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February 1995

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Constraints from Lepton Universality at the Z Peak on Unified Theories

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ABSTRACT

We suggest the use of a universality-breaking observable based on lepton asymmetries as derived from the left-right asymmetry and the $\tau$ polarization at the Z peak, which can efficiently constrain the parameter space of unified theories. The new observable is complementary to the leptonic partial width differences and it depends critically on the chirality of a possible non-universal Z-boson coupling to like-flavour leptons. The LEP/SLC potential of probing universality violation is discussed in representative low-energy extensions of the Standard Model (SM) that could be derived by supersymmetric grand unified theories, such as the SM with left-handed and/or right-handed neutral isosinglets, the left-right symmetric model, and the minimal supersymmetric SM.

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Supersymmetric (SUSY) grand unified theories (GUTs), such as the SUSY-$SU(5)$ model [1], have received much attention due to the recent observation that the prediction obtained for the electroweak mixing angle, $\sin^2 \theta_w(\equiv s_w^2)$, is in excellent agreement with its value measured experimentally. Another interesting feature is that the supersymmetric nature of a SUSY-GUT model has the tendency to drive the unification point to higher values than usual GUTs by one or two orders of magnitude, which prevents proton from decaying too rapidly. The low-energy limit of a SUSY-GUT scenario depends crucially on the possible representations of the chiral multiplets contained and the details of the breaking mechanism from the unification scale down to the electroweak one. For instance, the SM with right-handed neutrinos could be viewed as a conceivable low-energy realization of certain SUSY-GUTs, e.g. the SUSY versions of the models discussed in Ref. [2]. Such a minimal extension of the SM allows the presence of high Dirac mass terms without contradicting constraints on the light neutrino masses [3,4]. As an immediate consequence, the one-loop vertex function relevant for the lepton-flavour-violating decays of the Higgs [5] and $Z$ bosons [6] shows a strong quadratic dependence on the heavy neutrino mass, leading to rates that could be probed at the CERN Large Electron Positron Collider (LEP). For this purpose, an observable $U_{br}$ measuring deviations from lepton universality has been suggested in Ref. [7] in order to effectively constrain possible nondecoupling effects originating from heavy Majorana neutrinos.

In this note we introduce a universality-breaking observable based on lepton asymmetries at the $Z$ peak and discuss its phenomenological implications within the framework of three representative extensions of the SM that could naturally be derived by SUSY-GUTs: (i) the SM with left-handed and/or right-handed neutral isosinglets, (ii) the left-right symmetric model, and (iii) the minimal SUSY-SM. The new observable is complementary to $U_{br}$ and depends explicitly on the chirality of a possible non-universal $Zl\bar{l}$ coupling where $l$ denotes the charged leptons $e, \mu,$ and $\tau$.

In the limit $m_l \to 0$, the transition amplitude of the decay $Z \to l\bar{l}$ can generally be given by

$$T_l = \frac{ig_w}{2c_w} \varepsilon^\mu_L u_l \gamma^\nu [g_l^L P_L + g_R^R P_R] v_l,$$

(1)

where $g_w$ is the usual electroweak coupling constant, $P_L (P_R) = (1 - (+)\gamma_5)/2$, $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$, and $\varepsilon^\mu_L$ and $u_l (v_l)$ are the polarization vector of the $Z$ boson and the
Dirac spinor of $l$ ($\bar{l}$), respectively. Furthermore, we have defined $g^i_{L,R} = g_{L,R} + \delta g^i_{L,R}$, where $g_{L} = 1 - 2s^2_\theta$ and $g_{R} = -2s^2_\theta$ are the values at tree level, and $\delta g^i_{L,R}$ are obtained beyond the Born approximation. To the first nonvanishing order of perturbation theory, the above parametrization enables us to express the universality-breaking parameter $U^{(u)}_{br}$ [7] as follows:

$$U^{(u)}_{br} = \frac{\Gamma(Z \rightarrow l\bar{l}) - \Gamma(Z \rightarrow l'l')}{\Gamma(Z \rightarrow l\bar{l}) + \Gamma(Z \rightarrow l'l')} = \frac{g_{L}(\delta g^{\nu}_{L} - \delta g^{\nu}_{R}) + g_{R}(\delta g^{\nu}_{R} - \delta g^{\nu}_{L})}{g^2_{L} + g^2_{R}}. \tag{2}$$

