Quantum Electron Plasma, Visible and Ultraviolet P-wave and Thin Metallic Film

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Abstract

The interaction of the visible and ultraviolet electromagnetic P-wave with the thin flat metallic film localized between two dielectric media is studied numerically in the framework of the quantum degenerate electron plasma approach. The reflectance, transmittance and absorptance power coefficients are chosen for investigation. It is shown that for the frequencies in the visible and ultraviolet ranges, the quantum power coefficients differ from the ones evaluated in framework of both the classical spatial dispersion and the Drude – Lorentz approaches.

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1. Introduction

Studies of the electromagnetic waves interacting with tiny or nanoscale metallic objects attract a large attention for a long time \cite{1} – \cite{12}. Such investigations have not only theoretical interest but are aimed also on multiple practical applications. In these researches, the kinetic theory for the Fermi – Dirac electron gas is widely exploited. Using the theory, one takes into account the spatial dispersion of the electron plasma. And moreover, the theory of the electromagnetic waves interacting with flat metallic film was well developed for the case of specular electron reflection \cite{2} \cite{3}.

But almost always in these researches, the quantum wave properties of electrons in the electron plasma were disregarded. The main problem was in a getting acceptable both the longitudinal and the transverse dielectric functions (permittivities) of the quantum...
electron plasma \[5, 6, 13 \, \text{–} \, 16\]. However since the electrons in a metal obey in general the quantum laws, the quantum wave electron effects should contribute to interaction of light with a metal. It would be essential in case of the high frequency light and for the nanoscale metallic objects.

In the present paper, we study the interaction of the visible and ultraviolet electromagnetic P-wave with quantum degenerate electron plasma in the thin flat metallic film localized between transparent dielectric media. Being interested in the visible and ultraviolet light we consider the frequencies of order and larger than the electron plasma frequency. The values taken by us for investigation are the reflectance, transmittance and absorptance power coefficients. We study the quantum degenerate electron plasma with invariable relaxation time in case of specular electron reflection from the film surface. The longitudinal and transverse dielectric functions (permittivities) of the quantum electron plasma are taken in the Mermin approach \[6, 15, 16\]. We study the power coefficients as functions of frequency, of incidence angle and of quantum parameter. We compare also the coefficients with those obtained both in case of the Drude – Lorentz theory without spatial dispersion and in case of the classical degenerate electron gas approach accounting for the spatial dispersion.

2. The model and the power coefficients

Let us consider the thin flat metallic film of the width \(d\) placed between two isotropic nonmagnetic transparent dielectric media with the positive dielectric constants \(\varepsilon_1\) and \(\varepsilon_2\). So the light dispersion and absorption of these media are disregarded by us. We consider the electromagnetic wave is incident from the first dielectric medium on the film under the angle \(\theta\) from the surface normal (see fig. 1). Hence the second medium can be treated as a substrate.

We direct the \(Z\) axis perpendicular to the film surface towards the second dielectric medium. Let \(z = 0\) be the surface contacting with the first medium and therefore, the \(z = d\) is the second surface having contact with the second medium (fig. 1). We direct the \(X\) axis along the film surface in the incidence plane towards the wave propagation.

We study the P-waves i.e. the \(E\) vectors of the waves incident on, reflected from and transmitted through the film lie in the incidence plane. We direct the third \(Y\) axis in such a way that the rectangular system being the right-handed one. Then the \(H\) vectors of the waves are parallel to the \(Y\) axis.

The electric and magnetic fields of the waves in the \(z < 0\) space in the projections onto \(X\) and \(Y\) axes look as follows \[2\]:

\[
\begin{align*}
E_x(x, y, z, t) &= e^{i(k_1x - \omega t)} \left[ a_I e^{ik_1z} - a_R e^{-ik_1z} \right] \cos \theta, \\
H_y(x, y, z, t) &= e^{i(k_1x - \omega t)} \left[ a_I e^{ik_1z} + a_R e^{-ik_1z} \right] \frac{\sqrt{\varepsilon_1}}{Z_0}.
\end{align*}
\] (1)
Fig. 1. The metallic film between two dielectric media with $\varepsilon_1$ and $\varepsilon_2$, and the incident, reflected and transmitted waves.

