The predictive performance of liquidity risk

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Abstract: This paper assesses the explanatory power of the liquidity-risk-based pricing models relative to the Fama–French three-factor model (FF3) and the extensions to the FF3. We find that the liquidity-augmented capital asset pricing model (LCAPM) performs no worse but generally better than other models considered in describing liquidity risk and a variety of anomaly portfolios. Our finding remains intact relative to the troublesome portfolios related to small, value, and aggressive investment. This study highlights that liquidity risk is not negligible, which is in contrast to some recent findings that the price-impact-based liquidity risk factor contributes little to explain average returns.

Subjects: Economics; Finance; Business, Management and Accounting

Keywords: Liquidity risk; portfolio returns; model performance

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PUBLIC INTEREST STATEMENT

A large number of empirical tests show that liquidity risk has the ability to explain the expected stock returns. However, due to the multi-dimensional characteristics of liquidity risk, liquidity risk factors constructed with different liquidity proxies display different abilities to capture portfolio returns. This paper mainly examines the predictive performance of two influential liquidity-risk-based models in comparison with other popular asset pricing models for a series of portfolios. Our results indicate that the Liu (2006) liquidity-augmented capital asset model captures investment portfolio returns well, and therefore provides a reference for investment decision-making and financial economics research.
1. Introduction

There is little doubt that the Fama and French (1993) three-factor model (FF3) has been the most commonly used one in empirical research since its development. Recently, Fama and French (2015) extend their three-factor model to a five-factor model (FF5), which is likely to be popular again. However, both the Fama–French models do not consider liquidity risk in explaining the cross-section of average stock returns. This is because Fama and French (2015) find that the Pástor and Stambaugh (2003) factor, which is proposed to capture liquidity risk, adds little power to account for the cross-section of expected return. However, the Pástor–Stambaugh (PS) liquidity factor is constructed based on their price impact measure, which does not command a significant liquidity premium. There are other liquidity factors proposed in the literature and some of them is constructed with a liquidity proxy other than the price impact measure. For instance, the Liu (2006) liquidity risk factor is based on the trading-continuity measure of liquidity, which captures multi-dimensions of liquidity and generates a robust premium. Consequently, this paper explores whether liquidity risk is negligible and whether the liquidity risk models improve the explanatory power of expected stock returns.

Empirically, we first compare model performance in capturing liquidity risk. We examine the liquidity-risk-based models, the LCAPM and the PS model, and several popular asset pricing models, including the CAPM, the FF3, the Carhart (1997) momentum-extended FF3, and the FF5. The evidence shows that except for the LCAPM, none of the other models can explain the liquidity premium and the portfolio returns formed on the trading-continuity measure of Liu (2006). In particular, the FF5 shows the worst performance among all factor models as indicated by leaving a large number of portfolios unexplained. Moreover, for portfolios sorted by other liquidity measures, such as turnover, trading volume, price impact, and bid-ask spread measures, the LCAPM also exhibits consistent loadings on the liquidity factor and accounts for the liquidity premium that other models fail to explain. The PS model, on the other hand, shows limited improvement to the performance of the FF3. Similar to the finding of Fama and French (2015), the loadings on the PS liquidity factor are generally insignificant even with the portfolios formed on the Amihud (2002) price impact measure of liquidity. Accordingly, we next investigate the LCAPM performance in comparison with the non-liquidity-risk-based pricing models for explaining anomalies.

Following the Fama and French (2015) testing procedure, we evaluate the model performance under three sets of testing portfolios: the 25 size and book-to-market portfolios, the 25 size and momentum portfolios, and the 32 size, profitability, and investment portfolios. Although the testing portfolios are in favor of supporting the characteristics-based models such as FF3 and its extensions, the results show that the LCAPM does a good job in accounting for the portfolio returns. To a large extent, the LCAPM performs no worse but better than the PS model and non-liquidity-risk-based models considered. For instance, with the 25 size-momentum portfolios, the LCAPM outperforms all other models including the Carhart (1997) momentum-extended FF3. In addition, the LCAPM performs well in explaining the average returns of the “troublesome” portfolios related to small, value, aggressive investment, etc. The explanatory power of the LCAPM clearly lies in the liquidity factor that absorbs the patterns in average returns.

Our work reinforces the assertion that liquidity risk is important for asset pricing. The identification of the LCAPM as a preferable pricing model is helpful for investors to make investment decisions, and has important implications in finance research. Our results provide convincing evidence against some recent studies to ignore liquidity risk in asset pricing.

The rest of the paper is organized as follows. In Section 2, we describe the data and performance metrics used in empirical analysis. Section 3 investigates the model performance in capturing liquidity risk. Section 4 examines the ability of factor models in explaining anomaly returns. Section 5 concludes.
2. Data and performance metrics

2.1. Testing portfolios and asset pricing models

In this paper, we employ two groups of test assets in empirical analysis. The first group is liquidity-risk-based portfolios, which are used to compare the performance of factor models in capturing liquidity risk. We sort the NYSE/AMEX/ARCA common stocks into decile portfolios S (the most liquid decile) through B (the least liquid decile) based on the monthly returns of five different liquidity measures, namely, LM12, TO12, DV12, RV12, and BA12, where LM12 is the standardized turnover-adjusted number of zero daily trading volumes over the prior 12 months in Liu (2006); TO12 is the average daily turnover over the prior 12 months (where daily turnover is the ratio of the number shares traded on a day to the number of shares outstanding on the day); DV12 is the average daily dollar trading volume over the prior 12 months; RV12 is the Amihud (2002) price impact measure of liquidity, that is, the daily ratio of the absolute return on a day to the dollar volume on the day averaged over the prior 12 months; and BA12 is the average daily relative bid-ask spread over the prior 12 months. The second group of test assets are used to test factor models’ explanatory power for anomaly portfolios. We test three sets of portfolios, including the value-weighed monthly returns on the 25 size and book-to-market (Size-B/M) portfolios, the 25 size and momentum (Size-Mom) portfolios, and the 32 size, profitability and investment (Size-OP-Inv) portfolios.

