THE INTERGALACTIC MEDIUM

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About half a million years after the Big Bang, the ever-fading cosmic blackbody radiation cooled below 3000 K and shifted first into the infrared and then into the radio, and the smooth baryonic plasma that filled the Universe became neutral. The Universe then entered a "dark age" which persisted until the first cosmic structures collapsed into gravitationally-bound systems, and evolved into stars, galaxies, and black holes that lit up the Universe again. Some time between redshift of 7 and 15, stars within protogalaxies created the first heavy elements; these systems, together perhaps with an early population of quasars, generated the ultraviolet radiation that reheated and reionized the cosmos. The history of the Universe during and soon after these crucial formative stages is recorded in the all-pervading intergalactic medium (IGM), which is believed to contain most of the ordinary baryonic material left over from the Big Bang. Throughout the epoch of structure formation, the IGM becomes clumpy and acquires peculiar motions under the influence of gravity, and acts as a source for the gas that gets accreted, cools, and forms stars within galaxies, and as a sink for the metal enriched material, energy, and radiation which they eject. Observations of absorption lines in quasar spectra at redshifts up to 5 provide invaluable insight into the chemical composition of the IGM and primordial density fluctuation spectrum of some of the earliest formed cosmological structures, as well as of the ultraviolet background radiation that ionizes them.

1A review for the Encyclopedia of Astronomy and Astrophysics (Institute of Physics Publishing).
COSMOLOGICAL REIONIZATION

At epochs corresponding to $z \sim 1000$, the IGM is expected to recombine and remain neutral until sources of radiation develop that are capable of reionizing it. The detection of transmitted flux shortward of the Ly$\alpha$ wavelength in the spectra of sources at $z \sim 5$ implies that the hydrogen component of this IGM was ionized at even higher redshifts. There is some evidence that the double reionization of helium may have occurred later, but this is still controversial. It appears then that substantial sources of ultraviolet photons were already present when the Universe was less than 7% of its current age, perhaps quasars and/or young star-forming galaxies: an episode of pre-galactic star formation may provide a possible explanation for the widespread existence of heavy elements (like carbon, oxygen, and silicon) in the IGM, while the integrated radiation emitted from quasars is likely responsible for the reionization of intergalactic helium. Establishing the epoch of reionization and reheating is crucial for determining its impact on several key cosmological issues, from the role reionization plays in allowing protogalactic objects to cool and make stars, to determining the small-scale structure in the temperature fluctuations of the cosmic background radiation. Conversely, probing the reionization epoch may provide a means for constraining competing models for the formation of cosmic structures, and of detecting the onset of the first generation of stars, galaxies, and black holes in the Universe.

INTERGALACTIC HYDROGEN DENSITY

The proper mean density of hydrogen nuclei at redshift $z$ may be expressed in standard cosmological terms as:

$$\bar{n}_H = \left(\rho_{\text{crit}}/m_H\right)(1 - Y)\Omega_b(1 + z)^3$$

$$= (1.1 \times 10^{-5} \text{ cm}^{-3})(1 - Y)\Omega_b h^2 (1 + z)^3,$$

where $Y$ is the primordial He abundance by mass, $\rho_{\text{crit}} = 3H_0^2/(8\pi G)$ is the critical density, $\Omega_b = \rho_b/\rho_{\text{crit}}$ is the current baryonic density parameter, and $H_0 = 100 h \text{ km s}^{-1}\text{ Mpc}^{-1}$ is the present-day Hubble constant. Standard nucleosynthesis models together with recent observations of Deuterium yield $Y = 0.247 \pm 0.02$, and $\Omega_b h^2 = 0.0193 \pm 0.0014$. Thus,

$$\bar{n}_H = (1.6 \times 10^{-7} \text{ cm}^{-3}) \left(\frac{\Omega_b h^2}{0.019}\right) (1 + z)^3.$$  

As some of the baryons had already collapsed into galaxies at $z = 2 - 5$, the value of $\Omega_b h^2 = 0.019$ should strictly be considered as an upper limit to the intergalactic density parameter.
Because of the overwhelming abundance of hydrogen, the ionization of this element is of great importance for determining the physical state of the IGM. Popular cosmological models predict that most of the intergalactic hydrogen was reionized by the first generation of stars or quasars at $z = 7 - 15$. The case that has received the most theoretical studies is one where hydrogen is ionized by the absorption of photons, $H + \gamma \rightarrow p + e$ (as opposite to collisional ionization $H + e \rightarrow p + e + e$) shortward of 912 Å; that is, with energies exceeding 13.6 eV, the energy of the Lyman edge. The process of reionization began as individual sources started to generate expanding $H$ II regions in the surrounding IGM; throughout an $H$ II region, $H$ is ionized and He is either singly or doubly ionized. As more and more sources of ultraviolet radiation switched on, the ionized volume grew in size. The reionization ended when the cosmological $H$ II regions overlapped and filled the intergalactic space.

