Propagation of $B$ mesons in the atmosphere

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Abstract. Collisions of cosmic rays in the atmosphere may produce heavy hadrons of very high energy. The decay length of a $B$ meson of energy above $10^7$ GeV is larger than 1 km, implying that such a particle tends to interact in the air before it decays. We show that the fraction of energy deposited in these interactions is much smaller than in proton and pion collisions. We parameterize their elasticity and determine the average number of interactions and the atmospheric depth at the decay point for different initial energies. We find that the profile of a $3 \times 10^9$ GeV bottom shower may be very different from the profile of a proton shower of the same energy, defining either a very deep maximum, or two maxima, or other features that cannot be parameterized with a single Gaisser-Hillas function. Finally, we discuss under what conditions a bottom hadron inside the parent air shower may provide observable effects.

Keywords: bottom quark, hadrons, cosmic rays, propagation

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1 Introduction

Cosmic rays produce collisions in the upper atmosphere of energy well above the ones observed in particle colliders. Our knowledge of the hadron properties must then be extrapolated to such energies, a procedure that introduces some important uncertainties. These uncertainties are not only related to the appearance of new particles and interactions, but also to the transition to a regime where the properties of the standard particles could be substantially different. In particular, heavy hadrons that decay through weak interactions have typical decay lengths $\lambda_{0}^{\text{dec}} = 0.1–0.5$ mm. As their energy grows, Lorentz dilation will eventually make $\lambda_{0}^{\text{dec}}$ longer than the interaction length in the air, implying that they tend to collide before decaying. Obviously, we do not know how a charmed or a bottom hadron behaves inside a calorimeter, since in colliders they decay well before they can reach it. Although heavy-hadron interactions are of no interest there, they are processes that might occur in extensive air showers and could introduce unexpected effects. The production of standard heavy-hadrons in the atmosphere has been extensively studied in the literature (see [1] for a review), as its prompt decay is an important source of muons of very high energy (see also [2, 3]), however, their propagation has been basically ignored.

In a high-energy collision with a target at rest, a proton or a pion will break into several pieces, with each piece taking a similar amount of energy. The elasticity $z$ (fraction of energy taken by the leading hadron) and the multiplicity in such collisions will determine how the hadronic shower develops through the atmosphere, which is equivalent to 10 meters of water if crossed vertically. It is clear, however, that if the collision involves a charm or, especially, a bottom hadron the situation will be very different, as the piece carrying the heavy quark will take most of the initial energy after the collision. Therefore, qualitatively one expects lower inelasticities and, as a consequence, a different profile of deposited energy.

In this paper we will explore the propagation of $b$ hadrons in the atmosphere. In Section 2 we will follow the method applied for charmed hadrons in [4] to parameterize the elasticity of their collisions with air nuclei. In Section 3 we use CORSIKA [5] to find the profile of bottom showers of different energies. In Section 4 we discuss under what circumstances a bottom component inside an extensive air shower may give an observable effect. Section 5 is devoted to the summary and discussion.
2 Inelasticity in heavy-hadron collisions

In this section we will modify PYTHIA \cite{6} to simulate the collisions of a hadron \( H \) (containing a \( b \) quark, \( H = \Lambda_b, B \)) with matter. PYTHIA describes hadronic collisions of pions and protons, distinguishing two types of interactions: diffractive processes, where the two hadrons (as a whole) exchange momentum through pomerons, and non-diffractive or partonic ones, where gluons are exchanged between the partons in the colliding hadrons. The latter type includes both soft collisions of \( q^2 \leq 1 \) GeV and the hard ones in deep inelastic scatterings.

In PYTHIA non-diffractive processes dominate the inelastic cross section. For example, the cross section of a 10\(^9\) GeV proton or pion with a proton at rest is

\[
\sigma_{pp} = 94.1 \text{ mb} ; \quad \sigma_{\pi p} = 64.8 \text{ mb} ,
\]

where diffractive processes contribute just 30\% in pp collisions and 26\% in \( \pi p \) interactions. Therefore, we will study diffractive and partonic processes separately, then we will combine them to define a generic \( HP \) scattering, and finally we will discuss the collision with an air nucleus.

