SOLVING THE DISCREPANCY BETWEEN THE SEISMIC AND PHOTOSPHERIC SOLAR RADIUS

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ABSTRACT

Two methods are used to observationally determine the solar radius: One is the observation of the intensity profile at the limb; the other one uses $f$-mode frequencies to derive a “seismic” solar radius which is then corrected to optical depth unity. The two methods are inconsistent and lead to a difference in the solar radius of $\sim$0.3 Mm. Because of the geometrical extension of the solar photosphere and the increased path lengths of tangential rays the Sun appears to be larger to an observer who measures the extent of the solar disk. Based on radiative transfer calculations we show that this discrepancy can be explained by the difference between the height at disk center where $\tau_{5000} = 1$ ($\tau_{2500} = 2/3$) and the inflection point of the intensity profile on the limb. We calculate the intensity profile of the limb for the MDI continuum and the continuum at 5000 Å for two atmosphere structures and compare the position of the inflection points with the radius at $\tau_{5000} = 1$ ($\tau_{2500} = 2/3$). The calculated difference between the seismic radius and the inflection point is $0.347 \pm 0.006$ Mm with respect to $\tau_{2500} = 1$, and $0.333 \pm 0.008$ Mm with respect to $\tau_{5000} = 1$. We conclude that the standard solar radius in evolutionary models has to be lowered by $0.333 \pm 0.008$ Mm and is 695.66 Mm. Furthermore, this correction reconciles inflection point measurements and the seismic radii within the uncertainties.

Subject headings: astrometry — radiative transfer — Sun: fundamental parameters — Sun: photosphere

1. INTRODUCTION

The solar radius is a key parameter for understanding the Sun’s evolution and many astrophysical applications. Being our closest star, the Sun can be considered as our laboratory, allowing us to determine its radius with high precision. Over the past 25 years various groups have been dedicated to determining the solar radius by applying basically two approaches. First, there is the direct measurement of the photospheric radius. In earlier times its visual determination was carried out by an observer. Later, employing CCD-imaging devices, the visual radius determination was replaced by deriving the inflection point to the intensity profile of the limb. The error of both methods has been investigated in detail by Laclare et al. (1996), who derive a solar radius of 959.60°, which agrees well with 959.63° ± 0.10° or 695.99 ± 0.07 Mm (Allen 1976), commonly used for calibrating solar evolutionary models (Brown & Christensen-Dalsgaard 1998; Turk-Chièze & Murdin 2000). Second, Schou et al. (1997) and Antia (1998) determine the helioseismic radius by means of $f$-mode frequencies obtained from the Michelson Doppler Imager (MDI; Scherrer et al. 1995) on board the ESA/NASA spacecraft SOHO. Due to the high quality of the MDI observations the helioseismic determination is very precise and leads to a radius that is approximately 0.3 Mm smaller than the standard value by Allen (1976). Schou et al. (1997) conclude that the standard solar radius has to be decreased by this value. Finally, Brown & Christensen-Dalsgaard (1998) determine the radius from solar transit observations obtained with the High Altitude Observatory’s Solar Diameter Monitor (HAO SDM; Brown et al. 1982), combined with a detailed model of the data reduction procedure and limb intensity. The authors point out that there are significant uncertainties associated with the currently used radius value which are related to the definition of the solar limb adopted in the radius determinations and the reduction of the measured value to the photosphere. The authors indicate a height of about 0.3 Mm which “perhaps” explains the radius correction inferred from the $f$-mode frequencies. In the present Letter we determine the difference between the height at $\tau_{5000} = 1$ and $\tau_{2500} = 2/3$ at disk center and the distance of the inflection point from radiative transfer calculations for two wavelengths and atmosphere structures.

2. CALCULATIONS

With the radiative transfer code COSI we calculate the solar limb function for the MDI continuum intensity and the continuum at 5000 Å. COSI is a combination of a model atmosphere code in spherical symmetry, developed by Schmutz et al. (1989), and the spectrum synthesis program SYNSPEC, going back to Hubeny (1981) and further developed by Hubeny & Lanz (1992a, 1992b). The adaption to the solar atmosphere has been published in Haberreiter & Schmutz (2003). The model atmosphere code calculates the NLTE population numbers for a set of specified atomic levels by solving the radiative transport equations simultaneously with the equations for statistical equilibrium. The radiative transfer is solved along rays parallel to the central ray incident on a spherical distribution of the physical atmosphere structure (Mihalas et al. 1975). A spherical geometry allows one to calculate geometrically correctly the emerging intensity at the limb and line of sights beyond the solar limb.

