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To cite this article: S Goli et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 402 012155

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Coherent structures in the flow field generated by rigid flapping wing in hovering flight mode

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Abstract. Galilean invariant vortex identification methods and proper orthogonal decomposition (POD) are used to capture the coherent structures (CS) in the flow field generated by rigid wing in main flapping motion. Experiments are conducted for a square rigid wing at a flapping frequency \( f = 1.5 \) Hz. Tests are performed in water as a fluid medium in hover mode. The main flapping mechanism executes asymmetric lower-upper stroke of 1:3 ratio single degree of freedom motion. Two dimensional particle image velocimetry (PIV) technique is used to generate the phase locked velocity field at each discrete flapping angle by illuminating the mid chord plane of the wing. Three different Galilean invariant methods namely \( \lambda_2 \) criterion, \( Q \) criterion and \( \Delta \) criterion are used for vortex identification. These methods were found to be consistent in detecting swirling vortices or coherent structures (CS). Proper orthogonal decomposition is used to exhibit the most energetic modes of the flow field. The captured modes were identified to be in-connect with vortex identification methods. The combination of these tools were found to be more effective in comparison with velocity field data for achieving a deeper understanding of the complex flow produced by the flapping wing.

1. Introduction

Coherent structures (CS) are vortical motions that are spatially coherent and temporally evolve. The flow field generated by the flapping wing is rich in coherent structures with various scales and strengths. Due to flapping, these CS undergoes interaction, causing them to merge or split in the flow field. The interactions with fellow coherent structures or moving surface make the flow field very complex. The CS or vortical region represents low pressure region, which plays an important role in necessary force generation [1-6]. Therefore it is important to study various vortex identification methods to identify appropriate/suitable and effective methods to capture these structures. The study would be useful for the development of mechanical flyers, which have wide applications in military and civilian purposes.

It has always been challenging in fluid dynamics to define the term vortex. It has been mostly defined with intuitions and hand waving statements were made on how it looks, behaves and its classification. A vortex in Ref. [7] is considered as a tube whose surface consists of a vortex line. An example provided by Jeong and Hussain [8] that the vortex tube in a laminar pipe flow is not a vortex...
implies that existence of vortex tube does not ensure presence of a vortex. According to Lught [9], an intuitive definition of a ‘vortex’ is the multitude of material particles which rotate around a common center. Robinson [10] stated that vortex exists when instantaneous streamlines mapped into a plane, when viewed from a reference plane moving with center of the vortex core. The simplest tools to describe vortices are streamlines, path lines, helicity and vorticity. However, none of these tools are able to accurately distinguish the parallel shear movement and the actual rotational or swirl motion of the vortex. For example, let us consider the case of turbulent shear flows and complex unsteady flows. These flows have been found to be dominant with swirl motion or vortical region or coherent structures and also with heavy shear movement.

Different vortex identification methods have been proposed in the literature. Some of them are mentioned as follows. The Galilean invariant methods include Q criterion, which was proposed by Hunt et al. [11], Δ criterion by Chong et al. [12], λ₂ criterion by Jeong and Hussain [8], swirling strength by Zhou et al. [13], enhanced swirling strength by Cucitore et al. [14], cut-off value λ₂ by Miliou et al. [15], etc. Some of the Lagrangian based vortex identification methods are namely, Mz criterion proposed by Haller [16], delocalized unsteady vortex by Fuchs et al. [17], finite time Lapunov exponent (FTLE) [18, 41]. Though FTLE was not originally proposed for vortex identification but later on several studies showcased that it advocates in displaying vortices. Other category of vortex identification methods such as wavelet analysis and proper orthogonal decomposition have been proposed in the literature. Continuous wavelet analysis provides a decomposition of the velocity field into a band of scales with low to high frequencies through which the more energetic are considered to be coherent structures. Proper orthogonal decomposition extracts the most energetic flow modes from flow field information.

