Views, variety and celestial spheres

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Abstract

This paper describes a continuation of the program of causal views, in which the world consists of nothing but a vast number of partial views of its past. Each view is associated to an event, and is a representation of the immediate causal past of that event. These consists mainly of processes that transfer energy, momentum and other charges to it from its past events.

There is fundamentally no space or spacetime, just a large number of events, which are the causes of events to come. This is a development of energetic causal set theories, developed with Marina Cortes.

Momentum and energy are fundamental, and are conserved under their transformation from present events to future events. As a result Minkowski spacetime emerges, in a way that preserves causal relations. The locality of events as constructed in the emergent spacetime is a consequence of the conservation of energy-momentum fundamentally.

In this paper I propose that the views of events can be represented in terms of degrees of freedom on punctured two surfaces-each puncture corresponding to an immediate past event. This makes possible versions of the theory that are relativistically invariant.
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1 **Introduction**

The first question that physics asks is what the world is ultimately made of. A measure of the failure of positivism is that, after more than a century of dominance, what we really want to know is still the ontology of the world.

Newton gave us an ontology of point particles moving in an absolute space, subject to mutual forces at a distance. While Newton knew better, the sheer usefulness of Newtonian dynamics made it hard to disagree, only a few such as Descarte and Leibniz succeeded in formulating a viable alternative conceptually - based on the ideas of relationalism. But even armed with a more consistent philosophy, none was able to compete with Newton as a paradigm for exact description and prediction.

But beginning with Faraday and Maxwell, a new ontology did arise that was able to address the questions raised by Newton’s action at a distance: the ontology of fields. And since the
turn of the 20th century fields - both quantum and classical - have dominated our ontological thinking as well as our practices as physicists.

The picture of a universe of fields has inspired many discoveries, but its claims to give a fundamental ontology with a consistent foundation remains aspirational. A few quantum field theories are known to exist rigorously - and most of these live in 1 + 1 dimensions. The few exceptions are highly constrained by supersymmetry. None of the quantum field theories that make up the standard model are rigorous, in fact our understanding of the physics of quantum electromagnetic fields and nuclei are based on expansions with zero radius of curvature. Lattice gauge theory appears to be in more hopeful state until we remember that fermions double - making it impossible to describe those field theories that describe nature.

There is then a clear need to begin to explore the possibility of a new ontology for physics. This paper reports on a modest attempt to provide an alternative ontology that may be sufficiently different, and sufficiently rich, conceptually that it may have a chance to underlie and complete quantum mechanics, quantum field theory and general relativity.

Called the causal theory of views, this begins somewhat like the causal set theory in that we declare that what is real includes events and causal processes, by which they contribute to the catalyization of new events\(^1\). But our events are both sparser and more ephemeral. They do not exist in any familiar sense, (nothing does, if this new proposal is correct,) rather we perceive our events to be moments of transformation. What gets continually processed and transformed, is momentum and energy. So these have no stable existence-apart from the very important fact that they are conserved.

An event, while it lasts, is shaped by those impulses from its causal past-and this gives it a view of its near causal past. Each view is brief and incomplete. But the sum of them are the universe.

This is an ontology in which the universe continually recreates itself-as all it is is the collection of partial views of the causal pasts of the current events.

This paper reports progress in the construction of a theory that is meant to be an ultraviolet completion of general relativity and quantum theory, called the causal theory of views\(^1\, 2\). Having shown in previous papers how relativistic point particle dynamics (and also, of course, space) and non-relativistic quantum mechanics emerge, in different limits\(^1\, 2\), I present here a relativistic theory of causal views. By this I mean one whose action and equations of motion are invariant under lorentz transformations on momentum spaces\(^2\). The key to this is a relativistically invariant expression for the dynamics, based on an invariant measure of the difference between the views of two events (eqs 49, 17 and 42, respectively, below).

The main idea is that we live in a quantum universe made up of nothing but a vast number of partial views of its past. Each view is associated with an event, and is a representation of

\(^1\)Related models were proposed in [48, 47].

\(^2\)Of course, because there is no spacetime. Individual solutions including those that could be candidates to describe our own universe, break those and indeed all symmetries.
the immediate causal past of that event.

In this theory, space is not fundamental; nor is spacetime, although there is a constructive, Bergsonian notion of time, related to the idea of becoming, or the now, as a consequence of the continual creation of events. Energy and momentum and other conserved charges are, however, present in the fundamental picture, and each event has so much of each. These are transferred to that event’s antecedents in a way that is consistent with their conservation. These transfers are primarily what an event ”can know” about its causal past, and are coded in the views.

The collection of such views are the be-ables of the theory. There is a phase of the theory in which non-relativistic quantum theory is derived. This is the basis for the claim that these theories are non-local hidden variables theories[1, 2, 6, 7].

If space is not present initially, exactly how does it emerge? We make use of a mechanism for the emergence of space first discovered (as far as I know) in the relative locality and DSR papers[49, 50, 6, 7] which are also theories formulated in momentum space. The idea was then developed in the energetic causal set work.

The key point is simply that conservation of energy-momentum implies locality in a spacetime of the same dimension. We can draw a diagram of every interaction, which shows that processes embedding in a little shard of spacetime. The question is how to get the little shards to line up consistently to define an emergent spacetime. Once put that way we can find the necessary conditions, and construct classes of theories that satisfy them.

At the fundamental level, in the absence of space, how does nature decide which interactions are stronger and which are weaker? In [1, 2, 6, 7], a simple answer was proposed: measures of similarity of differences of views provides a suitable replacement for distant or nearby in space. So if the most important structure in a field theory is the derivative, in our theory the most basic operation is the comparison of views.

Roughly speaking, the more similar two views are, the more likely they are to interact. And the more similar they become, the more repulsive the force[6, 7, 49, 50].

