Gearbox bearing fault simulation using a finite element model reduction technique

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Abstract. The dynamics of a mechanical system such as a gearbox assembly comprising shafts, gears and bearings can be simulated using Lumped Parameter Models (LPMs). Finite Element Method (FEM) reduction techniques based on the Craig-Bampton method of Component Mode Synthesis (CMS) are useful in creating more accurate dynamic models. These models, despite having more degrees-of-freedom for the individual components than the LPM, make very much larger FE models computationally tractable. In this paper both these approaches, namely LPM and reduced FEM, are compared to create a dynamic model of a gearbox. Earlier simulation models (both LPM and combined LPM and reduced FEM) are further improved to better match the geometry of the bearing faults used in the experimental measurements, and the experimental results from a gearbox test rig. The dynamic model is used to simulate the vibration signals in the presence of localised inner and outer race faults. The new results show better correspondence with the measured signals, in particular with respect to the detailed response to entry and exit from the fault, which can be used to determine fault size. The paper highlights the plausibility of fault simulation in Machine Condition Monitoring (MCM) where a large amount of data can be gathered without experiencing large numbers of actual failures or carrying out costly and time consuming experiments until failure with seeded faults. The simulation data can be used to train neural networks to automate the diagnostic and prognostic processes.

1. Introduction

Machines utilizing geared transmissions are found extensively in all types of engineering industries. A typical gearbox is characterized by the complex dynamic interaction among the individual components such as gears, shafts, bearings and the casing. Rolling element bearing failure as a result of fatigue is one of the most frequent causes of machine breakdown. An effective machine condition monitoring (MCM) system should not only be able to detect and diagnose the faults but also be able to predict the expected remaining useful life (RUL) of the component – also known as prognosis of faults.

Dynamic modeling is an effective tool to generate vibration signals in the presence of bearing defects. Fault simulation allows us to vary the location and size of the faults and to generate large amounts of data without the need for the costly and time consuming experiments or real life failures. The gathered data can in principle be used to train neural networks to automate the diagnostic and prognostic processes.

Lumped Parameter Models (LPMs) are widely used to represent the dynamics of the gearbox system and to study the behaviour of gears and bearings in the presence of nonlinearities and
geometrical faults [1, 2]. However, the flexibility of the casing, which is an important consideration in lightweight structures (such as helicopter gearboxes), cannot be taken into account due to the limited number of degrees of freedom (DOFs) in the LPMs. This results in poor spectral matching over a wide frequency range, and does not reproduce the interaction of internals and casing, in particular with extended faults [2]. A first approach to solving this problem was to generate the forces at the bearings and gear mesh points and apply these to the frequency response functions (FRFs) derived from the full FE model of the casing [3]. This gave improved spectral matching, but did not properly represent the interaction of the bearings and casing for extended faults.

The limitations of the LPM model can be addressed by combining the LPM with a reduced finite element (FE) model of the casing [4, 5]. An FE model reduction technique based on the Craig-Bampton method of Component Mode Synthesis (CMS) can be used to extract greatly reduced mass and stiffness matrices of the gearbox components based on a small number of spatial DOFs (where interactions occur) and very efficient modal DOFs for the rest. The Craig-Bampton method has the advantage of effectively determining the frequency range over which accurate simulations are made, by specifying the number of modes for the modal DOFs. The reduced mass and stiffness matrices are imported into the dynamic model of the gearbox previously developed using Simulink® [1]. The flexibility of the casing is accounted for in the combined LPM - reduced FE model as it contains more DOFs for the gearbox casing, resulting in the improved representation of the interactions of the bearing faults with the flexible casing. The forces derived from this model were again applied to the casing FRFs.

This paper highlights the fault simulation results using updated fault geometry (where the fault entry and exit curvatures are identical to the radius of the rolling element) for both the LPM and the combined model (LPM - reduced FEM of the casing). The simulations were carried out in the presence of bearing localised inner and outer race faults and compared with the measured signals. The acceleration signals were further investigated to identify a step response at the fault entry and an impulse response at the fault exit, the separation between the two being a measure of (half) the spall size [6].

