Fatal youth of the Universe: 
black hole threat 
for the electroweak vacuum during preheating 

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Abstract 
Small evaporating black holes were proposed to be dangerous inducing fast decay of the electroweak false vacuum. We observe that the flat-spectrum matter perturbations growing at the post-inflationary matter dominated stage can produce such black holes in a tiny amount which may nevertheless be sufficient to destroy the vacuum in the visible part of the Universe via the induced process. If the decay probability in the vicinity of Planck-mass black holes was of order one as suggested in literature, the absence of such objects in the early Universe would put severe constraints on inflation and subsequent stages thus excluding many well-motivated models (e.g. the $R^2$-inflation) and supporting the need of new physics in the Higgs sector. We give a qualitative argument, however, that exponential suppression of the probability should persist in the limit of small black hole masses. This suppression relaxes our cosmological constraints, and, if sufficiently strong, may cancel them. 

1 Introduction and summary 

The electroweak (EW) vacuum of the Standard Model of particle physics (SM) with top-quark mass, strong coupling constant and Higgs boson mass taken at the central measured values [1] is definitely unstable given the high-order quantum corrections [2, 3] to the Higgs potential. The decay of this vacuum proceeds via tunneling through the potential barrier to the true vacuum at subplanckian values of the Higgs field [4, 5]. As a result, a bubble filled
with the Higgs field in the catchment area of the true vacuum is produced: the field rolls
down towards the true vacuum while the bubble expands occupying more and more space.
Fortunately, this catastrophe is found to be extremely rare [4], so that our Universe lifetime
grossly exceeds its present age of 14 billion years [1].

The late Universe, either dominated by matter or cosmological constant, is safe for billions
of billions of successive human generations. The early Universe expansion most probably
was driven by some new physics, but the process was arranged in such a way that the Higgs
field had avoided escaping to the true vacuum. This requirement implies various constraints
on the pre-Big-Bang history of the Universe, including inflation, preheating and reheating
stages, which have been largely discussed in literature, see e.g. [6, 7].

Recently it has been suggested [8, 9, 10] that the situation changes completely in the
presence of small evaporating black holes. These objects were proposed to act as nucleation
sites for the bubbles of true vacuum dramatically increasing the rate of their formation. The
largest enhancement was expected in the case of the smallest-mass black holes which were
argued to kick the Higgs field over the energy barrier and into the abyss with the probability
of order one. Then every black hole at the last stages of its evaporation should produce an
expanding bubble of true vacuum around itself. Since we still live in the false vacuum, no
black hole had ever completely evaporated during the entire history of our Universe.

In this paper we turn this observation into model-independent cosmological bounds and
further discuss their possible model-dependent refinements. Trying to be maximally accurate,
we critically analyze the black hole induced processes described above. We find that the
proposed effect is most efficient if the size of the black hole is much smaller than the radius
of the true vacuum bubble forming around it. This apparent violation of locality rises doubts
in physical interpretation of the solutions of Ref. [10] and suggests that the probability $\mathcal{P}_{EW}$
of the induced decay, though naturally enhanced as compared to the case without black
hole, may have been overestimated in [10]. We therefore keep arbitrary $\mathcal{P}_{EW}$ throughout the
paper.

The SM provides no mechanisms to produce small black holes neither today nor in the
early Universe, definitely not during the primordial nucleosynthesis and at the succeeding
epochs. Any new physics operating at earlier times, if it leaves the EW vacuum metastable,
should be of the same kind producing only sufficiently large black holes which do not evaporate until now. We emphasize that in this case even cosmological models with primordial
black holes fully evaporated before the beginning of the primordial nucleosynthesis are ex-
cluded.
The earliest time when a black hole might be produced is preheating, the epoch right after inflation. The cosmological models with copious black hole production at this stage are rather exotic and have fine-tuned parameters. In generic models the black holes can be produced, though very inefficiently [11]. An example of the production mechanism is Jeans instability leading to collapse of primordial matter perturbations at the post-inflationary matter dominated stage [12]. In this case strong suppression [13, 14] of black hole formation is due to the fact that the typical spatial inhomogeneities originating from the flat-spectrum primordial Gaussian perturbations are too aspherical [15] and uneven to form black holes.

