SENSITIVITY OF LOW-ENERGY PARITY-VIOLATION TO NEW PHYSICS

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I review the new physics sensitivity of low-energy parity-violating (PV) observables. I concentrate on signatures of new tree-level physics in atomic PV with a single isotope, ratios of atomic PV observables, and PV electron scattering. In addition to comparing the new physics sensitivities of these observables with those of high-energy colliders, I also discuss the theoretical issues involved in the extraction of new physics limits from low-energy PV observables.

1 Introduction

From its discovery in the $\beta$-decay of $^{60}\text{Co}$ by Wu et al. in 1957, parity-violation (PV) in nuclear and atomic processes has played a central role in elucidating the structure of the electroweak interaction. By now, our gauge theory of that interaction – the SU(2)$_L \times$ U(1)$_Y$ Standard Model – has been tested in a wide variety of processes, ranging in energies from the eV scale to the 100 GeV scale. The agreement between experiment and the predictions of the Standard Model (SM) is impressive. In nearly all cases, there is accord at the 0.1% level or better. A striking illustration is the discovery of the top quark, whose measured mass falls within a rather narrow range predicted from global analysis of electroweak observables at the level of one-loop SM radiative corrections.\textsuperscript{1} There are, however, a few exceptions to this pattern of agreement, such as the value of the Cabbibo-Kobayashi-Maskawa matrix element $|V_{ud}|$ determined from nuclear $\beta$-decay. Analyses of superallowed $\beta$-decays imply a value for $|V_{ud}|$ differing from the SM unitarity requirement by nearly two standard deviations.\textsuperscript{2} Whether this discrepancy is due to a deficiency in the SM or an unknown systematic in $\beta$-decay analyses remains an open question. Despite this – and a few other – apparent disagreements, the SM works incredibly well
for most of what is observed experimentally.

These days, experiments at high-energy colliders (LEP, Tevatron, HERA) are engaged in searches for physics “beyond” the SM. The motivation for seeking such “new” physics is that the Standard Model is just that – a model. As well as it works in describing and predicting electroweak processes, it also leaves several questions unanswered. It requires as input, for example, 17 independent, experimentally-determined parameters, but doesn’t tell us why these parameters take on their measured values. Why, for example, is $m_e << m_\mu << m_\tau$ (hierarchy problem)? Similarly, the SM assumes elementary fermions and bosons to have no size or structure, but does not tell us why this assumption must be true. It incorporates PV as observed, but does not explain why parity is violated. When the electroweak and strong sectors of the SM are taken together, the couplings associated with each interaction do not meet at a common point when run to high scales $\mu$. This absence of unification is theoretically unsatisfying at best. Such questions and “loose ends” suggest that the SM is really a low-energy ($<<$ weak scale) effective theory of some more general framework – one which contains, in principle, physics we have not searched for with sufficient intention. Hence, there exists considerable interest in searching for this new physics.

What I hope to show in this chapter is how PV on a table top (atoms) and in low-energy colliders (electron scattering) is a powerful probe of possible new physics – and one which generally complements high-energy collider searches. The reason for PV’s continued relevance is the high precision with which measurements can be performed. In this respect, the benchmark experiment is the one in atomic parity-violation (APV) performed on cesium by the Boulder group. The experimental error in that measurement is an impressive 0.35%. Unfortunately, the interpretation of the cesium result requires input from atomic theory, for which the present uncertainty is $\sim 1.2\%$. Future progress in using APV to probe for new physics will require significant reduction in that uncertainty. Alternatively, several groups are pursuing measurements of ratios of APV observables for different atoms along the isotope chain. The atomic theory-dependence of these “isotope ratios”, $\mathcal{R}$, largely cancels out, leaving an atomic theory-free probe of new physics. The isotope ratios, however, display a significant dependence on the neutron number density, $\rho_n(r)$, of atomic nuclei. The neutron densities are not sufficiently well-determined experimentally, so one must rely on nuclear theory to compute them. Whether or not the nuclear theory uncertainty is sufficiently small to interpret $\mathcal{R}$ in terms of new physics remains an open question. As I will illustrate toward the end of the chapter, PV $e-e$ and $e$-hadron scattering offers the theoretically cleanest probe of new physics. It remains to be see whether $A_{LR}$, the “left-right” asym-
metry in PV electron scattering (PVES), can be measured with the precision needed to make it competitive with APV. The recent successes of the Jefferson Laboratory PV experiment suggests that the possibilities are promising.

In the remainder of the chapter, my discussion of these points is organized as follows. In Section II, I introduce some formal definitions and conventions needed for the subsequent discussion. In Section III, I consider a variety of low-energy PV observables and compare their generic sensitivities to new physics. In Section IV, I illustrate these sensitivities with different model scenarios for physics beyond the Standard Model. Section V contains a discussion of theoretical uncertainties which arise in the interpretation of the PV observables. Section VI summarizes my principal conclusions. Much of what I discuss here is also treated more fully in Refs. [5], [6], and I refer the reader to those papers for additional details.

2 Formalism

For purposes of probing new physics with PV, the quantity of interest is the weak charge, $Q_W$. It is the weak neutral current analog of the electromagnetic charge, $Q_{EM}$. The weak charge appears in the amplitude associated with Fig. 1a, where an electron interacts with another fermion by exchanging a $Z^0$ boson. If $V(f)$ and $A(f)$ denote the weak neutral vector and axial vector currents, respectively, of an elementary fermion $f$, then the PV part of the amplitude in Fig. 1a depends on either $A(e) \times V(f)$ or $V(e) \times A(f)$. The former is characterized by $Q_{Wf}^f$, the coupling of $V(f)$ to the $Z^0$ boson. When an electron interacts weakly with a system composed of several elementary fermions, such as an atomic nucleus, the weak charge of that system, $Q_{W}$, is just the sum of the

![Diagram](image-url)
of its constituents. In this respect, $Q_w$ behaves just like $Q_{em}$. However, because electroweak symmetry is broken below the weak scale ($\lesssim 250$ GeV), leaving only electromagnetic gauge invariance as a good symmetry, $Q_w$ is not conserved. Its value can be predicted by the SM, but that value may be altered by the presence of new physics. The EM charge, on the other hand, is protected from such new physics modifications by EM gauge invariance.

The new physics modification of $Q_w$ can arise in basically two ways: by physics which modifies the propagation of the $Z^0$ from the electron to the other fermion, and by the exchange of possible new, heavy particles between $e$ and $f$. Modifications of the former type are called “oblique”, whereas the latter induce so-called “direct” new physics corrections to $Q_w$. In general, low-energy PV constitutes a much more powerful probe – relative to other electroweak processes – of direct new physics than of oblique new physics. Consequently, in what follows, I will not treat oblique corrections and concentrate solely on new direct interactions. To that end, I remind the reader that at low-energies, one has $|q^2| << M^2_{Z}$, where $q_{\mu}$ is the momentum-transfer from the electron to the fermion (or nucleus) in Fig. 1a. Consequently, in a low-energy experiment, the process of Fig. 1a looks like it arises from a four fermion contact interaction, whose strength is proportional to $1/M^2_{Z}$ (Fig. 1b). Similarly, any new direct interactions, characterized by a heavy mass scale $\Lambda$ (e.g., the mass of another exchanged particle) will also look like a four fermion interaction whose strength is proportional to $1/\Lambda^2$. Consequently, we may write

$$Q_w = Q^{0}_{w} + \Delta Q_w \quad .$$

(1)

Here, $Q^{0}_{w}$ gives the contribution in the Standard Model while $\Delta Q_{w}$ indicates possible contributions from new interactions. We consider $Q_w$ to be generated by the low-energy effective Lagrangian

$$\mathcal{L} = \mathcal{L}^{PV}_{SM} + \mathcal{L}^{PV}_{NEW} \quad ,$$

(2)

where

$$\mathcal{L}^{PV}_{SM} = \frac{G_F}{2\sqrt{2}} g^\nu_{\mu} e \gamma^\mu e \sum_f g^f \bar{f} \gamma^\nu f$$

(3)

$$\mathcal{L}^{PV}_{NEW} = \frac{4\pi\kappa^2}{\Lambda^2} \bar{e} \gamma^\nu \gamma^5 e \sum_f h^f \bar{f} \gamma^\nu f \quad .$$

(4)

Here $g^f = Q^{\nu}_{f} = 2T^f_3 - 4Q_f \sin^2 \theta_W$ and $g^{\nu}_{A} = -2T^f_3$ are the tree level vector and axial vector fermion-$Z^0$ couplings in the SM, with $Q_f$ being the fermion’s EM charge and $T^f_3$ its weak isospin. The coupling $h^f$ characterizes the interaction
of the electron axial vector current with the vector current of fermion \( f \) for a given extension of the Standard Model. As above, \( \Lambda \) is the mass scale associated with the new physics, while \( \kappa \) sets the coupling strength. Generally speaking, strongly interacting theories take \( \kappa^2 \sim 1 \) while for weakly interacting extensions of the Standard Model one has \( \kappa^2 \sim \alpha \). For scenarios in which the interaction of Eq. (4) is generated by the exchange of a new heavy particle between the electron and fermion, the constant \( h^A_f = \tilde{g}^A_e \tilde{g}^f \), where \( \tilde{g}^A_e, \tilde{g}^f \) are the heavy particle axial vector (vector) coupling to the electron (fermion).

