Non-Reciprocity View of the MIMO Antenna Arrays in Transmitting and Receiving Modes Using the Maximized Unique Receiving Pattern Theory Resulted by Angle-Wise Array Factor

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ABSTRACT A new approach to employ MIMO array behavior in system design calculations by distinguishing the transmitting and receiving modes’ characteristics is presented. It shows that the conventional array formulation complies merely with the transmitting mode behavior of MIMO arrays, while the newly proposed formulation eases the estimation of the receiving mode behavior by introducing the Angle Wise Array Factor (AWAF). A novel theory of “Non-reciprocity view of transmitting and receiving modes in MIMO arrays” is being discussed along with “Pattern non-uniqueness in transmitting mode” and “Pattern uniqueness in receiving mode” theories. The grating lobe definition is also re-introduced, proposing that this definition is supposed to be reserved merely for transmitting mode.

INDEX TERMS Angle-wise array factor (AWAF), MIMO array, receiving mode of MIMO systems, transmitting mode of MIMO systems, grating lobe.

I. INTRODUCTION
Next-Generation Networks (NGNs), which are part of Multi-Input Multi-Output (MIMO) systems, are currently experiencing a breathtaking development pace in terms of enhancing throughput speed and decreasing the latency to be capable of supporting day-to-day demand growth of their target industry. Although NGNs are almost covering the new requirements, something is still missing: “Re-evaluating the assumptions that are considered to be known” to investigate whether they still comply with the fundamentals as defined years ago or need modifications to become compatible with the new technologies. In other words, updating the fundamental theories concurrent with introducing the new technologies is essential to speed up the development.

There are multiple known advantages of antenna arrays over the single antenna element. Shaping the pattern in an arbitrary form, steering the beam to a specific direction, enhancing the overall gain of the antenna system, lowering the RF power, and increasing the reliability of the system, are some of the array antenna system benefits, which are hard to meet with the systems based on single element antenna.

Until the early 1970s [1], all array systems were limited to a single pattern or single reconfigurable pattern, which were compatible with the array formulations introduced before even for end-to-end system design calculations. However, since then, increasing demand for higher data rate from one side (specifically for cellular networks and mobile devices) and highly expensive frequency spectrum from the other side forced the industry to consider better use of the spectrum while minimizing the number of equipment per wireless link, which led to the definition of MIMO systems. However, the race to achieve higher bitrate resulted in the non-direct compatibility of available array system theories with MIMO systems.

For instance, the current approach of calculating the end-to-end behavior of a MIMO system considers the receiving and transmitting behaviors identical (based on reciprocity). Thus, array factor formulation is being applied for both modes. However, depending on the architecture of transceiver design, RX and TX could have different behaviors. Therefore, since the array factor is only meant for the TX mode, the RX mode needs an updated formulation supporting the modern
signal processing features (which is called Angle Wise Array Factor or AWAF) to be a precisely versatile tool for receiving mode calculations, which is the main objective of this manuscript.

In this manuscript, apart from differentiating the TX/RX behaviors by proposing the AWAF, some other fundamentals of such incompatibility issues have been discussed, such as grating lobe definition. Note that redefining these fundamentals based on the proposed diverse behaviors will tackle the compatibility issue. Moreover, by applying the AWAF in the receiving mode, an innovative approach, namely “Maximized Unique Receiving Pattern”, is introduced that reduces the complexity of the receiver’s signal processing section while minimizing the processing time.

As Figure 1 shows, MIMO technology consists of diverse sections starting from processing unit to the array antenna elements, but what is intended to be discussed here is only antenna elements along with its processing unit (phase-shifting mechanism) as investigating the whole system’s features, like diverse multiple-access algorithms, is somehow out of the scope of this article. So, we are going to restrict the article to “MIMO Array Antenna System,” in brief, “Array,” with all belongings following a necessary review of fundamentals.

