Precision physics in jet processes

Frédéric A. Dreyer
Sorbonne Universités, UPMC Univ Paris 06, UMR 7589, LPTHE, F-75005, Paris, France
CNRS, UMR 7589, LPTHE, F-75005, Paris, France
CERN, PH-TH, CH-1211 Geneva 23, Switzerland

We examine properties of small-radius jets, focusing on phenomenological applications to the inclusive jet spectrum. We match fixed-order calculations with the leading-logarithmic resummation of the jet radius (LL\(R\)), and propose a new prescription to evaluate theoretical uncertainties for next-to-leading order (NLO) predictions. We also examine the \(R\)-dependent next-to-next-to-leading order (NNLO) corrections, and include them in our calculation. We discuss hadronisation corrections, which are derived from Monte Carlo generators. Finally, we assemble these elements and compare the ratio of the inclusive jet spectra at two \(R\) values obtained from our matched (N)NLO+LL\(R\) predictions to data from ALICE and ATLAS, finding improved agreement.

1 Introduction

Achieving highly precise predictions is an essential goal of modern collider physics. It is important in numerous contexts, notably: in Higgs physics, where accurate determination of couplings are now one of the main goals; in PDF extractions, whose uncertainties feed back into all other theoretical predictions; and in the determination of electroweak parameters. A large number of analyses at the LHC rely on the use of jets, and as such, an understanding of how limits on precision could be pushed in such processes would bring valuable insights.

We consider an archetypal hadron-collider jet observable, the inclusive jet spectrum, which can provide a useful case study with both experimental and theoretical challenges. We aim to investigate the \(R\)-dependence of jet spectra, focusing particularly on the small-radius limit.

2 Small-radius resummation

Building on previous work treating the resummation of leading logarithmic small-radius terms [1], it is straightforward to express the small-\(R\) resummed inclusive “microjet” spectrum from the convolution of the inclusive microjet fragmentation function, \(f_{\text{jet}}^{\text{incl}}(p_t/p'_t,t)\), with the leading order (LO) inclusive spectrum of partons with transverse momentum \(p'_t\) and flavour \(k\):

\[
\sigma^{\text{LL}_R}(p_t,R) = \frac{d\sigma^{\text{LL}_R}}{dp_t} = \sum_k \int_{p_t} dp'_t \frac{p'_t}{p_t} f_{\text{jet}}^{\text{incl}} \left( \frac{p_t}{p'_t}, t(R,R_0, \mu_R) \right) \frac{d\sigma^{(k)}}{dp'_t}. \tag{1}
\]

Here we use an evolution variable \(t\), defined by

\[
t(R,R_0, \mu_R) = \int_{R_0}^{R} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(\mu_R \theta/R_0)}{2\pi}, \quad b_0 = \frac{11C_A - 4T_R n_f}{6}, \tag{2}
\]

where \(R_0 \sim 1\) is an angular scale at which the small-\(R\) approximation becomes valid.

To verify the accuracy of the small-\(R\) approximation, we can compare the difference between \(R\) values obtained from a fixed-order calculation and the corresponding result obtained from the expansion of the resummed result given by Eq. (1). Fig. 1 (left) shows \(\frac{\sigma^{\text{NLO}_R}(p_t,R) - \sigma^{\text{NLO}_R}(R_{\text{ref}})}{\sigma^{\text{NLO}_R}(R_{\text{ref}})}\) as a function of \(R\) for both the small-\(R\) expansion and the exact fixed-order calculation (obtained with \texttt{NLOjet++} [2]), taking \(R_{\text{ref}} = 0.1\). We observe that the small-\(R\) approximation works very well for values of the jet radius observing \(R \leq 0.6\).

Furthermore, to examine subleading terms beyond NLO, we take a NLO 3-jet calculation, using the fact that

\[
\sigma^{\text{NNLO}}(R) - \sigma^{\text{NNLO}}(R_{\text{ref}}) = \sigma^{\text{NLO}_{3j}}(R) - \sigma^{\text{NLO}_{3j}}(R_{\text{ref}}). \tag{3}
\]
Comparing again the exact result with the expansion of the resummed spectrum, we can see in Fig. 1 (right) that there are important subleading contributions of the form $\alpha_s^n \ln^{n-1} \mu$. We will include a subset of these terms by matching the LL$_R$ resummation with the exact $R$ dependence up to NNLO.

Figure 1 – The exact and small-$R$ approximation for the $R$ dependence of the cross section, at NLO (left) and NNLO (right).

3 Matching to fixed order

To obtain phenomenological predictions, we combine the LL$_R$ resummation with fixed-order results. We achieve this by adopting a multiplicative matching scheme, given by

$$\sigma^{\text{NLO}+\text{LL}_R} = (\sigma_0 + \sigma_1(R_0)) \times \left[ \frac{\sigma^{\text{LL}_R}(R)}{\sigma_0} \times \left(1 + \frac{\sigma_1(R) - \sigma_1(R_0) - \sigma_1^{\text{LL}_R}(R)}{\sigma_0} \right) \right],$$

where $\sigma_i$ is the contribution of order $\alpha_s^{2+i}$ to the inclusive jet cross section, and the superscript LL$_R$ signals predictions obtained from the small-$R$ approximation. Because the two terms in separate brackets in Eq. (4) lead to K-factors going in opposite directions, there is a partial cancellation of higher order effects. This leads to unphysical cancellations of the scale uncertainties for certain values of the jet radius. Therefore, we propose an alternative method to estimate missing higher order uncertainties, which we obtain by evaluating the scale dependence independently in each term, and adding the resulting uncertainties in quadrature.

