The entropy of SAT problem

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Abstract: In this paper with two equivalent representations of the information contained by a SAT formula, the reason why string generated by succinct SAT formula can be greatly compressed is firstly presented based on Kolmogorov complexity theory. Then what strings can be greatly compressed were classified and discussed. The equivalence of computation and information was clearly stated in succession. In the last the entropy of SAT problem was computed based on universal probability. The experiment results showed the information gained by solving SAT problem was quite likely exponentially increased.

Key Words: Entropy, Kolmogorov Complexity, Universal probability, Boolean Satisfiability Problem, Circuits Complexity

1. INTRODUCTION

As we know, any function \( f: \{0,1\}^n \rightarrow \{0,1\} \) can be constructed from the elementary gates AND, OR, NOT and FANOUT [1]. Therefore, these constitute a universal set of gates for classical computation. There are \( 2^{2^n} \) possible functions in this function space.

By representing computation using circuits, it’s easy to show that some functions require very large circuit to compute [2].

Theorem 1: For every \( n>1 \), there exists a function \( f: \{0,1\}^n \rightarrow \{0,1\} \) that cannot be computed by a circuit \( C \) of size \( 2^n / (10n) \).

Since every circuit of size at most \( S \) can be represented as a string of \( 9 \cdot S \log S \) bit (e.g., using the adjacency list representation), the number of such circuits is at most \( 2^{9 \cdot S \log S} \). Setting \( S = 2^n / (10n) \), the number of circuits of size \( S \) is at most \( 2^{9 \cdot S \log S} \leq 2^{2^n \cdot 9n/10n} < 2^{2^n} \). Hence the number of functions computed by such circuits is smaller than \( 2^{2^n} \), implying that there exists a function that is not computed by circuits of that size. More careful calculation has been used to obtain a tighter bound of \((1-\varepsilon)2^n / n \) for every \( \varepsilon > 0 \) and even \( 2^{n(1+\log n/n-O(1/n))} \) [3].

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There is another way to phrase this proof [1]. Suppose that we pick a function $f : \{0,1\}^n \rightarrow \{0,1\}$ at random by picking for every one of the $2^n$ possible inputs $x \in \{0,1\}^n$ the value $f(x)$ in $\{0,1\}$ uniformly and independently. Since there are at most $2^{0.92^n}$ circuits of size at most $2^n/(10n)$, the union bound can be applied to conclude the probability that there exists such a circuit $C$ computing $f$ is at most

$$\frac{2^{0.92^n}}{2^{2n}} = 2^{-0.12^n}$$

(1)

Which tends very fast to zero as $n$ grows. It yields a stronger result than the Theorem 1: not only does there exist a hard function (not computed by $2^n/(10n)$ size circuits), but in fact the vast majority of functions form $\{0,1\}^n$ to $\{0,1\}$ are hard.

The Boolean Satisfiability Problem (abbreviated as SAT) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula. In other words, it asks whether the variables of a given Boolean formula can be consistently replaced by the values TRUE or FALSE in such a way that the formula evaluates to TRUE. If this is the case, the formula is called satisfiable. On the other hand, if no such assignment exists, the function expressed by the formula is identically FALSE for all possible variable assignments and the formula is unsatisfiable. SAT is one of the first problems that were proven to be NP-complete. And resolving the question whether SAT has an efficient algorithm is equivalent to the P versus NP problem [4]. As we can see, the SAT problem wants to know the formula, with the $2^n$ input, is or is not the one output the $2^n$ zeros (represents unsatisfiable). The SAT, in this regard, wants to know the information of the whole function which corresponds to $2^n$ inputs.

Instead of modeling Boolean circuits as labeled graphs, we can also model them as a straight-line program [1]. A program is straight-line if it contains no branching or loop operations (such as “if” or “goto”), and hence its running time is bounded by the number of instructions it contains. The equivalence between Boolean circuits and straight-line programs is fairly general and holds for essentially any reasonable programming language. The straight-line program can be obviously demonstrated with Boolean operations (OP). A Boolean straight-line program of length $T$ with input variable $y_1, y_2, \ldots, y_n \in \{0,1\}$ is a sequence of $T$ statements of with OP, where OP is either AND, OR, NOT and FANOUT. That’s to say, a SAT formula. So SAT formula and circuit are two equivalent ways to represent a function.

