Outlier Detection on Mixed-Type Data: An Energy-based Approach

Kien Do, Truyen Tran, Dinh Phung and Svetla Venkatesh
Centre for Pattern Recognition and Data Analytics
Deakin University, Geelong, Australia
dkdo@deakin.edu.au

Abstract. Outlier detection amounts to finding data points that differ significantly from the norm. Classic outlier detection methods are largely designed for single data type such as continuous or discrete. However, real world data is increasingly heterogeneous, where a data point can have both discrete and continuous attributes. Handling mixed-type data in a disciplined way remains a great challenge. In this paper, we propose a new unsupervised outlier detection method for mixed-type data based on Mixed-variate Restricted Boltzmann Machine (Mv.RBM). The Mv.RBM is a principled probabilistic method that models data density. We propose to use free-energy derived from Mv.RBM as outlier score to detect outliers as those data points lying in low density regions. The method is fast to learn and compute, is scalable to massive datasets. At the same time, the outlier score is identical to data negative log-density up-to an additive constant. We evaluate the proposed method on synthetic and real-world datasets and demonstrate that (a) a proper handling mixed-types is necessary in outlier detection, and (b) free-energy of Mv.RBM is a powerful and efficient outlier scoring method, which is highly competitive against state-of-the-arts.

1 Introduction

Outliers are those deviating significantly from the norm. Outlier detection has broad applications in many fields such as security [8,10,20], healthcare [32], and insurance [14]. A common assumption is that outliers lie in the low density regions [6]. Methods implementing this assumption differ in how the notion of density is defined. For example, in nearest neighbor methods (k-NN) [2], large distance between a point to its nearest neighbors indicates isolation, and hence, it lies in a low density region. Gaussian mixture models (GMM), on the other hand, estimate the density directly through a parametric family of clusters [21].

A real-world challenge rarely addressed in outlier detection is mixed-type data, where each data attribute can be any type such as continuous, binary, count or nominal. Most existing methods, however, assume homogeneous data types. Gaussian mixture models, for instance, require data to be continuous and normally distributed. One approach to mixed-type data is to reuse existing methods. For example, we can transform multiple types into a single type – the
process known as coding in the literature. A typical practice of coding nominal data is to use a set of binary variables with exactly one active element. But it leads to information loss because the derived binary variables are considered independent in subsequent analysis. Another drawback of coding is that numerical methods such as GMM and PCA ignore the binary nature of the derived variables. Another way of model reusing is to modify existing methods to accommodate multiple types. However, the modification is often heuristic. Distance-based methods would define type-specific distances, then combine these distances into a single measure. Because type-specific distances differ in scale and semantics, finding a suitable combination is non-trivial.

A disciplined approach to mixed-type outlier detection demands three criteria to be met: (i) capturing correlation structure between types, (ii) measuring deviation from the norm, and (iii) efficient to compute \cite{18}. To this end, we propose a new approach that models multiple types directly and at the same time, provides a fast mechanism for identifying low density regions. To be more precise, we adapt and extend a recent method called Mixed-variate Restricted Boltzmann Machine (Mv.RBM) \cite{31}. Mv.RBM is a generalization of the classic RBM – originally designed for binary data, and now a building block for many deep learning architectures \cite{12}. Mv.RBM has been applied for representing regularities in survey analysis \cite{31}, multimedia \cite{23} and healthcare \cite{22}, but not for outlier detection, which searches for irregularities. Mv.RBM captures the correlation structure between types through factoring – data types are assumed to be independent given a generating mechanism.

In this work, we extend the Mv.RBM to cover counts, which are then modeled as Poisson distribution \cite{27}. We then propose to use free-energy as outlier score to rank mixed-type instances. Note that free-energy is notion rarely seen in outlier detection. In RBM, free-energy equals the negative log-density up to an additive constant, and thus offering a principled way for density-based outlier detection. Importantly, estimation of Mv.RBM is very efficient, and scalable to massive datasets. Likewise, free-energy is computed easily through a single matrix projection. Thus Mv.RBM coupled with free-energy meets all the three criteria outlined above for outlier detection. We validate the proposed approach through an extensive set of synthetic and real experiments against well-known baselines, which include the classic single-type methods (PCA, GMM and one-class SVM), as well as state-of-the-art mixed-type methods (ODMAD \cite{15}, Beta mixture model (BMM) \cite{23} and GLM-t \cite{19}). The experiments demonstrate that (a) a proper handling mixed-types is necessary in outlier detection, and (b) free-energy of Mv.RBM is a powerful and efficient outlier scoring method, being highly competitive against state-of-the-arts.

