Symmetry limit properties of decay amplitudes with mirror matter admixtures

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Abstract

We extend our previous analysis on the symmetry limit properties of non-leptonic and weak radiative decay amplitudes of hyperons in a scheme of mirror matter admixtures in physical hadrons to include the two-body non-leptonic decays of $\Omega^-$ and the two photon and two pion decays of kaons. We show that the so-called parity-conserving amplitudes predicted for all the decays vanish in the strong flavor SU(3) symmetry limit. We also establish the specific conditions under which the corresponding so-called parity-violating amplitudes vanish in the same limit.

Keywords: mirror matter, mixing, symmetry breaking

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I. INTRODUCTION

Two interesting paths implement the violations of the conservation laws of parity and strong flavors. One is via the perturbative intervention of the $W^\pm$ and $Z^0$ vector mesons, as in the Standard Model (SM). The other one is via non-perturbative superpositions of different parity and flavor eigenstates in the physical states, eigenstates of the Hamiltonian and mass operators, which arise from the fact that these operators do not commute with the parity and flavor operators. This second path cannot be implemented in the minimal SM. However, it emerges quite naturally in extensions of the SM and, thus, may indicate the existence of some forms of new physics. It is therefore important to see if and how this second form of parity and flavor violation is relevant in low-energy physics. Through a systematic study we have established that, indeed, it can give significant contributions to the enhancement phenomenon observed in non-leptonic, weak radiative, and some rare decay modes of strange baryons and mesons [1–7]. This we have done with a phenomenological model we have referred to as manifest mirror matter admixtures in ordinary hadrons.

The transition operators in the amplitudes of these decays are the parity and flavor conserving strong $H_{st}$ and electromagnetic $H_{em}$ parts of the exact Hamiltonian $H$. Although both of them break the strong flavor symmetries, we have considered in Ref. [8] some of the consequences of assuming that they do not, i.e., the consequences on the decay amplitudes of the symmetry limit behavior of these two operators. Only the so-called parity-conserving amplitudes of non-leptonic and weak radiative decays of hyperons (NLDH and WRDH) were studied in Ref. [8].

In this paper we extend our previous analysis of Ref. [8] to include the two-body non-leptonic decays of the $\Omega^-$, the two photon decays of $K_L$ and $K_S$, and the two pion decays of $K$ mesons. We find that in the manifest mirror matter admixtures scheme, the so-called parity-conserving amplitudes in all these decays vanish automatically in the strong flavor SU(3) symmetry limit. This extends the result obtained in Ref. [8] for NLDH and WRDH. We also find in the present analysis the specific and systematic conditions under which the same conclusion is valid for the corresponding so-called parity-violating decay amplitudes.

In Section II we reproduce the expressions for the physical (mass eigenstates) hadrons with mirror matter admixtures obtained in our previous work, Ref. [1]. In Secs. III and IV we study NLDH and the two-body non-leptonic decays of $\Omega^-$, respectively. Section V is dedicated to WRDH. We complement our analysis with the inclusion of the $2\gamma$ decays of $K_L$ and $K_S$ in Sec. VI and of the two pion decays of $K$ mesons in Sec. VII. We present our conclusions in Sec. VIII.

II. MIRROR MATTER ADMIXTURES IN PHYSICAL HADRONS

Let us start with the expressions for the physical hadrons we shall use in terms of the mirror matter admixtures [1]. The SU(3) octets of spin-0 mesons and spin-1/2 baryons are given by

$$K_{ph}^+ = K_p^+ - \sigma \pi_p^+ - \delta' \pi_s^+ + \cdots,$$

$$K_{ph}^0 = K_p^0 + \sigma \left( \frac{1}{\sqrt{2}} \pi_p^0 + \frac{\sqrt{3}}{2} \eta_{ph} \right) + \delta' \left( \frac{1}{\sqrt{3}} \eta_{8s} + \frac{1}{\sqrt{3}} \eta_{1s} \right) + \delta' \left( \frac{1}{\sqrt{6}} \eta_{8s} + \frac{1}{\sqrt{6}} \eta_{1s} \right) - \frac{1}{\sqrt{3}} \eta_{1s} + \cdots,$$
\[
\pi_{ph}^+ = \pi_p^+ + \sigma K_p^+ - \delta K_s^+ + \cdots,
\]
\[
\pi_{ph}^0 = \pi_p^0 - \sigma \frac{1}{\sqrt{2}}(K_p^0 + \bar{K}_p^0) + \delta \frac{1}{\sqrt{2}}(K_s^0 - \bar{K}_s^0) + \cdots,
\]
\[
\pi_{ph}^- = \pi_p^- + \sigma K_p^- + \delta K_s^- + \cdots,
\]
\[
\bar{K}_{ph}^0 = \bar{K}_p^0 + \sigma \frac{1}{\sqrt{2}}(\pi_p^0 + \sqrt{3} \eta_{8p}) - \delta \frac{1}{\sqrt{3}}(\eta_{ls} + \eta_{8s}) - \delta' \frac{1}{\sqrt{6}}(\pi_s^0 + \eta_{8s} + \eta_{ls}) + \cdots,
\]
\[
\bar{K}_{ph}^- = \bar{K}_p^- - \sigma \pi_p^- + \delta' \pi_s^- + \cdots,
\]
\[
\eta_{8ph} = \eta_{8p} - \sqrt{3} \sigma \frac{1}{\sqrt{2}}(\eta_{ls} + \eta_{8s}) + \delta' \frac{1}{\sqrt{3}}(\eta_{ls} + \eta_{8s}) + \cdots,
\]
\[
\eta_{iph} = \eta_{ip} - (\delta - \delta') \frac{1}{\sqrt{3}}(\pi_s^0 - \bar{K}_s^0) + \cdots,
\]
and,
\[
p_{ph} = p_s - \sigma \Sigma_s^+ - \delta \Sigma_p^+ + \cdots,
\]
\[
n_{ph} = n_s + \sigma \frac{1}{\sqrt{2}}(\Sigma_s^0 + \sqrt{3} \Lambda_s) + \delta \frac{1}{\sqrt{2}}(\Sigma_p^0 + \sqrt{3} \Lambda_p) + \cdots,
\]
\[
\Sigma_{ph}^+ = \Sigma_s^+ + \sigma n_s - \delta' n_p + \cdots,
\]
\[
\Sigma_{ph}^0 = \Sigma_s^0 + \sigma \frac{1}{\sqrt{2}}(\Xi_s^0 + \eta_{8s} + \eta_{ls}) + \delta \frac{1}{\sqrt{2}}(\Xi_p^0 + \eta_{8p} + \eta_{ls}) + \cdots,
\]
\[
\Xi_{ph}^0 = \Xi_s^0 - \sigma \frac{1}{\sqrt{2}}(\Sigma_s^0 + \sqrt{3} \Lambda_s) + \delta' \frac{1}{\sqrt{2}}(\Sigma_p^0 + \sqrt{3} \Lambda_p) + \cdots,
\]
\[
\Xi_{ph}^+ = \Xi_s^+ - \sigma \Sigma_s^+ + \delta' \Sigma_p^+ + \cdots.
\]

