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Enhanced PI control and adaptive gain tuning schemes for distributed secondary control of an islanded microgrid

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Abstract
This paper develops an enhanced proportional-integral distributed control scheme (EPI-DCS) to regulate the frequency and voltage of a droop-controlled microgrid and share the power mismatch, simultaneously. The proposed EPI-DCS is designed by using the control Lyapunov function method and adding a new consensus-based term to the integrand dynamic of the conventional PI control. In the proposed distributed EPI-DCS, the distributed generation units intermittently exchange information with the neighbouring distributed generation units, through a communication network. Considering the communication network time delays, the stability of the proposed EPI-DCS is examined using the Lyapunov–Krasovskii linear matrix inequality conditions, and the maximum stable time delay is calculated. In order to stabilise the system for large destabilising time delays, an adaptive gain scheme is proposed. Effectiveness of the proposed adaptive EPI-DCS is validated by numerical simulations with detailed models of the components and the converters, including load change, distributed generation outage, and adaptive gain-scheduling against destabilising communication network time delays.

1 | INTRODUCTION

A microgrid, representing a scaled-down power grid, is a group of distributed generation (DG) and active loads [1], which enhances the reliability and efficiency of the conventional power system [2]. Subsequent to unprepared disturbances as well as designed schedules, the microgrid can be exploited in the islanded mode. The islanded mode, however, has lower equivalent inertia compared to the grid-connected mode and requires additional control loops to preserve the stability of the microgrid.

In the hierarchical control architecture of a microgrid, two- and three-level controls are common [3–6]. Typically, the primary level is decentralised and employs the droop control method to automatically share the power demand among the DGs [3–6]. However, the droop method causes an undesirable variation of the microgrid voltage and frequency from the nominal values. The voltage droop also establishes a deficient performance in reactive power-sharing, because of the impact of the output line impedance [7]. The secondary control level is adopted to compensate for these deviations. The implementation of the secondary level has either centralised, decentralised, or distributed architectures [3–7].

Although a centralised controller is straightforward for the design of optimal operation and power dispatch [2], the use of one control centre reduces the reliability of a microgrid. Moreover, limited scalability, privacy concerns, and high computational requirements are some additional disadvantages of a centralised controller [5]. The decentralised controllers use only local measurement and may not manage all the control variables effectively due to a lack of communication [8]. In contrast, the distributed architectures eliminate the dependency on a control centre and reduce the costs of high penetration of DGs [6]. In a distributed control scheme (DCS), each DG determines its control actions using local measurements and communicating with neighbouring DGs [6]. These advantages of the DCSs lead to its application in power management and secondary control of microgrids. The existing mainstream designs of the DCSs in...
the microgrids are the dynamic consensus for averaging and the leader-follower schemes.

A two-layer controller was designed to balance the power in a microgrid with distributed storage and renewables in [9]. For power balance, this scheme requires the measurement of load demand and DG generation. Therefore, communication among all the loads and the DGs is vital for its stability. Moreover, it is required in [9] to redesign the control law after a change in the configuration of the CN. These limitations are challenging for the distributed operation of the microgrid.

The need for load measurement is eliminated in droop-based DCSs [10, 11]. However, the robust DCS in [10] and [11] requires global data from the GPS for the stability and operation of the frequency control loop. Therefore, it resembles a centralised control scheme and suffers the same drawbacks. In [12], a frequency DCS was introduced and the average consensus method was utilized to estimate the active power mismatch of the microgrid. Moreover, a cooperative DCS was designed in [13] for voltage restoration, using the feedback linearization method and the optimal state-feedback controller. However, the regulations of the voltage and the frequency are not discussed in [12] and [13], respectively. Considering the inverter-interfaced DGs and the conventional synchronous generators, a secondary frequency DCS was implemented in [14] using a ratio-consensus algorithm to control the output active power of the DGs.

In addition to the average consensus method, the leader-following method implemented in [15, 16] is also widely used in the DCSs. In [15], the leader-following approach is utilized for the voltage and frequency controls of the voltage-source converters as well as the power control of the current-source converters. The leader-following cooperative secondary DCS was also used in [16] for power-sharing and regulation of the voltage and the frequency of the microgrid with asymptotic convergence. In [17], a finite time controller (FTC) was designed for a fast response of the frequency loop. The controller switches between two bounds and utilizes the discontinuous sign function, which may excite the high-frequency chattering and thus can cause instability. In [18], the input-output partial feedback linearization method is employed to design a robust frequency FTC and a super-twisting algorithm was developed to eliminate the chattering problem. Moreover, continuous approximations of the designed control loops were used to eliminate chattering in proportional FTC [19] and terminal sliding mode control [20] schemes.

To decouple the control loops, the convergence characteristics of the voltage and frequency control loops were designed independently using a finite-time controller [18–22]. The convergence characteristics of the frequency and the voltage control loops in [22] were finite-time and asymptotic, respectively. The frequency FTC in [22] was input-bounded with saturation constraints. However, it used the discontinuous sign function, which may excite the high-frequency chattering.