For simplicity, I do not consider contributions to \( \Delta Q_W \) arising from new scalar-pseudoscalar or tensor-pseudotensor interactions. I also do not consider \( V(e) \times A(f) \) interactions, as they do not contribute to \( Q_W \). Although the Standard Model \( V(e) \times A(f) \) interaction is suppressed due to the small value of \( g^A_A = -1 + 4 \sin^2 \theta_W \), resulting in an enhanced sensitivity to new physics of this type, one is at present not able to extract the \( V(e) \times A(f) \) amplitudes from PV observables with the level of precision attainable for \( Q_W \) (see, e.g., Ref. 8). Moreover, the hadronic axial vector current is not protected by current conservation from hadronic effects which may cloud the interpretation of the hadronic axial vector amplitude in terms of new physics. For the interaction of an electron with a heavy nucleus, such as cesium, the \( V(e) \times A(f) \) interaction is interesting from another standpoint. As I discuss briefly in the next section, this term receives a sizeable contribution from PV quark-quark interactions inside the nucleus. These PV hadronic effects are discussed in greater detail elsewhere in this book.

It is straightforward to write down the corrections to the weak charge of a given system arising from \( \mathcal{L}_{PV}^{NEW} \). Specifically, consider the nucleon and electron:

\[
\begin{align*}
\Delta Q^P_W &= \zeta(2h^u_V + h^d_V) \\
\Delta Q^N_W &= \zeta(h^u_V + 2h^d_V) \\
\Delta Q^e_W &= \zeta h^e_V ,
\end{align*}
\]

where

\[
\zeta = \frac{8\sqrt{2}\pi \kappa^2}{\Lambda^2 G_F} .
\]

To obtain a feel for the sensitivity of \( Q_W \) to the mass scale \( \Lambda \), we may consider a generic scenario in which the couplings \( g^A_e, h^A_f \) entering \( Q_W \) and \( \Delta Q_W \) are of the same order of magnitude. In this case, the fractional correction induced by new physics is

\[
\frac{\Delta Q_W}{Q_W^0} = \frac{8\sqrt{2}\pi}{\Lambda^2 G_F} .
\]
If a determination of $Q_W$ is made with one percent uncertainty, then the ratio in Eq. (9) must be $\leq 0.01$. Re-arranging the inequality leads to a lower bound on $\Lambda$:

$$
\Lambda \geq \left[ \frac{8\sqrt{2}\pi N^2}{0.01G} \right]^{1/2} \approx 20\kappa \text{ TeV} .
$$

(10)

In short, determinations of $Q_W$ at the one percent or better level probe new physics at the TeV scale for weakly interacting theories and the ten TeV scale for new strong interactions.

### 3 Observables

Before analyzing the possible impact different new physics scenarios may have on low-energy PV, it is instructive to consider the new physics sensitivities of different observables in general terms. Specifically, I will consider a general atomic PV observable for a single isotope, $A_{PV}(N)$; ratios involving $A_{PV}$ for different isotopes, $R$; and the left-right asymmetry for scattering polarized electrons from a given target, $A_{LR}$. Of these, the simplest is the atomic PV observable for a single isotope. In general, $A_{PV}(N)$ contains a term which varies with the nuclear spin, $A^{NSD}_{PV}(N)$, and a term independent of the nuclear spin, $A^{SID}_{PV}(N)$. These two terms arise from the way the PV electron-nucleus interaction causes atomic states of definite parity to mix. Because the $Z^0$ exchange interaction is a contact interaction in co-ordinate space (Eqn. (3) above), only atomic $S$ and $P$-states can be mixed by it. The mixing matrix element has the form:

$$
\langle P | \hat{H}_{atom}^{PV} | S \rangle = (iG_F/2\sqrt{2})\mathcal{N}C_{SP}(Z)\{Q_W
+\tilde{k}Q[\frac{3}{4}] \}
$$

where $\mathcal{N}$ is a calculable normalization factor, $C_{SP}(Z)$ is an atomic structure-dependent function, $Q$ contains relativistic and nuclear finite size corrections to the atomic wavefunctions at the origin, and $I$ and $F$ are the nuclear and atomic angular momenta, respectively. The constant $\tilde{k}$ receives two contributions. One arises from the $V(e) \times A(f)$ $Z^0$-exchange interaction. It is suppressed by the presence of $Q^e_W = -1 + 4\sin^2 \theta_W \sim -0.1$. The second arises from the exchange of a photon between the electron and nucleus, where PV nucleon-nucleon interactions generate an axial vector coupling of the photon to the nucleus. This coupling is known as the nuclear anapole moment. It is
proportional to the matrix element of the following operator

\[ \tilde{k}_{\text{anapole}} \propto \langle 0| \int d^3r \ r^2 \left[ J_{\text{EM}}^E + \sqrt{2\pi} |Y_2 \otimes J_{\text{EM}}^E|_{1\lambda} \right] |0\rangle \ . \]  

Here, \( J_{\text{EM}}^E \) is the nuclear EM current. Because of the factor of \( r^2 \) in the integrand of Eq. (12), \( \tilde{k}_{\text{anapole}} \) grows as \( A^{2/3} \), where \( A \) is the nuclear mass number. For this reason, as well as the presence of \( Q_w^e \) in the \( V(e) \times A(f) \) \( Z^0 \)-exchange amplitude, the anapole moment contribution to the second term of Eqn. (11) is the dominant one for a heavy atom like cesium. The physics of the anapole moment, and its connection to the PV hadronic weak interaction, is interesting in its own right. It is discussed elsewhere in this book.

As the presence of the angular momentum quantum numbers in Eq. (11) implies, the first term generates \( A_{\nu \nu}^{\text{NSTD PV}} \) and the second gives rise to \( A_{\nu \nu}^{\text{NSTD PV}} \). These two terms can be separately determined by observing different atomic hyperfine transitions (different combinations of \( F \) quantum numbers). For our purposes in this chapter, \( A_{\nu \nu}^{\text{NSTD PV}} \) is of primary interest. We may write it as

\[ A_{\nu \nu}^{\text{NSTD PV}}(N) = \xi Q_w = \xi \left[ Q_{w0}^0 + Z \Delta Q_{w}^w + N \Delta Q_{w}^N \right] \ , \]  

where

\[ Q_{w0}^0 = Z(1 - 4\sin^2 \theta_w) - N \]  

at tree level, where

\[ Z \Delta Q_{w}^w + N \Delta Q_{w}^N = \zeta \left[ (2Z + N)h_{w}^u + (2N + Z)h_{w}^d \right] \ , \]  

where

\[ \Delta Q_{w}^w = \zeta (2h_{w}^u + h_{w}^d) \]  

(16)

\[ \Delta Q_{w}^N = \zeta (2h_{w}^d + h_{w}^u) \ , \]  

(17)

as above and where \( \xi \) is an atomic structure-dependent coefficient. A determination of the latter generally requires theoretical knowledge of the relevant atomic wavefunction and, therefore, introduces theoretical uncertainty into the extraction of \( Q_w \). The relative sensitivity of \( A_{\nu \nu}(N) \) to new physics can be seen by rewriting \( Q_w \) as

\[ Q_w = Q_{w0}^0 \left[ 1 + \delta_N \right] \ , \]  

where

\[ \delta_N = \frac{(Z \Delta Q_{w}^w + N \Delta Q_{w}^N) / Q_{w0}^0} {Q_{w0}^0} \]  

\approx -\zeta \left[ \left( \frac{2Z + N}{N} \right) h_{w}^u + \left( \frac{2N + Z}{N} \right) h_{w}^d \right] \]  

\approx -\zeta \left[ (Z/N)(2h_{w}^u + h_{w}^d) + (2h_{w}^d + h_{w}^u) \right] \ ,
where the approximation $Q^0_w \approx -N$ has been made in light of the small value
for $1 - 4 \sin^2 \theta_W \approx 0.1$. From Eq. (19) we observe that for atoms having $Z \approx N$,
the weak charge is roughly equally sensitive to the new up- and down-quark
vector current interactions.

The use of the isotope ratios involving $A^N_{VP}(N)$ and $A^N_{VP}(N')$ largely
eliminates the dependence on the atomic structure-dependent constant $\xi$ and
the associated atomic theory uncertainty. I consider two such ratios:

$$R_1 = \frac{A^N_{VP}(N') - A^N_{VP}(N)}{A^N_{VP}(N') + A^N_{VP}(N)}$$  \hspace{1cm} (20)

and

$$R_2 = \frac{A^N_{VP}(N')}{A^N_{VP}(N)}.$$  \hspace{1cm} (21)

To the extent that $\xi$ does not vary appreciably along the isotope chain, one
has

$$R_1 = \frac{Q_w(N') - Q_w(N)}{Q_w(N') + Q_w(N)},$$  \hspace{1cm} (22)

$$R_2 = \frac{Q_w(N')}{Q_w(N)}.$$  \hspace{1cm} (23)

It is straightforward to work out the sensitivity of these ratios to new physics.
To this end, I write

$$R_i = R^0_i (1 + \delta_i),$$  \hspace{1cm} (24)

where

$$R^0_1 = \frac{Q^0_w(N') - Q^0_w(N)}{Q^0_w(N') + Q^0_w(N)}$$  \hspace{1cm} (25)

$$R^0_2 = \frac{Q^0_w(N')}{Q^0_w(N)}.$$  \hspace{1cm} (26)

give the ratios in the Standard Model and the $\delta_i$ give corrections arising from
new physics. Letting $N' = N + \Delta N$ and dropping small contributions contain-
ing $1 - 4 \sin^2 \theta_W$ one has

$$R^0_1 \approx \frac{\Delta N}{2N},$$  \hspace{1cm} (27)

$$R^0_2 \approx 1 + \frac{\Delta N}{N}.$$  \hspace{1cm} (28)
and

\[
\delta_1 \approx \zeta \left( \frac{2Z}{N + N'} \right) (2h_u^N + h_d^N) \quad (29)
\]

\[
\delta_2 \approx \zeta \left( \frac{Z}{N} \right) \left( \frac{\Delta N}{N'} \right) (2h_u^N + h_d^N) \quad (30)
\]

At first glance, the dependence of the \( \delta_i \) for \( i = 1, 2 \) on \( \Delta Q^{PV}_W = \zeta (2h_u^N + h_d^N) \) and not \( \Delta Q^N_W = \zeta (2h_u^N + h_d^N) \) may seem puzzling. As I point out in Ref. 6, the shifts containing \( \Delta Q^N_W \) in the numerator and denominator of each \( R_i \) cancel to first order in \( \zeta \). To illustrate, consider \( R_1 \), for example, where one has

\[
Q_w(N') - Q_w(N) \approx -N' + N + (N' - N)\Delta Q^N_W \quad (31)
\]

\[
= (N - N') \left[ 1 - \Delta Q^N_W \right]
\]

and

\[
Q_w(N') + Q_w(N) \approx -(N + N') + (N + N')\Delta Q^N_W + 2Z\Delta Q^{PV}_W \quad (32)
\]

\[
= -(N + N') \left[ 1 - \Delta Q^N_W - \left( \frac{2Z}{N + N'} \right) \Delta Q^{PV}_W \right]
\]

so that in the ratio, the dependence on \( \Delta Q^N_W \) cancels to first order. Because of this feature, the \( R_i \) are twice as sensitive to new physics involving \( u \)-quarks than to new physics which couples to \( d \)-quarks. The weak charge of a single isotope, on the other hand, has essentially the same sensitivity to \( u \)- and \( d \)-quark new physics.

