Chromomagnetic Instability and Gluonic Phase in Dense Neutral Quark Matter

Osamu KIRIYAMA

Institut für Theoretische Physik, J.W. Goethe-Universität, D-60438 Frankfurt am Main, Germany

We discuss the chromomagnetic instability problem in the 2SC/g2SC phases and explore the phase structure of the gluonic phase which is a candidate for the ground state of dense neutral quark matter.

§1. Introduction

The phase structure of hot and/or dense QCD is one of the most exciting current topics in the field of strong interactions. In particular, the properties of cold and dense quark matter are of great interest in astrophysics and cosmology; it is now widely accepted that, at moderate densities of relevance for the interior of compact stars, quark matter is a color superconductor and has a rich phase structure with important implications for compact star physics (for a recent review, see Ref. 1)).

Bulk matter in the interior of compact stars should be color and electrically neutral and be in \( \beta \)-equilibrium. In the two-flavor case, these conditions separate the Fermi momenta of up and down quarks and, as a consequence, the ordinary BCS state (2SC) is not always energetically favored over other unconventional states. The possibilities include crystalline color superconductivity\(^2\),\(^3\) and gapless color superconductivity (g2SC).\(^4\) However, the 2SC/g2SC phases suffer from a chromomagnetic instability, indicated by imaginary Meissner masses of some gluons.\(^5\) The instability related to gluons of color \( 4\)–\( 7 \) occurs when the ratio of the 2SC gap over the chemical potential mismatch, \( \Delta/\delta \mu \), decreases below a value \( \sqrt{2} \). Resolving the chromomagnetic instability and clarifying the nature of true ground state of dense quark matter are central issues in the study of color superconductivity.\(^6\)–\(13\)

We will describe the results of recent studies of a chromomagnetic instability and a gluonic phase\(^6\) (gluonic cylindrical phase II) in dense neutral quark matter.

§2. Model, formalism and numerical results

In order to study the chromomagnetic instability and phases with gluonic vector condensates, we use a gauged Nambu–Jona-Lasinio (NJL) model with massless up and down quarks:

\[
\mathcal{L} = \bar{\psi}(iD + \hat{m})\psi + G_D(\bar{\psi}i\gamma_5\epsilon^bC\psi^T)(\psi C i \gamma_5 \epsilon^b \psi) - \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu},
\]

where the quark field \( \psi \) carries flavor \( (i, j = 1, \ldots, N_f \) with \( N_f = 2 \) \) and color \( (\alpha, \beta = 1, \ldots, N_c \) with \( N_c = 3 \) \) indices, \( C \) is the charge conjugation matrix; \( (\epsilon)_{ij} = \epsilon^{ijk} \) and \( (\epsilon^b)_{\alpha \beta} = \epsilon^{b \alpha \beta} \) are the antisymmetric tensors in flavor and color spaces, respectively.
The diquark coupling strength in the scalar color-antitriplet channel is denoted by \( G_D \). The covariant derivative and the field strength tensor are defined as

\[
D_\mu = \partial_\mu - igA^a_\mu T^a, \quad F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu.
\] (2.2)

In NJL-type models without dynamic gauge fields, one has to introduce color and electric chemical potentials by hand to ensure color- and electric-charge neutrality. In \( \beta \)-equilibrated neutral two-flavor quark matter, the elements of the diagonal matrix of quark chemical potentials \( \hat{\mu} \) are given by

\[
\mu_{ur} = \mu_{ug} = \bar{\mu} - \delta\mu, \quad \mu_{dr} = \mu_{dg} = \bar{\mu} + \delta\mu,
\]

\[
\mu_{ab} = \bar{\mu} - \delta\mu - \mu_8, \quad \mu_{db} = \bar{\mu} + \delta\mu - \mu_8,
\] (2.3)

with \( \bar{\mu} = \mu - \delta\mu/3 + \mu_8/3 \) and \( \delta\mu = \mu_\epsilon/2 \).

