ON THE ORIGIN OF THE NOTION OF GW

ET CETERA

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Abstract. The notion of gravitational wave (GW) came forth originally as a by-product of the linear approximation of general relativity (GR). Now, it can be proved that this approximation is quite inadequate to a proper study of the hypothetic GW’s. The significant role of the approximations beyond the linear stage is emphasized.

1. - As it is well known, the linear approximation of GR (physically, the approximation for weak gravitational fields) has Minkowski spacetime as its (fixed) substrate [1], [2]. It resembles the e.m. Maxwell theory, and is only Lorentz invariant.

With respect to transformations of general co-ordinates, its energy tensor becomes a false (pseudo) tensor, which can be reduced to zero through a suitable change of reference system.

A celebrated by-product of the linearized version of GR is the notion of gravitational wave [1], [2]. Now, in 1944 Weyl [3] pointed out that, rigorously speaking, the gravitational field of the linearized version exerts no force on matter, i.e. is a “powerless shadow”. Indeed, a basic result of the Einstein-Infeld-Hoffmann method [4] tells us that, as Weyl [3] wrote, “the gravitational force arises only when one continues the approximation beyond the linear stage.” Even in the modern literature this fact is generally overlooked and, quite uncritically, the action on matter of a gravitational wave – e.g. of a plane wave – is formally computed.

In conclusion, the linear approximation of GR – which is the favourite relativistic doctrine of the GW hunters – is completely inadequate to an approximate treatment of the question of the GW’s.

On the other hand, if we continue the approximation beyond the linear stage (cf. [1] and [5]), we find that the radiation terms of the gravitational field can be destroyed by convenient co-ordinate transformations: this proves that the GW’s are only a product of a special choice of the reference frame, i.e. that they do not possess a physical reality [5], [6].

In the recent literature the above crucial role of the co-ordinate system is ignored. E.g., Itoh and Futamase have published a learned study on the third post-Newtonian equation of motion for relativistic compact binaries [8], [9]; their result is derived under the harmonic co-ordinate condition.

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Motivation for this research [8]: “One promising source of gravitational waves for those detectors [i.e. GEO600, LIGO, TAMA300] is a relativistic compact binary system in an inspiraling phase. The detectability and quality of measurements of astrophysical information of such gravitational wave sources rely on the accuracy of our theoretical knowledge about the waveforms. A high order, say, third- or fourth-order, post-Newtonian equation of motion for an inspiraling compact binary is one of the necessary ingredients to construct and study such waveforms [...].” Itoh writes (see the abstract of [9]): “Our resulting equation of motion admits a conserved energy (neglecting the 2.5 PN radiation reaction effect), is Lorentz invariant, and is unambiguous [...].”

These authors do not suspect that the radiation terms are frame dependent, and can be destroyed by a suitable change of co-ordinate system. Further, they are unaware that – as it is easy to prove – the motions of point masses interacting only gravitationally – as, e.g., the two compact stars of some binaries or the bodies of the solar system – happen along geodesic lines [6], and consequently an emission of GW’s is obviously impossible.

More radically, it can be proved that no “mechanism” exists in GR for the generation of GW’s [4].

2. - Only recently, through a kind letter of Prof. A. Gsponer, I have known the existence of the beautiful memoir by Weyl [3].

Since it seems that the astrophysical community is not aware of Weyl’s results, I think very useful to reproduce in an APPENDIX, at the end of the present paper, the “Introduction and Summary” and sects. 1, 2 of Weyl’s memoir, which are particularly relevant to our theme.

I avail myself of this opportunity for an apology: in 1999 I published a very short Note entitled “Deduction of the law of motion of the charges from Maxwell equations” [10]. Now, my result is contained in Weyl’s treatment of Maxwell theory, see sect. 2. of [3]. Weyl wrote that this theorem was “well known”. Yes, but only to the Blessed Few!

Acknowledgment. - I am very grateful to Prof. G. Morpurgo, who has called my attention to the research of Itoh and Futamase [8], [9].

