Two-dimensional ionospheric radio-tomography by nanosatellite constellation receiving signals from the GLONASS navigation system

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Abstract. In this paper, we consider an approach to radiotomographic monitoring of the ionosphere by a constellation of nanosatellites with receiving equipment for L-band navigation signals on its satellites. We choose the constellation orbit and the angular position of nanosatellites on the orbit to obtain an acceptable latitudinal resolution of measurements equal to 7.5 degrees. The final sounding geometry makes it possible to obtain the first measurements in less than 3.5 minutes, and the full cycle of measurements is obtained in 74 minutes, which makes it possible to reconstruct the electron density of the ionosphere in the orbital plane in the altitude range from 200 to 500 km.

1. Introduction

Over the past 30 years, with the development of navigation satellite systems, it has become possible to carry out remote sensing of the ionosphere in a wide range of different positions of transceiver systems and to reconstruct its structure based on computed tomography methods [1-24]. Today, the most widespread approach to radio-tomography of the ionosphere is using of ground receiving stations [25-26]. Another promising approach of the radio-tomography of the ionosphere is using of the receiving equipment installed on the low Earth orbit (LEO) satellite constellation [27-28]. The authors considered two signal registration schemes: reception by the satellites constellation of signals from global navigation satellite systems (GNSS); reception and transmission of signals by receiving and transmitting devices installed on the satellites constellation.

In modern cosmonautics, nanosatellites are more and more widely used in tasks related to space exploration. Such features of nanosatellites as low cost, short development time and unification of onboard systems have made this class of spacecraft extremely popular, especially among universities; for instance two nanosatellites for on-board systems testing were manufactured in Samara University: SamSat-218D [29] and SamSat-QB50 [30]. The current state of the development of technologies for the manufacture of electronic equipment allows using nanosatellites in the tasks of the ionosphere monitoring and studying the ionosphere.

In the case of monitoring the electron density of the ionosphere, the total electron content (TEC) is estimated, which is a linear integral of the electron density along the path of an electromagnetic wave propagation and is expressed in TEC units (1 TECU = 10^{16} \text{el m}^{-2}). The linear integral of the distribution function $f(x,y)$ along a straight line located at a distance $l$ from the origin and making an angle $\theta$ with the positive direction of the OX axis corresponds to the Radon image at the point $(l,\theta)$:
\[
\left[ \mathbf{R}_f \right](l, \theta) = p(l, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - l) \, dx \, dy.
\]

The estimates for the values of the linear integrals \( p(l, \theta) \) for a finite number of pairs \((l, \theta)\) are called a sinogram [31]. Figure 1a shows the electron density profile on the ring carrier in the plane of Earth's vertical cross section, the range of altitudes \( H \) in ionospheric ring lies from 200 to 500 km; Figure 1b shows the sinogram corresponding to this profile.

**Figure 1.** (a) Distribution of the electron density in the ionosphere on the ring carrier in the Cartesian coordinate system; (b) Sinogram of the electron density distribution.

The solution to the tomographic problem is an estimation \( f^*(x, y) \) of the function \( f(x, y) \) from a set of integral characteristics obtained from all possible angles, assuming that the exact value of \( p(l, \theta) \) is known for all \( l \) and \( \theta \) (the sinogram is completely known).

We propose to use LEO constellation of nanosatellites with on-board receiving equipment for L-band navigation signals of the Russian Global Navigation Satellite System GLONASS [32] to monitor the ionosphere.

2. **Problem statement**

The GLONASS orbital constellation consists of 24 satellites located in three medium-altitude near-circular orbits (eight satellites in orbit) with nominal values of altitude \(-19100\) km, inclination \(-64.8\) deg and period \(-11\) hours 15 minutes 44 seconds [32].

We choose an inclination of nanosatellite constellation equal 64.8 deg. The choice of the number of satellites in the constellation is determined by the desired spatial and temporal resolution of the obtained measurements. The choice of the orbit altitude is also dictated by the desired resolution of the obtained measurements.

