Enhancing the multi-encoder-based cutting force estimation along the stationary axis of a machine tool with multiple inertia dynamics

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Abstract
Wideband cutting force sensing is a key technology for process monitoring. Sensorless cutting force estimation using the internal servo information of a machine tool with ball-screw-driven stages has been studied owing to its high maintainability and ease of introduction. In the motor current-based method, the cutting force estimation along the stationary axis is challenging, and the estimation bandwidth is significantly limited owing to the low sensitivity of the motor current in the high-frequency range. The dual-inertia model-based load-side disturbance observer (LDOB) can estimate the cutting force along the stationary axis using the relative position obtained from the rotary and linear encoder. The linear encoder is installed relatively near the cutting point and has a high sensitivity in the high-frequency range. However, this approach is not applicable to machine tools with complicated structural dynamics. To address this challenge, we propose a cutting force estimation method along the stationary axis using the Kalman filter (KF) based on a multiple inertia model derived solely from the relative position signal. The dynamics, depending on the stage position of the feed drive, were modeled using linear interpolation. Through end milling tests, we confirmed that the cutting force estimation accuracy along the stationary axis of a machine tool with multiple inertia dynamics was significantly improved by the proposed method compared to the current and LDOB-based methods. Additionally, the wideband cutting force could be estimated using the proposed method for bandwidths up to 1000 Hz.

Keywords Process monitoring · Cutting force · Stationary axis · Encoder · Kalman filter

Abbreviations

| Symbol | Description |
|--------|-------------|
| \( a_1, a_2, a_3 \) | Coefficients of the quadratic function model of stiffness |
| \( b_1, b_2, b_3 \) | Coefficients of the quadratic function model of modal parameters |
| \( C_t \) | Damping coefficient of the transitional element |
| \( D_m \) | Damping coefficient of the rotational element |
| \( F_{cut} \) | Cutting force |
| \( F_h \) | Impulse force |
| \( I_a \) | Motor current |
| \( J_m \) | Total inertia of the motor, coupling, and ball-screw |
| \( K_s \) | Axial stiffness of the ball-screw |
| \( K_t \) | Torque coefficient of the servo motor |
| \( l \) | Pitch length of the ball screw |
| \( M_i \) | Total movable mass |
| \( m_i, c_i, k_i \) | Modal mass, damping coefficient, and stiffness of the \( i \)th eigenmode |
| \( N \) | Total number of modes |
| \( p, q \) | Interpolation coefficients |
| \( r^2 \) | Coefficient of determination |
| \( R \) | Transformation coefficient from rotational to translational motion \( (= l/2\pi) \) |
| \( T_s \) | Discrete sampling time |
| \( v \) | System noise |
| \( w \) | Measurement noise |
| \( x_i \) | Displacement of the \( i \)th eigenmode |

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Cutting force sensing technologies can be classified into external sensor–based and sensorless approaches. In the external sensor–based approach, using a piezoelectric dynamometer is the de facto standard because of its high accuracy and wide sensing bandwidth, which are crucial in the research and development phase [4]. However, it has not been practically used at production facilities because of its high cost, restriction in machining space, and thermal instability. To avoid restrictions in machining space, cutting force sensing technologies with piezoelectric force sensors [5], displacement sensors [6, 7], and accelerometers [8, 9] installed in spindle units have been developed. The receptance coupling technique has reduced the number of time-consuming tap tests by modeling the spindle-holder-tool dynamics affected by tool and/or holder changes [8–10]. The estimation accuracy of the cutting force has been improved by integrating various spindle-mounted external sensors [7–9]. Nevertheless, an increased risk of sensor failure and reduction in maintainability are inevitable when many additional sensors are installed inside the spindle units.

On the other hand, the sensorless cutting force estimation using the internal servo information of a machine tool has been studied for its high maintainability and ease of introduction. Altintas [11] estimated the cutting force based on the current of a DC servo motor by compensating for the friction force generated in the feed drive. Lee et al. [12] applied this methodology to an AC servo motor. These current-based methodologies can estimate the cutting force along the feed axis. However, the estimation accuracy deteriorates along the stationary axis owing to the arbitrary property of the motor current [13]. To address this challenge, Sato et al. [14] extracted the cutting force component from the motor current using a fast Fourier transform (FFT) and inverse FFT. The cutting force along the stationary axis was successfully estimated when the cutting force was larger than the static friction force. However, the bandwidth of current-based cutting force estimation is limited by the structural dynamic modes of a machine tool. Altintas and Aslan compensated for the disturbance transfer function of a current-based cutting force estimation system with an inverse digital filter generated by the tap-test-based frequency response function (FRF) [15, 16]. However, the bandwidth remained limited to 200 Hz owing to the low sensitivity of the motor current in the high-frequency region. It is difficult to generate an inverse filter with an infinite filtration power in the high-frequency region to cover the low sensitivity of the motor current. Additionally, it is challenging to decouple the rigid body and vibration modes of a feed drive from the measured FRF.

Cutting force estimation based on the disturbance observer (DOB) theory has been studied to improve the wideband cutting force estimation. In the DOB-based method, the cutting force is estimated considering both the
motor current and motor angle based on the single-inertia model. The inertial force of the feed drive is directly considered using the motor angle converted from the rotary encoder signal. Although the sensitivity of the motor current is lowered in the frequency range above the bandwidth of the position control, the encoder can directly measure fluctuations up to an extremely high frequency. Therefore, a wideband cutting force estimation can be developed using the encoder signal. Shinno et al. [17] successfully estimated the cutting force independent from the temperature with a DOB. Takei et al. [18] successfully estimated the cutting force at bandwidths up to 350 Hz with a DOB in the linear motor-driven stage, which has a simple structure and can usually be modeled as a single inertia system.

