Research regarding the influence of driving-wires length change on positioning precision of a robotic arm

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Abstract. The paper emphasise positioning precision of an elephant’s trunk robotic arm which has joints driven by wires with variable length while operating. The considered 5 degrees of freedom robotic arm has a particular structure of joint that makes possible inner actuation with wire-driven mechanism. We analyse solely the length change of wires as a consequence due inner winding and unwinding on joints for certain values of rotational angles. Variations in wires length entail joint angular displacements. We analyse positioning precision by taking into consideration equations from inverse kinematics of the elephant’s trunk robotic arm. The angular displacements of joints are considered into computational method after partial derivation of positioning equations. We obtain variations of wires length at about tenths of micrometers. These variations employ angular displacements which are about minutes of sexagesimal degree and, thus, define positioning precision of elephant’s trunk robotic arms. The analytical method is used for determining aftermath design structure of an elephant’s trunk robotic arm with inner actuation through wires on positioning precision. Thus, designers could take suitable decisions on accuracy specifications limits of the robotic arm.

1. Introduction
Accuracy is a requirement of high importance for many applications from industrial field. Some of the applications such as technical inspections and pick-and-place operations require also flexibility of the robotic arms. Thus, positioning accuracy and flexibility of robotic arms is a high priority at the moment.

Elephant’s trunk robotic arms driven by wires through inner structure are used generally for 3D positioning of small objects [1]. These types of robotic arm structure were made for manipulating in narrow spaces as the case of those used in different medical procedures [2, 3]. Accuracy optimisation of elephant’s trunk robotic arms is of interest for many researchers. Most of the conducted studies are focusing on inverse kinematics [4], robot arm rigidity [5], joint backlashes [6, 7] and joints friction [8]. Also, research teams have developed complex automatic devices for positioning correction and control of the end-effector [9, 10].

The influence of structural and kinematic parameters on positioning precision of elephant’s trunk robotic arms with wire-driven systems is generally determined using analytical methods. The length of wires used for actuating each five joints varies because of winding and unwinding on the surfaces from the joints interior. The length variation of wires causes positioning deviations of joints and, thus, could affect accuracy of similar robotic arms.

In this paper we present the joints structure with a particular design and one degree of freedom (1DOF). The analytical method used for precision positioning computation emphasizes the aftermath of wire length variation. The obtained results enable to establish the optimum structural and functional parameters in design phase according to the prescribed quality limits for the five degree of freedom (5DOF) robotic arm position precision to achieve accuracy.
2. Joint structure driven by wire with pulley type mechanism

The structure of joint used at an elephant’s trunk robotic arm has 1DOF and a particular design that enables actuation with wires along the structure interior through a pulley type mechanism. Inner actuation is advantageous because avoids wire interaction with any objects from exterior area of the robotic arm.

The pulley type mechanism comprises tensioned wires as a drive element. Joint structure consists of a link (1), a mechanical sleeve (2), a cylinder (4), an interior guiding element (5) and a lid (8), figure 1. The bottom side of link element (1) is connected to the cylinder (4) through a threaded pin (3). Joint lid (8) fixes the cylinder (4) to the mechanical sleeve (2) through a screw (7). The link (1) is fixed at the upper side to the mechanical sleeve of next joint through a threaded pin.

The actuating wires of each joint are fastened in joint cylinder (4) on diametrically opposed sides with two pins (6), one for the upper side of the active wire and one for the upper side of the passive wire. The two pins (6) pass through a slot guide in joint cylinder (2), figure 2, and, thus, employ the joint rotation with a value given by \( \theta_i \) \((i=1...5)\).

Wires that actuate first joint aren’t crossing the surfaces of any guiding element and, therefore, these wires have no length variation as an effect of the winding and unwinding. While operating, the particular robotic arm design causes to the driving wire of \( i+1^{\text{th}} \) joint a rolling up on interior surfaces of \( i \) number guiding elements from previously actuated joint. Thus, each active wire of \( i+1^{\text{th}} \) joint has length variation determined by the \( R_i \) radius of guiding element, figure 3. Triangle \( A_iO_iB_i \) formed by virtual crossing of active wire, figure 3, has segments length \( A_iB_i = B_iC_i = a_i \) defined by equation (1).

\[
\tan \frac{\theta_i}{2} = \frac{a_i}{R_i} \rightarrow a_i = R_i \cdot \tan \frac{\theta_i}{2} \quad (1)
\]

Whatever the direction of joints rotation, the length of driving-wire \( i \) between points \( A_i \) and \( C_i \) equals the sum of segments \( A_iB_i \) and \( B_iC_i \), equation (2).

\[
\overline{A_iB_i} + \overline{B_iC_i} = 2a_i = 2R_i \cdot \tan \frac{\theta_i}{2} \quad (2)
\]

