A RANDOM STRUCTURE FOR OPTIMUM CACHE SIZE DISTRIBUTED HASH TABLE (DHT) PEER-TO-PEER DESIGN.

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ABSTRACT. We propose a new and easily-realizable distributed hash table (DHT) peer-to-peer structure, incorporating a random caching strategy that allows for polylogarithmic search time while having only a constant cache size. We also show that a very large class of deterministic caching strategies, which covers almost all previously proposed DHT systems, cannot achieve poly-log search time with constant cache size. In general, the new scheme is the first known DHT structure with the following highly-desired properties: (a) Random caching strategy with constant cache size; (b) Average search time of $O(\log^2(N))$; (c) Guaranteed search time of $O(\log^3(N))$; (d) Truly local cache dynamics with constant overhead for node deletions and additions; (e) Self-organization from any initial network state towards the desired structure; and (f) Allows a seamless means for various trade-offs, e.g., search speed or anonymity at the expense of larger cache size.

1. Introduction and Motivation

In general, the structure of a DHT peer-to-peer network can be modeled as follows: Each content when introduced into the network is assigned a key. The key is uniformly chosen from a numerable set $R$ containing $N$ all possible keys. This set can be a subset of the $d$ dimensional hypercube $Z^d$ as in or simply the one dimensional set $1, ..., N$. Suppose for a moment that each node $I$ can provide the content $K_I$ when it is asked to. This assumption can easily be relaxed in future, but for now, it keeps the arguments simpler. Henceforth, we interchangeably use the key $K_I$ to refer to the physical node $I$ when there is no ambiguity.

Note that the fact that the node $I$ has content $K_I$ does not really mean it actually has cached the file $K_I$, it instead might refer the search to a place where the actual file can be downloaded. Nevertheless, we assume that the search for content $K_I$ is hit the answer when reached the node $I$.

The node $I$, besides having the content $K_I$, has the address of $d$ other nodes $C_K = \{K_1, ..., K_d\}$ called its cache content.

A caching scheme is a deterministic or randomized rule that determines the subset $C_K$ of $R$ that the node $K$ has to cache in order for the final searching scheme to be able to find the queries initiated by arbitrary nodes for arbitrary keys.

This scenario as explained later contains almost all proposed peer-to-peer structures so far.

1.1. Deterministic DHT Schemes: By a deterministic caching scheme, we mean a deterministic algorithm that determines which keys should be addressed in the

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key cache of the node $K$ based on $K$. In other words, the topological properties of the emerging network directly specifies the ability of the network to find queries for keys initiated at different nodes. Suppose $R$ is supplied with an addition operator +. We define a commuting deterministic caching strategy as a set of $d$ different elements of $R$, to be called $c_1, ..., c_d$, such that the cache of the node $K$ is $C_K = \{ K + c_1, K + c_d \}$. Note that although restricted at first glance, this definition includes almost all non-hierarchical structures proposed so far, such as Chord and CAD.

For Chord [4], the keys are one dimensional on a ring lattice. Each node has $\log(N)$ references to nodes with keys $N/2, N/4, N/8, 2, 1$ apart. Apparently, such a network can be searched in time $\Theta(\log(N))$. When a node $K$ starts a query to find the key $T$, it passes the query to any node in its cache table which is closer to the target. Proceeding this way, it is not hard to see that any query will receive the target in $O(\log(N))$ steps.

The content space of CAD [1], on the other hand has a $d$ dimensional hypercube topology. Any node has in its cache the address of its $2d$ neighbors. For $d = \log(N)$ it is easy to show that the search time is $O(\log(N))$.

**Result 1:** For a commutable deterministic caching scheme as above, suppose $d \leq \frac{\log(N)}{m \log(\log(N))}(1 + \log(\alpha)/\log(N))$, then starting at each arbitrary node $K$, at most $\alpha N$ of all nodes can be reached in less than $\log^m(N)$ steps.

