Time-dependent Currents of a Single-electron Transistor in Dissipative Environments

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Currents of the single-electron transistors driven by time-dependent fields via external dissipative circuits are investigated theoretically. By expressing the external circuit in terms of driven harmonic oscillators and using the reduced-density operator method, we derive time- and environment-dependent tunneling rates in the regime of sequential tunneling and present expressions for both displacement and tunneling currents with these tunneling rates. It is found that the dissipative environments affect tunneling currents in two ways: the determination of driving voltages at tunneling junctions and the depletion of particle-hole distribution functions. Considering a simple dissipative circuit, we discuss the effects of the environment on tunneling currents in both static and time-dependent cases.

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I. INTRODUCTION

There have been considerable interests in a single-electron transistor because of its potential applications. Since Coulomb blockade peaks in its conductance oscillation are very sensitive to a fraction of charges, it is expected to be one of promising candidates for a detector in measuring quantum states of quantum computation and information processing[1]. Recently, Schoelkopf et. al.[2] developed a high sensitive single-electron transistor. By introducing a LC-resonant circuit connected to a single-electron transistor, they drove the system in the radio-frequency regime to overcome 1/f noise and obtain a high sensitivity of detecting charges.

From the theoretical point of view, such a single-electron transistor is also very interesting because one should consider the influence of dissipative environments(LC-resonant circuit with a cable resistance) on current-voltage characteristics as well as effects of time-dependent external perturbations. In the presence of the dissipative environments, tunneling rates of quasiparticles are strongly affected because an additional energy is needed to excite the environments as shown in the case of single- and multi-junctions[3]. On the other hand, time-dependent perturbations affect phase coherence of quasiparticles in time and its effects manifest itself in photon-assisted tunneling: quasiparticles are also able to tunnel by absorbing or emitting photons[4, 5]. Thus, quasiparticles in the single-electron transistor have two different energy-exchange mechanisms when they tunnel through junctions and then, one should take into account these two mechanisms in calculating the tunneling rates simultaneously.

In this work we investigate effects of the dissipative environments and the time-dependent perturbations on current-voltage characteristics of a single-electron transistor by calculating the tunneling rate and present expressions for currents at each electrode. In calculating the tunneling rate we make two assumptions for our model of the single-electron transistor. Firstly, tunneling barriers are considered so opaque that quasiparticles in each electrode are well localized there and their motions can be described by separated Hamiltonians. In other word, the tunneling barriers have a resistance much larger than the resistance quantum $R_K = h/e^2$. Secondly, it is assumed that time between successive tunneling events is much larger than the charge relaxation time of the dissipative environments. This assumption makes the problem easy to treat the dissipative environments as heat reservoirs being in thermal equilibrium. However, we still treat quasiparticles confined in the region of a quantum dot as in non-equilibrium.

The paper is organized as follows. We first describe the Hamiltonian of the single-electron transistor in Section II. By expressing the dissipative environments in terms of driven harmonic oscillators, we give the Hamiltonian separating it into an interested system part and its environment, and derive time- and environment-dependent tunneling rates using a reduced-density operator method. In Section III, we present expressions for currents flowing in each electrode in terms of displacement and tunneling components. As an application of our expressions, tunneling currents driven by a simple dissipative circuit are examined in Section IV, and then, a brief summary is given in Section V.

II. HAMILTONIAN

In Fig. 1, we show a typical drawing of the single electron transistor driven by time-dependent voltages via possible dissipative elements. The Hamiltonian of the entire system is modeled by $\mathcal{H} = \mathcal{H}_{qp} + \mathcal{H}_{RLC}(t) + \mathcal{H}_T$ where the first two terms describe the motion of the system in the absence of tunnelings and the last one is the tunneling Hamiltonian. In the absence of tunneling, the system

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is viewed as just a simple electronic circuit (a lumped-circuit) because all tunneling junctions are considered as capacitors. Then, its motion is described by two independent degrees of freedom; microscopic and macroscopic variables.4

By the microscopic variables, we mean those of quasiparticles in conductors of Fig. 1. In our system, since quasiparticles relevant to tunneling are distributed in an island called a quantum dot and adjacent electrodes (source and drain), we denote their Hamiltonian as $\mathcal{H}_{qp} = \mathcal{H}_{dot} + \mathcal{H}_{lead}$ where $\mathcal{H}_{dot} = \sum_{\ell=1}^{n} \epsilon_\ell d_\ell^\dagger d_\ell + V_{ee}(d_\ell^\dagger, d_\ell)$ and $\mathcal{H}_{lead} = \sum_{\alpha=S,D} \sum_{n} \epsilon_{\alpha n} a_{\alpha n}^\dagger a_{\alpha n}$. Here, $d_\ell$ and $a_{\alpha n}$ ($d_\ell^\dagger$ and $a_{\alpha n}^\dagger$) are the annihilation (creation) operators for quasiparticles in the quantum dot with an energy $\epsilon_\ell$ and in the electrodes $\alpha$ with an energy $\epsilon_{\alpha n}$, respectively. In the quantum dot we denote the electron-electron interaction by $V_{ee}(d_\ell^\dagger, d_\ell)$ which in the most case is approximated by a simple Coulomb-blockade model $Q^2/2C_S$ with excess charges $Q$ and $C_S = C_D + C_S + C_G$. In the metallic electrodes, energies of quasiparticles are assumed to be measured relative to their chemical potentials and independent of time-dependent external perturbations by considering small frequencies not to excite plasmon.

