Inflation and dark energy arising from geometrical tachyons

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We study the motion of a BPS D3-brane in the NS5-brane ring background. The radion field becomes tachyonic in this geometrical set up. We investigate the potential of this geometrical tachyon in the cosmological scenario for inflation as well as dark energy. We evaluate the spectra of scalar and tensor perturbations generated during tachyon inflation and show that this model is compatible with recent observations of Cosmic Microwave Background (CMB) due to an extra freedom of the number of NS5-branes. It is not possible to explain the origin of both inflation and dark energy by using a single tachyon field, since the energy density at the potential minimum is not negligibly small because of the amplitude of scalar perturbations set by CMB anisotropies. However geometrical tachyon can account for dark energy when the number of NS5-branes is large, provided that inflation is realized by another scalar field.

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I. INTRODUCTION

The dynamics of time dependent backgrounds in string theory has been a challenging problem for long time. Recent progress on tachyon condensation by Sen ¹ has been very useful for studying such time-dependent backgrounds. The rolling tachyon played an important role in studying the decay of unstable D-branes as well as annihilation of brane-antibrane pairs. The Dirac-Born-Infeld (DBI) action ² was used, as an effective field theory description, for the dynamics of this tachyon. It was observed that the tachyon condenses, the unstable brane or the brane-antibrane pair can decay to form a new stable D-brane. This observation has given rise to the original idea of tachyon cosmology with the hope that the open string tachyon on the unstable brane can be the scalar field driving inflation ³ ⁴ (see also Refs. ² for the application of tachyon in cosmology). This idea was also generalized to the radion field, in the case of a brane moving towards an antibrane and vice versa. ⁵. But problems like incompatibility of slow-roll, too steep potential and reheating plagued the development of tachyon cosmology, though some of them could be solved via warped compactification ⁶ ⁷ of string theory. Inspite of several attempts to overcome the problems in open string tachyon cosmology, it seems unlikely that this tachyon field is responsible for inflation. Instead it is more likely to play a role as dark matter fluid ⁸.

Recently the DBI action has found a prominent role in the study of a different time dependent background. It describes the dynamics of a D-brane in the background of k coincident NS5-branes ⁹ where the D-brane is effectively a probe brane i.e., it probes the background without disturbing it. The reason behind this phenomena is that while the tension of the NS5-brane goes as 1/gs2 the tension of the D-brane goes as 1/gs where gs is the string coupling. Thus NS5-branes are much heavier than the D-branes in the regime of small string coupling. Geometrically this means that the NS5-branes form an infinite throat in space-time and the string coupling increases as we move towards the bottom. Being lighter, the probe brane is gravitationally pulled towards the NS5-branes. Since the D-brane preserves half of the supersymmetry which is different from the other half preserved by the NS5-branes in Type-II theory, as the probe brane comes nearer to the source brane the configuration becomes unstable. The radion becomes tachyonic and is the source for the instability. Kutasov showed that there is a map between the tachyonic radion field living on the world volume of the probe brane and the rolling tachyon associated with a non-BPS D-brane and thus the motion of the probe brane in the throat could be described by the condensation of the tachyon. The probe brane thus decays into tachyonic matter with a pressure that falls off exponentially to zero at late times. Furthermore, it was shown, by compactifying one of the transverse direction to the source branes, it is possible to obtain a potential which resembles the potential obtained by the use of techniques of string field theory. Thus it is believed that the tachyon has a geometrical origin.

The above situation has been extended to a different configuration of the source brane in Refs. ¹¹ ¹². In this set-up, these authors examined the motion of the probe brane in the background of a ring of NS5-branes instead of coincident branes and obtained several interesting solutions for the probe brane in the near horizon (throat) approximation. It is observed that the radion field becomes tachyonic when the probe brane is confined to one dimensional motion inside the ring. Following the cosmological applications of the original tachyon conden-
sation, the condensation of the geometrical tachyon has also naturally played a role in cosmology \[13\]. In this paper, we shall carry out detailed analysis for the role played by the geometrical tachyons in cosmology following the analysis of \[14\]. We evaluate the spectra of density perturbations generated in geometrical tachyon inflation and place strong constraints on model parameters by confronting them with recent observational data. We also show an interesting possibility to use geometrical tachyon as a source for dark energy.

This paper is organized as follows. In the next section we discuss the basic set-up for geometrical tachyon. In Sec. III we shall study inflation based upon geometrical tachyon and evaluate the spectra of scalar and tensor perturbations generated in geometrical tachyon inflation by confronting them with recent observational data. We also show an interesting possibility to use geometrical tachyon as a source for dark energy. We summarize our results in the final section.

