WINDS FROM NEUTRON STARS
AND STRONG TYPE I X–RAY BURSTS

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ABSTRACT

A model for stationary, radiatively driven winds from X–ray bursting neutron stars is presented. General relativistic hydrodynamical and radiative transfer equations are integrated from the neutron star surface outwards, taking into account for helium nuclear burning in the inner, dense, nearly hydrostatic shells. Radiative processes include both bremsstrahlung emission–absorption and Compton scattering; only the frequency–integrated transport is considered here. It is shown that each solution is characterized by just one parameter: the mass loss rate $\dot{M}$, or, equivalently, the envelope mass $M_{\text{env}}$. We found that, owing to the effects of Comptonization, steady, supersonic winds can exist only for $\dot{M}$ larger than a limiting value $\dot{M}_{\text{min}} \approx \dot{M}_{E}$. Several models, covering about two decades in mass loss rate, have been computed for given neutron star parameters. We discuss how the sequence of our solutions with decreasing $M_{\text{env}}$ can be used to follow the time evolution of a strong X–ray burst during the expansion/contraction phase near to the luminosity maximum. The comparison between our numerical results and the observational data of Haberl et al. (1987) for the bursts in 4U/MXB 1820-30 gives an estimate for both the spectral hardening factor and the accretion rate in this source.

Subject headings: hydrodynamics – radiative transfer – stars: individual (4U/MXB 1820-30) – stars: neutron – stars: winds – X–rays: bursts

Accepted for publication in the Astrophysical Journal
I. INTRODUCTION

Mass loss from stars is a well-known phenomenon and various models have been proposed to explain it in different situations. In the case of winds from hot stars, radiation pressure in lines is often assumed to be responsible for the outflow while for cooler stars radiation pressure on dust grains may be the dominating mechanism (see e.g. the review by Cassinelli 1979). A general feature of all steady-state models is that the flow is subsonic at small radii and supersonic far from the star, passing through a critical point which, however, not necessary coincides with the sonic point.

As far as winds from neutron stars are concerned, several mechanisms were considered in the attempt to model different phenomena. Winds from young and very hot neutron stars, for example, can be driven by absorption of high-energy neutrinos by protons and neutrons close to the stellar surface (Salpeter & Shapiro 1981; Duncan, Shapiro & Wasserman 1986). Under less extreme conditions, however, the outflow is commonly thought to be powered by radiation pressure. In the optically thick models of Meier (1982a, b, c) mass and energy are injected at some radius and the rates of injection are free parameters. For high energy input rates (well above the Eddington limit) most of the energy is converted into gas kinetic energy and a strong wind is produced, although velocities are never relativistic.

Highly relativistic winds were considered by Paczyński (1986), Goodman (1986) and Paczyński (1990) as models for γ-ray bursters at cosmological distances. The idea is that γ-ray bursts may result from the merging of the two components in a double neutron star binary. As in Meier’s models, energy and mass are injected at some arbitrary rate near the star surface and local thermodynamic equilibrium (LTE) is assumed to hold. Owing to the very high temperature, electron–positron pairs are formed and the pair pressure is taken
into account along with the radiation pressure. The apparent temperature of
the photosphere is in excess of $10^9$ K and the object exhibits itself as a strong
$\gamma$–ray source.

The existence of winds from neutron stars during strong X–ray bursts is
a well acknowledged fact (see e.g. Lewin, Vacca & Basinska 1984; Tawara et al. 1984; Tawara, Hayakawa & Kii 1984; Haberl et al. 1987; see also Lewin, Van Paradijs & Taam 1993 for a very recent review on X–ray bursters). For
a distant observer the wind phase lasts just the few seconds separating the
precursor from the main burst (Haberl et al.). Since the true beginning of the
burst is marked by the precursor, the dip observed in the fitting temperature
curve is an observational evidence of a photospheric expansion produced, most
probably, by a super–Eddington luminosity.

Steady–state winds from neutron stars were the object of several
investigations in the past decade. The possible existence of such winds
was explained by the fact that, during thermonuclear helium burning, the
temperature at the base of the accreted atmosphere rises above $10^9$ K, so that
the electron scattering opacity decreases below its Thomson value owing to
Klein–Nishina corrections. The radiation flux diffusing out of the hot region
may be therefore below the local Eddington value, but it may appear to exceed
the local Eddington limit in the outer layers which are cooler. The hydrostatic
equilibrium of these layers is then violated and a radiation–driven wind may
develop.

Earlier studies (Ebisuzaki, Hanawa, & Sugimoto 1983; Kato 1983; Quinn &
Paczyński 1985) assumed Newtonian gravity and dealt only with optically thick
outflows. General relativistic effects were taken into account by Paczyński and
Prószyński (1986), maintaining the diffusion approximation, and by Turolla,
Nobili & Calvani (1986), who investigated also the effects of Compton heating–
cooling in the wind. A more refined treatment of radiative transfer in the outflowing envelope was introduced by Joss & Melia (1987), accounting also for Compton scattering in an approximate way. Using the wind structure computed by Ebisuzaki et al. as a background for frequency-dependent radiative transfer, Lapidus (1991) has confirmed the qualitative scenario of drastic spectral softening during the photospheric expansion, finding a satisfactory agreement between the computed spectral softening factors and those observed by EXOSAT in 4U/MXB 1820–30. In a recent paper Titarchuk (1993) presented an analytical study of spectral formation during expansion and contraction phases of X-ray bursters and found that the spectrum in the expansion phase depends strongly on the temperature in the helium burning region.

The models of neutron star winds explored up to now contain, however, some drastic assumptions. Nearly all previous investigations, in fact, neglected the effects of dynamics on radiative transfer and used LTE plus diffusion approximation everywhere in the flow. As a result, these models imply a discontinuity in the angular distribution of the radiation field which switches from near-isotropy below the photospheric radius to radial streaming above it, where an optically thin region with constant outflow velocity is assumed to exist. Moreover the rate of energy injection at the base of the flow was assumed to be a free parameter, uncorrelated to the energy actually released by thermonuclear reactions during the burst phase.

In this paper we present a wind model which overcomes these major disadvantages. General relativistic radiative transfer in a differentially moving medium is properly treated using Thorne’s (1980) moment formalism, so that not only the effects of thermal but also of dynamical Comptonization are accounted for. Integration of the flow equations is pushed down to the stellar surface and covers also the dense, inner shells in which nuclear burnings
occur. As a consequence, the energy input for our solutions is self-consistently computed and the only free parameter is the total mass of the envelope $M_{\text{env}}$. We have found that the inclusion of Comptonization results in a lower limit for $\dot{M}$ (and, correspondingly for $M_{\text{env}}$) for steady, supersonic winds to exist, the lower bound being $\dot{M} \sim 6 \times 10^{17}$ g/s for a neutron star mass of $1.5M_\odot$. Several models with different values of $M_{\text{env}}$, corresponding to mass loss rates in the range $10^{17} \div 10^{19}$ g/s were computed. Our series of models with smoothly varying $M_{\text{env}}$ can be used to follow the time evolution of a X-ray bursting source during the envelope expansion/contraction phase. A comparison of our results with the observational data of 4U/MXB 1820-30 allowed us to estimate the hardening ratio and the initial envelope mass for the bursts analyzed by Haberl et al.

