A CLOSING TALK FOR A VERY NICE MEETING

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I will use this opportunity on the one hand to comment upon some of the many interesting results that have been presented at this meeting, on the other hand to discuss some new features that we have recently learned on the structure and properties of the Lund Model both with respect to the fragmentation of multigluon string states and the partonic cascades based upon perturbative QCD.

1 Introduction

I have been very much impressed about the quality and the amount of new results that have been presented at this meeting and I would like to start by applauding both the organisers and the speakers for excellent arrangements, entertainment and new insights.

It is evident that there is no way to comment upon all the results we have heard of during the five days and consequently I will be satisfied to take up a few of them and to briefly consider their significance. I apologize to all those that I am not going to mention.

After that I will in the spirit of a Closing Talk go over to those parts of the multiparticle dynamics that are close to my own heart. After a brief lamentation on our present way to treat QCD I will start by pointing out that there are strong indications of the existence of a dynamical scale between one and two GeV that ought to be taken into account in all considerations of a confined field theory.

After that I will briefly touch upon the new features that we have found in the structure of Lund String Fragmentation both with and without multigluon excitations. There is also another (and even more recent) finding dealing with the properties of the QCD perturbative cascades.

One of the major differences between the abelian QED and the non-abelian QCD is the fact that the QCD field quanta, the gluons, are charged and that consequently emission of a gluon implies that the currents are changed. The phase space in QED for multiple emission of photons is given by the properties of the original current (besides the effects of the recoils in the emission of “hard” photons).

But already the emission of a first QCD gluon in $e^+e^-$-annihilation means that the original ($q\bar{q}$) dipole is changed. As it happens the change is (to a
very good approximation) from one dipole to two (independent) dipoles, one
between the quark \((q)\) and the first gluon and the other between the gluon
and the anti-quark \((\bar{q})\). While the first dipole is at rest in the total cms the
two “new” dipoles are moving with respect to each other. This implies that
the combined phase space for emitting either from one or the other dipole
(this is what “independence” means) will cover a larger phase space region.
In rapidity space the increase can be described as an extra region of a size
corresponding to the logarithm of the squared transverse momentum of the
first gluon emission.

It is possible to describe the phase space from multiple gluon emission
in terms of a generalised rapidity, \(\lambda\) or as the length of a curve composed of
connected hyperbolas related to the generalised rapidity in the same way as
the length of a single hyperbola is related to ordinary rapidity. This curve is
defined in an infrared stable way from the partonic energy momenta and it has
(multi-)fractal properties with dimensions given by the so-called anomalous
dimensions of QCD.

The basic new result is that while the bremsstrahlung spectrum in QED
can be described as a constant density (given by \(\alpha/\pi\) with \(\alpha\) the finestructure
constant) in rapidity \(y\) and the logarithm of the (squared) transverse momen-
tum, \(\kappa\), the QCD spectrum can be described as a similar constant density in
terms of the the generalised rapidity \(\lambda\) and \(\kappa\). I will discuss the implications
of this result, thereby ending with the usual optimistic credo of a theoretical
multiparticle physicist: “There is still a lot of interesting things to be found
in phase space!” (at least if you chose the right space).

2 Some Remarks On What We Have Learned

We firstly learned about the running of the chinese accelerator BES from Dr
Xu Guofa and we are happy about their precise results on the ratio between
hadron- and muon production from \(e^+e^-\)-annihilation almost down to the
treshold. We know that this work has a significance for the basic param-
ters of the Standard Model. Personally I feel that precise results from the
baryon-antibaryon channels should provide very useful constraints on all the
multiparticle production models. As I understand it from discussions with
Marek Karliner there are strong indications that the isospin one channel is
dominating close to threshold, which would imply that the particle content in
these channels is not given by “ordinary” quark counting. Let me also say
that the energy region covered by BES is interesting in the sense that it covers
a good deal of the mass spectrum of the “clusters” used in HERWIG.

We also learned about the experiments on the CP-break from Dr
Anzivino. I remember the times when we knew that there was a mixing parameter $\epsilon$ (known to be of the order of $10^{-3}$ from the Fitch-Cronin experiment) that made a connection between the $K^0$ and $\bar{K}^0$ states possible. We also knew that there might be another CP breaking parameter $\epsilon'$ connecting the neutral kaons to the two and three pion decay channels. This parameter may be vanishing or at least much smaller and I do not know the cosmological implications of the beautiful experiments done now (NA48 and KTeV) with an $\epsilon'$ of the order $10^{-6}$. What is so impressing for me is that the resulting experimental numbers again are converging quickly just as they evidently are for the $g - 2$ experiment done over at Brookhaven. I am just waiting for the day when these precision experiments are going to finally break our peace!

Pedro Abreu discussed a longstanding question, i.e. whether there are color (re- or inter-)connections in QCD. We know of one case where our present theory indicates the existence of such a mechanism, i.e. the decay of a $B^0$ into a $J/\Psi$ plus anything. The production mechanism is described in terms of the decay of the $b$-quark into a (color singlet) $W$ and $\bar{c}$-quark and the subsequent decay of the $W$ into among other a $c$, that joins with the $\bar{c}$. It is evident that the $W$ is very, very far off-shell but it is nevertheless a fact that the size of the mechanism matches the experimental findings.

