PREDICTIONS FOR THE DIRAC CP VIOLATION PHASE IN THE NEUTRINO MIXING MATRIX

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Using the fact that the neutrino mixing matrix $U = U^\dagger_e U^\nu$, where $U_e$ and $U^\nu$ result from the diagonalisation of the charged lepton and neutrino mass matrices, we analyse the predictions based on the sum rules which the Dirac phase $\delta$ present in $U$ satisfies when $U^\nu$ has a form dictated by, or associated with, discrete flavour symmetries and $U_e$ has a “minimal” form (in terms of angles and phases it contains) that can provide the requisite corrections to $U^\nu$, so that the reactor, atmospheric and solar neutrino mixing angles $\theta_{13}$, $\theta_{23}$ and $\theta_{12}$ have values compatible with the current data.

Keywords: Neutrino mixing; Dirac Leptonic CP violation; Flavour Symmetries; Sum Rules.

1. Introduction

One of the major goals of the future experimental studies in neutrino physics is the searches for CP violation (CPV) effects in neutrino oscillations (see, e.g., Refs. 1, 2, 3, 4). It is part of a more general and ambitious program of research aiming to determine the status of the CP symmetry in the lepton sector.

In the case of the reference 3-neutrino mixing scheme, CPV effects in the flavour neutrino oscillations, i.e., a difference between the probabilities of $\nu_l \rightarrow \nu_{l'}$ and $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ oscillations in vacuum, $P(\nu_l \rightarrow \nu_{l'})$ and $P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$, $l \neq l' = e, \mu, \tau$, can be caused, as is well known, by the Dirac phase present in the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) neutrino mixing matrix $U$. If the neutrinos with definite masses $\nu_i$, $i = 1, 2, 3$, are Majorana particles, the 3-neutrino mixing matrix contains two additional Majorana CPV phases. However, the flavour neutrino oscillation probabilities $P(\nu_l \rightarrow \nu_{l'})$ and $P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$, $l, l' = e, \mu, \tau$, do not depend on the Majorana phases. Our interest in the CPV phases present in the neutrino mixing matrix is stimulated also by the intriguing possibility that the Dirac phase and/or the Majorana phases in $U$ can provide the CP violation necessary for the
generation of the observed baryon asymmetry of the Universe\textsuperscript{(510)} (see also, e.g., Refs. 10, 11).

In the framework of the reference 3-flavour neutrino mixing we will consider, the PMNS neutrino mixing matrix is always given by \( U = U_c \dagger U_\nu \), where \( U_c \) and \( U_\nu \) are 3 \( \times \) 3 unitary matrices originating from the diagonalisation of the charged lepton and the neutrino (Majorana) mass terms. We will suppose in what follows that \( U_\nu \) has a form which is dictated by, or associated with, symmetries (see, e.g., Refs. 12, 13). In the present article we consider the following symmetry forms of \( U_\nu \): i) tri-bimaximal (TBM)\textsuperscript{(13)} (or corresponding to the conservation of the lepton charge \( \nu_1 - \nu_\mu - \nu_\tau \) (LC))\textsuperscript{(12,24)}, ii) bimaximal (BM) ii) bimaximal (BM) (or corresponding to the conservation of the lepton charge \( L' = L_e - L_\mu - L_\tau \) (LC))\textsuperscript{(12,24)}, iii) golden ratio type A (GRA)\textsuperscript{(11,12)}, iv) golden ratio type B (GRB)\textsuperscript{(25)} and v) hexagonal (HG)\textsuperscript{(24)}.

For all these symmetry forms \( U_\nu \) can be written as

\[
U_\nu = \Psi_1 \hat{U}_\nu Q_0 = \Psi_1 R_{23}(\theta_{23}^e) R_{12}(\theta_{12}^e) Q_0 ,
\]