Further developments will be aimed to include frequency-dependent radiative transfer in our code to obtain a self-consistent determination of the emergent spectrum. Frequency-dependent calculations are needed also to access the effect of bulk motion Comptonization, which proved to be relevant in near-Eddington accretion onto neutron stars (Zampieri, Turolla & Treves 1993), and to provide a better determination of spectrum-averaged quantities which enter the dynamical equations, like the opacity coefficients and the radiation temperature. This work is now in progress and, when completed, the scheme will provide a tool for unambiguous determination of mass-radius relation for neutron stars from observational spectral data of powerful X-ray bursts.
II. THE MODEL

In the following we assume that the gas outflow is spherically symmetric and stationary. The neutron star rotation is neglected so that the gravitational field can be described by the vacuum Schwarzschild solution; $M_*$ and $R_*$ will denote the star mass and radius, respectively. We also ignore all the effects induced by the large-scale $B$–field associated with the neutron star. These hypotheses should be satisfied for X–rays bursters, which are commonly thought to contain old neutron stars, and were the starting point of previous investigations on this subject (see e.g. Kato 1983; Quinn & Paczyński 1985; Paczyński & Anderson 1986; Paczyński & Prószyński 1986; Paczyński 1990). All these studies dealt with radiative wind acceleration in an optically thick plasma and assumed that the only contribution to opacity comes from electron scattering. Moreover the rate of energy release by thermonuclear reactions at the base of the envelope, $\dot{E}$, was not computed self–consistently but, together with the mass loss rate $\dot{M}$, is a free parameter of the model. In this investigation we present neutron star wind models which overcome these limitations and account properly for both energy production by nuclear burnings and radiative transfer outside the diffusion regime. The capability of the model to handle radiative transfer under general conditions is indeed crucial since, contrary to what is often assumed, a large value of the scattering depth is not enough, by itself, to guarantee that LTE is established. This will also allow for a more natural specification of boundary conditions, which can be placed at radial infinity where the optical depth vanishes, avoiding the need to impose “artificial” conditions at the photosphere, as in Paczyński & Prószyński (1986) and Paczyński (1990).

The set of equations governing the dynamics of the gas and the transfer of radiation in spherical, stationary flows in a Schwarzschild gravitational field are discussed in the paper by Nobili, Turolla & Zampieri (1991, hereafter NTZ) to
which we refer for all details. Although explicitly written for spherical inflows, their equations hold the same for winds, just reversing the sign of the flow velocity and changing $\dot{M}$ into $-\dot{M}$. They are

$$w_1' + vw_0' + vw_2 \left( \frac{u'}{u} - 1 \right) + 2w_1 \left( 1 + \frac{y'}{y} \right) + \frac{4}{3}vw_0 \left( \frac{u'}{u} + 2 \right) =$$

$$= \frac{rR_g}{y} \left( s_0 + \frac{\rho_0 \epsilon_{nucl}}{c} \right)$$

(1)

$$w_2' + vw_1' + \frac{1}{3} w_0' + w_2 \left( 3 + \frac{y'}{y} \right) + 2vw_1 \left( \frac{u'}{u} + 1 \right) + \frac{4}{3} \frac{y'}{y} w_0 =$$

$$= \frac{rR_g s_1}{y}$$

(2)

$$(v^2 - v_s^2) \frac{u'}{u} - 2v_s^2 + \frac{1}{2y^2 r} - \frac{rR_g}{u(P + \varrho)} [((\Gamma - 1)s_0 - vs_1] = 0$$

(3)

$$\frac{T'}{T} - (\Gamma - 1) \frac{\varrho_0'}{\varrho_0} + \frac{rR_g s_0}{Bu(P + \varrho)} = 0$$

(4)

$$\frac{u'}{u} + \frac{\varrho_0'}{\varrho_0} + 2 = 0.$$  

(5)

Here and in the following $r = R/R_g$ is the adimensional radial coordinate in units of the gravitational radius, a prime denotes derivation with respect to $\ln r$, $v$ is the fluid velocity measured by a stationary observer in units of $c$, $y = \sqrt{(1 - 1/r)/(1 - v^2)}$ and $u \equiv yv$; all the other symbols have their usual meaning. The variable Eddington factor is assumed to be a given function of the scattering depth $\tau$

$$f_E = \frac{w_2}{w_0} = \frac{2}{3} \frac{1}{1 + \tau^n};$$

$n = 2$ was used in the numerical calculations. This approximation for the closure should guarantee that the radiation field is correctly computed up to $\sim 15\%$ (see Turolla & Nobili 1988). We have assumed the plasma to be a perfect, fully
ionized gas: $X$, $Y$ and $Z$ denote the mass abundances of hydrogen, helium and metals. The chemical composition is taken to be constant through the envelope (see the discussion in section III). The present set of equations differ slightly from those of NTZ inasmuch as we included also the rate of energy generation by nuclear reactions, $\epsilon_{nuc}$. In order to keep our treatment simple we just take into account $3-\alpha$ He–burning, although actual nuclear reaction networks in X–ray bursting neutron stars are much more complicated (see e.g. Taam 1985 for a review). The expression for the energy generation rate is then (see e.g. Clayton 1968)

$$\epsilon_{nuc} = 3.9 \times 10^{11} f (1 - X)^3 \rho_0^2 T_8^{-3} \exp \left( -\frac{42.94}{T_8} \right) \text{erg g}^{-1} \text{s}^{-1},$$

$$f \simeq \exp \left[ 2.76 \times 10^{-3} \rho_0^{1/2} T_8^{-3/2} \right].$$

The source moments $s_0$ and $s_1$ account for both free–free emission–absorption and Compton scattering. They can be derived from the expressions given in NTZ, replacing the complete hydrogen cooling function with bremsstrahlung emissivity

$$s_0 = \kappa_{es} \rho_0 w_0 \left[ \frac{\kappa^P_{ff}}{\kappa_{es}} \left( \frac{a T^4}{w_0} - 1 \right) + 4 \frac{kT}{m_e c^2} \left( 1 - \frac{T_\gamma}{T} \right) \right],$$

$$s_1 = -\kappa^R \rho_0 w_1,$$

where $\kappa^P_{ff}$ and $\kappa^R$ are the free–free Planck and Rosseland mean opacities. The radial dependence of the radiation temperature $T_\gamma$, needed to evaluate the Compton heating–cooling term in $s_0$, is obtained from the equation

$$\frac{T'_\gamma}{T_\gamma} = Y \left( 1 - \frac{T_\gamma}{T} \right)$$

(6)

where $Y = (4kT/m_e c^2) \max(\tau, \tau^2)$ is the Compton parameter (Park & Ostriker 1989, NTZ).
Equations (1)–(6) can be solved to get the run of all physical quantities characterizing the wind once boundary conditions and a set of constituent relations are given. Numerical problems, however, are going to arise in the dense wind region close to the star surface where the scattering depth, \( \tau \), becomes very large. Equation (6) shows, in fact, that \( T_\gamma \) must be extremely close to \( T \) for \( \tau \gg 1 \) and, if \( \tau \) is so large to produce an effective depth, \( \tau_{\text{eff}} = \left[ 3\tau_{ff}(\tau + \tau_{ff}) \right]^{1/2} \), larger than unity, also the radiation energy density will nearly equal \( aT^4 \). Under such conditions the differences \( 1 - T_\gamma/T \) and \( (1 - aT^4/w_0) \) can become dangerously close to machine precision, producing unbound errors. To avoid this possibility we decided to split the integration range into two parts: an outer region \( r_{fit} < r < r_\infty \) (region I), where equations (1)–(6) can be safely used, and an inner region \( r_* < r < r_{fit} \) (region II), where we adopted the diffusion limit. The value of the fitting radius \( r_{fit} \), where the two branches of the solution must be matched, is arbitrary but it must be chosen in such a way that equations (1)–(6) can be integrated without troubles and, at the same time, \( \tau_{\text{eff}}(r_{fit}) \gg 1 \) for diffusion and LTE to hold. The diffusion limit of equations (1)–(4) is readily obtained by imposing \( w_0 = aT^4 \) (see Flammang 1982, Turolla et al.) and yields

\[
\frac{(yT)'}{(yT)} + \frac{\ell}{8\alpha v_c^2 y r} = 0 \tag{7}
\]

\[
(v^2 - v_c^2) \frac{u'}{u} + v_c^2 \left( \frac{T'}{T} - 2 \right) + \frac{1}{2y^2r} \left( 1 - \frac{\kappa R}{\kappa_{es}} \frac{y\ell}{h} \right) = 0 \tag{8}
\]

\[
\ell' + \frac{y'}{y} \ell + \frac{m v_c^2}{y} \left[ \left( 12\alpha + \frac{3}{2} \right) \frac{T'}{T} - (1 + 4\alpha) \frac{\varrho_0'}{\varrho_0} \right] - \frac{2\kappa_{es} R^2 g_0 y v r^2}{y c^2} \epsilon_{nuc} = 0 \tag{9}
\]

\[
2\kappa_{es} R g_0 y v r^2 = \dot{m} \tag{10}
\]