We base the model calculations on two model atmosphere structures for the quiet Sun, the LTE structure by Kurucz (1991) and the NLTE structure “model C” by Fontenla et al. (2007). The radius at disk center where $\tau_{5000} = 1$ is set to the solar radius derived from seismic observations, $R_{\odot} = 695.68$ Mm, as given by Schou et al. (1997). The high-resolution Ni line profile has been convolved with the five MDI filter functions F0 to F4 used to determine the actual continuum intensity observed by MDI.
3. RESULTS

Figure 1a shows the mean intensity variation for the MDI continuum at the solar limb applying all five MDI filter functions, and the continuum at 5000 Å calculated with the LTE and NLTE model atmosphere structures. Distance zero refers to the “seismic” solar radius $R_\odot = 695.68$ Mm, i.e., $\tau_{5000} = 1$ at disk center.

For the exact determination of the inflection point we calculate the derivative $dI/dx$ ($x$ being the outward coordinate) of all four limb profiles, shown in Figure 1b. The two atmosphere structures agree well in the photosphere and lead to very similar intensity profiles. The derivative of the intensity is always negative and shows a local minimum at the inflection point. The distances of the local minima for all four limb profiles are given in Table 1. The mean value of the distance for the two atmosphere structures for $\lambda = 5000$ Å is $0.347 \pm 0.006$ Mm, where we used the difference between the two atmosphere structures as a criterion to derive the error introduced by the atmosphere structures. This result is close to the value of $0.370$ Mm estimated by Wittmann (1974) based on the atmosphere structure by Holweger (1967).

In evolutionary models the radius is typically defined as the point where the temperature equals the effective temperature, i.e., where $\tau_{\text{Ross}} = 2/3$. From our calculations we find that the height where $\tau_{\text{Ross}} = 2/3$ is in the mean $0.014$ Mm farther out than where $\tau_{5000} = 1$. Thus the correction of an inflection point measurement to the seismic radius is $0.333$ Mm. This correction explains the discrepancy between the modeled and observed $f$-mode frequencies. We conclude that the standard radius used in the evolutionary models has to be corrected by this value, leading to $695.66$ Mm.

4. DISCUSSIONS

For consistency we investigate whether recent inflection point measurements are compatible with the mean of seismic radii relating to $\tau_{\text{Ross}} = 2/3$, i.e., where the temperature equals the effective temperature, by applying this correction term. Table 2 gives the solar radii based on (1) IP measurements and (2) seismic radii. Note that radius by Brown & Christensen-Dalsgaard (1998) is determined from solar transit observations and has been corrected to $\tau_{\text{Ross}} = 2/3$. Therefore, we group it also as seismic radius. Furthermore, the authors applied a detailed limb analysis to the observations, which is in some ways similar to the work presented here, but specifically tailored to their observation. The mean values of the IP measurements and seismic radii deviate by $\sim 0.24$ Mm. In Table 3 we correct the mean radius values of the inflection point observation by $0.333$ Mm and compare the result with the mean seismic radius, leading to a difference of $0.09 \pm 0.17$ Mm. We conclude that the inflection point measurements and the seismic radius can be reconciled by applying the correction of $0.333$ Mm.

Our calculations further indicate that the position of the MDI continuum inflection point is $0.02$ Mm further out than for $5000$ Å. The radius value determined by Kuhn et al. (2004) (see Table 2) refers to the inflection point of the intensity profile observed by MDI. The difference of $0.02$ Mm cannot explain the somewhat lower radius value by Kuhn et al. (2004).

We emphasize that we do not consider any variation of the solar diameter over the solar cycle (Emilio et al. 2007; Thuillier et al. 2005; Kuhn et al. 2004; Toulmonde 1997; Parkinson et al. 1980), which however could explain some of the deviation of the radius values. Furthermore, our considerations are with respect to the quiet Sun.

The potential effect of granulation on the position of the inflection point is also worth mentioning. From observations and 3D simulations we know that the quiet Sun shows horizontal temperature inhomogeneities (Uitenbroek et al. 2007; Cheung et al. 2007; Stein & Nordlund 2006; Wedemeyer et al. 2004). As tangential rays on the limb pass through all the different components of the quiet-Sun atmosphere it is necessary to apply a mean structure of the quiet Sun for the cal-

![Graph](image-url)
calculation of the intensity profile, even more so as the lifetime of a granule is about 6 minutes (Hirzberger et al. 1999). The atmosphere structures employed in the present Letter are a representation of the averaged quiet Sun, because these structures are derived by reproducing a solar spectrum of a spatial and temporal mean of the quiet Sun. Thus, we implicitly account for the mean effect of granulation.

