Electricity and Reserve Pricing in Chance-Constrained Electricity Markets with Asymmetric Balancing Reserve Policies

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Abstract—Recently, chance-constrained stochastic electricity market designs have been proposed to address shortcomings of scenario-based stochastic market designs. In particular, the use of chance-constrained market-clearing avoids trading off in-expectation and per-scenario characteristics and yields unique energy and reserves prices. However, current formulations rely on symmetric control policies based on the aggregated system imbalance, which restricts balancing reserve providers in their energy and reserve commitments. This paper extends existing chance-constrained market-clearing formulations by leveraging node-to-node and asymmetric balancing reserve policies and deriving the resulting energy and reserve prices. The proposed node-to-node policy allows for relating the remuneration of balancing reserve providers and payment of uncertain resources using a marginal cost-based approach. Further, we introduce asymmetric balancing reserve policies into the chance-constrained electricity market design and show how this additional degree of freedom affects market outcomes.

I. INTRODUCTION

The deployment of renewable energy sources (RES) challenges the efficiency of wholesale electricity markets, which largely treat RES injections as deterministic and do not internalize their stochasticity rigorously. Although, as Hobbs and Oren discuss in [1], recent market design improvements have been “primarily incremental in nature,” benefiting from enhanced computational capabilities and supply/demand technologies. As a result of these incremental changes, market-clearing procedures have become increasingly complex and market outcomes “are not transparent and perhaps have contributed to decreases in trading activity” [1]. This lack of transparency inhibits meaningful interpretations of energy and reserve allocations and prices, i.e., there is no technically and economically sound intuition on (i) which resources drive the need for balancing reserve and how much they should pay for it, and (ii) which resources are most efficient to mitigate this stochasticity and how much they should be paid. This paper aims to develop a stochastic market design that allows for such interpretations and insights into the energy and reserve price formation under RES stochasticity.

Existing stochastic market designs rely on either scenario-based stochastic programming [2] or chance-constrained [3] dispatch models, which outperform deterministic benchmarks in terms of the total operating cost and the accuracy of reserve allocations [4]. In [5]–[7], a two-stage scenario-based stochastic programming framework is used for the day-ahead market-clearing optimization, which yields scenario-specific locational marginal prices (LMPs). Although these LMPs are useful to understand dispatch and price implications of each scenario, it is impossible to ensure cost recovery (i.e., each producer recovers its production cost from market outcomes) and revenue adequacy (i.e., the payment collected by the market from consumers is greater than the payment by the market to producers) simultaneously in each scenario and in expectation over all scenarios without welfare losses and relying on out-of-market corrections and uplift payments [8].

As an alternative to [5]–[8], the work in [9]–[13] developed a stochastic market design that internalizes the RES stochasticity by means of its statistical moments (e.g., mean and variance) and chance constraints from [3]. Although stochastic by design, the models in [9]–[13] yield deterministic reformulations that computationally outperform scenario-based formulations, see [3], and produce uncertainty- and risk-aware LMPs and reserve prices. Although these prices capture all uncertainty realizations assumed, they are scenario-agnostic, which guarantees cost recovery and revenue adequacy for convex markets [10], [13], as well as minimizes the uplift for non-convex markets [10]. Further, [10] shows the chance-constrained framework makes it possible to ensure the cost recovery for each uncertainty realization and in expectation without welfare losses. The qualitative analyses in [10]–[12] show that these LMPs do not explicitly depend on statistical moments and risk preferences of the market, while reserve prices explicitly depend on these parameters. Despite these computational and market design advantages relative to [5]–[8], the previously developed chance-constrained frameworks have several limitations. First, they typically assume that the RES stochasticity is symmetric, which does not hold in practice, where upward and downward reserve needs in fact vary [14], [15]. Second, while allowing for a nodal reserve allocation, they lack a nodal reserve pricing mechanism, thus preventing from fairly charging and remunerating those resources that drive the need for and provide balancing reserve, respectively.

This paper extends the market design originally proposed in [9]–[12] to accommodate asymmetric reserve provision, node-to-node reserve pricing, and provide techno-economic insights on the energy and reserve price formation process under uncertainty. Considering the asymmetric reserve provision leads to appropriately sized and allocated reserve requirements drawn from empirical RES statistics (e.g., moments), while the node-to-node reserve pricing mechanism leads to the transparent allocation of (i) uncertainty costs among RES resources and (ii) reserve payments among producers, thus incentivizing the
II. ASYMMETRIC CHANCE-CONSTRAINED OPF

Consider an electricity market operator that uses an optimal power flow (OPF) formulation to compute a least-cost generator schedule. As shown in [9]–[13], traditional deterministic OPF-based market designs (see [16] for reference) can be efficiently robustified against uncertain injections from RES by means of chance constraints. For this purpose, injections at every network node \( u \) that hosts an uncertain RES resource is modeled as a random variable \( (w_u) \) using a given forecast value \( (w_u) \) and a random forecast error term \( (\omega_u) \) as follows:

\[
\omega_u = w_u + \omega_u, \tag{1}
\]

Typically, \( \omega_u \) is assumed to be zero-mean and normally distributed, [3], [4], [9], [11]–[13], [17]. However, in practice, empirical measurements of solar and wind power forecast errors are often asymmetric and can not be captured well by a normal distribution, [14], [15]. As a result, the impact of forecast errors from nodes with significant asymmetries might be over- or under-estimated. For example, by inspecting day-ahead forecast and actual generation data recorded at two nodes in the ENTSO-E (European Network of Transmission System Operators for Electricity) in 2020, plotted in Figure 1, various deviations of the forecast error from a normal distribution become evident. While the forecast at the German node, Figure 1(a), is symmetric and may be parametrized as a normal distribution, the forecast at the Italian node, Figure 1(b), is strongly skewed to overestimate generator output and a normal distribution is unsuitable to model the forecast error distribution. Additionally, there exists a tendency of RES operators to report generation values lower than the day-ahead forecast; which could be a result of strategic forecast offering by the generator in order to obtain additional benefits from complex and not always transparent market regulations. See in [18] Section 6.6.]

