EqSpike: Spike-driven Equilibrium Propagation for Neuromorphic Implementations

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Abstract

Neuromorphic systems achieve high energy efficiency by computing with spikes, in a brain-inspired way. However, finding spike-based learning algorithms that can be implemented within the local constraints of neuromorphic systems, while achieving high accuracy, remains a formidable challenge. Equilibrium Propagation is a hardware-friendly counterpart of backpropagation which only involves spatially local computations and applies to recurrent neural networks with static inputs. So far, hardware-oriented studies of Equilibrium Propagation focused on rate-based networks. In this work, we develop a spiking neural network algorithm called EqSpike, compatible with neuromorphic systems, which learns by Equilibrium Propagation. Through simulations, we obtain a test recognition accuracy of 96.9% on MNIST, similar to rate-based Equilibrium Propagation, and comparing favourably to alternative learning techniques for spiking neural networks. We show that EqSpike implemented in silicon neuromorphic technology could reduce the energy consumption of inference and training by up to three orders of magnitude compared to GPUs. Finally, we also show that during learning, EqSpike weight updates exhibit a form of Spike Timing Dependent Plasticity, highlighting a possible connection with biology.

Introduction

Spike-based neuromorphic systems have, in recent years, demonstrated outstanding energy efficiency on inference tasks (1). Implementing the training of deep neural networks in such systems remains, however, a considerable challenge, as backpropagation does not apply directly to spiking networks and requires spatially non-local computations that go against the principles of neuromorphic systems. A large number of neuromorphic systems use the unsupervised and biologically-inspired Spike Timing Dependent Plasticity (STDP) learning rule because its weight updates, based on the relative timing of pre- and post-synaptic spikes, are spatially local and can be achieved with compact circuits in several technologies (2–11). Unfortunately, STDP weight updates do not minimize an overall objective function for the network, and the accuracy of STDP-trained neural networks remains below state-of-the-art algorithms based on the error backpropagation (12). Important research efforts therefore investigate how the error backpropagation algorithm can be mathematically modified to make it spatially local and appropriate for spiking neural networks (13–19). The derived learning rules consist of three factors, including two, classically, for the behaviours of pre and post neurons, plus an additional supervision factor, which makes them less compact to implement on neuromorphic chips, and possibly less energy efficient, than two-factor learning rules such as STDP (20).
In this work, we propose a different approach to training spiking neural networks with high accuracy while using a local, two-factors learning rule compatible with neuromorphic implementations and scalable to complex tasks. Instead of starting from a non-local algorithm such as backpropagation and modifying it to make it local, we start from a rate-based algorithm called Equilibrium Propagation (21) that is intrinsically local in space, and features key advantages for neuromorphic implementations (22, 23). Equilibrium Propagation theoretically applies to any physical system whose dynamics derive from an energy function. By minimizing the energy of such a system on data patterns, it can be made to relax towards states of minimal error prediction with respect to targets (21). The weight updates of Equilibrium Propagation match those of Back-Propagation-Through-Time (BPTT) in recurrent neural networks with static inputs (24), and it reaches high accuracy on image benchmarks such as CIFAR-10 (25). Equilibrium Propagation uses the same set of weights for the forward and backward pass, a feature that is not biologically plausible, but is interesting for neuromorphic computing as it decreases the number of synaptic devices to update, thus reducing the overall power consumption. Contrarily to backpropagation, Equilibrium Propagation uses the same computations in the forward and backward phases, which is another highly desirable feature for neuromorphic systems as it greatly simplifies the circuits. Equilibrium propagation is, however, originally a rate-based algorithm.