In summary, we claim the following contributions:

– Introduction of a new outlier detection method for mixed-type data. The method is based on the concept of free-energy derived from a recent method known as Mixed-variate Restricted Boltzmann Machine (Mv.RBM). The method is theoretically motivated and efficient.

– Extension of Mv.RBM to handle counts as Poisson distribution.
A comprehensive evaluation on synthetic and real mixed-type datasets, demonstrating the effectiveness of the proposed method against classic and state-of-the-art rivals.

2 Related Work

Outliers, also known as anomalies or novelties, are those thought to be generated from a mechanism different from the majority. Outlier detection is to recognize data points with unusual characteristics, or in other word, instances that do not follow any regular patterns. When there is very little or no information about outliers provided, which is common in real world data, the regular patterns need to be discovered from normal data itself. This is called unsupervised anomaly detection. A variant known as semi-supervised is when the training data is composed of just normal data [6].

Single Type Outlier Detection A wide range of unsupervised methods have been proposed, for example, distance-based (e.g., k-NN [2]), density-based (e.g., LOF [5], LOCI [25]), cluster-based (e.g., Gaussian mixture model or GMM), projection-based (e.g., PCA) and max margin (One-class SVM). Distance-based and density-based methods model the local behaviors around each data point at a high level of granularity while cluster-based methods group similar data points together into clusters [1]. Projection-based methods, on the other hand, find a data projection that is sensitive to outliers. A comprehensive review of these methods were conducted by Chandola et al. [6].

Mixed-Type Outlier Detection Although pervasive in real-world domains, mixed-type data is rarely addressed in the literature. When data is mixed (e.g., continuous and discrete), measuring distance between two data points or estimating data density can be highly challenging. A nave solution is to transform mixed-types into a single type, e.g., by coding nominal variables into 0/1 or discretizing continuous variables. This practice can significantly distort the true underlying data distribution and result in poor performance [15]. In order to handle mixed-type data directly, several methods have been proposed. LOADED [10] uses frequent pattern mining to define the score of each data point in the nominal attribute space and link it with a precomputed correlation matrix for each item set in continuous attribute space. Since there are a large number of item sets generated, this method suffers from high memory cost. RELOAD [24] is a memory-efficient version of LOADED, which employs a set of Nave Bayes classifiers with continuous attributes as inputs to predict abnormality of nominal attributes instead of aggregating over a large number of item sets.

Koufakou et al. [15] propose a method named ODMAD to detect outliers in sparse data with both nominal and continuous attributes. Their method first computes the anomaly score for nominal attributes using the same algorithm as LOADED. Points detected as outliers at this step are set aside and the
remaining are examined over continuous attribute space with cosine similarity as a measurement. In \cite{4}, separate scores over nominal data space and numerical data space are calculated for each data point. The list of two dimensional score vectors of data was then modeled by a mixture of bivariate beta distributions. Similar to other cluster-based methods, abnormal objects could be detected as having a small probability of belonging to any components. Although the idea of beta modeling is interesting, the calculation of scores is still very simple, which is $k$-NN distance for continuous attributes and sum of item frequencies for nominal attributes.

The work of \cite{33} adopts a different approach called Pattern-based Outlier Detection (POD). A pattern is a subspace formed by a particular nominal fields and all continuous fields. A logistic classifier is trained for each subspace pattern, in which continuous and nominal attributes are explanatory and response variables, respectively. The probability returned by the classifier measures the degree to which an instance deviates from a specific pattern. This is called Categorical Outlier Factor (COF). The collection of COFs and $k$-NN distance form the final anomaly score for a data example. Given a nominal attribute, POD models the functional relationship between continuous variables. The dependency between nominal attributes, however, is not actually captured. Moreover, when data only contains nominal attributes, the classifier cannot be created.

For all the methods mentioned above, their common drawback is that they are only able to capture correlation between a set of nominal and numerical attributes but not pair-wise correlations. The most recent work of Lu et al. \cite{19} overcomes the mentioned drawback and models the data distribution. They design a Generalized Linear Model framework accompanied with a latent variable for correlation capturing and an another latent variable following Student-t distribution as an error buffer. The main advantage of this method is that it provides strong a statistical foundation for modeling distribution of different types. However, the inference for detecting outliers is inexact and expensive to compute.

