Mass screening by dark matter in weak gravity in de Sitter space

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Unitary holographic encoding of particle positions by their wave functions defines information that scales linearly with the distance to the screen which naturally couples to dimensionless curvature. Gravitational lensing hereby decouples from the energy density of vacuum fluctuations. In the limit of weak gravity in de Sitter space, it gives rise to mass screening by a factor of $4\pi$ based on recent compelling evidence of Milgrom’s empirical relation. The origin of mass screening over large distances is apparently a new physical property of dark matter.

I. INTRODUCTION

Gravitational lensing is a key feature of general relativity. It is of foremost importance astrophysics and cosmology as a tool to probe the distribution of dark matter. In this application to weak gravity, there is increasing observational evidence for the existence of a new dimensional scale of acceleration

$$a_0 = \frac{cH_0}{2\pi} \approx 10^{-8} \text{cm s}^{-2}, \quad (1)$$

where $H_0$ denotes the Hubble constant and $c$ the velocity of light. It also appears in relation to cosmological constant $\Lambda$, and it reflect physics beyond general relativity and galaxy formation scenarios in ΛCDM. However, the premise of universal coupling of curvature to energy in general relativity is notoriously at odds with the divergent energy density of vacuum fluctuations, commonly referred to as the cosmological constant problem.

Here, we revisit gravitational lensing by considering unitary holographic encoding of particle wave functions. This is a first principle approach, that leads to a dimensionless formulation of curvature as a function of dimensionless distances to holographic screens. Unitarity is seen to give rise to minimum holographic information in the encoding of the particle location. Earlier studies focused on entropic constraints on massive particles falling into black holes [1], the thermodynamics of the equation of state of space-time [2] or massive particles passing through time like surfaces [21]. However, neither of these studies derives gravitational lensing explicitly from unitarity in holography of massive fields.

Gravitational lensing arising from unitary encoding of a particle’s probability to be inside or outside a holographic screen is seen to be free of a cosmological constant problem. This part of the theory is rigorous with no assumptions, based on unitarity with no reference to thermodynamics. Its generalization to the gravitational attraction between massive particles inevitably involves finite temperatures as discussed in recent proposals of entropic gravity [21] that extends to the gravitational attraction between black holes [20].

There is increasingly compelling evidence for Milgrom’s law giving rise to [1][2], that opens a new window to exploring possible connections between baryonic matter, dark matter and the observed cosmological constant based on thermodynamic considerations in de Sitter space. It implies a distinction between baryonic mass as inferred from the luminous and gas content in galaxies and its effective mass at large distances, where gravitational attraction is weak.

The plan of this paper is as follows. In §2, we give a dimensionless formulation of curvature in general relativity. The information required for unitary holography is described in §3, and correlated to dimensionless curvature in §4. In this formulation, black holes appear in the formation of event horizons to enforce holographic censorship, that prohibit resolving three-dimensional structures smaller than the Planck scale (§5). Implications for the interaction between dark matter and baryonic matter are given in §6.

II. DIMENSIONLESS CURVATURE $J$

The past domain of dependence $D^-$ of an observer is the region of space-time in the past which can causally influence its present along time-like or null trajectories (Fig. [1]). Viewed over some time interval $\Delta t$, its shape is a four-dimensional cone with three-dimensional region $B$ at its base and a boundary $N$ along the past light cone of the observer. We introduce two surface areas: $A_f$ of a wave front emitted emitted into the past from the observer, given by the 2-surface $\partial B$ in common with $N$, and $A_0$ of an equatorial 2-surface in $B$. In Minkowski space-time, $B$ is a solid sphere of radius $R = ct$, where $c$ denotes the velocity of light, whereby $A_f = 4\pi R^2$ and $A_0 = \frac{1}{2}A_f$. The ratio

$$J = \frac{A_0}{A_f} \quad (2)$$

represents a dimensionless curvature of the past domain of dependence.