where \( R_{23}(\theta_{23}^e) \) and \( R_{12}(\theta_{12}^e) \) are orthogonal matrices describing rotations in the 2-3 and 1-2 planes, respectively, and \( \Psi_1 \) and \( Q_0 \) are diagonal phase matrices each containing two phases. The phases in the matrix \( Q_0 \) give contribution to the Majorana phases in the PMNS matrix. The symmetry forms of \( U_\nu \) of interest, TBM, BM (LC), GRA, GRB and HG, are characterised by the same values of the angles \( \theta_{13} = 0 \) and \( \theta_{23}^e = -\pi/4 \), but correspond to different fixed values of the angle \( \theta_{12}^e \) and thus of \( \sin^2 \theta_{12}^e \), namely, to i) \( \sin^2 \theta_{12}^e = 1/3 \) (TBM), ii) \( \sin^2 \theta_{12}^e = 1/2 \) (BM (LC)), iii) \( \sin^2 \theta_{12}^e = (2+r)^{-1} \approx 0.276 \) (GRA), \( r \) being the golden ratio, \( r = (1+\sqrt{5})/2 \), iv) \( \sin^2 \theta_{12}^e = (3-r)/4 \approx 0.345 \) (GRB), and v) \( \sin^2 \theta_{12}^e = 1/4 \) (HG). The best fit values\textsuperscript{(25)} and 1\( \sigma \) errors of the three corresponding neutrino mixing parameters in the standard parametrisation of the PMNS matrix\textsuperscript{11}, which we will employ, read\textsuperscript{25,26}:

\[
\sin^2 \theta_{12} = 0.308^{+0.017}_{-0.017},
\]

\[
\sin^2 \theta_{13} = 0.0234^{+0.0020}_{-0.0019},
\]

\[
\sin^2 \theta_{23} = 0.437^{+0.033}_{-0.023},
\]

where the quoted values correspond to neutrino mass spectrum with normal ordering (NO); the values for spectrum with inverted ordering (IO) found in\textsuperscript{26} differ insignificantly. The minimal form of \( U_\nu \) of interest that can provide the requisite corrections to \( U_\nu \), so that the neutrino mixing angles \( \theta_{13}, \theta_{23} \) and \( \theta_{12} \) have values compatible with the current data, including a possible sizeable deviation of \( \theta_{23} \) from \( \pi/4 \), includes a product of two orthogonal matrices describing rotations in the 2-3 and 1-2 planes\textsuperscript{20}, \( R_{23}(\theta_{23}^e) \) and \( R_{12}(\theta_{12}^e) \). \( \theta_{23}^e \) and \( \theta_{12}^e \) being two (real) angles\textsuperscript{3} This leads to the following parametrisation of the PMNS matrix \( U \):

\[
U = R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e) \Psi R_{23}(\theta_{23}^e) R_{12}(\theta_{12}^e) Q_0 ,
\]

\footnote{For a detailed discussion of alternative possibilities see Ref. 27.}
where $\Psi = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$, and $\theta_{23}^\nu = -\pi/4$. Equation (5) can be recast in the form

$$U = R_{12}(\theta_{12}^\nu)\Phi(\phi)R_{23}(\hat{\theta}_{23})R_{12}(\theta_{12}^\nu) \hat{Q},$$

where we have defined $\Phi = \text{diag}(1, e^{i\phi}, 1)$, $\phi$ being a CPV phase, $\hat{\theta}_{23}$ is a function of $\theta_{23}^\nu$, $\sin^2 \theta_{23} = 1/2 - \sin^2 \theta_{23}^\nu \cos(\omega - \psi)$, and $\hat{Q}$ is a diagonal phase matrix. The phases in $\hat{Q}$ give contributions to the Majorana phases in the PMNS matrix. The angle $\hat{\theta}_{23}$, however, can be expressed in terms of the angles $\theta_{23}$ and $\theta_{13}$ of the PMNS matrix:

$$\sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e 3}|^2} = \frac{\sin^2 \hat{\theta}_{23} - \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}},$$

and the value of $\hat{\theta}_{23}$ is fixed by the values of $\theta_{23}$ and $\theta_{13}$.

2. Predicting the Dirac Phase in the PMNS Matrix

In the scheme under discussion the four observables $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and the Dirac phase $\delta$ in the PMNS matrix are functions of three parameters $\theta_{12}^\nu$, $\theta_{23}$ and $\phi$. As a consequence, the Dirac phase $\delta$ can be expressed as a function of the three PMNS angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$, leading to a new “sum rule” relating $\delta$ and $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$. Within the approach employed this sum rule is exact. Its explicit form depends on the symmetry form of the matrix $\hat{U}_\nu$, i.e., on the value of the angle $\theta_{12}^\nu$. For arbitrary fixed value of $\theta_{12}^\nu$ the sum rule of interest reads

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[ \cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) \left( 1 - \cot^2 \theta_{23} \sin^2 \theta_{13} \right) \right].$$

A similar sum rule can be derived for the phase $\phi$.

