Creep motion in a granular pile exhibiting steady surface flow

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We investigate experimentally granular piles exhibiting steady surface flow. Below the surface flow, it has been believed existence of a ‘frozen’ bulk region, but our results show absence of such a frozen bulk. We report here that even the particles in deep layers in the bulk exhibit very slow flow and that such motion can be detected at an arbitrary depth. The mean velocity of the creep motion decays exponentially with depth, and the characteristic decay length is approximately equal to the particle-size and independent of the flow rate. It is expected that the creep motion we have seen is observable in all sheared granular systems.

Granular materials exhibit behavior not seen in ordinary fluids or solids. Considerable efforts have been made for understanding them, but further studies of various phenomena in granular materials are still required. One of the typical behaviors of granular materials is avalanche behavior of a sand pile. In contrast to ordinary fluids, granular materials can form piles with a sloped surface. When the angle of the surface exceeds some critical value, the pile cannot sustain the steep surface and an avalanche occurs. Without careful observation, flow of particles in an avalanche appears to be limited to a surface layer, with a ‘frozen’ bulk region below. Many studies have been made under such an assumption without convincing experimental evidence. Indeed, it is difficult to detect the frozen bulk clearly because an avalanche is a transient behavior. If such a frozen region actually exists and is well separated from the flow layer, it should be possible to observe and definitively identify it even in the case of steady surface flow. With the purpose of making such an observation, we experimentally investigated particle movements in piles exhibiting steady surface flow.

Here we report unexpected experimental findings in such piles: Even the particles in deep layers are not frozen but exhibit very slow flow (creep), and such motion can be detected at arbitrary depth (Fig.1). The mean velocity of the creep motion decays exponentially with depth, and that the characteristic decay length is approximately equal to the particle-size and independent of the flow rate. This velocity profile differs from that for the surface flow, and it is in this sense that the flow of particles is separated into two regions. We believe that creep motion of the type we have seen should be observable in all sheared granular systems.

Our experiments were performed in a quasi-two dimensional system, as described by Fig.2. Alumina beads of diameter \(a\) were continuously fed into the gap (A) existing between two vertical, parallel plates separated by width \(W\). In this gap, the particles formed a triangularly shaped pile with rapid flow on the surface slope, pouring out of the system from the right side (B). The existence of the short wall on the right side eliminates the slipping of particles on the bottom of the cell. Its presence thus assures that the motion of particles we observe is strictly due to the flow on the slope of the pile. In order to maintain a steady surface flow on the pile, particles were continuously fed onto the pile from the left side using a “double hopper”, in which the lower hopper was continuously filled with particles by the upper one. The mouth of the lower hopper was placed in such a manner that it always contacted the top of the pile. This configuration allowed us not only to minimize the effect of the impact on the pile exerted by newly added particles but also to control the feeding rate by changing the height of the mouth of the lower hopper. With this configuration, the feeding rate was essentially controlled by the flow on the pile, and it could thus be closely matched to the outflux from the system. In this way a steady-state system could be established. The flow rate \(Q\) for each experimental run was determined by measuring the weight of the particles that poured from the right side of the gap per unit time. With this system, the important control parameters are the particle diameter \(a\), the flow rate \(Q\), and the width of the gap \(W\). In the present study, we report the results obtained for three different diameters, \(a = 1.1 \pm 0.1, 2.1 \pm 0.1\), and \(3.1 \pm 0.1\)mm, and the gap width, \(W = 10a\). We confirmed that qualitatively the same results are obtained for \(W/a = 5 - 40\).

As is seen in Figs.1, the surface of the pile is flat in the steady flow state, and its angle \(\phi\) with respect to the horizontal direction is close to that after the flow stops when the supply of particles is cut off, i.e., the angle of
repose. In this state, the mean velocity is approximately parallel to the surface, and its functional dependence on the depth $h$ (measured perpendicularly to the surface see Fig.2) is the same everywhere, except in the vicinity of left and right boundaries. Figure 3 displays the velocity profiles as functions of the depth $h$ on a semi-log scale, while the inset displays the same data on a normal scale. From the graph in the inset, the mean velocity $\langle v(h) \rangle$ appears to decay as the distance from the surface increases, and it appears to vanish at some finite depth. Although in previous studies it has been assumed that these deep layers are ‘frozen’, we found that the mean velocity is finite at all depths. We call the slow motion of particles in such deep region as the *creep motion*.

As is clearly shown in Fig.3, for deep layers (large $h$) exhibiting creep motion, the velocity profile assumes the form of simple exponential decay:

$$\langle v(h) \rangle = v_0 \exp(-h/h_c), \quad (1)$$

where $h_c$ is the characteristic length. While $v_0$ increases with the flow rate $Q$, $h_c$ is approximately equal to the particle diameter $a$ for all values of $Q$. The value of $h_c$ suggests that the creep motion is driven by events occuring on a particle-size scale.

Each velocity profile in Fig.3 is seen to bend at some depth. At such a depth, there approximately exists a boundary at which the particle motion changes from rapid surface flow to slow creep motion. As shown schematically in Fig.4, this boundary which is parallel to the surface, lies just above the upper edge of the right side wall. The value $h_0$ corresponds to the thickness of the surface flow, which increases with the flow rate $Q$. The particles below the boundary are jammed tightly due to the fixed wall existing below the surface downstream, while the upper surface flow is free from such obstruction. The mean velocity $\langle v \rangle$ was found to have a longer characteristic length for the rapid surface flow above the boundary than for the creep motion; i.e. the creep motion decays more rapidly as a function of $h$ than the surface flow.