In [23], the discrete-time realization of a robust secondary controller was investigated, considering the uncertain CN links with time-varying couplings, where the CN time delay is not investigated. The impact of the CN time delays and data packet loss on the performance of the distributed secondary control was covered in [24]. Considering non-uniform time delays, the stability conditions of the averaging consensus limited to the slowly-varying time delays were investigated in [25], using the Nyquist stability criteria. However, the control performance in the abovementioned controllers [8–23, 25], and adaptive DCS with feedback linearization [26] are compromised in the presence of large destabilising time delays which cause instability for certain constant gains [24, 25].

This paper proposes an enhanced proportional-integral-DCS (EPI-DCS) control scheme with a consensus dynamic at the integrand as well as an adaptive gain-tuning method for the large destabilising CN time delays, using the control Lyapunov function method. The main contributions of the paper are summarized as:

- Using a distributed scheme, the designed EPI-DCS is robust against the changes in the CN configuration and provides plug-n-play functionality against DG outage and CN failure.
- The proposed EPI-DCS improves the transient overshoot and increases the convergence speed, compared to the conventional proportional and PI controllers. Its advantage is more noticeable when dealing with destabilising time delays.
- The impact of CN time delay on the stability of the proposed EPI-DCS is examined using the Lyapunov–Krasovskii linear matrix inequality (LMI) condition.
- An adaptive gain-scheduling method is proposed, based on the CN time delays, to improve the resiliency of the secondary DCS against the large destabilising time delays and to prevent the consequent instability.

The proposed adaptive online gain tuning to overcome the destabilising large time delays has not been studied yet to the authors’ best knowledge. Moreover, the proposed adaptive EPI-DCS provides a distinct structure compared to the complicated time-delay controllers [18, 22, 25]. The designed scheme is less conservative on control parameters with respect to the LMI conditions in [18, 22, 26]. The performance of the proposed EPI-DCS is verified by numerical simulations in MATLAB/Simulink, using the detailed models of the converters and the distribution system. The transient performance, small- and large-signal stability, and adaptive gain-scheduling against destabilising time delays are studied. The results demonstrate the superiority of the proposed EPI-DCS in voltage/frequency restoration, robustness against CN configuration changes and DG outage, and adaptive adjustment of control gains with respect to CN time delays.

2 DROOP CONTROL AND GRAPH THEORY PRELIMINARIES

The hierarchical control architecture of an islanded microgrid is depicted in Figure 1. The secondary control is added as
supplementary control to the primary droop control where the reference voltage and frequency are added.

The primary control level is the droop control for automatic power-sharing control, which is modelled as [3]:

\[
\omega_i = \omega_{i,0} + \omega^+_i - \omega^-_i P_i
\]  

(1)

where \(\omega_i\) is the frequency of the grid; \(\omega_{i,0}\) is the nominal frequency of the grid; \(\omega^+_i\) and \(\omega^-_i\) are the droop gains; and, \(P_i\) is the active power output of the DG. \(\omega^+_i\) and \(\omega^-_i\) are the droop gains; and, \(P_i\) is the active power output of the DG.

The droop control determines the output voltage and frequency (i.e. \(v_{di}\) and \(\omega_i\)) of the controllable DGs. The inner controllers of the DGs consist of the voltage and the current regulation loops in dq reference frame [23], considering the output filter and the dc link voltage, as shown in Figure 1.

The CN can be represented as a digraph \(G(V, E, A)\), where \(V\) is the node set; \(E \subseteq V \times V\) is the edge set; and \(A = [a_{ij}]\) is the adjacency matrix. The CN links and the DGs conform to the CN and the DGs conform to the CN. The CN nodes and the DGs conform to the CN. The CN graph \(G\) consists of the CN graph \(G\) and the DGs conform to the CN. The CN graph \(G\) is the CN graph.

The Laplacian matrix of the CN graph \(G\) is calculated as \(L = D - A\), where \(D\) is the in-degree matrix of the CN graph \(G\). Define \(\Gamma = L + G\). If \(G\) is a strongly connected weighted digraph with Laplacian \(L_{\omega_{0,\omega}}\), then rank \((L) = n - 1\). Moreover, an undirected graph \(G\) is connected if and only if rank \((L) = n - 1\) [27].

### 3.1 Proposed frequency and power-sharing controllers

The frequency and the P-sharing errors are defined as:

\[
e_{\omega} = g_{\omega} (\omega - \omega_n) + \sum_{j \in \mathcal{N}_i} a_{ij} (\omega - \omega_j)
\]  

(4)

\[
e_P = \sum_{j \in \mathcal{N}_i} a_{ij} (n_{P,j} - n_{P,j})
\]  

(5)

The supplementary secondary control signal is designed as:

\[
n_{\omega,i} = \int (\psi_{\omega,i} + \psi_{\omega,i}) \, dt
\]  

(6)

which includes two distinct control terms for the regulation of frequency (i.e. \(\psi_{\omega,i}\)) and active power sharing (i.e. \(\psi_P\)).

