Nuclear incompressibility: An analytical study on leptodermous expansion

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Abstract

A comparative study of the liquid-drop model (LDM) type expansions of energy $E$ and compression modulus $K_A$ is made within the energy density formalism using Skyrme interactions. As compared to the energy expansion, it is found that, in the pure bulk mode of density vibration, the LDM expansion of $K_A$ shows an anomalous convergence behaviour due to pair effect. A least squares fit analysis is done to estimate the minimum error, one would expect even with synthetic data due to the inherent nature of the LDM expansion of $K_A$ as well as the narrow range of accessible mass number $A$, in the values of the various coefficients. The dependence of the higher-order coefficients like curvature and Gauss curvature on the coupling $\beta_c$ between the bulk and surface parts of the monopole vibrations is analytically studied. It is shown that the $K_A$ expansion including the dynamical effect ($A$ dependence of $\beta_c$) shows an ‘up-turn’ behaviour below mass number about 120, suggesting the inapplicability of the LDM expansion of $K_A$ over this mass region.

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I. INTRODUCTION

Nuclear matter incompressibility $K_\infty$ is of fundamental significance as it is an important ingredient in the nuclear equation of state, which influences several astrophysical phenomena as well as high energy heavy-ion collision processes. The nuclear breathing mode or isoscalar giant monopole resonance (GMR) is the most promising source of information on $K_\infty$.

However, the nuclear breathing mode actually determines the finite nuclear compression modulus $K_A$, and not $K_\infty$. Hence, to determine the value of $K_\infty$ one has to extrapolate from finite nuclei to nuclear matter, which is a highly non-trivial task. Inspired by the success of the nuclear mass formulae, a liquid-drop model (LDM) like approach is currently being adopted to extract $K_\infty$ from its finite nuclear value $K_A$. In this approach, one first expresses the finite nuclear compression modulus $K_A$ in terms of the experimental breathing-mode energies $E_{gmr}$ using the relation,

$$K_A = m < r^2 > E_{gmr}^2/\hbar^2 \tag{1}$$

where the experimental values of the mean square radius $< r^2 >$ are used; and then one supposes an LDM expansion of $K_A$ to be valid, in analogy to the semi-empirical mass formula;

$$K_A = K_v + K_s A^{-1/3} + K_e A^{-2/3} + K_{Cov} Z^2 A^{-4/3} + K_\beta \beta^2 \tag{2}$$

where $\beta$ is the asymmetry parameter. The volume coefficient $K_v$ determined from a fit to the available breathing mode energies is identified with $K_\infty$, which is strictly valid only in the scaling model.

Following this approach, Sharma et al [1, 2] determined $K_v$ to be about 300 MeV using their data on various Sn and Sm isotopes, and on those of $^{24}Mg$, $^{90}Zr$ and $^{208}Pb$, which was later contested by Pearson [3]. Subsequently, Shlomo and Youngblood [4] made an extensive least-squares fit analysis by taking into account the available set of data on breathing-mode numbering about 46, spread over a mass region $28 \leq A \leq 232$. They concluded that this complete data set is not adequate to determine the value of $K_\infty$ to
better than a factor of about 1.7 \[200 - 350 \text{ MeV}\]. Further, they also observed in their analysis that a free least-squares fit to Eq.(2) leads to errors exceeding 100% in all the coefficients. As observed by Shlomo and Youngblood, such large errors may arise partly due to the break-down of the scaling approximation over light nuclei, deformation effects etc. Another reason may be that the presently available data are inadequate for fixing all the parameters in Eq.(2). We feel that, possible slow convergence and correlations among the coefficients, will also contribute toward this error. However, it is not possible to pinpoint what fraction of the error arises out of the quality and inadequacy of the data, and that which arises due to the nature of convergence of the LDM expansion of $K_A$. It is therefore desirable to make an extensive theoretical study of the nature of convergence and the effect of correlations to arrive at a conclusion.

Studies in this regard have been attempted\[5-9\] over the years and have normally been done in the scaling model, since an analytical relation between the experimental breathing-mode energies and $K_A$ is available only within this model. But, it is known from the works of Brack and his collaborators\[10-12\] that there can exist several types of density vibration, the scaling mode being a particular one. It was also shown by them in a hydrodynamical approach that, the degree of coupling between the bulk and surface parts of the density vibrations, sensitively depends upon the mass number $A$.

Let us consider the ground-state density of a nucleus to be given by a Fermi-function

$$\rho(r) = \rho_o /[1 + e^{(r-R)/\alpha}]$$

where $\rho_o$, $R$ and $\alpha$ are the central density, half density radius and diffuseness parameters respectively. When it undergoes monopole vibrations, the density function will be compressed or decompressed. In this state, it is assumed that the density function can still be represented by a Fermi-function. Then, following Brack and Stocker\[11\], one has

$$\rho_c(r) = \frac{\rho_c}{1 + e^{(r-R)/\alpha_c}}$$

where $\rho_c$ and $\alpha_c$ are defined as

$$\rho_c = \rho_o q$$

$$\alpha_c = a \beta_c = a (\rho_c/\rho_o)^{\beta_c}$$

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The parameter $q$ represents the amount of compression or decompression, $\beta_c$ gives the degree of coupling between the bulk and surface parts of the density vibration and $\alpha_c$ is the corresponding value of the surface diffuseness. The scaling property of the half-density radius $R$ is defined by the number equation given by Eqs.(6,12). In literature[10-12], the mode of vibration pertaining to $\beta_c = -1/3$ is normally referred to as scaling mode of vibration. In realistic hydrodynamical calculations [11, 12], which describe the experimental data on breathing-mode reasonably well, it is found that $\beta_c$ can vary from -0.23, in the case of $^{208}$Pb, to -0.84 for $^{90}$Zr. Further, $\beta_c$ tends to $\pm \infty$ for lighter nuclei like $^{48}$Ca [$\beta_c = -24$] and $^{40}$Ca [$\beta_c = +9.5$], i.e., pure surface like mode.

More importantly, when one makes use of an LDM expansion of $K_A$ to fit the experimental GMR data, the various finite-size coefficients in the expansion(2), such as surface $K_s$ and curvature $K_c$, must be $A$-independent, as in the case of standard nuclear mass formulae. Due to the mass number dependence of the dynamical coupling parameter $\beta_c$, there is still a residual mass dependence in these coefficients obtained theoretically. The asymptotic value of $K_s$ obtained in the limit $A \rightarrow \infty$ is about $-2K_v$, and corresponds to the pure bulk mode $\beta_c=0$. This large, negative value of $K_s$ shall lead to a slower converging $K_A$-expansion as compared to the one obtained in the scaling model, where $K_s \approx -K_v$. Therefore, it is of interest and relevance to study the convergence properties of the LDM expansion of $K_A$ for this particular mode. This has been stated[11] to be a well converging series, which may not be conclusive as we shall see.

