Spin 3/2 Baryons and Form Factors in AdS/QCD

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Abstract

We study the 5D Rarita-Schwinger fields to describe spin 3/2 baryons in AdS/QCD. We calculate the spectrum of spin 3/2 baryons (Δ resonances) and their form factors, together with meson-baryon couplings from AdS/QCD. The transition form-factors between Δ and nucleon are evaluated. Both pion and rho meson couplings have the same origin in the bulk and hence unified. The numerical values for the meson-baryon transition couplings are consistent with the values obtained from other methods. We also predict the numerical values of some new couplings associated with Δ resonances.

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I. INTRODUCTION

Solving Quantum Chromodynamics (QCD) is a long outstanding problem in physics. Recently gauge-gravity duality or Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence \cite{1, 2} has been used to study the strong coupling regime of gauge theories from string theory. The qualitative and quantitative results obtained in this manner indicate that we may hope to get QCD from string theory.

AdS/CFT correspondence may be adapted to describe the low energy dynamics of QCD. In the simplest model known as AdS/QCD, the AdS spacetime is cutoff at a finite distance from the ultraviolet boundary in AdS/CFT \cite{3, 4}. For related works, see also \cite{5}. This amounts to placing an infrared brane at some position in the bulk of AdS spacetime. The hadronic spectra obtained from AdS/QCD is 10-15 % off the experimental values. This hints that AdS/QCD may be a tip of some more accurate description of QCD from string theory. It has been shown recently that the results for the QCD current-current two point functions from AdS/QCD are related to Migdal’s approach to hadronization \cite{6}. In mathematical terms Migdal’s approach amounts to Pade approximation. The other models of QCD from string theory \cite{7, 8} use the microscopic description in terms of intersecting D-branes in string theory and perhaps AdS/QCD is some limit of these models. In the D-brane setup the baryons are realized as instanton solitons of the low energy effective theory \cite{9, 10, 11}.

In this paper we shall be interested in the spectrum of spin 3/2 baryons (Δ resonances) and the couplings between mesons and baryons from AdS/QCD. The holographic description of spin 3/2 operators in the boundary theory is given by the Rarita-Schwinger fields in the bulk \cite{12, 13}. The spectrum of spin 3/2 baryons is obtained by the eigenvalue equations for the normalizable modes of bulk Rarita-Schwinger fields, subject to appropriate boundary conditions at both the infrared (IR) and ultraviolet (UV) branes, where the IR brane is introduced by hand to implement the confinement of quarks, setting the scale for AdS/QCD, while the UV brane will be located infinitesimally close to origin. In AdS/QCD the chiral symmetry breaking is encoded in the vacuum expectation value of bulk scalar, dual of the order parameter of chiral symmetry at the boundary.

The effect of Yukawa couplings on the spectrum is very small. We also find the numerical results for the coupling of pions and rho mesons with nucleons and spin 3/2 baryons. The
pions and rho meson couplings arise from the same terms in the bulk and this allows us to predict some couplings not known so far. The transition form factors between nucleons and $\Delta$ are evaluated numerically for various values of momentum $Q^2$.

The plan of the paper is as follows. In the next section we briefly overview the chiral dynamics and the spectrum of mesons from AdS/QCD [3]. We also discuss the spectrum of nucleons [13]. In next section we consider the spectra of spin 3/2 baryons. The effect of bulk Yukawa couplings is also considered. In the following two sections we calculate the meson-baryon transition couplings and the nucleon-$\Delta$ transition form factors from AdS/QCD. The last section is devoted to discussion and conclusions.

II. MESONS AND NUCLEONS IN ADS/QCD

We shall be interested in the low-energy chiral dynamics of hadrons from AdS/QCD, proposed as the holographic dual of QCD [3]. The global flavor currents in the boundary theory will correspond to the bulk gauge fields, according to AdS/CFT correspondence. The bulk scalar $X$, which transforms as a bifundamental under chiral symmetry group $SU(2)_L \times SU(2)_R$, acts as an order parameter for the chiral symmetry breaking. In the boundary theory the expectation value of its dual operator, $\bar{q}_L q_R$ can be identified with the quark condensate. The minimal action in the bulk suitable for chiral dynamics can be written as

$$S_{\text{AdS}} = \int dz d^4x \sqrt{G} \text{Tr} \left[ |D X|^2 - M_5^2 |X|^2 - \frac{1}{4 g_5^2} (F_L^2 + F_R^2) \right]$$

(1)

where $G$ is the determinant of 5-dimensional AdS metric,

$$d s^2 = \frac{1}{z^2} \left( - d z^2 + \eta_{\mu \nu} d x^\mu d x^\nu \right),$$

(2)

and $\eta_{\mu \nu}$ being the flat 4-dimensional metric with the signature $(+, -, -, -)$. The fifth coordinate $z$ is cut-off at the infra-red scale $z_m = 1/\Lambda_{QCD}$, corresponding to the confinement scale of QCD, and one has to regulate the bulk action at the ultraviolet (UV) boundary $z = \epsilon \to 0$. The covariant derivative is defined as $D_M X = \partial_M X - i A_{LM} X + i X A_{RM}$. The five dimensional gauge coupling $g_5$ is related to the rank of the gauge group (number of colours) in the boundary theory [3] and we take $g_5 = 2\pi$ (for $N_c = 3$ and for two flavors). The five dimensional mass of the scalar, $X$, is fixed by the scaling dimension, $\Delta_0$, of
the operator $\bar{q}_L q_R$ in the boundary theory as $M_5^2 = \Delta_0 (\Delta_0 - 4)$ in the unit of inverse AdS radius \[2, 14\].

The classical solution for the bulk scalar $X$ can be written as,

$$\langle X(z) \rangle = \frac{1}{2} M z + \frac{1}{2} \sigma z^3. \quad (3)$$

By the AdS/CFT duality, the coefficient of the non-normalizable mode is identified as the source for the boundary operator $\bar{q}_L q_R$ and the coefficient of the normalizable mode as its vacuum expectation value, $\sigma = \langle \bar{q}_L q_R \rangle$. In the chiral limit, $M \to 0$, therefore we can write $X = v(z) e^{i P(x,z)}$ with $v(z) = \frac{1}{2} \sigma z^3$ and $P(x,z)$ is proportional to the pion fields upon the Kaluza-Klein reduction to four dimensions and $\sigma$ becomes the order parameter for the chiral symmetry breaking. Then one can identify the pions as the Nambu-Goldstone bosons of the broken symmetry.

In the leading approximation, one can work with linearized equations for the gauge fields. In the unitary gauge a linear combination $P(x,z)$ and the fifth component of axial gauge fields $A_z, z \partial_z (A_z/z) - 2 v^2 g^2_5 P/z^2$, and the fifth component of the vector gauge field become infinitely massive and decouple from the theory, while the orthogonal combination of $P$ and the axial gauge fields remain as a physical degree of freedom \[13\] if

$$z \partial_z \left( \frac{A_z}{z} \right) - 2 \frac{v^2 g^2_5}{z^2} P = 0, \quad (4)$$

which then relates pions with the fifth component of axial gauge field, $A_z$. The four dimensional components of the vector and axial vector bulk gauge fields can be identified with the infinite tower of vector and axial-vector mesons respectively, upon Kaluza-Klein reduction.

Next we turn to the description of baryons in AdS/QCD, as studied in \[13\]. To describe spin 1/2 baryons in accordance with AdS/CFT correspondence, we introduce a bulk spinor whose normalizable modes will correspond to color-singlet spin-1/2 states in the boundary theory (QCD), while its non-normalizable modes will couple to those states as sources. A 5D bulk spinor has same degrees of freedom as 4D Dirac spinor, which contains both left-handed components and right-handed components, transforming differently under (flavor) chiral symmetry of QCD. Since the flavor symmetry of boundary theory corresponds to gauge symmetry in the bulk, we need a pair of spinors (say $N_1$ for the left-handed components and $N_2$ for the right-handed components) in the bulk to describe a single Dirac spinor of boundary theory.
Furthermore, to calculate the correlation functions of baryons, we need to introduce sources in the boundary that couple to baryons as

\[ \int d^4x \left( \bar{O}_L B_R + \bar{B}_L O_R + \text{h.c.} \right). \]  

(5)

Since the source that couples to a chiral component of baryon has an opposite chirality, the non-normalizable mode of \( N_1(N_2) \) becomes the source of the normalizable mode of \( N_2(N_1) \).

