $\langle \tilde{V}_Y \tilde{V}_X^* \rangle$) were calculated and integrated over some $\Delta t$. The resulting uncalibrated net Stokes parameters were computed as

$$I_{\text{uncal}}(t, \nu) = \langle \tilde{V}_X \tilde{V}_X^* \rangle + \langle \tilde{V}_Y \tilde{V}_Y^* \rangle,$$

$$Q_{\text{uncal}}(t, \nu) = \langle \tilde{V}_X \tilde{V}_X^* \rangle - \langle \tilde{V}_Y \tilde{V}_Y^* \rangle,$$

$$U_{\text{uncal}}(t, \nu) = \langle \tilde{V}_X \tilde{V}_Y^* \rangle + \langle \tilde{V}_Y \tilde{V}_X^* \rangle,$$

$$V_{\text{uncal}}(t, \nu) = i \left( \langle \tilde{V}_X \tilde{V}_Y^* \rangle - \langle \tilde{V}_Y \tilde{V}_X^* \rangle \right).$$

The Stokes parameters were converted to temperature units after calibrating for the transducer gain, $G_T(t, \nu)$, and noise temperature, $T_n(t, \nu)$, through the use of

$$I_{\text{cal}}(t, \nu) = \frac{1}{k_B \Delta \nu} \left[ \frac{\langle \tilde{V}_X \tilde{V}_X^* \rangle}{G_{T,X}} + \frac{\langle \tilde{V}_Y \tilde{V}_Y^* \rangle}{G_{T,Y}} \right] - \left( T_{n,X} + T_{n,Y} \right),$$

$$Q_{\text{cal}}(t, \nu) = \frac{1}{k_B \Delta \nu} \left[ \frac{\langle \tilde{V}_X \tilde{V}_X^* \rangle}{G_{T,X}} - \frac{\langle \tilde{V}_Y \tilde{V}_Y^* \rangle}{G_{T,Y}} \right] - \left( T_{n,X} - T_{n,Y} \right),$$

$$U_{\text{cal}}(t, \nu) = \frac{2}{k_B \Delta \nu} \frac{\text{Re} \left( \langle \tilde{V}_X \tilde{V}_Y^* \rangle \right)}{\sqrt{G_{T,X} G_{T,Y}}},$$

$$V_{\text{cal}}(t, \nu) = \frac{-2}{k_B \Delta \nu} \frac{\text{Im} \left( \langle \tilde{V}_X \tilde{V}_Y^* \rangle \right)}{\sqrt{G_{T,X} G_{T,Y}}}.$$

where the subscripts in $G_T(t, \nu)$ and $T_n(t, \nu)$ refer to polarizations $X$ and $Y$. By definition, the transducer gain is the ratio of the averaged power delivered to the load of the system and the maximum power available from the source. The derivation of the calibration equations are provided in Appendix A.

3.2. Network-theory Based Calibration

3.2.1. Correct Transducer Gain with S-Parameters

Conventionally, total-power experiments utilize a broadband noise source, such as a 50 $\Omega$ resistor, as an internal reference calibrator to correct for the absolute gain in the system. One example is the on-off reference load calibration for correcting the measured sky signal power with the reference load power, which in a simple
where $G_{FE}(\nu)$ and $T_{n,FE}(\nu)$ are the transducer gain and noise temperature of the FE electronics, whereas $G_{rcv}(\nu)$ and $T_{n,rcv}(\nu)$ are ones contributed by the receiver. Similar to calibration techniques which compare the unknown measurement to a reference signal to correct the systematics, such as the Dicke switching (Dicke 1982), both the transducer gain and noise temperature were assumed to be identical among both states over the short time interval between switching. Hence, $T_{ant}(\nu)$ in Equation (11) was solved for by substituting the gain and noise temperature from Equation (12) into (11) such that,

$$T_{ant}(\nu) = \left[ \frac{P_{ant}(\nu)}{k_B \Delta \nu} - \frac{P_{ref,L}(\nu)}{G_{FE}G_{rcv}} \right] + T_{ref,L}, \quad (13)$$

where $T_{n,FE}(\nu)$ and $T_{n,rcv}(\nu)$ canceled out between the two states. Similarly, experiments like EDGES and PRiZM are calibrated by switching continuously between the antenna and multiple reference loads (e.g., 50 Ω, 100 Ω, short, and open).

Nonetheless, assuming the gain and noise temperature to be constant between the two states is inaccurate. According to electrical network theory, for a two-port network device, like the FE of the CTP, the power gain can be described by the transducer gain, $G_T(\nu)$, of the device under test (DUT). In terms of the complex scattering (or $S$-) parameters, $G_T(\nu)$ is given by (Collin 2007),

$$G_T(\nu) = \frac{(1 - \Gamma_{src}^2) (1 - |\Gamma_{load}|^2)}{|(1 - S_{11}\Gamma_{src})(1 - S_{22}\Gamma_{load}) - S_{12}S_{21}\Gamma_{src}\Gamma_{load}|^2}, \quad (14)$$

where $S_{11}(\nu)$, $S_{12}(\nu)$, $S_{21}(\nu)$, and $S_{22}(\nu)$ are the complex components of the scattering matrix which describes the intrinsic properties of the DUT and can be measured with a Vector Network Analyzer (VNA) in the laboratory. Meanwhile the complex reflection coefficients of the DUT due to the source, $\Gamma_{src}(\nu)$, and the system’s load $\Gamma_{load}(\nu)$, depend on the impedance of the external devices being connected to it, where $\Gamma_i = (Z_i - Z_0)/(Z_i + Z_0)$ with the characteristic impedance $Z_0$ which is typically chosen to be a standard value such as 50 Ω.

