The chameleonic behaviour of the String theory dilaton is suggested. Some of the possible consequences of the chameleonic string dilaton are analyzed in detail. In particular, (1) we suggest a new stringy solution to the cosmological constant problem and (2) we point out the non-equivalence of different conformal frames at the quantum level. In order to obtain these results, we start taking into account the (strong coupling) string loop expansion in the string frame (S-frame), therefore the so-called form factors are present in the effective action. The correct Dark Energy scale is recovered in the Einstein frame (E-frame) without unnatural fine-tunings and this result is robust against all quantum corrections, granted that we assume a proper structure of the S-frame form factors in the strong coupling regime. At this stage, the possibility still exists that a certain amount of fine-tuning may be required to satisfy some phenomenological constraints. Moreover in the E-frame, in our proposal, all the interactions are switched off on cosmological length scales (i.e. the theory is IR-free), while higher derivative gravitational terms might be present locally (on short distances) and it remains to be seen whether these facts clash with phenomenology. A detailed phenomenological analysis is definitely necessary to clarify these points.

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To the memory of my father Paolo,
with love and gratitude.
## Contents

1. Introduction
2. Scalar-Tensor theories of gravitation
   2.A Jordan-Brans-Dicke models
   2.B Conformal transformation
      2.B.1 Scale transformation (Dilatation)
      2.B.2 Conformal transformation (Weyl rescaling)
3. The model
4. The scale invariant Lagrangian
   4.A Classical level
      4.A.1 Jordan frame
      4.A.2 Einstein frame
   4.B Quantum level
      4.B.1 Anomaly and dilaton coupling
5. A stringy solution to the cosmological constant problem
   5.A The origin of the dual mass
   5.B The nature of the backreaction
      5.B.1 The metric backreaction
      5.B.2 The dilatonic backreaction
   5.C Symmetry restoration
      5.C.1 The dilaton
      5.C.2 Matter particles (spin-0, spin-1/2) and Rarita-Schwinger spin-3/2 fields
      5.C.3 Gauge particles: spin-1
      5.C.4 The graviton
      5.C.5 The chameleonic effective actions
   5.D The correct Dark Energy scale
6. Discussion: the cosmological constant and non-equivalent frames
7. Conclusions
1. Introduction

Our Universe is currently in a state of accelerated expansion \[1–4\]. Many possibilities have been discussed in the literature to account for this acceleration (for a review see for example \[5\]). Among them, we can mention: 1) a modification of General Relativity at large distances \[6\]; 2) a backreaction of properly smoothed-out inhomogeneities \[7–9\]; 3) a negative-pressure Dark Energy (DE) fluid. As far as the last case is concerned, the nature of the DE is far from clear and its contribution to the cosmic energy budget is comparable, at present, with that of the dark matter component (DM), although their dynamical evolution during the cosmological history can be totally different ("coincidence problem"). DE could be a cosmological constant (for a review see \[10\]) or it could have a dynamic behaviour. Let us focus our attention on this last possibility. On the one hand, the standard scenario for dynamical DE is given by a scalar field which is rolling over a (almost flat) potential (i.e. the scalar field is ultralight). On the other hand, there are reasons to maintain a non-trivial coupling between the scalar field and matter, for instance: a) to solve, at least partially, the coincidence problem, a direct interaction between DM and DE has been discussed \[11–18\]; b) string theory suggests the presence of scalar fields (dilaton and moduli) that can be coupled to matter with a strength comparable to (or even higher than) the gravitational strength. Consequently, we would like to allow a direct interaction between matter and an ultralight scalar field. However this could be phenomenologically dangerous (violations of the equivalence principle, time dependence of couplings - for reviews see \[19, 20\]).

One possible way-out is to consider "chameleon scalar fields" \[21,22\], namely scalar fields coupled to matter (including the baryonic one) with gravitational (or even higher) strength and with a mass dependent on the density of the environment. On cosmological distances, where the densities are very tiny, the fields are ultralight and they can roll on cosmological time scales. On the Earth instead, the density is much higher and the field acquires a large enough mass (i.e. stabilization). In other words, the physical properties of this field, including its value, vary with the environment, thus the name chameleon.

Recently, Y. Fujii suggested a connection between the cosmological constant problem and a surprising globally massless but locally massive dilaton (i.e. a dual mass) \[23\] in the framework of Scalar-Tensor theories of gravitation (for a review see \[24\]). In the scenario proposed by Fujii, the dilaton \(\sigma\), the (pseudo)-Nambu-Goldstone boson of broken scale-invariance, is split into a background part \(\sigma_b\) and a fluctuating part \(\sigma_f\). Surprisingly, \(\sigma_f\) can acquire a large mass in the process of quantization, while \(\sigma_b\) can remain (almost) massless. Moreover, the effects of \(\sigma_f\) on \(\sigma_b\) (dilatonic backreaction) play a major role: the contribution to the vacuum energy coming from quantum diagrams with dilatons in the external legs is (naturally) too large and the difference with the cosmological constant scale is, in the proposal of Fujii, the observed effect of the dilatonic backreaction. In other words, Fujii suggested to exploit the dilatonic backreaction as counterterm in the renormalization of the dilatonic vacuum energy. However, several points are unsatisfactory about this approach:

- the theoretical origin of the model is not discussed.

- A globally massless but locally massive dilaton can be accepted and it is not forbidden by the renormalization program. It would be rewarding to show that (1) this result must be obtained and (2) it admits a simple description.

- It would be rewarding to connect the dilatonic backreaction mentioned by Fujii with the cosmological (i.e. metric) backreaction.
• A detailed mechanism to suppress the contribution to the vacuum energy coming from quantum diagrams with dilatons in the external legs is missing. In other words, in the proposal of Fujii we know that the dilatonic backreaction exists, but we do not know whether it is effective as counterterm for the dilatonic vacuum energy. Moreover, Fujii carried on his calculations only for the diagrams with one and two external dilatons: the remaining diagrams (present of course in infinite number) have not been calculated.

• The contribution to the vacuum energy coming from matter fields, gauge fields and gravitons is not discussed.

• The origin of the correct DE scale is unknown;

In our analysis, we will extend the Fujii’s model and we will overcome all its drawbacks:
1) we point out the stringy origin of our model. In particular, we start considering the S-frame string action in the strong coupling regime with a constant dilaton and, after a conformal transformation to the E-frame, we will obtain a string dilaton \( \sigma \) running (on cosmological distances) towards the region of weak coupling.

2) We suggest that a conceivable interpretation of the dual mass of the dilaton is simply a chameleonic behaviour of the field. This will be our final result in the effective potential of the theory in the E-frame and it provides a simple description of the dual mass. Remarkably, this result must be obtained in our approach.

3) We establish a connection between the dilatonic and the metric backreactions.

4) We show that scale-invariance is almost restored on cosmological distances (in the Einstein frame). In this way on cosmological distances we protect, on the one hand, the mass of the dilaton, on the other hand, the cosmological constant. Our argument is valid at all orders in perturbation theory. Naturally this symmetry principle guarantees that the dilatonic backreaction suppress the dilatonic vacuum energy (i.e. the renormalization of the dilatonic vacuum energy must be successful). Moreover, we suggest in this article to exploit the metric backreaction as a counterterm in the renormalization of the gravitational vacuum energy. Once again, our symmetry principle guarantees that the metric backreaction must suppress the gravitational vacuum energy. As far as matter and gauge fields are concerned, we will show that their contribution to the cosmological constant is properly suppressed.

5) The correct DE scale is recovered in the E-frame without unnatural fine-tunings, granted that we assume a proper structure of the string loop corrections in the S-frame in the strong coupling regime. However, at this stage, the possibility still exists that a certain amount of fine-tuning may be required to satisfy some phenomenological constraints. Moreover, the model in the E-frame is IR-free, while higher derivative gravitational terms might be present locally (on short distances) and it remains to be seen whether these facts clash with phenomenology. A detailed phenomenological analysis is definitely necessary to clarify these points.

It must be stressed that solving these problems in the framework of String Theory led us to a totally new solution to the cosmological constant problem: in our proposal the dilaton in the Einstein frame is parametrizing the amount of (scale) symmetry of the problem. Therefore, the chameleonic behaviour of the field implies that Particle Physics is the standard one only locally. All the usual contributions to the vacuum energy (from supersymmetry [SUSY] breaking, from axions, from electroweak symmetry breaking...) are extremely large with respect to the meV-scale only locally, while on cosmological distances they are suppressed.

The chameleonic behaviour of the string dilaton is relevant, not only because it is a new
stringy way to deal with crucial problems like dilaton stabilization and cosmological constant problem, but also for experimental reasons. Indeed, in the present period a very important source of comparison between high energy physics theories and experimental data comes from the Large Hadron Collider. However, we think that it is very important to dedicate attention also to experiments which can test physics beyond the SM in alternative ways with respect to the "main road" given by accelerator physics. The present paper reflects this approach: there is the possibility of searching for chameleon fields in current experiments, most notably in the optical set-up of GammeV [27,28]. Consequently, the chameleonic behaviour of the string dilaton is relevant to establish a connection between String Theory and experiments.

