Abstract—We propose a polar coding mechanism for distributed matrix multiplication. Polar codes provably achieve channel capacity and have the advantage of low encoding and decoding complexity. These aspects of polar codes enable a scalable scheme for hundreds of compute nodes in coded computation. We analyze the polarization phenomenon in the context of run times of compute nodes and characterize polarizing matrices over real numbers. We design a sequential decoder specifically for polar codes in erasure channels with real-valued input and outputs. The proposed coded computation scheme is implemented for a serverless computing platform and numerical results are provided. Numerical results illustrate that proposed coded computation scheme achieves significant speed-ups. Finally, experiments are conducted where the performance of the proposed coded computation technique is tested in solving a least squares problem using gradient descent.

I. INTRODUCTION

Matrix multiplication is usually one of the most computationally expensive steps in algorithms for solving problems in optimization, and other scientific computing. The scales of problems of interest in optimization, machine learning, and other fields have been growing drastically, leading to larger and larger matrix multiplications. Distributed computing platforms hence have been gaining popularity. Distributed computing platforms include High Performance Computing (HPC) clusters and serverless computing tools (such as AWS Lambda) as they recently have been proposed for use in scientific computing [5]. One problem that these distributed computing platforms have is that a portion of the workers is likely to finish their task very late, and these workers are often referred to as stragglers. There has been a recent line of work [6], [2], [9] in dealing with stragglers encountered in distributed computation using coding theory.

In this work, we investigate the application of polar codes in distributed matrix multiplication over real-valued input matrices. Polar codes [1] provably achieve channel capacity for symmetric binary-input discrete memoryless channels (B-DMC). Among other features that make polar codes a promising candidate for use in coded computation is that polar codes have low encoding and decoding complexity ($O(N \log N)$). In fact, it is one of the main motivations of this work that encoding and decoding both are low complexity in polar codes.

A straggler-resilient scheme designed for a serverless computing platform needs to be scalable if we are planning to employ a high number of compute nodes (which is the main advantage of using serverless computing in scientific computing). Two things become particularly important if the number of workers is in thousands. The first one is that encoding and decoding must be low complexity. The second one is that one must be careful with the numerical round-off errors if the inputs are not from a finite field, but instead are real numbers. To clarify this point, when the inputs are real-valued, we already have round-off errors due to the matrix multiplication operation itself, and encoding and decoding of the code used introduce additional round-off errors. Polar codes show superiority over many codes in terms of both of these aspects. They have low encoding and decoding complexity and both encoding and decoding involve optimally small number of subtraction and addition operations (no multiplications involved in encoding or decoding). In addition, they achieve capacity, and the importance of that fact in coded computation is that the number of worker outputs needed for decoding is asymptotically optimal.

A. Related Work

Coded matrix multiplication for HPC clusters has been proposed in [6] which is based on maximum distance separable (MDS) codes. [6] shows that it is possible to speed up distributed matrix multiplication by using MDS codes to avoid waiting for the slowest nodes. MDS codes however have the disadvantage of having high encoding and decoding complexity, which could be restricting in setups with large number of workers. [2] attacks at this problem presenting a coded computation scheme based on $d$-dimensional product codes. [9] presents a scheme referred to as polynomial codes for coded matrix multiplication with input matrices from a large finite field. This approach might require quantization for real-valued inputs which could introduce additional numerical issues. [3] and [10] are other works investigating coded matrix multiplication and provide analysis on the number of worker outputs required. In addition to the coding theory approaches, [4] offers an approximate straggler-resilient matrix multiplication scheme where the ideas of sketching and straggler-resilient distributed matrix multiplication are brought together.

Using Luby Transform (LT) codes, a type of rateless fountain codes, in coded computation has been recently proposed in [8], and later in [7]. The proposed scheme in [7] divides the overall task into smaller tasks (each task is a multiplication of a row of $A$ with $x$) for better load-balancing. The work [8] proposes the use of inactivation decoding and the work [7] uses peeling decoder. Peeling decoder has a computational complexity of $O(N \log N)$ (same complexity as the decoder we propose), however its performance is not satisfactory if
the number of inputs symbols is not very large. Inactivation decoder overcomes the performance issue at the expense of computational cost.

