Ekpyrotic Reheating and Fate of Inflaton

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It is shown that perturbative reheating can reach a sufficiently high temperature with small or negligible inflaton decay rate provided that the inflaton potential becomes negative after inflation. In our model, inflaton and dark energy field are two independent scalar fields, and, depending on the mass of the inflaton and its coupling to matter fields, there is a possibility that the remaining inflaton after reheating can become a dark matter candidate.

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1. Introduction

Modern cosmology[1, 2] based on the inflation demands three seemingly different types of particles, whose identities are still elusive. Inflaton that drives inflation is one, dark matter (DM) is another, and the third one is to explain the nature of dark energy. There have been attempts to unify these different fields for the sake of simplicity[3, 4, 5]. Some have tried to reduce the number of fields by relating some of them. For example, ref.[6] considers a connection between the inflaton and dark energy. In this paper, we propose a possibility to link the inflaton and DM\(^1\), while introducing a new mechanism of reheating.

This paper is organized as follows. In section 2, the basic structure of the model is described. In section 3, it is shown that the perturbative reheating can reach high enough temperature, which imitates the “ekpyrotic” phase. In section 4, since the entire disappearance of the inflaton may not be needed for the sufficient reheating, the possibility of remaining inflaton as a dark matter candidate is explained. Finally, in section 5, some discussions are provided.

2. The Model

The model we consider has two scalar fields: one is the inflaton that drives inflation, while the other is responsible for later expansion of the universe as dark energy. The total (effective) potential energy takes the form of

\[
V(\phi, \eta) = V_1(\phi) + V_2(\eta) \geq 0, \tag{1}
\]

where \(\phi\) is the inflaton and \(\eta\) is the (late time) dark energy field (DEF). \(\phi\) carries no Standard Model charges and interacts with fermions only in terms of Yukawa couplings of \(\lambda_f \phi \bar{f} f\), while \(\eta\) does not interact even with fermions. We assume that \(V_2(\eta)\) is invariant under \(\eta \to \eta + a\), where \(a\) is an arbitrary constant. Note that \(\phi\) and \(\eta\) do not interact with each other. An important assumption is that \(V_1(\phi)\) becomes negative at some point\(^2\), and \(V_2(\eta)\) becomes dominant at later time to sustain the expansion of the universe. We also assume that \(\eta\) remains massless to avoid any localization of dark energy, while inflaton \(\phi\) could become massive. Since we could actually take \(V_2(\eta)\) to be constant\[^3\,5\] for simplicity, although it can be more complicated, we

\[^1\text{See [7],[8] for different proposals.}\]

\[^2\text{A negative scalar potential is also considered in [9, 10], in which the negative part of the potential is identified as big crunch before the big bang. In our case, however, there is no crunch because of } \eta \text{ such that } V(\phi, \eta) \geq 0 \text{ always.}\]
Choose

\[ V_2(\eta) = -V_1(\phi_{\text{min}}) \]  

such that

\[
V(\phi, \eta) \begin{cases} 
= 0 & \text{if } \phi = \phi_{\text{min}}, \\
> 0 & \text{otherwise}. 
\end{cases}
\]  

This choice can actually avoid fine-tuning because the magnitude of the current cosmological constant becomes equivalent to the vacuum fluctuation of the inflaton around the true vacuum, which can be small enough.

Note that in quantum theories the potential energy cannot be shifted up or down arbitrarily because the minimum value (of the effective potential) has a definite physical meaning, unlike in some classical theories, where only relative values have physical meaning. For decoupled fields, in particular, their ground state values can be in principle separately measurable. Since the inflaton and DEF are two independent fields without a direct coupling, it is legitimate to assume their separate ground state energy levels and even constant parts of their potential energies should not be exchanged. If a different minimum value is chosen for the inflaton potential, the equation of state changes so that it becomes a different physical system.

