Axion Theory Review

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I review the axion physics with emphases on the couplings of the very light axion
and a possible realization in superstring models.

1 Why Axions?

Before 1975, QCD was described by the Lagrangian
\[ \mathcal{L} = -\frac{1}{2g^2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q} (i\gamma^\mu D_\mu - M_q) q. \]

But after 1975, the following $\bar{\theta}$ term is known to be present due to the discovery
of instanton solutions in nonabelian gauge theories
\[ \frac{\bar{\theta}}{16\pi^2} \text{Tr} F_{\mu\nu} F^{\mu\nu} \]
where $\bar{\theta} = \theta_{\text{QCD}} + \theta_{\text{QFD}}$ which violates CP. From the experimental bound on
the neutron electric dipole moment, $|\bar{\theta}|$ is required to be less than $10^{-9}$. This
is “the strong CP problem”.

The axion solution of the strong CP problem is to introduce a dynamical
field axion ($a$) so that $\bar{\theta}$ is settled very near 0. In this case, one can study
the following Lagrangian below the electroweak scale, after integrating out the
quark fields,
\[ \mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} + \bar{q} (i\gamma^\mu D_\mu - M_q) q + \frac{\alpha}{32\pi^2 F_a} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \frac{1}{2} \partial^\mu a \partial_\mu a. \]

Here, $\bar{\theta}$ turns out to be $\bar{\theta} = a/F_a$ where $F_a$ is called the axion decay constant.
In this model, $\bar{\theta}$ is a dynamical field. But, for a moment consider $\bar{\theta}$ as a
parameter of the theory. Denoting
\[ \{ F \tilde{F} \} = \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}, \]
\[ ^a \text{Talk presented at Dark Matter 98, Buxton, England, Sep. 7–11, 1998.} \]
we obtain the generating functional in Euclidian space, with CP violation introduced only in the $\bar{\theta}$ term, as

$$Z \propto \int [dA_\mu] \prod \text{Det}(\gamma^\mu D_\mu + m_i) \exp \left( - \int d^4x \left[ \frac{1}{4g^2} F^2 - i\bar{\theta}\{F\bar{F}\} \right] \right). \quad (4)$$

Noting that $\text{Det}(\gamma^\mu D_\mu + m_i) > 0$, one can prove the following inequality,

$$e^{-\int d^4x V[\bar{\theta}]} \equiv \left| \int [dA_\mu] \prod \text{Det}(\gamma^\mu D_\mu + m_i) e^{-\int d^4x (\frac{1}{4g^2} F^2 - i\bar{\theta}\{F\bar{F}\})} \right| \leq \int [dA_\mu] \left| \prod \text{Det}(\gamma^\mu D_\mu + m_i) e^{-\int d^4x (\frac{1}{4g^2} F^2 - i\bar{\theta}\{F\bar{F}\})} \right| = \int [dA_\mu] \left| \prod \text{Det}(\gamma^\mu D_\mu + m_i) e^{-\int d^4x \frac{1}{4g^2} F^2} \right|$$

$$= e^{-\int d^4x V[0]} \quad (5)$$

where we used Schwarz’s inequality. Thus, $V[\bar{\theta}] \geq V[0]$ for any $\bar{\theta}$ without CP violation introduced in the other sector. As a coupling, any $\bar{\theta}$ will be good, as any value of $\alpha_{em}$ is theoretically allowable.

The axion solution is to make $\bar{\theta}$ a dynamical field $\bar{\theta} = a/F_a$. Namely, $a$ has a kinetic energy term, without any term in the potential except the one coming from the gluon anomaly term in Eq. (2). It is possible to introduce this kind of pseudoscalar as a Goldstone boson. A Goldstone boson nicely fits into this scheme, since it does not have a potential. But for this interpretation to work, the current corresponding to the Goldstone boson must have an anomaly so that $(a/F_a)F\bar{F}$ arises, which is called the Peccei-Quinn mechanism.

In addition to the decay constant $F_a$, there is another fundamental number for axion: the domain wall number $N_{DW}$. It arises because $\bar{\theta}$ is periodic with the period $2\pi$ but the fields in the theory can have a phase $\bar{\theta}/N_{DW}$, which can be represented as $a$ and has a periodicity

$$a \equiv a + 2\pi N_{DW} F_a. \quad (6)$$

In the standard Big Bang cosmology, there will be a domain wall problem. But with the inflation with the reheating temperature below $F_a$, the domain wall problem disappears.

