EPR correlations without the EPR dilemma: a local scheme

A. Matzkin

Laboratoire de Spectrométrie physique (CNRS Unité 5588),
Université Joseph-Fourier Grenoble-1,
BP 87, 38402 Saint-Martin d’Hères, France

Abstract

A model for two entangled systems in an EPR setting is shown to reproduce the quantum-mechanical outcomes and expectation values. Each system is represented by a small sphere containing a point-like particle embedded in a field. A quantum state appears as an equivalence class of several possible particle-field configurations. Contrarily to Bell-type hidden variables models, the fields account for the non-commutative aspects of the measurements and deny the simultaneous reality of incompatible physical quantities, thereby allowing to escape EPR’s “completeness or locality” dilemma.
I. INTRODUCTION

In their celebrated paper [1], Einstein, Podolsky and Rosen (EPR) argued that quantum mechanics was incomplete on the ground that for entangled states the formalism predicts with certainty the measurement outcomes of noncommuting observables although they cannot have simultaneous reality. They argued the alternative to incompleteness was to make the reality of one particle’s properties depend on the measurement made on the other particle, irrespective of their spatial separation. EPR concluded: “no reasonable definition of reality could permit this” nonlocal action at-a-distance. In a seminal work [2], Bell showed that local models based on a distribution of hidden variables (HV) intended to complete quantum mechanics must satisfy an inequality involving averages taken over the hidden variable distributions. He also showed that in certain circumstances the average values of two-particle quantum observables violate these inequalities. However, it is seldom mentioned that Bell-type models are only a subset of the local models that can be envisaged. Indeed, Bell’s theorem [3, 4] is grounded on two important assumptions: (a) the HV ascribe a sub-quantum elementary probability for any 1 or 2-particle outcomes; (b) this probability factorizes into two single particle probabilities. These assumptions lead [5] to the existence of a joint probability function for all the observables entering the inequality (though there is no such probability according to quantum mechanics), thereby accounting for the ‘simultaneous reality’ appearing in the EPR dilemma. General arguments seem to indicate that these assumptions are needed to comply with the EPR requirements but are by no means necessary ingredients in order to enforce locality [6, 7, 8]. In this work we put forward a scheme compatible with quantum mechanical correlations but does not abide by the EPR dilemma. The model, developed for the prototypical spin-1/2 pair, describes each system by postulating a particle and a classical field. It is shown that different particle-field configurations yield the same probabilities for outcomes detection, even when the outcomes can be predicted with certainty. We will first put forward the model for a single particle. We will then naturally extend the model to the two-particle case, and show how, by introducing a correlation at the source, the model reproduces the quantum predictions that violate the Bell inequalities without involving action at a distance.
II. SINGLE FIELD-PARTICLE SYSTEM

Let a single spin-1/2 be represented by a field-particle system assumed to be composed of a small sphere, with the position of its center in the laboratory frame being denoted by \( \mathbf{x} \) and the internal spherical variables relative to the center of the sphere by \( \mathbf{r} \equiv (r, \theta, \phi) \) (see Fig. 1a). A classical scalar field \( F(\mathbf{r}) \) is defined on the spherical surface. The point-like particle sits still at a fixed (but unknown position) on the sphere. We are interested in measuring the polarization of the particle, i.e., its internal angular momentum projection along a given axis\(^1\). Let \( \varepsilon_b \) denote the polarization along an axis \( b \) making an angle \( \theta_b \) with the \( z \) axis. We assume that the possible outcomes \( \varepsilon_b = \pm 1 \) can be obtained, the result depending in a manner to be described below (i) on the region on which the field is defined and (ii) on the position of the particle. The elementary support of the field \( F \) is a hemispherical surface. The value of the field at any point depends on the projection of that point on the axis. Let \( \Sigma_{+a} \) denote the positive half-sphere centered on the axis \( a \) making an angle \( \theta_a \) with the \( z \) axis, and \( F_{\Sigma_{+a}} \) denote the field distributed on that hemisphere. \( F_{\Sigma_{+a}}(\mathbf{r}) \) is defined by

\[
F_{\Sigma_{+a}}(\mathbf{r}) = \begin{cases} 
\mathbf{r} \cdot \mathbf{a} e^{i\phi_a} / \pi R^2 & \text{if } \mathbf{r} \in \Sigma_{+a} \\
0 & \text{otherwise}
\end{cases},
\]

