Dimension of quantum channel of radiation in pure Lovelock black holes

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It is known that the emission rate of entropy from a Schwarzschild black hole is exactly the same as that of a one dimensional quantum channel \[ D \]. We calculate the dimension of entropy emission from a \( D \) dimensional pure Lovelock black holes. Our results indicate that the dimension of transmission for odd \( D \) dimensional space-times is equal to \( D \) and for even \( D \) dimensional space-times, the dimension of quantum channel becomes \( 1 + \epsilon(\Lambda) \), where \( \Lambda \) is cosmological constant. It is interesting that cosmological constant may put some constraint on dimension of quantum channel in even dimensional space-times. The effect of Generalized Uncertainty Principle (GUP) on the dimension of transmission of entropy for a Schwarzschild black hole is also investigated.

I. INTRODUCTION

Hawking published his idea about the radiation from a black hole as a tunneling process due to the vacuum fluctuations near the horizon \[ 2 \]. There are several methods to derive the Hawking radiation. In his original derivation, Hawking calculated the Bogoliubove coefficients between in and out states in a black hole background. The Damour-Ruffini method for deriving Hawking radiation is based on the relativistic quantum mechanics in the curved space time \[ 3 \]. Based on Hawking’s first description, Parikh and Wilczek supposed that a virtual particle pair is spontaneously produced near the horizon while the negative energy particle tunnels inward and the positive energy particle escapes to infinity \[ 4 \]. Christensen and Fulling also developed a method based on the calculation of Hawking fluxes \[ 5 \]. They determined the strength of the Hawking radiation flux using the trace of energy-momentum tensor. Also, it is known that the fluxes of Hawking radiation cancel gravitational anomaly at the horizon \[ 6, 7 \]. It is interesting that a black hole as an entropy emitter behaves as a one-dimensional information channel \[ 1 \]. Recently, Hawking radiation energy and entropy flux of a Schwarzschild black hole are viewed as a one-dimensional quantum channel using the Landauer transport model \[ 8 \]. This model was first applied to the electrical transport in mesoscopic physics and then used to study thermal transport. According to this model, the one-dimensional quantum channel connects two thermal reservoirs with different temperatures to cause thermal transportation. One reservoir is the black hole and the other is the environment outside the black hole. The entropy flow is equal to the expected value of the energy-momentum tensor. This method is applied to different black holes such as BTZ, Reissner-Nordstrom, Kerr, Kerr-Newmann, Kaluza-Klein and 5D black rings \[ 9, 11 \]. The Hawking radiation from all of these black holes can be seen as a 1D quantum channel.

We believe that these results should have a bearing on the findings of recent studies that maintain that space-time might be two dimensional near the Planck scale (for a review see \[ 12 \]). The objective of the present paper is to calculate the dimensionality of entropy transmission from pure Lovelock black holes \[ 13, 10 \].

In higher dimensions, the Einstein-Hilbert Lagrangian can be generalized to new Lagrangians which exhibit several unique properties. These Lagrangians involve the sum of products of curvature tensors with the indices contracted in a specific manner. We know that the curvature tensor involves second derivatives of the metric tensor, \( \partial^2 g \), and a term involving the product of curvature tensors that has a cubic term in \( \partial^2 g \). Nevertheless, it is possible to construct Lagrangians which lead to equations of motion that only involve up to second derivatives of the dynamical variables. These theories were introduced by Laczos and Lovelock \[ 17, 18 \].

After introducing pure Lovelock black holes, we will investigate the dimensionality of entropy transmission of these black holes. Our results show that in odd space time dimensions, the radiation from a \( D \) dimensional pure Lovelock black hole can be described by the \( D \) dimensional quantum channel. In even dimensions, an interesting phenomenon appears, namely, we obtain a relation between the dimension of the quantum channel and the cosmological constant \( \Lambda \). We also investigate the effect of the Generalized Uncertainty Principle (GUP) on the emission rate of entropy from a Schwarzschild black hole. It is believed that the Hisenberg Uncertainty Principle needs to be revised as it is no longer satisfactory for strong gravity regimes. The concept of Generalized Uncertainty Principle (GUP) was first put forward by Mead \[ 19 \] in 1964, and some other models were proposed later \[ 20 \]. Many authors have applied GUP to modify Quantum Mechanics and black hole relations \[ 21 \]. We will show that the emission rate of entropy for a Schwarzschild black hole as given by GUP deviates by a small factor from that given by the one dimensional channel.

