Evaluation of railway transportation efficiency based on super-cross efficiency

Xiuyuan Kuang
1School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China

15120757@bjtu.edu.cn

Abstract. The efficiency of railway transportation is an important index. It can measure the development of railway transportation enterprises, and the efficiency of railway transportation has become a hot issue in the study of railway development. Data envelopment analysis (DEA) has been widely applied to railway efficiency analysis. In this paper, BBC model and super-cross efficiency model are constructed by using DEA theory, taking the 18 Railway Bureau as the research object, with the mileage, the number of employees, locomotive number, average daily loading number as input indicators, the passenger turnover, freight turnover and transport income as output indicators, then calculated and evaluated comprehensive efficiency, pure technical efficiency and scale efficiency. We get that the super-cross efficiency is more in line with the actual situation. Getting the super-cross efficiency is more in line with the actual situation.

Keywords. efficiency of railway, BBC model, super-cross efficiency.

1. Introduction
The efficiency of railway transportation is an important index. It can measure the development of railway transportation enterprises. The issue of railway transport efficiency evaluation is a multi-input and multi-output complex system engineering, on the one hand, manpower, vehicles and mileage are needed, and on the one hand, it can also produce the output of services such as volume and turnover. DEA method is a multi-input and multi output efficiency evaluation method, which has been widely used in railway transportation, Xu Luni used the generalized DEA model to evaluate the transportation efficiency of Railway Bureau. At the same time, network DEA and cross efficiency and improved DEA model were also applied to railway transportation.

2. Models and methods

2.1. Traditional DEA method
The traditional DEA method is a nonparametric method for evaluating the relative effectiveness of multi-input and multi-output decision-making units, which is very effective in evaluating multi-input and multi-output problems. It does not need to define the functional relation between inputs and outputs, and does not need to unify the selection of evaluation indexes. It has the characteristics of
certain "black box" method. Moreover, the DEA method can effectively avoid the influence of subjective factors, and has absolute advantages in efficiency evaluation.

There are no decision units (DMU), each unit has m kinds of input and s kinds of output, the jth decision unit input and output vector are

\[
X_j = (X_{1j}, X_{2j}, \ldots, X_{mj})^T, j=1,2,\ldots,n;
\]

\[
Y_j = (Y_{1j}, Y_{2j}, \ldots, Y_{sjj})^T, j=1,2,\ldots,n;
\]

Efficiency Evaluation The BBC model is as follows:

\[
\min \{ \theta - \varepsilon(\theta^T S^- + \theta^T S^+) \}
\]

\[
\begin{aligned}
& \text{s.t.} \quad \sum_{j=1}^{n} X_{kj} \lambda_j + S^- = \theta X_0 \\
& \sum_{j=1}^{n} Y_{kj} \lambda_j - S^+ = Y_0 \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0, \quad j = 1,2,\ldots,n \\
& S^- \geq 0, S^+ \geq 0,
\end{aligned}
\]  

(1)

\( \varepsilon \) represents Non Archimedes infinitesimal; \( \theta \in (1,1,\ldots,1)^T \in \mathbb{E}_m, e=(1,1,\ldots,1)^T \in \mathbb{E}_s \). To solve the above model, to judge whether the j decision unit is DEA valid depends on the values of \( S^+ \) and \( S^- \), and the rules are as follows: when \( \theta = 1 \) and \( S^+=S^-=0 \), the DMU is DEA valid, The system consisting of n decision units yields is optimal when output Y0 get from the input X0.

