Jet measurements at DØ using a $k_T$ algorithm

V. Daniel Elvira$^a$ for the DØ Collaboration.

$^a$Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL, 60510-500, USA.

DØ has implemented and calibrated a $k_T$ jet algorithm for the first time in a $p\bar{p}$ collider. We present two results based on 1992-1996 data which were recently published: the subjet multiplicity in quark and gluon jets and the central inclusive jet cross section. The measured ratio between subjet multiplicities in gluon and quark jets is consistent with theoretical predictions and previous experimental values. NLO pQCD predictions of the $k_T$ inclusive jet cross section agree with the DØ measurement, although marginally in the low $p_T$ range. We also present a preliminary measurement of thrust cross sections, which indicates the need to include higher than $\alpha_s^3$ terms and resumption in the theoretical calculations.

1. INTRODUCTION

Until recently, only cone algorithms were used to reconstruct jets in $p\bar{p}$ [1] colliders. The cone algorithm used to reconstruct the 1992-1996 Tevatron data [2,3] presents several shortcomings:

- An arbitrary procedure must be implemented to split and merge overlapping calorimeter cones.
- An ad-hoc parameter, $R_{sep}$ [4], is required to accommodate the differences between jet definitions at the parton and detector levels.
- Improved theoretical predictions calculated at the next-to-next-to-leading-order (NNLO) in pQCD are not infrared safe, but exhibit sensitivity to soft radiation.

A second class of jet algorithms, free of these problems, was developed during the past decade [5,6,7]. These recombination algorithms successively merge pairs of nearby objects (partons, particles, or calorimeter towers) in order of increasing relative transverse momentum. A single parameter, $D$, which approximately characterizes the size of the resulting jets, determines when this merging stops. No further splitting or merging is involved because each object is uniquely assigned to a jet. There is no need to introduce any ad-hoc parameter, because the same algorithm is applied at the theoretical and experimental level. Furthermore, by design, clustering algorithms are infrared and collinear safe to all orders of calculation.

The DØ Collaboration has implemented a $k_T$ algorithm to reconstruct jets from data taken during the 1992-1996 collider run. This paper is a review of the associated measurements recently performed by the DØ experiment [8,9].

2. THE RUN I DØ DETECTOR

DØ is a multipurpose detector designed to study $p\bar{p}$ collisions at the Fermilab Tevatron Collider. A full description of the Run I DØ detector can be found in Ref. [10]. The primary detector components for jet measurements at DØ are the calorimeters, which use liquid-argon as the active medium and uranium as the absorber. The DØ calorimeters provide full solid angle coverage and particle containment (except for neutrinos or high $p_T$ muons), as well as linearity of response with energy and compensation ($e/\pi$ response ratio is less than 1.05). Transverse segmentation is $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$, with $\eta = -\ln \tan(\theta/2)$. The single particle energy resolutions for electrons ($e$) and pions ($\pi$), measured from test beam data, are approximately 15% and 50% respectively.
3. $k_\perp$ JET ALGORITHM

The DØ $k_\perp$ jet algorithm starts with a list of energy pre-clusters, formed from calorimeter cells, final state partons or particles. The angular separation between pre-clusters is $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} > 0.2$.

The jet reconstruction is performed in three steps:

1. For each object $i$ in the list, the algorithm defines $d_{ii} = p_T^2_i$, where $p_T$ is the momentum transverse to the beam. For each pair $(i, j)$ of objects, it also defines $d_{ij} = \min(p_T^2_i, p_T^2_j) \frac{\Delta \eta^2}{p_T^2}$, where $D$ is a resolution parameter.

2. If the minimum of all possible $d_{ii}$ and $d_{ij}$ is a $d_{ij}$, the algorithm replaces objects $i$ and $j$ by their 4-vector sum before going back to step 1. If the minimum is a $d_{ii}$, then $i$ is defined as a jet and removed from the list of objects.

3. While there is an object left in the list, the algorithm returns to step 1.

The final product of this process is a list of jets, separated by $\Delta R > D$ from each other. Subjets may be defined by re-running the $k_\perp$ algorithm from a list of pre-clusters in a given jet. Pairs of objects with the smallest $d_{ij}$ are merged successively until all remaining $d_{ij}$ satisfy $d_{ij} > y_{\text{cut}} p_T^2\text{jet}$. The resolved objects are called subjets, and the number of subjets within the jet is the subjet multiplicity $M$. For $y_{\text{cut}}=1$, the entire jet consists of a single subjet ($M=1$). As $y_{\text{cut}}$ decreases, the subjet multiplicity increases, until every pre-cluster becomes resolved as a separate subjet in the limit $y_{\text{cut}} \to 0$.