In view of the above discussions, we make an analytical study of the LDM expansion of $K_A$ derived with $\beta_c=0$. It is found that this particular expansion shows an anomalous convergence behaviour in the sense that certain higher-order terms are equal in magnitude and opposite in sign to the preceding lower-order terms in the expansion, resulting in, what is termed here as, ‘pair effect’. Secondly as the LDM expansion of $K_A$ is relatively new compared to the well established energy expansion, it is worthwhile to make a comparative study of both these expansions at a fundamental level. Thirdly, we also make a least-squares fit analysis to get an idea regarding the minimum error in the val-
ues of the various coefficients, one would expect even with synthetic data because of the inherent nature of the LDM expansion as well as, the inadequacy of the narrow range of $A$ to fix the higher-order coefficients. Finally, to ascertain the goodness of the $K_A$-expansion for the extraction of the various coefficients, we have studied the $\beta_c$ dependence of curvature $K_c$ and Gauss curvature $K_o$ coefficients, and thereby extending the pocket model calculations to higher-order coefficients. It is found that, the validity of the LDM expansion of $K_A$ in particular cases such as scaling and pure bulk mode, does not necessarily guarantee unambiguous extraction of $K_\infty$ in realistic situations. This is shown by taking the dynamical effect (the $A-$ dependence of $\beta_c$) into account, where the nuclear incompressibility $K_A$ is found to exhibit an ‘up-turn’ behaviour below mass number about 120, which may be of non-leptodermous origin.

II. THE MODEL

Here, we analytically derive the LDM expansion of nuclear incompressibility $K_A$ starting from first principles, and using the concept of leptodermous expansion of energy. The model considered here is essentially the same as the one given in Ref.[13], which we have now generalised to consider various modes of monopole vibrations.

We consider a symmetric system of $A$ fermions, without the Coulomb interaction, whose total energy $E$, in the framework of energy density formalism[14] is given by,

$$E = \int \varepsilon [\rho(r)] d^3r$$  \hspace{1cm} (5)

with the number constraint equation,

$$A = \int \rho(r) d^3r$$  \hspace{1cm} (6)

where,

$$\varepsilon(r) = \frac{\hbar^2}{2m} \tau(r) + v(r)$$  \hspace{1cm} (7)

For the kinetic part $\tau(r)$, we use the extended Thomas-Fermi functional[15] upto $O(h^2)$, which for a symmetric system is

$$\tau(r) = \frac{3}{5}(1.5\pi^2)^{2/3} \rho^{5/3} + \frac{1}{36} \frac{(\nabla \rho)^2}{\rho} + \frac{1}{3} \nabla^2 \rho$$  \hspace{1cm} (8)
For the potential part $v(r)$, we use the standard Skyrme forces, without the spin-orbit contribution,

$$v(r) = \frac{3t_0}{8} \rho^2 + \frac{t_3}{16} \rho^{\sigma+2} + \frac{3t_1 + 5t_2}{16} \tau \rho + \frac{9t_1 - 5t_2}{64} (\nabla \rho)^2$$ (9)

Since our objective is to make an analytical study, we do not attempt to solve the Euler-Lagrangian equations for self-consistent densities. Further, to systematically study the general nature of the LDM compressibility coefficients and their dependence on the coupling parameter $\beta_c$, it is desirable to parametrise the density function $\rho(r)$ to be a Fermi function. Therefore, the ground-state density function $\rho(r)$ for a nucleus is given as

$$\rho(r) = \frac{\rho_o}{1 + e^{\frac{r-R}{a}}}$$ (10)

where $\rho_o$, $R$, and $a$ are the central density, half-density radius and the diffuseness parameter respectively. In order to obtain analytical expressions for the various finite-size compressibility coefficients in terms of the coupling parameter $\beta_c$, we define the general density distribution corresponding to a compressed/decompressed state to be of the form given by Eq.(3). Thus, our whole derivation is of general nature, and the ground-state properties can be determined by imposing $q = \rho_c/\rho_o = 1$.

Normally, one expands the finite nuclear ground-state energy into volume, surface etc using Taylor’s series. Then, each coefficient in the expansion is calculated (for e.g., see [8]) making use of the semi-infinite nuclear matter approximation. Instead, we here arrive at the LDM expansion of $K_A$ using the generalised Sommerfeld lemma [14, 17]. One can then systematically express the total energy as a sum of volume, surface, curvature and Gauss curvature. Further, as we are interested in the convergence properties of the LDM expansion of $K_A$, we go upto Gauss curvature order in both the energy and compressibility expansions.

Then, the total energy $E$ at any state of compression/decompression given by Eq.(5) results in the leptodermous expansion of $E$ as,

$$E = E_v \frac{4 \pi R^3}{3} + E_s 4 \pi R^2 + E_c 8 \pi R + E_o 4 \pi$$ (11)
The terms $E_v$, $E_s$, $E_c$ and $E_o$ are the volume, surface, curvature and Gauss curvature contributions to the total energy $E$. Then, the half-density radius $R$ can be given in terms of $\rho_c$, by using Eq.(6) as

$$R \simeq r_{oc} A^{1/3} \left[ 1 - \frac{\pi^2 \rho_c^2}{3 r_{oc}^2 A^{2/3}} \right]$$

(12)

where $\frac{4\pi^3}{3} \rho_c = 1$. Substituting the above expression for $R$ in Eq.(11), and retaining terms up to the order of $A^{-1}$, we obtain

$$\frac{E}{A} = e_v^* + e_s^* A^{-1/3} + e_c^* A^{-2/3} + e_o^* A^{-1}$$

(13)

The expressions for the different coefficients in the above equation are

$$e_v^* = p_\alpha \rho_c^{2/3} + p_0 \rho_c + p_1 \alpha \rho_c^{5/3} + p_3 \rho_c^{\sigma + 1}$$

(14)

$$e_s^* = -(36\pi)^{1/3} \alpha_c (0.759 p_\alpha \rho_c - \frac{p_\beta}{2 \alpha_c} \rho_c^{1/3} + p_0 \rho_c^{4/3} + 1.359 p_1 \rho_c^2
- \frac{[p_2 + p_1 (\beta - \gamma)]}{6 \alpha_c^2} \rho_c^{4/3} - p_3 g_{3s}^\sigma \rho_c^{\sigma + 4/3})$$

(15)

$$e_c^* = 4 \alpha_c^2 (6\pi^2)^{1/3} ( (1.517 - \frac{\pi^2}{6}) p_\alpha \rho_c^{4/3} + \frac{p_\beta}{2 \alpha_c} \rho_c^{2/3} + (1.973 - \frac{\pi^2}{6}) p_1 \alpha \rho_c^{7/3}
+ (0.5 g_{3c}^\sigma - \frac{\pi^2}{6}) p_3 \rho_c^{\sigma + 5/3})$$