2.1 Diffractive \( HP \) collisions

Let us first describe the approach to simulate a diffractive \( HP \) collision with PYTHIA from the analogous light-hadron process. A hadron \( H \) with mass \( m_H \) contains a heavy core of mass \( m_b \approx 4.7 \) GeV. One may think of a proton as three clouds of mass \( m \approx 0.3 \) GeV associated to the three constituent quarks. Within that picture, a \( \Lambda_b \) baryon will consist also of three similar clouds, but with one of them having an additional electroweak core and a total mass \( m_b + m = 5.0 \) GeV. In a diffractive process, however, this heavy core will be invisible (the proton and the \( \Lambda_b \) clouds will look identical), since the interaction has a \( q^2 \) much smaller than 1 GeV\(^2\) and cannot resolve it. Therefore, the momentum exchanged through pomerons or other non-perturbative dynamics with the target should not depend on the electroweak core in \( H \).

More precisely, the light degrees of freedom in \( H \) carry just a fraction \( w \equiv (m_H - m_b)/m_H \) of the hadron energy \( E \). In a diffractive scattering, \( H \) will be seen by the target nucleon like a light hadron of energy \( wE \). Therefore, to estimate the momentum \( q^\mu \) absorbed by \( H \) we will just simulate with PYTHIA the collision of a proton (for \( H = \Lambda_b \)) or a pion (for \( H = B \)) of energy \( wE \) with the nucleon and assume that \( q^\mu \) is the same when the incident particle is the heavy hadron.

This momentum is all we need to simulate how \( H \) evolves after the diffractive collision (see \cite{4} for details). Once \( H \) absorbs \( q^\mu \), it becomes a system of mass \( M^* \), the critical parameter in the collision. If \( M^* < m_H + 1 \) GeV the process is quasi-elastic and \( H_{\text{dif}} \) will just decay into two bodies (e.g., \( H + \eta \)). For larger values of \( M^* \) the system is treated by PYTHIA \cite{7} like a string with the quantum numbers of \( H \). When \( H \) is a baryon the string may be stretched between a quark and a diquark or between a quark, a gluon and a diquark, whereas for a diffractive meson the string connects a quark and an antiquark or a quark, a gluon and an antiquark.

In Fig. 1 we plot the distribution of the fraction \( z \) of energy taken by the bottom hadron after a diffractive collision of a 10\(^9\) GeV \( \Lambda_0 \) or a \( B \) with a proton at rest. The average values are \( \langle z \rangle = 0.88 \) and \( \langle z \rangle = 0.90 \), respectively, which imply an inelasticity \( K = 1 - \langle z \rangle \):

\[
K_{\Lambda_b p}^{\text{dif}} = 0.12 ; \quad K_{B p}^{\text{dif}} = 0.10 .
\]
In comparison, we note that for proton and charged-pion collisions, the inelasticity obtained also with PYTHIA is substantially higher:

\[ K_{pp}^{\text{diff}} = 0.40 ; \quad K_{\pi p}^{\text{diff}} = 0.23. \]  \hspace{1cm} (2.3)

### 2.2 Partonic \( H p \) collisions

For non-diffractive collisions, we model \( H \) as a system with the same parton content as the corresponding proton or pion, but substituting a valence up quark \( (u_0) \) for a bottom quark. As in diffractive processes, we will associate a hadron \( H \) of energy \( E \) to a light hadron of energy \( (m_H - m_b)/m_H E \). If \( u_0 \) carries a fraction \( x \) of the proton or the pion momentum, we will change it for a \( b \) with momentum fraction \( x_b \)

\[ x_b = \frac{m_b}{m_H} + \frac{m_H - m_b}{m_H} x. \]  \hspace{1cm} (2.4)

In this way the excess of energy in \( H \) is carried entirely by the bottom quark, whereas the light partons in both hadrons \( (H \) and \( p \) or \( \pi \)) carry exactly the same amount of energy.