The proposed frequency controller is designed as,

\[
\begin{bmatrix}
\psi_{\omega,i} \\
\dot{z}_{\omega,i}
\end{bmatrix} = -\rho_{\omega,i} \begin{bmatrix}
k_{\omega,i} e_{\omega,i} + \mu_{\omega,i} z_{\omega,i} \\
\end{bmatrix}
\]  

(7)

and the proposed power-sharing controller is designed as,

\[
\begin{bmatrix}
\psi_P \\
\dot{z}_P
\end{bmatrix} = -\rho_P \begin{bmatrix}
k_{\omega,i} e_P + \mu_P z_P
\end{bmatrix}
\]  

(8)

where \(k_{\omega,i}, \mu_{\omega,i}, k_P, \mu_P, \gamma_P, \gamma_P, \) are real positive control gains; \(z_{\omega,i}\) and \(z_P\) are the integrands, and \(0 < \rho_P, \rho_P \leq 1\) are the proposed adaptive gain ratios, to be designed later.
Proposition 1. The proposed EPI-DCS regulates the drooped frequency of the microgrid to the nominal value and automatically shares the active power demand between the DGs. At the steady state, the system converges to:

$$\lim_{t \to \infty} \omega_i(t) = \omega_{n}^*$$

and

$$\forall i, j : \lim_{t \to \infty} n_i \bar{P}_j(t) = \lim_{t \to \infty} n_j \bar{P}_i(t)$$

From Equations (4)–(8), the errors and controllers in vector form are:

$$\mathbf{e}_\omega = [\epsilon_\omega] = \mathbf{G} (\omega - \omega_{n}^*) + \mathbf{L} \omega$$

$$\mathbf{e}_p = [\epsilon_p] = \mathbf{L} n_p \bar{P}$$

$$\mathbf{u}_\omega = \psi_\omega + \psi_p$$

$$\left\{ \begin{align*}
\dot{\psi}_\omega = & \psi_{\omega} = -\rho_\omega (\mathbf{K}_\omega \mathbf{e}_\omega + \mathbf{\mu}_\omega \mathbf{z}_\omega) \\
\dot{z}_\omega = & z_{\omega} = \rho_\omega (\mathbf{e}_\omega - \mathbf{\gamma}_\omega \mathbf{Lz}_\omega)
\end{align*} \right.$$  

$$\left\{ \begin{align*}
\dot{\psi}_p = & \psi_{p} = -\rho_p (\mathbf{K}_p \mathbf{e}_p + \mathbf{\mu}_p \mathbf{z}_p) \\
\dot{z}_p = & z_{p} = \rho_p (\mathbf{e}_p - \mathbf{\gamma}_p \mathbf{Lz}_p)
\end{align*} \right.$$  

where $\mathbf{K}_\omega$, $\mathbf{\mu}_\omega$, $\mathbf{\gamma}_\omega$, $\mathbf{K}_p$, $\mathbf{\mu}_p$, and $\mathbf{\gamma}_p$ are diagonal gain matrices; and $\rho_\omega$ and $\rho_p$ are adaptive gain ratio matrices.

3.2 Voltage regulation with the proposed EPI-DCS

Similar to Equations (11)–(12), the voltage and $Q$-sharing errors are:

$$\mathbf{e}_v = [\epsilon_v] = \mathbf{L} v + \mathbf{G} (v - v_{n}^*)$$

$$\mathbf{e}_Q = [\epsilon_Q] = \mathbf{L} n_Q \bar{Q}$$

where $v = [v_{\omega d}, v_{\omega q}]$ and $v_{n}^* = 1 \otimes v_{n}^*$ are the vectors of the output and the reference voltage, respectively.

The supplementary secondary control signal is designed as:

$$\mathbf{u}_v = \psi_v + \psi_Q$$

which includes two distinct control terms for regulation of voltage (i.e. $\psi_v$) and $Q$-sharing (i.e. $\psi_Q$).

The proposed voltage and $Q$-sharing controllers are:

$$\left\{ \begin{align*}
\dot{\psi}_v = & -\rho_v (\mathbf{K}_v \mathbf{e}_v + \mathbf{\mu}_v \mathbf{z}_v) \\
\dot{z}_v = & z_v = -\mathbf{e}_v + \mathbf{\gamma}_v \mathbf{Lz}_v
\end{align*} \right.$$  

$$\left\{ \begin{align*}
\dot{\psi}_Q = & -\rho_Q (\mathbf{K}_Q \mathbf{e}_Q + \mathbf{\mu}_Q \mathbf{z}_Q) \\
\dot{z}_Q = & z_Q = -\mathbf{e}_Q + \mathbf{\gamma}_Q \mathbf{Lz}_Q
\end{align*} \right.$$  

where $\mathbf{K}_v$, $\mathbf{\mu}_v$, $\mathbf{\gamma}_v$, $\mathbf{K}_Q$, $\mathbf{\mu}_Q$, and $\mathbf{\gamma}_Q$ are diagonal gain matrices; and $\rho_v$ and $\rho_Q$ are diagonal adaptive gain ratio matrices.

