Finiteness of 2D Topological BF-Theory with Matter Coupling

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Abstract. We study the ultraviolet and the infrared behavior of 2D topological BF-Theory coupled to vector and scalar fields. This model is equivalent to 2D gravity coupled to topological matter. Using techniques of the algebraic renormalization program we show that this model is anomaly free and ultraviolet as well as infrared finite at all orders of perturbation theory.

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1 Introduction

During the last decade the study of topological gauge theories provided deep insights into the topology and geometry of low dimensional manifolds. The central feature of topological field theories is that the observables only depend on the global structure of the space-time manifold on which the model is defined. There are in fact two different types of topological field theories: whether the whole gauge fixed action or just the gauge fixing part can be written as a BRS variation the model is of Witten-type or of Schwarz-type, respectively. An example of Witten-type models is the topological Yang-Mills theory, representatives of Schwarz-type models are Chern-Simons and BF theories. A common feature of many such field theories is the appearance of a so-called vector-like supersymmetry in the flat space-time limit. It is due to the energy-momentum tensor being BRS-exact. Its generators when anticommuted with the BRS operator close on translations.

We are in particular interested in a BF-model defined on a two dimensional space-time manifold. Such a model was shown to be equivalent to a two dimensional gravity, which has been discussed in connection with the Liouville theory \[2\]. Typically in two space-time dimensions the propagators of massless scalar fields are ill-defined \[4, 5\]. Due to the singular behaviour of the ghost propagator at long distances, an infrared regulator mass has to be introduced \[3\]. The infrared and ultraviolet finiteness of the two dimensional BF-model was already discussed in the realm of algebraic renormalization \[8\].

The present work is devoted to the investigation of an enlarged model with the inclusion of a topological matter interaction \[2\] in the context of the algebraic renormalization program \[5\]. The resulting model is characterized by an enlarged BRS symmetry. Moreover, we show the existence of a vector-like supersymmetry, which fact simplifies the investigation of the infrared and ultraviolet renormalizability of this model.

The paper is organized as follows. In section 2 we describe the model at the classical level and we display its BRS transformations as well as the vector-like supersymmetry transformations. In section 3 we prove the finiteness of the model. Finally, in section 4, we show that the model is anomaly free.
2 Definition of the model at the classical level

Let us first consider the BF model on a two dimensional flat manifold $\mathcal{M}$ endowed with an Euclidean metric $\eta_{\mu\nu}$. This field model possesses the following metric independent action:

$$\Sigma_{inv}^{(1)} = \frac{1}{2} \int_{\mathcal{M}} d^{2}x \, \varepsilon^{\mu\nu} F_{\mu\nu}^a \phi^a, \quad (1)$$

where $\varepsilon^{\mu\nu}$ is the completely antisymmetric Levi–Civita tensor (with $\varepsilon^{12} = +1$), $\phi^a$ is a scalar field, and the field strength $F_{\mu\nu}^a$ is given by

$$F_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + f^{abc} A_{\mu}^b A_{\nu}^c. \quad (2)$$

$A_{\mu}^a$ stands for the gauge field with gauge index $a$. $f^{abc}$ are the structure constants of the gauge group which is assumed to be a compact and simple Lie group with all fields belonging to its adjoint representation. The generators of the Lie algebra are chosen to be anti-hermitian, such that $[T^a, T^b] = f^{abc} T^c$ and $Tr(T^a T^b) = \delta^{ab}$.

The action $\Sigma_{inv}^{(1)}$ is invariant under the following infinitesimal gauge transformations

$$\delta(\theta) A_{\mu}^a = (\partial_{\mu} \theta^a + f^{abc} A_{\mu}^b \theta^c) \equiv (D_\mu \theta)^a, \quad \delta(\theta) \phi^a = -f^{abc} \phi^b \phi^c, \quad (3)$$

where $\theta^a$ is a local parameter, and $D_\mu$ is the covariant derivative. This model has already been studied in much detail [3]. In the present work we enlarge the model by introducing a set of $N$ vector fields $B_{\mu}^{a\alpha}$ and $N$ scalars $X^{a\alpha}$ [2], where the index $\alpha$ takes all values from 1 to $N$. The contribution of these new fields represents a matter interaction. It is given by the metric independent local functional

$$\Sigma_N = \int_{\mathcal{M}} d^{2}x \, \varepsilon^{\mu\nu} (D_{\mu} B_{\nu})^a X^a_{\alpha}. \quad (4)$$

Without loss of generality, we restrict ourselves to the case $N = 1$ implying that (4) now reads

$$\Sigma_{inv}^{(2)} = \int_{\mathcal{M}} d^{2}x \, \varepsilon^{\mu\nu} (D_{\mu} B_{\nu})^a X^a. \quad (5)$$

The full action $\Sigma_{inv} = \Sigma_{inv}^{(1)} + \Sigma_{inv}^{(2)}$ is in fact invariant under an additional symmetry [2] given by the transformations

$$\delta(\psi) \phi^a = -f^{abc} \psi^b X^c, \quad \delta(\psi) B_{\mu}^{a} = D_{\mu} \psi^a, \quad (6)$$

where $\psi^a$ is an infinitesimal parameter.

As usual, the quantisation procedure requires a gauge fixing. In the present case we use
a Landau-type gauge. Due to the fact that we have two gauge symmetries we need two sets of ghost fields with corresponding Lagrange multiplier fields: \((c^a, \lambda^a)\) are the Faddeev-Popov ghost fields with corresponding antighost fields \((\bar{c}^a, \bar{\lambda}^a)\), and \((b^a, d^a)\) are Lagrange multiplier fields enforcing the Landau gauge conditions. The gauge fixed action is then given by \(\Sigma_{inv} + \Sigma_{gf}\), where

\[
\Sigma_{gf} = s \int_M d^2x (c^a \partial_\mu A^{a\mu} + \bar{\lambda}^a \partial_\mu B^{a\mu}) = 
\int_M d^2x [b^a \partial_\mu A^{a\mu} + d^a \partial_\mu B^{a\mu}] + 
+ \int_M d^2x [\bar{c}^a \partial_\mu (D^\mu c)^a - \bar{\lambda}^a \partial_\mu (D^\mu \lambda)^a + f^{abc} \bar{\lambda}^a \partial_\mu (c^b B^{c\mu})].
\]

