Is minimal supergravity not below 1 TeV?

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The masses of Standard Model (SM) particles are generated through electroweak symmetry breaking (EWSB) at electroweak scale \( Q_{EW} = v_{\text{weak}} = 175 \text{ GeV} \) (the vacuum expectation value of a fundamental, isodoublet, “Higgs” scalar field). If new physics beyond SM exists, then the scale of new physics \( Q_{NP} \) should be \( \sim m_\tilde{S} \) (masses of the particles in new Physics). These two scales originate from breaking of two completely separate symmetries and the physics at these two scales are completely different. The effective Higgs potential in minimal supersymmetric SM (MSSM) with full one loop and two loop corrections shows renormalization group evolution (RGE) scale invariance, which implies that perturbation theory works well in calculation of the effective Higgs potential. But, the shape of the potential as a function of the neutral parts of the Higgs fields \( (H_0^u, H_0^d) \) changes with RGE scale \( Q_0 \) as the individual parameters change with \( Q_0 \). This makes the changes in Higgs masses at different scales. At the energy scale much above from \( Q_{EW} \) the effect of new physics beyond SM becomes more visible and the effect of the EWSB becomes largely suppressed by them. Again, at the lower energy scale much below from \( Q_{EW} \) the physics of EWSB does not become even visible. This implies that the accurate spectra through EWSB can be found by generating them only at the true EWSB scale \( Q_{EW} \) and the accurate spectra for new physics can be found by generating them only at \( Q_{NP} \). We find a dramatically large allowed parameter space in mSUGRA model with almost no bounds on universal scalar mass \( m_0 \), universal gaugino mass \( m_{1/2} \) and universal trilinear coupling \( A_0 \) when EWSB minima is evaluated at \( Q_0 = v_{\text{weak}} \) in contrary with the one where EWSB minima is evaluated at \( Q_0 = \sqrt{m_{tL}m_{tR}} \). As the squarks are \( \gtrsim 1 \text{ TeV}, m_h < 135 \text{ GeV} \) implies that sleptons might be much lighter and much below from 1 TeV.

I. INTRODUCTION

The mechanism of generation of masses for the standard model (SM) particles via electroweak symmetry breaking (EWSB) has now been completely realized by the discovery of Higgs particle by ATLAS and CMS experiments at the Large Hadron Collider (LHC). Its mass around 125.5 GeV [1, 2] puts the studies of physics beyond SM in an entirely new perspective. It corresponds to the mass of lightest CP-even Higgs boson in minimal supersymmetric standard model (MSSM) provided it has SM like coupling [3]. In minimal supergravity (mSUGRA) model [4] bound on \( A_0 < 0 \) for \( m_0 \gtrsim 1 \text{ TeV} \) (implying slepton masses \( \gtrsim 1 \text{ TeV} \)) are found in earlier studies due to the requirement of CP even Higgs mass \( (m_h) \) around 125 GeV. But, \( m_h < 135 \text{ GeV} \) implies that sparticle masses in MSSM should not exceed 1 TeV and \( m_h > M_Z \) implies the loop corrections from particles of new physics (sparticles) are very significant. There is no strong direct limit on the slepton masses from LHC and low mass sleptons below 1 TeV may give clean signals at International linear collider (ILC) as they may remain suppressed due to huge strong interaction background at LHC. As the hierarchical pattern is seen in the mass spectrum of SM and squarks masses are \( \gtrsim 1 \text{ TeV}; m_h < 135 \text{ GeV} \) strongly indicates that sleptons masses might be much below from 1 TeV. The bounds in earlier studies
are obtained evaluating EWSB minima at the renormalization group evolution (RGE) scale \( Q_0 = \sqrt{m_{tL} m_{tR}} \) and also generating spectra at this scale.

