Compactification and Supersymmetry Breaking in M-theory

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Abstract

Keeping N=1 supersymmetry in 4-dimension and in the leading order, we discuss the various orbifold compactifications of M-theory suggested by Horava and Witten on $T^6/Z_3$, $T^6/Z_6$, $T^6/Z_{12}$, and the compactification by keeping singlets under $SU(2) \times U(1)$ symmetry, then the compactification on $S^1/Z_2$. We also discuss the next to leading order Kähler potential, superpotential, and gauge kinetic function in the $Z_{12}$ case. In addition, we calculate the SUSY breaking soft terms and find out that the universality of the scalar masses will be violated, but the violation might be very small.

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1 Introduction

Recently, M-theory on $S^1/Z_2$ suggested by Horava and Witten [1] has received considerable attention. Its successes include: explanation of why Newton’s constant appears to be so small when the 4-dimensional grand unified gauge coupling $\alpha_{GUT}$ takes the experimental acceptable values [2], gaugino condensation and supersymmetry breaking [3], [4], [5], [6], [7], [8], [9], strong CP problem [10], [11], threshold scale and strong coupling effects [12], [13], proton decay [14], Casimir effect [15], compactification and low energy phenomenology consequences [16], [17], [18], [19], [20], [21], [22]. In addition, this approach may open the door to M-theory model building [23] and the testing M-theory models, because there exists a crucial magnitude difference in soft terms between the M-theory and weakly coupled string theory, although their formulas are similar.

As it is well known, a supergravity theory is specified by the two functions, the Kähler function

$$G = K + \ln|W|^2,$$

(1)

where $K$ is the Kähler potential and $W$ is the superpotential, and the gauge kinetic function $f$. However, almost all the compactifications of this scenario which have been done is just the simplest case with the Hodge-Betti numbers of the Calabi-Yau manifold $h_{(1,1)} = 1, h_{(2,1)} = 0$ which has just one modulus from the Calabi-Yau manifold. In this paper, we consider the various orbifold compactifications on $T^6/Z_3$, $T^6/Z_6$, $T^6/Z_{12}$ and the compactification by retaining singlets under $SU(2) \times U(1)$ symmetry, corresponding to the Hodge-Betti numbers are $h_{(1,1)} = 9, 5, 3, 2$ and $h_{(2,1)} = 0$ respectively, these models will have 9, 5, 3, 2 families respectively. The compactification should be done from eleven-dimension to five-dimension, then to the four-dimension, at leading order ($\kappa^{2/3}$); the Kähler potential, superpotential and gauge kinetic function should be the same as previous [20] but the definitions of the moduli are different, as pointed out in Ref [17]. The interesting point is that all above models are no-scale supergravity models [24]. To the higher order, we should include the M-theory correction. The 4-dimensional effective action of the M-theory can be expanded in powers of the two kinds of dimensionless variables [3]:

$$\epsilon_1 = \kappa^{2/3} \pi \rho / V^{2/3},$$

(2)

and

$$\epsilon_2 = \kappa^{2/3} (\pi \rho V^{2/3}).$$

(3)

For example, in the model with just two moduli S and T, to the order of $\kappa^{4/3}$, the relevant variable is $\epsilon_1$ (or $ReT/ReS$) defined above [4], [18]. In other words, when we choose the metric as follows:

$$g_{\mu
u}^{(11)} = e^{-\gamma} e^{-2\sigma} g_{\mu
u}^{(4)}; \quad g_{11,11}^{(11)} = e^{2\gamma} e^{-2\sigma}; \quad g_{mn}^{(11)} = e^{\sigma} g_{mn},$$

(4)
the expansion variable is $\kappa^{2/3} e^{-3\sigma}$. The theory has more than two variables when one consider more moduli [21, 22]. Here, we consider in detail the model with $Z_{12}$ symmetry for detail. Although the original Lagrangian suggested by Horava and Witten is generally constructed up to the terms at order of $\kappa^{2/3}$, we write down Kähler potential, superpotential and gauge kinetic function at the order of $\kappa^{4/3}$ (the detailed calculation will be appeared elsewhere [21]), which might be the part of M-theory correction. The expansion variables now are:

$$\epsilon_3 = \kappa^{2/3} e^{\gamma-(\sigma_1+4\sigma_2+4\sigma_3)/3}$$

$$\epsilon_4 = \kappa^{2/3} e^{\gamma-(4\sigma_1+\sigma_2+4\sigma_3)/3}$$

$$\epsilon_5 = \kappa^{2/3} e^{\gamma-(4\sigma_1+4\sigma_2+\sigma_3)/3}$$

where of $\gamma$ and $\sigma_i$ are defined in the following sections. In addition, if we did not require $Z_{12}$ symmetry but had four moduli: $S, T_1, T_2$ and $T_3$ in the 4-dimension, there will be 6 expansion variables [22]. Furthermore, we discuss the non-perturbative superpotential related to $S$ and $T_i$. Finally, we calculate the SUSY-breaking soft terms under the general assumption that SUSY is spontaneously broken by the auxiliary components of the bulk moduli superfields in the model. We also discuss dilaton-induced SUSY-breaking, because it is simple and it is special in several respects in previous considerations. We emphasize that, although the Kähler potential, superpotential and gauge kinetic function might have similar forms as before, the magnitude is much different after we consider the next order correction, so there can be sizable differences in the weakly coupled string and M-theory. These results might be helpful for future M-theory model building and phenomenology analysis, and one might find the differences in the future accelerator experiment.