**Proof:** Starting from node $K$, let’s count the number of different nodes that can be reached in at most $\log^m(N)$ steps. At each node $K$, the query can proceed any of the $d$ different links taking it to nodes $K + c_1, ..., K + c_d$. After $l$ steps, the position of the query is $K + a_1c_1 + ... + a_dc_d$ and $\sum_{i=1}^d |a_i| \leq l \leq \log^m(N)$ hence $a_l < \log^m(N)$. The different number of the final locations possible after $\log^m(N)$ steps is bounded by: $T = (\log^m(N))^d$. Equating this by $\alpha N$ and taking the logarithm from both sides we get: $d = \frac{\log(N)}{m \log(\log(N))}(1 + \log(\alpha)/\log(N))$. Hence, for $T$ less than this amount, at most the total number of all possible nodes the query can reach in $\log^m(N)$ steps is $\alpha N$. There is no deterministic caching scheme achieving an average search time of polylog(N) unless $d = \Theta(\frac{\log(N)}{\log(\log(N))})$.

**Corollary:** There is no commuting deterministic caching strategy that allows for local or global search in mean polylog time when the cache size is less than $\Theta(\log(N)/\log(\log(N)))$. Specifically there is no commutable deterministic caching scheme with constant cache size that allows for polylog search.

**Result 1** can be easily extended to any finite commutable set of operations rather than +. Note that in the proof, we only assumed that the set of operators each node can choose from, is finite and independent of the node position $K$. Note that a tree structure can be fit to this general scheme, however, each node chooses a constant number of operators from the total $\log(N)$ number of operators. Hence, a tree can achieve logarithmic search time with constant cache size. However, tree suffers from a serious problem, making it an improper structure for a p-2-p system. Since the cache selection rule depends on the key $K$, the structure is not symmetric and hence the traffic is not distributed evenly, in fact deleting just one node might
completely destroy a big number of routings paths.

2. Freenet: An attempt for a less structured design

In Freenet [3], the topology is a ring lattice. At the steady state, each node is meant to be connected to \(L\) nearby neighbors as well as a few far neighbors called shortcuts. There is no specific caching scheme as to what the far keys cached should be. Freenet is important for our discussion because it is among the few less structured designs that as will be shown shortly has many features in common with our proposed design.

The original Freenet design does not seem to be aware of the crucial importance of random shortcuts for the scalability of the system. There is no simulation regarding the scaling behavior of Freenet as a function of the cache size in hand, however the original simulations of Freenet show that a relatively large cache size of about 250 will fail to provide reasonable search time when there are almost 200000 contents in the network.

Based on the small world intuition, that is trying to have a cluster of keys around a single key and having random shortcuts apart from that key, modifications to Freenet have been proposed in [5]. They also conclude that \(O(\log^2(N))\) cache size is necessary to perform search in time \(O(\log^p(N))\) on their modified Freenet.

The problem with the Freenet is that, when pointed as the source of a key by another node, a node might not in fact have that key as the center of its cluster and hence might not be able to provide a suitable search path for the query.

We will show that this in fact might make the Freenet incapable of scaling as seems to be the case with the current Freenet.

**Freenet is not scalable:** Freenet, or at least with its modifications in [5], apart from its efforts to enforce anonymity, reads as follows: Each node has a seed key \(K\). This node might be called node \(K\). When being in the routing path for a key \(K\), the key \(K\) once found is cached if it is closer to the seed than another key already in the seed. The variation to allow for the shortcuts, allows a far key to replace a closer key with some probability \(q\). The intuition behind this is to have an emerging ring lattice network topology, where every node is connected to \(C\) neighboring nodes, while it has \(qC\) shortcut links.