Kolmogorov defined the algorithmic (descriptive) complexity of an object to be the length of the shortest binary computer program that describes the object [5]. It’s the intrinsic descriptive complexity of an object. To be specific, The Kolmogorov
complexity $K_U(x)$ of a string $x$ with respect to a universal computer $U$ is defined as the minimum length over all programs that print $x$ and halt.

$$K_U(x) = \min_{p : U(p) = x} l(p)$$

(2)

Thus, $K_U(x)$ is the shortest description length of $x$ over all descriptions interpreted by computer $U$. The expected length of the shortest binary computer description of a random variable is approximately equal to its entropy. So this notion of intrinsic complexity is computer independent. Besides that, The Kolmogorov theory is very beautiful and has produced many profound and useful results that wait for further mining. Kolmogorov complexity often gives us a framework that helps us understand not only computation but efficient computation and in the same time allows us to put quite different and complex concepts in a common framework.

In all the $2^n$ possible functions, we already know the vast majority are hard and have exponential complexity measured by circuit. But in practice we are only or more interested in those functions which can be implemented by succinct circuits, then what properties those functions will have? What makes them special? This question should be important and has both theory and practical value. We will try to answer this question by using Kolmogorov theory as a bridge.

This paper is organized as the follows. In Sec. II we focus on the reason why the string generated by succinct SAT formula can be greatly compressed. In Sec. III we concentrate on what strings can be greatly compressed. In Section IV the equivalence of computation and information was stated. In Section V the entropy of SAT problem was computed with universal probability. Section VI is devoted to summary and discussions.

II. TWO EQUIVALENT REPRESENTATIONS OF THE INFORMATION CONTAINED IN A SAT FORMULA

With the Kolmogorov complexity theory, the information contained in a specific SAT formula (function) can be represented by the following two equivalent ways. One is in the form of program (Program 1).

```
for i = 0 to $2^n - 1$
    With i as the input, compute the one bit output of a SAT formula;
    Print the output bit;
end
```

This program will print out this function’s corresponding $2^n$ output bit string. The only two variables in the program are $i$ and the specific SAT formula. The total length of this program is:

$$l(p) = c + \log 2^n + l(\text{SAT formula}) = c + n + O(P(n)) = O(P(n))$$

(3)

In which $l(\text{SAT formula})$ represents the length of SAT formula. As we know,
we’re only interested in those succinct formulas, that’s to say, their lengths are with polynomial(P) complexity, let us represent its length with $O(P(n))$ bit. It should be noted that the length of SAT formula should not be the direct length with that formula, if the formula is only seemingly exponential long but it can be generated by succinct program, then it is still succinct and short. It’s same for circuit representation, some circuit may have exponential size, but they have a succinct representation in terms of a Tuning Machine[1], which can systematically generate any required vertex of the circuit in polynomial time.

Now we have a program which can generate the corresponding string, but according to Kolmogorov complexity theory, we can never guarantee this program is the shortest one which can generate the same $2^n$ bit string. So we have the upper bound of its Kolmogorov complexity:

$$K(x_1, x_2, ..., x_n \mid 2^n) \leq O(P(n))$$

(4)

In the same time, Kolmogorov complexity theory told us that the probability that a string can be compressed by more than $k$ bits is no greater than $2^{-k}$.

**Theorem 2**: Let $x_1, x_2, ..., x_n$ be drawn according to a Bernoulli $\frac{1}{2}$ process. Then

$$(K(x_1, x_2, ..., x_n \mid 2^n) < 2^n - k) < 2^{-k}$$

(5)

The second form that can represent the information contained in a specific SAT formula is its $2^n$ corresponding output string. According to the Theorem 2, in all the possible $2^n$ output strings, the possibility of those strings that can be compressed over $k$ bits will not exceed $2^{-k}$.

Then similar conclusion with Theorem 1 can be derived directly: of all the possible $2^n$ functions (sequences), the length of vast majority corresponding SAT formulas are exponential long (hard), because the strings that can be greatly compressed (compressed from exponential long to P complexity long) are very rare.

In the same time, besides the above conclusion which Theorem 1 has already told us, we can get more useful conclusion. The SAT formula with P complexity cannot generate all the $2^n$ functions. It can only generate at most $2^{O(P(n))}$ different functions. Considering the fact that some formula can also be compressed, the equivalent functions generated will be less than that number. And the functions generated must have corresponding output strings that can be greatly compressed. These strings can be compressed by at least $2^n - O(P(n))$ bit and have at most the possibility of $2^{O(P(n))-2^n}$. With a view to the fact that $O(P(n))<<2^n$ when $n \rightarrow \infty$, we
call it greatly compressed.