One last remark is in order. Typically different conformal frames are equivalent at the classical level and this result is well-established in the literature (see for example [29]). In our analysis we will point out that this equivalence is lost at the quantum level and we will select the E-frame as the physical one. For further details on the (non)-equivalence of different conformal frames the reader is referred to [30–32] and references therein.

About the organization of this paper, in section 2. we describe Scalar-Tensor theories of gravitation; our model will be briefly summarized in section 3.; the scale-invariant part of our lagrangian will be further discussed in section 4.; we will describe our solution to the cosmological constant problem in section 5.; we will touch upon the non-equivalence of different conformal frames at the quantum level in section 6. In the final section we will draw some concluding remarks.

2. Scalar-Tensor theories of gravitation

In this section we will briefly review some aspects of Scalar-Tensor (ST) theories of gravitation following [24,33].

2.A Jordan-Brans-Dicke models

The Lagrangian of the original ST model by Jordan-Brans-Dicke (JBD) can be written in the form:

\[
L_{\text{JBD}} = \sqrt{-g} \left( \frac{1}{2} \xi \phi^2 R - \frac{1}{2} \eta g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_{\text{matter}} \right).
\]

(2.1)

\(\xi\) is a dimensionless constant and \(\epsilon = \pm 1\) (in particular \(\epsilon = +1\) corresponds to a normal field having a positive energy, in other words, not to a ghost). The convention on the Minkowskian metric is \((-++,+++)\). The first term on the right-hand side is called "nonminimal coupling term" (NM), it is unique to the ST theory and it replaces the Einstein-Hilbert term (EH) in the standard theory:

\[
L_{\text{EH}} = \sqrt{-g} \frac{1}{16\pi G} R.
\]

(2.2)

If we compare this last formula with the NM-term, we infer that in this theory the gravitational constant \(G\) is replaced by an "effective gravitational constant" defined by

\[
\frac{1}{8\pi G_{\text{eff}}} = \xi \phi^2,
\]

(2.3)

\footnote{For a review of possible extensions of the Standard Model - SM - of particle physics see [25]. For an overview of the testable new physics at the LHC see [26].}
which is spacetime-dependent through the scalar field \( \phi(x) \).

We stress that Jordan admitted the scalar field to be included in the matter Lagrangian \( L_{\text{matter}} \), whereas Brans and Dicke (BD) assumed not. For this reason the name “BD model” seems appropriate to the assumed absence of \( \phi \) in \( L_{\text{matter}} \).

In this paper we will be particularly interested in string theory, supposed to be one of the most promising theoretical models of unification, where a scalar field, often called dilaton (a spinless partner of the tensor metric field in higher dimensional spacetime) appears with the same coupling as had been shown by the ST theory. Twenty years after the pioneering works by Jordan, Fierz, Brans and Dicke we re-discover ST theory with a “top-down” approach suggested by string theory.

2.B Conformal transformation

2.B.1 Scale transformation (Dilatation)

Let us start with a global scale transformation in curved spacetime, namely:

\[
g_{\mu\nu} \rightarrow g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \text{or} \quad g_{\mu\nu} = \Omega^{-2} g^*_{\mu\nu},
\]

(2.4)

where \( \Omega \) is a constant, from which follows also

\[
g^{\mu\nu} = \Omega^2 g^{*\mu\nu}, \quad \text{and} \quad \sqrt{-g} = \Omega^{-4} \sqrt{-g^*}.
\]

(2.5)

If we have only massless fields or particles, we have no way to provide a fixed length scale, we then have a scale invariance or dilatation symmetry. Had we considered a fundamental field or particle having a nonzero mass \( m \), the inverse \( m^{-1} \) would have provided a fixed length or time standard and the above-mentioned invariance would have been consequently broken.

To implement this idea, let us introduce a real free massive scalar field \( \Phi \) (not to be confused with the dilaton), as a representative of matter fields:

\[
L_{\text{matter}} = \sqrt{-g} \left( -\frac{1}{2}(\partial \Phi)^2 - \frac{1}{2}m^2 \Phi^2 \right), \quad (\partial \Phi)^2 = g^{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi).
\]

(2.6)

We then find

\[
L_{\text{matter}} = \Omega^{-4} \sqrt{-g^*} \left( -\frac{1}{2} \Omega^2 (\partial \Phi)^2 - \frac{1}{2}m^2 \Phi^2 \right)
\]

\[
= \sqrt{-g^*} \left( -\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}m^2 \Phi^2 \right),
\]

(2.7)

with \( \Phi_* = \Omega^{-1} \Phi \).

Notice that we defined \( \Phi_* \) primarily to leave the kinetic term form invariant except for putting the * symbol everywhere. On the other hand, the mass term in the last equation breaks scale invariance.

2.B.2 Conformal transformation (Weyl rescaling)

The global scale transformation in curved spacetime as discussed above may be promoted to a local transformation by replacing the constant parameter \( \Omega \) by a local function \( \Omega(x) \), an arbitrary function of \( x \). This defines a conformal transformation, or sometimes called Weyl rescaling:

\[
g_{\mu\nu} \rightarrow g^*_{\mu\nu} = \Omega^2(x) g_{\mu\nu}, \quad \text{or} \quad ds^2 \rightarrow ds^2_* = \Omega^2(x) ds^2.
\]

(2.8)
According to the last equation, we are considering a local change of units, not a coordinate transformation. The condition for invariance is somewhat more complicated than the global predecessors.

Let us see how the ST theory is affected by the conformal transformation. We start with

$$\partial_\mu g_{\nu\lambda} = \partial_\mu (\Omega^{-2} g_{\nu\lambda}) = \Omega^{-2} \partial_\mu (\Omega^2 g_{\nu\lambda}) = \Omega^{-2} (\partial_\mu g_{\nu\lambda} - 2 f\mu g_{\nu\lambda}),$$

(2.9)

where $f = \ln \Omega, f_\mu = \partial_\mu f, f^{\mu\nu} = g^{\mu\nu} f_\nu$. We then compute

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\rho} (\partial_\nu g_{\rho\lambda} + \partial_\lambda g_{\rho\nu} - \partial_\rho g_{\nu\lambda}) = \Gamma^\mu_{\nu\lambda} - (f_\nu \delta^\mu_\lambda + f^\mu_\lambda \delta^\nu_\mu - f^\mu_{\nu\lambda}),$$

(2.10)

reaching finally

$$R = \Omega^2 (R_* + 6 \Box_* f - 6 g^{\mu\nu} f_\nu f_\mu).$$

(2.11)

Using this in the first term on the right-hand side of (2.1) with $F(\phi) = \xi \phi^2$, we obtain

$$L_1 = \sqrt{-g} \frac{1}{2} F(\phi) R = \sqrt{-g} \frac{1}{2} F(\phi) \Omega^{-2} (R_* + 6 \Box_* f - 6 g^{\mu\nu} f_\nu f_\mu).$$

(2.12)

We may choose

$$F \Omega^{-2} = 1,$$

(2.13)

so that the first term on the right-hand side goes to the standard EH term. We say that we have moved to the Einstein conformal frame (E frame). We have

$$\Omega = F^{1/2}, \text{ then } f = \ln \Omega, \quad f_\mu = \partial_\mu f = \frac{\partial_\mu \Omega}{\Omega} = \frac{1}{2} \frac{\partial_\mu F}{F} = \frac{1}{2} F' \partial_\mu \phi,$$

(2.14)

where $F' \equiv dF/d\phi$. The second term on the right-hand side of (2.12) then goes away by partial integration, while the third term becomes $-\sqrt{-g} (3/4)(F'/F)^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. This term is added to the second term on the right-hand side of (2.1) giving the kinetic term of $\phi$:

$$-\frac{1}{2} \sqrt{-g} \Delta g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \quad \text{with } \Delta = \frac{3}{2} \left( \frac{F'}{F} \right)^2 + \epsilon \frac{1}{F}.$$

(2.15)

If $\Delta > 0$, we define a new field $\sigma$ by

$$\frac{d\sigma}{d\phi} = \sqrt{\Delta}, \quad \text{hence } \sqrt{\Delta} \partial_\mu \phi = \frac{d\sigma}{d\phi} \partial_\mu \phi = \partial_\mu \sigma,$$

(2.16)

thus bringing (2.15) to a canonical form $-\left( 1/2 \right) \sqrt{-g} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma$. If $\Delta < 0$, the opposite sign in the first expression of (2.15) propagates to the sign of the preceding expression, implying a ghost.

