Research on structural damage identification method based on optical flow method and flexibility difference

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Abstract: Dynamic fingerprints of structures such as dynamic displacements, vibration patterns and frequencies are important parameters for structural health monitoring. In this paper, a structural damage identification method based on spatio-temporal sequence images is proposed for the shortcomings of traditional structural health monitoring systems such as high cost, discontinuous measurement points and mismatch between sensor lifetime and the lifetime of the structure itself, which uses a camera to collect the vibration information of the structure and adopts the optical flow method for spatio-temporal sequence. The method uses a camera to capture the structural vibration information, adopts the optical flow method to track the dynamic displacement of the structure in the images, obtains the full-domain displacement, vibration pattern and frequency of the structure, and uses Python to compile a flexibility difference calculation program to locate the structural damage according to the flexibility difference after the structural damage. The method was experimentally studied on a cantilever beam structure. The results show that the dynamic characteristics of the structure obtained by using the method in this paper match the results of the numerical simulation, and the maximum error of the structural frequency is 2.9%. The damage location of the structure (unit 9) can be precisely located according to the flexibility difference after damage. The method has the advantages of being convenient efficient, full information, low cost and non-contact, while its accuracy is comparable to that of the theoretical method, providing a new means of obtaining holographic dynamic properties of structures and locating structural damage.

1. Introduction
For a set of Structural Health Monitoring system, the foremost thing to accomplish the assessment of the health status and safety condition of the structure is to select reasonable sensors through the monitoring purpose and demand, and effectively obtain the first-hand data for data assessment. Choosing the right sensing technology not only improves the accuracy of data acquisition, but also the convenience of data acquisition, the quickness of data transmission and even revolutionises the thinking and methods of data processing and bridge condition assessment. At present, traditional health inspection has a number of shortcomings: the life of the sensor is shorter than the life of the civil structure, inconvenient to replace thus leading to the interruption of monitoring data; traditional sensors are usually arranged in the key position of the structure, can only achieve a single point measurement, the full field measurement in the sensor points are extensive, wiring complex; measurement process and the object to be measured in direct contact, resulting in load effect. In recent years, some new non-contact optical measurement means have been applied to structural health monitoring, such as digital correlation (DIC)[1], computerised dialysis imaging technology (sonic
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CT\cite{2}, machine vision measurement technology\cite{3}, GPS technology\cite{4}, etc. These methods can complete the measurement of target position, size, shape and other parameters without contacting the object to be measured.

Machine vision-based measurement is one of the most popular technologies because it does not require a laser light source, is not subject to magnetic field interference, and does not require complex interference light paths and other auxiliary devices. It has the following advantages: no contact with the object to be measured; no load effect; wide measurement range; and long-term stable operation. Machine vision measurement is the use of vision devices such as cameras as vision sensors to collect information, and then the use of computers to process and analyse this information to obtain structural parameters. The captured video is composed of structural images in a certain time sequence, which has a time-range; and each frame of the image has different intensity values due to the vibration of different pixel points, which has a spatial nature; through the combination of time and space information, important dynamic parameters such as vibration amplitude, fundamental frequency and mode can be obtained, which can deformation feature analysis, the comparative analysis of structural vibration characteristics provides a basis for structural damage identification and health state assessment. The research and application progress of structural displacement monitoring methods were reviewed by Ye Xiaowei\cite{5} et al. who described various measurement methods such as template matching method, feature point matching method and optical flow method, and the advantages and disadvantages of each method. The displacement signals of the cantilever beam structure were obtained using an image processing algorithm, and finally the modal parameters of the cantilever beam structure were estimated by the subspace discrimination method. The dynamic displacement measurement results of the marker points of the test bridge were obtained by Chu Xi \cite{7} et al. by decomposing the frame number of the digital images after vibration amplification, and then analysing the edge features of the marker points in the image sequence. A comparison with displacement transducer measurements and a percentage table demonstrated that the video dynamic displacement measurement method can be used for preliminary quantitative analysis of certain vibration characteristics of bridges. Xu Yang \cite{8} proposed an image-based method for identifying the corrosion state and fatigue life assessment of in-service ties. A probabilistic modelling method based on the corrosion process and the statistical characteristics of the apparent images was investigated to establish a statistical mapping between the corrosion feature space and the fatigue life control parameters.

