Dynamic evaluation of regional water resources carrying capacity based on set pair analysis and partial connection number

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ABSTRACT

In order to describe the micro motion between the connection number components and seek a more applicable evaluation model, quantitatively evaluate and analyze regional water resources carrying capacity (WRCC). Firstly, an evaluation index system and grade standards of regional WRCC were constructed. Then, a method for determining the connection number was proposed, which considered the micro motion between the connection number components in system structure. Finally, built an evaluation model based on set pair analysis (SPA) and partial connection number (PCN) that used subtraction set pair potential (SPP) to identify vulnerability factors, and identification results were compared with total partial connection number (TPCN). The model was applied to Huaibei City, Anhui Province, China. The results showed that: the WRCC grade value was between 2 and 3 that was poor; the support and regulation subsystem grade value was between 2 and 3, and the pressure subsystem grade value was between 1 and 2. SPP identified that the support force and regulation force subsystem were the vulnerable subsystems. Eight indexes such as water resources per capita, rate of ecological water consumption and density of population were the main indicators causing the poor WRCC, which were in good agreement with the local measured data. In addition, the SPP and TPCN are compared to further verify rationality of the connection number determination method and reliability of the identification results. The model established in this paper has strong applicability and can also be used for the dynamic evaluation of other resources, environment and ecological carrying capacity. The results in this study can provide a scientific basis for water resources management and decision-making.

Key words: Anhui Province, connection number, Huaibei City, PCN, SPA, SPP, WRCC evaluation

HIGHLIGHTS

- Based on partial connection number, a method for calculating connection number of regional water resources carrying capacity (WRCC) evaluation sample was proposed.
- Constructed a dynamic evaluation model of regional WRCC based on set pair analysis and partial connection number.
- Identified reasons for poor WRCC in Huaibei City, China, and compared the results with local measured data.

1. INTRODUCTION

Water resources are indispensable basic natural resources to support economic development and ecological environment construction (Peng et al. 2021). For decades, water resources have become a strategic resource restricting the development of today’s world, and gradually become a public social resource with economic nature (Dai et al. 2019; Chi et al. 2021). Corresponding to the important position of water resources is the unfavorable position of water security, such as human abuse, water environmental pollution, unreasonable development and utilization of water resources, which will bring about problems such as intensified contradiction between supply and demand of water resources, unbalanced space of water resources, etc., which seriously restrict regional sustainable development (Gao et al. 2021; Liu 2021). In view of the complexity and severity of water security, scholars all over the world have put forward some solutions (Wu et al. 2020; Zhao et al. 2021). Of these, WRCC can quantitatively measure and analyze water resources security from the characteristics and
interaction among water resources, socio-economic and ecological environment, which has become an important research field (Cui et al. 2018).

The original concept of ‘carrying capacity’ comes from mechanics in physics, which refers to the resistance of an object itself to external objects (Kessler 1994). With the development of human society, the concept of carrying capacity has been cited to other research fields, such as ecological carrying capacity (Abia et al. 2017). Park first put forward the concept of ecological carrying capacity in 1921 (Aboulfoutoh 2018). Since the 1960s, carrying capacity has been applied to the research of social, economic and natural system. In the early 1980s, UNESCO and FAO put forward the concept of resource carrying capacity (Ahmed et al. 2019). The concept of WRCC began in the late 1980s and originated from the research on sustainable utilization of natural resources. WRCC can be regarded as the maximum supporting capacity of water resources to population, ecological environment, economy and society with water resources as limiting factor, which is a comprehensive key index to measure water security (Zhang et al. 2010). When WRCC exceeds a certain threshold, it will seriously limit sustainable development of economy and society. Therefore, the research on the evaluation and diagnosis of regional WRCC has attracted more and more scholars’ attention, and has always been a key and challenging issue in the field of water resources security (Wu et al. 2020; Zhou et al. 2021). From the view that the elements of WRCC system form the carrying state, the evaluation and diagnosis of water resources carrying capacity is to construct the evaluation index systems, evaluation grade standards and evaluation model, then judge whether water resources can support the maximum economic and social development scale of the region without damaging the ecological environment. Finally, diagnose and identify the main reasons for the poor WRCC. This is not only the preliminary work of WRCC prediction, early warning and regulation, but also an important basis for scientific decision-making of water resources management and the formulation of disaster prevention and reduction measures (Cui et al. 2018). It will help to implement the strictest water resources management system and realize sustainable utilization of regional water resources and water security.

At present, the evaluation methods of regional WRCC mainly include: fuzzy comprehensive evaluation method (Liu et al. 2019; Mingxia 2020), principal component analysis method (Li & Meng 2017), system dynamics method (Ge et al. 2021), grey correlation method (Ge et al. 2020) and set pair analysis connection number method (Wang et al. 2009; Kang et al. 2019; Lele et al. 2021). SPA connection number method is widely used because it can deeply analyze and quantitatively express the rich information between evaluation sample data and evaluation grades (Men & Liu 2018), which provides an effective way for evaluation of WRCC and identification of vulnerability factors. However, for the application of SPA connection number method in the evaluation and diagnosis of WRCC. There are also the following problems: (1) The process of determining the connection number only considers the uncertainty between the evaluation index value and the evaluation grades in the macro state, and does not consider the micro movement between connection number components in system structure, that is, the dynamic evolution characteristics (Jinbo & Jixiang 2021; Thanh et al. 2021). (2) Due to the interaction between water resources system, socio-economic system and ecological environment system, there is a high degree of complexity and uncertainty. Currently, there is a lack of quantitative evaluation model with strong applicability (Li et al. 2021). (3) It is challenging to accurately identify and diagnose the vulnerability subsystem(s) and index(es) of WRCC. Whether the recognition results are consistent with the local measured data is an important way to judge and measure the effectiveness of recognition method. At present, there are few studies in this field (Cui et al. 2018).

Currently, China is one of the countries lacking water resources in the world. The per capita of water resources is less than 1/4 of world’s (Borgomeo et al. 2015). With the continuous development of China’s national economy and society, regional water resources security (Yang et al. 2019; Drangert 2021) and water resources vulnerability (Chhetri et al. 2020; Xiang & Li 2020) have become an important bottleneck factor seriously restricting sustainable development. Huaibei City in Anhui Province is a prefecture city in Anhui Province in East China. It is not only an important resource-based city in China, but also one of the 114 cities with serious water shortage in China. The total amount of regional water resources is small, and per capita water resources is less than 400 m$^3$, which indicating situation of water shortage is more severe. Therefore, accurately evaluate the grade of WRCC in Huaibei City; identification of vulnerability subsystem(s) and index(es) of WRCC system is very vital for scientific decision-making and water resources management. Based on the theory of SPA and PCN, this study intends to introduce a mobility matrix that can reflect and characterize the micro motion between components in structural system, so as to explore the dynamic balance mechanism between the different connection number components. Then, a method to determine the connection number is proposed. The dynamic quantitative evaluation model of regional WRCC based on SPA and PCN is constructed and applied to Huaibei City, Anhui Province. Meanwhile, diagnose and identify
the vulnerability subsystem(s) and index(es) of WRCC system. This study will provide technical support for regional water resources management in order to realize the sustainable utilization of water resources.