Here, we design a spiking, hardware-friendly version of Equilibrium Propagation, called EqSpike, compatible with current neuromorphic technologies achieving online learning (26–32). EqSpike is local in space and time: contrarily to backpropagation, neither error gradients nor activations need to be stored in external memories, and synapses can be directly updated through neural events. We simulate a fully connected network based on this architecture on the MNIST handwritten digits database. We obtain a test recognition accuracy of 96.9%, which compares favourably with spiking neural networks learning with backpropagation-derived methods, and on par with rate-based Equilibrium Propagation. We show that EqSpike can be implemented in silicon neuromorphic technology, and thus reduce the energy consumption of inference and training by up to three orders of magnitude compared to graphics processing units (GPUs). Finally, we also show that during learning the weight updates of EqSpike exhibit a form of STDP, yielding insights to its link to biology.

EqSpike: a hardware-friendly spiking version of Equilibrium propagation

Equilibrium Propagation is an algorithm for training convergent recurrent neural networks. Input neurons are clamped to a static input and all the other neurons, bi-directionally connected through synapses, evolve dynamically in time to reduce the energy of the network (21). The algorithm functions in two phases: a free phase and a nudging phase. In the free phase, performing inference, the network is let to reach equilibrium (Fig. 1a). Once this is done, inputs are kept clamped, and output neurons are nudged towards the desired output (Fig. 1b). During this nudging phase, the prediction error at the output layer is converted into a “force” acting upon output neurons and propagating to the rest of the system through time until a second equilibrium is reached. For training, synaptic values are updated by probing the neuron states after (21) or during (33) the nudging phase through a learning rule that has been shown theoretically and numerically to match the updates of Back-Propagation Through Time, the state-of-the-art algorithm for such recurrent neural networks (24). It has been shown recently that Equilibrium Propagation also reaches accuracy within 1% of BPTT with convolutional architectures on the CIFAR-10 dataset (25).

The original version of Equilibrium Propagation uses a rate-based formulation where dynamical neurons evolve smoothly in time. For rate-based neurons with leaky-integrate dynamics, the Equilibrium Propagation learning rule, when weights are continuously updated during the nudging phase, is (33):
\[
\frac{dW_{ij}}{dt} \sim \dot{\rho}_i \rho_i + \dot{\rho}_j \rho_j, \quad (\text{Eq.} 1)
\]

where \( W_{ij} \) is the synaptic weight connecting neurons i and j, and \( \rho_i, \rho_j \) are the rates of the two neurons. The reformulation of this rate-based learning rule to a spiking neural network is therefore the following: each time neuron i spikes, the weight should be updated by a quantity proportional to the derivative of the rate of neuron j, \( \dot{\rho}_j \) (first term in Eq. 1), and reciprocally.

**Figure 1 - EqSpike: spike-driven Equilibrium Propagation**

a) Schematic of the free phase in Equilibrium Propagation. b) Schematic of the nudging phase in Equilibrium Propagation. c) Illustration of the weight update implementation in EqSpike. d) Spiking rate of the neuron as a function of the amplitude of the input signal. e) Schematic of the rate acceleration computation.

We propose here a simple method, compatible with current electronic hardware, to implement this learning rule. It is illustrated in Fig. 1c in the form of a circuit, including spike detection elements at the output of each neuron, as well as dedicated blocks that extract the rate derivative from the spike trains of each neuron in real-time, in order to update synapses accordingly.

We use leaky-integrate-and-fire (LIF) neurons whose membrane potentials follow the standard equation (34):

\[
\frac{du_i}{dt} = -\gamma_{\text{LIF}} u_i + I_i(t), \quad (\text{Eq.} 2)
\]

where \( \gamma_{\text{LIF}} \) is the leak factor and \( I_i(t) \) the input signal of neuron i at time \( t \):

\[
I_i(t) = \sum_{j=1}^{N_{\text{neuron}}} W_{ij} \delta(t - t_j),
\]

where \( \delta \) is the Dirac function and \( t_j \) are the times at which neurons j spike.

