Some Speculations on the Ultimate Planck Energy Accelerators

A. Casher
Tel Aviv University

S. Nussinov
Tel Aviv University
and Institute for Advanced Study
Princeton, NJ 08540

1 Abstract

The inability to achieve in the present universe, via electromagnetic or gravitational acceleration, Planck energies for elementary particles is suggested on the basis of several, some relatively sophisticated, failed attempts. This failure is essential for schemes were the superplanckian regime for the energies of elementary particles is "Unphysical". The basic observation is that this failure to achieve superplanckian energies naturally occurs in our universe of finite age and horizon. It does tie up in a mysterious fashion these cosmological quantities and elementary physics parameters such as the masses of the lightest charged fermions.

2 Introduction

The Planck mass, \( m_P = (\hbar c/G_{\text{Newton}})^{1/2} \approx 10^{19} \text{ GeV} \) and corresponding length \( l_P = \hbar/m_Pc \approx 10^{-33} \text{ cm} \) or time \( t_P = l_P/c \), are of fundamental importance, marking the onset of strong non-renormalizable quantum gravity effects. Planck mass objects could be the end products of black holes emitting Hawking radiation [1], obtained when the radius, mass and temperature of the black hole become Planckian. At this point the semiclassical argument for the radiation – based in particular on the existence of a well defined horizon breaks down due to quantum metric–fluctuations. Also the total mass of the black hole, \( m_P \), disallows emission of further quanta with \( E \approx T \approx m_P \). Stable Planck mass objects have been offered [2] as a possible solution to the apparent unitarity (or information) crisis encountered if the black hole completely evaporates. It has been conjectured [3] that a reciprocal mechanism exists, which protects mini black holes from crossing the \( M_{BH} = m_P \) line downwards, and
prevents elementary particles (field quanta) from crossing the $m_{\text{particle}} = m_P$ line upwards. A possible realization for this mechanism calls for accumulation of states at $m = M = m_P$.

A new, radical, approach of P. Mazur\cite{4} attempts to side-step the problems of non-renormalizeability due to elementary super-planckian excitations and of collapsing mini-black holes, in a kinematic rather than a dynamic way. The Planck length is introduced into a new set of commutators $[x, y] \sim [z, t] \sim il_P^2$. The corresponding uncertainty relations $\Delta r \Delta t \geq l_P^2/2$ excludes the problematic domain of one gravitating quantum degree of freedom when the latter is inside its own Schwartzschild radius – just like $[x, p] = i\hbar$ and Heisenberg’s uncertainty avoids the singular spiralling to the origin of a radiating electron in the classical Coulomb problem. The (area) discretization implied by the new commutators suggests a finite difference analogue of Schrödinger’s equation for the self gravitating quantum bubble\cite{5} that indeed implements the above exclusion. The finite difference coordinate space equation implies periodicity in momentum space with a period $\sim m_P$. The Planck mass $m_P$ is the limiting value of momentum or energy of any (elementary) particle\cite{5}. This is clearly stronger than the assumption that no two oppositely moving transplanckian (elementary) particles can collide in an S wave to form a mini-black hole of mass $M_{BH} \geq m_P$.

3 The Implications of the Maximal P, E Postulate

The suggestion that one cannot boost an electron or proton to super-planckian energies flies in the face of Lorentz invariance, one of the best tested principles in nature. It has been argued\cite{4} that due to a discrete coordinate structure underlying space time at short $\approx l_P$ scales, there could be – as in ordinary crystals – “Umklapp” processes which “absorb” a reciprocal lattice momentum, keeping $E, P \leq m_P$. Nonetheless, the following gedanken experiment points at some difficulty. Suppose that a proton is accelerated inside some microscopic device $D_1$ (see fig 1) to $\gamma = 11$. The whole $D_1$ device in turn sits within a larger setup $D_2$, which boosts $D_1$ to $\gamma = 11$. $D_2$ in turn sits inside $D_3$, etc., etc. The device $D_{18}$ is then also boosted by $D_{19}$ to $\gamma = 11$, thus finally achieving transplanckian energies $E = \gamma^{19} GeV \approx 10^{20} GeV$ for our proton. To avoid this we need to assume that the last device $D_{19}$ “knows” that eighteen layers down, inside the innermost microscopic $D_1$, one proton is about to break the “Planck Barrier”. This possibility is even more unlikely than the Grimm brothers’ fairy tale about the sensitive princess who could not fall asleep due to one pea under seventeen mattresses. It would be truly paradoxical if space-time on large scales was completely flat and uniform. However, precisely due to gravity ($G_N > 0$) and finite propagation velocity ($c < \infty$), the universe is curved and only a finite horizon has opened up since the big bang. To maintain the “Planck Barrier”, an ultimate micro-macro connection may be at work. It should force the largest device $D_{19}$ in our gedanken experiment to be larger than the observed universe, and cause any alternative less massive

