Robust Cubature Kalman Filter for SINS/GPS Integrated Navigation Systems With Unknown Noise Statistics

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ABSTRACT Due to the vehicle’s severe maneuver and abnormal measurements of GPS in practical applications, the statistic of process noise in SINS/GPS integrated navigation system may be unknown and the measurement noise may not follow the Gaussian distribution, which results in a deteriorated performance for the conventional cubature Kalman filter. To address this issue, we propose in this paper a new robust cubature Kalman filter based on the adaptive information entropy theory. In the proposed filter, the process uncertainty and non-Gaussian measurement noise are simultaneously suppressed based on a new constructed cost function using the maximum correntropy and residual orthogonal principle based weighted least squares technology, which is independent of noise distribution and more insensitive to the non-Gaussian noise. Moreover, a multiple-channel adaptive strategy for the better process uncertainty suppression is given. Furthermore, some improvements are proposed to avoid the numerical problem and implement the proposed robust filter effectively. Extensive simulation and car-mounted experiment demonstrate that the proposed filter can achieve higher estimation accuracy and better robustness as compared with the related state-of-the-art methods.

INDEX TERMS Dynamic state estimation, robust estimation, cubature Kalman filter, SINS/GPS integrated navigation.

I. INTRODUCTION

SINS/GPS integration navigation system has achieved considerable penetration in the civilian and military field in recent years due to its widespread application in positioning and attitude determination [1]–[6]. The core of SINS/GPS integration is to obtain the optimal estimate of position, velocity, and attitude by designing a proper integration filter. Kalman filter is the most commonly used method for the data fusion of GPS and SINS. It can be verified that the standard Kalman filter is only suitable for linear Gaussian system, and the performance of the Kalman filter highly depends on exact prior knowledge of the system noise statistic characteristics [7]. However, the linear system does not really exist since the error model of the low-cost SINS is essentially nonlinear in practical applications such as the SINS/GPS integration navigation system [8]. Furthermore, the system process uncertainty and measurement noises are generated due to the vehicle’s severe maneuver and abnormal measurements of the velocity and position in SINS/GPS integrated navigation system [9], and the Gaussian assumption may be violated, resulting in severely breaking down the performance of the conventional KF-based GPS/INS integration system. Therefore, there is an urgent need to design an effective filter to better solve the nonlinear and non-Gaussian filtering problem, which is the main focus of our work.

Recently, various filters have been developed to deal with the process uncertain and non-Gaussian system, such as the strong tracking filter [10], [11], the model predictive based Kalman filter [12], particle filter (PF) [13], [14], Student’s t filter (STF) [15], multiple-model filter (MMF) [16]–[18], and Maximum Correntropy Criterion.
Kalman filter (MCC-KF) [19]–[24]. The strong tracking filter copes with the process uncertainty problem by employing the time-variant suboptimal fading factor matrix to the state prediction covariance matrix to maintain the residual orthogonal sequence. Unfortunately, the fading factor is determined empirically, making it a suboptimal or biased filtering solution. The model predictive based Kalman filter deals with the process uncertainty by comparing the filtering outputs with the actual measurements to estimate the dynamic model error while it has a slower convergence rate. The particle filter (PF) utilizes the sequential Monte Carlo (MC) technique to approximate the posterior probability distribution of the states with a set of the random particle [25]. It has been demonstrated that the particle filter is a high-performance solution for the non-Gaussian state estimation since the Monte Carlo sampling enables the proper approximation of any probability distribution when the set of the random particle is sufficiently large [26]. However, it is not suitable for real-time computation in the high-dimensional systems such as SINS/GPS integration navigation systems due to the large computational burden. The Student’s t filter copes with the non-Gaussian state estimation problem by modeling the non-Gaussian process and measurement noises as Student’s t distribution and then obtaining a close form solution for the filtering problem [15]. The performance of the Student’s t filter depends largely on the DOF parameters of the Student’s t distribution, especially for the case of moderate contaminated process and measurement noise. Nevertheless, it is difficult to determine the true values of DOF parameters in actual SINS/GPS integration navigation applications, degrading the estimation performance of the Student’s t filter [27]. The multiple-model filter (MMF) handles the non-Gaussian noise by approximating the non-Gaussian distribution with a finite sum of parallel Gaussian distribution that represents different modes; then, the state posterior probability density function can be modeled by a weighted sum of Gaussian, and the state value can be estimated exactly using a blank of Kalman states with a set of the random particle [25]. It has been demonstrated that the outlier-robust filter proposed in section 4. Finally, conclusions are drawn to approximate the posterior probability distribution of the proposed algorithm successfully simplify the derivation of the proposed algorithm substantially improve the computational cost and estimation accuracy.

The rest of the paper is organized as follows. The maximum correntropy criterion and the main structure of cubature Kalman filter are briefly reviewed in Section 2. Afterward, the new robust cubature Kalman filter is based on the newly defined cost function derived in section 3. Then, simulations and experiments for the SINS/GPS integration navigation system are conducted to evaluate the performance of the proposed method in section 4. Finally, conclusions are drawn in Section 5.
II. PRELIMINARIES

A. MAXIMUM CORRENTROPY CRITERION

Correntropy is defined as the similarity measure between two random variables [32]–[34]. Considering two scalar random variables X and Y, their correntropy is presented mathematically as:

$$V(X, Y) = \mathcal{E}[\kappa(X, Y)] = \int \kappa(x, y) dF_{XY}(x, y)$$  \hspace{1cm} (1)

where $\mathcal{E} [\cdot]$ is the expectation operator, $\kappa(\cdot, \cdot)$ is the Mercer’s type positive definite kernel function, and $F_{XY}(x, y)$ denotes the joint probability density function of X and Y. In this paper, we consider the kernel function is the popular Gaussian kernel, which is given by:

$$\kappa(x, y) = \sigma_0 (x - y) = \exp \left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$  \hspace{1cm} (2)

where $\sigma > 0$ denotes the kernel bandwidth of correntropy. From (2) we can see that the correntropy of random variable X and Y with the Gaussian kernel reach its limit if and only if $X = Y$. Since the joint distribution function is always unknown and only a finite number of data sample is available in many practical applications, the correntropy is always estimated by using the sample estimator with N data points as follow:

$$\hat{V}(X, Y) = \frac{1}{N} \sum_{i=1}^{N} \sigma_0 (x_i - y_i)$$  \hspace{1cm} (3)

where $(x_i, y_i)_{i=1}^{N}$ denotes the samples drawn from the joint probability density function $F_{XY}(x, y)$. Applying the Taylor series expansion to the Gaussian kernel yields:

$$V(X, Y) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} \sigma_0 (x - y)^{2n}$$  \hspace{1cm} (4)

From (4) we can see that the correntropy contains a weighted sum of all even order moments of the random variable difference X-Y, which make it possible to capture the higher-order information from the measurements with proper selected kernel bandwidth. When comparing with the famous minimum mean square principle that uses the second-order information (mean and variance) from measurements, the robust method is able to utilize the measurements’ higher-order information. That is why the robust filters are more suitable for the stochastic systems with contaminated non-Gaussian noises.

