A Novel Dynamical Approach To Relativistic Heavy Ion Collisions

Dai-Mei Zhou\textsuperscript{1,2}, S. Terranova\textsuperscript{1} and A. Bonasera\textsuperscript{1} \textsuperscript{*}

\textsuperscript{1} Laboratorio Nazionale del Sud, Istituto Nazionale Di Fisica Nucleare,

\textit{Via S. Sofia 44, I-95123 Catania, Italy}

\textsuperscript{2} Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079 China

Abstract

A transport model for ultra-relativistic nucleus-nucleus collisions based on the mean free path approach is proposed. The method is manifestly Lorentz invariant. We discuss some calculations for pp and AA collisions and compare to a previously proposed transport model and to data. We demonstrate that our approach gives a different impact parameter distribution already in pp collisions as compared to the previous one. The role of hadronization times is discussed. Comparison to data is reasonable and the model can be easily modified to take into account genuine many body effects and quantum statistics similarly to low energy heavy ion collisions.

PACS numbers: 25.75.-q, 12.38.Mh, 24.10.Lx

\textsuperscript{*} Email: bonasera@lns.infn.it
Relativistic heavy ion collisions (RHIC) offer a unique tool to explore matter states never explored before. In particular at high energy densities a transition from nuclear matter to the quark gluon plasma (QGP) is expected. Because of the high complexity of the problem two different worlds, traditionally known as particle and nuclear physicists, are joining their efforts to gather important information on the transition to the QGP. This implies that many techniques and physics developed in the two fields are merged to have a complete picture of the AA collisions. In fact, in these collisions we can disentangle the necessary input coming from elementary collisions with creation of new particles from the dynamics of quarks and gluons (partons), and the time evolution of the many body systems which is extremely complicated due to the initial number of particles in AA collisions which largely increases during the time evolution depending on the beam energy. It is of course necessary to have both aspects under control to understand the complex physics of the process. The purpose of this paper is to propose a new method of solution of the semiclassical transport equation which should describe (in some approximations which we will not discuss here) the many body part of the process. The kinetic approach used so far is essentially a generalization of transport codes proposed for low energy heavy ion collisions (known as Boltzmann Uehling Uehlenbeck (BUU), Vlasov (VUU)/ Landau (LV)). It is based on following the time evolution of each particle with a collision occurring if two of them come to a closest distance less than or equal to \( b_{\text{max}} = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}} \). Here \( \sigma_{\text{tot}} \) refers to the total cross section. The method we discuss in this work is known as Boltzmann Nordheim Vlasov (BNV) approach at low energies. It is based on the concept of the mean free path approach and we will discuss it in detail in the following section. We would like to stress here that even though the two approaches might be comparable as far as average quantities are concerned, their detailed description of the dynamics is quite different. In fact in the first method (I), a particle is treated as a black sphere and if a second particle comes within a radius defined from the elementary cross section, then they collide. In the method (II) discussed in this work, a probability of collision is defined (see below) and particles can collide according to this probability. Thus, for instance in pp collisions if \( N \) particles are produced at a given energy, within method I, \( N \) particles will be produced for \( 0 < b < b_{\text{max}} \), while in method II \( N(b) \) particles are produced at each \( b \) and \( N \) particles are obtained after averaging over \( b \). Thus even though the two methods might give the same average number their \( b \) dependence is quite different and this might be important especially
when one searches for very violent events, i.e. events at very high multiplicities presumably obtained in very central collisions.