II. GEOMETRICAL TACHYON

We begin with the background fields around \( k \) parallel NS5-branes of type II string theory. In this case the metric is given by \[1\]

\[
ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + F(x^n) dx^m dx^m,
\]

where the dilaton field \( \chi \) is defined as \( e^{2(\chi - \chi_0)} = F(x^n) \) and the three form field strength associated with the NS two form potential is \( F_{mnp} = \varepsilon_{mnp} \partial_\chi. \) Here \( F(x^n) \) is the harmonic function describing the position of the five-branes. For the five-branes at generic positions \( x_1, \ldots, x_k \), the harmonic function is found to be

\[
F = 1 + \ell_s^2 \sum_{j=1}^{k} \frac{1}{|x - x_j|^2},
\]

where \( \ell_s = \sqrt{\alpha'} \) is the string length. The five-branes are stretched in the directions \( (x^0, x^1, \ldots, x^5) \) and are localized in the \( (x^6, \ldots, x^9) \) directions. For coincident five-branes an SO(4) symmetry group of rotations around the five-branes is preserved. This gives rise to a throat geometry. But we are interested in considering the geometry obtained from the extremal limit of the rotating NS5-brane solution as discussed in \[14\] where the branes are continuously distributed along a ring of radius \( R \), which is oriented in the \( x^6 - x^7 \) plane in the transverse space. The above SO(4) symmetry is thus broken. The full form of the harmonic function in the throat region is then given by

\[
F = 1 + \frac{k \ell_s^2}{2 R \rho \sinh(y)} \left( \frac{\sinh(ky)}{\cosh(ky) - \cos(k\theta)} \right),
\]

where \( \rho, \theta \) parameterize the coordinates in the ring plane i.e. \( x^6 = \rho \cos(\theta), x^7 = \rho \sin(\theta) \) and the factor \( y \) is given by

\[
\cosh(y) = \frac{R^2 + \rho^2}{2 R \rho}.
\]

We introduce a probe \( Dp \)-brane at the center of the circle in the background of NS5-branes (see Fig. 1). As mentioned in the introduction, the probe brane will be attracted towards the ring of NS5-branes due to gravitational interaction. The dynamics of the probe brane can be described by the action (see \[10,11,12\] for details):

\[
S = -\tau_p \int d^{p+1} \zeta \sqrt{F^{-1} - (\dot{\rho}^2 + \dot{\theta}^2)},
\]

where \( \tau_p \) is the brane tension and \( \zeta \) is the world-volume directions of the probe brane. Here \( \sigma \) is the radial coordinate for \( x^8 \) and \( x^9 \) when expressed in polar coordinates.

The \( Dp \) brane will be pulled towards the circumference of the circle and for simplicity we consider the motion in the plane of the circle (\( \sigma = 0 \)). As discussed earlier, the radion field \( \rho \) becomes tachyonic and the action \[4\] gets mapped to a well known action describing the dynamics of the tachyon field living on the world volume of a non-BPS brane in type II string theory:

\[
S = -\int d^{p+1} \zeta \sqrt{V(\phi)} \sqrt{1 - \dot{\phi}^2},
\]

where the tachyon map is given by

\[
\phi(\rho) = \int \sqrt{F} d\rho,
\]

and the tachyon potential is given by

\[
V(\phi) = \frac{\tau_p}{\sqrt{F}}.
\]

In what follows we shall consider the case of \( p = 3 \). We look for the motion of \( D3 \) brane far away from the circumference of the circle where NS5-branes are distributed (\( \ell > 2\pi R/k \) -- distances much larger than brane separation). In this case the expression of harmonic function simplifies to

\[
F = \frac{k \ell_s^2}{\sqrt{(R^2 + \rho^2 + \sigma^2)^2 - 4 R^2 \rho^2}}.
\]

The motion of the probe brane in the plane of ring corresponds to \( \sigma = 0 \). In this case the Harmonic function yields

\[
F = \frac{k \ell_s^2}{R^2 - \rho^2}.
\]

From Eqs. \[3\] and \[9\] we obtain

\[
\phi(\rho) = \sqrt{k \ell_s^2} \arcsin(\rho/R),
\]
Most of the contribution to inflation comes from the top of the potential corresponding to the motion of D3 brane near the center of the circle.

