Statefinder and Om Diagnostics for Interacting New Holographic Dark Energy Model and Generalized Second Law of Thermodynamics

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In this work, we have considered that the flat FRW universe is filled with the mixture of dark matter and the new holographic dark energy. If there is an interaction, we have investigated the natures of deceleration parameter, statefinder and Om diagnostics. We have examined the validity of the first and generalized second laws of thermodynamics under these interactions on the event as well as apparent horizon. It has been observed that the first law is violated on the event horizon. However, the generalized second law is valid throughout the evolution of the universe enveloped by the apparent horizon. When the event horizon is considered as the enveloping horizon, the generalized second law is found to break down excepting at late stage of the universe.

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I. INTRODUCTION

In recent observations it is strongly believed that the universe is experiencing an accelerated expansion. The observation from type Ia supernovae [1] in association with Large scale Structure [2] and Cosmic Microwave Background anisotropies [3] have shown the evidences to support cosmic acceleration. This observations lead to a new type of matter which violates the strong energy condition i.e., $\rho + 3p < 0$. The energy density of matter responsible for such a condition to be satisfied at a certain stage of evolution of the Universe is referred to as dark energy. This mysterious dark energy with negative pressure leads to this cosmic acceleration. Also the observations indicate that the dominating component of the present universe is this dark energy. Dark energy occupies about 73% of the energy of our universe, while dark matter about 23% and the usual baryonic matter 4%. There are several candidates obey the property of dark energy, given by \(-\) quintessence [4], K-essence [5], tachyon [6], phantom [7], ghost condensate [8,9] and quintom [10], interacting dark energy models [11], brane world models [12], Chaplygin gas models [13], agegraphic DE models [14], etc.

Recently, a new DE candidate, based on the holographic principle, was proposed in [15]. By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe [16]. Li [17] argued that the total energy in a region of size $L$ should not exceed the mass of a black hole of the same size for a system with UV cut-off $\Lambda$. The holographic dark energy density is defined as $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$, where $M_p$ is the reduced Planck Mass $M_p \equiv 1/\sqrt{8\pi G}$ and $c$ is a numerical constant characterizing all of the uncertainties of the theory, whose value can only be determined by observations. Recently several works [19] have been done in holographic dark energy models, Gao et al [20] proposed to replace the future event horizon area with the inverse of the Ricci scalar curvature. So the length $L$ is given by the average radius of Ricci scalar curvature, $R^{-1/2}$, so that the dark energy $\rho_\Lambda \propto R$, where $R = -6(2H^2 + \dot{H} + \kappa)$. This model is known as the Ricci dark energy model [20]. They find that this model works fairly well in fitting the observational data, and it could also help to understand the coincidence problem. There are few works on this dark energy model [21]. For purely dimensional reasons Granda et al [22] proposed a new infrared cut-off for the holographic density which besides the square of the Hubble scale also contains time derivative of the Hubble parameter. In favor of this new term, the underlying origin of the holographic dark energy is still unknown and that the new term is contained in the expression for the Ricci scalar which scales as $L^{-1/2}$. So the holographic density is of the

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form: \( \rho_{\Lambda} \approx (\alpha H^2 + \beta \dot{H}) \). This is known as the new holographic dark energy \cite{22}. There are some works on the new version of the holographic dark energy model \cite{23}.

Bekenstein \cite{24} first considered that there is a relation between the event of horizon and the thermodynamics of a black hole. In this case, the event of horizon of the black hole is a measure of the entropy of it. The thermodynamics in de Sitter spacetime was first investigated by Gibbons and Hawking \cite{25}. In a spatially flat de Sitter spacetime, the event horizon and the apparent horizon of the Universe coincide and there is only one cosmological horizon. When the apparent horizon and the event horizon of the Universe are different, it was found that the first law and the second law of thermodynamics hold on the apparent horizon, while they break down if one considers the event horizon \cite{26}. At the apparent horizon, the first law of thermodynamics is shown to be equivalent to Friedmann equations \cite{27} if one takes the Hawking temperature and the entropy on the apparent horizon and the generalized second law of thermodynamics is obeyed at the horizon. In this context, there are several studies \cite{28-30} in thermodynamics for dark energy filled universe on apparent and event horizons. Here we consider that the flat FRW universe is filled with the mixture of dark matter and the new holographic dark energy \cite{22}. If there is an interaction, we shall investigate the natures of deceleration parameter, statefinder and \( Om(z) \) diagnostics during whole evolution of the universe. In the interaction scenario, the validity of generalized second law of thermodynamics will be examined on the event horizon.