**Restricted Boltzmann Machine** (RBM) is a probabilistic model of binary data, formulated as a bipartite Markov random field. This special structure allows efficient inference and learning \cite{11}. More recently, it was used as a building block for Deep Belief Networks \cite{12}, the work that started the current revolution of deep learning \cite{17}. Recently RBM has been used for single-type outlier detection \cite{9}.

3 Mixed-Type Outlier Detection

In this section, we present a new density-based method for mixed-type outlier detection. Given a data instance $x$ we estimate the density $P(x)$ then detect if the instance is an outlier using a threshold on the density:

$$-\log P(x) \geq \beta \quad (1)$$
for some predefined threshold $\beta$. Here $-\log P(x)$ serves as the outlier scoring function.

### 3.1 Density Estimation for Mixed Data

Estimating $P(x)$ is non-trivial in mixed-type data since we need to model correlation structures within-type and between-types. A direct correlation between-types demands a careful specification for each type-pair. For example, for two variables of different types $x_1$ and $x_2$, we need to specify either $P(x_1, x_2) = P(x_1)P(x_2 \mid x_1)$ or $P(x_1, x_2) = P(x_2)P(x_1 \mid x_2)$. With this strategy, the number of pairs grows quadratically with the number of types. Most existing methods follow this approach and they are designed for a specific pair such as binary and Gaussian \cite{7}. They neither scale to large-scale problems nor support arbitrary types such as binary, continuous, nominal, and count.

Mixed-variate Restricted Boltzmann Machine (Mv.RBM) is a recent method that supports arbitrary types simultaneously \cite{31}. It bypasses the problems with detailed specifications and quadratic complexity by using latent binary variables. Correlation between types is not modeled directly but is factored into indirect correlation with latent variables. As such we need only to model the correlation between a type and the latent binary. This scales linearly with the number of types.

Mv.RBM was primarily designed for data representation which transforms mixed data into a homogeneous representation, which serves as input for the next analysis stage. Our adaptation, on the other hand, proposes to use Mv.RBM as outlier detector directly, without going through the representation stage.

![Diagram of Mix-variate Restricted Boltzmann machines for mixed-type data. Filled circles denote visible inputs, empty circles denote hidden units. Multiple choices are modeled as multiple binaries, denoted by a filled circle in a clear box.](image-url)
3.2 Mixed-variate Restricted Boltzmann Machines

We first review Mv.RBM for a mixture of binary, Gaussian and nominal types, then extend to cover counts. See Fig. 1 for a graphical illustration. Mv.RBM is an extension of RBM for multiple data types. An RBM is a probabilistic neural network that models binary data in an unsupervised manner. More formally, let \( x \in \{0, 1\}^N \) be a binary input vector, and \( h \in \{0, 1\}^K \) be a binary hidden vector, RBM defines the joint distribution as follows:

\[
P(x, h) \propto \exp (-E(x, h))
\]

where \( E(x, h) \) is energy function of the following form:

\[
E(x, h) = - \left( \sum_i a_i x_i + \sum_k b_k h_k + \sum_{ik} W_{ik} x_i h_k \right)
\]

Here \((a, b, W)\) are model parameters.

For subsequent development, we rewrite the energy function as:

\[
E(x, h) = \sum_i E_i(x_i) + \sum_k \left( -b_k + \sum_i G_{ik}(x_i) \right) h_k
\]

where \( E_i(x_i) = -a_i x_i \) and \( G_{ik}(x_i) = -W_{ik} x_i \).

Mv.RBM extends RBM by redefining the energy function to fit multiple data types. The energy function of Mv.RBM differs from that of RBM by using multiple type-specific energy sub-functions \( E_i(x_i) \) and \( G_{ik}(x_i) \) as listed in Table 1. The energy decomposition in Eq. (3) remains unchanged.

| Func. \( E_i(x_i) \) | Binary | Gaussian | Nominal \( \log x_i! - a_i x_i \) | Count \( -W_{ik} x_i \) |
|-----------------------|--------|----------|--------------------------------|--------------------------|
| \( E_i(x_i) \)       | \(-a_i x_i\) | \(-a_i x_i - \sum_i a_i \delta(x_i, c) \log x_i! + a_i x_i\) | \(-W_{ik} x_i\) | \(-W_{ik} x_i\) |

Table 1. Type-specific energy sub-functions. Here \( \delta(x_i, c) \) is the identity function, that is, \( \delta(x_i, c) = 1 \) if \( x_i = c \), and \( \delta(x_i, c) = 0 \) otherwise. For Gaussian, we assume data has unit variance. Multiple choices are modeled as multiple binaries.

