Fermion superfluid with hybridized $s$- and $p$-wave pairings

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Ever since the pioneering work of Bardeen, Cooper and Schrieffer in the 1950s, exploring novel pairing mechanisms for fermion superfluids has become one of the central tasks in modern physics. Here, we investigate a new type of fermion superfluid with hybridized $s$- and $p$-wave pairings in an ultracold spin-1/2 Fermi gas. Its occurrence is facilitated by the co-existence of comparable $s$- and $p$-wave interactions, which is realizable in a two-component $^{40}$K Fermi gas with close-by $s$- and $p$-wave Feshbach resonances. The hybridized superfluid state is stable over a considerable parameter region on the phase diagram, and can lead to intriguing patterns of spin densities and pairing fields in momentum space. In particular, it can induce a phase-locked $p$-wave pairing in the fermion species that has no $p$-wave interactions. The hybridized nature of this novel superfluid can also be confirmed by measuring the $s$-wave and $p$-wave contacts, which can be extracted from the high-momentum tail of the momentum distribution of each spin component. These results enrich our knowledge of pairing superfluidity in Fermi systems, and open the avenue for achieving novel fermion superfluids with multiple partial-wave scatterings in cold atomic gases.

I. INTRODUCTION

Fermion superfluid is one of the central research topics in modern physics. In recent years, ultracold Fermi gases have emerged as an excellent platform for the study of fermion superfluid in the strong-coupling regime via the Feshbach resonance (FR) technique [1]. Apart from the widely explored $s$-wave FRs, the $p$-wave FRs have also been realized in ultracold Fermi gases of $^{40}$K [2, 3] or $^6$Li [4] atoms. The $s$- and $p$-wave FRs are associated with distinct fermion superfluids. Across an $s$-wave FR, the pairing superfluid undergoes a smooth crossover from the Bardeen-Cooper-Schrieffer (BCS) regime with weakly bound Cooper pairs to the Bose-Einstein condensation regime with tightly bound molecules [5, 6]. While across a $p$-wave FR, the pairing superfluid can go through a phase transition which is characterized by a change of the pairing field orientation with respect to the external magnetic field [7, 8]. Such a dramatic difference originates from their contrastive pairing symmetries: for the $s$-wave case, a Cooper pair is a spin singlet with isotropic orbitals regardless of the interaction strength; the $p$-wave case, however, features spin-triplet pairing with anisotropic orbitals, which renders the pairing very sensitive to the relative interaction strengths along different orbital orientations.

Near a $p$-wave FR, atom losses have been generally considered to prohibit a global equilibration throughout the system. However, a quasi-equilibration can still be reached at short time scales, as has been demonstrated

in a Bose-Einstein condensate following a quench into regimes with large $s$-wave scattering length and considerable atom losses [9]. Indeed, in a very recent experiment [3], a steady state with strong $p$-wave correlations has been achieved in a Fermi gas at a low temperature $T = 0.2T_F$ ($T_F$ is the Fermi temperature) within a time scale of $\sim 0.5$ ms. It is thus hopeful that pairing physics can be explored near a $p$-wave FR at lower temperatures, where a steady state with interesting pairing correlations

FIG. 1. (Color online). Feshbach resonances(FR) of $^{40}$K atoms. Black lines show the $p$-wave scattering volume $\nu$ (in unit of $a_0^3$, $a_0$ is Bohr radius) and red line shows the $s$-wave scattering length $a_s$ (in unit of $a_0$). The $s$-wave FR between atomic hyperfine states $|F = 9/2, m_F = -7/2\rangle \equiv |\uparrow\rangle$ and $|F = 9/2, m_F = -9/2\rangle \equiv |\downarrow\rangle$ occurs at $B = 202.1$ G (with a width of 8 G), which is very close to the $p$-wave FRs between two $|\uparrow\rangle$s at 198.3G and 198.8G (with a width of 0.5 G), respectively with orbital angular momentum $l = 1$, $m = \pm 1$ and $l = 1$, $m = 0$.

