Neutrinoless double beta decay in an

\[ SU(3)_L \otimes U(1)_N \] model

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We consider a model for the electroweak interactions with \( SU(3)_L \otimes U(1)_N \) gauge symmetry. We show that, it is the conservation of \( F = L + B \) which forbids massive neutrinos and the neutrinoless double beta decay, \((\beta\beta)_{0\nu}\). Explicit and spontaneous breaking of \( F \) imply that the neutrinos have an arbitrary mass and \((\beta\beta)_{0\nu}\) proceeds also with some contributions that do not depend explicitly on the neutrino mass.
I. INTRODUCTION

It is well known that the observation of neutrinoless double beta decay, will imply a new physics beyond the standard model. Usually, two kinds of mechanisms for this decay were independently assumed: massive Majorana neutrinos and right-handed currents [1].

In the latter case, neutrino mass is not required but, if the right-handed currents are part of a gauge theory, it has been argued that at least some neutrinos must have non-zero masses. If this is the case, the observation of \((\beta\beta)_0\nu\) would imply the existence of massive neutrinos, whether or not right-handed currents exist [2]. It is also well known that whatever mechanism generates the neutrinoless double beta decay, it also generates a Majorana mass term [3][4]. In this sense, the fundamental concept underlying that decay is the existence of massive neutrinos. However an important point is to find out what is the mechanism which triggers \((\beta\beta)_0\nu\) and this could not be the neutrino mass.

Models based on the gauge symmetry \(SU(3)_c \otimes SU(3)_L \otimes U(1)_N\) are interesting extensions of the Standard Model. It is possible, in this class of models to cancel anomalies if the number of families are divisible by the number of colors [5–7]. The new gauge sector of the model must be very massive because there are flavor changing neutral currents mediated by the new neutral gauge boson, \(Z'\). This implies that \(M_{Z'} > 40\) TeV [7].

Some time ago it was claimed that in this sort of models \((\beta\beta)_0\nu\) could occur even with massless neutrinos [8]. Here we return to this question, and show that the neutrinos could remain massless but not in a natural way.

Here, the word natural is used in the technical sense [9]. It means that the masses in a theory are finite and calculable if there is a zeroth-order mass relation which is invariant under arbitrary changes of parameters of the theory. In fact, in renormalizable theories a mass is either zero because some unbroken symmetry or arbitrary since it is necessary a counterterm in order to implement the renormalization program, leaving masses as free parameters of the theory. Hence, if a mass is zero at tree level, and there is no a respective counterterm, loop corrections cannot be divergent since there is no counterterm available to
cancel the infinities.

In fact, there are contributions to the no-neutrino $\beta\beta$ decay that do not depend explicitly on the neutrino mass, but it is not possible to keep the neutrino massless, at least not in the natural way we have discussed above. The present necessity of massive neutrinos has no relation with the bad high energy behavior of processes as $W^-V^- \rightarrow e^-e^-$ since in this models the doubly charged gauge boson $U^{-+}$ cancels the divergent part of such a process.

If one wants lepton number to be conserved, one must assume that the $V^\pm, U^{\pm\pm}$ gauge bosons carry lepton number. The lepton number conservation can also still be maintained in the Yukawa sector by assigning appropriate lepton number to the scalar fields too. This naive assignment of lepton number does not work because the vector bosons also couple to the quarks. A more appropriate quantum number is the lepto-baryon number, $F = L + B$ defined in Sec. II. It is the conservation of this quantum number which forbids massive neutrinos and $(\beta\beta)_0$. We add trilinear terms to the Higgs potential, Sec. III, which explicitly violate $F$ and in Sec. IV we consider the $(\beta\beta)_0$ decay. In Sec. V we show that the explicit $F$-violation implies also an spontaneously breaking of this quantum number and neutrinos with arbitrary mass.