However, it is more challenging to accurately estimate the wideband cutting force in a ball-screw-driven stage because of its multiple dynamic modes. For a full closed-loop controlled ball-screw-driven stage, Yamada and Kakinuma [19] proposed a cutting force estimation method using a multi-encoder-based disturbance observer (MEDOB [20]). The MEDOB method estimates the cutting force based on the dual-inertia model. It estimates the cutting force using not only the motor current and motor angle but also the stage position converted from the linear encoder signal. The linear encoder is installed near the cutting point and has a higher sensitivity in the high-frequency region compared to the rotary encoder. The MEDOB-based method can estimate the wideband cutting force along the feed axis when the feed drive can be accurately modeled as a dual-inertia system. However, it is difficult to estimate the cutting force that is less than the maximum static friction force along the stationary axis.

As a solution to the stationary axis, Yamada et al. [21] proposed a mode-decoupled cutting force estimation based on two mutually orthogonal modes of a dual-inertia model: the rigid body and vibration mode. A cutting force that is less than the maximum static friction force along the stationary axis can be estimated based on the vibration mode. In this method, the relative position between the motor and stage (i.e., the deformation of a ball screw) is used for the cutting force estimation. Furthermore, Yamato et al. [22] proposed a cutting force estimation method based on a load-side disturbance observer (LDOB [23]). The LDOB-based method estimates the cutting force using the relative position like the vibration mode-based method. It can accurately estimate the cutting force along the stationary axis when the feed drive can be accurately modeled as a dual-inertia system.

The MEDOB, vibration mode, and LDOB method are based on the dual-inertia model, typically used for modeling a full closed-loop controlled ball-screw-driven stage. However, when many mechanical units such as spindle units and a feed drive along different axes are installed on the stage, the stage often has the complicated structural dynamics. A thin-walled workpiece, which has multiple eigenmodes, also makes the stage dynamics complicated [24]. The dual-inertia model rarely represents the machine tool dynamics with sufficient bandwidths in such cases. Therefore, the bandwidths of the cutting force estimations based on MEDOB, vibration mode, and LDOB are limited. Although the bandwidths of these methods can be improved by increasing the order of the assumed model, additional position and angle measurement sensors corresponding to the number of model orders are required. Additionally, the installation locations of the additional sensors are not determined systematically. This approach is not suitable for use in production facilities.

Using the relative position is effective for the cutting force estimation along the stationary axis. However, the vibration mode and LDOB method are practically limited to extensions up to a dual-inertia system. The encoder-based cutting force estimation method that can be applied to the multiple inertia dynamics is needed for wideband cutting force estimation along the stationary axis. For machine tools with full closed-loop controlled ball-screw-driven stages, this study proposes a multi-encoder-based cutting force estimation method based on the multiple inertia model. The proposed method estimates the cutting force with a Kalman filter (KF) generated by the tap-test-based FRFs between the tool-tip force and the relative position obtained from the rotary and linear encoders (multi-encoders). Using the relative position can alleviate the difficulty of decoupling the rigid body and vibration modes from the FRF. The FRF of the ball-screw-driven stage also varies depending on the stage position [25]. Because this variance of the FRF cannot be ignored in cutting force estimation, the stage position dependency of the FRF is incorporated into the multiple inertia model using linear interpolation. The proposed method is applicable to the stationary axes of machine tools with multiple inertia dynamics and contributes to eliminating constraints of the model order existing in the vibration mode and LDOB-based method. In this paper, the cutting force estimation performance of the proposed method along the stationary axis was verified through end milling tests conducted on the compact machine tool. Note that the proposed method is supposed to be also effective for the machine tools with the large strokes, when servo motors do not have reduction gears whose damping characteristics lower the sensitivity of the rotary encoder.

This paper is organized as follows. In Sect. 2, the proposed cutting force estimation method using the multi-encoders is described; in Sect. 3, the experimental procedure for verifying the proposed method is explained. The results of the model parameter identification and cutting force estimation are reported and discussed in Sect. 4.
2 The multi-encoder-based cutting force estimation based on a multiple inertia model

2.1 Modeling of machine tool dynamics

For a machine tool with full closed-loop controlled ball-screw-driven stages, this study proposes a cutting force estimation method along the stationary axis using the relative position based on the multiple inertia model. The cutting force along the stationary axis including the DC component can be estimated using the relative position without friction compensation, even if the cutting force is less than the static friction force [21]. In the proposed method, the machine tool dynamics are modeled using the relative position. The relative position \( x_r \) is calculated using the motor angle \( \theta_m \) converted from the rotary encoder signal and stage position \( x_t \) converted from the linear encoder signal as follows:

\[
x_r = R \theta_m - x_t
\]  

