Application of the four-dimensional lattice spring model in direct shear testing of intact rock

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Abstract. Rock shear strength and its influencing parameters, such as cohesion and friction angle, are critical for both surface and underground project rock engineering designs. The application of direct shear testing is more commonly employed for rock joints than for intact rock due to several limitations, such as the nonuniform normal stress distribution. Hence, triaxial tests are more frequently used to determine the cohesion and friction angle of intact rock specimens. However, compared to the basic tools of the direct shear test, these tests require more complex experimental equipment, making the triaxial testing less accessible to laboratories in general. Therefore, it is important to develop a method for making intact rock direct shear test results more accurate for engineering applications. To evaluate and enhance the accuracy of intact direct shear tests, laboratory experiments on intact cubic marble specimens and consequent numerical simulation were performed using the four-dimensional lattice spring model (4D-LSM). The 4D-LSM is arguably more efficient than the conventional discrete element method since it uses direct calibration of microscale parameters from macroscale parameters while maintaining the ability to reproduce material fracture. In this study, a two-stage simulation was introduced in the 4D-LSM to represent the stages when the normal and shear loads are applied. To determine the failure state of the specimen, the Mohr–Coulomb failure criterion was implemented into the 4D-LSM. A bilinear stiffness model was used to reproduce the shear force-displacement curve from the experiments. Our results show that this method was able to reproduce numerical results that are similar to those obtained in experimental direct shear tests.

1. Introduction

Laboratory testing remains the most common method to determine the rock material shear strength parameters, such as cohesion and friction angle used in rock engineering applications [1]. Although alternative methods, such as preset angle shear tests, have been discussed in recent literature [2], the conventional direct shear and triaxial tests remain the most commonly practiced methods[3,4]. Triaxial tests are believed to be the best method to obtain internal shear strength parameters of intact rock, while direct shear tests are believed to be more suitable when dealing with the contact shear strength, such as rock joints and other discontinuities. However, since triaxial testing equipment is more complex than direct shear testing equipment, it would be beneficial if intact direct shear test results could also be utilized for further analysis and application, such as input parameters used in numerical modeling of the rock strength.
A wide range of numerical studies on direct shear of rock joints can be found in other literature. The finite element method (FEM) \([5]\) and various discrete element method (DEM) simulations \([6,7]\) were used to study the degradation of rock joint asperities, the failure mechanism of irregular rock joints, and the influence of boundary condition in these tests. The DEM has been one of the most common methods for simulating the mechanism of direct shear tests, especially with the use of the particle flow code (PFC). Some examples of rock joint research with the PFC includes the study on roughness parameter and peak shear strength \([8]\), implementation of smooth joint parameters for joint failure \([9]\), the influence of scale effect on rock joints \([10]\), and the mechanism of asperity shearing \([11]\). Through existing research, the drawbacks of each FEM and DEM simulation were also identified. Generally, the FEM cannot realistically display the failure process and fracturing of the rock material, and although this shortcoming can be solved using the DEM, a calibration process of macroscale and microscale parameters is still required.

Despite the extensive range of numerical works on rock joints under direct shear, the research on intact rock is relatively understudied. In existing studies, some significant challenges for direct shear of intact rock, which include the nonuniform distribution of normal stress found during experiments \([12,13,14]\) and the transition of mechanism from tensile to shear failure under influence of factors, such as normal stress \([15,16]\), have been reported. Nevertheless, this research aims to evaluate the capabilities of the newly developed four-dimensional lattice spring model (4D-LSM) proposed by Zhao \([17,18,19]\) in simulating the mechanism of the intact rock direct shear tests. Unlike traditional DEMs, the 4D-LSM allows for automatic calibration between macroscale and microscale parameters. Furthermore, as a substitute for the shear interaction, the pure central particle interaction in the fourth dimension in the 4D-LSM can separate the rigid-body rotation from shear deformation, thus recovering the Poisson’s ratio and naturally capturing the geometric nonlinearity. Since the rotational degrees of freedom and shear deformation are neglected by only considering the pure normal deformation, the 4D-LSM generates higher computing efficiency compared to the DEM. More details can be found in previous works on 3D-LSM and 4D-LSM development \([17,18,19]\).

In this work, direct shear tests were conducted on intact marble specimens using a common shear box machine under a range of different normal stresses and confinement. This experimental work was reproduced using numerical simulations adopting the 4D-LSM method. To represent different stages based on when the normal load and shear load were applied, a two-stage simulation for the experimental conditions was introduced. Furthermore, this method was able to approximately reproduce the shear stress-displacement curve of experimental results by adopting a bilinear stiffness model.

