Pion interferometry with higher-order cumulants†

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Abstract

We have measured second- and third-order cumulants in UA1 data (\(\bar{p}p\) collisions at 630 GeV/c). Rather than quoting numerical values for source parameters, we have used these in three checks to test the “quantum statistics” theory for consistency over these cumulants. In the process, we have found a method for folding theoretical correlation functions with experimental one-particle distributions. Our preliminary results appear to indicate that, for the specific tests performed, the data contradicts the theory.

1 Introduction

Pion interferometry has been a part of particle physics for several decades [1]. The main sub-branch of this science, concerning itself with Bose-Einstein correlations, endeavours to elicit information on the size, shape and temporal evolution of the source emitting pions. This is based on an analogy between optical intensity interferometry and quantum mechanical interference between incoherent pion amplitudes.

Measurements of correlations between identical particles contain, besides amplitude interference, a plethora of other effects such as coherence, decaying resonances, variations

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in impact parameter and momentum distribution, contamination by kaons and protons, final-state interactions etc. Understandably, a solid basis for subtracting all such effects has been singularly hard to create. Given the number and degree of theoretical and experimental uncertainties entering even second-order correlations, the corresponding higher-order measurements have received only scant attention: if it is hard to extract source parameters in an honest and unambiguous way from second-order correlation data, it probably becomes even harder for third order.

In the present paper, we elect to take a different approach: We measure higher-order correlations not so much with a view to extracting source parameters or “true” Bose-Einstein correlations, but in order to perform consistency checks. While few theorists have so far worked out the implications of their respective models for higher orders, this is in principle possible, and some examples of higher-order predictions exist [2, 3, 4, 5, 6]. If a given theory provides formulae for both second and higher orders, then these should apply to the corresponding data using the same parameter values throughout.

With the aid of rapidly-improving measurement technology, we are attempting to put such predictions to the test. While the results reported here are quite preliminary in nature, they hopefully point the way to more general and sophisticated testing of theories rather than just measuring their respective parameters. Falsifying theories is arguably the best (some would say the only) way of making progress in a confused situation [7].

Our tools for performing these consistency checks are cumulants and the correlation integral [8, 9, 10]. Cumulants, in subtracting out trivial lower-order contributions, have proven far more sensitive than the corresponding moments; their implementation in various forms of the correlation integral has, at the same time, improved statistical accuracy to a degree where such measurements have become meaningful.

2 Quantum statistics theory

The test we shall be reporting here is confined to one particular variable, the four-momentum difference $q_{ij} = [(\vec{p}_i - \vec{p}_j)^2 - (E_i - E_j)^2]^{1/2}$. For this variable, the second and third-order cumulants are [3, 11]

$$C_2(q) = \rho_2(q) - \rho_1 \otimes \rho_1(q),$$
$$C_3 = \rho_3 - \sum_{(3)} \rho_2 \otimes \rho_1 + 2 \rho_1 \otimes \rho_1 \otimes \rho_1,$$  

where the third order quantities are functions of the three pair variables $(q_{12}, q_{23}, q_{31})$. These cumulants, including the crossed “⊗” quantities and event-mixing normalizations can be found from data samples in a precisely prescribed algorithm [9].

The quantum statistics (QS) theory itself has a long and distinguished tradition [12, 13]; the version we concentrate on is based on analogies to quantum optics (for details, we refer the reader to Refs. [3, 5, 6]). Briefly, the main features of interest to us are:

a) The pion field is split up into a “coherent” and a “chaotic” part:

$$\Pi(x) = \Pi_0(x) + \Pi_{ch}(x).$$
b) The ratio of chaotically created pions to the total number of pions is embodied in the “chaoticity parameter”,

\[ p = \frac{\langle n_{ch} \rangle}{\langle n_{ch} + n_0 \rangle}. \]  

