Identification Method of Geometric Deviations for Multi-Tasking Machine Tools Considering the Squareness of Translational Axes

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Received: 18 February 2020; Accepted: 3 March 2020; Published: 6 March 2020

Abstract: Some methods to identify geometric deviations of five-axis machining centers have been proposed until now. However, they are not suitable for multi-tasking machine tools because of the different configuration and the mutual motion of the axes. Therefore, in this paper, an identification method for multi-tasking machine tools with a swivel tool spindle head in a horizontal position is described. Firstly, geometric deviations are illustrated and the mathematical model considering the squareness of translational axes is established according to the simultaneous three-axis control movements. The influences of mounting errors of the measuring instrument on circular trajectories are investigated and the measurements for the B axis in the Cartesian coordinate system and the measurements for the C axis in a cylindrical coordinate system are proposed. Then, based on the simulation results, formulae are derived from the eccentricities of the circular trajectories. It is found that six measurements are required to identify geometric deviations, which should be performed separately in the B axis X-direction, in B axis Y-direction, in C axis axial direction, and three times in C axis radial direction. Finally, a numerical experiment is conducted and identified results successfully match the geometric deviations. Therefore, the proposed method is proved to identify geometric deviations effectively for multi-tasking machine tools.

Keywords: geometric deviations; multi-tasking machine tools; identification method; squareness of translational axes

1. Introduction

In recent years, multi-tasking machine tools have become widely popular in industry because of their growing capabilities in performing complex motions and in reducing machining time and cost. Therefore, many researchers research their machining capabilities and processing technology [1–3]. Based on the basic configuration of a lathe or turning machine, multi-tasking machine tools are developed by equipping with a swivel tool spindle head, which can perform not only a turning operation but also a drilling or milling operation [4]. With the increase of the functionality and the number of simultaneously controlled axes, multi-tasking machine tools are difficult to achieve high machining accuracy and efficiency. Therefore, it is essential to investigate the factors affecting the accuracy of finished products which are machined by multi-tasking machine tools. There are two kinds of factors which could affect the machining accuracy, which are geometric errors of machines operating under no-load or quasi-static conditions and kinematic errors during processing. The
geometric errors of machines include the geometric errors of the components and the accuracy of assembly of machine tool executive units. The kinematic errors during processing may be caused by thermal distortion [5], chatter vibrations [6], cutting force or component stiffness [7]. The research in this paper is to establish an effective measurement method for identifying geometric errors of multi-tasking machine tools to improve the motion accuracy under a no-load condition.

Up to now, identification methods of the geometric deviations for five-axis machining centers by using the ball bar, the R-test, a touch-trigger probe or other measuring instruments have been proposed by many researchers. For example, J.R.R. Mayer et al. proposed five tests by the ball bar with a single setup to assess the axis motion errors of a trunnion-type A-axis [8]. W.T. Lei et al. used the ball bar to inspect motion errors of the rotary axes on five-axis machining centers. As a result, the servo mismatch of the rotary axes was successfully detected and the gain mismatch errors could be eliminated by tuning the velocity gains of the position control loops of all servo-controlled linear and rotary axes [9,10]. Tsutsumi et al. proposed an algorithm for identifying particular deviations relating to rotary axes in five-axis machining centers [11]. Tsutsumi et al. applied the ball bar to diagnose the motion accuracy of simultaneous four-axis control movements for identifying the eight deviations inherent to five-axis machining centers [12]. Tsutsumi et al. also investigated the kinematic accuracy of five-axis machining centers with a tilting rotary table by two different settings of the ball bar in simultaneous three axis motion [13]. They corrected the squareness deviations of three translational axes for identifying the geometric deviations inherent to five-axis machining centers with an inclined A-axis [14]. S.H. Yang et al. measured and verified the position-independent geometric errors of a five-axis machining center using the ball bar [15]. In addition, other researchers also used the ball bar to explore the measurement and identification methods for geometric errors of five-axis machining centers with a tilting rotary table [16–18] or in universal spindle head type five-axis machining centers [19]. On the other hand, R-test has been applied recently to investigate the geometric deviations identification method for the five-axis machining centers with a swiveling head [20,21]. Ibaraki et al. identified the kinematic errors of five-axis machining centers by developing a simulator and a set of machining tests [22,23]. The simulator graphically presented the influence of rotary axis geometric errors on the geometry of a finished workpiece measured by R-test [24–28]. Furthermore, other measuring instruments and methods are also developed to investigate the geometric deviations for five-axis machining centers. Ibaraki et al. applied a touch-trigger probe to calibrate the error map of the rotary axes for five-axis machining centers by means of on-the-machine measuring of test pieces [29]. J.R.R. Mayer et al. estimated all axis to axis location errors and some axis component errors of a five-axis horizontal machining center by probing a scale enriched reconfigurable uncalibrated master balls artefact [30]. Beñat Iñigo et al. proposed a new strategy to simulate the calibration and compensation of volumetric error in milling machines of medium and large size and laser trackers are used to optimize volumetric error calibration processes [31]. E.Diaz-Tena et al. studied a radical new ‘multitasking’ machine model to give a useful outcome regarding the sensitivity of the machine with respect to the feasible assembly errors or errors produced by light misalignments caused by the machine tool continuous use [32].

