Theoretical modeling and analysis of piezoelectric energy harvester with variable section overhanging beam

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Abstract. With the development of integrated circuits, the volume and power consumption of electrical equipment have decreased dramatically. As an excellent energy conversion method, piezoelectric energy harvester can capture the vibration energy of the environment, so as to realize the self-powered supply of micro and small power electronic devices. In this paper, a variable cross-section extended beam structure is considered as the structure form of piezoelectric energy harvester. The influence of structural parameters on the steady-state response of piezoelectric energy harvester is analyzed, which can improve the output voltage and collection efficiency of the energy harvester. The results show that the thickness of the beam has a great influence on the natural frequency of the piezoelectric energy harvester system, but has little influence on the output response. By changing the thickness of the beam, the low-frequency energy can be captured and the energy collection efficiency can be improved. The thickness of the beam piezoelectric layer has a great influence on the natural frequency of the piezoelectric energy harvester system, but has little influence on the output response. By reducing the thickness of piezoelectric layer, the efficiency of energy collection can be improved.

1. Introduction

With the rapid development of integrated circuits, micro wireless sensors are widely used in embedded systems, extreme environment detection and control systems, remote or field monitoring and other fields [1-2]. At present, most of the wireless sensors are powered by chemical batteries, which have the disadvantages of easy pollution, short life and high replacement cost. In recent years, people gradually try to capture energy from the surrounding environment to supply energy for microelectronic devices. The research of capturing energy from the surrounding environment has very important practical significance. There are mainly solar energy, wind energy, vibration energy and so on in the surrounding environment. Because the vibration energy is least affected by external factors, this paper will study the vibration energy collector. There are a large number of vibration sources in the surrounding environment. These vibration sources are mainly caused by motor, machine tool, ship, human movement, water flow, sea wave and wind. These vibration energy can be converted into electrical energy through positive piezoelectric effect or electromagnetic effect [3-6]. Capturing the vibration energy in the surrounding environment and realizing the self-energy supply of microelectronic devices have gradually attracted widespread attention [7-11]. Using piezoelectric materials to obtain energy from vibration motion is an important alternative to battery or other electronic equipment energy [12].

Stepped beam is very common in engineering. Recently, the piezoelectric energy harvester with
stepped variable cross-section beam has attracted more and more attention due to its perfect vibration performance. Wickenheiser et al. [13] used the transfer matrix method to solve the natural frequency and frequency response function of multi segmented piezoelectric cantilever beam, and compared the calculation results with the software simulation results. The results show that the transfer matrix method can be used to accurately predict the first several modes of complex piezoelectric beam structure with multiple discontinuities. Wang et al. [14] considered the discontinuity caused by the piezoelectric layer, divided the piezoelectric cantilever into two sections, and gave the calculation method of the natural frequency, vibration mode and forced vibration response of the system, which was verified by experiments. This modeling method can be extended to the theoretical modeling of the piezoelectric cantilever with multiple piezoelectric layers. Usharani et al. [15-16] proposed a broadband step cantilever piezoelectric energy harvester with intermediate support, and carried out theoretical and Experimental Research on it. The results show that reducing the thickness can increase the output voltage and power in a lower frequency range. Hajhoseini et al. [17] applied periodic stepped piezoelectric cantilever beam structure to vibration energy acquisition. The results show that low frequency broadband gap can be obtained by changing geometric parameters. Finally, it is concluded that compared with the uniform cantilever beam piezoelectric energy harvester, the periodic stepped overhanging beam piezoelectric energy harvester has three advantages, such as generating more electricity in a wide frequency range, absorbing vibration, light weight and so on.

2. Model establishment

Figure 1 shows an overhanging beam with total length $L$ and width $B$, containing an simply supported section of length $L_1$. The thickness of the substructure layer is $H_1$ and $H_2$. There are two piezoelectric layers of thickness $H_p$ and width $B$ respectively perfectly bonded on the top and bottom surfaces of the beam. In the model, the X and Z directions correspond to the piezoelectric Directions 1 and 3, and the polarization direction is Z direction. The polarization direction of the bicrystal piezoelectric layer is opposite. The piezoelectric energy harvester vibrates laterally under harmonic excitation, and the mechanical energy is converted into electrical energy due to the positive piezoelectric effect of piezoelectric material.

