Homogenous Observer-Based Second-Order Sliding Mode Guidance Law*

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A new homogenous observer-based finite-time convergent guidance law is proposed to intercept maneuvering targets without line-of-sight (LOS) angular rate information. The presented formulation is obtained via a combination of homogenous theory and second-order sliding mode (SOSM) method. Different from some existing observers, the proposed algorithm can estimate the lumped uncertainty and the LOS angular rate in an integrated manner. By virtue of the principle of SOSM, the undesired chattering is mitigated significantly without any sacrifice in performance. Detailed stability shows that the LOS angular rate can be stabilized in a small region around zero in finite time. Numerical simulations with some comparisons are carried out to demonstrate the superiority of the proposed method.

Key Words: Missile Guidance, Line-of-Sight Angular Rate Estimation, Homogenous Theory, Second-Order Sliding Mode, Finite-Time Convergence

1. Introduction

Although the widely used proportional navigation guidance (PNG) law is easy to implement and might suffice for many applications, its performance is not satisfactory and a significant miss-distance may result during realistic engagement for intercepting highly maneuverable targets.1,2) To obtain a small miss-distance, some elegant advanced guidance laws have been formulated based on modern control theory. The most well-known extension of the PNG law is the augmented PNG (APNG) law,3) which adds an augmentation term to counteract target maneuvers. Compared to the PNG law, the APNG law reduces required missile acceleration at the end of interception as well as the total required control effort throughout the whole engagement.3) To bring such an improvement, an accurate target maneuvering profile is required, as is often the case with some other modern guidance laws. This information, unfortunately, is difficult to obtain precisely due to the unpredictability and complexity of target maneuvers. To this end, it is of prime importance to design a high-performance observer to accurately estimate target maneuvering. Based on a suitable dynamics model, several feasible estimation algorithms have been designed and inserted in guidance laws as target maneuvering estimation blocks, such as time-delay control,4) linear disturbance observer5,6) inertial-delay control,7) extended state observer,8) finite-time convergence disturbance observer9) and references therein.

Besides target maneuver information, many advanced guidance laws require the information on LOS angular rate, which is usually difficult to measure for some radar seekers.10) Moreover, owing to the advantages of low cost, mechanical simplicity and small size, the strapdown seeker has been well researched during the last several years.11) However, for this kind of seeker, the LOS angular rate is highly coupled with the interceptor body rate and cannot be measured directly. To obtain the LOS angular rate, an extended Kalman filter was utilized to estimate the LOS angular rate with bearings-only measurement,12) but this filter usually encounters the problem of poor robustness and lack of target observability,13,14) which, in turn, lead to a slower convergence rate and inaccurate estimation. A higher-order sliding mode differentiator was used in He et al.15) as an alternative way to extract the LOS angular rate information from LOS angle measurement, but it is somewhat sensitive to measurement noise. Regarding unknown target maneuvers as bounded external disturbances, high-gain observer (HGO)10) and $H_{\infty}$ observer16) were proposed to estimate the LOS angular rate.

This paper focuses on designing robust guidance laws for intercepting maneuvering targets in the absence of LOS angular rate measurement. To this end, a novel state observer with finite-time convergence is proposed based on homogenous theory. Compared with the state-of-the-art target maneuver estimators and LOS angular rate observers mentioned above, the proposed observation algorithm not only provides real-time estimation of the lumped uncertainty, but also possesses the accurate LOS angular rate observation capability. With the estimated lumped uncertainty information and the reconstructed LOS angular rate, a novel smooth SOSM guidance law is then synthesized. Due to its inherent continuous property, the undesired chattering phenomenon in the classical sliding mode guidance law is mitigated significantly. With the help of Lyapunov stability criteria, the finite-time stability of the closed-loop guidance system is established in theory. Numerical simulations and comparisons with the HGO-based input-to-state stability guidance (HGO-ISSG) law demonstrate the effectiveness of the proposed formulation.

The rest of the paper is organized as follows. Some back-
ground information and preliminaries are stated in Sec. 2. In Sec. 3, a homogenous observer-based composite two-dimensional guidance law is proposed, followed by extension to a three-dimensional case presented in Sec. 4. Finally, some simulation results and conclusions are offered.

2. Background Information and Preliminaries

2.1. Problem formulation

Assume that the missile and target are point masses with constant flying velocities. Fig. 1 depicts a schematic view of the planar endgame geometry between the missile and target, where the subscripts \( M \) and \( T \) denote the missile and target, \( \gamma_M \) and \( \gamma_T \) the missile and target flight path angles, \( \lambda \) and \( r \) the LOS angle and relative missile-target range, \( V_M \) and \( V_T \) the missile and target velocities, and \( a_M \) and \( a_T \) the missile and target accelerations, which are assumed normal to their corresponding velocities, respectively.