where \( R \) is the transformation coefficient from rotational to translational motion. The machine tool dynamics are modeled through tap tests conducted under various stage positions to adapt to the dynamics depending on the stage position of the ball-screw-driven stage. Tap tests are conducted with the stage positioned at the anti-motor side \( (x_t = x_{t,a}) \), center point \( (x_t = x_{t,c}) \), and motor side \( (x_t = x_{t,m}) \), as shown in Fig. 1. The FRF from the impulse force at the cutting point \( (F_h) \) to the relative position \( (x_r) \) is measured by applying an impulse force at the cutting point, as shown in Fig. 2. Here, the impulse force at the cutting point \( (F_h) \) can be treated as the cutting force \( (F_{cut}) \). In this study, the influence of the cross-transfer functions related to the cutting forces in other directions was assumed to be negligible, because the direct transfer function was dominant in the target frequency range in the experimental setup. To further improve the cutting force estimation accuracy, it is necessary to consider the influence of the cross-transfer function. The measured FRFs through tap tests are fitted with the multi-degree-of-freedom (MDoF) curve fitting technique with a nonlinear least square method [26]. The modal parameters comprising the residues \( (a_{m,j}, a_{c,j}, a_{a,j}) \), damping ratios \( (\zeta_{m,j}, \zeta_{c,j}, \zeta_{a,j}) \), and natural angular frequencies \( (\omega_{m,j}, \omega_{c,j}, \omega_{a,j}) \) of \( i \) th mode \( (i = 1, 2, \ldots, N) \) at the motor side, center point, and anti-motor side are identified. In the previous study, the stage position dependency of the axial stiffness of a ball-screw was modeled as a quadratic function under the assumption that the viscous damping coefficient does not have the stage position dependency [22]. However, the viscous damping also has the stage position dependency because of the position dependency of the friction force generated in a feed drive [19]. Therefore, the stage position dependencies of the modal parameters are modeled using linear interpolation to consider the stage position dependency of the viscous damping in this study. Linear interpolation models for the modal parameters of the \( i \) th mode are constructed based on the identified modal parameters as follows:

\[
\begin{align*}
\alpha_i(x_t) & = p_{m,i,a}x_t + q_{m,i,a} & (x_{t,m} \leq x_t \leq x_{t,c}) \\
\zeta_i(x_t) & = p_{m,i,\zeta}x_t + q_{m,i,\zeta} & (x_{t,m} \leq x_t \leq x_{t,c}) \\
\omega_i(x_t) & = p_{m,i,\omega}x_t + q_{m,i,\omega} \\
\alpha_i(x_t) & = p_{a,i,a}x_t + q_{a,i,a} & (x_{t,c} < x_t \leq x_{t,a}) \\
\zeta_i(x_t) & = p_{a,i,\zeta}x_t + q_{a,i,\zeta} & (x_{t,c} < x_t \leq x_{t,a}) \\
\omega_i(x_t) & = p_{a,i,\omega}x_t + q_{a,i,\omega}
\end{align*}
\]  

Fig. 1 The definition of the stage position in a ball-screw driven stage

Fig. 2 The schematic of the procedure to measure the machine tool dynamics by a tap test
where \( \omega_i, \zeta_i \), and \( \alpha_i \) are the resonance angular frequency, damping ratio, and residue of \( i \) th mode \((i = 1, 2, \ldots N)\), respectively, considering the stage position dependency of modal parameters. The interpolation coefficients are derived as follows:

\[
P_{m,i,a} = \frac{a_{i}a_{j}}{s_{j}-s_{i}}, \quad q_{m,i,a} = \frac{x_{i}a_{j}-x_{j}a_{i}}{s_{j}-s_{i}}
\]

\[
P_{m,i,\varepsilon} = \frac{x_{i}a_{j}-x_{j}a_{i}}{s_{j}-s_{i}}, \quad q_{m,i,\varepsilon} = \frac{x_{i}a_{j}-x_{j}a_{i}}{s_{j}-s_{i}}
\]

\[
P_{m,i,\varepsilon} = \frac{x_{i}a_{j}-x_{j}a_{i}}{s_{j}-s_{i}}, \quad q_{m,i,\varepsilon} = \frac{x_{i}a_{j}-x_{j}a_{i}}{s_{j}-s_{i}}
\]

Based on Eq. (2), the transfer function representing the characteristics of the vibration modes is derived as follows:

\[
\frac{x_r(s)}{F_{cut}(s)} = \sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}
\]

(4)

The term representing the rigid body mode is not included in Eq. (4) because the rigid body motion of the feed drive does not appear in the relative position between the motor and stage. The machine tool dynamics are simply modeled as the superposition of vibration modes in a feed drive using the relative position. Because \( F_{cut} \) can be treated as partially constant when the sampling frequency is sufficiently high relative to the tooth passing frequency, the differential of \( F_{cut} \) can be treated as a system noise as follows [5]:

\[
F_{cut} = \nu
\]

(5)

From Eqs. (4) and (5), the continuous-time state-space model is constructed as follows:

\[
\begin{align*}
&x = Ax + Bu \\
y &= x_c = Cx + \omega
\end{align*}
\]

(6)

where \( y \) and \( \omega \) are the measured value and noise, respectively. The state vector and coefficient matrices are derived as follows:

\[
A = \begin{bmatrix}
0 & 1 & \cdots & 0 & 0 & 0 \\
-\omega_1^2 & -2\zeta_1 \omega_1 & \cdots & 0 & \alpha_1 \omega_1^2 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & -\omega_N^2 & -2\zeta_N \omega_N & \alpha_N \omega_N^2 \\
0 & 0 & \cdots & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
1 \\
\end{bmatrix}, \quad C = \begin{bmatrix}
1 & 0 & \cdots & 1 & 0 \\
\end{bmatrix}
\]

(7)

The relative position \((x_c)\) is obtained as the superposition of the eigenmode displacements. Therefore, the total displacement of all modes can be observed as the relative position converted from the rotary and linear encoder signals inside the state-space model presented in Eq. (6).

