Abstract

ZZ Ceti stars are pulsating white dwarfs with a carbon-oxygen core (or possibly ONe for the most massive stars) build up during the core helium burning (CHeB) and thermally pulsing Asymptotic Giant Branch (TP-AGB) phases. Through the interpretation of their pulsation periods by means of asteroseismology, details about their origin and evolution can be inferred. The whole pulsation spectrum exhibited by ZZ Ceti stars strongly depend on the inner chemical structure. At present, there are several processes affecting the chemical profiles that are still not accurately determined. We present a study of the impact of current uncertainties in the evolution of white dwarf progenitor on the expected pulsation properties and on the stellar parameters inferred from asteroseismological fits of ZZ Ceti stars. Our analysis is based on a set of carbon-oxygen core white dwarf models that are derived from full evolutionary computations from the ZAMS to the ZZ Ceti domain. We considered models in which we varied the number of thermal pulses, the amount of overshooting, and the carbon-alpha reaction rate within their uncertainties. We explore the impact of these major uncertainties in prior evolution on the chemical structure and expected pulsation spectrum. We find that these uncertainties yield significant changes in the g-mode pulsation periods being those found during the TP-AGB phase the most relevant for the pulsational properties and the asteroseismological derived stellar parameters of ZZ Ceti stars. We conclude that the uncertainties in the white dwarf progenitor evolution should be taken into account in detailed asteroseismological analyses of these pulsating stars.
in turn depend on both the amount of OV adopted during the TP-AGB phase and the poorly constrained efficiency of mass loss (Karakas & Lattanzio 2014).

2 Impact over the chemical structure and period spectrum

As a first step, we explore the impact of uncertainties linked to:

1. the occurrence of TP during the AGB phase of the progenitor star,
2. the occurrence of OV during core He burning,
3. the \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) reaction rate,

on the chemical structure and period spectra of template models characterized by \( M_* = 0.548, 0.837 M_\odot \), with H envelope mass of \( M_{\text{H}} \sim 4 \times 10^{-6} M_* \) at \( T_{\text{eff}} \sim 12000 \) K, and considered the pulsation period spectrum for modes with \( \ell = 1 \). All our evolutionary computations were performed with the LPCODE evolutionary code, while the pulsational properties of our models were computed with LP-PUL, both numerical tools developed in La Plata, Argentina.

2.1 Occurrence of TP in AGB

Because during this phase a pulse-driven convection zone is developed, the building of an intershell region and a double-layered chemical structure is expected at the bottom of the He buffer. The size of this region decreases as the star evolves through the TP-AGB, as consequence of the He-burning shell. The number of TP experienced by the progenitor star is uncertain and depends on the rate at which mass is lost during the TP-AGB phase, on the initial metallicity of progenitor star, as well as on the occurrence of extra-mixing in the core/He main chemical transition region. In the other hand, the expectations for the massive model are markedly different. Indeed, in this case the period differences are on average 3 s, resulting to the slight differences found in chemical profiles. In fact, no intershell is expected to occur at the ZZ Ceti domain.

2.2 Occurrence of OV during core He burning

Extra mixing episodes during core He burning strongly affect the final shape of the carbon-oxygen core. We performed a comparison between models in the ZZ Ceti stage, that comes from progenitor stars in which we considered the extreme situation of no OV allowed during core He burning (NOV model), and compared it to the one in which we considered a moderate OV (OV model). Fig. 3(a) show the inner chemical profiles at the ZZ Ceti stage. The ingestion of fresh He in a C-rich zone, due to the inclusion of OV, favour the consumption of C via the reaction \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \), with the consequent increase of \( ^{16}\text{O} \). The B-V frequency is notoriously affected by the steep variation in the carbon-oxygen profile left by core OV. Differences in the period spectra are shown in Fig. 3(b), revealing variations of 4 s on average, markedly lower than the period differences resulting from uncertainties in the TP-AGB phase, being the period differences resulting from the inclusion of OV, similar for both stellar masses.

2.3 Uncertainties in the \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) reaction rate

In this case, we computed the CHeB phase of the progenitors by considering three different values for the \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) reaction rate, and followed the evolution until the progenitor leave the TP-AGB phase. We considered the reaction rates provided by Angulo et al. (1999) (Nacre, our reference model), and the extreme high and low values from Kunz et al. (2002) (Kh and Kl models, respectively). These values for the reaction rate allow us to account for the uncertainties in the carbon-oxygen chemical profiles of WD. For core He burning temperatures \( \sigma_{\text{Kl}}/\sigma_{\text{Nacre}} \sim 0.55 \) and \( \sigma_{\text{Kh}}/\sigma_{\text{Nacre}} \sim 1.1 \), being the uncertainty factor of about 2. Fig. 4(a) shows the chemical profiles for the models at the ZZ Ceti stage whose progenitors were computed by adopting different reaction rates during CHeB. Differences in chemical structure are mainly found in the location of the O/C and O/C/He chemical transitions and in the abundances of the intershell
region, with the resulting change in the B-V frequency.

