Electron conduction within Landau level tails of medium-mobility GaAs / AlGaAs heterostructures

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Abstract

The temperature dependence of both components of the resistivity tensor $\rho_{xx}(T)$ and $\rho_{xy}(T)$ has been studied at $T \geq 4.2$ K within IQHE plateaux around filling factors $\nu=2$ and $\nu=4$ of medium-mobility GaAs/AlGaAs heterostructures. In the middle of the mobility gap standard activated conductivity has been found with activation energies $\Delta$ scaling well with $\hbar \omega_c/2$. At filling factors slightly below $\nu=2$ another contribution adds to the activated conductivity at $T \leq 12$ K. This additional contribution can be further enhanced at higher measuring d.c. currents. We suggest, that it arises due to enhanced electric field assisted tunneling across potential barriers separating localized states within the bulk of the sample. This effect contributes to the backscattering across the sample leading to an enhanced longitudinal conductivity. The additional contribution to $\sigma_{xx}(T)$ can be reasonably well fitted to the formula for the variable range hopping in strong magnetic fields indicating that the hopping can persist even at temperatures well above 4.2K.
1 Introduction

It is generally accepted that the integer quantum Hall effect (IQHE) can be understood in terms of localized and extended single electron states. While the latter form very narrow bands of a width $\Gamma$ centered at Landau level energies $E_N$, the former fill the mobility gap of a width $(\hbar\omega_c - \Gamma)$ and do not contribute to the longitudinal conductivity $\sigma_{xx}$ at least in the limit $T \to 0$.

Such a picture has been supported by the measurement of thermally activated electron transport in 2DEG structures in strong magnetic fields corresponding to an integer filling factor $\nu$, i.e. to the center of a mobility gap. It has been found that there is a finite temperature interval $\Delta T$, where the longitudinal conductivity $\sigma_{xx}(T)$ can be described by the Arrhenius law

$$\sigma_{xx}(T) = \sigma^o_{xx} e^{-\Delta/k_B T}$$

with the activation energy $\Delta \approx \hbar\omega_c/2$ indicating that the mobility edge $E_m$ virtually coincides with $E_N$ ($\Gamma \to 0$).

The expression (1) appears as a special form of the general expression for the temperature dependent conductivity $\sigma_{xx}(T)$ that reads

$$\sigma_{xx}(T) = \int \sigma_o(E) \frac{\partial f}{\partial E} dE$$

provided that the Fermi - Dirac distribution $f(E)$ can be approximated by the Boltzmann statistics. Since only the extended states at $E \approx E_N$ contribute to $\sigma_o(E)$, the approximation (1) should hold for $|E_F - E_N| >> k_B T$. If the Fermi energy $E_F$ lies in the middle of a sufficiently wide mobility gap (i.e. at $\nu=N$ with $N$ being a small even integer) this condition can hold even at $T \geq 10K$.

It has been found from the investigation of high-mobility heterostructures GaAs/AlGaAs [1], that the the prefactor in (1) reaches a universal value $\sigma^o_{xx} \approx e^2/h$ independent on the sample and the filling factor. Although good fits to (1) in a finite temperature interval have been reported for samples of widely different parameters [1, 2, 3, 4], the universality of the prefactor $\sigma^o_{xx}$ has been disputed both experimentally [1] and theoretically [4]. Recently, Polyakov and Shklovskii [6] have shown, that a universal temperature independent prefactor in (1) can be derived from the percolation theory in the limit of a long range impurity potential, which is typical for high-mobility modulation doped GaAs/AlGaAs heterostructures. They predict, that just in the center of a mobility gap this prefactor equals to $2e^2/h$ i.e. it is twice as large as that found experimentally by Clark [1].

Deviations from a linear Arrhenius graph (1) occur both at high and low temperatures. The high temperature downward curvature has been attributed either to a violation of the Boltzmann statistics or to the situation, where the electron mean free path becomes larger than the perimeter $p_T$ of the percolation loops [6]. At sufficiently low temperatures an upward curvature is usually observed.
and three basic explanations have been suggested. First, variable range hopping (VRH) among localized states at $E_F$ is expected to compete with the activated conduction at lowest temperatures [4, 7]. Non-ideal potential contacts [8] and a finite width $W$ of broadened Landau levels [2] can both cause the observed low temperature deviations from the simple activated conductivity described by (1).

