Assessment of the vehicle vibration loading with taking into account the dynamic stiffness of the leaf spring

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Abstract. New multilink models of leaf spring suspensions take into account a number of important features. First, we are talking about the features of the kinematics of suspension elements in the design scheme of the vehicle. Their account is provided by application of the differential equations of large displacements of bodies and also modeling of a leaf spring in the form of the links, connected by joints. Secondly, the nonlinearity of the hysteresis dynamic characteristics of the suspension is taken into account, with the help of which the dynamic stiffness of the leaf spring is considered. The dynamic stiffness is depending from the dry friction between spring leaves and the amplitudes of cyclic oscillations with a random external road forcing. Thirdly, the accuracy of the installation scheme of the leaf spring in the vehicle suspension is considered. It takes into account the peculiarities of fixing the leaf spring and their impact on the change in stiffness.

1. Introduction
The elastic, hysteresis characteristics of the leaf springs in the vertical direction represents the fixation of the loading and unloading processes, obtained in the form of the dependence “applied force – deformation of the elastic element” [1].
Currently, the problem of formation of nonlinear dynamic characteristics of leaf spring suspensions, taking into account the kinematics of the leaf spring in the vehicle suspension and the influence of the features of the leaf spring attachment to change its stiffness, remains relevant. A complete solution of this problem is possible only on the basis of differential equations of large displacements of bodies [2–8]. This approach implies a precise description in these equations the relative positions of bodies and their the angular orientation.

2. The expedience of considering the dynamic stiffness of the leaf spring
The elastic characteristic of the leaf spring in the selected limits of deviations of the vertical load and deformations from the static equilibrium position can usually be approximated by linear sections (BC and DA) of the external branches of the loading and unloading of the leaf spring and the nonlinear sections (AB and CD) of the half-cycle curves, forming a closed hysteresis loop (figure 1). The points, lying on these elements of the loop, at different moments of time, determines the corresponding values of vertical elasticity and dry friction parameters in limits of each deformation cycle ABCD. Moreover, at points A and C of the deformation direction change, the spring leaves are blocked by the maximum possible friction force for this cycle, which provides the ultimate increase of the leaf spring stiffness.
Figure 1. The hysteresis characteristic of the vertical elastic of the leaf spring: $P_{st}$ – the vertical static load on the leaf spring; $P_{fr}$ – the force of dry (interleaf) friction in the leaf spring; $\delta_{st}$ – the position of static equilibrium; $\delta_{c}$ – the amplitude of the cyclic oscillations; $f$ – the nominal static deflection.

Sections $AB$ and $CD$ reflects the processes of gradual unblocking of leafs during loading and unloading of the leaf spring. They corresponds to significant changes of the vertical elastic force, occurring in the process of deformations at first of the ends of the root leafs, and then other spring leafs. Upon completion of unblocking all the leafs, the spring characteristic is determined by the $BC$ and $DA$ sections of external branches of the loading and unloading.

During the calculations, the spatial dynamic model of the vehicle usually includes an idealized linear characteristic of the leaf spring vertical stiffness. It is constructed as a midline between the branches of the loading and unloading of the leaf spring and takes into account the constant stiffness, determined by the inclination angle of the midline to the abscissa axis.

However, a more correct approach to assessing the vertical stiffness of the leaf spring is to take into account the non-constant equivalent dynamic vertical stiffness of the leaf spring as the tangent of the angle of inclination of the segment $AC$ to the abscissa axis.

The equivalent stiffness of the leaf spring increases with enhancement of the dry friction between the leafs and reducing the amplitude of the cyclic oscillations. In addition, the reduction of this amplitude entails a decrease in the dissipative work of the dry friction forces, the value of which corresponds to the area of the hysteresis loop.

As a result, at small oscillation amplitudes, when the vehicle moves on the dynamometer road, there is a significant decrease in the damping ability of the leaf spring. This is accompanied by an increase in the equivalent stiffness of leaf spring and natural oscillation frequencies of the sprung and unsprung vehicle masses.

3. Description of leaf spring characteristic

In the process of analytical description of the nonlinear leaf spring characteristics should be set:

- parameters of the external characteristics of the leaf spring, which will be limited to the cyclic deformations of the leaf spring;
- parameters of the half-cycle curves in the dependence of the elastic force from the deformation of leaf springs during a half-cycle of deformation (the description of these curves allows to close the hysteresis loop);
- the parameter of insensitivity of the leaf spring model (the value of the elastic force at the considered step of calculation remains unchanged, if the change of the current deformation was modulo less, than the value of the specified parameter).
The external characteristic of the leaf spring is represented by the branches of loading and unloading processes, each of which is a broken line without breaks, including several sections. The corresponding branch of the external leaf spring characteristic on each of the sections is given as follows:

\[ P(\delta) = C_{ji}\delta + P_{ji}, \]  

(1)

\( C_{ji} \) – the static stiffness of the leaf spring on the \( j \)-th section of the characteristic;  
\( \delta \) – the current vertical deflection of the leaf spring;  
\( P_{ji} \) – the force of dry (interleaf) friction in the leaf spring on the \( j \)-th section of the characteristic;  
\( C_{ji}, P_{ji} \) – constant values for the selected \( j \)-th section of the leaf spring characteristic. For unloading branch \( i = 1 \), for loading branch \( i = 2 \).

