CP Violation and the Baryonic Asymmetry of the Universe

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Abstract:

The physics of electroweak baryogenesis is described with the aim of making the essentials clear to non-experts. Several models for the source of the necessary CP violation are discussed: CKM phases as in the minimal standard model, general two higgs doublet models, the supersymmetric standard model, $Z$ condensates, and the singlet majoron model. In a more technical section, a strategy is introduced for consistently treating quark dynamics in the neighborhood of the bubble wall, where both local and non-local interactions are important. This provides a method for deciding whether gluonic corrections wash out the electroweak contribution to the baryonic asymmetry in the minimal standard model.

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1 General Picture

A major challenge to particle theory and cosmology is to account for the tiny but non-zero value of the baryonic asymmetry of the universe (bau). The main purpose of this lecture was to give an understandable yet relatively complete overview of the subject for non-experts, with special emphasis on the possible sources of CP violation. Interestingly, it is easier to obtain reliable predictions from extensions of the minimal standard model (§2.1, 2.2) than from the minimal standard model (MSM – §2.3). The major difficulty is discussed in section §2.4. A method for overcoming the difficulty is outlined in the final section.

Comparing the results of the theory of nucleosynthesis to observations on the abundances of primordial light nuclei, the ratio of baryon number density to entropy is inferred to be \( \frac{n_B}{s} \sim (2 \times 4) \times 10^{-11} \). If the universe starts out with no net baryon number, \( B = 0 \), then baryon number violating processes are necessary in order for a non-zero baryonic density to be produced. A significant baryonic asymmetry can only be produced at a phase transition, because in thermal equilibrium detailed balance guarantees that no asymmetry can develop. It was natural that early work focussed on a possible GUT (Grand Unified Theory) phase transition origin for the bau, since by their very nature GUT multiplets contain both leptons and quarks and thus GUTs naturally have baryon number violating transitions mediated by the GUT gauge and Higgs particles.

However Kuzmin, Rubakov and Shaposhnikov showed in 1985 [2, 3] that for temperatures above the electroweak (ew) phase transition temperature, sphaleron processes occur at a sufficiently high rate to greatly reduce and possibly even wipe out a baryonic asymmetry produced at the GUT phase transition. A sphaleron is a thermal fluctuation in the electroweak gauge fields, which connects vacuum configurations of different winding number. It has similar effects to the instanton, which is a quantum tunneling event which
occurs (at a negligible rate\(^4\)) in the zero-temperature theory. The energy levels of electroweak doublets present in the Dirac sea (left-chiral quarks and leptons) shift in response to changing external gauge fields. Remarkably, when the Chern-Simons number (roughly speaking, the number of twists in the vector potential giving rise to the gauge fields) changes by one unit, such as occurs in the neighborhood of a sphaleron or instanton, one complete set of fermions in the Dirac sea is “promoted” into being real particles. This gives rise\(^5\) to an effective interaction producing or destroying one each of the electroweak gauge doublets – i.e., creating or annihilating 9 quarks (one left chiral quark of each of the three colors and three generations) and three left-chiral leptons (one from each generation). In the presence of a baryon excess, these interactions will proceed in a direction which reduces the free energy by converting part of the baryon excess to anti-leptons. Since the sphaleron conserves \(B−L\), if GUT phenomena produce a \(B−L\) excess and not simply a \(B\) excess, that will not be affected by the ew sphaleron. However in SU(5), the most popular GUT, \(B−L\) is conserved so that one must rely on various other effects to circumvent the sphaleron, or use a more complicated GUT.

While there are a number of ways that GUT baryogenesis can be made to work in spite of these sphaleron transitions, interest has shifted to studying the possibility that phenomena occurring at the ew phase transition produced the observed bau and that will be the main subject of this talk.

Production of a non-zero bau requires CP violation as well as baryon number violation, since if CP is a good symmetry, processes in which particles are replaced by antiparticles of the same chirality will occur with equal rates\(^6\). It is of course well known that nature does not fully respect CP symmetry, since the \(K^0\) has been observed to decay into both CP even and CP odd final states. At the very least, this requires the physical \(K^0_L\) to be a superposition

\(^2\)Chirality is the same as helicity for a massless particle, and the opposite of the helicity for a massless antiparticle. The CP conjugate of a left chiral particle is a left chiral antiparticle, etc.
of even and odd CP. In the minimal standard model (MSM) the CP violation observed in the kaon system is explained as arising from the existence of an explicitly CP-violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix – the matrix describing mixing between gauge and mass eigenstates of the quarks. If the only source of CP violation is the CKM phase, as is the only possibility in the minimal standard model, then the magnitude of all other occurrences of CP violation can be predicted because the parameters of the CKM matrix are fully determined, albeit with limited precision at the present time\(^3\).

While CKM CP violation is very popular theoretically, since it appears naturally in the minimal standard model, it is not excluded that even the CP violation seen in the kaon system actually arises from some other mechanism. One possibility which has been extensively studied is spontaneous CP violation, when the vacuum and other physical states are not CP eigenstates, even though the underlying Lagrangian may be CP invariant. In order for CP to be violated spontaneously the theory must be more complicated than the minimal standard model, requiring at least an additional Higgs doublet and the presence of certain interactions between the Higgs doublets. Of course, both CKM CP violation as well as other sources of CP violation can be simultaneously present in nature. One of the main motivations for building a “B-factory” – an \(e^+e^-\) collider optimized for producing large numbers of B mesons – is to determine whether CP violation in the B meson system is that which is expected if a CKM phase is responsible for the kaon CP violation\(^4\). The non-observation of neutron and electron electric dipole moments are powerful constraints on non-CKM CP violation.

\(^3\)A more detailed discussion of CKM CP violation will follow in section 2.3. For a review of experimental information on the CKM parameters, see \(\text{[6]}\).

\(^4\)Measurement of “direct” CP violation in the kaon system is of great interest, e.g., observation of a non-zero value of the parameter \(\epsilon'\) or certain rare K decays, however strong interaction effects are theoretically more difficult to handle than in the B system.
for the observed bau, or whether there must be another source of CP violation. In order to address the question, one must consider some particular mechanism for producing the baryonic asymmetry and do a quantitative calculation. Since this lecture is meant to introduce the ideas to a general audience rather than be an exhaustive review, I will focus on a baryogenesis mechanism which is applicable when the boundary between high and low temperature phases (the bubble wall) is thin compared to the scattering length of particles in the plasma. The mechanism in this case is simpler to understand physically than for the thick wall case, and furthermore, recent numerical work on the phase transition suggests that the wall is in fact thin[7]. In all mechanisms considered so far, the fundamental crucial feature is that quarks and leptons couple to the vacuum expectation value of the Higgs field in proportion to their mass, since the spontaneous breaking of the electroweak gauge symmetry is supposed to give rise to the masses of these particles.

Due to the interactions of the Higgs field with particles in the high-temperature plasma, the effective potential of the Higgs field is temperature-dependent. As the temperature of the expanding universe drops, the density of particles in the plasma decreases until finally the effective potential is just the zero temperature potential of the standard electroweak theory. If there were no physics beyond the minimal standard model, and if the Higgs mass were known, then all of the parameters of the T=0 theory would be fixed. The effective potential at finite temperature would also be fixed, although its computation is a highly non-trivial business when the Higgs mass is not light, since non-perturbative effects become important[8]. It is known that at very high temperature the free energy of the system is minimized when the vacuum expectation value (vev) of the Higgs field vanishes[9, 10]. On the other hand, in the T=0 theory the minimum energy occurs when the Higgs field has a vev of $\sim 250$ GeV. Thus the universe undergoes a phase transition at a temperature which turns out to be of order 100 GeV.
For sufficiently small Higgs mass, possibly consistent with the present bound $m_H \gtrsim 60$ GeV\[1\], the phase transition will be a first order transition, as is assumed in nearly all work on ew baryogenesis. Then the picture is that as the universe cools and hits the transition temperature, bubbles of the low-temperature phase form. These bubbles, in which the vev of the Higgs field is non-zero, expand and eventually fill the universe. The relevant CP violation is supposed to occur as a result of the dynamics of quarks or leptons interacting with the bubble wall – the region in which the Higgs field makes a transition from having zero vev to having a non-zero vev. Due to their thermal motion and the motion of the wall, quarks and leptons present in the high temperature plasma near the wall will encounter the bubble wall. Since quarks and leptons get their mass from their coupling to the Higgs vev, the more massive the particle, the higher the potential barrier it sees to its motion. For instance, particles with insufficient kinetic energy cannot penetrate into the low temperature phase and are totally reflected from the bubble wall. Thus CP violation in the interaction of a heavy particle with the Higgs field will generally be more efficient for producing a baryonic asymmetry than CP violation for a light particle.

