Measurement and simulation of the dynamic characteristics of plain and profiled annular seals

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Abstract. The performance and reliability of turbomachinery is often limited by shaft vibrations induced by fluid forces of (i) plain or profiled annular seals, (ii) pistons and (iii) journal bearings. The rotodynamic coefficients namely, stiffness, damping and inertia of plain annular seals are in general calculated using the bulk-flow approach developed by Childs. However, when applying the calculation method to profiled gaps, especially those with non-symmetrical profiles, large differences between simulation and experimental data are reported. Furthermore, reliable experimental data as well as large databases including experimental and simulation results from a single author are limited. To overcome this, the paper introduces a unique active magnetic bearing test rig, built and operated at the Technische Universität Darmstadt as well as the corresponding simulation method to determine the rotodynamic influence of annular seals. An extensive experimental study is carried out investigating plain, symmetrically profiled and non-symmetrically profiled annular seals within the relevant parameter range for turbulent flow in pumps. The results are compared to the simulation results showing a significant influence of profiled gaps on the dynamic characteristics in comparison to plain annular seals. Furthermore, it is shown that symmetric and non-symmetric profiles have a different influence on the dynamic quantities inertia, damping and stiffness.

1. Introduction

The performance and reliability of turbomachinery like modern feed-water pumps is often limited by shaft vibrations induced by fluid forces. Due to increasing demands for flexibility in modern power plants, today’s pumps are often operated at partial load under dynamic operating conditions. This operation poses a challenge in terms of service life and the vibrations generated within the pump. While at the nominal point the excitations from the hydraulics are present but small, the partial load operation of the pump results in increased vibrations due to flow separation and recirculation areas occurring [1]. Thus, damping is becoming an increasingly important design parameter, especially within annular gaps. In centrifugal pumps, two different annuli exist which have a very strong influence on the vibration behaviour: (i) journal bearings and (ii) annular seals.

The dynamic characteristics of plain journal bearings, which are often lubricated by oil under laminar flow conditions, are well understood. In general, the static and dynamic properties are obtained by Reynolds equation of lubrication theory [2]. This partial differential equation can even be solved analytically for special cases [3]. In contrast, the dynamic properties of annular seals have not yet been sufficiently explored. In particular, the influence of surface-profiled
geometries, such as labyrinth or honeycomb seals, has not yet been adequately understood. Especially the influence of a certain profile geometry on the rotodynamic coefficients stiffness, damping and inertia are of great importance. Furthermore, reliable experimental data, as well as large data bases including experimental and simulation results from a single author, are limited. To overcome this, the paper introduces on a unique active magnetic bearing test rig, built and operated at the Technische Universität Darmstadt as well as the corresponding simulation method to determine the rotodynamic influence of annular seals. An extensive experimental study is carried out investigating plain, symmetrically profiled and non-symmetrically profiled annular seals within the relevant parameter range for turbulent flow in pumps. The results are compared to the simulation results showing a significant influence of profiled gaps on the dynamic characteristics in comparison to plain annular seals.

In general, annular gaps are manufactured with larger diameters and are lubricated with low viscous fluids such as water or cryogenic liquids. Therefore, the annuli are operated at high Reynolds numbers resulting in turbulent flow conditions and significant inertia effects, cf. [1, 4–6]. With the presence of high axial pressure gradients, the flow is additionally superimposed by an axial flow component leading to the formation of the inertia driven Lomakin effect [7]. Therefore, annular seals in centrifugal pumps and turbomachinery are crucial in terms of dynamic behaviour of the system and cannot be neglected in rotodynamic designs [1, 8, 9].

The geometries of the sealing gaps are manifold and depend on different boundary conditions. In accordance with the requirements, plain or profiled seals are used. The dynamic characteristics of annular gaps are in general described by rotodynamic coefficients: (i) stiffness \( \tilde{K} \), (ii) damping \( \tilde{C} \) and (iii) inertia \( \tilde{M} \) (\( \Box \) indicates dimensional variables). The equation of motion of the annular gap yields

