The watt-balance operation: magnetic force and induced electric potential on a conductor in a magnetic field

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Abstract
In a watt-balance experiment, separate measurements of magnetic force and induced electric potential in a conductor in a magnetic field allow for a virtual comparison between mechanical and electrical powers, which leads to an accurate measurement of the Planck constant. In this paper, the macroscopic equations for the magnetic force and the induced electric potential are re-examined from a microscopic point of view and the corrective terms due to a non-uniform density of the conduction electrons induced by their interaction with the magnetic field are investigated. The results indicate that these corrections are irrelevant to the watt-balance operation.

(Some figures may appear in colour only in the online journal)

1. Introduction
The force \( F = BLI \) acting on a wire of length \( L \), constrained to be at rest in a magnetic flux density \( B \) and carrying the electrical current \( I \) orthogonal to \( B \), is derived by integrating the Lorentz force on the conduction electrons, under the assumptions of current and field uniformity [1]. Similarly, the electric potential \( U = BLu \) induced on a wire of length \( L \), moving at the velocity \( u \) orthogonal to \( B \), is derived by assuming a uniform charge-carrier density. If the force \( F \) counterbalances the weight \( mg \) of a mass \( m \) in the gravitational field \( g \), then by combining these equations and eliminating the geometric factor \( BL \), we obtain the equation \( mgu = U/I \), which virtually relates mechanical and electrical powers and allows \( m \) to be determined in terms of electrical quantities and, hence, of the Planck constant.

A number of subtleties have been dismissed in the previous analysis. Firstly, the Lorentz force acts on the free electrons, but the forces of constraint act on the ion lattice. The microscopic origin of the magnetic force has been investigated by many authors in order to resolve apparent inconsistencies [2–4]. The conclusion is that the Hall field is the means whereby the force on the conduction electrons is transferred to the positively charged ions. This conclusion avoids misinterpretations, where a magnetic field does work on the conductor. It has also been shown that the Hall field leads to the magnetic force \( BLI \) without violating the identity between the electrical and mechanical powers, once the Joule power dissipated in the conductor has been taken into account. Corrections to the Ampere force-law were also proposed [5, 6].

In the second place, the Hall field—together with the electron- and ion-plasma stiffness—makes the electron and ion densities non-uniform and it induces charge layers close to the wire surfaces orthogonal to the Hall field. Consequently, the electrical-current density is not uniform.

In the third place, in a moving conductor, the Lorentz force strains both the electron gas and the ions, thus originating a compensating electric field. But, since the Lorentz force is counteracted also by the electron and ion elasticity, the induced electric potential is not as high as expected.

These phenomena suggest a reanalysis of the \( mgu = U/I \) equation. A quantum rigorous description of the free electrons’ dynamics in the ion lattice, which includes the mutual interaction, or the integration of Boltzmann transport equations, is beyond the objectives of this paper and probably...
redundant in the estimation of the order of magnitude of the corrections to be carried out on the Ampere force-law. Therefore, we derived the magnetic force and the induced electric potential using a magnetohydrodynamical model proposed in [5]. Accordingly, the watt-balance coil is described by two overlapping isothermal compressible charged media, which are coupled to the electric and magnetic fields both external and self-induced. In both the cases, a coil either carrying an electrical current or moving in the magnetic field, this model predicts non-uniform charge distributions that are the sources of electrical fields in a direction transverse to both the magnetic field and the charge motions. Our study was prompted by the discrepancy between the Planck constant values measured in different watt-balance experiments [7, 8]; it is also a preliminary step to understanding the physics of a proposed cryogenic version of the experiment [9].

2. Watt-balance operation

The watt-balance experiment compares virtual electrical and mechanical power and the comparison is carried out in two steps [10].