4. CALIBRATION OF JET MOMENTUM

The uncertainty in the jet energy or momentum is the dominant error in almost every jet measurement at a hadron collider. The jet momentum calibration is described in Ref. [8,11]. The calibration at DØ accounts for detector effects like response, noise, and signal pile-up from previous crossings. It also removes the underlying event formed by the remnant soft partons (u.e.), and the contribution of multiple $p_T$ interactions. These corrections enter a relation between the momentum of a jet measured in the calorimeter $p_{\text{meas}}^\text{jet}$ and the “true” jet momentum $p_{\text{true}}^\text{jet}$:

$$ p_{\text{true}}^\text{jet} = \frac{p_{\text{meas}}^\text{jet} - p_O(\eta^\text{jet}, L, p_T^\text{jet})}{R^\text{jet}(\eta^\text{jet}, p_T^\text{jet})} $$

where the offset term $p_O$ corrects for u.e., noise, pile-up, and multiple interactions, while $R_{\text{jet}}$ corrects for the response of the calorimeter to jets. The true jet momentum is defined as the particle level jet momentum. A particle level jet is reconstructed from the final state particles, after hadronization but before interaction with the
calorimeter material. The calibration procedure follows closely that of the calibration of the fixed-cone jet algorithm

The fractional momentum resolution for \( k_\perp \) jets (\( D=1 \)) is determined from the measured \( p_T \) imbalance in dijet events. At 100 (400) GeV, the fractional resolution is 0.061 ± 0.006(0.039 ± 0.003). Within statistical and systematic uncertainties, there is not a significant difference between energy resolutions associated with \( k_\perp \) (\( D=1 \)) and cone \( R=0.7 \) jets.

5. PHYSICS RESULTS

DØ has performed a number of measurements using the \( k_\perp \) jet algorithm. These include a study of the structure of quark and gluon jets \([4]\), and measurements of the central (\(|\eta| < 0.5 \)) inclusive jet cross section \([5]\), and thrust distributions.

5.1. Subjet multiplicities

In LO QCD, the fraction of final state jets which are gluons decreases with \( x \sim p_T/\sqrt{s} \), the momentum fraction of initial state partons within the proton. For fixed \( p_T \), the gluon jet fraction decreases when \( \sqrt{s} \) is decreased from 1800 GeV to 630 GeV. We select gluon and quark enriched samples with identical cuts in events at \( \sqrt{s} = 1800 \) and 630 GeV to reduce experimental biases and systematic effects. Of the two highest \( p_T \) jets in the event, we select \( k_\perp (D = 0.5) \) jets with 55 < \( p_T \) < 100 GeV and \(|\eta| < 0.5 \).

There is a simple method to extract a measurement of quark and gluon jets on a statistical basis. If \( M \) is the subjet multiplicity in a mixed sample of quark and gluon jets, it may be written as a linear combination of subjet multiplicity in gluon and quark jets:

\[
M = f M_g + (1 - f) M_q \tag{2}
\]

The coefficients are the fractions of gluon and quark jets in the sample, \( f \) and \( (1 - f) \), respectively. Consider Eq. (2) for two similar samples of jets at \( \sqrt{s} = 1800 \) and 630 GeV, assuming \( M_g \) and \( M_q \) are independent of \( \sqrt{s} \). The solutions are

\[
M_g = \frac{(1 - f^{630}) M^{1800} - (1 - f^{1800}) M^{630}}{f^{1800} - f^{630}} \tag{4}
\]

where \( M^{1800} \) and \( M^{630} \) are the experimental measurements in the mixed jet samples at \( \sqrt{s} = 1800 \) and 630 GeV, and \( f^{1800} \) and \( f^{630} \) are the gluon jet fractions in the two samples. The method relies on knowledge of the two gluon jet fractions, which are extracted from the HERWIG 5.9 \([6]\) Monte Carlo event generator and used in Eqs. (2-4).

Figure 3 shows a comparison between the ratio measured by DØ , the HERWIG 5.9 result of \( r=1.91 \), the ALEPH \([7]\) value of \( r=1.7 \pm 0.1 \) (\( e^+ e^- \) annihilations at \( \sqrt{s} = M_Z \)), and the associated Monte Carlo and resummation prediction \([8]\). Good agreement is observed. All of the experimental and theoretical values for \( r \) are smaller than the naive QCD prediction value of 2.25 for the ratio of color charges. This is because of higher-order radiation in QCD, which tends to reduce the ratio from the naive value.