III. THE STRINGS THAT CAN BE GREATLY COMPRESSED

Since we are more interested in succinct SAT formulas, that is to say, their corresponding strings that can be greatly compressed, then let’s look what strings can be greatly compressed. As we knew, they are very rare and special. We classify them into two types based on the number of ones (or zeros) in the sequences. Suppose the string $x = \{x_1, x_2, \ldots, x_n\}$ and $\sum_{i=1}^{n} x_i = k$, there are $k$ ones in the string.

**Type 1** are those strings with $k = O(P(n))$, ($k \ll 2^{n-1}$), they can be greatly compressed according to Kolmogorov theory.

**Type 2** are those strings with $k \approx 2^{n-1}$ and meanwhile can be generated by short programs like Program 1.

As we noted, the vast majority of $2^n$ possible functions have $k \approx 2^{n-1}$. As for the type 2 strings, in fact, there are also numerous strings which can be generated by short programs, eg, $\pi, e, \sqrt{2}$ and etc, but we don’t know is whether these strings can be generated by programs in the form similar to Program 1 which has $2^n$ cycles. What we do know is that there exist type 2 strings. For example, the following strings can be generated by succinct SAT formula.

\[
0101010101010101010101010101010101010101\ldots \\
0011001100110011001100110011001100110011\ldots 
\]

Suppose $n = 2m$, if we perform the “>” operation between the first $m$ bit and the second $m$ bit, The function fulfill the problem of compare operation ($>$, $!=$, $\geq$, $\ldots$) can also be represented by succinct SAT formula.

However, in this paper we’ll lay stress on type 1 and neglect type 2 strings for two reasons: we don’t know there are how many type 2 strings. In the same time, those functions are unimportant in SAT problem.

As we know in intuition, it’s very easy to distinguish the function outputs $2^n$ zeros (we call it reference function) with the function outputs $2^n$ ones. In the same time it’s hard to distinguish the reference function with the function outputs $2^n - 1$ zeros and 1 one. The key problem here is how to measure the hardness (complexity) to distinguish the reference function and other different functions. We made a try to measure the complexity in the following way. Suppose we have two functions, one is the reference function, the other is the function outputs $2^n$ ones. They both can be represented by simple (P complexity long) SAT formula. In the beginning we cannot tell which is which. In order to distinguish them, what we need to do is to random choose one SAT formula, meanwhile random choose one input, then do the computation, the result will be either 0 or 1. In this way, we get 1 bit information. And this 1 bit information is enough to distinguish these two functions. We call that the
function outputs $2^n$ ones has 1/2 bit information difference with reference function. Now we consider the general cases, suppose the function to be distinguished with reference function outputs $k$ ones and $2^n - k$ zeros. We still random choose one SAT formula, meanwhile random choose one input, then do the computation, there are totally $2^n + 2^n = 2^{n+1}$ possible input-outputs, in which $k$ outputs are one. To find the first output one in $2^n + 2^n = 2^{n+1}$ space, the information we get will be $n + 1 - k = \log_2 \frac{2^{n+1}}{k}$ bit in order to distinguish them. Or we can think like this: we have to do the computation $\frac{2^{n+1}}{k}$ times (the expected value) before we meet output 1, but before that the output are all zeros, and these zeros can be compressed to only $\log \frac{2^{n+1}}{k}$ bit.

Now we can see the type 2 functions are unimportant in SAT problem because they are easy to be distinguished with reference function. While for the type 1 sequences, the situation is quite different because they contain all the hard functions to be distinguished with reference function.