The neuron integrates input signals until its membrane potential \( u_i \) overcomes a threshold value \( u_{\text{th}} \). At that moment, the neuron declares a spike, the threshold value \( u_{\text{th}} \) is subtracted from \( u_i \), and the neuron undergoes a refractory period of duration \( T_{\text{refract}} \). The values of the different parameters used in simulations are listed in Table 1, where time is expressed in units of \( T_{\text{refract}} \). Fig. 1d shows the
frequency of the LIF neuron as a function of the amplitude of the input signal $I$. As can be see, we have chosen the parameters in Table 1 so that the curve approximates a hard sigmoid with maximum frequency $f_{\text{max}} = 1/T_{\text{refract}}$. The transformation of input signals to a spike rate is indeed the equivalent of the activation function in artificial neural networks, a hard sigmoid in the original formulation of Equilibrium Propagation (21).

![Table 1 - Simulation parameters](image)

| Simulation timestep, $dt$ ($T_{\text{refract}}$) | $\gamma_{\text{LIF}}$ | $\gamma_{\text{L}}$ | $u_{\text{th}}$ (T_{\text{refract}}) | $\tau$ (T_{\text{refract}}) | $\eta_r$ | $N_{\text{filt}}$ | $\beta$ | $T_{\text{free}}$ (T_{\text{refract}}) | $T_{\text{nudge}}$ (T_{\text{refract}}) |
|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.0 | 0.1 | 1 | 50 | 3x10^{-6} | 10 | 0.5 | 75 | 100 |

The method that we use for extracting the rate acceleration $\dot{\rho}$ for each neuron is illustrated in Fig. 1e. A leaky-integrator with a leak factor $\gamma_{\text{L}}$ (following Eq. 2, without reset nor spikes), takes as input the spike train emitted by the neuron to which it is connected and outputs a slowly varying signal proportional to the rate of the neuron spike train: $V_{\text{L}} \sim \dot{\rho}/\gamma_{\text{L}}$ (35). To take the derivative, we delay this signal by a duration $\tau$, and subtract the actual value with the delayed value: $V_{\text{delay}} = V_{\text{L}}(t) - V_{\text{L}}(t - \tau) \approx \tau \frac{\partial V_{\text{L}}}{\partial t} \propto \tau/\gamma_{\text{L}} \dot{\rho}$. We then apply a low-pass filter for smoothing the variations. The filter is simulated using an average over $N_{\text{filt}}$ simulation steps: $\overline{x(t)} = \frac{1}{N_{\text{filt}}} \sum_{i=0}^{N_{\text{filt}}-1} x_i(t - i \, dt)$, where $dt$ is the simulation time step.

The output of the filter, $\overline{\dot{\rho}}$, is then multiplied by a learning rate $\eta_r$. This approach is hardware-compatible as LIF neurons, leaky integrators, delays and low pass filters are circuit elements that can be efficiently implemented in CMOS technology (36). The corresponding pseudo-code is given in Algorithm 1.

**Algorithm 1:** EqSpike learning procedure for one image

**Inputs:** input image, Model ($n_{\text{input}}, n_{\text{hidden}}, n_{\text{out}}$), Loss function, length Free phase $T_{\text{free}}$, length Nudging phase $T_{\text{nudge}}$, Parameters $\gamma_{\text{LIF}}, \gamma_{\text{L}}, u_{\text{th}}, \beta, \eta_r, \tau, N_{\text{filt}}, W_{ij}$

for $t < T_{\text{free}}$:

| free phase |
|---|
| for each neuron $j$ |
| Update membrane potential $u_j(\gamma_{\text{LIF}}, I_j)$ |
| if $u_j > u_{\text{th}}$ |
| Emit a spike ($t_j$) |
| Update $\rho_j(t_j, \gamma_{\text{L}})$ |

for $t \in [T_{\text{free}}, T_{\text{free}} + T_{\text{nudge}}]$:

| nudging phase |
|---|
| for each output neuron $o$ |
| Compute error $e_o$ |
| Nudge neuron: $u_o \leftarrow u_o - \beta \cdot e_o$ |
| for each neuron $k$ |
| Update $u_k(\gamma_{\text{LIF}}, I_k)$ |
if $u_k > u_{th}$:
    Emit a spike ($t_k$)
Update $\rho_k(t_k, \gamma_L)$
Compute smoothed: $\tilde{\rho}_k \left( (\rho_k(t_k), \rho_k(t_k - \tau)), ..., N_{filt} \right)$

for each synapse $w_{ij}$:
    ■ Update synapses
    if neuron j emits a spike:
        $w_{ij} \leftarrow w_{ij} + \eta_r \cdot \hat{p}_i$
    if neuron i emits a spike:
        $w_{ij} \leftarrow w_{ij} + \eta_r \cdot \hat{p}_j$

Return: Trained weights for input image: $W_{ij}$ and go to next image/next epoch.

As can be seen from the algorithm, the neuron rates, $\rho$, are computed even during the free phase. Indeed, the computation of $\rho(t)$ in the nudging phase requires knowing the values of $\rho$ at $t - \tau$, which may belong to the free phase when $t$ corresponds to the beginning of the nudging phase. Neurons, integrators and filters are initialized to zero at each new image.

**Full network simulations: recognition rate on handwritten digits database**

We now evaluate the performance of EqSpike on the MNIST handwritten digits classification task. The MNIST dataset contains 60,000 images for training and 10,000 images for test. Pixel values are normalized between 0 and 0.5 before being sent as fixed input signals $I$ to the input neurons. We consider a one-hidden layer bidirectional neural network, with 100 hidden neurons, resulting in the network topology 784-100-10. The loss function that the network attempts to minimize is the mean square error: $L(\hat{y}, y) = \frac{1}{M} \sum_{k=0}^{m} (\hat{\rho}_k - \rho_k)^2$, where the sum is performed over the output neurons. The target $\hat{\rho}_k$ corresponds to the maximum neuron frequency $f_{max}$ if the class is correct, and zero otherwise. During the nudging phase, the output neurons integrate the error derivative $(\hat{\rho}_k - \rho_k)$ multiplied by the nudging factor $\beta$ in addition to the currents that they receive from other neurons. In practice $\rho_k$ is the rate evaluated over 100 simulation time steps. The batch size is one. Parameters in Table 1 have been optimized through hyperparameter search.

The obtained train (orange) and test (blue) accuracies are shown Fig. 2 as a function of the number of training epochs, with the deviation over six runs in shadow color. Table 2 compares the results to BPTT and the version of Equilibrium Propagation closest to our implementation, called Continual Eq-Prop (33), trained with a batch size of one (see Methods for details).
Figure 2 - Recognition accuracy during training (orange) and test (blue) as a function of the number of epochs for MNIST, averaged over six runs.

|               | BPTT                          | Continual Eq-Prop             | EqSpike                      |
|---------------|-------------------------------|-------------------------------|------------------------------|
| **MNIST**     | Test: 97.11% ± 0.23%          | Test: 96.97% ± 0.12%          | Test: 96.87% ± 0.18%         |
|               | Train: 99.06% ± 0.15%         | Train: 99.8% ± 0.04%          | Train: 98.59% ± 0.03%        |
|               | (average on 6 runs)           | (average on 6 runs)           | (average on 6 runs)          |

Table 2 - Comparison between BPTT, C-EP and EqSpike on the same network architecture, with the same initialization procedure. Batch size = 1.

The test accuracy of EqSpike matches closely the accuracy of stochastic gradient descent through BPTT on the same network architecture, given the error margin. Fully-connected spiking neural networks trained on MNIST without conversion from a non-spiking neural network typically achieve recognition rates in the 96-98% range (13, 37–40). EqSpike, with its local, two-factors online learning rule therefore reaches accuracies on MNIST comparable to those of the latest models investigated for training spiking neural networks on hardware platforms. EqSpike also achieves results equivalent to the baseline given by Equilibrium Propagation. EqSpike therefore has the potential, like Equilibrium Propagation, to adapt to convolutional architectures and perform with good accuracy on more complex image benchmarks (25), with the additional advantage of being compatible with current neuromorphic technologies.