From a comparison of \( \delta_N \) with the \( \delta_i \), it is apparent that, for a given experimental precision, the isotope ratios are generally less sensitive to direct new physics than is the weak charge for a single isotope. This statement is easiest to see in the case of \( R_2 \), since \( \delta_2 \) contains the explicit factor \( \Delta N/N' \). Taking \( Z \approx N \) for the case of \( R_1 \), one finds that a single isotope is three times more sensitive to new physics which couples to \( d \)-quarks and 1.5 times more sensitive to the \( u \)-quark coupling. For new physics scenarios which favor new \( e - d \) interactions over \( e - u \) interactions, the weak charge for a single isotope constitutes a more sensitive probe.

An alternative method for obtaining \( Q_w \) is to scatter longitudinally polarized electrons from fixed targets. Flipping the incident electron helicity and comparing the helicity difference cross section with the total cross section filters out the PV part of the weak neutral current interaction. The resulting left-right asymmetry for elastic scattering has the general form

\[
A_{LR} = \frac{N_+ - N_-}{N_+ + N_-} \approx \frac{2M^{PV}_{SC}}{M_{EM}} \quad (33)
\]
Here, \( N_+ \) (\( N_- \)) are the number of detected electrons for a positive (negative) helicity incident beam; \( M_{EM} \) and \( M_{PV} \) are, respectively, the electromagnetic and parity-violating neutral current electron-nucleus scattering amplitudes; \( Q_{EM} \) is the nuclear EM charge; and \( F(q) \) is a correction involving hadronic and nuclear form factors. In general, the latter term can be separated from the term containing the charges by varying electron energy and angle. For elastic scattering, the charge term can be isolated by going to forward angles and low energies. In the case of PV Möller scattering, one has \( F(q) \equiv 0 \). The present PV electron scattering program at MIT-Bates, Mainz-MAMI, and the Jefferson Laboratory seeks to determine the \( F(q) \) for a variety of targets, with a special emphasis on contributions from strange quarks \(^{13}\)\(^{14}\)\(^{15}\). The status and progress of this program is discussed in other chapters of this book.

In order to see how Eq. (33) comes about, it is instructive to consider elastic scattering from a positive parity, spin-zero, isospin-zero nucleus like \(^{4}\)He or \(^{12}\)C. For this case, only the charge operator contributes to the scattering amplitudes. The weak charge operator for the nucleus is

\[
\hat{Q}_W = Q_u^w u^\dagger u + Q_d^w d^\dagger d + Q_s^w s^\dagger s + \cdots
\]

where \( q^\dagger q \) counts the number of quarks of flavor \( q \) in the nucleus at zero momentum transfer and where the \( + \cdots \) represent the contributions from heavy quarks. The weak charge operator can be re-expressed as follows:

\[
\hat{Q}_W = (Q_u^w - Q_d^w) \frac{1}{2} (u^\dagger u - d^\dagger d) + 3(Q_u^w + Q_d^w) \frac{1}{6} (u^\dagger u + d^\dagger d - 2s^\dagger s) + (Q_u^w + Q_d^w + Q_s^w) s^\dagger s
\]

neglecting the three heaviest quark flavors. The combination of currents in the first line is the isovector EM charge operator. Since the nucleus has isospin zero, this combination cannot contribute to the scattering amplitude. The combination in the second line is the isoscalar EM charge operator. Its contribution goes as

\[
3(Q_u^w + Q_d^w) F_c(q)
\]

where \( F_c(q) \) is just the EM form factor for the nucleus, with \( F_c(q) = Q_{EM} = \) the nuclear EM charge. The third term in Eq. (33) contributes

\[
(Q_u^w + Q_d^w + Q_s^w) F_s(q)
\]
where $F_s(q)$ is the strange quark’s vector current form factor for the nucleus. Since stable nuclei have no net strangeness, one has $F_s(0) = 0$ and $F_s(q) \sim q^2$ for small momentum transfer. The numerator in Eq. (33) for $A_{LR}$ is just proportional to the sum of (36) and (37). The denominator is proportional to $F_c(q)$, with the same constant of proportionality. Consequently, in the ratio, $F_c(q)$ cancels out of the first term, leaving $A_{LR}$ proportional to the combination

$$3(Q_w^u + Q_w^d) + (Q_w^u + Q_w^d + Q_w^s) \frac{F_s(q)}{F_c(q)}.$$  \hspace{1cm} (38)

This combination has just the form of the RHS of Eqn. (33), where the first term is $q$-independent ratio of nuclear weak and EM charges and the second term is the $q$-dependent form factor term. A similar line of reason applies to other targets. The only difference is that when $J \neq 0$ and/or $T \neq 0$, the nucleus can support more form factors and the $F(q)$ term in the asymmetry has a more complicated structure.

In order to compare the sensitivities of different scattering experiments to new physics, I specify the terms in Eq. (33) for the following processes: elastic scattering from the proton, $A_{LR}(1H)$; elastic scattering from $(J^\pi,T) = (0^+,0)$, $A_{LR}(0^+,0)$ nuclei; excitation of the $\Delta(1232)$ resonance, $A_{LR}(N \rightarrow \Delta)$; and Möller scattering, $A_{LR}(e)$. The corresponding charge terms are (neglecting Standard Model radiative corrections)

$$Q_w(1H)/Q_{EM}(1H) = (1 - 4\sin^2 \theta_W) \left[1 + \delta_P\right]$$  \hspace{1cm} (39)
$$Q_w(0^+,0)/Q_{EM}(0^+,0) = -4\sin^2 \theta_W \left[1 + \delta_{00}\right]$$  \hspace{1cm} (40)
$$Q_w(e)/Q_{EM}(e) = (-1 + 4\sin^2 \theta_W) \left[1 + \delta_e\right]$$  \hspace{1cm} (41)

while for the transition to the $\Delta$ one replaces the ratio of charges by the ratio of isovector weak neutral current and EM couplings:

$$Q_w(N \rightarrow \Delta)/Q_{EM}(N \rightarrow \Delta) \rightarrow 2(1 - 2\sin^2 \theta_W) \left[1 + \delta_\Delta\right]$$  \hspace{1cm} (42)

The new physics corrections $\delta$ are given by

$$\delta_P = \zeta (2h_v^w + h_v^d)/(1 - 4\sin^2 \theta_W)$$  \hspace{1cm} (43)
$$\delta_{00} = -3\zeta (h_v^w + h_v^d)/(4\sin^2 \theta_W)$$  \hspace{1cm} (44)
$$\delta_e = -\zeta h_v^e/(1 - 4\sin^2 \theta_W)$$  \hspace{1cm} (45)
$$\delta_\Delta = \zeta (h_v^w - h_v^d)/[2(1 - 2\sin^2 \theta_W)]$$  \hspace{1cm} (46)

The expressions for the various $\delta_i$ allow us to make a few observations regarding the relative sensitivities the corresponding observables to new physics.
Table 1: Relative sensitivities of PV observables to new physics, assuming $h_u^V = h_d^V$, tree-level values for the corresponding weak charges, and $\sin^2 \theta_W = 0.232$. The scale factor $f_i = \sqrt{\delta_i/\delta_N}$ can be used to scale mass bounds from the cesium APV bounds to the bounds for observable $i$ assuming the same precision for both $\delta_N$ and $\delta_i$. The numbers shown in the Table are obtained using the $\overline{\text{MS}}$ value $\sin^2 \theta_W = 0.232$ while neglecting radiative corrections.

| Correction | $\delta$ | Scale factor $f_i$ |
|------------|---------|-------------------|
| $\delta_N \approx 5.1\xi$ | 1       |                   |
| $\delta_1 \approx 1.9\xi$  | 0.6     |                   |
| $\delta_2 \approx 0.4\xi$  | 0.3     |                   |
| $\delta_p \approx 40\xi$   | 2.8     |                   |
| $\delta_{00} \approx 6.5\xi$ | 1.1    |                   |
| $\delta_e \approx 13\xi$   | 1.6     |                   |
| $\delta_p = 0$              | 0       |                   |

For this purpose, it is useful to take $h_u^V = h_d^V = 1$ and specify $\delta_N$ for the case of $^{133}\text{Cs}$. I also use cesium for the isotope ratios and take a reasonable range of neutron numbers: $N = 78$, $N' = 95$. In Table 1 I show the $\delta_i$ in units of $\xi$. The third column gives a scale factor $f$ defined as

$$f_i = \sqrt{\delta_i/\delta_N}.$$  \hspace{1cm} (47)

The factor $f_i$ can be used to scale the cesium APV limits on the new physics mass scale $\Lambda$ to those obtainable from any other observable when measured with the same precision as $Q_w(\text{Cs})$: $\Lambda(i) = f_i\Lambda(\text{Cs})$. Alternatively, the limits from any other observable will be the same as those from cesium when the precision is $f_i^2$ times the cesium uncertainty. The numbers shown in the Table are obtained using the $\overline{\text{MS}}$ value $\sin^2 \theta_W = 0.232$ while neglecting radiative corrections.

