Didactical design of vectors in mathematics to develop creative thinking ability and self-confidence of Year 10 students

D Adharini¹, * and T Herman²

¹ Mathematics Education Study Program, School of Postgraduates, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia
² Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia

* E-mail: dewimtk98@upi.edu

Abstract. This research was motivated by the low ability of creative thinking in mathematics and students' self-confidence. This study aims to compile a didactic vector mathematical design to develop students' creative thinking skills and self-confidence. This type of research is Didactic Design Research (DDR). The participants were 50 Year 12 students and 24 Year 10 science students. Data collection on learning obstacle was carried out on Year 12 students using tests, interviews, and textbook analysis. It was found that most students had didactical obstacles. Students had low creative thinking ability, and students with mathematical skills low had low self-confidence. The didactic design was implemented online twice for Year 10 students using the zoom meeting application and google classroom. Students' creative thinking skills develop well for questioning and guessing and product improvement. Most of the students' self-confidence is in good condition. Further research should provide an explanation of the related material before the test, make a didactic design using e-learning, add formulas for student activities, and provide examples and explanations for each completion step.

1. Introduction

According to Guilford [1], divergent thinking as an intellectual operation that is responsible for creative thinking, with characteristics; fluency, the ability to come up with many ideas; flexibility, the ability to make qualitatively diverse solutions; originality, the ability to produce rare and atypical ideas, and elaboration, the ability to develop ideas. Mathematical skills will be related to mathematical creativity. A student with creative thinking uses the mathematical knowledge obtained during learning to solve problems and can use new, and unusual strategies in solving math problems [2]. Bandura [3] states that self-confidence is the belief in one's ability to unite and mobilize the required motivation and resources, and bring up the actions needed, and bring up for what must be completed, or the following task guidance.

Vector is one of the specialization materials for Year 10 that must be mastered by students. Based on the research conducted by Affandi [4], it was found that there were still many students who had difficulty understanding vector material.

Brousseau [5] argues that learning obstacle can come from students' psychological (ontogenic learning obstacles), mistakes in conveying of teaching material (didactical obstacle), or because of students' understanding of an incomplete concept (epistemological learning obstacle) so that when presented in a different context, students have difficulty solving it.
Plomp [6], the didactic design is a learning design for designing, developing, and evaluating educational interventions (such as programs, teaching-learning strategies, and materials, products, and systems) as solutions to complex problems in educational practice. The didactic design was designed based on students' learning obstacles and learning trajectories that students go through on material in learning mathematics.

Based on the description above, the researcher conducted a study entitled "Didactical design of vectors in mathematics to develop creative thinking ability and self-confidence of Year 10 students."

2. Methods
This type of research is Didactical Design Research (DDR). Participants in this study were 50 Year 12 science students and 24 Year 10 science students. Data collection on learning obstacles was carried out on Year 12 students using tests, interviews, and textbook analysis. Implementation of the didactic design was carried out on Year 10 students online using the zoom meeting and google classroom applications. The implementation was held in two meetings in May 2020.

3. Result and Discussion
3.1 Learning Obstacles
Based on the results of completing the questions that have been done by the students, from the 50 participants observed, a learning obstacle was obtained based on Brousseau; first, students' psychological (ontogenically learning obstacle), second, mistakes in conveying of teaching material (didactical obstacle) and third, students' understanding of an incomplete concept (epistemological learning obstacle) as follows;

| Type of Obstacle               | Items |
|-------------------------------|-------|
| Ontogenic learning obstacle   | 1     |
| Didactic obstacle             | 36    |
| Epistemological learning obstacle | 2  |

Table 1. Students' learning obstacles.

Based on table 1, it was found that three students had oncogenic learning obstacles, which are numbers one, three, and nine, students had less thorough and incomplete in solving the questions. Didactic obstacle experienced, by most students, based on the test results given most students could not work on the questions. The students' epistemological learning obstacle was mostly for questions number eight and nine; this problem, the students solved the questions but were wrong.

Based on the results test, showed that most students experienced problems in completing the test. The results obtained by the students did not achieve the completeness determined by the school. A condition, if associated with students' creative thinking skills in mathematics, that is very low. Based on The Alberta Teachers' Association [7], the table 2 shows the conditions of students' creative thinking abilities. Based on table 2, it can be seen that students' creative thinking skills are low.