Extending Mv.RBM for Counts We employ Poisson distributions for counts \([27]\). The sub-energy sub-functions are defined as:

\[
E_i(x_i) = \log x_i! - a_i x_i; \quad G_{ik}(x_i) = -W_{ik} x_i
\]

Note that count modeling was not introduced in the original Mv.RBM work.

1 The original Mv.RBM also covers rank, but we do not consider in this paper.
Learning Model estimation in RBM and Mv.RBM amounts to maximize data likelihood with respect to model parameters. It is typically done by n-step Contrastive Divergence (CD-n), which is an approximate but fast method. In particular, for each parameter update, CD-n maintains a very short Markov chain (MCMC), starting from the data, runs for n steps, then collects the samples to approximate data statistics. The MCMC is efficient because of the factorizations in Eq. (5), that is, we can sample all hidden variables in parallel through \( \hat{h} \sim P(h | x) \) and all visible variables in parallel through \( \hat{x} \sim P(x | h) \).

For example, for Gaussian inputs, the parameters are updated as follows:

\[
\begin{align*}
    b_k &\leftarrow b_k + \eta \left( \bar{h}_k | x - \bar{\hat{h}}_k | \hat{x} \right) \\
    a_i &\leftarrow a_i + \eta (x_i - \hat{x}_i) \\
    W_{ik} &\leftarrow W_{ik} + \eta (x_i \bar{h}_k | x - \hat{x}_i \bar{\hat{h}}_k | \hat{x})
\end{align*}
\]

where \( \bar{h}_k | x = P(h_k = 1 | x) \) and \( \eta > 0 \) is the learning rate. This learning procedure scales linearly with \( n \) and data size.

Mv.RBM as a Mixture Model of Exponential Size In Mv.RBM, types are not correlated directly but through the common hidden layer. The posterior \( P(h | x) \) and data generative process \( P(x | h) \) in Mv.RBM are factorized as:

\[
P(h | x) = \prod_k P(h_k | x); \quad P(x | h) = \prod_i P(x_i | h) \tag{5}
\]

Here types are conditionally independent given \( h \), but since \( h \) are hidden, types are dependent as in \( P(x) = \sum_h P(x, h) \).

The posterior has the same form across types – the activation probability \( P(h_k = 1 | x) \) is sigmoid \( (b_k - \sum_i G_{ik}(x_i)) \). On the other hand, the generative process is type-specific. For example, for binary data, the activation probability \( P(x_i = 1 | h) \) is sigmoid \( (a_i + \sum_k W_{ik} h_k) \); and for Gaussian data, the conditional density \( P(x_i | h) \) is \( \mathcal{N}(a_i + \sum_k W_{ik} h_k; 1) \).

Since \( h \) is discrete, Mv.RBM can be considered as a mixture model of \( 2^K \) components that shared the same parameter. This suggests that Mv.RBM can be used for outlier detection in the same way that GMM does.

3.3 Outlier Detection on Mixed-Type Data

Recall that for outlier detection as in Eq. (1) we need the marginal distribution

\[
P(x) = \sum_h P(x, h),
\]

which is:

\[
P(x) \propto \sum_h \exp(-E(x, h)) = \exp(-F(x))
\]

where \( F(x) = -\log \sum_h \exp(-E(x, h)) \) is known as free-energy. Notice that the free-energy equals the negative log-density up to an additive constant:

\[
F(x) = -\log P(x) + \text{constant}
\]

Thus we can use the free-energy as the outlier score to rank data instances, following the detection rule in Eq. (1).
Computing free-energy Although estimating the free-energy amounts to summing over $2^K$ configurations of the hidden layer, we can still compute the summation efficiently, thanks to the decomposition of the energy function in Eq. (3). We can rewrite the free-energy as follows:

$$F(x) = \sum_i E_i(x_i) - \sum_k \log \left( 1 + \exp \left( b_k - \sum_i G_{ik}(x_i) \right) \right)$$  \hspace{1cm} \text{(6)}$$

This free-energy can be computed in linear time.

Controlling model expressiveness A major challenge of unsupervised outlier detection is the phenomenon of swamping effect, where an inlier is misclassified as outlier, possibly due to a large number of true outliers in the data [28]. When data models are highly expressive – such as large RBMs and Mv.RBMs – outliers are included by the models as if they have patterns themselves, even if these patterns are weak and differ significantly from the regularities of the inliers. One way to control the model expressiveness is to limit the number of hidden layers $K$ (hence the number of mixing components $2^K$). Another way is to apply early stopping – learning stops before convergence has occurred.

