Non-linear simulation of the aspherical deformation of piezo-glass membrane lenses including hysteresis and fabrication effects

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Abstract. In this paper we present and verify the non-linear simulation of an aspherical adaptive lens based on a piezo-glass sandwich membrane with combined bending and buckling actuation. To predict the full non-linear piezoelectric behavior, we measured the non-linear charge coefficient, hysteresis and creep effects of the piezo material and inserted them into the FEM model using a virtual electric field. We further included and discussed the fabrication parameters – glue layers and thermal stress – and their variations. To verify our simulations, we fabricated and measured a set of lenses with different geometries, where we found good agreement and show that their qualitative behavior is also well described by a simple analytical model. We finally discuss the effects of the geometry on the electric response and find, e.g., an increased focal power range from \(\pm 4.5\) to \(\pm 9\) m\(^{-1}\) when changing the aperture from 14 to 10 mm.

Keywords: adaptive fluid glass membrane lens, varifocal, spherical correction, simulation

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1. Introduction

Fluidic varifocal lenses provide a fast and efficient method to change the focal power in optical beam paths without mechanical movement, e.g. as used in novel adaptive scanning microscopes [1,2] or flow velocimetry [3]. There are two main types of adaptive lenses, that use different physical principles. The first class of lenses uses a controlled
change of the refractive index, such as lenses based on liquid crystals [4, 5] or acoustic pressure gradients [3]. The second type of lenses uses a change of the curvature between two media with different refractive index. Examples for this kind are glass- and polymer-membrane fluid lenses [7, 8, 9, 10], lenses based on dielectric electroactive polymers [11] and electrowetting lenses [12]. Each kind of lens type has its own advantages and disadvantages: Electrowetting lenses generally provide a very low wavefront error, but they are relatively slow [12]. Lenses with a polymer membrane and integrated actuation can offer a relatively compact setup but are yet not very fast, because of their soft membrane and a strong fluidic damping [13].

Correcting not only the defocus, but also spherical aberrations, requires a second device within the optical path/setup, e.g. deformable mirrors based on piezoelectric [14] or electrostatic actuation [15] or liquid crystal spatial light modulators [16].

The main advantage of our design [17] is that we are able to control both, the defocus and the spherical aberrations simultaneously. Using only one transmissive device instead of multiple transmissive and reflective devices, we are able to simplify the optical setup and make it more compact. The active element of our lens concept is an active piezo-glass sandwich membrane, where an ultra thin glass-membrane is glued in-between two piezo rings, which directly deform the membrane and hence change the focal length due to a transparent fluid (or polymer [18]) that is added below the membrane. Other piezo actuated glass membrane lenses, e.g. [9], can also change their focal length, but have only one degree of freedom and hence cannot correct aberrations.

In [17] we presented the basic concept of this lens concept with high resonance frequencies (>1 kHz) and a large aperture (12 mm clear aperture vs. 18 mm diameter). It achieved a focal power range of approximately ±4 m⁻¹ for a very compact design with the additional ability to tune the spherical behavior. In [19, 20] we increased the focal power range for the same geometric dimensions and materials to more than ±6 m⁻¹ by a modification of the actuators using an in-plane polarization and an induced pre-stress in the fabrication process. The aspherical behavior can be adjusted using two different actuation modes: On the one hand, the ”bending mode”, with one contracted and one expanded piezo ring, leads to a rather spherical deformation of the glass-membrane. On the other hand the so called ”buckling mode”, where both piezos contract, results in a more parabolic shape. Compared to polymer membrane fluid lenses, the stiff glass-membrane of our lens results in a short response time below 0.2 ms [21].

A similar configuration was later also used in [10] where the authors actuate a glass-membrane with a segmented piezo actuator to control also higher order aberrations such as astigmatism and coma. As the main purpose of this lens is the higher order aberration correction, they achieve only a focal power range of approximately ±0.5 m⁻¹ and operating frequencies of 200 Hz [22]. Similarly, their setup of using two glass membranes, two sets of actuators and an additional stiffening glass window is more bulky and complex than our configuration.

To fully understand the actuation principle, design lenses with desired properties or optimize the working range of the focal power and spherical tuning of the present lens, it
is essential to predict the expected surface deformation of the actuated glass-membrane. Hence, in this paper, we develop a numerical simulation using COMSOL Multiphysics to predict the deformation of the lens profile as a function of the applied electric signal. In particular, our combined bending and buckling actuation is intrinsically a non-linear effect, and we will also take into account the effects of the fabrication and of the non-linear response of the piezo material. The same actuator with a metal membrane instead of glass-membrane was later used in [23] for a micro pump system. The authors of that paper measured the radial contraction of the piezo rings and from that value predicted the center point deflection of the membrane. In contrast, we measured and modeled the full non-linear piezoelectric response of the material and studied not only the center deflection, but the entire surface deformation of the membrane.

In this paper we first describe the operating principle of the lens in detail in section 2 where we also show analytic approaches to approximate the voltage-dependend focal power depending on the geometric parameters. In section 3 we explain, how we set up a simulation and address issues such as hysteresis. Then, we describe the fabrication process and the measurements of the lens prototypes in sections 4 and 5. In section 6, we compare the results of the simulation to the measurement data. We finally conclude our results in section 7.

