Effects of Radiative Diffusion on Dynamical Corotation Torque in Three-dimensional Protoplanetary Disks

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Abstract

The dynamical corotation torque arising from the deformation of horseshoe orbits, along with the vortensity gradient in the background disk, is important for determining the orbital migration rate and direction of low-mass planets. Previous two-dimensional studies have predicted that the dynamical corotation torque is positive, decelerating inward planet migration. In contrast, recent three-dimensional studies have shown that buoyancy resonance makes the dynamical corotation torque negative, accelerating inward migration. In this paper, we study the dependence of the dynamical corotation torque on thermal transport, using three-dimensional simulations. We first show that our results are consistent with previous three-dimensional studies when the disk is fully adiabatic. In more realistic radiative disks, however, radiative diffusion suppresses buoyancy resonance significantly, especially in high-altitude regions, and yields a positive dynamical corotation torque. This alleviates the issue of rapid migration being caused by the negative dynamical corotation torque in adiabatic disks. Our results suggest that radiative diffusion, together with stellar irradiation and accretion heating, are needed to accurately describe the migration of low-mass planets.

\textit{Unified Astronomy Thesaurus concepts: Planetary migration (2206); Planetary-disk interactions (2204); Protoplanetary disks (1300); Radiative transfer (1335)}

1. Introduction

Gravitational interactions between a protoplanetary disk and its embedded planet have important consequences for the evolution of the disk–planet system (Kley & Nelson 2012). The planet induces gravitational wakes in the disk, and the gravitational interactions between the wakes and the planet promote angular momentum exchanges between them. If the total gravitational torque exerted on the planet by the wakes is negative (positive), the planet loses (gains) its orbital angular momentum and migrates radially inward (outward). The theory of planet migration has brought many insights to the understanding of planet formation (Paardekooper & Johansen 2018) and the architecture of observed planetary systems (e.g., Baruteau et al. 2014, 2016; Emsenhuber et al. 2021).

Planet migration is divided into three types, depending on the planet mass. Low-mass (typically Earth-sized) planets experience type I migration, which is mostly driven by spiral waves formed at the Lindblad resonances (Goldreich & Tremaine 1980; Bae & Zhu 2018). The density waves inside (outside) the orbital radius of the planet exert positive (negative) torque, which is called the Lindblad torque. Previous studies have shown that the total Lindblad torque is negative for typical disk parameters, which results in an inward migration (Ward 1997; Tanaka et al. 2002). If the planet mass is large enough (up to Jupiter mass), however, the planet can carve a gap near its orbit and receive reduced torque (Lin & Papaloizou 1979; Papaloizou & Lin 1984).

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Crida et al. 2006; Yun et al. 2019). This type of migration is termed type II, which occurs more slowly compared to type I migration.

Spiral waves are not the only source of gravitational torque: the material in the corotation region can also exchange angular momentum with a planet during its horseshoe turn (Ward 1991). This torque is referred to as the corotation torque, and depends on the radial gradients of vortensity and entropy in the disk (Baruteau & Masset 2008; Paardekooper & Papaloizou 2008). If the gradients are steep enough, the corotation torque can halt or even reverse the inward migration driven by the Lindblad torque (Paardekooper & Papaloizou 2009), although it may vanish after a few horseshoe libration periods, due to the process known as corotation torque saturation (Masset 2001; Ogilvie & Lubow 2003). This is because the corotation region is filled with various fluid elements with different libration periods, leading to phase mixing that tends to flatten the vortensity and entropy gradients. Diffusive processes, such as viscous dissipation and radiative transfer, can possibly prevent corotation torque saturation (Masset 2001).

The migration rate is also an important factor for the corotation torque. While the material crossing the co-orbital region of a migrating planet accelerates the ongoing migration, the material trapped inside the horseshoe region slows down the migration as it is comoving with the planet. When the difference, called the co-orbital mass deficit, between the mass flowing across the co-orbital region and the mass trapped inside the horseshoe region becomes comparable to or larger than the planet mass, the migration is referred to as type III. Type III migration is important for intermediate-mass (Saturn-sized) planets that can open a partial gap (Masset & Papaloizou 2003) or in disks with a steep surface density gradient (Pepliński et al. 2008).
In a disk with a vortensity gradient, the migration of a planet develops a vortensity contrast between the co-orbital region and the background disk. For migrating low-mass planets initiated by the Lindblad torque, this vortensity contrast is equivalent to the co-orbital mass deficit in type III migration (Paardekooper 2014). The corotation torque arising from the vortensity contrast is called the dynamical corotation torque. It can be directed either radially inward or outward, depending on the background vortensity gradient, affecting the type I migration of low-mass planets.

