A SUPERSPACE FORMULATION OF AN “ASYMPTOTIC” 
\( OSp(3, 1|2) \) INVARIANCE OF YANG-MILLS THEORIES.

Satish D. Joglekar*

Department of physics
Indian Institute of Technology, Kanpur
Kanpur, 208016, India

and

Bhabani Prasad Mandal†

Theory Group,
Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Calcutta- 700 064, India,

Abstract

We formulate a superspace field theory which is shown to be equivalent to the \( c - \bar{c} \) symmetric BRS/Anti-BRS invariant Yang-Mills action. The theory uses a 6-dimensional superspace and one \( OSp(3, 1|2) \) vector multiplet of unconstrained superfields. We establish a superspace WT identity and show that the formulation has an asymptotic \( OSp(3, 1|2) \) invariance as the gauge parameter goes to infinity. We give a physical interpretation of this asymptotic \( OSp(3, 1|2) \) invariance as a symmetry transformation among the longitudinal/time like degrees of freedom of \( A_\mu \) and the ghost degrees of freedom.

*e-mail address: sdj@iitk.ac.in

†Address for correspondence, e-mail :- bpm@tnp.saha.ernet.in
I. INTRODUCTION

Non-abelian gauge theories are endowed with local gauge invariance \[1\]. Local gauge invariance leads to relations between Green’s functions of gauge and or ghost fields collectively denoted by WT identities \[2\]. Formulation of gauge theories in covariant gauges necessitates inclusion of unphysical degrees of freedom corresponding to the longitudinal and the time like gauge fields. Unitarity of S-Matrix (whenever defined) requires that these modes do not contribute to the intermediate states in the cutting equations \[3\]. The contributions from such intermediate states are canceled by contributions from diagrams containing ghost intermediate states. This is demonstrated in gauge theories with the use of the (on-shell) WT identities.

Thus the cancellation of intermediate states coming from longitudinal \( \text{/} \) time like gauge degrees of freedom and the ghost degrees of freedom (we denote this set by \( R \)) together is one of the essential consequence of WT identities. These, in turn, follow from the BRS symmetry (or gauge invariance) \[4\]. This, in turn, suggests that there should be a formulation of BRS symmetry where the above set of \( R \) of degrees of freedom are explicitly linked together.

There exist many attempts to link \((A, c, \bar{c})\) fields together. In view of both the commuting and anti-commuting degrees of freedom involved, this points to a “Supersymmetric/Superfields” formulation. A number of superspace/superfield formulation have been written down which exhibit the BRS symmetry in terms of translations or rotations in superspace \[4,5\]. For a brief summary of superspace/superfields formulations and their comparison see comments in Ref. \[8\] and references therein.

The superspace formulation of Ref. \[7\] constructed superfields \( A(x, \theta, \bar{\theta}), c(x, \theta, \bar{\theta}) \) and \( \bar{c}(x, \theta, \bar{\theta}) \) by hand by ascribing the values of the additional components \( (A_{\theta}, A_{\bar{\theta}} \cdots \text{etc}) \) equal to the BRS/Anti-BRS variations \[8\] of these. They exhibited the BRS/Anti-BRS structure thereby. However as the structure of the superfields was restricted there one could not construct a full-fledged field theory of these superfields. The works of references \[5,9\] (and subsequent works) attempted to constructed a field theory of superfields in superspace.
Here the superfields were entirely unconstrained and the superrotations could be carried out in the formulation. In fact the BRS and Anti-BRS were identified with these superrotations and the corresponding WT identities understood as arising from these. These constructions had a broken $OSp(3,1|2)$ symmetry. While these superspace formulation exhibited the BRS/Anti-BRS structure, and the renormalization properties of gauge theories compactly and correctly. They treated the anti-ghost field asymmetrically (and as far as we know it is necessary to do this, to exhibit the renormalization properties in linear gauges. Moreover, the underlying $OSp(3,1|2)$ symmetry was broken one.

Following the motivations outlined earlier, we have attempted, in this work, a formulation that (i) is a superspace field theories as in Ref. (ii) treats gauge, ghost and anti-ghost fields together in one single supermultiplet. (iii) has an underlying formal $OSp(3,1|2)$ symmetry as the basis of construction as a limiting symmetry of the Lagrange density. (iv) has WT identities that formally imply that this symmetry becomes exact as gauge parameter $\eta \to \infty$ (v) corresponds to the Yang-Mills theory in one of its formulation. In fact we find that the superspace formulation presented here corresponds to the BRS/Anti-BRS invariant formulation of Baulieu and Thierry-Mieg with $\beta = 1$ ($c, \bar{c}$ symmetric case).

We interpret heuristically the last property in the following manner. We note (as done in sec. IIC) that as $\eta \to \infty$ the gauge boson propagator is dominated by the longitudinal and time like modes. Thus in this limit, the multiplet $(A, c, \bar{c})$ is dominated by just the set $R$ of extra modes which enter the unitarity discussion via WT identities. It is precisely in this limit, the $OSp(3,1|2)$ symmetry is becoming exact.

We now briefly present the plan of the paper. In Sec II, we shall review the underlying superspace /superfield structure and the $OSp$ group properties. We briefly discuss the BRS/Anti-BRS symmetric formulation of Reference. We also include a brief discussion on the mode structure of propagator as $\eta \to \infty$. In section III, we present the superspace formulation and show its equivalence to the BRS/Anti-BRS symmetric formulation with $\beta = 1$. In this section IV, we show that the generating functional $W[\bar{X}]$ is asymptotically ($\eta \to \infty$) invariant under the $OSp(3,1|2)$ group. In sec V, we elaborate on the physical
meaning of the result so obtained.

II. PRELIMINARY

A. BRS/Anti-BRS symmetric action

In this section, we shall review the known results on BRS and anti-BRS symmetries of effective action in gauge theories [8].

