AN ASTRONOMICAL VIEW-POINT ON THE EASTER DATE

Ovidiu Vaduvescu
PhD Student, York University, Toronto ON Canada
Research Associate, The Astronomical Institute, Bucharest Romania

ABSTRACT

Old algorithms are still in use nowadays setting the Easter date accordingly to some elements giving the Spring Equinox and the New Moon for a given year. At least two different approaches (by the Orthodox and Catholic Churches) provide different results, most of the times different than the actual moments given by the two astronomical phenomena. A new algorithm built accordingly to the Niceea rule and based on some simple astronomical formulas is presented. It is proposed to replace the old different approaches, in order to celebrate the Easter at the same common date. A Windows program is provided, together with a table listing the Easter date between the years 1950-2050 calculated using the Orthodox, Catholic and the Astronomical approaches.

1. INTRODUCTION

The rule promulgated in Niceea (Bitinia, Minor Asia) in the year 325 A.D. concerning the Easter date sets this holiday “On the Sunday which follows the 14-th day of the Moon which reaches this age on 21 March or immediately after that” (e.g. Levy, 1974). In this definition, by “the 14-th day of the Moon” one has to understand “the age 14 of the Moon” (i.e. the day 14 days after the New Moon).

It is very difficult to establish the origin of the first algorithms used to calculate the Easter date. It is supposed that The Holy Father Chiril, bishop of Alexandria, was the first who approached this problem between 430-440 A.D. The first systematic algorithm dates from the year 456 A.D. and might be assigned to Victorius d’Aquitania.

The elementary arithmetic (involving only summation, multiplication and division) represents the instrument to calculate the Easter date accordingly to the classical approach presented in many papers (e.g. Levy 1975). The elements required by this algorithm (Vaduvescu, 1993, Vaduvescu and Soulie, 1994) are expressed as “modulo” functions of some arithmetical progression terms.
The imperfection of this method consists in the fact that some entire numbers can not represent accurately, for long time periods, the natural phenomena which set the Easter date, namely the Vernal Equinox and the New Moon phase. Moreover, because of the Reformation of the Calendar in 1582, the elements of the algorithm were modified. While the Catholic Church approach defines these elements based on the Gregorian calendar, the Orthodox Church preserves in its algorithm the elements expressed in the old Julian calendar, correcting the result by a number of days equal to the difference between the two calendars (Vaduvescu and Soulie, 1994). Nevertheless, although more precise, the Gregorian calendar will generate in the year 4317 a delay of a day in comparison with the tropical time (BdL, 1970, 1971).

This is my third and probably the last paper dealing with the Easter date. It was originally written as a manuscript in 1994. While the second paper (Vaduvescu and Soulie, 1994) provides an algorithm to calculate the Easter using elements defined based on the Gregorian Calendar, the first paper (Vaduvescu, 1993) introduces the computational elements necessary to calculate the Easter by the Orthodox Church, it compares them with the Catholic Church approach, providing a program to calculate both. In the present paper we propose a new algorithm based on some classic astronomical formulae giving the Equinox date and the Moon phase, instead of using the old elements set to different calendars to calculate the Easter Date.

2. THE VERNAL EQUINOX

It is well known that setting the equinox each year on 21 March represents an approximation, this phenomenon varying in time, as a function of the moment when the longitude of the Sun is zero. Savoie (1988) showed that in the year 325 A.D. the equinox has fallen on 20 March, about around midday (UT). Therefore, an astronomical view-point on the rigorous treating of the problem would be to define the Easter date as a function of the equinox, and not of 21 March.

Accordingly with Newcomb’s solar theory (e.g. Meeus, 1984, Meeus, 1991), denoting by $JJ$ the Julian date, let $T$ be the following fraction:

$$T = (JJ - 2415020.0) / 36525$$

Then, the mean longitude of the Sun is given by the following truncated series:

$$L = 279^\circ.69668 + 36000^\circ.76892 T + 0^\circ.0003025 T^2$$  (1)
The mean anomaly of the Sun is:

\[ M = 358.47583 + 35999.04975 T - 0.000150 T^2 - 0.0000033 T^3 \]  

and the equation of the Sun center is:

\[
C = (1^\circ.919460 - 0^\circ.004789 T - 0^\circ.000014 T^2) \sin(M) + \\
+ (0^\circ.020094 - 0^\circ.0001 T) \sin(2M) + 0^\circ.000293 \sin(3M)
\]

Then, the true longitude of the Sun is:

\[
TL = L + C
\]

Because \( TL \) is a continuous and monotonous function of the \( JJ \), the method of the bisection or Newton's method for example can provide the moment denoted by \( JJ_e \) of the year when the mean longitude of the Sun is a multiple of 360°, considering for example an initial value \( JJ_0 \) corresponding to 21 March 0h UT.