However, all these studies require the setting of marker points on the structure for tracking and the extracted dynamic parameters are not integrated into the damage identification of the structure. Based on this, this paper uses Python to write a program to track any point of interest on the structure to obtain the displacement time curve of the structure, and uses MATLAB to write a program to extract the frequency and vibration pattern of the structure, which is combined with the damage recognition method of the diagonal difference of the flexibility matrix. A cantilever beam structure was tested in the laboratory to verify the feasibility of the method proposed in this paper.

2. Constraint equations for the optical flow field
The study of optical flow uses the time-domain variation and correlation of pixel intensity data in an image sequence to determine the 'motion' of the respective pixel positions. That is the relationship between the temporal variation of the image grey scale and the structure and motion of objects in the scene. In general, optical flow is caused by the relative motion of the camera, the motion of the target in the scene, or the joint motion of the two.

The prerequisite assumptions for motion tracking based on optical flow fields are that the sampling time interval of the image sequence is small and that the grey scale values of the same point on two adjacent images remain constant. At time $t$, let the grey value of the pixel point $(x, y)$ be $I(x, y, t)$, and at time $t + \Delta t$, the position of the pixel point be $(x + \Delta x, y + \Delta y)$, and the grey value be $I(x + \Delta x, y + \Delta y, t + \Delta t)$. According to the assumption of constant luminance:

$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$  \hspace{1cm} (1)
A Taylor expansion of the right-hand side of equation (1) at \((x, y, t)\), ignoring higher order minima above the second order yields:

\[
\frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0
\]  

(2)

Dividing both sides of equation (2) by \(\Delta t\), letting \(\Delta t\) converge to 0, we obtain:

\[
\frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0
\]  

(3)

Let \(u\) and \(v\) be the velocities of the point along the \(x\) and \(y\) directions respectively, then:

\[
u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}.
\]  

(4)

Let \(I_x = \frac{\partial I}{\partial x}, I_y = \frac{\partial I}{\partial y}, I_t = \frac{\partial I}{\partial t}\), then equation (3) can be written as:

\[I_x u + I_y v + I_t = 0\]

(5)

Assume:

\[
\nabla I \cdot U + I_t = 0
\]  

(6)

Equation (6) is the basic equation for the optical flow field, which is known as the constraint equation. Since the equation contains two unknown variables, \(u\) and \(v\), it is not possible to solve for \(u\) and \(v\) with using the constraint equation alone, so other additional constraints need to be introduced.

3. Flexural matrix damage difference

The flexibility matrix has great advantages over the stiffness matrix. The flexibility matrix can be obtained relatively accurately by using the first few orders of the structure's intrinsic frequency and modal information. The flexibility matrix is reciprocal to the stiffness matrix, and for multi-degree-of-freedom systems, the stiffness matrix of the structure is inverted to obtain the flexibility matrix. The modalities and frequencies of the lower order vibrations in the flexibility matrix have a strong influence in the flexibility matrix. In algorithms for damage identification from structural stiffness matrices, the lower order modal information has a large error in approximating the stiffness matrix, a disadvantage that can be avoided by using the flexibility matrix. In practice, the actual stiffness matrix can be approximated by measuring only the first few orders of frequencies and vibration patterns of the structure.

The intrinsic frequency \(\Lambda\) and the canonical vibration mode \(\Phi\) of the structure are known:

\[
\Phi^T M \Phi = I
\]  

(7)

Where as \(M\) is the mass matrix, then:

\[
\Phi^{-1} = \Phi^T M
\]  

(8)

Let the characteristic equation be:

\[
K \Phi = M \Phi \Lambda
\]  

(9)

Where \(K\) is the stiffness matrix, multiplying the two ends of equation (9) right by the two ends of equation (8), we get:

\[
K = M \Phi \Lambda \Phi^T M = M \left( \sum_{i=1}^{n} \omega_i \phi_i \phi_i^T \right) M
\]  

(10)
Where, \( \omega_i \), \( \varphi_i \) is the i-th-order intrinsic frequency and mass-normalised oscillation pattern, multiplying both ends of equation (8) left by the flexibility matrix \( F \) and right by \( \lambda^{-1} \) to obtain:

\[
\Phi \lambda^{-1} = FM \Phi
\]  

(11)

Swap the two ends of equation (7) and multiply right by two ends of equation (11) to obtain:

\[
\Phi \lambda^{-1} \Phi^TM = FM
\]  

(12)

Then:

\[
F = \Phi \lambda^{-1} \Phi^T = \frac{1}{\omega_i^2} \varphi_i \varphi_i^T
\]  

(13)

Define the change in flexibility after structural damage as:

\[
\delta F_i = \Delta F_{ui} - \Delta F_{di}, \quad i = 1, 2, \ldots, n,
\]  

(14)

\( \delta F_i \) and \( F_u \) are the flexibility matrices of the structure before and after damage. Clearly, among \( \delta F_i \) if there is a sudden change in the rate of change of flexibility at two adjacent nodes, the cell between these two points is the damage cell.