\[
\begin{bmatrix}
\tilde{F}_X \\
\tilde{F}_Y 
\end{bmatrix} = \begin{bmatrix}
\tilde{K}_{XX} & \tilde{K}_{XY} \\
\tilde{K}_{YX} & \tilde{K}_{YY}
\end{bmatrix} \begin{bmatrix}
\tilde{X} \\
\tilde{Y}
\end{bmatrix} + \begin{bmatrix}
\tilde{C}_{XX} & \tilde{C}_{XY} \\
\tilde{C}_{YX} & \tilde{C}_{YY}
\end{bmatrix} \begin{bmatrix}
\dot{\tilde{X}} \\
\dot{\tilde{Y}}
\end{bmatrix} + \begin{bmatrix}
\tilde{M}_{XX} & \tilde{M}_{XY} \\
\tilde{M}_{YX} & \tilde{M}_{YY}
\end{bmatrix} \begin{bmatrix}
\ddot{\tilde{X}} \\
\ddot{\tilde{Y}}
\end{bmatrix}. \tag{1}
\]

Here \( \tilde{F}_X \) and \( \tilde{F}_Y \) are the induced hydrodynamic forces of the annulus acting on the rotor, \( \tilde{X}, \tilde{X}, \tilde{X} \) and \( \tilde{Y}, \tilde{Y}, \tilde{Y} \) are the lateral movements of the rotor and their time derivatives. The 12 coefficients depend in general on 3 different properties of the annular gap flow: (i) the profiled or plain gap geometry, i.e. symmetric or non-symmetric profiles, the radius of the shaft \( \tilde{R} \), the mean gap height \( \tilde{h} \) and the gap length \( \tilde{L} \); (ii) the operating parameters of the turbomachinery and the resulting parameters of the gap, i.e. the eccentricity \( \tilde{e} \), the angle of misalignment \( \gamma \), the angular velocity of the shaft \( \tilde{\Omega} \), the mean axial velocity through the annulus \( \tilde{C}_z \) and the pre-swirl at the gap entrance \( \tilde{C}_\varphi|_{z=0} \); (iii) the characteristics of the fluid used for lubrication, i.e. fluid density \( \tilde{\rho} \) and dynamic viscosity \( \tilde{\eta} \)

\[
\tilde{K}_{ij}, \tilde{C}_{ij}, \tilde{M}_{ij} = f \left( \tilde{R}, \tilde{h}, \tilde{L}, \tilde{e}, \gamma, \tilde{\Omega}, \tilde{C}_z, \tilde{C}_\varphi|_{z=0}, \tilde{\rho}, \tilde{\eta} \right), \quad i, j = \tilde{X}, \tilde{Y}. \tag{2}
\]

Here, we focus mainly on annular seals operating at zero eccentricity \( \tilde{e} = 0 \) without misalignment, i.e. \( \gamma = 0 \). Within this operating range the matrices of stiffness, damping and inertia are skew-symmetric, i.e. \( \tilde{K}_{ij}, \tilde{C}_{ij}, \tilde{M}_{ij} = -\tilde{K}_{ji}, -\tilde{C}_{ji}, -\tilde{M}_{ji} \), resulting in a simplified equation of motion with the direct and cross-coupled stiffness \( \tilde{K}, \tilde{k} \), the direct and cross-coupled damping \( \tilde{C}, \tilde{c} \) and the direct and cross-coupled inertia \( \tilde{M}, \tilde{m} \)

\[
\begin{bmatrix}
\tilde{F}_X \\
\tilde{F}_Y 
\end{bmatrix} = \begin{bmatrix}
\tilde{K} & \tilde{k} \\
-\tilde{k} & \tilde{K}
\end{bmatrix} \begin{bmatrix}
\tilde{X} \\
\tilde{Y}
\end{bmatrix} + \begin{bmatrix}
\tilde{C} & \tilde{c} \\
-\tilde{c} & \tilde{C}
\end{bmatrix} \begin{bmatrix}
\dot{\tilde{X}} \\
\dot{\tilde{Y}}
\end{bmatrix} + \begin{bmatrix}
\tilde{M} & \tilde{m} \\
-\tilde{m} & \tilde{M}
\end{bmatrix} \begin{bmatrix}
\ddot{\tilde{X}} \\
\ddot{\tilde{Y}}
\end{bmatrix}. \tag{3}
\]