A throat approximation is used to get the exact expression above. This demands the following condition

$$\sqrt{k l_s} \gg R.$$  \hspace{1cm} (14)

We note that the reduced Planck mass, $M_p = 1/\sqrt{8\pi G}$, is related with the string mass scale, $M_s = 1/l_s$, via the dimensional reduction:

$$M_p^2 = \frac{v M_s^2}{g_s^2},$$  \hspace{1cm} (15)

where $g_s$ is the string coupling parameter and

$$v \equiv \frac{(M_s l)^d}{\pi} = \frac{1}{\pi} \left( \frac{l}{l_s} \right)^d.$$  \hspace{1cm} (16)

Here $l$ and $d$ are the radius and the number of compactified dimensions, respectively. In order for the validity of the effective theory we require the condition $l_s \ll l$, which translates into the condition

$$v \gg 1.$$  \hspace{1cm} (17)

### III. INFLATION

In a flat Friedmann-Robertson-Walker (FRW) background with a scale factor $a$, the evolution equations are given by

$$H^2 = \frac{1}{3M_p^2} \frac{V(\phi)}{\sqrt{1 - \phi^2}},$$  \hspace{1cm} (18)

$$\frac{\dot{\phi}}{1 - \phi^2} + 3H\dot{\phi} + \frac{V_\phi}{V} = 0,$$  \hspace{1cm} (19)

where a dot denotes a derivative in terms of the cosmic time $t$ and $H \equiv \dot{a}/a$ is the Hubble rate. From the above equations we obtain

$$\frac{\ddot{a}}{a} = \frac{V(\phi)}{3M_p^2 \sqrt{1 - \phi^2}} \left( 1 - \frac{3}{2} \dot{\phi}^2 \right).$$  \hspace{1cm} (20)

Hence the inflationary phase ($\dot{a} > 0$) corresponds to $\dot{\phi}^2 < 2/3$. Since the energy density and the pressure density of tachyon are given by $\rho = V(\phi)/\sqrt{1 - \dot{\phi}^2}$ and $p = -V(\phi)/\sqrt{1 - \dot{\phi}^2}$, the equation of state is

$$w_\phi \equiv \frac{p}{\rho} = \dot{\phi}^2 - 1.$$  \hspace{1cm} (21)

### A. Background

By using a slow-roll approximation, $H^2 \simeq V(\phi)/3M_p^2$ and $3H\dot{\phi} \simeq -V_\phi/V$, the number of e-foldings, $N = \ln a$, is

$$N = \int_0^t H dt = \int_{\phi_f}^\phi \frac{V^2}{M_p^2 V_\phi} d\phi = s \left[ \cos x_f - \cos x + \ln \left( \frac{\tan(x_f/2)}{\tan(x/2)} \right) \right],$$  \hspace{1cm} (22)

where the subscript “$f$” represents the values at the end of inflation and

$$s = \frac{\tau_3 R}{M_p^2}, \quad x \equiv \frac{\phi}{\sqrt{k l_s^2}}, \quad x_f \equiv \frac{\phi_f}{\sqrt{k l_s^2}}.$$  \hspace{1cm} (23)

The slow-roll parameter, $\epsilon \equiv -\ddot{H}/H^2$, is given by

$$\epsilon = \frac{1}{2s} \frac{\sin^2 x}{\cos^3 x},$$  \hspace{1cm} (24)

It is convenient to introduce the following quantities:

$$y \equiv \cos x, \quad y_f \equiv \cos x_f.$$  \hspace{1cm} (25)

Then $N$ and $\epsilon$ can be expressed in terms of $y$:

$$N = s \left[ y_f - y + \frac{1}{2} \ln \left( \frac{1 - y_f}{1 + y_f} \frac{1 + y}{1 - y} \right) \right],$$  \hspace{1cm} (26)

and

$$\epsilon = \frac{1}{2s} \frac{1 - y_f^2}{y^3}.$$  \hspace{1cm} (27)
The end of inflation is characterized by $\epsilon = 1$, which gives

$$y_f \equiv f(s) = \frac{1}{6s} \left[ g(s) + \frac{1}{g(s)} - 1 \right],$$

(28)

where

$$g(s) \equiv [54s^2 - 1 + 6s\sqrt{3(27s^2 - 1)}]^{1/3}. \tag{29}$$

From Eq. (28), we have

$$\ln \frac{1 + y}{1 - y} - 2y = \frac{2N}{s} - 2f(s) - \ln \frac{1 - f(s)}{1 + f(s)}. \tag{30}$$

One can not obtain analytic expression for $y$ in terms of $s$ and $N$. In order to find $y$ as a function of $s$ for a fixed $N$, it is convenient to take the derivative of Eq. (30),

$$\frac{dy}{ds} = \frac{1 - y^2}{2s^2} \left[ \frac{f'(s)f^2(s)}{1 - f^2(s)} - \frac{N}{s^2} \right]. \tag{31}$$

From Eq. (31) one can find a value of $y$ for a given $s$. For example we have $y = 0.99505941$ for $s = 30$ with $N = 60$. Then we get $y(s)$ by numerically solving Eq. (31) for a fixed $N$. In Fig. 2 we show $y$ versus $s$ for $N = 50, 60, 70$. The function $y$ gets smaller for larger $s$, which means that inflation can be realized even if the field $\phi$ is not very close to the top of the potential.

