Quantitative description of the joint surface roughness considering the influence of shear direction

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Abstract. The joint surface roughness in consideration of the influence of shear direction is quantitatively described. A natural joint produced by Brazilian splitting test is scanned, a digital elevation model that represents the morphological features of the joint is established. The two-dimensional roughness of joint profiles along the shear direction is analysed according to the parameters of root mean square ($Z_2$), joint roughness coefficient (JRC) and the statistical characteristics of dip angles, respectively. It is proved that the standard deviation of dip angles is linearly correlated with the value of JRC\textsuperscript{2D}. Further, the probability distribution of apparent dip angles along the shear direction is introduced to describe the shear-related three-dimensional joint roughness. Based on the scanning images of the joint morphology, the probability distribution of apparent dip angle varies with different shear directions.

1. Introduction
The morphological characteristic of rock joints plays an important role in the evaluation of shear behaviours of rock masses. The presence of rough joints reduces the integrity of the rock mass, inducing the reduction of its shear resistance. Shear characteristics of rough joints directly affect the stability of rock mass engineering, such as slopes, tunnels and underground caverns, during both construction and operation processes.

Since the 1960s, scholars have conducted extensive studies on the effects of joint roughness on shear properties in brittle materials using experimental methods, as a result of which, parameters that used to quantitatively describe the two-dimensional joint roughness have been proposed[1]. Barton[2] primitively quantified the two-dimensional joint roughness as JRC(joint roughness coefficient), which is considered to be correlated with the peak shear strength of the rock joint. Some of the latter studies on the determination of JRC\textsuperscript{2D} rely on the typical profiles proposed by Barton[3], but this method is subjective and it inevitably lies on the judgment of the observer; later, Barton[4] proposed the tilt testing method that quantitatively determines the JRC value based on a cubic jointed specimen with a rough joint striking along the tilt direction, but the operation of this test is complicated and it is not commonly used in studies. Besides that, with the help of highly-developed surface topography measurement technology and digital image analysis technology, scholars proposed several statistical parameters to illuminate the joint roughness quantitatively, such as root mean square $Z_2$[5], fractal dimension $D$ of the joint surface[6] and asperity micro-slope angle $\theta$[7]. In comparison with the formal methods in the determination of JRC value, the further-proposed statistical parameters, are clearly defined based on the statistical morphological properties of the joint and easily calculated, and can be further connected with JRC by empirical formula. However, the shear-related joint morphology is
highly dependent on the shear direction, which means that the angle in front of the asperity tip dominates the shear-related behaviour of the rock joint compared with the angle on the back. Under the premise of accurately describing the statistical characteristics of joint surfaces, it makes it possible to explore the effect of joint roughness on shear strength and surface damage distribution. Rock mass properties show great variability. In the experimental research process of joint shear properties, the quantitative description of joint roughness should take the shear direction into consideration[8]. Park[9] defined “active micro-slope angles” and “inactive micro-slope angles” to characterize micro surfaces of asperities based on the relative relationship between the inclination direction of asperities and the joint shear direction. It was noticed that the distribution of damage on the joint surface during the shear process is closely related to the distribution of the active micro-slope angles. Grasselli[7] proposed the concept of apparent dip angle to characterize the joint surface morphology based on a certain shear direction. It was proposed as a statistical parameter, which varies with the shear direction, to describe the three-dimensional joint shear morphology. Liu et al.[10] regarded the peak strength as being affected by the peak dilation angle, which means that the peak dilation angle is related to the joint roughness. Based on direct shear tests, it is proposed to use the characteristic apparent dip angle and the average height of asperities as parameters to derive the constitutive relationship between joint shear strength and joint morphology. Jiang et al.[11] proposed that the role of joint morphology in the shearing process is mainly reflected in the influence of the apparent dip angles of positive values along the shear direction. Dang et al.[12] used apparent dip angle as the parameter to describe joint roughness under shear, and numerically analyzed the stress concentration at the asperity surfaces in different stages of shearing by using FLAC3D. Under shear conditions, the joint surface stress mainly concentrates on the asperities with larger apparent dip angles. With the development of shear displacement, the stress concentration gradually expands to areas with smaller apparent dip angles.