The minimal bulk action for the spinors can be written as,

\[ S_N = \int dzd^4x \sqrt{G} \left[ i\bar{N}_1 e^M_A \Gamma^A D_M N_1 - m_5 \bar{N}_1 N_1 + (1 \leftrightarrow 2 \& m_5 \leftrightarrow -m_5) \right], \]

(6)

where \( e^A_M \) are the 5D vielbein, \( e^B_M \eta_{AB} = g_{MN} \), and \( \Gamma^A \) (\( A = 0, 1, 2, 3, 5 \)) are the 4 × 4 gamma matrices in 5D and the gauge and Lorentz covariant derivative \( D_M \) is given by

\[ D_M = \partial_M - \frac{i}{4} \omega^A\Sigma_{AB} - i(A^a_L)M \xi^a, \]

(7)

where \( \omega^A_B \) is the spin connection and \( \Sigma_{AB} = \frac{1}{2i} [\Gamma_A, \Gamma_B] \). The bulk spinor mass \( m_5 \) is fixed up to a sign by the conformal dimension, \( \Delta_1 \) of the boundary spinor as \( m_5^2 = (\Delta_1 - 2)^2 \) and the sign of spinor masses are chosen such that it preserves 4D parity \[13\].

When the interactions are turned off, the spinors \( N_1 \) and \( N_2 \) obey free Dirac equations in the bulk. To analyze the Dirac equations, it is convenient to use the chirality basis,

\[ \Gamma^5 = -i\gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \Gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \Gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (i = 1, 2, 3). \]

(8)

We first decompose the 5D spinor as

\[ N_i(x, z) = N_{iL}(x, z) + N_{iR}(x, z), \quad (i = 1, 2), \]

(9)

where \( \gamma^5 N_{iL} = N_{iL} \) and \( \gamma^5 N_{iR} = -N_{iR} \).

Let us consider the (free) Dirac equation for the spinor \( N_1 \) first,

\[ i e^M_A \Gamma^A \left( \partial_M - \frac{i}{4} \omega^B\Sigma_{BC} \right) N_1 - m_5 N_1 = 0. \]

(10)

Using the local Lorentz symmetry, we take \( e^A_M = \frac{1}{z} \eta^A_M \). Then the only non-vanishing components of the spin connections are

\[ \omega^{5A}_\mu = -\omega^{A5}_\mu = \frac{1}{z} \delta^A_\mu \quad (\mu = 0, 1, 2, 3). \]

(11)
The Dirac equations become

$$(z\gamma^5 \partial_z + z i \partial - 2 - m_5) N_1 = 0.$$  \hspace{1cm} (12)$$

It is convenient to Fourier-transform the bulk spinor as

$$N_{iL,R}(x, z) = \int_p f_{iL,R}(p, z) \psi_{iL,R}(p) e^{-ip \cdot x},$$  \hspace{1cm} (13)$$

where the 4D spinors satisfy

$$\not{p} \psi(p) = |p| \psi(p),$$  \hspace{1cm} (14)$$

where $|p| = \sqrt{p^2}$ for a time-like four-momentum $p$. The corresponding Dirac equation can be written as,

$$\left( \partial_z - \frac{2 + m_5}{z} \right) f_{1L} = -|p| f_{1R},$$

$$\left( \partial_z - \frac{2 - m_5}{z} \right) f_{1R} = |p| f_{1L},$$  \hspace{1cm} (15)$$

Near the UV boundary, $z = \epsilon$, we find

$$f_{1L} \simeq c_1 (1 + 2m_5) z^{2+m_5} + c_2 |p| z^{3-m_5}, \quad f_{1R} \simeq c_1 |p| z^{3+m_5} + c_2 (2m_5 - 1) z^{2-m_5},$$  \hspace{1cm} (16)$$

where $c_1, c_2$ are constants to be fixed by boundary conditions.

We first note that the spin-1/2 baryons should have massless modes in the spectrum to satisfy the 't Hooft anomaly matching condition when the chiral symmetry is restored or baryons do not couple to the chiral symmetry breaking order parameter. In AdS/QCD the bulk spinors should have normalizable zero modes ($|p| = 0$) when the interactions are turned off. We choose $m_5 > 0$ for $N_1$ to be consistent with the chirality of the operator in the boundary theory, which fixes the boundary conditions at UV and IR

$$f_{1L}(p, \epsilon) = 0, \quad f_{1R}(p, z_m) = 0.$$  \hspace{1cm} (17)$$

The remaining boundary conditions for $f_{1L}(x, z_m)$ and $f_{1R}(x, \epsilon)$ follow from the equations of motions, coming from the boundary term. The normalizable solutions for non-zero modes ($|p| \neq 0$) are given by

$$f_{1L,R}(p, z) \sim z^{5/2} J_{m_5 + \frac{1}{2}}(|p| z).$$  \hspace{1cm} (18)$$

The free spectrum with the boundary condition $N_{1R}(x, z_m) = 0$ is given by the zeros of the Bessel function $J_{m_5 + \frac{1}{2}}(|p| z_m)$. Since the AdS/CFT gives the relation, $\Delta = |m_5| + 2$ for Dirac
fields, we have $m_5 = 5/2$ for the canonical dimension $\Delta = 9/2$ if we neglect the anomalous dimension of spin-1/2 baryons.

Similarly one can discuss the bulk spinor $N_2$ whose normalizable modes correspond to the right-handed components of baryons at boundary. The sign of mass term for $N_2$ is opposite to that of $N_1$ and the boundary conditions are replaced by $N_{2R}(x, \epsilon) = 0$ and $N_{2L}(x, z_m) = 0$. Then one has the same spectrum as for the left-handed baryons, as it should be for QCD.

In QCD we know that the bulk of baryon mass comes from chiral-symmetry breaking. Since the condensate $\langle \bar{q}q \rangle$ is the order parameter of chiral symmetry breaking, the baryons should get mass through coupling to the condensate. In AdS/QCD [13] it was shown that this can be easily achieved by introducing Yukawa couplings between bulk spinors and bulk scalars,

$$\mathcal{L}_{Yukawa} = -g \left( \bar{N}_2 X N_1 + \bar{N}_1 X^\dagger N_2 \right).$$

(19)

The zero modes now acquire the mass via Yukawa couplings. Also the degeneracy of excited states between parity-even and parity-odd baryons is lifted by this term. One can estimate $g$ and $z_m$ by fitting the mass spectrum for the lowest-lying state (with the experimental value 938 MeV), which gives $g = 14.4$ and $z_m^{-1} = 205$ MeV. Then one can predict the mass spectrum for excited states and numerical values agree well with the experimental results. Furthermore, one can show that the pion coupling with excited baryons becomes smaller for more excited states such that chiral symmetry is effectively restored for highly excited states [13].

### III. SPIN 3/2 BARYONS ($\Delta$ RESONANCES)

Next to lowest-lying baryons are $\Delta$ resonances, which have spin 3/2 and also isospin 3/2. The ground-state $\Delta$ baryon has even parity and its mass is measured to be 1210 MeV with decay width about 100 MeV. Being the closest resonance to nucleons, $\Delta$ baryons are very important in studying the properties of nucleons such as nucleon potential, as the dominant decay channel of $\Delta$ resonance is $\Delta \rightarrow N\pi$. Recently the transition form-factors of $\Delta$-to-nucleons are accurately measured [15] and also studied intensively in lattice [16, 17] and in chiral perturbation theory [18]. In this paper we investigate the properties of $\Delta$ resonances in AdS/QCD, including the $\Delta$-nucleon transition form-factors. It will be interesting to compare our results with the experimental data and also with the lattice calculations.
A simple holographic description of spin 3/2 baryons in the boundary theory is given by the Rarita-Schwinger fields $\Psi_M$ in the bulk [12]. The action for the Rarita-Schwinger field in AdS$_5$ is given by

$$\int d^5x \sqrt{G} \left( i\bar{\Psi}_A \Gamma^{ABC} D_B \Psi_C - m_1 \bar{\Psi}_A \Psi^A - m_2 \bar{\Psi}_A \Gamma^{AB} \Psi_B \right),$$

(20)

where $\Psi_A = e^M_A \Psi_M$ and we used notations $\Gamma^{ABC} = \frac{1}{3!} \Sigma_{\text{perm}} (-1)^p \Gamma^A \Gamma^B \Gamma^C = \frac{1}{2} (\Gamma^B \Gamma^C \Gamma^A - \Gamma^A \Gamma^C \Gamma^B)$ and $\Gamma^{AB} = \frac{1}{2} [\Gamma^A, \Gamma^B]$. The Rarita-Schwinger equations in AdS$_5$ are then written in the following form,

$$i\Gamma^A \left( D_A \Psi_B - D_B \Psi_A \right) - m_- \Psi_B + \frac{m_+}{3} \Gamma_B \Gamma^A \Psi_A = 0,$$

(21)

where $m_\pm = m_1 \pm m_2$. The values of $m_1$ and $m_2$ correspond to those of spinor harmonics on $S^5$ of AdS$_5 \times S^5$ [19].

