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Abstract – The traditional nuclear shell model approach is extended to include many-body forces. The empirical Hamiltonian with a three-body force is constructed for the identical nucleons on the 0f_{7/2} shell. Manifestations of the three-body force in spectra, binding energies, seniority mixing, particle-hole symmetry, electromagnetic and particle transition rates are investigated. It is shown that in addition to the usual expansion of the valence space within the traditional two-body shell model, the three-body component in the Hamiltonian can be an important part improving the quality of the theoretical approach.

The many-body problem is central for modern physics. A path from the understanding of interactions between fundamental constituents to that of the diverse physics of the whole system is non-trivial and involves various entangled routes. Among numerous issues, the questions of the whole system is non-trivial and involves various fundamental constituents to that of the diverse physics A path from the understanding of interactions between

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Table 1: States in \(f_{7/2}\) valence space with spin and seniority listed in the first and second columns. A * denotes seniority mixed states in 3B\(f_{7/2}\) and the seniority shown reflects the seniority assignment in the 2B\(f_{7/2}\) model. In order to distinguish among the mixed states of the same spin-label (first column) we include an additional subscript reflecting the order in which they appear in the energy spectrum. Following are columns with data for \(N = 28\) isotones and \(Z = 20\) isotopes. Three columns for each type of valence particles list name and excitation energy, experimental binding energy, and energy from the three-body SM calculation discussed in the text. All energies are in MeV.

| Spin | \(\nu\) | Name | Binding | \(3Bf_{7/2}\) | Name | Binding | \(3Bf_{7/2}\) |
|------|-------|------|--------|-------------|------|--------|-------------|
| 0/2  | 1     | 48Ca | 0      | 0           | 40Ca | 0      | 0           |
| 7/2  | 1     | 49Sc | 9.162  | 9.753       | 41Sc | 8.360  | 8.4870      |
| 0    | 0     | 50Ti | 21.787 | 21.713      | 42Ca | 19.843 | 19.837      |
| 2    | 2     | 52Cr | 37.986 | 38.002      | 44Ca | 37.802 | 37.736      |
| 7/2  | 1     | 51Mn | 46.915 | 47.009      | 43Ca | 38.908 | 38.736      |
| 5/2  | 3     | 53Fe | 55.769 | 55.712      | 45Ca | 36.717 | 36.726      |
| 3/2  | 3     | 54Co | 60.833 | 60.893      | 47Ca | 39.993 | 40.014      |
| 7/2  | 2     | 56Ni | 67.998 | 67.950      | 49Ca | 37.938 | 37.846      |

The best experimentally explored systems are \(N = 28\) isotones starting from \(^{48}\text{Ca}\) considered as a core with protons filling the \(0f_{7/2}\) shell and the \(Z = 20\) \(^{40-48}\text{Ca}\) isotopes with valence neutrons. The experimentally known states identified with the \(f_{7/2}\) valence space are listed in table 1.

The three-body interactions influence nuclear masses and result in important monopole terms [4]. The violation of particle-hole symmetry is related to this. Within the traditional SM in single \(j\)-shell, with occupancy \(\Omega = 2j + 1\), this symmetry makes the spectra of the \(N\) - and \(\bar{N}\)-particle systems (\(\bar{N} = \Omega - N\)) identical, apart from a constant shift in energy. Indeed, the particle-hole conjugation \(C\) defined with \(\tilde{a}_{jm} = C_{\alpha\beta} \tilde{a}_{\alpha\beta}^{-1} = \alpha_{jm} a_{j-m}\) transforms an arbitrary \(n\)-body interaction into itself plus some Hamiltonian of a lower interaction rank \(H'_{n-1}\), as follows: \(\tilde{H}^{(n)} = \alpha_{jm} H'_{n-1}\). The \(n = 1\) case corresponds to the particle number \(\tilde{N} = -N + \Omega\). For the \(n = 2\) we obtain...
Table 2: Interaction parameters of 2B$\frac{7}{2}$ and 3B$\frac{7}{2}$ SM Hamiltonians determined with the least-square fit are given in keV.

