NON-THERMAL EMISSIONS FROM COOL CORES HEATED BY COSMIC RAYS IN GALAXY CLUSTERS

YUTAKA FUJITA and YUTAKA OHIRA
1

1 Department of Earth and Space Science, Graduate School of Science, Osaka University, 1-1 Machikaneyama-cho, Toyonaka, Osaka 560-0043, Japan; fujita@vega.ess.sci.osaka-u.ac.jp
2 Theory Centre, Institute of Particle and Nuclear Studies, KEK, 1-1 Oho, Tsukuba 305-0801, Japan

Received 2011 September 18; accepted 2011 November 16; published 2012 January 24

ABSTRACT

We study non-thermal emissions from cool cores in galaxy clusters. We adopted a recent model in which cosmic rays (CRs) prevail in the cores and stably heat them through CR streaming. The non-thermal emissions come from the interaction between CR protons and intracluster medium (ICM). Comparison between the theoretical predictions and radio observations shows that the overall CR spectra must be steep, and most of the CRs in the cores are low-energy CRs. Assuming that the CRs are injected through active galactic nucleus activities, we study the nature of the shocks that are responsible for the CR acceleration. The steep CR spectra are likely to reflect the fact that the shocks travel in hot ICM with fairly small Mach numbers. We also study the dependence on the CR streaming velocity. The results indicate that synchrotron emissions from secondary electrons should be observed as radio mini-halos in the cores. In particular, low-frequency observations (e.g., LOFAR) are promising. However, the steepness of the spectra makes it difficult to detect non-thermal X-ray and gamma-ray emissions from the cores. The low-energy CRs may be heating optical filaments observed in the cores.

Key words: cosmic rays – galaxies: clusters: general – galaxies: clusters: intracluster medium – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Clusters of galaxies are filled with hot X-ray gas or intracluster medium (ICM) with temperatures of ~2–10 keV. While the radiative cooling time of the ICM is longer than the age of the universe in most of the region in a cluster, the exception is the core, which is \( r \lesssim 100 \) kpc from the cluster center. If there is no heating source, the ICM in the core cools and a flow toward the cluster center should develop (a cooling flow). However, X-ray observations have denied the existence of massive cooling flows in clusters, which suggests that the cores should be heated by some unknown sources (e.g., Ikebe et al. 1997; Makishima et al. 2001; Peterson et al. 2001; Tamura et al. 2001; Kaastra et al. 2001; Matsushita et al. 2002). Since active galactic nuclei (AGNs) are often found in the cores, they are often thought to be the heating sources (e.g., Churazov et al. 2001; Quilis et al. 2001; Brüggen & Kaiser 2002; Basson & Alexander 2003). X-ray observations have actually revealed the interaction between AGNs and the ambient ICM (e.g., Fabian et al. 2000; McNamara et al. 2000; Blanton et al. 2001; McNamara et al. 2001; Mazzotta et al. 2002; Fujita et al. 2002; Johnstone et al. 2002; Kempner et al. 2002; Takizawa et al. 2003; Fujita et al. 2004). However, even if AGNs can produce enough energy to heat the core, the energy must deliberately be transported to the surrounding ICM in the core. For example, conventional mechanical heating such as the dissipation of weak shocks and sound waves often cause thermal instabilities (Fujita & Suzuki 2005; Mathews et al. 2006). Therefore, strong turbulence may essentially be required to hold the instabilities for such heating mechanisms.

Cosmic rays (CRs) may be another channel of transporting energy to the ICM (e.g., Tucker & Rosner 1983; Rephaeli 1987; Rephaeli & Silk 1995; Colafrancesco et al. 2004; Pfrommer et al. 2007; Jubelgas et al. 2008). Specifically, CR streaming has been studied as an energy transport mechanism (Rephaeli 1979; Böhinger & Mollif 1988; Loewenstein et al. 1991; Guo & Oh 2008). In this mechanism, CR streaming in the ICM excites Alfvén waves. The CRs interact and move outward with the waves. The \( PdV \) work done by the CRs effectively heat the ICM. Recently, using numerical simulations, we showed that the CR streaming can stably heat the core for a long time (Fujita & Ohira 2011, hereafter Paper I). The reason for the stability is that the CR pressure is insensitive to changes in the ICM and that the density dependence of the heating term is similar to that of radiative cooling. Moreover, CRs can prevail in the entire core and the heating is not localized around the source. The CRs may be provided in the core not only by AGNs but also through pumping by turbulence (Enßlin et al. 2011).