For the discussion in this work we will base our considerations on a parton cascade model for ultra-relativistic heavy ion collision [1] generally composed of the parton initial state, the parton evolution, the hadronization, and the hadron evolution [6]. There are two ways of creating the parton initial state. The parton initial state was composed of partons from the mini-jets production in nucleus-nucleus collision and the HIJING multiple mini-jet generator [7] was specified in [8, 9]. The parton initial state in [10, 11] was created via probability distributions first for the spatial and momentum coordinates of nucleons in colliding nuclei and then for the flavor and spatial and momentum coordinates of partons in nucleons. Our parton cascade model follows the former way, however, the JPCIAE multiple mini-jet generator [12] is used instead of HIJING. The JPCIAE multiple mini-jet generator for ultra-relativistic nucleus-nucleus collision is based on PYTHIA [13] which is a well known event generator for hadron-hadron collisions. Thus in this work we will study the two different approaches starting from the same numerical code which has been in case II opportunely modified as discussed below. An important ingredient of the approach is the hadronization time (ht). This is the time it takes for partons created after a collision to coalesce and form a new hadron. We will perform some calculations assuming a ht equal to zero and compare to calculations where the ht is nonzero. In the latter case any finite ht value will suppress secondary hadron-hadron collisions because, at high beam energies, the ht is time elongated, thus it might be larger than the reaction time. This is a very important effect because it suggests that we can have a finite hot region of quarks and gluons which will not coalesce into hadrons for a ht time boosted to the nucleus-nucleus CM frame.

Quantum statistics (i.e. Pauli and Bose statistics) are not included at present but they have been considered explicitly in [14] for Bose and [15] for Fermi statistics. In future work we plan to include the different statistics and the influence of many particle collisions [5] which could be very important because of the reached high densities.

I. FORMALISM

We follow the mean free path method as discussed in [5] and modified to include relativistic effects. Briefly, for each event, at each time step dt and for each ion i we search for the
closest ion \( j(i) \) in phase space i.e. we define the quantity

\[
\Xi_{ij} = \frac{r_{ij}}{v_{ij}dt}
\]

where \( r_{ij}, v_{ij} \), and \( dt \) are the relative distances, velocities (relativistic) and \( dt \) is the time step used in the calculations. Define a collision probability as:

\[
\Pi = \frac{v_{ij}dt}{\lambda} = \sigma \rho(r_i)v_{ij}dt,
\]

where \( \rho(r_i) \) is the local density calculated at each time step. Note that the quantities defined above are scalars and it is quite easy to show that they are Lorentz invariant. For instance in eq. (1) we have a distance \( (r_{ij}) \) divided by a distance \( (v_{ij}dt) \).

The physical meaning of the equation (1) is simple. We search for particles that are close in coordinate space and far away in velocity space, i.e. particles with opposite momenta. For instance at RHIC energies we have a relative velocity of the order of the speed of light \( c \) for particles of the target (T) and projectile (P) respectively and zero for both particles belonging to T or (P). The latter particles are automatically excluded from the first condition. Once the two closest particles have been found, we calculate the local density knowing the relative distance and the number of particles near the colliding couple. From the relative energy and the particles type we know the elementary cross sections and thus the probability eq. (2). A random number is taken and if smaller than the calculated probability, a collision occurs and it is modeled by Pythia if \( \sqrt(s) > 4\text{GeV} \) or some other value which we will specify when fitting pp data. Because of the probabilistic nature of the process discussed above we can have events where particles are very close and they do not collide, while it can happen that for smaller densities a collision occurs with the small probability \( \Pi \). The time step is chosen such that the probability is always small compared to 1 and to minimize the CPU time. Once these conditions are fulfilled a further decrease of the time step dt will not change the results but will increase linearly the CPU time. Notice that in our approach we have to pay the price of largely CPU time consuming calculations, proportional to \( N(t)^2 \) for zero \( \lambda_t \), where \( N(t) \) is the number of particles at time \( t \). Such a proportionality with \( N(t) \) decreases for finite \( \lambda_t \). This calculation time might become prohibitive at LHC energies where many particles are produced. In the following we will show if it is worth or not to follow this approach or stick to the previous one. A discussion of the implementation of the alternative approach is not given here since it has been largely
discussed in the literature [1, 6]. In both approaches the partonic system is hadronized by JETSET [13] after some $ht$. As we discussed above the role of $ht$ is crucial. In fact, if such a time is small the partons hadronize and can collide again. In the next section we will study the effects of rescatterings when the $ht$ is zero and/or very small. But if the $ht$ is finite, then, also because of Lorentz dilatations hadrons cannot be formed and there cannot be other hadron-hadron (hh) collisions for the $ht$ duration. In the limit of large $ht$ all the secondary collisions are suppressed and our approach reduces to Pythia folded with the number of first chance nucleon-nucleon collisions. We will also study the effects of finite $ht$ effects. We would like to stress at this stage, that even though hadrons are not formed the partons could still collide. For illustration purposes we will neglect those collisions in our calculations, however we would like to notice the following important points.

