High-sensitivity optical measurement of mechanical Brownian motion

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We describe an experiment in which a laser beam is sent into a high-finesse optical cavity with a mirror coated on a mechanical resonator. We show that the reflected light is very sensitive to small mirror displacements. We have observed the Brownian motion of the resonator with a very high sensitivity corresponding to a minimum observable displacement of \(2 \times 10^{-19} \text{ m}/\sqrt{\text{Hz}}\).

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Thermal noise plays an important role in many precision measurements [1]. For example, the sensitivity in interferometric gravitational-wave detectors is limited by the Brownian motion of the suspended mirrors which can be decomposed into suspension and internal thermal noises. The latter is due to thermally induced deformations of the mirror surface. Experimental observation of this noise is of particular interest since its theoretical evaluation strongly depends on the mirror shape and on the spatial matching between light and internal acoustic modes. It is also related to the mechanical dissipation mechanisms which are not well known in solids [10]. Mirror displacements induced by thermal noise are however very small and a highly sensitive displacement sensor is needed to perform such an observation.

Monitoring extremely small displacements has thus become an important issue in precision measurements and several sensors have been developed. A technique commonly used for the detection of gravitational waves by Weber bars is based on capacitive sensors [12]. Another promising technique consists in optical transducers. Reflexion of light by a high-finesse Fabry-Perot cavity is very sensitive to changes in the cavity length. Such a device can thus be used to monitor displacements of one mirror of the cavity, as it has been proposed for gravitational wave bar detectors where the mirror is mechanically coupled to the bar, or for the detection of Brownian motion in gravitational wave interferometers. In this letter we report a high-sensitivity observation of the Brownian motion of internal modes of a mirror. The sensitivity reached in our experiment is better than that of present sensors and comparable to the one expected in gravitational wave interferometers.

We use a single-ended Fabry-Perot cavity composed of an input coupling mirror and a totally reflecting back mirror. The intracavity intensity shows an Airy peak when the cavity length is scanned through a resonance, and the phase of the reflected field is shifted by \(\pi\). The slope of this phase shift strongly depends on the cavity finesse and for a lossless resonant cavity, a displacement \(\delta x\) of the back mirror induces a phase shift \(\delta \varphi_x\) of the reflected field on the order of

\[
\delta \varphi_x \simeq 8F \frac{\delta x}{\lambda},
\]

where \(F\) is the cavity finesse and \(\lambda\) is the optical wavelength. This signal is superimposed to the phase noise of the reflected field. If all technical noise sources are suppressed, the phase noise \(\delta \varphi_n\) corresponds to the shot noise of the incident beam

\[
\delta \varphi_n \simeq \frac{1}{2\sqrt{I}},
\]

where \(I\) is the mean incident intensity counted as the number of photons per second. The sensitivity of the measurement is given by the minimum displacement \(\delta x_{\text{min}}\) that yields a signal of the same order of magnitude as the noise

\[
\delta x_{\text{min}} \simeq \frac{\lambda}{16F\sqrt{I}}.
\]

One expects to be able to detect a displacement corresponding to a small fraction of the optical wavelength for a high-finesse cavity and an intense incident beam.

In our experiment the coupling mirror has a curvature radius of 1 meter and a typical transmission of 50 ppm (Newport high-finesse SuperMirror). The back mirror is coated on the plane side of a small plano-convex mechanical resonator made of silica. The coating has been made at the Institut de Physique Nucléaire (Lyon) on a 1.5-mm thick substrate with a diameter of 14 mm and a curvature radius of the convex side of 100 mm. The two mirrors are mounted in a rigid cylinder which defines the distance and the parallelism between them. The cavity length is close to 1 mm so that the TEM\(_{00}\) optical mode of the cavity has its waist in front of the back mirror with a size of 90 \(\mu\)m.

