Inverse Problem for Electromagnetic Propagation in Human Muscle Tissues: Frequency Dependent Approach

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Abstract
This paper is concerned with an electromagnetic interrogation method for human muscle tissues using stochastic inversion methodology. The frequency dependence of relative permittivity of human muscle tissues is modeled by Debye formula with multiple dielectric relaxations. The associated signal response model is given by a complex reflectance at the interface between free space and the human body. The problem considered here is to identify posteriori distribution of the dielectric parameters from the measurements by a reflective interferometer. A computational experiments using Hamiltonian Monte Carlo sampling are summarized.

1 Introduction
Recently, electromagnetic imaging is of considerable interest in the signal detection and the identification related to the development of new ceramic materials, the early detection of anomalies in polymer, metabolic functioning of human bodies, etc [1]. Specifically, measurements for reflectance of microwave tests can be employed in the evaluation of the structural integrity of composite dielectric materials. This paper is focused on the identification and parameter estimation of human muscle tissues that entail treatments of individual parameters as random variables to be estimated from data. Our inversion technique is proposed based on classical Bayesian approach using Markov Chain Monte Carlo sampling [2][3]. Previous efforts have been concentrated on problems of stochastic or statistical inverse problems that include some application to structural health monitoring [4] to [7]. In this paper, the method is effectively used for reconstructing a posteriori probability density functions of dielectric parameters for human muscle tissues.

The ability to interrogate of tissues has wide ranging applications to medical imaging. Low amplitude and low energy microwaves can pass through dielectric media without causing biological tissues. Dielectric relaxation measurements can analyze dielectric and electrical loss which provide useful and sensitive information on the human muscle tissues. Network analyzers are able to present a TDR view by transforming a frequency sweep to the time domain with a FFT.

2 Mathematical Model
We consider a scattering problem for the region of interest $Z_{ROI} = (0, z_2) \subset \mathbb{R}^3$ as illustrated in Fig. 1. A homogeneous dielectric material is assumed to be located at the interval $(z_1, z_2) \subset Z_{ROI}$. Let $H_y(t, z)$ be the magnetic field at the time $t \in (0, t_f)$ at the location $z \in Z_{ROI}$.

![Fig. 1: Geometry of physical domain](image_url)