The outline of this paper is as follows: In Sec. II, we review the method of Pendry and Bekenstein for maximum emission rate of entropy for the Schwarzschild black hole. Entropy emission rate from a BTZ black hole is considered in Sec. III. In Sec. IV, the dimensionality of
radiation from a pure Lovelock black hole is obtained. In Sec. V, we consider the effect of GUP on the emission rate of entropy from a Schwarzschild black hole.

II. ENTROPY EMISSION OF SCHWARZSCHILD BLACK HOLE IN (D)-DIMENSIONAL SPACE TIME

In this paper, we use the derivation of Pendry for the entropy emission rate $\dot{S}$ [22].

The dimensionality of the transmission system can be inferred from the exponent of the radiation power $P$ in the expression $\dot{S}(P)$. To evaluate this expression, we notice that $\dot{S} = \nu^2\frac{P}{T}$, where $T$ is temperature, $\nu$ is $\frac{d+1}{2}$ in the flat space time ( $d$ is dimension of space) and equals to another constant in the curved space time [23]. For unidirectional current of modes $P(T)$ has the following form

$$P(T) = \frac{\pi T^2}{12}$$  (1)

By eliminating $T$ between $P(T)$ and $\dot{S}(T)$, we have:

$$\dot{S} = \left(\frac{\pi P}{3}\right)^{1/2}$$  (2)

By repeating the above analysis in the 3-d space and using the Stefan-Boltzmann law ($P = \frac{\pi^2 T^4 A}{120}$), we have

$$\dot{S} = \frac{4P}{3T} = \frac{2}{3}\left(\frac{2\pi^2 AP^3}{15}\right)^{1/4}$$  (3)

As in the flat spacetime, the dimensionality of the transmission system can be inferred from the exponent of $P$ in the expression $\dot{S}(P)$ for the d space dimensions

$$\dot{S}(P) \propto P^{\frac{d}{d+1}}$$  (4)

So, the entropy transmission in a single photon polarization out of a closed hot black body surface, according to Eq. (4), is 3-dimensional.

For the radiation from a Schwarzschild black hole of mass $M$ in a $D$ dimensional space time, we must consider that we have $A = \frac{2\pi^{D-1}}{\Gamma(\frac{D}{2})} r_h^{D-2}$ and that the Hawking temperature is $T_H = \frac{\hbar}{2\pi^{D/2}} r_h^{-1}$. The Stefan-Boltzmann law in $D$ dimensional curved space time

$$P = \sigma_D AT^D$$  (5)

where, $\sigma_D = \frac{\hbar^2}{8\pi^2}\frac{\Gamma^{D/2}}{\Gamma(D+1)} k_B \zeta(D) \zeta(D)$ [24] and $\zeta$ is the average of the frequency dependent transmission factor ($T$) over the Planck spectrum [25]. By Eliminating $r_h$ between the equations for $A$ and $T$ and using Eq. (5) and Pendry's maximum entropy rate for power $P$ ($\dot{S} = \nu^2\frac{P}{T}$), we will have

$$\dot{S} = \beta P^{1/2}$$  (6)

where, $\beta = \frac{2\pi^{D-1}}{\Gamma(D/2)} \left(\frac{\hbar}{\Gamma(D+1)}\right)^2 \frac{D-1}{2} \pi^{D/2}$. According to Eq.(6), this formula shows that the entropy flow or information out of the Schwarzschild black hole in D-dimensional space time is one-dimensional. It is also known that the Hawking radiation (entropy flow) can be represented by the one dimensional Landauer transport model [8]. There are other evidences that are consistent with the idea that black holes act as a one dimensional quantum channel. It is shown that near the horizon and transverse to the t-x plain ($r_H = 2M + \frac{\hbar}{2\pi^2}$), dimensional quantities and excitations are redshifted away and each outgoing partial wave acts as a (1+1)-dimensional black body wave at Hawking temperature [26]. Also we can obtain entropy production ratio defined as bellow [27]:

$$R = \frac{ds}{ds_{BH}} = T_H \frac{\dot{S}}{E}$$  (7)

where $ds$ is the entropy carried by radiation from black hole to the environment and $ds_{BH}$ is the change of entropy of the black hole through radiation. For radiation into vacuum, the entropy production ratio in 1d space is 50% larger than that of 3d space. It is interesting that we may obtain a general upper bound on the thermal conduct [8].