B. Main Contributions

This work investigates the use of polar codes in coded computation. We identify that polar codes are a natural choice for coded computation if one is interested in using a large number of workers due to their low complexity encoding and decoding and the fact that they achieve capacity. Furthermore, we propose a sequential decoder for polar codes designed to work with real-valued data for erasure channels. The proposed decoder is different from successive cancellation (SC) decoder given in [1], which is for discrete data and is based on estimation using likelihoods. We also analytically justify the use of the polar code kernel that we used in this work when the inputs and outputs are not from a finite field but are real-valued. Next, this work investigates the polarization of distributions of worker run times and provides some numerical results to that end. This provides some important insights towards understanding polarization phenomenon in a setting other than communication. Furthermore, numerical results illustrate that the proposed coded computation scheme based on polar codes speeds up distributed matrix multiplication. We also provide numerical results showing that it is possible to achieve faster results by increasing redundancy in the context of an iterative algorithm involving matrix multiplication.

II. CODED COMPUTATION WITH POLAR CODING

A. System Model

In this work, we are interested in speeding up the computation of \( A \times x \) where \( A \in \mathbb{R}^{m \times n} \) and \( x \in \mathbb{R}^{n \times r} \) using the model in Fig. 1. The master node encodes the \( A \) matrix and communicates the encoded matrix chunks. Compute nodes then perform their assigned matrix multiplication. After compute nodes start performing their assigned tasks, the master node starts listening for compute node outputs. Whenever a decodable set of outputs is detected, the master node downloads the available node outputs and performs decoding, using the sequential decoder described in subsection II.E.

![Fig. 1. System model.](image)

Note that we assume \( x \) fits in the memory of a compute node. In coded computation literature, this is referred to as large-scale matrix-vector multiplication, different from matrix-matrix multiplication where the second matrix is also partitioned. However, if \( x \) does not fit in the memory of a compute node, it is possible to partition \( x \) into small enough chunks that fit in the memory and copy them one by one into the memory.

B. Code Construction

Consider the polar coding representation in Fig. 2 for \( N = 4 \) nodes. The channels \( W \) in Fig. 2 in this work are erasure channels because of the assumption that if the output of a compute node is available, then it is assumed to be correct. If the output of a node is not available (straggler node), then it is considered an erasure.

![Fig. 2. Polar coding for \( N = 4 \).](image)

Let \( D_{ij} \) denote the data for the node \( i \) in the \( j \)'th level. Further, let us denote the erasure probability of each channel by \( \epsilon \), and assume that the channels are independent. Using the notation from [1], the coding scheme in Fig. 2 does the following channel transformation: \( W^4 \rightarrow (W_{4}^{(0)}, ..., W_{4}^{(3)}) \) where the transformed channels are defined as:

\[
W_{4}^{(0)} : D_{00} \rightarrow (D_{03}, ..., D_{33}) \\
W_{4}^{(1)} : D_{10} \rightarrow (D_{03}, ..., D_{33}, D_{00}) \\
W_{4}^{(2)} : D_{20} \rightarrow (D_{03}, ..., D_{33}, D_{00}, D_{10}) \\
W_{4}^{(3)} : D_{30} \rightarrow (D_{03}, ..., D_{33}, D_{00}, D_{10}, D_{20}) \quad (1)
\]

The transformation for \( N = 4 \) in fact consists of two levels of two \( N = 2 \) blocks, where an \( N = 2 \) block does the transformation \( W^2 \rightarrow (W_{2}^{(0)}, W_{2}^{(1)}) \), which is shown in Fig. 3. We can now compute the erasure probabilities for the transformed channels. Consider the block for \( N = 2 \) in Fig. 3.

\[
Pr(D_{00} \text{ is erased}) = 1 - (1 - \epsilon)^2, \quad Pr(D_{01} \text{ is erased}|D_{00}) = \epsilon^2.
\]

![Fig. 3. Polar coding for \( N = 2 \).](image)

1Note that during encoding, the node values \( D_{ij} \) represent the data before it is multiplied by \( x \), and during decoding, \( D_{ij}'s \) represent the data after the multiplication by \( x \).
To compute the erasure probabilities for \( N = 4 \), note that \( D_{01}, D_{21}, D_{03}, D_{23} \) form an \( N = 2 \) block. Similarly, \( D_{11}, D_{31}, D_{13}, D_{33} \) form another \( N = 2 \) block. It follows that the erasure probabilities for the level 1 are: 
\[
Pr(D_{01} \text{ is erased}) = Pr(D_{11} \text{ is erased}) = 1 - (1 - \epsilon)^2, \\
Pr(D_{21} \text{ is erased} | D_{01}) = Pr(D_{31} \text{ is erased} | D_{11}) = \epsilon^2
\]
Now, we can compute the erasure probabilities for the input level:
\[
Pr(D_{00} \text{ is erased}) = 1 - (1 - \epsilon)^4, \\
Pr(D_{10} \text{ is erased} | D_{00}) = (1 - (1 - \epsilon)^2)^2, \\
Pr(D_{20} \text{ is erased} | D_{00}, D_{10}) = 1 - (1 - \epsilon^2)^2, \\
Pr(D_{40} \text{ is erased} | D_{00}, D_{10}, D_{20}) = \epsilon^4
\]

