Stopping Powers and Inelastic Mean Free Path of 100 eV to 30 keV Electrons in Zirconium Silicates

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INTRODUCTION

Zirconium oxide and zirconium silicates have received great attention and exhibit important nuclear applications as engineering materials for inert matrix fuels, actinide waste forms and targets for transmutation of radionuclides, due to its high chemical durability, high corrosion resistance, low hydrogen absorption, and excellent radiation stability [1-5].

The development of new zirconium oxides for use as radiation tolerant materials in advanced nuclear energy systems has resulted in materials with unique physical and chemical properties. Many experimental studies have reported on the effects of crack growth [4], high pressure steam [1], and irradiation of heavy ions [3], He nuclei [3], and protons [2] on the electronic, chemical, and structural properties of zirconium oxide. However, we found few studies devoted to the electron stopping power (SP) and inelastic mean free path (IMFP) properties of zirconium oxide and zirconium silicates for a wide range of energy loss functions (ELFs) up to 80 eV.

The electron SP and IMFP properties describe the traverses of energetic electron through matter under interactions with atomic orbital electrons and atomic nuclei. Through these interactions, the electrons may lose their kinetic energy through elastic or inelastic collisions. In an elastic collision the electron is deflected from its original path but no energy loss occurs, while in an inelastic collision the electron is deflected from its original path and part of its energy is transferred to an orbital electron or emitted in the form of bremsstrahlung production. Electrons lose their kinetic energy through various types of inelastic scattering processes described by SP, while the mean distance between two collisions is described by the IMFP [6-12]. These two fundamental quantities are essential importance in many fields of research, such as radiobiology, biomedical applications, radiation dosimetry, and the modeling of electron transport in matter for many other applications. For instance, to understand radiation effects in radiation dosimetry, SP values are required for the calculation of energy deposition of energetic electrons passing through biological tissues [6-9]. SP values have also been used in Monte Carlo simulations of electron transport relevant to electron-probe microanalysis, Auger-electron spectroscopy, and dimensional metrology in the scanning electron microscope [10-12].

In this study, we report electron SP and IMFP values calculated from the energy loss function (ELF) of zirconium oxide, silicon dioxide, and zirconium silicates for energies from 100 eV to 30 keV. The ELFs are obtained from the experimental reflection electron energy loss
spectroscopy (REELS) spectra. REELS is surface sensitive and the spectra carries information on the electronic structure of the material because the energy loss experienced by the incident electron depends on the electronic structure of the material. The spectra can easily be recorded over a wide energy-loss range [13-29]. The calculation of the ELF of zirconium oxide, silicon dioxide, and zirconium silicates from REELS spectra has already been performed by Tahir et. al. [15]. Subsequently, the calculated SPs were compared with the results of evaluating the nonrelativistic Bethe equation, while the IMFPs were compared with the results of Tanuma Powell Penn (TPP-2M) methods [9] and Ashley et. al. [30], which are available from a National Institute of Standards and Technology (NIST) database. The aim of this work are twofold, namely: first, to provide an alternative basic data of SPs and IMFPs for the study on the energy deposition of low-energy electrons transport through zirconium oxide and zirconium silicates; and second, to show that the method presented in this study might be a good one for evaluating the SP and IMFP of radiation tolerant materials in advanced nuclear energy applications.

**EXPERIMENTAL METHOD**

According to the dielectric response theory, the Penn statistical approximation, and the Born–Ochkur correction method which includes the exchange between the incident electron and the electrons in the medium, the electron inelastic differential cross section can be expressed as [7]:

\[
\frac{d^2\sigma}{d(h\omega) dq} = \frac{\gamma}{\pi \alpha_0 E} \frac{\omega}{\omega_q} \delta\left(\omega - \omega_q \frac{h\omega}{2m}\right) C_s d\omega' \tag{1a}
\]

where

\[
C_s = 1 - \frac{Q}{E + h\omega - h\omega} + \left(\frac{Q}{E + h\omega - h\omega}\right)^2, \quad Q = h\alpha q^2 / 2m \tag{1b}
\]

With Eq. (1) and over all allowed integration region, the SP and IMFP can be given as follows [7-9]:

\[
SP = -\frac{dE}{dS} = \frac{1}{2\pi \alpha_0 E} \int_0^{\pi/2} (h\omega) \Im[-1/\varepsilon(\omega)] \nu(\alpha) d(h\omega) \tag{2}
\]

\[
\lambda = \left(\frac{1}{2\pi \alpha_0 E} \int_0^{\pi/2} (h\omega) \Im[-1/\varepsilon(\omega)] \nu(\alpha) d(h\omega)\right)^{-1} \tag{3}
\]

Where \(E\) is the kinetic energy of the incident electron, \(\alpha_0\) is the Bohr radius, \(h\omega\) is the energy loss, \(\Im[-1/\varepsilon(\omega)]\) is the energy loss function, \(\nu(\alpha)\) and \(\nu(\alpha)\) are, respectively;

\[
\nu(\alpha) = \frac{2s}{(1 + \alpha)(1 + \alpha + s)} + \ln\left(\frac{1 - \alpha^2(1 + \alpha)}{1 - \alpha^2(1 + \alpha + s)}\right) \tag{4}
\]

\[
w(\alpha) = \frac{3\alpha^2 + 3\alpha + 1}{(1 + \alpha)^2} \ln\left(\frac{1 + \alpha - s}{1 + \alpha}\right) + \ln\left(\frac{1 - \alpha}{1 - \alpha - s}\right)
+ \frac{2\alpha^2 + \alpha}{(1 + \alpha)^2} \ln\left(\frac{1 + \alpha}{1 + \alpha + s}\right) + \frac{2\alpha s}{(1 + \alpha)^2(1 + \alpha + s)} \tag{5}
\]

With \(\alpha = h\omega / E\) and \(S = \sqrt{1 - 2\alpha}\)

Tan et. al. [7] used the Born-Ochkur exchange correction in the calculation of SPs and IMFPs of organic compound. The results of their calculations of SPs attain similar accuracy with Ashley exchange correction for all electron energy region and also agree well with Bethe-Bloch theory at high-energy region.

As shown by equation (2) and (3), the calculations of SP and IMFP require the energy loss function (ELF), which completely determines the probability of an inelastic scattering event, the energy loss, and the scattering angular distribution. The key problem for the calculation of SP and IMFP is deriving the ELF. The well-known Lindhard dielectric function is only applicable for a limited class of materials, namely the so called nearly-free electron materials such as Al or alkali metals, and is not valid for other materials such as noble metals, insulators, or organic compounds. On the other hand, the ELF in the long wave-length limit \(k \rightarrow 0\) approaches the optical ELF, and is calculated from experimental optical data, the refractive index and the extinction coefficients, which are available for a number of materials [7-9].

Tahir et. al. [13,16,18,20-25] have employed the semi-classical dielectric response model proposed by Tougaard and Yubero [26-29] in their calculation of ELF. The algorithms of this method have been implemented in the generally available QUEELS-\varepsilon(k,\omega)-REELS software package [29]. The validity and consistency of this method was extensively tested recently [21] and it has also previously been successfully used to obtain the ELFS and optical properties of ultrathin dielectric [15,16,18] semiconductor [17], polymer [22], metals [14,23], and transparent oxide films [20,21,24,25]. In Ref. [14] there is a detailed discussion of
experimental tests of the validity of the elastic scattering model.

In this model, the dielectric function \( \varepsilon(k,\omega) \) of the material describes all excitations. The dielectric function is a function of wave vector \( k \) and frequency \( \omega \) which are the only inputs for the calculations in this model. The dielectric function is given in terms of the energy loss function (ELF) \( \text{Im}(-1/\varepsilon) \) which is parameterized as a sum of Drude-Lindhard type oscillators, as described in Refs. [14, 15, 27-29]:

\[
\text{Im}\left\{ \frac{-1}{\varepsilon(k,\omega)} \right\} = \frac{\theta(\hbar\omega-E_i)}{\hbar^2\omega_{ka}^2 + \hbar^2\omega^2} + \gamma_i^2 \hbar^2\omega^2, \tag{6}
\]

where the dispersion relation is given in the form

\[
\hbar\omega_{ka} = \hbar\omega_{k0} + \alpha_i \frac{\hbar^2k^2}{2m}. \tag{7}
\]

Here, \( \alpha_i, \gamma_i, \hbar\omega_{k0}, \) and \( \alpha_i \) are the oscillator strength, damping coefficient, excitation energy and momentum dispersion coefficient of the \( i \)th oscillator, respectively, and \( \hbar k \) is the momentum transferred from the REELS electron to the solid. The dependence of \( \omega_{ka} \) on \( k \) is generally unknown, but we use Eq. (6) with \( \alpha_i \) as an adjustable parameter. The values of the momentum dispersion coefficient \( \alpha_i \) are related to the effective mass, e.g., \( \alpha_i \approx 0 \) for insulator and \( \alpha_i \approx 1 \) for metals [14,27-29]. The step function \( \theta(\hbar\omega-E_i) \) is included to describe the effect of the band gap energy \( E_g \) in semiconductors and insulators. Here, \( \theta(\hbar\omega-E_i)=0 \) if \( \hbar\omega<E_i \) and \( \theta(\hbar\omega-E_i)=1 \) if \( \hbar\omega>E_i \). The band gap was estimated from the onset value of REELS spectrum as shown in Fig. 1 [15].

For materials studied in this work, we used data for ELF from Ref. [15] in which the parameters are obtained by fitting the inelastic electron scattering cross section \( \lambda K_{\alpha\beta}(\hbar\omega) \) spectrum simulated with the QUEELS-\( \varepsilon(k,\omega) \)-REELS software to an experimental inelastic scattering cross section \( \lambda K_{\alpha\beta}(\hbar\omega) \). The parameters \( \alpha_i, \gamma_i, \hbar\omega_i, \) and \( \alpha_i \) of the oscillators are varied until a good agreement between the calculated and experimental inelastic cross sections is obtained. The oscillator strengths are adjusted to make sure that \( \varepsilon(k,\omega) \) fulfills the well-established Kramers-Kronig sum rule [14,27-29].

\[
\frac{2}{\pi} \text{Im}\left\{ \frac{1}{\varepsilon(k,\omega)} \right\} \frac{d(\hbar\omega)}{\hbar\omega} = 1 - \frac{1}{n_0^2}. \tag{8}
\]

Here, \( n_0 \) is the refractive index of the materials in the optical limit \( \hbar\omega \to 0 \). These parameters are listed in Table 1 in Ref. [15] for all considered materials.

