Performance Analysis of a Decode-and-Forward Based Mixed RF–FSO–VLC System

Rima Deka1 · Sanya Anees1

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Abstract
In this paper, the performance analysis of a cooperative radio frequency-free space optical communication-visible-light-communication (RF–FSO–VLC) system using decode-and-forward (DF) relays is presented. In the proposed model, the RF link acts as the core network, the FSO link is used for providing last mile connectivity for the indoor cell users communicating through VLC environment. The RF link is characterized by Nakagami-$m$-distributed fading, while the FSO link undergoes double generalized gamma distributed atmospheric turbulence and pointing errors and the VLC link is characterized by the Lambertian pattern of the LED. The signal at each DF relay is decoded, re-encoded and again decoded before sending to the next relay. The signal to the first relay is sent by an RF link. The received signal is then decoded, re-encoded and again decoded to send to the second relay via a FSO link. From the second relay the signal is transmitted to the end users present in the indoor environment via an VLC link. The closed-form expressions of the statistical characteristics of signal-to-noise-ratio of the DF based triple-hop system are evaluated using which various performance metrics, i.e., outage probability, bit error rate (BER), and capacity of the considered system are obtained. The numerical results show that the system’s performance gets severely affected by the RF fading, atmospheric turbulence, and pointing errors. The outage performance of the system decreases as field-of-view and the height of the light-emitting-diode (LED) increase. It is also seen from the results that the BER performance of the system is better in case of heterodyne detection as compared to direct-detection. It is also observed that the capacity of the system decreases when the effect of atmospheric turbulence and pointing error is high and it increases when the no. of LEDs increases.

Keywords Free space optics · Visible light communication · Radio frequency · Decode-and-forward · Outage probability · Bit error rate · Capacity

1 Introduction

Optical Wireless Communication (OWC) offers wireless connectivity using infra-red (IR), visible or ultra-violet bands and enables features like high bandwidth, low cost, and operation in license free spectrum [1]. The OWC is a promising communication technology which incorporates benefits of both wireless and optical communication. OWC systems can also be installed on those locations where wired communication is not feasible, providing last mile access and backhaul connectivity [2]. Apart from contributing a large and unlicensed band in comparison to radio spectrum, the VLC technology is clustered with numerous other benefits like availability, no electromagnetic interference, high bandwidth, low power consumption etc.

Visible light communication (VLC) carries information by modulating optical signal lying in the visible spectrum (400-700 nm) that is mainly used for illumination. The VLC systems often employ light emitting diodes (LEDs), for both illumination and data communications [3, 4] by replacing the florescent lamps. Laser diodes also can be used as a source for VLC system to transmit information. In [5], the effect of a rapidly changing driving current, due to data modulation, on the emitted light quality of illumination LEDs is experimentally investigated.

Sanya Anees has contributed equally to this work.

Rima Deka
rima.deka@iiitg.ac.in

Sanya Anees
sanya@iiitg.ac.in

1 ECE, Indian Institute of Information Technology Guwahati, Bongora, Guwahati, Assam 781015, India
In [6], the development of such VLC links for implementation in broadcasting networks featuring advanced modulation formats such as orthogonal frequency division multiplexing (OFDM) or carrier-less amplitude and phase modulation (CAP) in conjunction with equalization techniques are discussed. An adaptive MIMO-OFDM based VLC system is discussed in [7] where, spatial dimension is considered as an adaptation parameter. It discusses a joint MIMO mode selection and bit loading scheme to maximize the spectral efficiency (SE) while satisfying a given bit-error rate (BER) target. The performance analysis of non-orthogonal multiple access (NOMA) in a downlink VLC system is discussed in [8] where analytical expression of the system coverage probability and a closed-form expression of the ergodic sum rate applicable for arbitrary power allocation strategies are derived.

Free space Optical communication (FSO) represents terrestrial point-to-point OWC using lasers and photodetectors to carry out transmission in the IR frequency band. The applications of FSO include short-distance interconnects on integrated circuits, indoor communications, terrestrial outdoor links, and under water optical communication links [9–15]. The performance of an FSO system is mostly dependent on the weather conditions, path loss, atmospheric turbulence, and pointing errors. In order to investigate the effect of atmospheric turbulence in FSO link, various statistical channel models have been proposed in different research studies. Among them Log-Normal, Gamma-Gamma, and Negative exponential models are in high compliance with experimental results for weak, moderate to strong, and saturate atmospheric turbulence regime, respectively [16]. However recently proposed Malaga and Double Generalized Gamma (DGG) atmospheric turbulence models have been proposed in [17] and [18] can be used for entire turbulence regime, respectively. The FSO link also suffers from pointing errors, which arise by the misalignment between the transmitter and the receiver, caused by the mechanical vibrations present in the system due to wind, earthquakes, thermal expansion etc. [19].

Some solutions of reducing the effects of atmospheric turbulence and pointing errors proposed in the literature are radio-on-FSO system [12, 20–24], hybrid RF/FSO communication system [25–30] multiple-input-multiple-output (MIMO) system [31, 32], and FSO based cooperative communication system [33–35]. By applying different types of relaying techniques, the coverage area of the FSO systems can be increased and also improving the performance and capacity of the system. In [36] and [37], the performance analysis in terms of outage probability, BER, and capacity of a dual-hop asymmetric RF/FSO communication system is performed using amplify and forward (AF) and decode-and-forward (DF) relays, respectively, where, the RF link experiences Nakagami-\(m\) fading and FSO link experiences Gamma-Gamma turbulence. Analysis of a secure dual-hop mixed RF–FSO down link for simultaneous wireless information and power transfer system is presented in [38]. A triple-hop DF based mixed RF/FSO/RF system is analyzed in [39], where terrestrial stations communicate with the relays over RF links and the relays communicate with each other over an FSO link. The RF links are characterized by Nakagami-\(m\) fading and the FSO link is characterized by path loss, DGG distributed turbulence, and pointing errors. The analysis of indoor relay assisted hybrid RF/VLC system in order to improve the coverage of the wireless system is proposed in [40]. The expressions for the outage probability for a predetermined threshold signal-to-noise-ratio (SNR) and BER performance using pulse position modulation method are obtained. In [41], the outage and the BER performance of the hybrid VLC-RF system under DF and AF schemes are studied. In [42], a cooperative VLC system where an intermediate light source acts as a relay terminal is discussed. The performance evaluations demonstrate the superiority of full-duplex relaying over half-duplex counterpart especially for high modulation sizes. A hybrid system with multiple VLC access points (APs) and RF APs is designed and compared in [43]. In indoor environments, VLC APs can provide very high data rates whilst satisfying illumination requirements, and RF APs can offer ubiquitous coverage. It is assumed that the VLC system resources are fixed, and the study quantifies the spectrum and power requirements for a RF system, which after introduction to the VLC system, achieves better per user rate performance.

