Monte Carlo simulation of the irreversible growth of magnetic thin films

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Abstract

The growth of magnetic films with ferromagnetic interactions between nearest-neighbor spins is studied in \((d + 1)\)-dimensional rectangular geometries for \(d = 1, 2\). Magnetic films are grown irreversibly by adding spins at the boundaries of the growing interface. The orientation of the added spins depends on both the energetic interaction with already deposited spins and the temperature, through a Boltzmann factor. At low temperatures thin films, of thickness \(L\), are constituted by a sequence of well ordered magnetic domains. Spins belonging to each domain, of average length \(l_D \gg L\), have mostly the same orientation, but consecutive domains have opposite magnetization. Such kind of “spontaneous magnetization reversal” during the growth process has a short characteristic length \(l_R\), such that \(l_D \gg l_R \sim L\). At higher temperatures, a transition between ordered and disordered states is also observed. The emerging behavior is compared to that of the equilibrium Ising model.

1 Introduction

The preparation and characterization of magnetic nanowires and films is of great interest for the development of advanced microelectronic devices. Therefore, the study of the behavior of magnetic materials in confined geometries, e.g. thin films, has attracted both experimental, see e.g. [1, 2, 3, 4, 5, 6, 7], and theoretical, see e.g. [8, 9, 10, 11, 12], attention. From the theoretical point of view, most of the work has been devoted to the study of equilibrium properties of thin magnetic films [8, 9, 10, 11, 12]. In contrast, the aim of this work is to study the properties of thin
magnetic film growth under far-from-equilibrium conditions. Within this context, a useful model for the study of the growth of magnetic materials is the so-called Magnetic Eden Model (MEM) [13], based on the well known Eden model [14]. The latter has become an archetypical growth model due to both its simplicity and interesting properties. Eden growth starts from a single particle called the seed. One then proceeds to add a new particle on a randomly chosen unoccupied site in the immediate neighborhood (the perimeter) of the seed. The growth process then continues by randomly adding new particles to the perimeter of the previously formed cluster. Although this simple rule leads to the growth of compact clusters filling the Euclidean space, the self-affinity that characterizes the behavior of the growing interface is of much interest (see e.g. [15, 16]). The MEM, originally motivated by the study of the structural properties of magnetically textured materials, introduces an additional degree of freedom to the Eden model, namely the spin of the added particles [13]. More recently, the Eden growth of clusters of charged particles has also been studied [17].

Considering the MEM with spins having two possible orientations (up and down), one can start the growth of the spin cluster from a single seed having a predetermined orientation, e.g. a spin up seed, placed at the center of the two-dimensional square lattice, whose sites are labelled by their rectangular coordinates \((i, j)\). Then, the MEM’s growth process consists in adding further spins to the growing cluster taking into account the corresponding interaction energies. By analogy to the classical Ising model [18] one takes \(J\) as the coupling constant between nearest-neighbor (NN) spins \(S_{ij}\) and the energy \(E\) given by

\[
E = -\frac{J}{2} \sum_{\langle ij, i'j' \rangle} S_{ij}S_{i'j'},
\]

where \(\langle ij, i'j' \rangle\) means that the summation is taken over occupied NN sites. As we are concerned with spin \(-\frac{1}{2}\) particles, the spins can assume two values, namely \(S_{ij} = \pm 1\).

It is worth mentioning that, while previous studies of the MEM were mainly devoted to determine the lacunarity exponent and the fractal dimension of the set of parallel oriented spins [13], the aim of the present work is to study the growth of MEM films using extensive Monte Carlo simulations. In order to simulate thin film growth, our study is performed in confined (stripped) geometries which resemble recent experiments where the growth of quasi-one-dimensional strips of Fe on a Cu(111) vicinal surface [1] and Fe on a W(110) stepped substratum [7] have been performed. Also, in a related context, the study of the growth of metallic multilayers have shown a rich variety of new physical phenomena. Particularly, the growth of magnetic layers of Ni and Co separated by a Cu spacer layer has recently been studied [19].
Another goal of the present work is to compare the results obtained for the MEM with the well known behavior of the equilibrium Ising model [18, 20], an archetypical model in the study of phase transitions in equilibrium magnetic systems. The Ising Hamiltonian ($H$) is given by

