Dressed Collective Qubit States and the Tavis-Cummings Model in Circuit QED

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We present an ideal realization of the Tavis-Cummings model in the absence of atom number and coupling fluctuations by embedding a discrete number of fully controllable superconducting qubits at fixed positions into a transmission line resonator. Measuring the vacuum Rabi mode splitting with one, two and three qubits strongly coupled to the cavity field, we explore both bright and dark dressed collective multi-qubit states and observe the discrete \( \sqrt{N} \) scaling of the collective dipole coupling strength. Our experiments demonstrate a novel approach to explore collective states, such as the \( W \)-state, in a fully globally and locally controllable quantum system. Our scalable approach is interesting for solid-state quantum information processing and for fundamental multi-atom quantum optics experiments with fixed atom numbers.

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In the early 1950’s, Dicke realized that under certain conditions a gas of radiating molecules shows the collective behavior of a single quantum system. The idealized situation in which \( N \) two-level systems with identical dipole coupling are resonantly interacting with a single mode of the electromagnetic field was analyzed by Tavis and Cummings. This model predicts the collective \( N \)-atom interaction strength to be \( G_N = g_j \sqrt{N} \), where \( g_j \) is the dipole coupling strength of each individual atom \( j \). In fact, in first cavity QED experiments the normal mode splitting, observable in the cavity transmission spectrum, was demonstrated with an average \( N > 1 \) atoms in optical and microwave cavities to overcome the relatively weak dipole coupling \( g_j \). The \( \sqrt{N} \) scaling has been observed in the regime of a small mean number of atoms \( N \) with dilute atomic beams and fountains crossing a high-finesse cavity. In these experiments, spatial variations of the atom positions and Poissonian fluctuations in the atom number inherent to an atomic beam are unavoidable. In a different limit where the cavity was populated with a very large number of ultra-cold \( ^87 \)Rb atoms and more recently with Bose-Einstein condensates the \( \sqrt{N} \) nonlinearity was also demonstrated. However, the number of interacting atoms is typically only known to about \( \sim 10\% \).

Here we present an experiment in which the Tavis-Cummings model is studied for a discrete set of fully controllable artificial atoms at fixed positions and with virtually identical couplings to a resonant cavity mode. The investigated situation is sketched in Fig.1a, depicting an optical analog where three two-state atoms are deterministically positioned at electric field antinodes of a cavity mode where the coupling is maximum. In our circuit QED realization of this configuration (Fig.1b), three transmon-type superconducting qubits are embedded in a microwave resonator which contains a quantized radiation field. The cavity is realized as a coplanar waveguide resonator with a first harmonic full wavelength resonance frequency of \( \omega_r / 2\pi = 6.729 \) GHz and a photon decay rate of \( \kappa / 2\pi = 6.8 \) MHz. The qubits are positioned at the antinodes of the first harmonic standing wave electric field. The transition frequency between ground \( |g\rangle \) and first excited state \( |e\rangle \) of qubit \( j \), approximately given by \( \omega_j \approx \sqrt{E_{Cj} (E_{Jj} (\Phi_j) + E_{Cj}) / \hbar} \), is controllable through the flux dependent Josephson energy \( E_{Jj} (\Phi_j) = E_{Jmax} |\cos (\pi E_{Jj} (\Phi_j) / E_{Jmax})| \). Here \( E_{Cj} \) is the single electron charging energy, \( E_{Jmax} \) the maximum Josephson energy at flux \( \Phi_j = 0 \) and \( E_{Jmax} \) the magnetic flux quantum. Independent flux control of each qubit is achieved by applying magnetic fields with three external miniature current biased coils (Fig.2a) where we take into account all cross-couplings by inverting the full coupling matrix. Optical images of the investigated sample

![FIG. 1: Schematic of the experimental set-up. (a) Optical analog. Three two-state atoms are identically coupled to a cavity mode with photon decay rate \( \kappa \), atomic energy relaxation rate \( \gamma \) and collective coupling strength \( G_N \). (b) Schematic of the investigated system. The coplanar waveguide resonator is shown in light blue, the transmon qubits A, B and C in violet and the first harmonic of the standing wave electric field in red.](image-url)
are depicted in Fig. 2 b and c. The resonator was fabricated employing optical lithography and Aluminum evaporation techniques on a Sapphire substrate. All qubits were fabricated with electron beam lithography and standard Al/AlOx/Al shadow evaporation techniques. Table I states the individual qubit parameters obtained from spectroscopic measurements.

