Parametric study of nonlinear electrostatic waves in two-dimensional quantum dusty plasmas

S Ali\textsuperscript{1,2}, W M Moslem\textsuperscript{1,3,6}, I Kourakis\textsuperscript{1,4} and P K Shukla\textsuperscript{1,5}

1 Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
2 Department of Physics, Government College University, Lahore 54000, Pakistan
3 Department of Physics, Faculty of Education-Port Said, Suez Canal University, 42111, Egypt
4 Centre for Plasma Physics, Department of Physics and Astronomy, Queen’s University Belfast, Belfast BT7 1 NN, UK
5 School of Physics, University of KwaZulu-Natal, Durban 4000, South Africa

E-mail: shahid gc@yahoo.com, wmmoslem@hotmail.com, ioannis@tp4.rub.de and ps@tp4.rub.de

Abstract. The nonlinear properties of two-dimensional cylindrical quantum dust-ion-acoustic (QDIA) and quantum dust-acoustic (QDA) waves are studied in a collisionless, unmagnetized and dense (quantum) dusty plasma. For this purpose, the reductive perturbation technique is employed to the quantum hydrodynamical equations and the Poisson equation, obtaining the cylindrical Kadomtsev–Petviashvili (CKP) equations. The effects of quantum diffraction, as well as quantum statistical and geometric effects on the profiles of QDIA and QDA solitary waves are examined. It is found that the amplitudes and widths of the nonplanar QDIA and QDA waves are significantly affected by the quantum electron tunneling effect. The addition of a dust component to a quantum plasma is seen to affect the propagation characteristics of localized QDIA excitations. In the case of low-frequency QDA waves, this effect is even stronger, since the actual form of the potential solitary waves, in fact, depends on the dust charge polarity (positive/negative) itself (allowing for positive/negative potential forms, respectively). The relevance of the present investigation to metallic nanostructures is highlighted.

\textsuperscript{6} Author to whom any correspondence should be addressed.
1. Introduction

A dusty plasma is an electron–ion plasma with an additional component of charged dust particulates. The latter are found in various sizes, shapes, and charging states in different environments, e.g. in space, in astrophysical and laboratory plasmas [1, 2]. The dust particulates respond to the electric and magnetic fields according to their charge polarity which is negative or positive depending upon the charging process, viz. the electron–ion attachment on the dust surface from the background plasma, the photo-emission, the secondary electrons and thermionic emissions, etc. About 15 years ago, Rao et al [3] predicted theoretically the existence of an extremely low-phase speed (in comparison to the electron and ion thermal speed) electrostatic dust-acoustic (DA) wave in an unmagnetized dusty plasma. In the DA wave, the restoring force comes from the pressures of the inertialess electrons and ions, whereas the massive dust particulates provide inertia for maintaining the wave. Later, Shukla and Silin [4] investigated the dust-ion-acoustic (DIA) wave, in which the electrons follow the Boltzmann distribution, the ions provide inertia, and the dust particulates are assumed to form a fixed neutralizing background in the dusty plasma. Both the DA and DIA waves have been observed in research experiments [5]–[9]. A comprehensive study of the properties of nonlinear waves propagating in dispersive complex media, including the generic paradigms of the Korteweg–de Vries (KdV) and Kadomtsev–Petviashvili (KP) equations, of interest to us here, has been presented in a monograph by Belashov and Vladimirov [10]. Recently, Vladimirov et al [11] have discussed the properties of waves in dusty plasmas.

The motivation for studying the quantum mechanical plasma effects is provided by new developments in manufacturing electronics and other disciplines of physics [12]–[14]. The quantum plasmas are essentially characterized by low temperatures and high particle number densities, can be modeled by means of the Wigner–Poisson (W–P) and the Schrödinger–Poisson equations [15]. The quantum hydrodynamical (QHD) equations are basically obtained from the moments of the Wigner equations in velocity space [15, 16] and are valid for long wavelengths, i.e. \( k \lambda_{Fe} \ll 1 \). For metallic electrons, the Fermi length (\( \lambda_{Fe} \)) is of the order of an Ångstrom while the time period (\( \omega_{pe}^{-1} \)) is of the order of a femtosecond. The electron–electron collisions have been ignored [15, 17] for the reason of small coupling parameter due to large number densities. In our model, although the time period \( \omega_{pi}^{-1} (\omega_{pd}^{-1}) \) associated with the electrostatic
quantum DIA (QDIA) (quantum DA (QDA)) wave is much longer than the time period \((\omega_{pe}^{-1})\) of the Langmuir wave, even then, collisional effects are unimportant.

The QHD model has gained much importance in describing the tunneling phenomena and negative differential resistance in semiconductor physics [18]. Several collective modes have been investigated [19]–[24] in a collisionless quantum plasma, incorporating the Bohm potential and the Fermionic distribution of the charge carriers. Small amplitude waves and their instabilities are usually studied by the linear theory, but when the wave amplitude increases, nonlinearity comes into play, resulting in different types of nonlinear excitations, viz the solitons, shocks, vortices, etc. Recently, Haas et al [16] studied the linear and nonlinear properties of the quantum ion-acoustic (QIA) waves in a collisionless, unmagnetized very dense quantum plasma by using the one-dimensional (1D) QHD model.

A number of efforts have been made theoretically for investigating collective modes [25]–[30] in Cartesian coordinates in a quantum dusty plasma, because the electronic micro- and nano-devices may be contaminated by the presence of highly charged dust impurities. Extending the original work of Haas et al [16] on nonlinear QIA waves, quantum DA [29] and DIA [30] waves have recently been reported in the Cartesian coordinate system.

Several authors [31]–[34] have considered the effects of nonplanar geometries in different plasma models. For example, Sahu and Roychoudhury [35] studied the spherical and cylindrical QIA waves in a two-component unmagnetized plasma, analyzing numerically the geometric and quantum effects on the soliton pulses. Later, they also studied QIA shocks [35] in planar as well as in nonplanar geometries.