The next step is to promote the two gauge symmetries to the nilpotent and nonlinear BRS-symmetry of the gauge fixed action

\[
es A^a_\mu = (D^\mu c)^a, 
es B^a_\mu = (D^\mu \lambda)^a - f^{abc} c^b B^c_\mu, 
es \phi^a = -f^{abc} (c^b \Phi^c + \lambda^b X^c), 
es X^a = -f^{abc} c^b X^c, 
es c^a = -\frac{1}{2} f^{abc} c^b c^c, 
es \lambda^a = -f^{abc} c^b \lambda^c, 
es \bar{c}^a = b^a, \quad sb^a = 0, 
es \bar{\lambda}^a = d^a, \quad sd^a = 0, 
es s^2 = 0.
\]

The canonical dimensions and Faddeev-Popov charges of all fields introduced so far are listed in (Table 1).

|       | \(A^a_\mu\) | \(B^a_\mu\) | \(\phi^a\) | \(X^a\) | \(c^a\) | \(\lambda^a\) | \(\bar{c}^a\) | \(\bar{\lambda}^a\) | \(b^a\) | \(d^a\) | \(\partial_\mu\) |
|-------|-------------|-------------|-----------|--------|--------|-------------|----------|-------------|--------|--------|-------------|
| \(\text{dim}\) | 1           | 1           | 0         | 0      | 0      | 0           | 0        | 0           | 0      | 0      | 1           |
| \(\phi\) | 0           | 0           | 0         | 1      | 1      | -1          | -1       | 0           | 0      | 0      | 0           |
| \(\bar{\phi}\) | 0           | 0           | 0         | 0      | 0      | 0           | 0        | 0           | 0      | 0      | 0           |

Table 1: Dimensions and Faddeev-Popov charges of the fields.

Note, that the BRS-exact gauge fixing term introduces a metric. As a consequence the energy-momentum tensor is BRS-exact as well and the model possesses a further symme-
try carrying a vectorial index:

$$\delta_{\mu} A^a_{\nu} = 0, \quad \delta_{\mu} B^a_{\nu} = 0,$$
$$\delta_{\mu} \phi^a = -\varepsilon_{\mu\nu}\partial^\nu \bar{c}^a, \quad \delta_{\mu} X^a = -\varepsilon_{\mu\nu}\partial^\nu \bar{X}^a,$$
$$\delta_{\mu} c^a = A^a_{\mu}, \quad \delta_{\mu} \lambda^a = B^a_{\mu},$$
$$\delta_{\mu} \bar{c}^a = 0, \quad \delta_{\mu} \bar{\lambda}^a = 0,$$
$$\delta_{\mu} b^a = \partial_{\mu} c^a, \quad \delta_{\mu} d^a = \partial_{\mu} \bar{\lambda}^a.$$  \hspace{1cm} (9)

The transformations (9) define the so-called vector-like supersymmetry \[8]. The invariant action plus the gauge fixing part is indeed invariant under (9),

$$\delta_{\mu}(\Sigma_{\text{inv}} + \Sigma_{\text{gf}}) = 0.$$  \hspace{1cm} (10)

Additionally, the symmetries (8) and (9) give rise to the following on-shell algebra:

$$\{s, s\} = 0,$$
$$\{s, \delta_{\mu}\} = \partial_{\mu} + \text{equations of motion},$$
$$\{\delta_{\mu}, \delta_{\nu}\} = 0.$$  \hspace{1cm} (11)

In order to write down the Slavnov identity, which expresses the symmetry content of the model with respect to BRS, we couple the nonlinear BRS transformations to BRS-invariant external sources leading to

$$\Sigma_{\text{ext}} = \int_M d^2 x \{ \Omega^{\mu a}(s A^a_{\mu}) + L^a(s c^a) + \rho^a(s \phi^a) + \sigma^{\mu a}(s B^a_{\mu}) + \Lambda^a(s \lambda^a) + Y^a(s X^a) \},$$  \hspace{1cm} (12)

where \((\Omega^{\mu a}, L^a, \rho^a, \sigma^{\mu a}, \Lambda^a, Y^a)\) are the external sources \[1\] whose canonical dimensions and Faddeev-Popov charges can be read off (Table 2).

|       | $$\Omega^{\mu a}$$ | $$L^a$$ | $$\rho^a$$ | $$\sigma^{\mu a}$$ | $$\Lambda^a$$ | $$Y^a$$ |
|-------|---------------------|---------|-----------|---------------------|-------------|---------|
| dim   | 1                   | 2       | 2         | 1                   | 2           | 2       |
| $$\phi\pi$$ | -1               | -2      | -1        | -1                  | -2          | -1      |

Table 2: Dimensions and Faddeev-Popov charges of the external sources.

Typically the propagators of massless scalar fields are ill-defined in two space-time dimensions \[4, 5\]. In particular, the analysis of the infrared problem connected with the

\[3\] We will see in due course that the algebra of \(\delta_{\mu}\) with the BRS operator \(s\) closes on translations.

\[4\] Clearly, the presence of these external sources breaks the symmetry (9). This fact will be discussed later.
ghost-antighost propagator

\[ \langle \bar{c}^a c^b \rangle = \delta^{ab} \frac{1}{k^2}, \tag{13} \]

requires the introduction of a regulator mass \( m \) \(^5\) such that

\[ \langle \bar{c}^a c^b \rangle_m = \delta^{ab} \frac{1}{k^2 + m^2}. \tag{14} \]

Keeping the algebraic structure \((13)\) amounts to adding \(^5\) the following expression \(^5\) to the action

\[
\Sigma_m = \int_M d^2x \left( -(\tau_1 + m^2)\bar{c}^a c^a - \tau_2(b^a c^a + \frac{1}{2} f^{abc} \bar{c}^b c^c) + \tau_4^\mu \bar{c}^a A^a_{\mu} + \tau_4^\mu (b^a A^a_{\mu} - \bar{c}^a (D_{\mu}c)^a) \right) = s \int_M d^2x [\tau_2 \bar{c}^a c^a + \tau_4^\mu \bar{c}^a A^a_{\mu}], \tag{15} \]

The quantities \((\tau_1, \tau_2, \tau_3^\mu, \tau_4^\mu)\) are the new external sources with the following BRS transformation laws

\[
\begin{align*}
&s\tau_2 = -(\tau_1 + m^2), & s\tau_1 = 0, \\
&\tau_3^\mu = \tau_4^\mu, & s\tau_\mu^3 = 0, \tag{16}
\end{align*}
\]

and with dimensions and Faddeev Popov charges as given in (Table 3).

| \( \tau_1 \) | \( \tau_2 \) | \( \tau_3^\mu \) | \( \tau_4^\mu \) |
|---|---|---|---|
| dim | 2 | 2 | 1 | 1 |
| \( \phi \pi \) | 0 | -1 | 1 | 0 |

Table 3: Dimensions and Faddeev-Popov charges of the external sources \( \tau \).

