New approaches to the determination of the total cross section
O.V. Selyugin

aBLTPh, JINR, 141980 Dubna, Russia

New methods have been developed for extracting the parameters of diffraction scattering amplitude, $\sigma_{\text{tot}}$ and $\rho = \text{Re} F_N / \text{Im} F_N$, from experimental data. The latter determines these parameters with less errors and without the knowledge of the normalization coefficient. The impact of additional information from the measurement of the spin-dependence cross section on the determination of the basic characteristic of elastic scattering amplitude is examined. The new form of the connection between some characteristics of the analyzing power and magnitude of the $pp$—total cross section is presented.

1. Introduction

In reality, in experiment one measure $dN/dt$, as a result of which “experimental” data such as $\sigma_{\text{tot}}$, slope-$B$, $\rho$ are extracted from $dN/dt$ with some model assumptions. Some approaches are needed to extrapolate the measured quantities to $t = 0$. The elastic scattering data were constrained by several conditions: the imaginary part of the nuclear amplitude has an exponential form in the small $t$ region; the real and imaginary parts of the nuclear amplitude have the same $t$ dependence; spin contributions are neglected. Note that the value of $\rho$ heavily correlates with the normalization of $d\sigma/dt$. Its magnitude weakly impacts the determination of $\sigma_{\text{tot}}$ only in the case when the normalization is known exactly.

There is no any experiment on measurement of the values separately. But in some experiments, to reduce experimental errors, the magnitude of some quantities is taken from another experiment. Sometimes, it leads to contradiction between the basic parameters for one energy (for example, if we calculate the imaginary part of scattering amplitude, we can obtain a non-exponential behavior). It can lead to some errors in the analysis based on the dispersion relations.

The procedure of extrapolation of the imaginary part of the scattering amplitude is significant for determining $\sigma_{\text{tot}}$. The importance of the extrapolated contribution is seen from $\rho$ where the contribution to $\sigma_{\text{tot}}$ of $\sigma_{\text{obs}}$, the directly measured value, and of $\Delta \sigma_{\text{el}}$ and $\Delta \sigma_{\text{inel}}$, the extrapolated contributions of the elastic and inelastic cross sections, are shown at energies $\sqrt{s} = 30.6, 52.8$ and $62.7$ GeV. One can see that the growth of the total cross sections is due to $\Delta \sigma_{\text{el}}$ by 50% for $pp$ and nearly by 100% for $pp$ scattering.

The analysis of experimental data also shows a possible manifestation of spin-flip amplitudes at high energies $\rho$. The research into spin effects will be a crucial stone for different models and will help us to understand the interaction and structure of particles, especially at large distances. All this raises the question about the measure of spin effects in the elastic hadron scattering at small angles at future accelerators. Especially, we would like to note the programs at RHIC where the polarization of both the collider beams will be constructed.

To obtain the magnitude of $\rho$, we fix the differential cross sections either taking into account the value of $\sigma_{\text{tot}}$ from another experiment, as made by the UA4/2 Collaboration, or taking $\sigma_{\text{tot}}$ as a free parameter. If one does not take the normalization coefficient as a free parameter in the fitting procedure, his determination requires the knowledge of the behavior of the imaginary and real parts of the scattering amplitude in the range of small transfer momenta and the magnitudes of $\sigma_{\text{tot}}$ and $\rho$. 

*e-mail: selugin@thsun1.jinr.ru
2. The method of changing the sign

The differential cross sections measured in the experiment are described by the squared scattering amplitude. From the equation for the differential cross section one can obtain the equation for \( \rho \) for every experimental point - \( t_i \):

\[
\rho(s, t_i) = \frac{1}{ImF_N(s, t_i)} \left\{ ReF_c(s, t_i) + \frac{1}{\pi} \frac{d\sigma}{dt} (n - (ImF_c(s, t_i) + ImF_N(s, t_i))^2)^{1/2} \right\}.
\]

(1)

As the imaginary part of scattering amplitude is defined by \( ImF_N(s, t) = H \exp(B/2t) \) where \( H = \sigma_{tot}/(4\pi*0.389) \) it is evident from (1) that the real part depends on \( n, \sigma_{tot}, B \).