2. Operating principle and analytic approximation

The active part of the lens consists of two out-of-plane polarized piezo-rings that are bonded to an ultra thin glass-membrane as shown in figure 1a. Applying an electric field $E$ to the piezo rings leads to an induced strain and hence to a change of their diameter $D$ by

$$
\Delta D = d_{31} ED.
$$

The piezoelectric coefficient $d_{31}$ depends on the used piezo material and is also a function of the applied electric field [24]. If one applies opposite voltages to the upper and lower
piezo ring \( E_{\text{up}} = -E_{\text{low}} \), which leads to a expansion of one and a contraction of the other ring, the membrane deforms approximately spherically, as shown in figure 1 (b) as it is bent by the piezo rings at its outer boundary. In contrast, a contraction of both rings forces the glass-membrane to buckle out of the plane, leading to a more hyperbolic shape (figure 1 (c)).

To generate a lens effect, we combine the active piezo-glass-sandwich with an elastic fluid chamber, add a transparent fluid (paraffin oil, \( n = 1.48 \)) and seal the chamber with a glass substrate. The complete assembly is shown in figure 2.

![Figure 2: Cross-section through the lens model (to scale).](image)

As derived in [17], the curvature in the bending mode can be approximated by

\[
R^{-1} \approx s^{-1} d_{31} (E_{\text{up}} - E_{\text{low}}),
\]

(2)

where \( s \) is the distance of the neutral planes of the piezo sheets (see figure 1), roughly of the order of the membrane thickness plus one piezo thickness. In the (pure) buckling mode, the curvature for an inner piezo diameter of \( D_{\text{in}} \) will be

\[
R^{-1} \approx D_{\text{in}}^{-1} \sqrt{24 d_{31} \left( \frac{E_{\text{up}} + E_{\text{low}}}{2} \right)}.
\]

(3)

As a result, using the lensmaker’s equation for thin lenses, the focal power for the bending is

\[
f^{-1} = \Delta n R^{-1} \approx \Delta n s^{-1} d_{31} (E_{\text{up}} - E_{\text{low}}).
\]

(4)

Similarly, the focal power in the buckling mode is

\[
f^{-1} \approx D_{\text{in}}^{-1} \sqrt{24 d_{31} \left( \frac{E_{\text{up}} + E_{\text{low}}}{2} \right)}.
\]

(5)

The direction of the buckling can be chosen by first bending the membrane and then buckling it. All of these estimates only take into account the geometric changes and do not consider the forces in the piezo rings and the glass-membrane and including forces in sufficient detail would be hard to impossible on an analytic level. This is why we need reliable simulations to predict and optimize the behavior of the lens.
3. Simulation

We used *COMSOL Multiphysics* (Version 5.3a) with the multiphysics module “Piezoelectric effect” that combines the “Solid Mechanics” and the “Electrostatics” module to simulate the piezo electric deformation. The piezo electric material properties were set to the strain-charge form.

3.1. Simulation strategy

We activated the “geometric non-linearity” in the simulation as the buckling is an intrinsically non-linear effect. Because of its bistability, we simulated the lens at maximum bending deflection in both directions and used this as a starting point for the subsequent buckling simulation. Without this pre-deflection, the simulation does not show any buckling as an un-deflected ideal lens is in a metastable state that, however, is always distorted in practice. After verifying that the 2-dimensional rotationally symmetric model agrees with a full 3-dimensional model, we chose the former one as it reduces the computation time dramatically. As we have an enclosed volume inside the lens, we keep the volume in the fluid chamber constant using the ODE module with an internal pressure as a the control variable.

3.2. Lens model

We defined the geometry according to figure 3 with the following set of free parameters:

- Upper and lower piezo ring with inner and outer radii $D_{in}/2$ and $D_{out}/2$ and thickness $t_{piezo} = 105 \mu m$
- Ultra-thin glass-membrane with radius $D_{out}/2$ and thickness $t_{glass}$
- Glue layer between glass-membrane and piezo with thickness $t_{glue}$
- Glue edge with $t_{edge}$ and $w_{edge}$ at the corner, where the piezo rings are in contact with the inner part of the glass-membrane

The other dimensions of the elastic polyurethane fluid chamber that we keep constant are also shown in figure 3. We defined all mesh sizes to ensure at least two layers of mesh elements in any component of the lens. An example of the mesh is shown in figure 4.

3.3. Piezoelectric non-linearity

Because of the non-linearity of the piezoelectric charge coefficient $d_{31}$ and the hysteresis and creep effects, we did not simply use a constant $d_{31}$, but we needed to determine the charge coefficient as a function of the applied voltage and its voltage history. For this reason, we followed the protocol of [24] and manufactured a simple mono-morph bending beam with one $100 \mu m$ thick passive glass layer and one *Ekulit* piezo layer that we could simulate reliably. We applied the same electric fields as for the lens measurements (see section 5) to this beam, measured the beam curvature and compared it to the
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Figure 3: Half section of the model geometry with fully defined geometry parameters (rotated around the vertical dashed red line).

