Angular and energy correlations in Higgs decay

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Abstract
We discuss how correlations among momenta of the decay products may yield information about the CP-parity of the Higgs particle. These are correlations of decay planes defined by the momenta of pairs of particles as well as correlations between energy differences. Our study includes finite-width effects. In the narrow-width approximation the angular correlations coincide with previously reported results. The correlations between energy differences turn out to be a much better probe for CP determination than the previously suggested angular correlations, especially for massive Higgs bosons.

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When some candidate for the Higgs particle is discovered, it becomes imperative to establish its properties, other than the mass. While the standard model Higgs boson is even under CP, alternative and more general models contain Higgs bosons for which this is not the case. In specific models, production cross sections and branching ratios will be different [1] but it would clearly be valuable if one could establish its CP on more general grounds, independent of the production mechanism. We shall here discuss how decay distributions may shed light on its intrinsic parity, in particular, how one can determine whether it is even or odd under CP.

This situation is similar to the classical one of determining the parity of the $\pi^0$ from the angular correlation of the planes spanned by the momenta of the two Dalitz pairs [2, 3],

$$\pi^0 \rightarrow \gamma\gamma \rightarrow (e^+e^-)(e^+e^-). \quad (1)$$

In that case, as pointed out by Yang [2], the correlation of the planes is unambiguously determined by the parity-conserving $\pi^0\gamma\gamma$ and $\gamma e^+e^-$ couplings. Classically, the $\pi^0\gamma\gamma$ coupling is proportional to $\vec{E} \cdot \vec{B}$, where $\vec{E}$ and $\vec{B}$ are the electric and magnetic field strengths, respectively. In relativistic notation, this corresponds to the coupling $i\epsilon^{\mu\nu\rho\sigma} F_{\mu\rho} F_{\nu\sigma}$, where $F_{\mu\rho}$ is the electromagnetic field strength tensor. If we describe the photons by the vector field $A^\mu$, then, with $p_1$ and $p_2$ the momenta of the two photons, this $\pi^0\gamma\gamma$ coupling has the form [3]

$$i \, g_{\text{eff}}(p_1^2, p_2^2) \epsilon^{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma, \quad (2)$$

where $g_{\text{eff}}$ is some effective coupling that depends on the kinematics. For comparison, the coupling of the standard-model Higgs to vector bosons (of mass $m_V$) is given by

$$i \,(2^{1/4}2^{1/4}) \sqrt{G_F m_V^2 g^{\mu\nu}}, \quad (3)$$

with $G_F$ the Fermi constant.

In the electroweak interactions, parity is known to be violated. However, we shall here assume that parity violation is limited to the couplings between vector bosons ($W$ and $Z$) and fermions (quarks and leptons), as is the case in the standard model. Then it turns out that correlations similar to those of Dalitz pairs in $\pi^0$ decay determine the CP-parity of the Higgs boson.
In non-standard or extended models of the electroweak interactions, there exist “Higgs-like” particles having negative CP. An example of such a theory is the minimal supersymmetric model (MSSM) \[4\], where there is a neutral CP-odd Higgs boson, often denoted \(A^0\) and sometimes referred to as a pseudoscalar.

We shall consider the decay where a standard-model Higgs (H) or a ‘pseudoscalar’ Higgs particle (A) decays via two bosons \((W^+W^-\) or \(ZZ\)), to two non-identical fermion-antifermion pairs,

\[
(H, A) \rightarrow V_1V_2 \rightarrow (f_1\bar{f}_2)(f_3\bar{f}_4). \tag{4}
\]

The two vector bosons need not be on mass shell.

If we let the momenta \((q_1, q_2, q_3, \text{and } q_4)\) of the two fermion-antifermion pairs (in the Higgs rest frame) define two planes, and denote by \(\phi\) the angle between those two planes, then we shall discuss the angular distribution of the decay rate \(\Gamma\),

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\phi} \tag{5}
\]

and a related quantity, to be defined below, both in the case of CP-even and CP-odd Higgs bosons.

Related studies have been reported by \[5, 6\] in the context of correlations between decay planes involving scalar and (technicolor) pseudoscalar Higgs bosons. It should be noted that the present discussion is more general, since finite-width effects of the vector bosons are taken into account. This enables us to investigate the angular correlations for Higgs masses below the threshold for decay into real vector bosons. In addition, we discuss correlations between energy differences. It turns out that under suitable experimental conditions these correlations provide an even better signal for CP determination.