(16)

$$e_o^* = -4 \pi \alpha_c^3 ((2.602 - \frac{2 \pi^2}{3} 0.759) p_\alpha \rho_c^{5/3} + \frac{\pi^2}{6} \frac{p_\beta}{\alpha_c^2} \rho_c^2 + (4.423 - \frac{2 \pi^2}{3} 1.359) p_1 \alpha \rho_c^{8/3}
- \frac{\pi^2}{3} p_0 \rho_c^2 + \frac{[p_2 + p_1 (\beta - \gamma)]}{\alpha_c^2} \left( \frac{\pi^2}{18} + \frac{1}{3} \right) \rho_c^2 - (g_{3o}^\sigma - \frac{2 \pi^2}{3} g_{3s}^\sigma) p_3 \rho_c^{\sigma + 2})$$

(17)

where $p_\alpha = \frac{\kappa^2 \alpha \rho_c}{2 m}$, $p_\beta = \frac{\kappa^2 \beta}{2 m}$, $p_0 = \frac{3 \kappa_0}{8}$, $p_3 = \frac{\eta_1}{16}$, $p_1 = \frac{3 \kappa_1 + 5 \kappa_3}{16}$, $p_2 = \frac{9 \kappa_2 - 5 \kappa_3}{64}$, and $\alpha = \frac{3}{5} (1.5 \pi^2)^{2/3}$, $\beta = \frac{1}{36}$ and $\gamma = \frac{1}{3}$. The quantities $g_{3s}^\sigma$, $g_{3c}^\sigma$ and $g_{3o}^\sigma$ are calculated using the integral

$$\eta_{\nu}^{(k)} = (-1)^k \int_0^\infty u^k \left[ 1 + \frac{(-1)^k e^{-u \nu}}{(1 + e^{-u})\nu} - 1 \right] du$$

(18)

where $\nu = \sigma + 2$, and $g_{3s}^\sigma = \eta_{\nu}^{(0)}$, $g_{3c}^\sigma = 2 \eta_{\nu}^{(1)}$ and $g_{3o}^\sigma = \eta_{\nu}^{(2)}$. 

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It must be noted that all the coefficients in Eq. (13) are function of the ground-state values of central density $\rho_o$ and the diffuseness parameter $a$, and also are function of the compression variable $q$ and the coupling parameter $\beta_c$. $\rho_o$ and $a$ are normally determined using the energy minimisation criteria. Due to the finite compressibility of the nuclear fluid, both the quantities could be functions of the size of the system, characterised by their mass number $A$. Therefore, Eq. (13) can not be identified as the LDM expansion of the nuclear energies.

To extract the $A$-dependence of the energy coefficients in Eq. (13), we define $\rho_o(A) = \rho_\infty + \delta \rho$, where $\rho_\infty$ is the symmetric infinite nuclear matter saturation density. $\rho_o(A)$ can be determined using the saturation condition,

$$\left. \frac{d(E/A)}{d\rho} \right|_{\rho_o} = 0$$

Remembering that the above derivative is a total one, we define

$$\rho_o \left[ \frac{d(E/A)}{d\rho_c} \right]_{\rho_c = \rho_o} = \left[ \frac{d(E/A)}{dq} \right]_{q = 1}$$

The saturation condition given by Eq. (19) now becomes,

$$e'_v + e'_s A^{-1/3} + e'_c A^{-2/3} + e'_o A^{-1} + e'_f A^{-4/3} + e'_h A^{-5/3} = 0$$

(21)

Here, the prime denotes total differentiation of $e_i^*$'s with respect to the central density variable. To determine all the coefficients at the density $\rho_\infty$ of symmetric infinite nuclear matter, we use the definition

$$\rho_o = \rho_\infty + \delta \rho$$

(22)

where $\delta \rho$ contains all the possible finite-size effects.

Making Taylor’s expansion of all the coefficients in Eq. (21) around $\rho_\infty$ up to $O(\delta \rho)^2$, we obtain,

$$\delta \rho = - \left[ \frac{e'_s A^{-1/3} + e'_c A^{-2/3} + e'_o A^{-1}}{e'_v + e'_s A^{-1/3} + e'_c A^{-2/3} + e'_o A^{-1} + 0.5(e'_v + e'_s A^{-1/3} + \cdots) \delta \rho} \right]_{\rho_\infty}$$

(23)
As the above equation contains $\delta \rho$ on the right hand side also, we determined the same by an iterative method. The zeroth order expression of $\delta \rho$ is equal to the above equation without the bracketed term in the denominator.

To arrive at the LDM expansion of $E/A$ and $K_A$, we should be able to expand $\delta \rho$ in powers of $A^{-1/3}$. Unlike in Ref. [13], here we consider the influence of higher-order terms like $e''_{s}$, $e''_{c}$ etc by making a binomial expansion of the denominator in Eq.(23). Considering terms upto third order, we have,

$$
\left[ 1 + \left( \frac{e''_{s}}{e''_{v}} - \frac{e''''_{s} e''_{c}}{2(e''_{v})^2} \right) A^{-1/3} + \cdots \right]^{-n} \\
\simeq 1 + f_1(n)A^{-1/3} + f_2(n)A^{-2/3} + \cdots
$$

where,

$$
\begin{align*}
f_1(n) &= -n \left[ \frac{e''_{s}}{e''_{v}} - \frac{e''''_{s} e''_{c}}{2(e''_{v})^2} \right] \\
f_2(n) &= -n \left[ \frac{e''_{c}}{e''_{v}} - \frac{e''''_{c} e''_{v}}{2(e''_{v})^2} \right] \\
&- n \left[ \frac{e''''_{s} e''_{c} e''_{v}}{2(e''_{v})^3} - \frac{e''''_{s} e''_{v}^2}{6(e''_{v})^3} \right] \\
&+ \frac{n(n+1)}{2} \left[ \frac{e''_{s}}{e''_{v}} - \frac{e''''_{s} e''_{c}}{2(e''_{v})^2} \right]^2.
\end{align*}
$$

It may be recalled here that prime denotes total differentiation with respect to the central density variable for a fixed value of $\beta_c$. One then has for any arbitrary function $f \left[ \rho_c, \alpha_c(\rho_c) \right]$,

$$
\begin{align*}
f' &\equiv \left( \frac{df}{d\rho_c} \right) \bigg|_{\rho_o} = \frac{1}{\rho_o} \left( \frac{df}{dq} \right) \bigg|_{q=1} \\
f'' &\equiv \left( \frac{d^2 f}{d\rho_o^2} \right) \bigg|_{\rho_o} = \frac{1}{\rho_o^2} \left( \frac{d^2 f}{dq^2} \right) \bigg|_{q=1}
\end{align*}
$$

