Coupling Extraction From Off-Shell Cross-sections

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In this note, we present a novel method of extracting the couplings of a new heavy particle to the Standard Model states. Contrary to the usual discovery process which involves studying the on-shell production, we look at regions away from resonance to take advantage of the simple scaling of the cross-section with the couplings. We apply the procedure to the case of a heavy quark as an illustration.

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INTRODUCTION

The current experiments at the CERN LHC seek to expand our knowledge of the TeV scale – one of the crucial elements of this undertaking is the recent discovery of a new scalar particle [1] that has all the characteristics of the Standard Model (SM) Higgs boson. However, even with the inclusion of the Higgs, we know that the SM is not the whole story. Among its deficiencies is a lack of explanation of an underlying mechanism that generates the electroweak scale, an understanding of neutrino mass, and an explanation of the origin of dark matter in the universe. There have been many theories put forth to understand these issues – the so-called Beyond the SM (BSM) physics. These extensions usually involve new resonances that appear at the TeV scale. So far, no such heavy particle has been observed. But it is still important for us to be ready to understand the properties of such particles once they are discovered. Experimentally, this involves measuring two fundamental properties – their masses and their couplings to the SM states. This note addresses the latter issue.

Once a new particle is discovered, it may not be straightforward to extract its couplings to the SM states accurately in a hadronic collider like the LHC, where the momenta of the interacting partons are not known for any single event. Generally, at the LHC, to probe a new particle such as a new heavy quark, one looks for its QCD mediated pair production. Comparing its pair production cross-section with different decay modes one can extract information about ratios of the couplings (or equivalently the branching ratios (BRs)). From these ratios, in principle, it is possible to measure the values of these couplings if, along with all the dominant decay modes, the total decay width of this particle is also known. However, since unlike the BRs, measuring the width of a particle accurately can be quite difficult, this way of measuring couplings may lead to large uncertainties. In general it is more efficient to measure couplings from single production channels but these typically have smaller cross-sections. In this note we present a new method to extract couplings at a hadronic collider that depends only on the BRs but not the measured width and can be used with both pair production and single production channels. Before proceeding, we note that studies about constraining the Higgs width that are similar in spirit to this work have been proposed recently [2].

OFF-SHELL ANALYSIS AND COUPLING EXTRACTION

Discovery of a new particle entails looking at the on-shell production of a particle, designing invariant mass cuts to isolate regions close to the peak of the distribution. While this usually gives a good idea of the mass of the particle, information about couplings is hidden or lost. To see why, let us consider the single production of a heavy scalar $\Phi$ that decays to two SM fermions via $\lambda_{qq'}$ in a toy model, i.e., $pp \to \Phi \to qq'$. The method of production of $\Phi$ does not concern us here as we are more interested in its decay and the final state kinematics. The following analysis would hold even if $\Phi$ is produced in association with a different particle or produced in pairs. Let us start with the $\lambda_{qq'}$ dependent part of the amplitude:

$$M(p, \lambda_{qq'}) \propto \frac{\lambda_{qq'}}{(p^2 - M_{\Phi}^2) + i\Gamma_{\Phi}(\lambda_{qq'})M_{\Phi}},$$

where $p$ is the momentum of the intermediate $\Phi$. When $p^2 \approx M_{\Phi}^2$, the sensitivity to $\lambda_{qq'}$ is subdued by the $\lambda_{qq'}$ dependent width. In particular, in the narrow width approximation (NWA), we get [3]:

$$|M(p, \lambda_{qq'})|^2 \propto \frac{\pi \lambda_{qq'}^2}{M_{\Phi}\Gamma_{\Phi}} \times \delta(p^2 - M_{\Phi}^2),$$
making it clear that the coupling information gets hidden in the ratio \( \Gamma_{qq'}/\Gamma_\Phi \), i.e., the BR. In this approximation, if \( qq' \) is the only decay mode, then all information about \( \lambda_{qq'} \) is lost and if \( \Gamma_\Phi \) involves additional decay modes, the scaling of the cross-section with coupling is, in general, not simple. However, in the kinematic region where \( |p^2 - M_\Phi^2| \gg \Gamma_\Phi(\lambda_{qq'})M_\Phi \), the cross-section does scale like \( \lambda_{qq'}^2 \). Accessing this kinematic region can give us a meaningful way to extract \( \lambda_{qq'} \). This can be implemented with a very simple restriction on \( M(qq') \):