(4)

c) Much of the dynamics is contained within the normalized field correlator,

\[ d_{ij} \equiv \frac{\langle \Pi_{\text{ch}}(k_i) \Pi_{\text{ch}}(k_j) \rangle}{\left[ \langle \Pi_{\text{ch}}(k_i) \Pi_{\text{ch}}(k_i) \rangle \langle \Pi_{\text{ch}}(k_j) \Pi_{\text{ch}}(k_j) \rangle \right]^{1/2}}, \]  

(5)

(where \( \langle A \rangle = \text{Tr}(\rho A) \) is the ensemble average over states, weighted by the density matrix \( \rho \) which is closely related to the Fourier transform of the chaotic field source functions.

d) Working out two-point, three-point and higher-order averages, this theory of quantum statistics predicts unambiguously the normalized moments and cumulants of all orders.

When relative phases are neglected, the first three “QS cumulants” of interest are

\[ k_2 \equiv \frac{C_2}{\rho_1 \otimes \rho_1} = 2p(1 - p)d_{12} + p^2d_{12}^2, \]  

(6)

\[ k_3 \equiv \frac{C_3}{\rho_1 \otimes \rho_1 \otimes \rho_1} = 2p^2(1 - p)[d_{12}d_{23} + d_{23}d_{31} + d_{31}d_{12}] + 2p^3d_{12}d_{23}d_{31}, \]  

(7)

\[ k_4 \equiv \frac{C_4}{\rho_1 \otimes \rho_1 \otimes \rho_1 \otimes \rho_1} = \sum_{(24)} p^3(1 - p)d_{12}d_{23}d_{34} \]

\[ + 2p^4[d_{12}d_{23}d_{34}d_{41} + d_{12}d_{24}d_{34}d_{31} + d_{14}d_{42}d_{23}d_{31}], \]  

(8)

where the brackets under the sum indicate the number of permutations. These cumulants are functions of 1, 3 and 6 pair variables \( q_{ij} \) respectively. Note the combination of “ring”– and “snake”–like structures in the combinatorics.

While in principle calculable from a given density matrix, the correlator is usually parametrized in a plausible and/or convenient way. Specifically, the parametrizations we shall be testing are, in terms of the 4-momentum difference correlators \( d_{ij} = d(q_{ij}) \),

\[ \text{gaussian:} \quad d_{ij} = \exp(-r^2q_{ij}^2), \]  

(9)

\[ \text{exponential:} \quad d_{ij} = \exp(-rq_{ij}), \]  

(10)

\[ \text{power law:} \quad d_{ij} = q_{ij}^{-\alpha}. \]  

(11)

3 UA1 data

We have measured second- and third-order normalized cumulants using a sample of about 160,000 minimum bias events taken with the UA1 detector for \( p\bar{p} \) collisions at 630 GeV/c. For details of the detector and other experimental information regarding particle pairs, the reader is referred to Ref. [11] The following cuts were applied to this sample: \(-3 \leq \eta \leq 3, p_{\perp} \geq 0.15 \text{ GeV}, 45^\circ \leq \phi \leq 135^\circ \) (by means of this “good azimuth” cut, our statistics were reduced considerably but acceptance corrections due to the “holes” in the UA1 detector at small \( \phi \) were thereby avoided). Cumulants were calculated for positives and negatives separately and then averaged to yield “like-sign” values. No Coulomb corrections were applied.
4 Fits to the second order cumulant

In Figure 1, we show the second-order like-sign differential cumulant $\Delta K_2 = (\int \rho_2/ \int \rho_1 \otimes \rho_1) - 1$, where numerator and denominator are integrals over bins spaced logarithmically between $q = 1$ GeV and 20 MeV. Fits to the data were performed using the three parametrizations (9)–(11), either in the full QS form (6) or in a simple form $\Delta K_2 = pd$. All fits shown include, besides the free parameters $p$ and $r$ (or $\alpha$), an additive constant as free parameter. These additive constants, necessary because UA1 data is non-poisonious in nature, will be commented on further below. Best fit parameter values obtained were $p = 0.66 \pm 0.07$, $r = 1.16 \pm 0.05$ fm for the QS exponential and $p = 0.05 \pm 0.01$, $\alpha = 0.64 \pm 0.05$ for the QS power law. Goodness-of-fits were $\chi^2/\text{NDF} = 1.3$, 4.2 and 11.5 for QS power, exponential and gaussian respectively.