“Nil sapientiae odiosius acumine nimio”.
Seneca

References
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[2] A. Einstein, Berl. Ber., (1918) 154.
[3] H. Weyl, Amer. J. Math., 66 (1944) 591.
[4] L. Infeld and J. Plebanski, Motion and relativity (Pergamon Press, Oxford, etc) 1960, see in particular Chapt. VI.
[5] A. Scheidegger, Revs. Modern Physics, 25 (1953) 451.
[6] For many proofs of the phantasmatic nature of GW’s, see A. Loinger, arXiv:physics/0312149 v3 (11 February 2004), and the literature quoted there sub [6], in particular: A. Loinger, Spacetime & Substance, 3 (2002) 129 and 3 (2002) 145;
As it is known, the first theoretical proof of the physical non-existence of the GW’s was given by T. Levi-Civita in 1917; see his fundamental memoir in *Rend. Lincei*, **26**(1917) 381; an English version in *arXiv:physics/9906004* (June 2nd, 1999).

[7] As it is well known, the astrophysical community perseveres in the chase of Nothing, see e.g. the report on INTERNET (13 January 2004) by Szabolcs Márka, for the LIGO scientific collaboration, entitled “Search for the gravity wave signature of GRB030329/SN2003dh”. Its abstract is as follows: “One of the major goals of gravitational wave astronomy is to explore the astrophysics of phenomena that are already observed in the particle/electromagnetic bands. Among potentially interesting sources for such collaboration are gravitational waves searches in coincidence with Gamma Ray Bursts. On March 29th, 2003, one of the brightest ever Gamma Ray Burst was detected and observed in great detail by the broader astronomical community. The uniqueness of this event prompted our search as we had the two LIGO Hanford detectors in coincident lock at the time. We will report on the GRB030329 prompted search for gravitational waves, which relies on our sensitive multi-detector data analysis pipeline specifically developed and tuned for astrophysically triggered searches. WE DID NOT OBSERVE A GRAVITY WAVE BURST, WHICH CAN BE ASSOCIATED WITH GRB030329 [Capital letters by A.L.]. However, the search provided us with an encouraging upper limit on the associated gravity wave strain at the Hanford detectors.\[7\]

[8] Y. Itoh and T. Futamase, *Phys. Rev.* **D**, **68** (2003) 121501(R).
[9] Y. Itoh, *arXiv:gr-qc/0310029* v2 (12 February 2004).
[10] A. Loinger, *Nuovo Cimento A*, **112** (1999) 407.

**APPENDIX**

**HOW FAR CAN ONE GET WITH A LINEAR FIELD THEORY OF GRAVITATION IN FLAT SPACE-TIME?**

By HERMANN WEYL

(from: *Amer. J. Math.* **66** (1944) 591)

**Introduction and Summary.** G.D. Birkhoff’s attempt to establish a linear field theory of gravitation within the frame of special relativity\[2\] makes it desirable to probe the potentials and limitations of such a theory in more general terms. In thus continuing a discussion begun at another place\[3\] I find that the differential operators at one’s disposal form a 5 dimensional linear manifold. But the requirement that the field equations imply the law of conservation of energy and momentum in the simple form $\partial T_k^k/\partial x_L = 0$ limit these $\infty$ possibilities to $\infty$, which, however, reduce easily to two cases, a regular one ($L$) and a singular one ($L'$). The regular case ($L$) is nothing but Einstein’s theory of weak fields. Resembling very closely Maxwell’s theory of the electromagnetic field, it satisfies a principle of gauge invariance involving 4 arbitrary functions, and although its gravitational field exerts no force on matter, it is well suited to illustrate the role of energy and momentum, charge and mass in the interplay between matter and field. It might

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2*Proceedings of the National Academy of Sciences*, vol.**29** (1943), p.231.

3*Proceedings of the National Academy of Sciences*, vol.**30** (1944), p.205.
also help, though this is much more problematic, in pointing the way to a more satisfactory unification of gravitation and electricity than we at present possess. Birkhoff follows the opposite way: by avoiding rather than adopting the $\infty^2$ special operators mentioned above, his "dualistic" theory ($B$) destroys the bond between mechanical and field equations, which is such a decisive feature in Einstein’s theory.