This paper is organized as follows: Section II is dedicated to the fundamentals of array antenna systems and the relation between transmit/receive behavior of the single beam arrays, even for physically reconfigurable ones. Section III distinguishes the differences between the MIMO array and the conventional arrays in terms of definitions and practical deployment. Transmitting behavior of a MIMO array and grating lobe definition in transmitting mode are being reviewed in Sections IV and V, respectively, to get ready for defining the problem in Section VI, which discusses the receiving mode behavior of MIMO array along with some notes about M×N MIMO technology in Section VII. The theory with a practical solution to solve the problem is finally proposed in Section VIII, followed by a simple explanation about how M×N MIMO theory complies with this theory in Section IX. Processing architectures of MIMO array are also discussed in Section X, followed by a subtle point regarding the grating lobe existence in the receiving mode in Section XI. In section XII, additional benefits of using the proposed theory to simplify the transmitting mode calculations are given. Finally, Section XIII is reserved for the conclusion and future potential studies.

II. ARRAY CLASSIFICATION AND RADIATION PATTERN

The antenna array system is a combination of multiple antennas, spatially distributed, to achieve a specific function of pattern or gain with added power distribution, and real-time phase shifting to achieve some more features such as beam shaping, nulling, multiple beams, and/or beam steering. In terms of steering feature and pattern characteristics, as shown in Figure 2, we can classify the whole antenna array related to the present study into three categories, namely “Single Beam Array,” “Phased Array,” and “MIMO Array.”

The first category, namely the Single Beam Array, has a fixed feeding network. In other words, one signal is being generated by the synthesizer/DSP to feed all antenna elements through a designated feeding network. Since this system does not offer any flexibility to change the power or phase distribution of each antenna element, it only provides a fixed pattern. Note that the phase-shifting unit, which is usually through delay lines, is already integrated into the dividers/combiners of the feeding network itself. Figure 2a depicts a block diagram and sample pattern of a fixed pattern array. The simplicity of the design, compactness, and inexpensive fabrication process are the most important advantages of this category.

The second category, phased array, is theoretically similar to the first one with an adaptive, controllable phase-shifting mechanism for antenna elements. As the system utilizes elements with physical phase shifters, it can change the phase distribution, and consequently the main beam direction; but still, there is just one phase distribution at any time, which
means the beam can be steered merely to one specific direction at each moment regardless of the frequency. So, this category supports a single beam pattern with the flexibility to change the primary beam angle. Figure 2b illustrates the system diagram of a reconfigurable fixed pattern array along with its beam steering capability.

Since neither single pattern arrays nor phased arrays were able to support the rising demand for higher data throughput because of their natural single beam characteristic, the third category or MIMO array is introduced to uphold multiple beams on different frequency channels on different directions independently. Therefore, there is no need for the conventional multi-section of dividing/combining components since all elements are being excited independently with element-wise signals. Concerning the systems handling multiple beams concurrently, several techniques exist to generate, process, and handle multiple signals in MIMO systems such as sub-channeling the frequency spectrum in FDMA (Frequency-Division Multiple Access) [2], TDMA (Time-Division Multiple Access) [3], CDMA (Code-Division Multiple Access) [4], etc. The overall system diagram of a MIMO system and multi-beam capability is figuratively illustrated in Figure 2c.

Conventionally, all these categories were focused on studying the array in the transmitting mode under the assumption that the receiving pattern of the array can be easily estimated by applying the reciprocity theorem of electromagnetics (transmitting and receiving properties of the radiating systems are the same [5]). So, in short, reciprocity applies only where the same phase distribution is used in both modes. However, this is not an obligation. In fact, such similarity could negatively impact the full capability of offering some extra system features, which will be discussed in the upcoming sections.

Given this logic, since the first two categories are usually excited with one specific source through a fixed or timely variable feeding network, they can be categorized as passive networks leading to a single beam at any time step. Therefore, the transmitting pattern of the array system is also the receiving pattern, which means reciprocity works for these two types of array systems. However, based on the reasoning and explanations provided in this article, the MIMO array will be proven to be a non-reciprocal system. Thus, there is no obligation to have a direct relation between transmitting and receiving patterns of the array.

III. MIMO ARRAY VS. CONVENTIONAL ARRAYS

Providing the detailed explanations of the array classification in Section II, in terms of system design, there are two distinct differences between MIMO array and conventional arrays: 1) Direct signal generation for each antenna element. 2) The fact that the signal processing section with the antennas are jointly constructing the array system. As a result, the MIMO array is the only type that supports multiple concurrent beams. Now, as the main features and differences of MIMO array are discussed, we can move to discussing the whole characteristics of the MIMO array in both transmitting and receiving modes separately.