We consider the most general effective action in linear gauges given by Baulieu and Thierry-Mieg [8] that has BRS/anti-BRS invariance, when expressed entirely in terms of necessary fields $A, c, \bar{c}$ (and no auxiliary fields)

$$S_{\text{eff}}[A, c, \bar{c}] = \int d^4 x \left[ -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} - \sum_\alpha \frac{(\partial \cdot A^\alpha)^2}{2\eta} - \mathcal{L}_G \right]$$

(2.1)

with

$$\mathcal{L}_G = (1 - \frac{1}{2}\beta)\partial^\mu \bar{c} D_\mu c + \frac{\beta}{2} D^\mu \bar{c} \partial_\mu c - \frac{1}{2} \beta (1 - \frac{1}{2}\beta) \frac{\eta}{2} g^2 [f_{\alpha\beta\gamma} \bar{c}^\beta c^\gamma]^2$$

(2.2)

$$= \partial^\mu \bar{c} D_\mu c + \frac{\beta}{2} g f_{\alpha\beta\gamma} \partial^\mu A^\alpha \bar{c}^\beta c^\gamma + \frac{1}{8} \beta (1 - \frac{1}{2}\beta) \eta g^2 f_{\alpha\beta\gamma} \bar{c}^\beta c^\gamma f^{\alpha\xi} c^\gamma \bar{c}^\xi$$

(2.3)

Here we are assuming a Yang-Mills theory with a simple gauge group and introducing the following notations:

- Lie Algebra: $[T^\alpha, T^\beta] = i f_{\alpha\beta\gamma} T^\gamma$
- Covariant derivative: $(D_\mu c)^\alpha = D_\mu^\beta c^\beta = (-\partial_\mu \delta^\alpha \beta + g f_{\alpha\beta\gamma} A_\mu^\gamma) c^\beta$

$f_{\alpha\beta\gamma}$ are totally antisymmetric. Note here we have changed the convention for the covariant derivative just to bring it in line with notations of Ref. [5]. This action has the global symmetries under the following transformations
\[
\delta A_\mu^\alpha = (D_\mu c)^\alpha \delta \Lambda \\
\delta c^\alpha = -\frac{1}{2} gf^{\alpha\beta\gamma} c^\beta \bar{c}^\gamma \delta \Lambda \\
\delta \bar{c}^\alpha = \left( -\frac{\partial \cdot A^\alpha}{\eta} - \frac{1}{2} \beta gf^{\alpha\beta\gamma} c^\beta \bar{c}^\gamma \right) \delta \Lambda
\]

(2.4)

and anti-BRS:
\[
\delta A_\mu^\alpha = (D_\mu \bar{c})^\alpha \delta \Lambda \\
\delta \bar{c}^\alpha = -\frac{1}{2} gf^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma \delta \Lambda \\
\delta c^\alpha = \left( -\frac{\partial \cdot A^\alpha}{\eta} - \left( 1 - \frac{1}{2} \beta \right) gf^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right) \delta \Lambda
\]

(2.5)

In the anti-BRS transformations the role of \( c \) and \( \bar{c} \) are interchanged in addition to change in some coefficients. Note that \( \beta = 0 \) case yields the usual Faddeev-Popov action and \( \beta = 1 \) yields an action symmetric in \( c \) and \( \bar{c} \).

**B. Superspace, Superfields and Invariants**

We shall work in superspace formulation of Yang-Mills theory given in Ref. [5], which we briefly review in this section. The superspace formulation uses an underlying six-dimensional superspace described by superspace coordinate \( \bar{x}^i \equiv (x^\mu, \lambda, \theta) \) with \( \lambda, \theta \) are real Grassmannerian variables. Superfields and supersources are function of superspace coordinates. The superspace is endowed with a metric \( g_{ij} \), with only non zero components

\[
g_{00} = -g_{11} = -g_{22} = -g_{33} = -g_{45} = g_{54} = 1
\]

(2.6)

The infinitesimal orthosymplectric coordinate transformations which leaves the norm of the supervector, \( x^i g_{ij} x^j \) invariant are consists of (i) Six Lorentz transformations which leaves \( g_{\mu\nu} x^\mu x^\nu \) invariant. (ii) Three simplectic transformations which leaves \( \lambda \theta \) invariant and characterized by three infinitesimal parameter. (iii) and eight SUSY transformations given by

\[
x'^\mu = x^\mu + \epsilon^\mu a\lambda + \delta^\mu b\theta
\]
\[ \lambda' = \lambda + \delta^\mu x_\mu b \]
\[ \theta' = \theta - \epsilon^\mu x_\mu a \]

(2.7)

[Where \( \epsilon_\mu, \delta_\mu \) are arbitrary four vectors and \( a, b \) are real infinitesimal Grassmannians] generated by \( S_{4\mu} \) and \( S_{5\mu} \). \( S_{4\mu} \) generates transformations with \( \delta = 0 \) and \( S_{5\mu} \) generates transformations with \( \epsilon = 0 \). \( \frac{\partial}{\partial x_i} \) are transforms as a covariant vector under the \( OSp(3,1|2) \) transformations and they are given by

\[ \frac{\partial}{\partial x_i'} = \frac{\partial}{\partial x_i} + \epsilon^\mu a \frac{\partial}{\partial \theta} - \delta^\mu b \frac{\partial}{\partial \lambda} \]
\[ \frac{\partial}{\partial \lambda'} = \frac{\partial}{\partial \lambda} + \epsilon^\mu a \frac{\partial}{\partial x_\mu} \]
\[ \frac{\partial}{\partial \theta'} = \frac{\partial}{\partial \theta} + \delta^\mu b \frac{\partial}{\partial x_\mu} \]

(2.8)

and the vector superfields \( A_i(\bar{x}) \equiv (A_\mu(\bar{x}), c_4(\bar{x}), c_5(\bar{x})) \) also transform as covariants vectors under these \( OSp(3,1|2) \) transformations and given by

\[ A'_\mu = A_\mu + \epsilon_\mu ac_5 - \delta_\mu bc_4 \]
\[ c'_4 = c_4 + \epsilon^\mu a A_\mu \]
\[ c'_5 = c_5 + \delta^\mu b A_\mu \]

(2.9)

The transformations for the vector supersource \( \bar{X}^i(\bar{x}) \) are such that \( \bar{X}^i(\bar{x}) A_i(\bar{x}) \) remain invariant under \( OSp(3,1|2) \)

We define the scalar product as

\[ A \cdot B = A_i g^{ij} B_j \equiv A^i B_j \]

(2.10)

And the tensor invariants are defined as

\[ A \cdot BC \cdot D = C_i A_j B_k D_l g^{kj} g^{li} = T_{ij} T_{kl} g^{kj} g^{li} \]