The main observation of this letter is that in a generic inflationary model with post-inflationary stage of matter domination continuing long enough for some matter perturbations to grow and become non-linear, \( \frac{\delta \rho}{\rho} \sim 1 \), the black holes are formed in the region occupied by the visible part of the Universe. In fact, they are produced even if the fluctuations grow to \( \frac{\delta \rho}{\rho} \sim 0.1 \); in this case a few occasionally largest fluctuations of a given wavelength become nonlinear and collapse. Then transition to the true (subplanckian) vacuum is performed via the black hole induced mechanism described above. The only general way to stop black hole formation is early reheating which reduces the period of post-inflationary matter domination. For illustration we put a corresponding limit on the reheating temperature in the large-field inflationary models. Similar limits exist in other models where the perturbations become nonlinear at preheating.

In a nutshell, this paper demonstrates that the black holes inducing tunneling are important for cosmology. The rate of the induced processes, however, should be revised for obtaining robust cosmological constraints.

2 Long post-inflationary matter domination

Black holes of masses in a wide range can be formed in the early Universe from collapsing initial matter density perturbations. To this end the overdense regions should be squeezed by gravity within their own Schwarzschild radii [16, 17]. This happens most efficiently during matter dominated stage when the pressure preventing contraction vanishes and the black hole formation is mainly determined [11] by whether the overdense region is sufficiently spherically symmetric and smooth, or not.

An early matter dominated stage is generically realized in the inflationary models with massive inflaton. It begins right after inflation and lasts while the Universe is dominated by the oscillating inflaton field until reheating. During this stage the Hubble parameter \( H(t) \)
and matter (inflaton) density $\rho(t)$ are related by the Friedmann equation and depend on the scale factor $a(t)$ as

$$H^2(t) = \frac{8\pi}{3} G \rho \propto \frac{1}{a^3(t)},$$

where $G$ is the gravitational constant. The matter density contrasts with conformal momentum $k$ and subhorizon size $a/k \simeq R(t) \ll 1/H(t)$ grow linearly with the scale factor, $(\delta\rho/\rho)_k \propto a$, starting from the primordial value

$$(\delta\rho/\rho)_{k,i} \equiv \delta_i \sim 10^{-4}$$

at the horizon crossing $R_* = 1/H_*$. 

Let us begin with the situation when matter domination is long enough for some of the shortest modes to grow, decouple from the Hubble flow at turnaround entering the nonlinear regime, $(\delta\rho/\rho)_k \sim 1$, and then collapse forming clumps of the inflaton field. Some of the clumps happen to be sufficiently spherical and sufficiently smooth to further collapse into black holes.\(^1\) This process was investigated in Refs. [12, 13, 14, 11], where the probability of a given clump to be appropriate for collapsing into a black hole was estimated as

$$P_{BH} \approx 2 \times 10^{-2} \left(\frac{r_g}{R}\right)^{13/2},$$

with $R$ and $r_g$ denoting the size of the clump at turnaround and the gravitational radius of the resulting black hole. These quantities are related to the clump mass $M$ and matter (inflaton) density $\rho$ at turnaround by

$$r_g = 2GM, \quad M \approx \frac{4\pi}{3} \rho R^3.$$

Taking the matter density from the Friedmann equation (1), one obtains,

$$\frac{r_g}{R} \approx H^2 R^2 = \frac{a_*}{a} \approx \delta_i,$$

where the second relation accounts for the scale factor dependence of $H$ and $R$ between the Hubble crossing and turnaround, see (1). The third relation uses the fact that the contrasts grow linearly with the scale factor. Hence, the probability (2) is

$$P_{BH} \approx 2 \times 10^{-2} \delta_i^{13/2}.$$  

\(^1\)The time scale of this process is about the free-fall time in the Tolman solution and hence is of the order of the cosmological time scale determined by the Hubble parameter.
There are \((HR)^{-3} \simeq \delta_i^{-3/2}\) clumps of size \(R\) inside the Hubble volume at turnaround, hence, the probability to have a black hole in that region,

\[
P_{BH,\text{hor}} \approx 2 \times 10^{-22} \left(\frac{\delta_i}{10^{-4}}\right)^5,
\]

is very small.

On the other hand, our present-day Universe with the Hubble parameter \(H_0 \ll H\) has many such regions. Indeed, since their volumes grow as \(a^3\) starting from \(H^{-3}\), there are

\[
N_{\text{hor}} = \left(\frac{H}{H_0}\right)^3 \left(\frac{a}{a_0}\right)^3
\]

of them in the visible part of the Universe.

