Comparative study of elastic properties and mode I fracture energy of carbon nanotube/epoxy and carbon fibre/epoxy laminated composites

Jyotikalpa Bora and Sushen Kirtania

Abstract

A comparative study of elastic properties and mode I fracture energy has been presented between conventional carbon fibre (CF)/epoxy and advanced carbon nanotube (CNT)/epoxy laminated composite materials. The volume fraction of CNT fibres has been considered as 15%, 30%, and 60% whereas; the volume fraction of CF has been kept constant at 60%. Three stacking sequences of the laminates viz. [0/0/0/0], [0/90/0/90] and [0/30/–30/90] have been considered in the present analysis. Periodic microstructure model has been used to calculate the elastic properties of the laminated composites. It has been observed analytically that the addition of only 15% CNT in epoxy will give almost the same value of longitudinal Young's modulus as compared to the addition of 60% CF in epoxy. Finite element (FE) analysis of double cantilever beam specimens made from laminated composite has also been performed. It has been observed from FE analysis that the addition of 15% CNT in epoxy will also give almost the same value of mode I fracture energy as compared to the addition of 60% CF in epoxy. The value of mode I fracture energy for [0/0/0/0] laminated composite is two times higher than the other two types of laminated composites.

Keywords: Carbon nanotube, Carbon fibre, Finite element analysis, Elastic properties, Mode I fracture energy

Introduction

Discovery of carbon nanotubes (CNTs) by Iijima in 1991 brought a revolution in research in the field of nanotechnology [1]. CNTs have excellent mechanical, thermal and electrical properties [2]. The combination of excellent properties has made CNTs an ideal candidate for reinforcement in matrix for structural applications. For light weight polymers, CNT has emerged as an ideal reinforcement due to its excellent mechanical and thermal properties. The CNT-reinforced polymer composites possess high specific strength due to which they have become very compelling structural materials not only in aerospace which requires light weight but also in some other applications such as automotive, electronics, biomedical, sporting goods etc.[3]. The in-plane properties of structural composites are excellent. But due to the weak through thickness properties, laminated composites are prone to delamination which is the most common mode of failure for laminated composites. Kormanikova and Kotrasova [4] calculated the longitudinal Young’s modulus, transverse Young’s modulus, in-plane shear modulus and Poison ratio of a unidirectional laminate and reported that in a frame of the analytical homogenization, comparatively better results obtained from the periodic microstructure model. Recently, Kirtania and Chakraborty [5] determined the thermoelastic properties of CNT/epoxy composite by considering a representative volume element. Kageyama et al. [6] fabricated RF-MEMS switch and reported that the Au–Au/ CNT composite contact switch exhibits higher life cycle, reliability and power capacity than the Au–Au contact.
The interlaminar fracture toughness of laminated composites could be increased by using CNTs as reinforcements have been reported by different researchers. Zhou et al. [7] reported an increase in mode I fracture toughness of carbon fibre/epoxy laminate by 125% using micro-sized carbon fibre interleaf coated with functional nano-sized carbon nanotube. Li et al. [8] reported an increase in mode I fracture toughness by 174.81% of glass fibre reinforced polymer composite using CNT bucky paper as interleaf. Quan et al. [9] fabricated MW/CNT reinforced CF fabric/epoxy composite and reported that the addition of 1 wt% CNT in composite, the mode I fracture toughness increased by 25%.

It has also been reported that sometimes the addition of CNTs into the composite did not produce desired results. The CNT-reinforced composites fabricated using conventional methods such as dispersion, buckypapers etc. are sometimes unable to show the exceptional properties due to their reinforcing nano building blocks. This can be attributed to low volume fraction and agglomeration tendency of the CNTs [10]. To increase the volume fraction of CNTs without agglomeration tendency super aligned CNTs have been used by researchers using different techniques. Wardle et al. [11] proposed a method to fabricate CNT/epoxy composite with high volume fraction of CNT up to 22% using mechanical densification of vertically aligned CNT forests followed by capillary induced wetting. Wang et al. [10] reported that the waviness, entanglement and poor packing of CNTs also degrade properties of the CNT-based composites. To enhance the mechanical properties, they [10] have also proposed a method of drawing and stretching of the CNTs to reduce the waviness and to increase the volume fraction of CNT up to 46%. Nam et al. [12] fabricated aligned CNT/epoxy composites with a high volume fraction of CNT up to 63.4% by using a novel combination of stretch-drawing and press winding techniques.

