Information theoretic approach for accounting classification

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Abstract

In this paper we consider an information theoretic approach for the accounting classification process. We propose a matrix formalism and an algorithm for calculations of information theoretic measures associated to accounting classification. The formalism may be useful for further generalizations, and computer based implementation. Information theoretic measures, mutual information and symmetric uncertainty, were evaluated for daily transactions recorded in the chart of accounts of a small company during two years. Variation in the information measures due the aggregation of data in the process of accounting classification is observed. In particular, the symmetric uncertainty seems to be a useful parameter for comparing companies over time or in different sectors; or different accounting choices and standards.

Keywords: Information theory, Accounting, accounting classification, mutual information

1. Introduction

Information Theory [1] provides useful and unifying concepts that has been applied in many fields, including Physics, Engineering, Computer science, Statistics and Data Analysis, Linguistics, Marketing, Economics, and Complex Systems research in general [2–19]. Here, we are including Accounting as a potential field for interdisciplinary research where concepts and methods from information theory may have interesting applications. In
fact, Accounting is considered an information science used to collect, classify, and manipulate financial data for organizations and individuals \[20\]. Nevertheless, an approach based on information theory were reported only in some academic research from 1960s \[21–25\]. Specifically, in ref \[21\] the authors proposed the viewing of Accounting as a communication process; Lee and Bedford \[23\] proposed a communication channel model to describe accounting classification. However, as far as we know, neither applications using real data nor further research in the field were performed.

Accounting classification is a relevant subject for international accounting harmonization studies \[26–28\]. In \[26\], the author observed a high level of complexity of an accounting system due to the conceptual and methodological pluralism found in accounting classification attempts. By considering fifteen national systems it was performed a cluster analysis and applied a nonmetric multidimensional scaling technique, obtaining a two-dimensional map revealing similarities (dissimilarities) between the systems. In \[27\] and \[28\] the author investigated international differences in the way that countries and companies have responded to the International Financial Reporting Standards (IFRS). They conclude that some countries have entirely abandoned national accounting rules in favor of IFRS. It is also observed that different national systems of IFRS practices are emerging, and only further will be classified. Thus, a quantitative measure of information may provide a more objective way to compare different accounting systems.

As pointed out by Demski \[20\], there is an absence of modern information science into the Accounting curriculum. In the research domain, there are several examples involving Accounting that can be related to probability, and allowing for an information theoretic perspective \[29, 30\]. Actually, the value of information \[31\] and the information content of inside traders before and after the Sarbanes-Oxley Act of 2002 (SOX) \[32\] have been previously considered. However, no investigation on information based on Information Theory was done. Hence, if compared with applications in other areas, information theory was not enough explored in Accounting. Besides, the technological developments including the storage and access of data, computer time processing, and the evolutions of information systems, may allow the implementation and inclusion of results that previously were not possible. Therefore, attempting to develop new tools to support further research, it seems appropriate to review and further explore earlier studies.

The aim of this work is to further explore the information theoretic approach for accounting classification. A matrix formalism and an algorithm for
calculations of information theoretic measures are introduced. Although the objects were correctly defined in Ref. [23], a matrix formalism was avoided. Our formalism may be useful for further generalizations, and computer based implementations. The formalism is applied to evaluate the information theoretic measures in classifying the transactions of a small company. The mutual information and the symmetric uncertainty are obtained for each level of classification in the chart of accounts, allowing us to observe their variation due to aggregation of data in the process of accounting classification. To the best of our knowledge, this is the first calculation of information theoretic measures for an accounting classification process using an empirical data set. Moreover, we indicate that the symmetric uncertainty may be a useful parameter for comparing companies over time or in different sectors; or different accounting choices and standards.

The article is organized as follows: In section 2 some basic concepts of Information Theory are presented. In section 3 the work of Lee and Bedford connecting the processes of accounting classification and the theory of information is revisited. The matrix formalism and an algorithm for calculations are described in section 4. In section 5 information theoretic measures are evaluated considering the events registered in a five levels chart of accounts of a small company. In section 6 some concluding remarks are presented. Finally, the source code for the R software environment [33] is included as an appendix.