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can be probed before atom losses become dominant.

In this work, we investigate a new type of fermion superfluid, where the aforementioned two distinct pairing symmetries co-exist and hybridize with each other. Such a superfluid is facilitated by the presence of both s- and p-wave interactions with comparable strengths, which can be realized in Fermi gases of $^{40}$K atoms near a magnetic field of $B \approx 198$G [2]. As illustrated in Fig. 1, for the two hyperfine states $|F = 9/2, m_F = -7/2\rangle \equiv |\uparrow\rangle$ and $|F = 9/2, m_F = -9/2\rangle \equiv |\downarrow\rangle$, the s-wave (between $|\uparrow\rangle$ and $|\downarrow\rangle$) and the p-wave (between two $|\uparrow\rangle$s) FRs are sufficiently close to each other. This physical system offers a promising platform to investigate the effects of multiple partial-wave scatterings on the pairing superfluidity of fermions, as we will address in this work.

To characterize the novel hybridized fermion superfluid, we first consider the case of isotropic p-wave interactions. We show that in the hybridized superfluid, the spin densities and the pairing fields exhibit intriguing patterns in momentum space that are drastically different from those of a conventional superfluid. We find an induced p-wave pairing between the $|\downarrow\rangle$ states, which have no p-wave interactions. Interestingly, the phases of the p-wave pairing between the $|\downarrow\rangle$ states and that between the $|\uparrow\rangle$ states are locked to be conjugate with each other through the hybridization with the s-wave pairing. Such a hybridization can also manifest itself in the measurement of the s-wave and p-wave wave functions, which can be extracted from the high-momentum tail of the momentum distribution of each spin component. Finally, we point out the rich orbital structures of the hybridized superfluid states under typical experimental conditions with anisotropic p-wave interactions.

\section{II. FORMALISM}

We start from a two-channel Hamiltonian, $\Omega = H - \sum_{k,\sigma = \uparrow, \downarrow} \mu_\sigma N_\sigma$, for spin-1/2 fermions:

$$\Omega = \sum_{k,\sigma = \uparrow, \downarrow} (\epsilon_k - \mu_\sigma) a_{k\sigma}^\dagger a_{k\sigma} + \sum_{q,\alpha} (\epsilon_q^b + \epsilon_\alpha - 2\mu_\uparrow) b_{q\alpha}^\dagger b_{q\alpha}$$

$$+ \frac{U}{V} \sum_{k,\sigma = \uparrow, \downarrow} \sum_{p,\mu = \uparrow, \downarrow} a_{k\uparrow}^\dagger a_{p\downarrow}^\dagger a_{p\downarrow} a_{k\uparrow} + \frac{g(|k|)}{\sqrt{V}} K_\alpha b_{q\alpha}^\dagger \hat{a}_{+k} a_{-k} + h.c.,$$  \hspace{1cm} (1)

with $N_\uparrow = \sum_k a_{k\uparrow}^\dagger a_{k\uparrow}$, $N_\downarrow = \sum_k a_{k\downarrow}^\dagger a_{k\downarrow} + 2 \sum_{q,\alpha} b_{q\alpha}^\dagger b_{q\alpha}$. Here, $a_{k,\sigma}$ is the creation operator of spin-$\sigma$ atom with momentum $k$ and energy $\epsilon_k = k^2/(2M)$; $b_{q\alpha}^\dagger$ is the creation operator of a p-wave bosonic molecule with momentum $q$, kinetic energy $\epsilon_q^b = q^2/(4M)$, and detuning $\epsilon_\alpha$ (the direction of spin-polarization $\alpha = x, y, z$); $g(|k|) = g\theta(\Lambda - |k|)$ is the p-wave coupling between a bosonic molecule and two spin-$\uparrow$ atoms with relative momentum $k$ (here $\Lambda$ is the momentum cutoff, $\theta(x)$ is the Heaviside step function). The bare p-wave coupling $g$, the detuning $\epsilon_\alpha$, and the momentum cutoff $\Lambda$ are related to the scattering volume $\nu_\alpha$ and the effective range $k_0$ [10]:

$$\frac{1}{\nu_\alpha} = -\frac{6\pi \epsilon_\alpha}{M g^2} + \frac{2}{3\pi} \Lambda^3,$$  \hspace{1cm} (2)

$$k_0 = -\frac{12\pi}{M g^2} - 4 \pi \Lambda.$$  \hspace{1cm} (3)

