Current and spin correlations in
Fulde-Ferrell-Larkin-Ovchinnikov state of a s-wave superconductor

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Abstract. In an attempt to characterize the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase in a two dimensional s-wave superconductor, we investigate current-current and spin-spin correlations and compare them with those in the BCS phase. The existence of the FFLO phase is demonstrated using a weakly attractive fermionic Hubbard model in presence of a harmonic trap and a population imbalance, caused by a magnetic field. The lower and upper values of the magnetic field, between which the order parameter shows spatial modulations (owing to a finite center of mass momentum pairing), mark the extent of the FFLO phase. In this regime, the current-current correlations show spatial modulation similar to the behaviour of the order parameter, while the longitudinal spin-spin correlations show modulations with a period half of that of the order parameter in the absence of trap and a bimodal distribution in the presence of the harmonic confinement, thereby underlining a dissimilarity in behaviour between the spin and charge sectors.

1. Introduction
The study of superconductivity in presence of magnetic field commenced nearly a century ago with the works of Clogston and Chandrasekhar \([1]\). More abounding implications of the presence of an external magnetic field is elucidated by Fulde and Ferrel and by Larkin and Ovichinnikov \([2]\) where a possibility of finite momentum pairing between the different participation species of electrons is explored. Backed by weak and ambiguous evidences in various condensed matter systems, such as the heavy fermion compounds, organic superconductors and very recently pnictides, the progress made in the field of trapped ultracold atomic gases has raised the possibility of the realization of the FFLO phase \([3]\).

In this paper, we shall consider a population imbalanced atomic superfluid in presence of a harmonic trap which mimics the optical and various other trapping effects which find a mention in cold atom literature. Further we shall focus on the effect of the trap on properties of the ground state by reviewing different correlation functions that characterize the state.
2. Model and Formalism
The mean field decoupling of the interaction term in the attractive Hubbard model yields the effective Hamiltonian of the form,

\[ \mathcal{H}_{\text{eff}} = \sum_{ij,\sigma} \mathcal{H}_{ij\sigma}(c_{i\sigma}^{\dagger}c_{j\sigma} + \text{H.c.}) + \sum_i \left[ \Delta_i c_i^{\dagger}c_i - \Delta_i^* c_i^{\dagger}c_i \right] \]  \hspace{1em} (1)

here \( \Delta_i = -|U|\langle c_{i\uparrow}c_{i\downarrow} \rangle \) is the gap parameter for the fermionic superfluid. \( \mathcal{H}_{ij\sigma} = -t\delta_{\pm 1j} + (V_i - \mu - U\delta n_{i\sigma} + \sigma h)\delta_{ij} \) where \( \delta n_{i\sigma} = n_{i\sigma} - 1/2 \) with \( \langle n_{i\sigma} \rangle = \langle c_{i\sigma}^{\dagger}c_{i\sigma} \rangle \) and \( \bar{\sigma} = -\sigma \). Here \( V_i \) is assumed to be of the form \( V_i = V_0(r_i - r_0)^2 \) where \( V_0 \) is the strength of the trapping potential. All of \( U, h, \mu \) and \( V_0 \) are expressed in units of hopping strength, \( t \). Eq. (1) is hence diagonalized using Bogoliubov transformation which yields the gap parameter and density (and hence magnetization, \( m_\sigma = \langle n_{i\sigma} \rangle - \langle n_{i\bar{\sigma}} \rangle \), see Eq. (3)) in terms of the eigenvectors \( u_n(r_i) \) and \( v_n(r_i) \) at a temperature \( T \) as,

\[ \Delta_i = -|U| \sum_n |u_n(r_i)v_n^*(r_i)f(E_{n\uparrow}) - u_n(r_i)v_n^*(r_i)f(-E_{n\downarrow})| \] \hspace{1em} (2)

\[ \langle n_{i\sigma} \rangle = \sum_n \left[ |u_n(r_i)|^2 f(E_{n\sigma}) + |v_n(r_i)|^2 f(-E_{n\bar{\sigma}}) \right] \] \hspace{1em} (3)

where \( f(E_{n\sigma}) \) is the Fermi distribution function. \( \Delta_i \) and \( \langle n_{i\sigma} \rangle \) are obtained self-consistently from Eq. (2) and Eq. (3) at each lattice site.

3. Results
We first present a schematic diagram of the BCS and FFLO phases, both in the absence and presence of trap in Fig. 1. The FFLO phase is marked by a lower and a upper critical magnetic field values, \( h_{c1} \) and \( h_{c2} \) respectively and lie intermediate to the BCS and normal phases. These boundaries are obtained from the behaviour of the gap parameter obtained solving the first of Eq. (2). Fig. 1 shows that the values for \( h_{c1} \) and \( h_{c2} \) are 0.35 \( t \) and \( 0.5 \) in the absence of trap whereas it shifts to \( h_{c1} = 0.25t \) and \( h_{c2} = 0.4t \) when the trapping potential is switched on for \( |U| = 2.5t \) and \( \mu = -0.5t \).