2. Shannon entropy and the measure of information

In this section some basic definitions of Information Theory are presented. A complete and more detailed treatment may be found elsewhere [8, 9].

Given an event $A$ occurring with probability $P(A)$, it is possible to associate a number, $- \log_2 P(A)$, to quantify the information associated with the occurrence of $A$. This definition agrees with the intuitive idea that the information content of independent events is the sum of the information of each event. In order to quantify the information content of a set of events, Shannon introduced the concept of average amount of information or entropy [1].

**Definition 2.1** Given the set of events $X = \{x_1, x_2, \ldots, x_n\}$ with probabilities $\{P(x_1), P(x_2), \ldots, P(x_n)\}$, the entropy $H(X)$ associated to $X$ is
defined as the mean information of $X$:

$$H(X) = -\sum_{i=1}^{n} P(x_i) \log_2 P(x_i).$$  \hfill (1)

Entropy may be interpreted as a measure of the uncertainty associated to a set of random events. The unit of information using the logarithm function to base two is called bit. Since $P(x_i)$ may be zero, $H(X)$ could be indeterminate in the above definition so, when $P(x_i) = 0$, the value zero is assigned to $P(x_i) \log_2 P(x_i)$. For two or more sets of events described by a joint probability distribution, the joint entropy may be defined as follows.

**Definition 2.2** Given two sets $(X, Y)$ of random events, with the joint joint probability distribution $P(X, Y)$, the joint entropy between $X$ and $Y$ is defined as

$$H(X, Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 P(x_i, y_j),$$  \hfill (2)

where $n$ and $m$ are the total number of distinct events in the $X$ and $Y$ sets, respectively.

For independent events the joint probability distribution factorizes and the joint entropy becomes the sum of the entropy of each set of events, i.e., $H(X, Y) = H(X) + H(Y)$. Moreover, for joint distributions, if the information about one variable is conditioned to the information about another variable, it is useful to define the conditional entropy.

**Definition 2.3** Given two sets $(X, Y)$ of random events, the conditional entropy $H(X/Y)$ is defined as

$$H(X/Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 P(x_i/y_j),$$  \hfill (3)

where $P(x_i/y_j)$ is the conditional probability of a random variable $X$ assumes the value $x_i$, given that another random variable $Y$ has taken a value $y_j$.

The conditioned entropy will be useful in the accounting classification process since it involves the registration of an economic event in one account given that it comes from another account \[23\], i.e., a double entry system. The joint entropy may be expressed in terms of the conditional entropy as
\[ H(X, Y) = H(X/Y) + H(Y), \] and since \( H(X/Y) \leq H(X) \), it follows that \( H(X, Y) \leq H(X) + H(Y) \).

We conclude this section with the concept of mutual information \( I(X, Y) \), which is the amount of information that one random variable contains about another random variable.

**Definition 2.4** Given two sets \((X, Y)\) of random events, the mutual information \( I(X/Y) \) between \( X \) and \( Y \) is defined as

\[
I(X, Y) = H(X) - H(X/Y) = H(X) + H(Y) - H(X, Y). \tag{4}
\]

Mutual information is the reduction in the uncertainty of one random variable due to the knowledge of the other \([9]\). By considering the joint probability distribution the mutual information may be written as

\[
I(X, Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i)P(y_j)}. \tag{5}
\]

For independent events, the joint probability distribution factorizes as \( P(X, Y) = P(X)P(Y) \), and \( I(X, Y) = 0 \). Thus, mutual information may be used as a measure of the degree of association between random events. It is also useful to consider a normalized form of Mutual Information \([34]\), the symmetric uncertainty, which is basically a measure of correlation defined as

\[
U(X, Y) = 2 \frac{I(X, Y)}{H(X) + H(Y)}. \tag{6}
\]

Symmetric uncertainty lies between 0 and 1, and it is the information shared between \( X \) and \( Y \) relatively at all information contained in both \( X \) and \( Y \).

### 3. Accounting classification and information theory

The mathematical model for the accounting classification process and its connection with information theory was first introduced by Lee and Bedford \([23]\). The overall process can be formalized by means of matrix algebra as will be shown in this paper. The first step of an accounting process in a firm is the registration of economic events.

