GAMMA-RAY EMISSION FROM THE VELA PULSAR MODELED WITH THE ANNULAR GAP AND THE CORE GAP

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Received 2010 July 9; accepted 2011 January 29; published 2011 March 16

ABSTRACT

The Vela pulsar represents a distinct group of $\gamma$-ray pulsars. \textit{Fermi} $\gamma$-ray observations reveal that it has two sharp peaks (P1 and P2) in the light curve, with a phase separation of 0.42 and a third peak (P3) in the bridge. The location and intensity of P3 are energy dependent. We use the three-dimensional magnetospheric model for the annular and core gaps to simulate the $\gamma$-ray light curves and the phase-averaged and phase-resolved spectra. We found that the acceleration electric field along a field line in the annular gap region decreases with height. Emission at the high-energy GeV band originates from the synchro-curvature radiation (mainly curvature radiation) of accelerated primary particles, while the synchrotron radiation from secondary particles contributes somewhat to the low-energy $\gamma$-ray band (0.1–0.3 GeV). The $\gamma$-ray light curve peaks P1 and P2 are generated in the annular gap region near the altitude of null charge surface, whereas P3 and the bridge emission are generated in the core gap region. The intensity and location of P3 at different energy bands depend on the emission altitudes. The radio emission from the Vela pulsar should be generated in a high-altitude narrow region of the annular gap, which leads to a radio phase lag of $\sim 0.13$ prior to the first $\gamma$-ray peak.

Key words: gamma rays: stars – pulsars: general – pulsars: individual (PSR J0835−4510) – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

The Vela pulsar is the brightest point source in the $\gamma$-ray sky. The Vela pulsar at a distance of $d = 287_{-11}^{+15}$ pc (Dodson et al. 2003) has a spin period of $P = 89.3$ ms, characteristic age $\tau_c = 11$ kyr, magnetic field $B = 3.38 \times 10^{12}$ G, and rotational energy-loss rate $\dot{E} = 6.9 \times 10^{36}$ erg s$^{-1}$ (Manchester et al. 2005). It radiates multi-waveband pulsed emission from radio to $\gamma$-ray, which gives considerable insight into magnetospheric activities. High-energy $\gamma$-ray emission takes away a significant fraction of the spin-down luminosity (Thompson et al. 1999; Thompson 2001). The pulsed $\gamma$-ray emission from the Vela pulsar has been detected by many instruments, e.g., SAS 2 (Thompson et al. 1975), COS B (Grenier et al. 1988), the Energetic Gamma Ray Experiment Telescope (Kanbach et al. 1994; Fierro et al. 1998), Astro-rivelatore Gamma a Immagini LEggero (AGILE; Pellizzoni et al. 2009), and Fermi (Abdo et al. 2009, 2010b). The $\gamma$-ray profile has two main sharp peaks (P1 and P2) and a third peak (P3) in the bridge. The location and intensity of P3 as well as the peak ratio (P1/P2) vary with energy (Abdo et al. 2010b, 2010c). Because of the large $\dot{E}$, the Vela pulsar has a strong wind nebulae, from which the unpulsed $\gamma$-ray photons were detected by AGILE (Pellizzoni et al. 2010) and Fermi (Abdo et al. 2010a).

Theories for non-thermal high-energy emission of pulsars are significantly constrained by sensitive $\gamma$-ray observations by the \textit{Fermi} telescope. Four physical or geometrical magnetospheric models have previously been proposed to explain pulsed $\gamma$-ray emission of pulsars: the polar cap model (Daugherty & Harding 1994, 1996) in which the emission is generated near the neutron star surface; the outer gap model (Cheng et al. 1986a, 1986b; Romani & Yadigaroglu 1995; Zhang & Cheng 1997; Cheng et al. 2000; Zhang et al. 2004, 2007; Hirotani 2008; Tang et al. 2008; Lin & Zhang 2009) in which the emission is generated near the light cylinder; the two-pole caustic, or slot gap, model (Dyks & Rudak 2003; Muslimov & Harding 2003, 2004; Harding et al. 2008) in which the emission is generated along the last open field lines; and the annular gap model (Qiao et al. 2004a, 2004b, 2007; Du et al. 2010) in which the emission is generated near the null charge surface. The distinguishing features of these models are different acceleration regions for primary particles and possible mechanisms to radiate high-energy photons. Romani & Yadigaroglu (1995) modeled the $\gamma$-ray and radio light curves for the Vela pulsar with a larger viewing angle ($\zeta \sim 79^\circ$). In their outer gap model, the two $\gamma$-ray peaks are generated from the outer gap of one pole, whereas the radio emission is radiated from the other pole. However, the correlation of high-energy X-ray emission and the radio pulse shown by Lommen et al. (2007) is not consistent with this picture. Dyks & Rudak (2003) used the two-pole caustic model to simulate the $\gamma$-ray light curve for the Vela pulsar, which was further revised by Yu et al. (2009) and Fang & Zhang (2010) to explain the details of \textit{Fermi} GeV light curves. A bump appears in the bridge in the model for a large inclination angle, but the width and location of P3 were not yet well modeled.

