Research on Model Correction of Turbofan Engine Based on Quantum-behaved Particle Swarm Optimization

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Abstract. In order to solve the problem that the calculation result of mathematical model and the measured data of the whole machine deviate greatly due to the conditional simplification in the modeling process and engine component difference, an engine model correction method based on Quantum-behaved Particle Swarm Optimization (QPSO) is proposed. And the engine model calculation results are compared with the measured data of the whole machine performance. The results show that using the QPSO to modify the engine model can significantly improve the accuracy of the model. Before the correction, comparing the performance calculation result of the engine model with the measured data, the maximum error reaches 4.84%. After the correction, the accuracy of the model is greatly improved, and the maximum error is only 0.966%. The correction effect is good.

1. Introduction
The mathematical model of the aero-engine is an important part of the engine development process and is the basis of the engine control system and performance evaluation system. A good simulation model can greatly shorten the development cycle of the engine and save development costs. The accuracy of the model directly determines the accuracy of the model and its practical value [1]. Compared with the actual working process, many simplifications are made in the engine modeling process. In addition to the influence of the manufacturing process level, the gas path performance parameters output by the engine model are often quite different from the actual engine test data. Therefore, it is necessary to correct the mathematical model of the engine so that it can match the real data [2].

In the 1990s, scholars began to study correction method of aero-engine components characteristics and model parameter. Among them, Stamatis A first proposed a method for engine model correction, taking the minimum error of engine measured data and engine model output value as the objective function by iterating component characteristic correction coefficient [3]. Based on the engine simulation model, the appropriate correction factors are selected and added to component characteristics. The minimum error of engine measured data and engine model output value is taken as the objective function. Quantum-behaved Particle Swarm Optimization (QPSO) with high efficiency is to optimize the solution of the objective function to achieve the correction of the engine model.

2. Building an engine model
According to the aerodynamic thermodynamic relationship between the gas path structure and the cross sections of a turbofan engine, the calculated cross sections are divided as shown in Figure 1.
According to the design point parameters, the engine thermodynamic model is established. The input parameters are atmospheric temperature, pressure, height and Mach number. The output parameters of the model are the total temperature and pressure of each section, thrust, fuel flow and other performance parameters. The initial predicted values of engine model are low-pressure speed, low-pressure ratio function, high-pressure ratio function, turbine front temperature, high-pressure turbine flow and low-pressure turbine flow. The equilibrium equations corresponding to the solution process are high-pressure turbine flow balance, low-pressure turbine flow balance, power balance between high-pressure turbine and compressor, power balance between low-pressure turbine and fan, static pressure balance between inlet duct and outlet duct of mixed chamber entrance and area balance of nozzle, which are:

\[
W_g / W_g' - 1 = 0
\]

\[
W_{g5} / W_{g5'} - 1 = 0
\]

\[
L_{TH} / L_{CH} - 1 = 0
\]

\[
L_{TL} / L_{CL} - 1 = 0
\]

\[
P_{55} / P_{55'} - 1 = 0
\]

\[
A_k / A_k' - 1 = 0
\]

Where \( W_g \) is the calculated value of gas flow, \( W_g' \) is the interpolation result of characteristic data, \( L_C \) is the calculated power of fan and compressor, \( L_T \) is the calculated power of turbine characteristic data after interpolation, \( P_{55} \) and \( P_{55'} \) are the static pressure of inlet duct and outlet duct of mixed chamber entrance respectively. When the engine is in steady state, the model solving problem is transformed into solving a set of nonlinear equations composed of six common working equations [4]. In this paper, N+1 residual method is used to solve the problem iteratively. If the residual satisfies the requirement, the result is convergent and the model is solved.

