Mathematical modelling of ladle furnace thermal regime with bubble melt blowing by gas

N A Spirin, V S Shvidkii, Y G Yaroshenko and V V Lavrov
Ural Federal University n.a. the first President of Russia B.N. Yeltsin, 51 Lenina ave, Ekaterinburg, 620075, Russia
E-mail: n.a.spirin@urfu.ru

Abstract. The differential equations of energy of steel and gas with the 3-d rank of boundary conditions were used to evaluate the regularities of steel heating in a ladle furnace. The developed mathematical model, however, cannot be used as a control tool in real process time. The analogy between heating of the porous media in a fixed bed was applied to describe the heat transfer mechanism during the gas bubbles movement through the melt. This approach provides accurate results and with some modifications could be used as a part of the expert system to control ladle furnace operation.

1. Introduction
Production of a high quality steel is one of the challenging problems of iron and steel industry. Finishing, heating and alloying of steel in a ladle furnace before casting is a common practice at BOF and EAF steelmaking shops. The heat transfer between heating/steering gas and liquid steel plays a major role in metal treatment. Introduction of the parameter “ϕ” — the ratio of the volume of gas bubbles to volume of steel (melt) in a ladle furnace (m$^3$/m$^3$) allows to apply regularities of heat transfer between moving gas and fixed bed of burden materials at pelletizing machine or sinter strand to describe the heat transfer in a ladle furnace [1, 2]. With such an approach to the physical phenomena the mathematical solution of heat transfer problem in a ladle furnace is based on the definition of volumetric heat transfer coefficient “$\alpha_v$”[kJ/(m$^3$ K·sec)] between gas bubbles and surface of pseudo steel particles in 1 m$^3$ volume of the liquid bath. The volumetric heat transfer coefficient is defined by the following formula: 

$$\alpha_v = \alpha_f F,$$

where $\alpha_f$ (kJ/(m$^2$·K·sec)) is a heat transfer coefficient from gas to the surface of pseudo macro particles of liquid steel and $F$ (m$^2$/m$^3$) — is a surface of pseudo macro particles of liquid steel in 1 m$^3$ volume of the liquid bath. Parameter “F” also could be interpreted as a overall surface of gas bubbles in 1 m$^2$ of steel bath.

2. Mathematical model
The physical formulation of the problem of the heat transfer between gas bubbles and liquid bath is as follows. Gas bubbles are moving through 1 m$^2$ area of the steel bath with the velocity at empty cross-section of the ladle furnace $W_g = \phi \omega_g$, passing the distance $dy$ along the furnace height during the time period $d\tau$. Here $\omega_g$ — is actual gas velocity. Assuming constant gas velocity $\omega_g = (0.44+0.507)/2 = 0.475$ [3] and infinite heat conductivity of the liquid bath the change in enthalpy of the elemental volume of the gas could be described by the following differential equation:
\[
d^2 Q = c_g \rho_g \left( \frac{dt_g}{d\tau} \right) dy d\tau = \\
= c_g \rho_g \left( \frac{\partial t_g}{\partial y} + \frac{\partial t_g}{\partial \tau} \right) dy d\tau + c_g \rho_g \frac{\partial t_g}{\partial \tau} dy d\tau.
\]

Here \( \frac{\partial y}{\partial \tau} = \omega_g \) and \( \varphi \cdot \omega_g = W_g \), where \( \omega_g \) is a velocity of bubbles, m/s; \( c_g \) – gas specific heat, J/(kg⋅K); \( \rho_g \) – gas density, kg/m\(^3\). Taking into account these definitions equation (1) can be simplified:

\[
d^2 Q = c_g \rho_g \left( W_g \frac{\partial t_g}{\partial y} + \varphi \frac{\partial t_g}{\partial \tau} \right) dy d\tau.
\]

(2)

The same amount of enthalpy provided by gas to steel could be expressed by the following equation:

\[
-d^2 Q = \alpha_Y (t_g - t_{melt}) dy d\tau + d^2 Q_{loss}.
\]

(3)

Equalization of equations (2) and (3) assuming that heat exchange is an adiabatic process and \( d^2 Q_{loss} = 0 \) allows deriving the following equation:

\[
-\alpha_Y (t_g - t_{melt}) = c_g \rho_g \left( W_g \frac{\partial t_g}{\partial y} + \varphi \frac{\partial t_g}{\partial \tau} \right)
\]

(4)

Similar consideration of the heat transfer in a metal bath, assuming that the enthalpy of macro particle of steel changes only in time \( (d_t) \), allows to conclude that

\[
-\alpha_Y (t_g - t_{melt}) = c_{melt} \rho_{melt} (1 - \varphi) \frac{\partial t_{melt}}{\partial y}.
\]

(5)

where: \( c_{melt} = c_{phys}^{melt} + c_{chem} \). Here \( c_{phys}^{melt} \) is a mass heat capacity of fusion, J/(kg⋅K), and \( c_{chem} = \sum q_{chem} \left[ m_{melt} (t_g - t_{melt}) \right] \); \( \sum q_{chem} \) is a total thermal effect of oxidation reactions, J.