**Inference speed and energy**

As EqSpike is derived from a rate-based algorithm, it is interesting for neuromorphic applications to quantify the number of spikes needed to achieve inference, and the time needed to reach high accuracy. Operation with fewer spikes is more desirable, as it reduces both execution time and energy consumption.
**Figure 3** - Inference time: recognition accuracy on MNIST on the test dataset as a function of time multiplied by the maximum neuron frequency $f_{\text{max}}$. Orange line: recognition accuracy computed from the output neuron showing the highest rate. Blue line: recognition accuracy computed from the output neuron spiking first. Red, vertical dotted line: average time of first output spike.

Fig. 3 shows the inference results as a function of the execution time multiplied by the maximum frequency of neurons ($t \times f_{\text{max}}$). The orange line is the mean accuracy result over the whole test dataset, obtained by computing the spike rate of output neurons (as done for Fig. 2 and Table 2), through averaging in a time window $T_{\text{average}}$ of 100 simulation time steps ($T_{\text{average}} = 50/f_{\text{max}}$) and considering that the neuron with highest frequency encodes the output. This method computes the rate accurately at the expense of having to wait the time $T_{\text{average}}$ and letting output neurons spike multiple times. Spiking neural networks also offer the possibility to accelerate the computation and reduce the energy consumption by determining the output class from the first output neuron to spike. The blue line in Fig. 3 is the accuracy as a function of time, averaged over all images in the test dataset, obtained by considering that the first output neuron to spike encodes the output.

The red, vertical dotted line in Fig. 3 indicates the average time of the first spike at the output over all presented images, corresponding to $t \approx 3.5/f_{\text{max}}$. At $t \approx 10/f_{\text{max}}$, the accuracy of single-spike inference (blue curve) reaches 95.11% ± 0.78%, within 1.4% of the precise rate computation (orange curve). This result shows that even though the algorithm is originally rate-based, a single spike at the output suffices in most cases to determine the correct class with good precision, a feature which is highly attractive for energy-efficient inference on neuromorphic chips. It means that inference can be achieved in 100 μs for electronic neurons with a firing rate of 100 kHz, available in neuromorphic chips working in accelerated time compared to biology (26), and 1 μs for electronic neurons with a firing rate of 10 MHz that can be produced, for example, with emerging nanotechnologies (41). The corresponding throughputs are respectively 10k and 1M images/s, on par with current spiking neural network implementations (32, 42). As the network operations are fully parallel, these orders of magnitudes will be conserved for wider networks. Simulations of rate-based Equilibrium Propagation on deeper networks indicate that the convergence time increases by a factor of about eight for a network with four hidden layers compared to a network of one hidden layer as here (25). It should be
noted that in the current implementation we present static inputs to the network, which means that input neurons need to integrate these signals before they emit the first spikes that will then propagate to the next layers. The speed of inference could be increased in the future by presenting inputs directly encoded in spikes, for example sourced from neuromorphic vision sensors (42).

An estimation of the energy consumption of a spiking neural network on a neuromorphic silicon chip can be performed by counting the number of synaptic operations involved. Synaptic operations (SynOps) are defined as the total number of spikes transiting through synapses of the network. Frenkel et al show that a SynOp on a neuromorphic chip requires as little as 10 pJ (30). The total number of synaptic operations needed for inference depends on the targeted recognition precision and, therefore, on the duration of inference (Fig. 3). For EqSpike, the recognition rate saturates at $t \approx 10/f_{\text{max}}$. The corresponding number of SynOps is about 150,000 in average, which is a bit less but comparable to the number of SynOps needed at inference for Event-Driven Random BackPropagation (13). Considering 10 pJ/SynOps, each EqSpike inference could potentially consume 1.5 µJ. This means that testing the 10,000 images of the whole MNIST dataset could be achieved with a neuromorphic chip while consuming only 15 mJ, in other words, three orders of magnitude less than with a GPU (43).