However, identification methods for multi-tasking machine tools are seldom reported in this field. In fact, due to the special topological structure of the multi-tasking machine tools, the identification method of geometric deviations is different from that of five-axis machining centers, which are introduced in the aforementioned works. Therefore, the accuracy measurement method for multi-tasking machine tools has not clarified and standardized. This is still a critical issue to be solved in field of precision machining.

In this paper, geometric deviations of multi-tasking machine tools are investigated in two different coordinate systems. Simulation results clarify that the measurements for the B axis in Cartesian coordinate system and the measurements for the C axis in cylindrical coordinate system are proposed to eliminate the influence of mounting errors of the ball bar on circular trajectories. Moreover, the formulae and the identification procedures for geometric deviations are concluded in consideration with the squareness of translational axes. The numerical experiment is conducted to
verify that the proposed method is effective to identify the deviations accurately for multi-tasking machine tools.

2. Coordinate System and Geometric Deviations of Multi-Tasking Machine Tools

Figure 1 shows the schematic view of a multi-tasking machine tool with a swivel tool spindle head in horizontal position. The structural configuration of this multi-tasking machine tool can be expressed as w-C’bZYXB (C1)-t if it is displayed from the work spindle side (w) to the tool spindle side (t), through the bed (b).

![Figure 1. Schematic view of a multi-tasking machine tool with a swivel tool spindle head in horizontal position.](image)

Although there are five controlled axes in the multi-tasking machine tools, the number of geometric deviations is different from that of five-axis machining centers according to the axis configuration and the mutual motion of the axes. Figure 2 shows the geometric deviations and the relation of each axis of the considered machine. Based on the theory of form-shaping system for machine tools [33], the negligible deviations of each axis are deleted from the possible deviations in order from ① to ⑤ shown in Figure 2. Therefore, there are 10 geometric deviations related with two axes of rotation—B and C axes, and three geometric deviations between three translational axes—X, Y, and Z axes, which should be identified to improve the motion accuracy of multi-tasking machine tools. In Figure 2, δx, δy, and δz represent the positional deviations in X-, Y-, and Z-direction, respectively. Similarly, α, β, and γ represent the angular deviations around X-, Y- and Z-axis, respectively. The large suffixes indicate two neighboring axes. For example, δBT presents the positional deviation in X-direction of the tool spindle axis of rotation with respect to B axis origin. The variable αBT presents the squareness error of B axis with respect to the tool spindle axis of rotation about X axis.

The definitions of thirteen geometric deviations are summarized in Table 1 and illustrated in Figure 3. There are four coordinate systems, which are machine coordinate system (Ow-XYZ), B axis coordinate system (Ow-XyZb), C axis coordinate system (Oc-XcYcZc) and tool spindle axis coordinate system (Or-XrYrZr). The machine coordinate system is defined as the reference system, whose origin is the intersection of the rotational center of B axis and the rotational center of C axis when the geometric deviations are all zero and the command values for each axis are set to its initial values. Since B axis and C axis are not directly connected, the upper surface of the C axis table is set as the origin of the machine coordinate system for the convenience.
### Table 1. Symbols of geometric deviations and descriptions.

| Symbol  | Description                                                                 |
|---------|-----------------------------------------------------------------------------|
| $\delta_{x_{BT}}$ | X-direction offset of tool spindle axis of rotation with respect to B axis origin |
| $\delta_{z_{BT}}$ | Z-direction offset of tool spindle axis of rotation with respect to B axis origin |
| $\alpha_{BT}$ | Squareness error of B axis with respect to tool spindle axis of rotation about X axis |
| $\alpha_{XB}$ | Squareness error of B axis of rotation with respect to Z axis motion |
| $\beta_{XB}$ | Initial angular position error of B axis of rotation with respect to X (Z) axis motion |
| $\gamma_{XB}$ | Squareness error of B axis of rotation with respect to X axis motion |
| $\gamma_{XY}$ | Squareness error between X axis motion and Y axis motion |
| $\alpha_{YZ}$ | Squareness error between Y axis motion and Z axis motion |
3. Simultaneous Three-Axis Control Movements and Mathematical Model