To summarize, we propose that each event in the history of the universe is nothing but the information available there of its near term causal past. The more those views differ, the more diverse the world is. The universe then is governed by a law which aims to maximize the diversity of its views[2, 29].

This picture is also suggested by the basic twistor duality[20], according to which an event in spacetime is dual to the set of light rays that converge there[4].

We also may wonder if there is a relationship between these twistor inspired dualities and the celestial sphere[25], recently discussed as a possible structure relevant for quantum grav-

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3Despite various rumours to the contrary, there are not a few non-local hidden variables theories that reproduce predictions of quantum mechanics[8]-[17].

4However, to incorporate a causal structure we have to break some of the symmetries of twistor space. We hope to return to this in a future publication.
ity. We show that the new theory proposed here is a kind of inversion of the structure in asymptotically flat spacetimes, in which we abolish the external boundary, along with its one fictional global outside observer, and rediscover the inversion of these structures as the microstructure of each event in a quantum universe.

The one line summary of the message of this paper is that the views of celestial spheres, turned inside out by a large conformal transformation\(^5\), from an event, are the right degrees of freedom to describe closed quantum universes. Th right dynamics (ie the one that recovers non-relativistic quantum mechanics in an appropriate limit), is to extremize the variety or diversity of the views[29].

This is a very old idea, it can be found, for example, in Leibiz’s revolutionary The Monadology[19].

Newton, who was Liebniz’s contemporary, had what would be regarded today as a more conventional view: he imagined that the universe was made of mass points flying around at the command of local equations of motion[?]. There was one Observer who lived outside the universe and observed it without being observed.

That archaic picture of a universe that can be seen as a whole by an all powerful asymptotic observer, is clearly holding back progress.

The universe is not something that exists that we monitor. To be an observer is, I propose, the same thing as being part of the universe. To observe the universe you must be a participant in it.

By a view, I mean the information about the causal past of an event, which is coded in degrees of freedom at the event. But if an event has the puny structure of a ”structureless point,” this is no more than the values of a few fields, evaluated at that point. There is not much one can do with that. I find that Leibniz’s idea starts to get interesting when we blow each event up to a two-sphere, so that it has directional information built into it.

When I use the word “information” here I do not mean Shannon[51] or von Neumann [52] information. I mean Gregory Batesman information[53], defined as ”the difference that makes a difference.\(^6\)”

Here is the basic idea of Bateson: We consider making a minimal and local change in the value of some physical observable, \(x\) at a time, \(t_0\) from \(x_1\) to \(x_2\). A long time, \(T\) later, we compare the two evolutions, the one starting from \(x_2\) to that from \(x_1\).

In many cases the late time states are minimally different. It may be impossible to tell the two apart. This is typical for systems in thermal equilibrium. In this case no information has been conveyed. But there are other cases in which the two states are macroscopically distinct. Think of a light switch. We say that initial value of \(x\) carried information. The change from \(x_1\) to \(x_2\) was a difference that made a difference.\(^7\).

\(^5\)[24]
\(^6\)Gregory Bateson was an anthropologist and psychiatrist who also invented the concept of the double bind.
\(^7\)In statistical mechanics we speak of damage being done.
So what I mean by a quantum universe is a continually becoming causally related sequence of views of the causal pasts of events, where the view of event, which is equivalent to the name of the event, is a snap shot of the information arriving from the causal past to the two sphere of directions.

Furthermore the information coded on the $S^2$ about its causal past could be “classical”, such as the direction a certain pulse of energy-momentum is arriving from the past, or a more “quantum” description, such as a Bell state expressing entanglement of different regions of the past. W explore both in the third chapter below.

Interpretational issues, concerned with the nature of time, the structure of a theory of a closed system and quantum foundations, are discussed in [3, 4, 5]. As in other cases, the theory does not dictate its interpretation. In many instances views on foundational issues are not strongly constrained by the formalism, so you can describe these models using the concepts of which ever interpretation you prefer.

We proceed to review the main idea of the $CTV$.

## 2 Summary of the causal theory of views[1, 2]

It is important to emphasize that the Causal Theory of Views is first of all a new proposal for the ontology of the physical world.

- The universe is a process of continual becomings and transformations, made up of events and causal processes.
- There is fundamentally no space, no spacetime.
- Causal processes carry impulses of energy, momentum and other charges. This requires taking energy and momentum as primitive, operational concepts, that do not need space to define them. Indeed, they have perfectly good operational definitions in terms of calorimeters, photomultiplier tubes etc. We conjecture an inverse theorem which constructs an emergent dimensionality of space for each conserved quantity.
- In all processes involving transfer of energy-momentum it is conserved. Thus,

$$p^+_a = \sum_{M \in Past(J)} p^+_M$$

$$p^-_a = \sum_{N \in Fut(J)} p^-_N$$

(1)
• Events do not exist, they happen. They are initiated by the combination of two or more causal processes. Then they do one thing, which is to reshuffle the quanta of energy-momentum they receive and send them forth in new causal processes that will initiate a future event.

• These transformations and processes conserve energy, momentum and other charges.

• There is just one universe, it happens just once.