$$H = -\frac{J}{2} \sum_{\langle ij, i'j' \rangle} S_{ij} S_{i'j'},$$

where $\langle ij, i'j' \rangle$ means that the summation runs over all NN sites, $S_{ij} = \pm 1$ is the state of the spin at the site of coordinates $(i, j)$ and $J$ is the coupling constant ($J > 0$ for the ferromagnetic case). It should be pointed out that, in spite of the fact that Eq. (1) and Eq. (2) are similar, the MEM describes the irreversible growth of a magnetic material while the Ising model is suitable for the study of a magnetic system under equilibrium conditions, and hence these two models operate under extremely different conditions.

This paper is organized as follows: in Section 2 we give details on the simulation method, Section 3 is devoted to the presentation and discussion of the results, while our conclusions are finally stated in Section 4.

## 2 The Monte Carlo simulation method

The MEM in $(1 + 1)$–dimensions is studied in the square lattice using a rectangular geometry $L \times M$ with $M \gg L$. The location of each site on the lattice is specified through its rectangular coordinates $(i, j)$, $(1 \leq i \leq M, 1 \leq j \leq L)$. The starting seed for the growing cluster is a column of parallel oriented spins placed at $i = 1$. As described in the foregoing section, the MEM growth occurs by selectively gluing spins at perimeter sites. It should be noticed that previous studies of the MEM were performed using a single spin seed placed at the center of the sample [13]. In this way, previous simulations that followed this approach were restricted to rather modest cluster sizes, i.e. containing up to 8000 spins [13]. In contrast, the rectangular stripped geometry used in this work is suitable for the simulation of the growth of magnetic films and it also has significant technical advantages. Indeed, when the growing film interface is close to reach the limit of the sample $(i = M)$ one simply computes the relevant properties of the irreversibly frozen film’s bulk (in the region where the growing process has definitively stopped), and subsequently applies an algorithm such that the interface is shifted towards the lowest possible $i$–coordinate (while, at the same time, the useless frozen bulk is erased). By repeatedly applying this procedure the growth process is not limited by the $M$–value of the lattice. In the present work films having up to $10^9$ spins have been typically grown. The described procedure can straightforwardly be extended to higher dimensions. In fact, we have also studied the MEM in $(2 + 1)$–dimensions.
employing a $L \times L \times M$ geometry ($M \gg L$). Each site on the lattice is now identified through the rectangular coordinates $(i, j, k)$, $(1 \leq i \leq M, 1 \leq j, k \leq L)$, and the starting seed for the growing film is taken to be a plane of $L \times L$ parallel oriented spins placed at $i = 1$. The cutting-and-shifting algorithm is in this case also suitable in order to allow the growing film to acquire particles beyond the $i = M$ limit.

As already mentioned, the growth process of a MEM film consists in adding further spins to the growing film taking into account the corresponding interaction energies given by equation (1). A spin is added to the film with a probability proportional to the Boltzmann factor $\exp\left(\frac{-\Delta E}{k_BT}\right)$, where $\Delta E$ is the total energy change involved. It should be noted that at each step all sites of the growing perimeter are considered and the probabilities of adding up and down spins to them have to be evaluated. After proper normalization of the probabilities the growing site and the orientation of the spin are determined through a pseudo-random number generator. Throughout this work we set the Boltzmann constant equal to unity ($k_B \equiv 1$), we consider $J > 0$ (i.e., the ferromagnetic case) and we take the absolute temperature $T$ measured in units of $J$.