**Definition 3.1** An elementary economic event is defined as any activity that an accountant records. The economic events of a firm are represented by the set \( X = \{x_1, x_2, \ldots, x_R\} \) of elementary economic events.
The accounting classification involves the registration of economic events as a debit or a credit. Then, the events may have several possibilities of classification.

**Definition 3.2** The set \( Y = \{y_1, y_2, \ldots, y_S\} \) represents the possibilities of classification as a debit in one account \( a_i \) and a credit in another account \( a_j \). The maximum number of classifications in \( N_a \) accounts is given by \( S = N_a(N_a - 1) \), which is the number of permutations of \( N_a \) accounts taken two at a time.

The function of accountant is to designate, among \( N_a \) accounts, which one represent the structure of the financial state, with at least two accounts involved in each transaction [23]. With the definitions of the economic events set of a firm and the classification set it is now possible to define the accounting classification process.

**Definition 3.3** The accounting classification processes is a map \( f : X \rightarrow Y \) relating each economic event \( x_i \) to an element \( y_j \).

Theory of information is based in the concept of information content of a set of events described by a probability distribution. In order to quantify the information content of an account transaction one need to introduce the probability associated to the classification process. In Ref. [23] the probabilities were introduced as subjective numbers depending on the accountant decision. We remove the subjectivity by looking directly to the transactions frequencies registered by the company. These distributions of frequencies should reflect the effects of accounting standards (or even the accountant subjectivity). Then, the probability of an economic event is identified with its relative frequency, i.e.,

**Definition 3.4** The probability associate to an account transaction of an event \( x_i \) is given by \( P(x_i) = n_i/R \), where \( n_i \) is number of occurrences of the event \( x_i \) in the total number of events \( R \).

Indeed, such probabilities can be obtained from some information system. The set of economic events is organized in the so called chart of accounts [35], a listing of the accounts names \( a_i \) that a company has identified and made available for recording transactions in its general ledger. A company has the flexibility to tailor its chart of accounts to best suit its needs. A specific economic event in the chart of accounts is coded as a sequence of numbers. Figure [1] illustrates a typical structure of a chart of accounts with five lev-
els encoded in a string of numbers with five pieces separated by dots. At the last level (Level 5) each economic event receives a specific code. For example, from Figure 1 the event “Salaries and Wages Payable” is encoded as 02.01.03.001.00006. At the intermediate levels the aggregation increases until reaching the first level (Level 1), which is the most aggregated one containing the events grouped in the main accounts. The classification process can be analyzed at each level, by introducing the conditional probability that a given economic event $x_i$ is classified as $y_j$, namely $P(y_j/x_i)$.

![Figure 1: Illustration of a typical chart of accounts with five levels. Here, economic events are encoded by a sequence of numbers with five pieces separated by dots.](image)

The attribution of probabilities to the classification event of an account transaction permits to associate quantities such as entropy and mutual information to characterize the process of accounting classification at different levels. Then, at least formally, the accounting classification process can be analyzed as a process of gain and loss of information.

A communication channel is a system in which the output depends probabilistically on its input. It is characterized by the probability transitions $P(Y/X)$ that determines the conditional distribution of the output given the input. To summarize, the following association of accounting classification and information theory is possible: $H(X)$ is the entropy associated to the set
of economic events $X$, $H(Y)$ is the entropy associated to the classification of the economic events in the set of classifications $Y$, $H(X,Y)$ is the mutual entropy between the economic events and its classification as a debit or credit in the set $Y$, and $H(X/Y)$ is the entropy associated to the economic events $X$ given their classification $Y$. Finally, $I(X,Y)$ is the average information about $X$ conveyed through the channel of classification $Y$.

