A comment on the 4D antisymmetric tensor field model

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Abstract. We show the existence of a renormalizable local supersymmetry for the gauge fixed action of the four dimensional antisymmetric tensor field model in a curved background quantized in a generalized axial gauge. By using the technique of the algebraic renormalization procedure, we prove the ultraviolet finiteness of the model to all orders of perturbation theory.

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1 Introduction

In [1] the authors have shown that the four dimensional antisymmetric tensor field model, quantized in a curved background admitting Killing vectors, was anomaly free and finite to all orders of perturbation theory. In this work we generalize the results of [1] to be valid for manifolds not necessarily admitting Killing vectors.

In order to avoid the difficulties the authors met in [1, 2], we introduce a vector field $n^\mu(x)$ which will play the role of a generalized axial gauge vector in curved space-time. In fact, from the beginning we choose the manifold $M$, on which the four dimensional antisymmetric tensor field model is discussed, to have a trivial topology. In particular, this means that the gauge vector field $n^\mu(x)$ can be chosen to be nowhere vanishing.

In the present paper we show, using the algebraic renormalization techniques [3, 4, 5], that the model is anomaly free and finite to all orders of perturbation theory. In section 2 we describe the model as well as its gauge fixing. In section 3 we display the superdiffeomorphisms transformations. Section 4 is devoted to the off-shell analysis of the theory and finally the stability as well as the anomaly analysis are performed in section 5.

2 The model

We begin with the classical action of the four dimensional antisymmetric field model in curved space-time:

$$S_{inv} = \frac{1}{4} \int d^4x \varepsilon^{\mu \nu \rho \sigma} F_{\mu \nu}^a B_{\rho \sigma}^a,$$  \hspace{1cm} (2.1)

where $M$ is a curved manifold endowed with the Euclidean metric $g_{\mu \nu}$. $B_{\rho \sigma}^a$ stands for the antisymmetric tensor field whereas $F_{\mu \nu}^a$ is the field strength given by

$$F_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c,$$  \hspace{1cm} (2.2)

where $A_\mu^a$ is the gauge field. All fields are Lie algebra valued and belong to the adjoint representation of some compact semi-simple gauge group $G$ whose structure constants $f^{abc}$ are completely antisymmetric in their indices. The generators of the Lie algebra are chosen to be anti-hermitian and fulfilling $[T^a, T^b] = f^{abc} T^c$ and $\text{Tr}(T^a T^b) = \delta^{ab}$. Finally, $\varepsilon^{\mu \nu \rho \sigma}$ is the totally antisymmetric Levi-Civita tensor density\footnote{In fact the manifold was chosen to be asymptotically flat, having trivial topology and admitting Killing vectors.} of weight $+1$.

We denote by $g^{\mu \nu}$ the inverse of the metric and its determinant by $g$. Under diffeomorphisms, $\sqrt{g}$ behaves like a scalar density of weight $+1$ and the volume element $d^4x$ has weight $-1$. The Levi-Civita tensor density $\varepsilon^{\mu \nu \rho \sigma}$ has weight $+1$ and its inverse $\varepsilon_{\mu \nu \rho \sigma}$ carries weight $-1$.\footnote{We denote by $g^{\mu \nu}$ the inverse of the metric and its determinant by $g$. Under diffeomorphisms, $\sqrt{g}$ behaves like a scalar density of weight $+1$ and the volume element $d^4x$ has weight $-1$. The Levi-Civita tensor density $\varepsilon^{\mu \nu \rho \sigma}$ has weight $+1$ and its inverse $\varepsilon_{\mu \nu \rho \sigma}$ carries weight $-1.$}
The action (2.1) is invariant under the following two infinitesimal symmetries:

\[
\delta^{(1)} A^a_\mu = -\partial_\mu \theta^a - f^{abc} A^b_\mu \theta^c \equiv -(D_\mu \theta)^a , \\
\delta^{(1)} B^a_{\mu\nu} = f^{abc} \theta^b B^c_{\mu\nu} ,
\]

