On Field Theory of Open Strings, Tachyon Condensation and Closed Strings

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Abstract. I review the physical properties of different vacua in the background independent open string field theory.

One of the most popular candidates for field theory of open strings is the classical action proposed by Witten in 1985 [1] - cubic CS action. The first test that any string field theory action shall be subject to is to recover all tree level as well as loop amplitudes (which are independently known exactly from world-sheet approach) by standard field theory methods, and cubic action does produce correct tree level amplitudes [2]. It seems that if tree level amplitudes are recovered by unitarity one can reconstruct all perturbative, loop, amplitudes. But, it is known that one loop diagram in any field theory of open strings shall contain the closed string pole (the cylinder diagram can be viewed as one loop in open string theory, or equivalently as a tree level propagator for closed strings; thus there shall be a pole for all on-shell closed string momenta); from the point of view of open string field theory these new (closed string) poles violate unitarity since corresponding degrees of freedom are not (at least in any obvious way) present in the particle spectrum of classical lagrangian. This situation is similar to the one in anomalous gauge theory, but in the latter one can choose the representation for fermions such that anomaly cancels. In open string field theory case one can make arrangements when closed string poles decouple (for example in topological or non-commutative setup) but it seems very interesting to study other possibilities.

One can think of two options: 1. find the closed string degrees of freedom as “already being present” in given classical open string field theory lagrangian, or 2. introduce them as additional degrees of freedom (for example by adding corresponding string field together with its lagrangian plus the interaction term with open string field).

In the lines of the option 2 the solution to the above problem has been found many years ago by B. Zwiebach (see recent version [3] which also contains references on old work). In this approach one shall take care of the problem of multiple counting when closed string field and its lagrangian is added - the same Feynman

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diagram will come both from open and closed string sectors; thus one shall make
sure that each diagram is properly counted only once; the solution of this problem
is rather complicated and requires the detailed knowledge of the string amplitudes
to all loop order from world-sheet approach.

It is very interesting to explore option 1 instead. In order to do so one needs to
have a truly background independent open string field theory. Unfortunately such
a theory has not yet been written although the theory which doesn’t depend (at
least formally) on the choice of open string background is known [5, 6, 7, 8] (it
also passes the test of reproducing all tree level on-shell amplitudes in a very simple
way since corresponding action on-shell is given by world-sheet partition function
on the disk as it was explained in [6, 8]).

One might try to make option 1 more precise by exploring the idea of closed
string degrees of freedom being some kind of classical, solitonic, solutions of open
string field theory. More concretely: start with open string field theory (background
independent) and find the new background - closed string. This seems to be a
natural way of “reversing the arrow” which describes D-branes (open strings in
case of space-filling branes) as solitons in closed string theory [9].

The above line of thoughts suggests that we shall change the point of view
about branes in general and think about them as solitons in open string field theory
rather than in closed one. In fact, if one considers the maximal dimensional brane
(or brane-anti-brane system) it is very easy to think about lower-dimensional
branes as solutions of classical equations of motion for corresponding open string
field theory action in the formalism of background independent open string field
theory. The latter is defined via the action $S(t)$ - a functional on the space of
boundary conditions for bosonic string on the disc with critical points $t = t_\ast \rightarrow$
being the conformal boundary conditions. Since for the trivial bulk backgrounds
(Δ operator on world-sheet) mixed Dirichlet-Neumann conditions (D-branes) are
conformal (in fact in this case these are only conformal boundary conditions) they
shall correspond to critical points of space-time action for open strings [6, 10].

Another motivation for the search of closed strings in open string field theory
(at least for the present author) comes from Matrix Strings [11]. If we take the
soliton corresponding to $N$ D1-strings in open string field theory and look on the
dynamics of collective modes we find (in strong coupling for 2d theory on D1 and
large $N$) the spectrum of closed strings in the same space-time where open strings
live, thus we might ask the question whether these closed string degrees of freedom
are already present in original theory of open strings where we had D1’s as solitons
in the beginning. This observation also might help in making contact with option
1 described above. One shall mention that the search for closed strings in open
string field theory has a long history and was originated in [12]: in the above
context, more in the lines of current developments related to D-branes, the interest
has been revived in [10].