Proposition 2. The proposed EPI-DCS regulates the drooped voltage of the microgrid and automatically shares the reactive power demand among the DGs. In the proposed EPI-DCS, the compromise between exact voltage control and $Q$-sharing LMI conditions is established, instead of exact or average voltage regulation, since the latter is already widely studied.

The exact voltage regulation leads to inaccurate $Q$-sharing control [8]. In order to account for the effect of the line impedances and provide an accurate $Q$-sharing, the regulation of the average voltage was proposed in [15, 22, 24, 25]. Therefore, in the proposed EPI-DCS, the compromise between the voltage regulation (i.e. $\lim_{t \to \infty} v_i(t) = v_{n}^*$) and the $Q$-sharing (i.e. $\lim_{t \to \infty} n_Q \bar{Q}_i(t) = \lim_{t \to \infty} n_Q \bar{Q}_j(t)$) is achieved using Equations (19) and (20). The proposed voltage EPI-DCS reduces the $Q$-sharing error compared to the exact regulation of the voltage.

3.3 Lyapunov stability of the proposed controllers

The stability of the microgrid control loops under the proposed controllers are proved using the Lyapunov stability criteria. In the following, the stability of the proposed P-sharing controller is proved as an instance. The stability of the proposed frequency and voltage controllers as well as the proposed Q-sharing controller can be proved using the same approach, which is not included for brevity.

Considering $n_p \bar{P}_i$ as the P-sharing controller [16] (i.e. $\psi_p = n_p \bar{P}$) and Equation (15), the P-sharing error dynamic is:

$$\dot{e}_p = \mathbf{L} n_p \bar{P} = \mathbf{L} \psi_p = -\rho_p \mathbf{L} (\mathbf{K}_p \mathbf{e}_p + \mathbf{\mu}_p \mathbf{z}_p)$$  

It is noted that $\mathbf{K}_p$ and $\mathbf{L}$ (as well as $\mathbf{\mu}_p$ and $\mathbf{\gamma}_p$) are commutative as the gain matrices $\mathbf{K}_p$ and $\mathbf{\mu}_p$ are diagonal with positive entries. Considering the state variables $\mathbf{e}_p$ and $\mathbf{z}_p$, the closed-loop state-space model of the P-sharing control loop is
derived as:
\[
\frac{d}{dt} \begin{pmatrix} e_p \\ z_p \end{pmatrix} = -A_p \begin{pmatrix} e_p \\ z_p \end{pmatrix}, \quad A_p = \begin{pmatrix} \rho_p & 0 \\ 0 & \rho_p \end{pmatrix} \left( \begin{array}{cc} K_p L & \mu_p L \\ -I & \gamma_p L \end{array} \right)
\]
(22)

Considering the Lyapunov function \( V_p = \frac{1}{2} e_p^T e_p + \frac{1}{2} z_p^T z_p \) and substituting Equations (14) and (21) into \( V_p \) yields:
\[
\frac{d}{dt} V_p = e_p^T e_p + z_p^T z_p = -\left( e_p^T, z_p^T \right) A_p \left( \begin{array}{c} e_p \\ z_p \end{array} \right)
\]
(23)

Assume the PI control gains \( K_p, \mu_p, \gamma_p \) are designed so that \( A_p \) is symmetric and the LMI \( A_p \geq 0 \) is satisfied. This is the necessary condition for the stability of the proposed controller. Suppose \( \lambda_i \) are the eigenvalues of \( A_p \), and \( \lambda_m = \min(\Re(\lambda_i)) \) is the minimum real part of \( \lambda_i \). For the positive control gains and a connected CN, it is obvious that \( A_p \geq 0 \) and \( \lambda_m \geq 0 \). Consequently, from Equation (23) one concludes:
\[
\dot{V}_p \leq -2\lambda_m V_p
\]
(24)
which confirms the Lyapunov stability of the proposed controller.

From Equation (22), it is clear that using real positive control gains is sufficient for Lyapunov stability of the proposed EPI-DCS. While the structure of CN can be altered, the stability of the control loops is preserved as long as the altered CN graph contains a spanning tree [16, 22, 23, 25]. Therefore, establishing an intermittently connected CN (i.e. \( \Gamma > 0 \)) is required for the stability of the control loops.