A. LDM expansion of energy

Now, the LDM expansion of $E/A$ can be obtained in a straight-forward manner by performing Taylor’s expansion of each of the term in Eq.(13) around $\rho_{\infty}$ as,

$$
\frac{E}{A} = e''_{v}(\rho_{\infty}) + e''_{s}(\rho_{\infty}) A^{-1/3} + \cdots + e''_{s}(\rho_{\infty}) A^{-1}
$$
Then, by using Eqs.(23-24) in the equation (26), and grouping terms with same power of $A$, the complete LDM expansion of $E/A$ upto $O(A^{-1})$ is

$$
\frac{E}{A} = a_v + a_s A^{-1/3} + a_c A^{-2/3} + a_0 A^{-1}
$$

(27)

where the various LDM coefficients in the above equation are defined as,

$$
a_v = e_v^*(\rho_\infty)
$$

$$
a_s = e_s^*(\rho_\infty)
$$

$$
a_c = e_c^*(\rho_\infty) + (d_{11} e'_s + \frac{1}{2} (d_{22} e''_s))
$$

$$
a_0 = e_0^*(\rho_\infty) + (d_{11} e'_c + d_{12} e'_s) + \frac{1}{2} (d_{22} e''_s + d_{23} e''_v) + \frac{1}{6} (d_{33} e'''_v)
$$

(28)

It may be noted that unlike the leptodermous expansion given by equation (11), the LDM expansion of energy is an infinite series. This infinite nature comes due to the $A^{-1/3}$ expansion of $\delta \rho$, which is derived by making a binomial series. Therefore, we feel it is essential to go up to at least Gauss curvature order in the LDM expansions of energy and compressibility, while one studies their convergence properties.

The explicit expressions for the various factors $d_{11}, d_{12}$ etc are;

(i) $d_{11} = -\frac{1}{e_v''} e'_s$

$$
d_{12} = -\frac{1}{e_v''} (e'_c + e'_s f_1(1))
$$

$$
d_{13} = -\frac{1}{e_v''} (e'_0 + e'_c f_1(1) + e'_s f_2(1))
$$

(ii) $d_{22} = \frac{1}{e_v''} e''_s$

$$
d_{23} = \frac{1}{e_v''} (2 e'_s e'_c + e'_s^2 f_1(2))
$$

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The important point to be noted regarding these factors is that they all have an explicit dependence upon $K_\infty$, which in turn controls the convergence behaviour of the LDM expansion. However, in the case of energy, the two leading coefficients volume and surface are independent of these factors, and thereby $K_\infty$. Only the higher-order terms like curvature and Gauss curvature are dependent on $K_\infty$.

**B. LDM expansion of nuclear compression modulus $K_A$**

In the following paragraphs, we derive the LDM expansion of $K_A$. The finite nuclear compression modulus $K_A$ is calculated using the definition,

$$K_A = 9\rho_o^2 \left( \frac{d^2 E/A}{d\rho_c^2} \right)_{\rho_c=\rho_o} = 9\frac{d^2 E/A}{dq^2} |_{q=1}$$

for a given value of $\beta_c$.

Making use of equations (27-28) in the above definition, we have

$$K_A = K_v^*(\rho_o) + K_s^*(\rho_o)A^{-1/3} + K_c^*(\rho_o)A^{-2/3} + K_o^*(\rho_o)A^{-1}$$

where,

$$K_i^*(\rho_o) = 9\rho_o^2 \frac{d^2 e_i^*}{d\rho_c^2} |_{\rho_o} = 9\frac{d^2 e_i^*}{dq^2} |_{q=1} ; \quad i = v, s, c, \text{ and } o$$

Now, the LDM expansion of each of the term in Eq.(31) around $\rho_\infty$ as,

$$K_A = K_v^*(\rho_\infty) + K_s^*(\rho_\infty)A^{-1/3} + \cdots + K_o^*(\rho_\infty)A^{-1} + \left( K_v^{**}(\rho_\infty) + K_s^{**}(\rho_\infty)A^{-1/3} + \cdots + K_o^{**}(\rho_\infty)A^{-1} \right) \delta \rho$$

$$+ \frac{1}{2} \left( K_v^{***}(\rho_\infty) + K_s^{***}(\rho_\infty)A^{-1/3} + \cdots + K_o^{***}(\rho_\infty)A^{-1} \right) (\delta \rho)^2$$

$$+ \frac{1}{6} \left( K_v^{****}(\rho_\infty) + K_s^{****}(\rho_\infty)A^{-1/3} + \cdots + K_o^{****}(\rho_\infty)A^{-1} \right) (\delta \rho)^3 + \cdots$$

$$= d_{33} = -\frac{1}{e_v^*e_s^*e_c^*}.$$
Here, we have included all the terms up to $O(\delta\rho)^3$, so that correct estimates up to Gauss curvature order i.e. $O(A^{-1})$, can be obtained, which is desirable in this study of the convergence behaviour of the LDM expansion of $K_A$.

Then, by using Eqs.(23-24) in the equation (33) and grouping terms with same power of $A$, the complete LDM expansion of $K_A$ up to $O(A^{-1})$ is

$$K_A = K_v + K_s A^{-1/3} + K_c A^{-2/3} + K_o A^{-1}$$

(34)

where the various LDM coefficients in the above equation are defined as,

\[
K_v = K_v^s(\rho_\infty) \\
K_s = K_s^s(\rho_\infty) + (d_{11} K_v') \\
K_c = K_c^s(\rho_\infty) + (d_{11} K_c' + d_{12} K_v') + \frac{1}{2} (d_{22} K_v'') \\
K_o = K_o^s(\rho_\infty) + (d_{11} K_c' + d_{12} K_s' + d_{13} K_v') \\
+ \frac{1}{2} (d_{22} K_s'' + d_{23} K_v'') + \frac{1}{6} (d_{33} K_v''')
\]

(35)

The explicit expressions for the various factors $d_{11}, d_{12}$ etc are as given in Eq.(29). It may be noted that unlike in the case of energy expansion, all the finite-size coefficients ($K_s, K_c, K_o$) explicitly depend upon $K_\infty$ through the factors $d_{11}, d_{12}$ etc.

**III. RESULTS AND DISCUSSIONS**

In this section, we discuss the basic nature of an LDM expansion of energy and incompressibility. In addition, the dependence of the various compressibility LDM coefficients on the coupling parameter $\beta_c$ is also studied.

**A. Structure of the LDM coefficients and dependence on $\beta_c$**

The first and the important point is that in the case of energy, the bulk part totally decouples itself from all the surface effects due to the saturation condition $e'_v(\rho_\infty) = 0$, in the leading order $\delta\rho$, which is evident from Eq.(26). This, in turn, results in the two important LDM energy coefficients $a_v$ and $a_s$, given in Eq.(28), being pure in nature, in the sense that there are no contributions arising from $\delta\rho$. In contrast, in the case of
compressibility, the volume gets strongly coupled to all the surface effects even through the leading order in $\delta \rho$, and hence, the surface coefficient $K_s$ has an additional term $d_{11}K'_v$, which explicitly depends upon $K_\infty$ through $d_{11}$. This fact contributes to the essential difference between the LDM expansion of $E/A$ and $K_A$. As it can be seen from Eqs.(28) and (35), it also gives rise to additional terms in the higher-order coefficients in the expansion of $K_A$. With this observation, we now calculate the values of the various LDM coefficients in Eqs.(27) and (34).

