The interaction of a giant planet with a disc with MHD turbulence II: The interaction of the planet with the disc

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ABSTRACT
We present a global MHD simulation of a turbulent accretion disc interacting with a giant protoplanet of 5 Jupiter masses in a fixed circular orbit. The disc model had aspect ratio \( H/r = 0.1 \), and in the absence of the protoplanet a typical value of the Shakura & Sunyaev (1973) stress parameter \( \alpha = 5 \times 10^{-3} \). As expected from previous work the protoplanet was found to open and maintain a significant gap in the disc, with the interaction leading to inward migration of the protoplanet orbit on the expected time scale. No evidence for a persistent net mass flow through the gap was found. However, that may be because an extensive inner cavity could not be formed for the model adopted.

Spiral waves were launched by the protoplanet and although these appeared to be diffused and dissipated through interaction with the turbulence, they produced an outward angular momentum flow which compensated for a reduced flux associated with the MHD turbulence near the planet, so maintaining the gap.

When compared with laminar disc models, with the same estimated \( \alpha \) in the absence of the planet, the gap was found to be deeper and wider indicating that the turbulent disc behaved as if it in fact possessed a smaller \( \alpha \), even though analysis of the turbulent stress indicated that it was not significantly affected by the planet in the region away from the gap. This may arise for two reasons. First, unlike a Navier–Stokes viscosity with anomalous viscosity coefficient, the turbulence does not provide a source of constantly acting friction in the near vicinity of the planet that leads to steady mass flow into the gap region. Instead the turbulence is characterised by large fluctuations in the radial velocity, and time averaging of these fluctuations over significant time scales is required to recover the underlying mass flow through the disc. In the vicinity of the planet the disc material experiences high amplitude periodic perturbations on time scales that are short relative to the time scale required for averaging. Consequently the disc response is likely to be significantly altered relative to that expected from a Navier–Stokes model. Second, the simulation indicates that an ordered magnetic connection between the inner and outer disc can occur enabling angular momentum to flow out across the gap, helping to maintain it independently of the protoplanet’s tide. This type of effect may assist gap formation for smaller mass protoplanets which otherwise would not be able to maintain them.

There is also some evidence that magnetic connection between the circumstellar disc and material that flows into the protoplanet’s Hill sphere may lead to significant magnetic breaking of the resulting circumplanetary disc, thereby modifying the expected gas accretion rate onto forming gas giant planets.

Key words: accretion, accretion disks — MHD, instabilities, turbulence – planetary systems: formation, protoplanetary discs

1 INTRODUCTION

The recent and ongoing discovery of extrasolar giant planets has stimulated renewed interest in the theory of planet formation (e.g. Mayor & Queloz 1995; Marcy, Cochran, & Mayor 1999; Vogt et al. 2002). At the present time there are two main competing theories for the formation of giant gas–rich planets, both of which assume formation in a protostellar disc. In the first, giant planets are envisaged to form directly from the gravitational fragmenta-
tion of a young protostellar disc (e.g. Boss 2001). In the second, giant planets form by the gradual build-up of a rocky core through coagulation of solids, and this core undergoes significant gas accretion once the core mass reaches \( \sim 15 \) Earth masses (e.g. Bodenheimer & Pollack 1986; Pollack et al. 1996). In either scenario, the gravitational interaction between the protoplanet and the protostellar disc from which it forms will play an important role in subsequent evolution.

There have been a number of studies of disc–protoplanet interaction. In the standard picture a protoplanet exerts torques on a protostellar disc through the excitation of spiral density waves at Lindblad resonances (e.g. Goldreich & Tremaine 1979), and these waves carry with them an associated angular momentum flux. This angular momentum is deposited in the disc material where the waves are damped, leading to an exchange of angular momentum between protoplanet and disc. The planet exerts a positive torque on the disc that lies exterior to its orbit and a negative torque on the interior disc.

Previous studies of the interaction between a protoplanet and a viscous disc (Papaloizou & Lin 1984; Lin & Papaloizou 1993) indicate that a sufficiently massive protoplanet can open up an annular gap in the disc around the planet’s orbital radius provided: i) the tidal torque due to the planet is greater than the internal viscous torque; ii) the spiral density waves excited by the planet form shocks, ensuring that the torque exerted by the planet is applied locally to the disc (Ward 1997). For most protostellar disc models the protoplanet needs to be approximately a Jovian mass for gap formation to occur. Recent simulations (Bryden et al. 1999; Kley 1999; Lubow, Seibert, & Artymowicz 1999; D’Angelo, Henning, & Kley 2002) examined the formation of gaps by giant protoplanets, and also estimated the gas accretion rate onto the planets. The detailed form of the gaps, and by association the gas accretion rate onto the protoplanet, is found to be a function of the viscosity in the disc, as well as the planet mass and the disc vertical scale height, \( H \). The orbital evolution of a Jovian mass protoplanet embedded in a standard viscous protostellar disc model was studied by Nelson et al. (2000), who showed that gap formation leads to the creation of a low density inner cavity in which the planet orbits, and that continued interaction with the outer disc leads to inwards migration on a timescale of a few \( \times 10^5 \) yr. It was estimated that a Jovian mass protoplanet would increase its mass to \( \sim 3 \) Jupiter masses by accretion of gas from the disc for this time. The disc models in these studies all modelled the disc viscosity through the Navier–Stokes equation, using an \( \alpha \) model for the kinematic viscosity.