2. CONSTRUCTION OF REGIONAL WATER RESOURCES CARRYING CAPACITY EVALUATION MODEL BASED ON SET PAIR ANALYSIS AND PARTIAL CONNECTION NUMBER

A dynamic evaluation model of regional WRCC based on SPA and PCN was constructed by comprehensively adopting SPA, PCN and SPP method. The establishment process of this model generally included the following 6 steps, shown in Figure 1:

Step 1: determine the evaluation index system and evaluation grade standards of regional WRCC. According to principle that regional water resources support, pressure and regulation force interact to form regional WRCC, system structure analysis was carried out (Cui et al. 2018; Yifan 2021). Combined with the characteristics of the study area, expert consultation, literature research and other methods, the evaluation index system \( \{ x_{ijk} | j = 1, 2, \ldots, n_k; k = 1, 2, 3 \} \) and grade standards \( \{ s_{gj} | g = 1, 2, \ldots, n_g; j = 1, 2, \ldots, n_j \} \) of regional WRCC were established. The corresponding evaluation sample data set was recorded as \( \{ x_{ijk} | i = 1, 2, \ldots, n_i; j = 1, 2, \ldots, n_j; k = 1, 2, 3 \} \), where \( x_{ijk} \) was index \( j \) value in the \( k \)-th subsystem of the \( i \)-th sample; \( n_i \) was the number of evaluation samples; \( n_k \) was the number of evaluation indexes in the \( k \)-th subsystem, obviously \( n_1 + n_2 + n_3 = n_j \); \( n_g \) was the number of evaluation grades. Without losing generality, here taking three evaluation grades (i.e., \( g = 3 \)) for each evaluation index and \( g = 1, 2, 3 \) representing three different states for WRCC. \( G = 1 \) stood for ‘loadable’ of water resources, which meant strong WRCC. Regional water resources still had large carrying capacity, and its water supply was relatively good; \( g = 2 \) stood for ‘critical overloaded’ of water resources, which meant average WRCC. The development and utilization of regional water resources had a considerable scale, but there was still a certain potential for development and utilization. The supply and demand of water resources can meet the social and economic development of the region to a certain extent; \( g = 3 \) stood for ‘overloaded’ of water resources, which meant weak WRCC. The regional WRCC was close to the saturation value, and the potential for further development and utilization was small. In the long run, there will be a shortage of water resources, which restricts the coordinated development of regional social economy. Therefore, corresponding control measures should be taken in time.

![Figure 1](https://example.com/figure1.png)  
**Figure 1 |** Construction of the dynamic evaluation model of regional water resources carrying capacity based on set pair analysis and partial connection number.
Step 2: the single index connection number of WRCC evaluation sample was calculated. SPA was a connecting mathematical method proposed by Chinese scholar Zhao Keqin in 1989 (Zhao 1996). It can integrate the certainty and uncertainty of the problem into a unified conclusion and study the internal relationship between each elements of the system and the whole system. The basic idea of SPA is to quantitatively compare and analyze the close attributes of two sets \( \{x_{ijk}\} \) and \( \{s_{ij}\} \) in the similarity, difference and opposition under given problem background. Here, certainty includes similarity and opposition, and uncertainty is expressed as difference. Obviously, if the attributes of two sets are closer, the similarity is greater. By analyzing the characteristics of the two sets, SPA established a mathematical model of the relationship between certainty and uncertainty. Arithmetic expression of the ternary connection number was as follows (Xie & Guo 2018):

\[
u = a + bI + cf\]

(1)

where \( a, b, \) and \( c \) were the degree to which the evaluation index belonged to grade 1, 2, and 3, respectively. \( I \) was the difference coefficient, and \( f \) was the opposition coefficient, \( f = -1 \). Formula (1) established the basic model of SPA. For the given value \( x_{ijk} \) of the regional WRCC evaluation index, the connection number \( u_{ijk} \) was calculated through SPA between \( x_{ijk} \) (index value of the \( k \)-th subsystem of the \( i \)-th sample) and evaluation grade standard \( s_{ij} \).

For the index (positive index) whose evaluation grade \( g \) increased with the increase of index value \( x_{ijk} \) (i.e., the larger the index value, the better the carrying situation was), the calculation formula of ternary connection number was as follows (Wang et al. 2009):

\[
\begin{align*}
U_{ijk1} &= 1 \\
U_{ijk2} &= 1 - 2(s_{ij} - x_{ijk})/(s_{ij} - s_{ij}) \quad s_{ij} < x_{ijk} \leq s_{ij} \\
U_{ijk3} &= -1
\end{align*}
\]

(2)

\[
\begin{align*}
U_{ijk1} &= 1 - 2(x_{ijk} - s_{ij})/(s_{ij} - s_{ij}) \\
U_{ijk2} &= 1 \quad s_{ij} < x_{ijk} \leq s_{ij} \\
U_{ijk3} &= 1 - 2(s_{ij} - x_{ijk})/(s_{ij} - s_{ij})
\end{align*}
\]

(3)

For the index (negative index) whose evaluation grade \( g \) decreased with the increase of index value \( x_{ijk} \) (i.e., the smaller the index value, the better the carrying situation was), the calculation formula of ternary connection number was as follows (Wang et al. 2009):

\[
\begin{align*}
U_{ijk1} &= 1 \\
U_{ijk2} &= 1 - 2(s_{ij} - x_{ijk})/(s_{ij} - s_{ij}) \quad s_{ij} > x_{ijk} \geq s_{ij} \\
U_{ijk3} &= -1
\end{align*}
\]

(5)

\[
\begin{align*}
U_{ijk1} &= 1 - 2(x_{ijk} - s_{ij})/(s_{ij} - s_{ij}) \quad s_{ij} < x_{ijk} \leq s_{ij} \\
U_{ijk2} &= 1 \quad s_{ij} > x_{ijk} \geq s_{ij} \\
U_{ijk3} &= 1 - 2(s_{ij} - x_{ijk})/(s_{ij} - s_{ij})
\end{align*}
\]

