Detrimental Decoherence

Gil Kalai*
Hebrew University of Jerusalem and Yale University

June 15, 2008

Abstract

We propose and discuss two conjectures on the nature of information leaks (decoherence) for quantum computers. These conjectures, if (or when) they hold, are damaging for quantum error-correction as required by fault-tolerant quantum computation.

The first conjecture asserts that information leaks for a pair of substantially entangled qubits are themselves substantially positively correlated.

The second conjecture asserts that in a noisy quantum computer with highly entangled qubits there will be a strong effect of error synchronization.

We present more general conjectures for arbitrary noisy quantum systems.

*Research supported in part by an NSF grant, an ISF grant, and a BSF grant. This paper is a revision and an extension of the formal part of [28]. I am grateful to Dorit Aharonov, Michael Ben-Or, Greg Kuperberg, and Robert Raussendorf for fruitful discussions, and to many colleagues for helpful comments.
1 Introduction

Quantum computers are hypothetical devices based on quantum physics. A formal definition of quantum computers was pioneered by Deutsch [1], who also realized that they can outperform classical computation. Perhaps the most important result in this field and certainly a major turning point was Shor’s discovery [2] of a polynomial quantum algorithm for factorization. The notion of a quantum computer along with the associated complexity class BQP has generated a large body of research in theoretical and experimental physics, computer science, and mathematics. For background on quantum computing, see Nielsen and Chuang’s book [3].

Of course, a major question is whether quantum computers are feasible. An early critique of quantum computation (put forward in the mid-90s by Landauer [4, 5], Unruh [6], and others) concerned the matter of noise:

[N] The postulate of noise: Quantum systems are noisy.

The foundations of noisy quantum computational complexity were laid by Bernstein and Vazirani in [7]. A major step in showing that noise can be handled was the discovery by Shor [8] and Steane [9] of quantum error-correcting codes. The hypothesis of fault-tolerant quantum computation (FTQC) was supported in the mid-90s by the “threshold theorem” proved by Aharonov and Ben-Or [10], Kitaev [11], Knill, Laflame, and Zurek [12], and Gottesman [13]. The threshold theorem asserts that under certain natural assumptions of statistical independence on the noise, if the rate of noise (the amount of noise per qubit in one computer cycle) is a small constant, then FTQC is possible. It was also proved that high-rate noise is an obstruction to FTQC.

Footnote:
1 The idea of a quantum computer can be traced back to works by Feynman, Manin, and others, and this development is also related to reversible computation and connections between computation and physics that were studied by Bennett in the 1970s.
The study of quantum error-correction and its limitations, as well as of various approaches to fault-tolerant quantum computation, is extensive and beautiful; see, e.g., [14, 13, 15, 16, 17, 18, 19].

Concerns about noise models with statistical dependence are mentioned in several places, e.g., [21, 22], and specific models of noise that may be problematic for quantum error-correction were studied by Alicki, Horodecki, Horodecki, and Horodecki [23]. Current FTQC methods apply to even more general models of noise than those first considered, which allow various forms of time- and space-statistical dependence; see [24, 25, 26].

The purpose of this paper is to present two conjectures concerning decoherence for quantum computers which, if (or when) true, are damaging for quantum error-correction and fault-tolerance.

The first conjecture concerns entangled pairs of qubits.

[A] A noisy quantum computer is subject to error with the property that information leaks for two substantially entangled qubits have a substantial positive correlation.

We emphasize that Conjecture [A] refers to part of the overall error affecting a noisy quantum computer. We refer to this error as detrimental. Other forms of errors and, in particular, errors consistent with current noise models may also be present. (We conjecture that the effects of detrimental errors described by Conjectures [B] and [C] below cannot be remedied by additional errors of a different nature.)

Error synchronization refers to a situation where, while the error rate is small, there is a substantial probability of errors affecting a large fraction of qubits.

[B] In any noisy quantum computer at a highly entangled state there will be a strong effect of error synchronization.

We will now describe the structure of the paper. Section 2 gives more background on noise and fault-tolerance. In Section 3 we define a simple
class of errors that suffice to demonstrate our conjectures and to show the connection between Conjectures [A] and [B].

The main Section 4 is devoted to mathematical formulations of the above conjectures and we also discuss why these conjectures are damaging to quantum error-correction. In the Appendix, stronger versions of Conjecture [A] are formulated.

Section 5 discusses examples that may give the conjectured behavior and actual models of noise that may demonstrate this behavior. We first discuss error propagation of “unprotected quantum programs” as a prototype for detrimental errors. A main thesis of this paper is that “new errors” in the evolution of a quantum computer are modeled after the behavior of unprotected quantum circuits.

Section 6 discusses related aspects of computational complexity. While our conjectures appear to be damaging to current fault-tolerance methods based on quantum error correction, it is still plausible that under the assumption of low-rate errors conjectures [A] and [B] will allow log-depth quantum computation.

Section 7 discusses extensions of these conjectures to more general quantum systems and briefly touches on relations with classical noisy systems. Section 8 discusses the rate of noise. Section 9 concludes.

The conjectures of this paper can be regarded as proposed properties for error models for quantum computers (and, at a later stage, for more general quantum systems) that will cause quantum error-correction and FTQC to fail. Alternatively, the conjectures can be regarded as proposed consequences of lack of fault-tolerance in quantum systems. As such they can be relevant to the nature of decoherence of quantum physical systems in nature even if quantum computers are possible. At present, there are no clear examples of quantum error-correction or of quantum fault-tolerance in quantum processes in nature.

We list now (again in an informal manner) additional conjectures made
in this paper.

1. (Section 4): We will refer to a pure state of a quantum computer that up to a small error is induced by its “marginal distribution” on small sets of qubits as “approximately local.” (See Section 4 for a formal definition.)

[C] The states of noisy quantum computers are approximately local.

2. The next conjecture proposes an extension of the conjectures above to general quantum systems (Section 7).

[D] A description (or prescription) of a noisy quantum system at a state $\rho$ is subject to error given by a quantum operation $E$ that tends to commute with every unitary operator that stabilizes $\rho$.

(Here, “tends to commute” reflects a small bias towards commutativity which will be motivated and described further in Section 7.)

3. (Section 8) The following further conjecture has some bearing also on the rate of noise and can be regarded as a strong form of the postulate of noise itself.