We let \(g_V\) and \(g_A\) denote the vector and axial-vector parts of the couplings, such that the fermion-fermion-vector coupling is given by

\[
-\frac{i}{2\sqrt{2}}\gamma^\mu(g_V - g_A\gamma_5). \tag{6}
\]

In order to parametrize the vector and axial couplings, we define the couplings involving vector bosons \(V_1\) and \(V_2\) in terms of angles \(\chi_1\) and \(\chi_2\) as

\[
g_V^{(i)} \equiv g_i \cos \chi_i, \quad g_A^{(i)} \equiv g_i \sin \chi_i, \quad i = 1, 2. \tag{7}
\]
The only reference to these angles is through $\sin 2\chi$. Relevant values are given in table 1 [7]. Furthermore, $g_1$ and $g_2$ contain CKM matrix elements and electric and weak isospin charges relevant to the fermions in question. The couplings of $A$ and $H$ to the vector bosons are given by (2) and (3), respectively. In the massless fermion approximation, we find

$$d^8\Gamma_i = C_i \left[ X_i + \sin(2\chi_1) \sin(2\chi_2) Y_i \right] d\text{Lips}(m^2; q_1, q_2, q_3, q_4), \quad i = H, A,$$

with $d\text{Lips}(m^2; q_1, q_2, q_3, q_4)$ denoting the Lorentz-invariant phase space, $m$ the Higgs mass, and with the momentum correlations given by

$$X_H = (q_1 \cdot q_3)(q_2 \cdot q_4) + (q_1 \cdot q_4)(q_2 \cdot q_3),$$

$$Y_H = (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3),$$

$$X_A = -2[(q_1 \cdot q_2)(q_3 \cdot q_4)]^2 - 2[(q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3)]^2$$

$$+ (q_1 \cdot q_2)(q_3 \cdot q_4) \{(q_1 \cdot q_3) + (q_2 \cdot q_4)^2 + [(q_1 \cdot q_4) + (q_2 \cdot q_3)]^2\}$$

$$Y_A = (q_1 \cdot q_2)(q_3 \cdot q_4) [(q_1 - q_2) \cdot (q_3 + q_4)][(q_3 - q_4) \cdot (q_1 + q_2)].$$

The normalizations are given as

$$C_H = \frac{\sqrt{2}G_F m_V^4}{m} D(s_1, s_2) \quad \text{and} \quad C_A = \frac{g_{\text{eff}}^2(s_1, s_2)}{4m} D(s_1, s_2),$$

with

$$D(s_1, s_2) = \prod_{j=1}^{2} \frac{g_j^2}{(s_j - m_j^2)^2 + m_j^2 \Gamma_V^2} N_j,$$

and

$$s_1 \equiv (q_1 + q_2)^2, \quad s_2 \equiv (q_3 + q_4)^2.$$

Here, $N_j$ is a colour factor, which is three for quarks, and one for leptons. Finally, $m_V$ and $\Gamma_V$ denote the mass and total width of the relevant vector boson, respectively.

We first turn to a discussion of angular correlations. The relative orientation of the two planes is defined by the angle $\phi$,

$$\cos \phi = \frac{(\vec{q}_1 \times \vec{q}_2) \cdot (\vec{q}_3 \times \vec{q}_4)}{|\vec{q}_1 \times \vec{q}_2||\vec{q}_3 \times \vec{q}_4|}.$$
We find
\[
\frac{d^3\Gamma_H}{d\phi \, ds_1 \, ds_2} = \frac{8\sqrt{2}G_F m_H^4}{9} \frac{D(s_1, s_2) \sqrt{\lambda(m^2, s_1, s_2)}}{(8\pi)^6 m^3} \left[ \lambda(m^2, s_1, s_2) + 12s_1s_2 \right] (17)
\]
\[
+ \left( \frac{3\pi}{4} \right)^2 \sqrt{s_1s_2(m^2 - s_1 - s_2)} \sin(2\chi_1) \sin(2\chi_2) \cos\phi + 2s_1s_2 \cos 2\phi \right],
\]
\[
\frac{d^3\Gamma_A}{d\phi \, ds_1 \, ds_2} = \frac{g_{\text{eff}}^2(s_1, s_2) D(s_1, s_2) [\lambda(m^2, s_1, s_2)]^{3/2}}{(8\pi)^6 m^3} \left[ 4 - \cos 2\phi \right],
\]
where \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)\) is the usual two-body phase space function. The term \(Y_A\) of eq. (12) does not contribute in eq. (18).

