Inflaton mass in the $\nu$MSM inflation

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Abstract

We analyze the reheating in the modification of the $\nu$MSM (Standard Model with three right handed neutrinos with masses below the electroweak scale) where one of the sterile neutrinos, which provides the Dark Matter, is generated in decays of the additional inflaton field. We deduce that due to rather inefficient transfer of energy from the inflaton to the Standard Model sector reheating tends to occur at very low temperature, thus providing strict bounds on the coupling between the inflaton and the Higgs particles. This in turn translates to the bound on the inflaton mass, which appears to be very light $0.1 \text{ GeV} \lesssim m_I \lesssim 10 \text{ GeV}$, or slightly heavier then two Higgs masses $300 \text{ GeV} \lesssim m_I \lesssim 1000 \text{ GeV}$.

Key words: Inflation, Standard Model, reheating, nuMSM

1. Introduction

In $^{1,2}$ it was shown that within the Standard Model (SM) complimented with three right-handed neutrinos $N_I$ with the masses smaller than the electroweak scale one can simultaneously explain both the dark matter and the baryon asymmetry of the universe $^{1,2,3,4,5,6,7,10,11,12}$. This model dubbed as $\nu$MSM represents a particular realization of the seesaw extension of the SM and is fully consistent with the current experimental data from the light neutrino sector. However, generation of the proper Dark Matter abundance of the sterile neutrino is not simple during the thermal evolution of the Universe, and requires some amount of fine-tuning $^{13,14}$. Being very weakly coupled, sterile neutrinos do not reach thermal equilibrium, so an interesting possibility is to generate them before the beginning of the standard thermal history. In $^6$ such mechanism was proposed, where the $\nu$MSM model was extended by adding the inflaton field, which generates all the masses in the model and decays into the SM particles and sterile neutrinos after inflation,

$$\mathcal{L}_{\nu\text{MSM}} \rightarrow (\mathcal{L}_{\nu\text{MSM}[M_I \to 0]} - f_{II} \bar{N}_I^c N_I \chi + h.c.) + \frac{1}{2} (\partial_\mu \chi)^2 + |\partial_\mu \Phi|^2 - V(\Phi, \chi),$$

where $\Phi$ and $\chi$ are the Higgs and the inflaton fields correspondingly and

$$\mathcal{L}_{\nu\text{MSM}[M_I \to 0]} = \mathcal{L}_{\text{SM}} + \bar{N}_I^c \partial_\mu \gamma^\mu N_I - F_{\alpha I} L_{\alpha}^c N_I \Phi + h.c. \quad (2)$$

is the $\nu$MSM Lagrangian with all the dimensional parameters being put to zero. The potential $V(\Phi, \chi)$ is

\footnote{In order to avoid the domain wall problem a cubic term $\mu \chi^3$ can be introduced. It will be further assumed that $\mu \lesssim \sqrt{\alpha^2/\lambda v_{EW}}$. In that case such term has no influence on the dynamics of the model during the reheating stage, and the relation (4) for the values of the parameters considered in the Letter is not altered significantly either.}

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\[ V(\Phi, \chi) = \lambda \left( \Phi^4 - \frac{\alpha}{\chi} \right)^2 + \frac{\beta}{4} \chi^4 - \frac{1}{2} m_I^2 \chi^2 + V_0 , \]  

where \( V_0 = \frac{m_I^4}{\lambda} \) was introduced in order to cancel the vacuum energy. Expanding (3) around its vacuum expectation value one has the relation between the inflaton mass \( m_I \) and the Higgs mass \( m_H \):

\[ m_I = m_H \sqrt{\frac{\beta}{2\alpha}} . \]  

If \( \alpha > \beta/2 \) the inflaton mass is smaller then the Higgs mass and, therefore, the decay of the inflaton into the Higgs can only occur in a thermal bath. In what follows we will first concentrate on this particular case. Parameter \( \beta \) is fixed by the COBE normalization of the amplitude of scalar perturbations [15], \( \beta \simeq 1.3 \times 10^{-13} \). Pure quartic potential inflation is currently disfavored by the WMAP5 data [16] because of the too large predicted contribution of the zero mode of the inflaton to the scalar amplitudes ratio. However, if one allows non-minimal coupling of the inflaton to gravity [17] one can bring this potential in agreement with the data. This, in turn, will influence the bounds on the inflaton mass. We will discuss this in the end of the Letter.