[E.1] A noisy quantum system is subject to (detrimental) noise with the following property: the infinitesimal rate of noise at time $t$ (in terms of trace distance) is bounded from below by a measure of non commutativity between the operators describing the evolution prior to time $t$ and those describing it after time $t$.

[E.2] A noisy quantum computer with $n$ (logical) qubits is subject to (detrimental) noise, with the following property: For some $\kappa > 0$, the infinitesimal rate of noise (in terms of trace distance) is at least $\kappa n$.

For standard models of noise, the infinitesimal rate of noise in terms of trace distance scales up linearly with the number of qubits. However, in cases of highly entangled states, which, by Conjecture [B], would lead to
error-synchronization, conjecture [E.2] suggests that the rate of detrimental noise for individual qubits will scale up linearly with the number of involved qubits. Conjecture [E.2] runs counter to the possibility of “decoherence free subspaces,” which are possible for the standard noise models.

2 Quantum computers, noise, and fault-tolerance

The state of a digital computer having \( n \) bits is a string of length \( n \) of zeros and ones. As a first step towards quantum computers we can consider (abstractly) stochastic versions of digital computers where the state is a (classical) probability distribution on all such strings. Quantum computers are similar to these (hypothetical) stochastic classical computers and they work on qubits (say \( n \) of them). The state of a single qubit \( q \) is described by a unit vector \( u = a|0 > + b|1 > \) in a two-dimensional complex space \( U_q \). (The symbols \( |0 > \) and \( |1 > \) can be thought of as representing two elements of a basis in \( U_q \).) We can think of the qubit \( q \) as representing ‘0’ with probability \( |a|^2 \) and ‘1’ with probability \( |b|^2 \). The state of the entire computer is a unit vector in the \( 2^n \)-dimensional tensor product of these vector spaces \( U_q \)'s for the individual qubits. The state of the computer thus represents a probability distribution on the \( 2^n \) strings of length \( n \) of zeros and ones. The evolution of the quantum computer is via “gates.” Each gate \( g \) operates on \( k \) qubits, and we can assume \( k \leq 2 \). Every such gate represents a unitary operator on \( U_g \), the \( (2^k\)-dimensional) tensor product of the spaces that correspond to these \( k \) qubits. At every “cycle time” a large number of gates acting on disjoint sets of qubits operate.

Moving from a qubit \( q \) to the probability distribution on ‘0’ and ‘1’ that it represents is called a “measurement” and it can be considered as an additional 1-qubit gate. We will assume that measurements of qubits that amount to a sampling of 0-1 strings according to the distribution these qubits represent is the final step of the computation.
The postulate of noise is essentially a hypothesis about approximations. The state of a quantum computer can be prescribed only up to a certain error. For FTQC there is an important additional assumption on the noise, namely, on the nature of this approximation. The assumption is that the noise is “local.” This condition asserts that the way in which the state of the computer changes between computer steps is statistically independent, for different qubits. We will refer to such changes as “storage errors” or as “qubit errors.” In addition, the gates that carry the computation itself are imperfect. We can suppose that every such gate involves a small number of qubits and that the gate’s imperfection can take an arbitrary form, and hence the errors (referred to as “gate errors”) created on the few qubits involved in a gate can be statistically dependent. Of course, qubit errors and gate errors propagate along the computation.

The basic picture we have of a noisy computer is that at any time during the computation we can approximate the state of each qubit only up to some small error term $\epsilon$. Nevertheless, under the assumptions concerning the errors mentioned above, computation is possible. The noisy physical qubits allow the introduction of logical “protected” qubits that are essentially noiseless.

Our conjectures apply to the same model of quantum computers but they require a more general notion of errors. They require that the storage errors will not be statistically independent (in fact, they should instead be very dependent) or that the gate errors will not be restricted to the qubits involved in the gates and will be of sufficiently general form. (Note that the errors may also reflect the translation from this ideal notion of quantum computers to a physical realization.)
3 A simple form of error

3.1 Simple forms of error

Let $W_k$ be the error operation that amounts to changing the state of the $k$th qubit to $\rho_0$, the maximum entropy state. Let $I_k$ be the identity operation for the $k$th qubit. For a 0-1 vector $x = (x_1, x_2, \ldots, x_n)$ let $E_x = \bigotimes_{k=1}^n E_k$ where $E_k = W_k$ if $x_k = 1$, and $E_k = I_k$ if $x_k = 0$.

Let $\mathcal{D}$ be a probability distribution on 0-1 vectors of length $n$. We let $E_\mathcal{D} = \sum \mathcal{D}(x)E_x$ where the sum is taken over all 0-1 vectors $x$ of length $n$. We will refer to errors of the form $E_\mathcal{D}$ as simple.

Simple errors are useful in demonstrating the notions we discuss. The standard assumption regarding storage noise would mean, in this special case, that $\mathcal{D}$ is a product probability distribution.

A case of particular interest is when the probability distribution $\mathcal{D}(x)$ depends only on the number of ‘1’ s in the vector $x$. Another way to describe this special case is to consider the error (regardless of the number $n$ of qubits) as depending on a probability distribution $f$ on $[0, 1]$. We first choose a real number $t$, $0 \leq t \leq 1$, according to the distribution $f$ and then, for every $k$, change the state of the $k$th qubit to $\rho_0$, with probability $t$ (independently for different qubits).

3.2 The amount of error, error synchronization, and correlation

A measure for the amount of error (or information leak) for the $k$th qubit for the error operation $E_\mathcal{D}$ is just $p_k(\mathcal{D})$, defined as the probability that $x_k = 1$. For the special case $E_f$ the amount of error for every qubit is $R(f) = \int_0^1 f(t)dt$.

It is simple to describe the notion of error synchronization in this setting. For errors of the form $E_f$, error synchronization refers to a situation where
\( R(f) \) is small, but for some \( t >> R(f) \), \( \int^1_t f(t) dt \) is substantial. When the error is described by \( E_D \), error synchronization refers to the situation where \( p_k(D) \leq t \) for every \( k \) and a small real number \( t \), but the probability for \( x = (x_1, x_2, \ldots, x_n) \in D \), with \( x_1 + x_2 + \cdots + x_n > sn \), is substantial for \( s >> t \).

