Comment on "Insight into the Microscopic Structure of an AdS Black Hole from a Thermodynamical Phase Transition"

M. Kord Zangeneh,1,* A. Dehyadegari,1 and A. Sheykhi1,2,*†

1Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran
2Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran

Thermodynamic geometry analysis of interesting Letter [Phys. Rev. Lett. 115, 111302 (2015)] for charged AdS black holes, which is based on studying the Ruppeiner invariant behavior is not correct and the authors made a mistake in calculating this quantity. In the present Letter, we address the correct Ruppeiner scalar curvature and reveal the correct possible microscopic properties of 4-dimensional charged AdS black holes arise from it. Some of these properties have not been discussed in pointed out Letter.

Introduction.—In their interesting Letter [1], Shaowen Wei and Yu-Xiao Liu have introduced the number density of the black hole molecules as a measure for microscopic degrees of freedom of the black hole. Based on this, they have figured out some microscopic properties of the 4-dimensional charged AdS black hole as an example relying on the thermodynamic phase transition and thermodynamic geometry, specially the behavior of the Ricci scalar ($R$) of Ruppeiner geometry [2]. At first glance, the obtained Ricci scalar seems surprising since shows no divergence as one usually expects for black holes [3]. This motivates us to check whether the obtained Ricci scalar is correct. We observed that Ricci scalar is not correct as we guessed and therefore discussions and insights about microscopic structure of charged AdS black holes relying on this should be revised. It is well-known that the main tool for studying microscopic structure of a black hole is thermodynamic geometry specially the size and sign of the Ricci scalar of Ruppeiner geometry (Ruppeiner invariant) [4–6].

In this Letter, we address the correct Ruppeiner invariant of the 4-dimensional charged AdS black holes and disclose the correct properties of the microscopic structure of these black holes. Furthermore, we find that the positivity of black hole’s horizon temperature ($T$) implies an upper bound on number density of the black hole molecules, the fact that has not been pointed out in [1]. Finally, we depict $R - T$ diagram in order to get more insights about microscopic structure of 4-dimensional charged AdS black holes.

Thermodynamic geometry of AdS black holes.—Since Ricci scalar is a thermodynamic invariant, the Ruppeiner geometry defined in $(M, P)$ space by taking entropy $S$ as thermodynamic potential, can be rewritten in the Weihold energy form [3]

$$g_{\alpha\beta} = \frac{1}{T} \frac{\partial^2 M}{\partial X^\alpha \partial X^\beta},$$

in $(S, P)$ space in order to calculate it. Using the mass, $M = Q^2/2r_h + r_h/2 + 4\pi r_h^3 P/3$, and temperature $T = 1/4\pi r_h + 2r_h P - Q^2/4\pi r_h^3$ of 4-dimensional charged AdS black hole, where $r_h = \sqrt{S/\pi}$, one can obtain the Ruppeiner scalar curvature (Ricci scalar) as

$$R = \frac{2\pi Q^2 - S}{8PS^3 + S^2 - \pi SQ^2}.$$  \hspace{1cm} (2)

Furthermore, $R$ can be rewritten in terms of number density $n = 1/2r_h$ as

$$R = \frac{1}{3\pi} \frac{(n/n_c)^6 - 3(n/n_c)^4}{(n/n_c)^4 + 6(n/n_c)^2 + 3(P/P_c)},$$  \hspace{1cm} (3)

where we have set $Q = 1$, and $(n_c, P_c) = (1/2\sqrt{6}, 1/96\pi)$. Some interesting results can be concluded from Eq. (3). We know that the sign of $R$ determines the kind of intermolecular interaction for the thermodynamic system [7]. Positivity (negativity) of $R$ shows the dominance of repulsive (attractive) intermolecular interaction while $R = 0$ indicates there is no interaction as in the case of classical ideal gas. Moreover, the diverging behavior of $R$ may be occured either at absolute zero temperature or at critical points [3].

Let us first examine the cases where black hole molecules behave as classical ideal gas ones. This occurs in two cases where $R = 0$. The first case is when $n = \sqrt{3}n_c$ provided the denominator of (3) is not zero. The second case corresponds to very large black holes in a fixed AdS space when $n/n_c \sim 0$. It is notable to mention that in the latter case, $R$ approaches to zero while in former it is exactly zero. Now, let us turn to investigate the diverging behavior of Ruppeiner scalar curvature. The divergence of $R$ took place at absolute zero temperature $T = 0$ where the black hole is extremal. We can also gain insight about microscopic properties of charged AdS black hole in different cases by using Eq. (3). For $n/n_c < 1$, we encounter a negative $R$. This means that for large black holes, the possible molecules attract each other. In the case of $n/n_c > 1$, the sign of Ricci scalar (3) depends on the value of $P/P_c$ and can be positive, negative, zero or infinity (two latter cases have been investigated above). In this case, table I shows that we have negative scalar curvature (attractive intermolecular

*Electronic address: mkzangeneh@shirazu.ac.ir
†Electronic address: asheykhi@shirazu.ac.ir
interaction) for $1 < n/n_c < \sqrt{3}$ while $R > 0$ (repulsive intermolecular interaction) for $\sqrt{3} < n/n_c < \sqrt{n_0}$ where $n_0 = 3 + \sqrt{9 + 3P/P_c}$. Table I also shows that $n/n_c > \sqrt{n_0}$ is excluded because temperature is negative in this case.

**TABLE I:** The allowed ranges of $(n/n_c) > 1$.

| $n/n_c < \sqrt{3}$ | $\sqrt{3} < n/n_c < \sqrt{n_0}$ | $n/n_c > \sqrt{n_0}$ |
|---------------------|----------------------------------|----------------------|
| $R$                 | +                                | $n/n_c$              |
| $T$                 | $n/n_c$                          | $n/n_c$              |
| validity            | allowed                          | not allowed          |

The standard diagram which discloses the microscopic properties along coexistence curve and presents more insights into the microscopic structure of black holes is $R - T$ diagram [3, 4, 8]. We depict this diagram in Fig. 1. This figure shows that there exist a point under critical point where there is no gap in $R$ and thus small and large black holes have similar effective attractive interactions in this point exactly like critical point. Furthermore, Fig. 1 reveals that extremal small black holes behave like fermion gas near zero temperature [5] while extremal large black holes resemble ideal gas [9]. Moreover, there are attractive interactions between large black holes molecules while the type of molecular interactions change at $T = 0.76T_c$ for small black holes.

**Summary.**—In last part of interesting Letter [1], the Ricci scalar of Ruppeiner geometry has been calculated in order to study some thermodynamical and microscopic properties of 4-dimensional charged AdS black holes. The Ricci scalar obtained in [1] is not correct. In this Letter, we calculated the correct Ruppeiner scalar curvature ($R$) of 4-dimensional charged AdS black holes and revealed the correct thermodynamical and microscopic properties of these solutions. We also showed that the positivity of black hole temperature impose an upper bound on number density ($n$) of the black hole molecules which has been introduced as a measure for microscopic degrees of freedom of the black holes in [1]. Finally, we depicted $R - T$ diagram in order to get more insights about microscopic structures of 4-dimensional charged AdS black holes. This diagram shows that in addition to critical point there is another point at which the effective attractive interactions for small and large black holes are the same. Moreover, we concluded from this diagram that extremal small and large black holes behave like fermion gas near zero temperature and ideal gas respectively. We also found that the type of molecular interactions for small black holes change at $T = 0.76T_c$.

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