It is straightforward to repeat this procedure to find the erasure probabilities for the transformed channels in the cases of larger values of \( N \).

Knowing the erasure probabilities of channels, we now select the best channels for data, and freeze the rest (i.e., set to zero). The indices of data and frozen channels are made known to both encoding and decoding. Note that by ‘channel’, we mean the transformed virtual channels \( W_N^{(i)} \), not \( W \). We refer the readers to [1] for an in-depth analysis and discussion on polar codes.

A possible disadvantage of the proposed scheme is that the number of compute nodes has to be a power of 2. This could pose a problem if the number of workers we are interested in utilizing is not a power of 2. We identify two possible solutions to overcome this issue. The first one is to use a kernel of different size instead of 2. The second solution is to have multiple code constructions. For instance, let us assume that \( \epsilon = \frac{1}{2} \). We can divide the overall task into 2 constructions. We would compute the first task using 16 compute nodes (12 data, 4 frozen channels), and the second task using 4 compute nodes (3 data, 1 frozen channel), the first task is to multiply the first \( \frac{1}{2} m \) rows of \( A \) with \( x \), and the second task is to multiply the last \( \frac{1}{2} m \) rows of \( A \) with \( x \). This way we obtain two straggler-resilient tasks and we use all 20 workers.

C. Encoding Algorithm

After computing the erasure probabilities for the transformed channels, We choose the \( d = N(1 - \epsilon) \) channels with the lowest erasure probabilities as data channels. The remaining \( Nc \) channels are frozen. The inputs for the data channels are the row partitions of matrix \( A \). That is, we partition \( A \) such that each matrix chunk has \( \frac{m}{2} \) rows: 
\[
A = [A_1^T, A_2^T, \ldots, A_T^T]^T
\]
where \( A_i \in \mathbb{R}^{\frac{m}{2} \times n} \). For example, for \( N = 4 \) and \( \epsilon = 0.5 \), the erasure probabilities of the transform channels are calculated (using Eq. 2) to be \([0.938, 0.563, 0.438, 0.063]\). It follows that we freeze the first two channels, and the last two channels are the data channels. This means that in Fig. 2 we set
\[
D_{00} = D_{10} = 0^{\frac{m}{2} \times n} \quad \text{and} \quad D_{20} = A_3, \quad D_{30} = A_2.
\]

We encode the input using the structure given for \( N = 4 \) in Fig. 2. Note that instead of the XOR operation like in polar coding for communication, what we have in this work is addition over real numbers. This structure can be easily formed for higher \( N \) values. For instance, for \( N = 8 \), we simply connect the circuit in Fig. 2 with its duplicate placed below it. The way encoding is done is that we compute the data for each level from left to right starting from the leftmost level. There are \( \log_2 N + 1 \) levels in total and \( N \) nodes in every level. Hence, the encoding complexity is \( O(N \log N) \).

D. Channel

Channels in the communication context correspond to compute nodes in the coded computation framework. The \( i \)th node computes the matrix multiplication \( D_i \log_2 N \times x \). When a decodable set of outputs is detected, if the output of the \( i \)th node is not available, then \( i \)th node is considered an erasure.

E. Decoding Algorithm

The decoding algorithm, whose pseudo-code is given in Alg. 1 is designed to decode real-valued data. Note that the notation \( I_{D_{ij}} \in \{0, 1\} \) indicates whether the data for node \( i \) in level \( j \) (namely, \( D_{ij} \)) is known. In the first part of the decoding algorithm, we continuously check for decodability as compute node outputs become available. When the available outputs become decodable, we move on to the second part. In the second part, decoding takes place given the available downloaded compute node outputs. The second part makes calls to the function decodeRecursive, given in Alg. 2.