2. Direct shear test
2.1. Experimental setup
The direct shear test was conducted using a common direct shear test machine, (TJW-1000 electro-hydraulic) with a servo-control function and a maximum loading capacity and displacement of 10 MN and 50 mm, respectively (Figure 1a). The load was applied to the specimen by two vertical and horizontal actuators, which were confined between a top and bottom shear box. The shearing velocity was kept constant at 0.1 mm s\(^{-1}\) throughout the test. The shear and normal loads were obtained directly from horizontal and vertical load cells installed at each corresponding boundary, with a range of normal loads of 15, 20, 25, and 30 kN for each test. The normal stress and shear strength were calculated as

\[
\sigma_n = \frac{F_n}{A} \tag{1}
\]

\[
\tau = \frac{F_s}{A} \tag{2}
\]
where $\sigma_n$ is the normal stress, $F_n$ is the normal load vertical to the shear plane, $\tau$ is the shear strength, $F_s$ is the maximum shear load on the shear plane, and $A$ is the shear area. From the linear regression of shear strength-normal stress plots, the Mohr–Coulomb (M–C) strength criterion can be derived as

$$\tau = \sigma_n \tan \phi + c \quad (3)$$

where $\phi$ is the internal friction angle and $c$ is the cohesion.

2.2. Direct shear test results

For this study, marble specimens with cubic dimensions of 50 mm $\times$ 50 mm $\times$ 50 mm were prepared (Figure 1b). The properties of the marble are listed in Table 1 based on preliminary tests. To generate the normal stress, the servo-controlled normal loads were applied gradually onto the upper surface of the shear box, until the targeted normal stresses of 6, 8, 10, and 12 MPa were achieved before the shear load was applied at a constant velocity. Since variations in the specimen dimension were likely unavoidable, a variation of 0.1 MPa to 0.2 MPa was calculated. The direct shear test results at various normal stresses are presented in Table 2.

Based on the M–C regression from experimental results, the friction angle and cohesion of the marble were 44.85° and 15.73 MPa, respectively (Figure 1c).

**Table 1.** Marble properties

| Properties       | Value       |
|------------------|-------------|
| Compressive Strength | 121.38 MPa |
| Tensile Strength     | 6.1 MPa     |
| Young’s Modulus      | 59.7 GPa    |
| Poisson’s Ratio      | 0.274       |
| Density             | 2845 kg m$^{-3}$ |

**Table 2.** Direct shear results

| Normal Load (kN) | $\sigma_n$ (MPa) | $\tau$ (MPa) |
|------------------|------------------|--------------|
| 15               | 6.2              | 21.674       |
| 20               | 8.12             | 24.629       |
| 25               | 10.18            | 24.890       |
| 30               | 12.09            | 28.156       |

Based on the M–C regression from experimental results, the friction angle and cohesion of the marble were 44.85° and 15.73 MPa, respectively (Figure 1c).

![Figure 1. Direct shear experiment (a) machine, (b) specimens, and (c) M–C envelope](image-url)
3. Numerical method

3.1. The 4D-LSM

The 4D-LSM numerical method was adopted to reproduce the experimental results. As previously stated, the 4D-LSM is capable of simulating the failure process and fracture propagation of rocks at an arguably better computing efficiency than the DEM, which only considers pure normal deformation. In the 4D-LSM, a solid is represented as a system of lattice spring elements linked by springs between each center point. While the particles, like the DEM, adopt Newton’s second law, the interaction between two particles is represented as

\[ F_{ij} = ku_n n_{ij} \]  

(4)

where \( F_{ij} \) is the force from particle i to particle j, \( k \) is the lattice spring stiffness, \( n_{ij} \) is the normal vector between the two particles, and \( u_n \) is the spring deformation, which is calculated by

\[ u_n = |x_j - x_i| - |x_j^0 - x_i^0| \]  

(5)

where \( x \) is the particle position and \( x^0 \) is the initial position and \( |\cdot| \) represents the distance between the two particles.

In the 4D-LSM, a particle and its surrounding particles in a lattice structure are classified as a particle cluster. The stress of this particle is calculated from the deformation of the spring bonds of its neighboring particles as

\[ \sigma_{ij}^I = \frac{1}{2V^I} \sum_{f=0}^{N} f_{ij}^{f} n_{ij}^{f} t_{ij}^{f} \]  

(6)

where \( \sigma_{ij}^I \) is the stress tensor of particle, \( I, V^I \) is the represented volume, \( f_{ij}^{f} \) is the interaction force between particle I and its neighboring particles, \( n_{ij}^{f} \) is the normal vector between particle I and its neighboring particles, and \( t_{ij}^{f} \) is the initial spring length between particle I and its neighboring particles. The stress state of the spring bond is represented as

\[ \sigma_{ij}^{bond} = \frac{\sigma_i + \sigma_j}{2} \]  

(7)

while the spring model parametric base stiffness is represented as

\[ k = \frac{6VE}{\eta \sum l_i^2} \]  

(8)

where \( V \) is the 3D volume of the model, \( E \) is the input elastic modulus, \( \eta \) is the elastic modulus increase ratio derived from the 4D stiffness ratio and Poisson’s ratio when scaled into a 3D space, and \( l_i \) is the length of the \( i \)th spring.