By using the explicit expression of $F(\phi)$ we find

$$\Delta = (6 + \epsilon \xi^{-1}) \phi^{-2} \equiv \zeta^{-2} \phi^{-2},$$

(2.17)

which translates the condition $\Delta > 0$ into $\zeta^2 > 0$. We further obtain

$$\frac{d\sigma}{d\phi} = \zeta^{-1} \phi^{-1}, \quad \text{hence } \zeta \sigma = \ln \left( \frac{\phi}{\phi_0} \right), \quad \text{or } \phi = \xi^{-1/2} e^\zeta \sigma,$$

(2.18)
reaching also
\[ \Omega = e^{\zeta \sigma} = \sqrt{\xi \phi}. \] (2.19)

We finally obtain the lagrangian in the E frame:
\[ \mathcal{L}_{JBD} = \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} g^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma + L_{\text{matter}} \right). \] (2.20)

In the next section we will describe our model.

3. The model

Our starting point is the string-frame, low-energy, gravi-dilaton effective action, to lowest order in the \( \alpha' \) expansion, but including dilaton-dependent loop (and non-perturbative) corrections, encoded in a few “form factors” \( \psi(\phi) \), \( Z(\phi) \), \( \alpha(\phi) \), ..., and in an effective dilaton potential \( V(\phi) \) (obtained from non-perturbative effects). In formulas (see for example [34] and references therein):

\[
S = -\frac{M_s^2}{2} \int d^4 x \sqrt{-g} \left[ e^{-\psi(\phi)} R + Z(\phi) (\nabla \phi)^2 + \frac{2}{M_s^2} V(\phi) \right] - \frac{1}{16\pi} \int d^4 x \frac{\sqrt{-g}}{\alpha(\phi)} F_{\mu \nu}^2 + \Gamma_m(\phi, g, \text{matter})
\] (3.21)

Here \( M_s^{-1} = \lambda_s \) is the fundamental string-length parameter and \( F_{\mu \nu} \) is the gauge field strength of some fundamental grand unified theory (GUT) group (\( \alpha(\phi) \) is the corresponding gauge coupling). We imagine having already compactified the extra dimensions and having frozen the corresponding moduli at the string scale.

Since the form factors are unknown in the strong coupling regime, we are free to assume that the structure of these functions in the strong coupling region implies an S-frame Lagrangian composed of two different parts: 1) a scale-invariant Lagrangian \( \mathcal{L}_{SI} \). This part of our lagrangian has already been discussed in the literature by Fujii in references [23][24]; 2) a Lagrangian which explicitly violates scale-invariance \( \mathcal{L}_{SB} \).

In formulas we write:

\[ \mathcal{L} = \mathcal{L}_{SI} + \mathcal{L}_{SB}, \] (3.22)

where the scale-invariant Lagrangian is given by:

\[
\mathcal{L}_{SI} = \sqrt{-g} \left( \frac{1}{2} \xi \phi^2 R - \epsilon g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{4} f \phi^2 \Phi^2 - \frac{\lambda_\Phi}{4!} \Phi^4 \right).
\] (3.23)

\( \Phi \) is a scalar field representative of matter fields, \( \epsilon = -1, \, \xi \simeq 1, \, f < 0 \) and \( \lambda_\Phi > 0 \). One may write also terms like \( \phi^3 \Phi, \phi \Phi^3 \) and \( \phi^4 \) which are multiplied by dimensionless couplings. However we will not include these terms in the lagrangian\(^\dagger\) The symmetry breaking Lagrangian \( \mathcal{L}_{SB} \) is

\(^\dagger\)The terms with odd powers of \( \Phi \) can be removed by imposing symmetries of the strong interaction, while \( \phi^4 \) is assumed to be absent through a proper choice of the form factors in the strong coupling regime.
supposed to contain scale-non-invariant terms, in particular, a stabilizing (stringy) potential for \( \phi \) in the S-frame. For this reason we write:

\[
\mathcal{L}_{SB} = -\sqrt{-g}(a\phi^2 + b + c\frac{1}{\phi^2}).
\]

(3.24)

Happily, it is possible to satisfy the field equations with constant values of the fields \( \phi \) and \( \Phi \) through a proper choice of positive (but not fine-tuned) values of the parameters \( a, b, c \), maintaining \( f < 0 \) and \( \lambda_\Phi > 0 \). We made sure that \( g_s > 1 \) can be recovered in the equilibrium configuration and that, consequently, the solution is consistent with the non-perturbative action that we considered as a starting point.

Here is a possible choice of parameters (in string units): \( f = -4 \), \( \lambda_\Phi = 27 \), \( a = \frac{1}{9} \), \( b = c = \frac{1}{72} \).

In the equilibrium configuration we have \( \phi_0 = \frac{1}{2} \) and \( \Phi_0 = \frac{1}{3} \).

As we will see, the description in the E-frame will guarantee the presence of scale-invariance on cosmological distances. This will be the crucial element in our analysis to address the cosmological constant problem. Remarkably, even if we stabilize the dilaton in the S-frame, the conformal transformation to the E-frame will be non-trivial. This point needs to be further elaborated. Let us consider a stabilizing potential \( V(\sigma) \) for the dilaton in the S-frame and let us call \( \sigma_0 \) the value of the dilaton in the minimum of the potential. When we perform the conformal transformation, the minimum of the potential \( V(\sigma_0) \) will be multiplied by the conformal factor \( e^{-4\zeta\sigma} \) (which is constant). A different point of the potential, for example \( V(\sigma_1) \), will be multiplied by a different constant conformal factor (i.e. \( e^{-4\zeta\sigma_1} \)). Consequently, the function \( V(\sigma) \) will be multiplied by a non-constant function of the dilaton, namely, a non-trivial conformal factor given by \( \xi^{-2}\phi^{-4} = e^{-4\zeta\sigma} \). Therefore, the potential \( 3.24 \) will be mapped by the conformal transformation into an E-frame potential given by

\[
V = \xi^{-2}[a\phi^{-2} + b\phi^{-4} + c\phi^{-6}] = ae^{-2\zeta\sigma} + be^{-4\zeta\sigma} + ce^{-6\zeta\sigma}.
\]

(3.25)

Remarkably, the parameters \( a, b \) and \( c \) are positive and, consequently, the potential \( 3.24 \) is run-away towards the region of weak coupling. This is a first hint that different dynamical behaviours of the dilaton in different frames are not forbidden. We warn the reader that the “dictionary” of the conformal transformation is still valid (i.e. we still write \( \phi = \xi^{-1/2}e^{\zeta\sigma} \)), but, as we will see in the following sections, in this model a stabilized dilaton in one frame (S-frame) does not correspond to a stabilized dilaton in another frame (E-frame). At first glance this result seems to clash with the approach discussed in \([35]\), therefore, we will try to overcome this problem in a future work.

4. The scale invariant Lagrangian

In this section we will analyze \( \mathcal{L}_{SI} \) following \([23, 24]\).

4.A Classical level

4.A.1 Jordan frame

\( \mathcal{L}_{SI} \) is attractive because the coupling constant \( f \) is dimensionless, hence vesting scale invariance in the gravity-matter system. By applying Noether’s procedure we obtain the dilatation current as given by

\[
J^\mu = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \partial_\nu (\xi^{-2}\phi^2 + \Phi^2),
\]

(4.26)
which is shown to be conserved by using the field equations.

We summarize the main features of the scale-invariant part of the model in the Jordan frame (J-frame) as follows:

- the model is scale invariant;
- $\Phi$ is massless;
- there is a direct coupling between matter and scalar field.

4.A.2 Einstein frame

Since our intention is to take into account quantum effects, *regularization* will be necessary and whatever will be the regularization method we will choose, a scale will be introduced in the theory leading to a breaking of scale invariance. In the following we will consider the method of continuous dimensions and, for this reason, we start writing conformal transformation equations in D-dimensions. To proceed further, the lagrangian (3.23) will be rewritten in the E frame and *spontaneous* breaking of scale invariance will be discovered.

In D=2d dimensional spacetime a scalar field has canonical dimension $(d-1)$ and the conformal transformation looks like

$$\sqrt{-g} = \Omega^{-D} \sqrt{-g_*}.$$  \hfill (4.27)

In order to recover the usual Einstein-Hilbert term we impose

$$\xi \phi^2 = \Omega^{D-2}$$  \hfill (4.28)

that implies

$$\Omega = e^{\frac{\xi \phi}{2(d-1)}},$$  \hfill (4.29)

where the relation (2.18) and $M_\rho = 1$ have been exploited.

In this way we can rewrite the lagrangian (3.23) in the E frame as

$$\mathcal{L}_* = \sqrt{-g_*} \left( \frac{1}{2} R_* - \frac{1}{2} g_*^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \mathcal{L}_{\text{matter}} \right),$$  \hfill (4.30)

where $\mathcal{L}_{\text{matter}}$ turns out to be

$$\mathcal{L}_{\text{matter}} = -\frac{1}{2} g_*^{\mu\nu} D_\mu \Phi_* D_\nu \Phi_* - e^{\frac{\xi \phi}{2d-4}} \zeta \phi (\xi \zeta^{-1} f \frac{M_P^2 \Phi_*^2}{4} + \frac{\lambda \phi}{4!} \Phi_*^4)$$  \hfill (4.31)

and $D_\mu = \partial_\mu + \zeta \partial_\mu \sigma$.