(2.11)

where \( A^i B_i \) is a commuting quantity. Using the above definitions of scalar products we construct the following \( OSp \) invariant
quantities \((i)\) \(F_{ij}F_{kl}g^{ij}g^{li}\) \((ii)\) \(\partial^i[A_i\partial^jA_j]\) \((iii)\) \(\partial^i[A^j\partial_jA_i]\) \((iv)\) \(\partial^i[(\partial_iA^j)A_j]\). Where the superspace field strength tensor, \(F_{ij}\) is defined as

\[
F_{ij}^\alpha(\bar{x}) = \partial_iA_j^\alpha(\bar{x}) - A_i^\alpha \tilde{\partial}_j + gf^{\alpha\beta\gamma}A^\beta_i(\bar{x})A^\gamma_j(\bar{x})
\]

\(2.12\)

C. Mode structure of gauge propagator

The propagator in the linear gauges is given by

\[
i\Delta_{F_{\mu\nu}}(k, \eta) = \frac{-i}{k^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 + i\epsilon} (1 - \eta) \right]
\]

\(2.13\)

We imagine expanding the gauge field (in the momentum space) in the basis consisting of the transverse, the longitudinal and the time like degrees of freedom,

\[
A_\mu(k) = \sum_{i=1}^{4} \epsilon^{(i)}_\mu(k) a_{(i)}(k^2)
\]

\(2.14\)

with \(\epsilon^{(1)}_\mu(k)\) and \(\epsilon^{(2)}_\mu(k)\) are transverse degrees of freedom with

\[
\epsilon^{(i)}_0(k) = 0, \quad \vec{k} \cdot \epsilon^{(i)}(\vec{k}) = 0 \quad i = 1, 2
\]

\(2.15\)

and

\[
\epsilon^{(3)}_\mu(k) = \left(0, \frac{-\vec{k}}{|k|}\right); \quad \epsilon^{(4)}_\mu = (1, 0, 0, 0)
\]

\(2.16\)

We note the orthonormality properties,

\[
\epsilon^{(i)}_\mu \cdot \epsilon^{(j)} = -\delta^{ij} + 2\delta_{00}\delta_{ij} \quad (i, j = 1, 2, 3, 4)
\]

\(2.17\)

Then the gauge boson propagator

\[
\langle A_\mu(k) A_\nu(-k) \rangle = \sum_{i,j=1}^{4} \epsilon^{(i)}_\mu(k) \epsilon^{(j)}_\nu(-k) \langle a_{(i)}(k^2) a_{(j)}(k^2) \rangle
\]

\(2.18\)

We recall

\[
\epsilon^{(i)}_\mu(k) \epsilon^{(i)}_\nu(-k) = -(g_{\mu\nu} - \delta_{\mu0}\delta_{\nu0}) + \frac{k_\mu k_\nu (1 - \delta_{\mu0})(1 - \delta_{\nu0})}{k^2}
\]

\(2.19\)
We then find by comparison,

\[ \langle a(i)(k^2), a(j)(k^2) \rangle = \frac{\delta_{ij}}{k^2 + i\epsilon} \quad 1 \leq i, j \leq 2 \]

\[ \langle a_j(k^2), a_i(k^2) \rangle = \langle a(i)(k^2), a(j)(k^2) \rangle = 0 \quad 1 \leq i \leq 2; \quad 3 \leq j \leq 4 \]

\[ \langle a(3)(k^2), a(3)(k^2) \rangle = -(\eta - 1) \frac{|k|^2}{(k^2 + i\epsilon)^2} \]

\[ \langle a(4)(k^2), a(4)(k^2) \rangle = -\left[ 1 + \frac{(\eta - 1)|k|^2}{k^2 + i\epsilon} \right] \frac{1}{k^2 + i\epsilon} \]

\[ \langle a(3)(k^2), a(4)(k^2) \rangle = \frac{(\eta - 1)}{2} \frac{|k|k_0}{(k^2 + i\epsilon)^2} \]  (2.20)

Thus we see that as \( \eta \rightarrow \infty \), the correlation functions of modes containing \( a(3) \) (the longitudinal) or \( a(4) \) (the time like) go to \( \infty \), while those containing the transverse components remain unaltered. We now scale as,

\[ a(3) = \sqrt{\frac{\eta}{3}} \tilde{a}_3; \quad a(4) = \sqrt{\frac{\eta}{3}} \tilde{a}_4; \quad \tilde{a}_i \equiv a_i \quad i = 1, 2 \]  (2.21)

(The factor of \( \frac{1}{3} \) is for future convenience only.) Then all correlation functions \( \langle \tilde{a}_i(k^2), \tilde{a}_j(k^2) \rangle \) have \( \eta \)-independent limits. Then the expansion of the gauge field reads

\[ A_\mu(k) = \sum_{i=1}^{2} \epsilon_\mu^{(i)}(k) \tilde{a}_i(k^2) + \sqrt{\frac{\eta}{3}} \epsilon_\mu^{(3)} \tilde{a}_3(k^2) + \sqrt{\frac{\eta}{3}} \epsilon_\mu^{(4)} \tilde{a}_4(k^2) \]

\[ \equiv A^T_\mu + \sqrt{\frac{\eta}{3}} A^L_\mu + \sqrt{\frac{\eta}{3}} A^t_\mu \]  (2.22)

The relation above exhibits explicitly the \( \sqrt{\eta} \) factors that say that (after suitable normalization) the longitudinal and the time like components of a general gauge field become dominant as \( \eta \rightarrow \infty \). This remark will find application in See. V in the context of the supermultiplet structure of fields introduced.

### III. CONSTRUCTION OF SUPERSPACE ACTION

In this section, we shall present the construction of the superspace action which is equivalent to the Yang-Mills theory in its BRS/Anti-BRS invariant formulation with \( \beta = 1 \) [ See sec. IIA]. The building block of the superspace action is a covariant vector field
\( \bar{A}_i(\bar{x}) = (A_\mu(\bar{x}), c_4(\bar{x}), c_5(\bar{x})) \) (\( c_4 \) and \( c_5 \) will turn out to be related to the antighost field \( \bar{c} \) and the ghost field \( c \)). We shall also introduce commuting contravariant vector source \( \bar{X}^i(\bar{x}) \). Unlike Ref. [6] we don’t however need a scalar superfield and scalar supersource. As we shall see later, the Lagrange density turns out to have a graded structure as the gauge parameter \( \eta \to \infty \) (i) \( \mathcal{L}_0 \) is an OSp invariant action of \( O(\eta^0) \) (ii) \( \mathcal{L}_1 \) turns out to be also an OSp invariant, but of \( O(\eta^{-1}) \) and (iii) \( \mathcal{L}_2 \) is an Sp(2) invariant symmetry breaking term of \( O(\eta^{-2}) \). Explicitly\[1\]

\[
\begin{align*}
\mathcal{L}_0 &= \frac{1}{4} F_{ij} F_{kl} g^{kj} g^{li} \\
\mathcal{L}_1 &= \alpha \partial^i \left[ A_i \partial_j A_j \right] + \beta \partial^i \left[ A^j \partial_j A_i \right] + \gamma \partial^i \left[ A^j \partial_i A_j \right] \\
\mathcal{L}_2 &= \frac{k}{2} \bar{c}^a \partial^i \partial_i c_a \quad a = 4, 5
\end{align*}
\] 