A planar electromagnetic wave $H_y(t, z)$ normally incident on a dielectric material with faces parallel to the $x - y$ plane. We also define the electric field $E_z(t, z)$ and the electric flux density $D_z(t, z)$ in $(0, \infty) \times Z_{ROI}$. Those are polarized with oscillations in the $x - z$ plane. Throughout the paper, we assume that the medium is composed of nonmagnetic material. Under these assumptions, Maxwell’s equations in $Z_{ROI}$
can be represented by
\[
\frac{\partial E_s(t, z)}{\partial z} = -j\mu_0 \frac{\partial H_s(t, z)}{\partial t} \quad \text{in } [0, \infty) \times Z_{ROI}
\]
\[
\frac{\partial H_s(t, z)}{\partial z} = \frac{\partial D_s(t, z)}{\partial t} + \sigma_0 E_s(t, z) + J_y^s(t, z) \quad \text{in } [0, \infty) \times Z_{ROI}
\]
with the initial state
\[
E_s(0, z) = H_s(0, z) = 0 \quad \text{on } Z_{ROI}
\]
where \(\mu_0\) and \(\sigma_0\) denote the magnetic permeability of air and the electrical conductivity of the dielectric medium. To describe the behavior of the macroscopic electric polarization in the dielectric medium in human body, we assume that the polarization directly depend on the history of the electrical field. The dielectric polarization \(P(t, z) = \sum_{k=1}^{K} P_k(t, z)\) can be then given by dielectric response function with a displacement susceptibility kernel function \((g_\xi)_{k=1}^{K}\) (See [1] [8] for more details);
\[
P_s(t, z) = \int_0^\infty g(t - \xi) E_s(\xi, z) d\xi.
\]
In our formulation, the kernel function \(g\) is approximated by
\[
g(t) = \varepsilon_0 (\varepsilon_r - \varepsilon_\infty) \sum_{m=1}^{M} r_m \exp(-t/r_m)
\]
where \(\varepsilon_0\), \(\varepsilon_r\), and \(\varepsilon_\infty\) denote the electric permittivity of free space, the static relative permittivity, and the permittivity at the very high frequencies of the medium, respectively. In Eq. (5), \(g_\xi\) and \(r_m\) imply the weight constant and the carrier relaxation time to be normally unknown. Allowing the instantaneous component of the polarization to be related to the electric field by a dielectric constant, the resulting constitutive law can be written by
\[
D_s(t, z) = \varepsilon_0 \varepsilon_r E_s(t, z) + P_s(t, z).
\]
Let us define the Laplace transform of the electrical field \(E_s(t)\) by
\[
\tilde{E}_s(s) := \mathcal{L}[E_s(t)] = \int_0^\infty E(t, \cdot) \exp(-st) dt
\]
where \(s = \psi + j\omega\). Similarly we define
\[
\tilde{D}_s := \mathcal{L}[D_s(t)], \quad \tilde{P}_s := \mathcal{L}[P_s(t)],
\]
\[
\tilde{H}_s := \mathcal{L}[H_s(t)], \quad \tilde{J}_y^s := \mathcal{L}[J_y^s(t)]
\]
Then we obtain
\[
\frac{\partial \tilde{E}_s(s, z)}{\partial z} = -\mu_0 s \tilde{H}_s(s, z)
\]
\[
\frac{\partial \tilde{H}_s(s, z)}{\partial z} = -s \tilde{D}_s(s, z) - \sigma_0 \tilde{E}_s(s, z) - \tilde{J}_y^s(s, z).
\]
Noting that
\[
\tilde{P}_s(s, z) = \sum_{k=1}^{K} \frac{B_k}{1 + \tau_k s} \tilde{E}_s(s, z)
\]
\[
\tilde{D}_s(s, z) = \varepsilon_0 \varepsilon_r \tilde{E}_s(s, z) + \tilde{P}_s(s, z).
\]
Eq. (9) is rewritten by
\[
\frac{\partial \tilde{H}_s(s, z)}{\partial z} = -\varepsilon_0 s \tilde{E}_s(s, z)
\]
\[
\frac{\partial \tilde{H}_s(s, z)}{\partial z} = -\varepsilon_0 s \tilde{E}_s(s, z) + \tilde{J}_y^s(s, z)
\]
In the frequency domain, the electromagnetic propagation in human muscle tissues can be obtained by substituting \(s = j\omega\) into Eqs. (8) and (12). The frequency dependent model in a dielectric material can be expressed by
\[
\frac{\partial \tilde{E}_s(\omega, z)}{\partial z} = -\mu_0 j\omega \tilde{H}_s(\omega, z)
\]
\[
\frac{\partial \tilde{H}_s(\omega, z)}{\partial z} = -j\varepsilon_0 \varepsilon_r \tilde{E}_s(\omega, z) + \tilde{J}_y^s(\omega, z)
\]
where \(\varepsilon^*\) is the complex relative permittivity defined by
\[
\varepsilon^*_\infty(\omega, z) := \varepsilon_0 + \frac{\sigma_0}{\varepsilon_\infty} \sum_{m=1}^{M} \frac{g_m}{1 + i\omega \tau_m}
\]
for \(z_1 < z < z_2\)