There are variations to how the Freenet updates its cache contents mostly based on different engineering intuitions. As an example, the least referenced key might be substituted with a new key. There however is no reason why any such caching strategy should lead to a scalable search algorithm. Approximating the emergent structure of the Freenet, with a local connection of \(2C\) links to \(C\) neighbors on each side along with \(C'\) uniformly randomly chosen shortcuts we have the following result:

**Result 2:** The Freenet with above assumptions can perform search in time \(\leq \log^m(N)\) iff \((C + C') = \Theta(\frac{N^{1/2}}{\log^{m/2}(N)})\).

More precisely, take \(\varepsilon = \frac{(C + C')}{2 \log^{m/2}(N)}\), then for taking \(C = C'\), the probability of not finding the target in \(\log^m(N)\) steps is at most \(e^{-2\varepsilon^2}\). On the other hand for \(\varepsilon << 1\) regardless of the choice of \(C\) and \(C'\), the probability of finding the target in \(\log^m(N)\) steps is at most \(\varepsilon^2(1 + o(1))\).
Proof: We first prove the if part: Starting from any node, consider a region of size $2C\log^m(N)$ with the target in the middle. At each step of the walk the probability that there is no shortcut into this region is $p = (1 - \frac{2C\log^m(N)}{N})C'$ because there are $C'$ shortcuts at each node visited. Once a shortcut is found into this region, then the target is simply reached in at most the next $\log^m(N)$ steps by the local connections. This probability is bounded by $p \leq e^{-2C'C\log^m(N)/N}$, now if $CC' = \varepsilon^2 N/(2\log^m(N))$ the probability of not finding the proper link after $\log^m(N)$ steps is bounded by:

$$p \{\text{no link in } \log^m \text{ steps}\} = (1 - 2C/N)^{\log^m(N)C'} < e^{-2CC'\log^m(N)} = e^{-2\varepsilon^2}.$$  

Of course for this choice of $CC'$ the minimum cache size $C + C'$ is when the two are equal, that is $C = C' = \varepsilon^2 (N^{1/2}/\log^m(N))$ completing the first part of the proof.

To see that this is in fact also necessary, consider the contrary. Consider two sets of nodes that are more than $\log^m(N)$ apart. Hence to reach the target in $\log^m(N)$ steps, at least one shortcut should be made into the regions of width $C$ around the target. In the greedy search followed by Freenet, the query gets closer to the target at each step, however, since the shortcuts are uniform, this does not change the probability of finding a shortcut into the specified region. Hence in any case, the probability of finding a shortcut into that region in the first $L$ steps is $p = 1 - (1 - 2C/N)^{LC'}$. The product $\alpha = 2CC'L/N$ is bounded by the choice of $C = C' = \varepsilon^2 (N^{1/2}/\log^m(N))$ giving $\alpha < \varepsilon^2$ Expanding $p$ noting that $\alpha$ is very small:

$$p = 1 - (1 - \alpha + O(\alpha^2)) = \varepsilon^2(1 + o(1))$$ completing the proof.$\Box$

Hence unless a precise caching policy is not employed the Freenet does not seem to be capable of scaling with ad hoc caching schemes currently in use.

In fact the necessity of a cache size of the order of $O(N^{1/2})$ for a fast search can be tracked in the early simulations of the Freenet in which case a cache size of 250 was used. The search time shows exponentially abrupt increase as the system size approaches 40000 with 5 keys per node. Note that $\sqrt{100000} \approx 300$.

The importance of caching strategy is hence evident. We propose a practical peer-to-peer system based on precise reconsideration of a rather similar architecture.

3. A truly p-2-p random DHT structure

With discussions in previous sections, new paradigms for designing DHT peer-to-peer structures turn out to be necessary.

A p-2-p system (at least with homogenous members) should have the following characteristics:

1) Short routing paths: This seems to be the first desired characteristics of any p-2-p structure. A scalable design is the one with path length scaling logarithmically with the network size. From the four well known DHT designs, CHORD, TAPESTRY and PASTRY, have search time $O(\log(N))$, while CAN has search time $O(dN^{1/d})$ for a constant $d$.