and

\[
\delta^{(2)} A^a_\mu = 0 , \\
\delta^{(2)} B^a_{\mu\nu} = -(D_\mu \varphi_\nu - D_\nu \varphi_\mu)^a ,
\]

where \( \theta^a \) is the local gauge parameter and \( \varphi^a_\mu \) is a local vector parameter. \( D_\mu \) represents the covariant derivative. In order to fix the gauge consistently we use a generalized axial gauge type with a local vector \( n^\mu(x) \). In fact, we will quantize the model on a four dimensional manifold which is assumed to be topologically trivial and asymptotically flat. Therefore, we can choose \( n^\mu(x) \) to be a nowhere vanishing local vector. Hence, the gauge fixing part of the action, which is metric dependent and therefore destroys the topological character of the model, reads:

\[
S_{gf} = s \int d^4x \sqrt{g} \left( c^a g^{\mu\nu} n_\mu A^a_\nu + g^{\mu\alpha} g^{\nu\beta} \tilde{\xi}_\beta n_\alpha B^a_{\mu\nu} + \bar{\phi}^a g^{\mu\nu} n_\mu \xi^a_\nu \right) ,
\]

where the vector \( \xi^a_\mu \) is the ghost field for the symmetry (2.4), \( \phi^a \) is the ghost for the ghost \( \xi^a_\mu \) and \( c^a \) is the ghost for the symmetry (2.3). We collect the antighosts and the corresponding Lagrange multipliers in pairs \((\bar{c}^a, b^a)\), \((\bar{\xi}^a_\mu, h^a_\mu)\) and \((\bar{\phi}^a, \omega^a)\).

Contrary to [1], gauge-fixing the four dimensional antisymmetric tensor field model using the generalized axial gauge is much simpler than using the Landau gauge such that (2.5) takes a simple form (see the expression (2.19) in [1]). In the present case the extended nilpotent BRS-transformations read as

\[
\begin{align*}
sA^a_\mu &= -(D_\mu c)^a , \\
sB^a_{\mu\nu} &= -(D_\mu \xi^a_\nu - D_\nu \xi^a_\mu)^a + f^{abc} c^b_\mu B^c_{\mu\nu} - \varepsilon_{\mu\nu\rho\sigma} f^{abc} \sqrt{g} g^{\rho\alpha} g^{\sigma\beta} n_\alpha \tilde{\xi}_\beta \phi^c , \\
s\xi^a_\mu &= (D_\mu \phi)^a + f^{abc} c^b_\mu \xi^c , \\
s\phi^a &= f^{abc} c^b \phi^c , \\
s\xi^a &= \frac{1}{2} f^{abc} c^b \xi^c , \\
s c^a &= b^a , \quad sb^a = 0 , \\
s \xi^a_\mu &= h^a_\mu , \quad sh^a_\mu = 0 , \\
s \phi^a &= \omega^a , \quad s\omega^a = 0 , \\
s g_{\mu\nu} &= \hat{g}_{\mu\nu} , \quad s\hat{g}_{\mu\nu} = 0 .
\end{align*}
\]

3For different applications of the non-covariant gauges in flat space-time in the context of the algebraic renomalization see [4]. In the present work, however, we generalize the axial gauge to curved manifolds having a trivial topology.
The metric plays in (2.5) the role of a gauge parameter \([1]\) which we also let transform as a BRS-doublet as given in the last line of (2.6). Furthermore, to control the \(n_\mu\)-dependence of the theory we use the arguments of [6] and enlarge the BRS-transformations by allowing also a variation of the local vector \(n_\mu\):

\[
    s n_\mu = \chi_\mu, \quad s \chi_\mu = 0, \tag{2.7}
\]

and add the following term to the action

\[
    S_n = - \int d^4 x \sqrt{g} \left( \varepsilon^a g^{\mu\nu} \chi_\mu A^a_\mu + g^{\mu\nu} g^{\rho\beta} \xi^a_\beta \chi_\alpha B^a_\mu - \tilde{\phi}^a g^{\mu\nu} \chi_\nu \xi^a_\rho \right). \tag{2.8}
\]

