The Odd Meanderings of the IMF Across Cosmic Time

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Abstract. It is difficult to reconcile the observed evolution of the star formation rate versus stellar mass (SFR-M*) relation with expectations from current hierarchical galaxy formation models. The observed SFR-M*, relation shows a rapid rise in SFR(M*) from z = 0 → 2, and then a surprisingly lack of amplitude evolution out to z ∼ 6+. Hierarchical models of galaxy formation match this trend qualitatively but not quantitatively, with a maximum discrepancy of ∼×3 in SFR at z ∼ 2. One explanation, albeit radical, is that the IMF becomes modestly weighted towards massive stars out to z ∼ 2, and then evolves back towards its present-day form by z ∼ 4 or so. We observe that this redshift trend mimics that of the cosmic fraction of obscured star formation, perhaps hinting at a physical connection. Such IMF evolution would concurrently go towards explaining persistent discrepancies between integrated measures of star formation and present-day stellar mass or cosmic colors.

1. Introduction

There is currently much debate on whether the stellar initial mass function (IMF) is everywhere and everywhen invariant, as highlighted by this excellent conference. Direct evidence for IMF variations are scant and controversial (Bastian et al. 2010, and contributions to these proceedings by Meurer, Lee, and Hoversten). One possibility is that the cluster-intrinsic IMF is invariant, but sampling and/or truncation effects convolved with a varying cluster mass distribution yields a varying IMF when averaged over entire galaxies (see e.g. contributions on the IGIMF theory). Theoretically it is difficult to envision that, despite enormous variations in the physical conditions of star formation, the IMF somehow conspires to remain completely invariant (Kroupa 2002). But the sense in which the IMF should vary is not well understood, and hence more empirical constraints are needed.

Recently, a cosmological approach to constraining the IMF has gained traction. Here, the evolutionary properties of galaxies are studied to test whether they are consistent with an invariant IMF. Early efforts involved comparing the observed integral of cosmic star formation (which traces massive star evolution) with the present-day stellar mass density (which traces ∼1M⊙ stars). Such studies typically favored a cosmically-averaged IMF that was weighted towards more massive stars than the present-day Milky Way disk IMF (Madau et al. 1998, Baldry & Glazebrook 2003, Wilkins et al. 2008). Other approaches include studying the color evolution of passive galaxies (van Dokkum 2008), and applying additional constraints from extragalactic background light measures (Fardal et al. 2007). In general, such studies again favor an IMF more weighted towards massive stars to satisfy all constraints. That said, a well-known theorem (Bastian et al. 2010) states that any problem in galaxy formation can be resolved through a suitable
choice of IMF, and hence such arguments are often dismissed as the last resort of a
scoundrel. This fails to deter us theorists, who have been called far worse.

In Dáve (2008), we argued that the evolution of the star formation rate-stellar mass
(SFR−M*) relation may be suggestive of a time-varying IMF for typical star-forming
galaxies. At that time, reliable observations of SFR−M* only extended out to z ∼ 2.
Improved data has now constrained SFR−M* to z ∼ 7 (Stark et al. 2009; Labbé et al.
2010; González et al. 2010), providing an opportunity to extend such arguments to very
high redshifts. Concurrently, Herchel has provided improved bolometric constraints on
star formation in galaxies out to z ∼ 2, mitigating some of the systematic uncertainties
that may have affected earlier works. Theoretical work has also progressed, as toy
models have been developed that can explain SFR−M* evolution (Bouché et al. 2010),
at the cost of introducing arbitrary modifications to galaxy star formation histories.
With such fervent recent activity, the time is right to revisit our previous arguments.
This is the goal of these proceedings. The end result, we argue, is that the evidence still
favors some discrepancy in the SFR vs. M* evolution in galaxies that could be resolved
by a time-varying IMF, though to do so requires a modestly varying IMF that evolves
non-monotonically with redshift.

2. The SFR−M*, Relation

The SFR−M*, relation has recently received much attention as an important barometer
of galaxy evolution theory. Models, particularly cosmological hydrodynamic simula-
tions, have long predicted that there should be a tight and slightly sub-linear relation
between these quantities (e.g. Dáve et al. 2000; Finlator et al. 2006; Dáve et al. 2006)
that evolves slowly upwards with redshift (Dáve 2008). Observations over the last five
years have confirmed these trends out to z ∼ 1 (Noeske et al. 2007; Elbaz et al. 2007),
z ∼ 2 (Daddi et al. 2007), and beyond (González et al. 2010). In models, this trend
arises because cold and smooth accretion is the dominant accretion path for fueling
star formation at all epochs (Murali et al. 2002; Kereš et al. 2005; Dekel et al. 2009);
the tightness of SFR−M* empirically constrains the contribution from starbursts (e.g.
Noeske et al. 2007). The broad agreement between theory and observations is a key
success of current galaxy formation models.