We will then distinguish two types of \( H p \) partonic collisions: those where the bottom is an spectator (i.e., it is a light parton in \( H \) which hits a parton in the target proton), and processes where the \( b \) quark itself interacts. For the first case we first just simulate with PYTHIA the parton process using a light hadron, later we then substitute the spectator \( u_0 \) for the bottom quark. Intrinsic bottom interactions, on the other hand, have a much smaller cross section,

\[ \sigma_{b \text{ int}}^{Hp} = 0.8 \text{ mb}, \]  \hspace{1cm} (2.5)

than the processes with an spectator \( b \) quark,

\[ \sigma_{b \text{ spec}}^{\Lambda p} = 56.2 \text{ mb}; \quad \sigma_{b \text{ spec}}^{Bp} = 38.7 \text{ mb}, \]  \hspace{1cm} (2.6)
Figure 2. Elasticity \( (z) \) in \( \Lambda_{bp} \) (solid) and \( Bp \) (dashed) non-diffractive collisions.

but they imply collisions of higher inelasticity.

In Fig. 2 we plot the distribution of energy taken by the bottom hadron after a \( \Lambda_{bp} \) or a \( Bp \) non-diffractive collision. The average inelasticity is

\[
K_{n-diff}^{\Lambda_{bp}} = 0.22 ; \quad K_{n-diff}^{Bp} = 0.21 ,
\]

(2.7)

For comparison for protons and pions of the same energy is:

\[
K_{n-diff}^{pp} = 0.59 ; \quad K_{n-diff}^{\pi p} = 0.70 .
\]

(2.8)

2.3 Inelasticity in \( H_p \) collisions

We need to combine both types of collisions. We find that the total (inelastic) \( H_p \) cross section for a projectile of energy \( 10^9 \) GeV is

\[
\sigma^{\Lambda_{bp}} = \sigma_{diff}^{\Lambda_{bp}} + \sigma_{n-diff}^{\Lambda_{bp}} = 82.1 \text{ mb} ; \\
\sigma^{Bp} = \sigma_{diff}^{Bp} + \sigma_{n-diff}^{Bp} = 54.4 \text{ mb} ,
\]

(2.9)

where diffractive processes contribute 30% in \( \Lambda_{bp} \) collisions and 28% in \( Bp \) interactions. The cross sections for proton and pion collisions of the same energy are 14% and 19% larger, respectively.

Using the relative frequency of these processes to find the average inelasticity, we obtain (see Fig. 3)

\[
K_{\Lambda_{bp}} = 0.19 ; \quad K_{Bp} = 0.18 ,
\]

(2.10)

which is substantially lower than the inelasticity in proton and pion collisions,

\[
K_{pp} = 0.59 ; \quad K_{\pi p} = 0.70 .
\]

(2.11)
2.4 $H$–air collisions

The total cross section for the collision of $H$ with an atomic nucleus of mass number $A$ can be approximated as

$$\sigma^{HA} \approx A^{2/3}\sigma^{Hp},$$

(2.12)

where the factor of $A^{2/3}$ takes into account the screening between the nucleons inside the nucleus. For an averaged atmospheric nucleus of $A = 14.6$ this implies

$$\sigma^{\Lambda_b \text{air}} = 490 \text{ mb}; \quad \sigma^{B \text{air}} = 325 \text{ mb}. \quad (2.13)$$

The associated interaction length $\lambda_{\text{int}}^H = m_{\text{air}}/\sigma^{H \text{air}}$ in the atmosphere is therefore

$$\lambda_{\text{int}}^{\Lambda_b} = 49 \text{ g cm}^{-2}; \quad \lambda_{\text{int}}^{B} = 74 \text{ g cm}^{-2}, \quad (2.14)$$

which is a 14% and a 19% longer than those of a pion and a proton of the same energy, respectively.