In this paper we investigate the temperature dependent conductivity $\sigma_{xx}(T)$ beyond the breakdown of the IQHE in the vicinity of the lowest even filling factors as a function of the position within the mobility gap and of the measuring d.c. current $I$. The samples studied were two modulation doped GaAs/AlGaAs heterostructures of medium mobilities and rather high carrier concentrations, i.e. with parameters typical for the samples recommended for metrological applications of the IQHE. The current dependence of their longitudinal resistivity at half-integer filling factors, i.e. within the extended states in the middle of Shubnikov - de Haas peaks in $\varrho_{xx}(B)$, has been described elsewhere [9].

2 Experiments

Two different samples have been employed in this study. Both were made from wafers grown by MBE and patterned into Hall bar geometry with 3 equidistant pairs of potential contacts separated by $L = 550 \mu m$ along the channel of the width $w$. Sample A (referred to as CS60-9 in ref. [9]) had following basic parameters: $n_s(4.2K) = 5.4 \times 10^{15} m^{-2}$; $\mu(4.2K) = 39 T^{-1}$; $w = 400 \mu m$. The other sample denoted here as B (CS50-6 in ref. [9]) has been characterized by the values: $n_s(4.2K) = 3.8 \times 10^{15} m^{-2}$; $\mu(4.2K) = 24 T^{-1}$; $w = 100 \mu m$.

Both components of the resistivity tensor $\varrho_{xx}(T)$ and $\varrho_{xy}(T)$ have been simultaneously measured at temperatures between 4.2K and 85K using a computer controlled data acquisition system with a voltage resolution better than 100 $nV$. The measuring d.c. current $I$ has been varied within the interval $I = 1 - 100 \mu A$ and $I = 0.2 - 10 \mu A$ for the sample A and B, respectively. Different currents used for samples A and B correspond to their different widths $w$. Measurement was performed in a dynamical regime with the temperature continually changing in both directions. The rate $dT/dt$ has been so slow, that no hysteresis could by observed on the $\varrho_{xi}(T)$ ($i=x,y$) curves throughout the whole temperature range.

The conductivity $\sigma_{xx}(T)$ was then calculated from the expression

$$\sigma_{xx}(T) = \frac{\varrho_{xx}(T)}{\varrho_{xx}^2(T) + \varrho_{xy}^2(T)}$$

3 Results and discussion

One of the remarkable features seen in both samples studied is a pronounced asymmetry of the minimum in $\varrho_{xx}(B)$ around $\nu = 2$. This asymmetry is moreover
strongly dependent on current $I$, which is illustrated in Fig.1a and Fig.1b for the samples A and B, respectively. The resistivity minimum around $\nu=2$ is shown here in the interval of currents covering the transition from non-local to local resistivity discussed extensively elsewhere [3, 4]. While the upper (low-energy) edge of the Shubnikov - deHaas (SdH) peak corresponding to the spin-resolved Landau level $1\uparrow$ are shown to depend strongly on the current $I$, this is not the case for the high-energy edge of the peak $0\downarrow$ on the high magnetic field side of the minimum. The highest curves correspond to saturation currents where there is a perfect coupling between edge and bulk channels and further increase of $I$ leads to a suppression of the heights of both SdH peaks $1\uparrow$ (N=3) and $0\downarrow$ (N=2) due to the overheating of the electron gas [10].

An alternative explanation of this asymmetry of the line shapes of the individual spin-resolved subpeaks in $\rho_{xx}(B)$ for low-indexed Landau levels has been suggested by Haug et al. [11]. They have shown, that it can reflect an asymmetry in the density of states induced by a particular distribution of the attractive (ionized Si donors in AlGaAs source layer) and repulsive (residual acceptors in GaAs buffer layer) scatterers in samples. The situation observed here, where the resistivity in the high-field side of the $1\uparrow$ peak is much more sensitive to both the current and the temperature than the low-field side of the adjacent $0\downarrow$ peak is then a result of dominant contribution of remote attractive scatterers to the conduction, which is typical for all heterostructures with electron mobilities $\mu \geq 15T^{-1}$.

We indicate in Fig.1 the range of magnetic fields $B_k$ in the vicinity of the filling factor $\nu = 2$, where the temperature dependence of $\sigma_{xx}$ has been studied. All the fields $B_k$ lie in the range where, within the resolution of our d.c. method, $\rho_{xx}(4.2K) = 0$ and $\rho_{xy}(4.2K) = \hbar/2e^2$. This resolution is limited mainly by the voltage noise and by the stability of the current supply for the longitudinal and the Hall resistivity, respectively, and it reaches about 0.1 mΩ at highest currents used.

