RESONANT ANDREEV SCATTERING
IN PHASE-COHERENT, SUPERCONDUCTING NANOSTRUCTURES.

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ABSTRACT.

Analytic predictions for resonant transport in three generic structures are presented. For a structure comprising a normal (N) contact - normal dot (NDOT) - superconducting (S) contact, we predict that finite voltage, differential conductance resonances are destroyed by the switching on of superconductivity in the S-contact. In the weak coupling limit, the surviving resonances have a double-peaked line-shape. Secondly, we demonstrate that resonant Andreev interferometers can provide galvonometric magnetic flux detectors, with a sensitivity in excess of the flux quantum. Finally, for a superconducting dot (SDOT) connected to normal contacts (N), we show that the onset of superconductivity can increase the sub-gap conductance, in contrast with the usual behaviour of a tunnel junction.

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Recent advances in the fabrication of nanoscale structures have led to increasing interest in transport through resonant tunnel junctions and quantum dots[1,2]. In part this is due to the new physics associated with Coulomb blockade[3] and in part due to growing interest in quantum chaos[4-7]. The bulk of work in this area has focussed on normal structures, but more recently attention has turned to hybrid structures involving a superconducting component. It has been demonstrated experimentally that the energy gap of a superconducting dot is directly observable through Coulomb blockade experiments[8] and theoretical work on incoherent transport through such dots has been carried out[9]. In this Letter, we describe the effect of superconductivity on phase-coherent transport through resonant structures, in the limit that charging effects can be ignored. This limit should be experimentally accessible, because even intimate contact with a superconductor will not broaden states below the gap.

A range of new phenomena and fundamental problems involving coherent transport through resonant superconducting hybrids are expected to manifest themselves in a small number of generic structures. One such example is a “N-NDOT-S” structure comprising one or more normal (N) current carrying leads, in contact with a normal zero dimensional “dot” (NDOT), which in turn makes contact with a superconducting (S) lead. In contrast with the zero voltage limit, where a general multi-channel description of this structure is available[10], there currently exists only a single one-dimensional study of finite voltage, resonant transport[11], in which a $\delta$-function potential well, with a single localized state is introduced into a one-dimensional insulating barrier. A key prediction of this Letter is that finite-voltage conductance resonances are destroyed by the onset of superconductivity in such a structure. This is illustrated in figures 1 and 2, which show for a two-dimensional system, the conductance of a resonant structure plotted against the Fermi energy of the dot. Figures 1a and 2a show the conductance of a N-NDOT-N structure at zero and finite voltages respectively, while figures 1b and 2b show the conductance of the corresponding N-NDOT-S structure.
A second class of structures involves two (or more) separate superconductors S and S′, with respective order parameter phases \( \phi, \phi' \). The resonant energies of a N-SS′ or N-NDOT-SS′ composite will vary periodically with the phase difference \( \phi - \phi' \) [15-26] and in what follows we describe the resonant properties of such structures. A third class is formed when a normal lead makes contact with a superconducting dot (SDOT), which in turn makes contact with a second normal lead. In such structures, we demonstrate that the sub-gap conductance can increase when superconductivity is switched on, in contrast with the conventional behaviour of a N-S tunnel junction.

In what follows, we adopt a multi-channel scattering approach, based on current-voltage relations for phase-coherent scatterers written down in reference [27], which have been extended and re-derived in several papers [28-30]. In the absence of inelastic scattering, dc transport is determined by the quantum mechanical scattering matrix \( s(E, H) \), which yields scattering properties of quasi-particles of energy \( E \), incident on a phase-coherent structure described by a Hamiltonian \( H \). If the structure is connected to external reservoirs by open scattering channels labelled by quantum numbers \( n \), then this has matrix elements of the form \( s_{n,n'}(E,H) \). The squared modulus of \( s_{n,n'}(E,H) \) is the outgoing flux of quasi-particles along channel \( n \), arising from a unit incident flux along channel \( n' \). For channels belonging to current-carrying leads, with quasi-particles labelled by a discrete quantum number \( \alpha \) (\( \alpha = +1 \) for particles, \( -1 \) for holes), it is convenient to write \( n = (i,a,\alpha) \), where \( i \) labels the leads and \( a \) labels all other quantum numbers associated with the channels. Then as shown in [27,28], transport properties are determined by the quantity

\[
P_{i,j}^{\alpha,\beta}(E,H) = \sum_{a,b} |s_{(i,a),(j,b)}^{\alpha,\beta}(E,H)|^2,
\]

which is referred to as either a reflection probability \( i = j \) or a transmission probability \( i \neq j \) from quasi-particles of type \( \beta \) in lead \( j \) to quasi-particles of type \( \alpha \) in lead \( i \). For \( \alpha \neq \beta \), \( P_{i,j}^{\alpha,\beta}(E,H) \) is referred to as an Andreev scattering probability, while for \( \alpha = \beta \),
it is a normal scattering probability.

