Structured Light by linking together diffraction-resistant spatially shaped beams: “Lego-beams” †

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\textbf{Abstract} — In this paper we present a theoretical method, together with its experimental confirmation, to obtain structures of light by connecting diffraction-resistant cylindrical beams of finite lengths and different radii. The resulting “Lego-beams” can assume, on demand, various unprecedented spatial configurations. We also experimentally generate some of them using a computational holographic technique and a spatial light modulator. Our new, interesting method of linking together various pieces of light can find applications in all fields where structured light beams are needed, in particular such as optical tweezers, e.g. for biological manipulations, optical guiding of atoms, light orbital angular momentum control, holography, lithography, non-linear-optics, interaction of electromagnetic radiation with Bose-Einstein condensates, and so on, besides in general the field of Localized Waves (non-diffracting beams and pulses).

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1 Introduction

Structured Light \cite{1, 2, 3, 4, 5, 6} has been more and more studied, and applied in various sectors, like optical tweezers \cite{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}, optical guiding of atoms \cite{18, 19, 20, 21, 22, 23, 24}, imaging \cite{25}, light orbital angular momentum control and applications \cite{26, 27, 28, 29, 30, 31}, and photonics in general.

A rather efficient method to model longitudinally the intensity of non-diffracting beams is by the so-called Frozen Waves (FWs) \cite{1, 2, 3, 32, 33, 34, 35, 36}, obtained from superpositions of co-propagating Bessel beams, endowed with the same frequency and order. The resulting diffraction resistant beam, with a longitudinal intensity shape freely chosen \textit{a priori}, may then propagate, remaining confined, along the propagation axis $z$, or over a cylindrical surface (depending on the order of the constituting Bessel beams), while its “spot” size, and the cylindrical surface radius, can be as well chosen \textit{a priori}. In this way, it is possible to construct, e.g., cylindrical beams whose static envelopes possess non-negligible energy density in finite, well-defined spatial intervals only: so that they can be regarded as \textit{segments or cylindrical pieces of light}.

Aiming also at a greater control on the beam \textit{transverse} shape, another method was recently proposed \cite{24, 26}, where different-order FW-type beams are superposed, which possess appreciable intensities along different, but consecutive, space intervals: So that one ends with cylindrical structures of light endowed with different radii and located in different positions along the $z$ axis. This new method resulted efficient, incidentally, also for controlling the orbital angular momentum along the propagation axis \cite{26}.

Anyway, and interestingly enough, it is possible to join together in the same way even two FW-type beams \textit{bearing the same order}, by getting again a structure with two different-radius cylinders, each one in its own space interval. To this aim, it is sufficient that each equal-order FW possesses a different value of its central longitudinal wave number: which implies a different radius for the corresponding cylindrical structure. An advantage of using FWs with the same order is that the resulting beam intensity keeps its azimuthal symmetry, thus avoiding the intensity perturbations (asymmetries) that one
meets on the transverse plane where two different-order FWs connect to each other.

Indeed, in this work we extend—theoretically and experimentally—on a method of ours proposed in [24, 26], by explicitly considering superpositions not only of different order FWs, but also of FWs with the same order but different central longitudinal wavenumbers. Moreover, we also investigate the superposition of FWs whose longitudinal intensity values have non-zero values inside the same space interval: This allows us creating even coaxial light structures, or cylindrical structures e.g. with “emboli” (blockages), and so on.

We call “Lego-beams” all such structures of light. Our new method of linking together various “pieces of light” can find applications in all fields where structured light beams are needed, such as in particular optical tweezers, e.g. for biological manipulations, optical guiding of atoms, light orbital angular momentum control, holography, lithography, non-linear-optics, interaction of electromagnetic radiation with Bose-Einstein condensates, and so on, besides, of course, the field of Localized Waves (non-diffracting beams and pulses).