\footnote{Clearly, massive extended macroscopic objects are irrelevant, as the quantum gravity divergences come from concentrating $m_P$ energy densities within $l_P$ size. The proton is composite, and so could be the electron (and quark). We are assuming, however, finite compositeness: 3 quarks, finite # of” preons”, etc., which suffices for our purpose.}

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and/or more compact design to break due to finite strength of materials, (an issue involving \( \hbar > 0 \)) or collapse into a black hole.

### 4 Attempts to Build (Gedanken) Super-planckian Accelerators

By considering a few gedanken accelerating devices, we would like to suggest that \( E > m_P \) may be inaccessible in our universe.

(a) **Electromagnetic acceleration methods**

(1) **Laser Beam Acceleration:** An intense laser beam can accelerate charged particle to high energies by repeated Compton scattering. As the particle approaches the putative super-planckian regime, it becomes extremely relativistic. In the particle’s rest frame the photons will be strongly redshifted (by a \( \gamma^{-1} \) factor) and \( \sigma = \sigma_{\text{Thompson}} \approx \frac{\pi \alpha^2}{m^2} \) is appropriate. If we have a flux of energy \( \Phi_E \) the particle gains energy at a rate:

\[
\frac{dW}{dt} = \Phi_E \sigma (1 - \beta) = \Phi_E \frac{\pi \alpha^2}{m^2} \frac{m^2}{2m_p^2} \approx \Phi_E \frac{\alpha^2}{m_p^2}
\]

The key factor entering the rate is the relative velocity \( (1 - \beta) \approx \frac{1}{2\gamma} \). Thus even if we allow Hubble time acceleration \( t_H \approx 3.10^{17} \text{ sec} \), we need, regardless of the mass \( m \) of the accelerated particle, a minimal energy flux \( \Phi_E = \frac{m_p}{\alpha \gamma^2 t_H} \approx 3.10^{68} \text{ ergs/cm}^2/\text{sec} \). The corresponding E.M. energy density is \( U \approx 10^{58} \text{ ergs/cm}^3 \approx 3.10^{41} (\text{ ergs/cm}^2)^2 \). Using \( U \approx E^2/8\pi \) we find that the E field in the beam should exceed \( \approx 10^{34} \text{ Volts/cm} \). Such fields are completely untenable. The vacuum itself “sparks” and produces \( e^+e^- \) pairs at a rate: \( dN/dt dV \approx (eE)^2/16\pi^2 e^{-eE/m} \).

(2) **Linear Accelerators:** The synchrotron radiation losses for a Planck energy electron or proton in a circular orbit are prohibitive: even if we take \( R = r_{\text{Hubble}} \approx 10^{26} \text{ meters} \), we find from [7]:

\[
\delta W \text{ (Synchrotron - loss)} \approx \left( \frac{E}{m} \right)^4 \frac{1}{R}
\]

