Generation of cosmological large lepton asymmetry from a rolling scalar field

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We propose a new scenario to simultaneously explain a large lepton asymmetry and a small baryon asymmetry. We consider a rolling scalar field and its derivative coupling to the lepton number current. The presence of an effective nonzero time derivative of the scalar field leads to CPT violation so that the lepton asymmetry can be generated even in thermal equilibrium as pointed out by Cohen and Kaplan. In this model, the lepton asymmetry varies with time. In particular, we consider the case where it grows with time. The final lepton asymmetry is determined by the decoupling of the lepton number violating interaction and can be as large as order unity. On the other hand, if the decoupling takes place after the electroweak phase transition, a small baryon asymmetry is obtained from the small lepton asymmetry at that time through sphaleron effects. We construct a model in which a rolling scalar field is identified with a quintessence field.

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I. INTRODUCTION

The baryon number density dominates over the antiparticle number density in our Universe. The magnitude of the baryon asymmetry is mainly estimated by the two different methods. One is big bang nucleosynthesis (BBN). By comparing predicted primordial abundances of light elements (D, 3He, 4He and 7Li) with those inferred from observations, the baryon-to-entropy ratio \( n_B/s \) is estimated as \( n_B/s \sim (8-10) \times 10^{-11} \). The other is observations of small scale anisotropies of the cosmic microwave background (CMB) radiation. The baryon-to-entropy ratio inferred from recent results of the Wilkinson Microwave Anisotropy Probe (WMAP) roughly coincides with that inferred from BBN. However, the detailed analysis shows that the best fit value of the effective number of neutrino species \( N_\nu \) is significantly smaller than 3.0. Of course, \( N_\nu = 3.0 \) is consistent at \( \sim 2\sigma \) and such discrepancies may be completely removed after observations are improved and their errors are reduced. However it is probable that such small discrepancies are genuine and suggest additional physics in BBN and the CMB.

An interesting possibility to eliminate such discrepancies is the presence of a large and positive lepton asymmetry of electron type. Roughly speaking, such an asymmetry causes two effects on the predicted primordial abundance of 4He. The excess of electron neutrinos shifts the chemical equilibrium between protons and neutrons toward protons, which reduces the predicted primordial abundance of 4He. On the other hand, the excess of electron neutrinos also causes increase of the Hubble expansion, which makes the predicted primordial abundance of 4He increase. In practice, the former effect overwhelms the latter so that the presence of a large and positive electron type asymmetry decreases the predicted primordial abundance of 4He, which often solves the discrepancies as mentioned above. Furthermore, large lepton asymmetries are useful to realize the cool dark matter model of the large scale structure formation and the relic neutrino scenario of the extremely high energy cosmic rays. However, it is very difficult for such a large lepton asymmetry to be compatible with a small baryon asymmetry. This is mainly because, if we take the sphaleron effects into account, lepton asymmetry is converted into baryon asymmetry of the same order with the opposite sign.

There are several ways to overcome this difficulty. One possibility is to generate the large lepton asymmetry after electroweak phase transition but before BBN, which may be realized through oscillations between active neutrinos and sterile neutrinos. Another is to disable the sphaleron effects. It was pointed out that the presence of a large chemical potential prevents restoration of the electroweak symmetry. Based on this non-restoration mechanism, the generation of a large lepton asymmetry compatible with the small baryon asymmetry was discussed. March-Russell et al. discussed another possibility. In their model, a positive electron type asymmetry but no total lepton asymmetry, that is, \( L_e = -L_\mu > 0 \) and \( L_\tau = 0 \), is generated by the Affleck-Dine mechanism for some flat direction. Then, the small positive baryon asymmetry is produced through thermal mass effects of sphaleron processes. Recently, Kawasaki, Takahashi, and the present author discussed another possibility, in which a positive electron type asymmetry but negative total lepton asymmetry is produced by the Affleck-Dine mechanism and almost all the produced lepton numbers are absorbed into L-balls. A small amount of negative lepton charges is...
evaporated from the L-balls due to thermal effects. These are converted into the observed small baryon asymmetry by virtue of sphaleron effects. However, the remaining lepton charges are protected from sphaleron effects and released into thermal plasmas by the decay of L-balls before BBN.