For a probability distribution \( D \) and two qubits \( j \) and \( k \) let \( c_{jk}(D) \) be the correlation between the events \( x_j = 1 \) and \( x_k = 1 \). When we consider a block of qubits representing an error correction code, then (under some additional assumptions see; Section 4) Conjecture \([A]\) asserts that these pairwise correlations are high. For error operations of the form \( E_D \) it is easy to deduce error synchronization from high pairwise correlation:

**Lemma 1** Let \( t < 1/20 \) and \( s > 4t \). Suppose that \( D \) is a distribution of 0-1 strings of length \( n \) such that the \( p_i(D) \geq t \) and \( c_{ij}(D) \geq s \). Then

\[
\text{Prob}\left( \sum_{i=1}^n x_i > sn/2 \right) > st/4.
\]  

**Proof:** It is easy to see that the probability in question is minimized if \( D \) is symmetric with respect to permutations of the variables. Suppose that for such a symmetric distribution we want to minimize the pairwise correlation subject to the condition that the probability that \( x_1 + x_2 + \cdots + x_n > s/2 \) is at most \( st/4 \), and the probability that every \( x_i = 1 \) is at most \( t \). It is easy to see that we should choose the following distribution: with probability \( p_1 = ts/4 \) all \( x'_i \)s are 1. With probability \( p_2 \) we do the following: we choose uniformly at random a set \( R \) of size \( \lceil sn/2 \rceil \) and let \( x_i = 1 \) if \( i \) belongs to this set. We choose \( p_1 \) and \( p_2 \) so that the probability for \( x_i = 1 \) is \( t \). Finally, with probability \( 1 - p_1 - p_2 \) we choose all \( x_i = 0 \). Now, slightly modify this distribution as follows: with probability \( st/4 \), choose \( x_i = 1 \) for every \( i \); with probability \( 1 - 2t/s + t/2 - st/4 \), choose \( x_i = 0 \) for every \( i \); and with probability \( 2t/s - t/2 \), choose, (independently for different \( i \)s,) \( x_i = 1 \) with probability \( s/2 \).
The probability of each individual $x_i$ being 1 is still $t$ and the pairwise correlations did not decrease. And now we can easily compute the pairwise correlation and they turn out to be smaller than $s$. (As pointed out by Yuval Peres, this type of lemma falls under the known analysis of the Curie-Weiss model [29].

4 A mathematical formulation

In this section we give mathematical formulations for Conjectures [A], [B], and [C]. Our setting is as follows. We have a quantum computer running on $n$ qubits. The ideal (or “intended”) state of the computer is pure. We want to propose a picture for noisy quantum computation based on this basic model of a (noiseless) quantum computer. We assume that the actual state of the computer is close to the ideal state $\rho$. Our conjectures refer to the “new errors” (storage and gate errors) in one computer cycle.

The error can be described by a unitary operator on the computer qubits and the neighborhood qubits or by a quantum operation $E$ on the space of density matrices for these $n$ qubits. We will not give a specific model of detrimental error but rather describe some of its expected properties.

4.1 Two qubits

We first describe a measure of information leak. For a state $\rho$ of the computer and a set $A$ of qubits let $\rho|_A$ be the induced state on $A$.

Consider a quantum operation $E$. Note that when the state $\tau$ of the quantum computer is a tensor product pure state then for every set $A$ of qubits, $S(\tau|_A) = 0$. Here, $S(\tau)$ is the (von Neumann) entropy function; see, e.g., [3], Ch. 11. The information leak of the noise operator $E$ from the set of qubits $A$, w.r.t. $\tau$, can be measured by the entropy $S(E(\tau)|_A)$. For a tensor product state $\tau$ and a qubit $a$ define $L_E(a; \tau) = S(E(\tau)|_a)$; more generally,
for a set $A$ of qubits define

$$L_E(A; \tau) = S(E(\tau)|_A)).$$

We will now state mathematically a version of Conjecture [A]. Our setting is as follows. Let $\rho$ be the “intended” (“ideal”) pure state of the computer and consider two qubits $a$ and $b$. We use as the (rather standard) measure of entanglement between qubits at pure states

$$ENT(\rho; a, b) = S(\rho|_a) + S(\rho|_b) - S(\rho|_{\{a,b\}}).$$

As a measure of correlation of information leaks we use

$$EL_E(a, b; \tau) = L_E(a; \tau) + L_E(b; \tau) - L_E(\{a,b\}; \tau).$$

Conjecture [A] can be formulated as follows:

For every tensor product state $\tau$,

$$EL_E(a, b; \tau) \geq K(L_E(a; \tau), L_E(b; \tau)) \cdot ENT(\rho; a, b), \quad (2)$$

where $K(x, y)/\min(x, y)^2 \gg 0$ when $x$ and $y$ are positive and small. ($K(x, y) = 0$, when $\min(x, y) = 0$ and so relation (2) tells us nothing about noiseless entangled qubits.)

Below in Section 4.4 we will describe and motivate a stronger form of Conjecture [A] based on a different measure for entanglement. In the appendix we point out an alternative mathematical formulation for information leaks and describe an extension of Conjecture [A] to several qubits rather than two.

A simple extension that we would like to mention at this point is to pairs of qudits rather than pairs of qubits. The term qudit is used to denote a unit of quantum information in a $d$-level quantum system. Relation (2) extends to qudits without any change. This applies, in particular, to two disjoint sets of qubits in a quantum computer.
Remark: Consider two qudits $a$ and $b$, with $d$ and $d'$ possible levels respectively. The ideal pure state of this pair of qudits is represented by a $d$ by $d'$ matrix. Our conjecture (roughly) asserts that when the state is not represented by (or close to) a rank one matrix then neither is the error.

4.2 Error synchronization

A simple way to describe error synchronization is in terms of the expansion of the quantum operation $E$ in terms of multi-Pauli operators. A quantum operation $E$ can be expressed as a linear combination

$$E = \sum v^I P^I,$$

where $I$ is a multi-index $i_1, i_2, \ldots, i_n$, where $i_k \in \{0,1,2,3\}$ for every $k$, $v^I$ is a vector, and $P^I$ is the quantum operation that corresponds to the tensor product of Pauli operators whose action on the individual qubits is described by the multi-index $I$. The amount of error on the $k$th qubit is described by $\sum \{\|v^I\|_2^2 : i_k \neq 0\}$. For a multi-index $I$ define $|I| = |\{k : i_k \neq 0\}|$. Let

$$f(t) =: \sum \{\|v^I\|_2^2 : |I| = t\},$$

we can regard $\int_0^1 f(t)t$, the average over the qubits of the amount of error, as the error rate.