i) At the partons stage, collisions might occur among partons. One simple way to implement this would be to scale down the hh cross sections by the number of valence quarks (or of participant partons). But in this case the local density should be scaled up of the same number which in turn gives the same mean free path of hadrons, cf., eq.(2). Thus, in this approximation, we expect that including parton-parton collisions will have the same effects as having a $ht$ equal to zero, because the hh and parton-parton mean free paths are the same as discussed above.

ii) If at some stage we would decide to include parton-parton collisions then as a first step we have to check that our approach coupled to Pythia fits the experimental pp collisions for instance. This might lead to a redefinition of the Pythia parameters. In other words, even though we are not formally including collisions at the parton level, those might be implicitly included in the parametrization of the elementary collisions.

Important differences for the collisions (thus for the dynamics) are due to the statistics. In fact, if we have $ht$ equal to zero, our system is made up essentially of pions, while for finite $ht$ we have quarks and gluons. In the hadronic state essentially Bose Einstein statistics applies while in the partonic stage, both Bose and Fermi statistics apply. This, we feel will make an important distinction and will be discussed in detail in following works.

The mean free path method discussed above has been studied in detail at low energies and it has been shown to solve the Boltzmann eq. in the cases where an analytical solution is known [5]. Here we have generalized the approach to keep into account relativistic effects. The particles move on straight lines during collisions since we have not implemented any
force yet. For short we name the method proposed here as Relativistic Boltzmann equation (ReB).

II. HADRONIC PICTURE

The essential inputs of all transport approaches are the elementary cross sections. Of course the used models Pythia and Jetset are tailored to reproduce those elementary processes. In this paragraph we will show what is the influence of the way the collision partners are chosen and we will fix the essential parameters which enter the calculations. The physical picture of the collision is the following. The colliding partners are chosen as discussed in the previous section. Depending on their relative energy it is decided if the collision is elastic or inelastic. If it is inelastic, the number of produced partons and their energies and momenta are decided using Pythia. Those partons are randomly distributed in a sphere whose diameter is given by the original positions of the colliding hadrons. The partons are evolved on straight line trajectories and they cannot collide before the ht. After the ht the partons hadronize and the formed hadrons can collide again. Clearly in elementary collisions, if the ht is long enough there will not be secondary collisions, thus our approach for elementary collisions reduces essentially to Pythia. But, on the other hand if the ht is zero or very short, the produced hadrons will be still very close in phase space and they might collide again. In this case we adjust the parameters of Pythia such that the elementary cross sections are still reproduced. Notice that this is not a feature of our approach but it is common to other methods of solution of the transport equation such as JPCIAE or URQMD. Now, we discuss the role of the ht, which influences the rescatterings.

First, in figure (1) we show the multiplicity of produced charged particles in pp collisions vs. impact parameter b. As we discussed above since a collision is chosen in a probabilistic way there are events where a collision occurs and events where they do not occur (apart b=0 fm where a collision strictly occurs for each event since the density calculated from the relative distance of the 2 particles diverges). This explain the b dependence decrease of the multiplicity. Not only, we see that the Blacker, Edger and Larger\footnote{(BEL) effect is reproduced: for higher energies the elementary cross sections increases thus the number of involved impact parameters increases as well. This feature is of course contained in the original JPCIAE model as well since the same cross section (obtained from data) is used.} effect is reproduced: for higher energies the elementary cross sections increases thus the number of involved impact parameters increases as well. This feature is of course contained in the original JPCIAE model as well since the same cross section (obtained from data) is used.
FIG. 1: Particles production probability versus impact parameter in pp collisions at two C.M. energies, 200 and 2000 GeV, respectively.