The conservation law remains true even after the conformal transformation, but the symmetry is broken *spontaneously* due to the trick by which a dimensionful constant $M_\rho(= 1)$ has been "re-installed" in (4.31). The dilatation current in the E-frame takes the form:

$$J^\mu = \frac{1}{2} \sqrt{-g_* g_*^{\mu\nu}} \{ 2 \zeta^{-1} \partial_\nu \sigma + \partial_\nu + 2 \zeta (\partial_\nu \sigma) \} \Phi_*^2,$$  \hfill (4.32)

while the generator

$$Q = \int d^3x J^0$$  \hfill (4.33)
does not annihilate the vacuum due to the presence of the term linear in $\sigma$ in (4.32). In other words, scale invariance is \textit{spontaneously} broken and we write

$$Q|0\rangle \neq 0.$$  \hspace{1cm} (4.34)

In this context $\sigma$ is a Nambu-Goldstone boson, a dilaton.

In the classical theory, at this stage, we can put $D = 4$ or $d = 2$ in (4.31) and we obtain a dilaton $\sigma$ coupled to matter only through the derivative coupling included in $D_\mu$. This means that at the classical level, at least in the static and long-wavelength limit, we have no way to observe the dilaton by measuring forces between matter objects. Moreover, if we focus our attention on the last two terms in (4.31) putting $d = 2$ will give us a Higgs-like lagrangian. For this reason we now apply the same recipe as with the Higgs sector in the Standard Model, we split $\Phi_*$ into a vacuum expectation value (VEV) and a fluctuating field in the form

$$\Phi_* = v_\Phi + \tilde{\Phi}$$ \hspace{1cm} (4.35)

and we rewrite the last two terms in (4.31) as

$$-L'_0 = L_{\text{vac}} + \frac{1}{2} m^2 \tilde{\Phi}^2 + \frac{1}{2} \sqrt{\frac{\lambda_\Phi}{3}} m \tilde{\Phi}^3 + \frac{\lambda_\Phi}{4!} \tilde{\Phi}^4,$$  \hspace{1cm} (4.36)

where $m^2 = -\frac{f}{\xi} M_p^2$ and

$$L_{\text{vac}} = -\frac{3}{8} \frac{f^2}{\xi^2 \lambda_\Phi} M_p^4 \propto v_\Phi^4$$ \hspace{1cm} (4.37)

is a vacuum energy term.

It is noteworthy to summarize the main features of $\mathcal{L}_{ST}$ in the E-frame at the classical level:

- $\Phi_*$ has a constant mass;
- no potential is present for the dilaton, which remains strictly massless, one more signal that the breaking of scale invariance is only \textit{spontaneous} at the classical level. However, we will deal with \textit{explicit} breaking of scale invariance when we will quantize the theory in the following paragraphs: in that case we will also find a non-vanishing dilatonic potential (i.e. dilatonic mass);
- At the classical level the dilaton disappears for $d=2$ and the only direct coupling matter-dilaton is included in the derivative term $D_\mu$.

4.B Quantum level

In this section a direct coupling between matter and dilaton will re-emerge as the result of a quantum anomaly. In particular we are going to touch upon these points: 1) quantum effects and conformal anomaly; 2) explicit breaking of scale invariance and Pseudo-NG-boson nature of the dilaton.
4.B.1 Anomaly and dilaton coupling

If we substitute (4.35) and the expansion
\[ e^{2\frac{d-2}{d-1}\zeta\sigma} = 1 + 2\zeta\frac{d-2}{d-1}\sigma + ... \]  
(4.38)
in formula (4.31), we obtain the following lagrangian at first order in \( \sigma \):
\[ -L_1 = 2\zeta(d-2)\sigma \left( \frac{1}{2}m^2\Phi^2 + \frac{1}{2}\sqrt{\frac{\lambda\Phi}{3}}m\Phi^3 + \frac{\lambda\Phi}{4!}\Phi^4 \right) - \frac{3}{4}\zeta(d-2)\sigma \frac{f^2}{\zeta^2\lambda\Phi} M_p^4. \]  
(4.39)

In this way several different 1-loop diagrams can be constructed (Figure 1): we will consider an explicit calculation starting from diagram (c).

![Diagram](image)

(a) (b) (c)

Figure 1. Examples of 1-loop diagrams for the interaction \( \lambda\Phi\Phi^4 \), represented, together with the derived 3-vertex, by a filled circle. Heavy dotted lines are for \( \sigma \). This figure can be found in reference [33].

The one-loop diagram is divergent. Exploiting dimensional regularization the contribution given by the scalar loop is
\[ I_c = -i\zeta\lambda\Phi(D-4)(2\pi)^{-D}\int d^Dk \frac{1}{(k^2 + m^2)}. \]  
(4.40)

where the integral in D-dimensions is evaluated explicitly as
\[ I_c = \int d^Dk \frac{1}{(k^2 + m^2)} = i\pi^2(m^2)^{d-1}\Gamma(1-d). \]  
(4.41)

Remarkably (2-d) cancels out the pole at d=2 in \( \Gamma(1-d) \) leading to a finite result in 4 dimensions:
\[ (2-d)\Gamma(1-d) = \frac{1}{1-d} (2-d)\Gamma(2-d) \]
\[ = \frac{1}{1-d} \Gamma(3-d) \overset{d=2}{\longrightarrow} -1 \]  
(4.42)

and this is one of the possible ways in which an anomaly may present itself.
Adding up the contributions from the three diagrams in Figure 1 we obtain
\[ I_{\text{tot}} = I_a + I_b + I_c = \frac{1}{\pi^2} \zeta \lambda \phi m^2 \] (4.43)
or, in other words, the fundamental (anomalous) interaction vertex between dilaton and matter [23]:
\[ \mathcal{L}_{\Phi \Phi \sigma} = -\frac{1}{2 M_p} \tilde{\Phi}^2 \sigma. \] (4.44)

There are a number of consequences of this last formula. Here we mention:

- weak equivalence principle (WEP) is violated.
- Scale invariance is broken explicitly and the dilatation current is not divergenceless anymore. In particular we can write \( \partial_{\mu} J^\mu = \zeta^{-1} \mu^2 \sigma + \ldots \).
- According to the relativistic quantum field theory, the "anomaly-induced" interaction (4.44) with the matter field \( \Phi \) leads us to the contributions depicted in Figure 2. Naturally every diagram will give a (too) large contribution to the unrenormalized vacuum energy. In the proposal of Fujii [23], the difference between the (dilatonic) unrenormalized vacuum energy and the cosmological constant scale is due to the dilatonic backreaction. In other words, he split the dilaton in two components: \( \sigma = \sigma_b(t) + \sigma_f(x) \) and the effects of \( \sigma_f \) on \( \sigma_b \) are supposed to renormalize the vacuum energy and to render it compatible with the Dark Energy scale on cosmological distances. We will further elaborate on these issues in the following sections. It seems worthwhile to point out that the set of diagrams in figure 2 must be considered as an expansion to all orders in perturbation theory, namely as a complete expansion and not as a 1-loop contribution. In other words, since the interaction vertex between dilaton and matter has been obtained from a 1-loop calculation (from the conformal anomaly), every time we add one external leg we are taking into account an additional loop in the calculation.

\[ + \ldots \]

Figure 2. First three examples of the 1-loop diagrams for the quantum corrections due to the "anomaly-induced" matter coupling. Solid and dotted lines are for \( \Phi \) and \( \sigma \) respectively. This figure can be found in reference [23].

5. A stringy solution to the cosmological constant problem

As already mentioned in the Introduction, several points are unsatisfactory about the proposal of Fujii in reference [23], namely:

- the theoretical origin of the model is not discussed.
A globally massless but locally massive dilaton can be accepted and it is not forbidden by the renormalization program. It would be rewarding to show that (1) this result must be obtained and (2) it admits a simple description.

It would be rewarding to connect the dilatonic backreaction mentioned by Fujii with the cosmological (i.e. metric) backreaction.

A detailed mechanism to suppress the contribution to the vacuum energy coming from quantum diagrams with dilatons in the external legs is missing. In other words, in the proposal of Fujii we know that the dilatonic backreaction exists, but we do not know whether it is effective as counterterm for the dilatonic vacuum energy. Moreover, Fujii carried on his calculations only for the diagrams with one and two external dilatons: the remaining diagrams (present of course in infinite number) have not been calculated.

The contribution to the vacuum energy coming from matter fields, gauge fields and gravitons is not discussed.

The origin of the correct DE scale is unknown;

In this section we will overcome all these problems. The final outcome is a totally new (and stringy) solution to the cosmological constant problem.

