Optimal Demand Adjustment of Consumers with Various Appliances Using Dynamic Pricing

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Abstract: This paper deals with a demand adjustment problem of each consumer having appliances by an aggregator based on optimal pricing in a day-ahead electricity market. In this paper, we model consumers, generators, and aggregators and design a market mechanism, where they act to maximize their own profit based on the power price and decide the electricity supply and demand. The dual decomposition is applied to the market problem to maximize the social welfare, where the proposed algorithm decides an electricity price based on the exchange of information among market participants to solve each distributed problem for improving convergence. In addition, the convergence of this proposed method is proven using Lyapunov’s stability theorem. Finally, simulation results show that matching of supply and demand is achieved, while satisfying constraints on the consumption amount and the use time of each appliance, and the price determined using proposed price update algorithm converges to a certain price.

Key Words: optimal pricing, demand response, aggregator.

1. Introduction

In Japan’s energy policy so far, unidirectional supply side management have been carried out. However, in the Great East Japan Earthquake, restrictions on the energy supply and the vulnerability of centralized energy systems became clear, and securing the quality of electricity such as voltage and frequency has become a problem as the introduction of renewable energy expands. Against this backdrop, demand response (DR) that smartly changes consumption patterns in response to energy supply situation has attracted attention in recent years [1]. However, in the demand response, which is controlled from both the demand and supply side, as the number of consumers participating in the market transaction increases, the market form becomes complicated, and it becomes difficult to maximize the profit of each consumer. In order to solve this problem, the aggregator is expected to bundle consumers so that they are interpreted as a single large demand.

In this paper, we consider an optimal pricing problem for demand adjustment by the aggregator. A method of adjusting electricity supply and demand using electricity price is called dynamic pricing, and many researches have been devoted to the issue. Real-time pricing mechanism with incentive for participation based on a game-theoretic approach is proposed in [2]. The papers [3], [4] deal with the problem of real-time pricing using a nonlinear model, and [5] deals with the problem of determining the optimal pricing of electricity in the power market including the electricity storage system. As the papers focusing on aggregators,[6] proposed a demand adjustment method for the aggregator to allocate power reduction among consumers, and [7] uses a hierarchical market model assuming that the aggregator optimizes incentive to pay to consumers so that the profits are not compromised by changing the consumption behavior.

This paper deals with the demand adjustment of electric appliances had by consumers based on optimal pricing of electricity in a day-ahead market. Specifically, the market participants act to maximize own profit. The market problem is decomposed by dual decomposition, and we propose an algorithm to solve this problem by exchanging information among market participants while satisfying the agreement of supply and demand.

The content of the paper has been partially presented in our previous conference publication [8]. In this paper, we revise it using additional numerical simulation on convergence characteristics of prices among market participants, and the effectiveness of Theorem 1 and Proposition 1 is shown.

In the following section, the behavioral model of each market participant and the social welfare maximization problem are derived. Next, in Section 3, the pricing algorithm based on information exchange among market participants is discussed, and the convergence of proposed method is proved by using Lyapunov’s stability theorem. Finally, it is demonstrated by simulation that optimal adjustment of demand and agreement of supply and demand are achieved, and the price is converged.

2. Problem Formulation

In this section, we describe problem setting in this research. The market model of this paper is shown in Fig. 1.

We describe market participants that consumers, one aggregator, generators, and an independent system operator (ISO). An ISO transacts between the generators and an aggregator and matches power balance by adjusting electricity price $\lambda$. In addition, the aggregator gains the difference between the sale of electricity and the operation cost as the profit, and adjusts the supply that the aggregator purchases from the generators and the demand of consumers by the price $\mu$. 

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Each consumer has appliances with different properties. In order to express consumption behavior more realistically, we design the utility function that represents financial satisfaction of consumers obtained by using appliances in detail. Also, we consider a small-scale model that classifies the household that stays at home all day as type A, and the household that is vacant at home during daytime as type B.