In Eq. (2), the known phase-space corrections coming from the masses of the charged leptons $l$ and $l'$ have been subtracted. To make contact with the corresponding observable given in [8], one can easily derive the relation: $\Gamma(Z \rightarrow l\bar{l})/\Gamma(Z \rightarrow l'l') = 2U^{(u)}_{br} + 1$. On the other hand, in the massless limit of final leptons, the lepton asymmetry $A_t$ is given by

$$A_t = \frac{\Gamma(Z \rightarrow l_{L}\bar{l}) - \Gamma(Z \rightarrow l_{R}\bar{l})}{\Gamma(Z \rightarrow l\bar{l})} = \frac{g_{L}^2 - g_{R}^2 + 2(g_{L}\delta g^{\nu}_{L} - g_{R}\delta g^{\nu}_{R})}{g^2_{L} + g^2_{R} + 2(g_{L}\delta g^{\nu}_{L} + g_{R}\delta g^{\nu}_{R})}. \tag{3}$$

Note that $A_e$ measured at LEP should equal the left-right asymmetry, $A_{LR}$, obtained at the Stanford Linear Collider (SLC). In view of the recent discrepancy of about $2\sigma$ between LEP and SLC results [9] for $A_e$ and $A_{\tau}$, respectively, we suggest using a universality-breaking parameter involving lepton asymmetries

$$\Delta A_{ul} = \frac{A_t - A'_t}{A_t + A'_t} = \frac{1}{A_{t}^{(SM)}}(U^{(u)}_{br}(L) - U^{(u)}_{br}(R)) - U^{(u)}_{br} \tag{4}$$

where $A_{t}^{(SM)}$ is the SM lepton asymmetry and $U^{(u)}_{br} = U^{(u)}_{br}(L) + U^{(u)}_{br}(R)$, with $U^{(u)}_{br}(L) = g_{L}(\delta g^{\nu}_{L} - \delta g^{\nu}_{R})/(g^2_{L} + g^2_{R})$ and $U^{(u)}_{br}(R) = g_{R}(\delta g^{\nu}_{R} - \delta g^{\nu}_{L})/(g^2_{L} + g^2_{R})$. It should be stressed that requiring $U^{(u)}_{br} = 0$ does not necessarily imply $\Delta A_{ul} = 0$. Moreover, contrary to $A_{LR}$ considered in Ref. [10], our observable $\Delta A_{ul}$ does not depend explicitly on universal electroweak oblique parameters. In the following, we will calculate $\Delta A_{ul}$ [or equivalently $U^{(u)}_{br}(L)$ and $U^{(u)}_{br}(R)$] in three illustrative and minimal extensions of the SM as mentioned above.

(i) The SM with left-handed and/or right-handed neutral isosinglets. Such a model can be obtained by adding right-handed neutrinos to the field content of the SM. We adopt the notation of Ref. [3] for the charged- and neutral-current interactions. The coupling of the $W$ boson to a charged lepton $l$ and heavy Majorana neutrinos $N_i$ is proportional to the mixing $B_{IN_i}$. The corresponding $ZN_iN_j$ coupling is governed by the mixing matrix $C_{N_iN_j}$. 

