Lightning-Fast and Privacy-Preserving Outourced Computation in the Cloud

Ximeng Liu Member, IEEE, Robert H. Deng, Fellow, IEEE, Pengfei Wu, Student Member, IEEE, Yang Yang, Member, IEEE

Abstract—In this paper, we propose a framework for lightning-fast privacy-preserving outsourced computation framework in the cloud, which we refer to as LightCom. Using LightCom, a user can securely achieve the outsource data storage and fast secure data processing in a single cloud server different from the existing multi-server outsourced computation model. Specifically, we first present a general secure computation framework for LightCom under the cloud server equipped with multiple Trusted Processing Units (TPUs) which face the side-channel attack. Under the LightCom, we design two specified fast processing toolkits which allow the user to achieve the commonly-used secure integer computation and secure floating-point computation against the side-channel information leakage of TPUs, respectively. Furthermore, our LightCom can also guarantee access pattern protection during the data processing and achieve user private information retrieval after the computation. We prove that the proposed LightCom can successfully achieve the goal of single cloud outsourced data processing to avoid the extra computation server and trusted computation server, and demonstrate the utility and the efficiency of LightCom using simulations.

Index Terms—Privacy-Preserving; Secure Outsourced Computation; Homomorphic Encryption; Secret Sharing Technique; Against Side-channel Attack.

1 INTRODUCTION

The internet of things (IoT), embedded with electronics, Internet connectivity, and other forms of hardware (such as sensors), is a computing concept that describes the idea of everyday physical objects being connected to the internet and being able to identify themselves to other devices. With large numbers of IoT devices, huge amount of data are generated for usage. According to IDC[1], the connect IoT devices will reach 80 billion in 2025, and help to generate 180 trillion gigabytes of new data that year. A quarter of the data will create in real time, and 95% is to come from IoT real time data. With such large volume real-time data are generated, it is impossible for the resource-limited IoT devices to store and do the data analytics in time. Cloud computing, equipped almost unlimited power of storage and computing, provides diversity of services on demand, such as, storage, databases, networking, software, analytics, intelligence. With the help of cloud computing, 49 percent of data will be stored in public cloud environments by 2025[2]. Thus, it is unsurprisingly that the huge volume data generated by IoT devices are outsourced to the cloud for long-term storage and achieve real-time online processing.

Despite the advantages provided by IoT-cloud data outsourcing architecture, the individual IoT users are hesitated to the system for data storage and processing without any protection method. In the Internet of Medical Things example [1], patients’ wearable mHealth devices that always equipped with the biometric measurements sensors (such as heart rate, perspiration levels, oxygen levels) to record the physical sign of the patient. The hospital can use clients PHI decision-making model to automatically check a patients health status. If no protection method is adopted, patients’s physical sign can be capture by adversary. Moreover, the hospital model can be got by other third-party company to make profit. Use the traditional encryption technique can protect the data from leakage, however, the ciphertext lost the original meaning of the plaintext which cannot doing any computations.

Protecting the data and achieve the secure outsource computation simultaneously is an eye-catching field to solve the above problems. Currently, there are typically two aspects of techniques to achieve secure outsourced computation: theoretical cryptography solution and system security solution. For the cryptography point of view, homomorphic encryption [3] is considered as a super-excellent solution for the outsourced computation which allows the third-party to perform the computation on the encrypted data without reveal the content of the plaintext. Fully homomorphic encryption [3] can achieve arbitrary computation on the plaintext corresponding to the complex operations on ciphertext. However, the computation overhead is still tremendous which is not
fit for the piratical usage (e.g., it requires 29.5 s to run secure integer multiplication computation with a common PC [4]). Semi-homomorphic encryption [5], [6] only supports one types of homomorphic (e.g. additive homomorphic), can achieve complex data computation on the encrypted data with the help of extra honest-but-curious servers. But, the extra computation server will increase possibility of the information leakage. Recently, for the industrial community, trusted execution environment (TEE, such as Intel® Software Guard Extensions (SGX) and ARM TrustZone) is developed to achieve the secure computation which allows user-level or operating system code to define private regions of memory, also called enclaves. The data in the enclave are protected and unable to be either read or saved by any process outside the enclave itself. The performance of the TEE is equivalent to the plaintext computation overhead. Unfortunately, TEE easily faces the side-channel attack, and the information inside the enclave can be leaked to the adversary. Thus, an fascinating problem appears for creating a system to balance the usage of practical outsourced computation system and eliminate the extra information leakage risk: how can a single cloud securely perform the arbitrary outsourced computation without the help of extra third-party computation server or trusted authority, which interactions between the user and the cloud kept to a minimum.

In this paper, we seek to address the above-mentioned challenge by presenting a framework for lightning-fast and privacy-preserving outsourced computation Framework in a Cloud (LightCom). We regard the contributions of this paper to be six-fold, namely:

- **Secure Data Outsourced Storage.** The LightCom allows each user to outsource his/her individual data to a cloud data center for secure storage without compromising the privacy of his/her own data to the other unauthorized storage.

- **Lightning-fast and Secure Data Processing in Single Cloud.** The LightCom can allow in a single cloud equipped with multiple Trusted Processing Units (TPUs), which provides a TEE to achieve the user-centric outsourced computation on the user’s encrypted data. Moreover, the data in outside untrusted storage are secure against chosen ciphertext attack for long-term, while data insider TPUs can be protected against side-channel attack.

- **Outsourced Computation Primitive Combinable.** Currently, the outsourced computation methods focus on a special computation task, such as outsourced exponential computation. Different specific outsourced tasks are constructed with different crypto preliminary. Thus, the previous computation result cannot be directly used for the input of the next computation. Our LightCom can directly solve the problem with uniform design method which can achieve computation combinable.

- **No Trusted Authority Involved.** In most of the existing cryptosystem, trusted authority is fully trusted which is an essential party in charge of distributing the public/private keys for all the other parties in the system. Our LightCom does not involve an additional fully trusted party in the system which makes the system more efficient and practical.

- **Dynamic Key/Ciphertext Shares Update.** To reduce the user’s private key and data leakage risk during the processing, our LightCom randomly splits the key and data into different shares which are processed in different TPUs, cooperatively. To avoid long-term shares leaking for recovering the original secrets, our LightCom allows TPUs updating user’s “old” data/private-key shares into the “new” shares on-the-fly dynamically without the participation of the data user.

- **High User Experience.** Most existing privacy-preserving computation technique requires user to preform different pre-processing technique according the function type prior to data outsourcing. The LightCom does not need the data owner to perform any pre-processing procedure - only needs to encrypt and outsource the data to the cloud for storage. Thus, interactions between the user and the cloud kept to a minimum - send the encrypted data to the cloud, and received outsourced computed results in a single round.

**Motivation and Technique Overview.** As the sensitive information contained inside TPU can be attacked, our primary goal of the LightCom framework is to achieve secure computation in a single cloud without the help of an additional party. The idea is to let the data store in the outside storage, and achieve privacy-preserving computation insider TPU. The main challenges are how to achieve both secure data storage and data processing against side-channel attacks, simultaneously. To solve the previous challenge, we use a new Pailler Cryptosystem Distributed Decryption (PCDD) which can achieve semantic secure data storage. To prevent information leakage inside TPU, our LightCom uses one-time pad by adding some random numbers on plaintext of the PCDD ciphertext. Even the “padded” ciphertext for TPU enclave for decryption and process, the attacker still cannot get the original message of the plaintext. To achieve ciphertext decryption, our LightCom uses multiple TPUs, and each TPU only stores a share of the private key to prevent the user’s key leakage risk. Even some partial private key/data shares may leak to the adversary; our framework can successfully update these shares dynamically inside the TPU to make the leaked shares useless. More importantly, all the secure execution environment (called TPU enclaves) in TPUs

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3. https://software.intel.com/en-us/sgx
4. https://developer.arm.com/ip-products/security-ip/trustzone
5. https://software.intel.com/en-us/articles/intel-sgx-and-side-channels
6. https://www.arm.com/products/silicon-ip-security/side-channel-mitigation
are dynamically building and release for the secure computation in our LightCom framework, which can further decrease the information leak risk in the enclave.

2 PRELIMINARY

2.1 Notations
Throughout the paper, we use \( \|x\| \) to denote bit-length of \( x \), while \( L(x) \) denotes the number of element in \( x \). Moreover, we use \( pk_a \) and \( sk_a \) to denote the public and private keys of a Request User (RU) \( a \), \( sk_a^{(1)}, sk_a^{(2)} \) to denote the partial private keys that form \( sk_a \). \( m \) denote the encrypted data of \( P \). Let the additive secret sharing scheme (a.k.a. Distributed Decryption (PCDD) in our previous work) \( \text{UnS} \) to denote the encrypted data of \( P \) -out-of-\( P \) secret sharing scheme can be classified into the following two algorithms - Data Share Algorithm (Share) and Data Recovery Algorithm (Rec):

\[
\text{Share}(m) : \text{Randomly generate } X_1, \ldots, X_{P-1} \in \mathbb{G}, \text{the algorithm computes } X_P = m - (X_1 + \cdots + X_{P-1}), \text{and outputs } X_1, \ldots, X_P.
\]

\[
\text{Rec}(X_1, \ldots, X_P) : \text{With the shares } X_1, \ldots, X_P, \text{the algorithm can recover the message } m \text{ by computing with } m = X_1 + \cdots + X_P \text{ under group } \mathbb{G}.
\]

2.2 Additive Secret Sharing Scheme (ASS)
Give \( m \in \mathbb{G} \) (\( \mathbb{G} \) is a finite abelian group under addition), the additive secret sharing scheme (a.k.a. \( \mathcal{P} \)-out-of-\( \mathcal{P} \) secret sharing scheme) can be classified into the following two algorithms - Data Share Algorithm (Share) and Data Recovery Algorithm (Rec):

\[
\text{Share}(m) : \text{Randomly generate } X_1, \ldots, X_{P-1} \in \mathbb{G}, \text{the algorithm computes } X_P = m - (X_1 + \cdots + X_{P-1}), \text{and outputs } X_1, \ldots, X_P.
\]

\[
\text{Rec}(X_1, \ldots, X_P) : \text{With the shares } X_1, \ldots, X_P, \text{the algorithm can recover the message } m \text{ by computing with } m = X_1 + \cdots + X_P \text{ under group } \mathbb{G}.
\]

2.3 Additive Homomorphic Encryption Scheme
To reduce the communication cost of the LightCom, we used an Additive Homomorphic Encryption (AHE) scheme as the basis. Specifically, we use one of the AHE support threshold decryption called Paillier Cryptosystem Distributed Decryption (PCDD) in our previous work which contains six algorithms called Key Generation (KeyGen), Data Encryption (Enc), Data Decryption (Dec), Private Key Splitting (Keys), Partially decryption (PDec), Threshold decryption (TDec). The plaintext is belonged to \( \mathbb{Z}_N \) and the ciphertext is belonged to \( \mathbb{Z}_{N^2} \). The construction of the above algorithms can be found in Supplementary Materials Section C. Here, we introduce the two properties of the PCDD as follows: 1) Additive Homomorphism: Given ciphertexts \([m_1]\) and \([m_2]\) under a same public key \( pk \), the additive homomorphism can be computed by ciphertext multiplication, i.e., compute \([m_1] \cdot [m_2] = [m_1 + m_2] \). 2) Scalar-multiplicative Homomorphism: Given ciphertext \([m]\) and a constant number \( c \in \mathbb{Z}_N \), it has \([cm]^c = [cm] \).

With the two properties given above, we show that our PCDD have the polynomial homomorphism property, i.e., given \([x_1], \ldots, [x_n]\) and \( a_1, \ldots, a_n \), it has \([a_1 \cdot x_1 + a_2 \cdot x_2 + \cdots + a_n \cdot x_n] \leftarrow [x_1]^{a_1} \cdot [x_2]^{a_2} \cdots [x_n]^{a_n} \).

2.4 Mathematical Function Computation
In this section, we define the function which is used for data processing in our LightCom.