Based on the results of interviews with students, of the five students who were interviewed with good, medium, and low mathematical abilities, it was found that students with good and medium mathematical abilities had good self-confidence about mathematical thinking skills and students with low mathematical abilities had low self-confidence in their mathematical abilities.
3.2 Learning Trajectory
According to Aljupri [8], the hypothetical learning trajectory (HLT) consists of three parts, learning objectives, learning activities, and hypothesized learning processes that will occur. The purpose of learning vector material is to use an open-ended and contextual approach. It is hoped that students can develop creative thinking skills and self-confidence.

Table 2. The ability to think creatively in learning obstacle tests.

| Creative Factors | Test and Subtest | Questions Indicator | Question number | Number of Students Answered Correctly |
|------------------|------------------|---------------------|-----------------|---------------------------------------|
| Fluency          | Ask and Guess    | Given a vector image in two-dimensions, the student can solve the problem related to the scalar multiplication of two vectors in a two-dimensions. | 2                | 3                                      |
| Flexibility      | Requires the person to ask questions based on drawings on a page | Students can solve problems related to the angle between two vectors in three-dimensions | 4                | 0                                      |
| Originality      | Guessing Causes and Guessing Consequence Require the person to make guesses about the causes and consequences of happenings related to a drawing | Students can solve problems related to the nature of the multiplication of two vectors in three-dimensions | 6                | 1                                      |
| Fluency          | Product Improvement Activity The person thinks of as many ways as possible | Students can solve problems related to vector projection in three-dimensions | 10               | 3                                      |
| Flexibility      | Unusual Uses Activities The person devises as many uses as possible for objects | Students can solve problems related to vector projection in three-dimensions | 10               | 3                                      |
| Originality      | Just Suppose Activity The person predicts possible outcomes and consequences of an improbable situation | Presented with pictures, students can explain scalar projections in two-dimensions | 7                | 1                                      |

Learning activities are developed based on the learning obstacle obtained from tests, teacher and student interviews, and textbooks for Year 10 science. Based on the results, the students' creative thinking ability was still low. Based on the interview, it was found that students with less ability in mathematics had low self-confidence. Based on the existing conditions, a learning trajectory developed that can develop creative thinking skills and self-confidence. The learning trajectory is structured for two meetings. Learning activities with the sub-principal of two-vector multiplication scalar, and vector projection; students are given material that has been submitted is still related to the material to be studied, so students can develop their creative thinking skills because they are confident in their mathematical abilities. After all, the material is always repeated; it is expected that students become accustomed to and grow a sense of self-confidence in learning mathematics. Students then try to construct a formula used to solve the scalar multiplication problem of two vectors. When finding the formula, the goal is
that students can understand the process from which the formula is obtained is also training students to think creatively. For example, students ask and construct how vector formulas are obtained, these activities are also expected to develop students' self-confidence because students are given the final results of the formula so that students believe in what they are doing.

Furthermore, after students are sure and understand the existing formulas, students try to apply the formulas, given guidelines in solving problems so that students are more confident in solving problems. The problem is divided into easy problems, in this case, to develop self-confidence in students who have fewer math skills. Followed by complex mathematical problems, this is to develop students' creative thinking skills. After students are given problems related to the real problems of students' daily lives, the hope is that students can further develop their creative thinking skills because students have to create problems that are related to the current context related to the material. To find out students' creative thinking skills, students are asked to make conclusions about learning, even though the conclusion framework has been made is to find out whether students can explain from the given frame of mind what kind of conclusions students make.

The learning hypothesis that will occur is that students can develop creative thinking skills and self-confidence. That is made to be realized, and the teacher makes steps that can support the hypothesis. The actions taken by the teacher are by providing prerequisite material, students are asked to prove the formula to be used in solving vector problems, the problem is equipped with formula, making contextual problem, and conclusions given the frame of mind.