Here, \(\chi_\mu\) is a local anticommuting vector parameter. It turns out that the BRS-operator is nilpotent on-shell\(^4\):

\[
    s^2 B^a_\mu = -\varepsilon_{\mu\nu\rho\sigma} f^{abc} \frac{\delta (S_{inv} + S_{gf} + S_n)}{\delta g_{\rho\sigma}} \phi^c \quad \text{and} \quad s^2 = 0 \quad \text{for all other fields.} \tag{2.9}
\]

One can easily verify the BRS-invariance of the gauge fixed action, which obeys

\[
    s (S_{inv} + S_{gf} + S_n) = 0. \tag{2.10}
\]

We present the canonical dimensions and the Faddeev–Popov charges of all fields in Table 1.

| Field | \(A^a_\mu\) | \(B^a_\mu\nu\) | \(\phi^a\) | \(\xi^a_\mu\) | \(c^a\) | \(\xi^a_\sigma\) | \(c^a\) | \(\phi^a\) | \(b^a\) | \(h^a\) | \(\omega^a\) | \(n_\mu\) | \(\chi_\mu\) | \(g_{\mu\nu}\) | \(\hat{g}_{\mu\nu}\) |
|-------|-------------|--------------|-----------|-------------|-------|-------------|-------|-----------|-------|-------|-------------|-------|----------|-----------|--------|
| dim   |   1        |   2          |   0       |   1         |   0   |   2         |   3   |   3        |   2   |   3   |   0           |   0   |   0      |   0       |   0    |
| \(\phi\pi\) | 0          | 0            | 2         | 1           | 1    | -1          | -1   | -2        | 0     | 0     | -1          | 1     | 0        | 0         | 1      |

Table 1: Dimensions and Faddeev–Popov charges of the fields

3 Superdiffeomorphisms

As already shown in [1] for the Landau-type gauge, the four dimensional antisymmetric tensor field model possesses besides the BRS-symmetry and the invariance under diffeomorphisms a further invariance of supersymmetric-kind, namely the so-called superdiffeomorphisms. For these local transformations we propose:

\[
    \delta_{(\eta)} A^a_\mu = \varepsilon_{\mu\nu\rho\sigma} \eta^\nu \sqrt{g} g^{\alpha\beta} n_\alpha \xi^a_\beta, \\
    \delta_{(\eta)} B^a_\mu\nu = \varepsilon_{\mu\nu\rho\sigma} \eta^\rho \sqrt{g} g^{\sigma\alpha} n_\alpha c^a, 
\]

\(^4\)It should be mentioned that contrary to [1] our analysis using the generalized axial gauge gets simpler due to the fact that we have less fields.
\[
\begin{align*}
\delta (\eta)c^a &= -\eta^\mu A^a_{\mu}, \\
\delta (\eta)c^a &= 0, \\
\delta (\eta)\xi^a_{\mu} &= \mathcal{L}_\eta \bar{c}^a, \\
\delta (\eta)\xi^a_{\mu} &= \eta^\nu B^a_{\mu\nu}, \\
\delta (\eta)\xi^a_{\mu} &= -g_{\mu\nu}\eta^\nu\tilde{\phi}^a, \\
\delta (\eta)h^a_{\mu} &= \mathcal{L}_\eta \bar{c}^a + s(g_{\mu\nu}\eta^\nu\tilde{\phi}^a), \\
\delta (\eta)\phi^a &= \eta^\mu \xi^a_{\mu}, \\
\delta (\eta)\bar{\phi}^a &= 0, \\
\delta (\eta)\omega &= \mathcal{L}_\eta \bar{\phi}^a, \\
\delta (\eta)n^a_{\mu} &= 0, \\
\delta (\eta)\chi^a_{\mu} &= \mathcal{L}_\eta n^a_{\mu}, \\
\delta (\eta)g_{\mu\nu} &= 0, \\
\delta (\eta)\hat{g}_{\mu\nu} &= \mathcal{L}_\eta g_{\mu\nu}, \\
\end{align*}
\]

where \(\mathcal{L}_\eta\) represents the Lie derivative and \(\eta^\mu\) is the vector parameter of the transformations carrying ghost number +2. The resulting algebra between the BRS-operator and the superdiffeomorphisms closes on-shell:

\[
\{s, \delta (\eta)\} = \mathcal{L}_\eta + \text{equations of motion}, \\
\{\delta (\eta), \delta (\eta')\} = 0.
\]

At this stage one remarks that contrary to the case of [1] there is no constraint which requires the manifold to possess Killing vectors. Therefore, the underlying paper is a generalization of [1].

