Application of Roundness Test on Tires Based on Five – Point Cubic Smoothing Algorithm

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Abstract. The application of tire roundness test data processing, the simplified system principle of tire contour detection and the idea of five-point cubic smoothing processing are expounded. Based on the theory of the five-point cubic smoothing method and MATLAB software programming, an effective method for data processing of tire roundness detection is established. Comparing with the data obtained from the traditional median smoothing method, the superiority of the five-point cubic smoothing algorithm is obvious.

1. Introduction
For tires roundness detection data processing, the document [1] has achieved excellent progress by using the traditional median smoothing method to process data. The five-point cubic smoothing algorithm proposed in this paper is widely applied, and the document [2] is applied to the research area of flow prediction in reservoir pivotal projects. The document [3] applied it to the analysis fluorescence spectrum data and applied for the patent. In the gait analysis of exoskeleton robot, the five-point cubic smoothing algorithm is used to improve the prediction accuracy of the document [4]. The document [5] can effectively smooth the observation curve in the data processing of simulation experiments, based on the five point three smoothing algorithm. This paper will employ MATLAB software programming simulation, compared with the traditional smoothing method to further verify the superiority of the five-point cubic smoothing method in the tire roundness detection data processing [6-7].

2. System principles
2.1 Tire roundness detection system
In the tire roundness detection system, as shown in Figure1, when the tire is stuck in place, the spindle is locked, the upper rim is loosened, the inflatable pressure is stable, and the tire is rotated at low speed. Three laser displacement sensors are used to measure the pulsation value of the top and bottom of the tire side and the upper, middle and lower radial positions of the tire relative to the rotary center surface respectively. Finally, according to the measured results, the roundness data is obtained. The distribution and installation of the laser sensor is shown in Figure2.
2.2 Data processing step

Data was collected from the production workshop by laser sensor shown in Figure 3. From the graph, we can see that original data is not the ideal signal. It includes tire contour information, and environmental interference, electromagnetic interference, spindle rotation error, wheel rim installation eccentric error and so on. Data processing is completed by the host computer, which removes interference to the acquisition signal, extracts the tire roundness information and displays it appropriately. The data processing flow is shown in Figure 4.

In the detection of tire roundness, the spindle revolves at the steady speed of 120rpm. The roundness signal of the tire is mainly low frequency signal of about 2Hz, but it contains a lot of high frequency interference signals, and the low pass filter is designed to filter this part of high frequency interference. Considering the data after filtering and excluding abnormal values, it also includes low frequency interference signals such as tire bulging and concave patterns. In the paper, the smoothing fitting method is applied to process data, thereby getting the envelope of external contour accurately. The paper mainly verifies the optimization of the smoothing method in the application of tire detection data processing.
3. The five-point cubic smoothing algorithm principle

The collected discrete data \( x(nT_s) \) sequence is smoothed. Set (2N+1) equal interval points for sampling, such as \( x_{-N}, x_{-N+1}, x_{-N+2}, \ldots, x_{2}, x_{1}, x_{0}, x_{1}, x_{2}, \ldots, x_{N}, x_{N-1}, x_{N} \), the sampling value on this, \( y_{-N}, y_{-N+1}, y_{-N+2}, \ldots, y_{2}, y_{1}, y_{0}, y_{1}, y_{2}, \ldots, y_{N-2}, y_{N-1}, y_{N} \).

Let \( h \) be an equal interval sampling step, and transform \( t = (x - x_0)/h \), then the above (2N+1) equal interval points become: \( t_{-N} = -N, t_{-N+1} = -N + 1, t_{-N+2} = -N + 2, \ldots, t_{-2} = -2, t_{-1} = -1, t_0 = 0, t_1 = 1, t_2 = 2, \ldots, t_{N-2} = N - 2, t_{N-1} = N - 1, t_N = N \).

The derivation process of the five-point cubic smoothing method:

Assuming that polynomial of degree \( m \), \( y(t) = a_0 + a_1 t + \cdots + a_m t^m \), to smooth the result of sampled values. In order to smooth the discrete number of polynomials, we must determine a set of appropriate coefficients \( a_j \) (0, 1, 2, ..., \( m \)). All points \((t_i, y_i)\) are subdivided into polynomial with \( m \) times, there are (2N+1) equations, such as in formula (1).

\[
\begin{align*}
   a_0 + a_1 t_{-N} + \cdots + a_m t^m_{-N} - y_{-N} &= R_{-N} \\
   a_0 + a_1 t_{-N+1} + \cdots + a_m t^m_{-N+1} - y_{-N+1} &= R_{-N+1} \\
   \vdots \\
   a_0 + a_1 t_0 + \cdots + a_m t^m_0 - y_0 &= R_0 \\
   \vdots \\
   a_0 + a_1 t_{N-1} + \cdots + a_m t^m_{N-1} - y_{N-1} &= R_{N-1} \\
   a_0 + a_1 t_N + \cdots + a_m t^m_N - y_N &= R_N
\end{align*}
\]

Because smooth curves can't pass all points \((t_i, y_i)\), these equations are not equal to 0. According to the principle of least squares, the best coefficient \( a_j \) for (2N + 1) sets of data \((t_i, y_i)\) is to find the values of \( a_j \) that can make the sum of square sum of error \( R_j \) as the minimum. Let

\[
\sum_{n=-N}^{N} R^2_n = \sum_{n=-N}^{N} \left( \sum_{j=0}^{m} a_j t^j - y_n \right)^2 \Rightarrow \phi(a_0, a_1, a_2, \ldots, a_m)
\]

