CP violation in minimal supersymmetric standard model

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\textit{CP} violating phenomena predicted by the minimal supersymmetric standard model are discussed in a case where the \textit{CP} violating phases in SUSY sector are not suppressed. The electric dipole moments of the neutron and the electron are large, but can be smaller than their experimental upper bounds if the scalar quarks and leptons are heavier than a few TeV.

1. INTRODUCTION

Cabibbo-Kobayashi-Maskawa (CKM) matrix is a standard candidate for the origins of \textit{CP} violation within a framework of the standard electroweak theory. In the minimal supersymmetric standard model (MSSM)\textsuperscript{[4]} there appear some complex parameters if the supersymmetry (SUSY) is not an exact symmetry but is broken. So in addition to the phase of CKM matrix we have other sources of \textit{CP} violation in MSSM. It is well-known that these new \textit{CP} violating phases cause large electric dipole moments (EDMs)\textsuperscript{[3]}\textsuperscript{[4]} which might give a denial or a signal of the existence of SUSY.

Not only EDMs are the \textit{CP} violating phenomena expected in MSSM, but also \textit{T} or \textit{CP}–odd asymmetries in the production of SUSY particles, such as neutralinos and charginos. These asymmetries are caused by tree diagrams so that they would not be too small to be tested by the future collider experiments in which the SUSY particles might be eventually produced. In this report we shall discuss first EDMs and show that SUSY particles, especially squarks and sleptons, are as heavy as a few TeV if the phases of the SUSY parameters are of order unity. Secondly we shall discuss the \textit{T}–odd asymmetries and show that it is not impossible to observe the asymmetries as the manifestation of \textit{CP} violation originating from SUSY.

Before closing this section, for the definiteness we show the MSSM Lagrangian,

\begin{equation}
L = L_{\text{kin}} + L_{\text{gauge}} + L_F + L_S;
L_F = [(E^c Y_E L) H_1 + (D^c Y_D Q) H_1 + (U^c Y_U Q) H_2 + m_H H_1 \times H_2] F,
L_S = (\bar{E}^c \eta_E L) \tilde{H}_1 + (\bar{D}^c \eta_D Q) \tilde{H}_1 + (\bar{U}^c \eta_U Q) \tilde{H}_2 + \tilde{M}_{\tilde{H}}^2 \tilde{H}_1 \times \tilde{H}_2 + \frac{1}{2} \sum_{i=1}^3 \tilde{m}_i \lambda_i \lambda_i + \sum_{a,b} \tilde{M}_{ab}^2 \phi_a^* \phi_b, \quad (1)
\end{equation}

to fix our notation of the relevant parameters. We assume the grand unification of the fundamental interactions, and relate some of these parameters...
in (1) each other as 
\[ M_{ab}^2 = |m_{3/2}|^2 \delta_{ab}, \]
\[ \eta_f = A_m m_f^3 / 2 \]
\[ \tilde{M}_H^2 = B_m m_H^3 / 2 m_H, \]
\[ \tilde{m}_i = M_\lambda (i = 1, 2, 3), \]
(2)
to reduce the number of the parameters. The parameters \( A_m m_f^3 / 2 \), \( B_m m_H^3 / 2 m_H \), \( m_H \), and \( M_\lambda \) in (2) are complex in general, and can become sources of CP violation. However, all the phases are not physical, but two of them are physical. So in this report we take \( A_m m_f^3 / 2 \) and \( m_H \) to be complex, and \( B_m m_H^3 / 2 \) and \( M_\lambda \) to be real, for simplicity.

2. ELECTRIC DIPOLE MOMENTS

EDMs of the quarks (leptons) are given by the one-loop diagrams by exchange of the squarks \( \tilde{q} \) (sleptons \( \tilde{\ell} \)) and the charginos \( \omega_i \), the neutralinos \( \chi_j \), or the gluinos \( \tilde{g} \) (not for the lepton EDM). People think that the gluino loop gives the largest contribution to EDMs of the quarks because the strong coupling constant is much larger than the electroweak coupling constant. Its typical estimate commonly made for the \( u \)-quark is like
\[ d_u^G / e \simeq \frac{2\alpha_s}{9\pi\tilde{m}_3^2} \frac{\text{Im}(A_m m_f^3)}{m_u^3} I(\tilde{M}_q^2 / \tilde{m}_3^2), \]
\[ I(x) = \frac{1}{(1-x)^2} (1 + x + \frac{2x}{1-x} \ln x). \]
(3)
Assuming that the gluino mass \( \tilde{m}_3 \) and the squark masses \( M_q \) are 100 GeV, we would get the \( u \)-quark EDM of \( 10^{-22} e \cdot cm \). This is much larger than the present experimental upper bound of the neutron EDM, \( 10^{-25} e \cdot cm \) \( [5] \), so we might have to assume that the SUSY parameters are almost real. However fortunately or unfortunately, the SUSY particles such as squarks and gluinos have not yet been discovered, and their masses are larger than 100 GeV. Since, the heavier they are, the smaller EDM becomes, even if the SUSY phases are of order unity, sufficiently large SUSY particle masses can reduce EDM. Thus it is not a bad question to ask how heavy the SUSY particles should be to reduce EDM to the value smaller than its experimental bound.