In this paper, we study the non-thermal emission from the CRs that heat cool cores and the AGN activities that are responsible for the acceleration of the CRs. It is to be noted that non-thermal emissions from CR protons accelerated by AGNs in the cores have been studied by Fujita et al. (2007). However, they studied CR acceleration associated with a single AGN burst with an extremely large energy, and they did not consider the heating of the ICM by CR streaming. This paper is organized as follows. In Section 2, we explain our models on core heating and AGN activities that are responsible for the generation of CRs. In Section 3, we present the results of our calculations and compare them with observations. In Section 4, we discuss the implications of our results, and Section 5 is devoted to conclusions. We refer to protons as CRs unless otherwise mentioned.

2. MODELS

2.1. Cosmic-ray Distributions

In Paper I, we studied heating of a cool core by CRs injected through the activities of the central AGN. The CRs travel with Alfvén waves in the ICM. They amplify the waves, which heat the surrounding ICM. In this subsection, we briefly summarize the models to obtain CR and ICM distributions.
For simplicity, we assumed that the cluster is spherically symmetric. The flow equations are
\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0 ,
\]
(1)
\[
\frac{\partial (\rho u)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) = -\rho \frac{GM(r)}{r^2} - \frac{\partial}{\partial r} (P_e + P_c + P_B) ,
\]
(2)
\[
\frac{\partial e_g}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u e_g) = -P_e \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \kappa(T) \frac{\partial T}{\partial r} \right] - n_e^2 \Lambda(T) + H_{\text{st}} + H_{\text{coll}} ,
\]
(3)
\[
\frac{\partial e_c}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u e_c) = -P_e \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 D(\rho) \frac{\partial e_c}{\partial r} \right] - \Gamma_{\text{loss}} + \dot{S}_c ,
\]
(4)
where \( \rho \) is the gas density, \( u \) is the gas velocity, \( P_e \) is the gas pressure, \( P_c \) is the CR pressure, \( P_B \) is the magnetic pressure, \( G \) is the gravitational constant, \( M(r) \) is the gravitational mass within the radius \( r \), \( \kappa(T) \) is the coefficient for thermal conduction and \( T \) is the temperature, \( n_e \) is the electron density, \( \Lambda \) is the cooling function, \( H_{\text{st}} \) is the heating by CR streaming, \( H_{\text{coll}} \) is the heating by Coulomb and hadronic collisions, \( \dot{S}_c \) is the CR transport velocity, \( D(\rho) \) is the diffusion coefficient for CRs averaged over the CR spectrum, \( \dot{S}_\text{los} \) is the energy loss by Coulomb and hadronic collisions, and \( \dot{S}_c \) is the source term of CRs. Energy densities of the gas and the CRs are respectively defined as \( e_g = P_e/(\gamma_g - 1) \) and \( e_c = P_c/(\gamma_c - 1) \), where \( \gamma_g = 5/3 \) and \( \gamma_c = 4/3 \). In this paper, we do not treat models with thermal conduction, and thus \( \kappa = 0 \). The terms for radiative cooling \( \Lambda \), Coulomb collisions \( H_{\text{coll}} \), hadronic collisions \( H_{\text{coll}} \), diffusion \( D(\rho) \), and the energy loss \( \dot{S}_\text{los} \) are the same as those in Paper I.

The source term of CRs is given by \( \dot{S}_c \propto L_{\text{AGN}} \), where \( L_{\text{AGN}} \) is the energy injection rate from the AGN. We assume that \( L_{\text{AGN}} = \epsilon M c^2 \), where \( \epsilon \) is the parameter, \( M \) is the inflow rate of the gas toward the AGN, and \( c \) is the speed of light.