In the $z > d$ space, these fields look as

$$
\begin{align*}
E_x(x, y, z, t) & = e^{i(k_{2x}x-\omega t)}a_T \cos \theta' e^{i k_{2z}(z-d)}, \\
H_y(x, y, z, t) & = e^{i(k_{2x}x-\omega t)}a_T \frac{\sqrt{\varepsilon_2}}{Z_0} e^{i k_{2z}(z-d)}.
\end{align*}
$$

(2)

Here $\omega$ is the wave frequency, $k_{1x}$ and $k_{1z}$ are the $x$- and $z$-coordinates of the incident wave vector $k_1$ in the first dielectric medium, $k_{2x}$ and $k_{2z}$ are the same coordinates of the transmitted wave vector $k_2$ in the second dielectric medium:

$$
\begin{align*}
k_{1x} & = \frac{\omega}{c} \sqrt{\varepsilon_1} \sin \theta = k_{2x} = \frac{\omega}{c} \sqrt{\varepsilon_2} \sin \theta' ; \\
k_{1z} & = \frac{\omega}{c} \sqrt{\varepsilon_1} \cos \theta, \quad k_{2z} = \frac{\omega}{c} \sqrt{\varepsilon_2} \cos \theta'.
\end{align*}
$$

(3)

Further, the $a_R$, $a_I$ and $a_T$ denote respective complex electric field amplitudes of the incident, reflected and transmitted waves. The $c$ is the vacuum speed of light, $Z_0$ denotes the dimensional (in Ohm) vacuum impedance. And at the end, $\theta'$ is the narrow refraction angle into the second dielectric medium from the surface normal (fig. 1). Note that in the case of total internal reflection $\sin \theta' > 1$, the $\cos \theta'$ value is pure imaginary with the $\text{Im} \cos \theta' > 0$. So, the $\cos \theta'$ is evaluated according to the prescription following from (3):

$$
\cos \theta' = \begin{cases} 
\sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2} \sin^2 \theta}, & \sin \theta \leq \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}; \\
i \sqrt{\frac{\varepsilon_1}{\varepsilon_2} \sin^2 \theta - 1}, & \sin \theta > \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}.
\end{cases}
$$

(4)

In the film space $0 < z < d$, the electric and magnetic fields may be represented in the following manner:

$$
\begin{align*}
E_x(x, y, z, t) & = e^{i(k_{1x}x-\omega t)}[\alpha_1 E_x^{(1)}(z) + \alpha_2 E_x^{(2)}(z)], \\
H_y(x, y, z, t) & = e^{i(k_{1x}x-\omega t)}[\alpha_1 H_y^{(1)}(z) + \alpha_2 H_y^{(2)}(z)].
\end{align*}
$$

(5)
Here $\alpha_1$ and $\alpha_2$ are some constant coefficients, and $E_x^{(j)}(z)$ and $H_y^{(j)}(z)$ are the symmetric or antisymmetric modes of electric and magnetic fields $(j = 1, 2)$:

$$E_x^{(j)}(z) = (-1)^j E_x^{(j)}(d - z), \quad H_y^{(j)}(z) = (-1)^{j+1} H_y^{(j)}(d - z). \quad (6)$$

On the film surfaces $z = 0$ and $z = d$, one has the boundary conditions:

$$\begin{cases}
  E_x(x, y, -0, t) = E_x(x, y, +0, t), \\
  H_y(x, y, -0, t) = H_y(x, y, +0, t);
\end{cases} \quad (7)$$

$$\begin{cases}
  E_x(x, y, d - 0, t) = E_x(x, y, d + 0, t), \\
  H_y(x, y, d - 0, t) = H_y(x, y, d + 0, t).
\end{cases} \quad (8)$$

