CONSERVATIVE UPDATING

Matthew Kovach∗

February 2, 2021

Abstract: This paper provides a behavioral analysis of conservatism in beliefs. I introduce a new axiom, Dynamic Conservatism, that relaxes Dynamic Consistency when information and prior beliefs “conflict.” When the agent is a subjective expected utility maximizer, Dynamic Conservatism implies that conditional beliefs are a convex combination of the prior and the Bayesian posterior. Conservatism may result in belief dynamics consistent with confirmation bias, representativeness, and the good news-bad news effect, suggesting a deeper behavioral connection between these biases. An index of conservatism and a notion of comparative conservatism are characterized. Finally, I extend conservatism to the case of an agent with incomplete preferences that admit a multiple priors representation.

Keywords: Conservative updating, prior-bias, non-Bayesian updating, confirmation bias, representativeness, good news-bad news effect, multiple priors.

JEL: D01, D81, D9.

∗Department of Economics, Virginia Tech. E-mail: mkovach@vt.edu. I would like to thank Aurélien Baillon, Jonathan Chapman, Federico Echenique, Bart Lipman, Pietro Ortoleva, Kota Saito, Marciano Siniscalchi, Gerelt Tserenjigmid, and Leeat Yariv for helpful feedback. This paper was previously titled Sticky Beliefs: A Characterization of Conservative Updating and is based on chapter 3 of my dissertation at Caltech.
1 Introduction

Many papers in both economics and psychology have documented biases in belief updating (see, for instance, Camerer (1995), Kahneman and Tversky (1972), El-Gamal and Grether (1995)). A recurrent finding is that people tend to exhibit conservatism (Phillips and Edwards (1966), Edwards (1982), Aydogan et al. (2017), and Möbius et al. (2014)). A conservative updater only partially incorporates new information into her beliefs; hence she puts too much weight on her prior. Despite its prevalence, conservatism has yet to be behaviorally founded.

I introduce a novel behavioral postulate, Dynamic Conservatism, to capture conservatism within the framework of conditional preferences over acts (see Savage (1954) and Anscombe and Aumann (1963)). This axiom weakens Dynamic Consistency to accommodate “preference stickiness.” Put loosely, Dynamic Conservatism allows for violations of Dynamic Consistency only when the initial preference and the information are in conflict. Further, I show that conservative preferences are consistent with several well-known biases, including confirmation bias, the representativeness heuristic, and the good news-bad news effect.

For a deeper intuition behind Dynamic Conservatism, consider the following hypothetical of an agent’s reaction to information regarding climate change. Suppose this agent initially favors the use of coal for power, yet she concedes that using alternative energy is better if climate change is occurring. If she obtains information regarding the scientific consensus that climate change is occurring, would she now support using alternative energy over coal? If she is dynamically consistent (i.e., Bayesian), then the answer is yes because she has already conceded that alternative energy is better in that contingency. If she is conservative, she may still prefer coal. When there is a conflict between information (climate change news) and an agent’s initial preference (use coal), conservatism may result in a violation of Dynamic Consistency. Dynamic Conservatism rules out such violations.

Dynamic Conservatism restricts the agent so that she may violate Dynamic Consistency only in situations in which there is a conflict between the information and her initial preference. To illustrate, consider how our agent would feel if she had initially preferred alternative energy to coal. Now that the evidence for climate change is consistent with her initial preference, she must continue to prefer alternative energy. That is, no matter how conservative she is, it would be absurd for her to (i) initially support alternative energy, (ii) acknowledge that alternative energy is better if climate change is occurring, and then (iii) state that she supports coal upon receipt of information regarding the consensus that climate change is occurring. Dynamic Conservatism rules out reversals of this form.

Suppose the agent is a subjective expected utility maximizer (SEU) and her prior beliefs
are given by a probability distribution $\mu$. In this case, Dynamic Conservatism characterizes a subjective expected utility agent whose posterior beliefs after $A$, denoted $\mu_A$, are a convex combination of the prior and the Bayesian posterior:

$$\mu_A = \delta(A)\mu + (1 - \delta(A))B(\mu, A)$$

(1)

where $\delta(A) \in [0, 1]$ and $B(\mu, A)$ is the Bayesian update of $\mu$ given $A$. When $\delta(A) > 0$, the agent is reluctant to move away from her initial beliefs, and therefore she is conservative. When $\delta(A) = 0$, she is Bayesian; when $\delta(A) = 1$, she is unresponsive to the information. Consequently, $\delta(A)$ can be interpreted as her degree of conservatism (at $A$). In general, a collection of conditional preferences admits a conservative subjective expected utility representation when posterior beliefs are given by Equation 1 for each event.

An important feature of the representation in Equation 1 is the dependence of the degree of conservatism, $\delta(A)$, on the realized event: bias may be source-dependent. Because of source dependence, conservatism is able to capture belief dynamics consistent with confirmation bias (Nickerson (1998)), the representativeness heuristic (Kahneman and Tversky (1972)), and the good news-bad news effect (Eil and Rao (2011)). Although these biases have been thought of as distinct aspects of behavior, this finding suggests that there may be a core behavioral aspect of conservatism that unifies these biases.

I then provide an analysis of various aspects related to an agent’s degree of conservatism. Intuitively, $\delta(A)$ captures how conservative an agent is (at $A$). When $\delta(A)$ is event independent, $\delta(A) = \delta$ for some $\delta \in [0, 1]$, then the agent is described by a single behavioral parameter and $\delta$ is an index of conservatism. I establish a behavioral characterization of this case through Weak Consequentialism, a novel weakening of Consequentialism. I then provide a simple definition of comparative conservatism that rests upon the comparison of a binary act with a constant outcome. This provides an efficient method for ordering people by each one’s degree of conservatism.

I close by exploring the implications of conservatism when agents do not have precise beliefs. To do so, I consider a collection of possibly incomplete conditional preferences and impose an adapted version of Dynamic Conservatism on these preferences, which I call Unambiguous Dynamic Conservatism. In this setting, the agent has a set of possible beliefs and prefers an act, $f$, to another, $g$, when $f$ provides (weakly) greater expected utility than $g$ for every possible belief. Unambiguous Dynamic Conservatism characterizes a conceptually

---

1 Relatedly, $1 - \delta(A)$ might be thought of as a measure of her confidence in the new information. This interpretation will be useful for the discussion of source dependence in section 4.
similar representation to Equation 1: the agent’s set of posterior beliefs is derived by mixing the initial set of priors and the set of Bayesian posteriors. A crucial distinction from the subjective expected utility case is that now information may induce a genuine expansion of the set of beliefs. Thus subjectively ambiguous information may result in a structural change in behavior; an initially subjective expected utility agent may become ambiguity averse after information.

1.1 Outline and Related Literature

The setup and model are presented in section 2. Behavioral foundations are presented in section 3. The connection between conservatism and other belief biases is discussed in section 4. The analysis of degrees of conservatism is discussed in section 5. I present the extension to multiple priors and incomplete preferences in section 6. The remainder of this section discusses related literature.

While numerous experimental papers suggest that people do not update their beliefs in a Bayesian manner, relatively few provide an axiomatic analysis of non-Bayesian updating. Epstein (2006) provided one of the first axiomatic analysis of non-Bayesian updating. He utilized a setup of preferences over menus and modeled non-Bayesian updating as a temptation (à la Gul and Pesendorfer (2001)) to modify prior beliefs in response to an interim signal. This was extended to an infinite-horizon model by Epstein et al. (2008). Because of the menu-preference setting, beliefs are typically dependent on both the information and the option set. More recently, Ortoleva (2012) utilized the conditional preference approach to introduce the Hypothesis Testing Representation. In his model, a decision maker may be surprised by low probability events and, in response, adopt a new prior before applying Bayes’ rule. This representation captures “over-reaction” to information and is conceptually distinct from conservatism.²

The notion of conservative updating I characterize results in violations of Dynamic Consistency and Consequentialism. While violations of these properties are well documented, the causes of these violations are still not fully understood. Additionally, violations of Consequentialism are relatively understudied compared to violations of Dynamic Consistency. Violations of Dynamic Consistency and Consequentialism were documented by Dominiak et al. (2012) in an Ellsberg-style experiment. While these violations were more frequent among ambiguity averse subjects, they also occur among ambiguity neutral subjects.³ More

²Additionally, such a decision maker always satisfies Consequentialism, which is typically violated by conservative updating.
³Further, all subjects report a “loss of confidence” in their choices after information, with the greatest loss

4
recently, Shishkin and Ortoleva (2019) documented behavior consistent with violations of Consequentialism in an experimental setting.