In Refs. 28, 30 we have derived predictions for $\cos \delta$, $\delta$ and the rephasing invariant $J_{CP}$, which controls the magnitude of the CPV effects in neutrino oscillations, using the sum rule in eq. (8) and the measured values of the lepton mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$. In the present article we first summarise the predictions for these observables obtained in Refs. 28, 30 in a simplified analysis employing the best fit values (b.f.v.) and the $3\sigma$ allowed ranges of the three relevant neutrino mixing parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$. This is followed by a summary of the results of the statistical analysis of the predictions performed in Ref. 30, which is based on i) the current, and most importantly, ii) the prospective, uncertainties in the measured values of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$.

We note first that the predicted values of $\cos \delta$ vary significantly with the symmetry form of $\hat{U}_\nu$. For the best fit values of $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{13} = 0.0234$ and $\sin^2 \theta_{23} = 0.437$ found in Ref. 28, for instance, we get $\cos \delta = (-0.0906), (-1.16), 0.275, (-0.169) \text{ and } 0.445$ for the TBM, BM (LC), GRA, GRB and HG forms, respectively. For the TBM, GRA, GRB and HG forms these values correspond to $\delta = \pm 95.2^\circ, \pm 74.0^\circ, \pm 99.7^\circ, \pm 63.6^\circ$, respectively. The unphysical value of $\cos \delta$
in the BM (LC) case is a reflection of the fact that the scheme under discussion with BM (LC) form of the matrix $\hat{U}_\nu$ does not provide a good description of the current data on $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$. Physical values of $\cos \delta$ can be obtained, for instance, for the b.f.v. of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ if $\sin^2 \theta_{12}$ has a larger value. For, e.g., $\sin^2 \theta_{12} = 0.34$ allowed at $2\sigma$ by the current data, we have $\cos \delta = -0.943$, corresponding to $\delta = \pm 160.6^\circ$. Similarly, for $\sin^2 \theta_{12} = 0.32$, $\sin^2 \theta_{23} = 0.41$ and $\sin \theta_{13} = 0.158$ we have $\cos \delta = -0.978$, $\delta = \pm 168.1^\circ$.

The results quoted above imply that the measurement of $\cos \delta$ can allow to distinguish between the different symmetry forms of $\hat{U}_\nu$, provided $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ are known with a sufficiently good precision. Even determining the sign of $\cos \delta$ will be sufficient to eliminate some of the possible symmetry forms of $\hat{U}_\nu$.

It was also found in that the sum rule predictions for $\cos \delta$ exhibit strong dependence on the value of $\sin^2 \theta_{12}$ when the latter is varied in its $3\sigma$ experimentally allowed range (0.259–0.359). The predictions for $\cos \delta$ change significantly not only in magnitude, but also the sign of $\cos \delta$ changes in the TBM, GRA and GRB cases. In the case of $\theta_{23}^\prime = 0$, for instance, we get for the TBM form of $\hat{U}_\nu$ for the three values of $\sin^2 \theta_{12} = 0.308$, 0.259 and 0.359: $\cos \delta = (-0.114)$, $(-0.469)$ and 0.221, thus $\cos \delta = 0$ is allowed for a certain value of $\sin^2 \theta_{12}$. For the GRA and GRB forms of $\hat{U}_\nu$ we have, respectively, $\cos \delta = 0.289$, $(-0.044)$, 0.609, and $\cos \delta = (-0.200)$, $-0.559$, 0.138. Similarly, for the HG form we find for the three values of $\sin^2 \theta_{12}$: $\cos \delta = 0.476$, 0.153, 0.789.

In what concerns the dependence of the sum rule predictions for $\cos \delta$ when $\sin^2 \theta_{23}$ is varied in its $3\sigma$ allowed interval, $0.374 \leq \sin^2 \theta_{23} \leq 0.626$, the results we obtained for $\sin^2 \theta_{23} = 0.374$ and $\sin^2 \theta_{23} = 0.437$, setting $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ to their best fit values, do not differ significantly. However, the differences between the predictions for $\cos \delta$ obtained for $\sin^2 \theta_{23} = 0.437$ and for $\sin^2 \theta_{23} = 0.626$ are rather large — they differ by the factors of 2.05, 1.25, 1.77 and 1.32 in the TBM, GRA, GRB and HG cases, respectively.