(1)

\( R \) being the radius of the sphere and \( \phi \) the phase of the field; for simplicity we will take all the axes to be coplanar with \( z \) and the phase will be assumed constant over an entire hemisphere (the phase thus appears as a global additional degree of freedom of the field). The mean value of \( \mathbf{r} \cdot \mathbf{b} / \pi R^2 \) taken over \( \Sigma_{+a} \) is given by

\[
\langle F_{\Sigma_{+b}} + F_{\Sigma_{-b}} \rangle_{\Sigma_{+a}} \equiv \int_{\Sigma_{+a}} \frac{\mathbf{r} \cdot \mathbf{b}}{\pi R^2} d\hat{\mathbf{r}} = \cos (\theta_b - \theta_a),
\]

(2)

where \( d\hat{\mathbf{r}} \) denotes the spherical surface element for a sphere of radius \( R \) and we have set \( \phi_{\pm b} = 0 \). The only requirement we make on the particle’s position is that it must be embedded within the field: the particle cannot be in a field free region of the sphere.

When the polarization \( \varepsilon_b \) is measured we postulate that the measuring apparatus along \( b \) interacts with the field \( F_{\Sigma_{+a}} \). Let \([a+b]\) and \([a-b]\) denote the directions lying halfway between

\(^{1}\) From a physical standpoint, what we have called here the position of the particle should more properly be called the position of the particle’s angular momentum \( \mathbf{r}_0 \times \mathbf{p} \) relative to the center of the sphere. We will not make explicitly this distinction in this paper.
the axes \( a \) (of the distribution) and \( b \) or \(-b\) (of the measuring direction), with respective angles \((\theta_b + \theta_a)/2\) and \((\theta_b + \pi + \theta_a)/2\). We will assume that the field-apparatus interaction results in a rotation of the original pre-measurement field \( F_{\Sigma+a} \) toward both of the apparatus axes, \( F_{\Sigma+a} \rightarrow (F_{\Sigma+b} + F_{\Sigma-a})e^{i\phi+a} \) (Fig 1b); \( \phi+a \) is the phase of the original field and we will suppose the measurement does not introduce additional phases. A definite outcome \( \varepsilon_b = \pm 1 \) depends on which of the hemispheres \( \Sigma_{\pm b} \) the particle is after the interaction. In terms of the field, this probability is given by the relative value of the average of the rotated field \( F_{\Sigma+b} + F_{\Sigma-a} \) over the intermediate 'half-rotated' hemisphere \( F_{\Sigma[a\pm b]} \), yielding in accordance with Eq. (2)

\[
P_{F_{\Sigma+a}} (\varepsilon_b = +1) = \left| \left\langle F_{\Sigma+b} + F_{\Sigma-a} \right\rangle_{\Sigma[a\pm b]} \right|^2/N = \cos^2 \frac{\theta_b - \theta_a}{2} \tag{3}
\]

\[
P_{F_{\Sigma+a}} (\varepsilon_b = -1) = \left| \left\langle F_{\Sigma+b} + F_{\Sigma-a} \right\rangle_{\Sigma[a\pm b]} \right|^2/N = \sin^2 \frac{\theta_a - \theta_b}{2} \tag{4}
\]

with \( N \) being the sum of both terms, thereby recovering the probabilities of measurements made on a single spin-1/2, reading in the standard notation \( |\langle \pm b |+a \rangle|^2 \) (normalization will be implicitly understood in the rest of the paper). If \( b \) and \( a \) are taken to be the same, then one has \( \Sigma_{[a+a]} \equiv \Sigma_{+a} \) and \( P_{\Sigma_{+a}} (\varepsilon_a = \pm 1) = 1 \) and 0 respectively. Hence a field \( F_{\Sigma_{+a}} \) corresponds to a well-defined positive polarization along the \( a \) axis. In this case the symmetry axis of the field distribution coincides with the measurement axis and the system-apparatus fields interaction has no effect: the particle’s pre and post-measurement position remains within the same hemisphere \( \Sigma_{+a} \). The particle’s hemispherical position can be said to determine the outcome, i.e. \( P_{F_{\Sigma+a}} (\varepsilon_a = \pm 1) = P_{F_{\Sigma+a}} (r \in \Sigma_{\pm a}) = 1 \) or 0.