Being a reducible (axial) vector-spinor, the Rarita-Schwinger fields contain not only spin-3/2 components but also spin-1/2 components as well. In 4D, the extra spin-1/2 components can be projected out by a Lorentz-covariant constraint,

$$\gamma^\mu \Psi_\mu = 0.$$  

(22)

As in 4D, the following Lorentz-covariant constraint will project out one of the spin-1/2 components from the 5D Rarita-Schwinger fields

$$e^M_A \Gamma^A \Psi_M = 0,$$

(23)

which then gives $\partial^M \Psi_M = 0$ for a free particle if combined with equations of motion.

The 5D Rarita-Schwinger fields have one more extra spin-1/2 components, $\Psi_z$, if reduced to 4D. In the case of bulk vector fields, the 5th component constitutes the longitudinal component of massive spin-1 vector mesons of boundary theory or can be gauged away in the unitary gauge. However, in the case of bulk Rarita-Schwinger fields there is no gauge degree of freedom but we may choose $\Psi_z = 0$ to further reduce the extra spin-1/2 degrees of freedom, because there is no boundary extra spinor that it can be mapped into, and convert the Rarita-Schwinger equations to a set of Dirac equations for the remaining components,

$$\left( iz \Gamma^A \partial_A + 2i \Gamma^5 - m_- \right) \Psi_\mu = 0, \quad (\mu = 0, 1, 2, 3).$$

(24)

To describe the spin 3/2 baryons ($\Delta$ resonances) in AdS/QCD one has to introduce a pair of Rarita-Schwinger fields in the bulk, $\Psi_1^A$ (for the left-handed spin-3/2) and $\Psi_2^A$ (for
the right-handed spin-3/2), which obey the above Rarita-Schwinger equations. Similarly to
the case of spin-1/2 in the previous section, using the 4D Fourier decomposition of the 5D
spinors in the basis of chirality,

$$\Psi_{iL,R}(x,z) = \int \psi^A_{iL,R}(p,z) e^{-ip \cdot x},$$

we can obtain the solutions for spin-3/2 baryons. However, unlike with the spin-1/2 spinor,
we should impose boundary conditions such that there are no zero modes for spin 3/2 fields,
since the anomalies are already saturated by spin-1/2 baryons. The normalizable solutions
for non-zero modes are given by,

$$F_{1L,R}(p,z) \sim z^{5/2} J_{m-\frac{1}{2}}(|p|z).$$

The boundary condition suitable for the description of left handed spin 3/2 baryons is,

$$\Psi_{1R}(x,z_m) = 0.$$  \hspace{1cm} (27)

The free spectrum with the above boundary condition is given by the zeros of the Bessel
function $J_{m-\frac{1}{2}}(|p|z_m)$. As the AdS/CFT gives the relation $|m_-| = \Delta_{3/2} - 2$ for Rarita-
Schwinger fields of scaling dimension $\Delta_{3/2}$, we get $|m_-| = 5/2$ for $\Delta_{3/2} = 9/2$. For the
right-handed components of spin 3/2 baryons we have the similar boundary condition but
with $L \leftrightarrow R$ as the sign of mass term for $\Psi^2_M$ has to be changed, $m_- \leftrightarrow -m_-,$ giving the
same mass spectrum.

To see the effects of chiral symmetry breaking, we consider the Yukawa couplings,

$$\mathcal{L}_{Yukawa} = -g_{3/2} \bar{\Psi}^M M^3 \Psi_1 \Psi_1 + \text{h.c.}.$$  \hspace{1cm} (28)

We find that the effect of Yukawa coupling is very small for spin 3/2 baryons for a sizable
value of $g_{3/2}$ as the parameter $\sigma$, which is very small, appears with third power in Yukawa
couplings. Using a cut off $z_m^{-1} = 205\text{MeV}$ (same as in the nucleon case) and taking $g_{3/2} = 215$,
we obtain the $\Delta$ resonance masses as

$$0.90^{(1232)}, \quad 1.39^{(1700)}, \quad 1.67^{(1920)} \text{ GeV},$$

which differ by 13-27 % from the experimental results, denoted as superscripts.
IV. MESON-BARYON TRANSITION COUPLINGS

The meson-baryon transition couplings can be evaluated in the framework of AdS/QCD. The most interesting meson-baryon couplings are of pions and rho mesons with the nucleons. They arise from the same term in the bulk and hence unified. The couplings of pions and rho meson with baryons can be read off from the chiral Lagrangian. The lowest order terms in the 4-D chiral Lagrangian give the couplings of pions with nucleons as,

\[ \mathcal{L}_{\pi N} = -\frac{g_A}{f_\pi} \bar{\psi} \gamma^\mu \gamma^5 \partial_\mu \pi \psi, \]

where \( \pi(x) = \pi^a \tau_a \) is the pion field and \( g_A \) and \( f_\pi = 93 \text{MeV} \) being the nucleon axial couplings and the pion decay constant, respectively.

Using the equations of motion for \( \psi \), the leading term can be written as,

\[ 2i g_{\pi NN} \bar{\psi} \gamma^5 \pi \psi, \]

where \( g_{\pi NN} = g_A m_N / f_\pi \) is the Goldberger-Treiman relation and \( m_N \) is the nucleon mass. The experimental value of pion-nucleon coupling is measured to be \( g_{\pi NN} \approx 13.5 \) from the pion-nucleon scattering. The couplings of rho mesons with nucleons are given as

\[ \mathcal{L}_{\rho NN} = g_{\rho NN} \bar{\psi} \gamma^\mu V_\mu \psi + \frac{f_\rho}{4m_N} \bar{\psi} \sigma^{\mu\nu} \partial_\mu V_\nu - \partial_\nu V_\mu \psi, \]

where the second term is the magnetic type of coupling, known as nuclear tensor coupling.

As the \( \Delta \) resonance is parity-even and described by an axial-vector spinor, the couplings of pions and rho mesons with nucleons and \( \Delta \) are given by

\[ \mathcal{L}_{\pi \Delta} = -i \frac{g_{\pi N\Delta}}{f_\pi} \bar{\psi} \gamma^\mu \partial_\mu \pi \psi + i g_{\rho N\Delta} \bar{\psi} \gamma^5 V_\mu \psi, \]

Similarly we may write down the couplings of pions and rho mesons with \( \Delta \) resonances as

\[ \mathcal{L}_{\pi \Delta \Delta} = -\frac{g_A}{f_\pi} \bar{\psi} \gamma^\sigma \gamma^\mu \gamma^5 \partial_\mu \psi_\sigma + g_{\rho \Delta \Delta} \bar{\psi} \gamma^\sigma \gamma^\mu V_\mu \psi_\sigma, \]

where the first term can be rewritten as \( 2i g_{\pi \Delta \Delta} \bar{\psi} \gamma^5 \pi \psi_\sigma \), using the Goldberger-Treiman relation.