Similar analysis can be performed for the predictions for the cosine of the phase $\phi$ which in many theoretical models serves as a “source” for the Dirac phase $\delta$. The phase $\phi$ is related to, but does not coincide with, the Dirac phase $\delta$. This leads to the confusing identification of $\phi$ with $\delta$: the sum rules satisfied by $\cos \phi$ and $\cos \delta$ differ significantly. Correspondingly, the predicted values of $\cos \phi$ and $\cos \delta$ in the cases of the TBM, GRA, GRB and HG symmetry forms of $\hat{U}_\nu$ considered by us also differ significantly. This conclusion is not valid for the BM (LC) form: for this form the sum rules predictions for $\cos \phi$ and $\cos \delta$ are rather similar.

We next present results of the statistical analysis of the predictions for $\delta$, $\cos \delta$ and the rephasing invariant $J_{\text{CP}}$ performed in Ref. 30 in the cases of the TBM, BM (LC), GRA, GRB and HG symmetry forms of the matrix $\hat{U}_\nu$. In this analysis the latest results on $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$ and $\delta$ obtained in the global analysis of the neutrino oscillation data performed in were used as input. The aim was to derive the allowed ranges for $\cos \delta$ and $J_{\text{CP}}$, predicted on the basis of the
current data on the neutrino mixing parameters for each of the symmetry forms of \( \tilde{U}_\nu \) considered. For this purpose the \( \chi^2 \) function was constructed in the following way:

\[
\chi^2(\{x_i\}) = \sum_i \chi^2_i(\{x_i\}),
\]

with \( x_i = \{\sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23}, \delta\} \). The functions \( \chi^2_i \) have been extracted from the 1-dimensional projections given in Ref. 25 and, thus the correlations between the oscillation parameters have been neglected. This approximation is sufficiently precise since it allows to reproduce the contours in the planes (\( \sin^2 \theta_{23}, \delta \)), (\( \sin^2 \theta_{13}, \delta \)) and (\( \sin^2 \theta_{23}, \sin^2 \theta_{13} \)), given in Ref. 25, with a rather high accuracy. We calculated \( \chi^2(\cos \delta) \) by marginalising \( \chi^2(\{x_i\}) \) over \( \sin^2 \theta_{13} \) and \( \sin^2 \theta_{23} \) for a fixed value of \( \cos \delta \). Given the global fit results, the likelihood function,

\[
L(\cos \delta) \propto \exp \left( -\frac{\chi^2(\cos \delta)}{2} \right),
\]

represents the most probable value of \( \cos \delta \) for each of the considered symmetry forms of \( \tilde{U}_\nu \). The \( n\sigma \) confidence level (C.L.) region corresponds to the interval of values of \( \cos \delta \) in which \( L(\cos \delta) \geq L(\chi^2 = \chi^2_{\text{min}}) \cdot L(\chi^2 = n^2) \), where \( \chi^2_{\text{min}} \) is the value of \( \chi^2 \) in the minimum.

In Fig. 1 we show the likelihood function versus \( \cos \delta \) for NO neutrino mass spectrum from Ref. 30. The results shown are obtained by marginalising over all the other relevant parameters of the scheme considered. The dependence of the likelihood function on \( \cos \delta \) in the case of IO neutrino mass spectrum differs little from that shown in Fig. 1. As can be observed in Fig. 1 a rather precise measurement of \( \cos \delta \) would allow to distinguish between the different symmetry forms of \( \tilde{U}_\nu \) considered by us. For the TBM and GRB forms there is a significant overlap of the corresponding likelihood functions. The same observation is valid for the GRA and HG forms. However, the overlap of the likelihood functions of these two groups of symmetry forms occurs only at 3\( \sigma \) level in a very small interval of values of \( \cos \delta \). This implies that in order to distinguish between TBM/GRB, GRA/HG and BM (LC) symmetry forms, a not very demanding measurement (in terms of accuracy) of \( \cos \delta \) might be sufficient. The value of the non-normalised likelihood function at the maximum in Fig. 1 is equal to \( \exp(-\chi^2_{\text{min}}/2) \), which allows us to make conclusions about the compatibility of the symmetry schemes considered with the current global data. The results of this analysis for \( \cos \delta \) are summarised in Table 1.

