Sparse-view reconstruction of dynamic processes by neutron tomography

Hu Wang, Anders Kaestner, Yubin Zou, Yuanrong Lu, Zhiyu Guo

State Key Laboratory of Nuclear Physics and Technology & School of Physics, Peking University, Beijing 100871, China
Paul Scherrer Institute, Laboratory for Neutron Scattering and Imaging, Villigen, CH5232, Switzerland

Abstract

As for neutron tomography, hundreds of projections over the range of 0-180 degrees are required to reconstruct the attenuation matrix with the traditional filtered back projection (FBP) algorithm, and the total acquisition time can reach several hours. This poor temporal resolution constrains that neutron tomography is only feasible to investigate static or quasi-static process. Reducing the number of projections is a possible way to improve the temporal resolution, which however highly relies on sparse-view reconstruction algorithms. To assess the feasibility of sparse-view reconstruction for neutron tomography, both simulation and an experiment of water uptake from a piece of wood composite were studied, and the results indicated that temporal resolution of neutron tomography can be improved when combining the Golden Ratio scan strategy with a prior image-constrained sparse-view reconstruction algorithm-PICCS.

Keywords: a prior image, sparse-view, neutron tomography, dynamic process

1. Introduction

As a three-dimensional (3D) non-destructive testing technique, neutron tomography has been extensively used in many fields such as investigations of porous media[1], plants[2], cultural heritage[3], and fuel cells[4].

Traditionally, 3D distribution of the neutron attenuation coefficients of a sample is reconstructed using analytical methods such as the famous filtered back projection (FBP) algorithm[5], in which case hundreds of projections are
usually required. On the other hand, the intensity of neutron sources in the world, varying from \(10^5\) to \(10^{10}\) n/cm\(^2\)/s\([6]\), is relatively low, and the acquisition time for a single projection generally ranges from a few seconds up to several minutes depending on flux and pixel size. Thus, it usually takes several hours to acquire the projection data for a standard tomography. This relatively long acquisition time constrains that neutron tomography can only investigate static or quasi-static processes. For a process that changes faster than the acquisition time, such as the dynamic process of water distribution in operating fuel cells\([7]\) and hydrogen absorption by lanthanum pentanickel \((\text{LaNi}_5)\[8]\), the tomography reconstruction will be rendered useless due to motion artefacts. In other words, temporal resolution of neutron tomography using standard techniques is insufficient to follow such processes.

Generally, temporal resolution of neutron tomography can be improved in the following two ways: The first one is increasing the neutron intensity and thus reducing the exposure time for each projection, but this is not practically feasible. The second one is to cut down on the number of projections (sparse-view) or reducing the exposure time of every projection (low-dose), in which cases special reconstruction algorithms should be taken into account. In this paper, we focused on sparse-view reconstruction of neutron tomography.

Mathematically, sparse-view reconstruction is a highly ill-conditioned problem and is a hot topic in the field of medical X-ray computed tomography (CT) aimed at reducing the patient dose from irradiation. Iterative reconstruction algorithms are considered the most promising replacement of FBP for sparse-view reconstruction, because a priori information can be embedded into the iteration to regularize the reconstruction. The compressed sensing (CS) theory\([9]\) has attracted enormous attentions in signal processing since Donoho proposed it in 2006. According to the traditional Shannon/Nyquist criterion, \((\pi/2)N\) projections are required to get an accurate reconstruction for parallel-beam geometry, where \(N\) is the number of detector elements for every projection. However, CS theory has demonstrated that it is possible to reconstruct an image from highly under-sampled projections if the image is sparse or can be sparsely represented in a transformation space. Sidky\[10]\) combined the total variation (TV) transformation with CS and demonstrated that 20 noise-free projections (the degree of undersampling is 20) uniformly sampled in the range of 0-180 degree are sufficient to reconstruct the Shepp-Logan head phantom accurately. Unfortunately, the success of TV-based algorithms highly relies on the assumption that the image is piecewise constant. For a more complex image, over 100 projections are still required even within the CS framework\[11]\).

Regarding dynamic tomography, some more a priori information is available. Kaestner et al.\[12]\) employed the Golden Ratio scan strategy into the projection acquisition of neutron tomography, which suppressed motion artefacts notably and obtained a great flexibility to balance between spatial and temporal resolution during data processing. Reconstruction with more projections has a lower temporal resolution and vice versa. In the image reconstructed from sufficient projections satisfying the sampling condition, dynamic information is blurred, but static information is reconstructed without under-sampling artefacts. This low-temporal resolution image can serve as a priori image to regularize the sparse-view/high-temporal resolution reconstruction. GH Chen realized a prior image constrained CS (PICCS) method\[13]\), and S Leng\[14]\) further utilized the PICCS method in the reconstruction of medical dynamic X-ray tomography and concluded that it can tremendously improve the temporal resolution.