System of equations (4) and (5) should be supplemented by initial and boundary conditions to complete the mathematical model:

\[
y = 0, \ t_g = t_{g0}; \\
\tau = 0, \ t_{melt} = t_{melt} = 1100 \ ^\circ C.
\]

(6)

Thus, the complete mathematical model of heat transfer in the ladle furnace with the gas bubble blowing incorporates equations (4), (5) and (6).

3. Mathematical model transformation

The numerical solution of this system of equations could be done with following assumptions:

1. Gas bubbles are evenly distributed in a volume of ladle furnace, maintaining their spherical shape and having average diameter 13.57 mm [4];
2. Steel bath is stagnant and the bubbles velocity is constant along the height and equal to 0.475 m/s;
3. There is no chemical reactions in a melt and because of this \( c_{chem}^{melt} = 0 \);
4. The “F” value is simply determined by the parameter “ϕ” and the size of the bubble $R_{bub}$:

$$\varphi = \frac{\sum V_{bub}}{V_{melt}} = \frac{n \cdot 4}{3} \frac{R_{bub}^3}{V_{melt}},$$

(7)

$$F = \frac{\sum F_{bub}}{V_{melt}} = \frac{n \cdot 4 \pi R_{bub}^3}{V_{melt}},$$

(8)

where “n” is an instantaneous amount of bubbles in a bath; $\sum V_{bub}$ – is a total volume of bubbles, m$^3$; $\sum F_{bub}$ – is a total surface of bubbles, m$^2$.

Joint solution of equations (7) and (8) allows estimation of value “F”:

$$F = \frac{3\varphi}{R_{bub}} = \frac{3 \cdot 2 \cdot \varphi}{13.57 \cdot 10^{-3}} = 0.442 \cdot 10^3 \varphi.$$

(9)

The statistics of bubble’s parameters in a ladle furnace during bottom blowing is presented in papers [5, 6]. The amount of bubbles in a bath as a function of the height and time in the course of different technological functions of ladle furnace operation could be found in literature [7, 8]. These data are in good correlation with estimated parameters [9].

Let’s assume “ϕ” = 11.3 %, which correspondence with the real conditions. The values of various coefficients in a system of equations (4) - (6) are as follows (air is assumed as a stirring gas):

$$\alpha_v = \alpha_F \cdot F = 442 \cdot \varphi \cdot \alpha_F \quad \text{and} \quad \alpha_F = Nu \cdot \lambda / (2 \cdot R_{bub}).$$

Here “Nu” – is the Nusselt number:

$$Nu = 2 + 0.6 \cdot Re^{0.5} \cdot Pr^{0.33} \quad [10].$$

After substitution of numerical values of specific heat and conductivity coefficient at the average temperature of the $t_{melt} = 1200°C \quad [11]$, the following values of similarity numbers and heat transfer coefficient are obtained: $Re = 28.8$; $Pr = 0.724$; $Nu = 4.894$; $\alpha_F = 33.02$, W/(m$^2$K) and $\alpha_v = 442 \cdot \varphi \cdot 33.016 = 14593.3 \varphi$, W/(m$^3$K).

As a result instead of system of equation (4) - (6) the following two differential equations represent the heat transfer in a liquid bath of the ladle furnace [12]:

$$\frac{\partial t_g}{\partial \tau} + 0.475 \frac{\partial t_g}{\partial y} = \frac{14593.3}{1210 \cdot 1.27} \left( t_{melt} - t_g \right)$$

$$= 14593.3 \cdot \varphi \cdot \left( t_{g} - t_{melt} \right),$$

(10)

System of equations (10) was numerically resolved for the following values of determining parameters: $\varphi = 0.1, 0.15, 0.2, 0.5$; $t_{g0}$, °C = 1500, 1600, 1800, 2000; $\tau = 0$, $t_{melt}=t_{initial}=1100$ °C. This system of equations (10) needs to be converted to the standard type to use standard solutions and define dimensionless parameters:

$$\frac{\partial t_g}{\partial \tau} + 0.475 \frac{\partial t_g}{\partial y} = \frac{14593.3}{1210 \cdot 1.27} \left( t_{melt} - t_g \right) =$$

$$= 9.496518 \cdot \left( t_{melt} - t_g \right),$$

(11)

$$\frac{513.9 \cdot 7790 \cdot (1 - \varphi)}{4593.3 \cdot \varphi} \frac{\partial t_{melt}}{\partial \tau} = \left( t_g - t_{melt} \right).$$
or
\[
0.105302 \cdot \frac{\partial t_m}{\partial \tau} + 0.0500183 \cdot \frac{\partial t_m}{\partial y} = t_{melt} - t_g;
\] (12)

\[
\frac{(1 - \varphi)}{0.00364533} \cdot \frac{\partial t_m}{\partial \tau} = t_g - t_{melt};
\] (13)

\[
\frac{\partial t_m}{\partial \tau} \left( \frac{0.00364533 \cdot \varphi \cdot \tau}{1 - \varphi} \right) = t_g - t_{melt}.
\] (14)

