Invariant Procedure for Error Sensitivity Analysis Applied to Cyclic Creep Modelling

A A Kaygorodtseva$^{1,2}$, K V Zakharchenko$^1$, V I Kapustin$^1$ and A V Shutov$^{1,2,a}$

$^1$ Lavrentyev Institute of Hydrodynamics, Pr. Lavrentyeva 15, 630090, Novosibirsk, Russia
$^2$ Novosibirsk State University, Ul. Pirogova 1, 630090, Novosibirsk, Russia

E-mail: $^a$ alexey.v.shutov@gmail.com

Abstract. Cyclic creep, also known as ratcheting, is a progressive accumulation of inelastic strain under cyclic stress-controlled loading. This mechanical effect is of great importance for numerous applications. In engineering practice, phenomenological models of cyclic creep are calibrated against a limited set of macroscopic test data. Since the testing results are prone to systematic and non-systematic experimental errors, the impact of experimental errors on the quality of simulation has to be analysed. A simple inspection procedure is demonstrated and tested. Based on the Monte Carlo computations, it allows for analysis of error propagation through the simulation cycle. The focus of the paper is on the independence of the procedure from the chosen model parametrization. For demonstration purposes, cyclic creep of VT6 alloy is simulated. The corresponding macroscopic constitutive equations are based on the second Ohno-Wang model, combined with refined rule of isotropic hardening. Two different parametrizations are introduced to show that the procedure predicts the same results for both of them.

1. Introduction

Cyclic creep (ratcheting) is one of the limiting factors for long-term structural safety. Besides, the phenomenon of progressive strain accumulation has to be accounted for while modelling ductile damage accumulation, occurring under non-symmetric stress-controlled loading. Therefore, the corresponding models are used in many advanced constitutive equations (cf. references in [10]).

Since cyclic creep tests are expensive, only a limited set of test data is usually available. Meanwhile, due to increased accuracy requirements, the common trend lies in further refinement of material models, accounting for versatile mechanical phenomena. Typically, such models are calibrated against experimental data. High accuracy comes at a cost of using too many material parameters. Since the test data contain errors, a problem of error propagation through the modelling cycle has to be dealt with. For instance, the experimental errors affect the identified material parameters and thus the large-scale FEM simulations of practical problems. In this paper we implement a protocol of error-sensitivity analysis, proposed in [5]. The basic ingredients of the protocol are the Monte Carlo simulations, efficient optimization algorithms, and a mechanics-based metric in the space of material parameters. For cyclic creep models and available experimental data, the inspection protocol allows us to compute the error-sensitivity measure. If the sensitivity is large, then the parameter identification is ill-defined and the obtained material parameters are unreliable. Due to unacceptable error-sensitivity, such parameters should never be used for simulations of engineering structures [7, 5]. However, if the sensitivity is small, the simulation results are error-resistant.

The protocol of sensitivity analysis, proposed in [5], is based on a specific parametrization of the material model. The main goal of this paper is to demonstrate that, if properly carried out, the error-sensitivity analysis provides results independent of the chosen parametrization.
2. Experimental data for the VT6 alloy

For demonstration purposes we implement experimental data from [4]. The cyclic creep of uniaxial samples made from the titanium alloy VT6 is studied. Within each test, the stress-controlled loading encompasses four stages (figure 1). The cyclic loading stage contains 2400 cycles with a non-constant R-value. We denote the maximum stress amplitude of the third stage by $\sigma_{a,max}$ and the mean stress by $\sigma_m$.

Figure 1. Stress-controlled loading employed for material testing. Reproduced from [4].

Three tests with various loading parameters are considered. The relevant experimental data set is formed by measured maximum and minimum strains at each cycle. Since each test contains 2400 cycles, it provides 4800 real-valued data points. Following [4, 5], tests with $\sigma_m = 420$ MPa, $\sigma_{a, max} = 470$ MPa (figure 2(left)) and $\sigma_m = 635$ MPa, $\sigma_{a, max} = 255$ MPa (figure 2(right)) are implemented for model calibration. The remaining experiment with $\sigma_m = 530$ MPa, $\sigma_{a, max} = 360$ MPa (figure 2(middle)) is employed for model validation.