### 3.4 Transfer function of the proposed controller

In the following subsection, the transfer function of the proposed EPI-DCS is calculated and compared to the conventional PI control. The transfer function of the conventional PI control in \( s \)-domain is:
\[
\psi_\omega = -\left( K_\omega + \frac{1}{s} \mu_\omega \right) e_\omega
\]
(25)

In order to calculate the transfer function of the proposed frequency EPI-DCS, the proposed adaptive gain scheduling is deactivated and thus the gains are constant (i.e. \( \rho_\omega = I \)). With respect to \( \dot{z}_\omega = e_\omega - \gamma_\omega L z_\omega \) in Equation (14), \( z_\omega \) in \( s \)-domain is:
\[
\dot{z}_\omega = I + \frac{I}{s} \gamma_\omega L \end{equation}^{-1} e_\omega
\]
(26)
Substituting Equation (26) into Equation (15), \( \psi_\omega \) in \( s \)-domain is calculated as:
\[
\psi_\omega = -\left( K_\omega + \frac{1}{s} \mu_\omega \right) e_\omega
\]
(27)

Applying the Woodbury matrix identity (i.e. \( (I + \gamma_\omega L s)^{-1} = I - \gamma_\omega (s I + \gamma_\omega L)^{-1} L \)) to Equation (27) yields:
\[
\psi_\omega = -\left( K_\omega + \frac{1}{s} \mu_\omega \right) e_\omega + \left( -\mu_\omega \gamma_\omega (s I + \gamma_\omega L)^{-1} L \right) e_\omega
\]
(28)

The transfer function Equation (28) consists of two parts: the first part is the conventional PI controller similar to Equation (25), whereas the second part is a consensus-based term including a double integrator scheme and communication with neighbouring nodes.

The transfer function of the voltage and power loops with the proposed controllers can be calculated in an identical approach. The proposed EPI-DCSs feature the leader-following schemes [18, 22, 23, 25, 26]. The reference voltage and frequency values are determined by the virtual leaders, which resembles a pinning control scheme [13, 18, 23, 24].

### 3.5 Application of the Lyapunov–Krasovskii LMI condition for time-delay systems to tune the control gains

The stability margin of the control gains is determined using the Lyapunov stability criteria [28]. Although a higher control gain shortens the convergence time according to Equation (23), there are several limiting factors to employ large gains, such as the CN time delays and the control hierarchy.

For the control hierarchy, the inner loops must be two to five times faster than the outer loops [29]. For the distributed secondary controller communicating with neighbouring DGs, the CN time-delay is the other limiting factor with a negative impact on the stability of the system [25]. The stability of the proposed controllers in the presence of CN time delays is examined using the time-dependent Lyapunov–Krasovskii LMI condition for a delayed system [30].

Consider the time delay system Equation (29), in which \( A_1 \) and \( A_2 \) are the system state matrices; \( 0 \leq \tau \leq \bar{\tau} \) is the time delay; and \( \bar{\tau} \) is the maximum bound of the delay.
\[
\dot{x}(t) = A_1 x(t) + A_2 x(t - \tau)
\]
(29)

The system Equation (29) is stable if the matrices \( M_i > 0, \quad i = 1, 2, 3, 4 \) exist that satisfy the LMI conditions.
Equations (30) and (31) [30].

$$
\begin{bmatrix}
M_i^T A + A^T M_i & M_1 - M_2^T + \tau M_2 & -\tau M_2^T A_2 \\
0 & -M_1 - M_2^T + \tau M_2 & -\tau M_2^T A_2 \\
0 & 0 & -\tau M_2
\end{bmatrix} < 0
$$

(30)

where $A = A_1 + A_2$.

For the known state matrices $A_1$ and $A_2$, and the desired maximum delay $\bar{\tau}$, the LMIs Equations (30), (31) are feasibility problem. This stability condition can be applied for a system with fast-varying delays, since no restriction is applied on $\dot{\tau}$. It is noted that different values of allowable $\bar{\tau}$ can be calculated using distinct methods [30].

Considering $\Gamma = D - A + G$ and knowing that only $A$ is contributing to the CN time delays, the time delay model of the frequency loop under the proposed controller is:

$$
\begin{bmatrix}
\dot{\xi}_{\omega} (t) \\
\dot{\omega} (t)
\end{bmatrix} = -\begin{pmatrix}
\rho_{\omega} & 0 \\
0 & \rho_{\omega}
\end{pmatrix}
\begin{pmatrix}
K_\omega (D + G) \mu_\omega (D + G) - I \\
\gamma_\omega D
\end{pmatrix}
\begin{pmatrix}
e_{\omega} (t) \\
\omega (t)
\end{pmatrix}
+ \begin{pmatrix}
\rho_{\omega} & 0 \\
0 & \rho_{\omega}
\end{pmatrix}
\begin{pmatrix}
K_\omega A \mu_\omega A \\
\gamma_\omega A
\end{pmatrix}
\begin{pmatrix}
e_{\omega} (t - \tau) \\
\omega (t - \tau)
\end{pmatrix}
$$

(32)

The LMIs Equations (30), (31) are used to calculate the maximum allowable time delay for the stability of Equation (32) with $\rho_{\omega} = I$.