The ideal (effective) potential for the inflaton could be visualized numerically, for example, as

\[ V_1(\phi) = \frac{1}{e^{a(\phi - \phi_0)} + 1} + b \left( 1 - e^{-c\phi^2} \right) \left( \frac{1}{\phi^2} - \frac{d}{\phi} \right), \]  

where \(a, b, c, d, \phi_0\) are constants, and shown in Fig.1 and Fig.2. This provides a slow-roll potential initially, then becomes steep to change to negative toward a stable vacuum, and, finally, approaches zero asymptotically. The parameter \(\phi_0\) controls the number of e-foldings.
What we further need is $\phi_{\text{min}}$ should be close to $\phi_0$. Note that, in arbitrary units, for $b \ll 1$, i.e. assuming $b$ is small so that the second term is negligible in computing $N_e$, then

$$\phi_0 \simeq \frac{1}{a} \ln \left( \frac{N_e a^2}{8\pi} \right). \tag{5}$$

To get the needed e-foldings $N_e \simeq 60$, for example, for $a = 40$, we need $\phi_0 \simeq 0.21$. To get a reasonable $\phi_{\text{min}}$ close enough to $\phi_0$, we need to adjust $c, d$ properly. The natural unit expression can be obtained by assigning a suitable energy scale to $\phi_0$, etc., but it is not essential for our purpose.

3. “Ekpyrotic” Reheating

The universe cools off during inflation, so it needs to reheat to a sufficiently high temperature in the radiation-dominated era[1]. (See [11] for a recent review.) It is commonly known that the usual perturbative reheating cannot generate a sufficient temperature due to normally small coupling constant between the inflaton and matter fields. To overcome this, the idea of preheating is introduced in [12, 13], but it requires a coupling between the inflaton and another scalar field.\(^3\) However, if one can achieve enough reheating, the preheating is unnecessary and an extra scalar field can be avoided. In this paper, we shall propose a new perturbative reheating mechanism that can generate a sufficiently high reheating temperature.

A sufficiently high reheating temperature requires rapid transfer of the inflaton energy to radiation right after inflation. There are two ways of achieving this: one is to take advantage of the coherent nature of the inflaton (near the new vacuum), another is to take advantage of rapid growth of inflaton kinetic energy when the slow-roll potential turns steep[15]. Here we shall take the latter path, but it is also related to the former implicitly.

Consider the equation of state for the inflaton, $p_\phi = w_\phi \rho_\phi$, where

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V_1(\phi), \tag{6a}$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V_1(\phi), \tag{6b}$$

so that the equation of state parameter $w_\phi$ can be expressed as

$$w_\phi = \frac{\frac{1}{2} \dot{\phi}^2 - V_1(\phi)}{\frac{1}{2} \dot{\phi}^2 + V_1(\phi)}. \tag{7}$$

\(^3\)See [14] for a proposal of preheating with fermionic coupling. However, preheating cannot take place until the inflaton becomes massive.
As the inflaton evolves, $w_\phi$ is not constant but varies. If $V_1(\phi) \geq 0$, $1 \geq w_\phi > -1$ always. However, for $\rho_\phi \geq 0$, if (and only if) $V_1(\phi) < 0$, $w_\phi > 1$ is allowed. (If $\rho_\phi < 0$ for $V_1(\phi) < 0$, $w_\phi < -1$.) Note that, if $V_1(\phi) < 0$, $\rho_\phi$ can be as small as possible. Hence, as the inflaton loses kinetic energy, i.e. $\rho_\phi \to 0$, $w_\phi \gg 1$ can happen.

The possibility of $w_\phi \gg 1$ is the key to successful perturbative reheating in our model. Hence, we call it ekpyrotic reheating, borrowing the word from [10, 19]. In general, the inflaton can lose its energy by decaying into radiation, scattering with radiation, and/or annihilating into radiation. With taking all these processes into account as the source of damping, the evolution equation for the inflaton can be generalized for arbitrary $w_\phi$ as

$$\frac{\dot{\rho}_\phi}{w_\phi + 1} + (3H + \Delta_\phi)\rho_\phi + \Sigma_\phi \rho_\phi^2 = 0. \quad (8)$$