The failure state of the spring bond is determined by implementing the M–C criterion:
\[ f(\sigma_{ij}^{\text{bond}}) = f(\sigma_{1}^{\text{bond}}, \sigma_{3}^{\text{bond}}) = \begin{cases} (1 - \sin \phi)\sigma_{3}^{\text{bond}} - (1 + \sin \phi)\sigma_{1}^{\text{bond}} + 2c \cos(\phi) \leq 0 \\ \sigma_{1}^{\text{bond}} - \sigma_{t}^* \geq 0 \end{cases} \] (9)

It is important to note that in the 4D-LSM, compression is specified as negative stress, while tension is specified as positive stress, which is contrary to the universally accepted terms in rock mechanics. Details on the principles, development, and other applications of the 4D-LSM can be found in the works of Zhao et al. [17,18,19].

3.2. Two-stage simulation and boundary condition
To reproduce the experimental shear tests, a two-stage method was introduced based on the stages when the normal (Figure 2a) and shear loads (Figure 2b) were applied. Following the experimental procedure, the first stage aims to adapt the targeted normal stress or confinement by applying a servo-controlled normal load downward on the upper boundary of the cubic specimen (y-axis). The first stage is completed when particles reach the targeted normal load as well as the kinetic stability, which is indicated by a drop in kinetic energy to zero (Figure 2c). At the end of this first stage, the cubic sample was in a stable confined state, and the corresponding particle and boundary deformations were saved as an output file (Figure 2d).

**Figure 2.** Two-stage direct shear simulation: (a) confinement at first stage, (b) energy and load stability at end of the first stage, (c) particle deformation state after the first stage, and (d) shear at the second stage.
The first stage model output was used as the input for the second stage, which introduced the shear load. Therefore, at the initial time step in the second stage, the corresponding deformations at the end of the first stage were already implemented. Aside from the servo-controlled boundary condition of the normal load, the other boundaries were set as wall boundaries with nonlinear constitutive laws. The shear load was applied horizontally at the top half of the specimen (x-axis) at a velocity of 10 mm s\(^{-1}\). The time step for the simulation was set to 10 s to generate a displacement range of 10 mm–6 mm in each time step. By assigning a fixed boundary to the z-axis, the displacement in the z-axis was neglected throughout the simulation. Both the normal and shear loads were recorded from the reaction force on each corresponding normal and shear load boundary.

4. **Numerical results**

4.1. Direct shear test rotation moment calibration and results

As previously stated, the nonuniform normal stress distribution is one of the drawbacks of direct shear tests. The phenomenon due to the rotation experienced by the rock specimens after the shear stress is applied has been identified in previous research \([12,13,14]\). This rotation was seen not only in conventional direct shear tests, but also in research on ballast stone particles \([20]\), inclusion matrix systems \([21]\), and shearing under dynamic and cyclic loading conditions \([22]\). The rotation phenomenon was also evident in the 4D-LSM simulation (Figure 3a). To solve this problem, a moment correction method based on the rotational phenomenon observed in direct shear tests was introduced. This method calibrates the value of recorded normal stress in the experiment, and new shear strength-normal stress plots from the adjusted values provide an adjusted M–C regression, as well as adjusted cohesion and friction angle values. More details about this moment calibration have been provided by Pratomo et al \([23]\). The adjusted normal stress, \(\sigma_{n\text{-adj}}\), was calculated as

\[
\sigma_{n\text{-adj}} = \frac{F_n - \alpha 0.75 F_s}{A}
\]

where \(\alpha\) is a correlation coefficient introduced to account for other factors such as friction, which was neglected in this calibration. To obtain the suitable \(\alpha\), simulations were conducted for a normal load of 15 kN or normal stress of 6.2 MPa using new parameters obtained from \(\alpha\) values ranging from 0.1 to 1.0. The results summarised in Table 3 show that parameters with \(\alpha\) of 0.8 provide the closest approximation to the experimental value.

