Gravastar model in Randall–Sundrum braneworld

José D V Arbañil, Pedro H R S Moraes and Manuel Malheiro

1 Departamento de Ciencias, Universidad Privada del Norte, Avenida Guardia Peruana 890 Chorrillos, Lima, Peru
2 Departamento de Física, Instituto Tecnológico de Aeronáutica, Centro Técnico Aerospacial, 12228-900 São José dos Campos, São Paulo, Brazil
3 UNINA, Dipartimento di Fisica, Università degli Studi di Napoli Federico II, Napoli I-80126, Italy

E-mail: jose.arbanil@upn.pe, moraes.phrs@gmail.com and malheiro@ita.br

Received 23 April 2019, revised 10 September 2019
Accepted for publication 25 September 2019
Published 1 November 2019

Abstract

In this work we derive a gravastar model in Randall–Sundrum II braneworld scenario. Gravastars (or gravitationally vacuum stars) were proposed by Mazur and Mottola as systems of gravitational collapse alternative to black holes. The external region of the gravastar is described by a Schwarzschild space-time, while its internal region is filled by dark energy. In between there is a thin shell surface with ultrarelativistic matter (sometimes referred to as stiff matter). We obtain solutions for some physical quantities of the braneworld gravastars. Those are compared to original Mazur–Mottola results as well as with some gravastar solutions in alternative gravity theories. The consequences of the braneworld setup in gravastar physics is deeply discussed.

Keywords: $\rho$, $\sigma$, $\varepsilon$

1. Introduction

The Randall–Sundrum II braneworld model (RSII, for short) describes the Universe as a single brane embedded in a five-dimensional (5D) nonfactorizable background geometry [1]. It reproduces properly the Newtonian and General Relativity theories of gravity in the referred regimes [2–5].

RSII has been applied to different areas, yielding remarkable results. Several aspects of RSII cosmology were investigated, for example, in [6–9]. Some constrains to the braneworld quantities have been put from different approaches [10–13].
The astrophysics of stellar objects was also deeply analysed in RSII. Germani and Maartens have firstly shown that in RSII the vacuum exterior of a spherical star is not in general a Schwarzschild space-time, but presents radiative-type stresses generated by 5D graviton effects [14]. In [15], the 4D Gauss and Codazzi equations were solved for an arbitrary static spherically symmetric star. It was shown how the 4D boundary data should be propagated into the 5D bulk in order to get the full space-time geometry. Further properties of compact stars in RSII were studied in [16]. It was found a new branch of stellar configurations that can violate the general relativistic causal limit and that may have an arbitrarily large mass. Moreover, the properties of quark and hadronic stars were analysed in [17].

RSII has also been widely applied in the astrophysics of black holes (BHs), also yielding remarkable results. For instance, the properties of gravitational lensing by BHs in RSII were explored in [18]. In [19], the authors studied the process of gravitational collapse driven by a massless scalar field which is confined to the brane. Further BH analysis in RSII may be checked in [20–22].

In the year of 2016, the first detection of gravitational waves was reported [23] by the Advanced LIGO (Laser Interferometry Gravitational Wave Observatory) Team. It was claimed that the detected gravitational wave sign was generated at a redshift $z \sim 0.09$ by a BH binary system. Later in the same year, it was argued that the signal-to-noise and quality of the referred data were such that there was some room to alternatively interpret such an event, as a gravastar (gravitationally vacuum stars) binary system [24].

Gravastars were proposed in [25] by Mazur and Mottola as systems of gravitational collapse which are alternative to BHs. The external region of a gravastar is described by a Schwarzschild space-time, such that $p = \rho = 0$, with $p$ and $\rho$ being the pressure and matter-energy density, respectively, whereas its surface is a thin shell of ultrarelativistic matter, with $p = \rho$. Its internal region is filled by dark energy, with $p = -\rho$.