Most published references [1, 4, 8, 9] assume that the cross-coupled inertia terms are small in comparison to the direct inertia \( \tilde{m} \ll \tilde{M} \) and are therefore negligible, i.e. \( \tilde{m} = 0 \). Here, for the sake of completeness, the cross-coupled terms are considered just in case the profiled gap geometry has a significant impact.
Figure 1. Generic annular seal geometry with the radius of the shaft $\tilde{R}$, the mean gap height $\bar{h}$ and the gap length $\bar{L}$, the gap eccentricity $\bar{e}$, the angular velocity of the shaft $\bar{\Omega}$, the angular velocity of the precession movement $\bar{\omega}$, the mean axial velocity through the annulus $\bar{C}_z$, the pre-swirl at the gap entrance $\bar{C}_\varphi|_{z=0}$, the fluid density $\bar{\varrho}$ and dynamic viscosity $\bar{\eta}$.

2. Governing equation of the bulk-flow approach
The generic geometry of an annular seal is shown in figure 1.

The governing equations of the bulk-flow approach are recapped. The most popular analysis in terms of rotordynamic influence were carried out by Black, Childs and Nelson [10–14]. In the following, the equation are mainly based on the work of [14]. Regarding the generic gap geometry, the bulk-flow approach formulates the conservation equation for mass as well as the conservation equation for circumferential and axial momentum:

$$\frac{\partial \bar{h}}{\partial t} + \frac{1}{\bar{R}} \frac{\partial (\bar{h} \bar{C}_\varphi)}{\partial \varphi} + \frac{\partial (\bar{h} \bar{C}_z)}{\partial z} = 0,$$

$$\frac{\partial \bar{h}}{\partial t} \left( \frac{\partial \bar{C}_\varphi}{\partial t} + \bar{C}_\varphi \frac{\partial \bar{C}_\varphi}{\partial \varphi} + \bar{C}_z \frac{\partial \bar{C}_\varphi}{\partial z} \right) = -\frac{\bar{h}}{\bar{R}} \frac{\partial \bar{p}}{\partial \varphi} + \bar{\tau}_{\text{stat},\varphi} - \bar{\tau}_{\text{rot},\varphi},$$

$$\frac{\partial \bar{h}}{\partial t} \left( \frac{\partial \bar{C}_z}{\partial t} + \bar{C}_\varphi \frac{\partial \bar{C}_z}{\partial \varphi} + \bar{C}_z \frac{\partial \bar{C}_z}{\partial z} \right) = -\frac{\bar{h}}{\bar{R}} \frac{\partial \bar{p}}{\partial z} + \bar{\tau}_{\text{stat},z} - \bar{\tau}_{\text{rot},z}. \tag{4}$$

Here, $\bar{C}_\varphi$ and $\bar{C}_z$ are the averaged bulk velocities across the gap height,

$$\bar{C}_\varphi = \frac{1}{\bar{h}} \int_0^{\bar{h}} \bar{C}_\varphi \, d\bar{\varphi}, \quad \bar{C}_z = \frac{1}{\bar{h}} \int_0^{\bar{h}} \bar{C}_z \, d\bar{\varphi} \tag{5}$$

and $\bar{\tau}_{\text{stat},i}$, $\bar{\tau}_{\text{rot},i}$ $(i = \text{stat}, \text{rot})$ are the wall shear stresses at the stator and rotor surfaces.

By introducing the dimensionless variables

$$z := \frac{\bar{z}}{\bar{L}}, \quad t := \frac{\bar{t}}{\bar{L}} \bar{C}_z, \quad h := \frac{\bar{h}}{\bar{R}}, \quad \bar{L} := \frac{\bar{L}}{\bar{R}}, \quad \bar{\varphi} := \frac{\bar{\varphi}}{\bar{R}},$$

$$C_z := \frac{\bar{C}_z}{\bar{C}_z}, \quad C_\varphi|_{z=0} := \frac{\bar{C}_\varphi|_{z=0}}{\bar{\Omega} \bar{R}}, \quad \phi := \frac{\bar{\varphi}}{\bar{\Omega} \bar{R}}, \quad \text{Re}_\varphi := \frac{\bar{\varphi} \bar{\Omega} \bar{R} \bar{h}}{\bar{\eta}}, \tag{6}$$
the dimensionless bulk-flow equations yield
\[ \frac{\partial h}{\partial t} + \frac{L}{\phi} \frac{\partial (hC_\varphi)}{\partial \varphi} + \frac{\partial (hC_z)}{\partial z} = 0, \]