This study attaches importance to the quantitative description of joint roughness on the basis of a specific shear direction. JRC$^{2D}$ along the joint profiles are quantitatively described from the value of Z$^2$ according to the empirical relationship. The probability distribution characteristics of dip angles are also obtained to make a comparison with JRC$^{2D}$. Further, the probability distribution of dip angles is obtained to characterize the three-dimensional joint morphological features. It is proved that the distribution parameters of apparent dip angles are diverse with different shear directions, which confirms the necessity of taking shear direction into consideration in description of the joint roughness under shear.

2. Making process of the rough joint model
The Brazilian splitting method of rock specimens is introduced to generate tensile joints[10, 13, 14]. In this paper, in order to obtain the rough tensile joint, a 15cm cubic sandstone specimen is used for the Brazilian splitting process. A 3D Camega optical scanning system with a sampling interval of 25μm is employed to obtain the point cloud that describes the joint surface morphological features. To improve the morphology image, pre-processing operations of noise treatment and hole filling are applied. The scanned rough surface is shown in Figure 1. The image-based coordinates of a rectangular area with a size of 100×110mm in the middle of the scanned digital model are extracted and a DEM (digital elevation model) of the joint morphology is established as Figure 2 (in which the joint is expanded in the XOZ plane, and the elevation is represented by the y coordinate. The sampling interval is 0.78125mm).

3. Image-based measurements of the 2D joint morphological features
The morphology of rock joint surfaces has the characteristic of anisotropy, which causes great difficulties in the description of the three-dimensional joint roughness. Figure 3 shows the anisotropic features of the two-dimensional profiles of the joint along x-axis and z-axis, respectively. Therefore, the quantitative characterization of joint surface morphology, which takes the shear direction into consideration, is of great significance in the description of shear-induced joint roughness.
Figure 1. 3D scanning result of the sandstone joint

Figure 2. Digital elevation model of the natural rock joint

Figure 3. 2D morphology of the natural joint (represented by profiles parallel to x and z axes)
Suppose that the shearing direction is along the positive direction of z-axis, that is, the shearing vector is \( s=(0,0,1) \). Figure 4 shows the profiles every 5mm along the z-axis on the joint surface. According to the sampling interval during the image processing operations, it is obtained that each profile collects 129 elevation points that reflect the undulation of the joint surface. Based on the proposed parameters that contribute to the description of two-dimensional joint roughness, the root mean square \( Z_2 \) and the dip angle are selected as indexes of joint profile roughness under shear.

\[
Z_2 = \left\{ \frac{1}{n-1} \sum_{i=1}^{n} \left( \frac{Z_{i+1} - Z_i}{\Delta} \right)^2 \right\}^{1/2}
\]

where \( Z_i \) represents the height of the \( i \)-th point on the joint; \( n \) is the number of measuring points; \( \Delta \) is the sampling interval taken on the two-dimensional joint profile.

| profiles along z-axis of the joint surface |
|-------------------------------------------|
| \( x=47 \)                              |
| \( x=42 \)                              |
| \( x=37 \)                              |
| \( x=32 \)                              |
| \( x=27 \)                              |
| \( x=22 \)                              |
| \( x=17 \)                              |
| \( x=12 \)                              |
| \( x=7 \)                               |
| \( x=2 \)                               |
| \( x=3 \)                               |
| \( x=8 \)                               |
| \( x=13 \)                              |
| \( x=18 \)                              |
| \( x=23 \)                              |
| \( x=28 \)                              |
| \( x=33 \)                              |
| \( x=38 \)                              |
| \( x=43 \)                              |

Figure 4. Profiles along z-axis of the joint surface

3.1. Root mean square

Tse[5] proposed that the root mean square \( Z_2 \) is closely related to the frictional feature of rough joints. It is defined as:

\[
Z_2 = \left\{ \frac{1}{n-1} \sum_{i=1}^{n} \left( \frac{Z_{i+1} - Z_i}{\Delta} \right)^2 \right\}^{1/2}
\]
The parameter $Z_2$ is regressed with JRC as:

$$JRC = 32.2 + 32.47 \log Z_2$$  \hspace{1cm} (2)

According to the following empirical formula, the value of the two-dimensional JRC can be well quantified based on a set of elevations of sampling points on the joint profile.

### 3.2. Dip angle

Dip angle $\theta$ means the arctangent of the ratio between the height difference between adjacent points and the sampling interval on the profile:

$$\theta = \arctan \left( \frac{Z_{i+1} - Z_i}{\Delta} \right)$$  \hspace{1cm} (3)

Based on the concept of the dip angle, it is known that the statistical characteristics of $\theta$ on the profile represent the distribution features of uneven asperities along the shear direction.