(6)

where \( s_{ij}, s_{ij} \), \( s_{ij}, s_{ij} \) and \( s_{ij}, s_{ij} \) were the interval endpoint values of grade 1, grade 2 and grade 3, respectively; \( x_{ijk} \) was sample value and \( u_{ijk} \) was connection number. For positive index, \( s_{ij} < s_{ij} < s_{ij} < s_{ij} \), and for negative index, \( s_{ij} > s_{ij} > s_{ij} > s_{ij} \). Obviously, the connection number \( u_{ijk} \) can be used as a measure of the closeness between \( x_{ijk} \) and \( s_{ij} \); \( u_{ijk} \) was transformed into the relative membership degree \( u_{ijk}^* \) \( (0 \leq u_{ijk}^* \leq 1) \), and the calculation formula was as follows.
where $v_{ijk}^*$ was the relative membership degree, and it represented the relative membership degree of WRCC evaluation sample value $x_{ijk}$ belonging to grade 1, 2 and 3. $i = 1, 2, ..., n_i$; $j = 1, 2, ..., n_j$; $k = 1, 2, 3$; $g = 1, 2, 3$. In order to make the sum of relative membership degree $v_{ijk}^*$, $v_{ijk}^*$, and $v_{ijk}^*$ equal to 1; forming the connection number components, it can be processed by the following formulas (Jin et al. 2015):

$$v_{ijk} = \frac{v_{ijk}^*}{v_{ijk}^* + v_{ijk}^* + v_{ijk}^*}$$

(9)

$$u_{ijk} = v_{ijk1} + v_{ijk2}I + v_{ijk3}J$$

(10)

where $v_{ijk}^*$ was relative membership degree; $u_{ijk}$ was the single index connection number of WRCC evaluation sample and $v_{ijk1}$, $v_{ijk2}$, $v_{ijk3}$ were the single index connection number components; there was $v_{ijk1} + v_{ijk2} + v_{ijk3} = 1$. $I$ and $J$ had the same meaning as formula (1).

Step 3: constructed the migration matrix based on PCN and calculated the index value connection number of evaluation sample $i$ and subsystem $k$. On the surface, the single index value connection number components $v_{ijk1}$, $v_{ijk2}$ and $v_{ijk3}$ of the evaluation sample reflected the similarity, difference and opposition value. Meanwhile, from the background of WRCC evaluation, $v_{ijk1}$, $v_{ijk2}$ and $v_{ijk3}$ quantitatively described the degree of a single index value belonging to grade 1, 2 and 3, respectively, and coincidence degree expression of between sample values and evaluation grades in the complex system of WRCC was realized, which was a very important part in the comprehensive evaluation. It can be seen that the connection number method was a powerful tool which can deeply analyze and quantitatively express specific uncertainty problems, and it was more unique in the interpretation of physical connotation. However, it should be noted that there was mutual transformation and migration among the similarity, difference and opposition components of connection number. According to the philosophical principles of the universality of connection, the absoluteness of movement and the universality of contradiction, the components of connection number always had the similarity, difference and opposition levels and their mutual transformation and migration, thus forming the system structure. Obviously, in this system structure, the components of connection number were no longer independent, but influenced each other through the contradictory movement at the micro level. At present, most existing studies do not consider the relationship structure characteristics of mutual migration and transformation (dynamic evolution) between the connection number components after calculating the connection number components. Instead, the connection number component (such as maximum membership degree method, attribute recognition method and level eigenvalue method) is directly used for the comprehensive evaluation of practical problems. It can be seen that how to quantitatively describe and express the influence on the dynamic evolution of the connection number components in connection number system structure at the micro level; intuitively see the micro movement and migration of the system under the macro static state, so as to grasp the migration law more objectively and accurately, and deeply reveal the essential characteristics of the system. The PCN focuses on the quantitative description of the set pair system relationship structure at the micro level, which can be derived from the system level relationship of connection number components (Xie et al. 2014; Xie & Guo 2018). Taking the ternary connection number $u = a + bl + cf$ as an example, the partial positive connection number was as follows (Dingtian & Xiaoxi 2011):

$$\partial^+\mu = \partial^+a + \partial^+bl = \frac{a}{a+b} + \frac{b}{b+c}i^+$$

(11)

where $\partial^+a = a/a + b$ and $\partial^+b = b/b + c$ were called the partial positive connection number of connection number components $a$ and $b$, respectively; $i^+$ was difference coefficient. $\partial^+a$ can be considered that the current $a$ was originally at the level $b$, which was a positive migration from the level $b$ to $a$. $\partial^+b$ can be considered that the current $b$ was originally at the level $c$, which was a positive migration from the level $c$ to $b$. The partial positive connection number indicates that...
the set pair event has a positive development trend. The larger the value, the greater the positive development trend of the event. Similarly, the partial negative connection number was as follows (Dingtian & Xiaoxi 2011):

$$
\partial \mu = \partial b i - \partial c j = \frac{b}{a+b} + \frac{c}{b+c}
$$

(12)

where $\partial b = b/a + b$ and $\partial c = c/b + c$ were called the partial negative connection number of connection number components $b$ and $c$, respectively; $i$ was difference coefficient and $j$ was opposition coefficient. $\partial b$ can be considered that the current $b$ was originally at the level $a$, which was a negative migration from the level $a$ to $b$. $\partial c$ can be considered that the current $c$ was originally at the level $b$, which was a negative migration from the level $b$ to $c$. The partial negative connection number indicates that the set pair event has a negative development trend. The smaller the value, the greater the negative development trend of the events. The PCN revealed the contradictory movement of the connection number components at the micro level, and can quantitatively express the mutual migration and transformation of the connection number components in the set system structure. To sum up, the partial connection number, as an adjoint function of the connection number, can be expressed as the function $f(a, b, c)$ of the connection number components, realizing qualitative and quantitative characterization of the movement direction and increment size of the connection number components at macro static and micro level. Taking the structural system’s connection number components as the driving object and the ‘information energy’ stored in the connection number components as the driving force, and the migration to different opposition levels as the driving direction, the updating and iteration of the connection number components were realized, which made the determined connection number components more reasonable and deeply understand the micro motion law.

According to the above, referring to the basic concept and connotation of PCN, this paper applied PCN to depict the mutual migration of connection number components of WRCC evaluation sample, and then put forward a method to determine the connection number, hoping to realize the rationality and accuracy of WRCC evaluation; meanwhile, the vulnerability subsystem(s) and index(es) were identified. Migration matrix $X$: in $a, b$ and $c$, for the component $a$ of connection number, there was a micro dynamic movement from $a, b$ and $c$ to $a$, and it was obvious that the possibility of component $a$ transferring to itself was 1. Both components $b$ and $c$ had migration to component $a$, so the migration of components $b$ and $c$ to component $a$ needed to be considered. $\partial a$ reflected the migration rate of component $a$ to $b$, and $b \partial a$ is the migration value of component $a$ to $b$. $\partial b \partial a$ reflected the migration rate of component $c$ to $a$, that was, component $c$ first transformed to component $b$ and then to component $a$. Component $b$ was the bridge between components $a$ and $c$, and $c \partial a \partial b$ was the migration value of component $c$ to $a$. According to the above, the new similarity component transformed by the PCN was $a + b \frac{b}{a+b} + c \frac{b}{b+c}$. Similarly, for the connection number component $b$, there was a micro dynamic movement from components $a, b$ and $c$ to $b$, and the new difference component transformed by the PCN was $b + a \frac{b}{a+b} + c \frac{b}{b+c}$. For the connection number component $c$, there was also a micro dynamic movement from components $a, b$ and $c$ to $c$, and the new opposition component transformed by the PCN was $c + b \frac{c}{b+c} + a \frac{b}{a+b}$. The above process can be described by formula (13):

$$
R = UX = [a, b, c] \begin{bmatrix}
1 & b & b \\
\frac{a}{a+b} & \frac{b}{a+b} & \frac{b}{a+b+c} \\
\frac{b}{b+c} & \frac{b}{b+c} & 1
\end{bmatrix} = [a + b \frac{a}{a+b} + c \frac{b}{b+c}, b + a \frac{a}{a+b} + c \frac{b}{b+c}, c + b \frac{c}{b+c} + a \frac{b}{a+b} + c]
$$