3.1. Simultaneous Three-Axis Control Movements

Simultaneous three-axis control movements which include two linear axes and one rotary axis can be conducted by means of the ball bar both in cylindrical coordinate system and in Cartesian coordinate system. The motions are named according to the sensitive direction of the ball bar as radial direction, tangential direction, axial direction in cylindrical coordinate system, illustrated in Figure 4, and X-, Y- and Z-direction in Cartesian coordinate system, illustrated in Figure 5.

\[ \beta_{yz} \] Squareness error between Z axis motion and X axis motion
\[ \delta_{x_{c_z}} \] X-direction offset of C axis origin with respect to machine coordinate origin
\[ \delta_{y_{c_z}} \] Y-direction offset of C axis origin with respect to machine coordinate origin
\[ \alpha_{c_z} \] Parallelism error of C axis of rotation with respect to Z axis about X axis
\[ \beta_{c_z} \] Parallelism error of C axis of rotation with respect to Z axis about Y axis
3.2. Mathematical Model

Since the angular deviations are generally less than 1°, the small angle approximation \((\sin \theta \approx \theta, \cos \theta \approx 1, \text{ if the angle } \theta < 0.244 \text{ radians } (14^\circ), \text{ the relative error does not exceed 1%})\) is assumed and second order errors are neglected. Moreover, for small rotation matrices, the order of rotation matrix could be interchanged and it is possible to add and subtract matrices. Therefore, according to this minute rotation approximation theory, homogeneous transformation matrix (HTM) can be simplified in the mathematical model to express the center coordinates of both the tool spindle side ball (T-side ball) and the work spindle side ball (W-side ball) viewed from the machine coordinate system.

3.2.1. Determination of Center Coordinate \(T\) of the T-Side Ball Viewed from the Machine Coordinate System

When the distance from the rotational center of B axis to the center of the T-side ball is \(R_B\), the center coordinates \(T_T\) of the T-side ball in the tool spindle axis coordinate system are expressed by Equation (1).

\[
T_T = \begin{bmatrix} 0 & 0 & -R_B \\ 1 & 0 & 0 \end{bmatrix}^T \tag{1}
\]

As there are angular deviation \(\alpha_{BT}\) and positional deviations \(\delta_{x_{BT}}, \delta_{z_{BT}}\) between B axis and the tool spindle axis, the homogeneous transformation matrix \(M_{BT}\) from the tool spindle axis coordinate system to the B axis coordinate system is expressed by Equation (2) if the above simplified homogeneous coordinate transformation is used.

\[
M_{BT} = \begin{bmatrix} 1 & 0 & 0 & \delta_{x_{BT}} \\ 0 & 1 & -\alpha_{BT} & 0 \\ 0 & \alpha_{BT} & 1 & \delta_{z_{BT}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
\]

In the same way, the homogeneous transformation matrix \(M_{XB}\) between the X axis and B axis is defined as following.

\[
M_{XB} = \begin{bmatrix} 1 & -\gamma_{XB} & \beta_{XB} & 0 \\ \gamma_{XB} & 1 & -\alpha_{XB} & 0 \\ -\beta_{XB} & \alpha_{XB} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
\]

The T-side ball circularly moves around the B axis with radius \(R_B\). The circular motion of the T-side ball is expressed by the transformation matrix \(E_B\) shown in Equation (4), using the rotation angle \(\varphi\) of the B axis.

\[
E_B = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \varphi & 0 & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
\]

The squareness of the X, Y and Z translational axes can be expressed by the homogeneous transformation matrix \(M_{YZ}\) and \(M_{XY}\) based on the theory of form-shaping system, as followings.
If the translational motion controlled by the command values of X, Y and Z axis is respectively expressed by $E_X$, $E_Y$ and $E_Z$, the center coordinate of the T-side ball viewed from the machine coordinate system is denoted as following.

$$T = E_Z M_{YZ} E_Y M_{XY} E_X M_{XB} E_B M_{BT} T_T$$

### 3.2.2. Determination of Center Coordinate W of the W-Side Ball Viewed from the Machine Coordinate System

If the initial position of the center coordinate of the W-side ball in the C axis coordinate system is $W_C (x_{wC}, y_{wC}, z_{wC})$, the center coordinates $W$ of the W-side ball viewed from the machine coordinate system are calculated as following.