2) Small Number of Neighbors or Cache Size: The number of neighbors of
a node, meaning the number of keys it has the address of, is probably the second important issue in a DHT design. Particularly there are two issues in having large cache sizes. First the mere notion of space complexity which is more relevant to theoretical issues than practical ones because of the relative small price of storage capacity.

The more important issue is the overhead involved in updating these caches. P-2-P networks are very dynamic meaning that nodes continuously join and leave the network. In fact one might assume the query rate to be in the order of log-in rate, meaning that many nodes just log-in to make constant number of queries and then they log-off. Hence a small cache size is very important. The only structure with constant cache size known so far is CAN. Unfortunately CAN can not have polylogarithmic search time while still having a constant cache size.

3) Uniform Load Balance: The load of query routing should be as evenly spread as possible. At the very least, when queries are made from uniformly random nodes to uniformly random targets, the average load on all nodes should be equal. This (as well as many other reasons) excludes tree like structures as candidates for a DHT structure.

4) Patternless structure: A deterministic design has a very distinct connection pattern between its nodes. An attacker can simply follow the pattern to disable a certain node. Suppose as an example the CHORD design. An attacker now in node $K$, knows exactly the set of keys $K$ should have in its cache. It then only suffices to disable those keys for the node $K$ to be once and for all excluded from the system. Less patterns in the connections makes it less likely for an attacker to systematically attack the network. All deterministic systems can be easily disabled by a smart enough distributed attack.

5) Locally adjustable tradeoffs: A good p-2-p protocol should consider the fact that the same algorithm might need to be implemented in different working environments composing of potentially different characterizations. As an example, it might be desirable that the same system work in different problem size regimes or different dynamical situations.

With these goals in mind, we propose the first known randomized DHT structure with the following characteristics:

1) Randomized caching strategy with constant cache size $O(1)$.
2) Average search time of $O(log^2(N))$.
3) Guaranteed search time of $O(log^3(N))$.
4) Truly local cache dynamics with no overhead.
5) Global convergence from any initial network state.
6) Local adjustments can trade cache size for search speed.
7) Anonymity can be bought in price of larger cache size with only local considerations.
The following lemma due to Kleinberg is essential to our design structure:

**Lemma 1** (Kleinberg) [2]: Consider a lattice topology in $\mathbb{Z}^d$. Nodes are placed on the grid points. Each node is connected to all its $2d$ neighbors. Also each node has a shortcut link. The probability that the node $K$ has a shortcut to node $K'$ is proportional to the inverse of its $d$ dimensional Euclidean distance, that is $p(c_K = K') \propto \frac{1}{|K - K'|^d}$. A greedy search algorithm starting from a random node searching for a random target, will find the target in average time $O(\log^2(N))$. Also, for no other exponent in the probability rather than $d$ the average search will have polylogarithmic time.

**Proof:** Please refer to [2] for a complete proof of the case $d = 2$. For the case $d = 1$ however we sketch the proof as it proves relevant to our other considerations in the rest of the paper:

Divide the region from 1, ..., $N$ into $\log(N)$ distinct regions $X_i = \{2^i, ..., 2^{i+1} - 1\}$. Suppose the target and the current node possessing the query are a distance $r \in X_i$ apart. Then we say that the search in phase $i$. Each node receiving the query, will pass it to a node in its cache which is closest to the target.

The proof relies on the fact that at most $O(\log(N))$ steps are required for the query in each phase $i$ to reduce its phase by at least one. To see why, note the furthest distance from nodes in the phase $i$ to those in phase $i - 1$ is $2^i - 2^{i-1} = 2^{i-1}$. Also the normalizing constant is $c \log(N)$ for a proper $c$ meaning that the probability that a node in phase $i$ has a link to a node in phase $i - 1$ is at least $c/\log(N)$. There are then at least $2^{i-1}$ nodes in phase $i - 1$, hence the total probability that a node in phase $i$ has a link to the phase $i - 1$ or less is at least $c/\log(N)$.