|      | $N = 28$ | $Z = 20$ |
|------|----------|----------|
| $\epsilon$ | $-9827(16)$ | $-9753(30)$ | $-8542(35)$ | $-8487(72)$ |
| $V_0^{(2)}$ | $-2033(60)$ | $-2207(97)$ | $-2727(122)$ | $-2863(229)$ |
| $V_2^{(2)}$ | $-587(39)$ | $-661(72)$ | $-1347(87)$ | $-1340(176)$ |
| $V_4^{(2)}$ | $443(25)$ | $348(50)$ | $-164(49)$ | $-198(130)$ |
| $V_6^{(2)}$ | $887(20)$ | $849(38)$ | $411(43)$ | $327(98)$ |
| $V_7^{(2)}$ | $55(28)$ | $53(70)$ |
| $V_8^{(2)}$ | $-18(70)$ | $2(185)$ |
| $V_9^{(2)}$ | $-128(88)$ | $-559(273)$ |
| $V_{11}^{(2)}$ | $102(43)$ | $51(130)$ |
| $V_{13}^{(2)}$ | $122(41)$ | $272(98)$ |
| $V_{15}^{(2)}$ | $-53(29)$ | $-24(73)$ |
| RMS | $120$ | $80$ | $220$ | $170$ |

a monopole shift

$$\hat{H}^{(2)} = \hat{H}^{(2)} + (\Omega - 2N)M, \quad M = \frac{1}{\Omega} \sum (2L + 1)V_L^{(2)}.$$  \hfill (2)

Any $\hat{H}^{(1)}$ is proportional to $N$ and is thus a constant of motion, which explains the particle-hole symmetry for the two-body Hamiltonian. The $n \geq 3$ interactions violate this symmetry leading to different excitation spectra of $N$- and $\tilde{N}$-particle systems. The deviations from exact particle-hole symmetry are seen in the experimental data, table 1. The excitation energies of $\nu = 2$ states in $N = 2$ system are systematically higher than those in the 6-particle case pointing on a reduced ground-state binding.

The $j = \frac{7}{2}$ is the largest single-$j$ shell for which the seniority, the number $\nu$ of unpaired nucleons, is an integral of motion for any one- and two-body interaction [10,14]. It is established experimentally that seniorities are mixed [15,16]. Configurations beyond the single-$j$ shell [8,17–19] have been suggested to explain the effects; however, the possible presence of the three-body force must be addressed. In a single-$j$ shell the pair operators $T_{00}^{(2)}, T_{01}^{(2)}$ and the particle number $N$ form an $SU(2)$ rotational quasi-spin group. The quantum numbers $\nu$ and $N$ are associated with this group. The invariance under quasi-spin rotations relates states of the same $\nu$ but different particle numbers $N$. For example, excitation energies of $\nu = 2$ states are identical in all even-particle systems. The classification of operators according to quasi-spin leads to selection rules. The s.p. operators associated with the particle transfer permit seniority change $\Delta \nu = 1$. The reactions $^{51}$V($^3$He,d)$^{52}$Cr and $^{43}$Ca(d,p)$^{44}$Ca show seniority mixing as the $\nu = 4$ final states are populated [16,18]. The one-body multipole operators are quasi-spin scalars for odd angular momentum, and quasi-spin vectors for even. Thus, the $M1$ electromagnetic transitions do not change quasi-spin. In the mid-shell the quasi-vector $E2$ transitions between states of the same seniority are forbidden. The seniority mixing between the $\nu = 2$ and $\nu = 4$ pairs of $2^+$ and $4^+$ states is expected in the mid-shell nuclei $^{52}$Cr and $^{44}$Ca. Seniority can be used to classify the many-body operators $T^{(n)}_{LM}$ and the interaction parameters. The three-body interactions mix seniorities with the exception of interaction between $\nu = 1$ nucleon triplets given by the strength $V^{(3)}_{7/2}$.

