Permanent Magnet Synchronous Motors are Globally Asymptotically Stabilizable with PI Current Control

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Abstract

This note shows that the industry standard desired equilibrium for permanent magnet synchronous motors (i.e., maximum torque per Ampere) can be globally asymptotically stabilized with a PI control around the current errors, provided some viscous friction (possibly small) is present in the rotor dynamics and the proportional gain of the PI is suitably chosen. Instrumental to establish this surprising result is the proof that the map from voltages to currents of the incremental model of the motor satisfies some passivity properties. The analysis relies on basic Lyapunov theory making the result available to a wide audience.

Key words: Motor control, PI control, passivity theory, nonlinear control

1 Introduction

Control of electric motors is achieved in the vast majority of commercial drives via nested loop PI controllers [10, 11, 19]: the inner one wrapped around current errors and an external one that defines the desired values for these currents to generate a desired torque—for speed or position control. The rationale to justify this control configuration relies on the, often reasonable, assumption of time-scale separation between the electrical and the mechanical dynamics. In spite of its enormous success, to the best of our knowledge, a rigorous theoretical analysis of the stability of this scheme has not been reported. The main contribution of this paper is to (partially) fill-up this gap for the widely popular permanent magnet synchronous motors (PMSM), proving that the inner loop PI controller ensures global asymptotic stability (GAS) of the closed-loop, provided some viscous friction (possibly arbitrarily small) is present in the rotor dynamics, that the load torque is known and the proportional gain of the PI is suitably chosen, i.e., sufficiently high. The assumption of known load torque is later relaxed proposing an adaptive scheme that, in the spirit of the aforementioned outer-loop PI, generates, via the addition of a simple integrator, an estimate for it—preserving GAS of the new scheme.

Several globally stable position and velocity controllers for PMSMs have been reported in the control literature—even in the sensorless context, e.g., [2, 12, 22, 23] and references therein. However, these controllers have received an, at best, lukewarm reception within the electric drives community, which overwhelmingly prefers the aforementioned nested-loop PI configuration. Several versions of PI schemes based on fuzzy control, sliding modes or neural network control have been intensively studied in applications journals, see [9] for a recent review of this literature. To the best of our knowledge, a rigorous stability analysis of all these schemes is conspicuous by its absence.

The importance of disposing of a complete theoretical
analysis in engineering practice can hardly be overestimated. Indeed, it gives the user additional confidence in the design and provides useful guidelines in the difficult task of commissioning the controller. The interest of our contribution is further enhanced by the fact that the analysis relies on basic Lyapunov theory, using the natural (quadratic in the increments) Lyapunov function. Various attempts to establish such a result for PMSMs have been reported in the literature either relying on linear approximations of the motor dynamics or including additional terms that cancel some nonlinear terms, see [5, 6] and references therein—a standing assumption being, similarly to us, the existence of viscous friction.

The remainder of this paper is organised as follows. The models of the PMSM are given in Section 2. The problem formulation is introduced in Section 3. The passivity of the PMSMs incremental model and the PI controller are established in Section 4. The main stability results are provided in Section 5. Some simulation results are presented in Section 6. Finally, some concluding remarks and discussion of future research are given in Section 7.

Notation. For \( x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}, A > 0 \) we denote \( \| x \|^2 = x^\top x, \| x \|^2_A := x^\top A x \). For the distinguished vector \( x^* \in \mathbb{R}^n \) and a mapping \( C : \mathbb{R}^n \to \mathbb{R}^{n \times n} \), we define the constant matrix \( C^* := C(x^*) \).

2 Motor Models

In this section we present the motor model, define the desired equilibrium and give its incremental model.

