Codeword Extension for Polar Decoding and CNN-aided Adaptation

Naoya Tanuma\textsuperscript{1a)}, Yuta Goto\textsuperscript{1}, Kazuki Maruta\textsuperscript{2b)} and Chang-Jun Ahn\textsuperscript{1}
\textsuperscript{1)} Graduate School of Engineering, Chiba University
1-33 Yayoi-cho, Inage-ku, Chiba, 263-8522, Japan
\textsuperscript{2)} Academy for Super Smart Society, Tokyo Institute of Technology,
2-12-1, Ookayama, Meguro-ku, Tokyo, 152-8552, Japan
1) ntanuma@chiba-u.jp, b) kazuki.maruta@m.ieice.org

Abstract:

Decoder of polar coded bits requires the successive cancellation (SC). Its capability is known to be limited when the code length is shorter. Encoding block size for the polar coding is predetermined and the block is divided into two types: frozen bits that contain fixed zeros and information bits that contain data. This letter proposes a new decoding scheme that artificially extends the block length by concatenating the copied codeword at the receiver side, focusing on the polar codes’ generator matrix characteristics. Since it simply doubles the computation load, we additionally apply a convolutional neural network (CNN) to determine its applicability by observing the received signal power spectrum. Computer simulations verifies its effectiveness; improved bit error rate and reduced computation complexity.

Keywords: Polar codes, successive cancellation decoding, OFDM, convolutional neural network

Classification: Wireless communication technologies

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1 Introduction

Polar codes, proposed by Arikan [1], are one of the error correction codes and classified as the linear block codes. Polar codes use a phenomenon called channel polarization for encoding. It generates optimal channels by combining and splitting original channels into polarized channels. This process can attain the capacity of a binary-input discrete memoryless channel (B-DMC). Moreover, polar codes can be implemented with a simple encoder and a simple successive cancellation (SC) decoder in which complexity is known to $O(N \log N)$ where $N$ is code length. Therefore polar codes have been adopted as channel coding for control information part in the fifth generation (5G) mobile communication systems. However, the error correction performance of the simple SC decoder based polar codes is inferior to low-density parity-check (LDPC) codes and turbo codes [2]. To improve the performance of polar codes, various extensions have been investigated such as list decoding [2][3], convolutional neural network (CNN) [4] and deep neural network (DNN) [5]. However, small IoT devices, which have been the focus of much attention in recent years, are not capable of large amount of calculations; require decoding schemes with small computational complexity [6]. To address this issue, we propose an improved SC decoding by simply extending the received codeword [7]. It can improve the bit error rate (BER) performance without modifying the configuration of the SC decoder. Meanwhile, such codeword extension requires additional computation. We then apply CNN by observing the received signal power spectrum to determine the applicability of the codeword extension. The rest of this letter is organized as follows. Sect. 2 shows a SC decoding for polar codes. In Sect. 3, a SC decoding using extended codeword is presented. Sect. 4 presents computer simulation results and Sect. 5 concludes this letter.

2 Successive Cancellation Decoding for Polar Codes

Consider the $(N, K, u_{\bar{A}})$ polar codes, where $N$, $K$ and $u_{\bar{A}}$ are code length, coding rate and frozen bit indices, respectively. $u_{\bar{A}}$ is transmitted across $W_N$ and a channel output $y_1^N$ is obtained with probability $W_N(y_1^N | u_{\bar{A}}^N)$. The SC decoder observes $(y_1^N, u_{\bar{A}})$ and generates an estimate $\hat{u}_{\bar{A}}^N$ of $u_{\bar{A}}^N$. We may visualize the decoder as consisting of $N$ decision elements (DEs), one for each source element $u_i$; the DEs are activated in the order 1 to $N$. If $i \in \bar{A}$, the element $u_i$ is known. The $i$-th DE simply sets $\hat{u}_i = u_i$ and sends this result to all succeeding DEs. If $i \in A$, the $i$-th DE waits until it has received the previous decisions $\hat{u}_{i-1}^N$, and upon receiving them, computes the
likelihood ratio (LR)

\[ L_N^{(i)}(y_1^N, \hat{u}_i^{i-1}) = \frac{W_N^{(i)}(y_1^N, \hat{u}_i^{i-1} | 0)}{W_N^{(i)}(y_1^N, \hat{u}_i^{i-1} | 1)} \]  

(1)

and generates its decision as

\[ \hat{u}_i = \begin{cases} 
0, & \text{if } L_N^{(i)}(y_1^N, \hat{u}_i^{i-1}) \geq 1 \\
1, & \text{otherwise}
\end{cases} \]  

(2)

which is then sent to all succeeding DEs. This is a single-pass algorithm, with no revision of estimates. The complexity of this algorithm is determined essentially by the complexity of computing the LRs.