Suppose that the error rate is $a$. All noise models studied in the original papers of the “threshold theorem,” as well as some extensions that allow time- and space-dependencies (e.g., [26]), have the property that $f(t)$ decays exponentially (with $n$) for $t = (a + \epsilon)n$, where $\epsilon > 0$ is any fixed real number. In contrast, we say that $E$ leads to error synchronization if $f(\geq t)$ is substantial for some $t >> a$. We say that $E$ leads to a strong error synchronization if $f(\geq t)$ is substantial for $t = 1/2 - \delta$ where $\delta = o(1)$ as $n$ tends to infinity, and to very strong error synchronization if $f(\geq t)$ is substantial for $t = 3/4 - \delta$ where $\delta = o(1)$ as $n$ tends to infinity. A random unitary operator
on the qubits of the computer with or without additional qubits representing
the environment yields very strong error synchronization.

4.3 Conjectures [A] and [B], quantum error-correction,
and fault-tolerance

For a fault-tolerant quantum computer (for current methods of fault-tolerance),
when the number of qubits is large, for most pairs of qubits the errors at ev-
ery time of the computation, namely the gap between the intended state and
the actual state, will be almost statistically independent. This property is
assumed for the “new errors” (the storage errors and the gate errors com-
bined), both for the standard model of noise and for recent, more general
models of noise [26]. Remarkably, when this property is satisfied for the new
errors, and the error rate is small, fault-tolerant schemes allow us to keep
this property for the accumulated error.

I will now describe why Conjecture [A] (or rather an appropriate strength-
ening) is damaging for quantum error-correction and FTQC. We will first
consider two simplifying assumptions:

1. Measuring a qubit and looking at its content does not induce errors on
other qubits.

2. The error is of the simple form $E_D$.

Consider the state of a quantum computer that applies a fault-tolerant
computation. The state of the computer (or of large blocks of qubits of
the computer) is $t$-wise independent for a large value of $t$; hence every two
qubits are statistically independent and Conjecture [A] does not directly
apply. Consider an error-correcting code and let $s$ be the minimal number
of qubits whose state “determines” that of the others, so that once they are
measured and their values are “looked at” the state of the other qubits is
determined. When we measure and look at the values of $s - 1$ qubits, we see
a very strong dependence between every pair of the remaining qubits. Given our first assumption, measuring the other qubits does not affect the error on the two qubits we are interested in. Therefore, Conjecture [A] implies that the correlation of information leaks for every pair of qubits is substantial. If the error has the simple form $E_D$ then Lemma 1 asserts that there is a strong form of error synchronization and this will fail the quantum error-correction required for the threshold theorem.

In order to extend the above argument so that it will not be based on assumption (1) we propose in Section 4.4 below a stronger form of Conjecture [A] that relies on a notion of “emergent entanglement.”

Assumption (2) poses a serious limitation to the above argument, but I expect that reliance on assumption (2) is technical and that Lemma 1 extends to general forms of errors.

We now show that a certain simple error model that satisfies Conjecture [A] (and even the strong conjectures of the Appendix) and Conjecture [B] (and even the strongest version of error-synchronization) still allows the use of log-depth quantum computation, e.g., for polynomial time factoring.

The model is very simple. In each computer cycle with probability $1 - t$ nothing happens and with probability $t$ every qubit collapses to its maximum entropy mixed state. (In other words, the new errors are described by $E_D$, where $D$ is the distribution giving the all 0 vector probability $1 - t$, and the all 1 vector probability $t$.)

If we run a log-depth quantum circuit a polynomial number of times, one of the runs will work without any error. For an algorithm like factoring, if we run the algorithm including the quantum subroutine a polynomial number of times, at one of these times we will end up with a correct factoring that we will be able to check in polynomial time.

**Remark:** Following is a simple argument proposed by Kuperberg why even the simplest form of Conjecture [A] would not allow quantum computation at all. “If quantum computing is possible, then a quantum computer
could have prepared a state $S$ and then communicated it to the system that has the noise operation $E$. If it is true quantum computing, then $S$ can be secret from $E$, for reasons similar to those that make quantum key distribution possible. In this case $E$ can act on $S$ but it cannot otherwise depend on it.” The difficulty with this argument is that moving from a logical protected state $S$ to a physical realization of $S$ on a different device requires some computation and fault-tolerance and thus relies on assumptions regarding errors that we cannot assume. Still, Kuperberg’s proposed reduction can be useful. (The example above shows that we cannot expect such a strong statement as Kuperberg’s.)

4.4 Two qubits: emergent entanglement

We proceed to describe and motivate a stronger form of Conjecture [A].

The expression $S(\rho|_a) + S(\rho|_b) - S(\rho|_{\{a,b\}})$ was used as a measure of entanglement between two qubits. We would like to replace it by a measure that can be called “emergent entanglement,” which we are now going to define. This measure, denoted by $EE(\rho; a, b)$, captures (roughly) the expected amount of entanglement among the two qubits when we measure some other qubits, “look at the outcome,” and condition on all possible outcomes for the measurement. It appears to be related to Briegel and Raussendorf’s notion of “persistent entanglement” [39].

For every representation $\omega$ of $\rho|_{\{a,b\}}$ as a mixture (convex combination) of joint states

$$\rho|_{\{a,b\}} = \sum_{i=1}^{t} p_k \rho_k,$$

let

$$ENT_\omega(\rho; a, b) = \sum p_k ENT(\rho_k; a, b).$$

Define

$$EE(\rho; a, b) = \max ENT_\omega(\rho; a, b),$$
where the maximum is taken over all representations \( \omega \). (We can assume that \( \omega \) is a mixture of pure joint states.)

A strong form of relation (2) is

\[
EL(a, b) \geq K(L(a), L(b)) \cdot EE(\rho; a, b),
\]

where, as before, \( K(x, y)/\min(x, y)^2 \gg 0 \) when \( x \) and \( y \) are positive and small.

Using the measure for emergent entanglement appears to give the “right” formulation for Conjecture [A]. It may also be relevant for formulating our conjectures when the intended state is not necessarily pure. Aharonov [30] proved that quantum computers with mixed states can run arbitrary quantum computation without any entanglement between qubits of the computer. (However, these qubits will have high emergent entanglement.)