3 Results and discussion

Magnetic Eden films grown on a stripped geometry of finite linear dimension $L$ at sufficiently low temperatures show an intriguing behavior that we call spontaneous magnetization reversal. In fact, we have observed that long clusters are constituted by a sequence of well ordered magnetic domains of average length $l_D \gg L$. Figure 1(a) shows a snapshot configuration of the $(1+1)$−dimensional MEM where the phenomenon of spontaneous magnetization reversal can be recognized. Here the reversal occurring between a domain of spins up (on the left side) and other one constituted by spins down (on the right), as well as the interface between both domains, can be clearly observed. It should be noted that the well known phenomenon of field induced magnetization reversal in thin films [21, 22] is quite different from the spontaneous reversal reported here. In the present study the reversal occurs during the growth process and in the absence of any applied magnetic field. The reported phenomenon is essentially due to the small size of the thin film and it becomes irrelevant in the thermodynamic limit. Within our best knowledge this theoretical prediction has not yet been observed experimentally. However, it will certainly be very interesting to design and carry out suitable experiments in order to observe this phenomenon.

Let $l_R$ be the characteristic length for the occurrence of the spontaneous magnetization reversal. Since $l_R \sim L$, we then conclude that the phenomenon has two characteristic length scales, namely $l_D$ and $l_R$, such that $l_D \gg l_R \sim L$. Figure 1(b) shows the magnetization profile that corresponds to the spontaneous magnetization
Figure 1: Spontaneous magnetization reversal observed for $L = 32$ and $T = 0.26$ in the $(1 + 1)$-dimensional magnetic film. (a) Snapshot configuration that shows the collective orientation change: the left (right) domain is constituted by up (down) oriented spins. The snapshot corresponds to the bulk of the sample and the growing interface is not shown. (b) Magnetization profile associated to the upper configuration. The characteristic length for the occurrence of the magnetization reversal, $l_R$, is of the order of the lattice width, as marked in the figure.
Figure 2: Plots of $P(l_d; \Delta l_d)$ versus $l_d/L$ for $L = 16$ and different values of temperature and interval width, as indicated. The inset shows a plot of $(l_D/L)^{-1}$ versus $T$, also for a lattice of side $L = 16$. As expected, increasingly long domains tend to show up at lower temperatures.

reversal shown in figure 1(a), where

$$m(i, L, T) = \frac{1}{L} \sum_{j=1}^{L} S_{ij}$$  \hspace{1cm} (3)

is the mean column magnetization at the distance $i - 1$ from the seed, for a system of linear dimension $L$ at temperature $T$. In figure 1(b) one can clearly observe how abruptly the magnetization drops from $m = +1$ to $m = -1$ within a characteristic length $l_R$ of the order of $L$.

In order to investigate the dependence of the characteristic domain length $l_D$ on $L$ and $T$, let us define $P(l_d; \Delta l_d)$ as the probability for the formation of a domain of length between $l_d$ and $l_d + \Delta l_d$. Clearly, the average domain length $l_D$ mentioned above is the average value of $l_d$ taken over a sufficiently long magnetic film (i.e. $l_D \equiv \langle l_d \rangle$ for $l_F \gg l_D$, where $l_F$ is the film’s total length). Figure 2 shows plots of $P(l_d; \Delta l_d)$ versus $l_d/L$ for a fixed lattice size and different values of temperature. As expected, increasingly long domains tend to show up at lower temperatures. This behavior can be also observed in the plot of $(l_D/L)^{-1}$ versus $T$ shown in the inset of figure 2. Figure 3 shows a log-linear plot of $l_D^{-1}$ versus $L^{-1}$ for a fixed temperature and lattice sizes in the range $16 \leq L \leq 512$. From this figure it turns clearly out that the average domain length diverges as we increase the lattice size towards the thermodynamic limit. Therefore, as early anticipate the phenomenon of magnetization reversal is a finite size effect relevant for the growth of magnetig
films in confined geometries.