IV. MIMO ARRAY IN TRANSMITTING MODE AND NON-UNIQUENESS

A well-known method of estimating the array pattern is to employ the Array Factor (AF) concept. Array Factor gives the pattern by replacing each antenna element of the array (without disturbing the relative currents or positions) by an isotropic point source, neglecting the individual elements of the array [6]. Then, multiplying the pattern of a single antenna element with the AF, which is based on the phase and spatial distribution of the antenna elements, leads to the overall pattern of the array system in transmitting mode. The reason for stating “in transmitting mode” is that the phase distribution can be considered as known merely in transmitting mode as we aim to excite antenna elements with a specific one, while receiving signals could have any phase distribution regardless of our aimed targets. This is to be discussed in section VI. For simplicity, we consider a linear one-dimensional array depicted in Figure 3. So, the AF can be expressed as Equation (1) [6].

\[
AF(\theta) = \sum_{n=0}^{N-1} A_n e^{i(\beta d_{ref} \cos \theta + \alpha_n)}
\]  

In this formula, \(A_n\) is the signal magnitude of Element \(n\), \(\beta\) is the wavenumber, \(d_{ref}\) is the spacing between the \(n^{th}\) element and the reference element (which is equal to \((n-1)d\) in this example), \(\theta\) is the observation direction in TX mode, and \(\alpha_n\) is the initial phase of the TX excitation signal of the \(n^{th}\) antenna element.

Having one unique transmitting pattern to examine the overall performance of a system at all times is very worthwhile since system designers could include one unique transmitting pattern in their calculations; but, unfortunately, depending on the capability and accuracy of the phase-shifting algorithm, the number of different transmitting patterns varies and is always more than one. For instance, by applying different phase shifts in ascending/descending way to a 10-element array with a half-wavelength spacing, diverse beam directions could be achieved, as Figure 4 depicts three samples out of the whole likely patterns with relevant
phase distributions assuming the amplitude is constant for all signals.

As shown, there is no fixed pattern to be considered as the transmitting pattern of the MIMO array, which implicitly proves the non-uniqueness theory of the TX pattern. In brief, this theory says the non-uniqueness of the pattern in the transmitting mode is unavoidable, and system designers should consider all possible scenarios in the overall system calculations. However, by making some changes and a new formulation, a unique pattern can be introduced for the receiving mode. Sections VI to X will cover the whole concept and formulation of MIMO array in the receiving mode introducing the angle wise array factor and uniqueness theory.

V. GRATING LOBES OF MIMO ARRAY IN TX MODE

Depending on the spatial arrangement of the array’s elements and phase distribution, there is always a possibility of having multiple lower gain lobes similar to the main desired lobe in the overall pattern. Additional major lobes that rise to an intensity equal to that of the main lobe are called grating lobes, and most of the time, it is undesirable to have grating lobes [6] because it wastes the transmitting power in an unintended direction. As an example, one of the primary sources of having a grating lobe is the scenario of having the antenna elements spaced more than one-wavelength. Although we don’t intentionally intend to have a grating lobe, sometimes, especially in millimeter-wave frequencies, it is inevitable to consider multiple-wavelength spacing between elements due to the critically short wavelength of that spectrum.

VI. CHARACTERISTICS OF MIMO ARRAY IN RX MODE

As mentioned earlier, reciprocity is one of the most important properties of electromagnetic science for pattern measurement, and it implies transmitting and receiving patterns are the same and can be used interchangeably. However, this theory is applicable only for end-to-end passive systems. Getting back to our classification of the array, it is pellucid that the first two categories are purely passive in steady-state. However, by adding the active digital phase-shifting part, the MIMO system violates being an end-to-end passive system. Therefore, we can claim that the MIMO array is non-reciprocal, and the transmitting and receiving patterns are not supposed to be necessarily similar to each other. So, they must be measured separately.

Given the fact that there is no unique pattern for the transmitting mode (Section IV), is it reasonable to claim the same in the receiving mode? Does the non-uniqueness theory of the transmitting mode comply with the receiving mode too? Is there any way to measure the receiving pattern, while non-reciprocity prevents us from using AF (which is meant for the transmitting mode) in the receiving mode? These are the concerns that are going to be addressed in the subsequent sections.