Most of the literature reported on the evaluation of elastic properties and fracture toughness of CNT/epoxy composites at a low volume fraction of the CNT. But a very few analysis on CNT-reinforced laminated composites have been reported. Therefore, in the present study, the elastic properties (i.e. Young’s modulus, shear modulus and Poisson’s ratio), and mode I fracture energy of CNT/epoxy laminated composites have been determined with a high volume fraction of CNT up to 60%. Comparative studies between CNT/epoxy and CF/epoxy laminated composites have also been presented.

### Materials and method

In the present study, single-walled CNT (SWCNT) and carbon fibre (CF) have been considered as the reinforcing materials and epoxy has been considered as the matrix material. The properties of the fibre and the matrix materials [5, 13] used in the present analysis are listed in Table 1.

#### Evaluation of elastic properties of laminated composites

The elastic properties of CNT/epoxy and CF/epoxy composite laminates have been calculated analytically. Various analytical techniques of homogenization such as Reuss model, Voigt model and Periodic microstructure model are available for the evaluation of the elastic properties of the laminated composite. In the present study, the periodic microstructure model [4] has been used for calculation of elastic properties of the laminated composites. Each laminate is composed of four laminas of thickness 0.5 mm as shown in Fig. 1. The length and breadth of each lamina have been considered as 100 mm and 20 mm, respectively. The long cylindrical fibres are arranged periodically in a square array \((a_2=a_3)\) in the matrix material is shown in Fig. 2.

The random arrangement of fibres in the composite, results in a transversely isotropic material having five independent elastic constants. For transversely isotropic materials, there should be a plane in which the elastic properties are equal in all directions and this plane is called as “plane of isotropy”. In the present analysis, the plane 2–3 is considered as the plane of isotropy. Therefore, the subscripts 2 and 3, use in the symbol of elastic properties are interchangeable. These are reflected in the following Eqs. (2–4). These five independent elastic constants can be determined using the following relations [13].

\[
E_1 = C_{11}^* - \frac{2C_{12}^*}{C_{22}^* + C_{23}^*} \quad (1)
\]

\[
E_2 = E_3 = \frac{(2C_{11}^* C_{22}^* + 2C_{11}^* C_{23}^* - 4C_{12}^*) (C_{22}^* - C_{23}^* + 2C_{44}^*)}{3C_{11}^* C_{22}^* + C_{11}^* C_{23}^* + 2C_{11}^* C_{44}^* - 4C_{12}^*} \quad (2)
\]

\[
G_{12} = G_{13} = C_{66}^* \quad (3)
\]

\[
\nu_{12} = \nu_{13} = \frac{C_{12}^*}{C_{22}^* + C_{23}^*} \quad (4)
\]

#### Table 1 Properties of fibres and matrix materials [5, 13]

| Materials                  | Young’s modulus (GPa) | Poisson’s ratio |
|----------------------------|-----------------------|-----------------|
| Single walled carbon nanotube | 1000                  | 0.28            |
| Carbon fibre               | 230                   | 0.20            |
| Epoxy                      | 3.89                  | 0.37            |
where, \( C^{*}_{11}, C^{*}_{12}, C^{*}_{23}, C^{*}_{44} \) and \( C^{*}_{66} \) are the components of \( C^{*} \) matrix, which is a 6 \( \times \) 6 square symmetric matrix. The components of this \( C^{*} \) stiffness matrix are calculated using the following relations [13].

\[
C^{*}_{11} = \lambda_m + 2\mu_m - \frac{V_f}{D} \left[ \frac{S_3^2}{\mu_m^2} \frac{2S_6S_3}{\mu_m c} + \frac{S_6 - S_7}{\mu_m g c} + \frac{a^2 - b^2}{4c^2} \right]
\] (7)

\[
C^{*}_{12} = \lambda_m + \frac{V_f}{D} \left[ \frac{S_3}{2c\mu_m} - \frac{S_6 - S_7}{2c\mu_m g} - \frac{a + b}{4c^2} \right]
\] (8)

\[
C^{*}_{23} = \lambda_m + \frac{V_f}{D} \left[ \frac{aS_7}{2\mu_m g c} - \frac{ba + b^2}{4c^2} \right]
\] (9)