In this article, the PICCS method was employed to assess the feasibility of sparse-view reconstruction of dynamic neutron tomography aimed at improving the temporal resolution at maintained spatial resolution of the methodology. The article is organized as follows: Section 2 briefly introduces the PICCS method and Golden Ratio scan strategy; Section 3 describes the simulation work; Section 4 presents the experiment set-up and results, and Sections 5 gives a conclusion and outlook.

2. Method

In general, there are two types of reconstruction algorithms for tomography. The first one is analytic algorithms, such as FBP and FDK (named after the co-authors Feldkamp, Davis and Kress)\[15]\), which runs very fast and dominates the community of neutron tomography. However, these algorithms have severe restrictions on the projection data: the number of projections should satisfy the sampling condition, and the projections must be uniformly distributed over the scan interval. The second type is iterative algorithms that work better when the number of projections is severely under-sampled or irregularly acquired. Moreover, a priori information can be embedded into the iterations to improve the quality of the reconstruction.
2.1 SART

Mathematically, the tomography problem can be described by a set of linear equations:

\[
\sum_{j=1}^{N} \omega_{ij} x_j = p_i, \quad i = 1, 2, \ldots, M
\]  

(1)

where \( \omega_{ij} \) represents the contribution of \( j \)th pixel \( x_j \) to \( i \)th measurement \( p_i \). Equation (1) can be solved by the iterative Simultaneous Algebraic Reconstruction Technique (SART)[16] with the updating step as follows:

\[
x_j^{n+1} = x_j^n + \frac{1}{\sum_{i=1}^{N} \omega_{ij}} \sum_{i=1}^{N} \omega_{ij} \left( p_i - \sum_{j=1}^{N} \omega_{ij} x_j^n \right) \quad (j = 1, 2, \ldots, N)
\]  

(2)

2.2 TV

Iterative algorithms such as SART have advantages over analytic algorithms when the number of projections is limited, but severe streaking artefacts still exist in the reconstructed image even with iterative algorithms when the projections are severely under-sampled. CS-based methods have attracted much attention in suppressing the streaking artefacts. The CS theory is applicable only when an image or a particular transformation of the image is sparse. Generally, images in neutron tomography are not sparse, but most of the images can be sparsely represented after applying a discrete gradient operation defined as:

\[
\nabla x_{m,n} = \sqrt{(x_{m,n} - x_{m-1,n})^2 + (x_{m,n} - x_{m,n-1})^2}
\]  

(3)

where \( x_{m,n} \) is the grayscale of the pixel \((m, n)\).

Total Variation (TV) refers to the summation of all pixel values in the gradient image.

\[
x_{TV} = \sum_{m,n} |\nabla x_{m,n}| = \sum_{m,n} \sqrt{(x_{m,n} - x_{m-1,n})^2 + (x_{m,n} - x_{m,n-1})^2}
\]  

(4)

Mathematically, the TV-based image reconstruction is equivalent to the following constrained minimization problem:

\[
\min_{x} x_{TV}, \quad s.t. \quad \omega x = p
\]  

(5)

2.3 Golden Ratio scan strategy

For computed tomography, a large number of projections are acquired at many different projecting angles. Usually, these projections are scanned sequentially with a constant angle step, a degree for instance. The difference is that Golden Ratio scan strategy uses the Golden Ratio \( \varphi = \frac{\sqrt{5} - 1}{2} \) to determine the projecting angle series:

\[
\theta_i = \text{mod}(i \varphi \pi, \pi) \quad (i = 0, 1, 2 \ldots)
\]  

(6)

With this strategy, every two contiguous projections are nearly orthogonally separated in angular space, and every chronological contiguous subsequence of the projection data represents a complete coverage of the scan arc \([0, \pi]\), so that obtaining a great flexibility to balance between spatial and temporal resolution. Reconstruction with more projections has a lower temporal resolution and vice versa.