While the aforementioned theoretical studies are useful for understanding the physics behind planet migration and predicting the total torque (Paardekooper & Papaloizou 2009; Paardekooper et al. 2010), they have been limited to two-dimensional (2D) disks with simplified thermodynamics, using a locally isothermal or adiabatic equation of state. However, real protoplanetary disks are vertically and thermally stratified, with hotter surface layers when stellar irradiation dominates the temperature structure (Chiang & Goldreich 1997; Law et al. 2021), and they are subject to radiative diffusion. Finite disk thicknesses and diffusive processes can not only weaken planet–disk interactions (Tanaka et al. 2002; Miranda & Rafikov 2020), but can also introduce intrinsically three-dimensional (3D) processes, such as buoyancy resonance (Zhu et al. 2012), which is absent in 2D disks.

Similar to the Lindblad resonance, buoyancy resonance occurs in regions where the vertical oscillation frequency of a fluid element is equal to integral multiples of $\Omega - \Omega_p$, where $\Omega$ and $\Omega_p$ stand for the angular frequency of the gas and the planet, respectively. Since the oscillation frequency varies with height, and $\Omega$ is a function of radius, the density ridges induced by buoyancy resonance take the form of tilted planes (Zhu et al. 2012, 2015) and can exert torque comparable to the Lindblad torque for low-mass planets (Zhu et al. 2012; Lubow & Zhu 2014). More recently, McNally et al. (2020) have found that the buoyancy resonance in a 3D adiabatic disk can change the sign of the dynamical corotation torque compared to the 2D case of Paardekooper (2014), accelerating the inward migration.

However, all of the 3D studies mentioned above still consider a simple isothermal or adiabatic equation of state, so that they have been unable to capture the effects of radiative diffusion that can potentially change the behavior of the corotation torque (e.g., Paardekooper & Papaloizou 2008; Kley et al. 2009). For instance, it is well known that buoyancy resonance is dependent quite sensitively on the disk’s structure and thermodynamics (Zhu et al. 2012; Bae et al. 2021). Thermal diffusion is likely to dissipate the propagation of gravity waves (Vadas & Fritts 2005) and inhibit the growth of temperature perturbations (Bae et al. 2021), which tend to suppress buoyancy resonance. In order to properly assess the effect of buoyancy resonance on planet migration, then, it is necessary to run 3D simulations that account for the effect of thermal transport, due to radiative diffusion being included.

In this paper, we investigate the effect of buoyancy resonance on the migration of a low-mass planet embedded in a 3D inviscid disk with radiative diffusion. This work extends the 2D simulations of Paardekooper (2014), by including the vertical dimension, and also the 3D simulations of McNally et al. (2020), by incorporating radiative diffusion. We fix the disk parameters and planet mass, and we consider four models with or without radiative diffusion and with or without planet migration. By comparing the total torque exerted on the planet from each model, we quantify the effect of the radiative diffusion on the buoyancy resonance and the dynamical corotation torque.

To realize the thermal transport inside a disk, we adopt the flux-limited diffusion (FLD) method (Levermore & Pomraning 1981), which calculates the radiation flux based on the opacity and the radiative energy gradient. The FLD method has been used in various studies, including planet accretion (Ayliffe & Bate 2009), inner disk structure (Flock et al. 2016), and planet migration (Kley et al. 2009; Lega et al. 2014; Benítez-Llambay et al. 2015) in 3D disks. For example, previous studies have shown that the inclusion of radiative diffusion induces thermal perturbations, referred to as “cold fingers,” in the vicinity of the planet, acting as a source of additional torque (Kley et al. 2009; Lega et al. 2014). However, these studies considered viscous radiative disks, and did not focus on the buoyancy resonance that might have been suppressed by both viscosity and radiative diffusion. The present work is complementary to these studies, in that the former help us to understand how the buoyancy resonance in an inviscid disk is affected by the radiation transport.

This paper is organized as follows. In Section 2, we describe our numerical methods and model parameters. In Section 3, we compare the torques on fixed or moving planets in purely adiabatic and radiative disks. Then, we explore how the radiative diffusion changes the torques that are exerted on the planets. In Section 4, we discuss our results in terms of the radiative diffusion timescale, and present the results of planet migration in a polytropic disk where buoyancy resonance is absent. Finally, we conclude in Section 5.

## 2. Methods

### 2.1. Basic Equations

We model a protoplanetary disk as a 3D, non-self-gravitating, unmagnetized, inviscid, gaseous disk around a central star with mass $M_*=1M_\odot$. The disk has an embedded planet with mass $M_\text{p}$ initially at radius $r_\text{p}$. The planet is either forced to rotate at the fixed angular frequency $\Omega_\text{b}=(GM_*/r_\text{p}^3)^{1/2}$ or allowed to migrate radially inward, due to instantaneous gravitational torque. We choose spherical polar coordinates ($r$, $\theta$, $\phi$) in the frame rotating at $\Omega_\text{b}$. The equations of hydrodynamics that we solve read as:

\begin{align}
\frac{\partial p}{\partial t} + \nabla \cdot (\rho v) &= 0, \\
\frac{\partial \rho}{\partial t} + (v \cdot \nabla)v &= -\frac{\nabla \rho}{\rho} - \nabla \Phi - 2\Omega_\text{b} \times v + \Omega_\text{b}^2 R, \\
\frac{\partial e}{\partial t} + \nabla \cdot (ev) &= -p \nabla \cdot v + Q_{\text{rad}}.
\end{align}

