$g_{\rho\sigma\gamma}$ coupling constant in light cone QCD

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Abstract

The coupling constant $g_{\rho\sigma\gamma}$ is determined from light cone QCD sum rules. A comparison of our result with the ones existing in literature is presented.

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1 Introduction

The quark model describes hadrons successfully. In its simplest version mesons are interpreted as pure $\bar qq$ states. Scalar mesons might constitute an exception to this successful scheme and indeed their nature is not well established yet [1]. At present there goes great deal of investigation about the identity and interpretation of the lowest lying scalar resonances. One remarkable feature of these particles is that they appear to be rather wide [2]–[5]. Other peculiar property of these particles is the observation that they have very short lifetimes and large couplings to the hadronic channels such as $K\bar K$ or $\pi\pi$. It should be noted here that the mass of the scalar mesons varies in the region $(400−1200)\text{ MeV}$ [6].

Photoproduction of the $\rho^0$ meson with proton near threshold can successfully be described by a simple one meson exchange, more precisely, by exchange of a scalar $\sigma$ meson. An analysis of experimental data requires to know the $g_{\rho\sigma\gamma}$ coupling constant. This coupling constant is calculated in framework of the effective Hamiltonian approach and 3–point sum rules in [7, 8], which predict $g_{\rho\sigma\gamma} = 2.71$ and $g_{\rho\sigma\gamma} = 3.2 \pm 0.6$ at $m_{\sigma} = 0.5 \text{ GeV}$, respectively. Since this coupling constant is one of the important quantities in analyzing the hadronic processes involving $\sigma$ meson, its calculation in framework of the other approaches is needed in order to get a reliable conclusion.

In the present note we study the coupling constant $g_{\rho\sigma\gamma}$ in framework of the light cone QCD sum rules (more about QCD sum rules and its applications can be found in [9, 10]). In further analysis we assume that the content of $\sigma$ meson is a pure $\bar qq$ state (although this point is still under debate, see for example [11, 12]).

The coupling of the $\sigma$ meson to the scalar current $J^\sigma = \frac{1}{2}(\bar uu + \bar dd)$ can be parametrized in terms of a constant $\lambda_\sigma$:

$$\langle 0 | J^\sigma | \sigma \rangle = \lambda^\sigma .$$

(1)

In order to determine the coupling constant $g_{\rho\sigma\gamma}$ in framework of the QCD sum rules, we consider the following two point correlator function

$$\Pi_\mu = i \int d^4x e^{ipx} \left\langle 0 \left| T\left\{ J^\sigma(x) J^\rho(0) \right\} \right| 0 \right\rangle_\gamma ,$$

(2)

where $\gamma$ means external electromagnetic field, and $J^\rho$ and $J^\sigma$ are the interpolating currents with $\rho$ and $\sigma$ meson numbers. The physical part of the sum rules can be obtained by inserting a complete set of one meson states into the correlator:

$$\Pi_\mu = \sum \frac{\langle 0 | J^\sigma(x) | \sigma(p_2) \rangle}{p_2^2 - m_\sigma^2} \frac{\langle \sigma(p_2) | \rho(p_1) \rangle}{p_1^2 - m_\rho^2} \frac{\langle \rho(p_1) | J^\rho(0) | 0 \rangle}{p_1^2 - m_\rho^2} ,$$

(3)

where $p_2 = p_1 + q$ and $q$ is the photon momentum. The matrix element $\langle \rho(p_1) | J^\rho(0) | 0 \rangle$ in Eq. (3) is defined as

$$\langle 0 \left| J^\rho_\mu \right| \rho \rangle = m_\rho f_\rho \varepsilon^\rho_\mu ,$$

(4)

where $\varepsilon^\rho$ is the $\rho$ meson polarization vector. In general, the $\langle \sigma | \rho \rangle_\gamma$ matrix can be parametrized as

$$\langle \sigma(p_2) | \rho(p_1) \rangle_\gamma = e \left\{ F_1(q^2)(p_1 q)\varepsilon^\rho_\mu + F_2(q^2)\varepsilon^\rho q_\mu \right\} \varepsilon^\mu ,$$

where $e$ is the electromagnetic charge.
where \( \varepsilon \) is the photon polarization vector. In this parametrization of the matrix element \( \langle \sigma | \rho \rangle_\gamma \), we neglect the terms \( \sim q^\mu \) since \( q^\mu \varepsilon_\mu = 0 \). From gauge invariance we have

\[
q^\mu \left\{ F_1(p_1 q) \varepsilon_\mu + F_2(\varepsilon^\rho q)p_1^\rho \right\} = 0 .
\]  

Since photon is real in our case, we immediately get from Eq. (5)

\[
F_2(0) = -F_1(0) .
\]

So, the matrix element \( \langle \sigma | \rho \rangle_\gamma \) takes the following form:

\[
\langle \sigma | \rho \rangle_\gamma = e F(0) \left\{ (p_1 q) \varepsilon_\rho - (q \varepsilon^\rho)p_1^\rho \right\} \varepsilon^\mu .
\]  

