Monopulse lidar Earth surface sounding method

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Abstract. The paper studies the monopulse lidar method capabilities for sounding the reflection coefficient of the earth surface. The mathematical modelling shows that the monopulse lidar method allows us to reconstruct spatial distribution of the reflection coefficient under measurement noise in a good-sized angular coverage of lidar. The most efficient for reconstruction of the reflection coefficients based on the measurement data (in terms of both the minimum errors and the computation time) are the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm. The monopulse lidar monitoring method using the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm allows (with an error of estimating expansion coefficients from units of per cent to 10 - 40% under measurement noise of 3%) adequate reconstruction of spatial distribution of the reflection coefficient on the earth surface under conditions of strongly and nonlinearly changeable reflection coefficient.

1. Introduction
Currently, one of the most rapid ways to monitor the earth surface is the use of space- or airborne sounding equipment.

Laser remote sensing systems are useful for a lot of tasks [1-4] in the optical spectral band. In airborne laser systems an appropriate coverage is usually provided by angular scanning of the laser beam in the plane perpendicular to the flight direction.

The simpler equipment (without angular scanning) can be implemented using a monostatic oblique monopulse sounding system. In this case, special processing of the laser echo signal reflected from the sounded earth surface provides the appropriate coverage.

The paper conducts a capability analysis of the monopulse lidar method for sounding the earth surface (to monitor the health of forests plantations, oil pollution, etc.) using a diversity of data processing algorithms.

2. Problem description
A diagram of the monostatic oblique sounding is shown in figure 1 (optical axes of lidar source and receiver are aligned in one X0Z plane perpendicular to the aircraft flight direction).

An airborne lidar irradiates the earth surface at an angle to the normal (for simplicity, we assume that, further, a laser beam divergence angle and a field-of-view angle of the receiving optical system are equal).
We believe that the earth surface is averagely flat and locally Lambert’s. Then, the formula for time realization of the lidar signal (recorded in the diagram of the monostatic oblique sounding in figure 1) after a series of mathematical transformations can be represented, using the results of [5], as follows:

$$B(\tau) \approx \int_{a}^{b} R_{\text{ref}}(x) f(\tau - x)dx$$

(1)

where $\tau = \frac{c(t - t')}{2 \sin \theta}$; $\theta$ - sounding angle; $B(\tau)$ - time realization of a normalized lidar signal; $(a, b)$ - laser spot region on the earth surface; $R_{\text{ref}}(x)$ - spatial reflection coefficient distribution at the $x$-axis; $f(t)$ - lidar pulse form; $t'$ - pulse delay on the route “lidar - current point on the sounded surface - lidar”.

The expression (1) represents an integral equation of first kind. The task to reconstruct the spatial distribution of the reflection coefficient $R_{\text{ref}}(x)$ on the earth surface according to measurements of the time realization of a lidar signal belongs to ill-posed mathematical problems and demands for using the special solution methods [6]. A method of finding quasi-solutions [6] is one of the efficient ones to solve ill-posed mathematical problems.

The paper uses mathematical modeling to conduct a capability analysis of the monopulse lidar method for earth surface monitoring, using a variety of algorithms to find quasi-solutions under measurement noise and grossly changeable reflection coefficient of the earth surface.

3. Finding quasi-solutions to the problem of monopulse lidar Earth surface monitoring

Let’s represent $R_{\text{ref}}(x)$, being spatial distribution of the reflection coefficient, as a polynomial function

$$R_{\text{ref}}(x) = a_0 + a_1x + a_2x^2$$

(2)

where $a_0, a_1, a_2$ - unknown expansion coefficients.

To reconstruct the expansion coefficients $a_0, a_1, a_2$, according to data from lidar sounding, it is necessary to measure a lidar signal $B(\tau)$ at the certain moments of time $\tau_i$ and solve a system of equations:
\[ B_{\text{mod}}(\tau_1, a_0, a_1, a_2) = B_{\text{meas}}(\tau_1) \]

\[ B_{\text{mod}}(\tau_n, a_0, a_1, a_2) = B_{\text{meas}}(\tau_n) \]  \hspace{1cm} (3)

where \( B_{\text{mod}}(\tau_i, a_0, a_1, a_2) \) - lidar signal value from the model \( B(\tau) \) at the moment \( \tau_i \) (it depends on three unknown expansion coefficients \(-a_0, a_1, a_2\)); \( B_{\text{meas}}(\tau_i) \) - lidar signal value \( B(\tau) \) measured at the moment \( \tau_i \); \( n \) - the number of measurements of a lidar signal \( B(\tau) \).