\[
C^{*}_{22} = \lambda_m + 2\mu_m - \frac{V_f}{D} \left[ -\frac{aS_3}{2\mu_m c} + \frac{aS_6}{2\mu_m g c} + \frac{a^2 - b^2}{4c^2} \right]
\] (10)

\[
C^{*}_{44} = \lambda_m - V_f \left[ -\frac{S_3}{\mu_m} + (\mu_m - \mu_f)^{-1} + \frac{4S_7}{\mu_m(2 - 2\nu_m)} \right]^{-1}
\] (11)

\[
C^{*}_{66} = \mu_m - V_f \left[ -\frac{S_3}{\mu_m} + (\mu_m - \mu_f)^{-1} \right]^{-1}
\] (12)

The value \( D, a, b, c, g, \lambda \) and \( \mu \) can be calculated using the following relations [13].

\[
D = \frac{aS_3^2}{2\mu_m^2 c} - \frac{aS_6S_3}{\mu_m g c} + \frac{a(S_6^2 - S_7^2)}{2\mu_m g^2 c} + \frac{S_3(b^2 - a^2)}{2\mu_m c^2} + \frac{S_6(a^2 - b^2) + S_7(ab + b^2)}{2\mu_m g c^2} + \frac{(a^3 - 2b^3 - 3ab^2)}{8c^3}
\] (13)
\[ a = \mu_f - \mu_m - 2\mu_f v_m + 2\mu_m v_f \quad (14) \]

\[ b = -\mu_m v_m + \mu_f v_f + 2\mu_m v_m v_f - 2\mu_f v_m v_f \quad (15) \]

\[ c = (\mu_m - \mu_f)(\mu_f - \mu_m + \mu_f v_f - \mu_m v_m + 2\mu_m v_f - 2\mu_f v_m v_f) \quad (16) \]

\[ g = (2 - 2v_m) \quad (17) \]

\[ \lambda = \frac{E}{(1+v)(1-2v)} \quad \text{and} \quad \mu = G = \frac{E}{2(1+v)} \quad (18) \]

The values \( S_3, S_6 \) and \( S_7 \) can be calculated using the following relations \([13]\).

\[ S_3 = 0.49247 - 0.47603V_f - 0.02748V_f^2 \quad (19) \]

\[ S_6 = 0.36844 - 0.14944V_f - 0.27152V_f^2 \quad (20) \]

\[ S_7 = 0.12346 - 0.32035V_f + 0.23517V_f^2 \quad (21) \]

where, the fibres and the matrix are represented by the subscripts \( f \) and \( m \), respectively. Using the above relations the engineering constant \((E_1, E_2, G_{12}, v_{12} \text{ and } v_{23})\) of a 0° lamina can be calculated. The compliance matrix \([S']\) of a 0° lamina is a \(6 \times 6\) matrix \([13]\), which can be calculated using the above elastic constants. The compliance matrix \([S]\) is used for a laminate with any orientation of fibres, which can be calculated using the following relation \([13]\).

\[ [S] = [T]^T [S'] [T] \quad (22) \]

where, \([T]\) is the transformation matrix. The stiffness matrix \([C]\) for each lamina in a laminate can be calculated using following equation \([13]\).

\[ [C] = [S]^{-1} \quad (23) \]

The stiffness matrix of the laminate \([\mathcal{C}]\) with thickness \( t \) and having \( N \) number of laminas is calculated using following relation \([13]\).

\[ [\mathcal{C}] = \sum_{K=1}^{N} \frac{t_K}{t} [C_K] \quad (24) \]

where, “\( K \)” indicates \( K^{th} \) lamina of the laminate. Finally, the compliance matrix \([\mathcal{S}]\) of the laminate is calculated using following relation \([13]\).

\[ [\mathcal{S}] = [\mathcal{C}]^{-1} \quad (25) \]

The Young’s modulus, shear modulus and Poisson’s ratio of a laminate are calculated using following relations \([13]\),

\[ E_x = 1/S_{11}, \quad G_{yz} = 1/S_{44}, \quad v_{yz} = -S_{32}/S_{22} \]

\[ E_y = 1/S_{22}, \quad G_{xy} = 1/S_{66}, \quad v_{xy} = -S_{21}/S_{11} \quad (26) \]

where, \( S_{11}, S_{12} \) etc. are the components of the compliance matrix \([\mathcal{S}]\). MATLAB codes have been developed for calculation of elastic properties of the all laminated composites.

