A simple model to explain narrow nucleon resonances below the $\pi$ threshold

Thomas Walcher
Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany
(Dated: March 25, 2022)

Several experiments suggest the existence of narrow excited states in the nucleon below the $\pi N$ threshold. Taking all information together a series of states emerges at masses of 940, 966, 985, 1003, $m_4$, 1044, $m_6$, 1094 MeV above the nucleon ground state at 940 MeV with $m_4$ and $m_6$ missing. A model based on the excitation of collective states of the quark condensate is proposed explaining these states as multiple production of a “genuine” Goldstone Boson with a mass of $\approx 20$ MeV. The model also accounts in a natural way for the non-observation of $m_4$ and $m_6$. It suggests an explanation of the mass gap problem of the hadronic spectrum and a confinement mechanism.

I. INTRODUCTION

Recently two experiments reported about narrow exited states of the neutron below the $\pi$ threshold [1, 2, 3]. In the first [1] the reaction $pp \to p\pi^+ N^*$ has been investigated at three energies of $T_p = 1520, 1805$, and 2100 MeV. Three peaks about 5 MeV wide at 1004, 1044, and 1094 MeV were observed in the covered mass range $960 < m_{N^*} < 1170$ MeV with a statistical significance between 17 and 2 standard deviations. In the second experiment [2] also three peaks are found in a partially overlapping mass range in the $pd \to ppX_1$ reaction with $X_1 = N^* \to N \gamma$ at a $p$ energy of 305 MeV. From a measurement of the $pp$ the missing mass of $X_1$ at $966 \pm 2, 985 \pm 2$ and $1003 \pm 2$ MeV were determined with an experimental width of 5 MeV and about 6 standard deviations statistical significance. These peaks were assigned to “super narrow dibaryons” states, but can as well be interpreted as narrow excited nucleons $X^{++}$, states, but can as well be interpreted as narrow excited states $X_1$ of the nucleon. A third evidence comes from a measurement of the $\gamma p \to p\pi$ Compton scattering in the photon-energy range $60 < E_\gamma < 160$ MeV where a peak with an experimental resolution of 5 MeV at $\approx 1048$ MeV with 3.5 standard deviations is observed [3]. In the charge exchange reaction $pp \to pX^{++}$, however, no doubly charged excited nucleons $X^{++}$ could be found [4]. Since such narrow states in the nucleon are very surprising indeed, they meet a considerable scepticism.

In this paper these peaks are taken serious as narrow states of the nucleon and a model for their explanation is proposed. The model starts from the observation that the series of masses taking all experiments together and including the neutron ground state gives 940, 966, 985, 1003, $m_4$, 1044, $m_6$, 1094 MeV with $m_4$ and $m_6$ missing. If one completes the series with $m_4 = 1023$ MeV and $m_6 = 1069$ MeV, then the masses are equidistant within the errors with an average mass difference of $\delta m = 21.2 \pm 2.6$ MeV. This mass difference is close to the best guesses of two times the mass of the light current quarks $m_q = 8 \cdot 14$ MeV [4, 5]. Somewhat against the current thinking we hypothesize the existence of a light pseudo scalar meson ($J^P = 0^-$), i.e. a light “$\pi^+$”, with a mass of $m_{light\pi} = 2m_q = 21$ MeV. The basic idea is now that the series of excited states is due to the nucleon in its ground state plus $1, 2, 3, \ldots$ light $\pi$s as the quantum of excitation with the energy $m_{light\pi}$.