In this paper, we investigate the nonlinear properties of nonplanar QDIA and QDA waves in an unmagnetized, collisionless quantum dusty plasma using the QHD equations. By means of computational investigations, we examine the quantum diffraction, quantum statistical and geometrical effects on the profiles of the QDIA and QDA solitary waves. Our main objective is to highlight the role of static as well as mobile (negatively or positively charged) dust impurities on the low-frequency electrostatic waves propagating in a quantum dusty plasma.

The paper is organized as follows. In section 2, we present the quantum dusty plasma model. The cylindrical Kadomtsev–Petviashvili (CKP) equations for the QDIA and QDA waves are derived in sections 3 and 4, respectively, by incorporating transverse perturbations. The solitary wave solutions are then obtained and used to analyze the amplitudes and widths for different quantum and geometrical effects. The results are summarized in section 5.

2. The quantum dusty plasma model

We consider an unmagnetized, collisionless, quantum dusty plasma in a 2D cylindrical geometry. Assuming that the plasma species in a 2D Fermi plasma follow the pressure law \(p_\alpha = (m_\alpha V_{F\alpha}^2 / 2n_{\alpha 0})n_\alpha^2\), where \(m_\alpha\) is the mass of species \(\alpha\) (\(\alpha\) equals e for the electrons, i for the ions, and d for the dust particles), \(V_{F\alpha}\) is the Fermi speed, and \(n_\alpha\) is the particle number density with an equilibrium value \(n_{\alpha 0}\). In the present model, all the dust particulates are supposed to have a uniform size, spherical shape and a constant charging state \(Z_d\). The latter is valid in laboratory plasmas [36, 37], as well as in numerical simulations [38], when the dust charging time is much smaller than the dust plasma period.
3. Nonplanar QDIA solitary waves

First, we consider the nonlinear propagation of the QDIA solitary waves in an unmagnetized, collisionless, quantum dusty plasma whose constituents are inertialess electrons, mobile singly charged ions, and immobile negatively/positively charged dust particulates. The latter affect the dynamics through the charge-neutrality condition \( \mu_e = 1 + \beta \mu_d \), where \( \beta \) equals \(-1\) for negative dust particulates and \(+1\) for positive dust particulates, \( \mu_e = n_{e0}/n_{i0} \), and \( \mu_d = Z_0 n_{i0}/n_{d0} \).

3.1. Model equations

The nonlinear dynamics of the low-phase (viz \( V_Fi \ll V_Fp \ll V_{Fe} \)) electrostatic QDIA waves can be described by the QHD equations

\[
\frac{\partial \bar{n}_i}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( \bar{r} \bar{n}_i \bar{U}_i \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \bar{n}_i \bar{V}_i \right) = 0, \tag{1}
\]

\[
\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_i \frac{\partial \bar{U}_i}{\partial r} + \bar{V}_i \frac{\partial \bar{U}_i}{\partial \theta} + \frac{\bar{V}_i^2}{r} = -\frac{\partial \bar{\phi}}{\partial \bar{r}} \omega_c \bar{n}_i \bar{V}_i^2, \tag{2}
\]

\[
\frac{\partial \bar{V}_i}{\partial t} + \bar{U}_i \frac{\partial \bar{V}_i}{\partial r} + \bar{r} \frac{\partial \bar{V}_i}{\partial \theta} + \frac{\bar{U}_i \bar{V}_i}{r} = -\frac{1}{r} \frac{\partial \bar{\phi}}{\partial \theta} \omega_c \bar{n}_i \bar{V}_i^2, \tag{3}
\]

\[
\frac{1}{r} \frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \bar{\phi}}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2} \frac{\partial^2 \bar{\phi}}{\partial \theta^2} = \bar{n}_e \mu_e - \bar{n}_i - \beta \mu_d \tag{4}
\]

and

\[
\bar{n}_e = 1 + \bar{\phi} + \frac{H_i^2}{2} \bar{n}_e^{-1/2} \left[ \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \sqrt{\bar{n}_e}}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2} \frac{\partial^2 \sqrt{\bar{n}_e}}{\partial \theta^2} \right]. \tag{5}
\]

We are interested in low-frequency electrostatic DIA waves at quantum scales. Equations (1)–(5) are valid in the limits \( \bar{v}_{ei} \ll \bar{\omega}_pi \), \( \bar{v}_{in} \ll |\bar{v}_e| \ll \bar{v}_{en} \), and \( |\bar{v}_i| \bar{v}_{en} \ll V_{Fe}^2 \bar{\nabla}^2 \), where \( \bar{\nabla} = \hat{\bar{r}} \partial/\partial \bar{r} + (\hat{\bar{\theta}} \partial/\partial \bar{\theta}) \hat{\bar{\theta}} \). \( \bar{r} \) and \( \hat{\bar{\theta}} \) are the unit vectors along the radial and polar angle coordinates \((r, \theta)\), \( \bar{v}_{ej} = \bar{v}_{ej}/\bar{\omega}_{pi} \), \( \bar{v}_{en} = \bar{v}_{en}/\bar{\omega}_{pi} \), \( \bar{v}_{en} = \bar{v}_{en}/\bar{\omega}_{pi} \) and \( \bar{\omega}_pi = \bar{\omega}_pi \) are the normalized collisional and plasma frequencies, respectively. The quantum fluid model is a good approximation of the linearized W–P system in the long wavelength limit, i.e. \( k \lambda_{Fe} \ll 1 \), and can be obtained by taking the moments of the W–P system [15, 16].