(3.1)

To this we add the source terms

\[
\mathcal{L}_s = \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} [\bar{X}^i \bar{A}_i]
\] 

(3.2)

Under OSp(3,1|2) transformations \( \mathcal{L}_s \) changes at most by a total derivative. We then construct the generating functional

\[
W[\bar{X}] = \int \mathcal{D} A \exp i \int d^4 x \left[ \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_s \right]
\] 

(3.3)

where

\[
\mathcal{D} \bar{A} \equiv \prod_{i=0}^{5} \mathcal{D} A_i(\bar{x}) \equiv \prod_{i=0}^{5} \mathcal{D} A_i(x) \mathcal{D} A_{i,\lambda}(x) \mathcal{D} A_{i,\theta}(x)
\] 

(3.4)

In order to establish the equivalence of the above generating functional with that of the Yang-Mills theory, we carry out the integrations over the variables \( A_{i,\lambda}, A_{i,\theta} \) explicitly as in Ref. [3]. The procedure is very straightforward and hence we shall not present the details;

\[1\] The parameter \( \beta \) in \( \mathcal{L}_1 \) is not to be confused with \( \beta \) in the BRS /anti-BRS invariant action of Sec. IIA which will always be taken to be 1 in this work.
but only the final result. Omitting the source terms for the present (as these are not relevant to the equivalence) we find

\[ W[\bar{X}] = \int \mathcal{D}A_\mu(x) \mathcal{D}c_4(x) \mathcal{D}c_5(x) \exp i [S_0[A_\mu, c_4, c_5] + \text{Source terms}] \quad (3.5) \]

with omitting redundant terms in $A_{i,\lambda \theta}$.

\[
S_0[A, c_4, c_5] = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{(\alpha - \beta)^2}{2(2\alpha - \gamma - \beta)} (\partial \cdot A)^2 - \frac{\alpha - \beta}{1 - 2\gamma} [D_\mu c_4 \partial^\mu c_5 + \partial_\mu c_4 D^\mu c_5]
\]
\[
+ \frac{2\gamma}{1 - 2\gamma} D_\mu c_4 D^\mu c_5 - \frac{3(\gamma + \beta)}{2(\beta + \gamma + 1)} (gf c_4 c_5)^2 - \left[ (\alpha - \beta)^2 - \kappa \right] \partial_\mu c_4 \partial^\mu c_5 \quad (3.6)
\]

Comparing with Eq. (2.1), we see that $S_0$ of (3.6) is compatible with the action in Eq. (2.1) only if

\[
\text{coefficient of } (fA_\mu c_4)(fA_\mu c_5) = 0 \Rightarrow \gamma = 0 \quad (3.7)
\]

and

\[
\eta = \frac{2\alpha - \beta}{(\alpha - \beta)^2} \quad (3.8)
\]

Further, we use the freedom to define $c_4$ and $c_5$ to set

\[
c_5 = \frac{1}{\sqrt{2(\alpha - \beta)}} \bar{c}; \quad c_4 = \frac{1}{\sqrt{2(\alpha - \beta)}} \bar{c} \quad (3.9)
\]

then the two actions coincide if, further,

\[
-\frac{3\beta}{2(\beta + 1)} = \frac{2\alpha - \beta}{2} \quad (3.10)
\]
\[
\text{and } \kappa = (\alpha - \beta)^2 \quad (3.11)
\]

The quadratic equation of (3.11) has solutions

\[
\beta = (\alpha + 1) \left[ 1 \pm \sqrt{1 + \frac{2\alpha}{(\alpha + 1)^2}} \right] \quad (3.12)
\]

\[^2 A_{i,\lambda \theta} \text{ here are not dynamical fields and can be dropped (i.e. can be set to zero by hand) in future.}\]
Either values of $\beta$ would be acceptable for our purpose.

We shall see in sec. IV that the solution in (3.12) with -ve sign leads to a superspace Lagrange density that has asymptotic (i.e. as $\eta \to \infty$) symmetry; and hence we shall make this choice. Thus the equivalence of the two action with $\beta$ and $\kappa$ given in terms of (3.10) and (3.11) is established completely.

We shall, however, be particularly interested in a special case. We further use the freedom we have in choosing the free parameter $\alpha$ to let $0 < \alpha \ll 1$ then,

$$\beta = (\alpha + 1) \left[ 1 - \sqrt{1 + \frac{2\alpha}{(\alpha + 1)^2}} \right] \simeq \frac{\alpha}{\alpha + 1} \simeq -\alpha \quad (3.13)$$

Then the gauge parameter becomes

$$\eta = \frac{2\alpha - \beta}{(\alpha - \beta)^2} \simeq \frac{3\alpha}{(2\alpha)^2} = \frac{3}{4\alpha} \quad (3.14)$$

Thus, as $\alpha \to 0^+$, our superspace action represents the BRS/Anti-BRS action with the parameter $\beta$ in 2.2 set equal to 1 and $\eta \to \infty$. Further,

$$\kappa = (\alpha - \beta)^2 \simeq 4\alpha^2 \quad (3.15)$$

expressing all parameters in terms of $\eta$ (as $\eta \to \infty$),

$$-\beta \simeq \alpha \simeq \frac{3}{4\eta}, \quad \kappa \simeq 4 \cdot \frac{9}{16\eta^2} = \frac{9}{4\eta^2} \quad (3.16)$$

and the scaling of (3.9) are re-expressed as

$$c_5 = \sqrt{\frac{\eta}{3}} c, \quad c_4 = \sqrt{\frac{\eta}{3\bar{c}}} \quad (3.17)$$

