THE EXISTENCE OF SUPERLUMINAL PARTICLES IS CONSISTENT WITH THE KINEMATICS OF EINSTEIN’S SPECIAL THEORY OF RELATIVITY

GERGELY SZÉKELY

Abstract. Within an axiomatic framework of kinematics, we prove that the existence of faster than light particles is logically independent of Einstein’s special theory of relativity. Consequently, it is consistent with the kinematics of special relativity that there might be faster than light particles.

1. Introduction

The investigation of superluminal motion in relativity theory goes back (at least) to Tolman, see [26, p.54-55]. After showing that faster than light (FTL) particles travel back in time according to some observers, Tolman writes: “Such a condition of affairs might not be a logical impossibility; nevertheless its extraordinary nature might incline us to believe that no causal impulse can travel with a velocity greater than that of light.” It is interesting to note that Tolman has not claimed that relativity theory implies the impossibility of the existence of superluminal particles; he just claims that, if they exist, they have some “extraordinary” properties.

Since then a great many works dealing with superluminal motion have appeared in the literature, see, e.g., Matolcsi–Rodrigues [18], Mittelstaedt [19], Recami [20, 21], Recami–Fontana–Garavaglia [22], Seleri [23], Weinstein [28] to mention a few.

From time to time short-lived experimental results appear that suggest the existence of FTL objects. Recently, the OPERA experiment, see [9], raised the interest in the possibility of FTL particles. The trouble is that if there are FTL particles, then several branches of the tree that grew out of relativity theory die out, e.g., the ones that directly assume the nonexistence of superluminal objects. Therefore, in order to accept the existence of FTL particles we want to see extremely solid experimental evidence.

Even if the OPERA result turns out to be erroneous, the possibility will always be there that one day an experiment will prove the existence of FTL particles beyond any doubt. Thus the question arises whether the whole tree sprung out of relativity theory will die with the existence

\footnote{This observation is the basis of several causal paradoxes (i.e., seemingly contradictory statements) concerning FTL particles.}
In this paper, we show that the roots of the metaphorical tree surely survive any experiment proving the existence of FTL objects, since their existence is completely consistent with the kinematics of special relativity. In a forthcoming paper, we will show that even some parts of dynamics survive the existence of FTL particles.

We will show that the statement “there can be faster than light particles” is logically independent of the kinematics of special relativity. This means that we can add either this assumption or its opposite to the axioms of special relativity without getting a theory containing contradictions. This result is completely analogous to the fact that Euclid’s postulate of parallels is independent of the rest of his axioms (in this case two different consistent theories extending the theory of absolute geometry are Euclidean geometry and hyperbolic geometry).

The best framework for analyzing this kind of logical independence situations is the axiomatic framework of mathematical logic. Therefore, we will investigate this question in the framework of mathematical logic, and we will use model theory to show the existence of the required models.

Based on Einstein’s original postulates, we formalize the kinematics of special relativity within an axiomatic framework. We chose first-order predicate logic to formulate axioms of special relativity because experience (e.g., in geometry and set theory) shows that this logic is the best logic for providing axiomatic foundations for a theory. A further reason for choosing first-order logic is that it is a well defined fragment of natural language with an unambiguous syntax and semantics, which do not depend on set theory. For further reasons, see, e.g., [1, §Why FOL?], [7], [24, §11], [27], [29].

First in Section 2, we will recall a first-order logic framework of kinematics from the literature with a minor modification fitting it to formalize Einstein’s original postulates. Then in Section 3, we formulate Einstein’s original two postulates and supplement them with some natural ones (e.g., with the statement that different inertial observers see the same events) which were implicitly assumed by Einstein, too. It is also a benefit of using first-order logic that we have to reveal all our tacit assumptions.

Within this axiomatic framework, we formulate and prove our main result, namely that the existence of FTL particles is independent of the kinematics of special relativity, i.e., we prove that neither the existence nor the nonexistence of FTL objects follows from the theory, see Theorem 4.2. Consequently, it is consistent with the kinematics of special relativity that there are FTL particles, which can of course carry “information,” see, e.g., Section 5. In Section 6, we will discuss why the possibility of sending particles back to the past does not necessarily
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

lead to a logical contradiction even if we leave our safe framework of kinematics.

2. THE LANGUAGE OF OUR AXIOM SYSTEM

To formulate Einstein’s original informal postulates within first-order logic, first we have to fix the set of basic symbols for the theory, i.e., what objects and relations between them we will use as basic concepts.

Here we will use the following two-sorted\(^{2}\) language of first-order logic parametrized by a natural number \(d \geq 2\) representing the dimension of spacetime:

\[
\{ B, Q; \text{IOb}, \text{Ph}, +, \cdot, \leq, W \},
\]

where \(B\) (bodies) and \(Q\) (quantities) are the two sorts, \(\text{IOb}\) (inertial observers) is a one-place relation symbol and \(\text{Ph}\) (light signal emitted by) is a two-place relation symbol of sort \(B\), \(+\) and \(\cdot\) are two-place function symbols of sort \(Q\), \(\leq\) is a two-place relation symbol of sort \(Q\), and \(W\) (the worldview relation) is a \(d+2\)-place relation symbol the first two arguments of which are of sort \(B\) and the rest are of sort \(Q\).

Relations \(\text{IOb}(m)\) and \(\text{Ph}(p,b)\) are translated as “\(m\) is an inertial observer,” and “\(p\) is a light signal emitted by body \(b\),” respectively.

To speak about coordinatization, we translate \(W(m,b,x_1,x_2,\ldots,x_d)\) as “body \(k\) coordinatizes body \(b\) at space-time location \(\langle x_1,x_2,\ldots,x_d \rangle\),” (i.e., at space location \(\langle x_2,\ldots,x_d \rangle\) and instant \(x_1\)).

Quantity terms are the variables of sort \(Q\) and what can be built from them by using the two-place operations \(+\) and \(\cdot\), body terms are only the variables of sort \(B\). \(\text{IOb}(m), \text{Ph}(p,b), W(m,b,x_1,\ldots,x_d), x = y, \text{ and } x \leq y \) where \(m, p, b, x, y, x_1, \ldots, x_d\) are arbitrary terms of the respective sorts are so-called atomic formulas of our first-order logic language. The formulas are built up from these atomic formulas by using the logical connectives \(\neg\) (not), \(\land\) (and), \(\lor\) (or), \(\implies\) (implies), \(\iff\) (if-and-only-if) and the quantifiers \(\exists\) (exists) and \(\forall\) (for all).

