The high-scale SUSY seesaw: LHC vs low energy

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In this contribution we outline the correlation between intergenerational slepton mass splittings and low energy lepton flavour violation in supersymmetric type-I and type-III seesaws, and illustrate how the combination of these two sets of observables could strengthen or disfavour a high-scale seesaw as the explanation of neutrino masses and mixings. This contribution summarises part of the analysis presented in\textsuperscript{1,2}.

1 Introduction

Under the assumption that the seesaw is the unique source of lepton flavour violation (LFV) and that slepton masses are flavour universal at an energy scale higher than that of the seesaw, intergenerational slepton mass splittings at low energy (i.e. $m_{\tilde{\ell}_i} - m_{\tilde{\ell}_j}$) are tightly correlated with low energy charged LFV (cLFV). This is a direct consequence of their common origin in radiatively generated slepton flavour mixing\textsuperscript{3}. For the illustrative examples of the supersymmetric (SUSY) fermionic seesaws, we will show that the aforementioned correlations have the potential of probing these high-scale SUSY seesaws.

2 Supersymmetric fermionic seesaws

We will focus on seesaw realisations that rely on the exchange of heavy fermions $F$ to generate neutrino masses and mixings. At the fundamental level, the terms in the Lagrangian density responsible for neutrino masses read

\begin{equation}
Y_{\nu}^{ij}F_i L_j H_u - \frac{1}{2}M_{ij}F_i F_j,
\end{equation}

where $i,j$ are generation indices. If $L_i H_u$ is a weak singlet (triplet), $F_i$ must be a weak singlet (triplet) and we have the so-called type-I (type-III) seesaw mechanism. At low energies the fundamental Lagrangian is matched by an effective Lagrangian that has a dimension-5 operator resulting from integrating out the heavy fermions $F_i$ and which, after electroweak symmetry breaking (EWSB), generates neutrino masses and mixings. Approximately, we have

\begin{equation}
m_\nu \simeq -v_u^2 Y^{\nu T} M^{-1} Y^\nu,
\end{equation}

where $v_u$ is the vacuum expectation value of $H_u$. This relation suggests that a convenient way of parametrizing the neutrino Yukawa couplings $Y^\nu$ at the seesaw scale, while at the same time allowing to accommodate neutrino data, is given by\textsuperscript{4}

\begin{equation}
Y^\nu = \frac{i}{v_u} \sqrt{M^{\text{diag}}} R \sqrt{m_\nu^{\text{diag}}} U^{\text{MNS}^T},
\end{equation}

where $M^{\text{diag}}$ and $m_\nu^{\text{diag}}$ are the diagonal matrices of the Majorana matrices $M$ and $m_\nu$, respectively, and $U$ is the MNS matrix.

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where $U_{\text{MNS}}$ is the neutrino mixing matrix and $R$ is a complex orthogonal matrix that encodes mixings among heavy fermions; Eq. 3 is cast in a basis in which the charged lepton Yukawa couplings ($Y^l$) and $M$ are diagonal in generation space. Hereafter, we will work in this basis.

In order to preserve the attractive features of SUSY gauge coupling unification, we will embed the heavy fermion triplets in a SU(5) grand unified theory (GUT). The lowest dimensional representation that contains a weak triplet that is a singlet under SU(3)$\otimes$U(1) is a 24-plet whose decomposition under SU(3)$\otimes$SU(2)$\otimes$U(1) reads

$$24 = (1,3,0) \oplus (1,1,0) \oplus \text{coloured fields}.$$  \hfill (4)

Thus, the type-III implementation by means of 24-plets is accompanied by a type-I seesaw.

Our analysis for the type-I seesaw will be conducted in the minimal supersymmetric SM (MSSM) extended by three heavy right-handed (RH) neutrino superfields while in the 24-plet seesaw we will consider the supersymmetrized version of the Georgi-Glashow SU(5) model supplemented by three generations of 24-plet superfields. For details on the extended superpotential and SUSY soft-breaking sector we refer to the works\textsuperscript{1,2} upon which this contribution is based.

