tick: a Python library for statistical learning, with a particular emphasis on time-dependent modeling

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Abstract

tick is a statistical learning library for Python 3, with a particular emphasis on time-dependent models, such as point processes, and tools for generalized linear models and survival analysis. The core of the library is an optimization module providing model computational classes, solvers and proximal operators for regularization. tick relies on a C++ implementation and state-of-the-art optimization algorithms to provide very fast computations in a single node multi-core setting. Source code and documentation can be downloaded from https://github.com/X-DataInitiative/tick.

Keywords: Statistical Learning; Python; Hawkes processes; Optimization; Generalized linear models; Point Process; Survival Analysis

1. Introduction

The aim of the tick library is to propose to the Python community a large set of tools for statistical learning, previously not available in any framework. Though tick focuses on time-dependent modeling, it actually introduces a set of tools that allow to go way beyond this particular set of models, thanks to a highly modular optimization toolbox. It benefits from a thorough documentation (including tutorials with many examples), and a strongly tested API that brings to the scientific community cutting-edge algorithms with a high level of customization. Optimization algorithms such as SVRG (Johnson and Zhang, 2013) or SDCA (Shalev-Shwartz and Zhang, 2013) are among the several optimization algorithms available in tick that can be applied (in a modular way) to a large variety of models. An emphasis is done on time-dependent models: from the Cox regression model (Andersen et al., 2012), a very popular model in survival analysis, to Hawkes processes, used in a wide range of applications such as geophysics (Ogata, 1988), finance (Bacry et al., 2015) and more recently social networks (Zhou et al., 2013; Xu et al., 2016). To the best of our knowledge, tick is the most comprehensive library that deals with Hawkes processes, since it brings parametric and nonparametric estimators of these models to a new accessibility level.
2. Existing libraries

tick follows, whenever possible, the scikit-learn API (Pedregosa et al., 2011; Buitinck et al., 2013) which is well-known for its completeness and ease of use, which makes it the reference Python machine learning library. However, while scikit-learn targets a wide spectrum, tick has a more specific objective: implementing highly-optimized algorithms with a particular emphasis on time-dependent modeling (not proposed in scikit-learn). The tick optimization toolbox relies on state-of-the-art optimization algorithms, and is implemented in a very modular way. It allows more possibilities than other scikit-learn API based optimization libraries such as lightning\textsuperscript{1}.

A wide variety of time-dependent models are taken care of by tick, which makes it the most comprehensive library that deals with Hawkes processes for instance, by including the main inference algorithms from literature. Despite the growing interest in Hawkes models, very few open source packages are available. There are mainly three of them. The library \texttt{pyhawkes}\textsuperscript{2} proposes a small set of Bayesian inference algorithms for Hawkes process. \texttt{hawkes R}\textsuperscript{3} is a R-based library that provides a single estimation algorithm, and is hardly optimized. Finally, PtPack\textsuperscript{4} is a C++ library which proposes mainly parametric maximum likelihood estimators, with sparsity-inducing regularizations. However, PtPack is not interfaced with a user-friendly scripting language such as Python, which makes it less accessible to end-users for quick prototyping and experimenting on datasets. Moreover, as illustrated below, PtPack exhibits poor performance compared to tick.

3. Package architecture

The tick library has four main modules: tick.hawkes for Hawkes processes (see Section 4 for a detailed review), tick.linear.model with linear, logistic and Poisson regression, tick.robust for robust regression and tick.survival for survival analysis. Each of these modules provide simulation tools and learners to easily learn from data. Whenever possible, tick follows the scikit-learn API. The core of tick is made of easy to combine penalization techniques (proximal operators), available in the tick.prox module and several convex solvers, available in the tick.solver, to train almost any available model in the library, see Table 1 for a non-exhaustive list of possible combinations. An exhaustive list is available on the documentation web page\textsuperscript{5}, and is given in Figure 6 of the supplementary material.

4. Hawkes

Distributing a comprehensive open source library for Hawkes processes is one of the primary aims of the tick library: it provides many non-parametric and parametric estimation algorithms as well as simulation tools for many kernel types, that are listed in Table 2. This diversity of algorithms is illustrated in Figure 1 (with the associated Python code) in which we show how two kernels of different shapes are estimated by four different algorithms. A

\textsuperscript{1} http://contrib.scikit-learn.org/lightning
\textsuperscript{2} https://github.com/slinderman/pyhawkes
\textsuperscript{3} https://cran.r-project.org/web/packages/hawkes/hawkes.pdf
\textsuperscript{4} https://github.com/dunan/MultiVariatePointProcess
\textsuperscript{5} https://x-datainitiative.github.io/tick/
tick: a Python library for statistical learning

| Model                  | Proximal operator | Solver                        |
|------------------------|-------------------|-------------------------------|
| Linear regression      | L2 (Ridge)        | Gradient Descent              |
| Logistic regression    | L1 (Lasso)        | Accelerated Gradient Descent  |
| Poisson regression     | Total Variation   | Stochastic Gradient Descent   |
| Cox regression         | Group L1          | Stochastic Variance Reduced Gradient |
| Hawkes with exp. kernels | SLOPE             | Stochastic Dual Coordinate Ascent |

