Impact of nuclear structure on production of superheavy nuclei

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Abstract. The calculations performed with the modified two-center shell model reveal quite strong shell effects at $Z = 120 - 126$ and $N = 184$ as in the self-consistent mean-field treatments. If our prediction of the structure of heaviest nuclei is correct, than one can expect the production of evaporation residues $Z = 120$ in the reactions $^{50}$Ti+$^{249}$Cf and $^{54}$Cr+$^{248}$Cm in near future. The nuclear level densities were calculated for the nuclei of $\alpha$-decay chains containing $^{296}$, $^{298}$, $^{300}$120. The minima of the level density parameter clearly indicate the strong shell effect at $Z = 120$.

1. Introduction

An extension of the nuclear chart to the elements with $Z > 100$ and a study of their properties provide us an important insight into the determination of an island of stability. The experimental trend of nuclear properties ($Q_\alpha$-values and half-lives) as well as the production cross sections of superheavy elements (SHE) reveal an increasing stability of nuclei approaching $N = 184$, and indicate quite a large shell effects behind $Z = 114$ [1, 2, 3, 4, 5, 6, 7, 8]. This means that the predictions of relativistic and nonrelativistic mean-field models [9, 10, 11, 12] seem to be valid. In accordance with these self-consistent models a center of the island of stability is expected at $Z=120-126$ and $N=172$ or 184. In the $(N,Z)$–plane, the line, along which all new SHE were discovered in the actinide-based reactions with $^{48}$Ca beam, just approaches this region [1, 3].

The structure of superheavies crucially influences the evaporation residue cross sections in the actinide-based complete fusion reactions [13]. The difference in the nuclear properties predicted for heaviest nuclei with various microscopic models is caused by the scanty experimental data in this region. Nuclear models contain a number of parameters which are fixed for the best description of known nuclei. Existing microscopic–macroscopic approaches [14, 15, 16, 17, 18, 19, 20] supply the basis for the intensive calculations of the properties of heavy nuclei. These approaches differ by the parametrization of nuclear shape and by the single-particle potential used. In Refs. [19, 20] we proposed the microscopic–macroscopic approach based on the two-center shell model (TCSM) [21]. The parameters were set so to describe the spins and parities of the ground state of known heavy nuclei. This approach has been used in Ref. [22] to reveal the trends in the shell effects and $Q_\alpha$ values with $Z$ in heaviest nuclei.
The experimental evaporation residue cross sections $\sigma_{xn}$ in the $^{48}\text{Ca}$-induced complete fusion reactions do not depend strongly on the atomic number $Z$ of SHE and are on the picobarn level. As known, the cross section of compound nucleus formation strongly decreases with increasing $Z_1 \times Z_2$. Since the absolute value of evaporation residue cross section is ruled by the product of complete fusion cross section and survival probability, the loss in the formation probability of compound nucleus in actinide-based reactions can be compensated by the gain in the survival probability of SHE.

2. Stability of SHE produced in actinide-based complete fusion reactions

The cross section of the production of SHE as the evaporation residues in the $xn$-evaporation channel is written as a sum over all partial waves $J$

$$\sigma_{xn}(E_{c.m.}) = \sum_J \sigma_{fus}(E_{c.m.}, J) W_{xn}(E_{c.m.}, J),$$

$$\sigma_{fus}(E_{c.m.}, J) = \int_0^{\pi/2} \int_0^{\pi/2} d\cos\Theta_1 d\cos\Theta_2 \sigma_c(E_{c.m.}, J, \Theta_i) P_{CN}(E_{c.m.}, J, \Theta_i).$$

Here, the averaging over the orientations of statically deformed interacting nuclei ($\Theta_i$ ($i=1,2$) are the orientation angles with respect to the collision axis) is taken into consideration. For the correct description of the experimental data, the partial capture cross section $\sigma_c$, fusion $P_{CN}$ and survival $W_{sur}$ probabilities should be properly calculated [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