The corresponding equations, describing the relative motion dynamics, are formulated as

\[
\begin{align*}
\dot{r} &= V_T \cos(\gamma_T - \lambda) - V_M \cos(\gamma_M - \lambda) \\
\dot{\lambda} &= \left[ V_T \sin(\gamma_T - \lambda) - V_M \sin(\gamma_M - \lambda) \right] / r \\
\dot{\gamma}_M &= a_M / V_M \\
\dot{\gamma}_T &= a_T / V_T
\end{align*}
\]

Differentiating Eqs. (1) and (2) with respect to time yields

\[
\begin{align*}
\ddot{r} &= r \dot{\lambda}^2 + a_T r - a_M r \\
\ddot{\lambda} &= -2 \dot{\lambda} \frac{\dot{\lambda}}{r} + \frac{a_T \dot{r}}{r} - \frac{a_M \dot{r}}{r}
\end{align*}
\]

where \( a_T \triangleq a_T \sin(\lambda - \gamma_T) \) and \( a_M \triangleq a_M \sin(\lambda - \gamma_M) \) denote the target and missile acceleration components along the LOS, and \( a_T \triangleq a_T \cos(\lambda - \gamma_T) \) and \( a_M \triangleq a_M \cos(\lambda - \gamma_M) \) the target and missile acceleration components perpendicular to the LOS, respectively. Usually, acceleration along the missile’s velocity cannot be controlled in the terminal guidance phase, and therefore only the relative two-degree dynamics between the control input \( a_M \omega \) and the LOS angle \( \lambda \), i.e. Eq. (6), will be used in missile guidance law design. For dynamics, Eq. (6), the following two assumptions are made.

**Assumption 1.** During the time horizon of terminal guidance phase, there exist two positive constants \( r_{\text{min}} \) and \( r_{\text{max}} \), such that \( r_{\text{min}} \leq r \leq r_{\text{max}} \). The existence of \( r_{\text{max}} \) is natural while \( r_{\text{min}} \) is defined based on the target size.

**Assumption 2.** The lead angle between the missile velocity vector and the LOS satisfies \( |\lambda - \gamma_M| < \pi/2 \); if not, the seeker may lose the target due to the physical limitation of the seeker’s field-of-view.

According to the concept of proportional navigation guidance, zeroing the LOS angular rate will lead to a successful interception with zero miss-distance. The control interest here is to design a guidance law in such a way that the LOS angular rate can be stabilized in a small region around zero in finite time.

2.2. Some definitions and lemmas

**Definition 1.** Consider the following nonlinear system

\[
\dot{x} = f(x, t), \quad f(0, t) = 0, \quad x \in \mathbb{R}^n
\]

where \( f : U_0 \times R \rightarrow \mathbb{R}^n \) is continuous on \( U_0 \times R \), and \( U_0 \) is an open neighborhood of the origin \( x = 0 \). The system equilibrium is called finite-time stable if: 1) for any given initial time \( t_0 \) and initial state \( x(t_0) = x_0 \in U \), there exists a settling time \( T \), which is dependent on \( x_0 \), such that every solution of the system, Eq. (7), satisfies \( x(t) = x(t; t_0, x_0) \in U/[0] \); 2) the system (local) equilibrium \( x = 0 \) is Lyapunov stable with finite-time convergence in a neighborhood of the origin \( U \subset U_0 \). Furthermore, if \( U = \mathbb{R}^n \), the origin \( x = 0 \) is globally finite-time stable.

**Lemma 1.** Suppose there exists a \( C^1 \) smooth positive definite function \( V(x) \) for Eq. (7), which satisfies

\[
V(x) + c V^\beta(x) \leq 0
\]

with \( 0 < \beta < 1, c > 0 \). Then, the origin of the system is finite-time stable. Moreover, the settling time can be estimated as

\[
t_1 \leq V^{1-\beta}(x_0) / [c(1-\beta)]
\]

where \( V(x_0) \) is the initial value of \( V(x) \).

**Lemma 2.** Consider the following perturbed form of the system

\[
\dot{x} = f(x, t) + g(t)
\]

Suppose there exists a \( C^1 \) smooth positive definite function \( V(x) \) to satisfy Eq. (8) with \( 0 < \beta < 1/2, c > 0 \). Then, there exist \( \delta_0 > 0, L_0 > 0 \), and an open neighborhood \( D \) of origin such that for every continuous function \( g(t) \) with \( |g(t)| < \delta_0 \), there exists a finite time \( t_r \), the solution of Eq. (9) satisfies

\[
\|x\| \leq L_0 \delta_0^{\sigma_0}, \quad \sigma_0 = (1-\gamma_0)/\gamma_0, \quad \gamma_0 \in (0, 1/2)
\]

3. Observer-Based Finite-Time Convergent Guidance Law Design

In this section, a robust composite finite-time convergence guidance law without LOS angular rate is proposed using a homogenous observer and SOSM method. The finite-time stability analysis of the closed-loop guidance system is also presented.
3.1 Homogenous observer design

To reduce the sensing requirement, let a new lumped uncertainty be

\[
\Delta = \left[ -2\dot{\lambda} + a_{\tau,1} + a_M - a_M \cos(\lambda - \gamma_M) \right]/r,
\]

then, Eq. (6) can be rewritten as

\[
\dot{\lambda} = \Delta - a_M/r \quad (11)
\]