The basic mechanism in the thin wall case was introduced by Cohen, Kaplan and Nelson, who dubbed it the “charge-transport” mechanism. It can be thought of in two steps. First, particle scattering off the phase boundary acts to separate some quantum number correlated with left-chiral baryon or lepton number. This gives rise to a current of, lets say, left-chiral anti-baryon number toward the unbroken phase (and correspondingly, since baryon number is not violated in the interaction with the bubble wall, a left-chiral baryon number current toward the broken phase). Producing such a separation requires CP violation in the interaction of quarks with the bubble wall. Some of the proposals for the origin of this CP violation are discussed in the next section. Given, say, a net flux of left-chiral anti-baryon number into the unbroken phase, ew sphaleron transitions in the unbroken phase can reduce the
free energy by converting some of the excess left-chiral anti-baryons into left-chiral leptons. At this point there is an excess of baryon number in the broken phase and lepton number in the unbroken phase. If the ew sphaleron transitions are sufficiently suppressed in the broken phase, the excess of left-chiral baryons present in the broken phase will not be converted to anti-leptons. A net baryonic density will remain after the phase transition is complete, and is supposed to account for the observed bau. Note that since the ew sphaleron only acts on left chiral particles and antiparticles, simply producing a separation of left chiral baryon number is sufficient if it is large enough to withstand erasure by the strong sphalerons as will be discussed in section §2.2.

In order that sphaleron transitions in the low temperature phase do not equilibrate a baryonic excess created during the phase transition, the sphaleron rate in the low-temperature phase must be smaller than the expansion rate of the universe. The rate of sphaleron transitions in the broken-symmetry phase is approximately

$$\Gamma = T^4 \left( \frac{\alpha_W}{4\pi} \right)^4 N_{tr} N_{rot} \left( \frac{2E_{sph}(T)}{\pi T} \right)^7 \exp \left( -\frac{E_{sph}(T)}{T} \right)$$

(1)

where the factors $N_{tr} \simeq 26$ and $N_{rot} \simeq 5.3 \times 10^3$ are zero mode normalizations [12] and

$$E_{sph}(T) \approx 3M_W(T)/\alpha_W$$

(2)

is the effective sphaleron mass at temperature $T$. The sphaleron rate will be less than the expansion rate of the universe if

$$E_{sph}(T_c)/T_c > 45.$$  

(3)

In the MSM, two parameters fix the Higgs potential. The combination of them which determines the $T = 0$ vev is known, since it fixes $m_W$ at $T = 0$. If the mass of the Higgs were known, that would completely fix the remaining freedom in the MSM Higgs potential. The Higgs mass is not known yet, but
the requirement that the vev after the phase transition is large enough to satisfy (3) also constrains the remaining parameter of the low temperature theory. Taking the one-loop effective potential for the Higgs field in the MSM, together with the one loop approximation for the sphaleron rate, leads to the requirement $\langle \phi(T_c) \rangle \gtrsim 2.4 g_w T$ and to the bound $M_H < M_{\text{crit}} = 45 \text{ GeV}$, which is inconsistent with the present LEP limit of $\sim 60 \text{ GeV}$. Although the 1-loop approximations to the effective potential have been improved by resummation and calculation of higher order effects, it is now clear that the use of perturbation theory is inappropriate to extract this information. The reason is that in the high temperature theory, the Higgs vev itself provides an essential infra-red cutoff to perturbative calculations. Thus perturbative calculations are accurate in the broken phase, but break down in the vicinity of the unbroken phase minimum, where the vev vanishes. Recent lattice work indicates that the actual unbroken phase minimum is much lower relative to the broken phase minimum than indicated by perturbation theory, lowering the temperature of the phase transition and modifying the bounds on the Higgs mass. While it is numerically difficult to find the new bound on the Higgs mass, it seems to be consistent with the present experimental bound. In the MSM it is probably within reach of LEP II, and even in a two-Higgs doublet model it is likely that the lighter Higgs could be found at LEP II.

In the next section we will discuss possible mechanisms for creating a left-baryonic or leptonic current from quark or lepton interactions with the bubble wall, which is the main topic of this talk. However before continuing with that, it is important to emphasize the great quantitative uncertainty which surrounds the question of the conversion of a given left baryonic current into a net baryonic excess after the phase transition is complete. In ref. [15] the relation between the left baryonic current, $J_{CP}$, and the final baryonic density was found in quasi-equilibrium approximation to be $n_B = \frac{12}{5} J_{CP} f_{\text{sph}}$, where $f_{\text{sph}}$ is essentially the probability that an excess antiquark will experience a sphaleron transition before it diffuses into the broken
phase. $f_{\text{sph}} = 1$ when the dimensionless ratio $3D_B \Gamma / v^2$ is large compared to one; $\Gamma$ is the sphaleron transition rate, $D_B$ the baryonic diffusion constant, and $v$ the wall velocity. However if this dimensionless ratio is much less than one, $f_{\text{sph}}$ is just proportional to it. Unfortunately, estimation of $\Gamma$, the sphaleron rate in the unbroken phase, has some 4-orders-of-magnitude uncertainty due to its sensitivity to the non-perturbative physics mentioned in the previous paragraph. While ref. [2] and subsequent analytic and numerical work established that $\Gamma$ is greater than the expansion rate of the universe at the time of the ew phase transition, allowing sphaleron transitions to wipe out a baryonic excess created in a GUT phase transition, it is not clear yet whether $\Gamma > \frac{v^2}{3D_B}$. Thus in all models there is a substantial uncertainty in the predicted final asymmetry just due to the uncertainty in the sphaleron transition rate.

2 The basic source of CP violation

In this section I will describe a number of possible sources for the CP violation which produces the baryonic asymmetry, concentrating on the ones which seem of greatest interest. My goal is to elucidate the fundamental issues, especially the consequences of CP, C, and P invariance and gauge symmetries. This leads me to examine the various models from a somewhat different point of view than presented in the original papers, so the reader interested in the perspective of the authors of the ideas is urged to consult the original references. A number of new observations are made. In order to make the amount of material manageable, I concentrate exclusively on scenarios relevant to thin bubble walls, as seem to b presently favored theoretically[8, 14]. On account of the general audience at the lecture, many of the details which follow were not presented there.

It is useful to recall the transformation properties of vector and axial
vector fermionic currents under the discrete symmetries:

\[
\begin{align*}
C &: \quad J^\mu \rightarrow -J^\mu \\
J_5^\mu &\rightarrow +J_5^\mu \\
P &: \quad (J^0, J) \rightarrow (J^0, -\vec{J}) \\
(J_5^0, \vec{J}_5) &\rightarrow (-J_5^0, \vec{J}_5) \\
T &: \quad (J^0, \vec{J}) \rightarrow (J^0, -\vec{J}) \\
(J_5^0, \vec{J}_5) &\rightarrow (J_5^0, -\vec{J}_5)
\end{align*}
\]

Thus we immediately see that a baryonic density, \( J^0 \), violates C, CP, and CPT, explaining why generation of the bau requires C and CP violation in the fundamental theory, as well as a departure from thermal equilibrium, which can be thought of as “spontaneous” CPT violation. Note also that \( \vec{J}_L \) and \( \vec{J}_R \) (\( \equiv \vec{J} \mp \vec{J}_5 \)) are even under CP, odd under CPT, and transform into the negatives of one another under C.

As long as there are equal fluxes of quarks and antiquarks on either side of the wall, CP violation in the reflection process is needed to produce a left baryonic current, even though \( \vec{J}_L \) is even under CP. Some of the left-chiral lepton or baryon number separation mechanisms which have been considered are mentioned below. They are generally based on the quantum mechanical reflection of quarks or leptons from the interface between phases, such that the reflection probability for a particle and its CP conjugate differ. This arises when there is an interference between the reflection phase shift (the same for a particle and its CP conjugate) and a CP-violating phase in the reflection process coming, for instance, from a non-trivial phase in the coupling to the Higgs field. In this case, the full reflection amplitude for the particle is \textit{not} just a phase rotation of the amplitude for its CP conjugate and general it has a different magnitude: \( |ae^{i\phi} + be^{i\delta}| \neq |ae^{i\phi} + be^{-i\delta}| \), where \( \phi \) and \( \delta \) are the CP conserving and violating phases.

Even if CP is violated, CPT and unitarity imply that if the system is in thermal equilibrium no net chiral baryonic or leptonic current is established, because particles incident from opposite sides of the wall make canceling contributions to the current and have the same flux in equilibrium. Of course

\[5\text{See, e.g., §28 of ref. [17].}\]
during the phase transition the bubbles of low-temperature phase are expanding, so that there is a net flux of particles toward the unbroken phase and a baryonic or axial baryonic current can be established. We now turn to specific models for producing the chiral current, starting with the simplest models to analyze and progressing to the most difficult – the minimal standard model.

2.1 “Singlet Majoron” Model

The earliest charge transport scheme [18, 19] involved adding, a neutral heavy lepton, \( N_R \), to the MSM. CP is assumed to be maximally violated in its Higgs coupling, in such a way that a deficit of left-chiral lepton number develops in the unbroken phase and a corresponding excess in the broken phase. Then in the unbroken phase, sphaleron transitions reduce the free energy by converting left-chiral antileptons into left-chiral quarks. The baryonic asymmetry which arises in this mechanism is proportional to the mass-squared of the heaviest of the light neutrinos, \( \nu_{hl} \), because the Higgs coupling of the lepton, which contains the CP violation, is automatically proportional to its mass. According to the estimates of refs. [18, 19], consistency with the observed bau requires either the existence of a fourth generation or requires \( m(\nu_{hl}) > 1 \) MeV. As a result of direct laboratory mass limits on the three known neutrinos, only \( \nu_\tau \) can be so heavy. However if the atmospheric neutrino oscillation “hint” holds up and is explained by a \( \nu_\mu \leftrightarrow \nu_\tau \) oscillation, that would imply that \( m(\nu_\tau) \sim m(\nu_\mu) \lesssim 0.27 \) MeV, ruling out this baryogenesis mechanism without a fourth family. Even without anticipating confirmation of \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations, nucleosynthesis constraints exclude a neutrino in the required mass range unless its lifetime is \( \lesssim 100 \) s [20], so that this scenario seems rather implausible now unless there is a fourth family.