Since the $S$-parameters are fixed for the DUT, the transducer gain depends strongly on $\Gamma_{src}(\nu)$, which can be the reflection coefficient from either the antenna or the reference loads in the load-switching scheme. In other words, the transducer gain is, in fact, different when the system is connected to either the antenna or one of the calibration loads.

Furthermore, the $S$-parameters are functions of operating temperature. Most total-power experiments typically have the FE electronics exposed to the ambient temperature, this increases the difficulty in correcting the transducer gain as it fluctuates with the operating temperature. The CTP has implemented an active thermal control system at the FE, using thermoelectric Peltier cooler powered by a proportional-integral-derivative (PID) feedback circuit.

Instead of constraining $G_T(\nu)$, the CTP’s gain calibration relies on determining the four $S$-parameters intrinsic to the FE system. Using a VNA, we measured the $S$-parameters of the FE at different set temperatures over the PID controller’s range (~20-35°C) in the laboratory. We acquired a set of calibration coefficients through least-squares fitting the $S$-parameters as functions of temperature and frequency. These coefficients are subsequently used to compute the required $S$-parameters for calibrating $G_T(\nu)$ based on the in situ temperature metadata recorded during the observation.

In conjunction with the thermal stabilization, the CTP employs a strong reference tone injected into the RF chain at a constant frequency of $\nu_{tone} = 68$ MHz (from either a crystal oscillator) to determine additional gain variations during the observation. Instead of switching between the antenna and reference loads as in other experiments, a constant tone at a fixed frequency helps to prevent changes in the input impedance and thus the spectral response of the electronic devices throughout the observation. As illustrated in Figure 3, as the calibrating tone is injected to the main RF chain through a directional coupler, the original tone power is also monitored separately with a universal frequency counter (HP 5335A) after its power is transformed by a voltage-to-frequency converter. This allows the tone power to be measured more accurately in frequency counts. In principle, the power gain fluctuations in the RF chain can be corrected by adjusting the power level deviations in the tone channel sampled by the ADC against the tone value recorded by the frequency counter. This tone calibration scheme is currently being improved and its effectiveness will be further evaluated in upcoming observations.

3.2.2. Correct Noise Temperature with Noise Parameters

In analogy to the transducer gain, instead of determining the noise temperature is also a function of the input source impedance $Z_{src}(\nu)$, it can be approximated in terms of a set of four noise parameters which are intrinsic to the DUT just like the $S$-parameters.

According to noise theory, $T_{n}(\nu)$ reaches a minimum value at $T_{min}(\nu)$ when the source impedance is matched to an optimal value $Z_{opt}(\nu)$, which can be parametrized as (Engberg & Larsen 1995),

$$T_{n}(\nu) = T_{min} + \frac{4NT_0 \Gamma_{src} - \Gamma_{opt}}{|1 + \Gamma_{opt}|^2 (1 - \Gamma_{src})^2}, \quad (15)$$

where the dimensionless $N(\nu) = R_n(\nu)G_{opt}(\nu)$ is the product of the equivalent noise resistance $R_n(\nu) = \langle v_n^2 \rangle/(4k_B T_0 \Delta \nu)$ and the optimal source conductance $G_{opt}(\nu)$, with $\langle v_n^2 \rangle$ the mean-square noise-generator voltage and $T_0 = 290 K$.

The complex reflection coefficients of the source and optimal impedance are defined

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9. The system load is distinct from the reference loads at the input port of the DUT. By convention, the load impedance refers to the one of any device connected at the output port of the device.

10. This assumes that the device operates in the linear regime, for example, an active device like an LNA is not overdriven which results in gain compression into the nonlinear regime.

11. The standard noise temperature is conventionally defined to be 290 K so that $k_B T_0 \approx 4.00 \times 10^{-21}$ Ws.
as $\Gamma_{\text{src}}(\nu)$ and $\Gamma_{\text{opt}}(\nu)$ respectively. Including the real and imaginary parts of $\Gamma_{\text{opt}}(\nu)$, the set of four noise parameters needed to determine $T_n(\nu)$ for the CTP system is $\{T_{\min}(\nu), \text{Re}[\Gamma_{\text{opt}}(\nu)], \text{Im}[\Gamma_{\text{opt}}(\nu)], N(\nu)\}$. The details of determining the noise parameters is outside the scope of this article. Simply put, we measured a set of $T_n(\nu)$ for the FE system in the laboratory with a set of input reference samples consisting of simple passive electronic components (50 $\Omega$, 75 $\Omega$, 100 $\Omega$, RC-circuit, RL-circuit).

We then simultaneously fit the four noise parameters for the measured $T_n(\nu)$ with the corresponding $\Gamma_{\text{src}}(\nu)$ using the Python implementation of the Markov chain Monte Carlo (MCMC) sampler, emcee\textsuperscript{12}.

As shown in Figure 7, the spectral gradients of the $E$- and $H$-planes plotted side-by-side to their respective beams are showing large gradient variations with $|\partial_\nu F(\theta, \phi, \nu)| \leq 0.05$ (linear directivity unit per MHz). The significance of these variations is elaborated in Section 5.2.