In this work, the performance analysis of a DF based triple-hop RF–FSO–VLC cooperative system is presented. Since FSO and VLC technologies can be easily integrated with the existing RF technology, this proves as a boon of using the advantages of OWC systems in synergy with existing RF systems. The proposed system provides connectivity between the existing backbone RF network to the end users by the last mile FSO–VLC communication link. VLC uses LEDs for data transmission along with room illumination ensuring safety to human health without extra cost making VLC ideal candidate for indoor communication. The multiple LEDs are mounted on the ceiling of a room to transmit the data to the end users present in the room. The LEDs are uniformly distributed over the coverage area of the room. However the indoor VLC system must be connected to the outdoor network to establish communication. FSO has garnered significant attention as last mile access solution providing flexibility over optical fibers in terms of deployment and installation specially in topographically challenging locations. For example, in [44], authors have experimentally investigated cascaded FSO/VLC communication link with data aggregation and distribution. In [45], authors have statistically analysed the FSO/VLC link and its outage and BER performance under various channel conditions. In
[12], the performance for RF–FSO–VLC system for outage probability and BER for binary modulation technique considering erroneous detection by the relays are discussed. The different cascaded systems present in the literature are consist of either RF–FSO system or FSO–VLC system with different channel models in comparison to our system. In the FSO–VLC cascaded system, we can see the performance analysis in terms of outage probability and BER considering binary modulation technique for erroneous relaying. We have considered the most recent and generalized channel models for RF, FSO, and VLC link in this study. With this combination we have discussed the performance analysis considering all the modulation techniques. Also the ergodic capacity expression is derived and numerical results are carried out for it. The impact of the different channel parameters on the results of our system is also discussed in our study. In this study, the relays use the subcarrier-intensity modulation (SIM) technique to transmit the optical signals and the receiving nodes uses either heterodyne or intensity modulation/direct detection (IM/DD) technique. The RF link experiences Nakagami-\(m\) fading, the FSO link is modeled by DGG turbulence, path loss, and pointing errors, and VLC links are characterized by Lambertian pattern in the considered cooperative system. Our main contributions are as follows: (1) closed-form mathematical expressions for various statistical characteristics of SNR of the considered system, i.e, cumulative distribution function (CDF) and probability density function (PDF), (2) analytical expressions of the outage probability, BER for different modulation schemes, and the ergodic capacity of the considered system, and (3) study of the effect of the system and channel parameters on the performance metrics of the considered cascaded system.

2 System Model

A DF based triple-hop RF–FSO–VLC cooperative system is considered in this paper. The relay \(R_1\) receives the electrical signal transmitted by source (S) and decodes it as shown in Fig. 1. The signal received at \(R_1\) can be expressed as

\[
y_{x,r_1} = h_{x,r_1}s + e_{x,r_1},
\]

where, \(h_{x,r_1}\) represents Nakagami-\(m\) distributed channel gain of the RF link, \(s\) denotes the signal transmitted by source \(S\), \(e_{x,r_1}\) is the complex valued additive white gaussian noise (AWGN) with zero mean and \(\sigma_{x,r_1}^2\) variance. The SNR of the link is \(\gamma_{x,r_1} = |h_{x,r_1}|^2 \tilde{\gamma}_{x,r_1}\), where, \(\tilde{\gamma}_{x,r_1} = \frac{E_x}{\sigma_{x,r_1}^2}\) is the average SNR of the RF link and \(E_{x,r_1}\) is the transmitted signal power and the path loss is considered to be 1. \(R_1\) re-encodes the signal, then converts it to its corresponding optical signal thereby forwarding it to \(R_2\) over the FSO link by using SIM scheme. In SIM technique, the RF subcarriers are modulated based on the message signal, then with proper DC bias, the intensity of the optical source is altered. That is why optical signal can be modulated by any of the RF modulation schemes. At \(R_2\), the relay receives the optical signal from the FSO link and decode the corresponding electrical signal and converts it to an optical signal and then transmits it through visible light environment to the end users. The signal received at \(R_2\) is given by

\[
y_{r_{1,r}} = M_{r_{1,r}}y_{x,r_1}r + e_{r_{1,r}},
\]

where, \(\tilde{y}_{r_{1,r}}\) denotes the estimated signal at \(R_2\), \(M_{r_{1,r}}\) represents photo electronic conversion ratio of \(R_1\)-\(R_2\) link, \(e_{r_{1,r}}\) is the complex AWGN with zero mean and \(\sigma_{r_{1,r}}^2\) variance, and \(I_{r_{1,r}} = I_{r_1}I_{r_2}\) denotes the real valued irradiance of \(R_1\)-\(R_2\) links, in which \(I_{r_1}\) stands for the effect of path loss and is normalized to 1, \(I_{r_2}\) stands for the turbulence effect modeled by DGG, and \(I_{r_2}\) stands for the effect caused by pointing error, is considered to be Rayleigh distributed [45]. The instantaneous SNR of the considered link, \(\gamma_{r_{1,r}} = \tilde{M}_{r_{1,r}}\mu_{r_{1,r}}\). The average SNR, \(\tilde{\gamma}_{r_{1,r}} = \frac{E[r]}{E[I]}\mu_{r_{1,r}}\), where, the average electrical SNR is defined as, \(\mu_{r_{1,r}} = \frac{\alpha M [\text{dBm}]}{N_0 r}\), \(r\) denotes the type of detection method, i.e., for heterodyne detection, \(r = 1\) and for IM/DD, \(r = 2\). At the destination (D), the photodetector converts the optical signal into the electrical signal through heterodyne or IM/DD technique. The signal received at destination D is given by

\[
y_{r_{2,d}} = p h_d P x + e_{r_{2,d}},
\]

where, \(h_d\) is the DC channel gain of the LOS link between LED and \(t\)-th user, \(p\) is electrical to optical conversion ratio of \(R_2\)-D link, \(x\) is the decoded signal at D, \(P\), is the relay transmit power and \(e_{r_{2,d}}\) represents the complex AWGN with zero mean and variance \(\sigma_{r_{2,d}}^2\). The instantaneous SNR is given by \(\gamma_{r_{2,d}} = \tilde{h}_d^2 \tilde{\gamma}_{r_{2,d}}\), in which average electrical SNR is defined as, \(\tilde{\gamma}_{r_{2,d}} = \rho^2 P^2/\sigma_{r_{2,d}}^2\).
3 Channel Model

In this section, the channel models used for VLC and FSO links along with the various factors affecting them.