4. Numerical simulation analysis of cantilever beams

4.1 Finite element model of cantilever beam and calculation conditions

In order to verify the above method, this article uses ABQUAS to simulate a rectangular cantilever beam of 1.2m in length and 0.044m × 0.002m in cross-section (as shown in Fig. 1). The beam is made of steel, whose modulus of elasticity \( E \) is 210 GPa and density is 7850 kg/m³. The damage to the structure is located in unit \( \text{⑨} \) (as shown in Fig. 2). In this article, the stiffness reduction method is used to simulate three conditions with damage levels of 15%, 30%, and 45% respectively, and each damage condition is numerically simulated according to 16 units and 17 nodes.

![Fig. 1 Schematic diagram of the cantilever beam structure](image)

![Fig. 2 Schematic diagram of the damage unit](image)

4.2 Analysis of results

4.2.1 Inherent frequency

The vibration mode of the cantilever beam is mass normalized, and the flexibility matrix extracts the first three order frequencies and vibration patterns. The first three order inherent frequencies of the cantilever beam without damage and under different damage conditions are shown in Table 1. When unit 9 is damaged by 15%, the first order frequency is reduced by 0.0020Hz, the second order
frequency is reduced by 0.0706Hz and the third order frequency is reduced by 0.017Hz; when unit 9 is damaged by 30%, the first order frequency is reduced by 0.005Hz, the second order frequency is reduced by 0.1609Hz and the third order frequency is reduced by 0.04Hz; when unit 9 is damaged by 45%, the first order frequency is reduced by 0.0094Hz, the second order frequency is reduced by 0.3092Hz and the third order frequency is reduced by 0.074Hz.

Table 1 First 3 orders of inherent frequency

| Conditions      | Frequency | No damage | The damage of 15% | The damage of 30% | The damage of 45% |
|-----------------|-----------|-----------|-------------------|-------------------|-------------------|
| First order     | 1.0129    | 1.0109 1.0079 | 1.0035           |
| Second order    | 6.3477    | 6.2771 6.1868 | 6.0448           |
| Third order     | 17.776    | 17.759 17.736 | 17.702           |

4.2.2 Normalised vibration pattern

The cantilever beam vibration pattern uses the mass normalized vibration pattern, and the flexibility matrix extracts the first three orders of frequency and vibration pattern. The first three orders of normalized vibration patterns of the cantilever beam without damage and under different damage conditions are shown in Table 2 and Table 3.

Table 2 First three modes of normalized vibrations for undamaged and damaged 15% beam

| First three modes without damage | First three modes of 15 % damage |
|----------------------------------|----------------------------------|
| First order                      | 0                                |
| Second order                     | 0                                |
| Third order                      | 0                                |
| First order                      | 0.00708055                      |
| Second order                     | -0.0411479                      |
| Third order                      | 0.107071                       |
| First order                      | 0.0267249                       |
| Second order                     | -0.142109                       |
| Third order                      | 0.334629                       |
| First order                      | 0.0577345                       |
| Second order                     | -0.277406                       |
| Third order                      | 0.572174                       |
| First order                      | 0.0989299                       |
| Second order                     | -0.422742                       |
| Third order                      | 0.729077                       |
| First order                      | 0.149143                        |
| Second order                     | -0.555904                       |
| Third order                      | 0.747328                       |
| First order                      | 0.207226                        |
| Second order                     | -0.657815                       |
| Third order                      | 0.610118                       |
| First order                      | 0.272057                        |
| Second order                     | -0.713493                       |
| Third order                      | 0.342729                       |
| First order                      | 0.342557                        |
| Second order                     | -0.712802                       |
| Third order                      | 0.00477788                     |
| First order                      | 0.417696                        |
| Second order                     | -0.650844                       |
| Third order                      | 0.324806                       |
| First order                      | 0.496511                        |
| Second order                     | -0.527954                       |
| Third order                      | -0.566731                      |
| First order                      | 0.578117                        |
| Second order                     | -0.349237                       |
| Third order                      | -0.659798                      |
| First order                      | 0.661724                        |
| Second order                     | -0.123658                       |
| Third order                      | -0.574422                      |
| First order                      | 0.746652                        |
| Second order                     | 0.137276                        |
| Third order                      | -0.317768                      |
| First order                      | 0.832345                        |
| Second order                     | 0.421183                        |
| Third order                      | 0.0711624                      |
| First order                      | 0.918391                        |
| Second order                     | 0.716732                        |
| Third order                      | 0.536134                       |
Table 3 First three orders of normalised vibrations for damaged 30% and 15% beam

