Study of parity violating observables in few-nucleon systems

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Abstract. Parity violation in few-nucleon systems is studied using a nucleon-nucleon parity-violating (PV) potential derived within an effective field theory framework at next-to-next-to-leading order. The potential includes one- and two-pion exchanges, contact interactions and relativistic corrections and depends on six low-energy constants: the pion-nucleon coupling constant $\hbar_1^\pi$ and five parameters multiplying the independent contact interaction terms (with one four-gradient). This potential is used to study the $\vec{p}$-$p$ longitudinal asymmetry, the neutron spin rotation in $\vec{n}$-$d$ scattering, and the longitudinal asymmetry in the $^3\text{He}(\vec{n}, p)^3\text{H}$ reaction.

1. Introduction
A number of experiments aimed at studying parity violation in low-energy processes involving few nucleon systems are being completed or are in an advanced stage of planning at cold neutron facilities, such as the Los Alamos Neutron Science Center, the NIST Center for Neutron Research, and the Spallation Neutron Source at Oak Ridge. The primary objective of this program is to determine the fundamental parameters of the parity-violating (PV) nucleon-nucleon (NN) potential \cite{1}.

Until recently, the standard framework by which nuclear PV processes were analyzed theoretically was based on the use of potentials derived from meson-exchange mechanisms. In particular, the model proposed by Desplanques, Donoghue, and Holstein (DDH) \cite{2} over thirty years ago consisted of $\pi$, $\rho$, and $\omega$ exchanges, and contained seven unknown meson-nucleon coupling constants.

On the other hand, in recent years, a new, more systematic, approach based on a model-independent field-theoretic treatment of nuclear forces has been developed. This approach uses the chiral symmetry exhibited by quantum chromodynamics (QCD) to severely restrict the form of the interactions of pions among themselves and with other particles \cite{3}. In particular, the pion couples to nucleons by powers of its momentum $Q$, and the Lagrangian describing these interactions can be expanded in powers of $Q/\Lambda_\chi$, where $\Lambda_\chi \sim 1$ GeV specifies the chiral-symmetry breaking scale. As a consequence, classes of Lagrangians emerge, each characterized by a given power of $Q/\Lambda_\chi$ and each involving a certain number of unknown coefficients, the so
called low-energy constants (LEC’s), which are then determined by fits to experimental data (see, for example, the review papers [4, 5, 6, 7], and references therein).

This effective field theory (EFT) approach has been used also to describe the PV components in the NN interaction. One starts by assuming that these components derive from the weak interactions between the quarks inside the hadrons. As is well known, in the “Standard Model” the weak interactions contains both parity-conserving (PC) and PV components. The PC part contributes to the PC NN interaction and is of no interest, since its contribution is totally dominated by the strong (and electromagnetic) interaction. In contrast, the part of the weak interaction which violates parity (P) gives origin to a PV component in the NN potential. Note that the combination of charge conjugation and parity (CP) is known to be violated to a much lesser extent than P, and, therefore, it is reasonable to consider only P-violating but CP-conserving terms. Clearly, the weak Lagrangian at the quark level is not symmetric under chiral symmetry, however, one can construct the most general PV Lagrangian of nucleons and pions requiring that chiral symmetry be violated in the same way as is violated at quark level.

Following this approach, Kaplan and Savage in a pioneering work [8] wrote down a PV effective Lagrangian describing the interaction of pions and nucleons up to one derivative. This Lagrangian includes a “Yukawa” pion-nucleon interaction with no derivatives, multiplied by a parameter denoted as $h_\pi^1$ and known as the “weak pion-nucleon” coupling constant. It leads to a long-range, one-pion exchange (OPE) contribution to the PV NN interaction. Several PV experiments are devoted to reveal this component and to obtain a measurement of $h_\pi^1$. So far, however, the experimental evidence for its presence is inconclusive, for a review see Ref. [9]—see also a very recent study, using lattice simulations, to determine $h_\pi^1$ [10]. The Lagrangian derived by Kaplan and Savage includes five additional pion-nucleon interaction terms with one derivative, each of them multiplied by a different low-energy constant. However, they do not enter the PV NN interaction at either tree level or one loop [8].

After the work by Kaplan and Savage, there have been several studies of the PV NN interaction in EFT. The PV NN potential at next-to-next-to-leading (N^2LO) was derived for the first time by Zhu and collaborators [11, 12]. This potential includes the long-range OPE component, medium-range components originating from two-pion exchange (TPE) processes, and short-range components deriving from ten four-nucleon contact terms (the Authors of Ref. [12] noted that at low energy their ten contact interactions collapse into five independent operators, corresponding to the five S–P low-energy PV amplitudes [13]). In a series of other papers, Desplanques and collaborators., have also derived the contribution of the TPE diagrams at N^2LO [14, 15, 16] to study, in particular, PV effects in the capture reaction $^1\text{H}(\bar{n}, \gamma)^2\text{H}$. The expression of the TPE contribution obtained by Desplanques et al. is slightly different from that reported by Zhu et al. [12].

