Modulation of AGN gamma rays by interaction with X-rays from an accretion disk hot spot

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ABSTRACT

We have developed a model for the variability of gamma ray emission in jets of active galactic nuclei in which the variability arises as a result of photon-photon pair production interactions with X-rays emitted by a hot spot in the inner part of the accretion disk. As the hot spot orbits around the central engine, the amount of absorption varies periodically. Our model may account for the observed variability of TeV emission from Markarian 421 and other blazars detected by the EGRET instrument on the Compton Gamma Ray Observatory, as well as correlated variations of X-ray and TeV gamma flux from Markarian 421. Quasi-periodic variations observed in Markarian 421 enable us to place an upper limit on the black hole mass of $(2-30) \times 10^8 M_\odot$.

Key words: galaxies: active – quasars: jets – blazars: gamma ray emission, variability

1 INTRODUCTION

Two BL Lac objects, Markarian 421 and Markarian 501, have recently been detected at TeV energies (Punch et al. 1992, Petry et al. 1996, Quinn et al. 1996). The observations of Markarian 421 show that the emission is variable on different time scales from weeks to days (Kerrick et al. 1995, Macomb et al. 1996, Schubnell et al. 1996, Buckley et al. 1996), and even variability over a fraction of an hour has been observed (Gaidos et al. 1996). In multiwavelength observations of Markarian 421, simultaneous variations in X-rays and TeV $\gamma$-rays have been reported (Takahashi et al. 1996a,b, Buckley et al. 1996). Also, the X-ray flux shows quasi-periodic variability on a time scale of $\sim 1$ day superimposed on a $\sim 1$ week time scale decline of the emission (Takahashi et al. 1996a).

This last feature is not easily explained in the models which assume that X-rays and TeV $\gamma$-rays come from a relativistic blob moving along the jet. However, it could be explained by a model in which the quasi-periodicity is caused by the rotation of a massive star which enters the jet and creates a shock at some distance from its surface, as recently proposed by Bednarek & Protheroe (1996). Another natural explanation is possible if the X-rays originate in a relatively small hot spot rotating on the surface of the inner accretion disk. If the inner disk thickness increases with radius, quasi-periodic modulation of X-ray emission may be observed as the hot spot orbits around the black hole with the disk, and is viewed at different inclination angles. The hot spot emission may be also intrinsically collimated by the hot spot’s geometry (e.g. the walls of the hot cavity in the disk), or by an ordered magnetic field, and this may increase the level of modulation.

Since the TeV emission can not originate in the hot spot itself (because of strong absorption), it must come from the jet above the accretion disk. However the processes responsible for the X-ray and TeV $\gamma$-ray variable emission can still be physically linked. It is very likely that significant changes in the jet conditions are initiated by changes in the accretion conditions at the inner disk. For example, a sudden release of energy in the inner disk region can change drastically the viscosity parameter, propelling huge amounts of accreting matter, and frozen-in magnetic field, into the black hole and/or the jet. Such an energy release may be caused, for example by an explosive reconnection of the magnetic field in the inner part of the disk (e.g. Haswell, Tajima & Sakai 1992), or as a result of a thermonuclear explosion on the surface of a compact object (white dwarf, neutron star) which has been captured by the accretion disk. If this happens, the compact object can accumulate enough matter on its surface by accretion in the dense medium to initialize a thermonuclear outburst similar to those in scenarios proposed for nova or Type I supernovae explosions (e.g. MacDonald 1983, Kovetz & Prihalík 1994). The phenomena mentioned above can create a hot spot (or a hot hole) on the surface of the disk. The energetic particles in such a hot spot, bathed in the magnetic and radiation field, may emit X-ray photons with the spectrum observed during an outburst in Markarian 421. The possible occurrence of low photon energy outbursts in the inner accretion disk has been recently proposed by Böttcher & Dermer (1995) as a method for localizing the emission region of VHE $\gamma$-rays in active galactic nuclei (AGN) or...
obtaining an upper limit on the column density of matter around the disk if the low energy photons are scattered by this matter.

The TeV emission could be produced in the jet by any of the mechanisms suggested as being responsible for γ-ray production in blazars (e.g. Maraschi, Ghisellini & Celotti 1992, Mannheim & Biermann 1992, Dermer & Schlickeiser 1993, Sikora, Begelman & Rees 1994, Blandford & Levinson 1995, Bednarek, Kirk & Mastichiadis 1996, Bloom & Marscher 1996, Protheroe 1997). If the TeV γ-rays are scattered by the inner disk radiation, and put the limits on the inner disk temperature, in Sect. 4. The variability of TeV emission expected in such scenarios, and its possible relevance to γ-ray modulation in compact sources, e.g., VHE γ-ray blazars or galactic superluminal sources, is discussed in Sect. 5.