To check its influence on fit values, the data point at smallest $q$, being of doubtful quality, was excluded; the resulting fit values do not differ much from the full fit. We note that the QS exponential misses the last three points (apart from the point at $q = 20$ MeV) and that the power laws (single or QS) appear to do the best job. The gaussian fits are too bad to warrant further attention and will be neglected from here on. Similar conclusions were reached by UA1 earlier [14].

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It should be remarked that previous work has shown the utility of using logarithmic rather than linear binning: much of what is interesting in correlations happens at small $q$, and this region is probed better by using logarithmic bins.
5 Consistency checks with third order cumulants

As stressed already, we are interested not so much in obtaining numerical values for parameters but rather in using these to check the theoretical formulae (6)–(11) for consistency with the data. Three separate checks were performed: two based on approximations, the third involving a novel approach tentatively called “theory⊗experiment” which will be explained in Section 5.2.

5.1 Approximate checks

Third-order correlations and cumulants are functions of the three pair variables \((q_{12}, q_{23}, q_{31})\), so that the question arises how best to view such three-dimensional correlations. The easiest projection involves setting the pair variables equal \([3, 5, 15]\), \(q_{12} = q_{23} = q_{31} \equiv q\), so that Eq. (7) reduces to the simple formula

\[
k_3(q) = 6p^2(1 - p)d^2 + 2p^3d^3.
\]

(12)

Experimentally, however, the prescription of three mutually equal \(q\)’s is so restrictive as to make measurement impossible. The usual way out \([15, 16]\) has been to include all triplets whose mean of the three \(q_{ij}\)’s is equal to a given \(q\) while still applying Eq. (12) (the effect of this approximation has, to our knowledge, not been checked).

The second approximation involves setting \(p \equiv 1\) without restricting the pair variables. Fortuitously, Eqs. (7), (10) then become \(k_3 = 2 \exp[-r(q_{12} + q_{23} + q_{31})]\), so that a simple change to the “GHP sum” variable \(S = (q_{12} + q_{23} + q_{31})\) does the trick.

In Figure 2, we show the UA1 third-order cumulant \(\Delta K_3\) as a function of the GHP sum variable \(S\). The lower line represents the first approximation, i.e. formula (12) using the exponential parametrization (10) and best-fit values from \(\Delta K_2\) plus an arbitrary additive constant. (Similar approximations using the gaussian form (9) with the variable \(Q\) \(\equiv \sqrt{q_{12}^2 + q_{23}^2 + q_{31}^2}\) and equal pair \(q\)’s have been used before \([16]\).) The upper line, representing the second approximate check, was calculated by first fitting \(\Delta K_2\) with \(p=1\) and an QS exponential for \(d\) to obtain \(r = 0.89 \pm 0.02\) fm (not shown) and then importing this value into \(k_3(p=1) = 2 \exp(-rS)\).

We see that, in both cases, the theoretical curves lie well below the \(\Delta K_3\) data. Even an arbitrary shift by an additive constant does not improve the match because of the different shape of the curves as compared to the data points.

5.2 The “theory⊗experiment” method

The approximate consistency checks performed above are unsatisfactory for two reasons: first, because they rely on simplifications of the formulae which may be unwarranted, second, because they are suitable only for the exponential parametrization (or, using \(Q\), for the gaussian equivalent). As shown above, however, the data for \(\Delta K_2\), while not

\[\text{Note that the linear sum } S \text{ is quite distinct from the pythagorean sum variable } Q.\]
Figure 2: Approximate test predictions, compared to the third-order UA1 cumulant using GHP sum topology.