2.2 “Two Higgs Doublet” Model and Z condensation
2.2.1 General Picture

In the original two Higgs-doublet mechanism for baryogenesis, the top quark is assumed to couple to a Higgs field whose vacuum expectation value has a spatially varying phase inside the bubble wall\cite{21, 22, 23}. We can write its mass term as

\[ \bar{t}m(z)e^{i\gamma_5\theta(z)}t = \bar{t}[m(z)\cos\theta(z) + i\gamma_5m(z)\sin\theta(z)]t, \quad (5) \]

where \( \hat{z} \equiv \hat{3} \) is the inward pointing normal to the bubble wall throughout this paper. If \( \theta(z) \) is a constant then a global chiral transformation on the top quark field can be used to remove it, so it has no physical effect\footnote{Note that if it were not removed, the Dirac equation would look different for a particle and its CP conjugate. This serves as a reminder that one must be careful to always compute physical quantities such as the net chiral baryon current and not rely exclusively on apparent CP or parity asymmetry of the Dirac equation.}. However it is not sufficient for \( < \theta' > \equiv < \partial_z\theta(z) > \) to be non-zero, since this is parity and CP invariant. Hence physical consequences require a non-vanishing \( < \theta'' > \).

We can see this requirement directly as follows: a chiral transformation can be used to remove \( \theta \) in the asymptotic broken phase where it is constant, and it can be set to zero in the unbroken phase since with \( m(z) = 0 \) there is no CP violation independently of \( \theta \). Thus it must have a non-vanishing second derivative to have any CP violating physical effect.

Parity and CP violation in the interaction of the top quark with the vev will in general result in different reflection coefficients for left and right chiral top quarks, separating left chiral baryon number. Baryon number itself is not separated because \( \vec{J}_L + \vec{J}_R \) is C odd while \( \vec{J}_L - \vec{J}_R \) is C even, and this model violates CP but not C, unless higher order corrections from gauge interactions are also included. Thus the net baryon number in either phase is zero, with an excess of left chiral baryon number balanced by a deficit of right chiral baryon number. The desired result is to have the left-chiral baryon number, \( n_B^L - \bar{n}_B^L < 0 \) in the unbroken phase and \( > 0 \) in the broken phase.
phase, so that ew sphaleron transitions in the unbroken phase convert some of the excess left anti-baryon number to left lepton number.

We remarked above that thermal fluctuations in the SU(2) gauge fields with non-trivial topology produce the electroweak sphaleron which causes baryon and lepton number violating transitions. In a similar way, thermal fluctuations in the gluonic gauge fields with a non-trivial change in the Chern-Simons number produce the “strong sphaleron”, violating quark chirality. Since the strong sphaleron has a faster rate than the ew sphaleron and it equilibrates left- and right-chiral baryon number, it dilutes the expected final $n_B/s$ compared to the initial estimates\cite{23} which neglected this effect. In the last year several papers have appeared on this subject. It was found that including strong sphaleron and other effects reduces the final asymmetry by a large factor\cite{24, 25, 26}. Fortunately for this mechanism, this effect seems to be more or less compensated by a more careful treatment of diffusion\cite{27} so that the latest estimate\cite{27} still can be compatible with observation.

One can also use the two Higgs mehansim to produce a current of $\tau_L$’s, which are not troubled by the strong sphaleron. This works best if the leptonic Yukawa couplings are large and the relatively small $T = 0$ lepton masses arise because $< v_2 >$ is small at $T = 0$. Ref. \cite{28} concludes that this can be responsible for the observed bau with a $\tau$ Yukawa coupling which is a factor $\sim 10$ larger than in the MSM. Some investigation is required to see whether such a large Yukawa, producing a vehicle for chirality change even in the unbroken phase, could cause a dilution of the asymmetry as the strong sphaleron does.

### 2.2.2 Spatially Varying Phases and Gauge Invariance

One has to be careful in treating gauge invariance in the two-Higgs doublet model. First let us review in greater detail how CP violation in two-Higgs doublet baryogenesis works, and at the same time lay the groundwork for a
discussion of the MSM. The relation to $Z^0$ condensation will emerge naturally. Consider the part of the standard model Lagrangian which involves quarks:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_Y.$$  \hspace{1cm} (6)

In the “gauge” basis,

$$\mathcal{L}_G = \bar{Q}_L \mathcal{D} Q_L + \bar{U}_R \mathcal{D} U_R + \bar{D}_R \mathcal{D} D_R,$$  \hspace{1cm} (7)

and

$$\mathcal{L}_Y = \frac{g_W}{\sqrt{2} M_W} \{ \bar{Q}_i^j V^{ij} M_L^{ij} D_R^i \phi + \bar{Q}_L^i M_u^{ij} U_R^i \tilde{\phi} + h.c. \},$$  \hspace{1cm} (8)

where $\mathcal{D}$ is the appropriate covariant derivative, $Q_L^i$ are the left-handed quark doublets ($i$ is the generation index), $U_R^i$ and $D_R^i$ are the right handed quarks with electric charges $\frac{2}{3}$ and $-\frac{1}{3}$ respectively, $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and $M_u$ and $M_d$ are the diagonal mass matrices of the quarks. In this basis, the Lagrangian has been written in terms of the fields which are eigenstates of the gauge interactions, and the CKM CP violation is contained in a phase in the matrix $V$, relating the gauge eigenstates to the mass eigenstates. In the minimal standard model, $\tilde{\phi}$, the Higgs which gives mass to the charge $2/3$ quarks $= \epsilon_{ij} \phi_j^i$. In a general 2-Higgs doublet model, $\phi$ can be an independent doublet, or else the second doublet can be taken to decouple from the quarks altogether and the Yukawa couplings are as in the MSM. In a the minimal supersymmetric standard model (MSSM), supersymmetry requires that $\tilde{\phi}$ be a distinct field from $\phi$.

The condition that the vacuum energy be minimized fixes the magnitudes of the Higgs doublets and the relative phase between them. In the MSM, with a single Higgs doublet, one can always use the $SU(2) \times U(1)$ gauge freedom to make the non-vanishing vev be real. This is evident, since any doublet can be written in the form

$$\exp \left( i \frac{\sigma \cdot \xi(x, t)}{v} \right) \left( \begin{array}{c} 0 \\ \frac{v + \eta(x, t)}{\sqrt{2}} \end{array} \right).$$  \hspace{1cm} (9)
Taking $v$ to be the solution to the equation of motion for the Higgs field, one then identifies $\eta(x, t)$ as the field corresponding to the physical “Higgs” particle. At the critical temperature, when the potential has degenerate minima, the vacuum energy is minimized by a non-constant $v$, interpolating between the vevs in the unbroken and broken phases. We will work in the wall rest frame\footnote{No important physics is affected if the vev in the unbroken phase is taken to be extremely small but non-vanishing, so that the singularity in (9) for $v \to 0$ is not a source of problems for the arguments we wish to make.} where the vev is static.

The most general $T = 0$ potential normally considered in a two Higgs doublet model can be put in the form

$$V(\phi_1, \phi_2) = \lambda_1(\phi^*_1 \phi_1 - v_1^2)^2 + \lambda_2(\phi^*_2 \phi_2 - v_2^2)^2 + \lambda_3[(\phi^*_1 \phi_1 - v_1^2) + (\phi^*_2 \phi_2 - v_2^2)]^2 + \lambda_4[(\phi^*_1 \phi_1)(\phi^*_2 \phi_2) - (\phi^*_1 \phi_2)(\phi^*_2 \phi_1)] + \lambda_5[Re(\phi_1 \phi_2) - v_1 v_2 \cos \zeta]^2 + \lambda_6[Im(\phi_1 \phi_2) - v_1 v_2 \sin \zeta]^2,$$

with the $\lambda_i$’s real for hermiticity’s sake. This potential is general enough to encompass the MSSM potential, and is only restricted in a general non-susy model by having its dimension-4 terms invariant under the discrete symmetry $\phi_1 \to -\phi_1$, a standard method for avoiding large flavor changing neutral currents\footnote{When $v_1 v_2 \sin \zeta \neq 0$ this Lagrangian explicitly violates CP.}. Our first concern is to insure that the vevs $v_1$ and $v_2$ only break $SU(2) \times U(1) \to U(1)$ and not electromagnetism as well, since we used all our gauge freedom in making the first vev be electrically neutral and real. The presence of the $\lambda_4$ term will insure this for a large range of parameters, so we will henceforth assume that this has been guaranteed.

The minimum of this potential can be taken to occur at

$$<\phi_1> = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad <\phi_2> = \begin{pmatrix} 0 \\ v_2 e^{i\kappa} \end{pmatrix}. \quad (12)$$

As long as the Lagrangian is not invariant under a global redefinition of the relative phase between $\phi_1$ and $\phi_2$, which would allow the $\zeta$ dependence to be
removed, there is a physically significant relative phase between the vevs in the $T = 0$ theory. This could be called “vacuum CP-violation”, reserving the term “spontaneous CP-violation” for a situation in which the Lagrangian is CP invariant. In a supersymmetric theory $\lambda_5 = \lambda_6$ in tree approximation, so that the two terms in the potential containing $\zeta$ can be combined into a term proportional to $\phi_1^\dagger \phi_2 - v_1 v_2 e^{i\zeta}$ and there is no vacuum CP violation.