Due to the asymmetry mentioned above, it is not straightforward to find experimentally an exact center of the IQHE plateaux, because it does not necessarily coincide with the geometric center of the minimum in Fig.1, which is moreover slightly current-dependent. As a criterion, we adopted the temperature dependence of the Hall resistivity $\rho_{xy}(B_k, T)$, which is expected to be independent of $T$ at $\nu = 2$. These dependences have been drawn in Fig.2 for two different currents. It can be seen from Fig.2, that some degree of quantization persists in the sample up to temperatures about 40K.

The temperature dependence of the conductivity $\sigma_{xx}$ in the middle of the mobility gap is presented in the Arrhenius graph on Fig.3 for $\nu \approx 2$ and $\nu \approx 4$, the only minima where at $T=4.2K$ the IQHE was still complete. Activated conductivity can be detected below $T \approx 20K$. At $\nu = 2.01$, the fitted activation energy $\Delta = 9.22meV$ ($\Delta/k_B = 107K$) agrees well with the value of $\hbar\omega_c/2 = 9.18meV$ (106 K in temperature units), which confirms a purely activated conduction at
low enough current $I$. The agreement is not so good for $\nu = 4.08 (\Delta/k_B = 44.7K$ \text{vers.} $\hbar\omega_c/2k_B = 52.5K$), but it can be attributed to the fact, that we are not just in the middle of the mobility gap.

Weiss et al. \cite{3} suggested that a measurement of the activated conductivity at various fixed magnetic fields within the mobility gap can serve for a determination of the density of (localized) states on the Fermi energy $D(E_F)$ provided that it does not depend too strongly on the energy. This method requires a measurement of $\sigma_{xx}(T, B = B_k)$ in a rather dense set of precisely known fields $B_k$ around that corresponding to an integer filling factor. Furthermore, it works only within Landau level tails far from energies $E_N$. Within the model of ref. \cite{3}, the Fermi energy shifts due to a change of the field from $B_1$ to $B_2$ ($B_2 > B_1$) by

$$\delta E = \Delta(B_1) - \Delta(B_2) - \frac{\hbar}{2}(\omega_c,2 - \omega_c,1).$$ \hspace{1cm} (4)

But the variation of the Fermi energy corresponds approximately to a change of the carrier density

$$\delta n \approx \frac{\nu e}{2\pi\hbar}(B_2 - B_1).$$ \hspace{1cm} (5)

For $\nu = 2$, the density of states $D(E) = \delta n/\delta E$ can then be estimated from

$$D(E) \approx \frac{e}{\pi\hbar} \frac{(B_2 - B_1)}{[\Delta(B_1) - \Delta(B_2)] - \frac{e\hbar}{2m^*}(B_2 - B_1)}.$$ \hspace{1cm} (6)

This approach assumes, that the mobility edge coincides with the center of the Landau level $E_N$ and that it does not depend both on the temperature and the carrier concentration. Although this is probably oversimplified, we can at least roughly estimate from the activation energies $\Delta(B_k)$ measured at low currents near the middle of the mobility gap a mean density of states. We have found that $D(E_F) \leq 2 \times 10^{10}$ $meV^{-1}cm^{-2}$ within the interval of the halfwidth $\delta E \approx 2meV$ around the center of the mobility gap. It should be compared with the zero-field value of $D_o(E) = m^*/\pi\hbar^2 = 2.8 \times 10^{10}meV^{-1}cm^{-2}$. Outside the above mentioned energy interval, the value of $D(E_F)$ increases and the increase is markedly steeper on the low-energy side, i.e. at $\nu < 2$. Our samples thus apparently differ from the higher mobility sample studied by Weiss et al. \cite{3}, where a much lower and nearly constant $D(E_F)$ was reported over a half of the mobility gap.

Fitting to the law (1) provides not only the energy $\Delta$, but also the value of the pre-exponential factor $\sigma_{xx}^o$. For the data presented in Fig.3 we get $\sigma_{xx}^o = 1.84 \times e^2/h$ and $\sigma_{xx}^o = 1.17 \times e^2/h$ for $\nu \approx 2$ and $\nu \approx 4$, respectively. The values of the prefactor are however very sensitive to the exact position within the gap falling steeply down with increasing $|\delta \nu|$. This can be seen in Fig.4. Taking the field $B_2=10.68T$ as that corresponding to $\nu = 2.00$ (see Fig.2), we find that both curves in Fig.3 correspond to energies slightly above the middle of the gap ($\nu = 2.01$ and $\nu = 4.08$, respectively). Especially in the latter case it implies,
that the value of $\sigma_{xx}^0$ at $\nu = 4$ has to be markedly higher than $e^2/h$. Our data seems thus to be better explained by the theory of Polyakov and Shklovskii [6] for a long-range scattering, predicting that $\sigma_{xx}^0 \approx 2e^2/h$. It is systematically higher both than the experimental result of Clark [1] $\sigma_{xx}^0 \approx e^2/h$ and than the theoretical prediction $\sigma_{xx}^0 \leq e^2/h$ of Polyakov and Shklovskii [5] that should hold for samples with pure short-range scattering.