2 The method

Let us consider as exact solutions to the wave equation the following superposition:

$$
\Psi(\rho, \phi, z, t) = \sum_{\nu=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \psi_{\nu \ell}(\rho, \phi, z, t) \tag{1}
$$

with

$$
\psi_{\nu \ell}(\rho, \phi, z, t) = M_\nu \sum_{n=-N_{\nu \ell}}^{N_{\nu \ell}} A_{\nu \ell n} J_{\nu}(h_{\nu \ell n} \rho) e^{i\nu \phi} e^{i\beta_{\nu \ell n} z} \tag{2}
$$

where

$$
\beta_{\nu \ell n} = Q_{\nu \ell} + \frac{2\pi}{L} n, \tag{3}
$$

$$
h_{\nu \ell n} = \sqrt{k^2 - \beta_{\nu \ell n}^2}, \tag{4}
$$

$$
A_{\nu \ell n} = \frac{1}{L} \int_0^L F_{\nu \ell}(z) e^{-i\frac{2\pi}{L} n z} dz, \tag{5}
$$
and with \( k = \omega/c \) and \( \mathcal{M}_\nu = 1/[J_\nu(.)]_{max} \), quantity \([J_\nu(.)]_{max}\) being the maximum value of the \( \nu \)-order Bessel function of the first kind.

For a better comprehension of it, let us explicitly examine the solution given by Eq. (1). First, let \( \nu \) and \( \ell \) have fixed values, and then consider a term \( \psi_{\nu\ell}(\rho, \phi, z, t) \) of the series. Eq. (2) teaches us that such term is a FW \([1, 2, 3]\) of order \( \nu \), whose central longitudinal wavenumber \( (n = 0) \) is \( \beta_{\nu\ell_0} = Q_{\nu\ell} \), so that \( h_{\nu\ell_0} = \sqrt{k^2 - Q_{\nu\ell}^2} \); and the FW intensity shape \(|F_{\nu\ell}(z)|^2\) in the interval \( 0 \leq z \leq L \) results to be concentrated: (i) either on a cylindrical surface having radius \( \rho_{\nu} \approx s_\nu/h_{\nu\ell_0} \) [quantity \( s_\nu \) being the \( s \) value where \( J_\nu(s) \) gets its maximum value], in the case \( \nu \geq 1 \); or (ii) around the propagation axis \( z \), with a spot radius \( r_0 \approx 2.4/h_{0\ell_0} \), in the case \( \nu = 0 \). Here, we define \( \rho_0 \equiv 0 \). In any case, \(|\psi_{\nu\ell}(\rho = \rho_{\nu}, \phi, z, t)|^2 \approx |F_{\nu\ell}(z)|^2\).

Let us now examine the sum \( \sum_\ell \psi_{\nu\ell} \) entering Eq. (1). It represents a superposition of FWs of the same order “\( \nu \)”, but with different values of their central longitudinal wavenumbers \( \beta_{\nu\ell_0} = Q_{\nu\ell} \). Each FW (with the same order, let us repeat, but different \( \ell \)) possesses its own intensity longitudinal shape \(|F_{\nu\ell}(z)|^2\).

Finally, the last sum, that is, \( \sum_\nu \sum_\ell \psi_{\nu\ell} \), refers to superposition of different order FWs: as before, for each “\( \nu \)” we deal with FWs with different \( \ell \) values, that is, possessing different values of \( Q_{\nu\ell} \) for their central longitudinal \( \beta_{\nu\ell_0} \) and also possessing their own longitudinal intensity shapes.

Before going on, let us stress the type of light structures which can be properly built up by the solution we started from. We know that each \( \psi_{\nu\ell} \) in superposition (1) is a FW whose static intensity envelope can be regarded as a “piece” of light. Due to the interference present in any wave phenomena, when summing different FWs together, like \( \psi_1 \) and \( \psi_2 \), one does not obtain the mere sum of their intensity envelopes. For instance, as well-known,

\[
|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2| \cos \delta , \tag{6}
\]

where \( \delta \) is the phase difference between the two waves. To minimize the contribution associated with these phase differences, one has to avoid e.g. cases in which two different
FWs have relevant intensities in the same spatial region. We can choose, for instance, different FWs, $\psi_{\nu \ell}$, which in the region $0 \leq z \leq L$ have appreciable longitudinal intensity patterns, $|F_{\nu \ell}(z)|^2$, only in different, and consecutive, intervals. Even when minimizing the interference, it goes on existing; its critical effect being in the welding region: In other words, critical interference takes mainly place in the $z$-planes corresponding to the interval boundaries. A way out is choosing successive FWs with orthogonal polarizations, so that the double product in the last equation does vanish: This will be examined in future works.