On the other hand, macroscopic variables represent charges on capacitors and flux through inductors in the lumped-circuit. To describe their motion, we model the dissipative elements of $Z_D(\omega)$ and $Z_G(\omega)$ in Fig. 1 with different sets of simple harmonic oscillators (i.e. LC-circuits) following Caldeira and Leggett. Then, starting from the Lagrangian formulation where generalized coordinates are chosen to be $\phi(t) = \int dt e^i\psi(t)/\hbar$ for a voltage $v(t)$ across each capacitor, the Hamiltonian $\mathcal{H}_{RLC}(t)$ of the macroscopic part is expressed as,

$$\mathcal{H}_{RLC}(t) = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \sum_{m=1}^{M} \frac{g_m^2}{2e_c} \frac{\hbar^2}{2e_c} \{ \phi_1 + \phi_2 + \varphi_m - \psi_G(t) \}^2$$

$$+ \sum_{n=M+1}^{N+M} \frac{g_n^2}{2e_c} \frac{\hbar^2}{2e_c} \{ \phi_1 + \varphi_n - \psi_D(t) \}^2$$

where charges $Q_j$ and $q_n$ are conjugate to phases $\hbar\phi_j/e$ and $\hbar\varphi_j/e$, respectively, and satisfy the commutation relations of $[\phi_j, Q_j] = i\epsilon$ and $[\varphi_n, q_n] = i\epsilon$. The external perturbation $V_G(t)$ are treated as a classical field and involved in externally driving phases $\psi_D(t)$ and $\psi_G(t)$ as,

$$\psi_D(t) = \frac{e}{\hbar} \int_0^t d\tau V_D(\tau)$$

$$\psi_G(t) = \frac{e}{\hbar} \frac{C_D + C_S}{C_D} \int_0^t d\tau V_G(\tau).$$

The first two terms in Eq. 1 are additional charging energies on the capacitors connected to each electrode with effective capacitances $C_1 = C_D C_S/(C_D + C_S)$ and $C_2 = C_G^2 C_S/(C_D + C_S)$, whereas the remaining terms describe charging energies on capacitors with a capacitance $c_m$ and magnetic energies of inductors with a inductance $L_m$ in the dissipative elements, $Z_D(\omega)$ and $Z_G(\omega)$. In fact, the charges $Q_1$, $Q_2$, and $Q$ in $\mathcal{H}_{RLC}$ describe charges $Q_\alpha (\alpha = S, D, G)$ accumulated on a plate of the capacitor connected to the electrode $\alpha$ in Fig. 1 and they are related to each other by,

$$\begin{pmatrix}
Q_1 \\
Q_2 \\
Q
\end{pmatrix} =
\begin{pmatrix}
\frac{C_D}{C_D + C_S} & \frac{C_S}{C_D + C_S} & 0 \\
-\frac{C_S}{C_D + C_S} & -\frac{C_D}{C_D + C_S} & 1 \\
1 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
Q_D \\
Q_S \\
Q_G
\end{pmatrix}.$$  

(3)

Then, classical relaxation equations for the charges $Q_\alpha(t)$ are exactly recovered from the Heisenberg equation of motions. To do this, the impedances $Z_G(\omega)$ and $Z_D(\omega)$ at a angular frequency $\omega$ are anticipated to have sets of parameters $\{c_n, L_n\}$ and $\{c_m, L_m\}$ like,

$$\frac{1}{Z_G(\omega)} = \frac{C_D^2}{(C_D + C_S)^2 Z_G(\omega)} = \sum_{m=1}^{M} \frac{i\omega c_m}{1 - c_m L_m \omega^2},$$

$$\frac{1}{Z_D(\omega)} = \sum_{n=M+1}^{N+M} \frac{i\omega c_n}{1 - c_n L_n \omega^2}.$$  

(4)

The tunneling part of the Hamiltonian $\mathcal{H}_T$ may be given by,

$$\mathcal{H}_T = \sum_{\alpha \beta \ell} T_{\alpha \beta \ell} \phi_\alpha^\dagger \phi_\beta d_\ell e^{-i\phi_\beta} + H.c.$$  

(5)

where $T_{\alpha \beta \ell}$ denotes the element of tunneling matrix between a state $|k\rangle$ in the electrode $\alpha$ and a single particle state $|l\rangle$ in the quantum dot. Here, the phase operator $\phi_\alpha (\alpha = S, D)$ is defined in terms of $\phi_1$ and $\phi_2$ through the relation,

$$\begin{pmatrix}
\phi_D \\
\phi_S
\end{pmatrix} = A \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix} = \begin{pmatrix}
\frac{C_D}{C_D + C_S} & \frac{C_S}{C_D + C_S} \\
-\frac{C_S}{C_D + C_S} & -\frac{C_D}{C_D + C_S}
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}.$$  

(6)

Thereby, the operator $e^{i\phi_\alpha}$ increases excess charges by the elementary charge $e$ in the electrode $\alpha$ for every tunneling event because they satisfy the commutation relation $[\phi_\alpha, Q_\alpha] = i\epsilon$. 

FIG. 1: A typical drawing of the single-electron transistor is shown where time-dependent voltages are applied to a quantum dot via possible dissipative elements connected to drain and gate electrodes, respectively.
In reality, the form of the Hamiltonian $H = H_{QP} + H_{RLC}(t) + H_T$ is not adequate in obtaining expressions of currents because the time-dependence of $H_{RLC}(t)$ makes the problem complicated. To circumvent this, we perform a standard time-dependent unitary transformation with,

$$U(t) = \prod_{j=1}^{2N} \prod_{m=1}^{M} \exp\{i\tilde{u}_j(t)\phi_j + i\tilde{u}'_m(t)\varphi_m\} \exp\{-i\tilde{u}_j(t)Q_j/e - i\tilde{u}'_m(t)q_m/e\}. \quad (7)$$

Here, the sets of time-dependent functions $\{x_j(t), x'_j(t)\}$ and $\{u_m(t), u'_m(t)\}$ are chosen to rotate the Hamiltonian $H_{RLC}(t)$ into the form irrelevant of time, $H_{RLC}(0)$. By a straightforward calculation for the rotated Hamiltonian $H_R(t)$,

$$H_R(t) = U^\dagger(t)H(t)U(t) - i\hbar U^\dagger(t)\frac{\partial U(t)}{\partial t} \quad (8)$$

one can show that a time-independent form of $H_{RLC}(0)$ is obtained by choosing $u_j(t)$ just as the phase difference across capacitors forced by $\psi_D(t)$ and $\psi_G(t)$ in the absence of tunneling and $x_j(t) = C_j\dot{u}_j(t)/e^2$. That is, $u_j(t)$ is chosen by,