4. COMPARISON OF SIMULATION AND OBSERVATION

For the CTP, by combining the S-parameters and noise parameters, we compute the respective $G_T(\nu)$ and $T_n(\nu)$ for Equation (7)-(10) by substituting the antenna reflection coefficient, $\Gamma_{\text{ant}}(\nu)$, measured in the field with a VNA into $\Gamma_{\text{src}}(\nu)$. A summary of the calibration procedures can be reviewed in Figure 5.

4.1. Fiducial Model

To determine influences of the ground soil dielectric properties on the beam pattern of the CTP antenna when tilting the antenna toward the NCP, we compared the titled antenna to a zenith-pointing fiducial antenna model. The fiducial CST beam model consists of setting the ground screen of the sleeved dipole parallel to a finite ground soil slab underneath. To decouple effects of the obstruction from Earth’s horizon, the fiducial model is assumed to have the antenna placed at the Geographic North Pole (GNP) to observe the NCP at the zenith.

The resulting beams $F(\theta, \phi, \nu)$ are relatively smooth spatially, with apparent frequency dependence, as shown in the $E$-plane ($\phi = 0^\circ$) and $H$-plane ($\phi = 90^\circ$) of polarization $X$ in Figure 6. To further quantify the beam chromaticity of the fiducial beam, we computed the spectral gradients of the beam patterns $\partial_\nu F(\theta, \phi, \nu)$ evaluated at each fixed $(\theta, \phi)$. As shown in Figure 7, the spectral gradients of the $E$- and $H$-planes plotted side-by-side to their respective beams are showing large gradient variations with $|\partial_\nu F(\theta, \phi, \nu)| \leq 0.05$ (linear directivity unit per MHz). The significance of these variations is elaborated in Section 5.2.

Without loss of generality, by using this zenith-pointing beam at 82 MHz, the resulting net polarization from the PIPE simulation produces sinusoidal waveforms with twice-diurnal period in Stokes $Q_{\text{cal}}$ and $U_{\text{cal}}$ in Figure 8. Besides the expected second harmonic ($n = 2$), Fourier decomposition of these two Stokes parameters for multiple consecutive sidereal days has also identified a weak sixth harmonic ($n = 6$). Additionally, the Stokes $I_{\text{cal}}$ and $V_{\text{cal}}$, which should have been constant and zero for a Gaussian beam, now contain a fourth harmonic ($n = 4$). Evidently, these are artifacts are due to deviations of the realistic CTP beam from a smooth Gaussian beam, which imply that Stokes parameters together can provide a direct means to further characterize the beam systematics.

4.2. Distortions from Observing with a Tilted Dipole

With such a large beam from the CTP dipole, we have anticipated potential interactions between the ground soil and the antenna beam prior to deployment, especially when the CTP antenna had to be tilted at an angle of $\delta_{\text{tilt}} = (90^\circ - \text{Observer Latitude}) = 52^\circ$ relative to the horizontal ground to point at the NCP. Due to logistical constraints, we were only able to place the CTP on a shallow north-facing slope in attempt to alleviate some of the the ground interactions.

\textsuperscript{12} http://dfm.io/emcee/current/

\textsuperscript{13} https://www.cst.com

Fig. 5.— Block diagram for the data pipeline for acquisition (gray shaded boxes), calibration (blue shaded) and reduction (white).

![Fig. 5.— Block diagram for the data pipeline for acquisition (gray shaded boxes), calibration (blue shaded) and reduction (white).](image)

![Fig. 6.— Angular plots for the CST beam of the sleeved dipole when the antenna is set parallel to the ground.](image)

![Fig. 6.— Angular plots for the CST beam of the sleeved dipole when the antenna is set parallel to the ground.](image)
Figure 7.— (Left panels) 2D plots of the $E$- and $H$-planes for the linear directivity in the $X$-polarization of the fiducial CST beam model, $F_X(\theta, \phi, \nu)$, which has the finite ground screen parallel to the ground soil. (Right panels) 2D plots of the frequency gradient, $\partial_{\nu}F_X(\theta, \phi, \nu)$, of the beams on the left panels. The strong fringing structures due to interactions between the beam and ground are more apparent in the gradient plots, with $|\partial_{\nu}F(\theta, \phi, \nu)| \leq 0.05$ (linear directivity unit per MHz). By symmetry, the beam for $Y$-polarization, $F_Y(\theta, \phi, \nu)$, shares similar patterns and frequency structures.

Figure 8.— PIPE simulation result for the ground-parallel fiducial antenna beam at the GNP (Lat. = 90°N). (Left panels) Temporal waveforms of the Stokes parameters (top to bottom: $I_{cal}, Q_{cal}, U_{cal}, V_{cal}$) for multiple sidereal days (as shown here for 7 days and 14 cycles). (Right panels) Harmonic decomposition for the corresponding Stokes parameters on the left, note the strong twice-diurnal ($n = 2$) component for Stokes $Q_{cal}$ and $U_{cal}$. Note that, although minute, imperfections on the CST beam give rise of the $n = 4$ components for Stokes $I$ and $V$ as well as the $n = 6$ components for Stokes $Q$ and $U$.