Remembering that the axion is a dynamical $\bar{\theta}$, we can easily derive its interaction terms. For this, we rely on the effective field theory or calculate them explicitly in a given model.

As an introduction of axion, let us consider the simplest axion example, the heavy quark axion or KSVZ axion. The heavy quark obtains mass by the
VEV of a singlet complex Higgs field $\sigma$, and the Yukawa coupling is

$$\mathcal{L}_Y = \sigma \bar{Q} R Q_L + \text{h.c.}$$

(7)

The potential contains terms invariant under the following Peccei-Quinn transformation,

$$Q_L \rightarrow e^{-i\alpha/2} Q_L, Q_R \rightarrow e^{i\alpha/2} Q_R, \sigma \rightarrow e^{i\alpha} \sigma, \quad \theta \rightarrow \theta - \alpha.$$  

(8)

This symmetry is broken by the VEV of $\sigma$, $\langle \sigma \rangle = F_a / \sqrt{2}$. This produces a Goldstone boson, which is hidden in $D_\mu \sigma^* D^\mu \sigma$ in the original Lagrangian, whose kinetic energy term is

$$\frac{1}{2} \left(1 + \frac{\rho}{F_a}\right)^2 \partial_\mu a \partial^\mu a$$

(9)

where $\sigma = (F_a + \rho)e^{i\alpha}/F_a / \sqrt{2}$. Due to the nontrivial Peccei-Quinn transformation of the heavy quark, the corresponding current is not conserved at one-loop level but

$$\partial_\mu J^\mu = \frac{1}{16\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

(10)

where $F_{\mu\nu}^a$ is the gluon field strength. Thus, the Goldstone boson obtains mass through this anomalous term.

There are basically two methods to introduce the axion at a scale $\mu$:

(i) As a global symmetry, and

(ii) as a fundamental field.

But at low energy these two methods give the same result with the axion coupling as given in Eq. (3). Case (i) implies the Peccei-Quinn symmetry. When this symmetry is spontaneously broken, there appears the axion. Above the symmetry breaking scale $F_a$, there is just the global symmetry. Below the symmetry breaking scale, there appears the axion—the pseudo-Goldstone boson. Its fundamental nature can be either the pseudoscalar present in the Higgs field or a composite axion. Case (ii) involves a pseudoscalar in gravity theory, including the superstring axion. In this case, the decay constant is the gravity scale $M_{\text{compactification}}$.

The axion properties depend on $F_a$, $N_{DW}$, and couplings to matter. These couplings drastically differ for different axion models.

2 Axion Mass and Couplings

Below the scale $F_a$, we consider light fields of quarks, gluons, $a$. This is enough for the study of axion mass. The axion and gluon Lagrangian is

$$\mathcal{L} = \frac{1}{2} (\partial_\mu a)^2 + \text{(derivative terms of } a) + \left(\theta + \frac{a}{F_a}\right) \{F \tilde{F}\}$$

(11)
from which we can redefine the axion field so that the coefficient of \( \{ F \bar{F} \} \) becomes \( a/F_a \). In the KSVZ model, we minimally created this term at the high energy scale. The low energy physics at 1 GeV will include quarks. After introducing a warm-up example of one-flavor QCD, we will present the axion mass in the two-flavor QCD.

2.1 One flavor

The strong interaction Lagrangian is

\[
\mathcal{L} = -(m_u \bar{u}_R u_L + \text{h.c.}) + \frac{a}{F_a} \{ F \bar{F} \} + (\text{K.E. terms})
\]

which has the following fictitious chiral symmetry,

\[
u_L \rightarrow e^{i \alpha} \nu_L, \quad u_R \rightarrow e^{-i \alpha} u_R, \quad m_u \rightarrow e^{-2i \alpha} m_u, \quad a \rightarrow a + 2 \alpha F_a.
\] (13)

There is no chiral symmetry due to the mass term, but we considered this fictitious symmetry to track down the parameter (here \( m_u \)) dependence of the effective potential below the QCD chiral symmetry breaking scale. Below the chiral symmetry breaking scale, \( < \bar{u}u > = v^3 \), we can write an effective potential consistent with the symmetry (13),

\[
V = \frac{1}{2} m_u \Lambda^3 e^{i \theta} - \frac{1}{2} \lambda_1 \Lambda v^3 e^{i(n/\nu-\theta)} - \frac{1}{2} \lambda_2 m_u v^3 e^{i n/\nu} + \lambda_3 m_u^2 \Lambda^2 e^{2i \theta} + \lambda_4 \frac{e^v}{v^3} e^{2i(n/\nu-\theta)} + \cdots + \text{h.c.}
\] (14)

