Penguin amplitudes: charming contributions

M. Ciuchini\textsuperscript{a}, E. Franco\textsuperscript{b}, G. Martinelli\textsuperscript{b} and L. Silvestrini\textsuperscript{c}

\textsuperscript{a}INFN - Sezione di Roma III and Dipartimento di Fisica, Università di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy
\textsuperscript{b}INFN - Sezione di Roma and Dipartimento di Fisica, Università di Roma “La Sapienza”, P.le A. Moro 2, I-00185 Roma, Italy
\textsuperscript{c}Technische Universität München, Physik Department, D-85784 Garching, Germany

Abstract

We briefly introduce the Wick-contraction parametrization of hadronic matrix elements and discuss some applications to $B$ and $K$ physics.

In spite of the progresses in non-perturbative techniques, the computation of hadronic matrix elements is still an open problem, particularly when the final state contains more than one meson. In this case, methods based on Euclidean field theory, such as QCD sum rules or lattice QCD, have serious difficulties in computing physical amplitudes \cite{1,2}. Besides the standard parametrization of hadronic matrix elements in terms of $B$ parameters, it is useful for phenomenological studies to introduce a different parametrization based on the contractions of quark fields inside the matrix element. In the following, we briefly discuss the Wick-contraction parametrization introduced within the framework of non-leptonic $B$ decays in ref. \cite{3}.

To be concrete, let us illustrate how this parametrization works in few examples taken from $B$ physics. Consider the Cabibbo-allowed decay $B^+ \rightarrow D^0 \pi^+$. Only two operators of the $\Delta B = 1$ effective weak Hamiltonian contribute to this amplitude, namely

\begin{align}
\langle \bar{D}^0 \pi^+ | Q_1^{\Delta C=1} | B^+ \rangle &= \langle \bar{D}^0 \pi^+ | \bar{b} \gamma_\mu (1 - \gamma_5) d \bar{u} \bar{\gamma}_\mu (1 - \gamma_5) c | B^+ \rangle,
\langle \bar{D}^0 \pi^+ | Q_2^{\Delta C=1} | B^+ \rangle &= \langle \bar{D}^0 \pi^+ | \bar{b} \gamma_\mu (1 - \gamma_5) c \bar{u} \bar{\gamma}_\mu (1 - \gamma_5) d | B^+ \rangle. \end{align}

*Talk given by M. Ciuchini at KAON '99, June 21–26, 1999, University of Chicago, Chicago, IL, USA.

1
In this particularly simple example, the quark fields in the operators can be contracted only according to the emission topologies $DE$ and $CE$, shown in fig. 1. In the absence of a method for computing them, these contractions can be taken as complex parameters in phenomenological studies. The matrix elements can be rewritten as

$$
\langle \bar{D}^0 \pi^+ | Q_1^{C=1} | B^+ \rangle = CE_{LL}(d, u, c; B^+, \bar{D}^0, \pi^+) + DE_{LL}(c, u, d; B^+, \pi^+, \bar{D}^0),
$$

$$
\langle \bar{D}^0 \pi^+ | Q_2^{C=1} | B^+ \rangle = CE_{LL}(c, u, d; B^+, \bar{D}^0, \pi^+) + DE_{LL}(d, u, c; B^+, \pi^+, \bar{D}^0). \tag{2}
$$

The subscript $LL$ refers to the Dirac structure of the inserted operators. In general there are 14 different topologies \[3\] - \[5\]. Of course, in order to be predictive, one needs to introduce relations among different parameters given by dynamical assumptions based on flavour symmetries, chiral properties, heavy quark expansion, $1/N$ expansion, etc. This approach proves particularly useful for studying the $\Delta S = 1$ $B$ decays. For instance, let us consider the decay $B^+ \rightarrow K^+ \pi^0$. Its amplitude receives contributions from all the operators of the $\Delta B = 1$, $\Delta S = 1$ effective Hamiltonian. We consider only the matrix elements of operators which are both proportional to the largest Wilson coefficients $C_1$ and $C_2$ and leading order in the Cabibbo angle. They read

$$
\langle K^+ \pi^0 | Q_1^1 | B^+ \rangle = \langle K^+ \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) s \bar{c} \gamma^\mu (1 - \gamma_5) c | B^+ \rangle = DP_{LL}(c, s, u; B^+, K^+, \pi^0),
$$