### 3.3. 2D roughness of the profiles

In accordance with the coordinates of the profiles selected in , the distribution of elevation at each profile follows a Gaussian distribution. $Z_2$ and JRC$^{2D}$ are calculated from the elevation data based on equations (1)–(2). The statistical distribution of dip angle is analyzed according to equation (3). Table 1 illustrates the calculation details of the parameters in description of 2D roughness of the profiles.

| x coordinate of the profile (mm) | elevation distribution |  | dip angle distribution |
|----------------------------------|------------------------|---|------------------------|
|                                  | mean value $\mu$ (mm)  | standard deviation $\sigma$ (mm) | $Z_2$ | JRC$^{2D}$ mean value $\mu$ (°) | standard deviation $\sigma$ (°) |
| -47                              | 0.26                   | 0.76                           | 0.1502 | 5.47 | 1.23 | 8.42   |
| -42                              | 0.38                   | 0.98                           | 0.1557 | 5.97 | 1.60 | 8.65  |
| -37                              | 0.36                   | 1.26                           | 0.1190 | 2.19 | 1.84 | 6.50  |
| -32                              | 0.64                   | 1.41                           | 0.1301 | 3.45 | 1.92 | 7.10  |
| -27                              | 0.73                   | 1.53                           | 0.1483 | 5.29 | 2.15 | 8.09  |
| -22                              | 0.77                   | 1.54                           | 0.1533 | 5.76 | 2.18 | 8.37  |
| -17                              | 0.71                   | 1.28                           | 0.1299 | 3.42 | 1.93 | 7.12  |
| -12                              | 0.69                   | 1.25                           | 0.1221 | 2.55 | 1.77 | 6.72  |
| -7                               | 0.92                   | 1.51                           | 0.1361 | 4.08 | 2.42 | 7.37  |
| -2                               | 1.06                   | 1.51                           | 0.1246 | 2.83 | 2.79 | 6.52  |
| 3                                | 1.03                   | 1.34                           | 0.1370 | 4.17 | 2.75 | 7.29  |
| 8                                | 1.14                   | 1.48                           | 0.1279 | 3.20 | 2.32 | 6.89  |
| 13                               | 1.09                   | 1.83                           | 0.1445 | 4.92 | 2.10 | 7.89  |
| 18                               | 0.77                   | 1.99                           | 0.1417 | 4.64 | 2.70 | 7.57  |
| 23                               | 0.22                   | 1.90                           | 0.1339 | 3.84 | 2.35 | 7.22  |
| 28                               | -0.37                  | 1.75                           | 0.1463 | 5.10 | 2.59 | 7.87  |
| 33                               | -1.23                  | 1.60                           | 0.1469 | 5.15 | 2.26 | 8.03  |
| 38                               | -2.18                  | 1.54                           | 0.1381 | 4.29 | 1.85 | 7.60  |
| 43                               | -3.30                  | 1.23                           | 0.1182 | 2.09 | 1.22 | 6.63  |

The statistical characteristics of dip angles follow the Gaussian distribution. The mean values of dip angles in different profiles are relatively close, while the standard deviations of dip angles are quite dispersed. Figure 5 illustrates the fitting results between the standard deviations of dip angles and the values of JRC$^{2D}$ that are regressed by the parameter $Z_2$. The standard deviation of dip angles shows a linear relationship with JRC$^{2D}$, which means that the standard deviation of dip angles is capable of being occupied in description of the two-dimensional joint roughness.
4. Measurements of 3D joint morphological features

4.1. Apparent dip angle

Some scholars recognize that JRC-based parameters have certain limitations in describing the three-dimensional roughness of joints under shear, so new morphological parameters are proposed to represent the surface characteristics. The surface morphology of rough joints can be covered by a series of triangular elements. The angle between a line along the shear direction and the horizontal projection on the triangular element is the apparent dip angle of the asperity under the predetermined shear direction, as shown in Figure 6. Apparent dip angle plays an significant role in the shear properties of rough joints.