1. Maxwell’s theory of the electromagnetic field and the monistic linear theory of gravitation ($L$). Gauge invariance. Within the frame of special relativity and its metric ground form

$$ds^2 = \delta_{ik}dx_i dx_k = dx_0^2 - (dx_1^2 + dx_2^2 + dx_3^2)$$

an electromagnetic field is described by a skew tensor

$$f_{ik} = \partial \phi_k / \partial x_i - \partial \phi_i / \partial x_k$$

derived from a vector potential $\phi_i$ and satisfies Maxwell’s equations

$$\partial f_{ki} / \partial x_k = s^i \quad \text{or} \quad D_i \phi = \Box \phi_i - \partial \phi^i / \partial x_i = s_i$$

where $s^i$ is the density-flow of electric charge and

$$\phi' = \partial \phi'/\partial x_i, \quad \Box \phi = \delta^{pq}(\partial^2 \phi / \partial x_p \partial x_q).$$

The equations do not change if one substitutes

$$\phi_i^* = \phi_i - \partial \lambda / \partial x_i \quad \text{for} \ \phi_i,$$

$\lambda$ being an arbitrary function of the coordinates ("gauge invariance"), and they imply the differential conservation law of electric charge:

$$\partial s^i / \partial x_i = 0.$$ As is easily verified, there are only two ways in which one may form a vector field by linear combination of the second derivatives of a given vector field $\phi_i$, namely,

$$\Box \phi_i \quad \text{and} \quad \partial \phi'_i / \partial x_i \quad (\phi'_i = \partial \phi^i / \partial x_p).$$

The only linear combination $D_i \phi$ of these two vector fields which satisfies the identity $(\partial / \partial x_i)(D^i \phi) = 0$ is the one occurring in $\Box$,

$$D_i \phi = \Box \phi_i - \partial \phi'/\partial x_i.$$ Herein lies as sort of mathematical justification for Maxwell’s equations.

Taking from Einstein’s theory of gravitation the hint that gravitation is represented by a symmetric tensor potential $h_{ik}$, but trying to emulate the linear character of Maxwell’s theory of the electromagnetic field, one could ask oneself what symmetric tensors $D_{ik} h$ can be constructed by linear combination from the second derivatives of $h_{ik}$. The answer is that there are 5 such expressions, namely

$$\Box h_{ik}, \quad \partial h'_i / \partial x_k + \partial h'_k / \partial x_i, \quad h'' \delta_{ik}, \quad \partial^2 h / \partial x_i \partial x_k, \quad \Box h \cdot \delta_{ik}$$

where

$$h = h_p^p, \quad h'_i = \partial h_p^p / \partial x_p, \quad h'' = \partial^2 h_p^p / \partial x_p \partial x_q.$$
With any linear combination $\bar{D}_{ik} h$ of these 5 expressions one could set up the field equations of gravitation

\begin{equation}
\bar{D}_{ik} h = T_{ik}
\end{equation}

the right member of which is the energy-momentum tensor $T_{ik}$. In analogy to the situation encountered in Maxwell’s theory one may ask further for which linear combinations $\bar{D}_{ik}$ the identity

\begin{equation}
(\partial/\partial x_k)(\bar{D}_i^k h) = 0
\end{equation}

will hold, and one finds that this is the case if, and only if, $\bar{D}_{ik} h$ is of the form

\begin{equation}
\alpha \{\square h_{ik} - (\partial h'_i/\partial x_k + \partial h'_k/\partial x_i) + h'' \delta_{ik}\} + \beta (\partial^2 h/\partial x_i \partial x_k - \square h \cdot \delta_{ik})\}
\end{equation}