Figure 2. Testing results for the cyclic creep of the VT6 titanium alloy from [4].

3. Material model

Small strains are assumed here, so that large strain kinematics does not obscure the main points of the research. Due to the specific structure, the model can be extended to large strains using modelling steps reported in [6, 9, 11]. For macroscopic simulations of cyclic creep, a combined isotropic-kinematic hardening model is implemented. The isotropic hardening is modelled by the ansatz from [5]; the kinematic hardening corresponds to the second Ohno-Wang model. To adjust the accuracy and complexity of the modelling, two, three, and four evolving backstresses are implemented.

The total strain $\varepsilon$ is split into the thermal strain $\varepsilon_\theta$ and the mechanical strain $\varepsilon_m$:

$$\varepsilon = \varepsilon_\theta + \varepsilon_m. \quad (1)$$

The thermal part is isotropic: $\varepsilon_\theta = \frac{1}{3} \alpha (\theta - \theta_0) \cdot 1$, where $\theta_0$ is the reference temperature and $\alpha$ is the coefficient of volumetric thermal expansion. The mechanical part is split into the elastic strain $\varepsilon_e$ and the inelastic strain $\varepsilon_i$:

$$\varepsilon_m = \varepsilon_e + \varepsilon_i. \quad (2)$$

We denote by $N_{\text{branches}} = 2, 3, 4$ be the number of branches, introduced to model backstresses. Thus, $N_{\text{branches}}$ equals the number of implemented backstress tensors. Similar to [6], for each backstress we consider dissipative ($\varepsilon_{li}$) and conservative ($\varepsilon_{ie}$) components of the inelastic strain tensor:
\[ \mathbf{e}_l = \mathbf{e}_{tc} + \mathbf{e}_{tr} \quad \text{for all} \quad l = 1, \ldots, N_{\text{branches}} \]  

(3)

In order to make the approach thermodynamically consistent, the specific Helmholtz free energy per unit mass is introduced:

\[ \Psi = \Psi_c(\mathbf{e}_c) + \sum_{i=1}^{N_{\text{branches}}} \Psi_{\text{kin}}(\mathbf{e}_{tc}) + \Psi_{\beta}(\theta), \]  

(4)

\[ \rho \Psi_c(\mathbf{e}_c) = k \left( \mathbf{tr}(\mathbf{e}_c)^2 + \mu \mathbf{e}_c^D : \mathbf{e}_c^D \right), \]  

(5)

\[ \rho \Psi_{\text{kin}}(\mathbf{e}_{tc}) = \frac{c_i}{2} \mathbf{e}_{tc} : \mathbf{e}_{tc}^D, \quad l = 1, \ldots, N_{\text{branches}}, \]  

(6)

\[ \rho \Psi_{\beta}(\theta) = -c_0 \left( \theta \ln \frac{\theta}{\theta_0} - (\theta - \theta_0) \right). \]  

(7)

The material constants \( k > 0 \) and \( \mu > 0 \) stand for the bulk and shear moduli; \( c_i > 0 \) is the pseudo-stiffness, related to backstress \( l \); \( c_{0l} \) is the specific heat capacity per unit mass. Just as in [5], the stress-strain relation is provided by Hooke's law and the backstresses \( \mathbf{X}_l \) (\( l = 1, \ldots, N_{\text{branches}} \)) are evaluated by its analog on the substructural level:

\[ \mathbf{e} = \frac{\partial \Psi}{\partial \mathbf{e}_c} = k \mathbf{tr}(\mathbf{e}_m - \mathbf{e}_c) \mathbf{1} + 2 \mu (\mathbf{e}_m - \mathbf{e}_c)^D, \]  

(8)

\[ \mathbf{X}_l = \frac{\partial \Psi_{\text{kin}}}{\partial \mathbf{e}_{tc}} = c_i (\mathbf{e}_c - \mathbf{e}_{c l})^D, \quad l = 1, \ldots, N_{\text{branches}}, \]  