Nonetheless, one of the more viable ways to assess how the beam was interacting with the ground soil is through CST modeling. We rotate the CTP antenna model in the CME software to align with the deployed configuration in addition to including a finite ground soil slab underneath. The simulated beams are indeed showing corruptions on their smoothness. As shown in the angular plots in Figure 9 and the gradient beam maps in Figure 10 there are fringes in the CST beam across the band due to the interferometric effects.

This can be understood through image theory, as illustrated in Figure 11. When a horizontal dipole antenna locates above a finite ground screen at height $h$, it produces a single image at the same distance under the ground plane. However, when tilting the antenna and its ground screen toward the ground soil, the image below the soil and the one behind the antenna’s ground screen no longer overlap. The interferometric interactions between these images subsequently imprint the unwanted fringes onto the beams. In fact, this effect is dependent on the antenna’s directive gain since higher directive gain results in smaller FOV.

In addition to the beam distortions, since the CTP’s observing latitude is 38°N, the Earth’s horizon partially obscures the northern sky. This reduces the sky region which can be observed continuously over 24 sidereal hours. This alters the temporal waveforms of the Stokes parameters by adding high-order harmonics as sky regions which extend beyond the observer’s horizon will set and rise every day from the FOV.

4.3. Constraints on the Measured Polarization
As a consequence of the ground distortions on the beam and the obstructed FOV at 38°N, the PIPE simulation produces more complex waveforms for the Stokes parameters. The smooth sinusoidal Stokes parameters from the fiducial model in Figure 8 are replaced by periodic functions consisting of multiple harmonics as shown in Figure 12. There are still the twice-diurnal components, but their power are reduced and distributed among the high order terms.

Since the CTP was deployed outside the radio-quiet zone, we expected the observed spectrum to be contaminated with RFI. Although the BPF between 60-80 MHz has substantially reduced the RFI from the FM band, majority of the band was corrupted. Nonetheless, we were able to identify a small frequency region around 82 MHz to be the cleanest in the data. We proceeded to compute the Fourier decomposition for multiple days of data and extracted the harmonic components in that band. The extracted Stokes harmonics are compared to the PIPE simulation which has included the tilting, the horizon obstruction for 38°N, and CST beam for the ground interactions, as shown in Figure 13.

By computing the ratio of the magnitude of the $S_{Q,n}^r$ and $S_{U,n}^r$ to their respective noise floors, we arrive at signal-to-noise ratio (S/N) of 3.12 and 4.95 for the twice-diurnal components at 81.98 MHz. We define the noise floor in the PSD of Stokes as the sample mean of the clean channels, further evaluation is needed when the CTP is operating multiple days of data into a single data stream for the FFT computation.

\[
\mu_i = \frac{1}{(n_{\text{max}} - n_{\text{min}} + 1)} \sum_{n=n_{\text{min}}}^{n_{\text{max}}} S^r_{S_i,n},
\]

where $[n_{\text{min}}, n_{\text{max}}]$ are the range of components in PSD of the Stokes $S_i = \{Q, U\}$ which contain only noise. Our...
simulation suggests that no significant harmonics should appear for \( n \geq 10 \), hence we have chosen \([n_{\text{min}}, n_{\text{max}}] = [10, 15]\). With such S/N, we can confirm a tentative detection of the induced polarization at 81.98 MHz and few neighboring channels. Yet, we will need to verify whether these harmonics were created by the PIPE or other measurement artifacts.

### Table 1

Harmonic ratio test for \( S^{n}_{Q,n} \) and \( S^{n}_{U,n} \) between observed data and simulation at 81.98 MHz.

| Stokes PSD | \( n \)-ratio | Obs. \([\pm 0.50]\) | Sim. |
|------------|---------------|-----------------|-----|
| \( S^{1}_{Q,n} \) | 1:2 | 9.58 | 1.77 |
| | 2:3 | 1.11 | 0.87 |
| | 2:4 | 1.51 | 2.03 |
| \( S^{1}_{U,n} \) | 1:2 | 1.83 | 2.24 |
| | 2:3 | 3.21 | 1.14 |
| | 2:4 | 2.14 | 2.72 |

We compare the relative magnitude ratios among the observed low-order harmonics \( n = \{0, 1, 2, 4\} \) in Stokes \( Q_{\text{cal}}(\nu) \) and \( U_{\text{cal}}(\nu) \) to the simulation at 82 MHz, as shown in Figure 13. From the simulation, the ratio \( S_{U,2}/S_{U,3} \) is approximately 2.24. This is consistent with the ratio measured in the CTP data \((1.83 \pm 0.50)\) regardless of the slight difference in the scaling between the simulated and observation data. Meanwhile, \( S_{U,2}/S_{U,3} \) is 1.14 for simulation and 3.21 \pm 0.50 for CTP data.

However, the \( S_{Q,1}/S_{Q,2} \) ratio from the Stokes \( Q \) \((9.58 \pm 0.05)\) is less consistent to the simulation \((1.77)\). Since Stokes \( Q \) measures the linear polarization at 0° and 90° along each dipole, the higher Stokes \( Q \) ratio is consistent with the observed stronger signal on the Y-polarization in the data. As shown in Figure 3, the Y-polarization corresponds to a horizontally-oriented polarization of the incoming signal. This can be resulted from additional signal being scattered and reflected signal off the ground soil in front of the tiled antenna. Future work is needed to revisit this inconsistency.