Now the probability that in $L$ steps in phase $i$ no link is found to phase $i - 1$ is at most $c \log(N)^L$. Hence in average $O(\log(N))$ steps a link is found to the next phase. Since there are $\log(N)$ phases altogether, the query reaches phase 1 which has $\log(N)$ members in $O(\log^2(N))$ steps. The last phase can be traversed by only local links in $\log(N)$ steps at most.

**Result 3:** For any realization of the above scheme, there is a constant $a$ independent of $N$, such that any query almost surely reaches any target at time at most $a \log(N)$ as $N \to \infty$.

**Proof:** Considering the above proof, the probability of not finding any link into the next phase after $3\log(N)^2/c$ steps, for the $c$ defined in the Lemma 1, is bounded by $N^{-3}$, there are $N^2$ possible queries making the probability of any query having that problem arbitrary small. So each query will reach the destination almost surely in at most $(3/c)\log^3(N)$. □.

Consider a space of at most $N$ nodes as described earlier formed as a ring lattice topology. Hence each node has the address of two close nodes in its cache. Also each node $K$ has the address of another node $K$. The probability of the node $K$ to be chosen is proportional to $\frac{1}{|K - K|}$. Kleinberg’s theorem can be readily applied to this case to show that a greedy algorithm can search this network with time $O(\log^2(N))$. 
Result 4: There is a randomized caching strategy along with a local search that allows for average $O(\log^2(N))$ search time, while each node has only 3 neighbors.

Proof: Consider a DHT system employing the above caching strategy. That is a node having key $K$, also has the keys $K + 1$ and $K - 1$ and another key $K'$ chosen with probability $\propto \frac{1}{|K - K'|}$ among all other keys (all operations of mod $N$). One can easily verify that the ring lattice topology does not alter the above lemma much. The number of neighbors of any node is hence just 3. Lemma 1, shows that such a network topology equipped with a greedy search algorithm has average search time of $O(\log^2(N))$ and Result 3 also predicts a worse case search time of $O(\log^3(N))$. Hence with constant cache size, the search time is polylog, something proven impossible for at least a very large class of deterministic caching strategies.

Practically the above design will have the following specifications:

1) Topology: Each content is assigned a key uniformly randomly chosen from the set of all possible keys $1, ..., M$. This assumption might restrict the future expansion ability of the system. Nevertheless, choosing a very big $M$ does not add much to the computational costs of the system. In fact assigning 32 bit integers to the keys, naturally confines any design to a few billion keys.

At a particular time, there are $N$ nodes in the network. $N$ might be very smaller than $M$, however, the $N$ keys are assumed to be uniformly distributed in the $M$ possible places. The topology is then that of a ring with $M$ places where $N$ of them are occupied. Note that a physical node, might in fact contain more than one key. The important point is that for each key introduced, a proper shortcut as well as close keys should also be added to the cache contents. Different shortcuts due to different keys of one node, need not be differentiated, since following a closer shortcut can only facilitate the search procedure.

2) Node Arrivals: When a node with a certain key joins the network, it initiates a query for its own key to a bootstrap node through which it has joined the network. The query passes until reaches a node where it cannot pass the query to any node with a closer key. This node, has the closest key to the new key. The new node then makes two connections to this node, and a node immediately after that in order for its local connections.

Now, to make its shortcut, the node pretends as if the key space was not sparse. Node $K$ chooses a key $K'$ with probability $|K - K'|^{-1}/H$, where $H = \sum_{i=1}^{M} 1/i \sim \log(M)$. It then initiates a query for this key. Again, the closest key to this query is returned. The node then adds this query to its cache table.