About the organization of this section, we will proceed stepwise solving all the problems mentioned above (naturally we have already pointed out the stringy origin of our model in section 3):

1) In subsection 5.A we suggest that a conceivable interpretation of the dual mass of the dilaton is simply a chameleonic behaviour of the field. This will be our final result at the level of the effective potential of the theory in the E-frame and it provides a simple description of the dual mass. Remarkably, this result must be obtained in our approach.
2) In subsection 5.B we establish a connection between the dilatonic and the metric backreactions.
3) In subsection 5.C we show that scale-invariance is almost restored on cosmological distances (in the Einstein frame). In this way on cosmological distances we protect, on the one hand, the mass of the dilaton, on the other hand, the cosmological constant. Our argument is valid at all orders in perturbation theory. Naturally this symmetry principle guarantees that the dilatonic backreaction must suppress the dilatonic vacuum energy (i.e. the renormalization of the dilatonic vacuum energy must be successful). Moreover, we suggest in this article to exploit the metric backreaction as counterterm in the renormalization of the gravitational vacuum energy. Once again, our symmetry principle guarantees that the metric backreaction must suppress the gravitational vacuum energy. As far as matter and gauge fields are concerned, we will show that their contribution to the cosmological constant is properly suppressed.
4) In subsection 5.D the correct DE scale is recovered in the E-frame without unnatural fine-tunings. However, at this stage, the possibility still exists that a certain amount of fine-tuning may be required to satisfy some phenomenological constraints. A detailed phenomenological analysis is definitely necessary to clarify this point.

5.A  The origin of the dual mass

We suggest that a conceivable interpretation of the dual mass of the dilaton $\sigma$ is simply a chameleonic behavior of the field. This will be our final result at the level of the effective
potential in the E-frame and it provides a simple description of the dual mass. As we will see, in our approach this result must be present.

5.B The nature of the backreaction

In the Fujii's proposal, the backreaction must be considered as effects from $\sigma_f$ on $\sigma_b$. We now connect this dilatonic backreaction with the cosmological backreaction recently discussed in the literature (see [7–9]).

5.B.1 The metric backreaction

In the proposal of [7–9] the backreaction of properly smoothed-out inhomogeneities can account for an accelerated expansion of the Universe. If our intention is to describe the local inhomogeneities, we can start considering the Einstein equations for the metric. Since the equations are non-linear, typically the averaged equation will not correspond to the equation for the averaged metric. In more detail, if we consider a metric $g_{\alpha\beta}$ and we construct the Einstein tensor $G_{\mu\nu}(g_{\alpha\beta})$, in general we have:

$$<G_{\mu\nu}(g_{\alpha\beta})> \neq G_{\mu\nu}(<g_{\alpha\beta}>). \quad (5.1)$$

This difference between the Einstein tensor obtained from the smoothed-out metric and the smoothed out matter tensor represents the effect (i.e. the backreaction) of small-scale inhomogeneities on the dynamic at the smoothed-out scale. For a review see for example [36].

5.B.2 The dilatonic backreaction

Let us come back to the dilaton. Local inhomogeneities in the matter density will introduce a length scale into the problem and scale invariance will be broken. We point out that these fluctuations of the matter density are directly connected with the fluctuations of a chameleon: the minimum of the effective potential of a chameleon depends on the density of the environment. Consequently, the fluctuations of a chameleon will define the length scale for the averaging procedure. In this way we suggest a connection between the chameleon (dilatonic) backreaction and the cosmological (i.e. metric) one of references [7–9].

Let us consider now a diagram with N external legs in figure 2. This diagram is a function $F_N(\sigma)$ of the dilaton. Expectation values of quantum operators can be rewritten as spatial integrals weighted by the integration volume (see for example [37]). Following the same procedure of eq. 5.1 whatever will be the diagram of figure 2 we consider, we can write:

$$<F_N(\sigma)> \neq F_N(<\sigma>). \quad (5.2)$$

In this last equation $<F_N(\sigma)>$ is the contribution of the diagram to the unrenormalized vacuum energy (and it is extremely large), while $F_N(<\sigma>)$ enters directly in the effective potential for the dilaton, it takes into account the dilatonic backreaction and, in the proposal of Fujii, it is not forbidden to think that it is small on cosmological distances. Happily, in the next subsections we will point out that this turns out to be the case.

5.C Symmetry restoration

We now suggest that scale invariance is almost restored on cosmological distances in the E-frame and the dilaton in the E-frame is parametrizing the amount of scale invariance of the problem.
To illustrate this point, let us mention once again that the quantum dilatonic potential of formula 3.24 becomes run-away after the conformal transformation to the E-frame, because the conformal factor introduces an exponential suppression. Naturally, the larger is the value of $\sigma$, the more effective will be the suppression. Therefore, the question is: what is the VEV of $\sigma$? The standard approach to find this VEV would be to evaluate the effective action of the model exploiting the deWitt-Vilkovisky method (for an introduction on this subject see for example [38]). However, in the absence of a detailed analysis, but inspired by chameleon theories, we parametrize the length scales of the problem through the value of $\sigma$. In this way, we can consider $\sigma_b$ and $\sigma_f$ as two distinct (but related) objects corresponding to different length scales (see also section 5.C.5 below). Accordingly, we write $\sigma_b \neq \sigma_f$. It follows that a constant term in the J-frame, which explicitly violates scale invariance on all distances, will be exponentially reduced in the E-frame, but only for large values of $\sigma$. In other words, even if our starting point is a J-frame theory which is not scale-invariant, a constant term in the J-frame will be suppressed for large $\sigma$ after the conformal transformation to the E-frame.

Let us discuss this issue for the various fields:

- The spontaneous breakdown of scale-invariance introduced by the conformal transformation to the E-frame is negligible for large $\sigma$.

As far as the matter sector is concerned, $\Phi$ has finite constant value $\Phi_J$ in the J-frame and the conformal transformation introduces the exponential rescaling of the $\Phi^*$-field.

As far as the Einstein-Hilbert term is concerned, it is not scale invariant and we will now show that it is (almost) negligible for large $\sigma$. This result is based on two elements, namely: (1) the action 3.22 in the S-frame is the result of a non-perturbative calculation and (2) quantum loops can induce a kinetic term for gravitons (i.e. induced gravity mechanism [39]), see figure 3. Accordingly, we suggest to induce a S-frame kinetic term for gravitons through quantum diagrams. Therefore, the Einstein-Hilbert term in formula 4.30 is coming from quantum diagrams. However, these diagrams are suppressed in the E-frame for large $\sigma$, because all interactions are switched off for large $\sigma$ [40] (i.e. in the weak coupling regime of String theory). Consequently, the larger is $\sigma$ in the E-frame, the smaller is the Einstein-Hilbert term. We will further elaborate on this non-perturbative induced gravity idea in the following subsections.

Figure 3. The one-loop diagram generating the Ricci scalar for the gravitons. Wave lines denote gravitons, solid lines denote massive scalars/fermions. Vertical short lines on scalar/fermion propagators indicate that they are massive. This figure can be found in reference [6].
• For large $\sigma$, conformal anomaly is harmless and all the interactions are switched off. A scale-dependence of the conformal anomaly has already been discussed in the literature. For example, in the framework of AdS/CFT correspondence [41] (for a review see for example [42,43]), an increasing central charge as a function of the energy scale has already been discussed (the reader is referred to [44] and references therein). To the best of our knowledge, this should be a rather general property of conformal anomalies in agreement with c-theorems (for a review see for example [45]). In our model, this result is obtained through the exponential rescaling mentioned above. In more detail, we can write the anomalous coupling [4,44] as:

$$\frac{1}{2} I_{\text{tot}} \tilde{\Phi}^2 \sigma = \frac{1}{2} I_{\text{tot}} e^{-2\zeta \sigma} \Phi_f^2 \sigma,$$

(5.3)

where $\Phi_f$ is the fluctuating component of $\Phi$ in the J-frame and it is constant. We infer that for large values of $\sigma$ the coupling is suppressed.

A harmless conformal anomaly for large $\sigma$ will be a crucial element in our analysis, where, as we will show below, large values of $\sigma$ will correspond to large distance scales. A word of caution is necessary: conformal anomaly is not always harmless in Cosmology. Indeed, the trace of the energy momentum tensor enters in the chameleon equations and it can influence the dynamical behaviour of the field. For relativistic degrees of freedom it is generally assumed that this trace is vanishing: not so at all. The trace receives a contribution, on the one hand, when a particle species becomes non-relativistic, on the other hand, from trace anomaly. In particular, it was shown (see for example [46]) that the effective equation of state for a plasma of an $SU(N_c)$ group with coupling $g$ and $N_f$ flavours is given by:

$$1 - 3w = \frac{5}{6\pi^2} \left( \frac{g^2}{4\pi} \right)^2 \frac{(N_c + \frac{5}{3} N_f)(\frac{11}{3} N_c - \frac{5}{3} N_f)}{2 + \frac{4}{3} N_c N_f/(N_c^2 - 1)} + \mathcal{O}(g^5).$$

(5.4)

Remarkably, if we switch off the interaction in this last formula, we recover $w = 1/3$ and trace anomaly is globally harmless. In our stringy model, since the dilaton controls the strength of the interactions, all the interactions (including gravity) will be switched off for large $\sigma$ (i.e. weak coupling regime). Indeed, string theories imply a natural unification of the couplings: gauge and gravitational couplings in heterotic string theory always automatically unify at tree-level to form the string coupling constant $g_s$ (which is determined by the dilaton). Consequently, we are considering a free string theory when $\sigma$ is large. For this reason, in the remaining part of the paper we will always neglect conformal anomaly for large $\sigma$.