Some of the literature based on application of different vortex identification algorithms and their reported comparisons are discussed briefly below. Cucitore et al. [14] have proposed a non-local region or non-Galilean invariant technique called R. This quantity R was compared with three different local region or Galilean invariant methods Q, Δ and λ₂. It was also reported that methods based on local flow field analysis have been found to be consistent with each other but may lead to ambiguous results in some cases because vortices are non-local phenomena inherently. No concrete comments were made on disadvantages of these methods. Similarly, Chakraborty et al. [19] have reported that these three criteria are closely similar in displaying the vortices. It was remarked that these methods may result in miscalculating the vortex geometry or shape in the case of vortex-interactions. Graffieaux et al. [20] have attempted combination of vortex identification methods (Γ₁ and Γ₂) and POD for analysis of unsteady turbulent flow data generated by PIV measurements. Γ₁ method which is not Galilean invariant was proposed by Michard et al. [20, 21]. Based on Γ₁ method, they have proposed Γ₂, a Galilean invariant method. Dubief and Delcayre [22] reported that Q criterion is superior to pressure and ω iso-surfaces in detecting the coherent structures for turbulent flows on the basis of DNS and LES simulation data. It was also stated that Q criterion has a good agreement with λ₂ criterion in exhibiting the coherent structures and setting suitable threshold is an important concern in these methods. Berson et al. [23] have used the Galilean invariant vortex identification method called γ₂ for the flow field generated by two dimensional PIV measurements in a thermo acoustic resonator for unsteady flows. Chen et al. [24] have compared four vortex identification methods (Δ, λ₅, Q, and λ₂) for wall turbulence problem. Two-dimensional PIV measurements and DNS simulations have been used to extract the velocity field data. It has been concluded that all the four methods were equally capable of extracting the stronger vortices and a non-zero threshold is necessary for all the methods to eliminate various errors. Liu et al. [25] have proposed Ω-method by representing the ratio of vortical vorticity and whole vorticity inside the vortex core. This method has been compared with Q and λ₂ criteria for a DNS simulated boundary layer data. They have reported the advantage of Ω-method over the other two methods as they need various thresholds to capture vortices, which may lead to losing information of weak vortices. While for Ω-method, they have reported a particular value as 0.52 which can reasonably capture the features in flow for that particular problem. But to implement and use the Ω-method successfully, the limitations are found that velocity in the flow field has to be in non-
dimensional form or the value of the constant denoted in the formulation as $\varepsilon$ has to be adjusted accordingly. Galilean invariant vortex identification methods have been reported in flapping wing domain for experimental and numerical data [26-29]. POD analysis has been used widely in various applications but to a very limited extent in flapping wing studies. Durnaz et al. [30] have reported POD modes for a NACA airfoil in pitching motion for PIV generated data, while Liang et al. [31, 32] extracted POD modes for pitching and plunging aerofoil on DNS simulated data. Bidakhvidi et al. [33] used POD analysis for piezoelectric flapping motion for a two dimensional PIV data and Stanford et al. [34] for flexible clap and flip motions. Li et al. [35] have applied POD method for a numerically simulated fruit fly wing data.

In the present study, three different Galilean invariant vortex identification methods, namely, $\lambda_2$ criterion, Q criterion and $\Delta$ criterion, are used to capture the vortical regions in the flow field and its correlation with proper orthogonal decomposition is studied; using two dimensional PIV data generated by rigid square flapping wing at 1.5 Hz flapping frequency. Limited experiments have been reported in open literature on main flapping kinematics with asymmetric lower-upper stroke. Further, application of the above vortex identification techniques and POD for analysis of asymmetric main flapping motion is not reported earlier. Therefore, the authors’ feel that this would be a novel effort to analyze PIV data on asymmetric main flapping motion of rigid wing with some advanced diagnostic tools.

2. Experimental system

The main flapping mechanism reported in the present paper is built based on a four bar mechanism. The dimensions of the four bar mechanism were reported in Goli et al. [36]. The flapping mechanism executes 1:3 ratio in lower-upper stroke in flapping motion from horizontal line as shown in figure 1a. The flapping mechanism is mounted on a test stand and fully immersed in the water tank with free surface of water exposed to the atmosphere. The dimensions of the water tank are 0.9m×0.6m×0.79m. The flapping mechanism operates in the center of the tank, the walls of the water tank on three sides and free surface of water on the top are at sufficiently large distances so that the flow field in the region of interest around the wing would not be perturbed by boundary proximity effects.