II. THE $SU(3)_L \otimes U(1)_N$ MODEL

Let us first recall some points of the model of Ref. [7]. The representation content is the following: the leptons transform as triplets,

$$
\psi_{1L} = \begin{pmatrix} \nu_a \\ l_a \\ l^c_a \end{pmatrix}_L \sim (3,0),
$$

with $a = e, \mu, \tau$. In the quark sector we have the triplet

$$
Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3,\frac{2}{3}),
$$

(2.1)
for the left-handed fields, and singlets
\[ u_{1R} \sim (1, +\frac{2}{3}); \quad d_{1R} \sim (1, -\frac{1}{3}); \quad J_{1R} \sim (1, +\frac{5}{3}), \]
for the respective right-handed fields, and we have not introduced right-handed neutrinos.

The second and third families of quarks are in antitriplets \((3^*, -\frac{1}{3})\)
\[
Q_{2L} = \begin{pmatrix} J_2 \\ u_2 \\ d_2 \end{pmatrix}_L, \quad Q_{3L} = \begin{pmatrix} J_3 \\ u_3 \\ d_3 \end{pmatrix}_L.
\]
(2.4)
The respective right-handed quarks are also in singlets. In fact, two of the three quark generations, it does not matter which, transform identically in contrast to the third one. The model is anomaly free if we have equal number of triplets and antitriplets, counting the color of \(SU(3)_c\), and furthermore requiring the sum of all fermion charges to vanish. The anomaly cancellation occurs for the three generations together and not generation by generation. In Eqs. (2.2) and (2.4) the quarks, except the one with charge \(+\frac{5}{3}\), are linear combinations of the mass eigenstates.

For the first generation of quarks we have the following charged current interactions:
\[
\mathcal{L}_{Q_1W}^{CC} = -\frac{g}{\sqrt{2}} \left( \bar{u}_L \gamma^\mu d_\theta L W^\mu_+ + \bar{J}_{1L} \gamma^\mu u_L V^\mu_+ + \bar{d}_\theta L \gamma^\mu J_{1L} U^{--} + H.c. \right),
\]
(2.5)
and, for the second generation of quarks we have
\[
\mathcal{L}_{Q_2W}^{CC} = -\frac{g}{\sqrt{2}} \left( \bar{c}_L \gamma^\mu d_\theta L W^\mu_+ - \bar{s}_\theta L \gamma^\mu J_{2\phi L} V^\mu_+ + \bar{c}_L \gamma^\mu J_{2\phi L} U^{++} + H.c. \right).
\]
(2.6)
The charge changing interactions for the third generation of quarks are obtained from those of the second generation, making \(c \rightarrow t\), \(s \rightarrow b\) and \(J_2 \rightarrow J_3\). We have mixing only in the \(Q=-\frac{1}{3}\) and \(Q=-\frac{4}{3}\) sectors, then in Eqs. (2.3) and (2.4) \(d_\theta, s_\theta\) and \(J_{2\phi}\) mean Cabibbo-Kobayashi-Maskawa states in the three and two-dimensional flavor space \(d, s, b\) and \(J_2, J_3\) respectively. In the leptonic sector we have the charged currents
\[
\mathcal{L}_l^{CC} = -\frac{g}{\sqrt{2}} \sum_l \left( \bar{\nu}_l L \gamma^\mu l_L W^\mu_+ + \bar{\nu}_L \gamma^\mu \nu_L V^\mu_+ + \bar{\nu}_L \gamma^\mu l_L U^{++} + H.c. \right).
\]
(2.7)
In order to generate the quark masses, it is necessary to introduce the following Higgs scalars,

\[
\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^- \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix},
\]

transferring, under \(SU(3) \otimes U(1)\), as \((3, 0), (3, 1)\) and \((3, -1)\), respectively.

The lepton mass term transform as \((3 \otimes 3) = 3^* \oplus 6_s\), then we can introduce a triplet, like \(\eta\), but in this case one of the charged leptons remains massless and the other two are mass degenerate, or a symmetric antisextet \(S = (6_s^*, 0)\). We choose the latest one \([10]\) in order to obtain arbitrary mass for leptons.

