Meeting the Challenges of Modeling Astrophysical Thermonuclear Explosions: Castro, Maestro, and the AMReX Astrophysics Suite

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Abstract. We describe the AMReX suite of astrophysics codes and their application to modeling problems in stellar astrophysics. Maestro is tuned to efficiently model subsonic convective flows while Castro models the highly compressible flows associated with stellar explosions. Both are built on the block-structured adaptive mesh refinement library AMReX. Together, these codes enable a thorough investigation of stellar phenomena, including Type Ia supernovae and X-ray bursts. We describe these science applications and the approach we are taking to make these codes performant on current and future many-core and GPU-based architectures.

1. Introduction
Astrophysical explosions come in many flavors: gravitational and thermonuclear supernovae, unstable burning on the surface of compact objects, and explosive ignition of burning stages in stellar evolution. Accurate modeling of these events requires the coupling of hydrodynamics, gravity, thermonuclear reactions, and in some cases, radiation and magnetic fields. Further, these environments are characterized by a wide range of length scales, from the size of the star or binary system down to the burning zone width and dissipation scales. Temporal scales are equally impressive—stellar evolution occurs over 10s of millions to billions of years, the simmering phase leading up to explosions lasts hours or days to millennia, and the explosion can be over in seconds to hours. The radiation leakage, which leads to the observables we see lasts from hours to months.

No single algorithm meets all of the demands imposed by these events. Instead, we advance our understanding of these events by piecing together simulations of different phases of the evolution from different codes. Here we discuss our simulation codes, Maestro and Castro,
designed to perform three-dimensional models of the early subsonic evolution leading to runaway and the subsequent explosion, respectively. Together this suite of codes allows us to address many problems in stellar and nuclear astrophysics. We describe some of the design details, the current architecture of the code, and some applications below.

2. Science drivers and challenges

Our interests are thermonuclear explosions, including Type Ia supernovae (SNe Ia), X-ray bursts (XRBs), and novae. The basic ingredients for these events are thermonuclear energy release and a degenerate equation of state that allows a runaway to build without a pressure response. Most of the current models for these events are characterized by a long timescale “simmering” phase where reactions heat the star or layer and drive convection. Eventually, reactions become vigorous enough that a runaway takes place, perhaps with an accompanying burning front that spreads through the star.

2.1. Type Ia supernovae

Among the most significant open questions for SNe Ia is the identity of the progenitor. About 20 years ago, the community had mostly converged upon the single-degenerate scenario—a Chandrasekhar-mass C/O white dwarf that accretes from its companion, eventually leading to runaway at the center that burns through the star (see [1] for the state of the field at that time). Since then, a wealth of observations has indicated that there is a lot of diversity in SNe Ia, and searches for progenitor systems have strongly suggested that Chandra-mass white dwarfs cannot explain most SNe Ia. Today, merging white dwarfs (the double degenerate scenario) has perhaps become the most popular model. Other progenitors, like He burning on the surface of a sub-Chandra white dwarf, have also seen interest in explaining some of the observed diversity. See [2] for a review.

There are open questions in all of these scenarios that can be addressed through simulation. For the Chandra and sub-Chandra models, what is the distribution (spatial and temporal) of the hotspots set up by turbulent convection that give rise to burning fronts? A longstanding question with the Chandra model is whether a deflagration can transition to a detonation during the burning front propagation through the star. For the sub-Chandra double detonation model, it is not clear whether it is possible to create a detonation in the thin surface He layer. For double degenerates, it is still unresolved whether the burning takes place promptly or after some delay. However it proceeds, we need to avoid an accretion-induced collapse to a neutron star. For many of these investigations, we need to address numerical issues such as whether it is possible to accurately model the ignition of a detonation with the spatial resolution we can attain. These are some of the questions we seek to answer.

2.2. X-ray Bursts

X-ray bursts—the burning of accreted H/He on the surface of a neutron star—can be important probes of neutron star structure. Interpreting observations requires that we understand what we are seeing, which can be influenced by the products of the burning and how the burning spreads across the star. Many efforts have focused on different aspects of these events. One-dimensional models capture the energetics well and inform us about the nucleosynthesis [3]. Global models show the importance of rotation in confining the burning [4], while models inspired from atmospheric science can explore the vertical structure [5]. However, the resolution differences from the scale of the burning to a reasonable fraction of the neutron star surface has prevented detailed explorations of the burning and how it feeds back on the flame structure and propagation in resolved calculations. Algorithms and computing architectures are starting to reach the point where we can span the gaps in spatial scales between these calculations to
provide a better understanding of the ignition and propagation of the burning front, and the nucleosynthesis produced.

