Stationary solutions for the parity-even sector of the CPT-even and Lorentz-covariance-violating term of the standard model extension

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In this work, we focus on some properties of the parity-even sector of the CPT-even electrodynamics of the standard model extension. We analyze how the six non-birefringent terms belonging to this sector modify the static and stationary classical solutions of the usual Maxwell theory. We observe that the parity-even terms do not couple the electric and magnetic sectors (at least in the stationary regime). The Green’s method is used to obtain solutions for the field strengths $E$ and $B$ at first order in the Lorentz-covariance-violating parameters. Explicit solutions are attained for point-like and spatially extended sources, for which a dipolar expansion is achieved. Finally, it is presented an Earth-based experiment that can lead (in principle) to an upper bound on the anisotropic coefficients as stringent as $(\delta_{e-})^{ij} < 2.9 \times 10^{-20}$.

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I. INTRODUCTION

In recent years, investigations concerning Lorentz symmetry violation have been undertaken mainly in the context of the standard model extension (SME) developed by Colladay and Kostelecky, which incorporates Lorentz-invariance-violating (LIV) terms in all sectors of the usual standard model of the fundamental interactions. The abelian or electromagnetic sector of the SME is composed of a CPT-even and a CPT-odd part. The CPT-odd sector is represented by the Carroll-Field-Jackiw term, $\xi_{\rho\nu\phi}V^{\beta}A^{\beta}F^{\rho\nu}$, whose properties were first examined in Ref. [3]. The investigations on this electrodynamics have been performed in a broad part. The CPT-odd sector is represented by the Carroll-Field-Jackiw term, $\xi_{\rho\nu\phi}V^{\beta}A^{\beta}F^{\rho\nu}$, whose properties were first examined in Ref. [3]. The investigations on this electrodynamics have been performed in a broad perspective, addressing aspects as diverse as the consistency and quantization of the model [4], radiative corrections [5], classical solutions [6], Cerenkov radiation [7], cosmic background radiation [8], and other features [9]. More recently, the CPT-even sector, represented by the term $W^{\alpha\beta}_{\rho\nu}F^{\alpha\beta}F^{\rho\nu}$, has been investigated as well, embracing the study of small deviations of the Maxwell electrodynamics stemming from this term and some attempts of imposing upper bounds on the LIV parameters [10, 11, 12, 13, 14, 15, 16, 17].

The Lagrangian density of the CPT-even electrodynamics of the Standard Model Extension has the form

$$\mathcal{L} = -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{4}W_{\alpha\nu\rho\phi}F^{\alpha\nu}F^{\rho\phi} - J_aA^a,$$  \hspace{1cm} (1)

where the background tensor $W_{\alpha\nu\rho\phi}$ has the same symmetries as the Riemann tensor $[W_{\alpha\nu\rho\phi} = -W_{\nu\alpha\rho\phi}, W_{\alpha\nu\rho\phi} = -W_{\alpha\nu\rho\phi}, W_{\nu\rho\phi\alpha} = W_{\rho\phi\alpha\nu}]$ and a double null trace, $W^{\alpha\beta}_{\alpha\beta} = 0$, implying 19 components. This tensor $W_{\alpha\nu\rho\phi}$ can be written in terms of four $3 \times 3$ matrices $\kappa_{DE}, \kappa_{DB}, \kappa_{HE}, \kappa_{HB}$, defined in Refs. [13, 14] as

$$\kappa_{DE}^{jk} = -2W^{j0k0}, (\kappa_{HE})^{jk} = \frac{1}{2}\epsilon^{ijklm}W_{ijklm}, (\kappa_{DB})^{jk} = -\delta^{jk}\kappa_{DE}^{ii},$$  \hspace{1cm} (2)