![Figure 1. Model of overhanging beam with variable cross-section piezoelectric energy harvester.](image)

3. Parameter modeling

3.1. Lagrange function

The $S_{ax}(x,z,t)$ axial strain of the beam at the distance $z$ from the neutral axis can be expressed as [18]

$$S_{ax}(x,z,t) = -z\delta^2 w(x,t) / (\delta^2 x)$$

(1)

where $w(x,t)$ resents the transverse displacement on the mid-plane of the beam.

The deformation of isotropic metal substructure layer follows Hooke's law:
where \( T_{xx} \) and \( E_s \) are axial stress and elastic modulus of structural layer.

Due to the lateral vibration of the piezoelectric cantilever beam system, considering the piezoelectric effect of the piezoelectric material, the second kind of piezoelectric equation is adopted, and the constitutive relation of the piezoelectric layer can be expressed as \[ T_i = c_{11}E_s - e_{31}E_3, \quad D_3 = e_{33}S_3 + \varepsilon_3^sE_3 \] (3)

where \( T_i \) and \( S_i \) respectively are axial stress and axial strain of piezoelectric layer, \( c_{11} \) respents elastic modulus under constant electric field, \( e_{31} \) is piezoelectric coupling coefficient, \( E_3 \) and \( D_3 \) are eletric field intensity and electric displacement in z direction, \( \varepsilon_3^s \) is dielectric constant at constant strain.

The electric field distribution of the variable cross-section overhanging beam in series can be expressed as \[ E_3 = -\nu_{KL}(t) / (2H_p)(0 \leq x \leq L) ; \quad E_3 = -\nu_{2L}(t) / (2H_p)(L_2 \leq x \leq L) \] (4)

where \( \nu_{KL}(t) \) and \( \nu_{2L}(t) \) respectively are voltage at both end of load resistances.

The Lagrange function of the system is expressed as \[ \mathcal{L}(x,t) = T + W_e - U \] (5)

where \( T \) is the kinetic energy of the system, \( W_e \) is the electric potential energy of the system and \( U \) is the potential energy in the system.

3.2. Discretization of model space

Due to the sparsity of structural modes, when the first mode is closer to the excitation frequency, the piezoelectric energy harvester of variable cross-section overhanging beam works in the low-frequency vibration environment, so it plays a leading role in the displacement response of the structure. Therefore, the displacement function is truncated by Galerkin method \[ ]

According to the vibration theory, the lateral displacement of the piezoelectric beam can be written as \[ w(x,t) = W_1(x)q_1(t) \] (6)

where \( W_1(x) \) is the first mode shape, \( q_1(t) \) is generalized modal coordinates. The first mode shape function can be written as \[ W_1(x) = W_{11}(x) \quad x \in [0, L_1] ; \quad W_1(x) = W_{12}(x) \quad x \in [L_1, L] \] (7)

\[ W_{11}(x) = C_1 \cos(k_{11}x) + C_2 \cos(k_{12}x) + C_3 \cosh(k_{11}x) + C_4 \sinh(k_{11}x) \quad \text{mode 1} \] (8)

\[ W_{12}(x) = C_5 \cos(k_{12}x) + C_6 \cos(k_{13}x) + C_7 \cosh(k_{12}x) + C_8 \sinh(k_{12}x) \quad \text{mode 2} \]

The coefficients \( C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8 \) in mode shapes can be determined by boundary conditions and continuity conditions.

The modal frequencies of each section of the beam are the same, so the following equation exists \[ ]

\[ k_{11}^2 \sqrt{EI_1 / \rho A_1} = k_{12}^2 \sqrt{EI_2 / \rho A_2} = \omega_1 \] (9)

In order to realize mode decoupling and ensure the uniqueness of mode shape, the following orthogonal conditions are introduced to standardize the mode shape.

\[ \rho A_1 \int_0^{L_1} W_{m1}(x)W_{m1}(x)dx + \rho A_2 \int_{L_1}^{L} W_{m2}(x)W_{m2}(x)dx = \delta_{mm} \]

\[ EI_1 \int_0^{L_1} W_{m1}(x)\dot{W}_{m1}(x)dx + EI_2 \int_{L_1}^{L} W_{m2}(x)\dot{W}_{m2}(x)dx = \omega_1^2 \delta_{mm} \] (10)