Definition 1 (Deterministic Multiple-output Multivariable Functions). Let \( D = \{ (x_1, \ldots, x_v) : x_i \in \mathbb{G} \} \) be a subset of \( \mathbb{G}^v \). We define the deterministic multiple-output multivariable function as follows: (I) A multiple-output multivariable function \( f \) of \( v \) variables is a rule which assigns each ordered vector \((x_1, \ldots, x_v) \) in \( D \) to a unique vector \((y_1, \ldots, y_w) \), denote \((y_1, \ldots, y_w) = f(x_1, \ldots, x_v) \). (II) The set \( D \) is called the domain of \( f \). (III) The set \( \{ f(x_1, \ldots, x_v) \mid (x_1, \ldots, x_v) \in D \} \) is called the range of \( f \).

Note that the deterministic multiple-output multivariable function is the general case of the deterministic multiple-output single-variable function \((v = 1)\), deterministic single-output multivariable function \((w = 1)\), and deterministic single-output single variable function \((v = 1, w = 1)\). As all the functions used in our paper can be successfully executed by a polynomial deterministic Turing machine (See Supplementary materials Section A), we omit the word “deterministic” in the rest of the paper.

3 SYSTEM MODEL & PRIVACY REQUIREMENT

In this section, we formalize the LightCom system model, and define the attack model.

3.1 System Model
In our LightCom system, we mainly focus on how the cloud server responds to a user request on outsourced computation in a privacy-preserving manner. The system comprises Request User (RU) and a Cloud with Untrusted Storage (UnS) and Trusted Processing Units (TPUs) - see Fig. 1.

- A RU generates his/her public key, private key shares, and data shares. After that, the RU can securely outsource the public key and private/data shares to the clouds UnS for secure storage (See Fig. 1). Moreover, the RU can also request a cloud to perform some secure outsourced computations on the outsourced data, and securely retrieve the final encrypted results (See Fig. 2).
- A UnS of the cloud has ‘unlimited’ data storage space to store and manage data outsourced from
the registered RU. Also, the UnS also stores all the intermediate and final results for the RU in encrypted form.

- The TPUs of the cloud provides online computation ability for each RUs. Each TPU provides isolation secure computing environment for individual RU and can load RU’s data shares from UnS (See 2), perform certain calculations over the data shares (See 3), and then securely seal the data shares in UnS for storage (See 4). Note that one TPU cannot load other TPU’s sealed data which are stored in UnS.

3.2 Attack Model

In our attack model, the cloud is curious-but-honest party, which strictly follow the protocol, but are also interested in learning data belonged to the RUs. The UnS inside the cloud is transparency to both the cloud and the outsider passive attackers. Every TPU can provide a secure execution environment (a.k.a., TPU enclave) for a RU which is secure against the other RU, the cloud and outsider passive attackers. The inside non-challenge RUs and outside attackers can also be interested to learn challenge RU’s data. Therefore, we introduce three active adversaries $A_1^*$, $A_2^*$, $A_3^*$, which can simulate the malicious actions corresponding to the outside attackers, non-challenge RUs, UnS, respectively. The goal of these adversaries is to get the challenge RU’s plaintext or try to let the challenge RU get wrong computation result with the following capabilities:

1) $A_1^*$ acts as the outside attacker that may eavesdrop on all communication links and CP’s UnS, and try to decrypt the challenge RU’s encrypted data. 2) $A_2^*$ may compromise RUs, with the exception of the challenge RU, to get access to their decryption capabilities, and try to guess all plaintexts belonging to the challenge RU. 3) $A_3^*$ may compromise the TPU to guess plaintext values of all data shares sent from the UnS by executing an interactive protocol. Noting that the above adversaries $A_1^*, A_2^*, A_3^*$ are restricted from compromising (i) all the TPUs concurrently7, and (ii) the challenge RU.

4.4 The LightCom Design Method for the Single Functions

Our LightCom achieves the user data’s privacy during the efficiency in the outsourced cloud with three-dimensional protection: 1) secure storing in the untrusted cloud storage; 2) secure processing in TPUs against side-channel attack; 3) efficient and dynamic outsourced key and data shares updating. Specifically, to outsource the data to the cloud, the RU first initializes the system, uses the RU’s public key to encrypt the data and outsource these encryptions along with the system parameters to UnS for storage. To achieve the second-dimensional protection, our LightCom uses the data sharing-based secure computation method between TPUs which can resist the side-channel attacks even the PPCD ciphertexts are decrypted. After finishing the processing, the data are sent back to UnS for further processing to finish the corresponding functionality defined in the program, and the enclaves in TPUs are released. Moreover, to tackle the leaked private key and data shares, all the TPUs can jointly update these shares without the help of RU. Thus, the LightCom can classify into the following four phases.

1) System Initialize Phase: Firstly, the RU generates a public key $pk$ and private key is $sk$ of appropriate public key crypto-system, and then splits the private key $sk$ into $P$ shares $sk_i$ ($i = 1, \cdots, P$) with the Share algorithm. After that, for each TPU $i$ in the cloud, it initials an enclave $i$, builds a secure channel, and uploads the $sk_i$ to the enclave $i$ securely. Finally, the TPU $i$ uses the data sealing to securely stored the $pk_i$, $sk_i$ in to UnS.

2) Data Upload Phase: In the phase, the RU randomly separate the data $x_{1,1}, \cdots, x_{1, P} \in G$, such that $x_{1,1} + \cdots + x_{1,P} = x_{j}$ for $j = 1, \cdots, v$. Then, the TPU $i$ ($i = 1, \cdots, P$) creates the enclave $i$. After that, the RU defines the program $C_i$ for some specific computation function, builds a secure channel with TPU enclave $i$, remotely loads $x_{1,i}, \cdots, x_{v,i}, C_i$ into the enclave $i$, and securely seals $x_{1,i}, \cdots, x_{v,i}, C_i$ in the UnS. After that, TPU $i$ release enclaves $i$ for all the $i = 1, \cdots, P$.

3) Secure Computation Phase: The goal of the phase is to achieve the secure computation among the TPUs according to the user-defined program $C_i$. Thus, it works as follows:

- (3-I) Each TPU $i$ generates an enclave $i$. After that, all the TPUs build a secure channel with each other. Load sealed data $x_{1,i}, \cdots, x_{v,i}, pk, sk_i, C_i$ to enclave $i$ from UnS, and denote them as $S_i$.
- (3-II) TPUs jointly compute $(y_{1,1}, \cdots, y_{w,1} : \cdots : y_{1,P}, \cdots, y_{w,P}) \leftarrow GenCpt(S_1 : \cdots : S_P)$ according to the user-defined program $C_1, \cdots, C_P$.
- (3-III) All the TPUs jointly update the private key shares and data shares dynamically.

After the above computation, the TPU $i$ seals $y_{1,i}, \cdots, y_{w,i}$ into the UnS, and releases the enclave.

4) Data Retrieve Phase: If the RU needs to retrieve the computation results from the cloud, the TPU $i$ creates an enclave $i$, opens the sealed data $y_{1,i}, \cdots, y_{w,i}$ builds a secure channel with the RU, and sends the data shares

7. See the algorithm Seal and UnSeal in Section 5.4
8. Note that $P \geq 3$ TPUs are required in LightCom for the security consideration.

9. The construction of General Secure Function Computation Algorithm (GenCpt) can be found in section 5.3
back to RU. Once all the shares are sends to RU, the RU computes $y_j = \sum_{i=1}^{P} y_{j,i}$ for $j = 1, \ldots, w$.

### 4.2 The LightCom Design for Combination of the Functions

Our LightCom can support for single data outsourced with multiple function operations. The procedure is as follows:

1) **System Initialize Phase**: Same to the LightCom with single function in Section 4.1

2) **Data Upload Phase**: After the system initialize phase, the RU defines the program $C_{i,t}$ for TPU $i (i = 1, \ldots, P)$ with function computation step $t (t = 1, \ldots, \zeta)$ and randomly separates the data $x_{j,1,1}, \ldots, x_{j,1,P}$, such that $x_{j,1,1} + \cdots + x_{j,1,P} = x_j$ for $j = 1, \ldots, \zeta$. After that, the RU builds a secure channel with TPU enclave $i$, remotely loads $C_{1,i}, \ldots, C_{\zeta,i}, x_{1,1,i}, \ldots, x_{1,\zeta,i}$ into the enclave $i$, and securely seals these data in the UnS. After that, TPU $i$ release enclaves $i$ for all the $i = 1, \ldots, P$.  

3) **Secure Computation Phase**: The goal of the phase is to achieve the secure computation among the TPUs according to the user-defined program $C_{i,t}$ for function $t (t = 1, \ldots, \zeta)$. Thus, for each step $t$, the phase works as follows:

- (3-I) Each TPU $i$ generates an enclave $i$. After that, all the TPUs build a secure channel with each other. Load sealed data $x_{1,t,i}, \ldots, x_{N,t,i}, pk, sk_i, \ldots, C_{1,i}, \ldots, C_{\zeta,i}$ to enclave $i$ from UnS, and put them in a set $E_{i,i}$.

- (3-II) TPUs jointly compute $(y_{t,1,1}, \ldots, y_{t,t,1}, \ldots, y_{t',1,1}, \ldots, y_{t',t,1}) \leftarrow GenCpt(E_{i,i})$, according to the user-defined program $C_{i,1}, \ldots, C_{i,t}$.

- (3-III) All the TPUs jointly update the private key and data shares. If $t = \zeta$, the TPU $i$ seals $y_{1,\zeta,i}, \ldots, y_{N,\zeta,i}$ into the UnS, release the enclave. Otherwise, move to (3-IV) for further computation.

- (3-IV) Select $x_{1,t+1,1}, \ldots, x_{N,t+1,1}$ from the $y_{t,1,1}, \ldots, y_{t,t,1}$ for TPU $i$. Then, the TPU $i$ seals $x_{1,t+1,1}, \ldots, x_{N,t+1,1}$ into the UnS, release the enclave, and move to (3-I) for next step computation.

After the $t$ step is finished, the TPU $i$ seals the set $E_{i,j}$ into the UnS, and releases the corresponding enclave.

4) **Data Retrieve Phase**: After the computation, TPU $i$ new an enclave $i$, opens the sealed data $y_{1,\zeta,i}, \ldots, y_{N,\zeta,i}$ builds a secure channel with the RU, and sends these data back to the RU. Once all the TPU’s data are sent, the RU computes the result $y_j, \zeta = \sum_{i=1}^{P} y_{j,i}$ for $j = 1, \ldots, w$ to get the final results.

### 4.3 General Secure Function Computation Algorithm (GenCpt)

As the key component of the LightCom, the General Secure Function Computation Algorithm (GenCpt) are proposed to achieve the secure deterministic multiple-output multivariable function $F$ computation which is introduced in definition 1. Assume TPU $i (i = 1, \ldots, P)$ holds $x_{1,i,1}, \ldots, x_{\zeta,i}, y_{1,i,1}, \ldots, y_{N,i,1}$, $GenCpt$ can securely output $y_{1,i,1}, \ldots, y_{N,i,1}$ for each TPU $i$, such that $(y_{1,i,1}, \ldots, y_{N,i,1}) \leftarrow F(x_{1,i,1}, \ldots, x_{\zeta,i})$, where $x_{1,1} + \cdots + x_{j,1} = x_j$ and $y_{1,1} + \cdots + y_{N,1} = y_k$ for $j = 1, \ldots, \zeta, v = 1, \ldots, w$. The $GenCpt$ can be classified into offline/online stages and constructed as follows:

**Offline Stage**: Each TPU $i (i = 1, \ldots, P)$ creates an enclave $i$, loads the sealed keys $pk, sk_i$ and program $C_i$ into the enclave from the UnS, builds a secure channel with the other TPUs. With the help of homomorphic public key cryptosystem, all the TPUs collaboratively generate the shares of random numbers and put them into a set $R_i$. Note the shares in set $R_i$ cannot be known by all the other TPUs during the generation. After the above computation, each TPU $i$ seals the $R_i$ into the UnS, respectively.