Conservative updating, as defined in Equation 1, has recently been applied to cheap-talk games in Lee et al. (2019). They found that a conservative receiver may induce the sender to provide more accurate signals, and thus conservatism may be welfare improving. Hence, while my results show that conservatism leads to “mistakes” in individual choice (the agent violates both Dynamic Consistency and Consequentialism) there may be benefits from conservatism in other situations. de Clippel and Zhang (2020) extended the model of Bayesian persuasion (Kamenica and Gentzkow (2011)) to general non-Bayesian receivers (e.g., a patient who exhibits conservatism bias).

2 Model and Setup

There is a (nonempty) finite set $S$ of states of the world, a collection of events given by an algebra $\Sigma$ over $S$, and a (nonempty) set of consequences $X$. Let $F$ denote the set of functions $f : S \rightarrow X$, referred to as an act. Following a standard abuse of notation, for any $x \in X$, by $x \in F$ I mean the constant act that returns $x$ in every state. Lastly, for any $f, g \in F$ and for any $A \in \Sigma$, let $fAg$ denote the act that returns $f(s)$ when $s \in A$ and returns $g(s)$ when $s \in A^c \equiv S \setminus A$. Following the literature, I assume that $X$ is a convex subset of a vector space. Thus, mixed acts can be defined point-wise, so that for every $f, g \in F$ and $\lambda \in [0, 1]$, by $\lambda f + (1 - \lambda)g$ I mean the act that returns $\lambda f(s) + (1 - \lambda)g(s)$ for each $s \in S$.

I assume that the agent has preferences over $F$ conditional on her information. Formally, the agent has a collection of preference relations, $\{\succsim_A\}_{A \in \Sigma}$ over $F$, where $\succsim_A$ are her preferences after observing $A$. Let $\succ_A$ and $\sim_A$ represent the asymmetric and symmetric parts of $\succsim_A$. The case when the agent has no information is represented by $\succsim_S := \succsim$. For an event $A$, say that $A$ is $\succsim$-null (or simply null) if $fAg \sim g$ for any $f, g \in F$. Otherwise, $A$ is non-null. Let $\Sigma_+ \subset \Sigma$ denote the collection of non-null events.

Let $\Delta(S)$ denote the set of probability measures over $S$. Typical elements, $\mu, \pi \in \Delta(S)$, are called beliefs. For any probability $\mu$ and non-null event $A \in \Sigma_+$, define the Bayesian update of $\mu$ given $A$ by $B(\mu, A)(B) = \frac{\mu[B \cap A]}{\mu[A]}$ for $B \in \Sigma$. Finally, for any two utility functions of confidence occurring among subjects who violated both Dynamic Consistency and Consequentialism. This is consistent with the interpretation of $1 - \delta(A)$ as the degree of confidence in the information, with lower confidence leading to more violations.

It is not difficult to extend to an infinite state space by assuming $\Sigma$ is a $\sigma$-algebra and $F$ is the set of finite-valued $\Sigma$-measurable functions.

For instance, $X$ may be an interval of monetary prizes or, as in the classic Anscombe and Aumann (1963) setting, a set of lotteries over some prize space.
$u, v : X \to \mathbb{R}$, say $u \approx v$ if $u$ is a positive affine transformation of $v$.

**Definition 1.** Say that a collection of preferences $\{\succsim_A\}_{A \in \Sigma}$ admits a **conservative subjective expected utility** (conservative SEU) representation if there are a non-constant utility function $u : X \to \mathbb{R}$, a prior probability $\mu \in \Delta(S)$, and a function $\delta : \Sigma_+ \to [0, 1]$ such that for all $A \in \Sigma_+$,

$$f \succsim_A g \iff \sum_{s \in S} u(f(s))\mu_A(s) \geq \sum_{s \in S} u(f(s))\mu(s)$$

and

$$\mu_A = \delta(A)\mu + (1 - \delta(A))B(\mu, A).$$

**Example 1.** There are two payoff states, $P = \{R, B\}$ and two signals $\Theta = \{r, b\}$. Let $S = P \times \Theta$ and, slightly abusing notation, let $r = \{(R, r), (B, r)\}$ and $b = \{(R, b), (B, b)\}$ denote the events corresponding to an $r$ or $b$ signal, respectively. The prior $\mu \in \Delta(S)$ is given in Table 1 below.

|       | $r$ | $b$ |
|-------|-----|-----|
| $R$   | $\frac{4}{8}$ | $\frac{1}{8}$ |
| $B$   | $\frac{1}{8}$ | $\frac{2}{8}$ |

Table 1: Prior over $S = P \times \Theta$.

This example has a natural interpretation as a standard signaling experiment with prior over $P$ given by $\mu_P = (\frac{5}{8}, \frac{3}{8})$, and (asymmetric) signal accuracy $\sigma(r|R) = \frac{4}{5}$, $\sigma(b|B) = \frac{2}{3}$.

If the agent admits a conservative SEU representation, posterior beliefs after $r$ and $b$, written in terms of the marginal belief on $R$, are

$$\mu_r(R) = \delta(r)\frac{5}{8} + (1 - \delta(r))\frac{4}{5}$$

$$\mu_b(R) = \delta(b)\frac{5}{8} + (1 - \delta(b))\frac{1}{3}.$$ 

One feature of the representation is that $\delta$ is event (i.e., signal) dependent, allowing for a rich variety of other “biases” to emerge. For instance, when $\delta(r) < \delta(b)$, the agent’s beliefs exhibit confirmation bias in addition to conservatism. That is, because her conservatism bias is greater when she receives a signal that is counter to her “prior hypothesis” (i.e., $\mu(R) > \frac{1}{2} > \mu(B)$) she is more reluctant to incorporate “disconfirming information.” This and other biases are further discussed in section 4.
3 Behavioral Foundations

The first axiom imposes a subjective expected utility (SEU) representation at each information set. The conditions for this are well-established in the literature.

**Axiom 1** (Conditional SEU). For each \( A \in \Sigma \), \( \succsim_A \) admits a non-degenerate subjective expected utility representation.

Before introducing the conservatism axiom, for comparison, I first state the classic axioms of Dynamic Consistency and Consequentialism. An excellent discussion of both axioms is provided in Ghirardato (2002), and so I will only briefly discuss them.

**Axiom 2** (Dynamic Consistency). For any \( A \in \Sigma \), \( A \) non-null, and for all \( f, g \in \mathcal{F} \),

\[
fA g \succsim g \implies f \succsim_A g.
\]

**Axiom 3** (Consequentialism). For any \( A \in \Sigma \) and for all \( f, g \in \mathcal{F} \),

\[
f(s) = g(s) \text{ for all } s \in A \implies f \sim_A g.
\]

In essence, \( fA g \succsim g \) reveals that the decision maker believes she would abandon \( g \) for \( f \) if \( A \) occurred. Dynamic Consistency states that if this is the case, then \( f \) must be preferred to \( g \) after being told \( A \) has occurred. Consequentialism states that whenever two acts are identical within \( A \), the agent must be indifferent between them after \( A \). It is well known that Dynamic Consistency and Consequentialism, when combined with Conditional SEU, are necessary and sufficient for Bayesian updating (Ghirardato, 2002). Hence, these must be relaxed to allow for conservatism.