The homogeneous transformation matrix $M_{CZ}$ between the C axis and Z axis is expressed by Equation (8).

$$M_{CZ} = \begin{bmatrix} 1 & 0 & \beta_{CZ} & \delta x_{CZ} \\ 0 & 1 & -\alpha_{CZ} & \delta y_{CZ} \\ -\beta_{CZ} & \alpha_{CZ} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation of C axis around Z axis is defined by the transformation matrix $E_C$ when the rotation angle $\theta$ of the C axis is used.

$$E_C = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the center coordinate $W$ of the W-side ball viewed from the machine coordinate system is denoted as following.

$$W = M_{CZ} E_C W_C$$

### 3.2.3. Determination of the Initial Position $W_C$ of Center Coordinate of the W-Side Ball

The W-side ball is positioned based on the center coordinate of the T-side ball, so it is necessary to add the influence of geometric deviations to the initial position of the W-side ball.

The mounting position $W_C'$ of the W-side ball in the machine coordinate system is same with the center coordinate $T$ of the T-side ball in the machine coordinate system. Further, the initial position of the W-side ball is decided only by the command values of the translational axes without the B axis rotation. Therefore, it can be calculated by removing $E_B$ from the Equation (7), expressed as the following Equations (11) and (12) for setup of the B and C axes measurements, respectively.

When W-side ball is set for the B axis measurement,

$$W_C' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_C + R_B \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -\gamma_{xy} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\beta_{yz} & \alpha_{yz} & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\alpha_{xy} & 0 & 0 \\ -\beta_{xy} & \alpha_{xy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -\gamma_{xy} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\beta_{xy} & \alpha_{xy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\alpha_{xy} & 0 & 0 \\ -\beta_{xy} & \alpha_{xy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\alpha_{xy} & 0 & 0 \\ -\beta_{xy} & \alpha_{xy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

When W-side ball is set for the C axis measurement,
The transformation matrix from the machine coordinate system to the C axis coordinate system is performed by the inverse transformation of Equation (8). Moreover, when the W-side ball is mounted, the command value of the C axis is zero, so there is no need to consider the C axis rotation in the Equation (10). Therefore, the conversion from \( W' \) to \( W \) is expressed by Equation (13).

\[
W = \begin{bmatrix}
1 & 0 & -\beta_{CZ} & -\delta x_{CZ} \\
0 & 1 & -\alpha_{CZ} & -\delta y_{CZ} \\
0 & 0 & 0 & 1 \\
\end{bmatrix} W' 
\]

### 3.2.4 Calculation of the Difference \( \Delta L \) between Reference Length and Measured Length of Ball Bar

From the above equations, the relative distance \( L \) between the T-side ball and the W-side ball can be calculated by the center coordinate \( T(x_t, y_t, z_t) \) of the T-side ball and the center coordinate \( W(x_w, y_w, z_w) \) of the W-side ball when a command value is given to each axis. The relative distance \( L \) is different from the reference length \( L_B \) of the ball bar because of the existence of geometric deviations in the multi-tasking machine tools. Therefore, the ball bar length change amount \( \Delta L \) can be obtained from the relative distance \( L \) by subtracting the ball bar reference length \( L_B \) as written in Equation (14).

\[
\Delta L = \sqrt{(x_t - x_w)^2 + (y_t - y_w)^2 + (z_t - z_w)^2} - L_B 
\]

### 4. Simulation

Influence of each deviation on the eccentricity is investigated by using the above mathematical model. The commands separately given to each axis during simulation are shown in Table 2.

| Command X | \((L_a + R_a) \sin \varphi\) | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Command Y | 0               | \(L_a\)         | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) | \(R_a \sin \varphi\) | \(R_a \cos \varphi\) |
| Command Z | \(Z_c + (L_a + R_a) \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) |
| Command B | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) |
| Command C | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) | \(Z_c + R_a \cos \varphi\) |

Table 2. Commands given to each axis during simulation.

The simulation of the simultaneous three-axis control movements is conducted at the condition of \(L_a = 100 \text{ mm}\), \(Z_c = 100 \text{ mm}\), \(R_a = 200 \text{ mm}\), \(R_c = 50 \text{ mm}\). ±0.005 degrees and ±20 \( \mu \)m are given as angular deviations and positional deviations, respectively. Then, simulation results are obtained as

\[
R_{BT} = \sqrt{R_{BT}^2 + L_{BT}^2}; R_{CT} = \sqrt{R_{CT}^2 + L_{CT}^2}
\]
shown in Table 3. The dotted circle represents theoretical trajectory when there is no geometric deviation and the red or blue one represents changed trajectory affecting by geometric deviations. The figures show that if only one of the thirteen geometric deviations exist, the red or blue circular trajectory will appear and reflect the effect of the given deviation on the eccentricity. The blank part shows that there is no influence of the geometric deviation on trajectory. For example, for B axis radial direction measurement, when a value of +20 μm is given to δBT while other twelve geometric deviations are all zero, eccentricity of trajectory occurs in -X axis direction. On the contrary, when a value of −20 μm is given to δBT, eccentricity of trajectory occurs in +X axis direction.