When space is curved, (2) will be different from 1/4. Null-generators of $N$ are generally subject to geodesic deviation, described by a local rate of expansion $\theta$ of two neighboring null-geodesics as defined the integrals of null-tangents $k^b = dx^b/d\lambda$ satisfying the Raychaudhuri
A massive particle \( m \) may be detected at a separation \( a < R \) from \( O \). This preparation fixes the probability for \( m \) to be in or out of \( D^- \) in any subsequent measurement, described by the propagator of \( m \). The corresponding Shannon entropy \( I \) has a holographic encoding on \( \partial B \). (Right.) Shown is an equatorial cross section of \( B \) with surface area \( A_0 \) and the surface area \( A_f \) of \( \partial B \). Included are the projection of two generators of \( N \) from the center \( O \) past \( m \) up to this wave front \( \partial B \). \( A_0 \) provides a measure for phase space and \( \frac{1}{4}A_f \) provides a measure for the total entropy seen by outside observers. Due to \( I \geq 0 \), \( B \) has a dimensionless curvature \( J \equiv A_0/A_f \geq \frac{1}{4} \).

In the linearized limit, the Raychaudhuri equations is

\[
\frac{d\theta}{d\lambda} = -R_{ab}k^a k^b, \tag{3}
\]

where \( R_{ab} \) denotes the Ricci tensor. For time-symmetric three-dimensional hyper surfaces of constant time with unit normal vector field \( s^i \), we have \( \frac{\partial}{\partial \lambda} \)

\[
A_f = \int_{|x| \leq r} \theta \sqrt{T_0} d^3 x \tag{4}
\]

where \( d\Sigma \) refers to a surface element at constant coordinate radius \( r \).

To illustrate the above, let us consider curvature as described by the Einstein equations. In the Schwarzschild metric, \( (1) \) gives \( \theta = 2\alpha/r, \alpha = \sqrt{1 - 2M/r} \), whereby \( A_f = 4\pi r^2 \) recovers the familiar surface area of spheres with coordinate radius \( r \). Also, \( R_{ab} = 8\pi\rho u_a u_b \) for a matter density \( \rho \) with four-velocity \( u^b \). In geometrical units, \( (3) \) then gives \( \frac{\partial}{\partial \lambda} \)

\[
J_E = \frac{1}{4} \left( 1 + \frac{2m}{R} \right) \tag{5}
\]

for a total mass \( m \) in \( \rho \). The same result follows by expanding the Schwarzschild metric for \( r \gg M \). For a given \( R \), increasing \( m \) leads to a contraction of a wave front of given phase emanating from \( m \left( A_f < A_0 \right) \).

Next, consider a massive particle \( m \) inside \( D^- \) in a static position some distance \( a \) away from the observer. In this event, \( (4) \) gives \( \frac{\partial}{\partial \lambda} \)

\[
J_E = \frac{1}{4} \left( 1 + \frac{2ms}{R^2} \right), \tag{6}
\]

denotes the depth of \( m \) in \( D^- \) across the causal boundary \( \partial B \). Evidently, \( (6) \) recovers \( J = 1/4 \) and \( (5) \) in the limit as \( m \) approaches the boundary \( \partial B \) and, respectively, the observer. In general terms, a compact central distribution of mass maximizes the dimensionless curvature \( (2) \).

In the next section, we shall relate \( (2) \) to unitarity associated with particle wave functions.

### III. UNITARITY IN HOLOGRAPHY

Holography \( [8, 17] \) describes the idea of encoding of phase space in a volume by information on its surface. The surface may arbitrary, as when choosing a closed surface (a screen) that defines space inside and its complement outside.

Quite generally, a screen provides a two-dimensional cut, giving rise to a region inside and outside. According to the wave function of a massive particle (described by its propagator), a screen hereby introduces the probabilities \( P_i \) and \( P_o \) for the particle to be inside or outside, respectively, subject to unitarity

\[
P_o + P_i = 1. \tag{8}
\]

must be satisfied by any theory encoding a particle’s location by information on two-dimensional screens.

