Comparing and Combining Approximate Computing Frameworks

Saeid Barati  
Computer Science Department  
University of Chicago  
Chicago, USA  
saeid@cs.uchicago.edu

Gordon Kindlmann  
Computer Science Department  
University of Chicago  
Chicago, USA  
glk@cs.uchicago.edu

Hank Hoffmann  
Computer Science Department  
University of Chicago  
Chicago, USA  
hankhoffmann@cs.uchicago.edu

Abstract—Approximate computing frameworks configure applications so they can operate at a range of points in an accuracy-performance trade-off space. Prior work has introduced many frameworks to create approximate programs. As approximation frameworks proliferate, it is natural to ask how they can be compared and combined to create even larger, richer trade-off spaces. We address these questions by presenting VIPER and BOA. VIPER compares trade-off spaces induced by different approximation frameworks by visualizing performance improvements across the full range of possible accuracies. BOA is a family of exploration techniques that quickly locate Pareto-efficient points in the immense trade-off space produced by the combination of two or more approximation frameworks. We use VIPER and BOA to compare and combine three different approximation frameworks from across the system stack, including: one that changes numerical precision, one that skips loop iterations, and one that manipulates existing application parameters. Compared to a state-of-the-art evolutionary algorithm, we find that BOA explores 14× fewer configurations yet locates 35% more Pareto-efficient points.

I. INTRODUCTION

Approximation frameworks configure applications to operate within a trade-off space where the result accuracy is exchanged for other benefits, typically increased performance. Different approximation frameworks exist across the layers of the system stack. Some focus on the circuit level [1]–[6]. Others replace expensive hardware with approximations [7]–[10]. Still others exist at the programming language and compiler level [11]–[18].

As approximation methods proliferate, it is natural to question their interaction; especially:

• How to compare the trade-off spaces induced by different techniques? Comparing individual points in the trade-off space is easy; simply compare all framework’s performance at that point. Most approximation frameworks, however, produce a trade-off space—with a range of operating points—so we need techniques that compare frameworks across that entire range.

• How to combine different techniques’ trade-off spaces and locate Pareto-efficient points in the new space? The challenge is quickly locating more efficient configurations in the immense combined trade-off space, which is too big to search exhaustively.

VIPER and BOA: To compare approximation frameworks we propose VIPER: Visualizing Improved PErformance Ratios. While existing techniques use numerical comparisons [19]–[23] or simply display Pareto-optimal curves, VIPER produces a visual representation of the trade-off space. VIPER produces charts showing normalized performance for different frameworks across all possible accuracy loss ranges. A chart is divided into different, mathematically meaningful regions that show how much one framework out-performs others.

To combine frameworks, we propose BOA: Blending Optimal Approximations. BOA is a family of algorithms that locate Pareto-efficient points in the huge trade-off space produced by multiple frameworks. In its simplest version, BOA-simple searches the cross product of Pareto-optimal points from individual frameworks. BOA extensions, evaluate more of the search space—either deterministically or probabilistically—including more near-optimal points. BOA then returns the Pareto-efficient points from this search space.

Summary of Results: We consider two case studies. Both use prior approximation frameworks from across the system stack: Loop Perforation—a compiler technique (LP) [24], PowerDial—an application-level technique (PD) [25], and the Approximate Math Library—a library that changes numerical precision (AML) [18]. We use eleven applications covering domains from machine learning to image processing. Each application includes multiple inputs that we divide into training and test sets to evaluate whether combination methods produce statistically sound results on unseen inputs.

The first case study uses VIPER to compare Loop Perforation, PowerDial, and the Approximate Math Library. Loop Perforation simply discards loop iterations with no regard to original intent. PowerDial, in contrast, builds off approximations that already exist in the application; i.e., those envisioned by the original programmer. The Approximate Math Library approximates math functions (e.g., \( \exp \), \( \log \) and \( \sqrt{\cdot} \)) using variable Taylor series expansion. While these approximations work at different levels of the stack, VIPER allows us to quickly compare them and produces more intuitive visualizations than simply looking at Pareto-optimal curves.

The second case study combines the three approximation
techniques and locates Pareto-efficient points in this new, significantly larger trade-off space. We compare BOA to two state-of-the-art design-space exploration algorithms: Multiple Choice Knapsack Problem (MCKP) \cite{26} and Non-Sorting Genetic Algorithm (NSGA-II) \cite{27}. Compared to these two approaches, BOA achieves:

- **More efficient configurations**: BOA-simple produces 48.2\% and 35.1\% more Pareto-efficient points than MCKP and NSGA-II, respectively.
- **More reliable behavior on unseen inputs**: BOA-simple finds statistically meaningful Pareto-efficient configurations that are not sensitive to input data and are more likely to be efficient on an unseen set of inputs. That is, the correlation between BOA’s behavior on training (seen) and test (unseen) inputs is much higher than MCKP and NSGA-II.

Somewhat surprisingly, BOA produces much better results while using a much simpler search technique than the prior works to which it is compared. The fact that simpler search methods can produce better results is a key contribution of this work. The primary insight is that, empirically, we find that optimal combinations of approximation frameworks tend to involve configurations that are near-optimal for the individual frameworks. Therefore, BOA explores combinations derived from these points with high probability. In contrast, MCKP does not consider enough non-optimal configurations and gets stuck in local minima, while NSGA-II explores too many non-optimal configurations—avoiding the worst local minima, but also stopping short of the true optimal combinations. Thus, BOA’s method represents a compromise that works well for approximation frameworks, whose optimal combinations tend to be near the individually optimal points.

**Contributions:**

- Introduction of VIPER to visually compare approximation trade-off spaces over their entire range.
- Comparison analysis—based on VIPER—of the three approximation frameworks (Loop Perforation \cite{24}, PowerDial \cite{25} and Approximate Math Library \cite{18}).
- Proposal of variations of BOA for fast locating Pareto-efficient configurations in combined trade-off space.
- Open-source release of the VIPER and BOA tools.