At high temperatures the potential (12) receives corrections from the interactions of the Higgs bosons with fermions and gauge bosons and other Higgs particles in the plasma. The main effect is to introduce effective cubic self-interactions for $\phi_1$ and $\phi_2$, causing there to be two degenerate minima in the potentials for $|\phi_1|$ and $|\phi_2|$ without changing qualitatively the $\lambda_5$ and $\lambda_6$ terms. Thus inside a bubble wall, where $|\phi_1|$ and $|\phi_2|$ are changing from their unbroken to broken phase values, the equations of motion for $\phi_1$ and $\phi_2$ will in general produce a spatially varying relative phase between them.

Now let us discuss the coupling of the Higgs fields to the quarks, which is supposed to produce the quark masses and the CP violation in the top quark or $\tau$ lepton scattering from the bubble wall. Clearly, the $SU(2) \times U(1)$ gauge invariance which was used in the discussion above to make $<\phi_1(x)>$ real can be used to remove the phase from either $<\tilde{\phi}(x)>$, which by definition gives mass to the charge $2/3$ quarks, or $<\phi^0(x)>$, which gives mass to the leptons and the charge $-1/3$ quarks, even inside the bubble wall where the relative phase between them is changing. Yet this seems paradoxical because we would have argued that if the CP violation is in the top quark coupling, the bau thus generated would be $\sim (m_t/m_b = 40)^2$ times greater than if it were in the bottom quark coupling! The resolution of this puzzle leads one naturally to the subject of a $Z$ field condensate.
2.2.3 Z Condensate

Since we have excellent experimental evidence that Lorentz invariance is unbroken, theorists generally never allow a field with a non-trivial Lorentz behavior to have a vacuum expectation value. However during the ew phase transition, the high-temperature plasma provides a preferred Lorentz frame and the bubble walls break translation invariance. Thus one should consider the possibility that a vector potential normal to the wall can have a vacuum expectation value in the vicinity of the bubble wall. For simplicity, it is natural to assume that $SU(2) \times U(1) \rightarrow U(1)$, with electromagnetism remaining a good symmetry even inside the bubble wall. Then only a vev for the $Z^0$ gauge potential need be considered. The gauge invariant quantities are $\zeta$, the relative phase between the Higgs vevs $v_1 e^{i\theta_1}$ and $v_2 e^{i\theta_2}$, and $Z_{\mu}^{GI} \equiv Z_{\mu} - \frac{2}{g} \left( \frac{v_1^2 \partial_{\mu} \theta_1 + v_2^2 \partial_{\mu} \theta_2}{v_1^2 + v_2^2} \right)$. Thus a complete specification of the vacuum state requires not only specification of the magnitudes of the Higgs vevs and the relative phase between them, but also specification of the vev of $Z_{\mu}^{GI}$. When both are specified, a $T_3$ or hypercharge gauge change by an angle $\theta$ (say moving the phase of the vev from the Higgs coupled to the top quark into the Higgs coupled to the bottom quark) will not change the physical predictions of the theory since it will induce a gauge condensate proportional to $\partial_{\mu} \theta$, interaction with which produces the same effect as the Higgs phase. Alternatively, in a particular gauge, one must specify $Z_{\mu}$, $\theta_1$, and $\theta_2$ at all positions. The original discussions of the two Higgs doublet models [22, 23] implicitly assumed $Z_{\mu} = 0$.

By the symmetry of the problem, $Z_{\mu}$ can only have a non-vanishing component in the 3 or 0 direction. These can only have spatial derivatives in the $\hat{z}$ direction so that $\vec{B}^Z = \vec{\nabla} \times \vec{Z} = 0$. On the other hand, $\vec{E}^Z = \vec{\nabla} \cdot Z_0 - \partial_0 \vec{Z}$ can have a non-zero vev as long as either $<Z_0>$ is non-zero and varying with $z$ or $\partial_0 \vec{Z} \neq 0$. Turok and collaborators [29] recently have argued that the pure gauge condensate $<Z_3> \neq 0$ can generate new physics.
The first step in figuring out the physical relevance of various possible $Z$ condensates is to analyze their transformation properties under CP, since a CP even quantity will not contribute to the formation of a bau. Since the Lagrangian is not parity invariant, the gauge potentials cannot be characterized as vector or axial vector, but when they have a constant vev they can be assigned a definite CP since CP is conserved in the gauge interaction and the CP properties of the currents to which they couple are determined. From (4) we see that a constant $\langle Z_0 \rangle$ is CP odd while $\langle \vec{Z} \rangle$ is CP even. This means that even if $\langle \vec{E}^Z \rangle \neq 0$, it is CP even if it is a constant and therefore has no leading order effect in producing a baryonic asymmetry. For the case of interest when the condensates are spatially varying, we can consider their local Taylor series expansion in $z$. Then, e.g., $\langle Z'_3 \rangle$ is CP odd, etc.

Now let us turn to the proposal of a $\langle Z_3 \rangle$ condensate. As discussed above, gauge transforming a time-constant but spatially varying phase in the Higgs vev produces a non-vanishing $\langle Z_3 \rangle$ which is constant in time, so that the physics of such a condensate cannot be any different than the physics of the gauge-equivalent 2-Higgs doublet model with a vanishing $Z_3$ condensate. Nonetheless it may be advantageous to analyze the problem in a basis with $\langle Z_3 \rangle \neq 0$. The most naive expectation in this basis would be that the CP violating Bohm-Aharonov phase $\int Z_3 \cdot dz$ takes the place of the CP violating Higgs phase. However this may not be the case, since in the $\theta$ basis a non-vanishing $\theta''$ is required in order to have CP violation. In the $\langle Z_3 \rangle$ basis this would correspond to requiring $\langle Z'_3 \rangle \neq 0$.

Turok et al have recently argued\cite{29, 30} that in the presence of a non-vanishing $\langle Z_3 \rangle$ there is a regime of momenta in which fermions experience a CP violating “classical force” which acts like a “momentum filter”, and that this CP violation does not depend on quantum interference between some CP violating phase shift and a reflection phase shift. As we shall discuss in section §2.4, quantum mechanical interference effects may be destroyed by collisions present in the high-temperature plasma, so a mechanism which does not re-
quire interference is attractive. However there are still a number of features of this “classical force” proposal which need clarification. First of all, it should be emphasized that the starting point of using the Dirac equation implicitly assumes coherence of the wavefunction, so that conclusions following from such an analysis must be very carefully examined to make sure that the presumed coherence plays no essential role. Furthermore refs. [29, 30] explicitly drop terms in $Z_3'$ in their WKB argument, while the general reasoning above would suggest these terms are essential for actual CP violation\textsuperscript{9}. Moreover, it is clear in the $\theta$ basis that the presence of the CP-conserving mass is essential to having a genuine CP violation, while its relevance in the Turok et al discussion is obscure. In fact, in the usual analysis the spatially varying CP conserving mass is necessary to allow quantum interference with some CP-violating phase shift. Thus understanding the dependence on CP conserving mass in the $<Z_3>$ basis is necessary for substantiating the claim that the a qualitatively new, “classical force” has been discovered. An actual solution to the Dirac equation in the background of a specific $Z_3$ condensate which displays the properties envisaged in refs. [29, 30], allowing computation of a physical current, could clarify these issues.

Even if having a $Z_3$ condensate introduces nothing qualitatively new with respect to coherence issues, retaining it explicitly may be useful for determining the vevs in the vicinity of the bubble wall. Nasser and Turok\textsuperscript{31} emphasize the importance of the possibility of an instability in which a chiral top-quark pileup forms in front of the wall. The idea is that if some thermal fluctuation produces a locally non-vanishing $<Z_3>$, top quark energy levels would shift in such a way as to locally redistribute the $t_L$ and $t_R$ densities, which in turn enhances the $<Z_3>$. The mechanism has only to do with the gauge couplings of the top quarks, so it is independent of the Higgs sector.

\textsuperscript{9}For instance a $<Z_3>$ uniform in all space would just shift the zero of the energy for left and right handed particles and shift their thermal distribution functions, so the “momentum filter” effect would have no physical significance.
and occurs in the minimal standard model. Since the “seeding” fluctuation is random, the sign of the condensate and thus of its the contribution to the bau would vary from region to region. In any given region of bubble surface the effect could be large. In order to get a non-zero result after averaging over all regions, there must be some asymmetry in the amount of bubble surface with positive and negative $< Z_3 >$. In the MSM, GIM suppression (to be discussed in section §2.3) would not appear in the CP violation from a given bubble wall; instead the local bau production would resemble that of a two-Higgs doublet model. The GIM cancelation would manifest itself as a tendency for the $Z$ condensate on different regions of bubble surfaces to produce bau’s of opposite signs to a very high degree of accuracy. Thus the crucial issue becomes the dynamics of competition between regions.