The altitude of the constellation orbit is calculated by the following formula:

\[
h_{\text{orb}} = \left( \frac{\mu}{\omega_{\text{orb}}^2} \right)^{1/3} - R_{\text{Earth}},
\]

where \( \mu = 398602 \text{ km}^3 \text{ s}^{-2} \) is gravitational parameter of the Earth; \( R_{\text{Earth}} \) is mean radius of the Earth; \( \omega_{\text{orb}} \) is the orbital velocity of the satellite, which in turn is determined from the expression:

\[
\Delta u_g = \omega_g \left( \frac{e + \Delta u_g}{\omega_{\text{orb}}} \right),
\]
where $\Delta u_g$ the angular distance that the GLONASS satellite passes while the nanosatellite passes $\varepsilon + \Delta u_g$; $\omega_g$ is orbital velocity of GLONASS satellite; $\varepsilon = 45$ deg is the angular distance between GLONASS satellites in one orbital plane.

The minimum number of nanosatellites required for carrying out the simplest tomographic reconstruction using low-angle tomography methods [33] is determined by the need to obtain at least two intersecting tomographic projections, for this requirement one nanosatellite is enough. In this paper, we determine a number of satellites in constellation equal eight, which corresponds to the number of GLONASS satellites in one orbital plane, for a one-time TEC registration in the entire ionospheric ring in the orbital plane.

3. The choice of an orbit altitude

We choose the orbit altitude so that while the nanosatellite passes $\varepsilon + \Delta u_g$ deg, the GLONASS satellite passes $\Delta u_g$ deg, then the next nanosatellite receives a signal from the same GLONASS satellite. For repeatability of measurements, it is necessary that an integer number of $\Delta u_g$ fit into the angle $\varepsilon$. The $\Delta u_g$ parameter essentially determines the latitudinal resolution of the obtained measurements, the smaller the $\Delta u_g$, the better the resolution. Table 1 shows the results of evaluating the relationship between the desired latitudinal resolution and the orbit altitude when registering GLONASS navigation signals.

| n  | $\Delta u_g$ (deg) | $h_{orb}$ (km) |
|----|------------------|----------------|
| 9  | 5.00             | -883           |
| 8  | 5.63             | -484           |
| 7  | 6.43             | -3             |
| 6  | 7.50             | 590            |
| 5  | 9.00             | 1343           |
| 4  | 11.25            | 2340           |

Table 1 shows that the first three altitudes are not feasible. To receive navigation signals from GLONASS satellites, we choose the altitude of the nanosatellites constellation $h_{orb} = 590$ km with $\Delta u_g = 7.5$ deg, which is the best from the physically possible.

4. Results

We consider the area of reconstruction limited by the range of altitudes $H$ from 200 to 500 km, the choice of which is due to the fact that the layer of maximum electron density, which reflects the impacts of both natural and anthropogenic nature, in the overwhelming majority of cases lies within this range.

When sounding the ionosphere by GNSS satellites, it is possible to achieve any altitude resolution due to the sounding geometry. In this paper, an altitude resolution of 25 km is considered. The initial angular distances are calculated for a pair of nanosatellite and GLONASS satellite using the equation (1) (see appendix):

$$R_{orb} \sin (u_{lao} - u_g) = (R_{Earth} + h_g) \sqrt{1 + \left(\frac{R_{orb}}{R_g}\right)^2 - 2 \frac{R_{orb}}{R_g} \cos (u_{lao} - u_g)}.$$  \hspace{1cm} (1)

Figure 2 (a) shows the location of the radio paths for eight nanosatellites receiving GLONASS signals so that the radio paths pass through the altitude ranges $H$ from 200 to 500 km. Labels from g1...
to g8 represent navigation satellites, labels from ns1 to ns8 represent nanosatellites. Each nanosatellite receives signals from two GLONASS satellites, so for example ns1 receives signals from g3 and g7. Figure 2b illustrates the intersection of six ionospheric samples by radio paths in the polar coordinate system by pairs g1-ns3 and g4-ns2.