Figure 4: Example of the mesh and illustration of the boundary conditions. Inset: Detailed view of glue layer, glue edge and fluid chamber (polyurethane, red), of piezo rings (gray) and of the glass-membrane (blue).

simulation as done in [24]. That way, we found a modified electric field sequence shown in figure 5 that is needed as an input for the simulation to achieve the same curvature as the measured beam while using the standard matrix with \( d_{31} = 274 \text{ pm V}^{-1} \) for PZT-5H. This virtual electric field sequence now contains the non-linearity of the charge coefficient in addition to all hysteresis and creeping effects. This way we bypassed the need to implement \( d_{31} \) as a function of the electric field and furthermore to implement the unknown and complex hysteresis behavior, that is not natively implemented in COMSOL Multiphysics.

In figure 5 we find a hysteresis (a) from the asymmetry of the rising and falling slopes and short term creep (b) from the time-dependence when the physical voltage is fixed. We further find a voltage shift to positive values for the non-symmetric cycles (buckling, trajectory) which is most likely due to long-term creep (c).

3.4. Material properties and fabrication effects

The elastomer chamber is made from polyurethane Smooth-on ClearFlex 50 with a Young’s modulus of 2.22 MPa that we determined in a pull test at approximately 5% strain. We took the Young’s modulus of the Schott D263t glass-membrane \( E_{\text{glass}} \approx \)
Figure 5: Electric field applied in the measurement (solid line) and virtual (corrected) electric field applied in the simulation (dashed line) for the bending mode (left), the buckling mode (middle) and the working range trajectory (right).

72.9 GPa \[25\] and the Smooth-on CrystalClear 200 glue layer \( E_{CC200} \approx 1.38 \text{ GPa} \) from their material data sheets \[26\]. In \[24\], we determined the young’s modulus of the Ekulit PZT to \( E_{\text{piezo}} \approx 37 \text{ GPa} \).

We measured piezo thicknesses between 100 and 110 µm resulting in a mean value of \( t_{\text{piezo}} = 105 \mu m \). Similarly, the glue layers varied from 18 to 32 µm, so we used \( t_{\text{glue}} = 25 \mu m \). Besides that, we took into account a thermally induced pre-strain in the glass-piezo sandwich which results from the gluing process in the oven at a temperature of 50 °C and the laboratory room temperature of 24 ± 3 °C. This temperature difference of 26 °C leads to an internal stress in the membrane composite that effects its deflection. Again, we took the thermal expansion coefficient \( (\alpha_{\text{glass}} = 7.2 \times 10^{-6} \text{K}^{-1}) \) from the material data sheet \[25\]. The thermal expansion of the piezo is not given by the manufacturer, but similar PZT materials have coefficients in the range of 4 to \( 8 \times 10^{-6} \text{K}^{-1} \). We added this thermal initial strain to the piezo rings and to the glass-membrane in the simulation. After variation of the thermal expansion coefficient of the piezos in the simulation we found that the mean of the mentioned value range of \( 6 \times 10^{-6} \text{K}^{-1} \) fitted best to our experiments and was used further on. The effect of these parameters will also be discussed in detail in section 6.

4. Fabrication

As a piezo material, we detached the piezo sheets from Ekulit sound buzzers with the mentioned small thickness of \( t = 105 \mu m \) to achieve high focal powers referring to equation (4). They in fact showed better performance with a high piezoelectric coefficient of \( d_{31} \approx -487 \times 10^{-12} \frac{m}{V} \) \[24\] and a better surface quality than many directly available raw piezo foils. We structure the piezo rings and the 30, 50 and 70 µm thick
glass-membrane with an UV laser. At the edges of the piezo we additionally remove 100 µm of the electrode to avoid electric breakdown at high electric fields. To remove residues caused by the laser cutting process, we clean the piezo rings subsequently in an ultrasonic bath for 5 s in 25% HNO$_3$. After structuring, we glue the glass in between the piezo rings using hard polyurethane (CC200) and a system of vacuum chucks with alignment structures in an oven at 50°C. When cured, we glue this glass-piezo-sandwich to the elastic polyurethane fluid chamber (CF50), that we cast from a mold. We seal this chamber with the glass substrate using the same soft polyurethane as glue.

In this paper, we do not fill the fluid chamber with an optical oil, because we are interested in the purely mechanical deformation of the membrane. For this reason, all focal power calculations are based on a virtually filled lens, where we take into account the curvature of the lens and calculate the refractive power with an assumed refractive index of $n = 1.48$. The internal pressures that we found in the simulations were of the order of 20 Pa for maximum deflections. This pressure influences the curvature by a factor of approximately 3%, so we need to take it into account. Nevertheless, the compressibility of an ideal gas compared to an incompressible fluid in the chamber results in changes of less then 0.3% so that we can consider air and fluid to be approximately equivalent. After the gluing, the lenses are normally filled by inserting two syringes through the elastic material of the fluid chamber along two fluidically optimized channels designed for bubble-free filling. One syringe supplies the optical oil, the other one releases the air from the fluid chamber as shown in figure 6 a. Finally, we seal the holes by adding a drop of polyurethane. The finished lens (figure 6 b) is then packaged in a custom mount for the standard 30 mm cage system, shown in figure 6 c.

