Adiabatic Non-resonant Acceleration in Magnetic Turbulence and Hard Spectra of Gamma-Ray Bursts

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Received 2017 June 19; revised 2017 August 14; accepted 2017 August 26; published 2017 September 8

Abstract

We introduce a non-resonant acceleration mechanism arising from the second adiabatic invariant in magnetic turbulence and apply it to study the prompt emission spectra of gamma-ray bursts (GRBs). The mechanism contains both the first- and second-order Fermi acceleration, originating from the interacting turbulent reconnection and dynamo processes. It leads to a hard electron energy distribution up to a cutoff energy at the balance between the acceleration and synchrotron cooling. The sufficient acceleration rate ensures a rapid hardening of any initial energy distribution to a power-law distribution with the index $p \sim 1$, which naturally produces a low-energy photon index $\alpha \sim -1$ via the synchrotron radiation. For typical GRB parameters, the synchrotron emission can extend to a characteristic photon energy on the order of $\sim 100$ keV.

Key words: acceleration of particles – gamma-ray burst: general – turbulence

1. Introduction

The gamma-ray burst (GRB) prompt emission is closely related to the physics of particle acceleration and radiation. The origin of its spectral behavior, despite the empirical description (Band et al. 1993), has not been well understood. The observed low-energy photon index has a typical value $\sim -1$ (Preece et al. 2000; Kaneko et al. 2006; Nava et al. 2011; Zhang et al. 2011), which is difficult to reconcile with the standard model invoking the first-order Fermi acceleration and fast synchrotron cooling (Ghisellini et al. 2000; Preece et al. 2002; Kumar & Zhang 2015). Many attempts have been made to seek the solution to the problem (e.g., Brainerd 1994; Liang 1997; Mészáros & Rees 2000; Pe’er & Zhang 2006; Asano & Terasawa 2009; Asano & Mészáros 2011; Daigne et al. 2011; Uhm & Zhang 2014; Asano & Terasawa 2015; Asano & Mészáros 2016).

In either a Poynting-flux-dominated or a baryonic relativistic outflow, turbulence is inevitably present and participates in the electron acceleration process. The stochastic acceleration through resonant scattering with magnetic fluctuations has been used to explain the hard electron spectrum (e.g., Bykov & Mészáros 1996; Asano & Terasawa 2009; Asano & Mészáros 2011; Murase et al. 2012). Advances in turbulence theories (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999) provide new insight into the problem. Turbulent reconnection, which was put forward by Lazarian & Vishniac (1999) and numerically confirmed in both non-relativistic (Kowal et al. 2009, 2012b) and relativistic (Takamoto et al. 2015) plasmas, provides an efficient dissipation mechanism of the magnetic energy in the GRB outflow. Zhang & Yan (2011) invoked a moderately Poynting-flux-dominated GRB jet and collision-induced magnetic dissipation to interpret GRB prompt emission. This ICMART model envisages significant turbulent reconnection and reconnection-driven turbulence in the emission region of GRBs. Relativistic MHD simulations (Deng et al. 2015) and Monte Carlo simulations (Zhang & Zhang 2014) confirmed some features (e.g., efficient energy dissipation, existence of mini-jets, and their effects on the light curves) of the original model (Zhang & Yan 2011). The large emission radius invoked in the ICMART model allows a modification of the fast synchrotron cooling theory through invoking the decrease of the magnetic field in the emission region as the jet expands in space, which can reproduce the desired $\alpha \sim -1$ even for first-order-Fermi-accelerated electrons (probably through reconnection; Uhm & Zhang 2014).