### B. Perturbations

Let us consider a general perturbed metric about the flat FRW background [17]:

$$\text{ds}^2 = -(1 + 2\Lambda)\text{dt}^2 + 2a(t)B_{ij}\text{dx}^i\text{dx}^j + a^2(t)[(1 - 2\psi)\delta_{ij} + 2E_{,i,j} + 2h_{ij}]\text{dx}^i\text{dx}^j. \tag{32}$$

Here $A, B, \psi$, and $E$ correspond to the scalar-type metric perturbations, whereas $h_{ij}$ characterizes the transverse-traceless tensor-type perturbation. We introduce comoving curvature perturbations, $\mathcal{R}$, defined by

$$\mathcal{R} \equiv \psi + \frac{H}{\phi}\delta \phi, \tag{33}$$

where $\delta \phi$ is the perturbation of the field $\phi$.

By using a slow-roll approximation the power spectrum of curvature perturbations is estimated to be [18]

$$P_S = \left(\frac{H^2}{2\pi\phi}\right)^2 \frac{1}{Z_S}, \tag{34}$$

where $Z_S = V(1 - \phi^2)^{-3/2} \simeq V$. The spectral index of scalar perturbations is defined by $n_S - 1 \equiv \text{dln}P_S/\text{dln} k$, where $k$ is a comoving wavenumber. Then we obtain

$$n_S - 1 = 2(2\epsilon_1 - \epsilon_2 - \epsilon_3), \tag{35}$$

where

$$\epsilon_1 \equiv \frac{\dot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\dot{\phi}}{H\phi}, \quad \epsilon_3 \equiv \frac{\ddot{Z}_S}{2HZ_S}. \tag{36}$$

The amplitude of tensor perturbations is given by [18]

$$P_T = 8\left(\frac{H}{2\pi}\right)^2. \tag{37}$$

The spectral index, $n_T \equiv \text{dln}P_T/\text{dln} k$, is

$$n_T = 2\epsilon_1. \tag{38}$$

We obtain the tensor-to-scalar ratio, as

$$r = \frac{P_T}{P_S} = 8\frac{\dot{\phi}^2}{H^2}Z_S. \tag{39}$$

Using a slow-roll analysis the above quantities can be expressed by the slopes of the potential:

$$P_S = \frac{1}{12\pi^2M_p^2}\left(\frac{V^2}{V_{\phi\phi}}\right)^2, \tag{40}$$

$$n_S - 1 = -4\frac{M_p^2V_{\phi}^2}{V^3} + 2\frac{M_p^2V_{,\phi\phi}}{V^2}, \tag{41}$$

$$n_T = -\frac{V_{\phi}^2M_p^2}{V^3}, \tag{42}$$

$$r = 8\frac{V_{\phi\phi}^2}{V^3}, \tag{43}$$

where we reproduced the Planck mass for a later convenience. Equations [12] and 18 show that the same consistency relation, $r = -8\pi\Gamma$, holds as in the Einstein gravity [7, 19].

![FIG. 2: The function $y = \cos(\phi/\sqrt{M_p^2})$ versus the parameter $s$ given by Eq. (23). Each case corresponds to the number of e-foldings: $N = 70, 60, 50$ from top to bottom.](image-url)
C. Constraints on model parameters

For the potential \[ \Phi \equiv \frac{\sqrt{6}}{2} \left( \frac{\cos^2 x}{\sin x} \right) \]
the amplitude of scalar perturbations is given by

\[
P_s = \frac{k l_P^2 V_0^2}{12 \pi^2 M_p^6 \left( \frac{\cos^2 x}{\sin x} \right)} \frac{y^4}{s^2} = 12 \pi^2 k (l_s M_p)^2 \frac{y^4}{1 - y^2}. \tag{44}
\]

The COBE normalization corresponds to \( P_s \approx 2 \times 10^{-9} \) for the mode which crossed the Hubble radius 60 e-foldings before the end of inflation. Then this gives the following constraint:

\[
k(l_s M_p)^2 \approx 10^{9} \frac{s^2 y^4}{12 \pi^2 (1 - y^2)}. \tag{45}
\]

In Fig. 3 we plot \( k(l_s M_p)^2 \) in terms of the function of \( s \) for \( N = 60 \). This quantity has a minimum around \( s = 100 \), which gives a constraint

\[
k(l_s/l_p)^2 \gtrsim 10^{11}, \tag{46}
\]

where \( l_p = 1/M_p \) is the Planck length.