2.1 Standard dq model

The dynamics of the surface-mounted PMSM in the dq frame is described by [10, 20]:

\[
\begin{align*}
L_d \frac{d i_d}{dt} &= -R_s i_d + \omega L_q i_q + v_d \\
L_q \frac{d i_q}{dt} &= -R_s i_q - \omega L_d i_d - \Phi + v_q \\
J \frac{d \omega}{dt} &= -R_m \omega + n_p [(L_d - L_q) i_d i_q + \Phi i_q] - \tau_L
\end{align*}
\]

(1)

where \( i_d, i_q \) are currents, \( v_d, v_q \) are voltage inputs, \( \omega \) is the electrical angular velocity, \( 2p_n \) is the number of pole pairs, \( L_d > 0, L_q > 0 \) are the stator inductances, \( \Phi > 0 \) is the back emf constant, \( R_s > 0 \) is the stator resistance, \( R_m > 0 \) is the viscous friction coefficient, \( J > 0 \) is the moment of inertia and \( \tau_L \) is a constant load torque.

Defining the state and control vectors as

\[ x := \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix}, \quad u := \begin{bmatrix} v_d \\ v_q \end{bmatrix}, \]

the system (1) can be written in compact form as

\[ D \dot{x} + [C(x) + R]x = Gu + d, \]

where

\[
D := \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & \frac{l}{n_p} \end{bmatrix} > 0, \quad R := \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & \frac{R_m}{n_p} \end{bmatrix} > 0
\]

\[
C(x) := \begin{bmatrix} 0 & 0 & -Lq \omega x_2 \\ 0 & 0 & Lq \omega x_2 + \Phi \\ Lq \omega x_2 - (Lq \omega x_1 + \Phi) & 0 \end{bmatrix} = -C^T(x)
\]

\[
G := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad d := \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad u := \begin{bmatrix} 0 \\ \frac{2 \pi}{n_p} \end{bmatrix}
\]

Besides simplifying the notation, the interest of the representation above is that it reveals the power balance equation of the system. Indeed, the total energy of the motor is

\[ H(x) = \frac{1}{2} x^\top D x, \]

whose derivative yields

\[
\dot{H} = -x^\top R x + y^\top u - x_1 \frac{\tau_L}{n_p}
\]

(2)

where we used the skew-symmetry of \( C(x) \) and defined the currents as outputs, that is,

\[ y := G^\top x = \begin{bmatrix} i_d \\ i_q \end{bmatrix}. \]

The current-feedback PI design is analysed in this paper viewing it as a passivity-based controller (PBC)—a term that was coined in [15]—where the main idea is to preserve a power balance equation like the one above but now with a new stored energy and a new dissipation term. This objective is accomplished in two steps, the shaping of the systems energy to give it a desired form, i.e., to have a minimum at the desired equilibrium, and the injection of damping. The shaped energy function
that with S2 Unknown \( \tau_L \) but verifying the following (reasonable) assumption.

### Assumption 1
A positive constant \( \tau_L^{\text{max}} \) such that

\[
|\tau_L| \leq \tau_L^{\text{max}},
\]

is known.

Moreover, we assume that \( \omega \) is measurable and, besides knowing the parameters \( \Phi \) and \( R_m \), it is also assumed that \( L_d, L_q \) and \( J \) are known.

In this scenario, we consider the adaptive PI controller

\[
\dot{x}_c = \begin{bmatrix} x_1 \\ x_2 - \hat{x}_2^* \end{bmatrix},
\]

\[
u = -K_I x_c - K_P \begin{bmatrix} x_1 \\ x_2 - \hat{x}_2^* \end{bmatrix},
\]

with \( K_I, K_P > 0 \), where \( \hat{x}_2^* \) is an estimate of the reference \( q \)-current \( x_2^* \), generated from an estimator of the simple integral form

\[
\hat{x}_c = f(x, \chi),
\]

\[
\hat{x}_2^* = h(x, \chi),
\]

with \( \chi \in \mathbb{R} \), which is to be designed.

In both scenarios we want to prove that there exists a positive-definite gain matrix \( K_P^{\text{min}} \) such that the PMSM model (1) in closed-loop with the PI controller (6) or (7) with \( K_P \geq K_P^{\text{min}} \) has a GAS equilibrium at \( (x^*, x_2^*, \chi^*) \) for some \( x_2^* \in \mathbb{R}^2 \) and \( \chi^* \in \mathbb{R} \) such that \( h(x^*, \chi^*) = x_2^* \).