Nonetheless, by comparing the the relative power of the harmonics at 81.98 MHz in Figure 13 the overall harmonics determined in the observed data channels align well with the PIPE simulation. A more complete power ratio between harmonics for both Stokes \( Q_{\text{cal}}(\nu) \) and \( U_{\text{cal}}(\nu) \) are listed in Table 1.

Despite of the strong beam distortion and RFI contamination, the observation has put a tentative detection on the induced polarization which exhibits a twice-diurnal characteristics in a small selection of channels. The prototype has provided invaluable insights on the argumentation as well as observation approaches, which have inspired many of the simulation analysis. A follow-up experiment is necessary for further evaluating of the PIPE and its values in global 21 cm experiments. Based on the lessons learned, we are currently upgrading and redepolying the CTP system to the Green Bank Observatory (GBO) in West Virginia, USA, to operate in a more radio-quiet environment.

### 5. OTHER SYSTEMATICS

#### 5.1. Beam Pointing Error

The original CTP observation strategy relies on centering the antenna at the NCP to measure the dynamic modulation imprinted on the Stokes \( Q_{\text{cal}}(\nu) \) and \( U_{\text{cal}}(\nu) \) as the asymmetry in the foreground sources drifts in LST. The alignment of the antenna’s boresight at the celestial pole determines the accuracy of the twice-diurnal \((n = 2)\) terms. If the antenna beam is not concentric to the foreground region centered at the NCP, the detected waveforms for the net foreground polarization can appear to be “wobbling” about the boresight.

We took extreme care in aligning the CTP to the NCP by using the Polaris as the reference target at night time. To further understand the effects of the pointing error, we have simulated the PIPE for different off-boresight pointing angles. We compare simulation results of the Stokes spectra for \( n = \{0, 1, 2, 4\} \) at different off-boresight angles, ranging from \(-10° \) to \(30°\) with \(0°\) to be the perfect alignment with the NCP, negative angles when the antenna pointing approaches the horizon, and positive angles when the pointing approaches the zenith as illustrated in Figure 14.

The PIPE simulation shows that, within the range of \( \pm 5°\) about boresight, the Stokes spectra are distinguishable between \( n = 1 \) and \( n = 2 \). In another words, this analysis suggests that the antenna is still sensitive to resolve the PIPE’s twice diurnal for pointing accuracy of within \( \pm 5°\). Since the Polaris has a declination (DEC) of \(+89.25°\), which is less than \(1°\) off from the NCP (DEC = +90°) in the equatorial coordinate system, our alignment procedure is justified.

#### 5.2. Antenna Beam Chromaticity

Conventionally, the sky-averaged antenna temperature \( T_{\text{ant}}(\nu) \) is written as the beam-weighted value of the sky brightness temperature \( T_{\text{sky}}(\theta, \phi, \nu) \) (e.g., [Kraus 1986][Wilson et al. 2009]),

\[
T_{\text{ant}}(\nu) = \frac{1}{\nu} \int_0^{2\pi} \int_0^{\pi/2} T_{\text{sky}}(\theta, \phi, \nu) F(\theta, \phi, \nu) \sin \theta d\theta d\phi.
\]

Equation (17)
Consequently, despite the intrinsic spectral smoothness of the sky-averaged foreground spectrum, the observed sky spectrum is corrupted by the spectral variations in the beam patterns. This can cause either absorption or emission features to appear on the sky-averaged spectrum, thus further complicate the separation of the background 21 cm signal from the foreground.

Some of the global 21 cm experiments have attempted to use CEM software, like CST, HFSS, FEKO, to characterize the effects of beam chromaticity (e.g., Bernardi et al. 2015; Mozdzen et al. 2016). Meanwhile, others have tried to mitigate the chromaticity with optimized antenna designs to achieve smoother frequency response over a large frequency range, such as the blade antenna from EDGES II (50-100 MHz, Mozdzen et al. 2016) or the spherical monopole from SARAS 2 (110-200 MHz, Singh et al. 2017). However, decoupling the beam dependence from the beam-weighted measurement of $T_{\text{am}}(\nu)$ is a nonlinear process, mainly due to the lack of detailed spatial information of the sky and the beam patterns from a single total-power measurement.

Similarly, the Stokes measurements are also susceptible to spectral variations due to the antenna beams, as ground-parallel fiducial beam patterns shown in Figure 6. In Figure 15, PIPE simulation with the fiducial beam indicates that the second-harmonic Stokes spectra, $S_{Q,2}$ and $S_{U,2}$, can no longer simply track the power-law input model for the foreground with spectral index $\beta = 2.47$ as with the idealized Gaussian beams.

We quantify the spectral distortions imprinted by the fiducial beam patterns on the simulated Stokes spectra by computing an overall RMS value across the band for the relative error between the simulated foreground spectrum with the power law input model as,

$$\delta(S_{i,n},\nu) = 100\% \times \left| \frac{S_{i,n} - \tilde{S}_{i,n}}{\tilde{S}_{i,n}} \right|,$$

where $\tilde{S}_{i,n}$ is the power law spectrum, computed with

$$\tilde{S}^\nu_{S_{i,n}} = S^\nu_{S_{i,n}} \left( \frac{\nu}{\nu_0} \right)^{-\beta},$$

normalized at frequency $\nu_0$, which was chose to be the lower end of the band, at 60 MHz. This metric, though simple, is sufficient in that it becomes non-zero with the presence of beam chromaticity. The RMS errors of the second-harmonic Stokes spectra relative to the smooth power law are 13.6% and 7.0% for Stokes $Q$ and $U$ respectively. Recall that the spectral gradient for this beam is $|\partial_\nu F(\theta,\phi,\nu)| \geq 0.05$.