For asset pricing models, we examine the liquidity-augmented capital asset pricing model (LACAPM) of Liu (2006), the four-factor model of Pástor and Stambaugh (2003) (PS), the CAPM of Sharpe (1964) and Lintner (1965), the three-factor model of Fama and French (1993) (FF3), the momentum-extension model of Carhart (1997) (C4), and the five-factor model of Fama and French (2015) (FFS). Empirically, for each set of testing assets, we perform the following time-series regressions:

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \beta_{i,LF}LF_t + \epsilon_{i,t} \]  

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,PSF}PSF_t + \epsilon_{i,t} \]  

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \epsilon_{i,t} \]  

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \epsilon_{i,t} \]  

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,UMD}UMD_t + \epsilon_{i,t} \]  

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,MKT}R_{MKT,t} + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t + \beta_{i,LS}LSM_t + \epsilon_{i,t} \]

where \( R_{it} \) is the return on asset \( i \) in month \( t \); \( R_{ft} \) is the risk-free rate in month \( t \); \( LF_t \) is the liquidity factor of Liu (2006); \( PSF_t \) represents the traded liquidity factor of Pástor and Stambaugh (2003); \( MKT_t, SMB_t, \) and \( HML_t \) are the market, size and value factors in the three-factor model of Fama and French (1993), respectively; \( UMD_t \) is the momentum factor in Carhart (1997); \( SMB_t^2, RMW_t \) and \( CMA_t \) are the size, profitability and investment factors in Fama and French (2015).

We examine the US data and the sample period is from July 1968 to June 2017. The data on \( PSF \) and \( LF \) are obtained from the web pages of Stambaugh, and Liu, respectively. Other factor data and the anomaly portfolios (the 25 Size-B/M, the 25 Size-Mom, the 32 Size-OP-Inv) are obtained from Kenneth French's data library.
Table 1. Descriptive statistics

Panel A: Means, standard deviations, and t-statistics

|       | MKT  | SMB  | SMB* | HML  | UMD  | RMW  | CMA  | LF   | PSF  |
|-------|------|------|------|------|------|------|------|------|------|
| Mean(%) | 0.50 | 0.13 | 0.17 | 0.35 | 0.63 | 0.27 | 0.33 | 0.62 | 0.37 |
| Std(%)  | 4.52 | 3.09 | 3.03 | 2.90 | 4.32 | 2.24 | 2.00 | 3.44 | 3.40 |
| t-statistic | 2.67 | 1.05 | 1.33 | 2.95 | 3.52 | 2.97 | 4.05 | 4.38 |      |

Panel B: Spearman correlated matrix

|       | MKT  | SMB  | SMB* | HML  | UMD  | RMW  | CMA  | LF   | PSF  |
|-------|------|------|------|------|------|------|------|------|------|
| MKT   | 1.00 | 0.28 | 0.25 | −0.27| −0.12| −0.23| −0.34| −0.69| −0.04|
| SMB   | 1.00 | 0.98 | 0.15 | −0.15| −0.04| −0.29| −0.14| −0.19| −0.01|
| SMB*  | 1.00 | −0.06| −0.04| −0.27| −0.09| −0.17| −0.17|      |      |
| HML   | 1.00 | −0.14| −0.17| 0.69 | 0.35 | 0.03 |      |      |      |
| UMD   | 1.00 | 0.17 | 0.01 | −0.01| 0.12 | 0.05 |      |      |      |
| RMW   |     |      |      |      |      |      |      |      |      |
| CMA   |     |      |      |      |      |      |      |      |      |
| LF    | 1.00 | 0.41 | 0.01 |      |      |      |      |      |      |
| PSF   |     |      |      |      |      |      |      |      |      |

Table 1 displays summary statistics for monthly factor returns. From Panel A, the factor with the highest average return is UMD, 0.63% per month (t = 3.52), followed by LF, 0.62% per month (t = 4.38), and the market factor MKT, 0.5% per month (t = 2.69). PSF also earns a significant average return, 0.37% per month (t = 2.67). Over our sample period, MKT has the highest volatility, with a standard deviation of 4.52% per month, while CMA exhibits the lowest volatility, only 2% per month. From Panel B, the correlation coefficient between LF and PSF is tiny, just 0.03, which indicates that the two factors measure liquidity from different perspectives. In addition, LF has moderate correlations with HML and CMA, with correlation coefficients above 0.35. In contrast, PSF has weak correlations with all pricing factors, as indicated by correlation coefficients below 0.1.