4. Algorithm for information analysis

The accounting process of classification is, in this way, an information system in which the data from economic events are aggregated into specific accounts. During the accounting classification, the transmission of information is realized through a communication channel [21]. In order to obtain the information measures associated to the process of accounting classification we propose a matrix formalism and an algorithm for calculations. The steps to obtain the amount of information transmitted in this channel can be structured in the flowchart of Figure 2. The goal in applying this algorithm is to obtain the average information about $X$ conveyed through the channel by $Y$, namely $I(X,Y)$. We have assumed that $P_X$, the probabilities for the occurrence of economic events $X$, as well as $P_{Y/X}$, the probability that the events $X$ are classified as $Y$, are obtained from their frequencies of occurrence and supplied by some information system. All others quantities are obtained from these inputs. In the following the mathematical expressions and the matrix algebra to obtain the quantities are presented.

4.1. The probability block

This first algorithm block consists of, from the input probability matrices, determining additional probabilities matrices, suitable to be used in the determination of entropy functions (see Fig. 2). The first input is the $P_X$ matrix, with entries given by the probability of occurrence of each distinct economic event $x_i$, namely $P(x_i)$. In this way, considering the maximum number of economic events - $R$, $P_X$ can be written as an $1 \times R$ matrix

$$P_X = \begin{bmatrix} P(x_1) & P(x_2) & \cdots & P(x_R) \end{bmatrix}. \quad (7)$$

Another input in this algorithm is $P_{Y/X}$, a $R \times S$ matrix, with $S$ the maximum number of classifications, and entries given by the probability that an event
Figure 2: Flowchart to obtain information measures associated to accounting classification process.

\( x_i \) is classified as \( y_j \), i.e.,

\[
\begin{bmatrix}
P(y_1/x_1) & P(y_2/x_1) & \cdots & P(y_S/x_1) \\
P(y_1/x_2) & P(y_2/x_2) & \cdots & P(y_S/x_2) \\
\vdots & \vdots & \ddots & \vdots \\
P(y_1/x_R) & P(y_2/x_R) & \cdots & P(y_S/x_R)
\end{bmatrix}
\]

(8)

These two probabilities matrices, \( \mathbf{P}_X \) and \( \mathbf{P}_{Y/X} \), can be obtained from the information system, but more generally their elements can be viewed as parameters for theoretical studies on classification. In fact, the elements are related to the economic events frequencies, and the adopted accounting classification procedure.

Following the flowchart in Fig. 2, the \( 1 \times S \) probability matrix, \( \mathbf{P}_Y = [P(y_1) \ P(y_2) \ \cdots P(y_S)] \) can be calculated from the input by a matrix mul-
The last probability matrix to be determined in this block is the conditional probability matrix $P_{X/Y}$, a matrix defined by the product

$$P_{X/Y} = P_{X/P} P_{Y/X} P_{Y^{-1}}.$$ (10)

In Eq. (10), $P_{X/D}$ and $P_{Y^{-1}}$ are $R \times R$ and $S \times S$ diagonal matrices with entries $[P_{X/D}]_{i,j} = P(x_i) \delta_{i,j}$, $(i, j = 1, 2, ..., R)$; and $[P_{Y^{-1}}]_{i,j} = \delta_{i,j} / P(y_i)$, $(i, j = 1, 2, ..., S)$, respectively, and where $\delta_{i,j}$ is the usual Kronecker function, which is 0 unless $i = j$, when it is 1.

In order to simplify the notation, it is convenient to specify two additional matrices. The first one is the $R \times 1$ matrix given by

$$L_{X} = \begin{bmatrix} -\log_2 P(x_1) \\ -\log_2 P(x_2) \\ \vdots \\ -\log_2 P(x_R) \end{bmatrix}. \tag{11}$$

The other one is the $S \times R$ matrix given by

$$L_{XY} = \begin{bmatrix} -\log_2 P(x_1/y_1) & -\log_2 P(x_2/y_1) & \cdots & -\log_2 P(x_R/y_1) \\ -\log_2 P(x_1/y_2) & -\log_2 P(x_2/y_2) & \cdots & -\log_2 P(x_R/y_2) \\ \vdots & \vdots & \ddots & \vdots \\ -\log_2 P(x_1/y_S) & -\log_2 P(x_2/y_S) & \cdots & -\log_2 P(x_R/y_S) \end{bmatrix}. \tag{12}$$

When $P(x_i) = 0$ or $P(x_i/y_j) = 0$ the corresponding entries in (11) or in (12) are zero.