The pseudo scalar character of the light $\pi$ explains naturally the missing of the states #4 and #6. The experiment in ref. [1] detected the $p$ and $\pi^+$ in the $pp \to p\pi^+ N^*$ reaction in one spectrometer with a very small relative angle at the same time. Since the cross section peaks very strongly in forward direction with respect to the beam ($\theta \approx 5^\circ$), this means that the outgoing $p$ and $\pi$ carry no angular momentum and only odd parity states in the $N^*$ can be excited. On the other hand, in the $pd \to ppX_1$ reaction the proton on the left hand side was measured at $\theta_L = 70^\circ$ and the proton on the right hand side at angles of $\theta_R = 34^0, 36^0$, and $38^0$. Therefore, odd and even partial waves of the outgoing particles are possible and allow the population of odd and even parity states in $N^*$. The presence of the second nucleon in the $pd \to ppX_1$ reaction may provide the effect of the $\pi$ present in the first reaction. The peak at $m_{N^*} \approx 1048$ MeV in the $\gamma p \to p\pi$ reaction is a $J^P = 1/2^-$ state and due to an $E1$ transition dominating at low energies $E_\gamma$. All excited nucleons suggested in these experiments have positive or no charge $N^{*0,\pm}$ and can decay electromagnetically. On the other hand, the $N^{*++} = X^{++}$ in the experiment of ref. [5] could decay only weakly and had a survival time $\tau_{N^{*++}} \approx 10^{-3}$s. This feature was used to search for the $N^{*++}$ directly in a magnetic spectrometer, but as already mentioned with negative result. As will be discussed in section II the light $\pi$ couples only very weakly and has a small overlap with the usual $\pi$ making it unlikely to excite the $N^*$ states in the $pp$ charge exchange reaction.

However, it is well known that a light $\pi$ has never been observed as free particle and one has to give more justification for this proposal and reconcile it with the known physics. One fact of this known physics is the success of the constituent quark model (see e.g. ref. [4] and references therein). This model uses “constituent” quarks with the naive estimate of the quark mass $m_c \approx 350$ MeV, i.e. one third of the nucleon mass. Therefore, the $\pi$ had to have as any other meson at least a mass of $\gtrsim 700$ MeV. However, the $\pi$ observed in nature has a mass of 138 MeV. The model described in the following assumes that the $\pi$ observed in nature is not the Goldstone Boson, it is normally identified with, but a mixture of the “genuine” Goldstone Boson $\pi_G$, i.e. the light $\pi$ suggested above consisting of light current quarks, with a “constituent” $\pi_c$ made of constituent quarks with a mass of $m_{\pi_c} = 2m_c = 700$ MeV. It will turn out that the mixture of these states will reproduce the observed $\pi$ mass and explain in a natural way the so called “gap problem”, i.e. the large mass gap between the mass of the hadronic scale of $m_p \approx 2m_c$ and
the mass of the observed \( \pi \) (see e.g. [7]).

II. MODEL

The model follows the formulation of the schematic model of G.E. Brown[8] for the description of collective states in nuclei as particle-hole excitation and this paper adopts closely his notation. The Goldstone Boson is denoted by \( \pi_G \) with \( |q_{curr.},q_{curr.}\rangle = |m\rangle \) and the constituent pion by \( \pi_c \) with \( |q_{const.},q_{const.}\rangle = |n\rangle \). They represent particle-hole excitations of the QCD vacuum where \( m, n \) label the particle and \( i, j \) the hole states. The possible propagation of the particle-hole states are depicted in fig.1. The model assumes the suppression of the closed Fermion loop coupling to the particle-hole state inside the confinement cavity. This assumption will be discussed in section III, item 1). The particle-hole interaction mixes the particle-hole states as already mentioned. In order to find the energy eigenvalues \( E \) of the mixed states one has to solve the secular equation:

\[
\sum_{m,i} (m|H|n) c_{nj} = E c_{mi}
\]

For the determination of \( E \) one observes that only the left hand diagram in fig.1 contributes essentially. The right hand diagram is an exchange diagram propagating the energy equivalent to the mass difference of the constituent quark to the current quark. Since this difference is of the order 700 MeV the exchange matrix element will be small compared to the particle-hole creation and annihilation matrix element which does not propagate energy or momentum. It is now assumed that the \( S \) matrix represented by the particle-hole propagation is unitary. Consequently, the creation-annihilation matrix element factorizes according to:

\[
\langle mj|V|in\rangle = \lambda \langle mj|\tilde{V}|0\rangle \langle 0|\tilde{V}|in\rangle
\]

where \( \tilde{V} \) represents the quark transition at the vertices in fig.1. This factorization is familiar from the more complete RPA description of collective particle-hole excitations in nuclei. The factor \( \lambda \) takes care of the slight violation of unitarity due to the neglect of the exchange diagram. The absolute squares of the matrix elements \( |\langle mj|\tilde{V}|0\rangle|^2 = m_{mj} \) and \( |\langle 0|\tilde{V}|in\rangle|^2 = m_{ni} \) are just the masses of the particle-hole states.