Summary To sum up, Mv.RBM, coupled with free-energy, offers a disciplined approach to mixed-type outlier detection that meet three desirable criteria: (i) capturing correlation structure between types, (ii) measuring deviation from the norm, and (iii) efficient to compute.

4 Experiments

We present experiments on synthetic and real-world data. For comparison, we implement well-known single-type outlier detection methods including Gaussian mixture model (GMM), Probabilistic Principal Component Analysis (PPCA) [29] and one-class SVM (OCSVM) [20]. The number of components of PPCA model is set so that the discarded energy is the same as the anomaly rate in training data. For OCSVM, we use radial basis kernel with $\nu = 0.7$. GMM and PPCA are probabilistic, and thus data log-likelihood can be computed for outlier detection.

Since all of these single-type methods assume numerical data, we code nominal types using dummy binaries. For example, a $A$ in the nominal set \{A,B,C\} is coded as (1,0,0) and $B$ as (0,1,0). This coding causes some nominal information loss, since the coding does not ensure that only one value is allowed in nominal variables. For all methods, the detection threshold is based on the $\alpha$ percentile of the training outlier scores. Whenever possible, we also include results from other recent mixed-type papers, ODMAD [15], Beta mixture model (BMM) [4] and GLM-t [19]. We followed the same mechanism they used to generate outliers.
4.1 Synthetic Data

We first evaluate the behaviors of Mv.RBM on synthetic data with controllable complexity. We simulate mixed-type data using a generalized Thurstonian theory, where Gaussians serve as underlying latent variables for observed discrete values. Readers are referred to [30] for a complete account of the theory. For this study, the underlying data is generated from a GMM of 3 mixing components with equal mixing probability. Each component is a multivariate Gaussian distribution of 15 dimensions with random mean and positive-definite covariance matrix. From each distribution, we simulate 1,000 samples, creating a data set size 3,000. To generate outliers, we randomly pick 5% of data, and add uniform noise to each dimension, i.e., $x_i \leftarrow x_i + e_i$ where $e_i \sim \mathcal{U}$. For visualization, we use t-SNE to reduce the dimensionality to 2 and plot the data in Fig. 2.

Out of 15 variables, 3 are kept as Gaussian and the rest are used to create mixed-type variables. More specifically, 3 variables are transformed into binaries using random thresholds, i.e., $\tilde{x}_i = \delta(x_i \geq \theta_i)$. The other 9 variables are used to generate 3 nominal variables of size 3 using the rule: $\tilde{x}_i = \text{arg max}(x_{i1}, x_{i2}, x_{i3})$.

Models are trained on 70% data and tested on the remaining 30%. This testing scheme is to validate the generalizability of models on unseen data. The learning curves of Mv.RBM are plotted in Fig. 3. With the learning rate of 0.05, learning converges after 10 epochs. No overfitting occurs.

The decision threshold $\beta$ in Eq. (1) is set at 5 percentile of the training set. Fig. 4 plots the outlier detection performance of Mv.RBM (in F-score) on test data as a function of model size (number of hidden units). To account for random initialization, we run Mv.RBM 10 times and average the F-scores. It is apparent that the performance of Mv.RBM is competitive against that of GMM. The best
Fig. 3. Learning curves of Mv.RBM (50 hidden units) on synthetic data for different learning rates. The training and test curves almost overlap, suggesting no overfitting. Best viewed in color.

F-score achieved by GMM is only about 0.35, lower than the worst F-score by Mv.RBM, which is 0.50. The PCA performs poorly, with F-score of 0.11, possibly because the outliers does not conform to the notion of residual subspace assumed by PCA.

The performance difference between Mv.RBM and GMM is significant considering the fact that the underlying data distribution is drawn from a GMM. It suggests that when the correlation between mixed attributes is complex like this case, using GMM even with the same number of mixture components cannot learn well. Meanwhile, Mv.RBM can handle the mixed-type properly, without knowing the underlying data assumption. Importantly, varying the number of hidden units does not affect the result much, suggesting the stability of the model and it can free users from carefully crafting this hyper-parameter.

4.2 Real Data

For real-world applications, we use a wide range of mixed-type datasets. From the UCI repository[^2] we select 7 datasets which were previously used as benchmarks for mixed-type anomaly detection [4][10][19]. Data statistics are reported in Table 2.