The conjectures put forward by A. Sen [13, 14] made it possible to study
these questions in much more precise terms. For simplicity one considers the open
bosonic string in 26 dimensions ($D25$, or any $Dp, p < 26$) which contains tachyon
and is unstable. Three conjectures made by A. Sen are:

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1One shall note that adding closed string fields to open string field theory lagrangian is very
similar to the formalism of [4] for quantization of anomalous gauge theories.
1. Tachyon potential takes the form:

\[ V(T) = M f(T), \]

with \( M \)-mass of D-brane and \( f \)-universal function independent of the background where brane is embedded. The conjecture of Sen states that \( f(T) \) has a stationary point (local minimum) at some \( T = T_c < \infty \) such that

\[ f(T_c) = -1 \]

and thus \( M + V(T_c) = M(1 + f(T_c)) = 0 \).

2. There are soliton configurations on unstable D-branes which correspond to lower-dimensional branes.

3. New vacuum, at \( T_c \), is a closed string vacuum and in addition there are no open string degrees of freedom.

One might be tempting to amplify the Conjecture 3 a bit \([10]\) and claim that in properly defined open string field theory there shall be an expansion around new critical point which will describe the theory of closed strings (without open string sector; of course in this theory of closed strings we again can discuss open strings as solitonic branes). Apriori there is no reason to assume that such expansion should exist since the potential not necessarily shall be analytic, but one just can hope to see whole closed string sector and not just vacuum by starting from classical lagrangian for open strings. We will comment on this question at the end of this talk. One shall note that the picture described below together with the corresponding tachyon potential is very attractive from the point of view of applications of string theory to the phenomenology related to branes and also to stringy cosmology.

We will address these problems in the formalism of \([5, 6, 7, 8]\) and present the exact tachyon lagrangian up to two derivatives in tachyon field, which provides the important tool in verifying Sen’s conjectures; more detailed discussion and references can be found in \([15, 16, 17, 18]\) for the bosonic case and \([19]\) for superstring. The important questions related to the description of multiple-branes in the formalism of background independent open string field theory and unified treatment of RR couplings is studied in \([20]\).

Following \([5]\) one starts with world-sheet description of critical bosonic string theory on disk. Consider the map of the disk \( D \) to space-time \( M \):

\[ X(z, \bar{z}) : D \to M \]

In general one can consider any critical 2d CFT coupled to 2d gravity on the disk but it is interesting to study the particular case of critical bosonic string with flat 26 dimensional space-time \( M = R^{1,25} \).

Two-dimensional quantum field theory on the string world-sheet is given by the path integral:

\[ <...> = \int [dX][db][dc] \quad e^{-I_{0}(X,b,c)}... \]

\[^{2}\text{The same tachyon lagrangian, which we will present below for bosonic string and its analog for supersymmetric case was proposed in \([20]\) as a toy model that mimics the properties of tachyon condensation. Very impressive progress has been achieved in verifying Sen’s conjectures in the cubic string field theory of \([6]\); see contributions of A. Sen, B. Zwiebach and W. Taylor in the proceedings of this, Strings 2001, conference. One should note that the world-sheet approach to the problem was discussed previously in \([21]\).}\]
Define BRST operator through the current $J_{BRST}$ and contour $C$ (note this is a closed string BRST operator):

$$Q_{BRST} = \int_C J_{BRST}; \quad Q_{BRST}^2 = 0$$

Denote the limit when contour $C$ approaches the boundary $\partial D$ by $Q$:

$$Q = \int_{C \rightarrow \partial D} J_{BRST}.$$

Now we consider the local operator $\mathcal{V}(X, b, c)$ of the form

$$V = b_{-1} \mathcal{O}(X, b, c), \quad b_{-1} = \int_{C \rightarrow \partial D} v^i b_{ij} \epsilon^l_{ij} dx^k$$

and deform the world-sheet action:

$$I_{ws} = I_0 + \int_{C \rightarrow \partial D} \mathcal{V}(X(\sigma))$$

The simplest case is when ghosts decouple: $\mathcal{O} = cV(X)$. The boundary term in the action modifies the boundary condition on the map $X^\mu(z, \bar{z})$ from the Neumann boundary condition (this follows from $I_0$) $\partial_r X^\mu(\sigma) = 0$ to “arbitrary” non-linear condition:

$$\partial_r X^\mu(\sigma) = \frac{\partial}{\partial X^\mu(\sigma)} \int_{\partial D} V(X)$$

