1. INTRODUCTION

In an earlier paper (Liszt 2014), we considered the relationship between H_1 column density N(H_1) derived from the large-scale 21 cm H_1 sky surveys (Kalberla et al. 2005; Peek et al. 2011), and reddening E(B − V) as derived from far-IR dust emission by Schlegel et al. (1998, hereafter SFD98). We traced N(H_1) and E(B − V) around the sky at Galactic latitudes |b| = 20°−60° and considered data at lower column densities 0.015 ≤ N(H_1)/E(B − V) ≤ 0.075 mag, where the hydrogen should be in the form of neutral atoms and corrections to N(H_1) for saturation and H_2 formation are unimportant. We showed that the relevant value of the gas/reddening ratio is N(H_1)/E(B − V) = 8.3 × 10^{21} \text{cm}^{-2} \text{mag}^{-1}, considerably larger than the usually cited ratio N(H_1)/E(B − V) = 5.8 \times 10^{21} \text{cm}^{-2} \text{mag}^{-1} derived from optical/UV absorption measurements toward stars by Bohlin et al. (1978), where N(H) = N(H_1) + 2N(H_2). It is also larger than the values N(H_1)/E(B − V) = 4.8−5.2 \times 10^{21} \text{cm}^{-2} \text{mag}^{-1} that are consistently quoted for stellar reddening and Lyα absorption toward early-type stars (Bohlin et al. 1978; Shull & van Steenberg 1985; Diplas & Savage 1994).

In fact, there are two mysteries in the disparity between radio/IR and optical/UV derivations of N(H_1)/E(B − V). The first is the numerical discrepancy, which puts several 21 cm surveys and the work of SFD98 together on one side in opposition to Lyα measurements by several groups using IUE and Copernicus and stellar reddening on the other. The second is the very nearly constant value for N(H_1)/E(B − V) quoted for the stellar data, as opposed to the radio−IR relationship shown in Figure 1 of Liszt (2014) that had a very strong point of inflection to smaller N(H_1)/E(B − V) at E(B − V) ≥ 0.08 mag. In principle the inflection in the radio data could reflect either the influence of saturation of the 21 cm line profiles or the expected onset of H_2 formation as originally discovered in the UV absorption data (Savage et al. 1977). We noted a seemingly similar inflection in the IUE results of Diplas & Savage (1994), whereby much higher values of N(H_1)/E(B − V) were seen at E(B − V) < 0.1 mag (their Figure 4(a)), but this was not taken into account in their final result. It should also have been present in the earlier treatment of the IUE results by Shull & van Steenberg (1985), who derived very nearly the same numerical result as in the later work.

This work is largely concerned with understanding the change in slope of the radio−IR defined N(H_1)−E(B − V) relationship, unravelling the possibly competing effects of 21 cm H_1 line saturation and H_2 formation. In Section 2, we discuss independent measures of 21 cm H_1 optical depth as a function of reddening, which can be employed to show that saturation corrections to 21 cm measurements of N(H_1) are small at least until E(B − V) > 0.3 mag. In that case, only H_2 formation can explain the observed inflection. In Section 3, we show that rather conventional models of H_2 formation in a low-density diffuse molecular gas can reproduce the observed N(H_1)−E(B − V) relationship. In Section 4, we extend our analysis to lower Galactic latitude |b| = 9°−20° and show that there are progressively higher values of N(H_1)/E(B − V) at all E(B − V) as |b| declines. Section 5 is a brief summary and discussion.
small as 0.01 mag. Also missing at small values of \( E(B-V) \) is a downturn in \( N(H) \) that could have signalled the presence of an increasing fraction of warm ionized gas.

2.1. Systematic Variation of the 21 cm Optical Depth with Reddening

To understand the possible effects of saturation, we began by binning the data in reddening (averaging over all the data comprising Figure 1) and forming mean \( H_1 \) emission profiles as a function of reddening. Figure 2 shows some of these for 0.025 mag \( \leq E(B-V) \leq 0.33 \) mag and it is clear that imputing high optical depth to the profiles around \( E(B-V) = 0.1 \) mag would require spin temperatures below 30 K for which there is no support in such diffuse gas.