As Table 1 illustrates, $A_{LR}(ep)$ has by far the greatest generic sensitivity.

\textsuperscript{a}The most significant effect of radiative corrections appear in the Möller asymmetry.\hspace{1cm}
to new physics for a given level of error in the observables. The reason is the suppression of $Q^0_W$ for the proton, which goes as $1 - 4\sin^2 \theta_W$ at tree level. This suppression, however, renders the attainment of high precision more difficult than for some of the other cases, since the statistical uncertainty in $A_{LR}$ goes as $1/A_{LR}$. To set the scale, consider a 10% $A_{LR}(ep)$ measurement. Using the scale factor $f_P$ in Table 1, we see that such a measurement would be roughly comparable to the present cesium result in terms of new physics sensitivity. Given the performance of the beam and detectors at the Jefferson Lab, it appears that a future measurement of $A_{LR}(ep)$ with 5% or better precision could be feasible. Such a determination would yield new physics limits comparable to those from cesium APV should the atomic theory error be reduced to the level of the present experimental error. A 2.5% $ep$ measurement would strengthen the present APV bounds by a factor of two. Constraints of this level would be competitive with those expected from high energy colliders well into the next decade. Similarly, a 0.8% determination of the isotope ratio $R_1$ would give new physics limits comparable to those expected from cesium APV should the atomic theory error be reduced to the level of the present experimental error. A 2.5% $ep$ measurement would strengthen the present APV bounds by a factor of two. Constraints of this level would be competitive with those expected from high energy colliders well into the next decade. Similarly, a 0.8% determination of the isotope ratio $R_1$ would give new physics limits comparable to those expected from cesium APV should the atomic theory error be reduced to the level of the present experimental error. A 2.5% $ep$ measurement would strengthen the present APV bounds by a factor of two. Constraints of this level would be competitive with those expected from high energy colliders well into the next decade.

4 Model Illustrations

With these general features in mind, we may now consider the implications of specific models for new direct physics interactions. For illustrative purposes, I discuss three such scenarios: (a) the presence of a second “low-mass” neutral gauge boson in addition to the $Z^0$; (b) the possible existence of lepto-quarks; and (c) new interactions arising from fermion sub-structure.

A. Additional neutral gauge bosons.

The existence of one or more additional neutral gauge bosons is a natural consequence of theories inspired by superstring theory. Certain versions of superstring theory depend on the group structure $E_8 \times E_8$ (for details, see Refs. [22, 23]). When superstrings “compactify” from 26 to four dimensions, one of these $E_8$ groups is reduced to $E_6$. In the same way that the $SU(2)_L \times U(1)_Y$
gauge symmetry spontaneously breaks down to $U(1)_{EM}$ symmetry at the weak scale, giving the $W^\pm$ and $Z^0$ masses, it is possible that the $E_6$ symmetry spontaneously breaks down to sub-group $SO(N) \times U(1)$ or $SU(N) \times U(1)$ symmetries which ultimately break down to the $SU(2)_L \times U(1)_Y$ symmetry of the SM. At each occurrence of spontaneous symmetry breakdown, the gauge boson associated with the extra $U(1)$ symmetry acquires a mass. It is possible that at least one of the new gauge bosons is light enough to be observable at low-energies. The models which generate such bosons from $E_6$ spontaneous symmetry breaking are known as $E_6$ theories.

An alternative reason to postulate the existence of additional $Z$-bosons is provided by left-right symmetric theories. Such theories give a natural explanation for the observation of PV below the weak scale. According to the idea of LR symmetry, there exists an $SU(2)_R$ gauge symmetry in addition to the SM electroweak gauge symmetry. The $W^\pm_R$ and $Z_R$ bosons associated with this right-handed gauge group are much heavier than the SM gauge bosons, so right handed-interactions are much weaker. However, the $W^\pm_R$ and $Z_R$ may still be sufficiently light to generate very small corrections to low-energy observables – corrections which may become apparent when the experimental precision becomes sufficiently high.

If $Z'$ and $Z$ denote the "new" and Standard Model neutral gauge bosons, respectively, the existence of a light $Z'$ which mixes with the $Z$ is ruled out by $Z$-pole observables. The reason is that any new physics which mixes with the $Z$ would show up strongly at the $Z$-pole. In the event that the $Z-Z'$ mixing angle is $\approx 0$, however, LEP and SLC measurements provide rather weak constraints. Consequently, I consider the case of zero mixing. For the sake of illustration, consider first the $E_6$ analysis of Ref. 22, in which the different symmetry breaking scenarios can be parameterized by writing the $Z'$ as

$$Z' = \cos \phi Z_\psi + \sin \phi Z_\chi .$$

(48)

The $Z_\psi$ and $Z_\chi$ arise, for example, from the breakdown $E_6 \to SO(10) \times U(1)_\psi$ and $SO(10) \to SU(5) \times U(1)_\chi$. Since the multiplets of $SO(10)$ contain both $f$ and $\bar{f}$ for the leptons and quarks of the Standard Model, $C$-invariance implies that the $Z_\psi$ can have only axial vector couplings to these fermions. As a result, it cannot contribute at tree-level to low-energy PV observables. In the case of $SU(5)$, however, the left-handed $d$-quark and $e^+$ live in a different multiplet from the left-handed $\bar{d}$ and $e^-$, whereas the $u$ and $\bar{u}$ live in the same multiplet. The $Z_\chi$ correspondingly has both vector and axial vector couplings to the electron and $d$-quarks, and only axial vector $u$-quark couplings. In short, $E_6$ $Z'$ bosons yield $h_u^V = 0$ and $h_\psi^A, h_\chi^A \propto \sin \phi$. 


According to the notation of Eq. (4), we have for $E_6$ models

\[
\begin{align*}
\kappa^2 &= \alpha' \\
\Lambda^2 &= M_Z^2 \\
h_u^v &= 0 \\
h_d^v &= -h_e^v = \left[ \sqrt{15} \sin \phi \cos \phi / 3 \right] \\
\end{align*}
\]

(49) (50) (51) (52)

where $\alpha'$ is the fine structure constant associated with the new gauge coupling. Generally, one has

\[
\alpha' \lesssim \frac{5}{3} \frac{\alpha}{\cos^2 \theta_W} \approx 2.2\alpha
\]

(53)

Different models for the $Z'$ correspond to different choices for $\phi$. Examples include the $Z_\eta$ ($\tan \phi = -\sqrt{3}/5$) and the $Z_I$ ($\tan \phi = -\sqrt{5}/3$), where the latter is associated with an additional “inert” SU(2) gauge group not contributing to the electromagnetic charge. From the standpoint of phenomenology, it is worth noting the dependence of $h_u^v$ and $h_e^v$ on the value of $\phi$. For $\phi = \phi_c = \tan^{-1}(\sqrt{5/3})$, $h_u^v = 0 = h_e^v$. For $\phi > \phi_c$, $h_d^v > 0$. In the event that any of the $\delta_i$ is determined to be non-zero, the sign of the deviation would constrain the allowable range of $E_6$ models. From Eq. (19), we observe, for example, that $\delta_N$ is negative for $h_u^v = 0$ and $h_d^v > 0$. At the 1\(\sigma\) level, the most recent value of $\delta_N$ for cesium implies that $h_d^v > 0$, and therefore could not be explained models giving $\phi < \phi_c$. The model which gives the largest possible contribution to the weak charge is the $Z_\chi$, which corresponds to $\phi = 90^\circ$.

In left-right symmetric theories, the low energy gauge group becomes SU(2)$_L \times$SU(2)$_R \times$U(1)$_{B-L}$, where $B - L = 1/3$ for baryons and $-1$ for leptons. In the case of “manifest” left-right symmetry the SU(2)$_L$ and SU(2)$_R$ couplings are identical. For this case, a second low-mass neutral gauge boson $Z_R$ couples to fermions with the strength

\[
\begin{align*}
h_u^v &= -3 \alpha \left( \frac{\alpha}{4} - \frac{1}{6\alpha} \right) \\
h_d^v &= 3 \alpha \left( \frac{\alpha}{4} + \frac{1}{6\alpha} \right) \\
h_u^v &= 3 \alpha \left( \frac{\alpha}{4} - \frac{1}{2\alpha} \right)
\end{align*}
\]

(54) (55) (56)

where

\[
\alpha = \left( \frac{1 - 2\sin^2 \theta_W}{\sin^2 \theta_W} \right)^{1/2} \approx 1.53
\]

(57)

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With this set of couplings, the combination appearing in the correction to the proton’s weak charge is $2h_u^v + h_d^d \approx 0.012 << h_u^v, h_d^d$. Consequently, the sensitivities of the $R_1$ and $A_{L,R}(^4\text{He})$ are suppressed relative to their generic scale. The corresponding mass limits on $M_{Z_R}$ are weaker than those obtainable from cesium APV or $A_{L,R}(0^+,0)$.

In Table 2, I give the present and prospective for two species of additional neutral gauge bosons, the $Z_\chi$ and $Z_R$. In particular, we show lower bounds on the Fermi constant associated with the new gauge boson $Z'$, defined as

$$
\frac{G'_a}{\sqrt{2}} = \frac{g'^2}{8M_{Z'_a}^2},
$$

(58)

where $g'$ is the coupling associated with the additional $U(1)_a$ gauge group. Low-energy PV observables constrain the ratio $g'/M_{Z'_a}$ and do not provide separate limits on the mass and coupling. Consequently, the ratio of $G'_a/G_F$ characterizes the strength of a new $U(1)_\chi$ gauge interaction relative to the strength of the Standard Model. In general, mass bounds for the $Z'$ can be obtained from the limits on $G'$ under specific assumptions for $g'$. A comparison of such mass bounds is often instructive, so I quote such bounds in the final two columns of Table 2. Lower bounds on $M_{Z_\chi}$ are quoted assuming the maximal value for $g'$ as given by Eq. (53). In the case of LR symmetry models with manifest LR symmetry, one has $g' = g$. The corresponding mass limits for the $Z_R$ are given in the final column of Table 2. Since I only discuss the case of manifest LR symmetry above, I do not include bounds on $G'_R/G_F$.