We use the notation \(Q^n\) for the set of all \(n\)-tuples of elements of \(Q\). If \(\vec{x} \in Q^n\), we assume that \(\vec{x} = \langle x_1, \ldots, x_n \rangle\), i.e., \(x_i\) denotes the \(i\)-th component of the \(n\)-tuple \(\vec{x}\). Specially, we write \(W(m,b,\vec{x})\) in place of \(W(m,b,x_1,\ldots,x_d)\), and we write \(\forall \vec{x}\) in place of \(\forall x_1 \ldots \forall x_d\), etc.

We use first-order logic set theory as a meta theory to speak about model theoretical terms, such as models, validity, etc. The models of this language are of the form

\[
\mathcal{M} = \langle B, Q; \text{IOb}_\mathcal{M}, \text{Ph}_\mathcal{M}, +_\mathcal{M}, \cdot_\mathcal{M}, \leq_\mathcal{M}, W_\mathcal{M} \rangle,
\]

where \(B\) and \(Q\) are nonempty sets, \(\text{IOb}_\mathcal{M}\) is a unary relation on \(B\), \(\text{Ph}_\mathcal{M}\) is a binary relation on \(B\), \(+_\mathcal{M}\) and \(\cdot_\mathcal{M}\) are binary operations and \(\leq_\mathcal{M}\) is a binary relation on \(Q\), and \(W_\mathcal{M}\) is a relation on \(B \times B \times Q^d\).

\(^2\)That our theory is two-sorted means only that there are two types of basic objects (bodies and quantities) as opposed to, e.g., Zermelo–Fraenkel set theory where there is only one type of basic objects (sets).
Formulas are interpreted in $\mathcal{M}$ in the usual way. For precise definition of the syntax and semantics of first-order logic, see, e.g., [8, §1.3], [11, §2.1, §2.2].

We denote that formula $\varphi$ is valid in model $\mathcal{M}$ by $\mathcal{M} \models \varphi$. A set of formulas $\Sigma$ logically implies formula $\varphi$, in symbols $\Sigma \models \varphi$, iff (if and only if) $\varphi$ is valid in every model of $\Sigma$.

3. Axioms for special relativity

In this section, we formulate Einsteins original axioms in our first-order logic language above. Einstein has assumed two postulates in his famous 1905 paper [10]. The first was the principle of relativity, which goes back to Galileo, see, e.g., [13] or [25, pp.176-178], and it roughly states that inertial observers are indistinguishable by physical experiments, see, e.g., Friedman [12, §5].

Principle of relativity is strongly depend on the language in which it is formalized, see Remark 3.1. So to introduce a general version of the principle of relativity (and not just a kinematic one), let $\mathcal{L}$ be a many-sorted first-order logic language (of spacetime theory) containing at least sorts $B$ and $Q$ of our language of Section 2 and a unary relation $\text{Ob}$ on sort $B$. In this paper, $\mathcal{L}$ will be the language of Section 2 except in the introduction of $\text{SPR}_F$ below and in Proposition 4.1 way below.

Let $\mathcal{F}$ be a set of formulas of $\mathcal{L}$ with at most one free variable of sort $B$. This set of formulas $\mathcal{F}$ will play the role of “laws of physics” in the formulation of the principle of relativity theory. The free variable of sort $B$ is used to evaluate these formulas on observers and to check whether they are valid or not according to the observer in question. We will call $\mathcal{F}$ set of (potential) laws. Now we can formulate a principle of relativity for each set of laws $\mathcal{F}$ as the following axiom schema:

$\text{SPR}_\mathcal{F}$: A potential law of nature $\varphi \in \mathcal{F}$ is either true for all the inertial observers or false for all of them:

$$\{ \text{Ob}(m) \land \text{Ob}(k) \rightarrow [\varphi(m, \bar{x}) \leftrightarrow \varphi(k, \bar{x})] : \varphi \in \mathcal{F} \}.$$

Now for all set of laws $\mathcal{F}$ we have a principle of relativity. Let us highlight two important cases.

We call strong principle of relativity, and denote by $\text{SPR}^+$, the one when $\mathcal{F}$ is the set of all formulas of $\mathcal{L}$ with at most one free variable of sort $B$, see also [15, p.84]. $\text{SPR}^+$ is implied by existence of automorphisms of the model between any two inertial observers, see Proposition 4.1 and Theorem 2.8.20 in [15]. If $\mathcal{F} \subseteq \mathcal{G}$, then $\text{SPR}_\mathcal{G}$ is stronger than $\text{SPR}_\mathcal{F}$, i.e., $\text{SPR}_\mathcal{G} \models \text{SPR}_\mathcal{F}$. Specially, $\text{SPR}^+$ is stronger than any other $\text{SPR}_\mathcal{F}$.

We call existential principle of relativity, and denote by $\text{SPR}_\exists$, the one when $\mathcal{F}$ is the set of existential formulas of our language with at
most one free variable of sort \( B \). The importance of \( \text{SPR}_3 \) is that the existential formulas are in some sense the experimentally verifiable statements of a physical theory. Similarly, universal formulas correspond to the experimentally refutable statements of the theory. Analogously to \( \text{SPR}_3 \), we could introduce a universal version \( \text{SPR}_\forall \), too. However, it is straightforward to show that \( \text{SPR}_\forall \) and \( \text{SPR}_3 \) are logically equivalent by interchanging every universal formula \( \varphi(h, \bar{x}) \equiv \forall \bar{u} \psi(h, \bar{x}, \bar{u}) \) of \( \text{SPR}_\forall \) and existential formula \( \varphi^*(h, \bar{x}) \equiv \exists \bar{u} \neg \psi(h, \bar{x}, \bar{u}) \) of \( \text{SPR}_3 \).

**Remark 3.1.** The richer the language \( \mathcal{L} \) we choose to formulate the principle of relativity the stronger our axiom schema \( \text{SPR}^+ \) is. Therefore, if we add extra basic concepts (e.g., masses of bodies) to our language it becomes more difficult to prove something (e.g., the existence of FTL particles) is consistent with \( \text{SPR}^+ \).