The matrices $\kappa_{DE}, \kappa_{HE}$ contain the parity-even components and possess together eleven independent components, while $\kappa_{HE}, \kappa_{HB}$ possess together eight components and describe the parity-odd sector of $W_{\alpha\nu\rho\phi}$. Four tilde matrices and one trace element can be written as suitable combinations of $\kappa_{DE}, \kappa_{DB}, \kappa_{HE}, \kappa_{HB}$:

$$(\tilde{\kappa}_{e+})^{jk} = \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk}, \quad (\tilde{\kappa}_{e-})^{jk} = \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \frac{1}{3}\delta^{jk}\kappa_{DE}^{ii},$$  \hspace{1cm} (3)

$$(\tilde{\kappa}_{o+})^{jk} = \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk}, \quad (\tilde{\kappa}_{o-})^{jk} = \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk}, \quad \kappa_{tr} = \frac{1}{3}(\kappa_{DE})^{ii}. \hspace{1cm} (4)$$
From the eleven independent components of the matrices $\tilde{\kappa}_{e+}$, $\tilde{\kappa}_{o-}$, the five elements enclosed in $\tilde{\kappa}_{e+}$ are constrained by birefringence to the level of 1 part in $10^{32}$ (see Refs. [13, 14]), there remaining six non-birefringent ones (the trace element and the five components of the matrix $\tilde{\kappa}_{o-}$) to be constrained by other methods. From the eight elements of the parity-odd sector, five (contained in the matrix $\tilde{\kappa}_{o-}$) are tightly bounded by birefringence, there remaining only three components (belonging to $\tilde{\kappa}_{e+}$), which were parameterized as the $\kappa$ vector [12], written as $\kappa^j = \frac{1}{2} \varepsilon^{jpi} (\kappa_{DB})^p$. In some recent papers [16], the absence of Cerenkov radiation from ultrahigh-energy cosmic rays (UHECRs) has been used to state bounds at the level of 1 part in $10^{18}$ on the three nonbirefringent terms of $W_{\alpha\nu\rho\phi}$, belonging both to the parity-even and parity-odd sectors.

In Ref. [11], there was performed an analysis focused on the three non-birefringent components ($\kappa^j$) of the parity-odd sector of $W$ (the parity-even components were taken as null in order to isolate the parity-odd sector physics). The stationary classical solutions for the Maxwell electrodynamics modified by these three LIV coefficients were properly evaluated by means of the Green’s method. With these solutions, it was described a device able to yield a nice upper bound, $\kappa^j < 10^{-16}$, in the context of an Earth-based experiment.

The aim of the present work is to study the stationary aspects of the classical electrodynamics stemming from the parity-even sector of the tensor $W_{\alpha\nu\rho\phi}$. For that, we use the following parameterization $\kappa_{DE} = -\kappa_{HB}$, and we consider as null the parity-odd sector. The goal is to determine how the the six non-birefringent parity-even components modify the classical and stationary solutions for Maxwell electromagnetism. Certainly, the idea is also to use the results obtained to properly constrain the magnitude of the LIV coefficients.

This work is organized as follows. In Sec. II, we write the wave equations and modified Maxwell equations and apply the Green method in order to obtain the required stationary solutions. In Sec. III, we present our final remarks and describe a measurement device able to yielding an upper bound on the LIV parameter as stringent as $(\tilde{\kappa}_{e-})^j \leq 2.9 \times 10^{-20}$.

II. WAVE EQUATIONS AND STATIONARY CLASSICAL SOLUTIONS

Here, the focus is on the classical properties of the parity-even part of the tensor $W$, particularly on those produced by the six non-birefringent components (located in the matrix $\tilde{\kappa}_{e-}$ plus the trace element $\tilde{\kappa}_{tr}$). Hence, we take the parity-odd sector as null ($\kappa_{DB} = \kappa_{HE} = 0$) to isolate the physics of the even sector. Further, we adopt the following parameterization $(\kappa_{DE}) = - (\kappa_{HB})$, which implies $(\tilde{\kappa}_{e+}) = 0$, by considering the stringent bound imposed by birefringence data [13, 14]. Moreover, it implies the following relation for the non-birefringent components:

$$(\tilde{\kappa}_{e-})^{jk} = (\kappa_{DE})^{jk} - n \delta^{jk}, \quad n = \frac{1}{3} \delta^{jk} Tr(\kappa_{DE}).$$

(5)

In order to evaluate the classical solutions of this model, we write the wave equation for the four-potential

$$\Box A^\alpha - 2W^{\nu\rho\lambda} \partial_\nu \partial_\rho A_\lambda = J^\alpha,$$