Moreover, in the second scenario, \( K_P^{\text{min}} \) should not depend on \( \tau_L \), but only on the bound given in Assumption 1.

### Remark 2
As indicated in the introduction, in practice the reference value for \( x_2 \) is generated with an outer-loop PI around speed errors, that is,

\[
\dot{x}_c = \hat{x}_3
\]

\[
u = -a_I \chi - a_P \hat{x}_3,
\]

with \( a_I, a_P > 0 \). Unfortunately, the stability analysis of this configuration is far from obvious and we will need to propose another form for the functions \( f(x, \chi) \) and \( h(x, \chi) \) in (8).

### Remark 3
For the sake of completeness we also propose an estimator for the viscous friction coefficient \( R_m \), which generates a consistent estimate under an excitation assumption. See Subsection 5.3.

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\( ^2 \) As shown in Proposition 2, these additional assumptions are needed to design the estimator of \( \tau_L \).
4 Passivity Analysis

4.1 Dissipativity of the incremental model

In this section we give conditions under which the incremental model (4) satisfies a dissipation inequality of the form

$$\dot{U} \leq \epsilon |\tilde{y}|^2 + \tilde{y}^T \tilde{u}. \quad (10)$$

with

$$U(\tilde{x}) = \frac{1}{2} \|\tilde{x}\|_B^2. \quad (11)$$

for some $\epsilon \in \mathbb{R}$. If $\epsilon$ is negative it is then said that the incremental model of the system (1) is output strictly passive, if it is positive, then it is called output feedback passive, indicating the shortage of passivity [8, 14, 24].

Comparing (10) with the open-loop power balance equation (2) we see that, besides removing the term of extracted power, we have shaped the energy—assigning a minimum at the desired equilibrium $x^*$—and replaced the damping term $x^T R x$ by $\epsilon |\tilde{y}|^2$. Notice that, if $\epsilon$ is positive, it is easy to add damping selecting a control $\tilde{u} = -K_P \tilde{y}$, with $K_P = k_p I_2 > 0$. Indeed, this yields a damping term $-(k_p - \epsilon) |\tilde{y}|^2$, with $k_p > \epsilon$ we ensure $\dot{U} \leq 0$—whence, stability of the equilibrium. As explained in Remark 6, a more clever option is to add an integral action, yielding a PI.

**Lemma 1** Define the matrix

$$B := \begin{bmatrix} 2R_s + 2\epsilon & (L_d - L_q)x_3^* - L_d x_2^* \\ (L_d - L_q)x_3^* & 2R_s + 2\epsilon & 0 \\ -L_d x_2^* & 0 & 2 \frac{K_p}{\rho_p} \end{bmatrix},$$

for some $\epsilon \in \mathbb{R}$. If $B \geq 0$ the dissipation inequality (10) holds.

**PROOF.** Computing the derivative of (11) along the solutions of (4) we get

$$\dot{U} = -\tilde{x}^T [C(x) - C^*] x^* - \tilde{x}^T R \tilde{x} + \tilde{y}^T \tilde{u}$$

$$= -\frac{1}{2} \tilde{x}^T (B - 2\epsilon G G^T) \tilde{x} + \tilde{y}^T \tilde{u}$$

$$= -\frac{1}{2} \tilde{x}^T B \tilde{x} + \epsilon |\tilde{y}|^2 + \tilde{y}^T \tilde{u},$$

where we have used the fact that

$$[C(x) - C^*] x^* = \begin{bmatrix} 0 & -L_d x_3^* & 0 \\ L_d x_3^* & 0 & 0 \\ -L_d x_2^* & 0 & 0 \end{bmatrix} \tilde{x},$$

3 In [14, 24] the property of passivity of the incremental model is called shifted passivity.

to get the second identity and use the definition of $\tilde{y}$ given in (4) in the third identity. The proof is completed imposing the condition $B \geq 0$.