We generate outliers by either using rare classes whenever possible, or by randomly injecting a small proportion of anomalies, as follows:

- **Using rare classes**: For the KDD99 10 percent dataset (KDD99-10), intrusions (outliers) account for 70% of all data, and thus it is not possible to use full data because outliers will be treated as normal in unsupervised

[^2]: https://archive.ics.uci.edu/ml/datasets.html
learning. Thus, we consider all normal instances from the original data as inliers, which accounts for 90% of the new data. The remaining 10% outliers are randomly selected from the original intrusions.

- **Outliers injection**: For the other datasets, we treat data points as normal objects and generate outliers based on a contamination procedure described in [4][18]. Outliers are created by randomly selecting 10% of instances and modifying their default values. For numerical attributes (Gaussian, Poisson), values are shifted by 2.0 to 3.0 times standard deviation. For discrete attributes (binary, categorical), the values are switched to alternatives.

Numerical attributes are standardized to zero means and unit variance. For evaluation, we randomly select 30% data for testing, and 70% data for training. Note that since learning is unsupervised, outliers must also be detected in the training set since there are no ground-truths. The outliers in the test set is to test the generalizability of the models to unseen data.

| Dataset                  | No. Instances | No. Attributes |
|--------------------------|---------------|----------------|
|                          | Train | Test | Bin | Gauss | Nominal | Poisson | Total |
| **KDD99-10**             | 75,669 | 32,417 | 4 | 15 | 3 | 19 | 41 |
| **Australian Credit**    | 533 | 266 | 3 | 6 | 5 | 0 | 14 |
| **German Credit**        | 770 | 330 | 2 | 7 | 11 | 0 | 20 |
| **Heart**                | 208 | 89 | 3 | 6 | 4 | 0 | 13 |
| **Thoracic Surgery**     | 362 | 155 | 10 | 3 | 3 | 0 | 16 |
| **Auto MPG**             | 303 | 128 | 0 | 5 | 3 | 0 | 8 |
| **Contraceptive**        | 1136 | 484 | 3 | 0 | 4 | 1 | 8 |

**Table 2.** Characteristics of mixed-type datasets. The proportion of outliers are 10%.
**Models setup** The number of hidden units in Mv.RBM is set to $K = 2$ for the KDD99-10 dataset, and to $K = 5$ for other datasets. The parameters of Mv.RBM are updated using stochastic gradient descent, that is, update occurs after every mini-batch of data points. For small datasets, the mini-batch size is equal to the size of the entire dataset while for KDD99-10, the mini-batch size is set to 500. The learning rate is set to 0.01 for all small datasets, and to 0.001 for KDD99-10. Small datasets are trained using momentum of 0.8. For KDD99-10, we use Adam [13], with $\beta_1 = 0.85$ and $\beta_2 = 0.995$. For small datasets, the number of mixture components in GMM is chosen using grid search in the range from 1 to 30 with a step size of 5. For KDD99-10, the number of mixture components is set to 4.

![Free energy histogram over KDD99-10 test data](image)

**Fig. 5.** Outlier detection on the KDD99-10 dataset. (a) Histogram of free-energies. The vertical line separates data classified as inliers (left) from those classified as outliers (right). The color of majority (light blue) is inlier. Best viewed in color. (b) ROC curve (AUC = 0.914).

**Results** Fig. 5(a) shows a histogram of free-energies computed using Eq. (6) on the KDD99-10 dataset. The inliers/outliers are well-separated into the low/high energy regions, respectively. This is also reflected in an Area Under the ROC Curve (AUC) of 0.914 (see Fig. 5(b)).

The detection performance in terms of F-score on test data is reported in Tables 3. The mean of all single type scores is 0.66, of all competing mixed-type scores is 0.77, and of Mv.RBM scores is 0.91. These demonstrate that (a) a proper handling of mixed-types is required, and (b) Mv.RBM is highly competitive against other mixed-type methods for outlier detection. Point (a) can also be strengthened by looking deeper: On average, the best competing mixed-type method (BMM) is better than the best single-type method (OCSVM). For point (b), the gaps between Mv.RBM and other methods are significant:
| Dataset         | Single type | mixed-type |       |       |       |       |
|-----------------|-------------|------------|-------|-------|-------|-------|
|                 | GMM OCSVM   | PPCA       | BMM ODMAD | GLM-t |       |       |
| *KDD99-10*      | 0.42        | 0.54       | 0.55   | –     | –     | **0.71** |
| *Australian Credit* | 0.74       | 0.84       | 0.38   | **0.972** | 0.942 | –     | 0.90 |
| *German Credit* | 0.86        | 0.86       | 0.02   | 0.934 | 0.810 | –     | **0.95** |
| *Heart*         | 0.89        | 0.76       | 0.64   | 0.872 | 0.630 | 0.72  | **0.94** |
| *Thoracic Surgery* | 0.71      | 0.71       | 0.70   | **0.939** | 0.879 | –     | 0.90 |
| *Auto MPG*      | **1.00**    | **1.00**   | 0.67   | 0.625 | 0.575 | 0.64  | **1.00** |
| *Contraceptive* | 0.62        | 0.84       | 0.02   | 0.673 | 0.523 | –     | **0.91** |
| **Average**     | 0.75        | 0.79       | 0.43   | 0.84  | 0.73  | 0.68  | **0.91** |