In our current EqSpike implementation, the input layer is the one leading to most spikes and SynOps: with only 16% of illuminated pixels in average in MNIST, the input layer emits 87.5% of all spikes and 98.6% of SynOps occur between the input layer and the hidden layer. In this work, we did not focus on reducing the number of spikes with encoding, but a better encoding of the input may reduce considerably the energy consumption. Kheradpisheh et al have shown that with a temporal encoding, the total number of spikes in the network before the first output spike can be reduced to 200 with a hidden layer four times larger than our network (44). In our case, 678 spikes in total are emitted in average before the first output spike. An adaptation of EqSpike to temporal encoding is not straightforward, but this number could potentially be decreased in the future by reducing the encoding frequency of the input.

**Training speed and energy**

Training with EqSpike requires performing the free phase and then the nudging phase, during which synaptic weights are updated. A way to speed-up the training, and reduce the total number of SynOps, is to perform the nudging phase only on poorly classified examples, and skip the updates when the accuracy is satisfactory. We apply this strategy inspired from (32) using the criteria that the nudging phase is performed only when the difference between the target rate and the actual rate $(\hat{\rho}_k - \rho_k)$ at the output is above 1%. Fig. 4a shows the number of presented examples per epoch as a function of epoch number. In the last 20 epochs only approximately 15% of the training dataset still require a nudging phase.

For MNIST, we thus perform the free phase on all the dataset ($3 \times 10^6$ images for the 50 epochs), and the nudging phase on 489,000 images. Given the durations of each phase, we can estimate the training time to $T_{\text{training}} \approx 2.74 \times 10^9/f_{\text{max}}$. For electronic neurons with a firing rate of 100 kHz (26), this leads to $T_{\text{training}} \approx 45$ min, and for electronic neurons with a firing rate of 10 MHz (41) to $T_{\text{training}} \approx 30$ s. As our networks feature a fully parallel nature, these training times would be the same for much wider networks, and increased by a factor of about eight only with four hidden layers (25).
Figure 4 - Training performance a) Number of presented images per epoch versus epoch number. b) Number of spikes/neuron/image occurring during the two phases (nudge+free), as a function of the epoch. c) SynOps: number of spikes during both phases (nudge+free) as a function of the recognition accuracy.

As EqSpike is derived from a rate-based approach, it is interesting to compare the actual spiking rates of neurons in the network during training to their maximum frequency $f_{\text{max}}$. For neuromorphic applications, low overall rates are indeed desirable. Fig 4b shows the average number of spikes emitted by each neuron for an image presentation in the training dataset, as a function of the epoch. We found that for the training conditions of Fig. 2, there are in average 36 spikes/neuron/image. This means that neurons in the network spike in average with a frequency of the order of 20% of $f_{\text{max}}$, well below $f_{\text{max}}$, which is promising for neuromorphic implementations. Again this number could be reduced in the future by optimizing the encoding of input at the first layer.

Fig. 4c shows the numbers of synaptic operations needed for training as a function of recognition rate. The total number of synaptic operations after 50 epochs is of $4.23 \times 10^{12}$, which is of the same order of magnitude as Event-Driven Random Back-Propagation (13) for similar accuracy, and below training MNIST with BP based on (14). With 10pJ per SynOps (30) the training phase of EqSpike on a neuromorphic chip could consume as little as 42 J, again, 3 magnitude less than with a GPU (43).