Newness in our study with respect to the earlier works on new holographic dark energy listed in the references \cite{22} and \cite{23} lies on the following matters:

- Granda and Olivers \cite{23} studied the correspondence of new holographic dark energy proposed in \cite{22} with other candidates of dark energy like quintessence, tachyon, k-essence and dilaton in flat FRW universe. In our study instead of studying correspondence between new holographic dark energy with other candidates we have considered interaction of this dark energy with dark matter and studied various diagnostics with zero and non-zero interaction parameter \( \delta \).

- Karami and Fehri \cite{23} generalized the results of Granda and Olivers \cite{23} for non-flat universe. They have considered non-interacting scenarios. But in our work, we have considered interacting scenarios.

- Yu et al \cite{23} considered interaction between new holographic dark energy and pressureless dark matter. In the present paper we have not neglected the pressure of dark matter. In our study, we have studied the deceleration, statefinder and \( Om(z) \) diagnostics under the interactions. Whereas, Yu et al \cite{23} studied only the equation of state and fractional energy densities. Malekjani et al \cite{23} studied the statefinder diagnostics for new holographic dark energy. However, in our study we have considered statefinder diagnostics in interacting situation.

- In none of the studies listed in references \cite{22-23}, the laws of thermodynamics are investigated. In the present paper we have examined the validity of the first and generalized second laws of thermodynamics for the universe enveloped by event as well as apparent horizon. Moreover, interacting, non-interacting as well as single component models have been considered for all of the cases.

II. BASIC EQUATIONS AND SOLUTIONS

The metric of a spatially flat homogeneous and isotropic universe in FRW model is given by

\[
ds^2 = dt^2 - a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]
\]

(1)

where \( a(t) \) is the scale factor. The Einstein field equations are given by

\[
H^2 = \frac{1}{3} \rho
\]

(2)

and

\[
\dot{H} = \frac{1}{2} (\rho + p)
\]

(3)
where \( \rho \) and \( p \) are energy density and isotropic pressure respectively (choosing \( 8\pi G = c = 1 \)). Now consider our universe is filled with dark matter and new holographic dark energy. So we assume, \( \rho = \rho_m + \rho_X \) and \( p = p_m + p_X \). Here, \( \rho_m, p_m \) and \( \rho_X, p_X \) are respectively the energy density and pressure for dark matter and new holographic dark energy. We know that the dark matter has a negligible pressure, i.e., \( p_m \approx 0 \). But here we have taken into account the non-zero value of \( p_m \) i.e., value of \( p_m \) is very small.

The energy conservation equation is given by

\[
\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0 \tag{4}
\]

Now we consider the model of interaction between dark matter and the new holographic dark energy model, through a phenomenological interaction term \( Q \). Keeping into consideration the fact that the Supernovae and CMB data determine that decay rate should be proportional to the present value of the Hubble parameter. At this juncture, it should be stated why we are going for the interaction between dark energy and dark matter. The models with interaction between dark energy and dark matter have been studied extensively in literature [31]. In a study by Pavon and Zimdahl [31] it was demonstrated that any interaction between pressureless dark matter with holographic dark energy can solve the coincidence problem. Some major points favoring the interacting models are summarized below:

- Given the unknown nature of both dark matter and dark energy there is nothing in principle against their mutual interaction (Pavon and Zimdahl [31])
- The interacting situation is compatible with SNIa and CMB data (Olivers et al [31], Zhang [31])
- Introducing interaction can alleviate coincidence problem (Sheykhi [31], Hu and Ling [31], Sadjadi and Almohammadi [31])