A straightforward calculation using the recursive formulas gives

\[ L_N^{(2i-1)}(y_1^N, \hat{u}_i^{2i-2}) = L_N^{(i)}(y_1^{N/2}, \hat{u}_{i/2}^{2i-2} \oplus \hat{u}_{i/2}^{2i-2})L_N^{(i)}(y_1^{N/2}, \hat{u}_{i/2}^{2i-2} \oplus \hat{u}_{i/2}^{2i-2}) + 1 \]  

(3)

and

\[ L_N^{(2i)}(y_1^N, \hat{u}_i^{2i-1}) = L_N^{(i)}(y_1^{N/2}, \hat{u}_{i/2}^{2i-2} \oplus \hat{u}_{i/2}^{2i-2})^{1-2^{2i-1}} \cdot L_N^{(i)}(y_1^{N/2}, \hat{u}_{i/2}^{2i-2}) \]  

(4)

3 Proposed Scheme

The polar encoding is attained with a specified generator matrix whose size is determined according to the code length. Since this generation matrix is composed of the Kronecker product, there is a relationship between the generator matrix and the code length. The key feature of our proposal is to artificially extend the codeword block length by concatenating the copy of the codeword at the receiver side. The detailed procedure is presented as follows.

3.1 SC Decoding Using Extended Codeword

Encoding block size is assumed to be predetermined and the block is divided into two types: frozen bits that contain fixed zeros and information bits that have data. For the SC decoding, the data bits are determined in order from the top of the codeword including the frozen bits. Even if a bit at the index of frozen bits is determined to be 1, it is replaced with 0 to improve the subsequent decoding accuracy. The proposed method improves the BER performance by inserting zeroed bits that work as the frozen bits on the receiver side. Suppose the code length is eight, it can be exemplified as follows.

\[ y = xG, \]  

(5)

\[ G = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}. \]  

(6)
where $y$, $x$ and $G$ are the codeword, the source bits and the generator matrix, respectively. If the number of elements in $0$ and $x$ are equal, the codeword $y$ becomes $[y', y']$ when $x$ is replaced with $[0, x']$. Therefore, Eq. (5) can be rewritten as,

$$[y', y'] = [0, x']G. \quad (7)$$

This transformation is done using the properties of the polar code generation matrix, so that the conventional SC formula can be applied as well. In other words, the transmitter simply performs polar encoding without bit selection, the receiver concatenates the copied codewords and then performs SC decoding which derives $[0, x']$. Since $0$ part works as the frozen bits, the impact of the additive noise on decoding can be mitigate and its performance can be improved. The proposed scheme can be constructed by the following procedures.

1. Determine code length and information data length.
2. Select frozen and information bit positions by using ether density evolution [8] or the Gaussian approximation [9].
3. Calculate channel capacity of frozen bit positions using channel polarization.
4. Treat these channels as information bits and encode them.
5. Perform modulation, transmission, and reception.
6. The receiver decodes the received signal after copying the received signal to increase the code length.

### 3.2 CNN-aided Adaptation of Codeword Extension

The proposed scheme presented above is quite simple but can improve BER performance. However, the computation complexity is also doubled due to doubled codeword length. The proposed approach is quite effective in improving BER performance when the received codeword experiences a stringent fading channel. In other words, the conventional SC decoding is sufficient depending on the channel condition which independently varies. Based on the above empirical observations, we additionally introduce an adaptive decision framework that switches between the traditional SC decoder and the proposed codeword extension depending on whether an error is likely to occur or not. Focusing on the impact of the fading channel is directly reflected in the power spectrum of the received signal, this spectrum is exploited to train CNN. It can then be incorporated into the receiver to switch between decoding schemes and can suppress the increase of computation complexity. The overview of the proposed system is drawn in Fig. 1.