### 2.2 Multi-encoder-based cutting force estimation using a Kalman filter

The cutting force can be estimated through a state estimation using a Kalman filter based on the relative position signal and continuous-time state-space model as follows:

\[
\hat{x} = A\hat{x} + K(y - \hat{y})
\]

\[
= A\hat{x} + K(y - Cx)
\]

\[
= (A - KC)\hat{x} + Ky
\]

\[
\hat{F}_{cut} = C_0\hat{x}
\]

where \( C_0 = \begin{bmatrix} 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \) and \( K \) is the continuous Kalman gain matrix. \( \hat{F}_{cut} \) is the estimated cutting force. Equation (8) is discretized with a zero-order hold as follows:

\[
\hat{x}(k + 1) = \exp((A - KC)T_s)\hat{x}(k) + \left( \int_0^{T_s} \exp((A - KC)\tau)Kd\tau \right)y(k)
\]

\[
\hat{F}_{cut}(k) = C_0\hat{x}(k)
\]

(9)

where \( T_s \) denotes the discrete sampling time. The Kalman filter is the state observer that minimizes the state estimation error \((\hat{x} - x)\) due to the system and measurement noise. The Kalman gain matrix \((K)\) is identified by minimizing the covariance matrix of the state estimation error \((P = E[\hat{x}\hat{x}^T])\).

The covariance matrix of the state estimation error \( P \) is evaluated by solving the following Riccati equation [27]:

\[
P = AP + PA^T + BQ^TB - PC^TR^{-1}CP
\]

(10)

where \( Q = E[\nu\nu^T] \) and \( R = E[\omega\omega^T] \) are the covariance matrices of the system and measurement noise, respectively. The optimal Kalman gain matrix is calculated using the solution of Eq. (10) as follows:

\[
K = PC^TR^{-1}
\]

(11)

The cutting force is estimated based on Eq. (9) using the optimal Kalman gain matrix. The covariance matrix of the measurement noise \((R)\) is determined based on the variance of the relative position signal when the stage is stopped. The covariance matrix of the system noise \((Q)\) is tuned so that the gain characteristics between the measured and estimated force would become unity through tap tests.
3 Experimental procedure

3.1 Experimental setup

The 3-axis ball-screw-driven compact machine tool used to verify the performance of the proposed method is presented in Fig. 3. The pitch length of the ball screw ($l$) was 5 mm. The AC servo motors were used as actuators. The stage in each axis was positioned by a full closed-loop control using the rotary (resolution: 23 bit) and linear encoders (LIF481R, from HEIDENHAIN) on the machine. The sampling frequency of the control signals was set to 10 kHz. Because the Z-stage and spindle unit were connected on the Y-stage, the Y-stage had complicated structural dynamics, as discussed in Sects. 4.1 and 4.2. The cutting force estimation results of the proposed method were verified on the Y-stage. The cutting force measured by the dynamometer (type 9129A, from Kistler), which was installed on the X-stage, was used as the reference value to verify the performance of the proposed method.

3.2 Verification method of proposed cutting force estimation

To verify the proposed method, the current-based, LDOB-based, and proposed cutting force estimation methods were compared through end milling tests, as shown in Fig. 4. To implement the proposed method, the modal parameters of the Y-stage were identified through tap tests. Next, a linear interpolation model for the dynamics depending on the stage position of the ball-screw-driven stage was constructed. Subsequently, the covariance matrices of the system and measurement noise used in the Kalman filter were tuned through tap tests, and a Kalman filter-based cutting force observer was constructed. To implement the current and DOB-based methods, the model parameters of the dual-inertia model were identified through a sinusoidal sweep motor excitation test in the Y-stage. Then, the current and LDOB-based cutting force observers were constructed. Through end milling tests, the cutting force along the stationary Y-axis of the experimental setup was estimated using the proposed, current-based, and LDOB-based methods. The three methods were compared to verify the accuracy of the proposed method.

4 Results and discussion

4.1 Identification of model parameters used in cutting force estimation with LDOB

A sinusoidal sweep motor excitation test was conducted at the Y-stage of the experimental setup to identify the

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Fig. 3 Experimental setup

Fig. 4 The verification process for the proposed cutting force estimation
model parameters used in the cutting force estimation with the LDOB method. The excitation frequency was logarithmically swept from 10 to 1000 Hz. The FRFs from the motor thrust force \( (K_{m_y} \dot{x}_m) \) to the stage acceleration \( (\ddot{x}_y) \) and equivalent motor acceleration in transitional motion \( (\ddot{x}_{m,y} = R \ddot{\theta}_m) \) were calculated along the \( Y \)-axis. Then, the model parameters of the dual-inertia model were identified using the MDoF curve fitting technique with a nonlinear least-squares method [26], such that the sum of residual errors between the experimental and fitted FRFs would be minimized. The experimental and fitted FRFs are depicted in Fig. 5, and the identified parameters of the dual-inertia model are listed in Table 1. The modeled FRFs between the motor thrust force and encoder accelerations were not fitted with the experimental FRFs overall, as shown in Fig. 5. In particular, the dual inertia model could not accurately express the experimental FRF from the motor thrust force to the stage acceleration obtained through the linear encoder. This can be attributed to the multiple inertia dynamics of the \( Y \)-stage on which the \( Z \)-stage and spindle are installed. This modeling error of the dual-inertia model is not negligible. The cutting force estimation performance of the LDOB-based method is significantly limited in machine tools with multiple inertia dynamics.