3. COMMON YEAR OR LEAP YEAR

Let \( m \) be the year for which we search the Easter date. The number of the days in February, expressed in the Gregorian calendar, is \( 28 + nf \), where we denote by \( nf \) the following number:

\[
\begin{align*}
\text{if } [m]_{400} &= 0, \text{ then } nf = 1 \\
\text{if } [m]_{400} &\neq 0 \text{ and } [m]_{100} = 0, \text{ then } nf = 0 \\
\text{if } [m]_{100} &\neq 0 \text{ and } [m]_4 = 0, \text{ then } nf = 1 \\
\text{if } [m]_{100} &\neq 0 \text{ and } [m]_4 \neq 0, \text{ then } nf = 0
\end{align*}
\]

where \([x]_y\) represents the rest of the entire division of the entire numbers \( x \) by \( y \).

4. THE NEW MOON

The fraction of year corresponding to 1 March is:

\[
f = (59 + nf)/(365 + nf)
\]
by means of which the multiplication constant corresponding to the mean phase of New Moon is:

\[ k = \text{int}[(m + f - 1900.0) \times 12.3685] + 1 \]  

(7)

where by "int[x]" be denoted the entire part of the real x.

According to Meeus (1986), the moment of mean phase of the New Moon is:

\[ JJ_{NM} = 2415020.75933 + 29.53058868k \]  

(8)

which sets the moment of the first New Moon subsequent to 1 March.

5. ADDITIONAL ACCURACY

Formula (1) can be improved by taking into consideration planetary perturbations, while (8) by taking into account the aberration of the Sun and the real phases, according to Meeus (1986, 1991). Also, corrections of diurnal parallax and transformations of local time corresponding to the coordinates of Niceea could be added if necessary to the study. We do not insist on these.

6. THE 14-TH DAY OF THE MOON

If \((JJ_{NM}+14) \geq JJ_\alpha\), then this is the Paschal Moon, otherwise we are placed into the former cycle. So, in the second case, by assigning:

\[ JJ_{NM} = JJ_{NM} + 29.53058868 \]

we will be placed in the Paschal Moon.

Denoting by \(JJ_{1,1}\) the Julian date corresponding to 1 January, one calculates the expression:

\[ z_a = \text{int}[JJ_{NM} - JJ_{1,1}] + 1 \]  

(9)

which represents the day of the year corresponding to the Paschal New Moon.

7. THE NEXT SUNDAY
The first Sunday after the \( j \)–th day of an year \( m \) is given by:

\[
d = j + [2 - j - m_{ac}] \times 7
\]  

(10)

where by "\( m_{ac} \)" we denoted "the Catholic hand of the year" (the Catholic element equivalent to the Orthodox dominical letter element) given by:

\[
m_{ac} = [u + \text{int}(u/4) + \text{int}(c/4) - 2c] \times 7 + 1
\]  

(11)

where:

\[
c = \text{int}(m/100) \text{ and } u=[m]_{100}
\]

If \( m \) is a leap year (\( nf=1 \)), then \( m_{ac} \) is obtained by assigning to \( m_{ac} = (m_{ac} - 1) \).

Finally, the day of the year corresponding to the Easter is:

\[
z_P = z_a + 14 + [2 - z_a - m_{ac}] \times 7
\]  

(12)

which remains to be transformed into calendar date:

\[
\text{if } z_P > (90+nf), \text{ then the Easter falls on } (z_P - 90 - nf) \text{ April}
\]
\[
\text{if } z_P <= (90+nf), \text{ then the Easter falls on } (z_P - 59 - nf) \text{ March}
\]

(13)

8. EXAMPLE

Let \( m=1994 \). One gets \( JJ_0 = 2449432.5 \); \( JJ_{1,1} = 2449353.5 \) and with (1), (2), (3), (4), by means of the half interval method with \( JJ_1 = 2449432.0 \) after four recurrences, one gets \( TL=0.02381 \) and \( JJ_e = 2449432.375 \) (that is 20 March 21h UT, in comparison with 20 March 20.5h UT from the ephemeris). (5) yields \( nf = 0 \). The new moon is given by (6), (7) and (8): \( f = 0.1616438; \ k = 1165; \ JJ_NM = 2449423.895 \) (that is 12 March 9h UT, in comparison with 12 March 7h UT from the ephemeris). Because \( (JJ_{NM}+14) >= JJ_e \), we are placed in the searched cycle, then from (9) \( z_a=71 \). From (11), \( m_{ac} = 7 \), then accordingly with (12), \( z_P = 86 \). Consequently, the astronomical Easter falls on 27 March 1994, in comparison with 3 April the Catholic Easter or 1 May the Orthodox Easter.
9. PROGRAM, COMPARISON

We wrote a PC code in Delphi 4 for Windows based on the above algorithm which provides the Easter date between two limits, given by a starting year and a number of years. The executable is available online (Vaduvescu, 2004).