![Figure 6](image)

Figure 6: (a) Filling process of the lens, (b) close up of the fabricated lens and (c) packaged lens for the standard 30 mm cage system with electric contacts.

5. Characterization of focal power and aspherical behavior

We characterized the membrane deformation by measuring the surface with a chromatic confocal pen (Polytec, TopSens CL4/MG35) with a vertical accuracy of approximately 0.1 µm and a spot size of 12.3 µm and with a high-precision translation stage. To obtain the time-dependent surface profile, we preformed a pointwise measurement during a periodic actuation with a sensor frequency of 400 Hz.
We limited the electric field against the polarization to $-0.38 \text{ kV/mm}$, i.e., less than half of the coercive field strength of $0.78 \text{ kV/mm}$ \cite{24}. We similarly set a maximum electric field of $1.43 \text{ kV/mm}$ to avoid electrostatic breakdown.

We performed three different measurements that we will compare to the simulation results later. First, we applied opposite voltages for the upper and lower piezo ring to drive the lens in a pure bending mode with an amplitude of $0.38 \text{ kV/mm}$ with a symmetric sinusoidal wave with 1 Hz as a quasi static actuation. For the pure buckling mode, where we needed a simultaneous contraction of both rings, we applied a sinusoidal wave with 1 Hz between 0 and $1.43 \text{ kV/mm}$. In this case, however, we waited 30 min before starting a measurement with an overall measurement time of 120 min to avoid longtime creeping effects of the piezo.

To determine the full working range of the lens, we applied a voltage trajectory, where we cover several points of interest and outline the stable operating region. First, we pulled the membrane flat by expanding both piezo rings at $-0.38 \text{ kV/mm}$. Then, the voltage of the upper piezo was increased linearly to the maximum voltage ($1.43 \text{ kV/mm}$) to reach the maximum bending effect. For the maximum buckling in the next step, we increased the electric field of the lower piezo ring to the same value. Finally we lowered both electric fields to $-0.38 \text{ kV/mm}$, pulling the membrane flat again. For symmetry reasons we exchanged the upper and lower voltages and applied the cycle a second time. In principle, it is also possible to operate the lens in a metastable mode, where we bend it against the direction of the deformation in a buckled state. However for reasons of reliability we do not consider such modes in this paper. The entire trajectory takes 2 s. The voltages for all three measurement modes are shown in section 3.3.

The measurement data is then evaluated with a $4^{th}$ order rotationally symmetric fit:

$$z(x, y) = \alpha_0 + \alpha_{1,x} x + \alpha_{1,y} y + \alpha_2 r^2 + \alpha_4 r^4,$$

where $r = \sqrt{x^2 + y^2}$ with the lens center at $x = 0$ and $y = 0$. Using the lensmaker’s equation, the focal power is then

$$f^{-1} \approx 2 \Delta n \alpha_2.$$

Similarly, a measure for the aspherical behavior is given by the parameter $\alpha_4$. A conversion to Zernike polynomials can be realized by a straightforward linear transformation.

6. Results and discussion

6.1. Evaluation of measurement and simulation

To verify the simulation and to study the influence of geometric parameters, we fabricated a set of different lens designs, systematically varying the inner or outer diameter of the piezo rings and the glass-membrane thickness in comparison to our standard design with $D_{\text{out}} = 18 \text{ mm}$, $D_{\text{in}} = 12 \text{ mm}$ and $t_{\text{glass}} = 50 \mu\text{m}$. Table 1 gives an
overview over the different lenses. We evaluated the surface profile only over a diameter of 10.4 mm to avoid edge effects caused by glue that may flow onto the membrane as mentioned in section 3.2. For the lens with the inner diameter of 10 mm, we had to reduce the evaluated diameter slightly to 9.8 mm. In this case, there was only a small amount of glue causing a less then 100 µm glue edge, while for other lenses the glue edge was up to 400 µm wide.

Table 1: Overview over the measured lenses: The highlighted dimensions show the changed parameter and the color represents the corresponding color of the result graphs.

| Lens design | Outer piezo diameter | Inner piezo diameter | Glass-membrane thickness |
|-------------|-----------------------|----------------------|--------------------------|
|             | \(D_{\text{out}}\) / mm | \(D_{\text{in}}\) / mm | \(t_{\text{glass}}\) / µm |
| 1           | 18                    | 12                   | 50                       |
| 2           | 18                    | 12                   | 30                       |
| 3           | 18                    | 12                   | 70                       |
| 4           | 18                    | 10                   | 50                       |
| 5           | 18                    | 14                   | 50                       |
| 6           | 15                    | 12                   | 50                       |
| 7           | 21                    | 12                   | 50                       |

In the left graph of figure 7 we show the focal power as a function of the electric field \(E\) applied to the upper piezo ring for a pure bending deflection for the lenses 1 and 3. We clearly see an approximately linear behavior as predicted by (4) with a fitted slope of 1.02 and 1.06 V\(^{-1}\) compared to a prediction of 2.28 and 2.08 V\(^{-1}\) if we assume a distance between the neutral planes \(s\) of 205 µm for the 50 µm thick glass and 225 µm for the 70 µm thick glass as shown in table 2. This calculation neglects forces and only takes into consideration the deformation generated by the piezo rings. Up to a small negative pre-deflection in both lenses, the simulation reproduces the hysteresis behavior very well for the 70 µm membrane and has small deviations for the 50 µm membrane, most likely caused by the pre-deflection of the prototype and variations of the temperature and glue layer thickness.