In the mean-field approximation, the effective potential with constant gluonic vector condensates is given by

\[
\Omega_R = \Omega(\Delta, \delta\mu, \mu_a, \vec{A}^a; \mu, T) - \Omega(0, 0, 0, \vec{A}^a; 0, 0),
\] (2.4)

where

\[
\Omega = \frac{\Lambda^2}{4G_D} - \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \ln S^{-1}(p) - \frac{\mu_\epsilon^4}{12\pi^2} + \frac{g^2}{4} f^{abc} f^{ade} A^b_\mu A^e_\nu A^{d\mu} A^{e\nu}.
\] (2.5)

Here \( S^{-1}(p) \) is the inverse full quark propagator in Nambu-Gor’kov space,

\[
S^{-1}(p) = \begin{pmatrix}
\phi + (\bar{\mu} - \delta\mu \tau^3)\gamma^0 + gA^a T^a & -i\epsilon^{b\gamma_5} \Delta

-i\epsilon^{b\gamma_5} \Delta & \phi - (\bar{\mu} - \delta\mu \tau^3)\gamma^0 - gA^a T^a
\end{pmatrix},
\] (2.6)

where \( \tau^3 = \text{diag}(1, -1) \) is a matrix in flavor space. Following the usual convention, we have chosen the diquark condensate to point in the third direction in color space. In order to evaluate loop diagrams we use a three-momentum cutoff \( \Lambda = 653.3 \text{ MeV} \) throughout this paper.

The chromomagnetic instability related to gluons 4–7 and 8 can be viewed as tendencies toward the vector condensation of \( \phi_\mu \) and \( A^{6,8}_\mu \) respectively, where \( \phi_\mu = \sqrt{1/2}(A^4_\mu - iA^5_\mu, A^6_\mu - iA^7_\mu)^T \) is the doublet field with respect to the residual \( SU(2)_c \) symmetry in the 2SC/g2SC phases. Because of the \( SU(2)_c \) symmetry, we can choose \( A^6_\mu \) without loss of generality.

In Fig. 1, we plotted the phase diagram of homogeneous neutral two-flavor quark matter in the plane of the 2SC gap at \( \delta\mu = 0 \) (\( \Delta_0 \)) and temperature \( (T) \). (The parameter \( \Delta_0 \) is essentially the diquark coupling strength. We neglect color chemical potentials throughout, because the present work remains qualitatively unaffected by them.) We calculated the squared Meissner masses of gluons from the second derivative of the effective potential with respect to \( \langle \vec{A}^6 \rangle \) and \( \langle \vec{A}^8 \rangle \) and mapped out the unstable regions for gluons 6 and 8 on the phase diagram. In the unstable regions, it is natural to suggest that the true ground state is given by the global minimum of the effective potential, including the gluonic vector condensation. Note that a \( \langle \vec{A}^6 \rangle \)-condensed phase is called gluonic phase and a \( \langle \vec{A}^8 \rangle \)-condensed phase is gauge equivalent to the (single plane-wave) LOFF phase. \(^2\), \(^3\), \(^6\), \(^7\)
Fig. 1. Left: The phase diagram of a neutral two-flavor color superconductor in the plane of temperature and $\Delta_0$. At $T = 0$, the g2SC phase exists in the window $92 \text{ MeV} < \Delta_0 < 134 \text{ MeV}$ and the 2SC window is given by $\Delta_0 > 134 \text{ MeV}$. The results are plotted for $\mu = 400 \text{ MeV}$. Middle: The same as the left panel, but the unstable region for gluons 4–7 is depicted by the region enclosed by the thick solid line. At $T = 0$, the g2SC phase and a part of the 2SC phase ($92 \text{ MeV} < \Delta_0 < 162 \text{ MeV}$) suffer from the chromomagnetic instability. Right: The same as the left panel, but the unstable region for the 8th gluon is depicted by the region enclosed by the thick solid line.