The fluid in water tank is initially maintained in quiescent condition. When the wing is executing main flapping in such a medium it is in hover mode. The water is seeded with hollow glass particles having a mean diameter of 10μm and density of 1.1 g/cc. The percentage of seeding particles is approximately $10^{-5}$ (40 ml water laden with particles mixed with 426 liter of clean water). The experiments have been performed with nominal flapping amplitude of $80^\circ$ ($\phi_{\text{max}}$), with upper half stroke angle ($\phi_u$) of $60^\circ$ and lower half stroke angle ($\phi_l$) of $20^\circ$ from the horizontal line as shown in figure 1a, with zero wing pitch angle and at a flapping frequency of 1.5 Hz. During the experiments, the achieved flapping amplitude ($\phi_u = 59.5^\circ$ and $\phi_l = -14.5^\circ$) with wings attached to the main flapping mechanism is different from nominal flapping amplitude due to inertial loads. The details in this regard (stroke-wise) are shown in Table 2. The flapping amplitude also depends on parameters like wing geometry and size, wing material and operating conditions.

The PIV measurements have been performed after completing uninterrupted 10 minutes of wing flapping thus enabling the flow to achieve periodic state of unsteadiness which ensures statistically stationary flow field data. The tests have been conducted by illuminating the flow along the span of the wing, with the laser sheet aligned to the mid chord position of the wing as shown in figure 1b. Schematic of complete experimental setup is shown in figure 2. At each discrete flapping angle, $\phi$, three instantaneous velocity vector fields are obtained and mean of these fields has been calculated to find the phase locked average velocity vector field. Vortex identification analysis of flow field is based on the average velocity field while POD study is based on the instantaneous velocity fields of 2000 snapshots.

The PIV system consists of a Nd: YAG dual pulsed laser with maximum energy/pulse of 150mJ, wave length 1064 nm, laser pulse rate of 14.5 Hz, 1.5 mm laser sheet thickness. The double shutter CCD camera which is used to acquire the images has 1600×1200 pixel resolution at 32 fps. TSI
INSIGHT 3G™ software is used to process the captured images by a cross correlation technique with interrogation window size maintained at 24 pixel, 50% overlapping with a time delay of 400μs between two frames. The Nyquist method is used for grid generation, where the input images are divided into smaller spots for processing. Gaussian spot masking algorithm is used for conditioning the spots. FFT based correlation function and Gaussian peak equation are used for correlation mapping and peak identification respectively.

The dimensions of the square planform flapping wing are shown in Table 1. The wing is made of Perspex® acrylic material. In the present experiments wing deflection has been recorded and found to be negligible. Therefore it is considered as rigid.

Table 1. Wing details

| Span | Chord | Thickness | Aspect ratio |
|------|-------|-----------|-------------|
| 40 mm| 40 mm | 1.5 mm    | 1.0         |

Table 2. Flapping angle limits

| Flapping cycle         | Flapping angle, $\phi$ | Normalized flapping angle, $\phi^*$ |
|------------------------|------------------------|-------------------------------------|
| Beginning of DS        | 58.5°                  | 0                                   |
| End of DS              | -14.5°                 | 1                                   |
| Beginning of US        | -13.5°                 | 0                                   |
| End of US              | 59.5°                  | 1                                   |

Figure 1. Schematic of (a) front view of main flapping motion and (b) plan view of experimental set-up
3. Methodology

3.1 Galilean Invariant Vortex Identification

Generic definition of a ‘vortex’ is commonly associated with the rotating motion of fluid around a common centerline. In a planar fluid motion occurring in the x-y plane of Cartesian coordinate system it is defined by the vorticity \( \omega_z \), which measures the rate of local fluid rotation. Typically, the fluid circulates around the vortex, the speed increases as the vortex is approached and the pressure decreases. In a three dimensional flow field all three components \( (\omega_x, \omega_y, \omega_z) \) would exist in general which are defined by \( \nabla \times \mathbf{V} \), where \( \mathbf{V} \) is the three dimensional velocity field.

