Developing a hypothetical learning trajectory of fraction based on RME for junior high school

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Abstract. Preliminary research showed the learning trajectory of fraction in the school that could not facilitate students to reinvent fraction concepts. Therefore, this research aimed to produce a hypothetical learning trajectory (HLT) for teaching fraction using realistic mathematics education (RME). The HLT was developed with a design research that consisted of a cyclic process of preparing for the experiment, conducting classroom experiment, and performing retrospective analysis. This study involved 25 students SMPN 1 Timpeh, Indonesia. Data were collected through student activity sheets, videotaping, field notes, and interviews. The data analysis technique used descriptive analysis techniques. The result showed HLT for teaching fraction using RME that could be help all of levels students. This HLT helped students to reinvent fraction concepts through horizontal and vertical mathematical processes. Finally, the HLT facilitated students to make their own models from informal to formal and improved the interactions between students and teachers.

1. Introduction
Fraction is an important basic material, not only in learning mathematics, but also in studying science and other sciences [1]. Students who experienced difficulties in learning of fraction [2, 3]. Based on observations conducted at Junior High School (SMPN) 1 Timpeh in West Sumatra, one of the factors causing this subject considered difficult by students is because fraction is still taught abstractly, therefore students find it complicated to understand the basic concepts of fractions and fraction operations [4]. The learning flow used by the teacher has not been able to facilitate students in finding the concept of the fraction. The teacher only stick to the flow of learning path in the textbook, provides students with materials that have been completed so that students have difficulty understanding the concept of fractions, solving fraction and problems related to fraction.

Preliminary research showed the learning process that was carried out by following the order of the lessons in the textbook and centered of teacher. Students was actively involved through group learning, but the interaction created during the discussion that was not optimal. Learning rarely started from problems related to the knowledge that students already learnt. Learning that began with problems related to the knowledge would gave students an opportunity to rediscover the mathematical concepts that are being studied, and would make students more active and gave more opportunities to develop their mathematical thinking skills. [5] that stated mathematics learning in schools that still dominated by teachers and that mathematics learning outcomes in schools have not shown satisfactory results.

Hypothetical Learning Trajectory (HLT) is a learning flow consisting of learning objectives, learning activities, and learning process hypotheses to predict how students' thoughts and understanding will develop in the context of learning activities [6]. Some studies [7] [8] [9] showed...
that the Hypothetical Learning Trajectory (HLT) could improve students' understanding. Other studies have shown that HLT not only enhances student understanding, but also increases students' mathematical thinking skills, as research conducted by [5], which showed HLT with the RME approach helped teachers to develop viable local learning theories for a certain topic and helped students to develop reasoning ability and other mathematical thinking skills.

One approach that can be used in designing HLT is the Realistic Mathematics Education (RME) approach. RME is an approach in the mathematics learning process that contains constructive, interactive and reflective elements. RME mathematics is seen as human activity [5], [10], [11], [12]. Mathematics is not given to students in its final form, but students must be able to construct their own knowledge through solving contextual problems interactively, both formally and informally, so that students find themselves or with the help of the teacher to find out the truth of the answer.

There are several steps taken in designing the RME-based HLT, including conducting preliminary research which includes needs analysis, literature review, determining learning objectives, determining student learning activities, and learning process hypotheses. HLT consists of three components, namely learning objectives, learning activity assignments, and hypotheses about how students learn and how students think [6]. In the preparation of learning activities based on RME, these activities allow students to reinvent (mathematics) through teacher guidance [5]. RME-based activities pay attention to the characteristics of RME, namely: 1) the use of contexts, 2) the use of models 3) the use of students’ own production and constructions, 4) the interactive character of teaching process, and 5) the interviewments of various learning strands (integrated with various other learning topics) [5].

To design and develop an RME-based HLT in this study, a study of existing research was conducted. Research related to the development of learning design in the form of RME-based HLT has been carried out by many previous researchers, among others research by [13], in this study David C. Webb designed the RME-based HLT to introduce the Logarithmic Concept. David uses contextual problems in the form of the growth of the weight of a pony, the growth of E. coli bacteria. This contextual problem allows students to carry out informal mathematical processes, to subsequently increase the formal mathematical process by giving problems in mathematics. This informal and formal mathematical process helps students understand the concept of logarithms. The use of RME in David W. Cobb's study that designed RME-based learning focus on the student centered learning. Other research was conducted by [14]. This study emphasizes on the effect of RME on the conceptual understanding of students' linear programs. The type of research was Quasi experimental, non equivalent pretest and posttest control group design. This study used control and experimental classes. The results of this study indicate the use of RME by using contextual problems makes a better understanding of students' linear concept programs.