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = A^{-1} \begin{pmatrix} y_D(t) \\ y_S(t) \end{pmatrix}, \quad y_D(t) = \frac{e}{\hbar} \int_0^t dt' \left\{ \delta_{\alpha,D} - \frac{C_{D}}{C_{\Sigma}} V_{D}^0 - \frac{C_{G}}{C_{\Sigma}} v_{G}(t) + g_0(t) \right\} \quad (9)$$

where we separate the external perturbation into DC and AC parts, i.e., $V_\alpha(t) = V_D(t) + v_\alpha(t)$. Here, $g_\alpha(t)$ describes voltage differences at tunneling junctions exerted by the AC part of the voltage sources and is given in its Fourier component as,

$$\begin{pmatrix} C_\Sigma + C_G/C_\Sigma \\ -C_G/C_\Sigma \\ -C_A/C_\Sigma \\ C_A/C_\Sigma \end{pmatrix} Z^{-1} \begin{pmatrix} \tilde{g}_D(\omega) \\ \tilde{g}_S(\omega) \end{pmatrix} = \begin{pmatrix} \tilde{v}_D/Z_D \\ \tilde{v}_G/Z_G \end{pmatrix} \quad (10)$$

where $\tilde{v}_\alpha(\omega)$ is a Fourier component of the AC part of the external perturbation and the matrix $Z$ is defined as,

$$Z = A \begin{pmatrix} i\omega C_1 + Z^{-1}_D + Z^{-1}_G \\ Z^{-1}_G \end{pmatrix}^{-1} A^T \quad (11)$$

Finally, under the unitary transformation $U(t)$, the total Hamiltonian becomes,

$$H_R(t) = H_{dot} + H_{lead} + H_{RLC}(0) + H_T(t). \quad (12)$$

Here, $H_T(t)$ is the tunneling Hamiltonian rotated by the unitary transformation $U$ and now involves effects of the time-dependence perturbations as,

$$H_T(t) = \sum_{a=S,D} \sum_{kl} \left[T^a_{kl}(t)\eta_{ka}^{\dagger} e^{-i\phi_0} + H.c.\right] \quad (13)$$

with $T^a_{kl}(t) = T^a_{kl} e^{-i\phi_0(t)}$.

### A. Ensemble average

In order to evaluate an ensemble average of physical quantities, we use the reduced-density operator method. In this method, the entire system is divided into an interested system part being in non-equilibrium, a heat bath, and their interaction; $H_R = H_S + H_B + H_{SB}$. Then, the effective density operator describing the interested system is obtained by averaging the Liouville equation over the heat bath. Since the heat bath is considered as in thermal equilibrium, the ensemble average of physical quantities is expressed as the sum of expectation values between quantum states weighted with the reduced-density matrix.

In our case, we consider the system part as quasiparticles in the quantum dot, $H_S = H_{dot}$, and the heat bath as the combination of quasiparticles in the electrodes and the lumped-circuit, i.e., $H_B = H_{lead} + H_{RLC}(0)$. Then, the system and the heat bath are coupled by the time-dependent tunneling Hamiltonian $H_{SB}(t) = H_T(t)$. By adopting very opaque tunneling junctions (weak coupling of $H_{SB}$) such that a typical tunneling period is much larger than the charge relaxation time of the heat bath, one can treat the heat bath as being in thermal equilibrium. In this case, its density operator $\rho_B$ is proportional to $e^{-\beta H_B}$ at an inverse temperature $\beta$.

The density operator describing the interested system $\rho(t)$ is obtained by tracing the density operator $\rho_{tot}(t)$ for the entire system over the heat bath, $\rho(t) = tr_B[\rho_{tot}(t)]$, and its equation of motions is derived from the Liouville equation. In a non-Markovian form, $\rho(t)$ is given by\footnote{\[\frac{d\rho(t)}{dt} = \frac{1}{\hbar}(H_S, \rho(t)) + C(t).\]}

Here, the generalized scattering operator $C(t)$ stands for,

$$C(t) = \frac{1}{\hbar} \int_{-\infty}^t d\tau tr_B\left\{[H_{SB}(\tau), [\rho^I(\tau, t)\rho_B, H_{SB}(\tau, t)]] + O(\hbar^4) \right\} \quad (15)$$

where $\rho^I(t)$ and $H_{SB}(t)$ are their interaction pictures of $\rho(t)$ and $H_{SB}(t)$, respectively;

$$\rho^I(\tau, t) = e^{-iH_{SB}(t-\tau)/\hbar} \rho(\tau) e^{iH_{SB}(t-\tau)/\hbar}, \quad (16)$$

$$H_{SB}(\tau, t) = e^{-i(H_S + H_B)(t-\tau)/\hbar} H_{SB}(\tau) e^{i(H_S + H_B)(t-\tau)/\hbar}. \quad (17)$$

Here, the forth order contributions of the interaction Hamiltonian $H_{SB}(t)$ are ignored by taking into account opaque tunneling junctions. Then, with this density operator, one can express the ensemble average of an arbitrary operator $O$ in terms of relevant system operators by replacing the part of the heat bath with their equilibrium values. That is, $\langle O \rangle = tr_S[tr_B\{O tr_B(t)] = tr_S\{O_S \rho(t)\}$ where $tr_S$ means the average of the operators over the system and $O_S$ is a relevant system operator.
B. Expectation values for heat-bath operators

For the ensemble averages of physical quantities, one need to evaluate several time-correlations between heat-bath operators. First, for those of the RLC-circuit, the correlation between phase operators of $\text{tr}_B\{e^{-i\phi_0}e^{i\phi_n(t)}\rho_B\}$ is necessary to calculate its effect on tunneling. Here, $\phi_n(t)$ is the Heisenberg operator of $\phi_0$ with respect to $\mathcal{H}_{RLC}$. Since the RLC-circuit of Eq. (1) is considered as the sum of independent harmonic oscillators in equilibrium, this correlation function can be rewritten as

$$\text{tr}_B\{e^{-i\phi_0}e^{i\phi_n(t)}\rho_B\} = e^{\text{tr}_B\{i\phi_0\phi_n(t)\}}$$

Then, based on the linear response theory, one can show that the fluctuation of $J_n(t) \equiv \text{tr}_B\{\phi_0\phi_n(t)\rho_B\}$ is directly related to the dissipation of the RLC-circuit (Fluctuation-dissipation theorem). From response functions of the RLC-circuit of Eq. (1), together with Eqs. (3) and (4), $J_n(t)$ is given by,

$$J_n(t) = \int_{-\infty}^{\infty} d\omega \frac{Re Z'_\omega(\omega)}{\omega} \frac{e^{i\omega t} - 1}{1 - e^{-\hbar\omega\beta}}$$

where $Z'_\omega(\omega)$ is an effective impedance of the RLC-circuit seen from the tunnel junction and is equal to diagonal elements of the impedance matrix $Z$ of Eq. (11); $Z'_\omega(\omega) = Z_{11}(\omega)$ and $Z'_\omega(\omega) = Z_{22}(\omega)$, respectively. In reality, $Z'_\omega(\omega)$ has a slightly different form from the impedance seen from a tunneling junction $\alpha$ in Fig. 1. This is because the region of the quantum dot in Fig. 1 is independent of the RLC-circuit and does not contribute to the fluctuation in the absence of tunneling.