2.2. Performance metrics

Following empirical literature (e.g., Ahmed et al., 2019; Fama & French, 2015, 2016, 2018; Hou et al., 2015, 2017), we run time-series regressions of each set of testing portfolios on factor models, and assess model performance based on several metrics. Specifically, we examine the GRS-statistic of Gibbons et al. (1989), which is used to investigate whether the estimated intercepts from time-series regression of testing portfolios on a factor model are jointly zero; the average absolute intercept, \( A|\alpha_i| \), which is used to measure the average deviation of a set of testing assets from a given model; the ratio of the average of intercepts’ squared sample standard errors to the average squared intercept, \( \frac{A^2(|\alpha|)}{A^2|\alpha_i|} \), which is used to measure the proportion of unexplained alpha dispersion due to sample errors; the number of abnormal intercepts (alphas) generated by factor models under a given set of portfolios; and the average of time series \( R^2 \)'s, \( A(R^2) \). In addition, we present a new performance metric, the model's maximum square share ratio, \( sh^2(f) \). Different from other metrics, the model ranking based on \( sh^2(f) \) does not depend on the choice of testing assets.4

3. Model performance in capturing liquidity risk

In this section, we examine the model performance in capturing liquidity risk. First, we compare liquidity-risk-based models relative to the non-liquidity-risk-based models in capturing portfolios formed on the trading-continuity measure of Liu (2006). Then, we further test the ability of LCAPM to explain the liquidity risk measured by other liquidity measures.
3.1. Model performance under the LM12-sorted portfolios

Table 2 reports the performance metrics of all factor models for value-weighted and equally weighted portfolios sorted by the LM12 measure. From Panel A, most models are rejected by the GRS test, as indicated by their significant p-values associated with GRS statistics. The exceptions are the LCAPM and C4 models, which produce GRS statistics of 0.675 (p = 0.749) and 1.733 (p = 0.062), respectively. Among competing models, the LCAPM captures all LM12 value-weighted portfolios and dominates other factor models on most measures. Specifically, the LCAPM shows the smallest deviation from testing assets by generating the point estimate of 0.063% for A(aj). In terms of the As2(aj)/Aaj2 metric, the largest estimates of 1.479 indicate that the unexplained alpha dispersion from the LCAPM largely comes from sample errors. The LCAPM also produces the highest estimate of 0.129 for the sh2(f) metric. A large sh2(f) estimate implies the smaller pricing error and better model performance of the LCAPM. The PS does not show satisfied explanation for the LM12-sorted portfolios. In fact, the PS appears to perform no better than the FF3 based on most metrics. For example, the estimates of the GRS statistic and A(aj) of the PS model are higher than those of the FF3, and the As2(aj)/Aaj2 estimate of the PS is lower than that of FF3. In addition, the FF3, PS and C4 all produce A(R2) estimates at 0.836, which indicates that adding PSF or UMD is with little help to improve the explanation of FF3 for the LM12-based portfolios. Except for looking at the largest estimate of 85.1% for A(R2), the FF5 performs worse than the other models based on most metrics. It generates the largest estimate of 3.95 for the GRS statistic, the largest estimate of 0.185% for A(aj), and also leaves seven significant intercepts among 10 portfolios. Similarly, the results from the equally weighted LM12-based portfolios (Panel B of Table 2) show a consistent conclusion that the LCAPM appears to be the best performer among competing models.

3.2. Liquidity risk measured by other liquidity measures

The existing literature has developed several liquidity measures, focusing on capturing different dimensions of liquidity. In this subsection, we further examine the ability of LCAPM in explaining portfolios sorted by TO12, DV12, RV12 and BA12 measures. To save space, we only report results for the value-weighted portfolios.

Table 2. Model performance metrics on portfolios sorted by LM12 measure

| Model | GRS | p(GRS) | A(aj)(%) | As2(aj)/Aaj2 | A(R2) | sh2(f) | n |
|-------|-----|--------|----------|--------------|--------|--------|---|
| Panel A: LM12 value-weighted portfolios | | | | | | | |
| CAPM  | 2.433 | 0.008 | 0.177 | 0.185 | 0.806 | 0.012 | 3 |
| LCAPM | 0.675 | 0.749 | 0.063 | 1.479 | 0.822 | 0.129 | 0 |
| PS    | 2.746 | 0.003 | 0.163 | 0.157 | 0.836 | 0.049 | 6 |
| FF3   | 2.288 | 0.012 | 0.150 | 0.182 | 0.836 | 0.038 | 5 |
| C4    | 1.773 | 0.062 | 0.136 | 0.252 | 0.836 | 0.081 | 3 |
| FF5   | 3.950 | 0.000 | 0.185 | 0.165 | 0.851 | 0.104 | 7 |
| Panel B: LM12 equally-weighted portfolios | | | | | | | |
| CAPM  | 6.083 | 0.000 | 0.194 | 0.206 | 0.773 | 0.012 | 4 |
| LCAPM | 2.205 | 0.016 | 0.113 | 0.577 | 0.789 | 0.129 | 1 |
| PS    | 5.413 | 0.000 | 0.166 | 0.082 | 0.899 | 0.069 | 3 |
| FF3   | 5.743 | 0.000 | 0.167 | 0.079 | 0.898 | 0.038 | 3 |
| C4    | 4.344 | 0.000 | 0.130 | 0.122 | 0.900 | 0.081 | 2 |
| FF5   | 6.387 | 0.000 | 0.244 | 0.052 | 0.916 | 0.104 | 8 |