A detection of a particle at a location \( r \) has the common interpretation that it prepares the particle wave function \( \phi(r) \) to be centered at \( r \), described by its propagator (e.g. \( [24] \)),

\[
p(\Delta r) = \langle 0|\phi(r - \Delta r)\phi(r)|0\rangle \approx e^{-k\Delta r}, \quad k = \frac{2\pi mc}{\hbar}, \tag{9}
\]

asymptotically for large phase separations \( k\Delta r \gg 1 \). For \( m \) prepared at a depth \( 1 \ll ks \leq kR \) inside \( \partial B \),

\[
P_o(s) \approx \int_{0}^{s} e^{-k\Delta r} d\Delta r = 4\pi k^2 s^2 + 2ks + 1 \tag{10}
\]

is naturally minimal for a preparation of \( m \) at the origin when \( s = R \).

Given a spectral representation of the wave function on \( \partial B \), e.g., in spherical harmonics, the holographic projection of \( \phi(r) \) inside a surface must be sufficiently accurate
to be consistent with $P_w$. According to complex function theory, the resolution obtained by a spectral representation (of analytic functions) scales exponentially with the degree of the partial sum. The number of terms required hereby scales with $-\log P_w$, i.e., the Shannon information (log 2 times the number of bits)

$$I(s) \simeq -\log P_w \simeq 2ks,$$

(11)

neglecting logarithmic terms following [10]. It follows that $0 \leq I(s) \leq 2KR$ for $m$ inside $\partial B$. If $I(s)$ is embedded in the structure of space, it naturally couples to our dimensionless curvature $J$.

IV. COUPLING OF INFORMATION TO CURVATURE J

The two dimensionless quantities (2) and (11) at hand are naturally coupled. Consistent with general relativity [5, 6], the simplest possible coupling is a linear relationship,

$$J_I = \frac{1}{4} [1 + \kappa I(s)],$$

(12)

where, $\kappa$ is a coefficient independent of $s$.

We can interpret (12) by considering outside observers, who attribute a total entropy to $D^+$ in fixed proportion to the surface area $A_f$ of the causal boundary $\partial B$, independent of $s$ by causality when $m$ is inside. On the null-surface $N$, the entropy satisfies $S = \frac{1}{4}A_f$, in common with black hole event horizons in general relativity. With this causality constraint, $J$ in (12) is expressed in variations in phase space $A_0$ in response to variations in $I(s)$, whereby

$$\delta A_0 = \frac{1}{2} \kappa A_f k\delta s = 2\kappa A_0 k\delta s.$$  (13)

It follows that $D^+$ is dynamical, which responds to nearby massive particles to accommodate $P_i$, quantified by $I(s)$ as a function of the distance $s$ of $m$ to $\partial B$, measured in terms of the dimensionless phase according to (7) and (11). These properties are defined by the propagator with no reference to temperature whatsoever. In sharp contrast, [7, 21] postulate an entropy attributed to time-like screens (measurable by outside observers) based on dimensional analysis alone with no reference to unitarity. By Gibbs’ principle, furthermore, a (real or virtual) displacement gives a change in phase space in proportion to $\kappa A_0$, hereby an increasing function as $m$ approaches the centre in accord with $\delta A_0$.

The scaling $\kappa A_0 = \frac{1}{2}$ brings (12) in accord with general relativity, whereby

$$J_I = \frac{1}{4} \left[ 1 + \frac{I(s)^2}{2\pi R^2} \right]$$

(14)

has the same $s$-dependence as Einstein’s curvature (6),

$$J_I(s) = J_E(s).$$

(15)

In (14), the Planck sized surface element $l_p^2 = G\hbar/c^3$ serves to convert $[k]=cm^{-1}$ to $[k l_p^2] = cm$, where $c$ denotes the velocity of light and $G$ denotes Newton’s constant. In general relativity, therefore, gravitational lensing is a manifestation of an accurate encoding of $m$ in a past domain of dependence. This is expressed in a dimensionless curvature as a function of $P_i$ or $P_w$, that are equivalent by [5, 6].