Actually, it is also possible to obtain interesting light structures by superposing FWs with non-zero intensities over the same $z$-interval, provided that the corresponding cylindrical structures have rather different radii: Thus, obtaining interesting co-axial structures for light.

Everything results clearer from the theoretical, and experimental, examples presented in the next Section.

3 Theoretical Examples

In this section we apply our method to obtain some interesting structured optical beams. We adopt a frequently used wavelength: $\lambda = 632.8$ nm.

We are going to represent with enough details our first example of a “Lego-beam”: Namely, of a beam whose spatial structure consists of two adjacent cylindrical surfaces, 12cm and 16cm long, with corresponding radii of 128$\mu$m and 182$\mu$m subsequently linked each other; besides a coaxial, central light-segment having spot 15$\mu$m and length 28cm.

To get such a beam, we use our “Lego-beam”-type solution, Eq. (1), and consider only three FWs, one of order zero, and two more of the same order 10 (but with different values of their central longitudinal wave numbers). That is, by inserting in Eq. (1) the non-zero functions (FWs) $\psi_{00}$, $\psi_{100}$ and $\psi_{101}$, endowed in the interval $0 \leq z \leq L = 1$ m with the following longitudinal intensity shapes:
\[ F_{\nu \ell} = \delta_{\nu 0} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.48)] \]
\[ + 0.8 \delta_{\nu 10} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.325)] \]
\[ + 0.8 \delta_{\nu 10} \delta_{\ell 1} [H(z - 0.32) - H(z - 0.48)] , \]

where \( \delta_{pq} \) is the Kronecker delta function and \( H(.) \) is the Heaviside function. The corresponding \( Q_{\nu \ell} \) are evaluated on the basis of the wished values for the cylindrical surface radii and for the light-segment spot, resulting to be:

\[ Q_{00} = 0.999910 \frac{\omega}{c} \]
\[ Q_{100} = 0.999970 \frac{\omega}{c} \]
\[ Q_{101} = 0.999985 \frac{\omega}{c} \]

Then, the coefficients \( A_{\nu \ell n} \) can be easily obtained via Eq.(5); while the longitudinal and transverse wave numbers, \( \beta_{\nu \ell n} \) and \( h_{\nu \ell n} \), respectively, are got by Eqs(3,4). In the present example, it was enough to use 25 Bessel beams in each one of the considered FWs: that is, \( N_{\nu \ell} = 12 \).

Our result is represented by the following Figures. First, Figures 1(a), 1(b) and 1(c) show, separately, the intensities of the three FWs which are going to compose our beam (in a sense, show the pieces of the “Lego-beam”). Figure 2 shows the intensity, instead, of the resulting beam (that is, of the structured light resulting from the sum of the chosen FWs).

One can notice that the interferences among the three FWs got minimized by the fact that such FWs possess relevant intensities in different space regions.
Figure 1: Figures (a), (b) and (c) show, separately, the intensities of the three FWs which are going to compose our beam (in a sense, show the pieces of the “Lego-beam”)

Figure 2: The intensity is here shown of the resulting “Lego-beam” (that is, of the structured light resulting from the sum of the chosen FWs)

Let us show some further possibilities forwarded by our method, even if—for the sake of conciseness— we skip mathematical details: Namely, let us show in Figs.3 four further
Figure 3: Figures (a), (b), (c) and (d) depict four further interesting “Lego-beams” obtained by our method. They refer to cylindrical or threadlike structures (adjacent or coaxial in their location), having transverse sizes of tens of micrometers and longitudinal sizes of tens of centimeters: They being, therefore, highly resistant to diffracting effects. See also the text.

light structures obtained from Eq. (1).

The first one, Fig.3(a), depicts the intensity of the “Lego-beam” consisting in two cylindrical surfaces, of different (decreasing) radii, linked one to the other, while the second cylinder on its right side has a light segment acting as a “cork”. Figure 3(b) shows the intensity of a structured light beam formed by two subsequent, coaxial cylindrical surfaces: In this case two FWs with the same order (larger than zero, of course) were adopted. In Fig.3(c) one has a sequence of donut-shaped light structures, with a central coaxial (zero order) light line. At last, Fig.3(d) refers to the intensity of a structured beam, made of a cylindrical surface with two light “embuli” (blockages) inside it.