(9)

Here, \( \mathbf{A}^D = \mathbf{A} - \frac{1}{3} \mathbf{tr}(\mathbf{A}) \mathbf{1} \) is the deviatoric part of a tensor. As typical for models with backstresses, the effective (active) stress tensor \( \mathbf{\sigma}^\text{eff} \) is computed as

\[ \mathbf{\sigma}^\text{eff} = \mathbf{\sigma} - \sum_{l=1}^{N_{\text{branches}}} \mathbf{X}_l. \]  

(10)

By \( K > 0 \) denote material's yield stress, measured in uni-axial tension test. Let \( R \in \mathbb{R} \) be the isotropic hardening parameter, responsible for uniform expansion of the yield surface in the stress space. By \( f \) denote the viscous overstress. It is related to the scalar inelastic strain rate \( \lambda = ||\mathbf{\dot{e}}|| \) through the Perzyna law

\[ f := \left( ||(\mathbf{\sigma}^\text{eff})^D|| - \frac{2}{\sqrt{3}} (K + R) \right), \quad \lambda = \frac{1}{\eta} \frac{f_0}{f_0} \]  

where \( \eta > 0 \) and \( f_0 \) are viscosity parameters; \( f_0 = 1 \) MPa is introduced to obtain a non-dimensional term in the bracket (it is not a material constant).

Recent study [4] has shown that the popular Voce's rule of isotropic hardening fails to describe the cyclic creep of the VT6 alloy. To improve the accuracy of modelling, a new rule of isotropic hardening was suggested in [5]:

\[ R = \gamma s - \beta s_\varepsilon, \quad \text{where} \quad s = \sqrt[3]{\frac{2}{\sqrt{3}}} ||\mathbf{\dot{e}}||, \quad \dot{s}_\varepsilon = \sqrt[3]{\frac{2}{\sqrt{3}}} ||\mathbf{\dot{e}}^D||. \]  

(11)

Here, \( \gamma \) and \( \beta \) are material parameters. According to (12), the isotropic hardening \( R \) is a function of the Odqvist parameter \( s \) and the accumulated total strain \( s_\varepsilon \). This new rule contains the same number of parameters as the classical Voce’s rule. The parameter \( \gamma \) is conservative, since it contributes to increased strength. In initial conditions we assume that the material is isotropic in the as-received state. Thus, all the backstresses are zero at \( t = 0 \) and the corresponding strains are zero as well:

\[ \mathbf{e}_{l|t=0} = \mathbf{e}_{l l|t=0} = \ldots = \mathbf{e}_{4 l|t=1} = 0, \quad s_{l|t=0} = s_{d l|t=0} = 0. \]  

(13)

As typical in metal plasticity, normality rule is implemented to govern the inelastic flow:

\[ \mathbf{\dot{e}}_l = \lambda \left[ (\mathbf{\sigma}^\text{eff})^D \right]^m. \]  

(14)

For the kinematic hardening of second Ohno-Wang type (OW-II) we employ the following evolution equation

\[ \mathbf{\dot{e}}_{l l} = \left( \frac{2}{\sqrt{3}} ||\mathbf{\mathbf{X}}|| \right)^m \left( \mathbf{\dot{e}}_l : \mathbf{\mathbf{X}}_l \right) \mathbf{\mathbf{X}}_l / ||\mathbf{\mathbf{X}}||. \]  

(15)

Note that the parameter \( r_1 \) has a meaning of critical stress. The exponent \( m \) is another material parameter. For simplicity, it is the same for all the branches.

The evolution equation for the temperature is not described here. Thermal conductivity equation and examples of temperature evolution for the considered model can be found in [5]. Since the temperature increase measured in experiment is small (cf. [4]), the dependence of material constants on the
temperature is disregarded here. The proof of thermodynamic consistency of the model is presented in [5].