**Evaluation of mode I fracture energy of laminated composites**

Double cantilever beam (DCB) made from laminated composites has been used in the present analysis. Figure 3 shows the DCB specimen with an initial crack length of 30 mm. One end of the beam has been fixed and a load has been applied on the other end.

Convergence study has also been performed to decide the FE mesh in the FE model of DCB. For the present FE analysis, the number of elements has been chosen as 200, 400, 800 and 1600. It has been observed that the load fluctuation with displacement was too high for lower number of elements. After the 400 number of elements, there is a negligible variation of load fluctuations with displacement, but after the 1600 number of elements there is no variation of load fluctuation with displacement. Therefore, it could be concluded that the FE model is converged and for all the FE analysis this FE mesh size has been used.

The DCB has been modelled using FE method in ANSYS. PLANE182 element has been used for FE modelling of the laminas and INTER202 element has been used for FE modelling of the interface between two laminas in laminate. The number of PLANE182 elements and INTER202 elements are 1600 and 280, respectively. Therefore, the total number of elements in the FE model is equal to 1880. The total number of nodes in the FE model is equal to 2687. The plane strain condition has
been considered for both the elements. In the present study, the linear fracture criterion has been considered. Therefore, the linear material properties of INTER202 element have been used. Figure 4 shows the finite element mesh of DCB specimen along with boundary conditions.

Virtual crack closure technique has been used to find out the reaction force, displacement and the crack extension length. The variation of these reaction forces with corresponding displacements are then plotted to find out the area under the curve. Mode I fracture energy \( G_I \) of the DCB has been calculated using the following relation [14].

\[
G_I = \frac{\Delta A}{B \Delta a}
\]  

(27)

where, \( \Delta A \) is the area under the reaction force versus displacement curve, \( \Delta a \) is the crack extension length and \( B \) is the breadth of the specimen.

Results and discussion

The elastic properties i.e. Young’s modulus, in-plane shear modulus and Poisson’s ratio; and the mode I fracture energy \( G_I \) of the CNT/epoxy laminated composites have been calculated for different volume fractions of CNT but for CF/epoxy both the properties have been calculated for a constant CF volume fraction of 60%. Three different laminate stacking sequences have also been considered for the calculation of elastic properties and \( G_I \) of the CNT/epoxy and CF/epoxy laminated composites.

Elastic properties of the CNT/epoxy and CF/epoxy laminated composites

The elastic properties of the laminated composites have been calculated using Eq. (26). The elastic properties of CNT/epoxy laminated composite at different fibre volume fraction and three stacking sequence of laminates are listed in Table 2.

It could be observed from Table 2 that the transverse Young’s moduli \( (E_y \text{ and } E_z) \) are not significant compared

| Stacking sequence | Volume fraction of CNT (%) | Young’s modulus (GPa) \( E_x \) | Shear modulus (GPa) \( G_{xy} \) | Poisson’s ratio \( \nu_{xy} \) |
|-------------------|---------------------------|-------------------------------|---------------------------------|------------------|
| \[0/0/0/0\]       | 15                        | 153.31                        | 1.92                            | 0.35             |
|                   | 30                        | 302.73                        | 2.63                            | 0.33             |
|                   | 60                        | 601.57                        | 5.67                            | 0.31             |
| \[0/90/0/90\]     | 15                        | 79.85                         | 1.92                            | 0.025            |
|                   | 30                        | 155.65                        | 2.63                            | 0.016            |
|                   | 60                        | 309.98                        | 5.67                            | 0.017            |
| \[0/30/–30/90\]   | 15                        | 78.06                         | 1.92                            | 0.36             |
|                   | 30                        | 153.28                        | 2.63                            | 0.35             |
|                   | 60                        | 304.97                        | 5.67                            | 0.35             |

Table 3 Elastic properties of CF/epoxy laminated composite at 60% CF volume fraction

| Stacking sequence | Volume fraction of CF (%) | Young’s modulus (GPa) \( E_x \) | Shear modulus (GPa) \( G_{xy} \) | Poisson’s ratio \( \nu_{xy} \) |
|-------------------|---------------------------|-------------------------------|---------------------------------|------------------|
| \[0/0/0/0\]       | 60                        | 139.59                        | 5.44                            | 0.25             |
| \[0/90/0/90\]     | 60                        | 78.19                         | 5.44                            | 0.05             |
| \[0/30/–30/90\]   | 60                        | 73.78                         | 5.44                            | 0.38             |
to the longitudinal Young’s modulus ($E_x$) for [0/0/0/0] laminated composites. As expected for [0/90/0/90] laminated composite that precisely the same value of the $E_x$ and transverse Young’s modulus in the y-direction ($E_y$) has been observed from Table 2. The elastic properties of CF/epoxy composites have been calculated at a constant CF volume fraction of 60% for different stacking sequences of laminates are listed in Table 3.