that a Planck energy electron (or proton) loses all its energy in traversing only \( 10^{-25} \) (or \( 10^{-14} \)) of a turn! We should therefore consider (truly straight!) linear accelerators producing constant gain \( G = \frac{\Delta W}{\Delta x} \) with minimal radiation losses. Since the electromagnetic fields originate in surface charges and currents, \( G_{\text{max}} \approx \frac{eV}{A} = \frac{10 \text{ GeV}}{\text{meter}} \sim Ry/a_{\text{Bohr}} \approx \frac{1}{2} m_e^2 \alpha^3 \) is a maximal gain allowed. \( (R_y = 13.6 \text{ ev} \) is the Rydberg energy constant and \( a_{\text{Bohr}} = 0.55 A_0 \) is the Bohr radius). E fields of such a magnitude destroy materials by “skimming” electrons.
from the top of the Fermi sea. This implies that the minimal acceleration length required in order to achieve planckian energies is \( L_{\text{min}} = \frac{m_p}{G_{\text{max}}} \approx 10^{20} \text{ cm} \approx 30 \text{ Parsecs.} \) (Present technology is limited by discharge breakdown at local defects to \( G'_{\text{max}} \sim \text{few MeV} \text{ meter}^{-1} \) and the required \( L'_{\text{max}} \) is 100 K-Parsecs!). Such a long cylinder is unstable with respect to bending under tydal forces and/or self gravity. If bent slightly into a (roughly) circular arc of height \( h \), the prolongation is \( \Delta L \approx \frac{h^2}{3L} \) (see fig 2). The corresponding elastic energy is

\[
\Delta E_{\text{Ela}} \approx N(Ry) \left( \frac{\Delta L}{L} \right)^2 \approx \frac{M}{m_N} \frac{1}{2} m_e \alpha^2 \left( \frac{h^2}{3L^2} \right)^2 \tag{4}
\]

with \( Ry \approx \frac{1}{2} m_e \alpha^2 \) representing a bond energy \((\Delta L/L)^2 \text{Ry} \text{ is the penalty for } \Delta L \text{ stretch}\) and \( N = \frac{M}{m_N} \) [with \( M(m_N) \) the mass of the cylinder (nucleon) respectively] is the number of bonds. If \( \Phi''_G \approx \frac{1}{r_{\text{Hubble}}} \) is the local \( g \) gradient \( \frac{\hat{g}}{\alpha} \) then the tydal energy gain is:

\[
\Delta E_{\text{tyde}} = M \Phi''_G h^2 = \frac{M h^2}{r_H^2} \tag{5}
\]

Also the self gravity gain is:

\[
\Delta E_{S.G.} = G_N M^2 \Delta \left( \frac{1}{L} \right) \approx \frac{M^2 h^2}{m_P^2 L^3} \tag{6}
\]

To ensure stability we should keep \( \Delta E_{\text{Ela}} > \Delta E_{\text{tyde}} \), i.e. we need

\[
h \geq \frac{L^2 \sqrt{m_N/m_e}}{r_H \alpha} \geq 10^{16} \text{ cm} \tag{7}
\]

where we used \( L \geq L_{\text{min}} = 10^{20} \). A weaker bound \( h \geq 10^{11} \text{ cm} \) follows from \( \Delta E_{\text{Ela}} \geq \Delta E_{S.G.} \). Even the weaker bound already implies that the arc in fig (2-b) is \( \theta = \frac{h}{L} \approx 10^{-9} \text{ rad} \) and a curvature radius \( R \approx \frac{h}{\theta} \approx 10^{20} \text{ cm} \approx 10r_H \). From eq (5) and discussion thereafter, catastrophic synchrotron radiation again follows. In principle we could attempt to avoid the accelerator pipe, and have a prearranged set of \( N \) acceleration stations \( S_1...S_N \) each of mass \( m \), spaced by length \( l \) and each accelerating by \( \Delta E = m_P/N \). To ensure a straight, kinkless, path, we would need to correct each time in the \( (n+1)th \) station the momentum (and location) of the accelerated particle. Indeed if the “aperture” of each station is \( R \), the emerging particle will have a transverse momentum uncertainty \( \Delta p = \frac{h}{R} \) and the corresponding angular uncertainty is \( \Delta \theta = \frac{h}{R m_P} \). This leads to \( \Delta l_{n+1} \approx \Delta \theta \approx \text{Min} \left\{ \frac{h}{m_P}, \frac{l}{R}, R \right\} \) transverse uncertainty upon arrival at \( S_{n+1} \). To correct for this we may need to move the magnets, etc. at \( S_{n+1} \) by a similar amount. A light signal from \( S_n \) arriving at \( S_{n+1} \) a distance \( \delta l = l(1-\beta) = l \frac{m_e}{m_P^2} \) ahead of the accelerated particle could facilitate this providing \( \delta l \geq \Delta l_{n+1} \). This requires, however, \( l \approx m_P^2/m_e^2 R \approx (10^{45} - 10^{38})R \) for \( X = \text{electron, nucleon}, \) which is clearly unacceptable.