$U$ gives the bare s-wave interaction, which is related to the s-wave scattering length $a_s$ by the renormalization relation: $1/U = M/(4\pi a_s) - 1/V \sum_k 1/(2\epsilon_k)$, with $V$ the volume of the system. In this work we set $h = 1$ for convenience.

Based on the standard BCS theory, we define two pairing fields $\Delta_p = \frac{U}{V} \sum_k (a_{k\downarrow}^\dagger a_{k\uparrow} - a_{k\uparrow}^\dagger a_{k\downarrow})$ and $\lambda_q = \frac{g}{\sqrt{V}} (b_{q\alpha}^\dagger b_{q\alpha})$, which are respectively the pairing order parameters of the s- and the p-wave superfluids. In this work we consider zero total momentum for each pairing state [11], and denote $\Delta_p = 0 \equiv \Delta$, $\lambda_q = 0 \equiv \lambda$. The Hamiltonian can then be written as $\Omega = \sum_{\alpha \geq 0} \psi_{k\alpha}^\dagger H_k \psi_{k\alpha} + \text{Const.}$, where the vector operator $\psi_k = \left( a_{k,\uparrow}, a_{k,\downarrow}^\dagger, a_{k\downarrow}, a_{k\uparrow}^\dagger \right)^T$, and the matrix $H_k$ is given by:

$$\begin{pmatrix}
\epsilon_k - \mu_\uparrow & -2\theta(\Lambda - |k|)\lambda & 0 & \Delta \\
-2\theta(\Lambda - |k|)\lambda^* & -\epsilon_k + \mu_\uparrow & -\Delta^* & 0 \\
0 & -\Delta & \epsilon_k - \mu_\uparrow & 0 \\
\Delta^* & 0 & -\epsilon_k + \mu_\uparrow & 0
\end{pmatrix}$$

with $\lambda = \sum_\alpha k_\alpha^\alpha$. By diagonalizing the matrix as $H_k = S^\dagger H_k S \equiv \text{Diag}(E_{k1}, E_{k2}, E_{k3}, E_{k4})$ with the eigen operator $\psi_k = S^{-1}\psi_k \equiv \left( a_{k,\uparrow}, a_{k\downarrow}^\dagger, \beta_{k\downarrow}, \beta_{k\uparrow}^\dagger \right)^T$, $\Omega$ can be reduced to:

$$\Omega = \sum_{k=0}^4 \left[ \sum_{i=1}^4 E_{ki} \theta(-E_{ki}) + 2(\epsilon_k - \mu) \right]$$

$$+ \sum_{\alpha} \epsilon_\alpha - 2\mu - 2\hbar \left| a_{k\alpha} \right|^2 / g^2 - V |\Delta|^2,$$  \hspace{1cm} (4)

where we have used $\mu = (\mu_\uparrow + \mu_\downarrow)/2$, $\hbar = (\mu_\uparrow - \mu_\downarrow)/2$. The ground state of the system can then be determined by minimizing $\Omega$ in terms of $\Delta$ and $\lambda_\alpha$. Due to the gaugeable global phase for both the s- and the p-wave pairing fields, we set $\Delta$ to be real and denote the vector $\lambda \equiv (\lambda_x, \lambda_y, \lambda_z)$ as $\lambda = \bar{u} + i\bar{v}$ with $\bar{u} \cdot \bar{v} = 0$ [7].