That is,

\[
\sum_{n=-N}^{N} y_n t^k_n = \sum_{j=0}^{m} a_j \sum_{n=-N}^{N} t^{i+j}_n
\]

When \( N = 2, m = 3 \), noticed the relationship between \( N \) and \( t_i \), allows for equation (4) to be established.

\[
\begin{align*}
   &5a_0 + 10a_2 = y_{-2} + y_{-1} + y_0 + y_1 + y_2 \\
   &10a_1 + 34a_3 = y_1 - y_{-1} + 2(y_2 - y_{-2}) \\
   &10a_0 + 34a_2 = y_1 - y_{-1} + 4(y_2 - y_{-2}) \\
   &13a_2 + 34a_4 = y_1 - y_{-1} + 8(y_2 - y_{-2})
\end{align*}
\]

From equation (4), we get \( a_0, a_1, a_2, a_3 \) and then substitute them into equation (1). Letting \( t = 0, \pm 1, \pm 2 \), we get five-point cubic smoothing formula (5).
When the data points are large, in order to make it symmetrical, two data points at the initial are solved by formula (a) and (b) in the equation (5). Two data points at the end are solved by formula (d) and (e) in the equation (5). The middle points are smoothed by formula (c). This is equivalent to smoothing computation with three different least squares polynomials in every small subinterval. In the data acquisition system, data are mostly discrete points arranged in order of \( n = 1, 2, \ldots, N \). For the sake of unity, equation (3-5) is generalized and is rewritten.

In fact, the five-point cubic smoothing method makes use of the smoothing factor to achieve smoothness.

In the traditional smoothing data processing, the median method is most commonly used, which is equivalent to the "median method" + "arithmetic average method". The median method combines the advantages of the two methods. It can effectively overcome the fluctuating interference caused by accidental factors.

4. Comparison between the five-point smoothing method and the median method

4.1 Comparison principle

According to the principles of the two methods, we compare the differences, as shown in table 1.

|                  | Median smoothing          | Five-point cubic smoothing |
|------------------|---------------------------|---------------------------|
| N value          | 3-14                      | 5(Adjustable)             |
| Boundary point   | No consider               | Consider                  |
| Smooth way       | Arithmetic mean method    | Least square method       |
| Application      | High-frequency concussion system | Time domain and frequency domain |
| Occupy memory    | More                      | Less                      |
| Processing time  | Slow                      | Fast                      |

According to the theoretical comparison in the table, the five-point cubic smoothing method makes use of polynomial least squares approximation to smoothen the sampling points, and process the data more efficiently. It is better than the median method in the processing and memory.

4.2 Comparison of results after data processing

The target object measured by this system is a tire, which specification is 11.00R20, and the spindle speed is 120rpm, the sampling frequency is 600Hz. Taking the original voltage data of the change in the roundness of the diameter of the tire as an example, the data processing analysis and the comparison of the results are performed.

According to the formula (6), the voltage value is converted into a displacement value, \( K=21 \) is the conversion coefficient of voltage value and displacement.

\[
X = \frac{U}{K}
\]  

Where X-displacement (mm), U-laser sensor output analog voltage (mv)

The article will take the tire diameter data collection as an example, and edit the program in the MATLAB environment to construct the function (smooth5_3 or mean5_3) of the five-point cubic
smoothing method. When the smoothing process data, call function can smooth the data. The result of data processing is shown in Figure 5.

![Comparison of Smoothing Data on Tire Diameter Data](image)

(a) Comparison of some data processing results (b) Local magnification processing results

**Figure 5.** Comparison after smoothing

From Figure 5 (b), we can see that the median method and the five-point cubic method can achieve a smooth processing effect. However, when the original data waveform is raised or at an inflection point, the median value method processing seems to be somewhat difficult, the five-point cubic method processing can retain the original data waveform and smooth transitions, and play accurately smoothing effect. Based on this, the data fitting results of several data points on the edge are deduced. Hence, the MATLAB simulation processing speed is faster than the median value smoothing method.

### 4.3 Comparison after smooth fitting

According to the data processing flow, after smoothing the concave pattern, the envelope of the outer contour can be obtained. The tire profile data is displayed in polar coordinates as shown in Figure 6.

![Tire surface envelope after five-point cubic smoothing](image)

(a) The regular pattern of the tire surface (b) Tire surface envelope after five-point cubic smoothing
It is easy to find from Figure 6 that the envelope curve obtained after smoothing by the median value method fluctuates greatly, while the five-point cubic method is relatively smooth and can accurately obtain the tire surface envelope. After subsequent data settlement, the tire out-of-roundness information can be obtained accurately.

5. Conclusion
By putting forward the idea and verifying that the median smoothing method has many advantages and disadvantages in the tire roundness detection data, there is much more room for improvement. However, the five-point cubic smoothing proposed in this paper optimizes the traditional smoothing method. It has the desired characteristics of smaller memory usage, faster speed, and overall higher efficiency, while guaranteeing the accuracy of the data processing. The industrial test cycle is thus shortened and efficiency is improved.

6. References
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