2.2. Super-Cross efficiency method

For the m input and s output n decision units, the input and output vectors of the kth decision unit are \((x_k, y_k)\), where \(x_k=(x_{1k}, x_{2k}, \ldots, x_{mk})^T, y_k=(y_{1k}, y_{2k}, \ldots, y_{sk})^T\). Take the output index weight vector \( u=(u_1, u_2, \ldots, u_s) \), and the input index weight vector \( v=(v_1, v_2, \ldots, v_s) \), definition efficiency evaluation index is the total output and total input ratio of DMUK:

\[
h_{kk} = \frac{\sum_{r=1}^{n} u_r y_{kr}}{\sum_{r=1}^{m} v_r x_{kr}} = \frac{u^T y_k}{v^T x_k}, \quad k = 1,2,\ldots,n
\]  

(2)

Based on the super efficiency model, the definition of cross efficiency matrix:

\[
H = \begin{bmatrix}
h_{11} & h_{12} & \cdots & h_{1m} \\
h_{21} & h_{22} & \cdots & h_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
h_{n1} & h_{n2} & \cdots & h_{nn}
\end{bmatrix}
\]  

(3)

Main diagonal elements \( h_{kk} = (1, 2, \ldots, n) \) is self-evaluation, off-diagonal elements \( h_{kj} (j \neq k) \) is cross evaluation.

For every DMUK, The input and output weight vectors are determined by solving the following model:

\[
\max h_{kk} = \frac{u^T y_k}{v^T x_k}
\]

\[
\begin{aligned}
& h_{kj} = \frac{u^T y_j}{v^T x_j} \leq 1, j = 1,2,\ldots,n; j \neq k \\
& v^T x_k = 1 \\
& u \geq 0, v \geq 0
\end{aligned}
\]  

(4)
Through the calculation, we can get the super efficiency evaluation value of each decision unit and get the optimal weight, and then calculate the other element values in the cross efficiency matrix. The formula of the evaluation unit without \( h_k \leq 1 \) limit, so unit efficiency evaluation DEA effective value is bigger than 1, while the cross efficiency matrix of non-diagonal elements are in \([0,1]\), the number of cross efficiency matrix is constructed such that \( h_k = 1 \) is reduced a lot.

### 3. Model solving and analysis

This paper selects the relevant data of each railway administration in 2013 to evaluate and analyze the efficiency value. The data comes from China Railway Yearbook 2014. When DEA efficiency evaluation is carried out, the model's input and output indexes should be reasonably determined. In this paper, 18 railway administrations are selected as decision-making units, and the operating mileage, the number of locomotives, the number of employees and the average daily number of vehicles are taken as input indicators, and the freight turnover, passenger turnover and transportation revenue are taken as indicators of output. These indexes are more comprehensive, reliable and representative, and can objectively reflect the transportation efficiency of each enterprise.

#### 3.1. Solution and analysis of BBC model

Through the DEAP2.1 software, the transportation efficiency of each railway bureau is calculated, and the comprehensive efficiency (TE), pure technical efficiency (PTE), scale efficiency (SE) and slack variable S value of each railway bureau are obtained. As show in Table 1.

| Railway Bureau | TE  | PTE | SE  | benefits | \( S_1^- \) | \( S_2^- \) | \( S_3^- \) | \( S_4^- \) | \( S_1^+ \) | \( S_2^+ \) | \( S_3^+ \) | \( S_4^+ \) |
|----------------|-----|-----|-----|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Beijing        | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Taiyuan        | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Zhengzhou      | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Wuhan          | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Shanghai       | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Guangtong      | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Lanzhou        | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Urumqi         | 1   | 1   | 1   |          | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Xi'an          | 0.988 | 0.993 | 0.995 | irs     | 305.08   | 0        | 0        | 1386.0   | 0        | 0        | 14.03    | 0        |
| Jinan          | 0.957 | 1     | 0.957 | irs     | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Hohhot         | 0.954 | 0.989 | 0.965 | irs     | 3381.45  | 0        | 0        | 11327.4  | 234.38   | 48.07    | 0        |
| Nanchang       | 0.905 | 0.952 | 0.951 | irs     | 579.17   | 312.03   | 140.982  | 0        | 0        | 0        | 0        | 113.3    |
| Chengdu        | 0.892 | 0.896 | 0.996 | irs     | 0        | 0        | 0        | 13061.6  | 0        | 0        | 0        | 0        |
| Qingzang       | 0.875 | 1     | 0.875 | irs     | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| Kunming        | 0.721 | 0.923 | 0.781 | irs     | 0        | 28.84    | 0        | 0        | 8.39     | 34.10    | 0        |
| Nanning        | 0.72  | 0.804 | 0.897 | irs     | 0        | 26.82    | 0        | 0        | 0        | 0        | 0        |
| Shengyang      | 0.705 | 0.903 | 0.781 | drs     | 4285.75  | 298.11   | 0        | 68831.7  | 0        | 0        | 124.7    | 0        |
| Harbin         | 0.664 | 0.674 | 0.986 | irs     | 485.65   | 0        | 0        | 69072.5  | 0        | 59.11    | 0        |