Here \( \ell \) is the luminosity in units of \( L_E = 4\pi G M c/\kappa_{es} \), \( \dot{m} = \dot{M} c^2/L_E \) is the mass
loss rate in units of the critical rate, $\alpha = P_{rad}/P_{gas}$ and $v_c$ is the isothermal sound speed. Some terms which become important only for $v \sim 1$ were dropped in equations (7)–(9).

Numerical models show that temperature never exceeds $\sim 10^7$ K at the fitting radius so that no relativistic corrections both to the equations of state and to the opacities are needed in region I. On the other hand, since temperature and density at the base of the envelope must be high enough to make nuclear burning effective, $T(R_*) \sim 4 \times 10^9$ K, $\varrho_0(R_*) \sim 10^6$ g/cm$^3$ in the case of 3–$\alpha$ reactions, electrons will be partially degenerate and relativistic in the deep layers. The form of the equations of state used in the numerical code was obtained by least–square fitting to Chandrasekhar’s (1939) tables the two functions

$$q_1(\ln \varrho_0, \ln T) = \ln \frac{P}{\varrho_0 T}$$

$$q_2(\ln \varrho_0, \ln T) = \ln \frac{2U}{3P}$$

where $U = \varrho - \varrho_0 c^2$ and $q_1$, $q_2$ are double Chebyshev series. Truncating the sums to sixth degree gives an accuracy $\lesssim 5\%$. The assumption of neglecting electron conduction appears to be justified in our case. The “effective” opacity $\kappa$, including both conductive $\kappa_c$ and radiative contributions, has the form (see e.g. Clayton) $1/\kappa = 1/\kappa_c + 1/\kappa_{es}$. In the burning region, temperature and density are such that $\kappa_c > \kappa_{es}$ and this inequality becomes even stronger moving to larger radii. Klein–Nishina corrections to the scattering opacity were included, using Paczyński (1983) interpolating formula. We checked the validity of this expression by computing the Rosseland mean of the frequency–dependent Compton transport opacity of Shestakov, Kershaw & Prasad (1988). Agreement is within 10 % for $T \lesssim 4 \times 10^9$ K.

The actual form of the boundary conditions for the two sets of equations will be discussed in detail in the next section. Here we want to analyze
how boundary conditions should be placed for equations (1)–(6) if they were integrated in the whole range $r_* < r < r_\infty$, since this will make apparent the number of free parameters of our model. At radial infinity radiation must stream freely, hence we have to require that

$$w_0 = w_1 \quad \text{at } r = r_\infty.$$  

On the neutron star surface we assume that LTE is established and that the radiative flux goes to zero since all luminosity is generated by nuclear reactions above $r_*$. This amounts to ask that

$$w_0 = aT^4$$

$$T_\gamma = T \quad \text{at } r = r_*$$

$$w_1 = 0.$$  

A further condition has to be imposed at the sonic point where $v = v_s$ to ensure the regularity of the solution, thus leaving just one degree of freedom which can be the value of either velocity, density or temperature at a given radius, or, alternatively, the mass loss rate $\dot{M}$. 
III. RESULTS AND DISCUSSION

In this section we present the results of the numerical integration of the equations of radiative hydrodynamics for the wind case. All our models refer to a neutron star of mass $M_\ast = 1.5M_\odot$ and radius $R_\ast = 3R_g = 13.5$ km; the chemical composition of the outflowing material can be varied to explore its effects on the wind properties. The numerical code makes use of a relaxation technique (Nobili & Turolla 1988) and the integration range extends from $r = 3$ to $r = 10^5$; the upper limit is fixed essentially by the requirement that the scattering depth become low enough to make the radial streaming approximation reasonable. As we discussed in the previous section, the integration domain is split into two parts and equations (1)–(6) are used for $r > r_{fit}$ (region I) while the diffusion limit, equations (7)–(10), is assumed for $r < r_{fit}$ (region II); the complete solution is then obtained by fitting the two branches. The procedure is the following. First $r_{fit}$ is fixed and equations (1)–(6) are integrated for $r > r_{fit}$ with the conditions $T = T_{fit}$, $T_\gamma = T$, $w_0 = aT^4$, $v = \epsilon v_s$ at $r_{fit}$ and $w_0 = w_1$ at radial infinity; here $\epsilon \leq 1$ is a parameter and boundary conditions must be supplemented with the regularity condition at the sonic point where $v = v_s$. Once the solution in region I is known, equations (7)–(10) are solved in region II, subject to the conditions that $\dot{m}$, the velocity and the velocity gradient at $r_{fit}$ are those provided by the solution we have just computed and that $L$ vanishes at $r = r_\ast$. In this way all the variables and their derivatives are continuous at $r_{fit}$, with the exception of temperature. Finally, the continuity of $T$ is achieved by fine–tuning the parameter $\epsilon$ so that the final model is characterized only by the value of $T_{fit}$ or, equivalently, by the mass–loss rate $\dot{m}$. In practice we found numerically more convenient to keep $T_{fit}$ fixed, $T_{fit} = 2.5 \times 10^7$ K, and to vary $r_{fit}$; all solutions have $25 < r_{fit} < 150$. Since in our numerical code the exact form of the critical point condition is not
so crucial (see the discussion in NTZ), we just asked that \((yL)' = 0\) at \(v = v_s\).

A sequence of models was obtained, covering nearly two decades in mass loss rate from \(\dot{m} \sim 1\) up to \(\dot{m} \sim 100\). Chemical composition was assumed to be nearly solar, with mass abundances \(X = 0.6\), \(Y = 0.35\), and \(Z = 0.05\). The radial dependence of some physically significant quantities is presented in figures 1–5 for three characteristic values of \(\dot{m}\), namely: a) \(\dot{m} = 2.8\), b) \(\dot{m} = 50.2\), and c) \(\dot{m} = 102.3\). In figures 1a, b, c the run of bulk velocity, sound speed and density is presented; the crossing of the two velocity curves marks the sonic point. Terminal wind velocities are never relativistic and do not exceed \(\sim 3 \times 10^{-3} c \sim 1000\) km/s. Our values for \(v_\infty\) are systematically lower than those obtained by Paczyński & Prószyński, as it should be expected, because of the stronger coupling between matter and radiation when diffusion is assumed, and also of those presented by Joss & Melia.