3. Engine model correction method based on QPSO

3.1 Correction factor selection

The component characteristic data is the basis of the engine component level model. Whether the characteristic data conforms to the real situation of the engine is directly related to the accuracy of the engine model. Therefore, engine model correction usually improves the accuracy by correcting the component characteristic data. Based on the measured data of the intermediate state of the engine, combined with the initial guess type of the common working equation of the engine model, the selected correction factors are fan flow, fan efficiency, compressor flow, compressor efficiency, high and low pressure turbine efficiency, a total of six correction factors [5], recorded as: \( X = [x_i], i = 1,2...6 \).
3.2 Objective function construction

The engine steady-state model correction method is to correct the original characteristic data in the model calculation process by selecting an appropriate correction factor \( X \), so that the error between the output parameter and the measured performance parameter of the model is reduced. Therefore, the objective function of the model correction by selecting the measured performance parameters of the engine is:

\[
F(x_1, x_2, x_3, x_4, x_5, x_6) = \sum_{i=1}^{m} \left( \frac{y_{i,\text{cal}} - y_{i,\text{act}}}{y_{i,\text{cal}}} \right)^2
\]  

(7)

Where \( y_{i,\text{cal}} \) is the calculated value of the model parameters, \( y_{i,\text{act}} \) is the measured parameter, and \( m \) is the number of selected measured performance parameters. In this paper, the thrust \( F_N \), the fuel flow \( W_{FT} \), the turbine rear temperature \( T_{55} \), the turbine rear pressure \( P_{55} \), the compressor outlet temperature \( T_3 \), the compressor outlet pressure \( P_3 \), and the fan outlet pressure \( P_{22} \) are selected as objective parameters. According to the objective function, the engine model correction method is to solve \( \min F(X) \) by a suitable optimization algorithm.

3.3 QPSO optimization algorithm

The basic idea of the PSO is to have a group of \( M \) particles flying at a certain speed to find the optimal position. During the searching process, the flight speed is updated according to the historical optimum point \( P_{\text{best}} \) of a single particle and the historical optimum point \( G_{\text{best}} \) of a group particle, and the position is updated [6]. QPSO introduces quantum mechanics principle into PSO. By solving the Schrodinger equation, the wave function is obtained, and then the probability density function and probability of particles appearing at a certain point in space are obtained. Finally, the position of particles is measured by random sampling [7].

The specific process of the QPSO is:

(1) Algorithm initialization:

Determining the size of the group \( M \) and Randomly generating the initial position of all particles in the group:

\[
X^0 = (x^0_1, x^0_2, \ldots, x^0_M), x^0_i \in [x_{lb}, x_{up}]
\]

(8)

Initializing local optimal position \( P_{\text{best}} \) and global optimal position \( G_{\text{best}} \):

\[
P_{\text{best}}^0 = x^0_i
\]

\[
G^0 = \min \left\{ f\left( P_{i}^0 \right) \cdot L, f\left( P_j^0 \right) \cdot L, f\left( P_M^0 \right) \right\}
\]

(9)

(2) The \( k \)th generation evolution process:

Step1: Calculating the fitness from each particle position: \( f\left( x^k_i \right) \)

Step2: Updating \( P^k_i \) and \( G^k \):

\[
P^k_i = \begin{cases} P^{k-1}_i, & \text{if } f\left( x^k_i \right) \geq f\left( P^{k-1}_i \right) \\ x^k_i, & \text{if } f\left( x^k_i \right) < f\left( P^{k-1}_i \right) \end{cases}
\]

(10)

\[
G^k = \min \left\{ f\left( P^k_i \right) \cdot L, f\left( P_j^k \right) \cdot L, f\left( P_M^k \right) \right\}
\]

(11)

Step3: Calculating learning tendency point \( p^k \) and average best position \( m_{best}^k \):

\[
p^k_q = \phi^k_q P^k_q + \left[ 1 - \phi^k_q \right] G^k_j
\]

(12)

\[
m_{best}^k = \left( m_{best}^1, \ldots, m_{best}^M \right) = \left( \sum_{i=1}^{M} P^k_i / M, \sum_{i=1}^{M} P^k_{i+1} / M, \ldots, \sum_{i=1}^{M} P^k_M / M \right)
\]

