Acceleration of repetitive units in construction projects using line-of-balance and linear scheduling with singularity function

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Abstract. Construction managers usually demand to increase the speed of completion of projects to avoid the penalties, to get the bonus from the owner. They plan and control qualitative and quantitative measures for the successful projects. Generally acceleration measures are followed to increase the speed of delivery of the project. Repetitive units in projects plays an important role in modern world such that most of the residential villa projects have some repetitive nature. This paper proposes a method for the acceleration of repetitive units in project by line-of-balance and linear scheduling with singularity function. Line-of-balance method is used for the programming and control of repetitive projects. Linear schedule is focused on the time-amount relationship and progress of the work. Singularity functions were previously used for the analysis of structures and lately applied for scheduling. Linear schedules associate the information on amount of work and time for each activity. This paper put forward a relationship between production rate, duration of completion of project and number of units. It helps to determine the required production rate to complete a particular number of units in a given duration. The analysis of repetitive units in project has been done, and the general equations are formulated for connecting the duration of project, total number of units, duration required for any activity and number of units completed.

Keywords: Repetitive projects; acceleration; linear scheduling; singularity function; productivity; line-of-balance; production rate.

1. Introduction
The projects that contain identical or similar units are usually referred to as repetitive project units [1]. This exclusive property brings out the noticeable savings on time and cost by managing the crew allocation and different resources involved in repetitive project units. The continuity in work progress in repetitive units leads savings in time and cost through maintaining a fixed crew by possessing skilled labor and minimizing firing and hiring of labor. Continuous activities are pervasive in repetitive construction projects like residential villa units and pipeline units, which needs the sufficient supply of resources across the repetitive project units. Though managing work continuity develops a supplementary constraint when planning, scheduling and executing repetitive units in projects. Therefore, using of conventional planning and scheduling tools to handle repetitive units is antiquated extensively, which shows the requirement of finding out the new tools and some special techniques.

The critical path method (CPM) was created by Morgan R. Walker and James E. Kelley in the late 1950s as a scheduling modelling technique for project works. It has some major disadvantages over repetitive units. Firstly, to perform a project, CPM network needs a large number of activities which produces crucial visualization of the project. Second, the CPM does not assure managing of work continuation. Repetitive projects commonly have a less number of activities which helps proper
scheduling and acceleration techniques reasonably to make a better privilege in comparison with normal conventional project units. Consequently, different methods are available for achieving scheduling and acceleration operations in each stage of the projects. A linear optimization approach is proposed by Amraoui [2] for different scheduling problems. An advanced linear scheduling program with different production rate was developed by Duffy [3] for linear construction projects. The functioning of a new algorithm was validated [4] for linear scheduling with singularity function.

An algorithm was created by Ibrahim Bakry in 2014 for schedule updating, dynamic rescheduling and optimized acceleration of repetitive project units [5]. The exact progress on site was captured by schedule updating and dynamic rescheduling helps to obtain the repetitive nature of units in project to regulate the rest portion of the project. Scheduling of repetitive project units by using various crews is symbolic mission for project planners, who may manage the number, size of crews, productivity and their lag or lead. Linear scheduling of repetitive units with various crews could be done by using productivity scheduling method with the help of singularity function and line of balance method [6]. Presently, the repetitive nature of works can be analyzed by Linear Scheduling Method (LSM) and Line of Balance (LOB). These two techniques have complementary merits over other methods so that they can be related efficiently. The comparison of linear schedule with line-of-balance concepts for the construction industry was done by Yi Su [7]. A new methodology for modelling linear schedules with singularity functions were proposed by Lucko [8].

Linear schedule analysis with singularity functions have been used previously for problems related to the structural analysis and are lately utilized to scheduling, helped Lucko to develop a new integrated method [9]. Linear schedules gives the information on work amount for each activity and the corresponding time needed to complete the particular activity. The new method leads to the consolidation of linear schedules after considering all constraints and consequently, it generates the overall minimum project duration. Singularity function could be used as a new tool for integrated project management [10]. The aligned two dimension model of project execution specifications, including linear schedules, resource histogram, and cash flows, which contribute an analogous mathematical structure when considered as singularity function. Cumulative and non-cumulative types of the sub-system models are explored and it is found that non-cumulative functions are extracted from cumulative to evaluate the rate of change of project execution specifications. The three dimensional models are integrated from two dimensional models, 3D models give a knowledgeable illustration of the project [10]. An equivalent approach for linear scheduling from classic critical path method was demonstrated [11]. Hisham M was developed a new method for spatial scheduling model [12] with singularity function for analyzing the progress of different activities.