If $k = O(P(n))$, $(k \ll 2^n)$, the function of type 1 can be represented by a succinct SAT formula in disjunctive normal form (DNF). In order to compute a certain Boolean function $f(a)$, Consider its minterms $f^{(i)}(a)$, defined, for each such that $f(a^{(i)}) = 1$, as

$$f^{(i)}(a) = \begin{cases} 1 & \text{if } a = a^{(i)} \\ 0 & \text{otherwise} \end{cases}$$

Then the function $f(a)$ reads as follows:

$$f(a) = f^{(1)}(a) \lor f^{(2)}(a) \lor \ldots \lor f^{(k)}(a)$$

(6)

Where $f(a)$ is the logical OR of all $k$ minterms. If $a^{(1)} = 110100...001$, we have

$$f^{(i)}(a) = a_{n-1} \land a_{n-2} \land \bar{a}_{n-3} \land a_{n-4} \land \bar{a}_{n-5} \land \bar{a}_{n-6} \land \ldots \land \bar{a}_2 \land \bar{a}_1 \land a_0$$

The length of this SAT formula (6) is approximate $O(kn)$.

We have the following program [5] which can generate the Type 1 strings (Program 2).

Generate, in lexicographic order, all sequences with $k$ ones; Of these sequences, print the $l$th sequence.
This program will print out the required string. The only variables in the program are \( k \) (with known range \( \{0, 1, 2^n\} \)) and \( I \) (with conditional range \( \{1, 2, \ldots, C_2^k\} \)). The total length of this program is:

\[
l(p) = c + \log 2^n + \log C_2^k \leq c' + \log 2^n + 2^n H(\frac{k}{2^n}) + \frac{1}{2} \log 2^n
\]

(7)

Since \( C_2^k \leq \frac{1}{\sqrt{\pi 2^n}} 2^{2^n H_0(p)} \) , in which \( p = \frac{k}{2^n}, q = 1-p \).

We already use \( n = \log 2^n \) bit to represent \( k \), and \( \sum_{i=1}^{2^n} x_i = k \), The Kolmogorov complexity of this binary string \( x \) is bounded by [5]

\[
K(x_1, x_2, \ldots, x_{2^n}) \leq 2^n H(\frac{k}{2^n}) + \frac{1}{2} n + c
\]

(8)

In which \( H(p) = -p \log p - (1-p) \log (1-p) \). In this way, they can be compressed to at least \( \frac{1}{2} n + 2^n H(\frac{k}{2^n}) \) bit according to Kolmogrov complexity theory. With regard to the length of Program 1 which can generate the same sequence, for succinct SAT formula of type 1, now we have another upper bound of the same string’s Kolmogrov complexity. They should have the same level of complexity, especially when there are only two complexity classes, polynomial and exponential length, to be distinguished. We can get the following result:

\[
O(P(n)) = 2^n H(\frac{k}{2^n}) + \frac{1}{2} n + c \Rightarrow 2^n H_0(\frac{k}{2^n}) = O(P(n))
\]

(9)

Unfortunately we cannot get the theory analysis of equation (9), we did a little experiment. We just report the results that we think important. In Fig 1: the ordinate is \( y = 2^n H_0(\frac{k}{2^n}) \) while abscissa is \( n \). we can see clearly that \( y = 2^n H_0(\frac{k}{2^n}) \) increase linearly when \( k \) is fixed and \( k << 2^n \).

![Figure 1. The property of equation (9)](image)
Inspired by the Maxwell demon thought experiment, we mapped three classical problems in computer science into Maxwell model in thermodynamics [6]. The computation processes of these problems are then all the processes of entropy reduction. The limits of their required physical resource can be derived based on the second law of thermodynamics. In this way we discovered that these three classical problems can be measured by entropy reduction (information).

In the following we’ll try to demonstrate why entropy reduction can be used to measure the complexity of any computation. The reason will be delivered from three points of view: Turing machine (TM), mathematics and information physics.

It’s obvious that we can represent a Turing machines as a string: Just write the description of TM on paper, and encode this description as a sequence of zeros and ones. This string can be given as input to another TM. This simple observation is very profound since it blurs the distinction between software, hardware, and data [1]. In this way, software, hardware, and data have no difference, they are the same thing. Every string in \{0,1\}^* represents some Turing machine. As we can see, from the viewpoint of TM, all computation is nothing but a string, that’s to say: corresponded to the probability of that string occurrence, or equal to the information represented by the same string. We can draw an important and profound conclusion: computation is nothing different with information, they are equivalent.

