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Non-linear correction for the k-ω SST turbulence model

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Abstract. Non-linear correction for the k-ω SST model based on the WJ-BSL-EARSM model was developed and tested for a wide range of flows. For the basic turbulence flows the corrected model (SST-NL) results are close to those of the linear SST model and surpass WJ-BSL-EARSM ones. At the same time non-linear correction is capable to predict secondary motion in rectangle channels and prevents false corner separation for flows around a wing-body junction as well as the WJ-BSL-EARSM model.

1. Introduction
Despite well-known fundamental restrictions of the Linear Eddy-Viscosity RANS turbulence models (LEVM), they are still most widely used in industrial and aerospace applications. Such popularity of these models is due to the existence of time-proved and numerically stable models for them. Reynolds Stress Models (RSM), which in principle present a more general class of models with a wider range of applicability, could be considered as an alternative of LEVM. The advantages of RSM are revealed most significantly for turbulent flows with secondary motion induced by Reynolds stress anisotropy. One of the impressive examples of such flows is a flow around a wing-body junction where LEVM predict false corner flow separation and only RSM are able to capture correct flow topology. However for relatively simple flows accuracy of RSM is less than accuracy of the best linear models. Other than that the worse robustness and convergence of equations prevent wide spreading of these models.

One of the best turbulence LEVM in terms of robustness, computational cost and accuracy for simple flows is the k-ω SST model [1]. However this model is not able to prevent false corner separation. Therefore the aim of the present work is to resolve this issue and develop a non-linear correction for the SST model (SST-NL).

2. Non-linear correction for the SST model
The SST model contains two transport equations for turbulent kinetic energy, k, and specific dissipation rate, ω. In the original SST model the Reynolds stress tensor is computed using the Boussinesq relation

$$\tau_{ij} = \frac{2}{3} k \delta_{ij} - 2 \nu_t S_{ij}$$

(1)

where $\nu_t = \frac{a_1 k}{\max(a_1 \omega, SF_2)}$ is the eddy viscosity, $S_{ij}$ - strain rate tensor, $S = \sqrt{2S_{ij}S_{ij}}$ - strain rate invariant, $F_2$ - SST function, and $a_1 = 0.31$.

The proposed correction affects only the Reynolds stress tensor computation and consists of addition of non-linear terms to relation (1). The «constitutive relation» in the SST-NL reads as follows:
\[ \tau_{ij} = \frac{2}{3} k \delta_{ij} - 2 \nu \frac{1}{k} \left[ \frac{1}{Q} \left( S_{ij}^* - \Omega_{ij}^* \right) + \frac{N}{Q} \left( \frac{1}{2} \left( S_{ij}^* + \Omega_{ij}^* \right) - S_{ij}^* \right) \right] \]

where \( S_{ij}^* \) and \( \Omega_{ij}^* \) are the non-dimensional strain rate and vorticity tensors and other quantities are defined as:

\[ N = C_1^t + \frac{9}{4} \sqrt{2C_\nu} II_S, \quad Q = \frac{(N^2 - 2II_{\Omega})}{A_1}, \quad Q_t = \frac{Q}{6} (2N^2 - II_{\Omega}). \]

Note, that the non-linear terms in (2) are based on the WJ-BSL-EARSM [2] which is a modification of the WJ EARS [3].

3. Application Test Cases
This section presents a comparative assessment of the SST-NL model with «its parents», SST and WJ-BSL-EARSM, and consists of three main parts. The first part presents results of computations of basic two dimensional turbulence flows including wall-bounded flows and flows with turbulent boundary layer separation. The second shows comparison of the models for rectangle channels where secondary motion in the sharp corners takes place. Finally an example of a flow with a wing-body junction is discussed in the third part.

The simulations of all considered flows were carried out using ANSYS FLUENT and the proposed correction was implemented using UDF (User Defined Functions). The Coupled method is used for pressure-velocity coupling. The inviscid fluxes in the momentum and turbulence equations are approximated with the use of the Second Order Upwind scheme. For pressure the «Standard» interpolation (weighted interpolation based on central coefficients) is utilized and the gradients are approximated with the use of cell based Green-Gauss method. For all test cases computational meshes ensure grid-independent solution and convergence of iterations was achieved.