Consider the largest possible black holes formed right before the reheating: \(H = H_{\text{reh}}\) and \(a = a_{\text{reh}}\) in Eq. (7). At the hot stages the entropy in the comoving volume is conserved; parameter \(H_{\text{reh}}\) is related to the reheating temperature \(T_{\text{reh}}\) and the number of relativistic degrees of freedom \(g_{*,\text{reh}}\) by the Friedmann equation \(H^2_{\text{reh}} \simeq G g_{*,\text{reh}} T_{\text{reh}}^4\). Then, neglecting some numerical factors, one finds,

\[
N_{\text{hor}} \simeq g_{*,0} G^{3/2} \frac{T_{\text{reh}}^3}{H_0^3} \sqrt{g_{*,\text{reh}}} T_{\text{reh}}^3.
\]

This large number must be multiplied by (6) to estimate the number \(N_{BH,0}\) of the primordial black holes \textit{within the presently visible part of the Universe}. Assuming \(g_* \sim 100\), one obtains,

\[
N_{BH,0} = N_{\text{hor}} \times P_{BH,\text{hor}} \simeq \left(\frac{T_{\text{reh}}}{3 \times 10^{-4} \text{ GeV}}\right)^3 \left(\frac{\delta_i}{10^{-4}}\right)^5.
\]

Recall that \(a_{\text{reh}}/a_{\text{inf}} \gtrsim \delta_i^{-1}\) is assumed in this formula. The number (9) exceeds unity as far as the reheating temperature is above MeV scale, which is required for the successful primordial nucleosynthesis [1]. The number of the lighter black holes is even larger because the respective perturbations start to grow earlier. In particular, the smallest black holes are formed by the perturbations starting to grow immediately after inflation. Their number is given by Eq. (9) multiplied by \((\delta_i a_{\text{reh}}/a_{\text{inf}})^{3/2} > 1\).

In cosmological models with realistic preheating temperature \(T_{\text{reh}} \gtrsim 100\text{ GeV}\) many evaporating black holes are formed. Recently it was suggested [8, 9, 10] that the electroweak vacuum decays with enhanced rate\(^2\) \(e^{-E_b/T_{BH}}\) in the vicinity of small black holes, where

\(^2\)We consider realistic case of small \(E_b\) as compared to the black hole mass.
$E_b \sim 10^{12}$ GeV is the height of the energy barrier between the vacua and $T_{BH}$ is the Hawking temperature. This result is based on Euclidean calculations [10] in the Standard Model of particle physics beyond the thin-wall approximation. It is also supported by the semiclassical exercises in toy models describing thin-wall bubbles [18, 19, 20, 8, 9]. From the physical viewpoint, the result implies that the black hole, like a thermal bath, activates over-barrier transitions between the vacua with the Boltzmann suppression. The smallest black holes at the last stage of their evaporation have $T_{BH} > E_b$ and therefore destroy our vacuum with the probability of order one. Conversely, since we still live in the metastable vacuum, no black hole had ever completely evaporated within the visible part of the present Universe.

As is noted in the Introduction, there is no doubt in qualitative role of black holes catalyzing bubble formation. However, the applicability of the Boltzmann formula for the decay probability is questionable, as we explain in Sec. 4. We therefore denote by $\mathcal{P}_{EW}$ the probability of induced false vacuum decay during the last stages of black hole evaporation, the very probability that was claimed in literature [10] to be of order one.

One concludes that the EW vacuum is destroyed if 1) it is metastable; 2) the post-inflationary stage is matter-dominated and lasts long enough for the shortest-scale perturbations to enter the non-linear regime (i.e. $a(t)$ grows by a factor $\delta_i^{-1} \sim 10^4$); 3) inside the present horizon the perturbations form more than $\mathcal{P}_{EW}^{-1}$ primordial black holes which are light enough to evaporate down to the Planckian masses by now.