Let us consider the limiting case: \( s \gg 1 \). For a fixed value of \( N \), the r.h.s. of Eq. 45 approaches zero for \( s \to \infty \). Comparing this with the l.h.s. of Eq. 45, we find that \( y \to 0 \) for \( s \to \infty \). By carrying out a Taylor expansion around \( y = 0 \) and taking note that \( f(s) \) behaves as \( f(s) \approx (2s)^{-1/3} \), we get the relation \( y^3 \approx (3N + 1/2)/s \). Then we find

\[
k(l_s M_p)^2 \approx 10^9 \frac{s^2 y^4}{24 \pi^2 (3N + 1)^{4/3} s^{2/3}}. \tag{47}
\]

This means that \( k(l_s M_p)^2 \to \infty \) in the limit \( s \to \infty \).

From Eqs. 41, 42 and 43, we obtain

\[
\begin{align*}
    n_S - 1 &= -\frac{2}{s} \frac{2 - y^2}{y^3}, \\
    n_T &= \frac{11 - y^2}{s y^3}, \\
    r &= \frac{8}{s} \frac{1 - y^2}{y^3}.
\end{align*} \tag{49}
\]

For a fixed value of \( N \) one can know \( y \) in terms of the function of \( s \). In Figs. 4 and 5, we plot \( n_S \) and \( r \) versus \( s \) for several numbers of e-foldings. The spectrum of the scalar perturbations is red-tilted \( (n_S < 1) \). We find that \( n_S \) gets larger with the increase of \( s \). Recent observations show that \( n_S \) ranges in the region \( n_S > 0.93 \) at 2\( \sigma \) level \[20, 21\]. If the cosmologically relevant scales correspond to \( N = 60 \), the parameter \( s \) is constrained to be \( s > 29.3 \). This value is not strongly affected by the change of the number of e-foldings, since \( s > 30.0 \) for \( N = 50 \) and \( s > 28.9 \) for \( N = 70 \). Hence we have the following constraint:

\[
s \gtrsim 30. \tag{51}
\]

Since \( y \) behaves as \( y^3 \approx (3N + 1/2)/s \) in the limit \( s \to \infty \), Eqs. 45 and 50 yield

\[
\begin{align*}
    n_S &= 1 - \frac{8}{6N + 1}, \\
    r &= \frac{16}{6N + 1} \quad (s \to \infty). \tag{52}
\end{align*}
\]

This means that asymptotic values of \( n_S \) and \( R \) are constants. When \( N = 60 \), for example, we have \( n_S = 0.978 \) and \( r = 0.044 \). We checked that \( n_S \) and \( r \) actually approach these values in numerical calculations in Figs. 4 and 5.

The tensor-to-scalar ratio is constrained to be \( r < 0.36 \) at the 2\( \sigma \) level from recent observations \[21\]. The ratio \( r \) obtained in Eq. 52 corresponds to the maximum value, since \( r \) is a growing function with respect to \( s \) as illustrated in Fig. 6. Since \( r_{\max} = 0.053 \) even for \( N = 50 \), the tensor-to-scalar ratio is small enough to satisfy observational contour bounds for any values of \( s \). Hence tensor perturbations do not provide constraints on model parameters from current observations.

We note that the Hubble rate during inflation is approximately given by \( H \approx \frac{\tau_3 R}{(3 M_P^2 \sqrt{k l_s^2})^{1/2}} \). Meanwhile the effective mass squared of the tachyon is \( M^2 = V_{\phi \phi}/V = -1/k l_s^2 \). Hence under the condition \[51\] we find that

\[
|M| \lesssim H/\sqrt{10}. \tag{53}
\]

Then the tachyon mass is smaller than the Hubble rate, which is required to give rise to inflation.

Taking note that the brane tension \( \tau_3 \) is related with a string coupling \( g_s \) via \( \tau_3 = M_s^2/(2 \pi)^3 g_s \), the condition \[51\] gives

\[
g_s = s \frac{(2\pi)^3}{\sqrt{k R M_s}} \gtrsim 30 \frac{(2\pi)^3}{\sqrt{k R M_s}}, \tag{54}
\]
where we also used Eq. (14). In order for the effective theory of tachyon to be valid, we require that we are in a weak coupling regime ($g_s \ll 1$). Then one obtains the constraint: $\sqrt{k}RM_s \gg 10^4$. Combining this with the throat condition (11), we find

$$\sqrt{k}l_s \gg R \gg \frac{10^4}{\sqrt{k}}l_s.$$  \hfill (55)

This shows that the number of NS5 branes at least satisfies the condition

$$k \gg 10^4.$$  \hfill (56)

If we fix the parameter $s$, we can know the value $k(l_s/l_p)^2$ from the information of COBE normalization (see Fig. 5). For example one has $k(l_s/l_p)^2 \approx 10^{11}$ for $s = 100$. In this case the string length scale is constrained to be $l_s \approx 3 \times 10^4 l_p$ (or $M_s \approx 3 \times 10^{-4} M_p$) from Eq. (56). Since $l_s$ is expected to be larger than $l_p$, we also find that $k \lesssim 10^{11}$ from the condition $k(l_s/l_p)^2 \approx 10^{11}$. Hence the number of NS5-branes is constrained to be $10^3 \lesssim k \lesssim 10^{11}$ for $s = 100$.