In equation (1), the parameters should be chosen in such a way, that a hysteresis loop of the external characteristic is formed, the width of which is determined by the formula:

\[ P(\delta) = (C_{j2} - C_{ji})\delta + (P_{j2} - P_{ji}) > 0. \]  

(2)

The experimental half-cycle curves, as suggested by S M Voevodenko, are approximated as the sum of two exponents with certain weight coefficients:

\[ P(\delta) = V_1P(i)\exp\left(-\frac{A_1}{P(i)}|\delta - \delta(i)|\right) + V_2P(i)\exp\left(\frac{A_2}{P(i)}|\delta - \delta(i)|\right), \]  

(3)

\(|\delta - \delta(i)|\) – the deformation since the beginning of deformation on the current half-cycle;  
\( P(i) \) – the hysteresis loop width at the start of the current half-cycle deformation;  
\( V_1, V_2 \) – the weight coefficients of each of the exponents; \( V_1, V_2 > 0; \; V_1 + V_2 = 1; \)  
\( A_1, A_2 \) – values that determine the performance of each of the exponents; \( A_1, A_2 > 0 \).

Numerical parameters of specific leaf springs models in the construction of calculation vehicles schemes should be determined on the basis of data obtained during bench tests of leaf springs.

4. Enable of the multilink leaf spring model into the dynamic vehicle model

The stiffness of the leaf spring is determined by the methods of fixing its links to the bridge beam and to the frame brackets, a specific installation scheme on the vehicle. In addition, the reduction of the active length of the leaf spring, due to the fastening of the leaf spring to the bridge beam, entails an increase of the suspension stiffness.

In this regard, when included in the dynamic vehicle model, the leaf spring consists of several bodies, connected by joints with elastic-friction angular bonds. They are selected from the condition of equivalence of the spring vertical stiffness, taking into account the friction forces. This representation of the leaf spring can accurately describe the nonlinear characteristics of the vertical stiffness, taking into account the peculiarities of the spring kinematics.

The kinematic of the leaf spring, which has considerable stiffness and is designed for medium and heavy-duty vehicles, is provided by a three-body representation of the leaf spring (figure 2). The leaf springs of small stiffness, for vehicles of small loading capacity, should be modeled in five-body execution (figure 3). The new models of spring suspensions allows to take into account not only the parameters of the spring, but also to consider the influence of stiffness and damping characteristics of its fastening elements (spring earrings, rubber bushings).
5. The example of calculation taking into account the dynamic stiffness of the leaf spring

At the design stage, a calculated assessment of the impact of the level of random external road forcing was carried out. The movement of an all-metal van along the road section with an even cobble and along a section of a dynamometer road with the same speed of 60 km/h was considered (figure 4).

Modeling the dynamic characteristics of leaf springs was carried out as described above. Moreover, the static stiffness of the all leaf springs remained unchanged in the process of calculations in the “FRUND” software system [6, 7]. During the analysis of the spectral densities of vertical accelerations in several points of the van model, obtained at different levels of random external road forcing, it was possible to establish the following. For points, taken in the front of the van model, with a decrease of the level of external road forcing, there is a shift of the first low-frequency spectral maximum to the right, at a higher frequency (from 1.5 to 2.0 Hz, figure 5). This effect is caused by an increase in the equivalent dynamic stiffness of the front leaf spring, when the van moves along a segment of the dynamometer road, providing small amplitudes of external influences.

Thus, the nonlinearity of the hysteresis characteristics of the leaf springs are manifested at different
levels of external road influences, which indicates the feasibility of their inclusion in the calculated dynamic model of the vehicle.

![Figure 5. Spectral densities of vertical accelerations when driving an all-metal van at a constant speed of 60 km/h: a) on the frame crossbar, under the rear mounting of the power aggregate; b) on the body floor, above its third mounting; 1 – the movement of an all-metal van along the road section with an even cobble; 2 – the movement of an all-metal van along the road section of a dynamometer road.](image)

6. Conclusion
New multi-link models of leaf spring suspensions takes into account a number of important features. We are talking about the features of the kinematics of suspension elements in the design scheme of the vehicle. Their account is provided by application of the differential equations of large displacements of bodies and also modeling of a leaf spring in the form of the links, connected by joints.

The nonlinearity of the hysteresis dynamic characteristics of the suspension is taken into account, with the help of which the dynamic stiffness of the leaf spring is considered. The dynamic stiffness is depending from the dry friction between spring leaves and the amplitudes of cyclic oscillations with a random external road forcing. The accuracy of the installation scheme of the leaf spring in the vehicle suspension is considered. It takes into account the peculiarities of fixing the leaf spring and their impact on the change in stiffness.

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