In detail, however, it has been noted that SFR−M* predicted in models (either hy-
drodynamic or semi-analytic) fails to match the observed amplitude evolution (Elbaz et al.
2007; Daddi et al. 2007; Dáve 2008). This is shown in Figure 1 which shows the evo-
lution of the SFR−M* relation at a fiducial stellar mass of M* = 10^{10} M_\odot from two
cosmological hydrodynamic simulations (details of which are in Oppenheimer et al.
2010). The green curve shows results from a simulation including outflows with scal-
ings as expected for momentum-driven winds; this outflow model has proved successful
at matching a wide range of galaxy and IGM data from z ∼ 0–6. The blue curve shows
a simulation with no galactic outflows included, which strongly overpredicts the global
stellar mass formed (Oppenheimer et al. 2010), but nevertheless produces a SFR−M*,
relation that is not markedly different from the strong outflow case. This illustrates
that despite dramatic variations in predictions owing to different feedback models, the
SFR−M* relation is relatively insensitive to feedback (Dáve 2008; Dutton 2010). The
basic reason is that feedback suppresses star formation which likewise suppresses stel-
lar mass growth, moving galaxies downwards along the (nearly linear) SFR−M* rela-
tion.
The IMF Across Cosmic Time

Figure 1. The amplitude evolution of the SFR\(-M_\ast\) relation at $M_\ast = 10^{10} M_\odot$ from $z = 0 - 7$. Observations (circles) are shown from Elbaz et al. (2007) at $z = 0, 1$, and from González et al. (2010) at $z \geq 2$. Dashed lines indicate $\pm 0.3$ dex spread around the mean relation. Red dot roughly indicates the downward correction of UV-derived SFRs based on Herschel data. Simulation results are shown for our favored run employing momentum-driven winds (green solid) and a run with no galactic outflows (blue dotted).

Observations from Elbaz et al. (2007, at $z = 0, 1$) and González et al. (2010, $z \geq 2$) are shown as circles, and a typical 1σ observed scatter around the relation of $\pm 0.3$ dex is shown by the dashed lines. Note that this is not the statistical or systematic uncertainties in $M_\ast$ or SFR, which are typically smaller than 0.3 dex at low-$z$ but could be significantly larger at high-$z$. As can be seen, the observed SFR\(-M_\ast\) relation amplitude matches well at $z \sim 0$, and then the observed SFRs are increasingly larger going out to $z \sim 2$. At this point, there is an abrupt change in the trend, and the observed amplitude is essentially invariant out to $z \sim 7$. The simulations broadly predict a similar trend, but do not reproduce the abrupt change at $z \sim 2$.

The discrepancies are modest, especially considering the uncertainties involved, maximizing at $\sim \times 3$ at $z \sim 2$. But what is particularly puzzling is that at $z \sim 2$, the observed star formation rates can exceed the accretion rate into galaxy halos, which is difficult to accomodate in current models that favor continual gas supply (“cold streams”) to fuel star formation. The implication is that galaxies must store up large reservoirs of gas, and then something triggers its rapid conversion into stars. If star formation proceeds exponentially, then galaxies quickly move upwards along the SFR\(-M_\ast\) relation Maraston et al. (2010), preserving the (nearly) linear slope at a fixed amplitude. Current hierarchical galaxy formation models do not yield this behavior naturally. Bouché et al. (2010) noted that an ad hoc way to accomplish this is to explicitly prevent all star formation in halos with $M < 10^{11} M_\odot$, which accumulates a large reservoir that is then rapidly consumed once the threshold halo mass is crossed. But this is difficult to arrange physically, and the lack of any discernible transition in the galaxy population around these halo masses (as compared to, say, around $10^{12} M_\odot$) argues against this scenario. Hence matching the $z \sim 2+$ SFR\(-M_\ast$ relation appears to require a fundamen-
An alternative possibility is that there are significant systematics in observational measures of SFR and/or $M_*$. As discussed in Dave (2008), it is easier to accommodate overestimated SFR’s than underestimated $M_*$, since stellar mass is cumulative and there are lower-$z$ constraints that must be satisfied. The Daddi et al. (2007) $z \sim 2$ data rely on extinction-corrected UV SFRs, which recent Herschel data suggest overestimate the true (bolometric) SFRs measured from far-IR data by 50% or perhaps up to $\sim \times 2$. Such a reduction is indicated by the red point on Figure 1. This would alleviate the discrepancy somewhat but not completely, and in a sense makes it harder to reconcile since a key systematic has been mitigated. We note that 24µ-based SFRs have been shown to be very poor at $z \sim 2$ since PAH emission that dominates the rest-frame 8µ emission is poorly understood (e.g. Kelson 2010), but such SFRs are not the basis for the SFR–$M_*$ observations shown. Nevertheless, better data is required to pin all the systematics down, and a proper accounting may yet remove the discrepancy altogether.