In magnetohydrodynamic (MHD) turbulence, the turbulent reconnection efficiently relaxes tangled field lines and facilitates turbulent motions. Meanwhile, turbulent shearing motions stretch field lines and generate magnetic fluctuations via the turbulent dynamo (Xu & Lazarian 2016). Their nonlinear interactions regulate the dynamics of MHD turbulence and affect the acceleration of the electrons for which the second adiabatic invariant applies (Brunetti & Lazarian 2016). The adiabatic condition is easily satisfied in a strongly magnetized GRB outflow because either the gyroresonance scattering is absent with the particle Larmor radius below turbulence scales, or it is inefficient due to turbulence anisotropy (Yan & Lazarian 2002). It is the first-order Fermi process within each reconnection/dynamo region and the second-order Fermi process as particles stochastically encounter the reconnection/dynamo event. The stochastic nature originates from the balance between the annihilation and generation of magnetic fluxes in a trans-Alfvénic turbulence (Goldreich & Sridhar 1995, hereafter GS95). In this Letter, based on the modern understanding of the dynamical nature of MHD turbulence, we analytically solve the evolution of the electron energy distribution resulting from the above adiabatic acceleration in trans-Alfvénic turbulence (Section 2) and demonstrate that the resultant hard energy distribution entails a hard synchrotron spectrum at low energies in the prompt GRB phase, consistent with observations (Section 3).

2. Adiabatic Acceleration of Electrons in MHD Turbulence

2.1. Energy Spectrum of Electrons

We consider a turbulence regime with the magnetic and kinetic energies in equipartition. It is the trans-Alfvénic turbulence described by GS95 and has been numerically tested...
in both non-relativistic/low-$\sigma$ (Maron & Goldreich 2001; Cho et al. 2002) and relativistic/high-$\sigma$ (Cho 2005, 2014) cases. In trans-Alfvénic turbulence, the field line stretching process, i.e., turbulent dynamo (Cho et al. 2009; Xu & Lazarian 2016), driven by turbulent velocities, and the field line shrinking process driven by the turbulent magnetic reconnection (Lazarian & Vishniac 1999) coexist. These two opposing processes take place at the same rate over all the turbulent scales, with the overall magnetic flux conserved.

The electrons, when they are not subject to scattering by magnetic fluctuations, undergo the first-order Fermi acceleration in reconnection regions (de Gouveia dal Pino & Lazarian 2005; Lazarian & Opher 2009) and deceleration in dynamo regions as a consequence of the second adiabatic invariant, leading to a globally diffusive energy gain. It is similar to the process that moving particles are stochastically trapped between approaching “mirrors” and receding “mirrors” (Fluegge 1961).

The energy gain/loss within each turbulent eddy follows the first-order Fermi process. Despite the large energy change, we consider the Fokker–Planck equation as a valid description, since it yields the basically identical particle spectrum as that from the statistical approach independent of the energy increment (Schneider 1993).

The evolution equation of the energy distribution function is

$$\frac{\partial N}{\partial t} = a_2 \frac{\partial}{\partial E} \left( E \frac{\partial (EN)}{\partial E} \right) - (a_{1, \text{rec}} - a_{1, \text{dyn}}) \frac{\partial (EN)}{\partial E} + \beta \frac{\partial (E^2 N)}{\partial E},$$

where $N(E, t) dE$ is the number of electrons within the energy interval from $E$ to $E + dE$. The terms on the right-hand side represent the second- and first-order Fermi processes, and synchrotron loss, while the adiabatic expansion of the plasma and electron escape are neglected.

We consider that the turbulent eddies at the injection scale $l_{\text{tur}}$ of the GS95 turbulence, i.e., the typical energy-containing scale, dominate the reconnection/dynamo. Provided $r_f < l_{\text{tur}}$, where $r_f$ is the Larmor radius, the stochastic acceleration rate $a_2$, related to the comparable reconnection acceleration rate $a_{1, \text{rec}}$ and the dynamo deceleration rate $a_{1, \text{dyn}}$, is independent of particle energy. It is associated with the eddy turnover rate:

$$a_2 \sim a_{1, \text{rec}} \sim a_{1, \text{dyn}} \sim \frac{u_{\text{tur}}}{\xi_{\text{tur}}},$$

where $u_{\text{tur}}$ is the relativistic turbulent velocity at $l_{\text{tur}}$, and $\xi = \Delta E/E \sim \gamma_{\text{tur}}$ for highly relativistic turbulence and particles to account for the energy conversion efficiency, with the turbulence Lorentz factor $\gamma_{\text{tur}}$ (Fluegge 1961).