Figure 3: Third-order GHP max and GHP sum cumulants, together with theory\times experiment predictions from QS theory and parameter values from $\Delta K_2$. Filled circles represent UA1 data, triangles are predictions based on the QS power-law parametrization; squares are QS exponential predictions.
excluding an exponential form, would seem to prefer the power law — and the power law cannot be handled by these approximations. A seemingly better methodology emerges, surprisingly, from some considerations about normalization.

Theory and theorists usually work with infinitesimally differential normalized quantities; for example, the second-order normalized cumulant is often written down as

$$ R(\vec{k}_1, \vec{k}_2) = \frac{\rho_2(\vec{k}_1, \vec{k}_2)}{\rho_1(\vec{k}_1)\rho_1(\vec{k}_2)} - 1 = \frac{C_2(\vec{k}_1, \vec{k}_2)}{\rho_1(\vec{k}_1)\rho_1(\vec{k}_2)} $$

(13)

which is (implicitly) fully differential in the momenta $\vec{k}_1, \vec{k}_2$. Similarly, the normalized theoretical cumulants $k_i = C_i/\rho_1 \otimes \cdots \otimes \rho_1$ used above assume essentially perfect measurement accuracy and infinite statistics.

Experimentally, one can never measure fully differential quantities; rather, the numerator and denominator are averaged over some bin of finite size $\Omega$ (however small) before the ratio is taken; for example

$$ \Delta K_2(\Omega) = \frac{\int_{\Omega} C_2(q) \, dq}{\int_{\Omega} \rho_1 \otimes \rho_1(q) \, dq}, $$

(14)

which approaches the theoretical cumulant $k_2(q)$ only in the limit $\Omega \to 0$.

This observation can be converted into an exact prescription for folding a given theoretical normalized quantity with experimentally measured one-particle distributions. For simplicity, we take second order quantities as an example. Since trivially $C_2(q) = k_2(q) \rho_1 \otimes \rho_1(q)$, we can take $k_2$ from theory, $\rho_1 \otimes \rho_1$ from experiment and write exactly

$$ \Delta K_2(\Omega) = \frac{\int_{\Omega} k_2^{th}(q) \rho_1 \otimes \rho_1^{expt}(q) \, dq}{\int_{\Omega} \rho_1 \otimes \rho_1^{expt}(q) \, dq} = \frac{\int_{\Omega} C_2^{th \otimes expt}(q) \, dq}{\int_{\Omega} \rho_1 \otimes \rho_1^{expt}(q) \, dq}. $$

(15)

Correlation integral theory prescribes that

$$ \rho_1 \otimes \rho_1^{expt}(q) = \left\langle \sum_{i,j} \delta[q - Q^{ab}_{ij}] \right\rangle_b^a, $$

(16)

where $Q^{ab}_{ij} = [(\vec{p}_i)^a - (\vec{p}_j)^b]^2 - (E_i^a - E_j^b)^2]^{1/2}$ is the four-momentum difference between two tracks $i$ and $j$ taken from different events $a$ and $b$. Taking, for example, the QS cumulant (6) and the exponential parametrization (10), this leads to

$$ C_2^{th \otimes expt}(q) = \left\langle \sum_{i,j} \delta[q - Q^{ab}_{ij}][2p(1-p) \exp(-rQ^{ab}_{ij}) + p^2 \exp(-2rQ^{ab}_{ij})] \right\rangle_b^a, $$

(17)

which can be binned in $q$ or otherwise integrated. In passing, we observe that Eq. (15) reduces to the theoretical $k_2$ for infinitesimal $\Omega$ or for constant $\rho_1 \otimes \rho_1$ as required.