In the interacting situation, the two components do not satisfy the conservation equation separately. Rather an interaction term \( Q \) is introduced. This interaction term describes the energy flow between the two fluids. Therefore the conservation equation (4) becomes

\[
\dot{\rho}_X + 3H(\rho_X + p_X) = -Q \tag{5}
\]

and

\[
\dot{\rho}_m + 3H(\rho_m + p_m) = Q \tag{6}
\]

There are many different forms of \( Q \) in the literature [44]. Following Wei and Cai [44] we choose \( Q \) as

\[
Q = 3\delta H \rho_m \tag{7}
\]

where \( \delta \) is the interaction parameter. In principle \( \delta \) may be positive or negative. According to Wei and Cai [44] the cases with positive \( \delta \) have physically richer phenomena. If \( \delta = 0 \) then we get a non-interacting situation. Thus, in the present study we shall take \( \delta \geq 0 \).

The new holographic dark energy density with infrared cut-off [22,23] is given by

\[
\rho_X = 3(\alpha H^2 + \beta \dot{H}) \tag{8}
\]

where \( \alpha \) and \( \beta \) are constants to be determined. Now suppose the equation of state for the dark matter is given by \( p_m = w_m \rho_m \), where \( w_m \) is very small and so the equation (6) becomes

\[
\rho_m = \rho_m^0 a^{-3(1+w_m - \delta)} \tag{9}
\]

and consequently equation (2) reduces to
\[ H^2 = C \alpha \frac{2^{2(a-1)}}{\beta} + \frac{2 \rho_{m0} a^{-3(1+w_m-\delta)}}{2(1-\alpha) + 3\beta(1+w_m-\delta)} \] (10)

where \( \rho_{m0} \) is the present value of matter density (at \( a = 1 \)) and \( C \) is the arbitrary integration constant.

Now define,

\[ \tilde{H} = \frac{H}{H_0}, \quad \tilde{\rho}_m = \frac{\rho_m}{3H_0^2}, \quad \tilde{p}_m = \frac{p_m}{3H_0^2}, \quad \tilde{\rho}_X = \frac{\rho_X}{3H_0^2}, \quad \tilde{p}_X = \frac{p_X}{3H_0^2}, \quad \Omega_m0 = \frac{\rho_{m0}}{3H_0^2}, \quad \Omega_X0 = \frac{\rho_{X0}}{3H_0^2}, \quad f_0 = \frac{C}{H_0^2} \] (11)

where \( H_0 \) is the present value of the Hubble parameter, \( \tilde{H} \) is the Hubble expansion rate, current density parameters are \( \Omega_{m0} \) and \( \Omega_{X0} \) for matter and dark energy respectively. So from equation (2), we may conclude that

\[ \Omega_{m0} + \Omega_{X0} = 1 \] (12)

Now the relation between scale factor \( a \) and the redshift \( z \) is given by

\[ a = \frac{1}{1+z} \] (13)

Therefore the equations (9), (10), (8) and (5) yield to

\[ \tilde{\rho}_m = \Omega_{m0} (1+z)^{3(1+w_m-\delta)} \] (14)

\[ \tilde{p}_m = w_m \Omega_{m0} (1+z)^{3(1+w_m-\delta)} \] (15)

\[ \tilde{H}^2 = f_0 (1+z) \frac{2^{2(a-1)}}{\beta} + \frac{2\Omega_{m0}(1+z)^{3(1+w_m-\delta)}}{2(1-\alpha) + 3\beta(1+w_m-\delta)} \] (16)

\[ \tilde{\rho}_X = f_0 (1+z) \frac{2^{2(a-1)}}{\beta} + \frac{(2\alpha - 3\beta(1+w_m-\delta))\Omega_{m0}(1+z)^{3(1+w_m-\delta)}}{2(1-\alpha) + 3\beta(1+w_m-\delta)} \] (17)

and

\[ \tilde{p}_X = f_0 \frac{2(\alpha - 1) - 3\beta}{3\beta} (1+z) \frac{2^{2(a-1)}}{\beta} + \frac{(2\alpha - 3\beta(1+w_m-\delta))(\Omega_{m0}(1+z)^{3(1+w_m-\delta)})}{2(1-\alpha) + 3\beta(1+w_m-\delta)} \] (18)

