Analysis on the buckling performance of embedded co-cured composite sandwich plates under uniaxial compression condition

Shaoqing Wang1,∗, Anfu Guo1, Changsheng Zheng2 and Sen Liang3

1 School of Mechanical and automotive Engineering, Liaocheng University, Liaocheng, People’s Republic of China
2 Facility Horticulture Laboratory of Universities in Shandong, Weifang University of Science and Technology, Weifang, People’s Republic of China
3 School of Mechanical and automotive Engineering, Qingdao University of Technology, Qingdao, People’s Republic of China

∗Author to whom any correspondence should be addressed.
E-mail: 740174584@qq.com

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Abstract
The research aims at exploring the buckling behavior of embedded co-cured composite sandwich structure (ECCSS) under static load. In accordance with the research objective, an analytical model, which is based on the interlayer stress continuous theory to formulate governing differential equations for predicting the critical buckling loads of ECCSS, has been established. The theoretical solution that satisfies the boundary conditions is obtained using the principle of minimum potential energy. The results obtained by ANSYS software have been used to corroborate the correctness of the analytical model. In addition, effects of various structural parameters (such as ply angles and skin-to-core thickness ratio) on buckling behavior of ECCSS have been investigated.

1. Introduction

In consideration of its strong structure stability, high rigidity and great tensile strength, composite material has gained widespread application recently. It proves that the mechanical properties of composite materials can be further improved and optimized, and that depends on favorable designability [1–7]. ECCSS is a new type of pre-processing structure that consists of a composite layer with soft material layers embedded into the composite. Co-curing means that the curing reaction time of soft material is consistent with the curing time of resin at certain temperature.

A great many scholars have performed study on the buckling performance of isotropic and laminated sandwich plates and shells with a series of models, such as Galerkin method, Rayleigh–Ritz method, and numerical method [8, 9]. Subsequently, mainstream theories, including classical laminate theory (CLT), first-order shear deformation theory (FSDT), and higher–order shear deformation theories (HSDT), and a unified kinematic equation were put forward in succession [10–14]. Based on the energy method and deflection function appropriate, Heder et al [15] inferred the proximate buckling loads of a simply supported stiffened sandwich structure. As indicated by the finding, the proposed expressions are applicable for the computing of the buckling loads of stiffened structures. After examining the influence of lateral pressures on FEM and the reduction of buckling strength imposed on axially loaded sandwich cylindrical shell, Ohga et al [16] explained the reduced rigidity of axially loaded sandwich cylindrical shell under lateral pressure. Kheirikhah et al [17] paid attention to the natural vibration and buckling performance of SMA wire-supported composite sandwich plates. Zhang et al [18] later used the ANSYS software to forecast the modal loss factors affecting the embedded co-cured composite structures. In a survey on the flat-wise compression performance in innovative cellular structure setting, Spadoni et al [19] revealed the reinforced performance and potentials of chiral noncentrosymmetric pattern. Throughout analysis on linear and non-linear finite-element analysis, Vinod et al [20] verified the buckling and collapse performance of foam core sandwich columns. Experiment results conformed to the failure load forecasts made by the postbuckling analysis. Collapse loads derived from eigenvalue analysis seem to be less conservative. In the experiment, Frostig et al [21] carried out a correlation...
analysis on sandwich unidirectional panel with transversely flexible core in condition of buckling loads. Liang et al.[22, 23] explored the mechanical performance of a self-developed embedded co-cured co-extruded composite test piece under given temperature and humidity thresholds. As to the study on corrugated soft-core composite test piece, Kheirikhah et al.[24] focused on the bending and buckling performance of the material with laminated composite face sheets under uniaxial load conditions. Safaei et al.[25] diverted their focus to the buckling performance of lightweight polymeric nanocomposite sandwich plate in a thermal environment. The structure was supported by two-parameter elastic bases. The elastic foundation model developed by Aviles et al.[26] explained the buckling performance of foam core sandwich columns with embedded through-width face/core debond. Hadi et al.[27] created a new algorithm to compute the gross buckling loads of simply supported composite sandwich panel containing a hole. Zhang et al.[28] went into the influence of microscopic geometrical parameters on natural frequency and buckling performance. Shiau et al.[29] developed a high precision and higher-order triangular plate element appropriate for the buckling analysis of rectangular composite sandwich plate in a thermal environment. In line with the quasi-3D theory, Vo et al.[30] developed a new model to deepen analysis on free vibration and buckling performance of functionally graded (FG) sandwich beams applying finite element method. Singh et al.[31] concluded the performance of functionally graded material plate by sandwich sigmoid function under different boundary conditions. On the grounds of the irst-order shear deformation theory (FSDT), Davoud et al.[32] proposed an equivalent analytical approach to calculate the gross buckling performance of composite lattice sandwich shells under uniaxial compression conditions. As shown by the results, FSDT was more applicable for thick sandwich shells’ critical buckling load calculation owing to its high computing efficiency. Li et al.[33] studied the secondary buckling and failure behaviors of composite sandwich panels subjected to in-plane shear loading applying a quasi-conforming finite element method. Sobhy et al.[34] investigated the hygro-thermal buckling of porous FGM sandwich microplates and microbeams based on a new shear deformation theory and the modified couple stress theory. Keshtegar et al.[35] studied with the vibration, buckling and bending analyses of annular nanoplate integrated with piezoelectric layers at the top and bottom surfaces. Jing et al.[36] presented a new buckling optimal solution for curved composite panels applying sequential permutation search algorithm. Yang et al.[37] established a mathematical model of carbon fiber composite beams via the FSDT and solved the theoretical solution of free vibration of beam structure using the Galerkin method. Shareef et al.[38] investigated dynamic characteristics and optimum design of multi-walled carbon nanotubes-reinforced magnetorheological elastomer sandwich panel using the finite element theory.

