Wavelet Resonance Demodulation Method and Its Application in Fault Diagnosis of Railway Bearings

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Abstract. Resonance demodulation method other than other diagnosis methods, such as AR model method and wavelet transform method, is adopted by engineers to diagnose the faults of rotating machinery, but its effectiveness depends to a great extent on how to choose the center frequency and the bandwidth of the filter. Here a new resonance demodulation method based on wavelet is presented, which applies time-frequency transform on the acceleration signals collected and extracts the time-energy signal from the time-frequency spectrum instead of filtering the original acceleration signals and then extracting the envelope. The key part of the method is that a criterion in terms of the minimum of kurtosis factor is proposed and the time-energy signal, which is equivalent to envelope, can be automatically derived from the time-frequency spectrum according to the criteria. The method is applied to the analysis of vibration signals of rolling bearings with outer-race, inner ring faults and rolling element faults, and the results show that it performs effectively in fault diagnosis of freight car rolling bearings and performs more effectively than conventional fault diagnosis methods, say, demodulated resonance technology, to extract the fault characteristic.

1. Introduction
As a rotating machinery, the diagnosis of rolling bearings on freight cars was firstly implemented with the digital characteristics of time domain signals, such as kurtosis factor and the effective value of vibration amplitude. The basic idea is to set the threshold value in advance, and then diagnose whether the railway bearings exist fault by checking whether the numerical values extracted from the time domain signals exceed the threshold value. This method is closely dependent on the selection of the threshold value. However, the selection of threshold value relies highly on experience and changes according to the external environment. Therefore, this method can only provide a preliminary qualitative judgement.

Spectrum analysis [1] [2] could provide accurate fault diagnosis of Rolling Bearings in the early period. The traditional resonance demodulation method [3] [4] is the most successful method in spectrum analysis. Its physical mechanism is given in document [5]. The basic principle is to demodulate the low-frequency impulse signal to high resonance frequency, and thus keep it far away from the energy-intensive low-frequency signals. When a rotating machinery with local defects works, the corresponding shock response will be generated at rotating defect for each rotation. If the rotation frequency when bearing work is fixed, the shock response will also occur at a fixed frequency. By demodulating the fault signal and applying Fourier analysis, the composition of the corresponding fault frequency can be found. However, the determination of the center frequency of envelopment analysis and the selection of its bandwidth often impose restrictions for engineers to apply resonance demodulation method to conduct fault diagnoses of rolling bearing.
Because time-frequency analysis [6] allows the vibration signals to achieve time-frequency localization. With the increasing researches and the improvement on time-frequency analysis method, engineers have been continued using it, such as short-term Fourier transform, Wigner-Ville distribution, wavelet transform, and Hilbert Huang Transform (HHT) [7], on the fault diagnosis of rolling machinery. Among these time-frequency analysis methods, continuous wavelet transforms (CWT) have taken advantage because of its superior performance on depicting the impacts caused by local defects. In this paper, we propose the kurtosis factor minimization criterion, apply the criterion to extract the time-energy signal, which is equivalent to the envelope, from the time-frequency density obtained from continuous wavelet transform, and conduct time-frequency analysis and fault diagnose on the envelope.

2. Resonance Demodulation Method Based on Wavelet Transform
This section provides the resonance demodulation algorithm based on wavelet transform, then discuss the implementation of each step of the algorithm in detail.

(a) Obtain the time-frequency spectrum by carrying out the time-frequency transform on the vibration acceleration signals of fault bearing using continuous wavelet transform.

(b) Extract the maximum time-energy signal, which is similar to time edge, from the calculated time-frequency spectrum based on kurtosis factor minimization criterion.

(c) Apply Fourier analysis to transform the components extracted from time-energy signals, then verify the existence of faults and determine its type by observing the frequency of the signal displayed on the spectrum and the characteristics of the integral frequencies on the spectrum.