In the model, there exists a set of consumers $H$, and the consumer $i$ has a set of appliances $A_i$ such as air conditioners and electric vehicles. Also, there exists a set of generators $S$, and the generator $i$ has large and small power generation equipment. In this paper, we set the number of iterations of price adjustment as $k$ and the time as $t (t = 0, 1, \ldots, N-1)$, and consider a day-ahead market. The definitions of symbols used in this paper are shown in Table 1.

### Table 1 Definitions of symbols.

| Symbol | Description | Symbol | Definition |
|--------|-------------|--------|------------|
| $d(i)$ | Price by ISO | $\mu(t)$ | Price by aggregator |
| $s_i(t)$ | Supply of generator | $d_{ia}(t)$ | Demand of consumer |
| $D(t)$ | Demand of aggregator | $d_{ia}(t)$ | Demand of appliance |

#### 2.1 Behavior Model of Each Customer

##### 2.1.1 Utility function for each appliance

Appliances used by consumers are classified into the following three types, and the utility function is designed according to the nature of each appliance [9],[10].

1. Electric appliances such as lights and TVs can be controlled freely, and the profit obtained by using them depends on electricity consumption.

2. Electric appliances such as washers, electric vehicles (EVs), and dryers require a certain amount of electricity consumption in a day; the profit obtained by using them depends on the total electricity consumption.

3. The profit obtained by using electric appliances such as air conditioners (ACs) depends not only on electricity consumption but also temperature.

As shown in Fig. 1, it is assumed that the consumer $i$ possesses appliances such as AC and EV, belonging to a set $A_{i,p}$, where $p = 1, 2, 3$ represents the type of appliances.

##### 2.1.2 Type 1 (lights, TVs)

We assume that $A_{i,1}$ is the set of type 1 owned by consumer $i$. The assumption about the utility function $v_{i,a,1}(d_{ia,1}(t))$ is written by:

**Assumption 1** The utility function $v_{i,a,1}(d_{ia,1}(t))$ is $C^2$ and strictly concave on $[0, \infty)$.

The total utility that the consumer $i$ obtains from type 1 in a day is given by:

$$v_{i,a,1}(d_{ia,1}) = \sum_{t \in T_{ia,1}} v_{i,a,1}(d_{ia,1}(t)), \quad (1)$$

$$d_{min,1}(t) \leq d_{ia,1}(t) \leq d_{max,1}(t), \quad \forall t \in T_{ia,1}. \quad (2)$$

We characterize by a set of constraints on its demand vector $d_{ia,1} = [d_{ia,1}(0) \cdots d_{ia,1}(N-1)]^T \in \mathbb{R}^N$. Then, $d_{min,1}(t)$ and $d_{max,1}(t)$ represent the upper and lower bounds of power consumption at the time $t \in T_{ia,1}$, where $T_{ia,1}$ represents the set of usage times of type 1.

##### 2.1.3 Type 2 (washers, EVs)

As with type 1, let $A_{i,2}$ be the set of type 2 owned by consumer $i$. The utility of consumers obtained from Type 2 is bounded by the total amount of electricity consumed throughout the day. Then, the assumption about the utility function $v_{i,a,2}(d_{ia,2}(t))$ is written as:

**Assumption 2** The utility function $v_{i,a,2}(d_{ia,2}(t))$ is $C^2$ and strictly concave on $[0, \infty)$.

The utility obtained from type 2 is given by:

$$v_{i,a,2}(d_{ia,2}) = \sum_{t \in T_{ia,2}} v_{i,a,2}(d_{ia,2}(t)), \quad (3)$$

$$d_{min,2}(t) \leq d_{ia,2}(t) \leq d_{max,2}(t), \quad \forall t \in T_{ia,2}. \quad (4)$$

$$E_{ia} \leq \sum_{t \in T_{ia,2}} d_{ia,2}(t) \leq E_{ia}^max, \quad \forall t \in T_{ia,2}. \quad (5)$$

where $d_{ia,2} := [d_{ia,2}(0) \cdots d_{ia,2}(N-1)]^T \in \mathbb{R}^N$. Then, $d_{min,2}(t)$ and $d_{max,2}(t)$ represent the upper and lower bounds of power consumption at the time $t \in T_{ia,2}$, where $T_{ia,2}$ represents the set of usage times of type 2, and $E_{ia}^min, E_{ia}^max$ represents the upper and lower bounds of the total electricity consumption.