The prefactor $1/(w_\phi + 1)$ is derived from the equation of motion of the inflaton and eq.(7), with the assumption that $\phi$ is not too heavy so that it can be approximately treated as a perfect fluid. $\Delta_\phi$ in the second term represents the contribution from decay and scattering processes, and the dominant contribution is from the former such that

$$\Delta_\phi = \Gamma_\phi + \cdots, \quad (9)$$

where $\Gamma_\phi$ is the decay rate and the detail of the ellipsis for scattering is not important for our purpose. The third term represents the energy loss due to annihilation processes (see eq.(24)) and

$$\Sigma_\phi \equiv \frac{\langle \sigma v \rangle}{E_\phi} \quad (10)$$

with some energy scale parameter $E_\phi$. If $\Delta_\phi \neq 0$, the third term is negligible. However, if $\Delta_\phi = 0$, this should be taken into account. If $\phi$ is massive, $E_\phi = m_\phi$. If $\phi$ is massless, the average value $E_\phi = \rho_\phi(t)/n_\phi(t) \simeq \rho_\phi(t_i)/n_\phi(t_i) \propto T_i$ can be taken. In our model, the inflaton decay rate is solely given by Yukawa couplings of $\lambda_f \phi f \bar{f}$ as

$$\Gamma_\phi = \sum_f \frac{\lambda_f^2}{8\pi} m_\phi \left(1 - \frac{m_f^2}{m_\phi^2}\right)^{3/2}. \quad (11)$$

\footnote{We leave the fermion content generic because we do not yet know the particle content of light degrees of freedom at the energy scale where the reheating takes place. This could be most likely beyond the EW scale. Nevertheless, to be more specific, for our purpose, any isospin singlet fermions with or without U(1) charges, e.g. hypercharges, can satisfy our need. Such fermions can be easily accommodated in models beyond the SM. For example, the right-handed neutrino could be one, then the Yukawa coupling is a Majorana type. Another example can be isospin singlet Dirac fermions with hypercharges without generating anomalies, as long as their masses at the low energy are generated beyond the SM energy scale. These fermions subsequently annihilate into the SM particles via gravitational or U(1) interactions as the Universe evolves.}
Note that this decay process is dominated by the largest Yukawa coupling constant and smallest fermion mass. Typically, if $\phi$ is massive, decay process can occur because $\Gamma_\phi \neq 0$. However, if $\phi$ is massless, decay cannot happen directly and we have to take into account of other processes.

Note that eq.(8) is actually valid for time-dependent $w_\phi$. However, since most of reheating is to take place as the inflaton potential turns negative and $w_\phi$ at its maximum, where $\phi$ does not vary too much, $w_\phi$ will be assumed to be constant to solve eq.(8).

We shall first demonstrate the ekpyrotic reheating in the case of $\Delta_\phi \neq 0$. Since the annihilation term can be ignored for $\Delta_\phi \neq 0$, eq.(8) can be solved to yield

$$\rho_\phi(t) \simeq \rho_\phi(t_i) \left( \frac{a_i}{a} \right)^{3(w_\phi+1)} e^{-(w_\phi+1)\Delta_\phi(t-t_i)},$$

where $a_i \equiv a(t_i)$ and $w_\phi$ is the chosen maximum value. Note that $\rho_\phi > 0$ and this is an approximate solution since we assumed $w_\phi$ to be constant. This shows that $\rho_\phi$ decreases more rapidly if $w_\phi > 0$, compared to the case of $w_\phi = 0$. So, we can already anticipate that this will lead to a higher reheating temperature. The energy density of radiation $\rho_R$ satisfies

$$\dot{\rho}_R + 4H\rho_R = -\dot{\rho}_\phi - 3H(1+w_\phi)\rho_\phi - \dot{\rho}_\eta = (w_\phi+1)\Delta_\phi \rho_\phi - \dot{\rho}_\eta,$$

where we assume $\rho_\eta \ll \rho_\phi, \rho_R$ so that we can ignore $\rho_\eta$ terms in the following. Since $a(t) \sim t^{2/(3(w+1))}$, where $p_{tot} = w\rho_{tot}$ for the universe, such that

$$H \equiv \frac{\dot{a}}{a} = \frac{2}{3(w+1)} \frac{1}{t},$$

eq.(13a) with eq.(13b) leads to

$$0 = \dot{\rho}_R + \left\{ \frac{8}{3(w + 1)} \frac{1}{t} + (w_\phi + 1)\Delta_\phi \right\} \rho_R - \frac{(w_\phi + 1)\Delta_\phi}{6\pi G (w + 1)^2 t^2},$$

