Effects on the hadron propagators due to $k_\mu k_\nu$ terms in the vector meson propagator

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Abstract

In an approximation where the baryon current conservation is violated, the contribution of the $k_\mu k_\nu$ terms in the vector meson propagator may not vanish. Their effects on the baryon and meson spectral functions and on the consequences of self-consistency are studied in the relativistic self-consistent Hartree-Fock approximation by means of the $\sigma - \omega$ model. Two cases where the $k_\mu k_\nu$ terms are and are not neglected are compared. It is found that there is a marked change in the baryon spectral function which becomes more peaked in the latter case. Such a change remains even by a proper readjustment of parameters. The effects of self-consistency in the $\sigma - \omega$ model are qualitatively the same in both cases, though quantitatively there is some significant difference.
1 Introduction

If baryons couple only with $\omega$-mesons ($\omega$ case), Krein, Nielsen, Puff and Wilets [1] found that in the relativistic self-consistent Hartree-Fock (RSCHF) [2, 3] calculation of the renormalized baryon propagator, its spectral function $A_R(\kappa)$ can be negative for some real values of $\kappa$. They emphasized that this result is unacceptable. The spectral representation they considered is of the form

$$G(k) = -\int_{-\infty}^{+\infty} d\kappa A_R(\kappa) \frac{\gamma_{\mu}k_{\mu} + i\kappa}{k^2 + \kappa^2 - i\varepsilon}.$$ 

Since $A_R(\kappa)$ represents the probability that a state of mass $|\kappa|$ is created, it must be non-negative. They suggested that it might be due to the inadequacy of the HF approximation or the inconsistency of the theory. The $\omega$-meson propagator can be written as [4, 5]

$$D_{\mu\nu}(k) = (\delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2})\Delta_{\nu}(k) - i\frac{k_{\mu}k_{\nu}}{k^2(m_{\nu}^2 + \delta m_{\nu}^2)}, \quad (1a)$$

$$\Delta_{\nu}(k) = \Delta_{\nu}^{0}(k) + \Delta_{\nu}^{0}(k)\Pi_{\nu}(k)\Delta_{\nu}(k) = -i[k^2 + m_{\nu}^2 + i\Pi_{\nu}(k) - i\varepsilon]^{-1}, \quad (1b)$$

where $\delta m_{\nu}^2$ is the mass counterterm for the $\omega$-meson. In their calculation they have neglected all the terms proportional to $k_{\mu}k_{\nu}$ in the $\omega$-meson propagator on the basis of the baryon current conservation implied by the model for a rigorous calculation. Though this is a generally accepted approximation [6]
and indeed, such terms need not be taken into account if the baryon current conserves [7], their contribution in the RSCHF approximation is not zero and has to be studied [8]. It also indicates that the RSCHF approximation does not preserve the baryon current conservation.

In Ref. [8] we showed that the negative baryon spectral function mentioned above is caused by the $k_\mu k_\nu$ terms in the $\omega$-meson propagator. In Eq. (1a) the last longitudinal term is not renormalizable, thus if the $k_\mu k_\nu$ terms should be considered, it must be studied carefully. In order to take a proper account of the contribution of the $k_\mu k_\nu$ terms in $D_{\mu\nu}(k)$ a rule has also been proposed in [8]. Refs. [9, 12] have shown if in addition to the $\omega$-meson, other mesons like $\pi$, $\sigma$, and chiral $\pi - \sigma$ are considered, the baryon spectral functions in the RSCHF approximation can be regular for parameters of physical interest, even though the $k_\mu k_\nu$ terms in Eq. (1a) are neglected. Clearly this does not mean that the contribution of the $k_\mu k_\nu$ terms is not important when the baryon current conservation is violated, because it is related with the relative strength between different fields. For example, in the $\sigma - \omega$ model, if we adjust the coupling constants $g_\sigma^2$ and $g_\omega^2$, one finds that along with $g_\sigma^2/g_\omega^2$ becoming smaller the undesirable negative spectral function will appear again. So, it is desirable to assess the effect of $k_\mu k_\nu$ terms, even in cases where there are other mesons. The role of the $k_\mu k_\nu$ terms in the vector meson propagator has been studied in Ref. [6]. It was found that under the $G_D$ approximation
the effect of the $k_\mu k_\nu$ terms on the observable quantities is negligible. In this paper we would like to consider the other case where the baryon propagator incorporates the propagation of virtual baryons and antibaryons. We shall study the contribution of the $k_\mu k_\nu$ terms to the baryon and meson spectral functions and their influence on the effects of self-consistency by means of the rule suggested in Ref. [8]. We find that in the $\sigma - \omega$ model the regularity of the effects of self-consistency is almost the same as found in Ref. [9], and the contribution of the $k_\mu k_\nu$ terms to the meson spectral function is not very important. However, there is a marked change in the baryon spectral function which becomes more peaked. Moreover, we cannot remove such a change by a proper readjustment of parameters. This shows that violation of the current conservation law may cause quite significant effects.