There are three constants \( \alpha, \beta \) and \( f_0 \) to be determined from the above equations. For this purpose let us suppose that the equation of state for the new holographic dark energy is \( p_X = w_X \rho_X \). Now at present epoch (i.e., \( a = 1 \), i.e., \( z = 0 \)), we have

\[ \tilde{\rho}_{X0} = \Omega_{X0}, \quad \tilde{p}_{X0} = w_X \Omega_{X0} \] (19)

So from equations (17), (18) and (19), we obtain

\[ f_0 = 1 + \frac{2\Omega_{m0}}{3\beta(\delta + w_X - w_m) + \Omega_{m0}(-2 + 3\beta(w_m - w_X))} \] (20)
Fig. 1 shows the variation of $q$ against redshift $z$ for $\delta = 0$ (red line) $\delta = .01$ (green line), $w_m = .1$, $w_X = -.9$, $\Omega_{m0} = .23$ and $\beta = .5$. 

and

$$\alpha = 1 + \frac{3\beta}{2}(1 + w_X) + \left[\frac{3\beta}{2}(w_m - w_X) - 1\right] \Omega_{m0}$$  \hspace{1cm} (21)$$

So values of $\alpha$ and $f_0$ are given in term of the free parameter $\beta$, but it can be fixed by the behaviour of the deceleration parameter $q$. The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{a}{2H^2} \frac{dH^2}{da} = -1 + \frac{(1 + z) dH^2}{H^2} dz$$  \hspace{1cm} (22)$$

Using above results, we obtain

$$q = -\frac{f_0(1 - \alpha + \beta)(2(1 - \alpha) + 3\beta(1 + w_m - \delta))(1 + z)^{2(\alpha - 1)} + \beta(1 - 3w_m + 3\delta)\Omega_{m0}(1 + z)^{3(1 + w_m - \delta)}}{f_0\beta(2(1 - \alpha) + 3\beta(1 + w_m - \delta))(1 + z)^{2(\alpha - 1)} + 2\beta\Omega_{m0}(1 + z)^{3(1 + w_m - \delta)}}$$  \hspace{1cm} (23)$$

The evolution of deceleration parameter has been shown in figure 1 for $\delta = 0$ (red line) and $\delta = .01$ (green line), $w_m = 0.1$, $w_X = -0.9$, $\Omega_{m0} = 0.23$ and $\beta = 0.5$. The deceleration parameter decreases from 0.5 to -1 for $z$ decreases in the whole evolution. At higher redshifts the deceleration parameter is positive. This indicates the earlier decelerating phase of the universe. At lower redshifts, the deceleration parameter is negative that indicates the accelerated phase of the universe. The signature flip occurs at $z = 0.5$ for both $\delta = 0$ and $\delta = 0.01$. Thus, we understand that in the non-interacting as well as non-interacting situation it is possible to get the transition from decelerated to accelerated universe for model under consideration. For the above specified values, using equations (20) and (21), we find that $\alpha = 1.02$ and $f_0 = 0.71$.

III. STATEFINDER DIAGNOSTICS

Since there are various candidates for the dark energy model, we often face with the problem of discriminating between them, which were solved by introducing statefinder parameters. These statefinder diagnostic pair i.e., $\{r, s\}$ parameters are of the following form [32]:
Fig. 2 shows the variations of \( r \) with \( s \) for \( \delta = 0 \) (red line) \( \delta = .01 \) (green line), \( w_m = .1 \), \( w_X = -.9 \), \( \Omega_{m0} = .23 \) and \( \beta = .5 \).

These parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The statefinder is dimensionless and is constructed from the scale factor of the Universe and its time derivatives only. The parameter \( r \) forms the next step in the hierarchy of geometrical cosmological parameters after \( H \) and \( q \).