The same strategy can be used to regularize the propagator

\[ \langle \bar{\lambda}^a \lambda^b \rangle = \delta^{ab} \frac{1}{k^2}. \tag{17} \]

\(^5\)In fact, it would be sufficient to add to the action the quantity

\[
\Sigma_\tau = \int_M d^2x \left( -(\tau_1 + m^2)\bar{c}^a c^a - \tau_2(b^a c^a + \frac{1}{2} f^{abc} \bar{c}^b c^c) \right) = s \int_M d^2x [\tau_2 \bar{c}^a c^a],
\]

which guarantees \((14)\) and the BRS invariance of the total action. However, to maintain also the vector supersymmetry (allowing only linear breaking terms in the corresponding Ward identity as well as keeping the algebraic structure \((13)\), see below) we need the additional expression \( \int_M d^2x s(\tau_4^\mu \bar{c}^a A^a_{\mu}) \) containing the external sources \( \tau_3^\mu \) and \( \tau_4^\mu \).
Correspondingly, we have to add the following BRS exact expression to the action

\[ \Sigma'_m = \int_M d^2 x \left( -(\tau_1 + m^2)\bar{\lambda}^a \lambda^a - \tau_2(d^a \lambda^a + f^{abc} \bar{\lambda}^a c^b \lambda^c) + \tau_3^\mu \bar{\lambda}^a B^a_{\mu} + \right. \\
\left. + \tau_4^\mu [d^a B^a_{\mu} - \bar{\lambda}^a (D_{\mu} \lambda)^a + f^{abc} \bar{\lambda}^a c^b B^a_{\mu}] \right) \\
= s \int_M d^2 x [\tau_2 \bar{\lambda}^a \lambda^a + \tau_4^\mu \bar{\lambda}^a B^a_{\mu}] . \tag{18} \]

It follows that the propagator (17) is regularized according to

\[ \langle \bar{\lambda}^a \lambda^b \rangle_m = \delta^{ab} \frac{1}{k^2 + m^2} . \tag{19} \]

The total action, which is just the vertex functional at the classical level, becomes

\[ \Sigma^{(0)} = \Sigma_{inv} + \Sigma_{gf} + \Sigma_{ext} + \Sigma_{IR} ; \tag{20} \]

with

\[ \Sigma_{IR} = \Sigma_m + \Sigma'_m = s \int_M d^2 x \left( \tau_2 (\bar{c}^a c^a + \bar{\lambda}^a \lambda^a) + \tau_4^\mu (\bar{c}^a A^a_{\mu} + \bar{\lambda}^a B^a_{\mu}) \right) . \tag{21} \]

We are now ready to write down the non-linear functional Slavnov identity corresponding to the BRS invariance of the total action

\[ S(\Sigma^{(0)}) = \int_M d^2 x \left( \frac{\delta \Sigma^{(0)}}{\delta \Omega_{\mu}^{a\mu}} \frac{\delta \Sigma^{(0)}}{\delta A^a_{\mu}} + \frac{\delta \Sigma^{(0)}}{\delta \bar{\lambda}^a} \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} + \frac{\delta \Sigma^{(0)}}{\delta \rho^a} \frac{\delta \Sigma^{(0)}}{\delta \phi^a} + \frac{\delta \Sigma^{(0)}}{\delta \sigma^{a\mu}} \frac{\delta \Sigma^{(0)}}{\delta B^a_{\mu}} + \frac{\delta \Sigma^{(0)}}{\delta \Lambda^a} \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} + \right. \\
\left. + \frac{\delta \Sigma^{(0)}}{\delta \bar{Y}^a} \frac{\delta \Sigma^{(0)}}{\delta \bar{X}^a} + b^a \frac{\delta \Sigma^{(0)}}{\delta \bar{c}^a} + d^a \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} - (\tau_1 + m^2) \frac{\delta \Sigma^{(0)}}{\delta \tau_2} + \tau_3^\mu \frac{\delta \Sigma^{(0)}}{\delta \tau_4^\mu} \right) = 0 . \tag{22} \]

For later use, we derive from (22) the linearized version \( S_{\Sigma^{(0)}} \) of the non-linear BRS-operator:

\[ S_{\Sigma^{(0)}} = \int_M d^2 x \left( \frac{\delta \Sigma^{(0)}}{\delta \Omega_{\mu}^{a\mu}} \frac{\delta \Sigma^{(0)}}{\delta A^a_{\mu}} + \frac{\delta \Sigma^{(0)}}{\delta \bar{\lambda}^a} \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} + \frac{\delta \Sigma^{(0)}}{\delta \rho^a} \frac{\delta \Sigma^{(0)}}{\delta \phi^a} + \frac{\delta \Sigma^{(0)}}{\delta \sigma^{a\mu}} \frac{\delta \Sigma^{(0)}}{\delta B^a_{\mu}} + \frac{\delta \Sigma^{(0)}}{\delta \Lambda^a} \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} + \right. \\
\left. + \frac{\delta \Sigma^{(0)}}{\delta \bar{Y}^a} \frac{\delta \Sigma^{(0)}}{\delta \bar{X}^a} + b^a \frac{\delta \Sigma^{(0)}}{\delta \bar{c}^a} + d^a \frac{\delta \Sigma^{(0)}}{\delta \lambda^a} - (\tau_1 + m^2) \frac{\delta \Sigma^{(0)}}{\delta \tau_2} + \tau_3^\mu \frac{\delta \Sigma^{(0)}}{\delta \tau_4^\mu} \right) . \tag{23} \]