Until quite recently most models of viscous accretion discs used the Shakura & Sunyaev (1973) \( \alpha \) model for the anomalous disc viscosity. This assumes that the viscous stress is proportional to the thermal pressure in the disc without specifying the origin of the viscous stress (but assumed to arise from some form of turbulence). Work by Balbus & Hawley (1991) indicated that significant angular momentum transport in weakly magnetised discs could arise from the magnetorotational instability (MRI – or the Balbus–Hawley instability). Subsequent non linear numerical simulations performed using a local shearing box formalism (e.g. Hawley & Balbus 1991; Hawley, Gammie, & Balbus 1996; Brandenburg et al. 1996) confirmed this and showed that the saturated non linear outcome of the MRI is MHD turbulence with an associated viscous stress parameter \( \alpha \) of between \( \sim 5 \times 10^{-3} \) and \( \sim 0.1 \), depending on the initial magnetic field configuration. More recent global simulations of MHD turbulent discs (e.g. Armitage (1998); Hawley (2000); Hawley (2001); Steinacker & Papaloizou (2002)) confirm the picture provided by the local shearing box simulations.

In a companion paper to this one (Papaloizou & Nelson (2002) – hereafter paper I) we present the results of 3D global MHD simulations of turbulent discs initiated with zero net flux magnetic fields. We examine the behaviour of the discs as a function of varying the disc scale height, \( H \), the extent of the radial domain, the numerical resolution, and the initial magnetic field configuration (poloidal or toroidal). We find that these disc models produce volume averaged values for the stress parameter \( \alpha \sim 5 \times 10^{-3} \). We note that this value is typical of that expected in protostellar discs to maintain the canonical mass accretion rate of \( 10^{-8} \) M\(_{\odot}\) yr\(^{-1}\) onto T Tauri stars (e.g. Hartmann et al. 1998). We also consider the issue of time averaged estimates for the radial distribution of the turbulent quantities such as the time averaged stress parameter \( \alpha \) and the time averaged mass flux through the disc. Balbus & Papaloizou (1999) noted the need to consider time averages of these quantities in order to connect the behaviour of MHD turbulent discs models with standard viscous accretion disc theory. Indeed we find reasonable agreement between the expectations of viscous disc theory and the results of the simulations in paper I, when appropriate time averages are considered. One of the models presented in paper I (model E) was used as the basis for the work presented in this paper.

The calculations presented in this paper and in paper I assume that the equations of ideal MHD apply. Questions have been raised concerning the degree of ionisation in protostellar discs (e.g. Gammie 1996), and thus about the applicability of the equations of ideal MHD in the modelling of protoplanetary discs. This is a complex issue that has yet to be fully resolved, so we assume that the equations of ideal MHD are a valid approximation in these first papers on the interaction between turbulent discs and embedded protoplanets. Future work will address the issue of the ionisation fraction in protostellar discs, and the effects this may have on disc–planet interactions.

Significant work has already been done on examining the interaction between giant protoplanets and viscous protostellar discs using laminar \( \alpha \) disc models. The improvement and availability of high performance computing resources has now made it possible to develop global models of MHD turbulent discs in which the underlying mechanism responsible for angular momentum transport is explicitly calculated. The next important step in understanding the interaction between protostellar accretion discs and embedded giant protoplanets is to compute models of turbulent MHD discs interacting with giant protoplanets. This is the subject of the present paper.

The underlying disc model that we consider (described in detail in paper I) has a constant ratio of disc scale height to radius of \( H/r = 0.1 \), and a volume and time averaged value of the stress parameter \( \alpha = 5 \times 10^{-3} \). In order for a protoplanet to form a gap in such a disc, it must have a mass of at least 3 Jupiter masses (assuming the central stellar mass is 1 M\(_{\odot}\)) according to the thermal criterion (see...
section [3.2]). In order to be certain of generating a clean gap we consider a protoplanet with 5 Jupiter masses. We note that at the present time it is beyond our computational resources to consider thinner discs and lower mass planets. Protostellar discs are more typically expected to have \( H/r = 0.05 \), but to model such discs would require a doubling of the number of grid cells in each direction. We have found from conducting numerical experiments that zero net magnetic flux disc models require a numerical resolution similar to that we consider here in order to sustain MHD turbulence at a reasonable level, and to generate reasonable values of \( \alpha \). Ideally we would wish to perform simulations of discs with \( H/r = 0.05 \) and Jovian mass protoplanets. However, we believe that the results obtained will still provide a reliable picture of how protoplanets interact with turbulent discs.