2.4 PICCS
In dynamic tomography, every slice of the sample is sequentially scanned many times to reconstruct each frame of the dynamic process. If we acquire the projection data with Golden Ratio scan strategy, we can easily balance between spatial and temporal resolution. In the image reconstructed from sufficient projections satisfying sampling condition, dynamic information is blurred, but static information is reconstructed without under-sampling artefacts. This low-temporal resolution image can serve as a prior image to regularize the sparse-view/high-temporal resolution reconstruction. PICCS algorithm has been reported to realize a priori image-constrained sparse-view reconstruction. Mathematically, it’s equivalent to the following constrained minimization problem

$$\min_x \left[ \alpha (x - x_{\text{prior}})_{TV} + (1 - \alpha) x_{TV} \right], \quad \text{s.t.} \quad \omega x = p$$  \hspace{1cm} (7)

where $\alpha$ is a weighting parameter to balance the constraint from the target image and the subtraction image, and it’s set in the range of $[0,1]$. Particularly, the PICCS algorithm will degrade to the TV constrained algorithm when $\alpha$ is set to zero.

2.5 Implementation

Equation (7) was realized in an alternative manner and each iteration consists of two phases: The data fidelity term (on the right side) was firstly solved with the SART algorithm, and then the minimization term was solved with the gradient descent algorithm. In addition, a non-negative constraint was embedded between the two phases. The pseudocode of the algorithm is:

1. Initialization: $x^0 = 0, n = 1$
2. Solving the data fidelity term:

$$x_{j,n} = x_{j,n-1} + \frac{1}{y} \sum_{i=1}^{y} \omega_{ij} \left( p_i - \sum_{j=1}^{x} \omega_{ij} g_{j,n-1} \right) \quad \left( j = 1, 2, 3, \ldots, N \right)$$

3. Non-negative constraint

$$x^n = \begin{cases} x^n & \left( x^n > 0 \right) \\ 0 & \left( x^n < 0 \right) \end{cases}$$

4. Minimization term:
   a. Initialization: $m = 1, x^{0,m} = x^n$
   b. gradient descent algorithm:

$$d_A = \|x^n - x^{n-1}\|_2$$

$$v = \frac{\partial \left[ \alpha (x - x_{\text{prior}})_{TV} + (1 - \alpha) x_{TV} \right]}{\partial x}, \quad \tilde{v} = \frac{v}{\sum v}$$

$$x^{n,m} = x^{n,m-1} - \beta \cdot d_A \cdot \tilde{v}$$

   c. $m = m + 1$
   d. repeat step b,c, until $m > M1$
5. $x^n = x^{n,M1}, n = n + 1$
6. Repeat step 2, 3, 4, 5, until $n > N1$

In the pseudocode, $N1$ and $M1$ correspond to the number of iterations of the overall algorithm and the inner gradient descent algorithm, respectively. $\beta$ is a parameter to control the step length of the gradient descent. $\beta$ and $M1$ combines to effect the strength of the minimization term. The reconstructed image will be over-smoothing if $\beta$
or $M1$ is too large, and the minimization term will become useless if $\beta$ or $M1$ is too small. The balance of these parameters depends on the specific circumstances and is more like an art.

3. Simulation

Firstly, we designed a linear dynamic model, which combines three typical situations of a dynamic process: The change of size, the change of grayscale, and the change of position. The dynamic model includes 15 time steps with 256-pixel width, and Fig.1 displays some of them.

![Frame No.1,5,9,13 (a-d) of the dynamic model](image)

Fig.1 Frame No.1,5,9,13 (a-d) of the dynamic model with a grayscale display window of $[-0.1,1.0]$, and the image size is $256 \times 256$. As marked in red, the radius of disc No.1 changes from 8 to 22 pixels; the grayscale of disc No.2 changes from 0.1 to 1; the center of disc No.3 and No.4 (with a same radius of 15 pixels) changes 1 pixel for every two contiguous frames, from left to right and from up to down respectively; Part No.5 and No.6 represent the static information.

Secondly, we compared the difference of reconstructions between Golden Ratio scan strategy and sequential scan strategy. In both cases, 180 projections are captured. In addition, dynamic processes with different rate of change were taken into account as well. A larger displacement of disc No.3 during the process of data acquisition represents a faster dynamic process. For convenience, the effect of rotating time is neglected here.

![Comparison of reconstructions with different scan strategies](image)

Fig.2 Comparison of reconstructions with different scan strategies for dynamic processes with different rate of change. The displacement (unit in pixel) of disc No.3 during the process of data acquisition: (a) 15 (b) 6 (c) 3 (d) 2; (1): sequential scan; (2): Golden Ratio scan
Reconstructions using the SART algorithm are shown in Fig.2. Motion artefacts exist in the reconstructions with sequential scan strategy, and the faster the change rate of the dynamic process, the more severe the motion artefacts are. In contrast, the Golden Ratio scan strategy is capable to suppress motion artefacts; though the dynamic information was blurred, the blurring was restricted within the dynamic region and the static information was reconstructed very well.