In this paper, we discuss the EWSB in mSUGRA model and its radiative corrections. Then we justify that one should evaluate EWSB minima at the true EWSB scale. Finally, we obtain the allowed parameter space (APS) using this EWSB scale in the framework of mSUGRA model and then compare the new APS with those obtained from considering the EWSB scale at \( \sqrt{m_{tL} m_{tR}} \) (which is used in all mSUGRA spectrum generator packages available in literature and also used in finding post-LHC constraints in mSUGRA model [6–9]). We find a dramatically large allowed parameter space in mSUGRA model with almost no bounds on \( m_0, m_{1/2} \) and \( A_0 \) when EWSB minima is evaluated at \( Q_0 = v_{\text{weak}} \) in contrary with the one where EWSB minima is evaluated at \( Q_0 = \sqrt{m_{tL} m_{tR}} \).

II. ELECTROWEAK SYMMETRY BREAKING

A. Radiative corrections

The tree level scalar potential keeping only the dependence on the neutral Higgs fields:

\[
V_0 = (m_{H_u}^2 + \mu^2) |H_u^0|^2 + (m_{H_d}^2 + \mu^2) |H_d^0|^2 + m_3^2 (H_u^0 H_d^0 + \text{h.c.}) + \frac{g^2 + \beta^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2
\]

Here, both the tree level potential \( V_0 \) and its parameters are strongly RGE scale dependent. However, if we include loop corrections at all orders; in principle, the effective potential \( V_{\text{eff}} = V_0 + \Delta V \) should be RGE scale independent. Otherwise, the perturbation theory will not work and the physics with \( V_{\text{eff}} \) will no longer be valid. From the minimization criteria one can find

\[
\mu^2 = -\frac{m_{H_u}^2}{2} + \frac{m_{H_d}^2}{2} + \frac{\Sigma_d - (m_{H_u}^2 + \Sigma_u) \tan^2 \beta}{\tan^2 \beta - 1}, \tag{2}
\]

\[
m_3^2 = -\frac{1}{2} \sin 2\beta \left( m_{H_d}^2 + m_{H_u}^2 + 2\mu^2 + \Sigma_d + \Sigma_u \right) \tag{3}
\]

where,

\[
\Sigma_u = \frac{1}{2\nu_u} \frac{\partial \Delta V}{\partial \nu_u}. \tag{4}
\]

The value of \( \Delta V \) can be different at different scale and it is not necessary to evaluate this at the scale where it is minimum. Only the necessary criteria is \( V_{\text{eff}} \) should be scale invariant to make sure that the perturbation theory works at the scale where \( V_{\text{eff}} \) is evaluated. The addition of 1-loop and 2-loop corrections stabilizes \( V_{\text{eff}} \) with respect to RGE scale (from \( Q_{NP} \) to \( Q_{EW} \)) and the the parameters obtained from this minimization criteria \((\mu \text{ and } m_3^2)\) are also stabilized [17]. This implies that the perturbation theory works well and ignored amount of loop corrections higher than 2-loops does not become significant below \( Q_{NP} \) to \( Q_{EW} \). Here, one should also note that \( \Sigma_{u,d} \) can be large at any scale and may be even comparable with \( m_{H_u,d}^2 \); but, must be finite as the large finite values of derivative of \( \Delta V \) with respect to \( \nu_{u,d} \) do not mean the violation the scale invariance of \( V_{\text{eff}} \). The Higgs mass squared parameters \( m_{H_u}^2 \) and \( m_{H_d}^2 \) are not physical observable. The interaction of the the Higgs field with other particles is such that these parameters changes rapidly with RGE scale by a few order of magnitude from positive to negative value (e.g., \(+10^8 \text{ GeV}\) to \( -10^8 \text{ GeV}\) from GUT scale to EW scale for a typical set of input parameters). The rate of the change of loop corrections \( \Delta V \) with respect to \( \nu_u \) are also significantly different at different scales (see Fig. [1]). They change the shape of the Higgs potential significantly with the RGE scale. This reflects that the physics at different RGE scale are significantly different. They need not to be small or large from any physical reason. The Higgs masses are determined from the shape of the potential.