\section{III. PHASE DIAGRAM}

To capture the essential physics of a hybridized superfluid, we first consider the simple case of isotropic p-wave interactions, i.e., all $\epsilon_\alpha$ are equal. It follows that $\bar{u}$ and $\bar{v}$ have a simultaneous SO(3) rotational symmetry in the coordinate space. For convenience, we choose $\bar{u} = u\hat{z}$ and $\bar{v} = v\hat{x}$. In this case, the p-wave superfluid is always
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FIG. 2. (Color online). (a) Ground-state phase diagram in the (µ, h) plane with interaction parameters aσ = 25/|k0|, 1/νσ = 0, and the p-wave cutoff Λ = 0.75|k0|, µ, h are in the unit of εk = k2/2M, with k0 = |k0|/10. The diagram includes the vacuum (VAC), the normal phase (N) and various superfluid phases (S, P, S+P). The solid (dashed) lines are the phase boundaries for first-order (continuous) transitions. The magenta region in-between N and S is the breached pairing phase [14, 15, 18–22], which is only stabilized in a small region in the phase diagram with µ/εk0 ∈ (−0.5, −0.4) (here εk0 = 1/(Ma2) is the two-body binding energy). The background color denotes the spin polarization \( P = (n_up - n_down)/(n_up + n_down) \).

associated with the \( p + ip \) pairing symmetry, reminiscent of the \( A \) phase in \(^3\)He [16, 17].

In Fig. 2, we show a typical ground-state phase diagram in terms of the chemical potentials \((\mu, h)\) at the \( p \)-wave resonance \(1/\nu_u = 0\). Based on the parameters of \(^{40}\)K atoms near the \( B \approx 198G\) \( p \)-wave resonance with \( k_0 = -8 \times 10^8m^{-1}\) and \( a_\sigma = 3.15 \times 10^{-8}m^{-1} \) [2], we set the interaction parameter \(|k_0|a_\sigma = 25\) and the \( p \)-wave cutoff \( \Lambda = 0.75|k_0|\), and use \( k_u = k_0/10\) as the momentum unit. We can see that by adjusting \( \mu \) and \( h \), the system can exhibit various superfluid phases, including the purely \( s \)-wave superfluid with \( \Delta \neq 0\), \( u = v = 0\) (S), and the purely \( p + ip \) superfluid with \( \Delta = 0\), \( u = v \neq 0\) (P). Of particular interest here is the superfluid with co-existing \( s \)- and \( p \)-wave pairing orders: \( \Delta \neq 0\), \( u = v = 0\) (S+P), which emerges in-between S and P phases on the phase diagram. By examining the continuity property of the pairing amplitudes \((\Delta, u, v)\) and the spin densities across the phase boundaries, we find that the transition between S and S+P is continuous, while that between S+P and P is of first order. These phase transitions can be observed directly by measuring density profiles of a trapped gas. As suggested by Fig. 2, from the trap center to the edge there could exist different sequences of phases, for instance, \((S+P)\to P\) (for \(n_up > n_down\)) or \((S+P)\to S\) — Normal (for \(n_up < n_down\)). At the \((S+P)\to P\) phase boundary the spin densities change discontinuously, while at the \((S+P)\to S\) boundary the spin polarization continuously goes to zero.

IV. SUPERFLUID WITH HYBRIDIZED PAIRINGS

In the following, we focus on the exotic \( S+P \) superfluid state, where both the \( s \)- and \( p \)-wave pairings are present. As shown in Fig. 2, this state exists for a spin-imbalanced system with more spin-up than spin-down atoms. However, the characterization of this state is far beyond a simple superposition of the \( s \)-wave pairing between an equal number of different spins and the \( p \)-wave pairing in the remaining spin-up atoms. We will demonstrate that the two pairing fields are in fact coherently entangled with each other, which leads to dramatic physical consequences as shown below.