**Variation of longitudinal Young’s modulus**

The variation of longitudinal Young’s modulus ($E_x$) of CNT/epoxy composite has been calculated with volume fraction for different stacking sequence of laminates. Figure 5 shows that the variation of $E_x$ of CNT/epoxy composite with CNT volume fraction as compared to the $E_x$ of CF/epoxy composite at a constant CF volume fraction of 60% for [0/0/0/0] stacking sequence of laminate.

It has been observed from Fig. 5 that with the addition of only 15% CNT in epoxy will give a higher value of $E_x$ as compared to addition of 60% CF in epoxy. After 15% fibre volume fraction of CNT the value of $E_x$ for CNT/epoxy composite becomes significantly higher than that of CF/epoxy with 60% CF volume fraction. It has also been observed from Fig. 5 that the value of $E_x$ of CNT/epoxy becomes 330.95% higher than that of CF/epoxy at a constant fibre volume fraction 60%.

A similar type of analysis has also been performed for [0/90/0/90] and [0/30/–30/90] laminate configurations. Figures 6 and 7 show that the variation of $E_x$ of CNT/epoxy composite with CNT volume fraction as compared to the $E_x$ of CF/epoxy composite at a constant CF volume fraction of 60% for [0/90/0/90] and [0/30/–30/90] laminate configurations, respectively.

It can be seen from Fig. 6 and that with the addition of only 15% CNT in epoxy will give almost the same value of $E_x$ as compared to the addition of 60% CF in epoxy. A maximum increase of 296.43% has been observed for CNT/epoxy as compared to CF/epoxy for the same fibre volume fraction of 60% for [0/90/0/90] laminate configuration. But for [0/30/–30/90] laminate configuration the maximum increase of 313.33% has been observed for CNT/epoxy composite as compared to CF/epoxy composite with the same fiber volume fraction of 60%. Therefore, it could be concluded that a similar trend of percentage increase in $E_x$ with CNT volume fraction has been observed from Figs. 5, 6, 7 for all the stacking sequences of composite laminates. The strength of the laminated composites depends on the stacking sequence of different laminae in the laminate, therefore, for a particular volume fraction, the values of the percentage increase in $E_x$ are different.
Variation of transverse Young's modulus and in-plane shear modulus

It can be observed from Table 2 that the transverse Young's modulus and in-plane shear modulus have also been increased with increase in fibre volume fractions for all the three stacking sequences of composite laminates. It can also be observed from Tables 2 and 3 that for [0/0/0/0] laminate configuration the values of transverse Young's modulus and in-plane shear modulus of CNT/epoxy composite are almost same compared to CF/epoxy composite at a constant fibre volume fraction of 60%. In case of [0/90/0/90] laminated configuration the values of Young's modulus along x and y directions (Ex and Ey) of CNT/epoxy composite are significant compared to CF/epoxy composite for a constant fiber volume fraction of 60%. But for the same laminated configuration the other elastic properties of CNT/epoxy composite are not significant compared to CF/epoxy composite. For [0/30/–30/90] laminate configuration, the values of Ex, Ey and in-plane shear modulus in the x–y plane (Gxy) of CNT/epoxy composite are significant compared to CF/epoxy at a constant volume fraction of 60%.

The strength of a lamina depends on the orientation of fibers in the lamina. The strength of the laminated composites depends on the stacking sequences of different laminae in the laminate, since a laminate is fabricated by stacking different laminae. Therefore, the values of the elastic properties are also depends on the laminate configuration. In the present study, three stacking sequences of the laminates viz. [0/0/0/0], [0/90/0/90] and [0/30/–30/90] have been considered in which the fibers are mainly oriented in x and y directions. Therefore, for the above three stacking sequences of laminate configurations, the elastic properties of Ex, Ey and Gxy are more significant compared to the other elastic properties.

