Sakata model of hadrons revisited

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Abstract

46 years ago the quark model replaced the Sakata model as the standard explanation of the hadron structure. The major alleged defect of the Sakata model was its prediction of just too many types of particles, which have not been seen in experiments. However, this allegation was made without detailed consideration of the forces acting between sakatons. In this article we suggest a set of pairwise sakaton-sakaton and sakaton-antisakaton potentials that describe stability and masses of strongly interacting elementary particles in a good agreement with observations.

1 The Sakata model

Today it is universally accepted that hadrons are made of quarks ($q = u, d, s, c, \ldots$). The quark model forms the basis of quantum chromodynamics (QCD), which aspires to explain the nature of strong interactions. Almost all compound particles predicted by the quark model have been found in experiments. Moreover, all observed particles have natural quark assignments: Mesons are quark-antiquark bound state ($q\bar{q}$), and baryons are bound states of three quarks ($qqq$). In spite of their well-known achievements, the quark model and QCD have some questionable features. These theories make assumptions (fractional charges of quarks, color, gluons, confinement potentials, etc.), which cannot be directly observed and thus destined to remain suspect. Then it seems justified to explore other approaches to the explanation of hadrons masses and stability. One interesting proposal is the Sakata model [2], which was rather popular before the “quark era”. The Sakata model assumes that proton ($p$), neutron ($n$), $\Lambda^0$ and $\Lambda^+_c$...
are the true elementary particles\(^2\) also called *sakatons* (*σ*)\(^3\). To emphasize their similarity with quarks, we will denote the four fundamental sakatons by capital letters \(U, D, S, C\)\(^4\). Each sakaton has its corresponding antisakaton \((\bar{U}, \bar{D}, \bar{S}, \bar{C})\) with the same mass and spin and opposite values of the electric charge, baryon charge, strangeness, and charm.

### Table 1: Stable baryons with their quark and sakaton structures.

| Baryon | Quark structure | Sakaton structure | Exp. mass MeV/c\(^2\) |
|--------|-----------------|-------------------|----------------------|
| \(n\)  | \(udd\)        | \(D\) (down)      | 938                  |
| \(p\)  | \(uud\)        | \(U\) (up)        | 940                  |
| \(Λ^0\)| \(sud\)        | \(S\) (strange)   | 1116                 |
| \(Λ^+\)| \(cud\)        | \(C\) (charmed)   | 2285                 |

\(\Sigma^-\)  | \(sdd\)        | \(SDU\)           | 1197                 |
\(\Sigma^0\)   | \(sud\)        | \(S\bar{N}\bar{N}\) | 1193                 |
\(\Sigma^+\)   | \(suu\)        | \(S\bar{D}\)       | 1189                 |
\(\Xi^-\)      | \(ssd\)        | \(SS\bar{U}\)      | 1322                 |
\(\Xi^0\)      | \(ssu\)        | \(SS\bar{D}\)      | 1315                 |
\(\Xi^+\)      | \(csd\)        | \(CS\bar{U}\)      | 2471                 |
\(\Xi^{++}\)   | \(csu\)        | \(CS\bar{D}\)      | 2468                 |
\(\Xi^{++}\)   | \(ccd\)        | \(CC\bar{U}\)      | 3519                 |
\(\Omega^-\)   | \(sss\)        | \(SSS\bar{U}\bar{D}\)| 1672
\(\Omega^0\)   | \(css\)        | \(CSS\bar{U}\bar{D}\)| 2698
\(\Omega^{+}\) | \(ccs\)        | \(CCS\bar{U}\bar{D}\)| not seen
\(\Omega^{++}\)| \(ccc\)        | \(CCC\bar{U}\bar{D}\)| not seen

The Sakata model assumes that sakatons interact with each other via short-range (few femtometers) potentials. All non-elementary hadrons are bound states of two or more sakatons. Various possible combinations are summarized in Table 2. Nuclei are composed of \(U\) and \(D\) sakatons (protons and neutrons). Mesons are sakaton-antisakaton \((σ\bar{σ})\) bound states. Compound baryons are sakaton-sakaton-antisakaton \((σσ\bar{σ})\) or pentasakaton \((σσσ\bar{σ})\) bound states. Here we are interested only in baryons, which are stable with respect to strong decays. All of them are listed in Table 1. Their decays are caused by flavor-changing weak interactions and their masses are lower than the sums of masses of constituents. One example of an unstable baryon state

\(^2\)In this paper we will not discuss bottom and top particles, because full experimental picture is still lacking in those sectors.