The starting point for our description of resonant Andreev scattering is the following general formula for the transition probability between two different scattering channels of an open vector space $A$ representing the leads, attached to a closed sub-space $B$ representing the scatterer. For $n \neq n'$, the result, which we derive[31] for normal leads described by a real Hamiltonian, is

$$T_{nn'} = 4 \text{Trace} \left[ \Gamma(n)G_{BB}\Gamma(n')G_{BB}^\dagger \right]$$

(1),

where

$$G_{BB}^{-1} = g_B^{-1} - \sigma' - \sigma + i\Gamma$$

(2)

and the trace is over all internal levels of $B$. In these expressions, $\Gamma(n)$ is a Hermitian matrix of inverse lifetimes, $\Gamma = \sum_n \Gamma(n)$, $\sigma$ and $\sigma'$ are Hermitian self-energy matrices and $g_B$ is the retarded Green's function of sub-space $B$ when $H_1 = 0$. The above result has been cast in a form which resembles the Breit-Wigner formula [12,13], but is very general and makes no assumptions about the presence or otherwise of resonances.

During the past decade, the Breit-Wigner formula has been applied to a variety of problems involving resonant transport in normal-state structures [32-34]. For a normal-metallic conductor, under resonant conditions, where the level spacing is much greater than the broadening, Büttiker has presented a multi-channel derivation of the Breit-Wigner formula through a single resonant level [35]. This limit is recovered from equation (1) by restricting the trace to a single level. In what follows, we shall encounter situations in which, due to particle-hole symmetry, degenerate states can simultaneously resonate and therefore the more general formula (1) is required.

Consider now a N-NDOT-S structure, where the sub-space $B$ describes the NDOT. At zero energy, if the isolated normal dot is on-resonance, then particle-hole symmetry ensures that a degeneracy occurs. Hence in this example, the trace in equation (1) can be restricted to two terms and equation (2) reduces to an expression involving 2x2 matrices.
This yields

\[ G_{BB} = \frac{1}{d} \left( \frac{E - \epsilon_- - \Sigma_{--} + i\Gamma_-}{\sigma'_{--}} \right) \left( \frac{E - \epsilon_+ - \Sigma_{++} + i\Gamma_{++}}{\sigma'_{++}} \right) \]  

(3),

where \( \epsilon_{\pm} \) are the particle and hole levels closest to the quasi-particle energy \( E \), \( d = (E - \epsilon_+ - \Sigma_{++} + i\Gamma_{++})(E - \epsilon_- - \Sigma_{--} + i\Gamma_-) - |\sigma'_{+-}|^2 \) and we have written \( \Sigma = \sigma' + \sigma \).

In the presence of a single normal lead, this yields for the electrical conductance in units of \( 2e^2/h \) \([14,27,28]\),

\[ G = 8\Gamma_{++}\Gamma_{--}|\sigma'_{+-}|^2 \left| \frac{d}{|d|^2} \right|^2 \]  

(4)

First consider the zero energy limit (\( E=0 \)), where particle-hole symmetry implies that the two levels closest to \( E = 0 \) satisfy \( \epsilon_- = -\epsilon_+ \) and therefore a vanishing particle-level \( \epsilon_+ \) is accompanied by a degeneracy. In this limit, \( \Sigma_{--} = -\Sigma_{++} \) and \( \Gamma_- = \Gamma_+ \).

Hence

\[ G = \frac{8\Gamma_{++}^2\sigma'_{+-}\sigma'_{++}}{(\epsilon_+ + \Sigma_{++})^2 + \Gamma_{++}^2 + |\sigma'_{+-}|^2)^2} \]  

(5),

which was obtained in reference [10] for resonant transport at zero energy. If \( |\sigma'_{+-}|^2 = \Gamma_{++}^2 \), then a resonance will occur when \( \epsilon_+ + \Sigma_{++} = 0 \). Since this involves only a single condition on the \( \epsilon_+ \), one expects resonances to occur with approximately the same probability when the S-contact is replaced by a N-contact.