2.1 A view may be expressed as a quantum state on a celestial sphere

We define a four dimensional relativistic energy-momentum space, $\mathcal{M}$

$$p_a = (p_i, p_0 = \epsilon) \in \mathcal{M}, \quad i = 1, 2, 3$$

$\mathcal{M}$ is endowed with a lorentzian metric and connection, which may or may not be related. The simplest characterization of a view is merely a set of labeled points on a unit punctured $S^2$. This can be coded into an unordered list of punctures, each of which is labeled by the angles from which it approached, $(\bar{w}, w)$ together with the energy $\epsilon$. Each view is, in this version,

$$\mathcal{V}_I = \{(\bar{w}, w, \epsilon)\}.$$ (3)

If $q_a$ is a null one-form representing a photon’s state, it corresponds to a point on the celestial sphere

$$q_a = (1 + \bar{w}w, w + \bar{w}, -\nu(w - \bar{w}), 1 - \bar{w}w)$$ (4)

Under a lorentz transformation, $q_a$ transforms as a vector density

$$q_a = \frac{1}{cw + d} \Lambda_a^b q_b$$ (5)

$w$ and $\bar{w}$ transform as

$$w \rightarrow \frac{aw + b}{cw + d}, \quad \bar{w} \rightarrow \frac{\bar{a}\bar{w} + \bar{b}}{\bar{c}\bar{w} + \bar{d}}$$ (6)

with

$$ad - bc = 1$$ (7)

incoming $\epsilon > 0$ and outgoing $\epsilon < 0$.

We posit that a variable number of impulses $p^E_a$ interact in the formation of an event, $E$; and this requires that the energy-momenta carried by these impulses that form the event, (which are labeled $p^E_a$) combine to the single injection of energy and momentum, which endows the event.
Thus, the view of an event $E$ is taken from its causal past set, which must include at least the elementary processes, or impulsives, whose combination initiated the event.

$$\mathcal{V}_E = \{p_a \in \text{past}(E)\}$$ (8)

This is governed by a map:

$$\mathcal{M} \times \mathcal{M} \rightarrow \mathcal{M} : \phi_1 \otimes \phi_2 \rightarrow \mu$$ (9)

One can also imagine an event making a probe of the future by sending out a number of photons, such that the total energy is preserved. So the whole action of an event can be understood as a reorganization of the energy-momenta incoming from the past to a new way of dividing the total incoming energy.

$$\mathcal{M} \otimes \mathcal{M} \rightarrow \mathcal{M} \rightarrow \mathcal{M} \times \mathcal{M} : \phi_1 \otimes \phi_2 \rightarrow \mu$$ (10)

### 3 Relativistic measures of differences of views

There are different ways we can express the view of an event; each leads to a different definition of variety, and hence potential energy.

They share a common strategy, which is to use the technique of best matching[61, 62] to define differences between a pair of views. So next we have a review.

#### 3.1 Best matching and local gauge invariance

Our relational philosophy tells us that observables that depend on arbitrary choice of coordinates cannot be physical. One way to accomplish this is through the method of best matching, developed by Barbour and Bertotti[61, 62].

We want to compare the past causal views of a pair of events, which we will now call $E$ and $F$, without reference to any frame external to or common to them.

We want to treat as identical views that differ from each other by elements of a symmetry group $H$. That is, if $\sigma \in H$ does exist such that there are two views $E$ and $F$ such that,

$$F = \sigma \circ E$$ (11)

we consider the two views as identical, as there is nothing in their common internal relations that will distinguish them.

We start with a measure of difference, $d(E, F)$ between two views. Then we consider how this measure of difference is affected by the relative orientations, positions and motions, of the two.
Thus what we really want is to vary $\sigma$ to find the group action on $E$ that minimizes its distinctiveness from $F$. We call this a best matching configuration.

$$D^{rel}(E, F) = D(E, \sigma^* \circ F)d_{\min}$$

where $\sigma$ represents an action of the group $H$ on the space of possible views. and $\sigma^*$ is the element of $H$ that minimizes the difference between the two views.

There are a number of different strategies for doing this comparison. Each gives us a definition of difference on the set of punctures on the $S^2$, which is to say, in the space of views.

### 3.2 The view as an unordered set of incoming energy-momentum

The simplest view of an event $E$ is an unordered set of $n_E$ incoming energy momentum vectors,

$$v_E = \{q^{E\alpha}_a \ldots \}$$

Different events receive different numbers of pulses, $n_E$.

We next find a measure of the difference among pairs of these that is invariant under Lorentz transformations and permutations.

Let’s start by comparing two light-like energy-momentum vectors. We use the Minkowski metric on momentum space:

$$D[q^a, p_b] = -\frac{1}{2} (q^a - p_a)^2 = q_ap_bg^{ab}$$

This is Lorentz invariant, non-negative when they are both null incoming or both null outgoing. It vanishes when $p_a = q^a$, so it gives a meaning to how different the two null vectors are.

However this comparison makes use of a common reference frame. That is, when we use the form (14), we are implicitly thinking of comparing two views, made by a single observer. That is why we apply the same Poincare transformation to both of them.

Suppose we are in touch with a large number of observers scattered in spacetime. They each from time to time take a snapshot of their past view, and send them to us. But an evil child has gotten into the collection and stripped them all of their labels, so what is left is an unlabeled and randomly ordered pile of snap shots. Our job is to reconstruct the partial order of the views, so as to have the most likely history of our universe. unordered collection of snap shots taken.

Best matching is a technique for doing just this. It gives us a Poincare invariant way to give the snap shots a partial order in terms of differences and similarities of these views.
To do this, we need the freedom to Poincare transform either one, and take the minimum of the resulting differences

$$D[q_a|p_b]_{b.m.} = ((\sigma \cdot q_a)p_b g^{ab}|_{\text{minimum value over group action } \sigma}$$  \hspace{1cm} (15)

For example, we can use $\sigma$ to bring the two momenta onto alignment, so the best matched value is $q^0 p^0$ if they are massive, 0 if they are massless.

Now let us have $n$ incoming energy-momenta in each view. We label arbitrarily the energy-momentum vectors belong to the same view by indices $\alpha$ and $\beta$. The best matched value of the distinction is

$$D[[q^\alpha_a]||p^\beta_b]_{b.m.} = \sum_{\pi(\alpha,\beta...)} |(\sigma \cdot q_\alpha^a),p_\beta^b| g^{ab} \cdots |_{\text{best match}}$$  \hspace{1cm} (16)

We evaluate the difference measure over all possible permutations of one set, and, for each permutation, we scan the group on that side, looking for a set of pairings and orientations that lead to the greatest matching. We then define the value of the best matched [61, 62] to be the minimal difference between the two views, compared over all Poincare transformations and all permutations.

