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On the Cohomology of the Moduli space of Parabolic Connections

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Introduction

In this paper, we study the moduli space of logarithmic connections of rank 2 on \( \mathbb{P}^1 \setminus \{ \theta_1, ..., \theta_n \} \) with fixed spectral data. We compute the cohomology of such moduli space, and the computation will be used to extend the results of parabolic Hitchin equations of [1] to the case where the parabolic connections have five simple poles in \( \mathbb{P}^1 \).

Proposition 4.1

\[ \pm L_i \equiv f_i, O_{K,p}, q \]

Lemma 3.1

In this paper, we study the moduli space of logarithmic connections of rank 2 on \( \mathbb{P}^1 \setminus \{ \theta_1, ..., \theta_n \} \) with fixed spectral data. We compute the cohomology of such moduli space, and this computation will be used to extend the results of parabolic Hitchin equations of [1] to the case where the parabolic connections have five simple poles in \( \mathbb{P}^1 \).

Definition 2.1 A logarithmic connection is a triple \((E, \nabla, q)\), such that \( \nabla : E \to \mathcal{E}(\Omega^1) \) is a connection, where \( \mathcal{E} \) is a rank 1 vector bundle on \( \mathbb{P}^1 \), \( \nabla \) is a possible logarithmic connection, and \( \text{ord}_p \nabla(T) \leq 1 \) for the residue \( \text{ord}_p(T) \) at \( p \) has expression \( a_i = \sum_i b_i \).

Denote by \( \mathcal{M} \) the moduli stack of logarithmic connections on \( \mathbb{P}^1 \). We have the next proposition.

Proposition 3.2

We can extend the map (2) to \( M \to \mathbb{M}|_{\mathcal{M}} \) and this map is injective.

Cohomology of \( \mathcal{M} \)

Suppose \( n = 5 \). For computing the cohomology of \( \mathcal{M} \), we introduce some blowing-up of the Hilbert stack of objects \( \mathcal{O} \). Put \( L = \mathcal{O}(\mathcal{E}) \). Let \( \nu_i \) be the total space of the line bundle \( L \), then \( L_i \equiv L_i \). Fix \( \mathcal{O} \), the Hilbert stack of objects in \( \mathcal{O}(\mathcal{E}) \). Note that \( L_i \) is the Hilbert stack of \( \mathcal{O} \). Fix \( \mathcal{O} \), the moduli stack of \( \mathcal{O} \). Put \( \nu_i = \nu_i \). Let \( \mathcal{O} \), the moduli stack of \( \mathcal{O} \). Put \( \nu_i = \nu_i \).

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