4.2 Modeling machine tool dynamics through tap tests

To obtain FRFs from the cutting force \( (F_{\text{cut},y}) \) to the relative position \( (x_{c,y}) \) in the \( Y \)-direction, tap tests were conducted when the \( Y \)-stage was positioned at the motor side \( (x_{c,y} = 0 \text{ mm}) \), center point \( (x_{c,y} = 32 \text{ mm}) \), and anti-motor side \( (x_{c,y} = 64 \text{ mm}) \). Figure 6 illustrates the obtained FRFs at the three stage positions. The reliable bandwidth of the measured FRFs ranged from 50 to 1000 Hz. Many resonance points were confirmed in the gain characteristics of the FRFs, and the \( Y \)-stage clearly exhibited multiple-inertia dynamics. In addition, the gain characteristics of the FRFs varied depending on the stage position. This may be due to the stage position dependency of the dynamics in the ball-screw-driven stage, which is not negligible in the cutting force estimation. The obtained FRFs were fitted using the MDoF curve-fitting technique with a nonlinear least square method [26] to identify the modal parameters used in the proposed method. The fitting results for the FRFs at the three stage positions are shown in Fig. 7a–c. The gain and phase characteristics of the FRFs at each stage position were fitted reasonably from 50 to 1000 Hz, and the modal parameters at the three stage positions were identified. The coefficient of determination of the fitted model was calculated as follows [26]:

\[
r^2 = \frac{\| (H_{\text{meas}})^T (H_{\text{model}}) \|_F^2}{\{ (H_{\text{meas}})^T (H_{\text{meas}}) \} \cdot \{ (H_{\text{model}})^T (H_{\text{model}}) \}}
\]

Here, \( r^2 \) is the coefficient of determination. \( H_{\text{meas}} \) and \( H_{\text{model}} \) are the complex column vectors of the measured and modeled FRF dataset, respectively. When the stage was positioned at the motor side, center point, and anti-motor side, the coefficients of determination were 0.969, 0.981, and 0.970, respectively. It was confirmed that model-based FRFs fitted well with the measured FRFs at the three stage position under the high coefficients of determination.

![Fig. 5](image1.png)  
**Fig. 5** Experimental and fitted FRFs obtained from a sinusoidal motor swept excitation test

![Fig. 6](image2.png)  
**Fig. 6** FRFs depending on the stage position obtained from tap tests

| Table 1 Model parameters of the dual-inertia model |
|-----------------------------------------------|
| Total inertia of motor, coupling, and ball-screw | \( 6.8 \times 10^{-5} \) |
| \( J_m \) (kg \( \cdot \) m\(^2\)) | |
| Total movable mass \( M_t \) (kg) | 24 |
| Damping coefficient of rotational element | \( 8.1 \times 10^{-3} \) |
| \( D_m \) (N \( \cdot \) m \( \cdot \) s/rad) | |
| Damping coefficient of transitional element \( C_t \) (N \( \cdot \) s/m) | \( 4.2 \times 10^3 \) |
| Axial stiffness of ball-screw \( K_s \) (K/m) | \( 4.0 \times 10^7 \) |

![Table](image3.png)  
**Table 1** Model parameters of the dual-inertia model
A linear interpolation model for the stage position dependency of dynamics was constructed from the identified modal parameters based on Eq. (2). To verify the validity of the modeling method using linear interpolation, a tap test was performed under the stage position \((x_{r,y})\) of 48 mm where the tap tests for modeling were not performed. Figure 7d shows the linear interpolation-based and measured FRF when the stage position was 48 mm. The linear interpolation-based FRF fitted well with the measured FRF under the high coefficient of determination of 0.940. The linear interpolation-based method could model the stage position dependency of the dynamics. This is because the tap tests for modeling were performed at short intervals of 32 mm. Although the coefficient of determination under the stage position of 48 mm was slightly lowered compared to those under the stage positions used for modeling, this is because the dynamics having the stage position–dependent non-linearity was modeled using the linear model. To further improve the modeling accuracy, performing tap tests for modeling at shorter intervals is required. The validity of the linear interpolation–based method was demonstrated.

4.3 Comparison of modeling methods for dynamics depending on the stage position

The previous study showed that the axial stiffness of a ball screw and stage position had a relationship that can be modeled using a quadratic function. On the other hand, in the quadratic function-based method, the viscous damping coefficient was assumed to be constant, and its stage position dependency was not modeled for simplicity [22]. However, the viscous damping has the stage position dependency because of the position dependency of the friction force generated in a feed drive [19]. Therefore, the linear interpolation-based method for modeling the stage position dependency is rational, compared to the quadratic function-based method, because the linear interpolation-based method can model the position dependency of the viscous damping. In this study, the modeling accuracy of the linear interpolation-based method was compared with that of the quadratic function-based method.