As for the electrodes, the following particle and hole evolutions are necessary to evaluate tunneling currents,

$$\text{tr}_B\{a_{\alpha}(t)\rho_B\} = f_{FD}(\epsilon_{\alpha}) e^{-\gamma_{\alpha}|t|}$$

$$\text{tr}_B\{a_{\alpha}^\dagger(t)\rho_B\} = \{1 - f_{FD}(\epsilon_{\alpha})\} e^{-\gamma_{\alpha}|t|}$$

where $a_{\alpha}(t)$ is the Heisenberg operator of $a_{\alpha}$ with respect to $\mathcal{H}_{\text{lead}}$ and $f_{FD}(\epsilon) = 1/(1 + e^{\beta\epsilon})$ is the Fermi-Dirac distribution function. Here, the exponential decay of $e^{-\gamma_{\alpha}|t|}$ represents effects of tunneling on states in the electrodes. In reality, the heat bath as well as the quantum dot in our system are affected by tunneling. As a result, the evolutions of quasiparticles in the heat-bath are different from those in the isolated one and are usually represented by a finite life time. The exponential decay is inserted by hand to represent such an effect on states. The parameter $\gamma_{\alpha}$ is assumed to be the bare tunneling rate at the electrode $\alpha$,

$$\gamma_{\alpha}(\epsilon) = \frac{2\pi}{\hbar} \sum_k |T_{ik}|^2 \delta(\epsilon - \epsilon_{\alpha})$$

which is usually considered as a constant independently of an energy within a so-called wide-band limit; $\gamma_{\alpha}(\epsilon) = \gamma_{\alpha}$.}

C. Time- and environment-dependent master equation

Now, we evaluate the reduced-density operator of Eq. (14) in the basis representation. A simplified form is obtained when one expands the reduced-density operator in terms of many-body eigenstates ($\tau$ and $s$) of the quantum dot, i.e.,

$$\rho(t) = \sum_{rs} P_{rs}(t) |r\rangle \langle s|$$

Substituting this into Eq. (14) and then projecting it on one of diagonal components, the occupation probability $P_{ss}(t)$ at a state $|s\rangle$ is given as a balanced form,

$$\frac{dP_{ss}(t)}{dt} = \sum_{r,\alpha=+,-} \left[ \int_{-\infty}^{0} d\tau P_{rr}(t+\tau) \Gamma^{\alpha}_{rs}(t,\tau) - \int_{-\infty}^{0} d\tau P_{ss}(t+\tau) \Gamma^{\alpha}_{ss}(t,\tau) \right]$$

where the first term describes the increasing rate of the probability density by transitions from other states while the second term is a decay rate due to transitions to others. In deriving this result we disregard the contribution of off-diagonal components because their effects are the forth order of the interaction Hamiltonian, $O(\mathcal{H}_{SB}^2)$ and thus, the result does not contain coherent evolutions between many-body states which may be caused by external perturbations.

In Eq. (22), the memory kernels of $\Gamma^{\alpha}_{rs}(t,\tau)$ describe quasiparticle tunneling into $|+\rangle$ or from $|\Delta\rangle$ the quantum dot through the barrier $\alpha$ to result in the transition from a state $|r\rangle$ to another state $|\Delta\rangle$. The detailed forms of $\Gamma^{\alpha}_{rs}(t,\tau)$ are given by,

$$\Gamma^{\alpha}_{rs}(t,\tau) = \frac{Re}{\pi\hbar} \int_{-\infty}^{\infty} de \Lambda^{\alpha}_{rs}(\epsilon + E_{rs}^{\alpha\pm}) \exp \left\{ \gamma_{\alpha} - \frac{ie}{\hbar} \tau + J_{\alpha}(\pm\tau) - \frac{ie}{\hbar} \int_{t+\tau}^{t} dt' g_{\alpha}(t') \right\}$$

Here, $\Lambda^{\alpha\pm}(E)$ represent tunneling rates of quasiparticles into $|+\rangle$ or from $|\Delta\rangle$ the quantum dot with an energy gain of $E$ when there is neither dissipative elements nor alternating perturbations in addition to negligible collision-broadening. In this case, the tunneling rates $\Lambda^{\alpha\pm}_r(E)$ are reduced to the widely used formula (11):

$$\Lambda^{\alpha\pm}_r(E_{rs}) = \gamma_{\alpha} S^{\pm}_{rs} \left\{ 1 + \frac{1}{2} \mp f_{FD}(E_{rs}^{\alpha\pm}) \right\}$$

where coefficients $S^{\pm}_{rs}$ represent the selection rules of tunneling,

$$S^{+}_{rs} = \sum_l |(r | dl | s)^2|, \quad S^{-}_{rs} = \sum_l |(s | dl | r)^2|$$

and $E_{rs}^{\alpha\pm} = \pm(E_{s} - E_{r}) + (\delta_{\alpha,D} - \frac{\epsilon_{\alpha}}{eS}) eV_{D}^{0} - \frac{\epsilon_{\alpha}}{eS} eV_{G}^{0}$ are energy gains at a tunneling event with $E_{r}$ and $E_{s}$,
eigenenergies of states $|r\rangle$ and $|s\rangle$, respectively. On the other hand, the exponential part of the integrand in Eq. (27) includes the effects of the dissipative environment and the alternating perturbations. As indicated by the Tien-Gorden theory, the external source contributes the imaginary part in an argument of the exponential function via $g_\alpha(t)$, and thus affects phases of electronic states. Whereas, the term $J_{\alpha}(t)$ in $\Gamma_{\alpha}^{r,s}(t,\tau)$ are more easily understood by transforming into its Fourier components as,

$$e^{J_{\alpha}(t)} = \int_{-\infty}^{\infty} d\omega e^{i\omega t} P_\alpha(\omega). \quad \text{(27)}$$

Substituting this relation into Eq. (24), one can see that tunneling occurs with a weight of $P_\alpha(\omega)$ in the range between $\omega$ and $\omega + d\omega$ and corresponding energy gains become $\epsilon \pm \hbar \omega + E_{\alpha}^{r,s}$. Since the sum of $P_\alpha(\omega)$ over all frequency range is equal to one, $P_\alpha(\omega)$ can be interpreted as the probability density to exchange the energy $\hbar \omega$ between the system and its environment. By considering energy differences between tunneling events, one can see that $P_\alpha(\omega)$ in the positive(negative) frequency represents the probability to emit(absorb) photons to(from) the environment.