As a result, assuming normally distributed (symmetric) forecast errors in combination with a symmetric balancing reserve policy as in [3], [4], [9], [11], [12], [17], [19]–[24] can lead to ineffective and inefficient operating decisions and electricity prices. Therefore, to adequately capture possible forecast error asymmetries and improve the efficiency of balancing reserve quantification and allocation, this paper explicitly models negative and positive forecast errors (i.e., real-time energy deficit and surplus, respectively) and proposes an asymmetric, node-to-node balancing reserve policy. Sections II-A and II-B below describe the asymmetric model of uncertain RES injections and introduce the proposed balancing reserve policy, respectively. Section II-C derives the resulting market-clearing optimization.

A. Asymmetric Uncertainty Model

Consider random variable \( \omega_u \):

\[
\omega_u = \omega_u + \omega_u^*, \tag{2}
\]

where \( \omega_u \leq 0 \) and \( \omega_u^* \geq 0 \) are the negative and positive components of the forecast error such that \( \omega_u^* \omega_u^* = 0 \), i.e., \( \omega_u \) and \( \omega_u^* \) are mutually exclusive events. First, we assume that any expected systematic forecast error can be considered as a fixed parameter in the model (e.g., by attributing it to fixed load forecasts), so that the total imbalance (\( \omega_u \)) can always be corrected to be zero-mean. Second, from (2) and the linearity of the expectation operator, it follows that \( E[\omega_u] = E[\omega_u^* + \omega_u^*] = 0 \), i.e., \( E[\omega_u^*] = -E[\omega_u] = \mu_u \).

Figures 2(a)–(b) illustrate this representation, which allows for modeling asymmetric distributions.

In turn, mean \( \mu_u \) of the forecast error distributions can be obtained by considering them as distributions truncated at zero. Thus, for an estimated continuous distribution, it follows:

\[
\mu_u = E[\omega_u^*] = \int_{0}^{\infty} \omega_u f_{\omega_u}(\omega_u) d\omega_u, \tag{3}
\]

where \( f_{\omega_u} \) is the probability density function of \( \omega_u \).

B. Asymmetric Node-to-Node Reserve Balancing

To ensure that the power system remains balanced (i.e., generation equals demand at all times), controllable generators with balancing reserve capabilities react to any real-time imbalances \( \omega_u \) or \( \omega_u^*_u \) by increasing or decreasing their power outputs. We model this balancing reserve policy using asymmetric “node-to-node” balancing participation factors.
\(\alpha^-=\{0,1\}\) and \(\alpha^+=\{0,1\}\) that model the participation of a generator at bus \(i\) in compensating a negative and positive imbalance caused by a stochastic RES at bus \(u\). Note that for ease of notation, we assume that each node \(i\) hosts exactly one controllable generator and each node \(u\) hosts exactly one uncertain RES. The resulting uncertain power output \(p_i\) of each controllable generator is then given by:

\[
p_i = p_i - \sum_u (\alpha^-_{iu} \omega_u + \alpha^+_{iu} \omega_u^+), \quad \forall i
\]  

(4a)

\[
\sum_i \alpha^-_{iu} = 1, \quad \sum_i \alpha^+_{iu} = 1, \quad \forall u
\]  

(4b)

where \(p_i\) and \(\alpha^-_{iu}, \alpha^+_{iu}\in [0,1]\) are decision variables that define the power output and asymmetric participation factors for balancing participation for generator \(i\) and positive forecast errors,

Appendix A for details):

Thus, the capacity constraints (7) can be reformulated

subject to:

\[
\begin{align*}
(\lambda_i): & \quad p_i + \sum_{u\in U_i} w_u - D_i - \sum_{j\in C_i} f_{ij} = 0, \quad \forall i \\
(\psi_{ij}): & \quad f_{ij} - \frac{1}{X_{ij}} (\theta_i - \theta_j) = 0, \quad \forall i, j \in C_i \\
(\eta_{ij}): & \quad f_{ij} \leq T_{ij}, \quad \forall i, j \in C_i \\
(\chi_u): & \quad \sum_i \alpha^-_{iu} = 1, \quad \forall u \\
(\chi_u^+): & \quad \sum_i \alpha^+_{iu} = 1, \quad \forall u
\end{align*}
\]  

(9a)–(9f)

by means of the Chebyshev approximation, as the following equivalent and deterministic constraints in (8a) and (8b):

\[
p_i - M \cdot A_i + z_i S_i \leq \bar{P}_i, \quad \forall i
\]  

(8a)

\[
p_i + M \cdot A_i + z_i S_i \leq \underline{P}_i, \quad \forall i
\]  

(8b)

where the parameter \(z_i=\sqrt{(1-\epsilon_i)/\epsilon_i}\).