Clearly, this can be generalized to all possible moments and cumulants, independently of variable or integration topology. The procedure exemplified by Eq. (17) and its generalizations amounts to a Monte Carlo integration of a theoretical correlation function sampled
according to the experimental uncorrelated one-particle distribution; for this reason, we like to call it by the diminutive “Monte Karli” or “MK” for short. MK can, of course, be implemented only for fixed numerical values of the theoretical parameters, in this case $p$ and $r$. These must be determined either by more naive fitting methods (and then checked for consistency) or by a very cumbersome fitting procedure using the full event sample many times over.

In Figure 3, the results of implementing the MK prescription are shown. Besides the GHP sum topology used in (b), we show in (a) a separate analysis using the “GHP max” topology [10], which bins triplets according to the largest of the three variables, $\max(q_{12}, q_{23}, q_{31})$. Fit parameter values used for the respective power law and exponential MK points were taken from the naive QS fit to $\Delta K^2$ of Figure 1. (The consistency of this procedure was checked by inserting these parameter values back into the MK formulae for $\Delta K^2$ and finding agreement between UA1 data and MK predictions.) Again, all MK points shown are determined only up to an additive constant, so that the curves may be shifted up and down. It is again clear, though, that the shape of third-order cumulant data measured differs appreciably from that predicted by the QS formulae and parameter values from $\Delta K^2$. This conclusion holds independently of the topology used and of the functional form taken for $d$.

6 Discussion

Concerning the fits to $\Delta K^2$, we have concluded that the gaussian parametrization $d_{ij} = \exp(-r^2 q_{ij}^2)$ is quite unsuitable, while the exponential is better but not good. The best fit was obtained using either a simple or QS (double) power law. This confirms earlier results [14][17]. The fits were reasonably stable even when excluding the point at smallest $q$, so that the effect is not due to this last point.

Parameter values obtained from fits $\Delta K^2$ were then applied to third-order cumulant data in three different checks. Both the two approximations as well as the exact theory$\otimes$experiment (Monte Karli) prescription yielded predictions that did not match the data. The tests performed in this paper, namely checking three specific parametrizations (gaussian, exponential and power-law) within one specific variable $q$ for consistency between $\Delta K^2$ and $\Delta K^3$ appear to indicate that, under these specific conditions, the theory is contradicted by the data.

It should be clear, though, that this conclusion can at this stage be preliminary and limited in scope only, for the following reasons:

- The data shown is preliminary only and will have to await further checks such as acceptance corrections, full-azimuth studies, sensitivity to binning, etc.

- The most important caveat relates to the structure of the overall multiplicity distribution. The fact that UA1 data is not poissonian in nature [18] can be seen immediately at large $q$ where $\Delta K^2$ converges not to zero (as a poissonian cumulant would) but to $\approx 0.4$. The same holds for $\Delta K^3$. Theories, however, are almost universally based on an overall poissonian: as can be easily verified from Eqs. (8)–(8),
all cumulants tend to zero for large $q$. The policy followed here, namely reconciling poissonian theory with non-poissonian data by means of an additive constant in the cumulants, is a sensible but hitherto poorly-understood first step. The question of handling cumulants more adequately within a non-poissonian overall multiplicity distribution is presently being considered [19]. We also hope that our results may goad theorists into more careful consideration of their work with respect to the implicit poissonian normalization used in most theories. See also Ref. [20].

- Closely related to these additive constants is the question of correct normalization. Traditional lore in second order divides the density correlation function (moment) $\Delta F_2$ by an additional normalization factor $f$, taken as the moment at some large value of $q$. An alternative methods creates as many background pairs as necessary to achieve the limit of unity for $\Delta F_2$. While the third order moment can similarly be normalized to unity, the prescription fails for third order cumulants. A brief scan of the literature on third-order cumulants reveals that no adjustments were made for possible non-poissonian multiplicity structure. [3, 4, 5, 6, 9, 10, 11, 15, 21, 22, 25, 26, 28, 29] Finally, one may mention possible changes to the present application of the theory such as inclusion of relative phases in the correlators, possible non-gaussian source currents, modelling the momentum dependence [6] of $p$, variable transformations [26] in $d$ and so on.