5.3. **Beam Elongation**

Intrinsically, a dipole beam is elongated such that the beam widths between the $E$- and $H$-planes are not symmetric (as comparing Figure 16 to Figure 6). Initially, there were concerns on how the elongated beams can reduce the sensitivity of the twice-diurnal component of the induced Stokes $Q_{\text{net}}$ and $U_{\text{net}}$, hence a skirt structure with conductive mesh was implemented to enhance the beam symmetry.

![Fig. 16.— Similar to the fiducial model Figure 6, angular plots for the CST beam of the sleeved dipole when the antenna is set parallel to the ground but without the conductive skirt. The chromaticity is significantly less comparing to Figure 6. In the absence of the skirt, as expected from a simple dipole antenna, the $E$-plane beams are narrower than the $H$-plane beams due to the intrinsic elongation in the beam.](image)

![Fig. 17.— (Left panels) 2D plots of the $E$- and $H$-planes for X-polarization of the fiducial antenna beam, without the skirt. (Right panels) 2D plots of the frequency gradient, $\partial_\nu F_X(\theta,\phi,\nu)$, of the beams on the left panels. Without the skirt, the beam is much smoother, with $|\partial_\nu F_X(\theta,\phi,\nu)| \leq 0.02$.](image)
comparing Figure 17 with Figure 7). Further analysis of the PIPE simulation with the latter beams indicates that the compromise of the beam elongation on the sensitivity of the induced polarization is more tolerable than expected. For comparison, in Figure 18 the PIPE simulation with the sleeved dipole beams without the skirt suggests much smoother spectra in the induced polarization comparing to Figure 15. The RMS errors of $\delta(Q, 2)$ and $\delta(U, 2)$ have decreased to 2.9% and 4.6%, respectively.

Follow-up observations at the GBO will be made with and without a skirt on the antenna for further evaluation of this effect. Overall, the results suggest that the compromise in sensitivity for the Stokes spectra due to beam elongation is minimal, and in return one obtains a much smoother beam.

6. IMPLICATIONS ON 21 CM BACKGROUND SIGNAL EXTRACTION

In the last two sections, it is evident that the PIPE is more complex than one simulated with the Gaussian beam in NB17, with the presence of instrumental systematics like beam chromaticity and influences of surrounding ground soil on the antenna when tilted. The spectral dependence of the beam patterns not only affects conventional total-power measurement but also distorts the Stokes spectra, $S_Q$, and $S_U$. This has complicated the foreground removal strategy proposed in NB17, where one can simply scale and subtract the $S_Q$ or $S_U$ from the $S_T$ iteratively to uncover the underlying background 21 cm signal.

However, the greatest advantage of the full-Stokes measurement over the single total-power spectrum is the additional information on the beam behavior and other instrumental effects which are imprinted on the Stokes parameters temporally. By utilizing sophisticated pattern recognition algorithms, together with a priori information, such as foreground maps (from sky surveys), beam models (such as those from CEM software), and laboratory measurements ($S$- and noise parameters), to compile statistical training sets, we can improve the robustness and accuracy for extracting the 21 cm signal in the presence of realistic beam effects.

In fact, the singular-value decomposition (SVD) can calculate the eigen modes for the sky signal along with the systematics, then identify and separate the corresponding modes from the observed data to retrieve the signal of interest. As an example, we applied the SVD, as implemented in the Python code pylinex (Tauscher et al. 2018), to a large training set of 21 cm signal models generated with ares (Mirocha 2014), and beam-weighted foreground training set consisting of convolutions of CST beams with different antenna structure variations (e.g., thermal perturbations on antenna structure, ground soil properties, ground screen behaviors) with the 408 MHz Haslam map scaled down to the relevant frequencies. By accounting for the PIPE due to multiple rotation angles, the SVD simultaneously constrains the input induced polarized foreground and 21 cm background model. As illustrated in Figure 19, the SVD provides a more robust extraction of the background 21 cm signal when applied to all four simulated Stokes measurements than on the total-power measurement alone. We are developing models with increased realism to continue testing the feasibility of applying the SVD pipeline analysis on observational data. The upgraded CTP at GBO will provide a convenient testbed for such development effort.

7. CONCLUSION AND FUTURE WORK

A proof-of-concept prototype has been developed to validate the Projection-Induced Polarization Effect (PIPE) and its practicality in constraining the foreground spectrum, according to the framework described in Nhan et al. (2017). The Cosmic Twilight Polarimeter (CTP) is a testbed for implementing our network-theory based calibration schemes for the power transducer gain, $G_T(\nu)$ and noise temperature, $T_n(\nu)$. Instead of attempting to directly determine these two variables, both of which are functions of the input reflection coefficient $\Gamma_{in}(\nu)$, we measure two sets of intrinsic measurements for the system: $S$-parameters and noise parameters.

