The effect of irradiation on the Jeans mass in fragmenting self-gravitating protostellar discs

Duncan Forgan* and Ken Rice
Scottish Universities Physics Alliance (SUPA), Institute for Astronomy, University of Edinburgh, Blackford Hill, Edinburgh EH9 3HJ, UK

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ABSTRACT
When a self-gravitating disc is subject to irradiation, its propensity to fragmentation will be affected. The strength of self-gravitating disc stresses is expected to dictate disc fragmentation: as the strength of these torques typically decreases with increasing sound speed, it is reasonable to assume, to first order, that disc fragmentation is suppressed when compared to the non-irradiated case, although previous work has shown that the details are complicated by the source of the irradiation. We expand on a previous analysis of the Jeans mass inside spiral structures in self-gravitating discs, incorporating the effects of stellar irradiation and background irradiation. If irradiation is present, fragmentation is suppressed for marginally unstable discs at low accretion rates (compared to the no-irradiation case), but these lower accretion rates correspond to higher mass discs. Fragmentation can still occur for high accretion rates, but is consequently suppressed at lower disc surface densities, and the subsequent Jeans mass is boosted. These results further bolster the consensus that, without subsequent fragment disruption or mass-loss, the gravitational instability is more likely to form brown dwarfs and low-mass stars than gas giant planets.

Key words: accretion, accretion discs – methods: analytical – stars: formation.

1 INTRODUCTION
Discs around both supermassive black holes and young stars are thought to undergo phases in which they are self-gravitating. It is possible that if these discs are sufficiently unstable while they are self-gravitating, they may fragment into bound objects, providing a mode of planet formation in protostellar discs (Kuiper 1951; Cameron 1978; Boss 1997) or star formation in AGN discs (Levin & Beloborodov 2003; Nayakshin & Sunyaev 2005).

The conditions under which a self-gravitating disc is expected to fragment have been extensively studied. The most important criterion is the Toomre parameter (Toomre 1964)

\[ Q = \frac{c_s \kappa}{\pi G \Sigma} < 1.5 - 1.7, \]

where \( c_s \) is the local sound speed, \( \kappa \) is the local epicyclic frequency (equal to the angular velocity \( \Omega \) in Keplerian discs) and \( \Sigma \) is the disc surface density. This is a linear stability criterion – the critical values given above apply to non-axisymmetric perturbations (see e.g. Durisen et al. 2007).

Once the disc becomes gravitationally unstable, spiral waves are excited in the disc, producing stresses and providing heating through shocks. This stress can be described as a pseudo-viscosity in some cases (Shakura & Sunyaev 1973; Balbus & Papaloizou 1999; Lodato & Rice 2004; Forgan et al. 2011), allowing a simple description of angular momentum transport in the disc. The stress produced by the spiral structure can therefore be described by a Shakura–Sunyaev turbulent viscosity parameter \( \alpha \), and semi-analytic models of self-gravitating discs can be constructed at low computational cost (e.g. Clarke 2009; Rice & Armitage 2009). This pseudo-viscous approximation has been shown to fail if the disc is too massive or geometrically thick (Lodato & Rice 2005; Forgan et al. 2011), but it remains useful for moderately massive, geometrically thin discs.

A balance can be struck between the shock heating from the spiral arms and the local radiative cooling. Discs which achieve this balance are described as being marginally stable (Paczynski 1978). This allows a relationship between the local cooling time normalized to the local angular frequency (\( \beta_c = t_{\text{cool}} \Omega \)) and \( \alpha \):

\[ \alpha = \frac{4}{9 \gamma (\gamma - 1) \beta_c}. \]