Equations (1)–(5) have been normalized by using the parameters: \( \bar{n}_{e(i)} = n_{e(i)}/n_{e0}, \bar{r} = r \omega_{pi}/C_i, \bar{U}_{e(i)} = U_{e(i)}/C_i, \bar{V}_{e(i)} = V_{e(i)}/C_i, \bar{\phi} = e \bar{\phi}/2K_BT_{Fe}, \) and \( \bar{t} = t \omega_{pi}, \) where \( K_B \) is the Boltzmann constant, \( C_i = (2K_BT_{Fe}/m_i)^{1/2} \) is the ion-sound speed, \( \omega_{pi} = (4\pi e^2 n_{i0}/m_i)^{1/2} \) is the ion plasma frequency, \( \bar{U}_i \) and \( \bar{V}_i \) are the normalized components of the ion fluid velocity \( U_i \) in \( r \)- and \( \theta \)-directions, and \( \bar{\phi} \) is the normalized electrostatic potential. The second terms appearing in the right-hand side of equations (2) and (3) represent the fermionic behavior of the particles and play a significant role in metallic nanostructures where the electron Fermi temperature is much higher than the room temperature. On the other hand, the origin of the third term in equation (5) is the quantum Bohm potential involving the electron tunneling effect at nanoscales. The quantum statistical and diffraction effects can be seen through the nondimensional parameters \( \delta_i = T_{Fi}/T_{Fe} \) and \( H_i = \hbar \omega_{pi}/(2K_BT_{Fe} \sqrt{1 + \beta \mu_d}) \), (also, \( H_i = \hbar \omega_{pi}/(2K_BT_{Fe}) \)), respectively. The numerical value of \( H_i \) is usually less than unity for
Following the reductive perturbation method \[^2\), here \(\omega_{pe} = (4\pi e^2 n_{e0}/m_e)^{1/2}\) is the electron plasma frequency, \(T_{Fe} = (h^2/2m_e K_B)(3\pi^2 n_{e0})^{2/3}\) and \(T_{Vi} = (h^2/2m_i K_B)(3\pi^2 n_{i0})^{2/3}\) are the electron and ion Fermi temperatures, respectively, \(h\) is the Planck constant divided by \(2\pi\). Since, the ion mass is much larger than the electron mass, one can ignore the quantum diffraction effects of the ions \((H_i = 0)\) in equations (2) and (3) (cf equation (19) in \[^1\]).

3.2. Reduction to CKP equation

To study the nonlinear QDIA waves of small but finite amplitude, we drop all the ‘hat’ notation over reduced dependent and independent variables in equations (1)–(5) and expand the dependent variables \(n_e, n_i, U_i, V_i,\) and \(\varphi\) about their equilibrium values in power of \(\varepsilon\), which is a small parameter measuring the amplitude of the perturbation

\[
\begin{align*}
n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} \ldots, \\
n_i &= 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} \ldots, \\
U_i &= \varepsilon U_{i1} + \varepsilon^2 U_{i2} \ldots, \\
V_i &= \varepsilon^{3/2} V_{i1} + \varepsilon^{5/2} V_{i2} \ldots, \\
\varphi &= \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 \ldots.
\end{align*}
\]

(6)

Following the reductive perturbation method \[^{39}\], we express the independent variables into a moving frame in which the nonlinear wave moves at a phase-speed of \(\lambda_i\) (normalized with the ion–sound speed \(C_i\)) are

\[
\begin{align*}
\xi &= \varepsilon^{1/2} (r - \lambda_i t), \\
\eta &= \varepsilon^{-1/2} \theta, \\
\tau &= \varepsilon^{3/2} t.
\end{align*}
\]

(7)

Using (6) and (7) into equations (1)–(5), we obtain the lowest order in \(\varepsilon\) as

\[
\begin{align*}
n_{i1} &= -\frac{\varphi_1}{\delta_i - \lambda_i^2}, \\
U_{i1} &= -\frac{\lambda_i \varphi_1}{\delta_i - \lambda_i^2}, \\
\frac{\partial V_{i1}}{\partial \xi} &= \frac{1}{\lambda_i^2 \tau} \left( \frac{\partial \varphi_1}{\partial \eta} + \delta_i \frac{\partial n_{i1}}{\partial \eta} \right),
\end{align*}
\]

(8)

(9)

along with the phase speed selection rule

\[
\lambda_i = \left( \delta_i + \frac{1}{\mu_e} \right)^{1/2} \equiv \left( \delta_i + \frac{1}{1 + \beta \mu_d} \right)^{1/2}.
\]

(10)

It is worth mentioning here that the phase-speed of the QDIA waves is significantly affected by the quantum statistical effect and by the presence of positive/negative static dust particulates.

The next order in \(\varepsilon\) gives

\[
\begin{align*}
\lambda_i \frac{\partial n_{i1}}{\partial \tau} - \lambda_i^2 \frac{\partial n_{i2}}{\partial \xi} + \lambda_i \frac{\partial U_{i1}}{\partial \xi} + \lambda_i \frac{\partial n_{i1} U_{i1}}{\partial \xi} + \frac{U_{i1}}{\tau} + \frac{1}{\tau} \frac{\partial V_{i1}}{\partial \eta} &= 0, \\
\lambda_i \frac{\partial U_{i1}}{\partial \tau} - \lambda_i \frac{\partial U_{i1}}{\partial \xi} + \lambda_i U_{i1} \frac{\partial U_{i1}}{\partial \xi} - \lambda_i^2 \tau \frac{\partial U_{i2}}{\partial \xi} + \xi \frac{\partial \varphi_1}{\partial \xi} + \lambda_i \frac{\partial \varphi_2}{\partial \xi} + \delta_i \frac{\partial n_{i1}}{\partial \xi} + \lambda_i \tau \delta_i \frac{\partial n_{i2}}{\partial \xi} &= 0
\end{align*}
\]

(11)

(12)

and

\[
\frac{\partial^2 \varphi_1}{\partial \xi^2} - \frac{H_e^2 \mu_e}{4} \frac{\partial^2 \varphi_1}{\partial \xi^2} - \mu_e \varphi_2 + n_{i2} = 0.
\]