4. Parameter optimization
Since the strain rate is small in considered tests, we neglect the rate-dependence of the stress response. This justifies the assumption $\eta \to 0$, $m_{\text{perysta}} = 1$. The elastic parameters of VT6 are well-defined: $k = 98,037$ MPa, $\mu = 37,593$ MPa. Moreover, we fix the exponent of the OW-II model, which appears in (15): $m = 3$. Depending on the number of backstresses, the model contains 7, 9, and 11 unknown material constants. To be definite, we discuss here the calibration of the model with four backstresses: $N_{\text{branches}} = 4$. For brevity of notation, we put the unknown parameters into the vector:

$$\vec{p} = (\gamma, \beta, c_1, c_2, c_3, r_1, r_2, r_3, r_4, K).$$

(16)

| $N_{\text{branches}}$ | $\gamma$ [MPa] | $c_1$ [MPa] | $c_2$ [MPa] | $c_3$ [MPa] | $c_4$ [MPa] |
|------------------------|----------------|-------------|-------------|-------------|-------------|
| 2                      | 8.958.5        | 215,357     | 18,442      | -           | -           |
| 3                      | 8.784.8        | 505,988     | 10,913      | 70,794      | -           |
| 4                      | 8.749.9        | 617,033     | 9,115.1     | 161,954     | 44,035      |

| $N_{\text{branches}}$ | $\beta$ [-] | $r_1$ [MPa] | $r_2$ [MPa] | $r_3$ [MPa] | $r_4$ [MPa] | $K$ [MPa] |
|------------------------|-------------|-------------|-------------|-------------|-------------|-----------|
| 2                      | 3.6189      | 101.38      | 39,056      | -           | -           | 757.14    |
| 3                      | 3.6194      | 99,554      | 27,837      | 58,396      | -           | 712.70    |
| 4                      | 3.6210      | 71,046      | 24,572      | 58,998      | 44,196      | 699.89    |

Table 1. Identified material parameters.

The vector $\vec{p}$ does not include the viscosity $\eta$, since rate-independent stress response is assumed in the current study: $\eta \to 0$. Similar to [4] and [5], the experiments with $\sigma_m = 420$ MPa and $\sigma_m = 635$ MPa are implemented. Let $\overline{Ex^p} \in \mathbb{R}^{360}$ be the vector of the experimental data. Employing the OW-II model, the stress-controlled cyclic loading is simulated. The model response vector $\overline{Mod}(\vec{p})$ contains simulated minimum and maximum axial strains, stored in the same order as in $\overline{Ex^p}$. According to the standard identification procedure [1] we introduce the target function $\Phi(\vec{p})$; it reflects the deviation of the model predictions $\overline{Mod}(\vec{p})$ from $\overline{Ex^p}$:

$$\Phi(\vec{p}) = \left(\overline{Ex^p} - \overline{Mod}(\vec{p})\right) \cdot \left(\overline{Ex^p} - \overline{Mod}(\vec{p})\right).$$

(17)

The target function (17) is relatively simple, since it treats all the experimental data as equally important. However, in some cases more stable identification results can be obtained when working with weighting matrices [1, 7]. The required parameter vector $\hat{\vec{p}}$ is a minimizer of $\Phi(\vec{p})$. A stable nested identification procedure is described in [4] and [5]. The core of the initial identification procedure is the gradient-free Nelder-Mead method. After that, the identified vector of material constants is used as initial approximation for the Levenberg-Marquardt optimization. This special treatment is needed to ensure that the gradient of $\Phi$ is as close to zero as possible. Zero gradient is vitally important for the error-sensitivity analysis protocol, which is presented in the following. The optimized parameters $\hat{\vec{p}}$ are listed in table 1. The corresponding simulation results are shown in figure 3 together with the experimental data. Recall that two tests serve the parameter identification, and the remaining test is needed for model validation.
5. Protocol of error-sensitivity analysis

Here we use the method to estimate the sensitivity of identified parameters to experimental errors. The original ideas were discussed in [2, 3]. The improvement of the method based on mechanics-based metric was considered in [7, 8, 5].