To deduce the energy and the species of the $b$ hadron after the collision we follow [4] and distinguish between peripheral and central collisions. We assume that the spectrum in peripheral processes coincides with the one in $Hp$ collisions, whereas the average inelasticity in central processes is the typical in a partonic scattering increased by 10%. In addition, we will take equal frequency for both types of processes. The average inelasticity can then be estimated as

$$K_{H \text{air}} \approx \text{frac}12 K_{H \text{air}}^{\text{peri}} + \frac12 K_{H \text{air}}^{\text{cent}} \approx \left(\frac12 K_{Hp} + \frac12 1.1 K_{Hp}^{-\text{dif}}\right). \quad (2.15)$$

At $E = 10^9$ GeV, we obtain

$$K_{\Lambda_b \text{air}} \approx 0.21; \quad K_{B \text{air}} \approx 0.20. \quad (2.16)$$
For proton and pion collisions the same prescription gives an inelasticity

$$K_{p\text{air}} \approx 0.66; \quad K_{\pi \text{air}} \approx 0.78.$$  \hspace{1em}(2.17)

This 12% increase in $K$ when going from a proton to a nucleus target compares well with the results obtained by other authors \[8\].

In Fig. 4 we plot (thick lines) the final distribution $z$ of the fraction of energy taken by the heavy hadron after the collision of a $\Lambda_b$ or a $B$ with an air nucleus. We also plot (thin lines) our parameterization of the distribution as an exponential between 0 and 1:

$$f(z) = \frac{1}{N} e^{-a(z-z_0)^2},$$  \hspace{1em}(2.18)

where $N$ normalizes the distribution to 1. We obtain $a = 31.3$, $z_0 = 0.799$ and $N = 0.30$ for an incident $\Lambda_b$ baryon and $a = 37.0$, $z_0 = 0.800$ and $N = 0.28$ for an incident $B$ meson.

Finally, we plot in Fig. 5 the frequency of the different hadron species after the collision of a $10^9$ GeV $B$ meson with a proton at rest for different values of the elasticity $z$. We obtain a $B^0$ 42% of the times, $B^-$ mesons appear after 41% of the collisions, whereas the frequency of $B^0_s$ mesons and $\Lambda^0_b$ baryons is 9% and 7%, respectively. In $\Lambda_b$ collisions the approximate frequency of $(\Lambda_b, B^0, B^-, B^0_s)$ provided by PYTHIA is (68%, 14%, 14%, 4%).

3 Propagation in the atmosphere

The propagation of a long-lived $b$ hadron differs from that of a proton or a pion, due to its larger elasticity in collisions with air nuclei and its longer interaction length. Using the Monte Carlo code CORSIKA we analyze the energy deposition of a $b$ hadron produced in the upper atmosphere, detached from the development of its parent shower. We focus on $B^0$
mesons, as Λ_b baryons tend to convert into mesons after a few hadronic interactions. The \( B^0 \) meson may interact\(^1\) (according to the cross section and the inelasticity described in Section 2) or decay\(^2\) as it propagates, producing secondary showers at different atmospheric depths.

As an example, we will take a \( B^0 \) meson produced at a depth of \( X_0 = 100 \text{ g cm}^{-2} \) with a zenith inclination \( \theta = 60^\circ \). We consider different values of the initial energy of the \( B^0 \) meson, \( E_B \). For values of \( E_B \) of the order of \( 10^8 \text{ GeV} \), the \( B^0 \) mesons decay after few interactions and therefore look like a proton shower. When larger energies \( O(10^9 \text{ GeV}) \) are considered, the differences between \( B^0 \) and proton initiated showers become more evident.

To stand out the most relevant features of the showers produced by \( B^0 \) particles, we consider in this section average proton shower profiles. The next section, devoted to the detectability of bottom hadrons produced inside extensive air showers, will deal with individual fully-simulated showers to take into account the effect that shower fluctuations introduce in our analysis.