From Eq. (24), it is noted that effects of the dissipative environments on tunneling are two folds. The first is the determination of the voltage difference $g_\alpha(t)$ at each tunneling junction through Eq. (13). The other is the probability density $P_\alpha(\omega)$ which is determined by the characteristic impedance $Z_\alpha(\omega)$ of the environments via Eq. (19).

### III. Expressions for Currents

Now, we calculate currents in the electrodes in Fig. 1 where a positive current at each electrode is defined to flow into the quantum dot. The current flowing in each electrode $\alpha$ consists of two different contributions; tunneling currents of quasiparticles $I_{\alpha}^d(t)$ and time-variation of charges on capacitors in the lumped-circuit $I_{\alpha}^s(t)$ called displacement currents, i.e.,

$$I_{\alpha}(t) = I_{\alpha}^d(t) + I_{\alpha}^s(t). \quad \text{(28)}$$

Here, each component is calculated by time-derivatives for the ensemble average of particle numbers and charges:

$$I_{\alpha}^d(t) = \frac{e}{i\hbar} \frac{d}{dt} \langle N_{\alpha}\rangle_0 = \frac{e}{i\hbar} \langle \sum_k a_{k\alpha}^\dagger a_{k\alpha} \rangle_0$$

$$I_{\alpha}^s(t) = \frac{d}{dt} \langle Q_{\alpha}\rangle_0 \quad \text{(29)}$$

where $\langle \ldots \rangle_0$ means the average over the Hamiltonian of $\mathcal{H}$. With the above current expressions, the currents flowing into the quantum dot are conserved even for the system subject to the time-dependent perturbations, as emphasized by Büttiker in his recent work. This can be shown by calculating the time-derivative of $\langle Q \rangle$, which gives $d\langle Q \rangle/dt = \langle [Q, \mathcal{H}] \rangle/\hbar = I_{\alpha}^d(t) + I_{\alpha}^s(t)$ reflecting the fact that the increase of charges in the quantum dot is enabled by tunneling processes. Alternatively, since $d\langle Q \rangle/dt$ is the sum of all displacement currents out of the quantum dot from Eq. (8), we obtain the conservation of the currents, $\sum_{\alpha} I_{\alpha}(t) = \sum_{\alpha} \{I_{\alpha}^d(t) + I_{\alpha}^s(t)\} = 0$ with $I_{\alpha}^s(t) = 0$.

By solving the Heisenberg equations of motion, $d\langle Q_{\alpha}\rangle/dt = \langle [Q_{\alpha}, \mathcal{H}] \rangle/\hbar$, we can express the displacement currents in terms of the contributions of tunneling currents and external perturbations. The results are, in its Fourier components of $I_{\alpha}^d(\omega)$,

$$\left( \begin{array}{c} \frac{1}{i\omega C_G} + Z_D \varepsilon_D(\omega) \\ \frac{1}{i\omega C_G} + Z_G \end{array} \right) \left( \begin{array}{c} \tilde{I}_D^d(\omega) \\ \tilde{I}_G^d(\omega) \end{array} \right) = \left( \begin{array}{c} \tilde{v}_D(\omega) \\ -\tilde{v}_G(\omega) \end{array} \right),$$

$$\left( \frac{Z_D}{i\omega C_G} Z_D + \frac{Z_G}{1+i\omega C_G} \right) \left( \begin{array}{c} \tilde{I}_D^d(\omega) \\ \tilde{I}_G^d(\omega) \end{array} \right) = \left( \begin{array}{c} \tilde{v}_D(\omega) \\ -\tilde{v}_G(\omega) \end{array} \right), \quad \text{(30)}$$

where $\tilde{I}_{\alpha}^d(\omega)$ is a Fourier component of a tunneling current $I_{\alpha}^d(t)$. Here, the first term on the right-hand side is the contribution from alternating perturbations while the second terms are resulted from tunneling. Alternatively, the above results can be expressed in the equivalent circuit as shown in Fig. 2 by modeling the tunneling contributions as current sources. Then, once tunneling currents $I_{\alpha}^d(t)$ are known, the total currents at each electrode are determined by applying basic circuit rules to Fig. 2.

Now we evaluate the tunneling component of the total current in Eq. (28). From the Heisenberg equation of motion, $d\langle e N_{\alpha}\rangle_0/dt = \langle [e N_{\alpha}, \mathcal{H}] \rangle_0/\hbar$, the tunneling current $I_{\alpha}^d(t)$ is calculated as,

$$I_{\alpha}^d(t) = \frac{e}{i\hbar} \sum_{\alpha k l} \langle T_{kl} a_{k\alpha}^\dagger d e^{i\phi_{\alpha} - \text{H.c.}} \rangle_0$$

$$= \frac{e}{i\hbar} \sum_{\alpha k l} \langle T_{kl}(t) d a_{k\alpha} e^{-i\phi_{\alpha} - \text{H.c.}} \rangle_0$$

$$\equiv \langle \mathcal{I}_{\alpha}(t) \rangle \quad \text{(31)}$$

where in the second line $\langle \ldots \rangle$ denotes an ensemble average over the rotated system $\mathcal{H}_R$. By considering the
second order contribution of $\mathcal{H}_{SB}$ or only sequential tunneling processes, the expectation value of Eq. (31) can be rewritten in the reduced-density operator formalism as,

$$I^t_{\alpha}(t) = \frac{1}{i\hbar} \int t^- \int \tau d\mathbf{tr}_{SB} \{ \mathcal{I}_a(t)[\mathcal{H}_{SB}(\tau, t), \rho_{SB}(\tau, t)] \}$$