Next, we compared sparse-view reconstructions with different algorithms. 180 projections were captured with the Golden Ratio scan strategy, and the displacement of disc No.3 during the process of data acquisition is 15 pixels (corresponding to the case of Fig.2(a)). We used the reconstruction from 180 projections as a priori image to constrain sparse-view/high temporal resolution reconstructions. We added 1.4% gaussian noise (corresponding to a photon counting of 5000) to the projection data to test the capability of noise suppressing of the algorithms.

Fig.3 illustrates the reconstructions from the first subsequence of the projection data. Apparently, the comparison indicates that SART has advantages over FBP for sparse-view reconstruction. Nevertheless, in the case of highly under-sampled projections, some important information, such as the low-contrast disc No.2, was beyond recognition for the sake of under-sampling streaking artefacts if no prior information was embedded. On the contrary, the quality of reconstruction was dramatically improved when we embedded a priori image into the algorithm; the frame was reconstructed very well by the PICCS method even with only 12 views (the degree of undersampling is 34).

For quantitative analysis, we reconstructed 15 frames from the total 180 projections, with 12 projections to
reconstruct each frame, and compared the reconstructed diameter of disc No.1, the mean grayscale of disc No.2 (region in the red circle marked in Fig.3) and the center of disc No.3 to the reference value. Fig.4 displays the comparison and shows that the quantitative information of the dynamic model was nearly accurately reconstructed with the PICCS method.

![Fig.4 Comparison of reconstructed quantitative information to the reference value, (a) the diameter of disc No.1 (b) the mean greyscale of disc No.2 (c) the center of disc No.3. The blue and red line coincide in Fig.4a and the reconstructed center of disc No.3 has an error of 1 pixel.](image)

**4. Experiment**

The dynamic process of water uptake from a piece of wood composite was studied to evaluate the feasibility of the PICCS method aimed at improving the poor temporal resolution of neutron tomography. Totally, 455 projections were captured with Golden Ratio scan strategy at the cold neutron imaging beamline ICON[17] of Paul Scherrer Institut, Switzerland. The neutron flux was 3.1×10^7/cm²/s at the detector position with a collimation ratio of 343, and the exposure time for each projection was 20 s.

We selected a slice of the sample and reconstructed 15 frames from the 455 projections with 30 projections for each, and compared the reconstructions with different algorithms as presented in Fig.5. The reconstructions with FBP and SART have severe streaking artefacts caused by under-sampling. However, the PICCS algorithm suppressed these artefacts noticeably. The reference image reconstructed from 455 projections using the SART algorithm was also displayed in Fig.5 (d) for comparison. The image quality reconstructed from only 30 projections (the degree of undersampling is 32) is reasonably acceptable.

![Fig.5 First frame of the dynamic process reconstructed from 30 views with different algorithms (a) FBP (b) SART (c) PICCS (d) reconstruction from 455 projections using SART algorithm (the display window for the greyscales is [0.002, 0.015])](image)

Fig.6 presents some frames reconstructed with PICCS algorithm. From the frame sequences, we can recognise the dynamic water distribution in this slice and thus know how water was uptaken dynamically by the wood composite. For instance, the sign that the black hole in red circle No.1 and No.2 (marked in Fig.6 (a)) gradually shrunk as time went on indicates that water gradually filled this empty part of wood; the grayscale in red circle No.3 increased and indicates that water went through this part of the slice. All these dynamic information with this level of temporal resolution will be lost if we use standard neutron tomography reconstruction.
Fig. 6 Reconstructed frame sequences of the dynamic process, (a)-(h) corresponds to frame No.1, 3, 5, 7, 9, 11, 13, 15 (the display window for the greyscales is [0.002, 0.015])

5. Summary and outlook

The Golden Ratio scan strategy is capable to notably suppress motion artefacts and flexible to balance between spatial and temporal resolution during data processing. A low-temporal resolution image reconstructed from sufficient projections can serve as a priori image to regularize the sparse-view/high-temporal resolution reconstruction.

With the Golden Ratio scan strategy and PICCS algorithm, we proved that sparse-view reconstruction is practical to improve the temporal resolution of neutron tomography with both simulation and experiment. This effort will extend the capability of neutron tomography to study faster processes.

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