MEDIUM-MODIFIED FRAGMENTATION FUNCTIONS

Néstor Armesto(1), Leticia Cunqueiro(1), Carlos A. Salgado(1,2) and Wenchang Xiang(3)

1 Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15782 Santiago de Compostela, Spain
2 Dipartimento di Fisica, Università di Roma “La Sapienza” and INFN, Roma, Italy
3 Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

E-mail: nestor@fpaxp1.usc.es, leticia@fpaxp1.usc.es, carlos.salgado@cern.ch, wcxiang@physik.uni-bielefeld.de

Abstract. We extend the Sudakov formalism from vacuum to opaque media by supplementing the splitting functions with an additional term given by the medium-induced gluon radiation spectrum. We then solve the DGLAP evolution equations to obtain the medium-modified fragmentation functions. In this way, both the additional energy loss and the modification of the QCD evolution by the medium are included in a consistent manner. As a phenomenological application, we compute the suppression of the high-$p_T$ yields in heavy-ion collisions by convoluting the obtained fragmentation functions with the perturbative spectrum.

1. Modification of the vacuum splitting functions

A hard parton produced in a heavy ion collision will travel through the opaque medium losing virtuality till its eventual hadronization, which takes place in vacuum for large enough transverse momentum. The medium accelerates the QCD evolution of the parton by inducing successive soft gluon radiations. Under the BDMPS approximation[1], the spectrum of gluons emitted by the parent parton can be written as a function of two parameters which completely characterize the medium: the medium length $L$ and the transport coefficient $\hat{q}$ [2]. The properties of the medium are encoded in the transport coefficient, $\hat{q}$, which can be related to the average squared transverse momentum transferred to the parton per mean free path.

To compute medium modified fragmentation functions (MMFF), we extend the fact that the formalism [1, 2] provides the vacuum splitting functions in the collinear limit, $dI^{\text{Vac}}/dzdk_T^2 = \alpha_s P(z)^{\text{Vac}}_{a\bar{b}}$, to the medium [3], $dI^{\text{Med}}/dzdk_T^2 = \alpha_s P(z)^{\text{Med}}_{a\bar{b}}$. The total splitting function is the sum of both (see also [4] and [5]),

$$P_{ba} = P^{\text{Vac}}_{ba}(z) + P^{\text{Med}}_{ba}(z, t, \hat{q}, L).$$

See that eq. 1 is only valid at $z \to 1$ due to the use of the medium induced gluon spectra, which relies on high energy approximations. So, at this level, the only difference between our total $g \to gg$ and $q \to qg$ splittings will be still a Casimir color factor. To extend it to all $z$, we take the full LO splitting functions $P^{\text{Vac}}$. Small $z$ corrections are also introduced in $P^{\text{Med}}$, whose effect is found to be small (see fig. 1 below) at the level of the Sudakovs.
2. Sudakov Factors

The Sudakov form factor is defined in the standard way

\[
\Delta_a(t, t_0) = \exp\left(-\sum_{a_{\text{col}} \neq a} \int_{t_0}^{t} \frac{dt'}{t'} \int_{z_{\text{min}}(t')}^{1-z_{\text{min}}(t')} dz' \frac{\alpha_s(z) (1-z)t'}{2\pi} P_{ca}(z)\right),
\]

which can be interpreted as the probability for a parton \(a\) not to radiate resolvable partons when evolving between the scales \(t_0\) and \(t\). In fig. 1 we show the effect of supplementing the Sudakov form factor with a medium term in the splitting function as given by eq. (1). This is done for two energies \(E = 10\) GeV, of interest for RHIC, and \(E = 100\) GeV, of interest for the LHC. Increasing the value of the transport coefficient leads to an enhancement on the radiation and, hence, to more suppressed Sudakov factors. The Sudakov for gluons is approximately enhanced by the color factor 9/4 with respect to that for quarks.

![Figure 1](image)

**Figure 1.** Left: Sudakov form factors for a parent parton of energy \(E_{jet} = 10\) GeV. Upper and lower curves correspond to quarks and gluons respectively. We use different colors for different medium densities. Solid/dashed lines account for collinear and large \(x\) corrected splitting functions. Right: The same as in left plot but for a higher parent parton energy \(E_{jet} = 100\) GeV.

3. Medium-evolved FFs

DGLAP evolution can be written in terms of the Sudakov factors:

\[
\frac{D_{b}(x,t)}{\Delta_{a}(t_0,t)} = D_{b}(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{1}{\Delta_{a}(t_0,t')} \int_{x}^{1-z_{\text{min}}(t')} \frac{dz}{z} \alpha_s(z) (1-z)t' \sum_{b} P_{ba} D_{b} \left(\frac{x}{z},t'\right),
\]

so that the probability that a parton \(a\) fragments into a hadron at the scale \(t\) with an energy fraction \(z\) is equal to the probability that the parton has not radiated since it was produced at \(t_0\) (first term) plus the probability of having radiated at any intermediate scale \(t'\) (second term). We consider only tree flavors (u, d, s) and we use as initial conditions the KKP vacuum fragmentation functions [6] at \(t_0 = 2\) GeV^2. That is to say, \(D(x,t_0)^{\text{MED}} = D(x,t_0)^{\text{VAC}}\) meaning that we do not consider medium modifications of the hadronization. The medium modifies the evolution of the parton but hadronization takes place in vacuum at scale \(t_0\) and beyond.

