Dileptons from decay of vector mesons hadronized from a quark-gluon plasma

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Abstract

We present a simple model for the dimuon production from the decay of vector mesons which are hadronized from a baryon-free quark-gluon plasma. Continuous decay of vector mesons in the medium is known to enhance the dimuon production and in our model we can estimate the total yield of dimuons decayed from vector mesons through the rate equations for the number of vector mesons of interest. Muon pair production critically depends on the time duration of both the mixed and hadronic phases. Results are discussed in view of possible measurements in experiments.
I. INTRODUCTION

Recently dilepton production from vector meson decays in relativistic heavy-ion collisions has been drawing strong interest [1]. One of the reasons is that the properties of vector mesons such as masses or widths may change in a hot and dense medium as a result of the onset of chiral symmetry restoration [2] and dileptons under vector meson channels may serve as good probes of such changes [3,4].

The other reason for the interest is due to the fact that vector mesons continuously decay in the hot and dense medium and thus they may contain valuable informations on the spacetime evolution of the collision zone. Heinz and Lee [5] suggested to measure the number of dileptons under rho and omega meson peaks from which one can estimate the lifetime of the purely hadronic matter. Seibert et al. suggested to measure the slope of the transverse mass spectrum of dileptons as a thermometer of the QCD transition [4].

Or combining the above two mechanisms there may appear double dilepton peaks for phi or rho mesons as a consequence of the expected quark-gluon plasma phase transition. However, in studying the dileptons one relies only on the idea and still important quantities such as relative magnitude of the two peaks are not estimable.

In this paper we extend the model by Heinz and Lee [5] for the dilepton production from the purely hadronic matter to the case of the evolution of quark-gluon plasma and study the total yield of dileptons from the decay of vector mesons. This model was initially used to simultaneously explain the $J/\Psi$ suppression and enhancement of $\phi/(\rho+\omega)$ measured by NA35 from a purely hadronic matter and modified to include the free decay to study the muon pair production [3]. We believe it is a simple model for the number of resonant dileptons which allows easy modification to incorporate the change of vector meson properties in the medium.

For the dilepton production decayed from vector mesons, $V$, we need to know the time evolution of the number of vector mesons, $V$, which can be obtained assuming a model both for the dynamic evolution such as hydrodynamics and for the hadronization mechanism. Hydrodynamics becomes simple for a baryon-free quark-gluon plasma whose physical quantities are invariant under longitudinal Lorentz boosts [7] and for the hadronization scenario we assume that hadronization occurs as a first order phase transition maintaining chemical equilibrium. These assumptions lead to a constant rate of hadronization. By assuming model equations of state one can evaluate the number of vector mesons as a function of time starting from an initial conditions reached during the collisions at a given beam energy.

However, continuous decays of vector meson $V$ may make deviations from chemical equilibrium and in this paper, we construct rate equations for the time evolution of vector mesons, $V$, by considering the gain term due creation of $V$ via secondary collisions and loss terms due to annihilation through secondary collisions and free decay of $V$ together with the hadronization rate in the mixed phase. One can solve the rate equations for the particle species of interest assuming that other particle densities are determined from the thermal and chemical equilibrium. In this way we loose knowledge on the time scales of the evolution and will present our results as a function of the time durations of each phases by controlling the degree of equilibration, namely the ratio of gain and loss terms.

In section 2, the equilibrium hadronization of a Lorentz-boost invariant quark-gluon plasma is discussed. Hadronization at constant temperature implies a constant rate of
hadronization during the mixed phase. In section 3, considering secondary collisions among hadrons and the free decay of certain particle species, rate equations for the change in particle numbers are constructed and solved in both the mixed and hadronic phases. Muon production decayed from the particle of interest are formally obtained. In section 4, the results of our numerical calculations are presented and we show that dilepton production from rho meson decays depends critically on the time length of both the mixed and hadronic phase. We summarize in section 5.

II. EQUILIBRIUM HADRONIZATION OF A BARYON-FREE QUARK-GLUON PLASMA

In this section we discuss the hadronization rates of particles under the assumption that thermal and chemical equilibrium is maintained throughout the phase conversion of a baryon-free quark-gluon plasma. This assumption considerably simplifies the calculations, but our final argument does not qualitatively depend on it. The results depend crucially, however, on our assumption that the phase transition is of first order. This issue is still under debate [1].