In Fig. 6 we plot the averaged profile of 100 \( B^0 \) initiated showers (solid) and of the same number of proton showers (dashed). The initial energy for \( B^0 \) and protons is \( 3 \times 10^9 \text{ GeV} \). The average \( B^0 \) decay takes place at \( \langle X_f \rangle = 1190 \pm 340 \text{ g cm}^{-2} \), after \( \langle N_I \rangle = 16 \pm 4 \) interactions with air nuclei. These \( B^0 \) showers are different from a proton shower of the same energy in two important aspects. First, the energy deposition rate in the atmosphere is slower, and the shower maximum is reached later, around \( 1000 \text{ g cm}^{-2} \). Among the 100 showers generated, 52 reach their maximum beyond 1000 g cm\(^{-2}\). Also, secondary showers of energy \( 10^7 \text{–} 10^8 \text{ GeV} \) start deep in the atmosphere (as a result of the decay or the collisions of the heavy hadron), increasing the number of electromagnetic particles that reach the ground.

\(^1\)If it deposits an energy \( \Delta E = (1 - z)E \) in a collision, we will simulate there a CORSIKA pion shower of the same energy \( \Delta E \).

\(^2\)If it decays at a depth \( X \) we treat the decay using PYTHIA and inject the products at that depth using CORSIKA.
Figure 6. Average profile of a p (dashed) and a B (solid) shower for an initial energy of $3 \times 10^9$ GeV.

Figure 7. Anomalous profile of a $3 \times 10^9$ GeV $B^0$ shower (histogram). The solid line corresponds to a fit using a combination of two Gaisser-Hillas functions.

The profile of these showers, in general, cannot be well described by a single Gaisser-Hillas function.

In Fig. 7 we plot as an illustration an anomalous profile found among these 100 showers, namely a longitudinal development featuring a double peak topology. We obtain this feature
in 15 out of 100 simulations. In the same figure we plot (solid) a fit to this shower using the sum of two Gaisser-Hillas.

4 Detectability of $B$ mesons inside extensive air showers

$B$ mesons are not primary particles that may start an air shower. They will be produced in the collision of a primary cosmic ray or an energetic leading hadron in the shower with an air nucleus. Therefore, the $B^0$ shower described in the previous section will always be a component inside a parent shower. Its observability will then depend critically on the fraction $x_b$ of the energy that the $B$ meson carries when it is produced. Values $x_b \geq 0.1$ are interesting. In Fig. 8, we plot for the sake of the argument an extreme example where a CORSIKA-simulated proton primary with energy $E_p = 2 \times 10^{10}$ GeV and zenith angle $\theta = 60^\circ$ produces a very energetic $B^0$ meson in the upper atmosphere, carrying $E_B = 3 \times 10^9$ GeV ($x_b = 0.15$). The rest of the secondary particles produce a shower with energy $E_{p'} = 1.7 \times 10^{10}$ GeV that reaches a maximum around 800 g cm$^{-2}$. At ground level the electromagnetic component of this shower has been almost completely absorbed. The $B^0$ meson, produced at $X_0 = 100$ g cm$^{-2}$ and decaying at $X = 1748$ g cm$^{-2}$ (depth along the proton incoming direction), has a deep maximum and a rich electromagnetic component at the ground. We observe a long tail in the shower profile, with $dE/dx$ values close to the ground that are anomalous in a regular proton shower.

Based on the model described in this work, the simulations show that the inclusion of $b$ hadrons in the shower does not produce a striking deformation of the shower profile, but rather some features rarely found in the development of proton showers. Even so, these showers will be very often similar to those of protons. Thus, the search for a bottom component should be done on the basis of large statistics, trying to determine which are the observ-
ables that provide the maximal separation between proton showers and proton showers with bottom production.

To assess the observability of $b$ quarks inside inclined extensive air showers, we have produced several samples of fully simulated extensive air showers. These CORSIKA showers naturally include fluctuations in the longitudinal development of the cascades, which can mimic the effects produced by a bottom component. All in all, we have fully simulated several thousands of events for background and signal components: 2000 proton showers for each of the following energies $\log_{10}(E/\text{GeV})=\{10, 10.25, 10.5, 10.75\}$ and fixed zenith angle of $\theta = 60^\circ$ stand for our initial background sample. Per each of the former energies, another 2000 proton showers with a bottom component constitute the signal.