Any consistent solution to this equation requires $w = 1/3$ as it should be since the universe is to be dominated by radiation, and the necessary solution is given by

$$\rho_R(t) = \frac{3}{32\pi G t^2} - \rho_\phi(t_i) \frac{t_i^2}{t^2} e^{-(w_\phi+1)\Delta_\phi(t-t_i)}.$$

The first term leads to the maximum temperature for $t_i \sim M_{pl}/\sqrt{\rho_\phi(t_i)}$, which is given by

$$T_{\text{max}} \sim \rho_\phi(t_i)^{1/4}.$$
For the reheating temperature, from eq.(12) we obtain

\[ t \sim \frac{1}{(w_\phi + 1)\Delta_\phi} \tag{18} \]

such that

\[ T_{RH} \sim \sqrt{w_\phi + 1} \sqrt{M_{pl}\Delta_\phi}, \tag{19} \]

which is higher than the reheating temperature for \( w_\phi = 0 \) [1, 11]. Note that \( t \sim 1/\Delta_\phi \) is not proper in our model because it makes the reheating period rather too long for small \( \Delta_\phi \). Thus eq.(18) should be the correct one. Indeed, this reheating temperature can become sufficiently high if \( w_\phi \gg 1 \) even for small \( \Delta_\phi \), hence it justifies the word “ekpyrotic.”

If \( \Delta_\phi = 0 \), the annihilation term can no longer be ignored, then the solution to eq.(8) is given by

\[ \frac{1}{\rho_\phi(t)} \approx \frac{1}{\rho_\phi(t_i)} \left( \frac{t}{t_i} \right)^{2\alpha} + \frac{\alpha(w + 1)\Sigma_\phi}{2\alpha - 1} \left( \frac{t}{t_i} \right)^{2\alpha - 1}, \tag{20} \]

where \( \alpha \equiv (w_\phi + 1)/(w + 1) \) and \( \Sigma_\phi \) is taken as the average value. As \( w_\phi \gg 1 \), i.e. \( \alpha \gg 1 \), \( \rho_\phi(t) \) decreases because \( t/t_i > 1 \). Using eqs.(13a)(13b) for \( \alpha \gg 1 \), in this case, we obtain

\[ \rho_R \approx \frac{3}{8\pi G} H^2 - \rho_\phi \tag{21a} \]

\[ \approx \frac{1}{6\pi G(w + 1)^2 t^2} - \left( \frac{t_i}{t} \right)^{2\alpha} \frac{\rho_\phi(t_i)}{1 + \frac{1}{2}(w + 1)\Sigma_\phi t_i \rho_\phi(t_i)}. \tag{21b} \]

For \( t_i \sim M_{pl}/\sqrt{\rho_\phi(t_i)} \), the maximum temperature is eq.(17) as before. With

\[ \frac{1}{t} \sim \Sigma_\phi \rho_\phi(t_i) \sim n_\phi(t_i)\langle \sigma v \rangle, \tag{22} \]

the reheating temperature is

\[ T_{RH} \sim \left( M_{pl}\frac{1}{t} \right)^{1/2} \sim \sqrt{M_{pl}n_\phi(t_i)\langle \sigma v \rangle}, \tag{23} \]

which can be sufficiently high since \( n_\phi(t_i)\langle \sigma v \rangle \) is large, particularly, for massless inflaton. Note that this is true only if \( w_\phi \gg 1 \), hence ekpyrotic.

The reheating temperature can be constrained by CMB[16, 17, 18]. Ref.[18] claims \( T_{RH} \sim 10^6 \text{ GeV} \). In \( \Gamma_\phi \neq 0 \) case, this can be achieved with Yukawa coupling constant \( \lambda_f \sim 10^{-4} \) for massive inflaton with \( m_\phi \sim 1 \text{ TeV} \) (see eq.(11)) even for \( w_\phi = 0 \). For smaller Yukawa couplings, sufficiently large \( w_\phi \) can achieve the desired reheating temperature even for massless inflaton. This indicates that the reheating temperature given by eq.(19) (or eq.(23)) is actually sufficient.
and preheating is unnecessary in our model. If $\Gamma_\phi \geq H_0 \sim 10^{-26}$ sec$^{-1}$ such that the inflaton decays entirely during reheating, then $\phi$ cannot be DM. However, for $w_\phi \gg 1$, with sufficiently small Yukawa coupling constants, we can achieve $\Gamma_\phi < H_0$ so that there is a possibility that the remaining inflaton can become DM after ekpyrotic reheating.