$I_{ws}$ defines the family of boundary 2d quantum field theories on the disk. The action $S(\mathcal{O})$ is defined on this space (more precisely - on the space of $\mathcal{O}$’s) and is formally independent of the choice of particular open string background:

$$dS = < d \int_{\partial D} \mathcal{O} \{ Q, \int_{\partial D} \mathcal{O} >$$

Since $d\mathcal{O}$ is arbitrary all solutions of the equation $dS = 0$ correspond to conformal, exactly marginal boundary deformations with $\{ Q, \mathcal{O} \} = 0$, and thus to valid string backgrounds.

A very important question at this stage is to understand what is the space of deformations given by $V(X(\sigma))$. An obvious assumption (which is also a very strong restriction, see the comment at the end of this talk) is - $V$ can be expanded into “Taylor series” in the derivatives of $X(\sigma)$:

$$V(X) = T(X(\sigma)) + A_\mu(X(\sigma)) \partial X^\mu(\sigma) + C_{\mu\nu}(X(\sigma)) \partial X^\mu(\sigma) \partial X^\nu(\sigma) + D_\mu(X(\sigma)) \partial^2 X^\mu(\sigma) + ...$$

Thus the action now becomes the functional of coefficients: $S = S(T(X(\sigma)), A_\mu(X(\sigma)), ...)$. It is almost obvious that the above assumption singles out the open string degrees of freedom from very large space of functionals of the map $X^\mu(\sigma) - \partial D \rightarrow M$. The goal is to write $S$ as an integral over the space-time $X$ (constant mode of $X(\sigma)$: $X(\sigma) = X + \phi(\sigma), \int \phi(\sigma) = 0$) of some “local” functional of fields $T(X), A(X), ...$ and their derivatives.
In a more general setup one can introduce some coordinate system \( \{ t^1, t^2, \ldots \} \) in the space of the boundary operators - \( O = O(t, X(\sigma)); V = V(t, X(\sigma)) \):

\[
dO = \sum dt^i \frac{\partial}{\partial t^i} O(t), \quad dV(t, X(\sigma)) = \sum dt^i \frac{\partial}{\partial t^i} V(t, X(\sigma))
\]

At the origin, \( t^i = 0 \), we have an un-deformed theory and the linear term in the deformation is given by an operator \( \int_{\partial D} V_i \) of dimension \( \Delta_i \) from the spectrum:

\[
I_{ws} = I_0 + t^i \int_{\partial D} V_i + O(t^2) = I_0 + t^i \frac{\partial}{\partial t^i} \int_{\partial D} V(t)|_{t=0} + O(t^2)
\]

For the general boundary term \( \int_{\partial D} V \) one might worry that the two-dimensional theory on the disk is not renormalizable; it makes sense as a cutoff theory, but it turns out that if one perturbs by a complete set of operators from the spectrum the field theory action is still well-defined (see discussion at the end \([8]\); for the tachyon T and gauge field A world-sheet theory is obviously renormalizable). It has been proven that the action \([1.11]\) can written in terms of world-sheet \( \beta \)-function and partition function \([3, 8]\):

\[
S(t) = -\beta^i(t) \frac{\partial}{\partial t^i} Z(t) + Z(t)
\]

here \( \beta^i \) is the beta function for coupling \( t^i \), a vector field in the space of boundary theories, and \( Z(t) \) - the matter partition function on the disk.