The idea of spontaneous CP violation having opposite signs on different bubbles or different regions of the same bubble was introduced by Comelli et al[32] in the context of the MSSM (see next section). Using the usual simple description of critical bubble formation in terms of the surface tension and energy density between true and false vacuum, they estimated the final baryon to entropy ratio to be the locally produced value of $\frac{n_B}{s}$ times $\Delta F$ where $\Delta F$ is the difference in free energy of critical bubbles of the two types. They obtained $\Delta F = \frac{\Delta F(R_{crit})}{T} 3\Delta \sigma$ where $\Delta \sigma$ is the difference in surface tension for the two types of regions. From ref. [33], $\frac{\Delta F(R_{crit})}{T} \sim 130$. Nasser and Turok[31] give a qualitative discussion of the competition between phases of $< Z_3 >$ after bubbles collide, but it is not explicit enough to allow comparison with the Comelli et al estimate.

Work on this subject is only in its infancy and many important effects have not yet been considered. However if the general suggestion is correct, and a dynamical instability in top quark reflection provides an important contribution to the CP violating condensate on the bubble wall, predictions of conventional models may be drastically altered. The central difficulty in the analysis will become understanding the dynamics of the evolution of the
bubbles toward predominance of one sign of spontaneous CP violation over the other.

2.2.4 Minimal Supersymmetric Standard Model

The supersymmetric minimal standard model is a special case of a two Higgs doublet model, however as noted above, supersymmetry does not allow the Higgs self-couplings necessary for the vevs to have a non-trivial relative phase. It was shown in ref. [34] that at $T = 0$ loop corrections involving soft susy breaking produce these couplings and thus the possibility of spontaneous CP violation. Ref. [32] suggested that at high $T$ there can be spontaneous CP violation such that the effective potential in the low-temperature phase has two nearly degenerate minima, with phases of $\pm \pi$. Thus on roughly half the bubbles $\theta_1(z)$ will decrease from 0 to $-\pi$ in going from the unbroken to the broken phase, while in the other half it will increase from 0 to $+\pi$. On each bubble the local bau production should be comparable to that of a maximal 2-Higgs doublet model. Then they argue that a tiny explicit CP violation, easily consistent with the limits on the neutron edm, could produce a difference in surface tension on the two types of bubbles enough that the net bau could be consistent with observation[32].

In the MSSM there can also be explicit CP-violating phases in the couplings between gauginos (and higgsinos) and squarks, sleptons, and Higgs and gauge bosons. In the absence of some special circumstances, these phases must be very small in order not to be in conflict with the limits on the electric dipole moment of the neutron. Production of a bau via this explicit CP violation was investigated in the “spontaneous baryogenesis” scenario, appropriate when the bubble wall is thick[35]. It was concluded[35] that if the perturbative effective potential is required to produce a sufficiently strong phase transition (see section §4) and the neutron edm is not too large, only a very tiny portion of parameter space for the MSSM could give a large enough bau,
and then only under extremely optimistic assumptions. However in view of
the recent developments regarding the importance of non-perturbative effects
in the effective potential [8, 14] discussed in the first section, the parameters
of the MSSM should probably not be so severely constrained as was done
in [35]. Furthermore, the bubble wall seems likely to be better described
as thin rather than thick, so that explicit CP violation from the MSSM for
the thin wall regime needs to be studied. In this case, the explicit CP
violating phases in electroweak gaugino and higgsino couplings to the Higgs
vevs presumably produce a chiral higgsino and/or gaugino current. Since
higgsinos and electroweak gauginos have an \( SU(2)_L \) anomaly, they couple to
the sphaleron, so that a suitable asymmetry in these could in principle lead
to a baryonic excess. This scenario deserves quantitative investigation. If
it does not work, baryogenesis in the MSSM will, like in the \( Z \) condensate
scenario for the MSM, be dependent on understanding the difficult problem
of competition between phases and bubble evolution.

2.3 The Minimal Standard Model

It is natural to ask whether the CKM CP violation which is usually thought to
account for the CP violation seen in the kaon experiments can also account for
the bau. Since the MSM violates C as well as CP, a baryonic current and not
just an axial baryonic current can be produced by the asymmetry in reflection
probabilities, so that effects of the strong sphaleron are less problematic than
in two Higgs doublet models. However there is a simple argument which
indicates that CKM CP violation should have a practically negligible effect
in cosmology. To explain the argument we must first look more closely at
CKM CP violation. In this model, CP violation is due to a non-trivial phase
in the matrix which relates the quark eigenstates for coupling to the gauge
fields to those for coupling to the Higgs field$^{10}$. When there are just two
generations this is the familiar Cabibbo matrix, whose matrix elements can
be taken to be real and thus are CP conserving. However using the freedom
to perform gauge and global chiral rotations on the quark fields, Kobayashi
and Maskawa showed that if there are three families of quarks, a general
CKM matrix is described by three Euler-like angles, plus a single physically
significant phase, $\delta_{CP}$, which cannot in general be removed by rotations on
the quark fields. Not surprisingly, this phase can be rotated away if any pair
of the like-charge quarks are degenerate in mass by using the extra freedom
to rotate these indistinguishable quarks into one another. Similarly, if the
gauge and Higgs couplings of two generations are “aligned” (i.e., one or more
of the three CKM mixing angles vanishes) there is an additional freedom to
rotate them into one another, removing the CKM phase.

Now we can see why one might expect that CKM CP violation cannot
be responsible for generation of the bau. CP violation vanishes when any
pair of quarks is degenerate or any CKM angle vanishes, in what is known
as “GIM” (Glashow-Illipoulos-Maiani) cancellation. Thus it vanishes when

\[
\delta_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP}
\]

\[(m_t^2 - m_c^2)(m_t^2 - m_b^2)(m_t^2 - m_d^2)(m_t^2 - m_s^2)(m_t^2 - m_d^2)
\]

vanishes$^{11}$. Since this is a dimensional quantity, one might imagine that the
temperature is the natural dimension in the problem and estimate the dimen-
sionless “figure of merit” for the effective magnitude of CKM CP violation
during the ew phase transition to be $d_{CP}T^{-12} \sim 10^{-18}$, using experimental
constraints on the product of sines of the CKM angles and quark masses
($m_d \sim m_u \sim 0$, $m_s \sim 0.15$ GeV, $m_c \sim 1.6$ GeV, $m_b \sim 5$ GeV and $m_t \sim 175$
GeV) and $T \sim 100$ GeV at the ew phase transition.

$^{10}$Since quark masses arise from their couplings to the Higgs field, the latter eigenstates
are just the physical mass eigenstates in the zero-temperature theory.

$^{11}$The dependence on mass-squared differences rather than just mass differences is due
to the fact that the sign of a fermion mass is not physically significant.
This estimate is not legitimate if the dependence on quark masses is not perturbative in all the mass-squared differences, or if for some reason the relevant dimensional parameter setting the scale is not the temperature\(^{12}\). To assess the validity of the \(10^{-18}\) estimate, it is necessary to actually consider a specific mechanism of baryogenesis. However doing a correct computation for this problem is much more difficult than in the models discussed earlier, since in this case it is not enough to consider simply the quantum mechanical reflection of a single species of quark or lepton from the Higgs vev. It is necessary to simultaneously treat the reflection of all species of quarks, since with fewer than three generations there is no CKM CP violation. Furthermore the interaction of quarks with the Higgs vev cannot by itself produce a baryonic current in the MSM, since in the basis of mass eigenstates the CP violating phase appears only in quark interactions with charged gauge bosons. Thus quark interactions with \(W^\pm\)'s in the plasma, as well as with the vev of the neutral Higgs field, must be taken into account. However the interaction of a quark reflecting from the vev is a non-local interaction, while the interactions of the quarks with the gauge bosons in the plasma are local. Until recently (see section 3 below) a formalism had not been developed for dealing with this situation.

In order to get an idea of whether the bau produced in the MSM might be significantly larger than \(\sim 10^{-18}\), M. Shaposhnikov and I took the following approach\(^{37, 15}\). By working in the basis of quasiparticle excitations of the plasma, some of the effects of the interactions of the quarks with the gauge and Higgs particles in the plasma are taken into account\(^{13}\). Then we considered the quantum mechanical scattering of these quasiparticles from the Higgs vev. This approach is not strictly consistent from the standpoint

\(^{12}\)Shaposhnikov\(^{36}\) pointed out that it also ignores the modifications to the effective local CKM matrix arising from interactions of the quarks with gauge and Higgs particles present in the high temperature plasma.

\(^{13}\)The quasiparticles are basically the quarks, “dressed” by their interactions with particles in the high temperature plasma.
of perturbation theory, since it includes for instance the $O(g^2)$ effect of interactions with $W$’s in the plasma on the quasiparticle propagation, but not the effect at the same order of processes in which real $W$’s in the plasma scatter from the quarks, while both quarks and $W$’s may be reflecting from the wall. On the other hand, there is no obvious bias in neglecting the multi-body processes and no method was known for including them, so that it seemed a reasonable approach for getting an idea of the possible size of the CKM effect.