In the quadratic function-based method, the relationship between the modal stiffness of each mode and stage position was modeled as a quadratic function. The modal stiffness of \(i\) th mode can be modeled as a quadratic function as follows:

\[
k_i = a_{1,i}x_i^2 + a_{2,i}x_i + a_{3,i}
\]

(13)

The modal parameters of \(i\) th mode considering the position dependency of the modal stiffness can be derived as follows:

\[
a_i = \frac{1}{k_i} = \frac{1}{a_{1,i}x_i^2 + a_{2,i}x_i + a_{3,i}}
\]

(14)
\[ \omega_i = \sqrt{\frac{k_i}{m_i}} = \sqrt{\frac{a_{1i}x_i^2 + a_{2i}x_i + a_{3i}}{m_i}} \quad (15) \]

\[ \zeta_i = \frac{c_i}{2\sqrt{m_i k_i}} = \frac{c_i}{2\sqrt{m_i (a_{1i}x_i^2 + a_{2i}x_i + a_{3i})}} \quad (16) \]

where \( m_i \) and \( c_i \) are the modal mass and damping coefficient. Assuming that the stage position dependencies of the modal mass and damping coefficient are ignorable, Eqs. (14)–(16) can be transformed as follows:

\[ a_i = \frac{1}{b_{1a,i}x_i^2 + b_{2a,i}x_i + b_{3a,i}} \quad (17) \]

\[ \omega_i = \sqrt{b_{1o,i}x_i^2 + b_{2o,i}x_i + b_{3o,i}} \quad (18) \]

\[ \zeta_i = \frac{1}{\sqrt{b_{1\zeta,i}x_i^2 + b_{2\zeta,i}x_i + b_{3\zeta,i}}} \quad (19) \]

The model coefficients in Eqs. (17)–(19) were calculated from the modal parameters measured when the stage was positioned at motor side \((x_{z1} = 0\text{mm})\), center point \((x_{z2} = 32\text{mm})\), and anti-motor side \((x_{z3} = 64\text{mm})\).

To compare the linear interpolation-based method with the quadratic function-based method, a tap test was performed when the stage position \(x_{z1}\) was 48 mm where the tap tests for modeling were not conducted. The FRFs calculated based on the linear interpolation model and quadratic function model were compared with the measured FRF through the tap test. Figure 8 shows the comparison results of the FRFs. The coefficient of determination of the linear interpolation model was 0.940 and that of the quadratic function model was 0.916. The linear interpolation-based model and quadratic function model could approximately represent the measured FRF obtained from the tap test. However, the modeling accuracy of the quadratic function-based method was slightly lowered, compared to that of the linear interpolation-based method. This may be because the quadratic function-based method did not consider the position dependency of the viscous damping. The linear interpolation-based method could reasonably model the dynamics depending on the stage position, because the tap tests for modeling were performed at short intervals of 32 mm. The validity of the linear interpolation-based method for modeling the stage position dependent dynamics was supported by comparing it with the quadratic function-based method.

When the stroke of a feed drive in a machine tool becomes large, the influence of the stage position-dependent non-linearity of the viscous damping is expected to become stronger. The modeling accuracy of the quadratic function-based method may be further lowered in that case. On the other hand, the linear interpolation-based method is supposed to reasonably model the stage position dependency of dynamics by conducting tap tests for modeling at short intervals. Therefore, when the stroke of a feed drive becomes large, the linear interpolation-based method is expected to be more advantageous than the quadratic function-based method. However, the interval size of tap tests for modeling is required to be further verified in a machine tool with large strokes.

### 4.4 Tuning noise covariance matrices of the Kalman filter

The covariance matrices of the system noise \((Q)\) and measurement noise \((R)\) need to be determined for the state estimation of the Kalman filter. The covariance matrix of the measurement noise was calculated from the relative position signal along the \(Y\)-axis when the \(Y\)-stage was stopped. Subsequently, the covariance matrix of the system noise was tuned based on the impulse force estimation results of the tap tests conducted along the \(Y\)-axis. The impulse forces generated by the tap tests conducted at the three stage positions \((x_{y3} = 0, 32, 64\text{mm})\) were estimated based on the relative position using the proposed Kalman filter. The covariance matrix of the system noise was tuned such that the gain characteristics between the measured and estimated impulse forces became unity. The tuned gain characteristics at the three stage positions are presented in Fig. 9. Here, the gain characteristics of the LDOB and current-based impulse force estimation were also depicted in Fig. 9 for comparison. The impulse force in the \(Y\)-direction was estimated using the LDOB and current-based methods, as follows [19, 22]:

\[
\hat{F}_{h,LDOB,Y} = -M \ddot{x}_{t,y} - C \dot{x}_{t,y} + K x_{t,y}
\]

---

Fig. 8 Frequency response functions obtained from the linear interpolation model, quadratic function model, and tap test when the stage position \(x_{z1}\) was 48 mm

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\[ F_{h,LDOB,Y} = -M \ddot{x}_{t,y} - C \dot{x}_{t,y} + K x_{t,y} \quad (20) \]
The impulse force was overestimated in a wide frequency range by the LDOB owing to the modeling error of the dual-inertia model in the Y-stage, shown in Fig. 9. The current-based method underestimated the impulse force above approximately 200 Hz owing to the low sensitivity of the motor current in the high-frequency range. On the other hand, the proposed method approximately achieved ideal unity gain characteristics. The proposed method successfully estimated the impulse force from 50 to 1000 Hz in each stage position, although the estimation accuracy was lowered near the anti-resonance frequencies (430 Hz, 542 Hz, and 890 Hz), where the accuracy of the curve fitting deteriorated. This is because the multiple-inertia dynamics in the Y-stage were reasonably modeled through tap tests using the relative position signal, which had a high sensitivity in the high-frequency region. The position dependencies of the covariance matrices of the system and measurement noise were not confirmed. The tuned covariance matrices of the system and measurement noise are reported in Table 2. The proposed Kalman filter-based cutting force observer was constructed based on the tuned covariance matrices of the system and measurement noise.