Assuming the two first conditions are fulfilled, let us elaborate on the last condition implying, in particular, that $M_{BH} \lesssim M_c \equiv 10^{14}$ g, see e.g. [21]. The smallest-mass black holes are formed by the perturbations entering the horizon immediately after inflation. Computing the mass within the cosmological horizon $H^{-1}_{inf}$ at that time, we obtain,

$$\min(M_{BH}) \simeq (2GH_{inf})^{-1},$$

where the Friedmann equation (1) was used. These smallest black holes are harmless, $M_{BH} > M_c$, if the scale of inflation is low enough,

$$H_{inf} \lesssim (2GM_c)^{-1} \approx \text{GeV}.$$ 

This gives low energy density

$$\rho_{inf} \lesssim \frac{3}{32\pi G^3 M_c^2} \approx (2 \times 10^9 \text{GeV})^4$$

at the end of inflation. Note that the bound (12), (11), if applicable, is way stronger than the condition $H_{inf}/2\pi \lesssim 10^{11}$ GeV, see e.g. [6, 7], ensuring stability of the EW vacuum during inflation.
The models obeying (12) are viable: the primordial black holes produced at that stage are harmless until now (yet the catastrophe awaits us in the future). Recalling the assumption of long enough post-inflationary matter dominated stage, one can recast (12) as the limit on the reheating scale. In this case $H_{reh} \leq H_{inf}\delta_i^{3/2}$, and all models with $M_{BH} > M_c$ have low reheating temperature,

$$T_{reh} \sim \frac{H_{reh}^{1/2}}{(g_*)^{1/4}} \lesssim 10^6 \text{GeV} \times \left(\frac{\delta_i}{10^{-4}}\right)^{3/4}.$$  \hspace{1cm} (13)

This inequality can be used together with Eq. (12) to identify the cosmologically viable models.

On the other hand, if the energy density at the end of inflation is high and violates (12), some primordial black holes evaporate before the present epoch and may destroy the EW vacuum. Requiring the number of black holes (9) to be smaller than $\mathcal{P}_{EW}^{-1}$, we obtain a constraint on the reheating temperature,

$$T_{reh} \lesssim 3 \times 10^{-4} \text{GeV} \times \mathcal{P}_{EW}^{-1/3} \times \left(\frac{\delta_i}{10^{-4}}\right)^{-5/3}.$$  \hspace{1cm} (14)

If the probability of the induced decay is of order one, $\mathcal{P}_{EW} \sim 1$ \cite{10}, this inequality excludes all models violating (12), (13) and having long enough post-inflationary matter dominated epoch because the temperature (14) is too low for successful primordial nucleosynthesis. If $\mathcal{P}_{EW} < 1$ but not too small, such models are still severely constrained by Eq. (14). The constrained models include the $R^2$-inflation \cite{22} where the energy density at inflation is high, the reheating temperature is low \cite{23, 24}, and the scale factor grows by $10^7$ between these epochs. Generally, for the inflationary models with high energy scale, e.g. the large-field models, the only way to avoid the danger is to prevent formation of the inflaton clumps, so that the scale factor grows by a factor smaller than $10^4$ during preheating. This places the lower limit on the reheating temperature,

$$T_{reh} \gtrsim 5 \times 10^{12} \text{GeV} \times \frac{\rho_{inf}^{1/4}}{10^{16} \text{GeV}} \times \left(\frac{\delta_i}{10^{-4}}\right)^{3/4}.$$  \hspace{1cm} (15)

Viable models satisfy either this inequality or Eq. (14), or Eqs. (12), (13). In particular, Eq. (15) can be met in inflationary models with quartic scalar potential and large non-minimal coupling to gravity similar to the Higgs inflation \cite{25} where the reheating temperature is estimated to be higher \cite{26}.
3 Beyond the simplest scenario

In special cases e.g. when the parameters of a given model — reheating temperature, energy density at inflation, duration of the matter-dominated stage — are close to the critical values separating the EW-safe and EW-destroyed models, one would like to refine our estimates. In fact, the only situation where the refinement is definitely needed is when the post-inflationary matter-dominated stage is long but not long enough for the shortest perturbations to form clumps with $\delta \rho/\rho \sim 1$ everywhere in the early Universe. In this situation the gravitationally bound clumps still may be formed in a few places due to positive fluctuations even if the fluctuations are not yet non-linear on average, similar process happened in the late Universe when the first stars get ignited.