A further issue is to what extent detailed 3D simulations could lead to different results than semiempirical 1D atmosphere structures. Calculations by Allende Prieto et al. (2001) and Asplund et al. (2004) show that temporally and spatially averaged temperature structures derived from 3D convection simulations are not substantially different from 1D semiempirical models of the quiet Sun around optical depth unity. Moreover, atmosphere structures derived from the inversion of line spectra (Koza et al. 2006) are very close to both atmosphere structures derived semiempirically and those from 3D MHD simulations. Again, the region under consideration here is the height range $0.1 < \tau_{5000} < 5$. From this we conclude that for the case of 3D simulations we do not expect considerable differences from the result presented here.

5. CONCLUSIONS

From radiative transfer calculations with the spherical code COSI we determine a distance of $0.333 \pm 0.008$ Mm (0.347 $\pm$ 0.006 Mm) between the height where $\tau_{5000} = 2/3$ ($\tau_{5000} = 1$) at disk center and the position of the inflection point of the 5000 Å intensity profile at the limb. This correction explains the differences between the $f$-mode frequencies derived from model calculations and observations. Therefore, we conclude that the standard solar radius currently used in evolutionary models has to be corrected by 0.333 Mm and is 695.66 Mm.

The correction of 0.333 Mm reconciles the radius values derived from inflection point measurements and the “seismic” radius by 0.09 $\pm$ 0.17 Mm. Due to the increasing precision of the instruments it has become necessary that the community agree on a generally binding definition of the solar radius and that values derived via different techniques be corrected accordingly. We propose that the solar radius refers to $R_{\odot} = R(2/3)_{\text{eff}}$. Thus, we further suggest that the value of solar radius determined from inflection point measurements in the visible, i.e., at $\lambda = 5000$ Å, have to be generally lowered by 0.333 Mm to be comparable with the seismic radius. A detailed study of the position of the wavelength- and solar-cycle-dependent inflection point will be crucial for upcoming observations with the SODISM instrument on board Picard (Thuillier et al. 2006).

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TABLE 2
SOLAR RADII AND THEIR MEAN VALUES

| Authors | $R_\odot$ (Mm) | $\bar{R}_\odot$ (arcsec) | Description | $\bar{R}_\odot$ (Mm) |
|---------|--------------|--------------------------|-------------|------------------|
| Brown, T. M., & Christensen-Dalsgaard 1998 | 695.680 ± 0.030 | 959.20 ± 0.01 | MDI f-modes | 695.901 ± 0.098 |
| Brown et al. 2006 | 695.787 | 959.35 | GONG f-modes | 695.658 ± 0.140 |
| Wittmann 1997 | 696.715 ± 0.029 | 960.63 ± 0.04 | Obs. del Teide | |
| Emilio & Leister 2005 | 695.946 ± 0.029 | 959.57 ± 0.04 | SP astrobol | |
| Koza et al. 2006 | 695.740 ± 0.110 | 959.29 ± 0.15 | MDI | |
| Laclare et al. 1996 | 695.927 ± 0.036 | 959.62 ± 0.03 | McMath ST | |
| Neckel 1995 | 695.917 ± 0.043 | 959.53 ± 0.06 | SDS | |
| Sofia et al. 1994 | 695.917 ± 0.043 | 959.53 ± 0.06 | SDS | |
| Schou et al. 1997 | 695.658 | 0.171 | Standard value | 695.901 ± 0.098 |
| Allende Prieto et al. 2001 | 695.660 ± 0.030 | 959.20 ± 0.01 | MDI f-modes | 695.901 ± 0.098 |
| Antia 1998 | 695.508 ± 0.026 | 958.97 ± 0.04 | HAO SDM | |

Notes.—Solar radii and their mean values $\bar{R}_\odot$ derived by means of (1) the standard solar radius and recent inflection point measurements, and (2) values referring to the seismic radius. As the result given by Wittmann (1997) is more than 8 σ different from the mean, we excluded this value from our considerations.

TABLE 3
CORRECTIONS

| Quantity | Radius (Mm) |
|----------|-------------|
| $R_{\text{ip}}$ | 695.901 ± 0.098 |
| Correction | 0.333 ± 0.008 |
| Corrected $R_{\text{ip}}$ | 695.568 ± 0.098 |
| $R_{\text{ip}}$ | 695.658 ± 0.140 |
| $\Delta R$ | 0.090 ± 0.171 |

Notes.—Correction of 0.333 Mm applied to the mean of the inflection point measurements. The corrected radius value is compared with the mean of the seismic radii given in Table 2, referring to $\tau_{5000} = 2/3$. The result is consistent within the uncertainties.
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