Since the variation of Ez, Gyz and Gxz of CNT/epoxy composite are not significant compared to CF/epoxy composite for a constant CF volume fraction of 60%, therefore, the variation of Ey and Gxy have been presented for the present study. Figures 8 and 9 show the percentage increase/decrease of Ey and Gxy of CNT/epoxy composite with volume fraction compare to the CF/epoxy composite for a constant fiber volume fraction of 60% for [0/30/–30/90] laminated composite.

It can be seen from Figs. 8 and 9 that at 15% fibre volume fraction, the values of Ey and Gxy for CNT/epoxy is slightly lower than that of CF/epoxy with 60% fibre volume fraction. After 15% CNT volume fraction, the values of Ey and Gxy for CNT/epoxy significantly increase compared to CF/epoxy with 60% CF volume fraction. At 60% CNT volume fraction, Ey for CNT/epoxy becomes 270.58% higher compared to CF/epoxy with 60% CF volume fraction. Similarly, for Gxy an increase of 225.90% has been observed for CNT/epoxy as compared to CF/epoxy at a constant fiber volume fraction of 60%. A similar trend of percentage increase/decrease of Ey and Gxy of CNT/epoxy composite with volume fraction compare to the CF/epoxy composite for a constant 60% CF volume fraction have also been observed for others two types of laminated composites.

Kormanikova and Kotrasova [4] calculated the elastic properties of the unidirectional lamina as well as effective elastic properties for a [0/45/–45/90], laminate with a thickness of lamina equal to 0.125 mm. The elastic properties of [0/45/–45/90], laminate with 0.125 mm thickness of lamina have also been calculated to validate the present study and similar values of elastic properties have been found as reported by Kormanikova and Kotrasova [4].
Mode-I fracture energy of the CNT/epoxy and CF/epoxy laminated composites

Finite element analysis has been performed of DCB made from CNT/epoxy and CF/epoxy laminated composite. The elastic properties calculated using periodic micro-structure model has been used in FE analysis of the DCB in ANSYS. The variation of reaction force with displacement is shown in the following figures. Figure 10 shows the variation of reaction force with displacement at different fibre volume fractions for [0/0/0/0] laminate. It can be seen from Fig. 10 that the critical load is highest for [0/0/0/0] laminated CNT/epoxy composite at 60% CNT volume fraction. It can also be seen from Fig. 10 that the critical load is lowest for CF/epoxy laminated composite though the volume fraction of CF is 60%. Figures 11 and 12 show the variation of reaction force with displacement at different fibre volume fractions of DCB made from [0/90/0/90] and [0/30/−30/90] laminates, respectively. A similar trend has been observed for both of these laminate configurations as observed for [0/0/0/0] laminate configuration. The highest critical load has been observed at 60% CNT volume fraction for all the three laminate configurations.

The area under the reaction forces and corresponding displacements curve and crack extension length have been found from the FE analysis of the DCB specimens using virtual crack closure technique. Then mode-I fracture energy ($G_I$) has been calculated for the DCB made from all the laminated composites using Eq. (27). The crack extension length ($\Delta a$), the area under the reaction force vs. displacement curve ($\Delta A$), $G_I$ and percentage increase/decrease in $G_I$ have been listed in Table 4.

It can be seen from Table 4 that with the addition of 15% CNT in epoxy will give almost the same value of $G_I$ when 60% CF is added to epoxy for all the three types of stacking sequences of laminated composites. The maximum value of $G_I$ has been found to be 0.667 J/mm$^2$ for [0/0/0/0] laminated CNT/epoxy composite. An increase of $G_I$ has been found by 173.87% for CNT/epoxy composite compared to CF/epoxy composite at a constant
volume fraction of 60% for [0/0/0/0] stacking sequence of laminate. But for [0/90/0/90] and [0/30/−30/90] stacking sequences of CNT/epoxy composite laminates the increase in $G_I$ have been found 88.78% and 85.65%, respectively compared to CF/epoxy composite at 60% fibre volume fraction. It could be concluded that the percentage increase of $G_I$ for [0/90/0/90] and [0/30/−30/90] laminated composites are almost the same. Therefore, it could be concluded that the maximum increase in $G_I$ has been observed for [0/0/0/0] laminate configuration. The value of $G_I$ for [0/0/0/0] laminated composite is two times higher than the other two laminate configurations.