more appropriately \( \Phi''_G \approx \frac{v_{\text{escape}}}{R} \) with \( v_{\text{escape}} \) the escape velocity from a structure (galaxy, cluster of galaxies), etc... of size \( R \). Since, however, these proper motions such as the infall towards the Virgo cluster stand up against the Hubble flow \( \frac{v_{\text{escape}}^2}{R^2} \approx \frac{1}{r_H^2} \approx \Phi''_G \).
(3) Strong Compact and/or Large Scale Magnetic/Electric Fields

The failure of the linear accelerator to achieve planck energies can be traced to the fragility of matter which cannot sustain large $E$ fields $E \geq \alpha^3 m_e^2 \approx \text{Ry}/a_{\text{Bohr}}$. Neutron stars are made of much stronger “nuclear matter” and ideally could sustain $B$ fields up to $B_{\text{max}} \approx m_N^2 = 10^{19} \text{ Gauss}$, corresponding to energy densities $B^2/8\pi \approx m_N^2$. Due to the star’s rotation (or other effects), the charged particle accelerated sees an effective electric field: $E \approx v c B \leq B$ over the relevant scale $R$ (size of star, say). The maximal energy attainable therefore is

$$E_{\text{max}} \leq eBR.$$  (8)

The total mass of the system $M$ exceeds the magnetic contribution, $B^2 R^3$. If the system is not a black hole, then $R_{SW} \equiv \frac{M}{m_{\text{p}} c^2} < R$. Using $M \geq B^2 R^3$, we find that

$$E_{\text{max}} \leq m_{\text{Planck}}^{(9)}$$

i.e. any system producing E.M. fields all the way from neutron stars to galaxies, or any part of the universe, fails by a factor $\sqrt{\alpha} \approx \frac{1}{12}$ to obtain Planck energies.

The most energetic cosmic rays observed to date have energies $\approx 3.10^{11} \text{ GeV}$. These presumably are accelerated on large cosmological scale by weak magnetic fields, or on short neutron star scales. In the first case the maximal energy is limited to $\sim 3.10^{11} \text{ GeV}$ due to the collision with a $3^\circ$ background photons.

(4) Acceleration by Repeated Particle Collisions

Consider next the cascade of collisions sketched in fig 3. We start with many, $N = 2^k$, particles all with the same energy $E_0 > m$ and all momenta pointing roughly to a common point. Pairs of these particles collide: #1 with #2, #3 with #4, etc..., #2k-1 with #2k. All collisions are elastic at low center mass energies and the emerging particles, say #1 and #2, etc., have isotropic angular distributions in the CMS frame of the corresponding pair. This reflects in a uniform distribution of the energy of emerging particles $E_{2k}'$, say in the interval $0 \leq E_{2k}' \leq 2E_0$. Let us assume that $E_2' \geq E_1', E_4' \geq E_3', \ldots E_{2k}' \geq E_{2k-1}'$ (this can always be achieved by relabeling). The expectation values of the energies of each of the more energetic particles in a pair $E_2', E_4'$, etc. averaged over many complete “experiments” is then $\frac{3}{2}E_0$. We will make the (drastically!) simplifying assumption that $E_2'E_4'$...etc. do in fact always assume their average values, i.e. $E_2' = E_4' = \ldots E_{2k}' = \frac{3}{2}E_0$. We next arrange for particle 2', 6', etc. to collide. Under the same simplifying assumption we have for the next generation of particles emerging from this second (k=2) series of collisions with energies $E_4'' = E_4' = \ldots E_{2k}' = \left(\frac{3}{2}\right)^2 E_0$. We will label $E'' = E^{(2)}, E''' = E^{[3]}, \text{ etc.}$ This process continues for k-1 stages. At the last (kth) stage particle $2^{k[k-1]}$ with energy $\left(\frac{3}{2}\right)^{k-1} E_0$ collides with particle $(2^{k-1})^{k-1}$ with the same energy producing finally $E_f = \left(\frac{3}{2}\right)^k E_0 \geq m_{\text{p}}$.