To summarize, the superspace action with one free parameter $\eta$

$$\int d^4x \left\{ L_0 + \alpha \partial^i [\partial^j A_j A_i - A_i \partial_j A_i] + 4\alpha^2 c^a \partial^i c_a \right\} \quad (3.18)$$

( with $\alpha = \frac{3}{4\eta}$ and coefficients valid for $\eta$ large ) is equivalent in the superspace generating functional to the BRS/Anti-BRS symmetric action with gauge parameter $\eta(\to \infty)$. In the next section we shall establish the asymptotic OSp invariance for $W[K]$ in other words, the formal equation of the form

$$[W[X'] - W[X]] |_{X'_{\iota}=0} = 0 \left( \frac{1}{\eta} \right) \quad (3.19)$$
IV. $OSP(3,1|2)$ WT IDENTITIES

In this section, we shall consider the consequence of the $OSp(3,12)$ transformations on the source $\bar{X}^i(\bar{x})$ to obtain the WT identities for the broken $OSp(3,1|2)$ symmetry. The result is summarized by the statement which in effect says that $\eta \to \infty$ $W$ recovers $OSp(3,1|2)$ invariance under the conditions clarified under. It is also shown how this WT identity embodies exact BRS/anti-BRS symmetry in the form of the statements 4.28 and 4.29.

We begin with the generating functional

$$W[\bar{X}(\bar{x})] = \int D\bar{A}(\bar{x}) \exp\left\{ i \int d^4x \left[ \mathcal{L}_0[\bar{A}] + \mathcal{L}_1[\bar{A}] + \mathcal{L}_2[\bar{A}] + \mathcal{L}_s \right] \right\} \quad (4.1)$$

We perform an $OSp(3,1|2)$ rotation on the sources $\bar{X}^i$

$$\bar{X}^i(\bar{x}) \to \bar{X}''^i(\bar{x}) \quad (4.2)$$

with

$$\bar{X}''^i(\bar{x}) = \bar{X}^j(\Lambda^{-1}\bar{x})A^i_j \quad (4.3)$$

Under this transformation, we have the invariance

$$X^i(\bar{x})A_i(\bar{x}) = X''^i(\Lambda \bar{x})A_i'(\Lambda \bar{x})$$

$$= X''^i(\Lambda \bar{x})\tilde{\Lambda}^i_\mu A_j(\bar{x}) \quad (4.4)$$

[Here $\tilde{\Lambda}$ is defined in 2.9, in particular for $S_{4\mu}$ and $S_{5\mu}$ transformations]. Then using (4.4), we have

$$\int d^4x \mathcal{L}_s = \int \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} \left[ \bar{X}^i(\bar{x})A_i(\bar{x}) \right]$$

$$= \int \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} \left[ \bar{X}''^i(\Lambda \bar{x})\tilde{\Lambda}^i_\mu A_j(\bar{x}) \right] \quad (4.5)$$

In view of the $SO(3,1) \times Sp(2)$ invariance of the entire $S$, we expect new informations to emerge from the transformations associated with additional supersymmetries $S_{4\mu}$ and $S_{5\mu}$. Hence we now restrict ourselves to the $\tilde{\Lambda}$ of Eq. (2.9) given in Sec. IIB. We note now that
\[
\Lambda(\bar{x}) = (x^\mu + \epsilon^\mu a\lambda + \delta^\mu b\theta, \lambda + \delta^\mu x_\mu, \theta - \epsilon^\mu a x_\mu) \quad (4.6)
\]

and express

\[
\int d^4x L_s = \int d^4x \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} \left[ \bar{X}^\mu(\bar{x}) \tilde{\Lambda}_j^i A_j(\bar{x}) + \left\{ (\epsilon^\mu a\lambda + \delta^\mu b\theta) \partial_\mu \bar{X}^i(\bar{x}) + \delta \cdot xb\bar{X}^i_\lambda - \epsilon \cdot x a\bar{X}^i_\theta \right\} A_i(\bar{x}) \right]
\]

(4.7)

In the last term, we have used the infinitesimal nature of \(\epsilon^\mu\) and \(\delta^\mu\) to replace \(\bar{X}' \rightarrow \bar{X}\) and \(\tilde{\Lambda} \rightarrow 1\). Further,

\[
\int d^4x L_s(\bar{X}) - \int d^4x L_s(\bar{X}') = \int d^4x \left[ (\epsilon^\mu a\lambda + \delta^\mu b\theta) \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} (\partial_\mu \bar{X}^i(\bar{x}) A_i(\bar{x})) \right] + \int d^4x \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} \left[ \bar{X}^i_{\lambda\theta} \epsilon \cdot x A_{i,\theta} - \bar{X}^i_{\lambda \theta} \delta \cdot x A_{i,\lambda} \right] + \int d^4x \frac{\partial}{\partial \theta} \frac{\partial}{\partial \lambda} \left[ \bar{X}^\mu (\epsilon_x ac_5 - \delta_x bc_4) + \bar{X}^4 (\epsilon_x a A_{\mu} + \bar{X}^5 \delta^\mu b A_{\mu}) \right]
\]

(4.8)

Using (4.8) we can write down change in \(W[\bar{X}]\) under an infinitesimal OSp transformation (4.7)

\[
\delta W[\bar{X}] = W[\bar{X}] - W[\bar{X}'] = \langle \epsilon \rangle \int d^4x (\epsilon^\mu a\lambda + \delta^\mu b\theta) \sum S(\bar{x}) \frac{\delta W}{\delta S}
\]

\[
+ i \int d^4x \left[ \bar{X}^\mu_{\lambda\theta} [\epsilon \cdot xa A_{i,\theta} + \epsilon_x ac_5 - \delta_x bc_4 - \delta \cdot xb A_{i,\lambda}] + \bar{X}^4_{\lambda\theta} [\epsilon \cdot xac_4,\theta + \epsilon_x a A_{\mu} - \delta \cdot xbc_4,\lambda] \right. \\
+ \bar{X}^5_{\lambda\theta} [\epsilon \cdot xac_5,\theta + \delta^\mu b A_{i,\theta} - \delta \cdot xbc_5,\lambda] + \bar{X}^\mu_{\lambda \theta} [-\delta_x bc_4,\theta + \epsilon_x ac_5,\theta] - \bar{X}^4_{\lambda \theta} \epsilon_x a A_{i,\theta} - \bar{X}^5_{\lambda \theta} \delta^\mu b A_{i,\theta} \\
- \bar{X}^\mu_{\lambda \theta} [\epsilon_x ac_5,\lambda - \delta_x bc_4,\lambda] + \bar{X}^4_{\lambda \theta} [\epsilon_x a A_{i,\lambda} + \bar{X}^5 \delta^\mu b A_{i,\lambda}] + \left[ \epsilon^\mu a \partial_\mu \bar{X}^i + \delta^\mu b \partial_\mu \bar{X}^5 \right] A_i
\]