In this paper, we discuss another possibility, in which a spontaneous leptogenesis mechanism proposed by Cohen and Kaplan is used \[21\]. We consider a real scalar field, which slow rolls like a quintessence field and couples derivatively to the lepton number current. The presence of an effective nonzero time derivative of the scalar field leads to CPT violation so that the lepton asymmetry can be generated even in thermal equilibrium \[22\]. The produced lepton asymmetry varies with time according to the dynamics of the rolling scalar field. Then, the final lepton asymmetry is determined by the decoupling of the lepton number violating interaction and it can be as large as order unity. On the other hand, in the case that the lepton asymmetry grows with time, it can be small at the electroweak phase transition, which is converted into the observed small baryon asymmetry. In a similar way to \[24\] and \[25\], we can generate a positive electron type asymmetry but a negative total lepton asymmetry according to the coupling constants. Thus, a large positive electron type asymmetry and the small positive baryon asymmetry are realized simultaneously.

In the next section, after reviewing the spontaneous baryo/leptogenesis mechanism proposed by Cohen and Kaplan, we explain our scenario to produce the desired asymmetries. In Sec. III, we give a model as an example, in which a real rolling scalar field is identified with a quintessence field. Several authors have already discussed similar scenarios, in which only the present small baryon asymmetry is explained by a Nambu-Goldstone boson \[23\], a quintessence field \[24\], \[25\], and an inflaton in warm inflation \[26\], with their derivative couplings to a baryon current or a lepton current.

II. SPONTANEOUS LEPTOGENESIS AND LARGE LEPTON ASYMMETRY

First of all, we introduce the derivative coupling of a real rolling scalar field $\phi$ with the lepton number current $J_{i}^{\mu}$,

$$
\mathcal{L}_{\text{eff}} = \sum_{i} \frac{c_{i}}{M} f(\phi) \partial_{\mu} \phi J_{i}^{\mu},
$$

where $i$ denotes the generation, $c_{i}$ are coupling constants, $f(\phi)$ is a function of $\phi$, and $M$ is the cutoff scale, which is set to be the reduced Planck mass $M_{\text{Pl}} \sim 10^{18}$ GeV in this paper. Assuming that $\phi$ is homogeneous,

$$
\mathcal{L}_{\text{eff}} = \sum_{i} \frac{c_{i}}{M} f(\phi) \dot{\phi} n_{L}^{i} = \sum_{i} \mu_{i}(t) n_{L}^{i},
$$

where the effective time-dependent chemical potential $\mu_{i}(t)$ is given by

$$
\mu_{i}(t) = \frac{c_{i}}{M} f(\phi) \dot{\phi}.
$$

As pointed out by Cohen and Kaplan, this interaction induces CPT violation if the time derivative $\dot{\phi}$ is effectively nonzero, which generates the lepton asymmetry even in a state of thermal equilibrium. Then, in thermal equilibrium, the lepton asymmetry $n_{L}^{i}$, for $\mu_{i} < T$, is given by

$$
n_{L}^{i} = \frac{g}{6} T^{3} \left\{ \frac{\mu_{i}}{T} + \mathcal{O} \left( \frac{\mu_{i}}{T^{3}} \right) \right\},
$$

where $g$ represents the number of degrees of freedom of the fields corresponding to $n_{L}$. Since the entropy density $s$ is given by $s = \frac{4\pi}{3} g_{s} T^{3}$ with $g_{s}$ the total number of degrees of freedom for the relativistic particles, the ratio between the lepton number density and the entropy density is given by

$$
\frac{n_{L}^{i}}{s} \simeq \frac{15}{4\pi^{2} g_{s}} \frac{g_{i}}{T} \mu_{i} = \frac{15}{4\pi^{2} g_{s}} \frac{g_{i}}{MT} f(\phi) \dot{\phi}.
$$

The above generation mechanism is effective as long as the lepton number violating interaction is in thermal equilibrium. Thus, the final lepton asymmetry is determined by the decoupling temperature $T_{D}$ of the lepton number violating interaction. Such a lepton number violating interaction with the low decoupling temperature can be obtained in the context of the Zee model \[27\] or the triplet Higgs boson with its lepton number not equal to two. However, we do not specify the lepton number violating interaction in this paper. Instead, for our purpose, we have only to demand that the decoupling of the lepton number violating interaction takes place after the electroweak phase transition.

