Model-Based Wald Test for Adaptive Range-Spread Target Detection

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ABSTRACT For the problem of range-spread target detection, many adaptive detectors commonly estimate the covariance matrix of the disturbance by utilizing the training data without target information. However, in the limited-training case, the conventional detectors suffer significant performance degradation. This paper devises and assesses a model-based Wald detector by modeling the disturbance as an autoregressive (AR) process with unknown parameters, which is able to overcome the detection degradation caused by insufficient training data. Meanwhile, the Wald test reduces the computational complexity because the unknown parameters are only estimated by maximum likelihood criterion under hypothesis that the target exists. Remarkably the asymptotic expression for the probability of detection and false alarm shows the detector is asymptotically constant false alarm rate (CFAR) with respect to the disturbance covariance matrix. The performance evaluation, conducted by resorting to simulation data, has confirmed the effectiveness of the current proposal in comparison with the previously proposed detectors.

INDEX TERMS Adaptive detector, autoregressive process, range-spread target, Wald test.

I. INTRODUCTION

The problem of detecting a target under the disturbance with unknown covariance has attracted more and more attention in the past decades [1]–[7]. It is usually assumed that the true covariance is estimated by a set of training data without target [1]. For Kelly’s generalized likelihood ratio test (GLRT) [1], the maximum likelihood estimation (MLE) of all unknown parameters should be obtained separately under each hypothesis. Compared with GLRT, the adaptive matched filter (AMF) [3] reduces the computation by a two-step design: first, assuming that the covariance matrix of the disturbance is known, the test expression of GLRT is derived, and then the covariance matrix is replaced by its MLE estimated by the training data. Other adaptive detectors such as the Wald test [8], the adaptive beamformer orthogonal rejection test (ABORT) [6], the adaptive coherence estimator (ACE) [5], etc. have been proposed subsequently. More recently, adaptive detection for multichannel signals receives increasingly attention [9], [10]. In order to ensure the performance loss within 3dB from the optimum bound, the number of training data required is at least twice as much as the dimension of unknown covariance matrix according to [11]. However, this requirement is difficult to meet in realistic environment especially when it comes to heterogeneous scenarios or dense-target scenarios. Therefore, it has practical significance to carry out the research of detectors that can achieve notable detection performance improvement in the limited training-data case.

Exploiting the properties of disturbance can effectively solve the problem of detection performance degradation when there are only a small amount of training data available. In many radar scenarios, the practical disturbance can be modeled as an AR process of a relatively low order, in the range of 1 to 4 [12]. The AR model has been verified effective in improving detection performance [13]–[16]. In [13], an adaptive filtering method based on AR spectrum estimation is proposed to suppress the disturbance. Then, Kay has applied the AR model to his GLR detector assuming that the target was known [14], and Sheikhi et al. have derived an ARGLR detector under the assumption of known target scaling factor [15]. In addition, the structural features of the disturbance matrix, such as persymmetric structure [17]–[22], Toeplitz structure [23] and spectrum
symmetry structure [24], [25], are increasingly used to eliminate the influence of insufficient training data.

All of the aforementioned detectors are designed for point-like target detection. However, the target is resolved into numerous scattering centers with the improvement of radar resolution [26]–[29]. In [30], adaptive range-spread target detectors ARGLR-G and ARGLR-HTG are derived under the assumption of Gaussian and heterogeneous environment respectively. The heterogeneous environment here refers to that the data vector of each range cell shares a different covariance matrix. More recently, the AR model has been increasingly studied in range-spread target detection [31]–[33].

Motivated by all the aforementioned works, in this paper, we focus on the design of a range-spread targets detector in the presence of Gaussian disturbance with unknown covariance matrix. Since a uniformly most powerful (UMP) test does not exist for the detection problem at hand, it is reasonable to adopt a detection strategy with better performance or less complexity than those based on GLRT. To this end, we resort to the Wald test criterion, which is easier to be derived and implemented than the GLRT [8], [34]–[36]. Precisely, we model the disturbance as an AR process of relatively low order \( M \) by utilizing both the primary and training data and then derive a new Wald detector. The asymptotic analysis of the false alarm and detection probabilities shows that the newly derived detector is asymptotic CFAR. Finally, the performance assessment conducted by Monte Carlo simulation indicates that the proposed detector achieves performance improvement compared with existing ones. The proposed AR-Wald detector and its performance analysis represent the novel contributions of this paper.