(13)

\[\text{New Journal of Physics 10 (2008) 023007 (http://www.njp.org/)}\]
Eliminating the second-order quantities from equations (11)–(13) with the aid of (8) and (9), we obtain the CKP equation

$$\frac{\partial}{\partial \xi} \left( \frac{\partial \varphi_1}{\partial \tau} + A_i \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B_i \frac{\partial^3 \varphi_1}{\partial \xi^3} + \frac{\varphi_1}{2\lambda_i^2} \right) + \frac{1}{2\lambda_i^2 \eta^2} \frac{\partial^2 \varphi_1}{\partial \eta^2} = 0,$$

(14)

where

$$A_i = -\frac{3\lambda_i}{2(\delta_i - \lambda_i^2)} \quad \text{and} \quad B_i = \left( \frac{\delta_i - \lambda_i^2}{2\lambda_i} \right)^2 \left( 1 - \frac{H_c^2}{4} - \beta \frac{H_e^2 \mu_d}{4} \right).$$

(15)

The quantum mechanical effects are contained in the coefficients of the nonlinear and dispersion terms in CKP equation (14). It is noticed from equation (36) in [16] that for $H_c = 2$, the soliton solution disappears because the dispersion term disappears, yielding the formation of a shock. However, in our model, the dispersion coefficient $B_i$ is modified by the presence of positive/negative static dust particles. As a result, the disappearance of $B_i$ is now shifted from $H_c = 2$ to high values of $H_c$ due to the contribution of the parameters $\mu_d$ and $\beta$. For an electron–ion quantum plasma, we consider $n_{d0} = n_0 = \mu_d$ as well as $H_c = 2$, which may lead to a shock when $B_i \to 0$. Thus, we point out that the dispersion coefficient may change sign at high values of the quantum diffraction parameter [16], so the soliton stability is expected to be affected in this case; this agrees with our results here. Furthermore, instability to transverse perturbations [40] is known to be an important issue, which is somehow overseen in our simple model here.

### 3.3. Solitary wave solutions

To obtain a stationary solution of equation (14), we assume that [33] $\varphi_1 = \varphi_1(\chi, \bar{\tau})$, where

$$\chi = \xi - \frac{\lambda_i}{2} \eta^2 \tau \quad \text{and} \quad \bar{\tau} = \tau.$$ 

(16)

Thus

$$\frac{\partial}{\partial \xi} \rightarrow \frac{\partial}{\partial \chi}, \quad \frac{\partial}{\partial \eta} \rightarrow -\lambda_i \eta \frac{\partial}{\partial \chi}, \quad \frac{\partial}{\partial \tau} \rightarrow \frac{\partial}{\partial \bar{\tau}} - \frac{\lambda_i}{2} \eta^2 \frac{\partial}{\partial \chi}. $$

(17)

Using equations (16) and (17) into equation (14), we obtain the KdV equation

$$\frac{\partial \varphi_1}{\partial \tau} + A_i \varphi_1 \frac{\partial \varphi_1}{\partial \chi} + B_i \frac{\partial^3 \varphi_1}{\partial \chi^3} = 0,$$

(18)

where we have dropped the ‘hat’ in $\bar{\tau}$. By using the vanishing boundary condition for $\varphi_1$ and their derivatives up to the second order at $|\rho| \to \pm \infty$, we obtain the KdV pulse soliton solution

$$\varphi_1 = \varphi_{i0} \text{sech}^2 \left( \frac{\rho}{W_i} \right),$$

(19)

where

$$\varphi_{i0} = \frac{3U_0}{A_i}, \quad W_i = \left( \frac{U_0}{4B_i} \right)^{-1/2}. $$

(20)

Here, $\varphi_{i0}$ and $W_i$ are the maximum amplitude and the width of the QDIA soliton. Searching for constant profile solutions (solitary waves), we have taken $\rho = \chi - U_0 \tau$, where $U_0$ is the QDIA...
soliton speed. Combining equations (16), (19) and the stretching (7), the solution of the CKP equation (14) takes the form

$$\varphi_1 = \varphi_{i0} \text{sech}^2 \left( \frac{R_i}{W_i} \right), \quad (21)$$

where

$$R_i = \xi - \left( U_0 + \frac{\lambda_i \eta^2}{2} \right) \tau \equiv \varepsilon^{1/2} \left[ r - \left( \lambda_i + \varepsilon U_0 + \frac{\lambda_i \eta^2}{2} \right) t \right]. \quad (22)$$

Note that $\varphi_{i0} W_i^2 = 12 B_i / A_i \equiv \text{constant}$ (independent of $U_0$, though a function of $\lambda_i$). We also see that, increasing $U_0$ leads to an increase in $\varphi_{i0}$ and a decrease in $W_i$, so that taller solitons will be narrower and faster, according to the standard KdV picture [41, 42].

In order to determine the stability/instability [43], we express equation (18) in terms of an energy balance equation as

$$\left( \frac{d\varphi_1}{d\rho} \right)^2 + V(\varphi_1) = 0,$$

where the Sagdeev potential $V(\varphi_1)$ is given by

$$V(\varphi_1) = \frac{A_i}{3 B_i} \varphi_1^3 - \frac{U_0}{B_i} \varphi_1^2.$$  

For the existence of the solitary waves, we use the condition $d^2 V(\varphi_1) / d\varphi_1^2 < 0$ at $\varphi_1 = 0$, obtaining

$$\left| d^2 V(\varphi_1) / d\varphi_1^2 \right|_{\varphi_1 = 0} = -2 \frac{U_0}{B_i}.$$  

It is easy to see from the above equation that a stable solitary excitation exists for $U_0 / B_i > 0$.