Using (4b), (6), (8a), and (8b), the OPF problem with a deterministic equivalent of the chance-constrained generator dispatch and node-to-node asymmetric balancing reserve policy (OPF-N2N-AB) is formulated in Model 1. The objective function in (9a) minimizes the expected system generation cost and is subject to dc power flow constraints (9b)–(9e), asymmetric node-to-node balancing reserve adequacy requirements (9f)–(9g), and chance-constrained generator output limits (9h)–(9i). In the OPF-N2N-AB, the network buses are indexed by \(i\), the RES buses by \(u\in U_i\), and the power lines by the tuple \((i, j)\) with \(j \in C_i\); where \(C_i\) is the set of nodes directly connected to node \(i\). The nodal demand is given by \(D_i\) and the line reactance by \(X_{ij}\). The line flow is represented by \(f_{ij}\) and the nodal voltage angle by \(\theta_i\). Line capacity limits are set to \(T_{ij}\).

The decision variables of OPF-N2N-AB are collected in set \(\Xi = \{p_i, \alpha^-_{iu}, \alpha^+_{iu}, f_{ij}, \theta_j\}\). Greek symbols, in parenthesis, define dual multipliers associated with each constraint. Note that OPF-N2N-AB is a non-linear program (NLP) with second-order conic constraints. Specifically, objective (9a) contains bi-linear terms, which are dealt with in Section VII. However, the second-order conic solution space formed by constraints (9b)–(9i) is convex and, thus, enables the analytical derivations in the following sections.

III. PRICING ENERGY AND BALANCING RESERVE PROVISION

From the perspective of an electricity market operator, OPF-N2N-AB resembles a market-clearing problem computing the optimal (least-cost) generator dispatch and balancing...
reserve decisions $p_i^*$, $\alpha_i^*, \alpha_i^{+*}$ and efficient prices for energy $\pi^p$ and reserves $\pi^u$, $\pi^*$, respectively. More specifically, the price-quantity tuples $(\pi^p, p^p)$, $(\pi^u, \alpha^u)$, $(\pi^*, \alpha^{+*})$ are efficient, if they support a competitive equilibrium, i.e., they ensure \cite{17}:

1) The market clears at $p_i^* = \sum_{u \in G} w_u - \sum_{j \in G_i} f_{ij} = D_i$, $\forall i$,

$$\sum_{i \in G} \alpha_i^* = \sum_{i \in G} \alpha_i^{+*} = 1, \forall u$$

2) Prices $\pi^p$, $\pi^*$, and quantities $p_i^*$, $\alpha_i^{+*}$ maximize the profit of $i$ given by $\Pi_i = \pi^p p_i + \sum_u (\pi^u \alpha_i^u + \pi^+ \alpha_i^{+*}) - C_i$, $\forall i$, i.e., there is no incentive to deviate these market outcomes.

To derive these prices from OPF-N2N-AB, we use duality theory similar to the previous work on stochastic market designs with chance constraints \cite{11, 12, 17}. However, unlike in \cite{11, 12, 17}, the prices derived from OPF-N2N-AB will support asymmetric balancing reserve needs and account for node-to-node participation factors, thus enabling a more precise allocation and deployment of balancing reserve. Thus, we begin with the following proposition:

**Proposition 1.** Consider the OPF-N2N-AB model. Let $\lambda_i^*$, $\chi_u^*$ be the locational marginal prices (LMPs) for energy and asymmetric locational balancing prices (A-LBPs) defined as dual multipliers of constraints \cite{29, 31, 20} respectively. Then, these prices can be expressed as:

$$\lambda_i = 2c_{2,i} (p_i - M.A_i) + c_{1,i} + \delta_i - \bar{\delta}_i, \forall i \quad (10a)$$

$$\chi_u = \frac{1}{|G|} \sum_i [\mu_i \lambda_i + (\Sigma_i^u A_i) (2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i))], \forall u \quad (10b)$$

$$\chi_u^+ = \frac{1}{|G|} \sum_i [\mu_i \lambda_i + (\Sigma_i^u A_i) (2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i))], \forall u \quad (10c)$$

where $\Sigma_i^u$ and $\Sigma_i^+$ are the rows of the covariance matrix $\Sigma$ related to the asymmetric forecast errors $\omega_u$ and $\omega_u^+$, respectively.

**Proof.** Consider the first-order optimality conditions of \cite{9} for variables $p_i$, $\alpha_i^u$, and $\alpha_i^{+*}$:

$$2c_{2,i} (p_i - M.A_i) + \delta_i - \bar{\delta}_i - \lambda_i = 0, \forall i \quad (11a)$$

$$\Sigma_i^u A_i \left(2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i)\right) + \mu_i \left(2c_{2,i} (p_i - M.A_i) + \delta_i - \bar{\delta}_i\right)$$

$$- \chi_u = 0, \forall i, u \quad (11b)$$

$$\Sigma_i^u A_i \left(2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i)\right) - \mu_i \left(2c_{2,i} (p_i - M.A_i) + \delta_i - \bar{\delta}_i\right)$$

$$- \chi_u^+ = 0, \forall i, u \quad (11c)$$

Expression \cite{11a} immediately follows from separating $\lambda_i$ from \cite{11a}. Note that $\chi_u$ and $\chi_u^+$ are the dual variables of the nodal balancing reserve adequacy constraints in Eq. \cite{29} and \cite{31} and, therefore, are specific to the balancing needs of node $u$.