Let us consider the energy scale of geometrical tachyon inflation. From Eqs. (13) and (23) we find

$$V_0 = \frac{s}{k(l_s M_p)^2 M_p^4}. \hfill (57)$$

In Fig. 6 we plot the energy scale $V_0^{1/4}$ as a function of $s$ for the $N = 60$ e-foldings before the end of inflation. For example one has $V_0 \approx 7.9 \times 10^{-11} M_p^4$ for $s = 30$ and $V_0 \approx 6.2 \times 10^{-10} M_p^4$ for $s = 100$.

In the limit $s \gg 1$ we have

$$V_0 = \frac{24\pi^2 (3N + 1)^{-4/3}}{10^9} s^{1/3} M_p^4. \hfill (58)$$

For the e-foldings $N = 60$ this is estimated as $V_0 \approx 10^{-9} s^{1/3} M_p^4$. Hence $V_0$ gets larger with the increase of $s$ as shown in Fig. 6. From the requirement $V_0 \lesssim M_p^4$ one finds that $s$ is constrained to be $s \lesssim 10^{27}$. Then by using Eq. (17) with e-foldings $N = 60$ we find the bound for the quantity $k(l_s M_p)^2$, as $k(l_s M_p)^2 \lesssim 10^{27}$. Hence together with Eq. (10), we obtain

$$10^{11} \lesssim k(l_s/l_p)^2 \lesssim 10^{27}. \hfill (59)$$

One can not take the limit $k \to \infty$ when we use geometrical tachyon for inflation because of the condition of COBE normalization. In the context of dark energy, however, we do not have the restriction coming from the perturbations. Actually the amplitude of density perturbations should be negligibly small in the latter case, which means that $10^9$ factor in Eq. (17) is replaced for a very large value. Then $k(l_s M_p)^2$ can be very large, thereby giving very small $V_0$ compared to the value obtained in inflation. In Sec. V we shall study the case in which geometrical tachyon is used for dark energy.

**IV. REHEATING**

In this section we shall study the dynamics of reheating for the geometrical tachyon. The tachyon potential has a minimum with a non-vanishing energy density at the ring of $NS5$-brane, as we see below. The field oscillates around the potential minimum with a decreasing amplitude by cosmic expansion.
By substituting this for Eq. (5), we find the following

Then the Harmonic function (4) is approximately given

Let us consider an

Here \( \phi_0 \) is an integration constant. Then the tachyon potential is

Here \( \phi_0 \) corresponds to \( \phi_0 = (\pi/2)\sqrt{k l_s^2} \).

The two potentials (13) and (62) can be connected at \( \rho = R \) by studying the case in which NS5-branes are not smeared out around the ring. Let us consider an expansion around \( \rho = R \), i.e., \( \rho = R + \xi \) with \( |\xi| \ll R \). By substituting this for Eq. (4), we find the following relation

Then the Harmonic function (4) is approximately given by

From Eq. (5) and (9) we obtain

and

By taking the first term in Eq. (66) we find

One can connect two potentials (13) and (67) at \( \phi = \phi_0 + \epsilon_1 \sqrt{k l_s^2} \). By matching the potentials with the continuity condition of \( V(\phi) \) together with that of \( dV(\phi)/d\phi \), we obtain

In deriving these values we used the condition (50), under which the terms including \( k \) in the denominator in Eq. (57) are neglected. Then the tachyon potential around \( \rho = R \) is approximately given by

Two potentials (62) and (67) can be also matched at \( \phi = \phi_0 + \epsilon_2 \sqrt{k l_s^2} \). One easily finds that \( \epsilon_2 = \epsilon_1 \) together with \( \cos(k\theta) = (-1 + \sqrt{6})/2 \) under the condition of Eq. (50).

The dynamics of reheating

The energy density at the potential minimum is given by Eq. (58). The ratio of \( V_1 \) and \( V_0 \) is

From Eq. (5) and (9) we obtain

and

where

FIG. 6: The energy scale of inflation, \( V_0^{1/4} \), as a function of \( s \) for the 60 e-foldings before the end of inflation.