III. ENTROPY EMISSION OF BTZ BLACK HOLE

The line element of the BTZ black hole could be written as [28]:

$$ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta} + r^2(d\phi - \frac{J}{2r^2} dt)^2$$  (8)

where the lapse function is

$$\Delta = -M + \frac{\nu^2}{r^2} + \frac{J^2}{4r^2}$$  (9)

and $M, J$ are the mass and angular momentum of the BTZ black hole, respectively. The Hawking temperature and the area of the event horizon, (when $J = 0$) are given by:

$$T_H = \frac{1}{4\pi} \frac{d\Delta}{dr} |_{r_h}, \quad A = 2\pi r_h$$  (10)

By using Eq. (5) ($P = \sigma_3 AT^3$), and the equations for $A$ and $T$ in (10) we have:

$$P = \sigma_3 \frac{r_h^3}{8\pi^2}$$  (11)

If we find $r_h$ in terms of $P$ and substitute in equation for temperature yields: $T_H = \frac{\beta P^{1/4}}{(2\pi^3)^{1/4}}$. Finally by considering Pendry’s maximum entropy rate ($\dot{S} = \nu^2\frac{P}{T}$), we will obtain

$$\dot{S} = \xi P^{3/4}$$  (12)
where $\xi = \nu (2\sigma_3)^{1/4}(\pi l)^{1/2}$. We conclude that the entropy flow or information out of the BTZ black hole in 3-dimensional spacetime is 3-dimensional.

IV. ENTROPY EMISSION OF LOVELOCK BLACK HOLES

There are two distinct derivations for the gravitational dynamics. One straightforwardly follows from the Bianchi differential identity, which is the only geometric relation available. By taking the trace (contraction) of the Bianchi identity, the Einstein equation can be deduced as follows

$$G_{ab} = \kappa T_{ab} - \Lambda g_{ab} \quad T^a_{\ b;a} = 0 \quad (13)$$

where, $T_{ab}$ is the second rank symmetric tensor that represents the energy momentum distribution and $\kappa$ and $\Lambda$ are constants. In the second derivation of the gravitational dynamics, the variation on the Einstein-Hilbert Lagrangian leads to the divergence free Einstein equation, $G_{ab}$, and, subsequently, to the Einstein equation. The generalized Lagrangian in higher dimensions is a Lovelock Lagrangian, which takes the following form

$$L = \sqrt{-g}(\alpha_0 + \alpha_1 R + \alpha_2 (R^2 + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu
u}R^{\mu
u}) + \alpha_3 O(R^3))$$

where, $\alpha_0$ corresponds to the cosmological constant ($\Lambda$), $\alpha_1$ is a coupling constant that represents the standard Einstein-Hilbert term and the second order term i.e. $L_2 = R^2 + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu
u}R^{\mu
u}$ is precisely the quadratic Gauss-Bonnet term. $\alpha_n$ with $n \geq 2$ are designated the coupling constants of the higher order terms that represent ultraviolet corrections to Einstein theory.