4.5 Censorship

Here is a suggestion for an entropy-based mathematical formulation for Conjecture [C]. We remind the reader that in this section we always assume that the “ideal” state of the quantum computer (before the noise is applied) is a pure state. Some adjustments to our conjectures will be required when the ideal state itself is a mixed state.

Let \( \rho \) be a pure state on a set \( A = \{a_1, a_2, \ldots, a_n\} \) of \( n \) qubits. Define

\[
ENT(\rho; A) = -S(\rho) + \max S(\rho^*),
\]

where \( \rho^* \) is a mixed state with the same marginals on proper sets of qubits as \( \rho \), i.e., \( \rho^*|_B = \rho|_B \) for every proper subset \( B \) of \( A \).

Next, define

\[
\widehat{ENT}(\rho) = \sum \{ENT(\rho; B) : B \subset A\}.
\]

In this language a way to formulate the censorship conjecture is:
Conjecture [C]: There is a polynomial $P$ (perhaps even a quadratic polynomial) such that for any quantum computer on $n$ qubits, which describes a pure state $\rho$,

$$\hat{\mathrm{ENT}}(\rho) \leq P(n).$$

(4)

5 Examples and models

5.1 Unprotected quantum circuits

A basic example to have in mind (with some caveats that we will discuss below) is the example of “unprotected” quantum circuits. This goes back to Unruh [6] who studied error propagation for Shor’s algorithm. Take the standard model of statistically independent errors and suppose that the error rate is so small that it accumulates at the end of the computation to a small constant-rate error. It is instructive to see in this context that error synchronization is often created.

We emphasize that since fault-tolerant quantum computation handles well propagation of errors, a model for decoherence that supports Conjectures [A] and [B] (and [E.2], Section 8 ) should already exhibit [A] and [B] (and [E.2]) for the “new errors” — whether storage errors or gate errors.

A main (yet informal) thesis of this paper is that a noisy quantum computer is subject to a substantial amount of error that behaves like propagated error for unprotected quantum circuits.

There are no (definite) examples of quantum error-correction in nature, and this suggests that models of unprotected quantum circuits can suffice to represent the evolution of natural quantum processes. Therefore, a study of the evolution of unprotected quantum circuits is of independent interest.

Let me remark at this point that errors of unprotected quantum circuits leading to a state $\rho$ will exhibit systematic dependencies on $\rho$ of various forms. When we have a discrete set of gates we encounter errors that depend
discontinuously on \( \rho \). This is an issue that has been studied in the quantum information literature and is related to the Kitaev-Solovay Theorem, see [3]; Chapter 4 and Appendix 3. In this paper, we are interested in a different phenomenon, that of systematic smooth dependence of the errors on \( \rho \). (This justified our description of the errors in terms of quantum operations to start with.) For this purpose it seems reasonable to consider quantum circuits with a continuous set of gates.

We should offer a more precise definition of “unprotected quantum circuits.” A random circuit leading to a given state \( \rho \) or a random perturbation of a specific circuit leading to \( \rho \) (which still leads to \( \rho \)) may serve this purpose.

Next we should describe the model of noise. To start, we may consider the standard model of independent noise (even the very simple model, considered in Section 3, when every qubit collapses to a fixed state with a small probability, and these probabilities are statistical independent), and study the accumulation of errors when the noise for a computer cycle is very small.\(^2\)

### 5.2 Models

A basic remaining challenge is to present concrete models of noise that support our conjectures.

We Emphasize again that a model for decoherence that supports conjectures [A] and [B] should already exhibit [A] and [B] for the “new errors” — whether storage errors or gate errors\(^3\) or both — and thus be quite different from current models and current perceptions regarding noise. Models that

\(^2\)However, note that we conjecture that new errors in a process leading to a state \( \rho \) share properties with accumulative errors of an unprotected program leading to \( \rho \) and, therefore, taking the standard model of statistically independent errors to understand unprotected circuits can be regarded as a first approximation.

\(^3\)As mentioned, we should allow gate errors to “apply” also to qubits not involved in the gate. Allowing this may reflect several concerns expressed in the literature regarding the qubit/gate model such as the issue of “slow” gates [31].
satisfy our conjectures may be based on the storage errors (in a single computer cycle) being represented by a rather primitive (but quick) stochastic quantum program (or circuit). Here are a few additional points regarding concrete noise models that may be relevant.

1) Noise models satisfying our conjectures can be regarded as a further step in the direction considered recently by Aharonov, Kitaev, and Preskill [26] (and a few earlier works). In these works, interactions between nearby qubits (arranged on a grid) that lead to statistical dependence between the noise acting on them are considered and it is shown that the threshold theorem prevails if the independence assumption still applies to faraway qubits. However, interactions between nearby qubits may lead to dependencies between errors that are not covered by the assumptions of [26]. (Compare the remark about cluster states below.)

2) Klesse and Frank [32] described a physical system in which qubits (spins) are coupled to a bath of massless bosons and then reached (after certain simplifications) a noise model with error synchronization.

3) The earlier models suggested by Alicki, Horodecki, Horodecki, and Horodecki [23] appear to be relevant to our conjectures.

4) Let me also mention the relevance of cluster states defined by Briegel and Raussendorf (see [33]). The description of cluster states involves an array of qubits located on the vertices of a rectangular lattice in the plane (or in space). Cluster states are “generated” by local entanglement between pairs of nearby qubits on the lattice grid. They can be regarded as the quantum analogs of the Ising and Potts classical models.

Controlled creation and manipulation of cluster states can be important for building quantum computers. On the other hand, cluster states and the local processes leading to them can possibly serve as a basis for concrete models of detrimental decoherence.

5) A toy model for noise that neglects some of the effects we consider in the paper and brings others to an extreme form is the following. There
are no gate errors. Consider the graph $G$ whose vertices are the qubits and whose edges are qubits that occur in a gate. Edges are labeled by the gate imperfection. The storage error is described by $E_D$ where the probability distribution $D$ is given by an Ising model on the graph $G$ based on these gate imperfections. Can quantum error-correction prevail in such (low-rate) error model?

6 Computation complexity

Scott Aaronson’s interesting “Sure/Shor challenge” [34] asks for restrictions on feasible (physical) states for quantum computers that do not allow for polynomial time factoring of integers and at the same time do not violate what can already be demonstrated empirically. This looks like a difficult challenge. In a similar spirit, while it looks intuitively correct that our conjectures are damaging to quantum computation, proving it is not going to be easy.