The most important property of a baryon-free quark-gluon plasma which is created in the central region during relativistic heavy-ion collisions is the invariance of the rapidity distribution under longitudinal Lorentz boosts. This property makes the hydrodynamic description very simple [7]:

$$\frac{d^4S}{dx^4} = 0 \quad \text{and} \quad \frac{d^4N}{dx^4} = 0.$$  \hspace{1cm} (1)

Assuming only longitudinal expansion (which initially dominates in any case [ref]) it is convenient to use the rapidity $y$ and the longitudinal proper time $\tau$ as variables. Then above hydrodynamic equations for the perfect fluid implies

$$s(\tau)\tau = s(\tau_0)\tau_0, \quad \rho(\tau)\tau = \rho(\tau_0)\tau_0.$$  \hspace{1cm} (2)

Here $\rho$ can be the number density of any particle species in the system, e.g. the quark density or the baryon density, etc.

In other words, the spatial volume increases linearly in $\tau$ as a consequence of hydrodynamic expansion:

$$V(\tau) = \frac{\tau}{\tau_0}V(\tau_0).$$  \hspace{1cm} (3)

We assume Eq. (2) holds throughout the entire evolution of a quark-gluon plasma, namely through the mixed and hadronic phases until freeze-out.

As the system cools below due to its hydrodynamic expansion the confinement phase transition occurs. We assume that this phase transition is of first order. Conversion of a quark-gluon plasma to hadronic matter then is usually described by the mixed phase which is characterized in terms of the volume fraction $\alpha$, $\alpha = V_H/V_{\text{tot}}$. Here $V_H$ is the volume of the hadronic subphase in the mixed phase of volume $V_{\text{tot}}$. Then any density, e.g. entropy density, in the mixed phase can be written as
\[ s(\tau) = \alpha(\tau)s_H + (1 - \alpha(\tau))s_Q, \]  

(4)

where \( s_H \) and \( s_Q \) are the entropy densities in the hadronic and plasma subphases, respectively.

If the phase transition maintains chemical equilibrium, entropy and particle densities can be described via thermal distributions. For a baryon free system, temperature is the only parameter for the thermal distributions and moreover, temperature stays constant at \( T_c \) during the phase transition. Thus entropy densities, \( s_H \) and \( s_Q \), are constant during the phase conversion. Solving Eq. (4) for \( \alpha(\tau) \) and using Eq. (2), we get the time dependence of \( \alpha(\tau) \):

\[ \alpha(\tau) = \frac{\tau_H - \tau_Q}{\tau_H - \tau} \frac{\tau - \tau_Q}{\tau}, \]  

(5)

where \( \tau_Q \) and \( \tau_H \) are the proper times at the beginning and end of hadronization.

Eq. (5) implies that as the total volume of the mixed phase increases linearly in \( \tau \) according to Eq. (3), the plasma subvolume decreases linearly while the hadronic subvolume increases linearly.

\[ V_Q = (1 - \alpha(\tau))V_{\text{tot}}(\tau) = \frac{\tau_H - \tau}{\tau_H - \tau_Q}V(\tau_Q) \]

\[ V_H = \alpha(\tau)V_{\text{tot}}(\tau) = \frac{\tau_H}{\tau_Q} \frac{\tau - \tau_Q}{\tau_H - \tau_Q}V(\tau_Q) \]  

(6)

Since the assumption of the chemical equilibrium during the phase conversion implies that the particle densities are kept constant, the number of particles hadronized should also increase linearly in \( \tau \). This can be seen as follows: Since \( d^4N/dx^4 \) is the particle density with respect to the 4-volume and \( dx^4 = \tau d\tau dy ds \), the density of particle species \( V \) can be rewritten as

\[ \rho_V(\tau) = \frac{dN_V}{\tau dy ds} = \alpha(\tau)\rho_V^H(T_c) \quad \text{for} \quad \tau_Q < \tau < \tau_H. \]  

(7)

One should note that number density in Eq. (4), \( \rho_V(\tau) \), is a density with respect to the total volume of the mixed phase, while \( \rho_V^H(T_c) \) is the one with respect only to the hadronic subvolume. Namely, \( \rho_V(\tau) = \alpha(\tau)\rho_V^H(T_c) \). From Eq. (7) we see that \( dN_V/\tau dy ds \) indeed grows linearly in \( \tau \). The corresponding rate equation for the hadronization of particle species \( V \) can be written as

\[ \frac{dN_V}{\tau d\tau dy ds} = \frac{1}{\tau} \frac{\tau_H - \tau}{\tau_H - \tau_Q}\rho_V^H(T_c) \equiv \frac{1}{\tau} a_V, \]  

(8)

with

\[ a_V = \frac{\tau_H}{\tau_H - \tau_Q} \rho_V^H(T_c), \]  

(9)

which is constant during the mixed phase since \( \rho_V^H(T_c) \) is kept fixed by its value for the chemical equilibrium at a constant temperature at \( T_C \). For a baryon-rich quark-gluon plasma,
dynamics will be different from Eq. (2) and also $\rho_H(T_c)$ may vary as $T_C$ changes due to the reheating.