The value of $\sigma_c(E_{c.m.}, J, \Theta_i) = \frac{\pi h^2}{2\mu E_{c.m.}} (2J + 1) T(E_{c.m.}, J, \Theta_i)$ defines the transition of the colliding nuclei over the Coulomb barrier with the probability $T$ and the formation of dinuclear system (DNS) when the kinetic energy $E_{c.m.}$ and angular momentum $J$ of the relative motion are transformed into the excitation energy and angular momentum of the DNS. The capture (transition) probability $T(E_{c.m.}, J, (E_{c.m.}, J)) = (1 + \exp[2\pi(V_j(R_0, \Theta_i) - E_{c.m.})/h\omega_j(\Theta_i)])^{-1}$ is calculated with the Hill-Wheeler formula. The effective nucleus-nucleus potential

$$V_j(R, \Theta_i) = V_N(R, \Theta_i) + V_C(R, \Theta_i) + \hbar^2 J(J + 1)/2$$

is calculated as a sum of nuclear $V_N$, Coulomb $V_C$ and centrifugal interactions and approximated near the Coulomb barrier at $R = R_0$ by the inverted harmonic-oscillator potential with the barrier height $V_j(R_0, \Theta_i)$ and frequency $\omega_j(\Theta_i)$. In the entrance channel the moment of inertia is $\Sigma = \mu R^2$. The nuclear potential $V_N$ is calculated with the double-folding model using a nuclear radius parameter $r_0=1.15$ fm and a diffuseness $a=0.54$ fm for $^{48}\text{Ca}$ and $a=0.56$ fm for the actinide targets [33]. The quadrupole deformation parameters of actinides are taken from Ref. [34].

The DNS model [23, 24, 25, 26, 27, 28, 29, 30, 31, 32] is successful in describing the complete fusion reactions especially related to the production of heavy and superheavy nuclei. In the DNS model the compound nucleus is reached by a series of transfers of nucleons from the light nucleus to the heavy one. The dynamics of the DNS is considered as a combined diffusion in the degrees of freedom of the mass asymmetry $\eta = (A_1 - A_2)/(A_1 + A_2)$ ($A_1$ and $A_2$ are the mass numbers of the DNS nuclei) and of the internuclear distance $R$. The diffusion in $R$ occurs towards the values larger than the sum of the radii of the DNS nuclei and finally leads to the quasifission (decay of the DNS). After the capture stage, the probability of complete fusion

$$P_{CN} = \lambda_\eta^{Kr}/(\lambda_\eta^{Kr} + \lambda_{\eta_{sym}}^{Kr} + \lambda_R^{Kr})$$

depends on the competition between the complete fusion in $\eta$, diffusion in $\eta$ to more symmetric DNS and quasifission. This competition can strongly reduce the value of $\sigma_{fus}(E_{c.m.}, J)$ and, correspondingly, the value of $\sigma_{xn}(E_{c.m.})$. Since the initial DNS is in the conditional minimum
of potential energy surface, we use a two-dimensional (in $\eta$ and $R$ coordinates) Kramers-type expression for the quasistationary rates $\lambda^K_{\eta R}$ of fusion, $\lambda^K_{\eta_{sym}}$ of symmetrization of the DNS, and $\lambda^K_{\eta R}$ of quasifission through the fusion barrier in $\eta$, the barrier in $\eta$ towards symmetric DNS, and quasifission barrier in $R$, respectively. These barriers are given by the potential energy $U$ of the DNS which is calculated as the sum of binding energies $B_i$ of the DNS nuclei and of the nucleus-nucleus potential $V_{ij}$: $U = B_1 + B_2 + V_{ij}$. The binding energies for known and unknown nuclei are taken from Ref. [35] and Ref. [14] (the finite range droplet model), respectively. The uncertainty of calculated $P_{CN}$ is within the factor of 2.

The survival probability $W_{xn}(E_{c.m.}, J)$ estimates the competition between fission and neutron evaporation in the excited compound nucleus. In expression (1) the contributing angular momentum range is limited by $W_{xn}$. In the case of highly fissile SHE, $W_{xn}$ is a narrow function of $J$ different from zero in the vicinity of $J=0$ for all bombarding energies. The angular momentum dependence can be separated as

$$ W_{xn}(E_{c.m.}, J) = W_{xn}(E_{c.m.}, J = 0) \exp[-\sum_{i=1}^{n} \frac{\Delta B^\text{rot}_i}{T_1}] \approx W_{xn}(E_{c.m.}, J = 0) \exp[-\frac{J^2}{J_m^2(x)},$$