There is a clear difference among the curves presented in Fig.4. While on the high-energy (low-field) side the activated conductivity seems to persist even slightly outside the center, another contribution to the conductivity appears on the low-energy side, where the Fermi energy starts to move to the next fully occupied Landau level $0\downarrow$. Only at $T > 10K$ we come back to the common activated conductivity that persists then up to temperatures above 20 K. As it is shown in Fig.5, qualitatively the same contribution can be induced even in the very center of the gap by increasing substantially the measuring current $I$.

Moreover, we demonstrate in Fig.6, that if this extra contribution to $\sigma_{xx}(T)$ is already present at low currents and $\nu < 2$ it can be further enhanced by increasing the current $I$.

The most obvious explanation of such current-induced deviation from the Arrhenius graph at lower temperatures could be given in terms of an enhancement of the electron temperature above that of the lattice, which we actually measure. To estimate this contribution, we show in Fig.7 a part of the curves $\rho_{xx}(B)$ measured on sample A at several temperatures between $T = 1.2K$ and $T = 4.2K$ with $I = 1\mu A$ and compare them with the data taken at $T = 1.2K$ using measuring currents $I = 10\mu A$ and $I = 50\mu A$. We show here the minimum centered at $\nu = 3$, where the Shubnikov- de Haas curves are most sensitive to the temperature (there has been no measurable temperature dependence of $\rho_{xx}(B)$ at $\nu \approx 4$). It can be seen from the graph, that there is a measurable electron overheating at the current $I = 50\mu A$, but that it does not exceed (at least near the center of the gap at $\nu \approx 3$) about 0.3K at $T \approx 1.2K$. To explain the deviations from [1] observed in Fig.5 and 6. in terms of electron heating effects only, one would need an enhancement of $T$ by an order of magnitude higher. Moreover, it should occur at $T > 4.2K$, where the transfer of thermal energy from electrons to the lattice should be easier than at $T \approx 1.2$. We can thus conclude, that electron heating effects do not dominate in the data taken with currents up to 50 $\mu A$.

An upward curvature from a linear Arrhenius graph at lower temperatures has been observed in most studies of the activated conduction in the IQHE regime. The onset temperature for this deviation is apparently sample dependent. From the two commonly used explanations, we can probably exclude a spurious influence of non-ideal contacts [8]. First, this kind of deviation should lead to a saturation of $\sigma_{xx}(T)$ at a finite value of the conductivity at lowest temperatures, which is not seen in our samples at $T = 4.2K$. Second, a measure of the non-ideality of the contact is their resistance that has been checked in our samples to be of the order of 10 $\Omega$ at most, two orders of magnitude below those
reported by Komiyama et al. The third argument stems from the micro-
scopic description of non-ideal contacts introduced in ref. Such contacts
induce a non-equilibrium population in the outgoing edge channels, that persists
over macroscopic distances along the sample. This non-equilibrium between up-
permost edge channels plays a crucial role in our model explaining the current
dependence of the amplitude of Shubnikov - de Haas peaks in strong magnetic
fields. Sufficiently high currents remove such a non-equilibrium and possible
influence of the contact should thus be restricted to its immediate vicinity only.
A contribution due to contacts should thus be suppressed by a high current and
just the opposite can be seen in Fig. 6.

It is widely accepted, that at low enough temperatures variable range hopping
(VRH), i.e. tunneling of the electrons from the interval of the order of $k_B T$ among
the localized states at $E_F$, prevails over the activated conduction. The temper-
atures, where VRH in high mobility GaAs/AlGaAs samples becomes dominant
is expected to lie at $T \leq 1K$. It has been stressed by Polyakov and Shklovskii
that the actual form of the $\sigma_{xx}(T)$ is governed by the overall character of the
disorder present in a particular sample. In the case of a purely short-range scat-
tering no dependence of the type (1) should be observed in any finite temperature
interval. Only an inflection point at $T = T_1$ is expected to separate the VRH
contribution at $T < T_1$ from the conduction due to the broadened Fermi-Dirac
distribution at $T > T_1$. It can thus be expected, that the enhanced disorder in
samples of a lower quality would shift the interval of VRH conduction to higher
temperatures.