In the region of the creep motion, particles are jammed tightly. For such particles to move, the existence of voids is essential. In fact, local rearrangements of particles are often observed around such voids. In the creep region, we also observed plastic global deformation of the network formed by inter-particle connections. Voids are created occasionally by such deformation, while this deformation is maintained by shear stress resulting from the surface flow.

These observations, together with the realization that the characteristic length of the decay is on the order of the particle size, lead to the simple idea that the exponential decay of the velocity profile can be understood in terms of an analysis based on the void creation process. Let us now pursue this point. For the region displaying creep motion we consider virtual layers parallel to the surface with particle size thickness. Since the particles in these layers are severely obstructed, the mean velocity $v_n$ of the $n$-th layer (The index $n$ increases from the surface.) should be proportional to the production rate $P_n$ of voids: $v_n \propto P_n$. A void in a given layer is generated by the escape of a particle or particles to neighboring layers, while this escape is driven by the relative motion between the neighboring layers. Thus, we hypothesize that $P_n$ approximately satisfies $P_n \propto (v_{n-1} - v_n)$. This leads to $v_n = \alpha(v_{n-1} - v_n)$, with some constant $\alpha$ ($> 0$). This ends us to expect that $v_n$ is an exponentially decaying function of depth. We remark that these probabilistic considerations leave the unknown parameter $\alpha$. Dynamical considerations, on the other hand, should allow us to determine a value for such a parameter, and this should provide an understanding of the physics of the granular materials behind this simple result.

We have reported only the results for alumina spheres. Qualitatively, the same results were obtained for particles with different shapes and roughness: long seeds and irregularly shaped sand particles. This suggests that our results do not depend sensitively on shapes nor roughness of particles. The case of particles interacting through an attractive force and the case of heterogeneous particles are left as future problems. We would like to briefly note the effect of creep motion on the surface flow. Comparing the surface flow in presently investigated system with that on a truly frozen bulk (which can be modeled by a mono-layer of particles fixed on an inclined board), it is observed that flow in the former case is slower than that in the latter. This implies that flow on a creeping bulk experiences greater resistance than that on a frozen bulk. This is reasonable, because the surface flow itself is the source of the shear stress inducing the creep motion, and through this mechanism, energy is dissipated to the bulk below. We believe that the slow creep behavior reported in this letter is common to all sheared granular systems, regardless of the source of the driving force, and that our results elucidate one of the fundamental properties of sheared granular systems. While we studied steady states of granular piles, it is also interesting to consider the significance of our results with regard to unsteady phenomena, such as earthquakes and the strengthening of granular piles exhibiting stick-slip phenomena under shear. For example, the creep motion described by Eq. (1) might have some close relation with the logarithmic time dependences seen in these systems.

In summary, we have experimentally studied granular piles exhibiting steady surface flow. We found that there exist rapid surface flow and slow creep motion below this flow. The boundary between these two types of motion, across which the velocity profile changes continuously, is possible to be defined. We do not find a ‘frozen’ layer at any depth. Rather, we found that the velocity of the
creep motion decays exponentially according to Eq. (1) and that the characteristic length of this decay is on the order of the particle size and independent of the flow rate. We believe that the behavior found here and the velocity profile we have observed are specific to granular materials. We described a simple picture in terms of which to understand this velocity profile. While most of our experiments were performed with spherical particles, we found that similar creep motion exists also for systems consisting of non-spherical particles. This suggests that the behavior we have studied here is quite common (and perhaps universal) in granular systems.

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FIG. 1. Snapshots of a granular pile in a steady flow state. Particles were fed constantly from the surface of the left side, out of these figures. The particles used were monodisperse alumina spheres with diameter 1.1 ± 0.1mm (weight density 3.6 grams/cm³). All photographs were taken under the same conditions, with only the shutter speeds differing; (a)1sec, (b)1min, (c)1hour. (a) A boundary between the surface flow and the ‘frozen’ layers appears to exist at a depth of several particles below the surface. (b) The apparent frozen layer of (a) is seen to flow. Similarly, (c) reveals the flow of a thicker layer than that revealed by (b). These observations reveal that the apparently ‘frozen’ particles under the rapidly flowing surface are not stationary but slowly creep and that the layer in which this creeping motion can be detected grows as observation time increases.

FIG. 2. The geometry of the experiment. We used a transparent material (acrylic plate) for the front plate (400mm wide) to allow observation of particle motion and aluminum for the back plate to reduce the static electricity effect. The bottom and the left side of the gap were completely bounded by plates, whereas the right side was bounded only in its lower region (a region of height 20 mm).

FIG. 3. Mean velocity profiles $\langle v \rangle$ as functions of the depth $h$ from the pile's surface on a semi-log scale. The inset presents the same data on a normal scale. The horizontal axes represent the depth from $h_0$ normalized by the particle diameter $a$, where $h_0$ is the distance from the top of the fixed wall existing at the right side of the system to the surface of the particles flowing over it. (See Fig.4.) The vertical axes represent velocity normalized by the value at $h_0$. The values of the particle diameter $a$(mm) and the flow rate $Q$(grams/sec) for the experiments are 1 and 12 (squares), 1 and 43 (solid squares), 2 and 20 (circles), 2 and 50 (solid circles), 2 and 100 (triangles), 3 and 34 (plus signs), 3 and 58 (crosses). The broken line corresponds to $\exp(-0.72x)$.

FIG. 4. A schematic picture showing the coexistence of the two different types of motion. The boundary between them is parallel to the free surface, and its position seems to be determined by the height of the fixed wall existing downstream in our system.
T.S.Komatsu et al Figure 1.
T.S. Komatsu et al. Figure 2.
T.S. Komatsu et al. Figure 4.

The diagram shows a surface flow with a designation for creep motion and a label $h_0$. The text indicates an underlying analysis or explanation related to the figure.