3.3 Didactic design of vetor

The developed didactic design of vector consists of scalar multiplication of two vectors and vector projection. Begins with the title, were subjects, class/semester, material, sub-material, and time. Furthermore, essential competencies and indicators of competency achievement are given. Followed by the learning objectives and then entered the learning step. Learning steps consist of numbers, student activities, teacher questions, expected student reactions, teacher anticipation, and evaluation points. Jean Piaget [9] states that schemata development (a collection of schemata) occurs due to individual interactions with their environment. Based on Jean Pigeat about students' cognitive development, a didactic design of two-vector scalar multiplication consisting of 8 activities begins with an introduction containing prerequisite material. According to David Ausubel [10], meaningful learning is learning, accompanied by understanding, it will happen if the recently received information is closely related to existing concepts or knowledge. The prerequisite material consists of students being asked to recall drawing vectors in two and three dimensions. The introduction of the two-vector scalar multiplication formula consists of two activities, was the two-vector scalar multiplication formula without using angles and the two-vector scalar multiplication formula using the formula. Application of the two-vector scalar multiplication formula is planned because based on Vigotsky [11] on human development. Human development consists of actual development was the ability to solve problems independently, and potential development was the ability to solve problems guided by adults or collaborate with peers. The formula application consists of three activities: first; application of the scalar multiplication formula of two vectors using the angle, second; find the angle value in three dimensions; third; solve the problem using the scalar multiplication property of two vectors. Two-vector scalar multiplication material is in connection with contextual issues. This activity is planned because it is based on Bruner's learning theory [12]. The learning process consists of three steps. First, the concrete, given real problems close to students about social distancing, prevents the covid-19 pandemic. Second, the iconic, knowledge in the form of pictures or diagrams. Third, symbolic is manifested in the form of symbols; students are changing the model from real problems into symbols in mathematics and ended by concluding the material that has been studied.

The didactic design of vector projection consists of nine activities. It starts with an introduction, that is prerequisite material on finding angular values in two and three-dimensional spaces, introducing the vector projection formula, which consists of two activities. First, orthogonal scalar projection formula, second; orthogonal vector projection formula, vector projection formula application consists of four
activities. First, solving the problem of orthogonal scalar projection, secondly; solving the problem of orthogonal scalar projection in two-dimensional space followed by drawing the graph, third, solving the problem of orthogonal vector projection, fourth, solving the problem of orthogonal vector projection in three-dimensional space followed by describing the graph, linking vector projection material with real problems, and ending with conclusions related to vector projection material.

3.4 Implementation of Didactic Design of Vector
The implementation of the didactic design of vectors is manifested in the form of student worksheets consisting of two-vector scalar multiplication and vector projections. Student worksheet contains identity, basic competencies and indicators, learning objectives and instructions for doing. A student worksheet consists of questions.

| Creative Factors | Test and Subtest | Questions Indicator | Question number | Number of Students Answered Correctly |
|------------------|------------------|---------------------|-----------------|----------------------------------------|
| Fluency          | Ask and Guess    | Students can draw vectors in two-dimensional space | 1               | 18                                     |
| Flexibility      | Requires the person to ask questions based on drawings on a page | Students can draw vectors in three-dimensional space | 2               | 18                                     |
| Originality      | Guessing Causes and Guessing Consequence | Require the person to make guesses about the causes and consequences of happenings related to a drawing | 6               | 16                                     |
| Fluency          | Product Improvement Activity | Students can solve problems related to scalar multiplication of two vectors in three-dimensional space | 7               | 12                                     |
| Flexibility      | The person thinks of as many ways as possible | Students are asked to look for problems in everyday life related to the scalar multiplication of two vectors and solve them | 8               | 8                                      |
| Originality      | Unusual Uses Activities | Students can solve problems related to scalar multiplication of two vectors using the properties of the scalar product of two vectors | 7               | 12                                     |
| Fluency          | Just Suppose Activity | The person predicts possible outcomes and consequences of an improbable situation | 8               | 8                                      |
| Flexibility      | The person devises as many uses as possible for objects | Students can solve problems related to scalar multiplication of two vectors | 7               | 12                                     |
| Originality      |                        | Students can solve problems related to scalar multiplication of two vectors | 6               | 16                                     |

Two-vector scalar multiplication worksheets consist of 9 questions, (1) drawing in two-dimensional space, (2) drawing in three-dimensional space, (3) determining the scalar multiplication of two vectors, (4) determining the scalar multiplication formula for two vectors with angles, (5) application of two-vector scalar multiplication, (6) application of determining angles, (7) determining the length of the
vector using the scalar multiplication property of two vectors, (8) compiling the story problem and determining its solution, (9) Explaining the conclusion.