The second postulate of Einstein’s states that “Any ray of light moves in the stationary system of co-ordinates with the determined velocity \( c \), whether the ray be emitted by a stationary or by a moving body,” see [10]. We can easily formulate this statement in our first-order logic frame. To do so, let us introduce the following two concepts. The **time difference** of coordinate points \( \bar{x}, \bar{y} \in Q^d \) is defined as:

\[
\text{time}(\bar{x}, \bar{y}) := x_1 - y_1.
\]

(3)

The **squared spatial distance** of \( \bar{x}, \bar{y} \in Q^d \) is defined as:

\[
\text{space}^2(\bar{x}, \bar{y}) := (x_2 - y_2)^2 + \ldots + (x_d - y_d)^2.
\]

(4)

**AxLight:** There is an inertial observer, according to whom, any light signal moves with the same velocity \( c \), independently of the fact that which body emitted the signal. Furthermore, it is possible to send out a light signal in any direction (existing according to the coordinate system) everywhere:

\[
\exists mc \left[ \text{IOb}(m) \land c > 0 \land \forall \bar{x} \bar{y} \left( \exists p b \left[ \text{Ph}(p, b) \land \text{W}(m, p, \bar{x}) \land \text{W}(m, p, \bar{y}) \right] \leftrightarrow \text{space}^2(\bar{x}, \bar{y}) = c^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \right] \right]
\]

Einstein assumed without postulating it explicitly that the structure of quantities is the field of real numbers. We make this postulate more general by assuming only the most important algebraic properties of real numbers for the quantities.

**AxOField:** The quantity part \( \langle Q, +, \cdot, \leq \rangle \) is an ordered field, i.e.,

\[\]
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

- \( \langle Q, +, \cdot \rangle \) is a field in the sense of abstract algebra; and
- the relation \( \leq \) is a linear ordering on \( Q \) such that
  i) \( x \leq y \rightarrow x + z \leq y + z \) and
  ii) \( 0 \leq x \land 0 \leq y \rightarrow 0 \leq xy \) holds.

Axiom \textbf{AxOField} not only makes our theory more general, but also
opens a new research area investigating which algebraic properties of
numbers are needed by different spacetime theories, see also [6]. An
importance of this research area (as well as using \textbf{AxOField} instead of
the field of real numbers) lies in the fact that we cannot experimentally
decide whether the structure of physical quantities is really isomorphic
to the field of real numbers or not.

To axiomatize special relativity based on Einstein’s original postu-
lates, we have to explicitly state one more assumption which was as-
sumed implicitly by Einstein. This assumption connects the worldviews
of different inertial observers by saying that all observers coordinatize
the same “external” reality (the same set of events). By the event
occurring for observer \( m \) at coordinate point \( \bar{x} \), we mean the set of
bodies \( m \) coordinatizes at \( \bar{x} \):

\[ \text{ev}_m(\bar{x}) := \{ b : W(m, b, \bar{x}) \}. \quad (5) \]

\textbf{AxEv}: All inertial observers coordinatize the same set of events:

\[ \text{IOb}(m) \land \text{IOb}(k) \rightarrow \exists \bar{y} \forall b [W(m, b, \bar{x}) \leftrightarrow W(k, b, \bar{y})]. \]

From now on, we will use \( \text{ev}_m(\bar{x}) = \text{ev}_k(\bar{y}) \) to abbreviate the subformula
\( \forall b [W(m, b, \bar{x}) \leftrightarrow W(k, b, \bar{y})] \) of \textbf{AxEv}.

Basically we are ready formulating Einstein’s theory special relativity
within our framework of first-order logic. Nevertheless, let us introduce
two more simplifying axioms to make life easier.

\textbf{AxSelf}: Any inertial observer is stationary relative to himself:

\[ \text{IOb}(m) \rightarrow \forall \bar{x} [W(m, m, \bar{x}) \leftrightarrow x_2 = \ldots = x_d = 0]. \]

Axiom \textbf{AxSelf} makes it easier to speak about the motion of inertial
observers since it identifies the observers with their time-axises. So
instead of always referring to the time-axises of inertial observers we
can speak about their motion directly.

Our last axiom is a symmetry axiom saying that all observers use
the same units of measurement.

\textbf{AxSymD}: Any two inertial observers agree as to the spatial dis-
tance between two events if these two events are simultaneous
for both of them; and the speed of light is 1 for all observers:

\[ \text{IOb}(m) \land \text{IOb}(k) \land x_1 = y_1 \land x_1' = y_1' \land \text{ev}_m(\bar{x}) = \text{ev}_k(\bar{x}') \]
\[ \land \text{ev}_m(\bar{y}) = \text{ev}_k(\bar{y}') \rightarrow \text{space}^2(\bar{x}, \bar{y}) = \text{space}^2(\bar{x}', \bar{y}'), \text{ and} \]
\[ \text{IOb}(m) \rightarrow \exists p b [\text{Ph}(p, b) \land W(m, p, 0, \ldots, 0) \land W(m, p, 1, 1, 0, \ldots, 0)]. \]
Axiom \textbf{AxSymD} makes life easier (it simplifies the formulation of our theorems) because we do not have to consider situations such as when one observer measures distances in meters while another observer measures them in feet.

Let us now introduce an axiom system of special relativity as the collection of the axioms above:

\[
\text{SR} := \text{SPR}^+ + \text{AxLight} + \text{AxOField} + \text{AxEv} + \text{AxSelf} + \text{AxSymD}.
\]

In our axiomatic approach, we usually use the following version of the light axiom, which follows from \text{SPR}^+ and \text{AxLight}, see Proposition 3.2:

\textbf{AxPh}: For any inertial observer, the speed of light is the same everywhere and in every direction (and it is finite). Furthermore, it is possible to send out a light signal in any direction (existing according to the coordinate system) everywhere:

\[
\text{IOb}(m) \rightarrow \exists c_m \left[ c_m > 0 \land \forall \bar{x}, \bar{y} \left( \exists pb \left[ \text{Ph}(p, b) \land W(m, p, \bar{x}) \wedge W(m, p, \bar{y}) \right] \leftrightarrow \text{space}^2(\bar{x}, \bar{y}) = c_m^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \right] \right].
\]

Let us note here that \textbf{AxPh} does not require (by itself) that the speed of light is the same for every inertial observer. It requires only that the speed of light according to a fixed inertial observer is a positive quantity which does not depend on the direction or the location.

By \textbf{AxPh}, we can define the \textbf{speed of light} according to inertial observer \( m \) as the following binary relation:

\[
\text{c}(m, v) \overset{\text{def}}{=} v > 0 \land \forall \bar{x}, \bar{y} \left( \exists pb \left[ \text{Ph}(p, b) \land W(m, p, \bar{x}) \wedge W(m, p, \bar{y}) \right] \rightarrow \text{space}^2(\bar{x}, \bar{y}) = v^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \right).
\]

By \textbf{AxPh}, there is one and only one speed \( v \) for every inertial observer \( m \) such that \( \text{c}(m, v) \) holds. From now on, we will denote this unique speed by \( c_m \).

Let us now prove that Einstein’s light axiom formulated as \textbf{AxLight} and the principle of relativity \text{SPR}_F implies \textbf{AxPh} if the set of laws \( F \) contain a certain existential formula.