The limits in Table 2 lead to several observations. Primary among these is that low-energy PV already constrains the strength of new, low-energy gauge interactions to be at most a few parts in a thousand relative to the strength of the SU(2)$_L \times$ U(1)$_Y$ sector. When reasonable assumptions are made about new gauge couplings strengths, low-energy mass bounds now approach one TeV. The significance of these bounds becomes more apparent when a comparison is made with the results of collider experiments. The present 110 pb$^{-1}$ $p\bar{p}$ data set analyzed by the CDF collaboration yields a lower bound on $M_{Z_R}$ of 620 GeV, assuming manifest LR symmetry. The lower bound for $M_{Z_\chi}$ is 585 GeV, assuming no $Z_\chi$ decays to supersymmetric particles. The sensitivity of cesium APV already exceeds that of the Tevatron experiments. I wish to emphasize that collider experiments and low-energy PV provide complementary probes of extended gauge group structure. PV observables are sensitive to the vector couplings of the $Z'$ to fermions. For a model for which this coupling is small or vanishing (e.g., the $Z_\psi$ having $\phi = 0^\circ$ in Eq. (24)), PV observables will yield no significant information. Collider experiments, on the other hand, retain a sensitivity to such $Z'$ interactions. For models in which the $ffZ'$ coupling is
Table 2: Present and prospective limits on two species of additional neutral gauge bosons. The third column gives the ratio of fermi constants as defined in the text. The fourth and fifth columns give lower bounds on masses for the $Z_X$ and $Z_R$, respectively, assuming the precision given in column two. The Möller limits are derived without accounting for Standard Model radiative corrections.

| Observable                  | Precision | $G'_X/G_f$ | $M_{Z_X}$ (GeV) | $M_{Z_R}$ (GeV) |
|-----------------------------|-----------|------------|----------------|----------------|
| $Q_W$ (Cs)                  | 1.3%      | 0.006      | 730            | 790            |
| $R_1$                       | 0.35%     | 0.0016     | 1410           | 1520           |
| $Q_W(1^{\text{H}})/Q_{EM}(1^{\text{H}})$ | 0.3%     | 0.006      | 740            | 360            |
| $Q_W(0^+,0)/Q_{EM}(0^+,0)$  | 0.1%      | 0.002      | 1300           | 630            |
| $Q_W(e)/Q_{EM}(e)$          | 10%       | 0.010      | 580            | 285            |
| $Q_W(e)/Q_{EM}(e)$          | 3%        | 0.003      | 1100           | 520            |
| $Q_W(e)/Q_{EM}(e)$          | 1%        | 0.003      | 1100           | 520            |
| $Q_W(e)/Q_{EM}(e)$          | 7%        | 0.013      | 700            | 350            |
| $A_{LR}(N \rightarrow \Delta)$ | 1%       | 0.013      | 490            | 920            |

not suppressed, low-energy PV yields the most stringent bounds.

A look to the future suggests that PV could continue to play such a complementary role. Assuming the collection of 10 fb$^{-1}$ of data at TeV33, for example, the current Tevatron bounds on $M_{Z'}$ would increase by roughly a factor of two. The prospective sensitivity of cesium APV, assuming a reduction in atomic theory error to the level of the present experimental uncertainty, would exceed the collider sensitivity by $\sim 50\%$. Precise determinations of the isotope ratio $R_1$ or various PV electron scattering asymmetries could also yield sensitivities which match or exceed the prospective TeV33 reach. Only with the advent of the LHC or $\gtrsim 60$ TeV hadron collider will high-energy machines probe masses significantly beyond those accessible with low-energy PV.

Finally, Table 2 illustrates the model-sensitivity of different PV observables. For the models considered here, the mass bounds do not scale with the $f_i$ of Table 1 since $h^u_V \neq h^d_V$. Both the $Z'$ in $E_6$ and the $Z_{LR}$ couple more strongly to protons than neutrons. Consequently, both $R_1$ and $A_{LR}(1^{\text{H}})$ dis-
play weaker sensitivity to new gauge interactions than their generic sensitivities to new physics indicated in Table 1.

B. Leptoquarks

In early 1997, the H1\(^2\) and ZEUS\(^3\) collaborations reported the presence of anomalous events in high-\(|q^2|\) \(e^+p\) collisions at HERA. These events have been widely interpreted as arising from s-channel lepton-quark resonances with mass \(M_{LQ} \approx 200\) GeV. Such so-called leptoquarks are particles, either scalar or vector, which carry both lepton and baryon number. In \(e^+p\) collisions, a leptoquark would be formed in the s-channel when the \(e^+\) and quark annihilate into a leptoquark, which subsequently decays back into the \(e^+q\) pair (see Fig 2). The stringent limits on the existence of vector leptoquarks (LQ’s) obtained at Fermilab make scalar leptoquarks the favored interpretation of the HERA events. Although the results remain ambiguous and are open to alternative explanations, they are nonetheless provocative and suggest a consideration of LQ effects in low-energy PV processes. To that end, I consider general LQ interactions of the form

\[
\mathcal{L}_{LQ}^S = \frac{\lambda_S}{\sqrt{2}} \phi \bar{e} \sigma_{\mu\nu} q \bar{q}\sigma^{\mu\nu} e + h.c.
\]

\[
\mathcal{L}_{LQ}^V = \frac{\lambda_V}{\sqrt{2}} \bar{e} \gamma_\mu q \bar{q}\gamma^\mu e + h.c.
\]

where \(\phi\) and \(\phi^\mu\) denote scalar and vector LQ fields, respectively. For simplicity, we do not explicitly consider the corresponding interactions obtained from Eqs. (59) with \(L \leftrightarrow R\). The corresponding analysis is similar to what follows. Assuming \(M_{LQ}^2 >> |q^2|\), the processes of Fig. 2 give rise to the following PV interactions:

\[
\mathcal{L}_{PV}^S = \frac{\lambda_S}{2} \bar{e} q \gamma^\mu e - \bar{e} q \gamma^\mu q e + \bar{e} q \gamma^\mu q e
\]

\[
\mathcal{L}_{PV}^V = -\frac{\lambda_V}{2} \bar{e} q \gamma^\mu q e + \bar{e} q \gamma^\mu q e
\]

After a Fierz transformation, these become

\[
\mathcal{L}_{PV}^S = \frac{\lambda_S}{2\sqrt{2}} \bar{e} q \gamma^\mu e - \bar{e} q \gamma^\mu q e + \bar{e} q \gamma^\mu q e
\]

\[
\mathcal{L}_{PV}^V = -\frac{\lambda_V}{2\sqrt{2}} \bar{e} q \gamma^\mu q e + \bar{e} q \gamma^\mu q e
\]

In terms of the interaction in Eq. (4), we may identify

\[
\Lambda^2 = M_{LQ}^2
\]

\[
\kappa^2 = \lambda^2/16\pi
\]
and \( h_v^u = 1/2 \) (\( h_v^d = -1 \)) for scalar (vector) LQ interactions.

Assuming for simplicity that either a u-type or d-type LQ (but not both) contributes to low-energy PV processes, the results from cesium APV, together with Eqs. (63) and (64), yield the following 1σ limits on LQ couplings and masses:

\[
\lambda_S \leq \begin{cases} 
0.042 \ (M_{LQ}/100 \ \text{GeV}), & \text{u-type} \\
0.04 \ (M_{LQ}/100 \ \text{GeV}), & \text{d-type}
\end{cases}
\]  

(67)

and

\[
\lambda_V \leq \begin{cases} 
0.030(M_{LQ}/100 \ \text{GeV}), & \text{u-type} \\
0.028(M_{LQ}/100 \ \text{GeV}), & \text{d-type}
\end{cases}
\]  

(68)

The HERA results are rather insensitive to the value of the coupling \( \lambda_S \). Substituting the HERA value of \( \approx 200 \ \text{GeV} \) into (67) yields an upper bound of \( \lambda_S \lesssim 0.08 \). On general grounds, one might have expected \( \kappa^2 \sim \alpha \) or \( \lambda_s \sim 0.6 \).

The cesium APV results require the coupling for a 200 GeV scalar LQ to be about an order of magnitude smaller than this expectation. Alternatively, if one does not interpret the HERA results as a 200 GeV LQ and assumes \( \kappa^2 \sim \alpha \), the APV bounds on the scalar LQ mass are \( M_{LQ} > 1.5 \ \text{TeV} \). Table 3 gives comparable bounds on the LQ coupling-to-mass ratio for the other PV observables discussed in Section II. The bounds are characterized by the quantity \( \gamma_q \), defined as

\[
\lambda_S \leq \gamma_q \ (M_{LQ}/100 \ \text{GeV})
\]  

(69)

where \( q \) denotes the quark flavor.