4 The off–shell analysis

In order to describe the BRS-symmetry content consistently at the functional level, we introduce a set of external sources\(^5\) coupled to the nonlinear BRS-variations of the quantum fields:

\[
S_{ext} = \int d^4x \left[ \gamma^{\mu\nu a}(sB^a_{\mu\nu}) + \Omega^{\mu a}(sA^a_{\mu}) + L^a(sc^a) + D^a(s\phi^a) + \phi^{\mu a}(s\xi^a_{\mu}) \right] + \\
+ \frac{1}{2} \int d^4x \varepsilon_{\mu\nu\rho\sigma} f^{abc}A^{\mu a}_\lambda \omega^{\nu b}_{\sigma} \phi^c.
\]

We display the canonical dimensions and the Faddeev–Popov charges of the external sources in Table 2.

\(^5\)One has to note that the sources have weight +1.
Therefore, the complete action

\[ \Sigma = S_{inv} + S_{gf} + S_n + S_{ext} \]  

obeys the Slavnov identity:

\[ S(\Sigma) = \int d^4x \left[ \frac{\delta \Sigma}{\delta \gamma^{\mu\nu}} \frac{\delta}{\delta B_{\mu\nu}^a} + \frac{\delta \Sigma}{\delta \Omega^{\mu a}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma}{\delta D_a} \frac{\delta}{\delta \phi^a} + \frac{\delta \Sigma}{\delta \phi^a} \frac{\delta}{\delta \xi_a} \right] = 0 . \]  

For later use we introduce the linearized Slavnov operator \( S_\Sigma \):

\[ S_\Sigma = \int d^4x \left[ \frac{\delta \Sigma}{\delta \gamma^{\mu\nu}} \frac{\delta}{\delta B_{\mu\nu}^a} + \frac{\delta \Sigma}{\delta \Omega^{\mu a}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma}{\delta D_a} \frac{\delta}{\delta \phi^a} + \frac{\delta \Sigma}{\delta \phi^a} \frac{\delta}{\delta \xi_a} \right] . \]  

The introduction of external sources leads to a linearly broken Ward identity for the superdiffeomorphisms:

\[ W^{S}_{(\eta)} \Sigma = \Delta^{cl}_{(\eta)} , \]  

where

\[ W^{S}_{(\eta)} = \int d^4x \left[ \varepsilon_{\mu\nu\sigma} \eta^\rho \left( \sqrt{g} g^{\sigma\rho} g^{\sigma\beta} n_\alpha \xi_\beta - \gamma^{(\sigma\rho)} \right) \frac{\delta \eta^\sigma}{\delta A_\mu^a} - \eta^\mu A_\mu^a \frac{\delta}{\delta c^a} + \mathcal{L}_{\tau} \frac{\delta}{\delta b^a} + \right] . \]  

is the Ward operator for superdiffeomorphisms and

\[ \Delta^{cl}_{(\eta)} = \int d^4x \left[ -\gamma^{\mu a} L_{\mu}^a - \Omega^{\mu a} \mathcal{L}_{\tau} \gamma^{\mu a} + L^a \mathcal{L}_{\tau} c^a - D_a \mathcal{L}_{\tau} \phi^a + \phi^a \mathcal{L}_{\tau} \xi^a + \right] . \]
is the breaking which is linear in the quantum fields and therefore harmless at the quantum level.