Here, $\rho$ is the gas density, $t$ is time, $v=(v_r, v_\theta, v_\phi)$ is the velocity in the rotating frame, $P=(\gamma-1)e$ is the gas pressure with adiabatic index $\gamma=1.4$, $R$ is the cylindrical radius, and $e$ is the internal energy per unit volume. In Equation (2), $\Phi=\Phi_\text{a}+\Phi_\text{p}$ is the total gravitational potential, consisting of two parts: $\Phi_\text{a} = -GM_\ast/r$ is from the central star and $\Phi_\text{p}$ is due to the planet. We shall specify $\Phi_\text{p}$ in Section 2.3.

In Equation (3), $Q_{\text{rad}}$ is the heating rate per unit volume due to radiative diffusion. For an adiabatic disk, we take $Q_{\text{rad}}=0$ and consider only the compressional heating ($-p\nabla \cdot v$). For a radiative disk, we follow the two-temperature approach (e.g.,...
and the Rosseland mean opacity

\[ Q_{\text{rad}} = \rho c_\text{k} (c E_R - 4\sigma T^4), \]

where \( c_\text{k} \) is the Planck mean opacity, \( c \) is the speed of light, \( E_R \) is the local radiative energy density, \( \sigma \) is the Stefan–Boltzmann constant, and \( T \) is the gas temperature. Under the two-temperature FLD approximation, \( E_R \) evolves as

\[ \frac{\partial E_R}{\partial t} + \nabla \cdot \mathbf{F} = -Q_{\text{rad}}, \]

where \( \mathbf{F} \) is the radiative flux given by

\[ \mathbf{F} = -\lambda_{\text{lim}} \frac{c}{\rho c_\text{k}} \nabla E_R, \]

with the flux limiter \( \lambda_{\text{lim}} \) and the Rosseland mean opacity \( c_\text{k} \). Although \( c_\text{k} \) and \( c_\text{k} \) are different from each other in reality, taking them as equal is a first-order approximation (Bitsch et al. 2013): we adopt the opacity law \( \kappa = \kappa_0 T^\alpha \), with the coefficient \( \kappa_0 \) and the power indices \( a \) and \( b \), as given in the form given in Equation (9) of Kley (1989). We have tested the radiative diffusion module described above on various problems, including oscillatory migration with accretion heating in a 3D disk with uniform opacity (Chrenko & Lambrechts 2019), as presented in the Appendix, and confirmed that our implementation is accurate and reliable.

### 2.2. Method

We integrate the basic equations, using the modified FARGO3D code, in spherical polar coordinates (Masset 2000; Benítez-Llambay & Masset 2016). Our computational domain extends from \( \phi_{\text{min}} = 0.8 \) au to \( \phi_{\text{max}} = 4 \) au in the radial direction, from \( \phi_{\text{min}} = -\pi \) to \( \phi_{\text{max}} = \pi \) in the azimuthal direction, and from \( \phi_{\text{min}} = \pi/2 - \arctan(3H/r) \) to \( \phi_{\text{max}} = \pi/2 \) in the polar (or vertical) direction, where \( H \) is the disk scale height. We set up a nonuniform logistically spaced grid, with 805 cells in the radial direction, 3141 cells in the azimuthal direction, and 75 cells in the polar direction. The resulting grid spacing is \( \Delta r \approx 0.004 \) au at \( r = r_p \), \( \Delta \phi \approx 0.002 \) rad, and \( \Delta \theta \approx 0.002 \) rad, corresponding to 25 zones per scale height, i.e., \( H/(r \Delta \theta) = 25 \).

We impose periodic boundary conditions at the azimuthal boundaries (\( \phi = \pm \pi \)), while adopting reflection boundary conditions at the midplane (\( \theta = \pi/2 \)), as we are only modeling the upper hemisphere of the disk. At the upper polar boundary, we extrapolate \( \rho \) and \( v_r \) from the active to the ghost zones, using the initial disk profile (see Section 2.3), while adopting the continuous boundary condition for \( v_r \) and the reflection boundary condition for \( v_\phi \). For the internal energy density \( e \), we fix the temperature at the upper boundary to the initial profile throughout the simulation, assuming that the disk is constantly irradiated by the central star. At the radial edges, we apply the same boundary conditions for \( \rho \) and \( v_\phi \) as for the upper polar boundary, the reflecting boundary condition for \( v_r \), and the continuous boundary conditions for \( v_\phi \) and \( E_R \). Additionally, we impose wave-damping zones at the radial boundaries to prevent wave reflection (de Val-Borro et al. 2006; McNally et al. 2019).