The paper is organized as follows. In Section 2, we shall consider the coupled set of Dyson-Schwinger (DS) equations for the renormalized hadron propagators in the $\sigma - \omega$ model. The numerical results are given and discussed in Section 3. A summary is presented in Section 4.

2 The models and coupled Dyson-Schwinger equations

For the $\sigma - \omega$ model [10], the Lagrangian density is given by

$\mathcal{L}_{\sigma-\omega} = -\overline{\psi}(\gamma_\mu k_\mu + M)\psi - \frac{1}{2}(\partial_\mu \phi \partial_\mu \phi + m_s^2 \phi^2) + g_s \overline{\psi}\psi \phi$
\[-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}m_\psi^2 A_\mu A_\nu - ig_\psi \gamma_\mu \gamma_\nu A_\mu + \mathcal{L}_{\text{CTC}} \]  

(2)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \partial_\mu = \frac{\partial}{\partial x_\mu} \), \( x_\mu = (\vec{x}, ix_0) \), \( x^2 = x_\mu x_\mu = \vec{x}^2 - x_0^2 \) with \( x_0 \equiv t \), and the CTC means the counterterm correction introduced for the purpose of renormalization. We shall use the same notation as Refs. [8, 9]. The DS equations in the dressed HF scheme (Fig. 1) can be written in the following form:

(a) for baryon

\[ G(k) = G^0(k) + G^0(k) \Sigma(k) G(k) = -[\gamma_\mu k_\mu - iM + \Sigma(k)]^{-1}, \]  

(3a)

\[ \Sigma(k) = \Sigma_s(k) + \Sigma_v(k) \]  

(3b)

\[ \Sigma_s(k) = -g_s^2 \int \frac{d^4q}{(2\pi)^4} G(q) \overline{\Delta_s(k - q)} \Gamma_s(k, q, k - q) + \Sigma_s^{\text{CTC}}(k), \]  

(4a)

\[ \Sigma_v(k) = g_v^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\eta \overline{G(q)} D_\eta \lambda(k - q) \Gamma_\lambda(k, q, k - q) + \Sigma_v^{\text{CTC}}(k), \]  

(4b)

(b) for \( \sigma \)-meson

\[ \Delta_s(k) = \Delta_s^0(k) + \Delta_s^0(k) \Pi_s(k) \Delta_s(k) = -i[k^2 + m_s^2 + i\Pi_s(k) - i\epsilon]^{-1}, \]  

(5a)

\[ \Pi_s(k) = \chi g_s^2 \int \frac{d^4q}{(2\pi)^4} \text{Tr} \{G(k + q) \Gamma_s(k + q, k) \overline{G(q)} \} + \Pi_s^{\text{CTC}}(k); \]  

(5b)

(c) for \( \omega \)-meson

\[ D_{\mu\nu}(k) = D_{\mu\nu}^0(k) + D_{\mu\eta}^0(k) \Pi_{\eta\lambda}(k) D_{\lambda\nu}(k), \]  

(6a)

\[ D_{\mu\nu}^0 = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \Delta_\nu^0(k) \]  

(6b)
\[ \Pi_{\eta\lambda}(k) = -\chi g_G^2 \int \frac{d^n q}{(2\pi)^4} \text{Tr}[\gamma_\mu \bar{G}(k + q) \Gamma_{\lambda}(k + q, q, k) G(q)]. \] (6c)

In the above equations \( \Gamma_s \) and \( \Gamma_\lambda \) (denoted by a heavy dot in Fig. 1) are the \( \sigma \)-baryon and \( \omega \)-baryon vertex functions, respectively; \( n = 4 - \delta (\delta \to 0^+) \) and in Eq. (3) the Feynman prescription \( M \to M - i\varepsilon \) is understood. In this paper, we shall only consider \( \Gamma_s = 1 \) and \( \Gamma_\lambda = \gamma_\lambda \). We assume the following formula is identical.