On the other hand, if the functional $\Sigma$ is a solution of the Slavnov identity \((4.15)\), of the superdiffeomorphisms Ward identity \((4.17)\) as well as the Ward identity for diffeomorphisms

$$\mathcal{W}^D_\varepsilon \Sigma = 0 \quad , \tag{4.20}$$

where $\mathcal{W}^D_\varepsilon$ stands for the corresponding Ward operator

$$\mathcal{W}^D_\varepsilon = \int d^4x \sum_\varphi (\mathcal{L}_\varepsilon \varphi) \frac{\delta}{\delta \varphi} \ , \tag{4.21}$$

for all fields $\varphi$, then the following off-shell algebra holds:

$$\{S_\Sigma, S_\Sigma\} = 0 \quad , \quad \{\mathcal{W}^S_\eta, \mathcal{W}^S_{\eta'}\} = 0 \quad , \quad \{\mathcal{W}^D_\varepsilon, \mathcal{W}^D_{\varepsilon'}\} = -\mathcal{W}^D_{[\varepsilon, \varepsilon']} \quad , \quad \{S_\Sigma, \mathcal{W}^S_\eta\} = \mathcal{W}^D_\eta \quad , \quad \{\mathcal{W}^D_\varepsilon, \mathcal{W}^S_\eta\} = -\mathcal{W}^S_{[\varepsilon, \eta]} \quad ,$$

$$\{S_\Sigma, \mathcal{W}^D_\varepsilon\} = 0 \quad . \tag{4.22}$$

Here, we used the graded Lie brackets:

$$\{\varepsilon, \varepsilon'\}^\mu = \mathcal{L}_\varepsilon \varepsilon'^\mu \quad , \quad [\varepsilon, \eta]^\mu = \mathcal{L}_\varepsilon \eta^\mu \quad . \tag{4.23}$$

It is straightforward to convince oneself that the total action \((4.14)\) fulfills the gauge conditions

$$\frac{\delta \Sigma}{\delta y^\mu} = \sqrt{g} g^{\mu\alpha} n_\alpha A^a_\mu \quad ,$$

$$\frac{\delta \Sigma}{\delta h^a_\mu} = -\sqrt{g} g^{\mu\nu} g^{\nu\beta} n_\nu B^a_{\alpha\beta} \quad ,$$

$$\frac{\delta \Sigma}{\delta \omega^a} = \sqrt{g} g^{\mu\alpha} n_\alpha \xi^a_\mu \quad , \tag{4.24}$$

the following antighost equations

$$\frac{\delta \Sigma}{\delta \bar{c}^a} + \sqrt{g} g^{\mu\alpha} n_\alpha \frac{\delta \Sigma}{\delta \bar{b}^{a\mu}} = -s(\sqrt{g} g^{\mu\alpha} n_\alpha) A^a_\mu \quad ,$$

$$\frac{\delta \Sigma}{\delta \bar{\xi}^a_\mu} - \sqrt{g} g^{\mu\alpha} g^{\nu\beta} n_\nu \frac{\delta \Sigma}{\delta \bar{\gamma}^{a}_{\alpha\beta}} = s(\sqrt{g} g^{\mu\alpha} g^{\nu\beta} n_\nu) B^a_{\alpha\beta} \quad ,$$

$$\frac{\delta \Sigma}{\delta \bar{\phi}^a} - \sqrt{g} g^{\mu\alpha} n_\alpha \frac{\delta \Sigma}{\delta \bar{\omega}^{a\mu}} = s(\sqrt{g} g^{\mu\alpha} n_\alpha) \xi^a_\mu \quad . \tag{4.25}$$
and a further integrated constraint, namely the ghost equation
\[
G^a \Sigma = \Delta^a ,
\]
where
\[
G^a = \int d^4x \left( \frac{\delta}{\delta \phi^a} - f^{abc} \frac{\delta}{\delta b^c} \right) ,
\]
and
\[
\Delta^a = \int d^4x f^{abc} \left( \sqrt{g} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu b^\rho_n a^\gamma \mu b c \xi_{\beta} + D^b c^c + \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu b c} \rho_{\sigma c} \right) .
\]
Here, $\Delta^a$ is a linear breaking.

5 Proof of the finiteness

This section is devoted to discuss the full symmetry content of the theory at the quantum level, i.e. the question of possible anomalies and the stability problem which amounts to analyze all invariant counterterms.