### 2.3. Model Parameters

Following McNally et al. (2020), we initially consider a disk with temperature \( T = 306.6(\rho/1 \text{ au})^{-1} \), surface density \( \Sigma = 3.8 \times 10^{-4}(\rho/1 \text{ au})^{-0.5} \), \( \text{au}^{-2} \), and mean molecular weight \( \mu = 2.3 \text{ g mol}^{-1} \). Assuming that the disk is initially isothermal in the vertical direction and has a constant aspect ratio \( h \approx H/r = 0.05 \), the corresponding density profile is

\[ \rho(r, \theta) = \rho_0 (\sin \theta)^{-2.5 + h^2} \left( \frac{r}{1 \text{ au}} \right)^{-1.5}, \]

where \( \rho_0 = 3.032 \times 10^{-3} \text{ M}_\odot \text{ au}^{-3} \) is the equatorial density at \( r = 1 \) au (Masset & Benítez-Llambay 2016). We initially take \( v_r = v_\theta = 0 \), and set \( v_\phi \) to the values that ensure an exact balance between the centrifugal force and the pressure gradient force.

We consider a planet with mass \( M_p = 6.67 \text{ M}_\oplus \), corresponding to the mass ratio \( q \equiv M_p/M_d = 2 \times 10^{-5} \), and place it at \( r_p = 2 \) au initially. Similar to McNally et al. (2020), we treat the planet as a uniform-density sphere with radius \( r = 0.1 H(r_p) \). The gravitational potential of the planet is then

\[ \Phi_p = \begin{cases} -GM_p/d, & \text{for } d > \epsilon, \\ -GM_p(3e^2 - d^2)/(2d^3), & \text{for } d \leq \epsilon, \end{cases} \]

where \( d \equiv |r - r_p| \), with \( r_p(t) \) being the position vector of the planet at time \( t \).

To avoid transients in the flows caused by the sudden introduction of the planet, we slowly introduce the planet at \( r_p(0) = (r_p, \pi/2, 0) \), and increase its mass linearly with time from zero to \( M_p \) over 2000 \( \text{orb} \), where \( \text{orb} \equiv 2\pi/\Omega_0 = 2.83 \text{ yr} \) is the orbital time of the planet. For models without planet migration, we keep the planet position unchanged for the next 300 \( \text{orb} \). For models with planet migration included, however, the planet is allowed to move, as a result of the instantaneous gravitational torque that it receives. We note that the planet is subject to the combined gravitational potentials of the central star and the disk, while the disk is governed by stellar potential alone. The mismatch between the potentials of the planet and the disk can shift the resonances, overestimating the torque on the planet (Baruteau & Masset 2008). In order to alleviate this issue, we adopt the workaround given in FARGO3D, which considers only the nonaxisymmetric part \( \delta \Phi \equiv \rho - \langle \rho \rangle \) of the disk density when calculating the torque on the planet. Here, the angle brackets denote the azimuthal average, i.e., \( \langle \rho \rangle \equiv \int \rho d\phi/(2\pi) \).

In addition, the regions inside the Hill sphere of the planet are poorly resolved and often exhibit temporal fluctuations in the gas density. To remove the noisy contributions of the Hill sphere, we calculate the torque as

\[ \Gamma = GM_p \int_{-\infty}^{\infty} \left\{ \delta \Phi(r) \xi(d) \left( \frac{r}{|r - r_p|}\right)^2 \sin \theta dr d\theta d\phi, \right. \]

where \( \xi \) is the tapering function, defined as

\[ \xi(d) = \begin{cases} 0, & \text{for } d/r_H < 1/2, \\ \sin^2(d/r_H - 1/2), & \text{for } 1/2 \leq d/r_H \leq 1, \\ 1, & \text{otherwise}, \end{cases} \]

where \( r_H \equiv (q/3)^{1/3} r_p \) is the Hill radius (McNally et al. 2020).

For a radiative disk, we initialize the radiative energy density as \( E_R = 4\sigma T^4/c \). This makes the disk slightly out of
equilibrium. We thus relax the initial disk described above for $50 \, t_{\text{orb}}$ in the $r$–$\theta$ plane. The resulting equilibrium configuration differs from the initial one by less than 3% in $\rho$ and $T$.

3. Results

In this section, we will first show that the results of the adiabatic models agree with those from McNally et al. (2020), in terms of the torque histories. We will then present the results of the radiative models.

3.1. Adiabatic Disk

Figure 1 plots the torque exerted on the fixed planet (orange) and on the moving planet (green) embedded in the adiabatic disk: the thin and thick lines correspond to the instantaneous and time-averaged values over 50 $t_{\text{orb}}$, respectively. The torque is normalized by $\Gamma_0 \equiv (q/h)^2 \Sigma_p r_p^2 \Omega_p^2$, where $\Sigma_p$ is the disk surface density at the planet position (Tanaka et al. 2002). While the instantaneous torque exhibits rapid fluctuations, due to the vortices generated in the corotation region, the time-averaged torque undergoes small-amplitude oscillations and slowly converges to the quasi-steady values $\Gamma/\Gamma_0 \approx -2.01$ for the fixed planet and $\Gamma/\Gamma_0 \approx -2.99$ for the moving planet.