In addition, we will impose at the GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV) minimal supergravity (mSUGRA) inspired universality conditions for the SUSY soft-breaking sector of each model: a common mass $m_0$ for the scalars (including the scalar partners of the heavy fermions); a common mass for the gauginos, $M_1/2$; trilinear couplings given by $A_{u,d,l,\nu} = A_0 Y_{u,d,l,\nu}$ (where $Y^\nu \equiv Y^{24}$ in the case of the 24-plet seesaw\textsuperscript{2}). The values of $Y_{u,d}$ at $M_{\text{GUT}}$ are fitted to give the low energy quark masses and mixings, while $Y^l$ and $m_{\nu}^{\text{diag}}$ in Eq. 3 (with an additional overall factor of $\sqrt{5/4}$ for the 24-plet seesaw case) are fitted to reproduce the low energy lepton masses and mixings. In doing this we are taking as input for $U_{\text{MNS}}$ in Eq. 3 the low energy neutrino mixing matrix, exploiting that the running of the mixing angles is negligible.

The running from $M_{\text{GUT}}$ down to the seesaw scale will induce flavour mixing\textsuperscript{3} in the approximately flavour conserving slepton soft breaking terms due to the non-trivial flavour structure of $Y^\nu$ (implied by neutrino mixing and by possible heavy fermion mixing). This effect is more pronounced in the soft breaking terms involving slepton doublets since these have local interactions with the heavy fermions. At leading order, the flavour mixing induced by these radiative corrections is given by

$$\left(\Delta m_{L_i}^2\right)_{ij} = -a_F \frac{1}{8\pi^2} \left(3 m_0^2 + A_0^2\right) Y^{\nu \dagger} L Y^\nu \right)_{ij}, \hfill (5)$$

$$\left(\Delta A^i\right)_{ij} = -a_F \frac{3}{16\pi^2} A_0 Y^l_{ij} Y^{\nu \dagger} L Y^\nu \right)_{ij} \bigg; L_{kl} \equiv \log \left(\frac{M_{\text{GUT}}}{M_k}\right) \delta_{kl}, \hfill (6)$$

where $a_F = 1$ for the single seesaw and $a_F = 9/5$ for the 24-plet seesaw. The amount of flavour violation in the slepton sector is encoded in $(Y^{\nu \dagger} L Y^\nu)_{ij}$ which, as made explicit in Eq. 3, is related to heavy fermion masses and mixings, as well as to neutrino masses and mixings. If the seesaw scale is $\sim 10^{15}$ GeV, the neutrino Yukawa couplings are of $\mathcal{O}(1)$ and the radiative corrections will yield sizeable slepton flavour mixing, with potentially observable effects.

### 2.1 Low energy cLFV observables

An effect of sizeable slepton flavour mixing are potentially large new SUSY contributions to cLFV observables (through sneutrino-chargino and slepton-neutralino loops). For the case of cLFV radiative decays $\ell_i \rightarrow \ell_j \gamma$, a simple illustrative expression can be obtained using the leading-logarithm approximation

$$\frac{\text{BR}(\ell_i \rightarrow \ell_j \gamma)}{\text{BR}(\ell_i \rightarrow \ell_j \nu_j \nu_j)} = \frac{\alpha^3 \tan^2\beta}{G_F^2 m_{\text{SUSY}}^2} \left| a_F \frac{1}{8\pi^2} \left(3 m_0^2 + A_0^2\right) Y^{\nu \dagger} L Y^\nu \right)_{ij} \right|^2.$$  \hfill (7)
2.2 Intergenerational slepton mass splittings

Under the assumption that SUSY breaking generates flavour universal slepton soft-breaking masses, intergenerational slepton mass splittings may arise from left-right mixing after EWSB and/or renormalisation group (RG) running. In the MSSM both of them are proportional to the only source of flavour non-universality, $Y^\mu$. Neglecting RG corrections as source of intergenerational slepton mass splittings, one finds that the $\tilde{e}_L,\tilde{\mu}_L$ mass difference normalised to their average mass, $m_{\tilde{\ell}}$, is approximately given by

$$\frac{\Delta m_{\tilde{\ell}}}{m_{\tilde{\ell}}} (\tilde{e}_L, \tilde{\mu}_L) \approx \frac{m_{\mu}}{2m_{\tilde{\ell}}^2} \frac{(A_0 - \mu \tan \beta)^2}{0.35M_{1/2}^2 + M_Z^2 \cos 2\beta (-1/2 + 2 \sin^2 \theta_W)} . \quad (8)$$

