The Study of the Modeling simulation for the Rocket-Assisted Cartridge

Zhang Guodong
Naval Research Academy, Beijing 100161, China
380055262@qq.com

Abstract. In this paper, taking the rocket-assisted cartridge as the research object, systematic analyses and studies have been conducted on its ballistics characteristics. The whole ballistics is divided into three stages. Different ballistics models have been determined according to the different characteristics on each stage, the calculating programs of the processing method of the rocket-assisted cartridge have been programmed and the model of the data processing method and the reliability of calculation have been tested and proved by simulation results.

1. Introduction
Because the rocket-assisted cartridge has the characteristic of speeding up in the air, so the projectile has important advantages such as long firing distance, wide striking range and strong damage ability. Simulating analyses on the ballistics characteristics is an effective way to improve the rocket-assisted cartridge’s range and the precision of hitting.

2. The principle characteristics of rocket-assisted cartridge
The rocket-assisted cartridge is a projectile with a rocket engine at the end of the projectile to increase its range. When a projectile is fired, it moves in the bore like a conventional projectile. The engine does not work within a certain distance from the muzzle. After a certain distance, the rocket engine ignites, providing thrust for the projectile to increase the range. After the fuel is burned out, the engine shuts down and the projectiles move like regular shells[1][2]. Thus, the biggest difference between a rocket-assisted cartridge and a conventional projectile is that it has jet fuel, which creates a reaction when the jet fuel burns. The whole flight path of the rocket booster consists of three sections, as shown in the figure 1.

![Figure 1. A schematic diagram of the rocket-assisted cartridge motion](image)

The first trajectory is a passive trajectory: The launch point of the projectile is point $O$, flying freely from point $O$ to point $O_1$, and the rocket engine begins to work;
The second trajectory is an active trajectory: The rocket engine starts at point \( O \) and stops at point \( O_2 \) when the fuel is burned. Set the starting time of the active trajectory is \( t_1 \) and the ending time is \( t_2 \), then;

The third trajectory is another passive trajectory: If the projectile is used to attack a sea target, then the projectile from point \( O_2 \) to point \( C \) is the third trajectory. If the projectile is used to attack an air target, then the projectile from point \( O_2 \) to point \( O_p \) in the air is the third trajectory.

3. Assumption of the rocket-assisted cartridge ballistics model
When studying the movement of the rocket-assisted cartridge, the following hypotheses are made:\(^3\):

(1) The projectile is a particle with variable mass, and its total mass is the sum of the projectile and rocket fuel.
(2) The position of the center of mass of the dynamic rocket does not change with the burning of the rocket;
(3) The trajectory tangent line of the projectile's center of mass and the projectile's axis coincide, and the air resistance direction is opposite to the velocity direction;
(4) In the absence of wind, it is considered that the projectile moves in the standard ballistic atmosphere;
(5) As the projectile has a long range, the influence of gravity acceleration, earth rotation and coriolis force on the movement of the projectile must be taken into account during its movement.

4. Model Building of the rocket-assisted cartridge ballistics

4.1. The first trajectory ballistics model
The first trajectory is a passive trajectory. In this process, the projectile moves from launch point \( O \) to ignition point \( O_1 \), and the rocket engine is not working. Therefore, its motion characteristics are the same as those of conventional projectiles. The equations of motion of projectile particle in the first trajectory of rocket-assisted cartridge are:\(^4,5\):

\[
\begin{aligned}
\dot{u} &= \ddot{x} = -a \cdot \cos \theta \cos \psi + e_x \\
\dot{w} &= \ddot{y} = -a \sin \theta - g_T + e_y \\
\dot{q} &= \ddot{z} = -a \cdot \cos \theta \sin \psi + e_z
\end{aligned}
\]  

(1)

Among them, \( \dot{u} = \ddot{x} \), \( \dot{w} = \ddot{y} \), \( \dot{q} = \ddot{z} \), \( \psi \) is the drift Angle, \( \theta \) is the inclination Angle of projectile, and \( e_x \), \( e_y \), \( e_z \) are the corrected acceleration in three directions. The mathematical expression is:

\[
\begin{aligned}
e_x &= -x \cdot g_T(\varphi, y) / R(y) + 2w\Omega_z \\
e_y &= 2y \cdot g_T(\varphi, y) / R(y) + 2u\Omega_z \\
e_z &= f(t) - 2(w\Omega_z - u\Omega_y)
\end{aligned}
\]  

(2)

The initial conditions are: \( t = 0 \), \( x = 0 \), \( y = 0 \), \( z = 0 \), \( u = V_0 \cos \theta_0 \), \( w = V_0 \sin \theta_0 \), \( q = 0 \).

4.2. The second trajectory ballistics model
The rocket engine starts at point $O_1$, and ends at point $O_2$ when the fuel is burned and the engine stops working. During this period, the engine will provide thrust $P$ for the projectile, which consists of dynamic component $P_D$ and static component $P_S$. The expression is:

$$P = P_D + P_S = \mu(t)V_e + S_a (p_a - p_{atm})$$  \hspace{1cm} (3)$$

Among them, $P_D = \mu(t)V_e$ is the dynamic component, which is the reaction produced by the gas impulse, $V_e$ is the velocity of gas against the shell, $\mu(t)$ is the mass consumption rate of fuel, $\mu(t) = \left| \frac{dm}{dt} \right|$. $P_S = S_a (p_a - p_{atm})$ is the static component, which is caused by the pressure difference between gas pressure and atmospheric pressure. $S_a$ is the nozzle area, $p_a$ is the gas pressure, $p_{atm}$ is the atmospheric pressure.