I remember that Peter Zerwas came to Lund about twelve years ago and that he made a model with Gosta Gustafson and one of our students on the possible repercussions in the measurement of the $WW$-production in LEPII. I was scared by the very large effects that may occur if the $W$’s decay into two $(q_j \bar{q}_j)$, $j = 1, 2$ and the partons afterwards rearrange themselves with “the wrong partner”. (As soon as LEPII started we knew that this was not the case). In a paper (before LEPII), Sjöstrand and Khoze, in very great detail showed that unless we would move far away from the $W$-poles only non-perturbative possibilities could produce effects. They went on to calculate such effects in a semi-classical model with overlaps between the decaying string fields. Their results was that even with a thirty percent overlap and subsequent color reconnection these nonperturbative effects would only be of the order of $10 - 20\,\text{MeV}$ on the $W$ width, i.e. “below danger”. There have been further tries (Lönnblad, even partly before S.-K., and Gustafson and Häkkinen) but it seems that if there is an effect it is terribly difficult to find.

Ingelman and his collaborators have introduced color reconnections to explain “rapidity gap” events in Deep Inelastic Scattering, noting that “the wrong colors” may occur as $1/N_c^2 \simeq 10\%$ corrections (which are in the right ball park for the observations). I am myself worried about the possibilities because a large part of the structure would go out of our present models if the fields are allowed to change color all over the place. I have personally taken
recourse to the following argument.

We should remember that in QED the fields are given almost exclusively by a knowledge of the charges but this is no longer so for the confined QCD. In the Lund Model we have always taken the QCD fields as the primary objects. In every order of perturbation theory (as it is done in present day perturbative QCD) you may calculate the emitted charges and then obtain interference between the different color charge configurations. But if we were able to sum up the whole result then we may find superselection rules saying that the different color field configurations do not interfere.

Franz Mandl discussed his work with Brigitte Buschbeck on the properties of gluon jets. Their problem is again of a longstanding nature: do the gluon jets show the same particle content as the $q \bar{q}$ or $\bar{q}$ jets? The Lund gluon model, where the gluons are internal excitations acting on the string fields with a force twice that of the $q$ and $\bar{q}$ endpoints is actually an extreme variety of all possible gluon models. Montvay showed in a Physics Letter from 1979 that you may build models in which the force from the gluon could be anything from twice down to zero times the endpoint force. Then the gluon would not be able to “drag along” the force field all the way when it moves out. Instead there would be a kind of “polyp” dragged out behind it linking it to the field. When the PETRA-PEP machines started we learned that if there was such an effect then it would be small, i.e. the gluon force ought to be at least 180% of the endpoint force. Peterson and Walsh tried to make models in these days with production of “glueballs” and as I learned from Wolfgang Ochs he and Minkowski are still pursuing this question. We know that this is a very difficult subject (although there are persistent rumours that there may be something on the level of 1.5 $GeV$). It is nevertheless evident that Buschbeck-Mandl (who are some of the most careful people I know of) seem to find something fishy at the end of the gluon jets. There is another really careful man, Bill Gary, who talked later about the multiplicities in gluon and $(q\bar{q})$ jets, and confirmed that you may hardly ever come up to the famous ratio of 9/4. Maybe Bill can be coaxed to look also?

I would also like to briefly discuss the spin effects, which were considered in the talk by Dr Liang. Once upon a time I was invited to the High Table in an Oxbridge College and beside me there was this old physics professor. He was a nice man with lots of fun stories and I remember one of them in particular. Once he had done experiments on 10 $MeV$ protons and he and his collaborators then saw large polarisation effects. But he said: “When I told the theorists of that time of the results they told me that if I would be able to perform the experiments with 20 $MeV$ beams then these effects would go away!” Now it is a fact that we have gone up to energies many thousands
of times larger and we have looked all over and everywhere we look there are
the same very big polarisation effects! So why have theorists so often claimed
that such effects should go away? Because there is among many theorists
the hope for simplicity so that there should be a single channel amplitude
that will dominate in Asymptopia. In most theoretical work you find that
polarisation effects stem from interferences between different amplitudes and
therefore they “should” go away.

A phenomenological string dynamics person like me would say that a
confined force field always “ought to” produce polarisation. In this case there
should be a well-defined field direction, \(\vec{n}\), defined e.g. from the 3 to the 3
charge along the field. Therefore for every particle with a momentum \(\vec{p}\) there
is an axial vector \(\vec{A} = \vec{n} \times \vec{p}\) that may together with the spin vector \(\vec{S}\) be used
to construct a scalar term in the Hamiltonian \(\vec{S} \cdot \vec{A}\). We should consequently
expect transverse polarisation effects, i.e. out of the plane spanned by the
particle momentum and the field direction.