2 OPTICAL DEPTH OF GAMMA-RAYS IN THE X-RAYS EMITTED BY THE HOT SPOT

Let us assume that the X-rays are produced in a small hot spot, and the TeV γ-rays are injected from the blob at a distance x above the hot spot, and propagate at an angle ψ measured from the direction defined by the hot spot and the blob (see Fig. 1a). To obtain some idea about the possible level of absorption of TeV γ-rays in a real source, we assume that the hot spot emits X-rays with a broken power law spectrum (phot cm$^{-2}$ s$^{-1}$ keV$^{-1}$), with an exponential cut-off below ~ 20 keV, similar to that observed during an outburst in Markarian 421 (Takahashi et al. 1996a,b, Macomb et al. 1996, Buckley et al. 1996):

$$F_X(\epsilon) = \begin{cases} 0.187 \epsilon^{-1.9}, & \epsilon < 1.65 \text{ keV;} \\ 0.229 \epsilon^{-2.3} \exp(\epsilon/20 \text{ keV}), & \epsilon \geq 1.65 \text{ keV.} \end{cases}$$

The optical depth for a γ-ray photon with energy $E_\gamma$, measured from the point of injection to infinity, as a function of the injection angle ψ, is given by

$$\tau(E_\gamma, \psi) = d_s^2 \int \frac{(1 - \cos \chi)}{D^2} dH \int F_X(\epsilon) \sigma_{\gamma\gamma}(s) de,$$

where $d_s \approx 180$ Mpc is the distance to Markarian 421, $D = (1 + H^2 + 2H \cos \psi)^{-1/2}$, $\cos \chi = (H + \cos \psi)/D$, $H$ is the γ-ray propagation distance in units of $x$, $s = 2E_\gamma(1 - \cos \chi)$ is the center of momentum frame energy squared, $F_X(\epsilon)$ is given by Eq. (1), and $\sigma_{\gamma\gamma}(s)$ is the cross section for photon-photon pair production (Jauch & Rorlich 1980).
The results for three different γ-ray energies are shown in Fig. 2. It is clear that, for reasonable hot spot-blob distances, the interaction of TeV γ-rays with hot spot X-rays can be important. For example, if $x = 10^{14}$ cm γ-ray photons with energy $E_γ = 1$ TeV can escape only within a cone with opening angle $\psi \sim 2^\circ$. For $x = 10^{17}$ cm, this escape cone has opening angle $\sim 17^\circ$. Hence, if the X-rays are produced in a relatively small region on the accretion disk, then the escape of TeV γ-rays may be strongly dependent on the geometry of the system. In the next section we discuss a possible geometry for the emission regions of X-rays and VHE γ-rays which may occur in the central regions of AGN.

3 HOT SPOT ORBITING AROUND THE CENTRAL ENGINE

We now consider a more complicated scenario. Let us assume that there is a small region (hot spot) in the inner disk at a distance $r_s$ from the central black hole orbiting with angular velocity $\omega_s$ (see Fig. 1b). We neglect the hot spot dimension which is assumed to be much smaller than the characteristic distance scale, $r_s$. The angular velocity of the hot spot is determined by $r_s$ and the black hole mass, and is given by Kepler’s law

$$\omega_s = \frac{(GM)^{1/2}}{r_s^{3/2}} = c \frac{r_g/2}{r_s^{3/2}},$$

where $c$ is the velocity of light, and $r_g = 2GM/c^2$ is the Schwarzschild radius of a black hole with a mass $M$.

As mentioned above, we assume that the γ-rays are continuously injected from the stream of plasma moving along the jet, and $\alpha$ is the viewing angle with respect to the jet axis (Fig. 1b). They may also originate in a blob, or a sequence of blobs, moving along the jet. The TeV γ-rays move through the X-ray radiation field of the hot spot located on the disk surface at phase angle $\phi$ which depends on time,

$$\phi = \phi_0 + \omega_s t,$$

where $\phi_0$ is the initial phase of the hot spot (see Fig. 1b), and we neglect any inward radial motion of the hot spot. As discussed below, this leads to a periodic variation of the angle between the γ-rays propagating towards the observer and the X-rays from the hot spot leading to periodic variation of the γ-ray flux. Kepler’s law (Eq. 4) gives us the radius of the hot spot in terms of the black hole mass and the angular velocity,