\[
|M(qq') - M_\Phi| \geq \gamma M_\Phi ,
\]
which, for small \( \gamma \), translates to \( |p^2 - M_\Phi^2| > \Gamma_\Phi M_\Phi \) if \( \Gamma_\Phi < 2 \gamma M_\Phi \).

Our minimal assumptions at this point are that both \( M_\Phi \) and \( \text{BR}(\Phi \to qq') \) are known fairly accurately from experiments. Below, we outline the main steps that one could follow to obtain \( \lambda_{qq'} \).

1. The first step is to numerically simulate the process for many different values of \( \lambda_{qq'} = \lambda_{qq'}^i \) using \( \Gamma_\Phi \) obtained from the theoretically computed partial width and the experimentally measured BR as \( \Gamma_\Phi^i = \Gamma_{qq'}^{i}/\text{BR}(\Phi \to qq') \). This way, one can avoid using the measured width but rely on the BR that can be measured more accurately.

2. The regions outside the resonance can be accessed with an invariant mass cut on \( M(qq') \) defined by a new parameter \( \gamma_O \) that parametrizes the degree of off-shellness of the intermediate \( \Phi \):

\[
|M(qq') - M_\Phi| \geq \gamma_O M_\Phi .
\]
For each \( \lambda_{qq'}^i \) and for various choices of \( \gamma_O \), one could compute the following ratio:

\[
R(\lambda_{qq'}^i) \bigg|_{\gamma_R, \gamma_O} = \frac{\sigma_\Phi(\lambda_{qq'}^i, \gamma_O)}{\sigma_\Phi(\lambda_{qq'}^i, \gamma_R)} ,
\]
where \( \sigma_\Phi \) is the on-shell cross-section, computed using a cut that isolates the resonance:

\[
|M(qq') - M_\Phi| \leq \gamma_R M_\Phi .
\]
The advantage of working with the ratio \( R(\lambda_{qq'}^i) \) instead of either cross-section is that most uncertainties associated with the production mechanism involving pdfs etc. get cancelled in the ratio. But more importantly, any unknown coupling present in the production of \( \Phi \), i.e., \( pp \to \Phi \) gets cancelled in the ratio.\(^1\) Now, for each combination of \( \{\gamma_R, \gamma_O\} \), one can prepare a “calibration curve” (CC) by interpolating between \( R(\lambda_{qq'}^i) \)’s and thus prepare a family of such curves.

3. From the experiment one can measure the ratio \( R = R_{\text{exp}} \pm \Delta R_{\text{exp}} \) for all the \( \{\gamma_R, \gamma_O\} \) combinations considered to prepare the CCs. Here, we have accounted for errors in the cross-section measurements by a factor \( \Delta R_{\text{exp}} \).

In principle, one could simply match \( R_{\text{exp}} \) for any single combination of \( \{\gamma_R, \gamma_O\} \) with the corresponding theoretical CC and read off the coupling. However, there are some difficulties associated with this procedure. The efficiency of coupling extraction is not uniform for a particular CC since it depends on the steepness (or slope) of the curve. It is not clear how to select the optimal combination of \( \{\gamma_R, \gamma_O\} \) a priori. One could also take the average of the couplings extracted from different CCs but the issue of assigning proper weight factors for averaging may become an ambiguous issue. Instead, one can extract \( \lambda_{qq'} \) from a simultaneous fit using all the CCs. This can be done, e.g., by maximizing a likelihood function defined as:

\[
L = \prod_k \frac{1}{\sqrt{2\pi} \Delta R_{\text{exp}}^k} \exp \left[-\frac{(R^k(\lambda_{qq'}^i) - R_{\text{exp}}^k)^2}{2 \Delta R_{\text{exp}}^k} \right] ,
\]
where the index \( k \) runs over all the combinations of \( \{\gamma_R, \gamma_O\} \). The \( \lambda_{qq'} \) thus extracted will correspond to the value for which \( R(\lambda_{qq'}^i) \) best describes \( R_{\text{exp}} \) for all the \( \{\gamma_R, \gamma_O\} \) combinations.

