Pseudogap, Superconducting Gap, and Fermi Arcs in Underdoped Cuprate Superconductors

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Through the measurements of magnetic field dependence of specific heat in \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) in zero temperature limit, we find that the nodal slope \( v_\Delta \) of the superconducting gap has a very similar doping dependence of the pseudogap temperature \( T^* \) or value \( \Delta_p \). Meanwhile the maximum quasiparticle gap derived from \( v_\Delta \) is quite close to \( T^* \). Both indicate a close relationship between the pseudogap and superconductivity. It is also found that \( T_c \approx \beta v_\Delta \gamma_n(0) \), where \( \gamma_n(0) \) is the extracted zero temperature value of the normal state specific heat coefficient which is proportional to the size of the residual Fermi arc \( k_{arc} \). These observations mimic the key predictions of the SU(2) slave boson theory based on the general resonating-valence-bond (RVB) picture.

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Since the discovery of the cuprate superconductors, 18 years have elapsed without a consensus about its mechanism. Many exotic features beyond the Bardeen-Cooper-Schrieffer theory have been observed. One of them is the observation of a pseudogap in the electron spectral function near the antinodal points \((\pi,0)\) and \((0,\pi)\) at a temperature \( T^* \gg T_c \). In a conventional BCS superconductor, this gapping process occurs simultaneously with the superconductivity at \( T_c \). It has been heavily debated about the relationship between the pseudogap and the superconductivity in cuprates. One scenario assumes that the pseudogap \( \Delta_p \) marks only a competing or coexisting order with the superconductivity and it has nothing to do with the pairing origin. However another picture, namely the Anderson’s resonating-valence-bond (RVB) model (and its offspring) predicts that the spin-singlet pairing in the RVB state (which causes the formation of the pseudogap) may lend its pairing strength to the mobile electrons and make them to naturally pair and then to condense at \( T_c \). According to this picture there should be a close relationship between the pseudogap and the superconductivity.

In order to check whether this basic idea is correct, we need to collect the information for both gaps, especially their doping dependence. The pseudogap values \( \Delta_p \) (or its corresponding temperature \( k_B T^* \sim \Delta_p \)) and its doping dependence have been determined through experiments, but it turns out to be a very difficult job to determine the superconducting gap since both gaps entangle into each other in the superconducting state. One exception is left, however, in the small region of momentum space near the nodal point where the pseudogap is generally assumed to be zero above \( T_c \) and the superconducting gap opens below \( T_c \). Therefore to detect the weak gap information (or the gap slope \( v_\Delta = [d\Delta_s/d\phi]_{\text{node}}/h k_F \)) near nodal point in the zero temperature limit becomes highly desired. Previous results using, for example, angle-resolved photoemission (ARPES) or superfluid density seem to be inconclusive due to either temperature limitation (ARPES above 10 K) or unexpected difficulty in analyzing the data (e.g., a so-called Fermi liquid correction factor \( \alpha_{FL} \) is inevitably involved in analyzing the low temperature data of superfluid density).

In this paper, we report the evidence of a proportionality between the nodal slope \( v_\Delta \) of the superconducting gap and the pseudogap temperature \( T^* \). Remarkably the maximum quasiparticle gap derived from \( v_\Delta \) is also quite close to \( T^* \). We also find that \( T_c \) is controlled by both the gap slope \( v_\Delta \) and the size of the Fermi arcs \( (k_{arc}) \) in the underdoped normal state. Both observations are anticipated by the SU(2) slave boson theory based on the general RVB picture.

We determine the properties of the nodal quasiparticles by measuring the low temperature electronic specific heat. The \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) single crystals measured in this work were prepared by travelling solvent floating-zone technique. Samples with seven different doping concentrations \( p=0.063(T_c=9K) \), nominal \( x=0.063 \), post-annealed in \( \text{Ar} \) gas at 800°C for 48 hrs, 0.069 (\( T_c=12K \), as-grown sample with \( x=0.063 \), 0.075 (\( T_c=15.6K \), nominal \( x=0.07 \) and post-annealed in \( \text{O}_2 \) gas at 750°C for 12 hrs), 0.09 (\( T_c=24.4K \), as grown, \( x=0.09 \), 0.11 (\( T_c=29.3K \), as grown, \( x=0.11 \), 0.15 (\( T_c=36.1K \), nominal \( x=0.15 \), 0.22 (\( T_c=27.4K \), nominal \( x=0.22 \)) have been investigated. The quality of our samples has been characterized by x-ray diffraction, and \( R(T) \) data showing a narrow transition \( \Delta T_c \leq 2 \) K. The samples have also been checked by AC and DC magnetization showing also quite narrow transitions. The full squares in Fig.3 represent the transition temperatures of our samples. The heat capacity presented here was measured with the relaxation method based on an Oxford cryogenic system.
FIG. 1: Field dependence of $\Delta \gamma = [C(H) - C(0)]/T$ normalized by the data at about 12 T in zero temperature limit. It is clear that Volovik’s $\sqrt{H}$ relation describes the data rather well for all samples. The inset (a) shows the typical original data of $\Delta \gamma$ vs. $T$ for the underdoped sample $p = 0.069$. The inset (b) shows the same set of data as $\Delta \gamma/\sqrt{H}$ vs. $T$. One can clearly see that in zero temperature limit $\Delta \gamma/\sqrt{H}$ is a constant for all fields implying the validity of the Volovik’s relation $\Delta \gamma = A\sqrt{H}$.

