A Width Parameter Estimation Through Equivalent Rectangle Methodology for Hydraulic Modeling Applications

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DOI: https://doi.org/10.14796/JWMM.C493

ABSTRACT

EPA SWMM hydraulic modeling requires the estimation of different parameters allowing the determination of the basin's response upon a given precipitation event. Some physical parameters, such as area or perimeter, are measurable and can be accurately determined; however, other parameter estimation presents greater uncertainty, such as the width parameter.

For regular and uniform drainage areas, width parameter estimation is relatively simple; however, when a complete irregular basin analysis is required, width determination presents greater uncertainty, and its representativeness becomes complicated to define.

Width determination is idealized with the representation of a rectangle, where, for an equal area, a higher width will result in a faster response of the basin, while a lower width will result in a slower response of the basin. This paper attempts to estimate a representative value of width for a realistic, irregularly shaped basin by defining the equivalent rectangle, which takes into account the area, perimeter, and compactness index of the basin. The compactness index can be used to classify the basin by its shape. The shape of the basin is an indicator of how the precipitation histograms are temporally distributed and how the water flows through the basin, i.e., it defines the response speed of the basin, as the width parameter does in modeling.

The width parameter has a high sensitivity in the EPA SWMM modeling results; therefore, an inaccurate estimation of the parameter leads to unrepresentative results. For this reason, this study seeks to find an optimal methodology to reduce modeling uncertainty and achieve more accurate simulations of an irregular watershed's hydrological response.
1 INTRODUCTION

1.1 Background

EPA SWMM hydraulic modeling requires the estimation of different hydrological and hydraulic parameters, allowing the determination of the basin's response upon a given precipitation event. Some physical parameters, such as area or perimeter, are measurable and can be accurately determined; however, other parameters present greater uncertainty when it comes to their estimation, such as the width parameter.

According to the PCSWMM support site (CHI 2014), a subcatchment width parameter is usually set by first estimating a representative length of overland flow, then dividing the subcatchment area by this length (Figure 1).

\[ W = \frac{A}{L} \]  

(1)

where:

\[ W = \text{subcatchment width}, \]
\[ A = \text{subcatchment area}, \]
\[ L = \text{self-consistent measure of the representative flow path for the subcatchment}. \]

Figure 1 Estimating subcatchment width (CHI 2014).
2 METHODOLOGY

2.1 Width parameter estimation

When the drainage area is regular and uniform, width parameter estimation is relatively simple; however, when the analysis of a complete natural watershed is required, its geomorphological characteristics define irregular drainage areas, therefore the width determination presents greater uncertainty, and its representativeness becomes complicated to define (Figure 2).

![Figure 2 Possible width estimations for an example watershed in Costa Rica.](image)

**2.2 Equivalent rectangle methodology**

The equivalent rectangle is a geometric transformation that enables the representation of a watershed in the shape of a rectangle (Villón Béjar 2004). The rectangle must have the same area and perimeter as the watershed. This theoretical watershed, having the same area and perimeter, also shares the Gravelius index, the distribution of heights, which translates into the same
hypsometric curve, and the distribution in the terrain in terms of its coverage (Heras 1972).

An equivalent rectangle of the same area as the watershed is constructed, such that the smaller side is \( I \) and the larger side is \( L \). The contour lines are placed parallel to \( I \), respecting the natural hypsometry of the watershed (Heras 1972).

The compactness coefficient, or Gravelius index \( K_c \) (Equation 2), is the proportion between the perimeter of the basin and the circumference of a circle with the same area of the basin. The more irregular the shape of the watershed, the higher its compactness coefficient. A circular watershed has a minimum compactness coefficient of one. There is a greater tendency to crescents when this number is close to one.

\[
K_c = \frac{0.28 \times P}{\sqrt{A}} \tag{2}
\]

where:

- \( A \) = subcatchment area, and
- \( P \) = subcatchment perimeter.

\( A \) and \( P \) can be either metric or imperial units, as long as they are consistent (e.g., \( \text{km}^2 \) and \( \text{km} \), \( \text{ft}^2 \) and \( \text{ft} \)).

