Probe Noncommutative Space-Time Scale Using $\gamma\gamma \rightarrow Z$ At ILC

Xiao-Gang He$^{1,2}$, Xue-Qian Li$^2$

$^1$NCTS/TPE, Department of Physics, National Taiwan University, Taipei

$^2$Department of Physics, Nankai University, Tianjin

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Abstract

In the standard model production of on-shell Z boson at a photon collider (or Z decays into $\gamma\gamma$) is strictly forbidden by angular momentum conservation and Bose statistics (the Yang’s Theorem). In the standard model with noncommutative space-time this process can occur. Therefore this process provides an important probe for the noncommutative space-time. The $\gamma\gamma$ collision at the ILC by laser backscattering of the electron and positron beams offers an ideal place to carry out such a study. Assuming an integrated luminosity of 500 fb$^{-1}$, we show that the constraint which can be achieved on $\Gamma(Z \rightarrow \gamma\gamma)$ is three to four orders of magnitude better than the current bound of $5.2 \times 10^{-5}$ GeV. The noncommutative scale can be probed up to a few TeVs.
The property of space-time has fundamental importance in understanding the laws of nature. Noncommutative quantum field theory, which modifies the space-time commutation relations provides an alternative to the ordinary quantum field theory, may shed some light on the detailed structure of space-time. The idea that the space-time coordinates may not commute has a long history [1, 2]. In recent years, noncommutative space-time has found a natural origin in string theories [3, 4, 5]. A simple and commonly studied noncommutative quantum field theory is based on the following commutation relation of space-time,

\[ [\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}, \tag{1} \]

where \( \hat{x}_\mu \) is the noncommutative space-time coordinates. \( \Theta_{\mu\nu} \) is a constant, real, antisymmetric matrix, and has mass\(^{-2} \) dimension. The size of the \( 1/\sqrt{|\Theta_{\mu\nu}|} \) represents the noncommutative scale \( \Lambda_{NC} \).

In the literature there are extensive studies on the mathematical structure of noncommutative space-time, but less studies on measurable physical consequences of the noncommutative models for strong and electroweak processes [5]. To know if noncommutative space-time has anything to do with nature, emphasis should be placed more on physical consequences. Although there are several studies for the constraints on noncommutative scale \( \Lambda_{NC} \) [6, 7, 8], most of the analyses were based on \( U(1) \) noncommutative gauge theory which is not yet a consistent noncommutative model of the strong and electroweak interactions. At present the constraints are fairly loose with \( \Lambda_{NC} \) only being limited to larger than a few TeV [8]. The noncommutative scale may be accessible at near future colliders, such as the Large Hadron Collider (LHC) and the International Linear Collider (ILC). In this work based on consistent formulation of noncommutative standard model (NCSM), we study the process \( \gamma\gamma \to Z \) at ILC and new constraint on the noncommutative scale may be achieved.

In the SM, \( \gamma\gamma \to Z \) is strictly forbidden, if all particles are on-shell, by angular momentum conservation and Bose statistics (the Yang’s Theorem) [9], but can occur in noncommutative version of the standard model (NCSM). Note that if one of the photon is off-shell, there can be non-zero self coupling of the type \( Z\gamma\gamma^* \) [10] even without noncommutative space-time, and can be studied in process like \( p\bar{p} \to \gamma^* \to \gamma Z \to \gamma l\bar{l} \) [11]. Therefore production of on-shell \( Z \) boson at \( \gamma\gamma \) collider can provide an interesting test for the NCSM and therefore the nature of space-time. We will show that the constraint can be achieved on \( \Gamma(Z \to \gamma\gamma) \) is several orders of magnitude better than the present bound of \( 5.2 \times 10^{-5} \) GeV at 95% confidence level.
C.L. A_{NC} can be probed to a few TeV which is much better than that can be extracted from present bound on $\Gamma(Z \to \gamma\gamma)$.[13]

Quantum field theory based on the commutation relation in eq. (1) can be easily studied using the Weyl-Moyal correspondence, i.e., replacing the product of two fields $A(\hat{x})$ and $B(\hat{x})$ with NC coordinates by the star “*” product:

$$A(\hat{x})B(\hat{x}) \to \hat{A}(x) \ast \hat{B}(x) = \exp[i\frac{1}{2} \Theta_{\mu\nu} \partial_\mu^x \partial_\nu^y] A(x)B(y)|_{x=y}. \quad (2)$$

Here fields with and without ‘hat’ indicate the fields in noncommutative space-time and ordinary space-time, respectively.