For a model with two right-handed neutrinos, for example, we have

$$B_{IN_1} = \frac{\rho^{1/4} s_{L}^{\nu_1}}{\sqrt{1 + \rho^{1/2}}} \ , \ \ B_{IN_2} = \frac{i s_{L}^{\nu_2}}{\sqrt{1 + \rho^{1/2}}} ,$$

where $\rho = m_{N_2}^2 / m_{N_1}^2$ is the square of the mass ratio between the two heavy Majorana neutrinos $N_1$ and $N_2$ predicted in such a model, and $s_{L}^{\nu_i}$ is defined as $|\sum_{j=1}^{n_\xi} |B_{IN_j}|^2|$. Furthermore, the mixings $C_{N_iN_j}$ can be obtained by $\sum_{i=1}^{n_\xi} B_{IN_i}^* B_{IN_j} = C_{N_iN_j}$. The mixing angles $(s_{L}^{\nu})^2$ are directly constrained by low-energy and LEP data [12,13]. Although some of the constraints could be model-dependent, in our analysis we will use the conservative upper limits [13]: $(s_{L}^{\nu_1})^2$, $(s_{L}^{\nu_2})^2 < 0.01$, and $(s_{L}^{\nu_3})^2 < 0.06$. Another limitation to the parameters of our model comes from the requirement of the validity of perturbative unitarity that can be violated in the limit of large heavy-neutrino masses. A qualitative estimate for the latter may be obtained by requiring that the total widths, $\Gamma_{N_i}$, and masses of neutrino fields $N_i$ satisfy the inequality $\Gamma_{N_i}/m_{N_i} < 1/2$ [7]. Taking into account dominant and subdominant nondecoupling contributions when $\lambda_{N_i} = m_{N_1}^2 / M_W^2 \gg 1$ and $\rho = m_{N_2}^2 / m_{N_1}^2 \geq 1$ [7], we find that

$$U_{br}^{(u')} = U_{br}^{(u')} (L) = \frac{\alpha_w}{8\pi} \frac{g_L}{g_L^2 + g_R^2} \left( (s_{L}^{\nu_1})^2 - (s_{L}^{\nu_2})^2 \right) \left[ 3 \ln \lambda_{N_1} + \sum_{i=1}^{n_\xi} (s_{L}^{\nu_i})^2 \frac{\lambda_{N_i}}{(1 + \rho^{1/2})^2} \left( 3\rho + \rho - 4\rho^{1/2} + \rho^2 \ln \rho \right) \right] .$$

As expected, in $SU(2)_L \otimes U(1)_Y$ models, the nature of a possible universality breaking is pure left-handed.

Another attractive low-energy scenario is an extension of the SM inspired by certain GUTs [14] and superstring theories [15,16], in which left-handed neutral singlets in addition to the right-handed neutrinos are present. In this scenario, the light neutrinos are strictly massless to all orders of perturbation theory [14], when $\Delta L = 2$ operators are absent from the Yukawa sector. The minimal case with one left-handed and one right-handed chiral singlets can effectively be recovered by the SM with two right-handed neutrinos when taking the degenerate mass limit for the two heavy Majorana neutrinos in Eq. (6). In this way, we obtain

$$U_{br}^{(u')} (L) = \frac{\alpha_w}{8\pi} \frac{g_L}{g_L^2 + g_R^2} \left( (s_{L}^{\nu_1})^2 - (s_{L}^{\nu_2})^2 \right) \left[ 3 \ln \lambda_N + \sum_{i=1}^{n_\xi} (s_{L}^{\nu_i})^2 \lambda_N \right] .$$
In Table 1, we present numerical results for both scenarios discussed above by assuming $m_{N_1} \simeq m_{N_2} = m_N$, in which case Eq. (6) leads to Eq. (7). The present experimental upper bound on $U_{br}^{(w)}$ is $|U_{br}^{(w)}| < 5 \times 10^{-3}$ [8], which automatically sets an upper limit on $|\Delta A_u| \lesssim 3\%$ since $U_{br}^{(w)}(R) = 0$. The experimental bound on $\Delta A_u$ depends on the value of $A_{1}(SM)$ we use. In fact, it is $A_{LR} = 0.1637 \pm 0.0076$ [9] and $A_r(P_{r}) = 0.143 \pm 0.010$ ($A_r(P_{r}) = 0.135 \pm 0.011$) [17] from measurements at SLC and LEP, respectively. The present experimental non-vanishing value of $\Delta A_{\tau\nu}$ has become the subject of some recent theoretical works [18]. Our aim is to show the sensitivity of $\Delta A_{\tau\nu}$ to nonoblique flavour-dependent diagonal interactions of the $Z$ boson induced by new physics. Table 1 shows theoretical predictions for $\Delta A_u$ close to values of phenomenological interest.