**II. Motivation and Background**

**A. Approximation Across the System Stack**

Approximation frameworks reduce runtime (or energy) by allowing output quality degradation. Hardware approximation computes inexactly in return for reduced energy, area, or time \cite{2, 4, 28}. Many software approximation techniques allow specific software components to be replaced by approximate variants; e.g., skipping loop iterations or replacing math operations with Taylor-series expansions \cite{17, 18, 24, 25, 29–38}. Some approaches use machine learning to replace exact computation with a faster, less accurate learned variant \cite{5, 8, 9, 39–41}. Languages support approximation allowing specification of variants for key functionality and formal analysis of their effects \cite{11–13, 15, 16, 42–45}. Other mechanisms guarantee that approximate programs will maintain some quality or energy guarantees even if either program analysis \cite{46–48} or runtimes with formally analyzable dynamic adaptation \cite{37, 49–55}.

**B. Comparing Approximation Frameworks**

Our intuition is that some approximation frameworks will produce better results (e.g., higher performance for the same accuracy) than others in different situations. Hence, we need a method to compare frameworks across their full range of accuracy and choose the best one for a specific usage scenario. Points in these trade-off spaces correspond to configurations of the approximation framework. Point-by-point comparison is unfeasible since trade-off spaces include numerous configurations, many of which are not useful. Typically, only the Pareto-optimal points are useful for comparison.

As an example, we compare three approximation frameworks: PowerDial \cite{25}, Loop Perforation \cite{24, 31, 32}, and the Approximate Math Library \cite{18}. We pick these three frameworks because (1) they are either easily recreated or publicly available, requiring no specialized language or hardware support, and yet (2) they are representative of approaches applied at different levels of the system stack. PowerDial is an application-level approach that exploits existing trade-offs envisioned by the application developers. Loop Perforation creates approximate applications by applying a compiler transformation to selectively skip loop iterations. The Approximate Math Library changes computation, and while implemented in software, it is a good proxy for approximation techniques that change hardware arithmetic units. Figure 1 illustrates the trade-off spaces induced by these three approaches for the fluidanimate benchmark. Each point represents the normalized runtime and accuracy loss.

Comparison of approximation frameworks across their full range of accuracy is necessary, as not all users have the same accuracy requirements. We find, however, that looking at Pareto-optimal curves like those in Figure 1 is unsatisfying and rarely makes it obvious which approximation method is better across a range of operating points. Moreover, while for a certain range of accuracy one framework might perform better, another framework might produce higher performance
at different accuracy ranges. This motivates VIPER, a tool that allows users to tell—at a glance—which framework has the best performance for any range of accuracy loss.

C. Combining Approximation Frameworks

Figure 1 suggests that none of the three approximation frameworks is uniformly best. Furthermore, the fact that the three are broadly representative of approaches from different levels of the system stack motivates us to combine them for better accuracy/runtime tradeoffs than any individual framework. The challenge is that merging multiple frameworks leads to an enormous trade-off space, which is infeasible to explore exhaustively. Table 1 lists the number of points in the trade-off spaces of Loop Perforation, PowerDial, and Approximate Math Library for sample benchmarks. For example, the x264 benchmark takes up to 4 weeks to test all combined configurations with a single input. Considering multiple inputs should be tested for statistically sound results, the unfeasibility of exhaustive exploration is obvious.

More formally, combining approximation frameworks requires quickly locating Pareto-efficient configurations in the new, larger trade-off space. Exploring large trade-off spaces is well-studied and has produced two broad classes of approach. The first is carefully selecting and exhaustively searching a subset of the combined trade-off space [26], [56]. The second class intelligently traverses the entire combined trade-off space—not limiting the initial configuration combinations, but exploring a small number of the total [21], [57], [58]. Among these intelligent search techniques, NSGA-II—a genetic algorithm-based approach—has repeatedly outperformed other proposals [27], [58].

While prior work has proven effective for application-specific processor design, we find that it is not the best match for combining approximation frameworks. Specifically, heuristic exploration of genetic algorithms appear to cause two issues: (1) in an effort to avoid local minima, they produce less efficient combinations (see Section VI-B) and (2) they add too much randomization that leads to lower correlation between training and test inputs (see Section VI-E).

III. COMPARING APPROXIMATION FRAMEWORKS

A. Terminology

To produce performance/accuracy tradeoffs, any approximation framework must have one or more tunable parameters. The values assigned to the parameter set represent a configuration and the range of possible parameter settings is a configuration space. Each configuration represents a trade-off between performance and accuracy. The trade-off space (or design space) is the set of all possible trade-offs; i.e., the range of achievable performance and accuracy.

We consider large search spaces and often do not know the true optimal values for which we are searching. We therefore distinguish between Pareto-optimal—meaning we know that a point is on the true Pareto-optimal frontier—and Pareto-efficient—meaning a point on the estimated, unknown Pareto-optimal frontier. Thus, if we say a point is Pareto-efficient, it is better than all other points found so far, but we do not know that it is truly Pareto-optimal.

B. Numerical Comparisons

For large trade-off spaces, a point-by-point comparison is not possible. Therefore, prior work has introduced analytical methods for comparing trade-off spaces based on the number of Pareto-optimal—if the trade-off space is known—or Pareto-efficient—if the trade-off space is estimated—points from each framework.

A point in our accuracy-performance trade-off space is a 2D vector with runtime and accuracyLoss. Ideally, we would have zero run time and zero accuracy loss; i.e., instantaneously get a perfect answer, leading to:

Definition 1. **Objective Function**: Given points \( x_1 \) and \( x_2 \) in the objective to be minimized is \( f(x) \) where:

\[
\begin{align*}
& f(x_1) < f(x_2) \iff \text{accuracyLoss}(x_1) < \text{accuracyLoss}(x_2) \\
& \quad \text{and \ runtime}(x_1) < \text{runtime}(x_2)
\end{align*}
\]  

Points closer to the origin represent more efficient configurations. Given the objective function \( f(x) \), we determine if a point is more efficient than another by:

Definition 2. **Dominance**: Given points \( x_1 \) and \( x_2 \), we say:

\[
\begin{align*}
& x_1 \preceq x_2 \text{ (weakly dominates) if } f(x_1) \leq f(x_2) \\
& x_1 > x_2 \text{ (dominates) if } f(x_1) < f(x_2)
\end{align*}
\]

A point is Pareto-optimal if it is not dominated by any other point. A point is Pareto-efficient if we do not know of another point that dominates it. Figure 2(a) illustrates an example of dominance where point \( x_3 \) is dominated by point \( x_2 \).