In all measurements the magnetic field was applied parallel to c-axis. As also observed by other groups for La$-214$ system, the anomalous upturn of $C/T$ due to the Schottky anomaly of free spins is very weak. This avoids the complexity in the data analysis. Details about the sample characterization, the specific heat measurement, the residual linear term and extensive analysis are reported in a recent paper[4].

It has been widely perceived that the pairing symmetry in the hole doped cuprate superconductors is of d-wave with line nodes in the gap function. In the mixed state, due to the presence of vortices, Volovik[10] pointed out that supercurrents around a vortex core lead to a Doppler shift to the quasi-particle excitation spectrum. This will dominate the low energy excitation and the specific heat (per mol) behaves as $C_{\text{vol}} = A\sqrt{H}$ with $A \propto 1/v_\Delta$. This square-root relation has been verified by many measurements which were taken as evidence for d-wave symmetry, for example by specific heat[12, 13, 14, 16, 17, 18], thermal conductivity[19], tunnelling (to measure the Doppler shift of the Andreev bound states)[21, 22], NMR[22], etc. In this way one can determine the nodal slope ($v_\Delta$). Since the phonon part of the specific heat is independent on the magnetic field, this allows to remove the phonon contribution by subtracting the $C/T$ at a certain field with that at zero field, one has $\Delta \gamma = \Delta C/T = [C(H) - C(0)]/T = C_{\text{vol}}/T - \alpha T$ with $\alpha$ the coefficient for the quasiparticle excitations of a d-wave superconductor at zero field ($C_\nu = \alpha T^2$). In the zero temperature limit $\Delta \gamma = C_{\text{vol}}/T = A\sqrt{H}$ is anticipated.

In order to get $\Delta \gamma$ in the zero temperature limit, we extrapolate the low temperature data of $C/T$ vs. $T^2$ (between 2K to 4K) to zero K. The data taken in this way and normalized at 12 T are presented in the main panel of Fig.1. It is clear that the Volovik’s $\sqrt{H}$ relation describes the data rather well for all doping concentrations. This is to our surprise since it has been questioned whether the Volovik relation is still obeyed in the underdoped regime[10] especially when competing orders are expected to appear[23, 24, 25] and impurity scattering is present. We attribute the success of using the Volovik relation here to three reasons: (1) We use $\Delta \gamma = |C_H|e - C_H|e|/T$ instead of using $\Delta \gamma = |C_H|e - C_{\text{HLL}}|e|/T$. The latter may inevitably involve the unknown DOS contributions from other kinds of vortices (for example, Josephson vortices) when $H \perp C$. (2) The contribution from a second competing order to $\Delta \gamma$ may be small compared to the Volovik’s term in the zero temperature limit. (3) Single crystals today have much better quality leading to much weaker impurity scattering. To have a self-consistent check of the $\gamma/\sqrt{H}$ relation found in the zero temperature limit, we plot the data of $\Delta \gamma/\sqrt{H}$ vs. $T$ at finite temperatures. A typical example for the very underdoped one ($p = 0.069$) is shown in the inset (a) and (b) of Fig.1. One can see that in the low temperature region the data $\Delta \gamma/\sqrt{H}$ scale for all fields ranging from 1 T to 12 T, showing the nice relation $\Delta \gamma \propto \sqrt{H}$ for this sample in the zero temperature limit. From here one can also determine the prefactor $A$ in $\Delta \gamma = A\sqrt{H}$ (here for example, $A = 0.28$ for $p = 0.069$) and then compare backwards to the value determined from the data shown in the main panel leading to of course the same value. The same feature appears for all other doping concentrations. For clarity they will not be shown here.