The dots stand for other mixings (with strong flavors other than strangeness) that will not be relevant here. The subindices s and p refer to positive and negative parity eigenstates, respectively. δ, δ', and σ are the mixing angles. Each physical hadron is the mass eigenstate observed in experiment and our phase conventions are those of Ref. [9].
III. TWO-BODY NON-LEPTONIC DECAY AMPHITUDES OF HYPERONS

As mentioned in Sec. I, mirror matter mixings in hadrons lead to NLDH via the parity and flavor conserving strong part of the Hamiltonian. The transition amplitudes will be given by the matrix elements \( \langle M_{ph} B'_{ph} | H_{st} | B_{ph} \rangle \), where \( B_{ph} \) and \( B'_{ph} \) are the initial and final hyperons and \( M_{ph} \) is the emitted meson. Using the above mixings these amplitudes will have the form \( \bar{u}_{B'} (A - B' \gamma_5) u_B \), where \( u_B \) and \( u_{B'} \) are four-component Dirac spinors and the amplitudes \( A \) and \( B \) correspond to the parity-violating and the parity-conserving amplitudes of the \( W^\pm \) mediated NLDH, although with mirror matter mixings these amplitudes are both actually parity and flavor conserving. These amplitudes are given by [3]

\[
A_1 = \delta' \frac{3}{2} g_{n,p\pi^-} + \delta (g_{\Lambda,pK^-} - g_{\Lambda,\Sigma^+\pi^-}),
\]

\[
A_2 = -\frac{1}{\sqrt{2}} \left\{ -\delta' \sqrt{3} g_{n,n\pi^0} + \delta (g_{\Lambda,nK^0} - \sqrt{3} g_{\Lambda,\Lambda\pi^0} - g_{\Lambda,\Sigma^0\pi^0}) \right\},
\]

\[
A_3 = \delta (g_{\Sigma^-,nK^-} + \frac{3}{2} g_{\Sigma^-,\Lambda\pi^-} + \frac{1}{\sqrt{2}} g_{\Sigma^-,\Sigma^0\pi^-}),
\]

\[
A_4 = -\delta' g_{p,p\pi^+} + \delta \left\{ \sqrt{3} g_{\Sigma^+,\Lambda\pi^-} + \frac{1}{\sqrt{2}} g_{\Sigma^+,\Sigma^0\pi^-} \right\},
\]

\[
A_5 = -\delta' g_{p,p\pi^0} - \delta \left\{ \frac{1}{\sqrt{2}} g_{\Sigma^+,pK^0} + g_{\Sigma^+,\Sigma^0\pi^0} \right\},
\]

\[
A_6 = \delta' g_{\Sigma^-,\Lambda\pi^-} + \delta (g_{\Xi^-,\Xi^-K^-} + \sqrt{3} g_{\Xi^-,\Xi^-\Sigma^+\pi^-}),
\]

\[
A_7 = \frac{1}{\sqrt{2}} [\delta' (\sqrt{3} g_{\Lambda,\Lambda\pi^0} + g_{\Xi^-,\Lambda\pi^0}) + \delta (-g_{\Xi^-\Xi^-K^-} + \sqrt{3} g_{\Xi^-\Xi^-\Sigma^0\pi^0})],
\]

and

\[
B_1 = \sigma (-\sqrt{\frac{3}{2}} g_{n,p\pi^-} + g_{\Lambda,pK^-} - g_{\Lambda,\Sigma^+\pi^-}),
\]

\[
B_2 = -\frac{1}{\sqrt{2}} \sigma (\sqrt{3} g_{n,n\pi^0} + g_{\Lambda,nK^0} - \sqrt{3} g_{\Lambda,\Lambda\pi^0} - g_{\Lambda,\Sigma^0\pi^0}),
\]

\[
B_3 = \sigma (g_{\Sigma^-,nK^-} + \sqrt{\frac{3}{2}} g_{\Sigma^-,\Lambda\pi^-} + \frac{1}{\sqrt{2}} g_{\Sigma^-,\Sigma^0\pi^-}),
\]
\[ B_4 = \sigma(g_{p,n\pi^+} + \sqrt{\frac{3}{2}}g_{\Sigma^+,\Lambda\pi^+} + \frac{1}{\sqrt{2}}g_{\Sigma^+,\Sigma^0\pi^+}), \] (4)

\[ B_5 = \sigma(g_{p,p\pi^0} - \frac{1}{\sqrt{2}}g_{\Sigma^+,p\bar{K}^0} - g_{\Sigma^+,\Sigma^+\pi^0}), \]

\[ B_6 = \sigma(-g_{\Sigma^-,\Lambda\pi^-} + g_{\Xi^-\Lambda K^-} + \sqrt{\frac{3}{2}}g_{\Sigma^-,\Sigma^0\pi^-}), \]

\[ B_7 = \frac{1}{\sqrt{2}}\sigma(-\sqrt{3}g_{\Lambda,\Lambda\pi^0} - g_{\Sigma^0,\Lambda\pi^0} - g_{\Xi^0,\Lambda\bar{K}^0} + \sqrt{3}g_{\Xi^0,\Xi^0\pi^0}). \]

The subindices 1, \ldots, 7 correspond to \( \Lambda \rightarrow p\pi^-, \Lambda \rightarrow n\pi^0, \Sigma^- \rightarrow n\pi^-, \Sigma^+ \rightarrow n\pi^+, \Sigma^+ \rightarrow p\pi^0, \Xi^- \rightarrow \Lambda\pi^-, \) and \( \Xi^0 \rightarrow \Lambda\pi^0, \) respectively. The \( g \)-constants in these equations are Yukawa coupling constants (YCC) defined by the matrix elements of \( H_{st} \) between flavor and parity eigenstates, for example, by \( \langle M_p B'_p | H_{st} | B_s \rangle = g_{\Sigma^+,\Sigma^0\pi^+} \). The YCC in the \( B \)'s are the ordinary ones, while the YCC in the \( A \)'s are new ones. In the latter, the upper indices serve as a reminder of the parities of the parity eigenstates involved. We have omitted the upper indices in the \( g \)'s of the \( B \) amplitudes because the states involved carry the normal intrinsic parities of hadrons. However, because of our assumptions, all the ordinary and new YCC have common properties.