The rest of this paper is arranged as follows. Section II gives both the problem formulation and the description of the signal and disturbance models. Then, we give the design and asymptotic performance analysis of the AR-Wald detector in Section III and Section IV, respectively. The performance assessment of the AR-Wald detector is carried out in Section V. Finally, some conclusions are given in Section VI.

II. PROBLEM FORMULATION

The primary data is denoted as \( z_t \in \mathbb{C}^{N \times 1}, t = 1, \ldots, L \), which are collected from a coherent train of \( N \) pulses in the \( r \)th range cell under test, and the possible target is completely contained within those data. As custom, a training data set \( z_t \in \mathbb{C}^{N \times 1}, t = L + 1, \ldots, L + K \) is available, which does not contain any useful target echo. The current detection problem can be formulated as the following binary hypothesis data model

\[
H_0 : z_t = n_t, \quad t = 1, \ldots, L + K, \\
H_1 : z_t = \alpha_t p + n_t, \quad t = 1, \ldots, L, \\
z_t = n_t, \quad t = L + 1, \ldots, L + K
\]

where \( \alpha_t, t = 1, \ldots, L \) is unknown deterministic factor which accounts for both the target and the channel effects; \( p = [1, \alpha_2, \ldots, \alpha^N(L-1)]^T \) denotes the \( N \)-dimensional steering vector, where \( (\cdot)^T \) denotes the transpose operator, and \( \Omega \) is the normalized target Doppler; the disturbance vectors \( n_t, t = 1, \ldots, L + K \) are zero mean complex Gaussian vectors with unknown covariance \( R \). Then we assume that \( n_t \) can be modeled as an AR process of order \( M \) and parameters \( a \) and \( \sigma^2 \)

\[
n_t(l) = -\sum_{m=1}^{M} a(m)n_{t-m} + w_t(l) \quad l = 1, \ldots, N \tag{2}
\]

where \( a = [a(1), a(2), \ldots, a(M)]^T \) is the complex parameter vector of the AR process, \( w_t(l) \) is a sequence of zero mean complex white Gaussian noise with variance \( \sigma^2 \). Note that \( a \) and \( \sigma^2 \) are unknown deterministic parameters in this paper. Since the training data are independent and identically distributed (IID), the approximate joint probability density function (PDF) of the primary data and the training data under hypothesis \( H_t \) is [14], [32], [33]

\[
f(Z|\theta, H_t) = \frac{1}{(\pi \sigma^2)^{(N-M)(L+K)}} \cdot \exp \left\{ -\frac{1}{\sigma^2} \left[ \sum_{l=1}^{L} (u_t + X_t a - ic_t(q + Pa))^\dagger \times (u_t + X_t a - ic_t(q + Pa)) \right. \right. \\
\left. \left. + \sum_{l=L+1}^{L+K} (u_t + X_t a)^\dagger (u_t + X_t a) \right]\right\} \tag{3}
\]

where \( i = 0, 1 \), \( (\cdot)^\dagger \) denotes the conjugate transpose operation,

\[
X_t = \begin{pmatrix}
    z_t(M) & z_t(M-1) & \cdots & z_t(1) \\
    z_t(M+1) & z_t(M) & \cdots & z_t(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    z_t(N-1) & z_t(N-2) & \cdots & z_t(N-M) \\
\end{pmatrix}
\]

\[
P = \begin{pmatrix}
    p(M) & p(M-1) & \cdots & p(1) \\
    p(M+1) & p(M) & \cdots & p(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    p(N-1) & p(N-2) & \cdots & p(N-M) \\
\end{pmatrix}
\]

\[
u_t = [z_t(M+1), \ldots, z_t(N)]^T,
\]

\[
q = [p(M+1), \ldots, p(N)]^T, \quad t = 1, \ldots, L + K.
\]

III. THE AR-WALD DETECTOR DESIGN

To evaluate the Wald test statistics, the following parameters are introduced in advance:

\[
\theta_t = [\alpha_{t, R}, \alpha_{t, I}, \alpha_{t, 2, R}, \alpha_{t, 2, I}, \ldots, \alpha_{t, L, R}, \alpha_{t, L, I}]^T
\]

is a \( 2L \)-dimensional real column vector, where \( \alpha_{t, R} \) and \( \alpha_{t, I} \)
denote the real and imaginary part of $\alpha_t$ respectively, $t = 1, \ldots, L$.