(ii) The left-right symmetric model. This model extends the gauge sector of the SM by an extra isospin $SU(2)_R$ group. For simplicity, we have worked out the realistic case (d) in [19], in which the vacuum expectation values of the left-handed Higgs triplet $\Delta_L$ and that of $\phi^0_2$ in the Higgs bi-doublet vanish. The model can give rise to both a left-handed and a right-handed non-universal $Z\bar{l}l$ coupling. The expression for $U_{br}^{(w)}(L)$ coincides with the one given in Eq. (6), while the dominant nondecoupling contributions to $U_{br}^{(w)}(R)$ can be obtained by calculating the Feynman graphs shown in Fig. 1. In the applicable limit where the charged gauge bosons $W_{\pm}^R$ associated with the group $SU(2)_R$ and the charged Higgs bosons $h^{\pm}$ are much heavier than the $Z$ boson, we find

$$U_{br}^{(w)}(R) = \frac{\alpha_w}{8\pi} \frac{g_R}{g_L^2 + g_R^2} \left( B^{R}_{l_n, N_i} B^{R*}_{l_n, N_j} - B^{R}_{l_n, N_i} B^{R}_{l_n, N_j} \right) \sqrt{\lambda_N \lambda_{N_i}} \times \left[ \delta_{ij} F_1 + C^{L}_{N_i N_j} F_2 + C^{L*}_{N_i N_j} F_3 \right],$$  

where $F_1$, $F_2$, and $F_3$ are form factors given by

$$F_1 = 4s_{\beta}^2 \left[ I(\lambda_R, \lambda_R, \lambda_{N_i}) - I(\lambda_R, \lambda_h, \lambda_{N_i}) \right],$$

$$F_2 = 2 \left[ I(\lambda_R, \lambda_h, \lambda_{N_i}) + I(\lambda_R, \lambda_h, \lambda_{N_j}) - I(\lambda_{N_i}, \lambda_{N_j}, \lambda_R) \right] + s_{\beta}^2 \left[ L(\lambda_h, \lambda_h, \lambda_{N_i}) + L(\lambda_h, \lambda_h, \lambda_{N_j}) - L(\lambda_{N_i}, \lambda_{N_j}, \lambda_h) \right] + \frac{s_{\beta}^2}{c_{\beta}^2} \left[ L(\lambda_{N_i}, \lambda_{N_j}, \lambda_h) - L(0, \lambda_{N_j}, \lambda_h) - L(\lambda_{N_i}, 0, \lambda_h) + L(0, 0, \lambda_h) \right],$$

$$F_3 = - \frac{2}{\sqrt{\lambda_{N_i} \lambda_{N_j}}} \left[ L(\lambda_{N_i}, \lambda_{N_j}, \lambda_R) - L(0, \lambda_{N_i}, \lambda_R) - L(\lambda_{N_i}, 0, \lambda_R) + L(0, 0, \lambda_R) \right].$$
with $\lambda_R = 1/s_3^2 = M_R^2/M_W^2$ and $\lambda_h = M_h^2/M_W^2$. In Eq. (8), $B^R$ and $C^L$ (no left-right mixing is assumed) are mixing matrices parametrizing the couplings $W_RYN$ and $ZNN$, respectively. The loop functions $I$ and $L$ in Eqs. (9)-(11) may conveniently be defined as

$$I(a, b, c) = \int_0^1 \int_0^1 y \, dxdy / \{(a(1-x) + bx)y + c(1-y)\},$$

$$L(a, b, c) = \int_0^1 \int_0^1 y \, dxdy \ln\{(a(1-x) + bx)y + c(1-y)\}.$$.