Looking at the procedure of the Lemma 1, it is easy to see that this way the sparse nature of the key space does not prevent the efficient search provided that the keys are uniformly distributed. In fact, every length scale, is scaled by a factor $M/N$ which does not affect the analysis. In later sections, we will introduce a more natural and practical self-organizing algorithm, which allows for cache update as the network answers queries. This eliminates the need for making dummy queries for keys that might not exist, also does not require knowing $M$. 
3) **Node Departure:** Though the original scheme does not require links to be bidirectional, there are certain practical advantages in having a bi-directional link. The most important of all is that when a node decides to log-off, it can inform the nodes referring to it to look for another random key. A convenient way is to refer to a nearby node. A better way is to simply redo the procedure done for the connection. In any case it is natural to assume that a node is informed about the log-off of any of its connections.

4) **Refreshing:** Nodes can change their shortcuts completely without previous notice. Since never in the proof of Lemma 1, it is necessary for a node to be certain about where its links are before it receives the query. In fact the principle of differed decision is in the heart of the proof and nodes can dynamically change their shortcut connectivity. This way the traffic can be evenly distributed among all nodes avoiding hot spots due to the nodes receiving very far links. It is a good idea to have a refreshing rate in the order of the network dynamics time scale (the log-n log-off rate).

**Trading cache size for speed:** In this section we introduce the concept of nested shortcuts to allow for faster search times when more than constant cache size can be tolerated. The idea is to provide more than one level of shortcuts by aggregation of close keys and relabelling them. As an example, consider the following aggregation process:

Assign each set of nodes $N_1^i = \frac{(i - 1)N}{log(N)}$, $iN/log(N) - 1$ a new label $i$. Call $N_1^1$ the set $N_1^1, ..., N_1^{log(N)}$. For each set $N_1^i$, proceed the same way and define the sets $N_1^{1,1}, N_1^{1,2}, ...$ by aggregating the keys of each set, into smaller sets. The number of sets resulting from each division is the logarithm of the size of the upper set. This will continue until the number of elements in the lowest hierarchy is $1$ (or even $log(N)$). Now at each level of the hierarchy, a node with label $i$ will have a link to another node from the same hierarchy with label $j$ with probability proportional to $|i - j|^{-1}$. The normalizing constant is chosen according to the number of labels on that hierarchy. Among all nodes with label $j$, node $i$ will choose one in random.

**Result 5:** A greedy search on a structure made by the above procedure, results in search time $O(log(N)/loglog(N))$ while the cache size is $S < log(N)/log(log(N))$.

**Proof:** Suppose for a moment that only the links of the very first hierarchy existed. Having a small-world in the first hierarchy, after at most $O(log^2(N_1))$ steps, the query and the target will match on the first label, where $N_1$ is the number of the labels in the first hierarchy. Note that having other links can only speedup this process. From then on, the same arguments can be made for other hierarchies. The number of labels in each hierarchy is at most $log(N)$. Hence the total search time is $O(log^2(log(N)))W$ where $W$ is the total number of the labels. Now let's bound $W$ the depth of the hierarchy which in turn is the cache size necessary. To do this, start with the first hierarchy having $log(N)$ labels and $N_1 = N/log(N)$ members in each label. Taking the $log$ results in $log(N_1) = log(N) - log(log(N))$. Hence the members of the second label have $N_2 = \frac{N}{log(N)log(log(N))}$ members. So:
\begin{align*}
\log(N_2) & = \\
& = \log(N) - \log(\log(N)(\log(N) - \log(\log(N)))) \\
& = \log(N) - \log(\log(N)) - \log(\log(N) - \log(\log(N))) \\
& = \log(N) - \log(\log(N)) - \log(\log(N)) (1 - \frac{\log(\log(N))}{\log(N)}) \\
& = \log(N) - 2\log(\log(N)) - \log(1 - \frac{\log\log(N)}{\log(N)}) \\
& = \log(N) - 2\log(\log(N)) + o(1)
\end{align*}

Proceeding this way, for other steps, it's not hard to see that:

\begin{enumerate}
\item \(\log(N_i) = \log(N) - i \times \log\log(N) + o(1)\)
\end{enumerate}

Assuming \(N_i > 1\), that is \(\log(N_i) > 0\), it implies that:

\begin{enumerate}
\item \(i < \log(N)/\log(\log(N))\)
\end{enumerate}

Meaning that in at most \(W < \log(N)/\log(\log(N))\), the number of labels gets to 1.