To illustrate how these axioms are relaxed, recall the introductory example on the use of alternative energy versus coal. The act \( f \) is “use alternative energy,” \( g \) is “use coal,” and the event \( A \) is “climate change is occurring.” The agent concedes that alternative energy is better if climate change is occurring. Depending on the agent’s initial preference between \( f \) and \( g \), there are two relevant cases:

**Case 1**: \( f \succsim g \) and \( fA g \succsim g \),

**Case 2**: \( g \succ f \) and \( fA g \succsim g \).

In case 1, the preference for the fixed action \( f \) (over \( g \)) is consistent with the preference for the contingent action \( fA g \) (over \( g \)). In case 2, the preferences are in conflict. Dynamic Con-
sistency requires the agent to conclude that \( f \) is better than \( g \) in the conditional preference in both cases. However, intuition suggests that a conservative agent may violate Dynamic Consistency in case 2. Indeed, she does so because she does not fully believe \( A \) has occurred. Similarly, a conservative agent would never violate Dynamic Consistency in case 1 since she prefers \( f \) to \( g \) irrespective of \( A \)’s occurrence. The following axiom takes this intuition as the defining behavior of conservatism.

**Axiom 4 (Dynamic Conservatism).** For any \( A \in \Sigma_+ \) and for all \( f, g \in \mathcal{F} \),

\[
\begin{align*}
(i) & \quad f \succsim g \\
(ii) & \quad fAg \succsim g \\
\end{align*}
\]

\( \implies f \succsim_A g. \)

Further, if both (i) and (ii) are strict, then \( f \succ_A g. \)

Dynamic Conservatism requires that if the agent (i) prefers \( f \) to \( g \) ex-ante and (ii) forecasts that she would abandon \( g \) for \( f \) if \( A \) occurred, \( fAg \succsim g \), then she must prefer \( f \) to \( g \) conditional on \( A \). Therefore, while Dynamic Conservatism allows for some violations of Dynamic Consistency (e.g., the climate change example), it restricts violations to cases where the agent cannot be completely sure that she is making the right decision. Hence, we can think of Dynamic Conservatism as allowing for a form of “stickiness” or skepticism about new information. Seen through this lens, Dynamic Conservatism can be viewed as a cautious response when the reliability of information is (subjectively) uncertain.\(^6\) Consequently, Dynamic Conservatism permits an agent to state \( fAg \succsim g \) and \( g \succ_A f. \)

**Theorem 1.** The following are equivalent:

(i) The collection \( \{ \succsim_A \}_{A \in \Sigma} \) satisfies Conditional SEU and Dynamic Conservatism;

(ii) The collection \( \{ \succsim_A \}_{A \in \Sigma} \) admits a conservative subjective expected utility representation.

Further, if \( (u, \mu, \delta) \) and \( (u', \mu', \delta') \) both represent \( \{ \succsim_A \}_{A \in \Sigma} \), then (i) \( u' \) is a positive affine transformation of \( u \), (ii) \( \mu' = \mu \), and (iii) \( \delta'(A) = \delta(A) \) for all \( A \) such that \( A, A^c \in \Sigma_+ \).

Theorem 1 shows that when conditional preferences admit a SEU representation, Dynamic Conservatism is the precise behavioral content of conservative updating. Further, the

\(^6\)Another way of viewing Dynamic Conservatism is through a two-self interpretation. Condition (i) captures a self that does not learn, while condition (ii) captures a Bayesian self. Dynamic Conservatism states that when the selves agree, then the agent’s behavior necessarily reflects this agreement. When they disagree, then Dynamic Conservatism says nothing. The representation result, however, shows that the agent’s behavior must be governed by a “compromise” belief.
degree of conservatism, $\delta(A)$, is uniquely pinned down at essentially every $A$.\footnote{Dynamic Conservatism may be extended to any event $A \in \Sigma$, rather than merely the non-null events, $\Sigma_+$. When $A$ is null, $fAg \sim g$ for any $f, g$. It then follows from Dynamic Conservatism that $\succcurlyeq = \succeq_A$. Thus, conservative agents react to null events by completely ignoring them.}

**Example 1** (Continued). To further illustrate this result, recall our example with two payoff states $\mathcal{P} = \{R, B\}$ and two signals $\Theta = \{r, b\}$. As illustrated in Figure 1, Dynamic Conservatism ensures that the lower contour set of the conditional indifference curve passing through $f$ must contain the intersection of the lower contour sets determined by (i) the initial preference (e.g., $\mu$) and (ii) the preference that corresponds to performing Bayesian updating $\mathcal{B}(\mu, r)$. This intersection is shaded in blue. In essence, conservatism pulls the conditional indifference curve to to be more aligned with the initial preference.

![Figure 1: Indifference curves after the agent receives an $r$ signal. The (orange) dashed line corresponds to an indifference curve passing through $f$ for some $\delta(r) \in (0, 1)$.](image)

**Remark 1.** It is easy to see from Figure 1 that over-inference or “prior-neglect” may be captured by rotating the (orange) indifference curve away from $\mu$ and beyond $\mathcal{B}(\mu, r)$ (e.g., intuitively, this is captured by $\delta(r) < 0$). This would imply the existence of an act $g$, such that $f \succcurlyeq g$, $f \succ r g \succcurlyeq g$ but $g \succ r f$, violating Dynamic Conservatism. However, over-inference is difficult to capture in the conditional preference framework with a general state space. This is because $\delta(A) < 0$ results in negative values for the probabilities of certain states. Thus, the behavioral foundations of such behavior remain an open question.
4 Source Dependence and Belief Biases

When beliefs (or attitudes) about uncertainty vary with the source of that uncertainty, then beliefs (or attitudes) are said to be source-dependent. This section demonstrates that source-dependent conservatism captures belief dynamics consistent with confirmation bias, representativeness, and the good news-bad news effect. Source dependence might be related to familiarity with a source. An implication of this notion is that there may be increased skepticism about new sources, which is conceptually similar to the notion of source-dependent ambiguity attitudes (Abdellaoui et al. (2011) and Chew and Sagi (2008)). Alternatively, source dependence might be related to message complexity, whereby subjectively simpler messages are accepted more readily.\(^8\)

4.1 Confirmation Bias

Confirmation bias (see Nickerson (1998) for an excellent review) refers to a tendency to accept information that supports already believed hypotheses and to downplay conflicting information. Confirmation bias emerges from conservatism bias when the weight attached to the prior is greater for “disconfirming” news than for “confirming” news.

**Example 1** (Continued). There are two payoff states, \(\mathbb{P} = \{R, B\}\), and two signals, \(\Theta = \{r, b\}\), and the prior \(\mu\) is given in Table 1 below.

| \(\mu\) | \(r\) | \(b\) |
|---------|------|------|
| \(R\)   | 4/8  | 1/8  |
| \(B\)   | 1/8  | 2/8  |

If the agent admits a conservative SEU representation, posterior beliefs after \(r\) and \(b\), written in terms of the marginal belief on \(R\), are

\[
\mu_r(R) = \delta(r)\frac{5}{8} + (1 - \delta(r))\frac{4}{5}
\]

\[
\mu_b(R) = \delta(b)\frac{5}{8} + (1 - \delta(b))\frac{1}{3}
\]

When \(\delta(r) < \delta(b)\), the agent’s beliefs exhibit confirmation bias in addition to conservatism; when she receives a signal that is counter to her “prior hypothesis” (i.e., \(\mu(R) >

---

\(^8\)This reasoning is consistent with theories in psychology that posit that conservatism results from the difficulty of aggregating sources of information (Slovic and Lichtenstein, 1971) and noisy recollection (Hilbert, 2012).
\[ \frac{1}{2} > \mu(B) \], she is more reluctant to incorporate “disconfirming information.” I provide a behavioral characterization of confirmation bias in Proposition 2.

4.2 Representativeness

The representativeness heuristic (Kahneman and Tversky, 1972) refers to a tendency to react more strongly to signals that are representative of, or similar to, an underlying state. Consider a setup like the confirmation bias example, but suppose the agent has a similarity relation \( \succeq \) where \((A, a) \succeq (B, b)\) denotes \( a \) is more representative of \( A \) than \( b \) is of \( B \). If \((A, a) \succeq (B, b)\) implies \( \delta(E_a) \leq \delta(E_b) \), then a form of the representativeness heuristic is present. Beliefs depend on both the objective information provided by the signal and the degree to which the agent perceives that signal as representative of the payoff-relevant variables.