2.3. Requirements
These problems share common algorithmic requirements: strong coupling between hydrodynamics and burning, support for a general equation of state, self-gravity, including isolated boundary conditions, and long timescale evolution for the convective phases. All of these problems are inherently three-dimensional, as turbulence, fluid instabilities, and rotation affect the dynamics. Conservation is important as well, suggesting approaches that implement gravitational and rotation sources conservatively, and methods for improving angular momentum conservation.

3. AMReX Astrophysics Suite
Our suite of application codes is built on the AMReX block-structured adaptive mesh refinement (AMR) library. The basic programming model has AMReX managing the grid data-structures and parallel communications, and it calls the computational kernels on a patch-by-patch basis. The core library is written in C++ with computational kernels written in Fortran\(^1\)—this allows us to take advantage of the strengths of both languages. AMReX supports subcycling in time, which we use in Castro.

AMReX uses a hybrid MPI + OpenMP approach to parallelism. Distribution of grid patches to nodes using MPI provides a natural coarse-grained approach to distributing the computational work, while subdividing patches into tiles and using threads to parallelize over tiles using OpenMP provides effective fine-scale parallelization and amortizes thread overhead over large units of work. Additionally, tiling does not incur the large increase in metadata associated with using smaller patches in a flat MPI mode\(^2\) since we can use fewer, larger patches. This strategy is especially important for many-core architectures like the Intel Xeon Phi. Ongoing development is being done to support GPU offloading, using managed memory provided by the latest generations of GPUs. Figure\(^1\) shows an example of a 2-level grid.

\(^1\) Currently Maestro is written in pure-Fortran, but will be ported to the updated C++ AMReX framework this coming year.
There are many application codes built on AMReX, including those in combustion, multiphase flow, accelerator design, and microfluidics. In astrophysics, these include **Maestro** and **Castro** for stellar and nuclear astrophysics applications and **Nyx** for cosmological applications. Here we focus on the former two.

### 3.1. Maestro

**Maestro** is a low Mach number stellar hydrodynamics code designed for efficiently modeling convection in stars. **Maestro** decomposes the state variables into a one-dimensional hydrostatic base state and a three-dimensional Cartesian state that models the deviation from hydrostatic equilibrium. A constraint equation is derived by requiring that the pressure everywhere is close to the background hydrostatic pressure. The constraint acts to enforce instantaneous acoustic equilibration, effectively filtering soundwaves from the system, while retaining the compressibility effects due to the background stratification of the star and local heat release, as well as the hydrostatic adjustment of the star. In this fashion, it is more general than traditional anelastic methods. The timestep constraint for these equations depends only on the fluid velocity, not the sound speed, enabling much larger timesteps than compressible codes for highly subsonic flows.

The state is advanced using a second-order accurate projection method. Fluid quantities are advected using an unsplit Godunov method, with reactions incorporated via operator splitting. The provisional velocities are then projected onto the space that satisfies the divergence constraint. The projections require solving a variable coefficient elliptic equation, which is done numerically using a multigrid algorithm. A number of recent advances in low Mach number modeling have been incorporated into **Maestro**. **Maestro** has been applied to convection in the Chandrasekhar-mass model for SNe Ia, the sub-Chandra model for SNe Ia, XRBs, and convection in massive stars.

### 3.2. Castro

**Castro** is a fully-compressible radiation hydrodynamics code that supports arbitrary equations of state, nuclear reaction networks, and Poisson gravity using geometric multigrid. The main hydrodynamics scheme in **Castro** is an unsplit piecewise parabolic method. The radiation solver in **Castro** uses the flux-limited diffusion approximation for gray or multigroup radiation. The integration algorithm on the grid hierarchy is a recursive procedure in which coarse grids are advanced in time, fine grids are advanced multiple steps to reach the same time as the coarse grids and the data at different levels are then synchronized. The synchronization for self-gravity is similar to the algorithm introduced by. Recent developments in **Castro** include a spectral-deferred correction method of coupling hydrodynamics and reactions, a conservative gravity and rotation source formulation, and a retry mechanism to redo a step based on criteria evaluated during the integration.

**Castro** has been applied to core-collapse supernovae, radiative shock breakout in supernovae, the Chandra model for SNe Ia, the sub-Chandra SNe Ia model, and white dwarf mergers as a model for SNe Ia. For **Maestro** simulations that evolve from the subsonic regime to the sonic regime, we have demonstrated the ability to restart the calculations in **Castro** to continue the evolution into the sonic regime (in this case, for Chandra model SNe Ia).

### 3.3. StarKiller Microphysics

**Maestro** and **Castro** share the same microphysics, available as the StarKiller Microphysics GitHub
project\[2\]. This includes equations of state and nuclear reaction networks\[3\]. The reaction networks are written such that the rates and integration strategy are decoupled, allowing us to change the integration strategy for a given set of rates. They are also written to be threadsafe and with GPUs in mind (more on that below). The goal of Starkiller is to create a set of community microphysics solvers that can be used in a variety of nuclear astrophysics codes, not just those discussed here.