At finite energies, this result is drastically modified, because a resonance can now occur only if both \( (E - \epsilon_+ - \Sigma_{++} = 0) \) and \( (E - \epsilon_- - \Sigma_{--} = 0) \). The probability of simultaneously satisfying both of these conditions is small and therefore we predict that the breaking of the particle-hole symmetry at \( E \neq 0 \) destroys finite-voltage conductance resonances. From the form of the quartic energy denominator \( |d|^2 \), the small number of surviving resonances will have a non-Lorentzian line-shape. This is illustrated in figure 3, which shows plots of the Andreev reflection coefficient \( R_a = P_{11}^{-+}(E,H) \), for various values of coupling to the normal lead; figure 3(a) has the strongest coupling and 3(d) the weakest. All quantities are plotted as functions of quasi-particle energy \( E \). Hence
as well as a drastic reduction in the probability of finding finite-voltage resonances, we predict that when a resonance does occur, the usual Lorentzian line-shape is replaced by a double peaked structure, with relative peak heights determined by the difference between $\Gamma_+$ and $\Gamma_-$ at finite energies. The above predictions are confirmed by the results shown in figures 1 and 2, which were obtained from an exact numerical solution of the Bogoliubov - de Gennes equation, as outlined in reference 28. The inset of figure 2b shows numerical results for the fine scale structure of a surviving resonance, in agreement with the analytic prediction of figure 3.

When a composite N-SS' or N-NDOT-SS' structure is formed from two or more superconductors, with different order parameter phases, or from a single superconductor with an imposed phase gradient [36], transport properties can be significantly modified if the phase difference between two points is varied by $2\pi$[15-22]. In experimental realizations of such structures[23-26], the phase difference between two superconducting contacts is modulated by connecting the superconductors to a macroscopic, external superconducting loop, whose phase is controlled by an applied magnetic field. Sub-gap quasi-particles can penetrate only a distance of order the superconducting coherence length into the superconductor and therefore apart from controlling the phase, the macroscopic loop plays no role in determining the s-matrix of the region near the contacts. Since the electrical conductance is a periodic function of the phase difference $\eta$, with period $2\pi$ and since $\eta$ changes by $2\pi$ when the flux $\Phi$ through the macroscopic control-loop changes by a flux quantum $\Phi_0$, such structures are galvanometric detectors of flux, with a sensitivity comparable with that of a SQUID. We now highlight generic properties of resonant interferometers and predict that flux sensitivity can be significantly enhanced.

The starting point for this analysis is a sub-space $B$ containing one or more superconductors, with eigenstates $|f_\nu\rangle$ and eigenvalues $\epsilon_\nu$, which are periodic functions of some dimensionless parameter $\eta$, with period $2\pi$. In this example, since particle-hole degeneracy is lifted by intimate contact with the superconductors, the trace in equation
(1) reduces to a single term and the electrical conductance takes the form
\[ G(\eta) = \frac{8\Gamma_\nu(\eta+\eta-)\Gamma_\nu(\eta-\eta)}{[(E - \epsilon_\nu(\eta)) - \Sigma(\eta) + i\Gamma_\nu(\eta)]^2} \] (6).

Consider now the situation in which, at \( \eta = \eta_0 \), the resonance condition \( E - \Sigma(\eta_0) = 0 \) is satisfied, where \( \Sigma(\eta) = \epsilon_\nu(\eta) + \Sigma(\eta) \). Then expanding equation (6) about \( \eta_0 \) yields
\[ G(\eta) = \frac{8\Gamma_\nu(\eta_0+\eta-\eta_0)}{[\partial \Sigma(\eta_0)/\partial \eta_0]^{2} |\eta - \eta_0|^{2} + \Gamma^2_\nu(\eta_0)} \] (7).

This demonstrates that with varying \( \eta \), \( G \) exhibits a Lorentzian resonance of width \( \Gamma_\nu(\eta_0)/[\partial \Sigma(\eta_0)/\partial \eta_0] \).

For the case \( \eta = 2\pi \Phi/\Phi_0 \), noting that \( \Sigma(\eta) \) can vary by at most an amount of order \( \Delta_0 \) as \( \eta \) varies by \( 2\pi \), yields an upper bound for \( \partial \Sigma(\eta_0)/\partial \eta_0 \) of order \( \Delta_0/2\pi \). Hence in terms of the flux through the external control loop, the resonance width is greater than or of order \( \delta \Phi = 2\pi \Phi_0 \Gamma_\nu(\eta_0)/\Delta_0 \). For simplicity in the above analysis, we have considered only a single resonance and a normal lead with no closed channels; the latter merely shifts the position of the resonance, while the former may lead to the appearance of several resonances per flux quantum. If the temperature \( T \) is greater than \( \Gamma_\nu(\eta_0)/k_B \), then the resonance width will be of order \( \delta \Phi = 2\pi \Phi_0 k_B T/\Delta_0 \). For a device operating at 1 Kelvin, formed from a cuprate superconductor with a transition temperature of 100 Kelvin, this yields \( \delta \Phi \simeq \Phi_0/20 \).