In order to estimate the LOS angular rate and the lumped uncertainty in an integrated manner, we propose the following continuous observer

\[
\begin{align*}
\dot{z}_1 &= z_2 - \frac{k_1}{\varepsilon} |z_1 - \lambda|^{\alpha_1} \text{sgn}(z_1 - \lambda) \\
\dot{z}_2 &= z_3 - \frac{k_2}{\varepsilon^2} |z_1 - \lambda|^{\alpha_2} \text{sgn}(z_1 - \lambda) - \frac{a_M}{r} \\
\dot{z}_3 &= -\frac{k_1}{\varepsilon^3} |z_1 - \lambda|^{\alpha_3} \text{sgn}(z_1 - \lambda)
\end{align*}
\]

where \( k_1 > 0, k_2 > 0, k_3 > k_1/k_3, \alpha_1 = \alpha \in (0, 1), \alpha_2 = (2\alpha_1 + 1)/3, \alpha_3 = (\alpha_2 + 2)/3 \) and \( \varepsilon \in (0, 1) \) are the observer coefficients to be designed, and \( z_1, z_2 \) and \( z_3 \) are the estimates of \( \lambda, \dot{\lambda} \) and \( \Delta \), respectively. The first result of this paper is presented in the following theorem.

**Theorem 1.** For Eqs. (11) and observer, Eq. (12), there exist \( L > 0 \) and \( \sigma = (1 - \gamma)/\gamma, \gamma \in (0, 1/2) \), such that

\[
|z_1 - \lambda| \leq L\varepsilon^{3\sigma} \quad |z_2 - \dot{\lambda}| \leq L\varepsilon^{3\sigma-1} \quad |z_3 - \Delta| \leq L\varepsilon^{3\sigma-2} \quad (13)
\]

can be fulfilled in finite time.

**Proof.** Let \( e_1 = z_1 - \lambda, e_2 = z_2 - \dot{\lambda} \) and \( e_3 = z_3 - \Delta \) be the estimation error. Then it follows from Eqs. (11) and (12) that

\[
\begin{align*}
\dot{e}_1 &= e_2 - \frac{k_1}{\varepsilon} |e_1|^{\alpha_1} \text{sgn}(e_1) \\
\dot{e}_2 &= e_3 - \frac{k_2}{\varepsilon^2} |e_1|^{\alpha_2} \text{sgn}(e_1) \\
\dot{e}_3 &= -\frac{k_1}{\varepsilon^3} |e_1|^{\alpha_3} \text{sgn}(e_1) - \dot{\lambda}
\end{align*}
\]

Consider the following coordinate transformation

\[
\tau = t/\varepsilon, \quad \eta(t) = \varepsilon^3 \Delta(t), \quad w_i(\tau) = \varepsilon^{i-1} e_i(t), \quad i = 1, 2, 3 \quad (15)
\]

Then, Eq. (14) can be rewritten as

\[
\begin{align*}
\frac{dw_1}{d\tau} &= w_2 - k_3 |w_1|^{\alpha_3} \text{sgn}(w_1) \\
\frac{dw_2}{d\tau} &= w_3 - k_2 |w_1|^{\alpha_2} \text{sgn}(w_1) \\
\frac{dw_3}{d\tau} &= -k_1 |w_1|^{\alpha_3} \text{sgn}(w_1) - \eta
\end{align*}
\]

Ignoring the uncertain term \( \eta \), then Eq. (16) reduces to

\[
\begin{align*}
\frac{dw_1}{d\tau} &= w_2 - k_3 |w_1|^{\alpha_3} \text{sgn}(w_1) \\
\frac{dw_2}{d\tau} &= w_3 - k_2 |w_1|^{\alpha_2} \text{sgn}(w_1)
\end{align*}
\]

Consider \( f_w \) as a continuous vector field of Eq. (17). Then, for \( r_1 = 1, r_2 = (\alpha + 2)/3 \) and \( r_3 = (2\alpha + 1)/3 \), the vector field \( f_w \) satisfies

\[
\begin{align*}
\frac{dw_3}{d\tau} &= -k_1 |w_1|^{\alpha_3} \text{sgn}(w_1) \\
f_w(e^{(1)}_1, e^{(2)}_2, e^{(3)}_3) &= \begin{pmatrix}
\varepsilon^3 w_2 - k_3 |w_1|^{\alpha_3} \text{sgn}(w_1) \\
\varepsilon^3 w_3 - k_2 |w_1|^{\alpha_2} \text{sgn}(w_1) \\
-k_1 |w_1|^{\alpha_3} \text{sgn}(w_1)
\end{pmatrix}
\end{align*}
\]