The upper constraint on the value of \( \alpha \) comes from the requirement that radiative corrections do not spoil the flatness of the inflaton potential and is given by \( \alpha \leq 3 \times 10^{-7} \). This corresponds to the lower bound on the inflaton mass

\[ m_I \geq 0.07 \left( \frac{m_H}{150 \text{ GeV}} \right) \sqrt{\frac{\beta}{1.3 \times 10^{-13}}} \text{ GeV} . \]  

One should note that larger values of \( \alpha \) (leading to smaller inflaton masses) may also be possible, but then the analysis of the loop corrections to the effective potential of the inflaton becomes important.

The lower bound on \( \alpha \) comes from the requirement to have successful baryogenesis in \( \nu \)MSM [2]. To allow for efficient sphaleron conversion of the lepton asymmetry to baryon asymmetry requires the reheating temperature to be larger than roughly 150 GeV [18]. In [6] it was advocated that the resulting lower bound is \( \alpha > \beta \sim 10^{-13} \). Below we will argue that the lower bound is quite a bit stronger which leads to a narrow window for the inflaton mass.

\[ m_{\text{eff}, \Phi} \sim \alpha \lambda_0^2, \quad m_{\text{eff}, \chi} \sim \beta \lambda_0^2 . \]

If \( \alpha > \beta \) the corresponding contribution to the effective mass of the Higgs is larger. Therefore at early stages of the evolution the energy transfer into the Higgs particles is the dominating process. This is in accord with [19, 20], and can be inferred from the early time behavior of the number densities shown in Fig. 1. One could then expect that the whole energy of the inflaton field will be transferred exponentially fast to the Higgs particles [3]. Since the Higgs decay to the SM fields and their consequent thermalization are fairly fast compared to the Hubble rate one could then estimate the resulting reheating temperature as in [6]

\[ T_R \sim m_{\text{pl}} \left( \frac{\alpha^2}{g_\star \lambda} \right)^{\frac{1}{2}} , \]

which for \( \lambda \sim 0.1 \), the number of the SM d.o.f. \( g_\star \sim 10^2 \) and \( \alpha > \beta \) leads to the values of \( T_R \) which greatly exceed the freeze-out temperature of the sphaleron processes.

\[ \text{Notations } I \text{ and } H \text{ will be used throughout the Letter to represent the diagonalized excitations above the vacuum expectation value for } \chi. \text{ } I \text{ is the one mostly mixed with inflaton } \chi, \text{ and } H \text{ mostly mixed with the SM Higgs } \Phi. \]
The lattice results, however, show that such exponential energy transfer into the Higgs particles for a broad range of parameters terminates before any significant part of the inflaton zero mode energy is depleted. The reason for that is the large Higgs boson self-coupling \( \lambda \sim 0.1 \) which makes the re-scattering processes become important quite early. Unless the Higgs-inflaton coupling \( \alpha \) is fairly large the re-scatterings terminate the resonance when only a negligible part of the energy in the inflaton zero mode is depleted.\(^4\) On Fig. 2 one can see how the amount of the transferred energy depends on the value of the Higgs self coupling \( \lambda \) which we allowed to vary to small values just to demonstrate the importance of the re-scattering processes.