$\theta_s = [a_{R}^{T}, a_{I}^{T}, \sigma^2]^T$ is a $(2M+1)$-dimensional real column vector, where $a_R = \text{vec}(\text{Re}[a])$ and $a_I = \text{vec}(\text{Im}[a])$, $\text{vec}(\cdot)$ is the vectorization operator, $\text{Re}[\cdot]$ and $\text{Im}[\cdot]$ denotes the real part and imaginary part of complex vector, respectively.

$\theta = [\theta_r^T, \theta_s^T]^T$ is a $(2L + 2M + 1)$-dimensional real column vector.

Hence the Wald test for the problem at hand is the following decision rule [8]

$$
\hat{\theta}_{r,1}^T \left[ J^{-1}(\hat{\theta}_1)\right]_{\theta_r,\theta_r}^{-1} \hat{\theta}_{r,1} \overset{H_1}{\gtrless} \eta_{\text{AR-Wald}}
$$

(4)

where $\eta_{\text{AR-Wald}}$ is the detection threshold to be set according to the desired value of false alarm probability $P_f, \theta_1 = \hat{\theta}_{r,1}^T \hat{\theta}_{s,1}^T$ is the maximum likelihood estimate of $\theta$ under $H_1, J(\theta) = J(\theta_r, \theta_s)$ denotes the fisher information matrix which can be partitioned as

$$
J(\theta) = \begin{bmatrix}
J_{\theta_r,\theta_r}(\theta) & J_{\theta_r,\theta_s}(\theta) \\
J_{\theta_s,\theta_r}(\theta) & J_{\theta_s,\theta_s}(\theta)
\end{bmatrix}
$$

where

$$
J_{\theta_r,\theta_r}(\theta) = -E[\partial^2 \ln f(Z|\theta) / \partial \theta_r \partial \theta_r^T],
$$

$$
J_{\theta_r,\theta_s}(\theta) = -E[\partial^2 \ln f(Z|\theta) / \partial \theta_r \partial \theta_s^T],
$$

$$
J_{\theta_s,\theta_r}(\theta) = -E[\partial^2 \ln f(Z|\theta) / \partial \theta_s \partial \theta_r^T],
$$

$$
J_{\theta_s,\theta_s}(\theta) = -E[\partial^2 \ln f(Z|\theta) / \partial \theta_s \partial \theta_s^T],
$$

\[\partial / \partial \theta_r = \begin{bmatrix}
\partial / \partial \alpha_{1,R}, \partial / \partial \alpha_{1,I}, \ldots, \partial / \partial \alpha_{L,R}, \partial / \partial \alpha_{L,I}, \partial / \partial \alpha_{\sigma^2}
\end{bmatrix}^T
\]

denotes the gradient with respect to $\theta_r, f(\cdot)$ is the PDF for the data, $Z = [z_1, z_2, \ldots, z_{L+K}]$ denotes both the primary and the training data.

A. FISHER INFORMATION MATRIX

The desired $[J^{-1}(\theta)]_{\theta_r,\theta_r}$ in (4) is the $(\theta_r, \theta_r)$-part of the inversion of $J(\theta)$, namely,

$$
[J^{-1}(\theta)]_{\theta_r,\theta_r} = \left( J_{\theta_r,\theta_r}(\theta) - J_{\theta_r,\theta_s}(\theta) J^{-1}_{\theta_s,\theta_s}(\theta) J_{\theta_s,\theta_r}(\theta) \right)^{-1}
$$

(5)