For comparison, Figure 1 also plots the 2D Lindblad torque $\Gamma/\Gamma_0 \approx -2.96$ (Paardekooper & Papaloizou 2008; dashed line) as well as the Lindblad torque combined with the linear corotation torque $\Gamma/\Gamma_0 \approx -2.42$ (Paardekooper et al. 2010; dotted–dashed line), assuming that the disk is infinitesimally thin. It is interesting to note that the torque on the fixed planet in our 3D simulation is close to the Lindblad torque plus the linear corotation torque in the corresponding 2D disk. Although this appears to contradict the previous prediction, that the corotation torque is saturated in the 2D inviscid disk, the dynamics in the 3D disk may differ significantly from those in the 2D counterpart, changing the behavior of the corotation torque (see also Fung et al. 2015; Jiménez & Masset 2017).

A migrating planet additionally receives the dynamical corotation torque $\Gamma_{\text{hs}}$. Paardekooper (2014) has shown that $\Gamma_{\text{hs}}$ in a 2D thin disk amounts to

$$\Gamma_{\text{hs}} = 2\pi \left[1 - \frac{\omega_0(r_p)}{\omega_c} \right] \Sigma_p r_p^2 \frac{d\rho}{dt},$$

(11)

where $\omega_0(r_p)$ is the vorticity of the unperturbed disk at the planet position, and $\omega_c$ and $x_s$ are the characteristic vortensity and the half-width of the horseshoe region, respectively.\(^6\) Since the vortensity of our initial disk decreases radially outward as $\approx r^{-1}$, $\Gamma_{\text{hs}}$ would be positive for a planet migrating inward (i.e., $d\rho/dt < 0$), if $\omega_0(r_p) > \omega_c$. However, Figure 1 shows that the torque on the inwardly migrating planet in the 3D disk is more negative than that on the fixed planet. This implies that the dynamical corotation torque in the 3D disk is negative.

McNally et al. (2020) argued that the negative dynamical corotation torque inside the horseshoe region is caused by buoyancy resonance, which is absent in 2D disks. In a 3D disk, internal gravity waves or buoyancy waves are characterized by the Brunt–Väisälä frequency,

$$N(z) = \sqrt{\frac{g_c}{\gamma}} \frac{\partial}{\partial z} \ln \left(\frac{\rho}{\rho_0}\right),$$

(12)

with the vertical gravity $g_c = -d\Phi_{\phi}/dz$. The buoyancy waves experience the gravitational force of the embedded planet as they propagate vertically, and grow in amplitudes at the positions where $N(z) = \pm m \, (\Omega_p - \Omega)$, where the integer $m$ denotes the azimuthal wavenumber. This buoyancy resonance is analogous to the Lindblad resonance that excites and amplifies density waves at the locations where the epicycle frequency is equal to $\pm m \, (\Omega_p - \Omega)$ (Zhu et al. 2012). For waves with the azimuthal wavelength $\lambda = 2\pi/m$, the resonance

\(^6\) In a 2D thin disk rotating at angular frequency $\Omega_{\text{pl}}$, $\omega = |\nabla \times \mathbf{v} + 2\Omega_{\text{pl}}/\Sigma|$, where $\mathbf{v}$ is the velocity in the rotating frame.
The position can be calculated as a function of \( r \) and \( z \). The line of constant phase \( 2n\pi \) (for the integer \( n \)) then occurs at \( \phi = n\lambda \), or

\[
\phi = \frac{\pm 2n\pi}{(1 - 1/\gamma)^{1/2}} \left( \frac{\Omega_p - \Omega}{\Omega_k(R)} \right) \left( 1 + \frac{\dot{\rho}}{\rho} \right)^{1/2},
\]

where \( R = (r^2 + z^2)^{1/2} \) (Zhu et al. 2015).

Figure 2(a) plots the vertical velocity perturbations \( v_z \) created by the buoyancy resonance in the \( r-\phi \) plane averaged over \( t = 200-500 \ t_{\text{orb}} \) at \( z = 2H \) obtained from the fixed-planet adiabatic disk model. Equation (13) is also plotted as dashed lines for \( n = 1, \ldots, 5 \), which well trace the resonance positions in the simulation. Panels (b) and (c) of Figure 2 plot the distributions of \( v_z \) along the azimuthal direction at \( z/H = 1, 2 \) and \( r/r_p = 0.95, 1.05 \), showing that the buoyancy resonance is stronger in the regions closer to the planet and at higher-altitude regions. Note that the strongest perturbations lying close to the \( \phi = 0 \) line seen in Figure 2 are not due to buoyancy resonance, but are parts of the spirals produced by the Lindblad resonance.

Figure 2 gives the impression that the buoyancy resonance at \( |z|/H = 2 \) is rather weak inside the horseshoe regions bounded by \( |r - r_p| = 0.02r_p \). To explore the vertical dependence of the buoyancy resonance, Figure 3 plots the velocity perturbations in the \( r-z \) plane from the adiabatic fixed-planet model. Note that the regions with strong \( |v_z| \) move toward the planet as \( |z| \) decreases, affecting the corotation regions whose boundaries are drawn by the vertical dotted lines. We also note that the predicted resonance positions do not match well with the simulation results in low-\( |z| \) regions (see also Zhu et al. 2012, 2015; McNally et al. 2020). This may be because the Brunt–Väisälä frequency becomes vanishingly small as \( |z| \to 0 \), in which case the resonance positions may be compromised by sound waves and/or epicycle motions.