3.1 RF Link

The RF link is assumed to undergo Nakagami-\( m \) distributed fading, therefore, the instantaneous SNR of the RF link will be Gamma distributed with PDF given by [37]

\[
    f_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)} e^{-m/\gamma}, \quad (4)
\]

where, \( \gamma \) is the instantaneous SNR, \( \Gamma(\cdot) \) is a Gamma function, \( \bar{\gamma} \) is the average SNR of RF link, and \( m \) is the Nakagami parameter. The CDF can be obtained as

\[
    F_{\gamma}(\gamma) = \frac{1}{\Gamma(m)} \gamma \left( m, \frac{m}{\bar{\gamma}} \right), \quad (5)
\]

where, \( \gamma(s,x) = \int_0^x e^{-t} e^{-s} dt \) is the lower incomplete Gamma function.

3.2 FSO Link

As the FSO link is characterized by atmospheric turbulence and pointing errors, the PDF of the instantaneous SNR of FSO link is characterized by DGG turbulence and Rayleigh distributed pointing errors. The unified PDF of SNR of FSO link for both type of detection techniques, heterodyne detection and IM/DD, is written as [45]

\[
    f_{\gamma}(\gamma) = C_1 f_{\gamma}^{U}(\gamma) + C_2 f_{\gamma}^{L}(\gamma), \quad (6)
\]

where, \( C_1 = \frac{e^2 \sigma_\beta \tau^{-3} \beta_1 \beta_2 \alpha_1 \alpha_2}{2 \Gamma(1/2) \Gamma(\beta_1) \Gamma(\beta_2)} \), \( C_2 = \frac{e^2 \sigma_\beta \tau^{-3} \beta_1 \beta_2}{2 \Gamma(1/2) \Gamma(\beta_1) \Gamma(\beta_2)} \), and \( \beta_1 \) and \( \beta_2 \) are shaping parameters defining the turbulence-induced fading while \( \alpha_1, \alpha_2, \Omega_1, \Omega_2 \) are identified using the variance of the small and large scale fluctuations, \( \lambda \) and \( \sigma \) are positive integers such that \( \lambda = 2^{\sigma} \frac{\Omega_1}{\alpha_1}, \frac{\Omega_1}{\alpha_2}, \Delta(\sigma : \beta_1), \Delta(\lambda : \beta_2), \)

\[
    K_2 = \frac{\alpha_2 \lambda + \sigma}{\alpha_2}, \quad h = \frac{C_1 B_1}{(1+e^2)C_2^2}, \quad B_1 = \prod_{i=1}^{\sigma} \Gamma \left( \frac{1}{\alpha_1 \beta_1} + K_{n,1} \right), \quad K_{n,1}
\]

is the \( n \)-th term of \( K_{n,1} \), \( \Delta(\chi : y) = \frac{1}{\chi}, \frac{\chi+1}{\chi}, ..., \frac{\chi+y-1}{\chi} \). Now, the CDF of the SNR of the FSO link is given by

\[
    F_{\gamma}(\gamma) = \frac{e^2 \sigma_\beta \tau^{-3} \beta_1 \beta_2}{2 \Gamma(1/2) \Gamma(\beta_1) \Gamma(\beta_2)} \times \frac{G_{\gamma}^{\alpha_2,1,\alpha_2+1}}{\Gamma(\gamma) \Gamma(\beta_1) \Gamma(\beta_2)} \left( c \left( \frac{\gamma}{\mu}, \frac{\alpha_2}{\beta_1}, 1, k_3 \right) \right), \quad (7)
\]

where, \( c = r(\lambda + \sigma + 1), \quad k_3 = \Delta(\gamma : K_2), \quad \) and \( k_4 = \Delta(\gamma : K_1) \).

3.3 VLC Link

The VLC model is used for indoor cell connectivity, where the LED transmitter is placed at height \( L \), from the \( m_{th} \) end user, the angle of incidence \( \Psi \), with the angle of irradiance, \( \theta_e \), \( r_e \) is the maximum radius of a LED cell footprint, \( \phi_e \) is the FOV of the photodiode. The Euclidian distance \( d_t \) for the \( t_{th} \) user as shown in Fig. 2 is given by \( d_t = (r_t^2 + L^2)^{1/2} \) and \( \cos(\Psi) = \cos(\phi) = L/d_t \).

The PDF of channel gain can be derived using the change of variable method and is given by [8]

\[
    f_{\gamma}(h) = \frac{2}{r_t^2(m_t + 3)} e^2(m_t + 1) 2^{m_t+1} \Gamma(2/m_t+3)^{-1}, \quad (8)
\]

where, \( h_t = \frac{\alpha e m_t + 1}{r_t^2 + l^2 + 1}, r_t \) is the radius on the polar coordinate plane, \( \alpha = \frac{1}{2} A R_p U(\Psi), g(\Psi), A, \) is the photodetector area, \( R_0(\theta_e), \) is the Lambertian radiant intensity of the transmitting LED and is expressed as \( R_0(\theta_e) = \frac{\alpha e m_t + 1}{r_t^2 + l^2 + 1} \cos(\theta_e), U(\Psi) \) is the gain of the optical filter, \( g(\Psi) \) is the gain of optical concentrator and given as

\[
    g(\Psi) = \begin{cases} 
        \frac{n}{\sin^2 \phi_e} & \text{if } 0 \leq \Psi \leq \phi_e, \\
        0 & \text{if } \Psi > \phi_e.
    \end{cases}
\]

and, \( n \) denotes refractive index, \( m_t \) is the order of Lambertian emission defined as \( m_t = \frac{\alpha e}{n^2} (\cos(\theta_e/2)), \) with \( \theta_e/2 \), denoting the transmitter semi-angle at half power. The PDF of the instantaneous SNR, \( \gamma_v = \gamma^2 h_t^2 \), is given in [8] as
\[ f_{\gamma V}(\gamma) = \frac{1}{\gamma V} \frac{1}{r_e^2} \frac{1}{(m+1)} L^{m+1} \gamma^{m+1} \gamma^{-m+3}, \]  
(9)

where, \( \gamma V = \frac{P}{N_0 B} \), \( P \) is transmitted optical power, \( N_0 \) being noise spectral density and \( B \) is the base-band modulation bandwidth. The CDF is given by

\[ F_{\gamma V}(\gamma) = \frac{1}{r_e^2} (m+1) L^{m+1} \gamma^{2/m+3} \left( \frac{\gamma}{\gamma V} \right) + \left( 1 + \frac{L^2}{r_e^2} \right), \]  
(10)

Now, assuming that all VLC links are independent and identical, the CDF of the SNR for best possible VLC link is given by [8]

\[ F_{\gamma v}(\gamma) = \frac{1}{r_e^2} (m+1) L^{m+1} \gamma^{2/m+3} \left( \frac{\gamma}{\gamma V} \right) + \left( 1 + \frac{L^2}{r_e^2} \right)^N \]  
(11)

Now, by applying Binomial approximation \((a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k\) in (11) and rearranging the terms, the CDF of the SNR of the VLC link is derived as [8]

\[ F_{\gamma V}(\gamma) = \gamma V - N_1 N^{-1} \frac{r_e^2}{r_e^2} (m+1) L^{m+1} \gamma^{2/m+3}, \]  
(12)

where, \( \gamma V = (1 + \frac{L^2}{r_e^2}), \chi = \frac{1}{r_e^2} w(m+1) L^{m+1} \gamma^{2/m+3} \), and \( N \) is the number of VLC access points.