Clearly what we attempt here is a highly schematic (and idealized!) implementation of the abstract concept of “nested accelerators” of fig 1. However, as we show next, even this realization fails on physical, kinematical and geometrical grounds.
In order to achieve \( \gamma_{\text{final}} \approx 10^{22} - 10^{19} \) (for \( m = m_{\text{electron}} \) or \( m_{\text{nucleon}} \) respectively) we need \( \left( \frac{3}{2} \right)^k \geq 10^{20} \). For each energetic particle with energy \( E_{[k+1]} \) we have two colliding “parent” particles in the (kth) generation. Furthermore, we require some extra minimal number \( \nu \) of “guiding” or control “particles” needed to ensure that indeed the more energetic particles, say \( E_2' \) and \( E_4' \) will collide, rather than \( E_2' \) and \( E_1' \), \( E_2' \) and \( E_3' \), \( E_4' \) and \( E_1' \), \( E_4' \) and \( E_3' \). (Clearly without this extra guidance we will have a stochastic thermalized system.) It would seem highly conservative to assume \( \nu \geq 4 \). Already in this case the total number of particles \( N = (2 + \nu)^k \geq 6^k \geq 10^{20} \ln \frac{3}{2} \geq 10^{84} \) - exceeding the total # of particles in the observed universe. To avoid inelastic collisions, we need to have the momenta of colliding particles very parallel \( \theta \leq m/E \leq E_0/(E_0) \left( \frac{3}{2} \right)^l \leq \left( \frac{3}{2} \right)^{-l} \).

It is very difficult, under these circumstances, to ensure that only the “chosen” particles will be within interaction range \( b_0 \) so as to avoid simultaneous multiple collisions which will “dilute” the obtained energies by mixing in low energy particles. Even the demand that initially the \( N \) colliding particles are not within each others’ interaction range - a feature always implicitly assumed - is not trivial. It implies \( R_0 \geq \sqrt{N} b_0 \) with \( R_0 \) the transverse size of the initial, first generation, beam of particles. For \( b_0 \geq \text{fermi} \) the minimal purely nuclear interaction range, and \( N \geq 10^{82} \) the large minimal required number (\( \approx \) total # of particles) in the universe, we find \( R_0 \geq r_{\text{Hubble}} \) a coincidence embodying Dirac’s large number hypothesis. The transverse focusing (see the expression for \( \theta \) above) implies then that the length \( L \) of the experimental set up, \( L \gg R_0 \geq r_{\text{Hubble}} \).

5 Gravitational acceleration

Gravitation is, in many ways, the strongest rather than the weakest interaction. This is amply manifest in the gravitational collapse to a black hole which no other interaction can stop. Along with the definition of \( m_P \) this naturally leads us to consider gravitational accelerators, and the acceleration (or other effects) of black holes in particular. If a particle of mass \( \mu \) falls to a distance \( r \) from a mass \( m \), it obtains, in the relativistic case as well, a final velocity

\[
\beta_f = \sqrt{\frac{r_{\text{SW}}}{r}}
\]

with \( r_{\text{SW}} = \frac{G \mu m}{c^2} \), the Schwartzschild radius of the mass \( m \). In order to obtain in a “single shot” planckian energies, we need that \( \epsilon = 1 - \beta_f = \frac{1}{2\gamma_f} = \frac{\mu^2}{2m_P^2} \) or \( \epsilon = \frac{r - r_{\text{SW}}}{r_{\text{SW}}} = \frac{\mu^2}{2m_P^2} \). The last equation applies also if at infinity we have initially a photon of energy \( \mu \). Taking generically \( \mu = \text{GeV} = m_N \) and \( m = m(\text{neutron star}) = m_{\text{Chandrasekhar}} = 1.4m_o = \frac{m_P^2}{m_N^2} \) as the mass neutron star or black hole doing the acceleration, we find \( r - r_{\text{SW}} = l_P = 10^{-33}\text{cm} \). The distance of closest approach \( r \) must exceed \( r_0 \), the radius of the star (compact object) and hence we have \( r - r_0 = l_P \). Thus the system of interest is most likely a black hole. In turn the black hole will also capture the accelerating particle.