PHOTOIONIZATION EQUILIBRIUM

At every point in an optically thin, pure hydrogen medium of neutral density $n_{\text{HI}}$, the photoionization rate per unit volume is

$$n_{\text{HI}} \int_{\nu_{\text{L}}}^{\infty} \frac{4\pi J_\nu \sigma_{\text{H}}(\nu)}{h_\nu} d\nu,$$

(4)

where $J_\nu$ is the mean intensity of the ionizing radiation (in energy units per unit area, time, solid angle, and frequency interval), and $h_\nu$ is the Planck constant. The photoionization cross-section for hydrogen in the ground state by photons with energy $h_\nu \nu$ (above the threshold $h_\nu \nu_L = 13.6$ eV) can be usefully approximated by

$$\sigma_{\text{H}}(\nu) = \sigma_L (\nu/\nu_L)^{-3},$$

$$\sigma_L = 6.3 \times 10^{-18} \text{ cm}^2.$$  

(5)

At equilibrium, this is balanced by the rate of radiative recombinations $p + e \rightarrow H + \gamma$ per unit volume,

$$n_ep_p \alpha_A(T),$$

(6)

where $n_e$ and $n_p$ are the number densities of electrons and protons, and $\alpha_A = \sum \langle \sigma_n v_e \rangle$ is the radiative recombination coefficient, i.e. the product of the electron capture cross-section $\sigma_n$ and the electron velocity $v_e$, averaged over a thermal distribution and summed over all atomic levels $n$. At the commonly encountered gas temperature of $10^4$ K, $\alpha_A = 4.2 \times 10^{-13}$ cm$^3$s$^{-1}$.

Consider, as an illustrative example, a point in an intergalactic $H$ II region at (say) $z = 6$, with density $\bar{n}_H = (1.6 \times 10^{-7} \text{ cm}^{-3})(1 + z)^3 = 5.5 \times 10^{-5} \text{ cm}^{-3}$. The $H$ II region surrounds a putative quasar with specific luminosity $L_\nu = 10^{30} (\nu L/\nu)^2$ ergs s$^{-1}$ Hz$^{-1}$, and the point in question is at a distance of $r = 3$ Mpc from the quasar. To a first approximation, the mean intensity is simply the radiation emitted by the quasar reduced by geometrical dilution,

$$4\pi J_\nu = \frac{L_\nu}{4\pi r^2}.$$  

(7)
We then have for the photoionization timescale:

\[ t_{\text{ion}} = \left[ \int_{\nu_L}^{\infty} \frac{4\pi J_\nu \sigma_H(\nu)}{h_\nu} d\nu \right]^{-1} = 5 \times 10^{12} \text{ s}, \]  

(8)

and for the recombination timescale:

\[ t_{\text{rec}} = \frac{1}{n_e \alpha_A} = 5 \times 10^{16} \text{ s} \frac{\bar{n}_H}{n_e}. \]

(9)

As in photoionization equilibrium \( n_{\text{HI}} / t_{\text{ion}} = n_p / t_{\text{rec}} \), these values imply \( n_{\text{HI}} / n_p \simeq 10^{-4} \), that is, hydrogen is very nearly completely ionized.