The spectrum of dielectric media in human muscles involves multiple relaxation points. The feature of this model includes the relaxation time \(\tau_m\) and the weight coefficients \(g_m\) which means the contributions to the permittivity loss at each relaxation time. Let \(\tau_m = (\mu_0 \varepsilon_\infty)^{-1}\) be the characteristic time. Set the angular frequency \(\omega = k_c / \Delta t\) be the interrogation wavenumber with respect to the specific angular frequency \(\omega\). Setting as \(\eta_0 = \sqrt{\mu_0 / \varepsilon_\infty}\), the complex permittivity is replaced by
\[
\tilde{\varepsilon}(k) = \varepsilon_0 + \frac{\eta_0 \sigma_0}{2\pi k} + \sum_{m=1}^{M} \frac{\Delta \varepsilon_m}{1 + i(2\pi \tau_m)k}
\]
where
\[
\Delta \varepsilon_m = (\varepsilon_r - \varepsilon_\infty) g_m.
\]
The purpose of this study is to identify the unknown parameters \(\tau_m, \Delta \varepsilon_m\) through the measurements by electromagnetic microscopy. The reflective coefficient \(r\) at the plane interface between air and a dielectric medium is given by
\[
r(k) = \frac{1 - \sqrt{\varepsilon(k)} \Delta t}{1 + \sqrt{\varepsilon(k)} \Delta t}
\]
Then, by sweeping frequencies over ten decades, measurements provide the observable reflectance [9] given by
\[
R(k_i) = r(k_i) r(k_i), \quad k_i = f_i / c \quad \text{for } i = 0, 1, \cdots, N-1.
\]
Observation data can be performed in time domain and the collected data are transformed into the spectrum using the DFT.

3 Formulation of the Problem

Let $Q^M$ be a random vector on the probability triple $(\Omega, \mathcal{F}, P)$. Assuming that random variables $Q_i$ are mutually independent, the joint distribution of $Q^M$ is represented by

$$F_{Q}(Q^M) = \prod_{m=1}^{M} F_{Q_m}(q_m)$$

$$= \prod_{m=1}^{M} P(Q_m \leq q_m) \quad \text{for} \quad q_m \in \mathbb{R}^M.$$ (20)

The associated joint prior density functions for $Q^M$ can be represented by

$$\pi_{Q}(Q^M) = \prod_{m=1}^{M} \pi_m(q_m)$$ (21)

where

$$\pi_m(q_m) = \frac{dF_{Q_m}(q_m)}{dq_m} \quad \text{for} \quad i = 1, 2, \cdots, M.$$ (22)

Our inverse problem treated here is to identify the unknown parameter vector $q^M = [\tau_m, \Delta \epsilon_m]_{m=1}^M \in \mathbb{R}^{2M}$ based on the observable reflectance $[R(k_i)]_{i=0}^{N-1}$. Hence the prior density of unknown parameters $q_M$ can be represented by the product form:

$$\pi_{Q}(Q^M) = \prod_{m=1}^{M} \pi_m(q_m)$$

$$= \pi(\tau_1) \times \pi(\tau_2) \times \cdots \times \pi(\tau_M)$$

$$\times \pi(\Delta \epsilon_1) \times \pi(\Delta \epsilon_2) \times \cdots \times \pi(\Delta \epsilon_M).$$ (23)

It is a natural way that the real data $Y_d = [y_d(k_i)]_{i=0}^{N-1}$ are corrupted with measurement noise. Hence we assume that the residual term

$$\eta_i = y_d(k_i) - R(k_i, q^M)$$

has the statistical distribution:

$$\eta_i \sim \mathcal{N}(y_d(k_i) - R(k_i, q^M)) \quad \text{for} \quad i = 0, 1, \cdots, N - 1.$$ (24)

Suppose that sequence of measurement errors $[\eta_i]_{i=0}^{N-1}$ are mutually independent measurement. Then the likelihood ratio function can be represented by

$$L(y_0, y_1, \cdots, y_{N-1}|q^M) = \prod_{i=0}^{N-1} \pi(\eta_i) = \prod_{i=0}^{N-1} \pi(y_i - R(k_i, q^M)).$$ (25)

Thus our inverse problem is stated as follows:

(II): Given a prior distribution $\pi_{Q}(Q^M)$ and the collection of the measurement data $[y_d(k_i)]_{i=0}^{N-1}$, estimate posterior density $\pi(Q^M)$ of unknown quantities

$$q^M = [q_m, q_{m:M}]_{m=1}^M = [\tau_m, \Delta \epsilon_m]_{m=1}^M$$ (26)

with the equality constraint:

$$\hat{y}(k, q^M) = e^{\omega_0} + \frac{\eta_0 y_0}{2\pi k_1} + \sum_{m=1}^{M} \frac{q_{m:M}}{1 + i(2\pi c_k)k_m}.$$ (27)