On the other hand when \( b \) and \( a \) lie along different directions, the particle position cannot ascribe probabilities: the probabilities depend on the system and apparatus fields and \( \varepsilon_b \) only acquires a value \( \pm 1 \) after the system field has interacted with the apparatus and rotated toward the measurement axis. A straightforward consequence is that the measurements do not commute, and thus joint polarization measurements along different axes are undefined.

Since fields obey the principle of superposition, we can envisage superpositions of fields defined on different hemispheres. But fields defined on different hemispheres turn out to be equivalent to a field defined on a single hemisphere. Indeed it is easy\(^2\) to see that one can

---

\(^2\) As the reader will have noted, the fields are defined through a mapping of the Hilbert space rays onto the relevant hemispherical surface.
meaning that although the two fields on the right and left hand sides (hs) of Eq. (5) are different – they are not defined on the same hemispherical surfaces –, they lead to exactly the same predictions. Indeed, when measurements are made along any axis $b$ the averages of the left and right hs of Eq. (5) give the same result $\cos(\theta_u - \theta_a)$.

From the particle standpoint a definite field configuration implies a different behaviour: for the field on the rhs of Eq. (5), denoted $F_{rhs}$, the no-perturbation axis is $u$, not $a$, and the particle distribution cannot be uniform. Hence there is a probability function $p_{F_{rhs}}(\varepsilon_a = \pm 1, r) = 1$ or $0$ depending on whether $r \in \Sigma_{\pm u}$ and such that $P_{F_{rhs}}(\varepsilon_u = \pm 1)$ is recovered by integration over the particle distribution. For $b \neq u$ however there is no probability function $p_{F_{rhs}}(\varepsilon_b = \pm 1, r)$ hence $P_{F_{rhs}}(\varepsilon_b = \pm 1)$ cannot depend on $r$: there is no sub-field mechanism that determines the outcome. This is consistent with Eqs. (3)-(4) in which the field rotation does not allow to define joint probabilities of the type $P_{F_{rhs}}(\varepsilon_u = \pm 1 \cap \varepsilon_b = \pm 1)$; it can be shown instead that such joint probabilities would follow by allowing the particle position to determine probabilities for measurements along arbitrary axes [10]. In the specific case of measuring $\varepsilon_a$ in the field $F_{rhs}$, the system and apparatus fields must interfere in such a way as to obtain $P_{F_{rhs}}(\varepsilon_a = -1) = 0$, irrespective of the initial particle’s position. Finally let us introduce the fields $F_{\alpha(u)\pm}$ defined by

$$F_{\alpha(u)\pm}(r) = e^{i\frac{\pi}{2}}F_{\Sigma+u}(r) + F_{\Sigma-u}(r),$$

which obey the equivalence $F_{\alpha(u)\pm} \sim F_{\alpha(b)\pm}$ for any axes $u$ and $b$. We have

$$P_{\alpha(u)\pm}(\varepsilon_b = \pm 1) = |\langle F_{\alpha(u)\pm} \rangle|^2 = \frac{1}{2}$$

for any $b$, the average depicting Eq. (2) taken on the rotated hemispheres $\Sigma_{[u\pm b]}$ (for $F_{\Sigma+u}$) and $\Sigma_{[-u\pm b]}$ (for $F_{\Sigma-u}$). An interpretation in terms of the particle position can only be given for $b = u$ with elementary probabilities $p_{F_{\alpha(u)\pm}}(\varepsilon_u = \pm 1, r) = 1$ or $0$ depending on whether $r \in \Sigma_{\pm u}$. It is nevertheless possible to postulate additional sub-quantum probabilities provided they are consistent with the field averages. For example we will suppose for either of the fields $F_{\alpha(u)\pm}$ that

$$P_{\alpha(u)}(\varepsilon_b = 1| r \in \Sigma_{\pm u}) = \cos^2 \left( \frac{\theta_u - \theta_b}{2} + \frac{\pi}{2}(1 \mp 1) \right)$$
FIG. 1: (a) A system is represented by a point-like particle lying on the surface of a small sphere, on which a field is defined. The particle position $r_0$ is unknown; the field is defined on the hemispherical surface centered on the positive $a$ axis. (b) Post-measurement situation after the polarization of the system pictured in (a) has been measured along the $b$ axis, yielding an outcome $\varepsilon_b = +1$: the field has rotated and is now centered on $b$. (c) An initial 2-particle system has fragmented into 2 subsystems (at $x_1$ and $x_2$), each carrying a particle embedded in a field $F_{\alpha(u)\pm}$. As a result of the fragmentation the particle positions lie on opposite points of their sphere and the fields become effectively correlated between opposite hemispheres of each subsystem, as symbolized by the colouring.