Firstly, a balance is used to compare the weight $m_g$ with the force generated by the interaction between the electrical current $J$ flowing in the coil and the magnetic flux density $B$. Hence, using the pseudo-cylindrical coordinate system defined in figure 1, this balance is expressed as

$$mg - \hat{z} \cdot \int_0^L \int_0^{2\pi} \int_{r_0}^{r_0} r j(r, \varphi, \tau) \times B(r, \varphi, \tau) \, dr \, d\varphi \, d\tau = 0,$$

(1)

where $\hat{z}$ is the vertical direction, $-g \hat{z}$ is the gravitational field, $j(r, \varphi, \tau)$ is the current density, $r$, $\varphi$ and $\tau$ are pseudo-cylindrical coordinates along the coil wires, $r_0$ is the wire radius and $L$ is its length.

In the second step, the coil is moved and the electric potential induced at the coil ends,

$$U = \int_0^L \left[ u(r, \varphi, \tau) \times B(r, \varphi, \tau) \right] \cdot \hat{\tau} \, d\tau$$

$$= \int_0^L u(r, \varphi, \tau) \cdot \left[ B(r, \varphi, \tau) \times \hat{\tau} \right] \, dr,$$

(2)

is measured. It must be noted that, provided the end surfaces $\tau = 0$ and $\tau = L$ are equipotential, $U$ is independent of the integration path. Assuming this to be true, we can set $r = 0$ and evaluate (2) on the $r = 0$ coil axis.

If $j = I\hat{I}/(2\pi r_0^2)$ and $B$ are uniform and the coil velocity is the same everywhere and parallel to $\hat{z}$, that is, $u = u\hat{z}$, (1) and (2) can be rewritten as $mg = kI$ and $U = kl u$, respectively, where $k = BL$ is a geometric factor. By eliminating it, we obtain the measurement equation $mgu = UI$, which equates virtual mechanical and electrical powers. To derive this equation the fluxes linked in the dynamic and static phases must be rigorously equal. There are many effects which might make these fluxes different. However, the present analysis is concerned only with the current and field inhomogeneities.

3. Static phase

3.1. Magnetohydrodynamics equations

In order to estimate the Hall field and charge distribution in the watt-balance coil, we model a metal as proposed by Goedecke and Kanim [5]. Hence, we consider two interpenetrating, isotropic, homogeneous, compressible and isothermal charged media—(i) the lattice of ions and (ii) the plasma of free conduction electrons—confined in a stationary wire, where forces of constraint are applied to hold the ion lattice in place. As a result, there will be a lattice deformation and a change of the electron density. In a steady state, the relevant equations,

$$\nabla(n_e v_e) = 0$$

(3a)

$$- n_e e (E + v_e \times B) - \nabla p_e - \rho (n_e e)^2 v_e = -\mu_e g$$

(3b)

$$n_i e E - \nabla p_i + \rho (n_e e)^2 v_e = -\mu_i g$$

(3c)

are the continuity equation (3a), the momentum transfer equations (3b) and (3c), and the equations of state

$$\nabla n_i = (\partial n_i / \partial n_i) \nabla n_i = \xi \nabla n_i$$

(4a)

$$\nabla n_e = (\partial n_e / \partial n_e) \nabla n_e = K \nabla n_e$$

(4b)

Equation (3c) describes how the ion lattice deforms due to the forces per unit volume, but only the scalar part of the stress tensor—which is associated with the pressure—has been considered. This approximation corresponds to neglecting all the off-diagonal components of the strain tensor.

Since the free electron and ion number densities

$$n_e = n_0 (1 + \zeta_e)$$

(5a)

$$n_i = n_0 (1 + \zeta_i)$$

(5b)

deviate only by the small amounts $\zeta_{e,i}$ from the mean value $n_0$, $(\partial n_i / \partial n_i)$ means $(\partial n_i / \partial n_i) \nabla n_i$. In equations (3b) and (3c), the forces acting on the media are the Lorentz force (the term proportional to $B$ is missing for the ions because they are immobile), the pressure gradients, the force due to the electron scattering on the lattice ions and the gravity. For the sake of simplicity, we assume that the charge carriers are electrons and that there is one charge carrier per ion. In (3a)–(3c), $e$ is the elementary charge, $v_i$ is the electron drift velocity, the ion drift velocity is zero, $p_{e,i}$ are the electron and ion pressures, $K_e$ and $K_i = \frac{1}{2} \varepsilon_F n_0$ are the bulk moduli of the ions and free electrons, and $\varepsilon_F$ is the