**Online Stage**: For each TPU $i (i = 1, \ldots, P)$, loads the sealed random numbers set $R_i$, from offline stage into the enclave $i$. All the TPUs cooperatively compute and output the results $(y_{1,i,1}, \ldots, y_{N,i,1}) \leftarrow f_i(x_{1,i,1}, \ldots, x_{\zeta,i}, R_i)$, where $f_i$ is the combination of $+, \times$ for $Z_N$ and $\oplus, \wedge$ for $Z_2$ with specific functionality according to the program $C_i$.

### 4.4 Private Key Share Update against Side-Channel Attack

The private key shares are more sensitive and vulnerable, as the adversary can use the private key to decrypt the RU’s data in the untrusted storage if all shares of the private key are leaked by side channel attack. Thus, we should frequently update the key shares in the TPU enclave. The intuitive idea is to let the RU choose a new private key, separate the new private key into different key shares, update these key shares in the different individual enclaves, and update all the ciphertext with the new key. However, the above strategy has the main drawback: the RU has to be involved in the private/public key update phase which brings extra computation and communication cost. Thus, in this case, the RU needs to generate and update the public/private keys frequently which is impractical. Thus, we bring the idea of proactive secret sharing into the LightCom: keeps the public/private key unchanged, the TPU will periodically refresh the key shares without the participation of the RU. Mathematically, to renew the shares at period $t (t = 0, 1, 2, \ldots)$, we need to update the shares such that $\sum_{i=1}^{P} sk_i(t+1) = \sum_{i=1}^{P} sk_i(t) + \sum_{i=1}^{P} \sum_{j=1}^{P} \delta_{i,j}$.
where $\sum_{j=1}^{P} \delta_{i,j} = 0$, $\sum_{j=1}^{P} s_{k}^{(0)} = sk$ and $s_{k}^{(0)} = sk_i$ for $i = 1, \ldots, P$ (See Fig. 2 for example of private key update procedure with $P = 3$). The special construction is as follows:

![Fig. 2. Key Shares Update (example of $P = 3$)](image)

1) Each TPU $i (i = 1, \ldots, P)$ creates an enclave $i$. After that, TPU $i$ builds a secure channel with TPU $j$'s enclave ($j = 1, \ldots, P; j \neq i$).

2) TPU $i$ picks random numbers $\delta_{i,1}, \ldots, \delta_{i,P} \in \mathbb{G}$ such that $\delta_{i,1} + \ldots + \delta_{i,P} = 0$ under the group $\mathbb{G}$, and then sends $\delta_{i,j}$ to TPU enclave $j$.

3) After received $\delta_{i,j}$, TPU $i$ computes the new shares $s_{k_i}^{(t+1)} = s_{k_i}^{(t)} + \delta_{i,j}^{(t)} + \delta_{j,i}^{(t)} + \cdots + \delta_{P,i}^{(t)} \in \mathbb{G}$. After that, TPU $i$ erases all the variables which it used, except for its current secret key $sk_i$.

4.5 Data Shares Update against Side-Channel Attack

As data shares need to load to TPU for processing, the shares can be leaked to the adversary by side channel attack, and reconstruct the RU’s original data. Thus, we also need to dynamically update data. For each $t$, $x_i^{(t)}$ at period $t$ ($t = 0, 1, 2, \cdots$), such that $\sum_{i=1}^{P} x_i^{(t+1)} = \sum_{i=1}^{P} x_i^{(t)} + \sum_{j=1}^{P} \delta_{i,j}^{(t)}$, where $\sum_{i=1}^{P} x_i^{(0)} = x_1, x_i^{(0)} = x_i$, and $\sum_{j=1}^{P} \delta_{i,j} = 0$ for $i = 1, \cdots, P$. The construction is same to the private key share update method in section 4.4.

5 TPU-based Basic Data Shares Operations

In this section, we introduce some basic TPU-based data shares operations which can be used as the basis of LightCom.

5.1 Data Domain and Storage Format

Here, we introduce three data group domain for LightCom: $\mathbb{Z}_N = \{0, 1, \ldots, N - 1\}$, $\mathbb{D}_N = \{-[\frac{N}{2}], \ldots, 0, \ldots, \frac{N}{2}\}$, and $\mathbb{Z}_2 = \{0, 1\}$. As we use PCDD for offline processing and its plaintext domain is $\mathbb{Z}_N$, we define the operation $[x]_N$ which transforms data $x$ from group $\mathbb{Z}_N$ into the group $\mathbb{D}_N$, i.e.,

$$[x]_N \left\{ \begin{array}{ll} x, & 0 \leq x < N/2 \\ x - N, & N/2 \leq x < N. \end{array} \right.$$  

Moreover, the data $[x]_N$ in group $\mathbb{D}_N$ can be directly transformed into group $\mathbb{Z}_N$ with $x = [x]_N \mod N$.

It can be easily verified that group $\mathbb{D}_N$ and $\mathbb{Z}_N$ are isomorphism.

To guarantee the security of secret sharing, two types of data shares are used in LightCom, called integer share (belonged to $\mathbb{Z}_N$) and binary share (belonged to $\mathbb{Z}_2$). For the integer share separation, RU only needs to execute $\text{Share}(m)$, such that $m = m_1 + \cdots + m_P$, where $m, m_1, \cdots, m_P \in \mathbb{D}_N$. For the binary shares, RU executes $\text{Share}(m)$, such that $m = m_1 + \cdots + m_P$, where $m, m_1, \cdots, m_P \in \mathbb{Z}_2$. After that, RU securely sends integer share $m_i$ or binary shares $m_i$ to TPU $i$, and seals to UnS for securely storage.

5.2 System Initial and Key Distribution

The LightCom system should be initialized before achieving the secure computation. Firstly, the RU executes KeyGen algorithm, output public key $pk = (N, g)$ and private key is $sk = \theta$. Then, use KeyS to split key $\theta$ into $P$ shares $sk_i = \theta_i (i = 1, \cdots, P)$. After that, for each TPU $i$ in the cloud, it initializes an enclave $i$, builds a secure channel, and uploads the $sk_i$ to the enclave $i$ securely. Beside, the RU’s PCDD public key $pk$ and program $C_i$ for the specific function $F$ are needed to securely send to TPU $i (i = 1, \cdots, P)$. Finally, the TPU $i$ securely sends the $pk, sk_i, C_i$ into UnS. As all the parameters need to load to the TPU enclaves along with the data shares according the specific functionality, we will not specially describe it in the rest of the section.

5.3 Secure Distributed Decryption Algorithm (SDD)

Before constructing the TPU-based operation, we need first to construct the algorithm called Secure Distributed Decryption (SDD) which allows all the TPUs decrypt PCDD’s ciphertext. Mathematically, if enclave in TPU $\chi$ contains the encryption $[x]$, the goal of SDD is to output $x$ which contains following steps: 1) The TPU enclave $\chi$ establishes a secure channel with the other TPU enclave $i(i \neq \chi)$. Then, enclave $\chi$ sends $[x]$ to all the other enclave $i$. 2) Once received $[x]$, the TPU $i$ uses $\text{PDec}$ algorithm to get $CT_i$ and securely send $CT_i$ to enclave $\chi$. 3) Finally, the TPU $\chi$ securely uses $CT_i$ with $\text{TDec}$ algorithm to get $x$.

5.4 Secure TPU-based Data Seal & UnSeal

As TPU enclaves are only provide an isolated computing environment during the secure processing, the data in the TPU enclave needs to seal to UnS for long-term storage. Thus, we propose two algorithms called Seal and UnSeal to achieve.

Seal($x_i$): The TPU $i$ encrypts the data share $x_i$ into $[x_i]$, then uses hash function $H : \{0, 1\}^* \rightarrow \mathbb{Z}_N$ with input the $[x_i]$ associated with TPU $t$-time period private key share $sk_i^{(t)}$ to compute $S_i \leftarrow H([x_i][sk_i^{(t)}][ID_i][t])$, where $ID_i$ is the transaction identity for $[x_i]$. Then, TPU $i$ sends $[x_i]$ with $S_i$ to UnS for storage.
UnSeal([x], S_{t,i}) \text{ : The TPU } i \text{ loads } [x_i] \text{ with } S_{t,i} \text{ to the enclave } i, \text{ and computes } H([x]|s_{k_i}^{(t)}|ID_i|t) \text{ to test whether the result is equal to } S_{t,i}. \text{ If the equation does not hold, the algorithm stops and outputs } \bot. \text{ Otherwise, the TPU } i \text{ uses } SDD \text{ to get the share } x_i.

5.5 Random Shares Generation

The secret sharing based privacy computation requires one-time random numbers for processing. Before constructing the TPU-based computation, we design a protocol called Random Tuple Generation Protocol (RTG). The goal of RTG is to let TPUs cooperatively generate random tuple \(\tau_i^{(1)}, \ldots, \tau_i^{(\ell)} \in Z_2\) for each TPU \(i (i = 1, \ldots, P)\), such that \(r = -\tau_i^{(1)}2^{\ell-1} + \sum_{j=1}^{\ell-1} \tau_i^{(j)}2^{\ell-j-1}\) and \(\tau_i^{(j)} = r \oplus \tau_i^{(j)}\) and \(r = r_1 + \ldots + r_P\) holds, where \(\ell\) is the bit-length of random number \(r \in \mathbb{D}_N\). The RTG generates as follows:

1. The TPU \(i \in \{1, \ldots, P\}\) randomly generates \(\tau_i^{(1)}, \ldots, \tau_i^{(\ell)} \in Z_2\), encrypts them as \([\tau_i^{(1)}], \ldots, [\tau_i^{(\ell)}]\), and sends these ciphertexts to TPU \(i\).

2. The TPU \(i (i = 2, \ldots, P)\) generates \(\tau_i^{(1)}, \ldots, \tau_i^{(\ell)} \in Z_2\) and computes

\[
[t^{(j)}] \leftarrow ([t^{(j)}]1^{(1-t^{(j)})} \cdot ([1] \cdot [t^{(j)}]N-1)\{t^{(j)}\} = [t^{(j)}] \oplus \tau_i^{(j)}
\]

If \(i \neq P\), the TPU \(i\) sends \([\tau_i^{(1)}], \ldots, [\tau_i^{(\ell)}]\) to TPU \(i+1\).

If \(i = P\), the TPU \(P\) computes

\[
[r] \leftarrow ([t^{(\ell-1)}]N-2^{\ell-1} \cdot [t^{(\ell-2)}]N-2^{\ell-2} \ldots [t^{(1)}].
\]

3. For TPU \(i (i = 1, \ldots, P-2)\), randomly generates \(r_i \in \mathbb{D}_N\) and computes \([r] \leftarrow [r] \cdot [r_i], \text{ and sends } [r_i] \text{ to TPU } i-1\). Once TPU \(1\) gets \([r]\), uses \(SDD\) to get \(r_i\) and denoted \([r_i] \in \mathbb{D}_N\) as \(r_i\). After computation, each TPU \(i (i = 1, \ldots, P)\) holds randomly bits \(\tau_i^{(1)}, \ldots, \tau_i^{(\ell)} \in Z_2\) and integer \(r_i \in \mathbb{D}_N\).

5.6 Share Domain Transformation

5.6.1 Binary Share to Integer Share Transformation (BSI)

Suppose TPU \(i\) holds a bit share \(x_i \in Z_2\), where \(a_1 \oplus \cdots \oplus a_P = s \in Z_2\), the goal of the protocol is to generate a random integer share \(b_i \in Z_N\) for each TPU \(i\), such that \(b_1 + \cdots + b_P = s\). To execute BSI, the TPU \(i\) randomly generates \(b_i \in Z_N\), denotes \(x = b_i\) and \(s = a_i\), encrypts \(x\) as \([x]s\), and sends \([x]s\) to TPU \(i\). After that, the TPU \(i (i = 2, \ldots, P-1)\) generates \(b_i \in Z_N\) and computes

\[
[x]s \leftarrow [x]s1^{(1-a_i)} \cdot ([1] \cdot [s]N-1)a_i = [s \oplus a_i], [x]s \leftarrow [x] \cdot [b_i].
\]

and sends \([x]s\) to TPU \(i+1\). Once received the \([x]s\), TPU \(P\) computes

\[
[s] \leftarrow [s]1^{(1-a_P)} \cdot ([1] \cdot [s]N-1)a_P = [s \oplus a_P],
\]

\[
[b_P] \leftarrow [s] \cdot [x]N-1,
\]

and uses the \(SDD\) to decrypt and gets \(b_P\).