The CR transport velocity in Equation (4) is given by \( \ddot{u} = u + v_A \), where \( v_A = B/\sqrt{4\pi \rho} \) is the Alfvén velocity for a magnetic field \( B \), which evolves as \( B \propto \rho^{\gamma/3} \). The initial magnetic field at the cluster center is \( B_0 = 10 \mu G \). The wave energy \( U_A = \delta B^2/(4\pi) \), where \( \delta B \) is the magnetic field fluctuation, which is amplified by the \( Pdv \) work done by the CRs on Alfvén waves:
\[
\frac{\partial U_A}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 U_A \left( \frac{3}{2} u + v_A \right) \right] = \frac{u}{r} \frac{\partial U_A}{\partial r} - v_A \frac{\partial p_e}{\partial r} - H_{\text{st}} ,
\]
(5)
(McKenzie & Völk 1982; Börhringer & Morfill 1988). This equation is more correct than the one we adopted in Paper I (Equation (6) in that paper) because it is based on wave energy conservation. However, the results are not affected by this equation change (see Section 3). After the wave energy increases to \( U_A \sim U_M \), where \( U_M \) is the energy of the background magnetic field, the waves are expected to heat ICM through nonlinear damping (e.g., Ohira et al. 2009; Gargaté et al. 2010). Thus, we give the heating term for CR streaming as
\[
H_{\text{st}} = \Gamma v_A \frac{\partial p_e}{\partial r} ,
\]
(6)
(Völk et al. 1984; Kang & Jones 2006). We simply give \( \Gamma = U_A/U_M \) for \( U_A < U_M \) and \( \Gamma = 1 \) after \( U_A \) reaches \( U_M \). Note that if the wave damping is fast, the wave growth and decay may be balanced at \( U_A \ll U_M \).

2.2. Non-Thermal Emissions

Although by solving equations presented in Section 2.1 we can obtain the profile of the ICM and that of the CR pressure \( P_c(r) \) required to heat the core effectively (Paper I), we do not have information on the energy spectrum of the CRs. Thus, we need to specify the spectrum of the CRs to calculate the non-thermal emissions from the CRs.

We assume that the central AGN drives outgoing shock waves and form cocoons or bubbles inside them. In the following, we show a description of their evolution (position, velocity, and Mach number as functions of time). The shocks should inject CRs with varying efficiencies and spectra. At some moment this injection will be maximal (actually, this moment differs for different CR energy ranges). We only consider the efficiency and Mach number at this moment and fix these numbers by requesting them to reproduce the observed radio emission for the sake of simplicity, although there would be a more physical approach to calculate the injection evolution and the full injected CR spectrum, assuming the AGN energy release, timescale, and initial cocoon radius.

The CRs are accumulated in the core through the AGN activities. In Paper I, we studied continuous CR injection as a time average, although the supply of the CRs may be intermittent. Each activity of the AGN is approximated by an instantaneous explosion. Thus, the shock expands in the ICM like a supernova remnant in the Galaxy, and the shock velocity depends on the energy input from the AGN.

In Paper I, we obtained the profiles of the ICM density \( \rho(r) \), the temperature \( T(r) \), and the magnetic field \( B(r) \) at a given time. We approximate the density profile of the ICM by a power law:
\[
\rho_{\text{ICM}}(r) = \rho_{\text{in}}(r/r_{\text{in}})^{-\omega} ,
\]
(7)
where \( \rho_{\text{in}} \) is the ICM density at the inner boundary \( r_{\text{in}} = 5 \) kpc. Using a shell approximation (e.g., Ostriker & McKee 1988), the radius of the shock can be written as
\[
R_s = \xi \left( \frac{E_0}{\rho_{\text{in}} r_{\text{in}}^2} \right)^{1/(5-\omega)} \frac{r_{\text{st}}^{2/(5-\omega)}}{t_{\text{in}}} ,
\]
(8)
where
\[
\xi = \left( \frac{5-\omega}{2} \right)^{2} \frac{3}{4\pi} \frac{5}{9} \frac{\gamma_g - 1}{\gamma_g - 3 - \omega} \left( \gamma_g + 1 \right)^{1/(5-\omega)} .
\]
(9)
\( E_0 \) is the energy released by the AGN, and \( t_{\text{in}} \) is the time elapsed since the last energy input from the AGN. The velocity of the shock is given by
\[
V_e = \frac{d R_s}{dt_{\text{in}}} .
\]
(10)
The Mach number of the shock is given by \( M_s = V_e/c_s(R_s) \), where \( c_s \) is the sound velocity. Since we know the profile of the ICM temperature \( T(r) \), we can construct the profile of the sound velocity \( c_s(R_s) \). Therefore, if \( M_s \) and \( E_0 \) are given, the shock radius \( R_s \), velocity \( V_e \), and the time \( t_{\text{in}} \) that satisfy Equations (8) and (10) can be specified.