As already mentioned, the introduction of external sources breaks the vector-like supersymmetry. This fact is expressed by the following broken Ward-identity (WI) for the symmetry (8):

\[ V_{\mu} \Sigma^{(0)} = \Delta_{\mu}^{cl} , \tag{24} \]
where $V_\mu$ is the vector–like supersymmetry Ward operator

$$V_\mu = \int_{\mathcal{M}} d^2x \left( \varepsilon_{\mu \rho} \rho^a \delta \partial_\mu \phi^a - \varepsilon_{\mu \nu} [\Omega^\mu + \partial^\nu \phi^a - \tau_4^\nu \epsilon_a] \delta \partial_\mu \phi^a + A^a_\mu \delta \phi^a + (\partial_\mu \epsilon^a) \delta \partial_\mu \phi^a + L^a \delta \Omega^\mu \right)$$

$$+ \int_{\mathcal{M}} d^2x \left( B^a_\mu \delta \lambda^a + \varepsilon_{\mu \nu} [\sigma^\mu \rho^a \partial_\mu \phi^a - \Omega^\mu \partial_\mu \phi^a + \varepsilon_{\mu \nu} \Omega^\mu \partial_\mu \phi^a + \varepsilon_{\mu \nu} \Omega^\mu \partial_\mu \phi^a + \Lambda^a \delta \lambda^a \right)$$

$$- (\partial_\mu \tau_2) \delta \partial_\mu \lambda^a + \left[ \partial_\mu \tau_4^a - \delta \mu^a (\tau_1 + m^2) \right] \delta \partial_\mu \lambda^a \right).$$

Fortunately, the breaking term $\Delta^{cl}_\mu$ is linear in the quantum fields and 'insertions' of linear quantum fields are not renormalized by quantum corrections. It is given by

$$\Delta^{cl}_\mu = \int_{\mathcal{M}} d^2x \left( L^a_\mu \partial_\mu \phi^a - \rho^a \partial_\mu \phi^a - \Omega^\mu \partial_\mu \phi^a + \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a + \Lambda^a \partial_\mu \lambda^a$$

$$- Y^a \partial_\mu X^a - \sigma^\mu \partial_\mu Y^a + \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a + \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a + \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a$$

$$+ \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a + \varepsilon_{\mu \nu} \rho^a \partial_\mu \phi^a \right).$$

Furthermore, the total action $\Sigma^{(0)}$ turns out to be constrained by:

(i) 2 gauge conditions,

$$\frac{\delta \Sigma^{(0)}}{\delta \phi^a} = \partial^a \phi^a - \tau_2 \phi^a + \tau_4^a \phi^a,$$

$$\frac{\delta \Sigma^{(0)}}{\delta \phi^a} = \partial^a \phi^a - \tau_2 \phi^a + \tau_4^a \phi^a.$$ (27)

(ii) 2 ghost equations, obtained by commuting the gauge conditions with the Slavnov identity [3],

$$\mathcal{G}_1 \Sigma^{(0)} = \frac{\delta \Sigma^{(0)}}{\delta \phi^a} \frac{\delta \Sigma^{(0)}}{\delta \phi^a} + \left( \partial^a + \tau_4^a \right) \frac{\delta \Sigma^{(0)}}{\delta \phi^a} + \mathcal{G}_2 \Sigma^{(0)} = \frac{\delta \Sigma^{(0)}}{\delta \phi^a} \frac{\delta \Sigma^{(0)}}{\delta \phi^a} + \left( \partial^a + \tau_4^a \right) \frac{\delta \Sigma^{(0)}}{\delta \phi^a}.$$ (28)

(iii) 2 antighost equations,

$$\mathcal{G}_1 \Sigma^{(0)} = \Delta^{a}_1,$$

$$\mathcal{G}_2 \Sigma^{(0)} = \Delta^{a}_2.$$ (29)

where

$$\mathcal{G}_1^a = \int_{\mathcal{M}} d^2x \left( \frac{\delta}{\delta \phi^a} - f^{abc} \frac{\delta}{\delta \phi^b} - f^{abc} \frac{\delta}{\delta \phi^c} \right).$$ (30)

The corresponding breaking $\Delta^{a}_1$ is linear in the quantum fields

$$\Delta^{a}_1 = \int_{\mathcal{M}} d^2x \left( f^{abc} \left( - \Omega^{\mu \phi^a}_\mu + L^b \phi^c - \rho^a \phi^c - \Omega^{\mu \phi^a}_\mu + L^b \phi^c - \Omega^{\mu \phi^a}_\mu + L^b \phi^c - \Omega^{\mu \phi^a}_\mu + L^b \phi^c \right) + \left( \tau_1 + m^2 \right) \phi^a + \tau_2 \phi^a \right).$$ (31)
For the second antighost equation we have

$$G_a^2 = \int_\mathcal{M} d^2 x \left( \frac{\delta}{\delta \lambda^a} - f^{abc} \bar{\lambda}_b \frac{\delta}{\delta \bar{b}^c} \right),$$  \hspace{1cm} (32)$$

such that

$$\Delta_a^2 = - \int_\mathcal{M} d^2 x \left( f^{abc} \rho_b^c X_c - \Lambda^b_c \sigma^{by} A^a_{by} \right) - (\tau_1 + m^2) \bar{\lambda} - \tau_2 d^a. \hspace{1cm} (33)$$

Now, for an arbitrary functional $\Gamma$, depending on the same fields as the total action $\Sigma^{(0)}$, the corresponding linearized Slavnov operator $S_\Gamma$, the Ward operator for the vector-like supersymmetry $V_\mu$, and the two antighost operators $\bar{G}_1^a, \bar{G}_2^a$ yield the following nonlinear algebra:

$$S_\Gamma S(\Gamma) = 0,$$

$$S_\Gamma (V^a_\mu V^b_\nu) = \mathcal{P}^{a}_{\mu} \Gamma,$$

$$\{V^a_\mu, V^b_\nu\} \Gamma = 0,$$

$$S_\Gamma (\bar{G}_1^a \Gamma - \Delta_1^a) + \bar{G}_1^a S(\Gamma) = \mathcal{H}^a \Gamma,$$

$$S_\Gamma (\bar{G}_2^a \Gamma - \Delta_2^a) + \bar{G}_2^a S(\Gamma) = \mathcal{K}^a \Gamma,$$

$$V^a_\mu \bar{G}_1^a \Gamma - \Delta_1^a + \bar{G}_1^a (V^a_\mu \Gamma - \Delta_1^{cl}) = 0,$$

$$V^a_\mu \bar{G}_2^a \Gamma - \Delta_2^a + \bar{G}_2^a (V^a_\mu \Gamma - \Delta_1^{cl}) = 0.$$  \hspace{1cm} (34)