Note that expressions \cite{11b} and \cite{11c} have the same value for each generator $i$ and can be summed over $|G|$ generators, leading to $\sum_{i \in G} \chi_i = |G| \chi_u$ and $\sum_{i \in G} \chi_i^+ = |G| \chi_u^+$. Hence, dividing \cite{11b} and \cite{11c} by $|G|$ leads to \cite{10b} and \cite{10c}.

We now prove that the dual variables used as prices in Proposition \cite{11} and the primal dispatch decisions satisfy the conditions for a competitive equilibrium.

\footnote{For example, in a system with two RES $\Sigma_i = [\text{Var}(\omega_i), \text{cov}(\omega_i, \omega_j), \text{cov}(\omega_i, \omega_j), \text{cov}(\omega_i, \omega_j^2)].$}

### Model 2 GPM_i-N2N-AB

\text{max.} \quad \Pi_i = \pi^p p_i + \sum_u (\pi^u \alpha_i^u + \pi^+ \alpha_i^{+*}) - C_i \quad (12a)\

subject to:

\begin{align*}
(\delta_i): & \quad p_i - M.A_i + z_i S_i \leq \bar{T}_i, \quad \forall i \quad (12b) \\
(\delta_i): & \quad p_i - M.A_i + z_i S_i \leq -\bar{p}_i, \quad \forall i. \quad (12c)
\end{align*}

**Theorem 1.** Consider the OPF-N2N-AB model and let $\lambda_i^*$, $\chi_u^*$ and $\chi_u^{+*}$ denote the optimal value of the dual multipliers. Using $\pi^p = \lambda_i^*$, $\pi^u = \chi_u^*$ and $\pi^+ = \chi_u^{+*}$ as the LMPs and A-LBPs constitutes a competitive energy and balancing reserve equilibrium prices.

**Proof.** Each generator $i$ is modeled using a profit-maximizing problem (GPM$_i$) given in Model 2, which is abbreviated below as GPM$_i$-N2N-AB. The first-order optimality conditions of this optimization with respect to variables $p_i$, $\alpha_i^u$ and $\alpha_i^{+*}$ lead to the expressions \cite{13a, 13b, 13c}, respectively:

$$\lambda_i = 2c_{2,i} (p_i - M.A_i) + c_{1,i} + \delta_i - \bar{\delta}_i, \forall i \quad (13a)$$

$$\pi^u = \mu_i (2c_{2,i} (p_i - M.A_i) + c_{1,i} + \delta_i - \bar{\delta}_i) + (\Sigma_i^u A_i) \left(2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i)\right), \forall i, u \quad (13b)$$

$$\pi^+ = -\mu_i (2c_{2,i} (p_i - M.A_i) + c_{1,i} + \delta_i - \bar{\delta}_i) + (\Sigma_i^u A_i) \left(2c_{2,i} + \frac{z_i}{S_i} (\delta_i + \bar{\delta}_i)\right), \forall i, u \quad (13c)$$

Due to the equality of first-order optimality conditions in \cite{13a}–\cite{13c} and in \cite{10a}–\cite{10c}, it follows that the primal $(p_i, \alpha_i^u, \alpha_i^{+*})$ and dual $(\lambda_i, \chi_u^*, \chi_u^{+*})$ solution from the OPF-N2N-AB model, which satisfies constraints \cite{29} and \cite{31}, also solves the GPM$_i$-N2N-AB model for each generator, which maximizes $\Pi_i$. Hence, the prices obtained in Proposition \cite{11} and derived from the OPF-N2N-AB \cite{10a}–\cite{10c}, constitute a competitive equilibrium, i.e. $\pi^p = \lambda_i^*$, $\pi^u = \chi_u^*$ and $\pi^+ = \chi_u^{+*}$.

### IV. Stochastic Markets under Symmetric and System-wide Balancing Reserve Policies

Using the results derived in Section III we consider several particular cases that allow for relating the obtained prices to existing results in \cite{3, 4, 9, 11, 12, 17, 21, 24}. A summary of the relationship between the pricing for the different balancing reserve policies is provided in Table I.

#### A. Asymmetric System-wide Balancing – OPF-SW-AB

Instead of the node-to-node balancing reserve policy \cite{4a}, the proposed OPF-N2N-AB can be modified to use a balancing reserve policy based on the aggregated system-wide uncertainty as in, e.g., \cite{9, 11, 12, 17}. Thus, the system-wide uncertainty is given by:

$$\Omega = \sum_u \omega_u^w, \quad \Omega^+ = \sum_u \omega_u^{w+} \quad (14)$$

where in turn $\Omega^+$ and $\Omega$ are related as: 
and the the node-to-node participation factors used in the OPF-N2N-AB are set to system-wide participation factors by enforcing \(\alpha^*_u = \alpha^*_i\) and \(\alpha^*_{iu} = \alpha^*_{ui}\), which leads to \(A_1 = [\alpha^*_1, \alpha^*_2, \ldots, \alpha^*_n]\). Although this approach resembles the pricing mechanism from [9], [11], [12], [17], there is a notable distinction of this representation, because it still accounts for asymmetric distributions and asymmetric provision of balancing reserve.