A table giving the Astronomical Easter between the years 1950-2050 (calculated with the above program), the Orthodox Easter, and the Catholic Easter (both provided by the program in Vaduvescu, 1993) is shown in Table 1 in the Appendix.

Major differences can be seen from Table 1 in most cases between the Astronomical Easter date (supposed to be calculated using the most correct algorithm) and the Orthodox approach (which has at its base the Julian calendar).

From the 101 years analyzed, there are 36 cases (35.6%) for which some differences can be seen between the Astronomical Easter date and the Catholic algorithm (which employs the Gregorian calendar to calculate the elements used for Easter algorithm). No apparent periodicity can be seen. Interestingly, these differences represent only two intervals (in the sense Astronomic minus Catholic): minus one week, appearing in 21 cases (20.8% from the total 101 years), and plus four weeks (in the same sense) appearing in the rest 15 cases (14.8% cases).

REFERENCES

BdL, 1970, Etude des divers calendriers, in Annuaire du Bureau des Longitudes, Paris
BdL, 1971, Etude des divers calendriers, in Annuaire du Bureau des Longitudes, Paris
Levy, J. 1974, La date de Paques, in Annuaire du Bureau des Longitudes, Paris
Levy, J. 1975, La date de Paques, in Annuaire du Bureau des Longitudes, Paris
Meeus, J. 1986, Calculs astronomiques a l’usage des amateurs, Paris
Meeus, J. 1991, Astronomical Algorithms, Willmann-Bell, Virginia
Savoie, D. 1988, La date de l’equinoxe et le Concile de Nicee, in L’Astronomie, 102, Paris
Vaduvescu, O. 1993, Study on the Easter Date, in Anuarul Astronomic (The Astronomical Yearbook), Bucharest (in Romanian)
Vaduvescu, O. and Soulie, E. 1994, La date de Paques orthodoxe dans le calendrier gregorien, in Observations et Travaux, 40, 23
Vaduvescu, O. 2004, PC Windows program calculating the Astronomical Easter Date (PASTE3.EXE), available online here http://www.geocities.com/ovidiuv/astросл.htm
## APPENDIX

Table 1: Easter dates calculated between 1950-2050 (first column) based on the Orthodox (column 2), Catholic (column 3), and the Astronomical approach.

| YEAR | ORTHODOX | CATHOLIC | ASTRONOMIC |
|------|-----------|-----------|-------------|
| 1950 | 9 April   | 9 April   | 2 April     |
| 1951 | 29 April  | 25 March  | 22 April    |
| 1952 | 20 April  | 13 April  | 13 April    |
| 1953 | 5 April   | 5 April   | 29 March    |
| 1954 | 25 April  | 18 April  | 18 April    |
| 1955 | 17 April  | 10 April  | 10 April    |
| 1956 | 6 May     | 1 April   | 1 April     |
| 1957 | 21 April  | 21 April  | 14 April    |
| 1958 | 13 April  | 6 April   | 6 April     |
| 1959 | 3 May     | 29 March  | 26 April    |
| 1960 | 17 April  | 17 April  | 10 April    |
| 1961 | 9 April   | 2 April   | 2 April     |
| 1962 | 29 April  | 22 April  | 22 April    |
| 1963 | 14 April  | 14 April  | 14 April    |
| 1964 | 3 May     | 29 March  | 29 March    |
| 1965 | 25 April  | 18 April  | 18 April    |
| 1966 | 10 April  | 10 April  | 10 April    |
| 1967 | 30 April  | 26 March  | 26 March    |
| 1968 | 21 April  | 14 April  | 14 April    |
| 1969 | 13 April  | 6 April   | 6 April     |
| 1970 | 26 April  | 29 March  | 26 April    |
| 1971 | 18 April  | 11 April  | 11 April    |
| 1972 | 9 April   | 2 April   | 2 April     |
| 1973 | 29 April  | 22 April  | 22 April    |
| 1974 | 14 April  | 14 April  | 7 April     |
| 1975 | 4 May     | 30 March  | 30 March    |
| 1976 | 25 April  | 18 April  | 18 April    |
| 1977 | 10 April  | 10 April  | 3 April     |
| 1978 | 30 April  | 26 March  | 23 April    |
| 1979 | 22 April  | 15 April  | 15 April    |
| 1980 | 6 April   | 6 April   | 30 March    |
| 1981 | 26 April  | 19 April  | 19 April    |
| 1982 | 18 April  | 11 April  | 11 April    |
| 1983 | 8 May     | 3 April   | 3 April     |
| 1984 | 22 April  | 22 April  | 15 April    |
| 1985 | 14 April  | 7 April   | 7 April     |
| 1986 | 4 May     | 30 March  | 27 April    |
| 1987 | 19 April  | 19 April  | 12 April    |
| 1988 | 10 April  | 3 April   | 3 April     |
| 1989 | 30 April  | 26 March  | 23 April    |
| 1990 | 15 April  | 15 April  | 15 April    |
| 1991 | 7 April   | 31 March  | 31 March    |
| 1992 | 26 April  | 19 April  | 19 April    |
| 1993 | 18 April  | 11 April  | 11 April    |
| 1994 | 1 May     | 3 April   | 27 March    |
| 1995 | 23 April  | 16 April  | 16 April    |
| 1996 | 14 April  | 7 April   | 7 April     |
| 1997 | 27 April  | 30 March  | 27 April    |
| 1998 | 19 April  | 12 April  | 12 April    |
| 1999 | 11 April  | 4 April   | 4 April     |
| 2000 | 30 April  | 23 April  | 23 April    |
| 2001 | 15 April  | 15 April  | 8 April     |
Table 1 (continued): Easter dates calculated between 1950-2050 (first column) based on the Orthodox (column 2), Catholic (column 3), and the Astronomical approach.