(6)

which yields two differential equations, one for the scalar potential, and one for the vector potential,

$$[(1 + n) \partial_t^2 - (1 + n) \nabla^2 - (\tilde{\kappa}_{e-})^{ij} \partial_i \partial_j] A_0 + (\tilde{\kappa}_{e-})^{ij} \partial_i \partial_j A_j = \rho,$$

(7)

$$[(1 + n) \partial_t^2 - (1 - n) \nabla^2] A_i - 2n \partial_\rho \partial_\sigma A_0 - (\tilde{\kappa}_{e-})^{ij} \partial_i E_j - \epsilon_{ipj} (\tilde{\kappa}_{e-})^{jl} \partial_j B_l = j_i.$$

(8)

where we have used $E_j = -F_{0j}$, $B_i = \frac{1}{2} \epsilon_{ipj} F_{pj}$, $(\tilde{\kappa}_{e-})^{ij} = (\tilde{\kappa}_{e-})_{ij}$. At the stationary regime, such equations are read

$$[(1 + n) \nabla^2 + (\tilde{\kappa}_{e-})^{ij} \partial_i \partial_j] A_0 = -\rho,$$

(9)

$$[(1 - n) \nabla^2] A_i + \epsilon_{ipj} (\tilde{\kappa}_{e-})^{jl} \partial_j B_l = -j_i.$$  

(10)

These equations reveal that the electric and magnetic sectors are decoupled (in the stationary regime) in contrast with the electrodynamics of the parity-odd sector, in which these sectors are entirely entwined (see
Applying the differential operator $\epsilon_{\alpha\beta} \partial_\theta$ to Eq. (10), we obtain the following differential equation for the magnetic field

$$\left[ (1 - n) \delta_{\alpha\beta} - (\kappa_{c-})^{\alpha\beta} \right] \nabla^2 + (\kappa_{c-})^{\beta\gamma} \partial_\alpha \partial_\gamma B_\gamma = - (\nabla \times j)_\alpha.$$  \hspace{1cm} (11)

While the homogeneous Maxwell equations remain unmodified ($\nabla \times \mathbf{E} + \partial_\theta \mathbf{B} = 0, \nabla \cdot \mathbf{B} = 0$), the inhomogeneous ones (Gauss and Ampere law) are altered, taking the form

$$(1 + n) \nabla \cdot (\mathbf{E} - (\kappa_{c-})^{ij} \partial_j B_i) = \rho,$$  \hspace{1cm} (12)

$$(1 + n) \partial_\theta E_i - (1 - n) (\nabla \times B)_i + \epsilon_{ijk} (\kappa_{c-})^{ij} \partial_j B_i + (\kappa_{c-})^{ik} \partial_k E_q = - j_i.$$  \hspace{1cm} (13)

In the stationary regime, the latter equation provides

$$(1 - n) (\nabla \times B)_j - \epsilon_{ijk} (\kappa_{c-})^{ij} \partial_j B_i = j_i,$$  \hspace{1cm} (14)

which under the action of the operator curl operator ($\epsilon_{\alpha\beta\gamma} \partial_\gamma$) yields the same expression as Eq. (11).

**A. The Green’s function for the scalar potential**

The solution for the scalar potential may be obtained by the Green’s method. The Green’s function for Eq. (9) fulfills

$$\left[ (1 + n) \nabla^2 + (\kappa_{c-})_{ij} \partial_i \partial_j \right] G(\mathbf{r} - \mathbf{r'}) = \delta^3(\mathbf{r} - \mathbf{r'}),$$  \hspace{1cm} (15)

and the scalar potential is given as

$$A_0(\mathbf{r}) = - \int G(\mathbf{r} - \mathbf{r'}) \rho(\mathbf{r'}) d^3\mathbf{r'}. \hspace{1cm} (16)$$