Literature review shows that considerable studies concerning buckling load capacity have been made. In reality, the Zig-Zag model is the theoretical foundation of mainstream theories represented by FSDT, HSDT, and other theories. The Zig-Zag model is developed based on the segmented displacement theory. Wang et al.[39] established an analytical model for the stiffened composite structure based on the segmented displacement theory and obtained the critical buckling load of the structure. The influence of some parameters on the critical buckling load of the structure was discussed. However, the influence of the ply angle on the critical buckling load of the structure was not mentioned in the literature [39]. In our work, a buckling model of the ECCSS is established theoretically based on the interlayer stress continuous theory, rather than the segmented displacement theory. The theory in this paper is different from literature [39] in the model construction method. Under the premise of satisfying the computational accuracy, the control equations derived in this paper have fewer unknown variables, and the analytical theory has higher computational efficiency. The research sets interface shear stress between every two layers at the same level, and derives the strain energy of each stress component after building the connection between layers. The principle of minimum potential energy is later adopted to solve the buckling loads of ECCSS and investigate the influence of overall parameters on the structural buckling loads.

2. Theoretical analyses

In the quest of deriving the governing equation, the research proposes the following three hypotheses: firstly, positive strain perpendicular to the middle plane could be overlooked. Secondly, there is no slip between layers. Thirdly, elastic layers have the same constitutive parameters and density.

Figure 1 demonstrates the geometric shape and size of thin plate of which its length and width is the values a and b respectively. The thickness of soft material layer and composite material is $h_1$ and $h_2/2$ respectively. Therefore, the research establishes a rectangular coordinate system, and sets the origin of coordinate on the middle plane of the thin plate. The force and constraint of the thin plate are as shown in figure 2.
2.1. Stress calculation

Subject to aforementioned hypotheses, the research expresses laminate constitutive relation as below:

\[
\begin{bmatrix}
\sigma_x^{(k)} \\
\sigma_y^{(k)} \\
\tau_{xy}^{(k)} \\
\varepsilon_x^{(k)} \\
\varepsilon_y^{(k)} \\
\gamma_{xy}^{(k)}
\end{bmatrix} =
\begin{bmatrix}
Q_{11}^{(k)} & Q_{12}^{(k)} & Q_{16}^{(k)} & 0 & 0 \\
Q_{12}^{(k)} & Q_{22}^{(k)} & Q_{26}^{(k)} & 0 & 0 \\
Q_{16}^{(k)} & Q_{26}^{(k)} & Q_{66}^{(k)} & 0 & 0 \\
0 & 0 & 0 & Q_{44}^{(k)} & Q_{45}^{(k)} \\
0 & 0 & 0 & Q_{45}^{(k)} & Q_{55}^{(k)}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^{(k)} \\
\varepsilon_y^{(k)} \\
\gamma_{xy}^{(k)}
\end{bmatrix}
\]  

Parameter $Q_{ij}^{(k)}$ is the rigidity matrix on the kth layer.