We now evaluate the above couplings from the bulk Lagrangian and for this we need a proper identification of pions and rho mesons in the bulk. As mentioned in the section two, we identify the pions as the Nambu-Goldstone bosons of the broken chiral symmetry. We
write $X = v(z)e^{iP(x,z)}$ where $v(z) = \frac{1}{2}Mz + \frac{1}{2}\sigma z^3$ and $P(x, z)$ is related to $A_z(x, z)$ by the relation (11). In the unitary gauge a linear combination of $A_z$ and $P$ becomes the pion field when Kaluza-Klein reduced. We take without loss of generality

$$A_z(z, x) = c_\pi f_0(z)\pi(x), \quad (35)$$

where $\pi(x)$ is identified as the pion field. The function $f_0(z)$ can be determined as in [13, 20]. Similarly, the fifth component of vector gauge field gets decoupled in the unitary gauge and the 4D vector components are identified with the rho meson and its excited states. For the ground state we have

$$V^\mu(z, x) = c_\rho g_0(z)\rho^\mu(x), \quad (36)$$

where $g_0(z) = zJ_1(m_\rho z)$, determined from the wave equation for the vector gauge fields in the bulk. The normalization constants $c_\pi$ and $c_\rho$ are fixed by the requirement of canonical kinetic terms for pions and (4D) vector gauge fields, respectively.

### A. $\pi$NN/ $\rho$NN couplings

The pion (rho)-nucleon couplings can be estimated numerically from the bulk interactions involving gauge fields, bulk scalars, and the nucleons. The minimal gauge coupling and the Yukawa coupling with scalars are given as

$$\mathcal{L}_{\piNN} = \bar{N}_1\Gamma^zA_zN_1 - \bar{N}_2\Gamma^zA_zN_2 - g(\bar{N}_1XN_2 + \text{h.c.}). \quad (37)$$

Since we identify the four dimensional vector components of the bulk gauge fields with rho mesons, the rho meson couplings also arise from the bulk gauge couplings,

$$\mathcal{L}_{\rhoNN} = \bar{N}_1\Gamma^\mu V_\mu N_1 + \bar{N}_2\Gamma^\mu V_\mu N_2. \quad (38)$$

Using the Kaluza-Klein (KK) reduction of five dimensional spinors as $N_{iL,R}(p, z) = \sum_n f_{iL,R}(p, z)\psi_{L,R}^{(n)}(p)$ ($i = 1, 2$), we can write the four dimensional couplings for pions as

$$g^{(0)nm}_{\piNN} = -\int_0^z dz_0 \left[ f_0 \left( f_{1L}^{(n)\ast} f_{1R}^{(m)} - f_{1L}^{(m)\ast} f_{1R}^{(n)} \right) - \frac{g^2z^2}{2g^2_v(z)} \left( \frac{f_0}{z} \right)' \left( f_{1L}^{(n)\ast} f_{2R}^{(m)} - f_{2L}^{(n)\ast} f_{1R}^{(m)} \right) \right]. \quad (39)$$
where the prime denotes the derivative with respect to $z$. (From now on we will absorb the normalization constants $c_\pi$ into $f_0$ and $c_\rho$ into $g_0$.) Similarly the rho meson couplings can be written as

$$g^{(0)nm}_{\rho NN} = \int_0^{z_m} \frac{dz}{z^3} g_0(z) \left( f_{1L}^{(n)*} f_{1L}^{(m)} + f_{2L}^{(n)*} f_{2L}^{(m)} \right). \tag{40}$$

In addition to the minimal gauge interaction we should include the following (parity-invariant) magnetic gauge couplings in the bulk, as was done similarly in [20],

$$\mathcal{L}_{FNN} = i\kappa_1 \left[ \bar{N}_1 \Gamma^{MN} (F_L)_{MN} N_1 - \bar{N}_2 \Gamma^{MN} (F_R)_{MN} N_2 \right] + \frac{i}{2} \kappa_2 \left[ \bar{N}_1 X \Gamma^{MN} (F_R)_{MN} N_2 + \bar{N}_2 X' \Gamma^{MN} (F_L)_{MN} N_1 - \text{h.c.} \right]. \tag{41}$$

These terms contribute through the unknown coefficients $\kappa_1$ and $\kappa_2$. Using the KK decomposition for the five dimensional spinors as before, the additional contribution to pion couplings can be written as

$$g^{(1)nm}_{\pi NN} = - (m_N^{(n)} + m_N^{(m)}) \int_0^{z_m} \frac{dz}{z^3} f_0 \left[ \kappa_1 \left( f_{1L}^{(n)*} f_{1L}^{(m)} + f_{2L}^{(n)*} f_{2L}^{(m)} \right) - \kappa_2 v(z) \left( f_{1L}^{(n)*} f_{2L}^{(m)} - f_{2L}^{(n)*} f_{1L}^{(m)} \right) \right], \tag{42}$$

where $m_N^{(n)}$ denotes the nucleon mass of n-th excited state. (Note that the second term in (42) vanishes identically for the ground state nucleons or same excited states.) Similarly the rho meson coupling has an extra contribution,

$$g^{(1)nm}_{\rho NN} = -2 \int_0^{z_m} \frac{dz}{z^3} g_0(z) \left[ \kappa_1 \left( f_{1L}^{(n)*} f_{1L}^{(m)} - f_{2L}^{(n)*} f_{2L}^{(m)} \right) + \kappa_2 v(z) \left( f_{1L}^{(n)*} f_{2L}^{(m)} + f_{2L}^{(n)*} f_{1L}^{(m)} \right) \right]. \tag{43}$$

The tensor coupling of rho mesons is determined by the magnetic gauge coupling (41) as

$$f_{\rho}^{nm} = 4m_N \int_0^{z_m} \frac{dz}{z^3} g_0(z) \left[ \kappa_1 \left( f_{1L}^{(n)*} f_{1L}^{(m)} - f_{2L}^{(n)*} f_{2L}^{(m)} \right) + \kappa_2 v(z) \left( f_{1L}^{(n)*} f_{2L}^{(m)} + f_{2L}^{(n)*} f_{1L}^{(m)} \right) \right]. \tag{44}$$

For the rho and $\pi$ meson couplings we have two unknown parameters $\kappa_1$ and $\kappa_2$. Fitting the couplings $g_{\pi NN} = 13.5$, $g_{\rho NN} = -8.6$ of the ground state nucleons, we get $\kappa_1 = -0.98$ and $\kappa_2 = 1.25$, which then determine the tensor coupling $f_\rho = -19.6$ for the ground state nucleons, larger than the value, quoted in [21]. We predict all other couplings of rho and $\pi$ mesons with (excited) nucleons. Some are shown in Table II and III for two different sets of values for the fitting parameters $\kappa_1$ and $\kappa_2$, respectively. The numerical values are compared with the calculated ones from the chiral quark model (quoted from the reference [21]), denoted in the brackets.
TABLE I: Numerical result for $\kappa_1 = -0.98$ and $\kappa_2 = 1.25$ and we have used $\sigma = 0.85/z^3 m - m_q/z^2 m$ with $z_m^{-1} = 205\text{MeV}$.

| $g_{\pi NN}$ | 13.5 (13.5) | $g_{\rho NN}$ | -8.6 (2.8) | $f_\rho$ | -19.6 |
|--------------|-------------|--------------|------------|--------|-------|
| $g_{\pi NN}^{(1440)}$ | -20.19 (0.26 $g_{\pi NN}$) | $g_{\rho NN}^{(1440)}$ | 25.94 (-3.1) | $f_\rho^{(1440)}$ | -18.72 |
| $g_{\pi NN}^{(1535)}$ | -7.12 (0.49 $g_{\pi NN}$) | $g_{\rho NN}^{(1535)}$ | 4.45 (4.8) | $f_\rho^{(1535)}$ | -19.01 |

TABLE II: Numerical result for $\kappa_1 = -0.78$ and $\kappa_2 = 0.5$ and we have used $\sigma = 0.85/z^3 m - m_q/z^2 m$ with $z_m^{-1} = 205\text{MeV}$.

| $g_{\pi NN}$ | 11.54 (13.5) | $g_{\rho NN}$ | -3.42 (2.8) | $f_\rho$ | -21.10 |
|--------------|-------------|--------------|------------|--------|-------|
| $g_{\pi NN}^{(1440)}$ | -13.96 (0.26 $g_{\pi NN}$) | $g_{\rho NN}^{(1440)}$ | 19.8 (-3.1) | $f_\rho^{(1440)}$ | -20.18 |
| $g_{\pi NN}^{(1535)}$ | -5.45 (0.49 $g_{\pi NN}$) | $g_{\rho NN}^{(1535)}$ | 1.59 (4.8) | $f_\rho^{(1535)}$ | -20.49 |