|                  | First three modes of 30% damage | First three modes of 45% damage |
|------------------|----------------------------------|----------------------------------|
|                  | First order          Second order   Third order   First order          Second order   Third order   |
|                  | 0                   | 0               | 0               | 0                   | 0               | 0               |
| 0.0069821        | -0.0401674          | 0.109537        | 0.0068956       | -0.0393955          | 0.111584        |
| 0.0263547        | -0.139094           | 0.342395        | 0.0260291       | -0.136727           | 0.348842        |
| 0.0569373        | -0.27242            | 0.585575        | 0.056236        | -0.268525           | 0.596709        |
| 0.0975683        | -0.416889           | 0.74632         | 0.0963705       | -0.412362           | 0.760665        |
| 0.147098         | -0.551187           | 0.765109        | 0.145298        | -0.547642           | 0.779928        |
| 0.204394         | -0.65693            | 0.624394        | 0.201902        | -0.656543           | 0.636338        |
| 0.268352         | -0.719595           | 0.349528        | 0.265093        | -0.724913           | 0.355294        |
| 0.33791          | -0.729243           | 0.0008151       | 0.333821        | -0.742978           | -0.0023353      |
| 0.413059         | -0.667191           | -0.334673       | 0.408979        | -0.68084            | -0.342915       |

Table 3 (continued)

|                  | First order          Second order   Third order   First order          Second order   Third order   |
|------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 0.492598         | -0.536144            | -0.572678            | 0.489155            | -0.54313             | -0.577654            |
| 0.574896         | -0.352108            | -0.662983            | 0.572061            | -0.35473             | -0.665657            |
| 0.659172         | -0.123566            | -0.575803            | 0.656926            | -0.123732            | -0.576973            |
| 0.744754         | 0.13855              | -0.318153            | 0.743084            | 0.139399             | -0.318492            |
| 0.831094         | 0.422474             | 0.0711716            | 0.829993            | 0.423397             | 0.071166             |
| 0.917783         | 0.717446             | 0.536193             | 0.871248            | 0.717945             | 0.536235             |

4.2.3 Diagonal differences of the flexibility matrix

The results were calculated using the algorithm program written in Python. As seen in Fig. 3 and Fig. 4, it can be seen that there ‘s a mutation in the flexibility between nodes 8 and 10, indicating that there is damage to the ninth cell between nodes 8 and 10. The diagonal difference of the flexibility matrix is 0.000423, 0.00096 and 0.0017 when the damage is 15%, 30% and 45%, respectively. The change in flexibility at this damaged unit increases significantly as the damage increases.

Fig. 3 Diagonal differences of the flexibility matrix
5. Experimental study of structural damage identification

5.1 Test arrangement
To facilitate comparative analysis, a rectangular cantilever beam identical to that in 4.1 was chosen as the test object in this article. To eliminate the effect of self-weight on bending, the steel bar was placed upright with the upper end free and the lower end welded to the base (Fig. 5). A Sony (SONY FDR-AX700) 4K ultra-HD camera was used to capture the vibration process of the structure and obtain a spatio-temporal image sequence of the structure.

5.2 Test apparatus and parameters
In order to reduce the effects of light variations, background and other noise, no external lighting equipment was used and only natural light was utilised.

