Comment on “Effective thermal conductivity in thermoelectric materials” [J. Appl. Phys. 113, 204904 (2013)]

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In a recent article, Baranowski et al. [J. Appl. Phys. 113, 204904 (2013)] proposed a model that allegedly facilitates optimization of thermoelectric generators operation as these latter are in contact with hot and cold temperature baths through finite conductance heat exchangers. In this Comment, we argue that the results and analyses presented by these authors are misleading since their model is incomplete and rests on an inappropriate assumption derived from thermoelectric compatibility theory.

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I. INTRODUCTION

In a recent article [1], Baranowski and coworkers developed a model that describes heat transport inside thermoelectric generators (TEG) using an effective thermal conductivity $\kappa_{\text{eff}}$. The purpose of this model is to simplify the optimization of TEGs operation, especially when dissipative thermal couplings between the generator and the heat baths are considered. In this Comment, we show that the description of this model is incomplete and suffers from some inconsistencies as conditions for optimization are applied. We thus focus on two points: first, the dependence of $\kappa_{\text{eff}}$ upon the electrical working conditions; second, the use of the “$u = s$” condition related to the thermoelectric compatibility [2].

II. INFLUENCE OF ELECTRICAL WORKING CONDITIONS ON $\kappa_{\text{eff}}$

The introduction of an effective thermal conductivity is motivated by the need to describe the contribution of thermoelectric processes to the heat transport inside the TEG. This additional contribution is presented by Baranowski and coworkers as [1]: “heat generation/consumption due to secondary thermoelectric effects”.

We do not agree with the terminology generation/consumption and secondary effects. First, the term convective heat transport, explained in the next paragraph, is more appropriate to qualify the process studied in Ref. [1]: the heat consumption is related to power production, which remains small compared to the heat transferred between the hot and cold reservoirs, while the heat generation is rather associated with Joule heating, which is overlooked in the discussed article [1]. Second, as the existence of the additional convective heat flux, compared to Fourier conduction only for non-thermoelectric materials, is the main property of a TEG it should not be qualified as “secondary effect”: quite the contrary, it is the essence of the physical phenomenon occurring in this system.

We now describe the heat transport inside the TEG using a model we developed in Ref. [3]. In that article, we focused on the contribution of the heat carried by the global movement of charge carriers, each one carrying a quantity of heat $e\alpha T$, with $e$ being the elementary electrical charge, $\alpha$ being the Seebeck coefficient, and $T$ being the average temperature inside the TEG. Since this process of heat transport is associated with a macroscopic motion, the electrical current $I$ was related to a convection phenomenon. Using a force-flux formalism, we expressed thus total heat flux $q_h$ inside the TEG as the sum of a convective term and of the classical Fourier conduction:

$$q_h = \frac{e\alpha T I}{TE} + K \Delta T$$  \hspace{1cm} (1)

where $\Delta T$ is the temperature difference across the TEG, and $K$ is the thermal conductance of the TEG under open-circuit condition.

In order to derive the expression of the effective thermal resistance $\Theta_{\text{TE}}$ used in Ref. [1], we simply factorize the previous expression by $\Delta T$:

$$q_h = \left( \frac{e\alpha T I}{\Delta T} + K \right) \frac{1}{\Theta_{\text{TE}}} \Delta T$$  \hspace{1cm} (2)

The main interest of the above form is to highlight the dependence of $\Theta_{\text{TE}}$ on the electrical working conditions. This point, while being essential to optimize the system, was overlooked in Ref. [1]: the derivation of $\Theta_{\text{TE}}$ by Baranowski and coworkers implies that this quantity is uniquely defined and hence may lead to confusion.
As the main focus of Ref. [1], we express this quantity as a function of the cogical conductivity $\kappa_{\text{eff}}$, we express this quantity as a function of the load resistance $R_{\text{load}}$ which reflects the dependence on the electrical working conditions. From Eq. (19) of Ref. [1], we get:

$$\kappa_{\text{eff}} = \kappa \left(1 + \frac{ZT}{1 + R_{\text{load}}/R_{\text{in}}}\right)$$  \hspace{1cm} (3)

since the electrical current inside the TEG is $I = \alpha \Delta T/(R_{\text{load}} + R_{\text{in}})$ with $R_{\text{in}}$ being the TEG electrical internal resistance. In Eq. (3), $Z = \alpha^2/(R_{\text{in}}K)$ is the TEG power factor.