For the proton-proton scattering this equation has a remarkable property. If we expand the expression under the radical sign, we obtain

\[(n - 1)(ImF_c + ImF_N)^2 + n(ReF_c + ReF_N)^2.\]

(2)

As the real part of the Coulomb scattering amplitude is negative and the real part of the nucleon scattering amplitude is positive, it is clear that this expression will have a minimum situated on the scale of \( t \) independent of \( n \) and \( \sigma_{tot} \). As we know the Coulomb amplitude, we estimate the real part of the proton-proton scattering amplitude at this point. Note that all other methods give us the real part only in a sufficiently wide interval of the transfer momenta.

This method works only in the case of the positive real part of the nucleon amplitude and it is especially good in the case of large \( \rho \). So, it is interesting for the future experiment at RHIC.

3. The differential method (“tail of ghost”)

Let us examine the simplest gedanken case of \( pp \)-scattering and try to determine the sign of \( d\rho/dt \) depending on the difference of normalization \( n \) and \( n + \Delta n \) and \( \delta \sigma = \sigma_{true} - \sigma_{tot - prob} \).

There, \( \sigma_{true} \) is the magnitude of the true total cross section and \( \sigma_{tot - prob} \) is the magnitude of the total cross section which we take as our first approximation.

In first, let us suppose that in experiment we find true normalization of data \( dN/dt \) (so \( n = 1 \)) and try to find the derivate of the calculated \( \rho \) on \( t \) (\( d\rho/dt \)) with a small deviation of \( n \) (\( \delta n \)).

Write for that case our equation for \( \rho \) for two points: \( t_1 \), where \(-F_c = 2ImF_h\), and for \( t_2 \), where \(-F_c = ImF_h; \) and calculate \( \Delta \rho = \rho(t_2) - \rho(t_1) \) for normalization \( n + \Delta n \) with \( \Delta n << 1 \). So, for example, for \( t_2 \) we can input in (1)

\[
\frac{1}{\pi} \frac{d\sigma}{dt} (n + \Delta n) = 2ImF_h^2(1 - \rho + \rho^2)(1 + \delta n)
\]

(3)

and obtain, if we take \( \sigma_{tot - prob} = \sigma_{true} \), that the difference between the calculated \( \rho \) will be

\[
\Delta \rho(t_2, t_1) \simeq \Delta n/4
\]

(4)

and is a function of \( t \). It is clear that if we find the true magnitude of the total cross section and \( \delta n = 0 \), then \( \Delta \rho(t_2, t_1) \) will be equal to zero. The same calculations are carried out for \( \Delta \sigma_{tr} \) with \( \delta n = 0 \) and show that the sign of \( \Delta \rho(t_2, t_1) \) depends on the sign of \( \Delta \sigma_{tot} \). Then, if we will calculate \( \Delta \rho(t_2, t_1) \) with differences \( \Delta n \) and \( \sigma_{tot} \) we can determine the true magnitude of \( \sigma_{tot} \).

4. Connection between \( A_N \) and \( \sigma_{tot} \)

Let us examine the future pppp Experiment at \( \sqrt{s} = 500 GeV \), as an example. Elastic differential cross section will be regarded as having 2% statistical errors, according to the Proposal of the PP2PP Collaboration. Now, in fitting procedure we take into account the standard assumption for high energy elastic hadron scattering at small angles: the simple exponential behavior with slope \( B \) of the imaginary and real parts of the scattering amplitude; hadron spin-flip amplitude does not exceed 10% of the hadron spin-non-flip amplitude. The differential cross section was calculated using \( \sigma_{tot} = 63.5 mb; \rho = 0.15; B = 15.5 GeV^{-2} \) ( in variant I with 150 points and in variant II with 75 points) from \( t_0 = 0.00075 \) with \( \Delta t = 0.00025 GeV^2 \) and then put through a special random process using 2% errors. After that, the obtained ”experimental” data were fitted. The systematic errors were taken into account as the free parameter \( n \). The result is presented in Table 1 for three \( n = 1 \) fixed) or four parameters \( n \) - free).
ants with different assumptions about the hadron spin-flip amplitude is a slowly varying function of $t$ apart the kinetic factor, and we parametrize it as
\[ \phi^h_5 = \frac{\sqrt{|t|}}{m}(\rho k_2 + i k_1)Im\phi^h_1, \] (5)
where $\rho$, $k_1$, $k_2$ are slowly changing functions of $s$. The coefficients $k_1$ and $k_2$ are the ratios of the real and imaginary parts of the spin-flip to spin-non-flip amplitudes without the kinematic factor $\sqrt{|t|}$. As a result, the $A_N$ can be written as
\[ \frac{A_N}{8\pi P_B} \frac{d\sigma}{dt} = -Im\phi^h_1 \frac{\alpha}{m\sqrt{|t|}}\left(\frac{\mu - 1}{2} - k_1\right) \]
\[ + \frac{\sqrt{|t|}}{m} |\rho|Im\phi^h_1|^2 \Delta k, \] (6)
with $\Delta k = k_2 - k_1$. We examined also a few variants with different assumptions about the magnitudes of the imaginary and real parts of the hadron spin-flip amplitude at $\sqrt{s} = 500$ GeV. Variant $A_1$: $\phi^h_5 = 0$, so $k_1 = k_2 = 0$. Variant $B_1$: $k_1 = 0.1$, $k_2 = 0.15$. Variant $C_1$: $k_1 = 0.1$, $k_2 = -0.15$. Variant $C_3$ was made by a random procedure with errors twice smaller, so the errors equal $5\% \div 10\%$ (see, Table 2).