where \( m = (\alpha - 1)/3 < 0 \). It follows from Eq. (18) that \( f_w \) is homogenous of degree \( m = (\alpha - 1)/3 < 0 \) with respect to the dilation \( (1, (\alpha + 2)/3, (2\alpha + 1)/3) \). Moreover, from Routh-Hurwitz stability criterion, one can conclude that the polynomial \( s^3 + k_3s^2 + k_2s + k_1 \) is Hurwitz if \( k_1 > 0, k_3 > 0 \) and \( k_2 > k_1/k_3 \). Since the vector field \( f_w \) is linear with the characteristic polynomial \( s^3 + k_3s^2 + k_2s + k_1 \), the origin is an asymptotically stable equilibrium of \( f_w \). With these in mind, applying Theorem 7.1 of Bhat and Bernstein,\(^{10}\) one can conclude the origin of Eq. (17) is finite-time stable. During real interception, it is reasonable to assume that \( |\Delta(t)| < L_\Delta \), and thus one has \( |\eta(t)| < L\varepsilon^{3\sigma} \). Therefore, it follows from Lemmas 1 and 2 that, there exist two positive constants \( \mu, \sigma = (1 - \gamma)/\gamma \) such that the inequality

\[
\|w(\tau)\| \leq \mu(L\varepsilon^{3\sigma})^{\sigma}
\]

can be fulfilled in finite time, where

\[
w(\tau) = [w_1(\tau), w_2(\tau), w_3(\tau)]^T.
\]

Finally, following coordinate transformation, Eq. (15), it can be concluded

\[
|e_i| \leq \|w(\tau)\|^{e^{i+1}} = L\varepsilon^{3\sigma-i+1} \quad (19)
\]

This completes the proof.

**Remark 1.** The choice of \( \gamma \in (0, 1/2) \) aims to make \( \sigma = (1 - \gamma)/\gamma > 1 \), and therefore the error bounds can be tuned arbitrarily small since \( \varepsilon \in (0, 1) \). Furthermore, different from some existing observers,\(^{4,10,15,16}\) where only the target maneuvers or the LOS angular rate can be estimated, the proposed observer, Eq. (12), can synchronously estimate the lumped uncertainty and LOS angular rate from LOS angle measurement in a finite time.

3.2 Guidance law design

As the lumped uncertainty and LOS angular rate can be reconstructed by the observer, Eq. (12), a new guidance law without LOS angular rate information is designed as

\[
a_M = \left( r_3 + \rho_1 |z_2|^{1-\rho} \text{sgn}(z_2) + \rho_2 \right)
\]

\[
\dot{x} = \frac{z_2^{1-2/\rho} \text{sgn}(z_2)}{r}
\]

\[
\frac{dw_3}{d\tau} = -k_1 |w_1|^{\alpha_3} \text{sgn}(w_1) \quad (17)
\]

\[
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\]

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\]
where $\rho_1$, $\rho_2 > 0$ and $p > 2$ are design parameters. The term $r_{zz}$ is used to reject the lumped uncertainty $\Delta$, while the terms $\rho_1 |z_2|^{1-1/p} \text{sgn}(z_2)$ and $\rho_2 \dot{X}$ are used to guarantee the convergence of LOS angular rate.

**Remark 2.** If one set $p = 2$, the presented guidance law can be viewed as the classical super-twisting control law. However, the term $|z_2|^{1-1/p} \text{sgn}(z_2)$ in Eq. (19) with $p = 2$ is discontinuous while the proposed guidance law is continuous, and therefore chattering is effectively mitigated.

### 3.3. Stability analysis

The second result of this paper is summarized in the following theorem.

**Theorem 2.** Consider Eq. (11), the proposed guidance law, Eq. (19), with the observer, Eq. (12), will drive the LOS angular rate to converge into a small region around zero in finite time.

**Proof.** Since the missile-target relative range is bounded and the observer, Eq. (12), guarantees finite-time estimation, guidance law, Eq. (19), can be rewritten as

$$a_M = \left( \dot{r}_\Delta + \rho_1 |\dot{a}|^{1-1/p} \text{sgn}(\dot{a}) + \rho_2 \dot{X} + h \right)$$

$$\dot{X} = \frac{|\dot{a}|^{1-2/p} \text{sgn}(\dot{a})}{r}$$

where $h$ is a bounded lumped uncertainty. Substituting Eq. (20) into Eq. (12) gives

$$\dot{\hat{\omega}} = -\frac{h}{r} - \rho_1 |\dot{a}|^{1-1/p} \text{sgn}(\dot{a}) - \rho_2 \jmath \frac{|\dot{a}|^{1-2/p} \text{sgn}(\dot{a})}{r}$$

To simplify the stability analysis, define a new vector $\omega = [\omega_1, \omega_2]^T$ as

$$\omega_1 = \dot{\omega}$$

$$\omega_2 = -h - \rho_2 \int \frac{|\dot{a}|^{1-2/p} \text{sgn}(\dot{a})}{r} \, dt$$

Then, the dynamics, Eq. (21), can be transformed to

$$\dot{\omega}_1 = \frac{\omega_2}{r} - \rho_1 |\omega_1|^{1-1/p} \text{sgn}(\omega_1)$$

$$\dot{\omega}_2 = -\hat{h} - \rho_2 |\omega_1|^{1-2/p} \text{sgn}(\omega_1)$$

For Eq. (23), consider the following Lyapunov function candidate

$$V = \frac{\rho_2 p}{p-1} |\omega_1|^{(p-1)/p} + \frac{1}{2} \omega_2^2 + \frac{1}{2} \left( \rho_1 |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1) - \omega_2 \right)^2$$

