Quintessence Reissner Nordström Anti de Sitter Black Holes and Joule Thomson effect

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Abstract

In this work we investigate corrections of the quintessence regime of the dark energy on the Joule-Thomson (JT) effect of the Reissner Nordström anti de Sitter (RNAdS) black hole. The quintessence dark energy has equation of state as \( p_q = \omega \rho_q \) in which \(-1 < \omega < -\frac{1}{3}\). Our calculations are restricted to ansatz: \( \omega = -1 \) (the cosmological constant regime) and \( \omega = -\frac{2}{3} \) (quintessence dark energy). To study the JT expansion of the AdS gas under the constant black hole mass, we calculate inversion temperature \( T_i \) of the quintessence RNAdS black hole where its cooling phase is changed to heating phase at a particular (inverse) pressure \( P_i \). Position of the inverse point \( \{T_i, P_i\} \) is determined by crossing the inverse curves with the corresponding Gibbons-Hawking temperature on the T-P plan. We determine position of the inverse point verse different numerical values of the mass \( M \) and the charge \( Q \) of the quintessence AdS RN black hole. The cooling-heating phase transition (JT effect) is happened for \( M > Q \) in which the causal singularity is still covered by the horizon. Our calculations show sensitivity of the inverse point \( \{T_i, P_i\} \) position on the T-P plan to existence of the quintessence dark energy just for large numerical values of the AdS RN black holes charge \( Q \). In other words the quintessence dark energy does not affects on position of the inverse point when the AdS RN black hole takes on small charges.

1 Introduction

Astronomical observation shows the accelerating expansion of the Universe [1-3]. The origin of the acceleration comes from negative pressure which can be caused by two different factors: (a) the cosmological constant, (b) the quintessence hypothetical form of the dark energy. The latter cosmic source