4.2. The entropy block

By considering the matrices defined in the previous section, in this block the goal is to obtain the entropies $H(X)$ and $H(X/Y)$.

The a priori entropy of the source $H(X)$ is a number (in bits units) calculated by the matrix multiplication,

$$H(X) = P_X L_X. \tag{13}$$

The a posteriori entropy $H_{X/Y}$ is a $S \times 1$ matrix with entries given by

$$[H_{X/Y}]_{j,1} = [P_{X/Y}^T L_{XY}]_{j,j}, (j = 1, 2, ..., S), \tag{14}$$
and the superscript $T$ stands for transpose. Since the output symbols $\{y_s\}$ occur with probabilities $P_Y$, an average a-posteriori-entropy, can be obtained by the following matrix multiplication

$$H(X/Y) = P_Y H_{X/Y}. \quad (15)$$

This conditional entropy $H(X/Y)$ measures the average final uncertainty of $X$ after an observation of the output produced by the input. It is straightforward, by applying the proposed algorithm, to verify that these matrices multiplications presented here are equivalent to the mathematical expressions in [23].

4.3. The information block

Following the flowchart in Fig. 2 the next step is to obtain the average information about $X$ conveyed through the channel by $Y$, namely $I(X,Y)$, which is given by $I(X,Y) = H(X) - H(X/Y)$, and the normalized symmetric uncertainty $U(X,Y)$. These quantitative parameters may be useful to compare accounting classifications in situations, such as, different accounting standards, periods, chart of accounts levels, and so on.

5. Application

In order to apply the theory to a realistic situation, we have investigated all economics events registered during the period of two consecutive years in a Brazilian small company located at São Paulo State. The data set contains 2075 daily transactions, with 1356 registered in the first year and 719 in the second year. The transactions and their accounting entries were coded according to a Brazilian specific chart of accounts with five levels.

We obtain measures of information inherent to the structure of the company chart of accounts. The company we are considering has a code with five pieces to discriminate uniquely their transactions. Thus, the classification process can be analyzed according to five different levels of aggregation. Table 1 shows some economic events $x_i$, their relative frequencies $P(x_i)$, and their classification ($y_i$) according to the chart of accounts at the Level 5. In the table, a typical classification $y_i$ is given by a pair of accounts, specified by the debt and credit columns.

The probability of economic events $X$ are classified as $Y$ is a matrix characterized by all entries being either one or zero, and having one, and only
Table 1: List of some economic events $x_i$, their frequencies ($P(x_i)$) and classifications ($y_i$).

| $x_i$                      | $P(x_i)$ | Classification - $y_i$ | Debit                        | Credit                        |
|----------------------------|----------|------------------------|------------------------------|-------------------------------|
| 1- Unemployment Compensation | 0.0014   | 01.01.01.001.00001     | 01.01.01.002.00005           |                               |
| 2- Sales in the State      | 0.0167   | 01.01.01.001.00001     | 03.01.01.001.00001           |                               |
| 3- Sales to other States   | 0.0153   | 01.01.01.001.00001     | 03.01.01.001.00002           |                               |
| 4- Resale of goods         | 0.0264   | 01.01.01.001.00001     | 03.01.01.001.00004           |                               |
| 5- Sales return            | 0.0014   | 01.01.01.001.00001     | 03.01.01.002.00007           |                               |
| 6- General Services sales  | 0.0125   | 01.01.01.001.00001     | 03.01.01.004.00001           |                               |
| 7- Loan Agreement          | 0.0014   | 01.01.01.002.00005     | 02.01.02.003.00001           |                               |
| 8- Buildings and Constructions | 0.0014 | 01.03.02.001.00002     | 01.01.01.001.00001           |                               |
| 9- Equipment and machinery | 0.0042   | 01.03.02.002.00005     | 01.01.01.001.00001           |                               |
| 10- Vehicles               | 0.0014   | 01.03.02.002.00007     | 01.01.01.001.00001           |                               |
| 11- Computing devices      | 0.0042   | 01.03.02.002.00008     | 01.01.01.001.00001           |                               |
| ...                        | ...      | ...                    | ...                          | ...                           |
| 78- 13th months salary     | 0.0014   | 04.01.03.004.00001     | 01.01.01.001.00001           |                               |
| 79- 13th months salary fees| 0.0014   | 04.01.03.004.00001     | 02.01.03.002.00002           |                               |
| 80- 13th months salary taxes receivable | 0.0014 | 04.01.03.004.00001 | 02.01.03.002.00004 |
| 81- Water Costs            | 0.0042   | 04.01.03.004.00002     | 01.01.01.001.00001           |                               |
| 82- Accountants fee        | 0.0153   | 04.01.03.004.00005     | 02.01.03.001.00006           |                               |
| 83- Energy Costs           | 0.0056   | 04.01.03.004.00020     | 01.01.01.001.00001           |                               |
| 84- Office Supply          | 0.0042   | 04.01.03.004.00033     | 01.01.01.001.00001           |                               |
| 85- Telephony Costs        | 0.0097   | 04.01.03.004.00043     | 01.01.01.001.00001           |                               |
one, non-zero element in each row. At the level 5 in the chart of accounts code each classification \( y_i \) is associated to a single event. When the accounts are aggregated, a single classification may be used to classify more than one event. However, a specific event remains classified in only one \( y_i \), and in this situation we have a deterministic accounting channel.