It may be appropriate to point out that the above solution has been obtained in the region of parameter values where both the nonlinearity and dispersion coefficients in the KdV equation (18), i.e. $A_i$ and $B_i$ respectively, bear positive values. Indeed, this was the dominant case in realistic dusty plasma parameters; see the details and our discussion on the parameter set below. However, one sees in the definitions (15) that both coefficients may be either positive or negative, depending on the value of the pulse speed $\lambda_i$ (given by (10)) and on the intrinsic plasma parameter values (Fermi temperatures, relative density of electron-to-ions, i.e. essentially, on the dust concentration). For instance, for positive $\lambda_i$, $A_i$ becomes negative below $\sqrt{\lambda_i}$, while $B_i$ becomes negative if the electron-to-ion density ratio $\mu_e$ becomes higher than a critical value $\mu_{e,cr} = 4 / H_e^2$. We may thus add, for rigor, that in the case $A_i < 0$ and $B_i > 0$ the solution (19) represents a negative pulse (a negative potential excitation, implying from (8) a negative ion density variation, i.e. localized ion density depletion), which otherwise bears the same generic characteristics of the KdV soliton solution discussed above. On the other hand, if both coefficients are negative ($A_i < 0$ and $B_i < 0$), then the KdV equation (18) remains unchanged (i.e. with both coefficients positive) upon shifting $\chi$ to $-\chi$; thus, the above solution(s) remain(s) valid, though they now physically represent the propagation in the opposite direction. Thus, the class of the solutions obtained above cover the entire range of the plasma parameters.
3.4. Parametric study

It is clear that the propagation speed of the nonplanar QDIA solitary wave is modified by the effect of the angle dependence. As time and the polar angle vary, the cylindrical QDIA solitary wave will swerve, while propagating in the positive radial direction, as is obvious from figure 2. The dependence of the maximum amplitude $\varphi_{\text{d0}}$ and width $W_i$ on the equilibrium dust number density ($n_{\text{d0}}$) is more complex. First, it is important to note that changing $n_{\text{d0}}$ leads to a change in the phase-speed ($\lambda_i$) of the QDIA waves (see equation (10)), as well as in the electron concentration (via the charge-neutrality condition $\mu_e = 1 + \beta \mu_d$). Since the electron (ion) Fermi temperature depends upon the equilibrium electron (ion) number density, it can also be affected by $n_{\text{d0}}$ through the charge-neutrality condition. As a result, the quantum statistical ($\delta_i$) and diffraction ($H_e$) effects will vary with $n_{\text{d0}}$ concentration.

Based upon the above findings, we shall now investigate the effects of the relevant physical quantities, namely the dust concentration $\mu_d$ and the dust polarity $\beta(\pm)$ on the profiles of QDIA solitary waves expressed in terms of the radial and the polar angle coordinates ($r, \theta$).

We have used, as a starting point, a typical set of plasma parameter values for metallic nanostructures [15] (in the absence of dust), namely: $n_{d0} = 5.9 \times 10^{22} \text{cm}^{-3}$, $T_{\text{Fe}} = 6.4 \times 10^4 \text{K}$, and $\omega_{\text{pe}} = 1.38 \times 10^{16} \text{s}^{-1}$, for $\mu_d = 0$. However, for example, once the dust species density is determined, the values of $T_{\text{Fe}(i)}$, $\lambda_i$ and $H_e$ are subsequently computed, according to the above formulae, which also determine $A_i$ and $B_i$. In the plots, we have a low dust concentration, e.g. for $\mu_d = 0.00003$ (i.e. $\mu_e = 0.99997$ for negative dust and $\mu_e = 1.00003$ for positive dust), which leads to $\delta_i = 0.000544$, and a pulse speed $\lambda_i = 1.00026$ (through (10)), so we have $A_i = 1.500$ and $B_i = 0.417$. Obviously, by varying the dust concentration, we simultaneously modify all parameter values, which are then used in the plots below.

Figure 1 depicts the relevant plasma parameters, such as the quantum parameter ($H_e$, $H_i$), the electron Fermi temperature ($T_{\text{Fe}}$), the ratio of ion-electron Fermi temperatures ($\delta_i$), the electron plasma frequency ($\omega_{\text{pe}}$), and the phase-speed of the QDIA soliton ($\lambda_i$). We note that these all strongly depend upon the dust concentration and dust the polarity.

Figure 2 displays the QDIA soliton pulse for different values of $n_{d0}$, which now determines $T_{\text{Fe}}$ through the charge-neutrality condition (this refers, for example, to the solid curve in figure 2(a)). It is obvious that the amplitude of the soliton pulse decreases by increasing $n_{d0}$, resulting in an increase (decrease) of the electron Fermi temperature $T_{\text{Fe}}$ ($H_e$, respectively) for positive dust impurities, viz $A_i$ increases. For negative dust, the QDIA soliton represents a reverse behavior of figure 2(a), because now $A_i$ decreases. Physically, the increase of $T_{\text{Fe}}$ leads to an increase of the electron Fermi energy (viz $K_B T_{\text{Fe}} = E_{\text{Fe}} \equiv (h^2/2m_e)(3\pi^2 n_{d0})^{2/3}$), and as a result the ion Fermi energy should decrease to conserve the energy. The decrease of the ion Fermi energy will decrease the height of the soliton pulse.

It may be appropriate to note that, quite counter-intuitively, taller pulses are faster and wider for both + and — dust polarities; see figure 2. This seems to contradict the KdV scenario discussed above in terms of $U_0$ in (18) and (19). However, this qualitative result implies no discrepancy, as it stems from the perplex interplay between the speed variables $\lambda_i$ and $U_0$ (see that both KdV coefficients in (15) depend on $\lambda_i$). The same effect was witnessed in previous studies, where the CKP equation was solved via a KdV equation [33].

The electrostatic potential excitations ($\varphi_1$) are depicted in figure 3 against the radial and the polar angle coordinates ($r, \theta$) for different times. It is found that the propagation of the QDIA soliton pulse becomes shifted in the radial direction and accelerated due to (weak) angular effects.