For fragmentation to be successful, a density perturbation produced by a spiral arm should be able to grow until it is gravitationally bound. Therefore, the fragment should be able to cool efficiently to reduce pressure support due to thermal energy, and continue collapsing. As cooling becomes more efficient in a marginally unstable disc (i.e. as \( \beta_c \) decreases), \( \alpha \) must increase to redress the balance. A consensus has developed in recent years that there is a maximum value of \( \alpha \) that the disc can sustain before quasi-steady self-gravitating torques saturate: \( \alpha_{\text{crit}} \sim 0.06 \) (Gammie 2001; Rice, 2004).
Lodato & Armitage 2005). This is supported by numerical experiments (Cossins, Lodato & Clarke 2009) that confirm an inverse relationship between $\beta_c$ and surface density perturbations:

$$\left(\frac{\Delta \Sigma_{\text{rms}}}{\Sigma}\right) \propto \frac{1}{\sqrt{\beta_c}} \propto \sqrt{\alpha}.$$  \hfill (3)

Once the disc reaches a state such that $\alpha$ cannot increase to maintain thermal equilibrium, $\Delta \Sigma / \Sigma$ is typically of order unity (i.e. density perturbations become non-linear) and fragmentation can occur. Rapid cooling also helps to prevent destructive fragment–collision, as the initial fragment spacing will be such that collisions occur within a few orbital periods (Shlosman & Begelman 1989). Rapid cooling can then ensure that colliding fragments do not become unbound as a result of the collision. The combination of these criteria assures that fragmentation can only occur in the outer regions of protostellar discs, typically at radii greater than $r = 30–40$ au (Matzner & Levin 2005; Rafikov 2005; Boley et al. 2006; Whitworth & Stamatellos 2006; Stamatellos & Whitworth 2008; Clarke 2009; Forgan et al. 2009; Vorobyov & Basu 2010; Forgan & Rice 2011). In the case of AGN discs, this critical radius is approximately $r = 0.1$ pc (Levin & Beloborodov 2003; Levin 2007; Nayakshin, Cuadra & Springel 2007; Alexander et al. 2008).

This description, however, is incomplete. As fragmentation is consigned to the outer regions of self-gravitating discs, the local temperature is unlikely to be determined purely by gravitational stresses, and is likely to be governed by irradiation, either from the central object or from an external bath (e.g. envelope irradiation in embedded protostellar discs). In this regime, there is no longer a uniquely determined relation between $\alpha$ and $\beta_c$, as irradiation provides an extra term to the disc’s energy budget.

It is somewhat intuitive to assume that adding extra heating will push the disc away from marginal instability by increasing the sound speed, weakening self-gravity and therefore suppressing fragmentation. Maizner & Levin (2005) demonstrate this using an analytic prescription for protostellar disc formation and evolution from Bonnor–Ebert spheres, showing that fragmentation is typically inhibited for orbital periods lower than 20 000 yr. Rafikov (2009) also shows that fragmentation is inhibited at large orbital periods when the disc accretion rate is low.

Cai et al. (2008) studied the effect of envelope irradiation in grid-based hydrodynamic simulations with radiative transfer, showing that envelope irradiation suppresses the higher $m$ spiral modes of the gravitational instability [a result to some extent predicted by Boss (2002), although he disagrees on the role of convection in resisting this suppression].

Stamatellos & Whitworth (2008) compare stellar and background irradiation using smoothed particle hydrodynamics simulations. The background irradiation behaviour is similar to that of Cai et al. (2008) – stellar irradiation allows the outer disc regions ($r > 30$ au) to cool sufficiently rapidly to fragment according to the minimum cooling time criterion, but they no longer satisfy the Toomre $Q$ criterion.

More recently, Kratter & Murray-Clay (2011) have noted that weakening self-gravity is not generally an impediment to disc fragmentation if the full effects of mass infall are considered. As was shown by Stamatellos & Whitworth (2008), the cooling time criterion can be satisfied quite easily in irradiated discs (although maintaining a low $Q$ is difficult), and that low-mass discs in the irradiation-dominated regime may be made more susceptible to fragmentation given the correct infall rate. This is consistent with the Cai et al. (2008) finding that mild irradiation increases gravitational torques (and therefore increases the equilibrium mass infall rate).