Together with Gunnar Ingelman we presented in a Physics Letter from
1979 a simple model containing these features and we were successful in describing
the large \(\Lambda\) particle polarisations seen in proton fragmentation regions.
Consider the breakup of a stringlike force field with a (constant) energy den-
sity \(\kappa \simeq 1\) GeV/fm into a \((q\bar{q})\)-pair with transverse momentum \(\pm \vec{k}_\perp\). In this
way you have conserved momentum at the breakup. If the parton restmass
is \(\mu\) then you need in order to conserve the energy to produce the pair at a
distance \(2\ell = 2\mu_\perp/\kappa\) where \(\mu_\perp = \sqrt{\mu^2 + \vec{k}_\perp^2}\). But then the orbital angular
momentum is not conserved. It is easy to see that you obtain a vector \(\vec{L}\)
pointing out of the plane with a size \(L \simeq 2\ell k_\perp = 2\mu_\perp k_\perp/\kappa\), i.e. it is of the
order of unity for an average transverse momentum of size \(\simeq 0.3\) GeV. You
may then conserve the total angular momentum \(\vec{J} = \vec{L} + \vec{S}\) by polarising the
\((q\bar{q})\)-pair out of the plane oppositely to \(\vec{L}\)! In this way I have presented you
with a production mechanism for the pair corresponding to (spectroscopic
notation) a \(3P_0\) assignment, i.e. the pair is produced with vacuum quantum
numbers!

Some of the results shown by Dr Liang can be interpreted along these
lines. Let me present you with a kind of favorite experiment (some of my
experimental friends, S.O. Holmgren et al. did something similar already in
the end of the 70’s). Suppose that you would trigger on a \(\Lambda\) and a strange
vector meson in its neighbourhood, a \(K^*\) or a \(\Phi\). The \(\Lambda\) is from an \(SU(6)\)
point of view a simple object, essentially made up of an \(s\)-quark and a \((ud)_0\),
i.e. a combination of an up- and down-quark with vanishing spin and isospin.
Consequently polarisation of a \(\Lambda\) means polarisation of the \(s\)-quark! Further
the Λ-particle decays weakly and consequently reveals (and this is a large effect) its spin in the decay distribution. The vector meson decays via strong interactions, which do not differ between up and down but you can make use of the Λ decay to define the direction out of the plane. And my friends found very large transversity amplitudes in their studies. You should note that while the string field spanned between the charges provide longitudinal dynamics and the transverse momentum fluctuations in the breakup give the dynamics in one of the remaining space directions the polarisation as used in such an experiments will provide also information in the third direction!

We have also seen the (first) results of the RHIC experiments in all their glory! And we are amazed, both about the fact that the accelerator works so well and that the detectors are all up and running. And this is of course the most amazing: they are able to measure details in events containing thousands of central tracks! We note that RHIC is the first instrument able to produce a real central region in heavy ion physics, i.e. the rapidity range is large enough so that there is more than just the fragmentation regions from the target and the beam. Up to now it seems that the signals correspond to “simple” scaling up of the results from the SPS heavy ion program and there is no hints of a phase transition as of yet.

A general comment is that they see so much more strangeness than we have found in ordinary hadronic events (not to mention in the “ordinary” $e^+e^-$-annihilation events with their rather low particle density). One way to explain it is to claim that in thermal equilibrium at a sufficiently high temperature we expect that the strangeness degrees of freedom are filled just as well as the “ordinary” up and down ones. Another (which may basically be the same) is to claim that in a dense hadronic gas it is an effect of rescatterings. Actually we have from the wonderful data set at the $Z^0$ pole such a large statistical sample of events in $e^+e^-$-annihilation that we may study particle densities essentially above the ordinary ones. I have asked Klaus Hamacher to investigate whether the strangeness content increases with the multiplicity. There is at least in the Lund Model no way to obtain high energy densities along the fields (particle production in a Lund Model jet is still a few particles per rapidity unit if you use a rapidity variable along the jet).

Another generality is that all theoretical signals proposed for a quark-gluon plasma seem to be negative, i.e. there should be less $J/\Psi$ because of Debye screening, there should be less energetic jets because of rescattering of the gluons on the way out etc. I, for one, would feel happy to hear at least one positive signal proposal because what do we need a new state of matter to if it only contains negative features w.r.t. everything else?

Pino Marchesini introduced the subject of hard QCD. He told us on the
one hand that nowadays there exist in some cases exact perturbative calculations up to the order $\alpha_s^5$ (!). On the other hand the resulting comparisons with data lead in general to good results (at least after some help from the friends, if I may be impertinent—“little help” means adding some things and subtracting some others, I will come back to that later).

Pino, who is an honest gentleman also pointed to some places where there are clear deviations between the experimental findings and our present understanding of QCD. In particular he pointed to the so-called pedestal effect, i.e. the fact that there is much more activity in the rapidity neighbourhood of a large jet than in “minimum-bias events”. Including all the bremsstrahlung radiation from the in- and outgoing partons participating in a hard Rutherford scattering one can raise the background level a bit but there is still a factor of more than two lacking in the particle production. Consequently it is necessary to introduce, besides the hard Rutherfords something new and extra. Sjöstrand in PYTHIA has introduced multiple parton interactions but in order to raise the signal sufficiently much he had to introduce a kind of “bunching” of the partons so that if there is one hard interaction there are in general several. Pino briefly described another effort by himself and Bryan Webber inside the HERWIG scenario, to introduce “beam-line radiation”. Actually I remember that inside the very simple FRITIOF scenarium which we introduced a long time ago (and which admittedly did not contain all the sophistication of today’s QCD) there was no difficulty to obtain the pedestal.