Since equations of motion \( dS = 0 \) coincide with the condition that deformed 2d theory is exactly conformal we have

\[
\frac{\partial}{\partial t^i} S(t) = G_{ij}(t) \beta^j(t)
\]

with some non-degenerate Zamolodchikov metric \( G_{ij}(t) \) (the equation \( \partial S(t) = 0 \) shall be equivalent to \( \beta^i = 0 \)). In addition we see that on-shell

\[
S_{on-shell}(t) = Z(t)
\]

and as a result classical action on solutions of equation motion will generate correct tree level string amplitudes.

It is important to note that in general the total derivatives don’t decouple inside the correlation function and we have coupling dependent BRST operator \( Q(t) \). In fact the same contact terms contributes to \( \beta \)-function. More precisely one can fix the contact terms from the condition that the definition \([1.11]\) is self-consistent after contact terms are included:

\[
Q = Q(t); \quad < \ldots \int \partial_{\theta} \ldots > \neq 0
\]

\[
d \left[ dS = < d \int_{\partial D} O \{ Q, \int_{\partial D} O \} > \right] = 0
\]

If one assumes that \( Q \) is constant and total derivatives decouple - \([1.17]\) is a simple Ward identity; otherwise it is a condition which relates the choice of contact terms with \( t \) dependence of corresponding modes of \( Q \) \([3, 8]\). Usually, in quantum field theory one has to choose the contact terms (regularization) based on some (symmetry) principle (an example from recent years is the Donaldson theory which rewritten in terms of Seiberg-Witten IR description requires the choice of the contact terms based on Seiberg-Witten modular invariance together with topological
Q symmetry \([23]\) and dependence of \(Q\) on moduli is fixed from this consistency principle); here we have \((0.13), (0.17)\) as guiding principle.

The principle \((0.17)\) leads to the formula \((0.14)\) for the action with all non-linear terms included: \(\beta_i = (\Delta_i - 1)t_i + c_{ijk}t_j t_k + ...\) and in addition guarantees that Zamolodchikov metric in \((0.13)\) is non-degenerate. In second order all terms except those that satisfy the resonant condition \(\Delta_j - \Delta_i + \Delta_k = 1\) can be removed by redefinition of couplings; obviously these correspond to logarithmic divergences and thus if the theory is perturbed with the complete set of operators - it is logarithmic.

More generally - from Poincare-Dulac theorem (which of course is for the finite-dimensional case but we will assume it is correct for infinite-dimensional space also) one can show that all coefficients are cutoff-independent after an appropriate choice of coordinates is made. Let us repeat in regard to the choice of coordinates - as it follows from the above discussion, any choice of coordinates is good as long as equations of motion lead to \(\beta\)-function equations with invertible Zamolodchikov metric \(G_{ij}\).

First we turn on only tachyon: \(V(X(\sigma)) = T(X(\sigma))\). It is not difficult to find the action \(S(T)\) as an expansion in derivatives; for example - exact in \(T\) and second order in derivatives \(\partial\). We know the derivative expansion of \(\beta\) and \(Z\):

\[\beta^T(X) = [2\Delta T + T] + a_0(T) + a_1(T)(\partial T) + a_2(T)(\partial^2 T) + a_3(T)(\partial T)^2 + ...\]  

\[(0.18)\]

\[Z(T) = \int dX e^{-T} (1 + b(T))(\partial T)^2 + ...\]  

\[(0.19)\]

with appropriate conditions for unknown coefficients dictated by the perturbative expansion. In this concrete case the basic relation becomes:

\[S(T) = - \int dX \beta^T(X) \frac{\partial}{\partial T(X)} Z(T) + Z(T)\]  

\[(0.20)\]

The condition for \(\beta\) in \((0.18)\) to be an equation of motion for \(S(T)\) \((0.20)\) with \(Z\) given by \((0.19)\) in lowest order in \(T\) (around \(T = 0 \rightarrow 2\Delta T + T = 0\)) fixes the two derivative action (and relevant unknowns \(a_i(T), b(T)\)):

\[S(T) = \int dX [e^{-T}(\partial T)^2 + e^{-T}(1 + T)]\]  

\[(0.21)\]

with equations of motion, \(\beta^T\) and metric \(G = e^{-T}\) in \((0.15)\):

\[\partial T S = e^{-T} \beta^T = e^{-T} (2\Delta T + T - (\partial T)^2) = 0\]  

\[(0.22)\]

\[\beta^T = 2\Delta T + T - (\partial T)^2; \quad Z(T) = \int e^{-T}\]  

\[(0.23)\]