The "experiment" with $10\% \div 20\%$ errors determine the magnitudes of real and imaginary parts of the hadron spin flip almost with $100\%$ errors. The variant $C_3$, which reflects the experiment with $5\% \div 10\%$ errors, twice decreases the errors of the magnitude of $\sigma_{tot}$ and hadron spin-flip.

Let us make the fit of both data on the differential cross section and the analyzing power. The results are shown in Table 3. The added polarization data decrease the error in $\sigma_{tot}$ only by $10\%$ (from $1.25$ mb to $1.1$ mb). But the determination of the magnitude of the real and imaginary parts of the hadron spin-flip amplitude become three time better. It is to be noted that the variant $C_3$ leads to decrease of in the error of $\sigma_{tot}$ from $\pm 1.25$ mb to $\pm 0.9$ mb.

| $N$ | $\sigma_{tot}$ mb | $\delta_B$ | $\delta_\rho$ | $\delta_n$ |
|-----|------------------|------------|-------------|---------|
| I   | 63.54 \pm 0.12   | \pm 0.2    | \pm 0.008   | fix     |
| I   | 63.6 \pm 1.25    | \pm 0.3    | \pm 0.02    | \pm 0.04|
| II  | 63.5 \pm 0.25    | \pm 0.7    | \pm 0.01    | fix     |
| II  | 64.05 \pm 1.4    | \pm 1.0    | \pm 0.03    | \pm 0.05|

It is clear that the most important value is the coefficient normalization of the differential cross section. Its small errors lead to significant errors in the $\sigma_{tot}$. So, we see that the normalization of experimental data is the most important problem for the determination of $\sigma_{tot}$.

Lacking better knowledge, we assume that the hadron spin-flip amplitude is a slowly varying function of $t$ apart the kinetic factor, and we parametrize it as
\[ \phi^h_5 = \frac{\sqrt{|t|}}{m}(\rho k_2 + i k_1)Im\phi^h_1, \] (5)
where $\rho$, $k_1$, $k_2$ are slowly changing functions of $s$. The coefficients $k_1$ and $k_2$ are the ratios of the real and imaginary parts of the spin-flip to spin-non-flip amplitudes without the kinematic factor $\sqrt{|t|}$. As a result, the $A_N$ can be written as
\[ \frac{A_N}{8\pi P_B} \frac{d\sigma}{dt} = -Im\phi^h_1 \frac{\alpha}{m\sqrt{|t|}}\left(\frac{\mu - 1}{2} - k_1\right) \]
\[ + \frac{\sqrt{|t|}}{m} |\rho|Im\phi^h_1|^2 \Delta k, \] (6)
with $\Delta k = k_2 - k_1$. We examined also a few variants with different assumptions about the magnitudes of the imaginary and real parts of the hadron spin-flip amplitude at $\sqrt{s} = 500$ GeV. Variant $A_1$: $\phi^h_5 = 0$, so $k_1 = k_2 = 0$. Variant $B_1$: $k_1 = 0.1$, $k_2 = 0.15$. Variant $C_1$: $k_1 = 0.1$, $k_2 = -0.15$. Variant $C_3$ was made by a random procedure with errors twice smaller, so the errors equal $5\% \div 10\%$ (see, Table 2).