![Figure 2. Corrected subjet multiplicity for gluon and quark jets, extracted from DØ data.](image-url)
5.2. Central inclusive jet cross section

The inclusive jet cross section for $|\eta| < 0.5$, $d^2\sigma/(dp_T d\eta)$, was measured as $N/(\Delta\eta \Delta p_T \epsilon L)$, where $\Delta\eta$ and $\Delta p_T$ are the $\eta$ and $p_T$ bin sizes, $N$ is the number of jets reconstructed with the $k_\perp$ ($D=1$) algorithm in that bin, $\epsilon$ is the overall efficiency for jet and event selection, and $L$ represents the integrated luminosity of the data sample [9].

The fully corrected cross section for $|\eta| < 0.5$ is shown in Fig. 4, along with the statistical uncertainties. The fractional uncertainties for the different components are plotted in Fig. 5 as a function of the jet transverse momentum. The results are compared to the NLO pQCD prediction from jetrad [17], with the renormalization and factorization scales set to $p_T^{\text{max}}/2$, where $p_T^{\text{max}}$ refers to the $p_T$ of the leading jet in an event. The comparisons are made using parameterizations of the parton distribution functions (PDFs) of the CTEQ [18] and MRST [19] families. Figure 6 shows the ratios of (data-theory)/theory. The predictions lie below the data by about 50% at the lowest $p_T$ and by (10-20)% for $p_T > 200$ GeV. To quantify the comparison in Fig. 6, the fractional systematic uncertainties are multiplied by the predicted cross section, and a $\chi^2$ comparison, using the full correlation matrix, is carried out. Though the agreement is reasonable ($\chi^2$/dof ranges from 1.56 to 1.12, the probabilities from 4 to 31%), the differences in normalization and shape, especially at low $p_T$, are quite large. The points at low $p_T$ have the highest impact on the $\chi^2$. If the first four data points are not used in the $\chi^2$ comparison, the probability increases from 29% to 77% when using the CTEQ4HJ PDF.
The NLO predictions of the inclusive cross section for $k_T$ ($D=1$) and cone jets ($R=0.7$, $R_{\text{sep}}=1.3$) in the same $|\eta| < 0.5$ interval are within a few percent of each other in the $p_T$ range relevant in this analysis \[13\]. The measured $k_T$ cross section, however, is 37% (16%) higher than the published cone algorithm \[20\] at 60 (200) GeV. This difference in the cross sections is consistent with the measured difference in $p_T$ for cone jets matched to $k_T$ jets, as shown in Fig. 6. For the same energy clusters the $p_T$ of $k_T$ jets ($D=1$) is higher than the $E_T$ of associated cone jets ($R=0.7$). The difference increases with jet $p_T$, from about 5 GeV (or 5%) at $p_T \approx 100$ GeV to about 7 GeV (or 3%) at $p_T \approx 250$ GeV \[8\]. Fig. 6 proves, however, that the energy difference does not depend on the instantaneous luminosity associated with the sample. After offset subtraction, it is clear that $k_T$ jets are not contaminated by energy coming from pile-up, uranium noise, or multiple interactions.

The effect of final-state hadronization on reconstructed energy, which might account for the discrepancy between the observed cross section using $k_T$ and the NLO predictions at low $p_T$, and also for the difference between the $k_T$ and cone results, was studied using HERWIG (version 5.9) simulations. Figure 7 shows the ratio of $p_T$ spectra for particle-level to parton-level jets, for both the $k_T$ and cone algorithms. Particle cone jets, reconstructed from final state particles (after hadronization), have less $p_T$ than the parton jets (before hadronization), because of energy loss outside the cone.

In contrast, $k_T$ particle jets are more energetic than their progenitors at the parton level, due to the merging of nearby partons into a single particle jet. Including the hadronization effect derived from HERWIG in the NLO JETRAD predic-
tion improves the $\chi^2$ probability from 29% to 44% (31% to 46%) when using the CTEQ4HJ (MRST) PDF.

We have also investigated the sensitivity of the measurement to the modeling of the background from spectator partons through the use of minimum bias events, and found that it has a small effect on the cross section: at low $p_T$, where the sensitivity is the largest, an increase of as much as 50% in the underlying event correction decreases the cross section by less than 6%.

5.3. Thrust cross sections

Event shape variables have been extensively used in $e^+e^-$ and $ep$ collider experiments to study the spatial distribution of hadronic final states, to test the predictions of perturbative QCD, and to extract a precise value of the coupling constant $\alpha_s$. Over the last few years, they have attracted considerable interest, as they have proved to be a fruitful testing ground for recent QCD developments like resummation calculations and non-perturbative corrections.