All these examples refer to cylindrical or threadlike structures (adjacent or coaxial in their location), having transverse sizes of tens of micrometers and longitudinal sizes of
tens of centimeters: They being, therefore, highly resistant to diffracting effects.

4 Experimental Confirmations

Let us present in this Section a couple of experimental confirmations of our approach, by constructing two “Lego-beams” via a holographic setup that performs the optical reconstruction of computer generated holograms, sent electronically into a reflective Spatial Light Modulator (SLM), used in amplitude modulation mode, followed by a 4F spatial filtering system. Further details can be found in the caption of Fig.4.

We obtain the 2D CGH of a “Lego-beam” through the complex transmittance function (hologram) obtained from the desired resulting beam at the origin $\Psi(\rho, \phi, z = 0, t)$, with $\Psi$ given by Eq.(1). The hologram equation is expressed by [32, 33, 37, 38]:

$$H(x, y) = \frac{1}{2} \left\{ \beta(x, y) + \alpha(x, y) \cos \left[ \theta(x, y) - 2\pi(u_0 x + v_0 y) \right] \right\}$$

(9)

where $\alpha(x, y)$ and $\theta(x, y)$ are amplitude and phase, respectively, of the complex field $\Psi(\rho, \phi, z = 0, t)$, quantity $\beta(x, y) = [1 + \alpha^2(x, y)]/2$ being a bias function chosen as a soft envelope for the amplitude $\alpha(x, y)$. In order to make easier the separation of the different diffraction orders from the encoded complex field $\Psi(\rho, \phi, z, t)$, the off-axis reference plane wave $\exp[i2\pi(\xi x + \eta y)]$ is used, shifting, in the Fourier plane, the center of signal information to the spatial frequencies $u_0; v_0$.

The holographic setup used in the experimental reconstruction process of the CGH of our “Lego-beams” is shown in Fig.4. The coherent light from the He-Ne laser (632.8 nm) passes through the spacial filter SF and is collimated by lens L1; it is then reflected by mirror M1 and polarized by polarizer P1; when arriving at the beam splitter BS1, the light is directed to the reflective SLM (model LC-R 1080, Holoeye Photonics), where the CGH was sent electronically, and diffracted, proceeding to polarizer P2. To select the correct diffraction order of the reconstructed (diffracted) beam, a 4F filter is used after the SLM, composed of two lenses (L2 and L3) and an iris diaphragm (ID). Finally, the “Lego-beam” is acquired by the CCD camera (model DMK 41BU02.H, Imaging Source),
at subsequent locations along $z$.

Figure 4: The holographic setup used in the experimental reconstruction process of the CGH’s of the “Lego-beams”. Where: Laser is a He-Ne (632.8 nm) laser; SF is a spatial filter; L1, L2 and L2 are lens; CoF is a neutral density filter; M1 is a mirror; P1 and P2 are polarizers; BS1 is a beam splitter; Blocker is an optical blocker; Reflective SLM is a reflective Spatial Light Modulator; ID is an iris diaphragm; and, finally, CCD is a camera.

The first experimentally generated beam possesses its shape similar to that of the first theoretical example, but with different values for the radii of the cylindrical surfaces due to the limited resolution of our SLM. Namely, we generate a beam whose diffraction resistant spatial structure consists in two adjacent cylindrical surfaces, 12 cm and 16 cm long, with the corresponding radii 148 $\mu$m and 203 $\mu$m, sequentially linked one to the other; while it coaxially exists a “light segment” having a spot of 38 $\mu$m and a length of 28 cm.

We use, therefore, our “Lego-beam”-solution, Eq. (1), with three FWs: one zeroth-order FW, and two eighth-order FWs (the latter endowed with different values of their central longitudinal wave numbers). That is, in Eq. (1), the non-zero functions (FWs) $\psi_{\nu \ell}$ are: $\psi_{00}, \psi_{80}$ and $\psi_{81}$, possessing in the interval $0 \leq z \leq L = 0.6$ m the intensity longitudinal patterns:
\[ F_{\nu \ell} = 1.2 \delta_{\nu 0} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.48)] + \delta_{\nu 8} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.32)] + 1.1 \delta_{\nu 8} \delta_{\ell 1} [H(z - 0.32) - H(z - 0.48)] , \]

where \( \delta_{pq} \) is a Kronecker delta function, and \( H(,) \) an Heaviside function. The corresponding \( Q_{\nu \ell} \) values depend on the chosen cylindrical surface radii and on the light-segment spot, and result to be:

\[
Q_{00} = 0.9999800 \frac{\omega}{c} \\
Q_{80} = 0.9999785 \frac{\omega}{c} \\
Q_{81} = 0.9999885 \frac{\omega}{c} \tag{11}
\]

Again, coefficients \( A_{\nu \ell n} \) are evaluated via Eq.(5); while the longitudinal values, \( \beta_{\nu \ell n} \), and the transverse ones, \( h_{\nu \ell n} \), of the wave numbers are given by Eqs(3,4). With regard to the number of terms \( N_{\nu \ell} \) for each \( \psi_{\nu \ell} \), we adopted \( N_{00} = 14 \), \( N_{80} = 10 \) and \( N_{81} = 10 \), respectively.

Figura 5 shows the experimentally generated “Lego-beam”. On its upper right side, in smaller size, we reproduce its theoretical prediction. An excellent agreement is apparent, which confirms validity and applicability of our method.

Let us now pass to the second experimentally generated “Lego-beam”. We choose two coaxial cylindric light surfaces, in analogy to our theoretical Fig.3(b), wherein we used two equal-order FWs (while for the experiment we adopt two different-order FWs). More specifically, we want to generate two coaxial light-surfaces, having the same length 20 cm, but the different radii 45 \( \mu m \) and 360 \( \mu m \). In other words, let us now adopt the “Lego-beam”-solution, Eq.(1), with two FWs only, the first of order 2, and the second of order 22. Therefore, in Eq.(1) and in the interval \( 0 \leq z \leq L = 0.5 \) m, the non-zero functions (FWs) \( \psi_{\nu \ell} \) are \( \psi_{20} \) and \( \psi_{220} \), whose longitudinal intensity patterns are:
Figure 5: Intensity of the first “Lego-beam” generated experimentally, whose diffraction-resistant spatial structure consists in two adjacent cylindrical surfaces, 12 cm and 16 cm long, with the corresponding radii 148 \( \mu m \) and 203 \( \mu m \), sequentially linked one to the other; while it exists coaxially also a “light segment” having a spot of 38 \( \mu m \) and a length of 28 cm. On the top right corner, in smaller size, we reproduce the correspondent theoretical prediction.

\[
F_{\nu \ell} = \delta_{\nu 2} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.4)] \\
+ \delta_{\nu 22} \delta_{\ell 0} [H(z - 0.2) - H(z - 0.4)] ,
\]

(12)

where, again, \( \delta_{pq} \) is a Kronecker delta and \( H(.) \) a Heaviside function. The corresponding \( Q_{\nu \ell} \) values, depending on the chosen cylindrical surface radii, are:

\[
Q_{20} = Q_{220} = 0.9999770 \frac{\omega}{c} .
\]

(13)

As before, coefficients \( A_{\nu \ell n} \) are evaluated via Eq.\( [5] \), and the longitudinal and transverse wave numbers (\( \beta_{\nu \ell n} \) and \( h_{\nu \ell n} \)) are calculated by Eqs\( [3,4] \). For this example we use \( N_{20} = N_{220} = 18 \).
Figure 6: Intensity of the second “Lego-beam” generated experimentally. This beam is composed by two coaxial cylindric light surfaces having the same length 20 cm, but the different radii 45 µm and 360 µm. On the top right corner, in smaller size, we reproduce the correspondent theoretical prediction.

Figure 6 shows this second experimentally generated beam, and, on its right side, in smaller size, its theoretical prediction. Once more, an excellent agreement is found between theory and experiment; provided that it is kept in mind our discussion of the interference, between different FWs, present at the end of our Sec. 2.