We first calculate the $(2L \times 2L)$-dimensional matrix $J_{\theta_r,\theta_r}(\theta)$ and $2L \times (2M + 1)$-dimensional matrix $J_{\theta_r,\theta_s}(\theta)$, which has the form of

$$
J_{\theta_r,\theta_r}(\theta)(i,j) = -E \left[ \frac{\partial^2 \ln f(Z|\theta)}{\partial \theta_r(i) \partial \theta_r(j)} \right]
$$

(6)

and

$$
J_{\theta_r,\theta_s}(\theta)(i,j) = -E \left[ \frac{\partial^2 \ln f(Z|\theta)}{\partial \theta_r(i) \partial \theta_s(j)} \right]
$$

(7)

respectively, $1 \leq i, j \leq 2H$.

The first derivative of the $\ln f(Z|\theta)$ versus $\theta_r$ is

$$
\frac{\partial}{\partial \theta_r} \ln f(Z|\theta) = \begin{bmatrix}
\frac{\partial}{\partial \alpha_{1,R}} \ln f(Z|\theta), & \frac{\partial}{\partial \alpha_{1,I}} \ln f(Z|\theta), \\
\vdots, & \frac{\partial}{\partial \alpha_{L,R}} \ln f(Z|\theta), & \frac{\partial}{\partial \alpha_{L,I}} \ln f(Z|\theta)
\end{bmatrix}
$$

(8)

where

$$
\frac{\partial}{\partial \theta_r} \ln f(Z|\theta) = 2Re \left\{ (q + Pa)^\dagger [u_i + X_i \alpha - \alpha_i(q + Pa)] / \sigma^2 \right\}
$$

(9)

$$
\frac{\partial}{\partial \theta_r} \ln f(Z|\theta) = 2Re \left\{ \frac{(q + Pa)^\dagger [u_i + X_i \alpha - \alpha_i(q + Pa)]}{\sigma^2} \right\}
$$

(10)

Then, we take the second partial derivative of $\ln f(Z|\theta)$ with respect to $\theta_r$ and $\theta_s$, respectively. The result is given as

$$
\frac{\partial^2}{\partial \alpha_{1,R} \partial \alpha_{1,I}} \ln f(Z|\theta) = \frac{\partial^2}{\partial \alpha_{1,R} \partial \alpha_{1,I}} \ln f(Z|\theta) = 0
$$

(11)

$$
\frac{\partial^2}{\partial \alpha_{1,R} \partial \alpha_{2,I}} \ln f(Z|\theta) = \frac{\partial^2}{\partial \alpha_{1,R} \partial \alpha_{2,I}} \ln f(Z|\theta) = 0
$$

(12)

$$
\frac{\partial^2}{\partial \alpha_{2,R} \partial \alpha_{R}} \ln f(Z|\theta) = 2Re \left\{ \frac{(u_i + X_i \alpha - \alpha_i(q + Pa))^\dagger P \frac{\partial}{\partial \alpha_R}}{(q + Pa)^\dagger (X_i - \alpha_i P) \frac{\partial}{\partial \alpha_R}} / \sigma^2 \right\}
$$

(13)

$$
\frac{\partial^2}{\partial \alpha_{R} \partial \sigma^2} \ln f(Z|\theta) = -2Re \left\{ (q + Pa)^\dagger [u_i + X_i \alpha - \alpha_i(q + Pa)] / \sigma^2 \right\}
$$

(14)

where $t = 1, \ldots, L, \delta(\cdot)$ denotes the Kronecker delta function, $\alpha_R$ is the $i$th element of $\alpha_R$. By taking the mathematical expectation of (13) and (14), we can get $E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{R} \partial \alpha_{R}] = 0, E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{R} \partial \sigma^2] = 0$ as a consequence of $E[u_i + X_i \alpha - \alpha_i(q + Pa)] = 0$ and $E[X_i - \alpha_i P] = 0$. Similarly, after some calculation, we get

$E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{1,I} \partial \sigma^2] = 0$, $E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{1,I} \partial \alpha_{R}] = 0$, $E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{1,I} \partial \alpha_{1,I}] = 0$, $E[\partial^2 \ln f(Z|\theta) / \partial \alpha_{1,I} \partial \alpha_{1,I}] = 0$.