Promotion of the usual space-time coordinates $x_\mu$ to the noncommutative space-time coordinates $\hat{x}_\mu$ has very interesting consequences[14]. We denote the noncommutative gauge field to be $\hat{A}_\mu = \hat{A}_a^a T^a$ of a group with generators normalized as $Tr(T^a T^b) = \delta^{ab}/2$. In noncommutative space-time two consecutive local gauge transformations $\hat{\alpha}$ and $\hat{\beta}$ of the type $\hat{\alpha} \Psi = i\hat{\alpha} \ast \hat{\Psi}$, with the matter field $\hat{\Psi}$ transforming as a fundamental representation of the gauge group, is given by $(\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha) = (\hat{\alpha} \ast \hat{\beta} - \hat{\beta} \ast \hat{\alpha})$. This commutation relation is consistent with the $U(N)$ Lie algebra, but not consistent with the $SU(N)$ Lie algebra since it cannot be reduced to the matrix commutators of the $SU(N)$ generators. Also note that even with the $U(1)$ group the above consecutive transformations do not commute implying that the charge for a $U(1)$ gauge theory is limited to only three possible values which can be normalized to 1, 0, -1.

The above properties pose difficulties in constructing noncommutative standard model for the strong and electroweak interactions because the standard gauge group contains $SU(3)_C$ and $SU(2)_L$ which cannot be naively gauged with noncommutative space-time. Also the charges of $U(1)_Y$ are not just 1, 0, -1, but some of them are fractionally charged after normalizing the right-handed electron to have -1 hypercharge, such as, 1/6, 1/2, 2/3, -1/3 for left-handed quarks, left-handed leptons, right-handed up and down quarks, respectively. This is the so called charge quantization problem. However all these difficulties can be overcome with the use of the Seiberg-Witten (SW) map which maps noncommutative gauge field to ordinary commutative gauge field. A consistent noncommutative $SU(N)$ gauge theory can be constructed by expanding $\hat{\alpha}$ in powers of $\Theta$ with $\hat{\alpha} = \alpha + \alpha_{ab}^{(1)} : T^a T^b : + \ldots + \alpha_{a_1...a_n}^{(n-1)} : T^{a_1} ... T^{a_n} : ...$ to form a closed envelop algebra. Here the symbol ‘$: T^{a_1} ... T^{a_n} :$’ means totally symmetric in exchanging $a_i$. Detailed description of the method can be found
in Ref. [15]. One can then expand gauge and mater fields in powers of $\Theta$ to have a consistent $SU(N)$ gauge theory order by order in $\Theta$. To the first order in $\Theta$, one has

$$\hat{A}_\mu = A_\mu - \frac{1}{4} g_N \Theta^{\alpha\beta} \{ A_\alpha, \partial_\beta A_\mu + F_{\beta\mu} \}.$$  

(3)

The above gauge field would generate new terms in the interaction Lagrangian compared with the ordinary $SU(N)$ gauge theory. For example the term $- \frac{1}{2} Tr (F_{\mu\nu} F^{\mu\nu})$ in the Lagrangian for an $SU(N)$ gauge field will become, to the first order in $\Theta$ [15],

$$L = - \frac{1}{2} Tr F_{\mu\nu} F^{\mu\nu} + g_N \Theta^{\mu\nu} \frac{1}{4} Tr [F_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - 4 F_{\mu\rho} F_{\nu\sigma} F^{\rho\sigma}].$$  

(4)

The SW map can also cure the charge quantization problem by associating a gauge field $\hat{A}_\mu^{(n)}$ for the a matter field $\psi^{(n)}$ with $U(1)$ charge $gQ^{(n)}$, $\hat{A}_\mu^{(n)} = A_\mu - (gQ^{(n)}/4) \Theta^{\alpha\beta} \{ A_\alpha, \partial_\beta A_\mu + F_{\beta\mu} \}$, where $A_\mu$ is the gauge field of $U(1)$ in ordinary space-time. With help of the SW map a specific method to construct NCSM and grand unified theories have been developed [15, 16, 17, 18].