The one loop corrections \( \Delta V_1 \) in Landau gauge is given by [11]:

\[
\Delta V_1 = \frac{1}{64\pi^2} STr M^4 \left[ \ln(M^2/Q^2) - 3/2 \right] \tag{5}
\]

The contribution from stop quarks is given by:

\[
\Sigma_u(\tilde{t}_i) \sim \frac{3g_i^2}{16\pi^2} m_{\tilde{t}_i} \ln(m_{\tilde{t}_i}^2/Q^2) \tag{6}
\]
The loop corrections are very significant, without which the evaluation of parameters from minimization of the tree level potential may give even wrong results. These radiative corrections depend strongly on renormalization scale $Q$ and the contributions normally becomes negligible at $Q = \sqrt{m_{\tilde{t}} L m_{\tilde{t}} R}$.

B. EWSB scale

The weak scale $Q_{EW} = (\sqrt{2} \sqrt{G_F})^{-1} = 175$ GeV first entered physics, when Enrico Fermi constructed the current-current interaction description of $\beta$-decay and introduced the constant, $G_F$, into modern physics. The Standard Model identifies $Q_{EW}$ with the vacuum expectation value (VEV) of a fundamental, isodoublet, “Higgs” scalar field.

The extension of SM with supersymmetry solves completely the quadratic divergence problem in the 1-loop corrections to the Higgs squared mass parameter as there exists a scalar partner in supersymmetric extensions corresponding to each fermion in SM. The logarithmic corrections involve an energy scale $Q$. This is the RGE scale, where the effective Higgs potential is evaluated.

The minimization of the Higgs potential gives vacuum expectation value (VEV) of the neutral part of the Higgs fields. In MSSM model EWSB gives a relation among the parameters of Higgs potential as the VEV $\langle v\rangle_{\text{weak}} = \sqrt{\langle H^0_u \rangle^2 + \langle H^0_d \rangle^2}$ is already fixed from SM predictions. This VEV should also be used to generate the masses of SM particles and also the Higgs particles in extended SM with supersymmetry (e.g., MSSM).

The potential $V_{\text{eff}}(v_u, v_d)$ is RGE scale invariant over the whole range of $Q_0$ from $M_{\tilde{S}}$ to $Q_{EW}$. But, the shape of the potential as a function of $H^0_u$ and $H^0_d$ changes with RGE scale $Q_0$ as the individual parameters of $V_{\text{eff}}$ change with $Q_0$. The EWSB occurs only when $\mu^2$ becomes positive. Below this scale, it has been shown that the $\mu^2$ also remains scale invariant. Due to the change of the shape of Higgs potential, the Higgs masses generated at different scales are different.

The scale of new Physics ($Q_{NP} \sim m_{\tilde{S}}$) and the scale of EWSB minima $Q_{EW}$ originate from breaking of two completely separate symmetries and the physics at these two scales are completely different. At the energy scale much above from $Q_{EW}$ the effect of new physics beyond SM becomes more visible and the effect of the EWSB becomes largely suppressed. Again, at the lower energy scale much below from $Q_{EW}$ the physics of EWSB does not become even visible. Since the masses of Higgs particles in MSSM are generated through EWSB, the accurate
spectra for measurement at experiments can only be generated at $Q_{EW} \approx v_{weak}$.

If $m_S \sim$ a few hundred GeV, then the running of parameters up to $Q = v_{weak}$ is negligible and one can use $Q_{NP}$ as the weak scale. But, if $m_S \sim$ TeV, one cannot neglect the running of the parameters (particularly, $m^2_{H_u}$ and $m^2_{H_d}$) and the approximation of using $Q_{NP}$ as $Q_{EW}$ does not work. The value of $m_h$ is increased significantly when one evaluates EWSB at $Q_{EW} \approx v_{weak}$ (see Fig. 2). On the other hand, if one considers EWSB scale $Q_0 \sim$ TeV, EWSB also may not occur due to less running of $m^2_{H_d}$ and $m^2_{H_u}$ (EWSB requires $\mu^2$ positive).

The MSSM parameters, particularly, $m^2_{H_u}$, $m^2_{H_d}$ and Yukawa couplings strongly depend on the RGE scale. The running of these parameters depends strongly on all five input parameters while the other MSSM parameters (mainly, sleptons and squarks mass parameters) have relatively very small scale dependence. So, the generated masses of the particles, mainly the particles in the Higgs sector depend crucially on the scale where we generate the spectra.

