New results from the quantum statistical approach to parton distributions

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Outline

- **Basic procedure** to construct the statistical polarized parton distributions
- **Essential features** from unpolarized and polarized Deep Inelastic Scattering data
- **Special predictions** for the flavor structure of the sea
- **Helicity asymmetries** for $W^\pm$ production
- **Transverse momentum dependence (TMD) extension**: Transverse energy sum rule. Gaussian shape with no $x, k_T$ factorization. Melosh-Wigner effects mainly in low $x, Q^2$ region
- **Conclusions**
Collaboration with Claude Bourrely and Franco Buccella

- A Statistical Approach for Polarized Parton Distributions
  Euro. Phys. J. C23, 487 (2002)

- Recent Tests for the Statistical Parton Distributions
  Mod. Phys. Letters A18, 771 (2003)

- The Statistical Parton Distributions: status and prospects
  Euro. Phys. J. C41,327 (2005)

- The extension to the transverse momentum of the statistical parton distributions
  Mod. Phys. Letters A21, 143 (2006)

- Strangeness asymmetry of the nucleon in the statistical parton model
  Phys. Lett. B648, 39 (2007)

- How is transversity related to helicity for quarks and antiquarks in a proton?
  Mod. Phys. Letters A24, 1889 (2009)

- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach, Phys. Rev. D83, 074008 (2011)

- The transverse momentum dependent statistical parton distributions revisited
  Int. Journal of Mod. Phys. A28, 1350026 (2013)

- $W^\pm$ bosons production in the quantum statistical parton distributions approach
Our motivation and goals

- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some well known phenomenological facts and some QCD features
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- Will propose a quantum statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.

- Will incorporate some well known phenomenological facts and some QCD features

- Will parametrize our PDF in terms of a rather small number of physical parameters, at variance with standard polynomial type parametrizations

- Will be able to construct simultaneously unpolarized and polarized PDF: A UNIQUE CASE ON THE MARKET!

- Will be able to describe physical observables both in DIS and hadronic collisions

- Will make some very specific challenging predictions, from the behavior of unpolarized and polarized PDF, either in the sea quark region or in the valence region

- Will also consider the case of the elusive polarized gluon distribution
Use a simple description of the PDF, at input scale $Q_0^2$, proportional to 
$[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and *minus* sign for gluons, corresponds to a Bose-Einstein distribution. $X_{0p}$ is a constant which plays the role of the *thermodynamical potential* of the parton $p$ and $\bar{x}$ is the *universal temperature*, which is the same for all partons.

**NOTE:** $x$ is indeed the natural variable, since all the sum rules we will use are expressed in terms of $x$. 
Basic procedure

Use a simple description of the PDF, at input scale $Q_0^2$, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, plus sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and minus sign for gluons, corresponds to a Bose-Einstein distribution. $X_{0p}$ is a constant which plays the role of the thermodynamical potential of the parton $p$ and $\bar{x}$ is the universal temperature, which is the same for all partons.

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From the chiral structure of QCD, we have two important properties, allowing to RELATE quark and antiquark distributions and to RESTRICT the gluon distribution:
- Potential of a quark $q^h$ of helicity $h$ is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity $-h$, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.
- Potential of the gluon $G$ is zero, $X_{0G} = 0$. 
The polarized PDF $q^{\pm}(x, Q_0^2)$ at initial scale $Q_0^2$

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{AX_0^h x^b}{\exp[(x - X_0^h)/\bar{x}] + 1} + \frac{\tilde{A}x^b}{\exp(x/\bar{x}) + 1},$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}(-h)(x, Q_0^2) = \frac{\bar{A}(X_0^h)^{-1}x^\bar{b}}{\exp[(x + X_0^h)/\bar{x}] + 1} + \frac{\tilde{A}x^\bar{b}}{\exp(x/\bar{x}) + 1}.$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$).
Extra term is absent in $\Delta q$ and $q_v$ also in $u - d$ or $\bar{u} - \bar{d}$.
The additional factors $X_0^h$ and $(X_0^h)^{-1}$ are coming from TMD (see below).
The polarized PDF \( q^\pm(x, Q_0^2) \) at initial scale \( Q_0^2 \)

For light quarks \( q = u, d \) of helicity \( h = \pm \), we take

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\]

consequently for antiquarks of helicity \( h = \mp \)

\[
x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A}(X_{0q}^h)^{-1}x^\bar{b}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^\bar{b}}{\exp(x/\bar{x}) + 1}.
\]

Note: \( q = q^+ + q^- \) and \( \Delta q = q^+ - q^- \) (idem for \( \bar{q} \)).