$\mathcal{P}^{a}_{\mu}$ is the Ward operator for translations

$$\mathcal{P}^{a}_{\mu} = \int_\mathcal{M} d^2 x \sum_{\Phi_i} \partial^{a}_{\mu \Phi_i} \delta \frac{\delta}{\delta \Phi_i}, \hspace{1cm} (35)$$

where $\Phi_i$ represents collectively all the fields introduced so far. For the operator $\mathcal{H}^a$ we have to consider only the fields possessing a gauge index (represented by $\Theta^a$) such that

$$\mathcal{H}^a = \int d^2 x \sum_{\Theta} \left( - f^{abc} \Theta^b \frac{\delta}{\delta \Theta^c} \right).$$  \hspace{1cm} (36)$$

The operator $\mathcal{K}^a$ is given by

$$\mathcal{K}^a = - \int d^2 x f^{abc} \left( X^b \frac{\delta}{\delta e^c} + A^b \frac{\delta}{\delta B^c} + \delta \frac{\delta}{\delta \bar{b}^c} + c^b \frac{\delta}{\delta \lambda^c} + \bar{\lambda} \frac{\delta}{\delta \bar{c}^c} \right) + \sigma^{b}_{\mu} \frac{\delta}{\delta \phi^{\mu}} + \rho^{b}_{\mu} \frac{\delta}{\delta Y^{\mu}} + \Lambda^b \frac{\delta}{\delta L^c}. \hspace{1cm} (37)$$

Now, it is easy to check that the classical action is invariant under the symmetries expressed by $\mathcal{H}^a$ and $\mathcal{K}^a$, i.e.,

$$\mathcal{H}^a \Sigma^{(0)} = \mathcal{K}^a \Sigma^{(0)} = 0.$$  \hspace{1cm} (38)$$

If the functional $\Sigma^{(0)}$ is a solution of the Slavnov identity, $\mathcal{S}(\Sigma^{(0)}) = 0$, of the Ward identities (24) and (33), and of the two antighost equations (29), then, from the above
nonlinear algebra, we get the following linear algebra:

\[
\begin{align*}
\{S_{\Sigma^{(0)}}, S_{\Sigma^{(0)}}\} &= 0, \\
\{S_{\Sigma^{(0)}}, V_\mu \} &= \mathcal{P}_\mu, \\
\{V_\nu, V_\mu \} &= 0, \\
\{S_{\Sigma^{(0)}}, \bar{G}_1^a \} &= \mathcal{H}^a, \\
\{S_{\Sigma^{(0)}}, \bar{G}_2^a \} &= \mathcal{K}^a, \\
\{V_\mu, \bar{G}_1^a \} &= 0, \\
\{V_\mu, \bar{G}_2^a \} &= 0,
\end{align*}
\]  \tag{39}

So far we have regularized the infrared divergent propagators and analyzed the symmetries of the model as well as derived the constraints which the total action obeys. In the remaining part of the paper we will extend our analysis to the quantum level.

3 Search for counterterms

We devote this section to the discussion of the stability problem which amounts to analyze all possible invariant counterterms for the total action. In a first step one has to modify the classical action as

\[
\Sigma' = \Sigma^{(0)} + \Delta,
\]  \tag{40}

where \(\Delta\) stands for appropriate invariant counterterms. The total classical action \(\Sigma^{(0)}\) is a solution of the Slavnov identity (22), the vector–like supersymmetry WI (24), the two gauge conditions (27), the two ghost equations (28), and the two antighost equations (29) as well as the two Ward identities (38). The perturbation \(\Delta\) is an integrated, local and Lorentz invariant polynomial of dimension 2 and vanishing ghost number.

By studying the stability of the theory we are looking for the most general deformation of the classical action such that the functional \(\Sigma'\) still obeys all the constraints listed above. Then the quantity \(\Delta\) is subject to the following set of constraints

\[
\begin{align*}
\frac{\delta \Delta}{\delta b^a} &= 0, \quad (41) \\
\frac{\delta \Delta}{\delta d^a} &= 0, \quad (42) \\
\frac{\delta \Delta}{\delta \bar{c}^a} + (\partial^\mu + \tau^\mu_4) \frac{\delta \Delta}{\delta \Omega^a_\mu} + \tau_2 \frac{\delta \Delta}{\delta L^a} &= 0, \quad (43) \\
\frac{\delta \Delta}{\delta \bar{\lambda}^a} + (\partial^\mu + \tau^\mu_4) \frac{\delta \Delta}{\delta \sigma^a_\mu} + \tau_2 \frac{\delta \Delta}{\delta \Lambda^a} &= 0, \quad (44) \\
S_{\Sigma^{(0)}} \Delta &= 0, \quad (45)
\end{align*}
\]
The first two equations (41) and (42) signify that $\Delta$ is independent of the two Lagrange multiplier fields $b^a$ and $d^a$. The equation (43) implies that $\Delta$ depends on the fields $\Omega^{a\mu}$, $\bar{c}^a$ and $L^a$ only through the two combinations

$$\tilde{\Omega}^{a\mu} = \Omega^{a\mu} + \partial^\mu \bar{c}^a - \tau^4 \bar{c}^a,$$
$$\tilde{L}^a = L^a + \tau_2 \bar{c}^a.$$ (52)

Correspondingly, we deduce from equation (44) that the fields $\sigma^{a\mu}$, $\bar{\lambda}^a$ and $\Lambda^a$ appear in the expression of $\Delta$ only through the two combinations

$$\tilde{\sigma}^{a\mu} = \sigma^{a\mu} + \partial^\mu \bar{\lambda}^a - \tau^4 \bar{\lambda}^a,$$
$$\tilde{\Lambda}^a = \Lambda^a + \tau_2 \bar{\lambda}^a.$$ (53)

Finally, the constraints (45), (46) and (47) may be combined in a single Ward-operator $\delta$,

$$\delta = S_{\Sigma^{(0)}} + \xi^\mu V_\mu + \varepsilon^\mu P_\mu - \int_M d^2 x \xi^\mu \frac{\partial}{\partial \varepsilon^\mu}.$$ (54)

It is easy to check the nilpotency of the above defined operator $\delta$. The vectors $\xi^\mu$ and $\varepsilon^\mu$ are constant vectors of ghost numbers $+2$ and $+1$, respectively. Clearly, we get

$$\delta \Delta = 0,$$ (55)

which constitutes a cohomology problem. The nilpotency property of the operator $\delta$ implies immediately that any expression of the form $\delta \hat{\Delta}$ is a solution of (53), where $\hat{\Delta}$ is a local integrated polynomial of dimension 2 and ghost number $-1$. Therefore, the general solution of (53) is of the form

$$\Delta = \Delta_c + \delta \hat{\Delta},$$ (56)

where $\Delta_c$ is the nontrivial solution whereas $\delta \hat{\Delta}$ is called the trivial solution.