We infer that for large $\sigma$ scale invariance is restored in the E-frame. Remarkably, there are a number of consequences of this restoration of scale-invariance:

**Consequence 1.** $\sigma_b$ and $\sigma_f$ acquire different mass because the degree of divergence of their loop diagrams is different.

**Consequence 2.** We extend consequence 1 to all the diagrams (present of course in infinite number) shown in figure 2.
Consequence 3. For large $\sigma$ the (almost) symmetric configuration will be compatible neither with large couplings between dilaton and matter, nor with large mass scales, including the cosmological constant and the mass of the dilaton. Therefore, the total renormalized vacuum energy is run-away because scale invariance is restored for large $\sigma$.

From these three consequences, since we know that $\sigma$ gets a mass through the interaction with matter (encoded in formula 4.44), we infer that $\sigma_b > \sigma_f$ and, therefore, scale invariance is restored on cosmological distances. In more detail, the mass of the dilaton is obtained through a competition between the run-away branch of the potential and the matter branch. Therefore, the total unrenormalized vacuum energy must be positive. Had we considered a negative (total) unrenormalized vacuum energy, there would have been no competition between the run-away branch of the potential and the matter branch and this would have clashed with consequences 1 and 2.

Naturally we have an infinite number of different contributions to the cosmological constant. Needless to say, a vacuum energy term on short distances does not contribute to the Dark Energy. Therefore, the contributions to the vacuum energy which are relevant for the cosmological constant are global and not local. This last comment requires a more detailed discussion. Expectation values of quantum operators can be rewritten as spatial integrals weighted by the integration volume (see for example [37]). On the one hand, if we consider an integration volume much smaller than the volume of the Universe, we are dealing with a local quantity and the result will be the expectation value of the field (EV). On the other hand, if we consider the spatial integral of the field weighted by the volume of the visible Universe, we are dealing with a global (cosmological) object and the result is the vacuum expectation value of the field (VEV), which is the relevant one in the evaluation of the Feynman diagrams contributing to the cosmological constant. In this way, our solution to the cosmological constant problem is intrinsically linked to a dual nature of the concept of particle. The splitting of a particle in a background (global) part and in a fluctuating (local) part, that we already discussed for the dilaton, is extended in our proposal to all the fields. Whatever will be the Feynman diagram we consider, if our intention is to evaluate its contribution to the cosmological constant (i.e. to the global renormalized vacuum energy), we must (1) construct the diagram exploiting only the background part of the particles and (2) give a VEV to the external legs (if they are present). An interesting line of development will elaborate on this dual nature of the concept of particle starting from [38, 47, 48] and references therein. We discuss separately the different particles and their contribution to the cosmological constant in the following (sub)sections. This will require the analysis of three different categories of (global) Feynman diagrams: A) the diagrams with the particle in the external legs; B) the diagrams with the particle in the internal legs; C) bubble diagrams.

5.C.1 The dilaton

The dilaton in the external legs. Since the local dilatonic vacuum energy is very different from the DE scale, when we average the local dilatonic vacuum energies on large distances, an unacceptably large contribution is expected. This is the cosmological dilatonic vacuum energy before the renormalization and it does not correspond to the DE. In other words, if we construct a diagram of figure 2 exploiting only the background part of the fields, a very large (unrenormalized) vacuum energy will be obtained if we don’t give a VEV to the external dilatons. As already pointed out by Fujii, the difference between the unrenormalized dilatonic vacuum energy and the
DE scale is due to (dilatonic) backreaction effects. The global renormalized vacuum energy is obtained by giving a VEV to the background external particles ($\sigma_b$ in this case), see also formula 5.2. We claim that this strategy must be successful, because we know that scale invariance is restored for large $\sigma$.

**The dilaton in the internal legs.** It seems noteworthy that scale invariance will forbid a dilatonic force on cosmological distances. The interaction dilaton-matter is encoded in the conformal anomaly and it is globally harmless. We infer that the global dilaton is harmless in the internal legs. One last remark is in order. If we remember that on short distances the kinetic term of $\sigma_b$ is negligible with respect to the kinetic term of $\sigma_f$, we can conclude that $\sigma_b$ does not mediate a force between macroscopic objects. Consequently, as already pointed out by Fujii, only $\sigma_f$ mediates a dilatonic force.

**The 1-loop bubble diagram with a dilaton.** The careful reader may be worried by the presence of bubble diagrams (i.e. Feynman diagrams without external legs). In particular, we can construct a 1-loop bubble diagram with the dilaton field. Once again, the diagram with $\sigma_f$ will give a large contribution to the vacuum energy, but the relevant diagram for the evaluation of the dilatonic contribution to the cosmological constant is constructed with $\sigma_b$. What is its contribution? We point out that the bubble diagram represents a quantum breakdown of scale-invariance. In other words, it is a conformally anomalous contribution and, for this reason, it will be negligible on cosmological distances. This last point requires a more detailed discussion. In the plots of figure 2, the interaction vertex between dilaton and matter is the result of a conformal anomaly and it is globally harmless because the vertex is exponentially suppressed on large distances (see formula 5.3). It must be stressed that in the simplest bubble diagram this argument cannot be applied because there are no vertices. However, since the dilaton is running towards the weak coupling regime, all the interactions are switched off on cosmological distances and, therefore, we can neglect the contribution of the bubble diagram because the related conformal anomaly will be globally harmless (see the discussion about formula 5.4). We infer that the restoration of scale invariance on cosmological distances is not affected by the 1-loop dilatonic bubble and the cosmological constant remains under control.

### 5.C.2 Matter particles (spin-0, spin-1/2) and Rarita-Schwinger spin-3/2 fields

**Matter particles in the external legs.** The VEV of the matter fields is almost vanishing, because we can write $\Phi_e = \Phi_J e^{-\zeta \sigma}$.

**Matter particles in the internal legs.** In the case of internal legs, we don’t give a VEV to the fields. The matter contribution is still negligible, because global matter is not interacting.

**Bubble diagrams with matter.** Matter bubble diagrams give conformally anomalous contributions and, consequently, they are globally harmless.

Therefore, the quantum contributions to the global renormalized vacuum energy are under control. For this reason, the vacuum energy of matter fields discussed in section 4.A.2, namely $L_{\text{vac}} = -\frac{3}{8} \frac{f^2}{F^2} M_p^4$, can be safely neglected for large $\sigma$.

The contribution of spin-3/2 particles to the cosmological constant can be analyzed following
the procedure discussed for matter particles.

5.C.3 Gauge particles: spin-1

Gauge particles in the external legs. Even if spin-1 gauge fields are not affected by the conformal transformation \( A_\mu = A^\star_\mu \), see for example [31], this is not a problem, because the presence of unbroken fundamental symmetries (e.g. \( SU(3)_C \times U(1)_{em} \), Lorentz...) requires a vanishing VEV for the gauge fields and, therefore, external global gauge particles are harmless.

Gauge particles in the internal legs. Gauge interactions are switched off globally and, therefore, internal (global) gauge particles are harmless.

1-loop bubble diagram with a gauge field. Once again, this contribution is a conformal anomaly and, consequently, it is globally harmless.

5.C.4 The graviton

Let us start writing the complete E-frame metric as:

\[
g^{\mu\nu}_s = (g^{\mu\nu})_b + (g^{\mu\nu})_f. \tag{5.5}
\]

Let us consider the two metrics separately.

The local metric will produce a (large) local gravitational vacuum energy. If we divide the visible Universe into small bubbles, whatever will be the bubble we choose, the local gravitational vacuum energy will be much larger than the DE scale. We infer that the average value (over the volume of the visible Universe) of all these local gravitational vacuum energies inside the bubbles will be much larger than the meV scale (this is the same argument we discussed for the dilatonic backreaction). However, this is the unrenormalized global gravitational vacuum energy and it does not correspond to the DE. We suggest in this paper to exploit the metric backreaction as a counterterm in the renormalization process of the gravitational vacuum energy. In other words, we extend the proposal of Fujii from the dilatonic to the metric vacuum energy. We also claim that the metric backreaction will be able to properly suppress the cosmological gravitational vacuum energy, because scale-invariance is almost restored on large distances.