VII. RX MODE PATTERN OF M × N MIMO ARRAYS

Based on Equation (1), in the transmitting mode, it is clear that AF changes in terms of merely $\theta$ (targeting angle in TX mode), assuming that the initial phase distribution is constant over time. In other words, depending on the intended beam direction, values of the specific phase distribution ($\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_N$) will remain constant while sweeping $\theta$, regardless of the angle. Apart from this, a MIMO system could handle multiple patterns for transmitting mode employing techniques like Orthogonal Frequency-Division Multiplexing (OFDM) as in [7], [8], which are separable in terms of subcarrier frequency, time, etc. Therefore, mixing these two techniques justifies multiple transmitting and receiving beams defined by IEEE 802.11n M × N. By applying the same transmitting phase distribution to the received signals, the system is forced to have a receiving beam like the transmitting pattern ending up with one-to-one TX-RX beams. However, it is not necessary to follow this rule. A better way is proposed in the next section to remove the complexity of treating each set of signals based on their transmitting phase distribution, which leads to Maximized Unique Receiving Pattern Theory.

VIII. ANGLE WISE ARRAY FACTOR AND RECEIVING MODE PATTERN

On the one hand, we know that MIMO base-stations are dividing the whole coverage area by allocating different subcarriers while working in transmitting mode, and this implies that all devices which are establishing a connection with the base-station have already been notified with their dedicated frequency band. So, all target devices work on their allowed frequency range determined by the base station based on the information received earlier. On the other hand, since the targets could be present at any receiving direction ($\theta^\text{C}$) within the coverage area and the surrounding’s frequency spectrum has already been optimally arranged, instead of...
using multiple beams for the receiver, we could develop a
technique to treat all receiving signals same as each other
maintaining maximum gain for all of them. In short, by applying
different phase shifts to different sets of received signals,
we are just dividing a unique receiving beam into multiple
receiving beams separated by frequency, time, etc. Therefore,
this method is supposed to make the system able to maximize
the gain in all directions simultaneously regardless of sub-
channel frequency/time/modulation/coding. This is precisely
the modification we need to apply to the known AF to make
it compatible with the nature of MIMO systems.

In simple words, this technique is achievable through com-
pensating the phase of each signal individually (regardless of
the frequency band) by DSP before adding up the whole
signals together to maximize the overall signal level after
combination. The new formulation is named Angle Wise
Array Factor (AWAF) to address the resulting phase distri-
bution of the original received signals plus the independent
phase compensation terms used to enhance the combination
factor of the ultimate unique signal for extracting the data.
Equation (2) shows the new formula.

\[
AWAF(\theta_0) = \sum_{n=0}^{N-1} A_n e^{j\beta d_n cos\theta_0 + \alpha_n'(\theta_0)}
\]

\[= \sum_{n=0}^{N-1} A_n e^{j\beta d_n cos\theta_0 + \alpha_n'(\theta_0)} \tag{2}\]

In this equation, the first term, \(A_n e^{j\beta d_n cos\theta_0}\), represents the
amplitude and phase of the received signal at each element
normalized with respect to the leftmost element, \(\alpha_n'(\theta_0)\) is
the phase compensation term that maximizes the received
signal level, and \(\theta_0\) is the observation angle in RX mode.
By multiplying the AWAF to the single element pattern,
the MIMO array receiving pattern can be calculated. In the
ideal case, where the compensation accuracy is 100%, the
terms \(e^{j\beta d_n cos\theta_0}\) (received signal’s phase) and \(e^{j\alpha_n'(\theta_0)}\) (phase
compensation) cancel out each other, ending up with an
overall pattern of the single element antenna with a gain
enhancement equal to the peak gain of a conventional array
factor in the transmitting mode. However, in practice, depending
on the complexity of the algorithm employed for phase
measurement/compensation section, base-band signal band-
width, and sampling frequency of DSP, reaching to such
accuracy of precise compensation is almost impossible. So,