On Fig. 3 one can see the dependence of the total energy transferred into the Higgs field as a function of the inflaton–Higgs coupling \( \alpha \). One can draw the conclusion that parametric resonance effects only become important at \( \alpha \sim 10^{-7} \), which is too large a value. Thus, the reheating process proceeds by means of the simple decay of the inflaton (generated abundantly by parametric resonance) into the Higgs particle. This process will be analysed analytically in the next subsection, where we will advocate that this perturbative inflaton decay really reheat the Universe at lower values of the parameter \( \alpha \).

2.1. Light inflaton case (\( m_1 < 2 m_H \))

While the parametric resonance regime for the Higgs is terminated quite early, the fluctuations of the inflaton field continue to grow exponentially. Since the amount

\(^4\) For a potential without the inflaton mass term in a different part of the parameter space similar claims were made in [22].
with the Hubble expansion rate
\[ H = \frac{\dot{a}}{a} \sqrt{\frac{\rho_{\text{crit}}}{3}}. \]
For the thermal distribution of the inflaton particles this leads to the estimate
\[ T_R \approx \frac{\zeta(3)\alpha^2}{\pi^4} \sqrt{\frac{90}{y_4^2 m_{\text{Pl}}}}, \quad (8) \]
However, the distribution of the inflaton excitations may be generally, rather far from thermal equilibrium [21, 22]. Evolution of the occupation numbers of the inflaton modes was found to be self similar in [21, 22]
\[ n(k, \tau) = \tau^{-q} n_0(k\tau^{-p}), \quad (9) \]
where \( \tau \) is the conformal time, \( k \) is the comoving momentum, and \( p = 1/5 \) for three particle interactions and \( 1/7 \) for four particle interactions, \( q \approx 4p \). The only relevant for us property of the function \( n_0(k\tau^{-p}) \) is that the average momentum at later times is smaller than expected from the total energy density, \( p_{\text{avg}}/T \sim (m_{\text{Pl}}/T)^p \beta^{(3+p)/4} \), where \( T \sim \rho^{1/4} \) is now not a real temperature, but rather a parameter defining the energy density\(^5\) (cf. equilibration time description in [21, 22]). This enhances the \( 2I \rightarrow 2H \) cross section together with the \( I \) number density, increasing the estimate \((8)\) by a factor \((T/p_{\text{avg}})^3\). This leads to the increase of the equilibration temperature by a factor \( 10^5 \) for four particle interaction, \( p = 1/7 \), and by a factor \( 10^2 \) for three particle interaction, \( p = 1/5 \). Exact calculation of the equilibration temperature requires extensive numerical study, but, in any case, the expression \((8)\) should be considered as the lower bound, while \( 10^2 T_R \) is the upper (most conservative) bound.

Requiring that \( T_R > 150 \text{ GeV} \) we can obtain the lower bound on \( \alpha \)
\[ \alpha \geq 7.3 \times 10^{-8}, \quad (10) \]
for the thermal estimate \((8)\) and
\[ \alpha \geq 7 \times 10^{-10}, \quad (11) \]
for the most conservative estimate of non-thermal distribution of the inflaton\(^7\).

While the bound \((10)\) roughly coincides with the one at which the energy transfer to the Higgs field becomes effective enough to significantly deplete the zero mode of the inflaton (see Fig. 3) while the value given by \((11)\) is about two orders of magnitude smaller. We can, therefore, conclude that the upper bound on the inflaton mass is given by
\[ m_I \leq (0.14 \div 1.40) \left( \frac{m_H}{150 \text{ GeV}} \right) \sqrt{\frac{\beta}{1.3 \times 10^{-13}}} \text{ GeV}, \quad (12) \]
where the range corresponds to the thermal or the most conservative non-thermal estimates.