$\alpha$ and $\beta$ being arbitrary constants. In this case the field equations (5) entail the differential conservation law of energy and momentum

\begin{equation}
\partial T^k_i/\partial x_k = 0.
\end{equation}

With two constants $a, b$, ($a \neq 0, a \neq 4b$) we can make the substitution

\begin{equation}
h_{ik} \rightarrow a \cdot h_{ik} - b \cdot h \delta_{ik}
\end{equation}

and thereby reduce $\alpha, \beta$ to the values 1, 1, provided $\alpha \neq 0, \alpha \neq 2\beta$. Hence, disregarding these singular values, we may assume as our field equations

\begin{equation}
D_{ik} h \equiv \{\square h_{ik} - (\partial h'_i/\partial x_k + \partial h'_k/\partial x_i) + h'' \delta_{ik}\} + \{\partial^2 h/\partial x_i \partial x_k - \square h \cdot \delta_{ik}\} = T_{ik}.
\end{equation}

$D_{ik} h$ remains unchanged if $h_{ik}$ is replaced by

\begin{equation}
h^*_{ik} = h_{ik} + (\partial \xi_i/\partial x_k + \partial \xi_k/\partial x_i)
\end{equation}

where $\xi_i$ is an arbitrary vector field. Hence we have the same type of correlation between gauge invariance and conservation law for the gravitational field as for the electromagnetic field, and it is reasonable to consider as physically equivalent any two tensor fields $h, h^*$ which are related by (9).

The linear theory of gravitation ($L$) in a flat world at which one thus arrives with a certain mathematical necessity is nothing else but Einstein’s theory for weak fields. Indeed, on replacing Einstein’s $g_{ik}$ by $\delta_{ik} + 2\kappa \cdot h_{ik}$ and neglecting higher powers of the gravitational constant $\kappa$, one obtains (5), and the property of gauge invariance (9) reflects the invariance of Einstein’s equations with respect to arbitrary coordinate transformations$^4$.

By proper normalization of the arbitrary function $\lambda$ in (2) one may impose the condition $\phi'_i = 0$ upon the $\phi_i$, thus giving Maxwell’s equations a form often used by H. A. Lorentz:

\begin{equation}
\square \phi_i = s_i, \quad \partial \phi'_i/\partial x_i = 0.
\end{equation}

In the same manner one can choose the $\xi_i$ in (9) so that $\gamma_{ik} = h_{ik} - \frac{1}{2} h \cdot \delta_{ik}$ satisfies the equations

\begin{equation}
\partial \gamma^k_i/\partial x_k = 0 \quad \text{and}
\end{equation}

$^4$Cf. A. Einstein, Sitzungsber. Preuss. Ak. Wiss. (1916), p.688 (and 1918, p.154).
(12) \[ \Box \gamma_{ik} = T_{ik}. \]

In one important respect gauge invariance works differently for electromagnetic and gravitational fields: If one splits the tensor of derivatives \( \phi_{k,i} = \partial \phi_k / \partial x_i \) into a skew and a symmetric part,

\[
\phi_{k,i} = \frac{1}{2} (\phi_{k,i} - \phi_{i,k}) + \frac{1}{2} (\phi_{k,i} + \phi_{i,k}),
\]

the first part is not affected by a gauge transformation whereas the second can locally be transformed into zero. In the gravitational case all derivatives \( \partial h_{ik} / \partial x_p \) can locally be transformed into zero. Hence we may construct, according to Faraday and Maxwell, an energy-momentum tensor \( L_{ik} \) of the electromagnetic field,

\[
L^k_i = f_{ip} f^{pk} - \frac{1}{2} \delta^k_i (ff), \quad (ff) = \frac{1}{2} f_{pq} f^{qp},
\]

depending quadratically on the gauge invariant field components

\[ f_{ik} = \phi_{k,i} - \phi_{i,k}, \]

but no tensor \( G_{ik} \) depending quadratically on the derivatives \( \partial h_{ik} / \partial x_p \) exists, if gauge invariance is required, other than the trivial \( G_{ik} \equiv 0 \).