We define \(\Omega^+ = [\Omega \Omega^\top]\) and its expected value as

\[
M = \mathbb{E}[\Omega^+] = [-\nu \nu],
\]

where \(\nu = \sum_u \nu_u = \sum_u \mathbb{E}[\omega_u^*] = -\sum_u \mathbb{E}[\omega_u^\top]^\top\) is the expected value of the aggregated asymmetric forecast errors. Finally, the covariance matrix of \(\Omega^+\) is defined as \(\Sigma = \text{cov}(\Omega^+)\).

1) Generation Output: Under the asymmetric balancing reserve policy with system-wide participation factors, we replace Eqs. (4a) and (4b) with:

\[
\begin{align*}
p_i &= p_i - \alpha_i \Omega - \alpha^*_i \Omega^*, \\
(\alpha_i, \chi_i): & \quad 1 = \sum_i \alpha_i = \sum_i \alpha^*_i.
\end{align*}
\]

2) Energy and Balancing Prices: Following the procedure described in Proposition 1 we derive the following prices:

\[
\begin{align*}
\lambda_i &= c_{2,i} p_i - M \cdot A_i + c_{1,i} + \delta_i - \delta^*_i, \\
(\chi^-, \chi^+): & \quad 1 = \sum_i \chi_i = \sum_i \chi^*_i.
\end{align*}
\]

where \(\Sigma^-\) and \(\Sigma^+\) are the rows of covariance matrix \(\Sigma\) related to the asymmetric forecast errors (\(\Omega\) and \(\Omega^\top\)). Additionally, \(S_i = \sqrt{\Sigma i^\top \Sigma i}\). Note that the value of \(S_i\) in the OPF-SW-AB model and in the derived balancing reserve policies presented below does not equal to the value of \(S_i\) in the OPF-N2N-AB model. Even though the expression for its calculation is the same, the way in which the vector \(A_i\) and the matrix \(\Sigma\) are formed depends on the balancing reserve policy itself. Notably, the A-LBPs become dependent on the aggregated asymmetric uncertainty in the system and as such are equal for all the RES nodes, i.e., the generators are paid to balance the system imbalance independently of the sources contributing to this imbalance. Hence, relative to OPF-N2N-AB model, a system-wide balancing approach does not capture a one-to-one relationship between individual sources of uncertainty and reserve providers, which causes inefficient price signaling.

B. Symmetric Node-to-Node Balancing – OPF-N2N-AB

Another modification of OPF-N2N-AB is derived under the assumption that controllable generators provide balancing reserve using nodal but symmetric participation factors \(\alpha_{iu}\) for symmetric forecast errors \(\omega_u = \omega^*_u + \omega^*\) as in [21], [24]. Under these assumptions, we obtain \(\alpha_{iu} = \alpha^*_{iu} = \alpha^*_{ui}\) and \(A_1 = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{iL}]\). Random variable \(\omega\) is the vector of nodal forecast errors and \(\Sigma\) is the covariance matrix of the nodal forecast errors.

1) Generation Output: Using these assumptions, (4a), (4b), (6) can be replaced with:

\[
\begin{align*}
p_i &= p_i - \sum_u (\alpha_{iu} \omega_u), & \forall i \quad (17a) \\
C_i &= c_i(p_i) + c_{2,i} S_i, & \forall i, \quad (17b) \\
(\chi_u): & \quad 1 = \sum_i \alpha_{iu}, & \forall u \quad (17c)
\end{align*}
\]

Expression (17b) simplifies the expected balancing cost, which cancels out terms due to the symmetric forecast errors.

2) Probabilistic Capacity Limits: Similarly to the expected cost in (17b), the capacity limits do not consider the asymmetric nodal RES forecast errors, which makes it possible to recast them as second-order conic constraints, which only depend on the variances of the nodal forecast errors:

\[
\begin{align*}
(\delta_i): & \quad p_i + \chi_i S_i \leq T_i, & \forall i \quad (17d) \\
(\delta_u): & \quad -p_u + \chi_u S_u \leq -P_u, & \forall u \quad (17e)
\end{align*}
\]

C. System-wide Symmetric Balancing – OPF-SW-AB

The most studied chance-constrained electricity market design has system-wide, symmetric RES forecast errors and symmetric generation response firstly introduced in [9] and then applied in [11], [12], [17]. This case can be modeled within the proposed OPF-N2N-AB model, if \(\Omega\) is the system-wide RES forecast error, i.e., \(\Omega = \sum_u \omega_u\), \(A_1 = A^*_1 = [\alpha_i, \forall u]\) and \(S_1 = S_i = \alpha_i\), where \(S^2\) is the total sum over the covariance matrix, \(s^2 = e \Sigma e^\top\).

1) Generation Output: Using these assumptions, (4a), (4b), (5), (6) can be replaced with:

\[
\begin{align*}
C_i &= c_i(p_i) + c_{2,i} \alpha^2 S^2, & \forall i. \quad (18a) \\
p_i &= p_i - \alpha_i \Omega, & \forall i \quad (18b) \\
(\chi): & \quad 1 = \sum_i \alpha_i, \quad (18c)
\end{align*}
\]

2) Probabilistic Capacity Limits: Similarly to (17d)–(17e), the capacity limits are reduced to the linear constraints:
(20) can be derived by factorizing it in the following way:
\[
\chi_u = \frac{1}{|\mathcal{G}|} \sum_i \left( \sum_{v,w} \left( \sigma_{uw} \left( 2c_{2,i} + z_i \frac{\delta_i + \delta_v}{\alpha_i s} \right) \right) \right), \quad \forall u
\]