In connection with this figure, and with Eq. (6), let us add a particular comment: If the external cylinder stays in the region where \( \cos \delta \) is positive (negative), in the resulting beam its intensity will be higher (lower) than the internal cylinder’s. Such effects can be avoided, or at least reduced, by properly choosing the (greater or smaller) intensities of the initial cylinders so that in the final beam they result with the same intensity.
5 Conclusions

In this work we present a theoretical method in which “Frozen Wave”-type optical beams (of different orders and/or with different sets of longitudinal wave-number values) are suitably superposed, providing us with really innovative possibilities (called by us “Lego-beams”) in the important field of Structured Light.

Various theoretical examples are developed and, moreover, our method has been experimentally verified by the production of two Lego-beams via a holographic setup, that performs the optical reconstruction of computer generated holograms through a reflective Spatial Light Modulator (SLM) followed by a 4F spatial filtering system.

The study of structured light is known to have played an important role in several areas of optics and photonics; and our present results can find interesting applications in all sectors in which more sophisticated light beams are needed, such as optical tweezers, optical guiding of atoms, light orbital angular momentum control, imaging systems, remote sensing, light detection and ranging, microscopy, metrology, optical communications, quantum information, etc... All these technologies are growing, and mastering the various types of structured light is becoming more and more important [39].

For completeness’ reasons, let us be here a little more specific about the expected applications of our “Lego-beams”, e.g., in the four sectors of optical tweezers for (e.g.) biological manipulations, atoms guiding, non-linear optics, and interaction of electromagnetic radiation with Bose-Einstein condensates:

(i) Optical tweezers are known to be a highly valid instrument for confining and manipulating nano or micro particles, including biological “particles” like bacteria, cells, viruses, etc. The use of Non-Diffracting Beams caused many improvements [8, 9, 10, 11, 12, 13, 14, 15, 16, 17], at the point that it was regarded as revolutionary in a Nature’s paper [13]. Among their advantages, let us mention the possibility of simultaneously imprisoning many scatterers. Ability in the spatial modelling of non-diffracting beams, as by our “Lego-beams”, is certainly useful for optical micro-manipulations, generating new light structures that could not be imagined before: For instance, by linking
together a zero-order FW (devoid of orbital angular momentum) with a FW of order 1 or more (carrying orbital angular momentum), it becomes possible to create a confining region, wherein the particles do not receive angular momentum, followed by another confining region in which the particles get angular momentum from the optical field and start rotating.

(ii) Another sector where use of non-diffracting beams resulted useful and promising is the optical guiding of neutral atoms, when Bessel beams of order larger than zero (non-diffracting hollow beams) generate optical potentials with the same shape [18, 19, 20, 21, 22, 23, 24]. Again, the possibility of obtaining diffraction-resistant beams with various interesting, new spatial configurations (by connecting different FWs together), namely our “Lego-beams”, can lead to unprecedented optical potential configurations for guiding, or holding, neutral atoms.

(iii) Applications in material modification, more specifically on the creation of waveguide structures and microchannels for microfluidics through the process of laser-writing, where a femtosecond laser induces a material modification (e.g. refractive index modification) in the longitudinal direction. Studies [40, 41, 42, 43] have related good results for laser micromachining in glass with Bessel beams, obtaining microchannels with 2 $\mu$m of diameter and with high aspect ratios (up to 40). “Lego-beams” could be used for laser-writing waveguide structures and microchannels (for microfluidics) with a great variety of forms and, due their diffraction resistant characteristics, with long longitudinal lengths.

(iv) A fourth interesting sector would be that of examining (theoretically, and experimentally) the interaction of our “Lego-beams” with Bose-Einstein condensates. The Gross-Pitaevkskii equation –describing a weakly interacting Bose-Einstein condensate, at the limit of zero temperature– is known to possess a mathematical structure similar to the non-linear Schroedinger equation of optics. Such a mathematical equivalence suggests that, while in the non-linear Schroedinger equation the light is the wave propagating in a medium constituted of atoms, on the contrary in the Gross-Pitaevsky equation the atoms play the role of the wave (matter wave) and the light acts as the propagation medium [44]. It is tempting, in this context, to investigate in theory and experiments the effect of
“Lego-beams” structures on those condensates.

6  Acknowledgments

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