Finally we consider a superconducting dot with a uniform order parameter, connected to two normal leads. The eigenstates of a superconducting dot satisfy \( H_B|f_\nu⟩ = \epsilon_\nu|f_\nu⟩ \) where \( H_B \) is the Bogoliubov-de Gennes operator for the isolated dot. For a dot with a uniform real order parameter \( \Delta_0 \), if \( |\phi⟩ \) is an eigenstate of the normal dot satisfying \( H_0|\phi⟩ = \epsilon^0_\nu|\phi⟩ \) then the solutions of the Bogoliubov equation are of the form \( \epsilon_\nu = \sqrt{(\epsilon^0_\nu)^2 + \Delta^2_0} \), \( |f_\nu⟩ = \left( \begin{array}{c} f^+_\nu|\phi⟩ \\ f^-_\nu|\phi⟩ \end{array} \right) = \left( \begin{array}{c} u^+_\nu|\phi⟩ \\ u^-_\nu|\phi⟩ \end{array} \right) \), where \( |u^+_\nu|^2 = (1 + \epsilon^0_\nu/\epsilon_\nu)/2 \) and \( |u^-_\nu|^2 = (1 - \epsilon^0_\nu/\epsilon_\nu)/2 \). At zero energy, this yields for the two-probe conductance derived by Lambert[31,18],
\[ G = \frac{4|u^+_\nu|^2 \Gamma_1 \Gamma_2}{| - \epsilon_\nu + \sigma \epsilon^0_\nu + i(\Gamma_1 + \Gamma_2)|^2} \] (8),
where $\Gamma_i$ is the broadening due to contact with lead $i$.

If $\delta \epsilon$ is the level spacing of the normal-state dot, then equation (8) reveals that for $|\Delta_0| > \delta \epsilon \gg (\Gamma_1 + \Gamma_2)$ the contribution to the conductance from a single level is of order $G = \frac{2\Gamma_1 \Gamma_2}{|\Delta_0|^2}$. Hence for a large enough value of $\Delta_0/\delta \epsilon$, all resonances will be suppressed and switching on superconductivity will typically decrease $G$, a behaviour which is well-known for N-S tunnel junctions. However from the form of the denominator in equation (8), it is clear that for small values of $\Delta_0$, the switching-on of superconductivity can produce an anomalous increase in the conductance, provided $\sigma > \epsilon_{\nu}^0$. Since $\epsilon_{\nu}^0$ will be randomly spread between $-\frac{\delta \epsilon}{2}$ and $\frac{\delta \epsilon}{2}$ this suggests that the probability of a positive change is approximately $\frac{\sigma}{\delta \epsilon}$, a result which we have confirmed through numerical solution of the Bogoliubov equation for such structures[31].

We have presented a theoretical framework and general formulae for resonant transport through hybrid normal–superconducting nanostructures. For N-NDOT-S structures, we predict that finite-voltage resonances will be almost completely suppressed by the switching on of superconductivity and those that survive can have a double-peaked line-shape. Such non-Lorentzian resonances have been discussed in other contexts [39] and may generate non-exponential delay-time curves. This destruction of resonances implies that the ensemble averaged conductance decreases with increasing bias, a behaviour reminiscent of zero-bias anomalies in the sub-gap conductance of superconducting-semiconducting junctions[40,41]. We have also demonstrated that N-SS' structures can possess Lorentzian resonances on a scale much smaller than a flux quantum, which suggests that these may provide a new class of magnetometers with a sensitivity at least matching that of present-day SQUIDs. Finally we have shown that switching on superconductivity in N-SDOT-N structures can produce anomalous positive changes in the conductance.

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Figure Captions.

Figure 1. For quasiparticles of energy $E = 0$, the top graph shows the conductance $G$ when $\Delta_0 = 0$, as a function of the mean diagonal element $\epsilon_0$ of a 2-dimension tight binding Hamiltonian describing the NDOT. The band width of this Hamiltonian is 8 and the Fermi energy is $4 - \epsilon_0$. The lower graph shows corresponding results for Andreev reflection coefficient $R_a$, when the order parameter assumes a non-zero value.

Figure 2. As for figure 1, except that the energy now takes a non-zero, sub-gap value. The inset of figure 2b shows the fine structure of a typical surviving resonance.

Figure 3. Figures (a) to (d) show plots of equation (3) against energy $E$, for decreasing values of the coupling to the normal lead.
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