These results should be compared with local shearing sheet simulations of irradiated discs (Rice et al. 2011). Under irradiation, there is no longer a fixed fragmentation boundary using either $\beta_c$ or $\alpha$. Comparing to the non-irradiated case, irradiated discs can fragment for lower values of $\alpha$, but still requires rapid cooling.

With the straightforward criteria for fragmentation in the non-irradiated regime becoming less clear when irradiation is incorporated, it is instructive to consider other more generalized fragmentation criteria. We have developed such a criterion, based on the local Jeans mass inside a spiral wave perturbation (Forgan & Rice 2011). If a fragment is to collapse and become bound, it must have a mass greater than the local Jeans mass. By measuring how the Jeans mass evolves with time, we can ascertain whether regions of a self-gravitating disc are becoming more or less susceptible to fragmentation. From this criterion, we are able to estimate not only when a disc fragments, but what the initial mass of the fragment should be, an important initial condition to models such as the ‘tidal downsizing’ hypothesis of terrestrial and giant planet formation (Boley et al. 2010; Nayakshin 2010a,b, 2011; Michael, Durisen & Boley 2011).

This is somewhat similar to other work which has focused on the relationship between spiral arm width and the local Hill radius (Rogers & Wadsley 2012). These criteria reflect different competing influences: the Jeans criterion compares self-gravity to local pressure forces, whereas the Hill criterion compares self-gravity to local shear. The advantage of both these criteria is that they are easily generalized to cases difficult to describe or explain by traditional minimum cooling time/maximum stress criteria.

In this paper, we apply the Jeans criterion to 1D self-gravitating disc models where the effects of irradiation are accounted for. We compare these to models where irradiation is not present, to assess whether irradiation promotes or inhibits fragmentation. In Section 2, we outline the Jeans criterion and how the 1D disc models are constructed. In Section 3 we describe and discuss the results from the models, and in Section 4 we summarize the work.

2 METHOD

2.1 Calculating the Jeans mass in irradiated spiral arms

To calculate the Jeans mass inside a spiral density perturbation, we adopt the same procedure as described in Forgan & Rice (2011). We assume that the Jeans mass is spherical:

$$M_J = \frac{4}{3} \pi \left(\frac{\pi c_s^2}{G \rho_{\text{pert}}^0}\right) \frac{3}{2} \rho_{\text{pert}} = \frac{4}{3} \pi c_s^2 \left(\frac{c_s^2}{G^2 \rho_{\text{pert}}^0}\right)^{1/2}. $$  \hfill (4)

Under the thin disc approximation

$$\rho_{\text{pert}} = \Sigma_{\text{pert}}/2H = \Sigma \left(1 + \frac{\Delta \Sigma}{\Sigma}\right) / 2H.$$  \hfill (5)

If we assume that the disc is marginally stable, we can obtain

$$M_J = \frac{4 \sqrt{2} \pi}{3G} \left(\frac{Q^{1/2} c_s^2 H}{1 + \frac{\Delta \Sigma}{\Sigma}}\right).$$  \hfill (6)

In Forgan & Rice (2011), we used the empirical result of Cossins et al. (2009) to estimate the fractional amplitude $\Delta \Sigma / \Sigma$:

$$\left(\frac{\Delta \Sigma_{\text{rms}}}{\Sigma}\right) = \frac{1}{\sqrt{\beta_c}}.$$  \hfill (7)
As we are now considering irradiated discs, it is more appropriate to use the empirical relationship determined by Rice et al. (2011):

\[ \Delta \Sigma_{\text{inst}} \Sigma = 4.47 \sqrt{\alpha} \]

(8)

For the disc to be in thermal equilibrium, the radiative cooling, viscous heating and irradiation heating must balance. The dimensionless cooling time, \( \beta_c \), is modified thus as

\[ \beta_c = \left( \frac{(r + r^*) \Sigma^2 \Omega^2 \sigma_s(T^4 - T^4_{\text{irr}})}{\sigma_s(T^4 - T^4_{\text{irr}}) \Omega^2 \gamma (\gamma - 1)} \right), \]

(9)

where \( T_{\text{irr}} \) represents the temperature of the local irradiation field.