Due to the smallness of $Y^\mu$, the mass splitting between the first two slepton generations in the MSSM with mSUGRA boundary conditions was found\(^1\) to be always below the 0.01% level. In the presence of radiative corrections induced by the seesaw (which we take to be sufficiently heavy so that $Y^\nu \sim 1$), and assuming $(\Delta m_{\tilde{\ell}}^2)_{ij}$ as the dominant off-diagonal entry, the mass splitting between the mostly left-handed (LH) $i$ and $j$ sleptons is approximately

$$\frac{\Delta m_{\tilde{\ell}}}{m_{\tilde{\ell}}} (\tilde{\ell}_i, \tilde{\ell}_j) \approx \frac{(\Delta m_{\tilde{\ell}}^2)_{ij}}{m_{\tilde{\ell}}^2} \propto \sqrt{\text{BR}(\ell_i \to \ell_j \gamma)}. \quad (9)$$

Eq. 9 illustrates in a simple way the correlation of these high- and low-energy observables in the case of a single source of flavour violation in the lepton sector, $Y^\nu$.

3 Illustrative results and discussion

In the left panel of Figure 1 we display the correlation between slepton mass splittings and two low energy cLFV observables, namely $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu-e, \text{Ti})$, for the 24-plet seesaw. “Horizontal” isolines correspond to constant values of $M_{1/2}$ with the 24-plet mass scale ($M_{24}$) increasing from left to right. For further details see\(^2\). The decrease in $\text{BR}(\mu \to e\gamma)$ with increasing $M_{24}$ along “horizontal” isolines is due to the way the 24-plets affect the SUSY spectrum: sleptons become lighter as $M_{24}$ gets farther from $M_{\text{GUT}}$. The balance between reducing slepton flavour mixing and alleviating the loop mass suppression is such that the latter dominates slepton mediated $\mu-e$ transitions. In contrast, $\Delta m_{\tilde{\ell}} (\tilde{e}_L, \tilde{\mu}_L)/m_{\tilde{\ell}}$ increases with increasing $M_{24}$.

In the right panel of Figure 1, with a different underlying scan, we illustrate similar correlations for the type-I seesaw case. Leading to the figure we have performed a random scan over the RH neutrino mixing matrix. These results confirm the correlation between the two sets of observables and the dependence displayed in Eq. 9.

In the 24-plet seesaw, slepton mass splittings between the first two generations are expected\(^2\) to lie in the 0.1-1% level for a SUSY spectrum within LHC range and for the entire viable range of the 24-plet mass scale. In contrast, for the type-I seesaw we have found\(^3\) that mass splittings above the 0.1% level restrict the RH neutrino scale to lie above $10^{13}$ GeV.

Assuming that sleptons are discovered at the LHC, their mass splittings can be compared to the results available on cLFV observables and interpreted under the light of both a high-scale SUSY model and its seesaw extension. If the high-scale SUSY model can be inferred from a subset of the data (independently from a possible seesaw extension), then by combining the information on low energy cLFV observables with intergenerational slepton mass splittings the hypothesis of a particular seesaw extension of a high-scale SUSY model can be strengthened, if it accommodates both sets of observables without ad hoc sources of LFV or of intergenerational slepton mass differences; likewise it can be disfavoured, if accommodating both sets of observables is difficult.

Although in our analysis we have set $\theta_{13} = 0.1^\circ$, we stress, however, that the results shown in Figure 1 are only mildly sensitive\(^4\) to the value of $\theta_{13}$ and little changes would appear from
for first- and second-generation sleptons, the current uppers bounds and future sensitivities on $\mu \to e\gamma$ (as now experimentally measured). Indeed, slepton flavour mixing generated by a degenerate heavy fermion spectrum with a degenerate heavy fermion spectrum with $R$ = 1, as taken in the scan leading to the left panel results, has a small dependence on the size of $\theta_{13}$. Moreover, a scan over the parameter space of the heavy fermion mixing matrix, as was done for the right panel, hides the dependence of slepton flavour mixing on the precise value of $\theta_{13}$.

4 Conclusion

We have shown that if sleptons are discovered and their masses reconstructed to a tentative accuracy of 0.1% for first- and second-generation sleptons, the current upper-bounds and future results of low energy cLFV experiments have the potential to strengthen or disfavour the high-scale SUSY seesaw explanation of neutrino masses and mixings.

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$^b$This benchmark point is now excluded by the LHC results on the SM-like scalar boson mass; nevertheless, a P5-HM1 variant with large $-A_0$ is able to lift the mass so as to be compatible with the LHC without significantly affecting the conclusions, but at the expense of shifting the points upwards along the correlation slope.