\[ h \left( \frac{\partial C_\varphi}{\partial t} + \frac{L}{\phi} C_\varphi \frac{\partial C_\varphi}{\partial \varphi} + C_z \frac{\partial C_\varphi}{\partial z} \right) = -\phi h L \frac{\partial p}{\partial \varphi} + \tau_{\text{stat}, \varphi} - \tau_{\text{rot}, \varphi}, \]

\[ h \left( \frac{\partial C_z}{\partial t} + \frac{L}{\phi} C_\varphi \frac{\partial C_z}{\partial \varphi} + C_z \frac{\partial C_z}{\partial z} \right) = -h \frac{\partial p}{\partial z} + \tau_{\text{stat}, z} - \tau_{\text{rot}, z}. \] (7)

The wall shear stresses \( \tau_{\text{stat}, \varphi} \) and \( \tau_{\text{rot}, \varphi} \) are modelled using Hirs’s approach \([15]\) by means of the Fanning friction factor \( f = m Re^{-n} \). The components of the wall shear stresses are expressed using the resulting wall shear stress \( \tau_{\text{stat}}, \tau_{\text{stat}} \), which are directly opposed by the resulting velocities \( C_{\text{stat}}, C_{\text{rot}} \). This yield

\[ \tau_{\text{stat}, \varphi} = \tau_{\text{stat}} \frac{1}{\phi} C_\varphi \frac{C_{\text{stat}}}{C_z}, \quad \tau_{\text{stat}, z} = \tau_{\text{stat}} \frac{C_z}{C_{\text{stat}}}, \]

\[ \tau_{\text{rot}, \varphi} = \tau_{\text{rot}} \frac{1}{\phi} C_\varphi \frac{C_{\text{rot}}}{C_z}, \quad \tau_{\text{rot}, z} = \tau_{\text{rot}} \frac{C_z}{C_{\text{rot}}}, \] (8)

with

\[ \tau_{\text{stat}} = \frac{1}{2} L \psi C_{\text{stat}} \left( \phi C_\varphi^2 - 1 \right), \quad \tau_{\text{rot}} = \frac{1}{2} L \psi C_{\text{rot}} \left( \phi C_\varphi^2 - 1 \right), \]

\[ C_{\text{stat}} = \sqrt{C_z^2 + \frac{1}{\phi^2} C_\varphi^2}, \quad C_{\text{rot}} = \sqrt{C_z^2 + \frac{1}{\phi^2} (C_\varphi^2 - 1)^2}. \] (9)

The boundary conditions for the velocity and pressure at the seal inlet and outlet read

\[ p|_{z=0} = p_{\text{in}} - \frac{1 + \zeta|_{z=0}}{2h^2}, \quad p|_{z=1} = p_{\text{out}} - \frac{1 - \zeta|_{z=1}}{2h^2}, \quad C_\varphi|_{z=0} \in \mathbb{R}. \] (10)

Here, \( p_{\text{in}} \) and \( p_{\text{out}} \) are the pressures at the plenum before and after the annulus and \( \zeta|_{z=0}, \zeta|_{z=1} \) are the pressure loss and recovery coefficient respectively. The pre-swirl at the gap entrance can be varied to take any real number.

2.1. Perturbation analysis

For the solution of the system of partial differential equations a perturbation analysis is used. Here, the variables \( C_\varphi, C_z, p \) and \( h \) are developed using a first order perturbation expansion with small amplitudes \( \Delta \varepsilon \) around the static concentric equilibrium position

\[ C_\varphi = C_{\varphi, 0} + \Delta \varepsilon C_{\varphi, 1}, \quad C_z = C_{z, 0} + \Delta \varepsilon C_{z, 1}, \]

\[ p = p_0 + \Delta \varepsilon p_1, \quad h = h_0 + \Delta \varepsilon h_1. \] (11)