Research related to designing the HLT was also conducted by [15]. This study developed HLT in mathematics education research using Research Based Learning. In developing HLT, researchers used the Gravemeijer and Cobb models. HLT consists of three components, namely (1) the purpose of mathematics teaching for students, (2) learning activities, devices that are used in the learning process, and (3) a project of understanding the process of learning how to learn and strategies students that emerges and drives when learning activities are done in Class. The purpose of this study is to improve the ability of research and academic writing of prospective mathematics teachers. Other studies related to RME and HLT are research conducted by [16] about The Effect of Realistic Mathematics Education on Students' Achievements and Attitude in Fifth Grades Mathematics Courses, research [17] The Multivoicedness of the Game Playing: Exploring the Unfolding of Learning Trajectory in a Gaming Context at School, and research by [18] about the Developing specialist expertise: an unanticipated learning trajectory.

Based on the research conducted above, research in this paper focuses on the development of learning design in the form of RME-based learning pathways in the set material for VII grade students of Junior High School and Islamic Junior High School. In the preparation of the learning flow, this study also used the steps proposed by Gravemeijer and Cobb. In HLT, there are also three components proposed by Gravemeijer. By designing learning pathways with RME-based learning, it
will result on student-centered learning, develop students' ability to solve problems, the use of concepts within themselves to discover new concepts. Thus, RME-based learning design will be able to improve and develop students’ abilities.

2. Method
This research used design research with the development model of Gravemeijer. This design research aimed to develop HLT [7]. In this case, design research aimed to formulate, identify, and develop learning flows based on the students’ learning process and thinking processes. The result of research was not design that works but the underlying principles explaining how and why this design works [8]. There was three stages in HLT design, namely preparing for the experiment, classroom experiment, and retrospective analysis. In the Preparing for the experiment stage, at this stage, student learning activities was developed through hypothetical learning trajectory (HLT) that was dynamic and could adapted to students' thinking strategies that occur during the classroom experiment. To develop HLT, various literatures related to fractions are studied. Furthermore, the Classroom experiment stage, at this stage HLT is implemented which has been designed with the aim of exploring, knowing the strategies and thoughts of students in learning the concepts of Fractional Numbers. The Classroom experiment stage has two cycles, namely the first cycle Small group trials and cycle 2 namely large group trials. Finally, in the Retrospective analysis phase, all data obtained during the classroom experiment are analyzed. HLT is designed to be applied to students of grade VII of SMPN 1 Timpehin West Sumatra.

The instruments used were HLT validation sheets, observation sheets, interviews and interviews, field notes and videos. The data in the study are in the forms of primary data, namely validation data from each validator, the results of interviews to determine the use of HLT, student worksheets, and field notes. This data used to repair HLT that has been designed. Data collection techniques used in this study were documentation, interviews and field notes. Data analysis techniques were in the form of descriptive analysis techniques and qualitative analysis to describe the course of HLT in fraction learning.

3. Result and Discussion
At the early stages of research, carried out literature review was conducted and compiled a series of student activities (such as their hobbies, their preferences in learning, and their preference activities) to achieve the concept of fractions from the informal to the formal stages. Based on the analysis, we designed HLT for teaching fractions. The results of this design are discussed with the mathematics teacher or the teacher model which is then applied in one-to-one evaluations and small groups before being tested in field tests. After conducting a preliminary study, the researcher makes revisions or improvements to the design that have been made based on the results obtained and discussions with the teacher.

HLT designed for fraction that begins with comparing fractions, types of fractions, addition and subtraction of fractions, and multiplication and division of fractions. Fractional learning flow can be seen in Figure 1. HLT of fraction is divided into four parts, starting understanding the concept of fractions as parts whole relationship, determining fractions with the same value, comparing fraction, and determining the calculation of fraction operation. The implementation of HLT is designed for three times with seven activities that must be followed by students. In addition to learning activities, predictions of student answers and anticipation that will be made by the teacher on the flow of students' thinking in working on learning activities were also designed. In Activity 1, students are led to find the concept of fractions as part whole relationship with paper folding activities as the initial experience of finding fractions. Activity 2 leads students to find the value of fractions by comparing parts of one cake to another. This activity trains students to submit guesses and provide alternatives to an argument. Based on students' answers to activities 1 and 2, students can deduce the meaning and value of fractions.
Figure 1. Learning Flow Design of Fraction

Activity 3 and 4 are activities that aim to enable students to compare fractions. The context used for Activity 3 and 4 is a continuation of the story of the birthday event in Activity 1 and 2. Activity 3 contains a question that requires students to find the results of a comparison provided by the stories while Activity 4 contains a question that leads students to be able to compare themselves the results of problem solving. Activities 3 and 4 have a role to train students to find mathematical indicators and draw conclusions from a statement. Activity 5, 6 and 7 are activities that are aimed for students to find ways to solve or operate fractions. The context used for activity 5 is a continuation of the story of Salma's birthday event. The context used for activity 6 is the activity of making cookies done by Mrs. Mega and rice harvesting carried out by Mr. Yusuf in activity 7. In activity 5, students were asked to look for the amount and the remaining cakes received by each child. Activity 6 contains a question that asks students to do an addition operation. Activity 7 contains a question that asks students to do multiplication operations and divide the fractions. Activities 5, 6 and 7 have the role of training students to draw conclusions from a statement, propose allegations, give alternatives to an argument, and find a mathematical indicator. For operational learning, designed learning activities are transferred into student activity sheets (LAS).