Set $\mu(t) = \omega / t_k$, $\omega$ is the fuel mass, $t_k = t_2 - t_1$ is the fuel burning time, then the formula can be translated into:

$$P = \mu(t)V_{c\phi}$$ \hspace{1cm} (4)$$

Among them, $V_{c\phi} = V_e + S_a (p_a - p_{atm}) / \mu(t)$, which is the effective speed of rocket gas. For the given projectile and fuel, the value of $V_{c\phi}$ is related to the performance parameters of the rocket engine.

Since the mass of the rocket-assisted cartridge will change during the movement of the second trajectory, its air resistance $Q$ will also change accordingly. Its expression can be modified on the basis of existing conclusions, and the modified expression of air resistance $Q$ can be obtained as follows:

$$Q = ma$$

$$m = m_0 - \int_{t_1}^{t} \mu(t) dt$$

$$a = c\pi(h)F(V_e) = c\pi(h)G(V_e)W$$

$$c = \frac{i_p d^2 \times 10^3}{g_T m} = \frac{i_p d^2}{g_T} \cdot (m_0 - \int_{t_1}^{t} \mu(t) dt)$$

$$\pi(h) = \frac{\kappa \rho(h) \tau(h)}{\kappa \rho_{0N} \tau_{0N}} = \frac{p(h)}{p_{0N}}$$

In addition, the rocket-assisted cartridge also receives thrust $P$ in the same direction as the motion speed during the course of motion, so the motion state of the projectile is shown in figure 2:

Figure 2. A diagram of the state of rocket-assisted cartridge motion
Therefore, the motion equations of projectile particle in the second trajectory are modified as follows:

\[
\begin{align*}
\dot{u} &= \ddot{x} = -(P/m + a) \cdot \cos \theta \cos \psi + \epsilon_x, \\
\dot{w} &= \ddot{y} = -(P/m + a) \sin \theta - g_y + \epsilon_y, \\
\dot{q} &= \ddot{z} = -(P/m + a) \cdot \cos \theta \sin \psi + \epsilon_z.
\end{align*}
\]

Among them, \( u = \ddot{x} \), \( w = \ddot{y} \), \( q = \ddot{z} \), \( P \) is thrust, \( \psi \) is drift angle, \( \theta \) is projectile angle, \( \epsilon_x, \epsilon_y, \epsilon_z \) are corrected acceleration in three directions.

4.3. The third trajectory ballistics model

The engine of the rocket stops working at point \( O \), and the projectile moves in the air like a regular projectile until it hits the target. During this period, the motion equation of the projectile is the same as section 3.1. The initial condition is the state of the rocket at time \( t_s \).

In the air strike, if the projectile has a time fuse, it will explode at the set time. If a static explosion fuse is set, when the distance between the projectile and the target is sufficient to trigger the fuse, the projectile explodes. In a land or sea attack, an explosion occurs when a shell falls on the ground or sea.

5. Simulation on the ballistics model

Based on the above differential equation, a simulation program is written, in which the simulation parameters are:

Ignition time \( t_e = 10 \) s , fuel burning time \( t_k = 8 \) s , fuel quality \( \omega = 6 \) kg , calibre \( 130 \) mm , initial velocity \( V_0 = 875 \) m/s , projectile quality \( m = 28.52 \) kg , elastic coefficient \( i = 1.04 \) , angle of throw \( \theta_0 = 45^\circ \) , and gas jet speed \( V_e = 1800 \) m/s .

Running the simulation program, the trajectory comparison curve and velocity comparison curve of conventional projectile (AK-130 ship gun) and rocket-assisted cartridge are obtained, as shown in figure 3:

(a) Exterior trajectory curve
Figure 3. Contrast curves of 130mm ship gun and iso-caliber rocket-assisted cartridge

The comparison curves of 130mm ship gun and iso-caliber rocket-assisted cartridge were observed and the following conclusions were obtained:

1) Before ignition, the trajectories of the two are the same: the initial conditions are the same in the simulation process, and the motion states of the two projectiles are obviously the same without ignition.

2) After ignition, the velocity of the rocket-assisted cartridge rises rapidly: at the moment of ignition, the work done by the thrust generated by the burning of the rocket fuel is significantly greater than the negative work done by the combined force of gravity and resistance, so that the velocity of the rocket-assisted cartridge rises rapidly.

3) Compared with conventional guns, the rockets with the same caliber have a longer range, higher trajectory and longer flight time.

On this basis, different external trajectory curves can be obtained by adjusting parameters such as ignition time, ignition time, and propellant quality, which is helpful for studying the shape of external trajectory curve, projectile speed, and rocket range increment rate.

References

[1] Cao Hongjin etc. 2013 A Study on Optimizing Design of Trajectory for Glide Extended-Range Projectile *Journal of Sichuan military engineering* (vol 34) p 8-10

[2] DING Songhinetc. 2000 The Study on Horizon Gliding Range of Gliding Range-As-sited Projectile With Fins *Journal of projectiles, rockets, missiles and guidance* (vol 4) p 57-60

[3] Ji jingxin etc. 2014 Gliding Trajectory Design of Gliding Extended Range Projectile *Ship Electronic Engineering* (vol 34) p 46-48

[4] Zhang bo etc. 2014 Design and Research of Wind Tunnel Test for Deflectable Nose *Research and exploration in laboratory* (vol 33) p 18-25

[5] Zhao yixin etc. 2006 Study on Delayed Ignition Device for Rocket Extended-range *Journal of Projectiles, Rockets, Missiles and Guidance* (vol 26) p 423-425