Note that no bounds are given for the Müller asymmetry, as LQ’s do not contribute at tree level. As shown in Ref. \[6\], the leading contributions arise.
Table 3: Present and prospective limits on leptoquark interactions. Third and fourth columns give $\gamma_q$ for a $q$-type leptoquark, as defined in Eq. (69). Leptoquark sensitivity of Möller asymmetry does not behave according to Eq. (69), so that no limits on the $\gamma_q$ are attainable.

| Observable                  | Precision | $\gamma_u$ | $\gamma_d$ |
|-----------------------------|-----------|------------|------------|
| $Q_W$(Cs)                   | 1.3%      | 0.04       | 0.042      |
|                             | 0.35%     | 0.021      | 0.022      |
| $R_1$                       | 0.3%      | 0.04       | 0.028      |
|                             | 0.1%      | 0.023      | 0.016      |
| $Q_W$($^1$H)/$Q_{EM}$($^1$H) | 10%       | 0.05       | 0.036      |
|                             | 3%        | 0.028      | 0.02       |
| $Q_W$(0+, 0)/$Q_{EM}$(0+, 0) | 1%        | 0.033      | 0.033      |
| $Q_W(e)/Q_{EM}(e)$          | 7%        | --         | --         |
| $A_{LR}(N \rightarrow \Delta)$ | 1%       | 0.06       | 0.06       |

from the loop graphs of Fig. 3. The corresponding contributions to the PV effective $ee$ interaction are

$$M_{PV}^{(a)} = \left( \frac{\lambda_S}{16\pi M_{LQ}} \right)^2 \bar{e}_\mu e \bar{e}_\gamma \gamma_5 e$$

$$M_{PV}^{(b)} = \frac{\alpha Q_q}{12\pi} \left( \frac{\lambda_S}{M_{LQ}} \right)^2 \ln \frac{m_q}{M_{LQ}} \bar{e}_\mu e \bar{e}_\gamma \gamma_5 e$$

where $m_q$ and $Q_q$ are the intermediate state quark mass and E.M. charge. For $M_{LQ} = 100$ GeV, a 7% determination of the Möller asymmetry would yield

$$\lambda_s \leq 1.2$$

from graph (a) and

$$\lambda_s \leq \begin{cases} 1.14, & \text{d-type} \\ 0.78, & \text{u-type} \end{cases}$$

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from graph (b). The limits for a vector LQ are comparable. The prospective Möller bounds are about ten times weaker than those attainable with semi-leptonic PV. Any deviation of the Möller asymmetry from the Standard Model prediction is unlikely to be due to LQ’s.

C. Compositeness

The Standard Model assumes the known bosons and fermions to be point-like. The possibility that they possess internal structure, however, remains an intriguing one. Manifestations of such composite structure could include the presence of fermion form factors in elementary scattering processes or the existence of new, low-energy contact interactions. The latter could arise, for example, from the interchange of fermion constituents at very short distances. A recent analysis of $p\bar{p} \rightarrow \ell^+\ell^-$ data by the CDF collaboration limits the size of a lepton or quark to be $R < 5.6 \times 10^{-4}$ f when $R$ is determined from the assumed presence of a form factor at the fermion-boson vertex. More stringent limits on the distance scale associated with compositeness are obtained from the assumption of new contact interactions governed by a coupling of strength $g^2 = 4\pi$. Collider experiments yield $R \sim 1/\Lambda < 6 \times 10^{-5}$ f, where $\Lambda$ is the mass scale associated with new dimension six lepton-quark operators.
It is conventional to write the lowest dimension contact interactions as

$$\mathcal{L}_{COMP} = 4\pi \sum_{ij} \frac{\eta_{ij}}{\Lambda^{2}_{ij}} \bar{e}_i \gamma_{\mu} e_i \bar{q}_j \gamma_{\mu} q_j \quad ,$$  \hspace{1cm} (74)

where \( \Gamma \) is any one of the Dirac matrices and \( i, j \) denote the appropriate fermion chiralities (e.g., \( \bar{e}_L \gamma_\mu e_L \bar{q}_R \gamma_\mu q_R \) etc.). The quantities \( \eta_{ij} \) take on the values \( \pm 1, 0 \) depending on one’s model assumptions. In terms of the PV interaction of Eq. (4), the contribution from \( \mathcal{L}_{COMP} \) is

$$\pi \bar{e} \gamma_\mu \bar{q} \sum_q \left[ \frac{\eta_{RR}}{\Lambda^{2}_{RR}} - \frac{\eta_{LL}}{\Lambda^{2}_{LL}} + \frac{\eta_{RL}}{\Lambda^{2}_{RL}} - \frac{\eta_{LR}}{\Lambda^{2}_{LR}} \right] q^\mu q \quad .$$  \hspace{1cm} (75)

Writing this interaction in terms of a common mass scale \( \Lambda \) yields

$$\frac{\pi}{\Lambda^{2}} \bar{e} \gamma_\mu \bar{q} \sum_q \left[ \tilde{\eta}_{RR} - \tilde{\eta}_{LL} + \tilde{\eta}_{RL} - \tilde{\eta}_{LR} \right] q^\mu q \quad ,$$  \hspace{1cm} (76)

where

$$\tilde{\eta}_{ij} = \eta_{ij} \left( \frac{\Lambda}{\Lambda_{ij}} \right)^2 \quad .$$  \hspace{1cm} (77)

The correspondence with \( \mathcal{L}^{PV}_{NEW} \) is given by

$$\kappa^2 = \frac{1}{4}$$  \hspace{1cm} (78)

$$h_v^q = \tilde{\eta}_{RR} - \tilde{\eta}_{LL} + \tilde{\eta}_{RL} - \tilde{\eta}_{LR}$$  \hspace{1cm} (79)

It is worth noting that in the two previous model illustrations, one had \( \kappa^2 \sim \alpha \) or smaller. The present case with \( \kappa^2 \) on the order of unity corresponds to a new strong interaction.

On the most general grounds, one has no strong argument for any of the \( h_v^q \) to vanish. Consequently, low energy observables will generate lower bounds on \( \Lambda \). To compare with the recent \( p\bar{p} \) collider limits, consider the case of \( \tilde{\eta}_{LL} = \pm 1 \) and \( \tilde{\eta}_{RR} = \tilde{\eta}_{RL} = \tilde{\eta}_{LR} = 0 \). In this case, the cesium APV results yield

$$\Lambda_{LL} \geq 17.3 \text{ TeV}$$  \hspace{1cm} (80)

assuming \( h_v^u = h_v^d = -\tilde{\eta}_{LL} \). Regarding other low-energy PV observables, we note that the general comparisons made in Section III apply here. For example, a 10% measurement of \( \delta_P \) with PV ep scattering would yield comparable bounds, while a measurement of the isotopie ration \( R_1 \) with 0.8% precision would be required to obtain comparable limits. Were the cesium APV theory error reduced to the level of the present experimental error, or were a 2% determination of \( \delta_P \) achieved, the lower limit (80) would double.
Specific bounds from present and prospective measurements are given in Table 4 below:

As with other new physics scenarios, the present and prospective low-energy limits on compositeness are competitive with those presently obtainable from collider experiments as well as those expected in the future. The CDF collaboration has obtained lower bounds on $\Lambda_{LL}(eq)$ of 2.5 (3.7) TeV for $\tilde{\eta}_{LL} = +1 (-1)$\textsuperscript{32}. One expects to improve these bounds to 6.5 (10) TeV with the completion of Run II and 14 (20) TeV with TeV33. It is conceivable that future improvements in determinations of $Q_w$ with APV or scattering will yield stronger bounds that those expected from colliders. In the case of $\Lambda_{LL}(ee)$, Z-pole observables imply lower bounds of 2.4 (2.2) TeV for $\tilde{\eta}_{LL} = +1 (-1)$\textsuperscript{32}. The prospective Möller PV lower bounds exceed the LEP limits considerably.
5 Theoretical Uncertainties

The attainment of stringent limits on new physics scenarios from low-energy PV requires that conventional many-body physics of atoms and hadrons be sufficiently well understood. At present, the dominant uncertainty in $Q_W(Cs)$ is theoretical. A significant improvement in the precision with which this quantity is known would rely on corresponding progress in atomic theory. The issues involved in reducing the atomic theory uncertainty are discussed elsewhere in this book. In what follows, I discuss the many-body uncertainties associated with the other observables discussed above.

A. Isotope ratios

It was pointed out in Refs. 33, 34 that the isotope ratios $R_i$ display an enhanced sensitivity to the neutron distribution $\rho_n(r)$ within atomic nuclei, and that uncertainties in $\rho_n(r)$ could hamper the extraction of new physics limits from the $R_i$. To see how this $\rho_n$-sensitivity comes about, I follow Refs. 33, 34 and consider a simple model in which the nucleus is treated as a sphere of uniform proton and neutron number densities out to radii $R_P$ and $R_N$, respectively. In this case, one may express the weak charge as

$$Q_W = ZQ^P_W q_p + NQ^N_W q_n ,$$

where

$$q_p = (1/N) \int d^3 x \langle P | \hat{\psi}^\dagger_e(\vec{x}) \gamma_5 \hat{\psi}_e(\vec{x}) | S \rangle \rho_p(\vec{x})$$

$$q_n = (1/N) \int d^3 x \langle P | \hat{\psi}^\dagger_e(\vec{x}) \gamma_5 \hat{\psi}_e(\vec{x}) | S \rangle \rho_n(\vec{x})$$

The latter matrix element may be written as

$$\langle P | \hat{\psi}^\dagger_e(\vec{x}) \gamma_5 \hat{\psi}_e(\vec{x}) | S \rangle = N f(x) ,$$

where $f(0) = 1$. The effect of uncertainties in $\rho_p(\vec{x})$ – which are smaller than those in $\rho_n(\vec{x})$ – are suppressed in $Q_W$ since $q_p$ is multiplied by the small number $Q^P_W$. Consequently, we consider only $q_n$. In the simple nuclear model discussed above, one obtains

$$q_n = 1 - (Z\alpha)^2 f^N_2 + \cdots ,$$

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where
\[ f_2^N = \frac{3}{10} x_N^2 - \frac{3}{70} x_N^4 + \frac{1}{450} x_N^6 \] (86)
\[ x_N = R_N/R_P . \] (87)

Letting \( \delta_n^N \) and \( \delta_i^N \) denote the corrections to \( Q_W(N) \) and \( R_i \), respectively, we obtain
\[ \delta_n^N \approx - (Z\alpha)^2 f_2^N(x_N) \] (88)
\[ \delta_i^N \approx - (Z\alpha)^2 (N'/\Delta N) f_2^N(x_N) \Delta x_N \] (89)
\[ \delta_2^N \approx - (Z\alpha)^2 f_2^N(x_N) \Delta x_N , \] (90)

where \( \Delta x_N = (R_{N'} - R_N)/R_P \). Uncertainties in \( Q_W \) and \( R_i \) arise from uncertainties in the foregoing quantities. As shown in Ref. 6, if measurements of the \( R_i \) are to generate new physics limits comparable to those obtainable from \( Q_W(\text{Cs}) \) (assuming a future reduction of atomic theory error to the experimental error level) one will need \( \delta(\Delta x_N) \leq 0.004 \) for \( R_1 \) and \( \delta(\Delta x_N) \leq 0.02 \) for \( R_2 \). These requirements on \( \Delta x_N \) will differ for other atoms, such as Yb or Ba, depending on the values of \( N \) and \( \Delta N \). At present, there exist no reliable experimental determinations of \( x_N \) or \( \Delta x_N \), so that the interpretation of APV observables must rely on nuclear theory. It is conceivable that the theory uncertainty in \( x_N \) is 5% or better. Consequently, one could argue that even if the atomic theory error in \( Q_W(\text{Cs}) \) were reduced to the present experimental error, neutron distribution uncertainties should not complicate the extraction of new physics constraints. The situation regarding isotope shifts is more debatable.

