Design and Analysis of a Signal Denoising and Estimation System

Hassan Saadallah Naji , Hanan Badeea Ahmed

1 Department of Biomedical Engineering, College of Engineering, Kerbala University, Iraq
2 Department of Electronic Engineering, College of Engineering, University of Diyala, Iraq
* E-mail: hassan.saadallah@uokerbala.edu.iq

Abstract. Heart disease is one of the most common causes of death worldwide and electrocardiogram (ECG) signals are among the best tools currently available for diagnosing heart disease by offering accurate data about heart performance and the circulatory system. Unfortunately, the linear-quadratic problem often arises, which is a serious problem affecting the estimation of any linear dynamic system’s immediate state due to disruption with white noise due to the linear employment of information about state and white noise corruption. This can theoretically be addressed by using a Kalman filter as a recursive estimator. However, all biomedical applications that generate electrical signals are disposed to creating different types of noise with fixed coefficients filters, due to the random nature of these signals, and these can cause inaccuracies over time. In this paper, a double Kalman filter is suggested for ECG signal denoising and estimation. Additive White Noise added to the original ECG signal to form a noisy signal, and Kalman filter was used initially to improve the quality of the ECG signal and to offer high performance convergence while increasing the SNR. The second step involved employing Kalman filter to estimate the new denoised ECG signal during a given time period, as accuracy in time is very important in medical applications. The results were satisfactory, offering highly recognisable estimations.

1. Introduction

The leading cause of death globally is currently cardiac diseases, emphasizing the need to adopt a proper methodology to evaluate the cardiac condition of patients. ECG assessment is an active method in which electrocardiography (ECG) is used to assess the state of the patient's heart. The ECG registers the electrical signals (activity) generated across the cardiac cycle via electrodes positioned at different locations on the patient's body skin, offering a visual result the time domain; however, ECG readings are always noisy, and require denising by the application of signal processing tools. In biomedical engineering, signal processing approaches are a vital and active means of reducing noise and error, and this century has already witnessed a revolution in biomedical signal processing reaching at the level of practical application of signal processing and pattern analysis techniques. Typically, an ECG signal involves a P wave, a QRS complex, and a T wave, and any deviation in these parameters' estimates can indicate issues with the heart [1] Figure (1) shows a typical ECG signal.

The ECG signal is a sequence of waves and aberrations that represents the electrical action of the heart over time; it thus contains several different shapes of noise that may cause even ECG specialists to have difficulty making accurate identification of ECG signal properties. To remove various undesired frequency components from ECG signals conventional filters are often used. However, such static filters are not powerful enough to work well with noisy ECG signal [2].
Theoretically, a Kalman filter can be used as a recursive estimator to overcome the linear-quadratic problem, which is often the main challenge in approximating any linear dynamic system's instantaneous state, based on such systems containing noise interruptions from linear state information as well as interruptions caused by white noise. From the statistical perspective, the use of Kalman filter as a recursive estimator is the most favourable approach as it gives more convenient solution with regard to the determination of estimation error using the quadratic function. Such recursive estimators play an important role in the area of statistical estimation and were thus one of the most important statistical advancements of the twentieth century, being as important in estimation problems as silicon is in electronic systems [3]. The important feature of the filtering process used in the Kalman approach is the use of original ideal filtering on a split of components alongside the solution of the inversion problem; this makes it possible to signify the specified variables in the form of functions of the most convenient variables. Fundamentally, it predicts the independent variables in the form of reversed functions reliant on (the measurable) variables, using a dynamic process that requires repeat partial estimates [4].

2. Kalman Filter

The following mathematical equations offer the full description analysis for a steady-state Kalman filter:

\[
\begin{align*}
\hat{x}[n+1|n] &= A \hat{x}[n|n] + Bu[n] \\
\hat{x}[n|n] &= \hat{x}[n|n-1] + M(\gamma_v[n] - C\hat{x}[n|n-1])
\end{align*}
\]

For these equations:

- the predicted value of \( x[n] \) calculated up to \( yv[n-1] \)
- the predicted value is modified depending on the last calculated value of \( yv[n] \)
- \( M \) is the innovation gain
- \( u[n] \) is the original signal or input to be predicted
- \( yv[n] \) is the last calculated noise value.
- \( A, B, \) and \( C \) are the model matrices.