To answer this question we calculate the EDM of the neutron arising from the gluino, chargino, and neutralino loops taking maximal CP violating phases. The results are shown in Figs. 1–4.

Figure 1. (i) The chargino, (ii) neutralino, and (iii) gluino contributions to the neutron EDM for \( \tan \beta = 2, \tilde{m}_2 = |m_H| = 0.5 \text{TeV} \).

Table 1
| \( \tilde{m}_2 \) (TeV) | \( |m_H| \) (TeV) |
|----------------------|----------------|
| (i) 0.2             | 0.2            |
| (ii) 1.0            | 0.2            |
| (iii) 0.2           | 1.0            |
| (iv) 1.0            | 1.0            |

The Fig. 3 shows as a function of the squark mass the absolute values of EDMs by the chargino, neutralino, and gluino loops which contribute to the neutron EDM. It should be noted that EDM from the chargino is larger than EDM from the gluino in this case, which would be contrary to the naive expectation. The reason of this comes mainly from the GUT mass relation \( \alpha_2 / \alpha_3 \tilde{m}_3 = \tilde{m}_2 \) which we assumed in
the calculation. The large $SU(3)$ coupling $\alpha_3$, which could make $\bar{m}_2$ large, leads to the large gluino mass, which almost cancels the enhancement by the strong coupling. In addition to this, since the mixing between left-handed and right-handed squarks, which is needed for the EDM operator by the gluino loop, is small, the magnitude of the gluino contribution to the neutron EDM is suppressed to be smaller than that of the chargino.

The Fig. 2 shows the chargino contribution to the neutron EDM for the different values of the parameters, $\tan\beta$, $\bar{m}_2$, and $|m_H|$, as a function of the squark mass. In this parameter region the chargino contributes to the neutron EDM the most, and therefore its contribution can be taken to be approximately equal to the whole neutron EDM. From the figure it can be seen that, if the squark mass is larger than about 3 TeV for $\tan\beta = 2$ and 7 TeV for $\tan\beta = 10$, the predicted value can be smaller than the present experimental upper bound of $10^{-25}$ e·cm, even though the SUSY $CP$ phase is of order one. This is the light gaugino case.

Can the squarks be lighter than 3 TeV? Figs. 3 and 4 try to answer the question, and show the squarks can be really as light as 0.2 TeV if $\bar{m}_2$ and $|m_H|$ are heavier than about 2 TeV. However, since EDM does not decrease as rapidly as in Fig. 2 (even begins to increase when $|m_H|$ becomes large!), the light squark possibility would be denied if the experimental bound on the neutron EDM is reduced by one order of magnitude.

The electron EDM is shown in Fig. 5. Its present experimental upper bound is $10^{-26}$ e·cm [6]. If the slepton is heavier than about 1 TeV for $\tan\beta = 2$ and 4 TeV for $\tan\beta = 10$, the predicted value can be smaller than the present experimental limit.

3. $T(CP)$–ODD ASYMMETRY

In the previous section it was realized that the present experimental bounds on the neutron and electron EDMs do not exclude the possibility that the SUSY $CP$ phases are of order unity. What $CP$ violating phenomena will be expected to come out from these new phases in high energy collider experiments? One possibility is the $T$–odd asymmetry in the production and
The neutron EDM as a function of $\tilde{m}_2$ for $\tan \beta = 2$. The higgsino mass $|m_H|$ is 1 TeV for (a) and 3 TeV for (b). The squark mass $\tilde{M}_q$ is 0.2 TeV for (i) and 1 TeV for (ii).