However the b dependence of the multiplicity distribution is dramatically different. In fact JPCIAE gives a constant yield as function of b. For sake of simplicity in this work we tune the parameters entering the calculations to pp collisions at zero impact parameter. For instance in fig.2 we display some results for zero hadronization time. We stress the fact that zero ht implies that at each collision the produced particles are hadronized instantaneously, thus we could say that this approximation correspond to a purely hadronic transport model.

In the figure (2) we show the particles multiplicities in pp collisions from ReB calculations, compared to data (full circles) for various energy cutoffs and parameter sets entering Pythia. The fact that ReB results for strictly zero ht and drastically reducing the a and b parameters (which tune the strength of the interaction in Pythia) does not reproduce the data is due to secondary collisions which increase the yield especially of low pseudo-rapidity particles. In order to improve the agreement to data we increased the low energy cutoff for Pythia up to 30 GeV. Notice however that the energy cutoffs have the same effect of a larger ht, in fact what happens is that inelastic secondary collisions occurring at later times are
FIG. 2: Particles production versus pseudo-rapidity in pp collisions at 200 GeV C.M. energy for zero hadronization times.

suppressed by our choice of 30 GeV cutoff and this has a similar effect to increase the ht which avoids secondary collisions at later times. However as we will show in the following a finite ht or a large cutoff in the energy does not have the same effect in AA collisions. Thus we can already conclude that zero ht (or very small ht) in our model are already excluded from pp collisions. This become strikingly evident when comparing to data for Au+Au collisions at $\sqrt{s} = 200$ GeV, see figure(3). In fact the data [17] for pseudorapities less than roughly 3 units are largely overestimated especially for kaons. This suggests that those particles are produced in the model at later stages of the reaction from lower energies colliding hadrons.

III. INFLUENCE OF THE HADRONIZATION TIMES

As we stressed above after an elementary hh collision, quarks and gluons are created which eventually combine to give new hadrons. This process takes some time (ht). In this section we will show the effects of finite ht assuming that during such a time the partons cannot collide. This is of course an approximation since collisions might occur among partons. However, especially if we have a QGP, the possibility of colliding is quite different from the
case where there is no phase transition. Thus to simply scale the elementary $hh$ collisions by, for instance, the number of valence quarks could be too naive if the partons are embedded on a plasma. As we stressed above, the simple scaling of the cross sections goes together with a scaling of the local density which will result in a similar mean free path of hadrons and partons, thus in our framework, the results including parton collisions (neglecting quantum statistics and modified partons cross sections in a QGP) will be very similar to the results discussed in the previous sections with $ht$ equal to zero. Instead, blocking the possibility of partons inelastic collisions, might give some hint, through a comparison to data of the relevance of the missing dynamics during the partons stage. But again we have to stress that maybe some parton-parton collisions are implicitly included in the model through the fitting of the pp data.

There is not a definite prescription on what should be the $ht$ times both in pp and AA collisions. Thus we will treat it as a free parameter, but for each choice we make we will first fit the experimental pp data by tuning the Pythia parameters exactly as we did in the previous section. In figure (4) we plot the pseudo-rapidity distributions compared to UA5 data [16] (full circles).
FIG. 4: Particles production versus pseudo-rapidity in pp collisions at 200 GeV C.M. energy for different hadronization times.

Different ht values are displayed in the figure together with the set of Pythia adjusted parameters. Again a zero ht time does not fit the pp data, while finite values of the ht do a reasonable job. Notice also that the results of the fits are not so dependent on the ht if this is larger than 0.1 fm/c. In fact because of Lorentz time dilatation hadronization does not occur before the system has expanded thus suppressing secondary collisions. This is also demonstrated by the good agreement of those calculations to the Pythia results with the same parameter sets.

For illustration in fig.(5) we plot the result for the same pp collisions but with a ht \( \tau = \hbar/2m \), i.e. the Heisenberg principle, where m is the mass of the forming hadron. This choice was done to see if there are some differences when the ht depends on the produced particle type.