B. $\pi\Delta\Delta/\rho\Delta\Delta$ couplings

The $\pi\Delta$ and rho-$\Delta$ couplings can be estimated numerically from the bulk interactions involving gauge fields and $\Delta$ resonances,

$$\mathcal{L}_{\pi\Delta} = \overline{\Psi}_1^\mu \Gamma^z A_z \Psi_{1\mu} - \overline{\Psi}_2^\mu \Gamma^z A_z \Psi_{2\mu} - g_{3/2}(\overline{\Psi}_1^\mu X\Psi_{2\mu} + \text{h.c.}).$$  \hspace{1cm} (45)

Using the KK reduction for the five dimensional spinors $\Psi_{L,R}^\sigma(p,z) = \sum_n F_{L,R}^{(n)}(p,z)\psi_{L,R}^{(n)\sigma}(p)$, we can write the four dimensional couplings for pions as

$$g_{\pi\Delta}^{(0)nm} = -\int_0^{z_m} \frac{dz}{z^2} \left[ f_0 \left(F_{1L}^{(n)\star} F_{1R}^{(m)} - F_{2L}^{(n)\star} F_{2R}^{(m)}\right) \right. $$

$$\left. - \frac{3g_{3/2}z^2 v(z)}{2g_5^2} \left(F_{1L}^{(n)\star} F_{2R}^{(m)} - F_{2L}^{(n)\star} F_{1R}^{(m)}\right) \right].$$  \hspace{1cm} (46)

Similarly the rho meson couplings arise from the bulk gauge couplings,

$$\mathcal{L}_{\rho\Delta\Delta} = \overline{\Psi}_1^\nu \Gamma^\mu V_{\mu\nu} \Psi_{1\nu} + \overline{\Psi}_2^\nu \Gamma^\mu V_{\mu\nu} \Psi_{2\nu},$$  \hspace{1cm} (47)

which can be written as

$$g_{\rho\Delta\Delta}^{(0)nm} = \int_0^{z_m} \frac{dz}{z^2} g_0 \left(F_{1L}^{(n)\star} F_{1R}^{(m)} + F_{2L}^{(n)\star} F_{2R}^{(m)}\right).$$  \hspace{1cm} (48)

The numerical values from the above coupling are too small to account for the experimental values. The additional contributions to $\pi\Delta$ and rho-$\Delta$ couplings can arise from the
following magnetic type of couplings in the bulk, similarly to the couplings with (excited) nucleons,

\[
\mathcal{L}_{F\Delta} = i\kappa_3 \left[ \bar{\Psi}_M^1 \Gamma^{NP}(F_L)_{NP} \Psi_1 - \bar{\Psi}_M^2 \Gamma^{NP}(F_R)_{NP} \Psi_2 \right]
\]

\[
+ \frac{i}{2} \kappa_4 \left[ \bar{\Psi}_M^1 X^3 \Gamma^{NP}(F_R)_{NP} \Psi_2 + \bar{\Psi}_M^2 (X^+)^3 \Gamma^{NP}(F_L)_{NP} \Psi_1 - \text{h.c.} \right].
\]  

(49)

Using the KK mode decomposition for the spinors as before, the additional contribution for pion couplings can be written as,

\[
g^{(1)nm}_{\pi\Delta\Delta} = -(m_{(n)}^{(m)} + m_{\Delta}) \int_0^{z_m} \frac{dz}{z} \int_0 f_0 \left[ \kappa_3 \left( F_{1L}^{(n)*} F_{1L}^{(m)} + F_{2L}^{(n)*} F_{2L}^{(m)} \right) \right.
\]

\[
- \kappa_4 \left( F_{1L}^{(n)*} F_{2L}^{(m)} - F_{2L}^{(n)*} F_{1L}^{(m)} \right) \right],
\]  

(50)

and similarly for the rho meson couplings as,

\[
g^{(1)nm}_{\rho\Delta\Delta} = -2 \int_0^{z_m} \frac{dz}{z} \int_0 f_0 \left[ \kappa_3 \left( F_{1L}^{(n)*} F_{1L}^{(m)} - F_{2L}^{(n)*} F_{2L}^{(m)} \right) + \kappa_4 \left( F_{1L}^{(n)*} F_{2L}^{(m)} + F_{2L}^{(n)*} F_{1L}^{(m)} \right) \right].
\]  

(51)

We fix the value of \(\kappa_3\) and \(\kappa_4\) by fitting the coupling \(g_{\pi\Delta\Delta} = 20\) and \(g_{\rho\Delta\Delta} = 10.9\) for the ground state \(\Delta\)-baryons, respectively. We present the \(\pi\)-\(\Delta\) and rho-\(\Delta\) couplings in Table III.

| \(g_{\pi\Delta\Delta}\) | 20 | \(g_{\rho\Delta\Delta}\) | 10.9 |
|----------------------|----|-----------------|-----|
| \(g_{\pi\Delta\Delta}^{(1700)}\) | 43.78 | \(g_{\rho\Delta\Delta}^{(1700)}\) | 37.40 |
| \(g_{\pi\Delta\Delta}^{(1920)}\) | -79.94 | \(g_{\rho\Delta\Delta}^{(1920)}\) | 45.35 |

TABLE III: Numerical result for \(\kappa_3 = 0.07\) and \(\kappa_4 = 11.32\) and we have used \(\sigma = 0.85/z_m^3 - m_q/z_m^2\) with \(z_m = 205\) MeV.
C. $\pi N\Delta/\rho N\Delta$ couplings

The transition couplings of pions is determined from the gauge invariant couplings of gauge fields with nucleons and $\Delta$ resonances in the bulk. The Lagrangian is given by

$$\mathcal{L}_{FN\Delta} = \left[ \alpha_1 \left( \bar{\Psi}_1^M \Gamma^N (F_L)_{MN} N_1 - (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\alpha_2 \left( (\partial^M \bar{\Psi}_1^N)(F_L)_{MN} N_1 + (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\alpha_3 \left( \bar{\Psi}_1^M (\partial^N (F_L)_{MN}) N_1 + (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + \beta_1 \left( \bar{\Psi}_1^M \Gamma^{NP} (\bar{F}_L)_{MNP} N_1 - (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\beta_2 \left( (\partial^M \bar{\Psi}_1^N) \Gamma^P (\bar{F}_L)_{MNP} N_1 + (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + \tilde{\alpha}_1 \left( \bar{\Psi}_1^M \Gamma^N (F_L)_{MN} X N_2 + (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\tilde{\alpha}_2 \left( (\partial^M \bar{\Psi}_1^N)(F_L)_{MN} X N_2 - (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\tilde{\alpha}_3 \left( \bar{\Psi}_1^M (\partial^N (F_L)_{MN}) X N_2 - (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + \tilde{\beta}_1 \left( \bar{\Psi}_1^M \Gamma^{NP} (\bar{F}_L)_{MNP} X N_2 + (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + i\tilde{\beta}_2 \left( (\partial^M \bar{\Psi}_1^N) \Gamma^P (\bar{F}_L)_{MNP} X N_2 - (1 \leftrightarrow 2 \ & L \leftrightarrow R) \right) + \text{h.c.} \right],$$

(52)

where $\alpha$’s and $\beta$’s are unknown parameters. The same term also contributes to the four dimensional rho meson couplings. Using the KK reduction of five dimensional spinors as $\Psi_{L,R}(p,z) = \sum_n f^{(n)}_{iL,R}(p,z)\psi^{(n)}_{L,R}(p)$ for nucleons and $\Psi_{L,R}(p,z) = \sum_n F^{(n)}_{iL,R}(p,z)\psi^{(n)}_{L,R}(p)$ for $\Delta$ resonances, we can write the pion-nucleon-$\Delta$ couplings as,

$$g_{\pi N\Delta}^{nm} = -f_\pi \int_0^{z_m} dz \left[ \frac{f_0}{z^2} \left( \kappa (F_{1L}^{(n)*} f_{1R}^{(m)} + F_{2L}^{(n)*} f_{2R}^{(m)}) + \tilde{\kappa} v(z) (F_{1L}^{(n)*} f_{2R}^{(m)} - F_{2L}^{(n)*} f_{1R}^{(m)}) \right) + \frac{f_0}{z} \left( \alpha_2 ((\partial_z F_{1L}^{(n)*}) f_{1R}^{(m)} - (\partial_z F_{2L}^{(n)*}) f_{2R}^{(m)}) + \tilde{\alpha}_2 v(z) ((\partial_z F_{1L}^{(n)*}) f_{2R}^{(m)} + (\partial_z F_{2L}^{(n)*}) f_{1R}^{(m)}) \right) - \frac{f_0}{z} \left( \alpha_3 (F_{1L}^{(n)*} f_{1R}^{(m)} - F_{2L}^{(n)*} f_{2R}^{(m)}) + \tilde{\alpha}_3 v(z) (F_{1L}^{(n)*} f_{2R}^{(m)} + F_{2L}^{(n)*} f_{1R}^{(m)}) \right) + 2m_\Delta \frac{f_0}{z} \left( \beta_2 (F_{1L}^{(n)*} f_{1R}^{(m)} - F_{2L}^{(n)*} f_{2R}^{(m)}) + \tilde{\beta}_2 v(z) (F_{1L}^{(n)*} f_{2R}^{(m)} + F_{2L}^{(n)*} f_{1R}^{(m)}) \right) \right],$$