The dimensionless time can be derived from equation (13):
\[
Z = \frac{0.00364533 \cdot \varphi \cdot \tau}{1 - \varphi}.
\] (15)

The following two points [13] need to be addressed with respect to the equation (12):

- The first component of this equation becomes significant only at \( \tau < \varphi y/W_g \) or until gas will not reach the level “\( y \)”. At the real conditions of ladle furnace operation (\( H = 1.812 \) m; \( \tau_{\text{heating}} = 3.75 \) h = 13500 s; \( W_g = 0.475 \cdot \varphi \)), this component could be neglected;
- The dimensionless height of steel bath \( Y = y/0.0500183 \) unlike the dimensionless time does not depend on the number of bubbles.

Taking into account these comments, the equations (16) were derived to describe the distribution of gas and steel bath temperature along the ladle furnace height.

For the complete height of steel bath \( H = 1,812 \) m values of dimensionless height are as follows:

| \( y, \) m | 0 | 0.302 | 0.906 | 1.51 | 1.812 |
|----------|---|-------|-------|------|-------|
| \( Y    \) |   | 6.038 | 18.113| 30.19| 36.27 |

\[
\theta = \exp\left( -Y \right) \int_0^1 \left( \frac{2}{\sqrt{Y}} \right) \cdot \exp\left( -\zeta \right) d\zeta;
\]

\[
t_{\text{melt}} = 1100 + \left( t_{g0} - 1100 \right) \cdot \vartheta;
\]

\[
\theta = 1 - \exp\left( -Z \right) \int_0^1 \left( \frac{2}{\sqrt{Z}} \right) \cdot \exp\left( -\eta \right) d\eta;
\] (16)

\[
t_g = 1100 + \left( t_{g0} - 1100 \right) \cdot \theta.
\]

Here \( \vartheta = (t_{\text{melt}} - t_{\text{init}}) / (t_{g0} - t_{\text{init}}) \); \( \theta = (t_g - t_{\text{init}}) / (t_{g0} - t_{\text{init}}) \). The values of dimensionless time are determined by the set volume of bubbles. At \( \phi = 0.1 \) the value of \( Z \) equal to \( Z = 4.05037 \cdot \tau \).

| \( \tau, \) sec | 2700 | 5400 | 10900 | 13500 |
|----------|------|------|--------|-------|
| \( Z    \) |   | 1.094 | 2.19 | 4.374 | 5.468 |

4. Results of mathematical modeling
Mathcad is the most convenient package to solve system of equation (16). The dimensionless gas temperature distribution along the furnace height as a function of dimensionless time \( Z \) is presented in figure 1 for the fraction of gas bubbles \( \varphi = 0.1 \) and initial gas temperature \( t_{g0} = 1500 \) °C.
Figure 1 results allow to conclude that at assumed fraction of the gas bubbles in the bath ($\varphi=0.1$) enthalpy of the incoming stirring gas is sufficient only for heating bottom part of the steel bath – approximately 1/3 of the overall height.

The increase in gas bubbles fraction to $\varphi=0.2$ does not change significantly this conclusion. Even at initial gas temperature of 2000 °C the temperature of the gas on the top of the bath remains equal to the initial steel temperature.

At $\varphi=0.5$ the situation changes dramatically. The enthalpy of the gas in this case is sufficient to heat steel even at initial gas temperature $t_g=1500$ °C (figure 2).

In this case dimensionless time is equal $Z = 0.00364533 \cdot \tau$. It is evident that the limit of the gas volume is a function of the melt temperature at the top of the bath. The change in bath temperature in time along the furnace height at $\varphi = 0.5$ is presented in figure 3.