This successful scaling of $\Delta \gamma$ vs. $\sqrt{H}$ makes it possible to derive the pre-factor $A$, and one can further determine the gap slope $v_\Delta$. Fig.2(a) shows the doping dependence of the pre-factor $A$. For a typical d-wave superconductor, by calculating the excitation spectrum near the nodes, it was shown that[18]

$$A = \alpha_p \frac{4k_B^2}{3\hbar c} \sqrt{\frac{\pi}{\Phi_0}} \frac{nV_{\text{mol}}}{v_\Delta} \quad (1)$$

here $l_c = 13.28$ Åis the c-axis lattice constant, $V_{\text{mol}} = 58$ cm$^3$ (the volume per mol), $\alpha_p$ a dimensionless constant taking 0.5 (0.465) for a square (triangle) vortex lattice, $n = 2$ (the number of Cu-O plane in one unit cell), $\Phi_0$ the flux quanta. The $v_\Delta$ has then been calculated without any adjusting parameter (taking $\alpha_p = 0.465$) and shown in Fig.2(b). It is remarkable that $\Delta \gamma$ has a very similar doping dependence as the pseudogap temperature $T^*$, indicating that $v_\Delta \propto T^* \propto \Delta_p$. If converting the data $v_\Delta$ into the virtue maximum quasiparticle gap ($\Delta_\gamma$) via $v_\Delta = 2\Delta_\gamma/a$ with $a = 3.8\text{Å}$(the in-plane lattice constant), surprisingly the resultant $\Delta_\gamma$ value [shown by the filled squares in Fig.2(b)] is quite close to $T^*$ ($\Delta_\gamma \sim 1.3k_BT^*$).
It is important to stress that this result is obtained without any adjusting parameters. Counting the uncertainties in determining $T^*$ and the value of $\alpha_p$, this relation is remarkable since $\Delta_p$ and $T^*$ are determined in totally different experiments. Because $v_\Delta$ and $\Delta_p$ reflect mainly the information near nodes which is predominantly contributed by the superconducting gap, above discovery, i.e., $v_\Delta \propto T^* \propto \Delta_p$ (or $\Delta_q \sim k_B T^*$) strongly suggests a close relationship between the superconductivity and the pseudogap. A similar conclusion was drawn in underdoped YBa$_2$Cu$_3$O$_y$ by analyzing the low temperature thermal conductivity. Since the pseudogap is supposed to be caused by the formation of the RVB state, our results here point to a fact that the RVB singlet pairing may be one of the unavoidable ingredients for superconductivity.

In the following we will investigate what determines $T_c$. Bearing the doping dependence of $v_\Delta$ in mind, it is easy to understand that $v_\Delta k_F$ should not be a good estimate of the superconducting gap for underdoped samples. The basic reason is that the normal-state Fermi surfaces are small arcs of length $k_{\text{arc}}$ near the nodal points. The superconducting transition only gaps the Fermi arc. So the effective superconducting gap should be estimated as $\Delta_s \sim \frac{1}{2} v_\Delta k_{\text{arc}}$. From the normal state electronic specific heat $C_{\text{ele}} = \gamma_n T$, $\gamma_n \sim \frac{2\pi k_B^2 k_{\text{arc}}}{V_{\text{mol}}} n v_F$, we can obtain $k_{\text{arc}}$. Assuming $\Delta_s \sim k_B T_c$, we find

$$T_c = \frac{\alpha_s}{4 n k_B^2 V_{\text{mol}}} \frac{\hbar v_F c \gamma_n v_\Delta}{\eta} = \beta \gamma_n v_\Delta$$

where $\alpha_s$ is a dimensionless constant. $v_F$ is the nodal Fermi velocity normal the Fermi surface. The value of $\gamma_n(0)$ can be estimated from specific heat, or indirectly by ARPES or NMR. Here we take the values for $\gamma_n(0)$ summarized by Matsuzaki et al. and fit it (in unit of $mJ/mol K^2$) with a formula $\gamma_n = \zeta (p - p_c)^n$ yielding $\zeta = 182.6$, $p_c = 0.03$, $\eta = 1.54$. In Fig.3 we present the doping dependence of the really measured $T_c$ (filled squares) and the calculated value (open squares) by eq.(2) with $\beta = 0.7445 K^3 mols/Jm$. In underdoped region, the really measured and calculated $T_c$ values coincide rather well implying the validity of eq.(2). In the overdoped region, $\gamma_n$ will gradually become doping independent, therefore one expects $T_c \propto v_\Delta$. Using $\beta = 0.7445 K^3 mols/Jm$ and taking $\alpha_s = 13.8$, we get $v_\Delta = 2.73 \times 10^4 cm/s$ which is the so-called universal nodal velocity determined by ARPES. So the