We begin by studying the stability where in the first step we consider one-loop corrections. This requires the analysis of the most general counterterms for the total action and implies to consider the following perturbed action
\[
\Sigma' = \Sigma + \hbar \Delta ,
\]
where $\Sigma$ is the total action (4.14) and $\Sigma'$ is an arbitrary functional depending via $\Delta$ on the same fields as $\Sigma$ and satisfying the Slavnov identity (4.15), the Ward identity for the superdiffeomorphisms (4.17), the gauge conditions (4.24), the antighost equations (4.25), the ghost equation (4.26) and the Ward identity for the diffeomorphisms (4.20). The perturbation $\Delta$ collecting all appropriate invariant counterterms is an integrated local field polynomial of dimension four and ghost number zero.

Now we are searching for the most general deformation of the classical action such that the perturbed action $\Sigma'$ still fulfills the above constraints. Therefore the perturbation $\Delta$ has to obey the following set of equations:
\[
\frac{\delta \Delta}{\delta b^a} = 0 ,
\]
\[
\frac{\delta \Delta}{\delta h^a_\mu} = 0 ,
\]
\[
\frac{\delta \Delta}{\delta \omega^a} = 0 ,
\]
\[
\frac{\delta \Delta}{\delta c^a} + \sqrt{g} g^{\mu a} n_\alpha \frac{\delta \Delta}{\delta \Omega^a} = 0 ,
\]
The first three equations (5.30)–(5.32) imply that the perturbation \( \Delta \) does not depend on the multiplier fields \( b^a \), \( h^a_\mu \) and \( \omega^a \), whereas the equations (5.33)–(5.35) imply that the dependence of \( (\Omega^\mu a, \tilde{\epsilon}^a) \), \( (\gamma^{\mu\nu a}, \tilde{\xi}^a_\mu) \) and \( (\varrho^a, \tilde{\phi}^a) \) is given by the following combinations

\[
\tilde{\Omega}^{\mu a} = \Omega^{\mu a} - \sqrt{g} g^{\mu a}_\alpha n_\alpha \tilde{\epsilon}^a ,
\]

\[
\tilde{\gamma}^{\mu\nu a} = \gamma^{\mu\nu a} - \sqrt{g} g^{\mu a}_\alpha g^{\nu\beta}_\gamma (n_\alpha \tilde{\xi}^a_\beta - n_\beta \tilde{\xi}^a_\alpha) ,
\]

\[
\tilde{\varrho}^a = \varrho^a + \sqrt{g} g^{\mu a} n_\alpha \tilde{\phi}^a .
\]

The equations (5.36)–(5.38), as in reference [1], can be unified into a single operator \( \delta \):

\[
\delta = \mathcal{S}_\Sigma + \mathcal{W}^{S}_{(\eta)} + \mathcal{W}^{D}_{(\varepsilon)} + \int d^4x \left\{ \left[ \varepsilon, \eta \right]^\mu \frac{\delta}{\delta \eta^\mu} + \left( \frac{1}{2} \left[ \varepsilon, \varepsilon \right]^\mu - \eta^\mu \right) \frac{\delta}{\delta \varepsilon^\mu} \right\} 
\]

producing a cohomology problem

\[
\delta \Delta = 0 .
\]

It can be easily verified that the operator \( \delta \) is nilpotent

\[
\delta^2 = 0 .
\]

Therefore, any expression of the form \( \delta \hat{\Delta} \) is automatically a solution of (5.42). A solution of this type is called a trivial solution. Hence, the most general solution of (5.42) reads

\[
\Delta = \Delta_c + \delta \hat{\Delta} .
\]

Here, the nontrivial solution \( \Delta_c \) is \( \delta \)-closed \( (\delta \Delta_c = 0) \), but not trivial \( (\Delta_c \neq \delta \hat{\Delta}) \).