To determine the interaction parameters of $H_3$, we conduct a full least-square fit to the data points in table 1. This empirical method, which dates back to refs. [10,20], is a part of the most successful SM techniques today [21]. Our procedure is similar to a two-body fit in sect. 3.2 of ref. [22], but here the fit is nonlinear and requires iterations due to seniority mixing. In table 2 the resulting parameters are listed for the proton (fixed $N = 28$) system and neutron (fixed $Z = 20$) system. The two columns in each case correspond to fits without (2B$\frac{7}{2}$ left) and with (3B$\frac{7}{2}$ right) the three-body forces. The root-mean-square (RMS) deviation is given for each fit. The confidence limits can be inferred from variances for each fit parameter given within the parentheses.

The lowering of the RMS deviation is the first evidence in support of the three-body forces; for $Z = 28$ isotones it drops from 120 keV to about 80 keV. All three-body parameters appear to be equally important, excluding
any one of them from the fit raises the RMS by about 10%. In contrast, inclusion of a four-body monopole force based on $\nu = 0$, $L = 0$ operator led to no improvement. The fit parameters remain stable within quoted error bars if some questionable data points are removed. The energies resulting from the three-body fit are listed in table 1. Based on the RMS alone this description of data is quite good, even in comparison with the available large-scale two-body SM calculations in the expanded model space [19,23].

The proton and neutron effective Hamiltonians are different, table 2. The s.p. energies reflect different mean fields; and the two-body parameters, especially for higher $L$, highlight the contribution from the long-range Coulomb force. However, within the error bars the three-body part of the Hamiltonians appears to be the same which relates these terms to isospin-invariant strong force.

A skeptic may question some experimental states included in the fit, thus we conduct a minimal fit considering binding energies of ground states only, similar to ref. [10]. We include a seniority conserving part given by $V_{7/2}^{(3)}$ with $\nu = 1$ triplet operator $T_{2j}^{(3)} \sim a_{jm} T_{00}^{(2)}$. This interaction is the main three-body contribution to binding and is equivalent to a density-dependent pairing force [24]. In a single-$j$ model it can be treated exactly with a renormalized particle-number-dependent pairing strength $V_0^{(2)} = V_0^{(2)} + \Omega \frac{N-2}{12} V_j^{(3)}$. From relations in refs. [10,13], the ground-state energies with $\nu = 0$ or 1 are

$$E = \epsilon N + \frac{N-\nu}{\Omega-2} \left( \frac{\Omega-N-\nu}{2} V_0^{(2)} + (N-2+\nu) M' \right),$$

where $M' = M + \frac{N-2}{12} V_j^{(3)}$. With a linear least-square fit and eq. (3) we determine s.p. energy $\epsilon$, pairing $V_0^{(2)}$, monopole $M$, and 3-body interaction $V_{7/2}^{(3)}$ (3) using 8 binding energies. The results, shown in table 3, are consistent with the full fit in table 2, the repulsive nature of the monopole $V_j^{(3)}$ is in agreement with other works [25].

Next we concentrate on $^{52}$Cr, fig. 1. In addition to the 2B$^{f_{7/2}}$ and 3B$^{f_{7/2}}$ interactions from table 2 we perform large-scale SM calculations 2B$^{f_{7/2}P}$ (which includes $p_{1/2}$ and $p_{3/2}$) and 2B$fp$ (entire $fp$-shell, truncated to $10^7$ projected $m$-scheme states) using FPBP two-body SM Hamiltonian [26]. The 2B$^{f_{7/2}P}$ model and its results are very close to the more restricted SM calculations in ref. [23].

The seniority mixing between the neighboring $4^+_1$ and $4^+_2$ states leads to level repulsion. The observed energy difference of 400 keV is not reproduced by 2B$^{f_{7/2}}$ (84 keV). The discrepancy remains in some of the extended two-body models: 2B$^{f_{7/2}P}$ and [23] (200 keV). The full 2B$fp$ replicates the splitting, but possibly at the expense of excessive intruder admixtures which distort the spectrum, see also ref. [27]. The 3B$^{f_{7/2}}$ is the best in reproducing the experimental spectrum, fig. 1. The 3B$^{f_{7/2}}$ model predicts seniority mixing: the $\nu(4^+_1) = 2.82$ and $\nu(4^+_2) = 2.71$ are inferred from the expectation value of the pair operator $(T_{00}^{(2)} T_{00}^{(2)}) \sim (N-\nu)(2j + 3 - N - \nu)/(4j + 2)$. The $2^+_1$, however, is relatively pure with $\nu(2^+_1) = 2.006$.