**Figure 1.** A schematic representation of the FFLO phase is shown as a function of the magnetic field, \( h \) both in absence and presence of the trap for \( |U| = 2.5t \) and \( \mu = -0.5t \) (same for all plots). The boundaries of the FFLO phase are marked by \( h_{c1} \) and \( h_{c2} \) in the absence (solid line) and presence (broken lines) of the trap of strength, \( V_0 = 0.016t \). Same numerical value of \( V_0 \) is chosen for Figs. 2-5.

We now present our results for the gap parameter, \( \Delta_i \) both with and without trap. It may be noted that \( \Delta_i \) is homogeneous (BCS) for low values of magnetic fields (population imbalance being small) in the absence of trap, whereas the trapping effects lead to a maximum of \( \Delta_i \) at the trap center, hence decreases and finally vanishes away from the center (see Fig. 2 (a) and...
Such contrasting behavior is caused by the trapping effects of the harmonic potential that results in the accumulation of atomic densities around the center thereby leading to such an inhomogeneous distribution of $\Delta_i$ in real space. Next we compare $\Delta_i$ for the modulating (FFLO) phase. $\Delta_i$ modulates along one direction (x-axis) of the lattice without the trap while it modulates along the radial direction in the presence of trap, as shown in Figs. 2 (c) and (d). That is, far away from the trap center, the gap parameter undergoes a sign change confirming the presence of the FFLO phase.

![Figure 2](image)

**Figure 2.** (Colour online) Local pairing amplitude, $\Delta_i$ in BCS ((a) and (b)) and modulating FFLO ((c) and (d)) phases are shown. Figs. (a) and (c) are for cases without trap while (b) and (d) are in presence of trap. The system size is $32 \times 16$ in the absence of trap and $24 \times 24$ when the trapping potential is present and are same for all our results.

We proceed to discuss results on the longitudinal spin-spin correlations e.g. the local magnetization, $m_i$ (= $2\langle S^z_i \rangle$) presented in Fig. 3. While $m_i$ for the BCS phase, is zero understandably (being non-magnetic due to lack of unpaired particles) across the lattice (see Fig. 3(a) and (b)), it modulates with the period half of that of $\Delta_i$ in the FFLO phase when trapping effects are not included (Fig. 3(c)). The large values of magnetization occurs at nodal lines with broken pairs, whereas lattice sites with finite $\Delta_i$ correspond to weak magnetization, thereby leading to a phase difference between these two quantities. The magnetization profile in the presence of trap, exhibits a bimodal structure (as shown in Fig. 3(d)) since the magnetization is non-zero around the ring like nodal line. In other words, a small number of particles are squeezed into the inner core of the harmonic trap due to pair formation in the superfluid phase, while the unpaired (majority) carriers are pushed out to outside of the core. It may be worth mentioning that the transverse spin-spin correlations ($S^\pm_i$) are all the while zero.

Finally, we analyze the behavior of the paramagnetic current-current correlations $\Lambda_{ij}^{\alpha\beta} = \langle j_\alpha(r_i)j_\beta(r_j) \rangle$ in an environment of harmonic confinement where $j_\alpha$ is the component of the paramagnetic current density operator at site $r_i$ given by $j_\alpha(r_i) = it \sum_\sigma \left[ c_{i+\alpha,\sigma}^\dagger c_{i,\sigma} - c_{i,\sigma}^\dagger c_{i+\alpha,\sigma} \right]$. Here $r_i$ and $r_j$ denote lattice sites. $\Lambda_{ij}^{xx}$ (with $\alpha = \beta = x$) rapidly decays to zero in the BCS phase, while it modulates across the lattice and remains finite in the FFLO phase, irrespective of the value of the trapping potential as shown in Fig. 4 and 5. The response of the system to an external perturbation is more in the FFLO phase than that in the BCS phase and thus in turn reiterates the presence of stronger superconducting correlations in the homogeneous BCS phase. The likely reason for lower $\Lambda_{ij}^{xx}$ in the presence of trap must be because of the reduced mobility of the charge carriers due to localization effects.
4. Conclusions
In summary, we have explored the ground state of a population imbalanced Fermi gas on a two dimensional lattice in presence of a harmonic confinement. The current-current and spin-spin correlations obtained behave differently in the BCS and FFLO phases for both with and without trap. While the current-current correlations show spatial modulation similar to the behaviour of the order parameter, the longitudinal spin-spin correlations (obtained in the form of magnetization) show modulations whose period is half of that of the order parameter. We hope that experimental realization of such correlations will be possible in near future for bosonic systems, however computing them in fermionic systems is going to be difficult.

5. Acknowledgments
We thank CSIR and DST, India under the Grants - F.No:09/731(0048)/2007-EMR-I, No.03(1097)/07/EMR-II and SR/S2/CMP-23/2009.

References
[1] Clogston A M 1962 Phys. Rev. Lett. 9 266; Chandrasekhar B S 1962 Appl. Phys. Lett. 17
[2] Fulde P and Ferrell 1964 Phys. Rev. 135 A550; Larkin A I and Ovchinikov Y N 1964 Zh. Eksp. Teor. Fiz. 47 1136
[3] Sheehy D E and Radzihovsky L 2006 Phys. Rev. Lett 96 060401