Spike Timing Dependent Plasticity

We have shown that EqSpike transforms Equilibrium Propagation into an efficient algorithm for neuromorphic chips. We now highlight that it also brings Equilibrium Propagation closer to biological plausibility. Bengio et al have pointed out a connection between the Equilibrium Propagation learning rule of Eq. 1 and STDP (45). The STDP learning rule, illustrated in Fig. 5a, reinforces causality between the spikes of pre- and post-synaptic neurons in networks with unidirectional synapses. If the post-synaptic neuron spikes after the pre-, causality is observed, and the weight is increased. In the opposite scenario, the weight is decreased.
Let us consider a situation where the pre-synaptic neuron \( i \) spikes and the post-synaptic neuron \( j \) accelerates, as illustrated in Fig. 5b. According to the Eq-Prop learning rule, in a network with unidirectional synapses, 
\[
\frac{dW}{dt} \propto \dot{\rho}_j \rho_i = \dot{\rho}_{\text{post}} \rho_{\text{pre}}
\]
a positive weight update should be applied. Due to the acceleration of the post-neuron, there are less post-neuron spikes before the pre-neuron spike than after. Therefore, \( t_{\text{post}} - t_{\text{pre}} \) is positive in average, yielding a positive weight update through STDP.

**Figure 5 - STDP.** a) Illustration of the STDP learning rule; reproduction with data from (2). b) Illustration of the link between Eq-Prop and STDP learning rules; illustration reproduced from (45). c) STDP-like curve during EqSpike learning.

We have investigated if STDP-like weight updates did emerge during learning in our simulations. For this purpose, we monitored weight variations in synapses that connect input neurons and the hidden layer neurons. These synapses are unidirectional as input neurons are clamped to the input: their frequency does not vary. We used 100 images during the first epoch for the MNIST dataset. The curve in Fig. 5c shows the average weight updates in the first layer as a function of the time difference between post-synaptic neuron spikes and the average time of pre-synaptic neuron spikes in a window of 200 time steps before the post-synaptic neuron spike. The obtained curve, centred on zero, indeed exhibits an STDP-like shape. It has been obtained by filtering out very low frequencies below 0.05. Quiet neurons in the free phase indeed induce large weight updates at the beginning of the nudging phase, due to the sudden acceleration from zero to non-zero frequency, inducing an additional noise in the curve. It should be noted that biological STDP curves being frequency-dependent, they are also often obtained by focusing on a given range of frequencies (46). These results confirm the possible connection between STDP and Equilibrium Propagation pointed out in (45), and show that STDP-like behaviour can be obtained during learning without a direct implementation of the original, causality-based rule.

**Discussion**

Other versions of Equilibrium Propagation have been specifically designed to train spiking neural networks. O’Connor et al have designed a spiking network which, by construction, stochastically approximates the dynamics of its rate-based counter-part until reaching the same steady state with a minimal spikes communication budget between neurons (47). Their technique, successfully tested against MNIST, comes at the cost of combining predictive coding, sigma-delta modulation and adaptive step sizes at the neuron level. In comparison, our technique simply requires LIF neurons, a spike-count and a low-pass filter to implement the learning rule. Mesnard et al proposed a version of Equilibrium Propagation to train spiking networks that also makes use of low-pass filters to estimate firing rates. However, they only demonstrate their approach on a non-linear toy problem, and the implemented learning rule is not local in time (48). In contrast, we demonstrate the effectiveness of our fully event-based implementation on MNIST.
More generally, several spike-based approaches to backpropagation have been proposed. One method consists in smoothing the spikes as a function of time so that gradients can be backpropagated through time (18, 49). Another technique consists in gating the spikes propagating the error signals by surrogate derivatives, as done in SpikeGrad (50). Finally, Event-driven Random Backpropagation uses firing rates in the backward pass, as we do in this paper, and achieves similar performance on MNIST (13). In comparison with these approaches however, our implementation of EqProp does not require to compute activation derivatives. One other interesting approach is S4NN, which employs latency coding, where each neuron can spike at most once, and the output is encoded as the first neuron to spike (44). In contrast with rate-based coding, latency coding has the potential to save important energy in neuromorphic implementations. However, there is no clear indication of whether S4NN could scale to harder visual tasks, while rate-based Equilibrium Propagation was shown to train deep ConvNets on CIFAR-10 (25). One intrinsic limitation of EqSpike, however, may stem from the necessity for the system to reach a steady state before the gradient computation phase. Payeur et al recently proposed a spike-based approach where top-down error signals are encoded as spike bursts and are multiplexed with bottom-up feedforward signals so that the backward pass and the forward pass can occur simultaneously with minimal disruption (19). While they show that the rate-based counterpart of their algorithm works on CIFAR-10 and ImageNet, it comes at the cost of employing dendritic network topologies and specialized synapses. Indeed, as noted in the introduction, most backpropagation-derived local learning rules involve a third factor for supervision (20). In this regard, we believe that our EqSpike implementation of Equilibrium Propagation, with only two factors, achieves an optimal trade-off between circuitry complexity and performance.