The physics of our system is described by the Tavis-Cummings Hamiltonian [2]

\[ \hat{H}_{TC} = \hbar \omega_r \hat{a}^\dagger \hat{a} + \sum_j \left( \frac{\hbar}{2} \omega_j \hat{\sigma}_j^+ + \hbar g_j (\hat{a}^\dagger \hat{\sigma}_j^- + \hat{\sigma}_j^+ \hat{a}) \right), \]  

where \( g_j \) is the coupling strength between the field and qubit \( j \). \( \hat{a}^\dagger \) and \( \hat{a} \) are the creation and annihilation operators of the field, \( \hat{\sigma}_j^+ \) and \( \hat{\sigma}_j^- \) are the corresponding operators acting on the qubit \( j \), and \( \hat{\sigma}_j^z \) is a Pauli operator.

First we investigate the resonant coupling of the \( |g, g, g\rangle \otimes |\emptyset\rangle \) of the three-qubit/cavity system is prepared by cooling the microchip to a temperature of 20 mK in a dilution refrigerator.

On resonance (\( \omega_j = \omega_r \)) and in the presence of just one two level system (\( N = 1 \)), Eq. (1) reduces to the Jaynes-Cummings Hamiltonian [18]. The eigenstates \( |N, n \pm \rangle \) of this system in the presence of a single excitation \( n = 1 \) are the symmetric and anti-symmetric qubit-photon superpositions \( |1, 1\pm \rangle = 1/\sqrt{2} (|g, 1\pm e, 0\rangle) \) (Fig. 4 a) where the excitation is equally shared between qubit and photon. Accordingly, we observe a clean vacuum Rabi mode splitting spectrum formed by the states \( |1, 1\pm \rangle \) (Fig. 4 b). From analogous measurements performed on qubits B and C (not shown) we obtain the single qubit coupling constants \( g_j \) listed in Tab. I. The coupling strengths are virtually identical with a scatter of only a few MHz. The strong coupling of an individual photon and an individual two-level system has been observed in a wealth of different realizations of cavity QED both spectroscopically [15, 19, 20] and in time-resolved experiments [21, 22]. The regime of multiple excitations \( n \) which proves field quantization in these systems has been reported both in the time resolved results cited above and more recently also in spectroscopic measurements [23, 24, 25].

In a next step, we maintain qubit A at degeneracy (\( \nu_A = \nu_r \)), where we observed the one-photon one-qubit doublet (see left of Fig. 3c). Qubit B remains far detuned

\[
\begin{array}{cccc}
|Qubit\rangle & E_{C_j}/\hbar & E_{\text{max}}/\hbar & g_j/2\pi\text{(MHz)} \\
A & 283 & 224 & 83.7 \\
B & 287 & 226 & -85.7 \\
C & 294 & 214 & 85.1
\end{array}
\]

TABLE I: Qubit and qubit-resonator coupling parameters. The single electron charging energy \( E_{C_j} \), the maximum Josephson energy \( E_{\text{max}} \), extracted from spectroscopic measurements and the coupling strengths \( g_j \) obtained from resonator transmission measurements for qubits A, B and C.
low and red, high transmission) versus normalized external flux bias $\Phi/\Phi_0$ of qubit A. Dash-dotted white lines indicate bare resonator $\nu_r$ and qubit $\nu_A$ frequencies and dashed white lines are calculated transition frequencies $\nu_{g0,N,n}$ between $|g,0\rangle$ and $|N,n,\pm\rangle$. In general we expect $4N$ states for two qubits and one photon. The third state $\nu_{g0,3,1}$ serves the energies of three out of the number of qubits, with probability 1/2 each (Fig. 3b). Both states have a photonic component and can be excited from the ground state $|g,g,g\rangle\otimes|0\rangle$ by irradiating the cavity with light. These are thus referred to as bright states. In general we expect $N+n=3$ eigenstates for two qubits and one photon. The third state $|2,1d\rangle = 1/\sqrt{2} (|e,g\rangle - |g,e\rangle) \otimes |0\rangle$ with energy $\hbar\omega_r$ at degeneracy has no matrix element with a cavity excitation and is referred to as a dark state. Accordingly we observe no visible population in the transmission spectrum at frequency $\nu_r$ at degeneracy. In this regime the two qubits behave like one effective spin with the predicted $g_2$ coupling strength $g_2 = \sqrt{2} g_{AC}$ with $g_{AC} = 1/2 g_A^2 + g_C^2$, which is indicated by dashed black lines in Fig. 3d. This prediction is in very good agreement with our measurement.