Finally, Figure 1 shows that feedback does have a noticeable impact on the shape of SFR–$M_*$ evolution. In particular, the momentum-driven wind scalings suppress star formation in smaller halos more, resulting in less early star formation in better agreement with data. One might envision that an even stronger feedback model could completely reconcile the theory and observations. However, the model of Bouché et al. (2010) indicates that the feedback required is quite extreme, essentially suppressing all star formation even in fairly sizeable halos. Our experience is that models with significantly stronger feedback have substantial difficulties producing enough early star formation to match data on $z \sim 6$+ galaxies, the mass-metallicity relation, and the metallicity of the IGM at early epochs. Perhaps some compromise will eventually be found between theory and observations, and certainly the downward revision of observed $z \sim 2$ SFRs goes towards this. This probably remains the most likely solution to the SFR–$M_*$ discrepancy, but it is not the most interesting one.

3. **A Time-Varying IMF?**

A more interesting and far-reaching systematic that could lower inferred SFRs is if the IMF is weighted towards more massive stars at high redshift, either by being top-heavy or bottom-light. Here I use top-heavy to indicate a changing upper-end slope, and bottom-light to refer to the suppression of low-mass stars. If this is to be the solution, then the IMF must evolve with redshift towards more top-heavy/bottom-light at $z \sim 2$, and then reverse its trend back towards a normal IMF by $z \sim 4$. At $z \gtrsim 5$ the systematic effects in $M_*$ and SFR are currently sufficiently large that it would be premature to infer anything from the small discrepancies at those redshifts.

One possible form of IMF variation, highlighted in Dave (2008) and van Dokkum (2008), is that the IMF characteristic mass evolves with redshift. For a Kroupa IMF, i.e. $\frac{dN}{d\log M} \propto M^{-2}$ for $M < M_{\text{IMF}}$ and $\frac{dN}{d\log M} \propto M^{-1.3}$ for $M > M_{\text{IMF}}$, Dave (2008) showed that one evolutionary form that reconciles SFR–$M_*$ evolution out to $z \sim 2$ is $\dot{M}_{\text{IMF}} = 0.5(1+z)^{3-0.75z} M_\odot$. However, beyond $z \sim 2$ the IMF must evolve back towards the present-day form, and moreover Herschel data has shifted the $z \sim 2$ point downwards. A general form that would accommodate these trends is

$$\dot{M}_{\text{IMF}} = 0.5(1+z)^{3-0.75z} M_\odot \quad (1)$$
Figure 2. The IMF characteristic mass $\dot{M}_{\text{IMF}}$ in various IMF evolutionary forms.

The solid black line shows the form of $\dot{M}_{\text{IMF}}$ from $z = 0 \rightarrow 2$. Recent results have suggested the IMF returns to its present-day form by $z \sim 4$, motivating a different evolutionary form. The green dashed line, with $\dot{M}_{\text{IMF}} = 0.5(1+z)^{3-0.75} \, M_\odot$, follows the old form out to $z \sim 1.5$, and then returns towards the present-day IMF by $z = 4$, broadly accommodating current constraints.

from $z = 0 \rightarrow 4$, remaining at $0.5 M_\odot$ at $z \geq 4$. As shown in Figure 2, this roughly mimics the $\dot{M}_{\text{IMF}}$ evolution of $\dot{M}_{\text{IMF}}$ from $z = 0 \rightarrow 2$, and then turns over, reducing the characteristic mass by almost a factor of two at $z \sim 2$, and eventually returns to the present-day value at $z = 4$. This is purely an eyeball estimate, and furthermore this choice of redshift dependence is merely a convenient parameterization; there is no physics in it. In reality, the IMF should be tied to some physical property of galaxies that evolves with redshift, such as star formation rate or surface density.