The large side (Equation 3) and small side (Equation 4) equations respect the conditions of the equivalent rectangle since the characteristics of area and perimeter of the watershed must be preserved. Solving the equations and making use of the definition of the compactness index \( K_c \), we have:

\[
L = \frac{K_c \times \sqrt{A}}{1.12} \left[ 1 + \sqrt{1 - \frac{1.12^2}{K_c^2}} \right] \tag{3}
\]

\[
l = \frac{K_c \times \sqrt{A}}{1.12} \left[ 1 - \sqrt{1 - \frac{1.12^2}{K_c^2}} \right] \tag{4}
\]

where:

- \( A \) = subcatchment area,
- \( P \) = subcatchment perimeter,

\( A \) and \( P \) can be either metric or imperial units, as long as they are consistent (e.g., \( \text{km}^2 \) and \( \text{km} \), \( \text{ft}^2 \) and \( \text{ft} \)), and

\( K_c \) = compactness coefficient.

### 2.3 Study area

The Quebrada Seca-Burío watershed is located in the central valley of Costa Rica, Central America, specifically in the greater metropolitan area (GMA). It has an area of approximately 23 \( \text{km}^2 \) and the main water courses are Quebrada Seca, Burío River, and Aries Creek (Masís Campos and Vargas
Picado 2014). The extension of the watershed includes parts of the municipalities of San Rafael de Heredia, Barva, Heredia, Flores, Belén, and Alajuela (Figure 3). The mean slope of the watershed is 20% with an elevation gradient between 869 and 1626 masl (above sea level) (Chen et al. 2021) and its average elevation is 1120 masl. The geomorphology of the watershed presents a primarily volcanic origin, which was derived from high permeable soils (Masís Campos and Vargas Picado 2014; Weyand 2020), allowing aquifer recharge and controlling the surface runoff. However, the watershed currently has a primarily urban land cover, representing about 66% of the total area. This urban development has a direct impact on the aforementioned hydrological processes, reducing infiltration and increasing surface runoff.

The climate of the watershed is characterized by an average annual precipitation of around 2000 mm (Solano and Villalobos 2012; Masís Campos and Vargas Picado 2014), with an average temperature of 24.8 °C (Chaves Herrera et al. 2014). The watershed is located on the Pacific side of the country, which has a well-defined rainy season (Chen et al. 2021). The temporal distribution of precipitation in the area is characterized by high intensities and short durations.

Two control points were defined, resulting in three subwatersheds to evaluate. The first control point is the confluence of the Quebrada Seca River and the Burio River, and the second control point is located at the intersection of the Quebrada Seca with Highway 1 (Figure 3).
Watershed elevation distribution

The evaluation of the receiving body, in this case the Quebrada Seca, at the Highway 1 control point, requires the delimitation of its drainage area, which is shown as an irregular polygon with its respective elevation distribution (Figure 4). This same exact information can be shown with a regular shape, with the same area, perimeter, and height distribution, through the equivalent rectangle methodology (Figure 5).

![Figure 4 Elevation distribution of the Highway 1 subwatershed.](image)

![Figure 5 Elevation distribution of the Highway 1 subwatershed through the equivalent rectangle.](image)
2.4 Modeling for width parameter evaluation

The software PCSWMM Version 7.3.3095, based on EPA SWMM (Rossman and Huber 2015), was employed to simulate the rainfall–runoff generation. A secondary hydrological modeling was carried out using the software HEC-HMS Version 4.2 (Scharffenberg 2016) in order to compare the results of both models. The synthetic unit hydrograph of Snyder was employed as a transformation method (Snyder 1938).