An analytical expression for (20) can be derived by factorizing (17g) in the following way:
\[
\chi_u = \frac{1}{|\mathcal{G}|} \sum_i \left( \sum_{v,w} \left( \sigma_{uw} \left( 2c_{2,i} + z_i \frac{\delta_i + \delta_v}{\alpha_i s} \right) \right) \right), \quad \forall u
\]

Finally, rearranging the terms in (21e) to match (20) leads to:
\[
\beta_u = \frac{\sum_v \alpha_v \sigma_{uv}}{\sum_{v,w} \sigma_{vw}}, \quad \forall u.
\]

Parameter \( \beta_u \) does not depend on system parameters, primal and dual variables. Thus, the nodal balancing reserve price depends on the effect that the RES at node \( u \) has on the system-wide variance, i.e., the greater the introduced uncertainty by RES at node \( u \), the greater the price that it must pay for deviations from its forecast. Further, it follows that \( \sum_u \beta_u = 1 \), which makes it possible to interpret \( \beta_u \) as the contribution of the RES at node \( u \) to the overall system balancing reserve price.

Since the LBPs \( \chi_u \) carry information on the marginal balancing cost that the RES at node \( u \) adds to the system, they can be used to incentivize system-beneficial RES deployment strategies. For example, a RES investor would prefer to install their RES project at a node with low, or ideally negative, correlation with the existing RES, so payments for introducing uncertainty and increasing balancing reserve requirements will be lower, possibly leading to a neutral or even negative value of \( \beta_u \). The balancing reserve prices derived from the proposed framework highlight the value that the system as a whole derives from complementary RES generation, i.e., RES generators with a negative correlation to other RES, since a RES that compensates the imbalance of others provides load balancing reserve support and as such its balancing costs are lower.

VI. ANALYSIS OF RES COSTS

Typically, RES resources submit zero-price bids to electricity market operators, reflecting their zero or near-zero marginal production cost. However, they inflict a non-zero marginal cost on the system by driving the need for procuring additional balancing reserves. This section compares two approaches to quantify these costs by (i) using the balancing reserve payments to controllable generation resources and by (ii) using the marginal balancing cost. We show that both approaches can be equivalent.
node $u$ as $C_u^α$ and define it as the sum of payments to reserve providers:

$$C_u^α = χ_u \sum_i α_{iu}, \quad \forall u \quad (22a)$$

and since $\sum_i α_{iu}=1$ we obtain:

$$C_u^α = χ_u \sum_i \left(\sum_{uv} \alpha_{iv} σ_{uv} \frac{ω_{2i,S_i + z_i(\bar{u} + \bar{v})} S_i}{S_i} \right), \forall u \quad (22b)$$

The balancing reserve compensation cost $C_u^α$ paid by the RES at node $u$ depends on the uncertainty it introduces to the system ($σ_{u,v}$) and its correlation with other nodes $v$ ($σ_{u,v}$). If the RES at $u$ does not introduce uncertainty to the system, then $C_u^α = 0$.

**B. RES Costs as Marginal Balancing Costs**

Since the RES are modelled with a zero marginal operating cost, their revenue can be calculated as:

$$R_u = -\frac{∂L_{SN}}{∂w_u} w_u, \quad \forall u \quad (23)$$

where $L_{SN}$ is the Lagrangian function for the OPF-N2N-SB model under the node-to-node symmetric balancing reserve policy presented in Section [V-B]. If the standard deviation $σ_u$ of $ω_u$ is proportional to its forecast injection, i.e., $σ_u = κ_u w_u$, and $ζ_{uv}$ is the correlation coefficient between uncertain injections at nodes $u$ and $v$ [22]. Then, the marginal revenue for the RES resource can be computed as:

$$-\frac{∂L_{SN}}{∂w_u} = λ_u - \sum_i \frac{\sum_{uv} α_{iv} σ_{uv} λ_{iu} σ_u}{S_i} \frac{ω_{2i,S_i + z_i(\bar{u} + \bar{v})}}{S_i}, \quad \forall u \quad (24)$$

where the positive part of expression (24) can be interpreted as the active power price paid to the RES at node $u$, $π_u^+$, and the negative one as the cost associated with its stochastic generation, $c_u$. If the the RES at node $u$ introduces no uncertainty to the system, i.e., $κ_u=0$, then $c_u=0$. The operation cost of the RES at node $u$ is then given by:

$$C_u^w = c_u w_u, \quad \forall u \quad (25)$$

**C. Cost Equivalence between the Compensating and Marginal Balancing Costs**

The equivalence between $C_u^α$ and $C_u^w$ can be demonstrated by examining [25]:

$$C_u^α = c_u w_u$$

$$= w_u \sum_i \left(\sum_{uv} α_{iv} σ_{uv} λ_{iu} σ_u \frac{ω_{2i,S_i + z_i(\bar{u} + \bar{v})}}{S_i} \right), \forall u \quad (26a)$$

Next, using $σ_u = κ_u w_u$ and $σ_{uv} = ζ_{uv} σ_u σ_v$, it follows:

$$C_u^α = \sum_i \left(\sum_{uv} α_{iv} σ_{uv} \frac{ω_{2i,S_i + z_i(\bar{u} + \bar{v})}}{S_i} \right), \forall u \quad (26c)$$

where replacing $χ_u$ results in:

$$C_u^w = χ_u \sum_i α_{iu} = C_u^α, \quad \forall u \quad (26e)$$

As shown in Eq. (26e), the total operating cost of the RES resources is equivalent when computed from balancing reserve payments $C_u^α$ and marginal costs $C_u^w$. However, the latter holds only if the total operating cost of the RES resources is computed under the assumptions that the variances, $σ_{u,v}$, are proportional to the injected power and fixed node-to-node correlation coefficients, i.e., $σ_u = κ_u w_u$. If these assumptions do not hold, the total operating cost of the RES resources can only be computed via the balancing reserve payments in (26b).