2.2. Heavy inflaton case \((m_I > 2m_H)\)

In this case the inflaton mass allows for the direct decay of the inflaton into two Higgs particles. The corresponding decay rate is given by
\[ \Gamma(I \rightarrow 2H) = \frac{1}{2} \left( \frac{\alpha^3}{2\pi^2\beta} \right) m_H = \frac{\beta}{8\pi} m_I^4. \quad (13) \]
Comparing this rate with the Hubble parameter and requiring again for the reheating temperature \( T_R > 150 \text{ GeV} \) we get
\[ m_I < 440 \left( \frac{m_H}{150 \text{ GeV}} \right)^{4/3} \left( \frac{\beta}{1.3 \times 10^{-13}} \right)^{1/3} \text{ GeV}. \quad (14) \]
Of course, in the case \( \alpha \lesssim \beta/8 \) the generation of the cosmological perturbations is different from the case of pure quartic inflation. The Higgs field becomes relatively light and the parameter space of the model is modified. In particular, isocurvature fluctuations which one would generically expect in the two-field model have to be somehow suppressed. This will put the restriction on the allowed values \((\alpha, \beta)\). The analysis of this parameter space is very involved. One can expect, for example, that the parameter \( \beta \) can differ from its value in the case of pure quartic inflation. That is one of the reasons why the parametric dependence on \( \beta \) is kept in \((14)\)\(^8\).

3. WMAP constraints and non-minimal coupling

Finally let us discuss the constraints on the model from the WMAP data \([16]\). As was already mentioned in the inflationary regime the model is indistinguishable from the pure quartic potential inflation. One should then confront the fact that the amplitude of the tensor perturbations is too large. One possible resolution of this problem is to assume that the inflaton \( \chi \) has non-minimal coupling to gravity \([17]\). We will repeat here

\(^5\) \( m_{\text{Pl}} = 2.44 \times 10^{18} \text{ GeV} \) is the reduced Planck mass.

\(^6\) After thermalization into the SM particles \( T \) transforms into the real temperature, up to the change of the number of d.o.f.

\(^7\) Strictly speaking, one should also check if there is any kinematical suppression of the process. This may lead to \( O(1) \) corrections and is, in fact, beyond the precision of present estimates.

\(^8\) Note, however, that the dependence of the bound \((14)\) on \( \beta \) is rather mild.
the estimates following closely \cite{25,26,27}. We will take the following action as an example

\[
S = \int d^4x \sqrt{-g} \left[ - \left( \frac{m^2 + \xi \chi^2}{2} \right) R + \frac{1}{2} (\partial \hat{\chi})^2 + |\partial \Phi|^2 - V(\chi, \Phi) \right],
\]

where \( m \simeq m_{\text{Pl}} \). Even if the coupling \( \xi \) is zero at a tree level one can expect that it will be generated via radiative corrections. As it will be discussed below even for small values of \( \xi \) the coupling \( \beta \) will deviate from the one, obtained from the COBE normalization in the absence of the non-minimal coupling \( \beta|_{\xi=0} \sim 1.3 \times 10^{-13} \).

The bound on the tensor-to-scalar ratio comes from the perturbations generated at \( N \simeq 62 \) e-foldings (see, e.g. \cite{15}) before the end of inflation. In that regime the Higgs part of the model is not important and can be dropped to simplify the discussion. The inflaton part of \cite{15} as it appears in Jordan frame by means of the conformal transformation can be rewritten as (hat denotes transformed quantities)

\[
S_J = \int d^4x \sqrt{-g} \left[ - \frac{m_{\text{Pl}}^2}{2} \hat{R} + \frac{1}{2} (\partial \hat{\chi})^2 - U(\hat{\chi}) \right],
\]

where

\[
\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 \simeq 1 + \frac{\xi \chi^2}{m_{\text{Pl}}^2},
\]

and the new field \( \hat{\chi} \) is defined as

\[
\frac{d \hat{\chi}}{d \chi} = \sqrt{\frac{\Omega^2 + 6 \xi \chi^2/m_{\text{Pl}}^2}{\Omega^2}}.
\]

The new potential is given by

\[
U(\hat{\chi}) = \frac{\beta}{4\Omega(\hat{\chi})^4} \hat{\chi}^4(\hat{\chi}).
\]