\[ \hat{\Pi}_{\mu\nu}(k) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \hat{\Pi}_\nu(k), \] (6d)

which implies \( \hat{\Pi}_\nu(k) = \frac{1}{3} \sum_\mu \hat{\Pi}_{\mu\mu}(k) \). From Eq. (6d) one observes that the renormalized \( \Pi_{\mu\nu}(k) \) can be obtained from

\[ \Pi_\nu(k) = \hat{\Pi}_\nu(k) + \Pi_\nu^\text{CTC}(k). \] (6e)

\( \bar{G} (\bar{\Delta}, \bar{D}) \) denotes an appropriate expression chosen for the calculation of the baryon (meson) propagator in the self-energy. Just as in Ref. [9], we shall study the four schemes shown in Table 1, where the first column gives the name of each scheme, while the second and third explain how its \( \Sigma_s [\Sigma_\nu] \) and \( \Pi_s [\Pi_\nu] \) are obtained.

In the potential scheme P, \( \Sigma(P) \) and \( \Pi(P) \) are obtained by setting \( \bar{G} = -[\gamma_\mu k_\mu - iM_t]^{-1} \) where \( M_t \) is the true baryon mass and \( \bar{\Delta}_s = \Delta_s^0 (\bar{D} = D^0) \);

\( \Sigma(EP) \) and \( \Pi(EP) \) in the extended potential scheme EP are obtained by setting \( \bar{G} = G(P) \) and \( \bar{\Delta}_s = \Delta_s(P) (\bar{D} = D(P)) \);
To obtain $\Sigma(BP)$ and $\Pi(BP)$ for scheme BP, one sets $\tilde{G} = G(BP)$, $\Delta_s = \Delta_s^0 (\overline{D} = D^0)$, which implies that the baryon propagator has to be determined self-consistently;

For $\Sigma(FSC)$ and $\Pi(FSC)$, one sets $\tilde{G} = G(FSC)$, $\Delta_s = \Delta_s(FSC)$ ($\overline{D} = D(FSC)$), i.e. all the baryon and meson propagators are calculated self-consistently.

It is known [3, 10] that in the zero density case, the baryon self-energy $\Sigma(k) = \gamma_\mu k_\mu a(k^2) - iM b(k^2)$. For convenience of discussion, the case I (II) refers to neglecting (considering properly) the contribution of $k_\mu k_\nu$ terms in $D_{\mu\nu}$. For the $\sigma-\omega$ model we shall denote $a = a_s + a_\delta$, $b = b_s + b_\delta$ in case I, while $a = a_s + a_\delta + a_\Delta$ and $b = b_s + b_\delta + b_\Delta$ in case II, where the superscript 'δ' denotes the contribution of $\delta$-term in Eq. (1a), and the 'Δ' is the contribution of $k_\mu k_\nu$-terms in Eq. (1a). To fix the renormalization counterterms, we shall follow Ref. [8] and use the on-shell renormalization condition on baryon and the intermediate renormalization condition on mesons, which can be written as [9]:

$$\Sigma_\eta(k)|_{\gamma_\mu k_\mu = iM_t} = 0; \quad \frac{\partial}{\partial (\gamma_\mu k_\mu)} \Sigma_\eta(k)|_{\gamma_\mu k_\mu = iM_t} = 0; \quad (7a)$$

$$\Pi_\eta(k)|_{k^2 = 0} = 0; \quad \frac{\partial}{\partial k^2} \Pi_\eta(k)|_{k^2 = 0} = 0, \quad (7b)$$

where $\eta = s, v$. In addition, we shall also use $(\alpha(k^2), \beta(k^2))$ and $\rho(k^2)$ to denote the baryon and meson spectral weight functions, respectively. As is
well known, one has
\[ \alpha(k^2) = \frac{1}{\pi} \text{Im}[\frac{1 + a(k^2)}{D(k^2)}], \]
\[ \beta(k^2) = \frac{1}{\pi} \text{Im}[\frac{1 + b(k^2)}{D(k^2)}], \]
\[ D(k^2) = [1 + a(k^2)]^2 k^2 + [1 + b(k^2)]^2 M_t^2; \]
\[ \rho_{\eta}(k^2) = \frac{1}{\pi} \text{Im}[i \Delta_{\eta}(k^2)], \quad \eta = s, v \]

Using the renormalization conditions (Eqs. (7)), we can obtain the expressions of the self-energy and the spectral weight functions. For the nuclear matter as well as neutron matter, the explicit formulae of \( \Sigma_{\eta}(k^2) \) in \( \sigma - \omega \) model are the same as in [8, 9], while for the \( \Pi_{\eta}(k) \) in Eqs. (5, 6), \( \chi = 1 \) is for the neutron matter and \( \chi = 2 \) for the nuclear matter. So, Eqs. (2-6, 8, 9) and their explicit expressions yield the closed set of the renormalized DS equations used for our calculation.