B. THE TRADITIONAL CUBATURE KALMAN FILTER

According to the previous study, cubature Kalman filter (CKF) is an effective method to deal with the state estimation of high dimensional nonlinear system [35], [36]. Consider the non-linear discrete-time dynamic system with additive noise as follows:

$$\begin{align*}
\mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k
\end{align*}$$  \hspace{1cm} (5)

where $k$ is the discrete time index, $\mathbf{x}_k \in \mathbb{R}^{n \times 1}$ is the n-dimensional system state at time step $k$, $\mathbf{z}_k \in \mathbb{R}^{m \times 1}$ is the m-dimensional measurement vector at discrete time $k$; $f(\cdot)$ and $h(\cdot)$ are respectively the nonlinear dynamic model and measurement model. $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ are assumed to be process and measurement Gaussian noise sequences with zero means and variances $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$, respectively.

Generally, cubature Kalman filter mainly contains the following two steps: prediction and update.

**Prediction:**

1. Factorize the covariance and evaluate the cubature points $(i = 1, 2, \ldots, 2n)$

$$\mathbf{P}_{k-1|k-1} = \mathbf{S}_{k-1|k-1} \mathbf{S}_{k-1|k-1}^T$$  \hspace{1cm} (6)

$$\mathbf{X}_{i,k-1|k-1} = \mathbf{S}_{k-1|k-1} \xi_i + \hat{\mathbf{x}}_{k-1|k-1}$$  \hspace{1cm} (7)

where $n$ is the dimension of $\mathbf{x}_k$, $\xi_i$ is the $i$th cubature point, which are given by:

$$\xi_i = \begin{cases}
\sqrt{n} [1]_i, & i = 1, 2, \ldots, n \\
-\sqrt{n} [1]_i, & i = n + 1, n + 2, \ldots, 2n
\end{cases}$$  \hspace{1cm} (8)

where $[1]_i$ denotes the $i$th column vector of the points set [1]. For example, $[1] \in \mathbb{R}^2$ represents the following set of points:

$$\left\{ \left( \begin{array}{c}
1 \\
0
\end{array} \right), \left( \begin{array}{c}
0 \\
1
\end{array} \right), \left( \begin{array}{c}
-1 \\
0
\end{array} \right), \left( \begin{array}{c}
0 \\
-1
\end{array} \right) \right\}$$

2. Evaluate the propagated cubature points through the state equation:

$$\mathbf{X}_{i,k|k-1}^* = \mathbf{f} (\mathbf{X}_{i,k-1|k-1})$$  \hspace{1cm} (9)

3. Estimate the prior state and evaluate the corresponding error covariance matrix

$$\hat{\mathbf{x}}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \mathbf{X}_{i,k|k-1}^*$$  \hspace{1cm} (10)

$$\mathbf{P}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \mathbf{X}_{i,k|k-1}^* \mathbf{X}_{i,k|k-1}^T - \hat{\mathbf{x}}_{k|k-1} \hat{\mathbf{x}}_{k|k-1}^T + \mathbf{Q}_{k-1}$$  \hspace{1cm} (11)

**Update:**

1. Factorize the prior covariance and evaluate the cubature points $(i = 1, 2, \ldots, 2n)$

$$\mathbf{P}_{k|k-1} = \mathbf{S}_{k|k-1} \mathbf{S}_{k|k-1}^T$$  \hspace{1cm} (12)

$$\mathbf{X}_{i,k|k-1} = \mathbf{S}_{k|k-1} \xi_i + \hat{\mathbf{x}}_{k|k-1}$$  \hspace{1cm} (13)

2. Evaluate the propagated cubature points through the measurement equation

$$\mathbf{Z}_{i,k|k-1} = h (\mathbf{X}_{i,k|k-1})$$  \hspace{1cm} (14)

3. Estimate the priori measurement

$$\hat{\mathbf{z}}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \mathbf{Z}_{i,k|k-1}$$  \hspace{1cm} (15)
Estimate the covariance matrix, cross covariance matrix and compute the Kalman gain

\[
P_{zz,k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} Z_i, k|k-1 Z_i, k|k-1 \hat{z}_{k|k-1}^T + R_k
\] (16)

\[
P_{xz,k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} X_i, k|k-1 Z_i, k|k-1 \hat{z}_{k|k-1}^T
\] (17)

\[
K_k = P_{xz,k|k-1}/P_{zz,k|k-1}
\] (18)

Estimate the posterior state and the corresponding error covariance matrix

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1})
\] (19)

\[
P_{k|k} = P_{k|k-1} - K_k P_{zz,k|k-1} K_k^T
\] (20)

III. THE NEW ROBUST CUBATURE KALMAN FILTER

Traditional cubature Kalman filter performs well for the nonlinear Gaussian system. However, its performance may be severely degraded under the non-Gaussian noises, especially when the process and measurement information are all disturbed by non-Gaussian noise. To solve the problem, we utilize the maximum correntropy criterion and residual orthogonal principle-based weighted least square technology to derive a new robust cubature Kalman filter (RCKF) in this work. This new RCKF may perform better in the presence of both process uncertainty and non-Gaussian measurement noise since the correntropy contains the high order information of error and the residual orthogonal principle can extract the useful information in the residual error completely.

A. DERIVATION OF THE NEW ROBUST CUBATURE KALMAN FILTER

In this section, a new cost function special for the suppression of both process system uncertainty and non-Gaussian measurement noise is defined, and the new robust filter in the framework of CKF is further derived. Considering that the special case that the measurement equation of the nonlinear loosely couple SINS/GPS integration system is linear[2], we have

\[
z_k = H_k x_k + v_k
\] (21)

Denote \( \varepsilon = v_k \), with the covariance matrix of \( \varepsilon \) is given by:

\[
E \left[ \varepsilon \varepsilon^T \right] = R_k = B_k B_k^T
\] (22)

where \( B_k \) can be obtained by Cholesky decomposition, left multiplying both sides of (21) by \( B_k^{-1} \) yields the linear regress model:

\[
D_k = g(x_k) + \xi_k
\] (23)

where \( D_k = B_k^{-1} z_k, g(x_k) = B_k^{-1} H_k x_k \) and \( \xi_k = B_k^{-1} v_k \).