(13)

where $U = [a, b, c]$ was original connection number matrix and $R$ was the evaluation sample connection number matrix transformed by the migration matrix $X$. Substituting the single index connection number components $v_{ijk1}$, $v_{ijk2}$ and $v_{ijk3}$ in
formula (10) into formula (13):

\[
[\psi_{ijk1}, \psi_{ijk2}, \psi_{ijk3}] = \left[ \frac{1}{\psi_{ijk1} + \psi_{ijk2}}, \frac{\psi_{ijk2} + \psi_{ijk3}}{\psi_{ijk2} + \psi_{ijk3}}, \frac{\psi_{ijk3}}{\psi_{ijk3}} \right]
\]

\[
+ \frac{\psi_{ijk2}}{\psi_{ijk2} + \psi_{ijk3}} + \frac{\psi_{ijk3}}{\psi_{ijk3}} \right]
\]

\[
= \left[ \frac{\psi_{ijk1}}{\psi_{ijk1} + \psi_{ijk2}}, \frac{\psi_{ijk2}}{\psi_{ijk2} + \psi_{ijk3}}, \frac{\psi_{ijk3}}{\psi_{ijk3}} \right]
\]

\[
+ \frac{\psi_{ijk2}}{\psi_{ijk2} + \psi_{ijk3}} + \frac{\psi_{ijk3}}{\psi_{ijk3}} \right]
\]

\[
\text{Formula (14)}
\]

referring to formula (9), get the single index connection number \(u'_ijk\) of WRCC evaluation sample determined by the method in this paper:

\[
u'_ijk = \nu'_ijk1 + \nu'_ijk2 I + \nu'_ijk3 I
\]

\[
\text{Formula (15)}
\]

where \(\nu'_ijk1, \nu'_ijk2, \text{and} \nu'_ijk3\) were the single index connection number components of the WRCC evaluation sample obtained by the migration matrix transformation, and \(\nu'_ijk1 + \nu'_ijk2 + \nu'_ijk3 = 1\). Obviously, these components were obtained by the superposition of the macro static state and the micro motion state. According to formula (15), the index value connection number \(u'_i\) of the \(i\)-th evaluation sample can be calculated by formula (16) and the index value connection number \(u'_{ki}\) of the \(k\)-th subsystem of the \(i\)-th evaluation sample can be calculated by formula (17). Formulas were as follows (Xie et al. 2014):

\[
u'_ik = \nu'_ijk1 + \nu'_ijk2 I + \nu'_ijk3 I = \sum_{j=1}^{n_k} \nu_j \nu'_ijk1 + \sum_{j=1}^{n_k} \nu_j \nu'_ijk2 I + \sum_{j=1}^{n_k} \nu_j \nu'_ijk3 I (i = 1, 2, \cdots n_i; k = 1, 2, 3)
\]

\[
\text{Formula (16)}
\]

\[
u'_i = \nu'_ij1 + \nu'_ij2 I + \nu'_ij3 I = \sum_{j=1}^{n_k} \sum_{j=1}^{n_k} \nu_j \nu'_ij1 + \sum_{j=1}^{n_k} \sum_{j=1}^{n_k} \nu_j \nu'_ij2 I + \sum_{j=1}^{n_k} \sum_{j=1}^{n_k} \nu_j \nu'_ij3 I (i = 1, 2, \cdots n_i)
\]

\[
\text{Formula (17)}
\]

where \(\nu_j\) was the weight of evaluation index \(j\), which was determined by the fuzzy analytic hierarchy process based on accelerated genetic algorithm (AGA-FAHP) (Jin et al. 2015).

Step 4: calculated the WRCC grade of the \(i\)-th evaluation sample and each subsystem by using the level eigenvalue method. According to \(i\)-th evaluation sample and each subsystem index value connection number obtained in step 3, the grade value was calculated by using the level eigenvalue method (Zhang & Zhang 2011):

\[
h_{ik} = \sum_{k=1}^{3} \nu'_ik + g
\]

\[
\text{Formula (18)}
\]

\[
h_i = \sum_{k=1}^{3} \nu'_ik + g
\]

\[
\text{Formula (19)}
\]

where \(h_{ik}\) and \(\nu'_i\) were the WRCC grade value and connection number of the \(k\)-th subsystem of the \(i\)-th evaluation sample; \(h_i\) and \(\nu'_i\) were the WRCC grade value and connection number of the \(i\)-th evaluation sample.

Step 5: calculated the SPP based on index value connection number of the \(i\)-th evaluation sample and each subsystems of regional WRCC; distinguished the state development trend of each subsystem, and then identified the vulnerability subsystem(s) of WRCC system. Set pair potential is also an adjoint function of connection number. Its basic idea is how to transform the connection number which contains the information of uncertainty trend into the overall development trend of certainty, that is, to realize the transformation from uncertainty to certainty, so as to distinguish the development trend.
of the system. Common set pair potentials were: division set pair potential, generalized set pair potential and SPP. The SPP for ternary connection number \( u = a + bl + cf \) had a wide application range and profound physical meaning, focusing on how to reasonably allocate the difference degree component \( b \) to the similarity degree component \( a \) and the opposition degree component \( c \). The SPP of ternary connection number was (Cui et al. 2018):

\[
s'_i(u) = [a + ba] - [c + bc] = (a - c)(1 + b)
\]

where, \( s'_i(u) \) was SPP; \( a = v'_{ai} \), \( b = v'_{bi} \), \( c = v'_{ci} \), when calculating \( s'_i(u) \) of \( i \)-th evaluation sample; \( a = v'_{ai} \), \( b = v'_{bi} \), \( c = v'_{ci} \), when calculating \( s'_i(u) \) of each subsystem.