The current prediction and the time update together gives a new estimation for the state value at the next trial (one-step-ahead predictor). The reading update then regulates this estimation based on the new value of \( yv[n+1] \), as \( T \) the modification item is a function of the innovation such that,
\[ \text{Discrepancy} = y_v[n + 1] - C\hat{x}[n + 1|n] \quad (3) \]

The value of innovation gain \( M \), is selected to be the mid point of the predicted value \( y_v[n+1] \) and measured value in order to provide a decrease in the steady state covariance of the estimation error given the noise covariance:
\[ E(w[n]w'[n]) = Q \quad (4) \]
\[ E(v[n]v'[n]) = R \quad (5) \]
\[ N = E(w[n]v'[n]) = 0 \quad (6) \]

The one state-space model (Kalman filter) which represents adding both the time and measurement update, is thus
\[ \hat{x}[n + 1|n] = A(I - MC) \hat{x}[n|n - 1] + [B AM][u(n)] y_v[n] \]
\[ \hat{y}[n|n] = C(I - MC) \hat{x}[n|n - 1] + CMv_v[n] \quad (7) \]

The Kalman filter produces an optimum estimate of \( y[n] \), with \( [n|n - 1] \) as the filter state. [5]

3. Methodology

Any ECG signal denoising aims to reduce the signal to noise ratio (SNR) and mean absolute difference (MAD) of the original ECG signal. Here, data was used from the MIT-BIH arrhythmia database of which includes 48 half-hour extracts of two-channel ambulatory ECG readings, taken from 47 cases reviewed by the laboratory of BIH Arrhythmia from 1975 to 1979 [6].

A random selection of 23 readings was taken from a group of four thousand 24-hour ambulatory ECG readings collected from a diverse populace of inpatients (about 60%) and outpatients (about 40%) at Boston’s Beth Hospital; the residual readings were nominated from a similar group with less common but clinically significant arrhythmias that are not distributed randomly [6]. The employed signal is shown in figure (2), which simulates an ECG signal; this was subject to additive white noise (AWN), and the distorted ECG signal was then fed to a Kalman filter which was used to denoise the ECG signal data. The activity of this filter was intended to reduce the noise and error in the signal and as well as to estimate the level of error by predicting the values of covariances \( Q \) and \( R \). The proposed system is shown in figure (3).
A flowchart of proposed system is offered in Figure 4:

**Figure 2. The ECG Signal used [14]**

**Figure 3. The Proposed System**

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4. Denoising of ECG Signals Using Kalman Filters

ECG time series can become deformed due to artefacts; these are common, and sufficient information about them is required to prevent confusion of patients's ECG readings. Causes of artefacts include electrical interference by outside sources, electrical noise within the body, weak contacts, and machine

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**Figure 4. Flowchart of the Proposed System**

5. Denoising of ECG Signals Using Kalman Filters

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breakdowns [7]. If the linkage of the Kalman Filter is achieved under a real system, however, the Kalman Filter can be tuned precisely to such issues. The two important principal elements in the construction of the main core of the Kalman Filter, are determining the process disorder (noise) auto-covariance $Q$ and the measurement noise auto-covariance $R$.