In the present contribution, we report on a new derivation of the PV NN potential at N^2LO, aimed at clarifying the correct form of its TPE component. Moreover, the recent analysis of Ref. [17] has shown that there exist only five independent contact terms with one derivative. In view of these developments, it is worthwhile to perform new calculations of various PV observables which can be measured in $A \leq 5$ nuclear systems, using this updated PV potential.

Measurements are already available (or will become available in the near future) for the following PV observables: the longitudinal analyzing power in $\vec{p}p$ [18, 19, 20] and $\vec{p}\alpha$ [21] scattering, the photon asymmetry and photon circular polarization in, respectively, the $^1\text{H}(\bar{n}, \gamma)^2\text{H}$ [22]–[23] and $^1\text{H}(n, \gamma)^2\text{H}$ [24] radiative captures, and the neutron spin rotation in $\bar{n}\alpha$ scattering [25, 26, 27]. There is also a set of experiments which are currently being planned, including measurements of the neutron spin rotation in $\bar{n}p$ [25] and $\bar{n}d$ [28] scattering, and of the longitudinal asymmetry in the charge-exchange reaction $^3\text{He}(\bar{n}, p)^3\text{H}$, at cold neutron energies [29].

In this contribution we present a preliminary study of the $\vec{p}p$ longitudinal asymmetry, the
neutron spin rotation in $\vec{n}$-$d$ scattering, and the longitudinal asymmetry in the $^3\text{He}(\vec{n},p)^3\text{H}$ reaction.

2. The parity-violating potential
As discussed above, the weak interaction between quarks induces a PV NN potential. This potential can be constructed starting from an effective pion-nucleon Lagrangian including all terms which violate chiral symmetry in the same way as at the quark level. The general expression for the Lagrangian describing PV pion-nucleon interactions up to one 4-gradient has already been given by Kaplan and Savage [8]. Beyond pion-nucleon interaction terms, the PV Lagrangian at $O(Q)$ also involves four-nucleon contact terms, which represent the part of the potential due to the exchange of heavy mesons, the excitation of $\Delta$-resonances, etc. The number of independent four-nucleon contact terms (up to one 4-gradient) is 5, as shown in Ref. [17].

More recently, we have started a program to identify the minimal set of both pion-nucleon and contact interaction terms containing two or more 4-gradients, by imposing the constraints arising from discrete symmetries and Fierz identities. A preliminary analysis indicates that no new terms enter the PV potential at N$^2$LO (further details of this study will be reported elsewhere [31]). Therefore, the PV potential (at N$^2$LO) depends on 6 unknown coupling constants: $h^\perp_\pi$ plus the low-energy constants (LEC’s) multiplying the 5 independent contact interactions with one 4-gradient.

For clarity, we report below the expressions of the “leading” $\pi N$ interactions:

\[
L^{PC}_{\pi NN}(x) = -\frac{g_A}{2f_\pi} \overline{N}(x)\gamma^\mu\gamma^5\vec{\tau} \cdot \partial_\mu \overline{\pi}(x) N(x) + \ldots , \tag{1}
\]

\[
L^{PV}_{\pi NN}(x) = \frac{h^\perp_\pi}{\sqrt{2}} \overline{N}(x) \left( \vec{\tau} \times \overline{\pi}(x) \right)_z N(x) + \ldots , \tag{2}
\]

where $g_A \approx 1.26$, $f_\pi \simeq 93$ MeV is the pion decay constant, and $h^\perp_\pi$ is the PV $\pi N$ coupling constant. The different diagrams contributing to the PV NN potential at $O(Q)$ are shown in figure 1. Note that in our notation, the chiral counting is given as

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{diagrams.png}
\caption{Time-ordered diagrams contributing to the PV potential (only one time ordering is given). Nucleons and pions are denoted by solid and dashed lines. The solid dot represents a PV vertex.}
\end{figure}
(i) leading $\pi NN$ vertex: $PC \sim Q^{1/2}$, $PV \sim Q^{-1/2}$
(ii) leading $\pi NN$ vertex: $\sim Q^0$ for both PC and PV vertices
(iii) leading $NN \rightarrow NN$ vertex: $PC \sim Q^0$, $PV \sim Q^1$
(iv) loops: $Q^3$
(v) energy denominators without pions $\sim Q^{-1}$
(vi) energy denominators with pions $Q^{-2}$