### 2.2 The fragmentation criterion

To produce a generalized fragmentation criterion, we consider the time-scale on which the local Jeans mass changes. We define

\[ \Gamma_j = \frac{M_j}{\dot{M}_j} \Omega. \]

(10)

If this quantity is small and negative, then the local Jeans mass decreases rapidly, and fragmentation becomes favourable. Equally, if this quantity is small and positive, the local Jeans mass increases rapidly, and fragmentation becomes unlikely.

The value of \( \Gamma_j \) estimates the number of local orbital periods that will elapse before fragmentation becomes likely (if at all). In a steady-state disc in perfect thermodynamic equilibrium, \( \Gamma_j \to \infty \), and fragmentation will never occur. Discs that are not in equilibrium will assume non-infinite values of \( \Gamma_j \), and fragmentation is either likely or unlikely.

We assume \(-5 < \Gamma_j < 0\) for fragmentation, but this is yet to be established empirically. This is equivalent to requiring a region of the disc to fragment in less than five orbital periods from the moment of measuring \( \Gamma_j \). Substituting for \( H \) in equation (6) gives

\[ M_j = \frac{4 \sqrt{2} \alpha c_s^3}{3 G \Omega (1 + 4.47 \sqrt{\alpha})} \Omega^2 \frac{Q^{1/3} c_s^3}{\Omega}. \]

(11)

We can calculate \( M_j \) using the chain rule:

\[ M_j = \frac{\partial \dot{M}_j}{\partial \dot{c}_s} \dot{c}_s + \frac{\partial \dot{M}_j}{\partial \Omega} \dot{\Omega} + \frac{\partial \dot{M}_j}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial \dot{M}_j}{\partial Q} \dot{Q}. \]

(12)

We will assume \( Q = \dot{\alpha} = \dot{\Omega} = 0 \), and hence

\[ M_j = M_j 3 \dot{c}_s \dot{c}_s. \]

(13)

From equation 22 of Forgan & Rice (2011), we calculate

\[ \dot{c}_s = 1/2 \left( \frac{9 \gamma (\gamma - 1) \Omega}{4} - \frac{1}{\dot{\epsilon}_{\text{cool}}} \right), \]

(14)

and hence derive

\[ \Gamma_j = \left( \frac{3}{2} \left( -\frac{1}{\beta_c} + \frac{9 \gamma (\gamma - 1)}{4} \right)^{-1} \right). \]

(15)

As in Forgan & Rice (2011), we assume that the maximum value for \( \alpha \) saturates at some value \( \alpha_{\text{sat}} = 0.1 \), which is slightly higher than the canonical value of 0.06 (Gammie 2001; Rice et al. 2005). We take this value as a conservative estimate given current uncertainties regarding convergence of 3D simulations of fragmentation (Lodato & Clarke 2011; Meru & Bate 2011; Rice, Forgan & Armitage 2012). If the local value of \( \alpha < \alpha_{\text{sat}} \), then the system is in thermal equilibrium and \( \Gamma_j \to \infty \). Once the local \( \alpha \) reaches \( \alpha_{\text{sat}} \), it is fixed at this value, while \( \beta_c \) is allowed to decrease, reducing the local Jeans mass at a rate given by \( \Gamma_j \). Comparing \( \Gamma_j \) to the critical range of values given earlier then determines whether a fragment shall be formed.