In this paper, we focus on the $\gamma$-ray light curves at different bands and spectra of the Vela pulsar. In Section 2, we introduce the annular and core gaps and calculate the acceleration electric field in the annular gap. In Section 3, we model the multi-band light curves using the annular gap model together with a core gap. We identify the radio emission region and explain the radio lag prior to the first $\gamma$-ray peak. To model the Vela pulsar spectrum, we also calculate the $\gamma$-ray phase-averaged and phase-resolved spectra of both synchro-curvature radiation from the primary particles and synchrotron radiation from the secondary particles. In Section 4, we present conclusions and discussions.
2. THE ANNULAR GAP AND THE CORE GAP

2.1. Formation of the Annular Gap and the Core Gap

The open field line region of the pulsar magnetosphere can be divided into two parts by the critical field lines (see Figure 1). The core region near the magnetic axis is defined by the critical field lines. The annular region is located between the critical field lines and the last open field lines. For an anti-parallel rotator, the radius of the core gap ($r_{core}$) and the full polar cap region ($r_p$) are $r_{core} = (2/3)^{3/2} R (\Omega R/c)^{1/2}$ and $r_p = R (\Omega R/c)^{1/2}$, respectively (Ruderman & Sutherland 1975), where $R$ is the neutron star radius and $\Omega$ is the angular velocity ($\Omega = 2\pi P / P$ is the pulsar spin period). The radius difference of the annular polar region therefore is $r_{ann} = r_p - r_{core} = 0.26 R (\Omega R/c)^{1/2}$. It is larger for pulsars with smaller spin periods.

The annular acceleration region is negligible for older long-period pulsars, but very important for pulsars with a small period, e.g., millisecond pulsars and young pulsars. It extends from the pulsar surface to or even beyond the null charge surface (see Figure 1). The annular gap has a sufficient thickness of trans-field lines and a wide altitude range for particle acceleration. In the annular gap model, the high-energy emission is generated in the vicinity of the null charge surface (Du et al. 2010). This leads to a fan-beam $\gamma$-ray emission (Qiao et al. 2007). The radiation from both the core gap and the annular gap can be observed by one observer (Qiao et al. 2004b) if the inclination angle and the viewing angle are suitable.

2.2. Acceleration Electric Field

The charged particles cannot co-rotate with a neutron star near the light cylinder and must escape from the magnetosphere. If particles escape near the light cylinder, they have to be generated and move out from the inner region to the outer region. This dynamic process is always taking place, and a huge acceleration electric field exists in the magnetosphere. To keep the whole system charge-free, the neutron star surface must supply charged particles to the magnetosphere.

The annular and core gaps have particles with opposite sign flowing, which can lead to circuit closure in the whole magnetosphere. The potential along the closed field lines and the critical field lines are different (Xu et al. 2006). The parallel electric fields ($E_\parallel$) in the annular and core gap regions are opposite, as has been discussed by Sturrock (1971). As a result, $E_\parallel$ vanishes at the boundary (i.e., the critical field lines) between the annular and core regions and also along the closed field lines. The positive and negative charges are accelerated from the core and annular regions, respectively.

We now consider a tiny magnetic tube in the annular gap region. We assume that the particles flow out at a radial distance about $r_{out} \sim R_{LC}$ = 4.3 $10^3$ km, and that the charge density of flowing-out particles $\rho_0(r_{out})$ is equal to the local Goldreich–Julian (GJ) charge density $\rho_{GJ}(r_{out})$ (Goldreich & Julian 1969) at a radial distance of $r_{out}$. For any heights $r < r_{out}$, $\rho_0(r) < \rho_{GJ}(r)$. The acceleration electric field therefore exists along the field line and cannot vanish until approaching the height of $r_{out}$.

For a static dipole magnetic field, the field components can be described as $B_x = \frac{\mu_0 \cos \theta}{r^2} \hat{n}$, and $B_y = \frac{\mu_0 \sin \theta}{r^2} \hat{n}$. Here $\theta$ is the zenith angle in the magnetic field coordinate, and $B_0$ is the surface magnetic field. Thus, the magnetic field at height $r$ is $B(r) = \frac{\mu_0 R^3 \sqrt{3 \cos^2 \theta - 1}}{2 r^2}$. In the co-rotating frame, Poisson’s equation is

$$\nabla \cdot E = 4\pi (\rho_0 - \rho_{GJ}) \tag{1}$$

Because of the conservation laws of the particle number and magnetic flux in the magnetic flux tube, the difference between the flowing charge density and local GJ charge density at the radius $r$ can be written as