The characteristics of linear construction projects are demonstrated by Liu [13], appreciable exploration has been done on continuity of work and the productivity. Even though, major researches are dealt with single skilled crews employing on linear projects, avoiding the resilience of multi skilling in construction, which leads to increased resilience, productivity and continuity in work. A duration optimized model has put forward to implement the concept of multi skilling to integrate single or multi skilled crews to increase the progress of work. Spreadsheets are adequate computational tool used widely with large range of estimations [14]. A spreadsheet algorithm was developed by Agrama in 2011, connecting the logical lag between sequential non-typical repeated activities. The schedule times for sequential activities to one unit to another are summarized and displayed in a LOB graph. Resource constrained scheduling model for sequential repetitive units with time based production crews, are divided into space segments [15]. The length of the segment is unified by the rate of production whereas the crew units would sustain the same rate of production in each space segment.

This paper proposes a method to understand the mutual relations within project duration, completion rate and production rate of construction projects with repetitive nature of construction projects.
2. Linear scheduling method with singularity function

Linear scheduling method is a project management technique of creating a schedule that displays a time axis and an axis showing produced amount of work in a coordinate system. The values on both two axes are cumulative. LSM, a graphical scheduling method concentrating on continuous resource utilization in repetitive activities and shows a simple diagram to show location and time at which a certain crew will be working on a given operation. The evolution of sequential activities produces an assembly of inclined lines in the LSM graphical representation. The ratio of amount of work per time gives the productivity, which is inverse of the slope obtained from the graphical representation of LSM.

Associated concepts are recognized under different names, consisting of linear scheduling model by Harmelink and Rowings [16], repetitive scheduling method by Harris and Ioannou [17], linear scheduling method by Chrzanowski and Johnston [18], line of balance by Arditi and Albulak[19], time space scheduling method by Stradal and Cacha [20], velocity diagram by Dressler [21], vertical production method by O’Brien [22] and several others. Although each of these methods alter considerably in their terminology, focus and analysis steps. Even though commonly they have two dimensional nature of the schedule combines amount and time.

Linear scheduling method have some unique characteristics.
(a) They describe the repetitive nature of the construction work;
(b) They can be shown the progress of the work very easily;
(c) They give the sequence of different work activities which are easily understandable;
(d) They have fairly high level of details;
(e) They can be created and arranged in a shorter time period than different configurations;
(f) They show the maximum resource utilization and
(g) They minimize the intrusion in on-going procedure, including enlisting and terminating.

There are different types of projects when linear scheduling can be applied. They are geometrically horizontal linear projects like pipelines, highways and tunnels, vertical projects like towers and high-rise buildings and the projects having repetitive operations.

Singularity function, a discontinuous function which does not possess a derivative. These functions were initially used to study the beam issues in structural engineering, are loaded with different types of loads, which are placed at various or dispersed regions on the beams. Singularity functions are having several desirable properties which are best suitable for the analysis of linear schedule.

The following section proposes the sample model and terminology of singularity function to linear schedules of repetitive construction projects. Equation (1) provides the equation for singularity function for modeling linear schedules of sequential activities.

![Figure 1. Sample model for linear schedule (graphical representation)](image)
\[ t(a) = t_0 (a-0)^0 + \frac{t_1-t_0}{a_1-a_0} (a-0)^1 \]  \hspace{1cm} (1)

Where, \( t \) = time variable of an activity on y axis; \( a \) = amount variable on x axis; \( t_0 \) = y intercept; \( a_0 \) = x intercept and \( t_1, a_1 \) = pairs of coordinates.