**Under real systems measurement, the concept of noise is generally contained in the relevant estimates.** If the $Q$ value is particularly large, then sturdier measurement required for updates, as state estimates with high $Q$ values lead the filter to impose large changes in the variables’ real state causing the state variables to be affected by process noise. Generally, a higher $Q$ means a higher value of gain $K$ for a Kalman filter, allowing update of the estimates in a robust manner. The main rule for tuning states is thus to select a value for $Q$ that is as high as possible with no addition for any extra noise made to the state estimates [3]. To improve the signal quality and to reduce random noise as a part of the error, the most convenient filtering techniques are digital. Equation (1) can be rewritten as:

$$y(t) = u(t) + v(t)$$

(9)

Where, $y(t)$ stands for the noised ECG signal measured under time domain $t$, $u(t)$ expresses the ECG original signal measured under time domain $t$ and $v(t)$ represents the random noise affecting the original signal under the same time domain. $R$ random noise is thus considered to be an additive to the ECG original signal. Figure (4) shows an original ECG signal, as simulated in MATLAB. This simulation adds a zero-mean white Gaussian noise signal (AWGN) to the time-series of the original ECG signal and the resulting sequence is obtained after filtering with a Kalman filter. Figures 5 to 7 show the noisy ECG signal with three different variance values, while figures 8 to 10 show the ECG signal after the filtration process using three distinct groups of variances [(Q=1 & R=1), (Q=3 & R=2), Q=5 & R=1)]. The results suggest that the regulation of this filter can be done through tuning the $Q$ and $R$ parameters.

![Figure 5. Simulated ECG time-series](image1)

![Figure 6. Noisy ECG time-series (Q=1 & R=1)](image2)
Figure 7. Noisy ECG time-series (Q=3 & R=2)

Figure 8. Noisy ECG time-series (Q=5 & R=1)

Figure 9 Filtered ECG time-series (Q=1 & R=1)

Figure 10 Filtered ECG time-series (Q=3 & R=2)

Figure 11. Filtered ECG time-series (Q=5 and R=1)

Table 1 shows signal to noise ratio (SNR or S/N), expressed in decibels, for the ECG time series before and after filtration for three different combinations of Kalman filter parameters' (Q and R parameters). This shows the enhancements in the corresponding values of SNR after filtering using the Kalman filter as a recursive estimator.

Table 1 (SNR) Values of ECG Signal

| Variance Values | SNR Values |
|-----------------|------------|
| Q | R | SNR(before) | SNR(after) |

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4. Estimation of ECG Signals Using a Kalman Filter

From an estimation perspective, the term “filter” in the Kalman filter is unindicative of its action as an estimator. Theoretically, the WienerKolmogorov filter defines the statistical specifications of probability distributions to extract the estimation form sequence of observable data associated with noise signals. The process of filtering, which is done using the Kalman filter, then identifies two specific features. This separates the signal sequence from the optimal value of the signal, given the sum of the signal and noise. The main feature of Kalman filtering in this regard is that it can use in the original ideal filtering and separation of components of noisy signals as well as to determine solution to the inversion problems such that it provides the possibility of representation of the determined variables in form of functions of available variables.

In particular, it utilises practical relationships and to predict independent variables’ reversion functions in terms of measurable variables identified by dependency characteristics. These variables can thus be dynamic variables and only partially predictable.[4] Because of difficulty in obtaining superior estimations for Kalman filter auto-covariance matrices (Q and R, the practical implementation of such filters also be difficult. Comprehensive research has thus been undertaken to provide the optimum estimation covariance's matrices (Q and R) from the available data. From a practical point of view, however, a validated operational approach to estimate both covariance's matrices using the auto covarianceleast squares (ALS) technique, which basically depends on the time – lagged auto – covariance of conventional operating [9][10].

The software used for this practical calculation of the matrices of noise covariance in this case were GNU octave and MATLAB, using ALS technique available online under the GNU general public License [11].