$$\rho_0(r) - \rho_{GJ}(r) = \frac{\Omega B(r)}{2\pi c} (\cos \zeta_{out} - \cos \zeta) \tag{2}$$

where $\Omega = 2\pi P$ is the angular velocity, $P$ is the rotation period, and $\zeta$ and $\zeta_{out}$ are the angles between the rotational axis and the $B$ field direction at $r$ and $r_{out}$, respectively. Wang et al. (2006) found

$$\cos \zeta = \cos \alpha \cos \theta_{\mu} - \sin \alpha \sin \theta_{\mu} \cos \psi \tag{3}$$

where $\psi$ and $\theta_{\mu}$ are the azimuthal angle and the tangent angle (half beam angle) in the magnetic field coordinate, respectively. Combining Equations (1)–(3), we obtain

$$\nabla \cdot E = -\frac{\Omega B_0 R^3}{c r^3} \sqrt{3 \cos^2 \theta + 1} (\cos \zeta_{out} - \cos \zeta) \tag{4}$$

Substituting $\cot \theta_{\mu} = \frac{2\cos \theta_{\mu} - 1}{3 \cos \theta}$ (Qiao & Lin 1998) and $ds = \sqrt{(r d\theta)^2 + (dr)^2}$ into Equations (3) and (4), we can solve the equation for $\nabla \cdot E$ and calculate the electric field $E_\parallel$ along a magnetic filed line for $\psi = 0^\circ$, as shown in Figure 2 for the Vela pulsar. The electric field is huge in the inner region of annular gap and drops quickly when $r \sim R_{LC}$.

3. MODELING THE FERMI \gamma-RAY PROFILES AND SPECTRA OF THE VELA PULSAR

We reprocessed the Fermi data to obtain the multi-band light curves in the following steps. (1) Limited by the timing solution for the Vela pulsar$^3$ from the Fermi Science Support Center (FSSC), we reprocessed the original data observed from 2008 August 4 to 2009 July 2. (2) We selected photons of 0.1–300 GeV in the “Diffuse” event class within a radius of 2° of the Vela pulsar position (R.A. = 128°55', decl. = −45°75') and with a zenith angle smaller than 105°. (3) As done by Abdo et al. (2009, 2010b, 2010c), we used “fselect” to select photons of energy $E_{G\gamma}$ within an angle of $< \max[1.6, 3 \log_{10}(E_{G\gamma}) / 1.3]$ degrees from the pulsar position. (4) Using the tempo2 (Hobbs

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$^3$ http://fermi.gsfc.nasa.gov/ssc/data/access/lat/ephems/
et al. 2006; Edwards et al. 2006) with the Fermi plug-in, we obtained the rotational phase for each photon. (5) Finally, we obtained the multi-band γ-ray light curves with 256 bins, as presented in Figure 4 (red solid lines). Two sharp peaks have a phase separation of δφ ≈ 0.42. The ratio of P2/P1 increases with energy. A third broad peak appears in the bridge emission. The intensity and phase location of P3 vary with energy.

These observed features challenge all current high-energy emission models. A convincing model with reasonable input parameters for magnetic inclination angle α and viewing angle ζ should produce multi-band light curves of the Vela pulsar and explain the energy-dependent location of P3 as well as the ratio of P2/P1.

3.1. Geometric Modeling the Light Curves

Model parameters for both the annular and core gaps of the Vela pulsar should be adjusted for the particle acceleration regions where γ-ray emission is generated. The framework of the annular gap model as well as the coordinate details have been presented in Du et al. (2010), which can be used for simulation of the multi-band γ-ray light curves of the Vela pulsar. For this paper, we added the simulations for the core gap to explain P3 and bridge emission. We adopted the inclination angle α = 70° and the viewing angle ζ = 64°, which were obtained from the X-ray torus fitting (Ng & Romani 2008). The modeling was done as follows:

1. We first separated the polar cap region into the annular and core gap regions by the critical field line. Then, we used the open volume coordinates (r_{OVC}, ψ_s) to label the open field lines for the annular gap and the core gap, respectively. Here r_{OVC} is the normalized magnetic colatitude and ψ_s is the magnetic azimuth. We defined ψ_s = 0 for the plane of the magnetic axis and the spin axis as shown in Figure 1. For the annular gap, we defined the inner rim r_{OVC,AG} ≡ 0 for the critical field lines and the outer rim r_{OVC,CG} ≡ 1 for the last open field lines. For the core gap, we defined the outer rim r_{OVC,CG} ≡ 1 for the critical field lines and the inner rim r_{OVC,CG} ≡ 0 for the magnetic axis. We also divided both the annular gap (0 \leq r_{OVC,AG} \leq 1) and the core gap (0.1 \leq r_{OVC,CG} \leq 1) into 40 rings for calculation.