As to the symmetrical laminate, corresponding small deflection strain field under displacement should be expressed as below:

\[
\begin{align*}
\varepsilon_x &= \frac{\partial u}{\partial x} = -z \frac{\partial^2 w_0}{\partial x^2} \\
\varepsilon_y &= \frac{\partial v}{\partial y} = -z \frac{\partial^2 w_0}{\partial y^2} \\
\varepsilon_z &= \frac{\partial v}{\partial z} = 0 \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \frac{\partial^2 w_0}{\partial x \partial y}
\end{align*}
\]
Here is the equation calculating in-plane stress:

\[
\sigma_x^{(k)} = -z\left(Q_{11}^{(v)} \frac{\partial^2 w_0}{\partial x^2} + Q_{12}^{(v)} \frac{\partial^2 w_0}{\partial y^2} + 2Q_{16}^{(v)} \frac{\partial^2 w_0}{\partial x \partial y}\right)
\]

(3-a)

\[
\sigma_y^{(k)} = -z\left(Q_{12}^{(v)} \frac{\partial^2 w_0}{\partial x^2} + Q_{22}^{(v)} \frac{\partial^2 w_0}{\partial y^2} + 2Q_{26}^{(v)} \frac{\partial^2 w_0}{\partial x \partial y}\right)
\]

(3-b)

\[
\tau_{xy}^{(k)} = -z\left(Q_{16}^{(v)} \frac{\partial^2 w_0}{\partial x^2} + Q_{26}^{(v)} \frac{\partial^2 w_0}{\partial y^2} + 2Q_{66}^{(v)} \frac{\partial^2 w_0}{\partial x \partial y}\right)
\]

(3-c)

The constitutive relation of the core layer material, similar to other isotropic materials, should be expressed as below:

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} =
\begin{bmatrix}
Q_{11}^{(v)} & Q_{12}^{(v)} & 0 \\
Q_{12}^{(v)} & Q_{22}^{(v)} & 0 \\
0 & 0 & Q_{66}^{(v)}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
\]

(4)

\[
Q_{11} = \frac{E}{1 - \mu^2};
Q_{12} = \frac{\mu E}{1 - \mu^2};
Q_{22} = \frac{E}{1 - \mu^2} = Q_{11}^v;
Q_{66} = \frac{E}{2(1 + \mu)}
\]

Parameter \( E \) and \( \mu \) refer to the elastic modulus and Poisson's ratio of the core layer material respectively.

Introduce equation (3) into (5) to derive the following new equation:

\[
\sigma_x^v = -z\left(Q_{11}^v \frac{\partial^2 w_0}{\partial x^2} + Q_{12}^v \frac{\partial^2 w_0}{\partial y^2}\right)
\]

(5-a)

\[
\sigma_y^v = -z\left(Q_{12}^v \frac{\partial^2 w_0}{\partial x^2} + Q_{22}^v \frac{\partial^2 w_0}{\partial y^2}\right)
\]

(5-b)

\[
\tau_{xy}^v = -2zQ_{66}^v \frac{\partial^2 w_0}{\partial x \partial y}
\]

(5-c)

Supposing the force of subject \( f_x \) and \( f_y \) is 0, and spatially balanced differential equation is applicable for the solution of \( \tau_{xz} \) and \( \tau_{yz} \) as below:

\[
\frac{\partial \sigma_i^v}{\partial x} + \frac{\partial \tau_{xy}^v}{\partial y} + \frac{\partial \tau_{xy}^v}{\partial z} = 0 (i = uni, v)
\]

(6-a)
\[ \frac{\partial r_{xy}^{(i)}}{\partial x} + \frac{\partial r_{yx}^{(i)}}{\partial y} + \frac{\partial r_{yz}^{(i)}}{\partial z} = 0 (i = \text{uni}, \nu) \] (6-b)

Parameter \( \text{Uni} \) is the general orthotropic layer, and \( \nu \) is the soft material layer.