### 3 The numerical results

We shall use the following values for the coupling constants and masses:
\[ M_t = 4.7585\text{fm}^{-1}, \]
\[ m_s = 2.6353\text{fm}^{-1}, \quad m_v = 3.9680\text{fm}^{-1} \]
\[ g_s^2 = g_s^2/16\pi^2 = 0.5263, \quad g_v^2 = g_v^2/8\pi^2 = 1.3685 \]
The notation is the same as in Refs. [8, 9].

Now, we consider the \( \sigma - \omega \) model in case II, where the additional contribution of the \( k_{\mu}k_{\nu} \)-terms in Eq. (1a) is taken into account. Following the method and rule obtained in [8], we solve the coupled set of DS equations
by the method of iteration. The numerical results are shown in Figs. 2-4. In our calculation, we have studied three different schemes: schemes P, EP and FSC. The baryon spectral functions obtained from these three schemes are very close to each other. Though the self-consistency makes the peak of the resonance higher, as a whole its effect is not significant, just as found in case I [9]. However, comparing Fig. 2 with Fig. 8 in [9], one observes there is a great quantitative change in the functional behavior. The maxima of $\alpha(k^2)$ and $\beta(k^2)$ become more distinct and sharper in case II.

Let us designate $k^2 > (\langle \rangle - (M_t + m_c)^2$ as region I (II). We note $a_\nu$ and $b_\nu$ are real in region I and become complex in region II, so in region II their imaginary parts will also contribute. Let us fix $\mathcal{g}_v^2 = 1.3685$ and consider, for instance, the variation of $\alpha(k^2)$ with $\mathcal{g}_s^2$ (see Fig. 3). When $\mathcal{g}_s^2$ is small, there are two resonances which are located in region I and II, respectively. Eq. (8a) can be rewritten as $\alpha(k^2) = \frac{1}{\pi}\left\{ \frac{a_i D_r - (1 + a_r) D_i}{D_r + D_i} \right\}$, where the subscript ‘r’ and ‘i’ denote the real and imaginary parts of $a(k^2)$ and $D(k^2)$, respectively. Comparing with $(a_\nu, b_\nu)$, the values of $(a_s, b_s)$ is very small, and in region I the contribution of $(a_\nu, b_\nu)$ to the denominator of $\alpha(k^2)$ is larger than to the numerator which is mainly determined by the imaginary part of $(a_s, b_s)$, so the resonance is small. If $\mathcal{g}_s^2$ tends to zero, the resonance in region II becomes more and more like the resonance of $\alpha(k^2)$ in the $\omega$ case and the resonance in region I disappears (see [8]). As $\mathcal{g}_s^2$ becomes larger, $a_s$ and $b_s$
are big. In this case the combined contributions of \((a_s, b_s)\) and \((a_v, b_v)\) to the denominator and numerator of \(\alpha(k^2)\) are comparable in region I, thus the resonance becomes sharper. In region II along with \(|k^2|\) becoming larger \(|D(k^2)|\) is big, so \(\alpha(k^2)\) becomes small and the resonance in this region almost disappears, just as shown in Fig. 2.

For both cases I and II, we have readjusted the parameters under the condition that the spectral function \(\alpha(k^2)\) should be non-negative. We find that for the parameters of physical interest the resonance is always in region II for case I, while the stronger one is in region I for case II. Moreover, the resonance in case II is more distinct than that in case I. Our results show the difference between these two cases will remain even by the readjustment of the parameters. Thus, the contribution of \(k_{\mu}k_{\nu}\)-term is very important to the baryon spectral functions. Since \(\alpha(k^2)\) relates directly to the probability of occurrence of an excited baryon state, the contribution of \(k_{\mu}k_{\nu}\)-term in the \(\omega\)-meson propagator seems to make the possibility of forming a resonance baryon state greater.

From Fig. 4, it looks that the effect of self-consistency on mesons is discernible. However, there is no need to require self-consistency in the meson propagators, because in Figs. 2 and 4 the results of scheme EP and FSC are almost the same. Compared with Fig. 10 in [9] the self-consistent meson spectral function is larger, but this change is not great. It means the
contribution of $k_{\mu}k_{\nu}$-terms to $(\rho_s, \rho_v)$ is not important.

From Fig. 2, Fig. 3 in [9] and Fig. 2b in [8], one notes that there also exists a cancellation between the effects on the self-consistency due to the $\sigma$ and $\omega$ mesons in case II.