Putting all equations together one arrives at the secular equation:

\[
1 = \sum_{m,i} \frac{\lambda m_{mi}}{E - (\epsilon_m - \epsilon_i)} = \sum_{m,i} \frac{\lambda m_{mi}}{E - m_{ij}}
\]

For the two possible particle-hole states mixing here the secular equation takes finally the simple form:

\[
1 = \frac{\lambda m_G}{E - m_G} + \frac{\lambda m_c}{E - m_c}
\]

The solution of this equation is most easily visualized by a plot of the right hand side and the left hand side as a function of \( E \). This plot is shown in fig.2. The physical solution appears at \( E = m_\pi = 138 \text{ MeV} \) with the fitted \( \lambda = -0.94 \) if one chooses \( m_G = 21 \text{ MeV} \) and \( m_c = 700 \text{ MeV} \) as discussed above. Since \( \lambda \approx -1 \) this solution shows that the creation-annihilation diagram dominates indeed and that the particle-hole interaction is attractive as required.

FIG. 1: Graphical representation of particle-hole states.

FIG. 2: Graphical solution of the secular equation [8].
III. DISCUSSION

It is evident that the proposed identification of the Goldstone Boson having $J^P = 0^-$ explains the combined experimental results for the narrow nucleon states presented above. If the available energy is below the $\pi$-nucleon threshold one can just see the excitation of a series of “genuine” Goldstone Bosons in the nucleon spectrum. In other words the energy available below the $\pi$ threshold and the binding of $\pi_G$ to the nucleon cause a de-mixing of the Goldstone $\pi_G$ and the constituent $\pi_c$. Narrow mass steps could of course not be seen in the meson spectra since already the ground state of the mesons decay in normal $\pi$s and, therefore, the Goldstone $\pi_G$ mixes with the constituent $\pi_c$ through the energy available in the decay.

It is interesting to consider also the narrow dibaryon states claimed so far since they could have been naturally explained as narrow bound states composed of $NN^*$ or $N^*N^*$. A compilation of the references of these states and a mass spectrum derived from them is given in ref. [9]. It is striking that this spectrum of narrow dibaryon states shows a rather equidistant level spacing. Fig. 3 depicts this spectrum after ref. [9] together with a straight line fit to the masses. The level spacing is very close to two times the mass of ref. [9] had no bias to a hypothesis of equidistant dibaryon interval considered. It is important to note that the authors of ref. [9] had no bias to a hypothesis of equidistant dibaryon masses. The level spacing is very close to two times the mass

![Graph](image-url)

FIG. 3: The experimental dibaryon masses according to ref. [9] against the number of excitation quanta $n$.

of the assumed light Goldstone Boson though no attempt has been made to correct for the effect of the binding energy of the dibaryons. As already argued the proposed model implies that in most experiments only the natural parity states are excited so that the unnatural parity states are missing and the step width is indeed two times the mass of the excitation quantum, i.e. the light Goldstone Boson mass.

The model implies some noteworthy further conclusions:

1) The free $\pi$ observed in nature is not the true Goldstone Boson, but a mixed state of the “genuine” Goldstone Boson composed of current quarks with a mass $m_q \approx 10 \, MeV$ and the constituent $\pi_c$ composed of constituent quarks with a mass of $m_c \approx 350 \, MeV$. This idea represents a modification of the usual dynamical generation of the $\pi$ mass through coupling of the particle-hole to the quark condensate $\langle 0 | \sigma(0) | 0 \rangle$ in the Nambu-Jona-Lasinio (NJL) model (see e.g. ref. [10]). The NJL model does not know confinement and a coupling to the closed Fermion loop, i.e. to the quark condensate, is always possible. In order to explain the narrow $N^*$ states, however, one had to assume that the coupling of the particle-hole state to the closed Fermion loop disappears inside the confinement cavity. This meant that the quark condensate is “expelled” out of the nucleon leaving a cavity with a real vacuum inside as suggested by the “color dielectric model” (see e.g. ref. [11]).