3.1. Basic turbulence flows
The goal of these computations is to check the influence of the suggested modification on the accuracy of simple turbulent flow prediction. Three tasks were considered: zero pressure gradient boundary layer, backward facing step and axisymmetric diffuser.

The first case, Turbulent Boundary Layer (TBL) was experimentally investigated by Wieghardt and Tillmann [4] and was included into the AFOSR-IFP Stanford database of turbulent flows [5]. The corresponding incompressible flow was computed at the Reynolds number based on inlet velocity and length of the flat plate \( \text{Re}=10^5 \).

An experimental investigation of the second case, an incompressible flow downstream of a Backward-Facing Step (BFS), was carried out by Vogel and Eaton [6]. The Reynolds number based on the maximum inlet velocity and the step height \( H \) is equal to \( \text{Re} = 2.8 \times 10^4 \). Inlet boundary is placed \( 3.8H \) upstream, and the outlet boundary is located approximately \( 40H \) downstream from the step. In order to reproduce the experimental inflow conditions in the simulations, a precursor computation of the flow in the plane channel was carried out, and the inflow conditions were extracted from this computation at the cross-section where the experimental boundary layer displacement thickness have been matched.

The last test case, CS0 diffuser by Driver [7], presents an example of shallow separation from a smooth wall. The test case geometry consists of an axisymmetric diffuser caused by adverse pressure gradient with an internally mounted cylinder along the centerline. The Reynolds number based on the maximal inlet velocity and the inner cylinder diameter \( D \) is equal to \( 2.8 \times 10^3 \).

Figure 1 shows schemes of the considered flows and the skin friction coefficient distribution for the considered cases. One can see that for the TBL flow all considered models are almost identical but for
separated flows (BFS and CS0) the difference is significant. Namely, the linear and non-linear SST models are close to each other and are in better agreement with experimental data than the WJ-BSL-EARSM model, which predicts less separation for the backward-facing step or even no separation for the CS0 diffuser.

| TBL | BFS | CS0 diffuser |
|-----|-----|--------------|

![Figure 1.](image)

**Figure 1.** Schemes of the considered flows (upper row) and skin friction coefficient distribution for the TBL, backward-facing step and CS0 diffuser

3.2. **Flows in rectangular ducts**

Flows in rectangle ducts are examples of the essential influence of Reynolds stress anisotropy which initiates secondary flow motion. In some cases the secondary flow can result in a strong change in a flow structure and this effect is beyond the capabilities of linear eddy viscosity models to predict.

3.2.1. **Fully developed flow in square duct**

The flow is considered under the conditions of the DNS [8] at a Reynolds number based on the averaged friction velocity and the channel width \( H \) of 1200. The computations are carried out in a «2.5D mode», which includes momentum equations for all the three velocity components but assumes that their streamwise derivatives are zero. A constant streamwise pressure gradient is adjusted to meet the DNS Reynolds number. At the solid walls, the no-slip boundary conditions are imposed. The streamlines at XY section show that SST-NL is capable of predicting secondary flow as well as WJ-BSL-EARSM while the SST model fails (see Figure 2). Lateral velocity at diagonal line (X=Y) predicted by SST-NL is slightly closer to the DNS data than the WJ-BSL-EARSM solution (Figure 3a) and the SST model as well as other LEVM predicts no secondary motion. As a result streamwise velocity predicted by SST-NL is in better agreement with DNS data than the results of the SST model (Figure 3b).
### Figure 2.
Comparison of streamline velocity contours and secondary motion with the DNS data in the fully developed square duct flow.

| SST | SST-NL | WJ-BSL-EARSM | DNS |
|-----|--------|--------------|-----|
| ![Streamline Contours](image1) | ![Streamline Contours](image2) | ![Streamline Contours](image3) | ![Streamline Contours](image4) |