4.3 Good News-Bad News Effect

When subjects react differently to “types” of news, they exhibit the good news-bad news effect (Eil and Rao (2011), Möbius et al. (2014), and Charness and Dave (2017)). Source-dependent conservatism may also allow for this effect. Informally, a decision maker reacts more to good news when her degree of bias is smaller. Similar to the representativeness example, suppose \( E_a \succeq E_b \) denotes \( E_a \) is better news than \( E_b \). Then asymmetric updating occurs when \( E_a \succeq E_b \) implies \( \delta(E_a) \leq \delta(E_b) \). The reverse behavior, stronger reaction to bad news, is captured by reversing the inequality.

5 Degrees of Conservatism

5.1 A Conservatism Index

Theorem 1 allows for the degree of conservatism to depend on the information received. In many settings, this is useful as it allows for source-dependent reactions to news. However, it is often convenient to fully describe behavior with a single parameter, or an index of conservatism. Further, a constant degree of conservatism simplifies the task of identifying behavioral parameters from data and increases the predictive power of the model. I show that constant conservatism is characterized by a consistency condition linking conditional preferences across information sets that may be viewed as a weak form of Consequentialism.
Axiom 5 (Weak Consequentialism). For any $A, B, C \in \Sigma_+$ with $C \cap (A \cup B) = \emptyset$ and for all $f, y, z \in F$,

$$fCy \succeq_A z \iff fCy \succeq_B z.$$ 

To see how this is a weak form of Consequentialism, suppose $C \cap (A \cup B) = \emptyset$ and that Consequentialism holds. Consider $fCy$ for an arbitrary $f$. Since for all $s \in A \cup B$, $fCy(s) = fCy(s)$, it follows from Consequentialism that both $fCy \sim_A y$ and $fCy \sim_B y$. Thus, under Consequentialism, $f$ is irrelevant, and under ordinal preference consistency, Weak Consequentialism always holds. On the other hand, Weak Consequentialism does not impose indifference between $fCy$ and $y$ but requires a consistent relative preference; if $fCy$ is preferred to $z$ after $A$, then it is also preferred after $B$.

Proposition 1. Suppose the collection $\{\succeq_A\}_{A \in \Sigma}$ admits a Conservative SEU representation $(u, \mu, \delta)$. Then the following are equivalent:

(i) The collection $\{\succeq_A\}_{A \in \Sigma}$ satisfies Weak Consequentialism.

(ii) There is a unique $\delta \in [0, 1]$ such that $\delta(A) = \delta$ for all $A$ such that $A$ and $S \setminus A$ are non-null.

A key strength of this result is that now $\delta$ may serve as a simple index of conservatism bias and may be elicited with only a few questions.

5.2 Generalized Confirmation Bias

By weakening Weak Consequentialism to depend on the ex-ante likelihood of events, a behavioral characterization of conservative preferences that are consistent with confirmation bias may be obtained. To do so, I first define a qualitative likelihood ordering over events.

Definition 2. For any $A, B \in \Sigma_+$, say that $A$ is more likely than $B$, denoted $A \geq_1 B$ if for all $x, y \in X$, $x \succeq y$ implies $xAy \succeq xBy$.

Recall that Weak Consequentialism ensures a constant $\delta$ by precisely calibrating the agent’s willingness to bet on $C$ across $A$ and $B$. Note that this is independent of the ex-ante relative likelihood of $A$ or $B$. Confirmation bias, on the other hand, suggests that subjectively more likely events are incorporated more accurately. In other words, because $B$ is viewed as less likely than $A$, the agent is more willing to bet on $C$ after $B$ than after $A$. This is captured by the following behavioral axiom.
Axiom 6 (Generalized Confirmation Bias). For any $A, B, C \in \Sigma_+$ with $C \cap (A \cup B) = \emptyset$ and for all $x, y, z \in \mathcal{F}$ if $x \succ y$ and $A \succeq_l B$, then

$$xCy \succeq_B z \implies xCy \succeq_A z.$$ 

Proposition 2. Suppose the collection $\{\succeq_A\}_{A \in \Sigma}$ admits a Conservative SEU representation $(u, \mu, \delta)$. Then the following are equivalent:

(i) The collection $\{\succeq_A\}_{A \in \Sigma}$ satisfies Generalized Confirmation Bias.

(ii) $\mu(A) \geq \mu(B)$ if and only if $\delta(A) \leq \delta(B)$.

5.3 Comparative Conservatism

Intuitively, a more conservative agent is less responsive to information, which can be captured in the representation by a larger weight on the prior belief. Analogously, one person is more conservative than another if he places a larger weight on his prior belief than she does on her prior belief. This can be formally defined in terms of preferences over binary acts.

Definition 3. Say that $\{\succeq^1_A\}_{A \in \Sigma}$ is more conservative than $\{\succeq^2_A\}_{A \in \Sigma}$ if for all $A$ and all $x, y_1, y_2 \in X$ satisfying (i) $x \succ^i y_i$ and (ii) $xAy_1 \succeq^1_A z \iff xAy_2 \succeq^2_A z$ for all $z \in X$

$$xAy_1 \succeq^1_A z \implies xAy_2 \succeq^2_A z.$$ 

If both agents are Bayesian, then $xAy \sim^j_A x$ for any $x, y \in X$. A conservative agent, however, may worry about the low payoff $y$ on $A^c$. The more conservative the agent, the lower his certainty equivalent for a bet on $A$. Consequently, a more conservative agent places a lower value on $xAy$ than a less conservative agent. Hence, when agent 1 (he) is more conservative than agent 2 (she), his certainty equivalent is lower than hers, and so whenever he prefers to bet on $A$, so must she. Importantly, this definition does not require the agents to have the same initial beliefs, but only the same tastes over constant acts.\(^9\)

Proposition 3. Suppose $\{\succeq^i_A\}_{A \in \Sigma}^{i=1,2}$ admit Conservative SEU representations $(u_i, \mu_i, \delta^i_{i=1,2})$ where $u_1 \approx u_2$ and $\Sigma_+^1 = \Sigma_+^2$. Then the following are equivalent:

(i) $\{\succeq^1_A\}_{A \in \Sigma}$ is more conservative than $\{\succeq^2_A\}_{A \in \Sigma}$.

(ii) $\delta^1(A) \geq \delta^2(A)$ for every $A$ such that $A$ and $S \setminus A$ are non-null.

\(^9\)By allowing $y_1$ and $y_2$ to differ, we account for the different prior beliefs about the ex-ante likelihood of $A$. Accordingly, we may take $y_1 = y_2$ if and only if $\mu_1(A) = \mu_2(A)$. 

13
Proposition 3 may be useful when attempting to classify subjects based on their degree of conservatism because it shows that subjects may be compared with relatively few questions. Given an event of interest, the experimenter need only elicit (conditional) certainty equivalents for particular binary acts (on $A$). Under the assumption of constant conservatism (i.e., Weak Consequentialism holds), a single elicitation of a (conditional) certainty equivalent suffices to order all subjects.

6 Multiple Beliefs

Agents may struggle to come up with a single, probabilistic belief; they perceive a situation to be ambiguous.\footnote{See Gilboa and Marinacci (2011) for an excellent summary of the literature.} Under ambiguity, we often suppose the agent has multiple beliefs. To study conservatism with multiple beliefs, I suppose the agent has a collection of (incomplete) preferences, each of which admits a multiple-prior “unanimity” representation à la Bewley (2002).

Similar settings have been used to study objective versus subjective rationality (Gilboa et al., 2010), distinguish indecisiveness in tastes versus beliefs (Ok et al., 2012), and differentiate ambiguity perception and attitude (Ghirardato et al., 2004). Unlike in Gilboa et al. (2010), I am focused purely on the evolution of beliefs and so do not consider the secondary “subjective” preference relation. Because of this my results hold irrespective of the agent’s ambiguity attitude.