Figures 2a–c show the radial distribution of the gas and radiation temperatures, \(T\) and \(T_\gamma\). The filled dots mark the points where the matching between the inner (diffusive) and outer regions was achieved, in order to illustrate the smoothness of the numerical fitting. At the point where the curves of \(T\) and \(T_\gamma\) start to diverge, LTE between radiation and matter breaks down; at larger \(r\), \(T_\gamma\) stays constant since matter and radiation are decoupled. The behavior of \(T\) reminds qualitatively that one of steady atmospheres of X–ray bursters in the contraction phase (Lapidus, Sunyaev, & Titarchuk 1986; London, Taam, & Howard 1986) and is similar to that found by Joss & Melia, although their treatment of Comptonization is different from the present one. The decrease of \(T\) with \(r\) after matter has decoupled from radiation is halted by the Compton heating of the cooler electrons by the photons originating in the inner, much hotter layers. Such an effect is much more pronounced for low \(\dot{m}\) models because they exhibit a sufficiently large translucent \((\tau_{eff} < 1\)
and $\tau > 1$) region, contrary to high $\dot{m}$ ones. At even larger radii adiabatic cooling, due to $PdV$ expansion, becomes more efficient than Compton heating, and $T$ decreases again. We stress that, contrary to previous investigations, the temperature at the base of the flow, $T_b$, is now self-consistently determined, since we have taken into account nuclear burnings in the expanding envelope. $3-\alpha$ reactions actually keep $T_b \sim 3 \times 10^9$ K for all values of the mass loss rate, because of the strong temperature dependence of the reaction rate.

The radial run of luminosity, as measured by the comoving observer in Eddington units, is given in figure 3 for all three models. Luminosity rises from zero at $R = R_*$, reaches its maximum extremely close to the stellar surface, and then decreases at larger $r$ as radiative flux is converted into bulk kinetic energy. The larger $\dot{m}$ is, the higher the peak luminosity is, to cope with the larger flow inertia. Nuclear reactions can always produce the luminosity required to propel the wind because larger envelope masses result in higher densities which, in turn, enhance the nuclear reaction rate. Luminosity at infinity is always extremely close to the Eddington value, as indeed should be, since all the exceeding power is transferred to the matter flow.

Figures 4 and 5 illustrate the radial runs of optical depths and radiation moments just for model b), the overall behaviour being similar for the other models; for the sake of clarity only the outer region is presented. In figure 4, together with the scattering depth, $\tau$ and the effective depth, $\tau_{eff}$, the product $\tau v$ is also shown since this parameter gives a measure of bulk motion Comptonization (Payne & Blandford 1981; Nobili, Turolla & Zampieri 1993). Bulk motion Comptonization is, however, expected to be efficient only in regions where $\tau_{eff} \lesssim 1$, $\tau \gtrsim 1$ and $\tau v \gtrsim 0.1$; as can be seen from the graph, $\tau v \lesssim 0.1$ above the thermalization radius and dynamical effects are never dominant. The transition between the diffusive and the streaming regime is clearly visible in
figure 5: at large optical depth $w_0 \sim \tau w_1$ while, above the last scattering radius, $w_0 \simeq w_1$.

Although, as we already pointed out, each solution is characterized by the value of $\dot{M}$, it is much more physically meaningful to label models with the total mass contained in the “static” part of the envelope. We define $M_{\text{env}}$ as the mass contained in between the base of the nuclear burning shell (which is assumed to coincide with the neutron star surface) and the sonic radius, $R_s$,

$$M_{\text{env}} = 4\pi \int_{R_s}^{R_*} dR R^2 \rho_0 .$$

Calling this portion of the atmosphere “static” seems reasonable, since below the sonic radius dynamics does not produce major changes in the structure with respect to the hydrostatic case. The envelope mass as a function of $\dot{m}$ is shown in figure 6 for all computed models. The importance of $M_{\text{env}}$ is that, contrary to $\dot{M}$, it can be used to characterize both the pre–burst phase and the time evolution during the photospheric expansion/contraction phase. In fact, the value of the envelope mass when the expansion begins is proportional, assuming a constant mass transfer rate from the companion star, to the time between two successive bursts. Furthermore, the temporal evolution of a single strong burst with expansion, at least close to the luminosity peak, can be thought as a sequence of quasi–stationary models with gradually decreasing $M_{\text{env}}$. This decrease is mostly due to the fact that the nuclear burning shell moves outwards leaving the products behind (and out of the wind region); there is, in addition, a small decrease of $M_{\text{env}}$ in time because some mass is actually lost through the wind itself. The fact that radiative luminosity pushes the material only above the He–burning shell together with the thinness of the shell makes the constant composition assumption reasonable, although a composition gradient will be present across the burning region. We do not expect this to modify our
results significantly since the variation in the chemical composition will affect, at most, the inner two or three radial zones.

The fact that, in order to start the wind, a sufficient amount of material should be accumulated onto the neutron star surface is unanimously accepted. The expansion phase, however, according to observational data, lasts just \( \lesssim 10 \) seconds, so one has to face the problem of quenching the wind in quite a short time. It is usually assumed that the wind ceases when nearly all the nuclear fuel is exhausted and the comoving luminosity at the base of the envelope drops below a critical value, which can be derived from the energy conservation. The characteristic time scale of the process is then \( t_{\text{nuc}} = \epsilon Y M_{\text{env}} c^2 / \dot{E} \), where \( \epsilon = 6.1 \times 10^{-4} \) is the efficiency of 3–α reactions (see e.g. Clayton 1968) and \( \dot{E} \sim (1 + \dot{m}) L_E \) is the total (radiative plus advected plus kinetic) luminosity. Clearly, no lower limit for the mass loss rate follows from energetic considerations, and winds with arbitrarily low \( \dot{m} \) are possible. On the contrary, we have found that a lower limit for the mass loss rate definitely exists, and the presence of such a bound is due to Compton heating. This effect is analogous to the so called “preheating” limit in spherical accretion (see e.g. NTZ). As we already discussed, Compton heating is stronger for low \( \dot{m} \) and tends to isothermize the outflow at \( T \sim 10^7 \) K, inhibiting the decrease of sound speed with radius: as a consequence there may be problems for the flow to become supersonic, since the sonic point will move to larger and larger radii. To see this in more detail let us consider the momentum equation [eq. (3)]; since the velocity gradient must be positive in the subsonic region, it follows that

\[
2v_s^2 - \frac{1}{2r} + \frac{rR_g}{u(P + \varrho)} [(\Gamma - 1)s_0 - vs_1] < 0.
\]  

Condition (11) can be written in a more transparent form using the relation \( w_1 = c^2 \ell/(2\kappa_{es} r^2 R_g) \) and taking into account that above \( r_{\text{fit}} \), where
Comptonization is effective, true emission–absorption can be neglected, $P + \varrho \sim \varrho_0 c^2$ and $y \sim 1$

$$2\nu_s^2 + \frac{1}{2r}(\ell - 1) + \frac{2}{3} \frac{r_0 R_g w_0}{\dot{m}} \frac{4kT}{w_1 m_e c^2} \left(1 - \frac{T_\gamma}{T}\right) < 0. \quad (12)$$

The physical meaning of the various terms in this expression is straightforward: the first term accounts for the gas pressure force, the second one represents the effective radiative force while the last one is the first order correction, due to non–conservative scatterings, to the Thomson radiative force exerted by the outgoing radiation on electrons. The Compton correction can be either positive or negative, according to the value of $T_\gamma/T$. For $T_\gamma < T$ it acts like an extra thrust, and gives rise to the so called Compton rocket (O’Dell 1981; Cheng & O’Dell 1981). Under our conditions $T_\gamma > T$, and thus the Compton correction results in a braking force, since in the scattering of more energetic photons on relatively less energetic electrons, a part of energy transferred to electrons goes to increase the gas thermal energy. Although this effect tends to lower the radiative force, in a way similar to the decrease of the scattering cross–section in the Klein–Nishina regime, it is a completely different phenomenon. The region on the $(M_{env}, \dot{m})$ plane for the existence of stationary, supersonic solutions permitted by condition (12) lies below the full line in figure 7, which represents the values of $\dot{m}_{\text{min}}$ obtained equating expression (12) to zero. This expression was evaluated at the sonic point for the different numerical models we have computed. The dashed curve of figure 7 shows the actual values of $\dot{m}$ for the same solutions. Although numerical difficulties prevented us to reach the lower possible value for the mass loss rate where the two curves cross, there is no doubt that a crossing occurs at $\dot{m}_{\text{min}} \sim 1 / 3$. No stationary, supersonic winds can exist with $\dot{m} < \dot{m}_{\text{min}}$. In figure 8 the location of the sonic radius is plotted vs. $\dot{m}$: as it should be expected $r_s$ steeply increases for low enough values of
\( \dot{m} \). We note that, in the absence of Comptonization, the sonic radius would monotonically decrease for decreasing \( \dot{m} \), so the minimum in figure 8 marks the range of \( \dot{m} \) at which Compton heating starts to dominate.