**Coverage** quantifies the number of Pareto-efficient points produced by different techniques [57].

Definition 3. **Coverage** is the dominance ratio of the Pareto-efficient curves induced by two separate frameworks. If \( X \) and \( Y \) are two Pareto-efficient curves, and \( x \) and \( y \) represent points on them respectively, then:

\[
C(X, Y) = \frac{| \{ y \in Y | \exists x \in X : x \preceq y \} |}{|Y|}
\]

\( C(X, Y) = 1 \) means that all points in \( Y \) are weakly dominated by points in \( X \); i.e., all points of \( X \) provide lower runtime for the same accuracy loss than the points of \( Y \).

Figure 2(b) illustrates the coverage of curve \( X \) with respect to curve \( Y \). The point \( y_2 \) and \( y_3 \) on the curve \( Y \) are dominated by at least one point on the curve \( X \)—\( x_2 \), for example—therefore \( C(X, Y) = \frac{2}{3} \). In contrast, no point on \( X \) is dominated by a point on curve \( Y \) which means \( C(Y, X) = 0 \). By this metric, we consider \( X \) more efficient, but note that the curve \( Y \) extends through a larger range within the trade-off space; i.e., \( y_1 \) is a useful point which neither dominates nor is dominated by any points in \( X \). The coverage function is non-symmetric (\( C(X, Y) \neq C(Y, X) \)) and usually their sum does not equal 1 [20]. Hence, we need a metric that considers both coverage functions simultaneously:
Definition 4. Difference Of Coverage compares coverage for two different Pareto-efficient curves.

\[ \text{DOC}(X, Y) = C(X, Y) - C(Y, X) \]  

As a result, when \( \text{DOC}(X, Y) \geq 0 \), that fraction of \( Y \) points which are dominated by \( X \) is greater than \( X \) points that are dominated by \( Y \). Higher \( \text{DOC} \) implies one set is more efficient than the other. If \( \text{DOC}(X, Y) \) is close to zero both may provide the same efficiency. This metric is widely used by in multi-objective optimization problems [21–23].

C. VIPER

Numerical comparisons suffer from two major shortcomings. First, they do not show the full range of accuracy loss induced by each framework. Second, as seen in Figure 1, the best approximation framework varies as accuracy loss changes. Numerical metrics—like DOC—have limited expressiveness; Figure 2(b) shows that \( y_1 \) is a useful point, but DOC makes \( X \) look uniformly better than \( Y \).

As an alternative to numerical methods, researchers have used Pareto-optimal curves to compare frameworks, but this graphical evaluation has proven problematic [56], [59]. While curves may look compact, they can be different by orders of magnitude; e.g., when there is a steep slope and a large range covered, a small change in one dimension (e.g., accuracy loss) leads to a significant shift in the other (e.g., runtime).

Algorithm 1 VIPER.

```
Require: M, B ⊿ Lower convex hulls for framework M and baseline B
1: \( \text{MinX} = \text{Max}(\text{Min}(M.x), \text{Min}(B.x)) \) \( \triangleright \) lower bound
2: \( \text{MaxX} = \text{Min}(\text{Max}(M.x), \text{Max}(B.x)) \) \( \triangleright \) upper bound
3: \( \text{step} = (\text{MaxX} - \text{MinX})/1000 \)
4: for \( \text{accuLoss} = \text{MinX}; \text{accuLoss} < \text{MaxX}; \text{accuLoss} += \text{step} \) do
5:   \( M_i \leftarrow \text{find point on } M \text{ where } M_i.x < \text{accuLoss} < M_{i+1}.x \)
6:   \( B_j \leftarrow \text{find point on } B \text{ where } B_j.x < \text{accuLoss} < B_{j+1}.x \)
7:   \( \hat{y}_M \leftarrow \text{interpolate runtime between } M_i \text{ and } M_{i+1} \text{ where } x = \text{accuLoss} \)
8:   \( \hat{y}_B \leftarrow \text{interpolate runtime between } B_j \text{ and } B_{j+1} \text{ where } x = \text{accuLoss} \)
9:   \( \text{perfImprovRatio}(\text{accuLoss}) = \hat{y}_M / \hat{y}_B \)
end for
11: \( \text{NORMALIZE}(	ext{perfImprovRatio}) \) \( \triangleright \) limit the ratio to [0,1]
12: \( \text{return perfImprovRatio} \) \( \triangleright \) array of points
```

To provide an alternative visualization of approximation frameworks we introduce VIPER, which illustrates the relative performance of frameworks for any range of accuracy loss. Algorithm 1 explains how VIPER calculates the performance improvement ratio (PIR) of one framework \( M \) over a baseline \( B \). The PIR is the ratio of the frameworks’ performance at a given accuracy loss. To make the charts readable, PIR is in the range \([0,1]\). PIR = 1 means a configuration is the fastest in the space, while PIR = 0 is the slowest. A configuration with PIR = 0.6 achieves 60% of the maximum speedup.

First, VIPER finds the lower and upper bounds for the accuracy loss which defines the range of comparison. Then, this range gets divided by a parameterized granularity. We use a granularity of 1000 in this paper as larger values produced no benefit and smaller values make the charts less clear. For each \( \text{accuLoss} \) value, VIPER finds the corresponding runtime in both frameworks. Afterwards, we search for the nearest points on each lower convex hull where their accuracy loss is smaller than \( \text{accuLoss} \) (identified with \( M_i \) and \( B_j \) points respectively). Then, we interpolate the runtime of the specified \( \text{accuLoss} \) for both frameworks (named as \( \hat{y}_M \) and \( \hat{y}_B \)), and divide these interpolated runtime values to compute the performance improvement ratio. Finally, we normalize the ratio to \([0,1]\). When we compare more than two frameworks, the ratio is normalized to the lowest and highest among all.