We can use an alternative parametrization for the \( \rho \sigma \gamma \) vertex, i.e.,

\[
\mathcal{L}_{\text{int}} = \frac{e}{m_\rho} g_{\rho \sigma \gamma} \partial^\alpha \rho_{\mu \nu} (\partial_\alpha A_\beta - \partial_\beta A_\alpha) .
\]  

When we compare Eqs. (6) and (7) we see that

\[
F(0) = \frac{g_{\rho \sigma \gamma}}{m_\rho} .
\]

Using Eqs. (1), (3), (4) and (6), for the phenomenological part of the sum rules we get

\[
\Pi^\text{phen}_{\mu} = g_{\rho \sigma \gamma} \frac{\chi_{e}}{p_2^2 - m_\gamma^2} \frac{f_{\rho \varepsilon^\nu}}{p_1^2 - m_\rho^2} \left\{ (p_1 q) g_{\mu \nu} + p_1^\rho q_\nu \right\} .
\]  

Our next problem is the calculation of correlator function from QCD side. First of all we note that the structure of the correlator is such that perturbation contribution to it is absent due to the odd number of \( \gamma \) matrix. So, QCD part of the correlator contains only nonperturbative contribution. Calculations lead to the following result for the theoretical part of the correlator:

\[
\Pi^\text{theor}_{\mu} = \frac{4 \pi^2 \lambda_{e}}{4 \pi^2} (e_u - e_d) \left[ (q x) \varepsilon_\mu - (\varepsilon x) q_\mu \right] \frac{\chi_{\phi} + g_1 x^2}{x^4} ,
\]  

where \( \phi \) and \( g_1 \) twist--2 and twist--4 photon wave functions, respectively, defined as [13]–[15]:

\[
\langle \gamma(q)|\bar{q} \sigma_{\alpha \beta} q|0 \rangle = i e_q (\bar{q} q) \int_0^1 du e^{i u q x} \left\{ (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \left[ \chi_{\phi}(u) + x^2 (g_1(u) - g_2(u)) \right] \right.
\]

\[
+ \left. \left[ q x (\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x (x_\alpha q_\beta - x_\beta q_\alpha) \right] g_2(u) \right\} ,
\]  

where \( \chi \) is the magnetic susceptibility of the quark condensate, \( e_q \) is the quark charge.

The sum rules is obtained by equating the phenomenological and theoretical parts (in Eq. (10) it is necessary to perform Fourier transformation first) of the correlator. Performing double Borel transformations on the variables \( p_2^2 = p^2 \) and \( p_1^2 = (p + q)^2 \) on both sides of
the function used to subtract continuum, we get the following sum rules for the \( g_{\rho\sigma} \) coupling constant:

\[
g_{\rho\sigma} e^{-\frac{m_\rho^2}{M_1^2} + \frac{m_\sigma^2}{M_2^2}} = \frac{(e_u - e_d)\langle \bar{q}q \rangle}{f_\rho \lambda_\sigma} \left[ \chi\varphi(u_0) M^2 E_0(s_0/M^2) - 4g_1(u_0) \right], \tag{12}
\]

where

\[
E_0(x) = 1 - e^{-x},
\]

is the function used to subtract continuum, \( s_0 \) is the continuum threshold and

\[
u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2},
\]

where \( M_1^2 \) and \( M_2^2 \) are the Borel parameters in \( \rho \) and \( \sigma \) channels. Since we assume that \( m_\sigma \approx 700 \, \text{MeV} \), which is close to the \( \rho \) meson mass, we will set \( M_1^2 = M_2^2 = 2M^2 \) and hence \( u_0 = 1/2 \).

It follows from Eq. (11) that in order to determine the coupling constant given in Eq. (12), one needs to know the parameters \( f_\rho \) and \( \lambda_\sigma \). Leptonic decay constant of the \( \rho \) meson is known to have the value \( f_\rho = 0.18 \, \text{GeV} \) from experimental result of the \( \rho \to e^+e^- \) decay width [19]. The residue \( \lambda_\sigma \) is determined from sum rules obtained in [12]

\[
\lambda_\sigma^2 e^{-m_\sigma^2/M^2} = \frac{3M^4}{16\pi^2} \left\{ 1 - \left( 1 + \frac{s_0}{M^2} \right) e^{-s_0/M^2} \right\} \left[ 1 + \left( \frac{\alpha_s}{\pi} \right) \frac{17}{3} + \left( \frac{\alpha_s}{\pi} \right)^2 \cdot 31.864 \right] \\
- \left( \frac{\alpha_s}{\pi} \right) \left[ 2 + \frac{95}{3} \left( \frac{\alpha_s}{\pi} \right) \right] \int_0^{s_0/M^2} x \ln(x) e^{-x} dx + \left( \frac{\alpha_s}{\pi} \right)^2 \frac{17}{4} \int_0^{s_0/M^2} x [\ln(x)]^2 e^{-x} dx \\
+ \frac{3}{2} \langle m_\rho \bar{q}q \rangle + \frac{1}{16\pi} \langle \alpha_s G^2 \rangle - \frac{88\pi}{27} \langle \alpha_s \langle \bar{q}q \rangle \rangle^2 \\
- \frac{3\rho^2 M^6}{16\pi^2} e^{-\rho^2 M^2/2} \left[ K_0 \left( \frac{\tau^2 M^2}{2} \right) + K_1 \left( \frac{\tau^2 M^2}{2} \right) \right], \tag{13}
\]

where the last term describes the single instanton contribution, \( K_i \) are the modified Bessel functions and \( \tau \) is the instanton size for which we will use \( \tau = (0.6 \, \text{GeV})^{-1} \) (see [20]).