This table reports performance metrics for six asset pricing models under the LM12-sorted portfolios. For each factor model, we present the GRS statistic of the Gibbons et al. (1989), and the corresponding p-value (pGRS); the average absolute intercept, A(aj); the ratio of the average squared sample standard errors of the estimated intercepts to Aaj2; As2(aj)/Aaj2; the average of time series R2; A(R2); the model’s maximum squared Sharpe ratio, sh2(f); and the number of significant intercepts generated by factor models under each set of testing assets.
Table 3. The performance of LCAPM on portfolios classified by TO12, DV12, RV12 and BA12

|     | S   | D2  | D3  | D4  | D5  | D6  | D7  | D8  | B   | B−S |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Panel A: TO12-based portfolios |
| mean (%) | 0.28 | 0.37 | 0.43 | 0.44* | 0.48* | 0.58* | 0.56* | 0.43* | 0.54* | 0.56* | 0.28 |
| (0.96) | (1.48) | (1.87) | (2.07) | (2.42) | (2.98) | (3.07) | (2.46) | (3.32) | (3.61) | (1.26) |
| LCAPM-adjusted performance |
| α (%) | −0.09 | −0.12 | −0.07 | −0.03 | −0.05 | 0.03 | −0.14 | 0.03 | 0.04 | 0.13 |
| (−0.68) | (−1.22) | (−0.77) | (−0.42) | (−0.75) | (0.35) | (−0.31) | (−1.66) | (0.32) | (0.44) | (0.75) |
| β_{LF} | −0.38* | −0.16* | −0.09* | −0.08* | 0.03 | 0.08* | 0.17* | 0.18* | 0.15* | 0.21* | 0.59* |
| (−8.05) | (−4.61) | (−2.77) | (−2.94) | (1.20) | (2.72) | (5.46) | (5.81) | (5.04) | (5.94) | (9.30) |
| Panel B: DV12-based portfolios |
| mean (%) | 0.40* | 0.59* | 0.59* | 0.70* | 0.63* | 0.67* | 0.69* | 0.68* | 0.72* | 0.32* |
| (2.30) | (3.24) | (3.14) | (3.59) | (3.19) | (3.37) | (3.50) | (3.38) | (3.22) | (3.57) | (2.18) |
| LCAPM-adjusted performance |
| α (%) | −0.07 | 0.08 | 0.08 | 0.18* | 0.08 | 0.13 | 0.14 | 0.05 | −0.05 | −0.12 | −0.04 |
| (−1.51) | (1.24) | (1.12) | (2.29) | (0.99) | (1.40) | (1.50) | (0.43) | (−0.45) | (−1.01) | (−0.30) |
| β_{LF} | 0.03 | 0.06* | 0.04 | 0.07* | 0.07* | 0.09* | 0.19* | 0.29* | 0.46* | 0.44* |
| (1.46) | (2.60) | (1.70) | (1.28) | (2.20) | (1.97) | (2.48) | (4.89) | (6.76) | (10.68) | (8.05) |
| Panel C: RV12-based portfolios |
| mean (%) | 0.42* | 0.57* | 0.64* | 0.62* | 0.62* | 0.68* | 0.67* | 0.69* | 0.63* | 0.63* | 0.21 |
| (2.44) | (3.08) | (3.16) | (3.07) | (3.07) | (3.27) | (3.05) | (3.06) | (2.74) | (2.73) | (1.25) |
| LCAPM-adjusted performance |
| α (%) | −0.06 | 0.07 | 0.13 | 0.11 | 0.10 | 0.15 | 0.10 | 0.04 | −0.08 | −0.27* | −0.20 |
| (−1.28) | (1.09) | (1.78) | (1.44) | (1.12) | (1.63) | (0.95) | (0.34) | (−0.63) | (−1.97) | (−1.21) |
| β_{LF} | 0.05* | 0.03 | −0.00 | 0.00 | 0.03 | 0.02 | 0.05 | 0.14* | 0.21* | 0.46* | 0.41* |
| (2.47) | (1.69) | (−0.02) | (0.06) | (0.92) | (0.69) | (1.31) | (3.24) | (4.55) | (9.16) | (6.68) |
| Panel D: BA12-based performance |
| mean (%) | 0.51* | 0.61* | 0.55* | 0.50* | 0.54* | 0.64* | 0.54* | 0.54* | 0.65* | 0.51* | 0.00 |
| (2.49) | (2.99) | (2.72) | (2.49) | (2.56) | (3.04) | (2.55) | (2.45) | (2.83) | (2.26) | (0.01) |
| LCAPM-adjusted performance |
| α (%) | −0.04 | 0.04 | −0.01 | −0.07 | −0.06 | 0.09 | −0.06 | −0.10 | −0.00 | −0.35* | −0.30 |
| (−0.55) | (0.48) | (−0.20) | (−0.85) | (−0.62) | (0.97) | (−0.59) | (−0.90) | (−0.02) | (−2.50) | (−1.95) |
| β_{LF} | 0.04 | 0.07* | 0.06* | 0.08* | 0.09* | 0.04 | 0.10* | 0.14* | 0.16* | 0.43* | 0.39* |
| (1.40) | (2.68) | (2.36) | (2.75) | (2.73) | (1.17) | (2.83) | (3.36) | (3.20) | (8.45) | (6.84) |

Panel A to Panel D report the performance of LCAPM for portfolios formed on TO12, DV12, RV12, and BA12 measures. S denotes the highest-TO12, the highest-DV12, the lowest-RV12, and the lowest-BA12 portfolios (the most liquid decile); \( \beta \) denotes the lowest-TO12, the lowest-DV12, the lowest-RV12, and the highest BA12 portfolios (the least liquid decile); \( B − S \) denotes the difference between B and S. For each panel, we show decile portfolio excess returns, the intercept estimates of the LCAPM regressions (\( \alpha \)), and the slopes of LF (\( \beta_{LF} \)). Numbers in parentheses are t-statistics, and * denotes significance at the 5% level.

least liquid decile B. In contrast, there is not significant pattern across the decile returns for the BA12-classified portfolios. Among these liquidity-risk-based portfolios, only the DV12-classified portfolios produce significant liquidity premium at 0.32% (\( t = 2.18 \)) per month. The LCAPM well captures liquidity risk measured by these liquidity measures. For example, it captures the spread portfolio and all decile portfolios with intercepts close to zero for the TO12-classified portfolios. By examining the results from Panel B to D, the LCAPM also captures the spread portfolios as well as
most decile portfolios for the DV12-, RV12-, and BA12-sorted portfolios. In contrast, the untabulated results show that the PSF contributes little to the PS model’s explanatory power as the loadings on PSF are mostly insignificant for each set of liquidity-risk-based portfolios.