\[
\epsilon^\mu a \partial_\mu \bar{X}^i A_{i,\theta} + \delta^\mu b \partial_\mu \bar{X}^i A_{i,\lambda} - \epsilon^\mu a \partial_\mu (\bar{X}^4 c_{4,\theta} + \bar{X}^5 c_{5,\theta}) - \delta^\mu b \partial_\mu (\bar{X}^4 c_{4,\lambda} + \bar{X}^5 c_{5,\lambda}) \rangle \rangle \quad (4.9)
\]

Here we have dropped terms proportional to \(A^i_{\lambda\theta}\) (as these fields can be set to zero). The double bracket, \(\langle\epsilon\rangle\rangle\) has been used to denote that the expression inside it is actually inside the path integral.

We now evaluate \(\delta W[\bar{X}]\) for the “supersymmetry transformations” \(S_{4\mu}\) only; i.e. set \(\delta = 0\). We, further, note that the sources \(-\bar{X}^\mu_{\theta}\) are to generate the Green’s functions of the composite operator involved in the anti-BRS transformations. These are not required to evaluate the basic Green’s functions of the Yang-Mills theory, nor the BRS WT identities.
Hence, we evaluate (4.9) at $\bar{X}'_{\bar{\theta}} = 0$. [The spurious terms involving $\partial_{\mu}\bar{X}^i$ can also be set to zero as $\bar{X}^i$ are sources for $A_{\lambda\theta}$ which are redundant field].

Now, the first term on the right hand side of (4.9) (here $\sum_{S}$ goes over the sources $\bar{X}^i; \bar{X}_{\lambda}^i; \bar{X}_{\theta}^i; \bar{X}_{\lambda\theta}^i$) vanishes by the translational invariance of $W[\bar{X}]$ in $x^\mu$.

We organize the rest of the terms in $\delta W$ as

$$\delta W[\bar{X}]|_{\bar{X}'_{\bar{\theta}}=0}=\bar{X}^i(x)=<<i\int d^4x \left\{ \bar{X}_{\lambda\theta}^\mu (\epsilon \cdot x a_{\mu,\theta} + \epsilon_{\mu} c_5) + \bar{X}_{\lambda\theta}^4 (\epsilon \cdot x a_{4,\theta} + \epsilon_{\mu} a_{\mu}) + \bar{X}_{\lambda\theta}^5 (\epsilon \cdot x a_{5,\theta}) + \bar{X}_{\lambda\theta}^\mu \epsilon_{\mu} a_{\mu,\theta} - \bar{X}_{\lambda\theta}^4 \epsilon_{\mu} a_{\mu,\theta} \right\} >> \tag{4.10}$$

We shall simplify the expression on the right hand side employing the 6-D gauge invariance of $L_0 \tag{12}$. We consider the gauge transformations

$$\delta A_i = D_i(c_5 \epsilon \cdot x a) \tag{4.11}$$

and the consequent transformations

$$\delta A_{i,\theta} = \frac{\partial}{\partial \theta} [D_i(c_5 \epsilon \cdot x a)]; \quad \delta A_{i,\lambda} = \frac{\partial}{\partial \lambda} [D_i(c_5 \epsilon \cdot x a)] \tag{4.12}$$

Under (4.11) and (4.12) $L_0$ is gauge invariant

$$\int d^4x \left[ \delta A_i \frac{\delta S_0}{\delta A_i} + \delta A_{i,\theta} \frac{\delta S_0}{\delta A_{i,\theta}} + \delta A_{i,\lambda} \frac{\delta S_0}{\delta A_{i,\lambda}} \right] = 0 \tag{4.13}$$

We now invoke the equations of motion

$$\frac{\delta S_0}{\delta A_{\mu}^i} = -\bar{X}_{\lambda\theta}^\mu + (\alpha - \beta) \partial_{\mu}(c_{4,\theta} - c_{5,\lambda})$$

$$\frac{\delta S_0}{\delta c_4^i} = \bar{X}_{\lambda\theta}^4 + (\alpha - \beta) \partial^{\mu}(A_{\mu,\theta}) + \kappa \partial^2 c_5$$

$$\frac{\delta S_0}{\delta c_5^i} = \bar{X}_{\lambda\theta}^5 - (\alpha - \beta) \partial^{\mu} A_{\mu,\lambda} - \kappa \partial^2 c_4$$

$$\frac{\delta S_0}{\delta A_{\mu,\lambda}} = \bar{X}_{\theta}^\mu - (\alpha - \beta) \partial_{\mu} c_5$$

$$\frac{\delta S_0}{\delta c_{4,\lambda}} = \bar{X}_{\theta}^4 - 2 \beta c_{5,\theta}$$

$$\frac{\delta S_0}{\delta c_{5,\lambda}} = \bar{X}_{\theta}^5 + (\alpha - \beta) \partial \cdot A - (\alpha - \beta) c_{5,\lambda} + 2 \alpha c_{4,\theta}$$

$$\frac{\delta S_0}{\delta A_{\mu,\theta}} = -\bar{X}_{\lambda}^\mu + (\alpha - \beta) \partial_{\mu} c_4$$
\[
\frac{\delta S_0}{\delta c_{4,\theta}} = -\bar{X}^4_{,\lambda} + (\alpha - \beta)\partial \cdot A - (\alpha - \beta)c_{4,\theta} + 2\alpha c_{5,\lambda}
\]
\[
\frac{\delta S_0}{\delta c_{5,\theta}} = -\bar{X}^5_{,\lambda} + 2\beta c_{4,\lambda}
\] (4.14)

[It is understood that these Eqs are in double brackets.]