5.1. Stochastic model of noise

Typically, experimental data contain errors; we assume that the errors are stochastic and additive [1, 2, 7]. One of ingredients of the error-sensitivity protocol is the stochastic model of the experimental noise. Following [5] we replace the vector of given experimental data $\tilde{\mathbf{E}}_x$ with the noisy data $\tilde{\mathbf{E}}_{x,y}$:

$$\tilde{\mathbf{E}}_{x,y} = \mathbf{E}_x + \sum_{k=1}^{20} \sigma_k \cdot \mathbf{R}_{\text{Mod}'}(t_i), \quad \mathbf{R}_{\text{Mod}'}(t) = \sin\left(\frac{k \pi}{2} \right),$$

$$\sigma_k \in \mathcal{N}(0, \sigma^2).$$

In this model we take $\sigma_k \in \mathcal{N}(0, \sigma^2)$ as independent normally distributed random values with zero mean and the standard deviation $\sigma$. The systematic error modes are denoted as $\mathbf{R}_{\text{Mod}'}(t)$.

5.2. Mechanics-based metric for models of cyclic creep

To quantify the impact of experimental errors on identified parameters, a special mechanics-based metric was considered in [5]. It is used to measure the distance between two vectors of material constants. This distance function is based on simulations of a praxis-relevant stress-controlled loading, each simulation is carried out with its own vector of parameters. The distance between two vectors $\mathbf{x}(1)$ and $\mathbf{x}(2)$ is the maximum difference between the simulated strain histories:

$$\text{dist}(\mathbf{x}(1), \mathbf{x}(2)) = \max_{t \in [0, T_{\text{metric}}]} \left| \sigma_{11}(t, \mathbf{x}(1)) - \sigma_{11}(t, \mathbf{x}(2)) \right|.$$  

In this definition, $T_{\text{metric}}$ is the duration of the local stress-controlled loading; $\sigma_{11}$ is the theoretical prediction of the axial strain, evolving during the cyclic creep. Following [5], a stress-controlled swelling loading with asymmetry value $R_{\text{asym}} = \sigma_{\text{min}}/\sigma_{\text{max}} = 0$ and gradually increasing $\sigma_{\text{max}}$ is used to compute the distance.

In previous publications, a similar metric was defined for plasticity models [7]. However, the plasticity-related metric is computed as the difference between two simulated trajectories in the stress space. In cyclic-creep problems, the ratcheting model works opposite to plasticity models, providing the strain history as a function of the stress. This explains why the metric (19) is non-dimensional.

The mechanics-based metric (19) is superior to the conventional Euclidean metric for the following reasons:

- the distance remains the same upon reparametization of the ratcheting model;
- the distance disregards changes in unimportant material parameters;
- there is no issue of dimensions; no transition to non-dimensional parameters is needed;
- application-related metric can be created by applying relevant stress-controlled loading.

5.3. Quantified error-sensitivity

Recall that $\mathbf{x}$ is the vector of identified material constants; it minimizes the target function $\Phi$ for noise-free data. The error-sensitivity inspection protocol utilizes the quasi Monte Carlo computation (cf. [8]). The number of instances of noisy experimental data is denoted as $N_{\text{noise}}$. Each instance of noise is generated by the stochastic model (18) with $\sigma = 10^{-6}$. For $j$-th instance ($j = 1, 2, \ldots, N_{\text{noise}}$), we identify the vector of material constants $\tilde{\mathbf{x}}(j)$. To speed up the Monte Carlo computations, the vector

![Figure 3. Results of parameter identification and validation for the cyclic loading of VT6 titanium alloy.](image-url)
\( \bar{p}^{(j)} \) is evaluated by a special semi-analytical procedure [7, 8]. This procedure is based on the linearization of the model response near \( \bar{p}^{*} \); the procedure works properly only when the gradient of the target function \( \Phi \) is zero at \( \bar{p}^{*} \).