At this point it is useful to perform a comparison between the results obtained with the MEM and the well known behavior of the Ising model analyzing the interplay between broken symmetry and finite-size effects at thermal phase transitions. In ordinary thermally driven phase transitions, the system changes from a disordered state at high temperatures to a spontaneously ordered state at temperatures below some critical value $T_c$ where a second-order phase transition takes place. Regarding the equilibrium Ising model as the archetypical example, one has that, in the absence of an externally applied magnetic field ($H = 0$), the low temperature ordered phase is a state with non-vanishing spontaneous magnetization ($\pm M_{sp}$, the sign depending on the initial state). This spontaneous symmetry breaking is possible in the thermodynamic limit only. In fact, it is found that the magnetization $M$ of a finite sample formed by $N$ particles, defined by

$$M(T, H = 0) = \frac{1}{N} \sum_{i=1}^{N} S_i(T, H = 0) ,$$

(4)

can pass with a finite probability from a value near $+ M_{sp}$ to another near $- M_{sp}$, as well as in the opposite direction. Consequently, the magnetization of a finite system, averaged over a sufficiently large observation time, vanishes at every positive temperature, irrespective of the (finite) size of the sample. The equation $M(T, H = 0) \approx 0$ holds if the observation time ($t_{obs}$) becomes larger than the ergodic time ($t_{erg}$), which is defined as the time needed to observe the system passing from $\pm M_{sp}$ to $\mp M_{sp}$. Increasing the size of the sample the ergodic time increases too, such that in the
thermodynamic limit ergodicity is broken due to the divergence of the ergodic time, yielding broken symmetry. Since Monte Carlo simulations are restricted to finite samples, the standard procedure to avoid the problems treated in the foregoing discussion is to consider the root mean square (or the absolute) magnetization as an appropriate order parameter [23]. Turning back to the MEM, we find that the phenomenon of magnetization reversal (as shown in figure 1(a)) causes the magnetization of the whole film to vanish at every non-zero temperature, provided that the film’s length $l_F$ (which plays the role of $t_{obs}$) is much larger than $l_D$ (which plays the role of $t_{erg}$). Therefore, as in the case of the Ising model [23], in order to overcome shortcomings derived from the finite-size nature of Monte Carlo simulations we have measured the mean absolute column magnetization, given by

$$|m(i, L, T)| = \frac{1}{L} \sum_{j=1}^{L} |S_{ij}|.$$  (5)

In the stripped geometry used in this work the bias introduced by the linear seed (a starting column made up entirely of up spins) can be studied in plots of $|m(i, L, T)|$ versus $i$. It is found that $|m(i; L, T)|$ exhibits a transient growing period with a characteristic length of order $L$, followed by the attainment of a stationary regime. In addition, using several randomly generated seeds we could also establish that the system evolves into a given stationary state independently of the seed employed. Thus, the spin-up linear seed can be used throughout without loss of generality. This behavior has been observed throughout the range of interest studied in the present work, i.e. $16 \leq L \leq 1024$ and $0.2 \leq T \leq \infty$. So, the influence exerted on the spin system by the seed can be easily recognized and eliminated from our results just by disregarding the first $l_{Tr} = N.L$ columns, with $N$ ranging between 10 and 50. The given procedure of column averaging out from the transient region represents a significant advantage of the stripped geometry used for the simulation of the MEM, in addition to that already mentioned (see section II). In fact, when a single seed at the center of the sample is used, the definition of the average magnetization of the whole cluster is strongly biased by the cluster’s kernel orientation at the early stages of the growing process. Hence, it turns very difficult in this case to disentangle the stationary regime from the transient region. Moreover, film growth on planar substrata has the advantage that it can be implemented experimentally, e.g. via vapor deposition in vacuum, chemical deposition, electrodeposition, etc.