**Remark 4** Lemma 1 follows as a direct application of Proposition 1 and Remark 3 of [14], where passivity of the incremental model of general port-Hamiltonian systems with strictly convex energy function is studied. To make the present paper self-contained we have included a proof of the lemma.

**Remark 5** A dissipativity analysis similar to Lemma 1 has been carried out within the context of transient stability of power systems in [17], for synchronous generators connected to a constant voltage source in [25] and [3]. In all these papers the shifted Hamiltonian of [8], which in these cases boils down to the natural incremental energy function, is also used to establish stability conditions—that involve the analysis of positivity of a damping matrix similar to $B$.

4.2 Strict passivity of the PI controller

In this subsection we prove the input strict passivity of the PI controller. Although this result is very well-known [24, 26], a proof is given here for the sake of completeness.

**Lemma 2** Given any constant $y_c^* \in \mathbb{R}^2$, define the error signal $\tilde{y}_c := \tilde{y}_c - y_c^*$. The PI controller

$$\begin{align*}
\dot{x}_c &= u_c \\
y_c &= K_I x_c + K_P u_c,
\end{align*}$$

defines an input strictly passive map $u_c \mapsto \tilde{y}_c$, with storage function

$$H_c(\tilde{x}_c) := \frac{1}{2} \|\tilde{x}_c\|^2_{K_I}, \quad (12)$$

where $x_c^* := -K_I^{-1} y_c^*$. More precisely

$$\dot{H}_c = -\|u_c\|^2_{K_P} + u_c^T \tilde{y}_c.$$

**PROOF.** First, notice that, using the definition of $x_c^*$ in $y_c$ we have that

$$\tilde{y}_c = K_I \tilde{x}_c + K_P u_c. \quad (13)$$

Computing the derivative of $H_c$ along the trajectories of (6) yields

$$\dot{H}_c = \tilde{x}_c^T K_I u_c = u_c^T (\tilde{y}_c - K_P u_c),$$

where we have used (13) in the second identity, which completes the proof.
**Remark 6** The PI controller described above will be coupled with the PMSM via the (power-preserving) interconnection $u_c = y_c$ and $y_c = -u$. Lemma 2 shows the interest of adding an integral action: there is no need to know $u^*$ to implement the controller.

5 Main Results

5.1 Stability of the standard PI controller

**Proposition 1** Consider the PMSM model (1) in closed-loop with the PI controller (6), the integral gain $K_I > 0$ and the proportional gain $K_P = k_p I_2 > 0$. There exists a positive constant $k_p$ such that

$$k_p > K_P^\text{min}$$

ensures that $(x^*, x_r^*)$ is a GAS equilibrium of the closed-loop system. For non-salient PMSM, i.e., when $L_d = L_q$, the constant $K_P$ can be chosen such that

$$K_P^\text{min} > \frac{L_d^2}{4 R_m n_p \Phi (\tau_L + R_m |\omega^*|)^2} - R_s - R_s^2$$

**PROOF.** Summing up (11) and (12) define the positive definite, radially unbounded, Lyapunov function candidate

$$W(\tilde{x}, \tilde{x}_c) := U(\tilde{x}) + H_c(\tilde{x}_c).$$

Computing its derivative we get

$$W = -\frac{1}{2} \| \tilde{x} \|^2 + (\epsilon - k_p) |y| = -\frac{1}{2} \| \tilde{x} \|^2 R_d,$$

where we defined the matrix

$$R_d := \begin{bmatrix} 2R_s + 2k_p & (L_d - L_q)\omega^* & -L_d x_2^* \\ (L_d - L_q)\omega^* & 2R_s + 2k_p & 0 \\ -L_d x_2^* & 0 & 2 R_m / n_p \end{bmatrix}$$

From (17) we immediately conclude that if $R_d > 0$, then the equilibrium $(x^*, x_r^*)$ is globally stable. Moreover, invoking Krasovskii’s Theorem, we prove that the equilibrium is GAS because

$$\dot{x}(t) \equiv 0 \Rightarrow \dot{x}(t) \equiv 0.$$ 

The gist of the proof is then to prove the existence of the lower bound $K_P^\text{min}$ that ensures positivity of $R_d$.