Overall, the evidence shows that the LCAPM performs well in describing liquidity risk. In addition, the significant liquidity premium robust to competing models for the LM12-based portfolios implies that liquidity risk is an important source of asset pricing.

4. Model performance in explaining anomalies
Fama and French (2015, 2016) suggest that the liquidity risk contributes little to explain stock returns, as they find that the liquidity factor of Pastor and Stambaugh (2003) performs weakly in capturing anomaly portfolios. Therefore, a natural question is whether other liquidity models also show limited power to describe the cross-section of stock returns? In this section, we compare the ability of the LCAPM relative to some popular factor models in explaining a series of anomaly portfolios, including the 25 Size-B/M portfolios, the 25 Size-Mom portfolios, and the 32 Size-OP-Inv portfolios. Since the selected portfolios are constructed based on the same characteristics as the non-liquidity-risk-based factors, we expect that the related models should provide a good performance.

4.1. Model performance metrics
Table 4 reports the performance metrics of factor models for three sets of portfolios. Not surprisingly, all factor models examined are rejected by the GRS test for each set of portfolios. Consistent with our expectation, the C4 model with a momentum factor shows an outstanding performance for the 25 Size-Mom portfolios, and the FF5 model that is designed to capture portfolios associated with investment and profitability performs well on the 32 Size-OP-Inv portfolios. Turning to our main focus, the two liquidity models show distinguishable descriptions of anomaly portfolios. The LCAPM shows good performance for these testing portfolios. For example, the LCAPM generates smaller GRS statistics of 3.159, and produces only five significant intercepts for the 25 Size-B/M portfolios. Although the 25 Size-Mom portfolios prefer to the C4 model, the LCAPM dominates other models by producing the smallest abnormal intercepts of six out of 25 momentum-sorted portfolios. In addition, the LCAPM also well explains the 32 Size-OP-Inv portfolios by leaving only 11 significant intercepts. For each set of portfolios, we find that the LCAPM produces large estimates for $\frac{A^2(a)}{Aq^2}$ among competing models. The relatively high ratio of $\frac{A^2(a)}{Aq^2}$ indicates that the dispersion of LCAPM intercept comes largely from the sample measurement error of liquidity risk, not the true dispersion. In contrast, the PS produces similar estimates in magnitude for nearly all metrics as the FF3 model under each set of testing portfolios. This evidence confirms the argument of Fama and French (2015, 2016) that the liquidity factor of Pastor and Stambaugh (2003) contributes little to improve the explanatory power of the FF3.

4.2. Regression details
Next, we specifically analyze regression intercepts and factor loadings produced by factor models for portfolio returns. To save space, for each set of testing portfolios, we focus on comparing the performance of the LCAPM and FF5 models for the troublesome portfolios provided by the FF3, especially for small size portfolios.

4.2.1. The 25 Size-B/M portfolios
Table 5 reports intercepts and slopes from regressing factor models for the 25 Size-B/M portfolios. Across all factor models, the LCAPM and PS models produce the smallest number of significant intercepts, both five out of 25 portfolios. The FF3 produces six significant intercepts, and the FF5 generates seven significant intercepts. As in Fama and French (2015), a major problem for the FF3 is the extreme growth stocks. From Panel A, four portfolios in the lowest B/M quintile have significant FF3 intercepts, and the most troublesome is, $-0.56\%$ ($t = -5.6$), for the microcap extreme growth portfolio. The FF5 (Panel B) reduces the problem produced by the FF3. It produces intercept, $-0.33\%$ ($t = -3.54$), for the microcap extreme growth portfolio, but still leaves three portfolios unexplained in the lowest B/M quintile. The LCAPM (Panel C) shows better performance in explaining these problem portfolios. In the lowest B/M
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**Table 4. Model performance metrics on anomaly portfolios**

| Model               | GRS  | p(GRS) | A|sh| As|sh| A(R²) | sh²(f) | n  |
|---------------------|------|--------|---|---|---|---|-------|--------|----|
| Panel A: 25 Size-B/M portfolios |
| CAPM                | 4.402| 0.000  | 0.227 | 0.201 | 0.745 | 0.012 | 11    |
| LCAPM               | 3.159| 0.000  | 0.160 | 0.380 | 0.750 | 0.129 | 5     |
| PS                  | 3.576| 0.000  | 0.098 | 0.228 | 0.912 | 0.049 | 5     |
| FF3                 | 3.745| 0.000  | 0.101 | 0.215 | 0.912 | 0.038 | 6     |
| C4                  | 3.128| 0.000  | 0.091 | 0.283 | 0.913 | 0.081 | 6     |
| FF5                 | 3.143| 0.000  | 0.094 | 0.293 | 0.919 | 0.104 | 7     |
| Panel B: 25 Size-Mom portfolios |
| CAPM                | 4.557| 0.000  | 0.297 | 0.130 | 0.736 | 0.012 | 15    |
| LCAPM               | 3.865| 0.000  | 0.208 | 0.251 | 0.743 | 0.129 | 6     |
| PS                  | 4.414| 0.000  | 0.308 | 0.065 | 0.843 | 0.049 | 15    |
| FF3                 | 4.340| 0.000  | 0.307 | 0.066 | 0.843 | 0.038 | 15    |
| C4                  | 3.304| 0.000  | 0.127 | 0.231 | 0.911 | 0.081 | 8     |
| FF5                 | 3.752| 0.000  | 0.254 | 0.107 | 0.855 | 0.104 | 17    |
| Panel C: 32 Size-OP-Inv portfolios |
| CAPM                | 4.485| 0.000  | 0.258 | 0.129 | 0.756 | 0.012 | 18    |
| LCAPM               | 3.292| 0.000  | 0.194 | 0.249 | 0.759 | 0.129 | 11    |
| PS                  | 4.057| 0.000  | 0.183 | 0.121 | 0.865 | 0.049 | 20    |
| FF3                 | 4.166| 0.000  | 0.184 | 0.120 | 0.865 | 0.038 | 19    |
| C4                  | 3.459| 0.000  | 0.165 | 0.150 | 0.865 | 0.081 | 15    |
| FF5                 | 3.103| 0.000  | 0.101 | 0.306 | 0.891 | 0.104 | 9     |