The Green’s function in Fourier space is given as $G(r - r') = (2\pi)^{-3} \int d^3\rho \tilde{G}(\mathbf{p}) \exp[-i \mathbf{p} \cdot (\mathbf{r} - \mathbf{r'})]$, so that we obtain

$$\tilde{G}(\mathbf{p}) \simeq - \frac{1}{p^2} \left[ 1 - n - (\kappa_{c-})^{ij} \frac{p_i p_j}{p^2} \right],$$  \hspace{1cm} (17)

at first order in the LIV parameters. Remembering that the LIV coefficients are small, we used $\left[ 1 + n + (\kappa_{c-})^{ij} p_i p_j / p^2 \right]^{-1} \simeq \left[ 1 - (\kappa_{c-})^{ij} p_i p_j / p^2 \right]$. Carrying out the inverse Fourier transform, the Green’s function takes the following form:

$$G(\mathbf{r} - \mathbf{r'}) = - \frac{1}{4\pi} \left\{ (1 - n) \frac{1}{|\mathbf{r} - \mathbf{r'}|} + \frac{(\kappa_{c-})^{ij} (\mathbf{r} - \mathbf{r'})_i (\mathbf{r} - \mathbf{r'})_j}{2|\mathbf{r} - \mathbf{r'}|^3} \right\}.$$  \hspace{1cm} (18)

It presents a genuine Coulomb contribution screened by the factor $(1 - n)$ and a non-Coulomb contribution related to the LIV non-isotropic coefficients $(\kappa_{c-})_{ij}$. The overall behavior, $r^{-1}$, remains the same as happens in the Maxwell electrodynamics.

Using the Green function (18) and Eq. (10), the scalar potential due to a general charge distribution $[\rho(\mathbf{r'})]$ is

$$A_0(\mathbf{r}) = \frac{1}{4\pi} \left\{ (1 - n) \int d^3\mathbf{r'} \frac{\rho(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|} + (\kappa_{c-})^{ij} \frac{\int d^3\mathbf{r'} (\mathbf{r} - \mathbf{r'})_i (\mathbf{r} - \mathbf{r'})_j}{2|\mathbf{r} - \mathbf{r'}|^3} \rho(\mathbf{r'}) \right\}, \hspace{1cm} (19)$$

which implies the following electric field strength:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi} \left\{ (1 - n) \int d^3\mathbf{r'} \rho(\mathbf{r'}) \frac{(\mathbf{r} - \mathbf{r'})^i}{|\mathbf{r} - \mathbf{r'}|^3} - (\kappa_{c-})^{ij} \frac{\int d^3\mathbf{r'} \rho(\mathbf{r'}) (\mathbf{r} - \mathbf{r'})_j}{|\mathbf{r} - \mathbf{r'}|^3} \right. \hspace{1cm} (20)$$

$$+ 3 (\kappa_{c-})^{ij} \frac{\int d^3\mathbf{r'} \rho(\mathbf{r'}) (\mathbf{r} - \mathbf{r'})_j (\mathbf{r} - \mathbf{r'})_i}{2|\mathbf{r} - \mathbf{r'}|^3} \right\}.$$
With this expression, we may immediately evaluate the scalar potential and electric field strength for a point-like charge at rest \( \rho(\mathbf{r}') = q\delta(\mathbf{r}') \), yielding

\[
A_0(\mathbf{r}) = \frac{q}{4\pi} \left\{ (1-n) \frac{1}{r} + (\bar{\kappa}_{e-})^{ij} \frac{r_ir_j}{2r^3} \right\},
\]

\[
E^i(\mathbf{r}) = \frac{q}{4\pi} \left\{ \left( 1 - n + \frac{3}{2} (\bar{\kappa}_{e-})^{ij} \frac{r_ir_j}{r^3} \right) \frac{r_i}{r^3} - (\bar{\kappa}_{e-})^{ij} \frac{r_j}{r^3} \right\},
\]

The scalar potential and the electric field present a genuine Coulomb contribution, with the screening factor \(1-n\), and a non-Coulomb contribution related to the LIV coefficient \((\bar{\kappa}_{e-})^{ij}\). This latter term leads to variations of the scalar potential and electric field along a circular path around the point-like charge. Such an effect can be used to impose an upper bound on the LIV parameter \(s\), as will be described in Sec. III.