There are several issues becoming important in this case of early reheating. First, one naturally concentrates on the perturbations of the shortest wavelengths which enter the horizon and start to grow immediately after inflation. However, at the end of inflation both the expansion law and the scalar perturbation amplitude may deviate (and noticeably, see e.g. the case of the inflaton quartic potential [27]) from what one has for the reference modes exiting the horizon some 50-60 e-foldings before the inflation terminates. Likewise, the expansion right after inflation is not exactly like at the matter-dominated stage, so that the contrasts do not immediately approach the linear-with-scale-factor growth. Hence, it well may happen in a particular model that it is not the shortest mode which has the highest amplitude and happens to be the first to approach $\delta \rho/\rho \sim 1$ and enter the non-linear regime. Finally, the reheating is also not an instant process, and the expansion of the Universe during this period also departs from that at the pure matter-dominated stage slowing down the contrasts growth. All the aforementioned effects are model-dependent.

Second, to study the features of the inflaton clumps, one has to express the clump size and height in terms of the relevant parameters from the inflaton sector. This includes extracting the subhorizon modes and summing over all shorter-wavelength modes that contribute to the local spatial inhomogeneity of size $R$. The latter summing inherits some arbitrariness due to the choice of the window function which constrains the modes in the space to the region of spatial size $R$. The common choice is the top-hat filter function which gives the following dispersion of the density contrasts,

$$\langle \delta_R^2(t) \rangle \equiv \sigma_R^2(t) = \int_{H_a}^{k_{\text{max}}} \frac{dk}{k} P(k, t) \times \frac{9 j_1^2 (R k/a)}{(R k/a)^2},$$

with $j_1(x)$ being the spherical Bessel function and $P(k, t)$ representing the matter power
spectrum. The integration is performed over the subhorizon modes, $H \lesssim k/a$, with the upper limit referring to the shortest modes which exit the horizon at the very end of inflation, $k_{\text{max}} = H_{\text{inf}} a_{\text{inf}}$. Since the function $j_1(x)$ oscillates with decreasing amplitude at large $x$, Eq. (16) implies that the modes most relevant for the clump formation belong to the interval $H \lesssim k/a \lesssim 1/R$.

Third, to estimate the probability of forming a clump of size $R$ out of perturbations with dispersion $\sigma_R^2(t)$ one can exploit the Press–Schechter formalism [28]. At $\sigma_R^2 \ll 1$, i.e. before the perturbations become nonlinear, one finds,

$$P_{\text{clump}} = \int_{\delta_{c}}^{\infty} \frac{d\delta}{\sqrt{2\pi}\sigma_R} \exp\left(-\frac{\delta^2}{2\sigma_R^2}\right) \approx \frac{\sigma_R}{\sqrt{2\pi} \delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_R^2}\right), \quad (17)$$

where the threshold value $\delta_c \approx 1.686$ is obtained using the Tolman solution, see e.g. [29]. Thus, naively one would expect to multiply the probability (5) by the factor (17) accounting for the fact that while the perturbations of size $R$ are too small on average, there are few high fluctuations which might form the clumps. However, this is not the end of the story yet.

Forth, the clumps, formed by the modes which are the very first to become nonlinear, are not entirely homogeneous enough to allow black hole formation. Indeed, there are no (or very few) fluctuations of shorter wavelengths, and the density profile of the mode itself is not smooth enough to fuel black hole formation in the clump center. Therefore, the probability of finding the smooth enough configuration may receive additional suppression in this case as compared to Eq. (2).

Let us estimate how early the reheating must have been occurred in order that the present visible Universe stays in the electroweak vacuum with high probability. This means that within the present horizon there are less than $P_{\text{EW}}^{-1}$ regions which had collapsed into black holes during the early post-inflationary stage. To make the estimates as general as possible, we avoid including some model-dependent effects mentioned above. Namely, we assume matter-dominated expansion law keeping in mind that departures from this law can be accounted for on model-to-model basis. We also keep the dispersion $\sigma_R^2$ as a free parameter which should be computed via Eq. (16) in a given model. The other effects give small impact because they do not change the leading exponent in Eq. (17).