The mean column magnetization given by equation (3) is a fluctuating quantity that can assume $L+1$ values. Then, for given values of both $L$ and $T$, the probability distribution of the mean column magnetization ($P_L(m)$) can straightforwardly be evaluated, since it represents the normalized histogram of $m$ taken over a sufficiently large number of columns in the stationary region [24, 25, 26]. In the thermodynamic limit (lattice size going to infinite) the probability distribution ($P_\infty(m)$) of the order parameter of an equilibrium system at criticality is universal (up to rescaling of
the order parameter) and thus it contains very useful and interesting information on the universality class of the system [27, 28, 29]. For example, $P_L(m)$ contains information about all momenta of the order parameter $m$, including universal ratios such as the Binder cumulant [27]. Figure 4 shows the thermal dependence of $P_L(m)$ for a fixed lattice size ($L = 128$ in the present example) as obtained for the $(1 + 1)$—dimensional MEM. We can observe that at high temperatures $P_L(m)$ is a Gaussian centered at $m = 0$. As the temperature gets lowered, the distribution broadens and develops two peaks at $m = 1$ and $m = -1$. Further decreasing the temperature causes these peaks to become dominant while the distribution turns distinctly non-Gaussian, exhibiting a minimum just at $m = 0$. It should be pointed out that the emergence of the maxima at $m = \pm 1$ is quite abrupt. This behavior reminds us the order parameter probability distribution characteristic of the one dimensional Ising model. In fact, for the well studied $d$—dimensional Ising model [26, 30], we know that for $T > T_c$, $P_L(M)[31]$ is a Gaussian centered at $M = 0$, given by

$$P_L(M) \propto \exp \left( \frac{-M^2 L^d}{2T \chi} \right),$$

where the susceptibility $\chi$ is related to order parameter fluctuations by
\[
\chi = \frac{L^d}{T} \left( \langle M^2 \rangle - \langle M \rangle^2 \right).
\]  

(7)

Decreasing temperature the order parameter probability distribution broadens, it becomes non-Gaussian, and near \( T_c \) it splits into two peaks that get the more separated the lower the temperature. For \( T < T_c \) and linear dimensions \( L \) much larger than the correlation length \( \xi \) of order parameter fluctuations, one may approximate \( P_L(M) \) near the peaks by a double-Gaussian distribution, i.e.

\[
P_L(M) \propto \exp \left( \frac{-(M - M_{sp})^2 L^d}{2T \chi} \right) + \exp \left( \frac{-(M + M_{sp})^2 L^d}{2T \chi} \right),
\]

(8)

where \( M_{sp} \) is the spontaneous magnetization, while the susceptibility \( \chi \) is now given by

\[
\chi = \frac{L^d}{T} \left( \langle M^2 \rangle - \langle |M| \rangle^2 \right).
\]

(9)

From equation (6) it turns out that the Gaussian squared width \( \sigma^2 \) associated with high temperature distributions is very close to the 2nd moment of the order parameter, i.e.

\[
\sigma^2 \approx \langle M^2 \rangle.
\]

(10)

It should be noticed that this equation is a straightforward consequence of the Gaussian shape of the order parameter probability distribution and, thus, it holds for the MEM as well. From the well known one-dimensional exact solution for a chain of \( L \) spins [32] one can establish the relationship

\[
\chi = \frac{1}{T} \exp(2/T);
\]

(11)

then, equations (7) and (11) lead us to

\[
\langle M^2 \rangle = \frac{1}{T} \exp(2/T)
\]

(12)

(where it has been taken into account that \( \langle M \rangle = 0 \) due to finite-size effects, irrespective of temperature). From equations (10) and (12) we can see that the high temperature Gaussian probability distribution broadens exponentially as \( T \) gets lowered, until it develops delta-like peaks at \( M = \pm 1 \) as a consequence of a boundary effect on the widely extended distribution. It should be noted that for \( d \geq 2 \) this phenomenon is prevented by the finite critical temperature which splits the Gaussian, as implied by equation (8).
Figure 5: Data corresponding to the (2 + 1)−dimensional MEM: plots of the probability distribution $P_L(m'')$ versus $m''$ for the fixed lattice size $L = 16$ and different temperatures, as indicated in the figure. The occurrence of two maxima located at $m'' = \pm M_{sp}$ (for a given value of $M_{sp}$ such that $0 < M_{sp} < 1$) is the hallmark of a thermal continuous phase transition that takes place at a finite critical temperature.