From the expressions (19) and (20), for an arbitrary charge distribution, we express the scalar potential and the electric field in the dipolar approximation \( (|\mathbf{r} - \mathbf{r}'|^{-1} = r^{-1} + \mathbf{r} \cdot \mathbf{r}'/r^3) \):

\[
A_0(\mathbf{r}) = \frac{1}{4\pi} \left\{ (1-n) \left[ q + \frac{\mathbf{r} \cdot \mathbf{P}_e}{r^3} \right] + (\bar{\kappa}_{e-})^{ij} \left[ r_ir_j(q + 3\frac{\mathbf{r} \cdot \mathbf{P}_e}{r^2} - r_iP_{ej} - r_jP_{ei}) \right] \right\},
\]

\[
E^i(\mathbf{r}) = \frac{1}{4\pi} \left\{ (1-n) \left[ qr^2 - \frac{P_i^3}{r^3} - 3\frac{\mathbf{r} \cdot \mathbf{P}_e}{r^5}r^3 \right] - (\bar{\kappa}_{e-})^{ij} \frac{1}{r^3} \left[ qr_j - P_{ej} + 3\frac{\mathbf{r} \cdot \mathbf{P}_e}{r^2}r_j \right] \right\}
+ 3(\bar{\kappa}_{e-})^{ij} \frac{r_ir_j}{2r^5} \left[ qr^4 - P_i^3 + 5\frac{\mathbf{r} \cdot \mathbf{P}_e}{r^2}r^3 \right] - 3(\bar{\kappa}_{e-})^{ij} \frac{r_jP_{ej}}{r^5}r^3 \right\},
\]

where \( q = \int \rho(\mathbf{r}')d^3\mathbf{r}' \) and \( \mathbf{P}_e = \int \mathbf{r}'\rho(\mathbf{r}')d^3\mathbf{r}' \) is the electric dipole moment. The LIV terms in \((\bar{\kappa}_{e-})^{ij}\) break the radial symmetry giving a non-Coulomb behavior to the static solutions. Despite the large number of terms in these solutions, we verify that the electric field preserves the \(r^{-2}\) and the \(r^{-3}\) decaying behaviors for the monopole and dipole moments, respectively, as occurs in the pure Maxwell electrodynamics. Obviously, it is a consequence of the dimensionless character of the LIV coefficients.

### B. The Green’s function for the magnetic field

Now, we search for an explicit solution for the magnetic field. The Green’s function for the magnetic field equation of motion (11) is written as

\[
\left[ (1-n)\delta_{at} - (\bar{\kappa}_{e-})^{at} \right] \nabla^2 + (\bar{\kappa}_{e-})^{atl} \partial_a \partial_l \right] G_{lb}(\mathbf{r} - \mathbf{r}') = \delta_{ab}\delta^3(\mathbf{r} - \mathbf{r}').
\]

Using the Fourier representation and having much care in the tensor inversion procedure, we obtain in the momentum space

\[
\tilde{G}_{ab}(\mathbf{p}) = -\frac{1}{\mathbf{p}^2} \left\{ (1+n) \delta_{ab} + (\bar{\kappa}_{e-})_{ab} - (\bar{\kappa}_{e-})_{ab} \frac{p_ap_b}{\mathbf{p}^2} \right\}.
\]

Performing the inverse Fourier transformation, we attain the following expression:

\[
G_{ab}(\mathbf{r} - \mathbf{r}') = -\frac{1}{4\pi} \frac{1}{(|\mathbf{r} - \mathbf{r}'|} \left\{ (1+n) \delta_{ab} + \frac{(\bar{\kappa}_{e-})_{ab}}{2} + \frac{(\bar{\kappa}_{e-})_{ab} (\mathbf{r} - \mathbf{r}')_a (\mathbf{r} - \mathbf{r}')_b}{2 |\mathbf{r} - \mathbf{r}'|^2} \right\},
\]

with which the magnetic field is then written as

\[
B^i(\mathbf{r}) = -\int d^3\mathbf{r}' \ G_{ij}(\mathbf{r} - \mathbf{r}') (\nabla' \times \mathbf{J}(\mathbf{r}')).
\]
It leads to the explicit solution:

\[
B^i(r) = \frac{1}{4\pi} \left\{ \left(1 + n\right) \delta_{ib} + \frac{1}{2} (\tilde{\kappa}_{e-})_{ib} \right\} \int d^3r' \frac{(\nabla \times j(r'))^b}{|r - r'|} + \frac{(\tilde{\kappa}_{e-})^{ij}}{2} \int d^3r' \left[ \frac{(\nabla \times j(r'))^j}{|r - r'|^3} (r - r')_j (r - r')^i \right].
\] (29)