We assume that \( \xi \chi^2/m_{\text{Pl}}^2 \lesssim 1 \), where \( \chi_e \) is the value of the inflaton field at the end of inflation, so the contribution to the effective Plank mass vanishes after the inflationary period. In that case the dynamics of the model with the action \cite{15} after inflation is not different from that of the \nu\textit{MSM} model with the potential \cite{3}. This suggestion corresponds to \( \xi < 0.1 \), see \cite{21}. Following \cite{17,25} one can find that the first slow-roll parameter \( \epsilon \) is given by

\[
\epsilon = \frac{8m_{\text{Pl}}^4}{\chi^2(m_{\text{Pl}}^2 + \xi \chi^2(1 + 6 \xi))}.
\]

Slow-roll ends when \( \epsilon = 1 \). From that one can find that

\[
\frac{\xi \chi_e^2}{m_{\text{Pl}}^2} = \frac{1}{2(1 + 6 \xi)} \left( \sqrt{192 \xi^2 + 32 \xi + 1} - 1 \right) \approx 8 \xi + \mathcal{O}(\xi^2), \quad (\xi \ll 1).
\]

The number of e-foldings from the moment when the inflaton field has the value \( \chi_N \) till the end of inflation is given by

\[
N = \frac{1}{m_{\text{Pl}}^2} \int_{\chi_e}^{\chi_N} \frac{U}{(dU/d\chi)} \left( \frac{d \hat{\chi}}{d \chi} \right)^2 d \chi
\]

\[
= \frac{1}{8} \left[ \frac{\chi_N^2 - \chi_e^2}{m_{\text{Pl}}^2} (1 + 6 \xi) - 6 \ln \left( \frac{m_{\text{Pl}}^2 + \xi \chi_N^2}{m_{\text{Pl}}^2 + \xi \chi_e^2} \right) \right] .
\]

Since \( \xi \ll 1 \) one can find that with a good accuracy \( \chi_N \approx 2^{(N+1)/(1 + 6 \xi)} m_{\text{Pl}} \). The tensor-to-scalar ratio is given by \cite{16}
\[ r \equiv 16\epsilon = \frac{128m_{\chi N}^3}{\chi_N^4(m_{\chi N}^2 + \xi \chi_N^2(1 + 6\xi))} \approx \frac{16(1 + 6\xi)}{(N + 1)(8\xi(N + 1) + 1)}. \] (23)

One can see \([17]\) that roughly in the interval \(\xi = 0.001 \div 0.1\) this ratio satisfies the WMAP constraints. The value of the inflaton self-coupling as a function of \(\xi\) can be found from the COBE normalization \(U(\chi N)/\epsilon(\chi N) = (0.027m_{\text{Pl}})^4\). The corresponding behavior is shown in Fig.4 This introduces slight growth of \(\beta\) with \(\xi\), and thus increases all bounds simultaneously, which is demonstrated in Fig.5.

4. Conclusions

In Fig. 5 we combined the bounds on the inflaton mass we have found so far. We can conclude, therefore, that the mass of the inflaton in the \(\nu\)MSM inflation \([6]\) should be roughly in the range
\[ 0.1 \text{ GeV} \lesssim m_I \lesssim 10 \text{ GeV} \] (24)
in the case when it is light and in the range
\[ 300 \text{ GeV} \lesssim m_I \lesssim 1000 \text{ GeV} \] (25)
in the case when the inflaton-Higgs coupling is very small.

These bounds could be evaded in models with arbitrary scalar field potentials, but the fact of the strong lower bound from reheating on the coupling between the inflaton and the Higgs should remain rather universal.

Values of \(\xi\) larger then 0.1 (and, therefore larger lower and upper bounds on the inflaton mass) are also allowed as well. However, since the dynamics of the model at preheating may be strongly modified from the one we have studied in this Letter it is hard for us to make any statements in that case, and we leave this for future analysis.

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