In further numerical analysis we use the following values for the input parameters:

\[
\langle m_\rho \bar{q}q \rangle = (-0.82 \pm 0.1) \times 10^{-4} \, \text{GeV}^4, \quad \langle \alpha_s G^2 \rangle = (0.038 \pm 0.011) \, \text{GeV}^4, \quad \langle \alpha_s \langle \bar{q}q \rangle \rangle^2 = (-0.18 \pm 0.1) \times 10^{-3} \, \text{GeV}^4 \text{ (see for example [10]).}
\]

For the mass of the scalar meson we use \( m_\sigma \approx 700 \, \text{MeV} \). The dependence of \( \lambda_\sigma \) on Borel parameter \( M^2 \) at three values of the continuum threshold \( s_0 = (1.4; \ 1.6; \text{ and } 1.8) \, \text{GeV}^2 \) is presented in Fig. (1). Since the Borel parameter \( M^2 \) has no physical meaning, we must look for the so-called stability region where sum rules is practically independent of \( M^2 \). We observe from Fig. (1) that the stability window \( M^2 \) lies in the interval \( 1.2 \, \text{GeV}^2 < M^2 < 1.4 \, \text{GeV}^2 \) and we get

\[
\lambda_\sigma = (0.2 \pm 0.02) \, \text{GeV}^2, \tag{14}
\]
where the error can be attributed to the variation of the continuum threshold $s_0$, Borel parameter $M^2$ and the errors in the condensates $\langle \bar{q}q \rangle$ and $\langle G^2 \rangle$. It should be noted that if we had used the values of the input parameters given in [8] we get

$$\lambda_\sigma = (0.16 \pm 0.02) \text{ GeV}^2.$$ 

This difference arises from the omitted instanton contribution and terms in perturbative contribution in [8].

Having the values of $\lambda_\sigma$ and $f_\rho$, our next goal is calculating the coupling constant $g_{\rho\sigma\gamma}$ from Eq. (12). As is obvious from Eq. (12) the main input parameters of the sum rule is photon wave functions. We shall make use of the following expressions for the photon wave functions [13, 15]

$$\varphi(u) = 6u(1-u),$$
$$g_1(u) = -\frac{1}{8}(1-u)(3-u),$$

and for the magnetic susceptibility we use $\chi = -4.4 \text{ GeV}^{-2}$ [21].

In Fig. (2) we present the dependence of the coupling constant $g_{\rho\sigma\gamma}$ on the Borel parameter $M^2$ at three different values of the continuum threshold: $s_0 = 1.4 \text{ GeV}^2; 1.6 \text{ GeV}^2; 1.8 \text{ GeV}^2$. It follows from this figure that for the choices $s_0 = 1.4 \text{ GeV}^2$ and $s_0 = 1.8 \text{ GeV}^2$ the variation in the result is about $\sim 10\%$, i.e., the coupling constant can be said to be practically independent of the continuum threshold $s_0$. Furthermore the coupling constant seems to be insensitive to the variation of the Borel parameter $M^2$. Along these lines, we calculated the coupling constant $g_{\rho\sigma\gamma}$ and our final result is value

$$g_{\rho\sigma\gamma} = (2.2 \pm 0.4),$$

where all possible sources of uncertainties are taken into account, namely, errors coming from determination of $\lambda_\sigma$, from variation of the continuum threshold $s_0$, Borel parameters $M^2$, neglected twist–3 photon wave functions and errors in the values of the condensates.

Finally, we would like to present a comparison of our prediction on the coupling constant $g_{\rho\sigma\gamma}$ with the existing results in literature. Traditional 3–point QCD sum rules analysis predicts $g_{\rho\sigma\gamma} = (3.2 \pm 0.6)$ [8]. Our result is about 50% lower than this result, which can be attributed to the difference in the values of $\lambda_\sigma$. More over our result is slightly lower than the result obtained from effective Lagrangian approach, which predicts $g_{\rho\sigma\gamma} = 2.71$ [7].
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Figure captions

**Fig. (1)** The dependence of $\lambda_\sigma$ on Borel parameter $M^2$ at three different values of the continuum threshold $s_0 = 1.4 \, GeV^2; 1.6 \, GeV^2$ and $s_0 = 1.8 \, GeV^2$.

**Fig. (2)** The dependence of the $g_{\rho \sigma \gamma}$ coupling constant on Borel parameter $M^2$ at three different values of the continuum threshold $s_0 = 1.4 \, GeV^2; 1.6 \, GeV^2$ and $s_0 = 1.8 \, GeV^2$. 