VIPER then charts the PIR across the range of accuracy loss. The values for the baseline \( B \) will be a straight horizontal line. Values of \( M \) above that line indicate that \( M \) achieves higher performance for that accuracy loss. If the line for \( M \) stays above that for \( B \) for a greater range of accuracy loss, it means \( M \) has found more efficient configurations, on average. The color shading on the plot background indicates the highest performance method for that accuracy loss from the multiple frameworks. Therefore, if the plot’s background is dominated by a single color, the corresponding method provides the more efficient configurations. Thus, VIPER allows users to see at a (literal) glance, whether one approximation framework is clearly superior to another.

IV. CASE STUDY 1: COMPARING FRAMEWORKS

A. Experimental Setup

We use a dual socket Intel Xeon E5 server system with 20 physical cores at 2.9 GHz, hyperthreading, and 32 GB memory. Table I lists the used benchmarks, from Parsec 3.0 [60] and Rodinia 3.1 [61]. Table I also contains the description, type, application accuracy metric, and default run-time for each benchmark. Accuracy loss is the error relative to the most efficient configurations. Shorter runtime interprets as higher performance. blackscholes’s only tunable parameter is the number of prices to estimate and modifying it does not affect accuracy. Thus, PowerDial has no effect on blackscholes. Similarly, canneal, heartwall, kmeans, and x264 use math functions infrequently; the Approximate Math Library is not applicable to them.

We evaluate each suite across multiple inputs and compare the median across the inputs for this evaluation. In this section, we are evaluating known frameworks, thus we use the training inputs from Table I. In subsequent sections—where we present new techniques—we divide inputs into training and test and build combinations of frameworks using the training data and
then use the test data to ensure our selected combination works well on previously unseen data.

We evaluate three approximation frameworks. PowerDial (PD) transforms an application’s command line parameters into software knobs that are automatically manipulated to trade accuracy for performance [25]. Each application has tunable knobs, which can take different values, and an assignment of values to knobs is a configuration. Loop Perforation (LP) identifies perforatable loops whose iterations can be skipped to produce faster, but less accurate results [24]. A set of loops and perforation rates is a configuration. The Approximate Math Library (AML) substitutes math functions with a variable Taylor series expansion. A set of functions and their number of terms is the configuration.

B. Comparison by Difference of Coverage

To compare Loop Perforation and PowerDial, we calculate the average coverage function \((C(X, Y))\) from Eq. 3) across all benchmarks for both. On average, Loop Perforation covers only 0.3664 Pareto-optimal points of PowerDial, while PowerDial covers 0.4416 Pareto-optimal points of Loop Perforation. Thus, \(DOC(\text{LoopPerforation}, \text{PowerDial}) = -0.075\) which shows the slight superiority of Loop Perforation over Loop Perforation, on average. On the other hand, negative values of \(DOC(\text{AML}, PD) = -0.9135\) and \(DOC(\text{AML}, LP) = -0.929\) shows significant inferiority of Approximate Math Library against other frameworks, on average.

C. Comparison by Pareto-optimal Curves

Figure 3a illustrates the frameworks’ trade-off spaces. Each point represents a configuration. The y-axis is runtime normalized to the default configuration and the x-axis is the accuracy loss. Each framework’s Pareto-optimal curve is shown in the same color as the trade-off space. These plots highlight how configurations cover wide ranges of runtime and accuracy loss. While in some cases—e.g. canneal, kmeans, and srad—Pareto-optimal curves are easily to compare, in other benchmarks—e.g. particlefilter and swaptions—comparison is unfeasible. For fluidanimate, the Pareto-optimal curves intersect multiple times; the best approximation framework differs across the range of accuracy loss.

D. Comparison by VIPER

Just viewing the Pareto-optimal curves in Figure 3a provides limited intuition as differences are not always visible. We use VIPER to compare these frameworks in Figure 3b. The y-axis represents the performance improvement ratio (PIR), while the x-axis illustrates accuracy loss. The horizontal line represents Loop Perforation: points above that line mean the corresponding technique is faster than Loop Perforation. The backdrop color indicates the best method for an accuracy loss. VIPER provides the following insights:

- It illustrates how frameworks perform within a specific accuracy loss range. For instance, while PowerDial finds a higher performance configuration than Loop Perforation for bodytrack and canneal, its performance is worse for kmeans and x264 for most accuracies.
- While the distinction between frameworks is not clear in Figure 3a for streamcluster and swaptions, VIPER allows quick, obvious comparison.
- VIPER clearly illustrates the intersection of Pareto-optimal curves; e.g., in fluidanimate and srad.

Since VIPER only requires trade-off spaces to compare, it can be applied to any approximation frameworks regardless of system level. We believe VIPER provides clear insights, which are instantly visually recognizable and mathematically meaningful. VIPER is not a replacement for existing methods, but a complement that simplifies comparison.

V. COMBINING APPROXIMATION FRAMEWORKS

The prior section shows that none of Loop Perforation, PowerDial, or the Approximate Math Library is uniformly best. This observation motivates us to combine frameworks. At one level, this process is quite easy—just create a new trade-off space that is the cross product of all configurations in the original frameworks. The challenge, of course, is quickly locating the Pareto-efficient points in the resulting massive combined trade-off space.

We meet this challenge with the BOA family of search algorithms. All BOA methods select a subset of the combined trade-off space and exhaustively search that subspace space. The first algorithm, BOA-simple, only considers configurations in the cross-product of individual frameworks’ Pareto-optimal configurations. This technique produces a relatively small set of points to search, but may be subject to local minima if the approximation frameworks are not independent.

Unfortunately, most approximation frameworks are not independent. For example, Loop Perforation changes the number of loop iterations within an application; PowerDial may change convergence criteria. When we combine configurations from these frameworks, we find that some configurations that were Pareto-optimal when considering only the original frameworks are now far from optimal. Conversely, we empirically find that some configurations that were not Pareto-optimal in the original frameworks combine to be Pareto-optimal when we consider multiple frameworks together. These observations motivate us to expand BOA-simple to include more non Pareto-optimal configurations in combination.

BOA-flex expands the combined search space to consider the configurations that produce a trade-off within a user-defined threshold of Pareto-optimal. This technique searches more points and tends to find more efficient combinations, but it is deterministic. A common way to avoid local minima in large search spaces is expanding the exploration area with some form of randomization. We follow this approach with the last algorithm: BOA-prob, which probabilistically selects configurations from each individual framework to combine.
Specifically, it uses a sigmoid probability function, so that the closer points are to Pareto-optimal, the more likely they are to be included in the combined trade-off space. BOA-prob includes most of the same points as the other frameworks, but includes some outliers with small probability, making it more robust in the presence of local minima.