Definition 4. A preference relation $\succsim$ admits a multi-prior expected utility representation if there are a utility $u : X \to \mathbb{R}$ and a nonempty, closed, convex set of beliefs $\mathcal{M} \subseteq \Delta(S)$ such that for all acts $f, g \in \mathcal{F}$,

$$f \succsim g \iff \sum_{s \in S} u(f(s))\mu(s) \geq \sum_{s \in S} u(g(s))\mu(s) \text{ for every } \mu \in \mathcal{M}. \quad (2)$$

Suppose the agent has a collection of possibly incomplete preferences $\{\succsim^*_A\}_{A \in \Sigma}$ defined over $\mathcal{F}$. For an event $A$, say that $A$ is unambiguously non-null if for all $x, y$ such that $x \succ^* y$, $xAy \succ^* y$. Let $\Sigma^*_+ \subseteq \Sigma$ denote the set of unambiguously non-null events. For any set of probabilities $\mathcal{M}$ and any $A \in \Sigma^*_+$, the set obtained when each belief in $\mathcal{M}$ is updated by Bayes’ rule is denoted $B(\mathcal{M}, A) = \{\pi \in \Delta(S) \mid \pi = B(\mu, A) \text{ for some } \mu \in \mathcal{M}\}$. Lastly, for any set $Y \subseteq \Delta(S)$, let $\text{conv}(Y)$ denote the convex hull of $Y$. 
Axiom 7 (Conditional Multi-prior Expected Utility). For each $A \in \Sigma^*_+$, $\succeq^*_A$ admits a non-degenerate multi-prior expected utility representation. That is, there exists a pair $(u, \mathcal{M}_A)$ that represents $\succeq^*_A$ as in Equation 2.

As in the case of subjective expected utility, the conditions for such a representation are well-known in the literature and so are not re-stated.

The following axiom, Unambiguous Dynamic Conservatism, is precisely Dynamic Conservatism applied to the unambiguous preferences.

Axiom 8 (Unambiguous Dynamic Conservatism). For any $A \in \Sigma^*_+$, and for all $f, g \in \mathcal{F}$

$$f \succeq^* g \quad fAg \succeq^* g \quad \implies \quad f \succeq^*_A g.$$  

Further, if both (i) and (ii) are strict, then $f \succ^*_A g$.

The following theorem is the direct counterpart of Theorem 1 for multiple priors.

Theorem 2. The following are equivalent:

(i) The collection $\{\succeq^*_A\}_{A \in \Sigma}$ satisfies Conditional Multi-prior Expected Utility and Unambiguous Dynamic Conservatism.

(ii) There is a non-constant utility function $u: X \to \mathbb{R}$ such that $u_A = u$ for every $A \in \Sigma$, and for each $A \in \Sigma^*_+$,

$$\mathcal{M}_A \subseteq \text{conv} \left( \mathcal{M} \cup \mathcal{B}(\mathcal{M}, A) \right).$$  

In this case, we say the agent admits a conservative multi-prior representation.

As before, Unambiguous Dynamic Conservatism ensures that risk attitudes are unchanged by the information (i.e., $u$ is independent of $A$). However, Theorem 2 is considerably more general than Theorem 1, which corresponds to the special case where $\succeq^*$ and $\succeq^*_A$ are complete. An important point of departure is that Theorem 2 allows for an agent to expand her set of beliefs. For instance, this occurs when $\succeq^*$ is complete but $\succeq^*_A$ is not. At the same time, my result complements results by Ghirardato et al. (2008) and Faro and Lefort (2019), who found that imposing dynamic consistency on the unambiguous preference ensures prior-by-prior updating (e.g., $\mathcal{M}_A = \mathcal{B}(\mathcal{M}, A)$).
Example 2 (Single prior-multiple weights). Suppose $\mathcal{M} = \{\mu\}$ and for each $A \in \Sigma^*_+$ there exists a closed, convex set of weights $W_A \subset [0, 1]$ such that

$$\mathcal{M}_A = \{\pi \in \Delta(S) \mid \pi = \delta_A \mu + (1 - \delta_A) B(\mu, A) \text{ and } \delta_A \in W_A\}.$$  

In this case, the agent is SEU before information. However, because she is uncertain about the reliability of the information, her set beliefs expands. Her posterior beliefs are constructed via a set of “confidence weights” that she places on the news. In fact, it is straightforward to show that this example corresponds to precisely the case in which the initial preference is SEU.

Corollary 1. Suppose $\{\succsim_A^*\}_{A \in \Sigma}$ satisfies Conditional Multi-prior Expected Utility and Unambiguous Dynamic Conservatism and that the initial preference, $\succsim^*$, is complete. Then for every $A \in \Sigma^*_+$ there exists a closed, convex set of weights $W_A \subset [0, 1]$ such that

$$\mathcal{M}_A = \{\pi \in \Delta(S) \mid \pi = \delta_A \mu + (1 - \delta_A) B(\mu, A) \text{ and } \delta_A \in W_A\}.$$  

Example 3 (Multiple priors-single weight). Suppose for some map $\delta : \Sigma^*_+ \to [0, 1]$,

$$\mathcal{M}_A = \delta(A) \mathcal{M} + (1 - \delta(A)) B(\mathcal{M}, A).$$  

In this case, the agent has multiple beliefs, but she has a well-defined opinion of the news source. Thus she uses a single parameter to shift her beliefs after each event $A$.

Example 3 (Continued). Suppose there are two payoff states, $\mathbb{P} = \{R, B\}$, two signals $\Theta = \{r, b\}$, and $\mathcal{M} = \text{conv}(\{\mu, \mu'\})$, which are shown in Table 2 below.

|   | $r$  | $b$  |
|---|------|------|
| $R$ | 4/10 | 2/10 |
| $B$ | 1/10 | 3/10 |

|   | $r$  | $b$  |
|---|------|------|
| $R$ | 3/10 | 3/10 |
| $B$ | 2/10 | 2/10 |

Table 2: Priors $\mu$ and $\mu'$.

Both $\mu, \mu'$ generate the same marginals over $\mathbb{P}$: $\mu(R) = \mu'(R) = 3/5$. The distinction between $\mu$ and $\mu'$ is in how they treat signals. Under $\mu$ signals are informative, while under $\mu'$ they are not. If the agent admits a conservative multi-prior representation, posterior beliefs after $r$ and $b$, written in terms of the belief over payoff-states $\mathbb{P}$, are

$$\mathcal{M}_r = \text{conv} \left( \left\{ \left( \frac{4 - \delta(r)}{5}, \frac{1 + \delta(r)}{5} \right), (\frac{3}{5}, \frac{2}{5}) \right\} \right)$$  

16
\[ M_b = \text{conv} \left( \left\{ \left( \frac{2 + \delta(b)}{5}, \frac{3 - \delta(b)}{5} \right), \left( \frac{3}{5}, \frac{2}{5} \right) \right\} \right). \]

These belief sets are illustrated in Figure 2 in terms of the marginal belief on \( R \). Notice that \( R \) is ambiguous after the signal, save for the extreme case \( \delta(\theta) = 1 \), and the range is increasing as \( \delta(\theta) \) decreases. Consequently, the conditional certainty equivalent for a bet on \( R \) (e.g., a monetary act paying \$x if \( R \) and \$0 otherwise) depends on the both the agent’s degree of conservatism and her ambiguity attitude. In the case of ambiguity aversion (e.g., maxmin expected utility (MEU) representation (Gilboa and Schmeidler, 1989)), the agent may exhibit an “all news is bad news” effect. For more flexible models, such as the \( \alpha \)-maxmin model, this need not be the case.\(^{11}\) The certainty equivalents for a bet on \( R \) can be seen in Table 3. In the table, I vary \( \delta(\cdot) \) and ambiguity attitude; 0-maxmin (e.g., Gilboa and Schmeidler (1989)) is the most ambiguity averse and 1-maxmin is maximally ambiguity seeking. Certainty equivalents for intermediate values of \( \alpha \) may be directly calculated from these extreme cases.

| \( \delta(r) \) \( \backslash \) \( \alpha \) | 0   | 1   | \( \delta(b) \) \( \backslash \) \( \alpha \) | 0   | 1   |
|-----------------|-----|-----|-----------------|-----|-----|
| 0               | 0.6x| 0.8x| 0               | 0.4x| 0.6x|
| 1/2             | 0.6x| 0.7x| 1/2             | 0.5x| 0.6x|
| 1               | 0.6x| 0.6x| 1               | 0.6x| 0.6x|

Table 3: Certainty equivalents for the monetary bet paying \( x \)-utils in payoff state \( R \), otherwise, for an \( \alpha \)-MEU agent. The left panel reports values after \( r \), while the right reports values after \( b \).