Towards this end, we recall the following well-known (Schur complement) equivalence:

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0 \Leftrightarrow C > 0 \text{ and } A - B C^{-1} B^T > 0.$$

Directly applying this to $R_d$ with

$$A := \begin{bmatrix} 2R_s + 2k_p & (L_d - L_q)\omega^* \\ (L_d - L_q)\omega^* & 2R_s + 2k_p \end{bmatrix}, \quad B := \begin{bmatrix} -L_d x_2^* \\ 0 \end{bmatrix},$$

and $C := 2 R_m / n_p$, shows that $R_d > 0$ if and only if

$$(R_s + k_p) I_2 > \frac{\frac{n_p L_d^2 |x_2^*|^2}{2 R_m}}{(L_q - L_d)\omega^*} \begin{bmatrix} (L_q - L_d)\omega^* & 0 \\ 0 & (L_q - L_d)\omega^* \end{bmatrix}.$$ (18)

This proves the existence of $K_P^\text{min}$ such that, if (14) holds then $R_d > 0$. In case $L_d = L_q$, $K_P^\text{min}$ can be chosen as in (15).

5.2 An asymptotically stable adaptive PI controller

In applications the load torque $\tau_L$, and consequently $x_r^*$, are unknown. It is, therefore, necessary to replace its value above by an estimate, a task, that is accomplished in the proposition below.

**Proposition 2** Consider the PMSM model (1) verifying Assumption 1 in closed-loop with the adaptive PI controller (7) with the estimator

$$\dot{\chi} = -R_m \omega + n_p [(L_d - L_q) i_d i_q + \Phi i_q] - \ell (\chi - \omega), \quad \dot{\tau}_L = \ell (\chi - \omega), \quad \dot{x}_2 = \frac{1}{n_p \Phi} (\hat{\tau}_L + R_m \omega^*).$$ (19)

where $\ell > 0$. Fix the proportional gain as $K_P = k_p I_2 > 0$.

There exists a positive constant $K_P^\text{min}$—dependent only on $\ell$—such that (14) ensures that $(x^*, x_r^*, \chi^*)$, with $\chi^* := \frac{\ell}{n_p \Phi} + \omega^*$ is a GAS equilibrium of the closed-loop system.

**PROOF.** Similarly to the proof of Proposition 1, we first need to prove that $R_d > 0$. This follows immediately invoking (18) and noting that, from the definition of the equilibria in (3), we have

$$\frac{1}{n_p \Phi} (|\tau_L| + R_m |\omega^*|) \geq |x_2^*|. $$
Thus, a $k_p \min$ that depends only on $\tau_L^{\max}$, can readily be defined.

We proceed now to prove that the estimator (19) generates an exponentially convergent estimate of $\tau_L$. Defining the estimation error $e_{\tau L} := \hat{\tau}_L - \tau_L$, the error dynamics yields
\[
\dot{e}_{\tau L} = -\ell J e_{\tau L},
\]
which is clearly exponentially stable for all $\ell > 0$.

To simplify the presentation of the analysis of the overall error dynamics let us define the reference output error signal
\[
e_{y^*} := \tilde{y} - y^* = \begin{bmatrix} 0 \\ \tilde{x}_1^* - x_2^* \end{bmatrix} = \frac{1}{n_p \Phi} \begin{bmatrix} 0 \\ e_{\tau L} \end{bmatrix},
\]
which replaced in (7) yields
\[
\begin{align*}
\dot{\tilde{x}}_c &= \tilde{y} - e_{y^*}, \\
\dot{u} &= -K_i \tilde{x}_c - K_p (\tilde{y} - e_{y^*})
\end{align*}
\]
The closed-loop is then a cascaded dynamics of the form
\[
\begin{align*}
\dot{e}_{y^*} &= -\frac{\ell}{n_p \Phi} J e_{y^*} \\
\dot{\xi} &= f(\xi) + \begin{bmatrix} D^{-1} G K_P \\ -I_2 \end{bmatrix} e_{y^*},
\end{align*}
\]
with $\xi := \text{col}(\tilde{x}, \tilde{x}_c)$ and the dynamics $\dot{\xi} = f(\xi)$ has the origin as a GAS equilibrium.