**BOA-simple:** The simple version of BOA forms the cross-product of all Pareto-optimal configurations from the individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks. After executing on evaluation platform, BOA-simple returns the Pareto-efficient configurations found in individual frameworks.

**BOA-flex:** BOA-flex augments BOA-simple with a user-specified selection threshold, as shown in Algorithm 2. This threshold also removes some inconsistency that may arise due to experimental noise; i.e., it is possible that for high variance applications, the true Pareto-optimal configurations cannot be found with confidence, so adding the threshold makes the search more robust. Specifically, BOA-flex considers all configurations whose trade-off is within the user-specified threshold of a Pareto-optimal trade-off.

This threshold is specified in terms of normalized Euclidean distance. All trade-offs are normalized so that accuracy loss and runtime range from 0 to 1. Accuracy loss of 0 means the lowest quality. A runtime of 1 is the slowest execution time. A trade-off point is the output of executing a configuration and, is thus, a pair of accuracy loss and runtime. Having normalized all configurations accuracy loss and runtime, we can then compute the Euclidean distance between the trade-offs of two separate configurations. Given this definition, the threshold specifies how close to Pareto-optimal a trade-off must be for it to be included in the search. For example, the threshold is 0.05, and then the algorithm will include any configuration whose accuracy loss/runtime trade-off is within 5% of a Pareto-optimal point. If the threshold is zero, this algorithm is equivalent to BOA-simple.

**Algorithm 2 BOA-flex: expands search space by threshold.**

```plaintext
1: Combination = []
2: for f in frameworks do
3:   Pareto - opt ← Get-Pareto-Opt(f)
4: for Config C_j in Pareto - opt do
5:   for Config C_k in f do
6:     if NormalizedEuclideanDistance(C_j, C_k) ≤ threshold then
7:       Combination.append(C_j)
8: end for
9: end for
10: return Combination
```

**BOA-prob:** While BOA-flex expands the combined search space, it only considers additional configurations that are close to an individual framework’s Pareto-optimal curve. To make BOA even more robust to local minima, BOA-prob employs a sigmoid probability function to include a few points that are further from the individual frameworks’ Pareto-optimal curves:

$$S(C_j) = \frac{1}{1 + \exp(-\frac{\Delta + \gamma}{\gamma})}$$  (5)

where $\Delta$ is the normalized Euclidean distance between configuration $C_j$ and the nearest Pareto-optimal configuration. $\beta$ is the horizontal shift and $\gamma$ decides the curve’s smoothness. Algorithm 3 shows how BOA-prob uses Equation 5.

We use constants $\beta = 0.2$ and $\gamma = 0.01$ so there is a 92% chance of including the point that has $\Delta < 0.05$, and 50% chance of selecting the point with $\Delta < 0.1$. At $\Delta = 0.2$, there is less than 1% chance of including the point in the combination. If $\Delta = 0$, it means that $C_j$ is actually Pareto-optimal and BOA-prob includes it. The interdependent parameters $\beta$ and $\gamma$ control the size of combined trade-off space and the exploration time.

### VI. Case Study 2: BOA Evaluation

We compare variations of BOA to prior exploration techniques using the same experimental setup from Section IV-A.

| Benchmarks          | Accuracy Metric                  | Training inputs                                      | Test Inputs                                      | Runtime (sec) |
|---------------------|----------------------------------|------------------------------------------------------|-------------------------------------------------|---------------|
| Blackscholes        | Average Relative Error of Prices | 30 lists with 1M initial prices                      | 90 lists with 1M initial prices                  | 3.2           |
| Bodytrack           | Average Distance of Poses        | sequence of 100 frames                               | sequence of 261 frames                          | 3.1           |
| Canneal             | Average Relative Routing Cost    | 30 netlists with 400K+ elements                      | 90 netlists with 400K+ elements                  | 6.88          |
| Fluidanimate        | Distance between Particles       | 5 fluids with 100K+ particles                        | 15 fluids with 500K+ particles                   | 17.2          |
| Heartwall           | Average Relative Error of heart frames | sequence of 30 ultrasound images              | sequence of 100 ultrasound images                | 11.6          |
| Kmeans              | Distance between Cluster Centers | 30 vectors with 256K data points                     | 90 vectors with 256K data points                 | 3.1           |
| Particlefilter      | Distance between Particles       | sequence of 60 frames                               | sequence of 240 frames                          | 12.9          |
| Srad                | Image Diff (RMSE)               | 3 images with 2560*1920 pixels                       | 9 images with 2560*1920 pixels                   | 22.6          |
| Streamcluster       | Distance between Cluster Centers | 3 streams of 1K-100K data points                     | 9 streams of 100K data points                    | 30            |
| Swaptions           | Average Relative Error of Prices | 40 swaptions                                        | 160 swaptions                                    | 6.2           |
| x264                | Relative PSNR+Bittrate           | 4 HD videos of 200+ frames                           | 12 HD videos of 200+ frames                      | 7.7           |

1 For the purpose of time complexity analysis, we assume each approximation knob can take on two values only, however, in reality, parameters may be assigned a larger number of values.
We now split our inputs into training and test data sets as shown in Table 1. For each exploration technique, we first use the training inputs to find Pareto-efficient configurations, then we evaluate those points using the separate test data.

### A. Points of Comparison

We compare BOA to state-of-the-art approaches for locating Pareto-efficient points in large trade-off spaces:

- **MCKP:** The *multiple choice knapsack problem* variant of the classic knapsack problem has classes of items and must choose one item from each class. MCKP has been used to find Pareto-efficient processor designs in the performance-power space for application specific embedded processors [26]. We declare each framework to be a class. MCKP then selects the Pareto-optimal configurations of each class while keeping the default values for other classes. This creates a new, small trade-off space which can be searched with brute force.