**Figure 1.** Gas temperature as a function of steel bath height and dimensionless time $\varphi=0.1; t_g=1500$ °C.

**Figure 2.** Gas temperature as a function of steel bath height and dimensionless time $\varphi=0.5; t_g=1500$ °C.
The influence of parameters of blowing on average melt temperature was studied in time interval of 1.0-1.5 hours. Since dimensionless height of the steel bath \( Y = \frac{y}{0.0500183} \) does not depend on number of bubbles or their fraction \( \varphi \) and gas temperature at the entrance to the bath is \( t_{g0} \), the average bath temperature in all possible variants will be determined by the same formula:

\[
t_{\text{melt}} = \frac{1}{36.267} \int_{0}^{36.267} \exp(-\frac{\varphi}{Z}) \int_{0}^{2\sqrt{\varphi t_{g0}}} \exp(-\mu) d\mu d\xi.
\]

(17)

**Figure 3.** Bath temperature distribution as a function of bath height and time at \( \varphi = 0.5 \).

For the time of blowing \( \tau = 1.25 \) hour the integral for average melt temperature could be transformed to simple arithmetical expression

\[
t_{\text{melt}} = 1231.133 - 1614.243 \cdot \varphi - 0.072 \cdot t_{g0} + 765.789 \cdot \varphi^2 + 1.031 \cdot \varphi \cdot t_{g0} - 4.801 \cdot 10^{-9} \cdot t_{g0}^2, \degree C.
\]

(18)

Results of calculations are presented in figure 4, where each curve represents the average melt temperature.

**Figure 4.** Average bath temperature as a function of gas initial temperature and fraction of gas bubbles at \( \tau = 1.25 \) hour.
Based on results of figure 4 it is possible to conclude that the desirable average temperature of melt, for example, $t_{\text{melt}}^\text{assign} = 1250\, ^\circ\text{C}$, could be achieved at $\varphi=0.25$ and $t_{e0} = 2000\, ^\circ\text{C}$ or $\varphi=0.45$ and $t_{e0}=1500\, ^\circ\text{C}$.

Thus, if time of blowing is set, then it is easy to run calculations. For one hour blowing time the average bath temperature could be estimated by the following expression:

$$t_{\text{melt}} = 1206.764 - 1305.325 \cdot \varphi - 0.058 \cdot t_{e0} + 624.18 \cdot \varphi^2 + 0.831 \cdot \varphi \cdot t_{e0} + 3.14 \cdot 10^{-8} \cdot t_{e0}^2, \, ^\circ\text{C}. \tag{19}$$

For 1.5 hour blowing time solution of integral (17) gives similar to (18) and (19) expression (20):

$$t_{\text{melt}} = 1256.704 - 1952.495 \cdot \varphi - 0.084 \cdot t_{e0} + 932.594 \cdot \varphi^2 + 1.243 \cdot \varphi \cdot t_{e0} - 7.616 \cdot 10^{-7} \cdot t_{e0}^2, \, ^\circ\text{C}. \tag{20}$$

5. Conclusions

- The mathematical model of heat transfer between gas bubbles and liquid bath was developed based on approximation of well known Shuman problem of heat transfer between moving gas and the fixed layer of material.
- Influence of bubbling gas initial temperature and blowing time on distribution of the gas and bath temperature along the furnace height at different moments of time was studied and discussed.
- While numerical analysis was applied to ladle furnace semi steel refining, developed mathematical model and approach are absolutely applicable for analysis of gas bubble movement and temperature distribution in steelmaking ladle furnace.

References

[1] Novokreschenov S, Shvidkii V et al 2009 Proc. of Int. Conf. on Sci. Heritage of B.I. Kitaev (Ekaterinburg, Russia) pp 324–328
[2] Novokreschenov S, Shvidkii V et al 2013 Proc. of the Conf. on Modern Metallic Materials and Technology (S. Petersburg, Russia)
[3] Novokreschenov S et al 2013 Izvestia Vuzov. Non-ferrous Metallurgy 5 58–60
[4] Novokreschenov et al 2011 Izvestia Vuzov. Non-ferrous Metallurgy 4 61–62
[5] Gizatulin R 2017 Izvestia Vuzov. Ferrous Metallurgy 8 26–29
[6] Gizatulin R 2016 Bulletin of South-Ural’s University. Series “Metallurgy” 10(65) 63–68
[7] Sokovnin O, Zagoskin N and Zagoskin S 2012 Theoretical Foundation of Chemical Technology vol 46 5 540
[8] Protopopov E 2002 Izvestia Vuzov. Ferrous Metallurgy 4 9–13
[9] Zgukov V et al 2013 Izvestia Vuzov. Non-ferrous Metallurgy 3 58
[10] Gordon Y et al 1992 Heat Transfer Calculations for Metallurgical Furnaces (Moscow: Metallurgia) p 355
[11] Belousov V, Klevtsov A, Pribitkov I and Sborschikov G 1993 Heat Transfer and Heat and Power Engineering (Moscow: Metallurgia) p 336
[12] Chirkin V 1967 Thermo-physical Properties of Nuclear Technology Materials (Moscow: Atomizdat) p 474
[13] Telegin A, Shvidkii V and Yaroshenko Yu 2002 Heat- and Mass Transfer (Moscow: Akademkniga) 263