In particular, as expected from previous work, the protoplanet was found to open and maintain a significant gap in the disc, with the interaction leading to inward migration of the protoplanet orbit on the expected time scale. Spiral waves were launched by the protoplanet producing an outward angular momentum flow and reducing that provided by the MHD turbulence near the planet. The turbulent disc behaved as if it possessed a smaller \( \alpha \) than expected. The characteristic flow induced by the protoplanet was found to facilitate an ordered magnetic connection between the inner and outer disc enabling angular momentum to flow out across the gap. This effect provides an additional mechanism for maintaining the gap and may account for the apparently smaller \( \alpha \), along with the fact that the turbulence does not provide a source of constantly acting friction, unlike a Navier–Stokes viscosity.

The plan of the paper is as follows. In sections 2 and 2.1 we describe our initial model set up and numerical procedure. In section 2.2 we discuss the issue of time averaging the relevant disc quantities and the connection to standard viscous disc theory. In section 3.2 we describe the basic ideas of disc–protoplanet interactions, and our expected results based on this previous work. In section 3.3 we discuss the time dependent evolution of the model, and in section 3.4 we compare our results with those obtained from laminar viscous disc models. Finally in section 4 we discuss our results and draw our conclusions.

2 INITIAL MODEL SETUP

The governing equations for MHD written in a frame rotating with a uniform angular velocity \( \Omega_p \hat{k} \) with \( \hat{k} \) being the unit vector in the vertical direction are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + 2 \Omega_p \hat{k} \times \mathbf{v} = -\nabla p - \rho \nabla \Phi + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}. \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{3}
\]

where \( \mathbf{v}, P, \rho, \mathbf{B} \) and \( \Phi \) denote the fluid velocity, pressure, density, magnetic field, and potential, respectively.

The angular velocity \( \Omega_p \) corresponds to that of a planet in fixed circular orbit at radius \( D \). The perturbing planet thus appears stationary in the chosen reference frame. The potential \( \Phi \) contains contributions due to gravity and the centrifugal potential \( -(1/2)\Omega_p^2 r^2 \). We use a locally isothermal equation of state

\[
P(r) = c(r)^2 \cdot \rho, \tag{4}
\]

where \( c(r) \) denotes the sound speed which is specified as a fixed function of \( r \).

The disc model investigated may be described as a cylindrical disc (e.g. Hawley 2001) and was considered prior to the introduction of a perturbing planet in paper I as model E. This disc model was such that \( c(r)/r\Omega(r) = 0.1 \), so that the effective ratio of disc scale height to radius \( H/r = 0.1 \), and had a volume averaged value of the stress parameter \( \alpha \approx 5 \times 10^{-3} \). We adopt cylindrical coordinates \((z, r, \phi)\) based on the centre of mass of the central star plus protoplanet system. The number of grid cells used was \((N_z, N_r, N_\phi) = (370, 580, 60)\). The gravitational potential is due to the central star and internally orbiting planet. Thus we have

\[
\Phi = \frac{GM_p}{\sqrt{r^2 + D^2 - 2rD \cos(\phi - \phi_p) + b^2}} - \frac{1}{2} \Omega_p^2 r^2. \tag{5}
\]

The first term represents the gravitational potential due to the central star, with \( G \) being the gravitational constant and \( M_p \) being the stellar mass. The second term is the gravitational potential due to a planet in circular orbit at radius \( D \) and azimuthal angle \( \phi_p \) with a mass \( m_p \). The final term represents the centrifugal potential. To avoid a singularity in the planet potential we adopt a softening parameter \( b = Rh/2 \) where \( Rh = D(m_p/3M_p)^{1/3} \) is the Hill sphere radius of the protoplanet. This enables a Roche lobe filling atmosphere to be set up around the protoplanet before the gap construction process is completed. Material flowing in the neighbourhood of the protoplanet then has to move around it and cannot be accreted. We thus cannot simulate the accretion of gas onto the protoplanet in this work. In previous work on disc–planet interactions (e.g. Nelson et al. 2000), accretion of gas by a protoplanet is modelled my simply removing matter from within the Roche lobe at a specified rate. A technical difficulty arises in a simulation like the one presented here that includes magnetic fields, since one also has to consider what happens to the magnetic field when gas is accreted by the protoplanet. This is the primary reason why we do not explicitly calculate the gas accretion rate onto the protoplanet in this work.