3.1. Activity 1: Salma's 1st Birthday Event
The purpose of this activity is to understand fractions as part whole relationships. The context used for activity 1 is folding paper which is likened to be the base of a birthday cake and writing the fraction form. The initial answer given by students for question number 1 consists of one and the same answer strategy. The four students answered according to the predictions made. Student answers can be seen in Figure 2.
3.2. Activity 2: Salma’s Birthday 2
This activity aims to understand the concept of fractions worth. The teacher reminded students of the previous story about Salma’s birthday and asked students to observe the folded paper done before. Students solved the problem of cake distribution in a discussion. Student answers to activity 2 can be seen in Figure 3a and 3b.

From Figure 3a and 3b, there are differences in student answers to solving activity problems 2. Students use their thinking skills to determine the values of fractions. To ensure the correct answer the teacher asked students to discuss and determined the answer using origami paper. During the discussion, the teacher gave a probing question that led students to find the right answer. Finally, the teacher and students came to an agreement about the concept of fraction value.

3.3. Activities 3 and 4: Salma’s Birthday (continued)
This activity aims to understand the concept of fraction comparison. In this activity the teacher asked students to recall the story in activity 2 then asked students to solve the problem in the activity 3. Four students had the same version of the answer by directly comparing how many cakes that Linda would get, but in a different way of learning. Student answers in completing fraction comparisons can be seen in Figure 4.

From Figure 4, students' answers can be seen in determining fraction comparisons. The teacher asked students to explain the reason why students found a problem solving in activity 3. There were students who provided explanations by explaining the strategies used, and there were students who explained along with evidence of the answer. Together with the teacher, students concluded about the concept of comparative fractions. Based on activities 1, 2 and 3, it can be seen that students could find their own concept of fractions as part whole relationships, fraction value and comparison of fractions.

3.4. Activities 5, 6 and 7: the Salma Birthday Event 4

The purpose of this activity is that students are able to understand fraction operations. The teacher reminded students of the previous story about Salma's birthday and asked students to observe the next story. From the problems given, students had their own strategies in solving each problem. Problems given were related to simplifying fractions, addition, subtraction and multiplication and converting decimal fractions to percent. Some student answers can be seen in Figure 5.
From Figure 5a and 5b, it can be seen that students use their own strategies in finding solutions to the problems given. Based on the achievement of objectives obtained during the learning process, it shows that the HLT developed has illustrated the increase in student knowledge. The use of the best strategies shows that students have at each meeting can lead other students to change their mindset in finding solutions to problems at the next meeting.

The result showed that the HLT (with the activities of solving contextual problems in the HLT) were potential to facilitate the students to reinvent the concepts of fraction; the activities were well sequenced, the HLT suit the key principles and characteristics of RME; and the components in the HLT were well designed and consistent between one and another. The LT worked as intended during the try out. The students understood the contextual problems and they conducted ‘doing math’ activities without major obstacle. The probing questions that were prepared as the anticipations of students’ thinking and solutions also helped the students to achieved the goals of the activities. In addition, the time provided for doing the activities of solving contextual problems was well planned. These findings were in line with the previous findings which showed that LT and RME gave positive impact on students’ mathematics skills (see [1, 2])

4. Conclusion

Hypothetical Learning Trajectory (HLT) based on Realistic Mathematic Education (RME) on fraction material can be confirmed to be valid after a trial. The trials were a small class trial and a large class trial. The trials went through three stages including experimentation, reflection, and retrospective analysis. HLT of fraction starts with understanding a fraction and its value, comparing fractions and determining fraction operations in everyday life. Fractional HLT was produced through the application of 7 learning activities that are used to achieve four specific goals that must be achieved by students, namely fractions as part whole relationships, determining the value of fractions, comparing fractions and completing fraction operations. Hypothetical Learning Trajectory (HLT) based on Realistic Mathematic Education (RME) on valid fraction material in the form of LIT can be used as a guide for educators in carrying out learning and developing other learning flows. This study has limitations, because it is only tested in one class. Further researchers can conduct similar research with trials in various schools with diverse conditions and backgrounds of students.

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