By inserting the expansion in the governing equations as well as the boundary conditions a set of partial differential equations for the zeroth and first-order is derived. Here, the solution of the linear zeroth-order equation system yields the static pressure and tangential velocity at a given mean axial flow, whereas the solution of the first-order system of equations gives the dynamic flow field. Due to the concentric operation of the annular seal the zeroth-order equations are rotationally symmetrical. Therefore, the dependency in the circumferential direction vanishes. The now non-linear zeroth-order system of ordinary differential equations yields

\[ \frac{\partial (h_0 C_{z, 0})}{\partial z} = 0, \]

\[ h_0 C_{z, 0} \frac{\partial C_{\varphi, 0}}{\partial z} = \tau_{\text{stat}, \varphi, 0} - \tau_{\text{rot}, \varphi, 0}, \]

\[ h_0 C_{z, 0} \frac{\partial C_{z, 0}}{\partial z} = -h_0 \frac{\partial p_0}{\partial z} + \tau_{\text{stat}, z, 0} - \tau_{\text{rot}, z, 0}. \] (12)
The linearized first-order equation system analogously reads

\[
\frac{\partial h_1}{\partial t} + \frac{L}{\phi} \left( \frac{\partial h_0 C_{\varphi,1}}{\partial \varphi} + \frac{\partial h_1 C_{\varphi,0}}{\partial \varphi} \right) + \left( \frac{\partial h_0 C_{z,1}}{\partial z} + \frac{\partial h_1 C_{z,0}}{\partial z} \right) = 0,
\]

\[
h_0 \frac{\partial C_{\varphi,1}}{\partial t} + \frac{L}{\phi} \left( h_1 C_{\varphi,0} \frac{\partial C_{\varphi,0}}{\partial \varphi} + h_0 C_{\varphi,1} \frac{\partial C_{\varphi,0}}{\partial \varphi} + h_0 C_{\varphi,0} \frac{\partial C_{\varphi,1}}{\partial \varphi} \right) + \left( h_1 C_{z,0} \frac{\partial C_{z,0}}{\partial z} + h_0 C_{z,1} \frac{\partial C_{z,0}}{\partial z} + h_0 C_{z,0} \frac{\partial C_{z,1}}{\partial z} \right) = 0,
\]

\[
\left. \frac{\partial C_{z,1}}{\partial t} \right|_{t=0} + \frac{L}{\phi} \left( h_1 C_{z,0} \frac{\partial C_{z,0}}{\partial \varphi} + h_0 C_{z,1} \frac{\partial C_{z,0}}{\partial \varphi} + h_0 C_{z,0} \frac{\partial C_{z,1}}{\partial \varphi} \right) + \left( h_1 C_{z,0} \frac{\partial C_{z,0}}{\partial z} + h_0 C_{z,1} \frac{\partial C_{z,0}}{\partial z} + h_0 C_{z,0} \frac{\partial C_{z,1}}{\partial z} \right) = 0,
\]

\[
\left. \frac{\partial C_{z,1}}{\partial t} \right|_{t=0} = -\phi L \left( h_1 \frac{\partial p_0}{\partial \varphi} + h_0 \frac{\partial p_1}{\partial \varphi} \right) + \tau_{\text{stat},\varphi,1} - \tau_{\text{rot},\varphi,1},
\]

(13)

The gap function \( h \) in cartesian coordinates reads

\[
h = h_0 - \frac{\dot{X}(t)}{\bar{h}} \cos \varphi - \frac{\dot{Y}(t)}{\bar{h}} \sin \varphi.
\]

(14)

Therefore, \( \Delta\varepsilon h_1 = -x \cos \varphi - y \sin \varphi \). Assuming the perturbed variables are of the same form gives

\[
C_{\varphi,1} = C_{\varphi,1c} \cos \varphi + C_{\varphi,1s} \sin \varphi, \quad C_{z,1} = C_{z,1c} \cos \varphi + C_{z,1s} \sin \varphi,
\]

\[
p_1 = p_{1c} \cos \varphi + p_{1s} \sin \varphi.
\]

(15)

By introducing complex variables and assuming a motion of the form \( e^{i\omega t T} \) the perturbed axial and circumferential velocities as well as the perturbed pressure and gap height are of the form

\[
C_{\varphi,1} = C_{\varphi,1} e^{i\omega t T}, \quad C_{z,1} = C_{z,1} e^{i\omega t T},
\]

\[
p_1 = p_{1c} e^{i\omega t T}, \quad h_1 = h_{1c} e^{i\omega t T}.
\]

