A rectangular radio pulse propagation in a selectively absorbing medium

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Abstract. A pulse with rectangular envelope propagation, in which carrier frequency is close to the medium spectral absorption line’s one frequency, is considered. It is shown that when the signal carrier frequency is shifted relative to the spectral line centre, the primary interference and the response signal can lead to the total signal significant oscillations over time.

1. Introduction

In [1], general relations were obtained for a pulse propagating in a selectively absorbing medium. These results’ importance is associated with the fact that their application allows one to obtain answers to general questions related to the causality principle application and the light limiting the vacuum speed principle for the information transmission in physical systems (see [2-24]). In this paper, these general relationships are used to analyse a rectangular envelope signal propagation in a selectively absorbing medium.

2. Methods

The final formula for the signal $A(z,t)$ envelope $E(z,t) = A(z,t) \exp(-i\omega_0 t) + A^*(z,t) \exp(i\omega_0 t)$, derived in [1], is

$$A(z,t) = A^{(p)}(z,t) + A^{(a)}(\xi, t),$$

$$A^{(p)}(z,t) = A^{(0)}(t_s) \exp(ik_0 z),$$

$$A^{(a)}(\xi, t) = \exp(ik_0 z) \sum_{n=0}^{\infty} \frac{\alpha_n(\xi)}{n!(n-1)!} \int_{-\infty}^{t} (t - t_1)^{n-1} A^{(0)}(t_1) dt_1, \quad \tau_{s}(z,t) = t - \tau_{s}(z),$$

$$\tau_{s}(z) = z/c.$$
\( \alpha_2(\xi) = -\gamma_2^{\xi} + \gamma_1^{\xi} \xi^2 \)
\( \alpha_3(\xi) = -\gamma_3^{\xi} + 3\gamma_1^{\xi} \gamma_2^2 - \gamma_1^3 \xi^3 \)
\( \alpha_4(\xi) = -\gamma_4^{\xi} + (4\gamma_1^{\xi} \gamma_3^2 + 3\gamma_2^2) \xi^2 - 6\gamma_1^{\xi} \gamma_2^3 + \gamma_1^4 \xi^4 \)

where for the line Lorentzian profile \( \gamma_1 = 1, \gamma_2 = -2, \gamma_3 = 6, \gamma_4 = -24 \), \( \tau_f = 2/\Delta\Omega_{1/2} \) is the spectral line coherence time (\( \Delta\Omega_{1/2} \) is the spectral line full width at a level 1/2 of the maximum).

3. Results and discussions

To analyse a non-zero signal duration effect in time on the exciting field, we first consider a rectangular pulse propagation, in which carrier frequency coincides with the spectral line centre frequency. Let the signal time dependence at the initial point \( z = 0 \) have the unit amplitude and duration a rectangular pulse form \( T_1 \), that is

\[
A^{(0)}(t) = \begin{cases} 1, & t \in [0, T_1], \\ 0, & t \notin [0, T_1]. \end{cases}
\]

Then series (1) can be rewritten as (\( \eta(x) \) is the Heaviside function)

\[
A^{(n)}(\xi, t) = \sum_{n=1}^{\infty} \frac{\alpha_n(\xi)}{(n!)^2} \left[ \left( \frac{t_+}{\tau_f} \right)^n \eta \left( \frac{t_+}{\tau_f} \right) - \left( \frac{t_+ - T_1}{\tau_f} \right)^n \eta \left( \frac{t_+ - T_1}{\tau_f} \right) \right].
\]  

The calculations results performed for the spectral line Lorentzian form factor using formula (3) at the parameter value \( T_1 = 0.5\tau_f \) are shown in figure 1.

**Figure 1.** The signal amplitude dependence on time at the substance layer optical thickness different values \( \xi \) (the primary signal envelope is a rectangular pulse).
In figure 1, the ordinate shows the signal amplitude $A$ at the layer optical thickness in various values $\xi = 0.5, 1, 2, 4, 6$. The solid lines denote the numerical data, and the dashed line denotes the applying formula results (3). For the most part, the dashed lines merge with the solid ones; therefore, the agreement between the analytical and numerical results at the parameter values $T_i / \tau_i$ less than or of the unity order can be considered quantitative. Let us note two curious circumstances. First, the primary signal (rectangular pulse) is distinguishable against the response signal background, and their interference does not lead to the total signal oscillations in time. This is due to both signal's complex envelope realness and their carrier frequency identity. Second, the decrease rate in the response signal against the primary signal background is almost the same as at its end.