V. HOLOGRAPHIC CENSORSHIP

Our interpretation of the information in $J_I(s)$ for accurate localization of $m$ as a function of dimensionless distance $ks$ describes a requirement for holographic encoding [8, 17] of $2I(s) l_p^2$-sized bits on $\partial B$ with area $A_f = 4\pi R^2$. A maximal resolution results from the principle of holographic projection by interference using the Compton wave length

$$\lambda = \frac{\hbar}{mc}.$$  (16)

Local images appear by interference between waves propagating in different directions. Consider interference of a wave bend over some small angle $\theta$ with a background wave, described by

$$\psi = \cos kx + \cos k'x \simeq \cos kx + \cos (kx - \Delta \phi).$$  (17)

Distinguishable patterns appear for $\Delta \phi = \frac{1}{2}kx\theta^2 = 2\pi$ (mod $2\pi$) with associated angular period

$$\theta^2 = \frac{2}{\lambda},$$

(18)

where $x = s$ denotes the distance to a holographic screen. The encoding on the screen of the interference pattern $\psi$ can be used to create an image at $x$ with resolution $l = 2\theta s$, that is

$$\lambda s = l^2,$$

(19)

For a resolution not to exceed the Planck length, i.e., $l > l_p$, it follows that $s > R_S$,

$$R_S = \frac{2Gm/c^2}{\hbar},$$

(20)

for the location of $m$ to be encoded on the holographic screen: the holographic screen is not allowed to be smaller in radius than $R_S$. Event horizons provide holographic censorship, to avoid the possibility of imaging sub-Planck sized structures. Any attempt to do so requires a screen within an event horizon, taking it out of causal contact with an outside observer.

Black hole formation is seen to be entropically paradoxical. On the one hand, its entropy is defined by null surfaces, for which the entropy density is maximal as a general property of null-surfaces viewed by outside observers. On the other hand, the total surface area of the event horizon is to be viewed as minimal by closing upon itself and forming a compact 2-surface by nonlinear extension of the dimensionless curvatures [5, 6].
VI. COSMOLOGICAL IMPLICATIONS

The propagator for a vacuum in Minkowski space-time with full Poincare invariance satisfies

\[ p(\Delta r) = \langle 0 | \phi(r - \Delta r) \phi(r) | 0 \rangle \equiv 1 \]  

(21)

for all separations \( \Delta r \). The corresponding information is vanishing: no information is gained from detections in vacuum. Equivalently, the probability of detecting vacuum inside the screen is unit, which needs no information for encoding. In the absence of any differentiation in locations, \( J = 1 \), corresponding to a (vacuum) state with maximal symmetry. Thus, lensing hereby decouples from any properties that may be universally attributed to vacuum. In particular, lensing decouples from the divergent energy density of vacuum fluctuations. Any theory of gravity induced by encoding the propagator of matter fields, therefore, is free of a cosmological constant problem. This part of the theory is rigorous, as it follows from unitarity with no reference to thermodynamics.

Following [23, 21], we next consider the role of finite screen temperatures \( T_S \) in considering forces between point particles and black holes alike. Based on (15), we encounter forces

\[ F = T_S \frac{\Delta S_0}{\Delta a} = T_S A_f \frac{\Delta J_I}{\Delta a} \]  

(22)

by application of Gibb’s principle and (15). For \( T_S \), consider in a first approximation the postulate [21]

\[ M_b c^2 = \frac{1}{2} N k_B T_S, \]  

(23)

i.e., by equipartition of the baryonic energy \( M_b c^2 \) with \( N \) Planck-sized surface elements encoding our Shannon information in bits on a holographic surface with area \( A_f = 4\pi r^2 \). \( N = A_f / l_p^2 \). The corresponding Unruh temperature [18],

\[ k_B T_S = \frac{M_b c^2}{4\pi r^2} = \frac{\hbar a}{2\pi c}, \]  

(24)

indicates an associated acceleration \( a \), experienced by a particle on the screen attributed to entropic forcing. As in [21], the result is Newton’s law \( F = -GM_m/r^2 \). General relativity gives the unique covariant embedding of Newton’s law up to a cosmological constant - an integration constant as shown above. It therefore gives the four-covariant embedding of curvature by unitary holographic encoding of the wave function of particles. Modifications to Newton’s law may hereby derive from breaking Poincare invariance as in Casimir experiments [2, 12, 13, 25] or in weak gravity in a de Sitter universe.