For these outlined steps to work optimally, the process in question should have a large enough cross-section so that there are an appreciable number of events left over after the off-shell cuts. If the cross-section is small it may not be possible to measure \( R_{\text{exp}} \) reasonably accurately for many \( \{\gamma_R, \gamma_O\} \) combinations which in turn will increase the error in fitting. Also, the method works better with a broad resonance.

**ILLUSTRATIVE EXAMPLE**

To demonstrate the above method with an example, we consider a simple model with a new, heavy color triplet \( b' \) quark with electric charge \(-1/3\). Heavy fermions are part of many well motivated extensions of the SM \([4, 5]\).

We just make the simplifying assumption that it decays to \( tW \) via:

\[
L_{\text{int}} \supset \frac{g_W}{\sqrt{2}} \lambda_{tW} \bar{t}_L \gamma^\mu P_L b'_L W^+_\mu + \text{H.c.} ,
\]

In the Lagrangian above, \( \lambda_{tW} \) is the unknown coupling that we want to extract. At the LHC, if the \( b' \) is not too heavy, QCD mediated pair production will be the dominant production channel. There will also be the \( W \) mediated single production channel of \( b' \), like \( qq' \to b't \). Following the steps described before one can use the single production process to probe \( \lambda_{tW} \). However, as this process is weak interaction mediated, its cross-section will be quite small and it will be more so if \( \lambda_{tW} \) is also small.
FIG. 1: The invariant mass distribution of the $tW$ pair in the process $pp \rightarrow b'tW$ for different values of $\lambda_{tW}$ (events generated with with Madgraph5 [6]). Here, we fix $M_{t'} = 1$ TeV and BR$(b' \rightarrow tW) = 0.75$.

Hence, to take advantage of the enhanced cross-sections, here we make use of pair production and simply apply off-shell cuts to one of the two final state particles\footnote{In [4], one rough demonstration of the idea was already performed. It was shown, using a fixed $\gamma_O$ (called $\alpha_{cut}$ there), the single resonant production of a charge $5/3$ quark becomes sensitive to its coupling with $tW$.} - this reiterates our earlier point that this analysis depends only on the kinematics of the final state particles and we are free to choose the particular production process that would serve our purpose well. We note, however, that in our analysis below we will include all possible contributions to the $pp \rightarrow b'tW \rightarrow tW\gamma$ process, although the pair production will dominate.

In Fig. 1 we show the invariant mass distributions of the $tW$ pair in $pp \rightarrow b'tW$ for different values of $\lambda_{tW}$. As $\Gamma_{tW}$ increases with increasing $\lambda_{tW}$, the distribution spreads out and, with the increasing width, the height of the distribution decreases so that the area contained by the distribution remains approximately constant, i.e., the total cross-section is not very sensitive to $\lambda_{tW}$. But as can be seen, there are parts of the phase space that are sensitive to $\lambda_{tW}$. For example, if we look away from the mass peak, i.e., when the $tW$ pair is not coming from a close to on-shell $b'$, the cross-section becomes highly sensitive to $\lambda_{tW}$, in keeping with our discussion earlier. Of course, the price to pay for looking away from the resonance is the reduced cross-section compared to the resonant production. However, we can mitigate this to a large extent by considering “mixed states” where one $b'$ is produced resonantly.