In order to obtain a more inclusive distribution, we shall next integrate over the invariant masses of the two pairs. Thus,
\[
\frac{d\Gamma}{d\phi} = \int_0^{m^2} d s_1 \int_0^{(m-\sqrt{s_1})^2} d s_2 \frac{d^3\Gamma}{d\phi \, ds_1 \, ds_2}.
\]
Integrating over \(0 \leq \phi < 2\pi\) in the standard-model case, we confirm the result quoted in ref. [8] for Higgs decay into four fermions including finite-width effects.

We introduce the ratios
\[
\mu = \left( \frac{m_V}{m} \right)^2 \quad \text{and} \quad \gamma = \left( \frac{\Gamma_V}{m} \right)^2,
\]
and change variables according to \(x_1 = s_1/m^2\) and \(x_2 = s_2/m^2\). This enables us to define the integrals
\[
F(m) \equiv \int_0^1 \frac{d x_1}{(x_1 - \mu)^2 + \mu\gamma} \times \int_0^{(1-\sqrt{\pi})^2} \frac{d x_2 \sqrt{\lambda(1, x_1, x_2)}}{(x_2 - \mu)^2 + \mu\gamma} \left[ \lambda(1, x_1, x_2) + 12x_1x_2 \right],
\]
\[
A(m) \equiv \int_0^1 \frac{d x_1 \sqrt{x_1}}{(x_1 - \mu)^2 + \mu\gamma} \int_0^{(1-\sqrt{\pi})^2} d x_2 \frac{x_2 \sqrt{\lambda(1, x_1, x_2)}}{(x_2 - \mu)^2 + \mu\gamma} (1 - x_1 - x_2),
\]
\[
B(m) \equiv \int_0^1 \frac{d x_1 \, x_1}{(x_1 - \mu)^2 + \mu\gamma} \int_0^{(1-\sqrt{\pi})^2} d x_2 \frac{x_2 \sqrt{\lambda(1, x_1, x_2)}}{(x_2 - \mu)^2 + \mu\gamma}.
\]

The distributions of eq. (5) then take the form
\[
\frac{2\pi}{\Gamma_H} \frac{d\Gamma_H}{d\phi} = 1 + \alpha(m) \sin(2\chi_1) \sin(2\chi_2) \cos\phi + \beta(m) \cos 2\phi,
\]
\[
\frac{2\pi}{\Gamma_A} \frac{d\Gamma_A}{d\phi} = 1 - \frac{1}{4} \cos 2\phi.
\]
where
\[ \alpha(m) = \left(\frac{3\pi}{4}\right)^2 \frac{A(m)}{F(m)}, \quad \beta(m) = \frac{2B(m)}{F(m)}. \] (26)

The functions \( \alpha \) and \( \beta \) depend on the ratios of the masses of the vector bosons to the Higgs mass. They are given in fig.1, for values of \( m \) up to 1000 GeV. In the narrow-width approximation these decay correlations are identical to the ones reported in [5] and our analysis fully justifies this approximation.

However, our analysis is valid also below the threshold for producing real vector bosons, \( m < 2m_V \). A representative set of angular distributions are given in fig.2 for the cases \( H \rightarrow (l^+\nu_l)(b\bar{q}) \) and \( H \rightarrow (l^+l^-)(b\bar{q}) \) for \( m = 70, 150, 300 \) GeV and \( m = 70, 300 \) GeV, respectively. (Of course, jet identification is required for this kind of analysis.) With \( \phi \) being defined as the angle between two oriented planes, it can take on values \( 0 \leq \phi \leq \pi \).

There is seen to be a clear difference between the CP-even and the CP-odd cases. We observe that the distribution (24) becomes uncorrelated in the heavy Higgs limit, whereas the distribution (25) is independent of the Higgs mass.