Therefore, Equation (1) can be reduced to

$$\frac{\partial N}{\partial t} = a_2 \frac{\partial}{\partial E} \left( E \frac{\partial (EN)}{\partial E} \right) + \beta \frac{\partial (E^2 N)}{\partial E}.$$

After the substitution of the relations, $f = EN = \exp(-\epsilon E) u(x, \tau)$, $\epsilon = \beta/a_2$, $x = \ln E$, $\tau = a_2 t$, and some algebra, we derive

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} - \frac{E}{E_{\text{cf}}} \frac{\partial u}{\partial x},$$

where we define the cutoff energy corresponding to the balance between the stochastic acceleration and the synchrotron loss,

$$E_{\text{cf}} = \frac{a_2}{\beta} = \frac{3(m_e c^2)^2 a_2}{4 \sigma_T c U_B},$$

(5)

where $\sigma_T$ is the Thomson cross section, $c$ is the light speed, $m_e$ is the electron rest mass, and $U_B = B^2/(8\pi)$ is the magnetic energy density. Obviously, in the energy range $E \ll E_{\text{cf}}$, Equation (4) becomes a straightforward diffusion equation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2},$$

(6)

which allows us to analyze the time-dependent behavior of $N(E, \tau)$.

The general form of the solution to Equation (6) is (e.g., Evans 1998),

$$u(x, \tau) = \frac{1}{2\sqrt{\pi \tau}} \int_{y_1}^{y_2} \exp \left[ -\frac{(x - y)^2}{4\tau} \right] u(y, 0) dy,$$

(7)

with the initial functional form $u(y, 0)$ within the range $[y_1, y_2]$.

The Gaussian function shows that the energy distribution spreads out in energy space following $E \sim \exp(\pm 2\sqrt{\tau})$ (Melrose 1969). Within a finite range of $E$ with lower and upper limits $E_l(=\exp(y_1))$ and $E_u(=\exp(y_2))$, there is

$$E_u = E_l \exp(2\sqrt{\tau}u), \quad \tau_u = \frac{(\ln E_u - \ln E_l)^2}{4}.$$

(8)

After the time $\tau_u$, the initial spectral form is essentially smeared out within the range, the behavior of $u$ is independent of $x$, and thus the energy spectrum of electrons

$$N(E, \tau) = E^{-1} u(\tau) \exp \left( -\frac{E}{E_{\text{cf}}} \right)$$

(9)

has a universal form of $E^{-1}$ at $E < E_{\text{cf}}$. The synchrotron cooling has a negligible effect on the energy distribution in the lower energy range away from $E_{\text{cf}}$.

2.2. Examples for Different Initial Energy Distributions

1. Delta function. Starting from an initial point source of energy with $u(y, 0) = \delta(y - x_0)$, $y \in (-\infty, +\infty)$, $x_0 = \ln E_0$, $u(x, \tau)$ evolves as

$$u(x, \tau) = \frac{1}{2\sqrt{\pi \tau}} \exp \left[ -\frac{(x - x_0)^2}{4\tau} \right],$$

(10)

and thus

$$N(E, \tau) = E^{-1} \frac{1}{2\sqrt{\pi \tau}} \exp \left[ -\frac{(\ln E - \ln E_0)^2}{4\tau} \right] \exp \left( -\frac{E}{E_{\text{cf}}} \right).$$

(11)

The Gaussian component has a negligible contribution to the spectral form at a sufficiently large $\tau$.