In order to study the effect of self-consistency more carefully, we have drawn $\alpha(k^2)$ and $\beta(k^2)$ calculated for an intermediate $\nu_v^2 = 0.3400$ in Fig. 5. One observes that the effect of self-consistency is not significant, except for the region of the peaks, where it makes the peak value a little higher. On the whole, we may say that similarly to case I, the effects of self-consistency in case II are also not important.

For the neutron matter the result are almost the same as above.

4 Summary

In this paper the coupled set of DS equations in the $\sigma - \omega$ for two cases were solved self-consistently. The calculations show that in the $\sigma - \omega$ model, there is no need to require self-consistency in meson propagators and the self-consistency almost has no effect on the baryon propagator. Compared with case I, there is a distinct change in case II in the baryon spectral functions which become more peaked. Such a change cannot be removed even by a proper readjustment of parameters. Moreover, there also exists a cancellation between the effects of the self-consistency due to the $\sigma$ and $\omega$ mesons.
Our results show that in a calculation where the law of the baryon current (BC) conservation is violated, the contribution of the $k_\mu k_\nu$ terms is generally not negligible and may serve as a sign signifying the degree of its violence. If we assume the approximation made for the calculation is appropriate, though BC may not be conserved, then the $k_\mu k_\nu$ terms have to be considered (Way-$k_\mu k_\nu$). Clearly whether the results obtained by Way-$k_\mu k_\nu$ are acceptable has still to be confirmed by a more rigorous calculation where the laws incorporated in the model are respected. Since a bare baryon-meson vertex is used, the RSCHF approximation does not satisfy the Ward-Takahashi (W-T) identity. It has been pointed out in Ref. [6] that this is the main reason why the BC conservation is violated. Thus, if vertices consistent with the W-T identity are used (Way-WT), the $k_\mu k_\nu$ terms may be neglected. So far no numerical calculations along this line have been reported. However, we do agree with Ref. [2] that this is a procedure worthy of pursuing, as it will also tell whether and when the simpler Way-$k_\mu k_\nu$ or some other simple approximation may be a good substitute for Way-WT, because the latter is quite complicated except for some simple cases.

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Table

Table 1: Different calculation schemes.

| Name | $\Sigma_s(\Sigma_v)$ | $\Pi_s(\Pi_v)$ |
|------|-----------------------|-----------------|
| P    | $G_0^\Sigma$, $\Delta_s^u(D^u)$ | $G_0^\Sigma$ |
| EP   | $G(P)$, $\Delta_s(P)(D(P))$ | $G(P)$ |
| BP   | $G(BP)$, $\Delta_s^u(D^u)$ | $G(BP)$ |
| FSC  | $G(FSC)$, $\Delta_s(FSC)(D(FSC))$ | $G(FSC)$ |
Figure captions

**Fig. 1**: Diagrammatic representation of the different self-consistent (dressed) HF schemes. a. the baryon propagator; b. the \(\omega\)-meson propagator; c. the \(\sigma\)-meson propagator.

**Fig. 2**: The baryon spectral functions \(\alpha(k^2)\) and \(\beta(k^2)\) for the \(\sigma - \omega\) model in case II.

**Fig. 3**: The baryon spectral functions \(\alpha(k^2)\) and \(\beta(k^2)\) for the \(\sigma - \omega\) model in case II, left: \((g_s^2, g_v^2) = (0.0500, 1.3685)\); right: \((g_s^2, g_v^2) = (0.0100, 1.3685)\).

**Fig. 4**: The meson spectral functions \(\rho_\lambda(k^2)\) for the \(\sigma - \omega\) model in case II. top: \(\sigma\)-meson: \(\lambda = s\); bottom: \(\omega\)-meson: \(\lambda = v\).

**Fig. 5**: The baryon spectral functions \(\alpha(k^2)\) and \(\beta(k^2)\) for the \(\sigma - \omega\) model in case II, \((g_s^2, g_v^2) = (0.5263, 0.3400)\).
Δs = G + G^0 + G - G

D = D^0 + G - G

Δs = Δs^0 + G - G
\( g_s^2 = 0.0500 \)
\( g_v^2 = 1.3685 \)

\( \beta(k^2) \)

\( \alpha(k^2) \)

\( k^2 (\text{fm}^{-2}) \)
\[ \rho_v(k^2) \]

(a) P-Scheme
(b) EP-Scheme
FSC

\[ \rho_s(k^2) \]
\[ \bar{g}_v^2 = 0.3400 \]