Motivated by the cost function of weighted least square (WLS), we propose to use the residual orthogonal principle-based WLS method to deal with process uncertainty noise and the maximum correntropy criterion to handle the non-Gaussian measurement noise, and then construct the new cost function termed adaptive maximum correntropy criterion (AMCC) as follows:

\[
J_L (x_k) = \|x_k - \hat{x}_{k|k-1}\|^2 / (\lambda_k P_k|k-1)^{-1} + \sum_{i=1}^{m} \rho_{MCC} (e_{k,i})
\] (24)

where \( \|x\|_A^2 = x^T A x \) is the quadratic form with respect to \( A \); \( \hat{x}_{k|k-1} \) denotes the prediction of state \( x_k \) at time step \( k \), and \( P_{k|k-1} \) represents the corresponding error covariance; \( \lambda_k \) is the fading factor, which is used to strengthen the robustness of the proposed filter against the process uncertainty noise. \( \rho_{MCC} (e_{k,i}) = \left( 1 - \exp \left( -e_{k,i}^2 / 2\sigma^2 \right) \right) / \sqrt{2\pi}\sigma \), \( e_{k,i} = \bar{D}_{k,i} - g(x_{k,i}) \), \( e_{k,i} \) is the i-th element of \( e_k \), and \( m \) refers to the dimension of \( D_k \).

Under the new cost function defined above, the linear regress problem in (23) can be solved effectively and robustly by minimizing the following function:

\[
\hat{x}_k = \arg \min_{x_k} \left\{ \|x_k - \hat{x}_{k|k-1}\|^2 / (\lambda_k P_k|k-1)^{-1} + \sum_{i=1}^{m} \rho_{MCC} (e_{k,i}) \right\}
\] (25)

The optimal solution can thus be found by differentiating the cost function with respected to \( x_k \) as:

\[
(\lambda_k P_k|k-1)^{-1} (x_k - \hat{x}_{k|k-1}) - \sum_{i=1}^{m} \psi (e_{k,i}) \frac{\partial e_{k,i}}{\partial x_k} = 0
\] (26)

where \( \psi (e_{k,i}) = -\frac{\partial \rho_{MCC} (e_{k,i})}{\partial e_{k,i}} \), by defining the function \( C_{k,i} = \frac{\psi (e_{k,i})}{\rho_{MCC} (e_{k,i})} \) and dialog matrix \( C_k = \text{diag} \{C_{k,i}\}_i = 1, \ldots, m, \) we can rewrite equation (26) as follows:

\[
(\lambda_k P_k|k-1)^{-1} (x_k - \hat{x}_{k|k-1}) - H_k^T B_k^{-T} C_k e_k = 0
\] (27)

Substituting \( e_k = D_k - g(x_k) = B_k^{-1} (z_k - H_k x_k) \) into (27) yields:

\[
(\lambda_k P_k|k-1)^{-1} (x_k - \hat{x}_{k|k-1}) - H_k^T B_k^{-T} C_k B_k^{-1} (z_k - H_k x_k) = 0
\] (28)

Let \( \tilde{P}_{k|k-1} = \lambda_k P_{k|k-1} \tilde{R}_k = B_k C_k^{-1} B_k^T \) and \( \hat{x}_{k|k} = x_k \), we have:

\[
\tilde{P}_{k|k-1} (\hat{x}_{k|k-1} - \hat{x}_{k|k-1}) = H_k^T \tilde{R}_k^{-1} (z_k - H_k \hat{x}_{k|k})
\] (29)

By adding and subtracting the term \( H_k^T \tilde{R}_k^{-1} \bar{H}_k \hat{x}_{k|k-1} \) on the right side of (29) we get:

\[
\left( \tilde{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} \bar{H}_k \right) \hat{x}_{k|k} = \tilde{P}_{k|k-1}^{-1} (\hat{x}_{k|k-1} - \bar{H}_k \tilde{R}_k^{-1} H_k \hat{x}_{k|k-1}) + H_k^T \tilde{R}_k^{-1} (z_k - H_k \hat{x}_{k|k})
\] (30)

\[
\left( \tilde{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} \bar{H}_k \right) \hat{x}_{k|k} = (\tilde{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k) \hat{x}_{k|k-1} + H_k^T \tilde{R}_k^{-1} (z_k - H_k \hat{x}_{k|k-1})
\] (31)
Then, by left multiplying both sides of (31) by \( \left( \hat{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k \right)^{-1} \), we can obtain the following results:

\[
\hat{s}_{k|k} = \hat{s}_{k|k-1} + K_k \left( z_k - H_k \hat{s}_{k|k-1} \right) \quad (32)
\]

\[
K_k = \left( \hat{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k \right)^{-1} H_k^T \tilde{R}_k^{-1} \quad (33)
\]

Meanwhile, the corresponding posterior covariance matrix can be updated by:

\[
\tilde{P}_{k|k} = (I - K_k H_k) \hat{P}_{k|k-1} (I - K_k H_k)^T + K_k \tilde{R}_k K_k^T \nonumber \]

\[
= (I - K_k H_k) \hat{P}_{k|k-1} \quad (34)
\]

It is noted in (33) that the calculation of \( \tilde{R}_k \) requires the inversion matrix of \( C_k \), which may not exist when the measurement obtains extremely large value, that is when \( ||z_k|| \rightarrow \infty \), then \( C_{k,i} = \frac{\sigma^2 \varepsilon_i}{\varepsilon^2} \rightarrow 0 \), \( C_k = \text{diag} \left[ C_{k,i} \right] \rightarrow 0 \). In order to solve the problem, we adopt the innovation sequence real-time detection strategy. That is when the innovation sequence is abnormal, only the time update is conducted, otherwise, both time update and measurement update are conducted.

As one can also see that two \( n \times n \) and one \( m \times m \) inversion matrix calculation are required for the computation of Kalman gain in (33), since \( \hat{P}_{k|k-1} \in R^{n \times n} \), \( \left( \hat{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k \right)^{-1} \in R^{n \times n} \) and \( \tilde{R}_k \in R^{m \times m} \). Therefore, for the high-dimensional SINS/GPS integration navigation system, the proposed robust CKF will suffer from the large computational complexity. In order to reduce the computational complexity, we modify the Kalman gain in (33) as an alternate form as follows:

Similar to the measurement update of classical KF, we can prove that the following formulas hold [37]:

\[
K_k = \left( \hat{P}_{k|k-1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k \right)^{-1} H_k^T \tilde{R}_k^{-1} = \tilde{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} \quad (35)
\]

Substituting (34) into (35), we have:

\[
K_k = \left( I - K_k H_k \right) \hat{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} \quad (36)
\]

\[
\hat{P}_k = \left( I + H_k \tilde{R}_k \tilde{P}_{k|k-1} H_k^T \right) \tilde{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} = \tilde{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} \quad (37)
\]

With \( \tilde{P}_{k|k-1} = \tilde{P}_k \), (37) is transformed to the form as follows:

\[
K_k = \left( I + H_k \tilde{R}_k \tilde{P}_{k|k-1} H_k^T \right) \tilde{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} = \tilde{P}_{k|k-1} H_k^T \tilde{R}_k^{-1} \quad (38)
\]

\[
K_k = \frac{\tilde{P}_{k|k-1} H_k^T}{\tilde{R}_k + H_k \tilde{P}_{k|k-1} H_k^T} \quad (39)
\]

It can be obviously seen that only one matrix inversion is required in the alternated from of \( K_k \) in (39) when compared with the old form in (33), which can greatly reduce the computational cost in the high dimensional SINS/GPS integrated navigation system.