- **NSGA-II:** The non-dominated sorting-based multi-objective evolutionary algorithm (NSGA-II) explores large trade-off spaces to find Pareto-optimal configurations using an evolutionary genetic algorithm [27]. NSGA-II is the state-of-the-art for multiobjective optimization of embedded processors that navigate performance-power trade-offs, having been cited over

---

**Algorithm 3** BOA-prob: probabilistic search space expansion.

```python
Require: frameworks
1: Combination = []  # trade-off spaces of frameworks
2: for f in frameworks do
3:  Pareto – opt f ← Get-Pareto-Opt(f)  # Pareto-efficient configurations
4:  for Config Cj in Pareto – opt do
5:    for Config Ci in f do
6:      \[ S(C_i) = \frac{1}{1 + \exp(-\Delta)} \]
7:      \[ \Delta = \text{NormalizedEuclideanDistance}(C_i, C_j) \]
8:      r ← Random number between 0 and 1
9:      if \( r < S(C_i) \) or \( \Delta d = 0 \) then
10:         Combination.append(Cj)
11:     end if
12:   end for
13: end for
14: return Combination  # Set of points to explore
```
B. Comparison by Difference of Coverage

Recall from Section III that difference of coverage (DOC) implies the efficiency of one curve over another. Figure 4 displays the difference of coverage for various techniques over NSGA-II per benchmark and on average. The y-axis shows DOC(X, NSGA) which is DOC of exploration technique X over NSGA-II (see Section III-B). Negative values of DOC(X, NSGA) indicate that X does not find as many Pareto-efficient points as NSGA-II. Conversely, positive values imply that technique X provides that many more values that dominate NSGA-II. BOA-flex and BOA-prob on average locate 52.8-65.6% more Pareto-efficient configurations than NSGA-II. BOA’s superiority is due to its focus on the configurations that have been shown to be Pareto-optimal on individual frameworks. NSGA-II starts the exploration from a random set of points in the combined trade-off space and iteratively looks for more efficient points. MCKP uses the individual Pareto-optimal curves but keeps the rest of frameworks at default configurations. Since the frameworks are not fully independent, we empirically find the some configurations that were not Pareto-optimal in the original frameworks become part of a Pareto-efficient curve of combined trade-off space when we consider multiple frameworks together. By expanding BOA with threshold and probabilistic exploration, we search more points, resulting in more efficient configurations. In short, MCKP does not search enough combinations, while NSGA-II searches too many. By restricting the search to points likely to be near the Pareto-optimal frontier for individual frameworks, BOA achieves the right balance and the best empirical results. This data shows that, for approximate computations, BOA produces many more efficient configurations than prior state-of-the-art search techniques.

C. Comparison by VIPER

Figure 5a shows the Pareto-efficient points for each benchmark and search method. The y-axis shows runtime (normalized to the default configuration) and the x-axis represents accuracy loss. We use median runtime across test inputs. These figures display the range of runtime and accuracy loss that a method can achieve. For instance, NSGA-II and MCKP cannot provide normalized runtime less than 78.1% and 55.5% of the default configuration respectively for particlefilter.

Figure 5b illustrates the VIPER comparison of NSGA-II, MCKP, and different variations of BOA. The y-axis shows performance improvement ratio while the x-axis shows accuracy loss. We use NSGA-II as the baseline, so it is represented by a horizontal line. For the same accuracy loss, lines above that horizontal represent better (more efficient) configurations, and lines below represent configurations worse than those found by NSGA-II. For most applications, MCKP stays below the horizontal, meaning it is worse than NSGA-II. By comparing BOA-simple and MCKP performance improvement ratio lines, we see that BOA-simple outperforms MCKP.

From the VIPER plots we also find the maximum and minimum performance improvement over NSGA-II. Considering fluidanimate, the NSGA-II line is at 0.25 indicating that the maximum performance is 4× better than NSGA-II, and minimum performance is 25% worse. In fact, for every benchmark BOA-flex finds at least one configuration with higher performance for the same accuracy.

Whenever NSGA-II locates more Pareto-efficient points than BOA-simple, by expanding the Pareto-efficient configurations we reduce the performance improvement ratio gap. Benchmarks heartwall, kmeans, and particlefilter demonstrate how expanding the combined configurations provides higher Pareto-efficiency. In total, we find by increasing the threshold, lines of BOA-flex are above the NSGA-II line more than 95% of the time. These results provide visual confirmation that BOA not only finds a greater number of efficient points than prior techniques, but BOA’s points are also significantly better, representing much more efficient trade-offs. Furthermore, we believe this case study provides further evidence of VIPER’s value, as the VIPER charts are visually intuitive in Figure 5b, but the Pareto frontiers (Figure 5a) do not immediately show which framework is best at a given accuracy or by how much.

D. Exploration Time

The Pareto-efficiency of the located points depends on exploration time. While Figures 4 and 5 show that BOA produces better configurations than other techniques, it is important to know if that gain comes from exploring more points or from a better exploration strategy. Table II presents the number of configurations explored for each benchmark for different methods, including different thresholds for BOA-flex. To get an estimate of the time spent exploring the combined trade-off space for a specific benchmark, we can multiply the number of explored configurations by the average runtime (from the last row in Table I). Comparing NSGA-II and BOA-simple across all benchmarks, NSGA-II explores 2.05% of all possible configurations, while BOA-simple explores about 14× less. BOA-flex and BOA-prob only search the 0.682% and 0.692% of all possible configurations, respectively. These
Fig. 5: Comparison of MCKP, NSGA-II, and BOA with different thresholds. (a) shows comparison by Pareto-efficient frontiers and (b) shows comparison by VIPER.
results indicate that BOA not only finds better combinations of approximate frameworks, it does so with less searching.

Since MCKP only chooses configurations from individual Pareto-optimal curves rather than merging the configurations, the number of explored configurations stays very low. In the worst case, MCKP explores up to the sum of the Pareto-optimal points of PowerDial, Loop Perforation, and the Approximate Math Library. Unfortunately, while MCKP searches a small space, it is too small to find many useful points.

Table III shows the correlation coefficient ($R$-values) for accuracy loss for each exploration method per benchmark. Table IV shows the $R$-values for runtime. By harmonic mean, the behavior of configurations found during training data is a good predictor of test behavior.