The eccentricities occur by the following three reasons:

- The position of trajectory center is changed; for example, the effect of δxBT on eccentricity in case of the B axis radial measurement.
- The size of trajectory radius is changed; for example, the effect of δzBT on eccentricity in case of the B axis radial measurement.
- The shape of trajectory is changed; for example, the effect of βYZ on eccentricity in case of the B axis radial measurement.

Table 3. Effect of geometric deviations on the eccentricities of circular trajectories in cylindrical coordinate system and in Cartesian coordinate system.

|          | δxBT | δzBT | αXB | βXB | γXB | αBT | δxCZ | δyCZ | αCZ | βCZ | αYZ | βYZ | γXY |
|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Radial   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Tangential|      |      |      |      |      |      |      |      |      |      |      |      |      |
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Axial    |      |      |      |      |      |      |      |      |      |      |      |      |      |
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| X-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Y-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |
| B axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Z-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Radial   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Tangential|      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Axial    |      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| X-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Y-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |
| C axis   |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Z-direction|      |      |      |      |      |      |      |      |      |      |      |      |      |

Table 3. Effect of geometric deviations on the eccentricities of circular trajectories in cylindrical coordinate system and in Cartesian coordinate system.

4.1. Influence of Mounting Errors of Ball Bar on Circular Trajectories

Considering the influence of mounting errors of the T-side ball, the center offset of the T-side ball (xT, yT, zT) with respect to the tool spindle axis is added to the initial center coordinate T in the Equation (1), expressed as the following Equation (15). In the same way, the center offset of the W-side ball (xw, yw, zw) with respect to the C axis origin is added to the initial center coordinate WC in the Equation (13), expressed as the following Equation (16). The simulation is performed in two
coordinate systems at the condition that the offset is 20 μm and the obtained results are shown in Table 4.

\[ T_T = \begin{bmatrix} x_T \\ y_T \\ z_T \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -R_0 \\ 1 \end{bmatrix} \] (15)

\[ W_C = \begin{bmatrix} x_W \\ y_W \\ z_W \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -\beta_{CZ} & -\delta x_{CZ} \\ 0 & 1 & \alpha_{CZ} & -\delta y_{CZ} \\ \beta_{CZ} & -\alpha_{CZ} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} W_C' \] (16)

**Table 4. Influence of mounting errors of ball bar on the eccentricity of circular trajectories.**

|                      | \( x_W \) | \( y_W \) | \( z_W \) | \( x_T \) | \( y_T \) | \( z_T \) |
|----------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| B axis Radial        | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| Cylindrical coordinate system |
| B axis Tangential    | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| B axis Axial         | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| Cartesian coordinate system |
| B axis X-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| B axis Y-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| B axis Z-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| C axis Radial        | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| Cylindrical coordinate system |
| C axis Tangential    | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| C axis Axial         | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| Cartesian coordinate system |
| C axis X-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| C axis Y-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |
| C axis Z-direction   | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  | ![Image]  |

In Table 4, a dotted circle represents theoretical trajectory when there is no mounting errors of ball bar and the red circle represents changed trajectory affecting by one mounting error for each measurement. The blank indicates the trajectory has not changed. It is found that the eccentricity of circular trajectories in these two coordinate systems are strongly affected by the mounting errors of the T-side ball. Therefore, it is crucial to coincide the center of the T-side ball to the tool spindle before conducting measurements. However, the mounting errors of the W-side ball do not affect the eccentricity in Cartesian coordinate system for the B axis measurements, and in cylindrical coordinate system for the C axis measurements. Therefore, to eliminate the influence of mounting errors of the W-side ball on the eccentricities of circular trajectories, the B axis measurements in Cartesian coordinate system and C axis measurements in cylindrical coordinate system are proposed to identify the geometric deviations of multi-tasking machine tools.
4.2. Influence of Squareness of Translational Axes

The influence of squareness deviations of translational axes, \( \alpha_{YZ} \), \( \beta_{YZ} \), and \( \gamma_{XY} \), on the eccentricity of circular trajectories are shown at the last three columns in Table 3. It is observed that the squareness deviation \( \gamma_{XY} \) only affect the eccentricity of circular trajectory in case of the C axis Y-direction measurement in Cartesian coordinate system. Therefore, it is indispensable to conduct C axis Y-direction measurement for identifying the squareness deviations \( \gamma_{XY} \). However, measurement accuracy of the eccentricity for the C axis Y-direction is strongly affected by the mounting errors of W-side ball and T-side ball, shown in Table 4. Besides, \( \gamma_{XY} \) is very small theoretically and it is difficult to identify \( \gamma_{XY} \) correctly by my proposed identification method. Therefore, the identification for \( \gamma_{XY} \) will not be researched in this study.