Eq. (8) is a consequence of the gradual phase conversion but, for different hadronization scenarios, such as sudden hadronization of strange particles, the result might be qualitatively different.

In the mixed phase there will also be two-body collisions among the increasing number of hadrons and free-decays of those hadrons. In the next section we consider them explicitly to set up a rate equation and solve it to obtain a time dependence of the number densities of a vector meson $V$.

III. RATE EQUATIONS WITH SECONDARY COLLISIONS AND FREE-DECAY

In a system of many different particle species, collisions between particles create or annihilate certain particle species $V$. $V$ can decay freely also. Those processes change the number density of $V$ on top of the hydrodynamic evolution of the system. In this section we consider three types of processes which changes the number of vector meson $V$:

\begin{align*}
i + j & \rightarrow V + X, \quad l + V \rightarrow X, \quad V \rightarrow X.
\end{align*}

These processes represent production of vector meson $V$ through a collision between species $i$ and $j$, annihilation of $V$ via collisions with species $l$, and the free decay of $V$, respectively.

The rate equation for the number density of particle species $V$ per unit volume and time can be written as

\begin{equation}
\frac{dN_V}{d^4x} = \frac{a_V}{\tau} + \sum_{i,j} <\sigma v>_{ij}^{X,V} \rho_i(x)\rho_j(x) - \sum_{l} <\sigma v>_{l,V}^{X,V} \rho_l(x)\rho_V(x) - \Gamma_{tot}^{\tau} \rho_V(x),
\end{equation}

where the first term on the right hand side is the hadronization rate discussed in the previous section. The second and third terms are the gain and loss through the two body collisions and the last term is the loss due to the free decay as in Eq. (10). In Eq. (11) the scattering cross section $<\sigma v>_{ij}^{X,V}$ denotes the proper average over the momentum distribution of particles involved, and the density of particle species $i$, $\rho_i(x)$, depends on time through its rate equation. Due to the lack of detailed knowledge, however, we will solve Eq. (11) only for the vector mesons of interest and assume that the number densities for other particle species, which appear in the right hand side of Eq. (11), are determined from the chemical equilibrium.

Although Eq. (7) looks same in each phases except for the hadronization term, its $\tau$-dependence is different for the mixed and hadronic phase. Thus Eq. (11) should be solved separately in each phases, as discussed in the following subsections. It should also be noted that in the mixed phase $\rho_i(x)$ is the density of particle species $i$ with respect to the total volume of the mixed phase, as in Eq. (7).
A. Mixed phase

In the mixed phase, the particle densities with respect to the total volume can be written in terms of the densities relative to the hadronic volume as in Eq. (7). Let us define 
\[ \lambda_{V,M}^{P,M} \]
and 
\[ \lambda_{V,M}^{A,M} \]
as
\[ \lambda_{V,M}^{P,M} = \sum_{i,j} < \sigma v >_{i,j}^V \rho_i^V \rho_j^V, \lambda_{V,M}^{A,M} = \sum_l < \sigma v >_{l,V}^X \rho_l^H, \]  
(12)
where superscript \( M \) which denotes the "mixed phase" is used to distinguish similar quantities defined for the hadronic phase. In Eq. (12) all the densities are taken as those relative to the hadronic subvolume. For the hadronization maintaining chemical equilibrium, \( \rho_i^H = \rho_i^H(T_c) \) depends only on the transition temperature, and \( \lambda_{V,M}^{P,M} \) and \( \lambda_{V,M}^{A,M} \) depend only on the collision geometry, but not on the time.

Then, using the relation \( d^4x = \tau d\tau dy ds \), we can simplify the rate equation, Eq. (11) as
\[ \frac{dN_V}{dy ds} = a_V + \tau \alpha^2(\tau) \lambda_{V,M}^{P,M} - (\alpha(\tau) \lambda_{V,M}^{A,M} + \Gamma_{V}^{\text{tot}}) \frac{dN_V}{dy ds}, \]
(13)
Eq. (13) can be easily solved as
\[ \frac{dN_V}{dy ds}(\tau) = \tau c_{\alpha} \lambda_{V,M}^{A,M} \tau_Q e^{-(c_{\alpha} \lambda_{V,M}^{A,M} + \Gamma_{V}^{\text{tot}})\tau} \]
\[ \cdot \int_{\tau_Q}^{\tau} [a_V + c_{\alpha} \lambda_{V,M}^{P,M} \frac{(\tau - \tau_Q)^2}{\tau'}] |_{\tau'}^{\tau - c_{\alpha} \lambda_{V,M}^{A,M} \tau_Q e^{(c_{\alpha} \lambda_{V,M}^{A,M} + \Gamma_{V}^{\text{tot}})\tau'}} d\tau', \]
for \( \tau_Q < \tau < \tau_H \)  
(14)
where \( c_{\alpha} = \tau_H / (\tau_H - \tau_Q) \).