5.6.2 Integer Share to Binary Share Transformation (I2B)

Suppose TPU \(i\) hold an integer share \(a_i \in Z_N\), where \(a_1 + \cdots + a_P = s \in Z_2\), the goal of the I2B protocol is to generate a random bit share \(b_i \in Z_2\) for each TPU \(i\), such that \(b_1 + \cdots + b_P = s\). To execute \(I2B\), the TPU \(i\) lets \(y = a_i\), encrypts \(y\) as \([y]\), and sends the ciphertext to TPU \(2\) for computation. After that, when the TPU \(i (i = 2, \ldots, P)\) uses the share to compute \([y] \leftarrow [y] \cdot [a_i]\). If \(i \neq P\), TPU \(i\) sends \([y]\) to TPU \(i+1\). After that, denote \([s] \leftarrow [y]\), and each TPU \(i (i = P, \ldots, 2)\) generates \(b_i \in Z_2\) and computes

\[
[s] \leftarrow [s]1^{(1-b)} \cdot ([1] \cdot [s]N-1)b_i = [s \oplus b_i],
\]

and sends \([s]\) to TPU \(i-1\). Once received \([s]\), TPU \(i\) uses the \(SDD\) to decrypt \([s]\) and denotes the result \(s\) as \(b_1\).

6 TPU-based Secure Outsourced Computing Toolkits in the Cloud

In this section, we introduce and construct the commonly used secure outsourced binary and integer computation sub-protocols for a single cloud.

6.1 Secure Computation over Binary Shares

6.1.1 Secure Bit Multiplication Protocol (SBM)

The SBM can achieve plaintext multiplication on bit shares and output are bit shares, i.e., given two shares \(x_i, y_i \in Z_2\) \((i = 1, \ldots, P)\) for TPU \(i\) as input, SBM securely outputs \(x_i \in Z_2\) for TPU \(i\), such that \(\bigoplus_{i=1}^P x_i = (\bigoplus_{i=1}^P x_i) \land (\bigoplus_{i=1}^P y_i).

Offline Stage: All the TPUs initialize their enclaves and load the public parameters to UnS. For enclave \(1\), generate \(a_1, b_1 \in Z_2\), compute \(c = a_1 \cdot b_1 \in Z_2\). Encrypt \([a_1], [b_1]\) and \([c]\), and denote them as \([a], [b], [c]\), respectively. After that, TPU enclave \(i (i = 1, \ldots, P-1)\) sends \([a], [b], [c]\) to enclave \(i + 1\), TPU \(i + 1\) generates \(a_{i+1}, b_{i+1}\) and compute

\[
[a] \leftarrow [a] \cdot [c]N-1 = [a \cdot (1 - b)],
\]

\[
[b] \leftarrow [b] \cdot [c]N-1 = [b \cdot (1 - a)],
\]

\[
[c] \leftarrow [1] \cdot [a]N-1 \cdot [b]N-1 \cdot [c] = ([1-b] \cdot (1-a)],
\]

\[
[a] \leftarrow [a]^{(1-a_{i+1})} \cdot ([1] \cdot [a]N-1)a_{i+1} = [a \oplus a_{i+1}],
\]

\[
[b] \leftarrow [b]^{(1-b_{i+1})} \cdot ([1] \cdot [b]N-1)b_{i+1} = [b \oplus b_{i+1}].
\]

After the above computation, enclave \(i (i = P, \ldots, 2)\) randomly generates \(\epsilon_i \in Z_N\) and computes \([c] \leftarrow [c]^{(1-c)} \cdot ([1] \cdot [c]N-1)\epsilon_i = [c \oplus \epsilon_i].\) When the TPU 2 sends \([c]\) to TPU 1, the TPU 1 uses \(SDD\) to get \(\epsilon\) and denotes as \(\epsilon_1\). After the above computation, each enclave holds \(a_{i}, b_{i}, \epsilon_{i}\), which satisfies \(a_{1} \oplus \cdots \oplus a_{P} = a, b_{1} \oplus \cdots \oplus b_{P} = b,\)
6.1.2 Secure Bit-wise Addition Protocol (BAdd)

The BAdd description follows the TPU 1 holds bit shares $a_1^{(0)}, \ldots, a_1^{(1)}$ of $\ell$ bit-length integer $a$ and $t_1^{(0)}, \ldots, t_1^{(1)}$ of $\ell$ bit-length integer $t$. The goal is to compute $y = a + t$, where $y = \eta^{(0)} \cdot 2^{(0)} + \sum_{j=1}^{\ell-1} \eta^{(j+1)} \cdot 2^{(j+1)}$. $a = a_1^{(j)} + a_2^{(j)} + \ldots + a_P^{(j)}$, $\eta = \eta_1^{(j)} + \eta_2^{(j)} + \ldots + \eta_P^{(j)}$ and $t = t_1^{(j)} + t_2^{(j)} + \ldots + t_P^{(j)}$. The idea is simple and easy: use the binary addition circuit to achieve the addition, i.e., compute the integer addition as $y^{(j)} = a^{(j)} \oplus t^{(j)}$ and $c^{(j+1)} = (a^{(j)} \land t^{(j)}) \oplus (a^{(j)} \lor t^{(j)})$ for $j = 1, \ldots, \ell$.

The procedure of BAdd works as follows:

1) For each TPU $i (i = 1, \ldots, P)$ and each bit position $j = 1, \ldots, \ell$, all the TPUs jointly compute $y^{(j)} = a^{(j)} \oplus t^{(j)}$ and $c^{(j+1)} = (a^{(j)} \land t^{(j)}) \oplus (a^{(j)} \lor t^{(j)})$. After using the construction of SMB, it indeed computes $c^{(j)} = a^{(j)} \land t^{(j)}$.

2) Each TPU $i$ sets $y^{(j)} = 0$ and $c^{(j+1)} = \tilde{a}^{(j)}$. Then, for $j = 2, \ldots, \ell$, all TPUs jointly computes $(y^{(j-1)}) = \text{SMB}(\tilde{a}^{(j-1)}, c^{(j-1)})$.

Moreover, for each TPU $i$ locally computes $c^{(j)} = \tilde{a}^{(j-1)} \oplus c^{(j-1)}$ and $y^{(j)} = \tilde{a}^{(j)} \oplus c^{(j)}$.

and outputs $y^{(j)}$ for all $j$.

6.1.3 Secure Bit Extraction Protocol (BExt)

Suppose TPU $i (i = 1, \ldots, P)$ contains an integer share $u_i$, where $u = \sum_{i=1}^{P} u_i$. The goal of BExt is to output the bit extraction shares $u_1^{(0)}, \ldots, u_1^{(1)}$ for each TPU $i (i = 1, \ldots, P)$, where $u = -u^{(0)} \cdot 2^{(0)} + \sum_{j=1}^{\ell-1} u_j^{(j)} \cdot 2^{(j+1)}$ and $u^{(j)} = \bigoplus_{i=1}^{P} u_i^{(j)}$. The BExt also contains online/offline phase which describes as follows:

OffLine Stage: Execute RTG to get $t_1^{(0)}, \ldots, t_1^{(1)}$ and $r_1$ for party $i$. Then, all the TPUs need to jointly compute $a_1^{(0)}, \ldots, a_1^{(1)} \in \mathbb{Z}_2$, such that $a^{(i)} = a_1^{(i)} \oplus a_2^{(i)} = 0$. Firstly, TPU 1 randomly generates $a_1^{(0)}, \ldots, a_1^{(1)} \in \mathbb{Z}_2$ and let $t_1^{(j)} = a_1^{(j)}$ for $j = 1, \ldots, \ell$. After that, the TPU $i$ generates $a_1^{(0)}, \ldots, a_1^{(1)} \in \mathbb{Z}_2$, computes $t_1^{(j)} = a_1^{(j)} \oplus a_1^{(j)}$, and sends these ciphertexts to TPU $i + 1$. Once the $t_1^{(j)}, t_1^{(0)}$ are received, the TPU $P$ uses the SDD to decrypt, gets $t_1^{(0)}, t_1^{(0)}$ and denotes them as $a_1^{(0)}, \ldots, a_1^{(1)}$. After that, each TPU $i$ seals $t_1^{(0)}, \ldots, t_1^{(1)}$ and $r_1$ and $a_1^{(0)}, \ldots, a_1^{(1)}$ in UnS, respectively.

Online Phase: The TPU $i$ computes $v_i = u_i - r_i$, encrypts $v_i$ and sends $[v_i]$ to TPU $P$. After receiving all the encryptions, the TPU $P$ computes $v = \prod_{i=1}^{P} [v_i]$ and executes SDD to get the $v$ and computes $[v]_N$. Then, TPU $P$ generates its two's complement binary representation $v^{(0)} \cdot 2^0 + \ldots, v^{(\ell-2)} \cdot 2^{\ell-2}$ and computes $v^{(0)} \cdot \oplus v^{(0)} \cdot \oplus a^{(0)}$, where $j = 1, \ldots, \ell$. Other TPU $(i = 1, \ldots, P - 1)$ keeps other $v_i$ unchanged.

After that, all the TPUs jointly compute

$$(\bar{u}_1, \ldots, \bar{u}_P) \rightarrow \text{BAdd}(\bar{u}_1, \ldots, \bar{u}_P; r_1, \ldots, r_P),$$

where $\bar{u}_i = (u_1^{(0)}, \ldots, u_1^{(1)})$, $\bar{u}_i = (v_1^{(0)}, \ldots, v_1^{(1)})$, $\bar{v}_1 = (r_1^{(0)}, \ldots, r_1^{(1)})$. Finally, the BExt algorithm outputs $\bar{u}_i = (u_1^{(0)}, \ldots, u_1^{(1)})$ for TPU $i = 1, \ldots, P$.

6.2 Secure Integer Computation

6.2.1 Secure Multiplication Protocol (SM)

The SM achieves integer multiplication over integer shares, i.e., given shares $x_i, y_i (i = 1, \ldots, P)$ for TPU $i$ as input, SM securely outputs $f_i$ for TPU $i$, such that $\sum_{i=1}^{P} f_i = x \cdot y$, where data shares $x_i, y_i$ satisfy $x = \sum_{i=1}^{P} x_i$ and $y = \sum_{i=1}^{P} y_i$.

Offline Stage: All the TPUs initialize their enclaves and load the public parameters to the UnS. Then, for the enclave 1, it generates $a_1, b_1 \in \mathbb{D}_N$ and computes $z = a_1 \cdot b_1$, encrypts $[a_1], [b_1]$ and $[c]$, and lets them be $[a], [b], [c]$, respectively. After that, enclave $i (i = 1, \ldots, P - 1)$ sends $[a], [b], [c]$ to enclave $i + 1$, TPU $i + 1$ generates $a_{i+1}, b_{i+1}$ and computes $[c] = [a] \cdot [b] \cdot [a] \cdot [b]$.

After the computation, for $i = P, \ldots, 2$, TPU enclave $i$ generates $c_i \in \mathbb{D}_N$ and computes $[c] = [c] \cdot [c] \cdot [a]^{N-1}$. After the computation, the TPU 2 sends $[c]$ to TPU 1. Then, TPU 1 uses SDD to get $c$ and denotes the final result $[c]_N$ as $c_1$. After the above computation, each enclave hold $a_i, b_i, c_i$ such that $[a_1 + \ldots + a_P]_N = [a]_N$, $[b_1 + \ldots + b_P]_N = [b]_N$, $[c_1 + \ldots + c_P]_N = [c]_N$ and $c = a \cdot b \mod N$. After the computation, each TPU enclave $i$ seals $a_i, b_i, c_i$ to UnS for storage individually.