The algorithm proposed in section 4 was implemented using the R-Software [33], and the R-code is presented in Appendix A. The algorithm (flowchart in Fig. 2 and R-code in Appendix A) was applied to evaluate the information measures for the five levels in the chart of account. The results obtained for \( H(X/Y), I(X,Y), \) and \( U(X,Y) \) are presented in Table 2. In Figures 3-(a) and 3-(b) we plotted the mutual information \( I(X,Y) \) and the symmetric uncertainty \( U(X,Y) \), respectively, for the chart of accounts five levels. The decreasing in the mutual information value is expected according to data aggregation, the form of this decreasing is revealed in Figure 3. The company has less variety of economic events in the first year \( (R_1 = 45) \) compared to the second one \( (R_2 = 85) \), resulting in a lower entropy for the first year. The mutual information is useful if the company economic events diversification matters, mainly for classifications that are not too aggregated. Differences over years are expected if the company is not stable in their activities. The more aggregated classification, the closer are the values of mutual information. However, the study of more aggregate levels may be useful for improving financial statement analysis. So, for an analysis based on aggregated levels is convenient to use the symmetric uncertainty. Fig. 3-(b) shows that the symmetric uncertainty degree of association at the level 1 is stronger for the first year. In particular, the first level (level 1) of classification in the chart.

| CA levels | First year, \( H(X) = 4.005 \) bits | Second year, \( H(X) = 5.519 \) bits |
|-----------|-----------------------------|-----------------------------|
|           | \( H(X/Y) \) | \( I(X,Y) \) | \( U(X,Y) \) | \( H(X/Y) \) | \( I(X,Y) \) | \( U(X,Y) \) |
| 5         | 0.000 4.005 1.000 | 0.000 5.519 1.000 |
| 4         | 0.930 3.075 0.869 | 1.663 3.855 0.823 |
| 3         | 1.460 2.545 0.777 | 2.406 3.112 0.721 |
| 2         | 1.776 2.229 0.715 | 2.985 2.534 0.629 |
| 1         | 1.827 2.178 0.705 | 3.066 2.453 0.611 |
of accounts should contain only the main accounts, such as assets, liability, equity, and so on, which is common to all accounting standards. Then, the symmetric uncertainty value at the first level of classification may be used, in a financial statement analysis, to distinguish between classifications over years; or in more general situations as a parameter to compare different sectors; or different accounting choices and standards.