The charge assignment of \((6^*, 0)\) is the following:

\[
S = \begin{pmatrix} \sigma_1^0 & h_2^+ & h_1^- \\ h_2^+ & H_1^{++} & \sigma_2^0 \\ h_1^- & \sigma_2^0 & H_2^{--} \end{pmatrix}
\]

The quark-Higgs interaction is

\[
\mathcal{L}_Y = \bar{Q}_{1L}(G^u_{1\alpha} U_{aR} \eta + G^d_{1\alpha} D_{aR} \rho + G^i J_{1R}\chi) \\
+ \bar{Q}_{1L}(F^u_{1\alpha} U_{aR} \rho^* + F^d_{1\alpha} D_{aR} \eta^* + F^i_{ik} J_{kR}\chi^* + H.c. )
\]

where \(\alpha = 1, 2, 3, i, k = 2, 3\), and \(U_{aR} = u_{1R}, u_{2R}, u_{3R}, D_{aR} = d_{1R}, d_{2R}, d_{3R}\) and all fields are still symmetry eigenstates. Explicitly from Eq. (2.10) one has,

\[
-\mathcal{L}_{QY} = G^u_{1\alpha}(\bar{u}_{1L}\eta^0 + \bar{d}_{1L}\eta_1^- + \bar{d}_{1L}\eta_2^+)U_{aR} \\
+ G^d_{1\alpha}(\bar{u}_{1L}\rho^0 + \bar{d}_{1L}\rho^0 + \bar{J}_{1L}\rho^{++})D_{aR} \\
+ G^i(\bar{u}_{1L}\chi^- + \bar{d}_{1L}\chi^{--} + \bar{J}_{1L}\chi^0)J_{1R} \\
+ F^u_{1\alpha}(\bar{J}_{1L}\rho^{--} + \bar{u}_{1L}\rho^{0*} + \bar{d}_{1L}\rho^-)u_{aR} \\
+ F^d_{1\alpha}(\bar{J}_{1L}\eta_2^- + \bar{u}_{1L}\eta_1^+ + \bar{d}_{1L}\eta_0^0)D_{aR} \\
+ F^i_{1\alpha}(\bar{J}_{1L}\chi^{0*} + \bar{u}_{1L}\chi^{++} + \bar{d}_{1L}\chi^+)J_{kR} + H.c.
\]
The Yukawa interaction in the leptonic sector is

\[ \mathcal{L}_S = -\frac{1}{2} \sum_{ab} G_{ab} \bar{\psi}_{i a L} \psi_{j b L} S_{ij}, \]  

(2.12)

with \( \psi^c = C \psi^T \), being \( C \) the charge conjugate matrix. Explicitly we have

\[ 2\mathcal{L}_S = -\sum_{ab} G_{ab} [\bar{\nu}_c a L \nu_b L \sigma_0^1 + \bar{l}_c a L l_b L H_1^{++} + \bar{l}_b a R l_b L H_2^{-} + (\bar{\nu}_a R l_b L + \bar{\nu}_b R \nu_a L) h_2^+ \\
+ (\bar{\nu}_a R \nu_b L) \bar{h}_1^- + (\bar{\nu}_b R \nu_b L + \bar{\nu}_a R \nu_a L) \sigma_2^0] + H.c. \]  

(2.13)

If we impose that \( \langle \sigma_1^0 \rangle = 0 \), then the neutrinos remain massless, at least at tree level.

As we said in the last section, let us define the lepto-baryon number, which is additively conserved, as follows

\[ F = L + B, \]  

(2.14)

where \( L \) is the total lepton number \( L = \sum_i L_i, \) \( i = e, \mu, \tau \) and \( B \) is the baryon number. As usual \( B(l) = 0 \) for any lepton \( l \), \( L(q) = 0 \) for any quark \( q \)

\[ F(l) = F(\nu_l) = +1, \]  

(2.15)

and

\[ F(u_\alpha) = F(d_\alpha) = \frac{1}{3}, \quad F(J_1) = -\frac{5}{3}, \quad F(J_i) = \frac{7}{3}, \]  

(2.16)

where \( \alpha = 1, 2, 3 \) and \( i = 2, 3 \). From Eqs. \( (2.13) \) and \( (2.16) \) we see that if

\[ F(V^-) = F(U^-) = +2, \]  

(2.17)

the interactions \( (2.5), (2.6) \) and \( (2.7) \) conserve \( F \).