| YEAR | ORTHODOX | CATHOLIC | ASTRONOMIC |
|------|-----------|-----------|-------------|
| 2002 | 5 May     | 31 March  | 31 March    |
| 2003 | 27 April  | 20 April  | 20 April    |
| 2004 | 11 April  | 11 April  | 4 April     |
| 2005 | 1 May     | 27 March  | 24 April    |
| 2006 | 23 April  | 16 April  | 16 April    |
| 2007 | 8 April   | 8 April   | 1 April     |
| 2008 | 27 April  | 23 March  | 20 April    |
| 2009 | 19 April  | 12 April  | 12 April    |
| 2010 | 4 April   | 4 April   | 4 April     |
| 2011 | 24 April  | 24 April  | 17 April    |
| 2012 | 15 April  | 8 April   | 8 April     |
| 2013 | 5 May     | 31 March  | 31 March    |
| 2014 | 20 April  | 20 April  | 13 April    |
| 2015 | 12 April  | 5 April   | 5 April     |
| 2016 | 1 May     | 27 March  | 24 April    |
| 2017 | 16 April  | 16 April  | 16 April    |
| 2018 | 8 April   | 1 April   | 1 April     |
| 2019 | 28 April  | 21 April  | 21 April    |
| 2020 | 19 April  | 12 April  | 12 April    |
| 2021 | 2 May     | 4 April   | 28 March    |
| 2022 | 24 April  | 17 April  | 17 April    |
| 2023 | 16 April  | 9 April   | 9 April     |
| 2024 | 5 May     | 31 March  | 28 April    |
| 2025 | 20 April  | 20 April  | 13 April    |
| 2026 | 12 April  | 5 April   | 5 April     |
| 2027 | 2 May     | 28 March  | 25 April    |
| 2028 | 16 April  | 16 April  | 9 April     |
| 2029 | 8 April   | 1 April   | 1 April     |
| 2030 | 28 April  | 21 April  | 21 April    |
| 2031 | 13 April  | 13 April  | 6 April     |
| 2032 | 2 May     | 28 March  | 28 March    |
| 2033 | 24 April  | 17 April  | 17 April    |
| 2034 | 9 April   | 9 April   | 9 April     |
| 2035 | 29 April  | 25 March  | 22 April    |
| 2036 | 20 April  | 13 April  | 13 April    |
| 2037 | 5 April   | 5 April   | 5 April     |
| 2038 | 25 April  | 25 April  | 18 April    |
| 2039 | 17 April  | 10 April  | 10 April    |
| 2040 | 6 May     | 1 April   | 1 April     |
| 2041 | 21 April  | 21 April  | 21 April    |
| 2042 | 13 April  | 6 April   | 6 April     |
| 2043 | 3 May     | 29 March  | 26 April    |
| 2044 | 24 April  | 17 April  | 17 April    |
| 2045 | 9 April   | 9 April   | 2 April     |
| 2046 | 29 April  | 25 March  | 22 April    |
| 2047 | 21 April  | 14 April  | 14 April    |
| 2048 | 5 April   | 5 April   | 29 March    |
| 2049 | 25 April  | 18 April  | 18 April    |
| 2050 | 17 April  | 10 April  | 10 April    |