2.1 Continuous Wavelet Transform
Let $\Psi(t) \in L^2(R) \cap L'(R)$, then after translation and stretching according to the function below generate the family function \{\Psi_{a,b}(t)\}[8]:

$$\Psi_{a,b}(t) = a^{-1/2}\Psi\left(\frac{t-b}{a}\right), \quad b \in R, a > 0$$  (1)

is called as analytical wavelet or continuous wavelet. In Formula (1) \(a\) is scale parameter and \(b\) is time-position parameter, \(\Psi(t)\) is called basic wavelet or mother wavelet, which satisfies the following admissibility conditions:

$$K = \frac{1}{2\pi} \int_R \left|\hat{\Psi}(\omega)\right|^2 /|\omega|d\omega < \infty$$  (2)

In the formula above, $\hat{\Psi}(\omega)$ is the Fourier transform of $\Psi(t)$, which is:

$$\hat{\Psi}(\omega) = \int_R \Psi(t) \exp(-i\omega t) dt$$  (3)

From formula (2), $\hat{\Psi}(0) = 0$ can be obtained, after substituting it into (3), we get $\int_R \Psi(t) dt = 0$, the condition that the integral is zero or that the mean value is zero which mother wavelet satisfied.

The continuous-time wavelet of the continuous-time signal $x(t) \in L^2(R)$ is defined as

$$W(a,b) = \{x(t), \Psi_{a,b}(t)\} = a^{-1/2} \int_R x(t) \Psi_{a,b}^* (t) dt$$  (3)

In which, $\Psi_{a,b}^* (t)$ represents the conjugation of $\Psi_{a,b}(t)$.

For discrete time series $s_m$, let $t = m\Delta t, b = n\Delta t$, with $m, n = 1, 2, \cdots, N$, $N$ is sampling point numbers, $\Delta t$ is sampling interval, then from formula (3), we can obtain that the continuous wavelet of $s_m$ transform as follow
\[ W(a, n) = a^{-1/2} \sum_{m=1}^{N} x_m \Psi^* \left( \frac{(m-n)\Delta t}{a} \right) \] (4)

The time-scale density of discrete time series \( x_m \) is obtained from (4). More broadly, scales correspond to frequencies, and their relationships are as follows [9]:

\[ a = \frac{F_c}{F_a \Delta t} \] (5)

where \( F_c, \Delta t \) represent the center frequency of the wavelet and sample interval; \( F_a \) represents the frequency corresponding to scale \( a \), more precisely, called pseudo-frequency. Combining function (4) and (5), we know that \( W(a, n) \) also can represent the wavelet coefficients of the corresponding frequency \( F_a \) at the sampling point \( b = n\Delta t \). This paper takes

\[ F_a = i \Delta t / (\Delta t N), \quad i = 1, 2, \cdots N / 2 \] (6)

substitute into function (5) and get Scale \( a \).

As for the selection of mother wavelet, according to reference [10], we know that when the bandwidth of Morlet wavelet is narrow, it approaches the impulse function. In this paper, we choose Morlet wavelet as mother wavelet. The expression is as follow

\[ \Psi(t) = \exp(-t^2 / 2)\cos(5t) \] (7)

It is easy to verify that \( \hat{\Psi}(0) = \sqrt{2\pi} \exp(-25/2) < 10^{-5} \), although it does not strictly satisfy the admissibility conditions, it meet the requirement from the perspective of numerical value.

2.2 Kurtosis Factor Minimization Criterion

In this paper, we extract the time-energy signal from the time-frequency spectrum based on kurtosis factor minimization criterion. For discrete signals collected with time sampling \( x_k, k = 1, 2, \cdots N \) where \( N \) is the number of sampling points (i.e. the length of data), we calculate its wavelet transform according to function (4), and set the wavelet coefficient matrix as \( W(a, n) \). From the energy-preserving property of wavelet transform and the transformation relation (5), can get

\[ \sum_{m=1}^{N} x_k^2 = \sum_{m=1}^{N/2} \sum_{n=1}^{N/2} |W(F_a, n)|^2 C \] (8)

in above function, constant \( C = \frac{2\Delta t}{F_c K} \), where the definition of \( \Delta t, F_c, K \) can be find at function (5) and (2). Under the limitation of Heisenberg uncertainty principle, we cannot regard \( |W(F_a, n)|^2 C \) as instantaneous energy on plane \( (F_a, b) \), but as time-frequency energy density on plane \( (F_a, b) \).