We have also performed in Ref. 30 a similar statistical analysis of the predictions for the rephasing invariant \( J_{CP} \) in the cases of the TBM, BM (LC), GRA, GRB and HG symmetry forms of the matrix \( \tilde{U}_\nu \) considered. In this analysis we used as input the latest results on \( \sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23} \) and \( \delta \), obtained in the global analysis of the neutrino oscillation data performed in Ref. 25, and minimised \( \chi^2 \) for a fixed value of \( J_{CP} \). The obtained b.f.v. and 3\( \sigma \) ranges are given in Table 1. We have found, in particular, that the CP-conserving value of \( J_{CP} = 0 \) is excluded in the cases of the TBM, GRA, GRB and HG neutrino mixing symmetry forms, respectively, at approximately 5\( \sigma \), 4\( \sigma \), 4\( \sigma \) and 3\( \sigma \) C.L. with respect to the C.L. of the corresponding best fit value. These results reflect the predictions we have obtained...
for $\delta$, more specifically, the C.L. at which the CP-conserving values of $\delta = 0 \ (2\pi)$, $\pi$, are excluded in the discussed cases. We found also that the $3\sigma$ allowed intervals of values of $\delta$ and $J_{CP}$ are rather narrow for all the symmetry forms considered, except for the BM (LC) form. More specifically, for the TBM, GRA, GRB and HG symmetry forms we have obtained at $3\sigma$: $0.020 \leq |J_{CP}| \leq 0.039$. For the b.f.v. of $J_{CP}$ we have found, respectively: $J_{CP} = (-0.034), (-0.033), (-0.034)$, and $(-0.031)$. Our results indicate that distinguishing between the TBM, GRA, GRB and HG symmetry forms of the neutrino mixing would require extremely high precision measurement of the $J_{CP}$ factor.

In Fig. 2 we present the likelihood function versus $\cos \delta$ within the Gaussian approximation, i.e., using $\chi^2_G = \sum_i (y_i - \bar{y}_i)^2 / \sigma^2_{y_i}$, with $y_i = \{ \sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23} \}$, where we used the current b.f.v. ($\bar{y}_i$) of the mixing angles for NO neutrino mass spectrum given in Ref. 25 and the prospective 1$\sigma$ uncertainties ($\sigma_{y_i}$) in the determination of $\sin^2 \theta_{12}$ ($0.7\%$ from JUNO$^{31}$), $\sin^2 \theta_{13}$ ($3\%$ derived from an expected error on $\sin^2 2\theta_{13}$ of $3\%$ from Daya Bay, see Refs. 4, 32) and $\sin^2 \theta_{23}$ ($5\%$ derived from the potential sensitivity of NOvA and T2K on
Table 1. Best fit values (b.f.v.) of $J_{\text{CP}}$ and $\cos \delta$ and corresponding 3\(\sigma\) ranges (found fixing $\chi^2 - \chi^2_{\text{min}} = 9$) in our setup using the data from\textsuperscript{33} for NO neutrino mass spectrum. (From Ref. 30, where results for IO spectrum are also given.)

| Scheme | $J_{\text{CP}}/10^{-2}$ (b.f.v.) | $J_{\text{CP}}/10^{-2}$ (3\(\sigma\) range) | $\cos \delta$ (b.f.v.) | $\cos \delta$ (3\(\sigma\) range) |
|--------|---------------------------------|---------------------------------|----------------|----------------|
| TBM    | -3.4                            | $[-3.8, -2.8] \cup [3.1, 3.6]$ | -0.07          | $[-0.47, 0.21]$ |
| BM (LC)| -0.5                            | $[-2.6, 2.1]$                   | -0.99          | $[-1.00, -0.72]$ |
| GRA    | -3.3                            | $[-3.7, -2.7] \cup [3.0, 3.5]$ | 0.25           | $[-0.08, 0.60]$ |
| GRB    | -3.4                            | $[-3.9, -2.6] \cup [3.1, 3.6]$ | -0.15          | $[-0.57, 0.13]$ |
| HG     | -3.1                            | $[-3.5, -2.0] \cup [2.6, 3.4]$ | 0.47           | $[0.16, 0.80]$  |