Following the same procedure, we then flux tune qubit B through the already resonantly coupled states of qubits A, C and the cavity ($\nu_A = \nu_C = \nu_r$), (Fig. 3e). We observe the energies of three out of $N+n=4$ eigenstates, the fourth one being dark, for a range of flux values $\Phi_B$. Starting with the dark state $|2,1d\rangle$ at frequency $\nu_r$ and the doublet $|2,1\pm\rangle$ (left part of Fig. 3e), the presence of qubit B dresses these states and shifts the doublet $|2,1\pm\rangle$ down in frequency. Again one of these states turns dark as it approaches degeneracy where it is entirely mixed with qubit B. At degeneracy we identify two bright doublet states $|3,1\pm\rangle = 1/\sqrt{2} (|g,g,g\rangle\otimes|1\rangle \pm 1/\sqrt{2} (|e,g,g\rangle + |g,e,g\rangle) \otimes |0\rangle$ (Fig. 3c). The part of the states $|3,1\pm\rangle$ carrying the atomic excitation is a so called W-state, in which a single excitation is equally shared among

![Figure 3](Image 76x588 to 170x740)

**FIG. 3:** Vacuum Rabi mode splitting with one, two and three qubits. (a) Measured resonator transmission spectrum $T$ (blue, low and red, high transmission) versus normalized external flux bias $\Phi_A/\Phi_0$ of qubit A. Dash-dotted white lines indicate bare resonator $\nu_r$ and qubit $\nu_A$ frequencies and dashed white lines are calculated transition frequencies $\nu_{g0,N,n}$ between $|g,0\rangle$ and $|N,n,\pm\rangle$. Red line is a fit to two Lorentzians. (c) Resonator transmission spectrum $T/\max$ versus external flux bias $\Phi_C/\Phi_0$ of qubit C with qubit A degenerate with the resonator ($\nu_A = \nu_r$). (d) Transmission spectrum $T/\max$ at flux as indicated in (c). (e) Transmission spectrum versus flux $\Phi_B/\Phi_0$ with both qubits A and C at degeneracy ($\nu_A = \nu_C = \nu_r$). The white dashed line at frequency $\nu_{g0,3,1}$ indicates the dark state occurring at degeneracy. (f) Transmission spectrum $T/\max$ at flux as indicated in (e).

![Figure 4](Image 400x588 to 494x740)

**FIG. 4:** Level diagram representing the total energy of (a) one (b) two and (c) three qubits resonantly coupled to a single photon. Bare energy levels of the qubits $|g\rangle$, $|e\rangle$ and the cavity $|0\rangle$, $|1\rangle$ are shown in black. The bright dressed energy levels $|N,n,\pm\rangle$, with $N$ the number of qubits, $n$ the number of excitations and $\pm$ indicating the symmetry of the state, are illustrated in blue. The areas of the circles indicate the relative population of the bare states in the eigenstates $|N,n,\pm\rangle$. 

$\nu_{g0,3,1}$.
all $N$ qubits\cite{27}. Both $|3, 1\pm\rangle$ states are clearly visible in the transmission spectrum shown in Fig. 3f. In addition, there are two dark states $|3, 1d_1\rangle = \frac{1}{\sqrt{2}}(|e, g, g\rangle - |g, g, e\rangle) \otimes |0\rangle$ and $|3, 1d_2\rangle = \frac{1}{\sqrt{2}}(|g, e, g\rangle + |g, g, e\rangle) \otimes |0\rangle$ which do not lead to resonances in the transmission spectrum at degeneracy. In general all $N + n - 2$ dark states are degenerate at energy $h\omega$. The symmetries of the dressed three-qubit states are determined by the signs of the coupling constants $g_A \approx -g_B \approx g_C$. While our measurement is not sensitive to the sign of coupling, it is a simple consequence of the phase shift of the electric field mode by $\pi$ between the ends and the center of the resonator. Again, the observed transmission peak frequencies are in agreement with the calculated splitting of the doublet $G_3 = \sqrt{3}g_{ABC}$ (dashed black lines in Fig. 3f). Also at finite detunings the measured energies of all bright states are in excellent agreement with the predictions based on the Tavis-Cummings model (dashed white lines in Fig. 3 a,c,e) using the measured qubit and resonator parameters. We have also performed analogous measurements of all twelve one, two and three qubit anti-crossings (nine are not shown) and find equally good agreement.

In Fig. 3 all twelve measured coupling strengths (blue dots) for one, two and three qubits at degeneracy are plotted vs. $N$. Excellent agreement with the expected collective interaction strength $G_N = \sqrt{N}s_{ABC}$ (red line) is found without any fit parameters and $s_{ABC} = 84.8$ MHz.

Our spectroscopic measurements clearly demonstrate the collective interaction of a discrete number of quantum two-state systems mediated by an individual photon. All results are in good agreement with the predictions of the basic Tavis-Cummings model in the absence of any number, position or coupling fluctuations. The presented approach may enable novel investigations of super- and sub-radiant states of artificial atoms. Flux tuning on nanosecond timescales should furthermore allow the controlled generation of Dicke states\cite{28,29} and fast entanglement generation via collective interactions\cite{30,31}, not relying on individual qubit operations. This could be used for quantum state engineering and an implementation of Heisenberg limited spectroscopy\cite{32} in the solid state.