In order to have \( F \) also conserved in the Yukawa sector, Eqs. \( (2.11) \) and \( (2.13) \), we also assign to the scalar fields the following values

\[ -F(\eta_2^+) = F(\rho^{++}) = -F(\chi^-) = +2, \]  

\[ -F(H_2^-) = F(H_1^{++}) = F(h_2^+) = F(\sigma_1^0) = -2, \]  

(2.18)
and with all the other scalars fields carrying $F = 0$.

Although the process $W^-V^- \rightarrow e^-e^-$ occurs in this kind of model with the exchange of massless neutrinos, it does not imply the $(\beta\beta)_{0\nu}$ decay since the vertex $d_L\gamma^\mu u_LV_\mu^+$ is forbidden by the $F$-conservation. This symmetry also forbids a mixing between $W^-$ and $V^-$. Hence, we see that the $F$ symmetry must be broken in order to allow the $(\beta\beta)_{0\nu}$ to occur.

### III. THE SCALAR SECTOR

Let

$$V(\eta, \rho, \chi, S) = \mu_1^2\eta^\dagger \eta + \mu_2^2\rho^\dagger \rho + \mu_3^2\chi^\dagger \chi + \mu_4^2 Tr(S^\dagger S) + \lambda_1(\eta^\dagger \eta)^2 + \lambda_2(\rho^\dagger \rho)^2$$

$$+ \lambda_3(\chi^\dagger \chi)^2 + (\eta^\dagger \eta) \left[ \lambda_4(\rho^\dagger \rho) + \lambda_5(\chi^\dagger \chi) \right] + \lambda_6(\rho^\dagger \rho)(\chi^\dagger \chi)$$

$$+ \lambda_7 \left[ Tr(S^\dagger S) \right]^2 + \lambda_8 Tr(S^\dagger SS^\dagger S) + Tr(S^\dagger S)[\lambda_9(\eta^\dagger \eta)]$$

$$+ \lambda_{10}(\rho^\dagger \rho) + \lambda_{11}(\chi^\dagger \chi) + \lambda_{12}(\rho^\dagger \eta)(\eta^\dagger \rho) + \lambda_{13}(\chi^\dagger \eta)(\eta^\dagger \chi)$$

$$+ \lambda_{14}(\rho^\dagger \chi)(\chi^\dagger \rho) + \left( f_1\epsilon^{ijk}\eta_i\rho_j\chi_k + \frac{1}{2}f_2\rho_i\chi_jS^{ij} + f_3\eta_i\eta_jS^{ij} \right)$$

$$+ \frac{1}{3!}f_4\epsilon^{ijk}\epsilon^{lmn}S_{il}S_{jm}S_{kn} + H.c. \right).$$

This is the most general $SU(3) \otimes U(1)$ gauge invariant, renormalizable Higgs potential for the three triplets and the sextet. The constants $f_i, i = 1, 2, 3, 4$ have dimension of mass. It is possible to show that the potential (3.1) has a local minimum at the following vacuum expectation values (VEV) for the scalar neutral fields [11]

$$\langle \eta \rangle = \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \langle \rho \rangle = \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}, \quad \text{and}$$

$$\langle S \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v_H \\ 0 & v_H & 0 \end{pmatrix}.$$  

(3.2)
Since we have chosen \( \langle \sigma_0^0 \rangle = 0 \), the neutrinos do not gain mass at tree level. However, we can verify the naturalness of this choice. The situation is similar when a triplet is added to the Standard Model \([12]\). We will return to this point later.