Dr Behnke pointed to possible problems in heavy quark production in DIS. Firstly it seems that the present signal for beauty exceeds the next-to-leading order calculations by a factor of three. Secondly he needed contributions both from “ordinary” boson-gluon fussion and from “resolved” photons (the latter was large) to describe the charm content. The difference between the two mechanisms introduces different correlations between the $c$ and the $\bar{c}$ partons. I understand that the new experiments in the upgraded HERA2 will be able to study this problem.

Let me also mention a careful study of how to differ between quark- and gluon jets by Dr Yu Meiling and the (expected) findings reported by Dr Heaphy that a quark jet is slimmer than a gluon jet in accordance with the models. Finally let us note that $\alpha_s$ is running all over the place and that we saw a particularly beautiful picture of it from Dr Flagmeyer.

We were introduced in a nice way to our “oldest participant”, i.e. the session on Correlations and Fluctuations by Wolfram Kittel. Nick var Remortel and Wes Metzger discussed different features of Bose-Einstein correlations. In particular the correlation pattern of charged pions indicates a “radius” of the emission region close to $1 \text{ fm}$ independently of the reaction (besides heavy
ion reactions), while the neutral pions seem to stem from a region that is $1 - 2 \sigma$ deviations smaller. Further one finds a shorter transverse than longitudinal size in the two-particle correlations. Wes also pointed out that the true three-particle correlations among identical bosons were entirely given by a combination of the two-particle correlations (at least if one uses the variable $Q_{123}^2 = Q_{12}^2 + Q_{23}^2 + Q_{13}^2$ with $Q_{ij}^2 = -(p_i - p_j)^2$. On a direct question he was reluctant to provide future partitioning of the data into longitudinal and transverse directions and I understand that these measurements puts a very large strain on the person doing them). In the conventional “chaotic sources” description of the correlations there is a possible angle between the two-particle Fourier transform of the sources and the Nijmegen result is that this angle vanishes.

A more puzzling result was discussed by van Dalen, i.e. the fact that there are no traces of Bose-Einstein correlations between the two $W$’s at LEP II (although all the experiments now show the same signals for the correlations inside each $W$ as they show on the $Z^0$-pole in LEP I). A very large amount of work has been done in this connection because of the possibility pointed out in some theoretical studies that the correlations may make the width of the $W$ uncertain by possibly much more than the hoped-for 40 MeV (needed to constrain the Higgs mass). Although the model I have myself built (together with Werner Hoffman (1986) and Marcus Rigné (1996-98)) does not provide for such “inter-correlations” it is evident that the conventional model should expect them. The reason for the lack of inter-correlations may actually be that LEP II has been too successful in going up in energies (which some people have been pushing for very strongly in the hope of finding the Higgs). It is evident that it is the slow particles produced “in-between” the two $W$’s that would have been able to show an effect. The larger the energy produced in LEP II the further the $W$’s will move away from each other and therefore the smaller would be the effect.

We also heard in a very interesting aside a report on Very Long Baseline Interferometry as it is used in astronomy (I understand from Dr Gurvits that the astronomers have come along way from Hanbury-Brown and Twiss!). I will from the last day (my only excuse is that I was pretty tired after having written my transparences the night before) only mention one very interesting parametrisation for the structure functions provided in the talk by David Milstead. It has been known for a long time that the DGLAP mechanism (corresponding to emission chains with well-ordered virtualities-transverse momenta along the ladders) does a very good job to describe the structure functions. A typical behaviour stemming from the summing up of such chains would be $F_2 \sim \exp(C \sqrt{\log(1/x)})$ with $x$ the longitudinal scaling
variable and $C$ a slowly varying function of the measured virtuality $Q^2)$. The competing BFKL mechanism (corresponding to a possible going up and down in virtuality) would result in a typical behaviour like $F_2 \simeq D(Q^2) x^{-\lambda}$ with $\lambda$ the eigenvalue of a complicated equation, but (after some corrections) expected to be $\simeq 0.3 - 0.4$. Such a behaviour has been looked for extensively because of its possible relationship to the elusive “hard funny-P” or whatever people call the instigator of diffraction. Needless to say the BFKL mechanism has not been pinned down (although the jury is out still). One reason is that despite its wonderful performance the range of $x$-values accesible in HERA may be much too small. An estimate on the emission rate of gluons along the ladders would tell us that there are only two to four emissions available between $x \simeq 10^{-2}$ to $x \simeq 10^{-5}$. We can hardly expect that to be sufficient to drive the equations to a steady state (Gustafson provided a pedagogical way to see the emergence of BFKL in a simplified scenarium).