The theory involving only the first three terms is known as the Einstein-Gauss-Bonnet (EGB) theory. The remarkable property of the Lovelock Lagrangian is that it does not contain the squares of the second derivative so, the equation of motion remains quasi-linear. An analogue of the Riemann tensor has been introduced, which is a polynomial in the Riemann curvature. It also has been shown that it is possible to derive an analogue of $G_{ab}$ for higher dimensional gravities. The analogue of $G_{ab}$, i.e. a divergence free $H_{ab}$, can be obtained by a variation upon the Lovelock Lagrangian and by tracing the Bianchi derivative of a tensor which is homogeneous quadratic in the Riemann curvature \([13]\). The Lovelock curvature polynomial can be defined as follows

$$F^{(n)}_{abcd} = F^{(n)}_{abcd} - \frac{n - 1}{n(D - 1)(D - 2)} F^{(n)}(g_{ac}g_{bd} - g_{ad}g_{bc}),$$

$$F^{(n)}_{abcd} = Q_{ab}^{mn} R_{cdmn},$$

$$G^{ab}_{\ cd} = g^{ab}_{\ cd} + \cdots a_{db} R_{a_1 b_1} \cdots a_{db} c_{d_1} \cdots c_{d_n},$$

$$Q^{ab}_{\ cde} = 0 \quad (16)$$

The analogue of $n^{th}$ order Einstein tensor is as follows

$$G^{(n)}_{ab} = n(R^{(n)}_{ab} - \frac{1}{2} R^{(n)} g_{ab})$$

and

$$R^{(n)} = \frac{D - 2n}{n(D - 2)} F^{(n)}$$

where, $F^{(n)}$ is the Lovelock action polynomial. The dynamics of gravity in higher dimensions (the Einstein-Lovelock equation) would be

$$\sum \alpha_n G_{ab}^{(n)} = 0$$

where, $\alpha_0 = \Lambda$, $G_{ab}^{(0)} = g_{ab}$, $G_{ab}^{(1)} = G_{ab}$ is the Einstein tensor and $G_{ab}^{(2)} = H_{ab}$ is the Gauss-Bonnet analogue. We shall focus on the pure Lovelock equation given by

$$G_{ab}^{(n)} = \Lambda g_{ab}$$

The spherically vacuum solution of the above equation is

$$ds^2 = f(r) dt^2 - \frac{1}{f(r)} dr^2 - r^2 d\Omega^2_{D-2}$$

with

$$f(r) = 1 - r^2(\Lambda + \frac{\mu}{r^{D-2}})^{1/n}$$

where, $\mu$ is the black hole mass parameter, and $D$ is the spacetime dimension \([13, 15]\). It should be noted that $G_{ab}^{(n)}$ is non-zero in dimension $D > 2n$, hence the critical odd and even dimension are $D = 2n + 1$ and $D = 2n + 2$ respectively. The black hole temperature is computed by evaluating the expression $T = \frac{1}{\Omega} f'(r_h)$ to obtain \([16]\)

$$T = \begin{cases} \frac{1}{4\pi} \left( \frac{1 - \frac{1}{r_h}}{r_h} + \frac{D - 1}{D - 2} \right) & D = 2n + 1 \\ \frac{1}{4\pi} \left( \frac{1 - \frac{1}{r_h}}{r_h} + \frac{D - 1}{D - 2} \right) & D = 2n + 2 \end{cases}$$

Using $f(r) \big|_{r=r_h} = 0$, we can initially obtain the following expressions for $\mu$

$$\mu = \begin{cases} 1 - \Lambda r_h^{D-1} & D = 2n + 1 \\ r_h - \Lambda r_h^{D-1} & D = 2n + 2 \end{cases}$$

If we substitute the above expressions for $\mu$ in Eq.\([23]\), we will have

$$T = \begin{cases} \frac{-\Lambda r_h^{D-2}}{4\pi} & D = 2n + 1 \\ \frac{1 - \Lambda r_h^{D-1} - \frac{D-2}{(D-2)r_h}}{4\pi} & D = 2n + 2 \end{cases}$$

First it should be noted that in odd dimensions the cosmological constant must be negative $\Lambda = -\frac{(D-1)(D-2)}{2(2n)^{D-2}}$. We will calculate the dimensionality of the transmission of entropy or information, for the pure Lovelock black hole. For $D = odd$, substituting the Hawking temperature of the pure Lovelock black hole, i.e, Eq.\([25]\), and
the area of the black hole horizon \( A = \frac{2\pi^{D-1}}{\Gamma(\frac{D}{2})} r_h^{D-2} \), in the D-dimensional version of the Stefan-Boltzmann law \( (P = \sigma_D A T^4) \) yields the following expression for \( r_h \) in terms of \( P \)

\[
r_h = \alpha P^{1/(D-2)(D+1)}
\]  

(26)

where, \( \alpha = \left( \frac{1}{\sigma_D (D-1)(D-2)} \right)^{1/(D-2)(D+1)} \).