The new terms in eq. (4) when applied to the NCSM, will generate terms inducing $Z-\gamma-\gamma$ interaction. These terms can be parameterized as

$$L_{Z\gamma\gamma} = egZ_{\gamma\gamma} \Theta^{\alpha\beta} (8Z_{\mu\alpha} A_{\nu\beta} A^{\mu\nu} + 4A_{\mu\alpha} A_{\nu\beta} Z^{\mu\nu} - 2A_{\alpha\beta} A_{\mu\nu} Z^{\mu\nu} - Z_{\alpha\beta} A_{\mu\nu} A^{\mu\nu}).$$  

(5)

In NCSM, $g_{Z\gamma\gamma}$ is not uniquely determined due to the need of introducing a gauge field for each matter field with different $U(1)_Y$ charge to solve the charge quantization problem [15]. This is because that as summing over different $U(1)_Y$ gauge fields for all matter fields to give the kinetic energy, even though the first term in eq. (4) is fixed with the right normalization, the triple gauge field terms are not fixed. This problem may be solved by obtaining low energy NCSM from grand unified theories such as noncommutative $SO(10)$ [16], $SU(5)$ [17] grand unification and $SU(3)^3 = SU(3)_C \times SU(3)_L \times SU(3)_R$ trinification theories where there is no $U(1)$ charge quantization problem to start with. In noncommutative $SO(10)$ grand unification, due to the same reason this theory is anomaly free, the triple gauge coupling is automatically zero. Therefore in this model $\gamma\gamma \rightarrow Z$ cannot occur. Naively, non-commutative $SU(5)$ grand unification can fix the triple gauge boson couplings [17]. However, in this model, there are several different multiplets for fermion and Higgs representations, $\bar{5}$, 10, 24 and etc., one needs to associate different gauge fields with them which lead to a similar problem of non-uniqueness of triple gauge boson couplings for different $U(1)_Y$.
gauge fields in the NCSM\textsuperscript{16}. \(SU(5)\) is not truly a unified model in noncommutative space-time. Unique non-trivial triplet gauge boson couplings can be generated in noncommutative trinification model\textsuperscript{18}. In this model, the fermion and Higgs representations are all in the 27-representation of the gauge group resulting in fixed triple gauge boson couplings. The coupling \(e g_{Z\gamma\gamma} \) in \(SU(3)^3\) is given, at the unification scale, by\textsuperscript{18}

\[
e g_{Z\gamma\gamma} = - \frac{g_U}{16 \sqrt{15}} \sin \theta_W (1 + \frac{19}{4} \cos 2 \theta_W).
\]

Using the normalization \(g_Y = \sqrt{3/5} g_U\), and running down to energy scale \(\mu = m_Z\), we have \(e g_{Z\gamma\gamma} = -5.58 \times 10^{-3}\). In rest of the discussions we will use noncommutative \(SU(3)^3\) as an illustration to show how the limit on the noncommutative scale can be determined using \(\gamma\gamma \to Z\) at the photon collision mode of ILC.

The matrix element for on-shell \(\gamma(k)\gamma(k') \to Z(p)\) in momentum space after symmetrizing the two photons is given by

\[
M = -ieg_{Z\gamma\gamma} 16 \left[ k \cdot k' (k' \cdot \epsilon_Z^* \epsilon \cdot \Theta \cdot \epsilon' + \epsilon' \cdot \epsilon_Z^* k \cdot \Theta \epsilon' + \epsilon' \cdot \epsilon_Z^* k' \cdot \Theta \cdot \epsilon) + k \cdot \Theta \cdot k' (k' \cdot \epsilon \epsilon' \epsilon_Z^* - k \cdot \epsilon' \epsilon \cdot \epsilon_Z^* + \epsilon \cdot \epsilon' k \cdot \epsilon_Z^*) \right],
\]

where \(a \cdot \Theta \cdot b = a_{\alpha} \Theta^{\alpha \beta} b_{\beta}\).