We have evaluated a preliminary CTP observation with the aid of PIPE simulations based on simulated antenna beams for the NCP-tilting configuration at latitude of 38°N. Despite strong RFI contamination on most of the band, we were able to identify the second-harmonic components in Stokes $Q$ and $U$ in the observational data around 82 MHz. Their magnitudes are at least 3 and 5

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16 https://bitbucket.org/ktausch/pylinex
17 https://bitbucket.org/mirocha/ares
times stronger than the system noise, respectively. Relative ratios between harmonic orders of Stokes $Q$ and $U$ are consistent to the PIPE simulation as summarized in Table 1. A follow-up effort is underway aiming to measure the PIPE across the entire band (60-80 MHz) at GBO.

Besides the partial obstruction of the Earth’s horizon on the northern sky region, two major systematics are distortion effects on the antenna beam due to ground soil interaction when tilting and intrinsic beam chromaticity of the antenna design. With some simplified assumptions on the environmental and observational conditions, the simulations suggest that these two beam-related systematics can be mitigated as the following:

1. The smooth antenna beam can be distorted drastically due to the interferometric effect produced when the antenna interacts with the ground soil when it is being tilted at a large angle relative to the ground underneath. This can be mitigated by either elevating the antenna high off the ground, or placing it on a slope, such as that of a hill side with similar tilting angle as needed at either of the two celestial poles.

2. Although the conductive mesh skirt surrounding the CTP helps to keep the antenna beam symmetric ($E$- and $H$-planes have similar beam widths), it also introduces some unwanted spectral structures into the beams. According to the PIPE analysis with the fiducial beam including the skirt, the spectral gradients of the beam are of $|\beta_{\nu} F(\theta, \phi, \nu)| \geq 0.05$. As the result, the second-harmonic Stokes spectra, $S'_{Q,2}$ and $S'_{U,2}$, deviate from the expected power law of spectral index $\beta = 2.47$ by RMS errors of 13.6% and 7.0%, respectively. Meanwhile, without the skirt, these values reduce to $|\beta_{\nu} F(\theta, \phi, \nu)| \leq 0.02$, with RMS errors for $\delta(Q,2)$ and $\delta(U,2)$ are 2.9% and 4.6%. In other words, the beam is much smoother and the Stokes spectra are much closer to the input foreground spectra, with only minimal compromise due to the beam.

3. The Earth’s horizon obstruction on the experiment’s FOV can be reduced by relocating the instrument closer to one of the geographic poles. By doing so, also decrease the tilting angle of the antenna and help alleviate some of the interferometric fringing.

In this paper, we have also utilized the SVD-based pylimex code on the PIPE simulations to develop a more robust background signal extraction procedure than the simplistic 21 cm signal recovery scheme proposed in NB17. Instead of requiring precise absolute knowledge of the systematics, the SVD constrains the background 21 cm signal by decomposing the full-Stokes measurements into eigen modes, then remove the modes corresponding to ones of the training sets for the systematics. Preliminary analysis shows that it is plausible to robustly extract the background signal as long as training sets with a priori information on the foreground, antenna beam, and other systematics are sufficiently well defined. There are no fundamental limitations which will prevent the induced polarization approach from being applied to search for the global 21 cm signal.

From the lessons learned with the prototype, an upgraded version of the CTP and new systematics mitigation strategies are under development. The new experiment is expected to be deployed to the radio-quiet zone at the GBO for further evaluation by early next year. The effectiveness of using the induced polarization approach will then be analyzed using the pylimex with training sets specifically compiled for the CTP antenna beams and instrument systematics affecting the observed data.

Furthermore, to mitigate the ground distortion on the beam from the NCP-pointing configuration, we have considered experimenting with pointing the CTP at zenith when deployed at the GBO site. Although this configuration will diminish the advantage of the twice-diurnal signature of the foreground, further experimentation with the SVD analysis on PIPE simulation for this zenith-pointing configuration shows that pylimex can also differentiate the foreground and 21 cm signal from the temporal variations of the foreground from drifting over the instrument versus the constancy in time of the global 21 cm signal.

Although this may resemble conventional global 21 cm experiments, to the best of our knowledge, no other single-element total-power global experiment has incorporated polarimetry and SVD together. With the next upgrade of the CTP, we will be able to take full advantage of both techniques, the drift scan, and particularly the induced polarization, for the first time with observed data. Hence, the CTP presents a compelling alternative as an independent observational technique for the global 21 cm experiment, perhaps giving some insights on the absorption feature reported by EDGES II.

Moreover, there is also great potential for adopting the induced polarization approach on a future space-based mission since many of the ground-based challenges, such as ground interactions with the antenna beam and horizon obstruction, as well as ionospheric effects, will be eliminated. In fact, previous studies (e.g., Burns et al. [2012] 2017 Falcke et al. [2018] have suggested that the lunar far side is the optimal location for such an experiment due to its pristine radio-quiet environment within the inner solar system.

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where the $t, \nu$ are equivalent to the complex antenna voltages, $\tilde{E}$ also like to thank K. Makhija for his comments on the instrumentation description.