Note that $V$ is continuously differentiable, except on the set

$$\Omega = \{ (\omega_1, \omega_2) \in \mathbb{R}^2 | \omega_1 = 0 \}.$$

Moreover, $V$ can be rewritten as the following compact form

$$V = \omega^T P \omega$$

where

$$P = \frac{1}{2} \begin{bmatrix} \rho_2 p & \rho_2 \rho_1 \rho_1 - \rho_1 \\ -\rho_2 & 2 \end{bmatrix}$$

Since $\rho_2 > 0$ and $p > 2$, $V$ is positive definite and radially unbounded; that is,

$$\lambda_{\min}(P)||\omega||^2 \leq V \leq \lambda_{\max}(P)||\omega||^2$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix $(\cdot)$, respectively, and $||\cdot||$ denotes the vector 2-norm.

Computing the time derivative of $V$ along trajectories, Eq. (23), gives

$$\dot{V} = \left( \frac{\rho_2 p}{p-1} + \frac{1}{2} \rho_1^2 \right) \frac{2 p - 2}{p} |\omega_1|^{(p-2)/p} \text{sgn}(\omega_1) \dot{\omega}_1 + 2 \omega_2 \dot{\omega}_2$$

$$- \rho_1 |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1) \dot{\omega}_2 - \rho_2 \rho_1 \rho_1 p - \frac{1}{p} |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1)$$

$$= \left( \frac{\rho_2 p}{p-1} + \frac{1}{2} \rho_1^2 \right) \frac{2 p - 2}{p} |\omega_1|^{(p-2)/p} \text{sgn}(\omega_1) \dot{\omega}_2$$

$$- \rho_1 |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1) \dot{\omega}_2 - \rho_2 |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1)$$

$$\times \left( -\frac{\omega_2}{r} - \rho_2 |\omega_1|^{1-2/p} \text{sgn}(\omega_1) \right) - \rho_1 \frac{p-1}{p} |\omega_1|^{(p-1)/p} \omega_2$$

$$\times \frac{\omega_2}{r} - \rho_1 |\omega_1|^{1-1/p} \text{sgn}(\omega_1)$$

$$= -\frac{|\omega_1|^{(p-1)/p}}{r} \left( \rho_1 \rho_2 |\omega_1|^{(p-1)/p} + \frac{p-1}{p} |\omega_1|^{2(p-1)/p} \right)$$

$$- 2 \frac{p-1}{p} |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1) \dot{\omega}_2 + \frac{p-1}{p} |\omega_1|^{(p-1)/p}$$

$$- \left( 2 \omega_2 - \rho_1 |\omega_1|^{(p-1)/p} \text{sgn}(\omega_1) \right) \dot{h}$$

Assuming that $|\hat{h}| < \kappa$, where $\kappa$ is a small positive constant; then, it follows from Eq. (28) that

$$\dot{V} = -\frac{|\omega_1|^{(p-1)/p}}{r} \omega^T Q \omega + \kappa B \omega \leq -\frac{|\omega_1|^{(p-1)/p}}{r_{\text{max}}} \omega^T Q \omega + \kappa B \omega$$

where

$$Q = \left[ \begin{array}{cc} \rho_1 \rho_2 + \rho_1^2 \frac{p-1}{p} & -\rho_1^2 \frac{p-1}{p} \\ -\rho_1^2 \frac{p-1}{p} & \rho_1 \rho_1 \frac{p-1}{p} \end{array} \right], \quad B = \left[ \begin{array}{c} -\rho_1 \\ 2 \end{array} \right]$$

Since $\rho_1$, $\rho_2 > 0$ and $p > 2$, it is easy to verify that matrix $Q$ is Hurwitz. Moreover, from the fact that

$$\|\omega\| = \sqrt{|\omega_1|^{2(p-1)/p} + \omega_2^2} \geq |\omega_1|^{(p-1)/p},$$
one can imply that

$$|\omega|^\frac{1}{p} \geq \|\omega\|^\frac{1}{(p-1)}.$$  

Then, it follows from Eqs. (27) and (29) that

$$\dot{V} \leq -\frac{\|\omega\|}{r_{\text{max}}} \frac{\lambda_{\text{min}}(Q)[\|\omega\|^2 + \kappa\|B\|\|\omega\|]}{r_{\text{max}} - \kappa\|B\|} \|\omega\|  \tag{31}$$

$$\leq -\frac{\lambda_{\text{min}}(Q)[\|\omega\|(p-2)/(p-1) - \kappa\|B\|]}{r_{\text{max}} - \kappa\|B\|} \frac{V^{1/2}}{\sqrt{\lambda_{\text{max}}(P)}}$$

When

$$\lambda_{\text{min}}(Q)[\|\omega\|(p-2)/(p-1) - \kappa\|B\|] > 0,$$

Eq. (31) can be rewritten as

$$\dot{V} \leq -\frac{bV^{1/2}}{\sqrt{\lambda_{\text{max}}(P)}} \tag{32}$$

where

$$b = \lambda_{\text{min}}(Q)[\|\omega\|(p-2)/(p-1) - \kappa\|B\|] > 0.$$  

According to Lemma 1, the region

$$\|\omega\| \leq (\kappa\|B\|/r_{\text{max}}/\lambda_{\text{min}}(Q))^{(p-1)/(p-2)}$$  

can be reached in finite time, which also proves that the LOS angular rate will be stabilized in a small region around zero in finite time. This completes the proof.