The mirror motion is due to the excitation of internal acoustic modes which have been extensively studied for plano-convex resonators [13][22]. For a curvature radius of the convex side much larger than the thickness of

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the resonator, those modes can be described as gaussian modes confined around the central axis of the resonator. The intracavity field experiences a phase shift proportional to the longitudinal deformation of the resonator averaged over the beam waist and only compression modes which induce such a longitudinal deformation are coupled with the light. In the following we focus on the fundamental mode which has a waist equal to 3.4 mm and a resonance frequency close to 2 MHz.

We have measured the optical characteristics of the cavity. Its bandwidth is equal to 1.9 MHz and its free spectral range is equal to 141 GHz. These values correspond to a cavity length of 1.06 mm and a finesse of 37000. We also measured the reflection coefficient of the cavity at resonance to derive the transmission of the coupling mirror and the cavity losses. We found a transmission of 60 ppm and losses equal to 109 ppm.

![Diagram](image)

**FIG. 1.** Experimental setup. A light beam supplied by a frequency (F. stab.) and intensity (I. stab.) stabilized titane-sapphire laser is sent into a high-finesse cavity composed of a coupling mirror and a highly-reflecting back mirror coated on a mechanical resonator. Phase fluctuations of the reflected field are measured by homodyne detection. An auxiliary beam with modulated intensity (AOM) is used to optically excite the acoustic modes of the resonator. The light entering the cavity is supplied by a titane-sapphire laser working at 810 nm and frequency-locked to a stable external cavity composed of a coupling mirror and a highly-reflecting back mirror coated on a mechanical resonator. Phase fluctuations of the reflected field are measured by homodyne detection. An auxiliary beam with modulated intensity (AOM) is used to optically excite the acoustic modes of the resonator.

The amplitude of this force can be changed by varying the depth of the intensity modulation. The auxiliary laser beam is uncoupled from the cavity by frequency filtering due to an optical frequency shift of 200 MHz induced by the acousto-optic modulator and by spatial filtering due to an astigmatism of 60 ppm and losses equal to 109 ppm. Note that the incident power is low enough to neglect quantum effects of radiation pressure. Quantum noise induced by radiation pressure is less than 0.2%. We have found that the auxiliary beam has no spurious effect on the homodyne detection. The amplitude of this force can be changed by varying the depth of the intensity modulation. The auxiliary laser beam is uncoupled from the cavity by frequency filtering due to an optical frequency shift of 200 MHz induced by the acousto-optic modulator and by spatial filtering due to a tilt angle of 10° between the beam and cavity axes. We have checked that the auxiliary beam has no spurious effect on the homodyne detection.

The last part of the experimental setup is used to optically excite the mechanical resonator. A 500-mW auxiliary beam derived from the titane-sapphire laser is intensity-modulated by an acousto-optic modulator and reflected from the rear on the back mirror. A modulated radiation pressure force is thus applied to the resonator. The amplitude of this force can be changed by varying the depth of the density modulation. The auxiliary laser beam is uncoupled from the cavity by frequency filtering due to an optical frequency shift of 200 MHz induced by the acousto-optic modulator and by spatial filtering due to a tilt angle of 10° between the beam and cavity axes. We have checked that the auxiliary beam has no spurious effect on the homodyne detection.

Figure 2 shows the experimental result of the optical excitation. Each square is obtained for a different modulation frequency of the auxiliary laser beam around the expected frequency for the fundamental mode of the mechanical resonator. The power of phase modulation of the reflected field is normalized to the shot-noise level, independently measured by sending only the local oscillator in the homodyne detection. We have checked that the phase noise of the reflected field corresponds to the shot-noise level when the laser is out of resonance with the high-finesse cavity. Any deviation of the phase from the shot-noise level is thus due to the interaction of the light with the cavity. Such a deviation reflects the mirror motion and the resonance in figure 2 corresponds to...
the excitation of the fundamental acoustic mode of the resonator. The solid curve is a lorentzian fit which shows that the mechanical response has a harmonic behavior around the resonance frequency with a quality factor $Q$ of 44000.