2. Particles as centers of force, and the charge vector and energy-momentum tensor of a continuous cloud of substance. Conceiving a resting particle as a center of force, let us determine the static centrally symmetric solutions of our homogeneous field equations (11) and (15) \((s^i = 0, T_{ik} = 0)\). One easily verifies that in the sense of equivalence the most general such solution is given by the equations

(14) \[ \phi_0 = e/4\pi r, \quad \phi_i = 0 \quad \text{for} \ i \neq 0; \]

(15) \[ \gamma_{00} = m/4\pi r, \quad \gamma_{ik} = 0 \quad \text{for} \ (i,k) \neq (0,0); \]

\( r \) being the distance from the center. As was to be hoped, it involves but two constants, charge \( e \) and mass \( m \). The center itself appears as a singularity in the field. Indeed \( \phi_0 \) and the factor \( \phi \) in \( \phi x_a [\alpha = 1, 2, 3] \) must be functions of \( r \) alone, and the relations

\[
\Box \phi_0 = 0, \quad \partial \phi_a / \partial x_a = 0 \quad [\alpha = 1, 2, 3]
\]

implied in (10) then yield

\[ \phi_0 = a/r, \quad \phi = b/r^3, \quad \phi_a = - (\partial / \partial x_a)(b/r). \]

Substitution of \( \phi_a - \partial \lambda / \partial x_a \) for \( \phi_a \) with \( \lambda = -b/r \) changes \( \phi_a \) into zero. In the same manner (15) is obtained from the equations (11) & (12).

A continuous cloud of “charged dust” can be characterized by its velocity field \( u^i \ (u_i u^i = 1) \) and the rest densities \( \mu, \rho \) of mass and charge. It is well known that its equations of motion and the differential conservation laws of mass and charge result if one sets \( s^i = \rho u^i \) in Maxwell’s equations and lets \( T^k_i \) in (7) consist of the Faraday-Maxwell field part (16) and the kinetic part \( \mu u_i u^k \):

\[
\partial (\rho u^i) / \partial x_i = 0, \quad \partial (\mu u^i) / \partial x_i = 0; \quad \mu du_i / ds = \rho \cdot f_{ip} u^p.
\]
Since the motion of the individual dust particle is determined by \( \frac{dx_i}{ds} = u^i \) we have written \( d/ds \) for \( a^k \partial/\partial x_k \). In this manner Faraday explained by his electromagnetic tensions (flow of momentum) the fact that the active charge which generates an electric field is at the same time the passive charge on which a given field acts. At its present stage our theory \((L)\) accounts for the force which an electromagnetic field exerts upon matter, but the gravitational field remains a powerless shadow. From the standpoint of Einstein’s theory this is at it should be, because the gravitational force arises only when one continues the approximation beyond the linear stage. We pointed out above that no remedy for this defect may be found in a gauge invariant gravitational energy-momentum tensor. However, the theory \((L)\) explains why active gravity, represented by the scalar factor \( \mu \) in the kinetic term \( \mu u_i u_k \) as it appears in the right member \( T_{ik} \) of the gravitational equations \((5)\), is at the same time inertial mass: this is simply another expression of the fact that the mechanical equations \((7)\) are a consequence of those field equation.

We have seen that even in empty space the field part of energy and momentum must not be ignored, and thus a particle should be described by the static centrally symmetric solution of the equations

\[
D_i \phi = 0, \quad D_{ik} h - L_{ik} = 0
\]

(of which the second set is no longer strictly linear!). Again we find, after proper gauge normalization,

\[
\phi_0 = e/4\pi r, \quad \phi_1 = \phi_2 = \phi_3 = 0,
\]

and then

\[
\begin{align*}
\gamma_{00} &= m/4\pi r - 1/4(e/4\pi r)^2, \\
\gamma_{0a} &= 0, \\
\gamma_{\alpha\beta} &= -(e/4\pi r)^2 \cdot \left(x_\alpha x_\beta/4r^2\right) \quad [\alpha, \beta = 1, 2, 3].
\end{align*}
\]

As before, two characteristic constants \( e \) and \( m \) appear. At distance much larger than the “radius” \( e^2/4\pi m \) of the particle the gravitational influence of charge becomes negligible compared with that of mass.

[The remaining sections of the paper by Weyl are entitled: sect. 3. The singular case; sect. 4. Derivation of the mechanical laws without hypothesis about the inner structure of particles; sect. 5. Vague suggestions about a future unification of gravitation and electromagnetism; sect. 6. A free paraphrase of Birkhoff’s recent linear laws of gravitation (B).]