Now \( r \) and \( s \) can be written in terms of Hubble parameter \( H \) in the following forms:

\[
r = 1 + 3 \frac{\ddot{H}}{H^2} + \frac{\dot{H}}{H^3}
\]

and

\[
s = \frac{r - 1}{3 (q - \frac{1}{2})}
\]

Or, equivalently \([23]\)

\[
r = 1 + \frac{2a}{H^2} \frac{dH^2}{da} + \frac{a^2}{2H^2} \frac{d^2H^2}{da^2} = 1 - \frac{(1 + z) \frac{d\dot{H}^2}{dz} + (1 + z)^2 \frac{d^2\dot{H}^2}{dz^2}}{2H^2}
\]

and

\[
s = -\frac{4a \frac{d\dot{H}^2}{dz} + a^2 \frac{d^2\dot{H}^2}{dz^2}}{3(3H^2 + a \frac{dH^2}{da})} = -\frac{2(1 + z) \frac{d\dot{H}^2}{dz} - (1 + z)^2 \frac{d^2\dot{H}^2}{dz^2}}{3(3H^2 - (1 + z) \frac{dH^2}{dz})}
\]
Fig. 3 shows the variations of $\Omega m(z)$ against redshift $z$ for $\delta = 0$ (red line), $\delta = 0.01$, (green line), $w_m = 0.1$, $w_X = -0.9$, $\Omega_{m0} = 0.23$ and $\beta = 0.5$.

Thus using equation (16), we obtain

$$r = 1 + \frac{9}{2}(w_m - \delta)(1 + w_m - \delta) + \frac{f_0(2(1 - \alpha) + 3\beta(w_m - \delta))(2(1 - \alpha) + 3\beta(1 + w_m - \delta))}{2f_0\beta^2(2(1 - \alpha) + 3\beta(1 + w_m - \delta))(1 + z)^{(2(\alpha - 1))}}2^{(\alpha - 1)} + 2\beta^2\Omega_{m0}(1 + z)^3(1 + w_m - \delta)$$

and

$$s = \frac{2f_0(\alpha - 1)(2 - 2\alpha + 3\beta)(2(1 - \alpha) + 3\beta(1 + w_m - \delta))(1 + z)^{(2(\alpha - 1))} - 18\beta^2(w_m - \delta)(1 + w_m - \delta)\Omega_{m0}(1 + z)^3(1 + w_m - \delta)}{3f_0\beta(2 - 2\alpha + 3\beta)(2(1 - \alpha) + 3\beta(1 + w_m - \delta))(1 + z)^{(2(\alpha - 1))} + 18\beta^2\Omega_{m0}(1 + z)^3(1 + w_m - \delta)}$$

From (30) and (31), we see that $r$ cannot be expressed explicitly in terms of $s$. Figure 2 shows the variations of $r$ with $s$ for $\delta = 0.01$, $w_m = 0.1$, $w_X = -0.9$, $\Omega_{m0} = 0.23$ and $\beta = 0.5$. We see that due to evolution of the universe, $r$ decreases and $s$ increases from small positive value to $+\infty$ and the also increases from $-\infty$ to 0. The section of the plot with positive $r$ and $s$ gives the radiation phase of the universe. Then we find that $r$ is finite and $s \to -\infty$, which indicates dust stage. Also, at $r = 1$, we have $s = 0$. This indicates $\Lambda$CDM stage. Thus the transition from radiation to $\Lambda$CDM stage through dust stage is obtained under the two-component model with $\delta = 0$ and $\delta = 0.01$.

IV. $\Omega m$ DIAGNOSTIC

As a complementary to $\{r, s\}$, a new diagnostic called $\Omega m$ has been recently proposed [32], which helps to distinguish the present matter density contrast $\Omega_{m0}$ in different models more effectively. The new diagnostic of dark energy $\Omega m$ is introduced to differentiate $\Lambda$CDM from other DE models. The starting point for $\Omega m$ diagnostic is the Hubble parameter and it is defined as [32]

$$\Omega m(x) = \frac{h^2(x) - 1}{x^3 - 1}$$

where $x = z + 1$ and $h(x) = \frac{H(x)}{H_0} = \tilde{H}$. Thus $\Omega m$ involves only the first derivative of the scale factor through the Hubble parameter and is easier to reconstruct from observational data. For $\Lambda$CDM model, $\Omega m = \Omega_{0m}$
is a constant, independent of redshift \(z\). It provides a null test of cosmological constant. The benefit for \(\Omega_m\) diagnostic is that the quantity \(\Omega_m\) can distinguish DE models with less dependence on matter density \(\Omega_0\) relative to the EOS of DE \(w_X\) [32]. \(\Omega_m\) and statefinder diagnostics for GCG model and decaying vacuum model from cosmic observations has also been discussed in [33].

