Finite Temperature Meson Masses with Improved Quenched Wilson Fermions

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We present recent results for meson screening masses with quenched Wilson fermions above and below the confinement-deconfinement phase transition. The action used in the simulation is the Sheikholeslami Wohlert action. The quark masses are chosen to be in a range going from light quarks up to the mass of the charm quark. These results are compared with zero temperature Wilson and high temperature staggered data.

1. Introduction

Mesons in a wide range of the quark mass have been thoroughly investigated throughout the last years in quenched studies with various kind of actions by many groups and collaborations. These investigations have mostly concentrated on zero temperature physics to extract the mass spectrum. Some studies have been performed on high temperature meson screening masses a few years ago by some groups\cite{1–4} using quenched staggered quark fields.

This study has been made to examine the behaviour of the pseudoscalar and the vector meson screening mass in the quenched Wilson formulation slightly below and above the high temperature phase transition. Furthermore we computed the free quark propagator analytically to see whether the pion and the rho are approaching the high temperature limit of \( m_{\pi,\rho} = 2 \sqrt{(\pi T)^2 + m_q^2} \).

2. Simulation

2.1. Action

We have used the standard Wilson action for the pure gauge part

\begin{equation}
S_{\text{Gluon}} = \frac{6}{g^2} \sum_{x,\mu>\nu} \left( 1 - \frac{1}{N} \Re \Tr \begin{vmatrix} \mu \end{vmatrix}_{\mu\nu}(x) \right)
\end{equation}

and the Sheikholeslami Wohlert action\cite{5} for the fermion part setting the clover coefficient to \( C_{\text{sw}} = 1 \).

\begin{equation}
S_{\text{Clover}} = \frac{1}{2\kappa} \sum_{x,y} \Psi(x) \left\{ \begin{array}{c}
\left( \mathbb{I} - \frac{\kappa C_{\text{sw}}}{2} \sum_{\mu,\nu} \Im \begin{vmatrix} \mu \end{vmatrix}_{\mu\nu}(x) \sigma_{\mu\nu}(x) \right) \delta_{x,y} \\
- \kappa \sum_\mu \left[ (\mathbb{I} - \gamma_\mu) \delta_{x+\hat{\mu},y} \Psi(x) + (\mathbb{I} + \gamma_\mu) \delta_{x-\hat{\mu},y} \Psi(y) \right] \end{array} \right\} \Psi(y).
\end{equation}

2.2. Observables

We have used extended sinks for the meson correlation function the way Lacock et al.\cite{6} proposed in 1995. Hence we have been able to fit meson propagators over a much wider range than in the local case to get more reliable results. The most suitable distance between quark and antiquark has been between 3 and 8 lattice units depending on the quark mass and the lattice spacing.

We have computed the meson propagators by combining two quark propagators generated with equal or different \( \kappa \)-values to obtain a large set of masses \( m_q = \frac{1}{2} (m_{q_1} + m_{q_2}) \) with \( m_{q_i} = \ln(1 + \frac{1}{2}(\frac{1}{\kappa_i} - \frac{1}{\kappa_c})) \).

2.3. Technical details

The simulations were performed at temperatures somewhat below the transition temperature phase transition at about \( T = 0.9 T_C \) on
16^3 \times 8, 24^3 \times 8 and 32^3 \times 8 lattices at \( \beta = 6.0 \) and above the transition at about \( T = 1.2 \, T_C \) on a 24^3 \times 8 lattice at \( \beta = 6.2 \). The lattice spacings are \( a = 2.05(6) \text{GeV}^{-1} \) and \( a = 2.7(1) \text{GeV}^{-1} \) computed from zero temperature data obtained from the literature.[7–10]

We also performed simulations on zero temperature lattices[11] at \( \beta = 6.0 \) (16^3 \times 32) and \( \beta = 6.2 \) (24^3 \times 48) to make sure that we are consistent with other groups.

The number of configurations lies between 20 and 100 being 40 or 100 in most cases.

3. Results

3.1. \( T = 0.9 \, T_C \)

We first compare the finite temperature results near \( \kappa_c \) at \( T = 0.9 \, T_C \) with the zero temperature data (Fig. 1). The dotted and dashed lines are linear fits. For the pseudoscalar meson the line of the high temperature screening mass comes to lie nearly on top of the zero temperature line. The small deviation is possibly due to finite size effects. Both sets of data extrapolate to nearly the same point of vanishing meson mass.

For the vectormeson channel we observe a larger deviation between the high temperature and the \( T = 0 \) data, even if we shift \( \kappa_{T=0} \) and \( \kappa_{T=0,T_C} \) indicated by the vertical lines on top of each other. This trend is perhaps a bit more pronounced than in results obtained with staggered fermions [4].

3.2. \( T = 1.2 \, T_C \)

We then compare the finite temperature results at \( T = 1.2 \, T_C \) with the zero temperature data over a wide range of the quark mass going from light masses near the chiral limit up to masses near \( m_{\text{charm}} \) (Fig. 2). The zero temperature curves are fits to \( a + bx + cx^2 \) while the high temperature curves are polynomials to guide the eye.

In contrast to the results at \( T = 0.9 \, T_C \) the screening masses of the pseudoscalar and the vectormeson acquire a large shift upwards compared to the zero temperature data. The pion remains heavy even in the chiral limit by restoration of the chiral symmetry above the phase transition temperature.

The rhomass \( (m_\rho \approx 0.77a^{-1}) \) is still heavier than the pionmass \( (m_\pi \approx 0.64a^{-1}) \), but they both seem to approach the high temperature limit of two free propagating quarks (see 3.3).

This result is in agreement with staggered data on quenched meson screening masses [2].

3.3. The free field

On a finite lattice we have analytically computed the free fermion field which is believed to be the high temperature limit \( (\beta \to \infty) \). The summation over the internal momenta has been done numerically. The results are shown in figure 3.

The results for the 24^3 \times 8 lattice are indicated by the dashed dotted line and the infinite volume limit by the fat line. On the 24^3 \times 8 lattice the mass of the pion and the rho for \( m_q \to 0 \)
are $m_{\pi,\rho} \approx 0.85a^{-1}$ while the continuum value is $m_{\pi,\rho} = 2\pi/8 = 0.785a^{-1}$. Thus on finite lattices the high temperature limit is approached from above. The results of the numerical simulation are quite close to this number at $T = 1.2T_C$ already.

The behaviour is understood as the influence of the two quarks being separately propagating $\{2 \sqrt{(\pi T)^2 + m_q^2} \leq \sqrt{(\pi T)^2 + m_{q_1}^2 + (\pi T)^2 + m_{q_2}^2}, (m_q = 1/2(m_{q_1} + m_{q_2})) \}$. 

**Figure 2:** The mass of the pseudoscalar squared and the mass of the vectormeson as a function of $m_q = 1/2(m_{q_1} + m_{q_2})$ at $\beta = 6.2$ for $T = 1.2T_C$ (open symbols) and zero temperature (filled symbols).

In addition to that we have looked at the different behaviour of the degenerate and non-degenerate quark mass meson propagators. The non degenerate meson masses in the infinite volume limit (dotted line) lie significantly above the degenerate masses, which confirms the results of the simulations (squares and triangles). The dashed curve is a polynomial to guide the eye where the degenerate meson masses are expected to be.

**Figure 3:** The masses of the vectormeson for $T = 1.2T_C$ against the bare quark mass including the results for the free quark field ($\beta = \infty$) on a $24^3 \times 8$ lattice and the infinite volume limit with equal and different constituent masses.

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