\[
\omega_z = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)
\]

(1)

Vortex identification method proposed by Jeong & Hussain [8], is called as \( \lambda_2 \) criterion. Negative and positive values of \( \lambda_2 \) correspond to swirling and shearing regions of the flow respectively. The following equation for \( \lambda_2 \) has been reported in Ref. [40].

\[
\lambda_2 = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 - 4 \left( \frac{\partial u \partial v}{\partial x \partial y} - \frac{\partial u \partial v}{\partial y \partial x} \right)
\]

(2)

Hunt et al. [11] has used the second invariant of velocity gradient tensor \( \nabla \mathbf{u} \) with a condition that pressure should be lower than ambient value and proposed as Q criterion.

\[
Q = \frac{1}{2} \left( \| \Omega \|^2 - \| \Sigma \|^2 \right)
\]

(3)

Where \( \Omega \) and \( \Sigma \) are symmetric and anti-symmetric parts of \( \nabla \mathbf{u} \). \( S_{ij} = \frac{1}{2} \left( u_{ij} + u_{ji} \right) \) and \( \Omega_{ij} = \frac{1}{2} \left( u_{ij} - u_{ji} \right) \). Positive and negatives values of Q represent the swirl and shear regions respectively.

Chong et al. [12] have proposed the \( A \) criterion. They have suggested that the vortex core is a region with complex eigenvalues of the velocity gradient tensor \( \nabla \mathbf{u} \). This criterion proposes that those region of the flow which contain swirling motion could satisfy the following condition.

Figure 2. Schematic of experimental set-up
Where and e title should be capitalized with the rest in lower case. It should not be indented, as shown above.

### 3.2 Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (POD) was first introduced by Lumley [37] in fluid mechanics. It is aimed to obtain low dimensional approximate descriptions of high-dimensional processes. More details in this regard can be found in Ref. [38, 39]. Data analysis using POD is often conducted to extract ‘mode shapes’ or basis functions, from experimental or numerical data of high-dimensional systems, for subsequent use in Galerkin projections that yield low dimensional dynamical models. Suppose we wish to approximate a function \( z(x, y, t) \) over some domain of interest as a finite sum in the variables separated form

\[
z(x, y, t) \approx \sum_{i=1}^{M} a_i(t) \varphi_i(x, y)
\]

where \( x \) and \( y \) are the spatial coordinates, \( t \) is the time, \( M \) is the number of modes, \( a_i(t) \) are the eigenvalue temporal coefficients and \( \varphi_i(x, y) \) are the eigenvectors (or) mode shapes.

In case of PIV data, the \( x \)-component and the \( y \)-component of 2D instantaneous velocity vector data obtained at \( N \) grid points for \( M \) time steps are available as \( \bar{u} \) and \( \bar{v} \) respectively. The fluctuating part of these velocity components i.e. \( u, v \) are obtained by subtracting the time averaged velocity component from its corresponding instantaneous velocity component. These fluctuating velocity components can be written in a spatiotemporal form as

\[
[u] = [u^1 \ u^2 \ldots \ u^N]
\]

\[
u^i = [u_1^i \ u_2^i \ldots \ u_N^i \ v_1^i \ v_2^i \ldots \ v_N^i]^T
\]

The covariance matrix \( R \) is constructed as:

\[
[R] = \nu^T \nu
\]

The POD modes are computed by solving the eigenvalue problem of

\[
R \varphi_i = \lambda_i \varphi_i
\]

\( \lambda_i \) and \( \varphi_i \) are the \( i \)th eigenvalue and eigenvector respectively and the eigenvalues (or) eigenvectors are sorted in the following descending order

\[
\lambda_1 > \lambda_2 > \ldots > \lambda_M > 0
\]

The eigenvalues correspond to the energy associated with each mode. The POD mode \( \Psi_i \) can be obtained as:
The relative energy of each mode is defined as

\[ \psi_l = \frac{\sum_{m=0}^{M} \varphi_m^l u_m^l}{\| \sum_{m=0}^{M} \varphi_m^l u_m^l \|} \] (11)