A residual function \( \Delta(a_0, a_1, a_2) \) between the value from the model and the measured value of the signal is as follows:

\[ \Delta(a_0, a_1, a_2) = \sum_{i=1}^{n} [B_{\text{mod}}(\tau_i, a_0, a_1, a_2) - B_{\text{meas}}(\tau_i)]^2 \]  \hspace{1cm} (4)

The values of coefficients \(-a_0, a_1, a_2\), which vanish the residual function \( \Delta(a_0, a_1, a_2) \), are a solution of the system of equations (3). However, even with a little measurement noise there can be situations \([6]\) when the system of equations (3) has no solution.

One of the most efficient methods to solve such mathematical problems is a quasi-solution approach.

The quasi-solution approach to determine a spatial distribution \( R_{\text{ref}}(x) \) from the integral equation (1) is that for various values of the expansion coefficients \(-a_0, a_1, a_2\), the values from the model of a lidar signal \( B_{\text{mod}}(\tau_i, a_0, a_1, a_2) \) are calculated from the equation (1) in the range of values \(-a_0, a_1, a_2\) that meet the physical context of the equation. Then the expansion coefficients \(-a_0, a_1, a_2\) are fixed. They minimise the residual function \( \Delta(a_0, a_1, a_2) \) between the values from the model and the measured values of a lidar signal. The found expansion coefficient values determine a vector \( \vec{A} = (a_0, a_1, a_2) \), which is the quasi-solution of the system of equations (3)

\[ \Delta(\vec{A}) = \inf_{\vec{A}\in M} \sum_{i=1}^{n} [B_{\text{meas}}(\tau_i) - B_{\text{mod}}(\tau_i, a_0, a_1, a_2)]^2 \]  \hspace{1cm} (5)

where \( M \) - the range of values of expansion coefficients \(-a_0, a_1, a_2\), which meet the physical context of the problem; \( \inf_{\vec{A}\in M} \rho \) - infimum of the distance \( \rho \) value between lidar signal values from the model and measured values.

Figure 2 shows a view of the residual function \( \Delta(a_0, a_1, a_2) \) at \( a_2 = 0.025 \). The residual function has a «ravine» appearance, which makes the search for a global minimum not easy.

To find quasi-solutions (a minimum of the residual function \( \Delta(a_0, a_1, a_2) \)) the direct search method can be used according to values of coefficients \(-a_0, a_1, a_2\) (in the range of values, which meet a physical context). However, using the direct search method requires, as a rule, a vast amount of computation and can result in great mistakes in the solution obtained. Therefore there is a need to use more complicated, but fast algorithms to find solutions.

Presently, there are efficient searching algorithms, in which available decision-making mechanisms are developed. Being efficient in solving optimization problems \([7-12]\), these decision-making principles provide an effective fauna adaptation to the natural environment.
For quasi-solution search and comparative analysis of search algorithms efficiency, the paper deals with the exhaustive algorithm; the genetic algorithm; the particle swarm algorithm; the hybrid particle swarm algorithm and simulated annealing; the ant colony algorithm; the modified ant colony algorithm with shake; the hybrid continuous interacting ant colony algorithm.

Algorithms stopped working when the specific number of iterations has been performed, and the best current solution reached necessary truth.

Mathematical modeling has been conducted to study capabilities of the monopulse lidar method to monitor the earth surface.

**4. Mathematical modelling of the monopulse lidar method to monitor the earth surface**

In mathematical modeling the measurement number of recorded laser signal $n$ was specified within the range from 4 to 8. The values of expansion coefficients $a_0, a_1, a_2$ were specified in the following ranges: $a_0 = 0...0,5$, $a_1 = -0,75...0,75$, $a_2 = -0,5...0,5$. Search for solution was brought to stop either after a certain number of algorithm iterations (or generations), or when in the last 10 iterations (generations) a relative change (between iterations or generations) of the residual function is $10^{-10}$ at most. A value range $x$ was specified from 1 to 3 km. Measurement noise was considered as normal with zero-mean value. The root-mean-square (relative) value was specified within a range of $1 – 10\%$.

The pulse form of a lidar was supposed to be Gaussian: $f(\tau) \equiv \exp\left(-\frac{\tau^2}{R_s^2}\right)$, where $R_s$ - spatial length of a lidar pulse on the sounded surface; $R_s = \frac{ct_s}{2\sin \theta}$; $\tau_s$ - lidar pulse width.