which assuming $r$ is uniformly distributed is consistent with (7) given that

$$\sum_{\pm} P_{\alpha(u)}(\varepsilon_b = 1 | r \in \Sigma_{\pm u}) P_{\alpha(u)}(r \in \Sigma_{\pm u}) = P_{\alpha(u)}(\varepsilon_b = 1).$$

(9)

Note that Eq. (8) supplements Eq. (7) with a condition on the hemispherical position of the particle, but the latter does not determine the outcome (it is not an elementary probability).

III. TWO-PARTICLE SYSTEM

Assume now an initial two-particle system is fragmented into two subsystems flying apart in opposite directions. Each of the two particles, labeled 1 and 2, is embedded in a field defined on the surface of a small sphere. $x_1$ (resp. $x_2$) denotes the position of the subsystem 1 (resp. 2) sphere in the laboratory frame. The internal variables within each sphere are labeled by $r_1$ and $r_2$. As soon as the fragmentation process is completed, the positions of each point-like particle as well as the fields are fixed, the polarization of each system depending on the field distribution and the particle position on its spherical surface. We will choose the initial correlation to correspond to the compound having zero polarization at least along an axis $u$ (but see below), in view of reproducing the statistics for the two
spin-1/2 in the singlet state problem. Assuming the total polarization is conserved, the fields and particle positions must be initially correlated such that \( \varepsilon_1 u = -\varepsilon_2 u \). Let us start by examining the no-perturbation measurements. In this case the particle positions determine the outcomes, from which it follows that we must set

\[
\mathbf{r}_1 = -\mathbf{r}_2
\]

(10)
at the source. Assume subsystem 1 and 2 fields to be given by \( F_{\alpha(u)+} \) and \( F_{\alpha(u)-} \) defined above. The total field for the system is thus

\[
F_{T(u)}(\mathbf{r}_1, \mathbf{r}_2) = F_{\alpha(u)+}(\mathbf{r}_1)F_{\alpha(u)-}(\mathbf{r}_2).
\]

(11)

Single outcome probabilities \( P(\varepsilon_1, \varepsilon_2 u) = 1/2 \) are straightforwardly computed from the single subsystem field \( F^1 \) or \( F^2 \). On the other hand two outcome probabilities must take into account the particle correlation (10). It is thus impossible to obtain \( \varepsilon_1 u = \varepsilon_2 u \); since there are no measurement perturbations, \( \varepsilon_1 u = \pm 1 \) is associated with \( \mathbf{r}_1 \in \Sigma^1_{\pm u} \), implying \( \mathbf{r}_2 \in \Sigma^2_{\mp u} \) so only \( \varepsilon_2 u = \mp 1 \) can be obtained. The probabilities in this case read

\[
P(\varepsilon_1 u = \pm 1, \varepsilon_2 u = \mp 1) = P(\varepsilon_1 u = \pm 1)P(\varepsilon_2 u = \mp 1|\varepsilon_1 u = \pm 1) = \frac{1}{2}
\]

(12)

where the conditional probability is computed by way of the particle dependence as \( P(\mathbf{r}_1 \in \Sigma^1_{\pm u})P(\mathbf{r}_2 \in \Sigma^2_{\mp u} | \mathbf{r}_1 \in \Sigma^1_{\pm u}) \) and setting \( b = u \) in Eqs. (7)-(8). Note that these probabilities are not equal to those obtained by taking the relevant averages of \( F_{T(u)} \) (e.g., \( \langle F_{T(u)} \rangle_{\Sigma^1_{\pm u} \Sigma^2_{\mp u}} \) does not vanish). The reason is that \( F_{T(u)} \) does not take into account the particle correlation. It is possible nevertheless to identify the term correlating the fields consistent with Eq. (10) by rewriting Eq. (11) as

\[
F_{T(u)}(\mathbf{r}_1, \mathbf{r}_2) = F_{0(u)}(\mathbf{r}_1, \mathbf{r}_2) + e^{i\pi/2}F_{\mathbb{R}(u)}(\mathbf{r}_1, \mathbf{r}_2)
\]