In reality, the spectrum of accelerated CRs at the shock may change during the expansion of the cocoon. The spectrum is
probably flat, when the cocoon is young, and the shock velocity and the Mach number are large. Then it gradually steepens as the Mach number decreases, and CR acceleration ceases when the Mach number approaches \( M_\ast \sim 1 \). However, we consider a typical Mach number \( M_\ast \) around which most CRs are accelerated. In other words, we consider a typical spectrum of CRs that are accelerated when the injection of CRs becomes maximal. We treat \( M_\ast \) and \( E_\ast \) as parameters. The shock radius, velocity, and age when \( M_\ast = M_\ast \) are \( R_\ast = R_\ast \), \( V_\ast = V_\ast \), and \( t_\ast = t_\ast \), respectively. We also assume that the spectrum of CRs that are just accelerated at \( r \sim R_\ast \) has the form of

\[
N(p, r) \propto p^{-x} e^{-p/p_{\text{max}}},
\]

where \( p \) is the CR momentum, \( x \) is the index, and \( p_{\text{max}} \) is the cutoff momentum of the CRs. Since we already know CR pressure \( P_r(r) \), the normalization of relation (11) is determined by the relation

\[
P_r(r) = \frac{c}{3} \int_{P_{\text{max}}}^{\infty} \frac{p^2 N(p, r)}{\sqrt{p^2 + m_e c^2}} dp,
\]

where \( m \) is the proton mass.

The index is given by \( x = (r_b + 2)/(r_b - 1) \), where \( r_b \) is the compression ratio of the shock (Blandford & Eichler 1987), which is given by

\[
r_b = \frac{(y_\ast + 1) M_\ast^2}{(y_\ast - 1) M_\ast^2 + 1}.
\]

Since the cooling of CR protons is not effective, the maximum energy of protons corresponding to \( p_{\text{max}} \) is determined by the age of the shock and is represented by

\[
E_{\text{max}} \sim 1.6 \times 10^4 \left( \frac{V_\ast}{10^3 \text{ km s}^{-1}} \right)^2 \left( \frac{B_d}{10 \mu \text{ G}} \right) \left( \frac{t_\ast}{10^7 \text{ yr}} \right) \text{ TeV},
\]

where \( B_d \) is the downstream magnetic field at \( r = R_\ast \), and is given by \( B_d = r_B \) (Yamazaki et al. 2006; Fujita et al. 2007). That is, the background magnetic field \( B \) is amplified by the compression ratio \( r_b \) (Equation (13)). Although some particles may be accelerated to higher energies when the expansion velocity of the cocoon was larger, their contribution to the overall spectrum is expected to be small.

The CRs injected at \( r \sim R_\ast \) propagate in the ICM with Alfvén waves. Although adiabatic cooling may change \( E_{\text{max}} \), it does not change the index \( x \) in relation (11). Moreover, the results in Section 3 show that the CR spectra must be steep. Thus, the results are not sensitive to the value of \( E_{\text{max}} \). Therefore, we do not consider the adiabatic cooling for \( E_{\text{max}} \) and adopt relations (11) and (12) at any radius \( r \), although adiabatic cooling was considered when we calculated \( P_r \) in Paper I. Since we expect that thermal protons with higher energies are accelerated as CRs, we assume that the minimum momentum of the CRs is \( p_{\text{min}} = 4mc_{\text{sd}}^2 \), where \( c_{\text{sd}} \) is the sound velocity of the ICM downstream of the shock at \( r = R_\ast \), which is obtained from the Rankine–Hugoniot relations for a given \( c_s(r_\ast) \) and \( M_\ast \):

\[
c_{\text{sd}} = c_s \frac{2\gamma_\ast M_\ast^2}{(\gamma_\ast - 1)\sqrt{\gamma_\ast - 1}M_\ast^2 + 2(\gamma_\ast + 1)M_\ast^2}.
\]

In this way we have the CR spectrum at each radius for a given \( M_\ast \) and \( E_\ast \).