##### 2.1.4 Type 3 (ACs)

We assume that $A_{i,3}$ is the set of type 3 owned by consumer $i$, and the utility of type 3 depends on the electricity consumption and temperature. The room temperature at time $t \in T_{ia,3}$ is expressed as $T_{ia,3}^t$, the outdoor temperature is expressed as $T_{ia,3}^{out}$, and the lower and upper bounds of the temperature that the consumer feels comfortably are $T_{ia,3}^{conf,min}(t)$ and $T_{ia,3}^{conf,max}(t)$. Here, the assumption about $v_{i,a,3}(T_{ia,3}^t)$ is written as:

**Assumption 3** The utility function $v_{i,a,3}(T_{ia,3}(t))$ is $C^2$ and strictly concave on $[0, T_{ia,3}^{conf,min}(t))$.

The equation for temperature is as follows:

$$T_{ia}^{in}(t) = T_{ia}^{in}(t-1) + \alpha(T_{ia}^{out}(t) - T_{ia}^{in}(t-1)) + \beta d_{ia,3}(t), \quad (6)$$

$$T_{ia}^{conf,min}(t) \leq T_{ia}^{in}(t) \leq T_{ia}^{conf,max}(t), \quad \forall t \in T_{ia,3}. \quad (7)$$

where $\alpha$ and $\beta$ are parameters that represent the thermal characteristics. The second term of (6) is heat transfer, and the third term is heat efficiency, which means that $\beta > 0$ works as a heater and $\beta < 0$ works as a cooler. Using (6), the room temperature $T_{ia}^{in}(t)$ is expressed by:
\( T^{in}(t) = (1 - \alpha)^t T^{in}_{0} - (1 - \alpha)^{t+r} \alpha T^{conf}_{in}(\tau_t) \)
\[
+ \sum_{t=0}^{\tau_t} (1 - \alpha)^{t+r} \beta d_{in_3}(r).
\]

Therefore, the utility obtained from type 3 and the constraint condition are expressed by:
\[
V_{ia}(d_{ia_3}) = \sum_{t \in T_{ia_3}} V_{ia}(T^{in}_{ia}(t)),
\]
\[
T^{conf, min}_{ia}(t) \leq T^{in}_{ia}(t) \leq T^{conf, max}_{ia}(t), \quad \forall t \in T_{ia_3},
\]
\[
d^{min}_{ia_3}(t) \leq d_{ia_3}(t) \leq d^{max}_{ia_3}(t), \quad \forall t \in T_{ia_3},
\]
where \( d_{ia_3} := [d_{ia_3}(0) \cdots d_{ia_3}(N - 1)]^T \in \mathbb{R}^N \). Then, \( d^{min}_{ia_3}(t) \) and \( d^{max}_{ia_3}(t) \) represent the upper and lower bounds of power consumption at the time \( t \in T_{ia_3} \), where \( T_{ia_3} \) represents a set of usage times of type 3.

### 2.1.5 Behavior model of each consumer considering characteristics of appliance

The total utility of type 1, 2 and 3 is defined as:
\[
v_i(d_i) := \sum_{a \in \mathcal{A}_i} v_{ia_1}(d_{ia_1}) + \sum_{a \in \mathcal{A}_i} v_{ia_2}(d_{ia_2}) + \sum_{a \in \mathcal{A}_i} v_{ia_3}(d_{ia_3}).
\]

When the price for the consumer determined by the aggregator is \( \mu = [\mu(0) \cdots \mu(N - 1)]^T \in \mathbb{R}^N \), the consumer welfare function \( W_d(d, \mu) \) is defined as:
\[
W_d(d_i, \mu) := v_i(d_i) - \mu^t d_i,
\]
where \( d_i = d_{ia_1} + d_{ia_2} + d_{ia_3} \).