E. Input Sensitivity

Since exhaustive exploration is not feasible, we use training and test data to ensure robustness of BOA on unseen data. We show how well the behavior on training inputs predicts that on test inputs. For each search method, we take the normalized runtime and accuracy loss, compute a linear least squares fit of training data to test data, and compute the correlation coefficient of each fit. Higher correlation coefficients imply greater robustness; i.e. the behavior of configurations found during training data is a good predictor of test behavior.

Table III shows the correlation coefficient ($R$-values) for accuracy loss for each exploration method per benchmark. Table IV shows the $R$-values for runtime. By harmonic mean, BOA has higher consistency of accuracy loss and normalized runtime comparing to NSGA-II by 17% and 64% respectively. Since MCKP evaluates few configurations, its predictions are quite robust—one advantage of MCKP over other techniques.

While some benchmarks such as fluidanimate and streamcluster clearly stress the difference between training and test inputs. NSGA-II’s heuristic approach can select configurations for the training data that produce bad results on the test data. In contrast, BOA not only finds more efficient configurations, its results are also much more robust when applied to new inputs, producing uniformly high $R$-values. These results indicate that BOA is a sound method for combining approximation frameworks.

F. Combination Distribution

When BOA combines frameworks, it considers multiple configurations from each rather than choosing from one or two only. Table V includes the number of configurations BOA-simple selects from each framework to generate the new, combined trade-off space. As mentioned in Section IV-D, the Approximate Math Library is never better than Loop Perforation or PowerDial in any range of accuracy loss. However, BOA uses the Approximate Math Library in combination with Loop Perforation and PowerDial for 7 out of the 11 applications. These results show that there is real benefit to combining frameworks, as even the Approximate Math Library—which is uniformly the worst of the three techniques by themselves—contributes to Pareto-efficient points in the combined space found by BOA.

VII. Conclusion

A proliferation of approximation frameworks have recently appeared that exploit different configurable parameters to trade reduced accuracy for decreased resource consumption. This paper proposes methods for both comparing and combining different frameworks. VIPER is a visualization tool for comparing approximation frameworks across their entire range of available accuracies. We show this tool is useful for comparing existing approximation frameworks regardless of their type and applied system level. BOA is a family of algorithms that combine approximation frameworks and quickly locate Pareto-efficient configuration combination.

Acknowledgments: The effort on this project is funded by the U.S. Government under the DARPA BRASS program and by a DOE Early Career Award. Additional funding comes...
TABLE V: Combinations of approximation frameworks found by BOA.

| Benchmark | LP   | PD   | AML  | IOA-simple |
|-----------|------|------|------|------------|
| Blackholes| 4    | 5    | 1    | 30         |
| Bodytrack | 4    | 5    | 1    | 30         |
| Camrund   | 12   | 14   | 1    | 98         |
| Fluidanimate | 8   | 4    | 2    | 24         |
| Heartwall | 2    | 1    | -    | -          |
| Kmeans    | 4    | 8    | 2    | 32         |
| Particledi | 6    | 8    | 5    | 240        |
| Srad      | 2    | 8    | 5    | 48         |
| Streamcluster | 8    | 5    | 2    | 80         |
| Straglion | 11   | 5    | 6    | 330        |
| X264      | 9    | 5    | -    | 45         |

from the NSF (CCF-1439156, CNS-1526304, CCF-1823032, CNS-1764039).

REFERENCES

[1] K. V. Palem, “Energy aware algorithm design via probabilistic computing: From algorithms and models to moore’s law and novel (semiconductor) devices,” in Proceedings of the 2003 International Conference on Compilers, Architecture and Synthesis for Embedded Systems, ser. CASES ‘03. New York, NY, USA: ACM, 2003, pp. 115–116. [Online]. Available: http://doi.acm.org/10.1145/951310.951312

[2] A. Lingamneni, C. Enz, K. Palem, and F. Piguet, “Designing energy-efficient arithmetic operators using inexact computing,” Journal of Low Power Electronics, vol. 9, no. 1, pp. 141–153, 2013.

[3] A. Ingole, B. Maiti, J. Augustine, and K. Palem, “Does customizing inexactness help over simplistic precision (bit-width) reduction? a case study,” in Compilers, Architecture and Synthesis for Embedded Systems (CASES), 2015 International Conference on, Oct 2015, pp. 33–34.

[4] V. K. Chippa, S. Venkataramani, S. T. Chakradhar, K. Roy, and A. Raghunathan, “Approximate computing: An integrated hardware approach,” in 2013 Asilomar Conference on Signals, Systems and Computers, Nov 2013, pp. 111–117.

[5] S. Muralidhara, A. Roy, M. Hall, M. Garland, and P. Rai, “Architecture-adaptive code variant tuning,” SIGPLAN Not., vol. 51, no. 4, pp. 325–338, Mar. 2016. [Online]. Available: http://doi.acm.org/10.1145/2954679.2972411

[6] L. N. Chakrapani, B. E. S. Akgul, S. Cheemalavagu, P. Korkmaz, K. V. Palem, and B. Seshasayee, “Ultra-efficient (embedded) soc architectures based on probabilistic cmos (pmcmos) technology,” in Proceedings of the Design Automation Test in Europe Conference, vol. 1, March 2006, pp. 1–6.

[7] M. Samadi, D. A. Jamshidi, J. Lee, and S. Mahlke, “Paraprox: Pattern-based approximation for data parallel applications,” in ACM SIGARCH Computer Architecture News, vol. 42, no. 1. ACM, 2014, pp. 35–50.

[8] H. Esmaeilzadeh, A. Sampson, L. Ceze, and D. Burger, “Neural acceleration for general-purpose approximate programs,” in Proceedings of the 2012 45th Annual IEEE/ACM International Symposium on Microarchitecture, ser. MICRO-45. Washington, DC, USA: IEEE Computer Society, 2012, pp. 449–460. [Online]. Available: http://dx.doi.org/10.1109/MICRO.2012.48

[9] O. Temam, “A defect-tolerant accelerator for emerging high-performance applications,” in Proceedings of the 39th Annual International Symposium on Computer Architecture, ser. ISCA ’12. Washington, DC, USA: IEEE Computer Society, 2012, pp. 356–367. [Online]. Available: http://dl.acm.org/citation.cfm?id=2337159.2337200

[10] H. Esmaeilzadeh, P. Saeedi, B. N. Araabi, C. Lucas, and S. M. Fakhraie, “Neural network stream processing core (nnsp) for embedded systems,” in 2006 IEEE International Symposium on Circuits and Systems, May 2006, pp. 4 pp.–2776.