(53)
and similarly the rho-nucleon-$\Delta$ couplings as,

\[
g_{\rho N\Delta}^{\text{nm}} = \int_0^{z_m} dz \left[ \frac{g_0}{z^2} \left( \kappa \left( F_{1L}^{(n)*} f_{1R}^{(m)} - F_{2L}^{(n)*} f_{2R}^{(m)} \right) + \tilde{\kappa} v(z) \left( F_{1L}^{(n)*} f_{2R}^{(m)} + F_{2L}^{(n)*} f_{1R}^{(m)} \right) \right) \\
+ \frac{g_0'}{z} \left( \alpha_2 \left( \partial_z F_{1L}^{(n)*} f_{1R}^{(m)} + \partial_z F_{2L}^{(n)*} f_{2R}^{(m)} \right) + \tilde{\alpha}_2 v(z) \left( \partial_z F_{1L}^{(n)*} f_{2R}^{(m)} - \partial_z F_{2L}^{(n)*} f_{1R}^{(m)} \right) \right) \\
- \frac{g_0''}{z} \left( \alpha_3 \left( F_{1L}^{(n)*} f_{1R}^{(m)} + F_{2L}^{(n)*} f_{2R}^{(m)} \right) + \tilde{\alpha}_3 v(z) \left( F_{1L}^{(n)*} f_{2R}^{(m)} - F_{2L}^{(n)*} f_{1R}^{(m)} \right) \right) \\
- 2m_\Delta \frac{g_0''}{z} \left( \beta_2 \left( F_{1L}^{(n)*} f_{1R}^{(m)} + F_{2L}^{(n)*} f_{2R}^{(m)} \right) + \tilde{\beta}_2 v(z) \left( F_{1L}^{(n)*} f_{2R}^{(m)} - F_{2L}^{(n)*} f_{1R}^{(m)} \right) \right) \right],
\]

where $\kappa = \alpha_1 - 4 \beta_1$ and $\tilde{\kappa} = \tilde{\alpha}_1 - 4 \tilde{\beta}_1$.

The nucleon-$\Delta$ couplings with pions or rho-mesons have eight unknown parameters ($\kappa$, $\tilde{\kappa}_2$, $\alpha_{2,3}$, $\tilde{\alpha}_{2,3}$, $\beta_2$, $\tilde{\beta}_2$). We fix those parameters in section V where we calculate the nucleon-$\Delta$ transition form-factors, and we get $g_{\pi N\Delta} = 20.93(1.55g_{\pi NN})$ and $g_{\rho N\Delta} = 8.7(8.7)$. (The values in the bracket are quoted from the reference [21]). In principle we can calculate the transition couplings for the excited $\Delta$ resonances, but our hard-wall model does not seem to work.

### V. NUCLEON TO $\Delta$ TRANSITION FORM FACTORS

In this section we evaluate the nucleon to $\Delta$ transition form-factors in AdS/QCD and present the numerical values for the form factors. The nucleon-$\Delta$ electromagnetic and axial transition form-factors in four dimensions are extracted from the matrix elements of the vector and axial vector currents between $\Delta$ and nucleon states,

\[
\langle \Delta(p')|J_{\mu}^{EM}|N(p) \rangle = \bar{u}(p') O_{\sigma \mu}^{EM}(p, p') u(p),
\]

\[
\langle \Delta(p')|A_{\mu}^{3}|N(p) \rangle = \bar{u}(p') O_{\sigma \mu}^{(A)}(p, p') u(p),
\]

where $u(p')$ and $u(p)$ are the Rarita-Schwinger and nucleon spinors for $\Delta$ and nucleon of momentum of momentum $p'$ and $p$, respectively.

By the Lorentz invariance and the current conservation we expand the operators $O_{\sigma \mu}^{EM}$ [22] and $O_{\sigma \mu}^{(A)}$ [16], assuming the CP invariance, respectively as

\[
O_{\sigma \mu}^{EM} = \left[ G_1(q^2)(q_\sigma \gamma_\mu - q_\gamma q_\mu) + G_2(q^2)(q_\sigma P_\mu - q_\cdot P q_\sigma) + G_3(q^2)(q_\sigma q_\mu - q^2 q_\sigma q_\mu) \right] \gamma^5,
\]

\[
O_{\sigma \mu}^{(A)} = C_3^A(q^2)m_N(q_\mu \cdot q_\sigma q_\mu - q_\sigma q_\mu q_\mu') + C_4^A(q^2)m_N(q_\sigma q_\mu - q_\mu q_\sigma) + C_5^A(q^2) q_{\sigma q_{\mu}} q_{\mu q_{\sigma}}.
\]
where \( q = p' - p \) and \( P = (p' + p)/2 \). All \( C_i^A \)'s are dimensionless, but \( G_1 \) has the dimension of mass inverse and both \( G_2 \) and \( G_3 \) have the dimension of mass-inverse squared. The pion-nucleon-\( \Delta \) form factor, \( G_{\pi N \Delta} \), is related to \( C_i^A \) by the Goldberger-Treiman relation

\[
G_{\pi N \Delta}(q^2) = \frac{2m_N}{f_\pi} C_i^A(q^2), \tag{59}
\]

and \( \pi N \Delta \) coupling constant is given by \( g_{\pi N \Delta} = G_{\pi N \Delta}(0) \).

The vertex operator for the electromagnetic matrix elements eq. (57) can be also expressed in terms of the three Sachs form factors \([17, 22]\) as follow

\[
\mathcal{O}_{\sigma \mu}^{(EM)}(p', p) = G_{M1}(q^2) K_{\sigma \mu}^{M1} + G_{E2}(q^2) K_{\sigma \mu}^{E2} + G_{C2}(q^2) K_{\sigma \mu}^{C2}, \tag{60}
\]

where \( G_{M1}, G_{E2} \) and \( G_{C2} \) are magnetic dipole, electric quadrupole and Coulomb quadrupole form factors, respectively and

\[
K_{\sigma \mu}^{M1} = -\frac{3}{(m_\Delta + m_N)^2 - q^2} \frac{m_\Delta + m_N}{2m_N} i\epsilon_{\sigma \mu \alpha \beta} p^\alpha p^\beta; \tag{61}
\]

\[
K_{\sigma \mu}^{E2} = -K_{\sigma \mu}^{M1} + 6\Omega^{-1}(q^2) \frac{m_\Delta + m_N}{2m_N} 2i\gamma^5 \epsilon_{\lambda \alpha \beta} p^\alpha p^\beta \gamma^\lambda p_\beta; \tag{62}
\]

\[
K_{\sigma \mu}^{C2} = -6\Omega^{-1}(q^2) \frac{m_\Delta + m_N}{2m_N} i\gamma^5 q_\sigma (q^2 (p + p')_\mu - q \cdot (p + p') q_\mu, \tag{63}
\]

where

\[
\Omega(q^2) = \left[ (m_\Delta + m_N)^2 - q^2 \right] \left[ (m_\Delta - m_N)^2 - q^2 \right]. \tag{64}
\]

