I. INTRODUCTION

With the development of quantum thermodynamics, there is an increasing interest in the realization of thermal devices operating at the quantum limit. In particular, heat engines whose working substance consists of systems with finite levels of energy, such as two-level systems, can absorb from or deliver to their surroundings quantities of energy as small as their corresponding energy gaps. In contrast, bosonic working substances, modeled by quantum harmonic oscillators, have infinitely many levels, where higher and higher levels can be populated by increasing temperature. Comparative studies exploring the difference between a bosonic and a fermionic working substance demonstrate that there are advantages in considering working substances with finite levels for quantum engines. In particular, systems with finite energy levels, such as two-level systems, can exhibit stationary states with population inversion. As the operating conditions are kept the same for both environments, our results emphasize that the presented advantages stems from the quantum nature of the fermionic reservoir.

II. MODEL

In the present work, we consider a self-contained quantum refrigerator (SCQR) composed of three interacting qubits, each in contact with a specific thermal reservoir. This SCQR was first proposed in Ref. [25], in which the authors took into account only bosonic reservoirs. Recently, we investigated this SCQR operating with one of the reservoirs being a fermionic one at a negative temperature, see Ref. [30]. Here, as in Ref. [25], we approach the case in which qubits 1, 2, and 3 interact respectively with a thermal reservoir at a cold temperature \( T_c > 0 \), a thermal reservoir at a "room" temperature \( T_r > 0 \), and a thermal reservoir at a hot temperature \( T_h > 0 \) - see the schematic shown in Fig. 1. The device in question works like a refrigerator when \( T_1 - T_c < 0 \), where \( T_1 \) is the temperature of qubit 1. In this case, therefore, heat flows from the cold reservoir to qubit 1. However, considering the asymptotic state, this only occurs if the relations \( E_3 = E_2 - E_1 \), with \( E_k \) being the energy gap of qubit \( k \) \((k = 1, 2, 3)\), and \( T_c < T_r < T_h \) are satisfied [25].

We assume the weak coupling limit and the Markovian regime governing the dynamics of the SCQR, such that...
the master equation is \[26, 31\]

\[
\frac{d\rho}{dt} = -i [H_0 + H_{\text{int}}] + \sum_{k=1}^{3} \Gamma_{B,(F),k}^{-} \left( \sigma_{-,k} \rho \sigma_{+,k} - \frac{1}{2} \{ \sigma_{+,k} \sigma_{-,k}, \rho \} \right) + \sum_{k=1}^{3} \Gamma_{B,(F),k}^{+} \left( \sigma_{+,k} \rho \sigma_{-,k} - \frac{1}{2} \{ \sigma_{-,k} \sigma_{+,k}, \rho \} \right). \tag{1}
\]

Here, the free qubits Hamiltonian $H_0$ and the three-body interaction Hamiltonian $H_{\text{int}}$ are given by

\[
H_0 = \frac{1}{2} E_1 \sigma_{z,1} + \frac{1}{2} E_3 \sigma_{z,3} + \frac{1}{2} E_3 \sigma_{z,3} \tag{2}
\]

and

\[
H_{\text{int}} = g (\sigma_{-,1} \sigma_{+,2} \sigma_{-,3} + \sigma_{+,1} \sigma_{-,2} \sigma_{+,3}), \tag{3}
\]

where $\sigma_{z,k}$ is the $z$ Pauli operator for qubit $k$, $g$ is the coupling constant, and $\sigma_{-,k}$ ($\sigma_{+,k}$) is the lowering (raising) Pauli operator for qubit $k$. Note that Eq. (1) governs the dynamics of either bosonic [31] and fermionic [23, 24, 26] thermal reservoirs: if qubit $k$ is interacting with a bosonic (fermionic) thermal reservoir, $\Gamma_{B,k}^{-} = \gamma_k (1 + n_{B,k})$ ($\Gamma_{F,k}^{-} = \gamma_k (1 - n_{F,k})$) and $\Gamma_{B,k}^{+} = \gamma_k n_{B,k}$ ($\Gamma_{F,k}^{+} = \gamma_k n_{F,k}$), where $\gamma_k$ is the dissipation rate and $n_{B,k} = 1/(e^{E_k/T_k} - 1)$ ($n_{F,k} = 1/(e^{E_k/T_k} + 1)$) is the average excitation number, being $\phi_1 = c, \phi_2 = r$, and $\phi_3 = h$. Note that since for bosons $\Gamma_{B,k}^{-} = \gamma_k (1 + n_{B,k})$, $\Gamma_{B,k}^{+} = \gamma_k n_{B,k}$ and for fermions the average excitation number is limited to 0.5 for positive temperatures, then $\Gamma_{F,k}^{-}$ and $\Gamma_{F,k}^{+}$ is always greater than $\Gamma_{F,k}^{-}$ and $\Gamma_{F,k}^{+}$. To obtain the asymptotic state of Eq. (1) we used the quantum optics toolbox [32, 33].