We found that in this approximation there is indeed a phenomenon for which the perturbative estimate above is inapplicable. In the thermal plasma there is a spectrum of quark momenta normal to the bubble wall. For a quark of mass $m$, a fraction $\sim m/T$ of its phase space corresponds to total reflection. When the quark is strongly or totally reflected, its interaction with the Higgs field is not at all perturbative. In particular, there is a region of momenta for which the strange quark is totally reflected but the down quark is not, in a fraction $\sim m_s/T$ of the total strange quark phase space. In this region, the GIM cancellation which is at the heart of the small result of the perturbative estimate is partially evaded, and one finds a result which could be consistent with observation\textsuperscript{14}. Solving the differential equations for the matrix of reflection coefficients numerically, and attempting to include errors from all sources, we estimated\textsuperscript{15}

$$n_B/s \approx (10^{-9} - 10^{-12}) \, v \, f_{sph} \, f_{3d}.$$  \hspace{1cm} (14)

where $f_{3d}$ is the error introduced by having done a 1-dimensional calculation

\textsuperscript{14}One can find the $s \leftrightarrow d$ and $s \leftrightarrow s$ quark reflection amplitudes analytically in thin wall approximation, perturbatively in the CKM mixing angles. The quantity which replaces $d_{CP}T^{-12}$ as the dimensionless measure of CP violation turns out to be\textsuperscript{15}

$$\Delta(\omega) = -2 \left( \frac{\sigma_0 T^2}{\sqrt{\omega M^2}} \right)^3 \frac{m_1^4 m_2^2 m_3^2 m_{12} m_{23} m_{13} s_{12} s_{13} s_{23}^2 \sin CP}{m_{12}^2 m_s^2} \, \text{Im}(r_s),$$

where $\text{Im}(r_s) \sim 1$ for $\omega$ such that the $s$–quark is totally reflected: $\omega \sim 50$ GeV. This energy is much larger than the strange quark mass due to thermal contributions to the quasiparticle mass gap. See ref. \textsuperscript{15} for a discussion of the limitations of this expression for $\Delta$.  

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rather than a three dimensional one, which we estimated to be in the range $10^{-2} < f_{3d} < 10^{+2}$. As noted above, the sphaleron efficiency factor $f_{sph}$ enters the asymmetry calculation in any model. It is estimated\(^\text{14}\) to be in the range $10^{-4} - 1$, while the bubble wall velocity has been estimated\(^{38, 39, 40, 41}\) to be $v \sim 0.1 - 0.9$. Thus one can see that if circumstances are favorable, and the quasi-particle-reflection approximation for computing the baryonic current is a reasonable guide, that CKM CP violation could be responsible for the observed bau, $\frac{n_B}{n} \sim (2 - 4) \times 10^{-11}$.

However it was stressed in ref. \(^{15}\) that for the kinematic region of importance in this mechanism, the quantum mechanical reflection of quasiparticles does not provide a satisfactory description of the problem since the penetration length of a totally-reflecting strange quark into the broken phase is much larger than the strong interaction collision length of the quasi-particle. The implications of this will be discussed below in section §2.4. Another aspect of the calculation of ref. \(^{15}\) which should be examined critically is the possibility of what could be called a “GIM conspiracy”, in which there is a cancellation between processes in the purely electroweak theory, for instance between processes involving real and virtual $W$’s, and/or those involving real and virtual heavier quarks. As an example, $W$’s, Higgses, and top quarks outside the bubble are mostly reflected back into the unbroken phase when they hit the bubble wall. Thus in the plasma rest frame there is a net current of $W$’s, Higgses, and tops toward the unbroken phase. If the average momentum transfer when left-chiral antiquarks scatter from one of these particles, summed over boson charges and quark flavors, is different than it is for left-chiral quarks, this would provide another mechanism for generation of a baryonic current which could cancel or add to the contribution from direct reflection. Like the issue of quasi-particle scattering which we discuss next, this is an aspect of the baryogenesis problem which requires simultaneous

\(^{15}\)See, e.g., ref. \(^{15}\)
treatment of reflection and particulate interaction\textsuperscript{[15]}. A method for dealing with such a situation will be presented in §3.

As a final remark on baryogenesis in the MSM, note that the phase transition may be much more violent than indicated by the perturbative calculations. If bubble-expansion produces a region of “compression” in the surface of the bubble, with the Higgs vev taking on much larger values than expected from the kink-solution to the perturbative effective potential, even purely perturbative MSM CP violation might account for the bau. Since it depends (see eqn (13)) on the twelfth power of the vev, a factor of 10 or so increase in the vev inside the wall could produce a bau of the right order-of-magnitude. This speculation is completely unmotivated, but provides additional incentive for making a quantitatively accurate theory of the phase transition.

2.4 Breakdown of the Quantum Reflection Approximation

In order for the validity of the quantum reflection approximation in the high temperature plasma to be guaranteed, collisions with other particles in the plasma must be irrelevant. This is assured if the collision length is large compared to the wall thickness, and also compared to the penetration length of a totally reflecting particle. For the quasiparticles of the high temperature plasma, the mean-free-path for gluonic collisions is estimated to be\textsuperscript{[17]} \[ \lambda_{\text{inel}} \sim (0.15g_s^2T)^{-1} \sim 1/20 \text{GeV}^{-1}, \] or up to a factor of five larger for low-momentum quasi-particles when Debye screening is taken into account. Thus for wall thicknesses of order a few \( T^{-1} \), with \( T \sim 100 \text{ GeV} \), the former condition may be approximately satisfied. The latter condition is more problematic for light quarks. The calculation of reflection ampli-

\textsuperscript{16}When quark reflection is not incorporated, the momentum-transfer asymmetry is negligible because the perturbative argument applies.

\textsuperscript{17}[15] and references therein.
tudes in the presence of total reflection relies on the boundary condition that the rising exponential solution be discarded as unphysical since it is non-normalizable. This boundary condition is appropriate if the particle which is totally reflecting has a sufficiently small scattering probability off other particles in the medium that it is unlikely to collide before the magnitude of the rising exponential is significantly greater than 1. This is clearly satisfied if $Im(p_z)\lambda >> 1$, where $p_z$ is the $z$-component of the momentum in the forbidden region and $\lambda$ is its mean-free-path. However although $|p_z| \sim 50$ GeV for the quasiparticle, the imaginary part of its momentum in the broken phase is $\sim m(T)$, the product of the Yukawa coupling of the quasiparticle and the vev in the broken phase. For the strange quark $m_s(T) \sim 10^{-3}T$ so if the relevant mean free path is $\lambda_{inel}$ for gluonic collisions given above, the product $m_s(T)\lambda_{inel} \sim 1/200 - 1/40$. Thus as stressed in [37, 15] for light quarks the quantum reflection approximation cannot be used without additional justification. On the other hand the approximation should be good for top quarks, for which $m(T) \sim 175$ GeV $v_1(T)/v_1(0)$, unless $v_1(T)/v_1(0)$ is very small [8]. It may also be good for the $\tau$ lepton, if the vev of the Higgs coupled to the $\tau$ is large compared to its low temperature value [30], since the collision length is greater by a factor $\sim (\alpha_{QCD}/\alpha_w)^2 \sim 10$.

When the collision length is not large compared to the penetration length of the wave function into the forbidden region, it means that the physical quantities of importance need to be computed directly, without discussing reflection of single-particle states. Indeed, the whole notion of the reflection probability becomes meaningless because there is no experimental way to prepare a particle far from the wall and determine its probability of reflection. Finding or not finding an outgoing particle with the opposite momentum obviously is not the relevant criterion in a plasma consisting of large numbers of particles in random motion. As an example, computation of the “snow-
plow” effect, in which quarks pile up in front of the bubble wall impeding its expansion, is straightforward for sufficiently heavy quarks because when $M^{-1} \ll \lambda$, quantum mechanical reflection is the dominant mechanism since the collision length is small compared to the penetration length. On the other hand, the drag on the bubble in a theory with 100 species of quarks having a mass $M/10 \lesssim \lambda^{-1}$ could be comparable, although in this case it would be a complicated process with several mechanisms being important. For instance in addition to losing momentum as a result of their interaction with the bubble wall, the quarks also collide with W’s (which are efficiently reflected) and lose net momentum due to the W flow away from the approaching bubble.

In the absence of a theory of how to treat these complicated interactions, two groups\cite{42,43} have tried to make models to determine the result of a complete theory. Both groups retain the use of the single-particle QM reflection approximation, although the details of their models differ. In the quasi-particle approach, collisions give rise to a “lifetime” or “damping rate” for the quasi-particle, corresponding to an imaginary part in the quasiparticle propagator of order $\gamma \sim 0.15 g_s^2 T^{-1}$. Gavela et al\cite{12} solve the Dirac equation with this imaginary part to find the reflection coefficients of the “decaying” quasiparticles. In order to avoid a manifest contradiction with unitarity, they incorporate a “source” for the quasi-particles which replaces them at a rate which keeps the system in equilibrium. Using this model, they find that the contribution of quasi-particle reflection to the baryonic current $J_{CP}$ is many orders of magnitude smaller than if there were no collisions.

While the existing calculations are clearly incapable of demonstrating that the MSM accounts for the observed bau, the authors of refs. \cite{12,13} go further and claim that their model calculations demonstrate conclusively that the MSM cannot be responsible for the bau. I believe that this conclusion is too strong. In a system where multi-body collisions are crucial (as in the snowplow drag computation mentioned above), the nature of the effect changes sufficiently that one cannot draw any \textit{a priori} reliable conclusions
from computing reflection coefficients, which is the approach used in refs. [12, 13]. Moreover, both refs. [12, 13] assume that the effect of inelastic collisions is to destroy the quantum coherence necessary to CP violation\footnote{In ref. [13] this is an explicit assumption and in ref. [12] it is implicit in their mechanism of maintaining a constant quasi-particle density even though quasi-particles decay.}. While they regard this assumption as self-evident, I will argue below that it may not be correct. Since it is the crucial physical question it should not be simply input as an assumption.