Table 3. Outlier detection F-score.

On average, Mv.RBM is better than the best competing method – the BMM (mixed-type) – by 8.3%, and better than the worst method – the PPCA (single type), by 111.6%. On the largest dataset – the KDD99-10 – Mv.RBM exhibits a significant improvement of 29.1% over the best single type method (PPCA).

5 Discussion

This paper has introduced a new method for mixed-type outlier detection based on an energy-based model known as Mixed-variate Restricted Boltzmann Machine (Mv.RBM). Mv.RBM avoids direct modeling of correlation between types by using binary latent variables, and in effect, model the correlation between each type and the binary type. We derive free-energy, which equals the negative log of density up-to a constant, and use it as the outlier score. Overall, the method is highly scalable. Our experiments on mixed-type datasets of various types and characteristics demonstrate that the proposed method is competitive against the well-known baselines designed for single types, and recent models designed for mixed-types. These results (a) support the hypothesis that in mixed-data, proper modeling of types should be in place for outlier detection, and (b) show Mv.RBM is a powerful density-based outlier detection method.

Mv.RBM opens several future directions. First, Mv.RBM transforms multiple types into a single type through its hidden posteriors. Existing single-type outlier detectors can be readily employed. Second, Mv.RBM can serve as a building block for deep architecture, such as Deep Belief Networks and Deep Boltzmann Machine. It would be interesting to see how deep networks perform in non-prediction settings such as outlier detection.

Acknowledgments

This work is partially supported by the Telstra-Deakin Centre of Excellence in Big Data and Machine Learning.
References

1. Charu C Aggarwal. Outlier analysis. In *Data Mining*, pages 237–263. Springer, 2015.
2. Fabrizio Angiulli and Clara Pizzuti. Fast outlier detection in high dimensional spaces. In *European Conference on Principles of Data Mining and Knowledge Discovery*, pages 15–27. Springer, 2002.
3. Yoshua Bengio, Aaron Courville, and Pascal Vincent. Representation learning: A review and new perspectives. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(8):1798–1828, 2013.
4. Mohamed Bouguessa. A practical outlier detection approach for mixed-attribute data. *Expert Systems with Applications*, 42(22):8637–8649, 2015.
5. Markus M Breunig, Hans-Peter Kriegel, Raymond T Ng, and Jörg Sander. LOF: identifying density-based local outliers. In *ACM sigmod record*, volume 29, pages 93–104. ACM, 2000.
6. Varun Chandola, Arindam Banerjee, and Vipin Kumar. Anomaly detection: A survey. *ACM computing surveys (CSUR)*, 41(3):15, 2009.
7. Alexander R De Leon and Keumhee Carri` ere Chough. *Analysis of Mixed Data: Methods & Applications*. CRC Press, 2013.
8. Christopher P Diehl and John B Hampshire. Real-time object classification and novelty detection for collaborative video surveillance. In *Neural Networks, 2002. IJCNN’02. Proceedings of the 2002 International Joint Conference on*, volume 3, pages 2620–2625. IEEE, 2002.
9. Ugo Fiore, Francesco Palmieri, Aniello Castiglione, and Alfredo De Santis. Network anomaly detection with the restricted Boltzmann machine. *Neurocomputing*, 122:13–23, 2013.
10. Amol Ghoting, Matthew Eric Otey, and Srinivasan Parthasarathy. Loaded: Link-based outlier and anomaly detection in evolving data sets. In *ICDM*, pages 387–390, 2004.
11. G.E. Hinton. Training products of experts by minimizing contrastive divergence. *Neural Computation*, 14:1771–1800, 2002.
12. G.E. Hinton and R.R. Salakhutdinov. Reducing the dimensionality of data with neural networks. *Science*, 313(5786):504–507, 2006.
13. Diederik Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.
14. Rob M Konijn and Wojtek Kowalczyk. Finding fraud in health insurance data with two-layer outlier detection approach. In *International Conference on Data Warehousing and Knowledge Discovery*, pages 394–405. Springer, 2011.
15. Anna Koufakou, Michael Georgiopoulos, and Georgios C Anagnostopoulos. Detecting outliers in high-dimensional datasets with mixed attributes. In *DMIN*, pages 427–433. Citeseer, 2008.
16. Christopher Kruegel and Giovanni Vigna. Anomaly detection of web-based attacks. In *Proceedings of the 10th ACM conference on Computer and communications security*, pages 251–261. ACM, 2003.
17. Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *Nature*, 521(7553):436–444, 2015.
18. Yen-Cheng Lu, Feng Chen, Yating Wang, and Chang-Tien Lu. Discovering anomalies on mixed-type data using a generalized student-t based approach. *IEEE Transactions on Knowledge and Data Engineering*, DOI:10.1109/TKDE.2016.2583429, 2016.
19. Yen-Cheng Lu, Feng Chen, Yating Wang, and Chang-Tien Lu. Discovering anomalies on mixed-type data using a generalized student-t based approach. 2016.