In the application to weak gravity in de Sitter space, we note that the screen temperatures [23] easily reach the thermal de Sitter temperature of the cosmological event horizon, \( T_{dS} = H_0 \frac{\hbar}{2\pi} \approx 3 \times 10^{-30} \text{K} \), where \( H_0 \approx 75 \text{km s}^{-1} \text{Mpc}^{-1} \) is the Hubble constant and \( k_B \) is the Boltzmann constant [6]. Scaling to galaxies of mass \( M = 10^{11} M_\odot \) gives a characteristic radius on the scale of a halo, \( R = \sqrt{R_g c H_0^{-1}} = 4.4 \left( \frac{M}{10^{11} M_\odot} \right)^{1/8} \text{kpc} \), where \( R_g = GM/c^2 \) denotes the gravitational radius of the central object. By (22) with \( T = T_{dS} \), we have a corresponding scale of acceleration \( a_{dS} = H_0 c \) produced by \( T_{dS} \) and \( \Delta S_0 = A_f \Delta J_I \) introduced above.

Observations on (1) appear to be sufficiently accurate to consider the discrepancy by a factor of 2\( \pi \) between \( a_{dS} \) and (1) to be significant. Specifically, (1) appears in a tight correlation between an observed acceleration \( a \) in excess of the acceleration \( a_N \) by baryonic mass alone, expressed in Milgrom’s empirical relationship

\[ a = \sqrt{\alpha a_0 a_N}. \]  

(25)

It accurately accounts for the baryonic Tully-Fisher relation (BTFR), the anti-correlation between baryonic and dark matter distributions as seen in high to low surface brightness galaxies, and an empirical bound on galaxy surface densities [6]. On scales larger than galaxies, however, a counter example is the apparently striking mismatch between dark matter and luminous matter in the Bullet cluster [5]. Hence, (25) should be viewed within the constraints of galaxy dynamics, and not necessarily beyond. On the scale of the Universe, (1) appears yet again in the surface gravity \( a_0/(2\pi) \) of the cosmological event horizon at Hubble radius \( R_H = c/H_0 \) with corresponding de Sitter temperature [6] \( k_B T_{dS} = \hbar H_0/(2\pi) \), where \( k_B \) denotes the Boltzmann constant. To leading order, the cosmological constant [10, 14, 17] can hereby be accounted for as entropic pressure [4].

The modification of (22) to entropic forces in a de Sitter background is \( F = [T] \Delta S_0 / \Delta a \), where \([T] = T_S - T_{dS}\) denotes the screen temperature \( T_S \) relative to \( T_{dS} \) due to its mass-energy content \( M \). Correspondingly, (23) is modified according to (e.g. (11) and references therein)

\[ M c^2 \frac{L_p^2}{2\pi r^2} \equiv T_S - T_{dS} = \sqrt{\frac{\hbar a}{2\pi c}}^2 + T_{dS}^2 - T_{dS}. \]  

(26)

The limits of Newtonian and weak gravity are hereby characterized by, respectively \( r^{-2} \) and \( r^{-1} \) force laws, obtained from \( a = a_M \) (\( a >> a_0 \)) and \( a = \sqrt{4\pi a_M a_0} \) (\( a << a_0 \)), where we put \( a_M \equiv GM/r^2 \). Here, \( \sqrt{4\pi} \approx 3.5 \) is significant by the accuracy of data supporting (25).

This discrepancy leads us to distinguish between the bare mass \( M_b \) measured locally (\( a >> a_0 \)) - the sum of the baryonic masses inside the screen - and the effective mass \( M \) which defines the total mass-energy of the information on the screen. In weak gravity, therefore

\[ M_S = \frac{1}{4\pi} M_b. \]  

(27)

It appears that dark matter screens baryonic matter over large distance in the limit of weak gravity. Conceivably, dark matter is fermionic (e.g. [3]).

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