After a certain algebraic effort, a dipolar expansion for the magnetic field is achieved as well, yielding

\[
B^i(r) = \frac{1}{4\pi} \left\{ \left(1 + n\right) \left( -\frac{m_i}{r^3} + \frac{3}{r^5} (r \cdot m) r^3 \right) - (\tilde{\kappa}_{e-})^{ib} \frac{m_b}{r^3} - (\kappa_{e-})_{pb} r_p r_b \left[ \frac{3}{2} \frac{m_i}{r^3} - \frac{15}{2} \frac{r \cdot m}{r^7} \right] \right\}
\] (30)

where we have considered a localized and divergenceless current density distribution \(j\), and \(m = \frac{1}{2} \int r' \times j(r') d^3r'\) is the magnetic dipole moment. In Eq. (30) the first term inside the parentheses is the usual Maxwell contribution, just corrected by the \((1 + n)\) factor. The terms that are proportional to the LIV coefficients, \((\tilde{\kappa}_{e-})^{ib}\), ascribe to the solution a directional dependence or anisotropic character. In principle, such a directional dependence could be used to impose an upper bound on the LIV parameters. In Sec. III, we show that the attained bound is not as restrictive as desired.

### III. FINAL REMARKS

We should now compare these parity-even stationary solutions with the parity-odd ones derived in Ref. [11]. At the stationary regime, the main difference is that now the electric and magnetic sectors are not coupled by the LIV tensor anymore. In the parity-odd case, a stationary current is able to produce an electric field as much as a static charge can generate a magnetic field. As such an interconnection does not appear in the present case, the manifestation of pure LIV electromagnetic effects (aside from Maxwell ones), as the production of magnetic field by a static point-like charge (see Ref. [11]), are absent. Now, the LIV effects appear as small corrections for the usual Maxwell’s electric and magnetic fields. Yet, the LIV effects can still be identified by means of suitable devices, as it is discussed below. Apart from this difference, the solutions of the parity-even and parity-odd sectors possess some similarities. Indeed, the electric field for a point-like charge (in both sectors) exhibits an asymptotic behavior as \(r^{-2}\), while a stationary current provides a magnetic field whose dipolar expansion is proportional to \(mr^{-3}\). This is ascribed to the dimensionless character of the tensor \(W\).

The attained magnetic field solution does not lead to good upper bounds on the magnitude of the parameters \(n\) and \((\tilde{\kappa}_{e-})^{ib}\) when we take as reference the Earth’s magnetic field. In fact, proceeding in a similar way as in Ref. [8], we assert that the LIV tensor must not imply a magnetic field contribution larger than \(10^{-4}\)G (otherwise it would be detected). From Eq. (30), we observe that the LIV terms are always proportional to \((m/r^3)\). Assuming that \(m\) represents the Earth’s magnetic dipole, and \(R_\oplus\) the Earth’s radius, it holds the following ratio \(m/R_\oplus^3 = 0.3\) G (see Ref. [2]). This procedure, however, implies a non-restrictive bound: \(n \leq 10^{-4}\).

A much better bound for the parameters \((\tilde{\kappa}_{e-})^{ij}\) can be attained from the expression for the scalar potential. The idea is to evaluate the scalar potential generated by a charged sphere in different outer points located at the same distance from the center of the sphere, observing the difference of potential induced by the non-Coulomb LIV term. The starting point is the expression for the potential generated by a conducting sphere of radius \(R\) and charge \(q\) (uniformly distributed over its surface), which can be achieved by replacing the charge density for a sphere, \(\rho(r') = q\delta(r' - R)/(4\pi R^2)\), in Eq. (19). Using Fourier integrations (see the appendix), the potential is (for \(r > R\))

\[
A_0(r) = \frac{1}{4\pi} \left\{ (1 - n) \frac{q}{r} + \frac{(\tilde{\kappa}_{e-})^{ab}}{2} \left[ r_ar_b \left( \frac{r^2 - R^2}{r^3} \right) \right] \right\}.
\] (31)