For a given CR proton spectrum, we calculate the radiation from it. Non-thermal emission originating from CR protons in the central region of clusters has been studied by several groups (e.g., Miniati 2003; Pfrommer & Enßlin 2004; Colafrancesco & Marchegiani 2008; Keshet & Loeb 2010). In this paper, we adopt the model of Fujita et al. (2009) in which they calculated non-thermal emissions from supernova remnants. We consider the synchrotron, bremsstrahling, and inverse Compton (IC) emissions from secondary electrons created through the decay of charged pions that are generated through proton–proton collisions. IC emissions are created by electrons that scatter cosmic microwave background (CMB) photons. We also consider \( \pi^0 \)-decay gamma rays through proton–proton collisions. We do not consider emissions from primary electrons that are directly accelerated at the shock, because we did not calculate the distribution of the primary electrons in Paper I. Because of the short cooling time of electrons, emissions from primary electrons will disappear soon after their acceleration is finished (Fujita et al. 2007).

The photon spectra are calculated based on the radiation models of Fang & Zhang (2007). For the production of secondary electrons and \( \pi^0 \)-decay gamma-ray photons through proton–proton interactions, we use the code provided by Karlsson & Kamae (2008). The spectrum of the secondary electrons is given by

\[
N_e(E_e) = \frac{\gamma_e}{Q_e} \frac{E_e}{t_{\text{cool},e}(E_e)} \frac{Q_e(E_e)}{Q_e},
\]

where \( E_e \) is the electron energy, \( t_{\text{cool},e} \) is the cooling time of an electron, and \( Q_e \) is the production rate of the secondary electrons. For the cooling, we include synchrotron cooling, IC scattering, bremsstrahling, and Coulomb loss.

3. RESULTS

Since we replaced the equation for the wave energy \( U_h \) (Equation (6) in Paper I) with Equation (5), we recalculate the distributions of the ICM and CRs and show them in Figures 1 and 2. The cluster is initially isothermal with \( P_r = 0 \). The input parameters are the same as those of Model LCR0 in Paper I, and we simply refer to this model as LCR0 again. The gravitational
Ea (solid line), and non-thermal bremsstrahlung (dashed line) are of the secondary thermal bremsstrahlung is shown by the dot-dashed line. Observations are for potential adopted in this model is that of the Perseus cluster. The ICM temperature outside the core is \( 7 \text{ keV} \). The heating by CR streaming and the synchrotron radiation (dotted line), IC scattering (solid line), and non-thermal bremsstrahlung (dashed line) are of the secondary electrons. The \( \pi^0 \)-decay gamma rays are shown by the two-dot-dashed line. The thermal bremsstrahlung is shown by the dot-dashed line. Observations are for the Perseus cluster. Radio observations are shown by dots (Sijbring 1993; Gitti et al. 2002), and gamma-ray upper limits are shown by arrows (Ackermann et al. 2010; Aleksić et al. 2010).

(A color version of this figure is available in the online journal.)

Figure 2. Profiles of the ratios \( P_c/P_g \) (solid) and \( P_B/P_g \) (dotted) at \( t = 9 \) Gyr for Model LCR0.

The synchrotron radiation (dotted line), IC scattering (solid line), and non-thermal bremsstrahlung (dashed line) are of the secondary electrons. The \( \pi^0 \)-decay gamma rays are shown by the two-dot-dashed line. The thermal bremsstrahlung is shown by the dot-dashed line. Observations are for the Perseus cluster. Radio observations are shown by dots (Sijbring 1993; Gitti et al. 2002), and gamma-ray upper limits are shown by arrows (Acknowledgment et al. 2010; Aleksić et al. 2010).

(A color version of this figure is available in the online journal.)