Figure 2. (a) Location of radio paths for GLONASS satellites and nanosatellites (measurement duration 200 seconds); (b) Location of radio paths of pairs g1-ns3 and g4-ns2 in the polar CS with a latitude step of 50 km.

Table 2 presents another example: the angular positions of GLONASS satellite g1 and two nanosatellites ns3 and ns7, calculated by (1), and the corresponding perigee of the generated radio paths.

Table 2 shows that sounding of the altitude range $H$ from 200 to 500 km is performed in 200 seconds (3.5 minutes), the latitudinal sounding step during this time is 45 deg. Figure 3, in turn, shows the geometry of sounding with a latitudinal step of 7.5 deg, which is achieved due to the fact that while the nanosatellite passes six times along $\varepsilon + \Delta u_g$, each GLONASS satellite passes $\varepsilon$ and occupies the position of the neighboring satellite at the initial moment of time. The positions of the eight GLONASS satellites at six time stamps are shown (the black circles indicate the initial time stamp and the white circles indicate the others time stamp), the first digit next to the symbol t in the label is the number of the GLONASS satellite, and the second digit indicates the time stamp of this satellite. Table 3 shows the correspondence of GLONASS satellites to nanosatellites, the pairs of which form the required radio paths. The complete monitoring presented in Figure 3 and Table 4 is carried out in 74 minutes.

Figure 4 shows a section of the sinogram (Figure 1b) with the obtained volume of measurements for sounding geometry (Figure 2a) and sounding geometry (Figure 3).
Table 2. The angular positions of the GLONASS satellite and nanosatellite and the corresponding perigee of the formed radio paths.

| Δt (s) | \( u_{g1} \) (deg) | \( u_{ns3} \) (deg) | \( u_{ns7} \) (deg) | \( h_r \) (km) |
|--------|-----------------|-----------------|-----------------|-----------|
| 0      | 0.00            | 83.57           | -81.86          | 500       |
| 24     | 0.21            | 85.07           | -83.26          | 475       |
| 45     | 0.40            | 86.38           | -84.60          | 450       |
| 65     | 0.58            | 87.62           | -85.79          | 425       |
| 83     | 0.74            | 88.74           | -86.97          | 400       |
| 100    | 0.89            | 89.80           | -88.03          | 375       |
| 116    | 1.03            | 90.80           | -89.03          | 350       |
| 131    | 1.17            | 91.80           | -90.02          | 325       |
| 147    | 1.31            | 92.73           | -90.96          | 300       |
| 161    | 1.43            | 93.60           | -91.83          | 275       |
| 175    | 1.56            | 94.47           | -92.70          | 250       |
| 188    | 1.67            | 95.28           | -93.51          | 225       |
| 200    | 1.78            | 96.03           | -94.32          | 200       |

\( ^a \) Time stamp of measurement.

\( ^b \) The angular coordinate of the GLONASS satellite \( g1 \) at the moment of time stamp \( \Delta t \).

\( ^c \) Angular coordinate \( ns3 \) nanosatellite at time stamp \( \Delta t \).

\( ^d \) Angular coordinate \( ns7 \) nanosatellite at time stamp \( \Delta t \).

\( ^e \) Altitude of the pericenter of the GLONASS – nanosatellite radio path at the time stamp \( \Delta t \).