Through introducing equations (3) and (5) into equation (6), and applying stress boundary conditions and interlayer stress equivalent theory, the \( xz \)-direction shear stress of the skin layer and core layer can be expressed as below:

\[ \tau_{xz}^{(k)} = \frac{1}{2} A_{xz}^{(k)}(x, y) \left( h_z^2 - z^2 \right) \left[ h_z \leq z \leq \frac{h_z}{2} \right] \] (7)

\[ A_{xz}^{(k)}(x, y) = Q_{11}^{(k)} \frac{\partial^3 w_0}{\partial x^3} + \left( Q_{12}^{(k)} + 2Q_{66}^{(k)} \right) \frac{\partial^3 w_0}{\partial x \partial y^2} \]

\[ + 3Q_{16}^{(k)} \frac{\partial^3 w_0}{\partial x^2 \partial y} + Q_{36}^{(k)} \frac{\partial^3 w_0}{\partial y^3} \] (8)

\[ \tau_{xz}^{(v)} = \frac{1}{2} A_{xz}^{(v)}(x, y) \left( h_z^2 - z^2 \right) \left[ \left| z \right| \leq \frac{h_z}{2} \right] \] (9)

\[ A_{xz}^{(v)}(x, y) = Q_{11}^{(v)} \frac{\partial^3 w_0}{\partial x^3} + \left( Q_{12}^{(v)} + 2Q_{66}^{(v)} \right) \frac{\partial^3 w_0}{\partial x \partial y^2} \] (10)

Like aforementioned procedures, the \( yz \)-direction shear stress of the skin layer and core layer is as below:

\[ \tau_{yz}^{(k)} = \frac{1}{2} A_{yz}^{(k)}(x, y) \left( z^2 - h_z^2 + \frac{h_z^2}{4} \right) \left( h_z \leq \left| z \right| \leq \frac{h_z}{2} \right) \] (11)

\[ A_{yz}^{(k)}(x, y) = Q_{11}^{(k)} \frac{\partial^3 w_0}{\partial x^3} + 3Q_{16}^{(k)} \frac{\partial^3 w_0}{\partial x^2 \partial y} \]

\[ + \left( Q_{12}^{(k)} + 2Q_{66}^{(k)} \right) \frac{\partial^3 w_0}{\partial x \partial y^2} + Q_{36}^{(k)} \frac{\partial^3 w_0}{\partial y^3} \] (12)

\[ \tau_{yz}^{(v)} = \frac{1}{2} A_{yz}^{(v)}(x, y) \left( z^2 - h_z^2 + \frac{h_z^2}{4} \right) \left( \left| z \right| \leq \frac{h_z}{2} \right) \] (13)

\[ A_{yz}^{(v)}(x, y) = Q_{11}^{(v)} \frac{\partial^3 w_0}{\partial x^3} + \left( Q_{12}^{(v)} + 2Q_{66}^{(v)} \right) \frac{\partial^3 w_0}{\partial x \partial y^2} + Q_{36}^{(v)} \frac{\partial^3 w_0}{\partial y^3} \] (14)

2.2. Strain energy calculation

The in-plane strain energy of the skin layer is as below:

\[ U_d^{\text{uni}} = 2 \int_{z=a/2}^{z=b/2} \int_{y=0}^{b} \int_{x=0}^{a} \left( \frac{1}{2} \sigma_{x}^{(k)} \varepsilon_{x}^{(k)} + \sigma_{y}^{(k)} \varepsilon_{y}^{(k)} + \tau_{xy}^{(k)} \gamma_{xy}^{(k)} \right) dx dy dz \] (15)

The strain energy of the orthotropic skin layer in the \( xz \) direction and \( yz \) direction is as below:

\[ U_{xz}^{\text{uni}} = 2 \int_{z=a/2}^{z=b/2} \int_{y=0}^{b} \int_{x=0}^{a} \frac{1}{2} \tau_{xz}^{\text{uni}} \gamma_{xz} dx dy dz \] (16)

\[ U_{yz}^{\text{uni}} = 2 \int_{z=a/2}^{z=b/2} \int_{y=0}^{b} \int_{x=0}^{a} \frac{1}{2} \tau_{yz}^{\text{uni}} \gamma_{yz} dx dy dz \] (17)