A sophisticated accelerator using repeated sling shot kicks in a system of four black holes
was suggested by Unruh. This beautiful concept is best illustrated in the following simple two black holes context. Consider first two black holes of equal mass \( m_1 = m_2 = m \) at points \( P_1 \) and \( P_2 \) located at \(+L, -L\) along the \( z\) axis. A relativistic particle \( \mu \) (which could also be a photon) is injected with some relative impact parameter parallel to the \( z\) axis near \( z=0\). With an appropriate choice of the impact parameter \( b = b_0 \), the accelerated particle describes a “semi-circle” type trajectory around \( m_1 \) at \( P_1 \), and is reflected around this mass by an angle \( \theta = \pi \) exactly. Moving then along a reflected \((x \rightarrow -x)\) trajectory the particle \( \mu \) approaches the other mass \( m_2 \) at \( P_2 \), and is reflected there by \( \theta = \pi \) as well. The particle will eventually describe a closed geodesic trajectory bound to the two mass \( m_1, m_2 \) system. In reality the two masses move. For simplicity consider the case when the masses move towards each other with velocity \( \beta \). Transforming from the rest mass of \( m_2 \) say to the “Lab frame”, we find that in the reflection the energy of \( \mu \) is enhanced according to \( E_\mu \rightarrow E_\mu\sqrt{\frac{(1+\beta)}{(1-\beta)}} \). The last equation represents the boost due to the sling shot kick alluded to above. If we have \( N \) such reflections, the total boost factor is \( \sim (1+\beta)^N \). This overall boost could exceed \( 10^{19} \) for \( \beta = 1/3 \) if \( N \) exceeds 150. However, in this simple geometry the total number of reflections is limited by \( N = 1/\beta \). After more reflections, the two masses will either coalesce or reverse their velocity, leading now to a deceleration of the particle \( \mu \) upon each reflection. The total amplification of the initial energy is therefore limited in this case simply to a factor \( e \approx 2.7 \).

The Unruh set up involves, however, two additional heavier black holes \( M_1 = M_2 = M \) with \( m_1, m_2 \), revolving around \( M_1, M_2 \) respectively, in circular orbits of equal radius \( R \) and period \( T = 2\pi R/\beta \), with \( \beta \) the orbital velocity. The two orbits are assumed to lie in the \((x - z)\) plane with the centers of the circles at \((x, z) = (0, +L)\). The tops of the two circles, \( i.e. \) the points where \( x \) is maximal, define now the original reflection centers \( P_1, \ P_2 = (R, L), \ (R, +L) \). The oppositely rotating masses \( m_1 \) and \( m_2 \) are synchronized to pass at \( P_1 \) and \( P_2 \), respectively, at the same time – once during each period \( T \). Furthermore, the motion of the accelerated mass \( \mu \) is timed so as to have \( \mu \) at the extreme left point on its “Stadium Shaped” orbit \((x, z) = (0, -L - \rho)\) or, at the extreme right point \((x, z) = (0, L + \rho)\), at precisely the above times. This then allows us to achieve the desired sling shot boosts, repeating once every period \( T \). Note that \( 2T \) is now the period of the motion of the five body system \((M_1, M_2, m_1, m_2, \mu)\).

However, the inherent instability of this motion again limits the number \( N \) of periods (and of sling shot boosts) and, as pointed to us by B. Reznik, foils this ingenious device. The assumed hierarchical set-up \( L >> R >> r_{SW} = b_0 \) can be used to approximate the angular deflection of \( \mu \) while it is circulating around \( m_1 \), say by

\[
\theta = \int_{u_{min} = 0}^{u_{max}} \frac{du}{\sqrt{\frac{1}{b^2} - u^2(1 - 2GNmu)}}
\]

with \( b \) the impact parameter and \( u_{max} = 1/\rho \) corresponding to the turning point of closest approach. Independently of the exact (inverse elliptic function) dependence of \( \theta \) on \( b/r_{SW}, \rho/r_{SW} \), we expect that a fluctuation \( \delta \theta^0 \) around the optimal \( b^0 \), for which \( \theta \) equals \( \pi \), causes a corresponding fluctuation in \( \theta \): \( \delta \theta = \pi - \theta = c\delta \theta^0 /b^0 \), with the dimensionless constant \( c \) being of order one. The large distance \( L \) transforms this small \( \delta \theta \) into a new
impact parameter deviation, $\delta^1(b) = L\delta^1\theta$. The ratio between successive impact parameter is then given by $|\delta^1(b)| = (cL/b_0)\delta^0(b)$. After $N$ reflections, we have therefore

$$\delta^N(b) \simeq (L/b_0)^N\delta^0(b)$$ (12)