where $\Delta B^\text{rot}_i = \hbar^2 J(1+\frac{1}{2\eta_i^2} - \frac{1}{2\eta_0^2}) (\theta^i_{\text{g},s} \theta^i_{\text{s},s})$ is the moment of inertia in the ground state (at the saddle point) in $i$-th evaporation step, $\theta^i_{\text{g},s} \approx \theta^j_{\text{g},s} \approx \theta^s_{\text{g},s}$, $i \neq j$, $J_m(x) = \phi(x)J_m(x = 1)$, $J_m(x = 1) = (T_1/\hbar^2(2\theta^s_0) - \hbar^2/(2\theta^s_0))^{1/2}$ ($T_1$ is the thermodynamical temperature in In-channel) and $\phi(x) = (1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}})^{-1}$. Since in heaviest nuclei the nuclear structure is drastically changed when the nucleus moves from the ground state to the saddle point, the values of $\theta^i_{\text{g},s}$ and $\theta^i_{\text{s},s}$ are expected to be rather different in spite of small difference in the deformation parameters between the ground state and saddle point. Thus, $W_{xn}(E_{c.m.}, J)$ is cut at higher angular momenta by a gaussian-like factor. The width of this cutoff $J_m(x)$ weakly decreases with increasing $x$. For the reactions leading to SHE, $J_m(x = 1) = 10$ is used at $E_{c.m.}$ near the Coulomb barrier [31, 32]. $J_m(x = 1) = 15$ was used in the actinide region [23]. These values of $J_m$ correspond to the impact parameters less than 1 fm. In all heavy-ion complete fusion reactions above the Coulomb barrier, we have $J_m < J_{crit}$, where $J_{crit}$ is the critical angular momentum which restricts the capture cross section. Therefore, only a limited part of the angular momentum distribution of compound nucleus appreciably contributes to the evaporation residue cross section.

Using Eqs. (1) and (2), and replacing the sum over $J$ by the integral, we obtain the following approximate factorization:

$$ \sigma_{xn}(E_{c.m.}) = \sigma_{fus}^{eff}(E_{c.m.})W_{xn}(E_{c.m.}, J = 0),$$

where the effective fusion cross section

$$ \sigma_{fus}^{eff}(E_{c.m.}) = \frac{\pi \hbar^2}{\mu E_{c.m.}} \int_0^\infty \int_0^{\pi/2} \int_0^{\pi/2} \frac{dJ d\cos \Theta_1 d\cos \Theta_2}{\int_0^{\pi/2} d\cos \Theta_1 d\cos \Theta_2} J e^{-\frac{J^2}{J_m^2(x)}} P_{CN}(E_{c.m.}, J, \Theta_i)$$

contains the angular momentum dependence of survival probability. Using Eq. (3), one can extract the value of survival probability at the zero angular momentum from experimental cross section $\sigma_{xn}^{exp}(E_{c.m.})$ as

$$ W_{xn}(E_{c.m.}, J = 0) = \sigma_{xn}^{exp}(E_{c.m.})/\sigma_{fus}^{eff}(E_{c.m.}).$$

With the reduction to the zero angular momentum the survival probability becomes independent of the projectile-target combination.
The fusion probability and, correspondingly, the effective fusion cross section \( \sigma_{\text{fus}}^{\text{eff}}(E_{\text{c.m.}}) \) decreases by about 2 orders of magnitude with increasing the charge number of compound nucleus from \( Z=112 \) to \( Z=118 \). The strong fusion hindrance is due to a competition between complete fusion and quasifission in the DNS.