This table reports performance metrics for six asset pricing models under the value-weighted monthly excess returns on the 25 Size-B/M portfolios, the 25 Size-Mom portfolios, and the 32 Size-OP-Inv portfolios. For each factor model under a given set of portfolios, we present the GRSF-statistic of the Gibbons et al. (1989) and the corresponding p-value (p(GRS)), the average absolute intercept, A|sh|; the ratio of the average squared sample standard errors of the estimated intercepts to As|sh|; the ratio of the average squared sample standard errors of the estimated intercepts to As|sh|; the average of time series R², A(R²); the factor model’s maximum squared Sharpe ratio, sh²(f); and the number of significant intercepts generated by factor models under each set of testing assets.

The LCAPM shrinks the FF3 intercepts toward zero for the two extreme momentum portfolios, but still leaves two significant negative intercepts, −0.39% (t = −3.66) and −0.21% (t = −2.62), for the extreme loser portfolios, and two significant positive intercepts, 0.29% (t = 3.48) and 0.15% (t = 2.19), for the extreme winner portfolios. The FF5 performs worse to explain this momentum-based portfolios. In
### Table 5. Regression results for the 25 Size-B/M portfolios

| B/M | low  | 2    | 3    | 4    | high | low  | 2    | 3    | 4    | high |
|-----|------|------|------|------|------|------|------|------|------|------|
|     | α    |      |      |      |      | α    |      |      |      |      |
| small | -0.56 | -0.01 | -0.04 | 0.18 | 0.11 | -5.60 | -0.18 | -0.63 | 3.14 | 1.87 |
| 2    | -0.16 | 0.01  | 0.05  | 0.08 | -0.04 | -2.35 | 0.22  | 0.85  | 1.52 | -0.67 |
| 3    | -0.06 | 0.08  | -0.02 | 0.05 | 0.07  | -0.94 | 1.16  | -0.23 | 0.81 | 0.88 |
| 4    | 0.13  | -0.05 | -0.03 | 0.07 | -0.12 | 2.05  | -0.71 | -0.34 | 0.98 | -1.33 |
| big  | 0.15  | 0.07  | 0.00  | -0.24 | -0.16 | 3.10  | 1.11  | 0.06  | -3.65 | -1.56 |
|      |      |      |      |      |      | α    |      |      |      |      |
| small | -0.33 | 0.14  | -0.02 | 0.19 | 0.11 | -3.54 | 1.99  | -0.35 | 3.23 | 1.73 |
| 2    | -0.06 | -0.01 | -0.03 | 0.03 | -0.04 | -0.88 | -0.26 | -0.56 | 0.55 | -0.65 |
| 3    | 0.06  | 0.01  | -0.10 | -0.03 | -0.01 | 0.99  | 0.23  | -1.47 | -0.52 | -0.08 |
| 4    | 0.20  | -0.19 | -0.14 | 0.02 | -0.13 | 3.12  | -2.69 | -1.90 | 0.32 | -1.39 |
| big  | 0.09  | -0.06 | -0.08 | -0.27 | 0.01  | 1.97  | -1.02 | -1.12 | -3.91 | 0.08 |

Panel A: FF3-adjusted performance

|     |      |      |      |      |      |     |      |      |      |      |
|-----|------|------|------|------|------|-----|------|------|------|------|
| α   |      |      |      |      |      |     |      |      |      |      |
| small | -0.68 | -0.11 | -0.11 | 0.08 | 0.03 | -3.22 | -0.59 | -0.75 | 0.55 | 0.20 |
| 2    | -0.10 | 0.10  | 0.17  | 0.21 | 0.18 | -0.63 | 0.80  | 1.42  | 1.77 | 1.17 |
| 3    | -0.01 | 0.22  | 0.15  | 0.26 | 0.37 | -0.06 | 2.19  | 1.47  | 2.38 | 2.64 |
| 4    | 0.18  | 0.08  | 0.14  | 0.24 | 0.19 | 1.82  | 0.95  | 1.43  | 2.48 | 1.44 |
| big  | 0.00  | 0.09  | 0.08  | -0.08 | 0.15 | 0.03  | 1.37  | 0.86  | -0.67 | 1.00 |
|     |      |      |      |      |      |     |      |      |      |      |
| α   |      |      |      |      |      |     |      |      |      |      |
| small | 0.04  | 0.18  | 0.28  | 0.37 | 0.48 | 0.52  | 2.72  | 5.13  | 7.06 | 8.42 |
| 2    | -0.23 | -0.01 | 0.09  | 0.17 | 0.19 | -3.93 | -0.14 | 2.15  | 3.90 | 3.46 |
| 3    | -0.24 | -0.05 | 0.05  | 0.09 | 0.08 | -5.10 | -1.33 | 1.24  | 2.19 | 1.62 |
| 4    | -0.24 | -0.04 | 0.03  | 0.08 | 0.05 | -6.69 | -1.43 | 0.98  | 2.25 | 1.06 |
| big  | -0.04 | 0.01  | 0.06  | 0.12 | 0.05 | -1.46 | 0.31  | 1.68  | 3.03 | 0.96 |