Therefore, the optimization problem that maximizes the welfare of each consumer is written as:
\[
\max_{d_i} W_d(d_i, \mu)
\]
\[
s.t. \quad d^{min}_{ia}(t) \leq d_{ia}(t) \leq d^{max}_{ia}(t), \quad \forall t \in T_{ia}, \quad \forall a \in \mathcal{A}_i,
\]
\[
d^{min}_{ia_1}(t) \leq d_{ia_1}(t) \leq d^{max}_{ia_1}(t), \quad \forall t \in T_{ia_1}, \quad \forall a \in \mathcal{A}_i,
\]
\[
d^{min}_{ia_2}(t) \leq d_{ia_2}(t) \leq d^{max}_{ia_2}(t), \quad \forall t \in T_{ia_2}, \quad \forall a \in \mathcal{A}_i,
\]
\[
E^{max}_{ia} \leq \sum_{t \in T_{ia_3}} d_{ia_3}(t) \leq E^{max}_{ia},
\]
\[
\forall t \in T_{ia_3}, \quad \forall a \in \mathcal{A}_i,
\]
\[
T^{conf, min}_{ia}(t) \leq T^{in}_{ia}(t) \leq T^{conf, max}_{ia}(t), \quad \forall t \in T_{ia_3}, \quad \forall a \in \mathcal{A}_i,
\]
\[
d^{min}_{ia_3}(t) \leq d_{ia_3}(t) \leq d^{max}_{ia_3}(t), \quad \forall t \in T_{ia_3}, \quad \forall a \in \mathcal{A}_i.
\]

### 2.2 Behavior Model of the Aggregator

In this paper, the aggregator is an organization that profits from purchasing electricity from generators through an ISO and selling the electricity to consumers. We define the operation cost on power trading as a cost function \( f(D(t)) \) and power purchase amount \( D(t) \) of the aggregator, and set the following assumption about the cost function:

**Assumption 4** The cost function \( f(D(t)) \) is \( C^2 \), strictly convex, and monotonically increasing on \([0, \infty)\).

The aggregator welfare function \( W_{DA}(D, \mu, \lambda) \) is defined as:
\[
W_{DA}(D, \mu, \lambda) := \sum_{t=0}^{N-1} \left[ (\mu(t) - \lambda(t))D(t) - f(D(t)) \right],
\]
where \( D := [D(0) \cdots D(N - 1)]^T \in \mathbb{R}^N \) and \( \lambda := [\lambda(0) \cdots \lambda(N - 1)]^T \in \mathbb{R}^N \). Therefore, the optimization problem that maximizes the welfare of the aggregator is written as:
\[
\max_D W_{DA}(D, \mu, \lambda)
\]
\[
s.t. \quad D^{min}(t) \leq D(t) \leq D^{max}(t), \quad \forall t \in T,
\]
where \( D^{min}(t) \) and \( D^{max}(t) \) are upper and lower bounds on the aggregator power purchase amount.

### 2.3 Behavior Model of Each Generator

Let \( c_i(s_i(t)) \) be the cost function of the supplier \( i \) which represents the cost of generating electricity and \( s_i(t) \) be the power supply amount of the generator \( i \). The following assumption are made about the cost function:

**Assumption 5** The cost function \( c_i(s_i(t)) \) is \( C^2 \), strictly convex, and monotonically increasing on \([0, \infty)\).

The generator welfare function \( W_s(s_i, \lambda) \) is defined as:
\[
W_s(s_i, \lambda) := \sum_{t=0}^{N-1} \left[ \lambda(t)s_i(t) - c_i(s_i(t)) \right],
\]
where \( s_i := [s_i(0) \cdots s_i(N - 1)]^T \in \mathbb{R}^N \). Therefore, the optimization problem that maximizes the welfare of each generator is written as:
\[
\max_{s_i} W_s(s_i, \lambda)
\]
\[
s.t. \quad s_i^{min}(t) \leq s_i(t) \leq s_i^{max}(t), \quad \forall t \in T_i,
\]
where \( s_i^{min}(t) \) and \( s_i^{max}(t) \) are upper and lower bounds on the power supply amount of the generator \( i \).