In the case that the number of snapshots or incoming impulses in each view are not equal, we consider all ways to distribute the smaller set amongst the larger.

$$D(E,F) = D[[q^\alpha_a]||p^\beta_b]_{b.m.} = \sum_{\pi(\alpha,\beta...)} |(\sigma \cdot q_\alpha^a),p_\beta^b| g^{ab} \cdots |_{bm2}$$  \hspace{1cm} (17)

3.3 The view as a state in a boundary Chern-Simons theory.

When we go from the classical theory to the quantum theory the values of the degrees of freedom are promoted to functionals of possible values. So, in the "classical" theory we have just described, the incoming causal processes arrives from the past at an event, $E$, from a direction specified by a point $(\bar{w},w)$ on the $S^2$. In the previous picture this corresponds to a null vector in Minkowski space, and the full view from $E$ is the unordered collection of these null vectors.

The best matching classes of the collections of null vectors give as we ust showed a language for views suitable for constructing relativistic equivalence classes of views.

If we take the non-relativistic limit that set of vectors reduces to a set of timelike vectors- and these were the basis for demonstrating the emergence of non-relativistic quantum mechanics in [1, 2].
Next we construct a way to recognize the views that allows us to keep more topological information. The basic idea is that the set of null directions \((\bar{w}, w, \epsilon)\) now become punctures, which we then use to parameterize a set of Hilbert spaces on the punctured two spheres.

Note that I will call the first correspondence ”classical” and the second ”quantum”, but they are both elements of the construction of a classical theory.

The second class of states assign to each state a wavefunction \(\psi(z)\) on the punctured \(S^2\). From a non-relativistic perspective, we could expand these functions in terms of representations of \(SO(3)\). So this would seem to bring us back to the \(SU(2)\) spin networks of \(LQG\)[38].

\[
(\bar{w}, w)\text{on the } S^2 \to (j, m)
\] (18)

But we can do more. With the understanding that the conformal two sphere carries a representation of the \(3 + 1\) Lorentz group, whose action is isomorphic to the 2 dimensional conformal group, we may expand the wavefunctions in terms of \(SL(2, C)\) representation labelled spin networks. There are both compact and non-compact representations.

But we shouldn’t stop here. To encode the magnitude of the energy-momentum impulses we need to represent the energy \(\epsilon\) in terms of a further extension, to the Poincare group representation theory.

We want to add the translations so the \(\epsilon\)’s now indicate how the states are elements of a representation of translations. So we have gone from

\[
(\bar{w}, w, \epsilon)\text{on the } S^2 \to (j, m, \epsilon)
\] (19)

which are representation labels for the \(3 + 1\) dimensional Poincare group.

So we may have a spin foam model constructed from the representation theory of \(Poincare(1, 3)\), as described in [63].

The events are now intertwinors of \(Poincare(1, 3)\), with a specified number of incoming and outgoing zero mass states.

We each have a view, or views, one for each at each moment. This is precisely information about our recent causal past, projected onto a sphere in momentum space. Our view, is in fact a two sphere on which is projected photons and other quanta coming from our causal past. So it is very natural to formulate a theory of views, as we have sketched it here, as a theory in which each event is blown up to a sphere.

### 3.4 The ladder of dimensions

Indeed, this idea sits right in the centre of some of the most important ideas about quantum spacetime geometry to have been studied these last few decades.
I am referring to that very influential concept in quantum gravity and combinatorial topology which is called the ladder of dimensions. The basic idea, enunciated by Crane[s][31], is as follows.

The top dimension - in our case 4 - is a topological quantum field theory on a four-manifold $\mathcal{M}$. This will be a $BF$ theory for some Lie group (or quantum group), $G$.

We are interested in the case where $\mathcal{M} = \Sigma \times R$

There are no non-dynamical fields on $\mathcal{M}$-because the partition function of the $BF$ theory can be shown to be independent of the choice of triangulation used to regulate it. So there are very few bulk physical observables.

Down one dimension, however, there are induced physical degrees of freedom living on the boundary,

$$\partial(\Sigma \times R) = (\partial \Sigma) \times R + \Sigma_+ - \Sigma_-$$  \hspace{1cm} (20)

These are described by a $G$ Chern-Simons theory on the three dimensional boundaries[34].

The two manifold $\Sigma$ may have boundaries as well. Or we may introduce two dimensional boundaries by choosing a surface $B$ which splits $\Sigma$ into two. The axioms of Atiyah and Segal[72] for topological quantum field theory posit that for each embedding of a two dimensional surface $B$ into a compact three manifold, $\Sigma$, that splits that three manifold into two halves, $\Sigma_+$ and $\Sigma_-$ there is a Hilbert space, $H_B$. For every topological three manifold $\Gamma^+$ that has $B$ as its boundary, ie $B = \partial \Sigma^+$, there is a state $\Psi \in H_B$. Let us assume that $B$ is oriented so it has two sides, $B^+$ and $B^-$. There is a conjugate map, $\dagger$

$$\dagger : B^+ \rightarrow B^-$$  \hspace{1cm} (21)

We can use $\dagger$ to construct an inner product, given a state $|\psi\rangle \in H_B^+$, and a dual state, $<\phi| \in H_B^-$, the inner product is naturally defined

$$<\psi|\phi> \in C$$  \hspace{1cm} (22)

This is then an invariant of the topology of $\Sigma$ because it is invariant under the choice of the splitting surface, $B$.

Let us call this structure the ladder theory. The ladder theory is not quantum gravity-nor is it a model of quantum gravity in four spacetime dimensions. All of its degrees of freedom and physical observables live on the three dimensional boundary. There is what has been called a boundary observables algebra[34].

