Composite panels design based on post-buckling state with combined loading

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Abstract. Buckling is possible for thin composite skins of light aircraft wing high-lift devices loaded above limit level. Aim of this work is to create a method for orthotropic panels minimal thickness definition with possible post-buckling behavior considering residual strength support. Hinge supported composite panels loaded in compression and shear jointly are considered. Method based on analytic solution of non-linear problem and design method by post-buckling state is suggested in the paper. In order to explain peculiarities of above-mentioned method examples of panels design loaded with tangential force supporting strength and stability are shown in this work. It is demonstrated that for panels minimal thickness calculation by strength, stability, and post-buckling behavior conditions corresponding linear, second- and third-degree equations in unknown thicknesses are derived. In order to develop design method by post-buckling state with combined loading analytic solution of non-linear problem using Bubnov-Galerkin method is shown in the paper. Chai criterion including all components of acting forces and stress critical values for residual strength has been used for composite package based on the conditions of combined loading. Variants of developed applied methods of design based on post-buckling state with only tangential force acting and with combined loading by compression and shear flows are shown in this work.

1. Introduction
Buckling is possible for skins of light and medium aircrafts wing high-lift devices loaded above limit level. In this case verification analysis and design applied methods should consider limitations in stability and possible panels post-buckling behavior if using initial geometrically non-linear correlations. At early design stages it is reasonable to have analytical solutions of above-mentioned problems. It should also be noted that according to modern regulatory requirements [1] composite panels may have defects and damages. According to requirements of circular [1] BVIDs (Barely Visible Impact Damages which are allowable manufacturing damages) with which structure must maintain ultimate loads during whole lifetime should be considered.

Investigations devoted to load-bearing composite panels analysis and design [2-16] should be mentioned. Monographs [2-4] presenting contemporary theories and composite panels stability and post-buckling behavior investigation results hold unique position.

At early design stages it is reasonable to have analytical solutions of stability problems and post-buckling behavior estimation. In this work we shall consider applied method defining orthotropic panel minimal thickness with post-buckling behavior considering combined loading with compression and shear flows in case of hinge support based on composite structures design method by post-buckling behavior [17-19].
In order to explain this method [17-19] in details we shall consider peculiarities of panels design methods based on post-buckling state and their position in general theory of thin-wall structures design. Table 1 shows design ratios for thickness definition of orthotropic rectangular panel of a*b geometrical parameters loaded with tangential forces as an example. This problem may be a problem of spar wall design for example. When designing based on static strength conditions with given shearing force Q, panel width (or spar wall height) b and allowable stress \( \tau_a \) we have known expression for unknown thickness \( \delta \) definition (table 1). In this case allowable stress \( \tau_a \) corresponds to critical stress values in terms of residual strength which are defined by the structure developer based on composite material characteristics, wall multi-layer package layout and allowable damage values.

Similar ratio can be obtained for case of design with limitations in stability. In this case we have quadratic dependence of unknown panel thickness and allowable stress \( \bar{\tau} \) (table 1).

It is necessary to analytically solve geometrically non-linear problems for panels design method based on post-buckling state. In the examined case (table 1) there is an expression for shear membrane stress determination with tangential stress (flow) \( \sigma_y(q_y) \) acting and with panel columnar deflection with amplitude \( f \). Equation in panel thickness \( \delta \) is also obtained from analytical solution of geometrically non-linear problem using Bubnov-Galerkin method. Providing attainment tangential stress critical in residual strength \( \bar{\tau}_{xy} \) in this case above mentioned equation system is reduced to cubic expression in panel thickness. It should be noted that similar dependencies for case with only normal compression longitudinal force acting are shown in investigations [17-19].