The contribution of the VRH in Landau level tails has been calculated by Ono
under the assumption that the magnetic field causes a Gaussian localization of
the electron wave function on the scale given by the magnetic length $\ell_c = \sqrt{\hbar/eB}$. His result reads

$$\sigma_{xx}^{\text{VRH}}(T) = \frac{e^2}{k_B T} \gamma_o e^{-\left(\frac{T_o}{T}\right)^{1/2}}$$

where $\gamma_o$ is a material parameter depending on electron-phonon coupling and $T_o$
is a critical temperature given by

$$T_o = \frac{C}{k_B D(E_F)\ell_c^2} ; \quad C \approx 1$$

It has been found that although the expression (7) fits the data at lowest
temperatures well, the density of states calculated from the fitted critical tem-
peratures $T_o$ on the basis of (8) becomes unrealistically high. Ebert et al.
reported values of $D(E_F)$ by a factor 36 higher than the zero field density
$D_o(E) = m^*/\pi\hbar^2$.

The theory by Ono leading to eq. (7) assumes a finite density of states at
$E_F$ and unperturbed wave functions of isolated impurities in the form $\psi(r) \sim$
exp[-r^2/2\ell_c^2]. It has been criticised by Polyakov and Shklovskii \[13\], who provide another expression for VRH conductivity in the form
\[
\sigma_{xx}^{\text{VRH}}(T) = \sigma_o e^{-(T_1/T)^{1/2}},
\]
which relies on the existence of a Coulomb gap at \(E_F\) and assumes, that tails of the wave function have a simple exponential form \(\psi(r) \sim \exp[-r/\xi]\) (\(\xi\) being the localization length) due to multiple scattering of a tunneling electron \[13\]. The basic difference between (7) and (9) is in the expression for the critical temperature; instead of (8) they have got for \(T_1\) the formula \[13\]
\[
T_1(\nu) = C_1 \frac{e^2}{k_B \epsilon(\nu)}
\]
with \(\xi(\nu)\) denoting the localization radius of the states on the Fermi energy for a given \(\nu\), \(\epsilon\) the dielectric constant and with \(C_1 \approx 6\) in two dimensions.

Our data is not sufficient to distinguish between (7) and (9). Both expressions are formally the same, because the fitting of experimental data to (9) by Koch et al. \[14\] gives \(\sigma_o \sim 1/T\). But while the fitting of our data to (7) provides us with \(T_o\) leading to reasonable values of \(D(E_F)\) (see below), critical temperatures \(T_1\) obtained from fitting the same data to (9) are by at least one order of magnitude higher than those discussed by Polyakov and Shklovskii \[13\]. We suggest, that the formula (7) is therefore more relevant to our samples.

We present in Fig.8 the data measured on the sample A with the saturation current \(I = 50\mu A\) \[9\] at a few magnetic fields \(B_k\) around \(\nu = 2\). At \(T \leq 12K\) the data can be fitted to the formula (7). This has not to be a convincing argument (we have shown in Fig.5a, that the same data can be nearly equally well fitted to the Arrhenius law \[11\]), let us mention, however, that the temperatures \(T_o\) obtained give very reasonable values of the density \(D(E_F)\) in our case. From the curves for \(B_k=10.63T\) and \(B_k=10.73T\), that correspond to an immediate vicinity of the center \(\nu = 2\) we obtain \(D(E_F) = 1.0 \times 10^{10}meV^{-1}cm^{-2}\) and \(D(E_F) = 1.4 \times 10^{10}meV^{-1}cm^{-2}\), respectively. These values agree with those determined from the activated conductivity by the method of Weiss et al. \[3\] and also a steep increase of \(D(E_F)\) further from the center is similar here. The values of the pre-exponential factor \(\sigma^*_xx = e^2\gamma_0/k_B\) strongly decrease if the field \(B_k\) is shifted with respect to this corresponding to \(\nu = 2\). It is hard to understand why the quantity \(\gamma_0\) being a material parameter which should be a function of the electron-phonon coupling strength only \[12\] would depend so strongly on the field \(B_k\) and/or on the density of states \(D(E_F)\). The corresponding changes of the filling factor are so small, that any pronounced changes of the density of states at \(E_F\) can hardly be expected.

Even the conductivity measured by a low current \(I\) at \(\nu < 2\) can be at \(T \leq 12K\) reasonably well approximated by formula (7), as it can be seen from Fig.9. An enhancement of the current \(I\) without changing the filling factor results
there in an apparent suppression of the critical temperature $T_0$ which means in terms of (3) an enhancement of the density $D(E_F)$. This can be understood since high measuring currents lead to a strong electric (Hall) field across the sample which may result in an additional broadening of the Landau levels.