\textbf{Proposition 3.2.} Let \( F \) be set of laws containing formula \( \exists pb \left[ \text{Ph}(p, b) \wedge W(h, p, \bar{x}) \wedge W(h, p, \bar{y}) \right] \). Then

\[
\text{SPR}_F + \text{AxLight} \models \text{AxPh} \land \forall m \left( \text{IOb}(m) \land \text{IOb}(k) \rightarrow c_m = c_k \right).
\]

\textbf{Proof}. By \text{SPR}_F, we get that

\[
\text{IOb}(m) \land \text{IOb}(k) \rightarrow \left( \exists pb \left[ \text{Ph}(p, b) \wedge W(m, p, \bar{x}) \wedge W(m, p, \bar{y}) \right] \leftrightarrow \exists pb \left[ \text{Ph}(p, b) \wedge W(k, p, \bar{x}) \wedge W(k, p, \bar{y}) \right] \right).
\]
The existence of FTL particles is consistent with SR by axiom \textbf{AxLight}, there are an inertial observer \( m \) and a positive quantity \( c \) such that:

\[
\exists pb [\text{Ph}(p, b) \land W(m, p, \bar{x}) \land W(m, p, \bar{y})] \\
\iff \text{space}^2(\bar{x}, \bar{y}) = c^2 \cdot \text{time}(\bar{x}, \bar{y})^2
\]

Therefore, for all inertial observer \( m \) exists a light signal moving through \( \bar{x} \) and \( \bar{y} \) if \( \text{space}^2(\bar{x}, \bar{y}) = c^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \), i.e., formula

\[
\exists c \left[ c > 0 \land \forall m \bar{x} \bar{y} \left( \text{IOb}(m) \to \exists pb [\text{Ph}(p, b) \land W(m, p, \bar{x}) \land W(m, p, \bar{y})] \iff \text{space}^2(\bar{x}, \bar{y}) = c^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \right) \right]
\]

follows from \( \text{SPR}_F \) and \textbf{AxLight}. This formula implies both axiom \textbf{AxPh} and formula \( \forall mk [\text{IOb}(m) \land \text{IOb}(k) \to c_m = c_k] \). \( \blacksquare \)

Since the sets of laws corresponding to \( \text{SPR}_+ \) and \( \text{SPR}_\exists \) contains formula \( \exists pb [\text{Ph}(p, b) \land W(m, p, \bar{x}) \land W(m, p, \bar{y})] \), Proposition 3.2 implies the following:

\textbf{Corollary 3.3.}

\[
\begin{align*}
\text{SPR}_+ + \textbf{AxLight} & \models \text{AxPh} \land \forall mk [\text{IOb}(m) \land \text{IOb}(k) \to c_m = c_k], \\
\text{SPR}_\exists + \textbf{AxLight} & \models \text{AxPh} \land \forall mk [\text{IOb}(m) \land \text{IOb}(k) \to c_m = c_k].
\end{align*}
\]

It is easy to see there are great many models of \textbf{SpecRel} which are not models of \textbf{SR}. Therefore, our theory \textbf{SpecRel} is more general than \textbf{SR} by Corollary 3.3, i.e., the following is true:

\textbf{Corollary 3.4.}

\[
\text{SR} \models \text{SpecRel} \quad \text{and} \quad \text{SpecRel} \not\models \text{SR}.
\]

In relativity theory, we are often interested in comparing the world-views of two different observers. To characterize the possible relations between the worldviews of inertial observers, let us introduce the \textbf{worldview transformation} between observers \( m \) and \( k \) (in symbols, \( w_{mk} \)) as the binary relation on \( Q^d \) connecting the coordinate points where \( m \) and \( k \) coordinate the same events:

\[
w_{mk}(\bar{x}, \bar{y}) \iff \forall b [W(m, b, \bar{x}) \leftrightarrow W(k, b, \bar{y})]. \quad (6)
\]

Map \( P : Q^d \to Q^d \) is called a \textbf{Poincaré transformation} iff it is an affine bijection having the following property

\[
\text{time}(\bar{x}, \bar{y})^2 - \text{space}^2(\bar{x}, \bar{y}) = \text{time}(\bar{x}', \bar{y}')^2 - \text{space}^2(\bar{x}', \bar{y}') \quad (7)
\]

for all \( \bar{x}, \bar{y}, \bar{x}', \bar{y}' \in Q^d \) for which \( P(\bar{x}) = \bar{x}' \) and \( P(\bar{y}) = \bar{y}' \).

Theorem 3.5 shows that even our (more general) axiom system \textbf{SpecRel} perfectly captures the kinematics of special relativity since it implies that the worldview transformations between inertial observers are the same as in the standard non-axiomatic approaches.
**Theorem 3.5.** Let \( d \geq 3 \). Assume SpecRel. Then \( w_{mk} \) is a Poincaré transformation if \( m \) and \( k \) are inertial observers.

For the proof of Theorem 3.5, see [6]. For versions of Theorem 3.5 using a similar but different axioms systems of special relativity, see, e.g., [1], [2], [3].

**Corollary 3.6.** Let \( d \geq 3 \). Assume SpecRel. Then \( w_{mk} \) is a Poincaré transformation if \( m \) and \( k \) are inertial observers.

The **worldline** of body \( b \) according to observer \( m \) is defined as:

\[
wl_m(b) := \{ \bar{x} : W(m, b, \bar{x}) \}.
\]

**Corollary 3.7.** Let \( d \geq 3 \). Assume SpecRel or SR. The \( wl_m(k) \) is a straight line if \( m \) and \( k \) are inertial observers.

### 4. Independence of FTL bodies of SR

Before we show that the existence of FTL objects (bodies) is independent of SR, let us first prove a useful connection between SPR\(^+\) and the automorphism group of the models of our language.

**Proposition 4.1.** Let \( L \) be a language on which SPR\(^+\) is formalized. Let \( M \) be a model of our language \( L \). If for all inertial observers \( m \) and \( k \), there is an automorphism of \( M \) fixing the quantities and taking \( m \) to \( k \), then SPR\(^+\) is valid in \( M \).

**Proof.** Let \( \varphi(h, \bar{x}) \) be an arbitrary formula with only free variables \( h \) of sort \( B \) and \( \bar{x} \) of sort \( Q \). To prove SPR\(^+\), we have to prove that

\[
M \models \forall mk \bar{x} \left( \text{IOb}(m) \land \text{IOb}(k) \rightarrow [\varphi(m, \bar{x}) \leftrightarrow \varphi(k, \bar{x})] \right). \tag{9}
\]

Let \( m \) and \( k \) be inertial observers and let \( \text{Aut}_{mk} \) be the automorphism taking \( m \) to \( k \) and fixing the quantities. This automorphism exists by our assumption. Let \( \bar{x} \) be such that \( M \models \varphi(m, \bar{x}) \). Then \( M \models \varphi(\text{Aut}_{mk}(m), \text{Aut}_{mk}(\bar{x})) \) since \( \text{Aut}_{mk} \) is an automorphism of \( M \). Since \( \text{Aut}_{mk}(m) = k \) and \( \text{Aut}_{mk}(\bar{x}) = \bar{x} \), we have that \( M \models \varphi(k, \bar{x}) \).