Remark 1: As one can observe that, different from the conventional maximum correntropy criterion (MCC) adopted in [29], [31] which is only suitable for the suppression of non-Gaussian measurement noise under the condition that the process distribution is Gaussian, our proposed AMCC use the advantage of residual orthogonal principle based weighted least square technology to further improve the robustness against both of the process uncertainty and non-Gaussian measurement noise without the consideration of process distribution, which can be appropriately applied to the loosely coupled SINS/GPS integrated navigation system with the time-varying process uncertainty.

B. PARAMETER DETERMINATION OF THE PROPOSED RCKF

To implement the proposed RCKF, two parameters i.e. the fading factor \( \lambda_k \) and the kernel bandwidth \( \sigma \) need to be determined.

1) FADING FACTOR DETERMINATION

The fading factor \( \lambda_k \) is determined according to the innovation sequence orthogonality principle i.e. \( E \left[ \varepsilon_{k+j} \varepsilon^T_k \right] = 0 \) [38], [39], and the sufficient condition of \( E \left[ \varepsilon_{k+j} \varepsilon^T_k \right] = 0 \) is as follows:

\[
\tilde{P}_{k|k-1} H_k^T - \tilde{K}_k V_k \equiv 0 \quad (40)
\]

where \( V_k = (z_k - H_k \hat{x}_{k|k-1}) (z_k - H_k \hat{x}_{k|k-1})^T \) denotes the residual error sequence covariance matrix. Using \( \tilde{K}_k = \tilde{P}_{k|k-1} H_k^T \left( H_k \tilde{P}_{k|k-1} H_k^T + \tilde{R}_k \right)^{-1} \) in (40) gives

\[
\tilde{P}_{k|k-1} H_k^T - \tilde{P}_{k|k-1} H_k^T \left( H_k \tilde{P}_{k|k-1} H_k^T + \tilde{R}_k \right)^{-1} V_k \equiv 0 \quad (41)
\]

Simplify equation (41) yields:

\[
H_k \tilde{P}_{k|k-1} H_k^T = V_k - \tilde{R}_k \quad (42)
\]

Using \( \tilde{P}_{k|k-1} = \lambda_k \tilde{P}_{k|k-1} \) in (42) yields

\[
H_k \lambda_k \tilde{P}_{k|k-1} H_k^T = V_k - \tilde{R}_k \quad (43)
\]

With the velocity and position integration mode adopted in this paper, the system observation matrix \( H_k \) can be therefore expressed as follows [40]:

\[
H_k = \begin{bmatrix} 0_{6 \times 3} & 1_{6 \times 6} & 0_{6 \times 12} \end{bmatrix} \quad (44)
\]

In the conventional fading factor determination method, the time-varying scaling fading factor \( \lambda_k \) is obtained by calculating (43) as described in [41]. However, in practical high-dimensional integration navigation application, a single scaling fading factor is unable to modify all the error state with abnormal estimation since the estimation degree of the multiple channel error states differs from each other. In order to improve the adaptive adjustment capability for the different channels of the multi-dimensional GPS/INS integrated navigation systems, a multiple-channel adaptive strategy with the time-varying multiple fading factor \( \lambda_{k \in \{1, 2, \ldots, n\}} = \text{diag} \left\{ 1, 1, 1, \lambda_{4,k}, \lambda_{5,k}, \ldots, \lambda_{9,k}, \ldots, 1 \right\} \) is introduced in this paper, where \( \left\{ \lambda_{4,k}, \lambda_{5,k}, \ldots, \lambda_{9,k} \right\} \) is utilized to adjust the state of velocity error and position error that can
be directly observed, whereas the other elements of $\lambda_k$ are all set as one due to their unobservabilities. Here we define:

$$\Delta_k = H_k \lambda_k$$

$$= \begin{bmatrix}
\lambda_{4,k} & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_{5,k} & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_{6,k} & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_{7,k} & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{8,k} & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{9,k}
\end{bmatrix}_{6 \times 12}$$ (45)

$$G_k = P_{k|k-1}H_k^T$$ (46)

$$N_k = V_k - \tilde{R}_k$$ (47)

Using (45) - (47) in (43) yields

$$\Delta_k G_k = N_k$$ (48)

i.e.

$$\begin{bmatrix}
\lambda_{4,k} & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_{5,k} & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_{6,k} & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_{7,k} & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{8,k} & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{9,k}
\end{bmatrix}_{6 \times 12} G_k = N_k$$ (49)

Employing the method of least squares in (49) yields:

$$\lambda_{4,k} = \frac{\sum_{j=1}^{6} G_k [4,j]N_k [1,j]}{\sum_{j=1}^{6} G_k^2 [4,j]}$$ (50)

Similarly, we obtain $\lambda_{5,k} \sim \lambda_{9,k}$, and the calculation of proposed multiple suboptimal fading factor $\lambda_{i,k}$ can be summarized as

$$\lambda_{i,k} = \frac{\sum_{j=1}^{6} G_k [i,j]N_k [i-3,j]}{\sum_{j=1}^{6} G_k^2 [i,j]}$$ (51)

In this section, the convergence of the proposed algorithm is analyzed using Lyapunov stability through the following proposition.

Proposition 1: Consider the nonlinear system described by (5). A sufficient condition to ensure the convergence of the proposed algorithm is as follows:

$$\alpha_k R_k^{-1} - \alpha_k R_k^{-1} F_k^{-1} \alpha_k + R_k^{-1} H_k P_k H_k^T R_k^{-1} \leq 0$$

And

$$F_k = \left(F_{k-1} P_{k-1} F_{k-1}^{-1}\right)^{-1} - \beta_k F_{k-1} - P_{k-1} \leq 0$$

Proof: Denote by $\hat{x}_k$ the state estimation, by $\tilde{x}_{k|k-1}$ the one-step state prediction. The state estimation error $\tilde{x}_k$ and state prediction error $\tilde{x}_{k|k-1}$ are respectively defined as follows:

$$\tilde{x}_k = x_k - \hat{x}_k$$

$$\tilde{x}_{k|k-1} = x_k - \tilde{x}_{k|k-1}$$ (53)

From (53) we can obtain the innovation $\varepsilon_k$ as follows:

$$\varepsilon_k = H_k \tilde{x}_{k|k-1}$$ (54)