### 4.5 Effectiveness of modeling dynamics depending on stage position

An end milling test was conducted to verify the effectiveness of modeling the dynamics depending on the stage position using linear interpolation. The experimental conditions are presented in Table 3. The end milling test was conducted by feeding the X-stage when the Y-stage was stopped at the anti-motor side. The cutting force along the stationary Y-axis was estimated by the Kalman filter based on two approaches: using modal parameters considering the stage position dependency by linear interpolation and using modal parameters obtained when the stage was positioned at the center point (i.e., using modal parameters not considering the stage position dependency). The estimated cutting forces based on the two patterns were compared with the cutting force measured using a dynamometer. Low-pass filters (LPFs) were applied to the estimated and measured cutting force signals to limit the bandwidth to the modeled frequency range.

The cutting force estimation results for the two patterns are shown in Fig. 10. In the time domain, the estimated cutting force based on the modal parameters considering the stage position dependency was closer to the measured cutting force than that based on the modal parameters without considering the stage position dependency. In the frequency domain, the spectrum errors of the DC and 200 Hz components decreased by 60% and 51%, respectively, owing to the modeling of the stage position dependency. While the gain characteristics of the FRF were relatively small at 200 Hz, the influence of the position dependency on the cutting force estimation became stronger at 200 Hz than that at other frequencies, as shown in Fig. 6. Additionally, because the DC component is mainly estimated based on the stiffness of the model, the estimation accuracy of the DC component improves when the modeled FRF is closer to the actual FRF. For these reasons, the DC and 200 Hz components of the cutting force were estimated more accurately by modeling the stage position dependency. The improvement in the cutting force estimation accuracy achieved by modeling the stage position dependency was experimentally confirmed.

![Fig. 9 Gain characteristics of the impulse force estimation using the motor current-based, LDOB-based and proposed methods obtained from tap tests conducted under various stage positions: a At x_{t,y} = 0 mm, b at x_{t,y} = 32 mm, and c at x_{t,y} = 64 mm](image)

\[
\hat{F}_{h,\text{CUR},y} = \frac{1}{R} K_{I_{\text{ref}}} y_{\text{a},y}
\]

(21)

Table 3 Experimental conditions for verifying the modeling dynamics depending on the stage position

| Feed direction | +X |
| Estimation axis | Y |
| Cutting tool | Square end mill (φ6.0mm, 2 flute) |
| Workpiece material | Al alloy (A7075) |
| Position of Y-stage x_{t,y} (mm) | 64 |
| Spindle speed (min⁻¹) | 3000 |
| Depth of cut (min) | Radial : 1.5; axial : 1.5 |
| Immersion type | Quarter immersion down milling |
| Feed per tooth (mm/tooth) | 0.03 |
| Cutoff freq. of LPF (Hz) | 1000 |

Table 2 Tuned covariance matrices of system and measurement noise

| Covariance matrix of system noise \( Q \) | \( [5.0 \times 10^6] \) |
| Covariance matrix of measurement noise \( R \) | \( [1.8 \times 10^{-17}] \) |
4.6 Verification of proposed cutting force estimation method

End milling tests under various stage positions and spindle speeds were conducted to verify the proposed cutting force estimation method. The experimental conditions are reported in Table 4. The cutting force along the stationary $Y$-axis generated by feeding the $X$-stage was estimated using the motor current-based, LDOB-based, and proposed methods. Then, the cutting forces estimated by the three methods were compared with the cutting force measured by the dynamometer. The estimated and measured cutting force signals were low-pass filtered to limit the bandwidth to the modeled frequency range.

Figures 11 and 12 show the cutting force estimation results in the time and frequency domains at spindle speeds of 4500 min$^{-1}$ and 6000 min$^{-1}$, respectively. The motor current-based method could not estimate the cutting force.

Table 4 Experimental conditions for verifying the proposed method. (a) Common conditions and (b) spindle speeds and positions of $Y$-stage in end milling tests

| (a) | (b) |
|---|---|
| **Feed direction** | +X |
| **Estimation axis** | $Y$ |
| **Cutting tool** | Square end mill ($\phi 6.0$mm, 2 flute) |
| **Workpiece material** | Al alloy (A7075) |
| **Depth of cut (mm)** | Radial : 1.5; axial : 1.5 |
| **Immersion type** | Quarter immersion down milling |
| **Feed per tooth (mm/tooth)** | 0.03 |
| **Cutoff freq. of LPF (Hz)** | 1000 |
| **Test name** | Spindle speed (min$^{-1}$) | Position of $Y$-stage $x_{cs}$ (mm) |
| Test 1 | 4500 | 32 |
| Test 2 | 6000 | 0 |
| Test 3 | 9000 | 64 |
| Test 4 | 15,000 | 48 |
components of the tooth-passing frequencies (i.e., 150 Hz at 4500 min\(^{-1}\) and 200 Hz at 6000 min\(^{-1}\)) owing to the dynamic structural modes in the low frequency region. Although the motor current had a sensitivity less than approximately 200 Hz, as shown in Fig. 9, the influence of the dynamics in the low-frequency region remained prominent. On the other hand, the cutting force components of the higher harmonics, namely, above 200 Hz, were scarcely sensed using the motor current-based method. The LDOB-based method overestimated the cutting force components in the relatively high-frequency region. This can be attributed to a modeling error in the dual-inertia model regarding the \(Y\)-stage with multiple-inertia dynamics, as shown in Figs. 5 and 9. In addition, the LDOB-based method could not estimate the DC component of the cutting force. In the LDOB-based method, the DC component of the cutting force is estimated using the relative position signal based on the axial stiffness of the dual-inertia model [22]. However, when the modeling error in the dual-inertia model was large, owing to the complicated structural dynamics, the cutting force estimation of the DC component was difficult using the LDOB. On the other hand, the proposed method successfully estimated the wideband cutting force independent from the stage position, as shown in Figs. 11 and 12. The error ratios of the root mean square errors (RMSE) to the force amplitudes in the current and LDOB-based methods were 39.9% and 74.2% at a spindle speed of 4500 min\(^{-1}\) and 32.4% and 62.4% at a spindle speed of 6000 min\(^{-1}\), respectively. The error ratios of the proposed method at spindle speeds of 4500 min\(^{-1}\) and 6000 min\(^{-1}\) were 18.5% and 22.3%, respectively. The proposed method achieved an improved estimation accuracy compared with the conventional current and LDOB-based methods. This is because the multiple inertia dynamics in the \(Y\)-stage with the stage position dependency was modeled reasonably up to 1000 Hz using the relative position signal, which has a high sensitivity in the high-frequency range. Furthermore, the proposed method could estimate the DC component of the cutting force using the relative position signal based on the stiffness of the model.