Therefore, the following half of (9) holds

\[
M \models \forall mk \bar{x} \left( \text{IOb}(m) \land \text{IOb}(k) \rightarrow [\varphi(m, \bar{x}) \rightarrow \varphi(k, \bar{x})] \right) \tag{10}
\]

since \( m \) and \( k \) were arbitrary inertial observers and \( \bar{x} \) was an arbitrary sequence of quantities. By interchanging \( m \) and \( k \), we get the converse direction. So \( M \models \text{SPR}^+ \); and this is what we wanted to prove.

For model theoretic characterizations of SPR\(^+\) based on existence of automorphism connecting the worldviews of inertial observers, see Theorem 2.8.20 in [15].

Now let us prove that the existence of FTL objects (bodies) is independent of special relativistic kinematics. To do so, let us introduce a formula of our language stating that there is a superluminal body.
\( \exists \text{FTLBody} \): There is an inertial observer according to who a body moving faster than the speed of light:

\[ \exists \bar{m} \bar{x} \bar{y} [\text{Ob}(\bar{m}) \land W(\bar{m}, \bar{x}) \land W(\bar{m}, \bar{y}) \land \text{space}^2(\bar{x}, \bar{y}) > c_m^2 \cdot \text{time}(\bar{x}, \bar{y})^2]. \]

Since we will prove that something does not follow from certain axioms, the more axioms we assume the stronger our theorem will be. Therefore, to get a stronger result, let us introduce an axiom ensuring the existence of several inertial observers.

\( \text{AxThExp} \): Inertial observers can move along any straight line of any speed less than the speed of light:

\[ \exists h \text{Ob}(h) \land \forall m \bar{x} \bar{y} (\text{Ob}(m) \land \text{space}^2(\bar{x}, \bar{y}) < c_m^2 \cdot \text{time}(\bar{x}, \bar{y})^2 \rightarrow \exists k [\text{Ob}(k) \land W(m, h, \bar{x}) \land W(m, h, \bar{y})]). \]

**Theorem 4.2.**

\( \text{SR} + \text{AxThExp} \not\models \neg \exists \text{FTLBody} \) and \( \text{SR} + \text{AxThExp} \not\models \exists \text{FTLBody} \)

In the proof of Theorem 4.2 we will use the following concepts. The identity map is defined as:

\[ \text{Id}(\bar{x}) := \bar{x} \text{ for all } \bar{x} \in Q^d. \] (11)

The time-axis is defined as the following subset of the coordinate system \( Q^d \):

\[ \text{t-axis} := \{ \bar{x} \in Q^d : x_2 = \ldots = x_d = 0 \}. \] (12)

We think of functions as special binary relations. Hence we compose them as relations. The composition of binary relations \( R \) and \( S \) is defined as:

\[ R \circ S := \{ (a, c) : \exists b [R(a, b) \land S(b, c)] \}. \] (13)

Therefore,

\[ (g \circ f)(x) = f(g(x)) \] (14)

if \( f \) and \( g \) are functions. Let \( H \) be a subset of \( Q^d \) and let \( f : Q^d \rightarrow Q^d \) be a map. The f-image of set \( H \) is defined as:

\[ f[H] := \{ f(\bar{x}) : \bar{x} \in H \}. \] (15)

The inverse binary relation of \( R \) is defined as:

\[ R^{-1} := \{ (a, b) : R(b, a) \}. \] (16)

**Proof.** We are going to prove our statement by constructing two models \( \mathfrak{M}_1 \) and \( \mathfrak{M}_2 \) of \( \text{SR} \) such that in \( \mathfrak{M}_1 \) there are FTL bodies and in \( \mathfrak{M}_2 \) there are no FTL bodies. There will be only a slight difference in the two constructions. Therefore, we are going to construct the two models simultaneously.
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

Let $\langle Q, +, \cdot, \leq \rangle$ be the ordered field of real numbers. Let us introduce the following sets, which will be unary relations on set $B$ after we will define $B$:

$$
\text{IOb} := \{ \text{Poincaré transformations of } Q^d \}, \quad \text{Ph} := \{ \text{lines of slope 1 in } Q^d \}, \quad \text{and } \text{IB} := \{ \text{lines in } Q^d \}.
$$

Let binary relation $\text{Ph}$ be defined as:

$$
\text{Ph}(p, b) \iff \text{Ph}(p) \land b = p.
$$

The only difference between the construction of models $\mathcal{M}_1$ and $\mathcal{M}_2$ is in the definition of the set of bodies. In models $\mathcal{M}_1$ and $\mathcal{M}_2$, the respective sets of bodies are defined as:

$$
B_1 = \text{IOb} \cup \text{Ph} \cup \text{IB} \quad \text{and} \quad B_2 = \text{IOb} \cup \text{Ph}.
$$

First we are going to give the worldview of observer $\text{ld}$.

$$
W(\text{ld}, \text{ld}, \bar{x}) \iff x_2 = \ldots = x_d = 0,
$$

for any other inertial observer $m$

$$
W(\text{ld}, m, \bar{x}) \iff \bar{x} \in m[t\text{-axis}],
$$

and for any body $b$ which is not an inertial observer, i.e., $b \in B \setminus \text{IOb}$

$$
W(\text{ld}, b, \bar{x}) \iff \bar{x} \in b.
$$

Now the worldview of observer $\text{ld}$ is given. From the worldview of $\text{ld}$, we construct the worldview of another inertial observer $m$ as follows:

$$
W(m, b, \bar{x}) \iff W(\text{ld}, b, m(\bar{x})
$$

for all body $b \in B$, see Figure 1. Now models $\mathcal{M}_1$ and $\mathcal{M}_2$ are given.