Step 6: calculated the SPP based on single index value connection number of regional WRCC, and distinguished the state development trend of each index, and then identified the vulnerability index(es) of WRCC system. To illustrate the reliability of this method, TPCN was calculated and compared with the SPP. Meanwhile, the identification results are compared with the local measured data to investigate whether they are consistent, and there are few quantitative studies in this field. The TPCN was as follows (Xie & Guo 2018):

\[
p_1(u) = \frac{a}{(a + b)} + \frac{b}{(b + c)}I_1 + \frac{b}{(a + b)}I_2 + c/(b + c)
\]

where, \( p_1(u) \) was TPCN; \( a = v'_{ai} \), \( b = v'_{bi} \), \( c = v'_{ci} \), \( I_1 = |a/(a + b)|/|a/(a + b) + b/(b + c)| \), \( I_2 = -|c/(b + c)|/|b/(a + b) + c/(b + c)| \) and \( f = -1 \).

According to the theory of SPA (Zhao 1996), The set pair potential of connection number reflected the development trend of water resources carrying state on the macro certainty level. Obviously, there was \( s'_i(u) \in [-1, 1] \), so the SPP can be divided into five equal parts, and the corresponding interval and situation were shown in Table 1.

Among them, the index with SPP at \((-1, -0.2)\) (i.e., counter potential situation or partial counter potential situation) can be considered as the vulnerability index(es) of regional WRCC, which needed to be regulated.

3. EVALUATING AND DIAGNOSING WATER RESOURCES CARRYING CAPACITY IN HUAIBEI, CHINA

Huaibei City is a prefecture-level city in Anhui Province in East China, an important resource-based city in China, and a new modern industrial city. It was built in 1960, built due to coal and developed with coal. By the end of 2019, it has jurisdiction over Xiangshan District, Duji District, Lieszhan District and Suixi County, and five provincial development zones, covering an area of 2,741 km², and a permanent population of about 2,187,000. The industrialization rate is 39%, and the urbanization rate of permanent residents is 65.9%. Huaibei City is located in the semi humid monsoon climate zone of warm temperate zone, with distinct seasons, mild climate and moderate rain. The annual average precipitation is about 832 mm, but the annual variation is large and the distribution is uneven in the year. The maximum annual precipitation is 3.0 times of the minimum annual precipitation; The precipitation in June to September accounts for 65% of the year. Huaibei City is one of the 114 serious water shortage cities in China. The total water resources in the region are less than 400 m³ per capita, and the water shortage situation is more severe. Therefore, it is very important to evaluate the WRCC level and identify the vulnerability subsystem(s) and index(es) of WRCC system accurately.

The dynamic evaluation model of regional WRCC based on SPA and PCN was applied to Huaibei City, Anhui Province. According to the physical formation mechanism of WRCC and referring to the existing research results, the evaluation index system and evaluation grade standards were constructed (Cui et al. 2018), and the weight was determined by AGA-FAHP (Jin et al. 2015), shown in Table 2.

The evaluation sample data of WRCC in Huaibei City came from Anhui statistical yearbook. Combined with Table 2, data for each index were brought into formulas (2)–(10) to obtain the single index connection number of the evaluation sample. It should be pointed out that there was micro motion in the connection number components under the macro state, and the
components can migrate and transform with each other. Therefore, to achieve the certainty and accuracy of the evaluation results, it was necessary to consider the micro motion. The single index new connection number of the evaluation sample can be obtained by bringing each component of formula (10) into formula (14). In order to have a more comprehensive and profound understanding of WRCC in Huaibei City, Anhui Province, firstly, the support force subsystem, regulation force subsystem and pressure force subsystem of WRCC were evaluated and analyzed. The single index connection number components of formula (15) were weighted by formula (16) to get the connection number $v_{ikg}$ of each subsystem, shown in Tables 3–5; then the WRCC system was evaluated and analyzed, meanwhile the single index connection number components of formula (15) were weighted by formula (17) to obtain the connection number $v_{og}$ of evaluation samples. The change of WRCC evaluation grade value and connection number components from 2010 to 2015, are shown in Table 6 and Figures 2 and 3.

According to Tables 3–6 and Figures 2 and 3:

Table 2 | Evaluation index system, evaluation grade standards, evaluation index weight and subsystems of regional WRCC

| Water Resources Carrying Capacity System | Evaluation index | Weight | Grade 1 (Loadable status) | Grade 2 (Critical status) | Grade 3 (Overloaded status) |
|------------------------------------------|------------------|--------|---------------------------|--------------------------|-----------------------------|
| Water resources carrying support force subsystem ($B_1$) | $C_1$ water resources per capita ($m^3$/person) | 0.1332 | ≥ 1670 | 1670–1000 | < 1000 |
| | $C_2$ production modulus of water resources ($10^4 m^3/km^2$) | 0.1332 | ≥ 80 | 80–50 | < 50 |
| | $C_3$ water supply per capita ($m^3$/person-year) | 0.1056 | ≥ 450 | 450–350 | < 350 |
| | $C_4$ rate of vegetation coverage (%) | 0.028 | ≥ 40 | 40–25 | < 25 |
| Water resources carrying regulation force subsystem ($B_2$) | $C_5$ rate of water resources utilization (%) | 0.0396 | ≤ 40 | 40–70 | > 70 |
| | $C_6$ gross domestic product per capita ($10^4 yuan/person$) | 0.0792 | ≥ 24840 | 24840–6624 | < 6624 |
| | $C_7$ standard rate of sewage discharge (%) | 0.0596 | ≥ 90 | 90–70 | < 70 |
| | $C_8$ standard rate of water function area (%) | 0.0792 | ≥ 95 | 95–70 | < 70 |
| | $C_9$ rate of ecological water consumption (%) | 0.0632 | ≥ 5 | 5%–1% | < 1 |
| Water resources carrying pressure force subsystem ($B_3$) | $C_{10}$ daily domestic water consumption per capita ($L/(person\cdot day)$) | 0.0792 | ≤ 70 | 70–180 | > 180 |
| | $C_{11}$ water consumption per $10^4$ yuan ($m^3/10^4$ yuan) | 0.0582 | ≤ 100 | 100–400 | > 400 |
| | $C_{12}$ water consumption per $10^4$ yuan of value-added by industry ($m^3/10^4$ yuan) | 0.0484 | ≤ 50 | 50–200 | > 200 |
| | $C_{13}$ density of population (person/km$^2$) | 0.0288 | ≤ 200 | 200–500 | > 500 |
| | $C_{14}$ rate of urbanization (%) | 0.0454 | ≤ 50 | 50–80 | > 80 |
| | $C_{15}$ water consumption per mu for agricultural irrigation ($m^3/mu$) | 0.0192 | ≤ 250 | 250–400 | > 400 |