If $c_{1}$ is positive but the total sum of $c_{i}$ is negative, positive electron type asymmetry but negative total lepton asymmetry is realized. Depending on the values of the scalar field $\phi$ and its time derivative $\dot{\phi}$, the absolute magnitude of the lepton asymmetry can be as large as order unity. On the other hand, the lepton asymmetry at the electroweak phase transition can be as small as the order of $10^{-10}$ according to the dynamics of the scalar field. Then, a part of the lepton asymmetry at that time is converted into the baryon asymmetry through the sphaleron processes, which can be estimated as \[14\]

$$
\eta = \frac{n_{B}}{s} \simeq -\frac{8}{23} \sum_{i} \frac{n_{L}^{i}}{s} \bigg|_{T_{\text{EW}}} \approx -c \frac{30}{23\pi^{2} g_{s} MT} f(\phi) \dot{\phi} \bigg|_{T_{\text{EW}}},
$$

where $c = \sum_{i} c_{i}$, $T_{\text{EW}}$ represents the temperature at the electroweak phase transition, and we have assumed the standard model with two Higgs doublets and three generations. Thus, the large positive electron type asymmetry and the small positive baryon asymmetry are obtained.
III. QUINTESSENTIAL BARYO/LEPTOGENESIS MODEL

In this section, as an example, we consider the case in which the real scalar field is identified with a quintessence field. Though a lot of quintessence models are proposed \[28, 29, 30, 31, 32, 33, 34, 35, 36, 37\], we take a k-essence model \[38, 39\], in which the quintessence field can naturally have the derivative coupling to the lepton current.

One of the candidates to realize the k-essence model is the gauge-neutral massless scalar fields present in string theory. The low-energy effective action \( S_{\text{eff}} \) has the following form \[37\]:

\[
S_{\text{eff}} = \frac{1}{6\kappa^2} \int d^4x \sqrt{-g} \left\{ -B_\mu(\phi) \dot{R} - B_\mu^{(0)}(\phi) (\nabla \phi)^2 + \alpha' \left[ c_1 B_\mu^{(1)}(\phi) (\nabla \phi)^4 + \sum_i c_2^{(i)} \psi_i \dot{D} \psi_i \right] + \cdots \right\} \tag{7}
\]

where \( \kappa = 8\pi G/3 \), \( \phi \) is the dilaton or the moduli, and \( \psi_i \) are the leptons, respectively. \( B_\mu(\phi) \) are the coupling functions and could take the complicated forms in the strong coupling regime. In the Einstein frame where \( g_{\mu\nu} = B_\mu(\phi) g_{\mu\nu} \), the effective action becomes

\[
S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{R}{6\kappa^2} + p(\phi, \nabla \phi) \right] \tag{8}
\]

with

\[
p(\phi, \nabla \phi) = p_\phi(\phi, \nabla \phi) + p_d(\phi, \nabla \phi). \tag{9}
\]

The function \( p_\phi(\phi, \nabla \phi) \) depends on only \( \phi \) and \( X \equiv (\nabla \phi)^2/2 \), and is given by

\[
p_\phi(\phi, X) = K(\phi)X + L(\phi)X^2 \tag{10}
\]

with

\[
K(\phi) = \frac{1}{6\kappa^2} \left[ 3 \frac{B_\phi^{(2)}(\phi)}{B_\phi^{(0)}(\phi)} - 2 \frac{B_\phi^{(0)}(\phi)}{B_\phi(\phi)} \right],
\]

\[
L(\phi) = \frac{2\alpha'}{3\kappa^2} c_1 B_\phi^{(1)}(\phi). \tag{11}
\]

On the other hand, the function \( p_d(\phi, \nabla \phi) \) is given by

\[
p_d(\phi, \nabla \phi) = \sum_i M_i(\phi) \partial_i \phi J_i^\mu \tag{12}
\]

with

\[
M_i(\phi) = \frac{\alpha'}{4\kappa^2} \sum_i c_2^{(i)} \frac{B_\phi^{(2)}(\phi)}{B_\phi^{(0)}(\phi)}. \tag{13}
\]

The above derivative coupling to the lepton current is obtained after integration by parts and the prime denotes \( d/d\phi \).