The form of the trivial counterterm is restricted by dimension and ghost number requirements. Since the fields $(\phi, X)$ both have dimension and ghost number zero, an arbitrary
combination of these fields may appear many times in the counterterm. We denote the most general and possible combination of \((\phi, X)\) by \(f_\alpha[\phi, X]\), such that

\[
f_\alpha[\phi, X] = \sum_{\{n_i\},\{m_i\}=0}^\infty \beta^\alpha_{n_i,m_i} \left( \prod_{i=0}^{\infty} \phi^{n_i} X^{m_i} \right), \tag{57}\]

where \(\{n_i\}\) and \(\{m_i\}\) are understood as \(\{n_0, n_1, \ldots\}\) and \(\{m_0, m_1, \ldots\}\), respectively. \(\beta^\alpha_{n_i,m_i}\) are, of course, constant coefficients to be determined. Actually, the most general trivial counterterm \(\delta \hat{\Delta}\) reads

\[
\delta \hat{\Delta} = \delta \int_M d^2 x \text{Tr} \left( \rho f_1 + Y f_2 + \tau_2 f_3 + \bar{\Omega}^\nu f_4 A_\nu f_5 + \varepsilon_{\mu \nu} \bar{\Omega}^\mu f_6 A_\nu f_7 + \bar{\sigma}^\nu f_8 A_\nu f_9 \\
+ \varepsilon_{\mu \nu} \bar{\sigma}^\mu f_{10} A_\nu f_{11} + \bar{\Omega}^\nu f_{12} B_\nu f_{13} + \varepsilon_{\mu \nu} \bar{\Omega}^\mu f_{14} B_\nu f_{15} + \bar{\sigma}^\nu f_{16} B_\nu f_{17} \\
+ \varepsilon_{\mu \nu} \bar{\sigma}^\mu f_{18} B_\nu f_{19} + (\partial^\nu \bar{\Omega}_\nu) f_{20} + \varepsilon_{\mu \nu} (\partial^\mu \bar{\Omega}^\nu) f_{21} + (\partial^\nu \bar{\sigma}_\nu) f_{22} + \varepsilon_{\mu \nu} (\partial^\mu \bar{\sigma}^\nu) f_{23} \\
+ \tau_4 f_{24} \bar{\Omega}_\nu f_{25} + \varepsilon_{\mu \nu} \tau_4 f_{26} \bar{\Omega}^\nu f_{27} + \tau_4 f_{28} \bar{\sigma}_\nu f_{29} + \varepsilon_{\mu \nu} \tau_4 f_{30} \bar{\sigma}^\nu f_{31} \\
+ \bar{L} f_{32} c f_{33} + \bar{\Lambda} f_{34} c f_{35} + \bar{L} f_{36} \lambda f_{37} + \bar{\Lambda} f_{38} \lambda f_{39} \right). \tag{58}\]

In fact the expression \((58)\) may depend on the vector parameters \(\xi^\mu\) and \(\varepsilon^\mu\) which are not present in the total action \((20)\). For this reason we have to arrange for the trivial counterterm to be independent of these two constant vectors. In other words, we require that \(\hat{\Delta}\) has to be invariant under the vector–like supersymmetry and translation Ward operators. A lengthy and detailed analysis results in the expression for the trivial counterterm given below:

\[
S_{\Sigma[0]C_t} = S_{\Sigma[0]} \int_M d^2 x \text{Tr} \left( \beta^1 (\bar{\Omega}^\nu A_\nu + \rho \phi - \bar{L} c) + \beta^2 (\bar{\sigma}^\nu A_\nu + Y \phi - \bar{\Lambda} c) + \beta^3 (\bar{\Omega}^\nu B_\nu + \rho X - \bar{L} \lambda) + \beta^4 (\bar{\sigma}^\nu B_\nu + Y X - \bar{\Lambda} \lambda) \right), \tag{59}\]

where the \(\beta^i, i = 1, \ldots, 4\) are arbitrary constants.

Now we turn to the computation of the nontrivial counterterms \(\Delta_\varepsilon\) in \((56)\). In a first step we introduce a filtering operator \(\mathcal{N}\) such that

\[
\mathcal{N} = \int_M d^2 x \left( \sum_f \frac{\delta}{\delta f} \right), \tag{60}\]

where we have assigned to each field homogeneity degree one. The quantity \(f\) in \((60)\) stands for all fields (including also \(\varepsilon^\mu\) and \(\xi^\mu\)). The operator \(\mathcal{N}\) leads to the decomposition of the operator \(\delta\) as

\[
\delta = \delta_0 + \delta_1. \tag{61}\]

\(^6\) Our aim is to renormalize the theory defined by \((20)\) which is independent of \(\xi^\mu\) and \(\varepsilon^\mu\).
The nilpotency of the operator $\delta$ implies now that

$$\delta^2 = \{\delta_0, \delta_1\} = \delta_1^2 = 0.$$  \hfill (62)

The operator $\delta_0$ does not increase the homogeneity degree, whereas $\delta_1$ increases the homogeneity degree by one unit. Now it is evident that from $\delta \Delta = 0$ we get

$$\delta_0 \Delta = 0, \quad \delta_1 \Delta = 0,$$  \hfill (63)

where

$$\begin{align*}
\delta_0 &= \int_{\mathcal{M}} \left( dA^a \frac{\delta}{\delta A^a} + dA^a \frac{\delta}{\delta \rho^a} + d\phi^a \frac{\delta}{\delta \Omega^a} + d\hat{\Omega}^a \frac{\delta}{\delta \hat{\rho}^a} \right) + \\
&+ \int_{\mathcal{M}} \left( dB^a \frac{\delta}{\delta B^a} + dB^a \frac{\delta}{\delta \hat{Y}^a} + dX^a \frac{\delta}{\delta \hat{\sigma}^a} + d\hat{\sigma}^a \frac{\delta}{\delta \hat{\Lambda}^a} \right) + \\
&+ \int_{\mathcal{M}} d^2x \left( - \tau_1 \frac{\delta}{\delta \tau_2} + \tau_3 \frac{\delta}{\delta \tau_4} - \xi \frac{\delta}{\delta \xi} \right). 
\end{align*}$$  \hfill (64)