$$
\langle K^+ \pi^0 | Q_2^1 | B^+ \rangle = \langle K^+ \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) s \bar{c} \gamma^\mu (1 - \gamma_5) c | B^+ \rangle \tag{3}
$$
The penguin contractions \( CP \) and \( DP \) are shown in fig. 1. We stress the difference between penguin operators, which we have neglected here, and penguin contractions, which can contribute to the matrix elements of any operator. This kind of non-perturbative contributions, called “charming penguins” in refs. [3], could dominate \( \Delta S = 1, \Delta C = 0 \) \( B \) decays because other contributions are either proportional to the small Wilson coefficients \( C_3 - C_{10} \) or are doubly Cabibbo suppressed, as in the case of the factorizable emission topologies of \( Q_{1,2}^6 \). A detailed analysis of non-leptonic \( B \) decays in this framework can be found in refs. [4, 6, 7]. The presence of “charming penguin” contributions are likely to make the naïve factorization approach fail in describing this class of decays.

A different, but related parametrization of hadronic matrix elements has been recently proposed in ref. [5]. In this approach, the parameters are the suitable combinations of Wick contractions and Wilson coefficients which are renormalization scale and scheme independent. In this way, the relations among contractions enforced by the renormalization group equations are explicit. Besides, the phenomenological determination of the parameters do not depend on the choice of the Wilson coefficients. On the other hand, imposing relations among parameters based on dynamical assumptions may be more involved.

Matrix-element parametrizations are less useful when applied to \( K \) decays, because there are few decay channels to fix the parameters and test the assumptions. \(^1\) In addition, chiral relations allow the connection between matrix elements with one pion and those with two or more pions in the final states, the former being calculable with lattice QCD. However, a reliable lattice determination of \( \langle \pi \pi | Q_6 | K \rangle \), the dominant contribution to \( \varepsilon'/\varepsilon \) [8], is presently missing [4].

Let us apply the Wick-contraction parametrization to \( K \rightarrow \pi\pi \) and verify whether there could be a connection between the longstanding problem of the \( \Delta I = 1/2 \) rule and a large value of the matrix element of \( Q_6 \), as suggested by the recent measurement of \( \varepsilon'/\varepsilon \). In terms of the Buras-Silvestrini parameters [5], the amplitudes \( K \rightarrow \pi\pi \) with definite isospin are

\[
\text{Re} A_2 \sim \frac{1}{3} (E_1 + E_2),
\]

\(^1\)Indeed, in the case of \( K \rightarrow \pi\pi \), there are only two complex amplitudes corresponding to the \( \pi\pi (I = 0, 2) \) final states.
\[ \text{Re}A_0 \sim \left( -\frac{2}{3}E_1 + \frac{1}{3}E_2 - A_2 + P_1' + P_3' \right), \]
\[ \text{Im}A_0 \sim - (P_1 + P_3), \]
where \( E_{1,2} \) are the emission parameters, \( A_2 \) is built with annihilations and

\[
\begin{align*}
P_1 & \equiv \sum_{i=2}^{5} (y_{2i-1} \langle Q_{2i-1} \rangle_{CE} + y_{2i} \langle Q_{2i} \rangle_{DE}) \\
& \quad + \sum_{i=3}^{10} (y_i \langle Q_i \rangle_{CP} + y_i \langle Q_i \rangle_{DP}) + \sum_{i=2}^{5} (y_{2i-1} \langle Q_{2i-1} \rangle_{CA} + y_{2i} \langle Q_{2i} \rangle_{DA}), \\
P_3 & \equiv \sum_{i=2}^{5} (y_{2i-1} \langle Q_{2i-1} \rangle_{DA} + y_{2i} \langle Q_{2i} \rangle_{CA}) \\
& \quad + \sum_{i=3}^{10} (y_i \langle Q_i \rangle_{CPA} + y_i \langle Q_i \rangle_{DPA}),
\end{align*}
\]  
are the penguin-like parameters. The notation \( \langle Q_i \rangle_{CE} \) refers to the connected emission with the insertion of the operator \( Q_i \), etc.