Of course, if $\int w \wedge tr(R \wedge R)$ is zero, then the next leading order correction will be zero and there may be no strong coupling in the hidden sector. It may be also possible to break SUSY by Scherk-Scherk mechanism [1, 2] because of the $Z^{HW}_{2}$ symmetry in this scenario. However, the length of the eleven-dimension could not take very small values because we should use the acceptable $\alpha_{GUT}$, $m_{GUT}$ and $m_{pl}$. The possible range of $\pi \rho$ is from $10^{13}$ GeV to $10^{15}$ GeV [24] when we use $m_{GUT} = 2.0 \times 10^{16}$ GeV estimated by extrapolating from the measurements at LEP and elsewhere. If so, the results discussed in Section 2 will not be changed in the next leading order ($\kappa^{4/3}$).

Obviously, the discussion in this manuscript is not the general case, the general cases of dilaton/moduli induced SUSY-breaking and the M-theory corrections to previous string no-scale supergravity [24] are under investigation.

This manuscript is organized as follows: in Section 2, we calculate the Kähler potential, superpotential and gauge kinetic function at order of $\kappa^{2/3}$. We generalize the result to the order of $\kappa^{4/3}$ in $Z_{12}$ symmetry case in Section 3. In Section 4, we discuss its the gaugino condensation and corresponding superpotential. In Section 5, we discuss its SUSY-breaking soft terms. Our conclusions are given in section 6.
2 SUGRA of the order $\kappa^{2/3}$

The four-dimensional effective action in M-theory scenario has been calculated in the simplest case with just $S$ and one $T$ fields. As previously, the strategy consists in a dimensional reduction of the 11-dimensional Lagrangian on $S^1/Z_3$. A consistent truncation is performed by keeping only singlets or invariance with respect to some suitable group, isomorphic to a subgroup of the rotation group of the internal manifold and then compactified on $S^1/Z_2$ to obtain a N=1 supersymmetry theory. For example, the 4-dimensional effective actions [17] are obtained by keeping only the singlets under the action of $SU(3)_D = SU(3)_1 + SU(3)_2$, in which $SU(3)_1$ is a subgroup of the rotation group $SO(6)$ or $SU(4)$ of the internal coordinates $X_I$, where $I=5, ..., 10$ in 11-dimension, and $SU(3)_2 \times E_6 \supset E_8$ where $E_8$ is the gauge group in the observable sector or boundary.

Using methods in Ref. [5, 17, 26], we generalize this strategy to obtain other N=1, D=4 effective actions in M-theory. With the boundary gauge group $G = E_8 \times E_8'$, one can obtain a wide range of the N=1 supergravity models with families in the 27 representation of $E_6 \supset E_8$. What has been already done is the one family model which keeps only the $SU(3)_D$ singlets [17] or just keeps two moduli: $S$ and $T$: one can compactify on the orbifolds by restriction to the subgroup of $SU(3)$, and then obtain other models. The N=1 “maximal model” with 9 families is obtained by choosing $Z_3$ symmetry. The lower families model can be obtained by a suitable truncation of the “maximal model”. As pointed in Ref. [17], to order $\kappa^{2/3}$, the result is the same as previous result except the definition of the dilaton and the moduli. After some exercise, one can obtain the following 4-dimensional Kähler potential, superpotential and the gauge kinetic function.

(I) $Z_3$ symmetry. This is the center $Z_3$ of the $SU(3)_D$. The model contains nine 27 of families which are triplets under the horizontal gauge $SU(3)$ symmetry. The massless fields in 5-dimension are the gravitational multiplet, the universal hypermultiplet and the eight vector multiplets, corresponding to the Hodge-Betti number is $h_{1,(1)} = 9$ and $h_{1,(2)} = 4$; and the observable gauge group is $SU(3) \times E_6$ (the hidden sector gauge group is $E_8$). In the Einstein frame after rescaling, the $Z_3$ invariant metric tensor in 5-dimension is:

$$g_{\mu\nu}^{(11)} = G^{-1/3}g_{\mu\nu}^{(5)}; \quad g_{11,11}^{(11)} = G^{-1/3}g_{11,11}^{(5)}; \quad g_{ij}^{(11)} = g_{ij} ,$$

and in 4-dimension is:

$$g_{\mu\nu}^{(5)} = e^{-\gamma}g_{\mu\nu}^{(4)}; \quad g_{11,11}^{(5)} = e^{2\gamma} ,$$

where the $G$ is the determinant of the metric in the compact 6-dimension space. The shape of the orbifold is described by the nine parameters $g_{ij}$ where $i, j = 1, 2, 3$.