(16)

Here, \( i = \sqrt{-1} \) is the imaginary unit, \( \omega \) is the dimensionless precession frequency \( \omega = \tilde{\omega}/\tilde{\Omega} \) and \( T = \tilde{\Omega} L / \tilde{C}_z \). With the assumed motion the time dependency as well as the dependencies in the circumferential direction of the first order equation system vanish. In combination with the boundary conditions of the first-order

\[
p_1|_{z=0} = \frac{-1 + \zeta|_{z=0}}{h_0|_{z=0}} C_{z,1}|_{z=0}, \quad p_1|_{z=1} = \frac{-1 - \zeta|_{z=1}}{h_0|_{z=1}} C_{z,1}|_{z=1}, \quad C_{\varphi}|_{z=0} = 0,
\]

(17)

the system of ordinary differential equations can be solved. The perturbed reaction forces are then calculated by

\[
F_{X,1} = -\frac{\varepsilon}{\zeta_{\text{res}}} \int_0^1 \int_0^{2\pi} p_{1c} \cos \varphi \, d\varphi \, dz, \quad F_{Y,1} = -\frac{\varepsilon}{\zeta_{\text{res}}} \int_0^1 \int_0^{2\pi} p_{1s} \sin \varphi \, d\varphi \, dz.
\]

(18)

The reaction forces are made dimensionless using the pressure difference across the annulus \( \Delta \bar{p} \), i.e. \( F = F / (2RL\Delta \bar{p}) \), whereas \( \zeta_{\text{res}} \) is the resulting pressure loss coefficient across the annulus.
$\zeta_{\text{res}} := 2\Delta\tilde{p} \left( \frac{\delta \tilde{C}_z^2}{\bar{\rho} \tilde{C}_z^2} \right)_{z=0}$. To extract the rotordynamic coefficients the cartesian reaction forces $F_{X,1}, F_{Y,1}$ are transformed into a polar coordinate system with the radial and tangential force component $F_{\text{rad},1}, F_{\text{tan},1}$. The coefficients are then extracted by applying a second-order polynomial fit at different precession frequencies

$$
\frac{F_{\text{rad},1}}{\varepsilon} = M\omega^2 - C\omega - K, \quad \frac{F_{\text{tan},1}}{\varepsilon} = -m\omega^2 - \omega C + k.
$$

### 3. Experimental setup

The experimental investigations are carried out using a worldwide unique test bench. The test rig uses two magnetic bearings to (i) support the rotor as well as an inherent (ii) force and (iii) displacement measurement system. The test rig allows an investigation of plain and profiled annular gaps within the relevant parameter range for turbulent flow in pumps.

As illustrated in figure 2 the test rig consists of the following 5 components: (i) two active magnetic bearings (AMB) supporting the rotor; (ii) the inlet; (iii) the gap module; (iv) the outlet and (v) two mechanical seals to seal the test rig against the ambient area.

Compared to conventional bearings such as ball or journal bearings used by several authors [1, 16, 17], magnetic bearings have the advantage of a completely contactless and thus frictionless support of the rotor. In addition, using AMBs has the advantage of an inherent force and displacement measuring system as well as the ability to displace and excite the shaft at user defined frequencies. Due to these advantages the use of active magnetic bearings is ideal for determining the static and dynamic characteristics of annular gap flows. To measure the induced hydrodynamic forces within the annulus, the AMBs are equipped with 8 hall sensors each, measuring the magnetic flux density $\tilde{B}$ within the air gap between rotor and stator. The corresponding force of each pole $\tilde{F}_{H,j}$ (pole surface $\tilde{A}$, magnetic field constant $\tilde{\mu}_0$) yields