Now in our interacting new holographic dark energy model, we obtain

\[
\Omega_m(z) = \tilde{H}^2(z) - 1 = f_0(1 + z)^{\frac{2(\alpha - 1)}{\beta}} + \frac{2\Omega_m(1 + z)^{3(1 + w_m - \delta)}}{2(1 - \alpha) + 3(1 + w_m - \delta)} - 1
\] (33)

In figure 3, we plot the evolution of \(\Omega_m(z)\) against redshift \(z\) corresponding to \(\delta = 0\) (red line), \(\delta = .01\), (green line), \(w_m = .1\), \(w_X = -9\), \(\Omega_m0 = .23\) and \(\beta = .5\). It may be seen that \(\Omega_m(z)\) increases as \(z\) decreases, so \(\Omega_m(z)\) increases due to evolution of the universe in the interacting model. According to reference [32], positive slope of \(\Omega_m(z)\) suggests phantom \((w < -1)\) and negative slope of \(\Omega_m(z)\) suggests quintessence \((w > -1)\). In figure 3 we find that the \(\Omega_m(z)\) is characterized negative slope indicating quintessence like behavior in the presence of interaction as well as non-interaction.

V. FIRST LAW OF THERMODYNAMICS

In this section we are going to examine the validity of the first law of thermodynamics on the event as well as on the apparent horizon in the interacting situation under consideration. The first law of thermodynamics to the apparent horizon in the FRW universe has been studied in the reference [34]. The thermodynamics of the de Sitter universe was considered in the reference [35], where it was shown that de Sitter universe experiences accelerated expansion and has only one cosmological horizon analogous to the black hole horizon.

In reference [36] it was disclosed that the first law of thermodynamics holds in the physically relevant part of the accelerating universe enveloped by the dynamical apparent horizon, while does not hold in the region enveloped by the cosmological event horizon. First we examine the validity of the first law of thermodynamics on the event horizon whose radius is given by [26]

\[
R_h = a \int_t^\infty \frac{dt}{a} = -\frac{1}{(1 + z)H_0} \int_z^{-1} \frac{dz}{\tilde{H}}
\] (34)

Differentiating with respect to cosmic time \(t\), we obtain

\[
\dot{R}_h = H R_h - 1
\] (35)

The temperature and the entropy on the event horizon are given as [28,34]

\[
T_h = \frac{1}{2\pi R_h}, \quad S_h = \frac{\pi R_h^2}{G} = 8\pi^2 R_h^2, \quad (8\pi G = 1)
\] (36)

The amount of the energy crossing on the event horizon is

\[
- dE_h = 4\pi R_h^2 H T_\mu \nu k^\mu k^\nu dt = -8\pi R_h^3 H \dot{H} dt
\] (37)

Validity of the first law of thermodynamics means validity of the equation [34]

\[
T_h dS_h = -dE_h = -8\pi R_h^3 H \dot{H} dt
\] (38)

In the present interaction, the Hubble’s parameter \(H\) is of the form obtained in equation (10). In figure 4 we present \(T_h \dot{S}_h + 8\pi R_h^3 H \dot{H}\) against redshift \(z\) when the universe if filled with only new holographic dark energy. It is observed in this figure that the said quantity is staying at negative level throughout the
Fig. 4 shows the variations of $X = T_h \dot{S}_h + 8\pi R_h^3 \dot{H} \dot{H}$ against redshift $z$ for only new holographic dark energy model (without dark matter) and fig. 5 shows the variations of $X = T_h \dot{S}_h + 8\pi R_h^3 \dot{H} \dot{H}$ against redshift $z$ for $\delta = 0$ (blue line), $\delta = .01$ (green line), $w_m = .1$, $w_X = -.9$, $\Omega_m = .23$ and $\beta = .5$.

evolution of the universe. This indicates that the first law of thermodynamics is not valid on the event horizon when the universe is considered to be filled with only new holographic dark energy. In figure 5 we plot $T_h \dot{S}_h + 8\pi R_h^3 \dot{H} \dot{H}$ against redshift $z$ when there is an interaction between new holographic dark energy and dark matter. Here also we observe that the first law of thermodynamics fails to be valid on the event horizon.