Looking at the pure buckling mode the right graph of figure 7, we find an approximate square-root shape dependency, which was predicted by (5) for the measurement and the simulation. The simulation of the 50 µm membrane agrees in a wide range to the measurement, whereas the 70 µm shows a small shift in the offset voltage, which corresponds to an internal stress, e.g. due to a deviation in the curing temperature.

Furthermore, we find a smooth transition from a non-buckled state to the buckled state in the measurement. For the simulation we find a sudden onset of the buckling between 0 and 0.1 kV / mm for the 50 µm membrane and between 0.5 and 0.8 kV / mm for the 70 µm membrane. The difference in the onset of buckling may be due to the fact that the simulation assumes an initial perfectly flat membrane, which is not the case in a real system due to a small pre-deflection or unevenness in the membrane after gluing. The dashed line shows the fit of the function \(f^{-1} = a\sqrt{E - E_0}\) comparing the measurement
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Figure 7: Simulated (solid) and measured (dotted) focal power as a function of the applied electric field (top electrode) for the pure bending mode (left) and the pure buckling mode (right): Lens with $D_{\text{out}} = 18$ mm, $D_{\text{in}} = 12$ mm and $t_{\text{glass}} = 50\,\mu\text{m}$ (black) and $t_{\text{glass}} = 70\,\mu\text{m}$ (green). Fit according to analytical model shown as dashed line.

to the analytic evaluation in [5] and introducing an offset electric field $E_0$ as a starting point for the buckling. The fit was evaluated from the central third of the measurement to neglect the transition zone and the zone where high forces dominate. The results given in table 2 match again within a factor of approximately 2. The significant difference in the offset electric field results from the larger buckling threshold load in the thicker membrane. The slightly negative $E_0$ for the thin membrane results from the combination of thermal pre-stress, the long-term creep in the piezo (see 5) and the critical buckling load of the membrane.

In figure 8, we show the working range of the focal power and aspherical coefficient for the voltage trajectory described in section 3 and the corresponding simulation result. Again, the simulation reproduces the behavior of these two different designs reasonably well with a small asymmetry towards positive focal powers and smaller aspherical parameters for both membranes. We also fabricated a lens with a 30 $\mu\text{m}$ thick membrane, but it showed a very strong pre-deflection of $0.35\,\text{m}^{-1}$ and also a strong asymmetry as the thin membrane is difficult to handle and is more affected by uneven piezo sheets and fabrication imperfections, so we did not include it in the data analysis. However, we see in fig. 8 that it promises a great improvement in the aspherical tuning region.

Reducing the inner diameter on the other hand increases the maximal focal power range as shown in figure 9 in agreement with [5] (see table 2). The deviation between the analytical estimate and the experimental result decreases with increasing inner diameter (and fixed outer diameter) as the amount of piezo material increases and hence the effect of the glass-membrane stiffness decreases. There is also a small increase in the offset electric field for decreasing inner diameters, as smaller inner diameters result in a smaller aspect ratio of the glass-membrane and hence more resistance to buckling. We define the aspect ratio as the width of the glass-membrane $D_{\text{in}}$ divided by its thickness $t_{\text{glass}}$. The simulation matches very well for all the designs taking into
account the asymmetry of the measurements. In the trajectory on the right graph, we see that, while the focal power increases with smaller \( D_{in} \), the aspherical tuning range shifts to more hyperbolic values. This comes from the fact that the wider piezo rings resist more the bending deformation of the glass-membrane, so the lens profile becomes more hyperbolic. While we see a relatively strong deviation for the smallest aperture, this one also had some fabrication asymmetry (vertical offset).

Figure 9: Comparison of measurement (dotted) and simulation (solid) for a varying inner diameter \( D_{in} \) of the piezo rings (10, 12 and 14 mm). Focal power in the buckling mode (left) and aspherical parameter and focal power in the maximal trajectory (right).
Finally, when changing outer diameter (figure 10), we find that a reduction of the outer diameter, i.e., a smaller amount of active piezo material causes less available force and therefore less buckling deflection (magenta). Secondly, while the trend is not as clear as for the different inner diameters, the wider rings seem to result also in larger aspherical parameters, i.e., a more hyperbolic behavior (blue).

Figure 10: Comparison of measurement (dotted) and simulation (solid) for a varying outer diameter $D_{\text{out}}$ of the piezo rings (15, 18 and 21 mm).