We calculated the various LDM energy coefficients using the analytical expressions given in Eq.(28) for the three Skyrme forces SkM*, SkA and S3. In this calculation, only the curvature $a_c$ and Gauss curvature $a_o$ coefficients have a $\beta_c$ dependence. We found that both these coefficients are almost invariant with respect to $\beta_c$. Therefore, we have used $\beta_c = 0$ while calculating the energy coefficients, as is normally done. Values obtained for the LDM energy coefficients using Eq.(28) are given in Table I for the three Skyrme forces. The values obtained here for the various LDM coefficients agree qualitatively well with those found in literature [15]. In regard to the nuclear curvature, it may be noted that the semi-classical ETF models [18, 19] give a value of about 10 MeV, as against the value of about zero, determined [20] from experiments. This is the so-called nuclear curvature energy anomaly, which is yet to be resolved.

Similarly, we calculate the values of compressibility coefficients using Eq.(35). In this calculation, we have shown our results for three particular values of $\beta_c$ in Table II. The ratio $|K_s/K_v|$ is also tabulated. It can be seen that as $\beta_c$ decreases from $\beta_c = 0$ to $\beta_c = -1/2$, $|K_s/K_v|$ also decreases. This behaviour of $K_s$ is in agreement with that of the pocket model [10]. Further, we find the scaling model result $K_s \simeq -K_v$ is found to be well satisfied for $\beta_c = -1/3$.

In our study we have also calculated the higher-order coefficients like $K_c$ and $K_o$ for different values of $\beta_c$. The value of $K_c$ remains more or less unchanged with respect to $\beta_c$. Due to this, as $\beta_c$ becomes more and more negative, curvature effect becomes as important as the surface. This may be due to the fact as $\beta_c$ becomes more and more negative, the
finite-size effects play a progressively important role in the breathing vibrations of a nucleus. Further, from the values of all finite-size compressibility coefficients like \( K_s, K_c \) etc given in Table II, it can be seen that \( \beta_c \geq 0 \) will lead to a relatively slow converging series compared to the other negative values of \( \beta_c \). In other words, as \( \beta_c \) becomes more and more positive, \( |K_s/K_v| \) increases.

It may be mentioned here that in Ref. [13], we had calculated the coefficients using \( \beta_c = 0 \) and neglecting terms like \( e_s' A^{-1/3} \), \( e_c' A^{-2/3} \) etc in the denominator of Eq.(23). Now retaining those terms and performing binomial expansion as given in Eq.(24), we find that the values of \( K_c \) and \( K_o \) obtained with \( \beta_c = 0 \) get substantially modified. For reasons discussed in the introduction, we presently study the convergence properties of the leptodermous expansion of \( K_A \) with \( \beta_c = 0 \), and also make a comparative study of the LDM expansions of energy and compressibility.

**B. Convergence behaviour and Pair effect**

To study the convergence of the LDM expansion of both energy and compressibility, it is necessary to evaluate exactly the values of energy \( E \) and compressibility \( K_A \) corresponding to symmetric (N=Z) systems, which will then be compared with the sum of all the terms given by the right hand side of Eqs.(27) and (34) respectively. We determine the total energy numerically using Eqs.(5-10) and then, \( K_A \) can be obtained using Eq.(30) with \( \beta_c = 0 \). Here, the diffuseness parameter \( a \) is held fixed at its semi-infinite nuclear matter (SINM) value. The values so obtained for four representative nuclei in the range \( 40 \leq A \leq 200 \) are presented in Tables III and IV for the energy and compressibility respectively for Skyrme forces SkM*, SkA and S3. It can be seen that in the case of energy, both the results agree extremely well, i.e. up to about second decimal place, and even, in the case of compressibility the agreement is quite good. This result implies that the convergence of the LDM expansion of \( K_A \) is almost as good as the LDM expansion of \( E/A \).

To understand this inference in a better way, we have presented in Tables V and VI, the contributions of the successive terms in the energy and compressibility expansions.
respectively, for the three forces and, for two representative mass numbers $A=40$ and $200$. It can be seen that in the case of energy, the successive terms decrease by a factor of about 5, giving rise to a rapidly converging series. In contrast to this, we find in the case of compressibility, [see Table VI], the Gauss curvature term $K_0A^{-1}$ is of the same order as the curvature term $K_cA^{-2/3}$. In fact, for values of $A$ below about 150, $K_0A^{-1}$ even overshoots $K_cA^{-2/3}$. Same is also found to be true in our calculation for the higher-order terms $K_hA^{-5/3}$ and $K_fA^{-4/3}$. Further, it is interesting to note that $K_cA^{-2/3}$ and $K_0A^{-1}$ are almost equal in magnitude, but opposite in sign, which is also the case with $K_f$ and $K_h$. Hence, we find that the terms of higher order than the surface one in the LDM expansion of $K_A$ cancel pairwise. This *pair* effect gives rise to a misleading conclusion regarding the importance of higher-order terms in the LDM expansion of $K_A$, unless otherwise investigated. Thus, the LDM expansion of $K_A$ in the case of pure bulk mode shows an anomalous behaviour, in the sense that $K_0A^{-1} \simeq -K_cA^{-2/3}$, in contrast to the rapidly converging energy expansion.

In addition, it may be mentioned here that the convergence properties of the LDM expansion of $K_A$ in the case of scaling mode has been widely studied[6-9], going up to order of curvature term. In our study, including terms up to Gauss curvature order, we arrive at similar conclusions. Although, *pair* effect is not observed in the scaling case, the rate of convergence is still found to be relatively slow as compared to the energy expansion.

Further, we would like to state that the above results were found to remain valid when one uses more realistic ETF functionals including $h^4$ terms, and with a generalised Fermi-function for $\rho(r)$. In view of this new feature in the convergence behaviour of the LDM expansion of $K_A$, the question arise: whether we can extract the various coefficients in the LDM expansion of $K_A$ using a least squares fit analysis, which may manifest the effect of correlations amongst the coefficients.

**IV. LEAST SQUARES FIT ANALYSIS**

In the earlier section, we have obtained the exact values of energy $E/A$ and compressibility $K_A$ numerically and also, the values for the various LDM coefficients in their
expansions for SkM*, SkA and S3 forces, and have established their goodness. Now, in this section, we would like to determine these coefficients again from a least-squares fit to the exact values of $E/A$ and $K_A$ (referred to as synthetic data) for a set of nuclei. A comparison of the two sets of values will then demonstrate clearly the reliability of the LDM expansions of $E/A$ and $K_A$ for the extraction of nuclear matter properties and, the surface properties of finite nuclei. This will also establish the convergence behaviour of both the expansions.

A. For symmetric case

Here, we attempt to determine all the coefficients in the LDM expansion of $K_A$ pertaining to symmetric systems (N=Z) by making a free least squares fit to the numerically calculated values of $K_A$ using Eqs.(5-10,30), for 210 nuclei in the mass range $40 \leq A \leq 250$. In Tables VII and VIII, we have presented the results so obtained for $E/A$ and $K_A$ respectively, for the SkA force only, whose value of $K_\infty$ lies in between those of SkM* and S3 forces. The corresponding error in all the coefficients calculated using the standard method [21] are also given. The first row shows the exact values of the coefficients obtained in the previous section, which are presented for comparison.