$$r_s = (GM)\frac{1}{3} \omega_s^{-2/3},$$

which gives

$$r_s = 1.4 \times 10^{14} M_8^{1/3} \text{ cm},$$

for an orbital period of $\sim 1$ day, i.e. corresponding to the observed quasi-periodic variability. However, this hot spot orbital radius must be at least that of the minimum inner accretion disk radius, $\sim (0.5 – 3)r_g$, for maximally rotating Kerr and Schwarzschild black holes respectively, giving

$$r_s > (1.5 – 9) \times 10^{15} M_8 \text{ cm}$$

Hence we obtain a strict upper limits to the mass of the black hole in Markarian 421 of $M \leq 2 \times 10^9 M_\odot$ (Schwarzschild black hole) or $M \leq 3 \times 10^9 M_\odot$ (Kerr black hole).

The optical depth for γ-ray photons can be calculated as a function of $r_s$, $\alpha$, $E_\gamma$, and $l$,

$$\tau(E_\gamma, \alpha) = \frac{d^2}{c \tau_s} \int \frac{1 - \cos \chi}{D^2} dH \int F_\gamma(e) \sigma_{\gamma\gamma}(s) de,$$

where it is convenient to express all distances in units of $r_s$: $H = h/r_s$, $L = l/r_s$. The values in Eq. 8 are given by,

$$D = (1 + H^2 + L^2 + 2HL\sin \alpha \cos \phi)^{-1/2},$$

and

$$\cos \chi = (H + L \cos \alpha – \sin \alpha \cos \phi)/D.$$
Figure 4. Transmission probability as a function of phase $\phi$ of the hot spot. The $\gamma$-rays are injected at a distance $l$ and at angle $\alpha$. (a) Dependence of transmission probability on the hot spot phase for 1 TeV $\gamma$-rays, propagating at $\alpha = 7^\circ$, for $r_s = 10^{14}$ cm, and for $\log(l/r_s) = 1$ (dotted line), 1.5 (full line), and 2. (dashed line). (b) Dependence of transmission probability on $\gamma$-ray energy: $E_\gamma$ (TeV) = 0.32, 1 TeV (full line), and 3.2 TeV (dashed line), with $r_s = 10^{14}$ cm, $\alpha = 7^\circ$, and $\log(l/r_s) = 1.5$. (c) Dependence on the injection angle: $\alpha = 5^\circ$ (dotted line), $7^\circ$ (full line), and $9^\circ$ (dashed line), with $E_\gamma = 1$ TeV, $r_s = 10^{14}$ cm, and $\log(l/r_s) = 1.5$. (d) Dependence on the orbital radius of the hot spot: $r_s = 3 \times 10^{13}$ cm (dotted line), $10^{14}$ cm (full line), $3 \times 10^{14}$ cm (dashed line), with $E_\gamma = 1$ TeV, $\alpha = 7^\circ$, and $\log(l/r_s) = 1.5$.

In order to gain an impression about the possible level of modulation of the VHE $\gamma$-ray emission, we compute the optical depth at $E_\gamma = 1$ TeV for two critical locations of the hot spot in its orbit around the black hole: $\phi = 0^\circ$, and $180^\circ$. The results are shown in Fig. 3 as a function of the distance of the $\gamma$-ray injection point from the black hole $l$, for four different angles of observation $\alpha$. If the hot spot is located at phase $\phi = 0^\circ$, and $\alpha$ is large, then the optical depth drops off monotonically with distance. The dependence of the optical depth on $l$, for when the hot spot is located at $\phi = 180^\circ$, shows characteristic cusps at distances which correspond to the $\gamma$-ray photons and the X-ray photons from the hot spot moving such that the angle between their directions is very small. The location of the cusps shifts to smaller $l$ with increasing $\alpha$. The optical depths for $\phi = 0^\circ$ and $\phi = 180^\circ$ converge at large distances $l$ from the black hole because the angles between interacting photons become comparable. The largest modulation of the $\gamma$-ray signal is expected at distances for which the differences between the full and dashed lines, computed for the same value of $\alpha$, are highest, and have $\tau \gg 1$ for $\phi = 0^\circ$ (full lines) and $\tau \ll 1$ for $\phi = 180^\circ$ (dashed lines).