Nevertheless David provided in his last transparency a parametrisation of $F_2$ as $F_2 \simeq D \exp(-a \log(x) \log(Q^2/Q_0^2))$ with $D$ a constant $\simeq 0.18$, $a \simeq 0.048$ and $Q_0(\simeq \Lambda) \simeq 0.26$ GeV. This would mean that the experimentalists are seeing a BFKL shape but with a “running” $\lambda$-value, i.e. it looks as if $\lambda \simeq E/\alpha_s(Q^2)$ with $E$ a constant! I should immediately say that David only presented the data and pointed out that the $\lambda$ parameter was a linear function of $\log(Q^2)$. But I cannot help but to take it all the way (although the data shown were only for $Q > 1.5$ GeV)! Whatever the cause for the remarkably simple parametrisation it is not the “ordinary” BFKL because in that case the $\lambda$ parameter is proportional to $\alpha_s$.

3 Things Dear to My Heart

3.1 A Lamentation and A Scale for The Border region

QCD is doubtlessly the greatest intellectual challenge and adventure of my generation of physicists and it is so different from the field theories we discussed in Lund in the 60’s! The gauge field self-interaction implies e.g. that the imaginary part of the polarisation tensor changes sign compared to anything we had seen when we summed over a positive definite metric for the asymptotic states. (This is a complex way to say that the running coupling vanishes at large $Q^2$ but at least older people may understand the surprise). The perturbation theory that Feynman and his contemporaries provided us with certainly works in QED with its small effective coupling and the vacuum fluctuations can (after renormalisation) be handled (and even understood!) just as small deviations from the no-particle state. But we know from the
very beginning that the corresponding treatment of QCD must lead to all kinds of problems and they certainly do! For every quantity that is calculated we need to introduce “resummations of leading or subleading quantities”. I am impressed about the technical skill that has been developed but I also have a feeling that the physics is slipping away from us. Maybe we are in the future only going to hear signals every now and then from this community in the same way as we now do from the lattice people.

The problems that we are facing now are, however, inside the border region between where present day perturbation theory (with all its extras) is expected to work and the region where we truly know that non-perturbative models are necessary. So let us start to try to define it. Inside the Lund Model (and frankly I expect that the results are pretty general) there are two scales that seem to be the same. The first is the scale where we can at first hope to be able to disentangle “real” gluons, i.e. objects that will stick out of the confining force field. The second scale is the size of the momentum transfers between the hadrons produced in the fragmentation process.

To see the first scale I will make use of an argument similar to the Landau-Pomeranchuk formation time. In that case the question is: “when can we differ between the state of a single (charged) electron and a state containing the electron and a photon?” They argued that if we assume that the electron moves along the z-axis and the photon has energy $e$ and tranverse momentum $k_{\perp}$ (for definiteness along the x-axis) then we can boost to a frame where the photon only has momentum along the x-axis. In that frame the formation time must correspond to at least a wave-length $\tau \simeq 2\pi/k_{\perp}$ because you cannot “see” a wave before that. In the earlier frame that corresponds to the “formation time” $2\pi e/k_{\perp}^2$, and with the $k_{\perp}$ exchanged for some suitable “virtuality scale” it works as an estimate of what quantum mechanics and relativity means in a general case (e.g. in the Lund Model it is always the slowest particles that are first disentangled in the “inside-out” cascade).

To see the confinement scale suppose that we consider a gluon moving transverse to the force field with energy and momentum equal to $k_{\perp}$. Then the gluon can move out at most the length $\ell = k_{\perp}/2\kappa$ (the force on the gluon is in the Lund Model twice the string constant $\kappa$). In order to be “real” this must correspond at least to a wave length, i.e. $\ell \geq 2\pi/k_{\perp}$ or $k_{\perp}^2 \geq 4\pi\kappa \simeq 2.5 \text{ GeV}^2$. (The width of the string field should be $\ell \simeq \sqrt{\pi/\kappa}$ and this is also obtained from the “ordinary” tunneling arguments). At first sight this seem to be a large scale but it is interesting to note that it is the same scale as will occur for the momentum transfers between the particles in the fragmentation process (and it is to compare to something else twice the (inverse) slope of the Regge trajectories).
The Lund Fragmentation Model is based upon the Area Law, i.e. the (non-normalised) probability to produce $N$ particles with the energy-momenta $\{p_j\}$ given the total energy momentum $P_{tot}$ is

$$dP_N = \prod_{j=1}^{N} N_j dp_j \delta(p_j^2 - m_j^2) \delta(\sum p_j - P_{tot}) \exp(-bA)$$  \hspace{1cm} (1)$$

with $N$ a normalisation constant and $A$ the area of the string before it decays. We have often interpreted this in accordance with Fermi’s Golden Rule as the phase space of the final state particles multiplied by the squared transition matrix element. In a paper [hep-ph/9910374] Fredrik Söderberg and I reinterpreted the result in Eq. (1) as a product of transition operators (in quantum mechanics it would be density operators) one for each particle between the momentum transfer states $\{q_j\}$ such that $p_j = q_{j-1} - q_j$. We found that the operators can be diagonalised as

$$< q_j | O | q_{j-1} > = \sum_n g_n(b\Gamma_j)\lambda_n(bm_j^2)g_n(b\Gamma_{j-1})$$  \hspace{1cm} (2)$$

in terms of the two-dimensional harmonic oscillator eigenfunctions ($\Gamma \equiv -q^2$) and with the eigenvalues $\lambda_n$ analytical continuations of the $g_n$ into time-like arguments ($p_j^2 = m_j^2$).