Since the impulse response signal generated by the fault bearing only contained in high frequency range, we can operate only on the part of high frequency with high time-frequency energy. The time-frequency energy density newly obtained is defined as follow:

\[ EP(\omega_j, k) = |W(F_a, n / 4, k)|^2 C, \quad j = 0, 2, \cdots N / 4, k = 1, 2, \cdots N, \] (9)

Similar to the definition of time edge, when \( \omega_j \) is fixed, \( EP(\omega_j, k) \) is defined as a time energy signal corresponding to frequency \( \omega_j \), and its kurtosis factor is calculated as:
\begin{equation}
K_i(\omega_j) = \frac{\sum_{k=1}^{n} (EP(\omega_j, k) - \bar{m})^4}{\left( \sum_{k=1}^{n} (EP(\omega_j, k) - \bar{m})^2 \right)^2} = \frac{\sum_{k=1}^{n} (EP(\omega_j, k) / C - \bar{m})^4}{\left( \sum_{k=1}^{n} (EP(\omega_j, k) / C - \bar{m})^2 \right)^2}
\end{equation}

In the function above, \( \bar{m} = \sum_{k=1}^{N} EP(\omega_j, k) / N \), \( \bar{m} = \sum_{k=1}^{N} (EP(\omega_j, k) / C) / N \). From equation (10) it can be seen that the value of constant \( C \) is not required for calculating kurtosis factor. The parameter \( \omega^* \) can be taken as:

\begin{equation}
\omega^* = \arg \min_j K_i(\omega_j),
\end{equation}

then its envelope is calculated according to function (12).

\begin{equation}
E(k) = EP(\omega^*, k), k = 1, 2, \cdots N
\end{equation}

2.3 Thinning Fourier Technique

In applications of engineering, although the envelope spectrum and theoretical defect frequency of defective bearings have relatively clear corresponding relations, because the frequency resolution of the signal is restrained by the sampling frequency and the length of the sampling data, it is still difficult for us to distinguish fault features when multiple defect frequencies occur and if they are all gathering in low frequency range. In order to diagnose bearing defects accurately, it is necessary to thin the envelope spectrum of the signal. Here, we apply the discrete time Fourier transform (DTFT) [12] [13] to thin the spectrum, without increasing the sampling length of data, continuous spectrum curves are obtained by Fourier series theory to eliminate the influence of frequency resolution.

Assume the sampling frequency of the time-energy signal \( E(k) \) obtained from function (12) is \( f_s \), then its Fourier series can be written as

\begin{equation}
E(k) = \frac{a_0}{2} + \sum_{j=1}^{N} \left( a_j \cos(2\pi j k / N) + b_j \sin(2\pi j k / N) \right), k = 1, 2, \cdots N
\end{equation}

The Fourier coefficient in the function above is

\begin{equation}
a_j = \frac{2}{N} \sum_{k=1}^{N} E(k) \cos(2\pi j k / N)
\end{equation}

\begin{equation}
b_j = \frac{2}{N} \sum_{k=1}^{N} E(k) \sin(2\pi j k / N)
\end{equation}

According to Nyquist sampling theorem, Fourier series \( E(k) \) contains continuous information in the range of \( 0 \sim f_s / 2 \). Therefore, if the variable \( j \) in function (14) and function (15) are seen as continuous variables in the range of \( [1, N / 2] \) and are expressed as \( f \), then function (14) and function (15) can be written as

\begin{equation}
a(f) = \frac{2}{N} \sum_{k=1}^{N} E(k) \cos(2\pi kf / f_s)
\end{equation}

\begin{equation}
b(f) = \frac{2}{N} \sum_{k=1}^{N} E(k) \sin(2\pi kf / f_s)
\end{equation}

For railway freight car bearings with rotational speed between 300 rpm and 310 rpm, the failure frequency corresponding to inner ring fault is highest, generally around 60Hz, so when thinning frequency, it is enough to calculate the components within 300Hz.
3. Practical Examples in Fault Diagnosis of Freight Car Bearings

The 197726 bearing vibration data provided by the Locomotive and Rolling Stock Research Institute of China Academy of Railway Science Co., Ltd. will be taken as the research object to verify the effectiveness and practicality of the proposed resonance demodulating method based on wavelet transform in the fault diagnosis of freight car bearing. Firstly, the bearing vibration acceleration data is used to diagnose the faults of freight car bearing enclose (outer-race and inner ring), and the result is compared with the result carried out by traditional resonance demodulation method. Then, the same method is conducted in the fault diagnosis of freight car bearing rollers. Under normal condition, the vibration signal sampled according to time and its corresponding spectrum diagram are given to illustrate that the misdiagnosis will not occur when using the new proposed method to diagnose the fault of freight car bearing.