Online Stage: TPU $i$ loads the $a_i, b_i, c_i$ into the enclave $i$. Then, compute $X_i = x_i - a_i$ and $X_i = y_i - b_i$. Securely send $X_i$ and $Y_i$ to other enclave $j (j = 1, \ldots, P; j \neq i)$. After receiving other $X_j$ and $Y_j$, the each TPU $i$ computes $X = \sum_{j=1}^{P} X_j$ and $Y = \sum_{j=1}^{P} Y_j$. After that, for each TPU $i (i = 1, \ldots, P - 1)$, compute $f_i = [c_i] + X_i Y_i$ and sends $f_i$ to TPU $P$ for TPU $P$, compute $f_P = [c_P] + X_P Y_P$. Here, we denote the protocol as $(f) \rightarrow \text{SM}(x, y)$.}

6.2.2 Secure Monic Monomials Computation (SMM)

The SMM protocol can achieve monic monomials computation over integer shares, i.e., given a share $x_i (i = 1, \ldots, \ell)$ and a public integer number $k$ for TPU $i$ as input, SMM securely outputs $f_i$ for TPU $i$, such that $\sum_{i=1}^{P} f_i = x^k$, where data shares $x_i$ satisfy $x = \sum_{i=1}^{P} x_i$. Finally, the SMM algorithm outputs $f_i = (u_1^{(0)}, \ldots, u_1^{(1)})$ for TPU $i = 1, \ldots, P$. 
The construction of the SMM is as follows: Denote $k$ as binary form $t_\ell, \ldots, t_1$. Initialize the share $f_i \leftarrow x_i$ for each TPU $i$. For $j = \ell - 1, \ldots, 1$, compute $(f^*) \leftarrow \text{SM}(f, \langle f \rangle)$. If $t_j = 1$, compute $(f) \leftarrow \text{SM}(f^*, \langle x \rangle)$. Otherwise, let $(f) \leftarrow (f^*)$. Here, the algorithm outputs $(f)$ and denotes the protocol as $(f) \leftarrow \text{SM}(x, k)$.

6.2.3 Secure Binary Exponential Protocol (SEP)

The SEP can achieve exponential over binary shares with a public base, i.e., given a binary share $t_i \in \mathbb{Z}_2$ ($i = 1, \ldots, P$) and a public integer $\beta$ for TPU $i$ as input, SEP securely outputs an integer share $f_i \in \mathbb{Z}_N$ for TPU $i$, such that $\sum_{i=1}^{P} f_i = \beta^\alpha$, where $\alpha = \bigoplus_{i=1}^{P} t_i$.

**Offline Stage**: All the TPUs initialize their enclaves and load the public parameters to the UnS. Then, for the enclave 1, it generates $a_1 \in \mathbb{Z}_2$, encrypts $a_1$ as $[a_1]$, and lets it be $[a]$. After that, each TPU $i$ ($i = 1, \ldots, P - 1$) sends $[a]$ to enclave $i + 1$, TPU $i + 1$ generates $a_{i+1} \in \mathbb{Z}_2$, computes $[a] \leftarrow [a]^{(1-a_{i+1})} \cdot ([1] \cdot [a]^{-1})^{a_{i+1}} = [a + a_{i+1}]_i$.

Once $[a]$ is received, TPU computes $[b] = [a]^\beta \cdot ([1] \cdot [a]^{-1}) = [\beta^\alpha + (1-a) \cdot [a]]^\beta = [\beta^\beta]^\beta$ and $[b^*] = [a] \cdot ([1] \cdot [a]^{-1})^\beta = [a + \beta(1-a)]^\beta = [\beta^\beta]$. After the computation for $i = P, \ldots, 2$, TPU $i$ generates $b_i, b_i^* \in \mathbb{D}_N$ and computes $[b] = [b] \cdot [b_i]^{-1}$ and $[b^*] = [b^*] \cdot [b_i^*]^{-1}$. After the computation, the TPU 2 sends $[b]$ and $[b^*]$ to TPU 1, and TPU 1 uses SDD to get $b, b^*$ and denote them as $b_1$ and $b_1^*$ respectively. After the above computation, each TPU $i$ holds $a_i, b_i$, which satisfies $a_1 \oplus \cdots \oplus a_P = a, b_1 \oplus \cdots \oplus b_P = \beta^\alpha, b_1^* \oplus \cdots \oplus b_1^* = \beta^{1-a}$. After the computation, each TPU $i$ seals $a_i, b_i$ to UnS for storage individually.

**Online Stage**: TPU $i$ loads the data share $t_i$ and random shares $a_i, b_i$ into its enclave. Then, TPU $i$ locally computes $X_i = t_i \oplus a_i$. Securely send $X_i$ to other enclave $j$ ($j = 1, \ldots, P; j \neq i$). After receiving other $X_j$, each TPU $i$ locally computes $X = \bigoplus_{i=1}^{P} X_i$ and $f_i \leftarrow ([b_1]^{-X} \cdot (b_1)^i \cdot [X])_N$. We can easily verify that $\sum_{i=1}^{P} f_i = \beta^{(1-a)(\text{length of } a) + a(1-\text{length of } a)} = \beta^\alpha$. Here, we denote the protocol as $(f) \leftarrow \text{SEP}_2(x, \beta)$.

6.2.4 Secure Integer Exponential Protocol (SEP)

The SEP can achieve exponential over integer shares with a public base, i.e., given an integer share $x_i \in \mathbb{D}_N$ ($i = 1, \ldots, P$) and a public integer $\beta$ for TPU $i$ as input, SEP securely outputs shares $f_i \in \mathbb{D}_N$ for TPU $i$, such that $\sum_{i=1}^{P} f_i = \beta^\alpha$, where data shares $x_i$ satisfy $x = \sum_{i=1}^{P} x_i$ and $x$ is relative small positive number with $\ell$ bit-length.

i) Compute $(f_1, \ldots, f_P) \leftarrow \text{BExt}(x_1, \ldots, x_P)$, where $f_i = ([f_i])^{(i)} \cdot \langle x \rangle$ for TPU $i = 1, \ldots, P$, and $f = \bigoplus_{i=1}^{P} f_i^{(i)}$, and $x = \sum_{j=1}^{\ell} f^{(j)} 2^{j-1}$.

ii) Execute $(f) \leftarrow \text{SEP}_2([f]^{(i)}, \beta)$. For $j = 2, \ldots, \ell$, compute $(f^*_j) \leftarrow \text{SEP}_2([f^{(j)}], \beta), (f_j^*) \leftarrow \text{SMM}((f_j^*), 2^{j-1})$.

13. $\beta$ is a small positive number which satisfies $gcd(\beta, N) = 1$.

6.2.5 Secure Comparison Protocol (SC)

The SC can securely compute the relationship between integer $u$ and $v$, where each TPU $i$ holds shares $u_i$ and $v_i$, where $u = u_1 + \cdots + u_P, v = v_1 + \cdots + v_P$. The construction of SC is as follows:

i) Each TPU $i$ ($i = 1, \ldots, P$) locally computes $w_i = u_i - v_i$. After that, all TPUs jointly compute $(\overline{w_1}, \cdots, \overline{w_P}) \leftarrow \text{BExt}(w_1, \cdots, w_P)$.

ii) As we use two complement binary representation, the most significant digit of $u - v$ will reflect the relationship between the $u$ and $v$. After the above computation, TPU $i$ outputs $w_i^{(i-1)} \in \overline{w_i}$. The most significant digit $w_i^{(i-1)}$ of $w = \sum_{i=1}^{P} w_i$ decides the relationship of $u$ and $v$, specifically, if $\sum_{i=1}^{P} w_i^{(i-1)} = 0$, it denotes $u \geq v$. Otherwise, it denotes $u < v$.

6.2.6 Secure Equivalent Protocol (SEQ)

The goal of secure equivalent protocol SEQ is to test whether the two values $u, v$ are equal or not by giving the shares of the two values $\langle u \rangle, \langle v \rangle$. Mathematically, given two shares $\langle u \rangle$ and $\langle v \rangle$, SEQ outputs the shares $\langle f \rangle$ for each TPU $i$ ($i = 1, \ldots, P$) to determine whether the plaintext of the two data are equivalent (i.e. test $u = v$).

i) All the TPUs jointly calculate $(\langle t_1 \rangle \leftarrow \text{SC}(\langle u \rangle, \langle v \rangle); \langle t_2 \rangle \leftarrow \text{SC}(\langle v \rangle, \langle u \rangle))$.

ii) For each TPU $i$, it computes $f_i = t_{1,i} \oplus t_{2,i}$ locally, and outputs $f_i \in \mathbb{Z}_2$.

6.2.7 Secure Minimum of Two Number Protocol (Min$_2$)

The TPU $i$ ($i = 1, \ldots, P$) stores shares $\langle x \rangle$ and $\langle y \rangle$ of two numbers $x$ and $y$. The Min$_2$ protocol outputs share $B$ of minimum number $B$, s.t., $B = \text{min}(x, y)$. The Min$_2$ is described as follows:

i) All the TPUs can jointly compute $(u) \leftarrow \text{SC}(\langle x \rangle, \langle y \rangle); (u) \leftarrow \text{BZI}(u); (X) \leftarrow \text{SM}(\langle x \rangle, \langle u \rangle)$.

ii) The TPU $i$ computes locally and outputs $B_i = y_i - X_i$.

6.2.8 Secure Minimum of H Numbers Protocol (Min$_H$)

The goal of Min$_H$ is to get the minimum number among $H$ numbers. Given the shares $x_1, \ldots, x_H$ for TPU $i$, the goal is to compute the share $x_i$ for TPU $i$ such that $x^* \text{ stores the minimum integer value among } x_1, \ldots, x_H$, where $x^* = \sum_{i=1}^{P} x_i^*; x_j = \sum_{i=1}^{P} x_{i,j}$ for $j = 1, \ldots, H$. The Min$_H$ executes as follows: Each TPU $i$ puts $x_{1,i}, \ldots, x_{H,i}$ into a set $S_i$. If $\mathcal{L}(S_i) = 1$, the share remaining in $\mathcal{L}(S_i)$ is the final output. Otherwise,
the protocol is processed according to the following conditions.

- If $\mathcal{L}(S_i) \mod 2 = 0$ and $\mathcal{L}(S_i) > 1$, set $S'_i \leftarrow \emptyset$; 2) for $j = 1, \ldots, \mathcal{L}(S_i)/2$, compute
  \[
  (x_j) \leftarrow \min((x_{2j-1}), (x_{2j})),
  \]
  and add $x_{j,i}$ to the set $S'_i$; 3) clear set $S_i$ and let $S_i \leftarrow S'_i$.

- If $\mathcal{L}(S_i) \mod 2 \neq 0$ and $\mathcal{L}(S_i) > 1$, take out the last tuple $x_{\mathcal{L}(S_i)-1,i}$ from set $S_i$ s.t., $\mathcal{L}(S_i) \mod 2 = 0$. Run the above procedure ($\mathcal{L}(S_i) \mod 2 = 0$ and $\mathcal{L}(S_i) > 1$) to generate set $S'_i$. Put $x_{\mathcal{L}(S_i)-1,i}$ into a set $S'_i$ and denote $S'_i \leftarrow S_i$.

After computation, each set $S_i$ in TPU $i$ only contains one element and we denote it as $x_i^\ast$. Thus, we denote the algorithm as $\langle x \rangle \leftarrow \max_H((x_1), \cdots, (x_H))$.

6.3 Security Extension of Integer Computation

The above secure computation only considers the data privacy. Two types of information can be leaked to the adversary: 1) the access pattern of function’s input, and 2) the access pattern of RU’s result retrieve. Here, we give two security extension to achieve access pattern hiding and private information retrieve, respectively.