The GAS proof is completed invoking Theorem 1 of [18] that shows that the cascaded system is globally stable, which implies that all trajectories are bounded. GAS follows immediately from the well-known fact [21] that the cascade of two GAS systems is GAS if all trajectories are bounded.

5.3 A globally convergent estimator of $R_m$

In the lemma below we show that it is possible to add an adaptation term to estimate the friction coefficient $R_m$, that is usually uncertain, provided some excitation conditions are satisfied.

**Lemma 3** Consider the mechanical equation in (1) and the gradient estimator
\[
\dot{R}_m = \gamma \phi (z - R_m \phi),
\]
with $\gamma > 0$ an adaptation gain and the measurable signals
\[
z := \frac{\beta p^2}{(p + \alpha)^2} [J \omega] + \frac{\beta p}{(p + \alpha)^2} [n_p (L_q - L_d) i_d i_q - n_p \Phi i_q] \\
\phi := \frac{\beta p}{(p + \alpha)^2} [\omega],
\]
where $p := \frac{d}{m}$ and $\alpha, \beta > 0$. The following equivalence holds true
\[
\phi \notin L_2 \iff \lim_{t \to \infty} |\hat{R}_m(t) - R_m| = 0,
\]
with $L_2$ the space of square integrable functions.

**PROOF.** Applying the filter $\frac{2p}{(p + \alpha)^2}$ to the mechanical equation in (1), recalling that $\tau_L$ is constant, and using the definitions (23) yields the linear regression model
\[
z = R_m \phi + \epsilon_t
\]
where $\epsilon_t$ is an exponentially decaying term stemming from the filters initial conditions, which can be neglected without loss of generality. Replacing the equation above in (22) yields the error equation
\[
\dot{e}_{R_m} = -\gamma \phi^2 e_{R_m},
\]
where $e_{R_m} := \hat{R}_m - R_m$ is the parameter estimation error. The proof is completed integrating (24).

**Remark 7** As always in estimation problems some kind of excitation on the signals must be imposed to guarantee convergence. In our case it is the condition of non-square integrability of $\omega$, which is weaker than the more classical persistence of excitation assumption—in which case the convergence of the parameter error is exponential.

**Remark 8** An alternative to the estimators presented above is to add a nonlinear integral action to compensate for both unknowns $\tau_L$ and $R_m$ as done in [4]. In any case, both options considerably complicate the control law, a scenario that is beyond the scope of this paper. Also, although it is possible to carry out the stability analysis of the combination of the estimators of $\tau_L$ and $R_m$, we avoid this discussion for the aforementioned reason.

6 Simulation Results

The objective of the simulations is to verify numerically the performance of the proposed controllers under different gains and external signals. First, we consider a constant speed reference and load input, then, to illustrate the tracking ability of the load torque estimator,
we propose the time-varying profiles depicted in Figs 1 and 2. In all cases we took the PI gains as $K_p = k_p I_2$ and $K_i = k_i I_2$.

The following scenarios were considered.

(C1) Inner-loop PI (6) with known $\tau_L$ and $R_m$, considering the cases of $k_p \geq k_p^{\text{min}}$ and $k_p < k_p^{\text{min}}$ and constant speed reference and load torque.

(C2) Adaptive inner-loop PI (7) with load torque estimator (19) for the load torque of Fig. 2 and the speed reference of Fig. 1.

(C3) Adaptive inner-loop PI (7) with load torque (19) and viscous friction coefficient (22) estimators for the time-varying profiles of Figs. 1 and 2.

(C4) Inner-loop PI (6) with outer-loop PI in speed error (9) for the time-varying profiles of Figs. 1 and 2.

In simulations we use the motor data provided by [13] with $R_m = 0.02 \text{ Nm}$. The motor parameters are given in Table 1.