Neglecting annihilations, as suggested by the large-\( N \) counting or by CPS+chiral symmetries \[\text{[10]}, \] we are left with four parameters and three measured quantities. It is unlikely that \( \text{Re}A_0 \) is dominated by emissions, since the large ratio \( \text{Re}A_0/\text{Re}A_2 \) would require large cancellations between \( E_1 \) and \( E_2 \), see eq. (4). Therefore, in the most natural scenario, both \( \text{Re}A_0 \) and \( \text{Im}A_0 \) are dominated by penguin parameters. Notice that \( P_1 \) and \( P_1' \) are different, so that no parametric relation between \( \text{Re}A_0 \) and \( \text{Im}A_0 \) can be established. However, the following relation holds

\[ P_1' = z_1 \langle Q_1 \rangle_{DP} + z_2 \langle Q_2 \rangle_{CP} + P_1 (y \to z), \]

where \( y_i \) and \( z_i \) are the Wilson coefficients of the 3-flavour effective weak Hamiltonian. Given this relation, we can envisage a dynamical mechanism to connect the two parameters. Let us assume that \( P_1' \gg P_1 (y \to z) \) and that \( \langle Q_{1,2} \rangle_{DP} \) and \( \langle Q_{5,6} \rangle_{DP} \) share the same enhancement. \[\text{[1]}\] Arguments may be provided to assume that \[\text{[1]}\]

\[ f = \langle Q_1 \rangle_{DP} \sim 1/N_c \langle Q_2 \rangle_{CP} \sim - \langle Q_5 \rangle_{DP} \sim -1/N_c \langle Q_6 \rangle_{CP}. \]  

\(^2\)We found that these two assumptions are compatible.
By using the experimental value of $\Re A_0$ and factorizing the emission contractions, we extract $f$, from which we derive

$$B_1 = -9, \quad B_2 = 7.5, \quad B_5 = B_6 = 1.5.$$  \hspace{1cm} (8)

It is interesting that the same mechanism enhances the $B$ parameters entering $\Re A_0$ by a factor of $\sim 10$ and those of $\Im A_0$ by a factor of $\sim 2$, as required by the theoretical calculations to explain the experimental data.

Alternatively, we could assume $P_1' \sim P_1(y \to z)$ in eq. (6), namely that everything comes from the penguin operators $Q_5$ and $Q_6$. This is the old suggestion of SVZ \cite{ref13}. Using perturbative coefficients, it is possible to show that this requires $B_6 \sim 20$ in order to fit $(\Re A_0)_{\exp}$. Such a large value is excluded by the measurement of $\varepsilon'/\varepsilon$.

To summarize, a connection between the enhancement of $\Re A_0$ and a large value of $\varepsilon'/\varepsilon$ cannot be established without some assumption on the long-distance dynamics. We have presented a simple example, which assume penguin-contraction dominance, that shows the correct pattern of enhancements. In this respect, models could give some insight, but quantitative predictions may prove hard to produce. Hopefully, non-perturbative renormalization and new computing techniques will help overcoming the problems which prevent the lattice computation of $\Re A_0$ and $B_6$ \cite{ref14}.

M.C. and L.S. thank A. Buras for useful discussions and excellent steaks, beer and particularly desserts. G.M. looks forward to acknowledging the same in the future.

References

[1] B.Y. Blok and M.A. Shifman, Sov. J. Nucl. Phys. \textbf{45} (1987) 522.

[2] L. Maiani and M. Testa, Phys. Lett. \textbf{B245} (1990) 585.

[3] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. \textbf{B501} (1997) 271.

[4] M. Ciuchini \textit{et al}., Nucl. Phys. \textbf{B512} (1998) 3.

[5] A.J. Buras and L. Silvestrini, \texttt{hep-ph/9812392}.

[6] M. Ciuchini \textit{et al}., Nucl. Instrum. Meth. \textbf{A408} (1998) 28.
[7] M. Ciuchini, R. Contino, E. Franco and G. Martinelli, Eur. Phys. J. C9 (1999) 43.

[8] A.J. Buras, these proceedings.

[9] D. Pekurovsky and G. Kilcup, hep-lat/9812019.

[10] C. Bernard, T. Draper, A. Soni, H.D. Politzer and M.B. Wise, Phys. Rev. D32 (1985) 2343.

[11] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, in preparation.

[12] S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, Nucl. Phys. B514 (1998) 63; \textit{ibid.} B514 (1998) 93; T. Hambye, G.O. Kohler and P.H. Soldan, hep-ph/9902334; T. Hambye, G.O. Kohler, E.A. Paschos and P.H. Soldan, hep-ph/9906434; H.-Y. Cheng, hep-ph/9906403; A.A. Belkov, G. Bohm, A.V. Lanyov and A.A. Moshkin, hep-ph/9907333; J. Bijnens, these proceedings.

[13] A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Sov. Phys. JETP 45 (1977) 670.

[14] T. Blum, these proceedings.