[11] J. Bornholt, T. Mytkowicz, and K. S. McKinley, “Uncertain; t.;: A first-order type for uncertain data,” ACM SIGPLAN Notices, vol. 49, no. 4, pp. 51–66, 2014.

[12] A. Kansal, S. Saponas, A. B. Brush, K. S. McKinley, T. Mytkowicz, and R. Ziola, “The latency, accuracy, and battery (lab) abstraction: Programmer productivity and energy efficiency for continuous mobile context sensing,” in Proceedings of the 2013 ACM SIGPLAN International Conference on Object Oriented Programming Systems Languages & Applications, ser. OOPSLA ’13. New York, NY, USA: ACM, 2013, pp. 661–676. [Online]. Available: http://doi.acm.org/10.1145/2509136.2509541

[13] A. Sampson, W. Dietl, E. Fortuna, D. Gnanapragasam, L. Ceze, and D. Grossman, “Enerj: Approximate data types for safe and general low-power computation,” in Proceedings of the 32Nd ACM SIGPLAN Conference on Programming Language Design and Implementation, ser. PLDI ’11. New York, NY, USA: ACM, 2011, pp. 164–174. [Online]. Available: http://doi.acm.org/10.1145/1993498.1993518

[14] T. Oh, H. Kim, N. P. Johnson, J. W. Lee, and D. I. August, “Practical automatic loop specialization,” SIGPLAN Not., vol. 48, no. 4, pp. 419–430, Mar. 2013. [Online]. Available: http://doi.acm.org/10.1145/2499368.2451161

[15] J. Ansel, Y. L. Wong, C. Chan, M. Olszewski, A. Edelman, and S. Amarasinghe, “Language and compiler support for auto-tuning variable-accuracy algorithms,” in Proceedings of the 9th Annual IEEE/ACM International Symposium on Code Generation and Optimization. IEEE Computer Society, 2011, pp. 85–96.

[16] W. Baek and T. M. Chilimbi, “Green: A framework for supporting energy-conscious programming using controlled approximation.”
[50] M. Maggio, A. V. Papadopoulos, A. Filieri, and H. Hoffmann, “Automated control of multiple software goals using multiple actuators,” in Proceedings of the 2017 11th Joint Meeting on Foundations of Software Engineering, ESEC/FSE 2017, Paderborn, Germany, September 4-8, 2017, 2017, pp. 373–384. [Online]. Available: https://doi.org/10.1145/3106237.3106247

[51] A. Filieri, H. Hoffmann, and M. Maggio, “Control strategies for self-adaptive software systems,” ACM Trans. Auton. Adapt. Syst., vol. 11, no. 4, pp. 24:1–24:31, 2017. [Online]. Available: https://doi.org/10.1145/3024188

[52] A. Farrell and H. Hoffmann, “MEANTIME: achieving both minimal energy and timeliness with approximate computing,” in 2016 USENIX Annual Technical Conference, USENIX ATC 2016, Denver, CO, USA, June 22-24, 2016., 2016, pp. 421–435.

[53] A. Filieri, M. Maggio, K. Angelopoulos, N. D’Ippolito, I. Gerostathopoulos, A. B. Hempel, H. Hoffmann, P. Jamshidi, E. Kalyvianaki, C. Klein, F. Krikava, S. Misailovic, A. V. Papadopoulos, S. Ray, A. M. Shariifloo, S. Shervtsov, M. Ujma, and T. Vogel, “Control strategies for self-adaptive software systems,” ACM Trans. Auton. Adapt. Syst., vol. 11, no. 4, pp. 24:1–24:31, 2017. [Online]. Available: https://doi.org/10.1145/3024188

[54] S. Wang, C. Li, H. Hoffmann, S. Lu, W. Sentosa, and A. I. Kistijantoro, “Understanding and auto-adjusting performance-sensitive configurations,” in Proceedings of the Twenty-Third International Conference on Architectural Support for Programming Languages and Operating Systems, ASPLOS 2018, Williamsburg, VA, USA, March 24-28, 2018., X. Shen, J. Tuck, R. Bianchini, and V. Sarkar, Eds. ACM, 2018, pp. 154–168. [Online]. Available: https://doi.org/10.1145/3173162.3173206

[55] C. Hankendi, A. K. Coskun, and H. Hoffmann, “Adapt&cap: Coordinating system- and application-level adaptation for power-constrained systems,” IEEE Des. Test, vol. 33, no. 1, pp. 68–76, 2016. [Online]. Available: https://doi.org/10.1109/MDAT.2015.2463275

[56] T. Givargis, F. Vahid, and J. Henkel, “System-level exploration for pareto-optimal configurations in parameterized systems-on-a-chip,” in Computer Aided Design, 2001. ICCAD 2001. IEEE/ACM International Conference on, Nov 2001, pp. 25–30.

[57] G. Palermo, C. Silvano, and V. Zaccaria, “Respir: A response surface-based pareto iterative refinement for application-specific design space exploration,” IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 28, no. 12, pp. 1816–1829, Dec 2009.

[58] E. Zitzler, M. Laumanns, and L. Thiele, “Spea2: Improving the strength pareto evolutionary algorithm,” Tech. Rep., 2001.

[59] J. Knowles and D. Corne, “The pareto archived evolution strategy: a new baseline algorithm for pareto multiobjective optimisation,” in Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on, vol. 1, 1999, p. 105 Vol. 1.

[60] C. Bienia, “Benchmarking modern multiprocessors,” Ph.D. dissertation, Princeton University, January 2011.

[61] S. Che, M. Boyer, J. Meng, D. Tarjan, J. W. Sheaffer, S. H. Lee, and K. Skadron, “Rodinia: A benchmark suite for heterogeneous computing,” in Workload Characterization, 2009. IISWC 2009. IEEE International Symposium on, Oct 2009, pp. 44–54.