The first two parts of the expression of $\delta_0$ are given in terms of forms where,

$$\begin{align*}
A^a &= A^a_{\mu} dx^\mu, \\
\hat{\Omega}^a &= \varepsilon_{\mu\nu} \hat{\Omega}^a_{\mu} dx^\nu, \\
\hat{L}^a &= \frac{1}{2} \varepsilon_{\mu\nu} \hat{L}^a_{\mu} dx^\mu dx^\nu, \\
\hat{\rho}^a &= \frac{1}{2} \varepsilon_{\mu\nu} \rho^a_{\mu} dx^\mu dx^\nu, \\
B^a &= B^a_{\mu} dx^\mu, \\
\hat{\sigma}^a &= \varepsilon_{\mu\nu} \hat{\sigma}^a_{\mu} dx^\nu, \\
\hat{\Lambda}^a &= \frac{1}{2} \varepsilon_{\mu\nu} \Lambda^a_{\mu} dx^\mu dx^\nu, \\
\hat{Y}^a &= \frac{1}{2} \varepsilon_{\mu\nu} \hat{Y}^a_{\mu} dx^\mu dx^\nu, \\
\end{align*}$$  \hfill (65)

and $d$ is the exterior derivative $d = dx^\mu \partial_\mu$.

By looking to the expression (64) one easily recognizes that the fields $\tau_1, \tau_2, \tau_3, \tau_4, \varepsilon^\mu$ and $\xi^\mu$ transform as $\delta_0$ doublets which means that they are out of the cohomology of $\delta_0$ [7]. In order to solve the cohomology problem (63) we write $\Delta$ as an integrated local polynomial:

$$\Delta = \int_{\mathcal{M}} f^0_2,$$  \hfill (66)

such that $f^0_2$ has form degree 2 and ghost number 0. The use of Stoke’s theorem, the algebraic Poincare lemma [4] and the anticommutator relation $\{\delta_0, d\} = 0$ lead to the set of descent equations

$$\begin{align*}
\delta_0 f^2 + df^1 &= 0, \\
\delta_0 f^1 + df^2 &= 0, \\
\delta_0 f^2 &= 0. 
\end{align*}$$  \hfill (67)
Due to dimension and ghost number requirements the most general expression for $f_0^2$ is given by

$$f_0^2 = \sum_{n_{ij}, m_{ij}, l_{pq}, k_{pq} = 0}^{\infty} \sum_{j, q = 1}^{\infty} \alpha_{jq} \text{Tr} \left[ c \prod_{i=1}^{\infty} \left( \phi^* n_{ij} X^{m_{ij}} \right) c \prod_{p=1}^{\infty} \left( \phi^*_{pq} X^{k_{pq}} \right) \right] + \sum_{n_{ij}, m_{ij}, l_{pq}, k_{pq} = 0}^{\infty} \sum_{j, q = 1}^{\infty} \beta_{jq} \text{Tr} \left[ c \prod_{i=1}^{\infty} \left( \phi^* n_{ij} X^{m_{ij}} \right) \lambda \prod_{p=1}^{\infty} \left( \phi^*_{pq} X^{k_{pq}} \right) \right] + \sum_{n_{ij}, m_{ij}, l_{pq}, k_{pq} = 0}^{\infty} \sum_{j, q = 1}^{\infty} \gamma_{jq} \text{Tr} \left[ \lambda \prod_{i=1}^{\infty} \left( \phi^* n_{ij} X^{m_{ij}} \right) \lambda \prod_{p=1}^{\infty} \left( \phi^*_{pq} X^{k_{pq}} \right) \right].$$

(68)

$\alpha_{jq}, \beta_{jq}$ and $\gamma_{jq}$ stand for constant and field independent coefficients. The upper indices of the fields $\phi$ and $X$ are just integer exponents required by locality. To solve the descent equations (67) we follow the same strategy as in [8]. We define the operator

$$\delta_0 = \int_M \left( 2\rho \delta \frac{\delta}{\delta c} + A \frac{\delta}{\delta c} + \Omega \frac{\delta}{\delta \phi} + 2\hat{L} \frac{\delta}{\delta \Omega} + 2\hat{Y} \frac{\delta}{\delta B} + B \frac{\delta}{\delta \lambda} + \sigma \frac{\delta}{\delta \sigma} \right),$$

(69)

which, when commuted with the operator $\delta_0$, gives translations

$$[\delta_0, \delta_0] = \int_M \left( d\Psi \frac{\delta}{\delta \Psi} \right).$$

(70)

Recall that we are now working in the space generated by the fields which belong to the cohomology of $\delta_0$. These fields are denoted by $\Psi$. In other words, the operator $\delta_0$ appearing in equation (70) is restricted to this space where the $\delta_0$ doublets are absent. One can easily show [8] that the solution of the descent equations (67) is given by

$$f_2^0 = \frac{1}{2} \delta_0 \delta_0 f_0^2.$$ 

(71)

$$\int_M f_2^0$$

is then the nontrivial solution of the cohomology problem (63). A direct investigation shows that each monomial in (68) leads (after applying on it $\delta_0^2$) to a polynomial depending on the ghost fields $c$ and $\lambda$. But this is forbidden by the two constraints (48) and (49) which are valid at each homogeneity degree. In other words the nontrivial solution of the $\delta_0$ cohomology in the space constrained by (48) and (49) is zero. From this we deduce that the cohomology of the whole operator $\delta$ is empty. So, the most general solution of (64) takes the form

$$\Delta = S_{\Sigma^{(0)}} c_l.$$ 

(72)

The restriction coming from the two antighost equations (48) and (49) eventually implies the vanishing of all constant coefficients in (59).
Thus we have shown that the constraint system \((41-49)\) forbids any deformations of the classical action. Furthermore, if the symmetries of the model are anomaly free, then the symmetry content of the model at the classical level is also valid in the presence of quantum corrections. The analysis of anomalies is the subject of the next section.