Real length of wires crossing between tangent points \( A_i \) and \( C_i \) is the length of arc \( A_iC_i \) which is given by equation (3).
Length variation of a wire, $\Delta L_{i+1}$, is denoted by the difference between virtual and real length of wire section which is crossing from tangent point $A_i$ to tangent point $C_i$ on guiding elements surface previously to actuated joint, equation (4). In addition, length variation of $i+1^{th}$ driving wire, from $i+1^{th}$ joint, depends on $k$ rotated joints previously to the analysed joint.

\[ \Delta L_{i+1} = 2R_i \cdot \sum_{i=1}^{k} \frac{\theta_i}{2} \cdot \frac{2\pi R_i}{360^\circ} \cdot \sum_{i=1}^{k} |\theta_i| = 2R_i \left( \sum_{i=1}^{k} \frac{\theta_i}{2} \cdot \frac{2\pi R_i}{360^\circ} \cdot \sum_{i=1}^{k} |\theta_i| \right) \]  

Wires length variations $\Delta L_{i+1}$ presume rotating deviations $\Delta \theta_{i+1}$ as described by equation (5).

\[ \Delta \theta_{i+1} = \frac{360^\circ}{\pi \cdot r_i} \cdot \Delta L_{i+1} \]  

3. Inverse kinematics of 5DOF elephant’s trunk robotic arm

The inverse kinematic model of elephant’s trunk robotic arm with 5DOF is determined using homogenous transformation matrices. The position of distal-end point $P(x_6, y_6, z_6)$ is defined towards each world coordinate system $O_iX_iY_iZ_i$ of joint $i$, figure 4, as described below.

**Figure 4.** Crossing of driving-wires through interior structure of elephant’s trunk robotic arm.

Reference system transformation from coordinate systems $O_{i+1}X_{i+1}Y_{i+1}Z_{i+1}$ to $O_iX_iY_iZ_i$ is defined by equation (6). Matrix $[T_{i+1,i}]$ denotes transition from coordinate systems $O_{i+1}X_{i+1}Y_{i+1}Z_{i+1}$ to $O_iX_iY_iZ_i$ using equation (7). Column vectors $[x_i, y_i, z_i]$ and $[x_{i+1}, y_{i+1}, z_{i+1}]$ represent position vectors of coordinate systems $O_iX_iY_iZ_i$ and $O_{i+1}X_{i+1}Y_{i+1}Z_{i+1}$ respectively. Matrix $[R_{i+1,i}]$ and column vector $[P_{i+1,i}]$ denotes orientation and position of coordinate systems $O_{i+1}X_{i+1}Y_{i+1}Z_{i+1}$ towards $O_iX_iY_iZ_i$ respectively.
\[
\begin{align*}
[x_i, y_i, z_i] &= \left[R_{r+1,i}\right] [x_{r+1}, y_{r+1}, z_{r+1}] \\
\left[R_{r+1,i}\right] &= \begin{bmatrix} R_{r+1,i} & P_{r+1,i} \\ 0 & 1 \end{bmatrix} \\
[R_{r+1,i}] &= \begin{bmatrix} n_{r+1,i}^x & n_{r+1,i}^y & n_{r+1,i}^z & a_{r+1,i}^r \\
n_{r+1,i}^x & n_{r+1,i}^y & n_{r+1,i}^z & a_{r+1,i}^r \\
n_{r+1,i}^x & n_{r+1,i}^y & n_{r+1,i}^z & a_{r+1,i}^r \\
0 & 0 & 0 & 1 \end{bmatrix} \\
\begin{bmatrix} P_{r+1,i} \end{bmatrix} &= \begin{bmatrix} p_{r+1,i}^x & p_{r+1,i}^y & p_{r+1,i}^z \end{bmatrix} = [0 0 l_i]
\end{align*}
\]

Position of \( O_X, Y, Z_i \) coordinate systems towards \( O_{r+1}, X_{r+1}, Y_{r+1}, Z_{r+1} \) coordinate systems is obtained using equations (6-9) and, after multiplication of matrices, it is described by equations (10-12).

\[
\begin{align*}
x_i &= n_{r+1,i}^x \cdot x_{r+1} + s_{r+1,i}^y \cdot y_{r+1} + a_{r+1,i}^r \cdot z_{r+1} + p_{r+1,i}^r = x_{r+1} \\
y_i &= n_{r+1,i}^y \cdot x_{r+1} + s_{r+1,i}^y \cdot y_{r+1} + a_{r+1,i}^r \cdot z_{r+1} + p_{r+1,i}^r = \cos \theta_{r+1} \cdot y_{r+1} - \sin \theta_{r+1} \cdot z_{r+1} \\
z_i &= n_{r+1,i}^z \cdot x_{r+1} + s_{r+1,i}^z \cdot y_{r+1} + a_{r+1,i}^r \cdot z_{r+1} + p_{r+1,i}^r = \sin \theta_{r+1} \cdot y_{r+1} + \cos \theta_{r+1} \cdot z_{r+1} + l_i
\end{align*}
\]

Position of \( O_{r+1}, X_{r+1}, Y_{r+1}, Z_{r+1} \) coordinate systems towards \( O_i, X_i, Y_i, Z_i \) coordinate systems is obtained using equations (10-12) according to equations (13-15).

\[
\begin{align*}
x_i &= x_i \\
y_i &= \cos \theta_{i} \cdot y_i + \sin \theta_{i} \cdot z_i - \sin \theta_{i+1} \cdot l_i \\
z_i &= -\sin \theta_{i} \cdot y_i + \cos \theta_{i+1} \cdot z_i + \cos \theta_{i+1} \cdot l_i
\end{align*}
\]

Position of \( O_6, X_6, Y_6, Z_6 \) coordinates system is given by default, \( x_0 = 0 \), \( y_0 = 0 \) and \( z_0 = 0 \). In addition, position of \( O_6, X_6, Y_6, Z_6 \) coordinate system can be obtained by replacing (10-12) in equations (13-15). After several simplifying calculation, we obtain equations (16-18). The former describe distal-end position of point \( P \) in world coordinate system that varies according to the values of structural parameters \( l_i \), \( l_i \), and functional parameter \( \theta_i \).

\[
\begin{align*}
x_6 &= 0 \\
y_6 &= \frac{l_0}{\sin \theta_i + \cos^2 \theta_i} \cdot \cos \sum_{i=0}^4 \theta_{i+1} - l_i \cdot \sin \sum_{i=0}^4 \theta_{i+1} - l_i \cdot \sin \sum_{i=3}^4 \theta_{i+1} - l_4 \cdot \sin \theta_5 \\
z_6 &= \frac{l_0}{\sin \theta_i + \cos^2 \theta_i} \cdot \sin \sum_{i=0}^4 \theta_{i+1} - l_i \cdot \cos \sum_{i=0}^4 \theta_{i+1} - l_i \cdot \cos \sum_{i=2}^4 \theta_{i+1} - l_4 \cdot \cos \theta_5 - l_5
\end{align*}
\]

In inverse kinematics analysis we know position coordinates of distal-end point \( P \). Thus, equations (17-18) don’t have a unique solution because of redundant structure. Methods applied for solving redundancy of elephant’s trunk robotic arms were studied by numerous research teams which recommended different simplifying procedures for unique solutions determination [6].

Deviations of positioning precision, \( \Delta x_6 \), \( \Delta y_6 \) and \( \Delta z_6 \), caused by all five angular deviations, \( \Delta \theta_i \), are denoted by equation (19). If not corrected, these deviation positions entail positioning errors which define accuracy of the robotic arm.

\[
\begin{align*}
\Delta x_6 &= \sum_{i=1}^5 \frac{\partial \Delta x_6}{\partial \theta_i} \Delta \theta_i \\
\Delta y_6 &= \sum_{i=1}^5 \frac{\partial \Delta y_6}{\partial \theta_i} \Delta \theta_i \\
\Delta z_6 &= \sum_{i=1}^5 \frac{\partial \Delta z_6}{\partial \theta_i} \Delta \theta_i
\end{align*}
\]
4. Influence of driving-wire length variation on positioning precision

Determining positioning precision as an aftermath of the influence of driving-wires length variation is necessary for setting proper corrections in programming algorithm of the 5DOF elephant’s trunk robotic arm. The analysis consists in substitution of known structural and functional parameters from given equations, in section 3, with numerical values to indentify unwanted positioning deviations of distal-end.

Wires length variation is determined by numerical values of structural parameters: interior radius of guiding element - \( R_i = 1.5 \text{ mm} \), radius of joint cylinder - \( r_j = 16 \text{ mm} \), rotation - \( \theta_i = \pm 23^\circ \), length between joints - \( l_0 = 50 \text{ mm} \) and \( l_i = 50 \text{ mm} \), figure 4. For the elephant’s trunk robotic arm, with specified structural and functional parameters, we obtain wires length variations at maximum values depending on the number on joints \( k \) previously rotated to the analysed joint as described by diagrams in figure 5. These variations of wires length lead to rotational deviations \( \Delta \theta_i \) which vary and depend, also, on the joints number \( k \) previously rotated to analysed joint, figure 6.

![Figure 5](image1.png)  
**Figure 5.** Wires length variation depending on \( k \) rotated joints previously to analysed joint.