**Remark 3.** By tuning the observer and guidance law design parameters, one can obtain

$$\|\omega\| \leq (\kappa\|B\|/r_{\text{max}}/\lambda_{\text{min}}(Q))^{(p-1)/(p-2)}$$

Furthermore, since $p > 2$, a sufficiently large value of $(p-1)/(p-2)$ can be achieved. Therefore, the region

$$\|\omega\| \leq (\kappa\|B\|/r_{\text{max}}/\lambda_{\text{min}}(Q))^{(p-1)/(p-2)}$$

can be tuned very close to zero by proper parameter selections.

4. **Extension to Three-Dimensional Case**

In real interceptions, the target-missile relative motion occurs in a three-dimensional environment. Consider the spherical LOS coordinate system $(r, \theta, \phi)$ shown in Fig. 2, where the origin is fixed at the gravity center of the missile. Based on the principle of kinematics, the missile-target relative motion dynamics can be described as follows:\n
$$\ddot{r} = r\dot{\phi}^2 + r\dot{\theta}^2 \cos^2 \phi + a_{T_r} - a_{M_r}$$  

$$\ddot{\theta} = \frac{2r\dot{\phi}^2}{r} + 2r\dot{\phi} \cos \phi \tan \phi + \frac{a_{T\theta}}{r \cos \phi} - \frac{a_{M\theta}}{r \cos \phi}$$  

$$\ddot{\phi} = \frac{2r\dot{\phi}^2}{r} - \phi^2 \sin \phi \cos \phi + \frac{a_{T\phi}}{r} - \frac{a_{M\phi}}{r}$$  

As stated earlier, only the acceleration normal to the missile’s velocity can be adjusted. Hence, only Eqs. (35) and (36) are used in guidance law design. One may observe from Eq. (35) that $\phi = \pm \pi/2$ is a singular point, which will make the missile acceleration unbounded. However, it has been shown in Yan and Ji\(^{10}\) that this problem can be avoided by properly choosing the inertial reference coordinate system during the terminal guidance phase to guarantee $\phi \in (-\pi/2, \pi/2)$. The purpose of designing a guidance law here is to nullify the LOS angular rates $\dot{\theta}$ and $\dot{\phi}$ in finite time.

Define

$$d_\theta = (-2\dot{\theta}^2 + 2\dot{\phi}\dot{\theta} \tan \phi + a_{T\theta}/r \cos \phi)/r$$

and

$$d_\phi = (-2\dot{\phi}^2 \sin \phi \cos \phi + a_{T\phi}/r \cos \phi)/r$$

as two lumped uncertainties. Then, Eqs. (34) and (35) can be rewritten as

$$\ddot{\theta} = d_\theta - a_{M\theta}/(r \cos \phi)$$

$$\ddot{\phi} = d_\phi - a_{M\phi}/r$$

Note that the preceding two expressions have a similar form to that of Eq. (12), thus, the proposed guidance law, Eq. (19), can be applied to a three-dimensional case as

$$a_{M\theta} = \cos \phi \left[ r\tilde{d}_\theta + \rho_1 \left| \tilde{\theta} \right|^{1-\rho} \left\{ \left| \tilde{\theta} \right| \operatorname{sgn}(\tilde{\theta}) + \rho_2 \tilde{\varphi} \right\} \right]$$

$$\ddot{d}_\theta = \frac{\left| \tilde{\theta} \right|^{1-\rho} \left\{ \left| \tilde{\theta} \right| \operatorname{sgn}(\tilde{\theta}) + \rho_2 \tilde{\varphi} \right\}}{r}$$

$$a_{M\phi} = \tilde{d}_\phi + \rho_1 \left| \tilde{\phi} \right|^{1-\rho} \left\{ \left| \tilde{\phi} \right| \operatorname{sgn}(\tilde{\phi}) + \rho_2 \tilde{\varphi} \right\}$$

$$\ddot{d}_\phi = \frac{\left| \tilde{\phi} \right|^{1-\rho} \left\{ \left| \tilde{\phi} \right| \operatorname{sgn}(\tilde{\phi}) + \rho_2 \tilde{\varphi} \right\}}{r}$$

where $\tilde{d}_\theta$, $\tilde{d}_\phi$, $\tilde{\theta}$ and $\tilde{\phi}$ denote the estimations of $d_\theta$, $d_\phi$, $\dot{\theta}$ and $\dot{\phi}$, respectively.

5. **Simulations**

The performance of the proposed composite guidance law developed in this paper is evaluated in this section via 3D
nonlinear numerical simulation, where the missile is assumed to intercept a weaving target.