To obtain an exact equality, we introduce here unknown diagonal matrix $\alpha_k = diag(\alpha_{1,k}, \alpha_{2,k}, \ldots, \alpha_{n,k})$ and obtain:

$$\alpha_k \varepsilon_k = H_k \tilde{x}_{k|k-1}$$ (55)

Expanding $x_k$ by Taylor series about $\hat{x}_{k-1}$ yields:

$$x_k = f(\hat{x}_{k-1}) + \nabla f(\hat{x}_{k-1}) \tilde{x}_{k-1} + \frac{1}{2} \nabla^2 f(\hat{x}_{k-1}) \tilde{x}_{k-1}^2 + \cdots + w_k$$ (56)

where we define $F_{k-1} = \nabla f(\hat{x}_{k-1})$. Similarly, we can obtain the one step prediction state $\tilde{x}_{k|k-1}$ and one step prediction error $\tilde{x}_{k|k-1}$ according to [42].
To describe the first order linearization error, we introduce the unknown time-varying diagonal matrix \( \beta_k = \text{diag}(\beta_{1,k}, \beta_{2,k}, \ldots, \beta_{\ell,k}) \) and obtain:

\[
\tilde{\mathbf{x}}_{k|k-1} = \beta_k \mathbf{F}_{k-1} \tilde{\mathbf{x}}_{k-1}
\]  

(57)

The candidate Lyapunov function \( \mathbf{V}_k \) is defined as:

\[
\mathbf{V}_k = \tilde{\mathbf{x}}_k^T \mathbf{P}_k^{-1} \tilde{\mathbf{x}}_k
\]  

(58)

Our objective here is to derive conditions under which \( \{\mathbf{V}_k\}_{k=2} \) is a decreasing sequence \( \mathbf{V}_k - \mathbf{V}_{k-1} \leq 0 \). Combining (32) and (53), we obtain

\[
\tilde{\mathbf{x}}_k = \tilde{\mathbf{x}}_{k|k-1} - \mathbf{P}_k \mathbf{P}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \tilde{\mathbf{x}}_{k|k-1} - \mathbf{P}_k \mathbf{P}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{R}_k^{-1} \mathbf{e}_k
\]  

(59)

where \( \mathbf{e}_k = (\mathbf{z}_k - \mathbf{H}_k \tilde{\mathbf{x}}_{k|k-1}) \).

From [37], we can prove the following formulas hold:

\[
\mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} = \mathbf{P}_k \mathbf{H}_k^T \mathbf{H}_k \mathbf{R}_k^{-1} \mathbf{H}_k^T \mathbf{R}_k^{-1}
\]  

(60)

\[
\mathbf{P}_k^{-1} = \mathbf{P}_k^{-1} \mathbf{H}_k^T \mathbf{R}_k^{-1} + \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k
\]  

(61)

Combining (58)-(61), we have

\[
\mathbf{V}_k = \left( \tilde{\mathbf{x}}_{k|k-1} - \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{e}_k \right)^T \mathbf{P}_k^{-1} \left( \tilde{\mathbf{x}}_{k|k-1} - \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{e}_k \right)
\]  

(62)

Substituting (61) into (62), the candidate Lyapunov function \( \mathbf{V}_k \) becomes

\[
\mathbf{V}_k = \mathbf{V}_k|k-1 + \tilde{\mathbf{x}}_k^T \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \tilde{\mathbf{x}}_{k|k-1} - \tilde{\mathbf{x}}_k^T \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{e}_k + \mathbf{e}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \tilde{\mathbf{x}}_{k|k-1} + \mathbf{e}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{e}_k
\]  

(63)

With

\[
\mathbf{V}_k|k-1 = \tilde{\mathbf{x}}_{k|k-1}^T \mathbf{P}_k^{-1} \tilde{\mathbf{x}}_{k|k-1}
\]  

(64)

To ensure \( \{ \mathbf{V}_k \}_{k=2} \) a decreasing sequence, we have

\[
\mathbf{V}_k - \mathbf{V}_{k-1}
\]

\[
= \mathbf{V}_k - \mathbf{V}_{k|k-1} + \mathbf{V}_{k|k-1} - \mathbf{V}_{k-1}
\]

\[
= \varepsilon_k \left[ \mathbf{a}_k \mathbf{R}_k^{-1} \mathbf{a}_k - \mathbf{a}_k \mathbf{R}_k^{-1} \mathbf{R}_k^{-1} \mathbf{a}_k + \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \right] \mathbf{e}_k
\]

\[
+ \mathbf{e}_k^T \left( \mathbf{F}_{k|k-1} \mathbf{F}_{k|k-1}^T \right)^{-1} \mathbf{F}_{k|k-1} \mathbf{F}_{k|k-1}^T \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1}
\]

\[
\times \tilde{\mathbf{x}}_{k|k-1} \leq 0
\]  

(65)

A sufficient condition to ensure that is

\[
\mathbf{a}_k \mathbf{R}_k^{-1} \mathbf{a}_k - \mathbf{a}_k \mathbf{R}_k^{-1} \mathbf{R}_k^{-1} \mathbf{a}_k + \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \leq 0
\]

And

\[
\mathbf{F}_{k|k-1} \mathbf{F}_{k|k-1} \mathbf{F}_{k|k-1}^T \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \leq 0
\]  

(66)
where \( \varphi^e, \delta v^e, \delta p^e \) denote the attitude error, velocity error and position error expressed in the n-frame, respectively. \( b_x \) and \( b_y \) denote the static biases of tri-axial gyroscope and accelerometer resolving in the b-frame, respectively. \( \delta b_x \) and \( \delta b_y \) are respectively the dynamic biases of tri-axial gyroscope and accelerometer expressed in the b-frame. The nonlinear system error equation of SINS can be formulated as:

\[
\begin{align*}
\dot{\varphi}^e &= C_w \left[ (I - C^p_n) w^p_n + \delta w^p_n - C^p_n \delta w^p_{ib} \right] \\
\dot{\delta v}^e &= \left( I - C^p_n \right) C^p_f b_f + C^p_n \delta b_f + \delta v^e \times (2w^p_e + w^p_{en}) \\
\dot{\delta \lambda} &= \frac{R_N + h}{\delta V_N} \sec L + \delta L \frac{V_E \sec L}{R_N + h} \tan L \\
\delta \hat{L} &= \frac{R_N + h}{V_U} \\
\delta \dot{b}_g &= 0 \\
\delta b_y &= 0 \\
\delta v^e &= -\frac{1}{\tau_g} \delta b_g + \eta_g \\
\delta b_f &= -\frac{1}{\tau_f} \delta b_f + \eta_f
\end{align*}
\]  

(67)

where \( C_w \) denotes the transformation matrix from angle rate to Euler angle, \( C^p_n \) the attitude rotation matrix from n-frame (ideal navigation frame) to p-frame (actual navigation frame), \( C^p_f \) the attitude rotation matrix from the body frame to actual navigation frame, \( w^p_{ib} \) the angle measured by the gyroscopes in the body frame, \( v^e \), the specific force measured by the accelerometer in the body frame, \( w^p_e \) the rotation velocity of a-frame with respected to b-frame expressed in c-frame. \( V_E, V_N, V_U \) are respectively the velocity component in east, north and up direction. \( L, \lambda, h \) denote the latitude, longitude, and height above the earth surface, respectively. \( R_N \) is the normal radius. \( \tau_g \) and \( \tau_f \) are respectively the correlation time of 1st-order Markovian process for gyroscope and accelerometer. \( \eta_g \) and \( \eta_f \) are the zero-mean Gaussian white noise process.