One can trade cache size for the same increase in the speed. This generalization might prove useful in applications where faster than \(\log^2(N)\) search is necessary. The limit of \(O(\log(N))\) cache size and search time is the same as that can be achieved by deterministic means as well. However, the randomized nature of the caching allows for more robust and less likely to be attacked networks. Also, all decisions are totally local, and at each level there are very many nodes from which the shortcut can be chosen (in fact in the \(i\)th hierarchy almost \(N \frac{\log(N)}{\log(N)}\) different nodes can be chosen. Though probably of less practical importance, this scheme shows the ability of the design to be adapted to different working regimes.

**Multiple contents and Anonymity:** So far we assumed each node has exactly one key, with which we named the node itself. As might have become clear, this assumption can readily be relaxed. Any physical node can have multiple content keys. Each content however should have its neighbor keys as well as a shortcut key chosen according to the proper distribution around that content.

Any search for any key in this new system can only be faster than the case where only one content was present.

On the other hand different contents need not really represent totally unique keys. Same key might be cached in many nodes. Turning to the idea used in Freenet to ensure a degree of anonymity one can think of the following algorithm:

A key once found would be cached, as if it was their own content, on all the nodes in the search path. Of course the node caching it must also provide the nearby neighbors as well as the proper shortcut for this new key. The search is really said to be finished when such a key is found.

The anonymity is insured by increasing the average cache size of the nodes.
4. LOCAL CACHE UPDATES AND TOPOLOGY CONVERGENCE

In previous sections, it is assumed that nodes are able to make shortcuts in the content space to keys that are a distance $r$ apart with probability $\propto 1/r$. We investigate the possibility of performing such task locally.

If the size of the key space is known a priori, meaning that the maximum capacity of the network is fixed, then each node joining the network can initiate a query for a random key it chooses from the $1/r$ distribution around its own key. If that key does not exist, the closest key is cached. To take care of new nodes joining, this procedure can be repeated with an appropriate rate.

The limitation for knowing $N$, comes from the fact that the distribution $1/r$ is not normalizable for $N \to \infty$.

In this section we examine a more natural approach. The network gradually organizes itself to take the form of the desired topology as more queries are being answered by the system. Our idea is based on the assumption that the queries for a key $K$ are initiated by randomly chosen nodes in the key space. A node $S$ starts a query for the target node $T$. The key $T$ can be a very popular key who receives many requests in time, however, the nodes initiating the request for that key can be assumed to be uniformly chosen from the network.

Now, as the system works, a node $K$ answers a request from the node $T$. Suppose $K$ already has key $L$ as its shortcut in the cache. Upon answering the request of the node $T$, it replaces $L$ with $T$ with probability $\frac{|K-L|}{|K-L|+|K-T|}$. Otherwise it keeps the key $L$. Similar idea is suggested in [5] as an intuitive answer. Here we prove the validity of this intuition by the following theorem:

**Result 6:** Repeating this procedure the probability of having the key $K'$ in the cache tends to be $\propto \frac{1}{|K-K'|}$.

**Proof:** We define a Markov chain as follows: If the distance of the cache from the node’s own key $K$ is $x$, we say that the Markov chain is in state $x$.

A step is made when a new sample with distance $y$ is received. Then the chain walks to state $y$ with probability $x/(x+y)$, otherwise it stays in the same state.