Here, comparing this with Eq. (31), one can see that the part of the commutator is resulted from the evolution of the total density $\rho_{SB}(t)$. By replacing heat-bath operators with their equilibrium values and substituting the reduced-density operator in the basis representation, finally we obtain the tunneling current at the electrode $\alpha$ as,

$$I^t_{\alpha}(t) = \frac{e}{\hbar} \sum_{rs} \int_{-\infty}^0 dt \{ \Gamma^{\alpha-}_{rs}(t, \tau) - \Gamma^{\alpha+}_{rs}(t, \tau) \} P_{rs}(t + \tau) \quad \text{(33)}$$

As in the case of the occupation probabilities, off-diagonal contributions are also ignored because their effects are the forth order of $\mathcal{H}_{SB}$.

Equations (24), (26), and (33) are main results of our work. Based on the master equations of Eq. (24) and the memory kernel of Eq. (24), the displacement and tunneling currents flowing in each electrode of Fig. 1 can be calculated using Eqs. (30) and (33), respectively.

### A. Expressions in the Fourier space

For numerical calculations, it is useful to introduce the Fourier transform of the memory kernel, Eq. (24). When external perturbations are periodic functions with an angular frequency $\omega_A$, we define Fourier components as,

$$\tilde{\Gamma}^{\alpha\pm}_{rs}(l, m) = \frac{\omega_A}{2\pi} \int_0^{2\pi/\omega_A} dt \int_{-\infty}^0 d\tau \exp \left\{ -i\omega_A t + im\omega_A \tau \right\} \tilde{\Gamma}^{\alpha\pm}_{rs}(t, \tau) \quad \text{(34)}$$

Then, by writing the external perturbations in their Fourier components,

$$\exp \left\{ -\frac{i\epsilon}{\hbar} \int_0^t dt' g_\alpha(t') \right\} = \sum_{n=-\infty}^{\infty} e^{in\omega_A t} \eta_\alpha(n) \quad \text{(35)}$$

we obtain $\tilde{\Gamma}^{\alpha\pm}_{rs}(l, m)$ as,

$$\tilde{\Gamma}^{\alpha\pm}_{rs}(l, m) = \gamma_\alpha S^\pm_{rs} f_{lm}^\alpha(E_{lm}^{\pm}) \quad \text{(36)}$$

Here, effective quasiparticle(+) and hole(−) distributions $f_{lm}^\alpha(E)$ are defined by,

$$f_{lm}^\alpha(E) = \frac{\delta_{l,0}}{2} \pm \int d\omega P_{\alpha}(\omega) \left[ \eta_\alpha(n) \eta_{\alpha}^*(n - l) \psi_\alpha(z) + \{ l, m \to -l, -m \}^* \right] \quad \text{(37)}$$

where

$$z = \frac{1}{2} \pm \frac{\hbar \gamma_\alpha \beta}{2\pi} + \frac{\beta}{\pi i} \{ E - (m + n) \hbar \omega_A \pm \hbar \omega \}$$

and $\psi_\alpha(z)$ is a digamma function. Without the dissipative elements and the alternating perturbations in addition to negligible collision-broadening $\gamma_\alpha$, $f_{lm}^{\alpha\pm}(E)$ are reduced to $\frac{1}{2}(1 \mp \tanh(\frac{E}{\hbar \beta}))$, the Fermi-Dirac distributions for particles(+) and holes(−). However, in general, $f_{lm}^{\alpha\pm}$ is deviated from the Fermi-Dirac form due to the dissipative environments(described by $P_a$) as well as the finite life time of quasiparticles (described by $\gamma_\alpha$).

In terms of $\tilde{\Gamma}^{\alpha\pm}_{rs}(l, m)$, the occupation probabilities and tunneling currents are calculated as,

$$i\omega_A \tilde{P}_{rs}(l) = \sum_{rs \xi, \eta, \pm} \left\{ \tilde{P}_{rs}^\alpha(m) \tilde{\Gamma}_{rs}^{\alpha\pm}(l - m, m) \right\}$$

$$- \tilde{P}_{rs}(m) \tilde{\Gamma}_{rs}^{\alpha\pm}(l - m, m) \quad \text{(38)}$$

and

$$I^t_{\alpha}(t) = \sum_{rs \xi, \eta, \pm} \tilde{P}_{rs}^\alpha(m) \tilde{\Gamma}_{rs}^{\alpha\pm}(l - m, m) \quad \text{(39)}$$

where we also expand $P_{rs}(t)$ and $I^t_{\alpha}(t)$ in their Fourier series; $P_{rs}(t) = \sum_{l} e^{i\omega_A t} \tilde{P}_{rs}(l)$ and $I^t_{\alpha}(t) = \sum_{l} e^{i\omega_A t} \tilde{I}_{\alpha}(l)$. In special case of $P_{rs}(\omega) = \delta(\omega)$, i.e. without dissipative elements in the circuit, we find that the above results together with the tunneling rates of Eq. (30) are similar to the formalism developed by Bruder and Schoeller [2].

### B. Time-convolutionless form

Evaluating the integral of Eq. (24), we derive another form of the memory kernel,

$$\Gamma^{\alpha\pm}_{rs}(t, \tau) = \gamma_\alpha S^\pm_{rs} \left[ \delta(\tau) + \frac{\csc \left( \frac{\pi \tau}{\hbar \beta} \right)}{\hbar \beta} \exp \{ \gamma_\alpha \tau \} \right] \quad \text{(40)}$$

with $\tau \leq 0$. Since this function decays exponentially from $\tau = 0$, we now expand $P_{rs}(t + \tau)$ in Taylor series at $\tau = 0$ to calculate Eqs. (24) and (33). By collecting the leading contributions, the occupation probabilities and tunneling currents are shown to be,

$$\frac{dP_{rs}(t)}{dt} = \sum_{\tau, \xi, \eta, \pm} \left[ P_{rs}(t) \Gamma^{\alpha\pm}_{rs}(t) - P_{ss}(t) \Gamma^{\alpha\pm}_{sr}(t) \right] + O(\mathcal{H}_{SB}^2) \quad \text{(41)}$$