### 3.2. Radiative Disk

Now we present the results of the simulations with the radiative disk. The temporal changes of the torques on the fixed and moving planets are plotted in Figure 1 as the red and purple lines, respectively. Again, light lines correspond to instantaneous torques, while thick lines give the values averaged over \( 50 \ t_{\text{orb}} \). The total torques at the end of the simulation are \( \Gamma/\Gamma_0 \approx -1.93 \) on the fixed planet and \( \Gamma/\Gamma_0 \approx -1.82 \) on the moving planet.

Let us first focus on the torque acting on the fixed planet in the radiative disk. Lega et al. (2014) have shown that in addition to the Lindblad and corotation torques, radiative diffusion introduces thermal perturbations in the vicinity of the planet, which are known as “cold fingers.” In the absence of radiative diffusion, the gas on horseshoe orbits is heated by compression as it approaches the planet, and cools down by rarefaction as it moves away from the planet. The radiative diffusion takes away the internal energy from the compressed region, which in turn results in the region being further compressed compared to the adiabatic case. This leaves elongated structures, namely cold fingers, near the planet, after the U-turns of the streamlines. The cold fingers are apparent in Figure 4, which plots the difference of the gas density at \( z = 0 \)
buoyancy resonance compared to the adiabatic case shown in Figure 2.

Figure 6 compares the azimuthally averaged torque density \( \langle d^2 \Gamma / (r^2 \sin \theta dr d\theta) \rangle \) between the adiabatic and radiative disks. The torque density map is dominated by two lobes, corresponding to positive (at \( r < r_p \)) and negative (at \( r > r_p \)) Lindblad torques. The corrugations of the isotorque contours reflect the effects of buoyancy resonance (McNally et al. 2020). Note that one of the corrugations resides inside the corotation region at \( z \approx 0.07 \) au, indicating that the buoyancy resonance affects the corotation torque exerted on the planet. Weaker corrugations in the radiative disk suggest that the radiative diffusion suppresses buoyancy resonance. In addition, the corotation torque that is saturated in the adiabatic disk becomes unsaturated in the radiative disk (e.g., Baruteau & Masset 2008; Masset & Casoli 2010).

That the radiative diffusion weakens the buoyancy resonance implies that vertical gas motions in the radiative disk are significantly reduced compared to those in the adiabatic disk. In this situation, one can expect the dynamical corotation torque on a migrating planet in the radiative disk to be driven by the vortensity gradient, as in 2D disks (Paardekooper 2014; McNally et al. 2020). Assuming that Equation (11) is responsible for the difference in the torques on moving and fixed planets in the radiative disk, one can calculate the effective characteristic vortensity \( \omega_{c,\text{eff}} \). Figure 7 plots \( \omega_{c,\text{eff}} \) relative to \( \omega_0(r_p,0) \), the vortensity at the initial planet position in the unperturbed disk, as a function of time. Note that \( \omega_{c,\text{eff}} \) is within 3% of \( \omega_0(r_p,0) \) throughout the simulation, which is consistent with Paardekooper (2014), who argued that \( \omega_c = \omega_0(r_p,0) \) when the corotation region migrates with a planet in an inviscid 2D disk. This supports the understanding that the positive dynamical corotation torque in our radiative disk is most likely caused by the vortensity gradient of the disk.

4. Discussion

We first discuss the effect of radiative diffusion on buoyancy resonance. If a disk is optically thin, the gas will cool down by emitting thermal photons that escape the disk easily. If a disk is instead optically thick, as in our models, thermal photons are absorbed and emitted multiple times before escaping the disk. In the latter case, the thermal transport timescale is comparable to the diffusion timescale in the vertical direction:

\[
\tau_d = \frac{H^2}{D} = \frac{3c_V\kappa\rho^2H^3}{16\sigma T^3},
\]

where \( D = 16\sigma T^3/(3c_V\kappa\rho^2) \) is the diffusion coefficient and \( c_V \) is the specific heat of the gas (Malygin et al. 2017; Bae et al. 2021). If \( \tau_d \) is shorter than the buoyancy timescale \( \tau_b \equiv 1/N(z) \), one can expect the buoyancy resonance to be erased by radiative diffusion. Since \( \tau_d \propto \rho^2 \) in our vertically isothermal disks decrease faster than \( \tau_b \propto z^{-1}(1 + z^2/R^2)^{3/2} \) with increasing height, this occurs in the regions with high \( |z| \). The critical thermal diffusion coefficient is given by

\[
D_{\text{crit}} = \left( \frac{\gamma - 1}{\gamma} \right)^{1/2} \Omega \nu \left( 1 + \frac{z^2}{R^2} \right)^{-3/2},
\]