The rotation speed of the freight car bearing is generally between 300rpm and 310rpm. According to the fault characteristic frequency theory, the fault frequencies of the outer ring, inner-race and bearing rollers are 59.40Hz to 61.38Hz, 45.60Hz to 47.12Hz and 18.60Hz to 19.22Hz. If the bearing rollers fails, because it is subjected to the impact modulation of the inner and outer ring failure, its fault frequency presents double times frequency characteristic, that is, the corresponding frequency occurs between 37.20Hz and 38.44Hz.

3.1 Inner ring fault diagnosis

A particular vibration acceleration signal of a rolling bearing of freight car with an inner ring fault is shown in figure 1. As we can see that there is no significant periodic shock signal found.

![Figure 1. The vibration signal of a freight car rolling bearing with an inner ring fault](image)

Besides, most amplitude of the vibration acceleration signals shown in Figure 1 are between [-1,1] except for a few points, which has virtually no differences compared with the amplitude of normal vibration acceleration signals. Therefore, it is impossible to diagnose the bearing fault by calculating the effective value of the vibration acceleration of rolling bearings. By using the proposed resonance demodulation method based on wavelet transform, we conduct continuous wavelet transform on the bearing vibration acceleration signals shown in Fig. 1. The time-frequency distribution diagram obtained is shown in Fig. 2a), and we then extract the time-energy signal according to minimization kurtosis factor criterion. Finally, the spectrogram is obtained by using thinning Fourier technique, which is shown in Fig. 2b).
It can be seen from the time-frequency diagram shown in Fig. 2a) that there are several periodic impacts which are noted as A2 to A5, and the two adjacent impact time intervals are exactly equal to the impact period corresponding to the inner-race fault. The time interval between A1 and A2 is equal to twice the impact period. Besides, the energy at the frequency corresponding to the inner ring fault and its corresponding integral multiple frequency is relatively large as can be seen from the Fig. 2b), which means that the inner ring fault exists.

Meanwhile, we use traditional resonance demodulation method to analyze the same group of signals. Firstly, the data is band-pass filtered, then the envelope of the band-pass filtered data is calculated. Finally, thinning Fourier method which has been described in section 1.3 is applied to process the envelope. If the band-pass frequencies are taken as 7~7.9KHz and 4~5.5KHz, its corresponding spectrograms which are obtained by thinning Fourier technique can be seen from Fig. 3a) and b). The band-pass frequencies of 7~7.9KHz taken in Fig. 3a) is randomly selected while the 4~5.5KHz in Fig. 3b) is obtained through extensive experimental data. By comparison, it can be found that the calculation result of Fig. 3b) is better than the result of Fig. 3a).

The results of Fig. 3a) and b) show that the effectiveness of the traditional resonance demodulation method highly depends on band pass frequencies chosen in bandpass filter. If the center frequency and the pass band has been not properly chosen, the faults of freight car bearing are probably unable to be diagnosed. For example, the filter band of Fig. 3a) is 7~7.9KHz, the fault frequency can be found and its corresponding energy is relatively large. However, the energy of its integral times frequencies is relatively small, or even don’t exist. But if the pass band is chosen properly, the energy could be more concentrated on the fault frequency and its integral times frequencies as in Figure 3 b). By comparing
the Fig. 3a) and 3b) we can know that repeatedly trails has to be taken on the traditional resonance demodulation method in order to obtain a similar result obtained by the wavelet transform based resonance demodulation method. At this very situation, the pass band is chosen as 4~5.5KHz because the impact energy is concentrated on the 4~5.5KHz as can be seen from Fig. 2a).

3.2 Diagnosis of outer-race fault

The vibration signal of a freight car bearing with outer ring fault is shown in Fig. 4. Its time-frequency diagram and spectrogram which are calculated by wavelet transform based resonance demodulation can be seen from Fig. 5a) and 5b). Meanwhile, the spectrograms calculated by traditional resonance demodulation method when the bandpass filtering frequency is 6~7.5KHz and 4~5.5KHz respectively, can be seen from Fig. 6a) and 6b).