The probability of a perturbation to form a black hole is given by the product of Eqs. (17) and (2). Strictly speaking, this probability should be summed over all spatial scales $R$ permitted in Eq. (16). However, it is natural to assume that the leading contribution comes from the smallest perturbations with $R(t) \approx a(t)/a_{\text{inf}} H_{\text{inf}}$ entering the horizon right after
inflation and immediately starting to grow. We consider the case when the average height \( \sigma_R \simeq a_{reh} \sigma_{R,inf}/a_{inf} \) of these perturbations remains small at reheating, where \( \sigma_{R,inf} \sim \delta_i \) is its value at the end of inflation. Nevertheless, the highest perturbations collapse into structures of mass \( M \approx (2G H_{inf})^{-1} \). Consequently, for the probability to form a black hole we obtain (instead of (5)),

\[
P_{BH} \simeq 5 \times 10^{-3} \times \sigma_{R,inf} \left( \frac{H_{reh}}{H_{inf}} \right)^{11/3} \exp \left( -\frac{\delta_c^2}{2\sigma_{R,inf}^2} \left( \frac{H_{reh}}{H_{inf}} \right)^{4/3} \right). \tag{18}
\]

Note that although \( \sigma_{R,inf} \sim \delta_i \), the numerical factor between these quantities must be estimated accurately, as it enters the exponent. The probability (18) accounts for somewhat suppressed amplitude of the initial perturbations at the smallest scales and for the inhomogeneity of the clump forming a black hole.

Following the lines of Sec. 2 one finds for the probability to have a black hole inside the horizon volume at reheating (cf. Eq. (6)),

\[
P_{BH,hor} \simeq \frac{P_{BH}}{(H_{reh} R)^3} \simeq 10^{-2} \times \sigma_{R,inf} \left( \frac{H_{reh}}{H_{inf}} \right)^{8/3} \exp \left( -\frac{\delta_c^2}{2\sigma_{R,inf}^2} \left( \frac{H_{reh}}{H_{inf}} \right)^{4/3} \right). \tag{19}
\]

Finally, the number of completely evaporated black holes inside the visible part of the present-day Universe reads,

\[
N_{BH,0} \simeq 10^{64} \left( \frac{T_{reh}}{5 \times 10^{12} \text{ GeV}} \right)^3 \left( \frac{\sigma_{R,inf}}{10^{-4}} \right)^{8/3} \left( \frac{H_{reh}}{H_{inf}} \right)^{8/3} \exp \left( -\frac{\delta_c^2}{2\sigma_{R,inf}^2} \left( \frac{H_{reh}}{H_{inf}} \right)^{4/3} \right), \tag{20}
\]

which replaces Eq. (9). Recall that this expression is valid only in the case of relatively short preheating with \( a_{reh}/a_{inf} \lesssim \sigma_{R,inf}^{-1} \sim 10^4 \); in the opposite case one should use constraints from Sec. 2. As we expected, Eq. (20) is exponentially sensitive to the ratio of inflation and reheating scales \( H_{inf} \) and \( H_{reh} \simeq (G g_{s,reh})^{1/2} T_{reh}^2 \) and weakly depends on \( T_{reh} \) in the prefactor. Requiring that no more than \( P_{EW}^{-1} \) black holes are formed, \( N_{BH,0} P_{EW} < 1 \), we express \( H_{reh}/H_{inf} \) from this inequality and obtain a constraint

\[
\frac{H_{reh}}{H_{inf}} \gtrsim 3 \times 10^{-5} \times \left( \frac{\sigma_{R,inf}}{10^{-4}} \right)^{3/2}, \quad \frac{a_{reh}}{a_{inf}} \lesssim 10^3 \times \left( \frac{10^{-4}}{\sigma_{R,inf}} \right), \tag{21}
\]

where \( \sigma_{R,inf} \sim \delta_i \sim 10^{-4} \) and \( T_{reh} \sim 5 \times 10^{12} \text{ GeV} \) are used in the prefactor of Eq. (20) in accordance with Eq. (15) and we assume that \( P_{EW} \) is not too small. Equations (21) slightly refine the naive condition \( a_{reh}/a_{inf} \lesssim 10^4 \) of no black hole formation and the respective constraint (15) on the reheating temperature which takes the form

\[
T_{reh} \gtrsim 3 \times 10^{13} \text{ GeV} \times \frac{1/4}{10^{16} \text{ GeV}} \times \left( \frac{\sigma_{R,inf}}{10^{-4}} \right)^{3/4}. \tag{22}
\]
The models violating this inequality or Eq. (21) are excluded.