In order to get insight for this complicated looking solution and get numerical values, let us define a ratio \( R_{V}^0 \) between the gain and loss terms of \( V \) due to secondary collisions and free-decay as
\[ R_{V}^0 = \frac{\tau \alpha^2(\tau) \lambda_{V,M}^{P,M}}{(\alpha(\tau) \lambda_{V,M}^{A,M} + \Gamma_{V}^{\text{tot}}) \tau \rho_V(\tau)} = \frac{\lambda_{V,M}^{P,M} \tau_H \frac{\tau - \tau_Q}{\tau_H - \tau_Q}}{\lambda_{V,M}^{A,M} \tau \rho_V(\tau)} \frac{1}{\Gamma_{V}^{\text{tot}}} \frac{1}{\alpha \lambda_{V,M}^{A,M}}. \]
(15)
\( R_{V}^0(\tau) \) is a \( \tau \)-dependent parameter, which controls the degree of balance between the gain and loss terms. For \( R_{V}^0(\tau) = 1 \), we expect that \( dN/dy ds \) should linearly increase as a function of \( \tau \). In our numerical estimation for the Eq. (14) and thus for the number of dimuons, we will approximate \( R_{V}^0(\tau) \) as its value at \( \tau = \tau_H \).
\[ R_{V}^0(\tau_H) = \frac{\lambda_{V,M}^{P,M} \tau_H - \tau_Q}{\lambda_{V,M}^{A,M} \tau_H a_V} \frac{1}{\Gamma_{V}^{\text{tot}}} \frac{1}{\alpha \lambda_{V,M}^{A,M}}. \]
(16)
This approximation is mainly due to the lack of the detailed knowledge on the parameters involved, namely \( \lambda_{V,M}^{P,M} \) and \( \lambda_{V,M}^{A,M} \). However, it allows our estimation of the number of vector
mesons $V$ and the corresponding dimuons produced in the mixed phase from the analysis of the purely hadronic experimental data. This point will be discussed later.

Writing $R_V = R^0_V (1 + \frac{\Gamma^{tot}_V}{\lambda_V})$, we eliminate $\lambda^{P,M}_V$ in terms of $R_V$ in Eq. (14).

\[
\frac{dN_V}{dyds}(\tau) = a_V \tau^c a^{-A,M}_{\lambda} \tau^Q \exp\left(-c_{a} a^{A,M}_V + \Gamma^{tot}_V\right) \tau \\
\cdot \int_{\tau Q}^{\tau} \left[1 + R_V \lambda^{A,M}_V \frac{(\tau' - \tau Q)^2}{\tau'}\right]^{\tau - c_{a} a^{A,M}_V \tau Q} \exp\left(c_{a} a^{A,M}_V + \Gamma^{tot}_V\right) \tau' d\tau' 
\]

(17)

B. Hadronic phase

Worked out for the mixed phase, time evolution of particle species $V$ in the hadronic phase can be obtained from Eq. (11) omitting the term for the hadronization, i.e. the first term in the right hand side. However, one should note that in the hadronic phase particle density is governed only from the expansion of the system and inversely proportional to $\tau$ as in Eq. (2), while in the mixed phase linear increase in the hadronic subvolume during the phase conversion and constant rate of hadronization makes the particle density in the hadronic subvolume constant.

From the fact that

\[
\frac{dN_V}{dyds} = \tau \rho_V(x) = \text{const.} 
\]

in the hadronic phase and with the definition of $\lambda^H$’s for the hadronic phase as

\[
\lambda^{P,H}_V = \sum_{i,j} < \sigma v >^{V,X}_{i,j} (dN/dyds)_i (dN/dyds)_j \\
\lambda^{A,H}_V = \sum_{l} < \sigma v >^{X}_{l,V} (dN/dyds)_l, 
\]

we obtain the rate equation for the particle density of $V$ in the hadronic phase [5] as

\[
\frac{d}{d\tau} \left(\frac{dN_V}{dyds}\right) = \frac{1}{\tau} \lambda^{P,H}_V - (\frac{1}{\tau} \lambda^{A,H}_V + \Gamma^{tot}_V) \frac{dN_V}{dyds}, 
\]

(20)