Violations of the quasi-spin selection rules are observed in nuclei [12,15,16,28,29]. The two-body shell models

| $N = 28$ | $Z = 20$ |
|---|---|
| $\epsilon$ | $-9703(40)$ | $-9692(40)$ | $-8423(51)$ | $-8403(55)$ |
| $V_0^{(2)}$ | $-2354(80)$ | $-2400(110)$ | $-3006(120)$ | $-3105(156)$ |
| $M$ | $1196(40)$ | $1166(50)$ | $-823(55)$ | $-876(76)$ |
| $V_{7/2}^{(3)}$ | $-$ | $18(20)$ | $-$ | $31(31)$ |
| RMS | $50$ | $46$ | $73$ | $65$ |

Table 3: Interaction parameters for the minimal $f_{7/2}$ SM determined with the linear least-squared fit of 8 binding energies. The variances for each parameter are shown within parentheses. The two columns for isotopes and isotones are fits without and with the three-body term.
Table 4: B(E2) transition summary on $^{52}$Cr expressed in units $e^2$ fm$^4$, the last row being the quadrupole moment of the first $2^+$ state in units $e$ fm$^2$. The data is taken from [30]. The zeros in the second column for the $2Bf_{7/2}$ model are the results of the mid-shell seniority selection rules. In this model the states $2_1^+, 2_2^-, 4_1$, and $4_2$ have seniorities $2, 4, 4,$ and $2$, respectively, see table 1.

|        | $2Bf_{7/2}$ | $2Bf_{7/2}p$ | $2Bf_p$ | $3Bf_{7/2}$ | Experiment |
|--------|-------------|--------------|---------|-------------|------------|
| $2_1 \rightarrow 0^{+*}$ | 118.0 | 118.0 | 118 | 117.5 | 83 $\pm$ 15(a),(b) |
| $4_1 \rightarrow 2_1$ | 130.4 | 122.5 | 105.8 | 73.2 | 69 $\pm$ 18 |
| $4_2 \rightarrow 2_1$ | 0 | 0 | 0 | 0 | 0 $\pm$ 0.05 |
| $4_2 \rightarrow 4_1$ | 125.2 | 59.3 | 2.6 | 0.5 | 0.5 |
| $2_1 \rightarrow 0$ | 0 | 0.003 | 0.9 | 0.5 | 0.5 |
| $2_2 \rightarrow 2_1$ | 119.2 | 102.2 | 101.9 | 117.1 | 150 $\pm$ 35 |
| $2_2 \rightarrow 4_1$ | 0 | 10.8 | 34.4 | 19.9 | |
| $2_2 \rightarrow 4_2$ | 57.8 | 7.2 | 5.2 | 38.7 | |
| $6 \rightarrow 4_1$ | 108.9 | 86.2 | 56.3 | 57.8 | 59 $\pm$ 20(a) |
| $6 \rightarrow 4_2$ | 0 | 9.3 | 27.6 | 51.1 | 30 $\pm$ 10(a) |

| $Q(2^{+}_1) ~ e$ fm$^2$ | 0 | $-13.0$ | $-13.4^{(c)}$ | $-2.4$ | $-8.2 \pm 1.6^{(d)}$ |

(c) In $2Bf_{7/2}p$ and $2Bf_p$ models we use 0.5 (neutron) and 1.5 (proton) effective charges. The overall radial scaling is fixed by $B(E2, 2_1 \rightarrow 0)$.  
(a) The lifetime error bars are used.  
(b) There are conflicting experimental results on the lifetime; here we use DSAM (HI, xγ) data from ref. [30], which is consistent with [31].  
(c) Other independent large-scale shell model calculations obtain similar values for the quadrupole moment for the $2^+_1$ state: $-12.3$ [32], $-15.0$, $-16.2$ and $-17.5$ [33], all in units $e$ fm$^2$.  
(d) The experimental data is from [34].