Redefining all neutral scalars as \( \varphi = v_\varphi + \varphi_1 + i \varphi_2 \), except for \( \sigma_0^0 \), we can analyze the scalar spectrum. For simplicity we will not consider relative phases in the vacuum expectation values. Requiring that the shifted potential has no linear terms in any of the \( \varphi_{1,2} \) components of all neutral scalars we obtain in the tree approximation the constraint equations:

\[
\begin{align*}
\mu_1^2 + 2\lambda_1 v_\eta_1^2 + \lambda_4 v_\rho^2 + \lambda_5 v_\chi^2 + 2\lambda_9 v_H^2 + f_1 v_\eta^{-1} v_\rho v_\chi &= 0, \\
\mu_2^2 + 2\lambda_2 v_\rho^2 + \lambda_4 v_\eta^2 + \lambda_6 v_\chi^2 + 2\lambda_{10} v_H^2 + f_1 v_\eta v_\rho^{-1} v_\chi + f_2 v_\chi v_H v_\rho^{-1} &= 0, \\
\mu_3^2 + 2\lambda_3 v_\chi^2 + \lambda_5 v_\eta^2 + \lambda_6 v_\rho^2 + 2\lambda_{11} v_H^2 + f_1 v_\eta v_\rho v_\chi^{-1} + f_2 v_\rho v_H v_\chi^{-1} &= 0, \\
\mu_4^2 + 4\lambda_7 v_H^2 + 2\lambda_8 v_\eta^2 + \lambda_9 v_\rho^2 + \lambda_{10} v_\chi^2 + 2\lambda_{11} v_H^2 + f_2 v_\rho v_H v_\chi^{-1} &= 0, \\
&= 0, \\
&Im f_i = 0, \quad i = 1, 2, 3, 4,
\end{align*}
\]

and the mass matrix in the \( \eta_1^-, \eta_2^-, \rho^-, \chi^-, h_1^-, h_2^- \) basis is

\[
\begin{pmatrix}
A_1 & A_2 & -F_3 c & 0 & 0 & -F_3 a \\
A_2 & A_3 & 0 & 0 & -F_2 & 0 \\
-F_3 c & 0 & B_1 & B_2 & -F_3 a & 0 \\
0 & 0 & B_2 & B_3 & 0 & -F_2 b \\
0 & -F_2 & -F_3 a & 0 & C_1 & C_2 \\
-F_3 a & 0 & 0 & -F_2 b & C_2 & C_1
\end{pmatrix}
\]

Where

\[
\begin{align*}
A_1 &= F_1 b a^{-1} - \lambda_{12} b^2, \quad A_2 = F_1 - \lambda_{12} a b, \quad A_3 = (F_1 a + F_2 c) b^{-1} - \lambda_{12} a^2, \\
B_1 &= F_1 b a^{-1} - \lambda_{13}, \quad B_2 = F_1 b - \lambda_{13} a, \quad B_3 = (F_1 a + F_2 c) b - \lambda_{13} a^2, \\
C_1 &= F_2 b c^{-1}, \quad C_2 = -F_3 a^2 c^{-1},
\end{align*}
\]
and we have defined the dimensionless constants $F_i = f_1/v_\chi$, $a = v_\eta/v_\chi$, $b = v_\rho/v_\chi$ and $c = v_H/v_\chi$. The mass matrix in Eq. (3.5) has just two Goldstone bosons and implies a mixing among all charged scalars, hence the physical charged scalars are linear combinations of $\eta_i^-,h_i^-(i = 1, 2)$, $\rho^-$ and $\chi^-$ which have no well defined value of $F$. As quarks $u,d$ interact according Eq. (2.11) with $\eta_1^−, \rho^−$ and the last fields are linear combinations of mass eigenstates we see that the diagram in Fig. 1 is possible even if the neutrino were massless.

**IV. THE $(\beta\beta)_{0\nu}$ DECAY**

The $F$ symmetry is softly broken by the $f_{3,4}$ terms in the scalar potential (3.1). As we said in the last section, the physical singly charged scalar, in the present case, are not eigenstates of $F$. Then, if $\Phi_1^-$ in Fig. 1 is one of the scalar mass eigenstates, in general

$$\phi_i^- = \sum_{ij} a_{ij} \Phi_j^-,$$

(4.1)

with $\phi_i^- = \eta_1^−, \eta_2^−, \rho^−, \chi^−, h_1^-, h_2^-$ and $a_{ij}$ are mixing parameters.