2. Gearbox test rig

The dynamic simulation model was based on the UNSW gearbox test rig (figures 1 and 2) which was built initially to study the effect of gear faults on transmission error [7]. The test rig consists of a single stage gearbox (in this case using spur gears with 1:1 ratio and 32 teeth on each gear) and is driven primarily by a 3-phase electric motor, but with circulating power via a hydraulic pump/motor set. The input and output shafts are arranged in parallel and each shaft is supported by two double row self-aligning ball bearings. The flywheels are used to reduce the fluctuations of the input and output shaft speeds with couplings to attenuate the shaft torsional vibration.

The bearings under test (figure 3) were double row self-aligning (Koyo 1205) with a contact angle of 0°, a ball diameter of 7.12 mm and a pitch diameter of 38.55 mm. A localised fault (notch) of 0.8 mm width was introduced into the bearing inner and outer race using electric spark erosion (figures 4 and 5) spanning over half of the race width so that only one set of balls was impacting with the fault.

Both the simulations and the measurements were carried out in the presence of localised inner and outer race faults with a 50 Nm load at 10 Hz shaft frequency. The vibration signal was recorded by positioning an accelerometer on top of the gearbox casing above the defective bearing. The signals were sampled at 65 536 Hz. The output of the torque transducer was verified through calibration and was used to measure the torque at the input shaft. The approximate ball pass frequency of the inner race (BPFI) was 71.1 Hz and that of outer race (BPFO) was 48.9 Hz.
3. Dynamic simulation – Lumped Parameter Model (LPM)

The dynamic simulation of the gearbox test rig was carried out based on a 34 DOF model developed earlier [1]. The model accounts for time varying nonlinearity of gear and bearing stiffness and random slippage in the bearings. The model is capable of simulating bearing faults (both localised and extended faults in the inner and outer race) in addition to spalls and cracks in the gears. Only two degrees of freedom were used to represent the casing, a low frequency rigid body mode and a high frequency resonance at 15 kHz excited by the bearing faults. The LPM model was found to be adequate to simulate localised bearing faults [1]. However, the limited representation of the casing resulted in the poor spectral matching over a wide frequency range and lack of interaction with the bearing faults (experienced in practice) mainly in case of extended faults. The spectral matching was improved considerably by applying the generated forces to the casing FRFs.
4. FE model reduction

We have previously applied a number of reduction techniques to this problem, and found that the Craig-Bampton (CB) method [8] was most appropriate because it uses a combination of spatial coordinates defining interactions between the internals and the casing (where the interaction is non-linear and time-varying), and modal coordinates for the rest giving a minimum total number of DOFs for a given frequency range.

In this method, the CB modes of the structure are derived assuming that the master degrees of freedom are held fixed. It enables defining the frequency range of interest by retaining only the modes up to a defined upper limit. Note that the low order modes are greatly affected by the restraints applied to the master DOFs, but boundary conditions have little effect on the frequency of the high order modes defining the frequency range. The decomposition of the model into both physical DOFs (master DOFs) and modal coordinates allows the flexibility of connecting the finite elements to other substructures, while maintaining a reasonably good result within a specified frequency range. Our earlier studies had shown that the simple LPM gave good results at high frequencies, and the interaction problems were primarily in the mid frequency range.

5. Combined LPM - reduced FE model

The FE model of the gearbox casing (with 104340 DOFs) is shown in figure 7 and was modelled using both solid and shell elements. The support of the casing by rubber pads was simulated using spring elements at the corners of the casing. The model has been compared with experimental modal testing and validated for lower frequency modes [9]. The centre node of each bearing was selected as a master degree of freedom which enables connecting the reduced model of the casing with the LPM model of the internals, thereby capturing the flexibility of the casing.