We begin with the determination of the nontrivial solution of (5.42). For this purpose we introduce a filtering operator \( \mathcal{N} \):

\[
\mathcal{N} = \int d^4x \sum_\varphi \varphi \frac{\delta}{\delta \varphi} ,
\]

where \( \varphi \) stands for all fields, including \( n_\mu, \chi_\mu, \varepsilon^\mu \) and \( \eta^\mu \). To all fields we assign the homogeneity degree 1. The filtering operator induces a decomposition of \( \delta \) according to

\[
\delta = \delta_0 + \delta_1 .
\]
The operator \( \delta_0 \) does not increase the homogeneity degree while acting on a field polynomial. On the other hand, the operator \( \delta_1 \) increases the homogeneity degree by one unit. Furthermore, the nilpotency of \( \delta \) leads to
\[
\delta_0^2 = 0, \quad \{ \delta_0, \delta_1 \} = 0, \quad \delta_1^2 = 0.
\]

Hence, we obtain from (5.47) the following relation
\[
\delta_0 \Delta = 0,
\]
which yields a further cohomology problem. The usefulness of the decomposition (5.46) relies on a very general theorem [3] stating that the cohomology of the complete operator \( \delta \) is isomorphic to a subspace of the cohomology of the operator \( \delta_0 \). The cohomology of \( \delta_0 \) is easier to solve than the cohomology of \( \delta \). The operator \( \delta_0 \) acts on the fields as follows:
\[
\begin{align*}
\delta_0 A^a_\mu &= -\partial_\mu c^a, \\
\delta_0 c^a &= 0, \\
\delta_0 \phi^a &= 0, \\
\delta_0 \Omega^{\mu a} &= \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} \partial_\nu B^{\rho \sigma}_a, \\
\delta_0 L^a &= -\partial_\mu \tilde{\Omega}^{\mu a}, \\
\delta_0 g_{\mu \nu} &= \hat{g}_{\mu \nu}, \\
\delta_0 n_\mu &= \chi_\mu, \\
\delta_0 \varepsilon^\mu &= -\eta^\mu, \\
\delta_0 \hat{\gamma}^{\mu a} &= \varepsilon^{\mu \nu \rho \sigma} \partial_\nu A^{\rho \sigma}_a, \\
\delta_0 D^a &= -\partial_\mu \tilde{g}^{\mu a}, \\
\delta_0 \eta^\mu &= 0, \\
\delta_0 \tilde{\Omega}^{\mu a} &= \partial_\nu \tilde{\gamma}^{\mu a}, \\
\delta_0 \tilde{\gamma}^{\mu a} &= \varepsilon^{\mu \nu \rho \sigma} \partial_\nu A^{\rho \sigma}_a, \\
\delta_0 \tilde{\Omega}^{\mu a} &= \partial_\nu \tilde{\gamma}^{\mu a}, \\
\delta_0 \tilde{\gamma}^{\mu a} &= \varepsilon^{\mu \nu \rho \sigma} \partial_\nu A^{\rho \sigma}_a,
\end{align*}
\]

We notice that the quantities \( g_{\mu \nu}, \hat{g}_{\mu \nu}, n_\mu, \chi_\mu, \varepsilon^\mu \) and \( \eta^\mu \) transform under \( \delta_0 \) as doublets, being therefore out of the cohomology [3, 7]. The nontrivial solution \( \Delta_c \) can now be written as integrated local field polynomial of form degree four and ghost number zero:
\[
\Delta_c = \int \omega^0_4,
\]
where \( \omega^p_q \) is a field polynomial of form degree \( q \) and ghost number \( p \). Using the Stoke’s theorem, the algebraic Poincaré lemma [7] and the relation \( \{ \delta_0, d \} = 0 \), where \( d \) represents the nilpotent exterior derivative \( (d^2 = 0) \), we obtain the following tower of descent equations:
\[
\begin{align*}
\delta_0 \omega^0_4 + d \omega^1_3 &= 0, \\
\delta_0 \omega^1_3 + d \omega^2_2 &= 0, \\
\delta_0 \omega^2_2 + d \omega^3_1 &= 0, \\
\delta_0 \omega^3_1 + d \omega^4_0 &= 0, \\
\delta_0 \omega^4_0 &= 0.
\end{align*}
\]

The tower of descent equations (5.51) has been solved in [1], where it was shown that \( \omega^4_0 \) takes the following form:
\[
\omega^4_0 = u \phi^a \phi^a + v f^{abc} c^a c^b \phi^c,
\]

\( \Delta_c = \int \omega^0_4, \)
with \( u \) and \( v \) being some constant coefficients. In \([1]\), the authors showed by using the equation (5.39) that both \( u \) and \( v \) vanish.