The "experiment" with $10\% \div 20\%$ errors determine the magnitudes of real and imaginary parts of the hadron spin flip almost with $100\%$ errors. The variant $C_3$, which reflects the experiment with $5\% \div 10\%$ errors, twice decreases the errors of the magnitude of $\sigma_{tot}$ and hadron spin-flip.

Let us make the fit of both data on the differential cross section and the analyzing power. The results are shown in Table 3. The added polarization data decrease the error in $\sigma_{tot}$ only by $10\%$ (from $1.25$ mb to $1.1$ mb). But the determination of the magnitude of the real and imaginary parts of the hadron spin-flip amplitude become three time better. It is to be noted that the variant $C_3$ leads to decrease of in the error of $\sigma_{tot}$ from $\pm 1.25$ mb to $\pm 0.9$ mb.

| $N$ | $\sigma_{tot}$ mb | $\delta_\rho$ | $\delta k_1$ | $\delta k_2$ |
|-----|------------------|-------------|-------------|---------|
| $A_1$ | 63.5 \pm 3.4     | \pm 0.08    | \pm 0.07    | \pm 0.05|
| $A_2$ | 63.46 \pm 3.8    | \pm 0.15    | \pm 0.06    | \pm 0.1 |
| $B_1$ | 63.5 \pm 3.8     | \pm 0.09    | \pm 0.07    | \pm 0.11|
| $B_2$ | 62.7 \pm 4.      | \pm 0.3     | \pm 6.3     | \pm 5.6 |
| $C_1$ | 63.4 \pm 3.6     | \pm 0.09    | \pm 0.07    | \pm 0.11|
| $C_2$ | 63.5 -fix        | fix         | \pm 0.015   | \pm 0.011|
| $C_3$ | 63.9 \pm 1.83    | \pm 0.05    | \pm 0.037   | \pm 0.035|

| $N$ | $\sigma_{tot}$ mb | $\delta_\rho$ | $\delta k_1$ | $\delta k_2$ |
|-----|------------------|-------------|-------------|---------|
| $A$  | 63.4 \pm 1.1     | \pm 0.02    | \pm 0.02    | \pm 0.02|
| $C$  | 63.4 \pm 1.1     | \pm 0.02    | \pm 0.02    | \pm 0.03|
| $C_3$ | 63.2 \pm 0.9     | \pm 0.015   | \pm 0.014   | \pm 0.02|

5. Connection between $t_{max}$ of $A_N$ and $\sigma_{tot}$

As noted above, most uncertainty in the determination of $\sigma_{tot}$ using the measurement of $A_N$ came from the error in the beam polarization which plays the role of a normalization factor of the differential cross section. The point of maximum of $A_N$ is independent of the magnitude of the beam polarization. So, it allows us to use this value for the extraction of the magnitude of $\sigma_{tot}$. Here is the formula (the case $B_1$)
\[ \sigma_{tot} = \frac{9.776\alpha}{t_{max}}[\sqrt{3} - \frac{8}{\mu - 1}(\rho I - R) - (\rho - \alpha\varphi)], \]
where $I = k_1$ and $R = \rho k_2$, and these coefficients are unknown. Its determination will depend on the magnitude of beam polarization. To reduce the impact of the hadron spin-flip amplitude, it was proposed to used the new value $t_{max2}$, the place of the maximum of $d\sigma/dt A_N^2$.

The derivation of this value gives (the case $A_1$)
(7)

$$\sigma_{tot} = 8\pi 0.39(\text{mb}/\text{GeV}^2) t_{max}^2.$$  

In the case of the exponential behavior of the scattering amplitude, one obtains (the case A2)

$$\sigma_{tot} = 9.776\alpha[1/t_{max}^2 - B/2].$$  

where B is the slope of the differential cross section. In our previous work we showed that it could be obtained from the measurement of $A_N$ of some ratio of the real and imaginary part of the hadron spin-flip amplitude which is independent of the magnitude of the beam polarization $\rho(k_2 - k_1)/(1 - k_1)$. Using this ratio, we can obtain the relation between $\sigma_{tot}$ and the point of maximum $A_N$ in the form (the case C2)

$$\sigma_{tot} = 9.776\alpha(\frac{1}{t_{max}^2} + B)[\frac{\sqrt{3 + 4\rho^2}}{1 + \rho^2}]$$

$$8\rho\Delta k$$

$$\frac{(\mu - 1)(1 + 2\rho)(1 - k_1)}{(\mu - 1)(1 + 2\rho)(1 - k_1)} - (\rho - \alpha\varphi).$$  