In Fig. 1 the extracted values of \( W_{3n} \) and \( W_{4n} \) with Eq. (5) deviates from the expected magic proton number \( Z=114 \). This indicates an increase of the stability of SHE beyond \( Z=114 \). The experimental error-bars result the error-bars in the deduced \( W_{xn} \). Since the fission barrier is determined by the shell correction energy, the absolute value of the shell correction energy is expected to be increased with \( Z \). The shell correction energy strongly depends on that how the neutron and proton numbers of the compound nucleus are closed to the magic proton and neutron numbers. The found experimental trend of the \( Q_{\alpha} \)-values in \( \alpha \)-decay chains also indicates the monotonic increase of the amplitude of the ground state shell correction energy with charge number in the region \( Z=112-118 \) [4]. One can expect an increasing stability of nuclei approaching the closed neutron \( N=184 \) shell. However, in Fig. 1 \( W_{3n}(^{296}_{140}\text{Lv}) < W_{3n}(^{297}_{179}\text{Cm}) \). This probably indicates that \( Z=114 \) is not a proper proton magic number and the next doubly magic nucleus beyond \( ^{208}_{82}\text{Pb} \) is the nucleus with \( Z \geq 120 \). The shell closure at \( Z \geq 120 \) may influence stronger on the stability of the SHE than the sub-shell closure at \( Z=114 \). Note that the experimental uncertainties seem to be too small to overcome the trends presented in Fig. 1.

Figure 1. The survival probabilities of SHE in 3\( n \)-and 4\( n \)-channels, extracted with Eq. (5) and experimental \( \sigma_{\text{exp}}^{\text{eff}} \) from Refs. [1], as functions of mass number of the compound nucleus. For the reaction \(^{48}\text{Ca}+^{238}\text{U}\), the experimental \( \sigma_{\text{exp}}^{\text{eff}} \) from GSI [4] is used as well.

3. Microscopic-macroscopic approaches based on the TCSM
The microscopic-macroscopic approaches provide a powerful tool for systematic calculations and predictions which are important for the experiments planned. We use the shape parametrization adopted for the TCSM [21] for finding the single-particle levels at the ground state of nucleus. The mirror symmetric shape parametrization used in this model effectively includes all even multipolarities. Calculating the quadrupole and hexadecapole moments, one can find the
The relationship between the deformation parameters used in the TCSM and the parameters of quadrupole $\beta_2$ and hexadecapole $\beta_4$ deformation used in the models of Refs. [14, 15, 16, 17, 18]. The ground state of the nucleus is resulted from the calculation of the potential energy surface as a function of deformation parameters [20]. Low-lying states in odd-mass nuclei are essentially determined by the unpaired nucleon. The contribution of an odd nucleon, occupying a single-particle state $|\mu\rangle$ with energy $e_\mu$, to energy of a nucleus is described by the one-quasiparticle energy $E_\mu = \sqrt{(e_\mu - \varepsilon_F)^2 + \Delta^2}$. Here, the Fermi energy $\varepsilon_F$ and the pairing-energy gap parameter $\Delta$ are calculated with the BCS approximation. The values of $\Delta$ obtained in our calculations differ from those in Refs. [14, 15, 16, 17, 18] within 0.1 MeV.

In the Nilsson-Strutinsky approach used, the dependencies of the parameters of $l_{s}$ and $l^2$ terms on $A$ and $N - Z$ are modified [20] for the correct description of the ground state spins and parities of known odd actinides. As found, this modification weakly influences the potential energy surface. The order of calculated single-particle levels seems to be close to the one in Ref. [36] for the same values of quadrupole and hexadecapole moments. Although we did not fit the parameters of the model to describe precisely the nuclear binding energies $B(Z, A)$ consisting of macroscopical and microscopical parts, the calculated $Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z - 2, A - 4)$ values for the ground state to ground state $\alpha$-decay differ within 0.3 MeV from the experimental data that is comparable with other approaches.

To demonstrate the quality of our calculations with the TCSM, the calculated energies of one-quasineutron states for $^{245}$Cm are compared in Fig. 2 with the experimental data [37]. The discrepancy in energy with well established states does not exceed 300 keV that is quite satisfactory. The Nilsson (asymptotic) quantum numbers $[Nn\Lambda]$ are assigned to each state. In addition, we calculated the one-quasiparticle states with the Quasiparticle-phonon model (QPM) [36] and Skyrme-Hartree-Fock-Bogolyubov (SHFB) approach with indicated Skyrme forces [38] (Fig. 2). One can see that all approaches provide the similar quality of the description of the experimental data. Therefore, they can be used to analyze the structure of heavy nuclei which are not well studied yet. The performed modification of the TCSM for nuclear structure calculations seem to be well confident.

![Figure 2](image-url)  
**Figure 2.** Comparison between the experimental (exp.) [37] and calculated (th.) one-quasineutron spectra for the nucleus $^{245}$Cm. The calculations within the TCSM, QPM, SHFB with SLy4 and SkP parameterizations are presented.