It can be seen from Tables VII and VIII that, in the case of both energy and compressibility, the volume term is quite well determined and its value progressively improves with the inclusion of higher-order terms. Same is almost true for the surface coefficients. And, the higher-order terms such as curvature and Gauss curvature are relatively ill-determined. This is presumably due to correlations amongst the various coefficients, which is clear from Eqs.(28) and (35). Thus, we find that in the case of symmetric systems without the Coulomb effect, the principal coefficients like volume compressibility $K_v$ and surface compressibility $K_s$ can be extracted reliably by means of a least square fit.

B. Inclusion of Coulomb and asymmetry effects

The Coulomb force plays an important role in real nuclei, and also, because of its influence in the extraction of $K_\infty$ from the breathing-mode data as shown by Pearson
and Shlomo & Youngblood [4], it is worthwhile to consider the effect of Coulomb interaction in our present analysis.

To investigate this, we repeat our calculation including the Coulomb force and asymmetry effect, for many realistic nuclei lying within the mass region $40 \leq A \leq 250$. In this calculation of total energy (5), we suppose the neutron and proton density distributions at the ground-state to be of the form,

$$\rho_l(r) = \frac{\rho_{ol}}{1 + e^{\frac{r-R_l}{a_l}}}; \quad l = n, p \quad (36)$$

where the various parameters are as defined in Eq.(10), and are determined by energy minimisation criteria. Also, for the sake of simplicity, we have used the same value of diffuseness parameter for both proton and neutron density distributions, $a_n = a_p$. The Coulomb energy is given as $E_{Coul} = 0.6Z^2e^2/R$, where we have considered the nucleus to be a uniformly charged sphere of radius $R$ as is normally done [6, 8]. One can then calculate $K_A$ using the total energy expression (5) including asymmetry and Coulomb effects in Eq.(30).

Now, we make a least squares fit to the so-obtained theoretical values of $K_A$ numbering 210. The results so obtained from the fit for, the case of compressibility is given in Table IX for the SkA force. We find that, in the 4-parameter fit involving $K_v, K_s, K_{Coul}$ and $K_\beta$, the extraction of all these coefficients, except for the symmetry compressibility $K_\beta$, are reliable. The discrepancies in $K_v$ and $K_s$ as compared to the exact values are about 4% and 12% respectively. With the introduction of a surface-asymmetry $K_{s\beta}$ term, i.e. in the 5-parameter fit, the estimate for $K_\beta$ improves, however, at the cost of important coefficients like $K_v, K_s$ and $K_{Coul}$. To see the effect of a curvature term, we made a 5-parameter fit ($K_v, K_s, K_{Coul}, K_\beta$ & $K_c$) to the 210 model data on $K_A$. As it can be seen, introduction of a curvature term as a free parameter somewhat spoils the whole fit, introducing a maximum of discrepancy of about 40% in $K_v$.

Therefore, from our analysis using synthetic data, it can be concluded that inspite of the correlations among various coefficients, it may be possible to extract $K_\infty$. However, it must be mentioned that even with such synthetic data, inclusion of higher-order terms
like $K_c$ and $K_{s\beta}$ as free parameters in the fit, lead to poorer determination of the volume coefficient $K_\infty$, and also other coefficients. Thus, the present theoretical study clearly points out the goodness and limitations of the LDM expansion of $K_A$ and the extent of error inherently present in this approach, in regard to the determination of $K_\infty$.

V. DYNAMICAL EFFECTS

Normally, the convergence properties of the LDM expansion of $K_A$ are mostly studied within the scaling model. This is because, only within this model, one can easily relate $K_A$ to the experimental data on GMR. However, in general, the mode of density vibrations is neither scaling-like nor pure bulk-like. Hence, we need to examine the convergence properties of $K_A$—expansion taking into account this dynamical effect.

To do so, firstly, we need to relate to the GMR data for any mode of monopole vibrations. In other words, we need to find a general empirical relation analogous to Eq.(1), and thereby, one can obtain experimental $K_A$ values from the GMR data without making the scaling assumption. Once $K_A$ values are known from the GMR data, one may then use Eq.(2) to extract $K_\infty$. Hence, in the following, we address two aspects: Firstly, how can one obtain a general relation between $K_A$ and experimental $E_{gmr}$?, Secondly, how does $K_A$ behave under the most general conditions?

In the appendix, we discuss the above mentioned first aspect using the hydrodynamical approach. This justifies our study of the $K_A$—expansion in the generalised situation, i.e. without using the scaling assumption. With this, we now focus upon the second aspect, i.e. the general behaviour of $K_A$ taking into account the $A$—dependence of $\beta_c$, in the following.

Within our analytical model, we calculate realistic values of $K_A$ using the general expression

$$K_A(\beta_c) = K_v + K_s(\beta_c)A^{-1/3} + K_c(\beta_c)A^{-2/3} + K_{Coul}Z^2A^{-4/3} + K_\beta \beta^2$$

(37)

where $K_{Coul}$ & $K_\beta$ are respectively the Coulomb and asymmetry compressibility coefficients and the asymmetry parameter $\beta = (N - Z)/A$. The values for $K_{Coul}$ and $K_{sym}$
obtained with SkM* force are $-4.70$ MeV and $-349.0$ MeV. In the calculation of $K_{C_{ou}}$, we have considered only the direct Coulomb term $a_C = 0.6e^2/r_o$. Realistic values of $K_s$ and $K_c$ are obtained as follows.

For a given nucleus $(A,Z)$, the optimum value of $\beta_c$ that will give rise to an excitation energy $\hbar \omega$ close to the experimental value can be obtained using\[11\],

$$\beta_c = \bar{\beta} - \sqrt{\bar{\beta}^2 + 1} \quad (38)$$

where $\bar{\beta}$ is related to $A$ as

$$\bar{\beta} = 0.685A^{1/3} - 2.15 \quad (39)$$

So, for a given $A$ and the corresponding $\beta_c$ obtained from Eq.(37), we have calculated $K_s$ and $K_c$ for the SkM* force using Eq.(38). The values thus obtained for $K_s$ and $K_c$ are plotted as a function of $\beta_c$ in Figs.(1) and (2) respectively. It can be seen that as $\beta_c$ varies from $-0.6$ to $0.0$, $K_s/K_v$ varies from from about $-0.5$ to $-2$. On the other hand, variation of $K_c$ is not so prominent. Further, it is interesting to note that value of $K_c$ shows a minimum at about the scaling model value ($\beta_c = -1/3$). An important point to be noted is that since $\beta_c$ is A-dependent, and $K_s$ & $K_c$ are $\beta_c-$dependent, the surface and curvature compressibility coefficients have a residual mass dependence. To obtain the unique $A-$independent value of $K_s$ and $K_c$, one should take the limit $A \rightarrow \infty$, which leads to $\beta_c = 0$. The asymptotic value of $K_s$ thus obtained is nothing but the value $K_s \simeq -2K_v$, as noted earlier\[2\]. And, the true asymptotic value of curvature compressibility coefficient is the value of $K_c$ obtained with $\beta_c = 0$ plus the contribution coming from the $\beta_c$ dependence of $K_s$.