Finally, Zoppo et al (22) and Kendall et al (23) have proposed using Equilibrium Propagation for training neuromorphic hardware. However, their implementations remain either rate-based or current-based, and are therefore not directly compatible with spiking neuromorphic chips. EqSpike, on the other hand, could be trained directly on reconfigurable neuromorphic systems with online learning such as SpiNNaker (27) and Loihi (29). It could also be sped up by building on dedicated hardware in analog or digital CMOS (10, 26, 28, 30, 32). Emerging nanotechnologies such as memristive synapses and nanoscale spiking oscillators are compelling candidates to scale up neuromorphic hardware due to their small size, their speed and their low energy consumption (51–55). These technologies are typically prone to imperfections such as the device-to-device variability, cycle-to-cycle variability or the non-linearity in the conductance-to-voltage response, which are known to considerably jeopardize learning in memristive neural networks (31, 56). Our paper implicitly assumes that the underlying memory technology at use would be linear, deterministic and identical across different synapses. Further study should be done to propose a fully end-to-end circuit to implement EqSpike and investigate its resilience to the memristive device imperfections mentioned.

Conclusion

In this work we present a new algorithm for spiking neural networks, EqSpike, compatible with neuromorphic systems, and achieving good performance on MNIST. We show that EqSpike implements the learning rule of Equilibrium Propagation locally and autonomously. The gradients are computed by the dynamics of the system and the weights are modified by a spike and the addition of only one block to the neuron. This can lead to spiking neuromorphic systems that do not need an external circuit to compute the error gradients given by backpropagation and learn autonomously, simply by presenting inputs and nudging the outputs according to errors. Our method obtained results on MNIST close to backpropagation through time and Equilibrium Propagation, two state-of-the-art algorithms. Moreover, because EqSpike is based on Equilibrium Propagation, the performance on
more complex task, like CIFAR-10, could be similar. The number of synaptic operations to obtain these results in MNIST show we can obtain theoretically three order of magnitude less energy consumption than a GPU and the same magnitude of time with high frequency neurons. The inference, after training the network, can be accelerated by waiting for the first output spike rather than compute the highest rate, with only a small loss of accuracy.

Finally, we show that the weight updates of EqSpike share similarity with STDP during learning, raising the question of a possible biological plausibility of the algorithm. In average, the modification of weight is proportional to the spike timing. This could permit to implement synapses in neuromorphic hardware by emergent nano-devices with a STDP-like behavior, to obtain a lower power consumption and higher surface density.

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Methods:

Parameters used for benchmarking EqSpike to BPTT and C-EP

Our rate-based benchmark with C-EP and BPTT use the prototypical models introduced in Ernoult et al (33). The table below describes the hyperparameters used for the simulations.

| Alg    | Topology   | Activation | T1  | T2  | beta | Learning rates   |
|--------|------------|------------|-----|-----|------|-----------------|
| MNIST  | BPTT       | 784-100-10 | hardsigm | 30  | 15   | NA              | 0.003 – 0.0015 |
| MNIST  | C-EP       | 784-100-10 | hardsigm | 30  | 15   | 0.5             | 0.003 – 0.0015 |

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