A. Matching tachyon potentials

The tachyon potential (13) can be used inside the ring of the NS5-branes (\( \rho < R \)). Outside the ring (\( \rho > R \)) the harmonic function is given by

From Eq. (5) we find

where \( \phi_0 \) is an integration constant.
where we used Eq. (70). Hence the energy density $V_1$ is suppressed by the factor $1/\sqrt{k}$ compared to the energy scale of inflation.

In Fig. 7 we plot the tachyon potential for $s = 10^2$ and $k = 10^6$, which is obtained by using the matching condition given in the previous subsection. We recall that the energy scale $V_0$ is determined by COBE normalization, e.g., $V_0 = 6.2 \times 10^{-10} M_p^4$ for $s = 10^2$. When $k = 10^6$ the energy scale $V_1$ at the potential minimum is $10^{-3}$ times smaller than $V_0$. In Fig. 8 we plot the evolution of the tachyon field $\phi$ together with its equation of state $w_\phi$ for $s = 10^2$ and $k = 10^6$. The field oscillates around the potential minimum and eventually settles down at $\phi = (\pi/2)\sqrt{M_T^2}$. The equation of state of the tachyon approaches the one of cosmological constant ($w_\phi = -1$), as is illustrated in Fig. 8.

Unless the energy scale $V_1$ is negligibly small compared to $V_0$, this energy density comes to dominate the universe in radiation or matter dominant era, which disturbs the thermal history of the universe. We note that the number of branes is constrained to be $k \lesssim 10^{27} (M_s/M_p)^2$ from Eq. (79). For example one has $V_1 \simeq 3 \times 10^{-14} V_0$ for $M_s = M_p$. Since $V_0$ is larger than $10^{-10} M_p^4$ from Fig. 8, we find $V_1 \gtrsim 10^{-24} M_p^4$. This energy scale is still too high to explain the late-time acceleration of the universe which is observed today. This situation does not change much even if the string energy scale is smaller than $M_p$ to realize larger $k$ (We note that it is not natural to consider the case in which $M_s$ is far below $M_p$).

In the context of open-string tachyon it was shown in Ref. 22 that there is a violent instability of tachyon fluctuations for the potential with a minimum, e.g., $V(\phi) = (1/2)m^2(\phi - \phi_0)^2$. In a flat FRW background the each Fourier mode of the perturbation in $\phi$ satisfies the following equation of motion

$$\frac{\ddot{\phi}_k}{1 - \phi^2} + \left[ 3H + \frac{2\dot{\phi}\phi}{(1 - \phi^2)^2} \right] \dot{\phi}_k + \left[ \frac{k^2}{a^2} + (\ln V)_\phi \right] \phi_k = 0, \quad (73)$$

where $k$ is a comoving wavenumber. One easily finds that the $(\ln V)_\phi$ term exhibits a divergence at $\phi = \phi_0$ for the potential $V(\phi) = (1/2)m^2(\phi - \phi_0)^2$.

One may worry that there is a similar instability for geometrical tachyon around the potential minimum, but this is not the case. For the potential (71) we find

$$(\ln V)_\phi = 2c[1 - c(\phi - \phi_0)]^2 / [1 + c(\phi - \phi_0)^2]^2, \quad (74)$$

where $c = [2 + \cos(k\theta)]/6k^2$. This means that there is no divergence in the denominator at $\phi = \phi_0$, thereby showing the absence of a violent instability around the potential minimum.

In spite of the fact that the violent growth of perturbations can be avoided in our model, we need to find out a way to avoid that the tachyon energy density overdominates the universe after inflation. This may be possible if there exists a negative cosmological constant which almost cancels with the energy density $V_1$ as in Ref. 7.
V. DARK ENERGY

The discussion in sections III and IV shows that the energy scale at the potential minimum is too large not to disturb the thermal history of the universe if we use the information of density perturbations generated during inflation. Alternatively let us consider a scenario in which inflation is realized by a scalar field other than the geometrical tachyon. In this case we are free from the constraint coming from the COBE normalization. We recall that the COBE normalization is used in order to derive the upper limit of $k$ in Eq. (59). This upper bound for $k$ is the reason why the energy density at the potential minimum overdominates the universe during radiation or matter dominant era. If the geometrical tachyon is not responsible for inflation, we do not have such an upper limit for $k$. This shows that it may be possible to explain the origin of dark energy if the value of $k$ is very large.