To extract $\lambda_{tW}$, we follow the steps outlined before. Assuming $M_{t'}$ and BR$(b' \rightarrow tW)$ are known from experiments, we generate events for $pp \rightarrow tW\gamma$ in Madgraph5 [6] (including both $b'$ and SM contributions) for different values of $\lambda_{tW}$ where we compute $\Gamma_{tW}$ as $\Gamma_{tW} = \Gamma_{tW}/\text{BR}(b' \rightarrow tW)$. For this example, we set $M_{t'} = 1$ TeV and BR$(b' \rightarrow tW) = 0.75$ and generate events for the 14 TeV LHC. Since in this case we are considering pair production, rather than using the cut defined in Eq. 4, we modify it to include two cuts on the $tW$ pairs, one on-shell and one off-shell:

\begin{align}
\text{I. } |M(t_1W_p) - M_{t'}| \leq \gamma_R M_{t'} , \\
\text{II. } |M(t_2W_q) - M_{t'}| \geq \gamma_O M_{t'} ,
\end{align}

where $p, q = \{1, 2\}$ or $\{2, 1\}$ and the numbers imply that the particles are $p_T$-ordered. Condition (I) reconstructs the resonant $b'$ while condition (II) accesses the $\lambda_{tW}$ sensitive off-shell region. Our expression for $\mathcal{R}(\lambda_{tW})$ is given by:

\begin{align}
\mathcal{R}(\lambda_{tW}) \bigg|_{\gamma_R, \gamma_O} &= \frac{\sigma_{tW\lambda_W}(\lambda_{tW}^I, \gamma_R, \gamma_O)}{\sigma_{b'b'}(\lambda_{tW}^I, \gamma_R)} ,
\end{align}

where $\sigma_{b'b'}(\lambda_{tW}^I, \gamma_R)$ is now computed by applying cuts as in Eq. 9a to reconstruct two resonant $b'$s. We produce a family of CCs for $\mathcal{R}$’s with various combinations of $\{\gamma_R, \gamma_O\}$. In Fig. 2 we plot $\sigma_{tW\lambda_W}$ (including SM and BSM contributions) with respect to $\lambda_{tW}$ before and after cuts for three different choices of $\{\gamma_R, \gamma_O\}$. We also show $\sigma_{b'b'}$ in the same figure. The total cross-section, i.e., $\sigma_{tW\lambda_W}$ without any cut is mostly insensitive to $\lambda_{tW}$ – it remains almost constant for smaller $\lambda_{tW}$ (where NWA is good), but slightly decreases in the higher $\lambda_{tW}$ region (mostly) because of the larger widths. In this example the conditions 9a and 9b cuts become competitive in nature as they are applied on different $tW$ pairs simultaneously. Applying the cut in Eq. 9a we reconstruct the resonant $b'$, i.e., we accept the $tW$ pairs that fall inside the $\gamma_R$ mass window and so an increasing $\gamma_R$ means accepting more events (larger cross-section). Whereas, with Eq. 9b we look away from the peak (we accept the $tW$ pairs that fall outside the $\gamma_O$ mass window) and so...
\(\lambda_{tW}^{\text{test}}\) and the corresponding values obtained by maximizing the likelihood function defined in Eq. 7. The errors in the fitted values correspond to 1\(\sigma\).

| \(\lambda_{tW}^{\text{test}}\) | \(\lambda_{tW}\) obtained |
|-----------------------------|--------------------------|
| 0.08125                    | 0.083 ± 0.0020           |
| 0.17338                    | 0.171 ± 0.0020           |
| 0.32785                    | 0.328 ± 0.0021           |
| 0.43597                    | 0.435 ± 0.0026           |

TABLE I: The different choices for \(\lambda_{tW}^{\text{test}}\) and the corresponding values obtained by maximizing the likelihood function defined in Eq. 7. The errors in the fitted values correspond to 1\(\sigma\).