2. Rather than following the conventional assumption of the uniform emissivity along an open field line when modeling the light curves (Dyks & Rudak 2003; Harding et al. 2008; Fang & Zhang 2010) for both the annular and core gaps, we assumed that the γ-ray emissivities I(θ_s, ψ_s) along one open field line have a Gaussian distribution, i.e.,

\[ I(\theta_s, \psi_s) = I_0(\theta_p, \psi_s) \exp \left[ \frac{-(C(\theta_s, \psi_s) - C_0(\theta_p, \psi_s))^2}{2\sigma_\Lambda^2} \right]. \]  

where \( C(\theta, \psi_s) = \int_0^{\delta} \sqrt{\sigma^2 + (d\theta/d\psi_s)^2} d\theta \) is the arc length of the emission point on each field line counted from the pulsar center, \( \sigma_\Lambda \) is the arc length to the emission region on each open field line in the annular gap or the core gap in units of \( R_{LC} \), and \( C_0(\theta_p, \psi_s) \) is the arc length for the peak emissivity spot \( P(\theta_p, \psi_s) \) on this open field line. In principle, the peak position \( P(\theta_p, \psi_s) \) is dependent on the acceleration electric field and the emission mechanism. Based on our one-dimensional calculation of the acceleration field (see Figure 2) and later the emissivity (see Figure 8), the peak emission approaches the null charge surface. The height \( r_{p,AG} \) for the emission peak on open field lines can be related to the height of the null charge surface \( r_N(\psi_s) \) by

\[ r_{p,AG}(\psi_s) = \lambda \kappa r_N(\psi_s) + (1 - \lambda)kr_N(0), \]  

where \( \kappa \) is a model parameter for the ratio of heights, and \( \lambda \) is a model parameter describing the deformation of emission location from a circle (see details in Lee et al. 2006). The emission peak position “P” on each field line can be uniquely determined, i.e., \( \theta_p = \arcsin \left( \frac{r_{p,AG}/r_N(\alpha, \psi_s)}{r_N(\alpha, \psi_s)} \right) \), where \( r_N(\alpha, \psi_s) \) is the field line constant of the open field line with \( \psi_s \). Figure 3 shows the variations of \( r_{p,AG}(\psi_s) \) and \( r_N(\psi_s) \) with \( \psi_s \). The minimum is at \( \psi_s = 0^\circ \) near the equator and the maximum at \( \psi_s = \pm180^\circ \) near the rotation axis.

The peak emissivity \( I_p(\theta_p, \psi_s) \) may follow another Gaussian distribution against \( \theta \) for a bunch of open field lines (Cheng et al. 2000; Dyks & Rudak 2003; Fang & Zhang 2010), i.e.,

\[ I_p(\theta_p, \psi_s) = I_0 \exp \left[ -\frac{(\theta_p - \theta_{cp}(\psi_s))^2}{2\sigma_\theta^2} \right], \]  

where \( I_0 \) is a scaled emissivity, \( \sigma_\theta \) is a bunch scale of \( \theta \) (in units of rad) for a set of field lines of the same \( \psi_s \), \( \theta_{cp} \) is used to label a field line in the pulsar annular regions, \( \theta_{cp} = (\theta_N(\psi_s) + \theta_{p,PS})/2 \) (i.e., \( r_{OVC}(\psi_s) = 0.5 \)) is the central field line among those field lines with \( \psi_s \). As seen above, we used two different Gaussian distributions to describe the emissivity on open field lines for the annular and core gaps. The model parameters were independently adjusted to best fit the observed γ-ray light curves. In the core gap, we assumed the height of emission peak
Figure 3. Height of the emission peak $r_p$ of the Vela pulsar in the annular gap model and the height of null charge surface $r_N$, calculated with an inclination angle $\alpha = 70^\circ$, $\kappa = 0.7$, and $\lambda = 0.9$. Note that $r_N$ and $r_p$ are symmetric around the magnetic axis in the magnetic frame. The projected $r_p$ is always within the light cylinder. We define $\psi_s = 0^\circ$ for the meridian between the magnetic axis and the equator in the plane of the spin axis and magnetic axes.

$\mathbf{r}_{p, CG} = \mathbf{r}_{p, AG}$, where $\mathbf{r}$ is a model parameter. We adopted two different $\sigma_{\theta, CG}$ for the core gap because of the different acceleration efficiencies for field lines in the two ranges of $\psi_s$. We will use $\sigma_{\theta, AN}$ for the annular region and $\sigma_{\theta, CG}$ for the core region.