Solution of Eq. (20) is

\[
\frac{dN_V(\tau)}{dyds} = \frac{dN_V(\tau_H)}{dyds} \exp\left[-\Lambda_V(\tau)\right] + \lambda^{P,H}_V \exp\left[-\Lambda_V(\tau)\right] \int_{\tau_H}^{\tau} \frac{d\tau'}{\tau'} \exp\left[\Lambda_V(\tau')\right] \\
\text{for} \quad \tau_H < \tau < \tau_f, 
\]

(21)

where $\Lambda_V$ is defined as

\[
\Lambda_V(\tau) = \lambda^{A,H}_V \ln \frac{\tau}{\tau_H} + \Gamma^{tot}_V (\tau - \tau_H). 
\]

(22)
Similarly for the mixed phase (Eq. (13)), we define in the hadronic phase the ratio \( R^0_V \) between gain and loss terms for particle species \( V \) due to rescattering and free-decay as
\[
R^0_V = \frac{\lambda^P_H}{(\lambda^A_H + \tau \Gamma^{tot}_V)(dN_V/dyds)}.
\]

Loss due to free decay of particle \( V \) is included in Eq. (23), which is omitted in the definition of \( R^0_V \) in Ref. [5]. The free decay term is not negligible compared to other terms and inclusion of this term is consistent when we consider the effect of free decay terms.

We will make approximation in the final calculation by taking the value for \( R^0_V \) at \( \tau = \tau_H \) in Eq. (23). Then from that fact that \( \lambda^{A,M}_V = \lambda^A_H / \tau_H \) at \( \tau = \tau_H \), it is easy to check that \( R^0_V \) defined in Eq. (15) and Eq. (23) are single valued at \( \tau = \tau_H \).

Defining \( R_V = R^0_V(\tau_H)(1 + \frac{\tau_H \Gamma^{tot}_V}{\lambda^A_H}) \), as in the mixed phase, Eq. (21) becomes
\[
\frac{dN_V(\tau)}{d\tau} = \frac{dN_V(\tau_H)}{d\tau} \left[ \exp[-\Lambda_V(\tau)] + R_V \lambda^A_H \exp[-\Lambda_V(\tau)] \int_{\tau_H}^{\tau} \frac{d\tau'}{\tau'} \exp[\Lambda_V(\tau')] \right]
\]

After breakup of the system, particles decay freely and we have the exponential decay law
\[
\frac{dN_V}{d\tau} = \frac{dN_V(\tau_f)}{d\tau} \exp[-\Gamma^{tot}_V(\tau - \tau_f)] \quad \text{for} \quad \tau_f < \tau.
\]

C. Production of muon pairs

Having the time dependence of the abundance of vector meson \( V \) in all phases we can calculate the dilepton production as
\[
\frac{dN^{l^+l^-}_V}{d\tau} = \Gamma^{l^+l^-}_V \int d\tau \frac{dN_V}{d\tau}(\tau)
\]
\[
= \Gamma^{l^+l^-}_V \left[ \int_{\tau_Q}^{\tau_H} d\tau \frac{dN_V}{d\tau}(\tau) + \int_{\tau_H}^{\tau_f} d\tau \frac{dN_V}{d\tau}(\tau) + \frac{1}{\Gamma^{tot}_V} \frac{dN_V}{d\tau}(\tau_f) \right],
\]
where \( \Gamma^{l^+l^-}_V \) is the partial decay width for the the \( l^+l^- \) dilepton channel and \( \Gamma^{tot}_V \) is the total decay width of \( V \). The first and second terms in Eq. (27) are production in the mixed and hadronic phases and the last term is the number of muon pairs decayed after the freeze-out. It should be emphasized that in p+p collisions or when thermalized dense system is not formed during the heavy-ion collisions, we have only the last term in Eq. (27) for the production of muon pairs.

Importance of the first two terms can be visualized from an idealized case when the gain and loss terms balance each other perfectly, namely for \( R^0_V(\tau) = 1 \). In this case the number of particle \( V \) in the mixed phase is a linearly increasing function of \( \tau \) and that in the hadronic phase is a constant:
\[
\frac{dN_V}{dyds}(\tau) = \frac{\tau - \tau_Q}{\tau_H - \tau_Q} \frac{dN_V}{dyds}(\tau_H) \quad \text{for} \quad \tau_Q < \tau < \tau_H
\]
\[
\frac{dN_V}{dyds}(\tau_H) = \frac{dN_V}{dyds}(\tau_H) \quad \text{for} \quad \tau_H < \tau < \tau_f.
\] (28)

Thus we have for the number of muon pairs
\[
\frac{dN_{V^{l^-}l^+}}{dyds} = B_V \frac{dN_V}{dyds}(\tau_H) \left[ \frac{\Gamma_{V^t}^{\text{tot}}}{2} (\frac{1}{2} \Delta \tau_M + \Delta \tau_H) + 1 \right],
\] (29)

where \( B_V = \frac{\Gamma_{V^t}^{l^-}l^+}{\Gamma_{V^t}^{\text{tot}}} \) is the branching ratio of \( V \) in vacuum for the \( l^-l^+ \) dilepton channel and \( \Delta \tau_M = (\tau_H - \tau_Q) \) and \( \Delta \tau_H = (\tau_f - \tau_H) \) are the time durations of mixed and hadronic phases, respectively.