The in-plane strain energy of the core layer is as below:

\[ U_p^{\gamma} = 2 \int_{z=a/2}^{z=b/2} \int_{y=0}^{b} \int_{x=0}^{a} \left( \frac{1}{2} \sigma_{x} \varepsilon_{x} + \sigma_{y} \varepsilon_{y} + \tau_{xy} \gamma_{xy} \right) dx dy dz \] (18)

The strain energy of the core layer in the \( xz \) direction and \( yz \) direction is as below:

\[ U_c^{\gamma} = 2 \int_{z=a/2}^{z=b/2} \int_{y=0}^{b} \int_{x=0}^{a} \frac{1}{2} \left( \tau_{xz}^{\gamma} \right)^2 \] (19)
Parameter \( G \) is the shear modulus of the core layer.

The total strain energy of the ECCSS should be expressed as below:

\[
U = U_p^{uni} + U_{xz}^{uni} + U_{yz}^{uni} + U_p^{v} + U_{xz}^{v} + U_{yz}^{v}
\]  

(21)

2.3. Calculation of critical buckling load

The following equation is the displacement function satisfying simply supported boundary conditions.

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right)
\]

(22)

Wherein internal force is

\[
N_x = -P_x, \quad N_y = 0, \quad N_{xy} = 0
\]

(23)

Parameter \( N_x \) and \( N_y \) refer to Axial Pressure and \( N_{xy} \) indicates the shear load.

The work done during buckling process is as below:

\[
W = -\frac{1}{2} \iint_A \left[ N_x \left( \frac{\partial w}{\partial x} \right)^2 + N_y \left( \frac{\partial w}{\partial y} \right)^2 + 2N_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dxdy
\]

(24)

Introduce equations (22) and (23) into equation (24) to derive the new equation:

\[
W = \frac{P_x}{2} \int_0^a \int_0^b \left( \frac{\partial w}{\partial x} \right)^2 dxdy
\]

\[
= \frac{P_x}{2} \int_0^a \int_0^b \left[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m\pi x}{a} \right] \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \cos \left( \frac{m\pi x}{a} \right) \left( \frac{n\pi y}{b} \right)^2 dxdy
\]

(25)

\[
W = \frac{\pi^2 b}{8a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n^2}{a^2} C^2 m^2 = R_1 W
\]

(26)

\[
W = \frac{\pi^2 b}{8a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C^2 m^2
\]

(27)

The principle of minimum potential energy can be consulted to infer the buckling loads of the ECCSS:

\[
\frac{\partial (U - W)}{\partial C} = 0
\]

(28)

\[
R_1 = \frac{\partial U}{\partial C} / \frac{\partial W}{\partial C}
\]

(29)

3. Results and discussion

Next, the buckling analyses of ECCSS are studied for validation of the analytical model using the ANSYS software. Later, the parametric studies are carried out to illustrate the effects of various parameters on the buckling behaviors of the structure.

The buckling critical load of the ECCSS was calculated applying ANSYS software. The material parameters of the skin layer and core layer are shown in tables 1 and 2, respectively. The material parameters of ECCSS are chosen the same as those given in Ref. [39]. We set \( a = 400 \text{ mm}, \ b = 400 \text{ mm}, \) and the core layer thickness \( h_c = 0.1 \text{ mm}. \) The thicknesses of the upper and lower skin layers are equal, and the value \( h_f \) represents the sum of the thicknesses of the upper and lower skin layers. SHELL181 element is chosen for the research. The four sides of the ECCSS are simply supported. The buckling behavior of specimens under uniaxial compression is investigated. The load conditions are shown in figure 2. The applied load is the unit load, so the eigenvalue obtained by using ANSYS software represents the critical buckling load. When the value \( h_f \) is 2.12 mm and 2.25 mm respectively, the critical buckling load of the ECCSS is calculated in table 3.

When the ANSYS software is used to calculate the critical buckling load of the ECCSS, the types and sizes of the elements are the main factors which affect the calculation accuracy. After the cell type is determined, as the cell size decreases, the calculation accuracy will increase, but the time required for the calculation will also increase. The convergence analysis of the FE model is carried out below, so that it can not only ensure the calculation accuracy but also improve the calculation efficiency. As suggested by table 3, more elements have
been introduced to derive a constant value. Despite the introduction of more computing components in structural critical buckling load, computing results are barely affected, and the time of computing is prolonged. When the element size is 4 mm, the results calculated by the ANSYS software appear to converge. ANSYS software altogether employs 10,000 elements (100 horizontal elements and 100 longitudinal elements), that is, the size of the element is 4 mm, for the verification of the theoretical model in our work.