1Generally, in 5-dimension, the theory contains one gravitational multiplet, $(h_{1,1} - 1)$ vector multiplets, and $(h_{2,1} + 1)$ hypermultiplets.
The massless modes of the three form in 4-dimension is $C_{5\mu\nu}$ and $C_{5i\bar{j}}$. The Kähler potential for 4-dimensional SUGRA is

$$K = -\ln [S + \bar{S}] - \ln det [T_{ij} + \bar{T}_{ij} - 2C_{i}^{\alpha a}C_{j}^{\bar{\alpha} \bar{a}}],$$

where

$$S = G^{1/2} + i24\sqrt{2}D,$$

and

$$T_{ij} = e^\gamma G^{-1/6}g_{ij} - i6\sqrt{2}B_{ij} + C_{i}^{\alpha a}C_{j}^{\bar{\alpha} \bar{a}},$$

with $\frac{1}{3!} G_{\mu\nu\rho5} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma D$, and $C_{5i\bar{j}} = iB_{ij}$. The gauge kinetic function for the observable and hidden sector is

$$f_{\alpha\beta} = S \delta_{\alpha\beta},$$

and the superpotential $W$,

$$W = c \ g_c \ e^{ijk} (\epsilon_{mni} d_{abc})C_{i}^{\alpha a}C_{j}^{\beta b}C_{k}^{\gamma c},$$

where $c$ is just a constant and $g_c = \frac{2\pi \rho \lambda^2}{k^3}$. And $\lambda^2 = 2\pi(4\pi\kappa^2)^{2/3}$. In addition, $\epsilon_{ijk}$ is the antisymmetric tensor of SU(3), $d_{abc}$ is the symmetric tensor of the 27 of the $E_6$, $a \in 27$ of $E_6$.

The “maximal” nine family Kähler manifold defined from above is

$$SU(1,1) \times SU(3,3 + 3n) \to SU(3) \times SU(3 + 3n) \times U(1),$$

where $n=27$.

(II) $Z_6$ symmetry. $Z_6$ symmetry is generated by the $3 \times 3$ matrix:

$$M = \text{diag}(-e^{i2\pi/3}, -e^{i2\pi/3}, e^{i2\pi/3})$$

where $e^{i2\pi/3}$.

The $Z_6$ invariance keeps only singlets and triplets under $SU(2)_D \supset SU(3)_D$. The model contains five 27 families. In other words, the massless fields in 5-dimension are gravitational multiplet, the universal hypermultiplet and four vector multiplets, corresponding to the Hodge-Betti number is $h_{(1,1)} = 5$ and $h_{(2,1)} = 0$, and the observable gauge group is $SU(2) \times U(1) \times E_6$ (the hidden sector gauge group is $E_8$). In the Einstein frame after rescaling, the $Z_6$ invariant metric tensor in 4-dimension is

$$g_{\mu\nu}^{(11)} = e^{-\gamma} G^{-1/3} g_{\mu\nu}^{(4)} ; \ g_{11,11}^{(11)} = e^{2\gamma} G^{-1/3} ,$$

One may have a normalization factor $f$ [17], but here, we put it as 1; otherwise one should write:

$$f_{\alpha\beta} = fS \delta_{\alpha\beta}.$$
\[ g_{ij}^{(11)} = g_{i\bar{j}}, \quad g_{3\bar{3}}^{(11)} = g_{3\bar{3}} \delta_{i\bar{j}} , \] (19)

where \( i, j = 1, 2 \) and \( l = 1, 2, 3 \); \( G \) is the determinant of the metric in the compact 6-dimension space. The shape of the orbifold is described by the five parameters \( g_{ij} \) where \( i, j = 1, 2 \) and \( g_{3\bar{3}} \). The massless modes of the three form in 4-dimension is \( C_{5\mu\nu} \) and \( C_{5\bar{3}} \). The Kähler potential for 4-dimensional SUGRA is

\[ K = -\ln [S + \bar{S}] - \ln \det [T_{ij} + \bar{T}_{ij} - 2C_i^{ma}C_j^{\bar{m}\bar{a}}] - \ln [T_3 + \bar{T}_3 - 2C^aC^{\bar{a}}] , \] (20)

where

\[ S = G^{1/2} + i24\sqrt{2}D , \] (21)

and

\[ T_{ij} = e^\gamma G^{-1/6}g_{ij} - i6\sqrt{2}B_{ij} + C_i^{ma}C_j^{\bar{m}\bar{a}} , \] (22)