The calculated mass excesses $M_{th}$, neutron separation energies $B_n$, shell corrections $\delta E_{sh} = -B_f$, and $Q_\alpha$–values are listed in Ref. [22] for nuclei with $105 \leq Z \leq 126$. We treat only the isotopes of superheavy nuclei which can be reached in complete fusion reactions with available
projectiles and targets. As seen in Fig. 3, the calculated $Q_\alpha$ are in a good agreement with the available experimental data. The shell at $N = 162$ is less pronounced in our calculations than in Refs. [14, 15, 16, 17, 18] The shell effects at $Z = 114$ and $N = 172 – 176$ provide rather weak dependence of $Q_\alpha$ on $N$. The strong role of the shell at $N = 184$ is reflected in the well pronounced minimum of $Q_\alpha$. For comparison, the $Q_\alpha$–values predicted in Ref. [14] are shown in Fig. 4. As in our case, the dependence of $Q_\alpha$ on $N$ becomes weaker at $N = 172 – 176$ with the data of Refs. [14, 15, 16, 17, 18]. The small role of $N = 184$ and $Z = 120 – 126$ is seen in Fig. 4. The phenomenological model [39] results no shell effects at $N = 162$ and at $N = 172 – 176$. However, as in our calculations, it provides a strong evidence of the shell closure at $N = 184$.

![Figure 3](image1)

**Figure 3.** Calculated $\alpha$–decay energies (symbols connected by lines) are compared with available experimental data (symbols) [1, 2, 4, 40] for nuclei with $Z \geq 108$.

![Figure 4](image2)

**Figure 4.** $\alpha$–decay energies (symbols connected by lines) calculated with the macroscopic-microscopic model [14] for nuclei with $Z \geq 108$.

The value of survival probability strongly depends on $B_f - B_n$, the difference between the height $B_f$ of the fission barrier and the neutron separation energy $B_n$. At fixed charge number the predicted values of $B_n$ steadily decrease in the region of $N \geq 170$ with increasing $N$. The values of $B_n$ predicted with different models vary within 0.5 MeV and the shell effects or $B_f$ cause the difference in the dependencies of $B_f - B_n$ on $N$ in Figs. 5 and 6.

The shell corrections prevent the fission of superheavies. The value of $B_f$ is mainly determined by the amplitude of the shell correction in the ground state for nuclei with $Z \geq 105$. As a result, $B_f$ strongly depends on the neutron and proton numbers of the compound nucleus, especially, on how close they are to the magic numbers. As seen in Fig. 6, the model [14] provides the closed proton shell at $Z = 114$ and the fission barrier grows with $N$ up to $N = 178 – 180$. At fixed neutron number and $Z > 114$, the height of the fission barrier decreases with increasing deviation of $Z$ from 114.
As seen in Fig. 5, our macroscopic-microscopic approach provides the shell at $Z = 114$. However, the shell effects at $Z = 120 - 126$ are rather strong. The fission barrier increases when $N$ approaches $N = 184$. Since for nuclei with $Z = 120 - 126$ the values of $Q_\alpha$ are minimal at $Z = 120$ (Fig. 3) where the fission barriers are rather high (Fig. 5), the nuclei with $Z = 120$ and $N = 180 - 184$ are expected to be the most stable nuclei beyond those with $Z = 114$ and $N = 176 - 178$. The shell closure at $Z = 120$ is expected in accordance with the relativistic mean-field model.

Figure 5. The isotopic dependence of the value of $B_f - B_n$ with our data. The fission barrier $B_f$ is assumed to be an absolute value of the shell correction in the ground state of the nucleus. The results for the isotopes related to the indicated charge number $Z$ are shown by symbols connected by lines.