The initial conditions required in Eqs. (34)–(36) are selected as: 1) missile-target initial relative range: 3000 m; 2) missile-target initial relative velocity: −300 m/s; 3) initial LOS angles: θ(0) = π/6 rad, φ(0) = 0 rad; 4) initial LOS angular rates: ˙θ(0) = 0.05 rad/s and ˙φ(0) = 0.025 rad/s. Actually, the missile acceleration achieved cannot be arbitrarily large due to its physical limitations. In these simulations, the missile acceleration command is bounded by 100 m/s². The missile acceleration is obtained through a first-order autopilot with its time constant being 0.1 s. The target performs the following weaving maneuvers:

\[ a_{Tr} = 30 \sin(0.25 \pi t) \, \text{m/s}^2 \]
\[ a_{Tp} = 50 \sin(\pi t/3) \, \text{m/s}^2 \]
\[ a_{T\phi} = 50 \sin(0.5\pi t) \, \text{m/s}^2 \]

In order to make a better illustration, the HGO-ISSG law, which also requires no information on LOS angular rate is performed in simulations for the purpose of comparison. It has been shown in Yan and Ji that the HGO-ISSG law has better performance than the adaptive sliding mode guidance (ASMG) law. Therefore, comparing with HGO-ISSG law is meaningful.

The HGO for estimating LOS angular rate is defined as

\[ \dot{\hat{\theta}} = \hat{\theta} + 2(\theta - \hat{\theta})/\varepsilon_h \]
\[ \ddot{\hat{\theta}} = -a_{M\theta}/(r \cos \phi) + (\theta - \hat{\theta})/\varepsilon_h^2 \]
\[ \dot{\hat{\phi}} = \hat{\phi} + 2(\phi - \hat{\phi})/\varepsilon_h \]
\[ \ddot{\hat{\phi}} = -a_{M\phi}/r + (\phi - \hat{\phi})/\varepsilon_h^2 \]

where \( \varepsilon_h \) is a small positive constant to be designed. The HGO-ISSG law is defined as

\[ a_{M\theta} = -2\dot{\theta} \cos \phi + r \left( \frac{1}{2m_1 r^2} + K \right) \ddot{\theta} \cos \phi \]
\[ a_{M\phi} = -2\dot{\phi} + r \left( \frac{1}{2m_2 r^2} + K \right) \ddot{\phi} \]

where \( m_1 > 0, m_2 > 0 \) and \( K > 0 \) are design parameters. The design parameters for these two guidance laws are listed in Table 1, in which the parameters for the HGO-ISSG law are taken from Yan and Ji.

Figures 3 and 4 present the response curves of LOS angular rate under these two guidance laws. It can be observed that the performance of the proposed guidance law in driving the LOS angular rate to zero is superior compared to the HGO-ISSG law. Figures 5 and 6 report the lumped uncertainty estimation obtained by the proposed observer, where the lumped uncertainty in \( \theta \) direction means \( r\delta_\theta \) and the lumped uncertainty in \( \phi \) direction means \( r\delta_\phi \). One can see that the proposed observer accurately estimates the lumped uncertainty whatever the target maneuver is. The LOS angular rate estimation performance under the proposed observer is depicted in Figs. 7 and 8, while the LOS angular rate estimation performance under HGO is given in Figs. 9 and 10. From these four figures, one can see that the LOS angular rate estimations under HGO diverge at the end of the conflict, while the proposed observer can track the real LOS angular rate truthfully. In addition, since high gains are adopted in HGO to suppress the effect of the unknown target maneuvers, the LOS angular rate estimations under HGO are somewhat oscillating during the final phase of interception. Moreover, since the proposed SOSM guidance law is inherently continuous, the undesired chattering phenomenon is avoided completely, which can be seen in Figs. 11 and 12. Although the initial estimations of the lumped uncertainty in Fig. 5 and the LOS rate in Fig. 7 show some fluctuation, the low-pass property of missile autopilot can filter out this undesired phenomenon in acceleration response, which can be seen in Fig. 11.

To evaluate the performance of the proposed guidance law in realistic engagement, a white noise with zero mean and 0.5 mrad standard deviation was added to the inertial LOS angle measurement, including LOS angle measurement noise in body frame and onboard gyro measurement noise.

### Table 1. Design parameters of two guidance laws.

| Observer           | Design parameters |
|--------------------|-------------------|
| HGO-ISSG law (41)  | \( \varepsilon_h = 0.005 \), \( K = 1 \), \( m_1 = 0.00005 \), \( m_2 = 0.00005 \) |
| Proposed law (19)  | \( k_1 = 8 \), \( k_2 = 12 \), \( k_3 = 6 \), \( a = 0.6 \), \( \rho_1 = 600 \), \( \rho_2 = 600 \), \( p = 2.1 \) |

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The response curves of the LOS angular rate under this condition are presented in Figs. 13 and 14. One can see that the proposed guidance law can force the LOS angular rate to converge into a small region around zero in finite time. In addition, the performance of the proposed guidance laws in driving the LOS angular rate to zero is superior to that of the HGO-ISSG law. Figures 15 and 16 present the missile acceleration responses. These two figures also show that the proposed observer is less sensitive to LOS angle measurement noise than the HGO.
this paper. The presented guidance law is chatter-free and re-
mogenous theory and SOSM methodology is proposed in

6. Conclusion

A new finite-time convergence guidance law based on ho-
mogenous theory and SOSM methodology is proposed in
this paper. The presented guidance law is chatter-free and re-
quires no information on target maneuver or LOS angular
rate. A key feature of the proposed homogenous observer lies
in that it not only provides real-time estimation of the lumped
uncertainty, but also possesses the capability of LOS angular
rate observation. Comparisons with the HGO-ISSG law
under different scenarios demonstrate the superiority of this
composite guidance scheme.