Furthermore, because of the importance of subleading $\alpha_s^n \ln R$ terms highlighted in Fig. 1 (right), it is important to include them. To this end, we construct a stand-in for the full NNLO result, denoted NNLO$_R$, which contains the complete $R$ dependence

$$\sigma^{\text{NNLO}_R}(R, R_m) \equiv \sigma_0 + \sigma_1(R) + [\sigma_2(R) - \sigma_2(R_m)].$$

Here $R_m$ is an arbitrary angular scale, which we will take as $R_m = 1$. We then extend the NLO matching scheme described in Eq. (4) up to NNLO to obtain matched NNLO$_R$+$\text{LL}_R$ predictions [3].

4 Non-perturbative effects

In order to compare our predictions with data, it is important to consider the impact of non-perturbative effects. The two main sources of non-perturbative corrections are: hadronisation, which is the transition from parton-level to hadron-level; and underlying event (UE), which corresponds to multiple interactions of the partons in the colliding protons.
Their dependence on the jet radius is very different, therefore it is useful to examine them separately: hadronisation is enhanced as $1/R$ at small radii, while the shift in jet $p_t$ from UE scales as $R^2$. The correction factors derived from 6 different Monte Carlo tunes is shown in Fig. 2 as a function of $R$, both for hadronisation (left) and UE (right).

We include non-perturbative effects by rescaling the perturbative spectra with factors derived from the average of several Monte Carlo tunes.

![Figure 2](image-url)  
Figure 2 – Non-perturbative corrections factors derived from Monte Carlo event generators, for hadronisation (left) and underlying event (right).

## 5 Comparison to data

We can now compare our predictions with existing inclusive jet data from the ALICE and ATLAS experiments.

The ATLAS data [4] is at centre-of-mass energy $\sqrt{s} = 7$ TeV, with two values of the jet radius: $R = 0.4$ and $R = 0.6$. To best evaluate the compatibility of our predictions with the experimental data, we take the ratio of the inclusive jet spectra at the two $R$ values. This allows us to study directly the $R$ dependence, as a number of experimental and theoretical uncertainties cancel in the ratio. In Fig. 3, we show the result of this comparison. Here we observe a much better agreement of the experimental data with the NNLO$_{(R)}$ based predictions compared with the plain NLO.

![Figure 3](image-url)  
Figure 3 – Comparison of theoretical predictions with data from the ATLAS collaboration [4].

We also compare our results with inclusive jet data from the ALICE collaboration [5], taken at $\sqrt{s} = 2.76$ TeV with $R = 0.2$ and $R = 0.4$. Taking again the ratio of these spectra, we show...
a comparison with our theoretical predictions in Fig. 4. We can see substantial effects beyond NLO from the resummation and the NNLO terms, with the NNLO+LL$_{R}$ seemingly providing the best match for the experimental data.

![Graph 1](image1)

**Figure 4** – Comparison of theoretical predictions with data from the ALICE collaboration [5].

## 6 Conclusion

We provided a detailed study of small-$R$ effects in inclusive jet spectra, considering $R$-dependent contributions up to NNLO which where matched with the LL$_{R}$ resummation. We studied non-perturbative effects and included them as corrections factors for the comparison with experimental data. Our work suggests that using multiple $R$ values can provide a powerful handle on systematic uncertainties. As such, we encourage experimental collaboration to use at least three different $R$ values for their jet measurements: $R = 0.2 - 0.3$, where UE is suppressed, $R = 0.4$, and $R = 0.6 - 0.7$ where hadronisation is suppressed. The computer code used for this study, as well as additional figures, are available online [6].

## Acknowledgements

This work has been done in collaboration with Mrinal Dasgupta, Gavin Salam and Gregory Soyez, and was supported by the ILP LABEX (ANR-10-LABX-63) supported by French state funds managed by the ANR within the Investissements d’Avenir programme under reference ANR-11-IDEX-0004-02.

## References

1. M. Dasgupta, F. Dreyer, G. P. Salam and G. Soyez, JHEP **1504** (2015) 039 doi:10.1007/JHEP04(2015)039 [arXiv:1411.5182 [hep-ph]].
2. Z. Nagy, Phys. Rev. D **68** (2003) 094002 doi:10.1103/PhysRevD.68.094002 [hep-ph/0307268].
3. M. Dasgupta, F. A. Dreyer, G. P. Salam and G. Soyez, arXiv:1602.01110 [hep-ph].
4. G. Aad *et al.* [ATLAS Collaboration], JHEP **1502** (2015) 153 Erratum: [JHEP **1509** (2015) 141] doi:10.1007/JHEP02(2015)153, 10.1007/JHEP09(2015)141 [arXiv:1410.8857 [hep-ex]].
5. B. Abelev *et al.* [ALICE Collaboration], Phys. Lett. B **722** (2013) 262 doi:10.1016/j.physletb.2013.04.026 [arXiv:1301.3475 [nucl-ex]].
6. M. Dasgupta, F. A. Dreyer, G. P. Salam and G. Soyez, [http://microjets.hepforge.org](http://microjets.hepforge.org).