### Figure 3.
Lateral and streamwise velocity profiles on the diagonal of the square duct.

### 3.2.2. Flow in a rectangular diffuser

Flow in the rectangular diffuser (Figure 4a) [9,10] is characterized by strong separation which is very sensitive to turbulence modelling. The Reynolds number based on the bulk velocity and height of the duct, \( H \), is equal to \( 10^4 \). In accordance with the experimental setup, the inlet flow is considered as fully developed flow. The velocity and turbulence characteristics in the inlet plane are specified using precursor computation of the developed rectangular channel flow with the same turbulence model as for the main computation. The no-slip boundary conditions are specified at the diffuser walls.

The pressure coefficient \( C_p = \frac{(p - p_{ref})}{\frac{1}{2} \rho u_b^2} \) at the centerline of the bottom flat wall is presented at Figure 4b (\( p_{ref} \) is the pressure at the first experimental point; \( u_b \) is a bulk velocity in the inlet channel). One can see that the results of the SST-NL and WJ-BSL-EARSM are close to each other and better agree with the experimental data than the linear SST model.

| Rectangular diffuser (a) | Pressure coefficient (b) |
|--------------------------|--------------------------|

### Figure 4.
Rectangular diffuser (a) and comparison of computed and measured pressure distributions along centerline of the bottom wall (b).
3.3. Flow around a wing in wind tunnel with no-slip side walls.

The flow around an A-Airfoil wing mounted between sidewalls was experimentally investigated in the ONERA F2 wind tunnel [11] at a Reynolds number based on the wing chord $C$ of $2.1 \times 10^6$. No corner separation from the wing-sidewall junction was observed at $\alpha \leq 16^\circ$ in the experiment while LEVM models tend to predict this separation. The incompressible flow was considered at $\alpha$ from $13^\circ$ to $16^\circ$. The height of the tunnel ($L_y = 3C$) and wing span ($L_z = 2.3C$) correspond to experimental wind tunnel parameters (see Figure 5). Uniform velocity and turbulent characteristics are specified at the inlet boundary. Top and bottom tunnel walls are modelled as slip walls. The beginning of the no-slip side walls ($2.25C$ upstream from the leading edge) ensures the same boundary layer thickness on the side wall near the leading edge as it was observed in the experiment. No-slip boundary conditions are specified on the airfoil surface.

Streamlines over the wing suction side show flow structure and separation size near wing-body junction (Figure 6). The SST model predicts corner separation for all considered angles of attack. At $\alpha = 13^\circ$ this separation is very small but at $\alpha \geq 13^\circ$ it exceeds half of the tunnel width. For both non-linear models the corner separation is absent. Moreover one can see that with increasing $\alpha$ the separation appears near the wing center and then spread on the entire wing suction side.

![Figure 5. Computational domain used for the A-Airfoil test case.](image)

| $\alpha$ (deg) | SST | SST-NL | BSL-EARSM |
|----------------|-----|--------|-----------|
| 13             | ![Streamlines](image) | ![Streamlines](image) | ![Streamlines](image) |
| 14             | ![Streamlines](image) | ![Streamlines](image) | ![Streamlines](image) |
| 15             | ![Streamlines](image) | ![Streamlines](image) | ![Streamlines](image) |
| 16             | ![Streamlines](image) | ![Streamlines](image) | ![Streamlines](image) |

![Figure 6. Visualization of streamlines near wing-sidewall junction.](image)
4. Conclusions
A non-linear correction for the k-ω SST model was developed and the corrected model (SST-NL) was examined. Tests show that the SST-NL predicts basic turbulence flows as well as the SST model and surpasses the WJ-BSL-EARSM model. For flows in rectangle ducts, where secondary flow leads to a strong changes in the flow, results of the SST-NL and WJ-BSL-EARSM are close to each other and are in better agreement with the experimental data than the linear SST model results. Finally, the SST-NL model as well as WJ-BSL-EARSM model prevents false corner separation for flow around a wing-body junction. So, for the considered flows SST-NL exceeds both «parents», namely SST and WJ-BSL-EARSM.

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