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has an equation of state as $p_q = \omega \rho_q$ in which the subscript $q$ denotes to the word ‘quintessence’ and $\omega$ is barotropic index such that $-1 < \omega < -\frac{1}{3}$ [4-7]. Origin of the quintessence can be made from dynamical scalar fields [8,9]. The border state of the quintessence namely $\omega = -1$ covers the cosmological constant regime. The cosmological constant affects on horizon of the black holes. For instance its effect on spherically symmetric static metric is well known as Schwarzschild-de Sitter black hole [10]. While the acceleration effects of the Universe caused by the quintessence is studied in [11,12]. Many authors are studied the effects of the quintessence dark energy on the black holes. For instance, Chen et al are studied effects of the quintessence dark energy on the Hawking radiation and quasi normal modes of black holes [13,14]. According to the original work [15] we know that in the extended phase space, the cosmological constant treats as thermodynamic pressure $P = -\frac{\Lambda}{8\pi}$ and its conjugate quantity acts as thermodynamic volume $V = (\frac{\partial M}{\partial P})_{S,Q}$ where $S, Q, M$ are the entropy, the charge and the mass of an AdS black hole respectively. AdS background is a vacuum de Sitter space time where the cosmological constant has negative values. See also [16] where Kubiznak and Mann are studied the phase structure of the quintessence RNAdS black hole in the extended phase space. In general, thermodynamic volume is different with a geometrical volume of the black holes but for RN type they become similar [17]. Black hole mass $M$ treats also as enthalpy of the AdS black hole in the extended phase space [18-21]. Readers can be follow relationship between the extended phase space with holographic heat engines perspective for charged AdS black holes in ref. [22]. Charged AdS black holes treat as liquid-gas system. For instance a RNAdS black hole behaves as the Van der Waals liquid gas with first order phase transition [23, 24]. Cai et al are studied P-V criticality in the extended phase space of Gauss-Bonnet black holes in AdS space time [25]. They obtained a P-V criticality and the large-small black hole phase transition just for a 5 dimensional black hole with spherical horizon. There is not obtained the P-V criticality for Ricci flat and hyperbolic Gauss Bonnet black holes in 5 dimension. Heat engine of a charged AdS black hole related to a small-large black hole phase transition is also studied in ref. [26]. See also [27] where the influence of the quintessence dark energy is studied on the holographic thermalization of the gravitational collapse. Superconductor formation of a quintessence RNAdS black hole is studied also in ref. [28] by using the holographic framework. Two point correlation functions which are made from bilinear quantum matter field operators, are
well known as non-local observable. Their thermodynamical properties are obtained to be similar to the entanglement entropy [29-34] in which the authors showed both of them take on similar non-equilibrium thermalization behavior. Authors of the works [35-42] showed that the entanglement entropy and two point functions behave as the similar superconductor phase transition and in refs. [43,44] is shown that they have similar cosmological singularity. Authors of [45] are studied phase structure of the quintessence RNAdS black hole by the nonlocal observable such as holographic entanglement entropy and two point correlation function. They obtained Van der Waals-like phase transition. In short, we can use entanglement entropy approach as a good probe to study the aspects of holographic superconductors and investigate phase transitions more deeply. Existence of the small-large AdS black hole phase transition comes from Hawking-page phase transition [46] where a large AdS black hole is stable while a small AdS black hole is unstable because of quantum matter interactions. This phenomena leads us to study cooling-heating regime of an AdS black hole. Instability of small AdS black holes reach to a stable gas in AdS background finally. Usually the gas exhibits with a Joule-Thompson (JT) expansion for which gas at a high pressure reaches to a low pressure under the constant enthalpy (the black hole mass). Ökcü and Aydiner, studied the JT expansion for AdS charged black hole and obtained some similarities and differences with Van der Waals fluid [47] and they extended their work to a Kerr-AdS black hole in ref. [48]. In general the JT expansion of a gas is happened at constant enthalpy which for a black hole is its mass. JT expansion [49] is an actual iso-enthalpic thermodynamical experiment in which a thermal system exhibits with a thermal expansion via molecular interaction. The JT coefficient $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H$ is a suitable parameter which determines cooling or heating phase of the system. In a gas expansion with temperature $T$, the pressure decreases, so the sign of $\partial P$ is negative by definition. Hence we can consider two different state by defining inverse temperature $T_i$ which is computed from the equation $\mu_{JT}(T_i) = 0$: If $T < T_i (T > T_i)$ for which $\partial P < 0$ by definition, then the JT effect cools (warms) the gas with $\partial T < 0(> 0)$ and so $\mu_{JT} > 0(< 0)$. When the gas takes on the inverse temperature $T_i$ it will be have an inverse pressure which we call here $P_i$. Cooling-heating process is happened at the inverse point $(T_i, P_i)$ (see figures 1,2,3) on the T-P phase space for a gas expansion because of molecular interaction with a constant enthalpy. For instance we have $\mu_{JT} = 0$ for ideal gas because of no interaction between the molecules [49]. In the present work we study quintessence dark energy
effects on the JT expansion of a RNAdS black hole under the conditions of the constant mass and the constant charge. In short one can infer by looking to diagrams of the figures 1,2,3 that output of our work is: (a) location of the inverse point \((T_i, P_i)\) is obtained by crossing diagrams of the inverse curve \(T_i(P_i)\) and diagrams of the state equation \(T(P)\). Its position is located where the temperature of the AdS black hole gas takes on its maximum value. We should notice that the inverse point is a critical point on the T-P phase space where the quintessence RNAdS black hole heating phase separates with its cooling phase. (b) The JT expansion is happened at high (low) pressure-temperature for small (large) fixed charge and mass parameter in absence and presence of the quintessence dark energy effect. (c) The quintessence dark energy effects become negligible on the JT expansion of the AdS gas for small numerical values of the black hole charge but should be considered more for large numerical values of the charge.

Organization of the work is as follows. In Section 2 we introduce RNAdS black hole metric surrounded with quintessence dark energy by according to the work presented by Kiselev [50] (see also [45]). In section 3 we calculate the JT coefficient of the quintessence RNAdS black hole and determine its cooling and heating phase. Section 3 denotes to conclusion and outlook.