\[ T_3 = e^\gamma G^{-1/6}g_{3\bar{3}} - i6\sqrt{2}B_3 + C^aC^{\bar{a}} , \] (23)

where \( \frac{1}{4}GG_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma D, C_{5\bar{j}} = iB_{ij} \) and \( C_{5\bar{3}} = iB_3 \); here, \( i, j, m = 1, 2 \). The gauge kinetic function for the observable and hidden sector is:

\[ f_{\alpha\beta} = S \delta_{\alpha\beta} , \] (24)

and the superpotential \( W \) is

\[ W = c \ g_e \epsilon_{ij} (\epsilon_{mn} d_{abc})C_i^{ma}C_j^{nb}C^c , \] (25)

where \( \epsilon_{ij} \) is the antisymmetric tensor of SU(2).

The five family Kähler manifold defined from above is

\[ \frac{SU(1,1)}{U(1)} \times \frac{SU(2,2+2n)}{SU(2) \times SU(2+2n) \times U(1)} \times \frac{SU(1,1+n)}{SU(1+n) \times U(1)} , \] (26)

where \( n=27 \).

(III) \( Z_{12} \) symmetry. \( Z_{12} \) symmetry is generated by the \( 3 \times 3 \) matrix:

\[ M = \text{diag}(ie^{i2\pi/3}, -ie^{i2\pi/3}, e^{i2\pi/3}) , \] (27)

The model contains three 27 families. In other words, the massless fields in 5-dimension are gravitational multiplet, the universal hypermultiplet and two vector multiplets, corresponding to the Hodge-Betti number is \( h_{(1,1)} = 3 \) and \( h_{(2,1)} = 0 \), and the observable gauge group is \( U(1) \times U(1) \times E_6 \) (the hidden sector gauge group is \( E_8 \)). In the Einstein frame after rescaling, the \( Z_6 \) invariant metric tensor is:

\[ g_{\mu\nu}^{(11)} = e^{-\gamma-2(\sigma_1+\sigma_2+\sigma_3)/3}g_{\mu\nu}^{(4)} ; \quad g_{11,11}^{(11)} = e^{2\gamma-2(\sigma_1+\sigma_2+\sigma_3)/3} ; \quad g_{ij}^{(11)} = e^{\sigma_i} \delta_{ij} , \] (28)
where \( i, j = 1, 2, 3 \). The shape of the orbifold is described by the three parameters: \( \sigma_i \). The massless modes of the three form in 4-dimension is \( C_{5\mu\nu} \) and \( C_{5\bar{i}\bar{j}} \). The Kähler potential for 4-dimensional SUGRA is:

\[
K = -\ln [S + \bar{S}] - \sum_{i=1}^{3} \ln [T_i + \bar{T}_i - 2C^a_i C^{\bar{a}}_i],
\]

where

\[
S = G^{1/2} + i24\sqrt{2}D,
\]

and

\[
T_i = e^{-\gamma - (2\sigma_1 + 2\sigma_2 + 3\sigma_3)/3} - i6\sqrt{2}B_i + C^a_i C^{\bar{a}}_i,
\]

where \( \frac{1}{4!} e^{2(\sigma_1 + \sigma_2 + \sigma_3)} G_{\mu\nu\rho\delta} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma D \), \( C_{5\bar{i}\bar{j}} = iB_i \). The gauge kinetic function for the observable and hidden sector is:

\[
f_{\alpha\beta} = S \delta_{\alpha\beta}
\]

and the superpotential \( W \) is

\[
W = c g_{d \, abc} C^a_1 C^b_2 C^c_3.
\]

The three family Kähler manifold defined from above is

\[
\frac{SU(1, 1)}{U(1)} \times \left( \frac{SU(1, 1 + n)}{SU(1 + n) \times U(1)} \right)^3,
\]

where \( n = 27 \). Also note that, three families have charges \((1, 1)\), \((-1, 1)\), \((0, -2)\) under two \( U(1) \), respectively.

(IV) \( SU(2) \times U(1) \) symmetry. This model contains two 27 families. In other words, the massless fields in 5-dimension are gravitational multiplet, the universal hypermultiplet and the one vector multiplet, corresponding to the Hodge-Betti number is \( h_{(1,1)} = 2 \) and \( h_{(2,1)} = 0 \), and the observable gauge group is \( U(1) \times E_6 \) (the hidden sector gauge group is \( E_8 \)). In the Einstein frame after rescaling, the \( SU(2) \times U(1) \) invariant metric tensor is:

\[
g_{\mu\nu}^{(11)} = e^{-\gamma -(4\sigma_1 + 2\sigma_2 + 3\sigma_3)/3} g_{\mu\nu}^{(4)}; \quad g_{11,11}^{(11)} = e^{2\gamma -(4\sigma_1 + 2\sigma_2 + 3\sigma_3)/3},
\]