In [26], it was shown that it is possible to discern a gravastar from a BH of the same mass due to their different quasi-normal modes. The problem of ergoregion instability for the viability of gravastars was also investigated in [27, 28]. In [29], the possibility of distinguishing BHs and gravastars using the properties of their accretion disks was considered. Observational constraints were put in gravastars from well-known BH candidates [30]. Further observational distinguishment between BHs and gravastars were discussed in [31], in which it was argued that high-resolution very-long-baseline-interferometry observations can contribute on this regard in near future.

Moreover, it should be remarked that by means of the usual Tolmann–Oppenheimer–Volkoff equation, it was shown that gravastars material content cannot be described by perfect fluids [32]. Instead, they should have anisotropic pressures.

On this regard, it is well-known that the 5D set up of RSII and other braneworld models induce anisotropy in brane objects, such that it might be interesting and valuable to investigate gravastars in the braneworld. In the present paper we will obtain and investigate gravastars solutions in RSII.

2. Braneworld field equations

2.1. Basic equations

The field equations of RSII in terms of an effective energy-momentum tensor read [14, 33, 34]

$$G_{\mu \nu} = k^2 T_{\text{eff}}^{\mu \nu}.$$  (1)
with $k^2 = 8\pi G$, $G$ is the newtonian gravitational constant, the speed of light $c = 1$ and such that the effective total energy density and pressure, anisotropic stress and energy flux read, respectively:

$$\rho^{\text{eff}} = \rho + \frac{1}{2\lambda} \left( \rho^2 + \frac{12}{k^2} \mathcal{U} \right),$$  \hspace{1cm} (2)

$$p^{\text{eff}} = p + \frac{1}{2\lambda} \left[ \rho(p + 2p) + \frac{4}{k^2} \mathcal{U} \right],$$  \hspace{1cm} (3)

$$\pi^{\text{eff}}_{\mu\nu} = \frac{6}{k^4 \lambda} \mathcal{P}_{\mu\nu},$$  \hspace{1cm} (4)

$$q^{\text{eff}}_{\mu} = \frac{6}{k^4 \lambda} \mathcal{Q}_{\mu},$$  \hspace{1cm} (5)

with $\lambda$ being the brane tension and the bulk cosmological constant was taken such that the brane cosmological constant is null. Moreover, $\mathcal{U}, \mathcal{Q}_\mu$ and $\mathcal{P}_{\mu\nu}$ represent respectively the nonlocal energy density, the nonlocal energy flux and the nonlocal anisotropic stress.

For a static spherically symmetric space-time, the nonlocal energy flux and nonlocal anisotropic stress become:

$$Q_\mu = 0,$$  \hspace{1cm} (6)

$$\mathcal{P}_{\mu\nu} = \mathcal{P} \left( r_\mu r_\nu - \frac{1}{3} h_{\mu\nu} \right),$$  \hspace{1cm} (7)

with $r^\mu$ being a unit radial vector and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$, such that $g_{\mu\nu}$ is the metric and $u_\mu$ is the four-velocity.

### 2.2. Static structure equations

With the aim of describing the properties of a spherically symmetric static fluid distribution, it is considered the line element in Schwarzschild coordinates:

$$ds^2 = -A^2(r)dt^2 + B^2(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$  \hspace{1cm} (8)

with $A(r)$ and $B(r)$ being metric potentials.

The nonzero components of the Einstein’s field equations on the brane for the metric above are:

$$\frac{1}{r^2} \left[ \frac{A''}{A} + \frac{A'}{rA} + \frac{A'B'}{AB} - \frac{B'}{rB} \right] = k^2 \rho^{\text{eff}},$$  \hspace{1cm} (9)

$$\frac{1}{r^2} \left[ \frac{A''}{A} + \frac{A'}{rA} - \frac{A'B'}{AB} + \frac{B'}{rB} \right] = -k^2 p^{\text{eff}}_r,$$  \hspace{1cm} (10)

$$\frac{1}{B^2} \left[ \frac{A''}{A} + \frac{A'}{rA} \right] = k^2 p^{\text{eff}}_t,$$  \hspace{1cm} (11)