III. RESULTS

To compare the SCQR operating in the different configurations involving bosonic and fermionic reservoirs, we start by fixing the energies $E_1 = 1$, $E_2 = 5$, and $E_3 = 4$; the temperatures $T_c = 1, 1.5, 2$, and $T_r = 2$; the coupling constant $g = 10^{-2}$; and the dissipation rates $\gamma_1 = \gamma_2 = \gamma_3 = g$. Next, we let $T_h$ vary from $10^{-1}$ to $10^{3}$.

Fig. 2(a) shows the temperature difference $T_1 - T_c$ versus $T_h$ (on logarithmic scale) for the SCQR working in a bosonic environment for the cold temperatures $T_c = 1$ (dotted green line), $T_c = 1.5$ (dashed red line), and $T_c = 2$ (solid blue line). As said before, cooling occurs when $T_1 - T_c < 0$. Similarly, Fig. 2(b) shows $T_1 - T_c$ as a function of $T_h$ for the same cold temperatures, but now the SCQR is surrounded by fermionic reservoirs. In Fig. 2(a), $T_1 - T_c$ decreases to a minimum value and then increases until it stabilizes at a negative value close to zero, while, in Fig. 2(b), $T_1 - T_c$ stabilizes at its minimum value. Thus, the SCQR in the fermionic environment has an advantage over the bosonic one, as its efficiency in cooling qubit 1 does not decrease at higher values of $T_h$. Furthermore, under fermionic reservoirs, qubit 1 reaches lower minimum temperature values than when under bosonic reservoirs, as can be seen from the difference $T_1 - T_c$, which is more negative for the fermionic environment (compare Figs. 2(a) and 2(b)).

According to our numerical simulations, when considering the bosonic environment, the minimum values for $T_1$ are $T_1 = 0.95$ (when $T_c = 1$), $T_1 = 1.41$ (when $T_c = 1.5$), and $T_1 = 1.87$ (when $T_c = 2$). On the other hand, when considering three fermionic reservoirs, since the values for $T_1$ continue to decrease with increasing $T_h$, we take the minimum value for $T_1$ when $T_h = 100$. These minimum values are $T_1 = 0.82$ (when $T_c = 1$), $T_1 = 1.09$ (when $T_c = 1.5$), and $T_1 = 1.29$ (when $T_c = 2$). By considering $T_c$ as a reference we can then calculate the cooling percentage $(|T_1 - T_c|/T_c) \times 100$ to compare how much the fermionic and bosonic reservoirs cools qubit 1 - see Tab. I, where 3B (3F) stands for three bosonic (fermionic) reservoirs.

Tab. I shows a significant difference in the cooling percentage for the two sets of reservoirs: it is always higher when using three fermionic reservoirs, thus clearly showing that the fermionic environment is far more efficient in decreasing the temperature $T_1$ than the bosonic one. Also, this percentage is better the higher the reference
temperature $T_c$, such that for $T_c = 2$, it can reach up to more than four times the value reached using only bosonic reservoirs. For lower values of the cold temperatures $T_c$, the percentage difference decreases but using fermionic reservoirs, the cooling when $T_c = 1$ is still more than double that of the case of bosonic reservoirs alone. It is worth mentioning that by fixing the SCQR parameters as we did, there is a limit to cooling qubit 1. As we found numerically, the corresponding lowest cooling percentage reached by qubit 1, irrespective of the type of reservoir used, occurs when $T_c \sim 0.48$. For temperatures lower than $T_c \sim 0.48$, $T_1 - T_c > 0$, meaning that the SCQR no longer works. Also, the percentage of cooling decreases more and more as $T_c$ approaches 0.48 for both reservoirs. However, the percentage of cooling when using fermionic reservoirs remains higher, as shown in Tab. II.

As we have seen, for fixed parameters we cannot cool down qubit 1 to zero absolute. However, there is a strategy to keep up cooling toward zero absolute, which is to isolate qubit 1 from its environment. This condition, obtained by imposing $\gamma_1 \to 0$ or equivalently $\Gamma_{B(F),1}^\dagger \to 0$ and $\Gamma_{B(F),1}^\dagger \to 0$ in (1), allows us to obtain the following analytical solution for the temperature of qubit 1:

$$T_1 = \frac{T_c}{1 + \frac{E_2}{E_1} \left(1 - \frac{T_c}{T_h}\right)}, \quad (4)$$

from which we can see that, if we let $E_2/E_1 \to \infty$, then $T_1 \to 0$. This result, obtained in Ref. [25], shows that there is no fundamental limit to cool down to zero absolute, provided we can perfectly isolate qubit 1.

So far we have considered fermionic reservoirs for all qubits in the SCQR. Other possibilities include the cases of combinations of bosonic and fermionic reservoirs. In fact, considering the fermionic reservoir as a quantum resource, it may be interesting to consider cases where only one or two fermionic reservoirs are used. For this, it is necessary to consider which qubit the fermionic reservoir is associated with. Let us use a notation in which B (F) denotes the bosonic (fermionic) reservoir and the order in which it appears in the sequence indicates which qubit the fermionic reservoir is attached to. For example, the sequence BFB indicates that qubit 1 is subjected to a bosonic reservoir, qubit 2 to a fermionic reservoir, and the third qubit to a bosonic reservoir. Next, we investigate all configurations numerically and grouped the results in Tab. III, ordering from highest to lowest percentage of cooling and following the same procedure as in the previous tables, i.e., we took the minimum value for $T_1$.