We start from the ground state wave function:

\[
|\Psi_G\rangle = \prod_{k>0} \prod_{i=1}^4 \gamma_{k,i} |\text{vac}\rangle,
\]

with \( \gamma_{k,i} = \theta(E_{k1})a_{k,i}^\dagger + \theta(-E_{k1})a_{k,i} \), \( \gamma_{k,2} = \theta(-E_{k2})a_{k,-i}^\dagger + \theta(E_{k2})a_{k,-i} \), \( \gamma_{k,3} = \theta(E_{k3})\beta_{k} + \theta(-E_{k3})\beta_{k}^\dagger \), \( \gamma_{k,4} = \theta(-E_{k4})\beta_{k} + \theta(E_{k4})\beta_{k}^\dagger \). For any given \( k \) and \( E_{ki} \), one can expand \( \gamma_{k,i} \) in terms of \( \{a_{k,i}, a_{k,i}^\dagger\} \), and finally we arrive at a general form of the wave function:

\[
|\Psi_G\rangle = \prod_{k>0} \left( \sum_{i=1}^4 \left( a_{k,i} + \nu_{k,i}^{\uparrow} a_{k,i}^\dagger + \nu_{k,i}^{\downarrow} a_{k,-i}^\dagger a_{k,-i} + \nu_{k,i}^{\uparrow\downarrow} a_{k,i}^\dagger a_{k,-i}^\dagger a_{k,-i} a_{k,i} \right) |\text{vac}\rangle \right).
\]
FIG. 3. (Color online). Contour plots of pairing fields and density distributions in the momentum space ($k_y = 0$) for the hybridized superfluid phase at $\mu = E_u$ and $h = 0$. (a) is for $s$-wave pairing; (b1) and (b2) are the real and imaginary parts of $p$-wave pairing between spin-$\uparrow$; (c1) and (c2) are the real and imaginary parts of the induced $p$-wave pairing between spin-$\downarrow$. (d) Schematics for the mechanism of induced $p$-wave pairing with a locked phase. (e1, e2) momentum distributions of total density $n(k)$ and spin density $\delta n(k)$. All momenta are in the unit of $k_u$, and all densities are in unit of $1/k^3_u$.

$a_{-k,\uparrow}^\dagger$ with a phase $e^{i\phi_k} (\phi_k = \arctan(k_z/k_x))$ inbetween; when they individually couple to $a_{-k,\downarrow}^\dagger$ and $a_{k,\downarrow}$ by $s$-wave pairing, the phase is effectively transferred to these spin-$\downarrow$ operators, and finally a conjugate phase ($e^{-i\phi_k}$) is produced, which gives rise to the induced $p - ip$ pairing between spin-$\downarrow$ atoms. The mutual phase locking between the two $p$-wave pairing fields is intrinsic to the $s$- and $p$-wave hybridized pairings, regardless of the symmetry of the $p$-wave interaction (isotropic or anisotropic in orbitals).

We note that the intriguing phenomenon of induced pairing is a unique feature of our system. It is fundamentally different from the co-existence of pairing correlations reported in previous studies. For example, the co-existence of $s$- and $p$-wave pairings has been investigated in a fermion superfluid with long-range dipole-dipole interactions, which naturally contain all partial-wave components [23], or in a spin-imbalanced Fermi gas with $s$-wave interactions [24]. In these cases, the $p$-wave pairing is either spin independent [23], or driven by spin/density fluctuations and thus quite small in strength [24]. In our system, the induced $p$-wave pairing is due to the interplay of the $s$- and $p$-wave interactions, which gives rise to an appreciable pairing strength (see Fig. 3(c1,c2)).

The hybridized nature of the $S+P$ state is also reflected in the momentum-space density distributions $n(k) = n_{\uparrow}(k) + n_{\downarrow}(k)$ and $\delta n(k) = n_{\uparrow}(k) - n_{\downarrow}(k)$ in the $(k_{\perp}, k_y)$ plane, with $k_{\perp} = \sqrt{k_z^2 + k_x^2}$ (see Fig. 3(e1,e2)). At $k = 0$, the $p$-wave pairing field vanishes, and the remaining $s$-wave pairing order requires $\delta n(0) = 0$. The largest spin imbalance occurs at a finite momentum $k_{\perp} \sim k_u$, where the $p$-wave pairing, and through hybridization, the $s$-wave pairing as well, show the strongest amplitudes (see Fig. 3(a,b1,b2)).