The request that the outflow can be described by our stationary model places also an upper limit on both \( \dot{m} \) and the total envelope mass, \( M_0 \), at the time the wind starts. Keeping in mind that \( M_{\text{env}} \) is the mass between the base of the burning shell and the sonic radius, the variation per unit time of the total envelope mass (which includes the nuclear processed material laying below the burning shell) is just \( dM_{\text{tot}}/dt = -\dot{M} \). We assume a simple relation between \( M_{\text{env}} \) and \( M_{\text{tot}} \) of the form \( M_{\text{env}} = M_{\text{tot}}(1 - t/t_{\text{nuc}}) \), which just states that on a timescale \( t_{\text{nuc}} \) all the helium will be burned out; furthermore we approximate the \( \log \dot{m} - \log M_{\text{env}} \) dependence with a linear law, \( \dot{m} = A M_{\text{env}}^{\alpha} \). Expressing the initial differential equation in terms of \( M_{\text{tot}} \) only, we get the solution

\[
M_{\text{tot}} = M_0 \left\{ 1 - BM_0^\alpha \left[ 1 - \left( 1 - \frac{t}{t_{\text{nuc}}} \right)^{\alpha+1} \right] \right\}^{1/(1-\alpha)}.
\]

For a model to be stationary the mass lost in the wind must be much smaller than \( M_{\text{tot}} \), which implies that \( BM_0^\alpha \ll 1 \). In our case, the limiting initial mass turns out to be \( M_0 \sim 1.8 \times 10^{26} \) g, corresponding to a maximum mass loss rate \( \dot{m} \sim 630 \), or, \( \dot{M} \sim 1.2 \times 10^{20} \) g/s. All models we computed are below this critical value of \( \dot{m} \). Moreover, even if the \( M_0 \) exceeds the above mentioned limit, after an initial, high mass loss, unstationary phase during which our model cannot be applied, the wind will enter the parameter range where the outflow can be reliably treated as a stationary one.

The summary of our series of models is given in table 1, where some global quantities are presented, namely \( M_{\text{env}} \), \( v_\infty \), photospheric and last scattering radii, \( R_{\text{ph}} \) and \( R_{\text{es}} \) where \( \tau_{\text{eff}} = 1 \) and \( \tau = 1 \) respectively, matter temperature at \( R_{\text{ph}} \), \( T_{\text{ph}} \), the characteristic timescale for expelling the whole envelope, \( t_{\text{wind}} = \).
$M_{env}/\dot{M}$, and the nuclear timescale, $t_{nuc}$. Observations show that the timescale of the expansion phase, over which the luminosity stays nearly constant around the maximum (i.e. near to the Eddington value), is about few seconds. The values of $t_{nuc}$ in Table 1 are indeed of the right order of magnitude, and it should be also taken into account that $t_{nuc}$ is an upper limit for the duration of the expansion, because not all the helium may actually be burned out. As it appears from the table, $t_{wind}$ is much longer than $t_{nuc}$, showing that the decrease of the envelope mass in time is due to nuclear burning, the mass lost in the wind being less important by far.

In order to access the relevance of chemical composition on the global properties of our solutions, a series of nearly pure helium models has been computed, $X = 0.05$, $Y = 0.90$ and $Z = 0.05$. Results are presented in table 2. In general, for the same $\dot{m}$, the envelope tends to be more compact with respect to the one with solar composition, all relevant radii are smaller and also $M_{env}$ is lower. Variations, however, are within a factor 2 and timescales are very nearly the same.

Although no frequency dependent calculations are presented here, the comparison between our results and spectral data of X–ray bursts can actually yield some useful informations. To illustrate this let us refer to the EXOSAT observations of 4U/MXB 1820-30 as presented by Haberl et al. This source is a binary with an 11 minute orbital period, consistent with a scenario in which the secondary is a low–mass, helium–rich star (Rappaport et al. 1987). Since we expect the transferred material to be nearly pure helium, our helium–rich set of models will be used. Fitting the observed bursts spectra with a planckian law gives the color temperature $T_{col}$, which can be used to derive an estimate of the envelope radius $R_{col}$, via the relation $L = 4\pi R_{col}^2\sigma T_{col}^4$. While the possible anisotropy of the radiation emitted during a burst, due to the
presence of the accretion disk, may be relevant in evaluating the bolometric luminosity (see e.g. Lapidus & Sunyaev 1985), it does not influence spectral data. The variation of $R_{col}$ with time is taken as an indication of the envelope expansion and successive contraction during the burst. It should be noted, however, that, while the previous argument is correct, $R_{col}$ is not directly related to any physically meaningful radius and no “color” radius can be extracted out of our models. The radius which does have an evident physical meaning, as far as spectral formation is concerned, is the photospheric radius $R_{ph}$ since it is here that the blackbody spectrum originates, with a temperature equal to the matter temperature $T_{ph}^m$; clearly the emergent spectrum will not be planckian because of Compton scatterings in the outer, translucent layers. The $T_{ph}^m$–$R_{ph}$ relation for our helium models is shown in figure 9; $\dot{M}$ decreases moving to higher temperatures along the curve. As it should be expected, our data deviate from the analytical law but the trend is the same and, moreover, the observed increase in time of the color temperature (see e.g. Haberl et al. figure 4) corresponds to an increase of $T_{ph}^m$ for decreasing $\dot{M}$ in our data, providing further evidence that the time evolution of the wind can be mimicked as a succession of stationary models with decreasing $M_{env}$.

Our results can also be used to construct a $R_{col}$–$T_{col}$ plot, in the same way as with observational data, and this enables us to derive an estimate of the spectral hardening, without any need of frequency dependent calculations. In fact, by introducing a hardening factor $\gamma = T_{col}/T_{ph}^m$, we have

$$4\pi R_{col}^2 \sigma (\gamma T_{ph}^m)^4 = L,$$

(13)

where $L$ can be safely assumed to be the Eddington luminosity. The value of $\gamma$ can be derived asking that