This procedure clearly defines a Markov chain with $N$ states. For any finite number of states $N$ a stationary probability distribution for the process exists. Let's call the stationary probability of residing in state $x$, $p_x$. We will find this stationary solution considering the flow-in flow-out of an arbitrary state $x$. The flow coming out of the state $x$, is the probability of being in $x$ (i.e. $p_x$) times the probability of moving out of it in the next step.

The flow into the state $x$ is the probability of moving to the state $x$ in the next move. Equating the two:

$$p_x \sum_{i=1,...,N} \frac{1}{N} \frac{x}{x+i} = \frac{1}{N} \sum_{j=1,...,N} p_j \frac{j}{x+j}$$

Hence, evidently $p_i = (1/c) \times \frac{1}{i}$ satisfies the equation for all $x$ where $c = \sum_{i=1}^{N} 1/i$ is the normalizing constant. Since the stationary solution is unique the system always converges to the proper distribution.

Meaning that no matter what the initial keys assigned to the nodes during the bootstrap section are, the system eventually forms towards the desired solution as
soon as enough random queries are made. Another interesting practical feature of this scheme is that the nodes are naturally informed from the entrance of new nodes, when those nodes make queries. The new nodes are then automatically put in the searching routes by being added to the cache of the target.

5. Simulations

To prove that the caching system in fact can be used as a superior structure for a peer-to-peer network, we have provided the following simulation. Starting form two nodes, a node joins the network and is assigned a random key. It is then placed between the two nearest keys already in the network, meaning that it caches their address as the immediate neighbors, the neighbors also update their caches to recognize the new node. It is also assigned a random key from the nodes previously in the network as its shortcut. This key can be the node contacted by the new node as the bootstrap. After $N$ nodes joined the network in this fashion, at each time step two nodes are randomly chosen from the network and one initiates a query for the other. Upon reception of the query by the target, it replaces its cache with the key of the requester with the mentioned probability. After each time step, 100 queries are made to random nodes in the network to find the average search time.

The average search time is depicted in FIG.1. As clearly seen in the figure, the system totally reaches the steady when the number of queries made is in the order of the number of nodes meaning that almost all nodes have had the chance to update their cache at least once. This is a totally satisfying result, meaning that each node has to answer in the order of one queries before the network topology is formed. A precise formulation of the settling problem is the subject of a subsequent paper.

6. Conclusion

We showed that there is in fact little hope for deterministic caching strategies in peer-to-peer networks to provide logarithmic search time with constant cache sizes. Also their deterministic structure makes them vulnerable to attacks. Our proposed system can provide search times in the order of $O(\log^2(N))$ with cache size of only 3. We also showed that the original Freenet lacks scaling, however our proposed strategy provably provides scalable and highly efficient peer-to-peer networks. It can also be used as a quick modification into Freenet original caching protocol. Through out the derivations, routings to different nodes are regardless of what shortcuts the receiving node has. This enables nodes to change their shortcuts without prior notice. Hence even during a single query search, the path might constantly change, making it very hard to track the paths. Also, the shortcuts are always being refreshed as new queries are received by a node without producing any overhead traffic. The symmetry of the protocol results in a naturally balanced load on all nodes. The randomized nature of the connections as well as their dynamic, prevents an attacker from fragmenting the network. Our future work is towards implementing a practical peer-to-peer network based on the architecture proposed in this rather theoretical paper by taking into account the
more detailed practical considerations such as duplicate keys and failure handling.

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Our Scheme
Random Shortcut

Figure 1. Average search time from the bootstrap phase until the system settles down to its steady state. The total number of nodes are 1000 for the top figure and 3000 for the bottom one. Included in each figure is the search time for a system with a random shortcut for comparison. The total key space is 10000. Keys are uniformly randomly chosen. Each node (key) knows the address of the nearest neighbors as well as one shortcut. Queries are in randomly chosen pairs. The cache is updated through the rule in Result 6. As enough queries are made (about twice the total number of nodes) the system settles down. The average search time at steady state is around 21 for $N = 1000$ and 28 for $N = 3000$. 