Figure 5 shows the thermal evolution of the probability distribution as obtained for the (2 + 1)−dimensional MEM, using a lattice of side $L = 16$. For high temperatures, the probability distribution corresponds to a Gaussian centered at $m'' = 0$. At lower temperatures we observe the onset of two maxima located at $m'' = \pm M_{sp}$ ($0 < M_{sp} < 1$), which become sharper and approach $m'' = \pm 1$ as $T$ is gradually decreased. These low-temperature probability distributions clearly reflect the occurrence of the magnetization reversal effect already discussed for the case of (1 + 1)−dimensional magnetic films.

Figure 6 shows the location of the maximum of the probability distribution as a function of temperature for both $(d+1)$-dimensional MEM models (with $d = 1, 2$) where we consider only maxima located at $m, m'' \geq 0$, since the distributions are symmetric around $m = m'' = 0$. After inspection of figure 6, it becomes apparent the different qualitative behaviors of both systems. In fact, while for the $d = 2$ case we observe a smooth transition from the $m''_{max} = 0$ value characteristic of high temperatures to nonzero $m''_{max}$ values that correspond to lower temperatures, the curve obtained for $d = 1$ shows, in contrast, a Heaviside-like jump. The latter case reflects a behavior which is similar to that observed simulating the equilibrium Ising model in 1d.

In order to carry out a quantitative comparison between both models, we have also evaluated the average absolute magnetization ($\langle |M_{Ising}| \rangle$) for chains of the same
length $L$ that the columns of the $(1 + 1)$—dimensional MEM. Figure 7 shows log-linear plots of $\langle |M_{\text{Ising}}| (L, T) \rangle - \langle |m_{\text{MEM}}| (L, T) \rangle$ versus $L^{-1}$ obtained for two different values of $T$. It becomes evident that the different $L$—dependent values of the magnetization are finite size effects observed as a consequence of the strips used. Such effects vanish in the thermodynamic limit.

In contrast to the $(1+1)$—dimensional case, the behavior exhibited by the $(2+1)$—dimensional MEM (e.g., as displayed by figures 5 and 6) is the signature of a thermal continuous phase transition that takes place at a finite critical temperature. It should be noticed that this transition involves the entire system, that may be either in the disordered phase or in the ordered one, depending on temperature. The broken symmetry at a finite critical temperature $T_c$ implied by the thermal continuous phase transition can be explained in terms of the broken ergodicity that occurs in the system when we tend to the thermodynamic limit ($L \to \infty$) making use of the temperature dependence exhibited by the order parameter distribution function. In fact, if we introduce an ergodic length $l_{\text{erg}}$ defined by $l_{\text{erg}} \equiv l_D$, where $l_D$ is the characteristic length of MEM’s domains, we can carry out a complete analogy with the Ising model by associating $l_{\text{erg}}$ to $t_{\text{erg}}$ (the Ising model ergodic time) and the above mentioned film’s total length $l_F$ to the Ising model observation time $t_{\text{obs}}$. In this way, we encounter that excursions of $m''$ from $m'' = +M_{sp}$ to $m'' = -M_{sp}$ and vice versa occur at length scales of the order of $l_{\text{erg}}$. When the film’s total length becomes larger and larger ($l_F \gg l_{\text{erg}}$) the whole film’s magnetization is averaged to zero. Furthermore, $l_{\text{erg}}$ diverges as the strip’s width becomes larger and larger, as
Figure 7: Comparison of results corresponding to the $(1 + 1)$–MEM and the $d = 1$ Ising model: log-linear plots of $\langle |M| \rangle (L, T) - \langle |m|_{MEM} \rangle (L, T)$ versus $L^{-1}$ for $T = 0.5$ and $T = 1.0$. The lines are guides to the eye. Hence, differences in the magnetization due to finite-size effects appear to vanish in the thermodynamic limit.

shown in figure 3, and again broken symmetry arises as the consequence of broken ergodicity.