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Figure 1. The pseudo-cylindrical coordinate system used in (1) and (2); $\tau$ runs over the wire length, $r$ and $\varphi$ are polar coordinates in the wire cross-section.
Fermi energy. According to the Drude model of electrical conduction, the frictional-force density is \( \rho (n_0 e)^2 v_e \), where \( \rho \) is the electrical resistivity. The density of the electrical current is \( j = -n_z e v_z \). We also considered the self-weight of the free electrons and ions; \( g = -g z \) is the acceleration due to gravity and \( \mu_e \) and \( \mu_i \) are the free electron and ion mass densities. Using (5a) and (5b) in (3a)–(3e), we obtain

\[
\nabla v_e + \nabla (\xi_e v_e) = 0 \tag{6a}
\]

\[-n_0 e (E + v_e \times B) - \kappa_e \nabla \xi_e - \rho (n_0 e)^2 v_e = -\mu_e g \tag{6b}\]

\[n_0 e E - \kappa_i \nabla \xi_i + \rho (n_0 e)^2 v_e = -\mu_i g, \tag{6c}\]

where the terms multiplied by \( \xi_e \) and \( \xi_i \) have been neglected, leaving only those terms multiplied by their gradients.

The electric field \( E = E_0 + E \) and magnetic flux density \( B = B_0 + B \) include the external fields, \( E_0 \) and \( B_0 \), as well as the fields generated by the charge and current distributions, \( E \) and \( B \). They are expressed by the Maxwell equations

\[
\nabla E = n_0 e (\xi_i - \xi_e) / \epsilon_0 \tag{7a}
\]

\[\nabla \times E = 0 \tag{7b}\]

\[\nabla \times B = -\mu_0 n_0 e v_e \tag{7c}\]

\[\nabla B = 0 \tag{7d}\]

### 3.2. Equation solution

As shown in figure 2, we consider a rectilinear wire having rectangular \( b \times a \) cross-section in the y-z plane and extending from \(-L/2\) to \(L/2\) in the x direction. An electric current flows in the positive x direction, having a density \( j = j_0 [1 + \iota(z)] \), \( 0, 0, 0 \) and an associated drift velocity of the free electrons \( v_e = -j / (n_0 e) \) and external field \( E_0 = [E_0, 0, 0] \). The wire is in an external magnetic flux density \( B_0 = [0, B_0, 0] \) pointing in the y direction. We assume that all quantities in (6a)–(6c), and (7c) and (7d) depend only on \( z \); strictly speaking, this corresponds to assuming an infinite extension of the wire in the x and y directions, in order to call on invariance arguments.

The magnetohydrodynamics and field equations are

\[
E_0 = \rho j_0 \tag{8a}
\]

\[j_0 B_0 - n_0 e \xi_e - \kappa_e \partial_z \xi_e = \mu_e g \tag{8b}\]

\[n_0 e \xi_e - \kappa_i \partial_z \xi_i = \mu_i g \tag{8c}\]

\[\epsilon_0 \partial_z \xi_e - n_0 e (\xi_i - \xi_e) = 0, \tag{8d}\]

where the continuity equation (6a) is identically satisfied. Equation (7c) has been omitted, i.e., the magnetic field generated by the current distribution has been neglected. The effect of this field is supposed small, but the real motivation of this approximation is that, otherwise, (8a)–(8d) can be solved only numerically. The first equation, expressing the equilibrium of the \( x \) components of the forces acting on the free electrons, is Ohm’s law. In the case of a cylindrical geometry, the magnetohydrodynamics equations can be solved with equivalent results using the same approximations.