This answer was deduced from the consistency condition for the expansion \((0.18), (0.19)\) and basic relation \((0.20)\), but one can compute it directly from world-sheet definition, practically repeating the computations (in this particular case) leading to general relation \((0.14)\). We first write the world-sheet path integral only in terms of boundary data \(\int dX(\theta)e^{-I_{ws}} = \int dX d\phi(\theta)e^{-I_{ws}}\) (we use the notation \(H(\theta, \theta') = \frac{1}{2} \sum_k e^{i(k(\theta - \theta'))}|k|\); bulk part decouples since arbitrary map \(X(z, \bar{z})\) can
be written as the sum of two terms: one has zero value on the boundary, another coincides with $X(\sigma)$ on boundary and is harmonic in the bulk):

\begin{equation}
X(\theta) = X + \phi(\theta); \quad \int \phi(\theta) = 0;
\end{equation}

\begin{equation}
I_{ws} = \int d\theta d\theta' X^\mu(\theta)H(\theta, \theta')X_\mu(\theta') + \int d\theta T(X(\theta))
\end{equation}

\begin{equation}
= T(X) + \int \phi^\mu(\theta)[H(\theta - \theta')\delta_{\mu\nu} + \delta(\theta - \theta')\partial_\mu \partial_\nu T(X)]\phi^\nu(\theta') + O(\partial^3 T(X))
\end{equation}

In the two derivative approximation contribution comes only from:

\begin{equation}
Z(T) = \int dX e^{-T(X)}d\alpha e[H + \partial^2 T]^{-\frac{1}{2}}
\end{equation}

This can be computed exactly. For $\partial_\mu \partial_\nu T = 3\partial^\alpha \partial_\alpha T$ it is \footnote{In fact (1.27) can be used only for two-derivative approximation for the action since the contribution of the last term in (0.26) will mix with higher order terms coming from $\Gamma$ function in (0.24), after integration by parts due to the presence of universal exponential $e^{-T}$ factor in the action; this is very similar to what happens in the attempts to write non-abelian version of Born-Infeld action.} (in some regularization)

\begin{equation}
Z = \int dX e^{-T(X)} \prod_\mu \sqrt{\partial^2 T(X)e^{3\partial^2 T(X)}\Gamma(\partial^2 T(X))}
\end{equation}

and in the two-derivative approximation this gives \footnote{If we turn on other fields from (0.12) it immediately follows from corresponding expression of the type (0.24) that only tachyon will condense; see discussion in [16].}:

\begin{equation}
Z(T) = \int dX e^{-T(X)}(1 + b(T))(\partial T)^2
\end{equation}

\begin{equation}
\beta^T = 2\Delta T + T
\end{equation}

\begin{equation}
b(T) = 0, \quad a_0(T) = a_1(T) = a_2(T) = a_3(T) = 0
\end{equation}

Thus the action (0.14) is:

\begin{equation}
S(T) = \int dX[e^{-T}(2(\partial T)^2 + e^{-T}(1 + T)]
\end{equation}

with equations of motion:

\begin{equation}
e^{-T}(T + 4\Delta T - 2(\partial T)^2) = 0
\end{equation}

Now we see that Zamolodchikov metric $G$ becomes :

\begin{equation}
G(\delta_1 T, \delta_2 T) = \int dX e^{-T}\delta_1 T \delta_2 T - 2(\partial_\mu \delta_1 T)(\partial_\mu \delta_2 T))
\end{equation}

and equations of motion (0.30) can be written in this approximation as $G\beta = 0$:

\begin{equation}
e^{-T}(1 + 2\Delta - 2\partial_\mu \partial T \partial_\mu + ...) (2\Delta T + T) = 0
\end{equation}