| Experiment   | Cohesion (MPa) | Friction Angle (°) | Shear Load (kN) | Shear Strength (MPa) |
|--------------|----------------|--------------------|-----------------|----------------------|
| 0 (no adjustment) | 15.73          | 44.85              | 34.99           | 14.00                |
| 0.1          | 17.07          | 46.84              | 36.56           | 14.62                |
| 0.2          | 18.62          | 48.90              | 38.58           | 15.43                |
| 0.3          | 20.44          | 51.05              | 40.32           | 16.13                |
| 0.4          | 22.57          | 53.22              | 42.56           | 17.02                |
| 0.5          | 25.08          | 55.38              | 45.09           | 18.04                |
| 0.6          | 28.02          | 57.43              | 47.90           | 19.16                |
| 0.7          | 31.37          | 59.19              | 50.97           | 20.39                |
| 0.8          | 34.95          | 60.35              | 54.73           | 21.89                |
| 0.9          | 38.16          | 60.24              | 59.10           | 23.64                |
| 1.0          | 39.58          | 57.25              | 64.98           | 25.99                |
When the input parameters in the 4D-LSM corresponded to the actual experimental values ($\alpha = 0$), the numerical results underestimated the experimental values at each normal stress range with an average error or deviation of 37.18%. After implementing the adjusted parameter values corresponding to $\alpha = 0.8$, the average error reduced significantly to a value of 6.75% (Figure 3b). This emphasizes the importance of normal stress calibration in obtaining adjusted cohesion and friction angle values for further numerical analysis.

![Figure 3](image1)

**Figure 3.** (a) Rotation in 4D-LSM direct shear and (b) comparison between experimental and numerical results with and without the application of moment correction

In terms of the failure mechanism. The shear failure mechanism observed in the experiments (Figure 4a) was also similar to that produced in the numerical simulation (Figure 4b), indicating a horizontal failure plane along the center of the sample.

![Figure 4](image2)

**Figure 4.** Example of direct shear failure mechanism in (a) experiments and (b) 4D-LSM

4.2. **Bilinear stiffness model**

To further improve the direct shear capabilities of the 4D-LSM, a bilinear stiffness model were introduced (Figure 5a). In common direct shear results, the force-displacement curves indicate a bilinear transition between the initial loading stage and the elastic stage (Figure 5b). In the initial loading stage, a lower stiffness was observed, indicating the process of microcrack closure. After a predetermined
critical shear displacement \((dU_n)\), the elastic stage was observed with the corresponding stiffness of \(k_{n2}\).

![Bilinear stiffness model and common shear force-displacement curves](image)

**Figure 5.** (a) Bilinear stiffness model and (b) common shear force-displacement curves of direct shear

Therefore, in the initial loading stage \((d < dU_n)\), the force is calculated as

\[
F = k_{n1} d \\
F = \beta k_{n0} d
\]

where \(k_{n1}\) represents the initial loading stiffness, \(d\) is the shear displacement, and \(\beta\) is the initial loading stiffness coefficient used to calibrate \(k_{n0}\), the numerical input stiffness. Furthermore, the elastic stage force \((d \geq dU_n)\) is calculated as

\[
F = k_{n2}(d - dU_n) + k_{n1}dU_n \\
F = \gamma k_{n0}(d - dU_n) + \beta k_{n0}dU_n
\]

where \(k_{n2}\) represents the elastic stiffness, \(dU_n\) is the critical displacement obtained from experimental results where the stiffness transition occurs, and \(\gamma\) is the elastic stiffness coefficient used to calibrate \(k_{n0}\).

Through several parameter simulations, the shear force-displacement of the marble was found to be well represented by values of \(\beta = 0.00075\), \(\gamma = 0.02\), and \(dU_n = 0.35\) mm. These stiffness parameters were used in further direct shear testing simulations under each normal stress value. To obtain and reproduce the maximum experimental shear force, additional adjustments, such as increasing the input cohesion value, were made at simulations under each experimental normal stress. The simulation results indicate that the experimental shear force-displacement curves could be reproduced (Figures 6a–d). However, it is important to note that during the experimental data recording, most results show a large initial loading duration, which is probably due to the contact closure process between the compression machine and the shear box.
Figure 6. Direct shear numerical and experimental results at a normal stress of (a) 6.2 MPa, (b) 8.12 MPa, (c) 10.18 MPa, and (d) 12.09 MPa

Conclusion
In this study, a 4D-LSM was used to simulate direct shear tests on intact cubic marble specimens. A two-stage simulation was introduced to represent the application of normal and shear loads during the experiment. The boundary conditions and the corresponding failure mechanism obtained in the experiments were reproduced using this method. Considering the rotation phenomenon observed in direct shear tests, further adjustments of shear strength parameters based on the moment correction method were made. The results show that the experimental shear strength can be reproduced with a significantly reduced error from 37.18% to 6.17%. Finally, a bilinear stiffness model with stiffness parameters representing the initial loading stage and the elastic deformation stage of shear loading was adopted in this study. The 4D-LSM was shown to be capable of roughly reproducing the shear force-displacement curves as observed in the experiments.

Acknowledgments
The first author and the last author would like to acknowledge the Australia Awards Scholarship by the Department of Foreign Affairs and Trade and the support from the Australian Research Council. The corresponding author and other authors would like to acknowledge the support from the China National Key R&D Program (Grant No. 2018YFC0406804) and the China National Natural Science Foundation (Grant No. 11772221).
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