(13)

where \( F_0 \) and \( F_{\mathbb{R}} \) are given by

\[
F_{0(u)}(\mathbf{r}_1, \mathbf{r}_2) = F^1_{\Sigma^1_{\pm u}}(\mathbf{r}_1)F^2_{\Sigma^2_{\mp u}}(\mathbf{r}_2) + F^1_{\Sigma^1_{\mp u}}(\mathbf{r}_1)F^2_{\Sigma^2_{\pm u}}(\mathbf{r}_2)
\]

(14)

\[
F_{\mathbb{R}(u)}(\mathbf{r}_1, \mathbf{r}_2) = F^1_{\Sigma^1_{\pm u}}(\mathbf{r}_1)F^2_{\Sigma^2_{\mp u}}(\mathbf{r}_2) - F^1_{\Sigma^1_{\mp u}}(\mathbf{r}_1)F^2_{\Sigma^2_{\pm u}}(\mathbf{r}_2).
\]

(15)

It is easy to show that \( F_{0(u)} \) cannot contribute to the probabilities by repeating the reasoning involving no-perturbation measurements. On the other hand \( F_{\mathbb{R}(u)} \) respects by construction the particle correlation (10) and the probabilities can be computed from the fields averages.
\[ \langle F_\alpha \rangle_{\Sigma^1_{+a} \Sigma^2_{-a}} \text{ (which are equal) and } \langle F_\alpha \rangle_{\Sigma^1_{+a} \Sigma^2_{+a}} = 0 \]. Note that the particle labels as well as the field indices can be interchanged in the definition (11) of \( F_T \).

Let us now investigate measurements along arbitrary directions \( a \) for particle 1 and \( b \) for particle 2. Probabilities for a single subsystem are immediately obtained from the subsystem’s field \( F^{1,2}_{\alpha(u)\pm} \) yielding 1/2 for any arbitrary direction. To compute correlations for two outcomes, say \( \varepsilon^1_a = 1, \varepsilon^2_b = 1 \), the averages involving \( F_T \) must again be supplemented with the correlation (10). This can be done by employing the equivalence relations \( F_{\alpha(u)\pm} \sim F_{\alpha(a)\pm} \) in Eq. (11), yielding \( F_{T(u)} \sim F_{T(a)} \). The probability is then computed as \( P(\varepsilon^1_a = 1) P(\varepsilon^2_b = 1|\varepsilon^1_a = 1) \) by writing as in Eq. (12) single subsystem probabilities in terms of the particle positions:

\[ P_{T(a)}(\varepsilon^1_a = 1, \varepsilon^2_b = 1) = P_{\alpha(a)+}(r_1 \in \Sigma^1_{+a}) P_{\alpha(a)-}(\varepsilon^2_b = 1| r_2 \in \Sigma^2_{-a}) \]

\[ = \frac{1}{2} \sin^2 \left( \frac{\theta_b - \theta_a}{2} \right) \]  

(16)

(17)

where we have used Eqs. (7) and (8). We can obviously reach the same result by employing \( F_{\alpha(u)\pm} \sim F_{\alpha(b)\pm} \) in Eq. (11) yielding

\[ P_{T(b)}(\varepsilon^1_a = 1, \varepsilon^2_b = 1) = P_{\alpha(b)-}(r_2 \in \Sigma^2_{+b}) P_{\alpha(b)+}(\varepsilon^1_a = 1| r_1 \in \Sigma^1_{-b}). \]  

(18)

Both computations hinge on employing the form of the field that does not perturb one of the measurements: this is necessary in order to be able to compute conditional statements. As in the single particle system case [see below Eq. (5)] each particular realization of an equivalence class gives rise to different, incompatible, accounts: Eq. (16) specifies that \( r_1 \in \Sigma^1_{+a} \) while assuming the field configuration is \( F_{T(a)} \) whereas Eq. (18) indicates that \( r_1 \in \Sigma^1_{-b} \) when the field is \( F_{T(b)} \). The direct computation of \( P_{T(u)}(\varepsilon^1_a = 1, \varepsilon^2_b = 1) \), without resorting to an equivalent configuration, cannot rely on conditional statements since both measurements involve perturbations\(^3\). The probability can be computed by obtaining the correlated averages of the fields rotated by the interaction for each measurement. As in the no-perturbation case \( F_{0(u)} \) is the field encapsulating the correlation (10) while \( F_{0(u)} \) does not contribute to the probabilities. This can be seen by noting that for the outcomes \( \varepsilon^{1,2}_{a,b} = 1 \) the averages \( \langle F_{0(u),N(u)} \rangle \), giving \( \cos \left( \frac{\theta_b - \theta_a}{2} \right) \) and \( \sin \left( \frac{\theta_b - \theta_a}{2} \right) \) for \( F_{0(u)} \) and \( F_{N(u)} \) respectively do

