Kaon condensate with trapped neutrinos and high-density symmetry energy behavior.

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Abstract

Effects of the neutrino trapping and symmetry energy behavior are investigated in the framework of the chiral Kaplan-Nelson model with kaon condensation. Decrease in the condensation threshold during deleptonization if found to be generic regardless uncertainties in the nucleon-kaon interactions and symmetry energy. Quantitatively however, differences are shown to be important.

Key words: neutron stars, symmetry energy, kaon condensate
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1. Introduction

Neutron stars are born in core-collapse supernova explosions \cite{1,2,3,4,5,6}. First minutes of their life is a period of rapid evolution \cite{7}. This is protoneutron star (PNS) stage, where matter is transformed to final cold catalyzed matter \cite{8} state: one of the greatest mysteries in astrophysics \cite{9}. Among other matter with kaon condensates, proposed by Kaplan and Nelson \cite{10,11}, is very intriguing possibility. Unfortunately, existing experimental and observational data is still unable to select one, correct model \cite{1}. Therefore further investigation of effects present under particular assumption on high-density matter model is in place.

Possibly, only nearby core-collapse supernova explosion will allow researchers to collect enough data, mainly in neutrino channel (cf. Fig. 3 in \cite{12}), to find clear signatures of particular model. However, neutrinos are trapped in protoneutron stars. Therefore, it is required to study nuclear matter with two-parameter (baryon number $n_B$ and lepton number $Y_L$) model at least.

Deleptonization in the first seconds after PNS is born causes decrease in lepton number from value typical for initial pre-supernova ,,Fe'' matter $Y_{Le} \approx 0.4$ \cite{13} down to some numerically small value, e.g. $Y_{Le} = 0$ in pure neutron matter model. Meanwhile neutrinos escape outer PNS region, but in central, high-density core we may assume quasistatic evolution of matter parameterized with decreasing $Y_{Le}$. We have dropped all finite temperature effects for simplicity, however they are potentially equally important \cite{9}. Generally, effects of decreasing temperature are smaller than decreasing lepton number and act in similar direction on critical kaon condensation density.

2. Model with kaon condensate

Energy density for matter with kaon condensate is given by:

$$\varepsilon(n_n, n_p, \theta, \mu_e, \mu_{\nu_e}, \mu_K, \mu_\mu, \mu_{\nu_\mu}) = \varepsilon_{F_n} + \varepsilon_{F_p} + \varepsilon_{F_{\nu_e}} + \varepsilon_{F_{\nu_\mu}} + \varepsilon_{F_\mu} + \varepsilon_{\text{int}} + \varepsilon_{\text{kaon}}.$$
where variables on the left hand are: \( n_n, n_p \) – neutron and proton density, \( \theta \) – amplitude of the kaon condensate \( \mu \) – chemical potentials. Right hand has been decomposed into: \( \varepsilon_F \) – Fermi sea energies, \( \varepsilon_{\text{Kaon}} \) – kaon-nucleon interaction and \( \varepsilon_{\text{int}} \) – nucleon interaction including symmetry energy.

Nucleon part can be rewritten as:

\[
\varepsilon(u, x) = \frac{3}{5} E_F^0 n_0 u^{5/3} + u n_0 (1 - 2 x)^2 S(u) \quad (2)
\]

where \( E_F^0 = 36.885 \text{ MeV} \). Standard parameterization \((n_0 = 0.16 \text{fm}^{-3})\) has been used:

\[
\begin{align*}
 n_p &= x n_B \quad (3) \\
 n_n &= (1 - x) n_B \quad (4) \\
 n_B &= u n_0 \quad (5)
\end{align*}
\]

and \( S(u) \) is the symmetry energy.

Leptons contribute to the total energy:

\[
\varepsilon_F = \frac{g_e \mu_e^4}{8 \pi^2}, \quad (6)
\]

where \( g_e=2 \) and \( g_\nu = 1 \) (see, however, footnote 2).

Generalization to other lepton families is trivial. However, we have not included \( \mu \) and \( \tau \) flavors, as their contribution is minimal \([15,17]\).

Contribution to the total energy from the kaon condensate can be computed from the Kaplan–Nelson chiral formalism \([14,15]\):

\[
\varepsilon_{\text{Kaon}} = \frac{\mu^2 f^2 \sin^2 \theta}{2} + (\cos \theta - 1) \times \\
\times [n_B x \Sigma_{Kp} + n_B (1 - x) \Sigma_{Kn} - f^2 m_K^2]
\]

where, \( m_K = 493, 7 \text{ MeV} \) and \( f = 93 \text{ MeV} \) is kaon mass and pion decay rate, respectively. Constants \( \Sigma_{Kp} \) and \( \Sigma_{Kn} \) define kaon-proton and kaon-neutron interaction strength. Due to large uncertainties they may be replaced with single quantity \( \Sigma_{KN} \) placed between 168 and 520MeV. Some authors use \( a_3 m_s \) instead:

\[
\Sigma_{KN} = -\frac{1}{2} (a_1 m_s + 2 a_2 m_s + 4 a_3 m_s) \quad (8)
\]

where \( a_1 m_s = -67 \text{ MeV} \), \( a_2 m_s = 134 \text{ MeV} \).