In this paper we consider the time dependent evolution of a turbulent disc together with a planet with \( m_p \) equal to 5 Jupiter masses. We initially immerse the planet at \( r = 2.2D \) and \( \phi_p = \pi \) into a fully turbulent disc model covering the full azimuthal domain of \( 2\pi \). The initial model we adopt is model E that has been described in paper I. When a planet is immersed directly into such a model we found that large scale transients occurred which prevented the efficient and quick formation of a gap. It was not possible to follow such transients to a conclusion in a run of reasonable duration. To avoid such problems we carved out a gap in the density profile centred on the planet’s radial
location before inserting the planet. This had the effect of perturbing the disc from a steady state, so it was allowed to relax until a steady state had been reattained. The planet was then inserted into the disc and the calculation initiated with the time $t$ being reset to zero. We comment that in carving out the initial gap the magnetic field was not altered. This means that the magnetic energy in the initial gap would be significantly larger than expected for relaxed turbulence under conditions of zero net flux. However, the runs were subsequently continued for about 100 planet orbits which is significantly longer than the time required for adjustment of the local turbulence (see paper I). Further, after such times, the magnetic stresses appeared to have reached a steady pattern and to be largely associated with magnetic field concentrated into wakes which were absent when the planet was initially inserted. Thus the behaviour of the magnetic field in the gap region is not determined by the conditions prevailing when the gap was set up.

A test run was performed in the absence of an embedded protoplanet to ensure that the turbulent disc relaxed to a state with the expected magnetic energy once an initial gap had been carved out. The results of this run showed that the disc returns to its normal turbulent state within a few orbits.

The form of the azimuthally averaged density profile located at the vertical midplane of the disc at the stage when the planet is introduced is plotted in figure 1.

As in paper I, units are adopted such that $r_1 = 1$ and $GM = 1$, where $r_1$ is the radius of the inner radial boundary of the computational domain. We note that a single simulation of this type requires of order 30,000 CPU hours on an Origin 3800 parallel processing facility.

2.1 Numerical procedure

The numerical scheme that we employ is based on a spatially second–order accurate method that computes the advection using the monotonic transport algorithm (Van Leer 1977). The MHD section of the code uses the method of characteristics constrained transport (MOCCT) as outlined in Hawley & Stone (1995) and implemented in the ZEUS code. The code has been developed from a version of NIVANA originally written by U. Ziegler (Ziegler & Rudiger 2000).

3 NUMERICAL RESULTS

Before describing results in detail we discuss some average quantities used in their description.

3.1 Vertically and horizontally averaged stresses, angular momentum transport and external torques

For a global coarse grained description of the flow, as in paper I, we use quantities that are both vertically and azimuthally averaged over the $(\phi, z)$ domain (e.g. Hawley 2000) and in some cases an additional time average. Adopting the same formalism and notation as in paper I, we write the vertically and azimuthally averaged continuity equation in the form

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \left( \frac{\partial \left( r \Sigma \Psi r \right)}{\partial r} \right) = 0,$$

where the disc surface density is given by

$$\Sigma = \frac{1}{2\pi} \int \rho dz d\phi.$$

We adopt the Shakura & Sunyaev (1973) $\alpha$ stress parameter appropriate to the total stress as defined in paper I. Then the vertically and azimuthally averaged azimuthal component of the equation of motion can be written in the form

$$\frac{\partial (\Sigma J)}{\partial t} + \frac{1}{r} \left( \frac{\partial \left( r \Sigma J \right)}{\partial r} \right) + \frac{\partial \left( r^2 \alpha P / \rho \right)}{\partial r} = - \int \rho \frac{\partial \Phi}{\partial \phi} \frac{dz dz}{2\pi}. \tag{8}$$

Here $j = r v_{\phi}$ is the specific angular momentum and the term on the right hand side represents the azimuthally and vertically integrated torque acting on the disc (absent in paper I) which is here due to the internally orbiting planet.

Note that this form is identical to that obtained in standard viscous disc theory (see Balbus & Papaloizou 1999).

Equations (3) and (4) may be used with or without a time averaging procedure in addition to horizontal and vertical averaging (see paper I).

3.2 Disc–Planet Interactions and Gap Formation

In the usual picture of disc–protoplanet interactions, the presence of a protoplanet orbiting in a gaseous accretion disc leads to the excitation of spiral density waves at Lindblad resonances, which propagate from the locations where they are launched (e.g. Goldreich & Tremaine 1979). These spiral waves carry with them an associated angular momentum flux that is deposited in the disc where the waves are damped, either through shocks or viscous effects.

Previous work on the interaction between a giant protoplanet and a viscous protoplanetary disc indicates that an annular gap is expected to form in the vicinity of the protoplanet provided that two criteria are satisfied (e.g. Lin & Papaloizou 1993). The first criterion (the so-called viscous criterion) requires that

$$m_p > 40 \alpha \left( \frac{H}{R} \right)^3,$$

which arises from the requirement that tidal torques induced by the protoplanet exceed viscous torques in the disc. The second criterion (the so-called thermal criterion) requires that

$$m_p > 3 \left( \frac{H}{R} \right)^4,$$

and comes from the requirement that the spiral waves introduced into the disc flow by the planet be non linear so that spiral shocks form (Ward 1997).