### B. Simulation Result

In this simulation, the performance of the proposed RCKF is illustrated in the problem of SINS/GPS integrated navigation with process uncertainty and non-Gaussian measurement noise. The process model and measurement of SINS/GPS are formulated as follows:

\[
\begin{align*}
{x}_k &= f({x}_{k-1}) + {v}_{k-1} \\
{z}_k &= {H}_k {x}_k + {o}_{k-1}
\end{align*}
\]

where the state vector is selected as (66), \( f(.) \) is the nonlinear dynamic model function of (67). The observation vector \( z_k = \begin{bmatrix} V_{GPS} - V_{SINS}, L_{GPS} - L_{SINS}, h_{GPS} - h_{SINS}, \lambda_{GPS} - \lambda_{SINS} \end{bmatrix}^T \), where the subscripts (GPS and SINS) denote the velocity and geographical position obtained from GPS and SINS, respectively. \( H_k = [I_{6x3}, I_{6x6}, 0_{6x12}] \) is the observation matrix. \( v_{k-1} \) and \( o_{k-1} \) are respectively the zero mean process uncertainty and non-Gaussian measurement noise, and the process uncertainty and non-Gaussian measurement noise are generated according to [44]:

\[
\begin{align*}
{v}_k &\sim N(0, Q_k) \text{ w.p.0.9} \\
{w}_k &\sim N(0, R_k) \text{ w.p.0.9}
\end{align*}
\]

(68)

(69)

where \( N(\mu, \Sigma) \) denotes the Gaussian distribution with mean \( \mu \) and covariance matrix \( \Sigma \), \( Q_k \) and \( R_k \) are the nominal covariance matrices from the Gaussian distribution, w.p. denotes “with probability”. Equation (68) and (69) mean that 90% of process and measurement noise are drawn from Gaussian distribution with nominal covariance \( Q_k \) and \( R_k \), and 10% of process and measurement noise are drawn from the Gaussian distribution with severely increased covariance. With this mixed-Gaussian distribution, the process uncertainty and non-Gaussian measurement noise can be obtained.

In our simulation scenario, the vehicle runs along the trajectory as shown in Figure 1. The total simulation time is 500s.

In this simulation, the sample intervals of GPS and SINS are respectively 0.1s and 0.001s. The initial position is \( \text{PosNo}= [38.014deg, 836m, 112.445deg]^T \), initial velocity is \( \text{VelNo}= [12m/s, 0, 0]^T \) and initial attitude is \( q_0^0 = [1, 0, 0, 0]^T \). The constant drift of gyroscope and accelerometer are respectively set as \( \epsilon_b = [2degh, 24degh, 24degh]^T \) and \( V_b = [5mg, 5mg, 5mg]^T \). The initial state vector and the associated covariance are set as \( \hat{x}_0^0 = 0_{21x1} \) and \( P_{00} = \text{diag}(0.0088rad, 0.023rad, 0.0088rad, 0.15ms, 0.15ms, 0.15ms, 3m, 10m, 3m, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 12mg, 12mg, 12mg, 0.0012deg/s, 0.0012deg/s, 0.0012deg/s, 0.0012deg/s, 0.1mg, 0.1mg, 0.1mg]^T \). The initial value of the nominal process noise covariance matrix and measurement noise covariance matrix are respectively set as \( Q_k = \text{diag}(0.3deg/sqrt(Hz), 0.3deg/sqrt(Hz), 0.3deg/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz), 0.3m/s^2/sqrt(Hz)^2 \) and \( R_k = \text{diag}(0.15m/s, 0.15m/s, 0.15m/s, 3m, 10m, 3m, 0.15m/s, 0.15m/s, 3m, 10m, 3m, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 0.25deg/s, 12mg, 12mg, 12mg, 0.0012deg/s, 0.0012deg/s, 0.0012deg/s, 0.0012deg/s, 0.1mg, 0.1mg, 0.1mg)^2 \). To further stress the performance, our proposed method is compared with the existing cubature Kalman filter (CKF) [35], maximum correntropy square-root cubature Kalman filter (MCSCKF) [31], H-infinity filter (H-infinity) [45], Huber’s M filter (Huber’s M) [46], robust Student’s t based nonlinear filter (RSTNF) [47] and interacting multiple model filter (IMM) [48]. Among which, the IMM and RSTNF can deal with both of the non-Gaussian process and measurement noise, while the H-infinity, Huber’s M and MCSCKF are only for the non-Gaussian measurement noise. In the RSTNF, the dof parameter, turning parameter and the iteration number are chosen as: \( v = 5, \tau = 5 \) and \( N = 10 \). In the MCSCKF, the kernel size is set as: \( \sigma = 3 \). The proposed filter and the existing filters are all coded with MATLAB and carried out
FIGURE 2. True trajectory of the vehicle.

We compare the overall estimation performance of the proposed filter and existing filters based on two metrics, i.e. the RMSEs and the averaged RMSEs (ARMSE) of velocity and position for the SINS/GPS integrated navigation system, which are respectively defined as follows:

\[ \text{RMSE} = \sqrt{\frac{1}{M} \sum_{s=1}^{M} (x_k^s - \hat{x}_k^s)^2} \]  
\[ \text{ARMSE} = \sqrt{\frac{1}{MT} \sum_{k=1}^{T} \sum_{s=1}^{M} (x_k^s - \hat{x}_k^s)^2} \]

where \( x_k^s \) and \( \hat{x}_k^s \) denote the true and the estimated value of velocity and position at time \( k \) of the \( s \)-th Monte Carlo run, respectively. \( M = 100 \) and \( T = 500000 \) are respectively the entire number of Monte Carlo runs and total simulation samples.