According to the data in the table, the 18 railway bureaus are divided into three parts according to the comprehensive efficiency, the technical efficiency and the scale efficiency. The comprehensive efficiency is the first part (DEA is effective). From the table we can see that the Beijing Railway Bureau, Taiyuan Bureau, Zhengzhou Bureau, Shanghai Bureau and other eight railway bureau comprehensive efficiency value of 1, which means the eight railway bureaus have reached DEA effective.

The second part is purely technical efficiency, removed the eight Railway Bureau which in the first part, Jinhua Bureau and the Qinghai Tibet Railway Bureau’ purely technical effective is valid, so the
input and output are not the problem, but the scale efficiency is invalid, the scale is increasing, the scale of the enterprise is the main cause of the comprehensive efficiency invalid, expand the scale of enterprises, increase investment can bring more output.

The third part is scale efficiency, besides the above mentioned railway bureau, There are eight railway bureau’ scale efficiency is invalid. Xi'an Bureau, Hohhot Bureau, Nanchang Bureau and other seven bureau of the scale efficiency is increasing, that is, the size of the enterprise did not reach the optimal, it should increase investment; Shenyang Bureau of the scale is reduced, it means the part of the input is invalid, Bureau should be appropriate to reduce investment.

3.2. Solution and analysis of super-cross efficiency

By using Matlab to compute the algorithm of super-cross efficiency model, the relevant data are filled in and calculated, and the efficiency values of each railway station are obtained, then compared with the efficiency of the BBC model. As show in Table 2:

| Railway Bureau | super-cross efficiency | rank | BBC efficiency | rank |
|----------------|------------------------|------|----------------|------|
| Taiyuan        | 1.017894               | 1    | 1              | 1    |
| guangtie       | 0.947656               | 2    | 1              | 1    |
| Urumqi         | 0.896444               | 3    | 1              | 1    |
| shanghai       | 0.874128               | 4    | 1              | 1    |
| wuhai          | 0.866644               | 5    | 1              | 1    |
| zhangzhou      | 0.832233               | 6    | 1              | 1    |
| jinan          | 0.825006               | 7    | 0.957          | 10   |
| Xi'an          | 0.818644               | 8    | 0.988          | 9    |
| lanzhou        | 0.811161               | 9    | 1              | 1    |
| beijing        | 0.804389               | 10   | 1              | 1    |
| chengdu        | 0.761994               | 11   | 0.892          | 13   |
| hohhot         | 0.702311               | 12   | 0.954          | 11   |
| Qingzang       | 0.638911               | 13   | 0.875          | 14   |
| nanchang       | 0.631906               | 14   | 0.905          | 12   |
| nanning        | 0.613683               | 15   | 0.72           | 16   |
| shenyang       | 0.591906               | 16   | 0.705          | 17   |
| kunming        | 0.583944               | 17   | 0.721          | 15   |
| Harbin         | 0.491822               | 18   | 0.664          | 18   |

From the table we can see the efficiency of super-cross efficiency obtained value efficiency rankings and solving the traditional DEA model value ranking is basically the same, and the decision-making unit with the original efficiency value of 1 can be sorted. At the same time, the efficiency values under the super-cross efficiency model are generally lower than those obtained by the traditional DEA model, this is because the traditional DEA model does not limit the choice of weights, and the decision-making units tend to choose the weights which are most conducive to their own efficiency values. Therefore, the evaluation results are too optimistic. After introduction of the evaluation of other decision unit to the decision unit, this mutual evaluation mechanism of super-cross-evaluation method makes its efficiency value is lower than the traditional efficiency value, and the ranking fluctuates significantly, so that the ranking has changed greatly.