The reduced mass and stiffness matrices of the casing (obtained using the Craig-Bampton based CMS reduction method) were combined with the LPM model in the Simulink environment. The combined model was created by re-writing the equations of motion at the connecting nodes. The LPM model of the internals had 22 DOFs. The reduced casing model had 124 DOFs which were made up of 24 master DOFs corresponding to 4 bearing centre nodes (each with 6 DOFs) plus 100 modes with master DOFs fixed. This resulted in a new combined model with a total of 146 DOFs (46 physical DOFs and 100 modal or generalised coordinates).
In order to improve the accuracy of the responses of the combined LPM - reduced FE model and extend this to a higher frequency range, the forces from the LPM - reduced casing model were extracted and convolved with the impulse response of the casing (obtained from the FRFs), as described in [3].

Figure 7. FE model of gearbox casing.

6. Updated fault geometry
The validity of vibration signals obtained from the seeded bearing faults depends on the realistic representation of the fault geometry in the simulation. The earlier models (figure 8) assumed sudden loss of contact as the rolling element entered the spall region and instant regaining of the contact when exiting the fault resulting in the large impulsive force [10]. In an attempt to model arbitrary fault geometries, in particular for extended faults, the geometry was specified as a function of roll angle by defining the depth of the spall as a function of rolling element position, and this was attempted for local faults [11] but modelling errors were later discovered. In fact, these models resulted in two impulses which were incorrectly believed to be the fault entry and exit, and it was subsequently found that the spacing between the two impulses was not related to the fault width. The dynamic model analysed in this paper includes a further update of the fault geometry where the curvature at both the entry and exit corresponds to the radius of the rolling element (figure 9). Note that the two curves are actually circular (corresponding to the radius of the rolling element), but the depth is distorted.

Figure 8. Fault geometry used in earlier simulations. Figure 9. Updated fault geometry.

Studies have indicated that there are two parts to the vibration signal from a local spall in a bearing, the first originating from the entry of the rolling element into the fault (de-stress) and the second due to exit from the fault (re-stress). The resulting acceleration can normally be described as a low frequency step response at the entry since the sudden change in curvature at the entry represents a step in acceleration. At the spall exit, the centre of the rolling element would have to change direction suddenly resulting in a step in velocity or an impulse in acceleration [6].

7. Results and Discussion
The dynamic simulation models namely the LPM and the 146 DOF (combined LPM - reduced FE) model developed previously and described in the earlier sections, were analysed using the updated
fault geometry (figure 9) where the entry and the exit curvatures of the fault correspond to the
diameter of the rolling element. For all models, the Simulink simulation was used to estimate forces at
the junction points (bearings and gear teeth) and these were then applied to the full size FE model of
the casing to estimate responses over a wider frequency range than that for the forces. For example,
the LPM only had a limited number of resonances of the internals, plus the two of the casing, while
the reduced FE model only included casing resonances up to 4 kHz (for generating the forces) but the
casing resonances up to 20 kHz were used for the estimated responses.

7.1. Localised inner race fault
The comparison of the Power Spectral Density (PSD) spectra for the Test data, LPM and 146 DOF
models is shown in figures 10a, 11a and 12a. The presence of localised inner race fault results in a dB
increase between the good and the faulty bearing in the high frequency region for all three cases. The
(corresponding squared envelope spectra clearly identify BPFI harmonics of 71 Hz (Ball Pass
Frequency Inner Race) which correlate well with the test results [figures 10b, 11b and 12b]. The
modulating sidebands of 10 Hz shaft frequency are clearer in the simulation models than in the test
data. The PSD spectra do not match well between 2-4 kHz, and the reason for this is being
investigated. It has little effect on the high frequency responses to localized faults, which are the
primary objective of this paper. At high frequency, the LPM appears to give a better correspondence
with the measured data.