The scale in \(\alpha_s\) is taken to be the transverse momentum of the emission \(Q^2 = z(1-z)t\). The virtuality is evolved between \(t_0 < t < 4E^2\) and the infrared cut is chosen as \(t_0/2t < z(t) < 1 - t_0/2t\). For the vacuum, we have checked that this method reproduces the KKP results better than 40% in the \((z, Q^2)\) region of interest.
In fig. 2 we show our results for the MMFF for charged pions or $\pi^0$s with the modified DGLAP evolution (3). The behavior of our MMFF with different parton energies and medium parameters, and for different parton types is shown. A clear suppression at large $z$ and enhancement at small $z$ can be seen. These two features grow with increasing scale, medium length and transport coefficient and, as expected, they are more pronounced for gluons than for quarks. The fact that these effects are larger for smaller parent parton energies can be understood qualitatively from the fact that the energy loss becomes more and more energy-independent with increasing energy.

Previous calculations of the MMFF were based on the quenching weights (QW) [7]. This method had some limitations concerning energy and momentum conservation. The MMFF were calculated shifting the vacuum ones and there was no evolution in virtuality, nor was there a unified description of medium and vacuum. Within our new approach this limitations are overtaken: we modify the QCD evolution and vacuum and medium are evolved together. Furthermore, it can be shown that by ignoring the evolution in virtuality and by taking the energy of the jet as the only scale, the old QW procedure is recovered. This must be seen as a proof of consistency.

4. Inclusive particle spectra and nuclear modification factor

The total inclusive cross section at LO can be written as a factorization of the long distance non perturbative elements (PDFs,FFs) and short distance perturbative terms (elementary partonic cross sections):

$$\sigma^{AB\rightarrow h} = f_A(x_1, Q^2)f_B(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{1\rightarrow h}(z, Q^2). \quad (4)$$

At high enough virtuality, medium modifications enter only the non perturbative terms. We simply solve previous equation introducing our new evolution-corrected FFs.

In the upper plot of fig. 3 we show our reference results for pp collisions at $\sqrt{s} = 200$ GeV. Below, in the same fig. 3, we plot our results for the nuclear modification factor (defined as $R_{AA} = \frac{d\sigma}{dydq^2}(pdf + EKS[10] + MMFF) / \frac{d\sigma}{dydq^2}(pdf + VACFF)$ ) in Au-Au collisions at a fixed length of $L = 6$ fm. All the three scales (factorization, renormalization, fragmentation) are set equal to the fragmenting parton momentum. The value of the transport coefficient which better describes the data is $\hat{q} \sim 1$ GeV$^2$/fm. This value is in agreement with the findings in [7], where also a fixed medium length was used. Once a realistic geometry is taken into account, this value is known [8] to increase. We do not attempt here to make a fit to experimental results but just to check that this new procedure, which explicitly conserves energy momentum at each splitting, results on values of $\hat{q}$ compatible with previous calculations.

Acknowledgments

We thank the organizers for this interesting meeting. CAS is supported by the 6th Framework Programme of the European Community under the Marie Curie contract MEIF-CT-2005-024624, and NA and LC by Ministerio de Educación y Ciencia of Spain under project FPA2005-01963 and by Xunta de Galicia (Consellería de Educación). NA also acknowledges financial support by Ministerio de Educación y Ciencia of Spain under a contract Ramón y Cajal.

References

[1] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B 484 (1997) 265.
[2] U. A. Wiedemann, Nucl. Phys. A 690 (2001) 731.
[3] A. D. Polosa and C. A. Salgado, Phys. Rev. C 75, 041901 (2007).
[4] X. N. Wang and X. f. Guo, Nucl. Phys. A 696 (2001) 788.
[5] N. Borghini and U. A. Wiedemann, arXiv:hep-ph/0506218.
[6] B. A. Kniehl, G. Kramer and B. Potter, Nucl. Phys. B 582 (2000) 514.
Figure 2. Upper plots: Medium-modified Fragmentation Functions for gluons (left), u,d-quarks (middle) and s-quarks (right plot) as a function of the energy fraction carried by the hadron, $z$. The parent parton energy is 10 GeV and the medium length is $L = 2$ fm. At each of the three different scales ($Q^2 = 2, 19$ and $400$ GeV$^2$ in black, green and red respectively) we plot the FFs at three different medium densities: vacuum(solid), $\hat{q} = 1$ GeV$^2$/fm(dashed) and $\hat{q} = 10$ GeV$^2$/fm (dotted). Lower plots: The same but for a parent parton energy of $E_{jet} = 100$ GeV and scales ($Q^2 = 2, 137$ and $40000$ GeV$^2$ in black, green and red respectively)

[7] C. A. Salgado and U. A. Wiedemann, Phys. Rev. D 68 (2003) 014008; R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, JHEP 0109 (2001) 033; M. Gyulassy, P. Levai and I. Vitev, Phys. Lett. B 538 (2002) 282.
[8] K. J. Eskola, H. Honkanen, C. A. Salgado and U. A. Wiedemann, Nucl. Phys. A 747 (2005) 511; A. Dainese, C. Loizides and G. Paic, Eur. Phys. J. C 38 (2005) 461.
[9] K. J. Eskola and H. Honkanen, Nucl. Phys. A 713 (2003) 167.
[10] K. J. Eskola, V. J. Kolhinen and C. A. Salgado, Eur. Phys. J. C 9 (1999) 61.
Figure 3. Up: Our reference vacuum $pp \rightarrow \pi^0$ spectra compared to PHENIX data at $\sqrt{s} = 200$ GeV. Choosing the fragmentation scale to be the parton (solid) or the hadron (dashed) transverse momentum results in a different normalization factor $K$ [9]. Down: Our results for the nuclear modification factor at a fixed medium length of 6 fm and for different transport coefficients compared to preliminary PHENIX data [11]

[11] M. Shimomura [PHENIX Collaboration], Nucl. Phys. A 774 (2006) 457.