Very remarkably quantum gravity in three dimensions is a TQFT-as was first discovered by Ponzonno and Regge[74].

Even more remarkably, there are several ways that the ladder theory may be disordered or constrained to yield a quantization of general relativity in four spacetime dimensions with three dimensional boundaries.
3.5 Quantum gravity and spin networks

One way to do this is to disorder the partition function by embedding spin networks for $G$ into the 3 dimensional bulk\cite{32,33}, $\Sigma^\delta$. These end with punctures on the 2 dimensional boundary. These inherit the labels on the spin network graphs, which are $G$ representations. That is to say we extend the previous correspondences. Now we assign a Hilbert space, $H_{B^\delta, j^\delta}$ to every punctured two surface, $B^\delta, j^\delta$. And a state in that Hilbert space is assigned to every embedding of a spin network in $\Sigma^+$ that ends on the punctures, matching irreducible representations, $j^\delta$ - up to topological deformations of the embeddings that leave the punctures fixed.

Given the correspondence arising from QG between representations and intertwinors of $SU(2)$ and area and volume these states have natural interpretations as triangulated three geometries.

When $G$ is chosen to be a local symmetry or gauge group for general relativity, the resulting partition function can be shown to be a quantization of general relativity\cite{26,34,35}. These are called spin-foam models. One of the things that is wonderful about them is that one can continue to use the boundary observables. In some cases they inherit new meanings from the map between the TQFT and gravity. For example the non-Abelian electric fluxes now carry geometrical observables such as areas and volumes.

In some cases the partition functions have ambiguities; this can happen because there are physical observables that depend on chirality, which is not represented by the one-dimensional structures of the spin networks. These require modification of the spin networks. The edges of the spin networks may be blown up into tubes. There are new degrees of freedom which are winding numbers of the tubes.

The end points of those lines, on the spatial boundaries, which were points, are now blown up into circles. To represent these we must extend $G$ to give it structure that can transform non-trivially under diffeomorphisms of these circles as well as under parity transformations.

When $G$ is compact we can extend it to a quantum group at root of unity. Or we extend the group to a loop algebra (or Kac-Moody algebra) the group of mappings of a circle into a Lie group. The Virasoro group, which is the group of mappings of circles onto circles, centrally extended, acts at those circles because it is a subgroup of the extension of $G$, giving us new physical degrees of freedom, which can be represented by a CFT such as the $WZW$ theory.

This structure fits nicely into the triangulations of four dimensional manifolds that we use in quantum gravity. The dual of the triangulation is a graph in a three dimensional bulk. The tetrahedra form the bulk and dual to each tetrahedron is a three dimensional spin network. Each tetrahedron is dual to an intertwiner. Its four triangles each are dual to a shared boundary of two tetrahehedra, which in turn is dual to an edge connecting the node.

\footnote{A $G$ spin network is an abstract graph, whose edges are labeled by irreducible representations of $G$ and whose vertices are labeled by intertwiners of the incident representations. When embedded in a manifold as just described it is also called a spin network.}
in each tetrahedra. Every four-simplex is bounded by 5 tetrahedra. That four simplex then represents an event in which \( n \) tetrahedra are replaced by the \( 5 - n \) tetrahedra.

In this way we get a quantum theory of gravity based on a lorentz group gauge symmetry, \( G = SL(2,C) \) or \( SO(3,1) \). This is close, but it is not exactly what we want. The models based on dynamical causal structure need to label the punctures by relativistic four momenta, \( p_n^I \). So do the celestial spheres.

There are at least two ways to do this

- Extend the gauge group to the Poincare group.
  
  This requires a good dose of the infinite dimensional unitary representations of the Lorentz and Poincare groups, so we postpone further discussion of it to a later publication.

- Extend the group to the deSitter or Anti de Sitter group.

\[
G = SL(2,C) \to SU(2,2) \tag{23}
\]

Construct the theory for the deSitter group. Then take \( \Lambda \) to zero, if needed.

### 3.6 The Chern-Simons boundary action

The second road is technically simpler, so we start with that: We work in a spinorial version of the MacDowell-Mansouri formulation of general relativity[26]. Everything we do we work with Lorentz spacetime. This is a small modification of a \( B \wedge F \) theory[27] for the double cover of the Desitter (or Ads) group: \( SU(2,2) \)

The novelty of this formalism is that we add a twist to the idea that general relativity is a constrained topological field theory, by making that theory a gauge theory of \( SU(2,2) \), where the constraints that introduce local degrees of freedom break the gauge group down to \( SL(2,C) \).

As a result, the frame field one form \( e^{AA'} \) is expressed as components of the \( SU(2,2) \) connection, in \( \frac{SU(2,2)}{SL(2,C)} \) that exists because \( SU(2,2) \) is broken spontaneously to \( SL(2,C) \). i.e. the metric geometry is a Higgs field- an order parameter that marks the spontaneous breaking of \( SU(2,2) \) to \( SL(2,C) \).