New Journal of Physics 10 (2008) 023007 (http://www.njp.org/)
The effect of the dust concentration and polarity on: (a) the electron quantum diffraction parameter, (b) the ion quantum diffraction parameter, (c) the electron Fermi temperature, (d) the ratio of ion–electron Fermi temperatures, (e) the electron plasma frequency, and (f) the phase speed of QDIA soliton. The solid curve is for positive dust ($\beta = 1$) and the dashed curve is for negative dust ($\beta = -1$). Note that the values of [15] are obtained in the absence of dust, as expected.

Figure 4 displays the electrostatic potential excitations as a function of the radial and time coordinates for different angular values. We note that the QDIA soliton pulse deviates in the radial direction with the increase of time.

4. Nonplanar QDA solitary wave

Now, we consider the propagation of the QDA solitary wave in an unmagnetized, collisionless, quantum dusty plasma comprised of inertialess electrons and ions, as well as mobile (negatively/positively) charged dust particles. In this case, the charge neutrality condition at
the equilibrium is expressed as \( \mu_e = \mu_i + \beta \), where \( \mu_e = n_{e0}/Zdn_{d0} \), and \( \mu_i = n_{i0}/Zdn_{d0} \). Recall that \( \beta = +1/-1 \) denotes the dust charge polarity. The nonlinear dynamics of the low-phase (viz \( V_{Fd} \ll V_p \ll V_{Fe}, V_{Fi} \)) electrostatic QDA wave in a 2D cylindrical geometry is governed by

\[
\frac{\partial \bar{n}_d}{\partial \bar{t}} + \frac{1}{r} \frac{\partial}{\partial r} \left( \bar{r}\bar{n}_d \bar{U}_d \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \bar{n}_d \bar{V}_d \right) = 0, \tag{23}
\]

\[
\frac{\partial \bar{U}_d}{\partial \bar{t}} + \bar{U}_d \frac{\partial \bar{U}_d}{\partial \bar{r}} + \bar{V}_d \frac{\partial \bar{U}_d}{\partial \theta} - \frac{\bar{V}_d^2}{r} = -\beta \frac{\partial \bar{\phi}}{\partial \bar{r}} - \delta_d \frac{\delta \bar{n}_d^2}{2 \bar{n}_d \bar{r}}, \tag{24}
\]

\[
\frac{\partial \bar{V}_d}{\partial \bar{t}} + \bar{U}_d \frac{\partial \bar{V}_d}{\partial \bar{r}} + \bar{V}_d \frac{\partial \bar{V}_d}{\partial \theta} + \frac{\bar{U}_d \bar{V}_d}{r} = -\beta \frac{\partial \bar{\phi}}{\partial \theta} - \delta_d \frac{\delta \bar{n}_d^2}{2 \bar{n}_d \bar{r} \bar{\theta}}. \tag{25}
\]
the ratio between the dust and ion Fermi temperatures, the electron and ion Fermi temperatures, and replaced by in the same way as in equation (11).

The expansions of the dependent variables are the normalized components of the dust fluid velocity $\mathbf{U}_d(r, \theta)$ in $r$ and $\theta$ directions. We have neglected the quantum diffraction effects ($H_d = H_i = 0$) of the dust particles and the ions, owing to their large mass compared to the electrons mass. We are interested in investigating the nonlinear properties of nonplanar QDA waves in which the dust hydrodynamics is studied at a very slow timescale compared to the electrons and ions. The expansions of the dependent variables $n_d$, $U_d$, and $V_d$ are carried out in the same way as in equation (6), whereas the phase speed ($\lambda_c$) of the QDA waves is to be replaced by $\lambda_d$ (normalized by $C_d$) for the QDA wave. Using the procedure mentioned in the previous section, one can solve the set of equations (23)–(28) to obtain the CKP equation

$$\frac{\partial}{\partial \xi} \left( \frac{\partial \varphi_1}{\partial \tau} + A_d \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B_d \frac{\partial^3 \varphi_1}{\partial \xi^3} + \frac{\varphi_1}{2\tau} \right) + \frac{1}{2\lambda_d \tau^2} \frac{\partial^2 \varphi_1}{\partial \eta^2} = 0,$$

Equations (24), (25) and (27) are valid for $\tilde{v}_{ej} \ll \tilde{v}_{pj}$, $\tilde{v}_{dn} \ll \tilde{v}_e$, $\tilde{v}_{in}$, and $|\tilde{\delta}_e| (\tilde{v}_{en} = \tilde{v}_{en}/\tilde{\omega}_{pd}), \tilde{v}_{ei} = \tilde{v}_{ei}/\tilde{\omega}_{pd}$, and $\tilde{\omega}_{pj} (= \tilde{\omega}_{pj}/\tilde{\omega}_{pd})$ are the normalized collisional and plasma frequencies, respectively. $\delta_d = T_{pd}/(Z_d T_{Fi})$ is the ratio between the electron and ion Fermi temperatures, and $H_e = \tilde{h}/\omega_{pe}/(2K_B T_{Fi}\sqrt{\mu_e + \beta})$ represents the ratio between the plasmon energy of the electrons and the thermal energy of the ions. In this case, we have used the normalization $n_{a} = n_\alpha/n_{a0}$, $\tilde{r} = r \omega_{pd}/C_d$, $\tilde{U}_d = U_d/C_d$, $\tilde{V}_d = V_d/C_d$, $\tilde{\varphi} = \varphi/2K_B T_{Fi}$, and $\tilde{t} = t \omega_{pd}$, $C_d = (2Z_d K_B T_{Fi}/m_d)^{1/2}$ is the dust acoustic speed, $\omega_{pd} = (4\pi Z_d^2 e^2 n_{a0}/m_d)^{1/2}$ is the dust plasma frequency, $\tilde{U}_d$ and $\tilde{V}_d$ are the normalized components of the dust fluid velocity $U_d(r, \theta)$ in $r$ and $\theta$ directions. We have neglected the quantum diffraction effects ($H_d = H_i = 0$) of the dust particles and the ions, owing to their large mass compared to the electrons mass. We are interested in investigating the nonlinear properties of nonplanar QDA waves in which the dust hydrodynamics is studied at a very slow timescale compared to the electrons and ions. The expansions of the dependent variables $n_d$, $U_d$, and $V_d$ are carried out in the same way as in equation (6), whereas the phase speed ($\lambda_c$) of the QDA waves is to be replaced by $\lambda_d$ (normalized by $C_d$) for the QDA wave. Using the procedure mentioned in the previous section, one can solve the set of equations (23)–(28) to obtain the CKP equation