In table 2, we show an overview of the fitted parameters. We see that the linear fit of the bending mode and the analytical estimate deviate by a factor of approximately 2, which may be due to the fact that we did not take into account forces and assumed a spherical deformation. We further assume in all estimates a typical large-field charge coefficient $d_{31} = -4.87 \times 10^{-12} \text{ m/V}$ [23], which does not apply for small and negative electric fields. The different geometries show relatively little variations, with the exception of lens 7 which in fact had an atypical asymmetric behavior in the bending mode with a large pre-deflection. The relatively (compared to the estimate) large coefficient of the 70 µm membrane may result from a better transfer of the deformation from the boundary to the center in the thicker membrane and the smaller curvature of the 14 mm lens may have the same reason, that the lens is bent at the side but remains relatively flat at the center.

We have a similar approximate factor in the buckling coefficient. Here, we see the clear correlation with the inner diameter as predicted by (5) and a small trend towards larger coefficients, the wider the piezo ring becomes ($(D_{\text{out}} - D_{\text{in}})/2$).

The offset voltage shows first of all negative values, which are simply due to the strain in the piezo caused by the long-term creep (figure 5). The various values can then be explained with two effects: A smaller aspect ratio of the membrane (lenses 3 and 4) results in an increased buckling threshold as expected from plate theory. Secondly, an increasing amount of piezo material (compare lens 1 to lenses 6 and 7) decreases the offset because it creates larger forces to overcome the critical buckling load.
Table 2: Overview over the fitted coefficients of the measured lenses

| Lens | \( D_{\text{out}}/D_{\text{in}}/t_{\text{glass}} \) in \( \text{mm/mm/µm} \) | Bending analytical \( 1/\text{mV} \) | Bending fit \( 1/\text{mV} \) | Buckling analytical \( \sqrt{1/\text{mV}} \) | Buckling fit \( \sqrt{1/\text{mV}} \) | \( E_0 \) fit \( 1/\text{mm} \) |
|------|----------------|--------|--------|--------------------|--------------------|--------|
| 1    | 18 / 12 / 50   | 2.28   | 1.02   | 9.01               | 5.06               | -0.07  |
| 2    | 18 / 12 / 30   | 2.53   | -      | 9.01               | -                 | -      |
| 3    | 18 / 12 / 70   | 2.08   | 1.06   | 9.01               | 5.77               | 0.75   |
| 4    | 18 / 10 / 50   | 2.28   | 0.99   | 10.81              | 7.92               | 0.09   |
| 5    | 18 / 14 / 50   | 2.28   | 0.89   | 7.72               | 3.85               | -0.30  |
| 6    | 15 / 12 / 50   | 2.28   | 1.02   | 9.01               | 5.22               | 0.12   |
| 7    | 21 / 12 / 50   | 2.28   | 0.69   | 9.01               | 5.60               | -0.12  |

6.2. Effects of the fabrication

In the above simulations, we included some estimated mean fabrication parameters: The thickness of the glue layer, the glue flowing onto the membrane and the thermal stress of the gluing process. As they cannot be controlled perfectly, it is important to know how they affect the behavior of the lens.

In figure 11 we find that a thinner glue layer results in a higher focal power in the bending mode as expected from (4) where a thinner glue layer reduces the distance \( s \) of the piezos. It also increases the aspherical tuning range for small focal powers. On the other hand, a thinner glue layer decreases the focal power in the buckling mode and also decreases the working range for the focal power and the aspheric tuning (see figure 11 right).

Figure 11: Effect of glue layer thickness variation for the bending mode (left), buckling mode (center) and aspherical working range (right).

In figure 12 we find that an increase of the glue edge, caused by additional glue flowing out of the glue layer while gluing, has nearly no effect on the bending mode, but it increases the focal power of the buckling mode as a wider glue layer has a similar effect as a decrease of the inner piezo diameter \( D_{\text{piezo}} \), resulting in a higher focal power by (5). There is similarly a small increase in the aspherical tuning range with increasing
glue edge as shown in the right graph of figure 12. When taking into consideration that the lens is a full 3-dimensional setup, an asymmetry of the glue edge also results in an asymmetry of the surface curvature and therefore higher aberrations.

![Figure 12: Effect of the glue edge (value of width and height) for the bending mode (left), buckling mode (center) and aspherical working range (right).](image)

Finally, in figure 13 we find that a higher difference between the thermal expansion coefficients of the thin glass ($\alpha_{\text{glass}} = 7.2 \times 10^{-6} \text{K}^{-1}$) and the piezo (varied between $5.4 \times 10^{-6}$ and $7.2 \times 10^{-6} \text{K}^{-1}$) or a higher curing temperature leads to a reduction of the focal power in the bending mode by over 30%. Furthermore, the buckling deflection is reduced as well, in particular due to an increasing buckling threshold. As the same effect occurs also in variations of the operating temperature, it may hence be very important to find a glass/piezo combination with similar expansion coefficients.