2. Power-law function. Given an initially steeper spectrum with the power-law index $p_0 > 1$,

$$N(E, 0) = C E^{-p_0} \exp \left( -\frac{E}{E_{\text{cf}}} \right), \quad E \in (E_l, E_u),$$

(12)
that is, \( n(y, 0) = C \exp[(1 - p_0)y] \), where \( C \) is an arbitrary constant, we can obtain the evolving spectrum by using Equation (7):

\[
N(E, \tau) = E^{-1} \frac{C}{2\sqrt{\pi \tau}} \exp \left( -\frac{E}{E_{\text{cf}}} \right) \int_{y_0}^{\infty} \exp \left( -\frac{1}{4\tau} \left( \ln E - y \right)^2 \right) \exp[(1 - p_0)y] dy.
\]

Its asymptotic form at a short time \( (\tau \ll 1) \) is

\[
N(E, \tau) = E^{-\frac{p_0}{1 - p_0}} \frac{C}{2\sqrt{\pi \tau}} \exp \left( -\frac{E}{E_{\text{cf}}} \right),
\]

which is governed by the initial power-law shape at \( E < E_{\text{cf}} \). Its long-time \( (\tau \sim \tau_{\text{lu}}) \) asymptotic expression is

\[
N(E, \tau) = E^{-1} \frac{C(E_{\text{lt}}^{-1} - p_0 - E_{\text{lt}}^{-1} - p_0)}{2(1 - p_0)\sqrt{\pi \tau}} \exp \left( -\frac{E}{E_{\text{cf}}} \right),
\]

which again recovers the universal \( E^{-1} \) power-law distribution at \( E < E_{\text{cf}} \).

In Figure 1, we display the electron energy distributions of the synchrotron photon number spectrum (Equation (4)). As illustrative examples, we adopt a delta function at \( E_0/E_{\text{cf}} = 2.4 \times 10^{-3} \) in Figure 1(a), a power-law function with \( C = 1 \) and \( p_0 = 2 \) over the entire energy range presented in Figure 1(b), and set \( p = 1 \). Figures 1(c) and (d) show the asymptotic analytical solutions in the low-energy limit at a specified \( \tau \), which agree well with the numerical results. As expected, irrespective of the initial spectral form, the distribution broadens and shifts in energy space and eventually conforms to the universal power-law shape \( E^{-1} \) at \( E < E_{\text{cf}} \) over the timescale of \( \tau_{\text{lu}} \approx 20 \) (Equation (8)).

To examine the effect of synchrotron cooling on the energy distribution, in Figure 2 we present the results with an initial delta function and different \( C \) values at \( \tau = 10 \). Notice that unlike in Figure 1 where \( E \) is normalized by \( E_{\text{cf}} \), here we use the normalization \( E/E_0 \) to show the change of the cutoff energy with varying \( C \). As \( C \) increases, the spectral cutoff moves to a lower energy, but the distribution below the cutoff energy is unaffected and remains the \( E^{-1} \) form.

3. Synchrotron Emission of the Accelerated Electrons

The above adiabatic acceleration of electrons arising in trans-Alfvénic turbulence leads to a hard electron energy distribution (see Equation (9)), corresponding to

\[
N(\gamma_e) \sim \gamma_e^{-p} \exp \left( -\frac{\gamma_e}{\gamma_{e,\text{cf}}} \right), \quad p = 1,
\]

where \( p \) is the power-law index, \( \gamma_e \) is the electron Lorentz factor, and \( \gamma_{e,\text{cf}} = E_{\text{cf}}/m_e c^2 \).

According to the relation between \( p \) and the index \( \alpha \) of the synchrotron photon number spectrum (Rybicki & Lightman 1979), there is

\[
N(\nu) \sim \nu^\alpha \exp \left( -\left( \frac{\nu}{\nu_e} \right)^{\frac{1}{p}} \right), \quad \alpha = -\frac{p + 1}{2} = -1,
\]

where the \( \delta \)-function approximation for the single-electron spectrum is made. This is the typical low-energy photon spectral index of GRBs (Preece et al. 2000; Kaneko et al. 2006; Nava et al. 2011; Zhang et al. 2011).