A Proofs

A.1 Preliminary Results

Consider the following two properties for a binary relation \( \succcurlyeq \) on \( F \).

\(^{11}\)By adapting axioms introduced by Gilboa et al. (2010), or more recently by Frick et al. (2020), one may characterize an \( (\alpha) \)-maxmin representation consistent with \( \{z^A_\alpha\} \). This additional structure on the agent’s ambiguity attitude generates sharper predictions regarding her willingness to bet on various events.
C-Completeness: For any $x, y \in F$, either $x \succeq y$ or $y \succeq x$.

Monotonicity: If $f(s) \succeq g(s)$ for all $s \in S$, then $f \succeq g$.

**Lemma 1.** Consider a collection of preferences $\{\succeq_A\}_{A \in \Sigma}$ such that (i) $\succeq$ satisfies C-Completeness and Monotonicity and (ii) the collection satisfies Dynamic Conservatism. Then for all $A \in \Sigma$ such that $A$ is non-null, and any $x, y \in X$, $x \succeq y \iff x \succeq_A y$.

**Proof.** Since $\succeq$ is complete for constant acts, suppose $x \succeq y$. By Monotonicity of $\succeq$ this is equivalent to $xAy \succeq y$ for all $A$. Then by Dynamic Conservatism, $x \succeq_A y$. Suppose that $x \succeq_A y$ but $y \succ x$. Then it follows from Monotonicity and the fact that $A$ is non-null that $yAx \succ x$. From Dynamic Conservatism it follows that $y \succ_A x$, a contradiction. Hence $x \succeq y$. □

**Lemma 2.** Suppose $\preceq^*$ admits a multi-prior expected utility representation $(u, M)$. For each $A \in \Sigma$ and all $f, g \in F$,

$$fAg \succeq^* g \iff fAh \succeq^* gAh \text{ for all } h \in F.$$ 

**Proof.** Let $A$ be any event and let $f, g, h$ be any acts in $F$.

$$fAg \succeq^* g \iff \sum_{s \in A} u(f(s))\mu(s) + \sum_{s \in S \setminus A} u(g(s))\mu(s) \geq \sum_{s \in A} u(g(s))\mu(s) + \sum_{s \in S \setminus A} u(g(s))\mu(s) \text{ for all } \mu \in M$$

$$\iff \sum_{s \in A} u(f(s))\mu(s) \geq \sum_{s \in A} u(g(s))\mu(s) \text{ for all } \mu \in M$$

$$\iff \sum_{s \in A} u(f(s))\mu(s) + \sum_{s \in S \setminus A} u(h(s))\mu(s) \geq \sum_{s \in A} u(g(s))\mu(s) + \sum_{s \in S \setminus A} u(h(s))\mu(s) \text{ for all } \mu \in M$$

$$\iff fAh \succeq^* gAh$$

This lemma only relies on $A$ being non-null, and it in fact holds for more general state spaces. □

**Lemma 3.** Suppose $\preceq^*$ admits a multi-prior expected utility representation $(u, M)$. For each $A \in \Sigma^*_+$ and all $f, g \in F$, say $f \succeq_A^* g$ if $fAh \succeq^* gAh$ for some $h$. Then $\succeq_A$ admits a multi-prior expected utility representation $(u, B(M, A))$. 18
Proof. By Lemma 2, \( \succeq_A \) does not depend on the choice of \( h \). It then follows that \( f \succeq_A^* g \) if and only if for every \( \mu \in \mathcal{M} \)

\[
\sum_{s \in A} u(f(s))\mu(s) + \sum_{s \in S \setminus A} u(h(s))\mu(s) \geq \sum_{s \in A} u(g(s))\mu(s) + \sum_{s \in S \setminus A} u(h(s))\mu(s)
\]

\[\iff \frac{1}{\mu(A)} \sum_{s \in A} u(f(s))\mu(s) \geq \frac{1}{\mu(A)} \sum_{s \in A} u(g(s))\mu(s)\]

\[\iff \sum_{s \in A} u(f(s))\pi(s) \geq \sum_{s \in A} u(g(s))\pi(s) \text{ for all } \pi \in \mathcal{B}(\mathcal{M}, A),
\]

where \( \mathcal{B}(\mathcal{M}, A) = \{ \pi \in \Delta(S) \mid \pi = \mathcal{B}(\mu, A) \text{ for some } \mu \in \mathcal{M} \} \).

\[\square\]

Note that when \( \succeq^*_A \) is complete, then we have the case of subjective expected utility and \( \mathcal{M} \) and \( \mathcal{B}(\mathcal{M}, A) \) are singleton sets.

A.2 Proof of Theorem 1

Proof. Necessity is clear so only sufficiency is shown. By Conditional SEU, there exists a \((u_A, \mu_A)\) for each \( A \in \Sigma \) that represents \( \succeq_A \). Further, by Dynamic Conservatism, preferences satisfy ordinal preference consistency (see Lemma 1): \( x \succeq y \) if and only if \( x \succeq_A y \). Hence we may assume \( u = u_A \) for all \( A \). Further, as \( X \) is convex, it is without loss to suppose \([-1, 1] \subset u(X)\), as \( u \).

For each \( A \in \Sigma_+ \), define the binary relation \( \succeq_A \) on \( \mathcal{F} \) by \( f \succeq_A g \) if and only if \( fAg \succeq g \). Then by Lemma 3, \( \succeq_A \) has an expected utility representation \((u, \mathcal{B}(\mu, A))\).

Next, define the set \( D_A := \{ \pi \in \Delta(S) \mid \delta \mu + (1 - \delta)\mathcal{B}(\mu, A) \text{ for } \delta \in [0, 1] \}. \) By Dynamic Conservatism, it follows that \( \mu_A \in D_A \). Suppose not, then as \( D_A \) and \( \{ \mu_A \} \) are closed, convex sets, there exists a separating hyperplane \( a \in \mathbb{R}^{|S|} \) so that \( \mu_A \cdot a > \hat{\mu} \cdot a \) for all \( \hat{\mu} \in D_A \). Let \( \bar{z} = \max_{\mu \in D_A} \hat{\mu} \cdot a \) and let \( \bar{a} = a - \bar{z}(1, \ldots, 1) \). Then

\[
\mu_A \cdot \bar{a} \geq 0 \iff \hat{\mu} \cdot \bar{a} \text{ for all } \hat{\mu} \in D_A.
\]

We may suppose that \( \bar{a} \in [-1, 1]|^S|, \) since we can always multiply both sides of (4) by \( \epsilon > 0 \). Further, there are acts \( f, g \) such that \( u(g(s)) - u(f(s)) = \bar{a}(s) \) for every \( s \in S \). Consequently,

\[
\sum_{s \in S} \mu_A(s)u(g(s)) > \sum_{s \in S} \mu_A(s)u(f(s))
\]

(5)
and
\[ \sum_{s \in S} \hat{\mu}(s)u(f(s)) \geq \sum_{s \in S} \hat{\mu}(s)u(g(s)) \] for all \( \hat{\mu} \in D_A. \) \hspace{1cm} (6)