To solve (8a)–(8d), we impose the boundary conditions

\[
\int_{-a/2}^{a/2} \xi_e(z) \, dz = 0, \tag{9}
\]

which express that \( n_0 \) is the mean number density, and

\[\xi_e(-a/2) = \xi_e(a/2) = 0, \tag{10}\]

because the wire has no net charge and it is assumed to extend to infinity in the x and y directions. With these boundary conditions, the solutions of (8a)–(8d) are

\[
\xi_e(z) = \frac{(B_0 j_0 + \mu_i g)K_i}{n_0 e (K_i + K_e)} \left[ 1 - \frac{\cosh (\kappa z)}{\cosh (\kappa a / 2)} \right], \tag{11a}
\]

\[
\xi_i(z) = \frac{B_0 j_0 - \mu_i g}{\kappa (K_i + K_e)} \left[ K_z - \frac{\xi_0 \sinh (\kappa z)}{K_e \cosh (\kappa a / 2)} \right], \tag{11b}
\]

\[\xi_i(z) = \frac{B_0 j_0 - \mu_i g}{\kappa (K_i + K_e)} \left[ K_z - \frac{\xi_0 \sinh (\kappa z)}{\cosh (\kappa a / 2)} \right], \tag{11c}\]

where

\[
\kappa = \sqrt{\frac{K_e + K_i}{K_e K_i} n_0 e / \epsilon_0}, \tag{12a}\]

\[\xi_0 = \frac{B_0 j_0 + \mu_i g}{B_0 j_0 - \mu_i g}, \tag{12b}\]

\[\mu = \frac{\mu_i K_e - \mu_e K_i}{K_i}, \tag{12c}\]

and \( \mu = \mu_i + \mu_e \). The dimensionless—in square brackets—Hall field and the free electron and ion densities across the
wire are shown in figure 3; the numerical values of the model parameter are given in table 1.

Equation (11a) shows two modifications of the textbook equation $E_i = B_0 j_0 / (n_0 e) = R_H B_0 j_0$, where $R_H$ is the Hall coefficient [11]. The first and most important modification, scales $R_H$ by the ratio of the bulk modulus of the ions to the sum of the bulk moduli of the free electron and ions, $K_i/(K_e + K_i)$. Furthermore, $E_i$ depends on the depth in the wire (see figure 3, left). Neglecting this $z$ dependence, which is relevant only within sheaths whose thickness is of the order of $\kappa^{-1} \approx 10^{-10} \text{m}$, the Hall coefficient is $R_H = 1/(n_0 e)$ only if $K_e/K_i \to 0$. The opposite $K_i/K_e \to 0$ limit yields a zero Hall coefficient. Since the ion bulk modulus is not much larger than electron-gas one, the modification predicted by (11a) is significant. The reason why $E_i$ depends on the ratio $K_e/K_i$ is that the Lorentz force is counteracted by both $E_i$ and the electron-gas stiffness; a low stiffness gives this task to the Hall field, a high stiffness does not require the Hall field contribution. As regards the ion lattice, by examining (8c), we observe that a null stiffness imposes a null Hall field whereas an infinite stiffness leads to the maximum Hall field.

According to the textbook formula, for monovalent transition metals such as Cu, Ag and Au, the product $R_H n_0 e$ should be identically one. However, the experimentally determined values are smaller than unity, as shown in table 2; the corrective term $K_i/(K_e + K_i)$ accounts for this discrepancy, though less effectively for gold.

The second, minor, modification accounts for the different effects of self-weight on the electron gas and ion lattice; also in the absence of the magnetic field, an electric field originates across the conductor. In fact, since the electron-gas and ion stiffnesses are different, the differential strain due to gravity splits the positive and negative charge distributions and generates an electric field. Owing to the layout of a watt balance, this field and the Hall fields are collinear.

As shown in figure 3 (right), both the free electron and ion densities increases linearly along the $z$ direction. These increases correspond to the strains caused by the compressive stresses originated by the Lorentz force (free electrons) and the Hall field (ion lattice); opposite strains are induced by the gravity. At the $z = \pm a/2$ surfaces, charge layers are created that are the sources of the Hall field.