The argument for modest optical depths may be made quantitative by considering the variation of measured 21 cm optical depth \( \Sigma_{H_1} = \int \tau(H) d\nu \) (units of \( \text{km s}^{-1} \)) with reddening first discussed by Liszt et al. (2010) using a combination of their own more recent Very Large Array data and that measured earlier by Dickey et al. (1983). Liszt et al. (2010) showed that there is a strong, nearly linear relationship between \( \Sigma_{H_1} \) and \( E(B-V) \) but with scant data at higher Galactic latitude and with much scatter and sparse data coverage at \( E(B-V) < 0.3 \) mag.

This situation is alleviated by inclusion of the new results of Roy et al. (2013) as shown in Figure 3.

The error-weighted regression line in Figure 3 is \( \log \Sigma_{H_1} = 1.183 \pm 0.019 + (1.057 \pm 0.034) \log E(B-V) \) or \( \Sigma_{H_1} = 14.07 E(B-V)^{1.074} \), using all the datapoints shown at \( E(B-V) > 0.02 \) mag. The ratio \( \Sigma_{H_1}/E(B-V) \) changes by only 45% over the range 0.02 \( \leq E(B-V) \leq 3 \) mag.

\( H_1 \) absorption (hence the presence of the cold neutral medium) is not consistently detected below \( E(B-V) = 0.02 \) mag. This was noted by Kanekar et al. (2011) who described the lack of \( H_1 \) absorption for \( N(H) < 2 \times 10^{20} \text{cm}^{-2} \).

\(^1\) For sightlines in common between the two data sets, the value from Liszt et al. (2010) has been retained.
This implies a ratio \( N(H\text{I})/E(B-V) = 10^{22} \text{ cm}^{-2} \text{ mag}^{-1} \) in keeping with the values found in our work but a more quantitative estimate can be derived from the table of \( \Xi_{H\text{I}} \) and optical-depth-corrected \( N(H\text{I}) \) of Roy et al. (2013), from which it is found that \( (N(H\text{I})/E(B-V)) = 7.7 \pm 1.4 \times 10^{21} \text{ cm}^{-2} \text{ mag}^{-1} \).

Note that Figure 3 appears to validate the use of \( E(B-V) \) from SFD98 up to rather higher values and at rather lower Galactic latitudes than are usually believed to be reliable; see the discussion in the original work. Unlike the original discussion in Liszt et al. (2010) the relationship between \( \Xi_{H\text{I}} \) and \( E(B-V) \) is demonstrated over a wide range of Galactic latitudes \( 1^\circ -60^\circ \).

2.2. Saturation Correction to \( N(H\text{I}) \)

A general saturation correction can be determined by constraining the derived column densities \( N(H\text{I}) \) to conform to the empirical \( \Xi_{H\text{I}}-E(B-V) \) relationship, where the free parameter connecting \( N(H\text{I}) \) to \( \Xi_{H\text{I}} \) is the spin temperature that is used to convert the observed brightness temperature profiles to \( N(H\text{I}) \). From the single power law for the \( \Xi_{H\text{I}}-E(B-V) \) relationship and the inflected variation of \( N(H\text{I}) \) with \( E(B-V) \) in Figure 1 it may be inferred that there is a variation in the mean spin temperature with increasing \( E(B-V) \), actually a decline leading to a saturation correction that increases with \( E(B-V) \) as expected.

We began by deriving mean \( T_{sp} \) values from binned \( H\text{I} \) profiles (Figure 2) across the range of \( E(B-V) \), assuming the power-law \( \Xi_{H\text{I}}-E(B-V) \) relationship shown in Figure 3: these mean \( T_{sp} \) are shown in the upper panel of Figure 4. Then we fit a smooth function to the variation of \( T_{sp} \) with \( E(B-V) \) and applied that to all \( H\text{I} \) profiles individually to correct them for the optical depth implied by their known reddening. This process is self-consistent in reproducing the power-law \( \Xi_{H\text{I}}-E(B-V) \) relationship when applied to the data at large. The power-law \( \Xi_{H\text{I}}-E(B-V) \) relation breaks down at \( E(B-V) < 0.02 \text{ mag}, N(H\text{I}) < 2 \times 10^{20} \text{ cm}^{-2} \), leading to an underestimation of the mean \( T_{sp} \), but no correction for saturation is needed at such small \( N(H\text{I}) \) anyway.