and

$$I^t_{\alpha}(t) = \sum_{rs} P_{rs}(t) \left\{ \Gamma^{\alpha-}_{rs}(t) - \Gamma^{\alpha+}_{rs}(t) \right\} \quad \text{(42)}$$
where $\Gamma_{rs}^{\alpha\xi}(t) = \int_{-\infty}^{0} d\tau \Gamma_{rs}^{\alpha\xi}(t, \tau)$. The next contributions are the fourth order of the interaction Hamiltonian $H_{SB}(t)$ (These results can be also derived starting from the time-convolutionless solution of the density operator[3]) and, neglecting them in the spirit of sequential tunneling, the results are now time-convolutionless. In the Fourier space, the time-convolutionless results read as,

$$\tilde{i}_A^{\alpha\xi}(l) = \sum_{r \alpha, \xi} \sum_{s \xi} \left\{ \tilde{P}_{rr}(m) \Gamma_{rs}^{\alpha\xi}(l-m,0) - \tilde{P}_{ss}(m) \Gamma_{sr}^{\alpha\xi}(l-m,0) \right\}$$

(43)

and

$$\tilde{I}_A^{\alpha\xi}(l) = e \sum_{rsm} \tilde{P}_{rr}(m) \left\{ \Gamma_{rs}^{\alpha\xi}(l-m,0) - \Gamma_{sr}^{\alpha\xi}(l-m,0) \right\}$$

(44)

C. Adiabatic limit

As shown in Eq. (40), $\Gamma_{rs}^{\alpha\xi}(t, \tau)$ is dominant around $\tau = 0$ and thus, for a slowly varying external field ($\omega_{\alpha} \ll \gamma_{\alpha} + \pi/\hbar\beta$) it can be further approximated as,

$$\Gamma_{rs}^{\alpha\xi}(t, \tau) = \gamma_{\alpha}s_{rs}^{\pm} \left[ \delta(\tau) \mp \frac{\text{csch}(\pi \tau/\hbar\beta)}{\hbar\beta} \right]$$

$$\text{Im} \ e^{\gamma_{\alpha} \tau + i(\varepsilon_{\alpha}(t) + E_{rs}^{\alpha\xi})\tau/\hbar + J_{\alpha}(\pm\tau)}$$

$$\text{Re} \ \frac{\text{csch}(\pi \tau/\hbar\beta)}{\hbar\beta} \int_{-\infty}^{\infty} de \ N_{rs}^{\alpha\xi}(e + \varepsilon_{\alpha}(t) + E_{rs}^{\alpha\xi})$$

$$\exp \left\{ \gamma_{\alpha} \tau - \frac{i\varepsilon_{\alpha}}{\hbar} \tau + J_{\alpha}(\pm\tau) \right\}$$

(45)

Namely, in this limit, energy states in the quantum dot are merely modulated by the external perturbations. Furthermore, since these modulations are much slower than the equilibrated rate of $\gamma_{\alpha} + \pi/\hbar\beta$ due to tunneling and temperature, one can treat the problem as a static one with an additional DC-bias voltage of $g_{\alpha}(t)$ at each instant. Then, the occupation probabilities may be determined from a static balance relation,

$$0 = \sum_{r \alpha, \xi = +,-} \left[ P_{rr}(t) \Gamma_{rs}^{\alpha\xi}(t) - P_{ss}(t) \Gamma_{sr}^{\alpha\xi}(t) \right]$$

(46)

IV. APPLICATIONS OF FORMALISM

As applications of our results, we now examine time-dependent currents in a single-electron transistor based on Eqs. (23), (24), and (33). To understand $I-V$ characteristics easily, we consider a simple circuit of Fig. 3 where a sinusoidal voltage of $v_D(t) = v_D^0 \cos(\omega_D t)$ is driven at a drain through a resistor $R_0$ and a static voltage is applied on a gate without any impedance. The electron-electron interaction is assumed by a Coulomb blockade model; $V_{ee} = Q^2/2C_S$. For well-separated peaks of the Coulomb blockade oscillation, a charging energy ($E_C = e^2/2C_S = 100\hbar\gamma_0$) is chosen to be much larger than a broadening due to tunneling $\hbar\gamma_0$ and a thermal energy $k_B T = \kappa_{\alpha} T$ considered here ($\gamma_0$ is a unit for a tunneling rate). We also assume symmetric barriers in the drain and sources, and denote their tunneling rates with $\gamma_D = \gamma_S = \gamma_0/2$. Although we assume the symmetric tunneling barriers, the voltage differences $g_{\alpha}(t)$ across each tunneling junction are given in the Fourier space as well as the characteristic impedances $Z_{\alpha}(o)$ are different from each other because of a special geometry of our circuit, so called, a common-source geometry. In this geometry, the voltage difference of the tunnel junction to the source is mainly determined by the gate voltage $V_G$ while the junction to the drain depends largely on the drain voltage $V_D$.

A. Effects of the dissipative element $R_0$

As mentioned in the previous section, the first role of the dissipative elements is the determination of the AC voltages $g_{\alpha}(t)$ across each tunnel junction. According to Eq. (40), $g_{\alpha}(t)$ for the circuit of Fig 3 are given by, in their Fourier components,

$$\begin{pmatrix} \tilde{g}_D(\omega) \\ \tilde{g}_S(\omega) \end{pmatrix} = \begin{pmatrix} C_S + C_G & -C_D \\ -C_D & C_S + i\omega(C_G + C_S) \end{pmatrix} \begin{pmatrix} \tilde{v}_D(\omega) \\ \tilde{v}_D(\omega) \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & \gamma_D \\ -\frac{1}{\gamma_D} \end{pmatrix} \begin{pmatrix} \tilde{v}_D(\omega) \\ 1 + i\omega C_D R_0 \end{pmatrix}$$