Figure 3. Spectra calculated based on Model LCR0 with \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \). The synchrotron radiation (dotted line), IC scattering (solid line), and non-thermal bremsstrahlung (dashed line) are of the secondary electrons. The \( \pi^0 \)-decay gamma rays are shown by the two-dot-dashed line. The thermal bremsstrahlung is shown by the dot-dashed line. Observations are for the Perseus cluster. Radio observations are shown by dots (Sibbing 1993; Gitti et al. 2002), and gamma-ray upper limits are shown by arrows (Ackermann et al. 2010; Aleksić et al. 2010).

(A color version of this figure is available in the online journal.)

Figure 4. Same as Figure 3 but for (a) \( M_{st} = 1.8 \) and (b) \( 4.0 \).

(A color version of this figure is available in the online journal.)

the ICM density profile is assumed to be \( \omega = 1 \), which is a good approximation for \( r \leq 70 \text{ kpc} \) (Figure 1(b)). The distance to the cluster is 78.4 Mpc, which is the one for the Perseus cluster. In this figure, we take \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \); we first give \( E_a \), and then adjust \( M_{st} \) in order to be consistent with radio observations for the mini-halo in the Perseus cluster (Sijbring 1993; Gitti et al. 2002). Since the Mach number \( M_{st} \) is fairly small, the CR spectrum is steep (\( x = 2.2 \)). The maximum energy of the CRs is \( E_{max} = 1.5 \times 10^5 \text{ TeV} \), the radius and age of the shock are \( R_{st} = 22 \text{ kpc} \) and \( t_{st} = 6.0 \times 10^6 \text{ yr} \), respectively. The spectrum of thermal bremsstrahlung is shown for comparison.

The slope of the synchrotron and IC scattering spectra at the higher energy side can be explained by a simple calculation. The slope of the energy spectrum of secondary electrons is the same as that of protons (\( x = 3.2 \)) if radiative cooling is not effective. However, cooling by synchrotron radiation and IC scattering increases the slope by one and it becomes \( x' = 4.2 \) (e.g., Sarazin 1999). The spectral indices of the synchrotron emission and IC scattering are represented by \( \alpha = (x' - 1)/2 = 1.6 \) (e.g., Rybicki & Lightman 1979), which are consistent with those in Figure 3 (\( f_\nu \propto \nu^{x'} \)).

We found that the results for \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{61} \text{ erg s}^{-1} \) are almost the same as those for \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \). In the former case, the shock radius and age are \( R_{st} = 59 \text{ kpc} \) and \( t_{st} = 1.4 \times 10^7 \text{ yr} \), respectively. Although the maximum energy of the CRs is increased (\( E_{max} = 2.7 \times 10^5 \text{ TeV} \)), the steep CR spectrum or the large \( x \) obscures the effect. This means that the radiation from the cool core is insensitive to the strength of an AGN activity (\( E_a \) for a given \( P_c(r) \)). Figure 4 shows the spectra when \( M_{st} = 1.8 \) and \( 4.0 \) for \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \). The indices are \( x = 3.8 \) and 2.3, respectively. Compared with the results of \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \), the non-thermal emissions are weaker (stronger) when \( M_{st} \) is smaller (larger), and the synchrotron radio emission is inconsistent with the observations. The results are very sensitive to the value of \( M_{st} \) for a given \( P_c(r) \). Basically, changing \( E_a \) and \( M_{st} \) correspond to changing \( E_{max} \) and \( x \), respectively.