To see if different choices of the ht give noticeable differences already in pp collisions, we plot in figure (6) various hadronic ratios vs. rapidity at the same energy as above. The data, when available, is given by the full circles with error bars. We see that our results do a similar job of the original Pythia approach (diamonds) for \( \pi^-/\pi^+ \) and \( K^-/K^+ \) ratios while we do not reproduce well the \( \bar{p}/p \) ratio.

Let us now turn to the Au+Au collisions at RHIC. In the different panels of fig.7, we
FIG. 5: Particles production versus pseudo-rapidity in pp collisions at 200 GeV C.M. energy for 'Heisenberg' hadronization times.

display the results of the calculations (open symbols) using different ht for pions (top panels) and kaons (bottom panels). The experimental data [17] is also displayed. The discrepancy with the data is striking especially at low pseudo-rapidity. No noticeable differences are found for the two first different prescriptions for the ht. When using a very large value of the ht (which implies first chances collisions only) the agreement is poor as well. If we compare to the result for zero ht, cf. fig.2, we notice that we have largely improved the description of the data especially for pions and \( \eta > 2 \). However, the results for lower pseudo-rapidities are not as good as those of fig.2. The difference among the calculations is not only the ht but the low energy cutoff for Pythia. Recall that for zero ht we increased the value of the energy cutoff to 30 GeV while for finite ht the smallest possible value for Pythia5.7 is used, i.e. 4 GeV. Therefore, as in the zero ht case the low energy cutoff was responsible for decreasing the low \( \eta \) yield, it could be that the 4 GeV cutoff in Pythia is responsible as well for the bad reproduction of the data for lower \( \eta \). This called for extra work of our postdocs who had to implement a new subroutine with a parametrization of elementary collisions data for energies below 4 GeV and above the energy for pion production. This was finally accomplished and the final result for finite ht is displayed in figs.8-11. In fact, first we had to fit the pp data again, (see one example in fig.8) and then we compared to the Au+Au
FIG. 6: Different hadron ratios versus pseudo-rapidity in pp collisions at 200 GeV C.M. energy for different hadronization times.

case(fig.9-11). The agreement to data is now better both for pions and kaons production, even though we slightly overestimate the data at small \( \eta \)s. The way we have reached this result suggests that low \( \eta \) particles are produced from hh collisions at lower energies and later times. Also, by comparing to fig.2, one might try to use smaller values of the \( h_t \) to fit the data. However, a reproduction of the data by fitting some parameter is not the purpose of this work. In fact, before comparing in detail to data we would like to add first some missing ingredients (quantum statistics, cooperative effect etc.). More on this point plus a discussion on the time development of the reaction is given in the next section.

IV. TIME EVOLUTION OF THE COLLISIONS

Even though the model is still at its infant stage we can discuss some of its features in more details also to see where and possibly what we have to do to improve it. We found that the model is very sensitive to the input elementary collisions which we parametrized from very low energy up to the threshold for Pythia on which we rely for the description of the elementary data at higher energies. Our strategy was in any case to fit the Pythia parameters to pp data when available and then turn to the AA collisions case. One surprising result
for us (but probably known to the authors of [2]) was that low $\eta$ particles come from the final stages of the reaction at longer times. This is similar to evaporation from a compound nucleus where the lowest energy particles (for instance neutrons) are emitted last. We have seen already that low $\eta$ particles come from colliding hadrons having a C.M. energy less than 4 GeV. This is also rather surprising to us since we are dealing with a Au+Au collision at 200 GeV and now we want to see more in detail the various time stages of the reaction.

In figure 9 we display the pseudo-rapidity distribution at various times of the collision for the 'Heisenberg' ht. We see that the high pseudo-rapidity tail (i.e. $\eta > 5$) saturates already in less than 1 fm/c, while lower pseudo-rapidipseudo-rapidityty particles are produced at times larger than 5 fm/c. It is obvious to think now that if these particles are created at such late times, they might not carry any relevant information about the QGP.