4. Inflaton as Dark Matter

Let us now consider the case in which the remaining inflaton can become DM after successful reheating. Once $\phi$ becomes massive, $\phi$ no longer behaves like a (uniform) fluid such that $w_\phi = 0$ is possible. In our model, the inflaton DM is basically Yukawa interacting scalar DM. (See [20] for another example.) We assume that Yukawa couplings are flavor conserving so that we do not have to worry about $f \to f\phi$ process for massive $\phi$. Then, the relic density can be computed in terms of the usual Boltzmann eq. for annihilation/creation process[21]

$$\dot{n}_\phi + 3Hn_\phi = -\langle \sigma v \rangle (n_\phi^2 - n_{\text{EQ}}^2).$$

(24)

Let $Y \equiv n_\phi/s_E$ and $x \equiv m_\phi/T$, where $s_E$ is the total entropy density of the universe and $T$ is the photon temperature, then, generalizing ref.[22] for arbitrary $w$, the Boltzmann eq. can be expressed as

$$\frac{dY}{dx} + c_w \frac{Y}{x} = -\frac{\pi}{45G} \frac{g_T^{1/2} m_\phi}{x^2} \langle \sigma v \rangle (Y^2 - Y_{\text{EQ}}^2),$$

(25)

where

$$g_T^{1/2} \equiv \frac{h_{\text{eff}}}{\sqrt{g_{\text{eff}}}} \left(1 + \frac{1}{3} \frac{d \ln h_{\text{eff}}}{d \ln T}\right),$$

(26a)

$$\rho = g_{\text{eff}}(T) \frac{\pi^2}{30} T^4,$$

(26b)

$$s_E = h_{\text{eff}}(T)(1 + w) \frac{\pi^2}{30} T^3,$$

(26c)

and

$$c_w \equiv \frac{4}{w + 1} - 3 = \begin{cases} 0, & \text{for radiation-dominated era}, \\ 1, & \text{for matter-dominated era}. \end{cases}$$

(27)

Note that the prefactor $(1 + w)$ in $s_E$ is due to $s_E \propto (\rho + p)$.

A relativistic derivation of $\langle \sigma v \rangle$ is given in [22], which reads in the lab frame

$$\langle \sigma v \rangle = \frac{2x}{K_2(x)} \int_0^{\infty} d\epsilon \sigma v_{\text{lab}} \sqrt{\epsilon}(1 + 2\epsilon) K_0(2x\sqrt{1 + \epsilon}),$$

(28)
where $\epsilon \equiv \frac{s}{4m^2} - 1$ is the average kinetic energy per mass for the Mandelstam variable $s$, and $K_n(x)$ is the modified Bessel function of the second kind of order $n$. Note that this formula works as long as two incident particles are collinear. If $\phi$ is sufficiently massive as in our case, we can use non-relativistic expansion of $\sigma v_{\text{lab}}$ in powers of $\epsilon$ as

$$
\sigma v_{\text{lab}} = \sum_{n=0} a_n \frac{\epsilon^n}{n!}
$$

such that

$$
\langle \sigma v \rangle = a_0 + \frac{3a_1}{2x} + \mathcal{O}(x^{-2}) \equiv \sum_{n=0} \frac{\langle \sigma v \rangle_n}{x^n}.
$$