![Figure 6](image2.png)  
**Figure 6.** Variation of angular deviation depending on \( k \) rotated joints previously to analysed joint.

Variations of rotational deviations \( \Delta \theta_i \) pass to position of distal-end, \( P(x_6, y_6, z_6) \), causing variations of positioning precision. The 1DOF joints from considered elephant’s trunk robotic arm are rotating to X axis, which entail a null value of positioning variation on X axis (\( \Delta x_6 = 0 \)). Thus, positioning precision employs variation of parameters \( \Delta y_6 \) and \( \Delta z_6 \), figure 7, that describe positioning variation on Y and Z axes respectively. We obtained diagram from figure 7 by replacing the known parameters from equation (19) and plotting our data in equivalent MatLab program.

![Figure 7](image3.png)  
**Figure 7.** Positioning precision variation.
In the analysis we obtain variations of wires length with maximum value of 32 $\mu m$ which renders maximum rotational deviations at joint positioning of 14’. Positioning deviations that resulted from these rotational deviations reach to maximum values of -0.16 mm on Y axis and of 0.30 mm on Z axis.

It can be noted from variation diagrams of deviations $\Delta y_6$ and $\Delta z_6$, figure 7, that deviations values are negative on direction of Y axis and positive on direction of Z axis. Let us assume that on Y axis direction it is horizontal plane, and on Z axis direction it is vertical plane. Therefore, distal-end position is closer to the base arm on horizontal plane direction and further from the base arm on vertical plane direction than the given 3D position. Knowing this effect of wire length variation on positioning precision, it can be made proper adjustments in programming algorithm of the robotic arm to achieve minimum positioning errors.

5. Conclusion
Variation of driving-wires length is different and particular to each joint from the elephant’s trunk robotic arm. This paper emphasise the wire length variations as a consequence of winding and unwinding of driving-wires on circular surface from guiding elements. These variations are (directly) proportional to the angle of rotation $\theta_i$. Also, from equation (4) results the proportional influence of interior radius value ($R_i$) from guiding element on wire length variation. Number of joints previously actuated to analysed joint is another factor that participates to the increase in length of wires.

A method of compensating position deviations is by electronic corrections applied in programming algorithm of the robotic arm. In addition, there are dynamic loads causing elastic deformation of the wires used for joints actuation of considered elephant’s trunk robotic arm. The wires length variations along with elastic deformations determine positioning deviations which are of interest for future research. Therefore, elephant’s trunk robotic arms driven by wires through structure interior require further analysis of dynamic wires behaviour. It is necessary to take appropriate actions which minimise positioning errors resulted from both sources, and, thus, to achieve optimum accuracy.

The analysis method of wire length variation, presented in this paper, helps designers in taking suitable decisions regarding the optimum structural and functional parameters of elephant’s trunk robotic arms and its technical specifications.

References
[1] Bercan N and Diaconescu D, 1995, Wire mechanisms for robots, Sibiu (Publisher University of Sibiu).
[2] Walker I D, 2013, Continuous Backbone "Continuum" Robot Manipulators, ISRN Rob., 19.
[3] Sánchez A, Poignet P, Dombre E, Menciassi A and Dario P, 2014, A design framework for surgical robots: Example of the ARAKNES robot controller, Rob. Aut. Sys. 62, 1342-52.
[4] Ahmmad S M, Khan R, Rahman M M and Billah M, 2013, Position control of a four link hyper redundant robotic manipulator, Asian J. Sc. Research 6/1, 67-77
[5] Zhang Z J and Zhang Y N, 2013, Equivalence of different-level schemes for repetitive motion planning of redundant robots, ACTA Automatica Sinica 39/1, 88-91.
[6] Zheng L and Ruxu D, 2013, Design and Analysis of a Bio-Inspired Wire-Driven Multi-Section Flexible Robot, Int. J. Adv. Rob. Sys. 10, 209-219.
[7] He B, Wang Z, Li Q, Xie H and Shen R, 2013, An analytical method for the kinematics and dynamics of a multiple-backbone continuum robot, Int. J. Adv. Rob. Sys. 10, 84-96.
[8] Xu K and Simaan N, 2006, Actuation compensation for flexible surgical snake-like robots with redundant remote actuation, submitted to: IEEE Int. Conf. Rob. Aut., 4148-54.
[9] Ning J and Wörgötter F, 2011, Control system development for a novel wire-driven hyper-redundant chain robot, 3D-trunk: submitted to IEEE/ASME Trans. Mech., 1083-93.
[10] Heidari S, Piltan F, Shamsodini M, Heidari K and Zahmatkesh S, 2013, Design new nonlinear controller with parallel fuzzy inference system compensator to control of continuum robot manipulator, Int. J. Contr. Aut. 6/4, 115-134.