The SU(2) symmetry limit of the YCC leads to the equalities

\[ g_{\Sigma^+,\Lambda\pi^+} = g_{\Sigma^0,\Lambda\pi^0} = g_{\Sigma^-,\Lambda\pi^-} = g_{\Lambda,\Sigma^0\pi^0}; \]

\[ g_{\Sigma^+,\Sigma^+\pi^0} = g_{\Sigma^-,\Sigma^0\pi^-} = g_{\Sigma^+,\Sigma^0\pi^+}; \]

\[ g_{\Sigma^+,\Sigma^+\pi^0} = g_{\Sigma^-,\Sigma^0\pi^-} = g_{\Sigma^+,\Sigma^0\pi^+}; \] (5)

\[ g_{\Sigma^+,\Sigma^0\pi^+} = g_{\Sigma^-,\Sigma^0\pi^-} = g_{\Sigma^+,\Sigma^0\pi^+}; \]

\[ g_{\Lambda,\bar{K}^0\pi^-} = g_{\Lambda,\bar{K}^0\pi^-} = g_{\Xi^0,\Xi^0\pi^0} = \frac{1}{\sqrt{2}}g_{\Xi^-,\Xi^0\pi^-}; \]

\[ g_{\Xi^-,\Lambda K^-} = g_{\Xi^0,\Lambda K^0}; \]

Similar relations are valid within each set of upper indices, e.g. \( g_{\Sigma^+,\Sigma^0\pi^+} = -g_{\Sigma^-,\Sigma^0\pi^-}, \) etc. when SU(2) symmetry is applied to the new YCC. In the SU(3) limit one also has [9]
\[ g_{p,p^0} = g, \quad g_{\Sigma^+,\Lambda\pi^+} = g_{\Lambda,\Sigma^+\pi^-} = -\frac{2}{\sqrt{3}} \alpha g, \]

\[ g_{\Sigma^+\Sigma^+,\pi^0} = 2(1 - \alpha)g, \quad g_{\Sigma^0,pK^-} = -g_{\Xi^0,\Xi^0\pi^0} = (2\alpha - 1)g \]  

(6)

\[ g_{\Lambda,pK^-} = \frac{1}{\sqrt{3}}(3 - 2\alpha)g, \quad g_{\Xi^-\Lambda K^-} = \frac{1}{\sqrt{3}}(4\alpha - 3)g. \]

The connection between \( \alpha \) and \( g \) and the reduced from factors \( F \) and \( D \) are \( \alpha = D/(D+F) \) and \( g = D + F \).

As a first approximation we shall neglect isospin violations, i.e., we shall assume that \( H_{st} \) is an SU(2) scalar. However, we shall not neglect SU(3) breaking. From Eqs. (5) one obtains for the \( A \)'s and \( B \)'s the results:

\[ A_1 = \delta' \sqrt{\frac{3}{2}} g_{p,p^0}^{p,sp} + \delta(g_{\Lambda,pK^-}^{s,ss} - g_{\Lambda,\Sigma^+\pi^-}^{s,pp}), \]

\[ A_2 = -\frac{1}{\sqrt{2}}[\delta' \sqrt{\frac{3}{2}} g_{p,p^0}^{p,sp} + \delta(g_{\Lambda,pK^-}^{s,ss} - g_{\Lambda,\Sigma^+\pi^-}^{s,pp})], \]

\[ A_3 = \delta(\sqrt{\frac{2}{3}} g_{\Sigma^0,pK^-}^{s,ss} + \sqrt{\frac{3}{2}} g_{\Sigma^+,\Lambda\pi^+}^{s,pp} + \frac{1}{\sqrt{2}} g_{\Sigma^+,\Sigma^+,\pi^0}^{s,pp}), \]

(7)

\[ A_4 = -\delta' \sqrt{\frac{2}{3}} g_{p,p^0}^{p,sp} + \delta(\sqrt{\frac{3}{2}} g_{\Sigma^0,pK^-}^{s,ss} - \frac{1}{\sqrt{2}} g_{\Sigma^+,\Sigma^+,\pi^0}^{s,pp}), \]

\[ A_5 = -\delta' g_{p,p^0}^{p,sp} - \delta(g_{\Sigma^0,pK^-}^{s,ss} + g_{\Sigma^+,\Sigma^+,\pi^0}^{s,pp}), \]

\[ A_6 = \delta' g_{\Sigma^+,\Lambda\pi^+}^{p,sp} + \delta(g_{\Xi^-\Lambda K^-}^{s,ss} + \sqrt{3} g_{\Xi^0,\Xi^0\pi^0}^{s,pp}), \]

\[ A_7 = \frac{1}{\sqrt{2}}[\delta' g_{\Sigma^+,\Lambda\pi^+}^{p,sp} + \delta(g_{\Xi^-\Lambda K^-}^{s,ss} + \sqrt{3} g_{\Xi^0,\Xi^0\pi^0}^{s,pp})], \]

and

\[ B_1 = \sigma(-\sqrt{3} g_{p,p^0}^{p,sp} + g_{\Lambda,pK^-} - g_{\Lambda,\Sigma^+\pi^-}), \]

\[ B_2 = -\frac{1}{\sqrt{2}} \sigma(-\sqrt{3} g_{p,p^0}^{p,sp} + g_{\Lambda,pK^-} - g_{\Lambda,\Sigma^+\pi^-}), \]

\[ B_3 = \sigma(\sqrt{2} g_{\Sigma^0,pK^-}^{s,ss} + \sqrt{\frac{3}{2}} g_{\Sigma^+,\Lambda\pi^+}^{s,pp} + \frac{1}{\sqrt{2}} g_{\Sigma^+,\Sigma^+,\pi^0}^{s,pp}), \]

(8)
\[ B_4 = \sigma(\sqrt{2}g_{p, p\pi^0} + \sqrt{\frac{3}{2}}g_{\Sigma^+, \Lambda^+} - \frac{1}{\sqrt{2}}g_{\Sigma^+, \Sigma^+ \pi^0}), \]

\[ B_5 = \sigma(g_{p, p\pi^0} - g_{\Xi^0, pK^-} - g_{\Xi^+, \Xi^+ \pi^0}), \]

\[ B_6 = \sigma(-g_{\Xi^+, \Lambda^+} + g_{\Xi^-, \Lambda^+} + \sqrt{3}g_{\Xi^0, \Xi^+ \pi^0}), \]

\[ B_7 = \frac{1}{\sqrt{2}}\sigma(-g_{\Xi^+, \Lambda^+} + g_{\Xi^-, \Lambda^+} + \sqrt{3}g_{\Xi^0, \Xi^+ \pi^0}). \]