5. Identification Procedures and Validity

5.1. Mathematical Expressions between Eccentricities and Geometric Deviations

Mathematical expressions between eccentricities of circular trajectories and geometric deviations are summarized in Table 5 according to the simulation results. \( ex \), \( ey \) and \( ez \) represent the components of eccentricities in X-, Y- and Z-direction, respectively. The subscripts indicate the type of measurements, for example, CA and BY represent measurements of the C axis axial direction and B axis Y-direction, respectively. Among the eccentricity, the positional deviation appears as eccentricity is, while the angular deviation, multiplied by coefficient \( Z_{C} \), \( R_{B} \) or \( R_{C} \), appears in eccentricity. The deviation is positive if the direction of the deviation is identical with the direction of the eccentricity. On the contrary, the deviation is negative.

Table 5. Mathematical expressions between eccentricities and geometric deviations.

|       | \( ex \)                  | \( ey \)                  | \( ez \)                  |
|-------|---------------------------|---------------------------|---------------------------|
| B: X-direction | \( \delta_{x_{BT}} \)     | —                         | \( \delta_{x_{BT}} - R_{B} \beta_{x_{B}} - R_{B} \beta_{y_{Z}} \) |
| B: Y-direction | \( \delta_{x_{BT}} \)     | —                         | \( R_{B} \beta_{x_{B}} + R_{B} \alpha_{x_{Y}} \) |
| B: Z-direction | \( R_{B} \beta_{x_{B}} - \delta_{x_{BT}} \) | \( \delta_{x_{BT}} \) | —                         |
| C: Radial | \( -\delta_{x_{BT}} + R_{B} \beta_{x_{B}} + \delta_{x_{CZ}} + Z_{C} \beta_{x_{CZ}} + R_{B} \gamma_{x_{Z}} \) | \( -\delta_{x_{BT}} + \delta_{x_{CZ}} + Z_{C} \alpha_{x_{CZ}} - R_{B} \alpha_{x_{Y}} - R_{B} \alpha_{x_{T}} + \delta_{x_{CZ}} - Z_{C} \alpha_{x_{CZ}} \) | —                         |
| C: Tangential | \( R_{B} \gamma_{x_{Z}} \) | \( R_{B} \gamma_{x_{Z}} \) | —                         |
| C: Axial | \( R_{C} \beta_{x_{CZ}} - R_{C} \beta_{y_{Z}} \) | \( R_{C} \alpha_{x_{CZ}} - R_{C} \alpha_{y_{Z}} \) | —                         |

It is found that the expressions of the C axis tangential measurement have only opposite arithmetic signs comparing to those of the C axis radial measurement, and the expressions of the B axis X-direction measurement is similar with those of the B axis Z-direction measurement. Furthermore, it has been known that a pitch error of the worm gear affects the eccentricity of circular trajectory measured in the tangential direction. Thus, the C axis tangential measurement and B axis Z-direction measurement are not discussed to identify geometric deviations in this study.

In summary, considering the squareness of translational axes, twelve geometric deviations for multi-tasking machine tools can be identified by measuring the eccentricities of circular trajectories of the B axis X-direction, B axis Y-direction, C axis radial direction and C axis axial direction.

5.2. Formulae Considering Squareness of Translational Axes to Calculate Geometric Deviations

To determine 12 variables from eight expressions obtained from above four sensitive direction measurements, measurement conditions should be devised as following.
Firstly, as only one geometric deviation affects the X axis component of eccentricity in the B axis X-direction and Y-direction measurements, two deviations $\gamma_{XB}$ and $\delta_{ZBT}$ are directly identified based on the corresponding eccentricities measured from the B axis X-direction and Y-direction, expressed as Equations (17) and (18).

$$\gamma_{XB} = \frac{e_{XBY}}{R_B}$$  \hspace{1cm} (17)

$$\delta_{ZBT} = e_{BX}$$       \hspace{1cm} (18)