20. Larry M Manevitz and Malik Yousef. One-class SVMs for document classification. *Journal of Machine Learning Research*, 2(Dec):139–154, 2001.

21. Geoffrey J McLachlan and Kaye E Basford. Mixture models, inference and applications to clustering. *Statistics: Textbooks and Monographs, New York: Dekker, 1988*, 1, 1988.

22. T.D. Nguyen, T. Tran, D. Phung, and S. Venkatesh. Latent patient profile modelling and applications with mixed-variate restricted Boltzmann machine. In *Proc. of Pacific-Asia Conference on Knowledge Discovery and Data Mining (PAKDD), Gold Coast, Queensland, Australia, April 2013*.

23. T.D. Nguyen, T. Tran, D. Phung, and S. Venkatesh. Learning sparse latent representation and distance metric for image retrieval. In *Proc. of IEEE International Conference on Multimedia & Expo*, California, USA, July 15-19 2013.

24. Matthew Eric Otey, Srinivasan Parthasarathy, and Amol Ghoting. Fast lightweight outlier detection in mixed-attribute data. Technical Report, OSU–CISRC–6/05–TR43, 2005.

25. Spiros Papadimitriou, Hiroyuki Kitagawa, Phillip B Gibbons, and Christos Faloutsos. Loci: Fast outlier detection using the local correlation integral. In *Data Engineering, 2003. Proceedings. 19th International Conference on*, pages 315–326. IEEE, 2003.

26. Leonid Portnoy, Eleazar Eskin, and Sal Stolfo. Intrusion detection with unlabeled data using clustering. In *In Proceedings of ACM CSS Workshop on Data Mining Applied to Security (DMSA-2001*. Citeseer, 2001.

27. R. Salakhutdinov and G. Hinton. Semantic hashing. *International Journal of Approximate Reasoning*, 50(7):969–978, 2009.

28. Robert Serfling and Shanshan Wang. General foundations for studying masking and swamping robustness of outlier identifiers. *Statistical Methodology*, 20:79–90, 2014.

29. M.E. Tipping and C.M. Bishop. Probabilistic principal component analysis. *Journal of the Royal Statistical Society: Series B*, 61(3):611–622, 1999.

30. T. Tran, D. Phung, and S. Venkatesh. Thurstonian Boltzmann Machines: Learning from Multiple Inequalities. In *International Conference on Machine Learning (ICML)*, Atlanta, USA, June 16-21 2013.

31. T. Tran, D.Q. Phung, and S. Venkatesh. Mixed-variate restricted Boltzmann machines. In *Proc. of 3rd Asian Conference on Machine Learning (ACML)*, Taoyuan, Taiwan, 2011.

32. Truyen Tran, Dinh Phung, Wei Luo, Richard Harvey, Michael Berk, and Svetha Venkatesh. An integrated framework for suicide risk prediction. In *Proceedings of the 19th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 1410–1418. ACM, 2013.

33. Ke Zhang and Huidong Jin. An effective pattern based outlier detection approach for mixed attribute data. In *Australasian Joint Conference on Artificial Intelligence*, pages 122–131. Springer, 2010.