Closer inspection of Fig. 3 allows us to choose the parameters in this scenario ($r_s$, $l$, $\alpha$) for which absorption effects can significantly modulate the 1 TeV $\gamma$-ray signal with the orbital period of the hot spot. In order to show the level of possible modulation, we compute the transmission probability for $\gamma$-ray photons ($e^{-\tau}$), selecting as a basic example the case where $E_\gamma = 1$ TeV, $\log(l/r_s) = 1.5$, and $r_s = 10^{14}$ cm. We investigate the dependence of the transmission probability as a function of phase $\phi$ for different values of the parameters mentioned above. Fig. 4a shows...
the level of modulation for 1 TeV γ-rays injected at angle α = 7°, for three distances from the black hole: \( \log(l/r_s) = 1.0, 1.5, \) and 2.0. At \( l = 10r_s \) (dotted line in Fig. 4a), the modulation is very strong and, in fact, the 1 TeV γ-rays can only escape when the hot spot is close to phase \( \phi = 180° \). At larger distances the level of modulation is lower (a factor of \( \sim 15 \) for \( l \approx 30r_s \); full line in Fig. 4a), and becomes negligible at \( l = 100r_s \) (dashed line).

The dependence of the transmission probability on the γ-ray energy is shown in Fig. 4b. The transmission probability for low energy γ-rays is high, but the level of modulation of the γ-ray signal by the orbital period of the hot spot is low. For example, for \( E_\gamma = 0.32 \) TeV, the modulation is only by a factor of \( \sim 3 \), in comparison to a factor of \( \sim 15 \) at 1 TeV and much higher at 3.2 TeV. The transmission probability depends strongly on the angle of γ-ray injection (see curves for \( \alpha = 5°, 7°, \) and 9° in Fig. 4c). However, the general pattern of modulation of the γ-ray signal with orbital phase is conserved. In Fig. 4d we show the dependence of the transmission probability on the orbital radius of the hot spot for \( E_\gamma = 1 \) TeV, \( \alpha = 7°, \) and \( \log(l/r_s) = 1.5. \) As expected, when the transmission probability is lowest (for a more compact source; see dotted line for \( r_s = 3 \times 10^{13} \) cm) the level of modulation of the γ-ray signal with the orbital period is highest.

One must keep in mind that, in the present scenario, the variable signal will be multiplied by the γ-ray emission efficiency which is likely to decline with the distance along the jet axis. However, the effects described above may be irrelevant if the radiation from the accretion disk in which the hot spot is located, is too intense. In the next section we shall discuss possible absorption of the TeV γ-ray signal in the accretion disk radiation.

## 4 ABSORPTION OF VHE GAMMA RAYS IN THE DISK RADIATION

Within the central regions of the AGN, interactions with photons directly from the accretion disk can be important (Becker and Kafatos 1995, Bednarek 1996; see Bednarek 1993 for an earlier discussion of γ-ray escape from the radiation field of an accretion disk surrounding a neutron star in an X-ray binary source). In order to check for which accretion disk parameters the modulation effects of VHE γ-ray emission by the X-rays from the hot spot, discussed in Sect. 3, are not swamped by absorption in the accretion disk radiation, we compute the optical depth, \( \tau \), for γ-rays in the disk radiation (e.g. Bednarek 1993, Protheroe & Biermann 1996). We assume the surface of the accretion disk emits black body radiation with temperature varying with radius as \( T(r) = T_{in}(r/r_{in})^{-3/4} \), where \( T_{in} \) is the temperature at the inner disk radius \( r_{in} \) (Shakura & Sunyaev 1973). The VHE γ-rays can escape if they are injected at distances farther than the so-called the “radius of the γ-sphere”, \( r_\gamma \), which is defined by the following condition (e.g. Blandford & Levinson 1995, Bednarek 1996),

\[
\tau(E_\gamma, \alpha, r_\gamma) = 1,
\]

where \( \tau \) is the optical depth to photon-photon pair production for γ-rays propagating at angle \( \alpha \), measured from the disk axis, through the radiation field of the accretion disk.

The radius of the γ-ray photosphere, expressed in units of \( r_{in} \), is shown in Fig. 5 as a function of γ-ray energy for different inner disk temperatures and radii. The angle of γ-ray injection has been fixed at \( \alpha = 7° \) because our results for the modulation of the γ-ray signal presented in Figs. 4(a)-(d) have been shown mainly for this angle. As expected, the radius of γ-ray photosphere increases with γ-ray energy, and also with intensity of the radiation field of the disk (i.e. higher \( T_{in} \) and/or \( r_{in} \)). For the modulation of VHE γ-rays by interaction with X-rays from an accretion disk hot spot to take place the emission point of VHE γ-rays must be outside the γ-ray photosphere, i.e \( l > r_\gamma(E_\gamma) \).