| \( N_{\text{branches}} \) | \( \gamma \)  | \( \beta \)  | \( c_1 \)  | \( c_2 \)  | \( c_3 \)  | \( c_4 \)  | \( r_1 \)  | \( r_2 \)  | \( r_3 \)  | \( r_4 \)  | \( K \)  |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2               | 0.0192 | 0.0249 | 1.50  | 1.30  | -     | -     | 0.445 | 0.522 | -     | -     | 0.00777 |
| 3               | 0.0208 | 0.0253 | 0.0256 | 3.34  | 4.56  | -     | 0.764 | 1.61  | 1.77  | -     | 0.00872 |
| 4               | 0.0209 | 0.0251 | 0.00239 | 6.98 | 5.83  | 2.15  | 0.507 | 3.58  | 6.26  | 8.90  | 0.0240  |

The stochastic of the parameter cloud in the \( \beta \)-cloud is shown in table 2. This table summarizes the variances of normalized material parameters \( \text{Var}(p_j/p_j^*) \cdot 10^4 \).

Table 2. Variances of normalized material parameters \( \text{Var}(p_j/p_j^*) \cdot 10^4 \).

| \( N_{\text{branches}} \) | \( \gamma \) | \( \beta \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( r_1 \) | \( r_2 \) | \( r_3 \) | \( r_4 \) | \( K \) |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2               | 0.0192 | 0.0249 | 1.50  | 1.30  | -     | -     | 0.445 | 0.522 | -     | -     | 0.00777 |
| 3               | 0.0208 | 0.0253 | 0.0256 | 3.34  | 4.56  | -     | 0.764 | 1.61  | 1.77  | -     | 0.00872 |
| 4               | 0.0209 | 0.0251 | 0.00239 | 6.98 | 5.83  | 2.15  | 0.507 | 3.58  | 6.26  | 8.90  | 0.0240  |

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| \( N_{\text{branches}} \) | \( \gamma \) | \( \beta \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( r_1 \) | \( r_2 \) | \( r_3 \) | \( r_4 \) | \( K \) |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2               | 0.0192 | 0.0249 | 1.50  | 1.30  | -     | -     | 0.445 | 0.522 | -     | -     | 0.00777 |
| 3               | 0.0208 | 0.0253 | 0.0256 | 3.34  | 4.56  | -     | 0.764 | 1.61  | 1.77  | -     | 0.00872 |
| 4               | 0.0209 | 0.0251 | 0.00239 | 6.98 | 5.83  | 2.15  | 0.507 | 3.58  | 6.26  | 8.90  | 0.0240  |

The stochastic of the parameter cloud in the \( \gamma \)-cloud is shown in table 2. This table summarizes the variances of normalized material parameters \( \text{Var}(p_j/p_j^*) \cdot 10^4 \).

6. Invariance of the protocol under model’s reparametrization

To test the invariance of the proposed error-sensitivity inspection protocol, we introduce an alternative parametrization of the material model. Recall that the OW-II model is parametrized by the vector \( \hat{p} = (\gamma, \beta, c_1, c_2, c_3, c_4, r_1, r_2, r_3, r_4, K) \) when \( N_{\text{branches}} = 4 \). The new vector \( \check{y} \in \mathbb{R}^{11} \) of material parameters is given by

\[
\begin{align*}
y_1 &= \gamma/K, \\
y_2 &= \beta/K, \\
y_3 &= r_1/c_1, \\
y_4 &= r_2/c_2, \\
y_5 &= r_3/c_3, \\
y_6 &= r_4/c_4,
\end{align*}
\]

\[
\begin{align*}
y_7 &= r_5, \\
y_8 &= r_6, \\
y_9 &= r_7, \\
y_{10} &= r_8, \\
y_{11} &= K.
\end{align*}
\]

For smaller number of backstresses, the vector \( \check{y} \) has a similar structure. For instance, when \( N_{\text{branches}} = 2 \) we have \( \check{p} = (\gamma, \beta, c_1, c_2, r_1, r_2, K) \) and

\[
\begin{align*}
y_1 &= \gamma/K, \\
y_2 &= \beta/K, \\
y_3 &= r_1/c_1, \\
y_4 &= r_2/c_2, \\
y_5 &= r_3, \\
y_6 &= r_4, \\
y_7 &= K.
\end{align*}
\]

Obviously, there is a one-to-one correspondence between vectors \( \check{p} \) and \( \check{y} \). The transition between \( \check{p} \) and \( \check{y} \) in describing the material properties is called reparametrization. Now, the following fundamental question arises: Is the error-sensitivity analysis presented in previous sections invariant under the reparametrization of the material model?