**VII. NUMERICAL EXPERIMENTS**

In this section, the proposed OPF-N2N-AB model and its modifications from Section [V] are evaluated on a modified IEEE 118-bus case [12]. The experiments were conducted using Julia 1.53, Jupyter 0.21.6 and Mosek [25] on an Intel i7-1165G7 processor clocked at 2.80GHz with 16 GB of RAM. The code and data supplement can be downloaded from [26].

**A. Test Case and Stochastic Data**

For this case study, we add 11 utility-scale wind farms to the IEEE-118 test system as the RES resources. Table I indicates their placement in the system. Forecasts and forecast error distributions are emulated from real data of onshore wind.
farms in different European regions collected between January 1st and October 1st of 2020 from [27]. Table II summarizes the normalized wind power data.

To study the RES impact at different penetration levels, we create four renewable penetration scenarios by scaling the nominal installed capacity of each wind farm by 50%, 100%, 200%, and 400%. In these scenarios, the available generation from all wind farms covers 4.8%, 9.7%, 19.4%, and 38.8% of total demand, respectively.

Every wind farm was assigned an independent normal distribution \( \omega_u \sim N(0,\sigma_u) \) with \( \sigma_u \) being estimated from the data in [27]. For asymmetric balancing reserve policies, we generate truncated normal distributions \( \omega_u^+ \) and \( \omega_u^- \) from \( \omega_u \) with the following parameters:

\[
\sigma_u^+ = \sigma_u \sqrt{\frac{2\pi - 4}{2\pi}}, \quad \mu_u^+ = \sigma_u \sqrt{\frac{2}{\pi}}
\]

We set the confidence level of chance constraints at \( (1-\epsilon) = 0.99 \), obtaining \( z_i = \sqrt{(1-\epsilon)/\epsilon} \approx 9.95 \) with the Chebyshev inequality as in [17].

### B. Solution Approach

Chance-constrained market clearing procedures as proposed in [9]–[12] can be solved in a single or two-step procedure using off-the-shelf solvers. However, solving OPF-N2N-AB and its modifications shown in Section [VII] requires dealing with bilinear terms in the objective function. Therefore, we use McCormick envelopes to convexify the bilinear terms in \( \Phi \).

The obtained envelopes are then sequentially tightened around the operating point produced at the previous iteration to improve accuracy. The proposed sequential solution approach based on using off-the-shelf solvers is described below:

- **Step 1.** Obtain a solution to a convex approximation of the original OPF-N2N-AB problem.
- **Step 2.** Tighten the convex approximation around Step 1’s solution by using a decreasing scalar factor to reduce the distance between its obtained operation point and the current bounds.
- **Step 3.** Stop if the convex approximation is good enough.

If not, update and return to Step 1 and repeat.

This solution approach has two advantages. First, the convexified problem always yields a feasible solution to the OPF-N2N-AB optimization, because the convexification only affects the objective function. Second, an optimal solution \( \hat{x}^+ \) of the convexified problem provides an upper bound to the objective of the original non-convex problem, i.e., \( C(\hat{x}^+) \leq C(x^+) \), where \( C(\cdot) \) is the objective function of OPF-N2N-AB, and \( x^+ \) is an optimal solution to the original non-convex OPF-N2N-AB version. On the other hand, the optimal objective value of the convexified problem \( \hat{C}(\hat{x}^+) \) provides a lower bound to the objective of the original non-convex problem, i.e., \( \hat{C}(\hat{x}^+) \leq C(x^+) \).

Therefore, we can observe that \( \hat{C}(\hat{x}^+) \leq C(x^+) \leq C(\hat{x}^+) \). However, while \( C(x^+) \) cannot be computed directly, both values \( \hat{C}(\hat{x}^+) \) and \( C(\hat{x}^+) \) can be obtained directly. Thus, the stopping criteria threshold from Step 3 is set by the approximation accuracy \( 100(\hat{C}(\hat{x}^+)) / C(x^+) - 1 \), representing an optimality gap. Once the approximation accuracy is below this threshold, we can derive the prices using the convexified OPF-N2N-AB version.

### C. Comparison of Balancing Reserve Policies

1) **System-wide and node-to-node balancing**: Table III summarizes the main results for our base scenario with a renewable penetration level of 9.7%.

Our results show that symmetric, zero-mean balancing yields the same objective value in both system-wide and node-to-node policies. Moreover, by inspecting dual values \( \chi \) and \( \chi_u \) we observe that \( \sum_u \chi_u = \chi \) as shown analytically in Eq. (20) above. Furthermore, the dependence of \( \chi_u \) on the variance \( \sigma_u \) is illustrated in Figure 3. As such, a node-to-node balancing reserve policy provides better pricing signals by reflecting the influence the forecast error variance introduced by the RES has on the price paid for its balancing.