Next, we move to the computation of the trivial counterterms which are constrained by the dimension, the ghost number and the weight requirements. The most general trivial solution can be constructed as follows

\[
\hat{\Delta} = \int d^4x \left( \alpha_1 \tilde{\chi}^{\mu a} A^a_{\mu} + \alpha_2 \tilde{\gamma}^{\mu a} B^a_{\mu} + \alpha_3 L^a c^a + \alpha_4 \tilde{\epsilon}^{\mu a} \varepsilon^a \right) + \alpha_5 D^a \phi^a + \\
+ \alpha_6 \tilde{\gamma}^{\mu a} A^a_{\mu} c^c + \alpha_7 \tilde{\gamma}^{\mu a} B^a_{\mu} c^c + \alpha_8 \tilde{\gamma}^{\mu a} \partial A^a_{\mu} + \alpha_9 \tilde{\gamma}^{\mu a} A^a_{\mu} \partial \phi^a + \\
+ \alpha_{10} \tilde{\gamma}^{\mu a} B^a_{\mu} \partial \phi^a + \alpha_{11} \tilde{\gamma}^{\mu a} c^c \partial \phi^a + \alpha_{12} D^a \partial \phi^a + \alpha_{13} \tilde{\gamma}^{\mu a} \partial \phi^a + \\
+ \alpha_{14} \frac{1}{\sqrt{g}} \tilde{\gamma}^{\mu a} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{15} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{16} \frac{1}{\sqrt{g}} f^{abc} d_{\nu a} g_{\nu a} \tilde{g}^{\mu a} \tilde{g}_{\nu a} + \alpha_{17} \tilde{g}^{\mu a} A^a \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{18} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} \tilde{g}^{\mu a} \tilde{g}^{\nu a} g_{\nu a} A^a_{\mu} + \alpha_{19} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} \tilde{g}^{\mu a} \tilde{g}^{\nu a} A^a_{\mu} + \\
+ \alpha_{20} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} A^a_{\mu} + \alpha_{21} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} B^a_{\mu} + \alpha_{22} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{23} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{24} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{25} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{26} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{27} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{28} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{29} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{30} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{31} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{32} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{33} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{34} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{35} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{36} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{37} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{38} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{39} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{40} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{41} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{42} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{43} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{44} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{45} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{46} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \\
+ \alpha_{47} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{48} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{49} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} + \alpha_{50} \frac{1}{\sqrt{g}} \tilde{g}^{\nu a} g_{\nu a} .
\]

The trivial counterterm\(^6\) may depend on the quantities \( \eta^\mu \) and \( \varepsilon^\mu \) which do not appear in the total action (4.14). For this reason we demand the expression \( \delta \Delta \) to be independent of the parameters \( \eta^\mu \) and \( \varepsilon^\mu \). In fact, after a tedious computation \( \Delta \) reduces to an expression which is forbidden by (5.39). Thus, all of the coefficients \( \alpha_i , i = 1 , \ldots , 50 \) vanish.

Therefore, we have shown that the total action \( \Sigma \) does not admit any deformations at the

\(\text{Page 10}\)
quantum level.

The last step in our analysis is devoted to the discussion of the existence of possible breaking of the symmetries at the quantum level. By using the same arguments as in [1] and under the assumption that the quantum action principle is also valid in the case of non-covariant gauges [4], one can easily show that the symmetries of the model do not admit any anomalies and therefore, are valid at the quantum level. This completes the proof of finiteness of the four dimensional antisymmetric tensor field model to all orders of perturbation theory, quantized on a topologically trivial and asymptotically flat manifold.

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