Let us now turn to a discussion of energy differences. In order to project out the \( Y_A \)-term of eq. (12), we multiply (8) by the energy differences \( (\omega_1 - \omega_2)(\omega_3 - \omega_4) \) before integrating over energies. In analogy with eq. (8), we introduce
\[ d^4\tilde{\Gamma}_i = C_i \left[ \tilde{X}_i + \sin(2\chi_1)\sin(2\chi_2)\tilde{Y}_i \right] d\text{Lips}(m^2; q_1, q_2, q_3, q_4), \] (27)

with
\[ \tilde{X}_i = X_i(\omega_1 - \omega_2)(\omega_3 - \omega_4), \] (28)
\[ \tilde{Y}_i = Y_i(\omega_1 - \omega_2)(\omega_3 - \omega_4), \] (29)

for \( i = H, A \). The distribution takes the form
\[ \frac{2\pi}{\Gamma} \frac{d\tilde{\Gamma}}{d\phi} = 1 + \frac{\kappa(m)}{\sin(2\chi_1)\sin(2\chi_2)} \cos \phi, \] (30)

in the CP-even case, whereas there is no correlation in the CP-odd case; i.e. \( \kappa(m) = 0 \) and in this case the term \( \tilde{X}_A \) of eq. (28) does not contribute. Here,
\[ \kappa(m) = \frac{1}{2} \left( \frac{3\pi}{16} \right)^2 \frac{K(m)}{F(m)}, \] (31)
with

\[ J(m) \equiv \int_0^1 \frac{dx_1 x_1}{(x_1 - \mu)^2 + \mu \gamma} \int_0^{(1-\sqrt{x_1})^2} dx_2 \frac{x_2 \left[ \lambda(1, x_1, x_2) \right]^{3/2}}{(x_2 - \mu)^2 + \mu \gamma}, \]  

(32)

\[ K(m) \equiv \int_0^1 \frac{dx_1 \sqrt{x_1}}{(x_1 - \mu_1)^2 + \mu_1 \gamma_1} \times \int_0^{(1-\sqrt{x_1})^2} dx_2 \frac{\sqrt{x_2} \left[ \lambda(1, x_1, x_2) \right]^{3/2}}{(x_2 - \mu)^2 + \mu \gamma} (1 - x_1 - x_2). \]  

(33)

The function \( \kappa \) is given in fig.3, for values of \( m \) up to 1000 GeV. In the narrow-width approximation

\[ \kappa(m) = \frac{1}{16} \frac{\alpha(m)}{\beta(m)}. \]  

(34)

The correlation is significant for any Higgs mass, and in particular \( \kappa(m) \propto m^2 \) in the heavy-Higgs limit. A representative set of distributions (30) are given in fig.4 in the cases \( H \to (l^+\nu_l)(b\bar{c}) \) and \( H \to (l^+l^-)(b\bar{b}) \) for \( m = 70, 300, 500 \) GeV and \( m = 70, 300 \) GeV, respectively. We see that the energy-weighted distribution is a much more sensitive probe for CP determination than the purely angular distribution of eq. (5). In this latter case, we compare an uncorrelated distribution with a strongly correlated one. Moreover, for \( V = Z \) the sin\( 2\chi \)-factors provide an enhancement in the energy-weighted correlations.

This research has been supported by the Research Council of Norway.

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Table 1: The factors $\sin 2\chi$ that give the relative importance of the axial couplings.

| $V$ | $f$     | $\sin 2\chi$ |
|-----|---------|---------------|
| $W$ | any     | 1             |
| $Z$ | $e, \mu, \tau$ | 0.1393       |
| $Z$ | $u, c, t$ | 0.6641       |
| $Z$ | $d, s, b$ | 0.9349       |
Figure captions

Fig. 1. The coefficients $\alpha$ and $\beta$ of the angular correlations in eq. (24), for a CP-even Higgs of mass $m$.

Fig. 2. Characteristic angular distributions of the planes defined by two Dalitz pairs for CP-even Higgs particles decaying via two $W$’s and two $Z$’s, compared with the corresponding distribution for a CP-odd Higgs particle (denoted A). Different Higgs masses are considered in the CP-even case.

Fig. 3. The coefficient $\kappa$ of the energy-weighted angular correlation of eq. (30), for a CP-even Higgs of mass $m$. For a CP-odd Higgs, $\kappa = 0$.

Fig. 4. Characteristic energy-weighted distributions for CP-even Higgs particles decaying via two $W$’s and two $Z$’s, compared with the corresponding distribution for a CP-odd Higgs particle (A).
Figure 2
Figure 4