where \( \Lambda \) is the QCD scale, \( \theta = a/F_a \), and \( \lambda_i \) are couplings. Assuming that the minimum is at \( < a > = < \eta > = 0 \), we obtain the following mass matrix, neglecting \( O(m_u^2) \) and \( O(e^{2i \theta}) \) terms,

\[
M^2 = \begin{pmatrix}
\lambda \Lambda v + \lambda' m_u v, & -\frac{\lambda \Lambda v^2}{F_a} \\
-\frac{\lambda \Lambda v^2}{F_a}, & -m_u \Lambda^3 + \frac{\lambda \Lambda v^3}{F_a} - \frac{\lambda' m_u^2 \Lambda^2}{F_a}
\end{pmatrix}
\] (15)

where \( \lambda \) and \( \lambda' \) are newly defined couplings. Then it has the following determinant

\[
\text{Det} M^2 = \frac{m_u \Lambda v}{F_a} (\lambda \Lambda v^3 - \lambda \Lambda^3 - \lambda' m_u \Lambda^2).
\] (16)

For \( F_a \gg (\text{other mass parameters}) \), we obtain \( M^2_\eta = (\lambda \Lambda + \lambda' m) v \) and the axion mass

\[
m_a^2 = \frac{m_u \Lambda}{F_a} \left( \frac{\lambda \Lambda v^3}{\lambda \Lambda + \lambda' m} - \Lambda^2 \right).
\] (17)

If this turns out to be negative, we are in the wrong vacuum, and should choose \( \theta = \pi \). The above axion mass formula shows the essential feature: it is suppressed by \( F_a \), multiplied by the quark mass, and the rest is the strong interaction parameter.
Two flavor

For the two flavor case, we have the following \( U(1)_u \times U(1)_d \) chiral symmetry

\[
\begin{align*}
    u_L &\to e^{i\alpha} u_L, & u_R &\to e^{-i\alpha} u_R, & m_u &\to e^{-2i\alpha} m_u, \\
    d_L &\to e^{i\beta} d_L, & d_R &\to e^{-i\beta} d_R, & m_d &\to e^{-2i\beta} m_d, \\
    \theta &\to \theta + 2(\alpha + \beta).
\end{align*}
\]

(18)

Following the same procedure as before, we diagonalize \( 3 \times 3 \) mass matrix, and obtain

\[
m_a = \frac{m_u F_\pi}{F_a} \frac{\sqrt{Z}}{1 + Z} \simeq 0.6 \times 10^7 \left( \frac{\text{GeV}}{F_a} \right) \text{eV}
\]

(19)

where \( Z = m_u/m_d \simeq 0.56. \)

The KSVZ model

The Yukawa coupling has the form \( \sigma \bar{Q}_R Q_L \) where \( \sigma \) is a singlet Higgs field and \( Q \) is a heavy quark. Below the scale \( F_a \), the interaction Lagrangian is given in Eq. (9). The axion current is

\[
J_\mu = v \partial_\mu a - \frac{1}{2} \bar{Q} \gamma_\mu \gamma_5 Q + \frac{1}{2(1 + Z)} (\bar{u} \gamma_\mu \gamma_5 u + Z \bar{d} \gamma_\mu \gamma_5 d - (\cdot \cdot \cdot))
\]

(20)

where the last term is added in the process of \( a, \pi^0, \eta \) diagonalization.

The DFSZ model

The Lagrangian is supposed to be

\[
\mathcal{L} = \sigma \bar{H}_1 H_2 + \bar{u}_R u_L H_2 + \bar{d}_R d_L H_1 + \bar{e}_R e_L H_1 + \cdots + \text{h.c.}
\]

(21)

where most of \( a \) resides in the phase of \( \sigma \), but \( H_1 \) and \( H_2 \) contain small components of \( a \). Thus there exists the tree level axion-electron-electron coupling,

\[
\left(2x/(x + x^{-1})(a/F_a) m_e \bar{e} i \gamma_5 e, \right. \text{where } x = \langle H_2^0 \rangle / \langle H_1^0 \rangle. \left. \text{The current is} \right)
\]

\[
J_\mu = v \partial_\mu a + \frac{x^{-1}}{x + x^{-1}} \sum_i \bar{u}_i \gamma_\mu \gamma_5 u_i + \frac{x}{x + x^{-1}} \sum_i \bar{d}_i \gamma_\mu \gamma_5 d_i + (\cdots)
\]