The implications of this extension can be worked out in any first order version or extension of general relativity. For the convenience of the reader who may not be familiar with the exotic Plebanski formalism[35], and is more likely to have met the Palatini variables[36]. We work here with the latter variables.
The theory is based on an $SU(2, 2)$ connection, $\mathcal{A}^{IJ}$, which decomposes as,

\[
\begin{align*}
\mathcal{A}^{AB} &= \mathcal{A}^{(AB)} = A^{AB} \quad \text{chiral } SU(2)_L \text{ connection} \\
\mathcal{A}^{A'B'} &= \mathcal{A}^{(A'B')} = A^{A'B'} \quad \text{chiral } SU(2)_R \text{ connection} \\
\mathcal{A}^{AA'} &= \sqrt{\Lambda} \quad e^{AA'} \quad \text{the frame field one-forms}
\end{align*}
\]  

(24)

We see that this approach requires a non-vanishing cosmological constant, $\Lambda$.\footnote{The $I, J \ldots = (AA') = (010'1')$ are four component Dirac spinor indices, and we are taking advantage of the two to one map between $SO(5)$ and $SU(2, 2)$.}

The corresponding components of the curvatures are

\[
\begin{align*}
\mathcal{F}^{AB} &= dA + A^2 + \Lambda e^{AA'} \wedge e^{B'} \\
\mathcal{F}^{A'B'} &= dA^{A'B'} + A^{A'B'} \wedge A \Lambda e^{AA'} \wedge e^{B'} \\
\mathcal{F}^{AA'} &= \sqrt{\Lambda}(de^{AA'} + A^{A} \wedge e^{BA'} + A^{A'} \wedge e^{AB'}) = \mathcal{D} \wedge e^{AA'} = \mathcal{T}^{AA'}
\end{align*}
\]  

(26) (27) (28)

where $\mathcal{T}^{AA'}$ is the torsion tensor.

The action of the $SU(2, 2)$ BF topological theory is

\[
S = -i \int_{\Sigma \times R} \left( \frac{1}{g^2} B^{IJ} \wedge \mathcal{F}_{IJ} - \frac{1}{e^2} \mathcal{F}^{IJ} \wedge \mathcal{F}_{IJ} \right) + \int_{\partial \Sigma \times R} \frac{k}{4\pi} Y_{CS}(SU(2, 2))
\]

\[
= -i \int_{\Sigma \times R} \frac{1}{g^2} \left[ B^{AB} \wedge F_{AB} - B^{A'B'} \wedge F_{A'B'} - B^{AB'} \wedge F_{AB'} - 1n^2 + B^{AA'} \wedge \mathcal{D}e_{AA'} \right] + \frac{1}{e^2} (F^{AB} \wedge F_{AB} + F^{A'B'} \wedge F_{A'B'} + \mathcal{D}e^{AA'} \wedge \mathcal{D}e_{AA'}) + \Lambda e^A \left[ \frac{4}{g^2} + \frac{4}{e^2} \right] + \int_{\partial \Sigma \times R} \frac{k}{4\pi} Y_{CS}(SU(2)_L) - \frac{k}{4\pi} Y_{CS}(SU(2)_R) + \frac{k}{4\pi} e^{AA'} \wedge \mathcal{D}e_{AA'}
\]

The boundary term is

\[
S_B^{BCS} = \frac{k}{4\pi} \int_{S^2 \times R} \left( Y_{CS}(SU(2)_L) - Y_{CS}(SU(2)_R) + e^{AA'} \wedge \mathcal{D}e_{AA'} \right)
\]  

(30)

One breaks the symmetry by adding Lagrange multipliers which enforce

\[
B^{AB} = e^{AA'} \wedge e^{B'}
\]  

(31)
After we break the symmetry by implementing these constraints, the action is then the Palatini action plus topological and boundary terms.

\[
S_{\text{Palatini}} = -i \int \Sigma \left( \frac{1}{g^2} + \frac{4}{e^2} \right) \left\{ \epsilon^{AA'} \wedge e_{A}^{B} \wedge F_{AB} - \epsilon^{A'}A \wedge e_{A}^{B'} \wedge F_{A'B'} \right\} - \Lambda e^{A} \left( \frac{2}{g^2} + \frac{4}{e^2} \right) + \frac{1}{g^2} B^{AA'} \wedge \mathcal{D} e_{AA'} + \frac{e^2}{2} (F^{AB} \wedge F_{AB} + F^{A'B'} \wedge F_{A'B'}) + (\mathcal{D} e^{AA'} \wedge \mathcal{D} e_{AA'}) \\
+ \int_{\partial \Sigma} \left( \frac{k}{4\pi} Y_{CS}(SU(2)_L) - \frac{k}{4\pi} Y_{CS}(SU(2)_R) + \frac{k}{4\pi} e^{AA'} \wedge \mathcal{D} e_{AA'} \right)
\]

(32)

Let us look at the first variation and make sure the action is functionally differentiable.

\[
\delta S = \int_{M} [(EOM)_{AB}] \delta A^{AB} + (EOM)_{A'B'} \delta A^{A'B'} + (EOM)_{AA'} \delta e^{AA'} \\
+ \int_{\partial M} \left( \left( \frac{1}{g^2} \Sigma_{AB} - \frac{k}{2\pi} \right) F_{AB} \right) \wedge \delta A^{AB} + \left( \frac{1}{g^2} \Sigma_{A'B'} - \frac{k}{2\pi} \right) F_{A'B'} \wedge \delta A^{A'B'} \\
+ \left[ \frac{k}{2\pi} - \frac{1}{e^2} \right] \delta e_{AA'} \wedge \mathcal{D} \wedge e_{AA'}
\]

(33)

For the action to be functionally differentiable, the boundary term of the variation must vanish.

As was first shown in [34, 37, 39] there are a number of ways to accomplish this. There are subtleties in each signature.

We discuss here only the basics of the Lorentzian signature[37].

There is a set of boundary conditions that leave the edge modes for \( e^{AA'} \), \( a^{AB} \) and \( a^{A'B'} \), (the pull back of the forms into the timeike or null boundaries) all free,

\[
\frac{1}{g^2} \Sigma_{AB} = \frac{1}{e^2} \wedge \left( -\frac{k}{2\pi} \right) F_{AB} \\
\frac{1}{g^2} \Sigma_{A'B'} = \left( \frac{1}{e^2} - \frac{k}{2\pi} \right) F_{A'B'} \\
0 = \left( \frac{1}{e^2} - \frac{k}{2\pi} \right) \mathcal{D} \wedge e_{AA'}
\]

(34)

This affirms that we must have a boundary term which includes the energy and momentum, i.e. one based on the Poincare or deSitter groups.