Given the present theoretical situation, a model-independent determination of \( \rho_n(\vec{x}) \) is desirable. To that end, PVES may prove useful. The basic idea is that the \( F(q) \) term in Eq. (33) depends on the Fourier transform of \( \rho_n(r) \) among other things. By measuring the \( q \)-dependence of \( A_{LR} \), then, one may be able to extract enough information on \( \rho_n(r) \) to reduce the corresponding uncertainties in the \( R_i \). To illustrate the possibility, I consider a \((J^\pi, T) = (0^+, 0)\) nucleus, such as \( \text{Ba}^{138} \), noting that the isotopes of barium are under consideration for future APV isotope ratio measurements. As noted in Refs. 36, 5, the PV asymmetry for \((0^+, 0)\) nuclei may be written as
\[ - \left[ \frac{4\sqrt{2}\pi\alpha}{G_F|q|^2} \right] A_{LR} = Q_W^P + Q_W^N \int d^3x \ j_0(qx) \rho_n(\vec{x}) \] (91)

Since \( |Q_W^P/Q_W^N| < < 1 \), and since \( \rho_p(\vec{x}) \) is generally well determined from parity conserving electron scattering, \( A_{LR} \) is essentially a direct “meter” of the Fourier
transform of \( \rho_n(\vec{x}) \). At low momentum-transfer \((q_{R,N,P} << 1)\) this expression simplifies:

\[
- \left[ \frac{4\sqrt{2\pi\alpha}}{G_F q^2} \right] A_{LR} \approx \frac{N}{Z} \left[ 1 + \frac{q^2}{6} \left( \frac{R_N^2}{Z} - \frac{R_P^2}{N} \right) \right]
\]

so that a determination of \( R_N \) is, in principle, attainable from \( A_{LR} \).

In a realistic experiment PVES experiment, one does not have \( q_{R,N,P} << 1 \); larger values of \( q \) are needed to obtain the requisite precision for reasonable running times\cite{36}. In Ref.\cite{36}, it was shown that a 1% determination of \( \rho_n(\vec{x}) \) for \(^{208}\text{Pb}\) is experimentally feasible for \( q \sim 0.5 \text{ fm}^{-1} \) with reasonable running times. An experiment with barium is particularly attractive. If the barium isotopes are used in future APV measurements as anticipated by the Seattle group, then a determination of \( \rho_n(\vec{x}) \) for even one isotope could reduce the degree of theoretical uncertainty for neutron distributions along the barium isotope chain. Moreover, the first excited state of \(^{138}\text{Ba}\) occurs at 1.44 MeV. The energy resolution therefore required to guarantee elastic scattering from this nucleus is well within the capabilities of the Jefferson Lab.

**B. Hadronic Form Factors**

From the form of Eq. (33), it is clear that a precise determination of \( Q_W \) from \( A_{LR} \) requires sufficiently precise knowledge of the form factor term, \( F(q) \). Since a measurement can never be performed at kinematics for which \( F(q) = 0 \), namely, \( q^2 = 0 \), it will always generate a non-zero contribution to a PVES determination of \( Q_W \). As discussed elsewhere in this book, the \( F(q) \) term is presently under study at a variety of accelerators, with the hope of extracting information on the strange quark matrix element \( \langle N(p')|\bar{s}\gamma_{\mu}s|N(p)\rangle \).

The latter is parameterized by two form factors, \( G_E^{(s)} \) and \( G_M^{(s)} \). The other form factors which enter \( F(q) \) are known with much greater certainty than are the strange quark form factors. An extraction of \( Q_W \) from \( F(q) \) requires at least one forward angle measurement\cite{37}. The kinematics must be chosen so as to minimize the importance of \( F(q) \) relative to \( Q_W \) while keeping the statistical uncertainty in the asymmetry sufficiently small. These competing kinematic requirements – along with the desired uncertainty in \( Q_W \) – dictate the maximum uncertainty in \( F(q) \) which can be tolerated. Since \( A_{LR}(^{1}\text{H}) \) generally manifests the greatest sensitivity to new physics, I illustrate the form factor considerations for PV ep scattering.

Since \( Q_{EM}^p = 1 \), the ep asymmetry has the form

\[
A_{LR} = a_0 \tau \left[ Q_W^p + F^p(q) \right],
\]

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In a realistic analysis of \( A_{LR} \) for heavy nuclei, the effects of electron wave distortion must be included in the analysis of \( A_{LR} \). For a recent distorted wave calculation, see Ref.\cite{37}. 

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where $a_0 \approx 3.1 \times 10^{-4}$ and $\tau = |q^2|/4m_s^2$. The form factor contribution is given at tree level in the Standard Model by

$$F^p(\tau) = - \left[ \frac{G_E^n (G_E^n + G_E^{(s)}) + \tau G_M^n (G_M^n + G_M^{(s)})}{[(G_E^n)^2 + \tau (G_M^n)^2]} \right] , \quad (94)$$

where $G_{E,M}^n$ denote the proton or neutron Sachs electric or magnetic form factors. Since $Q_{EM}^n = 0$ and since the proton carries no net strangeness, both $G_E^n$ and $G_E^{(s)}$ must vanish at $\tau = 0$. Consequently, one may write $F^p(\tau)$ as

$$F^p(\tau) = \tau B(\tau) . \quad (95)$$

The function $B(\tau)$ will carry a non-trivial $\tau$-dependence if the $\tau$-dependence of the strange-quark form factors differs from the behavior observed for the nucleon EM form factors. As noted in Refs. 5 and 19, however, any determination of $Q_{EM}^P$ must be made at such low-$\tau$ that only $B(\tau = 0)$ enters the analysis, where

$$b \equiv B(0) = (1 - \mu_p)\mu_n - \mu_p \mu_s - \rho_s \quad (96)$$

and where

$$\rho_s = \frac{dG_E^{(s)}}{d\tau} \bigg|_{\tau=0} \quad (97)$$

$$\mu_s = G_M^{(s)}(0) \quad . \quad (98)$$

The low-$\tau$ form of the asymmetry may be written as

$$\frac{A_{LR}}{a_0 \tau} = Q_{W}^P + b\tau + O(\tau^2) \quad . \quad (99)$$

Extraction of $Q_{W}^P$ in this kinematic regime requires that the the uncertainty in $b$ be minimized.

To set a scale for uncertainty in $b$, consider a 10% determination of $Q_{W}$, which is roughly comparable to the present 1.3% determination of $Q_{W}(Cs)$.

Consider also possible measurements at scattering angles $\theta = 10^\circ$ and $\theta = 5^\circ$. Roughly one month of running could yield a 10% determination of $Q_{W}^P$, provided one maintains $\tau \geq 0.011 \ (\theta = 10^\circ)$ or $\tau \geq 0.003 \ (\theta = 5^\circ)$. From Eq. (93) we infer the corresponding maximum uncertainty in $b$: $\delta b \leq 0.7$ for $\theta = 10^\circ$ and $\delta b \leq 2.5$. These numbers scale with the precision desired for $Q_{W}^P$.

The present program of PVES measurements at Jefferson Lab and the MAMI facility at Mainz suggests that a determination of $b$ at this level is

cThese choices correspond roughly to present ($\theta = 12^\circ$) or prospective ($\theta = 6^\circ$) Jefferson Lab capabilities.

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experimentally feasible. The HAPPEX collaboration has reported a 15% determination of the $ep$ asymmetry at $\tau = 0.14$ \cite{HAPPEX}, which translates into an uncertainty in $B(\tau)$ of 0.3. The translation of these results into uncertainties in $b$ requires some care, and perhaps an additional measurement. If $B(\tau)$ varies gently with $\tau$ then one may infer small uncertainties in $b$ from those in $B(\tau)$. Since a gentle $\tau$-dependence cannot be assumed \textit{a priori}, additional measurements to constrain the non-leading $\tau$-dependence of $B(\tau)$ may be necessary. Nonetheless, the experimental needed to sufficiently constrain $b$ appears possible.

C. Dispersion Corrections

The discussion in this chapter has implicitly relied upon a one vector boson exchange (OVBE) approximation of the electroweak amplitudes contributing to various processes. A realistic analysis of precision observables must take into account contributions beyond the OVBE amplitude. In the case of electron scattering, these contributions are generally divided into two classes: Coulomb distortion of plane wave electron wavefunctions and dispersion corrections. The former can be treated accurately for electron scattering using distorted wave methods \cite{DWF}. The dispersion correction, however, has proven less tractable.