By the above definition of $W$, if $m$ and $k$ are inertial observers, then

$$
W(m, k, \bar{x}) \quad \text{holds iff} \quad m(\bar{x}) \in k[t\text{-axis}],
$$

and if $m \in \text{IOb}$ and $b \in B \setminus \text{IOb}$, then

$$
W(m, b, \bar{x}) \quad \text{holds iff} \quad m(\bar{x}) \in b.
$$
The worldview transformations between inertial observers \( m \) and \( Id \), i.e., \( w_{mId} = m \) by equation (21). Therefore, the worldview transformation between inertial observers \( m \) and \( k \) is \( m \overset{+}{z} k^{-1} \), i.e.,

\[
w_{mk} = m \overset{+}{z} k^{-1}
\]  

(27)
since \( w_{mk} = w_{mId} \overset{+}{z} w_{Idk} \) and \( w_{Idk} = (w_{kId})^{-1} \) by the definition of the worldview transformation (6). Specially, the worldview transformations between inertial observers are Poincaré transformations in these models (as Theorem 3.6 requires it). Hence \( w_{mk} \) is a bijection for all inertial observers \( m \) and \( k \).

Now we are going to show that \( M_1 \) and \( M_2 \) are models of axiom system SR. It is clear that axiom AxOField is valid in these models since the ordered field of real numbers is an ordered field.

Axiom AxEv is valid in models \( M_1 \) and \( M_2 \) since worldview transformations between inertial observers are bijections by (28).

It is clear that axiom AxSelf is valid in these models by the definition of worldview relation \( W \) since

\[
W(m, m, \bar{x}) \iff m(\bar{x}) \in m[t\text{-axis}]
\]  

(17)
\[
\iff \bar{x} \in t\text{-axis} \iff x_2 = \ldots = x_d = 0.
\]  

It is clear that AxLight is valid in models \( M_1 \) and \( M_2 \) by the construction the worldview of inertial observer \( Id \). Moreover, the speed of light is 1 for observer \( Id \). Since Poincaré transformations take lines of slope 1 to lines of slope 1, the speed of light is 1 according to every inertial observer, which is the second half of AxSymD.

Any Poincaré transformation \( P \) preserves the spatial distance of points \( \bar{x}, \bar{y} \in Q^d \) for which \( x_1 = y_1 \) and \( P(\bar{x})_1 = P(\bar{y})_1 \). Therefore, inertial observers agree as to the spatial distance between two events if these two events are simultaneous for both of them. We have already shown that the speed of light is 1 according to each inertial observer in models \( M_1 \) and \( M_2 \). Consequently, axiom AxSymD is also valid in these models.

By Lemma 1.1 to prove that axiom SPR\(^+\) is valid in models \( M_1 \) and \( M_2 \), it is enough to show that, for all inertial observers \( m \) and \( k \), there is an automorphism \( Aut_{mk} \) fixing the quantities and taking \( m \) to \( k \).

Let us fix two arbitrary inertial observers \( m \) and \( k \). Let \( Aut_{mk} \) be the following map

\[
Aut_{mk}(h) := h \overset{+}{z} w_{mk} = h \overset{+}{z} m^{-1} \overset{+}{z} k
\]  

(29)
for all inertial observers \( h \).

\[
Aut_{mk}(q) := q \quad \text{and} \quad Aut_{mk}(b) := w_{mk}[b]
\]  

(30)
for all quantity \( q \) and for all body \( b \) which is not an inertial observer. Since \( w_{mk} \) is a bijection, it is clear that \( Aut_{mk} \) is also a bijection of \( M_1 \)
and $\mathfrak{M}_2$. It is clear that $\text{Aut}_{mk}(m) = k$ since $\text{Aut}_{mk}(m) = m \cdot m^{-1} = k$. It is also clear that $\text{Aut}_{mk}$ is fixing $Q$ by its definition. Now we show that $\text{Aut}_{mk}$ is an automorphism. Since $\text{Aut}_{mk}$ is fixing the quantities, $x \leq y$, etc. hold iff $\text{Aut}_{mk}(x) \leq \text{Aut}_{mk}(y)$, etc. hold.

Let $h$ be a body. We have to show that $\text{IOb}(h)$ holds iff $\text{IOb}(\text{Aut}_{mk}(h))$ holds. This is so because $\text{Aut}_{mk}(h) = h \cdot m^{-1} = k$ is a Poincaré transformation iff $h$ is a Poincaré transformation.

Let $p$ and $b$ be bodies. We have to show that $\text{Ph}(p, b)$ holds iff $\text{Ph}(\text{Aut}_{mk}(p), \text{Aut}_{mk}(b))$ holds. Relation $\text{Ph}(p, b)$ holds iff $\text{Ph}(p)$ and $p = b$ hold. Since transformation $w_{mk}$ is a bijection taking lines of slope 1 to lines of slope 1 (since it is a Poincaré transformation), $\text{Ph}(p)$ holds iff $\text{Ph}(\text{Aut}_{mk}(p)) = \text{Ph}(w_{mk}[p])$ holds. Therefore, relation $\text{Ph}(p, b)$ holds iff $\text{Ph}(\text{Aut}_{mk}(p), \text{Aut}_{mk}(b))$ holds since $p = b$ holds iff $w_{mk}[p] = w_{mk}[b]$ holds.

Let $h$ be an inertial observer, $b$ be a body and $\vec{x}$ be a coordinate point. We have to show that relation $W(h, b, \vec{x})$ holds iff relation $W(\text{Aut}_{mk}(h), \text{Aut}_{mk}(b), \text{Aut}_{mk}(\vec{x}))$ holds. There are two cases: $b \in \text{IOb}$, and $b \in B \setminus \text{IOb}$. If $b \in \text{IOb}$, then

\[
W(\text{Aut}_{mk}(h), \text{Aut}_{mk}(b), \text{Aut}_{mk}(\vec{x})) \iff W(h \cdot w_{mk}, b \cdot w_{mk}, \vec{x}) \iff (h \cdot w_{mk})(\vec{x}) \in (b \cdot w_{mk})[\text{t-axis}] \iff W_{mk}(h(\vec{x})) \in w_{mk}[b[\text{t-axis}]] \iff h(\vec{x}) \in b[\text{t-axis}] \iff W(h, b, \vec{x}).
\]

If $b \in B \setminus \text{IOb}$, then

\[
W(\text{Aut}_{mk}(h), \text{Aut}_{mk}(b), \text{Aut}_{mk}(\vec{x})) \iff W(h \cdot w_{mk}, w_{mk}[b], \vec{x}) \iff (h \cdot w_{mk})(\vec{x}) \in w_{mk}[b] \iff W_{mk}(h(\vec{x})) \in w_{mk}[b] \iff h(\vec{x}) \in b \iff W(h, b, \vec{x}).
\]

Therefore, by Lemma 11 axiom $\text{SPR}^+$ is valid in the models.