The relative energy of each mode is defined as

Relative Energy = \( \frac{\text{Eigenvalue of a particular mode}}{\text{Sum of eigenvalues of all modes}} \times 100 \) (12)

4. Results and Discussion

4.1 Galilean Invariant Vortex Identification

The genesis of the flow field for rigid wing in main flapping motion is summarized as follows. Since the intention of the present paper is to identify effective vortex identification methods and their connection with POD modes of the coherent structures, detailed discussion of the flow field at every discrete flapping angle is avoided. At the beginning of a stroke the relatively stronger bound vortex rolls up on the leeward side of the wing which is continuously fed by the wing root flow. It gradually stretches across the wing forming the spanwise flow and eventually rolls up at the wing tip to form a strong wing-tip vortex in the in-board direction. An added mass is induced towards the leeward surface of the wing due to suction which strengthens the spanwise flow on one hand and transforms into the residual flow to form a counter rotating vortex on the other. They are at times accompanied by weaker secondary vortex structures. In order to provide a generic picture of the evolution of the flow field for downstroke and upstroke, schematic diagrams are provided in figure 3. The red and black colour in the figure indicates clockwise and counter-clockwise direction of the vortex structures respectively.

Figures 4 and 5 show velocity field, vorticity with the three different vortex identification criteria at the beginning of downstroke and upstroke of flapping cycle respectively at flapping frequency \( f = 1.5 \) Hz. In these figures beginning of downstroke is at flapping angle \( \phi = 58.5^\circ \) or normalized flapping angle \( \phi^* = 0 \) and beginning of upstroke is at \( \phi = -13.5^\circ \) or \( \phi^* = 0 \). In vorticity figures, clockwise vorticity (CW), which satisfies the condition \( \omega_z < 0 \), has been represented by dotted contours and counter clockwise vorticity (CCW), which satisfies the condition \( \omega_z > 0 \), has been represented by continuous contours. While for vortex identification criteria, dark and white colour in figures correspond to swirling and shearing regions of the flow respectively. WTV1 and WTV2 are the wing-tip vortices generated during the beginning of downstroke and upstroke respectively. FACW and FACCW vortices are generated by the residual vorticity left from previous strokes.

From the velocity field, the wing-tip vortices have been noticed along with strong shear flow across the wing. From vorticity field the predominant vortices have been noticed as WTV1 and FACW forming a pair which are opposite in direction at the beginning of downstroke and similarly WTV2 and FACCW form a pair, at beginning of upstroke of flapping cycle. WTV1 and WTV2 have been observed in the inboard direction. Along with these predominant vortices, small pockets of vortices have been observed in the flow field. From vortex identification criteria the swirling and shear zones have been identified and in which swirling vortices have been found to be stronger. It is worth mentioning that the three vortex identification criteria have been found to produce similar and consistent results in identifying the required vortex rich zones which was not possible with velocity and vorticity field alone. The main challenge in applying the three criteria successfully is suitable threshold adjustment, which reflects on the quality of visualization and may lead to loss of information. From figures 4d and 4e, it appears to be over estimating the vortical regions in comparison with 4c; while in figure 5e, it was underestimating the required regions. The above two instances were cited by intentionally applying improper threshold adjustment to attract necessary attention. The quantitative information of highest to lowest magnitudes of each criterion have been
compared in correlation with spatial locations of their occurrence in the flow field and found to be identical in capturing the structures. The magnitudes of each criterion are found to be different which is quite obvious. In addition, the combination of contours based on vorticity superimposed with vortex identification methods has been found to be better in extracting the flow features. From vorticity, we get the direction of vortex while the vortex identification methods provide vortical regions in the flow field.