above which the buoyancy resonance is suppressed: \( D_{\text{crit}} \sim 8.5 \times 10^{16} \) cm\(^2\) s\(^{-1}\) at \( r = r_p \) and \( z = H \).
Figure 8 plots the distribution of $t_d/t_b$ in the $r$–$z$ plane of our model disk. Clearly, $t_d/t_b \lesssim 1$ at $z/H \gtrsim 2$. This suggests that the suppression of the buoyancy resonance by radiative diffusion is considerable in a thin layer with $z \gtrsim 2H$. At $z/H = 1$, however, $t_d/t_b \sim 220$, meaning that the radiative diffusion is too slow to affect the buoyancy resonance. This is entirely consistent with Figure 5, in that the radiative diffusion weakens the buoyancy resonance more strongly at higher $|z|$: only the primary mode
with $n = 1$ is visible at $z/H = 2$, while modes with higher $n$ are erased almost completely.

In Section 3.2, we argue that the radiative diffusion negates the negative dynamical corotation torque arising from buoyancy resonance. To see whether the negative dynamical corotation torque is really due to buoyancy resonance, we construct a vertically polytropic disk with $N(z) = 0$ (e.g., Nelson et al. 2013), and run simulations with a fixed or moving planet. We take an adiabatic equation of state and do not consider radiative diffusion.

Figure 9 plots the vertical velocity perturbations at $z/H = 2$ from the fixed-planet model in the polytropic disk at $t = 500 \tau_{\text{orb}}$. Compared to Figure 2, the polytropic disk does not possess structure relating to buoyancy resonance. Figure 10 plots the temporal variations of the torques on the fixed and moving planets in the polytropic disk. The fact that the torque on the fixed planet is more negative than that on the moving planet indicates that the dynamical corotation torque is positive. This is similar to the behavior in the radiative disk, but is opposite to the case with the adiabatic disk shown in Figure 1.

McNally et al. (2020) have shown that the buoyancy resonance in a 3D adiabatic disk provides a negative dynamical corotation torque, resulting in the fast inward migration of a planet. However, the radiative diffusion significantly reduces the negative dynamical corotation torque, by weakening the buoyancy resonance. Figure 11 compares the temporal changes of the semimajor axis $a_p$ of a planet migrating due to the torque that it receives in a radiative or adiabatic disk. The planet released at $t = 200 \tau_{\text{orb}}$ moves radially inward by $\sim 0.1 a_p$ and $\sim 0.175 a_p$ over the subsequent $300 \tau_{\text{orb}}$ in the radiative and adiabatic disks, respectively, showing that radiative diffusion slows down the inward migration by 57%. This suggests that the inclusion of radiative diffusion helps to alleviate the issue of rapid migration in adiabatic disks, as reported by McNally et al. (2020).

Finally, we remark on a few caveats to our simulations. First, our model disks have a surface density that decreases monotonically with radius in a power-law fashion ($\Sigma \propto r^{-1/2}$). This precludes the possibility of a protoplanet trap being created when the surface density has a radial jump (Masset et al. 2006). Even with a radial jump in the surface density profile, this would be unlikely to have a dramatic impact on the planet migration in our radiative disk. This is because a protoplanet trap generally results from the dependence of the corotation torque on the local surface density gradient (Masset et al. 2006; Baillié et al. 2016), while the corotation torque is saturated in our inviscid radiative disk. Nonetheless, the surface density profile can still affect the migration of a migrating planet via the dynamical corotation torque, which depends on the vortensity gradient in the initial disk.

Second, while our models incorporate radiative diffusion in protoplanetary disks, they are still missing a few crucial factors that could make the disks more realistic. The buoyancy resonance is dependent on the Brunt–Väisälä frequency $N(z)$, which is determined by the vertical disk structure. While we do not consider irradiation from the central star (Bitsch et al. 2013), this would primarily heat the optically thin high-$|z|$
regions, modifying $N(z)$ and changing the strength of the buoyancy resonance there (Bae et al. 2021). Our current models also do not consider the accretion luminosity of an embedded planet. It is well known that by heating the gas close to the planet, the accretion luminosity gives rise to a torque called heating torque (Benítez-Llambay et al. 2015). In addition, the density and temperature fluctuations due to the accretion luminosity would alter the behavior of the buoyancy resonance, which could affect the dynamical corotation torque. Moreover, the disk material near the planet could lead to a flow instability from the accretion luminosity were the disk to have a nonuniform opacity, which could potentially affect the buoyancy resonance as well (Chrenko & Lambrechts 2019). Additionally, the disk in our simulation only considers radiative cooling as a cooling process, but infrequent collisions between gas molecules and dust grains in the surface layers could dominate the thermodynamics of a real disk (Bae et al. 2021). When infrequent gas–dust collisions play a role, the buoyancy resonances will become strong again. To assess the effect of the buoyancy resonance on the planet migration in more realistic situations, then, one would have to include stellar irradiation, the accretion luminosity, and the effects of infrequent gas–dust collisions in the thermal evolution of protoplanetary disks.