| E (eV) | SiO\textsubscript{2} | ZrO\textsubscript{2} | SiO\textsubscript{2} | ZrO\textsubscript{2} |
|-------|----------------|
|       | SP | SP | IMFP | IMFP |
| 100   | 3.1164 | 2.603 | 8.85 | 9.231 |
| 200   | 3.1207 | 2.8568 | 11.2038 | 12.2565 |
| 300   | 3.2053 | 2.517 | 14.0763 | 15.5254 |
| 400   | 2.2206 | 2.123 | 16.9597 | 18.0734 |
| 500   | 1.9381 | 1.8557 | 19.7855 | 20.2886 |
| 600   | 1.7236 | 1.7026 | 22.5499 | 22.5639 |
| 700   | 1.5553 | 1.5701 | 25.284 | 24.8435 |
| 800   | 1.4195 | 1.4565 | 27.9177 | 27.1086 |
| 900   | 1.3076 | 1.3587 | 30.5334 | 29.3524 |
| 1000  | 1.2135 | 1.274 | 33.1107 | 31.5729 |
| 1500  | 0.9024 | 0.9794 | 45.5415 | 42.3425 |
| 2000  | 0.7268 | 0.8013 | 57.418 | 52.6567 |
| 3000  | 0.5301 | 0.5969 | 80.0941 | 72.3337 |
| 4000  | 0.422 | 0.4808 | 101.8031 | 91.132 |
| 5000  | 0.3527 | 0.4051 | 122.8389 | 109.3111 |
| 6000  | 0.3042 | 0.3514 | 143.3645 | 127.019 |
| 7000  | 0.2682 | 0.3113 | 163.4829 | 144.3502 |
| 8000  | 0.2403 | 0.28 | 183.2643 | 161.3696 |
| 9000  | 0.218 | 0.2548 | 202.7597 | 178.1243 |
| 10000 | 0.1998 | 0.2341 | 222.0075 | 194.65 |
| 15000 | 0.1423 | 0.1683 | 315.397 | 274.6469 |
| 16500 | 0.1313 | 0.1556 | 342.6885 | 297.9771 |
| 18000 | 0.122 | 0.1449 | 369.7128 | 321.0614 |
| 19500 | 0.114 | 0.1356 | 396.4967 | 343.9247 |
| 21000 | 0.107 | 0.1275 | 423.0624 | 366.5873 |
| 22500 | 0.1009 | 0.1204 | 449.2426 | 389.0667 |
| 24000 | 0.0955 | 0.1141 | 475.6107 | 411.3776 |
| 25500 | 0.0907 | 0.1084 | 501.6231 | 433.5327 |
| 27000 | 0.0864 | 0.1034 | 527.4774 | 455.5431 |
| 28500 | 0.0825 | 0.0988 | 553.1842 | 477.4184 |
| 30000 | 0.0789 | 0.0946 | 578.7527 | 499.1671 |

### RESULTS AND DISCUSSION

The SPs and IMFPs of zirconium oxide, silicon dioxide, and zirconium silicates have been determined...
The IMFP values for SiO$_2$ shown in Fig. 2 for energies from 100 eV to 30 keV. In Fig. 2, we compare our results for ZrO$_2$ and SiO$_2$ to the SPs and IMFPs from the nonrelativistic Bethe equation for solid [9] and to the TPP-2M results determined using Tanuma Powell Penn (TPP-2M) formula. We also compare our IMFP results for SiO$_2$ with Ashley et al. [30] which incorporates the exchange correction in their calculation but unfortunately there is no reference data for ZrO$_2$. The SP values for SiO$_2$ show good agreement with the predictions of the nonrelativistic Bethe-theory; however, our SP for ZrO$_2$ is about 10% lower. The nonrelativistic Bethe-theory obtains the SPs without considering the exchange correction and gives accurate value at energies higher than 10 keV. The TPP-2M results for IMFP without exchange correction also shown in Fig. 2 for comparison. The IMFP values for SiO$_2$ show good agreement with TPP-2M and Ashley, while for ZrO$_2$ our IMFP values are about 25% larger than those obtained from TPP-2M. The calculation of IMFP by using TPP-2M ignored the effect of surface elastic scattering and interference between surface and bulk excitation [9, 15].
Fig. 3. The stopping power (SP) and the inelastic mean free path (IMFP) of \((\text{ZrO}_2)_x\text{SiO}_2)_{1-x}\) \((x=0, 0.25, 0.50, 0.75, \text{and } 1.0)\) in this study as calculated by using the equation (2) and (3) from ELF determined from quantitative analysis of REELS spectra.

CONCLUSION

The SPs and IMFPs of Zr-silicates have been obtained for electron energies from 100 eV to 30 keV by using ELF from the quantitative analysis of REELS spectra in the modified Born–Ochkur equations. The values of SP and IMFP indicate that the ZrO\(_2\) has a strong effect on the electronic properties of Zr-silicates for ZrO\(_2\) contents of 50% and 75% in the silicates. However, for the ZrO\(_2\) content of 25%, the SiO\(_2\) has a strong effect on electronic properties of Zr-silicates. In summary, we have demonstrated that the applied procedure for using the ELF obtained from quantitative analysis of REELS spectra provides us with a straightforward means to determine the SPs and IMFPs of alloys for electron energies from 100 eV to 30 keV.

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