with primes denoting radial derivatives. The functions $\rho^{\text{eff}}, p^{\text{eff}}_r$ and $p^{\text{eff}}_t$ are given by the equalities:
\[ \rho_{\text{eff}} = \rho \left(1 + \frac{\rho}{2\lambda}\right) + \frac{6U}{k^4\lambda}, \]  
\[ P_{\text{eff}}^r = p + \frac{\rho}{2\lambda} (\rho + 2p) + \frac{2U}{k^4\lambda} + \frac{4P}{k^4\lambda}, \]  
\[ p_{\text{eff}}^t = p + \frac{\rho}{2\lambda} (\rho + 2p) + \frac{2U}{k^4\lambda} - \frac{2P}{k^4\lambda}. \]  
From \( p_{\text{eff}}^r \neq p_{\text{eff}}^t \), it can be understood that the effects of the extra dimensions produce anisotropy in the fluid contained in the star.

For our purposes, it will also be of great importance to know the covariant derivative of both the energy-momentum tensor and effective energy-momentum tensor on the brane. The conservative equations \( \nabla^\nu T_{\mu\nu} = 0 \) and \( \nabla^\nu T_{\mu\nu}^{\text{eff}} = 0 \) for the metric (8) read, respectively

\[ p' + \frac{A'}{A} (\rho + p) = 0, \]  
\[ U' + \frac{2A'}{A} (2U + P) + 2p' + \frac{6P}{r} = -\frac{k^4}{2} \rho' (\rho + p). \]  
We remark here, for the sake of completeness, that \( \rho(r), p(r), P(r) \) and \( U(r) \), as well as \( A(r) \) and \( B(r) \), depend on the radial coordinate only.

### 3. Gravastar in the Braneworld

#### 3.1. General remarks

The static structure of the gravastar under study is envisaged in the following form: the interior of the object is surrounded by a thin shell of ultra-relativistic fluid, while the outer space-time is described by a vacuum exterior solution. The three regions aforementioned are structured considering the following equations of state:

- Interior: \( 0 \leq r < r_1; \ p = -\rho \),
- Shell: \( r_1 < r < r_2; \ p = \rho \),
- Exterior: \( r_2 < r; \ p = \rho = 0 \),

with \( r_1 \) and \( r_2 \) being the interior and exterior radii of the gravastar, respectively.

In addition, with the aim of comparing our results to those obtained by Mazur and Mottola [25], we consider \( U = 0 \) in both the interior and shell of the gravastar. Moreover, we regard that the outer space-time is described by a Schwarzschild vacuum solution.

#### 3.2. Interior of the gravastar

First of all, it is important to say that in the interior region of the gravastar, from equation (15), \( p = -\rho = -\rho_0 = \text{cte} \).

Now, following [25], we define the potential metric \( B^2 \) in such a form that:

\[ B^{-2} = \left(1 - I_0^2 r^2\right), \]  
with \( I_0 \) being a constant. Considering equations (17) in (9), one obtains that \( I_0 \) and \( \rho_0 \) are connected through
\[ I_0 = \sqrt{\frac{k^2}{3} \left(\rho_0 + \rho_0^2\right)}. \]  

(18)

On the other hand, by replacing the nonlocal pressure

\[ \mathcal{P} = -\left(\frac{k^2 \lambda}{2r}\right) \frac{k_1}{1 - k_1 B} B', \]  

(19)

with constant \( k_1 \), in the sum of the equations (9) and (10), we can obtain an equation that relates the two potential metrics \( A \) and \( B \) as

\[ A = k_2 B^{-\frac{1}{k_1}}, \]  

(20)

where \( k_2 \) represents an integration constant. From equation (20), for \( k_1 = 0 \) we determine that the interior region is described by the de Sitter metric.

We note that another form for the function \( \mathcal{P} \) can be found by integrating equation (16), resulting in

\[ \mathcal{P} = \frac{k^3}{A r^3}, \]  

(21)

with constant \( k_3 \).