Figure 4 at bottom shows the derived saturation correction as a multiplicative correction to the values of \( N(H\text{I}) \) derived from the mean \( H\text{I} \) profiles in the limit of zero optical depth, \( N(H\text{I}) = 1.823 \times 10^{18} \text{cm}^{-2} \int T_d dv \) with the integral expressed in units of K-\text{km s}^{-1}. Note that the \( T_{sp} \) variation and the corrections shown are relevant only to the data that was considered. A different data set might require a different variation of \( T_{sp} \) with \( E(B-V) \) (see Section 4 below) and the magnitude of the correction that must be applied depends not only on \( T_{sp} \) but on the \( H\text{I} \) profile itself. When the profile integral is very small, even very small \( T_{sp} \) do not result in a significant correction to the optically thin value of \( N(H\text{I}) \). As well, the derived \( T_{sp} \) at a given \( E(B-V) \) might be very different for different data sets without implying significantly different correction factors at that \( E(B-V) \). The point is that \( \Xi_{H\text{I}} \propto N(H\text{I})/T_{sp} \) by definition but \( N(H\text{I}) \) depends on \( T_{sp} \) only when the optical depth is high.

In any case, the correction factor relevant to the data set shown in Figure 1 is below 20% for \( E(B-V) < 0.5 \text{ mag} \). In Liszt (2014), we used a constant \( T_{sp} = 145 \text{ K} \) but the important ramifications of the data are the same. There is no appreciable correction to \( N(H\text{I}) \) at small \( E(B-V) \), leading to a reliable value for \( N(H\text{I})/E(B-V) \). The correction for saturation is not responsible for the inflection in the plot of \( N(H\text{I})/E(B-V) \), which must be ascribed to the onset of H\text{2} formation.

The relevance and accuracy of using a so-called isothermal correction to \( N(H\text{I}) \) given the optical depth absorption profile has recently been examined by Chengalur et al. (2013), who derive correction factors comparable to ours. The correction is described as isothermal because a single \( T_{sp} \) is applied at each velocity to emission that is a blend of contributions from different gas phases. The correction does an excellent job of bounding even very large errors in \( N(H\text{I}) \) that may occur at \( \Xi_{H\text{I}} = 1-10 \text{ km s}^{-1} \) when the optical depth is unknown. The method adopted here is broader because we use an implied optical depth integral \( \Xi_{H\text{I}} \) to derive a single value of \( T_{sp} \) across an entire line profile but the conclusions are the same.

3. THE INFLUENCE OF H\text{2} FORMATION ON \( N(H\text{I}) \)

The empirical correction factors derived in Section 2 were applied to the data in Figure 1 but a significant gap remains at \( E(B-V) \gtrsim 0.1 \text{ mag} \) between the observed, corrected \( N(H\text{I}) \) and the straight line \( N(H\text{I})/E(B-V) = 8.3 \times 10^{21} \text{ cm}^{-2} \text{ mag}^{-1} \) that is applicable below \( E(B-V) = 0.08 \text{ mag} \) where H\text{2} formation does not occur (Bohlin et al. 1978). That is also the line of zero molecular fraction, and superposed on the data are (dashed) lines of constant molecular hydrogen fraction \( f_{H\text{2}} = 1-N(H\text{I})/N(H\text{I}) \), indicating very high molecular fractions at high \( E(B-V) \).