The $\exists h \text{IOb}(h)$ part of axiom $\text{AxThExp}$ is valid in models $\mathfrak{M}_1$ and $\mathfrak{M}_2$, since there are Poincaré transformations (e.g., $\text{Id}$ is one). Since $\text{SPR}^+$ is valid in $\mathfrak{M}_1$ and $\mathfrak{M}_2$, it is enough to prove the second part of $\text{AxThExp}$ only for observer $\text{Id}$ instead of for all inertial observer $m$. To do so, let $\vec{x}$ and $\vec{y}$ be coordinate points such that $\text{space}^2(\vec{x}, \vec{y}) < \text{time}(\vec{x}, \vec{y})^2$. It is easy to see that there is a Poincaré transformation $k$ for which $\vec{x}, \vec{y} \in k[\text{t-axis}]$. Therefore, by the definition of worldview transformation and by (22), $W(\text{Id}, k, \vec{x})$ and $W(\text{Id}, k, \vec{y})$ hold for inertial observer $k$ as $\text{AxThExp}$ requires it.

It is clear by the constructions of the models that in $\mathfrak{M}_1$, there are FTL bodies and in $\mathfrak{M}_2$ there is no FTL body. So neither $\exists \text{FTLBody}$ nor $\neg \exists \text{FTLBody}$ follows from $\text{SR}$, and this is what we wanted to prove.■
By Theorem 1.2, the existence of FTL bodies is independent of the theory of special relativity. Specially, it is consistent with axiom system SR that there are FTL objects (bodies). Because SR is based on the formulation of Einstein’s original postulates of special relativity, we have formally proved that the kinematics of Einstein’s special theory of relativity is consistent with the existence of FTL particles.

That $\exists \text{FTLBody}$ is logically independent of SR is completely analogous to that the axiom of parallels is independent of the rest of the axioms of Euclidean geometry or to that the continuum hypothesis is independent of Zermelo-Fraenkel set theory. It means that we can (consistently) extend theory SR both with assumption $\exists \text{FTLBody}$ or $\neg \exists \text{FTLBody}$. However, until we have a good reason to assume either of them, it is better to work without these extra assumptions.

5. Sending information back to the past

One of the nontrivial consequences of the possibility of sending out FTL particles is that, in some sense, we can send signals back to the past. More precisely, when two inertial observers are moving relative to each other, then both observers can send out a superluminal signal to the other such that, according to the other, the event of sending out of the signal happened later than the event of receiving of the signal. To see this fact, let us consider two observers moving relative to each other with speed $0.5c$. Let us call one of these observers “stationary observer” and the other “moving observer.” The worldview of the “moving observer” is drawn into the worldview of the “stationary observer” in Figure 2. Let the “moving observer” send out a signal with speed $3c$ at event A. It can be read from the figure that this signal reach the “stationary observer” at event B which is earlier than event A according to the “stationary observer.”

If we can send bodies (particles) back to the past, then we can also send any information back bit by bit. This can easily be done by sending or not sending a signal back in time at regular intervals; and the lack of signal means 0 and the presence of the signal means 1 in the given bit. Using this protocol, we can send arbitrary long information back to the past.

The possibility of sending back signals or even particles to the past foreshadows the possibility of deriving a logical contradiction. In the next section, we are going to show why this is not necessarily so even if we leave the language of the kinematics.

6. Why does not the possibility of sending particles back to the past lead to a logical contradiction?

To explain why the existence of FTL objects does not necessarily lead to a logical contradiction, let us consider the following thought experiment. Let a neutrino cannon fire a superluminal projectile which is
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

Figure 2. Sending a superluminal signal back to the past: a signal of speed $3c$ which is sent out by the “moving observer” (moving with speed $0.5c$) at event A reaches the “stationary observer” at event B which is earlier according to the “stationary observer.”

reflected back from a moving mirror such that it hits the cannon right before it has shot the projectile out, see Figure 3. The argument usually continues as follows. If the cannon shoots the projectile at event A, the projectile destroys the cannon at event C before event A. Therefore, the cannon cannot shoot the projectile at event A. This argument seemingly contradicts the plausible assumption that our cannon can shoot its FTL projectile anytime at any direction. So at first sight we have derived a logical contradiction from the existence of FTL particles.

However, if we think it over more carefully, we see that when we calculated (speculated) the trajectory of the neutrino beam we did not take into consideration of the causal effect of the projectile coming back from the “future.” If we take this casual effect into consideration, we can tell a logically consistent story in which the neutrino cannon hits itself before the ignition of the projectile. For example, we send out the mirror; and before we shoot the cannon to destroy itself, a neutrino projectile hits the cannon (at event C) and because of this damage the cannon shoots a neutrino projectile (at event A) which reflects back from the mirror (at event B) so that it becomes the one that hit the cannon (at event C).

This story is a perfectly consistent story of the

---

4According to the “mirror” event A is also later than event B. So it would be better to tell the story with another cannon in place of the mirror which shoots a projectile with speed $5c$ in the direction of the first cannon when it detects the projectile of the first cannon at event B.
self shooting cannon and a possible candidate of the resolution of this "cannon paradox."

These kinds of violation of causality do not lead to a logical contradiction by themselves, the same way as similar violations of causality, namely CTCs, do not lead to logical contradiction in general relativity. For a detailed resolution of similar causal paradoxes, see, e.g., Lossev–Novikov [14], Recami [20], Selleri [23]. Typically, these kinds of possibilities of causality violation only mean that we have to be more careful when reasoning about causality because we have to take into account the causal effects coming from the “future.”

It is important to note that the “cannon paradox” story above was told only in a framework of kinematics, in which we cannot calculate (just speculate) the effects of the collisions. Within the kinematics of special relativity the existence of FTL objects are perfectly consistent with the theory, see Theorem 4.2 at p.16. However, kinematics does not tell us anything about the interactions of particles. Therefore, within kinematics we can only say that the projectile meets the cannon at event C, but we cannot say anything about the “damage” the projectile makes to the cannon.

Therefore, to fully investigate the “cannon paradox” above, we have to put it into a richer logical framework in which we can speak about the interactions, too. It is a future task to investigate this “cannon paradox” within an axiomatic theory of relativistic dynamics of FTL particles. Our axiomatic framework of relativistic dynamics, see [4], [17], [24 §5], is suitable for investigating FTL dynamics and it is a possible candidate for an initial framework for this investigation.
7. Anyway, inertial observers cannot move FTL

We have seen that the kinematics of special relativity says nothing about the existence of FTL particles in general. It is important to note that even $\text{SpecRel}$ (which is more general than $\text{SR}$ by Proposition 3.2) implies that no inertial observer can move faster than or with the speed of light, see [5, Thm.2.1]. For similar results on the impossibility of existence of superluminal observers, see, e.g., [1], [2], [3], [16].