Manifestation of three-body forces in $f_{7/2}$-shell nuclei

The last row in table 4 compares the quadrupole moment of the first $2^+$ state in $^{52}$Cr between models and experiment. It is quoted in units of $e$ fm$^2$. Because of the seniority selection rule the quadrupole moment is exactly zero in the $2Bf_{7/2}$ model; the small seniority mixing between $2^+$ states in the $3Bf_{7/2}$ model leads to the quadrupole moment of a correct sign but it is lower than the observed one. The results from extended two-body models in table 4 and in the literature [32,33,35] improve the picture. However, strong configuration mixing is needed to generate seniority mixing which is inconsistent with $g$ factor measurements [27] and results in a quadrupole moment higher than observed. This again invites the possibility of an additional three-body Hamiltonian.

In table 5 proton removal spectroscopic factors are compared between theoretical models and experiment $^{51}$V($^3$He, d)$^{52}$Cr [15]. The $3Bf_{7/2}$ model is good in its description of observation especially for the $4^+_1$ and $4^+_2$ states. It was pointed out in ref. [15] that the spectroscopic factors for the $4^+_3$ states probe the $\nu = 2$ component, and, thus, their sum within the $f_{7/2}$ valence space is 4/3; this result is consistent with the observation [15] but does not support too much configuration mixing from outside the $f_{7/2}$ shell which would reduce the spectroscopic factors and their sum.

To conclude, the study of nuclei in $0f_{7/2}$ shell shows evidence for three-body forces. We extend the traditional shell model approach by including three-body forces into consideration, we find that a successful set of interaction parameters can be determined with an empirical fitting procedure. With a few new parameters a sizable improvement in the description of experimental data is obtained.
Table 5: Proton removal spectroscopic factors. The experimental data is taken from $^{31}$V($^{3}$He,$d$)$^{12}$Cr reaction [15]. Within error bars of about 0.1 this data is consistent with other results [30].

|          | 2B$f_{7/2}$ | 2B$f_{7/2}p$ | 2B$f_{p}$ | 3B$f_{7/2}$ | Exp |
|----------|-------------|--------------|-----------|-------------|-----|
| 0$^+$    | 4.00        | 3.73         | 3.40      | 4.00        | 4.00|
| 2$^+_1$  | 1.33        | 1.14         | 0.94      | 1.33        | 1.08|
| 4$^+_1$  | 0.00        | 0.13         | 0.34      | 0.63        | 0.51|
| 4$^+_2$  | 1.33        | 1.11         | 0.70      | 0.71        | 0.81|
| 6$^+$    | 1.33        | 1.28         | 1.28      | 1.33        | 1.31|

The apparent hierarchy of contributions from one-body mean-field, to two-body, to three-body and beyond is significant; it assures the possibility of high-precision configuration-interaction methods in restricted space and supports ideas about renormalization of interactions. The three-body forces observed in this study appear to be isospin invariant. The new Hamiltonian with three-body forces describes well the observables that are sensitive to such forces and to seniority, and particle-hole symmetries that are violated in their presence. A good experimental agreement is obtained for the features of spectra, for electromagnetic $E2$ transition rates, and for spectroscopic factors. While good agreement with experiment can also be obtained by configuration mixing within some advanced large-scale two-body shell model calculations, it appears that the three-body effective interaction has a somewhat different manifestation which can be experimentally identified. In particular, with the three-body force, the seniority forbidden transitions become allowed without generating excessively large quadrupole moment or, in the case of spectroscopic factors, without loosing too much strength away from a single-$j$ shell. Unlike the fit to experimental data discussed in this work, it appears to be difficult to fit the large-scale shell model results with a single-$j$ Hamiltonian containing a three-body force. This again suggests that configuration mixing in not necessarily equivalent to a three-body force. The work in this direction is to be continued. It is important to conduct similar phenomenological investigations for other mass regions and model spaces; on the other side, renormalization techniques that would link fundamental and phenomenological forces [5] have to be searched for.

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