(22)

where \((\cdots)\) imply the contribution coming from the process of \( a, \pi^0, \eta \) diagonalization as given in Eq. (20).
2.5 the $a - \gamma - \gamma$ coupling

This feeble coupling of axions can be probed by Sikivie’s cavity experiments. From the fundamental theory, one can calculate the $a\gamma\gamma$ coupling, $\bar{c}_{a\gamma\gamma}$. When going through the chiral symmetry breaking, it obtains an additional contribution, and the total coupling $c_{a\gamma\gamma}$ is given by

$$c_{a\gamma\gamma} = \bar{c}_{a\gamma\gamma} - \frac{2}{3} \frac{1}{Z} = \bar{c}_{a\gamma\gamma} - 1.92$$

for $Z = 0.6$, and

$$\bar{c}_{a\gamma\gamma} = \frac{E}{C}, \quad E = \text{Tr}Q_{\text{em}}^2, \quad \delta_{ab}C = \text{Tr}\lambda_a\lambda_bQ_{\text{PQ}}.$$  

Note that for the KSVZ model, $C_3 = -1/2, C_8 = -3, E_3 = -3e_Q^2$, and $E_8 = -8e_Q^2$. Thus, $\bar{c}_{a\gamma\gamma} = 6e_Q^2$ and $(8/3)e_Q^2$ for color triplet and octet quarks, respectively. For the DFSZ model, $C_3 = N_g, E = (8/3)N_g$ where $N_g$ is the family number. The DFSZ model is distinguished how electron obtains mass. If it gets mass through $H_1, H_2,$ and $H_3$ coupling, respectively, then they can be called $(d^c, e)\epsilon$, $(u^c, e)\epsilon$, and non-unification models, respectively. These couplings are listed in Table 1.

|                      | KSVZ                      | DFSZ                      |
|----------------------|---------------------------|----------------------------|
|                      | $e_{R}$                   | $c_{a\gamma\gamma}$      | $x$ (unif) | $c_{a\gamma\gamma}$ |
| $e_{R} = 0$          | -1.92                     | any ($d^c$)               | 0.75       |
| $e_{R} = -1/3$       | -1.25                     | $1$ ($u^c$)               | -2.17      |
| $e_{3} = 2/3$        | 0.75                      | $1.5$ ($u^c$)             | -2.56      |
| $e_{3} = 1$          | 4.08                      | $60$ ($u^c$)              | -3.17      |
| $e_{8} = 1$          | 0.75                      | $1$ (non)                 | -0.25      |
| $(m, m)$             | -0.25                     | $1.5$ (non)               | -0.64      |
| (1, 2)               | -0.59                     | $60$ (non)                | -1.25      |

Under the assumption that these very light axions are the dark matter of our galaxy, we can predict the detection rate in the Sikivie type detector. The model predictions and the experimental bounds are compared in Fig. 1. In reality, there can be many heavy quarks which carry nontrivial PQ charges. There are also two Higgs doublets in supersymmetric models. For example, in superstring models, there are more than 400 chiral fields, in which there appears in most cases heavy quarks. Therefore, if the PQ symmetry is spontaneously broken in superstring models, then it must contain both the KSVZ and the DFSZ aspects. If a string model is given, one can calculate $c_{a\gamma\gamma}$. 
Fig. 1. Comparison of several $c_{\alpha \gamma}$'s and experimental bounds.
3 Astrophysical and Cosmological Bounds

The astrophysical and cosmological bounds allow a window of $F_a$,

$$10^{9-10} \text{ GeV} \leq F_a \leq 10^{12} \text{ GeV}.$$  \hspace{1cm} (25)

The astrophysical bounds come from the requirement that the axions produced at the core of a star do not take out too much energy compared to the takeout by neutrinos. If the axion coupling is too small, the axions are not produced copiously in the star and hence even though they do not have a chance to be blocked by matter in the star, it will be allowed. This condition translates to the lower bound on the axion decay constant. The most stringent bound comes from SN1987A. The upper bound around $F_a \simeq 10^{12} \text{ GeV}$ comes from the cold axion energy density of the universe.