There are several observables which characterize the shape of an event. To be calculable by perturbation theory, these quantities must be infrared safe, i.e., insensitive to the emission of soft or collinear gluons. A widely used variable that meets this requirement is the thrust, defined as

$$T = \max_n \sum_i |\vec{p}_i \cdot \hat{n}| / \sum_i |\vec{p}_i|$$

where the sum is over all partons, particles or calorimeter towers in the event. The unit vector $\hat{n}$ that maximizes the ratio of the sums is called the thrust axis. The values of thrust range from $T=0.5$ for a perfectly spherical event, to $T=1$ for a pencil-like event, when all emitted particles are collinear. In this latter case, the thrust axis lies along the direction of the particles.

In most of the kinematic range, $e^+e^-$ and $ep$ collider experiments report good agreement of event shape distributions with $O(\alpha_s^2)$ pQCD corrections to the lowest order QED diagram that governs the interaction. Fixed order QCD calculations, however, fail when two widely different energy scales are involved in the event, leading to the appearance of large logarithmic terms at all orders in the perturbative expansion. This happens in the limit of the two jet back-to-back configuration, when $T \to 1$. This case is handled by the resummation technique, which identifies the large logarithms in each order of perturbation theory and sums their contributions to all orders. DELPHI reports excellent agreement of thrust distributions in $Z \to$ hadrons once resummation and hadronization corrections are added to the $O(\alpha_s^3)$ QCD prediction.

In a hadron collider, it is convenient to introduce “transverse thrust”, $T^T$, a Lorentz invariant quantity under $z$-boosts, which is obtained from Eq. (5) in terms of transverse momenta:

$$T^T = \max_n \frac{\sum_i |\vec{p}_T_i \cdot \hat{n}|}{\sum_i |\vec{p}_T_i|}$$

Transverse thrust ranges from $T^T=1$ to $T^T=2/\pi$ (||cos $\theta||$) for a back-to-back and an isotropic distribution of particles in the transverse plane, respectively. To minimize systematics associated with the busy environment in a $p\bar{p}$ collider, we use only the two leading jets in the event, reconstructed with a $k_T D=1$ algorithm, rather than using all calorimeter towers. Other particles in the event are inferred from the angular distribution of the two leading jets.

The measurement of the dijet transverse thrust, $T^T_2$ presents a good opportunity to test resummation models, as well as the recently developed NLO pQCD three-jet generators. $T^T_3$ is binned in terms of $H_{T3}$, defined as the scalar sum of the transverse momenta of the three leading jets of the event $p_{T1} + p_{T2} + p_{T3}$. $H_{T3}$ is an estimator of the energy scale of the event. The lowest order at which pQCD does not give the trivial result $T^T_2=1$ is $O(\alpha_s^3)$, corresponding to up to three parton jets in the final state. A $O(\alpha_s^3)$ calculation, like JETRAD, does not cover the whole physical range of $T^T_2$. A $O(\alpha_s^3)$ prediction will also fail at $T^T_2 \to 1$, where soft radiation in dijet events contribute large logarithms which need to be resummed.

Figures 1 and 10 show thrust cross sections as a function of $1 - T^T_2$ for the lowest and highest $H_{T3}$ bins: 160-260 GeV and 430-700 GeV.

The error bars are statistical, and the systematic uncertainties are in the 15-30% range. The
higher the value of $1 - T_2^T$, the more dominant is the contribution of high order terms in $\alpha_s$. This explains the disagreement between the data and the $\alpha_3^s$ prediction JETRAD in the high $1 - T_2^T$ range. For example, for $T_2^T$ between $\sqrt{2}/2$ and $\sqrt{3}/2$, the $\alpha_4^s$ terms contribute to LO. When we expand the range of thrust around unity using a logarithmic scale, we verify that JETRAD also fails, suggesting the need to resum higher order terms to improve agreement. Table 1 displays the agreement probability between the DØ data and JETRAD, which decreases sharply as we incorporate data points in the high and low $1 - T_2^T$ range. The probabilities are calculated using the full covariance error matrix in a $\chi^2$ test.

| $1 - T_2^T$ Range | $\chi^2$ | # d.o.f. | Prob. (%) |
|-------------------|----------|----------|-----------|
| 0 - 0.1           | 10       | 10       | 42        |
| 0 - 0.12          | 13       | 11       | 30        |
| 0 - 0.14          | 42       | 12       | 0.004     |
| $10^{-2.4} - 0.063$ | 2.7     | 5        | 75        |
| $10^{-3} - 0.063$  | 3.8      | 6        | 71        |
| $10^{-4} - 0.063$  | 95       | 7        | 0         |