More generally, the quark condensate as the leading order parameter of the spontaneously broken symmetry is a theoretical concept and is not yet experimentally established [12].

2) The Goldstone Boson does not interact at all in the chiral limit $m_q \to 0$ (see e.g. ref. [12]) and it is reasonable to assume that the “genuine” Goldstone Boson $\pi_G$ interacts only very weakly. This assumption explains why the narrow states have not been observed in less sensitive Compton scattering experiments than that of ref. [9] as already remarked in ref. [13]. Due to this very weak coupling $\pi_G$ will have very little influence in dispersion relations, as e.g. for the form factors, or on polarizabilities of the $\pi$ and nucleon.

The chiral loops in the framework of the Chiral Perturbation Theory (ChPTh) (see e.g. ref. [13]) have, however, to be reconsidered. Since the $\pi$ mass is an effective parameter in ChPTh and only requested to be small compared to the hadronic scale $m_\rho \approx 700 \, MeV$ it appears possible that a consistent picture also with $\pi_G$ emerges. In this modification of ChPTh the coupling constants and low energy constants had to be changed. It could also be that the $\pi$ mass in ChPTh is a more delicate parameter than realized so far. For observables with external $\pi$s one had to take the asymptotic mass $m_\pi = 138 \, MeV$ with strong coupling. In the chiral loops one either has to modify the low energy constants for the “genuine” Goldstone $\pi$s with $m_G \approx 20 \, MeV$ or had to take the effective $\pi$s with $m_\pi$ produced by a self consistent condensate of $\pi_G$.

3) The binding energy of a system of two current quarks $q\bar{q}$ by the familiar one gluon exchange potential $V(r) = (4/3) \alpha_s(Q^2)/r$ to $\pi_G$ is $\epsilon_B = -2 \, MeV$ if one uses the saturation value $\alpha_s = 0.5$ for $Q \leq 2 \, GeV/c$ suggested by the analysis of jet shapes [14]. This value of $\alpha_s$ is low in comparison to the running coupling value and means that beside $\Lambda_{QCD}$ a second scale is introduced. Consequently, the $\pi_G$ would be a relatively large object with a size estimated by the Compton wave length $\lambda_{Compton} = \hbar/21 \, MeV/c \approx 10 \, fm$. This means that the correlation length of chiral symmetry would be much larger than the best estimate of the confinement radius of $R_{conf.} \approx 1 \, fm$ [14]. Therefore, the overlap of the $N^* = (n \cdot \pi_G)N$ states with the ground state of the nucleon is small and together with the very weak $\pi_G$ coupling gives a further reason why it is so difficult to see the $\pi_G$ in
experiments.

4) The mechanism proposed in this paper provides for a natural explanation of the “gap problem”. It is, however, unclear how this relates to deeper considerations of this problem.

5) The existence of a light Goldstone Boson would also provide a mechanism for the quark confinement. A free non-conflined quark would be naturally a constituent quark, i.e. one dressed with gluons and a mass of $m_c \approx 350 \text{ MeV} + m_q$. The binding energy $\epsilon_B$ of a system of $q, \bar{q}$, by the mentioned one gluon exchange potential is $\epsilon_B = -32 \text{ MeV}$ if one uses the saturated value $\alpha_s = 0.5$ of ref. [3]. This is certainly a lower bound of $|\epsilon_B|$. If $|\epsilon_B|$ is larger than the mass $m_G$ of the “genuine” Goldstone Boson a quark could never escape since before it did, as many Goldstone Bosons as energy is available would be produced, effectively taking away the energy of the quark above the ground state. Since the Goldstone Bosons interact only very weakly with the colored constituents the influence on the quark binding potential would be weak. Before, however, such a picture can be established self consistent calculations including all effective degrees of freedom, i.e. the constituent quarks, the Goldstone Bosons and the gluons had to be performed.