The disc model that we employ in this work is characterised by a value of $H/R = 0.1$ and a volume averaged value of $\alpha \approx 0.005$ (see paper I). It is thus expected that the presence of a 5 Jupiter mass protoplanet will lead to gap clearing. Previous numerical work on disc protoplanet interactions (Nelson et al. 2000) suggests that gap formation in a disc in which the gas can accrete onto a central star will lead to the eventual formation of a low density inner cavity.
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3.3 Time dependent evolution of the model

The disc model together with planet was run for 2040 time units. This corresponds to \( \sim 324 \) orbits at \( r = 1 \), about 100 orbits at \( r = 2.2 \) (where the planet is located), and about 17 orbits at \( r = 7.2 \). We found that the initial gap considerably deepened and accretion onto the central parts occurred producing something like a central cavity seen in previous work (e.g. Nelson et al. 2000). We note that this is unable to become very extensive in the present work due to our adoption of an inner boundary layer which prevents mass from flowing through the inner boundary. The behaviour of the azimuthally averaged value of \( \rho(r) \) at the disc midplane throughout the run is indicated in figure 1.

Throughout the run MHD turbulence is sustained in regions of the disc not strongly affected by the planet. However, the planet has a large enough mass to strongly perturb the disc locally generating outward and inward propagating density waves as well as extending the initial gap as expected from the discussion in section 3.2.

The magnetic energy in the Keplerian domain expressed in units of the volume integrated pressure is given as a function of time in figure 2. A value of \( \sim 0.01 \) is maintained throughout. This is similar to what is found for models without the planet, and indicates that the perturbing presence of the protoplanet does not have a strong effect on the dynamo operating globally within the disc. However, there is evidence that the protoplanet affects the turbulence locally, creating an ordered field where the material passes through the spiral shocks in the gap region.

The magnetic energy in the Keplerian domain expressed in units of the volume integrated pressure as a function of time. Note that the value of \( \sim 0.01 \) is very similar to that obtained in the absence of the protoplanet.

The stress parameter \( \alpha \) volume averaged over the Keplerian domain is plotted as a function of time. The upper curve is derived from the total stress while the lower curve is derived from the magnetic contribution.

As in paper I we explore the behaviour of time averages of the stress parameter \( \alpha \) and its variation with radius. Only in this way can a connection with earlier theories of gap formation in viscous discs be made. A time average of the stress parameter \( \alpha \) taken between times \( t = 1650.0 \) and \( t = 2040.0 \) is plotted as a function of dimensionless radius in figure 4. Both the total stress parameter \( \alpha \) and the magnetic contribution to it become large in the vicinity of the planet \( r < 3 \) where the gap is. This arises even though the actual magnetic stress \( T_m = B_r B_\phi / 4\pi \) decreases in the gap region once the gap is formed, as indicated by figure 5, because the pressure \( P \) in the gap decreases by an even larger fraction (note that magnetic stress parameter \( \alpha = -T_m / P \)). Further away beyond \( r > 5 - 6 \), values similar to those pertaining to the disc without a planet are attained, indicating that the

\[ \text{Figure 1. The azimuthally averaged density at the vertical midplane of the disc is plotted as a function of radius at } t = 0 \text{ (solid line). The remaining curves correspond to the same physical quantity after times of } t = 675.6 \text{ (dotted line), } t = 1351.2 \text{ (dashed line), and } t = 2047.28 \text{ (dot-dashed line). These times correspond to } 0, 33, 66, \text{ and } 100 \text{ orbits of the protoplanet, respectively. Note that the initial gap has been deepened and widened considerably. Also some accretion into the central regions has occurred.} \]

\[ \text{Figure 2. Magnetic energy in the Keplerian domain expressed in units of the volume integrated pressure as a function of time. Note that the value of } \sim 0.01 \text{ is very similar to that obtained in the absence of the protoplanet.} \]

\[ \text{Figure 3. The stress parameter } \alpha \text{ volume averaged over the Keplerian domain is plotted as a function of time. The upper curve is derived from the total stress while the lower curve is derived from the magnetic contribution.} \]
planet does not have a significant effect on the turbulence outside of the gap region.

Inspection of equation (8), however, suggests we should consider the non advected part of the angular momentum flux which is proportional to $r^2 \times$ stress or $\Sigma r^2 \alpha P/\rho$ when discussing the disc evolution. We plot time averages of the angular momentum fluxes in figures 5 and 6. The time averages for these are over the time intervals $1650 < t < 1920$ and $1650 < t < 2040$ respectively and they both show the same behaviour indicating a stable pattern of behaviour has been established relatively quickly.