3. To derive the “photon sky-map” in the observer frame, we first calculated the emission direction of each emission spot $n_{\psi_s}$ in the magnetic frame; then used a transformation matrix $T_{\gamma}$ to transform $n_B$ into $n_{\gamma}$ spin in the spin frame; and finally, used an aberration matrix to transform $n_{\gamma}$ to $n_{\text{observer}} = \{n_x, n_y, n_z\}$ in the observer frame. Here $\phi_0 = \arctan(n_y/n_x)$ and $\zeta = \arccos(n_z/\sqrt{n_x^2 + n_y^2 + n_z^2})$ are the rotation phase with respect to the spin rotation axis and the viewing angle for a distant, nonrotating observer. The detailed calculations for the aberration effect can be found in Lee et al. (2010).

4. We added the phase shift $\delta \phi_{\text{ret}}$ caused by the retardation effect, so that the emission phase is $\phi = \phi_0 - \delta \phi_{\text{ret}}$. Here there is no minus sign for $\phi_0$ because of the different coordinate systems between our model and the outer gap model (Romani & Yadigaroglu 1995).

5. The “photon sky-map,” defined by the binned emission intensities on the ($\phi, \zeta$)-plane, can be plotted for 256 bins (see Figure 4). The corresponding light curves cut by a line of sight with a viewing angle $\zeta = 64^\circ$ are therefore finally obtained. For the viewing angle $\zeta = 64^\circ$, any magnetic inclination angles of $\alpha$ between 60$^\circ$ and 75$^\circ$ in the annular gap model can produce light curves with two sharp peaks and a large peak separation (e.g., 0.4–0.5), similar to those observed. The emission from the single pole is favored for the Vela pulsar in our model.

The modeled light curves are presented in Figure 4 (black solid lines), with the model parameters listed in Table 1. Emission of P1 and P2 comes from the annular gap region in the vicinity of the null charge surface, and P3 and bridge emission come from the core gap region. The higher energy P3 emission (>3 GeV) comes from a lower height, whereas the lower energy $\gamma$-ray emission comes from a higher region. In the annular gap region, higher energy emission is mostly generated in a higher region. Nevertheless, the $\gamma$-ray emission heights are above the lower bound of the height determined by $\gamma - B$ absorption (Lee et al. 2010).

The peak emission comes from different field lines and emission heights in the annular gap. The deformation of the radiation beam is related to the high value of geometric factor $\lambda$ as discussed in Du et al. (2010). Owing to the aberration and retardation effects, the enhanced gamma-ray emission in the outer rim of the photon sky-map makes the peak very sharp, especially for P2. For the Vela pulsar, the high inclination angle of about $\alpha = 70^\circ$ is important to getting the observed two sharp peaks with a large separation.

3.2. Radio Lag

With well-coordinated efforts for a pulsar timing program, Abdo et al. (2010c) determined the phase lag between radio emission and $\gamma$-ray light curves. The radio pulse comes earlier by a phase of $\sim 0.13$ (see Figure 5).

Radio emission might be generated in the two locations of a pulsar magnetosphere. One is the traditional low-height polar cap region for long-period ($P \sim 1$ s) pulsars (Ruderman & Sutherland 1975). The other is the outer magnetospheric region with high altitudes near the light cylinder (Manchester 2005). For the polar cap region, the low radio emission height leads to a small beam, which probably does not point to an observer for the Vela pulsar. Ravi et al. (2010) propose that the radio emission from young pulsars is radiated in a high region close to the null-charge surface, i.e., a similar region for $\gamma$-ray emission. This is somewhat similar to our annular gap model, in which the radio emission originates from a higher and narrower region than that of the $\gamma$-ray emission.

The modeled radio and $\gamma$-ray light curves in the two-pole annular gap model are shown in Figure 5. The region for the radio emission is mainly located at a height of $\sim R_{LE}$ on certain filed lines with $\psi_s = -138^\circ$. Our scenario of radio emission for the Vela pulsar is consistent with the narrow stream of hollow-cone-like radio emission (Dyks et al. 2010). According to our model, not all $\gamma$-ray pulsars can be detected in the radio band, and not all radio pulsars can have a $\gamma$-ray beam toward us.

3.3. $\gamma$-ray Spectra for the Vela Pulsar

Abdo et al. (2010b) achieved high quality phase-resolved spectra (P1, P2, and both low and high-energy P3) and the phase-averaged spectrum of the Vela pulsar. The observed
\( \gamma \)-ray emission is believed to originate from the curvature radiation of primary particles (Tang et al. 2008; Harding et al. 2008; Meng et al. 2008). Here, we use the synchro-curvature radiation from primary particles (Zhang & Cheng 1995; Cheng & Zhang 1996; Meng et al. 2008) as well as the synchrotron radiation from secondary particles to calculate the \( \gamma \)-ray phase-averaged and phase-resolved spectra of the Vela pulsar.