From the above results one readily obtains the equalities:

\[ A_2 = -\frac{1}{\sqrt{2}}A_1, \quad A_5 = \frac{1}{\sqrt{2}}(A_4 - A_3), \quad A_7 = \frac{1}{\sqrt{2}}A_6, \quad (9) \]

\[ B_2 = -\frac{1}{\sqrt{2}}B_1, \quad B_5 = \frac{1}{\sqrt{2}}(B_4 - B_3), \quad B_7 = \frac{1}{\sqrt{2}}B_6. \quad (10) \]

These are the predictions of the \(|\Delta I| = 1/2\) rule [10, 11]. That is, mirror matter mixings in hadrons as introduced above lead to the predictions of the \(|\Delta I| = 1/2\) rule, but notice that they do not lead to the \(|\Delta I| = 1/2\) rule itself. This rule originally refers to the isospin covariance properties of the effective non-leptonic interaction Hamiltonian to be sandwiched between strong-flavor and parity eigenstates. The \(I = 1/2\) part of this Hamiltonian is enhanced over the \(I = 3/2\) part. In contrast, in the case of mirror matter admixtures \(H_{st}\) has been assumed to be isospin invariant, i.e., in this case the rule should be called a \(\Delta I = 0\) rule.

It must be stressed that the results (9) and (10) are very general: (i) the predictions of the \(|\Delta I| = 1/2\) rule are obtained simultaneously for the \(A\) and \(B\) amplitudes, (ii) they are independent of the mixing angles \(\sigma, \delta, \text{ and } \delta'\), and (iii) they are also independent of particular values of the YCC. They will be violated by isospin breaking corrections. So, they should be quite accurate, as is experimentally the case.

Now, we assume that \(H_{st}\) is an invariant operator, an SU(3) invariant in the case of Eqs. (7) and (8). We replace the symmetry limit values of the \(g\)'s, Eqs. (6), in (7) and (8). In this case one obtains

\[ A_1 = -\sqrt{2}A_2 = \frac{1}{\sqrt{3}}\left\{ \delta' 3g^{p, sp} + \delta \left[(3 - 2\alpha)g^{s, ss} + 2\alpha g^{s, pp}\right]\right\}, \]

\[ A_3 = \delta\sqrt{2}(2\alpha - 1)(g^{s, ss} - g^{s, pp}), \quad (11) \]

\[ A_4 = -\sqrt{2}(\delta' g^{p, sp} + \delta g^{s, pp}), \quad A_5 = \frac{1}{\sqrt{2}}(A_4 - A_3), \]

\[ A_6 = \sqrt{2}A_7 = \frac{1}{\sqrt{3}}\left\{ -\delta'2\alpha g^{p, sp} + \delta \left[(4\alpha - 3)g^{s, ss} - 3(2\alpha - 1)g^{s, pp}\right]\right\}, \]
\[ B_1 = B_2 = \cdots = B_7 = 0. \] (12)

The so-called parity-conserving amplitudes automatically vanish in the SU(3) symmetry limit [8].

For the so-called parity-violating amplitudes to vanish in the same limit, one observes in Eq. (11) that two conditions must be met, namely

\[-g^{p,s} = g^{s,p} = g^{s,s} \quad \text{and} \quad \delta' = \delta.\] (13)

That is, when these two conditions are satisfied one has

\[ A_1 = A_2 = \cdots = A_7 = 0. \] (14)

**IV. TWO-BODY NON-LEPTONIC DECAY AMPLITUDES OF \( \Omega^- \)**

We shall study now the decays \( \Omega^- \to \Xi^- \pi^0 \), \( \Omega^- \to \Xi^0 \pi^- \), and \( \Omega^- \to \Lambda K^- \), not considered before [8]. They are described by a Lorentz invariant amplitude of the form

\[ \langle M(q)B'(p')|H_{st}|B(p)\rangle = \bar{u}(p')(B + \gamma^5 C)q^\mu u_\mu(p), \] (15)

where \( u \) and \( u_\mu \) are Dirac and Rarita-Schwinger spinors, respectively, and \( B \) and \( C \) are \( p \)-wave (parity-conserving) and \( d \)-wave (parity-violating) amplitudes, respectively. \( H_{st} \) is the transition operator and \( B(p), B'(p') \), and \( M(q) \) represent \( s = 3/2, s = 1/2 \) baryons, and a pseudoscalar meson, respectively.

In our approach the transition operator \( H_{st} \) is the strong flavor and parity conserving part of the Hamiltonian responsible for the two-body strong decays of the other \( s = 3/2 \) resonances in the decuplet where \( \Omega_s^- \) belongs to.

To obtain explicit expressions for \( B \) and \( C \) we need the mirror admixtures in \( \Omega_{ph}^-, \pi_{ph}^0, \pi_{ph}^-, K_{ph}^-, \Lambda_{ph}, \Xi_{ph}^0, \) and \( \Xi_{ph}^- \). We take them from our previous work [4],

\[ \Omega_{ph}^- = \Omega_s^- - \sigma \sqrt{3} \Xi_s^- + \delta' \sqrt{3} \Xi_p^+ \pi^0 + \cdots, \] (16)

and Eqs. (1) and (2).

The properties of \( H_{st} \) discussed above, lead to

\[ B(\Omega^- \to \Xi^- \pi^0) = -\sigma(\sqrt{3}g_{\Xi^- \Xi^0}^{s,sp} + \frac{1}{\sqrt{2}}g_{\Xi^- \Xi^- K^0}^{s,sp}), \]

\[ B(\Omega^- \to \Xi^0 \pi^-) = \sigma(-\sqrt{3}g_{\Xi^0 \Xi^-}^{s,sp} + g_{\Xi^0 \Xi^- K^-}^{s,sp}), \] (17)

\[ B(\Omega^- \to \Lambda K^-) = \sigma(-\sqrt{3}g_{\Xi^- \Lambda K^-}^{s,sp} + \sqrt{3}g_{\Xi^- \Xi^0 K^-}^{s,sp}), \]

\[ C(\Omega^- \to \Xi^- \pi^0) = \delta' \sqrt{3}g_{\Xi^- \Xi^-}^{p,sp} - \delta \frac{1}{\sqrt{2}}g_{\Xi^- \Xi^- K^0}^{s,ss}. \]
\[ C(\Omega^- \to \Xi^0 \pi^-) = \delta' \sqrt{3} g_{\Xi^* \Xi^0 \pi^-}^{s,sp} + \delta g_{\Omega^- \Xi^0 K^-}^{s,ss}, \quad (18) \]

\[ C(\Omega^- \to \Lambda K^-) = \delta' \sqrt{3} g_{\Xi^* \Lambda K^-}^{p,sp} + \delta \sqrt{\frac{3}{2}} g_{\Omega^- \Xi^0 K^-}^{s,pp}. \]