As we have mentioned already, the extra contribution to the activated conductivity (but not the activated conductivity itself) can be fitted both to (4) (see Fig.5) and (with nearly the same accuracy) to the expression (7). It is however hard to interpret the activation energy $\Delta^* \approx \Delta/2$ obtained in the latter case. Such a fit gives also very low values of the pre-exponential factor $\sigma_{xx}^0 \approx 0.01 \times e^2/h$, which are not compatible with any of the existing theoretical descriptions of the activated conduction. The fact, that our data can be fitted to an expression of the type (7) therefore indicates, that a contribution of hopping among the localized states around the middle of the mobility gap persists to much higher temperatures than those reported before [2, 7].

In the Büttiker - Landauer picture of the IQHE, a finite longitudinal conductivity $\sigma_{xx}$ appears once a backscattering between the two edges of the sample occurs. This happens if there is a mechanism capable to transfer an electron injected by one current contact into the edge channels on one side of the sample across the bulk composed of localized states only into the edge channels on the opposite side before it can arrive into the other current contact. At low measuring currents a gradient of electrochemical potential starts to develop near the edges without influencing the potential far from the edge [13]. If, however, the current exceeds some (sample dependent) critical value, a finite gradient of the electrochemical potential develops throughout the sample. The potential distribution across an IQHE sample at low and high sample currents has been recently calculated by Cage and Lavine [16]. They found that at high currents $I$ a so-called charge-redistribution potential $V_r(y)$ adds to common confining potential $V_c(y)$ that gives rise to the edge channels. Far from the edges, $V_r(y)$ changes linearly with the coordinate $y$ and its slope is proportional to $I$. The charge-redistribution potential $V_r(y)$ should provide a "driving force" for the transfer of the charge across the bulk of the sample.

We suggest that the enhanced contribution to the conductivity beyond the temperature induced breakdown of the IQHE observed near the middle of the mobility gap around $\nu = 2$ can be due to enhanced tunneling through the potential barriers separating localized states at $E_F$. It can be connected both with the thermally assisted tunneling if the density of localized states is sufficiently high and with the transversal electric (Hall) voltage $U_{H}^T \sim I$ developing within the bulk of the sample with a finite conductivity $\sigma_{xx}(T)$. The latter gives rise to an electric field assisted tunneling, which further promotes backscattering at higher measuring currents $I$. Due to this enhanced tunneling, the temperature dependence of the conductivity contains a contribution of the hopping conductivity up to temperatures $T \approx 12K$. This temperature corresponds to a thermal energy of 1 meV, which is about 10% of the halfwidth of the mobility gap in
our experimental condition. At still higher temperatures, activated conductivity
starts to prevail again and it could be observed up to \( T \approx 25K \) where the Boltz-
mann statistics can not be used any more and a downward deviation from the
Arrhenius graph starts to develop.

4 Conclusions

The temperature dependence of the longitudinal conductivity \( \sigma_{xx} \) has been stud-
ied on samples with well developed IQHE plateaux in magnetic fields around the
centers of highest mobility gaps corresponding to filling factors \( \nu = 2 \) and \( \nu = 4 \).
At low measuring currents \( I \) pure activated conductivity described by the ex-
pression (1) has been observed in the center of the mobility gaps. The activation
energies \( \Delta \) obtained by fitting the data to the Arrhenius law (1) scale well with
the half-width \( \hbar \omega_c/2 \) of the interval between adjacent Landau levels. The prefac-
tor \( \sigma^0_{xx} \) in (1) depends markedly on the exact position of the Fermi level
\( E_F \) with respect to the center of the mobility gap. At integer filling factors it approaches
the value of about \( 2e^2/h \). This value is consistent with the theoretical predictions
of Polyakov and Shklovskii [6] for samples with dominating long-range scatter-
ing, but it does neither support the experimental findings of Clark [1] nor the
prediction \( \sigma^0_{xx} \leq e^2/h \) calculated for a pure short-range scattering [5].

Further from the center of the mobility gap, another contribution to \( \sigma_{xx}(T) \)
develops at \( T \leq 12K \). At low enough currents, this contribution can be seen on
the high-field (low-energy) side of the center only, where the density of localized
states increases more steeply than on the low-field side. This asymmetry of \( D(E) \)
can be of the same origin as that reported by Haug et al. [11].