$$\gamma T_{ph}^m |_{\dot{m}_{min}} = T_{col} |_{max}$$
where $T_{\text{col}}|_{\text{max}}$ is the maximum observed color temperature, which is $\approx 3$ keV for the bursts in 4U/MXB 1820-30 analyzed by Haberl et al. We emphasize that the choice of the last point as the fiducial one is based on the existence of a minimum value of $\dot{M}$ which is assumed to be reached at the end of the expanding envelope phase. The hardening factor obtained in such a way turns out to be $\gamma \sim 1.53$ and, since $\gamma > 1$, we expect a genuine hardening of the spectrum as radiation propagates through the extended spherical shell $R_{ph} < R < R_{es}$. This result does not contradict the previous finding of Lapidus who obtained a softening (rather than hardening) factor $\sim 0.25 \div 0.7$, solving the frequency dependent transfer problem on a fixed hydrodynamical background with a solar chemical composition. This is because, in his investigation, the softening factor was defined as $\gamma_{\text{soft}} = T_{\text{col}}/T_{\text{eff}}$, $T_{\text{eff}}$ being the effective temperature at the neutron star surface, $T_{\text{eff}} \sim 2$ keV $\gtrsim T_{ph}^m \sim 0.3 \div 2$ keV. The scaling between the two factors is just $T_{\text{eff}}/T_{ph}^m \sim 1 \div 7$, in agreement with $\gamma/\gamma_{\text{soft}} \sim 2 \div 6$. The ratio $T_{\text{col}}/T_{\text{eff}}$ is, typically, $\sim 1.5$ in a static atmosphere (see e.g. London et al.) and then abruptly drops to $\lesssim 0.7$ at the onset of the wind phase, but in both situations the blackbody spectrum, produced at the thermalization radius, will be then hardened by Comptonization. In figure 10 the curve derived from equation (13), with $\gamma = 1.53$, is superimposed to the data of branch $b$ of Haberl et al. figure 4, here represented by the shaded area. We restrict our attention to branch $b$ because it can be taken as representative of the quasi–stationary wind phase which is modeled by our solutions. Within this framework, it is natural to interpret the intersection between our curve and the left border of the shaded area as the point in the parameter space where the quasi–stationary wind phase begins. The intersection point corresponds to a model with $\dot{m} \sim 90$, $M_{\text{env}} \sim 9 \times 10^{23}$ g and the latter value may be assumed as the total mass of the envelope at the onset of the burst, $M_0$. As can be seen from table 1, the final
envelope mass, corresponding to the minimum possible $\dot{m}_i$, is $\sim 2 \times 10^{21}$ g $\ll M_0$, so that nearly all the helium is burned out. The time required for this process is $\sim t_{nuc} \sim 10$ s which is close to the observed duration, $\sim 5$ s, of the quasi-stationary phase. Taking the interburst time $\Delta t = 1.1 \times 10^4$ s, we get an estimate of the neutron star accretion rate: $\dot{M}_{acc} \sim 10^{-6} M_\odot/yr$. The present estimate for $\dot{M}_{acc}$ turns out to be quite high in comparison with the values discussed in connection with model neutron star atmospheres with nuclear burnings (see e.g. Ayasli & Joss 1982; Fushiki & Lamb 1987). A lower value for $\dot{M}_{acc}$ can be obtained relaxing the hypothesis that $\gamma$ is same for all the wind models. Of course there is no physical reason to expect this to be true. It is reasonable, in fact, to assume that the hardening ratio increases with decreasing envelope mass since low $\dot{M}$ models have a more extended, hotter scattering region (see e.g. figure 2) where Comptonization is more effective. In this framework the value of $\gamma$ we have computed should be the maximum one and a lower limit for the initial envelope mass can be obtained assuming the initial model to have $\gamma = 1$, in which case $T_{ph}^m = T_{col} \sim 0.5$ keV. Data in table 2 show that now $M_0 \sim 5 \times 10^{23}$ g which gives $\dot{M}_{acc} \sim 5 \times 10^{-7} M_\odot/yr$, about half our previous estimate. Still lower accretion rates could be obtained if $\gamma < 1$ at the beginning of the quasi-stationary contraction phase, although a softening of the spectrum with respect to the blackbody at $T_{ph}^m$ seems implausible. We note that the actual dependence of $\gamma$ on $M_{env}$ does not affect the previous conclusions which rely only on the given initial value of the hardening factor. More severe uncertainties in the determination of $\dot{M}_{acc}$ stem from observational data. In the case of 4U 1820-30, Haberl et al. reported that, during the first 3 seconds of each burst, the blackbody fitting to the observed spectrum was rather poor. Since the contraction phase begins just after $\sim 1$ s, we expect the largest errors to affect the minimum color temperature which is the key parameter in selecting the
starting wind model. From the discussion in Haberl et al. about the fitting of early spectra, we surmise that a value of $T_{\text{col}}$ higher than 0.5 keV could be more appropriate. These authors state that, at the onset of the contraction phase, spectra show a broad maximum and were well fitted by a power law plus a bremsstrahlung. Such spectra are typically produced in rather dense, expanded envelopes with little or no Comptonization and a substantial free–free emission outside the thermalization radius, like what is expected in our more massive wind models. If $T_{\text{col}}$ has to serve as a measure of the photospheric temperature, as in our case, a fit of the exponential tail would be more significant, since it is this part of the spectrum which originates at the thermalization radius, and will give a larger $T_{\text{col}}$. The resulting accretion rate can be much reduced because $M_{\text{env}}$ decreases rather steeply with $1/T_{\text{ph}}^m = \gamma/T_{\text{col}}$. For instance, assuming $T_{\text{col}}^{\text{min}} \sim 1$ keV and $\gamma = 1$, we get $\dot{M}_{\text{acc}} \sim 10^{-7} M_\odot/\text{yr}$, which is still rather high. On the other hand, an application of the same technique to all other burst sources with photospheric expansion (Lapidus, Nobili & Turolla 1994) produced much lower values of the accretion rate, $\dot{M}_{\text{acc}} \sim 10^{-8} \div 10^{-9} M_\odot/\text{yr}$, supporting the current idea that 4U 1820-30 is a peculiar object.

Although for all present estimates $\dot{M}_{\text{acc}}$ is highly supercritical, no significant release of gravitational luminosity, $L_{\text{acc}} \sim G M_\ast \dot{M}_{\text{acc}}/c^2 R_\ast$, occurs in the interburst phase because the gas has no time to cool. The inner part of the flow, in fact, is optically thick and the appropriate cooling time is the adiabatic time (see e.g. Bildsten 1993)

$$t_{\text{cool}}^{\text{ad}} \sim \frac{c_p \kappa_{\text{es}}(\rho_0 r)^2}{caT^3} \sim 10^{11} \left( \frac{\rho_0}{10^6 \text{ g}} \right)^2 \left( \frac{T}{10^8 \text{ K}} \right)^{-3} \text{ s},$$

where $c_p$ is the specific heat at constant pressure. Since $\rho_0 T^{-3/2}$ is nearly constant, the cooling time can be computed using the values of $\rho_0$ and $T$ which correspond to the helium ignition at the base of the accretion flow. For $\dot{m} \sim 90$,
it is $\rho_0 \sim 10^6 \text{ g cm}^{-3}$, $T \sim 6 \times 10^9 \text{ K}$, and we get

$$t_{cool}^{ad} \sim 5 \times 10^5 \text{ s} \gg t_{acc} \sim 10^4 \text{ s}.$$ 

It follows, then, that the heat produced by the conversion of gravitational potential energy can not be radiated away in a time $t_{acc}$ and must go to increase the gas internal energy. This in turn implies that the accretion process can not be regarded as stationary. Only a small fraction of $L_{acc}$ is expected to escape to infinity while the progressive heating of the inner gas layers produces, at the end, the helium flash. It is either this fraction of $L_{acc}$ or the stationary hydrogen burning on the surface of the neutron star (see Ayasli & Joss, Taam et al.) which are responsible for the observed persistent X–ray luminosity, $\sim 0.1L_E$. The fact that the persistent luminosity has been observed to be higher ($\sim L_E$) when the source was burst inactive (see e.g. Stella et al. 1984) could be explained in terms of a lower accretion rate, for which the process is stationary. Under such conditions the flow has time to radiate away all the gravitational energy and a larger luminosity can be produced with a smaller $M_{acc}$.

A simple argument can be used to estimate the initial temperature at the base of the envelope needed to ignite the helium after a given amount of material is accreted. If we assume that the inner accretion layers are in hydrostatic equilibrium and the gas is heated adiabatically, it can be shown that $M_{env} \propto T^{5/2}$. At the beginning of the accretion process the envelope mass is $\sim 2 \times 10^{21}$ g, and $T$ should be $\sim 10^9$ K, in order to reach $\sim 6 \times 10^9$ K when $M_{env} \sim 9 \times 10^{23}$ g. This implies that the deeper layers should cool from $\sim 2 \times 10^9$ K, which is the value of $T_b$ when the expanded phase ends, to $10^9$ K in the burst decay time.