\(^3\) Employing \( P_{\alpha(a)+}(r_1 \in \Sigma^1_{+a}) P_{\alpha(a)-}(\varepsilon^2_b = 1| r_2 \in \Sigma^2_{-a}) \) along with Eq. (8) does not ensure the correlation is taken into account, since one may have \( r_1 \in \Sigma^1_{+a} \) and \( r_2 \in \Sigma^2_{-a} \) without \( r_2 = -r_1 \). It is only when at least one of the measurements is not perturbed that such an inference can be made.
not depend on \( u \). This implies that \( F_0 \) and \( F_\aleph \) form separately equivalence classes, i.e. we have

\[
F_{0(u)} \sim F_{0(a)} \quad \text{and} \quad F_{\aleph(u)} \sim F_{\aleph(a)}
\]

(19)

for any axes \( u \) and \( a \). Using Eq. (19) it can be established that \( F_0 \) does not contribute to the probabilities\(^4\). Hence \( P_{T(u)} \) is given by \( P_{\aleph(u)} \)

\[
P_{\aleph(u)}(\varepsilon^1_a = 1, \varepsilon^2_b = 1) = \left| \left\langle F^1_{\Sigma_{[u+a]}} \right\rangle_{+a} \left\langle F^2_{\Sigma_{[-u+b]}} \right\rangle_{+b} - \left\langle F^1_{\Sigma_{[-u+a]}} \right\rangle_{+a} \left\langle F^2_{\Sigma_{[-u+b]}} \right\rangle_{+b} \right|^2,
\]

(20)

where the term between the \( |...| \) is the explicit expression of \( \left\langle F_{\aleph(u)} \right\rangle \). Of course, it is possible (and simpler) to use Eq. (19) and compute \( P_{\aleph(a)} \) or \( P_{\aleph(b)} \) instead of \( P_{\aleph(u)} \) (expressions similar to Eqs. (16) and (18) are obtained – only the field indices need to be changed despite \( F_\aleph \) being defined jointly over the two subsystems).

IV. DISCUSSION AND CONCLUSION

The present dual field-particle model reproduces the EPR correlations without the need to invoke non-locality (i.e. action at a distance): a measurement carried out on one subsystem does not modify the field or the particle position of the other system. A striking feature is that although the total field \( F_T \) is separable, the effective field \( F_\aleph \) is a non-separable function. Non-separability does not involve nor imply non-locality (recall that non-separable functions are not exceptional in classical physics\(^5\)) but is necessary in order to account for field correlations between hemispheres encapsulating the particle correlation (10). The field configuration, as well as the particle positions, are set at the source, in the intersection of the past light cones of each subsystem’s space-time location and are modified locally by the measurement process\(^6\). Several differences between our model and Bell-type LHV models

---

\(^4\) This can be seen by computing first \( \left\langle F_0 \right\rangle \) for the outcomes \( \varepsilon^1_a = 1 \) and \( \varepsilon^2_b = 1 \) which we know to vanish from the no-perturbation case (use \( F_{0(a)} \) and \( F_{0(b)} \) respectively). The same averages can be computed instead from \( F_{0(b)} \) and \( F_{0(a)} \) implying that terms such as \( \left\langle F^1_{\Sigma_{[\pm b+a]}} \right\rangle_{+a} \left\langle F^2_{\Sigma_{[\pm b+a]}} \right\rangle_{+a} \) must be put to zero by hand to take the particle correlation into account. But these same terms also appear when computing \( P(\varepsilon^1_a = 1, \varepsilon^2_b = 1) \) from \( \left\langle F_0 \right\rangle \) with \( F_{0(a)} \) and \( F_{0(b)} \).

\(^5\) For example the classical action for a multi-particle system is a non-separable function in configuration space.