**APPENDIX**

**A. CALIBRATION FOR STOKES ANTENNA TEMPERATURE**

To derive the formulae to convert the correlation spectra into equivalent temperature units, we first note that the complex $E$-fields of each polarization in the coherence matrix,

$$C(t, \nu) = \begin{pmatrix} E_X E_X & E_X E_Y \\ E_Y E_X & E_Y E_Y \end{pmatrix}_{(t, \nu)},$$

are equivalent to the complex antenna voltages, $\tilde{V}_X(t, \nu)$ and $\tilde{V}_Y(t, \nu)$, which are acquired from Fourier transforming the sampled antenna voltages. Meanwhile, the power transducer gain $G_T(t, \nu)$ is defined as the square of the voltage gain $g_T(t, \nu)$, i.e., $G_T = g_T^2$. The noise power $P_n(v, t_i)$ from the FE signal path can be written as the absolute square of the complex noise voltage $\tilde{V}_n(t, \nu)$, and $P_n(t, \nu) = |\tilde{V}_n(t, \nu)|^2 = k_B \Delta \nu T_{\text{ant}}(t, \nu)$ with the Boltzmann constant $k_B$. Hence the monochromatic $E$-fields of both polarizations can be parametrized as,

$$E_X(t, \nu) = \tilde{V}_X(t, \nu) = g_{T,X}(t, \nu) \left[ \tilde{V}_{\text{ant},X}(t, \nu) + \tilde{V}_{n,X}(t, \nu) \right],$$

$$E_Y(t, \nu) = \tilde{V}_Y(t, \nu) = g_{T,Y}(t, \nu) \left[ \tilde{V}_{\text{ant},Y}(t, \nu) + \tilde{V}_{n,Y}(t, \nu) \right],$$

where $\tilde{V}_{\text{ant}}(t, \nu)$ is the antenna voltage signal of each polarization when observing the sky, where $P_{\text{ant}}(t, \nu) = |\tilde{V}_{\text{ant}}|^2 = k_B \Delta \nu T_{\text{ant}}(t, \nu)$. With these two equations, we can write the correlation terms in the coherence matrix as,

$$\langle \tilde{V}_X \tilde{V}_X^* \rangle = G_{T,X} \left( \langle \tilde{V}_{\text{ant},X} \tilde{V}_{\text{ant},X}^* \rangle + \langle \tilde{V}_{n,X} \tilde{V}_{n,X}^* \rangle \right),$$

$$\langle \tilde{V}_Y \tilde{V}_Y^* \rangle = G_{T,Y} \left( \langle \tilde{V}_{\text{ant},Y} \tilde{V}_{\text{ant},Y}^* \rangle + \langle \tilde{V}_{n,Y} \tilde{V}_{n,Y}^* \rangle \right),$$

$$\langle \tilde{V}_X \tilde{V}_Y^* \rangle = \sqrt{G_{T,X} G_{T,Y}} \left( \langle \tilde{V}_{\text{ant},X} \tilde{V}_{\text{ant},Y}^* \rangle + \langle \tilde{V}_{n,X} \tilde{V}_{n,Y}^* \rangle \right),$$

where the $(t, \nu)$ notation has been suppressed for the ease of reading, and $(\ldots)$ represents an ensemble average. The equivalent antenna temperatures resulting from these auto- and cross-powers can be written as

$$T_{\text{ant},kl}(t, \nu) = \frac{\langle \tilde{V}_{\text{ant},k} \tilde{V}_{\text{ant},l}^* \rangle}{k_B \Delta \nu},$$

where the subscripts $\{k, l\}$ correspond to the $X$ or $Y$ polarizations.

Subsequently, the calibrated Stokes parameters, which were computed with the auto-spectra and cross-spectra, can be written in temperature unit as the following,

$$I_{\text{cal}}(t, \nu) = T_{\text{ant},XX}(t, \nu) + T_{\text{ant},YY}(t, \nu),$$

$$Q_{\text{cal}}(t, \nu) = T_{\text{ant},XY}(t, \nu) - T_{\text{ant},YX}(t, \nu),$$

$$U_{\text{cal}}(t, \nu) = T_{\text{ant},YY}(t, \nu) + T_{\text{ant},XX}(t, \nu),$$

$$V_{\text{cal}}(t, \nu) = i \left[ T_{\text{ant},XY}(t, \nu) - T_{\text{ant},YX}(t, \nu) \right]$$

$$= \frac{2}{k_B \Delta \nu} \left[ \frac{\text{Re} \left( \tilde{V}_{\text{ant},X} \tilde{V}_{\text{ant},Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Re} \left( \tilde{V}_{n,X} \tilde{V}_{n,Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Re} \left( \tilde{V}_{\text{ant},X} \tilde{V}_{n,Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Re} \left( \tilde{V}_{\text{ant},Y} \tilde{V}_{n,X}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} \right],$$

$$= \frac{-2}{k_B \Delta \nu} \left[ \frac{\text{Im} \left( \tilde{V}_{\text{ant},X} \tilde{V}_{\text{ant},Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Im} \left( \tilde{V}_{n,X} \tilde{V}_{n,Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Im} \left( \tilde{V}_{\text{ant},X} \tilde{V}_{n,Y}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} - \frac{\text{Im} \left( \tilde{V}_{\text{ant},Y} \tilde{V}_{n,X}^* \right)}{\sqrt{G_{T,X} G_{T,Y}}} \right],$$

where $\Delta$ and $\Delta$ are the frequencies of interest.
Since the sky signal ($\tilde{V}_{\text{ant}}$) and the electronic noise ($\tilde{V}_e$) are uncorrelated both within the same polarization and among the two polarizations, time averaged values of the cross terms cancel out. Hence, the calibrated Stokes parameters reduce to Equations (7)-(10).

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