The SFR--$M_*$ relation does not constrain the detailed form of IMF variation. Other plausible ways to reconcile the SFR--$M_*$ would be to make the upper-end slope of the IMF more shallow with time (e.g. Wilkins et al. 2008). In detailed studies of lensed Lyman Break ($z \sim 2 - 3$) galaxies, the upper end of the IMF does not appear to be substantially different (Quider et al. 2009) though current constraints may not rule out mild changes that may resolve the SFR--$M_*$ discrepancy. Also, constraints on the IMF in $z \sim 2$ sub-millimeter galaxies indicate that it is consistent with a Chabrier (2003) IMF, perhaps slightly favoring a more top-heavy one, but clearly inconsistent with dramatic departures from a Salpeter slope as postulated in e.g. the semi-analytic models of Baugh et al. (2005). On the other hand, the ratio of H$\alpha$ to UV emission in local star-forming galaxies correlates strongly with its star formation rate and/or surface density (Meurer et al. 2009; Lee et al. 2009), for which one interpretation is that the IMF has a shallower upper-end slope where there is more concentrated star formation. Since $z \sim 2 - 3$ galaxies ubiquitously show more concentrated star formation, this could empirically imply a top-heavy IMF in such systems. It remains to be seen if simply applying constraints from local galaxies would yield the required evolution owing purely to the size and star formation rate evolution of typical galaxies, but this should be investigated.
An IMF that returns to normal at very high redshifts is also preferred by available constraints at those epochs. For instance, current models assuming a normal IMF are able to form enough stellar mass in very early \((z \sim 6+)\) objects as observed (e.g. Finlator et al. 2010), but a significantly bottom-light or top-heavy IMF would create difficulties. Another early-universe constraint comes from old globular clusters formed at high-\(z\), which are consistent with a present-day IMF. Hence there is a growing consensus that the IMF in the early universe is actually quite similar to today’s. Of course, Population III (i.e. very metal poor) stars are expected to have a quite top-heavy IMF, but it is unlikely that these contribute significantly to early galaxies observed to date (e.g. Davé et al. 2006; Finlator et al. 2010).

The central question is, what could cause the IMF to depart away from the present-day IMF in such a fashion? The non-monotonic evolution is particularly challenging to understand. This is a more a question for star formation theorists, but there is one possible intriguing connection: The required IMF evolution roughly mimics the cosmic fraction of obscured star formation as determined by Bouwens et al. (2009, see their Figure 11), which peaks at \(z \sim 2 – 3\) and falls off to higher and lower redshifts. Hence if there was something about an obscured mode of star formation that systematically caused the IMF to be weighted towards massive stars, this may yield the desired trend. A possible local test would then be to look in nearby ULIRGs, where some controversial claims of a top-heavy IMF have been made (Rieke et al. 1980, 1993). Obviously this connection is highly tenuous, but may provide an avenue for further exploration.

4. Cosmically-Averaged Star Formation

The IMF variation required to reconcile theory and observations of SFR\(-M_*\) also broadly reconciles the observed cosmic star formation and stellar mass growth histories (Davé 2008; Wilkins et al. 2008), and extragalactic background light constraints (Fardal et al. 2007). Clearly these quantities are all interrelated, so it isn’t terribly surprising that the same IMF fixes all of them, but it provides a useful consistency check. Another approach (see Wilkins, these proceedings) is to use the cosmic SED to constrain the IMF; once again, this shows a mismatch between the integrated cosmic star formation history and the present-day mean cosmic color in the analogous sense that there is too little near-IR light relative to the UV/optical light together with expectations from the observed cosmic star formation history. Present-day integrated measures do not necessarily require an evolving IMF, but rather only a cosmically-averaged IMF that differs from the local one. However, it would be puzzling if our locally-measured IMF was different than the (invariant) IMF over the rest of cosmic time.

Conversely, differential measures of stellar mass growth and star formation rate could imply an evolving IMF; and some claims to that end have been made (Wilkins et al. 2008; Davé 2008). However, such claims are controversial. A particularly strong argument was forwarded by Reddy & Steidel (2009, also see these proceedings), who noted that by making careful corrections for dust and incompleteness, it is possible to reconcile the global star formation rate and stellar mass density evolution at \(z \sim 2\). This would not counter arguments based on individual galaxies such as the SFR\(-M_*\) discrepancy, but would make observations of cosmic star formation and mass growth internally self-consistent at \(z \sim 2\).