an increasing \(\gamma_{O}\) results in reduced cross-section. With this in mind, it becomes easier to understand the curves with the \(\{\gamma_{R}, \gamma_{O}\}\) cuts. For example, for a fixed \(\gamma_{R}\), we expect the cross-section to go down with increasing \(\gamma_{O}\). This we can see by comparing the curves with \(\{0.10, 0.01\}\) and \(\{0.10, 0.10\}\) – the second one is smaller than the first for all values of \(\lambda_{tW}\). Similarly for a fixed \(\gamma_{O}\), if we decrease \(\gamma_{R}\) we should get a smaller cross-section which can be seen from the curves with \(\{0.10, 0.01\}\) and \(\{0.04, 0.01\}\). We also observe that with increasing \(\lambda_{tW}\), the \(\sigma_{tW}^{\text{test}}\) curves first increase then decrease. This is because for a fixed \(\gamma_{R}\), as we increase \(\lambda_{tW}\) (and hence \(\Gamma_{\nu}\)), we miss more and more \(tW\) pairs that come from the resonant \(b'\), i.e., reconstruction efficiency of the resonant \(b'\) decreases and as a result the cross-section decreases. This also explains why the \(\sigma_{\nu}^{\text{test}}\) curves, for which we are reconstructing two resonant \(b'\)'s, fall (and fall faster compared to \(\sigma_{tW}^{\text{test}}\) curves) with increasing \(\lambda_{tW}\). When the coupling \(\lambda_{tW} \to 0\), the difference between the \(\gamma_{R}\) curves and the total cross-section is accounted for by the SM background.

To see how the method performs we have considered four different test couplings \(\lambda_{tW} = \lambda_{tW}^{\text{test}}\) and use \(R(\lambda_{tW}^{\text{test}})\) in place of \(R_{\text{exp}}\) (Eq. 10), assuming all \(R(\lambda_{tW}^{\text{test}})\)'s have uniform 10% errors for all combinations of \(\{\gamma_{R}, \gamma_{O}\}\). We have considered 8 different values of \(\gamma_{R}\) between 0.02 and 0.16 and 10 different values of \(\gamma_{O}\) between 0.01 and 0.10. We present the results of our analysis in Table I. We see that we are able to extract the couplings with percent level accuracies. We note, however, that ours is a parton level analysis. Doing full detector simulations and considering SM backgrounds properly are likely to modify the extraction efficiency. These issues are currently under investigation [7].

DISCUSSIONS

In this note, we presented a novel method of extracting the couplings of new, heavy states to SM particles. The essential point underlying the method is a rather simple observation that when a particle decays inside a collider, the invariant mass distribution of its decay products retains information of the coupling involved in the decay. With the off-shell cuts, this coupling extraction procedure actually becomes sensitive to the shape of the invariant mass distribution. The method is largely free of modelling assumptions apart from the fact it uses theoretically computed partial decay width to avoid the use of measured total width of the new particle, and thus avoids the errors associated with width measurement. Of course, computing the total width using the theoretically computed partial width and the experimentally measured BR means that some errors could come from these sources and ideally one should account for both. However, since generally these errors are expected to be much less severe than the ones in measuring widths experimentally, we neglect these for the time being. Similarly we neglect any error in measuring the mass of the new particle. The method does not depend on exact production mechanism of the new particle and can be used with single or pair productions. The coupling extraction method outlined here is completely general and applicable to any new fermionic or bosonic state. It also has the advantage of being independent of collider details, or the actual values of the mass and the BRs of the particle in question.

In this note we have ignored the issue of considering the complete SM backgrounds which in reality is a very crucial one. Since realistic background can only be considered in a case by case basis, inclusion of simple parton level background in the example shown should only be considered as an illustration. However, we hope that after the discovery of a new particle, its background will also be fairly well-known and thus can be dealt with. Since our aim here is simply to outline the methodology, we postpone a more complete demonstration with detector level simulations for both signal and background to a future publication.

Since the invariant mass distributions of the decay products are independent of the sign of a coupling and the exact nature of the vertex, this method as described can not be used to extract these. One could, however, improve upon this method to extract other information. To extract any coupling the cross-section of the process in question needs to be large enough to start with so we have enough events left after all the cuts. It is interesting to contrast this method with the usual discovery procedure. If the new state has a significant width (comparable to its mass), discovery is rendered more difficult as we would not have the typical sharp peak. On the other hand, a large width guarantees that regions away from resonance do not have negligibly small cross-sections and hence coupling extraction becomes more feasible.

Of course, one need not restrict attention to new physics alone. In principle, this method can be applied to extract the SM CKM matrix element \(V_{td}\) from top pair production. In fact, it can be applied to extract any coupling, be it SM or BSM, as long as enough events remain after the off-shell cuts.
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