The maximum torque load of Assumption 1 is chosen 70% higher than the rated value, which corresponds to $\tau^{\text{max}}_L = 4.6 \text{ Nm}$. It should be noted that, for the nominal electrical speed $\omega_n = 104.72 \text{ rad/sec}$ and this conservative value of $\tau^{\text{max}}_L$, the minimal proportional gain which provides $R_d > 0$ is $k_p^{\text{min}} = -2.32$. Implied that the incremental model of the motor is passive—that is, $\epsilon$ in Lemma 1 is negative—and, consequently, it can be stably regulated setting $u = u^*$. Obviously, for robustness reasons, a closed-loop PI is preferred.

Figs. 3 and 4 show the effect of increasing the gain $k_p$ that, as expected, improves the convergence rate. Although of no practical interest, the simulation with negative $k_p$ is presented to corroborate the theoretical result. In this respect, numerical simulations show that the motor becomes unstable for $k_p < -5.8$. In Fig. 5 both PI gains are increased obtaining a much faster response—notice the difference in time scales. In all simulations the difference in time scales between the electrical and the mechanical dynamics is clearly apparent.

The transients for the adaptive PI with load torque estimator (19) are shown in Figs. 6 and 7. Here we use the time-varying torque profile of Fig. 2. In Fig. 7 the gain $\ell$ is taken higher than in Fig. 6, that as expected from (20), leads to a faster convergence of $\tau_L$ to zero and smaller speed errors. The next test, shown in Fig. 8, illustrates the system behaviour for the varying speed reference given in Fig. 1 with the same load input and gains as in the previous scenario. As seen from the figure the adaptive PI controller provides good performance and all the errors $\hat{\tau}_L$, $\hat{\omega}$, and $i_{dq}$ converge to zero fast.

Simulations for the adaptive PI (7), (19) equipped with the estimate of viscous friction coefficient $\hat{R}_m$ generated by (22) are shown in Figs. 9 and 10 for two different gain settings and the time-varying profiles of Figs. 1 and 2. In both cases $R_m(0) = 0.005$, which is 25% of the actual value. While the choice of parameters of Fig. 9 is suitable for the simultaneous estimation of $\tau_L$ and $R_m$, reducing the constant $\ell$ and changing the bandwidth of the filter $\frac{dp}{(p+\alpha)^2}$ has a deleterious effect. Indeed, as shown in Fig. 10 there is a static error in both estimators in the interval $t \in (0.6, 1) \text{ sec.}$, and it is not until the appearance of the speed reference change at $t = 1 \text{ sec.}$ that the estimators recover their alertness. This observation underscores, on
one hand, the need of excitation indicated in Lemma 3 and, on the other hand, the importance of selecting suitable tuning gains for the estimators.

Fig. 11 illustrates the transients of the system with the standard PI (6) and outer-loop PI around speed errors (9) that, as discussed throughout, is often used in practice. In order to compare the efficiency with the adaptive PI we use the same gains $k_p = 15$ and $k_i = 2000$ for the current regulation as in the previous test (Fig. 9). The torque load and speed references are also the same. The gains $a_p = a_p I$ and $a_i = a_i I$ of the outer-loop speed controller are tuned to attain similar current and voltage levels. Comparing Figs. 9a and 11a one can see that proposed controller with guaranteed GAS ensures a faster speed regulation with lower overshoot when the load changes.

Fig. 12 shows the deleterious effect of increasing the gains $a_p$ and $a_i$ of the outer-loop PI. Indeed, although this results in a better transient behaviour of the speed error, it yields unrealistic overshoots both in motor currents and voltages, shown in Figs. 12b and 12c, correspondingly.

7 Conclusions and Future Research

We have established the practically interesting—even if not surprising—result that the PMSM can be globally regulated around a desired equilibrium point with a simple (adaptive) PI control around the current errors, provided some viscous friction is present in the rotor dynamics and the proportional gain of the PI is suitably chosen. The key ingredient to establish this result is the proof in Lemma 1 that the incremental model of the PMSM satisfies the dissipation inequality (10). Our main results are established with simple calculations and invoking elementary Lyapunov theory with the natural—quadratic in the increments—Lyapunov functions.