By substituting all the results mentioned above into (5) and (7), we have $J_{\theta_r,\theta_r}(\theta) = 0, J_{\theta_r,\theta_s}(\theta) = 0$.

$$
\left( J^{-1}(\hat{\theta}) \right)_{\theta_r,\theta_r}^{-1} = \frac{2}{\sigma^2} (q + Pa)^\dagger (q + Pa) I_{2L \times 2L}
$$

(15)
B. THE MLE OF AR PARAMETERS

It can be seen from (3) that the maximum likelihood estimation of $\alpha_t$ under hypothesis $H_1$ can be obtained by minimizing $J(\alpha_t)$ with respect to $\alpha_t$, where

$$J(\alpha_t) = (u_t + X_t a - \alpha_t (q + Pa))^\dagger (u_t + X_t a - \alpha_t (q + Pa))$$

By taking the derivative of (16) with respect to $\alpha_t$ and equating to zero, we get the MLE of $\alpha_t$ given $a$

$$\hat{\alpha}_t = \frac{(q + Pa)^H (u_t + X_t a)}{(q + Pa)^H (q + Pa)}$$

substituting (17) into $f(Z|a, H_1)$, yields

$$\max_{\sigma^2} f(Z|a, H_1)$$

$$= \left( \frac{\pi e}{(N-M)(L+K)} \right)^{\frac{1}{2}} \times \left[ \sum_{t=1}^{L} (u'_t + X'_t a)^\dagger (u'_t + X'_t a) + \sum_{t=L+1}^{L+K} (u_t + X_t a)^\dagger (u_t + X_t a) \right]^{-(N-M)(L+K)}$$

which is the maximum with respect to $\alpha_t$ and $\sigma^2$, where $u'_t = Hu'_t$ and $X'_t = HX'_t$. Here $H$ is the projection matrix defined as

$$H = I - \phi \phi^\dagger$$

with $\phi = [1, e^{i2\lambda}, \ldots, e^{i(N-M-1)\lambda}]^T$, where $I$ is the $(N-M) \times (N-M)$ identity matrix. Then by minimizing

$$J(a) = \sum_{t=1}^{L} (u'_t + X'_t a)^\dagger (u'_t + X'_t a) + \sum_{t=L+1}^{L+K} (u_t + X_t a)^\dagger (u_t + X_t a)$$

with respect to $a$, we obtain

$$\hat{\alpha}_1 = - \left( \sum_{t=1}^{L} X'_t HX_t + \sum_{t=L+1}^{L+K} X'_t X_t \right)^{-1} \left( \sum_{t=1}^{L} X'_t Hu_t + \sum_{t=L+1}^{L+K} X'_t u_t \right)$$

Finally, by substituting (15), (17) and (20) into the decision rule (4), we obtain the AR-Wald detector after simplification (21), as shown at the bottom of the next page.

It can be seen that the computational cost of the proposed AR-Wald is $O((L+K)M^2N)$ when $N \gg M$.

IV. ASYMPTOTIC PERFORMANCE ANALYSIS

We now use Kay’s theorem to derive the asymptotic property of the AR-Wald [14], [37]. For the case of large data records (i.e. $N$ approaches infinite), the Wald detector has the identical asymptotic distribution as the GLRT

$$T_{AR-Wald} \sim a \left\{ \chi^2_{2L}, \chi^2_{2L}(\lambda), H_0, H_1 \right\}$$

FIGURE 1. The performance of the AR-Wald detector and the conventional Wald detector for $P_{fa} = 10^{-4}, N = 16, L = 8$ and various $K$.