In actuality, however, this argument ends up merely pushing the problem to lower redshifts. To see this, Reddy & Steidel (2009) notes that their incompleteness correc-
tions result in 57% of the present-day stellar mass already having been formed by \( z \sim 2 \). Conversely, current observations of the cosmic star formation history imply that only \( \sim 30\% \) of total cosmic star formation occurs prior to \( z = 2 \) (e.g. Hopkins & Beacom 2006; Fardal et al. 2007) — this is independent of assumed IMF, so long as it is time-invariant. Hence the growth from \( z = 2 \rightarrow 0 \) of stellar mass does not match expectations from the observed star formation history, in the sense (once again) that star formation rates are measured to be too high. Hence one can “fix” the problem at \( z \sim 2 \) and have difficulty with evolution to \( z = 0 \), or one can “fix” the problem at \( z = 0 \) and then have difficulty reconciling \( z \sim 2 \) observations. This argues for systematics in SFR or \( M_* \) measures that change with redshift, be they related to the IMF or not.

5. Conclusion

There is still no definitive evidence for IMF variations across cosmic time. That said, the issues motivating serious consideration of such variations have yet to be fully resolved. New data on the SFR–\( M_* \) relation out to \( z \sim 7 \) suggests that, if IMF variations are to explain the discrepancy with theoretical expectations, deviations from the present-day IMF must maximize at \( z \sim 2 \) and then lessen to higher redshifts, returning to a standard IMF by \( z \sim 4–5 \). A possible form that would roughly yield this is a Kroupa IMF with a characteristic (turnover) mass evolving as \( M_{\text{IMF}} = 0.5(1 + z)^{0.75} M_\odot \).

While the required non-monotonic IMF evolution is not straightforward to understand physically, a quantity showing analogous behavior is the cosmic fraction of obscured star formation, hinting at a possible physical connection.

The form of the IMF variation remains difficult to pin down. The best (albeit controversial) evidence for local variation is in the form of a changing upper-end slope with star formation rate. The high star formation rates of high-\( z \) galaxies would then naturally result in a more top-heavy IMF. But direct observations of \( z \sim 2–3 \) galaxies disfavor significant changes in the upper-end IMF slope. Meanwhile, the evolutionary form suggested by Davé (2008) (or, similarly, by van Dokkum 2008) towards more bottom-light with redshift is more difficult to test directly. It is also possible that the intrinsic IMF is invariant, but the larger typical sizes of molecular clouds in high-\( z \) galaxies results in a galactic-averaged IMF that yields proportionally more massive stars. An IMF weighted towards massive stars at intermediate redshifts would also go towards resolving the persistent discrepancies between integrated measures of cosmic star formation history and present-day measures of stellar mass or cosmic light.

In the near future, surveys such as the Spitzer Extragalactic Deep Survey (PI G. Fazio) and the Hubble CANDELS survey (PIs S. Faber and H. Ferguson) will provide unprecedented multiwavelength samples of galaxies out to \( z \sim 6 \) and beyond. Although this will not necessarily pin down systematic effects in SFR or \( M_* \) measures, it will enable tests of different galaxy formation scenarios with greater statistical precision. For instance, the “reservoir” scenario where gas consumption proceeds exponentially in galaxies predicts a significantly different evolution to high-\( z \) of the stellar mass function and halo occupancy than hierarchical galaxy formation models. Hence these surveys will be critical for testing current galaxy formation models, thereby indirectly testing ideas such as IMF evolution. We look forward to engaging in such comparisons.

Acknowledgments. The author would like to thank Ben Oppenheimer, Kristian Finlator, Dušan Kereš, Neal Katz, Avishai Dekel, and Nicolas Bouché for helpful discussions. Also thanks to Lee et al. (2010) for a stimulating meeting!
References