In our simulation, heavy quarks are produced only in the first interaction of the incoming proton with an air nucleus, and not in subsequent interactions of the leading or secondary hadrons with the air: their effect in the shower profile will be small due to their smaller energy. We have taken the cross section for bottom production in $p$-air collisions in [9] (they use a dipole picture in the Color Glass Condensate formalism, including saturation effects). It is found, for example, that at $10^{10}$ GeV this cross section is around 25 mb, with the bottom component carrying more than 1% of the initial energy (i.e. $x_{\text{th}} > 0.01$) in a 25% of the collisions. Therefore, in these showers there would be around 1% probability to obtain a bottom component of energy above $10^8$ GeV in the first proton interaction. In our analysis we will take a total set of $n$ showers, where $S_0 = 0.01n$ showers have produced bottom hadrons (with the energy distribution in [9]) and $B_0 = 0.99n$ are regular proton showers. The propagation of the bottom component in these showers is performed according to the results obtained in Section 3.

A thorough examination on an event by event basis of the simulated samples revealed a set of observables, related to the longitudinal development of the electromagnetic part of the shower, that help discriminating signal from background. The most relevant are:

- The logarithm of the number of electromagnetic particles reaching the ground.
- The difference between the depths beyond $X_{\text{max}}$ at which the shower is at 40% and 35% of the shower maximum.
- The difference between the depths before $X_{\text{max}}$ at which the shower is at 70% and 75% of the shower maximum.
- The full width at 5% of the profile maximum.
- The central moments of 3rd and 5th order of the shower profile.
- The depth beyond $X_{\text{max}}$ at which the curvature of the Gaisser-Hillas fit to the profile changes.

To enhance the separation power between signal and background, we trained a Fisher discriminant using the former observables [10]. Applying this discriminant and maximizing the value of $S/\sqrt{S+B}$, where $S$ and $B$ are the number of selected signal and background events, we obtain a signal selection efficiency $E_{S}^{\text{sel}} = 0.165$ and a background rejection efficiency $E_{B}^{\text{rej}} = 0.992$. Thus, the number of selected signal and background events would be $S = 0.00165n$ and $B = 0.00792n$, respectively. This means that, according to our model, to get a $3\sigma$ evidence for production of $b$ hadrons in extensive air showers (with energy above $\approx 10^{10}$ GeV and zenith angle between $55^\circ$ and $65^\circ$) would require to collect around $3 \times 10^4$ extensive air showers.
5 Summary and discussion

Hadrons containing a $b$ quark become long-lived at energies above $10^7$ GeV, and they tend to collide in the air before decaying. We have shown that the inelasticity in those collisions is almost four times smaller than in light-hadron scatterings. As a consequence, these hadrons can cross a large fraction of atmosphere keeping a significant fraction of energy. For example, a $3 \times 10^9$ GeV $B$ meson has an average of 16 interactions before decaying at $X_f \approx 1200$ g cm$^{-2}$. The final energy of the meson is around $10^7.5$ GeV, and it will be all deposited close to the ground. We find that the profile of these showers frequently presents a very deep shower maximum and occasionally a double maximum.

Although $b$ hadrons are rarely produced in extensive air showers, we have also discussed whether they can provide a significant deviation once included in the shower development. We have shown that if the heavy meson decay occurs deep enough in the atmosphere the profile could reveal such a feature. A multivariate analysis of observables defined on the shower electromagnetic profile shows that, according to our model, a very large number of events need to be collected in order to get an evidence of $b$ hadron production in ultra-high energy cosmic rays.

Despite this result, we think that the production and propagation of heavy hadrons should be included in Monte Carlo codes like AIRES [11] or CORSIKA [5], which simulate extensive air showers of energy up to $10^{11}$ GeV. Such an effort seems necessary to establish on a solid ground the expected frequency of events with late energy deposition and the best strategy in the search for observable effects.

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