## 2 Quintessence RNAdS Black Holes

Applying additivity and linearity conditions on the quintessence dark energy stress tensor one can infer [50]

\[
T^i_t = T^r_r = \rho_q, \tag{2.1}
\]

\[
T^0_\theta = T^\phi_\phi = -\frac{1}{2} \rho_q (3\omega + 1) \tag{2.2}
\]

where \(\rho_q(r) = -\frac{a}{2} \frac{2\omega}{1 + \omega}\). \(\omega\) is state parameter and \(a\) is the normalization factor related to the density of quintessence. For quintessence (phantom) regime of the dark energy the barotropic index will have \(-1 < \omega < -\frac{1}{3}\) \((\omega < -1)\) and so positivity condition of the energy density \(\rho_q\) is satisfied by choosing \(a > 0\)(\(a < 0\)). Applying (2.1) and (2.2) and units \(4\pi G = 1\), Kiselev solved the Einstein-Maxwell metric equation \(G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \{T_{\mu\nu}^{(\text{maxwell})} + T_{\mu\nu}^{(\text{quintessence})}\}\) in the presence of the cosmological constant \(\Lambda\). He obtained series solutions for metric of the spherically symmetric static black hole surrounded by the quintessence dark energy (see Eq. 18 in Ref. [50]). Leading
order part of his series solution, reduces to the quintessence RNAdS black hole metric which is given by (see Eq. 21 in Ref. [50].)

\[ ds^2 \approx -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \] (2.3)

with

\[ f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2} - \frac{a}{r^{3\omega+1}} \] (2.4)

where \( M \) is the black hole mass. \( l \) is radius of the AdS space time which is related to the cosmological constant with negative value which for ‘n’ dimensional AdS space time become \( \Lambda = -\frac{(n-1)(n-2)}{2l^2} < 0 \) [27,51]. One can obtain modified mass \( \tilde{M} = M + \frac{a}{2} \) for ansatz \( \omega = 0 \), modified charge \( \tilde{Q}^2 = Q^2 - a \) for ansatz \( \omega = \frac{1}{3} \) and modified AdS radius \( \frac{1}{\tilde{l}^2} = \frac{1}{l^2} - \frac{1}{a} \) for ansatz \( \omega = -1 \). In general, one can infer that the free quintessence generates the horizon of the black hole if \( 0 < 3\omega + 1 < 1 \) for which

\[ -\frac{1}{3} < \omega < 0 \] (2.5)

and generates the AdS radius \( l \) if \( -2 < 3\omega + 1 < -1 \) where

\[ -1 < \omega < -\frac{2}{3} \] (2.6)

Kiselev is also calculated affects of the quintessence dark energy on the Gibbons-Hawking temperature of the RNAdS black hole in ref. [50] where the quintessence decreases temperature of the RNAdS black hole which vanishes for \( a = (8M)^{3\omega+1} \). Phase structure of the above black hole solution is investigated by applying the nonlocal observable in ref. [45]. The authors obtained a Van der Waals phase transition for the black hole metric (2.4) in the holographic framework. They check the equal area law for the first order phase transition and critical exponent of the heat capacity for the second order phase transition against different values of \( \omega \). They also discuss the effect of the barotropic parameter \( \omega \) on the phase structure of the nonlocal observables. Now we study JT expansion of the quintessence RNAdS black hole as follows.

### 3 Joule Thompson effect

As we said in the introduction section the Hawking and the Page showed in ref. [46] that there is a critical temperature where a large stable black
hole in AdS space time reaches to thermal gas in the AdS background. This predicts a large-small black hole phase transition. According to this work some authors took action to study thermodynamic properties of AdS black holes due to the AdS/CFT correspondence [52,53,54] in which the black holes thermodynamics is identified with the dual strongly coupled CFT in the boundary of the bulk AdS space time. Thermodynamic property of an AdS black hole is quite different from those of the asymptotically flat or de Sitter space time. Large AdS black holes are thermodynamically stable while small ones are unstable. Instability of the small AdS black holes exhibit the Joule Thomson expansion of the AdS gas under the condition of constant enthalpy. The JT expansion of the gas has an important coefficient $\mu_{JT}$ which determines cooling and/or heating phase of iso-enthalpic expansion of the gas. In general, the JT coefficient is called by definition as [49]

$$\mu_{JT} = \left( \frac{\partial T_b}{\partial P} \right)_H = \frac{V}{C_P} \left( \frac{T_b}{T_i^b} - 1 \right)$$

(3.1)

in which $C_P = \left( \frac{\partial H}{\partial T_b} \right)_P$ is heat capacity at constant pressure and $T_i^b = V \left( \frac{\partial T}{\partial V} \right)_P$ is inversion temperature for which $V$ is thermodynamic volume of the gas (see introduction). One can infer $\mu_{JT}(T_i^b) = 0$ where there is no molecular interaction of the gas. Applying the thermodynamic equation $H = PV + U$ one can rewrite the equation (3.1) as

$$\mu_{JT} = \frac{T_b - T_i^b}{P + \left( \frac{\partial U}{\partial V} \right)_P}$$