Figure 3. Location of radio paths for nanosatellites and GLONASS satellites for \( \Delta \mu = 7.5 \) deg.
Table 3. Coupling of nanosatellites and GLONASS satellites.

| Position | Δt, (s) | Satellite numbers in the LEO constellation, which correspond to two GLONASS satellites |
|----------|---------|---------------------------------------------------------------------------------------|
| 1        | 0 – 200 | 3 + 7 4 + 8 5 + 1 6 + 2 7 + 3 8 + 4 1 + 5 2 + 6 |
| 2        | 843 – 1043 | 4 + 8 5 + 1 6 + 2 7 + 3 8 + 4 1 + 5 2 + 6 3 + 7 |
| 3        | 1686 – 1886 | 5 + 1 6 + 2 7 + 3 8 + 4 1 + 5 2 + 6 3 + 7 4 + 8 |
| 4        | 2529 – 2729 | 6 + 2 7 + 3 8 + 4 1 + 5 2 + 6 3 + 7 4 + 8 5 + 1 |
| 5        | 3372 – 3572 | 7 + 3 8 + 4 1 + 5 2 + 6 3 + 7 4 + 8 5 + 1 6 + 2 |
| 6        | 4215 – 4415 | 8 + 4 1 + 5 2 + 6 3 + 7 4 + 8 5 + 1 6 + 2 7 + 3 |

Figure 4. Sinograms corresponding to sounding geometry (a) 3.5 minutes measurement case; (b) 74 minutes measurement case.
5. Discussion and Conclusion
The proposed constellation of eight nanosatellites makes it possible to probe the ionosphere and solve a tomographic problem with a latitudinal resolution of 7.5 deg. An increase in the number of nanosatellites in a constellation in six times to 48 will reduce the time taken to obtain measurements from 74 to 3.5 minutes (the limitation of 3.5 minutes is due to the duration of the measurements (Table 2)).

Nevertheless, the proposed data registration scheme has the following features:

- mutual precessing of the orbits of the LEO and GNSS satellites leads to a change of planes, the angles between which are 120 deg; measurements are provided within ±30 deg deviation angle from orbital plane, the period of these measurements is about 19 days and period of resumption measurements is about 19 days as well;
- the need to constantly maintain the angular distance between the nanosatellite and the GLONASS satellite, which will change with the mutual precessing of the orbits and needs to be estimated on board of the satellite.

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Appendix
From the following formulas it is possible to calculate the initial angular distance between the GLONASS satellite and the nanosatellite. Let us write the equation of a straight line passing through two given points in Cartesian coordinates:

\[
(y_1 - y_2)x + (x_2 - x_1)y + (x_1y_2 - x_2y_1) = 0,
\]

or in standard form:

\[Ax + By + C = 0.\]

Substituting the coordinates of the points in the polar coordinate system:

\[
\begin{align*}
x_1 &= R_x \cos u_1 \\
y_1 &= R_y \sin u_1
\end{align*}
\]

and:

\[
\begin{align*}
x_2 &= R_x \cos u_2 \\
y_2 &= R_y \sin u_2
\end{align*}
\]

into the equation of the straight line in the standard form (A2), it is obtained:

\[
\begin{align*}
A &= R_x \sin u_1 - R_y \sin u_1 \\
B &= R_x \cos u_1 - R_y \cos u_1 \\
C &= R_x R_y (\cos u_1 \sin u_2 - \cos u_2 \sin u_1)
\end{align*}
\]

Substituting the obtained relations in the expression connecting the normal and standard form of the straight line equation, we obtain:

\[
p = -\frac{C}{\sqrt{A^2 + B^2}} = -\frac{R_{\text{orb}}}{R_g} \frac{\sin (u_1 - u_2)}{\sqrt{1 + \left(\frac{R_{\text{orb}}}{R_g}\right)^2 - 2 \frac{R_{\text{orb}}}{R_g} \cos (u_1 - u_2)}} = R_{\text{Earth}} + h_v.
\]

The final expression for calculating latitude arguments is:
\[
R_{orb} \sin(u_2 - u_1) = (R_{Earth} + h_r) \sqrt{1 + \left(\frac{R_{orb}}{R_g}\right)^2 - 2 \frac{R_{orb}}{R_g} \cos(u_1 - u_2)}.
\]  

(A7)

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