### Table 1. Numerical values of the model parameters in the case of a copper wire.

| Parameter | Value |
|-----------|-------|
| $\epsilon_0$ | $8.9 \times 10^{-12} \text{F m}^{-1}$ |
| $\epsilon_F$ | $7 \text{eV}$ |
| $g$ | $9.81 \text{m s}^{-2}$ |
| $\mu$ | $8960 \text{kg m}^{-3}$ |
| $L$ | $1 \text{m}$ |
| $a$ | $10^{-2} \text{m}$ |
| $\rho$ | $1.56 \times 10^{-6} \text{Ω m}$ |
| $\kappa_0$ | $2.2 \times 10^{10} \text{m}^{-1}$ |
| $\theta, B$ | $1 \text{T m}^{-1}$ |
| $j_0$ | $10^8 \text{A m}^{-2}$ |

### 3.3. Magnetic force

In (8a)-(8d) the mean current density $j_0$ is an external constraint. The simplifications made do not allow the current distribution $\iota(z)$ to be determined. However, by slightly twisting the mathematical rigour, at least for an order-of-magnitude estimate, we can use the Drude model of electrical conduction and assume that the $x = \text{const}$ end faces of the wire are equipotential to write the current density as $j_0(1 + \zeta_e)$. Therefore, the magnetic force is

$$F = B_0 L I_0 \left[ 1 + \frac{1}{a} \int_{-a/2}^{a/2} \beta(z) \zeta_e(z) \, dz \right],$$

where $B_0(1+\beta)$ is the magnetic flux density, $B_0$ is the mean flux density and $I_0 = abj_0$ is the current flowing in the wire. Since only the current inhomogeneities orthogonal to the magnetic field have been considered, only the vertical field-gradient can produce a force correction. As shown by equation (11c), $\zeta_e(z)$ is an odd function and only the vertical gradient $B_z = \partial_z B$ contributes to (13); hence,

$$F \approx B_0 L I_0 \left[ 1 + \frac{B_z}{a} \int_{-a/2}^{a/2} \zeta_e(z) \, dz \right] \approx B_0 L I_0 \left[ 1 + \frac{a^2(B_0 j_0 - \mu g) B_z}{12(K_e + K_i)} \right].$$

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Figure 3. Dimensionless Hall field—$n_0(K_i + K_e) e^2 / (\hbar m_e)$, left—and variations of the free electron and ion densities—$e K_i, e K_e$, right—across a wire in the magnetic flux density $B_0$ and carrying the electrical current $j_0$. 

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This equation predicts a correction to the $B_0L_0$ expression of the magnetic force. In the case of a typical watt-balance experiment, the magnetic force is of the order of 10 N. Hence, with a single copper ring of $L = 1$ m circumference and a flux density $B_0 = 1$ T, the required current is 10 A; if the dimensions of the wire cross-section are $a = 10$ mm and $b = 1$ mm, the current density is $j_0 = 1$ A mm$^{-2}$. A pessimistic estimate of the vertical gradient of the magnetic flux density is $B_z = 1$ T m$^{-1}$. With the numerical values of the remaining model parameters summarized in table 1, the associated relative correction is $4 \times 10^{-11}$—give or take ten per cent due to the gravitational load $\mu \gamma$. Thankfully, this figure makes the correction irrelevant to the watt-balance operation.