This behaviour may be understood in the context of gap formation theory as applied to laminar discs with Navier–Stokes viscosity (e.g. Bryden et al. 1999). Consider equation (8) under the assumption of a disc in a quasi steady state with $\tau_r = 0$. Suppose also there is a completely empty gap in which the planet is located. Then equation (8) predicts that the angular momentum flux tends to a constant value at large distances from the planet. The flux is zero within the gap where there is no material, and is generated by the tidal torque term at the disc edge near the planet.

The behaviour of the fluxes illustrated in figures 5 and 6 can be related to this expectation. We see that the total flux dips to a minimum at the location of the planet orbit and rises to an asymptotic value at large distances. The contribution from the Reynolds’ stresses initially rises, representing the generation of spiral waves by the protoplanet in the region of the gap wall and beyond, and reaches a maximum close to the outer 1:2 Lindblad resonance location, beyond which there is no further generation of spiral waves by a planet on a circular orbit. Beyond this region the contribution from the Reynolds’ stresses decreases as these waves are dissipated. The magnetic contribution rises from a small but non zero value near the planet to make most of the contribution at large radii. There the behaviour is that of an unperturbed turbulent disc. The persistence of the magnetic stresses through the gap may be significant in that this can allow for the transmission of angular momentum from one side of the gap to the other without needing the contribution of a tidal torque from the planet. This may enhance the gap formation process with respect to what may be expected from non magnetic laminar disc theory (see section 3.4).

The angular momentum fluxes plotted in figures 5 and 6 indicate a net outward value supplied by the planet’s tide. This results in inward migration as has been verified by direct calculation of the torques acting on the planet itself. The torque, $\dot{J}$, arising from the disc beyond the planet orbital radius indicates an inward migration time defined by $\dot{J}_{\text{planet}} / \dot{J} \approx 2 \times 10^4$ planetary orbits, for a disc mass normalisation such that there exist two Jupiter masses interior to
the orbital radius of the planet $r_p = 2.2$. Interestingly, the viscous timescale at the planet location defined by the usual expression $t_v = r^2/(\alpha H^2 \Omega) \simeq 1.03 \times 10^4$ planetary orbits. It is clear that the planet undergoes inward migration at a rate that is in broad agreement with the expectations of type II migration theory [Lin & Papaloizou (1986, 1993); Nelson et al. (2000)].

As in the unperturbed turbulent disc described in paper I, the radial velocity shows large temporal fluctuations about a small mean value. However, in this case when the planet is present the mean values tend to be smaller giving some indication that planetary tides balance viscous transport. Time averages of the product of the vertically and azimuthally averaged values of $\Sigma/L_z$ and $v_r$, in arbitrary units, are plotted as a function of dimensionless radius in figure 7. Here $L_z$ is the extent of the vertical domain. The averages are taken between time $t = 1650.0$ and $t = 2040.0$ while for the dotted curve it is taken between $t = 1650.0$ and $t = 1920.0$.

Figure 7. Time averages of the product of the vertically and azimuthally averaged values of $\Sigma/L_z$ and $v_r$ in arbitrary units are plotted as a function of dimensionless radius. For the solid curve the average is taken between time $t = 1650.0$ and $t = 2040.0$ while for the dotted curve it is taken between $t = 1650.0$ and $t = 1920.0$.

3.4 Comparison with laminar viscous disc models

In addition to running the full MHD turbulent disc model, we have also run some 2D laminar $\alpha$ disc models for the purposes of comparison. A 2D model was run with $\alpha = 0$, along with an $\alpha = 5 \times 10^{-3}$ case. For initial conditions in these 2D models, we took the midplane density distribution of the 3D turbulent model just prior to the addition of the planet, switched off the magnetic field, and introduced the required value of $\alpha$ in the Navier–Stokes viscosity. The implementation of the Navier–Stokes viscosity is described in Nelson et al. (2000).

On a qualitative level differences arise between the turbulent and laminar disc models. The most striking visual difference is that the spiral waves excited in the turbulent disc have a more ‘blurred’ appearance, whereas they appear to be more sharply defined in the laminar models [see figures 6, 10, and Nelson et al. (2000)]. Also, an inspection of the flow topology in the close vicinity of planet shows some differences. In the turbulent disc case, there appears to be little sign of the usual circulating flow around the planet for material in the Hill sphere (see figure 13), whereas the laminar models do show the expected circulating flow pattern. This may indicate that magnetic linkage between the protostellar disc and the circumplanetary disc that forms when material flows into the protoplanets Hill sphere leads appear much less sharply defined than in the laminar disc case (see Nelson et al. 2000 and figure 16).

In the region of the wakes the disc and its turbulence are strongly perturbed. Magnetic field vectors in the disc midplane in the neighbourhood of the planet are illustrated on different size scales in figures 8 and 17, along with corresponding density plots for the equivalent regions in figures 10 and 12. An inspection of the magnetic field vectors indicates that these tend to line up along the location of the wakes but in a somewhat broadened region slightly behind the shocks. In this way an ordered structure appears to be imposed on the flow and magnetic field by the protoplanet. The magnetic stress is largely communicated in these ordered regions as shown by figure 13, which shows a contour plot of the magnetic stress in the vicinity of the planet.