**Table 1.** Design ratios defining orthotropic rectangular panels thickness with shear [17-19].

| Conditions for panels design | Correlations for stress calculation | Correlations for panels minimal thickness definition |
|------------------------------|------------------------------------|--------------------------------------------------|
| Static strength              | \( \tau = \frac{Q}{\delta b} \)    | \( \delta = \frac{Q}{\tau_a b} \)               |
| Stability                    | \( \bar{\tau}_{xy} = K_t \left( \frac{\delta}{b} \right)^2 \), \( K_t = \frac{2\pi^2}{12} \times \) | \( \delta^2 = b^2 \frac{\bar{\tau}}{K_t} \) |
| \( \theta = \sqrt{\frac{E_x E_y}{(E_x \mu_{xy} + 2G_{xy})}} \), [20] | \( \tau_{xy} = -f^2 A_3 - p_{xy}^{\,*} \) | \( \delta^3 + \frac{8G_a B_{ab}}{D_{ab}} \bar{\tau}_{xy} b^2 \) |
| Post-buckling state          | \( f^2 B_{ab} \delta + D_{ab} \delta^3 = \frac{2a_2^p}{s^2} q_{xy}^{\,*} \) | \( \delta^3 + \frac{8G_a B_{ab}}{D_{ab}} \bar{\tau}_{xy} b^2 \) |

*) rf. expressions (5), (11) and (12) and corresponding designations in this work

Thus, air of this investigation is to develop method of orthotropic panel design based on post-buckling state with limitations in residual strength considering loading with compression and shear flows. It is reasonable for this method to be based on analytical solution of geometrically non-linear problem and to allow expert estimations of composite panels thickness at early design stages when implementing obtained correlations in some mathematic package.

In order to develop design method following peculiarities should be considered. First, when examining post-buckling behavior with combined loading it is important to estimate strength with one
and the same wave formation parameters. That is, it is desirable to have one and the same bending function when examining different loading components. Secondly generally when designing panels flows acting on the panels are known, not stress. Thirdly let us assume that allowable by residual strength stress is known.

2. Statement of problem

Let us examine orthotropic rectangular panel loaded with compression and tangential flows. It is assumed that all above mentioned forces change in proportion to on

In order to develop applied method providing skin post-buckling behavior possibility it is necessary to consider geometrically non-linear correlations and to choose “comfortable” bending type which allows us to get analytical solution of problem with combined loading. Following bending type is used when examining stability with combined loading in investigation [20]:

\[ W = f \cdot \sin \frac{\pi y}{b} \sin \frac{\pi(x - \alpha y)}{s}, \]

with \( \alpha \) being tangent of wave angle with columnar deflection, \( s \) being distance between node lines.

This type of bending was used in analytical solution of composite panel post-buckling shear behavior problem [17]. Equation (1) is used for solution of problem of panel post-buckling behavior with combined loading. Geometrically non-linear equation of orthotropic panel strain compatibility is transformed into [21]:

\[ L_1(F) - L_2(W) = 0, \]

with

\[ L_1(F) = \frac{1}{E_y} \frac{\partial^4 F}{\partial x^4} + \left( \frac{1}{G_{xy}} \frac{2 \mu_{xy}}{E_x} \right) \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{1}{E_x} \frac{\partial^4 F}{\partial y^4}, \]

\[ L_2(W) = \left( \frac{\partial^2 W}{\partial x \partial y} \right)^2 - \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2}, \]

hereinafter: \( E_x, E_y \) being elasticity modulus in x and y axes directions; \( G_{xy} \) being shear modulus in composite package plane; \( \mu_{xy} \) being Poisson ratio describing reduction along x axis with extension along y axis; orthotropy condition \( E_x \mu_{xy} = E_y \mu_{yx} \), \( F \) being Airy stress function defined by following correlations

\[ \sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}. \]

Geometrically non-linear von Karman equation is presented as

\[ L_4(W) - L_3(F, W) = 0, \]

with

\[ L_4(W) = \frac{1}{\alpha} \left[ D_{11} \frac{\partial^4 W}{\partial x^4} + 2 D_{12} \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 W}{\partial y^4} \right], \]

\[ L_3(F, W) = \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 W}{\partial x \partial y}, \]

Further on in order to obtain analytical solution we shall use Bubnov-Galerkin method. As soon as we substitute bending (1) into expression (2) we can obtain an equation solution of which is Airy stress function of following form

\[ F = \frac{f^2}{32} \left( \frac{1}{G_a} \left( \frac{s}{b} \right)^2 \cos \frac{2\pi(x - \alpha y)}{s} + \left( \frac{b}{s} \right)^2 E_x \cos \frac{2\pi y}{b} \right) - \frac{p_{x} y^2}{2} - \frac{p_{y} x^2}{2} + p_{xy}, \]

Herein designated as

\[ G_a = \frac{1}{E_y} + \left( \frac{1}{G_{xy}} - \frac{2 \mu_{xy}}{E_x} \right) \alpha^2 + \frac{\alpha^4}{E_x}, \]

\[ \alpha = \frac{1}{E_x} + \left( \frac{1}{G_{xy}} - \frac{2 \mu_{xy}}{E_x} \right) \frac{\alpha^2}{E_x}. \]
\( p_x, p_y, p_{xy} (q_x, q_y, q_{xy}) \) being analysis stress (flows) acting on the composite panel.