The value of $U_{\nu \nu}(R)$ depends on many kinematic variables, i.e., the masses of heavy neutrinos (in our numerical estimates we fix them to be $m_N = 4$ TeV), the $W_R$-boson mass ($M_R$), and the charged Higgs mass ($M_h$). Typically, we have set $(s_{\nu \nu}^L)^2 = 0.05$ and $(s_{\nu \nu}^R)^2 = 0.01$. As can be seen from Table 1, despite the fact that $U_{\nu \nu}(R)$ could be unobservably small of the order of $10^{-3}$ in this model, $\Delta A_{\nu \nu}$ can be as large as 10% well within the experimental reach of LEP and SLC.

(iii) The minimal supersymmetric SM. In this minimal scenario, nonvanishing values for $U_{\nu \nu}^{(W)}(L)$ and $U_{\nu \nu}^{(W)}(R)$ can be induced by left-handed and right-handed scalar leptons (denoted as $\tilde{l}_L$, $\tilde{l}_R$) as well as scalar neutrinos. To generate a non-zero non-universal $Z\tilde{t}\tilde{l}$ coupling, it is sufficient that two left-handed or right-handed scalar leptons, say $\tilde{l}$ and $\tilde{t}$, are not degenerate. In addition, we will consider the SUSY limit of the gaugino sector, where only explicit SUSY-breaking scalar-lepton mass terms are present. Only two neutralinos, the photino $\tilde{\chi}$ and the "ziggsino" $\tilde{\chi}$ with mass $m_{\chi} = M_Z$, will then contribute as shown in Fig. 2. "Ziggsino" is a Dirac fermion composed from degenerate Majorana states of a zino $\tilde{z}$ (the SUSY partner of the $Z$ boson) and one of the higgsino fields. For the sake of illustration, we will further assume that only one scalar lepton $\tilde{l}$ is relatively light whereas the others, e.g. $\tilde{t}$, are much heavier than $M_Z$. Since the decoupling theorem in softly broken SUSY theories will be operative [20], we neglect quantum effects of $\tilde{t}$. A straightforward calculation then gives

$$U_{\nu \nu}^{(W)}(L) = -\frac{\alpha_w}{8\pi} \frac{g_4^4 \cos 2\theta_L}{g_L^2 + g_R^2} \left[ g_R^2 \int_0^1 \int_0^1 dxdy \ln \left( 1 - \frac{\lambda_Z}{\lambda_{t_L}} yx(1-x) \right) \right. \left. + \lambda_Z \int_0^1 \int_0^1 dxdy y \ln \left( \frac{\lambda_{t_L} y + \lambda_Z (1-y - y^2 x(1-x))}{\lambda_{t_L} y + \lambda_Z (1-y)} \right) \right],$$

$$U_{\nu \nu}^{(W)}(R) = -\frac{\alpha_w}{8\pi} \frac{g_4^4 \cos 2\theta_R}{g_L^2 + g_R^2} \left[ \int_0^1 \int_0^1 dxdy \ln \left( 1 - \frac{\lambda_Z}{\lambda_{t_R}} yx(1-x) \right) \right].$$
Here, $\lambda_Z = M_Z^2/M_W^2$, $\lambda_{L} = m_L^2/M_W^2$, $\lambda_{R} = m_R^2/M_W^2$, and $\theta_L$ ($\theta_R$) is a mixing angle between the two left-handed (right-handed) scalar leptons $\tilde{l}_L$ ($\tilde{l}_R$) and $\tilde{l}_R$ ($\tilde{l}_L$). As has been displayed in Table 1, numerical estimates reveal that the universality-violating observables $U_{br}$ and $\Delta A$ are predicted to be no much bigger than $10^{-3}$. However, in SUSY-GUTs, numerous new fields and the corresponding SUSY partners are in general present, yielding group theoretical factors and quantum effects larger than the minimal SUSY-SM. For example, in SUSY models with right-handed neutrinos, one may get enhancements coming from the SUSY Yukawa sector, when the charged higgsinos couple to leptons and scalar neutrinos. One may therefore expect that $\Delta A_{re}$ could reach an experimentally accessible level $\sim 10^{-2}$.