Using the values of $K_s$ and $K_c$ calculated for the SkM* force with the optimum values of $\beta_c$ in Eq.(38), we plot in Fig.(3), the so-determined $K_A(\beta_c)/K_v$ values as a function of $A^{-1/3}$. For sake of comparison, we have shown in Fig.(4) the values of $K_A(\beta_c)/K_v$ obtained for three particular values of $\beta_c$. It may be mentioned that, for $A \geq 250$, the Coulomb force is switched off and $N=Z$. It can be clearly seen from Fig.(3) that $K_A$ shows an ‘ up - turn ’ behaviour as against the nice linear behaviour found for the three
particular values of $\beta_c$ in Fig.(4). This up-turn behaviour or change in the slope may be suggesting the onset of the breakdown of the leptodermous expansion of $K_A$ below mass number approximately 120. Indications for such an increasing nature of $K_A$ can also be found from the hydrodynamical calculations [11].

In analysing real data, one should indeed expect such an ‘up-turn’ behaviour as against the nice linear behaviour obtained under scaling or pure bulk mode assumption, which suggests that higher-order coefficients like $K_c$ become important over medium and low mass regions. Because of this ‘up-turn’ behaviour, it may be difficult to consistently determine all the parameters in Eq.(2) from a fit to the presently available few tens of data on GMR, which in-turn shall impair the extraction of the important quantity, $K_\infty$.

VI. CONCLUSION

In conclusion, the LDM expansion of $K_A$ in the pure bulk mode, shows an anomalous convergence behaviour due to pair effect, as compared to the widely studied scaling mode. It is also found that the $K_A$– expansion in both the cases is relatively quite slow, as compared to the energy expansion. However, $K_\infty$ can be reliably extracted in these specific cases. In realistic situations, one also encounters modes of density vibrations, other than the scaling and pure bulk modes, depending upon the mass region under consideration. When this dynamical effect is taken into account, the nuclear compression modulus shows an ‘up-turn’ behaviour below mass number about 120, suggesting the inapplicability of the LDM expansion of $K_A$ over this mass region.
APPENDIX

Here an attempt is made to obtain a generalised empirical relation between the nuclear compression modulus $K_A$ and the experimental breathing-mode energies $E_{gmr}$. Until now, a well-defined relation $E_{gmr} = \sqrt{\hbar^2 K_A / (m < r^2 >)}$, is available only in the case of scaling model.

Brack and Stocker using a variational hydrodynamical approach found that, the mass paramter $B(\beta_c, A)$ shows a regular behaviour with respect to both coupling paramter $\beta_c$ and mass number $A$. Using their results displayed in Table 1 of Ref., we have plotted $B(\beta_c, A)/m < r^2 >$ as a function of $A^{-1/3}$ in Fig.(5). It can be seen that the general mass paramter $B(\beta_c, A)$ expressed in terms of the scaling model value $m < r^2 >$ varies quite smoothly with respect to $A$. Hence, $B(\beta_c, A)$ can be expressed as

$$B(\beta_c, A) \sim m < r^2 > f(A)$$

where $f(A) = c_0 + c_1 A^{-1/3} + \cdots$ can be a polynomial in $A^{-1/3}$. The crucial function $f(A)$ can be determined from hydrodynamical calculations, which will be reported elsewhere. Once $f(A)$ is known, one can determine $K_A$ from GMR data using the relation

$$\hbar \omega = \sqrt{\frac{K_A(\beta_c)}{B(\beta_c, A)}} \approx \sqrt{\frac{K_A(\beta_c)}{m < r^2 > \cdot f(A)}}$$

Thus, a generalised expression relating $K_A$ and $E_{gmr}$, analogous to the well-known scaling relation, is proposed. This proposition is on sound footing as the hydrodynamical studies are successful in describing the GMR data.
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TABLE CAPTIONS

**Table I:** Values of the various coefficients in the LDM expansion of the energy (27) for the Skyrme forces SkM*, SkA and S3 using the local ETF functional with $\hbar^2$ terms and a pure Fermi-function for the density distribution (3). All quantities are in MeV.

**Table II:** Values of the various coefficients in the LDM expansion of the compressibility (34) for the Skyrme forces SkM*, SkA and S3 using the local ETF functional with $\hbar^2$ terms and a pure Fermi-function for the density distribution. Three values of $\beta_c$ have been used. All quantities are in MeV.

**Table III:** Comparison of the exact values of the total energy per nucleon $E/A$ (5) with the analytically determined values using the LDM expansion of $E/A$, given by Eq. (27), for four representative mass numbers $A$, and for the Skyrme forces SkM*, SkA and S3. All quantities are in MeV.

**Table IV:** Comparison of the exact values of the finite nuclear compression modulus $K_A$ (30) obtained using $\beta_c = 0$ with the analytically determined values using the LDM expansion of $K_A$, given by Eq. (34), for four representative mass numbers $A$, and for the Skyrme forces SkM*, SkA and S3. All quantities are in MeV.

**Table V:** Values of the different terms contributing to the total energy per nucleon obtained using the Skyrme forces SkM*, SkA and S3, for two representative mass numbers $A$. All quantities are in MeV.

**Table VI:** Values of the different terms contributing to the nuclear incompressibility
obtained using the Skyrme forces SkM*, SkA and S3 with $\beta_c = 0$, for two representative mass numbers $A$. All quantities are in MeV.

**Table VII:** Values of the parameters obtained from a least-squares fit to the exact values of energy per nucleon $E/A$ obtained for symmetric systems using the SkA force. The number of data points used is 210, in the mass region $40 \leq A \leq 250$. The first row gives the exact values for the various coefficients obtained in our analytical model. All quantities are in MeV.

**Table VIII:** Values of the parameters obtained from a least-squares fit to the exact values of nuclear incompressibility $K_A$ obtained for symmetric systems using the SkA force with $\beta_c = 0$. The number of data points used is 210, in the mass region $40 \leq A \leq 250$. The first row gives the exact values for the various coefficients obtained in our analytical model. All quantities are in MeV.

**Table XI:** Values of the parameters obtained from a least-squares fit to the exact values of nuclear incompressibility $K_A$ obtained for asymmetric systems with Coulomb interaction, using the SkA force and taking $\beta_c = 0$. The number of data points used is 210, in the mass region $40 \leq A \leq 250$. The first row gives the exact values for the various coefficients. Value of $K_{s,\beta}$ obtained within the scaling model is taken from Ref.[8]. All quantities are in MeV.
FIGURE CAPTIONS

FIG. 1 Values of the mass parameter $B(\beta_c, A)$ expressed in terms of $m < r^2>$, obtained in a hydrodynamical calculation\[1\] with SkM* force is plotted versus $A^{-1/3}$, where $m$ is the nucleon mass and $< r^2 >$ is the root mean square radius.