A realistic task would be to show that our conjectures exclude fault tolerance based on linear quantum error-correction, e.g., deriving relations (2) and (4) (or even (5)) for any form of “protected qubits” obtained by linear quantum error-correction.

A more ambitious goal than excluding quantum linear error-correction would be finding a reduction of noisy quantum computation (with detrimental errors) to the computational power of log-depth quantum circuits. (This will still fall short of Aaronson’s challenge in view of Cleve and Watrous [35], who gave a polynomial algorithm for factoring that requires, beyond classical computation, only log-depth quantum computation.) Reductions to log-depth quantum computation are known under the standard assumptions on noise, for reversible quantum computation [15]. For certain noise models, when the error rate is above 45% [20], it is known that the noisy quantum computer can be simulated by a classical computer.
When we insist on small error rate it may well be the case that log-depth quantum circuits represent the true complexity power of quantum computers with detrimental errors. As we already pointed out in Section 4.3, for a log-depth circuit such that the storage (and gate) errors demonstrate perfect error synchronization running the computation a polynomial number of times, with high probability there will be no errors in one of the runs. If we replace a given log-depth circuit by a larger one capable of correcting standard errors we may reach polynomial size (or quasi-polynomial size) circuits that are immune to low-rate errors of the kind considered in this paper. (Conjecture [E.2] below may be damaging also to log-depth quantum computation.)

7 Extensions

7.1 General quantum systems

The purpose of Section 4 was to describe formally the conjectures on decoherence of quantum computers based on the basic model of such a computer. In the context of general quantum systems these conjectures are thus somewhat arbitrary. (In particular, we always talk about Hilbert spaces of dimensions $2^m$.)

The main idea behind the conjectures is that the error-independence assumption (for different qubits) amounts to an extremely strong dependence of the errors on the tensor product structure of the Hilbert space describing the state of the computer. It can be useful to suggest and examine formulations of our conjectures that do not depend on the tensor product structure of the Hilbert space in question.

We want to consider quantum physical systems described by a complex Hilbert space $V$. Our conjectures suggest that if $E$ represents the error for state $\rho$ and $E'$ represents the error for state $U(\rho)$, for a unitary operator $U$
on $V$, then $E'$ will be “close” to $U^{-1}EU$. In particular, this implies that if $U(\rho) = \rho$ then $E'$ is “close” to $U^{-1}EU$; hence $UE$ is “close” to $EU$. In other words, $E$ and $U$ “tend” to commute if $U(\rho) = \rho$.

Here is an attempt at a formal conjecture. We will restrict our attention to the special case where the error is described by a quantum operation $E$ which is a convex combination of unitary operators.

[D] There is an $\alpha > 0$ such that a prescription (or description) of a noisy quantum system at a state $\rho$ is subject to error $E$ having the property that for every unitary operator $U$ such that $U(\rho) = \rho$

$$\|EU - UE\| \leq (1 - \alpha)\sqrt{2}. \quad (5)$$

Here we do not insist that the prescribed (or described) state be pure, and we refer to the Hilbert-Schmidt norm.

Several remarks are in place. First, Greg Kuperberg pointed out that at a thermodynamics equilibrium a certain limiting error $E$ will actually commute with every $U$ that stabilizes $\rho$. We can regard Conjecture [D] as a statement referring to non equilibrium thermodynamics.\(^4\)

Second, while a generic form of noise (say, a generic unitary operator with prescribed rate) indeed leads to error synchronization and is damaging to quantum error-correction, such a noise appears unrealistic. Here, the condition on the noise is nongeneric, and rather it is the standard assumption on noise that leads (for highly entangled states) to generic commutativity behavior between the noise operation and the state of the computer.\(^5\)

Third, it will be interesting to examine Conjecture [D] for noisy adiabatic models of quantum computers; see, e.g. [36].

\(^4\)In this context, the works (and even the small controversy) on quantum analogs of “Onsager’s regression theorem” come to mind.

\(^5\)Going back to the issue of approximating arbitrary matrices up to rank one matrices, note that if $A$ is a random $n$ by $n$ matrix and $D$ is any rank one matrix then with high probability $\|AD - DA\| \geq (1 - o(1))\sqrt{2}$. 
7.2 Classical noise

Conjectures [A] and [B] were originally formulated in [28] also for “natural” noisy classical correlated systems. For example, the analog of [A] asserts that in a noisy system the errors for two highly correlated elements tend to be substantially correlated. Because of the heuristic (or subjective) nature of the notion of noise in classical systems (and of the notion of probability itself), such a formulation, while of interest, leads to several difficulties.

Understanding noise and the study of de-noising methods span wide areas. (For example, in machine learning we can see the example where text and speech represent respectively the intended (ideal) and noisy signals.) Certain statistical methods of de-noising are based on assumptions that run counter to [A]. However, our conjectures are in agreement with insights asserting that such statistical de-noising methods will leave a substantial amount of noise uncorrected. Moreover, “natural” examples of noisy highly correlated classical systems exhibit a moderate degree of dependence, much less than the sort of dependence required for quantum error-correction, and appear to be in agreement with Conjecture [C].

8 The nature of noise and the rate of errors

Up to now, we have assumed that the error rate (per qubit, in each computer cycle) is small and fixed. Trying to understand systematic relations between the error rate (for individual qubits) and the intended state $\rho$ of the quantum computer may be of interest.

In this section we assume that an evolution of a noisy quantum computer (or a more general noisy quantum system) is described for a certain time interval. The intended state of the system is pure for the entire time, and we assume, as before, that for the entire time, the actual state of the computer is close to the intended state. Up to now, we took a “snapshot” at a single computer cycle and measured the error-rate for individual qubits. Here we
will consider the infinitesimal noise rate in terms of trace distance.

**[E.1]** A noisy quantum system is subject to (detrimental) noise with the following property: the infinitesimal rate of noise at time $t$ (in terms of trace distance) is bounded from below by a measure of noncommutativity between the operators describing the evolution prior to time $t$ and those describing it after time $t$.

Conjecture [E.1] is an attempt to describe in a more concrete form the postulate of noise itself. Conjecture [E.1] seems similar to models of decoherence (brought to my attention by Michael Khasin) where the decoherence is described as the effect of several noncommuting noise operations, and the rate of decoherence is related to “uncertainty measures” for these noise operators.