Secondly, when the distance between the center of the W-side ball and the workbench surface of the C axis is changed from $Z_C$ to $Z_C'$, new eccentricities $e_{XCR}'$ and $e_{YCR}'$ are obtained by measuring the radial direction of the C axis again. Then, another six geometric deviations $\delta_{XZ}, \beta_{ZB}, \beta_{YZ}, \alpha_{ZC}, \alpha_{YZ}$ and $\alpha_{XB}$ are calculated by the following Equations (19)–(24).

$$\delta_{XZ} = e_{XCR} + e_{ZBX} - \frac{Z_C'(e_{XCR}' - e_{XCR})}{Z_C'-Z_C}$$ \hspace{1cm} (19)

$$\beta_{ZB} = \frac{e_{XCR}' - e_{XCR}}{Z_C'-Z_C} - \frac{e_{XCA}}{R_C}$$ \hspace{1cm} (20)

$$\beta_{YZ} = \frac{e_{XCR}' - e_{XCR}}{Z_C'-Z_C} - \frac{e_{XCA}}{R_C}$$ \hspace{1cm} (21)

$$\alpha_{ZC} = \frac{e_{YCR} - e_{YCR}'}{Z_C'-Z_C}$$ \hspace{1cm} (22)

$$\alpha_{YZ} = \frac{e_{YCR} - e_{YCR}'}{Z_C'-Z_C} + \frac{e_{YCA}}{R_C}$$ \hspace{1cm} (23)

$$\alpha_{XB} = \frac{e_{XBY}}{R_B} - \frac{e_{YCA}}{R_C} - \frac{e_{YCR} - e_{YCR}'}{Z_C'-Z_C}$$ \hspace{1cm} (24)

Thirdly, when the radius value is changed from $R_B$ to $R_B''$, new eccentricities $e_{XCR}''$ and $e_{YCR}''$ are obtained from the C axis radial measurement. The remaining deviations $\delta_{ZBT}, \alpha_{XZ}$ and $\delta_{YCR}$ are calculated by the following Equations (25)–(28).

$$\delta_{XZ} = e_{XBY} + \frac{e_{XCR}'}{R_B} - \frac{e_{XCR}''}{R_B'' - R_B}$$ \hspace{1cm} (25)

$$\beta_{ZB} = \frac{e_{XCR}'}{R_B'' - R_B} - \frac{e_{XCR}'}{Z_C'-Z_C} + \frac{e_{XCA}}{R_C}$$ \hspace{1cm} (26)

$$\alpha_{ZC} = \frac{e_{YCR}'}{R_B'' - R_B} - \frac{e_{XBY}}{R_B}$$ \hspace{1cm} (27)

$$\delta_{YCR} = \frac{R_B(e_{YCR}' - e_{YCR}'')}{R_B'' - R_B} + \frac{Z_C'e_{YCR} - Z_C'e_{YCR}'}{Z_C'-Z_C}$$ \hspace{1cm} (28)

5.3. Measurement to Identify Geometric Deviations

To identify 12 geometric deviations based on the eccentricity of circular trajectory controlled by simultaneous three-axis motions, the following six measurements by means of the ball bar are indispensable, as shown in Figure 6.

Step 1: Four measurements, which are the B axis X-direction, B axis Y-direction, C axis axial direction and C axis radial direction, are conducted at the condition of $L_B$, $Z_C$ and $R_B$.

Step 2: The C axis radial measurement is conducted for the second time at the condition of $L_B$, $Z_C'$ and $R_B$. 

Step 3: The C axis radial measurement is conducted for the third time at the condition of \( L_B, Z_C' \) and \( R_B'' \).

Figure 6. Measurement procedures to identify twelve geometric deviations.

As a result, a ball bar measurement should be conducted once in the B axis X-direction, B axis Y-direction, C axis axial direction, and three times in C axis radial direction. After that, including the squareness of translational axes, 12 geometric deviations can be calculated by Equations (17)–(28).

5.4. Validity of the Proposed Identification Method

Numerical experiments are performed to confirm the validity of the formulae for geometric deviations. At first, by using a random number table, the initial values of geometric deviations are selected and substituted into the mathematical model. Six numerical experiments are conducted at \( R_C = 50 \text{ mm}, R_B = 360 \text{ mm}, R_B'' = 410 \text{ mm}, Z_C = 300 \text{ mm} \) and \( Z_C' = 340 \text{ mm} \). Table 6 summarizes the eccentricities obtained at each measurement. Finally, the eccentricities are substituted into the above formulae to calculate the geometric deviations. The differences between the initial values and the identified values are shown in Table 7.