For Yukawa coupling $\lambda_f \phi \bar{f} f$, the cross section for $\phi \phi \rightarrow \bar{f} f$ in the CE frame is given by\(^5\)

$$
\sigma_{\phi \phi \rightarrow \bar{f} f} = \frac{\lambda_f^4 p_f}{16 \pi p_i E_i^2} \left[ 1 + \frac{(m^2_{\phi} - 4m^2_f)^2}{m^4_{\phi} - 4m^2_{\phi}m^2_f + 4m^2_fE_i^2} - \frac{2E_i^2 - (m^2_{\phi} - 4m^2_f)}{2p_i p_f} \ln \left( \frac{2E_i^2 - m^2_{\phi} + 2p_ip_f}{2E_i^2 - m^2_{\phi} - 2p_ip_f} \right) \right],
$$

where $p_f \equiv (E_i^2 - m_f^2)^{1/2}$. Expanding this according to eq.(29), we can obtain

$$
a_0 = a_{\text{CE},0} = \frac{\lambda_f^4}{8\pi} \frac{\alpha_f^2(10 - \alpha_f^2)(1 - \alpha_f^2)^{1/2}}{m^2_{\phi}},
$$

where $\alpha_f \equiv m_f/m_{\phi}$. As $\alpha_f \rightarrow 0$, $a_0 \rightarrow 0$ and the next nonvanishing expansion coefficient is

$$
a_1 = a_{\text{CE},1} - a_{\text{CE},0} = \frac{\lambda_f^4}{3\pi m^2_{\phi}}.
$$

If $\phi$ is massless, which is relevant for the reheating process, this cross section develops an infrared singularity, which can be regularized by introducing a minimal cutoff initial energy.

With eq.(30), eq.(25) now reads, in the leading order of $x$,

$$
\frac{dY}{dx} + c_w \frac{Y}{x} = -A_n \frac{Y^2 - Y_{\text{EQ}}^2}{x^{2+n}},
$$

where $Y_{\text{EQ}}$ is to be suppressed by $e^{-x}$, $n$ accounts the leading nonvanishing term, and

$$
A_n \equiv \frac{1}{3} \sqrt{\frac{\pi}{5G}} g_{1/2}^{1/2} m_{\phi} \langle \sigma v \rangle_n.
$$

---

\(^5\)For the purpose of the dark matter relic density, effective Yukawa couplings between the inflaton and low energy fermions can be generated at higher orders after the EW symmetry breaking in terms of, e.g., the gauge interactions between isospin singlet fermions and other SM-like fermions.
Taking average value of $g_F^{1/2}$ for an approximation and integrating from $x_F$ to $x_0$, i.e. from the freeze-out to the present, we can obtain a solution to this equation as

$$\frac{x_0^{n+1}}{Y_0} \simeq \left( \frac{x_F^{n+1}}{Y_F} + \frac{A_n}{1 + n + c_w} \right) \left( \frac{x_0}{x_F} \right)^{1+n+c_w}, \quad (36)$$

where we assume $x_0/x_F = T_F/T_0 \gg 1$.

If the density evolution overlaps from radiation-dominated to matter-dominated era, we should solve eq.(34) consecutively from $x_F \rightarrow x_c \rightarrow x_0$, where $x_c$ is for the cross-over from radiation-dominated to matter-dominated era. Then, the current relic density can be better approximated as

$$\frac{x_0^{n+1}}{Y_0} \simeq \left( \frac{x_F^{n+1}}{Y_F} + \frac{A_n}{n + 1} \right) \left( \frac{x_0}{x_F} \right)^{n+1} \left( \frac{x_0}{x_c} \right), \quad (37)$$

where $x_0/x_c = T_c/T_0 \sim 10^4$.

If $\langle \sigma v \rangle$ is large enough, the freeze-out temperature can be determined by

$$H(t_F) = n_{EQ} \langle \sigma v \rangle, \quad (38)$$

where, for large $x$,

$$n_{EQ} = \frac{g_f T^3}{2\pi^2} x^2 K_2(x) \simeq \frac{g_f T^3}{2\pi^2} \sqrt{\frac{\pi}{2}} x^{3/2} e^{-x}. \quad (39)$$

Then, assuming freeze-out takes place during the radiation-dominated era, the freeze-out value $Y_F$ is given by

$$Y_F = Y_{EQ}(x_F) = \frac{n_{EQ}(x_F)}{s_E(x_F)} \simeq \frac{15}{\sqrt{2\pi}} \frac{\sqrt{g_{\text{eff}}(T_F)}}{h_{\text{eff}}(T_F)} \frac{x_F^{1+n}}{M_{\text{pl}} m_\phi \langle \sigma v \rangle_n}. \quad (40)$$