In figures 10-11 we display the pseudo-rapidity distribution for a ht equal to 1 and 100 fm/c. The difference with the previous case is striking even though both ht fit the elementary pp data. The effect of the largest ht is to block all secondary collisions, thus the result given
FIG. 8: Particles production versus pseudo-rapidity in pp collisions at 200 GeV C.M. energy with no cutoff in the elementary cross sections parametrization.

In the figure is coming from first chances collisions only and it is similar to the one of fig. 9 obtained in the first fm/c of the reaction. Thus a large ht is equivalent say to Pythia folded for the possible number of first chance collisions as calculated in the Eikonal approximation for instance [1].

In order to discuss the possibility of QGP formation, its lifetime and if it reaches equilibrium or not we define a sphere around the C.M. of the system of radius 2 fm, similarly to low energy [5] and UrQMD calculations [20]. In fact, if a QGP is formed it should happen in such a sphere first, also if equilibrium is reached (at least in our model) we should get isotropy in momentum space at least in a central region. In order to see this we look at the energy density and ‘transverse’ energy density in such a central region. We define 

$$\epsilon_T = \sum \sqrt{\frac{3}{2}(p_x^2 + p_y^2) + m^2},$$

where p and m are the momenta of the particles, x and y the directions perpendicular to the beam axis z, and the numerical factor is included in order to have \(\epsilon_T = \epsilon\) if equilibrium is reached. The sum runs over all the particles within the central sphere. In figure (12) we plot the energy densities vs. time for pp and d+Au collisions. Two features are worth noticing. First the transverse energy density is not much different for the two systems and second equilibrium is almost reached at times of the order of 5 fm/c but at low energy density values (less than 0.5 GeV/fm³).
FIG. 9: Particles production versus pseudo-rapidity in Au+Au collisions at 200 GeV C.M. energy at different times.

The situation is dramatically different for Au+Au collisions (figure 13) at the same RHIC energy. Now the reached energy densities are much larger and equilibrium is reached at 4 fm/c similar to UrQMD calculations [20]. On similar grounds we can calculate how many hadrons and partons are in the central cell at different times, fig.14. Here the partons are calculated from the number of produced particles at each time and that have lived less than the ht.

At first the number of hadrons (baryons) decreases because they have collided and formed new particles. After some ht partons coalesce and are counted as hadrons again. At this stage they could collide again and form new particles. The number of partons increases quickly with time and it seems to saturate already at 3 fm/c. There is a stage of non equilibrium (kinetic) but after 4 fm/c the system is equilibrated. At this time the ratio of partons to hadrons is about 10 and this could be considered as a QGP state. At later times the ratio decreases and we could consider this as a mixed state of partons and hadrons. Of course this effect is simply due to the finite ht since no (first order) phase transition is explicitly included in the model yet. Nevertheless, the model suggests a mixed phase in the early stages of the reaction, together with the possibility of reaching thermal equilibrium. At later times (not plotted), the number of partons and hadrons decreases because of the expansion of the
system. Still some collisions occur at low energies which produce new hadrons (especially pions). This stage is of course not so interesting but it could be potentially dangerous since it might wash out all the signals, for instance large fluctuations in D-mesons productions, intermittency etc., of the plasma. For completeness, in the same figure we have plotted the total number of partons and hadrons in the entire space (which never reaches kinetic equilibrium). Here one sees that the number of partons remains constant for a relatively long time while the number of hadrons increases with time. This is due to the interplay of the finite ht, the newly created partons and the hadronization and it is an entirely dynamical process.

