Vibration characteristics of thermal insulation tile with material and geometric nonlinear effects

Xinghong Zhao¹, Wei Xia¹, ², Shengping Shen¹

¹State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi’an Jiaotong University, Xi’an 710049, China.
²Shaanxi engineering laboratory for vibration control of aerospace structures, Xi’an Jiaotong University, Xi’an 710049, China.

Abstract. Thermal protection system (TPS) composed of ceramic insulation tile and strain-isolation pad (SIP) tends to be destroyed by excessive inner strain arises from aerodynamic load or substructure deformation/vibration. Both material nonlinearity and geometric nonlinearity affect the static and dynamic strength of TPS, and the nonlinear effects of SIP need to be considered in the strength analysis. By considering both the material nonlinearity and geometric nonlinearity of SIP, the effect of external static load on the dynamic characteristics of tile-SIP system is investigated. Firstly, a theoretical model is established to calculate the natural frequency of tiles under static load with material and geometric nonlinearities. The analysis procedure is separated into two steps. The first step is the nonlinear analysis to determine the static large deformation. The second step is the linearized dynamic analysis on the base of static deformation with small vibration amplitude assumption. The complexity is reduced by introducing a nonlinear coefficient parameter. The theoretical results have good agreement with COMSOL simulation. Numerical results show that large deformation is found in the flexible SIP when uniform static load applied on the surface of the tile, which introduces significant material and geometric nonlinearities and changes the dynamic characteristics of the tile. 9% increase of intrinsic frequency is found for the static load of 80kPa when considering material nonlinearity, while 16% increase of intrinsic frequency is found when considering both material and geometric nonlinearities. It is concluded that the uniform static load has important effect on the nature frequency of tile-SIP systems when introducing the material and geometric nonlinearities into the calculation. The nonlinearities of the SIP need to be considered in the tile-SIP dynamic strength analysis.

1. Introduction
The ceramic insulation tile is cemented on the strain isolating pad (SIPs) and placed in the aluminum shell of the aircraft as a whole to form a typical thermal protection system for high-speed aircraft (TPS) [1,2]. The ceramic insulating tile is light in weight. The advantages of reusability [3-5], but at the same time ceramic is a kind of brittle material, the critical strain of failure is very small [6, 7]. In order to prevent the occurrence of excessive strain, the ceramic heat insulation tile is designed in blocks, leaving a certain gap between the tile and the tile. And the strain-isolating pad with low elastic modulus was
added as a buffer between the ceramic insulating tile and the base wall [8]. Ceramic thermal insulation tile is not the main bearing load structure, but pneumatic, acoustic vibration, debris impact and other causative loads are transferred to the base bearing structure through ceramic insulation tile-strain isolation pad. In the strength analysis of ceramic thermal insulation tile, special attention should be paid to the pneumatic and acoustic vibration of the thermal insulation tile system. Dynamic strength problem under the combined action of substrate deformation [10-12]. Due to the great difference in elastic modulus between ceramic insulating tile and strain isolating pad, the deformation of the structure is mainly concentrated on the strain-isolating pad with lower modulus. In addition, the strain-isolation pad is generally very thin (thickness of 0.2mm-0.5mm [13, 14]). It is easy to introduce a variety of nonlinear effects (geometric nonlinearity, material nonlinearity, etc.) due to large deformation. The natural vibration characteristics of ceramic thermal insulation tile often deviate from the results of linear vibration theory [17, 18]. In this paper, the natural vibration characteristics of ceramic thermal insulation tile strain isolating pad system under uniform static load are studied. Considering the material nonlinearity and geometric nonlinearity of the strain-isolating pad, the influence equation of uniform static load on the vibration natural frequency of rigid thermal insulation tile on flexible substrate is established. The research results are useful for perfecting the dynamic strength analysis theory of rigid-flexible composite structure. It is meaningful to guide the structure design of ceramic thermal insulation tile.

2. Mechanical Model of Thermal Insulation Tile-strain Isolating Pad

A single ceramic thermal insulator-strain isolation pad system is shown in figure 1. The strain isolation pad is clamped on a rigid base structure. The upper surface of the strain-isolating pad is cemented with the ceramic insulating tile. Since the elastic modulus of the ceramic insulating tile is much larger than that of the strain-isolating pad, this paper assumes that the ceramic insulating tile is a rigid body. The elastic deformation of the strain-isolating pad on the upper surface of ceramic thermal insulation tile under uniform static load is analyzed.