Now we examine the validity of the first law of thermodynamics on the apparent horizon. Radius of the apparent horizon for the FRW universe is given by

$$R_a = \frac{1}{\sqrt{H^2 + \frac{k}{\pi^2}}} \quad (39)$$

As we are considering flat FRW universe, $k = 0$ and consequently $R_a = 1/H$. On the apparent horizon, temperature $T_a = H/2\pi$ and entropy $S_a = 8\pi^2/H^2$. The obvious consequence is

$$T_a \dot{S}_a + 8\pi R_a^3 \dot{H} \dot{H} = 0 \quad (40)$$

Thus, the first law of thermodynamics is always valid on the apparent horizon irrespective of the interaction. Thus, the above revelations are consistent with the observations made in the reference [36].

VI. GENERALIZED SECOND LAW OF THERMODYNAMICS

Importance of examining the validity of the generalized second law of thermodynamics in the accelerating universe driven by dark energy have been emphasized by a plethora of literatures [37-42]. Benkenstein [24] assumed that there is a relation between the event horizon and the thermodynamics of a black hole and consequently the second law of thermodynamics was modified to the generalized second law of thermodynamics. In references like [38] and [39], the validity of the generalized second law of thermodynamics has been studied on the event horizon. According to the generalized second law, entropy of matter and fluids inside the horizon plus the entropy of the horizon do not decrease with time [37, 38]. Using a specific model of dark energy, the generalized second law as defined in the region enveloped by the apparent horizon as well as in the event horizon was examined in [43], where it was found that it is obeyed in the case of the universe enveloped by the apparent horizon, not on the event horizon. In this section, we shall examine the validity of the generalized second law for both event and apparent horizon.
First we consider the validity of the generalized second law of thermodynamics considering the event horizon as the enveloping horizon of the universe. To show the validity of the generalized second law of thermodynamics we start with Gibb’s equation \[ T_h dS = pdV + d(\rho V) \] where, the volume of the sphere is \( V = \frac{4}{3} \pi R_h^3 \). Using (34) - (37), the rate of change of total entropy is obtained as

\[
\dot{S}_{total} = \dot{S} + \dot{S}_h = 16\pi^2 R_h \left[ - (1 + z) H_0^2 R_h^2 \frac{d\dot{H}}{dz} + H_0 \dot{H} R_h - 1 \right]
\]

The generalized second law will be valid if

\[
\dot{S} + \dot{S}_h \geq 0 \quad \text{i.e.,} \quad (1 + z) H_0^2 R_h^2 \frac{d\dot{H}}{dz} \geq H_0 \dot{H} R_h - 1
\]

In figure 6 we have plotted \( \dot{S} + \dot{S}_h \) against redshift \( z \) considering that the universe is filled with the new holographic dark energy only i.e. there is no dark matter. In this situation we find that the \( \dot{S}_{total} \) is positive at late stage of the universe i.e. at low redshift. In figure 7 we have plotted \( \dot{S}_{total} \) against \( z \) in the interacting (\( \delta \neq 0 \)) as well as non-interacting (\( \delta = 0 \)) situations. In both of the cases it is observed that the generalized second law of thermodynamics breaks down excepting very late stage of the universe.