Panel B: FF5-adjusted performance

|     |      |      |      |      |      |     |      |      |      |      |
|-----|------|------|------|------|------|-----|------|------|------|------|
| α   |      |      |      |      |      |     |      |      |      |      |
| small | -0.55 | -0.02 | -0.03 | 0.18 | 0.11 | -5.46 | -0.26 | -0.58 | 3.29 | 1.92 |
| 2    | -0.16 | 0.00  | 0.04  | 0.07 | -0.03 | -2.27 | 0.06  | 0.72  | 1.33 | -0.52 |
| 3    | -0.05 | 0.07  | -0.03 | 0.04 | 0.06 | -0.80 | 1.01  | -0.48 | 0.58 | 0.80 |
| 4    | 0.12  | -0.06 | -0.05 | 0.04 | -0.12 | 1.87  | -0.82 | -0.60 | 0.63 | -1.39 |
| big  | 0.14  | 0.06  | -0.00 | -0.24 | -0.15 | 3.03  | 0.99  | -0.01 | -3.57 | -1.41 |
|     |      |      |      |      |      |     |      |      |      |      |
| α   |      |      |      |      |      |     |      |      |      |      |
| small | -0.03 | 0.02  | -0.01 | -0.03 | -0.01 | -1.14 | 0.73  | -0.49 | -1.61 | -0.66 |
| 2    | -0.01 | 0.03  | 0.02  | 0.03 | -0.02 | -0.61 | 1.52  | 1.19  | 1.83 | -1.39 |
| 3    | -0.02 | 0.03  | 0.05  | 0.04 | 0.02 | -1.31 | 1.36  | 2.40  | 2.25 | 0.71 |
| 4    | 0.03  | 0.02  | 0.06  | 0.07 | 0.02 | 1.71  | 1.06  | 2.60  | 3.53 | 0.65 |
| big  | 0.01  | 0.02  | 0.01  | -0.01 | -0.04 | 0.46  | 1.13  | 0.63  | -0.65 | -1.38 |

The table reports regression results for the monthly excess returns on the 25 Size-B/M portfolios. Panel A shows the regression intercepts and their t-statistics from the FF3 model. Panel B shows the regression intercepts and their t-statistics from the FF5 model. Panel C shows the regression intercepts, factor loadings on LF, and their t-statistics from the LCAPM model. Panel D shows the regression intercepts, factor loadings on PSF, and their t-statistics from the PS model.
| Panel A: FF3-adjusted performance |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |    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Table 7. Regression results for the 32 Size-OP-Inv portfolios

| OP → | Small  |  |  |  | Big  |  |  |  |  |  |  |  |
|------|--------|---------------|---------------|---------------|--------|---------------|---------------|---------------|---------------|---------------|---------------|
|      | low | 2 | 3 | high | low | 2 | 3 | high | low | 2 | 3 | high |
| 
| Panel A: FF3-adjusted performance |  |  |  |  |  |  |  |  |  |  |  |  |
| α | t(α) | α | t(α) | α | t(α) |  |  |  |  |  |  |  |
| Low Inv | −0.20 | 0.05 | 0.35 | 0.28 | −2.06 | 0.65 | 3.92 | 2.96 | −0.01 | 0.15 | 0.20 | 0.22 | −0.10 | 1.75 | 2.26 | 2.35 |
| 2 | 0.10 | 0.11 | 0.18 | 0.17 | 1.22 | 1.78 | 2.98 | 2.20 | −0.33 | 0.02 | 0.19 | 0.25 | −3.42 | 0.20 | 2.69 | 3.09 |
| 3 | −0.30 | 0.21 | 0.14 | 0.27 | −3.55 | 3.55 | 2.42 | 3.75 | −0.11 | 0.02 | 0.01 | 0.11 | −1.20 | 0.23 | 0.07 | 1.30 |
| High Inv | −0.85 | −0.26 | −0.06 | −0.05 | −7.73 | −3.28 | −0.87 | −0.69 | −0.23 | −0.24 | −0.01 | 0.20 | 2.34 | 2.63 | −0.17 | 2.16 |
| Panel B: FF5-adjusted performance |  |  |  |  |  |  |  |  |  |  |  |  |
| α | t(α) | α | t(α) | α | t(α) |  |  |  |  |  |  |  |
| Low Inv | −0.04 | −0.06 | 0.15 | 0.06 | −0.52 | −0.80 | 1.83 | 0.72 | 0.03 | −0.04 | −0.03 | −0.05 | 0.36 | −0.56 | −0.38 | −0.61 |
| 2 | 0.19 | 0.01 | 0.05 | −0.01 | 2.69 | 0.12 | 0.94 | −0.24 | −0.21 | −0.01 | 0.08 | 0.04 | 2.23 | −0.13 | 1.17 | 0.55 |
| 3 | −0.19 | 0.19 | 0.05 | 0.11 | −2.35 | 3.28 | 0.96 | 2.00 | 0.04 | 0.03 | −0.08 | −0.04 | 0.50 | 0.33 | −1.07 | −0.51 |
| High Inv | −0.43 | −0.24 | −0.07 | −0.12 | −5.18 | −3.27 | −1.17 | −2.22 | 0.12 | −0.15 | 0.05 | 0.26 | 1.48 | −1.66 | 0.55 | 3.04 |
| Panel C: LCAPM-adjusted performance |  |  |  |  |  |  |  |  |  |  |  |  |  |
| α | t(α) | α | t(α) | α | t(α) |  |  |  |  |  |  |  |
| Low Inv | −0.16 | 0.14 | 0.46 | 0.39 | −0.80 | 0.98 | 3.11 | 2.61 | 0.17 | 0.23 | 0.23 | 0.17 | 1.75 | 2.33 | 2.23 | 1.69 |
| 2 | 0.15 | 0.18 | 0.23 | 0.26 | 0.93 | 1.54 | 2.09 | 2.10 | −0.22 | 0.06 | 0.17 | 0.21 | −2.05 | 0.71 | 2.19 | 2.34 |
| 3 | −0.28 | 0.24 | 0.20 | 0.32 | −1.79 | 1.93 | 1.75 | 2.68 | 0.01 | 0.12 | 0.03 | −0.03 | 0.06 | 1.29 | 0.38 | −0.37 |
| High Inv | −0.81 | −0.19 | 0.05 | 0.03 | −4.05 | −1.35 | 0.42 | 0.23 | −0.15 | −0.17 | −0.06 | 0.11 | −1.40 | −1.76 | −0.61 | 1.04 |