We point out that the comparison of our solutions with the data of 4U/MXB1820-30 was primarily intended as a test on the viability of our model.
and no attempt was made here to really fit the observational data by varying the free parameters of the model, i.e. the neutron star mass, radius and the chemical composition of the outflowing gas.

IV. CONCLUSIONS

We have presented a new model for stationary winds from neutron stars which accounts properly for the relevant radiative processes, and handles correctly the radiative transfer in all regimes, from diffusion to radial streaming. Unlike previous investigations on this subject, the energy input rate is not treated as a free parameter, but is consistently derived from thermonuclear helium burning at the base of the envelope. In accordance with generally accepted scenarios, the energy released in excess of the Eddington luminosity is converted into the kinetic energy of the outflowing envelope, so that the radiative flux seen by a distant observer is always very close to the Eddington value. At the present stage only the frequency–integrated problem was solved. Frequency–dependent calculations are in progress and will be published later. We have found that each model is characterized by only one free parameter: either the mass loss rate $\dot{M}$, or the total envelope mass $M_{\text{env}}$. It is shown that, due to the effects of Comptonization, there exists a lower limit for $\dot{M}$, i.e. stationary winds can exist only with $\dot{M}$ larger than $\dot{M}_{\text{min}} \approx \dot{M}_{E}$. We discussed how the sequence of our models may be used to mimic the temporal evolution of a strong X–ray burst during the expansion/contraction phase. Matching of our models with observational data of 4U/MXB 1820-30 results in a spectral hardening factor $\gamma \sim 1.5$, in accordance with the theoretical prediction that a genuine hardening of the spectrum occurs as radiation propagates from the photosphere outwards. We were able, also, to get an estimate of the accretion rate from the companion star between two successive strong type I bursts in this source,
\[ \dot{M}_{\text{acc}} \sim 10^{-7} \div 10^{-6} \, M_\odot/\text{yr}. \]

Further work should be aimed to compute of a whole grid of wind models, varying \( M_* \), \( R_* \), chemical composition, and to include a more complete treatment of nuclear reactions. In any case, in order to follow the burst evolution outside the quasi–stationary phase a full, time dependent approach is needed.

**ACKNOWLEDGMENTS**

We thanks A. Fabian, M. Rees, P. Podsiadlowski for useful discussions and an anonymous referee for some helpful comments. One of us (I. L.) gratefully acknowledges financial support from Consiglio Nazionale delle Ricerche (Gruppo Nazionale di Astronomia) and Royal Astronomical Society. He is also indebted to the Department of Physics, University of Padova, for kind hospitality during his stay.
REFERENCES

Ayasli, S., & Joss, P.C., 1982, ApJ, 256, 637

Bildsten, L. 1993, ApJ, to appear

Cassinelli, J.P. 1979, ARA&A, 17, 275

Chandrasekhar, S. 1939, Stellar Structure, (New York: Dover)

Cheng, A.Y.S., & O’Dell, S.L. 1981, ApJ, 251, L49

Clayton, D.D. 1968, Principles of Stellar Evolution and Nucleosynthesis, (New York: Mc Graw–Hill)

Duncan, R.C., Shapiro, S.L., & Wasserman, I. 1986, ApJ, 309, 141

Ebisuzaki, T., Hanawa, T., & Sugimoto, D. 1983, PASJ, 35, 17

Flammang, R.A. 1982, MNRAS, 199, 833

Fushiki, I., & Lamb, D. Q. 1987, ApJ, 323, L55

Goodman, J. 1986, ApJ, 308, L47

Haberl, F., Stella, L., White, N.E., Priedhorsky, W.C., & Gottwald, M. 1987, ApJ, 314, 266

Joss, P.C., & Melia, F. 1987, ApJ, 312, 700

Kato, M. 1983, PASJ, 35, 33

Lapidus, I.I., & Sunyaev, R.A. 1985, MNRAS, 217, 291

Lapidus, I.I., Sunyaev, R.A., & Titarchuk, L.G. 1986, Pis’ma Astron. Zh. 12, 918 (Sov. Astron. Lett., 12, 383 (1987))

Lapidus, I.I. 1991, ApJ, 377, L93

Lapidus, I., Nobili, L., & Turolla, R. 1994, ApJ submitted

Lewin, W.H.G., Vacca, W.D., & Basinska, E. 1984, ApJ, 277, L57

Lewin, W.H.G., Van Paradijs, J., & Taam, R.E. 1993, Space Sci. Rev., 62, 223

London, R.A., Taam, R.E., & Howard, W.E. 1986, ApJ, 306, 170

Meier, D.L. 1982a, ApJ, 256, 681

Meier, D.L. 1982b, ApJ, 256, 693
Meier, D.L. 1982c, ApJ, 256, 706
Nobili, L., & Turolla, R. 1988, ApJ, 333, 248
Nobili, L., Turolla, R., & Zampieri, L. 1991, ApJ, 383, 250
Nobili, L., Turolla, R., & Zampieri, L. 1993, ApJ, 404, 686
O’Dell, S.L. 1981, ApJ, 243, L147
Paczyński, B. 1983, ApJ, 267, 315
Paczyński, B. 1986, ApJ, 308, L43
Paczyński, B., & Anderson, N. 1986, ApJ, 302, 1
Paczyński, B., & Prószyński, M. 1986, ApJ, 302, 519
Paczyński, B. 1990, ApJ, 363, 218
Park, M.–G., & Ostriker, J.P. 1989, ApJ, 347, 679
Payne, D.G, & Blandford, R.D. 1981, MNRAS, 196, 781
Quinn, T., & Paczyński, B. 1985, ApJ, 289, 634
Rappaport, S., Nelson, L., Joss, P., & Ma, C.–P. 1987, ApJ, 322, 842
Salpeter, E.E., & Shapiro, S.L. 1981, ApJ, 251. 311
Shestakov, A.I., Kershaw, D.S. & Prasad, M.K. 1988, J. Quant. Spectrosc. Radiat. Transfer, 40, 577
Stella, L., Kahn, S.M., & Grindlay, J.E. 1984, ApJ, 282, 713
Taam, R.E. 1985, Ann. Rev. Nucl. Particle Sci., 35, 1
Taam, R.E., Woosley, S.E., Weaver, T.A., & Lamb, D.Q. 1993, ApJ, 413, 324
Tawara, Y., et al. 1984, ApJ, 276, L41
Tawara, Y., Hayakawa, S., & Kii, T., 1984, PASJ, 36, 845
Thorne, K.S. 1981, MNRAS, 194, 439
Titarchuk, L. 1993, ApJ, submitted
Turolla, R., Nobili, L., & Calvani, M. 1986, ApJ, 303, 573
Turolla, R., & Nobili, L. 1988, MNRAS, 303, 573
Zampieri, L., Turolla, R., & Treves, A. 1993, ApJ, 419, 311
FIGURE CAPTIONS

Figure 1. Bulk velocity (continuous line), sound speed (dashed line) and density (in g/cm$^3$, dotted line) versus radius for the models with $\dot{m} = 2.8$ (a), $\dot{m} = 50.2$ (b) and $\dot{m} = 102.3$ (c); both scales are logarithmic and a filled dot marks the fitting radius.

Figure 2. Same as figure 1 for matter (continuous line) and radiation (dashed line) temperatures.

Figure 3. Luminosity, as measured by the comoving observer in Eddington units, versus radius for $\dot{m} = 102.3$ (continuous line), $\dot{m} = 50.2$ (dashed line) and $\dot{m} = 2.8$ (dotted line).