Therefore, the Hawking temperature of the pure Lovelock black hole for \( D = \text{odd} \) can be written as follows

\[
T = \frac{(D-1)(D-2)}{4\pi l^2} \alpha^{D-2} P^{1/D+1}
\]

(27)

Finally, the Pendry’s maximum entropy rate \( (\dot{S}(P)) \) for the pure Lovelock black hole can be obtained as

\[
\dot{S} = \nu \frac{P}{T} = \nu \frac{4\pi l^2}{(D-1)(D-2)} \alpha^{2-D} P^{\frac{1}{D+1}}
\]

(28)

Eq.(28) shows that in an odd \( D \) dimensional spacetime, entropy emission from a pure Lovelock black hole is \( D \) dimensional. In odd dimension pure Lovelock gravity has a similar behavior as a \( \text{BTZ} \) black hole [14]. In both of \( \text{BTZ} \) black hole and pure lovelock black hole in odd dimensions the dimension of quantum channel is obtained to be the same as spacetime dimensions.

For \( D = \text{even} \), we will have the following equation for the radiation power \( P \)

\[
P = \gamma r_h^{-2}(1 - \Lambda(D - 1)r_h^{D-2})^D
\]

(29)

where, \( \gamma = \frac{\Gamma \sigma_T}{(2\pi)^{D-2} \Gamma(D/2)} \). From the above equation, we can obtain the following relation for \( r_h \) in terms of \( P \)

\[
r_h = (\gamma^{-1} P)^{-1/(2+\epsilon)}
\]

(30)

where, \( \epsilon = -D \ln r_h (1 - \Lambda(D - 1)r_h^{D-2}) \). Therefore, we can write \( (\dot{S}(P)) \) as follows

\[
\dot{S} = \nu \frac{P}{T} = \nu (2\pi)(D-2) \gamma^{D-2} P^{\frac{1}{D+1}}
\]

(31)

Since in even dimensions, the cosmological constant can be negative, as well as radiation from the event horizon, the radiation from the cosmological horizon should be considered. In this way, we may calculate the dimension of quantum channel related to the cosmological horizon that leads to \( 1 + \epsilon = -D \ln r_c (1 - \Lambda(D - 1)r_c^{D-2}) \), where \( r_c \) is the position of cosmological horizon. According to the above equation, entropy emission from a pure Lovelock black hole in even dimensions is different from that of a Schwarzschild black hole by a factor of \( \epsilon \). Since Schwarzschild black hole in four dimensional spacetime is pure Lovelock case with \( n = 1 \) and \( \Lambda = 0 \) [14] the dimension of quantum channel for Schwarzschild black hole is \( 1 + \epsilon \) with \( \epsilon(\Lambda = 0) = 0 \). So equation

(31) is consistent with the result of section II about Schwarzschild black hole. Also the channel should be exactly one dimensional for all even dimensional Lovelock black holes with \( \Lambda = 0 \). It is interesting that the value of the cosmological constant may put some constraints on the dimension of the quantum channel or vice versa:

\[
\Lambda_D = \frac{1 - \frac{\gamma}{\pi l^2}}{\left(\frac{D-1}{D} r_h^D\right)}
\]

By considering a particular value for the cosmological constant, one should be able to have black holes of various sizes and specific values of the dimension of radiation quantum channel. As a special case, if we assume that the dimensionality of transmission is an integer number; \( 1 + \epsilon = a = 2, 4, \ldots, D \), then the connection between the cosmological constant and the dimension of radiation quantum channel can be obtained as follows:

\[
\Lambda_D = \frac{1 - \frac{\gamma}{\pi l^2}}{(D-1)r_h^D}
\]

This analysis suggest a possible connection between the cosmological constant and a quantum theory of gravity. Recent numerical and analytical studies suggest that the dimension of spacetime may be spontaneously reduced to two dimensions near the Planck scale [12]. For even dimensional spacetimes, our results support these suggestions. However, odd dimensional pure Lovelock black holes are new objects with a possibly different behavior near the Planck scale. It is interesting to further study odd dimensional pure Lovelock black holes as they may really stay \( D \) dimensional near the Planck scale.