6. Concluding remarks

In this work a matrix formalism for the information theory formulation of the accounting classification process was presented. It was proposed an algorithm generalizing the procedure of Lee and Bedford \[23\] for the calculation of information theoretic measures in the accounting classification process. The
algorithm provides a matrix procedure suitable for software implementation integrated to information systems. The algorithm was applied to evaluate information theoretic measures, mutual information and symmetric uncertainty, for daily transactions recorded by a small company during two years. We have verified an information loss inherent to aggregation of levels in the chart of accounts. To the best of our knowledge, this is the first calculation of information theoretic measures for the accounting classification process of a company. In particular, the symmetric uncertainty at the first level of classification in a chart of accounts seems to be a useful parameter for comparing companies over time or in different sectors; or different accounting choices and standards. Furthermore, the accounts at the chart of accounts first level are commonly used to form financial and economic indexes to characterize the firm. Since the symmetric uncertainty contains the proportion of information shared between the main accounts and all the economic events, it can be used itself as a global index associated to the company. The relation between symmetric uncertainty and the financial and economic indexes deserves a further study and will be addressed in a future work.

It is worthwhile to mention that the probabilities used by the proposed procedure were entirely based on observed frequencies. On the other hand, the probabilities can be considered as parameters in a theoretical analysis of different classification standards and may be a useful quantitative tool in the searching for the \textit{a priori} most adequate level of classification.

We hope that this work may contribute to highlight Accounting as an interesting field for interdisciplinary research, and also to renew and stimulate the application of information theoretic tools in the accountancy practice.

\textbf{Appendix A. Algorithm in R language}

In the following we transcribe the \textit{\texttt{R}} code implemented to obtain the mutual information. Input files can be obtained by e-mail to the authors.

\begin{verbatim}
# Algorithm for mutual information calculation
#
# The inputs are in the csv file -----
freqs <- read.table("inputs.csv",header=TRUE,sep=";")
attach(freqs)
Tot <- length(freqs)
Tot
\end{verbatim}
# Tot are the number of columns in the dataframe "freqs"
# The Probability Block ---------------
# Px: the probability of economic events,
# the first dataframe column:
R <- length(Freqx)
R
Px <- matrix(Freqx,nrow=1,ncol=R,byrow=TRUE)
#
# PyGx Probability of y given x,
# The last S dataframe columns:
S = Tot - 1
S
PyGx <- data.matrix(freqs[,2:Tot])
#
# Py: probability of classification y
Py <- Px %*% PyGx
#
# Additional Matrices: Pxd e Pydm1
Pxd <- diag(Freqx)
Freqy <- colSums(Py)
Freqy2 <- 1/Freqy
Pydm1 <- diag(Freqy2)
#
# PxGy: probability of x, given y
Mult1 <- PyGx %*% Pydm1
PxGy <- Pxd %*% Mult1
#
# Additional matrix: Lx
FreqxLog2m <- -log2(Freqx)
Lx <- matrix(FreqxLog2m,nrow=R,ncol=1)
#
# Additional matrix: Ly
FreqyLog2m <- -log2(Freqy)
Ly <- matrix(FreqyLog2m,nrow=S,ncol=1)
#
# Transpost of PxGy
PxGyT <- t(PxGy)
#
# Additional matrix: Lxy
Lxy <- PxGy
for (i in 1:R){
  for (j in 1:S){
    if (Lxy[i,j] != 0){Lxy[i,j] <- -log2(Lxy[i,j]) }
  }
}
#
# The Entropy Block ---------------------
# H(x): Entropy for the economic events
Hx <- Px %*% Lx
print(Hx,digits=12)
#
# H(y): Entropy for classifications (not used)
Hy <- Py %*% Ly
print(Hy,digits=12)
#
# Conditional entropy (vector), HxGy
HxGy <- diag(PxGyT %*% Lxy)
#
# Conditional entropy (average), H(X/Y):
HxBy <- Py %*% HxGy
print(HxBy,digits=12)
#
# The Information Block -----------------
# Ixy: the mutual information
Ixy <- Hx - HxBy
print(Ixy,digits=12)
#
# Uxy: Symmetric uncertainty
Uxy = 2.0*Ixy/(Hx+Hy)
print(Uxy,digits=12)
#
detach(freqs)
rm(freqs,i,j,R,S,Tot,Hx,Hy,HxGy,HxBy,Ixy,Uxy,Mult1)
rm(Px,Pxd,PxGy,Lxy,PxGyT,Lx,Ly,Py,Pydm1,PyGx)


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