Algorithm 1: Decoding algorithm.

\[
\text{Input: Indices of the frozen channels} \quad y = A \times x \\
\text{Result:} y = A \times x
\]

\[
\text{Part I} \\
\text{Initialize} \quad I_{D_{i,0}} = [I_{D_{0,0}}, I_{D_{1,0}}, \ldots, I_{D_{N-1,0}}] = [0, \ldots, 0] \\
\text{while} \quad I_{D_{i,0}} \text{ not decodable do} \\
\quad \text{update} \quad I_{D_{i,0}} \\
\quad \text{checkDecodability}(I_{D_{i,0}}) \\
\text{end} \\
\text{Part II} \\
\text{Initialize an empty list} \quad y \\
\text{for} \quad i \leftarrow 0 \quad \text{to} \quad N - 1 \quad \text{do} \\
\quad D_{i,0} = \text{decodeRecursive}(i, 0) \\
\quad \text{if} \quad \text{node } i \quad \text{is a data node then} \\
\quad \quad y = [y; D_{i,0}] \\
\quad \text{end} \\
\quad \text{if} \quad i \mod 2 = 1 \quad \text{then} \\
\quad \quad \text{forward prop} \\
\quad \quad \text{for} \quad j \leftarrow 0 \quad \text{to} \quad \log_2 N \quad \text{do} \\
\quad \quad \quad \text{for} \quad l \leftarrow 0 \quad \text{to} \quad i \quad \text{do} \\
\quad \quad \quad \quad \text{compute} \quad D_{ij} \quad \text{if unknown} \\
\quad \quad \text{end} \\
\quad \text{end} \\
\quad \text{end}
\]

\[
\text{checkDecodability is a function that checks whether it is possible to decode} \\
\text{a given sequence of indicators (identifying availability of outputs). This works} \\
\text{the same way as part II of Algorithm 1, but differs in that it does not deal} \\
\text{with data, but instead binary indicators.}
\]
fying the linearity property be used in proving Theorem 1.

min(1 1

A kernel \( K \) is a polarizing kernel if and only if the following conditions are both satisfied: 1) Both \( K_{11} \) and \( K_{22} \) are nonzero, 2) \( K_{11} \) and \( K_{22} \) are both assumed to be nonzero. Hence this completes the proof that a kernel \( K \) satisfying the given two conditions is a polarizing kernel.

**Theorem 1:** Of all possible \( 2 \times 2 \) polarizing kernels, the kernel \( F_2 = [1 0 1 0] \) results in the fewest number of computations for encoding real-valued data.

**Proof:** By Lemma 1, we know that for \( K \) to be a polarizing kernel, it must be invertible and must have both \( K_{12} \) and \( K_{22} \) as nonzero. For a \( 2 \times 2 \) matrix to be invertible with both second column elements as nonzero, at least one of the elements in the first column must also be nonzero. We now know that \( K_{12} \) and \( K_{22} \) are both nonzero in a polarizing kernel \( K \). It is easy to see that having all four elements of \( K \) as nonzero leads to more computations than having only three elements of \( K \) as nonzero. Hence, we must choose either \( K_{11} \) or \( K_{21} \) to be zero (it does not matter which one). It is possible to avoid any multiplications by selecting the nonzero elements of \( K \) as ones. Hence, both \( K = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) and \( K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) are polarizing kernels and lead to the same amount of computations, which is a single addition. This amount of computations is the minimum possible as otherwise \( K \) will not satisfy the condition that a polarizing kernel must have at least 3 nonzero elements.

**IV. Numerical Results**

**A. Polarized Computation Times**

Fig. IV-A illustrates how the empirical CDFs of computation times polarize as we increase the number of nodes \( N \). Fig.
is the empirical CDF of the computation times of AWS Lambda nodes, obtained by having 500 AWS Lambda nodes run the same function. The other plots in Fig. IV-A are generated assuming that there are $N$ nodes with i.i.d. run time distributions (the CDF of which is plotted in Fig. IV-A(a)), and show the resulting transformed CDFs of the polarized computation times. Note that the empirical CDFs move away from each other (i.e. polarize) as $N$ is increased and we obtain better and worse computation time distributions.

whose CDF is plotted in Fig. IV-A(a). Further, $\epsilon$ scheme for the AWS Lambda platform and now present the

\[ CDF: Pr(t \geq T_{\epsilon}) \]

A function, showing that for large $N$ increases, the distributions converge to the dirac delta function, showing that for large $N$ values, the decodability times become deterministic.