Kaon density is \([9]\):

\[
n_K = \mu_K f^2 \sin^2 \theta + n_B \left( \frac{1}{2} x + \frac{1}{2} \right) (1 - \cos \theta) \quad (9)
\]

Minimalization of the energy must include conservation of the baryon number, electric charge and lepton numbers if the neutrinos are trapped:

\[
n_n + n_p = n_B \quad (10a)
\]

\[
n_p = n_e + n_K^- + n_\mu \quad (10b)
\]

\[
x_i + Y_{\nu_i} = Y_L \quad (10c)
\]

Baryon number density is a free parameter. Charged lepton fractions are denoted as \( x_i \equiv n_i/n_B \), neutrinos as \( Y_{\nu_i} \). Conservation of the electric charge is ensured using kaon chemical potential as a Lagrange multiplier:

\[
\varepsilon = \varepsilon - \mu (n_p - n_e - n_K^- - n_\mu). \quad (11)
\]

Electroweak and strong interactions allow reactions:

\[
\begin{align*}
 n &\leftrightarrow p^+ + e^- + \bar{\nu}_e \quad (12a) \\
 n &\leftrightarrow p^+ + K^- \quad (12b) \\
 n &\leftrightarrow p^+ + \mu^- + \bar{\nu}_\mu \quad (12c)
\end{align*}
\]

and respective chemical potentials obey:

\[
\begin{align*}
 \mu_n - \mu_p &= \mu_e - \mu_{\nu_e} \quad (13a) \\
 \mu_n - \mu_p &= \mu_\mu - \mu_{\nu_\mu} \quad (13b) \\
 \mu_n - \mu_p &= \mu_K \quad (13c)
\end{align*}
\]

If neutrinos escape freely, one can put simply \( \mu_{\nu_i} \equiv 0 \) and we have left with only one independent chemical potential equal for all negative electric charge particles.

### 3. Kaon condensation without neutrinos

If neutrinos escape freely (old neutron star case) we have only one driving parameter: baryon density. Energy is minimalized numerically solving system of equations:

\[
\frac{\partial \varepsilon(x, \mu, \theta)}{\partial \theta} = \frac{\partial \varepsilon(x, \mu, \theta)}{\partial x} = \frac{\partial \varepsilon(x, \mu, \theta)}{\partial \mu} = 0
\]

Functions \( x(u), \theta(u), \mu(u) \) are immediate result of calculations. Other properties, e.g. \( n_e(u), n_K(u) \) and EOS can be then easily obtained. Condensation threshold is defined as a maximum density where still \( \theta(u) = 0 \). Numerical results are presented for \( u \) up to 12 for three values \( \Sigma_{KN} = 168, 344, 520 \text{ MeV} \) values covering entire considered range of values for this parameter.

Larger \( \Sigma_{KN} \) (i.e smaller \( a_3 m_s \)) gives stronger kaon-nucleon interaction and lower condensation threshold (Fig. 3). Both amplitude of the condensate and proton fraction (Fig. 2, 3) exhibit asymptotic behavior. This is typical if symmetry energy is constant or growing with density. Chemical potential for kaons and electrons (Fig. 4) can reach large

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1 Equivalent values are: \( a_3 m_s = -134, -222, -310 \text{ MeV} \).
(μ < 100 MeV) negative values if Σ_{KN} is large, and muon flavor will be produced. Overall contribution from muons is however small [17,15]. As lepton number is not conserved electron/positron fraction can be relatively high. This behavior is completely changed with neutrino trapping, as we explain in the next section.

4. Kaon condensate and neutrino trapping

If neutrinos are trapped then properties of the dense matter are numbered by the two parameters: baryon density \( n_B \) and lepton number density \( Y_{Le} \). In principle we have three separate lepton numbers, but initially only \( Y_{Le} \) is not identically zero. Therefore, due to large muon and taon masses we may safely restrict to electron lepton number conservation alone\(^2\).

Lepton number conservation can be introduced into eq. (11) in the following manner. We rewrite (10c) using chemical potentials:

\[
\mu_e^3 + \frac{1}{2} \mu_{\nu_e}^3 = 3 \pi^2 n_B Y_{Le}. \tag{15}
\]

Eq. (13a) minus (13c) gives:

\[
\mu_K = \mu_e - \mu_{\nu_e}. \tag{16}
\]

Eq. (16) and (15) are used to derive \( \mu_e \) and \( \mu_{\nu_e} \) as a functions of \( \mu_K, n_B \) and \( Y_{Le} \). Now, minimalized function is:

\[
\tilde{\varepsilon}(Y_{Le}, n_B, x, \theta, \mu_K) = \varepsilon_{F_{n}} + \varepsilon_{F_{p}} + \varepsilon_{K_{\alpha}} + \varepsilon_{F_{\nu_e}}(\mu_K, Y_{Le}) - \mu_K [n_p - n_K - n_e(\mu_K, Y_{Le})].
\]

Expression above is explicite very complex due to presence of the third order radicals resulting from (15).