To show that one should not dismiss multi-body processes as “obviously” incoherent, let us consider in greater detail how the baryonic current results from the interference between CP-violating and CP-conserving phases. The idea is to first look at some particular quark reflection process (specifying, for instance, the momentum and angle of incidence and flavor of the quark) and then examine the contribution to $J_{CP}$ of multigluon processes with a similar quark current. If all such processes contribute to $J_{CP}$ with the same sign, the multigluon processes would not tend to cancel the contributions of the simple reflection, and the quantum mechanical reflection approximation could give an estimate which is reliable up to order $\alpha_s$ corrections.

Begin by considering a simple quark reflection with some average baryonic current, $J_A$. Let $A$ denote the amplitude for this process, to zeroth order in $g_w$. Due to the interaction of the quarks with the bubble wall, $A$ will in general be complex even at tree level. The amplitude for the antiparticle process will also be $A$ so the two contributions to $J_{CP}$ cancel. Now consider the lowest order corrections to this amplitude which have CKM CP violation, namely the original diagrams “decorated” with two charged $W$’s. Taking for the time being the CKM phase $\delta_{CP} = 0$, the full amplitude will be $A(1 + ae^{i\phi})$, where $a \sim g_w^4$ is defined to be a positive real number. The phase $\phi$ is non-zero because the masses of the quarks in the intermediate state are different than in the leading term and therefore lead to a different reflection
phase shift. When the CKM phase $\delta_{CP}$ is reinstated, the amplitude will be $A(1 + ae^{i(\phi + \delta_{CP})})$; the amplitude for the same process in which the quarks are replaced with antiquarks of the same chirality is $A(1 + ae^{i(\phi - \delta_{CP})})$. The net contribution to the baryonic current from the quark and antiquark processes will be proportional to $J_A \sin \phi \sin \delta_{CP}$. Now consider a similar process, but with some number $n$ of external gluons, which makes a contribution $J_B$ to the baryonic current with the same sign as the original $J_A$. Parametrically, $J_B \sim g_s^{2n} J_A$. When the virtual two-$W$ corrections to this process are included, we can write its amplitude as $B(1 + be^{i(\phi' + \delta_{CP}}))$ for the particle and antiparticle process respectively. When we add this to the contribution from the first process we find a total baryonic current $(J_A \sin \phi + J_B \sin \phi') \sin \delta_{CP}$. Since the virtual short distance $W$ exchange is the same in the two cases and $\phi$ and $\phi'$ originate from interaction with the bubble wall, we expect $b \sin \phi' \sim a \sin \phi$ and thus both processes contribute with the same sign to the net baryonic current, even though the amplitudes $A$ and $B$ are obviously incoherent.

We have seen that as long as the $W$ exchanges in the two processes are short distance effects, the multi-gluon processes will contribute coherently to $J_{CP}$ even though there is obviously no phase relation between the amplitudes involving different numbers of gluons. However when the $W$’s in the loops go on shell there are additional phases from the loop integrals and one can no longer argue that $b \sin \phi' \sim a \sin \phi$. This suggests that the flavor decoherence length of the strange quark, $\lambda_{fd} \sim \frac{\alpha_s M_W^2}{\Delta y_{inel} m^{el}} \lambda_{inel} [\text{F}^2]$, may be the relevant length scale determining when the quasiparticle reflection approximation can be used for estimating the baryonic current.

The discussion given above is very crude and could not convince anyone that gluonic effects do not wash out a CKM contribution to the bau. Clearly one needs a systematic approach to the problem, which can consistently keep track of the relative signs of the contributions of different processes to the net baryonic current in order to tell what the net effect of the multitude of different processes will be. However this example shows that one should not
make the a priori assumption of refs. [42, 43] that the multi-body processes contribute with random signs to the net baryonic current relative to one another. In order to discard with confidence the MSM as the source of the bau, one must make a first principles calculation of the effect of multi-body processes. An outline of a method to do this is given in the next section.

3 Field Theory in the Background of the Bubble Wall

In this section, I will describe a framework for systematically studying aspects of the physics which involve both local particulate scattering and non-local quantum mechanical reflection. The material is technical and was therefore only covered in summary form in the lecture at the “Trends in Astroparticle Physics” workshop. It is essential for reliably treating problems in which the collision length is comparable to or smaller than $m^{-1}$, as is the case in the MSM model in the important region of strange quark total reflection.

3.1 General Discussion

The ultimate quantity of interest for determining the bau produced during the electroweak phase transition is the expectation value of the baryonic charge in the broken phase, long after the wall has passed. In the plasma rest frame it is the 0-th component of the four-vector

$$< J_\mu(x) > = \sum_i < \bar{\Psi}^i(x) \gamma_\mu \Psi^i(x) > ,$$

(15)

where the superscript labels each type of quark. Thermal and non-equilibrium effects are included by taking the appropriate ensemble average, innocuously denoted $< ... >$. Up to now this density has been computed for the charge-transport mechanism by the procedure described in the previous sections:
1. Find the reflection coefficients for quarks and antiquarks of both chiralities from the bubble wall.

2. Find the current $J_{CP}$, by determining the contribution of each species and chirality of quark and antiquark to the current of interest, assuming fluxes from each side of the wall corresponding to equilibrium statistical distributions coming from the two phases, boosted to the wall rest frame.

3. Consider the action of the sphaleron on the system with the current $J_{CP}$ flowing into the unbroken phase to determine the final baryonic density.

A conceptually better approach is to instead develop field theory in the background of the non-constant vev. In perturbative approximation, the current gets a non-vanishing contribution already from a simple quark loop in the two-Higgs doublet baryogenesis model, but is only non-vanishing at 3-loops with CKM CP violation, as will be shown below. The effect of quark and gauge and Higgs boson scattering from the bubble wall is included non-perturbatively by using propagators in the background of the changing vev. A given higher order diagram for gets a non-vanishing contribution already from a simple quark loop in the two-Higgs doublet baryogenesis model, but is only non-vanishing at 3-loops with CKM CP violation, as will be shown below. The effect of quark and gauge and Higgs boson scattering from the bubble wall is included non-perturbatively by using propagators in the background of the changing vev. A given higher order diagram for corresponds to a number of physical processes which can occur in the background of the wall. For instance the diagram with a quark loop and two exchanged $W$'s represents quarks interacting with virtual $W$'s while propagating to or from the wall, as well as the multibody reflection process $Q_i + W \rightarrow Q_j + W$. Thus this approach naturally overcomes the difficulty with the quasi-particle reflection approach, of not being a consistent expansion in the coupling by not including multibody scattering-with-reflection\textsuperscript{20}. Equally importantly, it allows the effect of \textsuperscript{20}This is not a conceptual problem for the two-Higgs doublet model because in that case CP violation is present simply from the interaction with the vev, so all higher order corrections can be ignored without losing the effect.
higher-order gluonic interactions to be reliably assessed because local particulate scattering as well as non-local reflection from the wall can both be included.

In order to obtain a non-zero value for $< J_0 >$, sphaleron interactions must be included. In principle, they can be treated as a (space dependent) insertion in the fermion line. The dependence of the sphaleron rate on the local Higgs vev means that the insertion is small or negligible in the broken phase. Its value on a given quark line will depend on the chemical potentials of the other left-chiral quarks and leptons in the local environment. The steady state of the entire system should in principle be determined self-consistently, fixing the wall velocity and individual particle distribution functions for every flavor of quark and antiquark, lepton and antilepton, gauge and Higgs particle, as a function of position relative to the wall, in terms of each other. However a much simpler approach, which employs the approximation already in use for charge-transport calculations, is to ignore sphaleron insertions in the calculation of $< J_\mu(x) >$ and deviations of the particle distribution functions from what they would be if the vev were constant with its local value. Then given $J_{CP} \equiv < \vec{J}(x) >$, one can take into account the sphaleron processes and diffusion through the bubble wall by using the results developed in section 5 of ref.[15] relating $n_B$ to $\frac{12}{5} J_{CP} f_{sph}$.

To implement this approach, one must first find the Greens functions for free quarks and gauge and Higgs particles in the background of the changing vev. The quark propagators in the background of the changing Higgs field depend in a non-trivial way on the quark Yukawa couplings even in the unbroken phase, since the solutions to the Dirac equation in the presence of the vev contain the reflection amplitudes for scattering from the Higgs field. For instance for a step function vev, the reflection amplitude for a scalar is $\frac{k-l}{k+l}$ where $k$ and $l$ are the momenta in the unbroken and broken phases. The propagators also contain a non-trivial CP-conserving phase because the reflection coefficient is complex for at least some incident
The zero-temperature momentum-space quark propagator in a theta function background vev is given in ref. [44]. In the presence of the wall, the scattering solutions in the gauge and Higgs boson sector are changing superpositions of the asymptotic fields and one must retain a finite wall thickness if one wishes all the asymptotic states to have finite mass. Analytic solutions are given in ref. [45]; this allows the free field operators of the standard model bosons to be written down and their propagators to be determined. Since energy and momentum parallel to the bubble wall is conserved, but translation invariance perpendicular to the wall is lost, calculations are simplest in a mixed representation in which propagators depend on $z$, $z'$, $\vec{k}_\perp$ and $\omega$, where $z$ and $z'$ are the initial and final coordinates normal to the bubble wall. Equilibrium finite temperature perturbation theory works just as usual and finding the temperature Green’s functions is straightforward. The program is nearly complete to this point for the standard model and will be reported elsewhere[46]. Note that it is entirely different from the approach of Gavela et al[42] which considers only quasi-particle reflection and thus does not reap the benefits of the field theoretic formalism.