$$\frac{\partial}{\partial \xi} \left( \frac{\partial \varphi_1}{\partial \tau} + A_d \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B_d \frac{\partial^3 \varphi_1}{\partial \xi^3} + \frac{\varphi_1}{2\tau} \right) + \frac{1}{2\lambda_d \tau^2} \frac{\partial^2 \varphi_1}{\partial \eta^2} = 0,$$
Figure 5. (a) The QDA solitary electrostatic potential $\varphi_1$ is plotted against $\rho$ for different values of $H_e = 0.37$ (solid thick curve) and $H_e = 0.52$ (dashed curve) with $\mu_e = 4 \times 10^6$, $\lambda_d = 4 \times 10^{-4}$ and $\mu_e = 2 \times 10^6$, $\lambda_d = 7 \times 10^{-4}$, respectively, for positive dust ($\beta = 1$), and for negative dust ($\beta = -1$) having $U_0 = 0.1$. The values of $\lambda_d$ are obtained consistently from (30), in all cases. (b) The QDA solitary excitation phase speed $\lambda_d$ is depicted against the electron to dust charge concentration ratio $\mu_e$.

where

$$A_d = -\frac{3\beta \lambda_d}{2(\delta_d - \lambda_d^2)}, \quad B_d = \frac{(\delta_d - \lambda_d^2)^2}{2\beta^2 \lambda_d} \left(1 - \frac{H_e^2(\mu_i + \beta)}{4\delta_e^2}\right),$$

and

$$\lambda_d = \left[\delta_d + \frac{\delta_e^2 \beta^2}{\mu_i (1 + \delta_e) + \beta}\right]^{1/2}.$$

Applying the transformation (16) and (17) into (29) we obtain, once again, a KdV equation in the form of (18) (upon a trivial index shift from ‘i’ to ‘d’ in the coefficients). We finally obtain the localized soliton solution as

$$\varphi_1 = \varphi_{d0}\text{sech}^2\left(\frac{\rho}{W_d}\right),$$

Here, $\rho = \chi - U_0\tau$ ($\chi$ was defined in (16)), $U_0$ is the QDA soliton speed, $\varphi_{d0} = 3U_0/A_d$ is the maximum amplitude, and $W_d = 1/(U_0/4B_d)^{1/2}$ is the width of the QDA soliton. Note, for rigor, that we have assumed a positive value of the coefficient $B_d$ of equation (29) here, considering both positive and negative values of $A_d$ in the numerical plots.

The QDA potential excitations $\varphi_1$ are plotted against $\rho$ in figure 5(a) for different values of the quantum diffraction effects. It is found that the quantum diffraction effects lead to an increase in the amplitudes and the widths of the soliton pulse in case of (positive/negative) charged dust impurities. It is also noted that the strength of the potential excitations of the QDA soliton is relatively smaller than the QDIA soliton. The phase speed $\lambda_d$ is observed, in figure 5(b), to increase with higher dust concentration for both positive and negative dust cases.

5. Summary

To summarize, we have presented the nonlinear properties of nonplanar QDIA and QDA waves in a very dense Fermi dusty plasma, composed of electrons, ions and charged dust impurities.
By employing the reductive perturbation method, a 2D CKP-type equation has been derived, and its stationary localized solutions have been obtained. We have numerically examined the effects of the quantum statistics and quantum diffraction on the electrostatic potential excitations, in terms of the radial and polar angle coordinates, by varying relevant physical parameters. It is found that the amplitudes and widths of the nonplanar QDIA and QDA waves are significantly affected by the quantum tunneling effect. In particular, addition of dust to a quantum plasma is seen to modify the charge balance (via an associated electron density depletion, for negative dust, or vice versa), and thus affects the propagation characteristics (including a strong modification of the phase speed) of localized (dust-) ion-acoustic excitations. In the case of the low-frequency DA waves this effect is even stronger, since the actual form of potential solitary waves, in fact, depends on the dust charge polarity (positive/negative) itself (allowing for positive/negative potential forms, respectively).

The present results should be helpful for understanding the nonlinear electrostatic structures in metallic nanostructures involving charged particles in nanomaterials. It is observed that though the time period, viz the inverse ion (dust) plasma frequency $\omega_{pi}^{-1}$ ($\omega_{pd}^{-1}$), of the electrostatic QDIA (QDA) wave is much longer as compared to the time period ($\omega_{pe}^{-1}$) of the Langmuir wave [15, 17], even then, collisional frequencies can be neglected in comparison with the plasma frequencies. The occurrence of dense quantum plasmas has also been recognized in dense astrophysical objects [13] (e.g. white dwarfs, neutron stars and supernovae), as well as in the next generation laser-based plasma compression schemes [14, 44]. In such systems, the typical particle number densities (temperature) may reach $10^{32}$ and $10^{28} \text{m}^{-3}$ ($T_{Fe} = 10^5 \text{K}$), leading to numerical values of the quantum parameter $H_e \sim 1$ and $H_e \sim 10^{-2}$, respectively. Thus, $H_e$ is usually assumed to be less than unity for laboratory or technological plasmas, and $H_e \gtrsim 1$ for superdense astrophysical plasmas [21].

Acknowledgments

S A thanks Fernando Haas for useful discussions. He also acknowledges partial financial support from the Deutscher Akademischer Austausch Dienst. W M M is grateful to the Alexander von Humboldt-Stiftung (Bonn, Germany) for financial support. I K acknowledges support from the German Research Society (Deutsche Forschungsgemeinschaft, DFG) under the Emmy-Noether Programme (grant SH 93/3-1). The authors are also grateful to the referees for a number of useful suggestions.