Next, let’s verify that the selected types and sizes of the elements are correct. Using the parameters of the literature \[39\], the upper and lower panel parameters are \(E_l = 26.2 \text{ GPa}, E_r = 26 \text{ GPa}\), \(G_{xy} = 3.9 \text{ GPa}, G_{xz} = 3.7 \text{ GPa}, G_{yz} = 3.7 \text{ GPa}\), and \(\mu_{xy} = 0.33, \mu_{xz} = \mu_{yz} = 0.4\). The core material parameters are \(E_c = 15.5 \text{ MPa}\), \(\rho_l = 1.080 \text{ kg m}^{-3}\), and \(\mu = 0.498\). The values \(a\) and \(b\) are 100 mm and 200 mm respectively. The critical buckling loads of the structure were calculated by using ANSYS software. SHELL181 element is chosen for the research. The length and width of the elements are 4 mm, respectively. The four sides of the sandwich panel are simply supported. The load conditions are shown in figure 2, and the applied load is the unit load. The convergence analysis of the FE model is carried out. The results obtained by the ANSYS software are compared with the results of literature \[39\] and shown in table 4.

Table 4 shows comparison of critical buckling loads obtained by different methods. It can be seen from table 4 that the critical buckling loads obtained by the two methods are relatively close, which proves the correctness of the types and sizes of the elements.

The verified element type and size are used in the calculation of critical buckling load in ANSYS software, and the results obtained by ANSYS calculation are compared with the results obtained by the analytical method in this paper. The critical buckling loads of ECCSS with different thickness-to-length ratios are as shown in table 5. The ANSYS model is shown in figure 3. The buckling mode of ECCSS is shown in figure 4. The international system of units is adopted, and the results solved by ANSYS software in table 5 are converted into millimeter units.

| Element divisions X \(\times\) Y | Total number of elements | Buckling load (N mm\(^{-1}\)) | \(h/a = 1/180\) | \(h/a = 1/170\) |
|-------------------------------|--------------------------|-------------------------------|-----------------|-----------------|
| 31 \(\times\) 31               | 961                      | 5.7629                        | 6.8285          |
| 34 \(\times\) 34               | 1156                     | 5.7588                        | 6.8237          |
| 37 \(\times\) 37               | 1369                     | 5.7556                        | 6.8199          |
| 40 \(\times\) 40               | 1600                     | 5.7530                        | 6.8169          |
| 45 \(\times\) 45               | 2025                     | 5.7497                        | 6.8129          |
| 50 \(\times\) 50               | 2500                     | 5.7472                        | 6.8100          |
| 58 \(\times\) 58               | 3364                     | 5.7443                        | 6.8067          |
| 67 \(\times\) 67               | 4489                     | 5.7422                        | 6.8042          |
| 80 \(\times\) 80               | 6400                     | 5.7403                        | 6.8020          |
| 100 \(\times\) 100             | 10000                    | 5.7386                        | 6.8000          |
Table 5 exhibits comparison of buckling loads obtained by ANSYS software and our theoretical solution. Regardless of the gap between theoretical value and ANSYS software-based value, the two values are basically the same as shown in table 5. The correctness of theory is proved by the numerical results derived by ANSYS software. It should be noted that the material parameters in tables 1 and 2 will be used in subsequent critical buckling load analyses.

Table 4. Critical buckling loads of sandwich panel.

| $h_i/h_f$ | Ref. [39] | ANSYS |
|-----------|-----------|-------|
| 6.9       | 16.240    | 15.087|
| 8         | 13.590    | 12.662|
| 9.5       | 11.080    | 10.356|
| 11.8      | 8.705     | 8.1664|
| 15.1      | 6.600     | 6.2159|

Figure 4. Buckling mode of ECCSS. (a) $h/a = 1/100$; (b) $h/a = 1/110$; (c) $h/a = 1/120$; (d) $h/a = 1/130$; (e) $h/a = 1/140$. 
3.1. Effect of ply angles on the critical loads

Critical loads under different ply angles are computed respectively to probe into the influence of ply angles on ECCSS critical loads. The figure below demonstrates the computing results.