The OPE diagram (a) gives the lowest order (LO) contribution (of order $Q^0$). Explicitly

$$V_{a_1a_2a_1a_2}^{(a),\text{LO}} = \left(\frac{g_i h_1^i}{\sqrt{2}F_\pi}\right) \left(\frac{1}{4M^2}\right) (\tau_1 \times \tau_2)_z \frac{ik \cdot (\sigma_1 + \sigma_2)}{k^2 + m_\pi^2},$$  \hspace{1cm} (3)

where $F_\pi = 2f_\pi$ and $a_i \equiv \{p_i, s_i, t_i\}$ ($a_i' \equiv \{p_i', s_i', t_i'\}$) specifies the initial (final) momentum, and spin and isospin projections, respectively, of particle $i$. Above $k = p_i' - p_1 = -(p_2' - p_2)$ and $(\sigma)_{s_1', s_1} \equiv \sigma_1$, etc. There is no contribution of order $Q^0$, but there are several contributions of order $Q^1$ (namely, at $N^2\text{LO}$) coming from the diagrams depicted in figure 1:

$$V_{a_1a_2a_1a_2}^{(a),\text{REL}} = \left(\frac{g_i h_1^i}{\sqrt{2}F_\pi}\right) \left(\frac{1}{4M^2}\right) (\tau_1 \times \tau_2)_z \frac{1}{k^2 + m_\pi^2} \times$$

$$\left\{-4ik^2k \cdot (\sigma_1 + \sigma_2) + k \cdot (\sigma_1 (k \times K) \cdot (\sigma_2 + k \cdot (k \times K) \cdot \sigma_1) \right\},$$  \hspace{1cm} (4)

$$V_{a_1a_2a_1a_2}^{(d)} = -\frac{g_i h_1^i}{\sqrt{2}F_\pi} \left(\frac{4\pi^2F_\pi}{F_\pi}\right) (\tau_1 \times \tau_2)_z ik \cdot (\sigma_1 + \sigma_2) L(k),$$  \hspace{1cm} (5)

$$V_{a_1a_2a_1a_2}^{(f+g+g')} = -\frac{g_i h_1^i}{\sqrt{2}F_\pi} \frac{g_2^2}{4\pi^2F_\pi} \left[4(\tau_1 + \tau_2)_z ik \cdot (\sigma_1 \times \sigma_2) L(k) \right.$$  

$$\left. + (\tau_1 \times \tau_2)_z ik \cdot (\sigma_1 + \sigma_2) (H(k) - 3L(k)) \right],$$  \hspace{1cm} (6)

$$V_{a_1a_2a_1a_2}^{(CT)} = \frac{C_1}{\Lambda^2 m^2_\pi} K \cdot (\sigma_1 - \sigma_2) + \frac{C_2}{\Lambda^2 m^2_\pi} ik \cdot (\sigma_1 \times \sigma_2)$$

$$+ \frac{C_3}{\Lambda^2 m^2_\pi} (\tau_1 \times \tau_2)_z ik \cdot (\sigma_1 + \sigma_2) L(k) + \frac{C_4}{\Lambda^2 m^2_\pi} (\tau_1 + \tau_2)_z K \cdot (\sigma_1 - \sigma_2)$$

$$+ \frac{C_5}{\Lambda^2 m^2_\pi} \left(3\tau_1 \times \tau_2 - (\tau_1 \cdot \tau_2) K \cdot (\sigma_1 - \sigma_2) \right),$$  \hspace{1cm} (7)

where $M$ is the nucleon mass, $\Lambda = 4\pi f_\pi$, $K = (p_1 + p_1')/2$, and

$$s = \sqrt{k^2 + 4m^2_\pi}, \hspace{1cm} L(k) = \frac{1}{2k} \ln \left(\frac{s+k}{s-k}\right), \hspace{1cm} H(k) = \frac{4m^2_\pi}{s^2} L(k).$$  \hspace{1cm} (8)

The term $V_{a_1a_2a_1a_2}^{(a),\text{REL}}$ is a relativistic correction coming from diagram (a), $V_{a_1a_2a_1a_2}^{(d)}$ and $V_{a_1a_2a_1a_2}^{(f+g+g')}$ are TPE contributions, and $V_{a_1a_2a_1a_2}^{(CT)}$ is the contribution coming from the contact interactions. The contributions of diagrams (b) and (c) vanish after integrating over the loop variable, while those of diagrams (e) and (h) (vertex corrections) can be reabsorbed by a redefinition of the coupling constant $h_1^i$. More details on the derivation of the potential will be reported elsewhere [31].

In order to transform this potential to $r$-space, we have to multiply the expressions reported above by a cutoff, which is be chosen to be

$$f_A(k) = \exp\left(-\frac{(k/\Lambda)^4}{4}\right),$$  \hspace{1cm} (9)
where $\Lambda = 500 \div 700$ MeV is a cutoff parameter. With such a choice (a function which depends on $k$ only), the resulting potentials are local. The potential contains 6 unknown parameters, the pion-nucleon PV coupling constant $h^1_{\pi}$ and 5 LEC’s multiplying the contact interactions. Of course, the potential also depends on the cutoff parameter $\Lambda$.