Table 3 | Evaluation grade value and SPP of support force subsystem of WRCC in Huaibei City from 2010 to 2015

| Years | $v_{ik1}$ | $v_{ik2}$ | $v_{ik3}$ | $v_{ik1}$ | $v_{ik2}$ | $v_{ik3}$ | $v_{ik1}$ | $v_{ik2}$ | $v_{ik3}$ | Level eigenvalue method | SPP $s_i$ | Situation |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----------------|--------|-----------|
| 2010  | 0      | 0.277  | 0.723  | 0      | 0.335  | 0.665  | 2.665  | −0.888 | Counter potential |
| 2011  | 0      | 0.264  | 0.736  | 0      | 0.321  | 0.679  | 2.679  | −0.897 | Counter potential |
| 2012  | 0      | 0.289  | 0.711  | 0      | 0.346  | 0.654  | 2.654  | −0.880 | Counter potential |
| 2013  | 0      | 0.287  | 0.713  | 0      | 0.343  | 0.657  | 2.657  | −0.883 | Counter potential |
| 2014  | 0      | 0.301  | 0.699  | 0      | 0.355  | 0.645  | 2.645  | −0.874 | Counter potential |
| 2015  | 0      | 0.279  | 0.721  | 0      | 0.336  | 0.664  | 2.664  | −0.887 | Counter potential |
The values of the similarity component, difference component and opposition component changed after the transformation of mobility matrix. Taking the connection number of evaluation sample in 2015 as an example, the original connection number is 0.219, 0.360 and 0.421, which were transformed into 0.209, 0.393 and 0.398. The similarity component and opposition component decreased, while the difference component increased. If the value of the component decreased in the connection number structure system which was not independent of each other, there must be an increasing component. PCN can excavate the micro motion between components and quantify it. It made people realize the result of microscopic movement in macroscopic state.

The grade value of WRCC in Huaibei City was between 2 and 3, that is, between critical overloaded and overloaded status; and indicating WRCC was poor, which was consistent with SPP and its reflected situation (both in partial counter potential). Reference (Cui et al. 2018) reported the construction of SPA evaluation model and its application to WRCC.

Table 4 | Evaluation grade value and SPP of regulation force subsystem of WRCC in Huaibei City from 2010 to 2015

| Years | $v_{a1}$ | $v_{a2}$ | $v_{a3}$ | $v_{b1}$ | $v_{b2}$ | $v_{b3}$ | Level eigenvalue method | SPP $s_1$ | Situation                |
|-------|----------|----------|----------|----------|----------|----------|-------------------------|-----------|-------------------------|
| 2010  | 0.092    | 0.478    | 0.430    | 0.097    | 0.475    | 0.428    | 2.331                   | -0.488    | Partial counter potential|
| 2011  | 0.129    | 0.469    | 0.402    | 0.133    | 0.468    | 0.399    | 2.266                   | -0.390    | Partial counter potential|
| 2012  | 0.142    | 0.478    | 0.380    | 0.148    | 0.478    | 0.374    | 2.226                   | -0.334    | Partial counter potential|
| 2013  | 0.145    | 0.485    | 0.370    | 0.149    | 0.486    | 0.365    | 2.215                   | -0.320    | Partial counter potential|
| 2014  | 0.156    | 0.491    | 0.353    | 0.161    | 0.488    | 0.351    | 2.190                   | -0.283    | Partial counter potential|
| 2015  | 0.162    | 0.490    | 0.348    | 0.168    | 0.487    | 0.345    | 2.177                   | -0.263    | Partial counter potential|

Table 5 | Evaluation grade value and SPP of pressure force subsystem of WRCC in Huaibei City from 2010 to 2015

| Years | $v_{a1}$ | $v_{a2}$ | $v_{a3}$ | $v_{b1}$ | $v_{b2}$ | $v_{b3}$ | Level eigenvalue method | SPP $s_1$ | Situation                |
|-------|----------|----------|----------|----------|----------|----------|-------------------------|-----------|-------------------------|
| 2010  | 0.432    | 0.404    | 0.164    | 0.413    | 0.421    | 0.166    | 1.753                   | 0.351     | Partial identical potential|
| 2011  | 0.444    | 0.408    | 0.148    | 0.424    | 0.426    | 0.150    | 1.726                   | 0.391     | Partial identical potential|
| 2012  | 0.464    | 0.386    | 0.150    | 0.439    | 0.409    | 0.152    | 1.713                   | 0.404     | Partial identical potential|
| 2013  | 0.460    | 0.388    | 0.152    | 0.434    | 0.411    | 0.155    | 1.721                   | 0.394     | Partial identical potential|
| 2014  | 0.465    | 0.380    | 0.155    | 0.439    | 0.404    | 0.157    | 1.719                   | 0.395     | Partial identical potential|
| 2015  | 0.465    | 0.377    | 0.158    | 0.438    | 0.402    | 0.160    | 1.722                   | 0.390     | Partial identical potential|

(1) The values of the similarity component, difference component and opposition component changed after the transformation of mobility matrix. Taking the connection number of evaluation sample in 2015 as an example, the original connection number is 0.219, 0.360 and 0.421, which were transformed into 0.209, 0.393 and 0.398. The similarity component and opposition component decreased, while the difference component increased. If the value of the component decreased in the connection number structure system which was not independent of each other, there must be an increasing component. PCN can excavate the micro motion between components and quantify it. It made people realize the result of microscopic movement in macroscopic state.

(2) The grade value of WRCC in Huaibei City was between 2 and 3, that is, between critical overloaded and overloaded status; and indicating WRCC was poor, which was consistent with SPP and its reflected situation (both in partial counter potential). Reference (Cui et al. 2018) reported the construction of SPA evaluation model and its application to WRCC.

Table 6 | Sample evaluation grade value and SPP of WRCC in Huaibei City from 2010 to 2015

| Years | $v_{a1}$ | $v_{a2}$ | $v_{a3}$ | $v_{b1}$ | $v_{b2}$ | $v_{b3}$ | Level eigenvalue method | SPP $s_1$ | Situation                |
|-------|----------|----------|----------|----------|----------|----------|-------------------------|-----------|-------------------------|
| 2010  | 0.191    | 0.368    | 0.441    | 0.185    | 0.397    | 0.418    | 2.233                   | -0.326    | Partial counter potential|
| 2011  | 0.204    | 0.362    | 0.434    | 0.196    | 0.393    | 0.411    | 2.215                   | -0.299    | Partial counter potential|
| 2012  | 0.214    | 0.366    | 0.420    | 0.205    | 0.398    | 0.397    | 2.192                   | -0.269    | Partial counter potential|
| 2013  | 0.213    | 0.367    | 0.420    | 0.203    | 0.399    | 0.398    | 2.195                   | -0.272    | Partial counter potential|
| 2014  | 0.218    | 0.370    | 0.412    | 0.208    | 0.401    | 0.391    | 2.183                   | -0.257    | Partial counter potential|
| 2015  | 0.219    | 0.360    | 0.421    | 0.209    | 0.393    | 0.398    | 2.190                   | -0.264    | Partial counter potential|
evaluation in Anhui Province. The results showed that the WRCC in Northern Anhui Province was relatively severe. By calculating the index number connection number, index value connection number and their geometric average connection number, then using the level eigenvalue method, the grade values of WRCC in Huaibei City were 2.33, 2.30 and 2.32, respectively. This was in good agreement with the results of this paper, and the relative error was less than 5%. In addition, the reduction of water resources in Northern Anhui was the reason for its long-term overload. Reference (Gao et al. 2013) reported that the evaluation model of comprehensive development and utilization of water resources was constructed by matter-element analysis method and applied to Xiangshan District, Duji District, Lieshan district and Suixi County of Huaibei City. The results showed that the evaluation grade value of Xiangshan district was 3; the WRCC was saturated (i.e., overloaded status); the potential for further development and utilization was small, and the contradiction between supply and demand of water resources was prominent. The evaluation grade value of Duji district and Lieshan district was 2, which indicated that the scale of water resources development and utilization was large, and the potential of water resources utilization was small (i.e., the critical overloaded status). The evaluation grade value of Suixi County was 1, which indicated that the potential and trend of further development and utilization of water resources were optimistic (i.e., the loadable status). The largest area of Huaibei City in Suixi County has few industrial
enterprises and small population concentration, so the demand for water resources is small. It can be seen that the overall WRCC of Huaibei City was between grade 2 and 3, that is, between the critical overloaded and overloaded status, which was basically consistent with the research results of this paper.