The first two terms in Eq. (29) is the muon production in the mixed and hadronic phases, respectively, and the last term is those decayed after freeze-out. For the ideal case of perfect balance between gain and loss terms the production of muon pairs in the mixed and hadronic phases relative to those decayed after freeze-out can be simply estimated by comparing the magnitude of \( \Gamma_{V^t}^{\text{tot}}(\frac{1}{2} \Delta \tau_M + \Delta \tau_H) \) with 1. Please note the factor 1/2 in the contribution from the mixed phase which is due to the average of the linearly increasing number of vector mesons \( V \).

Having large total decay width, \( \Gamma_{\rho}^{\text{tot}} = 0.77 \text{ fm}^{-1} \), \( \rho \) mesons produce more dileptons in the medium than after freeze-out for relatively small lifetimes of a few fermi, while for other particles, e.g. for \( \omega \) mesons whose total decay width is \( \Gamma_{\omega}^{\text{tot}} = 0.043 \text{ fm}^{-1} \) the contributions from the mixed and hadronic phases becomes significant only when \( (\frac{1}{2} \Delta \tau_M + \Delta \tau_H) \geq 20 \text{ fm} \).

For a purely hadronic system one can estimate the lifetime of the hot matter, \( \Delta \tau_H \), if the number of vectors mesons \( V \) at freeze-out is known and the number of dileptons from them are measured. Or if we take the ratio of muon pair production from \( \rho \) and \( \omega \), for which we expect to be 1 as in the \( p + p \) collisions due to the small mass difference and the same quark contents, we can get valuable information of \( \Delta \tau_H \), as suggested in Ref. [5].

Even though the dependence of the muon production on the lifetime of the mixed phase is weaker by half than that in the hadronic phase, muon pairs decayed from the \( \rho \) mesons in the mixed phase may be the dominant one when \( \Delta \tau_Q \) is much larger than \( \Delta \tau_H \). This could be the case when the hadronization transition is of first order. During the first order phase transition the mixed phase is expected to be long because large amount of entropy carried by gluons should convert to the entropy of hadrons. In this case we expect large enhancement of dimuons under the rho peak.

If the mass shift of vector mesons due to restoration of symmetry occurs, those dimuons produced in the medium will appear at different invariant mass as suggested in ref. [4,3]. However, what we deal in this paper is the total number of muon pairs regardless of their invariant mass and is not affected from the possible mass shift.

**IV. RESULTS**

Eq. (29) is a result for the perfect balance between the gain and loss terms, i.e. \( R_V^0(\tau) = 1 \) for any \( \tau \). For the evaluation of Eqs. (17), (24), and (28) in general, values for the quantities \( \lambda_{V^t}^{A,M} \), \( \lambda_{V^t}^{A,H} \) and \( R_V(\tau) \) should be determined together with the time scales, \( \tau_Q \), \( \tau_H \), and \( \tau_f \).
Following the same argument by Ref. [3], one can estimate \( \lambda_{V}^{A,H} \) for rho, omega and phi mesons from the data for \( J/\Psi \) and \( \phi/(\rho + \omega) \) ratio, both measured in 200 A GeV O+U and S+U collisions by NA38. Underlying assumption of the argument is that both the suppression of \( J/\Psi \) and enhancement of \( \phi/(\rho + \omega) \) ratio can be simultaneously described by the secondary collisions among hadrons through the rate equations, Eq. (24) for a purely hadronic matter.

When we rewrite \( \lambda_{V}^{A,H} \) as

\[
\lambda_{V}^{A,H} = \gamma_{V} \lambda_{J/\Psi}^{A,H},
\]

where

\[
\gamma_{V} = \frac{\sum <\sigma v>_{V}^{X} (dN_{i}/dyds)}{\sum <\sigma v>_{i,J/\Psi}^{X} (dN_{i}/dyds)},
\]

\( \gamma_{V} \) involves only ratios of suitably weighted absorption cross sections and it does not depend on the details of the nuclear collision dynamics. From the analysis of NA38 data we have \( \lambda_{J/\Psi}^{A,H} = 0.9 \) and \( \gamma_{V} \) can be approximated as the ratio of cross sections involved, \( \gamma_{\rho} = \gamma_{\omega} \sim 12 \). Thus we get \( \lambda_{V}^{A,H} = \lambda_{\omega}^{A,H} \sim 11 \). One can also estimate \( \lambda_{V}^{A,M} \) from the relation \( \lambda_{V}^{A,M} = \lambda_{V}^{A,H}/\tau_{H} \), as both quantities are approximately independent of \( \tau \).