(13)
Step4: Updating each dimension component $x_{ij}^{k+1}$ of the particle position vector:

$$x_{ij}^{k+1} = p_{ij}^{k} \pm \beta m_{best}^{k} - x_{ij}^{k} \ln \left( 1 / u_{ij}^{k} \right)$$ (14)

$$p_{ij}^{k} = \phi_{ij}^{k} P_{ij}^{k} + \left[ 1 - \phi_{ij}^{k} \right] G_{j}^{k}$$ (15)

$$m_{best}^{k} = \sum_{i=1}^{M} P_{ij}^{k} / M$$ (16)

Where $\phi$ and $u$ are random numbers uniformly distributed between (0,1), and the contraction-expansion factor $\beta$ is an important parameter of QPSO.

Step5: Testing algorithm termination test:

It is checked whether the updated fitness value corresponding to $G^{k}$ reaches the termination requirement or the search total algebra reaches the maximum algebra limit, and if it is satisfied, it terminates; otherwise, let $k=k+1$ and return to Step1.

Combined with the engine model to calculate the solution process and the real test data, the flow chart of using the QPSO to modify the engine model is shown in Figure 2.

**Figure 2.** Modification of engine model flow chart using QPSO algorithms

### 4. Results and analyses of simulation

#### 4.1 Model calculation results before correction

Taking an engine for an example, the general model of the engine is established according to its design point parameters, and its specific performance parameters are calculated under standard atmospheric conditions. Compared with the measured performance of the whole engine, the results are shown in Table 1. It can be seen from the table that there is a significant deviation between the calculated value of the model and the measured value of the engine, and the calculation error of the thrust is the maximum error, which is 4.84%.

| Performance parameter | $T3$ | $T55$ | $P55$ | $FN$ | $WFT$ | $P22$ | $P3$ |
|-----------------------|-----|-----|-----|-----|-----|-----|-----|
| Relative error        | 1.73% | -2.11% | 1.55% | 4.84% | -2.10% | -0.20% | -0.98% |
| Absolute error        | 1.73% | 2.11% | 1.55% | 4.84% | 2.10% | 0.20% | 0.98% |

#### 4.2 Modified model calculation results

The QPSO described above is used to optimize the objective function of the engine correction model. The particle group is set to 30, and the maximum number of iterations is 500. The optimized correction factor is calculated as $X= (0.934, 1.0314, 0.999, 1.016, 0.934, 0.931)$. The particle group fitness changing process is shown in the Figure 3. The minimum value is 1.2044e-04.
The correction factor obtained by the QPSO optimization solution is brought into the engine model for calculation, and the calculated value of the intermediate state performance parameter of the modified engine model is obtained. Compared with the measured value of the engine, the results are shown in Table 2. It can be seen from the table that the calculated value of the model has less deviation from the measured value of the engine, and the maximum error is only 0.966%.

### Table 2. Comparisons between calculated and measured values of the model before correction

| Performance parameter | T3   | T55  | P55  | FN   | WFT  | P22  | P3   |
|------------------------|------|------|------|------|------|------|------|
| Relative error         | 0.034%| 0.366%| -0.054%| 0.097%| -0.274%| -0.171%| -0.966%|
| Absolute error         | 0.034%| 0.366%| 0.054%| 0.097%| 0.274%| 0.171%| 0.966%|

The errors between calculated values of engine model and measured data before and after modification are compared. It can be seen that compared with the engine model before correction, the calculated accuracy of the corrected model performance parameters is significantly improved.

### 5. Conclusion

The aerodynamic thermodynamic simulation model of a turbofan engine is established in this paper, and the engine model correction method based on Quantum-behaved Particle Swarm Optimization is proposed. The engine model calculation results are compared with the measured engine data. Before the correction, the maximum error between the calculated value of the engine model and the measured data reached 4.84%. After the correction, the accuracy of the model is greatly improved, and the maximum error is only 0.966%. The correction effect is good.

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