(3) The support force subsystem and regulation force subsystem grade value were between 2 and 3 (2.645 ≤ support force subsystem grade value ≤ 2.679, 2.177 ≤ regulation force subsystem grade value ≤ 2.331), and the pressure subsystem grade value was between 1 and 2 (1.713 ≤ pressure force subsystem grade value ≤ 1.753). From the perspective of subsystems, SPP of the support force subsystem was in the interval [−1, −0.6], and its trend was counter potential from 2010 to 2015; SPP of the regulation force subsystem was in the interval [−0.6, −0.2], and its trend was partial counter potential; SPP of the pressure subsystem was in the interval (0.2, 0.6), and its trend was partial identical potential, which showed that the vulnerability subsystems which caused the poor WRCC in Huaibei City were support force subsystem and regulation force subsystem. To make the formulation of control measures more targeted and realize the sustainability of water resources, It is important to accurately find the vulnerability index(es) that led to the critical overloaded and overloaded status, then carry out targeted control to alleviate and solve the current problem of poor WRCC.

Therefore, it was necessary to further diagnose and identify the vulnerability index(es) of WRCC system for regulation. The transformed components $v'_{ijk1}$, $v'_{ijk2}$ and $v'_{ijk3}$ were brought into the formulas (20) and (21) to calculate the average value of SPP and TPCN of WRCC evaluation samples for Huaibei City, shown in Table 7. The annual variation of SPP and TPCN of each index was shown in Figures 4–6.

According to Table 7 and Figures 4–6:

(1) In the support force subsystem of WRCC: the set pair potential of the four evaluation indexes of water resources per capita, production modulus of water resources, water supply per capita and rate of vegetation coverage changed in the interval [−1, −0.6] in 2010–2015, which were counter potential and consistent with the trend of TPCN. It can be seen that the vulnerability indexes of the support force subsystem in Huaibei City were the above four indexes. Among them, water resources per capita, production modulus of water resources and water supply per capita are significantly correlated with the amount of water coming from the area, while the situation of the three indexes were counter potential, indicating that the local water resources are scarce and there was a water shortage phenomenon. According to the water resources bulletin, affected by natural rainfall, topography and other factors, the distribution of water resources in Anhui Province had obvious North-South differences. Taking 2015 as an example, the water resources per capita of Huaibei City was basically consistent with the research results of this paper.

Table 7 | SPP average value of ternary connection number and its situation of WRCC evaluation samples in Huaibei City from 2010 to 2015

| Evaluation indexes                                      | SPP average value | Situation        | TPCN average value |
|---------------------------------------------------------|-------------------|------------------|--------------------|
| $C_1$ water resources per capita (m³/person)              | −0.942            | Counter potential| −1.191             |
| $C_2$ production modulus of water resources (10⁴ m³/km²) | −0.868            | Counter potential| −1.025             |
| $C_3$ water supply per capita (m³/(person · year))       | −0.833            | Counter potential| −0.964             |
| $C_4$ rate of vegetation coverage (%)                    | −0.808            | Counter potential| −0.923             |
| $C_5$ rate of water resources utilization (%)            | −0.758            | Counter potential| −0.845             |
| $C_6$ gross domestic product per capita (10⁴ yuan/person) | 0.599             | Identical potential | 0.648             |
| $C_7$ standard rate of sewage discharge (%)              | −0.747            | Counter potential| −0.828             |
| $C_8$ standard rate of water function area (%)           | −0.376            | Partial counter potential | −0.360             |
| $C_9$ rate of ecological water consumption (%)           | −0.805            | Counter potential| −0.923             |
| $C_{10}$ daily domestic water consumption per capita (L/(person · day)) | 0.356             | Identical potential | 0.364             |
| $C_{11}$ water consumption per 10⁴ yuan (m³/10⁴ yuan)    | 0.790             | Identical potential | 0.894             |
| $C_{12}$ water consumption per 10⁴ yuan of value-added by industry (m³/10⁴ yuan) | 0.674             | Partial identical potential | 0.770             |
| $C_{13}$ density of population (person/km²)             | −0.804            | Counter potential| −0.915             |
| $C_{14}$ rate of urbanization (%)                        | 0.307             | Partial identical potential | 0.290             |
| $C_{15}$ water consumption per mu for agricultural irrigation (m³/mu) | 0.940             | Identical potential | 1.186             |
(located in the north of Anhui Province in China), Hefei City (located in the middle of Anhui Province in China) and Huangshan City (located in the middle of Anhui Province in China) were 276.2 m³, 628.08 m³ and 10914.21 m³, respectively. The water resources per capita of Huangshan City is about 17 times that of Hefei city and 39 times that of Huaibei City. It is noticed that the water resources per capita of Huaibei city changed from 243.41 m³ to 313.18 m³ in 2010–2015, which were about 24.13% (243.41/1008.85 × 100%) and 24.47% (313.18/1279.78 × 100%) of the water resources per capita of Anhui Province in the same period, respectively. In addition, after the last ten days of September 2010, there were 61–73 consecutive days of no effective rainfall days, so autumn and winter droughts occurred in some areas of northern Huaibei. In the three years from 2011 to 2013, the average annual rainfall of Huaibei City was 653.6 mm, 773.2 mm and 729.5 mm, respectively, which did not reach the provincial average annual rainfall of 1,064.4 mm, 1,173.8 mm and 1,025.4 mm, accounting for 61.41% (653.6/1064.4 × 100%), 65.87% (773.2/1173.8 × 100%) and 71.28% (729.5/1025.4 × 100%), respectively. In 2014, Huaibei City had the least surface water resources in Anhui Province, and the surface water

**Figure 4** | Subtraction set pair potential and total partial connection number of each index of support force subsystem in Huaibei City from 2010 to 2015. ($C_1$ = water resources per capita, $C_2$ = production modulus of water resources, $C_3$ = water supply per capita $C_4$ = rate of vegetation coverage. △ represented SPP, □ represented TPCN).