3.4. Extension to a coil of wire

A last detail must be examined. In practice, the balance coil is made by winding up many wire turns and, owing to the ohmic voltage drop, a potential difference is set up between subsequent turns. Therefore, the boundary condition (10) becomes

$$E_z(-a/2) = E_z(a/2) = E_d,$$

where $E_d$ is the electric field in the gap between subsequent coil-windings, and the solutions of (8a)–(8d) become

$$E_z(z) = \left(\frac{B_0j_0 + \bar{\mu}g}{n_0e(K_i + K_e)}\right) \left[1 - \frac{\Xi_1 \cosh(\kappa x/2)}{\cosh(\kappa x/2)}\right],$$

$$\zeta_z(z) = \frac{B_0j_0 - \mu g}{\kappa (K_i + K_e)} \left[\kappa z - \frac{\Xi_2 \sinh(\kappa x)}{\cosh(\kappa x/2)}\right],$$

$$\zeta_i(z) = \frac{B_0j_0 - \mu g}{\kappa (K_i + K_e)} \left[\kappa z - \frac{\Xi_2 \sinh(\kappa x)}{\cosh(\kappa x/2)}\right],$$

where

$$\Xi_2 = \frac{B_0j_0 + \bar{\mu}g - \bar{q}E_d}{B_0j_0 - \mu g},$$

$$\Xi_1 = \frac{B_0j_0 + \bar{\mu}g - \bar{q}E_d}{B_0j_0 + \bar{\mu}g},$$

and

$$\bar{q} = \frac{n_0e(K_e + K_i)}{K_i}.$$

As regards the electron density, the only change with respect to (11b) is the additional term $\bar{q}E_d$ in the new scale factor $\Xi_2$ of the charge sheaths, which is necessary to ensure that the only internal field is Hall’s one. Since the surface charge layers do not contribute significantly to the integral in (14), the given expression for the magnetic-force correction is unchanged. A correction reduction of a few orders of magnitude is consequential to the reduction in the wire height.

4. Dynamic phase

4.1. Solution of the magnetohydrodynamics equations

Let us now consider the coil moving in the vertical direction with a constant velocity $u$. The wire is assumed to extend to infinity in the $x$ direction, but the magnetic field is zero outside the interval $[-L/2, +L/2]$. In addition, we assume that all the relevant quantities depend only on $x$. The magnetohydrodynamics and field equations are

$$n_0e(uB_0 - \frac{\Xi_z}{K_i}) - K_e \frac{\partial}{\partial x} \zeta_z = 0 \quad (20a)$$

$$n_0e(E_x - uB_0) - K_i \frac{\partial}{\partial x} \zeta_i = 0 \quad (20b)$$

$$\frac{\partial}{\partial x} E_x - n_0e(\zeta_z - \zeta_e) = 0 \quad (20c)$$

where

$$\int_{-L/2}^{+L/2} \zeta_e(x) \, dx = 0,$$

because $n_0$ is the mean number density in the interval $[-L/2, +L/2]$, and

$$E_x(-L/2) = E_x(L/2) = 0,$$

because outside that interval the electric potential is constant. With these boundary conditions, the solutions of (20a)–(20c) are

$$E_x(x) = uB_0 \left[1 - \frac{\cosh(\kappa x)}{\cosh(\kappa L/2)}\right],$$

$$\zeta_e(x) = \frac{n_0e(uB_0)}{\kappa K_i} \frac{\sinh(\kappa x)}{\cosh(\kappa x/2)},$$

$$\zeta_i(x) = -\frac{\kappa e}{K_i} \zeta_e(x).$$

The dimensionless—in square brackets—induced field and the free-electron and ion densities along the wire are shown in figure 4; the numerical values of the model parameters are given in table 1. As shown in figure 4 (right), the free electron and ion lattice are shifted in opposite directions to generate charge layers at the $\pm L/2$ ends of the wire, which are the sources of the induced electric field.

4.2. Induced electric potential

The induced electric potential is obtained by integrating $E_x(x)$ along the wire axis. Actually, this integration can be confined within the interval $[-L/2, L/2]$, because outside this interval there is no magnetic field to generate a Lorentz force counteracting an electric field. Hence,

$$U = \int_{-L/2}^{+L/2} E_x(x) \, dx = uB_0L \left[1 - \frac{2 \tanh(\kappa L/2)}{\kappa L}\right].$$

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Table 2. Measured Hall coefficients $R_{e0n_e}$ compared with the ones estimated by (11a). The measured values are from [11].