### 2.4 Market Model

For the simplification of the problem, we make the following assumption about the grid:

**Assumption 6** We ignore the loss of power in the transmission and distribution system, and do not consider the transmission capacity.

We consider a centralized system that considers the power network as one player, and the problem of concentration system is solved by dual decomposition. As a result, it becomes possible to calculate the problem by distributing the problem of the centralized system to that of each consumer, the aggregator, and each generator. The purpose of the market is to maximize the social welfare, and the social welfare function \( W_{all}(d_i, D, s_i) \) is defined as:
\[
W_{all}(d_i, D, s_i) := \sum_{i \in \mathcal{H}} v_i(d_i) - \sum_{t=0}^{N-1} f(D(t)) - \sum_{t=0}^{N-1} \sum_{i \in \mathcal{H}} c_i(s_i(t)).
\]
Therefore, the optimization problem that maximizes the social welfare is written as:

\[
\begin{align*}
\max_{d_i, D, s_i} & \quad W_{all}(d_i, D, s_i) \\
\text{s.t.} & \quad (15) - (20), (23), (26), \\
& \quad \sum_{i \in S} s_i(t) - D(t) = 0 \forall t, \\
& \quad D(t) - \sum_{i \in H} d_i(t) = 0 \forall t. 
\end{align*}
\]

(28)

The constraints (29) and (30) represent the agreement of the electricity amount. From the above, rewrite to the min-max dual problem using (29) and (30), and Lagrange multipliers

\[
\lambda = [\lambda_0(0) \cdots \lambda_0(N-1)]^T \in \mathbb{R}^N, \mu = [\mu_0(0) \cdots \mu_0(N-1)]^T \in \mathbb{R}^N:
\]

\[
\begin{align*}
\min_{d_i, D, s_i} & \quad W_{all}(d_i, D, s_i) \\
& \quad + \sum_{i = 0}^{N-1} \lambda_0(t) \left( \sum_{i \in S} s_i(t) - D(t) \right) \\
& \quad + \sum_{i = 0}^{N-1} \mu_0(t) \left( D(t) - \sum_{i \in H} d_i(t) \right)
\end{align*}
\]

(31)

s.t. (15) - (20), (23), and (26).

3. Price Update Algorithm Based on the Exchange of Information among Market Participants

In this paper, we propose a distributed algorithm to decide the electricity price and amount of the next day by exchanging information among market participants for improving convergence. Based on the min-max dual problem (31), we define the extended Lagrangian function \( L(t) \) for the optimization problem using \( \gamma, \epsilon > 0 \) as:

\[
\begin{align*}
L(d_i, D, s_S, s_L, \lambda_0, \mu_0) &= \sum_{i \in H} v_i(d_i) - \sum_{i = 0}^{N-1} f(D(t)) - \sum_{i = 0}^{N-1} c_S(s_S(t)) + c_L(s_L(t)) \\
& \quad + \lambda_0^T (s_S + s_L - D) + \mu_0^T \left( D - \sum_{i \in H} d_i \right) \\
& \quad - \frac{\gamma}{2} \sum_{i = 0}^{N-1} \left\| s_S(t) + s_L(t) - D(t) \right\|^2_2 \\
& \quad - \frac{\epsilon}{2} \sum_{i = 0}^{N-1} \left\| D(t) - \sum_{i \in H} d_i(t) \right\|^2_2.
\end{align*}
\]