Using (4.14) in (4.13), we obtain
\[
<< -i \int d^4x \left[ D_{\mu}(\epsilon \cdot xac_5)\bar{X}^\mu_{,\lambda\theta} - D_4(\epsilon \cdot xac_5)\bar{X}^4_{,\lambda\theta} - D_5(\epsilon \cdot xac_5)\bar{X}^5_{,\lambda\theta} + \frac{\partial}{\partial \theta}[D_{\mu}(\epsilon \cdot xac_5)]\bar{X}^\mu_{,\lambda\theta}
\right.
\]
\[
\left. \frac{\partial}{\partial \bar{\theta}}[D_{\mu}(\epsilon \cdot xac_5)]\bar{X}^4_{,\lambda\theta} + \frac{\partial}{\partial \bar{\theta}}[D_5(\epsilon \cdot xac_5)]\bar{X}^5_{,\lambda\theta} \right]
\]
\[
= O(\alpha, \bar{X}^i_{,\mu})
\] (4.15)

We now subtract (4.15) from (4.10) to obtain
\[
\delta W[X]|_{X_{a_0} = X_{i_0}} = << i \int d^4x e_i \cdot xa \left\{ (A_{\mu,\theta} + D_{\mu}c_5)\bar{X}^\mu_{,\lambda\theta} - (c_{4,\theta} - D_4c_5)\bar{X}^4_{,\lambda\theta} - (c_{5,\theta} - D_5c_5)\bar{X}^5_{,\lambda\theta}
\right.
\]
\[
\left. -\bar{X}^4_{,\lambda\theta}\epsilon^\mu_a A_{\mu} - \frac{\partial}{\partial \bar{\theta}}(D_{\mu}c_5)\bar{X}^\mu_{,\lambda} + \frac{\partial}{\partial \bar{\theta}}(D_4c_5)\bar{X}^4_{,\lambda} + \frac{\partial}{\partial \bar{\theta}}(D_5c_5)\bar{X}^5_{,\lambda} - \bar{X}^4_{,\lambda\theta}\epsilon^\mu_a A_{\mu,\theta} \right\} >>
\] (4.16)

Now we recall the equation of motion
\[
<< A_{\mu,\theta} + D_{\mu}c_5 + (\alpha - \beta)\partial_{\mu}c_5 - \bar{X}^\mu_{,\theta} >> = 0
\] (4.17)
\[
<< c_{5,\theta} - D_5c_5 + 2\beta c_{5,\theta} - \bar{X}^4_{,\theta} >> = 0
\] (4.18)

and
\[
\frac{\partial}{\partial \bar{\theta}}(D_5c_5) = -2f^{\alpha\beta\gamma}c_{5,\theta}^\beta c_{5}^\gamma = 0
\] (4.19)

Which can be obtained by using (4.18) at \(X_{i_0} = 0\). We further have the equation of motion of \(c_{4,\theta}\) and \(c_{5,\lambda}\).
\[
<< c_{4,\theta} + c_{5,\lambda} + gc_{4,\theta}c_5 + \beta(c_{4,\theta} + c_{5,\lambda}) - \frac{\bar{X}^4_{,\lambda} - \bar{X}^5_{,\theta}}{2} >> = 0
\] (4.20)
\[
<< (2\alpha - \beta)(c_{4,\theta} - c_{5,\lambda}) + (\alpha - \beta)\partial \cdot A - \frac{\bar{X}^5_{,\theta} - \bar{X}^4_{,\lambda}}{2} >> = 0
\] (4.21)

Subtracting (4.21) from (4.20) and setting \(\bar{X}^5_{,\theta} = 0\) we obtain
\[ << c_{4,\theta} - D_4 c_5 >> \mid \bar{X}^\nu_{\beta} = 0 \]
\[ = -\beta << c_{4,\theta} + c_{5,\lambda} >> + (2\alpha - \beta) << c_{4,\theta} - c_{5,\lambda} >> + (\alpha - \beta) << \partial \cdot A >> = O(\alpha) \quad (4.22) \]

Further, using (4.17) and (4.18), we obtain (at \( \bar{X}^i_{\bar{\theta}} = 0 \))
\[ \frac{\partial}{\partial \theta} (D_\mu c_5) = O(\alpha) \quad (4.23) \]

[ We recognize in (4.19) and in (4.23) the usual BRS invariance statement of \( \frac{1}{2} \) fcc and \( D_\mu c \). ]

We further recall the equation of motion of \( c_4 \)
\[ << -D_\mu^{2\alpha} (A_{\mu,\theta} + D_\mu c_5)^{\alpha} + f^{\alpha\beta\gamma} c_4 (2c_5^\alpha + g f^{\alpha\beta\delta} c_5^\eta c_5^\delta) - g f^{\alpha\beta\gamma} c_5^\alpha (c_{4,\theta} + c_{5,\lambda} + g f^{\alpha\beta\delta} c_4^\eta c_5^\delta) \]
\[ -\bar{X}^4_{\lambda\theta} - (\alpha - \beta) \partial_{\mu} A_{\mu,\theta} + \kappa \partial^2 c_5 >> = 0 \quad (4.24) \]

On account of (4.17), (4.18) and (4.22) used successively in the left hand side of (4.24) these terms vanish at \( \bar{X}^i_{\bar{\theta}} = 0 \). We thus conclude,
\[ << \bar{X}^4_{\lambda\theta} \epsilon^\mu a A_{\mu} >> = << O(\alpha) >> \quad (4.25) \]

Using (4.17), (4.22), (4.18), (4.25), (4.23), (4.21), (4.19) and (4.21) in the successive terms on the right hand side of (4.16) we obtain,
\[ \delta W[\bar{X}]|_{\bar{X}^i_{\bar{\theta}} = 0 = \bar{X}^i} = << O\left(\frac{1}{\eta}\right) >> \quad (4.26) \]

We could have alternatively considered the symmetry associated with \( S_\eta \mu \) transformations (\( \delta \neq 0, \epsilon = 0 \)) in (4.9). In view of the overall \( Sp(2) \) symmetry of the formulation, we will obtain the analogous relation
\[ \delta W[\bar{X}]|_{\bar{X}^i_{\lambda} = 0 = \bar{X}^i} = O\left(\frac{1}{\eta}\right) \quad (4.27) \]

The relations (4.26) and (4.27) are statements of formal \( OSp(3,1|2) \) symmetry as \( \eta \rightarrow \infty \). These contain in them the consequences of BRS and anti-BRS invariance. These consequences can be obtained in a manner analogous to the argument following Eq. (19) of the
Ref. [10] (See also [12] for alternative procedure for the entire derivation). They result in equations

\[ \frac{\partial W}{\partial \theta} \bigg|_{X_\mu^i=0=X_i} = O \left( \frac{1}{\eta} \right) \]  

(4.28)

\[ \frac{\partial W}{\partial \lambda} \bigg|_{X_\mu^i=0=X_i} = O \left( \frac{1}{\eta} \right) \]  

(4.29)

for BRS and anti-BRS symmetry respectively.