As already mentioned, TPE diagrams have been calculated earlier by Zhu et al. [12] and Desplanques et al. [15]. We find that the expressions reported above are in agreement with those of Ref. [12], but for the contribution deriving from diagram (d), which in Ref. [15] contains a factor $g^3_A$ instead of $g_A$. This is probably due to the fact that in calculation of Ref. [15] the authors have Z-type TPE diagrams in place of the “triangle” diagram (d).

In order to estimate the magnitude of the various LEC’s, we compare the chiral potential with the DDH model [2], for which there are a number of studies available in the literature. The DDH model contains a OPE contribution identical to that given in Eq. (3). The authors of Ref. [2] proposed a “reasonable range” of $h^1_{\pi}$ values, and also a “best choice” value:

$$h^1_{\pi} \approx 4.56 \times 10^{-7}, \quad \text{reasonable range} : 0 \leq h^1_{\pi} \leq 11.4 \times 10^{-7} \quad (10)$$

Furthermore, by assuming that the contact interactions represent exchanges of heavy mesons, we can also estimate the magnitude of the LEC’s $C_i$, $i = 1, \ldots, 5$. In practice, one compares the expression of $V^{(CT)}$ given in Eq. (7) with the DDH components coming from exchanges of $\rho$- and $\omega$-mesons in the approximation $|k| \ll m_\rho, m_\omega$. Using the values for the PV $\rho$ and $\omega$ coupling constants entering the DDH model, as estimated in Ref. [30] from an analysis of the longitudinal asymmetry in $\vec{p}\vec{p}$ scattering, we find

$$C_1 \approx 0.3 \times 10^{-7}, \quad C_2 \approx 13 \times 10^{-7}, \quad C_3 \approx -0.4 \times 10^{-7}, \quad C_4 \approx -0.8 \times 10^{-7}, \quad C_5 \approx 0.7 \times 10^{-7} \quad (11)$$

suggesting that, while $C_2$ is rather large, the remaining LEC’s are of order $1 \times 10^{-7}$.

3. Results

In this section, we study several PV observables using our PV potential. We start off by analyzing the longitudinal asymmetry in $\vec{p}\vec{p}$ scattering, and then study the neutron spin rotation in $\vec{n}\vec{d}$ scattering and the longitudinal asymmetry in the $^3\text{He}(\vec{n}, p)^3\text{H}$ reaction.

3.1. The $\vec{p}\vec{p}$ longitudinal asymmetry

The longitudinal asymmetry $A_{\vec{p}\vec{p}}^{pp}$ in the collision $\vec{p}\vec{p}$ is defined as

$$A_{\vec{p}\vec{p}}^{pp}(\theta, E) = \frac{\sigma_{+1/2}(\theta, E) - \sigma_{-1/2}(\theta, E)}{\sigma_{+1/2}(\theta, E) + \sigma_{-1/2}(\theta, E)}, \quad (12)$$

where $\sigma_{m}(\theta, E)$ is the differential cross section for an incident proton beam of energy $E$ (in the laboratory system) and longitudinal polarization $m = \pm 1/2$, while $\theta$ is the scattering angle. The observable actually measured in the experiment is defined as (the “average” longitudinal asymmetry)

$$\bar{A}_{\vec{p}\vec{p}}^{pp}(E) = \frac{\int_{\theta_{1} \leq \theta \leq \frac{\pi}{2}} d\theta \int_{\theta_{1} \leq \theta \leq \frac{\pi}{2}} d\theta \sigma(\theta, E) A_{\vec{p}\vec{p}}^{pp}(\theta, E)}{\int_{\theta_{1} \leq \theta \leq \frac{\pi}{2}} d\theta \sigma(\theta, E)}, \quad \theta_{1} \approx 15^0 \quad (13)$$

where

$$\sigma(\theta, E) = \frac{1}{2} \left( \sigma_{+1/2}(\theta, E) + \sigma_{-1/2}(\theta, E) \right) \quad (14)$$

To compute this observable, we need the matrix elements of the isospin operators entering the PV potential between $|pp\rangle \equiv |T = 1, T_z = +1\rangle$ states:

$$\langle T = 1, T_z = +1 | (\tau_1 \times \tau_2) z | T = 1, T_z = +1 \rangle = 0 \quad (15)$$

$$\langle T = 1, T_z = +1 | (\tau_1 + \tau_2) z | T = 1, T_z = +1 \rangle = 2 \quad (16)$$

$$\langle T = 1, T_z = +1 | 3\tau_1 z \tau_2 - \tau_1 \cdot \tau_2 | T = 1, T_z = +1 \rangle = 2 \quad (17)$$
Thus, the contributions of the OPE terms $V^{(a),\text{LO}}$ and $V^{(a),\text{REL}}$ vanish and the constant $h^1_{\pi}$ can contribute only through the potential terms associated with TPE diagrams. Among the contact terms, that multiplying $C_3$ does not contribute, while the terms that multiply $C_1$, $C_4$ and $C_5$ differ only for the isospin operator. Taking into account the matrix elements in Eqs. (15)–(17), the longitudinal asymmetry can be expressed as

$$\bar{A}_z^p(E) = a_0(E)h^1_{\pi} + a_1(E)C'_1 + a_2(E)C_2,$$  \hspace{1cm} (18)

where

$$C'_1 = C_1 + 2C_4 + 2C_5,$$  \hspace{1cm} (19)

and $a_0(E)$, $a_1(E)$ e $a_2(E)$ are numerical coefficients independent from the values of the LEC’s (however, they depend on $\Lambda$). The observable $\bar{A}_z^p(E)$ has been measured by several groups. In this contribution we consider only the experiments performed at $E \equiv E_{\text{lab}} = 13.6$ MeV [18], 45 MeV [19], and 221 MeV [20], where $\bar{A}_z^p$ has been obtained with a reasonable accuracy:

$$\bar{A}_z^p(13.6\text{MeV}) = (-0.97 \pm 0.20) \times 10^{-7};$$

$$\bar{A}_z^p(45\text{MeV}) = (-1.53 \pm 0.21) \times 10^{-7};$$

$$\bar{A}_z^p(221\text{MeV}) = (+0.84 \pm 0.34) \times 10^{-7}. $$  \hspace{1cm} (20)

The numerical values of the coefficients $a_0(E)$, $a_1(E)$ and $a_2(E)$ for these three energies, calculated using the chiral PC NN potential derived by Entem & Machleidt [32] at next-to-next-to-next-to-leading order (I-N3LO model), and our chiral PV potential, are reported in table 1, for three values of the cutoff parameter $\Lambda$.

In principle, one could fix the three unknown parameters $h^1_{\pi}$, $C'_1$, and $C_2$ by requiring that the longitudinal asymmetry in Eq. (18) reproduces the experimental values in Eq. (20). However, as can be seen from table 1, the values of $a_i$ at low energy scale as $\sqrt{E}$, since the longitudinal asymmetry is dominated by the contribution of $S$-waves—that is, the experimental data at $E = 13.6$ MeV and 45 MeV do not provide independent constraints—and therefore, in practice, the number of independent equations reduces to two.

It is therefore necessary to fix the value of one of the constant to determine the remaining two. In the following, we assume that the $h^1_{\pi}$ value is in the “reasonable range” given in Eq. (10),

\begin{table}[h]
\centering
\caption{Coefficients $a_i(E)$ at the three energies of the experimental data, for three choices of the cutoff parameter $\Lambda$. The calculations have been performed using the PC I-N3LO NN potential model [32] and the PV potential described in the text.}
\begin{tabular}{ccc}
$E$ [MeV] & $a_0$ & $a_1$ & $a_2$
\hline
\multicolumn{4}{c}{$\Lambda = 500$ MeV}
\hline
13.6 & 0.2567 & -0.0861 & -0.2227
\hline
45 & 0.5667 & -0.1832 & -0.4517
\hline
221 & -0.2443 & -0.0864 & 0.1771
\hline
\multicolumn{4}{c}{$\Lambda = 600$ MeV}
\hline
13.6 & 0.2130 & -0.0704 & -0.1991
\hline
45 & 0.5010 & -0.1522 & -0.4210
\hline
221 & -0.2079 & -0.0525 & 0.1640
\hline
\multicolumn{4}{c}{$\Lambda = 700$ MeV}
\hline
13.6 & 0.1618 & -0.0616 & -0.1705
\hline
45 & 0.3995 & -0.1331 & -0.3669
\hline
221 & -0.1236 & -0.0287 & 0.1203
\end{tabular}
\end{table}
Table 2. Coefficients $C'_1$ and $C_2$ for different values of $h^1_\pi$ and $\Lambda$ determined to reproduce the experimental values of $A_{pp}$ at 45 and 221 MeV.

| $\Lambda$ [MeV] | $C'_1$ | $C_2$ | $C'_1$ | $C_2$ | $C'_1$ | $C_2$ |
|-----------------|--------|-------|--------|-------|--------|-------|
| 500             | $4.56 \times 10^{-7}$ | 0      | $4.00 \times 10^{-7}$ | -3.11 | 18.95 |
| 600             | $9.98 \times 10^{-7}$ | 10.03 | $4.22 \times 10^{-7}$ | -3.48 | 18.45 |
| 700             | $11.4 \times 10^{-7}$ | 10.66 | $5.91 \times 10^{-7}$ | -3.53 | 17.86 |

Figure 2. (Color online) Proton-proton longitudinal asymmetry as a function of the laboratory energy $E$ for three values of the cutoff $\Lambda$. The calculation is performed using the EFT potential with $h^1_\pi = 4.56 \times 10^{-7}$ and the values of constants $C'_1$ and $C_2$ listed in Table 2.

see the discussion at the end of the previous section. In particular we perform the calculations for three values of the coupling constant $h^1_\pi$:

(i) $h^1_\pi = 4.56 \times 10^{-7}$ ("best choice")

(ii) $h^1_\pi = 0$ (minimum value of the "reasonable range")

(iii) $h^1_\pi = 11.4 \times 10^{-7}$ (maximum value of the "reasonable range")

The values of $C'_1$ and $C_2$, corresponding to the three choices of $h^1_\pi$ and determined so as to reproduce the experimental asymmetries at 45 and 221 MeV, are reported in Table 2. Inspection of the table suggests that the dependence on $\Lambda$ is reasonable: the values of the LEC’s are close to 1, as it would naively be expected in the present EFT at order $Q/\Lambda \chi$. We also note that the obtained values for $C'_1$ e $C_2$ corresponding to the case $h^1_\pi = 4.56 \times 10^{-7}$ are in reasonable agreement with those reported in Eq. (11).

Finally, the longitudinal asymmetries calculated for $h^1_\pi = 4.56 \times 10^{-7}$ and three choices of $\Lambda$ are reported, as functions of energy, in figure 2. The experimental asymmetry at $E = 13.6$ MeV is automatically reproduced, even though the corresponding experimental datum was not used for determining the LEC’s. This confirms the fact that this measurement does not provide
any additional information with respect to those carried out at 45 and 221 MeV. For different energies than those used to determine the constants $C'_1$ and $C_2$, the asymmetries calculated with the various values of $\Lambda$ are slightly different. This “spread” in practice gives an indication of the “theoretical uncertainty” inherent in the EFT approach. We note however that the spread is very small, in particular, it is less than the experimental uncertainty. A similar conclusion is also obtained for the other values of $h^1_\pi$, as can be seen from figures 3 and 4. Note that for $E \approx 150$ MeV, $A^p_{\pi} z$ shows some sensitivity to the different values of $h^1_\pi$.

3.2. $\vec{n}$-$d$ spin rotation

A beam of transversally polarized neutrons passing through a slab of matter exhibits a spin rotation due to PV interactions with the nuclei in the medium. The observable of interest is the spin rotation angle $\phi$ about the neutron direction of motion. In first order perturbation theory in the weak interactions, the neutron spin rotation per unit-length of matter traversed, $d\phi/dz$, in $\vec{n}$-$d$ scattering is given by [33, 34]

$$\frac{1}{\rho} \frac{d\phi}{dz} = \frac{1}{3 v_{\text{rel}}} \text{Re} \sum_{m_n m_d} \epsilon_{m_n} (-) \langle p\hat{z}; m_n, m_d | V^{\text{PV}} | p\hat{z}; m_n, m_d \rangle^{(+)} ,$$

where $\rho$ is the density of deuterons (a parameter under control of the experimenter), $V^{\text{PV}}$ denotes the PV nuclear potential, $| p\hat{z}; m_n, m_d \rangle^{(-)}$ and $| p\hat{z}; m_n, m_d \rangle^{(+)}$ are the $n$-$d$ scattering states with incoming-wave (-) and outgoing-wave (+) boundary conditions and relative momentum $p = p\hat{z}$ taken along the spin-quantization axis, i.e. the $\hat{z}$-axis, and $v_{\text{rel}} = p/\mu$ is the magnitude of the relative velocity, $\mu$ being the $n$-$d$ reduced mass. The expression above is averaged over the spin projections $m_d = \pm 1, 0$ of the deuteron, however, the phase factor $\epsilon_{m_n} \equiv (-)^{1/2-m_n}$ is $\pm 1$ depending on whether the neutron has $m_n = \pm 1/2$.

The $\vec{n}$-$d$ ingoing and outgoing wave functions have been calculated with the HH method [35]. The PC part of the potential is the I-N3LO NN potential [32] with the inclusion of the three-nucleon (3N) force derived from a chiral EFT at N$^2$LO [36], with the constants fixed in Ref. [37]
(N-N2LO model). The PV potential used in Eq. (21) is the chiral potential discussed in the previous section. We present the calculated angles in table 3, for the usual choices of $h_\pi^1$ and $\Lambda$ values, and the corresponding values of $C_1'$ and $C_2$ reported in table 2, by selecting the case $C_1 = C_1'$, $C_{3,4,5} = 0$. Several comments are in order. 1) This observable is dominated by the contribution of the LO OPE potential, and therefore is directly proportional to the value of $h_\pi^1$. Indeed, a measurement of this observable could provide an independent verification of the value of $h_\pi^1$, extracted from a measurement of the photon asymmetry in the radiative capture $^1\text{H}(\vec{n}, \gamma)^2\text{H}$, presently considered the best experiment for extracting this LEC. 2) The contributions of the other terms (due to TPE or contact interactions) are rather small. For $h_\pi^1 = 4.56 \times 10^{-7}$ and $h_\pi^1 = 11.4 \times 10^{-7}$, the contribution of these medium- and short-range terms is about $5\%$. Clearly, for values of $h_\pi^1$ close to zero the observable would be very small, and the tiny contribution would come entirely from contact interactions. 3) For the same reason, the observable is not very sensitive to other choices for the LEC’s $C_i$, $i = 1, 3, 4, 5$. A sensitivity study of $\phi$ varying these LEC’s (but keeping fixed the combination $C_i' = C_1 + 2C_4 + 2C_5$ to the value determined by the $pp$ longitudinal asymmetry as discussed above) has been performed. In this study we have chosen several combinations of $C_i = \pm 1$, $i = 3, 4, 5$ and we have found that the spin rotation angle varies less than $5\%$.

3.3. The $^{3}\text{He}(\vec{n}, p)^3\text{H}$ longitudinal asymmetry

For ultracold neutrons, the longitudinal asymmetry $A_z^{^{3}\text{He}}$ for the reaction $^3\text{He}(\vec{n}, p)^3\text{H}$ (defined in analogy with Eq. (12)), is given by $A_z^{^{3}\text{He}} = a_z \cos \theta$ [38], where $\theta$ is the angle between the outgoing proton momentum and the neutron beam direction. The coefficient $a_z$ can be expressed in terms of products of $T$-matrix elements involving three PC and three PV transitions (see Ref. [38] for more details). Such $T$-matrix elements are calculated with the HH method [35], using the same strong interaction Hamiltonian model as for the $n - d$ calculation, namely the I-N3LO [32] NN potential in combination with the N-N2LO 3N force model. The HH calculation is a challenging one, for two reasons. The first is the coupled-channel nature of the scattering

Figure 4. (Color online) Same as figure 2, but for $h_\pi^1 = 11.4 \times 10^{-7}$.
Table 3. Spin rotation for unit-length of matter traversed $d\phi/dz$ for $\bar{n}$-d scattering in units of $10^{-7}$ rad cm$^{-1}$ at zero energy assuming a liquid deuterium density of $\rho = 0.4 \times 10^{29}$ atoms cm$^{-3}$. The calculations are performed using the I-N3LO NN plus the N-N2LO 3N potentials for the PC interaction, and the chiral PV potential model discussed in this paper for various choices of the pion-nucleon coupling constant $h^1_{\pi}$ and the cutoff parameter $\Lambda$. The corresponding values of the LEC’s $C_i$, $i = 1, \ldots, 5$ are discussed in the text. In the columns labeled “OPE/LO” we have reported the spin rotation angle calculated by retaining in the PV potential only the LO OPE contribution, while in the columns labeled “FULL” we have included all terms.

|        | $h^1_{\pi} = 4.56 \times 10^{-1}$ | $h^1_{\pi} = 0$ | $h^1_{\pi} = 1.14 \times 10^{-1}$ |
|--------|----------------------------------|-----------------|----------------------------------|
| 500    | 0.1028                           | 0.0987          | 0.0000                           |
| 600    | 0.1028                           | 0.1022          | 0.0000                           |
| 700    | 0.1019                           | 0.1055          | 0.0000                           |

Table 4. Same as table 3, but for the coefficient $a_z$ (in units of $10^{-7}$) describing the $^3$He$(\bar{n}, p)^3$H longitudinal asymmetry (preliminary results).

|        | $h^1_{\pi} = 4.56 \times 10^{-1}$ | $h^1_{\pi} = 0$ | $h^1_{\pi} = 1.2 \times 10^{-1}$ |
|--------|----------------------------------|-----------------|----------------------------------|
| 500    | $-0.551$                         | $-0.544$        | 0.0000                           |
| 600    | $-0.554$                         | $-0.578$        | 0.0000                           |
| 700    | $-0.546$                         | $-0.584$        | 0.0000                           |

problem: even at vanishing energies for the incident neutron, the elastic $n$-$^3$He and charge-exchange $p$-$^3$H channels are both open. The second is the presence of $J^p = 0^\pm$ resonant states between the $p$-$^3$H and $n$-$^3$He thresholds, which slows down the convergence of the expansion, and requires a large number of HH basis functions in order to achieve numerically stable results. Further discussion of this aspect of the calculations will be presented elsewhere [31]. Here we will present preliminary results for $a_z$, obtained using not fully converged calculations.