The velocity field within the Hill sphere of the planet is shown in the left hand panel of figure 14 for the turbulent disc. An equivalent plot for a laminar disc is shown in the left hand panel of this figure. The material that enters the Hill sphere of the protoplanet and becomes gravitationally bound to it is normally expected to circulate around it by virtue of its angular momentum. In the calculation presented here the protoplanet does not accrete gas, and is modelled as a softened point mass. Consequently it is expected that material that enters the Hill sphere will form a hydrostatic ‘atmosphere’ around the planet that is supported through pressure and rotation. In figure 14 we would expect to see circulation occurring in a clockwise fashion for both the MHD turbulent disc and the laminar disc. However, it is apparent that the circulating pattern has been disrupted in the magnetised disc, indicating that magnetic breaking may have been responsible for this (see section 3.3). The expected circulation is observed in the non magnetic run performed at the same numerical resolution as shown in the right hand panel of figure 14.
Figure 8. A typical contour plot for the midplane disc density at time $t = 1680.0$. The gap, planet and wakes it causes in the disc are apparent. The underlying turbulence has the effect of slightly blurring this picture relative to those obtained from simulations performed using laminar accretion discs with anomalous Navier–Stokes viscosities.

to magnetic breaking. This effect may be highly significant for determining the accretion rate of gas onto the proto-planet through the circumplanetary disc, and indicates that accretion rates obtained from non-magnetic treatments of the problem may be quite different. We note, however, that higher resolution simulations will be required to examine this effect in detail, and will be the subject of a future publication. In addition to these qualitative differences, there are also some important quantitative differences.

The azimuthally averaged surface density distribution for these models, plus the midplane density distribution for the turbulent model, is plotted in figure 8. Each of the models have been run for a total time of $\simeq 2050$ time units, corresponding to $\sim 100$ planetary orbits. It is clear that the $\alpha = 0.0$ 2D model (dashed line) has the deepest and widest gap, as expected. It is also apparent that the turbulent model (dotted line) has a deeper and wider gap than the 2D $\alpha = 5 \times 10^{-3}$ model, even though a volume averaged estimate for the underlying turbulent disc model yields an effective $\alpha \simeq 5 \times 10^{-3}$ (see figure 18 in paper I). Thus, the turbulent model behaves as if it has a somewhat smaller $\alpha$ than reasonable estimates suggest it has. There are two
Figure 9. This figure shows the magnetic field vectors ($B_r, B_\phi$) in the disc midplane plotted in the $r-\phi$ plane at time $t = 2040$. Note that the field is compressed and ordered in the vicinity of the wakes induced by the protoplanet.

Figure 10. This figure shows the midplane density in the vicinity of the planet for an area that is the same as for the magnetic field plot in figure 9.
**Figure 11.** This figure shows the magnetic field vectors \((B_r, B_\phi)\) in the disc midplane plotted in the \(r-\phi\) plane at time \(t = 2040\) for an area that is somewhat smaller than that plotted in figure 9. It is clear that the field is ordered and compressed in the vicinity of the wakes induced by the protoplanet.

**Figure 12.** This figure shows density in vicinity of planet for area equivalent to magnetic field plot in figure 11.
probable reasons for this. First, a Navier–Stokes viscosity with anomalous viscosity coefficient provides a source of constantly acting friction in the disc, such that it can induce a steady mass flow into the gap region. The turbulence, however, does not operate as a constant source of friction that generates steady inflow velocities. Instead it generates large velocity fluctuations that may be much larger than the underlying inflow velocity arising from the associated angular momentum transport. Results presented in paper I indicate that a process of time averaging the turbulent velocity field

\[ \text{Radius} \]
\[ \text{Azimuthal Angle } \phi \]
Figure 16. This plot shows the vertical midplane density distribution for the turbulent disc model (first panel), a 2D laminar disc model with the Navier-Stokes $\alpha = 5 \times 10^{-3}$ (second panel), and a 2D laminar disc model with $\alpha = 0$ (third panel). It is apparent that the spiral waves excited in the turbulent disc are ‘blurred’ by the effects of the turbulence, whereas those in the laminar disc model cases remain more sharply defined.