When we consider a quantum computer with $n$ qubits whose evolution is restricted to a Hilbert space $V$ (which can be a subspace of the entire $2^n$-dimensional space of pure states for all the qubits) we can expect that the rate of noise (in terms of trace distance) will be bounded below by $\kappa \log(\text{dim} \ V)$. This appears to be in agreement with the behavior for unprotected noisy quantum circuits.

**[E.2]** A noisy quantum computer with $n$ (logical) qubits is subject to (detrimental) noise, with the following property: For some $\kappa > 0$, the infinitesimal rate of noise (in terms of trace distance) is at least $\kappa n$.

For standard models of noise, the infinitesimal rate of noise in terms of trace distance scales up linearly with the number of qubits. So Conjecture [E.2] is in agreement with the standard assumptions on the rate of noise. (The rate of noise is a small constant per qubit per one computer cycle). However, there are two important differences.

1) For strong forms of error-synchronization (like the model in Section 4.3) the amount of error in terms of trace distance, is sublinear in the number of qubits. Therefore, in cases of highly entangled states, which, by Conjecture
[B], would lead to error-synchronization, conjecture [E.2] suggests that the rate of detrimental noise for individual qubits will scale up (even linearly) with the number of involved qubits.

2) Ordinary models of noise enable the existence of “decoherence-free subspaces.” Conjecture [E] asserts that, in contrast, the (detrimental) decoherence rate for a subspace representing a small number of “protected logical qubits” is “intrinsic,” depends on the space of operators acting on this subspace along the evolution of the computer, and, in term of trace-distance, scale up linearly with the number of qubits.

**Remark:** When we wish to prescribe an evolution of a quantum system up to a small error, a lower bound on the error rate at an intermediate state \( \rho \) may depend not only on the process leading to \( \rho \) from the initial state but also on the process leading from it to the terminal state. For example, we can expect that the noise for two faraway entangled photons will be independent, and therefore the rate of detrimental decoherence in this case is zero. Note that this is consistent with Conjecture [E.1]. Any intervention to bring the two photons back together in order to carry out additional joint operations is expected to introduce strong correlation between their errors.

More generally, the point is this: Consider an intended pure-state evolution \( \rho_t, 0 \leq t \leq 1 \) of a quantum computer, and a noisy realization \( \sigma_t, 0 \leq t \leq 1 \). Assuming that \( \sigma \) is close to \( \rho \) for the *entire* time interval, may create dependence of the infinitesimal noise at an intermediate time \( t \) on the entire evolution of \( \rho \).

### 9 Conclusion

If (or when) true, our conjectures on the nature of information leaks (decoherence) for quantum computers are damaging to the possibility of storing and manipulating highly entangled quantum qubits. The conjectures do not contravene quantum mechanics nor, to the best of my knowledge, established
physics phenomena. Neither do our conjectures contravene the feasibility of classical forms of error-correction and fault-tolerant computation.

Testing these conjectures empirically may be possible for quantum computers with a relatively small number of qubits. The conjectures might also be refuted by constructions of highly stable qubits based on strong entanglement, such as stable non-Abelian anyons [17, 37, 38].

References

[1] D. Deutsch, Quantum theory, the Church-Turing principle and the universal quantum computer, Proc. Roy. Soc. Lond. A 400 (1985), 96–117.

[2] P. W. Shor, Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer, SIAM Rev. 41 (1999), 303-332. (Earlier version, Proceedings of the 35th Annual Symposium on Foundations of Computer Science, 1994.)

[3] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000.

[4] R. Landauer, Is quantum mechanics useful?, Philos. Trans. Roy. Soc. London Ser. A 353 (1995), 367–376.

[5] R. Landauer, The physical nature of information, Phys. Lett. A 217 (1996), 188–193.

[6] W. G. Unruh, Maintaining coherence in quantum computers, Phys. Rev. A 51 (1995), 992–997.

[7] E. Bernstein and U. Vazirani, Quantum complexity theory, Siam J. Comp. 26 (1997), 1411-1473. (Earlier version, STOC, 1993.)

[8] P. W. Shor, Scheme for reducing decoherence in quantum computer memory, Phys. Rev. A 52 (1995), 2493–2496.

[9] A. M. Steane, Error-correcting codes in quantum theory, Phys. Rev. Lett. 77 (1996), 793–797.
[10] D. Aharonov and M. Ben-Or, Fault-tolerant quantum computation with constant error, STOC '97, ACM, New York, 1999, pp. 176–188.

[11] A. Y. Kitaev, Quantum error correction with imperfect gates, in Quantum Communication, Computing, and Measurement (Proc. 3rd Int. Conf. of Quantum Communication and Measurement), Plenum Press, New York, 1997, pp. 181–188.

[12] E. Knill, R. Laflamme, and W. H. Zurek, Resilient quantum computation: error models and thresholds, Proc. Royal Soc. London A 454 (1998), 365–384, quant-ph/9702058.

[13] D. Gottesman, Stabilizer codes and quantum error-correction, Ph. D. Thesis, Caltech, 1997, quant-ph/9705052.

[14] D. Aharonov and M. Ben-Or, Polynomial simulations of decohered quantum computers, 37th Annual Symposium on Foundations of Computer Science, IEEE Comput. Soc. Press, Los Alamitos, CA, 1996, pp. 46–55.

[15] D. Aharonov, M. Ben-Or, R. Impagliazo, and N. Nisan, Limitations of noisy reversible computation, 1996, quant-ph/9611028.

[16] A. R. Calderbank and P. W. Shor, Good quantum error-correcting codes exist, Phys. Rev. A 54 (1996), 1098–1105.

[17] A. Kitaev, Topological quantum codes and anyons, in Quantum Computation: A Grand Mathematical Challenge for the Twenty-First Century and the Millennium (Washington, DC, 2000), pp. 267–272, Amer. Math. Soc., Providence, RI, 2002.

[18] E. Knill, Quantum computing with realistically noisy devices, Nature, 434 (2005), 39-44. Earlier version, quant-ph/0410199.

[19] A. Razborov, An upper bound on the threshold quantum decoherence rate, Quantum Information and Computation, 4 (2004), 222-228. quant-ph/0310136.

[20] H. Buhrman, R. Cleve, N. Linden, M. Lautent, A. Schrijver, and F. Unger, New limits on fault-tolerant quantum computation, 47th Annual IEEE Symposium on Foundations of Computer Science (FOCS’06), pp. 411–419. quant-ph/0604141.
[21] J. Preskill, Quantum computing: pro and con, *Proc. Roy. Soc. Lond. A* 454 (1998), 469-486, quant-ph/9705032.