We divide the annular gap region into 40 rings and 360 equal intervals in the magnetic azimuth, i.e., in total \( 40 \times 360 \) small magnetic tubes. A small magnetic tube has a small area \( A_0 \) on
the neutron star surface. From Equation (2), the number density of primary particles at height \( r \) is \( n(r) = \frac{Q B(r)}{2 \pi e c} \cos \zeta_{\text{out}} \), where \( c \) is the speed of light, and \( e \) is the electric charge. The cross-section area of the magnetic tube at \( r \) is \( A(r) = B_0 A_0 / B(r) \). Therefore, the flowing particle number at \( r \) in the magnetic tube is

\[
\Delta N(r) = A(r) \Delta s \frac{Q B(r)}{2 \pi e c} \cos \zeta_{\text{out}}, \tag{8}
\]

where \( \Delta s \) is the arc length along the field.

The accelerated particles are assumed to flow along a field line in a quasi-steady state. Using the calculated acceleration electric field shown in Figure 2, we can obtain the Lorentz factor \( \gamma \) of the primary particle from the curvature radiation reaction

\[
\gamma = \left( \frac{3 \rho_r^2 E_\parallel}{2 e} \right)^{1/2} = 2.36 \times 10^7 \rho_r^{0.5} E_\parallel^{0.25} \rho_r^{0.6}, \tag{9}
\]

where \( \rho_r \) is the curvature radius in units of 10^7 cm and \( E_\parallel \) is the acceleration electric field in units of 10^6 V cm^{-1}. The pitch angle \( \beta \) of the primary particles flowing along a magnetic field line is (Meng et al. 2008)

\[
\sin \beta \approx \beta \approx \frac{\eta \gamma m_e c^2}{e B(r) \rho_r} \tag{10}
\]

where \( \eta \lesssim 1, m_e \) is the electron mass, and \( \rho \) is the curvature radius. The characteristic energy \( E_c^{syn-cur} \) of synchro-curvature radiation (Zhang & Cheng 1995; Meng et al. 2008) is given by

\[
E_c^{syn-cur} = \frac{3}{2} \frac{\hbar c \gamma^2}{\rho} \left( \frac{r_B}{\rho} + 1 - \frac{3 \rho}{r_B} \right) \cos^2 \beta + \frac{3 \rho^2}{r_B} \cos^2 \beta + \frac{\rho^2}{r_B} \sin^4 \beta, \tag{11}
\]

where \( r_B = \frac{\gamma m_e c^2}{e B(r)} \) is the cyclotron radius of an electron, and \( \hbar \) is the reduced Planck constant.

The energy spectrum \( dN/d\gamma \) of the accelerated primary particles is unknown. Harding et al. (2008) assumed it follows a broken power-law distribution for pairs with indexes of \(-2.0\) and \(-2.8\) (see their Equation (47)). Here we assume the primary particles in the magnetic tube follow one power law \( dN/d\gamma = N_0 \gamma^{\Gamma} \) with an index of \( \Gamma = -2.4 \). Here, \( N_0 \) can be derived by integration into the equation above using Equations (8) and (9). The \( \gamma \)-ray spectrum emitted by the primary particle can be calculated by (Meng et al. 2008)

\[
F(E_\gamma) = \frac{E_\gamma^2}{2 \pi \Delta \Omega d^2} \frac{dN}{dE_\gamma dt} = \frac{\sqrt{3} \gamma^2}{2 \pi \Delta \Omega d^2} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} dN/d\gamma E_\gamma d\gamma, \tag{12}
\]

where \( \Delta \Omega \) is the solid angle of the \( \gamma \)-ray beam, \( h \) is the Planck constant, \( x = E_\gamma / E_c^{syn-cur} \), and \( G(x) = \int_x^{\infty} K_{5/3}(z) dz, K_{5/3}(z) \) and \( K_{2/3}(x) \) are the modified Bessel function. The order of 5/3 and 2/3, and \( rc \) and \( Q_2 \) are given by

\[
rc = \frac{e^2}{\sqrt{(r_B + \rho) \Omega_0^2 + r_B \omega_B^2}}, \quad \Omega_0 = \frac{c \cos \beta}{\rho}, \quad \omega_B = \frac{e B(r)}{\gamma m_e c}, \quad Q_2 = \frac{1}{r_B} \left( \frac{r_B^2 + r_B \rho - 3 \rho^2}{\rho^3} \cos^2 \beta + \frac{3 \rho^2}{r_B} \cos^2 \beta + \frac{1}{r_B} \sin^4 \beta \right), \tag{13}
\]

respectively.

The secondary particles can be generated with a large multiplicity \( (10^3-10^4) \) via the \( \gamma-B \) process in the lower regions of the annular and core gaps near the neutron star surface. Here, we assume the energy spectrum of secondary particles follows a power law, with an index of \( \Gamma_{\text{sec}} = -2.8 \) and a multiplicity of \( M_{\text{sec}} \sim 1000 \). The pitch angle of pairs increases due to the cyclotron resonant absorption of the low-energy photons (Harding et al. 2008). The mean pitch angle of the secondary particles is about 0.06, adopted from Equation (10) with a slightly large factor \( \eta \gtrsim 1 \) owing to the effect of cyclotron resonant absorption. The synchrotron radiation from secondary particles contributes somewhat to the low-energy \( \gamma \)-ray emission, e.g., \( \lesssim 0.3 \) GeV.

We further checked the optical depth \( \tau_{\gamma-B} \) of the \( \gamma-B \) absorption (Lee et al. 2010)

\[
\tau_{\gamma-B}(r) = \frac{1.55 \times 10^7 r}{E_\gamma} K_{1/3} \left( \frac{2.76 \times 10^6 r_{5/2} \rho_{1/2}}{B_{0,12} R_3 E_\gamma} \right); \tag{13}
\]

here \( E_\gamma \) is in units of MeV and \( B_{0,12} \) is in units of 10^{12} G. We found that the Fermi \( \gamma \)-photons of the Vela pulsar with an
energy of $<50$ GeV always have a $\tau_{\gamma-B} \ll 1$ if the emission height is greater than a few hundred kilometers.

To reduce the computation time, we calculate the synchro-curvature radiation at the “averaged emission-height” for three components, P1, P2, and P3, of the $\gamma$-ray light curve of the Vela pulsar. For P1, the emission height is about $0.62 R_{LC}$ on the field line of a magnetic azimuth $\psi = -110^\circ$; for P2, the emission height is $0.75 R_{LC}$ on the field line of $\psi = 131^\circ$; and for P3, the emission height is about $0.28 R_{LC}$ on the field line of $\psi = -104^\circ$. We compute $E_{||}$ for the three peaks and adjust the minimum and maximum Lorentz factor for primary particles $\gamma_{\text{pri}}^{\text{min}}$ and $\gamma_{\text{pri}}^{\text{max}}$, and secondary particles $\gamma_{\text{2nd}}^{\text{min}}$ and $\gamma_{\text{2nd}}^{\text{max}}$, and the $\gamma$-ray beam angle $\Delta\Omega$ to fit the $\gamma$-ray spectra for the Vela pulsar.

We fit the phase-averaged spectrum and phase-resolved (P1, P2, low-energy P3, and high-energy P3) spectra of the Vela pulsar as shown in Figures 6 and 7. The best-fit parameters for phase-resolved and phase-averaged spectra are similar as expected and are listed in Table 2. The maximum Lorentz factor of primary particles $\gamma_{\text{pri}}^{\text{max}}$ is consistent with that obtained from the curvature radiation balance of the outer magnetosphere models given by Abdo et al. (2010b). The modeled spectra are not sensitive to $\gamma_{\text{pri}}^{\text{min}}$ or $\gamma_{\text{2nd}}^{\text{min}}$, but quite sensitive to $\gamma_{\text{pri}}^{\text{max}}$ which is chosen around the value of the steady Lorentz factor given by Equation (9). The solid angle $\Delta\Omega$ was always, for simplicity, assumed by many authors to be 1. We adjust it as a free parameter around 1 for different phases.

The synchro-curvature radiation from primary particles is the main origin of the observed $\gamma$-ray emission, while the synchrotron radiation from secondary particles can contribute to the lower energy band to improve the fitting. The peak ratios of P1 and P2 shown in Figure 7 are roughly consistent with observations except for the band of 0.3–1.0 GeV (cf. Figure 4).
Table 2

| Peak | $\psi$ (°) | $R_e$ | $r$ | $\gamma_{\text{min}}^{\text{pri}}$ | $\gamma_{\text{max}}^{\text{pri}}$ | $\Delta\Omega$ | $\gamma_{\text{min}}^{\text{2nd}}$ | $\gamma_{\text{max}}^{\text{2nd}}$ |
|------|------------|-------|-----|-------------------------------|-------------------------------|---------------|------------------|------------------|
| P1   | −110       | 1.095 | 0.62| 0.50 × 10^7                  | 1.79 × 10^7                  | 0.11          | 8.45 × 10^7      | 1.31 × 10^6      |
| P2   | 131        | 1.278 | 0.75| 0.30 × 10^7                  | 2.65 × 10^7                  | 1.05          | 6.25 × 10^7      | 1.75 × 10^6      |
| P3   | −104       | 1.122 | 0.28| 0.35 × 10^7                  | 2.40 × 10^7                  | 1.09          | 3.05 × 10^7      | 1.22 × 10^6      |

Notes. $R_e$ is the field line constant in units of $R\text{_{LC}}$, and $r$ is the emission height of a profile component.

Figure 7. Fitting the $\gamma$-ray phase-averaged spectrum for the Vela pulsar. The observed data were taken from Abdo et al. (2010b) and plotted as black points with error-bars. The spectra for three profile components and the total phase-averaged spectrum of the Vela pulsar are modeled from the synchro-curvature radiation from primary particles and synchrotron radiation from secondary particles.