The translations are generated by energy and momentum, thus we can label the punctures by energy and momentum, and complete the picture of a causal theory of views based on celestial spheres.

We then have on each event’s \( S^2 \) a set of \( n = n_{in} + n_{out} \) punctures, of which \( n_{in} \) are incoming and \( n_{out} \) outgoing.
The quantum field theory on each $S^2_{\alpha,j}\, \text{is based on a Hilbert space } H_{n_{\alpha,n_{\text{out}}}(\sigma)\ldots}$. We work with a left-right symmetric spin network basis[37], where the edges are labeled by representations of $SU_L(2), SU_R(2)$.

The irreducible reps are labeled by $(j_L, j_R, I_L, I_R)$. The reality conditions impose the physical representations are balanced $j_L = j_R$.

3.7 Some remarks

The next step is to define a difference operator $\hat{D}(E,F)$ on

$$\hat{D} : H_E \otimes H_F \rightarrow R_+.$$ (36)

We can use the inner product on Chern-Simons theory to measure similarity. A state of $G$ Chern-Simons theory on the $n_A$ punctured $S^2$, notated,

$$|A, n_A, (\bar{y}, y_i, r_i) >$$ (37)

lives in a Hilbert space $H_{n_A, (\bar{y}, y_i, r_i)}$.

This unphysical Hilbert space is dependent on the positions on the $S^2$'s of the $n_A$ punctures. We note that the punctures play a role analogous to the $n$ particle trajectories in the particle construction. As in that case we ask for the best matching to construct the inner product. The Hilbert spaces decompose according to the positive number of punctures $n_A$

$$\mathcal{H}_{S^2_{\alpha,j}} = \int_{S^2_{\alpha,j}} (\bar{d}y, dy) \mathcal{H}_{-\text{punctures}}(\bar{d}y, dy)$$ (38)

We then define the inner product by

$$< B, n_B r_j | A, n_A, , r_i > = < B | A > \delta_{n_A n_B} \delta_{r_j r_k}$$ (39)

the positions of the punctures don’t matter up to diffeomorphisms of $S^2$- punctures. This is reflected in a braid group symmetry $\mathcal{B}\nabla(n_A)$ acting on the Hilbert space $\mathcal{H}_{n_A}$.

If the group $G$ were compact, these Hilbert spaces would be of finite dimension. As it is for the lorentzian theory we would probably be constrained to work within the countably infinite unitary representations[73].

The Hilbert spaces decompose according to the positive number of punctures $n_A$

$$< A, n_A, (y_i, r_i) | \phi \circ \phi | B, n_B, (y_i, r_i) > = < B | A > \delta_{n_A n_B} \delta_{r_j r_k}$$ (40)

$$< B, n_B r_j | A, n_A, , r_i > = < B | A > \delta_{n_A n_B} \delta_{r_j r_k}$$ (41)
We can then define the difference in two views, view A on \( n_A \) punctures, which are in the past of \( E \), and view B on \( n_B \) punctures. For a quantum system it is natural to take the inverse of the inner product to be the difference. As in the classical description, we compare first the punctures, and then the fields.

\[
\mathcal{D}[E, F] = \frac{1}{\langle \psi_E | \psi_F \rangle^2}
\]

(42)

4  The dynamics

If the context is novel, the dynamics will be as much as possible conventional. We begin by defining potential energy, then an Hamiltonian.

4.1  The variety as potential energy

To make the potential energy, we sum the views- difference \( \mathcal{D}[E, F] \) over all pairs of causally unrelated events, \( <> \). Because it is based on a best matching procedure, which takes the best value over all pairs in the orbit of the Lorentz group action \( \mathcal{D}[E, F] \) the result is lorentz invariant. That a pair is not causally related is also Lorentz invariant. Thus, up to divergent terms, and effects of non-invariant cut offs, the potential energy is lorentz invariant.

\[
U = \sum_{I<>J} \mathcal{D}(I, J)_{\text{bestmatched}}
\]

(43)

The last step is to restrict to events \( I \) and \( J \) in the past of a third “observer event”, \( K \).

\[
\mathcal{D}(I, J; K)_{\text{bestmatched}}
\]

(44)

We then sum over all triples

\[
\mathcal{V}_{<>} = \sum_K \sum_{I<>J\in \text{Past}(K)} \mathcal{D}(I, J; K)_{\text{bestmatched}}
\]

(45)

We propose to call this the acausal variety. We will propose it as a measure of potential energy.

\[
gU = \mathcal{V}_{<>} = \sum_K \sum_{I<>J\in \text{Past}(K)} \mathcal{D}(I, J; K)_{\text{bestmatched}}
\]

(46)

g is a coupling constant.

We notice that the best matched value of a comparison is lorentz invariant, as it considers the differences scanning over the whole group. Additionally the criterion we use to pick out which pairs of events to be compared are not altered under the operations defined in.
One might want to call the potential energy non-local. I prefer to call it a-local because it makes no reference to space. The theory does have an emergent spatial geometry and relative to that the potential that derives from the variety remains non-local, as is discussed in [1, 2].

The Hamiltonian is then

\[ H_0 = gU \]  

In the non-relativistic limit, defined as kinetic energy dominated by some mass, \( m \), this exactly reproduces Bohm’s potential[9].

Note that in the non-relativistic limit a kinetic energy term will appear proportional to[1, 2].

In the full relativistic theory the kinetic energy terms are hidden in the constraints.

\[ K.E. = \frac{(p_i)(p_j)g_{ij}}{M} \]  

4.2 The dynamics of difference: the half- integral

We are ready now to define the dynamics of the theory.

We propose that the dynamics is defined by the following half integral.

The degrees of freedom that we integrate over are the causal structure, given by the partial order structure on the \( S^2 \)'s and the energy-momentum transferred in each causal relation.