(3.2)

for which

$$C_P = \left( \frac{\partial M}{\partial T_b} \right)_P = \frac{V}{T_b} \left[ P + \left( \frac{\partial U}{\partial V} \right)_P \right].$$

(3.3)

As we said in the introduction section the above JT equation can be applicable for an AdS black hole because of extension of the phase space. In the latter case the negative cosmological constant related to the AdS radius $l$ treats as thermodynamical pressure and its conjugate acts as thermodynamic volume and the black hole mass behaves as the enthalpy. According to the latter statements the pressure $P$ given in the above equations should correspond to the AdS radius of the 4 dimensional space time (2.3) as follows [17,19,55].

$$P = \frac{3}{8\pi l^2}.$$  

(3.4)
The above identity shows that the AdS radius \( l \) plays as a thermal variable. Location of exterior horizon \( r_+ \) of the black hole metric (2.3) is determined by choosing the largest root of the equation \( f(r_+) = 0 \) given by (2.4). To do so we can use (3.4) and write

\[
\frac{M}{2} = r_+ - \frac{a}{2r_+^3} + \frac{4\pi P}{3} r_+^3 + \frac{Q^2}{2r_+}
\]  

(3.5)

which by according to the work [19] means the AdS black hole enthalpy. In the thermodynamic perspective the equation (3.5) reads

\[
M = H = PV + U
\]

(3.6)

where \( V = \left(\frac{\partial M}{\partial P}\right)_{S,Q} = \frac{4\pi}{3} r_+^3 \) is the thermodynamic volume of AdS RN black hole surrounded with the quintessence dark energy (see the introduction section) and

\[
U(r_+) = \frac{r_+}{2} + \frac{Q^2}{2r_+} - \frac{a}{2r_+^{3\omega}}
\]

(3.7)

is its internal energy. Applying (2.4) and (3.5) one can calculate Gibbons-Hawking temperature of the quintessence-RNAdS black hole as

\[
T_b = \frac{f'(r)}{4\pi} \bigg|_{r_+} = \frac{1}{4\pi r_+^2} \left(3a\omega r_+^{-3\omega} + 8\pi P r_+^3 + r_+ - \frac{Q^2}{r_+}\right).
\]

(3.8)

To plot the isoenthalpic curves in T-P plane we need an explicit formula for \( T(P) \). To do so we should eliminate \( r_+ \) between (3.5) and (3.8). In general, without setting numerical values on the barotropic index \( \omega \), we can not obtain an analytic solution for \( T(P) \), while one obtain

\[
-8192P^3\pi^3 Q^6 - 432\pi^4 Q^4 T^4 + 9216P^2\pi^2 Q^6 a - 1728M^2 P\pi^3 Q^2 T^2
\]

\[+1152P\pi^3 Q^4 T^2 - 3456P\pi Q^6 a^2 - 864M^3\pi^3 T^3 + 648M^2\pi^2 Q^2 T^2 a
\]

\[+864M\pi^3 Q^2 T^3 - 432\pi^2 Q^4 T^2 a + 432Q^6 a^3 + 5184M^4 P^2 \pi^2 - 6912M^2 P^2 \pi^2 Q^2
\]

\[+1536P^2 \pi^2 Q^4 - 3888M^4 P \pi a + 5184M^2 P\pi Q^2 a - 1152P\pi Q^4 a + 729M^4 a^2
\]

\[+108M^2 \pi^2 T^2 - 972M^2 Q^2 a^2 - 108\pi^2 Q^2 T^2 + 216Q^4 a^2
\]

\[+72M^2 P \pi - 72P\pi Q^2 - 27M^2 a + 27Q^2 a = 0
\]