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References

1) Zarchan, P.: Tactical and Strategic Missile Guidance, 2nd Ed., AIAA,
Washington DC, 1998.
2) Zhou, D., Mu, C. D., and Shen, T. L.: Robust Guidance Law with L2
Gain Performance, J. Jpn. Soc. Aeronaut. Space Sci., 44 (2001), pp.
82–88.
3) Shima, T. and Golan, O. M.: Eko-atmospheric Guidance of an Accel-
erating Interceptor Missile, J. Franklin Inst., 349, 2 (2012), pp.
622–637.
4) Talole, S. E., Ghosh, A., and Phadke, S. B.: Proportional Navigation
Guidance Using Predictive and Time Delay Control, Control Eng.
Pract., 14, 12 (2006), pp. 1445–1453.
5) He, S., Lin, D., and Wang, J.: Robust Terminal Angle Constraint Guid-
ance Law with Autopilot Lag for Intercepting Maneuvering Targets,
Nonlinear Dyn., 81, 1-2 (2015), pp. 881–892.
6) Zhang, Z. X., Li, S. H., and Luo, S.: Composite Guidance Laws Based
on Sliding Mode Control with Impact Angle Constraint and Autopilot
Lag, Trans. Inst. Meas. Control, 35, 6 (2013), pp. 764–776.
7) Phadke, S. B. and Talole, S. E.: Sliding Mode and Inertial Delay Con-
trol Based Missile Guidance, IEEE Trans. Aerosp. Electron. Syst., 48,
4 (2012), pp. 3331–3346.
8) Zhu, Z., Xu, D., Liu, J., and Xia, Y.: Missile Guidance Law Based on
Extended State Observer, IEEE Trans. Ind. Electron., 60, 12 (2013),
pp. 5882–5891.
9) He, S. and Lin, D.: Continuous Robust Guidance Law for Intercepting
Maneuvering Targets, J. Jpn. Soc. Aeronaut. Space Sci., 58 (2015),
pp. 163–169.
10) Yan, H. and Ji, H. B.: Guidance Laws Based on Input-to-state Stability
and High-gain Observers, IEEE Trans. Aerosp. Electron. Syst., 48, 3
(2012), pp. 2518–2529.
11) Vergez, P. L. and McClendon, J. R.: Optimal Control and Estimation
for Strapdown Seeker Guidance of Tactical Missiles, J. Guid. Control
Dynam., 5, 3 (1982), pp. 225–226.
12) Battistini, S. and Shima, T.: Differential Games Missile Guidance with
Bearings-only Measurements, IEEE Trans. Aerosp. Electron. Syst., 50,
4 (2014), pp. 2906–2915.
13) Speyer, J. L., Kim, K. D., and Tahk, M.: Passive Homing Missile
Guidance Law Based on New Target Maneuver Models, J. Guid. Con-
trol Dynam., 13, 5 (1990), pp. 803–812.
14) Song, T. L.: Observability of Target Tracking with Bearings-only
Measurements, IEEE Trans. Aerosp. Electron. Syst., 32, 4 (1996),
pp. 1268–1472.
15) He, S., Wang, J., and Lin, D.: Composite Guidance Laws Using Higher
Order Sliding Mode Differentiator and Disturbance Observer, Proc.
Inst. Mech. Eng. Part G J. Aerosp. Eng., 2015, DOI: 10.1177/0954410015576365
16) Liao, F., Ji, H., and Xie, Y.: A Novel Three-dimensional Guidance

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Law Implementation Using Only Line-of-sight Azimuths, *Int. J. Robust Nonlinear Control*, 2015, DOI: 10.1002/rnc.3287

17) Bhat, S. P. and Bernstein, D. S.: Finite-time Stability of Continuous Autonomous Systems, *SIAM J. Control Optim.*, 38, 3 (2000), pp. 751–766.

18) Bhat, S. P. and Bernstein, D. S.: Geometric Homogeneity with Applications to Finite-time Stability, *Math. Control Signals Syst.*, 17, 2 (2005), pp. 101–127.

19) Edwards, C., Fridman, L., and Levant, A.: *Sliding Mode Control and Observation*, Springer, New York, 2014.

20) Yang, C. D. and Chen, H. Y.: Nonlinear H Infinity Robust Guidance Law for Homing Missiles, *J. Guid. Control Dynam.*, 21, 6 (1998), pp. 882–890.

21) Zhou, D., Mu, C. D., and Xu, W. L.: Adaptive Sliding-mode Guidance of a Homing Missile, *J. Guid. Control Dynam.*, 22, 4 (1999), pp. 589–594.

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