As we have defined the quantity \( R_{V}(\tau) \) in the previous section we can eliminate \( \lambda_{V}^{P,M} \), and \( \lambda_{V}^{P,H} \) in terms of \( R_{V}(\tau) \). We further approximate \( R_{V}^{0}(\tau) \), which is a function of \( \tau \), with its value at at \( \tau = \tau_{H} \), i.e. \( R_{V}^{0}(\tau) \sim R_{V}^{0}(\tau_{H}) \) and take it as a parameter which controls the degree of equilibration. Namely for \( R_{V}^{0} > 1 \) creation due to secondary collisions wins over the annihilation by the secondary collisions and free decay, and for \( R_{V}^{0} < 1 \), vice versa.

Time scales involved, \( \tau_{Q} \), \( \tau_{H} \) and \( \tau_{f} \), should be determined from the hydrodynamic equations together with a model equations of state for each phase. Instead of making models we rather satisfy ourselves using those time scales as variables.

It is convenient to define a ratio

\[
Y_{V} = \frac{dN_{V}(\tau)/dyds}{dN_{V}(\tau_{H})/dyds}.
\]

In Fig. 1 we have plotted \( Y_{V} \) for rho and omega mesons with arbitrarily chosen values, \( \tau_{Q} = 5 \) fm, \( \Delta \tau_{M} = (\tau_{H} - \tau_{Q}) = 20 \) fm, \( \Delta \tau_{H} = (\tau_{f} - \tau_{H}) = 35 \) fm. Solid lines are the results of the approximation \( R_{V}^{0}(\tau) \sim R_{V}^{0}(\tau_{H}) = 1 \) while the dotted lines are for the ideal case, i.e. Eq. (29). The approximation \( R_{V}^{0}(\tau) \sim R_{V}^{0}(\tau_{H}) = 1 \) underestimates the loss due to free decay and one should note the importance of this contribution especially for the rho mesons.

The unknown overall factor \( [dN_{V}(\tau_{H})/dyds] \) can be removed by taking a ratio of muon production from \( \rho \) to \( \omega \) mesons. In \( p + p \) collisions \( \rho \) and \( \omega \) mesons are produced in equal numbers, which can be understood from the small mass difference and same quark contents. Except for the medium effect on the number of the two vector mesons, it seems natural to approximate that \( [dN_{V}(\tau_{H})/dyds] \) for \( \rho \) and \( \omega \) are same. Then we get from Eq. (29) for the perfect balance

\[
X = \frac{dN_{\rho}^{int\tau-}/dyds}{dN_{\omega}^{int\tau-}/dyds} = \frac{B_{\rho}}{B_{\omega}} \left[ \frac{\Gamma_{\rho}^{tot}(\frac{1}{2}\Delta \tau_{M} + \Delta \tau_{H}) + 1}{\Gamma_{\omega}^{tot}(\frac{1}{2}\Delta \tau_{M} + \Delta \tau_{H}) + 1} \right]
\]

(33)
with $B_\rho/B_\omega = 0.65$.

In Fig. 2 we show calculated $X$ as a function of the lifetime of the mixed phase, $\Delta \tau_M = (\tau_H - \tau_Q)$ for two fixed values of $\Delta \tau_H = (\tau_f - \tau_H)$.

When $\Delta \tau_M/2 + \Delta \tau_H \ll 20 \text{ fm}$, we can approximate $X$ as

$$X \sim \frac{B_\rho}{B_\omega} \left[ \Gamma_{\rho}^{\text{tot}} \left( \frac{1}{2} \Delta \tau_M + \Delta \tau_H \right) + 1 \right],$$  

(34)

which is a linear function of $\Delta \tau_M/2 + \Delta \tau_H$.

Taking $\Delta \tau_M = 0$ in Eqs. (33) we get the ratio, $X$ for a purely hadronic system.

$$X = \frac{B_\rho}{B_\omega} \left[ \Gamma_{\rho}^{\text{tot}} \Delta \tau_H + 1 \right] \sim 0.65 (\Gamma_{\rho}^{\text{tot}} \Delta \tau_H + 1).$$

(35)

According to Eq. (35) measurement of $X$ larger than 0.65 implies the formation of a hot and dense medium in thermal equilibrium. If the hadronization of a quark-gluon plasma occurs as a first order phase transition, value of $X$ much larger than 0.65 is expected.