Imposing the constraints as \( \delta \) functionals in the measure of the half integral.

\[
Z(J) = \sum_{\text{processes } I \mid J} \left[ \prod_I dp_a \delta(C_I) \delta(Q_I) \prod_I \delta(P_I^a) e^{iH_0(p)} \right]
\]

The constraints are imposed as constraints in the measure of the half-integral.

This is the complete definition of the theory. There is no reference to space or spacetime at the level of the fundamental definition of the theory. As a consequence there is no \( \hbar \). There are no non-trivial commutation relations and no uncertainty principle.

4.3 Semiclassical limit: the emergence of Minkowski spacetime

Now we introduce Lagrange multipliers to exponentiate each of the constraints. There is a conservation law \( P_a^I \) for each event, so we write,

\[
\delta(P_a^I) = \int dz e^{iP_a^I} \]

\[
(50)
\]

19
The next step is finding the equations of motion. We will see that the semiclassical theory is sufficient to understanding how a classical spacetime emerges from this theory\(^{10}\),

\[
\mathcal{P}^I_a = \sum_{J < I} p^J_a - \sum_{K > I} p^I_a
\]

(51)

Take the first variation by \(p^K_a\) of the action and set it to zero.

\[
0 = \frac{\delta S^{\text{eff}}}{\delta p^K_a} = (z^a_J - z^a_K) + N p^I_b g^{ab} + g \frac{\delta\mathcal{V}}{\delta p^K_a}
\]

(52)

where \(g_{ab}\) is the metric on momentum space.

At first, ignore the potential term

\[
g \frac{\delta\mathcal{V}}{\delta p^K_a}
\]

(53)

It then follows that in the limit \(g \to 0\),

\[
(z^a_J - z^a_K)(z^b_J - z^b_K)g_{ab} = 0
\]

(54)

We see that the Lagrange multipliers \(z^a\) have become coordinates on an emergent Minkowski spacetime. We see also that \(g_{ab}\) is a conformal metric on spacetime and the intervals \(z^a_J - z^a_K\) are null. So we see that a conformal metric emerges on spacetime, which is just the inverse of the metric on momentum space.

## 5 Conclusions

Any proposal for a new physical ontology, faces huge challenges, even greater that those that confront attempts to understand quantum gravity or foundations within a more standard ontology.

The causal theory of views was proposed in [1, 2], using elements from previous models of spacetime; causal set theory, energetic causal sets, and is influenced by relative locality[49, 50], twistor theory[32], the study of amplitudes and loop quantum gravity[30]. How is it doing?

1. The framework of the ontology is clear but there are several subtle issues that are still being studied, in [3, 4].

2. The be-ables are views of an event—which is what can influence an event from that event’s causal past. We have several versions of this; these models differ by the mathematical framework used.

\(^{10}\)we assume the simplest case where the momentum is conserved linearly
3. Fundamentally there is no space or spacetime. Energy and momentum are fundamental; relativistic spacetime emerges as a consequence of the conservation of energy and momentum. We have seen how this happens in several contexts.

4. The dynamics is based on comparisons of how diverse are the views of pairs of events. The potential energy is related to the variety of the universe, which is the sum over all pairs of their differences.

5. We understand how to derive non relativistic quantum many body theory, due to the similarity of the variety and the Bohmian quantum potential.

6. We have a start on formulating relativistic theories of views. We see how special relativity can emerge readily, making use of the connection between energy-momentum conservation and locality in spacetime. But how will general relativity appear? One answer is to introduce parallel transporters into the causal processes, so the energy momentum that arrives need not be that which was sent.

7. The statistical physics of the theory described here is going to be very interesting. The statistics of the possible and actual states or transitions are very different in our context. Preliminary numerical studies by M Cortes of a simple model show new phases and new phase transitions[41]-[44]. The equilibrium ensemble is not reached during the lifetime of the universe. As a result, our universe is, we conjecture, very far from ergodic.

8. There is an expectation that underlies or motivates the study of asymptotic structures. Spacetime may be really weird when we probe it at very short distances, but if we could travel in an opposite direction, further and further away from us, at longer and longer wavelengths, it is ultimately more of the same, only more so. Our view is quite the opposite: far far away from us is going to be more and more quantum and, indeed, beyond quantum. There is no reason to expect a classical asymptopia, when IR/UV symmetry governs, going to the very very large gets to the same ”place” going very very small.

Which is to say that if we haven’t yet seen indications of IR/UV symmetry we have not yet begun to study real cosmology.

9. The Causal Theory of Views is also part of another research program, which is based on the idea that completions of background dependent theories eliminate their background dependence by a process which replaces dualities withtrialities.

This is based on an observation that in many cases of dualities in physics, the duality transformations are based on a fulcrum of background structure.

For example, UV/IR dualities or in general weak/strong dualities leave fixed a scale which defines which is which. Or the Born dualities are based on a fixed definition of time.
In these cases a deeper theory was found by eliminating the dual pairs’ dependence on non-dynamical background elements by elevating the duality to a triality in which each of the three elements is defined by the interaction of the other two[71].

This motivated my studies of cubic matrix models, which furnish many examples of such duality to triality moves. Indeed, string theory furnishes some beautiful examples of the passage from a background dependent to a background independent formulation which is cubic. (Think of the construction of actions for strings of the form of $S = Tr\Phi^3$.)

There are also many examples in the mathematical general relativity literature of the special role played by cubic actions, such as the Plebanski action. These cubic formulations are by far the simplest - since the actions are cubic the equations of motion are all quadratic equations. Many new unexpected results were made possible by adopting one of these cubic formulations.

10. This proposal raises a number of fundamental issues concerning quantum foundations and the nature of time, which are discussed elsewhere[3, 4, 5].

Much remains to do.

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