[Diagram of apparent dip angle of the asperity]

where the physical meaning of the symbols are shown as follows:

$\eta$  The shear plane
$\xi$  The plane perpendicular to the triangle element
$n$  Outward normal vector of the triangle element
$s$  The shear vector
$n_1$  The component of $n$ that parallel to the shear plane

Figure 5. Fitting result of $\sigma$ of dip angles and JRC\textsuperscript{2D}

Figure 6. Apparent dip angle of the asperity
The component of $n$ that perpendicular to the shear plane

$\theta$ Dip angle ($^\circ$)

$\alpha$ Azimuth angle measured clockwise from $n_1$ to $s$ ($^\circ$)

$\theta^*$ Apparent dip angle ($^\circ$)

The geometric calculation process of the apparent dip angle is shown as follows:

- The outward normal vector $n$ of the triangle element $ABC$:
  \[ n = AB \times AC \] (4)

- The component of $n$ that parallel to the shear plane:
  \[ n_1 = (a,0,c) \] (5)

- Azimuth angle measured clockwise from $n_1$ to $s$ ($^\circ$):
  \[ \alpha = \frac{n_1 \cdot s}{|n_1||s|} \] (6)

- Dip angle ($^\circ$):
  \[ \sin \theta = \cos \angle n,n_1 \] (7)

- Apparent dip angle ($^\circ$)
  \[ \tan \theta^* = -\tan \theta \cos \alpha \] (8)

If the shear vector $s$ is anti-parallel to $n_1$, which means that $\alpha=\pi$, the definition of apparent dip angle will degenerate into the two-dimensional concept as dip angle. According to the definition of apparent dip angle, it can be seen that in the process of shearing of the integrated joint surfaces, the surfaces that are squeezed together have the positive values of apparent dip angles, while the surfaces separated from each other correspond to negative values of apparent dip angles. It is proposed that surfaces that of negative apparent dip angles have no effect on the shearing behaviour in the course of the shearing process.

4.2. 3D roughness of the joint

The statistical distribution of apparent dip angles of the sandstone joint under the shear direction vector of $s=(0,0,1)$ is shown in Figure 7. This parameter follows a Gaussian distribution with an average value of 2.07° and a standard deviation of 8.12°.

![Figure 7. The statistical distribution of apparent dip angles of the sandstone joint](image)
Figure 8. Anisotropic characteristics of statistical parameters of apparent dip angles ((a) mean value; (b) standard deviation; (c) maximum)

Figure 8 illustrates the anisotropic characteristics of statistical parameters of apparent dip angles that cover the joint surface. Polar angles symbolize angles between observation directions and the shear vector. The dispersion of mean values under all observation directions is insignificant (in the range of 0±3.5°); at the same time, the absolute values of the mean angles corresponding to the two angles on the polar coordinate axis are equal, and the signs are opposite, which proves that the determination of the shear direction directly affects the statistical law of apparent dip angles. The degree of anisotropy of standard deviation is not quite apparent, the standard deviation parallel to the shear direction is the smallest (8.12°), and the standard deviation perpendicular to the shear direction is the largest (9.14°). It should be noticed that the maximum values of apparent dip angles, some of which exceed 80° under certain directions, represent an extremely small number of apparent dip angles. Therefore, the statistical data is not intentionally reflected in the probability distribution histogram.

In accordance with the linear relationship between the standard deviation of the distribution of dip angles and JRC\textsuperscript{2D} in the description of the two-dimensional joint roughness of the natural joint, the standard deviation of the distribution of apparent dip angles can be used to describe the three-dimensional joint roughness.

5. Conclusions
This study focuses on the quantitative description of the morphological features of natural joints. It contains several aspects:

1. The Brazilian splitting method of cubic rock specimens is introduced to generate a tensile joint of sandstone. 3D scanning methodology is employed to visualize the rough surface morphology of the joint. A digital elevation model is established to digitalize the morphological features of the rough surface.

2. The 2D joint morphological features of the joint are analyzed based on the coordinate information of the digital elevation model. Root mean square $Z_r$ and JRC$^{2D}$ are used as parameters to describe the roughness of joint profiles along a specific shear direction.

3. The statistical distributions of dip angles of the joint profiles are obtained. The standard deviation of the distribution of apparent dip angles is proved to be of linear relationship with JRC$^{2D}$.

4. In accordance with the linear relationship between the standard deviation of the distribution of dip angles and JRC$^{2D}$, the standard deviation of the distribution of apparent dip angles is utilized in the further description of three-dimensional joint roughness. The joint roughness shows an anisotropic characteristic in the quantitative description of morphologic features. Therefore, the quantitative description of joint roughness under shear conditions should take the shear direction into consideration.

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