We should note that we do not consider infall in this analysis, although we do assume that the discs reach a steady state where the disc accretion rate \( \dot{M} \) is the appropriate value to process material to maintain a constant disc mass (i.e. the local rate of change of surface density \( \Sigma = 0 \)). From the above, we could suppose that a disc experiencing infall such that \( \Sigma > 0 \) may give \( \Gamma_j < 0 \), and as such irradiation could encourage fragmentation provided that \( Q \) also does not increase. We leave investigation of this possibility to future work.

It is unclear from this analysis whether irradiation should drive or stop fragmentation – consequently, it is instructive to test this using semi-analytic disc models.

### 2.3 Simple irradiated discs with local angular momentum transport

As in Forgan & Rice (2011), we construct simple disc models in the same manner as Levin (2007) and Clarke (2009) to evaluate the dependence of the Jeans mass on disc parameters. We fix the disc accretion rate

\[ \dot{M} = \frac{3 \pi \alpha c_s^2 \Sigma}{\Omega} \]

(16)

to be constant for all radii, and impose an outer disc radius. Assuming marginal instability (\( Q = 2 \)) and thermal equilibrium then fixes the disc surface density profile. If thermal equilibrium demands \( \alpha > \alpha_{\text{sat}} \), then \( \alpha \) is set to \( \alpha_{\text{sat}} \), and \( \Gamma_j \) can then become a finite, negative quantity. If, or when, \(-5 < \Gamma_j < 0\), the disc model is assumed to fragment, and the local Jeans mass is calculated. We fix the star mass at 1 \( M_{\odot} \) and consider four scenarios:

(i) no irradiation \( (T_{\text{irr}}(r) = 0 \text{ K}) \),
(ii) envelope irradiation, \( T_{\text{irr}}(r) = 10 \text{ K}, \) at all radii,
(iii) envelope irradiation, \( T_{\text{irr}}(r) = 30 \text{ K}, \) at all radii and
(iv) irradiation from the central star.

For the third case, we use the radiation field expression given by Hayashi (1981):

\[ T_{\text{irr}}(r) = 280 \text{ K} \left( \frac{M_*}{1 \text{M}_{\odot}} \right)^{1/2} \left( \frac{r}{1 \text{au}} \right)^{-1/2}. \]

(17)

For \( \alpha_{\text{sat}} = 0.1 \) and \( \gamma = 5/3 \), we can actually determine the maximum value of \( \beta_c \) at which fragmentation occurs (i.e. where \( \Gamma_j = -5 \)), which we find to be

\[ \beta_c < 2.2. \]

(18)

Note that the conventional critical cooling time for this value of \( \gamma \) is \( \beta_c = 3 \). We have however assumed a value of \( \alpha_{\text{sat}} \) around 60 per cent larger than the usual value, so our slightly smaller value of \( \beta_c \) compares sensibly to the typically used cooling time criteria. Altering the critical value of \( \Gamma_j \) instead would also allow \( \beta_c \) to be increased to a more standard value, but this would permit discs to fragment on time-scales longer than might be considered physical.

In any case, the resulting fragment masses are only very weakly sensitive to these considerations.
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3 RESULTS

3.1 Disc profiles

Fig. 1 shows 2D contours of the disc-to-star mass ratio \( q \) in all four scenarios, given the accretion rate and the disc outer radius. The differences between the non-irradiated case (top left in Fig. 1) and the models discussed in Forgan & Rice (2011) are due to a different selection of \( Q \) (in this paper, we assume \( Q = 2 \), and in the previous work we assumed \( Q = 1 \)), but the same broad features remain: a high disc-to-star mass ratio is required to maintain a self-gravitating disc with a modest accretion rate and reasonable outer disc radius. For example, in the non-irradiated disc, an accretion rate of \( 10^{-7} \, M_\odot \, yr^{-1} \) and outer radius of \( r_{\text{out}} = 30 \, \text{au} \) demand a disc mass of \( 0.119 \, M_\odot \).