### 4 Computer Simulation

#### 4.1 Simulation Parameters

The effectiveness of the proposed scheme is verified through a computer simulation. We compared the BER performance of the proposed schemes (Prop. SC and Prop. SC with CNN) with the conventional scheme (Conv. SC). Computer simulation is conducted based on the QPSK-modulated orthogonal frequency division multiplexing (OFDM) transmission. The codeword length is 2048 and the coding rate is $1/2$. 

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Random bit interleaving is also applied to the codeword. Transmission bandwidth is 20 MHz. The channel model follows 15 paths Rayleigh fading with exponential decay, its interval is 50 ns and the maximum Doppler frequency is 10 Hz. The number of Fast Fourier transform (FFT) points is 64 and the cyclic prefix (CP) has 16 samples. The number of subcarriers is also 64, and 16 OFDM symbols make up a transmission frame.

Let the CNN learn the sequence of power spectra, i.e. spectrogram, to switch decoding schemes. CNNs are used when $\text{Eb}/\text{No}$ is 16 dB or higher. We prepared two CNNs trained with $\text{Eb}/\text{No} = 15$ dB (defined as CNN1) and 20 dB (CNN2), respectively. CNN1 is used for $16 \leq \text{Eb}/\text{No}[\text{dB}] < 20$ and CNN2 is used for $20 \leq \text{Eb}/\text{No}[\text{dB}]$.

### 4.2 Simulation Results

Figure 2 shows the BER performance. The proposed SC decoding exhibits improved BER better than the conventional by about 4 dB at $\text{BER}=10^{-4}$ basis. This result confirms that the proposed codeword extension with the SC decoder can reduce the impact of receiver noise contributing to BER improvement. Moreover, we compare the CNN-aided proposed SC decoding with the conventional scheme. Although the CNN-aided adaptation slightly imposes a penalty, it can maintain superior BER characteristics to the conventional by about 3 dB. Here, the probability that the codeword extension has been applied, $P$, is 22% at $\text{Eb}/\text{No} = 16$ dB and 6.8% at $\text{Eb}/\text{No} = 20$ dB, respectively.

Table I shows the computation complexity of each decoding scheme. It is defined as the number of complex multiplications required for the polar decoding. The computational quantities, $C$, including the CNN-aided adaptation are estimated as follows:

$$C = C_{\text{Conv}}(1 - P/100) + C_{\text{Prop}}P/100 + C_{\text{CNN}}$$

where $C_{\text{Conv}}$, $C_{\text{Prop}}$, and $C_{\text{CNN}}$ denote computational complexities for the conventional SC decoding, the proposed codeword extension, and the classification of spectrum by CNN, respectively. From Table I, we can see that the computation complexity of the proposed SC decoding is about twice as large as that of the conventional
one. By introducing the CNN-aided adaptation, the complexity can be alleviated by about 25\% at \( \text{Eb}/\text{No} = 16 \text{ dB} \), and by about 35\% at \( \text{Eb}/\text{No} = 20 \text{ dB} \).

![Figure 2](image.png)  
**Fig. 2.** BER performance (Prop. SC with CNN vs. Conv. SC and Prop. SC).

| Schemes                              | Computation complexity |
|--------------------------------------|------------------------|
| Conventional SC                      | 22528                  |
| Proposed SC by codeword extension    | 49152                  |
| CNN classification                   | 8200                   |
| Proposed SC with CNN (Eb/No=16 dB)   | 36585                  |
| Proposed SC with CNN (Eb/No=20 dB)   | 32538                  |

**Table I.** Computation complexity comparison

5 Conclusion

This paper proposed the new polar decoding scheme by exploiting features of the generator matrix. Although this approach is computationally intensive, it can reduce the impacts of additive noise and fading channel to improve the BER performance of polar coded OFDM systems. To reduce the computation load, we additionally proposed the adaptive decision framework using CNN, to switch decoding schemes. From the simulation results, our overall proposed approach can improve BER performance by 3 dB compared to the conventional SC decoding while reducing the computation complexity by up to 35\%.