(32)

where \( s_S := [s_S(0) \cdots s_S(N-1)]^T \in \mathbb{R}^N \) and \( c_S(s_S(t)) \) are the power generation amount and the cost function of the small power generator at the time \( t \) and \( s_L := [s_L(0) \cdots s_L(N-1)]^T \in \mathbb{R}^N \), and \( c_L(s_L(t)) \) are the power generation amount and the cost function of the large power generator at the time \( t \). Based on the extended Lagrangian function (32), each market participant updates its information with the following formula in a distributed manner. Each consumer calculates the electricity demand that maximizes its own profit according to (33). And based on the information of the demand, the aggregator calculates the power purchase amount using (34). And based on the information of the power purchase amount, the small generator calculates the supply using (35), the large generator calculates the supply using (36). Here, (33)-(36) are shown below,

\[
d^k_{i+1} = \arg \max_{d_i} v_i(d_i) - \mu^T \left( D - \sum_{i \in H} d_i \right), \\
\]

(33)

\[
D^{k+1} = \arg \max_D \mu_0^T D - \sum_{i = 0}^{N-1} f(D(t)) \\
\quad - \frac{\gamma}{2} \sum_{i = 0}^{N-1} \left\| s_S(t) + s_L(t) - D(t) \right\|^2_2 \\
\quad - \frac{\epsilon}{2} \sum_{i = 0}^{N-1} \left\| D(t) - \sum_{i \in H} d_i \right\|^2_2,
\]

(34)

\[
s^k_{S+1} = \arg \max_{s_S} s_S - \sum_{i = 0}^{N-1} c_S(s_S(t)) \\
\quad - \frac{\gamma}{2} \sum_{i = 0}^{N-1} \left\| s_S(t) + s_L(t) - D(t) \right\|^2_2, \\
\]

(35)

\[
s^k_{L+1} = \arg \max_{s_L} s_L - \sum_{i = 0}^{N-1} c_L(s_L(t)) \\
\quad - \frac{\gamma}{2} \sum_{i = 0}^{N-1} \left\| s_S(t) + s_L(t) - D(t) \right\|^2_2. \\
\]

(36)

The above (33)-(36) imply that the Lagrange multipliers \( \lambda_0 \) and \( \mu_0 \) equals the price \( \lambda \) and \( \mu \). Based on the information of (33)-(36), the ISO and the aggregator update the price as follows.

1. If the aggregator’s power purchase amount and supply amount do not match, the ISO updates the price using the following formula and tells the aggregator and the generator again:

\[
\lambda^{k+1} = \lambda^{k} - \gamma (s^{k+1} + s^{k+1} - D^{k+1}). \\
\]

(37)

Also, if the purchase amount of power of the aggregator does not match the total demand amount of the consumers, the aggregator updates the price using the following formula and tells the consumer again:

\[
\mu^{k+1} = \mu^{k} - \epsilon (D^{k+1} - \sum_{i \in H} d^{k+1}_i). \\
\]

(38)

2. Repeat until the demand and supply match.

Next, we analyze convergence of the price of the algorithm using Lyapunov’s stability theorem. The residual supply and demand balance between generators and the aggregator is defined as \( \delta := s^k_S + s^k_L - D \), and the residual supply and demand balance between the aggregator and consumers is defined as \( \tilde{\delta} := D - \sum_{i \in H} d^k_i \). The following theorem holds with respect to the convergence of the price:

**Theorem 1** Assume that assumptions 1-6 hold. With the optimal price as \( \lambda^* \), the price \( \lambda \) converges to the optimal price \( \lambda^* \) when the step width \( \gamma > 0 \) satisfies the following formula:

\[
0 < \gamma < \frac{2((\lambda^* - \lambda^*')\delta^{k+1})}{\delta^{k+1} + 2 \sum_{j = 1} \delta^{k+1}}. \\
\]

(39)
Therefore, it is shown that the updated price \( \bar{\lambda} \) converges to its optimal price \( \lambda^* \).