Just for fun let me point out that (almost) everything you can do with the (free) plain-wave momentum eigenstates $\exp(ikx)$ you can also do with the harmonic oscillator wave functions. As an example, Fredrik and I proved that the total integrals of the $dP_N$ over the $N$-particle phase space

$$\int dP_n = R_N(s) \text{ with } s = P_{tot}^2$$  \hspace{1cm} (3)$$

can be expanded in a simple way

$$\int \frac{ds R_N(s)}{s + u} = \sum \lambda^N L_n(bu)$$  \hspace{1cm} (4)$$

in terms of the (two-dimensional) harmonic oscillator correspondences to the Hermite polynomials, the Laguerre polynomials. You may remember the nice lecture by Engel on the Sommerfeld-Watson transform done on the partial wave (this corresponds to Laplace polynomials) sums of the elastic amplitude and its relations to Regge trajectories. You can do exactly the same transform on the sum over all the multiplicities of $R_N$ in Eq. (3), $\sum R_N = R$ by means of the sum in Eq. (4) to obtain that $R(s) \simeq s^a$ where the parameter $a$ is the correspondence to a Regge intercept in the Lund Model.
The parameter $a = a(N, bm^2)$ is phenomenologically close to 0.5 and it also regulates the behaviour of the average momentum transfer, $\Gamma$, size in the Lund Model. The inclusive $\Gamma$ behaviour is $\propto \Gamma^a \exp(-b\Gamma)$ which implies an average size $<\Gamma> = (1 + a)/b \simeq 2.5$ GeV, i.e. the same size as the minimum size of the "real" gluons defined above! Maybe the occurrence of the harmonic oscillator wave functions, that evidently contains information of confinement in the string field or some similar functions should play a major role in the future work on confinement in QCD?

3.2 On Some Further Developments Based on the Area Law

I have in earlier talks at these conferences pointed out that if we interprete the Area Law as the phase space times the square of a transition matrix element then there is an obvious candidate for such a matrix element, i.e. it is a Wilson loop operator evaluated over the confined string field during its breakup. The reason is that the particles in the Lund Model are produced over a region of the field, i.e. a (color-singlet) hadron stems from a quark from one breakup vertex and an anti-quark from an adjacent vertex and the field in between. In order to keep to gauge invariance it is then necessary to have a gauge connector between the vertices $\exp(i \int gA^\mu dx_\mu)$.

Using the same argument for all the adjacent vertices we find that the whole state must be endowed with $\exp(i \oint gA^\mu dx_\mu) = \exp(i \xi A)$, where the Wilson loop goes over the field region with the (breakup) area $A$. The parameter $\xi$ has a real part given by the string constant (this is used in lattice gauge calculations). In a decay situation $\xi$ must also have an imaginary part corresponding to “absorption”. In the Kramers-Kronig interpretation the imaginary part of the dielectricity is related to the pair production rate. I have in my book (“The Lund Model”) presented detailed calculations and I feel that the size of the Lund Model parameter $b$ in the Area Law $b \simeq 0.6$ GeV$^{-2}$ fits well inside that picture.

If this is taken seriously then we have a model with a matrix element containing a complex phase. Such a phase will be noticable e.g. if there are two identical bosonic particles produced in the state because then the matrix element must be symmetrical in the particle variables. It turns out (in the simple case when the string state stems from an original ($q\bar{q}$) pair and there are no gluonic excitations) that the same state can be produced if the two particles are exchanged in the production process with all the rest of the state unchanged. We then obtain in an easily understood notation the total matrix element $\mathcal{M} = \mathcal{M}_{12} + \mathcal{M}_{21}$. The square of the matrix element $\mathcal{M}$ will contain both the possibility to make the production in the order (12) or in the order
(21) (with everything else the same). But there will also be an interference term $I = \cos(\Delta A) / \cosh(B\Delta A+C\delta(p_{1\perp}))$ with $\Delta A$ the area difference between the two configurations and $\delta(p_{1\perp})$ the necessary change in the $p_{1\perp}$ generation according to the tunneling mechanism in the Lund Model. Further $B$ is a (small) and $C$ an essentially larger parameter.

You should note that $\Delta A$ is solely an energy momentum space quantity in this interpretation. It is given by $\Delta A = (p_1-p_2)\cdot\Delta$ with $\Delta$ the (space-like) energy-momentum content in the string produced between the two identical particles with energy-momenta $p_1$ and $p_2$ (counted in inverse string constant units). It is evident that the interference pattern is in this model sensitive to the energy momentum region inside which the quantum numbers of the particles are locally compensated.