$$
\tilde{F}_{H,j} = \frac{\tilde{A}}{2\tilde{\mu}_0} \tilde{B}_{j}^2, \quad j = 1..8.
$$
Due to the dependence of the magnetic flux density on the position of the rotor within the magnetic bearing the hall sensors have to be calibrated. This is done using a modified iterative process initially developed by Krüger [18]. The calibration is carried out using the known rotor weight and the location of the centre of gravity on the rotor as a calibration standard. With an unloaded, non-rotating shaft, the measured force of the magnetic bearing must, at each point within the bearing, output the weight as well as the centre of gravity of the rotor. After calibration, the measurement uncertainty of the force measurement yields $\delta \bar{F} = \pm 0.035 \bar{F}$. The displacement of the rotor inside the AMB is measured using 4 eddy current sensors with an uncertainty of $\delta X_{\text{pos}} \leq 6 \mu m$. To monitor the temperature of each AMB, each is equipped with two PT100 temperature sensors. The inlet is specially designed to generate pre-swirled flows upstream from the annulus. The fluid is injected tangentially into the inlet. By dividing the flow into two parts, gap and bypass volume flow, it is possible to vary the circumferential velocity component in before the annulus continuously, cf. figure 2. To measure the circumferential velocity component at the gap entrance a pitot tube is used. The test rig is capable of generating a pre-swirl in the range of $C_r \bigg|_{z=0} = 0..1.4$. The supply pressure is measured at the inlet of the gap module by using 4 wall pressure taps equally spaced around the annulus ($\delta \bar{p}_{\text{in}} \leq 0.02 \text{bar}$). The pressure difference across the annulus is measured by a differential pressure sensor with an absolute uncertainty of $\delta \Delta \bar{p} \leq 0.1 \text{bar}$.

The test rig is designed for pressure differences of up to 20 bar. To seal the test rig against the environment, the fluid path is sealed with two mechanical seals. These allow rotation of the shaft as well as other relative movements with respect to the stator, e.g. the dynamic excitations to identify the characteristics of the annular gap flow.

4. Identification procedure

Similar to the identification process within the bulk-flow approach, the rotordynamic coefficients are determined on the basis of a circular excitation of the shaft at a user defined eccentricity $\varepsilon$ and precession frequency $\omega$.

First, the rotor is centred and aligned inside the annulus. This is done by moving the rotor from the centre of the magnetic bearings until contact between rotor and stator occurs. The occurring contact force $\bar{F}_C$ is measured by the magnetic bearings and increases exponentially at occurring rotor-stator contact. Therefore, it can be used as a positioning criterion. In order to avoid an inclined position of the shaft in the annulus, the centre of gravity of the rotor is also calculated from the measured forces. If the calculated centre of gravity at contact is unequal to the known centre of gravity of the shaft, the shaft and stator axis are inclined to each other by the angle $\gamma$. The quotient of the centres of gravity indicates whether the contact point is closer to magnetic bearing A or magnetic bearing B. Finally, the position of the shaft is corrected until the centre of gravity at contact and the centre of gravity of the shaft coincide. The method is carried out over the entire circumference of the stator. By applying a circular fit to the stored positions at contact the final determination of the centric position of the shaft is found.

Second, the operating conditions are set and the rotor is excited using the magnetic bearings. The data acquisition is done using the software LabVIEW with a sample rate of 3340 Hz over a period of 1 s. Figure 3 shows an example of such an excitation (left) and the corresponding induced dimensionless hydrodynamic reaction forces (right). Clearly visible here is the imprinted circular motion of the shaft at a related orbital frequency $\omega = 1.8$ due to the excitation of the magnetic bearing as well as the measured resulting circularity of the induced forces. In accordance to the method used at the bulk-flow approach, the induced forces are transformed into a polar coordinate system with the radial and tangential force component $F_{\text{rad},1}, F_{\text{tan},1}$, cf. figure 4. By means of a second order polynomial fit the dimensionless rotordynamic coefficients stiffness $K_j := \left( \tilde{K}_j \tilde{h} \right) / \left( \tilde{L} \tilde{R} \Delta \bar{p} \right)$, damping $C_j := \left( \tilde{C}_j \tilde{h} \tilde{\Omega} \right) / \left( \tilde{L} \tilde{R} \Delta \bar{p} \right)$ and inertia $M_j := \left( \tilde{M}_j \tilde{h} \tilde{\Omega}^2 \right) / \left( \tilde{L} \tilde{R} \Delta \bar{p} \right)$ are identified.
\[ L = 1; \psi = 4.2\% \]
\[ Re\varphi = 3810; \omega = 1.8 \]
\[ C_{\varphi} \bigg|_{x=0} = 0.5 \]

Figure 3. left: orbit of the rotor centre during excitation; right: associated induced hydrodynamic force of the annular seal.

\[ -F_{\text{TANG}} = \varepsilon(-k + C\omega + m\omega^2) \]
\[ -F_{\text{RAD}} = \varepsilon(K + \omega - M\omega^2) \]

Figure 4. left: induced radial force; right: induced tangential force.