Stress in midsurface of buckled orthotropic panel is defined using following equations

\[
\sigma_x = \frac{\partial^2 F}{\partial y^2} = -f^2 \Delta_1 - p_x, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} = -f^2 \Delta_2 - p_y, \quad \tau_{xy} = \frac{\partial^2 F}{\partial x \partial y} = f^2 \Delta_3 - p_{xy},
\]

with

\[
\Delta_1 = \frac{\pi^2}{8} \left( \frac{\alpha^2}{G_y b^2} \cos 2\pi(x - \alpha y) + \frac{E_y}{s^2} \cos \frac{2\pi y}{b} \right),
\]

\[
\Delta_2 = -\frac{\pi^2}{8} \frac{1}{G_y b^2} \cos 2\pi(x - \alpha y), \quad \Delta_3 = \frac{\pi^2}{8} \frac{\alpha}{G_y b^2} \cos \frac{2\pi(x - \alpha y)}{s}.
\]

Further on applying Bubnov-Galerkin method procedure considering geometrical non-linearity to equation (3)

\[
\left[ L_3 (F, W) - L_4 (W) \right] \sin \frac{\pi y}{b} \sin \frac{\pi(x - \alpha y)}{s} \ dx \ dy = 0,
\]

following equation is derived with \( f \neq 0 \)

\[
\frac{\pi^2}{s^2} \left[ \frac{\pi^2 f^2}{16G_y} \left( \frac{\alpha^2}{b^2} + \frac{E_y}{s^2} \right) + p_x \right] + \left[ \frac{\pi^2 f^2}{16G_y} + p_y \right] \left( \frac{\pi^2}{b^2} + \frac{\alpha^2}{s^2} \right) + \frac{\alpha \pi^2}{s^2} \left( \frac{\pi^2 f^2}{8b^2 G_y} + 2p_{xy} \right) \frac{\pi^2}{\delta s^2} - D_{11} \frac{\pi^4}{b^4} - 2D_2 \left( \frac{\pi^2}{b^2} + \frac{\alpha^2}{s^2} \right) \pi^2 = 0
\]

In case of combined loading it is assumed that all loading component change in proportion to one parameter: \( q_x^p = p_x \delta, \ q_y^p = p_y \delta, \ q_{xy}^p = p_{xy} \delta \) being analysis flows acting on composite panel.

In case of combined loading it is assumed that all loading component change in proportion to one parameter: \( q_x^p = p_x \delta, \ q_y^p = p_y \delta, \ q_{xy}^p = p_{xy} \delta \). Then wave formation critical parameters in case of buckling are generally defined numerically using equation system based on equation (7)

\[
\hat{\partial} \lambda / \hat{\partial} \alpha = 0, \hat{\partial} \lambda / \hat{\partial} s = 0,
\]

In some cases stability problems are solved with two flows fixation, for example, \( q_x \) and \( q_y \), with further \( q_{xy} \) flow minimization in parameters \( \alpha \) and \( s \).

Further on for optimal design problem solution strength criterion considering all components of acting forces are necessary. Let us use Tsai criterion [22]

\[
\frac{\sigma_x^2}{\sigma_s} - \frac{\sigma_x \sigma_y}{\sigma_s \sigma_y} + \frac{\tau_{xy}^2}{\tau_{xy}} = 1,
\]