This extra contribution can be induced even in the middle of the mobility
gap by increasing the current \( I \) up to the values where the non-local conduction
[3, 10] is suppressed and decoupling between the edge and bulk channels has
been removed. From the temperature dependence of \( \varrho_{xx}(B) \) measured at \( T \leq
4.2K \) with various currents we can conclude, that this is not a simple electron
overheating effect.

The additional contribution to \( \sigma_{xx}(T) \) can be formally fitted both to the
expression for the activated conduction (1) and to the formula (7) derived for
the variable range hopping in strong magnetic fields. In the first case we obtain
activation energies \( \Delta^* \) that do not scale with the separation of adjacent Landau
levels and that are substantially smaller than "true" activation energies \( \Delta \). Pre-
exponential factors \( \sigma^0_{xx} \) in (1) become very small, well below any theoretical
prediction for the activated conductivity.

Fitting to (7) gives densities of states \( D(E_F) \) calculated from (8) that agree
well with those estimated from the activation energies \( \Delta \) according to ref. [3].
Both methods give for the density just in the middle of the mobility gap values
\( D(E_F) \approx (1 - 2) \times 10^{10} meV^{-1} cm^{-2} \), which seems to be rather high but not unrea-
sonable if compared with the zero-field density \( D_0(E) = 2.8 \times 10^{10} \text{meV}^{-1} \text{cm}^{-2} \).

We suggest that the additional contribution to \( \sigma_{xx}(T) \) can be a result of an enhanced backscattering of electrons from one edge of the sample to the other one. The fact, that the data can be reasonably well fitted to an expression derived for the variable range hopping in strong magnetic fields, seems to indicate that it is the tunneling through the potential barriers separating localized states at \( E_F \) that contributes to the backscattering. At high enough densities of the localized states \( D(E_F) \) such a tunneling can apparently lead to a hopping conductivity that can be seen even at temperatures well above 4.2K. In addition to thermally assisted tunneling, the current dependence of our data indicates that it can be an electric field assisted tunneling driven by the transversal electric field arising due to the Hall voltage developing across the bulk of the sample once its conductivity becomes finite.

Upon increasing the current \( I \) injected into the sample, an accumulation of the charge in the edge channels gives rise to a charge-redistribution potential \([10]\) that adds to the common confining potential. At high enough currents, a nearly constant potential gradient develops across the sample. We believe that this gradient further enhances the probability of crossing the potential barriers within the bulk. It supports the backscattering between the edges which manifests itself in an enhanced longitudinal conductivity or resistivity of the sample at higher currents.

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Figure captions

Fig. 1 : Longitudinal resistivity $\varrho_{xx}(B)$ at $T = 4.2K$ for the sample A (a) and B (b) in the vicinity of the minimum corresponding to the filling factors $\nu \approx 2$. Measuring currents $I$ cover the interval where the transition from non-local to local conduction [9,10] occur for these samples. The curves correspond to following currents: (a) .... $I = 1\mu A$ (○); $I = 5\mu A$ (▽); $I = 10\mu A$ (●); $I = 20\mu A$ (□); $I = 50\mu A$ (△). (b) .... $I = 0.2\mu A$ (○); $I = 1\mu A$ (▽); $I = 2\mu A$ (●); $I = 5\mu A$ (□); $I = 10\mu A$ (△). Vertical dashed lines indicate the range of magnetic fields $B_k$ around $\nu = 2.00$ where the temperature dependence of the resistivity $\varrho_{xx}(T)$ and $\varrho_{xy}(T)$ has been studied.

Fig. 2 : Temperature dependence of the Hall resistivity $\varrho_{xy}(T)$ in the middle of the second IQHE plateaux measured on the sample A. Just in the center of the plateaux Hall resistivity should be independent of the temperature and given by $\varrho_{xy} = h/2e^2$.

(a) .... $I = 1\mu A$; ○... $B_1 = 10.48T$ ($\nu = 2.04$), □... $B_2 = 10.63T$ ($\nu = 2.01$), ▽... $B_3 = 10.73T$ ($\nu = 1.99$), ●... $B_4 = 10.83T$ ($\nu = 1.97$).

(b) .... $I = 50\mu A$; ○... $B_1 = 10.53T$ ($\nu = 2.03$), □... $B_2 = 10.63T$ ($\nu = 2.01$), ▽... $B_3 = 10.73T$ ($\nu = 1.99$), ●... $B_4 = 10.83T$ ($\nu = 1.97$).

