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Statistics and gap distributions in random Kakutani partitions and multiscale substitution tilings. (English) Zbl 07584838
J. Math. Anal. Appl. 516, No. 2, Article ID 126535, 14 p. (2022)

Summary: We study statistics of tiles in random incommensurable Kakutani sequences of partitions in $\mathbb{R}^d$. We provide explicit formulas that illustrate the dependence on the combinatorial structure, the volumes of the participating tiles and the entropy of the partitions in the underlying random substitution system. These improve previous results for non-random Kakutani partitions and multiscale substitution tilings, and imply a gap distribution formula for Delone sets associated with multiscale substitution tilings of the real line.

MSC:
62-XX Statistics
52C23 Quasicrystals and aperiodic tilings in discrete geometry
05C22 Signed and weighted graphs
11K36 Well-distributed sequences and other variations
37B52 Tiling dynamics

Keywords:
tile frequencies; gap distributions; incommensurable substitution systems; Kakutani splitting procedure; multiscale substitution tilings; random partitions

Full Text: DOI arXiv

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Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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