Under uniform static load, considering the thickness and side length of the strain-isolating pad, the shear strain is negligible. The stress-strain relationship of the strain-isolating pad based on isotropic material assumption is as follows:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{bmatrix} = 2G \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z
\end{bmatrix} + \lambda \theta
\]  

(1)
In which, Lame constant \( G = \frac{E}{2(1+v)} \), \( \lambda = \frac{vE}{(1-2v)(1+v)} \),

\( v \) is Poisson’s ratio \( \theta = \varepsilon_x + \varepsilon_y + \varepsilon_z \), it is the first strain invariant.

If the effect of in-plane deformation is further neglected, the stress-strain relationship of the strain-isolating pad on the thickness direction is further ignored.

\[
\sigma_z = P(v)E\varepsilon_z \tag{2}
\]

In the formula

\[
P(v) = \frac{2G + \lambda}{E} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)},
\]

\( E \) is Yang modulus. In the theory of online elasticity, \( E \) is a constant, but considering that the strain isolation pad is made of a thin layer of viscoelastic material, the material has a nonlinear characteristic, and its nonlinear behavior can be described by the following formula:

\[
E = K \left(1 + \varepsilon_z^{-1}\right) \tag{3}
\]

\( K \) is a constant and can be determined according to the material test results. Therefore, the stress-strain relationship of the strain-isolation pad considering the material nonlinearity under uniform static load is as follows:

\[
\sigma_{0z} = P(v)K \left(\varepsilon_{0z} + \varepsilon_{0z}^0\right) \tag{4}
\]

The strain-displacement relationship is as follows:

\[
\varepsilon = \left(\nabla u^T + \nabla u + (\nabla u)^T\nabla u\right)/2 \tag{5}
\]

In the formula, the high order term \( (\nabla u)^T\nabla u \) is Geometric nonlinear term. When deformation is not considered, the term can be ignored. The linear strain-displacement relationship is as follows:

\[
\varepsilon = \left(\nabla u^T + \nabla u\right)/2 \tag{6}
\]

3. Theory of Non Linear Vibration

The equation of motion of free vibration for ceramic insulator-strain isolating pad system can be written as follows:[19]:

\[
-\rho \omega^2 u = \nabla \cdot F \sigma, \quad i\omega = \varphi \tag{7}
\]

In which, \( F \) is a deformed gradient matrix (considering geometric nonlinearity) or a unit matrix \( I \) (not taking into account geometric nonlinearity), \( \sigma = 2G \varepsilon + \lambda \theta I \). Assuming that the structure does free vibration on the basis of static deformation, and the dynamic deformation is small, then the static prestress can be obtained. Assuming that the structure is subjected to free vibration on the basis of static deformation, and that the dynamic deformation is small. Then static prestressing can be performed \( \sigma_0 \) (Z direction prestress is \( \sigma_{0z} \)) and prestrain \( \varepsilon_0 \) (z direction \( \varepsilon_{0z} \)). On the basis of dynamic stress and strain, the stress-strain relationship is as follows:

\[
\sigma = \sigma_0 + 2G (\varepsilon - \varepsilon_0) + \lambda \theta I \tag{8}
\]
It is important to note that there is no shear strain component in prestrain, but there is shear strain component in dynamic strain. Static load can change the natural frequency of free vibration of the system by influencing the prestress and prestrain of the strain-isolating pad. In this paper, two kinds of nonlinear working conditions are considered to study the effect of static load on the vibration of the ceramic insulating-strain isolating pad system. The effect of dynamic natural frequency on the material nonlinearity is studied. (2) the nonlinear geometric nonlinearity of the material.

3.1. Influence of Material Nonlinearity on Natural Frequency of Vibration

Considering only the material nonlinearity, the equation (6)–(8) can be established simultaneously, taking the z direction equation and neglecting the small quantity can be obtained.

$$-\rho \omega^2 u_z = P(v) \frac{\partial E(\varepsilon_z - \varepsilon_{0z}) + \sigma_{0z}}{\partial z} \tag{9}$$

The formula (3)–(4) is replaced by a simplified version:

$$-\rho \omega^2 u_z = P(v) K \left(-\varepsilon_{0z}(n-1)\varepsilon_z^{n-2} + 1 + n\varepsilon_z^{n-1}\right) \frac{\partial^2 u_z}{\partial z^2} \tag{10}$$

According to the second step of the small deformation hypothesis $\varepsilon_z \rightarrow \varepsilon_{0z}$, the formula can be approximately reduced to:

$$-\rho \omega^2 u_z = P(v) K(1+\varepsilon_z^{n-1}) \frac{\partial^2 u_z}{\partial z^2} \tag{11}$$

The scaling coefficient associated with the nonlinearity of the material can be set as $R(\varepsilon_{0z})$, $\omega$ is in direct ratio with $R(\varepsilon_{0z})$

$$R(\varepsilon_{0z}) = \sqrt{1+\varepsilon_z^{n-1}}, \quad \omega = \omega_0 R(\varepsilon_{0z}) \tag{12}$$

It can be seen that under the assumption of material nonlinearity, the influence of prestrain on the natural frequency of the free vibration of ceramic thermal insulating-strain isolating pad system is affected by the influence of prestress.

3.2. Influence of geometric Nonlinearity on Natural Frequency of Vibration

If the geometric nonlinearity and the material nonlinearity are considered at the same time, we can combine the formulas (5) (7) and (8), only take the z direction equation and ignore the small quantity.

$$-\rho \omega^2 u_z = P(v) \frac{\partial}{\partial z} \left[1 + \frac{\partial u_z}{\partial z} \right] E(\varepsilon_z - \varepsilon_{0z}) + \frac{\sigma_{0z}}{P(v)} \tag{13}$$

Replace "4" with "simplified", and constantly reduce the number of "high" to "small":

$$...$$
According to the hypothesis of small deformation of vibration, the formula can be approximately reduced to:

$$\begin{align}
-\rho \omega^2 u_z & \approx P(\nu) K \frac{\partial^2 u_z}{\partial z^2} \\
& \left(1 + 3(\varepsilon_z) + \frac{3}{2}(\varepsilon_z)^2 + n(\varepsilon_z)^{n-1} + \frac{3(n+1)}{2}(\varepsilon_z)^n + \frac{n+2}{2}(\varepsilon_z)^{n+1} - \varepsilon_{0z}(n-1)(\frac{1}{2}(\varepsilon_z)^n + (\varepsilon_z)^{n+1})\right)
\end{align}$$

(14)

The scaling coefficient $T(\varepsilon_{0z})$, which can be assumed to be correlated with the material nonlinearity and geometric nonlinearity, $\omega$ is directly proportional to that of the material $^T(\varepsilon_{0z})$.

$$\begin{align}
T(\varepsilon_{0z}) &= \sqrt{1 + 3(\varepsilon_{0z}) + (\varepsilon_{0z})^{n-1} + \frac{n+5}{2}(\varepsilon_{0z})^n} \\
\omega &= \omega_0 T(\varepsilon_{0z})
\end{align}$$

(15)

(16)

It can be seen that under the assumption of material nonlinearity and geometric nonlinearity, the influence of static load on the natural frequency of free vibration of ceramic thermal insulating-strain isolating pad system is affected by the influence of prestress and prestrain.

To sum up, considering that the uniform force load applied on the upper surface of the tile is linearly related to the inner SIP, and the $pS_{tile} = \sigma_{0z}S_{SIP}$, the $S_{tile}$ and $S_{SIP}$ on behalf of tile and SIP surface area, introduce parameter $k_s = S_{tile}/S_{SIP}$, thus

$$\begin{align}
\sigma_{0z} &= pS_{tile} / S_{SIP} = pk_s
\end{align}$$

(17)

Simultaneous (17) and (6), dimensionless load can be obtained $P/K$ and steady state $\varepsilon_{0z}$ corresponding relation:

$$\begin{align}
\frac{P}{K} = \frac{1}{k_s} P(\nu) \left(1 + \varepsilon_{0z}^{n-1}\right)\varepsilon_{0z}
\end{align}$$

(18)

It can be obtained that only corresponding $\varepsilon_{0z}$ to the dimensionless load $P/K$ can be obtained. The correlation formula (12)(16)of theoretical prediction $\omega$ under nonlinear influence can be described as follows.

$$\begin{align}
\omega &= \omega_0 R(\varepsilon_{0z} (P/K)) \\
\omega &= \omega_0 T(\varepsilon_{0z} (P/K))
\end{align}$$

(19)

(20)
Formula (19) is the natural frequency of structural vibration in consideration of material nonlinearity, formula (20) is the natural frequencies of structural vibration considering material nonlinearity and geometric nonlinearity are considered simultaneously. Simply given dimensionless loads $p/K$ and basic frequency without load $\omega_0$, the theoretical value of each case $\omega$ can be obtained.

4. Numerical Results
Ceramic insulation tile thickness $h_{tile} = 5.08\text{cm}$, length of a side $l_{tile} = 15.24\text{cm}$, (SIP) thickness $h_{SIP} = 0.41\text{cm}$, length of a side $l_{SIP} = 12.7\text{cm}$. Elastic Modulus of strain-isolating cushion $K = 1.5 \times 10^6 \text{ Pa}$, Material nonlinear index $n = 1.5$, Poisson ratio $\nu = 0.36$, parameter $P(\nu) = 1.6807$, parameter $k = l_{tile}^2/l_{SIP}^2 = 1.44$, The vibration natural frequencies of the system under different uniform static loads can be calculated. In order to verify the calculation method in this paper, the finite element software is used at the same time. COMSOL5.2 The finite element model is shown in the diagram, in which the thermal insulation tile and the SIP are modeled as a whole and the bottom surface of the SIP is fixed. The elastic modulus of the middle tile is much larger than that of the strain isolation pad. The tetrahedron element is used in the mesh and the strain isolation pad is locally refined.