With the above amplitude we obtain

\[
\sigma(\gamma\gamma \to Z, s) = 6\pi^2 m_Z \Gamma(Z \to \gamma\gamma) \frac{1}{m_Z^2} \delta(s - m_Z^2),
\]

\[
\Gamma(Z \to \gamma\gamma) = \frac{4}{3} \alpha_{em} g_{Z\gamma\gamma}^2 m_Z^5 (\Theta_T^2 + \frac{7}{3} \Theta_S^2),
\]

where \(\Theta_T^2 = \theta_{01}^2 + \theta_{02}^2 + \theta_{03}^2\) and \(\Theta_S^2 = \theta_{12}^2 + \theta_{13}^2 + \theta_{23}^2\). The expression for \(\Gamma(Z \to \gamma\gamma)\) agrees with that obtained in Ref.\textsuperscript{13}.

We would like to emphasize the fact that in the usual SM the process \(\gamma\gamma \to Z\) is strictly forbidden makes it possible to consistently test models which are based on noncommutative space-time and expanded to the first order in \(\Theta\) in terms of the cross section of \(\gamma\gamma \to Z\). It is interesting to remark that if a process has SM contribution, there can be two types of terms proportional to \(\Theta^2\) for cross section or decay rate, one from the square of the interaction expanded to the first order in \(\Theta\) and another from interference of the usual SM and second order term in \(\Theta\) in NCSM. One then needs to expand the NCSM to second order in \(\Theta\) to have a consistent test. This can in principle be carried out using the SW map described earlier.
but would complicate the analysis. Tests of NCSM to the first order in $\Theta$ are, however, still possible in this case if one uses observable which is at the first order in $\Theta$, such as some asymmetries in polarization of spin or certain angular distribution of a particular particle involved. Here we concentrate on the simple on-shell process $\gamma\gamma \rightarrow Z$.

Convolting the energies of the two photon beams produced by using the laser backscattering technique on the electron and positron beams in an $e^+e^-$ collider with center of mass frame energy $\sqrt{s}$, we have

$$\sigma_c = \int_{x_{\text{min}}}^{x_{\text{max}}} dx_1 \int_{x_{\text{min}}}^{x_{\text{max}}} dx_2 \sigma(\gamma\gamma \rightarrow Z, x_1x_2s)F(x_1)F(x_2) = I(m_Z^2/s)6\pi^2m_Z^3\Gamma(Z \rightarrow \gamma\gamma)/m_Z^4,$$

where

$$I(y) = \int_{y/x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x}F(x)F(y/x),$$

with $y = m_Z^2/s$, and $x_{\text{max}} = \xi/(1 + \xi)$ with $\xi = 2(1 + \sqrt{2})$. The function $F(x)$ is given by

$$F(x) = \frac{1}{D(\xi)} \left( 1 - x + \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right),$$

$$D(\xi) = (1 - \frac{4}{\xi} - \frac{8}{\xi^2}) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}.$$

Note that the function $I(y)$ is a function of $m_Z^2/s$ only. The model-dependent part resides purely in the expression for $\Gamma(Z \rightarrow \gamma\gamma)$. In Fig. 1 we show $I(y)$ as a function of $y$. We see that for a large range of $m_Z^2/s$, $I(y)$ is sizeable. The ILC of energy between 120 to 250 GeV can be very useful for the purpose of studying $\gamma\gamma \rightarrow Z$. When energy becomes higher the cross section goes down. If there is a $Z'$ particle with a mass of a few hundred GeV, ILC of energy around several hundred GeV to one TeV would be an excellent place to look for $Z'$ via the process $\gamma\gamma \rightarrow Z'$.