Following [2] – [4], we consider the dimensionless surface impedance for the P-wave modes on the $z = 0$ surface $(j = 1, 2)$:

$$Z_P^{(j)} = \frac{1}{Z_0} \frac{E_x^{(j)}(+0)}{H_y^{(j)}(+0)}. \quad (9)$$

Substituting (11) and (5) into (7), (2) and (5) into (8), taking into account the property (6) of the modes and using the surface impedance (9), after elimination of similar multipliers one gets the following system:

$$\begin{cases}
  (a_I - a_R) \cos \theta = \alpha_1 E_x^{(1)}(+0) + \alpha_2 E_x^{(2)}(+0), \\
  (a_I + a_R) \sqrt{\varepsilon_1} = \alpha_1 \frac{E_x^{(1)}(+0)}{Z_P^{(1)}} + \alpha_2 \frac{E_x^{(2)}(+0)}{Z_P^{(2)}}, \\
  a_T \cos \theta' = -\alpha_1 E_x^{(1)}(+0) + \alpha_2 E_x^{(2)}(+0), \\
  a_T \sqrt{\varepsilon_2} = \alpha_1 \frac{E_x^{(1)}(+0)}{Z_P^{(1)}} - \alpha_2 \frac{E_x^{(2)}(+0)}{Z_P^{(2)}}.
\end{cases} \quad (10)$$

After elimination of the values $\alpha_j E_x^{(j)}(+0) (j = 1, 2)$ from the system (10), one comes to the system

$$\begin{cases}
  a_T = U_p^{(1)} a_I - V_p^{(1)} a_R, \\
  a_T = -U_p^{(2)} a_I + V_p^{(2)} a_R.
\end{cases} \quad (11)$$

Here we denoted $(j = 1, 2)$:

$$U_p^{(j)} = \frac{\cos \theta - Z_P^{(j)} \sqrt{\varepsilon_1}}{\cos \theta' + Z_P^{(j)} \sqrt{\varepsilon_2}}, \quad V_p^{(j)} = \frac{\cos \theta + Z_P^{(j)} \sqrt{\varepsilon_1}}{\cos \theta' + Z_P^{(j)} \sqrt{\varepsilon_2}}. \quad (12)$$

Now we turn to the definition of the reflectance $R$, transmittance $T$ and absorptance $A$ power coefficients. The first two of them are merely the following ratios [17][18]:

$$R = \frac{|\langle S_z \rangle_R|}{|\langle S_z \rangle_I|}, \quad T = \frac{|\langle S_z \rangle_T|}{|\langle S_z \rangle_I|}. \quad (13)$$

Here $\langle S_z \rangle$ is the time averaged energy flux density, or Poynting, vector, projected onto the $Z$ axis:

$$\langle S_z \rangle = \frac{1}{2} \text{Re}(E \times H^*) \cdot e_z, \quad (14)$$
where \( * \) denotes the complex conjugation and \( \mathbf{e}_z \) is the unit vector towards the Z axis direction. In the equations (13), the \( I, R \) and \( T \) subscript letters stand for the incident, reflected and transmitted waves respectively. For the P-wave, the expression (14) can be transformed to the following one:

\[
\langle S_z \rangle = \frac{1}{2} \text{Re}(E_x H_y^*) .
\]  

(15)

We substitute the expressions (1) and (2) to the (15) and select the terms related to \( \langle S_z \rangle_I, \langle S_z \rangle_R \) and \( \langle S_z \rangle_T \). Then we substitute these terms to the equations (13) and take into account the positivity of the dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \). And one arrives at the following expressions for the reflectance \( R \) and transmittance \( T \) power coefficients:

\[
R = \left| \frac{a_R}{a_I} \right|^2 , \quad T = \text{Re} \left( \frac{\cos \theta' \sqrt{\varepsilon_2}}{\cos \theta \sqrt{\varepsilon_1}} \right) \left| \frac{a_T}{a_I} \right|^2 .
\]  