In Fig. 7, the energies of lowest two-quasiproton states for the nuclei of $\alpha$-decay chains of the SHE $^{308,310,312}126$ are calculated within our model. While for nuclei with $110 \leq Z \leq 118$ the first two-quasiproton states have energies smaller than 1.21 MeV, in nuclei with $Z = 120$ and $Z = 126$ the lowest two-quasiproton state are at about (1.68–1.91) MeV and (1.54–1.59) MeV, respectively. This indicates the larger gap in the proton single-particle spectra of nuclei with $Z = 120$ and 126. So, the proton shell effects become stronger beyond $Z = 114$. The $\alpha$-decay chain starting from $^{298}120$ will likely terminate at $^{282}$Cn by spontaneous fission [1]. The $\alpha$-decay chain starting from $^{296}120$ probably will terminate at $^{284}$Fl by spontaneous fission.
Figure 7. Calculated energies of the lowest two-quasiproton states in the indicated nuclei of $\alpha$-decay chains of $^{308}_{126}$ (solid squares), $^{310}_{126}$ (open circles), and $^{312}_{126}$ (open triangles) nuclei.

4. Intrinsic level densities in heaviest nuclei

Employing the saddle-point method, the intrinsic level density $\rho(U)$ of nucleus with $Z$ protons, $N$ neutrons, and excitation energy $U$ is expressed as

$$\rho = \frac{\exp[S(\beta, \lambda_Z, \lambda_N)]}{(2\pi)^{3/2}\sqrt{D}}.\quad (6)$$

Here, $S$ is the entropy, $\beta = T^{-1}$ is the inverse temperature, $\lambda_Z$ and $\lambda_N$ are the chemical potentials for protons and neutrons, respectively, and $D$ is the determinant of the matrix comprised of the second derivatives of the entropy [41, 42]

$$D = \begin{vmatrix}
\frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} \\
\frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 \\
\frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 & \frac{\partial^2 S}{\partial \beta \partial \mu_N}
\end{vmatrix}\quad (7)$$

with $\mu_k = \beta \lambda_k$ ($k = N, Z$).

In the superfluid model adopted here, the entropy $S$ is expressed as

$$S = 2 \sum_{k=Z,N} \sum_{\nu} \left\{ \ln[1 + \exp(-\beta E_{k\nu})] + \frac{\beta E_{k\nu}}{1 + \exp(\beta E_{k\nu})} \right\},\quad (8)$$

where proton ($k = Z$) and neutron ($k = N$) quasi-particle energies $E_{k\nu} = \sqrt{(\varepsilon_{k\nu} - \lambda_k)^2 + \Delta_k^2}$ are calculated using the single-particle levels $\varepsilon_{k\nu}$ obtained with the TCSM. The Fermi energies $\lambda_k$ and correlation functions $\Delta_k$ ($k = N, Z$), are calculated at the thermodynamic equilibrium of nucleus. At given temperature $T$, they are determined by solving the system of equations

$$Z = \sum_{\nu} \left( 1 - \frac{\varepsilon_{Z\nu} - \lambda_Z}{E_{Z\nu}} \tanh[\frac{1}{2}\beta E_{Z\nu}] \right),$$

$$N = \sum_{\nu} \left( 1 - \frac{\varepsilon_{N\nu} - \lambda_N}{E_{N\nu}} \tanh[\frac{1}{2}\beta E_{N\nu}] \right),$$

$$\frac{2}{G_Z} = \sum_{\nu} \frac{\tanh[\beta E_{Z\nu}/2]}{E_{Z\nu}},$$

$$\frac{2}{G_N} = \sum_{\nu} \frac{\tanh[\beta E_{N\nu}/2]}{E_{N\nu}}.\quad (9)$$
where $G_Z$ and $G_N$ are the constants of pairing interaction [43]. Here, the sums run over all the single-particle levels considered.

The total $E_{Z,N}$ and excitation $U$ energies of the nucleus at temperature $T$ are calculated as

$$E_{Z,N}(T) = \sum_{k=Z,N} \left\{ \sum_{\nu} \varepsilon_{kv} \left( 1 - \frac{\varepsilon_{kv} - \lambda_k}{E_{kv}} \tanh \frac{1}{2} \beta E_{kv} \right) - \frac{\Delta_k^2}{G_k} \right\},$$

$$U = E_{Z,N}(T) - E_{Z,N}(0).$$

(10)

Intrinsic level density is strongly influenced by nuclear shell structure. For magic or nearly magic nuclei, the level density is smaller than for mid-shell nuclei at the same excitation energy. This effect is related to the large single-particle spacings in the nuclei with the closed shell in the ground state. So, irregularities of the single-particle spectra are responsible for the shell corrections and peculiarities of the intrinsic level density. Note that the expectation of strong shell corrections at $Z=120$ is consistent with the predictions of relativistic shell models [9, 10, 11, 12] as well as with the recent calculations with modified Nilsson potential [44].