The mechanism has been termed “the string symmetrisation (or coherence) scheme” by Eddi de Wolfe and the resulting formulas has similarities (albeit it is a very different dynamical mechanism) to the ordinary scheme where the basic assumption is that the production region is completely chaotic or incoherent. It is, however, well-known that the Low theorem seem to work for the description of photon emission in the wavelength region used for the study of the interference pattern for the identical hadrons. Low’s theorem is certainly based upon coherence and as I have said it before there are no reasons to believe that there are incoherent sources inside a multiparticle production region similar to those occurring for photon emission from a large star surface. The model describes the properties mentioned in the session on Correlations and Fluctuations (the fact that the neutral pions (may) have smaller size parameter than the charged is because they can in the Lund Model be produced adjacent in rank, the fact that longitudinal and transverse sizes are different is due to the size of the governing parameters etc.) I would, however, like to go on and briefly describe what we learned from a recent study of the multigluon states.

I have (together with two great graduate students, Sandipan Mohanty and Fredrik Söderberg) written one paper \footnote{hep-ph/0106185} on the Lund Area Law for multigluon states and when I write this there are more to appear, hopefully within a month or two. The work has been done because the well-known JETSET Monte Carlo will for multigluon states produce particles with only an approximate implementation of the Lund Area Law, i.e. inclusively it works very well but to study string coherence and other structure we evidently need the precise area.

The final results can be briefly described for $e^+e^-$-annihilation events:

A Given a partonic state (e.g. from a perturbative cascade) with the par-
tonic energy-momentum vectors \( k_1, k_2 \ldots k_n \) in color order then we can construct a four-vector valued curve, the directrix curve \( A_\mu \), by laying them out in order. The string surface used in the Lund Model as a model for the QCD force fields is a minimal surface and it is consequently completely described by its boundary curve which turns out to be just this directrix curve. Therefore the breakup of the string can be just as well described “along the directrix curve” as “on the string surface”.

**B** The fragmentation process corresponds to the production of another curve, the \( X \)-curve, with the hadron energy momenta \( \{ p_j \} \) laid out in rank-order. The relationship between the \( A \) and the \( X \)-curves can be described as the formation of an area in between them, composed out of four-cornered “plaquettes”. Each plaquette is bordered by a particle momentum (along the \( X \)-curve), a piece of the directrix \( \delta A \) and two (time-like) vectors \( x \) such that

\[
x_{j-1} + \delta A_j = x_j + p_j
\]

The area of each plaquette correspond to the (sub)areas related to each particle production (as in the transfer operators mentioned in connection with Eq.(2)). The length of the vectors \( x_j \) fulfil \( x_j^2 = \Gamma_j \) in that equation).

Although this description looks very abstract you can intuitively consider it so that each particle obtains its energy momentum both from a “new” part of the directrix, \( \delta A_j \) and also from the “remaining” energy momenta of the “earlier” parts of the directrix through the \( x_{j-1} \). The “new” remainder is then brought forward to the next production by the vertex vector \( x_j \).

### 3.3 Further Developments Based upon the Structure of the Parton Cascades and the Fragmentation Process

There is no time to cover the many delightful properties we obtain but I would like to mention two further features of the model.

**C** There is a limiting situation corresponding to a vanishing mass-value. In that case the \( X \)-curve goes over to a continuous curve, the \( \mathcal{X} \)-curve that is characteristic of the particular partonic state but also in a very precise way corresponds to the “average” hadronic \( X \)-curve obtained from the stochastical fragmentation process.

**D** The vectors \( x \) goes over to the time-like tangents of the \( \mathcal{X} \)-curve reaching to the directrix. They quickly approach a constant length, \( m_0 \). The \( \mathcal{X} \)-curve in this way looks like a set of connected hyperbolas, spanned
on the “distance” $m_0$ between the lightlike parton energy momenta in the directrix. In this way $m_0$ may be considered as a “resolution scale” for the partonic directrix curve. The area between the directrix and the $\mathcal{X}$-curve is given by $m_0^2 \lambda (m_0)$.

This generalised rapidity $\lambda (m_0)$ that comes out of the Lund Area Law fragmentation process also comes out of (at least one of) the perturbative parton cascades, viz. the Lund Dipole Cascade Model as it is implemented in the Monte Carlo program ARIADNE! (I believe that there are correspondences in both HERWIG and JETSET). As I have talked about this model at earlier meetings I will be very brief.

We firstly note the two basic formulas for QCD bremsstrahlung. The dipole bremsstrahlung formula for the inclusive production of gluons is

$$dn = \alpha_{\text{eff}} dy \frac{d^2 k_{\perp}}{k_{\perp}} (\text{Pol} - \text{sum})$$  \hspace{1cm} (6)

(where $(\text{Pol} - \text{sum})$ is the coupling of the spins of the emitters and the gluon. It is close to unity except for collinear gluon emissions). The emission of two gluons from an original $(q \bar{q})$-dipole is factorisable and can be written (besides a small correction)

$$dn(qg_1g_2\bar{q}) = dn(qg_1\bar{q})(dn(qg_2g_1) + dn(g_1g_2\bar{q}))$$  \hspace{1cm} (7)