Implementation of didactic design of vector using online media (zoom and google class). Based on the results of student completion, it can be seen that students' creative thinking abilities for the scalar multiplication of two vectors as showed in table 3. Based on table 3, It was found that students' creative thinking skills developed well for ask and guess, guessing, and consequently, product improvement and using unusual activities, and not yet developing for predictive activity. The ability to think creatively for vector projection material can be seen as follows:

**Table 4. Creative thinking ability vector projection material.**

| Creative Factors      | Test and Subtest             | Questions Indicator                                                                 | Question number | Number of Students Answered Correctly |
|-----------------------|------------------------------|-------------------------------------------------------------------------------------|-----------------|---------------------------------------|
| • Fluency             | Ask and Guess               | Students can write the scalar multiplication formula for two vectors and determine the angle value in two-dimensional space | 1               | 21                                    |
| • Fluency             | Requires the person to ask questions based on drawings on a page |                                                                                     |                 |                                       |
| • Fluency             | Guessing Causes and Guessing Consequence | Students can determine the angle value in three-dimensional space                      | 2               | 8                                     |
| • Fluency             | Require the person to make guesses about the causes and consequences of happenings related to a drawing |                                                                                     |                 |                                       |
| • Fluency             | Product Improvement Activity | Students can solve orthogonal scalar projection problems in two-dimensional space     | 6               | 14                                    |
| • Fluency             | The person thinks of as many ways as possible | The person devises as many uses as possible for objects                              |                 |                                       |
| • Fluency             | Unusual Uses Activities     | Students can solve the problem of orthogonal vector projection in three-dimensional space | 8               | 10                                    |
| • Fluency             | The person predicts possible outcomes and consequences of an improbable situation | Students are asked to look for problems in everyday life related to vector projections and solve them | 9               | 11                                    |

Based on table 4, It can be seen that students' creative thinking skills are well developed for ask and guess and product improvement and have not developed to guessing and consequently, use unusual and predictive activities.

Based on tables 3 and 4 students' thinking skills develop well to ask and guess and improve the product because more than half of them can complete it.

**Table 5. Self-confidence attitude score.**
Interval class \((144-36)/4 = 27\). The value of the self-confidence attitude interval is as follows.

| Formula | Score |
|---------|-------|
| \(4 \times 36 = 144\) | Highest |
| \(1 \times 36 = 36\) | Lowest |

The following are the conditions for students' self-confidence:

| Number | Answer Score | Number of students | Scale |
|--------|--------------|--------------------|-------|
| 1      | 143 - 117    | 0                  | Very good |
| 2      | 116 – 90     | 23                 | Good   |
| 3      | 89 – 63      | 3                  | Fair   |
| 4      | 62 - 36      | 0                  | Poor   |

Based on Table 7, it can be concluded that most of the students' self-confidence is in good condition.

3.5 Revised Didactic Design of Vector

Mulyana [8] states that all data obtained from the previous phase is analyzed in this phase. The data analysis compares the anticipated HTL before the learning experiment, and the actual activity occurs, followed by an analysis of the possible causes, and a synthesis of the possibilities that can be done to improve the HTL used in the next cycle.

Based on implementation didactic design, in general, students experienced the first difficulty; to drawing in three-dimensional space, for the teacher's response, it is necessary to provide a detailed example and explanation until students understand, second; Many students write the formula wrong so that in student activities it is necessary for each step of the activity to display a formula to make it easier for students to complete their activities, third; students have difficulty in making story questions that link between the problems given in the pictures and the material, here there needs to be an improvement in the teacher's response, that is the teacher provides examples and solutions so that students can modify the examples given by the teacher.