Fig. 4(b) shows the differences in the pulsation spectrum due to the uncertainty in the $^{12}$C + $\alpha$ reaction rate, by taking as reference the model computed with the Nacre value. In particular, for the low-mass model, appreciable differences are expected for both low and high radial-order modes, with average variations of 4 s, while in the case of the massive model (not showed here), small period differences are found for modes with radial orders $k < 23$. These small variations reflect the smooth behaviour of the chemical profile at the outermost part of the carbon-oxygen core. But, for radial orders $k > 23$, appreciable differences are found, that stem from the fact that some modes of high radial order $k$ are sensitive to the chemical structure of the core.

3 Impact over asteroseismological determinations

As we have seen before, whether the progenitor star evolves through the TP-AGB or not, strongly impact the period spectrum of pulsating DA WDs, specially in the case of the low-mass ZZ Ceti stars.

For our study of the impact of the thermal pulses on the stellar parameters derived through asteroseismology, we developed a grid of evolutionary models with final masses in the range $0.5349 \lesssim M_{\text{wd}}/M_\odot \lesssim 0.6463$. During the progenitor evolution we forced the evolutionary models to abandon this stage by enhancing the stellar mass loss rate at two different stages: previous to the onset of the first thermal pulse, and at the end of the third thermal pulse (0TP and 3TP models, respectively). We want to mention that, it is during the first
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Figure 3: Inner chemical profiles for models computed with an OV parameter $f = 0.016$ (OV) and with $f = 0$ (NOV) during core He burning (left panel) and the period differences for $k$ fixed, for the low mass model.

Figure 4: Same as Fig. 3 but for models computed with three different $^{12}\text{C}+\alpha$ reaction rates during CHeB (left panel) and the corresponding period differences (right panel).

TP that the main chemical features of the intershell and the double layered regions emerge. In the case of low mass stars, we do not expect the occurrence of a large number of TP, so considering 3 TP is enough for capturing the essential of the chemical structure arising from TP-AGB phase. For higher stellar masses, we expect a larger number of TP to take place, but as shown before, see also [De Gerónimo et al. (2017)], the period spectrum expected at the ZZ Ceti stage is not markedly affected by the number of additional TP's experienced by the progenitor star.

To study the impact of the uncertainties in the $^{12}\text{C}+\alpha$ reaction rate on the asteroseismological-derived stellar parameters, we considered a stellar grid of models with masses $0.523 \lesssim M_{\text{wd}}/M_\odot \lesssim 0.90$, for which we take into account, during CHeB phase, both low- and high-values of the rate for the $^{12}\text{C}+\alpha$ nuclear reaction from the work of [Kunz et al. (2002)] obtaining two sets of models ($K_l$ and $K_h$, respectively).

In addition, we considered H-envelopes thinner than those predicted from evolutionary computations, ranging $-9 \lesssim \log(M_H/M_{\text{wd}}) \lesssim -4$.

$$\phi = \phi(M_*, M_H, T_{\text{eff}}) = \frac{1}{N} \sum_{i=1}^{N} \frac{[\Pi^i_{\text{th}} - \Pi^i_{\text{obs}}]^2 A_i}{\sum_{i=1}^{N} A_i}$$  \hspace{1cm} (1)

As a first approach, we performed period-to-period fits to 1000 artificially-generated pulsating DA WD\footnote{Our artificially-generated pulsating DA are composed by 3-random generated periods, with values representative of ZZ Ceti stars.} That is, we selected the best fit model for each set of models, 0TP and 3TP (or $K_l$ and $K_h$) according to the minima of the quality function $\phi$ [Eq. (1)] — the
best period-fit model. By making histograms with the value of the differences in the asteroseismologically-derived stellar parameters and adjusting to them Gaussian function we found that the sigma (σ) values are very close to the observational errors for those quantities (Tremblay et al., 2011), see table 1 (for more details, see De Gerónimo et al., 2018).

Finally, we contrast the previous results with those derived from asteroseismological fits to some selected real ZZ Ceti stars. We considered those ZZ Ceti stars with modes previously identified as low- \( T_{\text{eff}} \) or hot ZZ Ceti stars depending on the value of the pulsation period with the highest amplitude. For the selection of the best fit models we have taken into account:

- the models minimize the quality function given by Eq. 1
- \( T_{\text{eff}} \) and \( \log(g) \), as close as possible to the spectroscopic determinations.

The results of the asteroseismological period-to-period fits to the real ZZ Ceti stars are illustrated in Figs. 5 and 6. The figures show the deviations found in the asteroseismologically-derived stellar parameters as given by the two sets of evolutionary sequences. In the X-axis we plot the percentage of the deviation in effective temperature, in the Y-axis the deviation in stellar mass while in color scale the absolute difference in \( \log(M_H/M_{\text{wd}}) \). From the figures we warn that these results are consistent with those found previously. Although we might expect a certain link between the magnitude of the deviations found on the different stellar parameters, we did not find a clear pattern. An important result is that some hot- and intermediate-effective temperature ZZ Ceti stars are sensitive to the thickness of the hydrogen envelope.