Evidently, the functions presented in (19) and (21) must be equal, thus, we find that the constants \( k_1 \) and \( k_3 \) are related through:

\[ k_1 = \frac{k_3^3}{k_3^3 - k^2 \lambda I_0^2 (1 - r^2 I_0^2)^\frac{1}{k_3}}. \]  

(22)

In order to obtain regular solutions we need to have:

\[ k^2 \lambda I_0^2 (1 - r^2 I_0^2)^\frac{1}{k_3} \neq k_3^3. \]  

(23)

From equation (22), note that if \( k_3 = 0 \) then \( k_1 = 0 \) and if \( \lambda \to \infty \) then \( k_1 = 0 \) \((k_3 = 0)\). In both cases we derive \( \mathcal{P} = 0 \), indicating that the Mazur–Mottola case [25] is obtained.

3.3. Shell

Such as aforementioned, we consider that the pressure and energy density of the fluid contained in the shell are related through \( p = \rho \).

In order to determine \( \mathcal{P} \), we replace (15) in (16) yielding:

\[ \mathcal{P} = -\frac{k^4}{3} \rho^2. \]  

(24)

Now, as well as it is considered by Mazur and Mottola [25], let us introduce a dimensionless variable \( \xi \) as

\[ \xi = k^2 r^2 \rho. \]  

(25)

Replacing equation (25) together with equations (24) and (15) into (9) and (10), respectively, these can be rewritten as:

\[ \frac{dr}{r} = \frac{d \left( B^{-\frac{2}{3}} \right)}{1 - \frac{1}{\xi} - \xi - \frac{\xi^2}{2 \lambda k^2}}, \]  

(26)
\[
\frac{d \left( B^{-2} \right)}{B^{-2}} = - \left[ 1 - \frac{1}{B^2} - \xi - \frac{\xi^2}{6 \lambda k r^2} \right] \frac{d \xi}{\xi}. \tag{27}
\]

We can note that it is difficult to obtain analytical solutions from these field equations. Nevertheless, this can be achieved by taking into account that in the thin shell limit, \(0 < B^{-2} \ll 1\). Under this limit, the integration of equation (27) yields

\[
B^{-2} \approx \epsilon \left[ \frac{\xi^2}{6 \lambda k r^2} + \xi + 1 \right] \frac{d \xi}{\xi}, \tag{28}
\]

where \(\epsilon\) is an integration constant. Moreover, since \(B^{-2} \ll 1\) we need that \(\epsilon \ll 1\) also.

Finally, by making use of equations (26) and (28), we obtain:

\[
dr \approx -\epsilon r \left[ \frac{\xi^2}{6 \lambda k^2 r^2} + \xi + 1 \right] \frac{d \xi}{\xi^2}. \tag{29}
\]

### 3.4. Exterior of the gravastar

For this region, we consider that \(U = P = 0\). This ensures that the exterior space-time is described by the Schwarzschild vacuum metric. Thus, the gravastar outer space-time is depicted by the line element:

\[
d s^2 = - F dr^2 + F^{-1} d r^2 + r^2 \left( d \theta^2 + \sin^2 \theta d \phi^2 \right), \tag{30}
\]

with

\[
F = 1 - \frac{2 MG}{r}, \tag{31}
\]

where \(M\) represents the total mass of the gravastar.

### 4. Junction conditions

As previously shown, the study of gravastars involves an inner region and an outer region separated by a shell of matter which we shall denominate \(\Sigma\). In order to realign the physical and geometric quantities of the inner and outer regions with the magnitudes of surface, we will use the conditions of continuity of Israel–Darmois \([37, 38]\). They say that the metric coefficients are continuous in \(\Sigma (r = R)\), but their derivatives are not continuous at this point.

It is possible to determine the surface energy-momentum tensor with the help of the Lanczos equation \([39]\):

\[
S^i_j = \frac{1}{8 \pi} \left( \kappa^j_i - \delta^j_i \kappa^k_k \right), \tag{32}
\]

with the Latin indexes running as \(i, j = t, \theta, \phi\). The factor \(\kappa_{ij}\) depicts the discontinuity in the extrinsic curvature \(K_{ij}\), with \(\kappa_{ij} = K^+_ij - K^-ij\), where the signs \(-\) and \(+\) correspond respectively to the interior and exterior regions. The extrinsic curvature is defined by:

\[
K^\pm_{ij} = -n^\pm_\beta \left( \partial e^\beta_i e^\delta_j + \Gamma^\delta_\mu\nu e^\mu_i e^\nu_j \right), \tag{33}
\]
with \( \epsilon^\mu = \frac{\partial x^\mu}{\partial \xi^i} \), where \( \xi^i \) represents the coordinate on the shell, \( n^\mu_+ \) depicts the normal vector to the surface and \( \Gamma^\beta_{\mu
u} \), refers to the Christoffel symbols.

Once considered \( S_i^j = \text{diag}(\sigma, -v, -v) \), with \( \sigma \) and \( v \) being respectively the surface energy density and the surface pressure, the Lanczos equation can be placed on the form:

\[
\sigma = -\frac{1}{4\pi} \kappa^\theta_\theta,
\]

(34)

\[
v = \frac{1}{8\pi} (\kappa^i_i + \kappa^\theta_\theta).
\]

(35)

Using equations (17), (31) and (34), the thin shell mass can be found using the equality:

\[
m_s = 4\pi R^2 \sigma = -R \sqrt{1 - \frac{2M}{R}} + R \sqrt{1 - \frac{I_0^2 R^2}{R^2}}.
\]

(36)

Considering (18), from equation (36) we obtain that the total mass of the gravastar is given by:

\[
M = \frac{R}{2} - \frac{R}{2} \left[ \sqrt{1 - \frac{k^2 R^2}{3}} \left( \rho_0 + \frac{\rho_0^2}{2\lambda} \right) - \frac{m_s}{R} \right]^2.
\]

(37)

It is important to mention that if we consider \( \lambda \to \infty \), equation (37) is reduced to the same equation than those found in [35] and [36], in their particular cases.

5. **Some physical features of the model**

It is important to highlight that the shell of the gravastar is limited by the interfaces \( R_1 = R \) e \( R_2 = R + \epsilon \), thus connecting the inner space-time with the outer space-time. In order to analyze the principal physical characteristics of the matter in the shell, some definitions used by Mazur and Mottola [25] are considered throughout this section.

5.1. **The proper thickness of the shell**

The proper thickness of the shell is determined from

\[
\ell = \int_{R_1}^{R_2} B \, dr.
\]

(38)

Using (28) and (29) in the equation above, it becomes

\[
\ell \approx \epsilon^{1/2} R \int_{\xi_1}^{\xi_2} \xi^{-3/2} d\xi \approx 2 \epsilon^{3/2} R,
\]

(39)

where we notice that \( \ell \) is very small in relation to \( R \). This last result is the same found by Mazur and Mottola [25]. In this way we understand that the 5D bulk has no effects on the proper thickness of the shell.

5.2. **Energy within the shell**

The energy inside the thin shell is determined as
\[ \mathcal{E} = 4\pi \int_{R_1}^{R_2} \rho^{eff} r^2 dr. \]  
(40)

Considering equations (25) and (29) in (40), we obtain
\[ \mathcal{E} = \frac{4\pi \epsilon R}{k^2} \int_{\xi_1}^{\xi_2} \left[ 1 + \frac{\xi}{2\lambda k^2 r^2} \right] \left[ 1 + \frac{1}{\xi} + \frac{\xi}{6\lambda k^2 r^2} \right] d\xi, \]  
(41)

which integrated yields
\[ \mathcal{E} = \frac{4\pi \epsilon R}{k^2} \left[ \ln(\xi) + \frac{1}{9} \left[ \frac{2}{2\lambda R^2 k^2} + 1 \right] \xi + \frac{1}{2\lambda R^2 k^2} \xi^2 + \frac{1}{2\lambda R^2 k^2} \xi^3 \right] \bigg|_{\xi_1}^{\xi_2}. \]  
(42)

Once \( \epsilon << 1 \), the energy \( \mathcal{E} \) until first order of \( \epsilon \) is given by
\[ \mathcal{E} \simeq \frac{8\pi \epsilon^2 R}{k^2} \left[ 1 + \left( \frac{1}{2} + \frac{2\xi_1}{3} \right) \frac{1}{2\lambda R^2 k^2} + \frac{\xi_1^2}{6} \left( \frac{1}{2\lambda R^2 k^2} \right)^2 \right]. \]  
(43)

The resulting nonlinear equation provides information about the energy within the shell. It indicates that the energy is directly proportional to \( \epsilon^2 \). Note that the energy in the shell, equation (43), is larger than the one derived by Mazur and Mottola [25]. This is due to the effects of the 5D bulk on the brane which helps to increase the energy within the shell. In comparison with an alternative gravity gravastar, such as the one found in \( f(R,T) \) gravity for instance, it can be noted a stronger dependence of the energy with \( \epsilon \) in the braneworld model with the obtained by \( f(R,T) \) gravity [35].

### 5.3. Entropy of the shell

We calculate the entropy in the shell as
\[ S = 4\pi \int_{R_1}^{R_2} s \cdot r^2 B dr, \]  
(44)

where \( s \) represents the local specific entropy density, given by:
\[ s = \alpha \frac{k_B}{\hbar} \sqrt{\frac{p}{2\pi G}} \]  
(45)

with \( \alpha \) being a dimensionless constant, \( k_B \) representing the Boltzmann constant and \( \hbar = h/(2\pi) \) where \( h \) is the Planck constant.

Considering equations (25), (28) and (29) in (44), (44) becomes:
\[ S \simeq \frac{\alpha k_B}{\hbar G} \epsilon^{1/2} R^2 \ln \left( \frac{\xi_1}{\xi_2} \right). \]  
(46)

As in [25], by taking into account that \( \xi_1/\xi_2 = 1 + \mathcal{O}(\epsilon) \) as well as equation (39), we have that equation (46) yields:
\[ S \simeq \frac{\alpha k_B R\xi}{2\hbar G}. \]  
(47)

From equation (47), we can see that the entropy depends directly on the proper thickness of the shell. This result is equal to the one derived by Mazur and Mottola [25]. This shows that the 5D bulk does not affect the entropy of the shell. Comparing with gravastars obtained in
$f(R,T)$ gravity [35], we note that the entropy has a substantial dependence on $\epsilon$ as $\epsilon^2$ while in $f(R,T)$ gravity, it goes approximately with $\epsilon^3$.

6. Discussion

With recent advances in gravitational wave observational astronomy, several studies of gravastars as possible sources of gravitational radiation have been made. Let us briefly review some of those important contributions.

In [40], it was deeply discussed how the presence or absence of an event horizon can produce qualitative differences in the gravitational waves emitted by ultracompact objects. In [41], it was shown that the gravitational sign emitted by a gravastar could provide a unique signature of the horizonless nature of such an object. In [42] it was discussed how a measurement of the tidal deformability from the gravitational-wave detection of a compact-binary inspiral can be used to constrain gravastars.

A further alternative to detect gravastars was proposed in [43], through their gravitational lensing. Naturally, the recently reported results regarding the M87 BH shadow [44] can also in near future help us to distinguish BHs and gravastars.

We have constructed in the present paper gravastar solutions in RSII. Besides [35, 36], gravastars have also been constructed in alternative gravity theories in [45, 46]. Alternative gravity theories have been mostly used to try to account for the cosmological dark sector of the universe. In fact, it is possible, through modified gravity, to describe the galactic and cosmological scales of the universe without dark matter and dark energy [47–56].

Although RSII has been proposed as an alternative to the hierarchy problem, it also works pretty well in cosmology. On this regard, besides [6–9], one can also check [57, 58].

The investigation of gravastars in braneworld scenarios can give us some new insights on both the geometrical and physical features of these objects and the braneworld setup itself.