No matter how complex it is, any computation is nothing but a mapping (function) between two sets in mathematics. Any function with input of \(n\) bit and output of \(m\) bit:

\[ f : \{0,1\}^n \rightarrow \{0,1\}^m \]

The key problem now is how to measure the complexity of a specific function. In the above defined problem space, there will be at most \(2^{2^{n \cdot m}}\) mappings. If we have no information about the specific function, the best assumption we can make is that they are maximum entropy distributed, and in this circumstance, they are uniform distributed with the same possibility. Of course later we’ll see this hypothetical uniform distribution is wrong, their distribution should be perfectly replaced by universal probability distribution. It’s well known that when it comes to distribution and possibility, entropy is no doubt the best candidate to measure its complexity. The entropy before we know the exact mapping is \(k \cdot ln 2^{2^{n \cdot m}} = k 2^n m \cdot ln 2\). The entropy will become 0 after we obtain the specific mapping. In this process we’ll acquire the entropy reduction, \(k 2^n m \cdot ln 2\), of this specific function. That is to say, acquire the information of \(2^n m\) bit. When we perform one computation, the \(n\) bit input is determined, the information we acquire will be \(m\) bit. In this regard, the one computation (mapping, function) can be measured by negentropy and is equivalent to
the information of $m$ bit. In this way, the computation and the information have nothing different, so they are equivalent. Or we can say information and computation are merely the different representations of negentropy.

On the other side, information physics provided another theory: reversible computation. Indeed, any irreversible function: $f : \{0,1\}^n \rightarrow \{0,1\}^n$ can be embedded into a reversible function. Based on this theory, many researchers in information physics think it is possible to build a reversible computer without energy consumption.

If the theory of reversible computation is completely correct, in our opinion, it’ll lead to an obvious illogicality: everything happened (all computation was finished) but nothing changed (reversible). So there must be something been neglected in this theory that need to be amended. As we understand it, the above reversible computer is not truly, completely reversible because it still have to record the computation result in memory (“print the result” in the Bennett’s description) which need energy consumption according to Landauer principle. In fact if we take the minimum memory needed to record the computation result into account, this specific amount of energy consumption is very important, the energy consumption will exactly the same with that we use to measure the complexity of a problem [6]. Just as indicated by Landauer. Neither $kT$ (thermodynamics) nor the uncertainty principle (quantum mechanics) leads to unavoidable minimum energy dissipation requirements for computation. The only limit imposed by physics is memory, because in a finite universe (quite likely), nature is unlikely to provide unlimited memory.

The unique purpose of any computation is to acquire information. On the other hand, to obtain information must be a process (Otherwise it cannot have entropy reduction, change must correspond to certain process) and this process must be a process of computation because everything happened in the universe is doing computation. In this regard, computation and information are the same things. The reversible computation told us the computation can be done reversible. That is to say. The only physical resource needed was the memory to record the computation result. In this way, same with information, Computation definitely can be measured with entropy reduction: the information gained or the minimum memory needed to record the result.

V. THE ENTROPY OF SAT PROBLEM

Suppose that a computer is fed a random program. The probability distribution on the output strings is far from uniform[5]. Under the computer-induced distribution, simple strings are more(exponentially) likely than complicated strings of the same length. This motivated the definition of universal probability distribution on strings as follows:

The universal probability of a string $x$ is:

$$P_U(x) = \sum_{p : U(p) = x} 2^{-|p|} = \Pr(U(p) = x)$$

which is the probability that a program randomly drawn as a sequence of fair coin
flips \( p_1, p_2, \ldots \) will print out the string \( x \). This probability is universal in many senses. It can be considered as the probability of observing such a string in nature [5]; the implicit belief is that simpler strings are more likely than complicated strings.

Kolmogrov complexity and universal probability have equal status as universal algorithmic complexity measures. Their relation is very simple:

\[
P_U(x) \approx 2^{-K(x)}
\]  

(11)

This is especially interesting since \( \log \frac{1}{P_U(x)} \) is the ideal codeword length (the Shannon codeword length) with respect to the universal probability distribution \( P_U(x) \).

In the same time, Chaitin’s mystical, magical number, \( \Omega \), which has some extremely interesting properties.

\[
\Omega = \sum_{p:U(p)\text{ halts}} 2^{-l(p)}
\]  

(12)

Since the programs that halt are prefix-free, their lengths satisfy the Kraft inequality, and hence the sum above is always between 0 and 1.