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**FIG. 2:** (a) Doping dependence of the pre-factor $A$ determined in present work (full circles). Here the point at $p=0.19$ was adopted from the work by Nohara et al. on a single crystal. (b) Doping dependence of the pseudogap temperature $T^*$ (open symbols) summarized in Ref.3 and our data $v_\Delta$ (solid line). $T^*$-susceptibility refers to the pseudogap temperature determined from the maxima in the static susceptibility, and $T^* - \rho$ to the temperature at which there is a slope change in the DC resistivity. Above $T^* - \rho$ the resistivity has a linear temperature dependence. The full squares represent the calculated virtue maximum quasi-particle gap $\Delta_p$ derived from $v_\Delta$ without any adjusting parameters. Surprisingly both set of data are close to each other ($\Delta_p \sim 1.3 k_B T^*$) although they are determined in totally different experiments. This result implies a close relationship between the pseudogap $\Delta_p$ and the superconducting gap slope $v_\Delta$.

**FIG. 3:** Doping dependence of the really measured superconducting transition temperature $T_c$ (full squares) and that calculated by $T_c = \beta T_\Delta \gamma_n(0)$ (open squares) with $\beta = 0.7445 K^3 mols/Jm$. The solid line represents the empirical relation $T_c/T_{c_{\text{max}}} = 1 - 82.6(p - 0.16)^2$ with $T_{c_{\text{max}}} = 38 K$. The inset shows the doping dependence of $\gamma_n$ derived from specific heat (open squares), ARPES (open circles), and Knight shift (up-triangles). The solid line is a fit to the data with $\gamma_n = \zeta (p - p_c)^\eta$. The data with $\gamma_n = \zeta (p - p_c)^\eta$ yield $\zeta = 182.6$, $p_c = 0.03$, $\eta = 1.54$. In underdoped region, the really measured and calculated $T_c$ values coincide rather well implying the validity of eq.(2). In the overdoped region, $\gamma_n$ will gradually become doping independent, therefore one expects $T_c \propto v_\Delta$. Using $\beta = 0.7445 K^3 mols/Jm$ and taking $\alpha_s = 13.8$, we get $v_\Delta = 2.73 \times 10^4 cm/s$ which is the so-called universal nodal velocity determined by ARPES. So the
energy scale of the superconductivity is not given by \( v_{\Delta} \hbar k_F \sim \Delta_p \), but by \( \frac{1}{2} v_{\Delta} \hbar k_{\text{arc}} \) or more precisely by eq. (2).

To have a framework about the experimental results, in the following, we will review one particular explanation based on the slave-boson approach. Within the SU(2) slave-boson theory, the pseudogap metallic state is viewed as a doped algebraic spin liquid (ASL). A doped ASL is described by spinons (neutral spin-1/2 Dirac fermions) and holons (spinless charge-e boson) coupled to a U(1) gauge field. Due to the attraction between the spinons and the holons caused by the U(1) gauge field, a spinon and a holon recombine into an electron at low energies. Due to the spin-charge recombination, the pseudogap metallic state is described by electron-like quasiparticles at low energy. Since the binding between the spinon and the holon is weak, the large pseudogap near the anti-nodal points (\( \pi, 0 \)) and (0, \( \pi \)) is not affected. So the Fermi surface of the recombined electrons cannot form a large closed loop. A simple theoretical calculation suggests that the Fermi surface of the recombined electrons forms four small arcs near the nodal points (\( \pm \pi/2, \pm \pi/2 \)). Thus the SU(2) slave boson theory contains two important features: the pseudogap due to spin singlet pairing and the Fermi arcs due to the spin-charge recombination. The superconductivity arises from the condensation of the quasiparticles on the arcs, thus one expects that \( T_c \) is proportional to the gap on the Fermi arc: \( k_B T_c \sim \frac{1}{2} v_{\Delta} \hbar k_{\text{arc}} \), instead of the pseudogap \( \Delta_p \) near the anti-nodal points. Meanwhile, since the spin pairing is responsible for both the pseudogap \( \Delta_p \) near the anti-nodal points and the superconducting gap slope \( v_{\Delta} \), one finds that \( v_{\Delta} \) is proportional to \( T^* (\propto \Delta_p) \) or \( \Delta_p \sim k_B T^* \) in the simplest version of the slave-boson theory. These are exactly what we found in the experiment.

In summary, the Volovik’s relation of the d-wave pairing symmetry has been well demonstrated by low temperature specific heat in wide doping regime in \( La_{2-x}Sr_xCuO_4 \). From here the nodal slope \( v_{\Delta} \) of the superconducting gap is derived and is found to follow the same doping dependence of the pseudogap \( \Delta_p \). Meanwhile it is found that the superconducting transition temperature \( T_c \) is controlled by \( v_{\Delta} \gamma_n(0) \) instead of \( v_{\Delta} \). Both observations are consistent with the SU(2) slave boson theory based on the general RVB picture.

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