4 Statistical Characteristics of Instantaneous SNR of the DF Based Triple-Hop RF–FSO–VLC System

4.1 Cumulative Distribution Function

For the DF based mixed RF–FSO–VLC cooperative system, the end-to-end SNR is defined as [37],

\[ \gamma_{eq} = \min(\gamma_R, \gamma_F, \gamma_V), \]  
(13)

where, \( \gamma_R \) is the instantaneous SNR of the RF link, \( \gamma_F \) is the instantaneous SNR of the FSO link, and \( \gamma_V \) is the instantaneous SNR of the VLC link. The CDF of the end-to-end system is defined as,

\[ F_{\gamma_{eq}}(\gamma) = 1 - [(1 - F_{\gamma R}(\gamma) ) \times (1 - F_{\gamma F}(\gamma))(1 - F_{\gamma V}(\gamma))]. \]  
(14)

By substituting (5), (7), and (12) in (14), the closed-form expression for the CDF of instantaneous SNR of the considered system is obtained as

\[ F_{\gamma_{eq}}(\gamma) = 1 - \left( 1 - \frac{1}{m} \frac{m \gamma}{\gamma V} \right) \times \left( 1 - \frac{c^2 \sigma_\gamma - 0.5 \beta_\gamma - 0.5 (2\pi)^{1-\frac{2(m+1)}{m+3}} \beta_\gamma + \beta_\gamma^2 \right) \times G^{n,1}_{r+1,a+1} \left( \gamma \frac{\gamma}{\mu_r} \frac{\alpha_r}{\gamma V} \frac{\gamma}{\mu_r} \frac{\gamma}{\mu_r} \right) \times \left( 1 - \gamma V + N_1 N^{-1} \left( \frac{r_e^2}{r_e^2} \right) \right). \]  
(15)

4.2 Probability Density Function

From (14), the PDF of the instantaneous SNR for the DF based triple-hop system can be obtained by differentiating (14) and then by substituting PDF and CDF of the instantaneous SNR of the corresponding RF, FSO, and VLC links as

\[ f_{\gamma_{eq}}(\gamma) = \frac{m \gamma}{\gamma V} \left( 1 - \frac{m \gamma}{\gamma V} \right) \left( 1 - \frac{m \gamma}{\gamma V} \right) \times \left( 1 - \frac{c^2 \sigma_\gamma - 0.5 \beta_\gamma - 0.5 (2\pi)^{1-\frac{2(m+1)}{m+3}} \beta_\gamma + \beta_\gamma^2 \right) \times G^{n,1}_{r+1,a+1} \left( \gamma \frac{\gamma}{\mu_r} \frac{\alpha_r}{\gamma V} \frac{\gamma}{\mu_r} \frac{\gamma}{\mu_r} \right) \times \left( 1 - \gamma V + N_1 N^{-1} \left( \frac{r_e^2}{r_e^2} \right) \right). \]  
(16)

5 Performance Analysis

In this section, using the expressions of statistical characteristics of the SNR of the considered system discussed in previous section, the performance of the DF based cascaded RF–FSO–VLC communication system is evaluated.

5.1 Outage Probability

The considered RF–FSO–VLC system will be in outage if system’s SNR falls below predetermined threshold value.
The outage probability, \( P_{\text{out}} \), of a communication system is defined as follows:

\[
P_{\text{out}} = P_{\gamma < \gamma_{th}} = F_{\gamma}(\gamma_{th}).
\]  

(17)

Using (14), the outage probability for the considered cooperative system is obtained as:

\[
P_{\text{out}} = 1 - \left( 1 - \frac{1}{m} \gamma \left( m, \frac{\gamma_{th}}{\gamma_{th}} \right) \right) \times \left( 1 - \frac{e^{2\sigma_{\theta} - 0.5 \beta_2 - 0.3(2\pi)^{1-\frac{\nu}{2}} \rho_{\theta} + \beta - 2}}{\sqrt{2\pi}} \right)
\times G_{r+1, m+1}^{\nu, 1} \left( \frac{\gamma_{th}}{\mu_r} x \right) \left( 1, k, 0 \right) \times \left( 1 - v_{1}^{N} + N_{r}^{N-1} x \left( \frac{\gamma_{th}}{\gamma_{th}} \right)^{-1/m+3} \right).
\]

(18)

The outage probability of the considered system depends on the fading parameter of the RF link 'm', atmospheric turbulence parameters '\( \beta_1 \)' and '\( \beta_2 \)', and pointing errors of the FSO link, and different VLC parameters like FOV of the photodetector, height of the LED's 'L', no of LED's 'N'.

### 5.2 Bit Error Rate

In this subsection, the BER of the DF based triple-hop RF–FSO–VLC system is derived.

#### 5.2.1 Binary Modulation Techniques

The generalized expression of average BER for various binary digital modulation techniques can be expressed in as [46]

\[
P_e = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma)F_{\gamma}(\gamma)\,d\gamma.
\]  

(19)

where, \( p \) and \( q \) are the BER modulations parameters describing various binary modulation techniques as shown in Table 1. Substituting \( F_{\gamma}(\gamma) \) from (14) in (19), the expression for the BER of the RF–FSO–VLC system is given by

\[
P_{e,\text{BPSK}} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \times \left[ 1 - \left( (1 - F_{\gamma}(\gamma))(1 - F_{\gamma}(\gamma))(1 - F_{\gamma}(\gamma)) \right) \right] \,d\gamma.
\]  

(20)

By substituting \( F_{\gamma}(\gamma) \), \( F_{\eta}(\gamma) \), and \( F_{\varepsilon}(\gamma) \) from (5), (7), and (12) in (20) and using the series expansion of the lower incomplete Gamma function, i.e., \( \gamma(m, \frac{\nu}{\gamma}) \) and considering the fact that the finite summation of incomplete gamma function (from 0 to \( m - 1 \)) is valid for integer values of \( m \), the expression of BER can be written as

\[
P_{e,\text{BPSK}} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \times [1 - (R_{1} - R_{2}F_{\gamma}(\gamma) - R_{3}F_{\eta}(\gamma)F_{\varepsilon}(\gamma))d\gamma.
\]  

(21)

where, \( R_{1} = \exp (-\frac{\nu}{\gamma}) \sum_{k=0}^{m-1} \left( \frac{\nu}{\gamma} \right)^{k} \). By simplifying (21), the BER is given by

\[
P_{e,\text{BPSK}} = I_{1} + I_{2} + I_{3} + I_{4} - I_{5},
\]  