For a purely hadronic system we get from Eq. (35) $\Delta \tau_H$ by measuring the ratio of dileptons from $\rho$ to $\omega$ mesons. Thus careful analysis of muon production in the $\rho$ and $\omega$ channels may reveal valuable informations even on the formation of the hadronic fireball as suggested by Heinz and Lee [5]. In Ref. [5] the dependence of $X$ on the total transverse energy, $E_T$, is investigated. In this paper, however, we merely note the fact that as the beam energy increases, the initial temperature of the hadronic fireball increases and so does $\Delta \tau_H$.

For the beam energy just above that needed to make a deconfinement transition, a state of mixed phase with small $\Delta \tau_M$ will be formed and as the beam energy increases, so rapidly does $\Delta \tau_M$ with more or less saturated $\Delta \tau_H$. The dependence of $(\Delta \tau_M/2 + \Delta \tau_H)$ on the beam energy will be different from that of $\Delta \tau_H$ at lower energies.

For high enough beam energies a quark-gluon plasma will be made and $\Delta \tau_M$ and $\Delta \tau_H$ should be functions of the initial temperature of the QGP. Thus as the beam energy increases there will be two characteristic changes in the behavior of $X$ at the energies of barely forming a state of mixed phase and a quark-gluon plasma.

As in the case of the average hadronic $p_T$ spectra as a function of the beam energy, characteristic changes in slopes of $X$ as a function of the beam energy can serve as a signature of QGP. Further studies are needed to investigate how drastic will be the change.

Under the debate is the change of mass of vector mesons in the hot and dense medium due to the restoration of chiral symmetry, as discussed in the last section. If the mass of vector mesons decreases due to chiral restoration in the hot and dense medium, dileptons decayed during the mixed phase will peak at smaller invariant mass.

In this case we may have in the invariant mass spectrum a new peak from the mixed phase at the mass corresponding to the hadronization transition, $m_V(T_C, \rho_B)$ together with a peak due to the free decay after freeze-out. Dileptons decayed in the hadronic phase whose temperature and baryon density change continuously will spread out in the mass region between $m_V(T_C, \rho_B)$ and $m_V$ in the free space and can hardly be distinguished from the background. However, it should be stressed that even when the mass of vector mesons $V$ changes in the hot and dense medium, our results so far are valid as we have considered the
total number of muons pairs decayed from vector mesons $V$ in each stages of the system but not the invariant mass spectrum.

There may also a change of the decay width of vector mesons in the hot and dense medium [8–10]. In this case we need to put $\tau$- dependent $\Gamma_{V}^{\text{tot}}(\tau)$ in every equations and the relation between $\tau$ and the temperature and baryon density should be determined from the dynamics of the fireball.

V. SUMMARY

We explore interesting aspects of muon pair production in the relativistic heavy-ion collisions and have shown that careful measurement of muon pairs under $\rho$ and $\omega$ peaks may reveal valuable informations on the formation of quark-gluon plasma.

Especially, the ratio of muon pairs from $\rho$ to $\omega$ mesons, $X$, carries the information on the lifetime of a fireball either in the hadronic or quark-gluon plasma phase. Characteristic changes in $X$ as a function of increasing beam energies should serve as a signature of formation of a quark-gluon plasma.

Analysis of muon pair production decayed from vector mesons is made by solving rate equations which is constructed by considering a hadronization rate, creation and annihilation of particles via secondary collisions, and free-decays. For the hadronization rate of vector mesons of interest we consider a baryon-free quark-gluon plasma which is invariant under Lorentz boost, and assume equilibrium hadronization at a constant temperature. This implies a constant rate of hadronization during the mixed phase.

However, inside a hot medium broadening of mass due to frequent collisions will spread the muon peaks under rho mesons and this broadening of rho peak in the dimuon spectrum may offset the increase of the rho peak to make to measurement of the increase hard [8–10]. Studies of the collisional broadening are needed for any realistic estimation of the enhancement of dimuons under the rho meson peak from the medium.

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FIGURES

Fig.1: Number of vector mesons per unit rapidity and transverse area as a function of $\tau$. Solid lines are the results of the approximation $R^0_V(\tau) \sim R^0_V(\tau_H) = 1$ while the dotted lines are for the ideal case, i.e. Eq. (29).

Fig.2: Ratio of muon production from $\rho$ to $\omega$ mesons as a function of lifetime of the mixed phase. Here time length of hadronic phase is fixed as 5 fm.
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