**Figure 5** | Subtraction set pair potential and total partial connection number of each index of regulation force subsystem in Huaibei City from 2010 to 2015. ($C_5$ = rate of water resources utilization, $C_6$ = gross domestic product per capita, $C_7$ = standard rate of sewage discharge, $C_8$ = standard rate of water function area, $C_9$ = rate of ecological water consumption. △ represented SPP, □ represented TPCN).
resources in 2015 continued to decrease compared to 2014. The above analysis indicated that the identification results of the evaluation model were consistent with the local measured data, which showed the accuracy of the method to determine the connection number and the reliability of the evaluation model. Rate of vegetation coverage increased gradually from 2010 to 2015, but it was still counter potential in 2015, which was also an important reason for the poor WRCC. The set pair potential of water supply per capita first increased and then decreased, which showed that the construction capacity of water conservancy facilities, water supply capacity, management capacity of water transmission and distribution process needed to be further improved. It is worth noting that the large-scale water conservancy project of diverting water from the Yangtze River to the Huaihe River under construction will provide available water resources for the relatively water deficient Northern Anhui and further solve the problem of water shortage.

(2) In the regulation force subsystem of WRCC: The set pair potential of rate of water resources utilization, standard rate of sewage discharge and rate of ecological water consumption changed in the interval \([-1, -0.6]\), and the situation was counter potential for a long time, which were the main indexes causing the poor WRCC in Huaibei City. Among them, the set pair potential of rate of water resources utilization continued to decline from 2010 to 2013, which may be related to the drought disaster caused by the low annual average rainfall in Huaibei City, resulting in over exploitation of groundwater. Reference (Zhu 2013) reported that Huaibei City was a semi-arid area of Anhui Province in eastern China. It used to be an area rich in groundwater resources. For many years, the urban human living water and development of industry and agriculture mainly depended on the groundwater resources. After the 1980s, with the growth of population and the development of industry and agriculture, the over exploitation of groundwater had led to the decline of groundwater level, other environmental and social problems (groundwater level from +36 m to \(-12\) m). Although the set pair potential of rate of ecological water consumption increased slowly, it was counter potential. The set pair potential of standard rate of water function area showed an upward trend from 2010 to 2015, but it was still partial counter potential in 2015, indicating that standard rate of water function area was an important indicator of poor WRCC in Huaibei City. The set pair potential of gross domestic product per capita showed an overall upward trend, and the situation fluctuated in the identical potential and partial identical potential, which was a factor to improve and enhance the WRCC. The variation trend of TPCN was the same as that of SPP, which showed that SPP was reasonable to diagnose vulnerability factors.

(3) In the pressure force subsystem of WRCC: In 2010–2015, the set pair potential of density of population changed in interval \([-1, -0.6]\), and the situation was counter potential. According to the water resources bulletin, the density of population of Huaibei City in Anhui province changed from 754.99 person/km² in 2010 to 777.59 person/km² in 2015, indicating that it was the main factor of poor WRCC. The set pair potential of rate of urbanization gradually decreased from 0.462 in 2010 to 0.171 in 2015, with the trend of transition from partial potential to equilibrium potential, which was an important factor to enhance the WRCC.
factor of poor WRCC. The trend of daily domestic water consumption per capita, water consumption per 10^4 yuan and water consumption per mu for agricultural irrigation was identical potential, which can not be considered as the cause of the poor WRCC. The set pair potential of water consumption per 10^4 yuan of value-added by industry gradually increased from −0.008 in 2010 to 0.812 in 2015, which was an important factor to improve the WRCC of Huaibei City. It was related to the promotion of water saving, the use of water-saving products and the overall improvement of workers’ operation technology.

The above analysis results showed that the method of determining the connection number proposed in this paper was reasonable. The evaluation model of WRCC based on SPA and PCN can examine the micro motion between the connection number components, and quantitatively express the micro motion, so as to make the evaluation results more objective and accurate; the vulnerable subsystems and indexes causing poor WRCC in Huaibei City were accurately identified. This method can describe the microscopic movement of the connection number more deeply, had strong explanatory power, and the recognition result was reliable. It can quantitatively distinguish the relative deterministic carrying status and its development trend under the current macro state.

4. CONCLUSION

In this paper, an evaluation index system and the evaluation grade standard of the regional WRCC were constructed by the principle that the regional water resources support force, regulation force and pressure force interact to form the regional water resources carrying status. Then, according to the basic principle and connotation of SPA and PCN, a method to determine the connection number was proposed. Finally, a dynamic evaluation model of regional WRCC based on SPA and PCN was constructed and applied to Huaibei City, Anhui Province. The following conclusions can be obtained:

(1) In 2010–2015, the evaluation grade value of water resources capacity of Huaibei City was between 2 and 3, and it was in the critical overloaded and overloaded status for a long time. The vulnerable subsystem which caused the poor water resources carrying capacity of Huaibei City was the support force subsystem and regulation force subsystem. The vulnerability indexes that caused the poor WRCC were: water resources per capita, production modulus of water resources, water supply per capita and rate of vegetation coverage in the support force subsystem. The water resource shortage was very obvious in Huaibei City. Rate of water resources utilization, standard rate of sewage discharge, rate of ecological water consumption in the regulation subsystem and density of population in the pressure subsystem need to be artificially regulated to alleviate or solve the current situation of poor. In addition, gross domestic product per capita and water consumption per 10^4 yuan of value-added by industry were two important indicators to improve the WRCC of Huaibei City. The above evaluation and diagnosis results were in good agreement with the existing research results, indicating that the construction of the evaluation model was reasonable and effective.

(2) The migration matrix constructed based on the PCN principle can quantitatively express the balance mechanism of micro motion between the connection number components, and clearly see the micro evolution under the macro state, thus the connection number can be determined. Based on the components of connection number determined by the method in this paper, the vulnerability subsystems and indexes of Huaibei WRCC system can be accurately identified by SPP; the trend of SPP was consistent with that of the TPCN, and the identification results were in good agreement with the local measured data. It can be seen that the method of determining connection number proposed in this paper was reasonable and effective. The constructed model can accurately identify the vulnerability subsystems and indexes, which is an intelligent method of ‘system state trend analysis’.

(3) The evaluation model of regional WRCC based on SPA and PCN, the status of regional WRCC was evaluated, and the vulnerable subsystems and indexes causing the poor WRCC were diagnosed and identified, which has strong adaptability. It provided a new way for the determination of connection number, the dynamic analysis of WRCC system, the judgment of its development trend, and had the value of popularization and application in water resources research and decision-making management.

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CONFLICT OF INTEREST

The authors have declared no conflict of interest.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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