4.1. Influence of Material Nonlinearity on Natural Frequency of Vibration
Only considering material nonlinearity, take $p/K$ dimensionless load as an independent variable, parameters can be given $P(\varepsilon_{\text{eq}})$. The corresponding natural frequencies of vibration are obtained.

Fig. 2 FEM model and mesh.
Fig. 3 The influence of nonlinear material elasticity on the first four nature frequencies of tile-SIP system.

It can be seen that the calculated results in this paper are basically consistent with the results of finite element numerical simulation, and the deviation of the first four natural frequencies is not more than 2. The larger the load is, the higher the natural frequency of vibration is. When the dimensionless load is 4/15, The first four natural frequencies are linear. On the basis of the natural frequency corresponding to $p=0$, the minimum increase is 6 (first order) and the maximum is 10 (fourth order). Thus, it is necessary to consider the influence of material nonlinearity in the dynamic strength analysis of thermal insulation tile even if the external load is in the normal range.

4.2. Influence of geometric Nonlinearity on Natural Frequency of Vibration
At the same time, considering material nonlinearity and geometric nonlinearity, dimensionless loads are taken into account $p/K$ as an independent variable, we can get $T(\epsilon_s)$. And we eventually get the affected $\omega / \omega_n$. Figure 4 shows the curve of the first 4 order dimensionless natural frequencies with the increase of the load. The dashed line in the diagram is the result of this method, and the hollow point is the finite element simulation result corresponding to the dimensionless load of 2/15, 4/15, 6/15, 8/15, respectively.
Fig. 4 The influence of material and geometric nonlinearities on the first four nature frequencies of tile-SIP system.

It can be seen that the calculated results in this paper are basically consistent with the results of finite element numerical simulation, and the first four order natural frequency deviations are not more than 3. The load is larger because the effects of material nonlinearity and structural geometric nonlinearity are taken into account at the same time. When the dimensionless load is 4/15, the first four natural frequencies increase by a minimum of 10 (the first order and the maximum by 12 (the fourth order)) on the basis of the linear value of \( p=0 \).

For the sake of comparison, figure 5 shows the curves of the base frequency of the natural vibration (the first natural frequency) of the ceramic thermal insulating-strain isolation pad system with uniform static load, taking into account the material nonlinearity and the geometric nonlinearity, respectively. The real line takes into account both the material nonlinearity and the geometric nonlinearity, while the dashed line takes into account only the material nonlinearity. It can be seen that when the load is 80Kpa, the dimensionless load \( p/K \approx 0.053 \). The natural vibration fundamental frequency calculated by the material nonlinear model increases by 9 on the basis of the linear fundamental frequency, and the fundamental frequency calculated by the material geometric nonlinear model increases by 16 percent on the basis of the linear fundamental frequency, so it is applied to the thermal insulation tile. The uniform static load of the variable isolation cushion system will be subjected to the large deformation of the flexible strain isolation pad. The material nonlinear effect and geometric nonlinear effect are introduced, which have a significant influence on the natural vibration characteristics of the thermal insulation tile.
Fig. 5 The influence of nonlinear elasticity on the intrinsic frequency of tile-SIP system under uniform static load.

5. Conclusion
In this paper, the theory of analysis of the natural vibration characteristics of ceramic thermal insulation / strain isolating pad system under uniform static load is developed, which takes into account the material nonlinearity and geometric nonlinearity. A model for calculating the vibration natural frequency of thermal insulation tile on flexible substrate under uniform static load is established. The numerical results are in good agreement with the results of COMSOL finite element simulation. The main conclusions are as follows:

1. Because the elastic modulus of ceramic thermal insulation pad and strain isolation pad are very different, the deformation of the structure is mainly concentrated on the strain isolation pad with lower modulus. In this paper, when the uniform static load is 80Kpa, the deformation of the strain isolation pad reaches more than 5%. This will lead to a variety of nonlinear effects, such as large structural deformation, material nonlinearity, and so on.

2. The large deformation of flexible strain isolation pad under uniform static load increases the stiffness of the structure and leads to the increase of the natural frequency of vibration of the thermal insulation tile. In this paper, when the uniform static load is 80Kpa, Only the fundamental frequency of material nonlinear structure is increased by 9 parts, and the fundamental frequency of structure is increased by 16 parts considering material nonlinearity and geometric nonlinearity.

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