4 Inverse Methodology

From the Bayes formula, the posterior probability density function has the representation

$$\pi(Q^M) \propto L(y_0, y_1, \cdots, y_{N-1}|Q^M)\pi_{Q}(Q^M).$$ (28)

An estimation algorithm can be performed by sampling procedures for the posteriori distribution density $\pi(Q^M)$ from which sample paths can be drawn using Markov chains. There are many alternative algorithms of MCMC, such as Metropolis-Hasting (MH), Gibbs sampling methods, etc (See, [3]). Our prior efforts on material characterizations exist using independent Metropolis-Hastings [4], gPC Galerkin method [5], and the stochastic spline Galerkin method [6]. Those have been performed using time domain approach (the so-called "snapshot method"). The crucial aspects on our previous efforts involved spending the heavy computational times for practical implementation of the sampling procedures. As a result, the asymptotic behavior of the sampling process is not especially well suited for identifying multi-dimensional unknown quantities. Recently, an estimation scheme using Hamiltonian Monte Carlo method (HMC) was applied to the inverse problem for Dobye model for composite materials [7]. The method is a Metropolis method making use of gradient information to reduce random walk behavior. HMC has the advantages on the asymptotic behavior of MCMC sampling [10]. In HMC, the state space $z = q^M$ is augmented by momentum variables $p$. Let us define the Hamiltonian

$$H(z, p) = E(z) + K(p)$$ (29)

where $K$ is a kinetic energy. Then the algorithm is used to create samples from the joint density

$$\hat{p}_H(z, p) = \frac{1}{C_H} \exp \{-H(z, p)\}$$

$$= \frac{1}{C_H} \exp \{-E(z)\} \exp \{-K(p)\}.$$ (30)

The continuous form of the sampling mechanism can be described by the differential rule

$$\left\{ \begin{array}{l}
\dot{z} = p \\
\dot{p} = -\partial E(z)/\partial z
\end{array} \right.$$ (31)

The the explicit form of $E$ in our inversion is described by

$$E(z) = \log C + \log \pi_r (Y_d - Y(z)) + \log \pi_z(z)$$ (32)

where $C$ denotes the normalizing factor and where

$$Y(z) = [R(k_i, z)]_{i=0}^{N-1}.$$ (33)

In this paper, the same scheme is adopted to the problem for human muscle tissues.
5 Computational Experiments

5.1 Forward Analysis

Gabriel et al. [11][12] carried out an experimental study on a large number of biological tissues spanning the frequency range from 10Hz to 20GHz. Figure 2 depicts the model reflectance for $N = 100$ using the parameter values in Table 1 for the case of $M = 1, 2, 3, 4, 5$. Throughout the experiments, we set as

$$\epsilon_m = 4.3, \sigma_0 = 0.0762.$$  

The wavenumber was taken as

$$k_i = 10^{\log(f_L) + \log(f_H)/\log(N)}$$

for $i = 0, 1, 2, \cdots, N$.

In the experiments, we set as

$$f_L = 10, \quad f_H = 5 \times 10^{10}, \quad \text{and} \quad N = 10000.$$  

| $m$ | $\tau_m^{-1}$ | $\Delta \epsilon_m$ |
|-----|----------------|-----------------|
| 1   | 69 [Hz]        | $8 \times 10^7$ |
| 2   | 43 [kHz]       | 81900           |
| 3   | 670 [kHz]      | 11900           |
| 4   | 230 [MHz]      | 32              |
| 5   | 20 [GHz]       | 46              |

The simulated reflectance in human muscle model for the case $M = 1, 2, 3, 4, 5$.