![Figure 3](image-url). Schematic diagram of flow features for (i) downstroke and (ii) upstroke of flapping cycle (red and black color indicates clockwise and counter-clockwise direction respectively).
Figure 4. Beginning of downstroke of flapping cycle (a) velocity field, (b) vorticity contours, (c, d, and e) vorticity with $\lambda_2$ criterion, Q criterion, $\Delta$ criterion respectively. Schematic of experimental set-up.
Figure 5. Beginning of upstroke of flapping cycle (a) velocity field, (b) vorticity contours, (c, d, and e) vorticity with $\lambda_2$ criterion, Q criterion, $\Delta$ criterion respectively.
4.2 Proper orthogonal decomposition

The amount of kinetic energy associated with each POD mode is proportional to corresponding eigenvalue. Therefore, by arranging the obtained eigenvalues in decreasing order, it is ensured that the first few modes are the most significant in terms of energy content and dominant flow structures. The present POD study is based on 2000 snapshots of the flow field generated by rigid flapping wing at flapping frequency 1.5 Hz. In-house MATLAB code has been developed for POD analysis.

The velocity field data spanning 2000 snapshots is ensemble averaged to obtain the mean velocity field which is shown in figure 6a. This mean velocity field is subtracted from the velocity field of each snapshot providing time varying velocity snapshot which are used to generate the POD covariance matrix and subsequently solve the eigenvalue problem to extract POD modes. The three dominant POD modes are shown in figures 6b–6d with the mode number and percentage of energy content mentioned at the top right corner of each figure. It is important to note that the length and direction of the vectors in each POD mode doesn’t have any physical meaning unless they are multiplied by their corresponding POD temporal coefficients. Similar information has been reported and discussed in Ref. [38].

In figure 6a, the mean velocity flow field shows two flow regions near the wing tip which are visible above and below the horizontal axis for the flapping mechanism (Figure 1a). Along the horizontal line, the mean flow velocity is purely along the positive x direction. Above this line, the mean flow starts turning towards positive y direction and points away from the wing tip. Below the horizontal line the mean flow turns towards negative y direction. Due to the 1:3 ratio lower-upper stroke asymmetry in flapping motion, the lower and upper flow regions scale approximately in that ratio.

Mode 1 captures high velocity region that is swept by the wing during its downstroke and upstroke which is referred to as the added mass effect. This consists of bound vorticity across the wing surface, which enables formation of wing-tip vortices (WTV1 and WTV2) at the beginning of respective new strokes.

As Mode 1 is the most energetic mode, this proves the fact that the kinematics of the wing has a major impact on the flow field structures. It may be noted that the vortex identification methods discussed in the earlier section are mainly deployed for effective capture of coherent structures of various scales. The bulk flow dragged by the wing can even be identified using velocity or vorticity information and does not need invoking the more sophisticated vortex identification schemes. Since the first mode connects with the bulk flow induced by the wing, it has been compared with phase locked velocity field data at a few discrete flapping angles (figure 7) showing a close connect.

Mode 2 captures the major vortical structures developed in the flow field. Vortex structures similar to the wing-tip vortices (WTV1 and WTV2) developed during the beginning of downstroke and upstroke as observed through vortex identification in figures 4 and 5 are visible in the modal structure at corresponding phase locations. FACW and FACCW which have been noticed in the above figures are not clearly captured in the modal structure, but they appear in designated spatial locations. Figure 8 shows vorticity with λ criterion which captured these flow features at discrete flapping angles during downstroke and upstroke and has a good agreement.

POD mode 3 captures the wing-tip vortices in the vicinity of the wing with lesser clarity and flow bifurcation is clearly observed along the wing span which appears to be of opposite nature in terms of rotation; it either drags the flow away or towards the wing, apparently because of a strong added mass effect. The modal structure has certain resemblance with modes 1 and 2. This mode shows the bulk flow in mid sweeping region of wing which was not distinctly captured in mode 1.

Additionally, when ensemble averaging of phase locked velocity field data was performed for flapping angle range of \( \phi = 58.5^0 \) to \( 44.5^0 \) and \( \phi = -13.5^0 \) to \( -0.5^0 \) during downstroke and upstroke respectively, the mean velocity and vorticity with λ criterion as shown in figures 9a and 9b were obtained. Black and red lines in the figures indicate downstroke and upstroke wing orientations respectively. This covers \( 14^0 \) and \( 13^0 \) of wing sweeping region from the beginning of downstroke and upstroke respectively. It is worth noting that these figures have a close resemblance with the mode 2
structure. The flapping angles were chosen by scrutinizing the velocity field data to check the range over which wing-tip vortices (WTV1 and WTV2) seem to be dominant. Similar attempt has been made to capture the bulk flow by choosing flapping angle range from \( \phi = 36.5^0 \) to \(-14.5^0\) and \( \phi = 27.5^0 \) to \(59.5^0\) during downstroke and upstroke respectively, and the mean flow field obtained is shown in figure 9c. The bulk flow region captured through ensemble averaging was found to be similar to the high velocity region captured in mode 1, while in remaining regions of the flow it appears to be overestimating.