The constants \( g_{B,B'M}^{s,sp} \) are the Yukawa strong couplings observed in the strong two-body decays of \( s = \frac{3}{2} \) resonances. The constants \( g_{B,B'M}^{s,sp}, g_{B,B'M}^{s,ss} \) and \( g_{B,B'M}^{s,pp} \) are new, because they involve mirror matter. As for NLDH, they are defined by the matrix elements of \( H_{st} \) between flavor and parity eigenstates. In the isospin limit they are related by

\[ g_{\Omega^- \Xi^0 K^-} = g_{\Omega^- \Xi^- K^0} ; \]

\[ g_{\Xi^* \Xi^0 \pi^-} = -\sqrt{2} g_{\Xi^* \Xi^- \pi^-} ; \quad (19) \]

In the SU(3) limit one also has

\[ g_{\Xi^* \Xi^- \pi^-} = -\frac{1}{\sqrt{6}} g_{\Omega^- \Xi^- K^0} ; \]

\[ g_{\Xi^* \Lambda K^-} = \frac{1}{\sqrt{2}} g_{\Omega^- \Xi^- K^0} ; \quad (20) \]

where the indices \( s \) and \( p \) may be dropped out.

In the explicit amplitudes (17) and (18) we can now replace the isospin equalities (19). Eqs. (17) and (18) become

\[ B(\Omega^- \to \Xi^- \pi^0) = -\sigma (\sqrt{3} g_{\Xi^* \Xi^- \pi^0} + \frac{1}{\sqrt{2}} g_{\Omega^- \Xi^- K^0}), \]

\[ B(\Omega^- \to \Xi^0 \pi^-) = \sigma (\sqrt{6} g_{\Xi^* \Xi^- \pi^-} + g_{\Xi^- \Omega^- K^0}), \quad (21) \]

\[ B(\Omega^- \to \Lambda K^-) = \sigma (-\sqrt{3} g_{\Xi^* \Lambda K^-} + \sqrt{\frac{3}{2}} g_{\Omega^- \Xi^- K^0}), \]

\[ C(\Omega^- \to \Xi^- \pi^0) = \delta' \sqrt{3} g_{\Xi^* \Xi^- \pi^0}^{s,sp} - \delta \frac{1}{\sqrt{2}} g_{\Omega^- \Xi^- K^0}^{s,ss} ; \]

\[ C(\Omega^- \to \Xi^0 \pi^-) = -\delta' \sqrt{6} g_{\Xi^* \Xi^- \pi^-}^{s,sp} + \delta g_{\Omega^- \Xi^- K^0}^{s,ss} ; \]

\[ C(\Omega^- \to \Lambda K^-) = \delta' \sqrt{3} g_{\Xi^* \Lambda K^-}^{p,sp} + \delta \sqrt{\frac{3}{2}} g_{\Omega^- \Xi^- K^0}^{s,pp} ; \quad (22) \]

We have omitted the indices \( s \) and \( p \) in the \( g \)'s of the \( B \) amplitudes because the states involved carry the normal intrinsic parities of hadrons.
One readily sees that the equalities

\[
\frac{B(\Omega^- \to \Xi^-\pi^0)}{B(\Omega^- \to \Xi^0\pi^-)} = \frac{C(\Omega^- \to \Xi^-\pi^0)}{C(\Omega^- \to \Xi^0\pi^-)} = -\frac{1}{\sqrt{2}}
\]  

are obtained. This is the $|\Delta I| = 1/2$ rule in these decay modes and applies to both decay amplitudes. The mirror admixtures in $\Omega^-$ lead to the results of this rule. However, it must be recalled that in this approach these results are really a $\Delta I = 0$ rule, since isospin is assumed unbroken.

If we now assume the SU(3) limit, we can replace (20) in the amplitudes (21) and (22). In this case one obtains

\[
B(\Omega^- \to \Xi^-\pi^0) = B(\Omega^- \to \Xi^0\pi^-) = B(\Omega^- \to \Lambda K^-) = 0,
\]

\[
C(\Omega^- \to \Xi^-\pi^0) = -\frac{1}{\sqrt{2}} C(\Omega^- \to \Xi^0\pi^-) = -\frac{1}{\sqrt{2}} (\delta' g_{\alpha-,K^0}^{s,sp} + \delta g_{\alpha-,K^0}^{s,pp}),
\]

\[
C(\Omega^- \to \Lambda K^-) = \sqrt{\frac{3}{2}} (\delta' g_{\alpha-,K^0}^{s,sp} + \delta g_{\alpha-,K^0}^{s,pp}).
\]

We observe from these equations that, as was the case in NLDH, the so-called parity-conserving amplitudes vanish automatically in this limit, as previously reported [4]. Now, one can also observe for the so-called parity-violating amplitudes that

\[
C(\Omega^- \to \Xi^-\pi^0) = C(\Omega^- \to \Xi^0\pi^-) = C(\Omega^- \to \Lambda K^-) = 0
\]

when the same conditions of Eq. (13) are met.

V. WEAK RADIATIVE DECAY AMPLITUDES OF HYPERONS

Next, we shall discuss the application of the above scheme to the weak radiative decays of hyperons [2]. In this case, these transitions receive contributions due to the matrix elements of the parity and flavor conserving electromagnetic part $H_{em} = iJ_{em}^\mu A_\mu$ of the Hamiltonian between the physical baryons with mirror matter admixtures, Eqs. (2). $A_\mu$ represents the photon field operator and $J_{em}^\mu$ is the ordinary electromagnetic current operator. This operator is — and this must be emphasized — a flavor-conserving Lorentz proper four-vector. The radiative decay amplitudes we want are given by the usual matrix elements $\langle \gamma, B_{ph}^l | H_{em} | B_{ph}^r \rangle$, where $B_{ph}$ and $B_{ph}^r$ stand for hyperons. The hadronic parts of the transition amplitudes are then given by:

\[
\langle p_{ph} | J_{em}^\mu | \Sigma^+ \rangle = \bar{u}_p [\sigma (f_{2s}^p - f_{2s}^{\Sigma^+}) - (\delta' f_{2s}^{sp} + \delta f_{2s}^{ps}) \gamma^5] i\sigma^{\mu \nu} q_\nu u_{\Sigma^+}
\]