Extra term is absent in \( \Delta q \) and \( q_v \) also in \( u - d \) or \( \bar{u} - \bar{d} \).

The additional factors \( X_{0q}^h \) and \( (X_{0q}^h)^{-1} \) are coming from TMD (see below)

For strange quarks and antiquarks, \( s \) and \( \bar{s} \), use the same procedure which leads to

\[
x s(x, Q_0^2) \neq x\bar{s}(x, Q_0^2) \quad \text{and} \quad x \Delta s(x, Q_0^2) \neq x\Delta\bar{s}(x, Q_0^2) \quad (\text{Phys. Lett. B648, 39 (2007)}).
\]

For gluons we use a Bose-Einstein expression given by

\[
xG(x, Q_0^2) = \frac{AGx^bG}{\exp(x/\bar{x}) - 1},
\]

with a vanishing potential and the same temperature \( \bar{x} \). For the polarized gluon distribution \( x\Delta G(x, Q_0^2) \) we take a similar expression at initial scale (positive for all \( x \))
Essential features from the DIS data

From well established features of $u$ and $d$ extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X_{0u}^+ + X_{0u}^- > X_{0d}^+ + X_{0d}^-$
- $\Delta u(x) > 0$, therefore $X_{0u}^+ > X_{0u}^-$
- $\Delta d(x) < 0$, therefore $X_{0d}^- > X_{0d}^+$. 
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- \( \Delta u(x) > 0 \), therefore \( X_{0u}^+ > X_{0u}^- \)
- \( \Delta d(x) < 0 \), therefore \( X_{0d}^- > X_{0d}^+ \).

So we expect \( X_{0u}^+ \) to be the largest potential and \( X_{0d}^+ \) the smallest one. In fact, from our fit we have obtained the following ordering

\[
X_{0u}^+ > X_{0d}^- \sim X_{0u}^- > X_{0d}^+.
\]

This ordering has important consequences for \( \bar{u} \) and \( \bar{d} \), namely...
Essential features from DIS data

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from Pauli exclusion principle. This was already confirmed by the violation of the Gottfried sum rule (NMC).

- $\Delta \bar{u}(x) > 0$ and $\Delta \bar{d}(x) < 0$, a PREDICTION from 2002, in agreement with polarized DIS (see below) and has been more precisely checked at RHIC-BNL from $W^\pm$ production, already in active running phase (see below).
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- Note that since $u^-(x) \sim d^-(x)$, it follows that $\bar{u}^+(x) \sim \bar{d}^+(x)$, so we have

$$\Delta \bar{u}(x) - \Delta \bar{d}(x) \sim \bar{d}(x) - \bar{u}(x),$$

i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions ($\Delta \bar{u}$ and $\Delta \bar{d}$ contribute to about 10% to the Bjorken sum rule).
Very few free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x, Q^2)$, $F_2^n(x, Q^2)$, $xF_3^\nu N(x, Q^2)$ and $g_{1,p,d,n}^1(x, Q^2)$, in correspondance with ten free parameters for the light quark sector with some physical significance:

* the four potentials $X_{0u}^+$, $X_{0u}^-$, $X_{0d}^-$, $X_{0d}^+$,
* the universal temperature $\bar{x}$,
* and $b$, $\bar{b}$, $\tilde{b}$, $b_G$, $\tilde{A}$. 
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* the universal temperature $\bar{x}$,
* and $b$, $\bar{b}$, $\tilde{b}$, $b_G$, $\tilde{A}$.

We also have three additional parameters, $A$, $\tilde{A}$, $A_G$, which are fixed by 3 normalization conditions:

$$u - \bar{u} = 2, \quad d - \bar{d} = 1$$

and the momentum sum rule.

There are several additional parameters to describe the strange quark-antiquark sector and for the gluon polarization. We use the constraint $s - \bar{s} = 0$.

We note that potentials become smaller for heaviest quarks and since $X_{0s}^- > X_{0s}^+$, we will have $\Delta s < 0$ like for $d$-quarks.
Some data on $F_2^D(x, Q^2)$, $F_2^P(x, Q^2)$
Effect of the evolution on the diffractive term

Both terms become comparable for small $x$, as soon as $Q^2$ takes off.
Some data on $F_2^n(x, Q^2)/F_2^p(x, Q^2)$
Some data on anti-neutrino

CCFR/NuTeV 305GeV
Anti-Neutrino

$\frac{1}{E} \frac{d^3\sigma}{dx dy} \times 10^{-38} \text{ cm}^2/\text{GeV}$

- $x = 0.015$
- $x = 0.045$
- $x = 0.08$
- $x = 0.125$
- $x = 0.175$
- $x = 0.225$
- $x = 0.35$
- $x = 0.45$
- $x = 0.55$
- $x = 0.65$

$y$ range: 0 to 1

New results from the quantum statistical approach to parton distributions – p. 13/37
A global view of the unpolarized parton distributions

\[ Q^2 = 10 \text{GeV}^2 \]

\[ x f(x, Q^2) \]

\( u_v, d_v \) agree well, but \( G \) grows a bit faster at low \( x \)

\[ H1 \text{ and ZEUS} \]

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ x f(x, Q^2) \]

New results from the quantum statistical approach to parton distributions – p. 14/37
$\bar{s}$ agrees well but $\bar{u}$, $\bar{d}$ grow slower and we expect $\bar{d} > \bar{u}$ for all $x$
Unpolarized sea parton distributions from ATLAS + HERA

It grows slower than the data
The predicted $d/u$ ratio versus $x$
The predicted charge asymmetry from BBS, PLB 726, 296 (2013)
The $s(x, Q^2) + \bar{s}(x, Q^2)$ from HERMES (2013)

Comparison PLB666 with HERMES (2013)

$s$-quark distributions mainly in the low $x$ region. Not compatible with HERA data
Important issue: $\bar{d}/\bar{u}$ at large $x$ and high $Q^2$

We look forward to the results of E906
Important issue: $\bar{d}/\bar{u}$ at large $x$ and high $Q^2$

Ratio of $W^\pm$ cross sections: Another possible way to access it
A global view of the polarized parton distributions

\[ Q^2 = 10 \text{GeV}^2 \]

\[ x \Delta f(x, q^2) \]

\[ x(\Delta u + \Delta \bar{u}) \]

\[ x(\Delta d + \Delta \bar{d}) \]

BBS

DSSV

New results from the quantum statistical approach to parton distributions – p. 22/37
A compilation of data on $A_1^p(x, Q^2)$
A compilation of data on $A_1^p(x, Q^2)$.

Comparison with models (thanks to D. Flay)

New results from the quantum statistical approach to parton distributions – p. 24/37
A compilation of data on $A_1^n(x, Q^2)$

New results from the quantum statistical approach to parton distributions – p. 25/37
A compilation of data on $A_1^n(x, Q^2)$

Comparison with models (thanks to D. Flay)
The elusive gluon polarization $x \Delta G(x, Q^2)$

It seems to be concentrated in the low $x$-region. PHENIX and STAR plan to access it.
The ratio $\Delta G(x, Q^2)/G(x, Q^2)$

Comparison with recent COMPASS data
Helicity asymmetry in $W^\pm$ production at BNL-RHIC

Consider the processes $\vec{p}p \rightarrow W^\pm + X \rightarrow e^\pm + X$, where the arrow denotes a longitudinally polarized proton and the outgoing $e^\pm$ have been produced by the leptonic decay of the $W^\pm$ boson. The helicity asymmetry is defined as $A_L = \frac{d\sigma_+ - d\sigma_-}{d\sigma_++d\sigma_-}$. Here $\sigma_h$ denotes the cross section where the initial proton has helicity $h$.

For $W^-$ production, the numerator of the asymmetry is found to be proportional to

$$\Delta \bar{u}(x_1, M_W^2) d(x_2, M_W^2)(1 - \cos\theta)^2 - \Delta d(x_1, M_W^2) \bar{u}(x_2, M_W^2)(1 + \cos\theta)^2,$$

where $\theta$ is the polar angle of the electron in the c.m.s., with $\theta = 0$ in the forward direction of the polarized parton. The denominator of the asymmetry has a similar form, with a plus sign between the two terms of the above expression. For $W^+$ production, the asymmetry is obtained by interchanging the quark flavors ($u \leftrightarrow d$).

We first show below the results of the calculations of the helicity asymmetries, versus the charged-lepton pseudo-rapidity and for a clear interpretation some explanations are required. At high negative $\eta_e$, one has $x_2 >> x_1$ and $\theta >> \pi/2$, so the first term above dominates and the asymmetry generated by the $W^-$ production is driven by $\Delta \bar{u}(x_1)/\bar{u}(x_1)$, for medium values of $x_1$. Similarly for high positive $\eta_e$, the second term dominates and now the asymmetry is driven by $-\Delta d(x_1)/d(x_1)$, for large values of $x_1$. So we have a clear separation between these two contributions.
Helicity asymmetry in $W^\pm$ production at BNL-RHIC (BBS, PLB 726, 296 (2013))

Comparison with preliminary STAR data
Transverse momentum dependence (TMD) of the PDF

How to introduce the TMD of the PDF?