Figures 3а and 3b show examples of specified and reconstructed spatial distributions of the reflection coefficient of the earth surface.

Figure 3 shows specified spatial distributions of the reflection coefficient of the earth surface $R_{ref}(x)$ and reconstructed ones (based on data «measured» by the lidar method with a noise root-mean-square value of 3 % and the number of measurements $n$ being equal to 8). The spatial length of the lidar pulse $R_s$ was assumed to be equal to 1.5 km.
Figure 3. Examples of specified and reconstructed spatial distributions $R_{\text{amp}}(x)$.

Solid lines indicate the specified (model-based) spatial distributions of the reflection coefficient (Figure 3a: $a_0=0.49; a_1=0.45; a_2=-0.2$; figure 3b: $a_0=0.1; a_1=0.05; a_2=0.025$), while dotted lines indicate the reconstructed spatial distributions.

Tables 1 and 2 show the relative errors of reconstruction of the expansion coefficients in per cents for a variety of finding quasi-solution algorithms, including exhaustive search ($10^{-3}$ sample search for each parameter) with the measurement noise of 3%. Tables 1 and 2 also present the computation time (computation time, certainly, depends on the processor characteristics, but the values in these tables show a relationship between the computation amounts for different algorithms).

Table 1 presents the errors of reconstruction of the expansion coefficients $a_0, a_1, a_2$ for the spatial distribution shown in figure 3a, and table 2 does the same for the distribution in figure 3b. Description field in these tables is as follows: 1 – exhaustive algorithm; 2 - genetic algorithm; 3 - particle swarm algorithm; 4 - hybrid particle swarm algorithm and simulated annealing; 5 - ant colony algorithm; 6 - modified ant colony algorithm with shake; 7 - hybrid continuous interacting ant colony algorithm.

The mathematical modelling outcomes given in tables 1, 2 and in figure. 3 show that the monopulse lidar method allows us to reconstruct spatial distribution of the reflection coefficient under measurement noise in a good-sized angular coverage of lidar.

The most efficient for reconstruction of the reflection coefficients based on the measurement data (in terms of both the minimum errors and the computation time) are the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm.

The monopulse lidar monitoring method using the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm allows (with an error of estimating expansion coefficients from units of per cent to 10 - 40% under measurement noise of 3%) adequate reconstruction of spatial distribution of the reflection coefficient on the earth surface under conditions of strongly and nonlinearly changeable reflection coefficient.

Table 1. Errors (%) of reconstruction of expansion coefficients $a_0, a_1, a_2$ for data in figure 3a.

| Parameter | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| $a_0$     | 4.5   | 8.9   | 0.7   | 9.1   | 6.6   | 0.3   | 4.5   |
| $a_1$     | 4.0   | 9.2   | 0.3   | 8.6   | 6.7   | 4.6   | 3.9   |
| $a_2$     | 2.0   | 4.9   | 0.3   | 4.2   | 3.6   | 4.9   | 1.9   |
| Computation time | 8h 31m | 0h 51m | 0h 50m | 0h 26m | 0h 22m | 0h 25m | 0h 44m |
Table 2. Errors (%) of reconstruction of expansion coefficients $a_0, a_1, a_2$ for data in figure 3b.

| Parameter | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-----------|-----|-----|-----|-----|-----|-----|-----|
| $a_0$     | 40.0| 29.8| 32.6| 6.1 | 18.3| 45.6| 44.3|
| $a_1$     | 90.0| 69.8| 78.8| 14.0| 43.6| 99.0| 99.8|
| $a_2$     | 44.0| 34.5| 39.9| 4.4 | 21.0| 22.4| 49.2|
| Computation time | 8h 10m | 1h 35m | 0h 46m | 0 h 25m | 0h 27 m | 0 h 25 m | 0 h 47 m |

5. Conclusion

The paper studies the monopulse lidar method capabilities for sounding the reflection coefficient of the earth surface. The mathematical modelling shows that the monopulse lidar method allows us to reconstruct spatial distribution of the reflection coefficient under measurement noise in a good-sized angular coverage of lidar. The most efficient for reconstruction of the reflection coefficients based on the measurement data (in terms of both the minimum errors and the computation time) are the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm. The monopulse lidar monitoring method using the hybrid particle swarm algorithm and simulated annealing and the ant colony algorithm allows (with an error of estimating expansion coefficients from units of per cent to 10 - 40% under measurement noise of 3%) adequate reconstruction of spatial distribution of the reflection coefficient on the earth surface under conditions of strongly and nonlinearly changeable reflection coefficient.

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