Plots in Fig. 6 are provided for comparison and show the decodability time histograms for LT codes with peeling decoder. The degree distribution is the robust soliton distribution as suggested in [7]. Fig. 5 and 6 show that polar codes achieve better decodability times than LT codes. Furthermore, LT codes have the disadvantage that decodability is not guaranteed for a given number of input symbols $N$ (an output time of 150 seconds in Fig. 6 shows that decodability is not achieved).

C. Coded Computation on AWS Lambda

We have implemented the proposed coded computation scheme for the AWS Lambda platform and now present the results illustrating its performance. The implementation is written in Python. The compute nodes are AWS Lambda nodes each with a memory of 1536 MB and a timeout limit of 300 seconds. We have utilized the Pywren module [5] for running code in AWS Lambda. The master node in our implementation is a Macbook Pro laptop with 8 GB of RAM.

Given in Fig. 7 is a plot showing the timings of compute nodes, and of the decoding process, where we compute $A \times x$ where $A \in \mathbb{R}^{38400 \times 3000000}$ and $x \in \mathbb{R}^{3000000 \times 20}$ using $N = 512$ compute nodes, and taking $\epsilon = 0.25$. The vertical axis represents the numbers we assign to the compute nodes. Encoding process is not included in the plot, as we believe that it is usually the case for many applications that we multiply the same $A$ matrix with different $x$’s. Hence for an application that requires iterative matrix multiplications with the same $A$ but different $x$’s, it is sufficient to encode $A$ once, but decoding has to be repeated for every different $x$. Therefore, the overall time would be dominated by the matrix multiplications and decoding, which are repeated many times unlike encoding which takes place once.
For the experiment shown in Fig. 7, each compute node performs a matrix multiplication with dimensions $(100 \times 3000000)$ and $(3000000 \times 20)$. A matrix with dimensions $(100 \times 3000000)$ requires a memory of 2.3 GB. It is not feasible to load a matrix of this size into the memory at once; hence, each compute node loads $1/10^{th}$ of its corresponding matrix at a time. After computing $1/10^{th}$ of its overall computation, it moves on to perform the next $1/10$ of its computation. In Fig. 7, the red stars show the times each compute node starts running. Green stars (mostly invisible because they are covered by the blue stars) show the times compute nodes finish running. Blue stars indicate the times that the outputs of compute nodes become available. Green (vertical) line is the first time the available outputs (shown as blue stars) become decodable. After the available outputs become decodable, we cannot start decoding right away; we have to wait for their download (to the master node) to be finished, and this is shown by the magenta line. Magenta line also indicates the beginning of the decoding process, and lastly the black line indicates that decoding is finished.

We observe that there are straggler nodes that finish after decoding is over, showing the advantage of our coded computation scheme over an uncoded scheme where we would have to wait for all nodes to finish. The slowest compute node seems to have finished around $t = 200$, whereas the result is available at around $t = 150$ for the proposed coded computation. Furthermore, we observe that the process after the first time the outputs become decodable is dominated by the data download (rather than the decoding itself). A remedy for this problem would be to implement the master node as a node in AWS as this would make it much faster to access S3.

D. Application: Gradient Descent on a Least Squares Problem

We now use the proposed coded matrix multiplication scheme in solving a least squares problem via gradient descent. The problem that we focus on in this subsection is given as:

$$\text{minimize } \|Ax - y\|_2^2$$  \hspace{1cm} (6)

where $x \in \mathbb{R}^{n \times r}$ is not necessarily a matrix. Let $x^* \in \mathbb{R}^{n \times r}$ be the optimal solution to (6), and let $x_i^*$ represent the $i$'th column of $x^*$. Note that if $x$ is a matrix, then the problem (6) is equivalent to solving $r$ different least-squares problems with the same $A$ matrix: minimize $\|Ax_i - y_i\|_2^2$. The solution to this problem is the same as the $i$'th column of $x^*$, that is, $x_i^*$. When $r$ is large (and if we assume that $A$ fits in the memory of each worker), one way to solve (6) in a distributed setting is by assigning the problem of minimizing $\|Ax_i - y_i\|_2^2$ to worker $i$. This solution however would suffer from possible straggler nodes, and assumes that $A$ is small enough to fit in the memory of a worker. Another solution, the one we experiment with in this subsection, is to use gradient descent as in this case we are no longer restricted by $A$ having to fit in the memory. Furthermore, we can apply our coded computation technique for computing the matrix multiplications in a distributed setting, which would make the process straggler-resilient.