Some topics of current research are the following.

- From the theoretical viewpoint the main drawback of the results reported in the paper are the requirement of existence, and knowledge, of the friction coefficient $R_m$. As shown in Lemma 3 the requirement of knowing $R_m$ can be relaxed—at the price of complicating the controller and requiring some excitation conditions. However the assumption of $R_m > 0$ seems unavoidable if we want to preserve a a simple PI structure, see Remark 8. It should be underscored, however, that from the practical viewpoint, the assumption that the mechanical dynamics has some static friction—that may be arbitrarily small—is far from being unreasonable.

- As discussed in [27] in the context of power systems, the absence of the outer-loop PI significantly deteriorates the transient performance of the inner-loop PI. A similar situation appears here for the PMSM. Unfortunately, the analysis of the classical outer-loop PI in speed errors (9) is hampered by the lack of a convergence proof of the estimation error.

- In the case of $L_d \neq L_q$ torque can be made even larger by an additional reluctance component $r_d^* \neq 0$. The implications of this choice on the passivity of the incremental model remains to be investigated.

- The extension of the result to the case of salient PMSM is also very challenging—see [7] for the corresponding $a/3$ model.

- The lower bound on the proportional gain can be computed invoking the physically reasonable Assumption 1. However, the reference value for $i_q$ is dependent on $\tau_L$. As shown in Proposition 2 this problem can be solved using an adaptive PI, at the high cost of knowledge of the PMSM model parameters.

- Experimental results of PI current control abound in the literature and experiments of an observer, similar to (19), may be found in [20]. However, it would be interesting to validate experimentally the performance of the proposed adaptive PI and, in particular, investigate how it compares with the classical outer-loop speed PI (9).

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Fig. 3. Transients in the system with inner-loop current PI (6): $k_p = k_{p_{\text{min}}}$ and $k_i = 100$

Fig. 4. Transients in the system with inner-loop current PI (6): $k_p = -5 < k_{p_{\text{min}}}$ and $k_i = 100$

Fig. 5. Transients in the system with inner-loop current PI (6): $k_p = 20 > k_{p_{\text{min}}}$ and $k_i = 4000$

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Fig. 6. Transients in the system with adaptive current PI (7), (19) and load torque of Fig. 2: \(k_p = 15, k_i = 2000, \ell = 0.1\)

Fig. 7. Transients in the system with adaptive current PI (7), (19) and load torque of Fig. 2: \(k_p = 15, k_i = 2000, \ell = 20\)
Fig. 8. Transients in the system with adaptive current PI (7), (19), load torque of Fig. 2 and speed reference of Fig. 1: $k_p = 15$, $k_i = 2000$, $\ell = 20$

Fig. 9. Transients in the system with adaptive current PI (7) with estimators of load torque (19) and resistance (22), load torque of Fig. 2 and speed reference of Fig. 1: $k_p = 15$, $k_i = 2000$, $\ell = 20$, $\alpha = \beta = 300$, $\gamma = 200$, $\hat{R_m}(0) = 0.005$ Nm
Fig. 10. Transients in the system with adaptive current PI (7) with estimators of load torque (19) and resistance (22), load torque of Fig. 2 and speed reference of Fig. 1: \( k_p = 15, k_i = 2000, \ell = 10, \alpha = \beta = 1300, \gamma = 500, \hat{R}_m(0) = 0.005 \text{ Nm} \)

Fig. 11. Transients in the system with inner-loop current PI (6) and outer-loop speed PI (9), load torque of Fig. 2 and speed reference of Fig. 1: \( k_p = 15, k_i = 2000, a_p = 0.03 \) and \( a_i = 1.1 \).

Fig. 12. Transients in the system with inner-loop current PI (6) and outer-loop speed PI (9), load torque of Fig. 2 and speed reference of Fig. 1: \( k_p = 15, k_i = 2000, a_p = 0.2 \) and \( a_i = 18 \).
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