The RMSEs of velocity, position and attitude obtained from the six existing filters and the proposed filter are shown in Figure 2-4. It is obvious that the CKF has the poorest accuracy since the Gaussian filtering (i.e., the KF, EKF, UKF or CKF) are not suitable for the nonlinear systems with unknown or uncertain noise. In contrast, MCSCKF, H-infinity, Huber’s M perform better than CKF. This is due to the fact that the maximum correntropy method, min-max estimation methods, and M-estimation method exploited in MCSCKF, H-infinity, Huber’s M can effectively deal with the non-Gaussian measurement noise. It is worth noting that due to the influence of kernel bandwidth selection, the MCSCKF exhibits worse performance as compared with H-infinity filter and Huber’s M filter. Additionally, as the MCSCKF, H-infinity, Huber’s M are sensitive to the time-varying process noise, these algorithms results in poor RMSEs of velocity, position and attitude as compared with RSTNF and IMM. This agrees with the fact that RSTNF and IMM can simultaneously cope with the process uncertainty and non-Gaussian measurement noise by utilizing the student’s t and multiple-model technology. Moreover, it can also been observed from Figure 2-4 that the proposed filter has smaller RMSEs of attitude, velocity and position than the existing CKF, MCSCKF, H-infinity filter, Huber’s M, RSTNF and IMM, demonstrating the inherited advantages of adaptive maximum correntropy criterion.

The implementation time of the CKF, MCSCKF, H-infinity filter, Huber’s M, RSTNF, IMM and the proposed filter for a single step run are respectively 0.408ms, 0.430ms, 0.412ms, 0.419ms, 0.459ms, 0.759ms and 0.415ms. Therefore, taking both the estimation accuracy and computational efficiency into account, we can conclude that the proposed RCKF can achieve better performance with the compromised
computational cost when compared with the existing state-of-art method.

In order to discuss how different values of the kernel bandwidth in the proposed RCKF method impact the estimated accuracy of the SINS/GPS integration navigation system, we evaluated the five values of the kernel bandwidth \( \sigma = 2, 3, 4, 5, 10 \). The ARMSEs of velocity and position from the proposed filter with different kernel bandwidths \( \sigma = 2, 3, 4, 5, 10 \) are shown in Table 1. It can be seen from Table 2 that the proposed filter with the kernel bandwidth range from 2 to 10 outperforms the CKF. In particular, the proposed filter exhibits the best performance when \( \sigma = 3 \), and too large or too small value of \( \sigma \) will decline the filtering performance. Furthermore, we can also see that the proposed filter with adaptive kernel bandwidth \( \sigma = ||z_k - H_k \hat{x}_k|k-1|| R_k^{-1} \) can also obtain the compromised filtering accuracy.

To verify the superiority of our proposed multiple-channel adaptive strategy for the process uncertainty suppression, we use the robust cubature Kalman filter with the traditional fading factor proposed in [38], [41], [49] as comparison. The comparison result is given in Figure 5, where the ARMSE of velocity and position are computed. It is noted that the newly proposed multiple-channel adaptive strategy has much smaller ARMSE than the traditional fading factor. The ARMSE of velocity and position obtained using the
TABLE 1. ARMSEs of RCKF with different kernel bandwidths.

| Filters | PosN (m) | PosU (m) | PosE (m) | VelN (m/s) | VelU (m/s) | VelE (m/s) |
|---------|----------|----------|----------|------------|------------|------------|
| CKF     | 0.167    | 0.297    | 0.121    | 0.025      | 0.036      | 0.032      |
| RCKF ($\sigma = 2$) | 0.103    | 0.228    | 0.116    | 0.023      | 0.014      | 0.025      |
| RCKF ($\sigma = 3$) | 0.087    | 0.186    | 0.101    | 0.019      | 0.012      | 0.018      |
| RCKF ($\sigma = 4$) | 0.103    | 0.242    | 0.115    | 0.022      | 0.014      | 0.022      |
| RCKF ($\sigma = 5$) | 0.116    | 0.261    | 0.118    | 0.021      | 0.016      | 0.026      |
| RCKF ($\sigma = 10$) | 0.119    | 0.278    | 0.120    | 0.023      | 0.021      | 0.029      |
| RCKF (adaptive) | 0.106    | 0.245    | 0.116    | 0.022      | 0.015      | 0.023      |

proposed multiple-channel adaptive strategy are significantly improved over those obtained using the traditional fading factor, which are 32.8%, 43.9%, 43.8%, 64.7%, 37.8% and 46.7%, respectively. Thus, according to figure 5, the robust cubature Kalman filter with our proposed multiple-channel adaptive strategy exhibits much better robustness than with the traditional fading factor under the condition of process uncertainty.

C. CAR-MOUNTED FIELD TEST FOR SINS/GPS INTEGRATION NAVIGATION

1) EXPERIMENT SETUP AND DESCRIPTION

In this section, car-mounted experiments for SINS/GPS integrated navigation system are performed to validate the effectiveness and superiority of the proposed algorithm in this paper. The car-mounted experimental platform is exhibited in Figure 6, and it is composed of our self-made miniature SINS/GPS integrated navigation system and a SINS/GPS-based high-accuracy reference integration navigation system. The high-accuracy integration navigation system is an integration of an LCI-1 tactical grade IMU (inertial measurement unite), a Propak satellite receiver, and two GPS antennas, and is used to provide the reference attitude, velocity, and position of high-accuracy for our self-made SINS/GPS integrated navigation system. The attitude, velocity, and position accuracy provided by the high-accuracy reference system are 0.01deg, 0.05m/s, and 0.1m, respectively. The sensor specification of our self-made SINS/GPS integration navigation system is listed in Table 2. Additionally, the experiment was conducted by the advanced navigation system research group of the North University of China (Taiyuan, China), and the total test time is 512s. The test trajectory is presented in Figure 7. The initial velocity and position of GNS/SINS are obtained directly from GPS measurement, the initial level attitude information is acquired from the coarse alignment results of SINS through the measurement of accelerometer, and the initial heading information is received from the GPS velocity vector. In the experimental test, the car moves along a dump road, and the GPS always works abnormally due to the occlusion of trees and buildings. The measurement error of GNSS velocity is exhibited in Figure 8, demonstrating that the GNSS measurement noise cannot satisfy the Gaussian distribution. Therefore, the car-mounted experiment can be utilized to verify the performance of the proposed filter against the process uncertainty noise and non-Gaussian measurement noise.
2) PERFORMANCE COMPARISON WITH DIFFERENT ROBUST FILTERING ALGORITHMS

In this section, the proposed RCKF is compared with cubicature Kalman filter (CKF) [35], maximum correntropy square-root cubicature Kalman filter (MCSCKF) [31], H-infinity filter (H-infinity) [45], Huber’s M filter (Huber’s M) [46], robust Student’s t based nonlinear filter (RSTNF) [47], and interacting multiple model filter (IMM) [48] in the car-mounted experiment to evaluate the overall performance for SINS/GPS integrated navigation system. The parameter configuration of the proposed filter and existing filters are set as follows. In the RSTNF, the dof parameter, turning parameter, and the iteration number are chosen as: \(\nu = 0.2, \tau = 5\) and \(N = 5\). In the MCSCKF, the kernel size is set as: \(\sigma = 10\). In our proposed robust cubicature Kalman filter, the kernel size and the turning parameter are set as: \(\sigma = 10\) and \(\varepsilon = 6e - 1\). The other parameters are the same as those in the simulation.