Using the relation between the excitation energy $U$ and entropy $S$,

$$a = S^2/(4U),$$

(11)

one can extract the level density parameter. The value of $a(U)$ smoothly approaches the asymptotic value with increasing $U$. The growth of $a$ with $U$ is sharper for the isotopes with smaller shell corrections.

We found that the best fit of the calculated intrinsic level density with the Fermi-gas expression

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}} \exp 2\sqrt{aU},$$

(12)

is achieved if one uses the level density parameter (11) and excitation energy $U$, where $S$ and $U$ are calculated as shown above.

The level density parameters $a$ for the nuclei with $Z \geq 100$ are of especial interest to look for the position of the next proton shell closure beyond $Z = 82$. Employing the method of the extraction of the level density parameter, we analyze the dependencies of $a$ on $A$, $Z$, and $N$ for three $\alpha$-decay chains containing the SHE $^{296,298,300}_{120}$ which could be synthesized in near future with available projectiles and targets. The results are presented in Fig. 8.

The lower and the middle parts of Fig. 8 present, respectively, the dependence of the shell corrections $\delta E_{sh}$ and level density parameters $a$ on the atomic number $A$ for superheavy nuclei considered. The data of Ref. [22] are used for $\delta E_{sh}$. One can see the strong correlation of these dependencies. The larger negative shell corrections result in the decrease of the value of $a$ with respect to the neighborhood nuclei. At $Z=108$ and 120 there are minima of $a$ in all chains. This reflects quite strong proton shell effects at $Z=108$ and 120. At $Z=120$, the minima of $a$ are due to the neutron shell at $N=184$.

At small excitation energy ($U=10$ MeV), one can use the parameterizations $a \approx A/(12 - 14)$ MeV for the nuclei with $Z < 116$ and $a \approx A/(14 - 17)$ MeV for the nuclei with $Z > 116$. At higher excitation energy ($U=60$ MeV), the correlation between $\delta E_{sh}$ and $a$ is destroyed. Based on the study of the dependencies of $a$ on $\delta E_{sh}$ and $U$, one can use the following parametrization [45] of the level density parameter:

$$a(A, U) = \tilde{a}(A) \left[ 1 + \frac{1 - \exp\{-U/E_D\}}{U} \delta E_{sh} \right],$$

(13)
where $\tilde{a}(A)$ is the parameter smoothly depending on $A$. It defines $a$ at large excitations when the shell effects are washed out. By analyzing the level density parameters with the Eq. (11), the value of the damping parameter $E_D'=17.3$ MeV is found. The corresponding asymptotic level density parameter $\tilde{a}(A)$ can be fitted with the following functions [45]:

$$\tilde{a}(A) = \alpha A + \beta A^2,$$

where the constants $\alpha=0.118$ MeV$^{-1}$ and $\beta=-0.53 \times 10^{-4}$ MeV$^{-1}$ are found with the least square method. These values are close to those proposed in Ref. [45].

5. Impact of SHE properties on evaporation residue cross sections

Using our predictions of nuclear properties, we calculated the evaporation residue cross sections in the reactions $^{48}$Ca,$^{50}$Ti,$^{54}$Cr,$^{58}$Fe,$^{64}$Ni+$^{238}$U,$^{244}$Pu,$^{248}$Cm,$^{249}$Cf (Fig. 9). In comparison to our previous calculations with the mass table of Ref. [14], in Fig. 9 the values of $\sigma_{ER}$ decreases slower with increasing $Z$. The stronger shell effect revealed here for nuclei with $Z > 118$ result larger survival probabilities and larger values of $\sigma_{ER}$. In the reactions $^{48}$Ca+$^{238}$U,$^{248}$Cm,$^{249}$Cf the experimental values of $\sigma_{3nER}$ are 0.5–2.5 pb, about 1 pb, and 0.5 pb [1], respectively. Thus, the difference between the calculated and experimental $\sigma_{ER}$ are within the experimental and theoretical uncertainties. A good description of existing data allows us to be confident in the predictions for the reactions with heavier projectiles.