\(^3\)See Table 1. We use symbol \(N\) do denote collectively \(U\) and \(D\) sakatons. For example, \(NN\bar{S}\) means either \(UU\bar{S}\) or \(DD\bar{S}\).
omitted in Table 1 is the $\Delta^{++}(=UU\bar{D})$ particle whose mass is 1232 MeV/c$^2$. This is higher than the sum of masses of dissociation products $p(940) + \pi^+(140)(=U+U\bar{D})$. Therefore $\Delta^{++}$ is a metastable resonant state in our model. The calculation method adopted in this work (see section [2]) can deal only with true bound states, therefore we will not discuss the $\Delta^{++}$ and other resonances.

The Sakata model avoids some problems characteristic for the quark model. The fundamental constituents of the Sakata model – the sakatons – are readily observable as normal baryons with integer charges, so there is no need for additional assumptions about "confinement". There is also no need to introduce "hidden" degrees of freedom, such as color and gluons. The short-range character of sakaton potentials means that strong interactions satisfy the important property of cluster-separability [4], similar to electromagnetic and gravitational forces.

| sakaton content | particle type | examples | antiparticle |
|-----------------|---------------|----------|--------------|
| $\sigma$        | baryon        | $p, n, \Lambda, \Lambda^+$ | $\bar{\sigma}$ |
| $N N' \ldots N''$ | nucleus      | deuteron($=UD$) | $NN' \ldots N''$ |
| $\sigma\bar{\sigma}$ | meson        | $K^+ (=U\bar{S})$ | $\sigma\bar{\sigma}$ |
| $\sigma\sigma\bar{\sigma}$ | baryon      | $\Sigma^- (=SDU)$ | $\sigma\sigma\bar{\sigma}$ |
| $\sigma\sigma\sigma\bar{\sigma}$ | tetrasakaton | unstable? | $\sigma\sigma\sigma\bar{\sigma}$ |
| $\sigma\sigma\sigma\sigma\bar{\sigma}$ | baryon      | $\Omega^- (=SSSU\bar{D})$ | $\sigma\sigma\sigma\sigma\bar{\sigma}$ |

The biggest problem of the Sakata model is that it seemingly predicts more types of particles than actually observed. Certain $\sigma\sigma\bar{\sigma}$ combinations, which look acceptable from the point of view of the Sakata model, have not been seen in experiments. This refers, for example to $NN\bar{S}$ and $UD\bar{S}$ baryons with strangeness $+1$[4]. Furthermore, the simplest sakaton assignment of the $\Omega^-$ baryon (baryon number $=1$, charge $=-1$, strangeness $=-3$) is in the form of a pentasakaton $\Omega^- (=SSSU\bar{D})$. Then, from the principle of isotopic invariance, it seems that analogs of the $\Omega^-$ particle should also exist, such as $\Omega^- (=SSSU\bar{U})$ and $\Omega^0 (=SSSU\bar{D})$. Why haven’t they been seen in experiments?

In order to answer these and other questions, it is important to have a realistic model of interactions between sakatons. The goal of this paper is to suggest an approximate set of pairwise sakaton-sakaton and sakaton-antisakaton potentials and to calculate masses of their bound states - mesons and baryons.

*Reports about discovery of the exotic baryon $\Theta^+ (=U\bar{D}S)$ are not credible [1].
2 Computational model and results

Matumoto and co-authors established [5, 6, 7, 8, 9] that masses of hadrons can be roughly calculated from the assumption of strong attraction in sakaton-antisakatons pairs (i.e., one $\sigma - \bar{\sigma}$ bond contributes about 1275-1740 MeV to the binding energy) and equally strong sakaton-sakaton and antisakaton-antisakaton repulsions. Binding energies of mesons are very high (above 1GeV), because only the $\sigma - \bar{\sigma}$ attraction contributes there. Much lower binding energies are expected in 3-sakaton $\sigma\sigma\sigma$ and in pentasakaton $\sigma\sigma\sigma\sigma\sigma$ baryons. In the former case two attractive interactions $\sigma - \bar{\sigma}$ are balanced by one repulsion $\sigma - \sigma$. In the latter case there are 6 attractions vs. 4 repulsions. Tetrasakatons $\sigma\sigma\sigma\sigma$ are not likely to be stable because the number of repulsive and attractive pairs is equal in this case. Some instructive studies of multiparticle systems with pairwise interactions can be found in [10, 11]. They suggest that stability of multi-sakaton states may depend on a delicate balance of masses of the constituents and shapes of their interaction potentials.

The approximate non-relativistic Hamiltonian describing an $N$-sakaton system can be written as

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i<j} V_{ij}(r_{ij})$$

(1)

where $m_i, p_i, r_{ij} = |r_i - r_j|$ are masses and momenta of the sakatons and their relative distances, respectively. Interactions between sakatons were modeled as superpositions of two Yukawa potentials

$$V_{ij}(r) = A_{ij} z_i z_j \frac{e^{-\alpha_{ij} r}}{r} + B_{ij} \frac{e^{-\beta_{ij} r}}{r}$$

(2)

where $z_i = +1$ for sakatons and $z_i = -1$ for antisakatons.

All calculations were performed using the stochastic variational method of Varga and Suzuki [12, 13, 14]. The FBS computer program was obtained from the CPC Program Library (Queen’s University of Belfast, N. Ireland) and slightly modified to fit our needs. This program solves the non-relativistic stationary Schrödinger equation and yields accurate energies and wave functions of the ground and few excited states for systems of several (typically, 2-6) quantum particles interacting via pairwise potentials. Only states with the lowest total spin ($s = 0$ for mesons and $s = 1/2$ for baryons) and zero orbital momentum were considered here. In all calculations masses of sakatons were fixed as $m(N) = 940$ MeV/$c^2$, $m(S) = 1116$ MeV/$c^2$, $m(C) = 2285$ MeV/$c^2$.

The equality of masses of the $U$ and $D$ sakatons and the assumption that their interactions with other sakatons are the same (see Table 4) imply that all calculated masses are invariant with respect to replacements in which all $U$ sakatons are changed to $D$ and all $D$ sakatons are simultaneously changed to $U$. 

$^5$The equality of masses of the $U$ and $D$ sakatons and the assumption that their interactions with other sakatons are the same (see Table 4) imply that all calculated masses are invariant with respect to replacements in which all $U$ sakatons are changed to $D$ and all $D$ sakatons are simultaneously changed to $U$. 

$^4$
Internally in the code these masses were expressed in units of the proton mass $940\text{ MeV}/c^2$. Distances were measured in femtometers and energies in MeV. In this system of units $\hbar^2/m = 41.47$. The basis set selection procedure used iteration numbers $M_0 = 10, K_0 = 50$. Other computational parameters depended on the number of sakatons in the system as shown in Table 3. They were adjusted for the optimal balance between accuracy, convergence, and speed. The exact meaning of these parameters was explained in [12].

Table 3: Computational parameters for the FBS code. $b_{\text{min}}/b_{\text{max}}$ are minimum/maximum values of nonlinear parameters in Gaussian basis functions.

| Number of sakatons | Basis set size, $K$ | $b_{\text{min}}$ (fm) | $b_{\text{max}}$ (fm) |
|-------------------|-------------------|-----------------|-----------------|
| 2                 | 50                | $10^{-6}$       | 10              |
| 3                 | 250               | $10^{-6}$       | 10              |
| 4                 | 300               | $10^{-6}$       | 10              |
| 5                 | 500               | $10^{-6}$       | 100             |

Table 4: Optimized parameters of the potentials [2]. $A$ and $B$ are measured in MeV·fm; $\alpha$ and $\beta$ are in fm$^{-1}$.

| Interaction | $A$  | $\alpha$ | $B$  | $\beta$ |
|------------|------|---------|------|--------|
| $N - N$    | 617.8| 0.091   | 92.14| 0.359  |
| $U - D$    | 570.2| 0.091   | 25.7 | 0.094  |
| $N - S$    | 530.0| 0.108   | 14.0 | 0.49   |
| $S - S$    | 446.7| 0.118   | 42.1 | 0.444  |
| $N - C$    | 397.5| 0.102   | 14.0 | 0.49   |
| $S - C$    | 340.8| 0.12    | 46.1 | 0.444  |
| $C - C$    | 317.0| 0.118   | 24.1 | 0.484  |

Our major goal is to optimize parameters $A, \alpha, B, \beta$ of the potentials [2]. The optimization was performed in two steps. In the first step we fitted parameters relevant to interactions of $U, D, S$ sakatons. The training set contained 24 species shown in Table 5. They included 4 ground states of mesons, 4 stable baryons, and 16 states, which are supposed to be unstable. The goal was to reproduce experimental masses of the 8 stable species as close as possible and, at the same time, do not allow the binding energy of the 16 unstable species to become positive. In the second step we froze the $U - D - S$ parameters obtained above and varied interactions $C - N$, $C - S$, and $C - C$ using the training set in Table 6. This set included $C$-containing particles: 3 mesons, 3 stable charmed baryons, and 17 unstable species. The final optimized
values of parameters $A, \alpha, B, \beta$ are given in Table 4. Plots of the optimized $U - D$ and $U - \bar{D}$ (same as $\bar{U} - D$) potentials are shown in Fig. 1. Potentials for other pairs of sakatons have qualitatively similar shapes. These interactions demonstrate rather strong attraction of $\sigma - \bar{\sigma}$ pairs and repulsion of $\sigma - \sigma$ and $\bar{\sigma} - \bar{\sigma}$ pairs in a qualitative agreement with Matumoto’s guesses.

The resulting masses of hadrons are shown in the third column of tables 5 and 6. The binding energies (B.E.) are in the 4th column and the lowest-energy dissociation products are in the 5th column.

Ideally, the binding energies of unstable tetrasakatons and baryons$^6$ must be equal to zero. In practice this can be achieved only with very large and diffuse basis sets, which allow the wave functions of dissociation products to separate widely, so that their repulsion is minimized. For computational reasons our basis sets were limited. This explains why some residual repulsion (reflected in negative binding energies from 0 to -5 MeV) remained for several dissociated unstable species. Extremely large negative binding energies of $U\bar{D}\bar{U}D$, $S\bar{U}D\bar{D}$, $SDU\bar{U}\bar{D}$, $CSU\bar{U}\bar{D}$ and $CSU\bar{U}\bar{D}$ are explained by the fact that they have converged to metastable dissociated configurations (=local minima) $U\bar{U} + D\bar{D}$, $S\bar{U} + D\bar{D}$, $SD\bar{D} + U\bar{U}$, $C\bar{S}\bar{D} + U\bar{U}$, and $C + S\bar{U} + U\bar{D}$, respectively.

The next step is to consider properties that have not been used directly in the fitting.

$^6$They are shown in the lower portions of Tables 5 and 6.
First, we looked at the two charmed baryons whose existence is predicted by the quark model and whose experimental confirmation is still lacking. These are the \( \Omega_c^+ \) and \( \Omega_{cc}^{++} \) particles. We found that \( \Omega_{cc}^+ (= CCSU\bar{D}) \) is stable with the mass of 4430 MeV/c\(^2\) and binding energy of 25 MeV with respect to the \( \Omega_{cc}^+ \rightarrow \Xi_c^+ + K^0 (= CC\bar{U} + S\bar{D}) \) dissociation channel. The calculated mass of \( \Omega_{cc}^{++} (= CCC\bar{U}\bar{D}) \) is 6099 MeV/c\(^2\), which means that this particle dissociates spontaneously as \( \Omega_{cc}^{++} \rightarrow \Xi_{cc}^{++} + D^+ (= CCC\bar{U} + C\bar{D}) \).