In order to find the gluonic/LOFF phases, we solved the gap equations for $\Delta$ and $\langle \vec{A}^a \rangle$ ($a \in (6,8)$) and the neutrality condition for $\delta \mu$ self-consistently and computed their free energies.\(^{14}\) (This has been done first by Hashimoto and Miransky.\(^{13}\)) We illustrate the comparison of the free energies of the gluonic/LOFF phases in Fig. 2. The result is plotted for $\mu = 400 \text{ MeV}$ and $T = 0$ as a function of $\Delta_0$. The gluonic phase exists in the window $66 \text{ MeV} < \Delta_0 < 162 \text{ MeV}^*$ and is energetically more favored than the LOFF phase in a wide range of the coupling strength. It is also quite interesting to note that the gluonic (LOFF) phase could be energetically more favored than the NQ phase for $66 \text{ MeV} < \Delta_0 < 92 \text{ MeV}^*$ ($65 \text{ MeV} < \Delta_0 < 92 \text{ MeV}$), though the chromomagnetic instability does not exist in the NQ phase. By computing the effective potential as a function of the vector condensates along the self-consistent solution of the gap equation for $\Delta$ and the neutrality condition for $\delta \mu$, we revealed that it indeed happens in the weak-coupling regime (see Fig. 3). In the following, we

\(^{*}\) The upper boundary of the gluonic (LOFF) phase is slightly larger than that of the unstable region for gluons 4–7 (8), because the transition from the gluonic (LOFF) phase to the stable 2SC phase is of first order.
Fig. 3. The effective potential of the gluonic (LOFF) phase as a function of $B = \langle gA^6 \rangle (q = \langle gA^8 \rangle / (2\sqrt{3}))$. Note that the effective potentials are calculated along the self-consistent solution of the gap equation for $\Delta$ and the neutrality condition for $\delta \mu$ and are measured with respect to the 2SC/g2SC/NQ phases at $B = 0$ ($q = 0$). The results are plotted for $\mu = 400$ MeV and $T = 0$.

The effective potentials are calculated along the self-consistent solution of the gap equation for $\Delta$ and the neutrality condition for $\delta \mu$ and are measured with respect to the 2SC/g2SC/NQ phases at $B = 0$ ($q = 0$). The results are plotted for $\mu = 400$ MeV and $T = 0$.

We extended our analysis to nonzero temperatures and obtained the schematic phase diagram shown in Fig. 4. As expected, the low-temperature regions of the 2SC/g2SC phases are replaced by the gluonic phase. We also find that the transition between the gluonic phase and the 2SC/g2SC phases is of second order or weakly first order. Furthermore, the gluonic phase wins against a part of the NQ phase and undergoes a strong first-order phase transition into the NQ phase. (Here, it should be mentioned that a cutoff artifact appears in the gluon sector of the effective potential. Therefore it may be impossible to distinguish the second-order transition from the weak first-order transition. However, the cutoff artifact is not significantly large, hence we could calculate the phase diagram.)

From the result shown in Fig. 4, we could imagine that a large region of the 2SC/g2SC/NQ phases in the intermediately coupled cold dense quark matter are replaced by the gluonic phase. To be more explicit and realistic, we used a three-flavor gauged NJL model and investigated the 2SC/g2SC/NQ phases in the $T-\mu$ plane.
Fig. 5. Left: The phase diagram of neutral three-flavor quark matter in $T$-$\mu$ plane, with the diquark coupling chosen so that the 2SC gap $\Delta_0 = 80$ MeV at $T = 0$, $\mu = 400$ MeV and $M_s \to \infty$. Note that only the 2SC/g2SC phases are included in the analysis. In the region enclosed by the thick solid line, the Meissner masses of gluons 4–7 are tachyonic. Right: The same as the left panel, but the gluonic phase is taken into account. The transition between the gluonic phase and the g2SC phase is of second order or weakly first order. On the other hand, the gluonic phase undergoes a strong first-order transition to the NQ phase.

phase diagram (left panel of Fig. 5). We used a (density-independent) fixed value of the strange quark mass ($M_s = 500$ MeV). The diquark coupling was chosen so that the 2SC gap $\Delta_0 = 80$ MeV at $T = 0$, $\mu = 400$ MeV and $M_s \to \infty$. It should be noted that only the 2SC/g2SC/NQ phases were included in this analysis. We also neglected the dynamical breakdown of chiral symmetry. Nevertheless, at least qualitatively, this phase diagram is consistent with those presented in Ref. 15). We again studied the gluonic phase in a self-consistent manner and obtained a phase diagram displayed in the right panel of Fig. 5. One can see that the unstable g2SC phase is replaced by the gluonic phase and, furthermore, the gluonic phase is favored over the stable NQ phase in the low-temperature regime.