[22] L. Levin, The tale of one-way functions, *Problems of Information Transmission (= Problemy Peredachi Informatsii)* 39 (2003), 92–103, cs.CR/0012023.

[23] R. Alicki, M. Horodecki, P. Horodecki, and R. Horodecki, Dynamical description of quantum computing: generic nonlocality of quantum noise, *Phys. Rev. A* 65 (2002), 062101, quant-ph/0105115.

[24] B. B. Terhal and G. Burkard, Fault-tolerant quantum computation for local non-Markovian noise, *Phys. Rev. A* 71 (2005), 012336.

[25] P. Aliferis, D. Gottesman, and J. Preskill, Quantum accuracy threshold for concatenated distance-3 codes, *Quant. Inf. Comput. 8* (2008), 181–244. quant-ph/0504218.

[26] D. Aharonov, A. Kitaev, and J. Preskill, Fault-tolerant quantum computation with long-range correlated noise, *Phys. Rev. Lett.* 96 (2006), 050504. quant-ph/0510231.

[27] G. Kalai, Thoughts on noise and quantum computing, 2005, quant-ph/0508095.

[28] G. Kalai, How quantum computers can fail, 2006, quant-ph/0607021.

[29] R. S. Ellis, *Entropy, Large Deviation, and Statistical Mechanics*, Springer, New York, 1985.

[30] D. Aharonov, Why we do not understand mixed state entanglement, working paper, 2006.

[31] R. Alicki, D.A. Lidar, and P. Zanardi, Are the assumptions of fault-tolerant quantum error correction internally consistent?, *Phys. Rev. A* 73 (2006), 052311, quant-ph/0506201.

[32] R. Klesse and S. Frank, Quantum error correction in spatially correlated quantum noise, *Phys. Rev. Lett.* 95 (2005), 230503.

[33] R. Raussendorf, D. E. Browne, and H. J. Briegel, Measurement-based quantum computation with cluster states, *Phys. Rev. A* 68 (2003), 022312.
10 Appendix

10.1 Another measure for information leaks

Given a quantum operation $E$, our measure

$$L(A) = L_E(A; \tau)$$

for the information leaks for a set $A$ of qubits depended on a pure tensor product state $\tau$. The two-qubits basic property of detrimental decoherence was made for every $\tau$ separately. For the stronger conjectures below we will continue to make the statements in terms of an auxiliary tensor product state $\tau$. We will write $L(A) = L_E(A; \tau)$ and similarly delete $E$ and $\tau$ from other definitions based on $L(A)$.

An alternative approach is as follows. Let $\psi$ be the state of the computer's qubits and the environment that is represented by a set $N$ of qubits. Let $U$
be a unitary operator of the computer and environment qubits representing
the noise. A standard measure of the information that the environment has
on the qubits in \( A \) is

\[
L'(A) = S(U(\psi)|_A) + S(U(\psi)|_N) - S(U(\psi)|_{A\cup N}).
\]

For our purposes we take \( \psi = \psi_0(A) \otimes \psi_1(N) \) where \( \psi_1(N) \) is any pure state
on the environment and \( \psi_0(A) \) is the mixed state of maximum entropy on \( A \).
I would expect that \( L'(A) \) can replace \( L(A) \) for the formulation of Conjecture
\[A\] and the stronger conjectures below.

10.2 More qubits

Here is a suggestion for an extension of the above conjecture from pairs of
qubits to larger sets of qubits. This suggestion goes beyond Conjectures \[A\]
and \[B\] and is related to strong error synchronization.

For a set \( A = \{a_1, a_2, \ldots, a_m\} \) of \( m \) qubits recall that

\[
ENT(\rho; A) = -S(\rho) + \max S(\rho^*),
\]

where \( \rho^* \) is a mixed state with the same marginals on proper sets of qubits
as \( \rho \), i.e., \( \rho^*|_B = \rho|_B \) for every proper subset \( B \) of \( A \).

Define in a similar way

\[
EL(\rho; A) = -L_E(\rho) + \max L_{E^*}(A),
\]

where \( E^* \) is a quantum operation that satisfies \( E^*|_B = E|_B \) for every proper
set \( B \) of \( A \).

Using these definitions we will extend our conjectures, given by relations
(2) and (3), from pairs of qubits to larger sets of qubits. Let \( \rho \) be an ideal
state of the computer and let \( A \) be a set of \( m \) qubits. Extending (2) we
conjecture that

\[
EL(\rho) \geq K_m ENT(\rho|_A). \quad (6)
\]
Here, \( K_m = K_m(\{L(a) : a \in A\}) \) is substantially larger than \( \min\{L(a)\}^2 \) and it vanishes when all the individual information leaks vanish.

Here again we further conjecture that for every representation \( \omega \) of the state \( \rho|_A \) as a convex combination \( \rho|_A = \sum p_k \rho_k \) of pure joint states,

\[
EL(A) \geq K_m \sum p_k ENT(\rho_k; A). \tag{7}
\]

**Remark:** The value of \( ENT(\rho; A) \) is intended to serve as a measure of the additional information when we pass from “marginal distributions” on proper subsets of qubits to the entire distribution on all qubits.

The additional conjectures of this section are meant to draw the following picture: we have an ideal notion of a quantum computer that has extraordinary physical and computational properties. Next come noisy quantum computers with an ideal notion of noise. If the noise rate is small then FTQC is possible. Next come noisy quantum computers that satisfy relation (2). For them, fault-tolerance will require controlling the error rate as well as \( K_2 \), which we expect to be much harder. This model is also an idealization as long as \( K_3 = 0 \) and so on. For such highly entangled states as those required in quantum algorithms, (7) will be more and more damaging for larger values of \( i \).

### 10.3 Mathematical challenges

We will mention now some mathematical challenges. It will be interesting to prove relation (4) based on relation (6), and to formulate and prove weak and strong forms of error synchronization based, respectively, on relations (2) and (6). A further goal would be to derive, based on the assumptions on noise for the physical qubits (relations (6) and (7)), the same relations as well as relation (4) for “protected” qubits, namely logical qubits represented by quantum error-correction. It will also be of interest to find the right general formulation of “tend to commute” as in relation (5) and to relate it to the
more concrete conjectures for quantum computers.