(A color version of this figure is available in the online journal.)

The high-energy P3 is generated in the relatively low height of the core gap, where the particles have higher acceleration efficiency than those for the low-energy P3, which leads to their difference in cutoff energy. The phase-resolved spectra for both high- and low-energy P3 can be explained in the synchro-curvature radiation from primary particles from the core gap, with little contribution from the synchrotron radiation of secondary particles because in general they have small pitch angles with respect to field lines and large curvature radius. However, the synchrotron radiation from secondary particles does contribute to the 0.1–0.3 GeV band for P1 and P2.

In Figure 8, we plot the emission fluxes of P1 and P2 components at 1, 3, and 8 GeV of the Vela pulsar against the emission height, which is not uniform along an open field line. The bump at a low height for high-energy $\gamma$-ray (e.g., $\gtrsim$8 GeV) is due mainly to the small curvature radius and large acceleration electric field there. In Section 3.1, we take a rough Gaussian distribution along the arc (Equation (5)) to describe the emissivity near the peak emission region, which is natural in our annular gap model and independent of the model parameters.

4. DISCUSSIONS AND CONCLUSIONS

The detailed features of $\gamma$-ray pulsed emission of the Vela pulsar observed by Fermi provide a challenge to current emission models for pulsars.

Figure 8. Emission at 1, 3, and 8 GeV for the P1 (top panel) and P2 (bottom panel) is not uniform along open field lines. It varies with height.

(A color version of this figure is available in the online journal.)

The charged particles cannot co-rotate with a neutron star near the light cylinder and must flow out from the magnetosphere. To keep the whole system charge-free, the neutron star surface must have the charged particles flowing into the magnetosphere. We found that the acceleration electric field $E_l$ in a pulsar magnetosphere is strongly correlated with the GJ density $\rho_{\text{GJ}}$ near the light cylinder radius $R\text{_{LC}}$, while $\rho_{\text{GJ}}$ at $(R\text{_{LC}})$ is proportional to the...
local magnetic field $B_{LC}$. The Fermi $\gamma$-ray pulsars can be young pulsars and millisecond pulsars, which have a high $B_{LC}$. This means that the acceleration electric field $E_0$ in a pulsar magnetosphere is related to the observed Fermi $\gamma$-ray emission from pulsars.

To understand the multi-band pulsed $\gamma$-ray emission from pulsars, we considered the magnetic field configuration and two-dimensional global acceleration electric field with proper boundary conditions for the annular and core gaps. We developed the three-dimensional annular gap model combined with a core gap to fit the $\gamma$-ray light curves and spectra. Our results reproduce the main observed features for the Vela pulsar. The emission peaks P1 and P2 originate from the annular gap region, and the P3 and bridge emission come from the core gap region. The location and intensity of P3 are related to emission height in the core region. The higher energy emission ($\sim 3$ GeV) comes from lower regions below the null charge surface, while the emission of energy of less than 3 GeV comes from the region near or above the null charge surface. Radio emission originates from a region higher and narrower than those for the $\gamma$-ray emission, which explains the phase lag of $\sim 0.13$ prior to P1, consistent with the model proposed by Dyks et al. (2010).

Synchro-curvature radiation is an effective mechanism for charged particles to radiate in the generally curved magnetic field lines in pulsar magnetosphere (Zhang & Cheng 1995; Cheng & Zhang 1996). The GeV band emission from pulsars is originated mainly from curvature radiation from primary particles, while synchrotron radiation from secondary particles makes some contributions to the low-energy $\gamma$-ray band (e.g., $0.1-0.3$ GeV). Moreover, contributions of curvature radiation from secondary particles and inverse Compton scattering from both primary particles and secondary particles may be ignored in the $\gamma$-ray band. The synchro-curvature radiation from primary particles and synchrotron radiation from secondary particles are calculated to model the phase-resolved spectra for P1, P2, and P3 of low- and high-energy bands and the total phase-averaged $\gamma$-ray spectrum.

In short, the $\gamma$-ray emission from the Vela pulsar can be well modeled with the annular and core gaps.

The authors are very grateful to the referee and Dr. Wang Wei for helpful comments. Y.D. thanks the COSPAR community for financial support to participate in the 11th COSPAR Capacity-Building Workshop on “Data Analysis of the Fermi Gamma-ray Space Telescope,” held in Bangalore, India, from 2010 February 8 to February 19. He also thanks Professor Biswajit Paul and the Raman Research Institute for their kind help, and Professor D. J. Thompson for fruitful discussions and the tempo2 Fermi plug-in. We also thank the pulsar groups of NAOC and of Peking University for useful conversations. The authors are supported by NSFC (10821061, 10778611, and 10833003) and the Key Grant Project of Chinese Ministry of Education (305001).

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