5.2 Inverse Analysis

The nonlinear hierarchical models used in the inversion technique are formulated as:

$$y_i \sim \text{Normal} \left( R(k_i; q^M), \sigma_e \right)$$

$$\sigma_e \sim \text{InvGamma}(\alpha, \beta),$$

$$\tau_m^{-1} \sim \text{Uniform} \left( \bar{m}_m, \bar{m}_m^U \right), \quad (m = 1, 2, \cdots, 5)$$

$$\Delta \epsilon_m \sim \text{Uniform} \left( \Delta \epsilon_L, \Delta \epsilon_U \right), \quad (m = 1, 2, \cdots, 5)$$

where all the statistical parameters of a prior distributions are given. Syntheses data were used by computing the forward problem provided with true parameters and by adding the Gaussian random noise to those data. Those were preassigned as

$$\sigma_e = 0.01, \quad \alpha = 1.0, \quad \beta = 1.0.$$  

In the experiments, the HMC sampling using R-stan language code was adopted to our inversion [13]. Figures 5 to 12 show the marginal posteriori density plots for the unknown parameters. Tables 2 to 3 summarize the estimated values of the unknown quantities.

Fig. 2: The simulated reflectance in human muscle model for the case $M = 1, 2, 3, 4, 5$.

Fig. 3: Recovered posterior density $\pi(\tau_1)$.

Fig. 4: Recovered posterior density $\pi(\tau_2)$.  

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Fig. 5: Recovered posterior density $\pi(\tau_1)$

Fig. 6: Recovered posterior density $\pi(\tau_4)$

Fig. 7: Recovered posterior density $\pi(\tau_5)$

Fig. 8: Recovered posterior density $\pi(\Delta \epsilon_1)$

Fig. 9: Recovered posterior density $\pi(\Delta \epsilon_2)$

Fig. 10: Recovered posterior density $\pi(\Delta \epsilon_3)$

Fig. 11: Recovered posterior density $\pi(\Delta \epsilon_4)$

Fig. 12: Recovered posterior density $\pi(\Delta \epsilon_5)$
6 Concluding Remarks

Stochastic inverse methodologies for dielectric components of composite materials were investigated. The physical model given by the Debye model was considered for identifying the time relaxation parameters in human muscle tissues. Nonlinear hierarchical regression models were constructed in order to implement MCMC sampling. The feasibility studies were demonstrated by computational experiments using the simulated data analysis. Feasibility studies were carried out illustrating the method with computational experiments using the same simulated data. The consistency of estimated values can be demonstrated by the estimated marginal densities from extracting sampling results which were able to provide the reliability and uncertainty quantification for the associated inverse solutions.

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| Table 2: Test Summary ($r_m$) |
|-----------------------------|
| $m$ | True | Mean | Standard Deviation |
|-----------------------------|
| 1  | $1.45 \times 10^{-2}$ | $1.12 \times 10^{-2}$ | $1.30 \times 10^{-2}$ |
| 2  | $2.33 \times 10^{-3}$ | $2.00 \times 10^{-3}$ | $3.28 \times 10^{-9}$ |
| 3  | $1.49 \times 10^{-6}$ | $1.43 \times 10^{-6}$ | $1.18 \times 10^{-10}$ |
| 4  | $4.35 \times 10^{-9}$ | $3.37 \times 10^{-9}$ | $2.99 \times 10^{-11}$ |
| 5  | $5.00 \times 10^{-11}$ | $5.49 \times 10^{-11}$ | $4.79 \times 10^{-13}$ |

| Table 3: Test Summary ($\Delta \alpha_m$) |
|-----------------------------|
| $m$ | True | Mean | Standard Deviation |
|-----------------------------|
| 1  | $8.00 \times 10^3$ | $8.88 \times 10^3$ | $1.13 \times 10^4$ |
| 2  | $8.19 \times 10^4$ | $9.00 \times 10^4$ | $1.50 \times 10^5$ |
| 3  | $1.19 \times 10^4$ | $1.30 \times 10^4$ | $1.06$ |
| 4  | $3.20 \times 10^5$ | $5.00 \times 10^5$ | $2.31 \times 10^{-2}$ |
| 5  | $4.60 \times 10^5$ | $5.00 \times 10^5$ | $4.63 \times 10^{-2}$ |