Next we verified that all 133 possible tetrasakaton \((\sigma\sigma\sigma\sigma)\) and baryon \((\sigma\sigma\sigma)\) species not presented in Tables 5 and 6 are unstable in our approach, as expected.

Other interesting pieces of information, which have not been involved in the fitting, are the meson excitation energies. Note that the strongly attractive \( \sigma\sigma \) potential (see

\begin{table}[h]
\centering
\begin{tabular}{ |c|c|c|c|c| }
\hline
\text{Particle} & \text{Sakaton structure} & \text{Mass (MeV/c}^2\text{)} & \text{B.E. (MeV)} & \text{Products} \\
\hline
\pi^0 & DD & 238/135 & 1642/1745 & \text{n+\pi/n+\pi} \\
\pi^+ & U\bar{D} & 142/141 & 1738/1739 & p+\pi/p+\pi \\
K^- & SU & 364/494 & 1692/1562 & \bar{p}+\Lambda^0/\bar{p}+\Lambda^0 \\
\eta & SS & 1095/548 & 1137/1684 & \Lambda^0+\bar{\Lambda}^0/\Lambda^0+\bar{\Lambda}^0 \\
\Sigma^+ & SUD & 1210/1189 & 48/67 & \Lambda^0+\pi^+/\Lambda^0+\pi^+ \\
\Sigma^0 & SU\bar{U} & 1260/1193 & 44/58 & p+K^-/\Lambda^0+\pi^0 \\
\Xi^0 & SSD\bar{D} & 1314/1315 & 166/295 & \Lambda^0+K^0/\Lambda^0+K^0 \\
\Omega^- & SSSUD & 1670/1672 & 8/136 & \Xi^-+K^0/\Xi^-+K^0 \\
\hline
\text{unstable} & UUU & 1179 & -1/0 & p+\pi^0/p+\pi^0 \\
\text{unstable} & UD\bar{U} & 1082 & 0/0 & p+\pi^-/p+\pi^- \\
\text{unstable} & UU\bar{S} & 1307 & -3/0 & p+K^+/p+K^+ \\
\text{unstable} & UD\bar{S} & 1307 & -3/0 & p+\bar{K}^0/p+\bar{K}^0 \\
\text{unstable} & SU\bar{S} & 1482 & -2/0 & \Lambda^0+K^+/\Lambda^0+K^+ \\
\text{unstable} & SS\bar{S} & 2211 & 0/0 & \Lambda^0+\eta/\Lambda^0+\eta \\
\text{unstable} & UDUD & 483 & -199/0 & \pi^++\pi^-/\pi^0+\pi^0 \\
\text{unstable} & SD\bar{U}\bar{D} & 606 & -101/0 & K^0+\pi^-/K^-+\pi^0 \\
\text{unstable} & SSD\bar{D} & 728 & 0/0 & K^0+K^0/\bar{K}^0+\bar{K}^0 \\
\text{unstable} & SSUU & 733 & -5/0 & K^-+K^-/K^-+K^- \\
\text{unstable} & SSS\bar{S} & 2192 & -2/0 & \eta+\eta/\eta+\eta \\
\text{unstable} & UDUDU & 1223 & 0/0 & p+\pi^-+\pi^-/p+\pi^-+\pi^- \\
\text{unstable} & SDD\bar{U}\bar{D} & 1499 & -147/0 & \Sigma^++\pi^-/\Sigma^0+\pi^0 \\
\text{unstable} & SSD\bar{D}\bar{U} & 1457 & -1/0 & \Xi^0+\pi^-/\Xi^-+\pi^0 \\
\text{unstable} & SSUUU & 1678 & 0/0 & \Xi^-+K^-/\Xi^-+K^- \\
\hline
\end{tabular}
\caption{Training set for no-charm particles.}
\end{table}
Table 6: Training set for particles with charmed sakatons