Figures 13 and 14 show the cutting force estimation results for relatively high spindle speeds of 9000 min\(^{-1}\) and 15,000 min\(^{-1}\). The motor current–based method scarcely sensed the cutting forces because the tooth passing frequencies (i.e., 300 Hz at 9000 min\(^{-1}\) and 500 Hz at 15,000 min\(^{-1}\)
exceeded 200 Hz, which is the approximate limit of the current-based cutting force estimation. The LDOB-based method significantly overestimated the cutting force owing to the modeling error in the dual-inertia model regarding the $Y$-stage. However, the proposed method could estimate the cutting force at relatively high spindle speeds, as shown in Figs. 13 and 14. Although the end milling test at a spindle speed of 15,000 min$^{-1}$ was conducted under a stage position that was not used in modeling ($x_{y} = 48$ mm), the proposed method successfully estimated the cutting force using the model considering the stage position dependency. The error ratios of the current and LDOB-based methods were 32.5% and 162% at a spindle speed of 9000 min$^{-1}$ and 26.2% and 160% at a spindle speed of 15,000 min$^{-1}$, respectively. The error ratios of the proposed method at spindle speeds of 9000 min$^{-1}$ and 15,000 min$^{-1}$ were 16.0% and 20.3%, respectively. The proposed method achieved a significantly improved estimation accuracy at relatively high spindle speeds. The proposed method could estimate a cutting force component with a bandwidth of 1000 Hz at a spindle speed of 15,000 min$^{-1}$ as shown in Fig. 14. It was experimentally confirmed that the proposed cutting force estimation method has a wide estimation bandwidth of up to 1000 Hz along the stationary axis.

5 Conclusion

For machine tools with full closed-loop controlled ball-screw-driven stages, this study proposes a cutting force estimation method along the stationary axis of a machine tool using a KF based on a multiple inertia model. The proposed method estimates the cutting force using the relative position between the motor and stage. The dynamics depending on the stage position of the ball-screw-driven stage were modeled as a multiple inertia system using linear interpolation. The proposed cutting force estimation method was experimentally verified through end milling tests. The following conclusions were drawn from this study:

1. This study proposed a KF-based cutting force estimation method along the stationary axis of a machine tool based on a multiple inertia model. The proposed method estimates the cutting force using the relative position converted from the rotary and linear encoder signal. Because the linear encoder had high sensitivity in the high-frequency region, the wideband multiple-inertia dynamics of a machine tool were accurately modeled through tap tests.

2. The dynamics depending on the stage position were modeled with linear interpolation through tap tests conducted when the stage was positioned at the motor side, center point, and anti-motor side of the feed drive. The cutting force along the stationary axis was estimated using the proposed method through an end milling test conducted when the stage was positioned at the anti-motor side. The effectiveness of the modeling stage position dependency was demonstrated.

3. End milling tests under various stage positions and spindle speeds were conducted to verify performance and accuracy of the proposed cutting force estimation method. The cutting force estimation accuracy was significantly improved by the proposed method compared with that achieved by the conventional current and LDOB-based methods in a stationary axis. This is because the proposed method accurately modeled the wideband dynamics using the relative position through tap tests. Furthermore, it was experimentally confirmed that the proposed cutting force estimation method had a wide estimation bandwidth of up to 1000 Hz.

In this paper, machine tool dynamics between a cutting point and multi-encoders were measured through tap tests. However, machine tool dynamics vary when a cutting tool or tool holder is changed. Considering the application to production facilities, it is time-consuming to re-do tap tests when a cutting tool or tool holder is changed. The receptance coupling method will improve the flexibility of the proposed method for
changes in a cutting tool and tool holder [10]. The receptance coupling method can combine the tap-test-based-FRFs between a spindle and multi-encoders with the analytically calculated FRFs of a cutting tool and tool holder using the finite element method or beam theory. The proposed method estimates the cutting force based on the combined FRFs between a cutting point and multi-encoders. This procedure will reduce the time and cost for tap tests when a cutting tool or tool holder is changed and increases the practicality of the proposed method. We plan to work on constructing this procedure in the future.

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**Declarations**

**Ethics approval** Not applicable.

**Consent to participate** Not applicable.

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