6.3.1 Achieve Input Access Pattern Hiding (APH)

As data are directly sealed in the UnS, the adversary may analyze the access pattern of UnS without knowing the function’s input. Suppose the system contains $H$ data $x_1^\ast, \ldots, x_H^\ast \in \mathbb{D}_N$. The data share $x_{j,i}$ are hold by each TPU $i$ ($j = 1, \ldots, H; i = 1, \ldots, \mathcal{P}$), such that $x_{j,1} + \cdots + x_{j,P} = x_j^\ast$. To achieve access pattern hiding, the homomorphic property of PCDD can be used. Specifically, the RU uploads $[a_1], \ldots, [a_H]$ to each TPU $i$, s.t., for a specific $1 \leq \gamma \leq H$, it has $a_{\gamma} = 1$, and other $j \neq \gamma$ and $1 \leq j \leq H$, it holds $a_j = 0$. Then, the goal of the algorithm is to securely select the shares of $x_{\gamma,j}$ from the input shares, and constructs as follows:

1) **Obviously select encrypted shares.** Each TPU initializes an enclaves. Then, for each TPU $i$ ($i = 1, \ldots, \mathcal{P}$), compute
  \[
  [b_i] \leftarrow ([a_1]^{x_{1,i}} \cdot [a_2]^{x_{2,i}} \cdots [a_H]^{x_{H,i}}) \mod N^2.
  \]

2) **Securely update share $[b_i]$ for TPU $i$.** Without any share update, the adversary can still know the access pattern once the ciphertexts are decrypted. Thus, all the shares should be dynamically updated before the decryption. The TPU $i$ picks random numbers $\delta_{i,1}, \ldots, \delta_{i,P} \in \mathbb{Z}_N$ such that $\delta_{i,1} + \cdots + \delta_{i,P} = 0 \mod N$, and then encrypts $\delta_{i,j}$ and sends $[\delta_{i,j}]$ to TPU envelope $j$. Once all the update shares are received, TPU $i$ computes
  \[
  [b_i^\ast] \leftarrow [b_i] \cdot [\delta_{1,i}] \cdot [\delta_{2,i}] \cdots [\delta_{P,i}] \mod N^2.
  \]

Finally, each TPU $i$ uses the SDD to get $b_i^\ast$ and denotes $[b_i^\ast]_N$ as the final share output.

6.3.2 Achieve Private Information Retrieve (PIR)

If the computation results is needed, the RU will let the TPU to send the data shares back via a secure channel. However, if one of the TPU has been compromised, even if the data cannot been known by the adversary, the retrieve access pattern has been leaked to the adversary.

Suppose the system contains $H$ data $x_1^\ast, \ldots, x_H^\ast \in \mathbb{D}_N$. The data share $x_{j,i}$ are hold by each TPU $i$ ($j = 1, \ldots, H; i = 1, \ldots, \mathcal{P}$), such that $x_{j,1} + \cdots + x_{j,P} = x_j^\ast$. Thus, to achieve the private information retrieve, the RU uploads $[a_1], \ldots, [a_H]$ to each TPU, s.t., for a specific $1 \leq \gamma \leq H$, it has $a_{\gamma} = 1$, and other $j \neq \gamma, 1 \leq j \leq H$, it holds $a_j = 0$. The goal of PIR is to let RU privately retrieve $x_{\gamma}$. Then, the algorithm computes among all TPU as follows:

1) For each TPU $i$, compute
  \[
  [b_i] \leftarrow ([a_1]^{x_{1,i}} \cdot [a_2]^{x_{2,i}} \cdots [a_H]^{x_{H,i}}) \mod N^2.
  \]

2) TPU 1 denotes $[b^\ast] \leftarrow [b_1]$, and sends $[b^\ast]$ to TPU 2. Then, each TPU $i = 2, \ldots, \mathcal{P}$, computes $[b^\ast] \leftarrow [b^\ast] \cdot [b_i] \mod N^2$. If $i = \mathcal{P}$, then send $[b^\ast]$ to RU. Otherwise, $[b^\ast]$ is sent from TPU $i$ to $i + 1$. Finally, RU uses the Dec to get the $b^\ast$, such that $x_{\gamma} = [b^\ast]_N$ is the output share.

6.4 Secure Floating Point Number Computation

6.4.1 Data Format of Floating-Point Number

To achieve the real number storage and computation, we can refer to the IEEE 754 standard to use Floating-Point Number (FPN) for real number storage. To support the LightCom, we change the traditional FPN and describe the FPN by four integers: 1) a radix (or base) $\beta \geq 2$; 2) a precision $\eta \geq 2$ (roughly speaking, $\eta$ is the number of “significant digits” of the representation); 3) two extremal exponents $e_{\min}$ and $e_{\max}$ such that $e_{\min} < 0 < e_{\max}$. A finite FPN $\tilde{a}$ in such a format is a number for which there exists at least one representation triplet $(m, e)$ with public parameters $\beta, \eta, e_{\min}, e_{\max}$, such that,

\[
\tilde{a} = m \cdot \beta^{e-\eta+1}.
\]

- $m$ is an integer which $-\beta^{\eta+1} \leq m \leq \beta^{\eta} - 1$. It is called the integral **significand** of the representation of $x$;
- $e$ is an integer such that $e_{\min} \leq e \leq e_{\max}$, called the **exponent** of the representation of $a$.

As only the **significand** and **exponent** contains sensitive information, we assume all the FPNs have the same public base $\beta = 10$, and use the fix bit-length to store the integer $m$. Thus, to achieve the secure storage, the RU only needs to random share the $\tilde{a}$ into $\tilde{a}_1 = (m_1, e_1), \ldots, \tilde{a}_P = (m_P, e_P)$, and sends $\tilde{a}_i$ to TPU $i$ for storage, respectively.

For the secure FPN computation, if all the FPNs are transformed with the same exponential, we can directly use secure integer computation method introduced in Section 6. Thus, the key problem to achieve the secure FPN computation is how to allow all the FPNs securely...
transformed with the same exponential. Here, we first construct an algorithm called Secure Uniform Computation (UNI) and then achieve the commonly-used FPN computations.

6.4.2 Secure Uniform Computation (UNI)
Assume each TPU \((i = 1, \ldots, P)\) stores into \(\hat{a}_{j,i} = (m_{j,i}, e_{j,i})\), the goal of UNI is to output \(\hat{a}_{j,i}^* = (m_{j,i}^*, e_{j,i}^*)\) for \(j = 1, \ldots, H\), and the construction of UNI can be described as follows:

i) All the TPUs jointly compute

\[
(e^*) \leftarrow \text{Min}_H(\langle e_1 \rangle, \ldots, \langle e_H \rangle).
\]

ii) Each TPUs locally computes \(\langle e_j \rangle = \langle e_j \rangle - \langle e^* \rangle\). As \(e_j - e^*\) is a relative small number, TPUs jointly executes \(10^{e_j-e^*} \leftarrow \text{SEP}(\langle e_j \rangle, 10)\) and \(\langle m_j^* \rangle \leftarrow \text{SM}(10^{e^*-e_j}, \langle m_j \rangle)\).

After computation, all the \(\langle a_j \rangle, \ldots, \langle a_H \rangle\) will transform to \(\langle a_j^* \rangle, \ldots, \langle a_H^* \rangle\) which shares the same \(e^*\), where \(\hat{a}_j = (\langle m_j^* \rangle, \langle e^* \rangle)\).

6.4.3 Computation Transformation
The secure floating-point number computation can be transformed into the secure integer computation protocols with the usage of UNI. Formally, given FPN shares \(\hat{a}_j = (\langle m_j \rangle, \langle e_j \rangle)\), (for \(j = 1, \ldots, H\)), we can first compute

\[
\langle \hat{a}_1 \rangle, \ldots, \langle \hat{a}_H \rangle \leftarrow \text{UNI}(\langle \hat{a}_1 \rangle, \ldots, \langle \hat{a}_H \rangle),
\]

where \(\langle \hat{a}_j \rangle = (\langle m_j \rangle, \langle e_j \rangle)\). Then,

\[
\langle y_1 \rangle, \ldots, \langle y_H \rangle \leftarrow \text{SIF}(\langle m_1^* \rangle, \ldots, \langle m_H^* \rangle),
\]

where SIF denote secure integer computation protocol designed in Section 6.4.2 and \(\langle y_1 \rangle, \ldots, \langle y_H \rangle\) can be either integer shares or binary shares according to the function type. If the SIF is the SC and SEQ, then the SIF output the binary share \(\langle y^* \rangle\) as the final output, and we denote these two algorithms as secure FPN comparison (FC) and secure FPN equivalent test protocol (FETQ). If the SIF is the SM, SMM, MinH and MinH, then the SIF outputs the integer share \(\langle y_1^* \rangle\), and denotes \(\langle y^* \rangle = (\langle y_1^* \rangle, \langle e^* \rangle)\) as the secure FPN’s output, and we denote above four algorithms as secure FPN multiplication (FMin), secure FPN monic monomials computation (FMM), secure minimum of two FPNs protocol (FMin2), and secure minimum of \(H\) FPNs protocol (FMinH), respectively. Specifically, for the multiple FPN addition (FAdd), given FPN shares \(\langle a_j \rangle = (\langle m_j \rangle, \langle e_j \rangle)\), (for \(j = 1, \ldots, H\)), we can first compute \(\langle a_1^* \rangle, \ldots, \langle a_H^* \rangle\) with the UNI, where \(\langle a_j^* \rangle = (\langle m_j^* \rangle, \langle e^* \rangle)\). Then, compute \(\langle y^* \rangle \leftarrow \sum_{j=1}^{H} \langle m_j^* \rangle\) and denote the final FPN addition result as \(\langle y \rangle = (\langle y^* \rangle, \langle e^* \rangle)\).

6.4.4 Secure Extension for FPN Computation
Similar to the secure integer computation, we have the three following extension for LightCom.

Access Pattern Hiding: As all the secure FPN computation can be transformed in to secure integer computation with the help of the UNI, we can also use the same method in section 6.3.1 to achieve input access pattern hiding for the secure FPN computation.

Achieve Private FPN Retrieve: In out LightCom, one floating point number can be securely stored as two integer numbers. Thus, we can use the method in section 6.3.2 to privately retrieve integer for twice to achieve the private FPN retrieve.

6.5 Functional Extension for LightCom
6.5.1 Non-numerical Data Storage and Processing
For the non-numerical data storage, the traditional character encodings with Unicode [8] and its standard Unicode Transformation Format (UTF) schemes can be used which maps a character into an integer. Specifically, for secure storage, use UTF-8 to map the character into 32-bit number \(x\), randomly splits \(x\) into \(x_1, \ldots, x_P\), such that \(x_1 + \cdots + x_P = x\), and sends \(x_i\) to TPU \(i\) for processing. In this case, all the non-numerical data processing can be transformed into secure integer computation which can be found in section 6. For the secure storage, each TPU \(i\) securely seals the share \(a_i\) into the UnS with the algorithm Seal in Section 5.4. Once the data shares are needed for processing, TPUs needs to use UnSeal algorithm to recover the message from UnS.

6.5.2 Extension of Multiple User Computation
All the secure computations in the previous section are designed for the single user setting, i.e., all the data are encrypted under a same RU’s public key. If all RUs want to jointly achieve a secure computation, each RU \(j(j = 1, \ldots, \psi)\) executes KeyGen to generate public key \(pk_j\) and private key is \(sk_j\) locally. Then, RU \(j\) uses KeyS to split key \(sk_j\) into \(P\) shares \(\langle sk_j \rangle\), and sends these shares to TPUs in the cloud. Assume RU \(j\)’s ciphertext \(\langle x_j \rangle_{pk_j}\) is securely stored in UnS, TPUs can get data shares \(\langle x_j \rangle\) with UnSeal and achieve the corresponding secure computations GenCpt in Section 4.3 with these shares.

7 Security Analysis
In this section, we first analyze the security of the basic crypto primitives and the sub-protocols, before demonstrating the security of our LightCom framework.

7.1 Analysis of Basic Crypto Primitives
7.1.1 The Security of Secret Sharing Scheme
Here, we give the following theorem to show the security of the additive secret sharing scheme.

Theorem 1. An additive secret sharing scheme achieves an information theoretic secure when the \(P\) participants can reconstruct the secret \(x \in G\), while any smaller set cannot discover anything information about the secret.

Proof. The shares \(X_1, \ldots, X_P\) are selected with random uniform distribution among \(P\) participants such that
X_1 + \cdots + X_p = m \in \mathbb{G}$. Even the attacker \( A \) holds \( P - 1 \) shares, (s)he can only compute \( x' = \sum_{i=1}^{P-1} X_i' \), where \( X_i' \) is selected from \( X_1, \cdots, X_p \). The element \( x \) is still protected due to the \( x = x' + X_p' \). Since random value \( X_p' \) is unknown for \( A \), it leaks no information about the value \( x \).

\[ \sum_{i=1}^{P-1} X_i' = x. \]

\[ \exists \text{time period} \ t, \ \text{secure according to the theorem } 1. \]

Theorem 2. A proactive additive secret sharing scheme achieves an information theoretic secure if satisfies the following properties: I. Robustness: The new updated shares are corresponding to the secret \( x \) (i.e., all the new shares can reconstructed the secret \( x \)). II. Secrecy: The adversary at any time period knows no more than \( P \) shares (possible a different shares in each time period) learns nothing about the secret.