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\begin{thebibliography}{99}
\bibitem{1} R. H. Dicke, \textit{Phys. Rev.} \textbf{93}, 99 (1954).
\bibitem{2} M. Tavis and F. W. Cummings, \textit{Phys. Rev.} \textbf{170}, 379 (1968).
\bibitem{3} G. S. Agarwal, \textit{Phys. Rev. Lett.} \textbf{53}, 1732 (1984).
\bibitem{4} S. Leslie, N. Shenvi, K. R. Brown, D. M. Stamper-Kurn and K. B. Whaley, \textit{Phys. Rev. A} \textbf{69}, 043805 (2004).
\bibitem{5} M. G. Raizen, R. J. Thompson, R. J. Brecha, H. J. Kimble and H. J. Carmichael, \textit{Phys. Rev. Lett.} \textbf{63}, 240 (1989).
\bibitem{6} Y. Zhu et al., \textit{Phys. Rev. Lett.} \textbf{64}, 2499 (1990).
\bibitem{7} F. Bernardot, P. Nussenzveig, M. Brune, J. M. Raimond and S. Haroche, \textit{Europhys. Lett.} \textbf{17}, 33 (1992).
\bibitem{8} J. J. Childs, K. An, M. S. Otteson, R. R. Dasari and M. S. Feld, \textit{Phys. Rev. Lett.} \textbf{77}, 2901 (1996).
\bibitem{9} R. J. Thompson, Q. A. Turchette, O. Carnal and H. J. Kimble, \textit{Phys. Rev. A} \textbf{57}, 3084 (1998).
\bibitem{10} P. Münstermann, T. Fischer, P. Maunz, P. W. H. Pinkse and G. Rempe, \textit{Phys. Rev. Lett.} \textbf{84}, 4068 (2000).
\bibitem{11} H. J. Carmichael and B. C. Sanders, \textit{Phys. Rev. A} \textbf{60}, 2497 (1999).
\bibitem{12} A. K. Tuchman et al., \textit{Phys. Rev. A} \textbf{74}, 053821 (2006).
\bibitem{13} F. Brennecke et al., \textit{Nature} \textbf{450}, 268 (2007).
\bibitem{14} Y. Colombe et al., \textit{Nature} \textbf{450}, 272 (2007).
\bibitem{15} A. Wallraff et al., \textit{Nature} \textbf{431}, 162 (2004).
\bibitem{16} R. J. Schoelkopf and S. M. Girvin, \textit{Nature} \textbf{451}, 664 (2008).
\bibitem{17} J. Koch et al., \textit{Phys. Rev. A} \textbf{76}, 042319 (2007).
\bibitem{18} E. Jaynes and F. Cummings, \textit{Proc. IEEE} \textbf{51}, 89 (1963).
\bibitem{19} A. Boca et al., \textit{Phys. Rev. Lett.} \textbf{93}, 233603 (2004).
\bibitem{20} G. Khitrova, H. M. Gibbs, M. Kira, S. W. Koch and A. Scherer, \textit{Nat. Phys.} \textbf{2}, 81 (2006).
\bibitem{21} M. Brune et al., \textit{Phys. Rev. Lett.} \textbf{76}, 1800 (1996).
\bibitem{22} M. Hofheinz et al., \textit{Nature} \textbf{454}, 310 (2008).
\bibitem{23} I. Schuster et al., \textit{Nat. Phys.} \textbf{4}, 382 (2008).
\bibitem{24} J. M. Fink et al., \textit{Nature} \textbf{454}, 315 (2008).
\bibitem{25} L. S. Bishop et al., \textit{Nat. Phys.} \textbf{5}, 105 (2009).
\bibitem{26} C. E. López, H. Christ, J. C. Retamal and E. Solano, \textit{Phys. Rev. A} \textbf{75}, 033818 (2007).
\bibitem{27} W. Dür, G. Vidal and J. I. Cirac, \textit{Phys. Rev. A} \textbf{62}, 062314 (2000).
\bibitem{28} J. K. Stockton, R. van Handel and H. Mabuchi, \textit{Phys. Rev. A} \textbf{70}, 022106 (2004).
\bibitem{29} C. E. López, J. C. Retamal and E. Solano, \textit{Phys. Rev. A} \textbf{76}, 034313 (2007).
\bibitem{30} T. E. Tessier, I. H. Deutsch, A. Delgado and I. Fuentes-Guridi, \textit{Phys. Rev. A} \textbf{68}, 062316 (2003).
\bibitem{31} A. Retzker, E. Solano and B. Reznik, \textit{Phys. Rev. A} \textbf{75}, 022312 (2007).
\bibitem{32} D. Leibfried et al., \textit{Science} \textbf{304}, 1476 (2004).
\end{thebibliography}