Eq. (6) is valid if $k_{\perp 1} > k_{\perp 2}$ or else the two gluons are exchanged. Note that the two “new” dipoles” are moving w.r.t. each other (and also remember the remarks in the Introduction). The coherence conditions limits the phase space (in this case just as energy momentum conservation would do!) so that in the dipole restframe (with the dipole squared mass equal to $s$) we obtain

$$k_{\perp } \cosh(y) \leq \frac{\sqrt{s}}{2}$$  \hspace{1cm} (8)

i.e. essentially the region $|y| < 1/2 \log(s/k_{\perp}^2)$ so that the rapidity range for a dipole emission is $\Delta y \simeq \log(s/k_{\perp}^2)$. If we denote the three partons after the first emission (1, 3) (the emitters) and 2 (the gluon) then the combined rapidity range for a second emission either from the dipole (12) or from the dipole (23) is

$$(\Delta y)_{\text{gen}} = (\Delta y)_{12} + (\Delta y)_{23} =$$  \hspace{1cm} (9)

$$\log(s_{12}/2k_{12}^2) + \log(s_{23}/2k_{23}^2) = \log(s/k_{12}^2) + \log(s_{12}s_{23}/4s_{12}k_{12}^2)$$  \hspace{1cm} (10)

The argument in the last logarithm is $s_{12}s_{23}/s = k_{12}^2$, i.e. an invariant definition of the first gluon transverse momentum squared. In this way the first
emission will increase the rapidity range for the second (the variable s is still the total cms mass squared. The factor 1/2 in each of the dipoles is due to the fact that only half of the first gluon goes into each dipole). If the first emission is “soft” or “collinear” then there is a correction to the formula so that we obtain

\[ \lambda(m_0^2) \equiv \log(s/m_0^2 + s_{12}s_{23}/4m_0^4) \]  

(11)
i.e. a nice interpolation between the emission from one and from two dipoles. (the sign \( \equiv \) is used because we will for factorisation purposes add a constant in the argument of the logarithm). This is then the \( \lambda \)-measure for a state with one gluon and it can be extended in an infrared safe way to the multigluon states. Then it coincides with the results from the length of the \( \mathcal{X} \)-curve that we obtained in the fragmentation process as I described it above.

I will end with a further very remarkable property of the \( \lambda \)-measure. The Dipole Cascade Model is built in such a way that you start with a single dipole and then you “go downwards” in transverse momentum until it breaks up into two dipoles (this means in technical language that there is a Sudakov form factor). Then you continue downwards all the time looking for new emissions thereby producing new dipoles. In that way the transverse momentum at every stage is both an ordering and a resolution parameter. In ARIADNE the process is implemented with a running \( \alpha_{\text{eff}} \), a precise implementation of the \( (\text{Pol} - \text{sum}) \) and finally with a local energy-momentum conservation so that the emitting dipole takes the recoil. A hard gluon emission will increase the phase space, i.e. the size of the \( \lambda \)-measure, thereby opening up for more and more softer gluon emissions. In this way the dipoles are spreading and moving away in different directions, although in the “ordinary” phase space with the rapidity e.g. defined along some thrust axis they will seem to be collimated along a set of jet directions. It is evidently interesting to look for the resulting “local” properties of the cascade, i.e. to look for the distribution of gluons (or equivalently dipoles) along the color flow, i.e. along the \( \mathcal{X} \)-curve and the corresponding \( \lambda \)-measure.

Due to the nice properties of the \( \lambda(m_0) \) function it is possible at every scale to define it as an independent sum of contributions one for each gluon that is resolved on the scale \( m_0 \):

\[ \lambda(m_0) = \sum_{j=1}^{n} (\Delta \lambda)_j(m_0) \text{ with } (\Delta \lambda)_j = \log(1 + x_{j-1}k_j/m_0^2) \]  

(12)
The great surprise is that the distribution in \( (\Delta \lambda) \) depends rather little upon the scale \( m_0 \) and it is the same for all events independently of thrust, sphericity or other global “excitation” variables! It has an average value around
three units and a width around one and looks very much like the well-known mathematical Γ-distribution, i.e. it has an exponential tail. Remembering the relationship between the λ-measure and an area it seems as if we are getting back a new area law, this time for the parton cascade! We are evidently at the moment investigating these properties in detail and as I write this I know much more than I knew when I gave the talk. But besides one feature this will be for another occasion.

The only feature I would like to mention is that this partitioning of the λ-measure into (∆λ) pieces has a direct reflection into the properties of the fragmentation process. Each piece (which you could call a generalised dipole region) decays independently of the others into a set of hadrons that are all in an essentially “planar” state, i.e. besides (small) transverse momentum fluctuations they all lie in a (1 + 1)-dimensional (time-like) subspace. This is just as in the original Lund Fragmentation Model stemming from a (q̄q)-state with no internal excitations. Therefore we can in each generalised dipole region do string symmetrisation in accordance with the model for Bose-Einstein correlations. You can look upon these regions as “coherent sources”. It seems to occur very seldomly that particles stemming from different such sources are so close in energy momentum that they should be able to interfere. This work is not finished yet but as it seems at the moment string coherence should give the right correlation pattern also for the multigluon states.

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