\[
\langle \Sigma^- | J_{em}^\mu | \Xi^- \rangle = \bar{u}_{\Sigma^-} [\sigma (-f_{2s}^{\Xi^-} + f_{2s}^{\Xi^-}) + (\delta' f_{2s}^{sp} + \delta f_{2s}^{ps}) \gamma^5] i\sigma^{\mu \nu} q_\nu u_{\Xi^-}
\]

\[
\langle n_{ph} | J_{em}^\mu | A_{ph} \rangle = \bar{u}_n \left\{ \sigma \left[ \sqrt{\frac{3}{2}} (-f_{2s}^{n} + f_{2s}^{\Lambda}) + \frac{1}{\sqrt{2}} f_{2s}^{n,A} \right] + \left[ \sqrt{\frac{3}{2}} (\delta' f_{2s}^{n} + \delta f_{2s}^{ps}) + \delta \frac{1}{\sqrt{2}} f_{2s}^{n,A} \right] \gamma^5 \right\} i\sigma^{\mu \nu} q_\nu u_{\Lambda}
\]  

(27)
In these amplitudes only contributions to first order in $\sigma$, $\delta$, and $\delta'$ need be kept. Each matrix element of $J_{em}^\mu$ between flavour and parity eigenstates is flavour and parity conserving and can be expanded in terms of charge $f_1(0)$ form factors and anomalous magnetic $f_2(0)$ form factors. We have used the generator properties of the electric charge, which require $f_{1ss}^p = f_{1ss}^{\Sigma^+} = 1$, etc. and also, since $s$ and $p$ baryons belong to different irreducible representations, $f_{1sp}^p = f_{1sp}^{\Sigma^+} = 0$, etc. Notice that the amplitudes (27) are all of the form $\bar{u}_B (C + D_{\gamma 5}) i\sigma^{\mu\nu} q u_B$, where the spinors $u_B$, $B = p, \Sigma^+$, etc. are ordinary four-component Dirac spinors and $q = p_{B'} - p_B$. $C$ is the so-called parity-conserving amplitude and $D$ is the so-called parity-violating one. We stress, however, that in this model both $C$ and $D$ are parity and flavor conserving.

In the SU(3) symmetry limit the anomalous magnetic moments $f_2$ are related by

$$f_2^{\Sigma^+} = f_2^p, \quad f_2^{\Xi^-} = f_2^{\Sigma^-}, \quad f_2^{\Xi^0} = f_2^n, \quad f_2^{\Sigma^0} = f_2^n, \quad f_2^{\Sigma^0 A} = \frac{\sqrt{3}}{2} f_2^n, \quad f_2^{\Sigma^0} = \frac{1}{2} f_2^n, \quad f_2^{\Lambda} = \frac{1}{2} f_2^n. \quad (28)$$

When Eqs. (28) are replaced into Eqs. (27) we obtain

$$C(\Sigma^+ \rightarrow p\gamma) = C(\Xi^- \rightarrow \Sigma^-\gamma) = C(\Lambda \rightarrow n\gamma) = C(\Xi^0 \rightarrow \Lambda\gamma) = C(\Xi^0 \rightarrow \Sigma^-\gamma) = 0, \quad (29)$$

$$D(\Sigma^+ \rightarrow p\gamma) = -\delta' f_{2sp}^p - \delta f_{2ps}^p, \quad D(\Xi^- \rightarrow \Sigma^-\gamma) = \delta' f_{2sp}^{\Sigma^-} + \delta f_{2ps}^{\Sigma^-}, \quad (29')$$

$$D(\Lambda \rightarrow n\gamma) = D(\Xi^0 \rightarrow \Lambda\gamma) = \sqrt{3} D(\Xi^0 \rightarrow \Sigma^0\gamma) = \sqrt{\frac{3}{2}} (\delta' f_{2sp}^{n\gamma} + \delta f_{2ps}^{n\gamma}), \quad (30)$$

From Eqs. (29) one sees that the same result of NLDH and $\Omega^-$ two-body non-leptonic decays for the so-called parity-conserving amplitudes is automatically obtained [8].

The so-called parity-violating amplitudes Eqs. (30) vanish in the SU(3) symmetry limit,

$$D(\Sigma^+ \rightarrow p\gamma) = D(\Xi^- \rightarrow \Sigma^-\gamma) = D(\Lambda \rightarrow n\gamma) = D(\Xi^0 \rightarrow \Lambda\gamma) = 0, \quad (31)$$

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when the conditions
\[ \delta' = \delta \quad \text{and} \quad f_{2ps}^B = -f_{2sp}^B \] (32)
are fulfilled. Notice that Eq. (32) is the equivalent of Eq. (13), except that now it is the \(H_{em}\) part of the Hamiltonian that is involved.

VI. TWO PHOTON DECAY AMPLITUDES OF \(K_L\) AND \(K_S\)

Let us now study the symmetry limit properties of the decays \(K_{L,S} \to \gamma \gamma\) [5, 6]. Our phenomenological model, based on parity and flavor admixtures of mirror matter in ordinary mesons of Eqs. (1), may contribute to the enhancement phenomenon observed in these decays via the electromagnetic part \(H_{em}\) of the Hamiltonian transition operator.

The physical mesons (mass eigenstates) of Eqs. (1) are used to form the short and long lived \(K\)’s, \(K_L, K_S\). Notice that the physical mesons satisfy \(\text{CP}\ K_0^0_{\text{ph}} = -\bar{K}_0^0_{\text{ph}}\) and \(\text{CP} \bar{K}_0^0_{\text{ph}} = -K_0^0_{\text{ph}}\). We can form the CP-eigenstates \(K_1\) and \(K_2\) as
\[ K_{1\text{ph}} = \frac{1}{\sqrt{2}}(K_0^0_{\text{ph}} - \bar{K}_0^0_{\text{ph}}) \quad \text{and} \quad K_{2\text{ph}} = \frac{1}{\sqrt{2}}(K_0^0_{\text{ph}} + \bar{K}_0^0_{\text{ph}}), \] (33)
the \(K_{1\text{ph}} (K_{2\text{ph}})\) is an even (odd) state with respect to CP. Here, we shall not consider CP-violation and therefore, \(|K_{S,L}\rangle = |K_{1,2}\rangle\).