The decay processes of the SUSY particles. In MSSM the asymmetry can appear at the tree level. To be definite, let us consider the production processes of two different charginos mediated by $Z$-boson,

$$e^+ e^- \rightarrow \omega_2^+ \omega_1^-,$$

and two different neutralinos,

$$e^+ e^- \rightarrow \chi_i \chi_j.$$

The charginos and the neutralinos are the mixed states of the $SU(2) \times U(1)$ gauginos and the higgsinos. To get them as the mass eigenstates, we have to diagonalize their mass matrices by unitary matrices which contain imaginary phases originating from the complex SUSY parameters. When we rewrite the gaugino and higgsino couplings to $Z$ by the mass eigenstates through the unitary transformations, there appear off-diagonal couplings of the charginos and neutralinos to $Z$. Their coupling constants are made from the complex elements of the unitary matrices. Moreover their relative phases can not be absorbed, and remain physical. Thus these off-diagonal couplings to $Z$ cause $T(CP)$ violation at the tree level, and the violation can be expected to be large enough to be observed.

First we discuss the $T$–odd asymmetry in the chargino production process (4). When the spin of $\omega_2$, $s_2$, is summed, but the spin of $\omega_1$, $s_1$ (which will be chosen such as it is perpendicular to the interaction plane), is not summed, the cross section becomes in the CM system of $e^+ e^-$ as

$$\sum_{s_2} d\sigma(e^+ e^- \rightarrow \omega_2^+ \omega_1^-)$$

$$= \frac{G_F^2 \ p \ M_Z^4}{2 \pi \sqrt{3} (S - M_Z^2)^2 + \Gamma_M^2 M_Z^2} \times$$

$$\left[ (f_L^2 + f_R^2)((G_{L}^{12})^2 + (G_{R}^{12})^2)(E_1 E_2 + p^2/3) + 2\Re(G_{L}^{12} G_{R}^{12*}) m_{\omega_1} m_{\omega_2} \right] + \text{sign}(s_1)(f_L^2 - f_R^2)\Im(G_{L}^{12} G_{R}^{12*}) \frac{\pi}{2} m_{\omega_2} p,$$

where $p$ is the magnitude of the momentum of the chargino, $E_{1,2}$ are the energies of $\omega_{1,2}$, $G_{L,R}^{12}$ are the coupling constants of the charginos to $Z$ defined as

$$J_L^Z = e \omega_1^\gamma_\mu [G_{L}^{11} P_- + G_{R}^{11} P_+] \omega_1$$

$$+ e \omega_2^\gamma_\mu [G_{L}^{22} P_- + G_{R}^{22} P_+] \omega_2$$

$$+ e \omega_1^\gamma_\mu [G_{L}^{12} P_- + G_{R}^{12} P_+] \omega_2$$
The asymmetry of these two states manifests a vector product \( p \times p_1 \) means.

The asymmetry of these two states is defined as \( \sum_{s_j} d\sigma(e^+e^\rightarrow \chi_i\chi_j) \) for the charged lepton EDMs of these parameter set in Table 3 by taking the squark and slepton masses for 3 TeV. Their values are smaller than the present experimental bounds [6].

The calculated asymmetry is shown in Fig. 6 as a function of the CM energy \( \sqrt{S} \). When the incident electron beam is unpolarized, the magnitude of the asymmetry is smaller than \( 10^{-2} \).

\begin{align}
&+ e\tilde{\omega}_2 \gamma_\mu [C_L^{21} P_- + C_R^{21} P_+] \omega_1, \\
&f_{L,R} \text{ are the coupling constants of the electron to } Z, \text{ and } \text{sign}(s_1) \text{ is defined by the three momentum of the incident electron, } p^-, \text{ and the three momentum of } \omega_1, p_1, \text{ as } \text{sign}(s_1) = \text{sign}(s_1 \cdot (p^- \times p_1)).
\end{align}

The term breaking \( T \) invariance is the last term in \( \delta \). Because it depends on \( s_1 \), we must observe at least a spin state of either chargino to detect the \( T \) violation effect. The positive sign of \( s_1 \) means a vector product \( s_1 \cdot (p^- \times p_1) \) is positive, and the negative sign of \( s_1 \) means \( s_1 \cdot (p^- \times p_1) < 0 \). Since these two states are transformed to each other by time reversal, the difference of the cross section between these two states manifests \( T \) violation at the first-order perturbation. The asymmetry of these two states