Particularly, here we have derived different gravastar physical parameters in the RSII, as Mazur and Mottola have done in standard gravity scenario. We noted that both the mass and shell energy are altered with respect to the Mazur–Mottola original results due to the presence of the brane tension. On the other hand, the proper thickness of the shell and the shell entropy are not altered due to RSII configuration. The present model can be used to investigate the possible 5D effects in some physical phenomenon that arise in the study of gravastars. An extent of the present formalism can be used to analyze the ergoregion instability in rotating gravastars [27, 28], since the projections on the brane could help the stability of gravastars against the ergoregion instability, such as they help the fluid pressure of compact stars to support more mass against the gravitational collapse [16, 17].

Let us further analyse the physical features of our model. It should also be interesting to compare our results with the present literature in alternative gravity gravastars, such as the model presented in [35] within the $f(R,T)$ gravity [59], for which $f(R,T)$ is a general function of the Ricci scalar $R$ and trace of the energy-momentum tensor $T$.

The proper thickness $\ell$ of the shell found in our model is directly proportional to $R$, the radius of the inner shell. It is also proportional to the thickness of the shell as $\ell \sim \epsilon^{1/2}$. Since $\epsilon << 1$, the latter proportionality indicates that $\ell \ll R$.

The entropy we have obtained is gradually increasing with respect to $\epsilon$, a result also obtained in $f(R,T)$ gravity [35]. On the other hand, the energy within the shell has a stronger dependence on $\epsilon$ as $\sim \epsilon^2$ when compared to the $f(R,T)$ gravity result, which reads $\sim \epsilon$.

To finalize we remark that our results retrieve the Mazur–Mottola results in the regime $\lambda^{-1} \to \infty$, as it is expected in braneworld features.
Acknowledgment

PHRSM would like to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Grant 2018/20689-7. The authors thank FAPESP for financial support under the projects 2013/26258-4.

ORCID iDs

José D V Arbañil © https://orcid.org/0000-0002-2623-4900
Pedro H R S Moraes © https://orcid.org/0000-0002-8478-5460