FIGURE 2. The performance of the AR-Wald detector and the AR-Rao detector for $P_{fa} = 10^{-4}, N = 16, L = 8$ and various $K$.
V. EXPERIMENTAL RESULT

The performance analysis of the newly derived AR-Wald detector will be carried out by using simulated data. To begin we resort to $100/P_{fa}$ independent Monte Carlo experiments to evaluate the threshold and $10/P_{fa}$ independent experiments to compute the $P_d$. The disturbance signal $n_t$ is generated from the AR process of order 2 (except specific explanation) with given parameters $a$ and $\sigma^2$. These parameters should be set to make sure the AR process is stable [33]. The signal to interference plus noise ratio (SINR) is defined by

$$
\text{SINR} = \frac{1}{L} \sum_{i=1}^{L} \left| \alpha_i \right|^2 p_i \, \mathbf{R}^{-1} \, p
$$

(25)

where $\mathbf{R}$ is the actual covariance matrix of the disturbance, which can be calculated using the AR parameters $a$ and $\sigma^2$ [38]. Both the previous derived Wald [27] and AR-Rao [33] tests for the range spread target detection are simulated for comparison. The test statistic of the previous Wald and AR-Rao detector which is asymptotic CFAR [33].

Thus following (22) and (24), we have derived the asymptotic probability distribution of the AR-Wald detector. The result indicates that the newly derived AR-Wald detector has the same asymptotic property as the previous AR-Rao detector which is asymptotic CFAR [33].

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where \( \hat{\alpha}_0 = - \left( \sum_{i=1}^{L+K} X_i^\dagger X_i \right)^{-1} \left( \sum_{i=1}^{L+K} X_i^\dagger u_i \right) \).

**A. INFLUENCE OF THE NUMBER OF TRAINING DATA**

In this subsection, we evaluate the influence of the number of training data on the detection performance of the AR-Wald detector. From Figure 1 and Figure 2, we can see that the detection performance of AR-Wald, Wald and AR-Rao are all improved with the increase of \( K \). This is because the estimation accuracy of unknown parameters improves as the number of training data increases.

In order to make sure the sample covariance matrix is not singular, the number of training data of the conventional Wald test is set to \( K > N \). It can be seen from Figure 1 that the conventional Wald experiences a significant performance loss unless there are sufficient training data (\( K \geq 2N \)) available, which is in agreement with [27]. The curves in Figure 1 also show that the newly derived AR-Wald detector has significant performance improvement compared with the Wald detector in limited-training case.

The curves in Figure 2 indicate that, compared with the previous derived AR-Rao detector, the AR-Wald detector has a small improvement (about 1.06dB for \( K = 16 \), 0.15dB...
for $K = 32$ at $P_d = 0.9$). The results highlight that the performance improvement is negligible for $K \gg N$. Moreover, when the SINR is greater than 10dB, the detection performance of the AR-Rao detector deteriorates with the increases of SINR for the case $K = 8$. This can be explained because both of the primary and the training data are needed to estimate the parameters under hypothesis $H_0$ and as a result, the estimation of AR parameters are polluted by the signal [39]. In a word, the AR-Wald detector outperforms another two detectors especially when there are only a small amount of training data available.

B. INFLUENCE OF THE NUMBER OF PULSES

In this subsection, we analyze the influence of the number of pulses $N$ on the detection performance of the AR-Wald detector. For the case $K = 0$, as is shown in Figure 3(a), the detection performance improves when the pulse number $N$ increases. From Figure 3(b), it can be seen that the detection performance of the AR-Wald detector for $N = 64$, $K = 0$ almost coincides with the case for $N = 16$, $K = 32$. It shows that we are able to remedy the performance degradation of the detector caused by insufficient training data by increasing the number of sample pulses.

C. INFLUENCE OF THE MULTIPLE DOMINANT SCATTERER (MDS) MODELS

In this subsection, we analyze the influence of the MDS models on the detection performance of the AR-Wald detector. There are three different MDS models taken into consideration: model1, model2 and model3. First of all, the sum of the energy of the scatterers, namely, the total energy of the target, is equal. For model1, we assume that the total energy is evenly distributed among 8 range cells. Model2 denotes the case that one of the range cells has the strongest energy with the rest of energy are uniformly distributed in other range cells. Model3 represents the target whose total energy is only distributed in 1 range cell, namely, the unresolved target, which could lead to the collapsing loss [40]. Note that the MDS model used in other simulations in this paper is model1. See Table 1 for details of each MDS model.

In Figure 4(a), the detection probabilities of the AR-Wald detector for $N = 16$, $L = 8$, $K = 8$ is plotted with three MDS models. It can be seen that the three different MDS models have little effect on the detection performance of the proposed detector. Meanwhile, the curves of model2 and model3 show that the AR-Wald detector does not suffer collapsing loss. The conclusion is also supported by the curves in Figure 4(b) for the case $N = 32$.