(47)

where a relatively large capacitance of the gate capacitor compared with those of the source and drain is assumed in the second relation. It is noted that for a small frequency compared to a cut-off frequency $1/C_D R_0$, the most part of $v_D$ is applied on the drain junction with a small fraction of $C_D/C_G$ on the source junction. However, above the cut-off frequency, $g_{\alpha}$ fall like $1/\omega$, therefore, a smaller fraction of $v_D$ is applied on the tunneling junctions.
The second role of the resistance $R_0$ appears in the broadening of quasiparticle distributions as in Eq. (47) through the probability density $P_\alpha$. From Eq. (44), $Z'_{\alpha}(\omega)$ becomes for the circuit of Fig. 3,

$$
\left( \begin{array}{c} Z_D'(\omega) \\ Z_S'(\omega) \end{array} \right) = \left( \begin{array}{c} \frac{1}{C_G^2} \\ \frac{R_0}{1 + i\omega C_D R_0} \end{array} \right). \tag{48}
$$

The impedance $Z_S'(\omega)$ is smaller than $Z_D'(\omega)$ by a factor of $C_D^2/C_G^2$ and, for $C_G \gg C_D$, we can readily set $P_S(\omega) \sim \delta(\omega)$ which implies that tunneling through the barrier to the source is irrelevant to the environment. In Fig. 4, we show the probability density $P_D(\omega)$ at the drain and corresponding particle-distribution function for various $R_0$ when there are no alternating perturbations. Starting from a $\delta$–function for $R_0 = 0$, the probability density $P_D(\omega)$ shows a Lorentzian shape in the region of positive frequencies and exhibits exponentially decaying behavior of $P_D(-\omega) = \exp\{-\hbar \omega \beta\} P_D(\omega)$ in the negative region. As the values of $R_0$ increases the shapes of $P_D(\omega)$ are found to become more broad together with shifted peak positions to a positive frequency. This means that when particles tunnel a barrier more energies should be transferred to the environment as $R_0$ increases. These results are also reflected in the particle(hole)-distribution functions as shown in Fig. 4-(b). As $R_0$ increases, the particle-distribution functions are largely depleted in the region of negative energies. These depletions are similar to the case as if it is a high temperature and a chemical potential is shifted to a negative energy. Thus, one can expect that, compared to results of $R_0 = 0$, tunneling currents are smeared out and start to flow at a higher drain voltage as the resistance $R_0$ increases.

**B. Tunneling currents under time-dependent fields**

As a simple example, we consider two degenerate states ($\epsilon_1 = \epsilon_2 = 0$) in the quantum dot and, therefore, two possible many-body states with energies of $E_s = s^2 E_C(s = 1, 2)$ are available for tunneling. In the static case, we obtain typical $I-V$ curves; step-like behavior and Coulomb blockade oscillations as a function of drain and gate voltages, respectively, for $R_0 = 0$ as shown in Fig. 5(dashed lines). Under the dissipative environments ($R_0 \neq 0$), the shape of the $I-V$ curves are found to be smeared out due to the depletion of the particle(hole) distribution functions(not shown in Fig. 5).

The smearing of the tunneling currents is also found for the case of time-dependent fields. Applying an alternating perturbation, fine structures in tunneling currents are developed because energy levels in the quantum dot are split into photon-side bands, $E_s + k \hbar \omega_A$ ($k =$integer). These split energy levels are well identified in a DC part of the tunneling current as a function of additional static drain voltages, especially in the case of $R_0 = 0$ (dotted line in Fig. 5-(a)). Namely, each step appears at the voltage which gives the chemical potential of the drain corresponding to one of energy levels, $E_s + k \hbar \omega_A$. Under the dissipative environments, this step-like behavior is washed out as well as the height of the steps is reduced as $R_0$ increases as shown in Fig. 5-(a)(solid lines). Different from the static case, the reduction of the tunneling currents can be caused by the decrease of voltage differences across tunneling barriers as well as the depletion of the particle distributions described by Eqs. (47) and (48), respectively. However, the results of Fig. 5 are mostly responsible for the latter effect of the environment in this numerical simulation because the cut-off frequency of $1/R_0 C_D$ is still much larger than the applied frequency $\omega_A$ even for $R_0 = 5k\Omega$. Thus, we find that the tunneling currents are smeared out nearly by the same amount in both static and time-dependent cases as $R_0$ increases.

For various values of $R_0$, we also examine a DC part of the tunneling current as a function of a static gate voltage $V_G^0$ with a static drain voltage being zero, and plot the results in Fig. 5-(b). Instead of a Coulomb blockade peak in the static case(dashed line), it is found that the tunneling currents have negative or positive values depending on the gate voltages, and the direction is abruptly altered around the peak position. This behavior is retained for lower frequencies of the alternating perturbation, even in adiabatic limit. In reality, the direction of the tunneling currents is easily inferred because the voltage difference across the tunneling junction to the

**FIG. 4:** We plot the probability density of $P_D(\omega)$ at the drain in (a) and corresponding particle-distribution function $f_{D,+}^0(E)$ in (b) for $R_0 = 0$(dotted lines), 1, 2, 3, and $5k\Omega$(solid lines) when there are no alternating perturbations.
source is mainly determined by the gate voltage as mentioned in the previous section. In the region of the negative (positive) tunneling currents, the chemical potential of the source is lower (higher) than an energy of a relevant quantum state in the dot and, therefore, electrons tunnel from (to) the quantum dot through the barrier connected to the source, and vice versa. Similar to the results as a function of a static drain voltage in Fig 5-(a), photon-side bands are manifested itself in steps apart from each other by $\hbar\omega_A$. We find that these steps are well resolved in the case of $R_0 = 0$ while they are relatively washed out as $R_0$ increases.

For valid applications of the time-convolutionless formalism we also calculate tunneling currents for the circuit in Fig. 3 based on Eqs. (11) and (12). By varying parameters within small tunneling rates, it is found that the time-convolutionless formalism give negligible differences from results obtained by the time-convolution forms.

V. SUMMARY

In summary, we have studied time-dependent currents of the single-electron transistor embedded in possible dissipative circuits and driven by time-dependent perturbations. In the regime of sequential tunneling, we present numerically tractable forms for both displacement and tunneling currents where the tunneling rates of Eq. (25) contain explicitly the influence of the dissipative environments and time-dependent perturbations. We find that the dissipative environments affect tunneling currents of the single-electron transistor in two ways: the determination of driving voltages at tunneling junctions and the depletion of particle-hole distribution functions at each electrode. Applying our formalism to a simple dissipative system and solving the problem numerically, we show how steps in tunneling currents developed by photon-side bands are smeared out as the system becomes more dissipative.

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