Proof. The data shares \( X_i^{(t)} \) in time period \( t \) are stored in party \( i \), s.t., \( \sum_{i=1}^{P} X_i^{(t)} = x \). Each party \( i \) generates shares \( \delta_i^{(t)} \), which satisfies \( \delta_1^{(t)} + \cdots + \delta_{P}^{(t)} = 0 \mod N \). Thus, the new shares denote \( X_i^{(t+1)} = X_i^{(t)} + \delta_i^{(t)} + \cdots + \delta_{P}^{(t)} \), and satisfy \( \sum_{i=1}^{P} X_i^{(t+1)} = \sum_{i=1}^{P} X_i^{(t)} + \sum_{i=1}^{P} \delta_i^{(t)} = x \) which the robustness property hold.

To guarantee the secrecy property, the data shares in time period \( t \) can achieve the information theoretic secure according to the theorem \( \square \). Even adversary can get \( P - 1 \) shares in each time period \( t \) \( (t \leq t^*) \), the adversary can compute \( x^{(t)} = x - X_p^{(t)} = \sum_{i=1,i\neq P} X_i^{(t)} \), where \( X_p^{(t)} \) is the non-compromised share in time period \( t \). The adversary \( A^* \) still can not get any information from \( x^{(1)}, \cdots, x^{(t)}, \delta_{P_i}^{(t)}, \cdots, \delta_{P_x}^{(t)} \) are independently and randomly generated and cannot be compromised by the adversary. Thus, the secrecy property holds. \( \square \)

7.1.2 The Security of PCDD

The security of our PCDD is given by the following theorem.

Theorem 3. The PCDD scheme described in Section 2.3 is semantically secure, based on the assumed intractability of the DDH assumption over \( \mathbb{Z}_{N^2}^{*} \).

Proof. The security of PCDD has been proven to be semantically secure under the DDH assumption over \( \mathbb{Z}_{N^2}^{*} \) in the standard model \( \square \).

7.2 Security of TPU-based Basic Operation

Theorem 4. The RTG can securely generate random shares against adversary who can compromise at most \( P - 1 \) TPUs, assuming the semantic security of the PCDD cryptosystem.

Proof. For each TPU \( i \) \( (0 \leq i < P) \), only the PCDD encryption \( [r^{(1)}], \cdots, [r^{(t)}] \) are sent to TPU \( i + 1 \). After that, the PCDD encryption \( [r] \) is sent from TPU \( i + 1 \) to \( i \). According to the semantic security of the PCDD (theorem \( \square \)), the TPU \( i + 1 \) cannot get any information from the ciphertext sent from TPU \( i \). Even the adversary can compromise at most \( P - 1 \) TPUs and get the shares \( r^{(1)}, \cdots, r^{(t)} \), (s)he cannot get the secret \( r^{(1)}, \cdots, r^{(t)}, r \) due to \( r^{(1)}, \cdots, r^{(t)}, r \) are unknown to adversary according to the security of Theorem \( \square \).

The security proof of the secure share domain transformation in section, secure binary shares operation in section, secure integer computation, and secure FPN computation are similar to the proof of theorem \( \square \). The security of above operations are based on the semantic security of the PCDD cryptosystem. Next, we will show that AHP and PIR can achieve its corresponding functionality.

Theorem 5. The AHP can securely achieve the access pattern hidden for the function input under the semantic security of the PCDD cryptosystem.

Proof. In the select share phase, all \( a_1, \cdots, a_H \) are selected and encrypted by RU, and are sent to TPUs for processing. It is impossible for the adversary to know the plaintext of the ciphertext due to the semantic security of PCDD. Also, the shares are dynamically update by computing \( x_i \leftarrow b_i + \delta_i,i + \delta_i,i + \cdots + \delta_i,p \mod N \). As \( \delta_i,i \) is randomly generated by TPU \( i \) and is sent from TPU \( j \) to TPU \( i \), it is hard for the adversary to recover \( b_i \) even adversary compromise the other \( P - 1 \) TPUs due to the secrecy of Theorem 2. Thus, it is still impossible for the adversary to trace the original shares with the update shares which can achieve the access pattern hidden. \( \square \)

Theorem 6. The PIR can securely achieve the private information retrieve under the semantic security of the PCDD cryptosystem.

Proof. In PIR, all \( a_1, \cdots, a_H \) are selected and encrypted by RU, and sent to TPUs for processing. After that, \( [b^*] \) is transmitted among TPUs. As all the computations in the PIR are executed in the ciphertext domain, it is impossible for the adversary to know the plaintext of the ciphertext due to the semantic security of PCDD, which can achieve the private information retrieve. \( \square \)

7.3 Security of LightCom

Theorem 7. The LightCom is secure against side-channel attack if \( t_c + t_p + t_d < P \cdot t_r \), where \( t_c, t_p \) and \( t_d \) are the runtime of secure computation GenCpt, private key update, and data share update, respectively; \( t_r \) is the runtime for attacker successfully compromising the TPU enclave; \( P \) is the number of TPUs in the system.

Proof. In the data upload phase, RU’s data are randomly separated and uploaded to TPUs via secure channel. According to theorem \( \square \) no useful information about the RU’s data are leaked to the adversary with compromising \( P - 1 \) TPUs enclaves. For the long-term storage, the data shares are securely sealed in the UnS with PCDD crypto-system. With the theorem \( \square \) we can find the encrypted data shares are semantically secure stored in the UnS.

In the secure online computing phase, all the ciphertext are securely load to the TPUs with UnSeal. Then, all
the TPUs jointly achieves the secure computation with the GenCpt. During the computing phase, the system attacker can launch the following three types of attacks: 1) compromise the TPU enclave: adversary can compromise a TPU enclave to get current data shares and private key shares with the time $t_s$; 2) stores the old private key shares: the adversary tries to recover the RU’s private key with current and old private key shares. 3) stores the old data shares and try to recover the RU’s original data: the adversary tries to recover the RU’s data with current and old data shares. To prevent first type of attack, RU’s data are separated and distributed among $P$ TPUs. Unless adversary can compromise all the TPU enclaves at the same time, $A$ can get nothing useful information from compromised shares according to theorem 1. Thanks to the secrecy property of proactive additive secret sharing scheme in Theorem 2, it is impossible for the adversary to recover the private key and RU’s data by getting compromised shares according to theorem 1. Thus, the adversary fails to attack our LightCom system if the data shares are successfully sealed in the UnS and all the TPU enclaves are released before the adversary compromises all the enclaves in the secure computation phase. In this case, the LightCom is secure against adversary side-channel attack if $t_c + t_p + t_d < P \cdot t_a$.

8 Evaluations

In this section, we evaluate the performance of LightCom.

8.1 Experiment Analysis

For evaluating the performance of the LightCom, we build the framework with C code under the Intel® Software Guard Extensions (SGX) environment as a special case of TPU, and the experiments are performed on a personal computer (PC) with 3.6 GHz single-core processor and 1 GB RAM memory (single-thread program are used) on virtual machine with Linux operation system. To test the efficiency of our LightCom, there are two types of metrics are considered, called runtime and security level (associate with PCDD parameter $N$). The runtime refers to the secure outsourced computation executing duration on server or user’s side in our testbed. The security level is an indication of the security strength of a cryptographic primitive. Moreover, we use SHA-256 as the hash function $H(\cdot)$ in LightCom. As the communication latency among CPUs is very low (use Intel® UltraPath Interconnect (UPI) with 10.4 GT/s transfer speed and theoretical bandwidth is 20.8 GB/s [14], we do not consider the communication overhead as a performance metric in our LightCom.

8.1.1 Basic Crypto and System Primitive

We first evaluate the performance of our basic operation of cryptographic primitive (PCDD cryptosystem) and basic system operations (Seal, UnSeal and SDD protocol). We first let $N$ be 1024 bits to achieve 80-bit security to test the basic crypto primitive and basic protocol. For PCDD, it takes 1.153 ms to encrypt a message (Enc), 1.171 ms for Dec, 1.309 ms to run PDec, 5.209 $\mu$s to run TDec. For the basic system operations, it takes 1.317 ms for Seal, 1.523 ms for UnSeal, and 1.512 ms for SDD ($P = 3$). Moreover, Seal, UnSeal and SDD are affected by the PCDD parameter $N$ and the number of TPUs $P$ (See Fig. 3(a) and Fig. 3(b) respectively). From the Figs. 3(a) and 3(b) we can that the parameter $N$ will affect greatly on the runtime and communication overhead of the protocols.

8.1.2 Performance of TPU-based Integer Computation

Generally, there are four factors that affect the performance of TPU-based integer computation: 1) the number of TPUs $P$; 2) the PCDD parameter $N$; 3) the bit-length of the integer $\ell$; 4) the number of encrypted data $H$. In Fig. 4(a)-4(e) we can see that the runtime of all the protocols increase with $P$. It is because more runtime are needed and more data in online phase and random numbers in offline phase are required to process with extra parties. Also, we can see that the runtime of all the TPU-based integer computations increase with the bit-length of $N$ from Table 1. It is because the running time of the basic operations (Enc and Dec algorithms of PCDD) increases when $N$ increases. Moreover, in Fig. 1(f)-1(k) the performance of RTG, SMM, BAdd, BExt, SEP, SC, SEQ, Min2, MinH, UNI are associated with $\ell$. The computational cost of above protocols are increased with $\ell$, as more computation resources are needed to process when $\ell$ increase. Finally, we can see that performance of APH and PIR are increased with $H$ in Fig. 1(l). It is because more numbers of PCDD ciphertexts cost more energy with the homomorphic and module exponential operations.

8.1.3 Performance of TPU-based FPN Computation

For the basic TPU-based FPN computation, there are four factors that affects performance of LightCom: 1) the...
number of TPUs; 2) the PCDD parameter \( N \); 3) the bit-length of the integer \( \ell \); 4) the number of encrypted data \( H \). The runtime trends of FPN computation protocols (e.g. FC, FEQ, FM, FMM, FMin2, FMinH) are similar to the trends of corresponding secure integer computation (e.g. SC, SEQ, SM, SMM, Min2, MinH), as the runtime of FPN computation is equal to the runtime of corresponding secure integer computation add the runtime of UNI.

### 8.2 Theoretical Analysis

Let us assume that one regular exponentiation operation with an exponent of \( ||N|| \) requires \( 1.5 \cdot ||N|| \) multiplications \([11]\). For PCDD, it takes \( 3 \cdot ||N|| \) multiplications for Enc, \( 1.5 \cdot ||N|| \) multiplications for Dec, \( 1.5 \cdot ||N|| \) multiplications for \( PDec, P \) multiplications for TDec, \( 1.5 \cdot ||N|| \) multiplications for CR. For the basic operation of LightCom, it takes \( 1.5 \cdot ||P|| \cdot ||N|| \) multiplications to run SDD, \( 3 \cdot ||N|| + t_{hash} \) multiplications for Seal, \( 1.5 \cdot ||P|| \cdot ||N|| + t_{hash} \) multiplications for UnSeal, \( O(\ell + ||P|| \cdot ||N||) \) multiplications for RTG, \( O(||P|| \cdot ||N||) \) multiplications for B2I, I2B. For the integer and binary protocol in LightCom, it takes \( O(||P|| \cdot ||N||) \) multiplications for off-line phase of \( SBB \) and \( SM \), \( O(||P|| \cdot ||N||) \) multiplications for off-line phase of \( BAdd, BExt, SC, SEQ, Min2, O(||P|| \cdot ||N||) \) multiplications for both off-line and online phase of \( SEP, O(HP ||N||) \) multiplications for off-line phase of \( APH \) and \( PIR, O([\log_2 H] \cdot ||P|| \cdot ||N||) \) multiplications for off-line phase of \( MinH \). For the FPN computation in LightCom, it takes \( O(HP ||N||) \) multiplications for off-line phase of \( FM, FMM, FC, FEQ, FMin2, \) and \( O([\log_2 H] \cdot ||P|| \cdot ||N||) \) multiplications for off-line phase of \( FMinH \). All the above protocols only need \( O(1) \) multiplications in online phase, which is greatly fit for fast processing.

### 9 Related Work

**Homomorphic Encryption.** Homomorphic encryption, allow third-party to do the computation on the ciphertext which reflected on the plaintext, is considered as the best solution to achieve the secure outsourced computation. The first construction of fully homomorphic encryption was proposed by Gentry in 2009 under the ideal lattices, which permits evaluation of arbitrary circuits over the plaintext \([12]\). Later, some of the new hard problems (such as Learning With Errors (LWE) \([13]\), Ring-LWE \([14]\) are used to construct the FHE which can greatly reduce the storage overhead and increase the performance of the homomorphic operations \([15, 16]\). However, the current FHE solutions and libraries are still not practical enough for the real-world scenarios \([17, 18]\). Somewhat homomorphic encryption \([19, 20]\) can allow semi-honest third-party to achieve the arbitary circuits with limited depth. The limited times of homomorphic operations are only restrict the usage scope of the application. Semi-homomorphic encryption (SHE) can only support additive \([21]\) (or multiplicative \([22]\) homomorphic operation. However, with the help of the extra semi-honest computation-aid server, a new computation framework can be constructed to achieve commonly-used secure rational number computation \([23]\), secure multiple keys computation \([24]\), and floating-point number computation \([25]\). The new framework can greatly balance the security and efficiency concerns, however, the extra server will still complex the system which brings more risk of information leakage.

**Secret Sharing-based Computation.** The user’s data in secret sharing-based (SS-based) computation are separated into multiple shares with the secret sharing technique, and each shares are located in one server to guarantee the security. Multiple parties can jointly together to securely achieve a computation without leaking the original data to the adversary. Different from the heavy-weight homomorphic operation, the SS-based computation \([26, 27, 28]\) can achieve the lightweight computation. Despite the theoretical construction, many real-word computation are constructed for practical us-
age, such as SS-based set intersection [29] and top-$k$ computation [30]. These basic computations can be used to solve data security problem in data mining technique, such as deep learning [31]. Emekçi et al. [32] proposed a secure ID3 algorithm to construct a decision tree in a privacy-preserving manner. Ma et al. [33] constructed a lightweight privacy-preserving adaptive boosting (AdaBoost) for the face recognition. The new secure natural exponential and secure natural logarithm which can securely achieve the corresponding computation computation to balance accuracy and efficiency. Although many of the privacy-preserving data mining techniques with secret sharing are constructed [34], [35], the SS-based computation still need to build secure channel among these parties. Moreover, the high communication rounds among the computation parties still become an obstacle for a large-scale application.

Intel® Software Guard Extensions. Intel® SGX is a kind of TEE which provides strong hardware-enforced confidentiality and integrity guarantees and protects an application form the host OS, hypervisor, BIOS, and other software. Although an increasingly number of real-world industry applications are securely executed in the untrusted remote platforms equipped with SGX, the SGX still faces side-channel attack to expose the information during the computation. Götzfried et al. [36] proposed a new attack called root-level cache-timing attacks which can obtain secret information from an Intel® SGX enclave. Lee et al. [37] presented a new side-channel attack cannelled branch shadowing which reveals fine-grained control flows in a SGX enclave. Bulck et al. [38] constructed two novel attack vectors that infer enclave memory accesses. Chen et al. [39] presented a new attack call SGXPECTRE that can learn secrets inside the enclave memory or its internal registers. Currently, three types of solutions are used to protect the side-channel attack: hardware method [40], [41], system method [42], [43], and application method [44], [45]. These methods can only guarantee some dimension of protection, and cannot be used for all-directional protection even against the unknown side-channel attack.

10 CONCLUSION

In this paper, we proposed LightCom, a framework for practical privacy-preserving outsourced computation framework, which allowed a user to outsource encrypted data to a single cloud service provider for securely data storage and process. We designed two types of outsourced computation toolkits which can securely guarantee the achieve secure integer computation and floating-

Fig. 4. Simulation results of LightCom
point computation against side-channel attack. The utility and performance of our LightCom framework was then demonstrated using simulations. Compared with the existing secure outsourced computation framework, our LightCom takes fast, scalable, and secure outsourced data processing into account.

As a future research effort, we plan to apply our LightCom in a specific applications, such as e-health cloud system. It allows us to refine the framework to handle more complex real-world computations.

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17

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Ximeng Liu (S’13–M’16) received the B.Sc. degree in electronic engineering from Xidian University, Xi’an, China, in 2010 and the Ph.D. degree in Cryptography from Xidian University, China, in 2015. Now he is the full professor in the College of Mathematics and Computer Science, Fuzhou University. Also, he is a research fellow at the School of Information System, Singapore Management University, Singapore. He has published more than 100 papers on the topics of cloud security and big data security including papers in IEEE Transactions on Computers, IEEE Transactions on Information Forensics and Security, IEEE Transactions on Dependable and Secure Computing, IEEE Transactions on Service Computing, IEEE Internet of Things Journal, and so on. He awards Minjiang Scholars Distinguished Professor, Qishan Scholars in Fuzhou University, and ACM SIGSAC China Rising Star Award (2018). His research interests include cloud security, applied cryptography and big data security. He is a member of the IEEE, ACM, GCF.

Robert H. Deng (F’16) is AXA Chair Professor of Cybersecurity and Professor of Information Systems in the School of Information Systems, Nanyang Management University since 2004. Prior to this, he was a principal scientist and a manager of Infocomm Security Department, Institute for Infocomm Research, Singapore. His research interests include data security and privacy, multimedia security, network and system security. He served/is serving on the editorial boards of many international journals, including the IEEE Transactions on Information Forensics and Security, and IEEE Transactions on Dependable and Secure Computing.
**Supplementary Materials**

### A. Deterministic Turing Machines

Turing machines are a model of computation which anything can be computed that can be computed by a Turing Machine. As all the function in our PVOA framework can be computed by Deterministic Turing Machine. The rigorous definition is defined as follows.

**Definition 2** (Deterministic Turing Machine). A Deterministic Turing Machine contains a tuple $(Q, \Sigma, \delta, s, h)$ where 1) $Q$ is a finite set of states which contains the states $s, q_{acc}, q_{rej}$. 2) $\Sigma$ is a finite alphabet which contains the symbol #. 3) Transition function $\delta : Q \times \{q_{acc}, q_{rej}\} \times \Sigma \rightarrow Q \times \Sigma \cup \{R, L\}$, where $L$ is left shift, $R$ is right shift. 4) $s \in Q$ is the start state, $q_{acc}$ is the accept state, $q_{rej}$ is the reject state.

Suppose $M$ is a deterministic Turing machine that halts on all inputs. Time complexity function $T_M : \mathbb{N} \rightarrow \mathbb{N}$ is defined as

$$T_M(n) = \max\{m|\exists w \in \Sigma^*, |w| = n \text{ such that the computation of } M \text{ on } w \text{ takes } m \text{ moves}\},$$

where numbers are coded in binary format. We call a Turing machine is polynomial if there exists a polynomial $p(n)$, such that $T_M(n) \leq p(n)$, for all $n \in \mathbb{N}$.

### B. Hard Problem

**Assumption 1.** (DDH assumption over $\mathbb{Z}_{N^2}^*$). For every probabilistic polynomial time algorithm $A$, there exists a negligible function $\text{negl}(\cdot)$ such that for sufficiently large $l$.

$$\Pr \left[ A(N, X, Y, Z_b \mod N) = b \right] = \frac{1}{2} = \text{negl}(l).$$

**Theorem 8.** Let $N$ be a composite modulus product of two large primes. Let $\mathbb{G}$ be the cyclic group of quadratic residues modulo $N^2$. The decisional Diffie-Hellman problem over $\mathbb{Z}_{N^2}^*$ (in $\mathbb{G}$) cannot be harder than factoring.

**Proof.** The detailed proof can be found in [9].

### C. Paillier Cryptosystem Distributed Decryption (PCDD)

In order to realize LightCom, our previous Paillier Cryptosystem Distributed Decryption (PCDD) [23] cryptosystem is used and works as follows:

- **KeyGen:** Given a security parameter $k$ and two large prime numbers $p, q$, where $L(p) = L(q) = k$, we have two strong primes $p', q'$, s.t. $p' = \frac{p - 1}{2}$ and $q' = \frac{q - 1}{2}$ (due to the property of the strong primes). We then compute $N = pq$ and $\lambda = \text{lcm}(p-1, q-1)$, define a function $L(x) = \frac{x - 1}{N}$, and choose a generator $g$ of order $(p-1)(q-1)/2$. The public key is $pk = (N, g)$, and the corresponding private key is $sk = \lambda$.

- **Encryption (Enc):** Input a message $m \in \mathbb{Z}_N$, the $\text{Enc}$ chooses a random number $r \in \mathbb{Z}_{N^2}^*$, and output ciphertext as $[m] = g^{mN\lambda} \mod N^2$.

- **Decryption (Dec):** Input a ciphertext $[m] \in \mathbb{Z}_{N^2}$ and the private key $sk$, the $\text{Dec}$ compute $[m]^{\lambda} = (1 + mN\lambda) \mod N^2$. Since $gcd(m, \lambda) = 1$, the plaintext $m$ can be recovered as $m = L([m]^\lambda \cdot \lambda^{-1} \mod N^2)$.

- **Private Key Splitting (KeyS):** Input the private key $\lambda$, the $\text{KeyS}$ separates $\lambda$ into $n$ shares such that $\lambda^1 + \cdots + \lambda^n \equiv 0 \mod \lambda$ and $\lambda^1 + \cdots + \lambda^n \equiv 1 \mod N$.

- **Partially decryption (PDec):** Once $[m]$ is received, with partially private key $\lambda_i$, the partially decrypted ciphertext $CT_i$ can be calculated as: $CT_i = [m]^{\lambda_i} \mod N^2$.

- **Threshold decryption (TDec):** Once $n$ decrypted ciphertexts $CT_1, \cdots, CT_n$ are received, the $\text{TDec}$ algorithm can calculates $T = \prod_{i=1}^{n} (CT_i) \mod N^2$, and $m = L(T \mod N^2)$.

**Given** $[x_1], \cdots, [x_n]$ and $a_1, \cdots, a_n$, we show that our PCDD have the polynomial homomorphism property (poly):

$$[a_1 \cdot x_1 + a_2 \cdot x_2 + \cdots + a_n x_n] \leftarrow [x_1]^{a_1} \cdot [x_2]^{a_2} \cdots [x_n]^{a_n}$$

Homomorphic Properties of DT-PKC: Here, we give three homomorphic properties of DT-PKC as follows:
1) **Additive homomorphism**: Given ciphertexts $[m_1]$ and $[m_2]$ under a same public key $pk$, the additive homomorphism can be achieved by ciphertext multiplication, i.e., compute $[m_1]_{pk} \cdot [m_2]_{pk} = \{(1 + (m_1 + m_2) \cdot N) \cdot h^{r_1 + r_2} \mod N^2, g^{r_1 + r_2} \mod N^2\} = [m_1 + m_2]_{pk}$.

2) **Scalar-multiplicative Homomorphism**: Given ciphertexts $[m]_{pk}$ and a constant number $c \in \mathbb{Z}_N$, it has $( [m]_{pk} )^c = \{(1 + m \cdot N)^c \cdot h^{cr_1} \mod N^2, g^{cr_1} \mod N^2\} = [cm]_{pk}$. Specifically, let $c = N + 1$ and we have $( [m]_{pk} )^{N-1} = \{(1 + (mN^2 - mN) \cdot h^{(N-1)r_1} \mod N^2, g^{(N-1)r_1} \mod N^2\} = [-m]_{pk}$.

Without any ambiguity, all the ciphertexts below are encrypted under the same public key $pk$, and we use the notion $[x]$ instead of $[x]_{pk}$. 