For $L > R > b_0$ we expect a large growth rate of the fluctuations in the impact parameter. The circular trajectory of $m$, with radius $R$, decays at a rate proportional to $\beta^5$ due to gravitational radiation. This limits $\beta$ and implies that we need a large number (of order 100) reflections to achieve $10^{19}$ energy enhancement. This implies that in order to avoid complete orbit deterioration for the accelerating particle $\mu$, i.e., to avoid $\delta^N(b) \simeq b_0$, we need unattainable initial precision $\delta b^0/b_0 = 10^{-100}$, which in particular exceeds the quantum uncertainty.

Ultra-relativistic acceleration can be achieved in the universe as a whole; the Hubble velocities of the most distant galaxies or oldest particles, $\beta_H$, are arbitrarily close to one in a sufficiently large and flat universe, though no collisions at Planck CM energies can be thus engineered. It is amusing to recall one “simple” context in which Planck energies may be achieved but not much exceeded – namely the evaporating mini-black holes. Indeed, as indicated in the Introduction, black holes emit all elementary quanta with a thermal spectrum. Only as $R_{BH} \rightarrow l_P$ does $T_{BH} \rightarrow m_P$, so that a Planckian black hole would have generated Planck energy quanta except for the fact that at this point $M_{BH} \rightarrow m_P$ as well, and energy conservation prevents achieving super-planckian energies.

If as it emits the last quanta, the center of mass of the black hole had appreciable boost, say $\gamma \geq 3$, then one might expect that Hawking quanta emitted in this direction would have super-Planckian energies: $\gamma E_{\text{in rest frame}} \geq m_P$. The recoil momentum accumulated through the Hawking radiation is, at all stages: $P_{\text{Rec}} \approx m_P$, so that $\gamma_{\text{Rec}} \sim 1$, and no appreciable extra boost effect is expected. $P_{\text{Rec}} \approx m_P$ implies a recoil kinetic energy of the black holes $F_{\text{recoil}} \approx \frac{P_{\text{Rec}}^2}{2M_{BH}} = \frac{M_P^2}{2M_{BH}} = T_{BH}$ as required by equipartition. This fact features in a new approach to Hawking radiation which attempts to avoid any reference to transplanckian acceleration due to B. Reznik.

6 Summary and Further Speculations

The difficulty of conceiving even Gedanken super-planckian accelerators suggests that models without superplanckian elementary particles may be consistent. In particular models with light fermions may not allow acceleration to super-planckian energies when embedded into the present universe with its given Hubble radius. This could not happen in an infinite or sufficiently large open universe. However the universe with its specific global parameters does provide the required violation at the present time. Our discussion was clearly not exhaustive and the possibility that some ingenious suggestion (which we failed to realize), can actually lead to a planck accelerator is still open. A more comprehensive discussion is presently
under preparation and will address some further possibilities. Let us conclude with several comments and speculations.

a) Our considerations do not exclude theories with super-planckian energies and masses of elementary excitation. Indeed the very notion of what is an elementary particle may be profoundly revised in schemes such as string theory. Rather, all that is hinted is a possible consistency of theories where such a super-planckian regime is unphysical and is excluded.