### 3.6 Proposed adaptive gain-scheduling considering the CN time-delays

According to the LMIs Equations (30), (31), it is concluded that a control loop with constant parameters is ineffective against the destabilising time delay $\tau > \bar{\tau}$. In the following subsection, the proposed adaptive gain ratio $\rho_{\omega}$ is employed to stabilise the control loops against the destabilising time delay $\tau > \bar{\tau}$. The frequency control loop is proposed, whereas the same is applied to other control loops, which are not included for brevity.

Considering the time delay $\tau_{ij}$ in the communication link between nodes $i$ and $j$, the frequency error Equation (4) is rewritten as:

$$
e_{\omega_{ij}} (t) = g_j (\omega (t) - \omega^*_i) + d_i \omega_i (t) - \sum_{j \in N_i} a_{ij} \omega_j (t - \tau_{ij})
$$

(33)

Using the first-order Taylor series expansion of the time delay term $\omega_j (t - \tau_{ij}) \approx \omega_j (t) - \tau_{ij} \dot{\omega}_j (t)$, gives:

$$
e_{\omega} (t) = G (\omega (t) - \omega^*_i) + L \omega (t) + \tau \dot{\omega} (t)
$$

(34)

where $\tau = [\tau_{ij}]$ is the time delay matrix. Similarly, for $\dot{\omega}_{\omega} (t)$ in (14), the time delay term is approximated as:

$$
\dot{\xi}_{\omega} (t) = \rho_{\omega} \left( e_{\omega} (t) - \gamma_\omega L \omega (t) - \gamma_\omega \tau \dot{\xi}_{\omega} (t) \right)
$$

(35)

which results in,

$$
\dot{\xi}_{\omega} (t) = (I + \rho_{\omega} \gamma_\omega \tau) \rho_{\omega} \left( e_{\omega} (t) - \gamma_\omega L \omega (t) \right)
$$

(36)

Calculating $\dot{\omega}_{\omega}$ from Equation (33) as $\dot{\omega}_{\omega} (t) = \Gamma \omega (t) + \tau \dot{\omega} (t)$, considering $\dot{\omega} = \dot{\psi}_{\omega}$, and substituting Equations (14) and (15) into $\dot{\omega}_{\omega}$ yields:

$$
(I + \tau \rho_{\omega} K_{\omega}) \dot{\omega}_{\omega} = -\rho_{\omega} \left( \Gamma K_{\omega} e_{\omega} + \Gamma \mu_{\omega} \omega + \tau \mu_{\omega} \dot{\omega}_{\omega} \right)
$$

(37)

Substituting $\dot{\xi}_{\omega} (t)$ from Equation (35) into Equation (37) gives:

$$
\dot{\epsilon}_{\omega} = -\rho_{\omega} \left( I + A \tau \rho_{\omega} K_{\omega} \right)^{-1}
\begin{pmatrix}
(I + \tau \rho_{\omega} K_{\omega}) \dot{\omega}_{\omega} \\
(\Gamma K_{\omega} + A \tau \rho_{\omega} (I + \rho_{\omega} \gamma_\omega \tau)^{-1} \rho_{\omega} \gamma_\omega L) \omega_{\omega}
\end{pmatrix}
$$

(38)

**Proposition 3.** Adaptive gain tuning

The proposed adaptive gain tuning is designed as:

$$
\rho_{\omega} = \begin{cases}
1 & \text{if } \tau \leq \bar{\tau} \\
\min \left\{ -\Re \left( \lambda_{\omega} \right) \right\} & \text{if } \tau > \bar{\tau}
\end{cases}
$$

(39)

where $\bar{\tau}$ is calculated offline using the Lyapunov–Krasovskii LMIs Equations (30), (31); $\Theta = \omega$, $P$, $Q$, $Q$ indicates the system states; $\lambda_{\omega}$ is the eigenvalue of $A_{\omega}$; $\Re (\lambda_{\omega})$ is the real part of the eigenvalues of $A_{\omega}$; $A_{\omega}$ is the state matrix of the state $\Theta$ calculated as Equation (40) with respect to Equation (22).

$$
\begin{aligned}
A_{\omega} &= \begin{pmatrix}
K_{\omega} \Gamma \\
-\gamma L
\end{pmatrix}, & A_{p} &= \begin{pmatrix}
K_{p} L & \mu_{p} L \\
-I & \gamma_{p} L
\end{pmatrix} \\
A_{c} &= \begin{pmatrix}
K_{c} \Gamma \\
-\gamma L
\end{pmatrix}, & A_{q} &= \begin{pmatrix}
K_{q} L & \mu_{q} L \\
-I & \gamma_{q} L
\end{pmatrix}
\end{aligned}
$$

(40)

Applying the proposed adaptive gains Equation (39) to the state dynamics Equations (36) and (38) gives:

$$
\frac{d}{dt} \begin{pmatrix}
e_{\omega} \\
\omega_{\omega}
\end{pmatrix} = -T_{\omega} \begin{pmatrix}
0 \\
T_{\gamma}
\end{pmatrix}
\begin{pmatrix}
e_{\omega} \\
\omega_{\omega}
\end{pmatrix}
$$

(41)

where $T_{\omega} = (I + \beta A K_{\omega})^{-1}$ and $T_{\gamma} = (I + \beta A \gamma_{\omega})^{-1}$.
It is worth noting that all the eigenvalues of the state matrix of the closed-loop system Equation (41) are stable under the proposed EPI-DCS with positive gains, the proposed adaptive gains Equation (39), and an intermittently connected CN. The proposed adaptive gain tuning method improves the stability of the system against large destabilising time delays and prevents the consequent instability. Clearly, the proposed gain ratios Equation (39) are bounded with respect to the LMIs Equations (30), (31).