\(^6\) The non-separable part of the field \( F_T \) that takes into account the correlations between both subsystems becomes irrelevant to describe the system once a measurement is made (since the correlations are broken at that point).
deserve to be pointed out. First, note that the probabilities are obtained from average field intensities, not from elementary probabilities averaged over HV distributions $\rho(\lambda)$. It is known that in general classical fields do not have to obey Bell-type inequalities [9]. Here, the fields (i) are not necessarily positive valued, (ii) can interfere, and (iii) define equivalence classes. Field measurements are non-commutative, whereas LHV models assume factorizable elementary probabilities $p(\varepsilon_1^a, \varepsilon_2^b, \lambda) = p(\varepsilon_1^b, \lambda)p(\varepsilon_2^a, \lambda)$, leading to the existence of global joint probabilities (e.g. $R(\varepsilon_1^a, \varepsilon_1^a', \varepsilon_2^b, \varepsilon_2^b')$) that in quantum mechanics can only be defined for commuting operators [5]. If the hemispherical fields were replaced by probability distributions for the particles, then the equivalence relations would not hold and the conditional probabilities appearing in Eqs. (16) or (18) would imply outcome dependence [10, 11]. The particles’ positions thus appear as pre-determined, and can play the role of hidden-variables, but they do not ascribe probabilities except in the absence of measurement perturbations. The field configurations can also be taken as hidden variables and they do ascribe probabilities but only as members of an equivalence class that does not give a more complete specification than afforded by the quantum-mechanical state.

These last remarks lead us back to the original EPR dilemma recalled in the Introduction. In a single particle system the field dynamics ensure that there is no pre-existing outcome as an element of reality, even when it is possible to make a prediction with unit probability (in this case too there is an infinity of possible field-particle configurations, the outcome arising from the interference between the system and the apparatus fields). For an arbitrary measurement axis, a definite field-particle configuration, even if known, would not give an elementary sub-quantum description of a measurement outcome; such a description is only possible by resorting to an equivalent, albeit fictitious, field-particle configuration in which there is no perturbation. In the two particle system, the additional constraint is that the particle positions as well as the effective fields on each sphere are correlated, allowing to infer one subsystem’s outcome once the other subsystem’s outcome is known. This inference, in terms of a sub-quantum description, also relies on the existence of an equivalence class providing an equivalent configuration characterized by a no-perturbation measurement along at least one axis. As a consequence the model denies the attribution of simultaneous reality to $\varepsilon_1^a$ and $\varepsilon_2^b$ on the ground that an observer has the choice of measuring $\varepsilon_1^a$ or $\varepsilon_1^b$ on particle 1 (this would imply that $F_{T(a)}$ and $F_{T(b)}$ be both realized as the system’s field which is impossible as noted above), although both conditional probabilities are unity. Thereby the
“simultaneous reality” branch of EPR’s dilemma – which is fulfilled by Bell-type models – is decoupled here from the issue of locality.

To sum up, we have given an explicit model in which a quantum state appears as an equivalence class comprising an infinity of possible field-particle configurations. The model can be said to ‘complete’ quantum mechanics (in the sense that it assumes an underlying reality relative to the quantum state) though it does not generally allow to give more complete and deterministic sub-quantum predictions. The model despite being local does not abide by Bell’s causality condition \([12]\) but nevertheless defuses the EPR dilemma while avoiding the type of probability ascription leading to Bell’s theorem.

[1] Einstein A, Podolsky B and Rosen N 1935 Phys. Rev. 47 777
[2] Bell J S 1964, Physics 1 195
[3] Clauser J F and Shimony A 1978 Rep. Prog. Phys. 41 1881
[4] Bell J S 1971 in B. d’Espagnat (Ed.) *Foundations of quantum mechanics*, Academic Press, New York, p. 171
[5] Fine A 1982 J. Math. Phys. 23 1306
[6] Jaynes E T 1989, in J. Skilling (Ed.), *Maximum-Entropy and Bayesian Methods*, Kluwer, Dordrecht p. 1
[7] Orlov Y F 2002 Phys. Rev. A 65, 042106
[8] Khrennikov A 2007, AIP Conf. Proc. Vol. 962 121
[9] Morgan P 2006 J. Phys. A 39 7441
[10] Matzkin A 2008 J. Phys. A 41 085303
[11] Matzkin A 2008 Phys. Rev. A 77 062110
[12] This condition assumes the existence of a complete set of probability-ascribing beables for each subsystem; see J.S. Bell, *Speakable and unspeakable in quantum mechanics* (Cambridge University Press, Cambridge, 2004), Chap. 24 “La nouvelle cuisine”.