(16)

Finding from the system (11) the ratios \( a_R/a_I \) and \( a_T/a_I \) and then substituting them into the equations (16), one obtains the final equations for the \( R \), \( T \) and also for the absorptance \( A \) (see also [12]):

\[
R = \left| \frac{U_p^{(1)} + U_p^{(2)}}{V_p^{(1)} + V_p^{(2)}} \right|^2 ,
\]  

(17)

\[
T = \text{Re} \left( \frac{\cos \theta' \sqrt{\varepsilon_2}}{\cos \theta \sqrt{\varepsilon_1}} \right) \left| \frac{U_p^{(1)} V_p^{(2)} - U_p^{(2)} V_p^{(1)}}{V_p^{(1)} + V_p^{(2)}} \right|^2 ,
\]  

(18)

\[
A = 1 - R - T .
\]  

(19)

If the two dielectric media are vacuum or air when \( \varepsilon_1 = \varepsilon_2 = 1 \), one has \( \theta' = \theta \) and the equations (17), (18) go over to the reflectance and transmittance power coefficients presented in papers [3, 4].

3. The surface impedance and the dielectric functions of the degenerate electron plasma

The surface impedance defined by equation (9), was evaluated in [3] for the flat metallic film in the case of specular electron reflections from the film surface. For the P-wave it looks as follows \(( j = 1, 2)\):

\[
Z_p^{(j)} = \frac{2i\Omega}{\beta W} \sum_n \frac{1}{Q_n^2} \left( \frac{Q_n^2}{\Omega^2 \varepsilon_{it}(\Omega, Q_n)} + \frac{(\pi n/W)^2}{\Omega^2 \varepsilon_{tr}(\Omega, Q_n) - (Q_n/\beta)^2} \right) .
\]  

(20)

Here \( \Omega, \beta, W, Q_n, Q_x \) are dimensionless variables and parameters:

\[
\Omega = \frac{\omega}{\omega_p} , \quad \beta = \frac{v_F}{c} , \quad W = \frac{\omega_p d}{v_F} ,
\]  

(21)

\[
Q_n = \sqrt{\left( \frac{\pi n}{W} \right)^2 + Q_x^2} , \quad Q_x = \frac{v_F k_x}{\omega_p} .
\]  

(22)
In (21) and (22) $\omega_p$ is the degenerate electron plasma frequency, $v_F$ is the electron Fermi velocity and the $k_x$ is the $x$-coordinate of the wave vector $k$. Further, the $\varepsilon_l(\Omega, Q)$ and $\varepsilon_{tr}(\Omega, Q)$ are the respective longitudinal and transverse dielectric functions (permittivities) of the electron gas. And summation in (20) is performed over all odd integers $n$ if $j = 1$ or over all even integers if $j = 2$:

$$
\begin{align*}
  j = 1 : & \quad n = \pm 1, \pm 3, \pm 5, \pm 7, \ldots; \\
  j = 2 : & \quad n = 0, \pm 2, \pm 4, \pm 6, \ldots.
\end{align*}
$$

It is convenient to use dimensionless variables in the units of the degenerate electron plasma. The dielectric functions of the quantum electron plasma at zero temperature with invariable relaxation time due to electron collisions, obtained in the Mermin approach involving the electron density matrix in the momentum space, look as follows [6, 15, 16]:

$$
\begin{align*}
  \varepsilon^{(qu)}_l(\Omega, Q) &= 1 + \frac{3}{4Q^2} \left( \frac{\Omega + i\gamma}{\Omega F(0, Q)} \right) + \frac{i\gamma}{\Omega + i\gamma} F(\Omega + i\gamma, Q), \\
  \varepsilon^{(qu)}_{tr}(\Omega, Q) &= 1 - \frac{1}{\Omega^2} \left( 1 + \frac{\Omega G(\Omega + i\gamma, Q) + i\gamma G(0, Q)}{\Omega + i\gamma} \right).
\end{align*}
$$