4. Conclusion

It can be concluded that there are some student learning obstacles in vector material. The first is the ontogenic obstacle. Students are not careful in solving the questions and questions are not resolved. The second is the didactical obstacle. Students can not work on the questions, and third; epistemological obstacles, students solve the questions but provide the wrong solution. Learning trajectory starts with presenting problems related to prerequisite material, constructing formulas related to material, presenting problems using formulas, presenting real/ contextual problems using formulas, and concluding the material. The didactic vector design consists of the title (subject, class/semester, material, material, and time), basic competencies and indicators, learning objectives, and learning steps. The learning steps consist of, no, student activities, teacher questions and expected student reactions, teacher anticipation, and evaluation points. The teacher's questions are completed with answers as a guide in anticipating student responses. Implementation of didactic design, the activity begins with praying, student attendance, and repeating material related to the material to be delivered and followed by
conveying basic competencies, learning indicators, and learning objectives. Furthermore, the rules are presented during the lesson. Entering the core of learning, the teacher asks students to write down names and classes, followed by giving students explanations to complete questions. The teacher has finished explaining students are asked to do and give students opportunities to ask questions if anything is unclear. The implementation results showed that students’ creative thinking skills developed well for the ability to ask, suspect and improve the product. Based on the questionnaire results, the student self-confidence in good condition. The implementation of didactic designs 1 and 2 shows that students experienced problems completing several activities due to incorrectly entering formulas, so it needed to be revised by adding formulas to some student activities. Many students cannot complete some activities, so it needs to be added to the teacher's anticipation by providing examples of questions and guiding any completion steps that are not understood.

Based on the conclusions that have been described, there are several suggestions for improvements to this research: first; before giving the test, it is better to explain the related material so that students have provisions to study the material again, second; This didactic design was made for normal face-to-face conditions so that the anticipation of student and teacher responses did not occur optimally. It is necessary to make a didactic design using e-learning, third; the teacher’s explanation complements the design by providing examples and guiding students for each step of completion. In student activities, formulas are included to assist students in completing activities.

5. References
[1] J.P. Guilford 1967 The nature of human intelligence (New York: Mc Graw - Hill)
[2] Wessels H 2014 Levels of mathematical creativity in model-eliciting activities J. Math. Model. Appl. 19 22–40
[3] Bandura A 1977 Social Learning Theory (Oxford, England: Prentice-Hall)
[4] Affandi Y 2014 Remediasi Kesalahan Belajar Siswa Tentang Vektor Dengan Pemberian Booklet Disertai Umpam Balik kelas X J. Pembelajaran dan Pendidik. Khatulistiwa 37 1–11
[5] Brousseau G 2002 Theory of Didactical Situations in Mathematics (Dordrecht: Kluwer Academic Publisher)
[6] Plomp T 2007 An Introduction to Educational Design Research (Netzdruk, Enschede the Netherlands: SLO)
[7] The Alberta Teachers' Association 2014 Three Examples - Torrance Tests of Creative Thinking (TCT) Figural p. 1–3.
[8] Mulyana E, Turmudi and Juandi D 2014 Model Pengembangan Desain Didaktis Subject Specific Pedagogy Bidang Matematika Melalui Program Pendidikan Profesi Guru J. Pengajaran Mat. dan Ilmu Pengetah. Alam 19 2 141
[9] Nurjannah 2018 Teori Belajar dalam Pembelajaran Matematika. [Online] Available: http://file.upi.edu/Direktori/FPMIPA/JUR_PEND_MATHEMATIKA/196511161990012-NURJANAH/Teori_belajar.pdf. [Accessed: 06-Jan-2020]
[10] Rachmawati T and Daryanto 2015 Teori Belajar dan Proses Pembelajaran yang Mendidik (Yogyakarta: Gava Media)
[11] Nurjannah A 2013 Pembelajaran Matematika Berdasarkan Teori Konstruktivisme Sosial (Vygotsky) [Online]. Available: https://amalianurjannah.files.wordpress.com/2013/05/10-pembelajaran-matematika-berdasarkan-teori-konstruktivisme-sosial-1.pdf. [Accessed: 08-Jan-2020]
[12] Shadiq F 2011 Penerapan Teori Belajar dalam Pembelajaran Matematika di SD Modul Mat. SD Progr. BERMUTU Penerapan 70

Acknowledgements
I would like to thank the principal, teachers, and students who have helped with this research.