As far as the global (cosmological) metric is concerned, we choose \((g^{\mu\nu})_b = g^{\mu\nu}_{FRW}\), where \(g^{\mu\nu}_{FRW}\) is the usual Friedmann-Robertson-Walker (FRW) metric. As already mentioned above, \(\sigma_b\) does not mediate a force between macroscopic objects. The same result is obtained in the case of the FRW-metric, because, in our approach the kinetic term for the gravitons is induced by quantum diagrams. In this way, since in our proposal all the interactions are switched off on cosmological length scales in the E-frame, the kinetic term for the global gravitons will be subdominant with respect to the kinetic term for the local gravitons. Therefore, the global metric \((g^{\mu\nu})_b\), like \(\sigma_b\), does not mediate a force between macroscopic objects.

This induced gravity approach requires a more elaborated discussion. It seems worthwhile to point out that, on the one hand, we exploit the induced gravity strategy not only in the E-frame, but also in the string one and, on the other hand, we are considering multiple quantizations. In other words, we quantized the theory more than once:

Quantization - Step 1. Our starting point in the S-frame is the strong-coupling stringy effective action [3.21] where the result of the non-perturbative quantum calculation is encoded in
the form factors, which have been properly chosen to produce the lagrangian \[3.22\]. Therefore, the Einstein-Hilbert term of formula \[4.30\] is induced by a non-perturbative quantum calculation, namely, the first step.

Quantization - Step 2. We considered \[4.30\] as a starting point for another quantization (see figure 1) and we discovered the presence of a conformal anomaly.

Quantization - Step 3. We exploited the anomaly induced interaction vertex \[4.44\] to quantize once again the theory (see figure 2).

Since the dilaton controls the strength of all interactions, the chameleonic behaviour of the field in the E-frame guarantees the simultaneous presence of two different elements after step 3, namely: (1) a (almost) negligible kinetic term for global gravitons (therefore, global gravity is non-dynamical) and (2) a non-negligible kinetic term for local gravitons obtained by applying the induced gravity strategy for the first time (step 1). The question is: what kind of gravitational theory should we expect locally? At this stage, if we proceed with the quantization beyond step 1, namely, if we apply the induced gravity idea to the lagrangian \[4.30\] it is not yet clear whether higher derivative gravitational terms are present locally. However, this discussion is not directly relevant for the cosmological constant problem and it will be addressed in a future work. Moreover, in our proposal all the interactions are switched off on cosmological length scales and it remains to be seen whether this fact clashes with large-scale phenomenology. A detailed phenomenological analysis is definitely necessary to clarify these points.

5.C.5 The chameleonic effective actions

In our proposal, the chameleonic behaviour of the dilaton is compatible, and it is the effective result, of a renormalization process which is carried on to all orders in perturbation theory. The success of this renormalization program is guaranteed by the restoration of scale-invariance.

It must be stressed that in our approach we do not evaluate the diagrams: we exploit the restoration of scale invariance on cosmological distances to protect the mass of the dilaton and the cosmological constant. Our argument is valid at all orders in perturbation theory.

Remarkably, the chameleonic behaviour of the dilaton and the splitting of the particles in global and local objects led us to a formulation of the theory which strongly depends on the choice of the length scale (parametrized by the value of the dilaton). This comment must be further elaborated. Let us start by describing the concept of effective theory in more detail. Let us consider a particle physics theory, that we will call fundamental, where the degrees of freedom (particles and fields) are described by a Lagrangian \(L_A\). Moreover, let us consider a certain energy scale \(M\) and let us suppose that \(L_A\) is able to correctly describe physical processes not only below \(M\), but also above \(M\). Now, if our intention is to describe physical phenomena below \(M\), we are free to describe these phenomena not only with the lagrangian \(L_A\), but also with another lagrangian \(L_B\), that we will call effective and that is obtained from the fundamental one by integrating out the heavy degrees of freedom. In other words, the particles with a mass \(m > M\) will be removed from the effective theory and \(L_B\) will be able to correctly describe physical phenomena below (but not above) the energy scale \(M\). If we are interested in the description of the physical phenomena above the scale \(M\), then the reintroduction of the heavy degrees of freedom will be mandatory. A well-known example is given by the SM (theory A) and the Fermi theory (theory B).
Let us now come back to the chameleon. As already mentioned above, we parametrize the length scales of the model through the dilaton $\sigma$. In particular, as we argued above, large values of $\sigma$ correspond to large distance scales (i.e., small mass scales). In this way we can construct two different lagrangians $L_A$ and $L_B$. On the one hand, the local particles can be considered as the degrees of freedom of a lagrangian $L_A$, describing physical phenomena below an energy scale $M_A$ fixed by $\sigma_f$. On the other hand, global particles can be considered as the degrees of freedom of a lagrangian $L_B$, describing physical phenomena below an energy scale fixed by $\sigma_b$ that we call $M_B \simeq H_0 < M_A$. $L_B$ can be considered as the effective theory valid in the deep infrared (IR) region, obtained from the (more fundamental) lagrangian $L_A$ by integrating out the heavy degrees of freedom. We infer that the running of the dilaton towards large values can be interpreted as the running of the theory towards the IR region and, in general, a shift of the dilaton towards larger (smaller) values will correspond to integrating out (in) degrees of freedom through the exponential prefactor $e^{-\zeta \sigma}$ already mentioned above. It is time to summarize our chameleonic lagrangians (in the E-frame).

The local one is written exploiting only local particles as:

$$L_A = \sqrt{-g_f^*} \left( \frac{1}{2} (R_f^*) - \frac{1}{2} (g_{\mu\nu}^*) \partial_\mu \sigma_f \partial_\nu \sigma_f + L_{\text{matter}} + V_{\text{eff}}(\sigma_f) + \ldots \right)$$

(5.6)

Here the dots include the gauge part of the theory and they might also include higher derivative (local) gravitational terms. $V_{\text{eff}}(\sigma)$ is the total chameleonic effective potential for $\sigma$ obtained by adding together (1) a run-away exponential branch and (2) a matter branch linear in $\sigma$, given by formula 4.44. $L_{\text{matter}}$ is given by:

$$L_{\text{matter}} = -\frac{1}{2} (g_{\mu\nu}^*) D_\mu \Phi_f^* D_\nu \Phi_f^* - \frac{1}{4} M_p^2 (\Phi_f^*)^4 + \frac{\lambda_f}{4!} (\Phi_f^*)^4$$

(5.7)

and $D_\mu = \partial_\mu + \zeta \partial_\mu \sigma_f$. Needless to say, $L_A$ is not scale invariant.

The global lagrangian is written exploiting only the background part of the fields and it is (almost) scale invariant. It is written as:

$$L_B = \sqrt{-g_{\text{FRW}}} \left( -\frac{1}{2} (g_{\mu\nu}^*_{\text{FRW}}) \partial_\mu \sigma_b \partial_\nu \sigma_b + V_{\text{eff}}(\sigma_b) + \ldots \right),$$

(5.8)

where the dots represent kinetic terms for massless global gauge fields. Therefore, the effective lagrangian of the model on cosmological distances corresponds (basically) to a free cosmological dilaton $\sigma_b$ in a FRW-background. A promising line of development will analyze further the consequences of this IR-free theory in Cosmology and gravitational Physics (also from the standpoint of the AdS/CFT correspondence).

5.D The correct Dark Energy scale

Naturally, our intention is to recover the correct Dark Energy scale. We will exploit the chameleonic behaviour of the dilaton. In the minimum of the effective potential we can write

$$V(<\sigma_b>) = \rho_m B(<\sigma_b>).$$

(5.9)

\footnote{It seems now worthwhile to point out that in our model the matter particles are not chameleons and they can be safely integrated out on cosmological distances.}
where $\rho_m$ is the matter energy density and $B(\sigma)$ is the usual function of a chameleonic model where the coupling is encoded. As previously mentioned in the Introduction, Dark Matter and Dark Energy give a similar contribution to the cosmic energy budget today. Consequently, the correct DE scale is given by the chameleon mechanism, granted that $B \simeq 1$. In the case of the coupling $4.44$, we identify the matter energy density in the E-frame with $\frac{1}{2}m^2\Phi^2$. Therefore we have

$$B = \frac{1}{M_p^2\pi^2}\zeta\lambda\sigma$$

and if we choose $\lambda \simeq \zeta \simeq 1$, the condition $B = 1$ requires $\sigma \simeq 1$. In the absence of a detailed analysis, we have no theoretical grounds to support this value of $\sigma$. Happily, however, this choice of parameters and fields is not a fine-tuning. Remarkably, our $B$-function depends on $\sigma$ in a linear way and, therefore, we can tolerate a certain deviation of the value of $\sigma$ in the minimum from the planckian scale.

6. Discussion: the cosmological constant and non-equivalent frames

The careful reader may be puzzled by some of our considerations and may ask a number of legitimate questions.

What about a vanishing backreaction for the dilaton? This would clash with the non-linear nature of a chameleonic theory.