**Proposition 1** Assume that assumptions 1-6 hold. With the optimal price as \( \mu^* \), the price \( \mu \) converges to the optimal price \( \mu^* \) when the step width \( \varepsilon > 0 \) satisfies the following formula:

\[
0 < \varepsilon < \frac{2(\mu^0 - \mu^*)^T \Delta}{\Delta^T + 2 \sum_{i=1}^k \delta^i}
\]

**Proof.** It can be proved in the same way as Theorem 1.

### 4. Numerical Verification

In this simulation, there are eight households of consumers, an aggregator, generators with large power generation facilities and generators with small power generation facilities in one area. Four households of type A are staying at home all day, and four households of type B go out from 8:00 to 18:00. Each household has five types of appliances (lighting, TV, EV, washer, and AC). At this time, it is assumed that \( a = 1 \) is lighting, \( a = 2 \) is TV, \( a = 3 \) is EV, \( a = 4 \) is a washer, and \( a = 5 \) is AC.

#### 4.1 Simulation Conditions

##### 4.1.1 Parameter and utility function of type 1

The utility function of type 1 is set as:

\[
v_{u_1}(t) = c_{ia} - b_{ia}(d_{ia}(t) - d_{ia}^{\text{pref}}(t))^2,
\]

where \( d_{ia}^{\text{pref}}(t) \) represents the power consumption that consumers can obtain the most utility at time \( t \), and \( b_{ia} \) and \( c_{ia} \) are positive constants. Lights are used by both types A and B for \( T_{0.1} = [18, . . . , 23] \), and TVs are used by type A for \( T_{0.1} = [6, . . . , 23] \) and by type B for \( T_{0.2} = [18, . . . , 24] \). Also, the upper and lower bounds of demand \( d_{in}^{\text{min}}, d_{in}^{\text{max}} \) Wh and optimal demand \( d_{in}^{\text{pref}} \) W of lightings and TVs are set as:

\[
\begin{align*}
&d_{in}^{\text{min}}, d_{in}^{\text{max}} = [200, 800], \quad d_{in}^{\text{pref}} = 600, \\
&d_{in}^{\text{min}}, d_{in}^{\text{max}} = [100, 500], \quad d_{in}^{\text{pref}} = 300.
\end{align*}
\]

##### 4.1.2 Parameter and utility function of type 2

The utility function of type 2 is set as:

\[
v_{u_2}(t) = b_{ia} \log \left( \sum_{i \in T_{ia}} d_{ia}(t) + 1 \right),
\]

EVs be charged by all consumers for \( T_{4.1} = [18, . . . , 24, 0, . . . , 7] \). We assume that washers are available for type A throughout the day, and type B only use it for \( T_{4.2} = [18, . . . , 24, 0, . . . , 7] \). Also, the upper and lower bounds of demand \( d_{in}^{\text{min}}, d_{in}^{\text{max}} \) Wh and total demand \( E_{in}, E_{in}^{\text{max}} \) Wh of EV and washer are set as:

\[
\begin{align*}
&d_{in}^{\text{min}}, d_{in}^{\text{max}} = [0, 1500], \\
&E_{in}, E_{in}^{\text{max}} = [5100, 6000], \\
&d_{in}^{\text{min}}, d_{in}^{\text{max}} = [0, 1000], \\
&E_{in}, E_{in}^{\text{max}} = [1600, 2500].
\end{align*}
\]

##### 4.1.3 Parameter and utility function of type 3

The utility function of type 3 is set as:

\[
v_{u_3}(T_{in}^c(t)) = c_{ia} - b_{ia}(T_{in}^c(t) - T_{in}^{\text{conf}}(t))^2.
\]

We use the temperature data of Tokyo on July 3, 2016 of the Japan Meteorological Agency every hour [11]. We assume that ACs are available for type A throughout the day, and type B can only be used for \( T_{4.3} = [18, . . . , 24, 0, . . . , 7] \). Also, the upper and lower bounds of demand \( d_{in}^{\text{min}}, d_{in}^{\text{max}} \) Wh, optimal temperature \( T_{in}^{\text{conf}} \), indoor initial temperature \( T_{in}^{\text{conf}}(-1)^c \), coefficient of heat transfer \( \alpha \) and thermal efficiency \( \beta \) are set as:

\[
\begin{align*}
&d_{in}^{\text{min}}, d_{in}^{\text{max}} = [0, 4000], \\
&[T_{15}^{\text{conf}}, T_{15}^{\text{conf}}] = [25, 28], \\
&T_{15}^{\text{conf}}(-1) = 27, \quad \alpha = 0.9, \quad \beta = -1.2 \times 10^{-3}.
\end{align*}
\]

