A Range Doppler Imaging Algorithm Based on the MSR Spectrum for the Multi-Aperutre Syntactic Sonar

Jinhua Lv, Jinsong Tang and Haoran Wu

1College of Electrical and Electronic Engineering, Wuhan Institute of Shipbuilding Technology, Wuhan, 430050, China
2College of Electronic Engineering, Naval University of Engineering, Wuhan, 430033, China

Abstract. For ignoring the azimuth variation of the “non-hop-go-hop” time, the existing imaging algorithm for multi-aperture have a poor imaging in moderate squint, so this paper proposes a multi-aperture range Doppler imaging algorithm based on the spectrum of method of series reversion. The exact range history is approximated once, and the error can be adjusted by the order of Taylor series expansion. Then, the analytical expression of the two-dimensional spectrum is obtained by the method of series reversion. In signal processing, the signal of the single aperture is handle with the squint range Doppler imaging algorithm, and the azimuth spectrum is superposed to each aperture, to obtain the image. Finally, the effectiveness and correctness of the algorithm is provided by the simulation experiments.

1. Introduction
In order to overcome the problem of high resolution and high mapping rate in synthetic aperture imaging system, multi-receiver technology is introduced. In SAS (Synthetic Aperture Sonar), the “stop-and-go” hypothesis for synthetic aperture radar is no longer valid. The individual sub-array signals are then processed using the RDA algorithm (Range Doppler Algorithm). In order to avoid the influence of the single subarray signal azimuth undersampling on the final image quality, it is necessary to perform azimuth spectrum expansion before the single subarray signal processing, increasing the number of points of the single subarray orientation. Therefore, after each sub-array signal processing is completed, the aliasing due to undersampling can be eliminated by means of azimuth spectrum superposition. Finally, the effectiveness and correctness of the proposed algorithm are proved by simulation experiments.

2. Strabismus multi-subarray SAS model

2.1. Precise distance history
In this section, we will establish the exact distance history of the squint SAS based on the geometric model of the squint SAS.
According to the geometric relationship of Figure 1, the exact distance history of P is obtained as:

\[ R^* (t; r) = R_r (t; r) + R_l (t; r) = \sqrt{r^2 + v^2 t^2 - 2vt \sin(\theta_q)} + \sqrt{r_i^2 + v^2 \left(t + t^*\right)^2 - 2v \left(t + t^*\right) r_l \sin(\theta_{qi})} \] (1)

In the formula, \( \sin(\theta_{qi}) \) is equal to \( \sin(\theta_q) - \cos(\theta_q) / r_l \) and \( r_l \) is equal to \( \sqrt{r^2 + d^2} \). The precise distance history of the point target P can also be obtained from the distance during which the acoustic signal propagates during transmission and reception:

\[ R^* (t; r) = ct^* \] (2)

According to formula (1) and formula (2), the exact distance history is solved as follows:

\[ R^* (t; r) = c \frac{B + \sqrt{B^2 + AC}}{A} \] (3)

In formula (3),

\[ A = c^2 - v^2 \] (4)

\[ B = tv^2 - vr_l \sin(\theta_{qi}) + c\sqrt{r^2 + v^2 t^2 - 2vt \sin(\theta_q)} \] (5)

\[ C = 2vtd_l \cos(\theta_{qi}) + d_l^2 \] (6)

For the fourth-order Taylor series expansion, the approximate distance history is obtained, and the approximate distance history is:

\[ R \left(t, r\right) = k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4 \] (7)

2.2. Point target echo signal model

After demodulating the echo signal received by the ith receiving array to the baseband, the expression is:
\[
ss(t,t,r) = p\left(\tau - \frac{R(t,r)}{c}\right) \cdot \omega_k(t) \cdot \exp\left\{j\pi k\left(\tau - \frac{R(t,r)}{c}\right)^2\right\} 
\cdot \exp\left\{-\frac{2\pi f_0}{c} R(t,r)\right\}
\]

(8)

\(\tau\) is the fast change time; \(p(.)\) is the envelope of the pulse; \(\omega_k(.)\) is the antenna weight; \(k\) is the frequency modulation slope; \(c\) is the sound speed; \(f_0\) is the carrier frequency.

3. Algorithm derivation
Figure 2 shows the main steps of the squint SAS range Doppler algorithm.

![Algorithm process](image)

3.1. Doppler center frequency shift
In the case of squint, the energy of the signal "winds" in the azimuth frequency domain.

\[f_{\Delta f} = \frac{1}{\lambda} \left| \frac{dR(t,r)}{dt} \right|_{t=0} = \frac{v \sin(\theta_x)}{\lambda} + \frac{v \sin(\theta_y)}{\lambda}
\]

(9)

\(-PRF/2 < f_{\Delta f} < f_{\Delta f} + PRF/2
\]

(10)
By multiplying the signal by a linear phase $\exp(-j2\pi f_{a}t)$, the problem of azimuth frequency domain energy "winding" can be solved.

3.2. Azimuth spectrum expansion

The data received by each receive array is downsampled data for the azimuth time domain signal. With the method shown in FIG. 3, the length of the azimuth spectrum can be increased by M times, and the requirements for the number of processing points of a single sub-array are satisfied. In Fig. 3, N is the number of pulses in the length of a synthetic aperture, and M is the number of receiving arrays.

![Figure 3. Azimuth spectrum extension](image)

As shown in FIG. 3, the signal subjected to the azimuth Doppler center frequency shift is subjected to azimuth Fourier transform to obtain azimuth spectrum signal. Then, the azimuth spectrum signals are copied and arranged in the order shown in Fig. 3.

3.3. SRC and residual phase correction

Since the SRC term has a weak dependence on the distance, its phase can be replaced by the phase at the reference distance. The residual phase is independent of the distance and azimuth frequencies and therefore has no effect on the focus of the target, but it causes the target intensity to vary with distance and can be compensated by phase multiplication.

3.4. RCMC and azimuth pulse pressure

In the distance Doppler domain, RCMC (Range Cell Migration Correction) is implemented by interpolation. The distance migration required to be corrected is:

$$
\Delta \tau = -\frac{\phi_{RW}(f_{r}, f_{a}; r)}{2\pi f_{r}} c^{-2r} \quad (11)
$$

From equation (11), the azimuth matching filter required for azimuth pulse pressure is:

$$
H_{a}(f_{a}) = \exp\{-j\phi_{RW}(f_{a}; r)\} \quad (12)
$$

3.5. Coherent superposition

After completing the above four steps, the distance modulation, the azimuth modulation, and the coupling of the distance and the azimuth have been eliminated. According to Figure 4, the signal of the i-th receiving array is represented as:
\begin{equation}
\begin{aligned}
\mathbf{S}(\tau,f_c,r) = \\
\begin{cases}
W(\nu)\text{sinc}|B(\tau-2r/c)| \exp\left[j2\pi f_c \frac{\Delta f}{v} (i-N+L)\right], N>i>N-L \\
W(\nu)\text{sinc}|B(\tau-2r/c)|, 0<i\leqslant N-L
\end{cases}
\end{aligned}
\end{equation}

Figure 4. The distribution of the receiving array

By coherently superimposing the signals of each receiving array, the complete SAS signal can be obtained as:

\begin{equation}
\begin{aligned}
\mathbf{sS}(\tau,f_c,r) = \sum_{i=1}^{N} \mathbf{sS}_i(\tau,f_c,r) \\
= A(f_c)W(\nu)\text{sinc}|B_\tau(\tau-2r/c)| \exp\left[j2\pi f_c \frac{\Delta d(L-1)}{2v}\right]
\end{aligned}
\end{equation}

Among them, \(A(f_c)\) is a complex envelope. The linear phase term in equation (14) is removed by phase multiplication. Then, an azimuth inverse Fourier transform is performed to obtain a two-dimensional time domain signal. At this point, the focused point target signal is:

\begin{equation}
\mathbf{ss}(\tau,t) = \text{sinc}\left(\tau - \frac{2r}{c} + \frac{2v\sin(\theta_c)}{c}t\right) \rho_c(t)
\end{equation}

According to equation (15), the target deviates from its original position, so the target position needs to be corrected. Multiply the phase factor by the distance frequency domain and then perform the inverse Fourier transform of the distance direction to obtain an accurate focused image of the point target:

\begin{equation}
\mathbf{ss}(\tau,t) = \text{sinc}\left(\tau - \frac{2r}{c}\right) \rho_c(t)
\end{equation}

4. Simulation results

In order to verify the validity of the proposed distance Doppler algorithm, simulation analysis is performed in this section. Table 1 shows the system parameters of the multi-subarray SAS. Figure 5 shows the simulation scenario for five point targets.

| Table 1. The parameters of the SAS system. |
|------------------------------------------|
| Parameter                                 | Value           |
| Carrier frequency of the transmitted signal | 150kHz          |
| Transmitter length                       | 0.08m           |
| Transmitted signal bandwidth             | 20kHz           |
| Receive subarray length                  | 0.08m           |
| Pulse width of the transmitted signal    | 20ms            |
| Platform speed                           | 2.5m/s          |
| Pulse repetition frequency               | 200ms           |
| Number of receiving subarrays            | 25              |
| Oblique viewing angle                    | 10°             |
Figure 5. Scene

Figure 6 shows the simulation results for the target in Figure 5 with an oblique viewing angle of 100. Figure 7 is a cross-sectional and azimuthal cross-sectional view taken at point A in Figure 6. Table 2 shows the measurement results of the distance resolution, the azimuth resolution, the distance PSLR (Peak Sidelobe Ratio) and the azimuth PSLR of the point target A at different oblique angles. The results of Fig. 6, Fig. 7 and Table 2 show that the proposed algorithm still has a good focusing effect under medium oblique viewing angle.

Figure 6. Simulation imaging result

Figure 7. The profile of target A
| Oblique viewing angle | 0°  | 5°  | 10° |
|----------------------|-----|-----|-----|
| Distance resolution  | 1 cells | 1 cells | 1 cells |
| Azimuth rate         | 1.1 cells | 1.3 cells | 2.3 cells |
| Distance PSLR        | -13.4dB | -13.4dB | -12.9dB |
| Orientation PSLR     | -17.1dB | -25.4dB | -34dB |

5. Conclusion
This paper proposes a distance Doppler algorithm for processing multiple sub-arrays of medium squint. Because this method directly approximates the history of precise distance, it needs to first approximate the non-stop and stop time, and then perform the method of approximating the distance between the two roots with smaller approximation error. In addition, the orientation of this paper can control the distance history error and the size of the two-dimensional spectral error by adjusting the order of the Taylor series.

Acknowledgements
National Natural Science Foundation of China (61671461)

References
[1] N. Gebert, G. Krieger, A. Moreira, “SAR signal reconstruction from non-uniform displaced phase centre sampling in the presence of perturbations,” [J]. Geoscience and Remote Sensing Symposium, 2005.
[2] G. A. Gilmour, “Synthetic aperture side-looking sonar system,” [J]. Journal of the Acoustical Society of America, vol.65, no.3, pp. 557-562, May 1978.
[3] H. J. Callow, “Signal processing for synthetic aperture sonar image enhancement,” [J]. Christchurch: University of Canterbury, 2003.
[4] W.J. Bonifant, “Interferometric synthetic aperture sonar processing,” [D]. Georgia Institute of Technology, Georgia, Atlanta, USA, 1999.
[5] Yang Haилиang, “Multi-receiver array synthetic aperture sonar imaging algorithm research,” [D]. Naval University of Engineering, Wuhan, 2009.
[6] YL Neo, FH Wong, IG Cumming, "A two-dimensional spectrum for bistatic SAR processing using series reversion," [J]. IEEE Geosci. Remote Sens. Letter, vol. 4, no. 1, pp. 93 –96, Jan. 2007.
[7] YL Neo, FH Wong, IG Cumming, "Processing of azimuth invariant bistatic SAR data using the range doppler algorithm," [J]. Geosci. Remote Sens, vol.46, no.1, pp.14-21, January 2008
[8] YL Neo, FH Wong, IG Cumming, “Focusing bistatic SAR data using the nonlinear chirp scaling algorithm,” [J]. IEEE Geosci. Remote Sens. Lett, vol. 46, no. 9, pp. 2493–2505, Sept. 2009.
[9] Zhang Xuebo, Tang Jinsong, Zhang Sen, “Multi-received synthetic aperture sonar distance Doppler imaging algorithm for fourth-order model,” [J].Journal of Electronics & Information Technology, vol.36, no.7, pp.1592 -1598, Jul.2014.
[10] I. G. Cumming, F. H. Wong, “Digital processing of synthetic aperture radar data: algorithms and implementation”, [J]. Norwood, MA: Artech House, 2005 pp236-247.