![Graphs showing the relationship between critical loads and ply angles for different dimensions.](image)

Figure 5. Relationship between $P_c$ and $\theta$.

| Table 5. Critical buckling loads of ECCSS  
| ($a = 400$ mm, $b = 400$ mm, $h_c = 0.1$ mm). |
| $h/a$ | ANSYS | Present |
|-------|--------|---------|
| 1/100 | 32.6594 | 32.0320 |
| 1/110 | 24.6578 | 23.9742 |
| 1/120 | 19.0714 | 18.4058 |
| 1/130 | 15.0531 | 14.4374 |
| 1/140 | 12.0888 | 11.5322 |
| 1/150 | 9.8535  | 9.35794 |
| 1/160 | 8.1387  | 7.69676 |
| 1/170 | 6.8000  | 6.40644 |
| 1/180 | 5.7386  | 5.38951 |

3.1. Effect of ply angles on the critical loads

Critical loads under different ply angles are computed respectively to probe into the influence of ply angles on ECCSS critical loads. The figure below demonstrates the computing results.

Figures 5(a)–(d) imply the less influence of ply angles on critical loads of ECCSS. Together with the increase of ply angle, structure critical load first increases and then decreases. Structure stability is optimized in case that...
the ply angle is $\pi/4$. This is due to the fact that the in-plane shear stiffness is maximized when the ply angle of the structure is $\pi/4$. Therefore, it has the strongest resistance to deformation when external loads are applied. With other parameters remaining unchanged, growing size of thin sheet decreases structure critical loads. The skin layers primarily play the role of the structural stability, so the stability of ECCSS can be reinforced by enriching the thickness of skin lays.

3.2. Effect of skin thickness to core thickness ratio on the critical loads

This section will talk about the influence of skin thickness-core thickness ratio ($h_f/h_c$) on structure critical loads. On the premise of preserving all the other parameters unchanged, the research progressively augments structure skin thickness-core thickness ratio to compute critical loads. Figure 6 shows the computing results.

Figures 6(a)–(d) manifest the concurrent growth of critical loads. Buckling load is extremely sensitive to the change of value $h_f/h_c$ in condition the value $h_f/h_c$ is preset as 55. It is due to the fact that the skin layer provides the in-plane stiffness and strength of the structure, which mainly bears the in-plane tensile and compressive stress and shear stress caused by tension and compression. When the thickness of the value $h_f/h_c$ increases, that is, when the value $h_f$ increases, the stiffness of the ECCSS also increases accordingly and the structural stability increases.

3.3. Effect of main elastic modulus ratio on the critical loads

Next, the research digs into the influence of prime elastic modulus ratio on structure critical loads. With other parameters remaining unchanged, it respectively augments structure elastic modulus ratio to compute structural critical loads. Figure 7 shows the computing results.

Thus it can be seen that increase of value $E_i/E_z$ as shown in figure 7 leads to the increase of critical loads. The Structural rigidity and stability can be greatly improved by increasing principal elastic modulus ratio. The
The resistance of the ECCSS to deformation is greatly enhanced. Therefore, the critical buckling loads of the ECCSS increase with the increase of the value \( \frac{E_1}{E_2} \).

4. Conclusions

For concluding the buckling performance of ECCSS, the paper refers to the minimum potential energy theory to infer structural critical loads. As proved by the test, theoretical computing results are well consistent with finite element computing results. Efforts have been made to explore the influence of different parameters on ECCSS buckling loads. The stability of the structure reaches the maximum in condition that the ply angle reaches \( \pi/4 \). Growing thickness of the skin layer can reinforce the carrying capacity of ECCSS. In view of its large principal elastic modulus ratio, these materials manifest strong applicability in composite and sandwich plates and favorable stability under different loads. Those plates in large size usually have poor stability in structure. In a word, the research lays a theoretical foundation for subsequent studies on the design theory of ECCSS carrying capacity.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Conflict of Interests

The authors hereby confirm that no conflict of interest exists for this article.

ORCID iDs

Shaoqing Wang © https://orcid.org/0000-0001-8375-3996

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