6) The lattice gauge theory allows for such a self consistent calculation, but it may miss the physics of the effective degree of freedom represented by the Goldstone Boson because its correlation length is, as argued, 10 $\text{fm}$ and all lattices used are much smaller. In view of the computers considered to be already to small for the lattices used so far, it is a good idea to add to the QCD on the lattice the Goldstone Boson as an effective degree of freedom from the outside. Work in this direction is in progress.

7) The model accounts in a trivial way for the “non phase transition” behavior in the mass spectrum of mesons [18], i.e. the smooth decrease of the mass splitting as a function of the current quark mass. Due to the large masses of heavy current quarks a meson made of heavy quarks is not a Goldstone Boson in the strict sense. But, the mixing of the current-quark meson with the constituent-quark meson in the framework of this model reproduces this aspect in a natural way.

8) The proposed light Goldstone $\pi$ represents an axial current. The excited narrow nucleon states below the $\pi$ threshold will, therefore, produce a parity violation in the $p(\vec{e}, e')N^*$ reaction. Since the $N^*$ states are not resolved experimentally from the ground state in the ongoing parity violation experiments, one has to be careful about the interpretation of their results and the conclusion about the strange quark content in the nucleon (for a review see e.g. [19]).

The author is indebted to Reinhard Beck, Dieter Drechsel, Jörg Friedrich, Stefan Scherer, Marc Vanderhaegen and Wolfram Weise for helpful discussions. This work has been supported by SFB 443 of the Deutsche Forschungsgemeinschaft (DFG) and the Federal State of Rhineland-Palatinate.

[1] B. Tatischeff, et al., Phys. Rev. Lett. 79, (1997) 601-604
[2] L.V. Fil'kov, et al., hep-ex/0006029v1 (23 Jun 2000), and to be published in EPJ A
[3] R. Beck, et al., nucl-th/0104070v1 (24 Apr 2001)
[4] R. Beck, et al., private communication and to be published
[5] S. Ram, et al., Phys. Rev. D49, (1994) 3120-3125
[6] J.F. Donoghue, E. Golowich, and B.R. Holstein, Dynamics of the Standard Model, Cambridge University Press 1992, pp. 327
[7] A.W. Thomas and W. Weise, The Structure of the Nucleon, Wiley-VCH, Berlin 2001
[8] G.E. Brown, Unified Theory of Nuclear Models and Forces, North Holland Publishing Company, Amsterdam 1967
[9] B. Tatischeff, et al., Phys. Rev. C59, (1999) 1878-1889
[10] U. Vogl and W. Weise, Part. Nucl. Physics 27, (1991) 195-272
[11] G. Colangelo, J. Gasser, and H. Leutwyler, Nucl. Phys. B603, (2001) 125-179
[12] A.I. L’vov and R.L. Workman, Phys. Rev. Lett. 81, (1998) 1346
[13] Y. Dokshitzer, hep-ph/0106348v1 (29 Jun 2001) and Phil. Trans. Roy. Soc. Lond. A359, (2001) 309
[14] B. Povh and Th. Walcher, Comments Nucl. Part. Phys. 16.2, (1986) 85-93
[15] K.-I. Kondo, Phys. Lett. B514, (2001) 335-345
[16] K.-I. Kondo, hep-ph/0110013 (30 Sep 2001)
[17] D.B. Leinweber and T.W. Thomas, Nucl. Phys. A684, (2001) 35-43
[18] N. Isgur, Summary talk at the Chiral Dynamics 2000 Workshop, New Port News, VA, 2000. Proceedings Ed. A. Bernstein, J. Goity, and U.-G. Meissner, World Scientific, Singapore, 2001
[19] D.H. Beck and R.D. McKeown, hep-ph/0102334 (27 Feb 2001), Ann. Rev. Nucl. Part. Sci. 51, (2001)