2) **Asymmetric balancing provision with varying amounts of renewable penetration**: Next, we study the asymmetric balancing reserve provision as described in Section VII-B with different renewable penetration levels. To facilitate the comparison, the results in Figure 4 are presented with the results obtained using the symmetric balancing policy (as in Section VII-C1 above). Figure 4 contains the best objective value found in the sequential process described in Section VII-B such that \( C(\hat{x}^+) \approx C(x^+) \). Further, it shows the payments collected from consumers \( (\lambda \Delta) \), payments made to wind farms \( (\Delta W) \) and conventional generators \( (\sum_u \Pi_u) \), respectively, for the different renewable penetration levels defined above.

With an increasing penetration of wind generation, system cost given by objective value \( C(x^+) \) are decreasing due to
the greater availability of low-cost energy from RES. This leads to notable differences in payments made to conventional generators. However, for low renewable penetration rates, the total cost barely change between balancing reserve policies.

Our experiments confirm the revenue adequacy property of the prices, i.e., $\lambda D = \lambda W + \sum_i \Pi_i$. Table IV shows consumer payments, generator revenues and generation costs using different balancing reserve policies. The difference in consumer payments between asymmetric and symmetric balancing markets is defined as $\Delta \lambda D$. The approximation error introduced by the convexification is captured in the term $\text{err}$. As summarized in Figure 4 and Table IV markets with asymmetric node-to-node balancing policies show that significant reductions in consumer payments are possible, representing up to 9% of cost savings.

These results point to two observations. First, the solution procedure in Section VII-B performs well for system-wide asymmetric balancing policies. Given a solution, the objective differs by less than 0.0026% from the exact objective. Second, the proposed node-to-node balancing reserve policies yield consistent reductions in consumer payments relative to the symmetric balancing reserve policy.

Figure 5 provides insights as to how prices are formed under different balancing policies and renewable penetration. Where $z^*$ represents the objective value, $\lambda D$ consumer payments, $\lambda W$ wind payments and $\sum_i \Pi_i$ the generators’ revenue.

### Table III

| Scenario | $\lambda D$ | $\Delta \lambda D$ [%] | $\text{err}$ [%] |
|----------|-------------|-------------------------|-----------------|
| SW       | 188965.12   | -0.013                 | 4.0e-5          |
| N2N      | 188940.56   | -0.005                 | 0.70217         |

### Table IV

| Scenario | $\lambda D$ | $\Delta \lambda D$ [%] | $\text{err}$ [%] |
|----------|-------------|-------------------------|-----------------|
| A-SW     | 188940.56   | -0.013                 | 4.0e-5          |
| N2N      | 188940.56   | -0.005                 | 0.70217         |
other balancing policies are proven to be particular cases of the asymmetric node-to-node regulation one, thus yielding greater operational costs.

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A. Expected Cost Derivation: Asymmetric Node-to-Node Balancing

Generator i’s cost can be expressed in terms of nodal directional imbalances by

\[ c_i(p_i) = c_{i1}(p_i)^2 + c_{i2}(p_i) + c_{i0}, \quad \forall i \] (A.1)

where

\[ p_i = p_i - \sum_u (\alpha_{iu}^+ \omega_u^+ + \alpha_{iu}^- \omega_u^-), \quad \forall i. \] (A.1a)

Let the vectors \( A_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i|M|}, \alpha_{i1}^+, \alpha_{i2}^+, \ldots, \alpha_{i|M|}^+]^T \), \( \Omega = [\omega_1, \omega_2, \ldots, \omega_n, \omega_1^+, \omega_2^+, \ldots, \omega_n^+]^T \), then the expected generation costs can be calculated as:

\[ E[c_i(p_i)] = E[c_i(p_i - \Omega \cdot A_i)] = c_i(p_i) + E[Y_{1,i}] + E[Y_{2,i}], \quad \forall i \] (A.2)

where

\[ Y_{1,i} = -(2c_{i1}p_i + c_{i1})X_{1,i}, \quad \forall i \] (A.2a)

\[ Y_{2,i} = c_{i2}X_{2,i}^2, \quad \forall i \] (A.2b)

\[ X_{1,i} = \Omega \cdot A_i, \quad \forall i. \] (A.2c)

Let \( \Omega = M \), and since for a random variable \( Z \),

\[ E[Z^2] = \text{Var}(Z) + E[Z]^2, \]

\[ E[Y_{1,i}] = -(2c_{i1}p_i + c_{i1})(M \cdot A_i), \quad \forall i \] (A.3a)

\[ E[Y_{2,i}] = c_{i2}(M \cdot A_i)^2 + \text{Var}[\Omega \cdot A_i], \forall i. \] (A.3b)

The variance of a linear combination can be calculated by

\[ \text{var} \sum_n (b_n Z_n)^2 = \sum_n [b_n b_m \text{cov}(Z_n Z_m)], \]

where \( b_n \) is a scalar and \( Z_n \) a random variable. Let \( \Sigma = \text{Var}[\Omega] \) be the non-trivial covariance matrix of the forecast errors, we can then rewrite (A.3b) using (A.4) as

\[ E[Y_{2,i}] = c_{i2}((M \cdot A_i)^2 + \|A_i \Sigma^{1/2}\|^2_2), \quad \forall i. \] (A.5)

Finally, by using expression (A.2) the expected costs are

\[ E[c_i(p_i)] = c_i(p_i) - (2c_{i1}p_i + c_{i1})(M \cdot A_i) + c_{i2}((M \cdot A_i)^2 + \|A_i \Sigma^{1/2}\|^2_2), \quad \forall i. \] (A.6)