### Table 1: σ values for the deviation in the stellar parameters for hot-, intermediate- \( T_{\text{eff}} \) artificially-generated ZZ Ceti. Values are strongly improved if \( M_H \) remain fixed

| \( \sigma_{T_{\text{eff}}} \) | Thermal pulses \( ^{12}C + \alpha \) |
|----------------|-----------------|
| hot            | 5.8%            |
| intermediate   | 4.5%            |
| cool           | 8%              |
| \( \sigma_{M_{\text{eff}}} \) (order of magnitude) |            |
| hot            | 7%              |
| intermediate   | 6%              |
| cool           | 11%             |

\[ \sigma (\text{order of magnitude}) = \begin{bmatrix} 0.5 & 0.6 \\ 0.6 & 0.74 \\ 2 & 2 \end{bmatrix} \]

4 Conclusions

Marked differences in the pulsational period spectrum are expected due to the uncertainties in the \( ^{12}C(\alpha, \gamma)^{16}O \) reaction rate and whether the progenitor star evolves or not through the TP-AGB phase. We find that these uncertainties during the progenitor evolution have a non-negligible impact on the resulting asteroseismologically-derived stellar parameters. We report that these uncertainties implies average deviations close to the spectroscopic errors, except for cool ZZ Ceti where the uncertainties are appreciably greater. In the other hand, for the mass of the H envelope we find deviations up to 2 order of magnitude in the case of low-\( T_{\text{eff}} \) ZZ Ceti models, while for hot- and intermediate-\( T_{\text{eff}} \) ZZ Ceti models, we find negligible differences. We want to highlight that these results are obtained by considering extreme situations, both for the values of the reaction rate as well as the evolution during the TP-AGB phase. We also find that the period spectrum of some hot and intermediate effective temperature ZZ Ceti stars are sensitive to the mass of the hydrogen envelope. Our main finding is that the uncertainties in prior WD evolution do affect both the period spectra of pulsating DA WD and WD asteroseismology, but the effects are quantifiable and bounded. In particular, differences found in the determinations of stellar mass, effective temperature and H envelope thickness due to the uncertainties studied, are within the typical spectroscopic errors. These results add confidence to the use of fully evolutionary models with consistent chemical profiles, and render much more robust our asteroseismological approach.

### References

Althaus L. G., Córsico A. H., Isern J., García-Berro E., 2010, A&A Rev., 18, 471

Angulo C., et al., 1999, Nuclear Physics A, 656, 3

Bischoff-Kim A., Østensen R. H., 2011, ApJ, 742, L16

Bischoff-Kim A., Østensen R. H., Hermes J. J., Provençal J. L., 2014, ApJ, 794, 39

Brickhill A. J., 1991, MNRAS, 251, 673

De Gerónimo F. C., Althaus L. G., Córsico A. H., Romero A. D., Kepler S. O., 2017, A&A, 599, A21

De Gerónimo F. C., Althaus L. G., Córsico A. H., Romero A. D., Kepler S. O., 2018, A&A, 613, A46

Dolez N., Vauclair G., 1981, A&A, 102, 375

Fontaine G., Brassard P., 2008, PASP, 120, 1043

Giammichele N., Fontaine G., Brassard P., Charpinet S., 2014, in Guzik J. A., Chaplin W. J., Hanlon J. D., eds, IAU Symposium Vol. 301, Precision Asteroseismology. pp 285–288, doi:10.1017/S1743921313014464
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Figure 5: Variations of $T_{\text{eff}}$, $M_{\text{wd}}$ and $M_{\text{H}}$ – in color scale – of selected ZZ Ceti stars resulting from the two sets of evolutionary sequences corresponding to the study of thermal pulses. Squares, circles, and triangles refer to hot, intermediate, and cool ZZ Ceti stars, respectively.

Giammichele N., Fontaine G., Brassard P., Charpinet S., 2016, ApJS, 223, 10
Goldreich P., Wu Y., 1999, ApJ, 511, 904
Herwig F., 2000, A&A, 360, 952
Karakas A. I., Lattanzio J. C., 2014, MNRAS, 31, e030
Kunz R., Fey M., Jaeger M., Mayer A., Hammer J. W., Staudt G., Harissopulos S., Paradellis T., 2002, ApJ, 567, 643
Romero A. D., Córsico A. H., Althaus L. G., Kepler S. O., Castanheira B. G., Miller Bertolami M. M., 2012, MNRAS, 420, 1462
Romero A. D., Kepler S. O., Córsico A. H., Althaus L. G., Fraga L., 2013, ApJ, 779, 58
Saio H., 2013, in European Physical Journal Web of Conferences. p. 05005, doi:10.1051/epjconf/20134305005
Tremblay P.-E., Bergeron P., Gianninas A., 2011, ApJ, 730, 128
Winget D. E., Kepler S. O., 2008, ARA&A, 46, 157
Winget D. E., van Horn H. M., Tassoul M., Fontaine G., Hansen C. J., Carroll B. W., 1982, ApJ, 252, L65
Figure 6: Same as Fig. 5 but for the study of the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate.