Bi-probability plots and PMNS compatibility studies with the T2K experiment

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Abstract. Bi-probability plots are a way of displaying the expected \( P(\nu_\mu \rightarrow \nu_e) \) and \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \) for long baseline neutrino experiments. They provide an easy visualisation of how \( \delta_{CP} \) and mass hierarchy affect neutrino probabilities, and allow theorists to easily compare their theory predictions to experimental data fit results. We present the electron (anti)neutrino appearance bi-probability plots from the most recent T2K data fit. We also present bi-probability plots from fits with an extra parameter, \( \beta \), that allows the fit additional freedom compared to the PMNS model and tests the compatibility of the T2K data with the PMNS model.

1. Introduction
Tokai to Kamioka (T2K) is a long baseline neutrino experiment that searches for the appearance of \( \nu_e(\bar{\nu}_e) \) and the disappearance of \( \nu_\mu(\bar{\nu}_\mu) \) in the \( \nu_\mu(\bar{\nu}_\mu) \) beam [1]. A narrow 0.6 GeV neutrino beam is generated at the J-PARC facility in Tokai in either \( \nu_\mu \) or \( \bar{\nu}_\mu \) mode. The near detector, ND280, measures the unoscillated neutrino beam, and the far detector, Super-Kamiokande, measures the oscillated neutrinos. A schematic diagram is shown in figure 1. Bi-probability plots allow for a more direct comparisons of sensitivities to \( \delta_{CP} \) and mass hierarchy between long baseline experiments. The experimental expectations and results can be plotted in a form of \( P(\nu_\mu \rightarrow \nu_e) \) against \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \) probabilities by converting the oscillation parameters into oscillation probabilities with the standard neutrino oscillation formula [2]. This also allows the theorists to compare their theory predictions to the experimental data without having to take all the experimental details into consideration (e.g. when calculating the expected neutrino event rates) - especially when non-PMNS models are considered. In these proceedings we present the standard T2K results from the oscillation analysis in a form of the bi-probability plots. We then introduce an additional parameter, \( \beta \), that gives the fit an extra degree of freedom by exploring a continuum of oscillation models. The latter produces results that reflects the data alone, without being constrained by the PMNS - allowing for the model comparisons against non-PMNS theories.

2. Bi-probability plots with PMNS model
The bi-probability plots constrained by the PMNS model are constructed using the output from the standard T2K oscillation analysis. The T2K Bayesian analysis produces a Markov Chain Monte Carlo (MCMC) chain, each MCMC step containing values for each oscillation and systematic (e.g. cross-sections, flux, detector models) parameter. Normally, these MCMC steps can be binned separately for each parameter, producing 1D posterior probability plots, or in
coupled, producing 2D posterior probabilities and credible interval contours. These 1D and 2D plots are a part of the standard results from the T2K oscillation analysis [3]. To present these results on the bi-probability space, we convert the oscillation parameters’ values for each MCMC step into $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ using the standard neutrino oscillation formula from the PMNS model. Figure 2 presents such plot, together with the 68% and 90% credible interval contours and the prediction ovals in yellow and orange for normal (NH) and inverted (IH) mass hierarchy, respectively.

The prediction ovals are drawn for the best-fit oscillation parameters, but with $\delta_{\text{CP}}$ varying between $\pm \pi$, and for the IH and the NH separately. They are flattened to almost straight lines due to the T2K baseline and neutrino energies involved. The binned posterior probability has two high-density areas that are perpendicular to the prediction ovals; they correspond to the NH (lower on $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ axis) and the IH. It can be clearly seen that there is more posterior probability in the NH. This plot is marginalized over all the systematic parameters, meaning the credible interval contours include both statistical and systematic errors. Theorists can now easily compare predictions from their novel non-PMNS models by producing $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ for 0.6 GeV neutrinos over T2K baseline and comparing against figure 2.

3. Bi-Probability plots beyond the PMNS model
Although theorists can now easily compare their models against the T2K results, these results are still constrained by the PMNS model. It is therefore desirable to produce a result on the bi-probability space that is independent of the oscillation model used - to represent the T2K data alone, just like the bi-event plots do. Such result could be also used to see if there are any tensions between the T2K data and the PMNS model. We introduce a new effective parameter, $\beta$, to the standard PMNS oscillation formula, giving an extra degree of freedom in the fit and allowing for a continuum of oscillation models. We multiply this parameter by $P(\nu_\mu \rightarrow \nu_e)$ and

![Figure 2. Bi-probability plot with the binned posterior probability density in blue (without the reactor constraint on $\theta_{23}$), 68% (dashed line) and 90% (solid black line) credible interval contours and the prediction ovals for NH (yellow) and IH (orange). Although the plot is for 0.6 GeV neutrino energy, it does not change much if close to T2K beam peak energy.](image-url)
divide it from $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, allowing it to be free in the sampler with a flat prior between $\beta=0$ and $\beta=\infty$:

$$P(\nu_\mu \rightarrow \nu_e) = \beta \times P_{PMNS}(\nu_\mu \rightarrow \nu_e)$$

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 1/\beta \times P_{PMNS}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e).$$

Figure 3 shows a T2K posterior probability from the free-\(\beta\) fit, with the same labels as in figure 2. We can see that the area explored has increased significantly. This is expected; as we move away from the PMNS model with the free \(\beta\), the combinations of probabilities (and the underlying oscillation parameters) can move away from the constraint given by the unitarity of the PMNS matrix. Theorists are now able to use this plot to compare their theories against the T2K results that represent the data, without assuming any oscillation model. Figure 4 compares the standard PMNS-constrained results and the free \(\beta\) results. From this plot alone it is clear there are no tensions between the two parametrizations, and therefore between the T2K data and the PMNS oscillation model. More data needs to be collected to draw stronger conclusions.

![Figure 3](image1.png)

**Figure 3.** Non-PMNS Bi-probability plot with the binned posterior probability density in blue, 68% and 90% credible interval contours and the prediction ovals for NH and IH.

![Figure 4](image2.png)

**Figure 4.** Comparison of the PMNS-constrained (\(\beta=1\)) and the non-PMNS (free-\(\beta\)) results. Given both 68% and 90% contours overlap, there are no tensions between the data and the PMNS model.

4. **Summary**

T2K results can now be presented in the form of bi-probability plots to show $P(\nu_\mu \rightarrow \nu_e)$ against $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ using the standard T2K Bayesian oscillation analysis output. These plots can be used by other experiments and theorists to compare their predictions against T2K without requiring the knowledge of the full systematic treatment in T2K. These plots are also available in a framework that is independent of the oscillation model - allowing for non-PMNS comparisons. Using this framework, no tensions between T2K data and the PMNS model were found.

**References**

[1] Abe K et al (T2K Collaboration) 2011 *Nucl. Instrum. Meth. A* 659 106

[2] Maki Z et al 1962 *Progress of Theoretical Physics* 28(5) 870

[3] Abe K et al (T2K Collaboration) 2017 *Phys. Rev. D* 96 092006