The proposed ILC energy will be in the range from several hundred GeV to TeV, and therefore can be an ideal place to study $\gamma\gamma \rightarrow Z$. We list reachable upper bounds (using current bound $5.2 \times 10^{-5}$ GeV$^2$ on the decay rate $\Gamma(Z \rightarrow \gamma\gamma)$) on signal event number and the decay rate in Table 1, assuming an integrated luminosity of 500 fb$^{-1}$. We see that if a theory gives $\Gamma$ for $Z \rightarrow \gamma\gamma$ close to the current upper limit of $5.2 \times 10^{-5}$ GeV, one would see more than $10^5$ events. If no events are seen, this information would be translated into a bound on the rate $\Gamma$ of $Z \rightarrow \gamma\gamma$ to be less than a few times of $10^{-10}$ GeV. This is much better than the constraint obtained before. Even assuming an efficiency as low as 1%,
FIG. 1: The function $I(y)$. The horizontal axis is $10^3 y$.

one can still set an upper bound of $\Gamma < 10^{-8}$ GeV which is still more than three orders of magnitude better than the current bound.

One can obtain the bound on the noncommutative scale $\Lambda_{NC}$ from the bound on the event rate for $\gamma\gamma \rightarrow Z$. We list the upper limits on the scales $\Lambda_S = 1/\sqrt{\Theta_2^S}$ and $\Lambda_T = 1/\sqrt{\Theta_2^T}$ in the last two rows of Table 1. We see that the noncommutative scale can be probed up to a few TeV. If the efficiency is lowered to 1%, the noncommutative scale can still be probed up to 1.5 TeV.

There are other three gauge boson interactions involving other particles, such as $\gamma$, $Z$, and gluon $g$, which are not present in the SM, but can exist in NCSM such as $Z - g - g$, $\gamma - g - g$, $\gamma - \gamma - \gamma$, $Z - Z - \gamma$ and $Z - Z - Z$. Experimental studies of other processes at LHC and ILC can also provide further information about noncommutative space-time extension of the SM. An interesting process similar to the process $\gamma\gamma \rightarrow Z$ discussed in this paper is gluon fusion into a $Z$-boson, i.e. $qq \rightarrow Z$ at LHC by studying $pp \rightarrow ZX$. Note that this process is not allowed in $SU(3)^3$ \cite{18} and nor in $SO(10)$ \cite{16}. It is of course much more complicated to carry out such a study to identify the noncommutative effects because $pp \rightarrow ZX$ can also occur in the usual SM. More detailed studies are needed to isolate the SM background and the NC effects.

To summarize, we have studied the process $\gamma\gamma \rightarrow Z$ which is strictly forbidden in the standard model of strong and electroweak interactions, but allowed in the noncommutative space-time standard model. We have shown that the $\gamma\gamma$ collision mode at the ILC by laser backscattering of the electron and positron beams with an integrated luminosity of 500 fb$^{-1}$ can obtain a constraint on $\Gamma(Z \rightarrow \gamma\gamma)$ three to four orders of magnitude better than the
| $\sqrt{s}$ (GeV) | 120 | 200 | 250 | 500 | 1000 |
|------------------|-----|-----|-----|-----|-----|
| $I(m_Z^2/s)$     | 0.397 | 0.333 | 0.275 | 0.120 | 0.043 |
| Upper limit on event number | $3.13 \times 10^5$ | $2.65 \times 10^5$ | $2.19 \times 10^5$ | $0.95 \times 10^5$ | $0.34 \times 10^5$ |
| Upper bound on $\Gamma/\text{GeV}$ | $1.66 \times 10^{-10}$ | $1.96 \times 10^{-10}$ | $2.38 \times 10^{-10}$ | $5.45 \times 10^{-10}$ | $1.51 \times 10^{-9}$ |
| $SU(3)^3$: $\Lambda_S$ (TeV) | 4.72 | 4.53 | 4.31 | 3.51 | 2.72 |
| $SU(3)^3$: $\Lambda_T$ (TeV) | 5.83 | 5.59 | 5.33 | 4.33 | 3.36 |

TABLE I: Upper limit on event number for $e^+ e^- \rightarrow \gamma \gamma \rightarrow Z$ (with current bound $5.2 \times 10^{-5}$ GeV on $\Gamma(Z \rightarrow \gamma \gamma)$, and upper bound on $\Gamma(Z \rightarrow \gamma \gamma)$). In obtaining these bounds the integrated luminosity is assumed to be 500 fb$^{-1}$.

current bound of $5.2 \times 10^{-5}$ GeV. The noncommutative scale can be probed up to a few TeV. The process $\gamma \gamma \rightarrow Z$ at ILC can be a very powerful test for NCSM.

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