(3.9)
for the cosmological constant regime of the quintessence \( \omega = -1 \) and
\[
- 131072\pi^3 Q^6 - 6912\pi^4 Q^4 T^4 - 13824\pi^3 Q^4 T^3 a - 27648 M^2 \pi^3 Q^2 T^2 \\
+ 73728 M P^2 \pi^2 Q^4 a + 18432 \pi^3 Q^4 T^2 - 7776 \pi^2 Q^4 T^2 a^2 - 13824 M^3 \pi^3 T^3 \\
+ 13824 M^3 Q^2 T^3 + 82944 M^4 P^2 \pi^2 - 10368 M^3 \pi^2 T^2 a - 110592 M^2 P^2 \pi^2 Q^2 \\
+ 1728 M^2 P \pi Q^2 a^2 + 12096 M \pi^2 Q^2 T^2 a + 24576 P^2 \pi^2 Q^4 - 10368 P \pi Q^4 a^2 \\
+ 729 Q^4 a^4 - 10368 M^3 P \pi a + 864 M^3 a^3 + 1728 M^2 \pi^2 T^2 + 11520 M P \pi Q^2 a \\
- 972 M Q^2 a^3 - 1728 \pi^2 Q^2 T^2 + 1152 M^2 \pi a - 108 M^2 a^2 - 1152 P \pi Q^2 \\
+ 108 Q^2 a^2 = 0
\]
(3.10)

for the quintessence regime of the dark energy \( \omega = -\frac{2}{3} \). Applying (3.8), one can obtain inversion temperature of the quintessence RNAdS black hole as follows.

\[
T_b^i = \frac{r_+}{3} \left( \frac{\partial T_b}{\partial r_+} \right)_{P=P_i} r_+ = r_+^i = \frac{1}{4 \pi r_+^3} \left( -a \omega (3 \omega + 2) r_+^{-3 \omega} + \frac{8 \pi P_i}{3} r_+^3 - \frac{r_+^i}{3} + \frac{Q^2}{r_+^i} \right).
\]

(3.11)

When the Gibbons-Hawking temperature (3.8) reaches to (3.11) as \( T_b = T_b^i \) for a constant inversion pressure \( P = P_i \) and particular radius \( r_+ = r_+^i \), we will have

\[
T_b^i = \frac{1}{4 \pi r_+^3} \left( 3 a \omega r_+^{-3 \omega} + 8 \pi P_i r_+^3 + r_+^i - \frac{Q^2}{r_+^i} \right).
\]

(3.12)

Subtracting (3.11) and (3.12) we obtain the following constraint condition.

\[
a \omega (3 \omega + 5) \frac{r_+^{-2 (3 \omega + 2)}}{4 \pi} + \frac{4}{3} \frac{P_i}{r_+} r_+^3 \frac{1}{3 \pi} + r_+^i \frac{1}{2 \pi} r_+^{-3} = 0.
\]

(3.13)

To plot inversion curves in T-P plane we must be eliminate \( r_+^i \) between (3.12) and (3.13). In general, without setting numerical values on the barotropic index \( \omega \), we can not obtain an analytic solution for \( T_b^i (P_i) \), but we will have

\[
- 8192 P_i^3 \pi^3 Q^4 + 6912 \pi^4 Q^2 T_i^4 + 9216 P_i^2 \pi^2 Q^4 a - 3456 P_i \pi Q^4 a^2 + 432 Q^4 a^3 \\
- 512 P_i^2 \pi^2 Q^2 + 384 P_i \pi Q^2 a - 32 \pi^2 T_i^2 - 72 Q^2 a^2 - 8 P_i \pi + 3 a = 0
\]

(3.14)

for \( \omega = -1 \) and

\[
- 8192 P_i^3 \pi^3 Q^4 + 6912 \pi^4 Q^2 T_i^4 + 3456 \pi^3 Q^2 T_i^3 a + 1152 P_i \pi^2 Q^2 T_i a
\]