More phenomenologically, the above Lagrangian can be obtained by imposing a real scalar field \( \phi \) on the shift symmetry as proposed in \[38, 39\],

\[
\phi \rightarrow \phi + CM_G, \tag{14}
\]

with \( C \) is a dimensionless parameter. Then, the Lagrangian depends on only \( \partial_\mu \phi \) so that \( \phi \) can naturally have the derivative coupling to the lepton current. The functions \( K(\phi), L(\phi), \) and \( M_i(\phi) \) may be associated with the breaking of such a symmetry.

After redefining the scalar field \( \phi \) such that

\[
\phi_{\text{new}} = \int d\phi \frac{L(\phi)^{1/2}}{[K(\phi)]^{1/2}} M_s^2, \tag{15}
\]

the function \( p(\phi, \nabla \phi) \) has a simple form,

\[
p(\phi, \nabla \phi) = g(\phi) \left( -X + \frac{X^2}{M_s^2} \right) + \sum_i h_i(\phi) \partial_\mu \phi J_i^\mu \tag{16}
\]

with

\[
g(\phi) = \frac{K^2(\phi)}{L(\phi)} \frac{1}{M_s^2},
\]

\[
h_i(\phi) = \frac{[K(\phi)]^{1/2} M_i(\phi)}{L^{1/2}(\phi) M_s^2}. \tag{17}
\]

Here the subscripts new are omitted and the constant \( M_s \) with the dimension one is introduced to give the new field \( \phi \) a correct dimension. Thus, the quintessence field \( \phi \) can have the derivative coupling to the lepton current as given in the previous section after replacing \( h_i(\phi) \) by \( c_1 f(\phi)/M \).

In \[32\], it is shown that, during the matter or radiation dominated epoch, the scaling solution exists for the function \( g(\phi) \) with the form of the inverse power law \( g(\phi) = (\phi/M_s)^{-\alpha} M_s^4 \). So, we adopt such a function as the form of \( g(\phi) \). Then, the scaling solution is given by

\[
\phi = \xi M_t^2 t, \quad \dot{\phi} = \xi M_s^2, \tag{18}
\]

where the coefficient \( \xi = \sqrt{2(1 - w_Q)/(1 - 3w_Q)} \), and \( w_Q = \rho_Q/\rho_Q \) is the equation of state for the quintessence field and is given by

\[
w_Q = \frac{(1 + w_B)\alpha}{2} - 1. \tag{19}
\]

Here \( w_B \) is the equation of state for the background matter or radiation. Requiring that \( w_Q < 0 \) during the matter dominated epoch, the exponent \( \alpha \) is constrained as \( \alpha < 2 \).

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\(^{1}\) It can be easily shown that the modification of the equation of motion for \( \phi \) due to the presence of the derivative coupling is negligible in our context.
In order to fix the parameter $M_s$, we require that the energy density of the quintessence field starts to dominate the energy density of the Universe recently and obeys the scaling solution until recently, which gives

\[ M_s \sim 10^{-43} \text{ GeV}. \]  

(20)

Then, the value of the quintessence field at BBN is given by

\[ \phi_{\text{BBN}} \sim 10^{43 - 4} \text{ GeV}. \]

(21)

Now, we discuss the spontaneous leptogenesis induced by the quintessence field $\phi$. To make the discussion concrete, we set $\alpha = 0.9$ as an example. In this case, $M_s \sim 10^{-3}$ GeV. The discussion applies to the other cases in the same way except the dependence on the parameters. Then, we assume that $f(\phi)$ has the following form:

\[ f(\phi) = \left( \frac{\phi M^5}{(\phi + M)^4 M^3_s} \right)^{\frac{1}{2}}. \]  

(22)

Such a coupling function may be obtained by the extension of the shift symmetry introduced in $[38, 39]$. This function $f(\phi)$ is roughly classified into two regions,

\[ f(\phi) \simeq \begin{cases} \left( \frac{\phi M^5}{M^4} \right)^{\frac{1}{2}}, & \text{for } \phi \lesssim M, \\ \left( \frac{\phi^3 M^2}{M^4} \right)^{\frac{1}{2}}, & \text{for } \phi \gtrsim M. \end{cases} \]  

(23)

For $M = M_G \simeq 10^{18}$ GeV, we can use the second region for the function $f(\phi)$ at least until BBN.