\[
g_{ij}^{(11)} = e^\sigma \delta_{ij}; \quad g_{3\bar{l}}^{(11)} = e^{\sigma_3} \delta_{3\bar{l}},
\]

where \( i, j = 1, 2 \) and \( l = 1, 2, 3 \). The shape of the orbifold is described by the two parameters: \( \sigma \) and \( \sigma_3 \). The massless modes of the three form in 4-dimension is \( C_{5\mu\nu} \) and \( C_{5\bar{i}\bar{j}} \). The Kähler potential for 4-dimensional SUGRA is

\[
K = -\ln [S + \bar{S}] - 2\ln [T + \bar{T} - 2C^a_1 C^{\bar{a}}_{1\bar{T}}] - \ln [T_3 + \bar{T}_3 - 2C^a_3 C^{\bar{a}}_3],
\]

where
where

\[ S = G \frac{1}{\sqrt{2}} + i 24 \sqrt{2} D, \]  

(38)

and

\[ T = e^{\gamma - (2\sigma + \sigma_3)/3 + \sigma} - i 6 \sqrt{2} B + C_T^a C_T^a, \]  

(39)

\[ T_3 = e^{\gamma - (2\sigma + \sigma_3)/3 + \sigma_3} - i 6 \sqrt{2} B_3 + C_3^a C_3^a, \]  

(40)

where \( \frac{1}{i} \epsilon^{\alpha\beta\gamma} G_{\mu\nu\rho} = \epsilon_{\mu\nu\rho} \partial^\sigma D, \) \( C_5^{a\bar{a}} = i B \) where \( i = 1, 2 \) and \( C_{53} = B_3. \) The gauge kinetic function for the observable and hidden sector is

\[ f_{\alpha\beta} = S \delta_{\alpha\beta} \]  

(41)

and the superpotential \( W \) is

\[ W = c g_c d_{abc} C_T^a C_T^b C_3^c. \]  

(42)

The two family Kähler manifold defined from above is

\[ SU(1,1) \times (SU(1+2) \times U(1))^2 \]  

(43)

where \( n = 27. \) In addition, \( C_T \) and \( C_3 \) have +1 and -2 \( U(1) \) charges, respectively.

### 3 Some SUGRA Terms of the order \( \kappa^{4/3} \)

One can not compute the full effective action to the order of \( \kappa^{4/3} \) because the original 11-dimensional theory is generally constructed up to terms of the order of \( \kappa^{4/3}. \) Here, we just consider order \( \kappa^{4/3} \) terms involving gauge or gauge matter fields, and do not consider higher derivative terms and terms of higher mass dimension [18].

To order \( \kappa^{4/3} \), we should consider the deformed Calabi-Yau manifold or orbifold in order to have \( N=1 \) supersymmetry in 4-dimension. Moreover, the effective action of M-theory can be analyzed in the simple case by expanding it in one dimensionless variable [8, 9, 18]:

\[ \epsilon_1 = \kappa^{2/3} \pi \rho / V^{2/3} \]  

(44)

To calculate the 4-dimensional effective action, one first expands the 11-dimensional action in powers of \( \kappa^{2/3} \) to obtain the compactification solution which is expanded in \( \epsilon_1. \) The subsequent Kaluza-Klein reduction of the 11-dimensional action for this compactification solution will lead to the desired 4-dimensional effective action. The compactification which has been down is following above line.
In the $Z_3$ symmetry, the M-theory correction will be very complicated, and one should also keep in mind that there are only three families in the low energy phenomenology. Therefore, in the following parts, we will only pay attention to the $Z_{12}$ case. In this case, the physical volume of the Calabi-Yau manifold is $V e^{\sigma_1 + \sigma_2 + \sigma_3}$, so we will have more expansion variables. The technical details will be given elsewhere [21]; here we just want to consider the general picture. We write down the Kähler potential, superpotential and gauge kinetic function. Notice that we can also write the $Z_{12}$ invariant metric as the following:

$$g^{(11)}_{\mu\nu} = e^{-\gamma - 2(\sigma_1 + \sigma_2 + \sigma_3)/3} g^{(4)}_{\mu\nu}; \quad g^{(11)}_{11,11} = e^{2\gamma - 2(\sigma_1 + \sigma_2 + \sigma_3)/3}; \quad g^{(11)}_{ij} = e^{\sigma_i} g^{f}_{ii} \delta_{ij},$$

where $i, j = 1, 2, 3$. $g^{f}_{ii}$ is a function of the coordinates in 6-dimensional compact space which is also satisfy $Z_{12}$ invariance and $V$ in above paragraph can be expressed as $V = \int d^6x \sqrt{g^f}$.