V. MODIFICATION TO BLACK HOLE RADIATION DUE TO GUP

One off the predictions of quantum gravity theories such as string theory, loop gravity, doubly special relativity etc., is the existence of a minimum measurable length or area. This has led to the so-called Generalized Uncertainty Principle or GUP [19–21]. To study the modification to the black hole radiation due to GUP, we should notice two corrections; the modification to Stefan-Boltzmann radiation law and the black hole temperature correction. Using the modified commutation relation of the form

\[
[x, p] = i(1 + bp^2)
\]

(32)

we have the modified Stefan-Boltzmann law for the single photon radiation channel as follows [29]

\[
\frac{P}{A} = \alpha T^4 - b \beta T^6
\]

(33)

where

\[
\alpha = \frac{\Gamma}{120}
\]

\[
\beta = \frac{\Gamma}{24} \left( \frac{18.5\pi^2}{15} + \frac{34\pi^4}{63} \right)
\]

(34)

For temperature correction along the lines of [30], we
have
\[ T_H = \frac{1}{8\pi M} \left(1 + \frac{b}{16M^2}\right) \] (35)

where, \( M \) is the black hole mass. By substituting \( A = 4\pi r_s^2 = 4\pi (2M)^2 \), we can eliminate \( M \) between radiation power \( P \) and entropy current \( \dot{S} \) (in the form of \( \dot{S} = \nu \frac{P}{T} \)) and calculate \( \dot{S}(P) \) as follows
\[
\dot{S} = \left(\frac{\nu^2 \Gamma \pi P}{480}\right)^{1/2} \left[1 + \frac{b}{1} \left(4.97\pi - \frac{5.78}{\pi}\right) P\right]
\] (36)
\[
\dot{S} = \left(\frac{\nu^2 \Gamma \pi}{480}\right)^{1/2} \frac{1}{P} \frac{b}{\ln P} \left(4.97\pi - \frac{5.78}{\pi}\right)
\] (37)

where
\[
\epsilon = -\frac{P}{\ln P} \frac{b}{1} \left(4.97\pi - \frac{5.78}{\pi}\right)
\] (38)

As a special case, if we assume an integer dimension for the transmission radiation, i.e. \( 1 + \epsilon = D = 1, 2, 3, 4 \), we can obtain a quantized value for \( b \) as follows
\[
b = \frac{\Gamma(D-1) \ln P}{P \left(\frac{480}{\pi^2} - 4.97\pi\right)}
\] (39)

So, we have a relation for \( b \) in terms of the radiation power \( P \) and the dimensionality of entropy transmission \( D \). So, an integer dimension of the entropy emission imposes some constraints on GUP.

VI. CONCLUDING REMARKS

Entropy emission from a \( D \)-dimensional Schwarzschild black hole behaves as a one dimensional quantum channel. In this paper, we considered the dimension of entropy emission from \( D \) dimensional pure Lovelock black holes. It was seen that in odd space time dimensions, the quantum channel of radiation is \( D \) dimensional. In even dimensions, cosmological constant put some constraints on the dimension of quantum channel. However, pure Lovelock black holes are the first example that radiate through D-dimensional quantum channel. Although Lovelock black holes are not in the framework of General Relativity but their study from this point of view has some new results. Finally, we considered the effect of the Generalized Uncertainty Principle (GUP) on the emission rate of entropy for a Schwarzschild black hole to find that it may deviate from its one dimensional behavior. It should be noted that we do not know if the dimension of quantum channel should be an integer or might be other values. Assuming an integer number for the dimension of quantum channel leads to some constraints on cosmological constant.

It may be concluded that a relation might exist between the dimension of quantum channel and recent findings about the dimensional reduction of spacetime near the Planck scale \( 12 \). We may expect that for pure Lovelock black holes such dimensional reductions does not appear in odd space time dimensions.

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