Beyond these caveats, the following points are of relevance to the interpretation of our results:

- No corrections for Coulomb repulsion [1] were included. We could argue that the same Coulomb effects that might shift $\Delta K_2$ data upward would increase $\Delta K_3$ data even more, since there are three pairs involved rather than one. Even more convincing is the fact that $\Delta K_3$ data rises more strongly than theoretical predictions even for large $q$ (several hundred MeV) where Coulomb repulsion is not expected to be important.

- Strictly speaking, the good power-law fit to $\Delta K_2$ in itself is inconsistent: QS theory requires $\lim_{q \to 0} d(q) = 1$, while the power-law parametrization diverges. Attempts to explain this in terms of variable transformations [26] or source size distributions [27, 28] may therefore provide useful starting points in explaining the discrepancies in $\Delta K_3$.

- Track mismatching can lead to strong correlation effects because the reconstruction program may split a single track into a closely correlated pair. A great deal of effort in early experimental intermittency studies went into creating clean “split-track-finding” algorithms [29] and these are included in our analysis. We have checked through additional small-$q$ cuts that mismatching does not appear to explain our strong rises in the cumulants.

- The UA1 sample consists of $\sim 15\%$ kaons and protons which cannot be distinguished from pions. The effect that these would have on $\Delta K_3$ is unclear.
• Resonances are known to increase $\Delta K_2$ at small $q$, the main effect in second order deriving from interference between “direct” pions and resonance-decay products. How and whether resonances would contribute to like-sign cumulants in third order (and for values of $q$ of several hundred MeV shown in $\Delta K_3$) is still quite mysterious.

• At this point, one could wonder whether it is wise to even attempt to eliminate resonances from hadron-hadron collision data: apart from the theoretical and technical difficulties, what dynamical information does the typical “size” $r < 1$ fm of a hypothetical “source” contain that is more important than a cascade structure containing resonances whose existence is beyond doubt? If the “source” is scarcely larger than a nucleon, then how can one speak of incoherent or even classical production of two pions? And if one eliminates long-lived resonances, then one would presumably still be left with the short-lived ones rather than the holy grail of an abstract quantum mechanical “source”.

The results of the present paper may appear, at first sight, to contradict the conclusion \[\footnote{We have also looked at like-sign correlations resulting from resonance decay chains such as ($\eta' \rightarrow \eta \pi^+ \pi^-$); ($\eta \rightarrow \pi^+ \pi^- \pi^0$); using PYTHIA with the Bose-Einstein routines switched off, we find no significant like-sign correlations in PYTHIA from resonances alone.}], based on an earlier UA1 paper \[\footnote{We have also looked at like-sign correlations resulting from resonance decay chains such as ($\eta' \rightarrow \eta \pi^+ \pi^-$); ($\eta \rightarrow \pi^+ \pi^- \pi^0$); using PYTHIA with the Bose-Einstein routines switched off, we find no significant like-sign correlations in PYTHIA from resonances alone.}], that QS theory was compatible with higher-order moments. The apparent discrepancy is explained by pointing out that 1) measurement techniques have improved considerably since then, 2) these techniques have permitted the present direct measurements of cumulants, which are considerably more sensitive than moments, and 3) even for these moment fits \[\footnote{We have also looked at like-sign correlations resulting from resonance decay chains such as ($\eta' \rightarrow \eta \pi^+ \pi^-$); ($\eta \rightarrow \pi^+ \pi^- \pi^0$); using PYTHIA with the Bose-Einstein routines switched off, we find no significant like-sign correlations in PYTHIA from resonances alone.], the radii were not quite constant but showed a systematic increase.

Bose-Einstein correlation measurements with a view to extracting source parameters are by now well-established in hadronic and heavy ion phenomenology. Our intention here was to show that consistency checks between cumulants of different orders might be a second route to learning something about the system: if by this method a given theory can be tested already on a qualitative rather than quantitative basis, then opportunities for feedback and improvement of such theories may expand.

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