As noted previously, $J_{CP}$ vanishes when fluxes from the broken and unbroken phases are equal as they are in thermal equilibrium. Thus it is not sufficient to use equilibrium finite temperature field theory to solve this problem. However the approximation conventionally used for charge-transport baryogenesis can be easily implemented. Particles incident from the broken phase are taken to have distribution functions appropriate to particles in equilibrium with the plasma deep in the broken phase, and correspondingly for those from the unbroken phase, all boosted to the wall rest frame. Although it may not be possible to put the corresponding “temperature” Green’s functions into a compact form, that is not necessary to performing

\footnote{In the thin wall limit it is complex for the regions of integration in which total reflection occurs: $k^2 < m^2$, while for finite wall thickness it can be complex even without total reflection.}
Given this framework, the importance of gluonic interactions can be assessed by computing the gluonic corrections to the leading contribution. Although this requires a 4-loop calculation for the MSM, the general question can be addressed in the 2-Higgs doublet model. Given an analytic solution to the Dirac equation with a spatially varying phase for the vev, one can find the Greens function for the quark in the 2-Higgs doublet model. Computing the $m_q \to 0$ part of the 1-gluon correction to the basic quark loop should be a feasible calculation. An indication of trouble from gluonic interactions would be the presence of collinear or soft logarithms. If these are detected, the techniques developed for studying Sudakov suppression in QCD can be extended to ascertain whether resummation of the higher order QCD corrections damps the leading order effect. In the absence of such logs or some other indication of a breakdown in perturbation theory when the height of the barrier is small, the strong corrections will be nothing but an order $\alpha_s$ correction to the leading electroweak result.

An important virtue of the approach advocated here is that it automatically includes all effects, systematically at any given order of perturbation theory, with the correct relative weights and coherence properties. It solves the problem of accounting for both coherent quantum mechanical reflection and the localized interactions of particles which are themselves being reflected and transmitted from the interface between low and high temperature phases.

### 3.2 Application to the MSM

Consider computing the expectation value of the current, using perturbation theory to whatever order is necessary. In order to produce a net non-vanishing result for $J_{CP}$, an interference is needed between $\delta_{CP}$ and the CP conserving phases appearing in propagators and in loop integrals (which can have a non-vanishing absorptive part due to the presence of real intermediate
states). In a 2-Higgs doublet model, the CP violating phase would already be present in the quark propagator, as well as a CP-conserving phase associated with the reflection process, so that a non-zero result for $J_{CP}$ should appear already in the basic quark loop. On the other hand, in the minimal standard model, a non-zero result for $J_{CP}$ arises first at 3-loop order. This can be seen as follows. Taking $T = 0$ quark mass eigenstates as our basis, quark interactions with the neutral Higgs are purely diagonal in flavor and thus the Green’s functions are flavor-diagonal. Unlike the two-Higgs doublet model with a spatially varying vev, the phases in the MSM propagators are exclusively CP-conserving. The couplings of the quarks to $W^{\pm}$’s and charged Higgs bosons are proportional to the CKM matrix $V$, and in this basis at least 2 $W^{\pm}$’s or charged Higgs bosons are required in the loop in order for there to be a non-trivial dependence on $\delta_{CP}$. With a single $W$ exchanged in the quark loop, the flavor structure is just $\text{tr}[VAV^{\dagger}B]$. The antiparticle contribution is given by replacing $V \rightarrow V^*$. But $\text{tr}[V^*AV^{\dagger}B] = \text{tr}[BVAV^{\dagger}]$ because $A$ and $B$ are diagonal in the basis we have chosen. This is equal to the particle contribution by the cyclic property of the trace, so there is no CP violation until next order.

Non-trivial flavor dependence, necessary if the GIM cancellation is to be evaded in the MSM, arises from the mass dependence of the quark Green’s functions and from Higgs vertices. In the quasi-particle reflection approximation, it was shown in ref. [15] that the cancellation is evaded in the region of momenta in which the $s$ but not $d$ quark is totally reflected. Now we can ask whether there could be a “GIM conspiracy”, with cancellations occurring between different processes in the purely electroweak theory. There are four distinct diagrams with two charged $W$’s exchanged in the quark loop, as well as diagrams with charged Higgs bosons replacing the $W^{\pm}$’s. In each of these diagrams the structure in flavor space is $\text{tr}[VAV^{\dagger}BVCV^{\dagger}]$, where
$A$, $B$, $C$, $D$ are diagonal but not proportional to the unit matrix\textsuperscript{22}. In the absence of the wall, the only way for $A$, $B$, $C$, $D$ to have a non-trivial flavor dependence is for there to be Higgs interactions on the quark lines between the $W^{\pm}$ vertices. But since Higgs interactions change the chirality of the quark, and the $W^{\pm}$'s couple only to left chiral quarks, one would need two Higgs interactions to change the chirality from $L$ to $R$ and back to $L$, so that $A$, $B$, $C$, $D \sim M^2$ and one arrives at the perturbative estimate. In fact, if the only source of flavor dependence is from the Yukawa couplings at Higgs vertices, one can see that there is no CP violation whatever at 3-loop order, as follows. In this case, since the propagators in the loops are being taken to be massless, for every contribution to $A$ there is an identical contribution to $C$ taking, e.g., $A$ and $C$ to be associated with the $D$–quark lines. But $\text{tr}[V A V^\dagger B C V^\dagger D] + \text{tr}[V C V^\dagger B V A V^\dagger D]$ is the same as the antiparticle contribution obtained by replacing $V \rightarrow V^*$, taking the transpose and using the cyclic property of the trace. This is the essential content of the argument of ref. \textsuperscript{47}.

In the actual problem, however, modifications in the quark propagators due to the presence of the wall introduce the possibility of a less suppressed dependence on some of the mass matrices between $V$’s and $V^\dagger$’s. For instance in a non-normal collision with the wall, angular momentum conservation does not require chirality flip as it does for normal incidence. At the same time, the reflection coefficients are still dependent on the height of the wall, and thus are flavor dependent. While it is possible to identify contributions which can have a sufficiently favorable dependence on quark masses to possibly account for the bau, a detailed investigation is necessary to estimate the final result. The complete calculation necessary to obtain a quantitatively accurate result and see whether the predicted sign is correct, is in principle straightforward but extremely difficult. Fortunately it should be possible to answer the most
urgent question, whether gluonic collisions obliterate the production of a baryonic or axial baryonic current, with a much easier calculation in the two-Higgs model.

3.3 Corrections to the electroweak sphaleron

Near the bubble wall, there can in principle be non-trivial CKM-dependent corrections to the fermionic lines in the effective vertex which lead to a different rate between sphalerons involving quarks and those involving antiquarks. Naively taking the ’t Hooft effective interaction in the basis in which gauge interactions are diagonal, and adding three Higgs to form a closed loop and gluons to fix the color, one can find diagrams which can interfere with the lowest order process. In the absence of the vev and the non-trivial mass dependence thus introduced into the quark propagation, it is easy to show that there is no CP violation arising from these corrections. However when the mass dependence is more subtle, this may not continue to be the case. Moreover the excess of top quarks in front of the wall, and the overall antisymmetry in flavor and color of the quarks in the sphaleron effective interaction, means that the sum over ways to insert the three Higgs into the flavor structure of the sphaleron is not a symmetric sum as it is in the usual trace over flavors. It does not seem particularly likely that including these corrections will give a significant contribution to the bau, however the possibility should be looked at more carefully before being discarded.

4 Summary

After reviewing in the first section the general issues of baryogenesis, the second section was devoted to discussing possible sources of the CP violation needed to produce the observed baryonic excess at the electroweak phase transition. The emphasis was on making the physical ideas of some of the
most interesting possibilities clear, not on giving a complete review of all models. Special attention is given to the relation between two-Higgs doublet models and the question of $Z$-condensation, and to the problems of baryogenesis in the MSM. The supersymmetric minimal standard model is seen to be surprisingly similar to the non-supersymmetric minimal standard model in spite of having a more complicated Higgs sector and other possible sources of CP violation. It was argued that in order to decide whether the MSM can be responsible for the bau, it is necessary to compute and include the contributions of multi-body processes, since pure quantum mechanical reflection is negligible for light particles in the high temperature plasma.

The final section outlines a new approach to treating dynamics near the bubble wall when both the non-local interaction of particles with the changing vev, and their interaction with other particles in the plasma, may be important. It provides a framework for settling the question of the importance of gluonic corrections to the electroweak processes in the MSM, and allows the issue of a possible GIM conspiracy to be investigated. The methodology presented here of doing perturbation theory in the background of the changing vev can also be used to obtain improved particle distribution functions in the neighborhood of the bubble wall\[10\].

No “bottom line” on electroweak baryogenesis is given. While a number of models have promise of explaining the observed baryonic asymmetry, it could also be the case that no model works. Many details regarding the phase transition, sphaleron rates, and bubble dynamics need to be understood better before any firm conclusion will be possible.

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