Two parameter family of solutions is presented in Fig. 5. Proton fraction strongly depends on trapped lepton number until kaon condensation threshold. This behavior depends somewhat on \( \Sigma_{KN} \), cf. Fig. 2. Fig. 1 illustrate lepton number conservation. Without kaons electrons are required by electric charge conservation. If condensate is present, negative charge is provided by preferred due to strong interactions kaons, and neutrinos begins to provide required lepton number amount.

The most important effect of deleptonization is decrease in kaon condensation threshold, cf. Fig. 3. Direction of the effect do not depend on \( \Sigma_{KN} \). Therefore, as kaons tends to „soften” EOS, deleptonization cause decrease in maximum neutron star mass. If PNS is born in stable state with large \( Y_{Le} \) then deleptonization may cause delayed collapse to a black hole if \( Y_{Le} \) reach small values long time (i.e. tens of seconds [5]) after core-collapse\(^3\).

Model with \( Y_{Le} \to 0 \) is clearly different from model without neutrino trapping (Fig. 4). Initially, for large values of \( Y_{Le} \) model works well, but transition to the free streaming regime requires solving transport equations rather than quasistatic evolution.

5. Symmetry energy effects

High density energy symmetry behavior is very important for neutron stars and core-collapse supernovae (Lattimer & Yamada at [1]) but is a source of the great uncertainty [17,18,19,20]. Two qualitative behavior has been found:

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\(^2\) Neutrino oscillation phenomenon indicates conservation of the total lepton number only. Therefore (10c) could be replaced with \( x_e + Y_{\nu_e} + Y_{\nu_\mu} + Y_{\nu_\tau} = Y_{Le} \equiv Y_{\nu_e} \), where we have put \( Y_{\nu_\mu} = Y_{\nu_\tau} = 0 \). As neutrinos have very small masses all three terms are identical and finally we get \( x_e + 3Y_{\nu_e} = Y_{Le} \). Only difference is “degeneracy factor”: \( g = 3 \) instead of \( g = 1 \) in eq. (6) for neutrinos.

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\(^3\) Neutron star has not been found in the remnant of the supernova 1987A [22]. Delayed collapse is the most probable explanation.
Fig. 2. Proton fraction for three values of the $\Sigma_{KN}$. Arrows indicate direction of the deleptonization effects.

Fig. 3. Deleptonization effects on the amplitude of the kaon condensate $\theta$. Threshold for condensation with trapped neutrinos is always higher and proportional to the lepton number density $Y_{Le}$.

Fig. 4. Electron fraction with and without neutrino trapping.

Fig. 5. Kaon chemical potential as a function of nuclear density.

- always growing (from mean field theories)
- decreasing at high densities (variational methods)

Mean field theory results can be approximated with [15]:

\[ m = 222 \text{ MeV} \]
\[ V_2 = a u, \quad V_2' = a \sqrt{u}, \quad V_2'' = a \frac{2u^2}{1+u} \quad (17) \]

where \( a = 17 \text{ MeV} \).

Fig. 6 compare mean field and variational results. In the latter, symmetry energy not only decrease, but can reach negative values and pure proton or pure neutron states are preferred.

Symmetry energy strongly influences on matter properties both below and above condensation threshold (Figs. 7-9).

General tendency to decrease condensation threshold is however unaffected, and amplitude is still growing with density (Fig. 10).

6. Conclusions

Decrease of the kaon condensation threshold during deleptonization has been shown to be universal in the considered class of models. Main source of the uncertainty is the high density behavior of the symmetry energy and value of the kaon-nucleon interaction parameter \( \Sigma_{KN} \). Decrease in the condensation threshold may cause newborn PNS to be unstable.
Fig. 9. The same as in Fig. 8 for the electron fraction.

\[ \Sigma_{\text{ne}} = 345 \text{ MeV} \]
\[ a, m = -222 \text{ MeV} \]
\[ Y_{\text{le}} = 0.2 \]

Fig. 10. Deleptonization effect on kaon condensate with symmetry energy are qualitatively similar to the case without symmetry energy, cf. Fig. 3.

as trapped lepton number is carried away from the neutrinospheres [9].

Increasing condensate volume finally may cause collapse to a black hole and immediate disappearance of the neutrino flux (effect observable for a next Galactic supernova) as neutrino spheres are swallowed under event horizon [21].

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