By substituting equation (6) into equation (5) and using the orthogonal condition of equation (10), the simplified Lagrange function is obtained as follows:
These coefficients are expressed as follows:
\[
\begin{align*}
\theta_1 &= (H + H_p)e_{31} B \left[ W_{11}'(L_1) - W_{11}'(0) \right] / 2 \\
\theta_2 &= (H + H_p)e_{31} B \left[ W_{12}'(L) - W_{12}'(L_1) \right] / 2 \\
\beta &= \rho A_p \int_{L_1}^{L} W_{11}(x) dx + \rho A_3 \int_{L_1}^{L} W_{12}(x) dx \\
M &= \rho A_p L_1 + \rho A_3 (L - L_1) + 2 \rho_p A_p L \\
C_{p1} &= e_{33}' B L_1 / (2H_p) \\
C_{p2} &= e_{33}' B (L - L_1) / (2H_p)
\end{align*}
\]

Substituting equation (11) into Lagrange equation:
\[
\frac{d}{dt} \left( \frac{\partial \ell}{\partial q_1} \right) - \frac{\partial \ell}{\partial q_1} = F(t) \\
\frac{d}{dt} \left( \frac{\partial \ell}{\partial \dot{q}_1} \right) - \frac{\partial \ell}{\partial \dot{q}_1} = Q_{R1}(t) \\
\frac{d}{dt} \left( \frac{\partial \ell}{\partial \dot{q}_2} \right) - \frac{\partial \ell}{\partial \dot{q}_2} = Q_{R2}(t)
\]

The electromechanical coupling governing equation of piezoelectric energy harvester with variable cross-section overhanging beam is obtained. Let the foundation excitation be harmonic acceleration, where \( w_b \) is acceleration amplitude and \( Z_b \) is excitation frequency. Instead of the electromechanical coupling governing equation of the piezoelectric energy harvester with variable cross-section overhanging beam, the following results are obtained.

\[
\begin{align*}
(\omega_1^2 - \omega_2^2 + 2j\xi_2\omega_1\omega_2)H - \theta_1 V_{R1} - \theta_2 V_{R2} &= -\beta Z_b \\
(j\omega_1 C_{p1} + 1 / R_1) V_{R1} + j\omega_1 \theta_1 H &= 0 \\
(j\omega_2 C_{p2} + 1 / R_2) V_{R2} + j\omega_2 \theta_2 H &= 0
\end{align*}
\]

4. Results and discussion

The system geometry and material parameters [19-20] are shown below. Beryllium bronze is used as the substructure layer, the material of piezoelectric layer is piezoelectric ceramic PZT-5H. Firstly, defining the thickness ratio of beam is \( e_1 \) and thickness ratio of piezoelectric layer is \( e_2 \), supposing the acceleration amplitude is \( Z_b \), the resistance value of load resistance is 10KΩ. The numerical simulation results are obtained by substituting the following parameters into equation (14).

\[
\begin{align*}
L_1 &= 50mm; L = 200mm; H_1 = 5mm; H_2 = 2mm; H_p = 1mm; B = 20mm; e_{31}' = 0.02; \rho = 8920kg \cdot m^{-3}; E = 106GPa; \\
\rho_p &= 7500kg \cdot m^{-3}; E_p = 60.6GPa; e_{31} = -16.6C \cdot m^{-2}; e_{33}' = 21nF \cdot m^{-1}; Z_b = 5m/s^2; e_1 = H_1 / H_2; e_2 = H_p / H_1
\end{align*}
\]

Figure 2. Influence of beam thickness ratio on piezoelectric energy harvester.
As can be seen from Figure 2, the influence of beam thickness ratio on the resonance frequency of the energy harvester system is very significant. The resonance frequency increases with the increase of beam thickness ratio, the peak voltage increases with the increase of beam thickness ratio, and the beam thickness ratio has little influence on the peak power density. As can be seen from Figure 3, the thickness ratio of piezoelectric layer has a very significant effect on the resonant frequency of the energy harvester system. The resonant frequency increases with the increase of the length ratio, the peak power density decreases with the increase of the thickness ratio of piezoelectric layer. The thickness ratio of piezoelectric layer has little effect on the peak voltage.

5. Conclusion

The results show that the thickness of the beam has a great influence on the natural frequency of the piezoelectric energy harvester system, but has little influence on the output response. The low-frequency energy harvester can be realized by changing the thickness of the beam. So the energy collection efficiency can be improved. The thickness of the piezoelectric layer has a great influence on the natural frequency of the piezoelectric energy harvester system, but has little influence on the output response. The thickness of piezoelectric layer can be reduced to achieve low-frequency energy harvester. By reducing the thickness of piezoelectric layer, the efficiency of energy collection can be improved.

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