![Figure 13: Effect of the thermal stress caused by different thermal coefficients and a curing temperature 26° above the measurement temperature for the bending mode (left), buckling mode (center) and aspherical working range (right).](image)

To identify the critical dimensions combinations we simulated the buckling onset depending on the aspect ratio (figure 14 left) and the focal power in the buckling mode for different piezo ring widths (figure 14 right). In both cases we ignored hysteresis and assumed a constant $d_{31} = -487 \times 10^{-12} \text{m V}^{-1}$. We find, as expected,
that the onset of buckling decreases with increasing aspect ratio as approximately \( E_0 \approx 3.14 \times 10^5 \frac{(d/t)^{-2} \text{ kV}}{\text{mm}} \). Hence, one needs to have a minimum aspect ratio of approximately \( d \gtrsim 100t \). Looking at the piezo width on the right of figure 14, we see that the effect of the increasing strength of the piezo starts to saturate once the radial cross section of the piezo, \( 2 t_{\text{piezo}} (D_{\text{out}} - D_{\text{in}}) \), equals the radial cross section of the glass membrane \( t_{\text{glass}} D_{\text{out}} \).

Figure 14: Left: Buckling onset \( E_0 \) as a function of the aspect ratio, simulated at a fixed piezo width with \( D_{\text{out}} = 18 \text{ mm} \) and aperture \( D_{\text{in}} = 12 \text{ mm} \). Right: Maximum focal power in the buckling mode as a function of the width of the piezo ring, \( (D_{\text{in}} - D_{\text{in}})/2 \), for a 50 \( \mu \text{m} \) glass membrane and 12 mm aperture.

7. Summary and conclusions

We successfully demonstrated the simulation of a non-linear piezo bending and buckling actuator at the example of a varifocal adaptive lens with spherical correction. We included the non-linear charge coefficient of the piezo into our simulation, including hysteresis and creep effects, by first measuring the curvature of a well-defined cantilever actuator and then modifying the voltage in the simulation accordingly. We found that the simplified analytical approach based on geometric deformation and neglecting forces matched our measured results within a factor of 2 and described the qualitative behavior.

As overall conclusions for the design, we find that in general, smaller apertures inversely proportionally increase the focal power range and thinner membranes increase the aspherical operating region. As we found that the buckling offset voltage scales quadratically with the aspect ratio of the membrane, thinner membranes also enable smaller apertures. The width of the piezo rings had relatively little effect, provided that they are wide enough to cause buckling, and there was in general little effect on the bending mode.

Furthermore, we analyzed the effects of fabrication uncertainties to develop a realistic simulation and provide the basis for optimizations of the lens. We found that changes in glue edges have a relatively small influence, that appears only in the large buckling displacement. The glue layer caused a small decrease in the focal power and
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aspherical tuning range in the bending mode, so one should try to minimize it, if possible. The thermal stress due to elevated curing temperatures, however, significantly affects the behavior: It greatly reduces the focal power in the bending mode and causes a small increase in the buckling onset, but also a small improvement in the aspherical tuning range at small fields. As temperature variations during operation have the same effect, it will be important to find material combinations with similar thermal expansion coefficients.

Based on these findings, we will in the future aim to improve the fabrication process to reliably fabricate lenses with a membrane thickness of only 30 µm, reduced thermal stress and minimal glue layer. To further modify the operating region, we will in the future need to study the effects of the lens chamber, that may be used to adjust the counter pressure and the mechanics of the lens.

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References

[1] N. Koukourakis, M. Finkeldey, M. Stürmer, C. Leithold, N. C. Gerhardt, M. R. Hofmann, U. Wallrabe, J. W. Czarske, and A. Fischer. Axial scanning in confocal microscopy employing adaptive lenses (cal). Opt. Express, 22(5):6025–6039, Mar 2014.
[2] K. Philipp, A. S, N. Koukourakis, A. Fischer, M. Stürmer, U. Wallrabe, and J. W Czarske. Volumetric hilo microscopy employing an electrically tunable lens. Opt. Express, 24(13):15029–15041, Jun 2016.
[3] J. Czarske, H. Radner, C. Leithold, and L. Bttner. Smart laser interferometer with electrically tunable lenses for flow velocity measurements through disturbing interfaces. Photonics, 2:1–12, 03 2015.
[4] Susumu Sato. Liquid-crystal lens-cells with variable focal length. Japanese Journal of Applied Physics, 18(9):1679, 1979.
[5] Nabeel A. Riza and Michael C. DeJule. Three-terminal adaptive nematic liquid-crystal lens device. Opt. Lett., 19(14):1013–1015, Jul 1994.
[6] A. Mermillod-Blondin, E. McLeod, and C. B. Arnold. High-speed varifocal imaging with a tunable acoustic gradient index of refraction lens. Opt. Lett., 33(18):2146–2148, Sep 2008.
[7] A. Pouydebasque, C. Bridoux, F. Jacquet, S. Moreau, E. Sage, D. Saint-Patrice, C. Bouvier, C. Kopp, G. Marchand, S. Bolis, N. Sillon, and E. Vigier-Blanc. Varifocal liquid lenses with integrated actuator, high focusing power and low operating voltage fabricated on 200mm wafers. Sensors and Actuators A: Physical, 172(1):280 – 286, 2011. Euroensors XXIV, Linz, Austria, 5-8 September 2010.
[8] F. Schneider, J. Draheim, R. Kamberger, P. Waibel, and U. Wallrabe. Optical characterization of adaptive fluidic silicone-membrane lenses. Opt. Express, 17(14):11813–11821, Jul 2009.
[9] K. H. Haugholt, D. T. Wang, F. Tyholt, W.E. Booij, and I. Johansen. Polymer lens, us 8199410 b2, granted patent. https://lens.org/035-398-745-324-177
[10] S. Bonora, Y. Jian, P. Zhang, A. Zam, E. N. Pugh, R. J. Jawadzki, and M. V. Sarunic. Wavefront
Non-linear simulation of the aspherical deformation of piezo-glass membrane lenses