But it is known that the topological defects such as axionic strings which is comparable to the cold axion energy density and axionic domain walls which overcloses the universe. However, these will not cause a serious problem in the inflationary universe paradigm. With supergravity inflation, anyway one needs the reheating temperature after inflation less than $10^9 \text{ GeV}$ which is supposed to be less than $F_a$. Thus the most robust bound on cosmological bound of $F_a$ comes from the cold axion energy density which is

$$\Omega_a h^2 = 0.13 \times 10^{0.4} \Lambda_{200}^{-0.7} f(\theta_1) m_a^{-1.18} N_{\text{DW}}^2.$$  \hspace{1cm} (26)

where $\Lambda_{200}$ is the QCD scale in units of 200 MeV, $f(\theta_1)$ is the initial value if $\theta$ at the starting moment of the axion rolling around $T \sim 1 \text{ GeV}$, and $m_a = m_a/10^{-5} \text{ eV}$. Note that superstring models have $N_{\text{DW}} = 1$. In general, the $F_a$ bound is customarily given as in Eq. (25). If $\Omega_a \simeq 0.3$, then we have a bound $F_a \simeq 5 \times 10^{11} \text{ GeV}$ and $m_a \simeq 1.2 \times 10^{-5} \text{ eV}$. Assuming that the cold dark matter in our galaxy is mostly the cold axion, the axion search experiment is going on, which was reviewed above.

4 Superstring Axion

The standard introduction of axion through spontaneous symmetry breaking of the Peccei-Quinn global symmetry is ad hoc. As we have seen, there are many ways to introduce axions. Among these, there exists a very interesting method. It is from string theory. Here, the very light axion must be present

$^b$For a high reheating temperature, Battye and Shellard\footnote{For a high reheating temperature, Battye and Shellard give $F_a < 4 \times 10^{10} \text{ GeV}$, but the Harari-Sikivie\footnote{estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.} estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.} give $F_a < 4 \times 10^{10} \text{ GeV}$, but the Harari-Sikivie\footnote{estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.} estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.\footnote{For a high reheating temperature, Battye and Shellard give $F_a < 4 \times 10^{10} \text{ GeV}$, but the Harari-Sikivie estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.} estimate of the axion string energy is smaller than the Battye-Shellard estimate by a factor 10–100.
Furthermore, the string theory—the anticipated theory of everything—must solve the strong CP problem if it is a physical theory. Therefore, it is worthwhile to see what is the problem in superstring axions.

The 10D string theory contains the bosonic degrees, \( G_{MN}, B_{MN} \) and \( \phi \) where \( \{M,N\} = 0, 1, 2, \ldots, 9 \). \( G_{MN} \) contains the graviton, \( \phi \) is called the dilaton, and the antisymmetric tensor \( B_{MN} \) contains the axion. When \( M \) and \( N \) are restricted to 4D indices \( \mu, \nu \), etc., it has one physical degree. The 4D field strength of \( B \) is denoted as \( H \), the three index antisymmetric tensor. The axion is the dual of \( H \),

\[
\partial^\sigma a \propto \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho}, \quad H_{\mu\nu\rho} \propto \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a. \tag{27}
\]

Why can we call this axion? It is because it has a coupling given in the form of Eq. (2). Let us see how it arises.

The field strength \( H \) is not just the curl of \( B \), but is made gauge invariant by adding Chern-Simmons terms,

\[
H = dB - \omega^0_{3Y} + \omega^0_{3L} \tag{28}
\]

where the Yang-Mills and Lorentz Chern-Simmons forms satisfy \( d\omega^0_{3Y} = \text{tr} F^2 \) and \( d\omega^0_{3L} = \text{tr} R^2 \). Therefore,

\[
dH = -\text{tr} F^2 + \text{tr} R^2. \tag{29}
\]

The reason, \( B \) transforms nontrivially under gauge transformation to cancel the Yang-Mills and Lorentz anomalies is the source of Eq. (29). Also, the Green-Schwarz term contains

\[
S_{GS} \propto \int (B \text{tr} F^4 + \cdots). \tag{30}
\]

Eq. (29), through the duality relation (27), gives the following equation of motion for \( a \),

\[
\partial^2 a = -\frac{1}{M_c} (\text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} - \text{Tr} R_{\mu\nu} \tilde{R}^{\mu\nu}) \tag{31}
\]

which can be obtained from an effective Lagrangian

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu a)^2 - \frac{a}{M_c} (\text{Tr} FF - \text{Tr} RR) \tag{32}
\]

where \( M_c \) is the compactification scale. This axion is called the model-independent axion (MIA). Any string models have this one.
Eq. (30) also has a coupling which allows to interpret another pseudoscalar $a'$ an axion. It depends on the compactification scheme. This model-dependent axion receives contributions to the superpotential from the world-sheet instanton effects, obtaining a large mass in general. Therefore, it is not useful for the solution of the strong CP problem. Recently, it has been pointed out that in the M-theory this model-dependent axion can have a room to play for the strong CP solution.