Using Eq. (20), the lepton asymmetry is estimated as

\[ \frac{n_L}{s} \sim \frac{c_1}{M_G T} f(\phi) \dot{\phi} \sim c_1 \left( \frac{M_s}{T} \right)^2 \propto T^{-2}. \]

(24)

By inserting $T_{\text{EW}} \sim 100$ GeV and $M_s \sim 10^{-3}$ GeV into the above equation, the total lepton asymmetry at the electroweak phase transition is given by

\[ \frac{n_L}{s} \bigg|_{T = T_{\text{EW}}} \sim c \times 10^{-10}. \]  

(25)

A part of the lepton asymmetry is changed into the baryon asymmetry through the sphaleron effects. Thus, taking $c = \sum_i c_i = -O(1)$, the present baryon asymmetry is given by

\[ \frac{n_B}{s} \sim 10^{-10}. \]

(26)

On the other hand, if the decoupling temperature $T_D$ of the lepton number violating interaction is nearly equal or lower than $T_{\text{BBN}} \sim 1 \text{ MeV}$, the lepton asymmetry at BBN is given by

\[ \frac{n_L}{s} \bigg|_{T = T_{\text{BBN}}} \sim c_i. \]  

(27)

Taking $c_1 = O(1)$, the electron type asymmetry becomes positive and of order unity at BBN.

Finally, we must check the present constraint on the derivative coupling given in Eq. (1) with those from laboratory experiments. For the time component, the coefficient $\mu(t_0)$ is constrained as $|\mu(t_0)| \lesssim 10^{-25}$ GeV $[40]$. Here $t_0$ is the present age of the Universe. In our model, $\mu(t_0)$ is given by

\[ |\mu(t_0)| \sim \frac{|c|}{M} \left( \frac{M^5}{\phi_0 M^2} \right)^{\frac{1}{2}} \sim 10^{-32} \text{GeV} \ll 10^{-25} \text{GeV} \]

(28)

for $|c| = O(1)$ and $M = M_G$.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we considered the large lepton asymmetry from a rolling scalar field. Considering the derivative coupling of the scalar field to the lepton number current, the presence of an effective nonzero time derivative of the scalar field leads to $CPT$ violation, which generates the lepton asymmetry even in thermal equilibrium. This lepton asymmetry changes with time. So, depending on the dynamics of the scalar field, it is possible that the lepton asymmetry is small at the electroweak phase transition but large at BBN. A part of the lepton asymmetry is converted into the baryon asymmetry with the opposite sign through sphaleron effects. We pointed out that by choosing the sign of the coupling constants properly, a large positive lepton asymmetry of electron type and a small positive baryon asymmetry can be realized simultaneously.

As an example, we considered the real rolling scalar field, which realizes the k-essence model. By considering the scaling solution, we showed that this model manifests just such asymmetries. However, the coupling function to the lepton number current is a bit complicated. This is mainly because the time derivative of the scaling solution is a constant irrespective of cosmic time. If we abandon the scaling solution, there may be evolution of the quintessence field, in which the magnitude of the time derivative of the quintessence field at BBN is much larger than that at the electroweak phase transition. For such evolution, the model may work, in which the coupling to the lepton number current is simple, that is, $f(\phi)$ $\equiv 1$. Such a possibility will be considered in a further publication.

Finally, we comment on the recent discussion of neutrino oscillations around BBN. It was pointed out that complete or partial equilibrium between all active neutrinos may be accomplished through neutrino oscillations in the presence of neutrino chemical potentials, depending on neutrino oscillation parameters $[41]$. In case of partial equilibrium, we need not change our scenario. Complete equilibrium may spoil our scenario. In that case, if we choose the coupling constants $c_i$ as $c_1 = -c_2 = -c_3$ for some symmetry reason, our scenario still works. In such
a case, no mixing takes place because of the cancellation, as pointed out in [11].

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