It should be noted that as in the case of equilibrium systems, in the present case various “effective” $L$-dependent critical temperatures can also be defined. In particular, we will define $T_{c1}(L)$ as the value that corresponds to $\langle |m''| \rangle = 0.5$ for fixed $L$, and $T_{c2}(L)$ as the temperature that corresponds to the maximum of the susceptibility for a given $L$, assuming that the susceptibility is related to order parameter fluctuations in the same manner as for equilibrium systems (as given by equations (7) and (9)). Then, we should be able to obtain $T_c$ from plots of $T_{cn}(L)$ versus $L^{-1}$ (for $n = 1, 2$), as it is shown in figure 8. Indeed, following this procedure we find that, for $L \to \infty$, both $T_{c1}(L)$ and $T_{c2}(L)$ extrapolate (approximately) to the same value, allowing us to evaluate the critical temperature $T_c = 0.69 \pm 0.01$ in the thermodynamic limit. Notice that $T_c$ depends on both the coordination number and the topological structure of the lattice. Furthermore, these properties are not uniquely determined by the dimensionality. So, the small value of $T_c$ obtained for the MEM, as compared to the Ising model on the square lattice with four nearest neighbors (NN) given by $T_{cIsing}^{Ising} = 2.27$ [32], reflects the fact that the effective number of occupied NN sites upon deposition of spins in the MEM ($\langle NN_{MEM} \rangle$) should be $\langle NN_{MEM} \rangle < 4$. Furthermore, the effective topological structure of the MEM compatible with the measured value of $T_c$ remains as an open question.
Figure 8: Plots of the effective finite-size critical temperatures $T_{cn}(L)$ versus $L^{-1}$ (for $n = 1, 2$) corresponding to the $(2 + 1)$-dimensional magnetic film. $T_{c1}(L)$ is defined as the value that corresponds to $\langle |m''| \rangle = 0.5$, while $T_{c2}(L)$ is the temperature that corresponds to the maximum of the susceptibility. The solid lines show the linear extrapolations that meet at the critical point given by $T_c = 0.69 \pm 0.01$.

4 Conclusions

In the present work we have studied the growth of magnetic films with ferromagnetic interactions between nearest neighbor spins in a $(d + 1)$-dimensional rectangular geometry (for $d = 1, 2$), using Monte Carlo simulations. For both dimensions the phenomenon of spontaneous magnetization reversal is observed at low temperatures. Indeed, MEM films grown at low temperatures are constituted by a sequence of magnetic domains, each of them with a well defined magnetization, such that the magnetization of adjacent domains is antiparallel. Further increasing the temperature causes the onset of disorder in the bulk of the domains. Subsequently, a rounded effective transition to a fully disordered state takes place. These pseudo “phase transitions” occur at film width ($L$) dependent effective critical temperatures. However, in the thermodynamic limit ($L \to \infty$), the $(1 + 1)$-dimensional MEM is not critical (the transition takes place at $T = 0$), while a true second-order phase transition is expected to occur at a finite temperature ($T_c = 0.69 \pm 0.01$) in the $(2 + 1)$-MEM. The observed behavior is reminiscent to that of the equilibrium Ising model, although it should be stressed that the MEM is a far-from-equilibrium growing system.

The finite size of the films causing magnetization reversal and the occurrence of effective order-disorder transitions may be undesired effects that shall be avoided in the preparation of high quality magnetic films. However, these shortcomings may
disappear if the film strongly interacts with the substrate where the actual growing process takes place. Further studies on the growth of magnetic films in the presence of surface magnetic films, that account for the interaction with the substrate, are under progress [33, 34].

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