$\sin^2 2\theta_{23}$ of 2\%, see Ref. 4, this sensitivity can be also achieved in future neutrino facilities as T2HK\textsuperscript{33}. The BM (LC) case is very sensitive to the b.f.v. of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ and is disfavoured at more than 2\(\sigma\) for the current b.f.v. found in\textsuperscript{33}. This case might turn out to be compatible with the data for larger (smaller) measured values of $\sin^2 \theta_{12}$ ($\sin^2 \theta_{23}$), as can be seen from Fig. 3 which was obtained for $\sin^2 \theta_{12} = 0.332$ (the best fit values of the two other mixing angles being kept intact). With the increase of the value of $\sin^2 \theta_{23}$ the BM (LC) form becomes increasingly disfavoured, while the TBM/GRB (GRA/HG) predictions for $\cos \delta$ are shifted somewhat to the left (right) with respect to those shown in Fig. 2. For, e.g., the best fit values of $\sin^2 \theta_{12} = 0.304$, $\sin^2 \theta_{13} = 0.0219$ and $\sin^2 \theta_{23} = 0.579$, found in Ref. 34 for IO neutrino mass spectrum, these shifts in $\cos \delta$ are approximately by 0.1.

The measurement of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ with the quoted precision will open up the possibility to distinguish between the BM (LC), TBM/GRB, GRA and HG forms of $\hat{U}_\nu$. Distinguishing between the TBM and GRB forms would require relatively high precision measurement of $\cos \delta$. Assuming that $|\cos \delta| < 0.93$, which means for 76\% of values of $\delta$, the error on $\delta$, $\Delta \delta$, for an error on $\cos \delta$, $\Delta (\cos \delta) = 0.10 (0.08)$, does not exceed $\Delta \delta \lesssim \Delta (\cos \delta)/\sqrt{1 - 0.93^2} = 16^\circ (12^\circ)$. This accuracy is planned to be reached in the future neutrino experiments like T2HK (ESSvSB)\textsuperscript{33}. Therefore a measurement of $\cos \delta$ in the quoted range will allow one to distinguish between the TBM/GRB, BM (LC) and GRA/HG forms at approximately 3\(\sigma\) C.L., if the precision achieved on $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ is the same as in Figs. 2 and 3.

3. Summary and Conclusions

In conclusions, we have derived in\textsuperscript{33} the ranges of the predicted values of $\cos \delta$ and $J_{\text{CP}}$ for the TBM, BM (LC), GRA, GRB and HG symmetry forms of $\hat{U}_\nu$, from a statistical analysis using the sum rule in eq. 8 obtained in\textsuperscript{23} and the current global neutrino oscillation data\textsuperscript{22}. The results of this analysis are summarised in Table 1 and in Fig. 1. We found, in particular, that in the TBM, GRA, GRB and HG cases, the best fit values of $J_{\text{CP}}$ lie in the narrow interval $(-0.034) \lesssim J_{\text{CP}} \lesssim (-0.031)$, while at 3\(\sigma\) we have $0.020 \leq |J_{\text{CP}}| \leq 0.039$. The predictions for $\delta$, $\cos \delta$ and $J_{\text{CP}}$...
in the case of the BM (LC) symmetry form of $\tilde{U}_\nu$, as the results of the statistical analysis performed by us showed, differ significantly: the best fit value of $\delta \approx \pi$, and, correspondingly, of $J_{CP} \approx 0$. For the $3\sigma$ range in the case of NO (IO) neutrino mass spectrum we find: $-0.026 (-0.025) \leq J_{CP} \leq 0.021 (0.023)$, i.e., it includes a sub-interval of values centred on zero, which does not overlap with the $3\sigma$ allowed intervals of values of $J_{CP}$, corresponding to the TBM, GRA, GRB and HG symmetry forms of $\tilde{U}_\nu$.

Finally, we have derived in [30] predictions for $\cos \delta$ using the prospective $1\sigma$ uncertainties in the determination of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ respectively in JUNO, Daya Bay and accelerator and atmospheric neutrino experiments (Figs. 2 and 3). The results thus obtained show that i) the measurement of the sign of $\cos \delta$ will allow to distinguish between the TBM/GRB, BM (LC) and GRA/HG forms of $\tilde{U}_\nu$, ii) for a best fit value of $\cos \delta = -1 (-0.1)$ distinguishing at $3\sigma$ between the BM (TBM/GRB) and the other forms of $\tilde{U}_\nu$ would be possible if $\cos \delta$ is measured with $1\sigma$ uncertainty of 0.3 (0.1).
The results obtained in the studies performed in Refs. 28, 30 show, in particular, that the experimental measurement of the Dirac phase $\delta$ of the PMNS neutrino mixing matrix in the future neutrino experiments, combined with the data on the neutrino mixing angles can provide unique information about the possible discrete symmetry origin of the observed pattern of neutrino mixing.

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