![Image](image1)

**FIG. 2.** Mechanical response of the resonator. Squares represent the power of phase modulation of the reflected field normalized to the shot-noise level. Each square corresponds to a different modulation frequency of the optical excitation, around the fundamental resonance frequency of the resonator. The solid curve is a lorentzian fit of the resonance. Vertical scale on the right represents the equivalent displacement in $m^2$.

As explained in the end of this paper we have calibrated the measured displacement and the resulting scale is shown on the right of figure 2. The displacement at resonance corresponds to an amplitude of $1.6 \times 10^{-15} m$. One can estimate the radiation pressure exerted by the auxiliary beam as $F_{rad} = 2\hbar k \delta I = 1.2 \times 10^{-9} N$ where $2\hbar k$ is the momentum exchange during a photon reflection and $\delta I$ is the intensity modulation. One thus finds that the mechanical susceptibility $\chi[\Omega]$ has a lorentzian shape around the mechanical resonance frequency $\Omega_M$

$$\chi[\Omega] = \frac{\chi_0}{1 - \Omega^2/\Omega_M^2 - i/Q},$$

with $\chi_0 = 3.2 \times 10^{-11} m/N$.

Figure 3 shows the phase noise spectrum of the reflected beam obtained with a resolution bandwidth of 1 Hz and for the same frequency range (500-Hz span around the fundamental resonance frequency). The auxiliary laser beam is now turned off (no optical excitation) and the resonator is at room temperature. The spectrum is obtained by an average over 1000 scans of the spectrum analyzer. It is normalized to the shot-noise level and the vertical scale is smaller than the one of figure 2. The thin line in figure 3 corresponds to a theoretical estimation of the thermal noise at 300 K by using the mechanical susceptibility $\chi[\Omega]$ derived from optical excitation (eq. 4). Note that there is no adjustable parameter and the excellent agreement with experimental data clearly shows that the peak observed in figure 3 corresponds to the thermal noise of the fundamental mode of the resonator.

We have calibrated the observed displacements by frequency modulation of the incident laser beam. The detuning between the laser and the cavity resonance is indeed only depends on the optical frequency and on the cavity length. A displacement $\delta x$ of the back mirror is thus equivalent to a frequency modulation $\delta \nu$ of the laser related to $\delta x$ by

$$\frac{\delta \nu}{\nu} = \frac{\delta x}{L},$$

where $\nu$ is the optical frequency and $L$ the cavity length. We can thus calibrate the observed displacements by measuring the frequency modulation which yields the same phase signal for the reflected field.

![Image](image2)

**FIG. 3.** Phase noise spectrum of the reflected field normalized to the shot-noise level for a frequency span of 500 Hz around the fundamental resonance frequency of the resonator. The peak reflects the Brownian motion of the resonator at room temperature. The thin line is a theoretical estimation of the thermal noise. Vertical scale on the right represents the equivalent displacement in $m^2/Hz$.

The frequency modulation of the laser beam is obtained by applying a sinusoidal voltage on the internal electro-optic modulator of the laser. We determine the amplitude $\delta \nu$ of modulation by locking the mode cleaner at half-transmission and by measuring the intensity modulation of the transmitted beam. This intensity modulation is proportional to the ratio $\delta \nu/\nu_{cav}$ between the amplitude of frequency modulation and the cavity bandwidth $\nu_{cav}$ of the mode cleaner. We have determined this bandwidth with a good accuracy by measuring the transfer function of the mode cleaner at resonance for an intensity-modulated incident beam.

![Image](image3)

**FIG. 4.** Power of phase modulation of the reflected field normalized to the shot-noise level as a function of the amplitude of the laser frequency modulation. The solid line is a linear fit of the data points represented by squares.