3.4. Open source and reproducibility
All of our simulation codes are open source and follow a fully open development model—the development git repos are hosted on GitHub\[4\] available for anyone to see and contribute to using issues and pull-requests. Additionally, we have mailing lists for discussions and asking for help. Several branches are used in our workflow. New changes are put into the development branch in each repo. Nightly regression testing is used to ensure that no new bugs were introduced. Once a month, development is merged into master. Finally, all source files, model files, input parameters, etc. for any published science results are also available in the code repos. When feasible, the git hashes for the published results are included in paper acknowledgments.

4. Parallel Performance and GPUs
A key design goal of our application codes is performance portability. We want the same kernels to run on clusters, manycore machines (e.g. Intel Xeon Phi), and GPU-based machines. Our development has balanced this need with architecture-specific optimizations to maximize code reuse.

Figure 2 shows strong scaling for Maestro on the 3-d XRB problem. This is a typical Maestro application\[1\], where burning can be a significant part of the overall evolution. We ran on both OLCF Titan and NERSC Edison. We see that the code scales well to $O(10^4)$ processors. The upturn at the end of the scaling curves on Titan reflect the change from 1 MPI task / 8 OpenMP threads per NUMA node to 1 MPI task / 16 OpenMP threads per compute node (2 NUMA nodes). The main limitation to the scaling at the moment is the multigrid solves used to enforce the projection (in particular the nodal solver). Also, Maestro currently uses a simpler fine-grained approach to parallelism where planes in the z-direction are divided among OpenMP

\[2\] https://github.com/StarKiller-astro/microphysics/
\[3\] Several of the reaction network righthand sides and the EOS were provided from Frank Timmes’ cococubed software instruments page http://cococubed.asu.edu/code_pages/codes.shtml We thank him for making them available.
\[4\] https://github.com/AMReX-Astro/
Figure 3. (left) Castro strong scaling on OLCF Titan for a pure hydro (Sedov w/ real EOS) problem. The different colors represent different base resolutions. We see excellent strong scaling for the single level runs. For the 1024\(^3\) run, we also ran with one level of refinement by a factor of 2 (triangles) or a factor of 4 (squares), and see good strong scaling. The variability (shown by the error bars) at high core counts shows we are becoming work-starved. These runs used the PGI 17.7 compilers. (right) Castro strong scaling on OLCF Titan for a hydro + Poisson gravity (wdmerger) problem. This problem uses a multipole solver to determine Dirichlet boundary conditions representing an isolated mass distribution, and then geometric multigrid to solve for the potential in the interior. Two coarse grid sizes are shown, and demonstrate great strong scaling. We also look at a single level of refinement (by a factor of 4) on top of this coarse grid. These runs used the Cray 8.5.7 compilers.

Figure 4. Castro scaling on new architectures. The left figure shows the Castro GPU proxy app (StarLord) on OLCF Summitdev. We see good scaling behavior to 64 GPUs in this test of a pure hydro problem (Sedov w/ real EOS). On the right, we explore performance on NERSC Cori (an Intel Xeon Phi platform).
ran in the 16 cores / node mode. We see excellent scaling behavior for the pure hydro problem to \( \mathcal{O}(10^5) \) cores. For the self-gravity problem, we scale very well with a uniform grid, but with a factor of 4 refinement for the AMR level on top of this, we observe degraded performance at higher core counts. This is a challenging problem, as only 3% of the domain volume is refined. For a given core count, there are a number of different combinations of MPI ranks and OpenMP threads we can use. In general, with multilevel problems, we found the best performance with fewer MPI ranks and more OpenMP threads.

Our latest focus has been on GPU ports of our application codes. A small proxy app, StarLord, was created from Castro with just the hydrodynamics and stellar equation of state. It uses a simple method-of-lines formulation of the hydrodynamics and advects 13 nuclear species in addition to the hydrodynamics. To offload work on GPUs, GPU support was added directly into AMReX. In AMReX, an iterator loops over all of the boxes at the same level of refinement and passes a data pointer into a Fortran kernel function where the work is done. For the simulation state data that resides in each box, we have modified the memory allocator so that it can use a CUDA allocator (mainly relying on managed memory). The domain iterator is configured to handle data motion to and from the device, so that the compute kernels can operate on data that is presumed to already be there, and the computation is decoupled from the memory management. Computation on the data can then be performed with OpenACC, CUDA Fortran, or (more recently) OpenMP 4.5. We have also built CUDA compute support into AMReX so that a compute kernel can be transparently operated on using CUDA Fortran without substantially modifying the kernels (and we anticipate using a similar strategy to use OpenACC and/or OpenMP in the future). This helps ensure that we can continue to maintain performance portability in our simulation codes, by decoupling the physics algorithms from the backend support used to implement them on various compute architectures.