From eq.(36) and eq.(40), assuming the density has evolved as if the entire period were radiation-dominated era, we can obtain

$$Y_0 = B \frac{x_F^{1+n}}{M_{\text{pl}} m_\phi \langle \sigma v \rangle_n}, \quad (41)$$

where

$$B \equiv \left[ \frac{\sqrt{2\pi}}{15} \frac{h_{\text{eff}}(T_F)}{\sqrt{g_{\text{eff}}(T_F)}} + \frac{\sqrt{\pi} \sqrt{g_{\text{eff}}}}{5 1 + n} \right]^{-1} \quad (42)$$

If we use eq.(37) to compensate the density evolution during the matter-dominated era,

$$Y_0 = B \left( \frac{x_c}{x_0} \right) \frac{x_F^{1+n}}{M_{\text{pl}} m_\phi \langle \sigma v \rangle_n}. \quad (43)$$
Note that $Y_0$ in this approximation is $10^{-4}$ times $Y_0$ of approximation assuming only radiation-domination. Then, with $h_{\text{eff}}(T_0) = 3.909$, $T_0 = 2.725$ K, and $H_0 = 2.133 \times 10^{-42} h$ GeV, $\Omega_{\text{DM}}$ is given by

$$\Omega_{\text{DM}} h^2 = 2.051 \times 10^8 \frac{m_\phi}{\text{GeV}} Y_0. \quad (44)$$

Comparing this to the current measured value $\Omega_{\text{DM}} h^2 = 0.1126 \pm 0.0036[23]$, we can obtain conditions on $m_\phi$ and $\lambda_f$. We will come back to this for a specific case later.

The above works only if the annihilation cross section is large enough to satisfy eq.(38). If it is too small so that eq.(38) cannot be satisfied at all, $n_{\text{EQ}}(x_F)$ should be used to determine the relic density. The freeze-out parameter $x_F$ for $\langle \sigma v \rangle \simeq 0$ can be determined from the following equation[22]:

$$0 \simeq \frac{K_1(x)}{x K_2(x)} - \frac{1}{x} \frac{d \ln h_{\text{eff}}(T)}{d \ln T}. \quad (45)$$

For large $x$, up to $x^{-2}$,

$$\frac{K_1(x)}{K_2(x)} \simeq \left(1 + \frac{3}{8x}\right) \left(1 + \frac{15}{8x} + \frac{105}{128 x^2}\right)^{-1} \simeq 1 - \frac{3}{2x} + \frac{345}{128 x^2}, \quad (46)$$

then eq.(45) leads to an equation for $x_F$

$$x^2 - \frac{3}{2x} + \frac{345}{128} - \xi m_\phi = 0, \quad (47)$$

where

$$\xi \equiv \frac{d \ln h_{\text{eff}}(T)}{dT} \quad (48)$$

can be approximated by taking a suitable numerical value. With eqs.(36)(39) and $c_w = 0$,

$$\Omega_{\text{DM}} h^2 = \frac{8\pi G h^2}{3 H_0^2} m_\phi n_\phi(t_0) = 7.814 \times 10^{44} \frac{m_\phi T_0^3}{(\text{GeV})^4} \frac{h_{\text{eff}}(T_0)}{h_{\text{eff}}(T_F)} 3/2 e^{-x_F}, \quad (49)$$

where $T_0 = 2.35 \times 10^{-13}$ GeV. Using $h_{\text{eff}}(T_0) = 3.909$ and $h_{\text{eff}}(T_F) \simeq 10$, $\xi = 0.83$ GeV$^{-1}$ and $x_F \simeq 30$ for $m_\phi \simeq 1$ TeV.