V. CONTACTS

Practically, the hybridized superfluid can also be recognized in the measurement of contact [25], a physical quantity connecting the microscopic two-body physics with the thermodynamics of a many-body system [25–30]. Experimentally, the $s$- and $p$-wave contacts have been successfully extracted in cold atoms experiments, respectively, in an unpolarized spin-1/2 Fermi gas [31–33], and in a fully polarized Fermi gas [3]. In our system with mixed $s$- and $p$-wave interactions, the contacts can be individually defined through the adiabatic relations...
under the grand canonical ensemble:

\[ \frac{\partial \Omega}{\partial (-1/a_s)} |_{\mu,h} = \frac{C_s}{4 \pi M} \quad (7) \]
\[ \frac{\partial \Omega}{\partial (-1/\nu_\alpha)} |_{\mu,h} = \frac{C^{(\alpha)}_{p,\nu}}{4 \pi M} \quad (8) \]
\[ \frac{\partial \Omega}{\partial \nu_0} |_{\mu,h} = \sum_\alpha C^{(\alpha)}_{p,R} / 4 \pi M \quad (9) \]

Here, \( C_s \) is the \( s \)-wave contact \([25–27]\), and \( C^{(\alpha)}_{p,\nu}, C^{(\alpha)}_{p,R} \) are the \( p \)-wave contacts \([28–30]\) respectively related to the scattering volume and the effective range. In the case of isotropic \( p \)-wave interactions with \( \nu_\alpha = \nu \), Eq. (8) can be replaced by \( \partial \Omega / \partial (-1/\nu) |_{\mu,h} = C_{p,\nu} / 4 \pi M \), with \( C_{p,\nu} = \sum_\alpha C^{(\alpha)}_{p,\nu} \).

The contacts defined in Eqs. (7-9) uniquely determine the high-momentum tail of the spin-\( \sigma \) (\( \sigma = \uparrow, \downarrow \)) momentum distribution \( n_\sigma(k) \). Following the standard derivations in Ref. [25–30], we obtain the following asymptotic behavior of \( n_\sigma(k) \) for \( k_F \ll k \ll \Lambda \) (\( k \equiv |k| \), \( k_F \) is the Fermi momentum):

\[ n_\uparrow(k) \to \frac{C_s}{k^4}, \quad (10) \]
\[ n_\downarrow(k) \to \frac{C_s}{k^4} + \sum_m 4 \pi |Y_{1m}(\hat{k})|^2 \left[ \frac{C^{(m)}_{p,\nu}}{k^4} + \frac{C^{(m)}_{p,R}}{k^4} \right]. \quad (11) \]

Here, \( \{C^{(m)}_{p,\nu/R}\} \,(m = 0, \pm 1) \) are the projections of \( \{C^{(\alpha)}_{p,\nu/R}\} \,(\alpha = x,y,z) \) in the magnetic angular momentum space. For the experimentally relevant cases, we have \( C^{(0)}_{p,\nu/R} = C^{(x)}_{p,\nu/R} \) and \( C^{(\pm 1)}_{p,\nu/R} = C^{(y)}_{p,\nu/R} \). Eqs. (10,11) show that the momentum distribution is highly asymmetric in spin. In particular, the distribution of the spin-\( \uparrow \) component exhibits a quite non-trivial high-momentum tail, which is due to the involvement of both the \( s \)- and the \( p \)-wave interactions. In contrast, the spin-\( \downarrow \) component follows the same \( 1/k^4 \) distribution as in the \( s \)-wave case, since the induced \( p \)-wave pairing within the spin-\( \downarrow \) species is purely a low-energy effect and does not generate additional high-momentum or short-range physics.

Given Eqs. (10,11), both the \( s \)- and \( p \)-wave contacts can be directly extracted from the time-of-flight measurement in experiments.