Figure 4. Electron scattering (continuous line) and effective (dashed line) optical depths for the model with $\dot{m} = 50.2$; the run of $\tau_v$ (dotted line) is also shown.

Figure 5. The run of radiation moments, $w_0$ (upper curve) and $w_1$ (lower curve), for the model with $\dot{m} = 50.2$.

Figure 6. The variation of $M_{env}$ versus $\dot{m}$.

Figure 7. The $\dot{m} - M_{env}$ relation for a sample of models (continuous line) together with the limit given by equation (12) (dashed line). Only the low $\dot{m}$ range is shown; see text for details.

Figure 8. The variation of sonic radius, $R_s$, versus $\dot{m}$.

Figure 9. The $T_{ph}^m - R_{ph}$ relation for helium solutions; the photosphere is defined by the condition $\tau_{eff} = 1$.

Figure 10. The comparison between the relation $4\pi R_{col}^2\sigma(\gamma T_{ph}^m)^4 = L$, for helium models with $\gamma = 1.53$, and the data of branch $b$ of Haberl et al. for 4U/MXB 1820-30 (shaded area).
Table 1

Characteristic Parameters for Selected Solar Composition Models

| $\dot{M}$ ($\dot{M}_E$) | $M_{env}$ ($10^{22}$ g) | $v_\infty$ ($10^{-3} c$) | $R_{ph}$ ($10^3$ km) | $R_{es}$ ($10^3$ km) | $T_{ph}^m$ (keV) | $t_{wind}$ (s) | $t_{nuc}$ (s) |
|------------------------|-------------------------|--------------------------|-----------------------|-----------------------|-----------------|----------------|-------------|
| 139.2                  | 195.8                   | 1.20                     | 19.37                 | 180.71                | 0.06            | 3157           | 10          |
| 124.7                  | 174.4                   | 1.23                     | 14.82                 | 158.69                | 0.08            | 2937           | 10          |
| 113.0                  | 156.7                   | 1.25                     | 11.74                 | 142.40                | 0.09            | 2668           | 10          |
| 102.3                  | 140.3                   | 1.28                     | 9.27                  | 127.60                | 0.11            | 2399           | 10          |
| 89.8                   | 121.3                   | 1.34                     | 6.67                  | 107.63                | 0.13            | 2156           | 10          |
| 79.1                   | 104.1                   | 1.48                     | 4.63                  | 86.77                 | 0.17            | 1920           | 10          |
| 68.7                   | 87.3                    | 1.66                     | 3.33                  | 67.61                 | 0.20            | 1688           | 9           |
| 59.3                   | 72.0                    | 1.81                     | 2.55                  | 53.75                 | 0.24            | 1460           | 9           |
| 53.9                   | 63.5                    | 1.92                     | 2.27                  | 46.53                 | 0.26            | 1345           | 9           |
| 50.2                   | 57.1                    | 2.00                     | 2.03                  | 41.97                 | 0.27            | 1242           | 8           |
| 40.2                   | 40.9                    | 2.21                     | 1.51                  | 31.12                 | 0.33            | 996            | 7           |
| 34.0                   | 30.9                    | 2.32                     | 1.22                  | 25.74                 | 0.37            | 847            | 7           |
| 30.4                   | 25.6                    | 2.42                     | 1.04                  | 22.49                 | 0.41            | 754            | 6           |
| 26.5                   | 20.0                    | 2.51                     | 0.88                  | 19.40                 | 0.45            | 672            | 5           |
| 19.9                   | 11.5                    | 2.81                     | 0.59                  | 13.80                 | 0.57            | 519            | 4           |
| 16.3                   | 7.8                     | 2.94                     | 0.46                  | 11.36                 | 0.66            | 455            | 3           |
| 14.3                   | 6.0                     | 3.04                     | 0.39                  | 10.00                 | 0.73            | 419            | 3           |
| 11.3                   | 3.9                     | 3.16                     | 0.31                  | 8.17                  | 0.85            | 385            | 2           |
| 8.8                    | 2.5                     | 3.17                     | 0.26                  | 6.94                  | 0.95            | 370            | 2           |
| 6.7                    | 1.7                     | 3.32                     | 0.20                  | 5.62                  | 1.09            | 364            | 2           |
| 5.9                    | 1.4                     | 3.32                     | 0.19                  | 5.26                  | 1.14            | 369            | 2           |
| 4.8                    | 1.1                     | 3.28                     | 0.18                  | 4.77                  | 1.18            | 396            | 1           |
| 2.8                    | 0.7                     | 3.33                     | 0.15                  | 3.69                  | 1.31            | 515            | 1           |
Table 2
Characteristic Parameters for Selected Helium Models

| \( \dot{M} \) (\( \dot{M}_E \)) | \( M_{env} \) (\( 10^{22} \) g) | \( v_{\infty} \) (\( 10^{-3} \) c) | \( R_{ph} \) (\( 10^3 \) km) | \( R_{es} \) (\( 10^3 \) km) | \( T_{ph}^m \) (keV) | \( t_{wind} \) (s) | \( t_{nuc} \) (s) |
|---|---|---|---|---|---|---|---|
| 130.3 | 137.3 | 1.51 | 3.55 | 47.92 | 0.23 | 2812 | 13 |
| 123.5 | 129.7 | 1.55 | 3.24 | 44.60 | 0.25 | 2692 | 13 |
| 116.3 | 121.5 | 1.56 | 3.00 | 42.15 | 0.26 | 2592 | 13 |
| 103.3 | 106.3 | 1.68 | 2.47 | 34.81 | 0.29 | 2361 | 13 |
| 92.9 | 94.1 | 1.77 | 2.06 | 30.05 | 0.33 | 2110 | 13 |
| 82.2 | 81.1 | 1.87 | 1.73 | 25.49 | 0.37 | 1874 | 12 |
| 70.1 | 66.6 | 1.97 | 1.43 | 21.04 | 0.41 | 1656 | 12 |
| 62.1 | 56.5 | 2.14 | 1.17 | 17.24 | 0.47 | 1424 | 11 |
| 56.0 | 48.9 | 2.16 | 1.06 | 15.68 | 0.50 | 1324 | 11 |
| 51.5 | 43.2 | 2.19 | 0.96 | 14.42 | 0.53 | 1220 | 10 |
| 40.5 | 29.5 | 2.42 | 0.70 | 10.54 | 0.64 | 961 | 9 |
| 34.8 | 22.5 | 2.50 | 0.57 | 8.99 | 0.71 | 835 | 8 |
| 30.9 | 17.9 | 2.57 | 0.49 | 7.91 | 0.78 | 750 | 7 |
| 24.5 | 11.0 | 2.88 | 0.35 | 5.78 | 0.97 | 579 | 5 |
| 20.3 | 7.1 | 2.91 | 0.28 | 4.94 | 1.09 | 513 | 4 |
| 16.2 | 4.0 | 2.90 | 0.22 | 4.19 | 1.24 | 455 | 3 |
| 12.4 | 2.0 | 2.88 | 0.17 | 3.50 | 1.46 | 405 | 2 |
| 10.4 | 1.3 | 2.82 | 0.15 | 3.20 | 1.57 | 399 | 1 |
| 7.1 | 0.6 | 2.74 | 0.12 | 2.70 | 1.81 | 395 | 1 |
| 6.6 | 0.5 | 2.73 | 0.11 | 2.61 | 1.86 | 396 | 1 |
| 5.9 | 0.4 | 2.68 | 0.11 | 2.54 | 1.89 | 412 | 1 |
| 4.8 | 0.3 | 2.60 | 0.10 | 2.42 | 1.97 | 442 | 1 |
| 4.2 | 0.2 | 2.56 | 0.10 | 2.31 | 2.03 | 467 | 1 |
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