In order to explain the origin of dark energy we require the condition

$$ V_1 \simeq 10^{-123} M_p^4. $$

By using Eq. (89) together with the matching condition, we find that $V_1 \simeq \tau_3 R/(kl_s)$. Hence the number of $NS5$-branes is constrained to be

$$ k \simeq 10^{123} \frac{\tau_3}{M_p^4} R / l_s, $$

When $\tau_3 = 10^{-10} M_p^4$ (around GUT scale) and $R = 10^2 l_s$, for example, one has $k = 10^{115}$. Since $R$ is at least greater than $l_s$ for the validity of the effective string theory, we find that $k \gtrsim 10^{123} \tau_3 / M_p^4$. Hence we require very large values of $k$ unless the brane tension $\tau_3$ is very much smaller than the Planck scale. We note that such large values of $k$ automatically satisfy the throat condition (14).

The energy scale $V_0$ is $\sqrt{k}$ times bigger than $V_1$ by Eq. (92). When $k = 10^{115}$ mentioned above, this corresponds to $V_0 \simeq 10^{-66} M_p^4$ (around TeV scale). Therefore the tachyon has a considerable amount of energy when it begins to roll down from the the top of the hill. However if the tachyon settles down at the potential minimum in the early universe (before the TeV scale), the energy density $V_1$ of the tachyon does not affect the thermal history of the universe until it comes out around present epoch as dark energy.

An important quantity which characterizes the amount of inflation is the quantity $s$ defined in Eq. (28). We found that $s$ is constrained to be $s \gtrsim 30$ from the observational data about the spectral index $n_S$ of scalar perturbations. The power spectrum approaches a scale-invariant one for larger $s$ (see Fig. 4). In the large $s$ limit the spectral index takes a constant value given by Eq. (79) ($n_S = 0.978$ for the e-foldings $N = 60$). The tensor to scalar ratio grows for larger $s$, but there is an upper limit $r_{\text{max}}$ given by Eq. (75). Since $r_{\text{max}} = 0.044$ for $N = 60$, this well satisfies the recent observational constraint: $r < 0.36$ at the 2$\sigma$ level.

The amplitude of scalar perturbations also places constraints on model parameters. We found that the number of $NS5$-branes satisfies the condition (10) for any value of $s$ (see Fig. 8). From the requirement that the energy scale during inflation does not exceed the Planck scale, we obtained an upper limit of the number of $NS5$-branes as given in Eq. (59). We note that the string coupling is weak ($g_s \ll 1$) provided that $\sqrt{k R / l_s} \gg 10^4$. Combining this with the throat condition $\sqrt{k l_s} \gg R$, we found another constraint: $k \gg 10^4$. This is well consistent with the condition (59).

In the vicinity of the ring of $NS5$-branes ($p \sim R$), the tachyon potential is approximately given as Eq. (72) by expanding a Harmonic function around the ring. This is connected to the potential inside and outside the ring by imposing matching conditions. We found that the energy scale at the potential minimum, $V_1$, is $1/\sqrt{k}$ times smaller than the energy scale, $V_0$, during inflation. If we use the constraint $k(l_s / l_p)^2 \lesssim 10^{27}$ coming from the condition for inflation, the energy scale $V_1$ can not be very small to explain dark energy. Although the tachyon exhibits oscillations after inflation, the equation of state approaches $w_\sigma = -1$ as illustrated in Fig. 5. The energy density of tachyon dominates during radiation or matter dominant era, thus disturbing the thermal history of the universe. Hence we need to find a way to reduce the energy density $V_1$ to obtain a viable tachyon inflation scenario.

Although it is difficult to explain both inflation and dark energy by using a single tachyon field, it is possible to make use of the geometrical tachyon scenario for dark energy provided that inflation is realized by another scalar field. In this case we do not have the upper limit given by Eq. (59), since the geometrical tachyon is not responsible for CMB anisotropies. We derived the condition (70) for the number of $NS5$-branes to realize the energy scale of dark energy observed today. Although $k$ is required to be very large to satisfy this constraint, it is interesting that the origin of dark energy can be explained by using the geometrical tachyon.

While we have considered the motion of the BPS $D3$-brane in the plane of the ring, it is possible to investigate the case in which its motion is transverse to the ring. Then the tachyon map yields a cosh-type potential, in which case we do not need to worry for the continuity condition around the ring. Since there remains an energy

VI. CONCLUSIONS

In this paper we have studied geometrical tachyon based upon the movement of a BPS $D3$-brane in the $NS5$-brane ring background. This model gives rise to a cos-type potential, which can lead to inflation as the tachyon rolls down toward a potential minimum. We carried out a careful analysis for the dynamics of inflation and resulting density perturbations.
density $\tau_3 R / \sqrt{k_3}$ at the potential minimum ($\phi = 0$), we expect that cosmological evolution may not change much compared to the model discussed in this paper. Nevertheless it is certainly of interest to investigate this case in more details to distinguish between two geometrical tachyon scenarios.

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