In conclusion, we have considered the observable $\Delta A_{\nu}$ in Eq. (4) based on lepton asymmetries, which can effectively show a discrepancy between the left-right asymmetry at SLC and $\tau$ polarization at LEP originating from new physics. $\Delta A_{\nu}$ is sensitive to the nature of chirality of a possible nonuniversal $Zll$ coupling and is hence complementary to $U_{br}$ discussed in [7]. To precisely demonstrate this, we have analyzed conceivable low-energy scenarios of unified theories, such as the SM with neutral isosinglets, the left-right symmetric model and the minimal SUSY model, which can induce sizeable values for $\Delta A_{re}$ at the experimental visible level of $5 - 10\%$, whereas non-SM signals through the usual observable $U_{br}$ may turn out to be rather small. As is seen from Table 1, the sign of $\Delta A_{re}$ will provide a discrimination among the various theoretical scenarios beyond the SM.

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Figure and Table Captions

Fig. 1: Feynman diagrams generating a right-handed non-universal $Zl\bar{l}$ coupling in the left-right symmetric model. Contributions from wave-function renormalization constants of the charged leptons have been considered but not displayed.

Fig. 2: Feynman graphs contributing to a left-handed and right-handed non-universal $Zl\bar{l}$ coupling. In addition, selfenergies of external leptons $l$ have been taken into account.

Tab. 1: Numerical estimates of the universality-breaking observables $U_{lr}^\gamma(L)$, $U_{lr}^\gamma(R)$, and $\Delta A_{re}$ in the context of models (i), (ii), and (iii) discussed in the text. We have used the value $s_w^2 = 0.232$. 
Table 1

| Gauge | Models | $U_{br}^e$ (L) | $U_{br}^e$ (R) | $\Delta A_{re}$ |
|-------|--------|----------------|----------------|-----------------|
| (i) $(s_L^e)^2$ | $m_N$ [TeV] | | | |
| 0.060 | 4.0 | $-1.5 \times 10^{-2}$ | 0 | $-9.1 \times 10^{-2}$ |
| 0.035 | 4.0 | $-4.0 \times 10^{-3}$ | 0 | $-2.4 \times 10^{-2}$ |
| 0.020 | 4.0 | $-2.0 \times 10^{-3}$ | 0 | $-1.2 \times 10^{-2}$ |
| (ii) $M_R$ [TeV] | $M_h$ [TeV] | | | |
| 0.4 | 5 | $-1.1 \times 10^{-2}$ | 7.7 $\times 10^{-3}$ | $-0.13$ |
| 0.4 | 25 | " " | 1.0 $\times 10^{-2}$ | $-0.14$ |
| 0.4 | 50 | " " | 1.5 $\times 10^{-2}$ | $-0.18$ |
| 1.0 | 5 | " " | 1.2 $\times 10^{-3}$ | $-0.16$ |
| 1.0 | 100 | " " | 3.2 $\times 10^{-3}$ | $-0.10$ |
| 1.0 | 1000 | " " | 6.0 $\times 10^{-3}$ | $-0.12$ |
| (iii) $\theta_L = 0$ | $m_l$ [GeV] | | | |
| $\theta_R = \frac{\pi}{2}$ | 45 | 1.1 $\times 10^{-4}$ | $-6.4 \times 10^{-5}$ | 1.2 $\times 10^{-3}$ |
| $m_{l_L} = m_{l_R}$ | 60 | 5.5 $\times 10^{-5}$ | $-3.1 \times 10^{-5}$ | 6.1 $\times 10^{-4}$ |
| $= m_l$ | 100 | 2.0 $\times 10^{-5}$ | $-1.1 \times 10^{-5}$ | 2.2 $\times 10^{-4}$ |

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Fig. 1