We can estimate a lower bound on the mass of $\phi_1^−$, by assuming that its contribution to $(\beta\beta)_{0\nu}$ is less than the amplitude due to massive Majorana neutrinos and vector bosons $W^-$ exchange. The latter amplitude is characterized by a strength which is proportional to

$$\frac{g^4 m_{\nu}^{eff}}{m_W^4 < p^2 >},$$

(4.2)

where $m_{\nu}^{eff} = |\sum_j U_{ij}^2 m_j|$ is the “effective neutrino mass” [1]. The experimental limit on $(\beta\beta)_{0\nu}$ decay rate imply that $m_{\nu}^{eff} < M_\nu = (1 - 2)\text{ eV}$ [2]; $< p^2 >$ is an average square 4-momentum carried by the virtual neutrino, its value is usually $(10\text{ MeV})^2$ [3]. On the other hand, the amplitude of the process in Fig. 1 is proportional to

$$\frac{(a_{11}a_{21})^2 G_{ud}^4 G_{ee}^4}{m_{\phi_1}^4 < p^2 >^2},$$

(4.3)

Then, assuming that Eq. (4.3) is less than Eq. (4.2) when $m_{\nu}^{eff} = M_\nu$, we have,

$$m_{\phi_1}^4 > \frac{(a_{11}a_{21})^2 G_{ud}^4 G_{ee}^4 \sqrt{2} < p^2 >^2}{32 G_F^2 M_\nu} \simeq (6, 9 \text{ TeV})^4 (a_{11}a_{21})^2 G_{ud}^4 G_{ee}^4.$$  

(4.4)
The factors in Eq. (4.3) arise as follows. In Eq. (2.11) the fields are symmetry eigenstates. It is possible to redefine the quark fields as

\[ q'_L = V^Q_L q_L, \quad q'_R = V^Q_R q_R \]  

with \( V^Q_{L,R} \) being unitary matrices in the flavor space and the primed fields denote mass eigenstates for the respective charge-\( Q \) sector. In Eq. (2.11) the interactions are as \( G_{ud} \bar{d}_L u_R \eta^-_1 \) with \( G_{ud} = (V_L^{(-\frac{2}{3})} G^u V_R^{(\frac{2}{3})})_{ud} \) and \( d, u \) are mass eigenstates. The coefficients \( G_{ee} \) appear in Eq. (2.13). As these mixing parameters in Eq. (4.4) can be very small it does not imply a strong lower bound on the scalar fields.

There are not contributions to \( (\beta\beta)_{0\nu} \) from trilinear Higgs interactions like \( \eta^-_1 h^-_2 H^{++} \). In models in which these contributions exist, they are negligible \([3]\) unless a neighboring mass scale \( (\sim 10^4 \text{GeV}) \) exist \([13]\).

V. CONCLUSIONS

As promised, we now consider the question of the neutrino masses. First at all, notice that if we forbid, by assuming a discrete symmetry \([10]\) the trilinear terms in \( f_{3,4} \) the mixing arisen from Eq. (3.5) is among \( \eta^-_1, \rho^-, h^-_1 \) and separately among \( \eta^-_2, \chi^-, h^-_2 \) and the \( F \) is conserved and \( (\beta\beta)_{0\nu} \) is forbidden. Then, it is important the fact that the \( F \) symmetry is softly broken by the trilinear terms \( f_{3,4} \) in the scalar potential (3.1).

As we have made \( \langle \sigma^0_L \rangle = 0 \), we can think that the neutrino masses vanish at tree level but that they are finite and calculable, in the sense of Sec. I. Let us consider this issue more in detail.

Besides (2.13) there is the additional Yukawa coupling between leptons and the triplet scalar \( \eta \),

\[ \mathcal{L}_{\eta} = \sum_{ab} f_{ab} \bar{\psi}^c_{aiL} \psi_{bjL} \epsilon^{ijk} \eta_k + H.c., \]  

\( a, b \) denote family indices, \( i, j \) denote \( SU(3) \) indices and \( \epsilon^{ijk} \) is the totally antisymmetric symbol. The Yukawa couplings \( f_{ab} \) must be antisymmetric, \( f_{ab} = -f_{ba} \), due to Fermi statis-
tics and the antisymmetric property of the charge conjugation matrix, $C$. Then, Eq. (5.1) connects leptons of different families. Typical terms of Eq. (5.1) read

$$(\bar{\nu}_e \mu_L - \bar{\nu}_L \nu_{\mu L})\eta_2^+, (\bar{\nu}_e \mu_L - \bar{e}_{L\nu_{\mu L}})\eta_1^-.$$ (5.2)

Next, due to Eqs. (2.13) and (5.2), the neutrinos gain finite masses through loop diagrams as in Figs. 2(a) and 2(b).