In our calculation, we consider program SuSeFLAV-1.2. It considers full one loop corrections together with two loop leading contributions $\mathcal{O}(\alpha, \alpha_t, \alpha_s^2)$ to the Higgs mass squared parameters following [18]. We have compared SuSeFLAV-1.2 with softsusy3.4.0 [8] for different sets of input parameters and find no serious significant change; similar changes in the spectra are observed with the changes in input parameters. For typical sets of mSUGRA input parameters we show in Fig. 2 (left) that the value of $\mu$ at $M_Z$ (evaluated through RG running from $Q_0$ to $M_Z$) is almost stable with EWSB scale $Q_0$, while the change in $m_h$ (evaluated at $Q_0$) with EWSB scale $Q_0$ is significantly large. It is clear that $\mu$ at $M_Z$ (determined from EWSB minima at RGE scale $Q_0$) remains almost unaltered with the change in $Q_0$. But, there is a significant change in $m_h$ with change in $Q_0$ (see Fig. 2 (right)).

The accurate spectra through EWSB can be found by generating them only at the true EWSB scale $Q_{EW}$ and the accurate supersymmetric spectra through supersymmetry breaking can be found by generating them only at the supersymmetry breaking scale $Q_{NP}$. In generation of spectra through EWSB (masses of the Higgs particles), one should run all MSSM parameters up to $Q_{EW}$, and in generation of the masses of sparticles one should use all MSSM parameters evaluated at $Q_{NP}$. Since $\mu$ is generated at $Q_{EW}$, one should take the RG running value of $\mu$ at $Q_{NP}$ from $Q_{EW}$ for calculating the masses of sparticles.

III. THE MSUGRA PARAMETER SPACE

We have compared the allowed mSUGRA parameter space for two cases of evaluation of EWSB minima: i) at $Q_0 = \sqrt{m^2_L m^2_R}$ and ii) $Q_0 = v_{weak} = \sqrt{(H_u)^2 + (H_d)^2}$. Here, we consider the only parameter space where one can generate $m_h = 125.5 \pm 0.5$ GeV. No other constraints are considered (neutralino may not be the lightest supersymmetric particle (LSP)).

We generate the spectra for the range of $m_0 = 100 - 3100$ GeV, $m_{1/2} = 100 - 3100$ GeV $A_0 = -3m_0$ to $+3m_0$, $\tan \beta = 3 - 63$ and $\text{sign}(\mu) = \pm 1$.

We find that a dramatically large allowed parameter space in mSUGRA model with almost no bounds on $m_0, m_{1/2}$ and $A_0$ when EWSB minima is evaluated at $Q_0 = v_{weak}$ in contrary with the one where EWSB minima is evaluated at $Q_0 = \sqrt{m^2_L m^2_R}$ (see Fig. 3). The parameter $m^2_{H_u}$ goes to larger negative value as one decreases the RGE scale. This helps to have successful EWSB providing $\mu^2$ positive and gives correct masses for Higgs particles by determining the exact shape of the Higgs potential at $Q_{EW}$.

IV. CONCLUSION

The LHC experiment puts strong lower bounds on squarks and gluino masses ($\gtrsim 1$ TeV) [19, 20], but low
values of slepton masses $\sim$ hundreds of GeV remained allowed in a model when it can produce mass of the Higgs particle at 125 GeV. It may be the fact that signals for low mass sleptons at LHC are not seen due to huge strong interaction background and it may give clean signal at ILC. As the hierarchical pattern is seen in the mass spectrum of SM and squarks masses are $\gtrsim$1 TeV, $m_h < 135$ GeV strongly indicates that sleptons masses might be much below from 1 TeV.

LHC data has restricted mSUGRA parameter space, but there is still enough region to search mSUGRA model and there is no absolute bounds on $m_0$, $m_{1/2}$ and $A_0$ in
mSUGRA model if EWSB minima is evaluated at the RGE scale equal to the true weak scale value.

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