FIG. 2 Values of the ratio of the surface compressibility coefficient $K_s$ to the nuclear matter incompressibility $K_v$ obtained in our analytical model using SkM* force is shown as a function of the coupling parameter $\beta_c$.

FIG. 3 Same as Fig. 2, but for the curvature compressibility coefficient $K_c$.

FIG. 4 Values of the ratio of finite nuclear compression modulus $K_A$ to $K_v$ obtained including the dynamical effect ($A-$ dependence of $\beta_c$) is shown versus $A^{-1/3}$. The force used is SkM*.

FIG. 5 Values of the ratio of finite nuclear compression modulus $K_A$ to $K_v$ obtained for three particular values of $\beta_c$ is shown versus $A^{-1/3}$. The force used is SkM*.
### Table I

|     | SkM* | SkA | S3  |
|-----|------|-----|-----|
| $a_v$ | -15.79 | -16.01 | -15.87 |
| $a_s$ | 19.08 | 19.95 | 18.90 |
| $a_c$ | 10.24 | 9.67 | 6.87 |
| $a_0$ | -12.21 | -11.42 | -7.24 |

### Table II

| Force   | $\beta_c$ | $K_s$(MeV) | $K_c$(MeV) | $K_o$(MeV) | $K_s/K_v$ |
|---------|-----------|------------|------------|------------|-----------|
| SkM*    | 0         | -406.1     | -109.9     | 568.1      | -1.87     |
| $(K_v = 217$ MeV) | -1/3      | -231.0     | -129.3     | 138.1      | -1.07     |
|         | -1/2      | -129.1     | -118.0     | -69.1      | 0.6       |
| SkA     | 0         | -484.6     | -123.7     | 595.0      | -1.84     |
| $(K_v = 263$ MeV) | -1/3      | -295.8     | -145.7     | 167.8      | -1.12     |
|         | -1/2      | -186.4     | -138.0     | -43.8      | -0.71     |
| S3      | 0         | -570.3     | -114.1     | 452.5      | -1.60     |
| $(K_v = 356$ MeV) | -1/3      | -389.6     | -140.1     | 156.2      | -1.10     |
|         | -1/2      | -285.1     | -142.7     | 5.26       | -0.8      |
### Table III

| A  | SkM* Exact | SkM* Eq.(27) | SkA Exact | SkA Eq.(27) | S3 Exact | S3 Eq.(27) |
|----|------------|--------------|-----------|-------------|----------|-----------|
| 40 | -9.72      | -9.64        | -9.70     | -9.64       | -9.96    | -9.94     |
| 100| -11.35     | -11.33       | -11.39    | -11.38      | -11.56   | -11.55    |
| 150| -11.93     | -11.92       | -12.00    | -12.00      | -12.12   | -12.12    |
| 200| -12.30     | -12.39       | -12.38    | -12.37      | -12.47   | -12.47    |

### Table IV

| A  | SkM* Exact | SkM* Eq.(34) | SkA Exact | SkA Eq.(34) | S3 Exact | S3 Eq.(34) |
|----|------------|--------------|-----------|-------------|----------|-----------|
| 40 | 103.4      | 102.7        | 126.9     | 125.9       | 191.3    | 190.3     |
| 100| 130.2      | 129.7        | 159.5     | 159.1       | 232.3    | 231.9     |
| 150| 140.4      | 140.1        | 172.0     | 171.7       | 247.4    | 247.1     |
| 200| 147.1      | 146.8        | 180.0     | 179.8       | 257.1    | 256.9     |
### Table V

| Force | A   | $a_v$ | $a_s A^{-1/3}$ | $a_c A^{-2/3}$ | $a_0 A^{-1}$ |
|-------|-----|-------|----------------|----------------|-------------|
| SkM*  | 40  | −15.79| 5.58           | 0.8755         | −0.3052     |
|       | 200 | −15.79| 3.26           | 0.2994         | −0.0611     |
| SkA   | 40  | −16.01| 5.83           | 0.8268         | −0.2855     |
|       | 200 | −16.01| 3.41           | 0.2828         | −0.0571     |
| S3    | 40  | −15.87| 5.53           | 0.5874         | −0.1810     |
|       | 200 | −15.87| 3.23           | 0.2009         | −0.0362     |

### Table VI

| Force | A   | $K_v$  | $K_s A^{-1/3}$ | $K_c A^{-2/3}$ | $K_0 A^{-1}$ |
|-------|-----|--------|----------------|----------------|-------------|
| SkM*  | 40  | 216.6  | −118.7         | −9.40          | 14.20       |
|       | 200 | 216.6  | −69.4          | −3.21          | 2.84        |
| SkA   | 40  | 263.3  | −141.7         | −10.58         | 14.88       |
|       | 200 | 263.3  | −82.9          | −3.62          | 2.98        |
| S3    | 40  | 355.5  | −166.8         | −9.76          | 11.31       |
|       | 200 | 355.5  | −97.5          | −3.34          | 2.26        |
### Table VII

| No. of Para. | $a_v$      | $a_s$     | $a_c$   | $a_0$    |
|--------------|------------|-----------|---------|----------|
| 2            | −16.0 ± 0.0003 | 19.95 ± 0.0003 | 9.67 ± 0.0003 | −11.42 ± 0.0003 |
| 3            | −16.2 ± 0.002 | 22.1 ± 0.002 | −0.30 ± 0.04 |          |
| 4            | −16.0 ± 0.0001 | 19.7 ± 0.001  | 11.6 ± 0.007 | −18.2 ± 0.01 |

### Table VIII

| No. of Para. | $K_v$    | $K_s$    | $K_c$    | $K_0$    |
|--------------|----------|----------|----------|----------|
| 2            | 256.4 ± 0.14 | −447.7 ± 0.69 | −125.8 ± 0.0003 | 604.4 ± 0.0003 |
| 3            | 269.0 ± 0.07 | −569.1 ± 0.65 | 282.1 ± 1.5 |          |
| 4            | 263.7 ± 0.24 | −494.0 ± 3.4  | −67.1 ± 15.8 | 531.2 ± 24.0 |
Table IX

| $K_v$     | $K_s$    | $K_\beta$ | $K_{\text{Coul}}$ | $K_c$    | $K_{s/\beta}$ |
|-----------|----------|-----------|--------------------|----------|--------------|
| 263.3     | −484.6   | −441.1    | −5.14              | −125.8   | ~ 875        |
| 252.3±2.0 | −428.4±5.6| −240.8±7.0| −4.7±0.19          |          |              |
| 237.5±5.8 | −391.2±14.7| −394.6±56.8| −3.28±0.55        |          |              |
| 368.7±17.3| −1108.4±100.5| −423.3±28.0| −9.4±0.71         | 1126.5±166.3| 1009.8±370.0 |
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9505031v1