\begin{align}
&\sum_{s_j} d\sigma(e^+e^\rightarrow \chi_i\chi_j) \\
&= \frac{G_F^2}{4\pi} \frac{p}{\sqrt{S}} \frac{M_Z^4}{(S - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} |O_Z|^2 \times \\
&\times \left[(f_L^2 + f_R^2)(E_i E_j + p^2/3 - \cos 2\delta_Z m_{\chi_i} m_{\chi_j}) \right] + \text{sign}(s_i)(f_L^2 - f_R^2) \frac{\pi}{4} \sin 2\delta_Z m_{\chi_j} p, \\
&\text{where } p \text{ is the magnitude of the momentum of the neutralino, } E_{i,j} \text{ are the energies of } \chi_{i,j}, \text{ and sign}(s_i) = \text{sign}(s_i \cdot (p^- \times p_1)). O_Z \text{ is the coupling constant of the neutralinos to } Z \text{ defined as}
\end{align}

\begin{align}
&L^Z_{\text{int}} = \frac{e}{\sin 2\theta_W} \chi_i \gamma_\mu (O_Z P_- - O_Z^* P_+) \chi_j Z^\mu; \\
&P_\pm = (1 \pm \gamma_5)/2,
\end{align}
and $\delta Z = \arg(O Z)$. Again we have the $T$ violating term in (6), and we can define an asymmetry

$$A_T = \frac{d\sigma(s_i \cdot (p^- \times p_i) > 0) - d\sigma(s_i \cdot (p^- \times p_i) < 0)}{d\sigma(s_i \cdot (p^- \times p_i) > 0) + d\sigma(s_i \cdot (p^- \times p_i) < 0)},$$

which quantifies how large the $T$ violation is. It should be noted that, differently from the chargino production, the final state of the process (5) is an eigenstate of $CP$ because of the neutralinos being the Majorana particles, so that the asymmetry (11) is a $CP$–odd asymmetry. Thus a non-vanishing value of (11) directly means the violation of $CP$. The calculated asymmetry is shown in Fig. 7 as a function of $\sqrt{S}$. The magnitude of the asymmetry is approximately the same as the chargino one. We also have calculated an asymmetry of the lepton pair produced by the decay of $\chi_j$, $A'_T$, which is more realistic than (8) and (11) because we only have to measure the momenta of the final state leptons, $p^\pm_D$, instead of the spin of $\chi_j$ being experimentally difficult to be measured. In Fig. 8 the result in the left-handedly polarized electron beam case is shown by dots.

The magnitude of $A'_T$ is the same order as $A_T$ near the threshold, but it rapidly decreases as $\sqrt{S}$ becomes large. This is because, as $\sqrt{S}$ becomes large, $E_i$ also becomes large so that $p^+_D$ and $p^-_D$ come to orient to the same direction. Since the $T$ violation term is proportional to $p^- \cdot (p^+_D \times p^-_D)$, $A'_T$ decreases as $\sqrt{S}$ gets large. Therefore, if one wishes to get a large asymmetry, the asymmetry should be measured near the threshold.

4. DISCUSSION

In this report we have viewed $CP(T)$ violation in MSSM. The present experimental bounds on the neutron and electron EDMs do not immediately rule out the possibility that the imaginary phases of the SUSY parameters have their natural values of $O(1)$. If this is the case, the $CP$ violation originating from MSSM could lead to $T(CP)$–odd phenomena in high energy experiments. If the charginos and neutralinos are produced in $e^+ e^-$ collisions, $T$–odd asymmetry would be observed in the angular distribution of the final decay products.

In the $T(CP)$-odd asymmetries discussed here two different mass eigenstates of the charginos or neutralinos are involved. In the pair production of the charginos or neutralinos of the same mass eigenstate, does the asymmetry appear? The answer is ‘yes’ for the charginos. The electric and the weak “electric” dipole moments of the charginos, $D_\omega$, are generated at one-loop level, which break $T$ invariance. These dipole moment terms give rise to $T$–odd asymmetry in the same chargino pair-production processes. However, from the dimensional grounds, $A_T$ in this case can be roughly estimated as $A_T \sim \sqrt{S} D_\omega/e$. The main contributions to $D_\omega$ are given by the loop diagrams involving the top quark, the top squarks, the $W$ bosons, and/or the Higgs bosons. The top or stop contribution to $D_\omega$ is proportional to the large top-quark mass. However, within MSSM the magnitude of $D_\omega$ is strictly restricted by the upper bounds of the neutron and electron EDMs, and it would be at most $10^{-20}\, e\, \text{cm}$, if $m_t \sim 150\, \text{GeV}$ and unless one of the scalar quarks is accidentally much lighter than the other scalar quarks and leptons. The contri-
butions from the W bosons, and the Higgs bosons are roughly estimated to be also $10^{-20}$ e·cm. Thus we could expect in $e^+e^- \rightarrow \omega_i \bar{\omega}_i A_T \sim 10^{-4}$ which would be too small for detection in the future $e^+e^-$ experiments.

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