A new puzzle for random interaction

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We continue a series of numerical experiments on many-body systems with random two-body interactions, by examining correlations in ratios of excitation energies of yrast \( J = 0, 2, 4, 6, 8 \) states. Previous studies, limited only to \( J = 0, 2.4 \) states, had shown strong correlations in boson systems but not fermion systems. By including \( J \geq 6 \) states and considering different scatter plots, strong and realistic correlations appear in both boson and fermion systems. Such correlations are a challenge to explanations of random interactions.

Nuclei show a remarkable array of behaviors at low excitation energy, notably collective motion for even-even nuclides\(^1\). Three sets of tools have emerged to shed light on collective motion: algebraic models, based on representations of low-dimension groups in many-fermion and many-boson systems\(^2\); precise characterizations of the nucleon-nucleon interaction and rigorous derivation of effective interactions\(^3\); and, paradoxically, studies of the behavior of random interactions\(^4\).

The study of random two-body interactions was originally applied to quantum chaos and statistical properties of compound nuclear states\(^5\). A few years ago numerical experiments showed, surprisingly, that random interactions could show spectral signatures of regular, collective behavior\(^4\). The first and foremost signature is that, out of an ensemble of randomly chosen two-body interactions, the ground state predominantly has angular momentum \( J = 0, 2, 4 \ldots \) even though such states are a small fraction (typically 2-10\%) of the many-body space. There are other signatures, which we will not review fully here.

Instead we focus on band structure, in particular vibrational and rotational bands, experimentally seen in many even-even nuclides and which in part lead to the liquid drop model and its quantized version the collective geometric model (a generalization of the Bohr Hamiltonian). The most obvious signature of bands are regular structures in the excitation energies of yrast \( J = 0, 2, 4 \ldots \) states: archetypal vibrational bands have excitation energy \( E_x(J) \propto J \) while rotational bands have \( E_x(J) \propto J(J+1) \). (A deeper, and no less important signature, are ratios of intraband E2 transitions strengths.) The simplest model of pairing, the seniority model, by contrast has the first excited \( J = 0, 2, 4, \ldots \) states degenerate.

Bijker and Frank\(^6\) found strong evidence for band structure in the interacting boson model (IBM) with random interactions through two pieces of evidence. First, they profiled, for an ensemble of random interactions, the frequency of the excitation energy ratio

\[
R_{42} \equiv \frac{E_x(J = 4)}{E_x(J = 2)}
\]

and found sharp peaks at \( R_{42} = 2 \) and 3.33, corresponding to vibrational and rotational bands. (The seniority model has \( R_{42} = 1 \).) More significantly, they made a scatter plot of \( R_{42} \) versus the ratio of E2 transition strengths \( B(E2: 4^+ \rightarrow 2^+)/B(E2: 2^+ \rightarrow 0^+) \), and found significant enhancements at the exact U(5) (vibrational) and SU(3) (rotational) limits. Bijker and Frank later analyzed these results in terms of a mean-field model\(^7\).

Fermion systems with random interactions do not show the same correlations as boson systems. Fig. 1 shows frequency distributions for \( R_{42} \) for several typical cases. Fig. 1(a) is for 10 identical particles in a \( 1p_{1/2}-1p_{3/2}-0f_{5/2}-0f_{7/2} \) or \( pf \) space, which, if one assumed a closed \( ^{40}\)Ca core, would correspond to \( ^{50}\)Ca; of course, the interaction is random and there is no \textit{a priori} constraint on the single-particle radial wavefunctions, so labelling this system as \( ^{50}\)Ca is simply for convenience. In this and all cases the interaction conserves angular momentum. Fig. 1(b) has 4 protons (\( \pi \)) and 4 neutrons (\( \nu \)) in a \( 1s_{1/2}-0d_{3/2}-0d_{5/2} \) or \( sd \) space, so we colloquially refer to it as \( ^{24}\)Mg; in this case we constrain the interaction to conserve isospin as well. For the final two panels of Fig. 1 we use single-\( j \) spaces, popular with many investigations of random interactions. Fig. 1(c) has 10 identical particles in a \( j = 21/2 \) space while 1(d) has 4 protons and 4 neutrons in a \( j = 13/2 \) space. These systems were chosen in order to have a large number of interacting particles and non-trivial dimensions of the many-body space (\( M \)-scheme dimension of about \( 10^4 \)), but still relatively small enough that a high-performance \( M \)-scheme shell model code\(^8\) can run thousands of cases. For all the plots in this paper we only select cases for which the ground state has \( J = 0 \).

The broad peaks at \( R_{42} = 1 \) in Fig. 1 is closest to a simple seniority model, and although other signatures of pairing can be found\(^9\), detailed investigations discourage interpretation as a simple pairing condensate\(^10\). Similarly, a Bijker-Frank plot of \( R_{42} \) versus ratio of E2 strengths shows no strong correlations (not shown here).

Although random interactions do not quantitatively reproduce experimental behavior, the results are striking enough to have spawned a mini-industry. Several attempts have been made to explain the results; see\(^11\)\(^12\) for some broad examples. Although these analyses yield some valuable and interesting insights, arguably none rise to the level...
of a comprehensive “theory” of random interactions complete with predicting new phenomena. Thus there is still room for empirical exploration.

In this short note, we show results of a new numerical experiment. In particular, we look at correlations with higher-\( J \) yrast states, which with few exceptions [10] have been paid scant attention so far. We define \( R_{62} \equiv E_x(J = 6)/E_x(J = 2) \), as well as \( R_{82} \), etc. in an obvious generalization to Eq. (1); in Figs. 2 and 3 we look at scatter plots of \( R_{62} \) vs. \( R_{42} \), and \( R_{82} \) vs. \( R_{42} \), respectively. We also show the loci for seniority (‘S’), vibrational (‘V’), and rotational (‘R’) limits. Although there are strong correlations for the energy spectrum in fermion systems, another signal of band structure are large, consistent quadrupole deformations. We looked for these by computing the quadrupole moments of the yrast \( J > 0 \) states and found no obvious correlations. We also looked at ratios of \( B(E2) \) strengths [13] and found only weak correlations.

Finally in Fig. 4 we show similar results for the interacting boson model with random interaction, using the program PHINT[14]. Similar strong correlations, not shown, occur even for \( J = 10 \). Here are the correlations are less surprising, given the results of Bijker and Frank.
FIG. 3: (Color online) Scatter plot of $R_{82}$ versus $R_{42}$ for (a) $(pf)^{10} (^{50}\text{Ca})$; (b) $(sd)^2(sd)^4 (^{24}\text{Mg})$; (c) $(21/2)^{10}$; and (d) $(13/2)^4(13/2)^6$. Also shown are locations for seniority (‘S’), vibrational (‘V’), and rotational (‘R’) limits.

FIG. 4: (Color online) Scatter plot of (a) $R_{62}$ versus $R_{42}$ and (b) $R_{82}$ versus $R_{42}$ for the interacting boson model with 16 bosons. Also shown are locations for seniority (‘S’), vibrational (‘V’), and rotational (‘R’) limits.

We have no broad explanation for these results, and certainly no quantitative explanation. One can invoke a mean-field picture, but given the presence of “geometric chaoticity” [10] it is surprising that only a select range of mean-fields can form. Other recent work [12] has shown that shell-model dynamics are dominated by a relatively few combinations of two-body matrix elements; why such select combinations give rise to collectivity or even pseudo-collectivity is not immediately clear and not addressed by those authors. For the moment we present these curious empirical phenomena as a provocative challenge to existing and future analyses of random interactions.

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