D. INFLUENCE OF THE ORDER $M$

In the derivation of the AR-Wald detector, we assume that the order $M$ is known. However, this parameter is usually unknown and needs to be estimated with both the primary and training data before detection. In this subsection, we assess the effect of the order $M$ on the detection performance when $L = 8$ and $K = 3$. The MDS model used here is model1.

The curves in both Figure 5(a) and (b) indicate that the performance of the AR-Wald detector gets worse as the order $M$ increases. In the case $N = 16$ shown Figure 5(a), the AR-Wald detector suffers a 0.8db performance loss for $M = 3$ and which is up to 1.5db for $M = 4$ with respect to its performance for $M = 2$ at $P_d = 0.8$. It can be seen that the performance loss caused by the mismatched order $M$, namely, $M = 3$ and $M = 4$, is more significant for small $N$ by comparing Figure 5(a) and (b). This can be explained that $M$ samples have been discarded in the derivation and calculation of the test statistic, which may be incorrect when the pulse number is small in respect to $M$.

Meanwhile, the mismatched order $M$ is negligible compared with other parameters when $N$ is large enough.

E. REAL DATA ANALYSIS

In order to further assess the performance of the newly derived detector, we resort to the real data of high-resolution range profiles (HRRPs) of Tiangong-1 measured by inverse synthesis aperture radar (ISAR). The original echo data of the target is shown in Figure 6(a), form which it can be seen that there are range migration in moving target. Firstly, the HRRPs are envelope aligned and autofocused to eliminate the effect of the target’s translational motion, and then the AR-Wald detector is used for detection. The aligned echo is shown in Figure 6(b). Since the target occupies 67 range cells, we set $L = 80$.

To assess the detection performance of the AR-Wald for various sample number $N$, we simulate the detection curves for $L = 80$, $K = 0$ with real data in Figure 7(a). It can be seen that there is no signal contamination (exist in AR-Rao detector [33]) in high SINR, which indicates that the newly derived AR-Wald detector outperforms the AR-Rao under high SINR. In Figure 7(b), the receiver operating characteristic (ROC) curves of the AR-Wald detector are plotted with

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**TABLE 1. MDS models.**

| Model number | Range cell number |
|--------------|------------------|
| 1            | 1/8              |
| 2            | 1/8              |
| 3            | 1/8              |
| 4            | 1/8              |
| 5            | 1/8              |
| 6            | 1/8              |
| 7            | 1/8              |
| 8            | 1/8              |

**FIGURE 6.** High-resolution range profiles of Tiangong-1 measured by inverse synthesis aperture radar. (a) Original, (b) Aligned.
AR-Wald detector achieves better detection performance over the AR-Rao detector. Other simulations indicate that the AR-Wald detector performs robust for large sample pulse number case even no training is available. Through the analysis of the real data, the detection results of the real data are consistent with those of the simulation data. The effectiveness of the proposed AR-Wald detector is proved by both the simulation data and the real data. However, the newly proposed AR-Wald detector has the problem of large computation, and our future work will focus on improving this problem. In addition, we will also consider the application of AR-Wald algorithm to the range-spread target detection in inhomogeneous disturbance.

VI. CONCLUSION

Aiming at the problem of range-spread target detection in Gaussian disturbance, we have proposed an adaptive detector by modeling the Gaussian disturbance as an AR process and exploiting the Wald test criterion. Then we have derived the asymptotic expression of the false alarm and detection probabilities, which show that the newly proposed AR-Wald detector is asymptotic CFAR. Simulation results demonstrate that the AR-Wald detector overcomes the dependence of conventional Wald detector on training data. Moreover, the various values of $L$ when $K = 0$ and SINR = 10dB. We can see that the performance of the AR-Wald detector is improved with the increase of $L$. In other words, improving the radar resolution can enhance the detection performance, which is in line with our cognition. Further analysis indicates that the results with real data and simulation data are consistent. Simulation results and the real data results show the effectiveness of the newly derived AR-Wald detection algorithm.

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