Figure 5 shows the surface brightness profiles for Model LCR0 at \( t = 9 \text{ Gyr} \) with \( M_{st} = 2.1 \) and \( E_a = 1 \times 10^{60} \text{ erg s}^{-1} \). The model generally reproduces the surface brightness profile observed in the radio band, although we did not intend to reproduce that when we calculated Model LCR0 in Paper I. In that figure, the surface brightness rapidly increases toward the cluster center for the synchrotron radio emissions because of the increase of the magnetic fields toward the cluster center (\( B(r) \propto \rho(r)^{2/3} \)). On the other hand, the profile for the...
IC emissions is relatively flat because electrons scatter CMB photons, which are uniformly distributed. The size of the region with high surface brightness is regulated by radiative cooling, because radiative cooling increases the ICM density and makes a cool core. On the other hand, CRs can fill the entire core with fast Alfvén waves (Paper I). Proton–proton interactions are effective in such a high-density region. The surface brightness for thermal bremsstrahlung slightly decreases at the cluster center because radiative cooling increases the ICM density and makes a cool core. We also study a less massive cluster. Figure 6 shows the spectra of the entire core ($r < 1$ Mpc) at $t = 9$ Gyr calculated using parameters of Model SCR0 in Paper I. For this model, we adopted the observed gravitational potential of the Virgo cluster. The efficiency of AGN energy input is $\epsilon = 1 \times 10^{-4}$. Although we recalculated the ICM and CR distributions, they are almost identical to those calculated in Paper I. The ICM temperature outside the core is $\sim 2$ keV. We take $M_{\text{IC}} = 2.1$ ($x = 3.2$) and $E_a = 1 \times 10^{59}$ erg s$^{-1}$. The distance to the cluster is set to be $16$ Mpc. We take $\omega = 0.7$, which is a good approximation for $r \lesssim 50$ kpc (Figure 11 in Paper I). The shock radius and age are $R_s = 21$ kpc and $t_{\text{sh}} = 6.3 \times 10^9$ yr, respectively. The maximum energy of CRs is $E_{\text{max}} = 3.3 \times 10^4$ TeV. The gamma-ray flux is much smaller than the upper limits for the Virgo cluster obtained with Fermi (Ackermann et al. 2010). The luminosity is sensitive to $M_\nu$ but not to $E_\nu$. The surface brightness profiles for this model are shown in Figure 7. The surface brightness is smaller than that in Figure 5.

4. DISCUSSION

We have studied the non-thermal spectra of cool cores heated by CR streaming. The results indicate that the Mach number of the shock that accelerate CRs must be small ($\sim 2$) to be consistent with radio observations at least for the Perseus cluster. We think that this is reasonable because the temperature and the sound velocity of the ICM is large and thus it is difficult for the cocoon shock to have a large Mach number. The small Mach number means that the CR spectrum must be steep. In Paper I, we did not specify the injection mechanism of CRs. We emphasize that even if CRs are injected by anything other than the cocoon, the spectrum must be steep for the given $P_c$.

Recently, Enßlin et al. (2011) indicated that the CR streaming velocity may be much larger than the Alfvén velocity $v_A$ in the hot ICM. This is because in high-$\beta$ plasma, where $\beta$ is the ratio of thermal to magnetic energy, waves may suffer stronger resonant damping by thermal protons. In this case, the sound velocity $c_s$ may be appropriate as the streaming velocity instead of $v_A$ (Holman et al. 1979; Enßlin et al. 2011). Thus, we simply replace $v_A$ by $c_s$ in Equations (4) and (5) and see what would happen. Figures 8 and 9 show the profiles of the ICM and CRs for the parameters of Model LCR0 except for the larger streaming velocity $c_s$; we refer to this model as Model LCRs. The ICM is stably heated by the CR streaming even in this model and the evolution of $M$ is not much different from that in Model LCR0. Compared with Figure 2, the fraction of CR pressure is small in the central region because of the larger streaming velocity and the escape of CRs. Since the ICM temperature is an increase function of radius (Figure 8), the sound velocity or the streaming velocity is also an increasing function. Thus, $P_c/P_g$
Figure 8. Same as Figure 1 but for the Model LCRs.
(A color version of this figure is available in the online journal.)

Figure 9. Same as Figure 2 but for Model LCRs.
(A color version of this figure is available in the online journal.)

tends to decrease outward fairly rapidly. Figures 10 and 11 are the same as Figures 3 and 5 but for Model LCRs. If we assume \( M_{st} = 2.1 \) as in the case of Model LCR0, non-thermal luminosities in Model LCRs are smaller than those in Model LCR0, because more CRs have escaped from the core with the high ICM density. Thus, we increase the Mach number and set it to be \( M_{st} = 2.4 \). We present the spectra and surface brightness in Figures 10 and 11. The synchrotron spectrum and the surface brightness are consistent with the observations.