The \( G_{M1}, G_{E2} \) and \( G_{C2} \) are related to \( G_i \)'s \([22]\) as, with \( \hat{m} \equiv m_N/(m_\Delta + m_N) \),

\[
G_{M1}(q^2)=\left[ \frac{(3m_\Delta + m_N) (m_\Delta + m_N) - q^2}{m_\Delta} G_1(q^2) + (m_\Delta^2 - m_N^2) G_2(q^2) + 2q^2 G_3(q^2) \right] \frac{\hat{m}}{3}; \tag{65}
\]

\[
G_{E2}(q^2)=\left[ \frac{(m_\Delta^2 - m_N^2) + q^2}{m_\Delta} G_1(q^2) + (m_\Delta^2 - m_N^2) G_2(q^2) + 2q^2 G_3(q^2) \right] \frac{\hat{m}}{3}; \tag{66}
\]

\[
G_{M1}(q^2)=\left[ 2m_\Delta G_1(q^2) + \frac{1}{2}(3m_\Delta^2 + m_N^2 - q^2) G_2(q^2) + (m_\Delta^2 - m_N^2 + q^2) G_3(q^2) \right] \frac{2\hat{m}}{3}. \tag{67}
\]

The ratios of the electric and Coulomb quadrupole amplitudes to the magnetic dipole amplitude, \( R_{EM} \) (EMR) and \( R_{SM} \) (CMR), are defined as

\[
R_{EM} = -\frac{G_{E2}(q^2)}{G_{M1}(q^2)}, \quad R_{SM} = -\frac{|q|}{2m_\Delta} \frac{G_{C2}(q^2)}{G_{M1}(q^2)}. \tag{68}
\]
given from the bulk action by taking the normalizable modes for the nucleon and \( \Delta \), and non-normalizable modes for (external) vector and axial vector gauge fields. Taking the axial gauge, where \( V_z = 0 = A_z \), we Fourier-transform the vector and axial vector gauge fields as

\[
V_{\mu}(x, z) = \int_q V_{\mu}(q) V(q, z) e^{-iq \cdot x}, \quad A_{\mu}(x, z) = \int_q A_{\mu}(q) A(q, z) e^{-iq \cdot x},
\]

with boundary conditions, \( V(q, \epsilon) = 1 \) at UV \( (z = \epsilon) \) and \( \partial_z V(q, z_m) = 0 \) at IR \( (z = z_m) \), and similarly for \( A(q, z) \). The 5D wave functions of the gauge fields, \( V(q, z) \) and \( A(q, z) \), are determined by solving the equations of motions. We decompose Rarita-Schinger fields into chirality basis as

\[
\Psi^\mu_i(z, x) = \int_p \left[ F_{iL}(p, z) u^*_L(p) + F_{iR}(p, z) u^*_R(p) \right] e^{-ip \cdot x}, \quad (i = 1, 2)
\]

and similarly for the nucleon fields but with \( f(p, z) \) and \( u(p) \) instead of \( F(p, z) \) and \( u^\mu(p) \).

By matching the operators from the 5D Lagrangian (52) with eq.'s (57) and (58), we easily read off the nucleon to \( \Delta \) transition form-factors and get

\[
G_1(Q^2) = \int dz \left[ \frac{V(q, z)}{z^2} \left( \kappa (F_{1L}^*(p', z) f_{1L}(p, z) - F_{2L}^*(p', z) f_{2L}(p, z)) \\
+ \tilde{\kappa} v(z) (F_{1L}^*(p', z) f_{2L}(p, z) + F_{2L}^*(p', z) f_{1L}(p, z)) \right) \\
+ \frac{2m_\Delta V(q, z)}{z} \left( \beta_2 (F_{1L}^*(p', z) f_{1L}(p, z) + F_{2L}^*(p', z) f_{2L}(p, z)) \\
+ \tilde{\beta}_2 v(z) (F_{1L}^*(p', z) f_{2L}(p, z) - F_{2L}^*(p', z) f_{1L}(p, z)) \right) \right],
\]

\[
G_2(Q^2) = -\int dz \left[ \frac{V(q, z)}{z^2} \left( (\alpha_2 + 2\beta_2)(F_{1L}^*(p', z) f_{1L}(p, z) + F_{2L}^*(p', z) f_{2L}(p, z)) \\
+ (\tilde{\alpha}_2 + 2\tilde{\beta}_2) v(z) (F_{1L}^*(p', z) f_{2L}(p, z) - F_{2L}^*(p', z) f_{1L}(p, z)) \right) \right],
\]

\[
G_3(Q^2) = -\int dz \left[ \frac{V(q, z)}{2z} \left( (\alpha_2 + 2\beta_2 + 2\alpha_3)(F_{1L}^*(p', z) f_{1L}(p, z) + F_{2L}^*(p', z) f_{2L}(p, z)) \\
+ (\tilde{\alpha}_2 + 2\tilde{\beta}_2 + 2\tilde{\beta}_3) v(z) (F_{1L}^*(p', z) f_{2L}(p, z) - F_{2L}^*(p', z) f_{1L}(p, z)) \right) \right].
\]
for vectors, and

\[ C_3^A(Q^2) = -m_N \int dz \left[ \frac{A(q, z)}{z^2} \left( \kappa (F_{1L}^*(p', z) f_{1L}(p, z) + F_{2L}^*(p', z) f_{2L}(p, z)) 
+ \tilde{\kappa} v(z) \left( F_{1L}^*(p', z) f_{2L}(p, z) - F_{2L}^*(p', z) f_{1L}(p, z) \right) \right) 
- \frac{2m_\Delta A(q, z)}{z} \left( \beta_2 \left( F_{1L}^*(p', z) f_{1R}(p, z) - F_{2L}^*(p', z) f_{2R}(p, z) \right) 
+ \tilde{\beta}_2 v(z) \left( F_{1L}^*(p', z) f_{2R}(p, z) + F_{2L}^*(p', z) f_{1R}(p, z) \right) \right) \right) 
+ \frac{2A(q, z)}{z} \left( \beta_2 \left( (\partial_z F_{1L}^*(p', z)) f_{1L}(p, z) - (\partial_z F_{2L}^*(p', z)) f_{2L}(p, z) \right) 
+ \tilde{\beta}_2 v(z) \left( (\partial_z F_{1L}^*(p', z)) f_{2L}(p, z) + (\partial_z F_{2L}^*(p', z)) f_{1L}(p, z) \right) \right) \right], \] (74)

\[ C_4^A(Q^2) = -m_N^2 \int dz \left[ \frac{A(q, z)}{z} \left( \left( \alpha_2 + 2\beta_2 \right) \left( F_{1L}^*(p', z) f_{1R}(p, z) - F_{2L}^*(p', z) f_{2R}(p, z) \right) 
+ \left( \alpha_2 + 2\tilde{\beta}_2 \right) v(z) \left( F_{1L}^*(p', z) f_{2R}(p, z) + F_{2L}^*(p', z) f_{1R}(p, z) \right) \right) \right], \] (75)

\[ C_5^A(Q^2) = \int dz \left[ \frac{\partial_z A(q, z)}{z^2} \left( \kappa \left( F_{1L}^*(p', z) f_{1R}(p, z) + F_{2L}^*(p', z) f_{2R}(p, z) \right) 
+ \tilde{\kappa} v(z) \left( F_{1L}^*(p', z) f_{2R}(p, z) - F_{2L}^*(p', z) f_{1R}(p, z) \right) \right) 
+ \frac{\partial_A(q, z)}{z} \left( \alpha_2 \left( (\partial_z F_{1L}^*(p', z)) f_{1R}(p, z) - (\partial_z F_{2L}^*(p', z)) f_{2R}(p, z) \right) 
+ \tilde{\alpha}_2 v(z) \left( (\partial_z F_{1L}^*(p', z)) f_{2R}(p, z) + (\partial_z F_{2L}^*(p', z)) f_{1R}(p, z) \right) \right) \right) 
- \frac{\partial^2 z A(q, z)}{z} \left( \alpha_3 \left( F_{1L}^*(p', z) f_{1R}(p, z) - F_{2L}^*(p', z) f_{2R}(p, z) \right) 
+ \tilde{\alpha}_3 v(z) \left( F_{1L}^*(p', z) f_{2R}(p, z) + F_{2L}^*(p', z) f_{1R}(p, z) \right) \right) \right) 
+ \frac{2m_\Delta \partial_A(q, z)}{z} \left( \beta_2 \left( F_{1L}^*(p', z) f_{1L}(p, z) - F_{2L}^*(p', z) f_{2L}(p, z) \right) 
+ \tilde{\beta}_2 v(z) \left( F_{1L}^*(p', z) f_{2L}(p, z) + F_{2L}^*(p', z) f_{1L}(p, z) \right) \right) \right], \] (76)

\[ C_6^A(Q^2) = m_N^2 \int dz \left[ \frac{A(q, z)}{z^2} \left( \left( \alpha_3 \left( F_{1L}^*(p', z) f_{1R}(p, z) - F_{2L}^*(p', z) f_{2R}(p, z) \right) 
+ \tilde{\alpha}_3 v(z) \left( F_{1L}^*(p', z) f_{2R}(p, z) + F_{2L}^*(p', z) f_{1R}(p, z) \right) \right) \right) \right], \] (77)

for axial vector, where \( \kappa = \alpha_1 - 4\beta_1, \tilde{\kappa} = \alpha_1 - 4\tilde{\beta}_1 \) and \( Q^2 = -q^2 \).