Why does the backreaction cancel the unrenormalized vacuum energy? A different result would clash with the restoration of scale-invariance at large distances. Accordingly, the effective potential must fall to zero on cosmological distances and this result is obtained taking into account all the quantum corrections. Backreaction is a crucial element to keep under control the contribution of the quantum diagrams to the cosmological constant, but this is true only for the diagrams containing the dilaton and the gravitons.

What about a small value of $\zeta\sigma_b$? This would render the suppression negligible. The exponential suppression is a crucial ingredient to restore scale invariance cosmologically and it depends on the value of $\zeta\sigma$. In the chameleonic literature an upper bound on the product $\zeta\sigma_{today}$ has been discussed in connection with Big Bang Nucleosynthesis (BBN) constraints on the variation of particle masses, see for example [46]. However, these bounds cannot be directly applied to our proposal, because our coupling function $B(\sigma)$ is linear. Therefore, it is not the usual exponential function exploited to discuss the bounds. This difference could play an important role in comparing the model with experimental constraints. At this stage, the possibility still exists that a certain amount of fine-tuning may be required to satisfy BBN constraints. A detailed phenomenological analysis is definitely necessary to clarify this point.

This approach to the cosmological constant problem is effective granted that you assume the presence of scale-invariance in the Lagrangian. As far as the S-frame Lagrangian is concerned, all that is really needed is a proper structure of the S-frame form factors in the strong coupling regime. This will guarantee the presence of a certain sector in the S-frame Lagrangian where the symmetry is present. We can start with a S-frame Lagrangian where scale-invariance is explicitly
broken and after the conformal transformation to the E-frame scale-invariance will be restored (or, strictly speaking, established) on cosmological distances. Consequently, the global Einstein frame lagrangian is (almost) scale-invariant. In our scenario, the string dilaton in the E-frame is the order parameter of the breaking of scale-invariance. Since the dilaton is a chameleon, Particle Physics will be the standard one only locally: the usual contributions to the vacuum energy (e.g. from SUSY breaking, from electroweak symmetry breaking, from axions...) will be extremely large locally, but exponentially reduced on large distances.

Are the Jordan and Einstein frames equivalent? In this model the dilaton and the Φ-field have constant values in the S-frame and this is due to the presence of a stringy lagrangian which is the result of a quantum calculation. Needless to say, the cosmological constant in the S-frame is much larger than the meV-scale. However, after the conformal transformation to the E-frame the cosmological constant is surprisingly under control (at the quantum level). These comments are pointing out a non-equivalence of different conformal frames at the quantum level and, in particular, the E-frame is selected to be the physical frame. We warn the reader that we did not evaluate the cocycle function (for an introduction see, for example, [49]), because the cocycle is basically a conformal anomaly, therefore, it is globally harmless and it will not clash with the restoration of scale invariance on cosmological distances in the E-frame. We infer that this correction does not modify the qualitative characteristics of the chameleonic potential in the E-frame and it can be neglected.

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Is it possible to avoid the conformal transformation? Can we make our calculations in one single (and physical) frame (namely the Einstein one)? The string frame is the natural frame in which the string effective action is written and we need a conformal transformation to establish a connection between string theory and the physical (Einstein) frame. Therefore, it is not surprising or strange to make a conformal transformation as an intermediate step of our procedure: string theory is suggesting us to use a non-physical frame (i.e. the string one).

Do we have a dynamical mechanism that drives the (φ, Φ) system towards the equilibrium position in the S-frame? No. If the initial condition of the system is characterized by non-fine-tuned values φ₀ and Φ₀, then it is easy to choose the parameters of the model to satisfy the field equations with those particular values of the fields.

Is this a successful chameleon model? A detailed phenomenological analysis is necessary to clarify this point and it will be discussed in a future work. However, in this paper we point out that, happily, in our model, the chameleonic force cannot modify the motion of planets. This point we mentioned last needs to be further elaborated. As already mentioned in the Introduction, a direct coupling between matter and an ultralight scalar field can be phenomenologically dangerous. Indeed, local limits on the coupling between matter and a scalar field are particularly stringent and they require a small coupling unless the scalar field is sufficiently heavy (i.e. stabilization). Typically, one requires the (local) mass of the chameleon to be larger than 10⁻³ eV. This condition can be satisfied in a high density environment such as the atmosphere, but in the solar system, where the density is much smaller, the chameleon can be very light.

In principle, there is also the possibility that the cocycle will restore scale invariance locally. In this case, whatever will be the length scale we choose in the problem, the scale symmetry will be present. However, in the absence of theoretical motivations to support this possibility, we think that a locally non-symmetric configuration is more generic.
it can mediate a long-range force and it may be responsible for an (unacceptable) distortion of planetary trajectories. To overcome this problem, the standard approach discussed in the chameleonic literature is to exploit the so called *thin-shell* mechanism. A body is said to have a thin-shell if a chameleon $\chi$ is approximately constant everywhere inside the body apart from a small region near the surface of the body. Large ($O(1)$) changes in the value of $\chi$ can and do occur in this surface layer or thin-shell. Inside a body with a thin-shell $\nabla \chi$ vanishes everywhere apart from a thin superficial layer. Since the chameleonic force is proportional to $\nabla \chi$, it is only that surface layer, or thin-shell, that both feels and contributes to the ‘fifth force’ mediated by $\chi$. Therefore, when the thin-shell mechanism is operative, the chameleon force between the Sun and the planets is very weak and the otherwise tight limits on such a long-range force are evaded \[21, 22\]. Remarkably, in a recent paper \[50\], the chameleonic behaviour of a string dilaton running towards the strong coupling region has been ruled out for an exponential coupling, because the thin-shell mechanism was absent and unacceptable deviations from standard gravity were predicted by the model. Let us now come back to our model and let us discuss the motion of planets. The anomaly induced interaction vertex between dilaton and matter has been exploited to construct the mass correction diagram of figure 2b, which is responsible for the stabilization of the field. The exponential factor $e^{-\zeta \sigma}$ plays a crucial role, on the one hand, to suppress the coupling to matter, on the other hand, to suppress the mass of the dilaton. Since in our model there is no cosmological constant fine-tuning, the mass scale in the potential is planckian and the exponential suppression must produce a hierarchy of many orders of magnitude to obtain a long-range force. We infer that in our model the chameleonic dilaton $\sigma$ is ultralight *if and only if* it is non-interacting with matter. Consequently, the dangerous set up given by a very light scalar field with a relevant coupling with matter is naturally avoided in our proposal and, for this reason, the planetary orbits are not modified by our chameleon.

7. Conclusions

In this paper the chameleonic behaviour of the string dilaton has been suggested and some of its consequences have been discussed in detail. In particular (1) we proposed a new stringy solution to the cosmological constant problem and (2) we pointed out a non-equivalence of different conformal frames at the quantum level. The correct Dark Energy scale is recovered in the E-frame without fine-tunings of the parameters and this result is robust against all quantum corrections, granted that we assume a proper structure of the S-frame form factors in the strong coupling regime. However, at this stage, the possibility still exists that a certain amount of fine-tuning may be required to satisfy some phenomenological constraints. Moreover, the theory is IR-free, while higher derivative gravitational terms might be present locally and it remains to be seen whether these facts clash with phenomenology. A detailed phenomenological analysis is definitely necessary to clarify these points.

In our approach to the cosmological constant problem, the string dilaton in the E-frame is the order parameter of the breaking of scale-invariance. Since the dilaton is a chameleon, Particle Physics will be the standard one only *locally*: the usual contributions to the vacuum energy (e.g. from SUSY breaking, from electroweak symmetry breaking, from axions...) will be extremely large *locally*, but exponentially suppressed on large distances.

To the best of our knowledge this was the first attempt of dealing with the cosmological constant problem from the standpoint of chameleon fields.

A chameleonic string dilaton, on the one hand, is a new stringy way of tackling crucial
problems like dilaton stabilization and cosmological constant problem, on the other hand, it is a major step forward to establish a connection between String Theory and current experiments (most notably GammeV [27,28]).

We conclude by summarizing some of the possible lines of development. One interesting project will study this model in the framework of AdS/CFT correspondence and its possible connections with the braneworld model of reference [35]. Moreover, the theory that we discussed in this paper and its phenomenological consequences should be further investigated, on the one hand, in Cosmology and gravitational physics (gravity is globally absent and we must be sure that this fact does not clash with phenomenology), on the other hand, in connection with the BBN constraints on \( \zeta \sigma \) (checking whether they can be faced without fine tuning). The dual nature of the concept of particle should be carefully investigated starting from [38,47,48] and references therein. The potential presence of higher derivative gravitational terms in the local lagrangian should be also studied considering the role played by multiple quantizations. One more future research direction should be mentioned. In our model a stabilized dilaton in the S-frame does not correspond to a stabilized dilaton in the E-frame and, at first glance, this result seems to clash with the approach discussed in [35]. These issues will be discussed in a future work.

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