the compensation factor should always be in the formulation.
Note that the main difference between conventional AF in
the transmitting mode and the AWAF in the receiving mode
is that the initial phase \((\alpha_n)\) of the excitation signals of ele-
ments are constant while sweeping the observation angle in
the transmitting mode, whereas, both observation angle and
phase compensation term \((\alpha_n'(\theta_0))\) are changing with respect
to the angle in receiving mode. Figure 5 demonstrates a visual
comparison of AF and AWAF in two different modes they are
supposed to be used. As an instance, using a simple phase
measurement through the time delay method, by sampling a
signal of 20 MHz bandwidth with a frequency of 200 MHz,
the worst-case phase-detection step can be calculated using
Equations (3)-(5), resulting in 36-degree steps. The ideal
phase and actual measured phase calculated using these equa-
tions are illustrated in Figure 6, which means the maximum
phase measurement error of the received signal, and conse-
sequently, the phase of the compensated signal is 36 degrees.
Therefore, \(\alpha_n'(\theta_0)\) and the actual phase of the received signal
cannot totally cancel out each other, which leads to a non-
maximized combination of the signals received by the array
elements.

\[if: \quad PDS = \frac{SSB\ Bandwidth}{\text{Sampling\ Frequency}} \times 360 \tag{3}\]

\[P_{Th} : \text{Actual Phase of the received signal} \tag{4}\]

\[\alpha_n'(\theta_0) = 360 - \left( \text{floor} \left( \frac{P_{Th} (\theta_0)}{PDS} \right) \times PDS \right) \tag{5}\]

Assuming a simple patch antenna element designed to
operate at 2.45 GHz as an instance, the traces depicted in
Figure 7 illustrate the result of the actual receiving pattern
of this example with different phase measurement accuracy.
As it is clear, even with a poor phase measuring accuracy of
90°, the receiving pattern will not be altered severely, and this
is one of the most important advantages of this technique.
Figure 8 also shows that the receiving pattern resulted from
this method is the single element pattern plus the array gain
in all directions. Therefore, depending on the compensation
accuracy, the resulting pattern will always remain within the
area between the ideal array and the single element patterns,
unless we intend to suppress the gain of a specific angular
region; that means there is always a possibility to compensate
the phase in a way that the elements’ signals could destruct...
each other in the case of intentionally we want to suppress a specific signal set like interference. In such cases, AWAF could decrease the gain even less than the single element’s gain, as depicted in Figure 8.

The whole reasoning and formulations mentioned above are devoted to support this idea: “Regardless of the number of possible transmitting patterns of any MIMO array, receiving pattern is always unique and predictable unless we intentionally aim otherwise.” In other words, since AWAF covers the whole signals received from targets in different observation angles at the same time, only one pattern can be naturally defined for MIMO array in the receiving mode, while the transmitting pattern could have multiple different patterns depending on the phase distribution of the array. Therefore, the non-uniqueness theory of the transmitting pattern is to be replaced with the uniqueness theory in the receiving mode.

Based on this theory, by applying AWAF in full phase compensation mode, ‘Maximized Unique Receiving Pattern of MIMO array’ can be calculated. Considering this fact, not only the complexity of dealing with multiple patterns is going to be bypassed, but also the number of different scenarios to find the worst-case scenario of transceiver performance is impressively reduced, leading to super-easy receiver predictions for end-to-end system design purposes. As the conventional M×N MIMO is supposed to have the same RX and TX patterns for each frequency channel, Figure 8 also compares the ideal RX pattern with one sample of TX patterns (broadside) resulted by in-phase excitation of the whole antenna elements, which proves that reciprocity in MIMO systems is strictly conditional to the system characteristics.

IX. COMPATIBILITY OF THE THEORY WITH M×N MIMO SYSTEMS

Because of the uniqueness theory introduced for the receiving mode, there might be some vague points regarding the current technologies which are manufactured using M×N MIMO technology. However, there is no conflict between these two concepts whatsoever. In other words, the unique receiving pattern can be separated into multiple patterns by applying diverse phase compensations based on the frequency/time of the received signal. For example, we can filter out the signals using frequency sub-channeling (as in FDMA) and apply different constant phase compensations based on a carrier frequency to achieve multiple different receiving patterns; but all of these beams are just a limited version of the unique receiving pattern proposed. In fact, in this case, since all the signals received via different directions are going to be compensated with a constant phase distribution instead of their specific required distribution in an angle wise manner, multiple receiving patterns are formed. While it seems that these patterns with different frequency bands are somehow independent in terms of how they have created, they are all restricted versions of the unique receiving pattern limited to a specific direction. Time can also have such an effect in the MIMO system (as in TDMA).