The calculation of $\sigma_{tot}$ by using these formulæ is shown in Table 4 for $\sqrt{s} = 52$ GeV and in Table 5 for $\sqrt{s} = 540$ GeV for different variants of the magnitude of the hadron spin-flip amplitude. For comparison, the variant C1 is show without taking into account of the contribution of the hadron spin-flip amplitude (the case C1)

$$\sigma_{tot} =$$

$$9.776\alpha(\frac{1}{t_{max}^2} + B)[\frac{\sqrt{3 + 4\rho^2}}{1 + \rho^2} - (\rho - \alpha\varphi)].$$  

6. Conclusions

The magnitudes of $\sigma_{tot}$, $\rho$ and slope - B have to be determined in one experiment and their magnitudes depend on each other. The normalizations of $dN/dt$ and $A_N$ are most important for the determination of these values. The new methods of extracting magnitudes of these quantities are required. Some of such ideas were shown in this talk. Additional information on $A_N$ slightly reduces the errors in the size of $\sigma_{tot}$. The presented new formulæ for the determination of the magnitude of $\sigma_{tot}$ using the place of $t_{max}$ of the CNI give sufficiently good results in a wide energy region and can be used as an additional method for the determination of $\sigma_{tot}$.  

Table 4. $\sigma_{tot}$ as function of $t_{max}$

| Input $\rho$ | $\rho$ | $\sigma_{tot}$ (mb) |
|-------------|-------|------------------|
| $k_1$, $k_2$ | B1 | tot |
| 0., 0. | 42.0 | 43.2 | 43.3 | 43.3 |
| 0.1, 0.2 | 41.9 | 42.1 | 42.2 | 43.3 |
| 0.2, 0.2 | 42.0 | 43.2 | 43.3 | 43.3 |
| 0.0, 0.2 | 42.0 | 41.0 | 42.1 | 43.4 |
| $\rho = 0.075$ | 0.0, 0.2 | 41.9 | 41.4 | 41.7 | 43.5 |

Table 5. $\sigma_{tot}$ as function of $t_{max}$

| Input $\rho$ | $\rho$ | $\sigma_{tot}$ (mb) |
|-------------|-------|------------------|
| $k_1$, $k_2$ | B1 | tot |
| 0., 0. | 62.1 | 63.5 | 63.3 | 63.5 |
| 0.2, 0.2 | 62.3 | 63.5 | 61.9 | 63.5 |
| 0.0, 0.1 | 62.8 | 61.8 | 61.5 | 63.7 |
| 0.1, 0.2 | 62.0 | 61.3 | 60.7 | 63.3 |
| 0.0, 0.2 | 62.2 | 59.3 | 58.5 | 63.7 |
| $\rho = 0.1$ | -0.1, -0.2 | 61.0 | 64.9 | 64.9 | 63.1 |
| 0.2, 0.2 | 62.0 | 63.5 | 63.6 | 63.6 |
| $\rho = 0.0$ | 0.2, 0.2 | 61.9 | 63.2 | 63.8 | 63.7 |

Acknowledgments. I would like to express my sincere thanks to the Organizing Committee and especially to R. Fiore and A. Papa for the kind invitation and the financial support at such remarkable Conference, and W. Gurin, B. Nicolescu and E. Predazzi for fruitful discussions.

REFERENCES

1. G. Carbonny, Nucl. Phys. B 254 (1985) 697.
2. O.V. Selyugin, Phys. Atom. Nuc. 62 (1999) 333.
3. O.V. Selyugin, in Proc. "Frontiers in Strong Interactions", ed. P. Chappetta, M. Hague, K. Kang, S.K. Kim, C. Lee (1997) 134.
4. P. Gauron, B. Nicolescu, O.V. Selyugin, Phys. Lett. B 390 (1997) 405.
5. O.V. Selyugin, in the Proc. of the Int. Con. Recent Advances in Hadron Physics", Seoul, ed. K. Kang, S.K. Kim, C. Lee (1997) 134.
6. N.H. Buttimore et al., Phys. Rev. D 59 (1999) 114010.