V. CONCLUSIONS

In this work we have introduced a novel transport approach to relativistic heavy ion collisions. Elementary collisions are chosen following a mean free path method already developed at low energies. The elementary process is dealt within the Pythia approach. The low energies (below 4 GeV C.M. energies) elementary collisions have been implemented as well.
Within this model we have shown that a purely hadronic approach fails in reproducing AA data and can reproduce the pp data at a price of a large (and unreasonable) energy cutoff in the elementary inelastic collisions. Finite hadronization times and a careful inclusion of all possible inelastic channels down to the pion threshold production gives a good description of pseudo-rapidity distributions both in pp and Au+Au data. The possibility of a finite ht hints to the formation of a mixed QGP and hadronic state after kinetic equilibrium is reached (i.e. after 5 fm/c). The comparison to data in this paper has been restricted to the highest RHIC energy at the moment. We plan to extend the calculations with the present model to other observables and different energies. Collective flow calculations are also of interest. A priority however is the possibility of including a phase transition in the model to see if the data (or which data) is sensitive to the phase transition. This we hope to accomplish within a Constrained Molecular Model (CoMD) for quarks degree of freedom recently developed [22].

Finally, Z.D.M. acknowledges the financial support from INFN and Department of Phys. University of Catania in Italy (where most of the work was performed) and NSFC (10347131) in China. This work is supported in part by the EU under contract CN/ASIA-LINK/008(094-791). We thank Prof. Ben-Hao Sa for discussions and for making his code.

FIG. 11: Particles production versus pseudo-rapidity in Au+Au collisions at 200 GeV C.M. energy at different times for proper formation time equal to 100 fm/c.
FIG. 12: Energy densities vs. time in a central cell of radius 2 fm in pp and d+Au collisions at 200 GeV C.M. and zero impact parameter.

FIG. 13: Same as figure (12) but for Au+Au collisions.
FIG. 14: Hadrons and partons produced versus time in the central cell for Au+Au collisions.

JPCIAE available to us.

[1] K. Geiger and B. Müller, Nucl. Phys. B 369, 600 (1992); K. Geiger, Phys. Rep. 258, 237 (1995).
[2] S. Bass, et al., Prog. Part. Nucl. Phys. 41, 255 (1998).
[3] J. Cugnon, T. Mitzulani, J. Meulen, Nucl. Phys. A 352, 505 (1981).
[4] G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).
[5] A. Bonasera, F. Gulminelli, and J. Molitoris, Phys. Rep. 243, 1(1994).
[6] Ben-Hao Sa, Wang Rui-Hong, Zhang Xiao-Ze, Zheng Yu-Ming, and Lu Zhong-Dao, Phys. Rev. C 50, 2614 (1994).
[7] M. Gyulassy and X.-N. Wang, Comput. Phys. Commun. 83, 307 (1994).
[8] Bin Zhang, Comput. Phys. Commun. 109, 193(1998).
[9] D. Molnár and M. Gyulassy, Phys. Rev. C 62, 054907 (2000).
[10] V. Börchers, J. Meyer, S. Gieseke, G. Martens, and C. C. Noack, Phys. Rev. C 62, 064903 (2000).
[11] S. A. Bass, B. Müller, and D. K. Srivastava, nucl-th/0212103
[12] Ben-Hao Sa, An Tai, Hui Wang and Feng-He Liu, Phys. Rev. C 59, 2728 (1999);
    Ben-Hao Sa and An Tai, Phys. Rev. C 62, 044905 (2000).
[13] T. Sjöstrand, Comput. Phys. Commun. 82, 74 (1994).
[14] G. M. Welke and G. F. Bertsch, Phys. Rev. C 45, 1403 (1992).
[15] Ben Hao Sa and A. Bonasera, Phys. Rev. C 70, 034904 (2004).
[16] G. J. Alner, et al., UA5 Collaboration, Z. Phys. C - Particles and Field 33, 1-6 (1986).
[17] I. G. Bearden, et al., BRAHMS Collaboration, nucl-ex/040350.
[18] I. G. Bearden, et al., BRAHMS Collaboration, [14] Phys. Lett. B607,42 (2005)).
[19] S. V. Afanasiev et al., NA49 Collaboration, Phys. Rev. C66, 054902 (2002).
[20] M. Belkacem et al., Phys.Rev. C58, 1727 (1998)
[21] S.Terranova, D.M. Zhou and A.Bonasera, [15] nucl-th/0412031
[22] S.Terranova and A.Bonasera, Phys.Rev.C70,024906 (2004).