Note that $\phi \to \bar{f}f$ requires $m_\phi > 2m_f$, i.e. $1/2 > \alpha_f$, while $\phi \phi \to \bar{f}f$ requires $m_\phi > m_f$, i.e. $1 > \alpha_f$. So, there are three possibilities:

$$\alpha_f \geq 1: \quad \sigma = 0, \quad \Gamma_\phi = 0, \quad (50a)$$

$$1 > \alpha_f \geq \frac{1}{2}: \quad \sigma \neq 0, \quad \Gamma_\phi = 0, \quad (50b)$$

$$\frac{1}{2} > \alpha_f: \quad \sigma \neq 0, \quad \Gamma_\phi \neq 0. \quad (50c)$$
Since we expect $m_\phi \gtrsim 1$ TeV, $\alpha_f \geq 1$ for all flavors cannot happen. So, the fate of the inflaton depends on these mass ranges as well as Yukawa coupling constants and there are three different cases:

- **Case I:** The inflaton is not a DM candidate and decays entirely before present day.
- **Case II:** The inflaton is a DM candidate and the relic density is given by eq.(49).
- **Case III:** The inflaton is a DM candidate and the relic density is given by eq.(44).

If there is at least one light fermion flavor with $\lambda_f \gtrsim 10^{-26}$ (i.e. $\Gamma_\phi \gtrsim 10^{-26}$ sec$^{-1}$), it is Case I. If $\lambda_f \lesssim 10^{-26}$ for all flavors, it is Case II. Case III is the case with some flavors with $1 > \alpha_f \geq \frac{1}{2}$ and other flavors with $\frac{1}{2} > \alpha_f$. A good example of Case III is the technicolor (TC) model with Yukawa interacting sterile scalars[24], in which technifermions have $\lambda_{TC} > 10^{-26}$ while quarks and leptons have $\lambda_{Q,L} \lesssim 10^{-26}$. Another possibility of Case III without TC is to allow the fourth generation of suitable mass with $\lambda_4 > 10^{-26}$.

Let us look into the TC case more in detail. For $m_\phi \simeq 1$ TeV, at least the lightest technifermion mass should be between 1 TeV and 500 GeV. Then, the density evolution is dominated by the annihilation process so that we can use eq.(43) to determine the relic density. For example, if we assume there is only one technifermion within the mass range with mass about 600 GeV, then $\lambda_f \simeq 3.4 \times 10^{-2}$ leads to $m_\phi \simeq 1$ TeV with $x_F \simeq 18$. If we use eq.(41) just for radiation-dominated era, we get $\lambda_f \simeq 0.38$ with $x_F \simeq 28$. Since the difference is significant enough, we believe that the relic density is better approximated by eq.(43). This is because a significant part of relic density evolution takes place during the matter-dominated era so that it should not be ignored. If we have more technifermions within the mass range, smaller Yukawa coupling constants can lead to $m_\phi \simeq 1$ TeV.

5. Discussions

In this paper, we have demonstrated that a successful cosmological model could be in principle achieved with just two sterile scalar fields. In cases II and III, the remainder of the inflaton after reheating can become a DM candidate. In this context, it may be possible that massive inflaton can trigger local clustering of matter even during the radiation-dominated era, hence it can become an early seed of structure formation. The other sterile scalar field as DEF behaves more like a free field and is uniformly distributed with its flat potential, whose magnitude is the same as the absolute value of the inflaton potential energy at a new
vacuum after inflation, so that the inflaton’s vacuum fluctuation after reheating can account for the magnitude of dark energy. We leave the computation of the amount of this vacuum fluctuation as a future task. Of course, we can make $\eta$ more dynamical by introducing a nontrivial potential energy and/or by adding sufficiently weak interactions to the inflaton, but due to the observational constraint indicating no dynamics[25], we think it is a reasonable assumption at this moment. However, it will be interesting to work out if any observable dynamics out of $\eta$, which we will also leave as a future work.

It will be also interesting to solve eq.(8) for time-dependent $w_\phi$ for comparison. We leave this also as a future work.

The inflaton potential we have used has an analogous form to the Fermi surface so that it could indicate a more fundamental origin of the potential. This could also explain proper scale for parameters introduced in the potential so that we can describe more detailed behavior of the inflaton. Also, even though we have not used the oscillatory nature of the inflaton field explicitly (eq.(8) is consistent with implicit usage of it[1]), we suspect there might be a connection between our ekpyrotic reheating and the fermionic preheating considered in ref.[14] to a certain level. It will be interesting to see if this is indeed the case.

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