Figure 4 shows the performance of StarLord on the Summitdev platform at OLCF. The highest GPU count (128) corresponds to 32 nodes on Summitdev. We see a nearly linear speedup with the number of GPUs, indicating good weak scaling on this machine. A single P100 GPU achieves a performance approximately 2.5 times that of the 20 Power8 cores. Efforts are underway to complete the port of the GPU developments into Castro. This figure also shows our performance on the Intel Xeon Phi manycore processors (using NERSC Cori). This is for the same problem as the self-gravity test on Titan. The Intel Xeon Phi chips in Cori have 68 cores than can be run with 1, 2, or 4 hardware threads each. For all these runs, we ran with 4 threads / core (272 threads / chip). We have not yet focused on optimizing Castro for the Intel Xeon Phi architecture.

A parallel effort is porting our microphysics—in particular reaction networks—to GPUs. Our strategy is to do the entire ODE integration on the GPU, i.e., the data for a patch of zones is passed to the GPU, all right-hand side and Jacobian evaluations and the timestepping itself is done on the GPU, and once the burning in all zones is completed, we access the data as needed on the CPU. To enable this, we ported our workhorse ODE integrator (VODE [33]) to CUDA Fortran. This required extensive rewrites of the internals of VODE, which was originally written using Fortran 77 syntax unsupported by CUDA Fortran. Accelerating VODE with CUDA Fortran has proven successful, and we now see significant performance gains with the CUDA version of our reaction networks, even for moderate-sized networks (a 13-isotope standard network). Figure 5 shows the speed-ups on a GPU vs. single CPU core. The main issue with scaling is running out of local stack memory per thread with larger networks. Reducing the memory footprint is a near-term goal for this work. This test problem also shows a lot of thread divergence due to the widely differing thermodynamic conditions in the zones that are burning.

5 This support is currently on a feature branch in the git repo, awaiting merge into development.
6 Summitdev consists of 2 IBM Power8 processors and 4 NVIDIA Pascal GPUs per node.
5. Some science results

Figure 6 shows some of our recent science simulations. The left panel is an image of convection in the helium layer on a sub-Chandra white dwarf. This is part of a study of the early stages of the double detonation SNe Ia model. Using Maestro, we are able to model the convection in the He layer for many turnover times and saw a range of outcomes depending on the mass of the white dwarf and He layer, including a both nova-like behavior where the entire layer runs way together and localized burning ignited in a small region \[15\]. We are performing further studies to characterize the ignition.

The middle figure shows convection in a H/He layer on a neutron star, as a model of the early burning in an XRB. This Maestro model was the first 3D model of convection for this problem \[18\]. This study showed that the convective field became fully turbulent, achieving a Kolmogorov spectrum, and the overall dynamics was very different than our earlier 2-d simulations. This calculation acts as a bridge to our next set of studies that will look at larger scales.

The rightmost image is the coalesced remains of the merger of a 0.9 \(M_\odot\) and 0.6 \(M_\odot\) WD performed with Castro. This used the developments from \[24\]. Our primary focus with this suite of simulations is understanding the numerical sensitivity of mergers and collisions on the burning that takes place. This work is ongoing.
6. Summary and future development

We have described our suite of astrophysics codes built on the AMReX block-structured adaptive mesh refinement framework. These codes were developed to model problems in stellar astrophysics spanning from low speed convection to explosive burning. A major theme of the codes is the open development model, with the code development done on GitHub and all problem files needed to recreate any science results freely available. Future development efforts for Maestro include higher-order hydrodynamics and time-integration and rotation. For Castro, we are investigating stronger coupling between hydrodynamics and reactions, new solvers (including MHD), and finishing the GPU port.

Acknowledgments

The work at Stony Brook was supported by DOE/Office of Nuclear Physics grant DE-FG02-87ER40317 and NSF award AST-1211563. The work at LBNL was supported by the DOE Office of Advanced Scientific Computing Research under Contract No, DE-AC02-05CH11231. An award of computer time was provided by the Innovative and Novel Computational Impact on Theory and Experiment (INCITE) program. This research used resources of the Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No, DE-AC05-00OR22725.

We appreciate the efforts of the OLCF (and in particular Fernanda Foertter) in organizing the GPU hackathons. This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No DE-AC02-05CH11231. The authors would like to thank Stony Brook Research Computing and Cyberinfrastructure, and the Institute for Advanced Computational Science at Stony Brook University for access to the high-performance LItred and SeaWulf computing systems, the latter of which was made possible by a $1.4M National Science Foundation grant (#1531492). The GPU work benefited from a grant of an Titan X Pascal board from NVIDIA Corporation through the GPU Grant Program. Visualizations were done using yt [34]. This research has made use of NASA’s Astrophysics Data System Bibliographic Services.

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