In table 4, we present the results of this preliminary calculation of $a_z$ using the PV chiral potential, with the choice $C_i = C'_i$ and $C_{3,4,5} = 0$. We observe that $A_{2,3}^{\text{He}}$ too is dominated by the contribution of the LO OPE potential. Naively, one expects that the most important contribution would come from the isoscalar operators. In fact, at this energy, the reaction proceeds mainly through the close $0^+$ and $0^-$ resonances, which are considered to have total isospin $T = 0$ [39]. Thus, the isoscalar operators in the PV potential should give the dominant contribution. However, the Coulomb interaction in the final state induces sizable isospin mixing configurations and, since the LO OPE term is the longest range term, it ultimately ends up giving the most important contribution (with the obvious exception where $h^1_{\pi}$ is close to zero). We also observe a rather large cancellation between the contributions coming from the TPE and the different contact interaction terms, while the contribution of $V^{(a)}_{\text{REL}}$ is always very tiny. For the case $h^1_{\pi} = 0$, $a_z$ is given mainly by the contribution of the isoscalar operator multiplying the LEC $C_2$.

We now discuss the dependence of $a_z$ on the LEC’s $C_{3,4,5}$. Due to the aforementioned cancellation among the various contributions of the contact potential, we find in this case greater sensitivity to these LEC’s, of the order of 20%. Therefore, the measurement of this observable could be very useful to extract them. We also observe that $a_z$ is not very sensitive to $\Lambda$, suggesting that contributions beyond N$^2$LO to the PV potential may not be significant.
4. Conclusions and perspectives
In this contribution, we have provided a preliminary account of an ongoing study of PV effects in few-nucleon systems, based on chiral EFT. First, we have presented a re-derivation of the PV potential at N^2LO, in order to clarify ambiguities present in previous studies. Second, we have carried out calculations of a number of PV observables in A = 2–4, based on chiral potentials for both the strong- and weak-interaction sector. We should note that there are slight inconsistencies in the strong-interaction potentials, adopted in the present study, which however could be easily removed. For example, the 3N force at N^2LO includes the parameters c_1, c_3, and c_4 which also enter the NN force. In the N-N2LO model, the values of these parameters have been taken from the NN force model derived in Ref. [40], rather than from the I-N3LO model by Entem and Machleidt.

At order Q our PV potential contains 6 low-energy constants. One can envisage, at least in principle, a suite of experiments involving A = 2–5 systems which would constrain, in fact over-constrain, these six LEC’s. The PV pion-nucleon coupling constant h_{1S} should be accurately determined in the near future by the NPDGAMMA experiment [23], aimed at the measurement of the photon asymmetry in the $^1H(\vec{n}, \gamma)^2H$ radiative capture. However, for electromagnetic processes, one has to include in the Lagrangian additional LEC’s needed to fix the strengths of PV two-body current operators of pion range [12], and this could complicate the extraction of h_{1S}. Alternatively, as discussed in this contribution, one could use the measurements of spin-rotation angles in $\vec{n}$-$p$ and $\vec{n}$-$d$ scattering. Such experiments are planned to be performed in a few years [25, 26, 28, 27]. In this contribution, the sensitivity of the $\vec{n}$-$d$ spin rotation angle to h_{1S} has been discussed in detail.

Once h_{1S} is known, the other LEC’s could be determined as follows. From the existing measurements of the $\vec{p}$-$p$ longitudinal asymmetry we can fix C_2 and the combination $C_1' = C_1 + 2C_4 + 2C_5$. We have also found that the the longitudinal asymmetry in the charge-exchange reaction $^3$He($\vec{n}$,p)$^3$H at ultracold neutron energies is sensitive to the $C_{3,4,5}$ LEC’s, and an experiment to measure it is being planned at the SNS facility at Oak Ridge [29]. Additional information could come from the old measurement of the $\vec{p}$-$\alpha$ longitudinal analyzing power at 46 MeV [21] and from the study of the neutron spin rotation in $\vec{n}$-$\alpha$ scattering, which is in progress [27]. However, an accurate theoretical analysis of these two reactions (which involve five nucleons) is still lacking, and this will certainly be the subject of future theoretical efforts.

Other PV observables which could be interesting to study both experimentally and theoretically are the photon asymmetries in $^2$H($\vec{n}$,\gamma)$^3$H and $^3$He(\vec{n},\gamma)$^4$He radiative captures. The normal PC captures are strongly suppressed: the experimental values for the corresponding cross sections [41, 42] are, respectively, almost 3 and 4 orders of magnitude smaller than measured in $^1$H($\vec{n}$,\gamma)$^2$H. One would naively expect relatively large PV asymmetries in these cases, possibly orders of magnitude larger than in the A=2 system. Clearly, accurate theoretical estimates for them could be useful in motivating our experimental colleagues to carry out these extremely challenging measurements. Work to construct PV electromagnetic transition operators using the same EFT described here is in progress (see, for example, Ref. [43] for the application of chiral EFT to derive PC electromagnetic transition operators).

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