Also shown in Figure 1 are the results of a model calculation of H\text{2} formation in a diffuse gas at low-density \( n(H) = 14 \text{ cm}^{-3} \). These are the same equilibrium heating/cooling/H\text{2} formation models we have used earlier in for instance Liszt (2007) to illustrate H\text{2} and CO formation, but now with the remainder H\text{I} shown on the vertical axis. Also, the models were calculated using the newly inferred value \( N(H\text{I})/E(B-V) = 8.3 \times 10^{21} \text{ cm}^{-2} \text{ mag}^{-1} \) that provides less dust shielding and extinction at a given \( N(H) \). Each leaf of the plot is for a separate cloud model having a central hydrogen column density \( N(H) \) differing by a factor 2\text{1/4} from its neighbors and the variation...
along each leaf represents the locus of column density seen at all impact parameters across the face of the model. The right-most points along each leaf correspond to sightlines passing closer to the center of the model and so would be observed with smaller probability in real observations.

In any case, the point is to demonstrate that although there seems to be little alternative to H$_2$ formation, that explanation also works in practice.

4. LATITUDE VARIATION

Figure 1 shows that the data at latitudes at $|b| > 20^\circ$ fit together into a coherent whole with a single message about $N$(H$\text{I}$)/$E(B - V)$ over the Galaxy and over a wide range of $E(B - V)$. This is not true of the sky at smaller $|b|$, as will be discussed now.

The left panel of Figure 5 shows the latitude variation of the mean reddening averaged around the sky. Below about 15$^\circ$ much of the sky is well-described by the cosecant law expected for a plane-parallel stratified medium. At higher latitudes the latitude dependence progressively steepens with, finally, a cubic dependence (actually, power law $\sim$3.1) for $|b| > 36^\circ$. Given these gradients we worried that the larger H$\text{I}$ beamsize of the LAB survey might have artificially increased the values we derived for $N$(H$\text{I}$)/$E(B - V)$. The disparity in beamizes (36$^\prime$ versus 6$^\prime$) would merely introduce scatter for a uniform sky, but the larger H$\text{I}$ beam has a slightly lower intensity-weighted mean $|b|$ when viewing a medium that is concentrated to the Galactic equator.

Numerical integration over the H$\text{I}$ beam using the gradients shown in Figure 5 suggested that the effect would not be important but as a test we recalculated our results comparing $N$(H$\text{I}$) with $E(B - V)$ measured 9$^\prime$ and 18$^\prime$ closer to the Galactic equator (ie 50% and 100% of the radius of the H$\text{I}$ beam). We found only that the mean $N$(H$\text{I}$)/$E(B - V)$ declined progressively below $|b| = 8^\circ$, by a maximum of 5% at $|b| = 4^\circ$. The vertical sky gradient should not have affected any of the conclusions drawn in this work.

In the right panel of Figure 5 we show the variation in the mean $N$(H$\text{I}$)/$E(B - V)$ at 0.015 $\leq E(B - V) \leq 0.075$ mag and over all $E(B - V)$.

Note that we derived a separate saturation correction at $|b| < 20^\circ$ where profiles are broader, with smaller peak brightness and integrated opacity at a given $N$(H$\text{I}$) or $E(B - V)$.4
Emission-corrected 353 GHz optical depth maps converted to N(H i) nearly linearly in E(B - V), with the solid black line giving the reddening maps of SFD98, based on previous work by other investigators. The reddening maps of SFD98, had, if anything, overestimated E(B - V), correcting the results of SFD98 in that manner would only exaggerate the effect discussed here. More recently the Planck reddening maps have appeared (Planck Collaboration et al. 2013), and they may tell a different story. Although the Zodiical Emission-corrected 353 GHz optical depth maps converted to reddening give the same result we derived, N(H i)/E(B - V) = 8.3 x 10^{21} cm^{-2} mag^{-1}, those maps are recommended for use only at larger E(B - V). By contrast, the Planck reddening maps based on QSO colors that are recommended for use below E(B - V) = 0.3 mag would have E(B - V)' = (E(B - V)_{SFD}+0.003 mag)/0.92. When such a transformation is used to rederive the present results, we find a high-latitude asymptote N(H i)/E(B - V) = 7.2 x 10^{21} cm^{-2} mag^{-1}, accounting for about half the effect noted, in the log sense. This transformation increasing the E(B - V) values of SFD98 at smaller E(B - V) would shift the curves in Figure 6 to the right in the manner required to reconcile them with a single value of N(H i)/E(B - V), as discussed in Section 4.

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