The linear form of $\beta$ for arbitrary $T(X)$ looks strange since we miss a possible higher order in $T$ (but second order in $\partial T$) terms which shall come from a 3-point function. In addition this metric is rather complicated and is not an obvious expansion of some invertible metric in the space of fields. Thus, according the principle for the
choice of coordinates, we need to choose new coordinates such that the metric is invertible. Such new variables are given by:

\[(0.33)\]

\[T \rightarrow T - \partial^2 T + (\partial T)^2\]

This modifies the \(\beta\) function (without changing its linear part) and metric to \((0.22), (0.23)\); in new coordinates action is given by:

\[S(T) = \int dX [e^{-T}(\partial T)^2 + e^{-T}(1 + T)]\]

The potential in this action has unstable extremum at \(T = 0\) (tachyon) and stable at \(T = \infty\). The difference between the values of this potential is 1, exactly as predicted by A. Sen in **Conjecture 1**.

In a new variable with the canonical kinetic term \(\Phi = e^{-\frac{T}{2}}\):

\[(0.34)\]

\[S(\phi) = \int [4(\partial \Phi)^2 - \Phi^2 \log \Phi] e^\Phi\]

(Interestingly this is also an exact action, see [15], for a \(p\)-adic string for \(p \rightarrow 1\)).

In the unstable vacuum \(T = 0\), \(\Phi = 1\); \(m^2 = -\frac{1}{2}\); in new vacuum - \(T = \infty\), \(\Phi = 0\) and one could naively think that \(m^2 = +\infty\), but since the action is non-analytic at this point the meaning is unclear.

\(\text{DN boundary conditions, } p \leq 25:\)

\[(0.35)\]

\[\partial_r X^a(\sigma) = 0, \quad a = 1, \ldots, p; \quad X^i(\sigma) = 0, \quad i = p + 1, \ldots, 26\]

are obviously conformal. We conclude that they give critical points of string field theory action. In addition since the value of classical action is always \(S(t_c) = Z(t_c)\) - we conclude that these solitons are in fact \(Dp\)-branes; e. g. one can take \([5]\).

\[(0.36)\]

\[T(X) = a + u_\mu (X^\mu)^2 \Rightarrow \partial_r X^\mu = u_\mu X^\mu; \quad u_i \rightarrow \infty, \quad u_a \rightarrow 0\]

This verifies the **conjecture 2**.

**Conjecture 3** is in the heart of the discussion we had in the beginning, and is also most difficult one. In order to address it we add the gauge field. The action in two derivative approximation can be constructed using the same basic relation \((0.14), [18]\). Here we will also introduce the (background) closed string fields \(G\) and \(B\) (for covariance):

\[(0.37)\]

\[S(G, B, A, T) =
\]

\[S_{\text{closed}}(G, B) + \int d^26 X \sqrt{G}[e^{-T}(1 + T) + e^{-T}||dT||^2 + \frac{1}{4} e^{-T}||B - dA||^2 + \cdots ]\]

One can choose a different regularization and obtain the action which is an expansion of \(\text{BI}\) action

\[
\int V(T) \sqrt{\text{det}(G - B + dA)}
\]

but it would lead to a complicated and not obviously non-degenerate metric, exactly like in purely tachyon case discussed above \((0.32)\).

In \(\Phi = e^{-\frac{T}{2}}\) coordinates:

\[(0.38)\]

\[S(G, B, A, \Phi) =
\]

\[S_{\text{closed}} + \int d^26 X \sqrt{G}[\Phi^2(1 - 2 \log \Phi) + 4||d\Phi||^2 + \frac{1}{4} \Phi^2||B - dA||^2 + \cdots ]\]
The latter immediately suggests the analogy with an abelian Higgs model for complex scalar and gauge field in angular coordinates $H e^{i \phi}, A$:

\[
S(H, \phi, A) = S_{YM}(A) + \int \left[ \lambda (H^2 - H_0^2)^2 \right] + |dH|^2 + H^2 |A - d\phi|^2
\]

with identifications:

\[
B \rightarrow A, \quad A \rightarrow \phi, \quad \Phi = e^{-\frac{H}{2T}} \rightarrow H
\]

Exactly like in a symmetric point for an abelian Higgs model where phase $\phi$ is not a good coordinate - in string theory the gauge field $A$ becomes ill-defined in new vacuum, $\Phi = 0, T = \infty$; the same is true for all open string modes; all open string degrees of freedom, except the zero mode of tachyon, $T_0$, are angular variables with $T_0$ being only the radial one. In addition, as it has been explained in [18], using the Zwiebach’s open/closed string field theory, in the symmetric vacuum when the closed string gauge symmetries are restored, all open string degrees of freedom are gauge parameters for closed string gauge transformation and thus disappear from the spectrum. Thus, as long as we know that new vacuum is invariant under closed string gauge transformation we can safely conclude that there are no open string degrees of freedom.