Figure 2(a,b) illustrate the diagram of the proposed EPI-DCS and the proposed adaptive gain tuning, respectively. At the $i^{th}$ DG, the gain ratios (i.e. $\rho_{\omega,i}, \rho_{P,i}, \rho_{V,i}, \rho_{Q,i}$) in Figure 2(a) are varied using the adaptive gain tuning in Figure 2(b). The maximum $\bar{\tau}$ is calculated offline using the LMIs Equations (30) and (31).

4 | NUMERICAL SIMULATIONS AND CASE STUDIES

The performance of the proposed adaptive EPI-DCS is examined through numerical simulations of a typical islanded microgrid shown in Figure 3, including 6 loads and 4 DGs. The specifications of the microgrid are listed in Table 1 [18]. The microgrid is modelled in MATLAB/Simulink using the SimPowerSystems Blockset. The detailed switching-based model of the 3-phase 3-level converters is utilized in the simulation studies.

The voltage and current regulators, shown in Figure 1 with the control parameters given in Table 1, are utilized as the inner control loops of the DGs. Several scenarios are studied, such as I–connection of a new load; II–outage of a DG; III–performance comparison of the proposed EPI-DCS with the conventional PI; and IV–performance of the proposed adaptive gain tuning against the destabilising time delays. The control gains of the proposed EPI-DCS in cases studies I, II, and IV are given in Table 2.

The CNs among the DGs are depicted in Figure 4. The CN topology of Figure 4(a) is used for all the cases except when DG 3 is forced outage in case 2, whereas the CN topology is altered to Figure 4(b) after DG 3 outage. Since only DG 1 is connected to the virtual leader, the pinning gains are $g_1 = 1$ and $g_2 = g_3 = g_4 = 0$ [16, 18].
4.1 Case 1: Small-signal stability under load change

In this case, the primary droop is activated at $t = 0.5$ s. Then, the proposed EPI-DCS is enabled at $t = 1.5$ s. Up to $t = 3$ s, exact voltage regulation is enabled and Q-sharing is ignored. The proposed Q-sharing controller is activated at $t = 3$ s. Moreover, Load 6 is connected at $t = 4.5$ s. The output frequency, voltage, active power, and reactive powers of the DGs are shown in Figure 5. As shown in Figure 5(a), the microgrid frequency deviates from its nominal value without the secondary control under the primary droop control. Enabling the proposed EPI-DCS restores the microgrid frequency and voltage to their nominal values and automatically shares the power demand among the DGs.

As shown in Figure 5(b,d), the exact voltage regulation is achieved between $t = 1.5$ s and $t = 3$ s. At $t = 3$ s, the proposed Q-sharing control is activated and the compromise between voltage regulation the Q-sharing is achieved using Equations (19) and (20), as pointed out in Proposition 2 in Section III. The proposed Q-sharing EPI-DCS reduces the Q-sharing error after $t = 3$ s for the exact voltage regulation. Under the proposed EPI-DCS, the voltage of the leader (i.e. DG 1) is set to the reference value and the voltages of the follower DGs are varied according to the Q-sharing controller and line impedances. As shown in Figure 5(b), the voltages all the DGs are kept within the practical range.

Moreover, when Load 6 is connected at $t = 4.5$ s, it is shown in Figure 5 that the proposed EPI-DCS automatically regulates the frequency and voltage and shares the new power demand based on the droop gains. Also, the voltage profile is regulated using the designed EPI-DCS, as shown in Figure 5(b). The outputs of the proposed secondary frequency and voltage controllers are shown in Figure 6(a,b), respectively.

4.2 Case 2: Large signal stability: disconnection of DG 3

In the following, the large signal stability of the proposed EPI-DCS is examined under converter outage. In the beginning, the microgrid system is under primary droop control. Then, the proposed EPI-DCS is activated at $t = 0.5$ s and DG 3 is suddenly disconnected at $t = 2.5$ s. The original CN topology before DG 3 outage is depicted in Figure 4(a), whereas it is changed to Figure 4(b) after DG 3 outage.

The performance of the proposed EPI-DCS in regulating the state variables after a sudden outage of DG 3 is shown in Figure 7. The results in Figure 7 confirm the resiliency of the proposed EPI-DCS to preserve the stability of the microgrid with diverse configurations of the CN under DG outage.