Data from the Santa Lucía rain gauge, located within the watershed, was used to characterize the precipitation events in the Quebrada Seca-Burío watershed. From the National Meteorological Institute of Costa Rica (IMN) database, the maximum daily annual rainfall time series from 1982 to 2020 was obtained. An extreme event analysis was conducted to estimate the expected precipitation for different return periods. This analysis was made with the R software package extRemes (Gilleland and Katz 2016), using the generalized extreme value distribution. To model the typical behavior of the storms that usually present on the watershed, the Santa Lucía rain gauge characteristic temporal distribution for extreme events was used (Murillo 1994).

The rainfall abstractions for both models were estimated using the National Resources Conservation Service (NRCS) method, formerly known as the Soil Conservation Service (SCS) method (Chow 1964). The curve number (CN) values of each land cover class were obtained from standard runoff curve number tables. Land cover classification in the Quebrada Seca-Burío watershed was carried out through the classification of satellite images from Landsat sensors (Figure 6).

Figure 6 Land cover classification for the drainage area of the Highway 1 control point.
The characterization parameters of the three watersheds are shown in Table 1.

| Parameter                | Subwatershed     | Highway 1 | Seca | Burio |
|--------------------------|------------------|----------|------|-------|
| Area (km²)               |                  | 18.96    | 5.43 | 7.70  |
| Perimeter (km)           |                  | 27.11    | 17.27| 20.18 |
| Main river length (km)   |                  | 12.59    | 3.56 | 10.00 |
| Compactness index (Kc)   |                  | 1.74     | 2.08 | 2.04  |

#### Equivalent rectangle

- Major side (km):  
  - Highway 1: 11.97
  - Seca: 7.95
  - Burio: 9.26
- Minor side (width) (km):  
  - Highway 1: 1.58
  - Seca: 0.68
  - Burio: 0.83

| Parameter                          | Highway 1 | Quebrada Seca | Burio River |
|------------------------------------|-----------|---------------|-------------|
| Average watershed elevation (masl) | 1144.37   | 1189.00       | 1223.00     |
| Maximum elevation (masl)           | 1620.00   | 1620.00       | 1620.00     |
| Minimum elevation (masl)           | 963.16    | 1064.00       | 1064.00     |
| Main river slope (%)               | 5.19      | 3.43          | 5.55        |
| Impervious area (%)                | 64.30     | 55.50         | 59.40       |

3 RESULTS AND DISCUSSION

### 3.1 Modeling for width parameter evaluation

Parameters such as runoff coefficient, runoff volume, peak discharge, and peak time were the main indicators to evaluate the results of both models. Modeling results for the three sub-watersheds from the EPA SWMM and HEC-HMS models are shown in Table 2.

| Parameter                          | Highway 1 | Quebrada Seca | Burio River |
|------------------------------------|-----------|---------------|-------------|
| Runoff coefficient                 | 0.54      | 0.66          | 0.52        |
| Volume (ML)                        | 1347.67   | 1638.05       | 368.95      |
| Peak discharge (m³/s)              | 186.90    | 180.40        | 59.80       |
| Peak time (min)                    | 60        | 90            | 40          |
The results achieved in both models were similar in terms of peak flow, runoff coefficient, and runoff volume. The runoff coefficient and runoff volume determined with the HEC-HMS model exceeded the value obtained using EPA SWMM by between 12% and 17%, while the variations in peak flows between the two models were between 4% and 7%. The peak time observed with EPA SWMM was lower for each of the drainage areas analyzed. In the case of the Highway 1 and Quebrada Seca subwatersheds, the peak time obtained with HEC-HMS exceeded the peak time obtained with EPA SWMM by 30 min. For the Burio River subwatershed, the EPA SWMM peak time was half of that determined by the HEC-HMS model. The main differences between the models occurred in the peak times, and this was due to the difference in the calculation of the travel time by each model. In the HEC-HMS model, the time lag for the peak time is calculated using an empirical relationship that is a function of the length of the channel and the length to the center of gravity of the basin; this relationship does not consider the roughness of the terrain. In the EPA SWMM model, in addition to the length of travel, the roughness coefficient of the terrain is also taken into account, which causes the flow to accelerate when passing through urban coverage such as roofs and pavements.