(22)

where,

\[
I_{1} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \,d\gamma,
\]

\[
I_{2} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \,d\gamma,
\]

\[
I_{3} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \,d\gamma,
\]

\[
I_{4} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \,d\gamma,
\]

\[
I_{5} = \frac{q^{p}}{2\Gamma(p)} \int_{0}^{\infty} \gamma^{p-1} \exp(-q\gamma) \,d\gamma.
\]

\[
I_{1} \text{ and } I_{2} \text{ are calculated as }
\]

\[
I_{1} = \frac{1}{2}.
\]  

(24)

and

\[
I_{2} = \sum_{k=0}^{m-1} R_{2} \left( \frac{\nu}{m + q\gamma} \right)^{(p+k)} \Gamma(p + k),
\]  

(25)

where, \( R_{2} = \frac{q^{p}}{2\Gamma(p)} \). By substituting (5) and (12) in (23), and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.0013.01)], \( I_{3} \) can be derived as

\[
I_{3} = \sum_{k=0}^{m-1} \left( R_{2} \gamma_{1}^{N} \left( \frac{\nu}{q\gamma + m} \right)^{k} \Gamma(p + k) - \sum_{k=0}^{m-1} R_{2} \right) \times \left( \frac{\nu}{q\gamma + m} \right)^{-\frac{n}{n+1}} \Gamma(p + k - \frac{1}{m_{1} + 3})
\]  

(26)
where, \( R_3 = N_0 \alpha N^{-1} \chi \left( \frac{1}{\gamma} \right)^{-\frac{1}{\alpha}} \). The integral of \( I_4 \) can be derived by putting (5) and (7) in (23), and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.0013.01)] as

\[
I_4 = C_1 \sum_{k=0}^{m-1} R_2 W_1^{(p+k)} \left( \frac{v}{W_1 \mu_r} \right)^y \left( \frac{1-(p+k)}{k_4, 0} \right).
\]

(27)

where, \( W_1 = -q + \frac{m}{\gamma} \). The integral of \( I_5 \) can be expressed by putting (5), (7), and (12) in (23), and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.0013.01)] as

\[
I_5 = C_1 \sum_{k=0}^{m-1} R_2 v^y W_1^{(p+k)} \left( \frac{v}{W_1 \mu_r} \right)^y \left( \frac{1-(p+k)}{k_4, 0} \right)
\]

\[
\times G_{r+1+1}^{u, 1+1} \left( \frac{v}{W_1 \mu_r} \right)^y \left( \frac{1-(p+k)}{k_4, 0} \right).
\]

(28)

where, \( R_4 = (-q + \frac{m}{\gamma})^{(p+k-\frac{1}{\alpha})} \) and \( R_5 = v^{\frac{m}{\gamma}} \). Now, by substituting (24), (25), (26), (27), and (28) in (21), the closed-form expression of BER is obtained as

\[
P_{e, \text{BPSK}} = \frac{1}{2} \left( 1 - \sqrt{\gamma} \right) \sum_{k=0}^{m-1} R_2 \left( \frac{\sqrt{\gamma}}{m+q\sqrt{\gamma}} \right)^{(p+k)}
\]

\[
\times \Gamma(p+k) - R_3 \left( \frac{\gamma}{q\gamma + m} \right)^{\frac{1}{\alpha}} \sum_{k=0}^{m-1} R_2 \left( \frac{\sqrt{\gamma}}{m+q\sqrt{\gamma}} \right)^{(p+k)}
\]

\[
\times \Gamma(p+k) - R_3 \left( \frac{\gamma}{q\gamma + m} \right)^{\frac{1}{\alpha}} \sum_{k=0}^{m-1} R_2
\]

\[
\times G_{r+1+1}^{u, 1+1} \left( \frac{v}{W_1 \mu_r} \right)^y \left( \frac{1-(p+k)}{k_4, 0} \right)
\]

\[
\times \left( \frac{v}{W_1 \mu_r} \right)^y \left( \frac{1-(p+k)}{k_4, 0} \right).
\]

(29)

5.2.2 MQAM

The generalized expression of the instantaneous BER of the M-QAM modulation scheme can be written as [49]

\[
P_e(\gamma) = 4\gamma M \sum_{j=1}^{\sqrt{2}} Q(b_j \sqrt{\gamma}).
\]

(30)

where, \( \gamma_M = \frac{1}{\log_2 M} (1 - \frac{1}{\sqrt{M}}) \) and \( b_j = (2j - 1) \sqrt{\frac{3}{M-1}} \). The average BER is expressed as,

\[
P_e = \int_0^{\infty} P_e(\gamma) f_\gamma(\gamma) d\gamma = \int_0^{\infty} F_e(\gamma) dP_e(\gamma).
\]

(31)

By substituting the values of \( F_e(\gamma) \) from (14) and (30) in (31), the expression of BER of the RF–FSO–VLC using M-QAM modulation scheme is given by

\[
P_{e, \text{MQAM}} = 4\gamma M \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) d\gamma.
\]

(32)

By simplifying the BER can be derived as

\[
P_{e, \text{MQAM}} = I_6 - I_7 + I_8 + I_9 - I_{10}.
\]

where,

\[
I_6 = 4\gamma M \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) d\gamma.
\]

(33)

\[
I_7 = 4\gamma M R_1 \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) F_{r} (\gamma) d\gamma.
\]

(34)

\[
I_8 = 4\gamma M R_1 \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) F_{r} (\gamma) d\gamma.
\]

(35)

\[
I_9 = 4\gamma M R_1 \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) F_{r} (\gamma) d\gamma.
\]

(36)

\[
I_{10} = 4\gamma M R_1 \sum_{j=1}^{\sqrt{2}} b_j \int_0^{\infty} \gamma^{-0.5} \exp \left( \frac{-b_j^2 \gamma}{2} \right) F_{r} (\gamma) d\gamma.
\]

(37)

\[
I_6 \text{ and } I_7 \text{ are calculated as}
\]

\[
I_6 = 4\gamma M
\]

(38)

and
\[ I_7 = \frac{4\gamma_M}{\sqrt{2\pi}} \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \left( \frac{2\gamma}{2m + \gamma b_j^2} \right)^{0.5+k} \Gamma(0.5 + k). \]  

(35)

By substituting (5) and (12) in (33), \( I_6 \) can be derived as
\[ I_6 = \frac{4\gamma_M}{\sqrt{2\pi}} \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \left( \frac{2\gamma}{2m + \gamma b_j^2} \right)^{0.5+k} \Gamma(0.5 + k) \]
\[ \times \left( \frac{2\gamma}{2m + \gamma b_j^2} \right)^{(0.5-\frac{1}{m+3})} \Gamma\left( \frac{0.5 - \frac{1}{m+3} + k}{} \right). \]

(36)