In the traditional DEA model, the efficiency value of eight railway bureau is 1 in the process of solving the comprehensive efficiency value, that is, the eight railway bureau are DEA valid, and it is impossible to judge the efficiency of these eight effective bureau. But using super efficiency cross efficiency model to calculate can be very light and effective to solve this problem, and get the
complete sorting of all decision-making units. Through the rank of super-cross efficiency, the top six decision-making units is Taiyuan, Guangzhou Railway Group, Urumqi, Shanghai, Wuhan and Zhengzhou bureau. There are two bureau’ ranking in the DEA valid is decline, Beijing and Taiyuan bureau shows that the traditional DEA model generates pseudo efficient decision units at the time of operation, and they are not optimal in terms of transportation efficiency. Lanzhou and the Xi’an bureau is the main transit station in west, so there is some competitive relationship, and the efficiency of Xi’an bureau is higher than Lanzhou bureau, it showed some investment is unreasonable, which is consistent with the reality.

Coal is the main source of the railway, Shanxi is the main origin of coal, so it ranked first; Guangzhou Railway Group is located in the eastern economically developed areas, with a large number of port logistics, an important source of electronic products, and people travel demand is high, so efficiency is high; Urumqi Bureau is the transit point of the Eurasian continent, and has a lot of mineral resources; Wuhan and Zhengzhou bureau is an important transfer station in the central part of China, with large volume of passenger traffic and freight traffic; Because of the geography, resource, economy and management level, the transportation efficiency of other bureau has not reached the optimum, and the sorting of super-cross efficiency is consistent with the arrangement of the Railway Bureau in reality. Therefore, the efficiency of the super-cross efficiency method is more reasonable and more realistic.

4. Conclusion
(1) In this paper, the traditional DEA method is used to evaluate and analyze the efficiency of the 18 railway bureaus. The comprehensive efficiency of most railway bureaus is invalid and the ineffective railway bureaus are analyzed. The vast majority of enterprises are not achieve the best, they should expand the scale of enterprises and increase the amount of input.

(2) This paper analyzes the comparison between the traditional DEA method and the super-cross efficiency method, the traditional DEA model does not limit the choice of weights, and the decision-making units tend to choose the weights which are most favorable to their own efficiency values, which makes the evaluation results too optimistic, and cannot compare the decision units with a efficiency value of 1. The efficiency of the decision-making unit has been applied to the decision-making unit, so that the final evaluation results can effectively avoid the self-evaluation is too blind and optimistic, and the final results of the decision-making unit is more comparable and reasonable, The super-cross efficiency method can give the decision-making unit efficiency ranking.

References
[1] Luni.Xu, Railway Transport Efficiency Analysis based on Generalize DEA model[D].Beijing Jiaotong University,2016
[2] Chunhao Li, Hang Su, Yijie Tong, Yonghe Sun, DEA cross efficiency evaluation model based on ideal decision making unit and reference solution strategy[J/OL], Chinese Management Science, 2015,23(02):116-122.(2015-03-02)
[3] Xin Mu, Xueqing Cheng, Xi Zhang, Yun Pu, Efficiency evaluation of Railway Heavy Haul Freight Cars Based on DEA cross efficiency model [J], Chinese Journal of Railway Science, 2014,35(01):130-134.[2017-09-24]
[4] Jinxian Wang, Cross efficiency evaluation method based on super efficiency DEA model [J]. Systems engineering,2009, 27(06):115-118.[2017-09-24]