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\[-216\pi Q^2 T_i a^3 - 512 P_i^2 \pi^2 Q^2 + 288 P_i \pi Q^2 a^2 - 27 Q^2 a^4 \]
\[-32 \pi^2 T_i^2 + 4 \pi T_i a - 8 P_i \pi a^2 = 0 \quad (3.15)\]
for \(\omega = -\frac{2}{3}\). If we set \(a = 0\) then the quintessence dark energy correction is removed from the RNAdS black hole and so our work reaches to results of the paper [47] (see figure 1). To study quintessence dark energy effects on the JT expansion of the RNAdS black hole we should choose \(a \neq 0\). Hence we use two different ansatz \(a = \frac{1}{4}\) for weak coupling quintessence (see figure 2) and \(a = 10\) for strong coupling quintessence (see figure 3). To plot diagrams of the inversion curves and the iso-enthalpic curves we should choose some numerical values for the mass \(M\) and the charge \(Q\) of the quintessence RNAdS black hole. We are free to choose numerical values of \(M\) and \(Q\). According to the work [47] we avoid particular hypersurfaces in T-P plane which exhibits with naked singularity. This leads to the condition \(M > Q\) (see also [56]). To compare our results with which ones are given in ref. [47] for RNAdS black hole in absence of the quintessence dark energy, we use \(M = 1.5, 2.5, 3\) for \(Q = 1; M = 2.5, 3, 3.5, 4\) for \(Q = 2; M = 13, 14, 15, 16\) for \(Q = 10\) and \(M = 22, 23, 24, 25\) for \(Q = 20\). Then we plot diagrams of inversion curves \((P, T_i)\) given by (3.14) and (3.15) in figures 1, 2 and 3 (see solid lines) for \(a = 0\), \(a = \frac{1}{4}\) and \(a = 10\) respectively. We plot isoenthalpic curves \((P, T)\) given by the equations (3.9) and (3.10) in figures 1, 2 and 3 (see discontinued lines) by regarding the above mentioned numerical values for \(a = 0\), \(a = \frac{1}{4}\) and \(a = 10\) respectively. According to definition of the JT coefficient (3.1), the black holes always cool (warm) above (below) the inversion curves (solid line in the figures 1,2,3) during the JT expansion. Diagrams in the figure 1 show cooling and or warming phase of a AdS RN black hole without the quintessence effect with arbitrary value for \(\omega\), the JT effect is happened for all states where \(M > Q\). Diagrams of the figure 2 (3) show that for weak (strong) quintessence effect \(a = \frac{1}{4}(10)\), the JT effect is happened when \(\omega = -1, -\frac{2}{3}\) for situations where \(M > Q\). Comparing the figures 1,2,3 we can infer that physical effects of the quintessence changes position of cooling-heating critical point in TP plane to the center \((T, P) \rightarrow (0, 0)\) just for large values of the black hole charge. In other words position of the cooling-heating critical point of a quintessence AdS RN black hole is sensitive more to the black hole charge.
4 Conclusion and outlook

In this work we considered quintessence dark energy effects on the JT expansion of the RNAdS black hole and determine regions where the black hole takes on the cooling-heating phase under the condition of constant mass. Our mathematical calculations predict that the quintessence dark energy affects more to large black holes $M > Q > 1$ where critical inversion point $(T_i, P_i)$ reaches to some smaller values. In this paper we assumed that the quintessence dark energy originates from classical dynamical scalar fields. In this case the black hole thermal entropy satisfies the Bekensten-Hawking theorem (entropy is equivalent to the horizon surface area).

As a future work we will extend the present work for the quantum conformal field theories of the AdS black holes. To do so we consider the JT expansion of the quantum RNAdS black holes surrounded with the quantum quintessence dark energy. In the latter case the quintessence dark energy originates from quantum scalar fields for which we should calculate its two point correlation function counterpart. It is called as non-local observable. Its dynamical effects appear in the conformal anomaly [57](see also [58,59]). Physical effects of the conformal anomaly leads to an additional logarithmic term for the black hole entropy by regarding an high-energy cutoff scale. On the other side this anomaly affects on the classical black hole metric and perturbs it. It causes to evaporate the black hole mass (see for instance [56,60,61,62] and references therein) but its final state reaches to remnant stable mini black hole. Hence its JT expansion can be challenging issue. One of applicable ways to do the work is to use 4 dimensional Lovelock gravity [63] and or Gauss-Bonnet higher order derivative metric action (see Also [25,34] and references therein).

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Figure 1: Diagrams of the inversion curves and the isoenthalpic curves of the RNAdS black hole without the quintessence effects \( a = 0 \).
Figure 2: Diagrams of the inversion curves and the isoenthalpic curves of the RNAdS black hole with the weak effects of the quintessence $a = \frac{1}{4}$. 
Figure 3: Diagrams of the inversion curves and the isoenthalpic curves of the RNAdS black hole with the strong effects of the quintessence $a = 10$. 
This figure "Acceptance_2.png" is available in "png" format from:

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