5. Conclusion

Dynamical corotation torque arises from the deformation of horseshoe orbits along with the vortensity gradient in the background disk. While previous 2D studies of planet migration have predicted that the dynamical corotation torque would counteract the inward migration (Paardekooper 2014; McNally et al. 2019), the recent 3D simulations of McNally et al. (2020) have shown that the buoyancy resonance in adiabatic disks can alter the vortensity inside the libration region and change the sign of the dynamical corotation torque for earth-mass planets, causing the planets to migrate even faster. To study how buoyancy resonance would behave in a more realistic radiative disk, in this paper we include radiative diffusion in an otherwise adiabatic disk, and investigate its effect on the buoyancy resonance and dynamical corotation torque. We adopt the FLD approximation, to realize radiation transfer, and we compare the torques from simulations of adiabatic and radiative disks with fixed or moving planets. The main results of this work can be summarized as follows.

1. In 3D adiabatic disks, the dynamical corotation torque is negative, consistent with the results of McNally et al. (2020).
2. The radiative diffusion suppresses the buoyancy resonance, making the dynamical corotation torque positive. The suppression of the buoyancy resonance is stronger in higher-altitude regions, since the diffusion timescale there is shorter than the buoyancy timescale.
3. In a vertically polytropic disk, in which buoyancy resonance is completely absent, the dynamical corotation torque is slightly positive, as well. This confirms that the negative dynamical corotation torque in the 3D adiabatic disk is indeed due to buoyancy resonance.
4. The radiative diffusion leads to the formation of cold fingers, consistent with the results of Lega et al. (2014), although their contribution to the total torque is minimal, because of the tapering function that is applied to the torque calculation in our simulations.

The radiative diffusion can alleviate the issue of rapid migration caused by the negative dynamical corotation torque seen in the adiabatic disks of McNally et al. (2020). This highlights the significance of radiative diffusion on buoyancy resonance or planet–disk interactions more generally. Future studies that include stellar irradiation and accretion heating will help to fully assess the importance of buoyancy resonance on type I migration.

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Appendix

Test of the Radiative Diffusion Module

We implement the two-temperature FLD method for radiative diffusion in FARGO3D, as described in Section 2. To test our implementation, we consider the migration of an accreting protoplanet, as studied by Chrenko & Lambrechts (2019). To make the situation identical to that in Chrenko & Lambrechts (2019), we include the viscous diffusion term with kinematic viscosity $\nu = 10^{15}$ cm$^2$ s$^{-1}$ as well as the accretion heating term in this test. The disk is initialized with the surface density $\Sigma = 484(r/1 \text{ au})^{-1/2} \text{ g cm}^{-2}$, with a constant aspect ratio $h = 0.05$ across the disk. The disk is radially extended from $r_{\text{min}} = 3.12 \text{ au}$ to $r_{\text{max}} = 7.28 \text{ au}$, and vertically from $\phi_{\text{min}} = 83^\circ$ to $\phi_{\text{max}} = 90^\circ$. The disk is resolved by $512 \times 64 \times 1382$ evenly spaced cells. We follow Bell & Lin (1994) in calculating the nonuniform opacity that depends on the local gas density and temperature.

Initially, a planet with mass $M_p = 3M_{\oplus}$ is forced to move on a circular orbit with an orbital radius of $a_p = 5.2 \text{ au}$ for the first 30 orbits, without any accretion or accretion heating. During this time, the disk adjusts itself to the gravitational potential of the planet, which is smoothed by a tapering cubic spline function, with the smoothing length being equal to the half-Hill radius. At $t = 30 \tau_{\text{orb}}$ the planet is allowed to accrete and emit related heat, whose luminosity is given by Equation (7) of Chrenko & Lambrechts (2019), with a mass-doubling timescale of $\tau = 100 \text{ kyr}$.

Figure 12 plots the temporal evolution of the total torque $\Gamma$. The mean torque for $20 \tau_{\text{orb}} < t < 30 \tau_{\text{orb}}$ is $\Gamma = -1.60 \times 10^{-6} \rho^2 \Omega^2$, but undergoes strong oscillations, which are caused by 3D distortions of the streamlines in the vicinity of the planet. The oscillation has a period of $1.9 \tau_{\text{orb}}$ and an amplitude of $1.24 \times 10^{-4} \rho^2 \Omega^2$, entirely consistent with the results of Chrenko & Lambrechts (2019). This proves that our radiative diffusion module reproduces the oscillatory accretion torque in a disk with nonuniform opacity.
Figure 12. Temporal variations of the total torque $\Gamma$ on an accreting protoplanet with mass $M_p = 3M_{\oplus}$. The accretion and related heating are turned on at $t = 30 \, t_{\text{orb}}$, after which $\Gamma$ exhibits oscillations with a period of $\sim 1.9 \, t_{\text{orb}}$ and amplitudes of $\approx 1.24 \times 10^{-5} \Omega_0^2 M_0^4$, in good agreement with the results of Chrenko & Lambrechts (2019).

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