Unlike a soft distribution with \( p > 2 \), for which the lower cutoff energy accounts for the characteristic electron energy and synchrotron frequency (e.g., Piran 2004; Zhang & Mészáros 2004), as regards a hard distribution, the upper cutoff energy \( E_{\text{cf}} \) is more significant as it dominates the electron energy density and may characterize the peak energy of the \( \nu F_\nu \propto \nu^{-\alpha}N(\nu) \) spectrum (Dai & Cheng 2001). \( E_{\text{cf}} \) depends on the acceleration mechanism (Equations (2) and (5)):

\[
E_{\text{cf}} = \frac{\xi u_{\text{tur}}}{l_{\text{tur}}^{\beta}} = \frac{6\pi \xi (m_e c^2)^2}{\sigma_T B^2 l_{\text{tur}}},
\]

where \( u_{\text{tur}} \) is approximately equal to \( c \) for relativistic turbulence.

In the case of reconnection-driven turbulence, the thickness of the turbulent region increases with time, as indicated by the numerical results in Kowal et al. (2017). Meanwhile, the magnetic field strength \( B \) decays with time (Pe'er & Zhang 2006; Uhm & Zhang 2014; Zhao et al. 2014). The comoving-frame \( B \) during the GRB prompt phase is estimated as (e.g., Zhang & Mészáros 2002)

\[
B = \frac{\mathcal{L}}{L/c} L^2 r^{-1} \Gamma^{-1},
\]

where \( L \) is the total outflow luminosity of the GRB, \( r \) is the distance of the emission region from the central engine, and \( \Gamma \) is the Lorentz factor of the outflow. We assume the following relation between \( l_{\text{tur}} \) and \( B \) to account for their anticorrelation:

\[
l_{\text{tur}} = l_0 \left( \frac{B}{B_0} \right)^{-\zeta}, \quad \zeta > 0,
\]

where \( l_0 \) and \( B_0 \) are normalization parameters. Then, \( E_{\text{cf}} \) in Equation (18) can be expressed as

\[
E_{\text{cf}} = \frac{6\pi \xi (m_e c^2)^2}{\sigma_T B_0^2 l_0} B^{\zeta - 2}.
\]

The corresponding electron Lorentz factor is

\[
\gamma_{e,\text{cf}} = \frac{6\pi \xi m_e c^2}{\sigma_T B_0^2 l_0} B^{\zeta - 2},
\]

and the emitted photon energy in the observer frame is

\[
E_{\text{obs}} = (h \nu_{\text{cf}})_{\text{obs}} = h \frac{eB}{m_e c} \gamma_{e,\text{cf}} \Gamma (1 + z)^{-1}
\]

\[
= \frac{\hbar m_e c^3 e}{\sigma_T B_0^2 l_0} \left( \frac{6\pi \xi}{\sigma_T B_0^2 l_0} \right)^2 (1 + z)^{-1} B^{2\zeta - 3},
\]

where \( h \) is the Planck constant, \( e \) is the electron charge, and \( z \) is the redshift. Inserting Equation (19), the above equation
The dependence of $E_{\text{obs}}$ on $\Gamma$, $L$, and $r$ is determined by the exact value of $\zeta$. Based on the empirical tight correlation among the peak energy, $\Gamma$, and $L$ suggested by observations (Liang et al. 2015), we adopt $\zeta \approx 2.1$. By assuming $\xi = 10^4$, $B_0 = 10^5$ G, and $l_0 = 2 \times 10^{10}$ cm together and using other typical parameters $\Gamma = 100 \Omega_2$, $L = 10^{52}$ erg s$^{-1}L_{52}$, and $r = 10^{15}$ cm $r_{15}$, Equation (24) gives

$$E_{\text{obs}} \approx 385 \text{keV} \left( \frac{1+z}{2} \right)^{-1} \Gamma^{-0.2} L_{52}^{0.6} r_{15}^{-1.2}.$$  

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Figure 1. (a), (b) Temporal evolution of the energy distribution of electrons from the numerical solution to Equation (4). (c), (d) The spectral distribution at a specified $\tau$; “$N_n$” and “$N_a$” denote the numerical and analytical results, respectively. The analytical results in (c) are from Equation (11), and in (d) are from Equation (14) for $\tau = 0.1$ and Equation (15) for $\tau = 10$.