is required over long time periods before these fluctuations can be averaged out to reveal the underlying mass flow. The disc material in the vicinity of the planet experiences periodic high amplitude perturbations induced by the planet on a time scale much shorter than the required averaging time scale, so that the disc response is expected to differ from that in the case of a disc with Navier–Stokes viscosity. A second plausible reason for the apparently lower $\alpha$ is that the existence of the magnetic field in the turbulent disc allows for field lines to connect across the gap region, and to enable angular momentum transport across the gap. In this way the magnetic field actually helps the planet to maintain the gap. Figure 17 shows an example of how magnetic structures are able to extend across the gap and thus maintain communication across the gap region. The stresses associated with these structures lead to outwards angular momentum transport, and thus assist in maintaining the gap. However, it should be borne in mind that the above remarks apply to one particular simulation involving a rather thick disc with $H/r = 0.1$ and a 5 Jupiter mass protoplanet. The ability
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Figure 17. This plot shows magnetic field vectors ($B_r$ and $B_\phi$) plotted in the $r-\phi$ plane of the disc vertical midplane showing field structure in the gap region. It is evident that magnetic field lines are able to cross the gap (located between $r \approx 1.8 - r \approx 3.0$) and thus transmit torques between the two sides. This effect may be partially responsible for making the gap appear to resemble one formed in a laminar disc with smaller effective stress parameter $\alpha$ when a protoplanet is present in a turbulent disc. This is because these magnetic stresses are able to help maintain the gap. Note that features of this type tend to be less prevalent in the azimuthal domain near to the planet, but are more apparent in the regions that are close to $\pi$ radians away from the planet.

Figure 15. This plot shows the vertical midplane density distribution for the turbulent disc model (dotted line), a 2D laminar disc model with the Navier-Stokes $\alpha = 0$ (dashed line), and a 2D laminar disc model with $\alpha = 5 \times 10^{-3}$. It is clear that the gap for the 2D $\alpha = 0$ disc is the deepest, followed by the full turbulent disc model, with the 2D $\alpha = 5 \times 10^{-3}$ being the least deep and wide.

of ordered magnetic field to traverse the gap may be much less effective in a thinner disc in which the gap structure tends to be sharper and deeper. Also the existence of significant disc material interior to the planet’s orbit may assist this process, and in a model with an extensive inner cavity it may be less efficient. Nonetheless the differences in gap structure found here suggest that the accretion rate onto a protoplanet in an MHD turbulent disc is likely to be less than previous estimates based on laminar $\alpha$ disc models indicate [e.g. Bryden et al. (1999); Kley (1999); Lubow, Seibert, & Artymowicz (1999); Nelson et al. (2000); D’Angelo, Henning, & Kley (2002)].

4 DISCUSSION AND CONCLUSIONS

We have presented a global MHD simulation of a turbulent disc interacting with a giant protoplanet in a fixed circular orbit. The disc model that we considered (described in detail in paper I) had $H/r = 0.1$, and in the absence of the protoplanet a volume and time averaged value of the stress parameter $\alpha = 5 \times 10^{-3}$. We considered a protoplanet with 5 Jupiter masses which from previous work was expected to open and maintain a significant gap. Although the available computational resources have limited the parameters
that could be adopted, we expect the essence of the disc–protoplanet interaction to be captured.

Using appropriate time averaged stress parameters and radial velocities, many of the phenomena seen in two dimensional laminar disc simulations could be demonstrated.

Spiral waves that produced an outward angular momentum flux through Reynolds’ stresses were launched by the protoplanet. As expected the existence of the waves reduced the magnitude of the stresses provided by the MHD turbulence so reducing the associated outward angular momentum flow and thus maintaining the gap. A net outward angular momentum flow from the planet orbit to the disc was seen which leads to inward migration of the protoplanet orbit on a time scale expected from type II migration theory.

When compared with laminar disc models, with the same estimated $\alpha$, the gap in the turbulent disc was found to be deeper indicating that the turbulent disc behaved as if it possessed a smaller $\alpha$. This was in spite of the fact that the turbulent disc appeared more effective at disrupting and dissipating the spiral waves. This behaviour may occur for two basic reasons. The first is that the turbulence does not provide a source of constantly acting friction in the disc, unlike a Navier–Stokes viscosity, and time averaging for significant periods is required before velocity fluctuations cancel out to reveal the underlying mass flow. The disc is periodically perturbed by the planet on time scales shorter than those required for the time averaging, indicating that the disc response is likely to differ from a disc with Navier–Stokes viscosity operating. The second reason is that ordered magnetic structures where found to be able cross the gap at azimuthal locations that were displaced from the planet by $\approx \pi$. Such a connection would enable an outward flux of angular momentum to flow across the gap and help to maintain it independently of the planetary torques, so, as far as gap properties are concerned making the disc appear to behave as if there was a smaller $\alpha$. We also found no evidence for a persistent mass flow through the gap from the outer to inner disc but we emphasize that this may change if an extensive deep inner disc cavity can be produced or if the disc is thinner. On the other hand it may persist for smaller protoplanets which make weaker gaps and inner cavities.

The behaviour of the material bound to the planet was found to behave differently in the case of a magnetised disc, with the circulating material within the planetary Hill sphere apparently experiencing magnetic breaking. This has implications for the structure and evolution of the circumplanetary disc that is expected to form during the latter stages of gas giant planet formation, and the mass accretion rate onto giant protoplanets. These issues require detailed investigation through additional simulations, and will be the subject of future publications.

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