Baldry, I., & Glazebrook, K. 2003, ApJ, 593, 258
Bastian, N., Covey, K. R., & Meyer, M. R. 2010, ARA&A in press, arXiv:1001:2965
Baugh, C. M., Lacey, C. G., Frenk, C. S., Granato, G. L., Silva, L., Bressan, A., Benson, A. J., & Cole, S. 2005, MNRAS, 356, 1191
Benson, A. J. 2010, Physics Reports
Bouché, N., Dekel, A., Genzel, R., Genel, S., Cresci, G., Frster Schreiber, N. M., Shapiro, K. L., Davies, R. L., & Tacconi, L. 2010, ApJ, 718, 1001
Bouwens, R. J., Illingworth, G. D., Franx, M., Chary, R.-R., Meurer, G. R., Conselice, C. J., Ford, H., Giavalisco, M., & van Dokkum, P. 2009, ApJ, 705, 936
Chabrier, G. 2003, PASP, 115, 763
Daddi, E., Alexander, D. M., Dickinson, M., Gilli, R., Renzini, A., Elbaz, D., Cimatti, A., Chary, R., Frayer, D., Bauer, F. E., Brandt, W. N., Giavalisco, M., Grogin, N. A., Huynh, M., Kurk, J., Mignoli, M., Morrison, G., Pope, A., & Ravindranath, S. 2007, A&A, 468, 33
Davé, R. 2008, MNRAS, 385, 147
Davé, R., Finlator, K. M., & Oppenheimer, B. D. 2006, MNRAS, 370, 273
Davé, R., Gardner, J., Hernquist, L., Katz, N., & Weinberg, D. 2000, ASPC, 200, 173
Dekel, A., Birnboim, Y., Engel, G., Freundlich, J., Goerdt, T., Mucuoglu, M., Neistein, E., Pichon, C., Teyssier, R., & Zinger, E. 2009, Nature, 457, 451
Dutton, F. C. D. A., Aaron A.; van den Bosch 2010, MNRAS, 405, 1690
Elbaz, D., Daddi, E., Le Borgne, D., Dickinson, M., Alexander, D. M., Chary, R.-R., Starck, J.-L., Brandt, W. N., Kitzbichler, M., MacDonald, E., Nonino, M., Popesso, P., Stern, D., & Vanzella, E. 2007, A&A, 468, 33
Fardal, M. A., Katz, N., Weinberg, D. H., & Davé, R. 2007, MNRAS, 379, 985
Finlator, K., Oppenheimer, B. D., & Davé, R. 2010, MNRAS
Finlator, K. M., Davé, R., Papovich, C., & Hernquist, L. 2006, ApJ, 639, 672
González, V., Labbé, I., Bouwens, R. J., Illingworth, G., Franx, M., Kriek, M., & Brammer, G. B. 2010, ApJ, 713, 115
Hopkins, A. M., & Beacom, J. F. 2006, ApJ, 651, 142
Kelson, B. P., Daniel D.; Holden 2010, ApJ, 713, 28
Kereš, D., Katz, N., Weinberg, D. H., & Davé, R. 2005, MNRAS, 363, 2
Kroupa, P. 2002, ASPC, 285, 86
Labbé, I., González, V., Bouwens, R. J., & et al. 2010, ApJL, 716
Lee, J., Seibert, M., Wyder, T., Neill, D., & Treyer, M. 2010, The Sedona Weekly, 20, 10
Lee, J. C., Gil de Paz, A., Tremonti, C., & et al. 2009, ApJ, 706, 599
Madau, P., Pozzetti, L., & Dickinson, M. 1998, ApJ, 498, 106
Maraston, C., Pforr, J., Renzini, A., Daddi, E., Dickinson, M., Cimatti, A., & Tonini, C. 2010, MNRAS
Meurer, G. R., Wong, O. I., Kim, J. H., & et al. 2009, ApJ, 695, 765
Murali, C., Katz, N., Hernquist, L., Weinberg, D. H., & Davé, R. 2002, ApJ, 571, 1
Noeske, K., Weiner, B. J., Faber, S. M., & et al. 2007, ApJL, 660
Oppenheimer, B. D., Davé, R., Kereš, D., Fardal, M., Katz, N., Kollmeier, J. A., & Weinberg, D. H. 2010, MNRAS, 406, 2325
Quider, A. M., Pettini, M., Shapley, A. E., & Steidel, C. C. 2009, MNRAS, 398, 1263
Reddy, N. A., & Steidel, C. C. 2009, ApJ, 692, 778
Rieke, G. H., Lebofsky, M. J., Thompson, R. I., Low, F. J., & Tokunaga, A. T. 1980, ApJ, 238, 24
Rieke, G. H., Loken, K., Rieke, M. J., & Tamblyn, P. 1993, ApJ, 412, 99
Stark, D., Ellis, R. S., Bunker, A., Bundy, K., Targett, T., Benson, A., & Lacy, M. 2009, ApJ, 697, 1493
van Dokkum, P. 2008, ApJ, 674, 29
Wilkins, S. M., Trentham, N., & Hopkins, A. M. 2008, MNRAS, 385, 687