The Field Kalman Filter (FKF), a Bayesian algorithm, which permits simultaneous estimation of the state parameters and covariance matrices of noise was proposed in [12]. The main algorithm for FKF has three key advantages, its complexity is relatively low, its formulation uses a recursive mode, and the algorithm's observed convergence is good as compared with other relevant algorithms. This promotes the feasibility of using FKF at least as a standby or as an alternative procedure for evaluating Auto-covariance Least-Squares methods. In this work, the results of estimation of covariance values were Q=0.98 and R=0.98, Q=2.89 and R=1.75, Q=4.91 and R=0.96, and the estimated ECG signals for the three covariance values are thus as shown in the figures 12 to 14:

|   |   |    |    |
|---|---|----|----|
| 1 | 1 | 3.5915dB | 15.2920dB |
| 3 | 2 | 2.7067dB | 17.5377dB |
| 5 | 1 | 1.1093dB | 22.8128dB |

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Figure 12. Filtered and estimated ECG time-series (Q=0.98 and R=0.98)

Figure 13. Filtered and estimated ECG time-series (Q=2.89 and R=1.75)

Figure 14. Filtered and Estimated ECG time-series (Q=4.91 and R=0.96)
5. Results and Analysis

The analysis of the proposed system was based on MATLAB simulation with a period of ECG time series measurement equal to 48 minutes and Kalman filter variances of $Q=1 \& R=1$, $Q=3 \& R=2$, and $Q=5 \& R=1$. The calculation of the mean absolute difference percentage (MAD%) of the proposed system before and after denoising and absolute difference percentage (AD%) of the ECG signal were thus evaluated. The evaluation of the absolute difference percentage (AD%) between the ECG filtered signal amplitude and the consistent actual ECG signal amplitude was done by applying Equation (10), with low MAD% values preferred [8]. The same procedure was then repeated for the estimated noise covariance matrices $Q_k$ and $R_k$, with estimated values $Q=0.98$ and $R=0.98$, $Q=2.89$ and $R=1.75$, and $Q=4.91$ and $R=0.96$, as shown in Table 2.

$$AD(t) = \frac{|ECG_{filtered}(t) - ECG_{actual}(t)|}{ECG_{actual}(t)} \times 100 \%$$

Here $AD(t)$ is evaluated as $AD\%$ at time t; $ECG_{filtered}$ (t) represents the filtered ECG signal amplitude at time t; $ECG_{actual}$ (t) is the actual ECG signal at time t, and finally N indicates the number of data points. The determination of the mean of all evaluated AD% values is the MAD% calculated by applying Equation (11):

$$MAD\% = \frac{\sum_{i=1}^{N} AD\%(t)}{N}$$

Table 3 shows the results for the three different groups of filter parameters (Q and R values) as filter variance values, in addition to the resulting equivalent values of the MAD values.

| Variance Value | MAD Values |
|----------------|------------|
| $Q$ | $R$ | $MADEq\%$ |
| 1 | 1 | 14.25% |
| 3 | 2 | 8.13% |
| 5 | 1 | 25.50% |
6. Conclusions

Accurate filtering of various noise distributions' shapes in ECG data allows this data to be employed in more complex manipulations offer more accurate diagnosis and detection of complex heart diseaseses. This work proposed applying a Kalman filter algorithm to reduce noise in ECG signals, and the resultant enhancements in the denoising of the ECG signals based on data set simulation showed that this could be accomplished successfully. The SNR of the denoised ECG signal varied with the Kalman filter covariance's Q and R values; thus the filter covariance's Q and R values must be adjusted precisely to ensure accurate results; moreover, a decrease in MAD% values occured where there was no further difference in the values of the two covariances in the Kalman filter. Nevertheless, the experimental results suggest that the suggested algorithm is very effective for the extraction of ECG signals from noisy data readings.

7. Future Work

This analysis can be used to provide enhancements in form matching with SNR variations from case to case were researchers make full use of the effects of changing the values of covariance's (Q and R) for the Kalman filter. To obtain precise results, the use of real ECG data which is likely to be more responsive to all forms of analyses is recommended and, the application of extended Kalman filter is proposed for enhancement of such work.

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| Table 3. MAD Values of the estimated and denoised systems |
|-----------------------------------------------|
| Variance Value | MAD Values |
|----------------|------------|
| Q  | R  | MADeq% |
| 0.98 | 0.98 | 12.35% |
| 2.89 | 1.75 | 7.11% |
| 4.91 | 0.96 | 23.44% |
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