Figure 4 shows the result of the calibration. We applied...
a sinusoidal voltage to the laser with different amplitudes at a frequency of 2 MHz. The horizontal axis represents the amplitude $\delta \nu$ of frequency modulation determined from the mode-cleaner cavity. The vertical axis corresponds to the power of phase modulation observed in the field reflected by the high-finesse cavity. Experimental results represented by squares are obtained with a 1-Hz resolution bandwidth of the spectrum analyzer and are normalized to the shot-noise level. The linear fit (solid curve in figure 3) has a slope equal to 2 as expected in log-log scales since the power of phase modulation must be proportional to the square of the frequency modulation. From equation (3) one can associate a displacement $\delta x$ to any observed phase modulation of the reflected field. In particular, the shot-noise level corresponds to a frequency modulation $\delta \nu$ min equal to $96 \, \text{mHz}/\sqrt{\text{Hz}}$. The smallest observable thermal displacement $\delta x_{\text{min}}$ which corresponds to the shot-noise level is thus equal to

$$\delta x_{\text{min}} [2 \, \text{MHz}] = \frac{I}{\eta} \frac{\delta \nu_{\text{min}}}{\nu} = 2.8 \times 10^{-19} \, \text{m}/\sqrt{\text{Hz}}. \quad (6)$$

This experimental result can be compared to the theoretical prediction. Equation (3) corresponds to a static analysis for a lossless cavity and for a perfect detection system. Cavity filtering at non zero frequency and losses reduce the theoretical sensitivity. The proper expression of the minimum displacement at frequency $\Omega$ is

$$\delta x_{\text{min}} [\Omega] = \frac{\lambda}{16 \pi \sqrt{T_c} \sqrt{\eta T_c}} \left( 1 + \frac{(\Omega/\Omega_{\text{cav}})^2}{A + T_c} \right)^{1/2}, \quad (7)$$

where $\eta$ is the quantum efficiency of the detection, $T_c$ the transmission of the coupling mirror, $A$ the cavity losses and $\Omega_{\text{cav}}$ the cavity bandwidth. The cavity behaves like a low-pass filter with a cutoff frequency $\Omega_{\text{cav}}$. We have thus performed another sensitivity measurement at the frequency of 500 kHz. We have found that the shot-noise level corresponds to a frequency modulation $\delta \nu_{\text{min}}$ of 68 mHz/$\sqrt{\text{Hz}}$ and the sensitivity $\delta x_{\text{min}}$ is then equal to

$$\delta x_{\text{min}} [500 \, \text{kHz}] = 2 \times 10^{-19} \, \text{m}/\sqrt{\text{Hz}}. \quad (8)$$

Both experimental values (eqs. 3 and 5) are in perfect agreement with theoretical values deduced from equation (3) with the parameters of the cavity (finesse $F = 37000$, coupler transmission $T_c = 60$ ppm, cavity losses $A = 109$ ppm, cavity bandwidth $\Omega_{\text{cav}}/2\pi = 1.9 \, \text{MHz}$, quantum efficiency $\eta = 0.91$, wavelength $\lambda = 810 \, \text{nm}$ and incident power $P = (hc/\lambda) T = 100 \, \mu\text{W}$). The discrepancy is less than 5%.

In conclusion, we have observed the Brownian motion of internal acoustic modes of a mirror with a very high sensitivity. This result demonstrates that a high-finesse cavity is a very efficient displacement sensor. The possibility to observe the thermal noise even far on the wings of the mechanical resonances opens up the way to a quantitative study of the spectral dependence of the Brownian motion. This would allow to discriminate between different dissipation mechanisms in solids. Let us emphasize that our device also allows to study with a very high accuracy the mechanical characteristics of the various acoustic modes (resonance frequency, quality factor, spatial structure, effective mass) and their coupling with the light. It is furthermore possible to obtain even larger sensitivities by increasing the finesse of the cavity or the incident light power. Mirrors with losses of the order of 1 ppm are now available and cavity finesses larger than $3 \times 10^5$ have been obtained [24]. For an incident power of 1 mW one would obtain a sensitivity better than $10^{-20} \, \text{m}/\sqrt{\text{Hz}}$.

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