| Element | $\epsilon_f$/eV | $10^3n_0/m^3$ | $K_e$/GPa | $K_i$/GPa | $R_{e0n_e}$ | $K_e/(K_i + K_e)$ |
|---------|----------------|---------------|-----------|-----------|-------------|------------------|
| Cu      | 7.10           | 8.49          | 140       | 63.4      | 0.67        | 0.68             |
| Ag      | 5.50           | 5.86          | 100       | 34.4      | 0.77        | 0.74             |
| Au      | 5.53           | 5.90          | 180       | 34.8      | 0.67        | 0.84             |

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Figure 4. Dimensionless electric field—$\frac{\varepsilon_0}{B_0 u}$, left—and free electron and ion densities—$\frac{\kappa K}{\mu_0 n_0 B_0}$, right—along a wire moving in the magnetic flux density $B_0$ with velocity $u$.

As (14) does for the magnetic force, (24) predicts a correction to the textbook expression of the induced electric potential. The predicted potential is smaller than $u B_0 L$; as shown in figure 4 (left), the reduction originates in the smooth transition of $E_x(x)$ from the bulk value $u B_0$, in the conductor part immersed in the magnetic field, to zero, in the part external to the field. With the parameter values given in table 1 and a single winding of $L = 1$ m length, the relative correction is $9.2 \times 10^{-11}$. Also in this case, the correction is irrelevant to the watt-balance operation. In a real coil, made by winding up many wire turns, the much greater $L$ value ensures that this correction is further scaled down by orders of magnitude.

5. Conclusions

The basic principles of operation of the watt-balance experiment have been examined from a microscopic viewpoint in order to verify the presence of corrective terms to the formulae for the magnetic force acting on a conductor in a magnetic field and for the electric potential induced by its motion. The model used describes the conductor as a plasma with interpenetrating compressible charged media: an ion lattice and a free-electron gas. In order to solve analytically the relevant magnetohydrodynamics equations an extreme simplification of the coil–magnet system is necessary, but the order of magnitude of the sought corrections should have been correctly estimated. These corrections are many order of magnitudes below the sensitivity of any present and future watt-balance experiment. Nevertheless, they shed light on the background of the watt-balance operation.

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References

[1] Fletcher K A, Iyer S V and Kinsey K F 2003 Some pivotal thoughts on the current balance Phys. Teach. 41 280–4
[2] Rostoker N 1951 Hall effect and ponderomotive force in simple metals Am. J. Phys. 20 100–7
[3] Mosca E P 1974 Magnetic forces doing work? Am. J. Phys. 42 205–7
[4] Redinz J A 2011 Forces and work on a wire in a magnetic field Am. J. Phys. 79 774–6
[5] Goedecke G H and Kanim S E 2007 The Hall effect in accelerated and stationary conductors Am. J. Phys. 75 131–8
[6] Sakai M, Honda N, Fujimoto F, Nakamura O and Shibata H 2010 A complementary study of the Hall electric field for generation of the force on current-carrying wire in a magnetic field Am. J. Phys. 78 160–9
[7] Steiner R L, Newell D B and Williams E R 2005 Details of the 1998 Watt balance experiment determining the Planck constant J. Res. Natl Inst. Stand. Technol. 110 1–26
[8] Steele A G, Meija J, Sanchez C A, Yang L, Wood B M, Sturgeon R E, Mester Z and Inglis A D 2012 Reconciling Planck constant determinations–watt-balance and enriched-silicon measurements at NRC Canada Metrologia 49 L8–10
[9] Picard A, Stock M, Fang H, Witt T J and Reymann D 2007 The BIPM watt-balance IEEE Trans. Instrum. Meas. 56 538–42
[10] Robinson I A 2012 Toward the redefinition of the kilogram: A measurement of the Planck constant using the NPL Mark II watt balance Metrologia 49 113–56
[11] Ashcroft N W and Mermin N D 1976 Solid State Physics (Philadelphia, PA: Saunders College Publishing) pp 11–15