**A. Numerical Results**

In order to perform the numerical calculation we set three parameters, \( z_m, m_q, \) and \( \sigma \), to

\( z_m = (205\text{Mev})^{-1}, m_q = 0.0023 \) and \( \sigma = 0.85/z_m^2 - m_q/z_m^2 \). We also determine the unknown
parameters
\[
\kappa = -179, \quad \bar{\kappa} = -733, \quad \alpha_2 = 18, \quad \bar{\alpha}_2 = 136, \\
\alpha_3 = 51, \quad \bar{\alpha}_3 = -148, \quad \beta_2 = -9, \quad \bar{\beta}_2 = -68,
\]
by setting the numerical values of our \(\pi N\Delta, \rho N\Delta\) coupling constants to be the same as the experimental values and \(N - \Delta\) form factors to be the same as the lattice QCD values at a fixed value of \(Q^2\), i.e.,

\[
g_{\pi N\Delta} = 1.55 g_{\pi N\Delta} = 20.93, \quad g_{\rho N\Delta} = 8.7, \quad [21], \\
C_3^A(0) = 0, \quad C_4^A(0) = 0, \quad C_5^A(0) = \frac{f_\pi}{2m_N} g_{\pi N\Delta} = 1.11, \quad C_6^A(0.15) = 2.266, \quad [16], \\
G_1(0) = 2.38 \quad G_2(0) = -0.56, \quad [22]. \quad (79)
\]
The numerical values of various form factors are tabulated and plotted below for different values of $Q^2$.

| $Q^2$ (GeV$^2$) | $C_3^A$ | $C_4^A$ | $C_5^A$ | $C_6^A$ | $G_{\pi N\Delta}$ |
|----------------|---------|---------|---------|---------|-----------------|
| 0.15           | 2.492   | 0.0018  | -4.866  | 2.266   | -98             |
| 0.34           | 3.635   | 0.0026  | -8.367  | 3.455   | -169            |
| 0.53           | 3.942   | 0.0027  | -9.749  | 3.668   | -197            |
| 0.71           | 3.880   | 0.0026  | -10.104 | 3.514   | -204            |
| 0.87           | 3.698   | 0.0024  | -10.008 | 3.265   | -202            |
| 1.04           | 3.450   | 0.0022  | -9.677  | 2.964   | -196            |
| 1.34           | 2.983   | 0.0018  | -8.830  | 2.446   | -178            |
| 1.49           | 2.759   | 0.0016  | -8.360  | 2.213   | -169            |
| 1.63           | 2.562   | 0.0015  | -7.920  | 2.014   | -160            |
| 1.77           | 2.377   | 0.0014  | -7.484  | 1.835   | -151            |
| 1.90           | 2.216   | 0.0012  | -7.098  | 1.684   | -143            |
| 2.03           | 2.056   | 0.0011  | -6.714  | 1.549   | -136            |
| 2.15           | 1.939   | 0.0010  | -6.394  | 1.437   | -129            |
| 2.40           | 1.699   | 0.0009  | -5.758  | 1.240   | -116            |

TABLE IV:
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$Q^2 \ (GeV^2)$ & $G_1$ & $G_2$ & $G_3$ & $G_{M1}$ & $G_{E2}$ & $G_{C2}$ & $R_{EM}$ & $R_{SM}$ \\
\hline
0.15 & -12.331 & -0.280 & 390 & -31.681 & -15.537 & 44.997 & -0.490 & 0.224 \\
0.34 & -15.515 & -0.147 & 205 & -39.060 & -18.232 & 5.983 & -0.467 & 0.036 \\
0.53 & -14.728 & -0.088 & 122 & -36.060 & -16.630 & -7.040 & -0.500 & -0.056 \\
0.71 & -13.160 & -0.058 & 81 & -33.232 & -14.545 & -11.288 & -0.438 & -0.116 \\
0.87 & -11.695 & -0.042 & 58 & -29.564 & -12.566 & -12.378 & -0.425 & -0.159 \\
1.04 & -10.245 & -0.030 & 42 & -25.951 & -10.694 & -12.288 & -0.412 & -0.196 \\
1.34 & -8.088 & -0.019 & 26 & -20.875 & -8.281 & -11.100 & -0.397 & -0.250 \\
1.49 & -7.198 & -0.015 & 21 & -18.786 & -7.331 & -10.348 & -0.390 & -0.273 \\
1.63 & -6.467 & -0.012 & 16.8 & -16.774 & -6.307 & -9.461 & -0.376 & -0.293 \\
1.77 & -5.822 & -0.010 & 13.9 & -15.183 & -5.581 & -8.725 & -0.368 & -0.311 \\
1.90 & -5.290 & -0.008 & 11.7 & -13.840 & -4.967 & -8.060 & -0.359 & -0.326 \\
2.03 & -4.816 & -0.007 & 19.92 & -12.644 & -4.432 & -7.442 & -0.351 & -0.341 \\
2.15 & -4.424 & 0.006 & 8.58 & -11.662 & -4.003 & -6.917 & -0.343 & -0.354 \\
2.40 & -3.724 & 0.005 & 6.42 & -9.892 & -3.243 & -5.934 & -0.328 & -0.378 \\
\hline
\end{tabular}
\caption{Table V:}
\end{table}
FIG. 1: This graph shows $C_3^A(Q^2)$ (black-square) and $C_4^A(Q^2)$ (red-circle) as a function of $Q^2$, respectively. $C_4^A(Q^2)$ has nearly zero-values for all $Q^2$. 
FIG. 2: The black-square and red-circle represent $C_5^A(Q^2)$ and $C_6^A(Q^2)$ as a function of $Q^2$, respectively. The blue-triangle and green-inverted triangle represent the lattice results for $C_5^A(Q^2)$ and $C_6^A(Q^2)$ of [16].

FIG. 3: The black-square is the numerical results of our model and the red-circle is the lattice results of [16] for pion-nucleon-$\Delta$ form factor, $G_{\pi N\Delta}$. 
FIG. 4: This graph shows $G_i(Q^2)$ as a function of $Q^2$.

FIG. 5: The graph of magnetic dipole (black-square), electric quadrupole (red-circle) and Coulomb quadrupole (blue triangle) form factors as a function of $Q^2$. 
FIG. 6: The graph of the ratios of the electric (black-square) and Coulomb quadrupole (red-circle) amplitudes to the magnetic dipole amplitude.

VI. SUMMARY AND CONCLUSIONS

The AdS/QCD has provided us a useful tool to calculate hadronic spectra and meson-baryons transition couplings. We considered the holographic description of spin 3/2 baryons (Δ resonances) in terms of bulk Rarita-Schwinger fields, which satisfy a set of Dirac equations, upon imposing constraints to project out their spin-1/2 components. Then the boundary conditions at the infra-red cut-off give the free spectra. We considered the effects of Yukawa couplings and they turn out to be very small for a sizable value of the coupling constant.

We also considered meson-baryon transition couplings in the bulk. In particular we considered pion-nucleon couplings. The same term also contribute to four dimensinal rho-nucleon couplings. The numerical results for the meson-baryon couplings from AdS/QCD are in good agreement with the the results obtained from other methods. The form factors for nucleon and Δ transitions are also evaluated numerically from AdS/QCD.
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