References

[1] Shukla P K and Mamun A A 2002 Introduction to Dusty Plasma Physics (Bristol: Institute of Physics Publishing)
Shukla P K and Mamun A A 2003 New J. Phys. 5 17

[2] Fortov V E, Ivlev A V, Khrapak S A and Morfill G E 2005 Phys. Rep. 421 1
Nambu M, Vladimirov S V and Shukla P K 1995 Phys. Lett. A 203 40
Shukla P K and Rao N N 1996 Phys. Plasmas 3 1760

[3] Rao N N, Shukla P K and Yu M Y 1990 Planet. Space Sci. 38 543

[4] Shukla P K and Silin V P 1992 Phys. Scr. 45 508

New Journal of Physics 10 (2008) 023007 (http://www.njp.org/)
[5] Barkan A, Merlino R L and D’Angelo N 1995 Phys. Plasmas 2 3563
[6] Pieper J B and Goree J 1996 Phys. Rev. Lett. 77 3137
[7] Prabhakara H R and Tanna V L 1996 Phys. Plasmas 3 3176
[8] Barkan A, D’Angelo N and Merlino R L 1996 Planet. Space Sci. 44 239
[9] Merlino R L, Barkan A, Thompson C and D’Angelo N 1998 Phys. Plasmas 5 1607
[10] Belashov V Y and Vladimirov S V 2005 Solitary Waves in Dispersive Complex Media (Berlin: Springer)
[11] Vladimirov S V, Ostrikov K and Samarian A A 2005 Physics and Applications of Complex Plasmas (London: Imperial College)
[12] Markowich P A, Ringhofer C A and Schmeiser C 1990 Semiconductor Equations (New York: Springer)
[13] Jung Y D 2001 Phys. Plasmas 8 2454
[14] Opher M, Silva L O, Dauger D E, Decyk V K and Dawson J M 2001 Phys. Plasmas 8 2454
[15] Marklund M and Shukla P K 2006 Rev. Mod. Phys. 78 597
[16] Manfredi G 2005 Fields Inst. Commun. 46 263
[17] Manfredi G and Haas F 2001 Phys. Rev. B 64 075316
[18] Gardner C 1994 SIAM J. Appl. Math. 54 409
[19] Shokri B and Rukhadze A A 1999 Phys. Plasmas 6 4467
[20] Manfredi G and Feix M 1996 Phys. Rev. E 53 6460
[21] Suh N, Feix M R and Bertrand P 1991 J. Comput. Phys. 94 403
[22] Garcia L G, de Oliveira L P L and Goedert J 2005 Phys. Plasmas 12 012302
[23] Marklund M 2005 Phys. Plasmas 12 082110
[24] Shukla P K and Eliasson B 2007 Phys. Rev. Lett. 99 096401
[25] Shukla P K and Eliasson B 2006 Phys. Rev. Lett. 96 245001
[26] Marklund M and Brodin G 2007 Phys. Rev. Lett. 98 025001
[27] Shaikh D and Shukla P K 2007 Phys. Rev. Lett. 99 125002
[28] Shukla P K, Stenflo L and Bingham R 2006 Phys. Lett. A 359 218
[29] Ali S, Moslem W M, Shukla P K and Kourakis I 2007 Phys. Lett. A 366 606
[30] Shukla P K and Stenflo L 2006 Phys. Lett. A 355 378
[31] Shukla P K 2006 Phys. Lett. 352 242
[32] Stenflo L, Shukla P K and Marklund M 2006 Europhys. Lett. 74 844
[33] Shukla P K and Ali S 2005 Phys. Plasmas 12 114502
[34] Moslem W M, Shukla P K, Ali S and Schlickeiser R 2007 Phys. Plasmas 14 042107
[35] Ali S and Shukla P K 2006 Phys. Plasmas 13 052113
[36] Ali S and Shukla P K 2007 Eur. Phys. J. D 41 319
[37] Ali S and Shukla P K 2006 Phys. Plasmas 13 022313
[38] Khan S A and Mushtaq A 2007 Phys. Plasmas 14 083703
[39] Mamun A A and Shukla P K 2001 Phys. Lett. A 290 173
[40] Mamun A A and Shukla P K 2002 Phys. Plasmas 9 1468
[41] Xue J-K 2003 Phys. Plasmas 10 3430
[42] Wang Y-Y and Zhang J-F 2006 Phys. Plasmas 13 42308
[43] Wang Y-Y and Zhang J-F 2006 Phys. Lett. A 352 155
[44] Sahu B and Roychoudhury R 2007 Phys. Plasmas 14 012304
Sahu B and Roychoudhury R 2007 Phys. Plasmas 14 072310
[45] Hazelton R C and Yadlowsky E J 1994 IEEE Trans. Plasma Sci. 22 91
[46] Praburam G and Goree J 1996 Phys. Plasmas 3 1212
[47] Choi S J and Kushner M J 1994 IEEE Trans. Plasma Sci. 22 138
[39] Washimi H and Taniuti T 1966 Phys. Rev. Lett. 17 996
    Yu M Y and Shukla P K 1977 Plasma Phys. 19 889
    Schamel H, Yu M Y and Shukla P K 1977 Phys. Fluids 20 1286
[40] Infeld E and Rowlands G 2000 Nonlinear Waves, Solitons, and Chaos (Cambridge: Cambridge University Press)
[41] Dauxois T and Peyrard M 2005 Physics of Solitons (Cambridge: Cambridge University Press)
[42] Moslem W M, Kourakis I, Shukla P K and Schlickeiser R 2007 Phys. Plasmas 14 102901
[43] El-Labany S K, Moslem W M, El-Taibany W F and Marklund M 2004 Phys. Scr. 70 317
[44] Malkin V M et al 2007 Phys. Rev. E 75 026404