In sum, there are axions in string models. The model-independent axion is the promising one toward a solution of the strong CP problem. But it has two problems to be solved:

(A) Decay constant problem–The MIA has the decay constant around $F_a \approx 10^{16}$ GeV which is too large for the axion cold dark matter scenario.

(B) Hidden sector problem–Popular supergravity models require a confining hidden sector, say $SU(N)_h$, at $\sim 10^{13}$ GeV scale. If so, MIA obtains a mass also from the hidden sector contribution. Because the hidden sector potential is so steep compared to the QCD potential that it will settle $\theta_h \approx 0$, but leaves $\theta_{QCD} \neq 0$. Therefore, one axion cannot solve the strong CP problem. Two axions are needed with the hidden sector.

Problem (A) seems to be solved in anomalous $U(1)$ models. In this case, the MIA becomes the longitudinal degree of the anomalous gauge boson. The gauge boson and MIA is removed at this gauge boson mass scale. At low energy, there remains a global PQ symmetry which can be broken at $\sim 10^{12}$ GeV. However, if the low energy theory maintains supersymmetry, it is very difficult to realize this scenario. The reason is that the supersymmetry condition with matter fields always breaks the global PQ symmetry at the gauge boson mass scale, and a remaining $U(1)$ symmetry (if any) turns out to be a gauge symmetry.

To solve (B), we need another axion or massless hidden sector fermion. One obvious choice is massless hidden sector gluino (h-gluino). The h-gluino will obtain mass eventually; but this case is different from the massless up quark case since h-gluino mass is generated by the h-gluino condensation. Massless h-gluino suggests a possible global symmetry; $U(1)_R$ symmetry. But there is no global symmetry in string models except that corresponding to the MIA. Therefore, the best we can do is to consider a discrete subgroup of $U(1)_R$ when we consider the massless h-gluino. Under certain assumptions, it is easy to calculate the axion dependence of the potential. For $Z_N$ subgroup of $U(1)_R$, we have $V \sim 10^{13-25}$ GeV for $N = 2$, $10^{-29-8}$ GeV for $N = 3$, $10^{-8-10}$ GeV for $N = 4$, $10^{-50-26}$ GeV, etc. Except $Z_2$ and $Z_4$, any discrete subgroup of $U(1)_R$ is phenomenologically acceptable. The schematic
behavior of the contributions to the potential is shown below. In general, we expect the following contributions to $V$,

\[
\begin{array}{cc}
\text{QCD} & \text{SU}(N)_h \\
\includegraphics[width=0.2\textwidth]{qcd contributed.png} & \includegraphics[width=0.2\textwidth]{su contributed.png}
\end{array}
\]

But with a massless h-sector gluino, the contributions will look like

\[
\begin{array}{cc}
\text{QCD} & \text{SU}(N)_h \\
\includegraphics[width=0.2\textwidth]{qcd contributed.png} & \includegraphics[width=0.2\textwidth]{su contributed.png}
\end{array}
\]

With sufficiently suppressed h-sector contribution ($Z_3$, $Z_5$, etc.), $\theta_{QCD}$ is settle at 0 but $\theta_h \neq 0$,

\[
\begin{array}{cc}
\text{QCD} & \text{SU}(N)_h \\
\includegraphics[width=0.2\textwidth]{qcd contributed.png} & \includegraphics[width=0.2\textwidth]{su contributed.png}
\end{array}
\]

Therefore, the QCD axion physics is the usual one, satisfying the condition for the cold dark matter. [Note added after the talk: In fact, if this scenario of making the hidden sector potential extremely shallow, one can obtain a reasonable value for a nonvanishing cosmological constant. The magnitude of the cosmological constant is roughly the height shown in the last cartoon.]

5 Conclusion

I reviewed briefly that the strong CP solution is guaranteed with a very light axion, its possible imbedding in superstring models, cosmological implication as a cold dark matter candidate, and possible detection of this very light axion in the cavity detectors assuming that the missing mass is mostly these axions. If discovered, its implication will be dramatic, since it will prove experimentally the whole idea of instanton, the collective axion oscillation, and maybe the superstring idea. On the other hand, if not discovered, the axion misalignment
angle may be very small so that there are not so much axions as needed by the missing mass, or the axion decay constant may be very large\cite{25,31}. Also, the strong CP problem must be solved outside the axion mechanism, which is however considered to be not so attractive\cite{4}.

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