Now, joining the neutrino lines in Figs. 2 by $\sigma_0^1$, we obtain the divergent contribution to $\langle \sigma_1^0 \rangle$ appearing in Fig. 3(a,b). This implies a counterterm and makes it impossible to maintain $\langle \sigma_1^0 \rangle = 0$, at least in a natural way. Hence, neutrinos gain an arbitrary small mass since we can always assume that $\langle \sigma_0^0 \rangle \approx 0$.

If $\langle \sigma_1^0 \rangle \neq 0$ there is a mixing in the charged vector sector $W^+ V^+$,

$$W^\pm = \alpha X_1^\pm + \beta X_2^\pm, \quad V^\pm = -\beta X_1^\pm + \alpha X_2^\pm, \quad \alpha^2 + \beta^2 = 1,$$ (5.3)

where $X_{1,2}^\pm$ are mass eigenstates. Hence the $(\beta\beta)_{0\nu}$ proceeds also as in Fig. 4, whithout a direct dependence on the neutrino mass, but this contribution is suppressed by the large mass of the vector boson $X_2^-$ or by the mixing parameters since $\beta \simeq 0$. If $\langle \sigma_1^0 \rangle \neq 0$, and assuming discrete symmetries to forbid the explicit violations in Eq. (3.1), we have spontaneously breakdown of the $F$ symmetry, since $\sigma_1^0$ carries $F = -2$, see Eq. (2.18), implying a like-Majoron Goldstone boson since the $\sigma_1^0$ belongs to a triplet under $SU(2)$ together with $h_2^-$ and $H_1^-$. The phenomenology of this Goldstone boson can be similar to that of the Majoron [14] and it deserve a more detail study. As $\langle \sigma_1^0 \rangle \neq 0$ we expect a deviation from the $\rho = 1$ value ($\rho = \cos^2 \theta_W M_Z^2 / M_W^2$), however as $\langle \sigma_1^0 \rangle$ is arbitrary small its contributions can be made negligible.

Summarizing: we see that $(\beta\beta)_{0\nu}$ proceeds in this model also as a Higgs bosons effect, with almost massless neutrinos. Recall that if we have forbidden all trilinears in Eq. (3.1) except those with $f_1, f_2$, neutrinos could remain massless but the mixing in the charged scalar sector is only among $\eta_1^-, \rho^-, h_1^-$ and $\eta_2^-, \chi^-, h_2^-$ separately. Hence the $(\beta\beta)_{0\nu}$ cannot occur as was shown in Ref. [7].
In this sort of model, there must be exotic hadrons formed by combinations such as $qqJ^2$ with $B = 3$, being $q$ any of the known quarks.

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Figure Captions

Fig. 1 Scalar contribution to the $(\beta\beta)_{0\nu}$, $G_{ud,ee}$ are Yukawa couplings and $a_{11,21}$ mixing parameters in the scalar sector.
Fig. 2 Finite contribution to the Majorana neutrino mass due to the Yukawa couplings in Eqs. (2.13) and (5.2) and the trilinear terms $f_3$ and $f_4$ in the scalar potential (3.1).

Fig. 3 Joining the neutrino lines by $\sigma_1^0$ in Fig. 2 we obtain a divergent contribution to $\langle \sigma_1^0 \rangle$.

Fig. 4 Contribution to the $(\beta\beta)_{0\nu}$ due the charged vector bosons exchange if $\langle \sigma_1^0 \rangle \neq 0$. $X_1^+$ is a linear combination of $W^+$ and $V^+$. See Eq. (5.3).