At high accretion rates, the lifetime of material in the disc becomes comparable to the orbital time-scale. For this reason, we truncate the upper limit of the contours by demanding that

\[
\frac{M_{\text{disc}}}{\dot{M}} > \frac{2\pi}{\Omega}, \tag{19}
\]

which essentially requires the disc to exist for at least five orbital periods at the given radius. Any disc model which does not satisfy this is discarded and not considered in the subsequent analysis. This explains the empty regions in the upper-right portion of each plot.

Comparing the non-irradiated case to the other three cases, we can see that the high-\( \dot{M} \) section of the parameter space (i.e. above \( \dot{M} \approx 10^{-6} \, M_\odot \, yr^{-1} \)) remains similar. The effect of irradiation becomes more apparent at lower accretion rates, forcing the equilibrium disc structure to be more massive for a given \( \dot{M} - r_{\text{loc}} \) locus. For example, in the case of background irradiation at 10 K, the \( 0.214 \, M_\odot \) contour at \( r = 150 \, \text{au} \) moves downwards from \( \dot{M} = 10^{-7} \) to \( \sim 2 \times 10^{-8} \, M_\odot \, yr^{-1} \). At \( T_{\text{irr}} = 30 \, \text{K} \), this contour intersects the \( x \)-axis at \( < 100 \, \text{au} \). Indeed, discs with radii 150 au and accretion rates of \( 10^{-7} \, M_\odot \, yr^{-1} \) have masses around \( 0.357 \, M_\odot \).

In the stellar irradiation case, the change is less striking – a \( 0.214 \, M_\odot \) disc with radius 150 au is still formed at an accretion rate of \( \sim 2 \times 10^{-8} \, M_\odot \, yr^{-1} \), but the lower \( \dot{M} \) region allows higher mass discs to have larger outer radii than the background irradiated cases.

Adding extra mass to the outer regions of self-gravitating discs will in general encourage fragmentation (Kratter et al. 2010a; Vorobyov & Basu 2010; Kratter & Murray-Clay 2011; Forgan & Rice 2012), so we might expect to see the Jeans criterion satisfied at lower accretion rates in the presence of irradiation. Conversely, increasing the disc sound speed will increase the typical fragment masses (if fragmentation can still be achieved).

3.2 Jeans masses

Fig. 2 shows the regions of parameter space in which the four scenarios can fragment, as well as the local Jeans mass at the point of fragmentation.
Figure 2. 2D contour plots of the Jeans mass inside a spiral perturbation, as a function of the steady-state 'pseudo-viscous' accretion rate $\dot{M}$ and radius in the disc $r$, for the four test cases considered in this work: without irradiation (top left), with background irradiation at 10 K (top right), with background irradiation at 30 K (bottom left), and with stellar irradiation (bottom right). Again note that the upper regions of the parameter space are excluded because the disc lifetimes are too short. Regions beneath the lowest contour are marginally unstable, but in thermal equilibrium, and do not fragment.

The non-irradiated case is qualitatively similar to that found by Forgan & Rice (2011), with the minimum Jeans mass also taking a similar value of $2.66 M_{\text{Jup}}$. Self-gravitating discs will not fragment at low disc radii (except at very high, probably unphysical accretion rates). There is a minimum accretion rate required for fragmentation even at large disc radii, corresponding in this case to around $7 \times 10^{-7} M_\odot \text{yr}^{-1}$. By comparison with Fig. 1, we can see that this sets a minimum mass at which discs can fragment at around $0.2 M_\odot$ for a disc outer radius of 50 au.