4 Search for anomalies

In order to describe possible breaking of the symmetries which characterize the model, one has to apply the quantum action principle (QAP) \([5]\). The latter allows to describe symmetry breaking in the following way:

\[
\delta \Gamma = \tilde{\Delta}, \quad (73)
\]

where \(\Gamma\) is the full vertex functional given by a power series in \(\hbar\). The QAP requires that the breaking \(\tilde{\Delta}\) is a local, integrated, Lorentz-invariant polynomial of dimension 2 and ghost number 1. The nilpotency of \(\delta\) leads again to a cohomology problem

\[
\delta \tilde{\Delta} = 0, \quad (74)
\]

which implies the solution,

\[
\tilde{\Delta} = \delta \tilde{\Delta} + \mathcal{A}, \quad (75)
\]

where \(\mathcal{A} \neq \delta \tilde{\Delta}\). The anomaly candidate \(\mathcal{A}\), as a solution of \((74)\), has to obey the constraints \((41)\) — \((44)\) as well as \((48)\) and \((49)\). In other words we have to solve the cohomology problem \((74)\) in the same space of functionals as the problem \((55)\), Hence, \(\mathcal{A}\) depends only on the fields: \(A^a, c^a, \rho^a, \varphi^a, \hat{\Omega}^a, \hat{L}^a, \lambda^a, B^a, \hat{Y}^a, X^a, \hat{\sigma}^a\) and \(\hat{\Lambda}^a\). In terms of forms the functional \(\mathcal{A}\) is a local integrated polynomial of form degree 2 and ghost number 1

\[
\mathcal{A} = \int_\mathcal{M} f_2^1. \quad (77)
\]

\(^8\) A non trivial statement is that the anomaly candidate has to fulfill the antighost equations. Due to the algebra \((39)\) we deduce

\[
\{\delta, \hat{G}^a\} = \mathcal{H}^a,
\]

\[
\{\delta, \hat{G}_2^a\} = \mathcal{K}^a. \quad (76)
\]

Furthermore, the quantum generating functional of vertex functions \(\Gamma = \Sigma^{(0)} + \mathcal{O}(\hbar)\) fulfills the two antighost equations and the two WI’s \((18)\), see below. This fact together with \((73)\) and \((74)\) shows that the anomaly candidate must obey the antighost equations.
By using the strategy of the previous section we get the following set of descent equations

\[
\delta_0 f^i_2 + df^i_1 = 0, \\
\delta_0 f^i_1 + df^i_3 = 0, \\
\delta_0 f^3_0 = 0.
\] (78)

The last equation in (78) has the general solution

\[
f^3_0 = \sum_{n_{ij}, m_{ij}, t_{pq}, k_{pq}, t_{gy}, h_{gy}, \alpha_{jgy}, \beta_{jgy}, \gamma_{jgy}, \pi_{jgy}} \sum_{j, q, y = 1}^{\infty} \alpha_{jgy} Tr \left[ c \prod_{i=1}^{\infty} (\phi^{n_{ij}} X^{m_{ij}}) c \prod_{p=1}^{\infty} (\phi^{l_{pq}} X^{k_{pq}}) c \prod_{g=1}^{\infty} (\phi^{t_{gy}} X^{h_{gy}}) \right] + (79)
\]

\[
+ \sum_{n_{ij}, m_{ij}, t_{pq}, k_{pq}, t_{gy}, h_{gy}, \alpha_{jgy}, \beta_{jgy}, \gamma_{jgy}, \pi_{jgy}} \sum_{j, q, y = 1}^{\infty} \beta_{jgy} Tr \left[ c \prod_{i=1}^{\infty} (\phi^{n_{ij}} X^{m_{ij}}) c \prod_{p=1}^{\infty} (\phi^{l_{pq}} X^{k_{pq}}) \lambda \prod_{g=1}^{\infty} (\phi^{t_{gy}} X^{h_{gy}}) \right] +
\]

\[
+ \sum_{n_{ij}, m_{ij}, t_{pq}, k_{pq}, t_{gy}, h_{gy}, \alpha_{jgy}, \beta_{jgy}, \gamma_{jgy}, \pi_{jgy}} \sum_{j, q, y = 1}^{\infty} \gamma_{jgy} Tr \left[ c \prod_{i=1}^{\infty} (\phi^{n_{ij}} X^{m_{ij}}) \lambda \prod_{p=1}^{\infty} (\phi^{l_{pq}} X^{k_{pq}}) \lambda \prod_{g=1}^{\infty} (\phi^{t_{gy}} X^{h_{gy}}) \right] +
\]

\[
+ \sum_{n_{ij}, m_{ij}, t_{pq}, k_{pq}, t_{gy}, h_{gy}, \alpha_{jgy}, \beta_{jgy}, \gamma_{jgy}, \pi_{jgy}} \sum_{j, q, y = 1}^{\infty} \pi_{jgy} Tr \left[ \lambda \prod_{i=1}^{\infty} (\phi^{n_{ij}} X^{m_{ij}}) \lambda \prod_{p=1}^{\infty} (\phi^{l_{pq}} X^{k_{pq}}) \lambda \prod_{g=1}^{\infty} (\phi^{t_{gy}} X^{h_{gy}}) \right],
\]

where the quantities \(\alpha_{jgy}, \beta_{jgy}, \gamma_{jgy}\) and \(\pi_{jgy}\) are constant coefficients.

By using the same arguments as before (see the last section) one can prove that all constant coefficients appearing in (73) must vanish. Of course, this is due to the constraints (48) and (49). Let us, for instance consider the special case where

\[
f^3_0 = \alpha Tr(c)^3 + \beta Tr(\lambda)^3 + \gamma Tr(c^2 \lambda),
\] (80)

It leads to the anomaly candidate

\[
\mathcal{A} = Tr \int_M \left( 3\alpha (\hat{\rho}c^2 + A^2 c) + 3 \beta (\hat{Y} \lambda^2 + B^2 \lambda) + \gamma (\hat{\rho} \{c, \lambda\} + \{c, A\} B + A^2 \lambda + \hat{Y} c^2) \right),
\] (81)

where \(\alpha, \beta\) and \(\gamma\) are constant coefficients.

However, it is easy to verify that this anomaly candidate (81) does not obey the two antighost equations (48) and (49) unless \(\alpha = \beta = \gamma = 0\). This means in particular that the nontrivial solution of (72) is zero. Then, the Slavnov identity as well as the WI for the vector-like supersymmetry transformations are anomaly free. Hence, they are valid at the full quantum level. Concerning the two antighost equations, one can prove their validity at the quantum level by simply following the arguments of [3]. Furthermore, the two gauge conditions, the two ghost equations and the two WI's (48) are also valid at the quantum level. This can be proven by simply using the strategy of [4].

As a conclusion, the model we analyzed in this paper is anomaly free and ultraviolet as well as infrared finite at all orders of perturbation theory.
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