Table 1: tick allows the user to combine many models, prox and solvers

| Non Parametric                        | Parametric                        |
|---------------------------------------|-----------------------------------|
| EM (Lewis and Mohler, 2011)           | Single exponential kernel         |
| Basis kernels (Zhou et al., 2013)     | Sum of exponentials kernels       |
| Wiener-Hopf (Bacry and Muzy, 2014)    | Sum of gaussians kernels (Xu et al., 2016) |
| NPHC (Achab et al., 2017)             | ADM4 (Zhou et al., 2013)          |

Table 2: Hawkes estimation algorithms implemented in tick

first use case for modeling high-frequency financial data is given in Figure 2, while a second use-case about propagation analysis of earthquake aftershocks can be found in Figure 4.

5. Benchmarks

We perform benchmark tests for both simulation and estimation of Hawkes processes (with exponential kernels) using tick, hawkes R (where only simulation is available) and PtPack, on respectively 2, 4 and 16 cores. In Figure 3 we compare computational times for simulation and fitting of Hawkes processes. The model fits are compared on simulated 16-dimensional Hawkes processes, with an increasing number of events: small=5 × 10^4, medium=2 × 10^5, large=10^6, xlarge= 5 × 10^7. We observe on this experiment that tick outperforms by several orders of magnitudes both hawkes R and PtPack, in particular for large datasets. Benchmarks against scikit-learn for logistic regression are also provided in Figure 5 from the supplementary material.

![Figure 1: Illustration of different kernels shapes and estimations obtained by tick on two 1D simulated Hawkes processes with intensity kernels displayed with dashed orange lines.](image-url)
Figure 2: Kernels norms of a Hawkes process fitted on high-frequency financial data from the Bund market (Bacry et al., 2016) where $P_u$ (resp. $P_d$) counts the number of upward (resp. downward) mid-price moves and $T_a$ (resp. $T_b$) counts the number of market orders at the ask (resp. bid) that do not move the price.

Figure 3: Computational timings of tick versus PtPack and hawkes R. tick strongly outperforms both libraries for simulation and fitting (note that fit graph is in log-scale). Third figure shows that tick benefits from multi-core environments to speed up computations.

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Figure 4: Analysis with Hawkes processes of earthquake propagation with a dataset from Ogata (1988). On the left we can see where earthquakes have occurred and on the right the propagation matrix, i.e. how likely a earthquake in a given zone will trigger an aftershock in another zone. We can observe than zone 1, 2 and 3 are tightly linked while zone 4 and 5 are more self-excited.
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Appendix

A. Speed comparison

We compare fitting results for binary logistic regression with tick and scikit-learn. These experiments are run on commonly used datasets described in Table 3. Note that Covtype has been standardized, hence the first two datasets IJCNN and Covtype are dense and the last four datasets are sparse. Two types of penalization have been tested: $\ell_1$ (Lasso) and $\ell_2$ (Ridge). In both cases the regularization parameter $\lambda$ has been set to $1/n$ where $n$ is the number of samples and we have left the default step-size for both libraries. Results are given in Figure 5. Overall, tick is slightly faster because it makes faster iterations: both libraries reach the same objective after each pass over the data but tick performs these computations faster. Also, $\ell_1$ penalization in high dimension is difficult for scikit-learn (see URL and KDD 2010) whereas tick handles it without any additional problem.
B. Package structure

The package structure is detailed in Figure 6. We retrieve all the following modules:

- **tick.hawkes**: Inference and simulation of Hawkes processes, with both parametric and non-parametric estimation techniques and flexible tools for simulation. It is split in three submodules: `tick.hawkes.inference`, `tick.hawkes.simulation`, `tick.hawkes.model`.

- **tick.linear_model**: Inference and simulation of linear models, including among others linear, logistic and Poisson regression, with a large set of penalization techniques and solvers.

- **tick.robust**: Tools for robust inference. It features tools for outliers detection and models such as Huber regression, among others robust losses.

- **tick.survival**: Inference and simulation for survival analysis, including Cox regression with several penalizations.

- **tick.prox**: Proximal operators for penalization of models weights. Such an operator can be used with (almost) any model and any solver.

- **tick.solver**: A module that provides a bunch of state-of-the-art optimization algorithms, including both batch and stochastic solvers.

- **tick.dataset**: Provides easy access to datasets used as benchmarks in tick.

- **tick.plot**: Some plotting utilities used in tick, such as plots for point processes and solver convergence.
Figure 6: Structure of tick package