| Particle | Sakaton structure | Mass (MeV/c^2) | B.E. (MeV) | Products |
|----------|------------------|---------------|------------|----------|
|          |                  | calc./exp.    | calc./exp. |          |
| $D^0$    | $CU$             | 2001/1897     | 1224/1328  | $\bar{p} + \Lambda_c^+ / \bar{p} + \Lambda_c^+$ |
| $D_s^+$  | $CS$             | 2587/1968     | 814/1433   | $\Xi_c^0 + \Lambda_c^+ / \Lambda_c^0 + \Lambda_c^+$ |
| $\eta_c$ | $C\bar{U}$       | 3339/2980     | 1231/1590  | $\Lambda_c^+ + \bar{\Lambda}_c / \Lambda_c^+ + \bar{\Lambda}_c$ |
| $\Xi_c^0$ | $CSU$           | 2606/2471     | 43/308     | $\Lambda_c^+ / K^- / \Lambda_c^+$ |
| $\Xi_{cc}^+$ | $CC\bar{U}$     | 4092/3519     | 194/636    | $\Lambda_c^+ + D^0 / \Lambda_c^+ + D^0$ |
| $\Omega_c^+$ | $CSS\bar{D}$   | 2852/2698     | 118/268    | $\Xi_c^+ + K^- / \Xi_c^+ + K^-$ |

| unstable | $UDC$            | 2944          | -3/0       | $p + D^- / p + D^-$ |
| unstable | $UU\bar{C}$      | 2945          | -4/0       | $p + \bar{D}^0 / p + \bar{D}^0$ |
| unstable | $SU\bar{C}$      | 3121          | -4/0       | $\Lambda_c^0 / \Lambda_c^0 + \bar{D}^0$ |
| unstable | $SS\bar{C}$      | 3705          | -2/0       | $\Lambda_c^0 + D_s^- / \Lambda_c^0 + D_s^-$ |
| unstable | $CU\bar{D}$      | 2427          | 0/0        | $\Lambda_c^+ + \pi^+ / \Lambda_c^+ + \pi^+$ |
| unstable | $CU\bar{U}$      | 2523          | 0/0        | $\Lambda_c^+ + 2\pi^0 / \Lambda_c^+ + 2\pi^0$ |
| unstable | $CSS$            | 3380          | 0/0        | $\Lambda_c^+ + \eta / \Lambda_c^+ + \eta$ |
| unstable | $CS\bar{C}$      | 4456          | -1/0       | $\Lambda_c^0 / \eta_c / \Lambda_c^0 / \eta_c$ |
| unstable | $CSS$            | 4872          | 0/0        | $\Lambda_c^+ + D_s^- / \Lambda_c^+ + D_s^+$ |
| unstable | $CU\bar{S}$      | 5625          | -1/0       | $\Lambda_c^+ / \eta_c / \Lambda_c^+ / \eta_c$ |
| unstable | $CU\bar{U}$      | 2365          | 0/0        | $D^0 / K^+ + \bar{D}^0 / K^+$ |
| unstable | $CUSS$           | 2953          | -2/0       | $D_s^+ + D_s^- / D_s^+ + K^+$ |
| unstable | $CSS\bar{S}$     | 3685          | -3/0       | $D_s^+ / D_s^- / \Lambda_c^+ + \eta / \eta_c$ |
| unstable | $CU\bar{D}\bar{U}$ | 2568     | -1/0       | $\Lambda_c^+ + \pi^+ + \pi^- / \Lambda_c^+ + \pi^0 + \pi^0$ |
| unstable | $CSU\bar{D}$     | 2850          | -102/0     | $\Xi_c^0 + \pi^+ / \Xi_c^+ + \pi^0$ |
| unstable | $CSUDD$          | 2794          | -46/0      | $\Xi_c^+ + \pi^+ / \Xi_c^+ + \pi^+$ |
| unstable | $CSSDD$          | 2970          | 0/0        | $\Xi_c^+ + K^- / \Xi_c^+ + K^-$ |