Substituting the expressions given in Eqs. (1), we obtain,
\[ K_{L_{\text{ph}}} = K_{L_p} + \sigma \pi_0^0 + \sqrt{3}\sigma \eta_8 p, \]
\[ K_{S_{\text{ph}}} = K_{S_p} + \frac{1}{\sqrt{2}}(2\delta + \delta')\eta_8 s + \delta' \pi_0^0 + \sqrt{\frac{2}{3}}(\delta - \delta')\eta_1 s, \] (34)
where the usual definitions \(K_{1p} = (K_p^0 - \bar{K}_p^0)/\sqrt{2}\) and \(K_{2p} = (K_p^0 + \bar{K}_p^0)/\sqrt{2}\) were used.

A very simple calculation, using Eqs. (34) and \(H_{em}\), gives the following contributions of mirror matter admixtures to the \(K_{L,S} \to \gamma \gamma\) amplitudes:
\[ F_{K_L\gamma \gamma} = \sigma F_{\pi_0^0 \gamma \gamma} + \sqrt{3}\sigma F_{\eta_8 p \gamma \gamma}, \] (35)
\[ F_{K_S\gamma \gamma} = \frac{1}{\sqrt{3}}(2\delta + \delta')F_{\eta_8 s \gamma \gamma} + \delta' F_{\pi_0^0 \gamma \gamma} + \sqrt{\frac{2}{3}}(\delta - \delta')F_{\eta_1 s \gamma \gamma}, \] (36)
where \(F_{K_L\gamma \gamma} = \langle \gamma | H_{em} | K_{L_{\text{ph}}}\rangle\), \(F_{K_S\gamma \gamma} = \langle \gamma | H_{em} | K_{S_{\text{ph}}}\rangle\), and \(F_{\pi_0^0 \gamma \gamma} = \langle \gamma | H_{em} | \pi_0^0\rangle\), etc.

Given that \(K_S\) and \(K_L\) are \(\text{CP} = +1\) and \(\text{CP} = -1\) pure states respectively, and because the two-photon state is a \(C = +1\) state, then \(K_S \to \gamma \gamma\) must go through a so-called parity-violating transition while \(K_L \to \gamma \gamma\) goes through a parity-conserving transition. In the first case the two-photon final state is \(P = +1\) while in the second one, \(P = -1\). However, as we can see from Eqs. (35) and (36), in the context of mirror matter admixtures all the contributions to both amplitudes are flavour and parity conserving.

In the SU(3) symmetry limit (U-spin invariance) one has
\[ F_{\eta_8 \gamma \gamma} = -\frac{1}{\sqrt{3}}F_{\pi_0^0 \gamma \gamma}. \] (37)
The amplitudes (35) and (36) become in this limit

\[ F_{K_L\gamma\gamma} = 0 \]  

and

\[ F_{K_S\gamma\gamma} = \sqrt{\frac{2}{3}}(\delta' - \delta) \left( \sqrt{\frac{2}{3}} \bar{F}_{\pi_0\gamma\gamma} - F_{\eta_1\gamma\gamma} \right). \]  

Once again, the so-called parity-conserving amplitude of \( K_L \rightarrow \gamma\gamma \) vanishes automatically.

The vanishing of the so-called parity-violating amplitude

\[ F_{K_S\gamma\gamma} = 0, \]  

requires only that \( \delta' = \delta \) of Eqs. (13) and (32) be satisfied. The reason for this simplification is that only one hadron is involved in these \( K_{L,S} \rightarrow \gamma\gamma \) decays.

\section{VII. \( K \rightarrow \pi\pi \) Decay Amplitudes}

Contributions of the parity and flavor admixtures in physical mesons to the \( K \rightarrow \pi\pi \) decay amplitudes are determined by the matrix elements of the flavor and parity conserving strong part of the Hamiltonian, \( H_{st} \) [7].

From expressions (34) of Sec. VI we obtain some important conclusions. Since the Hamiltonian \( H_{st} \) is by assumption isoscalar and also a flavour and parity conserving one, we notice that the physical state \( K_{S_{ph}} \) can only decay into two pions and not into three pions. In the latter case, the final state made out of three pions has total angular momentum equal to zero. The parity is odd, since each of the pions has negative parity, and then the Hamiltonian can not make the transition. In the case of having two pions in the final state, the transition is possible and proportional to the constants \( \delta \) and \( \delta' \). Similarly for the state \( K_{L_{ph}} \): it has to go to three pions and the amplitude is proportional to \( \sigma \). The above qualitative behavior is observed experimentally neglecting CP-violation effects.

The amplitudes for the decays \( K \rightarrow \pi\pi \) are denoted by \( A_{+0} = \langle \pi^+\pi^0_{ph}|H_{st}|K^+_{ph} \rangle \), \( A_{+-} = \langle \pi^0_{ph}\pi^-_{ph}|H_{st}|K^-_{ph} \rangle \), and \( A_{00} = \langle \pi^0_{ph}\pi^0_{ph}|H_{st}|K^0_{ph} \rangle \). After the substitution of the physical mass eigenstates given in Eqs. (1), we obtain explicitly

\[ A_{+0} = -\delta'\langle \pi^+\pi^0_{p}|H_{st}|\pi^+_{s} \rangle + \frac{1}{\sqrt{2}}\delta\langle \pi^+ K^0_{s}|H_{st}|K^+_{p} \rangle - \delta\langle K^+\pi^0_{p}|H_{st}|K^+_{p} \rangle \]

\[ A_{+-} = \frac{1}{\sqrt{3}}(2\delta + \delta')\langle \pi^+\pi^-_{p}|H_{st}|\eta_{ss} \rangle + \delta'\langle \pi^+\pi^-_{p}|H_{st}|\pi^0_{s} \rangle + \sqrt{\frac{2}{3}}(\delta - \delta')\langle \pi^+\pi^-_{p}|H_{st}|\eta_{ls} \rangle \]

\[ -\frac{1}{\sqrt{2}}\delta\langle K^+\pi^0_{p}|H_{st}|K^0_{p} \rangle - \frac{1}{\sqrt{2}}\delta\langle \pi^+ K^-_{p}|H_{st}|K^0_{p} \rangle \]

\[ A_{00} = \frac{1}{\sqrt{3}}(2\delta + \delta')\langle \pi^0\pi^0_{p}|H_{st}|\eta_{ss} \rangle + \delta'\langle \pi^0\pi^0_{p}|H_{st}|\pi^0_{s} \rangle + \sqrt{\frac{2}{3}}(\delta - \delta')\langle \pi^0\pi^0_{p}|H_{st}|\eta_{ls} \rangle \]

\[ + \frac{1}{2}\delta\langle \pi^0 K^0_{s}|H_{st}|K^0_{p} \rangle + \frac{1}{2}\delta\langle \pi^0 K^0_{p}|H_{st}|K^0_{p} \rangle + \frac{1}{2}\delta\langle K^0\pi^0_{p}|H_{st}|K^0_{p} \rangle \]
In the right hand side of these equations, the amplitudes are flavor and parity conserving. The two pions in the physical final state of these amplitudes are either in the $I = 0$ or in the $I = 2$ isospin configuration, since the $I = 1$ state is forbidden by the generalized Bose principle [12]. Also, there is no contribution from the $I = 2$ component since the interaction Hamiltonian is an isosinglet (so, amplitudes with a $\pi^0\pi^0$ becomes operative in the SU(3) symmetry limit and forces the matrix elements to vanish, as described above.