With \( \sigma_x \) (\( \sigma_y \) and \( \tau_{xy} \)) values hereinafter being membrane longitudinal (transverse and tangential) stress acting in the buckled panel; \( \sigma_x \) (\( \sigma_y \) and \( \tau_{xy} \)) being residual critical longitudinal (transverse and tangential) stress obtained during tests of specimen with defects of the 1st category.
3. Applied methods of panels design based on post-buckling state

a) Let us examine at first a simple case when only tangential flow $q_{xy}^p = p_{xy}\delta$ acts provided optimal thickness definition with attainment of critical stress in terms of residual strength obtained after 1-st category damage. In this case expression for tangential stress (5) is transformed taking tangential flow action into account

$$\tau_{xy}\delta = \delta \frac{f^2}{8} \frac{\pi^2}{G_y} \cos \frac{2\pi(x-\alpha y)}{s} - q_{xy}^p,$$

(10)

In order to attain critical residual stress $\tau_{xy}$ it is necessary to fulfill stipulation

$$\cos 2\pi(x - \alpha y)/s \to 1$$

in panel critical points where maximum stress can be. Non-linear equation (6) is transformed into

$$f^2 B_{apb} \delta + D_{apb} \delta^3 = \frac{2\alpha \pi^2}{s^2} q_{xy}^p,$$

(11)

with

$$B_{apb} = \frac{1}{s^2} \left[ \frac{1}{16G_y} \left( \frac{\alpha}{b} \right)^2 + \frac{E_x}{16s^2} \right] + \frac{\pi^2}{16G_y} \left[ \frac{1}{b^2} + \frac{\alpha^2}{s^2} \right] + \frac{\alpha}{s^2} \frac{1}{8b^2 G_y},$$

$$D_{apb} = \frac{E_x}{12(1 - \mu_{xy} \mu_{yx})} \frac{1}{s^2} + 2 \frac{\mu_{xy} E_x}{12(1 - \mu_{xy} \mu_{yx})} \frac{G_y}{6} \left[ \frac{1}{b^2} + \frac{\alpha^2}{s^2} \right] \frac{1}{s^2} + \frac{E_y}{12(1 - \mu_{xy} \mu_{yx})} \left[ \frac{1}{b^2} + \frac{\alpha^2}{s^2} \right]^2 + \frac{4\alpha^2}{b^2 s^2} \right].$$

Further on bending amplitude is expressed from equation (10) and is substituted into expression (11), and non-linear equation in unknown thickness is obtained

$$\delta^3 + \delta \frac{8G_y B_{apb} \tau_{xy} b^2}{D_{apb} \pi^2 \alpha} + \frac{q_{xy}^p}{\pi^2 D_{apb}} \left( \frac{8G_y B_{apb} b^2}{\alpha} - \frac{2\alpha \pi^2}{s^2} \right) = 0,$$

(12)

Wave formation critical parameters in case of buckling depend on geometrical dimensions and stiffness parameters ratios which are defined by composite layout and generally are defined numerically using correlations (8) from linear equation (7) with stipulation $q_x = 0, q_y = 0, q_{xy} \neq 0$.

b) Now let us examine case of combined action of longitudinal compression $q_x^p = p_x\delta$ and tangential flows $q_{xy}^p = p_{xy}\delta$. Expressions (5) are transformed considering flows action into

$$\sigma_x\delta = -\Delta \frac{f^2}{s^2} - q_x^p, \quad \tau_{xy}\delta = \Delta \frac{f^2}{s^2} - q_{xy}^p,$$

(13)

Let us write expression for strength criterion (9) taking multiplication by thickness into account

$$\frac{(\sigma_x\delta)^2}{\sigma_x^2} + \frac{(\tau_{xy}\delta)^2}{\tau_{xy}^2} = \delta^2$$

(14)

Non-linear equation (6) is transformed into

$$f^2 B_{apb} \delta + D_{apb} \delta^3 = \frac{\pi^2}{s^2} q_x^p + \frac{2\alpha \pi^2}{s^2} q_{xy}^p,$$

(15)

Let us perform following transformations of equations (13)-(15). At first, we shall substitute flows from (13) into the strength criterion (14). Further on bending amplitude is expressed from (15) and substitute it into the obtained at the previous stage correlation. Required non-linear equation for thickness definition can be written as
In this combined loading case wave formation critical parameters \((\alpha_{crit}, s_{crit})\) are as previously defined numerically using correlations (8) from linear equation (7) with stipulation \(q_x \neq 0, q_y = 0, q_{xy} \neq 0\). It should also be taken into account that due to \(1\Delta_1\) and \(3\Delta_2\) values type and definition in correlations (5) panel critical points coordinates where maximum stress may be can be obtained at the design stage based on the condition that \(\cos 2\pi (x - \alpha_{crit} y) / s_{crit} \rightarrow 1\).