Here the functions $F$ and $G$ are defined by the equations:

$$
\begin{align*}
  F(\Omega + i\gamma, Q) &= \frac{1}{r} \left[ B_1(\Omega_+ + i\gamma, Q) - B_1(\Omega_- + i\gamma, Q) \right] + 2, \\
  G(\Omega + i\gamma, Q) &= \frac{3}{16r} \left[ B_2(\Omega_+ + i\gamma, Q) - B_2(\Omega_- + i\gamma, Q) \right] + \\
  & \quad + \frac{9}{8} \left( \frac{\Omega + i\gamma}{Q} \right)^2 + \frac{3}{32} Q^2 r^2 - \frac{5}{8},
\end{align*}
$$

where the functions and variables

$$
\begin{align*}
  B_1(\Omega + i\gamma, Q) &= \frac{1}{Q^3} \left[ (\Omega + i\gamma)^2 - Q^2 \right] L(\Omega + i\gamma, Q), \\
  B_2(\Omega + i\gamma, Q) &= \frac{1}{Q^3} \left[ (\Omega + i\gamma)^2 - Q^2 \right]^2 L(\Omega + i\gamma, Q), \\
  L(\Omega + i\gamma, Q) &= \ln \frac{\Omega + i\gamma - Q}{\Omega + i\gamma + Q},
\end{align*}
$$

$$
\Omega_\pm = \Omega \pm \frac{1}{2} Q^2 r.
$$

And further, the dimensionless variable and parameters

$$
Q = \frac{v_F |k|}{\omega_p}, \quad \gamma = \frac{1}{\omega_p \tau}, \quad r = \frac{h\omega_p}{m_e v_F^2},
$$

where $\tau$ is the relaxation time owing to the electron collisions, $m_e$ is the effective mass of the conduction electrons and $h$ is the Planck constant.

Some words about the logarithm ratio $L(\Omega + i\gamma, Q)$ defined by (25). Here the complex logarithm is defined on the complex plane with the branching real negative half-line. And therefore, one has to evaluate the logarithm ratio according to the rule:

$$
L(\Omega + i\gamma, Q) = \frac{1}{2} \ln \frac{(\Omega - Q)^2 + \gamma^2}{(\Omega + Q)^2 + \gamma^2} + i \left( \arctan \frac{\Omega + Q}{\gamma} - \arctan \frac{\Omega - Q}{\gamma} \right).
$$
Using (27) and taking into account that $Q \geq 0$ and $\gamma \geq 0$ one can prove that the functions $F(0, Q)$ and $G(0, Q)$ are real.

Note that the dielectric functions (23), (24) in the classical limit $r \to 0$ go over to corresponding dielectric functions of the degenerate Fermi gas disregarding for the quantum wave electron properties called as classical spatial dispersion case \cite{2, 12}:

$$
\varepsilon_{l}^{(cl)}(\Omega, Q) = 1 + \frac{3}{Q^2} \left( 1 + \frac{\Omega + i\gamma}{2Q} L(\Omega + i\gamma, Q) \right) \times \\
\times \left( 1 + \frac{i\gamma}{2Q} L(\Omega + i\gamma, Q) \right)^{-1},
$$

(28)

$$
\varepsilon_{tr}^{(cl)}(\Omega, Q) = 1 - \frac{3}{4\Omega} \left( \frac{2(\Omega + i\gamma)}{Q^2} + B_1(\Omega + i\gamma, Q) \right).
$$

(29)

And also, in the static limit $Q \to 0$ the quantum dielectric functions (23), (24) as well as the classical spatial dispersion functions (28), (29), go over to the well-known classical Drude – Lorentz electron dielectric function without the spatial dispersion \cite{18}:

$$
\varepsilon_{l}^{(DL)}(\Omega) = \varepsilon_{tr}^{(DL)}(\Omega) = 1 - \frac{1}{\Omega(\Omega + i\gamma)}.
$$

(30)

Note that in the case of Drude – Lorentz dielectric function (30), the summation in (20) can be done exactly.