In the Einstein frame after rescaling, the Kähler potential for 4-dimensional SUGRA is

$$K = -\ln [S + \bar{S}] - \sum_{i=1}^{3} \ln [T_i + \bar{T}_i - 2C_{ia}^* C_i^a + 2\alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3]$$

$$- \frac{2}{3} \frac{1}{S + \bar{S}} \left( \sum_{j=1}^{3} \alpha_j (T_j + \bar{T}_j) \right) \sum_{i=1}^{3} \left( C_{ia}^* C_i^a / T_i + \bar{T}_i \right),$$

where

$$S = G^{1/2} + i24\sqrt{2} D,$$

and

$$T_i = e^{\gamma - (\sigma_1 + \sigma_2 + \sigma_3)/3 + \sigma_i} - i6\sqrt{2} B_i + C_{ia}^* C_i^a + \alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3,$$

The gauge kinetic functions for the observable and hidden sector sector are:

$$f_{\alpha\beta}^o = (S + \alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3) \delta_{\alpha\beta},$$

$$f_{\alpha\beta}^h = (S - \alpha_1 T_1 - \alpha_2 T_2 - \alpha_3 T_3) \delta_{\alpha\beta}.$$

Exactly speaking, we should write the gauge kinetic function as:

$$f_{\alpha\beta}^o = (S + \sum_{i=1}^{3} \alpha_i (T_i - C_{ia}^* C_i^a)) \delta_{\alpha\beta},$$

$$f_{\alpha\beta}^h = (S - \sum_{i=1}^{3} \alpha_i (T_i - C_{ia}^* C_i^a)) \delta_{\alpha\beta}.$$
and the superpotential $W$ is

$$W = c \, g_c \, d_{abc} \, C_1^a C_2^b C_3^c .$$

The three expansion variables are

$$\epsilon_3 = \kappa^{2/3} e^{-(\sigma_1 + 4\sigma_2 + 4\sigma_3)/3},$$

$$\epsilon_4 = \kappa^{2/3} e^{-(4\sigma_1 + \sigma_2 + 4\sigma_3)/3},$$

$$\epsilon_5 = \kappa^{2/3} e^{-(4\sigma_1 + 4\sigma_2 + \sigma_3)/3} .$$

Of course, if we chose the following metric:

$$g^{(11)}_{\mu\nu} = e^{-\gamma - 2(\sigma_1 + \sigma_2 + \sigma_3)/3} g_{(4)}^{\mu\nu}; \quad g^{(11)}_{11,11} = e^{2\gamma - 2(\sigma_1 + \sigma_2 + \sigma_3)/3}; \quad g^{(11)}_{i\bar{j}} = e^{(\sigma_i + \sigma_j)/2} g_{i\bar{j}}^{CY}$$

we will also have a model with moduli: $S$, $T_1$, $T_2$ and $T_3$. However, in this case, the Kähler potential and gauge kinetic function will be a little more complicated and we will have 6 expansion variables.

### 4 Effective Superpotential by the Gaugino Condensation

For the case of two moduli $S$ and $T$ in M-theory, the superpotential and gaugino condensation were analyzed. The arguments should not be changed when we consider $Z_{12}$ symmetry. The superpotential in M-theory is similar to that in the weakly coupled string theory because to the leading order and next to the leading order, their results are similar. In short, one can write the following superpotential as previously:

$$W = m_{11} (c + h \exp(-\frac{3}{2b_0 g_h}))$$

Where $m_{11}$ is the 11-dimensional Planck scale and $m_{11} = \kappa^{2/9}$, $b_0$ is the coefficient of the one-loop $\beta$ function in the hidden sector, and $h$ and $c$ are constants. Notice 11-dimensional Lagrangian of the Yang-Mills Fields in the hidden sector is

$$L_B = \frac{1}{2\pi(4\pi\kappa^2)^2} \int_{M^{10}} d^{10}x \sqrt{g} \frac{1}{4} F^a_{\alpha\beta} F^{a\alpha\beta}$$

4In order to keep the soft breaking terms not too large, one might consider $c = 0$, i.e., the following superpotential:

$$W = m_{11} \, h \, \exp(-\frac{3}{2b_0 g_h})$$
The 4-dimensional gauge coupling in the hidden sector is

\[ g_{h}^{-2} = \frac{V_{p}^{h}}{2\pi(4\pi\kappa^{2})^{\frac{3}{8}}} \quad (61) \]

Where \( V_{p}^{h} \) is the physical 6-dimensional volume in the hidden sector and \( V_{p}^{h} = Vf^{h} \). Assuming \( V^{-1/6} = m_{11} \), we have:

\[ g_{h}^{-2} = \frac{f^{h}}{2\pi(4\pi)^{\frac{3}{8}}} \quad (62) \]

Having the above information, one can obtain the non-perturbative superpotential:

\[ W = m_{11}(c + h \exp\left(-\frac{3}{b_{0}(4\pi)^{3/8}}(S - (\sum_{i=1}^{3} \alpha_{i}T_{i})))\right)) \quad (63) \]

Here, we would like to reemphasize one point. Although the above results are similar to those in the weakly coupled string because the low energy action in M-theory is similar to that of weakly coupled string, the difference between the M-theory and the weakly coupled string is the magnitude of the corrections. In M-theory, they are relatively very large.

5 Soft Terms

We would like to calculate the soft terms that arise in above case. We assume the auxiliary components \( F^{S} \) and \( F^{T_{i}} \) of the moduli superfields \( S \) and \( T_{i} \) could get non-vanishing vev and break SUSY; even though we do not know much about the details of SUSY breaking process, we can get the most important information: soft terms.

Apply the standard soft term formulas for the SUGRA model \[28, 29, 30\], one can calculate the soft terms straightforwardly. We do not consider the bilinear parameter \( B \) because it dependent on the particular mechanism which could generate the \( \mu \)-term.

First, we write the Kähler potential in another form:

\[ K = \hat{K} + \hat{K}_{ii}C_{ia}C_{i}^{a} \quad (64) \]

where \( i=1, 2, 3 \) and

\[ \hat{K} = -\ln [S + \bar{S}] - \sum_{i=1}^{3} \ln [T_{i} + \bar{T}_{i}] \quad (65) \]

and

\[ \hat{K}_{ii} = (2 + \frac{2}{3S + \bar{S}}(\sum_{j=1}^{3} \alpha_{j}(T_{j} + \bar{T}_{j})))\frac{1}{T_{i} + \bar{T}_{i}} \quad (66) \]
The tree level supergravity scalar potential is

\[ V(\phi, \phi^*) = (F_N \bar{N} F^M - 3e^G), \]  

(67)

where G is the Kähler function, \( K_{MN} \equiv \partial_N \partial_M K \). The auxiliary fields \( F^M = e^{G/2} K^{MP} G_P \) where \( K^{MP} \) is the inverse of the \( K_{PM} \). Consider \( C_i^a > 0 \) and gravitino mass \( m_{3/2} = e^{G/2} \), we can write \( F^S \) and \( F^T_i \) as the following:

\[ F^S = \sqrt{3} \sin \theta \ m_{3/2} C(S + \bar{S}) e^{-i\delta_S}, \]  

(68)

\[ F^{T_1} = \sqrt{3} \cos \theta \ \sin \beta \ m_{3/2} C(T_1 + \bar{T}_1) e^{-i\delta_{T_1}}, \]  

(69)

\[ F^{T_2} = \sqrt{3} \cos \theta \ \cos \beta \ m_{3/2} C(T_2 + \bar{T}_2) e^{-i\delta_{T_2}}, \]  

(70)

\[ F^{T_3} = \sqrt{3} \cos \theta \ \cos \beta \ m_{3/2} C(T_3 + \bar{T}_3) e^{-i\delta_{T_3}}, \]  

(71)

where \( C = (1 + \frac{V}{3m_{3/2}^2})^{1/2} \) and V is the tree level vacuum energy density. Therefore, the normalized soft gaugino masses and the un-normalized soft scalar masses, and the trilinear parameters are:

\[ M_{1/2} = \frac{F^S + \sum_{i=1}^{3} \alpha_i F^{T_i}}{(S + \bar{S}) + \sum_{j=1}^{3} \alpha_j (T_j + \bar{T}_j)}, \]  

(72)

\[ m_i^2 = (m_{3/2}^2 + V) \tilde{K}_{ii} \]  

\[-F^m(\partial_m \partial_n \tilde{K}_{ii} - \partial_m \tilde{K}_{ii} \frac{1}{\tilde{K}_{ii}} \partial_n \tilde{K}_{ii}) F^m, \]  

(73)

\[ A_{abc} = F^m(\tilde{K}_m - \sum_{i=1}^{3} \frac{1}{\tilde{K}_{ii}} \partial_m \tilde{K}_{ii}), \]  

(74)

where the non-zero \( \tilde{K}_m, \partial_m \tilde{K}_{ii} \) and \( \partial_m \partial_n \tilde{K}_{ii} \) are

\[ \tilde{K}_S = \frac{-1}{S + \bar{S}} , \]  

(75)

\[ \tilde{K}_{T_i} = \frac{-1}{T_i + \bar{T}_i} , \]  