![Figure 10a. Test data: PSD comparison. Inner race fault.](image)

![Figure 11a. LPM: PSD comparison. Inner race fault.](image)

![Figure 10b. Test data: Sq. envelope spectrum. Inner race fault.](image)

![Figure 11b. LPM: Sq. envelope spectrum. Inner race fault.](image)
7.2. Localised outer race fault

The comparison of the PSD spectra for the Test data, LPM and 146 DOF model is shown in figures 13a, 14a and 15a. The presence of the localised outer race fault results in a dB increase between the good and the faulty bearing in the high frequency region for all three cases. The corresponding squared envelope spectra clearly identify BPFO harmonics of 49 Hz (Ball Pass Frequency Outer Race) for both the simulation models and correlate well with the test results [figures 13b, 14b and 15b]. Here also there is a poor matching of the PSD spectra between 2-4 kHz. In this case at high frequency the reduced model (146 DOF) has a better correspondence with the measured data. The reasons for this are being investigated.

Figure 12a. 146DOF: PSD comparison. Inner race fault.

Figure 12b. 146DOF: Sq. envelope spectrum. Inner race fault.

Figure 13a. Test data: PSD comparison. Outer race fault.

Figure 13b. Test data: Sq. envelope spectrum. Outer race fault.
7.3. Fault entry and exit phenomena

The acceleration signals obtained from the measurements and simulations were further processed to enhance the step and impulse responses (at the entry and exit of the spall) using signal prewhitening. The entry into the fault in principle gives a low frequency step response while the impact on exit excites a much broader band impulse response. Prewhitening the signals using the cepstrum [12] (used in the current paper) or autoregressive (AR) method [6] increases the relative strength of the entry event although the two events still tend to be different in frequency content.

Figures 16-18 indicate the rolling element fault entry and exit events for a single pulse for the simulated and measured acceleration signals (LPM, 146 DOF combined model and test results). The fault entry and exit events are marked with arrows in the figures. The impulse at exit is clearly seen in the simulated results and correlates well with the test results for the localised inner and outer race faults. However, there is a better match of the step response of the LPM model with the step response observed in the test results than with the 146 DOF combined model. The step response in case of test data with outer race fault was found to be of poor quality. This indicates that further signal processing is required on the 146 DOF model and test data (with outer race fault) and will form the part of future work.

The results also confirm that the impact event takes place when the centre of the rolling element is located at half the spall width and the average time to impact is found to be approximately 0.5 ms for
the 0.8 mm fault, which is independent of the location of fault i.e. whether on inner or outer race, which agrees with theory.

**Step / impulse response at fault entry and exit**

Test (inner race fault) - Pre-whitened signal - Cepstrum method

![Figure 16a. Test data: Localised inner race fault.](image)

Test (outer race fault) - Pre-whitened signal - Cepstrum method

![Figure 16b. Test data: Localised outer race fault.](image)

Test (inner race fault) - Pre-whitened signal - Single pulse

![Figure 17(a). LPM: Localised inner race fault.](image)

Test (outer race fault) - Pre-whitened signal - Single pulse

![Figure 17(b). LPM: Localised outer race fault.](image)

Test (inner race fault) - 146DOF - Pre-whitened signal - Cepstrum method

![Figure 18 (a). 146DOF: Localised inner race fault.](image)

Test (outer race fault) - 146DOF (acc) - Pre-whitened signal - Cepstrum method

![Figure 18 (b). 146DOF: Localised outer race fault.](image)
8. Conclusions
This paper presents the dynamic simulation of a gearbox system using a lumped parameter model (LPM) and a combined LPM-reduced FE model of the casing. The Craig-Bampton method of component mode synthesis was used to reduce the FE model of the casing. The simulation models were analysed in the presence of localised inner and outer race faults in the bearing, with updated fault geometry. The results of simulated bearing fault frequencies obtained by envelope analysis correlate well with the test results and are in agreement with theoretical calculations. However, the comparison of PSD spectra shows poor matching between 2-4 kHz. Cepstrum prewhitening was carried out on the measured and simulated acceleration signals to enhance the step and impulse responses at the fault entry and exit. The LPM model provided a clear indication of these two events and the results confirm that the average time to impact corresponds to half the spall width. The methodology can be used to estimate the spall size which is an important monitoring parameter in the bearing prognostics.

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