Fig. 1 can accommodate a few stationary states that can be regarded as excitations of the ground-state meson. In Table 7 we show calculated masses of 3 lowest spherically symmetric ($J^P = 0^-$) meson states and compare them with experimental numbers where available. The same basis set was used for the ground and excited states. Obtained excitation energies of the order of several hundreds of MeV are roughly consistent with observed data. This gives us some confidence regarding the overall shape of the selected interaction potentials.
Table 7: Low mass states of some mesons with angular momentum quantum numbers $S = L = J = 0$. Masses are in MeV/c$^2$.

| Sakaton structure | $1^1S_0$ mass calc./exp. | $2^1S_0$ mass calc./exp. | $3^1S_0$ mass calc./exp. |
|-------------------|---------------------------|---------------------------|---------------------------|
| $DU$              | $\pi^-$ 142/140            | $\pi$(1300) 1480/1300     | $\pi$(1800) 1726/1816     |
| $SU$              | $K^-$ 364/494              | 1669                      | 1909                      |
| $SS$              | $\eta$ 1095/548            | $\eta$(1475) 1974/1476    | 2136                      |
| $CU$              | $D^0$ 2001/1897            | 2942                      | 3117                      |
| $CS$              | $D^+_s$ 2587/1968          | 3214                      | 3331                      |
| $CC$              | $\eta_c(1S)$ 3339/2980    | $\eta_c(2S)$ 4282/3637   | 4456                      |

3 Discussion

The most important lesson of the above calculations is that the Sakata model is qualitatively correct, at least in the part concerning masses of strongly interacting particles. With properly adjusted interaction potentials, this model correctly predicts the stability of those species, which are found stable in nature. On the other hand, the unbound combinations of sakatons are exactly those, which were not seen in experiments. The calculated masses of stable particles (see Tables 5 and 6) sometimes differ from experimental values by hundreds of MeV/c$^2$. For example, masses of baryons are systematically overestimated. However, such discrepancies are expected due to our use of simplified 2-particle potentials (2). One can expect that true sakaton interactions have a more sophisticated form.

For example, in our approach, $U - D$, $U - U$, and $D - D$ potentials are purely repulsive. This does not allow us to describe bound states like $UD$ (deuteron) or $UUDD$ ($\alpha$-particle). It seems plausible that these interactions (especially the $U - D$ potential) can be slightly modified so as to make them attractive at distances $\approx$1-2 fm (see broken line in Fig. 1). Then it might be possible to reproduce the bonding of protons and neutrons in nuclei. Such a possibility is especially exciting as it would allow us to describe the stabilities of mesons, baryons and nuclei within the same set of sakaton interactions.

Another missing piece is the absence of relativistic corrections that may include momentum-dependent, spin-orbit, spin-spin, and contact interactions. It is well-established that they can contribute up to several hundreds of MeV to the overall energy balance of mesons and nuclei.

One can also add to (2) terms which change the number and/or types of particles. For example, terms like\textsuperscript{7}

\textsuperscript{7}Here $u, \bar{u}, d, \bar{d}$ are annihilation operators for the $U, \bar{U}, D, \bar{D}$ sakatons, and $u^\dagger, \bar{u}^\dagger, d^\dagger, \bar{d}^\dagger$ are their creation operators.
\[ V_{\text{mix}} \propto u^\dagger \overline{u} d + d^\dagger \overline{d} u + \ldots \] (3)

are responsible for the mixing of \( UU \) and \( DD \) states and for the mass splitting between \( \pi^0 = \frac{1}{\sqrt{2}}(UU - DD) \) and \( \eta' \approx \frac{1}{\sqrt{2}}(UU + DD) \) mesons \[1\]. Without interaction (3) particles \( \pi^0 \) and \( \eta' \) have the same mass, while experimentally their masses are quite different: \( m(\pi^0) = 135 \text{ MeV}/c^2 \), \( m(\eta') = 958 \text{ MeV}/c^2 \). This indicates the significant role of terms like (3). Generally, one can also expect the presence of interactions that lead to the mixings \( NN \leftrightarrow SS \leftrightarrow CC \). Our neglect of these interactions may partially explain the overestimation of masses of \( \pi^0 \), \( \eta \), and \( \eta_c \) mesons.