X. RECEIVING MODE PATTERN IN TERMS OF MIMO ARRAY’S PROCESSING ARCHITECTURES

Depending on the amount and type of information/data available in the receiving signals, there are two diverse receiving architectures where each one has its pros and cons; but before getting deep into the details of these architectures, we need to clarify what we call information and what we call data. Generally, on a very classic classification, we can say receiving signals are supposed to give us some direct and indirect information following the desired application. Direct information, or data, implies the part of the information which is available through the whole signals with no need to compare two or more signals to each other, while indirect information, or in brief information, is a result of calculations based on comparing the signals received on elements. Extracting the data-frame of a wireless link and DOA (direction of arrival [9]–[11]) prediction (through phase comparison) are tangible examples of direct and indirect information, respectively. Even though the information is a result of element level signal
XI. GRATING LOBE ASSESSMENT IN RECEIVING MODE

As discussed in section V, grating lobes are part of the TX beam, which wastes the transmitting power in undesired directions with a bit lower gain comparing the main lobe. Therefore, the grating lobe has practical meaning in transmitting mode. However, since there is no priority between the targets located in different directions in the receiving mode of MIMO systems and this beam is supposed to cover the whole directions widely, we cannot define a part of the beam as main and another part as the grating lobe. In other words, the whole angles are of equal importance. Thus, no angular region can be neglected. Moreover, as proved earlier, since the general RX pattern does not have even a second lobe (similar to the single element pattern), the grating lobe definition seems incompatible with this mode. So, in general, we can suggest reserving a grating lobe definition for transmitting mode as there is no rational interpretation for it in receiving mode. It is also worth noting that a semi-equivalent definition of the grating lobe for the RX mode is the “Ambiguity Level of the Direction of Arrival Estimation.”

XII. APPLYING AWAF TO TRANSMITTING MODE FOR EASE OF CALCULATIONS

So far, we have seen that introducing a unique pattern for receiving mode has a lot of advantages in terms of simplification of the design and calculations; but, still, since the transmitting pattern is not unique, system designers must deal with multiple scenarios based on the desired direction. Because the source of generating $\alpha_n(\theta)$ (initial phases for TX mode) and $\alpha'_n(\theta_0)$ (compensation phase for RX mode) are the same; it seems reasonable to say that their accuracy is also the same. Apart from this fact, another important point is that the gain of an array in the desired direction is the critical factor of the TX pattern in calculating the link budget, SNR, etc. So, if we can gather peak gains of the TX patterns in one pattern (which is precisely the RX pattern based on AWAF), all different scenarios can be aggregated into one scenario. Therefore, all the calculations will be done by employing one equivalent (not true) pattern for TX mode and one unique, actual pattern for RX mode, which is far less complicated than dealing with multiple scenarios. Note that although this method could enhance the design remarkably, it should not be forgotten that replacing multiple actual TX patterns with a unique pattern will affect the overall interference and noise levels, so they are needed to be involved in the calculation using the conventional methods.

XIII. CONCLUSION

A distinct behavior of the MIMO array has been discussed. It has proven that transmitting and receiving patterns of MIMO array are not necessarily the same unless intentionally forced, using the transmitting phase distribution in the receiving mode with no change concerning the different angles. Also, it has been illustrated that in an ideal scenario, the receiver is supposed to be able to fully compensate the received signals’ phase to maximize the gain of an array in all directions, which resembles a unique receiving pattern similar to the single element with a higher gain. However, even if the phase-detection/compensation has not been accurately deployed, the RX pattern is to remain between the ideal receiving pattern and the single element pattern. System design simplifications resulted from the proposed formulation have also been addressed even for transmitting mode. It has also been
discussed that replacing a constant phase compensation in M×N MIMO system with the dynamic angle wise phase compensation could result in far better aerial coverage of MIMO array in terms of receiving gain flatness and SNR stability. Also, it could be used to suppress interference/noise to have higher SNR, which consequently leads to the higher data rate.

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