The steady state frequency, voltage, and output powers are depicted in Figure 7. The outputs of the proposed secondary frequency and voltage controllers are shown in Figure 8(a,b), respectively.

4.3 Case 3: Performance comparison of the proposed EPI-DCS with the conventional PI controller

In this case, the performance of the proposed EPI-DCS is compared with the conventional PI control in Equation (25). The control gains are set to $k_{\omega i} = k_{P_i} = 15$ and $\mu_{\omega i} = \mu_{P_i} = 60$. 

FIGURE 5 DC outputs when load increases (a) frequency, (b) voltage, (c) active power, (d) reactive power

FIGURE 6 Outputs of the proposed secondary controller when load increases (a) secondary frequency controlled, (b) secondary voltage controller
for both the proposed EPI-DCS and the conventional PI control. In order to examine the impact of the proposed consensus-based term (i.e. \( \gamma_{\omega}Lz_{\omega} \) and \( \gamma_{p}Lz_{p} \)) in the proposed EPI-DCS, the gains \( \gamma_{\omega} \) and \( \gamma_{p} \) are increased from 4 to 15 in the proposed EPI-DCS, whereas \( \gamma_{\omega} = \gamma_{p} = 0 \) in the conventional PI control. The secondary frequency and active power controllers are enabled at \( t = 1 \) s. As depicted in Figure 9, the proposed EPI-DCS improves the transient response of the average P-sharing error and the frequency, compared to the conventional PI control, by reducing the overshoot magnitude and the settling time. It is also worth noting that with the arbitrary PI gains and by increasing \( \gamma_{\omega} \) and \( \gamma_{p} \) from 4 to 15, the overshoot is further decreased, which verifies the advantage of adding the proposed consensus-based term (i.e. \( \gamma_{\omega}Lz_{\omega} \) and \( \gamma_{p}Lz_{p} \)).

4.4 Case 4: Performance of the proposed adaptive gain tuning against large destabilising varying time delays

In this case, the performance of the proposed EPI-DCS with the proposed adaptive gain tuning Equation (39) is evaluated. Moreover, its performance is compared to the conventional PI control enhanced with the proposed adaptive gain tuning. According to the Lyapunov–Krasovskii LMI conditions Equations (30) and (31) with the constant control gains given in Table 2, the CN time delay in Figure 10(a) causes instability in the control loops, which is also confirmed by the simulation results in Figure 10.

As illustrated in Figure 10(c,d), although the control loop with a constant gain is unstable for the variable time delays, the proposed adaptive gain scheduling stabilises both the proposed EPI-DCS and conventional PI control. The proposed adaptive gain scheduling scheme automatically decreases the control gains using the calculations given in subsection 3.6, to increase the maximum tolerable time delay of the control loop. As shown, the suggested adaptive gains improve the stability of both the EPI-DCS and the conventional PI control. However, the transient response with the proposed EPI-DCS is further improved in comparison with the conventional PI. The developed EPI-DCS lowers the overshoot and decreases the settling time compared to the conventional PI. It is worth mentioning that the control loop is unstable under the conventional PI controller without adaptive gain tuning.
FIGURE 10  Stabilising the control loop using the proposed adaptive gain tuning with the proposed EPI-DCS and the conventional PI control. (a) Assumed destabilising CN time delays, (b) DGs’ frequency, (c) average active power sharing error, (d) proposed adaptive variation of the control gain ratio (i.e. \( \rho_i \))

5  CONCLUSION

Sharing the power mismatch in a self-organised microgrid with a droop-based primary control results in deviation of the frequency and voltage from the nominal values. To restore the frequency and voltage through the secondary control level, a new robust EPI-DCS has been proposed. Moreover, the power mismatch is also shared among the DGs in a distributed droop-based scheme. The proposed architecture establishes a distributed cooperative approach, in which the DGs are communicating the required information with their neighbours through the CN.

In this paper, an enhanced PI control is developed using the control Lyapunov function method, for the secondary control level of a stand-alone microgrid. The proposed EPI-DCS is designed by adding a new consensus-based term to the integrand dynamic of the conventional PI control. In a distributed secondary control, the DGs exchange information through the CN with varying time delays. For stable operation, the maximum allowable CN time delay is calculated using the Lyapunov–Krasovskii LMI condition for the time-delayed systems. Based on that, an adaptive gain-scheduling method is proposed to improve the stability of the secondary control level against the destabilising CN time delays and prevent consequent instability.

The control gains are adaptively modified if the CN time delays are larger than the maximum allowable time delays.

The performance of the proposed adaptive EPI-DCS is validated by detailed models of the microgrid components. Simulation results confirm that the suggested EPI-DCS improves the transient response of the control loop, compared to the conventional PI control. Moreover, the developed adaptive gain tuning method stabilises the microgrid under destabilising CN time delays. With CN time delays, the transient responses of the control loops are further improved using the proposed EPI-DCS compared to the conventional PI controller.

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