At first it may sound plausible to assume that, if inertial particles can move with a certain speed, then inertial observers can also move with this speed. This assumption together with $\text{SpecRel}$ (or $\text{SR}$) would imply that there are no superluminal particles since $\text{SpecRel}$ (and hence $\text{SR}$) implies that there are no FTL inertial observers. The problem with this “natural” assumption is that (together with $\text{SpecRel}$ or $\text{SR}$) it also implies that there are no particles moving with the speed of light which directly contradict $\text{AxLight}$. So surprisingly this assumption is not plausible at all since it contradicts special relativity.

8. Concluding remarks

We have proved that the existence of superluminal objects is independent of (hence consistent with) the kinematics of special relativity, see Theorem 4.2. A future task is to investigate how far this independence result can be extended beyond kinematics. It is important to continue this investigation within an axiomatic framework of logic stating our axioms explicitly. Otherwise, there is a danger that some of our tacit assumptions will remain hidden and we will not see clearly the logical connection between our theory and the possibility of the existence of FTL particles.

9. Acknowledgment

I am grateful to Hajnal Andréka, Judit X. Madarász, and István Németi for the interesting discussion on the subject and their valuable comments and suggestions. This research is supported by the Hungarian Scientific Research Fund for basic research grants No. T81188 and No. PD84093.

References

[1] H. Andréka, J. X. Madarász, and I. Németi, with contributions from: A. Andai, G. Sági, I. Sain, and Cs. Tőke. *On the logical structure of relativity theories*. Research report, Alfréd Rényi Institute of Mathematics, Hungar. Acad. Sci., Budapest, 2002. http://www.math-inst.hu/pub/algebraic-logic/Contents.html.

[2] H. Andréka, J. X. Madarász, and I. Németi. Logical axiomatizations of spacetime. Samples from the literature. In A. Prékopa and E. Molnár, editors, *Non-Euclidean geometries*, pages 155–185. Springer-Verlag, New York, 2006.
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

[3] H. Andréka, J. X. Madarász, and I. Németi. Logic of space-time and relativity theory. In M. Aiello, I. Pratt-Hartmann, and J. van Benthem, editors, Handbook of spatial logics, pages 697–711. Springer-Verlag, Dordrecht, 2007.

[4] H. Andréka, J. X. Madarász, I. Németi, and G. Székely. Axiomatizing relativistic dynamics without conservation postulates. Studia Logica, 89(2):163–186, 2008.

[5] H. Andréka, J. X. Madarász, I. Németi, and G. Székely. A logic road from special relativity to general relativity. Synthese, pages Online–first: 1–17, 2011.

[6] H. Andréka, J. X. Madarász, I. Németi, and G. Székely. What are the numbers in which spacetime?, 2012. In preparation.

[7] J. Ax. The elementary foundations of spacetime. Found. Phys., 8(7-8):507–546, 1978.

[8] C. C. Chang and H. J. Keisler. Model theory. North-Holland Publishing Co., Amsterdam, 1990.

[9] OPERA collaboration. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam, 2011. arXiv:1109.4897v1 [hep-ex].

[10] A. Einstein. Zur Elektrodynamik bewegter Körper. Annalen der Physik, 17:891–921, 1905.

[11] H. B. Enderton. A mathematical introduction to logic. Academic Press, New York, 1972.

[12] M. Friedman. Foundations of Space-Time Theories. Relativistic Physics and Philosophy of Science. Princeton University Press, Princeton, 1983.

[13] G. Galilei. Dialogues Concerning Two New Sciences. Macmillan, New York, 1914, First published in 1638. Translated from the Italian and Latin into English by Henry Crew and Alfonso de Salvio. http://ebooks.adelaide.edu.au/g/galileo/dialogues/complete.html.

[14] A. Lossev and I. D. Novikov. The jinn of the time machine: nontrivial self-consistent solutions. Classical and Quantum Gravity, 9(10):2309, 1992.

[15] J. X. Madarász. Logic and Relativity (in the light of definability theory). PhD thesis, Eötvös Loránd Univ., Budapest, 2002.

[16] J. X. Madarász, I. Németi, and Cs. Tőke. On generalizing the logic-approach to space-time towards general relativity: first steps. In V. F. Hendricks et al., editors, First-Order Logic Revisited, pages 225–268. Logos Verlag, Berlin, 2004.

[17] J. X. Madarász and G. Székely. Comparing relativistic and Newtonian dynamics in first order logic. In Máté A., Rédei M., and F. Stadler, editors, The Vienna Circle in Hungary, pages 155–179, Wien, 2011. Springer-Verlag.

[18] T. Matolcsi and W. A. Rodrigues, Jr. The geometry of space-time with superluminal phenomena. Algebras Groups Geom., 14(1):1–16, 1997.

[19] P. Mittelstaedt. What if there are superluminal signals? The European Physical Journal B - Condensed Matter and Complex Systems, 13:353–355, 2000.

[20] E. Recami. Tachyon kinematics and causality: a systematic thorough analysis of the tachyon causal paradoxes. Found. Phys., 17(3):239–296, 1987.

[21] E. Recami. Superluminal motions? A bird’s-eye view of the experimental situation. Foundations of Physics, 31:1119–1135, 2001.

[22] E. Recami, F. Fontana, and R. Garavaglia. Special relativity and superluminal motions: a discussion of some recent experiments. Internat. J. Modern Phys. A, 15(18):2793–2812, 2000.

[23] F. Selleri. Superluminal signals and the resolution of the causal paradox. Foundations of Physics, 36:443–463, 2006.

[24] G. Székely. First-Order Logic Investigation of Relativity Theory with an Emphasis on Accelerated Observers. PhD thesis, Eötvös Loránd Univ., Budapest, 2009.
THE EXISTENCE OF FTL PARTICLES IS CONSISTENT WITH SR

[25] E. F. Taylor and J. A. Wheeler. *Spacetime Physics*. W. H. Freeman and Company, New York, 1997.

[26] R. C. Tolman. *The Theory of the Relativity of Motion*. University of California, Berkely, 1917.

[27] J. Viäänänen. Second-order logic and foundations of mathematics. *Bull. Symbolic Logic*, 7(4):504–520, 2001.

[28] S. Weinstein. Superluminal signaling and relativity. *Synthese*, 148:381–399, 2006.

[29] J. Woćński. First-order logic: (philosophical) pro and contra. In V. F. Hendricks et al., editors, *First-Order Logic Revisited*, pages 369–398. Logos Verlag, Berlin, 2004.

Alfréd Rényi Institute of Mathematics, of the Hungarian Academy of Sciences, Budapest P.O.Box 127, H-1364, Hungary

E-mail address: szekely.gergely@renyi.mta.hu.