Constraints (21) are valid if $\mathcal{P}_{EW}$ is larger than the critical value

$$\mathcal{P}_{EW,c} \sim 10^{-48} \left( \frac{T_{reh}}{5 \times 10^{12} \text{GeV}} \right)^{-3} \left( \frac{\sigma_{R,inf}}{10^{-4}} \right)^{-5}$$

(23)

which one can read off (20). Above this value $\mathcal{P}_{EW}$ enters logarithmically into the constraints. One concludes that the black holes are produced if the Universe stretches $10^3$ times (21) during preheating and the decay probability is not exceedingly low, $\mathcal{P}_{EW} > \mathcal{P}_{EW,c}$.

4 Do small black holes catalyze the EW vacuum decay?

As we see, the danger of black hole induced false vacuum decay imposes severe constraints on the inflationary models. Thus, it is natural to pay detailed attention to this process. In Sec. 2 we mentioned the arguments of Ref. [10] suggesting that the probability to form a bubble of true vacuum around an isolated black hole of mass $M_{BH}$ is suppressed by the loss of black hole entropy $\Delta S$ in the process, $\mathcal{P}_{EW} \propto e^{-\Delta S}$. If the bubble mass $E_b \equiv \Delta M_{BH}$ is much smaller than $M_{BH}$, this suppression reduces to the Boltzmann factor

$$\mathcal{P}_{EW} \propto e^{-E_b/T_H},$$

(24)

involving the black hole temperature $T_H \equiv (8\pi G M_{BH})^{-1}$. Then the transition to the true vacuum becomes unsuppressed at

$$T_H \geq E_b,$$

(25)

indeed implying that the black holes of sufficiently small mass catalyze decay of the EW vacuum.

Note, however, that the result (24) of Ref. [10] essentially relies on the interpretation of static critical bubbles surrounding the black holes as Euclidean instantons describing EW vacuum decay. To understand these solutions physically, consider the regime $E_b \sim \text{few} \times T_H$ when the probability (24) is relatively large yet exponentially suppressed, so that the semi-classical methods are applicable. In this case the size and mass $r_g, M_{BH}$ of the black hole and the respective parameters $R_b, E_b$ of the surrounding bubble are essentially different. Indeed, $M_{BH}/E_b \sim G M_{BH}^2 \gg 1$. Thus, back-reaction of the bubble on the background geometry is small. On the other hand, $R_b E_b \gg 1$ because the bubble is classical, and therefore $r_g/R_b \ll E_b/T_H \sim 1$. This means that the major part of the true vacuum bubble lives in
flat spacetime far away from the black hole. As a result, the solution of Ref. [10] coincides with the flat-space critical bubble up to small corrections.

Given the flat-space nature of the solutions in [10] at $E_b \sim T_H$, the drastic enhancement of the related probabilities looks surprising. Technically, the difference is related to the fact that the Wick-rotated Schwarzschild time $\tau = it$ is periodic i.e. takes values on the circle $0 < \tau \leq T_H^{-1}$. This property holds even in the spatial regions far away from the black hole where the spacetime is flat. As a consequence, the static bubbles of energy $E_b$ and any size have finite action $B = E_b/T_H$ and give contributions (24) to the Euclidean path integral. Likewise, the correlator of the quantum field $h(x)$ in the Euclidean Schwarzschild background coincides with the thermal correlator rather than with the vacuum one, even if it is computed far away from the black hole. Since we cannot pretend that the black hole changes physics in the distant parts of the Universe, the Euclidean Schwarzschild spacetime should be interpreted as describing a black hole surrounded by the infinite bath of temperature $T_H$ [30] rather than an isolated black hole in empty spacetime. Then the solutions obtained in [10] give the rate\(^3\) (24) of false vacuum decay activated by fluctuations in the infinite-size thermal bath of temperature $T_H$. In the regime (25) this process is sensitive to the bath itself rather than to the tiny black hole in the bubble center.

In reality, small-mass black holes are not isolated but surrounded by their own Hawking flux of temperature $T_H$. The energy density within this flux, however, decreases as $r^{-2}$ and becomes essentially lower than thermal at the distances of several Schwarzschild radii. The probability (24) is not applicable to false vacuum decay near such black holes unless their sizes are comparable to the sizes of the true vacuum bubbles, $r_g \sim R_b$. While in the latter case one expects to find enhancement\(^4\), no unsuppressed vacuum decay should occur in the regime (25).