We shall note that in an abelian Higgs model instead of assuming that background gauge field is a dynamical variable one can also choose the cartesian coordinates, but in the stringy case there is no possibility to do so since the “phase” (gauge field) and absolute value ($\Phi = e^{-\frac{H}{2T}}$) carry different space-time spin, thus we can only conclude that the theory is smooth and consistent with closed string modes becoming dynamical in new vacuum. This should be compared to the restriction made for the boundary functional $V(X(\sigma))$ in (0.12) - we see now that our assumption was too restrictive and one shall consider in addition truly non-local functionals which most likely shall correspond to dynamical closed string modes. At the same time one can introduce non-local string field theory wave-function:

\[
\Psi(X(\sigma)) = e^{-\int C T(X(\sigma)) + A_\mu(X(\sigma)) \partial X^\mu(X(\sigma)) + ...}
\]

which can be considered as a formal analog of complex scalar (cartesian coordinates) field and 2-form field $B$ gives natural connection on the space of such functionals (this $\Psi$ now depends on the choice of contour $C$ in space-time; the creation and annihilation operators of $A$ do not make sense anymore and only loop operator $\Psi$ describes the physical degree of freedom). It seems to be closely related to string field that enters in cubic string field theory [1]: consider wave-function/string field $\Psi(X(\sigma))$ given by world-sheet path integral for disk and divide it in two equal parts with the first half carrying the fixed boundary conditions $X(z, \overline{z})_{|D} = X_*(\sigma)$ and on the other half the operator inserted. One shall think about this path integral as a very non-linear and non-local change of variables from $V(X(\sigma))$ to $\Psi$ of cubic string field theory:

\[
\Psi_{V(X)}(X_*(\sigma)) = \int_{X_*(\sigma)} [dX] e^{-I_0(X) - I_0^* V(X(\sigma))}
\]

For tachyon zero mode it leads to $e^{-\frac{T}{2}} = 1 + T_0^{cubic}$. One can map the disk to “1/3 of pizza” (see Fig. 1) and glue three such wave-functions in order to get a cubic term in cubic CS string field theory which becomes the disk partition function with the operator inserted everywhere on the boundary of the disk (the second term in the action (0.14)); as far as first term with $\beta$-function it should
be possible to obtain from cubic string field theory action kinetic term \(\langle \Psi, Q \Psi \rangle\) via the conformal map of the disk to the half of the disk (note this \(Q\) is now the open string BRST operator as opposed to the one which enters in the definition of background independent string field theory action). This proposal is incomplete and needs serious investigation; unfortunately a more direct relation is difficult to demonstrate since we considered the restrictive situation when ghosts decouple from matter in the boundary perturbation.

One can wonder what is the description of the above space-time picture from the point of view of world-sheet theory on disk. Since condensation takes place for infinite value of tachyon field, \(T \to \infty\), we see that the typical size of the boundary which will contribute to world-sheet path integral has to be small and thus at the end-point of condensation the boundary of the disk shrinks to the zero size (see the Fig. 2). Thus there is no boundary anymore and the operator insertion at the boundary in fact becomes the insertion at the points on the bulk. More careful treatment shows that these operators, which by definition were the operators defined in the bulk and taken to the boundary, are integrated over the whole sphere (with Liouville factor properly included) and thus deformation on the boundary becomes deformation on the sphere. It is tempting to think that this is in fact the correct world-sheet way of thinking about the end-point of condensation and string field theory action for open strings \([0,1]\) becomes the action for closed string degrees of freedom.

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