**Table 6.** Eccentricities of circular trajectories in numerical experiments.

| Direction | B Axis (µm) | C Axis (µm) |
|-----------|-------------|-------------|
|           | X \( ^a \) | Y \( ^a \) | Axial \( ^a \) | Radial \( ^a \) | Radial \( ^b \) | Radial \( ^c \) |
| \( e_x \) | 16.63       | -11.24      | 0.29         | 25.73          | 26.89          | 28.97          |
| \( e_y \) | -           | -           | 0.05         | -19.59         | -20            | -23.26         |
| \( e_z \) | -4.02       | 10.05       | -            | -              | -              | -              |

\( ^a \) \( R_B = 360 \text{ mm}; Z_C = 300 \text{ mm}; \)
\( ^b \) \( Z_C' = 340 \text{ mm}; \)
\( ^c \) \( R_B'' = 410 \text{ mm}. \)

**Table 7.** Difference between initial values and identified values.

| | Initial Values | Identified Values | Difference |
|----------------|-----------------|---------------|------------|
| \( \delta x_{BT} \) (µm) | 11.00           | 10.96         | -0.04      |
| \( \delta z_{BT} \) (µm) | 17.00           | 16.63         | -0.37      |
| \( \alpha_{BT} \) (") | 7.63            | 7.69          | 0.06       |
| \( \alpha_{SB} \) (") | 3.51            | 3.44          | -0.07      |
| \( \beta_{SB} \) (") | 3.92            | 3.80          | -0.12      |
| \( \gamma_{SB} \) (") | 6.39            | 6.44          | 0.05       |
| \( \alpha_{YZ} \) (") | 2.27            | 2.32          | 0.05       |
| \( \beta_{YZ} \) (") | 4.74            | 4.79          | 0.05       |
From the results, it is found that the difference of angular deviations is less than 0.12 arcsecond and the difference of positional deviations is smaller than 0.37 µm. Therefore, 12 geometric deviations of multi-tasking machine tools, including the squareness deviations, αyz and βyz, can be identified correctly by using the above formulae.

6. Conclusions

In this paper, a method to identify geometric deviations which exist in multi-tasking machine tools on the basis of the trajectories of simultaneous three-axis control motions is investigated. The proposed method is applied to a multi-tasking machine tool with a swivel tool spindle head in horizontal position and the identification of the deviations is carried out. From the numerical experiments, the validity of the proposed identification method is clarified. Conclusions are summarized as following.

(1) The identification method for 12 geometric deviations, in which two squareness deviations of translational axes αyz and βyz are included, is proposed.
(2) From the simulation results, it is confirmed that in order to eliminate the influence of the mounting errors of the W-side ball on the eccentricities of the circular trajectories, measurements for the B axis should be performed in Cartesian coordinate system and those for the C axis should be performed in cylindrical coordinate system.
(3) Considering the squareness of translational axes, six measurements by means of the ball bar are necessary to identify twelve geometric deviations.
(4) The results of numerical experiments agree well with the given intentional deviations. Therefore, the influence of the analysis accuracy of the formulae on the identification could be considered to be negligible.

It can be concluded that the proposed identification method and the measurement procedure can be sufficiently utilized to identify geometric deviations for multi-tasking machine tools, in which the squareness of translational axes is taken into consideration.

At the next stage, the validity of the proposed method will be verified by an actual measurement of a multi-tasking machine tool. According to the proposed identification method, six measurements by using a ball bar will be conducted firstly and the eccentricities in each translational axis are obtained. Then, 12 geometric deviations of the considered machine tool will be calculated based on the formulae proposed in this paper. Finally, the measurements by a ball bar will be conducted again with compensation of deviations. If the eccentricities would disappear after the compensation, it can be concluded that the proposed method is effective to identify the geometric deviations accurately for multi-tasking machine tools.

Author Contributions: Conceptualization, M.T. and K.N. (Keichi Nakamoto); methodology, N.K. and M.T.; software, N.K., K.N. (Keisuke Nishizawa) and Y.Y.; validation, M.T. and K.N. (Keisuke Nishizawa); formal analysis, Y.Y. and K.N. (Keisuke Nishizawa); investigation, K.N. (Keisuke Nishizawa) and Y.Y.; resources, K.N. (Keiichi Nakamoto); data curation, N.K., K.N. (Keisuke Nishizawa) and Y.Y.; writing—original draft preparation, Y.Y.; writing—review and editing, K.N. (Keiichi Nakamoto) and M.T.; visualization, Y.Y. and M.T.; supervision, K.N. (Keiichi Nakamoto) and M.T.; project administration, K.N. (Keiichi Nakamoto); funding acquisition, K.N. (Keiichi Nakamoto). All authors have read and agreed to the published version of the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.
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