(b) It has been suggested that “Planck Scale Physics” induces effective interactions violating all global symmetries. A particular example is a \( \frac{1}{m_P} \Phi^+ \Phi \Phi^+ \Phi \) term where the \( \Phi \) bosons carry two units of lepton number. Such a term violates \( U(1), \ (B-L) \) and therefore endows the corresponding would-be massless Goldstone boson (Majoron) with a finite mass. A concrete mechanism involves the formation of a black hole in a collision of, say, \( \Phi^+ \Phi \), followed by the decay of the B.H. into \( \Phi \Phi \Phi^+ \), a final state with two units of lepton number. In this way the violation of the global quantum numbers traces back to the fundamental “No Hair Theorem” for black holes. Exactly as in the case of SU(5), where a virtual \( X, Y \) GUTS meson can mediate nucleon decay by generating effective four Fermi terms, the virtual “mini black hole” system was conjectured to induce the \( \frac{1}{m_P} \Phi^+ \Phi \Phi^+ \Phi \) term. The estimated resulting Majoron mass \( M_x \approx \text{KeV} \) is rather high.\[10\] Also Planckian black holes would constitute some irreducible environment and may require modification of quantum mechanics\[8\]. If, the whole super-planckian domain is inaccessible for elementary excitations, it is conceivable that such effects may not be there. Super-planck physics – even in terms of indirect low energy manifestation would then be completely absent.

(c) ’tHooft has been emphasizing \[9\] that understanding the elementary particle–black hole connection issue may lead to a more profound understanding of both field theory and gravity. Our approach attempts to realize the ’tHooft conjecture, but in a very different way. Rather than try to bridge the gap between particles and black holes, we elevate such a gap – or the upper bound \( E \leq m_P \) on the energy-momentum of any elementary excitation – to a fundamental postulate of the theory. The basic \( \Delta x \Delta p \geq \hbar \) postulates, allows a qualitative understanding of the spectra and structure of atoms. Our hope is that \( E, P \leq m_P \), will allow better understanding of elementary particles, cosmology, and their interrelations.

(d) As indicated above collisions of elementary particles at such energies do occur, but very rarely, with exponentially small probabilities. However, when such a collision occurs, there could be potentially dramatic repercussions. The newly formed mini black hole could be a region of a new expanding “baby universe” where the very basic fundamental parameters may be different. Indeed, we would like to suggest that a posteriori explain the rarity of such collisions, which spell the “end” of the present universe. The new universe still preserves the same gauge group \( SU(3) \times SU(2) \times U(1) \) or a corresponding \( SU(5) \) or \( SU(10) \), etc. GUTS group. The point is that these small length scales( \( R_{BH} \approx l_P \)) all symmetries are restored, and the “gauge hair” is a common link between the two universes. New fermion masses and even new fermionic degrees of freedom may arise in this process, but due to the “gauge memory” it is natural to assume the same gauge structure for them. This could then provide an “evolutionary” explanation for the repeating fermionic generations in the spirit of speculation by Nambu\[11\] and Coleman\[12\].
Indeed, as indicated by our estimates the new generations with lighter electrons and quarks could make it more difficult to achieve super-planckian energies again in the new universe, thus allowing the long lived present universe. If fermion masses stay fixed and the universe expands for ever (which appears to be observationally more favorable at the present time) then planck accelerators would become eventually possible. The above extremely heuristic notion, could potentially also evade this difficulty. A continuous change of fundamental constants with the expansion of the universe was suggested by the Dirac large number hypothesis. This clearly failed various experimental checks. Our conjecture that a universe which is one billion times older than the present universe would still fail to accelerate to planck energies due to the possible emergence of a fourth superlight generation is much harder to check.

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Figure Captions

1. The nested “Accelerator Within Accelerator” system designed to achieve super-planck energies.

2. A Gedanken super-planck linear accelerator of length $L$ and radius $R$. It is bent into a circular arc of angle $\Delta \theta$ (not indicated). The center of the chord to the arc is at a height $h$ below the middle of the arc.

3. A multiparticle collider designed to “Breed” – by repeated collisions and choices of the more energetic particles to collide in consecutive stages – a super-planck particle.

4. The Unruh accelerator. The two black holes $m_1 = m_2 = m$ go around say the stationary more massive $M_1 = M_2 = M$ black hole in circular orbits of radius $R$ and in opposite direction. The accelerating particle goes around in the oblong “stadium-like” trajectory of thickness $2b_0$ with $b_0$ the impact parameter. It gets the “sling-shot kicks” boosting its energy as it goes around $P_1, P_2$ at times $t, t + T$ with $m_1, m_2$ at $P_1, P_2$ respectively.
1st collision

k collision

$2^{k-1}$

$2^k$

$(2^k)'$