Fig. 3 : Temperature dependence of the longitudinal conductivity $\sigma_{xx}(T)$ for the sample A drawn in the Arrhenius graph for the fields $B = 10.63T$ (○) and $B = 5.24T$ (▽), corresponding to filling factors $\nu = 2.01$ and $\nu = 4.08$, respectively. Straight lines in the graph indicate the best fit to formula for activated conductivity (1). The parameters obtained by the fitting are:

$\sigma_{xx}^0 = 1.84e^2/h$; $\Delta/k_B = 107K$ and $\sigma_{xx}^0 = 1.17e^2/h$; $\Delta/k_B = 44.7K$ for $\nu = 2.01$ and $\nu = 4.08$, respectively.

Fig. 4 : Temperature dependence of the longitudinal conductivity $\sigma_{xx}(T)$ for sample A measured with $I = 1\mu A$ on both sides of the center of the mobility gap at $\nu = 2$ and fitted to the expression (1). □ .... $\nu = 2.04$, $\sigma_{xx}^0 = 0.71e^2/h$, $\Delta/k_B = 79.7K$; ● .... $\nu = 2.01$, $\sigma_{xx}^0 = 1.84e^2/h$, $\Delta/k_B = 107K$; ○ .... $\nu = 1.97$, $\sigma_{xx}^0 = 0.08e^2/h$, $\Delta/k_B = 41.6K$ ( for $T < 12K$ ).

Fig. 5 : Conductivity $\sigma_{xx}(T)$ measured in the center of the mobility gap at $\nu = 2$ using various currents $I$ and fitted to (1).

(a) .... sample A : ○ ... $I = 1\mu A$; $\sigma_{xx}^0 = 1.84e^2/h$, $\Delta/k_B = 107K$; ▽ ... $I = 50\mu A$, $\sigma_{xx}^0 = 0.02e^2/h$, $\Delta/k_B = 49.7K$ ( for $T < 12K$ );

(b) .... sample B : ○ ... $I = 0.2\mu A$; $\sigma_{xx}^0 = 2.31e^2/h$, $\Delta/k_B = 82K$; △ ... $I = 10\mu A$; $\sigma_{xx}^0 = 0.2e^2/h$, $\Delta/k_B = 28K$ ( for $T < 7K$ ).
**Fig. 6**: Conductivity $\sigma_{xx}(T)$ of the sample A at $\nu = 1.98$ for the currents $I = 1\mu A$ (□), $I = 10\mu A$ (∇) and $I = 50\mu A$ (●). The dashed line represents a linear extrapolation of the activated conductivity at $T \geq 12K$.

**Fig. 7**: Longitudinal resistivity of the sample A near the minimum corresponding to $\nu = 3$. Full lines correspond to the current $I = 1\mu A$ and to the temperatures $T = 1.23K$ (□), $T = 1.45K$ (⊙), $T = 1.81K$ (∇), $T = 2.11K$ (◇), $T = 2.42K$ (△), $T = 2.75K$ (+), $T = 3.11K$ (⊕) and $T = 4.23K$ (●). Dotted line is for $T = 1.23K$ and $I = 10\mu A$, the dashed line for the same temperature but $I = 50\mu A$.

**Fig. 8**: Fitting of the data on $\sigma_{xx}(T)$ measured in sample A using $I = 50\mu A$ at various filling factors to the expression (5) with $\sigma_{xx}^* = e^2\gamma_o/k_B$.

- ○ ... $\nu = 2.03$; $\sigma_{xx}^* = 0.007$, $T_o = 1116K$;
- □ ... $\nu = 2.01$; $\sigma_{xx}^* = 0.048$, $T_o = 1815K$;
- ∇ ... $\nu = 1.99$; $\sigma_{xx}^* = 0.030$, $T_o = 1347K$;
- ● ... $\nu = 1.97$; $\sigma_{xx}^* = 0.008$, $T_o = 740K$;
- ◇ ... $\nu = 1.95$; $\sigma_{xx}^* = 0.004$, $T_o = 458K$.

**Fig. 9**: Fitting of the data on $\sigma_{xx}(T)$ for the sample A at $\nu = 1.97$ to the expression (5). The parameters of the fit at temperatures $T \leq 12K$ are:

- ● ..... $I = 1\mu A$; $\sigma_{xx}^* = 0.018$, $T_o = 1116K$;
- □ ..... $I = 50\mu A$; $\sigma_{xx}^* = 0.008$, $T_o = 740K$. 
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Fig. 1
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Fig. 3
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Fig. 7
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Fig. 9

$\ln(\sigma_{xx} T)$ vs $T^{-1/2} [K^{-1/2}]$