By construction, \( \mu, \mathcal{B}(\mu, A) \in D_A \) and so by (6) it follows that \( fAg \succeq g, f \succeq g. \) However, by (5) \( g \succ_A f, \) which contradicts Dynamic Conservatism. Hence \( \mu_A \in D_A. \) As \( A \) was arbitrary, the preceding argument applies to any non-null \( A. \) It is standard to show that \( u \) is unique up to a positive, affine transformation and, since \( u(x) > u(y) \) for some \( x, y \in X, \) that \( \mu \) and \( \mu_A \) are also unique. Given uniqueness of \( \mu \) and \( \mu_A, \) it is trivial that there is a unique \( \delta(A) \) that satisfies Equation 1 whenever \( \mu(A) < 1. \) When \( \mu(A) = 1 \) (i.e., \( A^c \) is null), \( \mu = \mathcal{B}(\mu, A) = \mu_A \) and \( \succeq = \succeq_A. \) When \( A, A^c \) are both non-null, define \( \delta_1 = \delta_2 = \delta \) as the unique solution to

\[ \mu_A = \delta(A)\mu + (1 - \delta(A))\mathcal{B}(\mu, A). \]

When \( A^c \) is null, define \( \delta(A) \) arbitrarily. \qed

### A.3 Proof of Proposition 1

**Proof.** Theorem 1 shows that for each \( A, \) there is a \( \delta(A) \in [0, 1] \) satisfying the representation. Suppose Weak Consequentialism holds. It is sufficient to show that for any pair of non-null events \( A, B \in \Sigma, \) such that both \( \mu(A) < 1 \) and \( \mu(B) < 1, \) \( \delta(B) = \delta(A). \)

**Case 1:** \( (\mu(A \cup B) < 1) \). Fix any non-null \( C \) satisfying \( C \cap (A \cup B) = \emptyset \) and choose \( x, y, z \) such that \( xCy \sim_A z. \) By Weak Consequentialism, it follows that \( xCy \sim_B z. \) Since preferences admit a conservative SEU representation, it follows that

\[ \mu_A(C)u(x) + (1 - \mu_A(C))u(y) = u(z), \] \hspace{1cm} (7)
\[ \mu_B(C)u(x) + (1 - \mu_B(C))u(y) = u(z). \] \hspace{1cm} (8)

Since \( x, y \) are arbitrary, it is without loss to suppose that \( u(x) > u(y). \) Then, combining (7) and (8), it is clear that \( \mu_A(C) = \mu_B(C). \) Since \( C \cap (A \cup B), \) it follows that \( \mathcal{B}(\mu, A)(C) = 0 = \mathcal{B}(\mu, B)(C), \) and hence \( \mu_A(C) = \delta(A)\mu(C) \) and \( \mu_B(C) = \delta(B)\mu(C). \) Hence equality is true if and only if \( \delta(A) = \delta(B). \)

**Case 2:** \( (\mu(A \cup B) = 1) \). Suppose \( A \cap B \neq \emptyset. \) Then notice that \( A, A \cap B (B, A \cap B) \) satisfy the conditions of Case 1. It follows then that \( \delta(A) = \delta(A \cap B) = \delta(B). \) Now suppose \( A \cap B = \emptyset. \) Since there are at least three non-null events, without loss there exists some non-null set \( A' \) such that \( A' \subset A \) and \( \mu(A') < \mu(A). \) \footnote{Alternatively, there exists \( A' \) such that \( A \subset A' \) and \( \mu(A) < \mu(A'), \) but this is just a relabeling. Note that} Then we may again apply Case 1 to
$A, A', A \setminus A'$, showing that $\delta(A) = \delta(A') = \delta(A \setminus A')$. Further, since $A \cap B = \emptyset$, it follows that $\mu(A' \cup B) < 1$, and so $\delta(A') = \delta(B)$. □

A.4 Proof of Proposition 2

Proof. Let $A \succeq B$ and fix any non-null $C$ satisfying $C \cap (A \cup B) = \emptyset$. Choose any $x, y, z$ such that $x > y$ and suppose $xCy \sim_B z$. It follows that

$$\mu_B(C)u(x) + (1 - \mu_B(C))u(y) \geq \mu_A(C)u(x) + (1 - \mu_A(C))u(y). \quad (9)$$

Since $u(x) - u(y) > 0$, simplifying the above yields $\mu_B(C) \geq \mu_A(C)$. The result then follows simply from the fact that $\mu_B(C) = \delta(B)\mu(C)$ and $\mu_A(C) = \delta(A)\mu(C)$. The reverse direction is routine. □

A.5 Proof of Proposition 3

Proof. Suppose $\{\succ_A^1\}_{A \in \Sigma}$ admit representations $(u_i, \mu_i, \delta_i)_{i=1,2}$ where $u_1 \approx u_2$. Then without loss $u_1 = u_2 = u$. Suppose agent 1 is more conservative than agent 2. Note that $A$ and $A^c$ are $\succeq_i$-non-null if and only if $\mu_i(A) \in (0, 1)$. Pick some $x, y_1, y_2$ satisfying the conditions of Definition 3; then $u(x)\mu_1(A) + u(y_1)(1 - \mu_i(A)) = u(x)\mu_2(A) + u(y_2)(1 - \mu_2(A))$.

**Case 1:** $(\mu_1(A) = \mu_2(A))$. It is without loss to suppose $y_1 = y_2 = y$ for some $y$ and that $u(x) = 0$. Further, we may ignore the dependence of $\mu$ on $i$. If $\{\succ_A^1\}_{A \in \Sigma}$ is more conservative than $\{\succ_A^2\}_{A \in \Sigma}$, it follows that $\delta_1(A)(1 - \mu(A))u(y) \leq \delta_2(A)(1 - \mu(A))u(y)$, or $\delta_1(A) \geq \delta_2(A)$. The reverse direction is similar.

**Case 2:** $(\mu_1(A) \neq \mu_2(A))$. Note that

$$\mu_1(A) - \mu_2(A) = (1 - \mu_2(A)) - (1 - \mu_1(A)) \neq 0. \quad (10)$$

By hypothesis, $u(x)\mu_1(A) + u(y_1)(1 - \mu_i(A)) = u(x)\mu_2(A) + u(y_2)(1 - \mu_2(A))$ from which, when combined with (10), it follows that

$$(1 - \mu_1(A))[u(y_1) - u(x)] = (1 - \mu_2(A))[u(y_2) - u(x)]. \quad (11)$$

From $\{\succ_A^1\}_{A \in \Sigma}$ is more conservative than $\{\succ_A^2\}_{A \in \Sigma}$, we conclude that

$$V_A^1(x, Ay_1) = \delta_1(A)[\mu_1(A)u(x) + (1 - \mu_1(A))u(y_1)] + (1 - \delta_1(A))u(x)$$

such a pair of nested events must exist for at least one of $A$ or $B$. 

21
\[ \leq \delta_2(A)[\mu_2(A)u(x) + (1 - \mu_2(A))u(y_2)] + (1 - \delta_2(A))u(x) = V_A^2(xAy_1). \]

Simplifying the above expression yields
\[ \delta_1(A)(1 - \mu_1(A))u(y_1) - \delta_1(A)(1 - \mu_1(A))u(x) \leq \delta_2(A)(1 - \mu_2(A))u(y_2) - \delta_2(A)(1 - \mu_2(A))u(x), \]

which directly implies
\[ \delta_1(A)(1 - \mu_1(A))[u(y_1) - u(x)] \leq \delta_2(A)(1 - \mu_2(A))[u(y_2) - u(x)]. \] (12)

The result that \( \delta_1(A) \geq \delta_2(A) \) then follows by combining (12) with (11) and the facts that \( u(y_i) - u(x) < 0 \) and \( 0 < \mu_i(A) < 1 \) for \( i = 1, 2 \).

\[ \square \]

### A.6 Proof of Theorem 2

**Proof.** The proof of this theorem is similar to the proof of Theorem 1. First, for each \( A \in \Sigma \), there exists a pair \( (u_A, M_A) \) such that \( \succsim_A^* \) is represented by Equation 2. For any \( A \in \Sigma^*_+ \), define the binary relation \( \succeq_A^* \) on \( \mathcal{F} \) by \( f \succeq_A^* g \) if and only if \( fAg \succsim^* g \). Then by Lemma 3, \( \succeq_A^* \) has a multi-prior expected utility representation \( (u, B(\mathcal{M}, A)) \).