One may argue that Sakata’s assumption of a fundamental point-like proton must be wrong because, being probed by truly point-like electrons, the proton demonstrates a sizeable charge radius of 0.877 fm. However, this experimental fact can be accommodated within the Sakata model as well. To achieve that, one can assume the presence of particle-number-changing interaction terms like

\[ V_{\text{unphys}} \propto \overline{u} u^\dagger \overline{u} d + \overline{d} d^\dagger \overline{d} u + \overline{u} u^\dagger uu + \overline{u} d^\dagger du + \ldots \] (4)

in the Hamiltonian. In the classification of \[15\] these terms are called “unphys”. If they are present, then single “bare” proton states \( u^\dagger |0\rangle \) are not eigenstates of the total Hamiltonian. To make the theory sensible, one needs to perform a renormalization. If coefficient functions in the interaction (4) are properly chosen, then all loop integrals are finite, the renormalization effects are finite too, and the “bare” proton becomes “dressed” by a cloud of virtual pairs and pions, thus acquiring a non-zero size \[16\].

In spite of the deficiencies listed above, our results indicate a remarkable consistency between the quark and Sakata models: both models predict the same set of stable hadron states. This suggests that Sakata’s idea about the hadron structure has a non-vanishing fighting chance against the quark model. Further studies with more elaborate potentials would be certainly welcome. In addition to the masses of stable species considered here, these future studies should address resonances and scattering properties as well.

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References

[1] K. Nakamura, et al. (The Particle Data Group). The Review of Particle Physics. J. Phys. G, 37:075021, 2010.

\[8\] e.g., if they decay rapidly at large values of momenta; see Theorem 7.12 in \[15\]

\[9\] This does not apply to the \( \Omega^{++} \) particle whose quark content is \( ccc \). This particle appears unstable in our approach. The experimental confirmation of its existence is still lacking.
[2] S. Sakata. On a composite model for the new particles. Prog. Theor. Phys., 16:686, 1956.

[3] H. J. Lipkin. Lie groups for pedestrians. North-Holland, Amsterdam, 1966. 2nd edition.

[4] S. Weinberg. The Quantum Theory of Fields, Vol. 1. University Press, Cambridge, 1995.

[5] K. Matumoto. Some consequences of the compound hypothesis for elementary particles. Prog. Theor. Phys., 16:583, 1956.

[6] K. Matumoto, M. Nakagawa. On the structure of the elementary particles. Prog. Theor. Phys., 23:1181, 1960.

[7] K. Matumoto, S. Sawada, Y. Sumi, M. Yonezawa. Mass formula in the Sakata model. Prog. Theor. Phys. Suppl., 19:66, 1961.

[8] K. Matumoto. Remarks on the mass formula in the Sakata model. Prog. Theor. Phys., 25:1047, 1961.

[9] S. Sawada, M. Yonezawa. Mass levels of baryons and mesons. Prog. Theor. Phys., 23:662, 1960.

[10] A. Martin, J.-M. Richard, T. T. Wu. Stability of systems of three arbitrary charges: General properties. Phys. Rev. A, 52:2557, 1995.

[11] E.A.G. Armour, J.-M. Richard, K. Varga. Stability of few-charge systems in quantum mechanics. Phys. Rep., 413:1, 2005. http://arxiv.org/abs/physics/0411204v1.

[12] K. Varga, Y. Suzuki. Solution of few-body problems with the stochastic variational method. I. Central forces with zero orbital momentum. Comp. Phys. Comm., 106:157, 1997.

[13] K. Varga, Y. Suzuki. Precise solution of few-body problems with the stochastic variational method on a correlated Gaussian basis. Phys. Rev. C, 52:2885, 1995.

[14] Y. Suzuki, K. Varga. Stochastic variational approach to quantum-mechanical few-body problems. Springer-Verlag, Berlin, Heidelberg, 1998.

[15] E. V. Stefanovich. Relativistic quantum dynamics, 2005. http://www.arxiv.org/abs/physics/0504062v13.

[16] R. E. Wagner, M. R. Ware, Q. Su, R. Grobe. Space-time properties of a boson-dressed fermion for the Yukawa model. Phys. Rev. A, 82:032108, 2010.