In Fig. 4, we show \( C_s \) and \( C_{p,\nu} \) across the \( p \)-wave FR, for the cases of pure \( s \)-wave superfluid (S), pure \( p \)-wave superfluid (P), and hybridized superfluid (S+P). A distinguished feature of the S+P hybridized superfluid is that both \( C_s \) and \( C_{p,\nu} \) are finite, while we have \( C_s = 0 \) for P and \( C_{p,\nu} = 0 \) for S state. Moreover, we have checked that for the S+P state, the \( s \)-wave contact \( C_s \) sensitively depends on the variation of the \( p \)-wave scattering volume \( (\nu) \) even with the \( s \)-wave scattering length \( (a_s) \) fixed. Similarly, the \( p \)-wave contact \( C_{p,\nu} \) depends on the variation of \( a_s \) with \( \nu \) fixed. All these properties of contacts unambiguously confirm the co-existence and hybridization of the \( s \)- and the \( p \)-wave pairings. Thus the contacts

**VI. EFFECT OF ANISOTROPIC \( P \)-WAVE INTERACTION**

In the previous discussions, we have demonstrated the key features of the hybridized superfluid phase under an isotropic \( p \)-wave interaction. In \( ^{40}K \) Fermi gases, the \( p \)-wave interactions are typically anisotropic (see Fig. 1). In Fig. 5, we map out the phase diagram in the \((\mu, h)\) plane under a typical anisotropic \( p \)-wave interaction: \( 1/\nu_z = 0 \) and \( 1/\nu_x = 1/\nu_y = -k^2 \). The only essential difference from the isotropic case is that the \( p \)-wave pairing states here (both the P and the S+P phases) have richer inner structures. Namely, the original \( p \pm ip \) pairing (in Fig. 3) is now replaced by \( p_z \) \((u \neq 0, v = 0)\) or \( p_z + i \beta p_x \) \((\beta < 1, 0 < v < u)\) pairings, depending on the densities of two spin species. In this case, the hybridized superfluid phase (S+P) still occupies a considerable region in the phase diagram, and its key features, as discussed previously, are not qualitatively altered by the anisotropy of \( p \)-wave interactions.
FIG. 5. (Color online). Ground-state phase diagram in the \((\mu, h)\) plane for anisotropic \(p\)-wave interactions with \(1/\nu_x = 0\) and \(1/\nu_y = 1/\nu_z = -k_u^2\). The other parameters are the same as in Fig. 2.

VII. SUMMARY AND DISCUSSION

To summarize, we have revealed a new type of fermion superfluid in spin-1/2 Fermi systems with hybridized \(s\)- and \(p\)-wave pairings. We demonstrate various non-trivial properties of such a hybridization, such as the phase locking of the induced \(p\)-wave pairing, the novel structure of pairing fields in momentum space, and the co-existence of \(s\)-wave and \(p\)-wave contacts. These features can be detected from the measurement of number distribution in real and momentum space in two-component \(^{40}\)K Fermi gases with very close \(s\) and \(p\)-wave FRs near \(B \approx 198\,G\).

All the features of the hybridized superfluid indicate that it is a highly entangled state that unifies different pairing symmetries in a single system. Such an entanglement emerges on the many-body level (rather than two-body), and is therefore a collective phenomenon due to Cooper pairs. Moreover, such a superfluid can accommodate large spin magnetization in a considerably broad parameter regime (see Fig. 1), in contrast to previously discussed magnetized superfluids such as the Fulde-Ferrell-Larkin-Ovchinnikov state, whose stability region in three dimension is typically small on the phase diagram.

The discovery of the hybridized superfluid enriches our knowledge of pairing superfluidity in Fermi systems, and opens a promising avenue for achieving novel fermion superfluid with multiple partial-wave scatterings in cold atomic gases. For instance, in future studies, it is worthwhile to address the hybridized pairing in lower spatial dimensions, where the atom losses may be further reduced due to the much less singular two-body wave function at short range compared to the three dimensional case. Furthermore, as the \(p\)-wave interaction in lower dimensions can induce topological superfluid, the hybridization could be even more intriguing as it would involve not only different pairing symmetries, but also different topology classes.

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