## 5 CONCLUSION

We have shown that if the X-ray emission observed from Markarian 421 during the outburst stage (Takahashi et al. 1996a,b) originates on the surface of the accretion disk, then it creates a sufficiently strong radiation field for the absorption of VHE γ-rays provided they originate in the jet near the accretion disk (Sect. 2). We have discussed a simple geometrical model in which a small hot spot emitting X-rays orbits with the accretion disk around the central black hole, and the VHE γ-rays are injected at some distance along the jet at a small angle to its axis. We have shown that absorption effects on VHE γ-rays in the radiation field of a hot spot, with an X-ray spectrum similar to that of Markarian 421, can become very important for some sets of model parameters, i.e. angle of injection \( \alpha \), location of injection point \( l \), and radius of the orbit of the hot spot \( r_s \) (see Fig. 3). Absorption becomes stronger for higher energy γ-rays and may result in a change in the shape of the VHE γ-ray spectrum.

Modulation of the γ-ray emission with the orbital period of the hot spot is also very significant, and may result in simultaneous quasi-periodic oscillations of X-ray and TeV γ-ray emission. Note that \( \sim 1 \) day quasi-periodic variability has been detected by ASCA in X-rays from Markarian 421 (Takahashi et al. 1996a), and enables us to place an upper limit on the black hole mass for the present scenario of \( M \leq 2 \times 10^{6} M_\odot \) (Schwarzschild black hole) or \( M \leq 3 \times 10^{9} M_\odot \) (Kerr black hole). Modulation effects such as those described in the present work are expected provided the disk radiation itself does not produce a sufficiently intense radiation field for VHE γ-ray absorption. For the parameters discussed above, the disk radiation is not important if the disk inner temperature is less than \( \sim 2 \times 10^{4} K \) for a disk inner radius \( r_{in} = 10^{14} \) cm. The direct accretion disk radiation is observed in some quasars e.g., in quasar 3C 273, which is a γ-ray emitting active galaxy and has an inner disk temperature \( (2–3) \times 10^{6} K \) (Shields 1978, Malkan & Sargent 1982).

The disk radiation fields of the BL Lacertae objects, e.g. Markarian 421, are significantly weaker since their spectra are continuous, without strong evidence of emission lines or other thermal features.

The TeV γ-rays may also be absorbed on radiation scattered by matter distributed around the accretion disk (e.g. Sikora, Begelman & Rees 1994, Böatcher & Dermer 1995). However, these radiation fields, if important, are difficult to consider in detail because they depend on parameters which are not known precisely. In some sources, where the bulk of the infrared emission comes from a dusty molecular torus.

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surrounding the central engine, VHE emission from points below the top of the torus would be absorbed by photon-photon pair production on infrared radiation from the torus (Protheroe and Biermann 1996). However, in the absence of such a torus, or if the temperature of the torus is low, modulation of VHE γ-rays by interaction with X-rays from an accretion disk hot spot as described in the present paper should be possible.

The stream of relativistic jet plasma, or sequence of blobs, from which the VHE γ-rays might originate does not need to move rectilinearly along the jet axis. It may emerge from the region of the hot spot and move outwards from the disk following, e.g., helical magnetic field lines. Farther from the disk, the angular rotation of the plasma stream can be slower than that of the inner disk. In such a situation the period of the hot spot’s orbit, responsible for the modulation of observed X-ray flux in this scenario, may be shorter than the period of the TeV γ-ray modulation.

An interesting feature of the present scenario is that it predicts a time delay of the peak TeV γ-ray flux with respect to the peak X-ray flux. This is caused by the different paths to the observer from the X-ray and γ-ray emission regions. The delay depends on the distance of the γ-ray emitting region from the disk l, the radius of the hot spot’s orbit r_s, and the angle of observation α, and is given by

$$\Delta t = \left( r_s^2 + l^2 \right)^{1/2} |1 - \cos(\theta - \alpha)|/c,$$

(12)

where, tan θ = rs/l. For example, if r_s = 10^{14} cm, α = 7° and l = 30r_s, then the expected delay of the maximum in γ-ray flux is Δt ≈ 40 minutes, and for l = 10r_s the delay is Δt ≈ 6 minutes.

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