The authors have not found any kind of literature on the determination of mode I fracture energy of CNT/epoxy laminated composite. Due to this, the authors have not compared the present FE result of mode I fracture energy with any existing literature. But the trend of the variation of reaction vs. displacement curve of the present advanced CNT/epoxy composite are similar in nature to the conventional glass/epoxy unidirectional laminated composite as reported by Samborski et al. [15]. Elastic properties and mode I fracture energy of CNT/epoxy and CF/epoxy laminated composite have been calculated and then compared. In the future, the CF/epoxy laminated composites may be replaced by CNT/epoxy laminated composites. Therefore, the present study may provide valuable information to the future researcher about the importance of CNT-reinforced laminated composites for potential future applications since a few numbers of literatures have been reported on the fracture analysis of CNT/epoxy laminated composites.

### Conclusions

Elastic properties of CNT/epoxy and CF/epoxy laminated composites with three stacking sequences have been calculated analytically and mode-I fracture energy of double cantilever beams made from those laminated composites have been determined computationally using FE method. To understand the importance of the CNT-reinforced laminated composites over conventional laminated composite, a comparison has also been presented between CNT/epoxy and CF/epoxy laminated composites. Some of the important conclusions drawn from the present study are -

- The addition of only one-fourth amount of CNT in epoxy as compared to CF in epoxy will give almost the same values of longitudinal Young’s modulus ($E_x$) and mode I fracture energy ($G_I$).
- The $E_x$ of CNT/epoxy composite has been found 330.95% higher compared to the CF/epoxy composite at a constant fiber volume fraction of 60% for [0/0/0/0] stacking sequence of laminate.
- The variation of $E_y$, $E_z$ and $G_{xy}$ are significant compared to other elastic properties.
- The mode I fracture energy of CNT/epoxy composite has been determined 173.87% higher compared to the CF/epoxy composite at a constant fibre volume fraction of 60% for [0/0/0/0] stacking sequence of laminate.
- The percentage increase of $G_I$ for [0/90/0/90] and [0/30/−30/90] laminated composites are almost the same.
- The value of $G_I$ for [0/0/0/0] laminated composite is two times higher than the other two types of laminated composites.

### Table 4 Percentage increase/decrease in mode I fracture energy of the CNT/epoxy with volume fraction compared to CF/epoxy composite at a constant CF volume fraction of 60%

| Material | Laminate configuration | Volume fraction (% | $\Delta a$ (mm) | $\Delta A$ (N mm) | $G_I$ (J/mm$^2$) | % increase/decrease in $G_I$ (%) |
|----------|------------------------|--------------------|------------------|------------------|----------------|--------------------------------|
| CF/epoxy | [0/0/0/0]              | 60                 | 58.50            | 285.246          | 0.24380        | –                             |
| CNT/epoxy | [0/0/0/0]              | 15                 | 62.50            | 295.914          | 0.23673        | – 2.89                        |
|          |                        | 30                 | 53.50            | 402.931          | 0.37657        | 54.46                         |
|          |                        | 60                 | 46.00            | 614.285          | 0.66770        | 173.87                        |
| CF/epoxy | [0/90/0/90]            | 60                 | 56.50            | 239.351          | 0.21181        | –                             |
| CNT/epoxy | [0/90/0/90]            | 15                 | 55.25            | 235.798          | 0.21339        | 0.75                          |
|          |                        | 30                 | 57.75            | 296.463          | 0.25667        | 21.18                         |
|          |                        | 60                 | 52.50            | 419.856          | 0.39986        | 88.78                         |
| CF/epoxy | [0/30/−30/90]          | 60                 | 55.75            | 236.373          | 0.21199        | –                             |
| CNT/epoxy | [0/30/−30/90]          | 15                 | 57.00            | 240.932          | 0.21134        | – 0.30                        |
|          |                        | 30                 | 58.50            | 301.509          | 0.25770        | 21.56                         |
|          |                        | 60                 | 51.75            | 407.321          | 0.39354        | 85.64                         |
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Authors’ contributions
JB is an M.Tech student therefore he has worked under my guidance. He has developed MATLAB code as well as finite element analysis using ANSYS software. Whenever he has faced technical problems, as a supervisor I have helped him. Both authors read and approved the final manuscript.

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The author declare that we have no competing interest.

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