In addition, figure 4 (right) shows a comparison between the identification with and without presence of a cross-coupled inertia term \( m \). It is clearly visible that the cross-coupled terms are small compared to the direct inertia terms.

5. Results and discussion
The experiments were carried out at a constant pressure difference using three different surface profiled geometries: (i) plain annular gaps, (ii) symmetric profiles, i.e. saw-tooth pattern, (iii) non-symmetric profiles, i.e. borehole pattern. During the experiments the Reynolds number was varied as well as the gap length and the pre-swirl. Here, only the results for the general impact of surface profiling with varying Reynolds number at a gap length of \( L = 1 \) are presented.

Figure 5 shows the influence of surface profiling on the measured dimensionless rotodynamic coefficients for the operating conditions \( L = 1, \psi = 4.2\% \) and \( C_{\varphi} \bigg|_{x=0} = 0.5 \) compared to the corresponding bulk-flow calculations. Here, the marker gives the experimental results, whereas the lines indicated the calculated coefficients by means of the bulk-flow approach. First, considering the experimental results the effect due to symmetric and non-symmetric profiles mainly affect the direct and cross-coupled stiffness as well as the direct damping, cf. figure 5 (top and mid-left). The inertia coefficients are almost independent of the profiling. A symmetric surface profiling with borehole pattern leads to a reduction in the direct stiffness of
the measured geometries and an increase in the direct damping compared to the non-profiled gap. The coefficients of cross-coupled stiffness and damping were not significantly influenced. A profiling by means of symmetric profiles, i.e. saw tooth profile, resulted in a decreased direct and cross-coupled stiffness in comparison to the plain as well as to the non-symmetric profiled gap with hole pattern. In addition, a decrease in the main damping is shown with symmetric profiling. The cross-coupled damping remained unchanged. By comparing the experimental results to the bulk-flow calculation, a good agreement of the calculations with the results of the non-profiled plain sealing gap is observed. The occurring deviations from the experimentally determined stiffness, damping and inertia are in the order of 20%. Deviations of these orders of magnitude are also documented in the literature. When comparing the calculations to the profiled annuli the inadequacy of the bulk flow approach for the calculation of surface-profiled sealing gaps is obvious.

Finally, the influence of the profiles on the leakage of the sealing gap is investigated. Figure 6
\[ L = 1; \phi = 4.2\% \]
\[ C_{\phi restr} = 0.5 \]

**Figure 6.** Experimental results (marker) for the flow number of profiled and non-profiled annuli compared to the flow number calculated by means of the bulk-flow approach (lines).

shows the change in flow number \( \phi \) over the Reynolds number with different profiling (left) and the influence of the gap length for the sealing gap with borehole profile (right). First, examining the effects of profiling on the flow number, a significant reduction can be seen compared to the plain annulus. There is also a clear difference to the calculation results. This leads to an increased prediction of the flow coefficient in the bulk flow calculations. Comparing the different surface-profiled geometries, no significant deviations in the flow numbers are found. Finally, the change in flow number at reduced gap length is investigated. An increase in the flow number with decreasing gap length can be observed.

### 6. Conclusion

In summary, the following statements can be made:

- The construction of sealing gaps with surface profiled geometries such as borehole patterns and saw-tooth geometries leads to a clearly visible change in rotordynamic properties of the gap flow.
- The use of borehole profiles leads to a reduction of the direct stiffness and to an increased direct damping.
- The use of saw tooth geometries generally leads to a holistic reduction of the rotordynamic coefficients.
- The qualitative effects of the seal gap length can be represented by the bulk flow approach. However, the general differences become apparent when considering profiled geometries.
- The leakage of the sealing gaps is improved by the profiling. However, there is also a discrepancy in the predictions for profiled gaps due to the bulk-flow approach.

In conclusion, it can be stated that the predictions of the bulk flow approach for plain annular gaps are in good agreement with the experimental results. With surface profiling however, an increasing deviation can be seen. This leads to the necessity of extended or novel calculation approaches for the prediction of surface-profiled annular seals.

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