By substituting (5) and (7) in (33) and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.013.01)], \( I_6 \) can be derived as
\[ I_6 = C_1 \frac{4\gamma_M}{\sqrt{2\pi}} \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \mathcal{P}_1 \]
\[ \times G_{\nu,1+v}^{\nu,1+v} \left( cW_2 \right) \left| \Delta(v : s_1), 1, k_3 \right|, k_4, 0 \]  

(37)

\[ I_{10} = v_i^N C_1 \frac{4\gamma_M}{\sqrt{2\pi}} \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \times \mathcal{P}_1 \]
\[ \times G_{\nu,1+v}^{\nu,1+v} \left( cW_2 \right) \left| \Delta(v : s_1), 1, k_3 \right|, k_4, 0 \]  
\[ - Nv_i^{N-1} \times \left( \frac{1}{\gamma_i} \right)^{-\frac{1}{m+1}} C_1 \frac{4\gamma_M}{\sqrt{2\pi}} \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \times \mathcal{P}_2 \]
\[ \times G_{\nu,1+v}^{\nu,1+v} \left( cW_2 \right) \left| \Delta(v : s_2), 1, k_3 \right|, k_4, 0 \]  

(38)

Now, by substituting (34), (35), (36), (37), and (38) in (32), the closed-form expression of BER for M-QAM modulation technique is derived as
\[ P_{e,MQAM} = 4\gamma_M \sum_{j=1}^{\sqrt{2\pi}} \sum_{k=0}^{m-1} \frac{b_j \left( \frac{m}{\gamma} \right)^k}{k!} \]
\[ \times \left( \frac{2\gamma}{2m + \gamma b_j^2} \right)^{0.5+k} \Gamma(0.5 + k) \left[ 1 - v_i^N + Nv_i^{N-1} \right] \]
\[ \times \mathcal{P}_1 \left[ cW_2 \right] \left| \Delta(v : s_1), 1, k_3 \right|, k_4, 0 \]
\[ \times \mathcal{P}_2 \left[ cW_2 \right] \left| \Delta(v : s_2), 1, k_3 \right|, k_4, 0 \]  

(39)

### 5.2.3 MPSK

The instantaneous BER of the M-PSK modulation scheme can be written as [49]
\[ P_e(\gamma) = \frac{2}{\psi_M} \sum_{j=1}^{T} Q(a_j \sqrt{2\gamma}), \]  

(40)

where, \( \psi_M = \max(\log_2 M, 2), T = \max\left(\frac{M}{4}, 1\right) \) and \( a_j = \sin \left( \frac{(2j-1)\pi}{M} \right) \). By substituting the values of \( F_{\nu_0}(\gamma) \) from (14) and (40) in (31), the expression of BER of the RF–FSO–VLC using M-PSK modulation scheme is given by
\[ P_{e,\text{MPSK}} = \frac{2}{\sqrt{\psi_M}} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) \left( -R_1 - R_2 F_{\nu_0}(\gamma) - R_1 F_{\nu_1}(\gamma) \right) d\gamma. \]  

(41)

By simplifying the BER can be derived as
\[ P_{e,\text{MPSK}} = I_{11} - I_{12} + I_{13} + I_{14} - I_{15}, \]  

(42)

where
\[ I_{11} = \frac{2}{\psi_M} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) d\gamma, \]
\[ I_{12} = \frac{2R_1}{\psi_M} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) d\gamma, \]
\[ I_{13} = \frac{2R_1}{\psi_M} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) F_{\gamma_j} d\gamma, \]
\[ I_{14} = \frac{2R_1}{\psi_M} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) F_{\gamma_j} d\gamma, \]
\[ I_{15} = \frac{2R_1}{\psi_M} \sum_{j=1}^{T} a_j \int_{0}^{\infty} \gamma^{-0.5} \exp(-a_j^2 \gamma) F_{\gamma_j} d\gamma. \]

By substituting (5) and (12) in (43), \( I_{13} \) can be derived as
\[ I_{13} = \nu_i^N \frac{2}{\psi_M} \sum_{j=1}^{T} \sum_{k=0}^{m-1} a_j \left( \frac{m}{\gamma_j} \right)^k \left( \frac{\bar{\gamma}}{m + a_j^2 \bar{\gamma}} \right)^{0.5+k} \times \Gamma(0.5 + k). \]

By substituting (5) and (7) in (43), and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.0013.01)], \( I_{14} \) can be derived as
\[ I_{14} = \frac{2}{\psi_M} \sum_{j=1}^{T} \sum_{k=0}^{m-1} \left( \frac{m}{\gamma_j} \right)^k (a_j \lambda)^k \left( a_j^2 + \frac{m}{\gamma_j} \right)^{0.5+k} \times G_{\nu_i^{1+v}}^{w,+1} \left( c(W_3)^2 \right) \Delta(v \cdot s_j, 1, k_3) \]

where, \( W_3 = \left( \frac{m}{m + a_j^2 \bar{\gamma}} \right) \). The integral of \( I_{15} \) can be expressed by putting (5), (7), and (12) in (43), and using [47, Eq. (8.4.3.1)] and [48, Eq. (07.34.21.0013.01)].

By substituting (44), (45), (46), (47), and (48) in (42), the closed-form expression of BER for M-PSK modulation technique is derived in (49).

5.3 Ergodic Capacity

The ergodic capacity of a system is defined as [50, 51]
\[ C = \mathbb{E} \left[ \log_2 (1 + \delta \gamma) \right] = \int_{0}^{\infty} \log_2 (1 + \delta \gamma) f_{\gamma} (\gamma) d\gamma, \]
where, \( \delta \) is the free parameter which takes value '1' for heterodyne detection and '\( \frac{1}{2\Delta v} \)' for IM/DD detection. The ergodic capacity can be expressed in terms of the complementary CDF as [45]
\[ C = \int_{0}^{\infty} \frac{1}{\ln(2)} (1 + \delta \gamma)^{-1} F_{\gamma}^c (\gamma) d\gamma, \tag{51} \]

where, \( F_{\gamma}^c (\gamma) = 1 - F_{\gamma} (\gamma) \). Putting the values of \( F_{\gamma} (\gamma) \) from (14) the ergodic capacity of the system can be written as,

\[ C = \int_{0}^{\infty} (1 + \delta \lambda)^{-1} (1 - F_{\eta}(\gamma)) d\gamma. \tag{52} \]