Figure 2. Energy distribution with different $\epsilon$ values at $\tau = 10$. The vertical dashed lines indicate the positions of $E_{\text{cf}}$. Besides, we can also estimate the timescale for an initial energy distribution with the index $\rho_0$ to evolve to a hard
spectrum (Equations (2), (8), (19), and (20)):

\[ I_{E_0} = \frac{\tau_{ad}}{a_2} = \frac{1}{4c} \left[ \ln \left( \frac{E_2}{E_1} \right) \right]^2 \frac{I_0}{L_{\text{tur}}} \left( \frac{B}{B_0} \right)^{-\zeta} \]

\[ = \frac{1}{4c} \left( \frac{2}{c} \right)^{5/2} \left[ \ln \left( \frac{E_2}{E_1} \right) \right]^2 \frac{B_0^2 I_0}{L_{\text{tur}}} \tau_{\text{accel}}^{-\zeta} L^{-1/2} \tau_{\text{accel}}^{1/2}, \]

(26)

with \( \ln \left( \frac{E_2}{E_1} \right)^2 = 10^2 F \left( \frac{E_\mu}{E_\nu} \right)^2 \). Irrespective of the value of \( p_0 \) (which can be larger or smaller than one), after \( I_{E_0} \), the electron distribution index \( p \) approaches one under the effect of the adiabatic acceleration.

4. Discussion

We have applied the adiabatic acceleration mechanism in MHD turbulence, which was identified earlier by Brunetti & Lazarian (2016) and derived a robust electron energy distribution index \( p \sim 1 \) and a synchrotron low-energy photon index \( \alpha \sim -1 \), generally consistent with the observations. The estimated characteristic synchrotron emission energy with proper turbulence parameters required is in the sub-MeV regime, which is also consistent with the observations. A hard particle spectrum due to stochastic accelerations was also discussed within the GRB context by, e.g., Bykov & Mészáros (1996), Asano & Terasawa (2009), Asano & Mészáros (2011), Murase et al. (2012), Asano & Terasawa (2015), and Asano & Mészáros (2016). However, here we consider a different non-resonant acceleration related to the reconnection and dynamo processes in MHD turbulence and strictly derive \( p = 1 \) analytically.

Depending on the relation between the magnetic and turbulent kinetic energies, turbulence has various regimes. In the magnetic energy-dominated turbulence, the first-order Fermi acceleration during the turbulent reconnection (de Gouveia dal Pino & Lazarian 2005; Kowal et al. 2012a) can dominate the electron acceleration and shape the initial energy distribution. With the conversion of magnetic energy to turbulent kinetic energy, the acceleration process becomes globally stochastic in the trans-Alfvénic turbulence and rapidly flattens the electron energy distribution. To more realistically model the synchrotron spectrum, one should consider the interplay between the particle injection and acceleration, with synchrotron cooling incorporated self-consistently (S. Xu et al. 2017, in preparation).

Aside from the GRB prompt emission spectrum, observations of active galactic nuclei, blazars, and pulsar wind nebulae also reveal a hard electron distribution (e.g., Shen et al. 2006; Hayashida et al. 2015). The acceleration mechanism presented here is a promising candidate for interpreting the spectral hardness in various scenarios.

We thank the anonymous referee for insightful comments. S.X. is grateful for the support from the Strategic Priority Research Program of the Chinese Academy of Sciences (grant No. XDB23040000) and the Research Corporation for Scientific Advancement during her visit at the Aspen Center for Physics. S.X. thanks Yuanpei Yang for valuable discussions. This work is partially supported by the National Basic Research Program (973 Program) of China under grant No. 2014CB845800.

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