The velocity, position, and attitude error results obtained from the different algorithms are illustrated in Figures 9-11, and the corresponding RMSEs are listed in Table 3. It can be observed that the proposed filter has smaller RMSEs of velocity, position, and attitude compared to the existing robust filters. As indicated in Table 3, the RMSEs of velocity, position, and attitude from the proposed filter is reduced by 36% at least compared to the existing filters.

Large values of velocity and position error appear frequently with the CKF because CKF is specially designed for the Gaussian noise, enabling it to be more sensitive to the process uncertainty and non-Gaussian measurement noise. By contrast, the MCSCKF with the proper kernel bandwidth, H-infinity, and Huber’s M exhibit better performance than CKF due to their robustness against the non-Gaussian measurement noise provided by the maximum correntropy method, min-max estimation methods, and M-estimation method. Particularly, the min-max estimation method is superior to the M-estimation method and maximum correntropy method in the case of non-Gaussian measurement noise. Thus, the H-infinity filter has smaller RMSEs of position and velocity than MCSCKF and Huber’s M.
the above methods present an unsatisfactory performance under the condition of process uncertainty noise during the maneuvering stage on the dump road. This is because the statistical value of Gaussian process noise is changed owing to the vehicle’s severe maneuvering while MCSCKF and Huber’s M are not able to estimate the time-varying process noise. In contrast, the RSTNF and IMM using the student’s t and multiple-model method can further suppress the process uncertainty noise and achieve better performance than MCSCKF, H-infinity, and Huber’s M filter. Besides, our proposed robust filter is superior to the RSTNF and IMM, coinciding with the fact that our proposed adaptive maximum correntropy outperforms the student’s method and multiple-model filter technology when simultaneously dealing with the process and measurement noise. Large value of velocity error often appears with RSTNF since it uses inaccurate scale matrices and DOF parameters, which is difficult to determine in the real-time application. It is worth noting that
and achieve better performance than the existing robust filters with only a little additional computational time under the condition of vehicle’s severe maneuver and abnormal measurements in SINS/GPS integrated navigation system.

V. CONCLUSION

In this paper, a new robust cubature Kalman filter for the SINS/GPS integration navigation system is proposed, which exhibit strong robustness against the process uncertainty and non-Gaussian measurement noise induced by the vehicle’s severe maneuver and abnormal measurements of the velocity and position from GPS. In the proposed robust cubature Kalman filter, the advantage of residual orthogonal principle and higher order information of Gaussian correntropy are utilized to improve the robustness against the non-Gaussian measurement noise. Additionally, the multiple-channel adaptive strategy is employed to further enhance the filtering performance under the condition of process uncertainty. Moreover, some improvements are proposed to avoid the numerical problem and implement the proposed filter effectively. Simulations and car-mounted experiments indicate that the proposed new robust cubature Kalman filter can achieve high estimation accuracy and better robustness than the existing state-of-art methods against the process uncertainty and non-Gaussian measurement noise.

However, our proposed RCKF suffers from several weaknesses that need to be further investigated, though it presents significant advantages. First, our proposed RCKF is vulnerable to Gaussian kernel bandwidth. Thus, an adaptive strategy to choose proper Gaussian kernel bandwidth in each iteration step needs to be designed in future research. Second, the standard Gaussian kernel may not be the best for non-gaussian measurement noise in practical application. Therefore, we plan to develop the generalized Gaussian kernel to better deal with the above issue in future work. Note that when the solution domain of a given optimization problem is complex, finding global extremum point would be a challenge and there is a possibility that the proposed objective function gets trapped by the local optima. Hence, we will use the evolutionary optimization algorithms to tackle this problem in future work since they are derivative-free methods and also, mostly begin with random numbers as the initial population.

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TABLE 3. RMSEs of velocity and position from different algorithms.

| Filters  | RMSE (PosN, m) | RMSE (PosE, m) | RMSE (VelN, m/s) | RMSE (VelE, m/s) | RMSE (Pitch, deg) | RMSE (Roll, deg) |
|----------|----------------|----------------|------------------|------------------|-------------------|------------------|
| CKF      | 8.99           | 1.13           | 0.45             | 0.45             | 3.21              | 1.82             |
| MCSCKF   | 5.54           | 1.09           | 0.42             | 0.44             | 2.42              | 1.56             |
| H-infinity | 4.54         | 0.68           | 0.41             | 0.35             | 1.92              | 1.34             |
| Huber’s M | 4.88         | 0.51           | 0.35             | 0.33             | 1.81              | 1.27             |
| RSTNF    | 3.91           | 0.44           | 0.34             | 0.32             | 1.56              | 1.16             |
| IMM      | 1.36           | 0.36           | 0.33             | 0.30             | 1.07              | 0.95             |
| Proposed | 0.92           | 0.23           | 0.12             | 0.11             | 0.88              | 0.77             |

TABLE 4. Implementation times of different algorithms for a single step run.

| Filters  | Time (ms) |
|----------|-----------|
| CKF      | 0.410     |
| MCSCKF   | 0.432     |
| H-infinity | 0.413     |
| Huber’s M | 0.422     |
| RSTNF    | 0.461     |
| IMM      | 0.760     |
| Proposed | 0.416     |

the proposed filter and existing filters are almost consistent in the azimuth error, as exhibited in Figure 11(a). The reason is that the up-misalignment angle is unobservable in the loosely coupled SINS/GNSS integrated navigation, resulting in the divergence problem of azimuth error accumulating over time. Thus, the proposed filter is more robust against the process uncertainty and non-Gaussian measurement noise than the existing state-of-the-art filters.

To test the computational efficiency of the proposed RCKF, we coded it with MATLAB and run on a computer with Intel Core i5-3320 CPU at 2.60GHz. The implementation times of the proposed filter and existing filter are provided in Table 4, demonstrating that the IMM consumes much more time than the IMU sampling periods of 0.5ms in a single step run since there are several Kalman filters executing in parallel. Therefore, it is not suitable for real-time implementation. Moreover, the CKF has the fastest computing speed, followed by the H-infinity. As indicated in the results, our proposed RCKF have comparative computational efficiency with CKF and H-infinity filter, which are lower than the IMU sampling periods of 0.5ms. Regarding Huber’s M, MCSCKF, and RSTNF, although their execution time is large than that of RCKF, it is lower than the sample time. Hence, our proposed RCKF is computationally cost-effective and will be more celebrated for real-time applications.

Therefore, taking both accuracy and computational efficiency into account we can conclude that the proposed new robust cubature Kalman filter can simultaneously handle the process uncertainty and non-Gaussian measurement noise and achieve better performance than the existing robust filters with only a little additional computational time under the condition of vehicle’s severe maneuver and abnormal measurements in SINS/GPS integrated navigation system.
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