Recall that the surface impedance (20) is used in (12) to evaluate the power coefficients (17) – (19). In (20), the $Q_x$ variable is evaluated by substituting $k_{1x}$ by (3) in the (22) for $Q_x$ and taking into account the definitions (21). As a result, one gets:

$$
Q_x = \Omega \beta \sqrt{\varepsilon_1} \sin \theta.
$$

(31)

4. Numerical studies of power coefficients

We performed numerical studies of the reflectance $R$, transmittance $T$ and absorptance $A$ power coefficients for P-wave. These coefficients evaluated according to the equations (17) – (19) with use of (4), (12), (20), (22) and (31). First, we studied the power coefficients evaluated for the quantum plasma with the dielectric functions (23), (24). Then we compared these coefficients with those evaluated in the cases of the classical spatial dispersion approach with dielectric functions (28), (29) and in the Drude – Lorentz theory with the function (30). And at the end, we investigated the dependence of the power coefficients for the quantum plasma on the parameter $r$ in (26) called also as quantum parameter.

We have taken the following data appropriate for potassium \cite{4}: $v_F = 8.5 \cdot 10^5$ m/sec, $\omega_p = 6.61 \cdot 10^{15}$ sec$^{-1}$, the mass of conductance electron equals to the mass of the free electron, and also $\gamma = 10^{-3}$. Then using (21) and (26), we have adopted the dimensionless parameters $\beta = 2.83 \cdot 10^{-3}$ and $r = 1.07$.

At the fig. 2 we present numerical results for $R$ as function of $\Omega$ at two values $W = 10$ and $W = 50$ corresponding according to (21), to the film width $d = 1.286$ nm and
Fig. 2. The reflectance $R$ as function of $\Omega$ for quantum plasma at $\beta = 2.83 \cdot 10^{-3}$, $\gamma = 10^{-3}$,
$r = 1.07$, $\theta = 60^\circ$, $\varepsilon_1 = 1$ (air), $\varepsilon_2 = 2$ (quartz), $W = 10$ (left plot) and $W = 50$ (right plot).

$d = 6.43$ nm respectively. The first dielectric medium taken by us is an air with $\varepsilon_1 = 1$ and the second medium, or substrate, is a quartz having $\varepsilon_2 = 2$. We considered the $\Omega$ in the interval $0.5 \leq \Omega \leq 3$ related to the visible and ultraviolet light. One observes peaks near $\Omega = 1$ and when the $\Omega > 1$. The distances between peaks decrease with growth of the $W$. These results confirm the conclusions [3, 10] that the peaks are caused by oscillations of longitudinal plasmons in the metallic film and that the distances between peaks are of order $\pi/W$ or $\pi/d$.

Then we evaluate the power coefficients as functions of the incidence angle $\theta$. Results are shown at the fig. 3 for the same values $W$. One sees that the reflectance $R$ almost always increases, the transmittance $T$ decreases with growth of the $\theta$ and the curves for $R$ and $T$ are intersecting. But the absorptance $A$ first have growth then it decreases. It is obvious also that the angle of the $A$ maximum decreases with growth of $W$ or $d$ and this angle is very close to the angle of the intersection of curves $R$ and $T$.

The above results for quantum electron plasma reproduce a qualitative behaviour of the power coefficients for the classical degenerate spatial dispersion plasma [12].
Fig. 4. The reflectance $R$ (upper plots) and absorptance $A$ (lower plots) as functions of $\Omega$ at $\beta = 2.83 \cdot 10^{-3}$, $\gamma = 10^{-3}$, $r = 1.07$, $\theta = 60^\circ$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $W = 10$: 1 – quantum plasma (solid line), 2 – classical spatial dispersion approach (dotted line), 3 – Drude – Lorentz approach (dashed line).