The leading dispersion correction (DC) arises from diagrams of Fig. 4, where the intermediate state nucleus or hadron lives in any one of its excited states. More generally, box diagrams like those of Fig. 4 can be treated exactly for scattering of electrons from point like hadrons. When at least one of the exchanged bosons is a photon, the amplitude is prone to infrared enhance-
ments. For elastic PV scattering of an electron from a point-like proton, for example, the \(Z - \gamma\) amplitude contains infrared enhancement factors such as \(\ln|s|/M_Z^2\), where \(s\) is the \(ep\) c.m. energy. Such factors can enhance the scale of the amplitude by as much as an order of magnitude over the nominal \(O(\alpha)\) scale. Consequently, one might expect box graph amplitudes which depend on details of hadronic or nuclear structure to be a potential source of theoretical error in the analysis of precision electroweak observables.

Data on the electromagnetic dispersion correction for \(ep\) scattering is in general agreement with the scale predicted by theoretical calculations. The situation regarding electron scattering from nuclei, however, is less satisfying. Recent data \(^{12}\)\(^C(e,e')\) taken at MIT-Bates and NIHKEF, however, disagree dramatically with nearly all published calculations (for a more detailed discussion and references, see Ref. \(^{38}\)). An experimental determination of any electroweak DC (\(V = \gamma, V' = W^\pm, Z^0\)) is unlikely, and reliance on theory to compute this correction is unavoidable. As shown in Ref. \(^{6}\), the corresponding theoretical uncertainty is far less problematic for a determination of \(Q_w\) from PVES than for the extraction of information on the strange quark form factors. The arguments leading to this conclusion are instructive, and I repeat them here.

To this end, it is convenient to write the \((V,V')\) DC as a correction \(R_{VV'}\) to the tree level EM and PV neutral current amplitudes:

\[
M_{EM} = M_{EM,\text{tree}} [1 + R_{\gamma\gamma} + \cdots] \quad (100)
\]

\[
M_{PV,NC} = M_{PV,\text{tree},NC} [1 + R_{VV'} + \cdots], \quad (101)
\]

where \(\cdots\) denotes other higher order corrections to the tree level amplitude. Because \(M_{EM,\text{tree}} \propto 1/q^2\) while the \(\gamma\gamma\) amplitude contains no pole at \(q^2 = 0\), \(R_{\gamma\gamma}\) has the general structure

\[
R_{\gamma\gamma}(q^2) = q^2 \tilde{R}_{\gamma\gamma}(q^2) \quad (102)
\]

where \(\tilde{R}_{\gamma\gamma}(q^2)\) describes the \(q^2\) dependence of the \(\gamma\gamma\) amplitude and is finite at \(q^2 = 0\). Since the tree level NC amplitude contains no pole at \(q^2 = 0\), however, the PV DC’s do not vanish at \(q^2 = 0\). Using Eq. \((102)\) and expanding the PV corrections in powers of \(q^2\) yields

\[
\frac{A_{LR}}{a_0^\tau} = Q_w \left[ 1 + R_{WW}(0) + R_{ZZ}(0) + R_{Z\gamma}(0) \right] + \tilde{F}(q) \quad (103)
\]

where \(F(q)\) in Eq. \((33)\) is replaced by an effective form factor \(\tilde{F}(q)\):

\[
\tilde{F}(q) = F(q) + q^2 \left[ R_{WW}'(0) + R_{ZZ}'(0) + R_{Z\gamma}'(0) - \tilde{R}_{\gamma\gamma}(q^2) + \cdots \right] \quad (104)
\]
As before, $F(q)$ contains the dependence on hadronic and nuclear form factors discussed above.

From Eqs. (102-104) we can see that the entire $\gamma\gamma$ DC, as well as the sub-leading $q^2$-dependence of the $WW$, $ZZ$, and $Z\gamma$ DC’s, contribute to $A_{LR}$ as part of an effective form factor term, $\tilde{F}(q)$. Since $F(q) \sim q^2$ for low-$|q^2|$ at forward angles, the $\gamma\gamma$ DC contribution entering Eq. (33) will be experimentally constrained along with $F(q)$ when the form factor term $\tilde{F}(q)$ is kinematically separated from the weak charge term. Consequently, an extraction of $Q_W$ from $A_{LR}$ does not require theoretical computations of the $\gamma\gamma$ DC or of the sub-leading $q^2$-dependence of the other DC’s. A determination of the strange-quark form factors, however, will require such theoretical input.

In order to extract constraints on possible new physics contributions to $Q_W$, one must compute $R_{WW}(0)$, $R_{ZZ}(0)$, and $R_{Z\gamma}(0)$. The theoretical uncertainty associated with $R_{WW}(0)$ and $R_{ZZ}(0)$ is small, since box diagrams involving the exchange are dominated by hadronic intermediate states having momenta $p \sim M_W$. These contributions can be reliably treated perturbatively. The $R_{Z\gamma}(0)$ correction, however, is infrared enhanced and displays a greater sensitivity to the low-lying part of the nuclear and hadronic spectrum. Fortunately, the sum of diagrams 4a and 4b conspire to suppress this contribution by $g^c_{\gamma} = -1 + 4\sin^2\theta_W$. This feature was first shown in Ref. [39] for the case of APV. Here, I summarize the argument as it applies to both APV and scattering.

The largest contributions to the loop integrals for diagrams 4a and 4b arise when external particle masses and momenta are neglected relative to the loop momentum $\ell$. The integrands from the two loop integrals sum to give

$$\tilde{u}[\gamma_\alpha \not{\ell} \gamma_\beta (g^5_\nu + g^5_\rho) \not{u}] \gamma_\alpha u T^{\alpha\beta}(\ell) D(\ell^2) = 2i \epsilon_{\alpha\lambda\beta\mu} \ell^\lambda \tilde{u} \gamma^{\mu} (g^5_\nu + g^5_\rho) u T^{\alpha\beta}(\ell) D(\ell^2)$$

where

$$T^{\alpha\beta}(\ell) = \int d^4 x \exp i \ell \cdot x \langle 0 | J^\alpha_{EM}(x) J^\beta_{NC}(0) | 0 \rangle.$$  

$D(\ell^2)$ contains the electron and gauge boson propagators when external momenta and masses are neglected relative to $\ell$. The terms in Eq. (103) which transform like pseudoscalars are those containing the EM current and either (a) both the axial currents $\tilde{u} \gamma^\mu \gamma_5 u$ and $J^5_{NC}$ or (b) both the vector currents $\tilde{u} \gamma^\mu u$ and $J^\beta_{NC}$. The former has the coefficient $Q^c_W = -1 + 4\sin^2\theta_W$ and the latter has a the coefficient $g^c_\alpha = 1$. The dependence of these terms on
the spatial currents is given by ($\lambda = 0$ in Eq. (106))

\[
Q_{e}^e \text{ term} : \sim \bar{u} \gamma_5 u \cdot \left( \vec{J}_{EM} \times \vec{J}_{NC}^5 \right) \\
g_{A}^e \text{ term} : \sim \bar{u} \gamma_\mu u \cdot \left( \vec{J}_{EM} \times \vec{J}_{NC}^\mu \right).
\] (107) (108)

The hadronic part of the $Q_{e}^e$ term transforms as a polar vector, so that this term contributes to the $A(e) \times V($had$)$ amplitude. The hadronic part of the $g_{A}^e$ terms, on the other hand, transforms as an axial vector, yielding a contribution to the $V(e) \times A($had$)$ amplitude. Hence, only the $Q_{e}^e$ term contributes to $Q_{e}^e$ term in the asymmetry.

Since $Q_{e}^e \sim -0.1$, the contribution $R_{Z\gamma}(0)$ in Eq. (103) is suppressed with respect to the generic one-loop scale. Consequently, then, the extraction of new physics constraints from the first term in Eq. (103) will not be appreciably affected by large theoretical uncertainties in the computation of $R_{Z\gamma}(0)$. In the case of $A_{LR}(1\, H)$, however, both $R_{Z\gamma}(0)$ and the tree-level amplitude are proportional to $Q_{e}^e$, so that there exists no additional suppression of the former with respect to the latter. Nevertheless, the theoretical computations of the $\gamma\gamma$ dispersion correction for parity-conserving ep scattering appear to work reasonably well. Consequently, one would expect calculations of the $Z\gamma$ DC to be similarly reliable for PV ep scattering.

6 Conclusions

The realm of physics beyond the Standard Model offers rich possibilities for new discovery as well as fertile ground for the development of new theoretical scenarios. As experiments begin to probe this ground at TeV mass scales, low-energy PV can continue to play an important role in elucidating the larger framework in which the Standard Model must lie. Indeed, it is remarkable that experiments performed with eV or a few GeV energies may have significant statements to make about physics at the TeV scale. The motivation for pursuing the future of PV is undoubtedly high.

The future of low-energy PV presents several challenges to both experimentalists and theorists. Improvements in new physics sensitivity will require progress on any one of a number of fronts. If the 1\% cesium atomic theory uncertainty cannot be significantly improved upon, then the future of APV may rest with measurements of isotope ratios. The interpretation of the $R_{i}$, however, requires that nuclear theory and experiment achieve a more reliable determination of $\rho_{n}(r)$ than presently exists. In this respect, PVES may prove to be an effective complement to APV by providing a direct measurement.
of the neutron distribution. Precisely how such a measurement would constrain the nuclear theory error entering the interpretation of $R_i$ remains to be clearly delineated. From the standpoint of interpretability, PVES offers the theoretically “cleanest” probe of new physics. The theoretically most uncertain quantities entering $A_{L,R}$ are either suppressed or can be measured by exploiting their kinematic dependence. The prospects for improved new physics sensitivity is the highest with a forward angle measurement on the proton, though the approved Möller experiment at SLAC and a possible experiment with a $^4$He or $^{12}$C target would provide useful complements. It stands as a challenge to experimentalists to achieve the necessary precision.

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