In the following we’ll use the universal probability distribution to compute the entropy of SAT problem. Suppose that a computer is fed a succinct random program which can halt and in the same time output a \( 2^n \) long string \( x \). As we already knew, this succinct random program must correspond to some succinct function or SAT formula. This string \( x \) must can be greatly compressed and its Kolmogrov complexity \( K(x) < < 2^n \). The universal probability of this program (function, SAT formula) is 

\[
P_U(x) \approx 2^{-K(x)} \gg 2^{-2^n}.
\]

Now we only consider the type 1 succinct string \( X_k = \{x_1, x_2, \ldots, x_k\} \), and \( \sum_{i=1}^{2^n} x_i = k \).

With equation (8), their Kolmogrov complexity is:

\[
K(X_k) \leq 2^n H\left(\frac{k}{2^n}\right) + \frac{1}{2} n + c
\]  

(13)

\[
P_U(X_k) \approx 2^{-K(X_k)} \geq 2^{-2^n H\left(\frac{k}{2^n}\right) + \frac{1}{2} n + c}
\]  

(14)

The sum universal probability of all succinct functions is

\[
Pr_{\text{succ}} = \sum_{k=0}^{O(2^n)} C_k^2 P_U(X_k) < \Omega < 1
\]  

(15)

The entropy of all \( 2^{2^n} \) functions is:
\[
\text{Ent}_{\text{all}} = \sum_{k=0}^{2^n} \text{Ent}(k) = -\sum_{k=0}^{2^n} C_{2^n}^k \cdot P_U(X_k) \log P_U(X_k)
\]
\[
\geq -\sum_{k=0}^{2^n} C_{2^n}^k \cdot 2^{-2^n H(k \frac{k}{2^n})} \frac{1}{2}^{n-c} \log(2^{-2^n H(k \frac{k}{2^n})} \frac{1}{2}^{n-c})
\]

(16)

Figure 2 is the entropy of all functions. In this Figure, \( n = 10, c = 0 \). This figure is only schematic and the different value of \( c \) won’t change that.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The entropy of all functions}
\end{figure}

In fact, the equation (16) told us in the computation of the entropy, different functions should be given different weights, although the strings produced by vast majority functions have \( \sum_{i=1}^{2^n} x_i = 2^{n-1} \), but their universal probability are exponentially decreasing and their contribution to entropy are not decisive big. As for succinct functions, the opposite are true. So we can observe sharp increase when \( k \) is small.

The entropy of all succinct functions is:
\[
\text{Ent}_{\text{suc}} = -\sum_{k=0}^{\text{Op}(\text{p})} C_{2^n}^k \cdot P_U(X_k) \log P_U(X_k)
\]
\[
\geq -\sum_{k=0}^{\text{Op}(\text{p})} C_{2^n}^k \cdot 2^{-2^n H(k \frac{k}{2^n})} \frac{1}{2}^{n-c} \log(2^{-2^n H(k \frac{k}{2^n})} \frac{1}{2}^{n-c})
\]

(17)

Table 1. The entropy of succinct functions set

| \( n \) | \( \text{Ent}_{\text{all}} \) | \( \text{Ent}_{\text{suc}} \)_{p(n) = 0.01n^4} | \( \text{Ent}_{\text{suc}} \)_{p(n) = 0.001n^5} | \( \text{Ent}_{\text{suc}} \)_{p(n) = 0.0001n^6} |
|-----|---------------------|---------------------|---------------------|---------------------|
| 8   | 182.91             | 23.5                | 18                  | 14                  |
| 9   | 362.42             | 35.5                | 31.5                | 27.7                |
| 10  | 720.73             | 50.2                | 50.2                | 50.2                |
| 11  | 66.7               | 75.5                | 85.1                |                     |
Unfortunately we’re unable to get the theory analysis and cannot prove $\text{Ent}_{\text{suc}}$ is exponentially increasing although it seems that’s quite likely. In order to see this more clearly, we did some experiment. The result (see Table 1) is also rough and only schematic.

The above $\text{Ent}_{\text{suc}}$ is the entropy before the SAT is solved. To solve this problem is to decide the input random SAT formula (function, string) is or is not the one output $2^n$ zeros. The entropy($\text{Ent}_{\text{suc}}$) after the problem is solved will be 0. In the process of solving this problem, the information(negentropy) acquired is $-\text{Ent}_{\text{suc}} = \text{Ent}_{\text{suc}} - \text{Ent}_{\text{suc}}$.

VI. CONCLUSIONS

In this paper we have shown an interesting connection between Kolmogorov complexity and computational complexity. In our opinion, to measure the computational complexity from information perspective is feasible and may be more effective.

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