By substituting (5), (7), and (12) in (52), the expression for ergodic capacity can be written as,

\[ C = \int_{0}^{\infty} \frac{1}{\ln(2)} (1 + \delta \gamma)^{-1} [R_1 - R_i F_{\gamma}(\gamma) - R_i F_{\gamma}(\gamma) + R_i F_{\gamma}(\gamma) ] d\gamma. \tag{53} \]

\[ = I_{16} - I_{17} - I_{18} + I_{19}, \tag{54} \]

where,

\[ I_{16} = \int_{0}^{\infty} \frac{R_1}{\ln(2)} (1 + \delta \gamma)^{-1} d\gamma. \]

\[ I_{17} = \int_{0}^{\infty} \frac{R_i}{\ln(2)} (1 + \delta \gamma)^{-1} F_{\gamma}(\gamma) d\gamma. \]

\[ I_{18} = \int_{0}^{\infty} \frac{R_i}{\ln(2)} (1 + \delta \gamma)^{-1} F_{\gamma}(\gamma) d\gamma. \]

\[ I_{19} = \int_{0}^{\infty} \frac{R_i}{\ln(2)} (1 + \delta \gamma)^{-1} F_{\gamma}(\gamma) F_{\gamma}(\gamma) d\gamma. \tag{55} \]

By using the identities [48, Eq. (07.34.03.0271.01) and Eq. (07.34.03.0228.01)] to express \((1 + \delta \gamma)^{-1}\) and exponential function in terms of Meijer-G and applying formula [48, Eq. (07.34.21.0011.01)] to integrate \(I_{16}\) can be calculated as,

\[ I_{16} = \sum_{k=0}^{m-1} \frac{\xi^k}{m k! \ln(2)} G_{2,1}^{1,2} \left( \frac{\delta \gamma}{m} \right) \left| \begin{array}{c} 0, -k \ 0 \end{array} \right| \tag{56} \]

The integral of \(I_{17}\) can be derived by putting (12) in \(I_{17}\). Using [48, Eq. (07.34.03.0271.01) and Eq. (07.34.03.0228.01)] for Meijer-G equivalent conversion of \((1 + \delta \gamma)^{-1}\) and exponential function, and using [48, Eq.(07.34.21.0011.01)] for integration of two Meijer-G function, \(I_{17}\) is derived as,

\[ I_{17} = \sum_{k=0}^{m-1} \frac{\xi^k}{m k! \ln(2)} G_{2,1}^{1,2} \left( \frac{\delta \gamma}{m} \right) \left| \begin{array}{c} 0, -k \ 0 \end{array} \right| \]

\[ - \sum_{k=0}^{m-1} \frac{\xi^k}{m k! \ln(2)} N_{\gamma} N_{\gamma}^{-1} \frac{1}{\gamma^m} \]

\[ G_{2,1}^{1,2} \left( \frac{\delta \gamma}{m} \right) \left| \begin{array}{c} 0, -k, -1 \ 0, 0 \end{array} \right|. \tag{57} \]

Now, the closed form expression for \(I_{18}\) can be obtained by using [48, Eq. (07.34.03.0271.01) and Eq. (07.34.03.0228.01)] for equivalent Meijer-G conversion of \((1 + \delta \gamma)^{-1}\) and exponential function, and then converting each Meijer-G term into the Fox-H function [52] using [48, Eq. (07.34.26.0008.01)]. The integral of \(I_{18}\) can be obtained using [53, Eq. (2.3)] in terms of H-function as,

\[ I_{18} = \sum_{k=0}^{m-1} \frac{\xi^k C_{1}}{m k! \ln(2)^{m}} \]

\[ \times H_{0,1:1:1}^{0,1:1:1} \left( (\frac{k}{m})^{r+1}, 1, 1 \right) - \left( 0, 1 \right) \left( s_{3}, s_{4:5} \right) \frac{\delta \gamma}{m} \frac{c_{1}}{\mu, m}, \tag{58} \]

where, \( s_{1} = (k_{5}, 0)^{-1} \), \( s_{4} = (k_{4}, 0)^{-1} \), \( s_{5} = (0, 0)^{-1} \). The integral of \(I_{19}\) can be obtained by using [48, Eq. (07.34.03.0271.01) and Eq. (07.34.03.0228.01)] for equivalent Meijer-G conversion of exponential and \((1 + \delta \gamma)^{-1}\) function and then converting each Meijer-G term in to the Fox-H function [52] using [48, Eq. (07.34.26.0008.01)]. Now, the expression of \(I_{19}\) turns to be in terms of H-function by using [53, Eq. (2.3)] as,

\[ I_{19} = \sum_{k=0}^{m-1} \frac{\xi^k C_{1}}{m k! \ln(2)^{m}} \]

\[ \times H_{0,1:1:1}^{0,1:1:1} \left( (\frac{k}{m})^{r+1}, 1, 1 \right) - \left( 0, 1 \right) \left( s_{3}, s_{4:5} \right) \frac{\delta \gamma}{m} \frac{c_{1}}{\mu, m} \]

\[ - \sum_{k=0}^{m-1} \frac{\xi^k C_{1}}{m k! \ln(2)^{m}} \]

\[ \times H_{0,1:1:1}^{0,1:1:1} \left( (\frac{k}{m})^{r+1}, 1, 1 \right) - \left( 0, 1 \right) \left( s_{3}, s_{4:5} \right) \frac{\delta \gamma}{m} \frac{c_{1}}{\mu, m} \tag{59} \]
Table 2 Simulation parameters and their values

| Parameters | Values |
|------------|--------|
| Strong turbulence ($\alpha_1, \alpha_2, \beta_1, \beta_2, \Omega_1, \Omega_2$) | 1.8621, 1, 0.5, 1.8, 1.5074, 1 |
| Moderate turbulence ($\alpha_1, \alpha_2, \beta_1, \beta_2, \Omega_1, \Omega_2$) | 2.1690, 1, 0.55, 2.35, 1.5793 |
| Pointing errors $\epsilon$ | 1, 5.2 |
| $\theta_{1/2}$ | 60° |
| Nakagami-$m$ | 1.5 |
| FOV | 50, 70, 90 |
| Height of LED ‘L’ | 2.5m, 3.5m |
| Detection technique ‘y’ | 1, 2 |
| No of LED’s ‘q’ | 6, 12 |

\[
C = \sum_{k=0}^{m-1} \frac{1}{k!m(2)^m} \binom{m}{s_1} \binom{m}{s_2} \binom{m}{s_3} \binom{m}{s_4} \binom{m}{s_5} \binom{m}{s_6} \binom{m}{s_7} \binom{m}{s_8} \binom{m}{s_9} 
\]

By substituting the values of (56), (57), (58), and (59) in (52), the closed form expression for ergodic capacity of the considered cooperative system can be obtained (Table 2).

6 Numerical Results

In this section, the numerical results for the outage probability, the average BER, and the ergodic capacity for a DF based triple-hop RF–FSO–VLC system are discussed. Here, both strong and moderate cases of atmospheric turbulence scenarios are considered, i.e., strong and moderate [45]. For the VLC link, transmitter semi-angle at half power $\theta_{1/2}$ is set at 60° and number of used LED luminaries are considered to be 6. The RF link undergoes Nakagami-$m$ fading with parameter $m = 5$ and 5 in this cooperative communication system.