### 6. Conclusions

DØ has successfully implemented and calibrated a $k_\perp$ jet algorithm in a $p\bar{p}$ collider. Quark and gluon jets have a different structure consistent with the HERWIG prediction and previous experimental results from $e^+ - e^-$ colliders. The thrust cross section measurements indicate the need to include higher than $\alpha_3^s$ terms and resummation in the theoretical predictions. They also offer an excellent opportunity to test the recently developed NLO three jet generators [24,26]. The marginal agreement between the measurement of the particle level inclusive $k_\perp$ jet cross section with the $\alpha_3^s$ theory is opening a debate on mat-
ters such as hadronization, underlying event, and algorithm definition.

REFERENCES

1. J. Huth et al., in Proc. of Research Directions for the Decade, Snowmass 1990, edited by E.L. Berger (World Scientific, Singapore, 1992).
2. B. Abbott et al. (D0 Collaboration), Phys. Rev. D64, 032003 (2001).
3. T. Affolder et al. (CDF Collaboration), Phys. Rev. D64, 032001 (2001).
4. S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. D48, 3160 (1993).
5. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. D65, 52008 (2002).
6. S. Abachi et al. (D0 Collaboration), Nucl. Instrum. Methods in Phys. Res. A338, 185 (1994).
7. B. Abbott et al. (D0 Collaboration), Nucl. Instrum. Meth. A424, 352 (1999).
8. W. Giele et al. (Jet Physics Working Group), in QCD and Weak Boson Physics in Run II, edited by U. Baur, R.K. Ellis, D. Zeppenfeld (Fermilab, Batavia, IL, 2000).
9. S. Grinstein, Ph.D. thesis, Univ. de Buenos Aires, Argentina, 2001 (in preparation).
10. G. Marchesini, B.R. Webber, G. Abbiendi, I.G. Knowles, M.H. Seymour, and L. Stanco, Comp. Phys. Comm. 67, 465 (1992).
11. D. Buskulic et al. (ALEPH Collaboration), Phys. Lett. B346, 389 (1995).
12. M.H. Seymour, Phys. Lett. B378, 279 (1996).
13. W.T. Giele, E.W.N. Glover, and D.A. Kosower, Phys. Rev. Lett. 73, 2019 (1994).
14. H.L. Lai et al., Phys. Rev. D55, 1280 (1997).
15. A. D. Martin et al., Eur. Phys. J. C 4, 463 (1998).
16. B. Abbott et al. (DØ Collaboration), Phys. Rev. Lett. 86, 1707 (2001).
17. ALEPH Collaboration, D. Decamp et al., Phys. Lett. B234, 399 (1990); Phys. Lett B255, 623 (1991). DELPHI Collaboration, P. Aarnio et al., Phys. Lett. B240, 271 (1990). L3 Collaboration, B. Adeva et al., Phys. Lett. B237, 136 (1990) OPAL Collaboration, M.Z. Akrawy et al., Phys. Lett. B235, 389 (1990); Z. Phys. C47, 505 (1990). TASSO Collaboration, W. Bartel et al., Z. Phys. C33, 187 (1990).
18. OPAL Collaboration, K. Ackerstaff et al., Z.Phys. C75, 193 (1997). DELPHI Collaboration, P. Abreu et al., Z. Phys. C73, 229 (1997). H1 Collaboration, C. Adloff et al., Phys. Lett. B406, 256 (1997). K. Rabbertz and U. Wollmer [hep-ex/0008006]. G. J. McCance [hep-ex/0008009]. G.P.Korchemsky and S.Tafat [hep-ph/0007003]. Y. Dokshitzer [hep-ph/9911293]. O. Biebel, P. Movilla and S. Bethke, Phys. Lett. B459, 326 (1999); S. Catani, L. Trentadue, G. Turlock and B. Webber Phys. Lett. B263, 491 (1991). G.P.Korchemsky and G.Sterman, Nucl.Phys. B555, 335 (1999).
19. DELPHI Collaboration, P. Abreu et al., Z.Phys. C59, 21 (1993).
20. W.T. Giele and W.B. Kilgore, Phys. Rev. D55, 7183 (1997); W. Kilgore, W. Giele, ICHEP 2000, Osaka, Japan [hep-ph/0009193].
21. Z. Nagy, Phys. Rev. Lett. 88, 122003 (2002).