Thus, contrary to the statement of Baranowski and coworkers: “This model is especially powerful in that the value of $\kappa_{\text{eff}}$ does not depend upon the operating conditions of the TEG but rather on the transport properties of the TE materials themselves”, $\kappa_{\text{eff}}$ varies from $\kappa$ for open circuit condition ($R_{\text{load}} = \infty$) to $\kappa(1 + ZT)$ for closed-circuit condition ($R_{\text{load}} = 0$). This variation however does not lessen the power of the model. The explicit expression of the thermal properties dependence on electrical working conditions is even the key point to optimize the system: Several recent articles [4,8] deal with this issue and stress importance of the interdependence of electrical and thermal optimization. For example, the optimal TEG length to maximize output power proposed in Eq. (20) of Ref. [1] does not take account of electrical conditions: This expression is only valid for a specific value of the electrical load whereas Eq. (14) derived in the previous section that the effective thermal conductivity $\kappa_{\text{eff}}$ has to depend on the electrical working conditions. In Ref. [1], the authors restrain their analysis to the condition “$u = s$”, where $u$ and $s$ are respectively the relative current density and the compatibility factor as defined by the thermoelectric compatibility theory developed by Snyder and Ursell [2]. Since the relative current density is a function of the electrical current $I$, setting the value of $u$ is equivalent to defining the electrical working conditions. However, we highlight here that this choice does not correspond to the maximum power condition. The condition “$u = s$” is indeed associated with an efficiency maximization [2] and, despite the claim of the authors that, “if the TE element length is allowed to vary, it can be shown that the “$u = s$” condition also produces the maximum power possible for the given temperature gradient and transport properties”, power maximization is reached for an other value of the relative current density.

To illustrate this point we plot both the efficiency and the output power of a TEG as functions of the relative current density $u$ on Figure 1: the parameters are close to those used in Figure 8 of Ref. [1]. The curve found for the efficiency is, as expected, similar that of the Figure 8 of Ref. [1]; its maximum corresponds to the condition “$u = s$”. However, the maximum of the output power is obtained for $u \neq s$: It appears clearly that efficiency maximization and power maximization are two different optimization targets, which cannot be reached simultaneously, except when the TEG performances are much deteriorated, in which case both maximum efficiency and maximum power vanish [4]. Then, by setting “$u = s$”, Baranowski and coworkers do not maximize power contrary to what they claim.

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### III. DISCUSSION ON THE “$u = s$” CONDITION

As stressed by Su and coworkers [9], the ratio $\omega = \Theta_{\text{TE}}/\Theta_{\text{Hx}}$ should not be used as an optimization variable for the output power: this choice yields confusion since power maximization is obtained for an infinite $\omega$, if this latter variable’s variations are due to $\Theta_{\text{Hx}}$’s variations, with constant $\Theta_{\text{TE}}$. It amounts to stating that thermal contact resistances should be minimized. The condition $\Theta_{\text{TE}} = \Theta_{\text{Hx}}$ corresponds to power maximiza-
tion only if the thermal resistances of heat exchangers are not allowed to vary.

V. SUMMARY

While the model proposed in Ref. [1] is valuable to the theory of TEG optimization, the analyses based on it, made by Baranowski and coworkers, are misleading as they overlook the dependence of $\kappa_{\text{eff}}$ upon the electrical working conditions, in contrast with the recent literature. Moreover, the particular working condition used to maximize the output power, i.e., “$u = s$”, actually does not correspond to power maximization but to efficiency maximization which is a distinct optimization criterion.

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