At high accretion rates, the regions of parameter space susceptible to fragmentation remain roughly similar across all four cases, but the irradiated discs show suppression of fragmentation at lower accretion rates. The 10 K irradiation case shows fragmentation at accretion rates around a factor of 1.25 higher than the no-irradiation case. This increases to a factor of 5 as the background temperature is increased to 30 K. The stellar irradiation case displays similar behaviour at low radii, but at around 70 au we see discs with low accretion rates possessing sufficiently low optical depths for the irradiation to heat the mid-plane so that $T_{\text{irr}} \sim T$. While marginally unstable solutions still exist, fragmentation is suppressed at low accretion rates. At radii of $\sim 120$ au, accretion rates as high as $10^{-5} M_\odot \text{yr}^{-1}$ are required to produce a fragmenting disc.

The minimum Jeans mass is typically larger for the three irradiated cases: $4.1 M_{\text{Jup}}$ in the 10 K irradiation case, $15 M_{\text{Jup}}$ in the 30 K case and $11.2 M_{\text{Jup}}$ for the stellar irradiation case. Also, the rate at which the Jeans mass increases with disc radius is higher than in the non-irradiated case. This further supports the creation of low-mass binary star systems as opposed to single-star planetary systems (Clarke 2009).

In short, irradiated discs tend to produce more massive fragments. There are two reasons for this:

(i) the effective $\alpha$ generated by the instability is reduced in the presence of irradiation, and hence at a given $M - r$ locus, the discs are more massive, and

(ii) the steady state temperatures in irradiated discs are higher, providing extra pressure support against gravitational collapse.

Further, if we consider the expected isolation masses of these fragments (Lissauer 1987), then we expect irradiation to push fragments well into the brown dwarf/low-mass star regime (see also Kratter, Murray-Clay & Youdin 2010b).

3.3 Surface densities at fragmentation

We have established that irradiation suppresses fragmentation at lower accretion rates, and boosts disc masses. If this is the case, the surface density at the point of fragmentation should typically be higher. In Fig. 3, we again plot contours showing the Jeans mass of the fragments, but now study the $\Sigma - r$ parameter space. As expected, lower mass fragments form at lower surface densities, and the fragment mass tends to increase with $\Sigma$. The disc lifetime criterion described in the previous section excludes part of the parameter...
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Figure 3. 2D contour plots of the Jeans mass inside a spiral perturbation for (left) a disc irradiated at a background temperature of 10 K, and (right) irradiation from the central star, as a function of the disc surface density $\Sigma$ and radius in the disc $r$, for the four test cases considered in this work: without irradiation (top left), with background irradiation at 10 K (top right), with background irradiation at 30 K (bottom left), and with stellar irradiation (bottom right). Regions without contours do not possess a fragmenting solution.

We see that irradiation does not affect the general trend at high $\Sigma$, but the low-$\Sigma$ fragmentation is suppressed. This is seen weakly in the background irradiation case, and much more strongly in the stellar irradiation case. Low-surface-density regions will generally be optically thin, and hence have their temperatures set by irradiation, remaining too warm to produce fragments.

3.4 Cumulative distributions

Although a uniform parameter space in $\dot{M}$ and $r$ is not what nature presents to us, it is instructive to see the resulting probability distributions for fragment mass (and the disc-to-star mass ratio) that can be inferred if we assume that each $\dot{M} - r$ combination in the parameter space is equally likely. These can then be convolved with the true ($\dot{M}, r$) distributions (once known) to predict the observed distributions of fragment properties.

The left-hand panel of Fig. 4 shows the cumulative distribution of fragment masses found in this uniform parameter study. As low-$\Sigma$ regions do not fragment when irradiated, and disc masses are boosted, consequently the initial fragment masses are typically higher for the background and stellar irradiation cases (where the effect becomes significant for the stellar case). The peak of the mass distribution is shifted upwards by around a factor of 2 (from around $10 M_{\text{Jup}}$ to around $20 M_{\text{Jup}}$) compared to the case without irradiation.

With the surface density at fragmentation higher in the presence of irradiation, and the disc masses typically higher for a given $\dot{M} - r$ locus, one might assume that the typical disc-to-star mass ratio would itself be higher. In the right-hand panel of Fig. 4, we plot the cumulative distribution of disc-to-star mass ratios, assuming that all fragmenting discs extend to $r = 150$ au so as to make the comparisons consistent.