The outage probability of the mixed RF–FSO–VLC system for the heterodyne and direct detection techniques considering the conditions of strong, moderate turbulence for $m = 5$ and no of LED’s are 6 is shown in Fig. 3. It is seen from the figure that the outage probability of the system is
more while using direct compared to heterodyne detection. It is also seen from the figure that the outage probability of the system is higher in the strong turbulence environment as compared to moderate turbulence environment.

The outage probability of this mixed RF–FSO–VLC system for different values of FOV of the photodetector and different heights of the LED luminaries considering \( m = 5 \), and pointing errors \( e = 1 \) for strong turbulence condition is shown in Fig. 4. It is observed from the figure that the outage probability of the system decreases with the reduction in FOV of the photodetector as well as reduction in the height of the LED lights. This is due to the fact that, as FOV decrease, the power of the concentrator of the photodetector will increase, improving the outage performance of the considered cascaded system and a receiver with a wide FOV is susceptible to more ambient light noise as compared to a receiver with a narrow FOV.

Figure 5 shows the outage performance of the considered system for different values of RF fading parameter and pointing errors. This shows that the outage probability of the system increases when the pointing errors effect is high and the effect of RF fading is high. For example, at average SNR = 10 dB and \( m = 5 \), the outage probability of the considered system are \( 3.845 \times 10^{-1} \) and \( 4.633 \times 10^{-1} \), respectively. For \( m = 1.7 \) and \( e = 5.2, 1 \), the outage probability of the considered system are \( 4.24 \times 10^{-1} \) and \( 4.977 \times 10^{-1} \) for \( e = 5.1, 1 \), respectively. Again, for \( m = 1 \) and \( e = 5.2, 1 \), the outage probability of the considered system are \( 5.41 \times 10^{-1} \) and \( 5.997 \times 10^{-1} \) for \( e = 5.1, 1 \), respectively. It is also seen that the outage probability of the system is poor when the value of Nakagami-\( m \) parameter is less (i.e., the effect of RF fading is high) along with the value of pointing error '\( e \)' (the effect of pointing error is high). Also at high SNR we can say it from the figure that the effect of pointing error is more dominating than the effect of nakagami-\( m \) parameter.

but for higher SNR the system gives better outage performance for less value of RF fading parameter and weak

Fig. 5 Outage probability vs Avg. SNR of the DF based triple-hop RF–FSO–VLC system for different values of \( m \) and pointing errors \( e \)

Fig. 6 BER vs Avg. SNR of the DF based triple-hop RF–FSO–VLC system for both heterodyne and IM/DD detection considering different values of atmospheric turbulence

Fig. 7 Avg. BER vs. Avg. SNR of the triple-hop RF–FSO–VLC system for different modulation techniques considering perfect relaying
pointing error as compared to strong pointing error and weak RF fading parameter.

Figure 6 shows that the BER performance of the considered system for heterodyne detection and IM/DD considering different values of atmospheric turbulence, $L = 2.5 \text{ m}$, $m = 5$, pointing error $\epsilon = 1$, and FOV $= 60^\circ$. It is observed from the figure that the heterodyne detection provides a better performance compared to IM/DD. It can also be seen that, for moderate atmospheric turbulence the system provides better error performance.

In Fig. 7, the BER analysis for perfect relaying using different modulation schemes such as 8-QAM, 16-QAM and 16-PSK are presented considering $L = 2.5 \text{ m}$, $\text{FOV} = 60^\circ$, in the strong atmospheric turbulence with a pointing errors $\epsilon = 1$. It is clearly seen from the figure that MQAM gives better performance than the MPSK modulation techniques.

Furthermore, the ergodic capacity of mixed RF–FSO–VLC triple-hop communication system is evaluated in Fig. 8. It is observed that the capacity of the system decreases when the effects of pointing error and atmospheric turbulence are high. It is also seen that the capacity gap of the system is higher for strong atmospheric turbulence as compared to moderate atmospheric turbulence.

Figure 9 shows the ergodic capacity of the mixed RF–FSO–VLC system for different VLC parameters, i.e., for different heights of photodetector and no of LED’s considering strong turbulence condition. It is seen from the figure that the capacity of the system increases as the distance between the LED and photodetector decreases and the no of LEDs increases. For lower SNR, the capacity performance is more affected by height of photodetector and for higher SNR, the capacity performance is more affected by the no of LEDs.

7 Conclusion

The performance evaluation of the DF based cascaded RF–FSO–VLC communication system is proposed. The closed-form expressions for statistical characteristics of SNR of the considered system are obtained. These statistical characteristics of SNR are used to derive the analytical expressions for the outage probability, BER, and ergodic capacity of the DF based triple-hop communication system. The numerical results are obtained by varying different channel parameters such as RF fading parameter $m$, pointing error under the strong and moderate turbulence conditions for both the detection types, heterodyne and IM/DD. It is seen from the numerical results that the pointing errors and Nakagami-$m$ parameter in different atmospheric turbulence conditions affect the system performance. The indoor environment performance is dominated by the field of view of the detector and height at which LED luminaries are present. Such system can be used to connect the indoor user to provide last mile access to the end users. This kind of system can also be used to provide the campus connectivity as well as the enterprise connectivity. We have discussed the performance analysis of the cascaded RF–FSO–VLC system in terms of outage probability, BER, and ergodic capacity considering Nakagami-$m$ channel model for RF link and DGG for FSO link. Also the VLC link is characterized by the Lambertian pattern of the LED’s. The performance of the considered system can be carried out by considering different channel models for different links. Also optimizing the transmit...
power and see its impact on the different performance metrics along with its asymptotic results can be carried out in future work.

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Sanya Anees (M’16) received Masters (W/D) in Communication Engineering from University of Manchester, U.K., in 2011 and received Ph.D. from the Indian Institute of Teaching Delhi, in 2016. Her research interests include optical wireless communications, MIMO systems, and cooperative communications. Currently, she is working as an Assistant Professor in the Department of Electronics and Communication Engineering, Indian Institute of Information Technology Guwahati, Assam, India. She is reviewer of IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, IEEE Transactions on Aerospace and Electronic Systems, IEEE Access, IET Communications, and has reviewed various IEEE conference papers such as, ICC, Globecom, VTC, and WCNC. She has also served as TPC Member of IEEE NCC-2018, IEEE ICSC-2018, and IEEE CICT-2017. Recently she was awarded Early Career Research Award by SERB.
DST. In her Graduation, she has been awarded University Gold medal-2010 for being University Topper in Electronics and Communication Engg. Branch, Shri Rawatpura Sarkar Gold medal-2010 for being University Topper amongst students from Electronics and Communication Engineering, Computer Science, and Information Technology, and Prof. S. T. Chakravati Gold medal-2010 for being University Topper amongst students from Electronics and Communication Engineering, Electrical Engineering, Mechanical Engineering, and Civil Engineering.