The table reports regression results for the monthly excess returns on the 32 Size-OP-Inv portfolios. Panel A shows the regression intercepts and their t-statistics from the FF3 model. Panel B shows the regression intercepts and their t-statistics from the FF5 model. Panel C shows the regression intercepts, factor loadings on $L_F$, and their t-statistics from the LCAPM model.
the LCAPM captures the microcap portfolio with insignificant intercept 0.32% (t = 1.9), as the strongly positive LF slope, 0.33 (t = 5.21) absorbs the abnormal return of this extreme microcap portfolio. However, the nearly zero LF slopes are not helpful for explaining the other portfolios in the extreme winners in which three of four portfolios produce significant intercepts from 0.38% (t = 3.02) to 0.52% (t = 3.84). On the other hand, the LCAPM performs the best in explaining the small portfolios of stocks. In the smallest size quintile, the LCAPM leaves only one significant intercept, while the other models leave at least three portfolios unexplained.

4.2.3. The 32 Size-OP-Inv portfolios
Table 7 reports regression results for the 32 Size-OP-Inv portfolios constructed based on the investment and profitability characteristics. The FF3 has difficulty in capturing this set of portfolios with strong Inv and OP tilts. Out of 32 portfolios, the FF3 produces 19 significant intercepts, which are most located in the small size groups. The FF5 improves the description of average returns of the FF3 with the help of adding RMW and CMA factors. It captures most of the portfolios, and leaves only nine portfolios unexplained. The main difficulty for the FF5 is to explain small stock portfolios in the lowest OP and the highest Inv quartiles. From panel B, except the extreme intercept, −0.43% (t = −5.18), for the lowest OP and the highest Inv portfolio, two of the other three portfolios in the lowest OP quartile and the highest Inv quartile also produce significant intercepts. The LCAPM performs satisfactorily in describing this set of portfolios. It produces 11 significant intercepts out of 32 portfolios. In addition, for the troublesome portfolios of the FF5 in the small group, the LCAPM only leaves one significant intercept, −0.81% (t = −4.05), for the lowest OP and the highest Inv portfolios. Moreover, for the portfolio of big stocks in the highest OP and Inv quartile, the main problem of FF5 for the big stock, the intercept of the LCAPM is 0.11% (t = 1.04), versus the FF5 intercept 0.26% (t = 3.04).

Overall, the good performance of the LCAPM confirms the importance role of liquidity risk in explaining portfolio returns.

5. Conclusion
Motivated by the recent findings that liquidity has limited contribution in explaining stock returns, we revisit the explanatory power of two liquidity risk factors/models proposed by Pastor and Stambaugh (2003) and Liu (2006) relative to some traditional and popular factor models for a larger number of portfolios. Our evidence shows that the LCAPM of Liu (2006) reveals the consistent pattern to capture liquidity risk. In most cases, the LCAPM performs no worse but better than other liquidity-risk-based and non-liquidity-risk-based pricing models. The LCAPM also well explains the performance of anomaly portfolios, especially for the troublesome portfolios containing small stocks, value stocks, and so forth. Our study reinforces the view that liquidity risk is not negligible in asset pricing. Also, this paper identifies the LCAPM as a preferable model and has important implications for investment decision-making and empirical finance research.

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Notes
1. Some other studies reveal similar evidence that the Pastor–Stambaugh liquidity factor provides little improvement to explain average returns, e.g., Fama and French (2016), Hou et al. (2017), and Ahmed et al. (2019). Preprint submitted to Cogent Economics & Finance 12 July 2021.
2. The constructions of factors in these characteristics-based models are based on the same characteristics as the sorting variables in forming the testing portfolios.
3. Examples are Pastor and Stambaugh (2003), Sadka (2006), and Liu (2006). Taking into account liquidity risk in asset pricing is also consistent with the significant liquidity premium documented in the literature such as Amihud and Mendelson (1986), Datar et al. (1998), Brennan et al. (1998), Lesmond et al. (1999), Amihud (2002), Bekker et al. (2007), Hasbrouck (2009), and Corwin and Schultz (2012).
4. Fama and French (2018), Barillas and Shanken (2017), and (2020) give relevant proofs.
5. Tableout, we also examine the equally-weighted decile portfolios formed on the four liquidity measures, and the results are qualitatively similar.

Declaration of competing interest

The author(s) declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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