These equations can also be alternatively verified evaluating \( W[X] \) along the lines of Ref [5] and evaluating \( \frac{\partial W}{\partial \theta} \) and \( \frac{\partial W}{\partial \lambda} \) along the line of Ref [9,13] using BRS /anti-BRS symmetry of the resultant \( W \).

V. PHYSICAL MEANING OF OSP(3,1|2) INVARIANCE

We expand the multiplet \( \bar{A}_i(\bar{x}) \) explicitly as

\[ \bar{A}_i(\bar{x}) = \begin{pmatrix} A_\mu(\bar{x}) \\ c_4(\bar{x}) \\ c_5(\bar{x}) \end{pmatrix} \equiv \begin{pmatrix} \frac{1}{\sqrt{\eta}} A_\mu^T(\bar{x}) + \sqrt{\frac{2}{3}} A_\mu^L(\bar{x}) + \sqrt{\frac{2}{3}} A_\mu^T(\bar{x}) \\ \sqrt{\frac{2}{3}} \bar{c}(\bar{x}) \\ \sqrt{\frac{2}{3}} c(\bar{x}) \end{pmatrix} \]

\[ = \sqrt{\frac{\eta}{3}} \begin{pmatrix} A_\mu^L(\bar{x}) + A_\mu^T(\bar{x}) \\ \bar{c}(\bar{x}) \\ c(\bar{x}) \end{pmatrix} + \begin{pmatrix} A_\mu^T(\bar{x}) \\ 0 \end{pmatrix} \]

\[ \equiv \sqrt{\frac{\eta}{3}} \left[ \bar{A}_\mu^R(\bar{x}) + \sqrt{\frac{3}{\eta}} A_\mu^T(\bar{x}) \right] \]  

(5.1)

Here \( \bar{A}_\mu^R(\bar{x}) \), in particular, contains the fields corresponding to the set \( R \).

We further expand the transformation laws for fields under \( OSP(3,1|2) \) viz.

\[ A_\mu'(\bar{x}) = A_\mu(\bar{x}) - \delta_\mu bc_4(\bar{x}) + \epsilon_\mu ac_5(\bar{x}) - (\epsilon a \lambda + \delta b \theta) ^\nu \partial_\nu A_\mu(\bar{x}) \]

\[ - \delta \cdot xb A_{\mu,\lambda}(\bar{x}) + \epsilon \cdot xa A_{\mu,\theta}(\bar{x}) \]

\[ c_4'(\bar{x}) = c_4(\bar{x}) + \epsilon^\nu a A_\mu(\bar{x}) - (\epsilon a \lambda + \delta b \theta) ^\nu \partial_\nu c_4 \]

\[ - \delta \cdot xb c_{4,\lambda}(\bar{x}) + \epsilon \cdot xc_{4,\theta}(\bar{x}) \]
\[ c_5'(\bar{x}) = c_5(\bar{x}) + \delta^\mu b A_\mu(\bar{x}) - (\epsilon a \lambda + \delta b \theta)^\nu \partial_\nu c_5 \\
- \delta \cdot x b c_{5,\lambda}(\bar{x}) + \epsilon \cdot x a c_{5,\theta}(\bar{x}) \tag{5.2} \]

in powers of \( \eta \). We find that these read

\[
A_{\mu}^{R}(\bar{x}) = A_{\mu}^{R}(\bar{x}) + P_{\mu\nu}^{R}[-\delta_\mu b \bar{c}(\bar{x}) + \epsilon_\mu a c(\bar{x}) - (\epsilon a \lambda + \delta b \theta)^\nu \partial_\nu A_{\mu}^{R}(\bar{x}) \\
- \delta \cdot x b A_{\mu,\lambda}^{R}(\bar{x}) + \epsilon \cdot x a A_{\mu,\theta}^{R}(\bar{x})] + 0(\frac{1}{\sqrt{\eta}}) \\
\bar{c}'(\bar{x}) = \bar{c}(\bar{x}) + \epsilon a A_{\mu}^{R}(\bar{x}) - (\epsilon a \lambda + \delta b \theta)^\nu \partial_\nu \bar{c}(\bar{x}) \\
- \delta \cdot x b \bar{c}_{,\lambda}^{R}(\bar{x}) + \epsilon \cdot x a \bar{c}_{,\theta}(\bar{x}) + 0(\frac{1}{\sqrt{\eta}}) \\
c'(\bar{x}) = c(\bar{x}) + \delta^\mu b A_{\mu}^{R}(\bar{x}) - (\epsilon a \lambda + \delta b \theta)^\nu \partial_\nu c(\bar{x}) \\
- \delta \cdot x b c_{,\lambda}(\bar{x}) + \epsilon \cdot x a c_{,\theta}(\bar{x}) + 0(\frac{1}{\sqrt{\eta}}) \tag{5.3} \]

and

\[ A_{\mu}^{T}(\bar{x}) = A_{\mu}^{T}(\bar{x}) + 0(\sqrt{\eta}) \tag{5.4} \]

\([P_{\mu\nu}^{R} \text{ is the projection operator that projects away the transverse part}]. \) We note that as \( \eta \to \infty \), (5.3) refers to the transformations within the set \( A^R \) only.

Thus, in the limit \( \eta \to \infty \), the \( OSp(3,1|2) \) transformations, in particular, contain a set of symmetry transformations among the members of the redundant set \( R \). The WT identities are a particular consequence of these symmetries. A special consequence of the WT identities is the cancellation of the contributions from the set \( R \) in the intermediate states in the unitarity relations using the Cutkowsky rules [3].

In the present superspace formulation, we have an explicit construction of a set of symmetry transformations amongst this set \( R \); originating from the original \( OSp(3,1|2) \) transformation which, as we have shown, lead to WT identities in particular. Thus, this formulation can be looked upon as an explicit realization of that relationship that is expected to exist with in the fields of the set \( R \) that is ultimately known to lead to mutual cancellations in the cutting equations.

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