As in the proof of Theorem 1, since \( \succsim_A^* \) is complete on constant acts for every \( A \) it follows from Lemma 1 that for any \( A \in \Sigma^*_+ \), \( x \succsim y \) if and only if \( x \succeq_A^* y \) if and only if \( x \succ_A y \). Hence it is without loss to suppose that \( u = u_A \).

Next, let \( D_A := \text{conv}(\mathcal{M}, B(\mathcal{M}, A)) \). Since \( \mathcal{M} \) is a closed subset of \( \Delta(S) \), it is compact. Further since \( A \) is unambiguously non-null, \( B(\mathcal{M}, A) \) is closed and hence also compact. Further, they are both convex. Then \( D_A \) is also compact and convex.

Now, suppose for contradiction that \( \mathcal{M}_A \subseteq D_A \) is false. Then there exists some \( \tilde{\mu}_A \in \mathcal{M}_A \setminus D_A \). Following an argument similar to a Theorem 1, there exists a separating hyperplane \( a \in \mathbb{R}^{[S]} \) so that \( \tilde{\mu}_A \cdot a > \hat{\mu} \cdot a \) for all \( \hat{\mu} \in D_A \). Let \( \bar{z} = \max_{\hat{\mu} \in D_A} \hat{\mu} \cdot a \) and let \( \bar{a} = a - \bar{z}(1, \ldots, 1) \). Then
\[ \tilde{\mu}_A \cdot \bar{a} > 0 \geq \hat{\mu} \cdot \bar{a} \text{ for all } \hat{\mu} \in D_A. \] (13)

We may suppose that \( \bar{a} \in [-1, 1]^{[S]} \), since we can always multiply both sides of (13) by \( \epsilon > 0 \).
Further, there are acts $f, g$ such that $u(g(s)) - u(f(s)) = \overline{a}(s)$ for every $s \in S$. Consequently,

$$\sum_{s \in S} \tilde{\mu}_A(s) u(g(s)) > \sum_{s \in S} \tilde{\mu}_A(s) u(f(s))$$  \hspace{1cm} (14)$$

and

$$\sum_{s \in S} \hat{\mu}(s) u(f(s)) \geq \sum_{s \in S} \hat{\mu}(s) u(g(s)) \text{ for all } \hat{\mu} \in D_A.$$  \hspace{1cm} (15)$$

By construction, $\mathcal{M}, B(\mathcal{M}, A) \subset D_A$ and so by (15) it follows that $fAh \succ^* gAh$ and $f \succ^* g$. However, by (14) $f \not\succ^* A g$, which contradicts Unambiguous Dynamic Conservatism. Hence $\mathcal{M}_A \subset D_A$. 

\[ \square \]

References

Abdellaoui, M., A. Baillon, L. Placido, and P. P. Wakker (2011): “The rich domain of uncertainty: source functions and their experimental implementation,” American Economic Review, 101, 699–727.

Anscombe, F. J. and R. J. Aumann (1963): “A Definition of Subjective Probability,” The Annals of Mathematical Statistics, 34, 199–205.

Aydogan, I., A. Baillon, E. Kemel, and C. Li (2017): “Signal perception and belief updating,” Working paper.

Bewley, T. (2002): “Knightian Decision Theory. Part I,” Decisions in Economics and Finance, 25, 79–110.

Camerer, C. (1995): “Individual Decision Making,” in Handbook of Experimental Economics, ed. by A. Roth and J. Kagel, Princeton University Press.

Charness, G. and C. Dave (2017): “Confirmation bias with motivated beliefs,” Games and Economic Behavior, 104, 1–23.

Chew, S. H. and J. S. Sagi (2008): “Small worlds: Modeling attitudes toward sources of uncertainty,” Journal of Economic Theory, 139, 1–24.

de Clippel, G. and X. Zhang (2020): “Non-Bayesian Persuasion,” mimeo.

Dominik, A., P. Duersch, and J.-P. Lefort (2012): “A Dynamic Ellsberg Urn Experiment,” Games and Economic Behavior.

Edwards, W. (1982): “Conservatism in human information processing,” in Judgment under Uncertainty: Heuristics and Biases, ed. by D. Kahneman, P. Slovic, and A. Tversky, Cambridge University Press, 359–369.
Eil, D. and J. M. Rao (2011): “The Good News-Bad News Effect: Asymmetric Processing of Objective Information about Yourself,” *American Economic Journal: Microeconomics*, 3, 114–138.

El-Gamal, M. A. and D. M. Grether (1995): “Are People Bayesian? Uncovering Behavioral Strategies,” *Journal of the American Statistical Association*, 90, 1137–1145.

Epstein, L. G. (2006): “An Axiomatic Model of Non-Bayesian Updating,” *Review of Economic Studies*.

Epstein, L. G., J. Noor, and A. Sandroni (2008): “Non-Bayesian Updating: A Theoretical Framework,” *Theoretical Economics*.

Faro, J. H. and J.-P. Lefort (2019): “Dynamic objective and subjective rationality,” *Theoretical Economics*, 14, 1–14.

Frick, M., R. Iijima, and Y. L. Yaouanq (2020): “Objective rationality foundations for (dynamic) α-MEU,” *working paper*.

Ghirardato, P. (2002): “Revisiting Savage in a Conditional World,” *Economic Theory*.

Ghirardato, P., F. Maccheroni, and M. Marinacci (2004): “Differentiating ambiguity and ambiguity attitude,” *Journal of Economic Theory*, 118, 133–173.

——— (2008): “Revealed Ambiguity and Its Consequences: Updating,” in *Advances in Decision Making Under Risk and Uncertainty*, ed. by M. Abdellaoui and J. D. Hey, Springer Berlin Heidelberg, vol. 42, 3–18.

Gilboa, I., F. Maccheroni, M. Marinacci, and D. Schmeidler (2010): “Objective and Subjective Rationality in a Multiple Prio Model,” *Econometrica*, 78, 755–770.

Gilboa, I. and M. Marinacci (2011): “Ambiguity and the Bayesian Paradigm,” *Advances in Economics and Econometrics: Theory and Applications*.

Gilboa, I. and D. Schmeidler (1989): “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*.

Gul, F. and W. Pesendorfer (2001): “Temptation and Self-Control,” *Econometrica*.

Hilbert, M. (2012): “Toward a synthesis of cognitive biases: how noisy information processing can bias human decision making,” *Psychological Bulletin*, 138, 211–237.

Kahneman, D. and A. Tversky (1972): “Subjective Probability: A Judgment of Representativeness,” *Cognitive Psychology*.

Kamenica, E. and M. Gentzkow (2011): “Bayesian Persuasion,” *American Economic Review*, 101, 2590–2615.

Lee, Y.-J., W. Lim, and C. Zhao (2019): “Cheap Talk with Non-Bayesian Updating,” *working paper*.

Möbius, M. M., M. Niederle, P. Niehaus, and T. S. Rosenblat (2014): “Managing Self-
Confidence,” Working paper.

Nickerson, R. S. (1998): “Confirmation Bias: A Ubiquitous Phenomenon in Many Guises,” Review of General Psychology, 2, 175–220.

Ok, E. A., P. Ortoleva, and G. Riella (2012): “Incomplete Preferences Under Uncertainty: Indecisiveness in Beliefs versus Tastes,” Econometrica, 80, 1791–1808.

Ortoleva, P. (2012): “Modeling the Change of Paradigm: Non-Bayesian Reactions to Unexpected news,” The American Economic Review.

Phillips, L. D. and W. Edwards (1966): “Conservatism in a simple probability inference task,” Journal of Experimental Psychology, 72, 346–354.

Savage, L. J. (1954): The Foundations of Statistics, John Wiley and Sons.

Shishkin, D. and P. Ortoleva (2019): “Ambiguous Information and Dilation: An Experiment,” working paper.

Slovic, P. and S. Lichtenstein (1971): “Comparison of Bayesian and regression approaches to the study of information processing in judgment,” Organizational Behavior & Human Performance, 6, 649–744.