§3. Summary

We studied the chromomagnetic instability in the 2SC/g2SC phases at moderate density and at finite temperature. We calculated the Meissner masses squared of gluons 4–7 and 8 and mapped out the unstable regions on the phase diagram. In order to resolve the chromomagnetic instability, we investigated the phases with gluonic vector condensations, i.e., the gluonic phase and the single plane-wave LOFF phase. Using the gauged NJL model and the mean-field approximation, we computed the free energies of the gluonic/LOFF phases in a self-consistent manner and explored the phase structure:

- The gluonic phase is favored over the LOFF phase in a wide range of coupling strength.
- The effective potential shows a peculiar behavior as a function of the vector condensations. It is particularly interesting that, in the weak-coupling regime, the gluonic/LOFF phases could be energetically more favored than the chromomagnetically stable NQ phase.
- The strongly first-order transition between the gluonic phase and the NQ phase
Chromomagnetic Instability and Gluonic Phase

59

takes place. On the other hand, the transition from the gluonic phase to the stable g2SC is of second order or weakly first-order.

In addition to the two-flavor case, we also explored the 2SC/g2SC and the gluonic phases in neutral three-flavor quark matter. The resulting $T$-$\mu$ phase diagram shows clearly that currently known phase diagrams must be significantly altered.

As we have shown, the gluonic phase is energetically more favored than the unstable 2SC/g2SC phases. However, the project of finding the true ground state of neutral dense quark matter is still not complete. In order to draw a conclusion, we have to investigate LOFF phases with multiple plane waves and other types of the gluonic phase.$^{6,16}$ It is also indispensable to look at the chromomagnetic stability of those phases.$^{17}$

Although the result obtained from the (gauged) NJL models seems to be qualitatively reasonable, it should be unreliable quantitatively. Therefore, a novel technique is required to perform nonperturbative QCD calculations at relevant densities and to obtain a plausible QCD phase diagram.

Acknowledgements

I thank Dirk Rischke and Igor Shovkovy for fruitful discussions. I also thank the organizers of YITP international symposium on “Fundamental Problems in Hot and/or Dense QCD” for their invitation and warm hospitality. This work was supported by the Deutsche Forschungsgemeinschaft (DFG).

References

1) M. G. Alford, A. Schmitt, K. Rajagopal and T. Schäfer, arXiv:0709.4635.
2) A. I. Larkin and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47 (1964), 1136 [Sov. Phys. -JETP 20 (1965), 762.
3) M. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D 63 (2001), 074016.
4) I. Shovkovy and M. Huang, Phys. Lett. B 564 (2003), 205.
5) M. Huang and I. Shovkovy, Nucl. Phys. A 729 (2003), 835.
6) E. V. Gorbar, M. Hashimoto and V. A. Miransky, Phys. Lett. B 632 (2006), 305; Phys. Rev. D 75 (2007), 085012.
7) E. V. Gorbar, M. Hashimoto and V. A. Miransky, Phys. Rev. Lett. 96 (2006), 022005.
8) K. Fukushima, Phys. Rev. D 73 (2006), 094016.
9) O. Kiriyama, D. H. Rischke and I. A. Shovkovy, Phys. Lett. B 643 (2006), 331.
10) O. Kiriyama, Phys. Rev. D 74 (2006), 074019; Phys. Rev. D 74 (2006), 114011.
11) L. He, M. Jin and P. Zhuang, Phys. Rev. D 75 (2007), 036003.
12) R. Gatto and M. Ruggieri, Phys. Rev. D 75 (2007), 114004.
13) M. Hashimoto and V. A. Miransky, Prog. Theor. Phys. 118 (2007), 303.
14) O. Kiriyama, arXiv:0709.1083.
15) S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, Phys. Rev. D 72 (2005), 034004.
16) K. Rajagopal and R. Sharma, Phys. Rev. D 74 (2006), 094019.
17) M. Hashimoto and J. Jia, Phys. Rev. D 76 (2007), 114019.

M. Hashimoto, arXiv:0803.0175.