Interestingly, and contrary to what one might think, the best case does not occur when three fermionic reservoirs are used. As Tab. III shows, the greatest cooling range occurs for FFB case, i.e., when only qubit 1 and 3 are bound to fermionic reservoirs. Although the difference between the FFB and FFF configurations is small, it is still notable that the cooling percentage is higher when only two fermionic reservoirs are used instead of three.
Table III. Comparative values showing the cooling percentages \((T_h - T_c)/T_c\) × 100 for several reservoir configurations. Here, for example, BFF means qubit 1 bound to a bosonic reservoir and qubits 2 and 3 bound to fermionic reservoirs. Note that the highest percentage of cooling occurs for FFB configuration, meaning that qubit 1 is bound to a fermionic reservoir, qubit 2 is bound to a bosonic reservoir and qubit 3 is bound to another fermionic reservoir.

Table IV. Exchange rates \(\Gamma_{B(F),k}\) and \(\Gamma_{F(B),k}\) of the qubit \(k\) with their respective reservoirs. From this Table, we see that the lowest (highest) exchange rates occur for fermionic (bosonic) reservoirs. Note that the FBF configuration presents the smallest exchange rates for qubit 1. This explains why it is more effective for cooling qubit 1 - see main text. The temperatures used are \(T_c = 2\), \(T_e = 2\), and \(T_h\) varies to minimize the values of \(T_h\), as the SCQR behavior changes as shown in Figs. 2(a) and 2(b). The same pattern occurs if we use other values for \(T_c\).

In this regard, note that the FFB and BFF sequences, although each also contain only two fermionic reservoirs and one bosonic reservoir, they have a lower cooling percentage than that of the FBF sequence. For instance, for \(T_c = 2\), the cooling percentage of FFB is 29.72\%, which is higher than that for sequence FFB (25.09\%) and BFF (10.33\%). The explanation for this fact is given below. Remembering that the lowest temperature for qubit 1, which is the qubit we want to cool, occurs when it is completely isolated, it is to be expected, therefore, that when the exchange rates \(\Gamma_{F(B),1}\) and \(\Gamma_{F(B),1}\) of qubit 1 with its reservoir are the lowest possible, the cooling will take place more effectively, with perfect insulation being the best case. In Tab. IV we show the exchange rates \(\Gamma_{F(B),k}\) and \(\Gamma_{F(B),k}\), \(k = 1, 2, 3\), for the \(k\)-th qubit for temperatures \(T_c = 2\), \(T_e = 2\), and \(T_h = 10\). From Tab. IV we see that the \(\Gamma_{F(B),1}\) and \(\Gamma_{F(B),1}\) rates are the smallest whenever the sequence starts with F, and remain the smallest irrespective of the temperatures used, according to our simulations. Another relevant point to be considered, which we have already shown in Figs. 2(a) is that bosonic reservoirs have the disadvantage of making the cooling non-monotonic, and therefore less effective. Thus, obtaining better cooling percentages requires that the last reservoir be fermionic, as we also verified in our numerical simulations. The role of the nature of the second reservoir and its relevance to cooling percentages is quite complex. In our numerical simulations, we were able to identify that the best cooling percentages occur, whenever one bosonic reservoir and two fermionic reservoirs are used, in the sequence FBF - see Tab. III. Regarding other configurations with lower cooling percentages but yet involving two fermionic and one bosonic reservoir, note for example that the FFB sequence may have higher or lower cooling percentages than the BFF sequence depending on whether the temperature \(T_c\) is higher or lower than unity.

### IV. CONCLUSION

Recent studies on heat machines have used quantum reservoirs as a resource to obtain better performances both in engines and in refrigerators [12, 21, 22, 34]. For example, fermionic reservoirs have been explored in previous works, especially in their purely quantum characteristic of presenting population inversion [17, 20], which, in turn, is associated with negative effective temperatures [3, 18]. Here we explore the quantum nature of fermionic reservoirs without taking population inversion into account, such that we restrict to the domain of positive temperatures. Using a qubit-based refrigerator model proposed in Ref. [25], we show that, once the operating parameters of the refrigerator are fixed, the use of fermionic reservoirs allows to obtain better results, with respect to the cooling capacity, than the use of bosonic reservoirs. We have verified, for example, that when the qubit to be cooled cannot be perfectly insulated, the use of only fermionic reservoirs allows to reach lower temperatures than the use of only bosonic reservoirs. In addition, contrary to what might be thought, the cooling can be more effective, in the sense of obtaining a higher percentage of cooling, when instead of three, only two fermionic reservoirs are used. We show that an explanation of this somewhat unexpected result is due to the exchanged rates between qubit 1 and its reservoir as well as to the behavior of the asymptotic cooling of qubit 1 when subjected to different types of reservoirs. In summary, when the condition for perfect insulation cannot be reached, our results unequivocally demonstrate the superiority of the fermionic reservoir in the process of cooling qubits to the lowest possible temperatures.

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