correction and high-resolution in vivo oct imaging with an objective integrated multi-actuator
adaptive lens. *Opt. Express*, 23(17):21931–21941, Aug 2015.

[11] L. Maffli, S. Rosset, M. Ghilardi, F. Carpi, and H. Shea. Ultrafast allpolymer electrically tunable
silicone lenses. *Advanced Functional Materials*, 25(11):1656–1665.

[12] B. Berge and J. Peseux. Variable focal lens controlled by an external voltage: An application of
electrowetting. *The European Physical Journal E*, 3(2):159–163, Oct 2000.

[13] J. Draheim, F. Schneider, R. Kamberger, C. Mueller, and U. Wallrabe. Fabrication of a fluidic
membrane lens system. *Journal of Micromechanics and Microengineering*, 19(9):095013, 2009.

[14] M. Shaw, S. Hall, S. Knox, R. Stevens, and C. Paterson. Characterization of deformable mirrors for
spherical aberration correction in optical sectioning microscopy. *Opt. Express*, 18(7):6900–6913,
Mar 2010.

[15] L. Sherman, J. Y. Ye, O. Albert, and T. B. Norris. Adaptive correction of depth-induced aberrations
in multiphoton scanning microscopy using a deformable mirror. *Journal of Microscopy*,
206(1):65–71.

[16] C. Maurer, A. Jesacher, S. Bernet, and M. Ritsch-Marte. What spatial light modulators can do
for optical microscopy. *Laser & Photonics Reviews*, 5(1):81–101.

[17] M. C. Wapler, M. Strmer, and U. Wallrabe. A compact, large-aperture tunable lens with adaptive
spherical correction. In *2014 International Symposium on Optomechatronic Technologies*, pages
130–133, Nov 2014.

[18] M. C. Wapler, C. Weirich, M. Strmer, and U. Wallrabe. Ultra-compact, large-aperture solid
state adaptive lens with aspherical correction. In *2015 Transducers - 2015 18th International
Conference on Solid-State Sensors, Actuators and Microsystems (TRANSUDCERS)*, pages 399–
402, June 2015.

[19] F. Lemke, M. Stürmer, U. Wallrabe, and M. C. Wapler. Topological in-plane polarized piezo
actuation for compact adaptive lenses with aspherical correction. *ACTUATOR 2016, 15th
International Conference on New Actuators*, 2016.

[20] F. Lemke, Y. Frey, U. Wallrabe, and M. C. Wapler. Pre-stressed piezo bending-buckling actuators
for adaptive lenses. In *ACTUATOR 2018; 16th International Conference on New Actuators*,
pages 1–4, June 2018.

[21] M. C. Wapler and U. Wallrabe. Ultra-fast and compact varifocal lens. *Accepted at MEMS 2019*.

[22] D. Magrin, P. Favazza, S. Bonora, M. Quintavalla, M. Bergoni, and R. Ragazzoni. Multi-actuator
adaptive lens in astronomy: in lab test results. *volume 10703*, pages 10703 – 10703 – 7, 2018.

[23] A. Shabanian, F. Goldschmidtboeing, S. Vilches, H.-H. Phan, A. Bhat Kashekodi, P. Rajaelpour,
and P. Woias. A novel piezo activated high stroke membrane for micropumps. *Microelectronic
Engineering*, 158:26 – 29, 2016. Micro and Nano Technologies for Biology and Life Sciences.

[24] B. P. Bruno, A. Fahmy, M. Stürmer, U. Wallrabe, and M. C. Wapler. Properties of piezoceramic
materials in high electric field actuator applications. *Smart Materials and Structures*, 2018.

[25] Schott. Glass wafer specification. [https://www.schott.com/d/advanced_optics/7af2454e-555a-4071-9590-bb7b32e75fec/schott-glass-wafer-specification-english-28062018.pdf](https://www.schott.com/d/advanced_optics/7af2454e-555a-4071-9590-bb7b32e75fec/schott-glass-wafer-specification-english-28062018.pdf)

[26] Smooth-On. Crystal clear series. [http://www.smooth-on.com/tb/files/CRYSTAL_CLEAR_200_TB.pdf](http://www.smooth-on.com/tb/files/CRYSTAL_CLEAR_200_TB.pdf)