References

[1] Randall L and Sundrum R 1999 Phys. Rev. Lett. 83 4690
[2] Maartens R 2004 Living Rev. Relativ. 7 7
[3] Garriga J and Tanaka T 2000 Phys. Rev. Lett. 84 2778
[4] Figueiras P and Wiseman T 2011 Phys. Rev. Lett. 107 081101
[5] Kim Y, Lee C O, Lee I and Lee J 2004 J. Korea Astron. Soc. 37 1
[6] Ramírez E and Liddle A R 2004 Phys. Rev. D 69 083522
[7] Holanda R F L, Silva J W C and Dahia F 2013 Class. Quantum Grav. 30 205003
[8] Hebecker A and March-Russell J 2001 Nucl. Phys. B 608 375
[9] Meekins S and Whittingham I B 2014 J. Cosmol. Astropart. Phys. JCAP06(2014)018
[10] Tsujikawa S and Liddle A R 2004 J. Cosmol. Astropart. Phys. JCAP03(2004)001
[11] Liddle A R and Smith A J 2003 Phys. Rev. D 68 061301
[12] Yagi K, Tanahashi N and Tanaka T 2011 Phys. Rev. D 83 084036
[13] Moraes P H R S and Miranda O D 2014 Astrophys. Space Sci. 354 645
[14] Germani C and Maartens R 2001 Phys. Rev. D 64 124010
[15] Visser M and Wiltshire D L 2003 Phys. Rev. D 67 104004
[16] Lugones G and Arbañil J D V 2017 Phys. Rev. D 95 064022
[17] Lugones G and Arbañil J D V 2015 Astron. Nachr. 336 876
[18] Bin-Nun A Y 2010 Phys. Rev. D 81 123011
[19] Wang D and Choctuik M W 2016 Phys. Rev. Lett. 117 011102
[20] Abdolrahimi S, Cattoën C, Page D N and Yahgoobpour-Tari S 2013 Phys. Lett. B 720 405
[21] Abdolrahimi S, Cattoën C, Page D N and Yahgoobpour-Tari S 2013 J. Cosmol. Astropart. Phys. JCAP06(2013)039
[22] Tanahashi N and Tanaka T 2008 J. High Energy Phys. JHEP03(2008)041
[23] Abbott B et al 2016 Phys. Rev. Lett. 116 061102
[24] Chirenti C B M H and Rezzolla L 2016 Phys. Rev. D 94 084016
[25] Mazur P and Mottola E 2004 Proc. Natl Acad. Sci. USA 101 9545
[26] Chirenti C B M H and Rezzolla L 2007 Class. Quantum Grav. 24 4191
[27] Cardoso V, Paniz P, Cadoni P and Cavaglia M 2008 Phys. Rev. D 77 124044
[28] Chirenti C B M H and Rezzolla L 2008 Phys. Rev. D 78 084011
[29] Harko T, Kovács Z and Lobo F S N 2009 Class. Quantum Grav. 26 215006
[30] Broderick A E and Narayan R 2007 Class. Quantum Grav. 24 659
[31] Sakai N, Saida H and Tanaki T 2014 Phys. Rev. D 90 104013
[32] Cattoën C, Faber T and Visser M 2005 Class. Quantum Grav. 22 4189
[33] Shiomizu T, Maeda K and Sasaki M 2000 Phys. Rev. D 62 024012
[34] Maartens R 2000 Phys. Rev. D 62 084023
[35] Das A, Ghosh S, Guha B K, Das S, Rahaman F and Ray S 2017 Phys. Rev. D 95 124011
[36] Banerjee A, Rahaman F, Islam S and Govender M 2016 Eur. Phys. J. C 76 34
[37] Israel W 1966 Nuovo Cimento 44B 1
Israel W 1967 Nuovo Cimento 48B 463 (erratum)
[38] Darmois G 1927 Memorial des Science Mathematiques XXV, Fisticule XXV (Gauthier-Villars, Paris, France)
[39] Lanczos K 1924 Ann. Phys., Berlin 379 518
[40] Pani P, Berti E, Cardoso V, Chen Y and Norte R 2009 Phys. Rev. D 80 124047
[41] Pani P, Berti E, Cardoso V, Chen Y and Norte R 2010 Phys. Rev. D 81 084011
[42] Uchikata N, Yoshida S and Pani P 2016 Phys. Rev. D 94 064015
[43] Kubo T and Sakai N 2016 Phys. Rev. D 93 084051
[44] The Event Horizon Telescope Collaboration 2019 Astrophys. J. Lett. 875 L1
[45] Bhar P 2005 Astrophys. Space Sci. 354 457
[46] Rahaman F, Chakraborty S, Ray S, Usmani A A and Islam S 2015 Int. J. Theor. Phys. 54 50
[47] Capozziello S, Cardone V F and Troisi A 2007 Month. Not. R. Astron. Soc. 375 1423
[48] Zlosnik T G, Ferreira P G and Starkman G D 2007 Phys. Rev. D 75 044017
[49] Nojiri S and Odintsov S D 2006 Phys. Rev. D 74 086005
[50] Joyce A, Lombriser L and Schmidt F 2016 Ann. Rev. Nucl. Part. Sci. 66 95
[51] Kase R and Tsujikawa S 2019 Int. J. Mod. Phys. D 28 1942005
[52] Nojiri S and Odintsov S D 2005 Phys. Lett. B 631 1
[53] Woodard R 2007 Lect. Not. Phys. 720 403
[54] Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2006 Phys. Rev. D 73 084007
[55] Kase R and Tsujikawa S 2018 Phys. Rev. D 97 103501
[56] Amendola L, Polarski D and Tsujikawa S 2007 Phys. Rev. Lett. 98 131302
[57] Nozari K and Shoukrani M 2009 Mod. Phys. Lett. A 24 3205
[58] Barros B J and Nunes N J 2016 Phys. Rev. D 93 043512
[59] Harko T, Lobo F S N, Nojiri S and Odintsov S D 2011 Phys. Rev. D 84 024020