Let \( \check{y}^* \) be the vector of parameters corresponding to the optimal solution \( \check{p}^* \). In order to check the invariance, we repeat all the steps of the error-sensitivity analysis protocol, described above. Thus, a new cloud of parameters is computed in the \( \check{y} \)-space. Let \( \check{y}^{(j)} \) be the parameter vector, minimizing the new target function \( \Phi_{\text{reparam}}(\check{y}) \), containing \( j \)-th draw of noisy data. The variances of normalized parameters are summarized in table 3. Comparing tables 2 and 3, one can hardly see that they describe the same error-sensitivity.

Table 3. Variances of normalized material parameters \( \text{Var}(y_j/y_j^*) \cdot 10^4 \).

| \( N_{\text{branches}} \) | \( y_1 \) | \( y_2 \) | \( y_3 \) | \( y_4 \) | \( y_5 \) | \( y_6 \) | \( y_7 \) | \( y_8 \) | \( y_9 \) | \( y_{10} \) | \( y_{11} \) |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2               | 0.0252 | 0.0301 | 0.696 | 0.235 | -     | -     | 0.445 | 0.522 | -     | -     | 0.00776 |
| 3               | 0.0786 | 0.0803 | 5.02  | 0.403 | 1.60  | -     | 2.89  | 1.65  | 2.49  | -     | 0.0520  |
| 4               | 0.139  | 0.142  | 20.3  | 0.638 | 12.6  | 3.77  | 23.4  | 3.46  | 21.3  | 9.34  | 0.110  |

To obtain a useful information, the new cloud size is computed now similar to (20)

\[
\text{CloudSize}_{\text{reparam}} = \frac{1}{N_{\text{noise}}} \sum_{j=1}^{N_{\text{noise}}} \text{dist}(\check{y}^*, \check{y}^{(j)}) \quad N_{\text{noise}} = 10,000.
\]
The values of the CloudSize are listed in table 4. The table shows that with increasing number of backstresses, the parameters of the OW-II model exhibit increasing error-sensitivity; this indicates that the model is becoming overparametrized for large $N_{\text{branches}}$ (cf. [5]). For small clouds, the CloudSize does not depend on the chosen parametrization of the model. However, with increasing CloudSize, there is an increasing discrepancy between CloudSize and CloudSize$^{\text{reparam}}$. This discrepancy comes from the nature of the semi-analytical optimization procedure, used for finding $\hat{\tilde{\rho}}^{(j)}$ and from nonlinear relation between $\tilde{\rho}$ and $\tilde{y}$.

| $N_{\text{branches}}$ | $N_{\text{branches}} = 2$ | $N_{\text{branches}} = 3$ | $N_{\text{branches}} = 4$ |
|------------------------|---------------------------|---------------------------|---------------------------|
| CloudSize              | $6.3625 \cdot 10^{-6}$    | $2.8984 \cdot 10^{-4}$    | $7.5676 \cdot 10^{-4}$    |
| CloudSize$^{\text{reparam}}$ | $6.3626 \cdot 10^{-6}$    | $2.4866 \cdot 10^{-4}$    | $7.9584 \cdot 10^{-4}$    |

7. Conclusion

The protocol of error-sensitivity analysis is tested using a concrete set of experimental data for the VT6 alloy and the OW-II model. It is shown that the parameter CloudSize is invariant under reparametrization of the cyclic creep model, if the cloud size is not large. For larger error-sensitivities, some discrepancies may appear due to nonlinear relation between vectors $\tilde{\rho}$ and $\tilde{y}$ and owing to linearization used in the semi-analytical solution procedure.

The invariance of the error-sensitivity protocol indicates that CloudSize provides a reliable insight. This parameter quantifies the error propagation through the modelling cycle and assesses the robustness of a specific parameter identification procedure.

Acknowledgements

The research was supported by the Russian Science Foundation, project number 19-19-00126.

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