Now we turn to a comparison of the quantum power coefficients with those in the classical spatial dispersion and in the Drude – Lorentz approaches. Numerical results are presented at the fig. 4 for the reflectance $R$ and absorptance $A$ as functions of $\Omega$. We took again the $\Omega$ in the interval $0.5 \leq \Omega \leq 3$.

One sees that quantum results differ from the Drude – Lorentz ones. In the Drude – Lorentz approach, the power coefficients have only one peak in vicinity of $\Omega = 1$ related to the critical point of the dielectric function (30) whereas the quantum approach gives peaks both near $\Omega = 1$ and at $\Omega \gg 1$.

The power coefficients in the quantum approach differ also from those in the classical spatial dispersion approach. As in the quantum case, one has peaks in the classical theory. But the peaks in quantum and in classical approaches are not coinciding at $\Omega > 1$. And quantity of the classical peaks is greater than the quantum ones. So, one can conclude that the quantum wave effects of electrons in the electron plasma lead to a displacement, smoothing and vanishing of the most of the resonant peaks.

We made a more detailed investigation of the influence of the quantum effects on the power coefficients. Typical results are presented at the fig. 5 and at the fig. 6.
Fig. 5. The absorptance $A$ as function of $\Omega$ at $\beta = 2.83 \cdot 10^{-3}$, $\gamma = 10^{-3}$, $\theta = 60^\circ$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $W = 50$ for $r = 0.01$, 0.1, 0.5 and 1.07 values: 1 – quantum plasma (solid line), 2 – classical spatial dispersion approach (dotted line).

At the fig. 5, we show the quantum absorptance $A$ as function of $\Omega$ for various small quantum parameter values $r$ in comparison with the one in the classical spatial dispersion approach corresponding to the $r = 0$ case. One sees that as far as $r$ grows, the difference between quantum and classical results becomes stronger. And the disagreement starts at relatively large $\Omega$. Following [3] and taking into account the equations (20), (22) with (23) and (24), one can qualitatively explain such a behaviour by an influence of the values $rQ_n^2/2 \sim r(\pi n/W)^2$ on the dielectric functions for large enough integers $n$. Hence the peaks for the $\Omega > 1$ caused by contribution to the impedance (20) of terms with $\Re \varepsilon_l(\Omega, Q_n) = 0$ in case of the odd integers $n$, are shifted and disappear in quantum theory in comparison with the classical spatial dispersion approach.

The dependences of the power coefficients on the quantum parameter $r$ are shown at the fig. 6. One sees that the reflectance $R$ and the absorptance $A$ are much more sensitive to violation of the parameter $r$ than the transmittance $T$. And at some $r$ values these coefficients have a critical behaviour with peaks.
5. Conclusion

In this paper, we have studied numerically in the framework of the quantum degenerate electron plasma theory the interaction of the visible and ultraviolet P-waves with thin metallic film localized between two dielectric media. We have chosen for the investigation the reflectance, transmittance and absorptance power coefficients. We have considered the dielectric functions of the quantum completely degenerate electron plasma with invariable relaxation time in the Mermin approach. Results for the quantum power coefficients were compared with those both in the classical degenerate spatial dispersion approach and in the Drude – Lorentz one without spatial dispersion.

It has been detected that for the frequencies of order and larger than the electron plasma frequency and related to the visible and ultraviolet ranges, the power coefficients in quantum approach have the same qualitative behavior as in the classical spatial dispersion degenerate plasma theory. But the quantum power coefficients differ both from the classical spatial dispersion and from the Drude – Lorentz ones. The quantum wave effects of electrons in electron plasma lead to displacement, smoothing and vanishing of the classical degenerate plasma peaks. Difference between quantum and classical approaches becomes more visible at a growth of quantum parameter for relatively large frequencies. The quantum power coefficients especially the reflectance and absorptance, depend on the quantum parameter and these dependences have some critical points.

The obtained results can be used in the theoretical investigations of the quantum electron plasma as well as of the interaction of the electromagnetic wave with thin metallic objects. These results may have also practical applications for fine optical devices working with visible and ultraviolet light.

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