The gradient descent update rule for this problem is:

$$x_{t+1} = x_t - \mu (A^T Ax - A^T y),$$  \hspace{1cm} (7)

where $\mu$ is the step size (or learning rate), and the subscript $t$ specifies the iteration number.

We applied our proposed coded matrix multiplication scheme to solve a least squares problem (6) using gradient descent where $A \in \mathbb{R}^{20000 \times 4800}$ and $x \in \mathbb{R}^{4800 \times 1000}$. Fig. 8 shows the downward-shifted cost (i.e., $l_2$-norm of the residuals $\|Ax_t - y\|_2^2$) as a function of time under different schemes. Before starting to update the gradients, we first compute and encode $A^T A$ offline. Then, in each iteration of the gradient descent, the master node decodes the downloaded outputs, updates $x_t$, sends the updated $x_t$ to S3 and initializes the computation $A^T X_t$. In the case of uncoded computation, we simply divide the multiplication task among $N(1 - \epsilon)$ workers, and whenever all of the $N(1 - \epsilon)$ workers finish their computations, the outputs are downloaded to the master node, and there is no decoding. Then, master node computes and sends the updated $x_t$, and initializes the next iteration.

Note that in a given iteration, while computation with polar coding with rate $(1 - \epsilon)$ waits for the first decodable set of outputs out of $N$ outputs, uncoded computation waits for all $N(1 - \epsilon)$ nodes to finish computation. With this in mind, it follows that coded computation results in a trade-off between price and time, that is, by paying more, we can achieve a faster convergence time, as illustrated in Fig. 8. Note that in coded computation we pay for $N$ nodes, and in uncoded computation we pay for $N(1 - \epsilon)$ nodes. Using $\epsilon$ as a tuning parameter for redundancy, we achieve different convergence times.

V. Conclusion

We have proposed to use polar codes for distributed matrix multiplication, and discussed that the properties of polar codes could prove useful for the coded computation framework. We considered a centralized model with a master node and a large number of compute nodes, which we implemented for
the AWS Lambda platform. A sequential decoder algorithm acting on real-valued data is presented. We have identified and illustrated with a numerical example a trade-off between the computation price and convergence time for the gradient descent algorithm applied to a least-squares problem.

REFERENCES

[1] E. Arikan. Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels. IEEE Transactions on Information Theory, 55:3051–3073, 2009.

[2] T. Baharav, K. Lee, O. Ocal, and K. Ramchandran. Straggler-proofing massive-scale distributed matrix multiplication with $d$-dimensional product codes. IEEE International Symposium on Information Theory (ISIT), pages 1993–1997, 2018.

[3] S. Dutta, M. Fahim, F. Haddadpour, H. Jeong, V. R. Cadambe, and P. Grover. On the optimal recovery threshold of coded matrix multiplication. CoRR, abs/1801.10292, 2018.

[4] V. Gupta, S. Wang, T. A. Courtade, and K. Ramchandran. Oversketch: Approximate matrix multiplication for the cloud. CoRR, abs/1811.02653, 2018.

[5] E. Jonas, S. Venkataraman, I. Stoica, and B. Recht. Occupy the cloud: Distributed computing for the 99%. CoRR, abs/1702.04024, 2017.

[6] K. Lee, M. Lam, R. Pedarsani, D. Papailiopoulos, and K. Ramchandran. Speeding up distributed machine learning using codes. IEEE Transactions on Information Theory, 64(3):1514–1529, March 2018.

[7] A. Mallick, M. Chaudhari, and G. Joshi. Rateless codes for near-perfect load balancing in distributed matrix-vector multiplication. CoRR, abs/1804.10331, 2018.

[8] A. Severinson, A. G. i Amat, and E. Rosnes. Block-diagonal and Lt codes for distributed computing with straggling servers. IEEE Transactions on Communications, 2018.

[9] Q. Yu, M. A. Maddah-Ali, and A. S. Avestimehr. Polynomial codes: An optimal design for high-dimensional coded matrix multiplication. Adv. in Neural Info. Proc. Sys. (NIPS) 30, pages 4406–4416, 2017.

[10] Q. Yu, M. A. Maddah-Ali, and A. S. Avestimehr. Straggler mitigation in distributed matrix multiplication: Fundamental limits and optimal coding. CoRR, abs/1801.07487, 2018.