Next we consider the generalized second law of thermodynamics in the universe enveloped by apparent horizon \( R_a = 1/H \). Time derivatives on and within the apparent horizon are given by

\[
\dot{S}_a = -\frac{16\pi \dot{H}}{H^3} ; \quad \dot{S} = \frac{3\pi}{2H} (1 + 3w_x \Omega_x)(1 + w_x \Omega_x)
\]

So the rate of change of total entropy of the universe bounded by apparent horizon is given by

\[
\dot{S}_{total} = \dot{S} + \dot{S}_a = \frac{3\pi}{2H} (1 + 3w_x \Omega_x)(1 + w_x \Omega_x) - \frac{16\pi \dot{H}}{H^3}
\]
Fig. 8 shows the variations of the time derivative of total entropy against redshift \( z \) for only new holographic dark energy model (without dark matter) and fig. 9 shows the variations of the time derivative of total entropy against redshift \( z \) for \( \delta = 0 \) (blue line), \( \delta = .01 \) (red line), \( w_m = .1, w_X = -.9, \Omega_{m0} = .23 \) and \( \beta = .5 \). In these figures the entropies are calculated for the universe enveloped by the apparent horizon.

In figure 8 we have plotted \( \dot{S} + \dot{S}_A \) against redshift \( z \) considering that the universe is filled with the new holographic dark energy only i.e. there is no dark matter. In figure 9 we have plotted \( \dot{S}_{\text{total}} \) against \( z \) in the interacting (\( \delta \neq 0 \)) as well as non-interacting (\( \delta = 0 \)) situations. In these situations we find that the \( \dot{S}_{\text{total}} \) is always positive in the evolution of the universe. In both of the cases it is observed that the generalized second law of thermodynamics is always satisfied.

VII. DISCUSSIONS

In this work, we have considered that the flat FRW universe is filled with the mixture of dark matter and the new holographic dark energy proposed by [22]. We have considered here that there is an interaction between dark matter and dark energy. The solutions have been obtained in terms of redshift for a particular form of interaction term. The expressions of \( \alpha \) and \( f_0 \) are obtained in term of the free parameter \( \beta \), but it can be fixed by the behaviour of the deceleration parameter \( q \). Also in the current observations, for specific values of other parameters, we find that \( \alpha = 1.02 \) and \( f_0 = 0.71 \). If there is an interaction, we have investigated the natures of deceleration parameter, statefinder and \( Om \) diagnostics during whole evolution of the universe (figures 1-3). At the same time we have examined the cases of the interaction parameter \( \delta = 0 \) which implies the existence dark matter and dark energy in the form of a mixture without interaction. It has been observed that the nature of the various diagnostic parameters are not significantly influenced by the interaction. It has been observed from the \( r-s \) plot that the two-component model is able to explain the evolution of the universe from radiation to \( \Lambda \)CDM though dust stage. The negative slope of the \( Om(z) \) has indicated that the model behaves like quintessence. A signature flip for the deceleration parameter \( q \) has occurred thereby indicating the evolution of the universe from early deceleration to present acceleration.

In the next phase we have investigated the validity of the laws of thermodynamics considering apparent as well as event horizon as the enveloping horizon of the universe. When the new holographic dark energy is considered alone (without dark matter), it is observed that first law of thermodynamics is violated on the event horizon (figure 4) throughout the evolution of the universe. However, for this case the generalized second law of thermodynamics is valid in the late stage of the universe enveloped by the event horizon (figure 6). When the apparent horizon is considered as the enveloping horizon of the universe, the new holographic dark energy (without dark matter) satisfies the generalized second law throughout the evolution of the universe (figure 8). While considering the new holographic dark energy with dark matter we find that for \( \delta \neq 0 \), the first law breaks down throughout the evolution of the universe enveloped by the event horizon (figure 5) and generalized second law is valid only in the late stage of the universe enveloped by the event horizon (figure 7). However, figure 9
shows that the generalized second law is valid when we assume that the universe is enveloped by the apparent horizon. Similar behaviors are discerned for non-interacting (δ = 0) situations also. We have noted from figures 8 and 9 that the time derivative of total entropy has an increasing pattern for only dark energy model with the evolution of the universe enveloped by the apparent horizon, whereas for the two-component model is is decaying from higher to lower redshifts. However, when the event horizon is assumed as the enveloping horizon, the two and single-component models behave similarly for the time derivative of total entropy.

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