Figure 10 shows the relative energy distribution for first 20 POD modes. Highest energy has been captured by the first few modes while energy levels start attenuating rapidly with increase in mode number. Cumulative energy captured by the first three POD modes was found to be 19.47%. The quantitative details regarding the relative energy and cumulative energy are shown in Table 3 and Table 4 respectively.

From figure 10 and Table 3, it is evident that mode 1 has significantly higher energy content of 10.29% in comparison with all other modes. Modes 2 and 3 have 56% and 32% of energy with respect to mode 1. The energy levels in the remaining modes gradually decrease. It may be worth noting that the present study focuses over the first 3 modes to highlight the dominant flow features which account for approximately 20% of the energy content. The remaining 80% of the energy is recovered over large number of modes as shown in Table 4 and figure 11 which is predominantly distributed in the smaller structures.

![Figure 6. Mean velocity and three POD modes](image-url)
Figure 7. Velocity vector field during (a-c) downstroke and (d-f) upstroke of flapping cycle
Figure 8. Vorticity with $\lambda_2$ criterion during (a-c) downstroke and (d-f) upstroke of flapping cycle
Figure 9. Averaged flow field over discrete flapping angles

Figure 10. Relative energy distribution
Figure 11. Cumulative energy distribution

Table 3. Relative energy.

| Mode number | Relative energy (%) |
|-------------|---------------------|
| 1           | 10.29301            |
| 2           | 5.83406             |
| 3           | 3.34758             |
| 4           | 2.14146             |
| 5           | 1.78912             |

Table 4. Cumulative energy.

| Cumulative energy (%) | No. of modes required |
|-----------------------|-----------------------|
| 30                    | 11                    |
| 40                    | 27                    |
| 50                    | 53                    |
| 60                    | 130                   |
| 70                    | 304                   |
| 80                    | 593                   |
| 90                    | 1055                  |
| 99                    | 1838                  |

5. Conclusion
In the present paper two dimensional PIV measurements were reported for a rigid square wing executing asymmetric lower-upper stroke 1:3 ratio main flapping motion which is associated with hovering flight mode at a flapping frequency 1.5 Hz. Vortex identification methods, namely, λ2 criterion, Q criterion, Δ criterion captured the swirling and shear zones in which swirling vortices or
coherent structures were found to be stronger. The three criteria were found to produce similar and consistent results in identifying the required zones. The quantitative information of highest to lowest magnitudes of each criterion were compared in correlation with spatial locations of their occurrence in the flow field and found to exhibit identical capability in capturing the structures. The main challenge in applying the three criteria effectively was found to be threshold adjustment, which would reflect in visualization and may lead to loss of information. The combination of contours based on vorticity superimposed with vortex identification methods were found to be better in extracting the flow features. From vorticity, we get the direction of vortex while the vortex identification methods provide vortical regions in the flow field. The POD modes were found to be most energetic and they contain approximately 20% of total energy. The three POD modes have captured dominant flow structures like bulk flow (due to added mass effect), wing-tip vortices (WTV1 and WTV2) and residual coherent structures (FACW and FACCW) in the flow field. These flow structures were found to be similar in comparison with the vortex identification methods. Ensemble averaged velocity data over carefully selected phase orientation of the wings revealed close connection with the POD modes.

In the present study, POD analysis was carried out for complete flapping cycle which consists of downstroke and upstroke. As a future scope of study POD analysis for downstroke and upstroke can be carried out separately to extract more details related to the respective strokes and subsequently compare and contrast them. Through such procedure it is expected that coherent structures captured by POD and Galileian invariant vortex identification methods would have even better agreement.

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