(76)
\[ \partial_s \bar{K}_{ii} = \frac{-2}{3} \frac{1}{(S + \bar{S})^2} \left( \sum_{k=1}^{3} \alpha_k(T_k + \bar{T}_k) \right) \frac{1}{T_i + \bar{T}_i}, \]  
\hspace{1em} (77)  

\[ \partial_S \partial_S \bar{K}_{ii} = \frac{4}{3} \frac{1}{(S + \bar{S})^2} \left( \sum_{k=1}^{3} \alpha_k(T_k + \bar{T}_k) \right) \frac{1}{T_i + \bar{T}_i}, \]  
\hspace{1em} (78)  

\[ \partial_{\bar{T}_i} \partial_S \bar{K}_{ii} = \frac{2}{3} \frac{1}{(S + \bar{S})^2} \left( \sum_{j=1, j \neq i}^{3} \alpha_j(T_j + \bar{T}_j) \right) \frac{1}{(T_i + \bar{T}_i)^2}, \]  
\hspace{1em} (79)  

\[ \partial_{T_j} \partial_S \bar{K}_{ii} = \frac{-2}{3} \frac{\alpha_j}{(S + \bar{S})^2} \frac{1}{T_i + \bar{T}_i}, \]  
\hspace{1em} (80)  

\[ \partial_{T_j} \bar{K}_{ii} = \frac{2}{3} \frac{\alpha_j}{(S + \bar{S})} \frac{1}{T_i + \bar{T}_i}, \]  
\hspace{1em} (81)  

\[ \partial_{\bar{T}_i} \partial_{T_j} \bar{K}_{ii} = \frac{-2}{3} \frac{\alpha_j}{(S + \bar{S})} \frac{1}{(T_i + \bar{T}_i)^2}, \]  
\hspace{1em} (82)  

\[ \partial_{\bar{T}_i} \bar{K}_{ii} = -(2 + \frac{2}{3}) \frac{1}{(S + \bar{S})} \]  
\[ \left( \sum_{j=1, j \neq i}^{3} \alpha_k(T_j + \bar{T}_j) \right) \frac{1}{(T_i + \bar{T}_i)^2}, \]  
\hspace{1em} (83)  

\[ \partial_{\bar{T}_i} \partial_{\bar{T}_i} \bar{K}_{ii} = \left( 4 + \frac{4}{3} \right) \frac{1}{(S + \bar{S})} \]  
\[ \left( \sum_{j=1, j \neq i}^{3} \alpha_k(T_j + \bar{T}_j) \right) \frac{1}{(T_i + \bar{T}_i)^3}, \]  
\hspace{1em} (84)  

where in all above equations, \( j \neq i \). From the above calculation, we notice that, for the Yukawa coupling, the universal condition is satisfied. But it seems that we might not have the scalar mass universality. However, noticing these conditions may be reasonable: \(< T_1 > \sim < T_2 > \sim < T_3 > \) and \( \alpha_1 \sim \alpha_2 \sim \alpha_3 \), we conclude that scalar mass universality might be approximately satisfied or the violation of the scalar mass
universality may be very small. In addition we would like to consider the dilaton-
induced SUSY-breaking, because it is simple, and these boundary conditions for soft
terms may be obtained for any compactification of M-theory or weakly coupled string.
Moreover, these conditions are universal, gauge group and flavour independent. But,
now the soft masses obtained for the scalars might be negative, which lead to tachyons
unlike the previous results. We define $F^S = \sqrt{3} m_{3/2} (S + \tilde{S})$ and require $V = 0$. Then,
the normalized soft terms are:

$$M_{1/2} = \frac{\sqrt{3} m_{3/2} (S + \tilde{S})}{(S + \tilde{S}) + \sum_{j=1}^{3} \alpha_j (T_j + \tilde{T}_j)}$$

(85)

$$m_i^2 = m_{3/2}^2 - 3 m_{3/2}^2 (2 f_i - f_i^2) ,$$

(86)

$$A_{abc} = \sqrt{3} m_{3/2} (-1 + f_1 + f_2 + f_3) ,$$

(87)

where

$$f_i = \frac{2}{3} \frac{1}{K_i} \frac{1}{(S + \tilde{S})} \frac{1}{(T_i + \tilde{T}_i)} \left( \sum_{j=1}^{3} \alpha_j (T_j + \tilde{T}_j) \right) .$$

(88)

6 Conclusion

In the leading order, following the compactification line from 11-dimensional to 5-
dimension, then to 4-dimension, we discuss various orbifold compactifications of the
M-theory suggested by Horava and Witten on $T^6/Z_3$, $T^6/Z_6$, $T^6/Z_{12}$ and the comp-
actification by keeping singlets under $SU(2) \times U(1)$ symmetry, whose Hodge-Betti
numbers are $h_{(1,1)} = 9, 5, 3, 2$ and $h_{(2,1)} = 0$ respectively, then the compactification on
$S^1/Z_2$. Although the original Lagrangian is order $\kappa^{2/3}$, we also discuss next-leading
order Kähler potential, superpotential, and gauge kinetic function in the $Z_{12}$ case.
In addition, we discuss the non-perturbative superpotential related to the $S$ and $T_i$.
Finally, we calculate the SUSY breaking soft terms and find out that the universality
of the scalar masses will be violated, but the violation might be very small.

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