Regardless of the streaming velocity, the CR spectra in the cores must be steep, because if not, the luminosities are too large (Figure 4(b)); this is inconsistent with the small number of clusters in which radio mini-halos have been observed (Govoni et al. 2009) and the non-detection of gamma rays from clusters. Because of the steep spectra, future observations in the low-frequency radio band would be useful. Thus, cool cores would be promising targets for radio telescopes such as LOFAR. The number of mini-halos may increase as the sensitivities of radio telescopes are improved. In our model, we assumed that CRs are mostly accelerated when the Mach number of the shock is \( M_s \sim M_{st} \). For real clusters, however, we expect that the Mach number \( M_s \) decreases and that the CR spectrum at the shock steepens during the expansion of a cocoon. Thus, we expect that the spectral index should increase outward in the cluster, which has actually been observed in the radio band (Sijbring 1993; Gitti et al. 2002), although the interferometric nature of these measurements might result in smaller radio halos at higher frequencies (missing zero spacing problem). On the other hand, observations in other bands would be difficult in the near future (Figures 3 and 6). In the X-ray band, IC emissions should be observed (Figures 3 and 6). However, thermal emissions from cool cores are very bright, which makes it difficult for the non-thermal emission to be detected. For the Perseus cluster, Sanders et al. (2005) claimed the detection of non-thermal emission with a flux of \( 6.3 \times 10^{-11} \) erg cm\(^{-2}\) s\(^{-1}\) between 2 and 10 keV. However, the detection was not confirmed by later observations (Molendi & Gastaldello 2009; Eckert & Paltani 2009). Even with hard X-ray telescopes that will be launched in the near future such as NuSTAR and Astro-H, the detection may be difficult because of the low surface brightness (Figures 5 and 7). For the detection in the gamma-ray band, good angular resolutions as well as sensitivities are required, because gamma rays could also be emitted from the central AGNs (e.g., Abdo et al. 2009; Kataoka et al. 2010), which must be resolved.

The steep CR spectra mean that most of the CRs in cool cores have low energies. Thus, indirect studies may be useful. For example, optical filaments observed in cool cores may be heated by those CRs (see Ferland et al. 2009; Bayet et al. 2010).
We note that the CR heating is locally unstable, and that the filaments could be created through local thermal instabilities (Paper I). Moreover, our model does not require turbulence for stable heating. Thus, cool cores in which strong turbulence is not developing may be observed with detectors having high spectral resolutions such as Astro-H, while the detection of turbulence does not deny our model. Although we did not consider primary electrons, they may be accelerated at shocks in cores in spite of the low Mach numbers (Matsukiyo et al. 2011), and the emissions from them may be observed in some clusters.

Finally, we caution the reader that we did not consider energy-dependent diffusion of CRs because we do not know the actual diffusion coefficient in the ICM, especially away from the shock front (see Fujita et al. 2011). If CRs with higher energies escape from the core faster than those with lower energies, the energy spectrum could be steep (e.g., Fujita et al. 2009; Ohira et al. 2010, 2011). Moreover, we did not include the contribution of gamma rays from CRs accelerated at cosmological shocks and those from dark-matters (e.g., Totani 2004; Pinzke & Pfrommer 2010; Pinzke et al. 2011).

### 5. CONCLUSIONS

We have investigated non-thermal emissions from cool cores heated by CRs. For the distributions of CRs, we used the model in which the cores are stably heated by CR streaming. CR protons interact with ICM protons and produce secondary electrons and \( e^0 \)-decay gamma rays. We found that the CR spectra must be steep in order to be consistent with observations of a radio mini-halo. The steep spectra reflect the fact that the CRs are accelerated at shocks with low Mach numbers (~2) in hot ICM. We have also studied the dependence on the CR streaming velocity and found that the stronger shocks are required to be consistent with the observations for the larger CR streaming velocity. Since most of the CRs in cores have low energies, synchrotron emissions from them should be observed in low-frequency radio bands. Thus, the number of clusters that have radio mini-halos would increase as the sensitivities of radio telescopes increase. On the other hand, the detection in other bands such as the X-ray and gamma-ray bands would be difficult in the near future. The low-energy CRs could be studied by observing optical filaments that are often found in cool cores.

We thank the referee for useful comments. This work was supported by KAKENHI (Y.F.: 23540308, Y.O.: 21684014).