| $A_{+0}$ | $= -\delta(\frac{1}{\sqrt{2}}G^{p,ps}_{K^+,\pi^K_0} - G^{p,sp}_{K^+K^K_0})e^{i\alpha_0}$ |
| $A_{+-}$ | $= \frac{1}{\sqrt{3}} (2\delta + \delta')G^{s,pp}_{\eta_8,\pi^K_0} + \sqrt{\frac{2}{3}}(\delta - \delta')G^{s,pp}_{\eta_1,\pi^{0,\pi^K_0}}e^{i\alpha_1}$ |
| $A_{00}$ | $= \frac{1}{\sqrt{3}} (2\delta + \delta')G^{s,pp}_{\eta_8,\pi^{0,\pi^K_0}} + \sqrt{\frac{2}{3}}(\delta - \delta')G^{s,pp}_{\eta_1,\pi^{0,\pi^K_0}}e^{i\alpha_1}$ |

Above we have used the assumption that the strong coupling constants have the property, $(G_{M^1M^2}^{i,j,k})^{\text{CPT}} = G_{M^2M^1}^{i,j,k}$. Also, we have used the properties, $\langle j_1j_2m_1m_2|j_1j_2JM|$ = $(-1)^{j_1-j_2}\langle j_2j_1m_2m_1|j_2j_1JM|$ of the SU(2) Clebsch-Gordan coefficients to simplify the expressions for the amplitudes. The phase introduced for the final state interaction depends only on the total isospin of the final particles; it is for this reason that $A_{+-}$ and $A_{00}$ have the same phase factor.

From the Clebsch-Gordan Tables we get that in the SU(2) symmetry limit $G_{K^+,\pi^K_0} = \sqrt{2}G_{\eta_8,\pi^K_0}$, $G_{\eta_8,\pi^K_0} = G_{\eta_8,\pi^{0,\pi^K_0}}$ and $G_{\eta_1,\pi^{0,\pi^K_0}}$. We observe then from Eqs. (42) that in this limit we obtain the so-called $|\Delta I| = 1/2$ rule predictions: $A_{+0} = 0$ and $A_{+-} = A_{00}$. The so-called parity-violating amplitude $A_{+0}$ vanishes already in the SU(2) symmetry limit.

In the SU(3) symmetry limit, where $G_{\eta_8,\pi^{0,\pi^K_0}} = -(1/\sqrt{3})G_{\pi^{0,\pi^{0,\pi^K_0}}}$, the expressions for the remaining so-called parity-violating amplitudes $A_{+-}$ and $A_{00}$ take the form

| $A_{+-}$ | $= A_{00} = \frac{1}{3} (2\delta + \delta')G^{s,pp}_{\eta^{0,\pi^{0,\pi^K_0}}} + \sqrt{\frac{2}{3}}(\delta - \delta')G^{s,pp}_{\eta^{0,\pi^{0,\pi^K_0}}}e^{i\alpha_1}$. |

In this case, the vanishing of $A_{+-}$ and $A_{00}$ requires not only that $\delta' = \delta$ of conditions (13) or (32) be satisfied but it also requires the use of the generalized Bose principle [12] which becomes operative in the SU(3) symmetry limit and forces the matrix elements $\langle \pi\pi|H_{st}|\pi\rangle$ to vanish, as described above.

VIII. CONCLUSIONS

In this paper we have extended our previous analysis on the SU(3) symmetry limit properties of the so-called parity-conserving amplitudes of NLDH and WRDH [8] to all the amplitudes involved in these two groups, in the two-body non-leptonic decays of $\Omega^-$, and in the two photon and two-body non-leptonic decays of $K$’s.
The amplitudes have been obtained by direct application of expressions (1) and (2) for the physical hadrons with mirror matter admixtures. The contributions of these mixings were determined by the matrix elements of the parity and strong-flavor conserving strong and electromagnetic parts of the exact Hamiltonian.

The so-called parity-conserving amplitudes vanish automatically in the strong flavor SU(3) symmetry limit. So that we may restate our conclusion by saying that even if mixings with parity conserving but flavor violating eigenstates are allowed in physical hadrons (i.e., $\sigma \neq 0$) the mixing angle $\sigma$ will drop out of the matrix elements of the strong and electromagnetic parts of the Hamiltonian that contribute to non-leptonic, weak radiative, and two photon decays of strange hadrons in the strong flavor SU(3) symmetry limit.

The so-called parity-violating amplitudes vanish if $\delta' = \delta$ and a relative minus sign be present in the Yukawa couplings and transitions magnetic moments as shown in Eqs. (13) and (32). The vanishing of the $K_S \to \gamma\gamma$ amplitude requires only that $\delta' = \delta$. The vanishing of the so-called parity-violating $K \to \pi\pi$ amplitudes also requires $\delta' = \delta$ and the extended Bose-Einstein statistics to the $s = 0$ meson octet in the SU(3) symmetry limit. We restate our conclusion by saying that even if parity and flavor violating admixtures were allowed in physical hadrons the mixing angle $\delta' = \delta \neq 0$ will drop out of the matrix elements of the strong and electromagnetic parts of the Hamiltonian that contribute to the above decays in the strong flavor SU(3) symmetry limit.

Notice that the decays $K_{L,S} \to \mu^+ + \mu^-$ are automatically covered due to the central role that $K_{L,S} \to \gamma\gamma$ play in these decays, respectively.

One should contrast our previous results on the so-called parity-conserving amplitudes and our new results on the so-called parity-violating amplitudes with the existing theorems for the $W$-mediated non-leptonic and weak radiative decays of hyperons, which are referred to as the Lee-Swift [13] and Hara [14] theorems, respectively. Both these theorems state that in the flavor symmetry limit it is the corresponding parity-violating amplitudes that vanish. The theorem of Ref. [13] refers only to the baryon pole contributions to the parity-violating amplitudes of non-leptonic decays of hyperons. The theorem of Ref. [14] is limited to $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$. Our results here cover more cases than these two theorems and in this sense are more general.

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