##### 4.1.4 Cost function

The cost function is set as follows, where the coefficients \( b_D, b_S \), and \( b_L \) are positive constants:

\[
\begin{align*}
\text{Aggregator : } & f(D(t)) = b_D D^2(t), \\
\text{Small generator : } & c_S s_1(t) = b_S s_1^2(t), \\
\text{Large generator : } & c_L s_2(t) = b_L s_2^2(t).
\end{align*}
\]

Each parameter is shown in Table 2.
Table 2 Simulation parameters.

| Parameter          | Symbol | Value     |
|--------------------|--------|-----------|
| End time           | $N$    | 25        |
| Constant price     | $\lambda_f$ | $25 \times 10^{-3}$ |
| Step size          | $\gamma$ | $7.0 \times 10^{-7}$ |
| Step size          | $\epsilon$ | $9.0 \times 10^{-7}$ |

4.2 Simulation Result

In this section, we show that the optimal price that the demand and supply matches is decided by the proposed method. We also verify that consumers are adjusting the consumption of appliances while satisfying the constraints on the usage time and consumption of appliances based on the price. Figures 2-4 show the simulation results of the demand of type A, the demand of type B, and the total demand. In each Figure, the horizontal axis shows the time, the vertical axis shows the demand, and shows the consumption of each appliance. From the results, it can be seen that the consumer adjusts the demand while satisfying the constraint on the use of appliances. Figure 5 represents the price, and it is confirmed that the price is low in the time zone when the demand is low and the price is high in the time zone where the demand is high. Figures 6 and 8 represent the convergence property of price between generators and an aggregator, and the convergence property of price between an aggregator and consumers. From Figs. 6 and 8, the price decided by the proposed method converges to a certain price. Additionally, Figs. 10 and 12 represent relationship between $\gamma$, $\epsilon$, and the upper bounds of (39) and (47), and Figs. 11 and 13 are enlarged from Figs. 10 and 12. From the results, it can be seen that the values of $\gamma$ and $\epsilon$ do not exceed the upper bounds of (39) and (47). Therefore, the effectiveness of Theorem 1 and Proposition 1 is confirmed. Figures 7 and 9 represent
the transition of the supply-demand balance between generators and an aggregator, and the transition of the supply-demand balance between an aggregator and consumers, and it is confirmed that each supply-demand balance converges to 0.

The results in Figs. 2-13 are set by the step size $\gamma = 7.0 \times 10^{-7}$, but Figs. 14 and 15 show the convergence of the price $\lambda$ at 14 o’clock when the step size is changed from $\gamma = 7.0 \times 10^{-7}$ to $\gamma = 3.0 \times 10^{-6}$. Then, we compare with the result of the conventional pricing method using only the gradient method (54)-(57).

From Figs. 14 and 15, the convergence of the price can be maintained by increasing the step size in the proposed pricing method. Therefore, in the proposed algorithm, it can be confirmed that the price convergence speed can be improved.

5. Conclusion

In this paper, we deal with the adjustment of the demand amount of appliance based on optimal price determination on a day-ahead market. At first, we design market model where the generators, the aggregator, and the consumers exist. Especially with regard to the consumer model, the utility of consumers obtained by electricity consumption is distinguished by appliances. Therefore, in this paper, we consider the constraints on
the consumption amount and the use time of each appliance, and derive optimal consumption of each appliance. In addition, we apply the dual decomposition to the market problem that maximizes welfare of each market participant, and propose an distributed algorithm to solve this problem by information exchange between market participants. Furthermore, by using Lyapunov’s stability theorem, we prove the convergence of the price determined by the proposed algorithm. Finally, by simulation, we confirmed that this proposed method can achieve optimal adjustment of demand for appliances and improvement of the price convergence speed.

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