2.1 Advantages of linear scheduling with singularity function

Singularity functions are a group of functions with a few scientific properties that are alluring for the depiction and analysis of linear schedules. The linear schedule with singularity function have some valuable advantages over normal CPM method.

(a) They catch any adjustments in progress over time and amount.
(b) They can incorporate endlessly numerous segments of various behavior.
(c) They can be differentiated and integrated like regular functions.
(d) They can be scaled with any factor and are autonomous of units
(e) They provide the productivity directly.

2.2 Preparation of linear schedule

A residential villa project having 32 repetitive units was considered for the study of repetitive nature of construction projects. Each unit have ten sequential activities and each activity have specific duration. The linear schedule is prepared by assuming a constant amount of work has done for each activity, and the work is continuous with a buffer time of one day. The graph is plotted with start time and finish time of an activity, and the work is assumed to be continuous. A single crew is used throughout the project for the preparation of linear schedule. The linear schedule of the project is shown in figure 2.

![Figure 2. Linear schedule for repetitive units](image)

The singularity function equations based on equation (1) are demonstrated from equation (2) to equation (11) for ten sequential activities.

\[ t_1 = 0. (a-0)^0 + \frac{74}{1000} (a-0)^1 \]  \hspace{1cm} (2)
\[ t_2 = 76. (a-0)^0 + \frac{122}{1000} (a-0)^1 \]  \hspace{1cm} (3)
\[ t_3 = 124. (a-0)^0 + \frac{210}{1000} (a-0)^1 \]  \hspace{1cm} (4)
\[ t_4 = 212. (a-0)^0 + \frac{274}{1000} (a-0)^1 \]  \hspace{1cm} (5)
\[ t_5 = 276. (a-0)^0 + \frac{380}{1000} (a-0)^1 \]  \hspace{1cm} (6)
\[ t_6 = 382. (a-0)^0 + \frac{470-382}{1000}. (a-0)^1 \] (7)

\[ t_7 = 472. (a-0)^0 + \frac{538-472}{1000}. (a-0)^1 \] (8)

\[ t_8 = 540. (a-0)^0 + \frac{622-540}{1000}. (a-0)^1 \] (9)

\[ t_9 = 624. (a-0)^0 + \frac{816-624}{1000}. (a-0)^1 \] (10)

\[ t_{10} = 818. (a-0)^0 + \frac{904-818}{1000}. (a-0)^1 \] (11)

The first part of the second portion of equation (1) provides the slope of the line in the graphical representation of linear schedule. The first derivative of singularity function gives the slope and the inverse of slope provide the productivity rate. The values of slope and corresponding productivity rate are given in Table 1. The first derivative of the equation (2) to equation (11) gives the slope mathematically. Use of singularity function provides a mathematical authority to the linear schedule.

| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|---|---|---|---|---|---|----|
| Slope    | 0.074 | 0.048 | 0.086 | 0.062 | 0.104 | 0.088 | 0.066 | 0.082 | 0.192 | 0.086 |
| Productivity | 13.51 | 20.83 | 11.627 | 16.129 | 9.615 | 11.36 | 15.15 | 12.195 | 5.208 | 11.627 |

2.3 Transformation of linear schedule into line-of-balance

The linear schedule was converted into the form of line-of-balance diagram by fixing amount as constant and duration required to complete each activity is transformed from linear to bar chart model, called as line-of-balance diagram, shown in figure 3. Line-of-balance is best suited for the projects having repetitive nature [15]. The width of the bar for each activity in the LOB represents the duration of each activity to complete a single unit. LOB, an alternate method from linear schedule that provides the operation balance so that each activity is performed continuously. It provides the information about project duration and completion rate in an easiest way as a graphical representation.

![Figure 3. Line-of-balance of repetitive units](image-url)
the duration and \( t \) gives the duration of completion within the range of first and last unit, from figure 3, shown from equation (12) to equation (21).

\[
U_1 = 1 + \left( 31 \times \frac{t_1-10}{64-0} \right) \quad (12)
\]

\[
U_2 = 1 + \left( 31 \times \frac{t_2-66}{96-66} \right) \quad (13)
\]

\[
U_3 = 1 + \left( 31 \times \frac{t_3-84}{156-84} \right) \quad (14)
\]

\[
U_4 = 1 + \left( 31 \times \frac{t_4-124}{172-124} \right) \quad (15)
\]

\[
U_5 = 1 + \left( 31 \times \frac{t_5-140}{230-140} \right) \quad (16)
\]

\[
U_6 = 1 + \left( 31 \times \frac{t_6-172}{246-172} \right) \quad (17)
\]

\[
U_7 = 1 + \left( 31 \times \frac{t_7-220}{288-220} \right) \quad (18)
\]

\[
U_8 = 1 + \left( 31 \times \frac{t_8-246}{306-246} \right) \quad (19)
\]

\[
U_9 = 1 + \left( 31 \times \frac{t_9-270}{442-270} \right) \quad (20)
\]

\[
U_{10} = 1 + \left( 31 \times \frac{t_{10}-380}{456-380} \right) \quad (21)
\]

From the above equations (12) to (21), two equations are generalized to find out the number of units that can be completed for any activity with the time within the range of duration of completion of first unit and last unit (equation (22)) and to find out the duration of completion of any activity of any unit (equation (23)).

\[
U_i = 1 + \left[ (n-1) \times \frac{t_i-t_{i1}}{t_{in}-t_{i1}} \right] \quad (22)
\]

\[
T_i = t_{i1} + \left[ \frac{(U_n-U_1)x(t_{in}-t_{i1})}{n-U_1} \right] \quad (23)
\]

Where, \( U_i \) = number of units for a particular activity \( i \) is completed in the time \( t_i \) which must be within the range of duration required to finish first unit and last unit; \( T_i \) = duration of completion for any activity \( i \) corresponding to any unit; and \( t_{in} \) = duration of completion for activity \( i \) for \( n^{th} \) unit.

3. Acceleration of repetitive units

The productivity could be increased by managing slopes from line-of-balance representation. Practically these procedures were done by employing more product crews to the activities which are having less slope or by assigning overtime work for labors or by making double shift work or by providing the schedule for working in weekends. An example of four units was taken for the illustration. Each unit is having four continuous activities, the management of slopes would be analyzed by the criteria such that the lesser the slope value in linear schedule, the higher the rate of productivity. But the higher value of slope in LOB diagram gives the higher production rate. The LOB representation of four units of four activities are shown in figure 4 and the corresponding slopes are given in Table 2. On comparing the results of slopes from figure 4, the first activity has higher slope value so that the production rate of first activity is higher than other three activities. The duration of these three activities were reduced by keeping the number of units is constant, according to the above described methods such as employing more product crews.
Table 2. Slope for 4 activities

| Activity | 1   | 2    | 3    | 4   |
|----------|-----|------|------|-----|
| Slope (production rate) | 0.1 | 0.0556 | 0.0714 | 0.0625 |

Figure 4. LOB for four units

The production rates of all four activities were became approximately nearest values to maintain the same acceleration throughout the project. The modified LOB diagram shown in figure 5 and production rates are shown in table 3.

Table 3. Slope for 4 activities after acceleration.

| Activity | 1   | 2   | 3    | 4   |
|----------|-----|-----|------|-----|
| Slope (production rate) | 0.1 | 0.11 | 0.1428 | 0.125 |

If the owner needs to complete the project in a particular duration, the above described method leads to the trial and error method. So, a modified acceleration method is proposed in this work to satisfy the owner’s requirement. The acceleration of the project can be done by increasing the production rate. Completion rate or production rate is defined as the ratio between number of units completed and the duration of completion. The acceleration of repetitive units is the reduction of duration of completion between first unit and the last unit. After the completion of first unit, the acceleration procedures were done to optimize the duration of completion.

Figure 5. LOB for 4 units after managing slope
The following section proposes a new mathematical equation for applying the significance of repetitive units with the line-of-balance diagram of the construction project. Equation (24) provides the mathematical correlation of production rate with duration of completion of any unit and the duration of completion of first unit. This equation is valid when the difference of durations of last unit with the first unit is more than the difference of required durations of last unit and first unit. In the cases of differences of durations which are less than or equal to the required difference of durations, the production rate will be less than the initial production rate. The corresponding production rates for all activities, based on equation (24), from the duration of completion details, illustrated in table 4.

\[ P = \frac{N-1}{D-t} \]  

(24)

Where, \( P \) = production rate; \( N \) = number of units; \( D \) = duration of completion at any unit; and \( t \) = duration of completion for first unit.

The values of duration of completion of the activity of first unit is required to find out the production rate of an activity. The production rates of all ten activities were determined for 32 units by using the equation for production rate (equation (24)), are shown in table 4.

The above acceleration method works according to the production rate of final activity for a number of units. The production rates for 15 units are also determined for demonstrating the acceleration procedures in repetitive units using the equation (24). The values of production rates for 32 units and 15 units are approximately equal due to the same slopes.

The production rate of the construction project can be adjusted according to the requirement of owner or contractor, based on the equation (24). The initial duration of completion and production rate of last activity for 15 units are 414 days and 0.41 number of units per day respectively from the table 5. By taking an example, the owner wants to complete the project within 340 days and also the first unit should be completed in 310 days. The production rate for 15 units of the tenth activity obtained is 0.46 as shown in table 5. A difference of 0.05 in production rate is noted. For all the preceding activities, the corresponding change in production rate is applied to accelerate the total project by accelerating other activities together to achieve required duration of completion. A change of 0.05 is given to the all preceding activities of tenth activity. The corresponding changes in the production rate of activities are proposed the new duration of completion for all activities, which is capable of decreasing the duration of project.

This method can be used to accelerate all types of repetitive units in project whereas the criteria specified for the equation (24) mandatorily satisfied.

| Activity | Duration of completion | Duration for first unit | Production rate |
|----------|------------------------|-------------------------|-----------------|
| 1        | 74                     | 10                      | 0.48            |
| 2        | 96                     | 66                      | 1.03            |
| 3        | 156                    | 84                      | 0.43            |
| 4        | 172                    | 124                     | 0.65            |
| 5        | 230                    | 140                     | 0.34            |
| 6        | 246                    | 172                     | 0.42            |
| 7        | 268                    | 220                     | 0.65            |
| 8        | 306                    | 246                     | 0.52            |
| 9        | 442                    | 270                     | 0.18            |
| 10       | 456                    | 380                     | 0.41            |
Table 5. Accelerated production rate and corresponding changes in duration for 15 units

| Activity | Duration of completion | Duration for first unit | Production rate | Accelerated production rate | Duration of completion after acceleration | Duration for first unit after acceleration |
|----------|------------------------|-------------------------|----------------|-----------------------------|------------------------------------------|------------------------------------------|
| 1        | 40                     | 10                      | 0.47           | 0.52                        | 37                                       | 10                                       |
| 2        | 80                     | 66                      | 1.00           | 1.05                        | 77                                       | 63                                       |
| 3        | 118                    | 84                      | 0.42           | 0.47                        | 106                                      | 76                                       |
| 4        | 146                    | 124                     | 0.64           | 0.69                        | 136                                      | 115                                      |
| 5        | 180                    | 140                     | 0.35           | 0.40                        | 158                                      | 123                                      |
| 6        | 206                    | 172                     | 0.42           | 0.47                        | 185                                      | 154                                      |
| 7        | 242                    | 220                     | 0.64           | 0.69                        | 225                                      | 204                                      |
| 8        | 272                    | 246                     | 0.54           | 0.59                        | 249                                      | 226                                      |
| 9        | 348                    | 270                     | 0.18           | 0.23                        | 273                                      | 212                                      |
| 10       | 414                    | 380                     | 0.41           | 0.46                        | 340                                      | 310                                      |

4. Conclusion

The general equations and relationships, which connect duration of completion of project, production rate and number of units were formulated in this work. The major conclusions proposed are following:

(1) A mathematical correlation of production rate with duration of completion and number of units is formed.

(2) General equations are formed to find out the following:
   (a) In how many units a particular activity is completed in the specified time.
   (b) Duration of completion of an activity corresponding to any unit.

(3) Acceleration of repetitive units in projects can be done effectively with the help of linear schedule and line-of-balance.

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