The apparatus used in this test and its related parameters are detailed in Table 4.

| Parameters             | Name         | SONY FDR-AX40 |
|------------------------|--------------|---------------|
| Frame rate (fps)       | 100          |               |
| Resolution (pixel)     | 1920×1080    |               |

5.3 Test conditions
For comparative analysis, the test conditions are identical to those of the numerical simulation in 4.1. Since the crack depth is linearly related to the section stiffness, the crack depth is taken as the degree of damage. In the test, three damage conditions of 6.6 mm (15%), 13.2 mm (30%) and 19.8 mm (45%) are considered for the crack depth, as shown in Fig. 6.
6. Analysis of results

In this experiment, the cantilever beam was divided into 16 units with 17 measurement points. According to the principle of image measurement, 17 aliquot points were picked up as motion tracking targets (as shown in Fig. 7). The location of the crack is located near point 9. The displacement of the tracking point by the optical flow method is used to write a program to track the 16 equal points of the cantilever beam and obtain its displacement time range data; and the frequency of the structure is obtained by Fourier transform, and the vibration pattern of the structure is obtained by the method of modal subspace identification [9] compiling a Matlab program, and the experimental analysis flow (as shown in Fig. 8).

6.1. Displacement time course curves

Fig. 9 shows the displacement time traces of 17 measurement points in the full range of the cantilever beam vibration by the optical flow method, where (a), (b) and (c) are the image displacement time curves of measurement point 1, 9 and 17, respectively.
6.2. Inherent frequencies
The inherent frequencies of the first three orders of the cantilever beam without damage and under different damage conditions were obtained using the optical flow method as shown in Table 5. As can be seen from Table 5, the maximum error between the first order frequencies and those obtained from the numerical simulation is 1.9% and the maximum error in the second order frequencies is 1.07%. The maximum error for the third order frequency is 2.9%.

Table 5 First three orders of inherent frequency

|                  | No damage | The damage of 15% | The damage of 30% | The damage of 45% | Maximum percentage error (%) |
|------------------|-----------|-------------------|-------------------|-------------------|-------------------------------|
| First order      | 1.056     | 1.032             | 1.008             | 0.984             | 1.9                           |
| Second order     | 6.336     | 6.216             | 6.12              | 5.98              | 1.07                          |
| Third order      | 18        | 17.88             | 17.71             | 17.2              | 2.9                           |

6.3. Regularised vibration patterns
Fig. 10 shows the first three orders of vibration patterns of the cantilever beam for 30% of the damage obtained from the photogrammetry and numerical simulations. As can be seen from the figure, the photogrammetry results agree well with the numerical simulation results, with only a few points of offset. The reason for this may be the lack of camera accuracy and the drift of fewer points when tracking the dynamic displacement time curve of the points with using the optical flow method.
6.4. Diagonal differences of the flexibility matrix

Using Python to prepare the corresponding calculation program according to the above algorithm, the normalised relative displacements of the above measurement points were obtained by the calculation of the diagonal difference of the flexibility matrix, the results of which are shown in Fig. 11.

![Image of Fig. 11]  
Fig. 11 Diagonal difference of flexibility matrix

As can be seen from Fig. 11, the rate of change in flexibility at nodes 8 and 10 is much greater than the rate of change in flexibility at the other nodes. The diagonal difference in flexibility is 0.0269 for 15% damage, 0.03498 for 30% damage and 0.03921 for 45% damage. With the damage increasing, the diagonal difference in flexibility will increase. However, the diagonal difference in flexibility obtained from the image measurements was in error with the numerical simulations due to the lack of camera accuracy and the drift of the tracking points. The results show that it is still possible to identify structural damage by means of image measurement and that damage localisation can be achieved when the damage is large.

7. Conclusion

In this article, a structural damage identification method combining optical flow method and flexibility difference is proposed, which is based on the spatio-temporal image sequences of the structure. That is, the camera is used to acquire dynamic images of the structure and the optical flow method is used to track the global displacement time profile of the structure, thus obtaining the modal parameters of the structure. At the same time, the damage localisation of the structure is achieved by combining the flexibility difference, and the results are compared and analysed with the numerical simulation results. The results show that the method in this article agrees well with the numerical simulation method and has high accuracy. The main conclusions are as follows:

1. This article uses the optical flow method to obtain the displacement time curve of the structure by dynamic displacement tracking at any point on the structure, which solves the cumbersome problems of traditional measurement methods.
2. Using python to write the program to achieve the acquisition of the frequency and vibration pattern of the structure, the frequency error of the structure is within 5%, and the vibration pattern curve of the structure matches, which can meet the demand of structural health inspection.
3. Using Python to write a program to calculate the results of the diagonal difference of the flexibility matrix, the damage localization is achieved.
4. This article effectively combines machine vision with structural health inspection, providing a new way for non-contact structural health inspection.

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