With $^{50}$Ti beam the values of $\sigma_{ER}$ for the nuclei with $Z = 114 – 118$ are expected to be 5–10 times smaller than those resulted by $^{48}$Ca beam. The main reason for this is the decrease of $P_{CN}$ with mass asymmetry in the entrance channel of reaction. With $^{50}$Ti the nucleus $^{295}$120 is predicted to be produced with the cross section of 23 fb. In the $^{54}$Cr+$^{248}$Cm reaction the compound nucleus would have 3 neutrons more than in the $^{50}$Ti+$^{249}$Cf reaction. Therefore, the decrease of $P_{CN}$ is partly negated by the increase of $W_{sur}$ and the nucleus $^{298}$120 could be produced with the cross section of 10 fb. For the production of nuclei with $Z = 122 – 126$, $^{64}$Ni beam would lead to largest cross sections, 1–8 fb.
The calculations performed with the modified TCSM and the DNS model reveal quite strong shell effects at $N = 184$. The shell structure of heaviest nuclei predicted with macroscopic-microscopic model crucially depends on the parameters of single-particle potential and spin-orbit interaction. If our predictions of the structure of heaviest nuclei are correct, than one can expect the production of evaporation residues 120 in the reactions $^{50}\text{Ti}, ^{54}\text{Cr}, ^{58}\text{Fe}, ^{64}\text{Ni}+^{238}\text{U}, ^{244}\text{Pu}, ^{248}\text{Cm}, ^{249}\text{Cf}$ with the predicted properties of SHE. The excitation energies of compound nuclei are given in brackets.

\[ Q = -12.1\text{–}11.2\text{ MeV} \]

The evaporation residue cross-sections at the maxima of excitation functions and corresponding optimal excitation energies calculated with the mass table of Ref. [14] are presented in Fig. 10 for the reactions $^{50}\text{Ti}, ^{54}\text{Cr}, ^{58}\text{Fe}, ^{64}\text{Ni}+^{238}\text{U}, ^{244}\text{Pu}, ^{248}\text{Cm}, ^{249}\text{Cf}$. The values of $\sigma_{ER}$ decrease by about 2–3 orders of magnitude with increasing the charge number of the target from 92 to 98. The reactions with lighter targets are more favorable for larger $\sigma_{ER}$. The main reason of the fall-off of $\sigma_{ER}$ with $Z$ of the compound nucleus is the strong decrease of the fusion probability $P_{CN}$. Only the projectiles $^{50}\text{Ti}$ and $^{54}\text{Cr}$ results in the production cross-section of $Z = 114, 116, and 118 on the level of present experimental possibilities.

6. Summary

The calculations performed with the modified TCSM and the DNS model reveal quite strong shell effects at $Z = 120 – 126$. So, our macroscopic-microscopic treatment qualitatively leads to the results close to those in the mean-field treatments. The strong shell effect is at $N = 184$. The shell structure of heaviest nuclei predicted with macroscopic-microscopic model crucially depends on the parameters of single-particle potential and spin-orbit interaction. If our predictions of the structure of heaviest nuclei are correct, than one can expect the production of evaporation residues 120 in the reactions $^{50}\text{Ti}+^{249}\text{Cf}$ and $^{54}\text{Cr}+^{248}\text{Cm}$ with the cross sections $23$ and $10$ fb. The $Z = 120$ nuclei with $N = 178 – 182$ are expected to have $Q_{a}$ about $12.1–11.2$ MeV and lifetimes $1.7$ ms–$0.16$ s in accordance with our predictions. These $Q_{a}$ are in fair agreement with Ref. [39] and about 2 MeV smaller than in Refs. [14, 15, 16, 17, 18]. The experimental measurement of $Q_{a}$ for at least one isotope of $Z = 120$ nuclei would help us to set proper shell model for the SHE with $Z > 118$. Note that the definition of maxima of the excitation functions provides a good test for the predictions of the models as well.

Acknowledgments

This work was supported by RFBR and DFG. The IN2P3(France)-JINR(Dubna) and Polish-JINR(Dubna) Cooperation Programmes are gratefully acknowledged.
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