A question remains, however. Common knowledge suggests that black holes spit various field configurations, in particular, the bubbles of true vacuum, with Boltzmann-suppressed probability (24). This intuition is based on numerous semiclassical exercises with thin-wall bubbles, see e.g. Refs. [18, 19, 20], which did not rely on the Euclidean methods at all. One therefore can imagine a dynamical process where the high-temperature black hole produces a Higgs field bubble of initial size $r_g$. The bubble expands with nonzero velocity and eventually

\(^3\)Note that the quantum corrections to the Higgs potential should be computed on the same background with periodic $\tau$ as the leading-order semiclassical solutions. This procedure produces finite-temperature Higgs potential rather than the vacuum one. The probability of the EW vacuum decay receives additional suppression in this case [31].

\(^4\)In particular, due to smaller bubble energy in the black hole gravitational well [32].
reaches the critical size $R_b \gg r_g$. However, the Higgs field bubble in the SM has thick walls, it is formed by the field quanta with typical wavelength $R_b$. A configuration of this kind cannot originate from the local area of small size $r_g$ in the course of classical evolution. One therefore expects the probability of producing such a bubble to be exponentially suppressed as compared to Eq. (24).

To get a quantitative feeling of the suppression, we recall that the probability of emitting a particle of wavelength $R_b \gg r_g$ from the black hole involves, in addition to the thermal rate, a gray factor $\Gamma \propto (r_g/R_b)^2$ [33, 34]. The bubble of true vacuum containing $E_b R_b \sim 10^3$ quanta is therefore produced with the probability suppression $\Gamma^{E_b R_b} \propto (r_g/R_b)^{10^3}$ in addition to Eq. (24).

Depending on its size, this additional suppression does or does not make meaningless the cosmological bounds of the type considered in this paper. In particular, Eq. (9) shows that even in models with relatively low reheating temperature $T_{reh} \sim 10^9$ GeV like the $R^2$ inflation [23, 24] the number of primordial black holes in the visible part of the Universe (9) is relatively large, $N_{BH,0} \gtrsim 10^{37}$. When multiplied by the exponentially suppressed probability $P_{EW}$ of producing a bubble near each black hole, this number is still larger than one if $P_{EW} > P_{EW,c}$, cf. Eq. (23). Then the respective model (i.e. the $R^2$-inflation in our example) is excluded.

5 Conclusion

To summarize, recently suggested process of black hole induced false vacuum decay may exclude generic inflationary models with sufficiently long post-inflationary matter-dominated stages because the inflaton inhomogeneities grow, decouple from the Hubble flow, and a few of them form black holes catalyzing decay of the EW vacuum. The only general way to suppress this black hole formation is early reheating which stops gravitational contraction of the matter perturbations by nonzero pressure. Then the condition of having short enough preheating stage severely constrains the inflationary models and reheating mechanisms, see Fig. 1. Similar constraints exist in models with other than matter dominated expansion laws at preheating if the contrasts of matter perturbations grow sufficiently fast at this stage to approach the non-linear regime. If the constraints are not met, the scalar sector of the

\footnote{More accurately, this bound constrains duration of the matter-dominated epoch until production of relativistic particles which stop perturbation growth and black hole formation. By itself, thermalization is not needed for the radiation dominated stage to settle.}
Figure 1: Limits on the cosmological models with inflation scale $H_{inf}$ and reheating temperature $T_{reh}$. If the probability of induced vacuum decay is not too small, $P_{EW} > P_{EW,c}$, the value of $T_{reh}$ cannot be much lower than the temperature of the instantaneous reheating, see Eqs. (21). This leaves the narrow allowed (white) strip in the ($H_{inf}$, $T_{reh}$) plane overlapping with the experimentally allowed region $H_{inf} \lesssim 7 \times 10^{13}$ GeV [35, 36]. The second allowed (white) region in the lower left corner of the plot represents models with large primordial black holes which do not evaporate until now, see Eq. (11). In that case the danger awaits us in the future.

SM should be modified in a way to make the EW vacuum true, and that gives one more argument in favor of new physics in the Higgs sector. As we also argue, the process of black hole induced tunneling deserves further investigation, since the physics underlying it remains hidden and its probability may have been overestimated.

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