Until now, we have identified the signal carrier frequency and the spectral line frequency. Nevertheless, the proposed estimates can be used for a signal whose carrier frequency does not coincide with the line frequency, as long as the carrier shift is small compared to its frequency (but not necessarily small compared to the signal bandwidth). In this case, it is sufficient to simply include an additional phase factor in the expression for the signal complex envelope.

Therefore, the calculations next series (figure 2-4) was carried out to study the signal carrier frequency shift effect relative to the spectral gain line centre frequency. Let the signal time dependence at the initial point $z = 0$ have the form

$$E(t) = A_i^{(0)}(t) \exp(-i \omega t) + A_i^{(0)*}(t) \exp(i \omega t),$$

$$A_i^{(0)}(t) = \begin{cases} 1, t \in [0, T_i], \\ 0, t \notin [0, T_i], \end{cases}$$

$$\omega_i = \omega_0 + \Delta \omega.$$

Signal (4) is unit amplitude a rectangular pulse and duration $T_i$ with the carrier frequency $\omega_0$ (which does not necessarily coincide with the gain line centre frequency $\omega_0$).

To use relations (1), it is necessary to introduce the function $A^{(0)}(t)$ using the relation

$$A^{(0)}(t) = \begin{cases} \exp((-i \Delta \omega t), t \in [0, T_i], \\ 0, t \notin [0, T_i], \end{cases}$$

where $\Delta \omega = \omega_0 - \omega_0$ is the packet carrier frequency shift relative to the spectral line centre. Now series (1) can be rewritten as

$$A^{(n)}(\xi, t) = \sum_{n=1}^{\infty} \left[ \exp(-i \Delta \omega t) - \sum_{j=0}^{n-1} (-i \Delta \omega t)^j / j! \right] \eta(t) -$$

$$- \left[ \exp(-i \Delta \omega t) - \exp(-i \Delta \omega T_i) \sum_{j=0}^{n-1} (-i \Delta \omega (t - T_i))^j / j! \right] \eta(t - T_i) \frac{\alpha_n(\xi)}{n!(-i \Delta \omega \tau_i)^n}.$$  

The calculations results were carried out using formula (6) with the parameters' values $T_i = 0.5 \tau_i$ and $\Delta \omega = 6 \tau_i^{-1}$ are shown in figure 2. Similar calculations for the case of $T_i = 0.5 \tau_i$, $\Delta \omega = 18 \tau_i^{-1}$ and are shown in figure 3, and for the case of $T_i = 0.5 \tau_i$ and $\Delta \omega = 30 \tau_i^{-1}$ - in figure 4. In these figures, the numerical calculation data are indicated by solid lines and applying the formula (6) results are indicated by a dashed line. For the most part, the dashed lines merge with the solid ones; therefore, the agreement
between the analytical and numerical results at the parameter values \( t_s / \tau_l \) less than or of the unity order can be considered quantitative.

**Figure 2.** The signal amplitude dependence on time at the material layer optical thickness different values \( \xi \) (primary signal carrier shift \( \Delta \omega = 6\tau_l^{-1} \)).

**Figure 3.** The signal amplitude dependence on time at the material layer optical thickness different values \( \xi \) (primary signal carrier shift \( \Delta \omega = 18\tau_l^{-1} \)).
Figure 4. The signal amplitude dependence on time at the material layer optical thickness different values $\xi$ (primary signal carrier shift $\Delta \omega = 30 \tau_1^{-1}$).

4. Conclusion
The main conclusion that can be drawn from considering figures 2-4, is concluded that in the signal carrier frequency a shift case relative to the spectral line centre, the primary signal and the response signal interference can lead to the total signal significant oscillations over time. It can be verified that this is indeed interference by observing that temporal oscillations only occur in the area in which the primary signal and the response signal overlap (no oscillations are observed at the primary signal end). In addition, the oscillations' frequency corresponds to the primary signal and the response signal beat's time $\Delta \omega$, and their amplitude decreases under the response signal amplitude. From this viewpoint, it is not surprising that the oscillations' amplitude does not exceed either the primary signal amplitude or the response signal amplitude.

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