There are several possibilities

- Assume factorization and simple Gaussian behavior for the PDF

\[ q(x, k_T) = q(x) \frac{1}{\pi \mu_0^2} \exp\left[-\frac{k_T^2}{\mu_0^2}\right], \]

and also for the fragmentation function

\[ D(z, q_T) = D(z) \frac{1}{\pi \mu_D^2} \exp\left[-\frac{q_T^2}{\mu_D^2}\right]. \]

A naive assumption which has no theoretical justification

- No factorization: Covariant approach, derivative method

- No factorization: The statistical distributions for quarks and antiquarks
The parton distributions $p_i(x, k_T^2)$ of momentum $k_T$, must obey the momentum sum rule

$$\sum_i \int_0^1 dx \int x p_i(x, k_T^2) dk_T^2 = 1 ,$$

and also the transverse energy sum rule

$$\sum_i \int_0^1 dx \int p_i(x, k_T^2) \frac{k_T^2}{x} dk_T^2 = M^2 .$$

From the general method of statistical thermodynamics we are led to put $p_i(x, k_T^2)$ in correspondance with the following expression

$$\exp(-\frac{x}{\bar{x}} - \frac{k_T^2}{x \mu^2}) ,$$

where $\mu^2$ is a parameter interpreted as the transverse temperature.

So we have now the main ingredients for the extension to the TMD of the statistical PDF. We obtain in a natural way the Gaussian shape with NO $x, k_T$ factorization.
The quantum statistics distributions for quarks and antiquarks read in this case

\begin{align*}
    xq^h(x, k_T^2) &= \frac{F(x)}{\exp(x - X_{0q}^h)/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 - Y_{0q}^h) + 1}, \\
    x\bar{q}^h(x, k_T^2) &= \frac{\bar{F}(x)}{\exp(x + X_{0q}^{-h})/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 + Y_{0q}^{-h}) + 1},
\end{align*}

where

\begin{align*}
    F(x) &= \frac{Ax^{b-1}X_{0q}^h}{\ln(1 + \exp Y_{0q}^h)\mu^2} = \frac{Ax^{b-1}}{k\mu^2},
\end{align*}

because $Y_{0q}^h$ are the thermodynamical potentials chosen such that

\[
    \ln(1 + \exp Y_{0q}^h) = kX_{0q}^h,
\]

in order to recover the factors $X_{0q}^h$, introduced earlier. Similarly for $\bar{q}$ we have $\bar{F}(x) = \bar{A}x^{2b-1}/k\mu^2$. This determination of the 4 potentials $Y_{0q}^h$ can be achieved with the choice $k = 3.05$. Finally $\mu^2$ will be determined by the transverse energy sum rule and one finds $\mu^2 = 0.198\text{GeV}^2$. 

(TMD) in the statistical approach
The statistical distributions $u$ and $d$ vs $k_T$
Melosh-Wigner effects

So far in all our quark or antiquark TMD distributions, the label "$\hat{h}$" stands for the helicity along the longitudinal momentum and not along the direction of the momentum, as normally defined for a genuine helicity. The basic effect of a transverse momentum $k_T \neq 0$ is the Melosh-Wigner rotation, which mixes the components $q^\pm$ in the following way

\[
q^{+\text{MW}} = \cos^2 \theta \, q^+ + \sin^2 \theta \, q^- \quad \text{and} \quad q^{-\text{MW}} = \cos^2 \theta \, q^- + \sin^2 \theta \, q^+ ,
\]

where, for massless partons, $\theta = \arctan \left( \frac{k_T}{p_0 + p_z} \right)$, with $p_0 = \sqrt{k_T^2 + p_z^2}$.

It vanishes when either $k_T = 0$ or $p_z$ goes to infinity.

Consequently $q = q^+ + q^-$ remains unchanged since $q^{\text{MW}} = q$, whereas we have $\Delta q^{\text{MW}} = (\cos^2 \theta - \sin^2 \theta) \Delta q$. 

New results from the quantum statistical approach to parton distributions – p. 35/37
Predicted quark helicity distributions

The effect is relevant for small $Q^2$ and mainly in the low $x$ region.
Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All unpolarized and polarized distributions depend upon a small number of free parameters, with some physical meaning.
- New tests against experiments in particular, for unpolarized and polarized sea distributions, are very satisfactory. *s-quark distributions remain a problem*
- Gluon helicity distribution is concentrated in the low $x$-region. Need to be confirmed
- Good predictive power but some special features remain to be verified, specially in the high $x$-region.
- Extension to TMD has been achieved and must be checked more accurately together with Melosh-Wigner effects in the low $x$-region.