We see that the term in \((\tilde{\kappa}_{e-})^{ij}\) breaks the radial symmetry of the potential, implying potential variations along a circular path around the center. We now expand the term \((\tilde{\kappa}_{e-})^{ab} r_ar_b\) at the form

\[
(\tilde{\kappa}_{e-})^{ab} r_ar_b = (\tilde{\kappa}_{e-})^{11} \left[ (r_1)^2 - (r_3)^2 \right] + (\tilde{\kappa}_{e-})^{22} \left[ (r_2)^2 - (r_3)^2 \right] + 2 (\tilde{\kappa}_{e-})^{12} r_1 r_2 + 2 (\tilde{\kappa}_{e-})^{13} r_1 r_3 + 2 (\tilde{\kappa}_{e-})^{23} r_2 r_3,
\]
where we have used the traceless matrix

\[
(\tilde{\kappa}_{e-}) = \begin{pmatrix}
(\tilde{\kappa}_{e-})_{11} & (\tilde{\kappa}_{e-})_{12} & (\tilde{\kappa}_{e-})_{13} \\
(\tilde{\kappa}_{e-})_{12} & (\tilde{\kappa}_{e-})_{22} & (\tilde{\kappa}_{e-})_{23} \\
(\tilde{\kappa}_{e-})_{13} & (\tilde{\kappa}_{e-})_{23} & -(\tilde{\kappa}_{e-})_{11} - (\tilde{\kappa}_{e-})_{22}
\end{pmatrix}.
\]  

(32)

Then, we can conceive of an experiment to measure the electrostatic potential generated by a 1 C charged sphere of radius \(R\) (maintained in vacuum) in two distinct outer points, \(A\) and \(B\), located at the a circle of radius \(r > R\) on the \(x-y\) plane. We consider the points \(A\) and \(B\) symmetrically disposed in relation to the \(y\)-axis at the positions: \(A = r(\cos \phi, \sin \phi, 0)\), \(B = r(-\cos \phi, \sin \phi, 0)\). Then, the difference of potential between these points is simply

\[
\Delta A_0 = A_0(A) - A_0(B) = \frac{q}{4\pi} (\tilde{\kappa}_{e-})^{12} \sin 2\phi \frac{(r^2 - R^2)}{r^3},
\]

(33)

for \(r > R\). For \(\phi = \pi/4\) and a 1 C charge, such difference of potentials equal to

\[
\Delta A_0 = 9 \times 10^9 (\tilde{\kappa}_{e-})^{12} \frac{(r^2 - R^2)}{r^3}.
\]

(34)

For attaining the best bound, we should consider the maximum value of Eq. (33). So, it must be evaluated at the point \(r = R\sqrt{3}\), in which the expression \((r^2 - R^2)r^{-3}\) has a maximum. For a charged sphere of unitary radius \((R = 1m)\), we obtain \(\Delta A_0 = 3.46 \times 10^9 (\tilde{\kappa}_{e-})^{12} V\). Given the existence of sensitive methods for measurement of the potential able to detect slight variations of 1 part in \(10^{10}\) V, we can infer that the voltage difference of Eq. (33) cannot be larger than \(10^{-10}\) V, that is, \(3.46 \times 10^9 (\tilde{\kappa}_{e-})^{12} < 10^{-10}\). This condition leads to \((\tilde{\kappa}_{e-})^{12} < 2.9 \times 10^{-20}\). Choosing pairs of points on the planes \(y-z\) and \(x-z\), this upper limit holds equivalently for \((\tilde{\kappa}_{e-})^{23}\) and \((\tilde{\kappa}_{e-})^{13}\).

This device can also be used to set up an upper bound on the diagonal components \((\tilde{\kappa}_{e-})^{ii}\). For constraining \((\tilde{\kappa}_{e-})^{11}\), we take the points \(A\) and \(B\) in the positions: \(A = r(1,0,0), B = r(0,0,1)\). The difference of potential between these points is

\[
\Delta A_0 = A_0(A) - A_0(B) = \frac{q}{4\pi} (\tilde{\kappa}_{e-})^{11} \frac{(r^2 - R^2)}{r^3},
\]

(35)

which leads to the same bound obtained for the non-diagonal components: \((\tilde{\kappa}_{e-})^{11} < 2.9 \times 10^{-20}\). Choosing two points on the \(y-z\) plane, \(A = r(0,1,0), B = r(0,0,1)\), this bound can be stated to \((\tilde{\kappa}_{e-})^{22}\). Thus, we conclude that by means of this experiment it is possible to establish an upper bound as stringent as

\[
(\tilde{\kappa}_{e-})^{ij} < 2.9 \times 10^{-20},
\]

for the five non-isotropic components of the traceless matrix \((\tilde{\kappa}_{e-})^{ij}\). This is a nice bound for an Earth-based experiment, as good as the best bounds stated from astrophysical data analysis of UHECRs [16].