The resulting distributions depend on what fraction of the $\dot{M} - r$ parameter space permits fragmentation. From Fig. 2, we can see that the non-irradiated case permits fragmentation in the largest area of parameter space, and at the lowest accretion rates. As such, the disc masses required for fragmentation are the lowest, beginning at around $0.2 M_{\odot}$ (for the selected disc outer radius of 150 au). The 10 K irradiation case fragments at slightly higher $\dot{M}$, and boosts disc mass slightly for the same accretion rate, and as such shows a slightly higher cumulative fraction. The 30 K case fragments at noticeably higher accretion rates, as does the stellar irradiation case. However, stellar irradiation suppresses fragmentation at large radii and low accretion rates, preventing lower disc masses from fragmenting. This does not occur for the 30 K case, and as such its distribution is skewed towards even higher masses than the stellar irradiation case.

4 CONCLUSIONS

We have continued our investigation of the Jeans mass in spiral arm perturbations as a means of predicting disc fragmentation and
fragment masses (Forgan & Rice 2011), extending the formalism to include the effects of irradiation, either from a constant background or from stellar irradiation. We construct simple one-dimensional disc models, where the disc maintains thermal equilibrium between the heating due to self-gravitating spiral structures, irradiation and the radiative cooling. From this, we can discover the region of $M - r$ parameter space in which fragmentation is expected to occur. We can also determine the expected fragment mass as a function of disc parameters. We compare three scenarios: non-irradiated self-gravitating discs, self-gravitating discs in a uniform temperature bath and self-gravitating discs irradiated by the central object.

Adding irradiation alters the equilibrium disc mass for a given accretion rate and outer radius, requiring higher mass self-gravitating discs to exist for these parameters. We find that in general, adding irradiation will suppress fragmentation at low accretion rates compared to the non-irradiated case. This is a similar conclusion to that drawn by Rafikov (2009). This is not surprising, as the analysis used here is similar, the only difference being the precise application of the maximum stress condition for fragmentation.

Irradiated discs typically must sustain a higher accretion rate in order to fragment, and the local surface density at fragmentation is typically higher, as low-surface-density regions will not be gravitationally unstable due to the radiative heating. The fragment mass is typically boosted, and, as a result, disc fragmentation is more likely to form brown dwarfs and/or low-mass stars than gas giant planets (unless some action of the disc or central star can prevent the fragments growing to their isolation mass).

As this work investigates azimuthally averaged disc profiles, we are not able to address the possibility of irradiated stochastic fragmentation (cf. Paardekooper 2012), which suggests that regions which experience a small density perturbation can be irradiated in such a fashion that the local cooling time is reduced, and fragmentation could subsequently occur. This may be further assisted by the opacity regime in which the perturbation resides (Cossins, Lodato & Clarke 2010).

This stochastic fragmentation condition is equivalent to a local density perturbation having a reduced Jeans mass for a sufficient duration to allow self-gravitating collapse to produce a bound object. The key issue here is: can the local value of $Q$ also remain low? As we fix $Q$ when carrying out these models, we cannot answer this question. Future work should investigate the behaviour of the Jeans mass in numerical simulations, both with and without irradiation, to first establish how the Jeans mass evolves during a ‘realistic’ fragmentation event, and also to test whether stochastic fragmentation is a sensible outcome of Jeans mass evolution.

To some extent, we therefore agree with Kratter & Murray-Clay (2011): irradiated discs that possess high accretion rates can still be driven to fragmentation, i.e. more massive discs are required with sufficient infall to maintain them, as well as high surface densities and temperatures sufficiently low that gravitational instability can still arise. We would also agree with Stamatellos & Whitworth (2008) that the last of these three conditions is likely to be the greatest obstacle to the fragmentation of irradiated self-gravitating discs.

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