c) Now let us examine case of combined action of longitudinal compression \(q_x = p_x \delta, q_y = p_y \delta\) and tangential flows \(q_{xy} = p_{xy} \delta\). Equations (5) are transformed into

\[
\sigma, \delta = -\delta\Delta_1 f^2 - q_x, \sigma_y, \delta = -\delta\Delta_2 f^2 - q_y, \tau_{xy} \delta = \delta\Delta_3 f^2 - q_{xy},
\]

(17)

General strength criterion expression (9) is as follows

\[
\left(\frac{\sigma_x}{\sigma} + \frac{\sigma_y}{\sigma} + \frac{\tau_{xy}}{\tau_{xy}}\right)^2 = \delta^2.
\]

(18)

Non-linear equation (6) is presented as

\[
f^2 B_{\alpha \beta} \delta + D_{\alpha \beta} \delta^3 = \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy}.
\]

(19)

Further on performing like item 3(b) transformations we obtain following non-linear equation for optimal thickness definition

\[
-\Delta_1 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_x \right]^2
\]

\[
-\sigma_x \sigma_y \tau_{xy} - \Delta_1 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_x \right] \times
\]

\[
-\Delta_2 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_x \right] \times
\]

\[
-\Delta_2 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_x \right] +
\]

\[
-\Delta_2 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_y \right] +
\]

\[
-\Delta_2 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_y \right] +
\]

\[
-\Delta_2 \left[ \frac{\pi^2}{s^2} q_x + \left(\frac{\pi^2}{b^2 + \alpha^2 \pi^2 \frac{s^2}{2}}\right) q_y + \frac{2\alpha \pi^2}{s^2} q_{xy} - D_{\alpha \beta} \delta^3 \right] B_{\alpha \beta} - q_y \right] +
\]

\[
= \delta^2 \sigma_x \tau_{xy},
\]

(16)
It should also be noted here that in combined loading case wave formation critical parameters are defined numerically by correlations (8) from linear equation (7). Besides taking $\Delta_1$, $\Delta_2$, and $\Delta_3$ values type in correlations (5) into account it is also possible at the design stage to obtain coordinates of panel critical points where maximum stress may be based on the condition that $\cos 2\pi(x - \alpha_{\text{crit}}y)/s_{\text{crit}} \to 1$.

4. Applied skin design with post-buckling state in shear
Let us consider as an example orthotropic rectangular panel of KMU-4 with shear with following initial data: panel width $b = 100$ mm, layout $h_0 = 0.1$, $h_{45} = 0.7$, $h_{90} = 0.2$. Figure 1 shows panel bending shape in case of buckling. Figure 2 shows dependence $t(x, y) = \cos 2\pi(x - \alpha y)/s$ which shows tangential stress change nature.

5. Conclusion
Applied method for minimal thickness definition of orthotropic panel with post-buckling behavior and with compression and shear flows action is suggested in this investigation. This method is based on analytical solution of geometrically non-linear problem which is obtained using Bubnov-Galerkin method. General design problem is reduced to non-linear equation in unknown panel thickness providing attainment of stress critical combination in terms of residual strength.
Practical importance of suggested method consists in possibility of orthotropic panels thickness definition for post-buckling behavior at early design stages. Further method development can relate to examination of different variants of panels support and loading cases using corresponding expressions for panel bending.

Besides when designing medium thickness panels by post-buckling state it is reasonable to take not only membrane stress (5) into account but also bending stress. In this case it is necessary to add bending stress in extreme fibers to stress acting in midsurface [21]:

$$\sigma_x' = -z \left[ B_{11} \frac{\partial^2 W}{\partial x^2} + B_{12} \frac{\partial^2 W}{\partial y^2} \right], \quad \sigma_y' = -z \left[ B_{12} \frac{\partial^2 W}{\partial x^2} + B_{22} \frac{\partial^2 W}{\partial y^2} \right], \quad \tau_{xy}' = -2zB_{66} \frac{\partial^2 W}{\partial x \partial y},$$

with $B_{mn}$ being reduced stiffness parameters of composite structure, $z = \pm \delta/2$.

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