In highly urbanized areas, it is expected that the parameters related to the impervious elements of the catchment will control the hydrograph modeling, since the impervious surfaces generate most of the runoff (Fernandez et al. 2019). In the case of basins with predominantly urban land cover, it is recommended to estimate the time to peak with the EPA SWMM model since it is closer to the real value of the basin.

The width parameter in this study was used to determine the hydrological response of a watershed and compare it with another known model, but sometimes there is no information available to validate and calibrate the models. According to Nowogoński et al. (2019), the width value is a calibration parameter with a significant impact on the ability to obtain the correct adjustment of the simulation model to real conditions. When working with scarce information and in the absence of known hydrographs, the width parameter requires a stricter estimation to obtain results that are representative of the drainage area under study. In this case, the correct estimation of the width parameter is fundamental due to the sensitivity of this parameter on the results.

### 3.2 Modeling sensitivity analysis for Highway 1 subwatershed

In this study, a sensitivity analysis was performed to evaluate the variation in the resulting peak flow, based on the only modification of the width parameter in the model, keeping the other parameters at their original values.

The purpose of this analysis is to define the level of sensitivity that the width parameter has on the results of the model, in order to ratify the importance of the correct estimation of the parameter, since in several studies the width parameter has been determined as one of the parameters with the highest degree of sensitivity. (Chen et al. 2021; Smith et al. 2005; and Fernandez et al. 2019).

In this specific study, increasing the width parameter by 50% resulted in a 35.8% increase in peak flow. Doubling the width value resulted in a 71.9% increase in peak flow rate. Decreasing the width value by 50% reduced the peak flow rate by 25.8%. This means that if we increase the width of the watershed from 1583 m to 2375 m (a 50% increase) the peak flow will increase from 181.1 m$^3$/s to 245.9 m$^3$/s, which for a watershed of 18.9 km$^2$ could result in a significant variation in the water
level and compromise the hydraulic capacity of the river. The results of the analysis are shown in Figure 7.

![Figure 7 Peak flow versus width for sensitivity analysis.](image)

Model results are affected by several variables and parameters that determine the hydrologic response of a watershed. The changes in catchment area and width are sensitive parameters in the SWMM engine, which can increase or decrease the volume of runoff generated by EPA SWMM (Smith et al. 2005). According to Fernandez et al. (2019), the most sensitive parameters are the percentage of runoff routing from impervious to pervious areas and subcatchment width, confirming the sensitivity of the parameter investigated in this study. The sensitivity analysis performed by Fernandez et al. (2019) showed that (a) percentage routed, (b) characteristic width, (c) percentage of imperviousness, and (d) slope are the most sensitive parameters for calibration.

4 CONCLUSIONS

The use of the equivalent rectangle method allows an unbiased estimation of the width parameter, based on the physical characteristics of the drainage area that provides an objective approach of the hydrological response of the watershed.

The use of the method is recommended for small watersheds, with low order numbers, to avoid errors in the time lag. For larger watersheds, it is recommended to subdivide the drainage area into subareas that allow a better definition of hydrological responses.

The lack of flow measurements in the study area prevented the comparison of observed data with
the model results, therefore the study purpose and the use of the models is mainly to analyze the
influence of the width parameter on the hydrological response of the basin. Hence, the comparison
of the results obtained with EPA SWMM and HEC-HMS facilitates the analysis of two different
methodologies that converge in similar results for the hydrological response of the Quebrada Seca
watershed. However, differences are shown in time to peak, which is attributed to the roughness
coefficient consideration.

The width parameter is a parameter with high uncertainty and the results are very sensitive to its
modification.

ACKNOWLEDGMENTS

The publication of this article was funded by the Open Access Publishing Fund of Leibniz Universität
Hannover.

DAAD (German Academic Exchange Service) and CONARE (National Council of Rectors) are funding
the PhD. program for the student Jose Ricardo Bonilla Brenes through an ALECOSTA scholarship.

The authors would like to thank Computational Hydraulics International (CHI) and HydroPraxis for
access to PCSWMM through their educational grant program.

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