![Figure 4. Vibration signal of a freight car bearing with outer ring fault](image)

![Figure 5. Outer-ring fault diagnosis result by using wavelet transform based resonance demodulation method](image)

![Figure 6. The spectrogram of outer ring peeling fault by using traditional resonance demodulation method](image)
It can be seen from the vibration acceleration signal of the rolling bearing of the freight car in Fig. 4 that the signal has a very significant periodic impact, indicating that the outer ring of the bearing is severely peeled off. It can be seen from Fig. 5b) that the energy at the outer ring fault frequency and its integral multiple frequency is relatively large, the peak value is obvious, and as the multiple increases, the peak energy energy gradually decreases. This result indicates that the resonance demodulation method based on wavelet transform can diagnose the fault of the outer ring of the rolling bearing of the freight cars well. From the time-frequency distribution diagram shown in Fig. 5a), it can be clearly seen that the impact defect causes resonance of the entire high frequency band, and Fig. 6 shows that although the traditional resonance demodulation method can diagnose the outer ring failure of the rolling bearing of the freight cars, The diagnostic effect has a lot to do with the choice of the center frequency of the bandpass filter and its bandwidth. Comparing Fig. 6 and Fig. 5b), the calculation results of the resonance demodulation method based on wavelet transform are much better than those of the traditional resonance demodulation method.

### 3.3 Bearing rollers peeling failure and normal state diagnosis

The resonance demodulation method based on wavelet transform is used to analyze the bearing rollers fault of the railway freight cars. Compared with the traditional resonant demodulation method, the results similar to the those shown in previous two subsections can be obtained, which will not be repeated here. At the same time, correct diagnosis under normal working conditions also shows that there is no misjudgment phenomenon by using this method.

A vibration acceleration signal of a rolling bearing of a freight car with an outer ring fault is shown in Fig. 7. It is analyzed by a wavelet demodulation-based resonance demodulation method, and the calculated time-frequency distribution map and spectrogram are respectively shown in a) and b) of FIG.

![Figure 7. Vibration Acceleration signal of the rolling bearing of freight car with bearing rollers pealing fault](image)

![a) time-frequency spectrum](image)

![b) spectrogram](image)

**Figure 7. Vibration Acceleration signal of the rolling bearing of freight car with bearing rollers pealing fault**

**Figure 8. Diagnose result of bearing rollers peeling failure with wavelet transform-based resonance demodulation method**
Different from the above-mentioned inner and outer ring faults, even the most experienced engineers cannot see the periodic impact from the original signal of Fig. 7, and from the time-frequency distribution shown in Fig. 8a). However, by applying the kurtosis factor minimization criterion mentioned in this paper, the time-energy signal can be extracted from Fig. 8a), and then the time-frequency diagram shown in Fig. 8b) can be obtained by thinning Fourier analysis technique, and the bearing rollers failures can then be clearly displayed.

For the vibration acceleration signal of the normal railway freight car rolling bearing shown in Fig. 9, the analysis is performed using the wavelet transform-based resonance demodulation method, and the time-frequency distribution map and the spectrogram as shown in a) and b) of Fig. 8 are obtained. On the spectrogram shown in Fig. 8b), except for a prominent peak in the low frequency range (less than 20 Hz), no energy and obvious peaks can be seen in other frequency bands. Therefore, it can be seen from Fig. 8b) that the rolling bearing of the freight cars corresponding to the signal is in a normal working state, and proves that the use of the resonance demodulation method based on wavelet transform to diagnose the fault of the rolling bearing of the freight cars would cause no misjudgment.

![Figure 9] vibration acceleration signal of the normal railway freight car rolling bearing

![Figure 10] a) time-frequency distribution map  b) spectrogram

**4. Conclusions**

A resonance demodulation method based on wavelet transform is proposed, and the method is used to analyze and diagnose the inner and outer ring faults, bearing rollers faults and the normal state of railway freight car rolling bearings. The numerical results show that the method can not only effectively diagnose the various faults of railway freight rolling bearings, but also cause no misjudgment.

The resonance demodulation method based on wavelet transform borrows the idea of the traditional resonance demodulation method, but it is very different in the specific calculations. The method replaces the process of filtering original acceleration signals and the process of randomly selecting parameters in extracting the envelop with applying continuous wavelet transform in the time-sampling rolling bearing.
vibration signals and automatically extracting the time-energy signal. Thus avoid the difficulties in selecting the center frequency of band-path filter and in the selection of bandwidth.

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