As the isotropic component \(n\) does not break the spherical symmetry of the potential, this kind of experiment does not provide any way for bounding it. This component induces a slight screening on the Coulomb potential that may be interpreted as a charge screening. An experiment able to constrain \(n\) could be based on a charge or potential screening measurement. In this case, the major difficult is that the tiny LIV effect is disguised by the dominant Maxwell’s behavior, avoiding its isolation. So, the LIV effect stays limited by the experimental imprecisions of the device. A two-sided bound was recently stated for this coefficient in the context of quantum electrodynamics decay processes modified by this LIV parameter [17].

Finally, we should note that this work completes the calculation of the stationary solutions of Maxwell’s electromagnetism modified by the non-birefringent elements of the abelian CPT-even and LIV sector of the standard model extension, a task initiated in Refs. [10, 11].
APPENDIX A: EVALUATION OF THE SCALAR POTENTIAL GENERATED BY A CHARGED SPHERE

By starting from Eq. (19), the scalar potential is rewritten as

\[ A_0(r) = \frac{1}{4\pi} \left\{ (1 - n) \int d^3 r' \frac{\rho(r')}{|r - r'|} - \frac{(\tilde{\kappa}_{e-})^{ab}}{2} \partial_a I_b \right\}, \quad (A1) \]

where

\[ I_b = \int d^3 r' \frac{(r - r')_b}{|r - r'|} \rho(r'), \quad (A2) \]

and using \((\tilde{\kappa}_{e-})_{ii} = \text{tr}(\tilde{\kappa}_{e-}) = 0\), we show that

\[ (\tilde{\kappa}_{e-})^{ab} \frac{(r - r')_a (r - r')_b}{|r - r'|^4} = - (\tilde{\kappa}_{e-})^{ab} \partial_a \left[ \frac{(r - r')_b}{|r - r'|} \right]. \]

Knowing that the charge density for a charged sphere of radius \(R\) is \(\rho(r') = q \delta(r' - R) / (4\pi R^2)\), its Fourier transform is

\[ \tilde{\rho}(\mathbf{p}) = \int d^3 r' e^{i \mathbf{p} \cdot \mathbf{r}'} \rho(r') = q \frac{\sin(pR)}{pR}, \quad (A3) \]

with \(p = |\mathbf{p}|\). For evaluating the integral \(I_b\), we use the Fourier representation of the Coulomb potential

\[ \frac{1}{|r - r'|} = 4\pi \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{-i \mathbf{p} \cdot (r - r')}}{p^2}. \quad (A4) \]

By substituting in the Eq. (A2) and using (A3) we obtain, after some algebraic manipulations, the following expression for \(I_b\):

\[
I_b = 4\pi q R_k \frac{q}{R} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( \frac{1}{p^2} \frac{e^{-i \mathbf{p} \cdot \mathbf{r}} \sin(pR)}{p} \right) + 4\pi q R \partial_b \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( \frac{1}{p^2} \frac{e^{-i \mathbf{p} \cdot \mathbf{r}} \sin(pR)}{p^3} \right) - 4\pi q \partial_b \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( \frac{1}{p^2} \frac{e^{-i \mathbf{p} \cdot \mathbf{r}} \cos(pR)}{p^2} \right). \quad (A5)
\]

Solving these integrals, we obtain

\[ I_b = \int d^3 r' \frac{(r - r')_b}{|r - r'|} \rho(r') = \frac{q}{r} r_b - \frac{q R^2}{3r^3} r_b. \quad (A6) \]

Thus, finally, we obtain the scalar potential generated by the charged sphere of radius \(R\) for \((r > R)\)

\[ A_0(r) = \frac{1}{4\pi} \left\{ (1 - n) \frac{q}{r} + \frac{q (\tilde{\kappa}_{e-})^{ab}}{2} \left[ \frac{r_a r_b (r^2 - R^2)}{r^3} \right] \right\}. \quad (A7) \]

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