A source radiating ultraviolet photons at a finite rate cannot ionize an infinite region of space, and therefore there must be an outer edge to the ionized volume (this is true unless, of course, there is a population of UV emitters and all individual H II regions have already overlapped). One fundamental characteristic of the problem is the very small value of the mean free path for an ionizing photon if the hydrogen is neutral, \( (\sigma_L n_H)^{-1} = 0.9 \text{ kpc} \) at threshold, much smaller than the radius of the ionized region. If the source spectrum is steep enough that little energy is carried out by more penetrating, soft X-ray photons, we have one nearly completely ionized H II region, separated from the outer neutral IGM by a thin transition layer or ‘ionization-front’. The inhomogeneity of the IGM is of primary importance for understanding the ionization history of the Universe, as denser gas recombines faster and is therefore ionized at later times than the tenuous gas in underdense regions. An approximate way to study the effect of inhomogeneity is to write the rate of recombinations as

\[ \langle n_e n_p \rangle \alpha_A(T) = C \langle n_e \rangle^2 \alpha_A(T) \]

(10)

(assuming \( T \) is constant in space), where the brackets are the space average of the product of the local proton and electron number densities, and the factor \( C > 1 \) takes into account the degree of clumpiness of the IGM. If ionized gas with electron density \( n_e \) density filled uniformly a fraction \( 1/C \) of the available volume, the rest being empty space, the mean square density would be \( \langle n_e^2 \rangle = n_e^2 / C = \langle n_e \rangle^2 C. \)

The IGM is completely reionized when one ionizing photon has been emitted for each H atom by the radiation sources, and when the rate of emission of UV photons per unit (comoving) volume balances the radiative recombination rate, so that hydrogen atoms are photoionized faster than they can recombine. The complete reionization of the Universe manifests itself in the absence of an absorption trough in the spectra of galaxies and quasars at high redshifts. If the IGM along the line-of-sight to a distant source were neutral, the
resonant scattering at the wavelength of the Ly$\alpha$ ($2p \rightarrow 1s; h_P\nu_\alpha = 10.2$ eV) transition of atomic hydrogen would remove all photons blueward of Ly$\alpha$ off the line-of-sight. For any reasonable density of the IGM, the scattering optical depth is so large that detectable absorption will be produced by relatively small column (or surface) densities of intergalactic neutral hydrogen.

**GUNN-PETTERSON EFFECT**

Consider radiation emitted at some frequency $\nu_e$ that lies blueward of Ly$\alpha$ by a source at redshift $z_e$, and observed at Earth at frequency $\nu_o = \nu_e(1+z_e)^{-1}$. At a redshift $z$ such that $(1+z) = (1+z_e)\nu_\alpha/\nu_e$, the emitted photons pass through the local Ly$\alpha$ resonance as they propagate towards us through a smoothly distributed sea of neutral hydrogen atoms, and are scattered off the line-of-sight with a cross-section (neglecting stimulated emission) of

$$\sigma[\nu_o(1+z)] = \frac{\pi e^2}{m_e c}\phi[\nu_o(1+z)], \tag{11}$$

where $f = 0.4162$ is the upward oscillator strength for the transition, $\phi$ is the line profile function [with normalization $\int \phi(\nu)d\nu = 1$], $c$ is the speed of light, and $e$ and $m_e$ are the electron charge and mass, respectively. The total optical depth for resonant scattering at the observed frequency is given by the line integral of this cross-section times the neutral hydrogen proper density $n_{\text{HI}}(z)$,

$$\tau_{\text{GP}} = \int_0^{z_e} \sigma[\nu_o(1+z)]n_{\text{HI}}(z)\frac{d\ell}{dz}dz, \tag{12}$$

where $d\ell/dz = cH_0^{-1}(1+z)^{-1}[\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda]^{-1/2}$ is the proper line element in a Friedmann-Robertson-Walker metric, and $\Omega_M$, $\Omega_\Lambda$, and $\Omega_K = 1 - \Omega_M - \Omega_\Lambda$ are the matter, vacuum, and curvature contribution to the present density parameter. As the scattering cross-section is sharply peaked around $\nu_\alpha$, we can write

$$\tau_{\text{GP}}(z) = \left(\frac{\pi e^2 f}{m_e c\nu_\alpha}\right) \frac{n_{\text{HI}}}{(1+z)} \frac{d\ell}{dz}. \tag{13}$$

In an Einstein-de Sitter ($\Omega_M = 1$, $\Omega_\Lambda = 0$) Universe, this becomes

$$\tau_{\text{GP}}(z) = \frac{\pi e^2 f}{m_e H_0\nu_\alpha} \frac{n_{\text{HI}}}{(1+z)^{3/2}} = 6.6 \times 10^3 h^{-1} \left(\frac{\Omega_b h^2}{0.019}\right) \frac{n_{\text{HI}}}{\bar{n}_{\text{H}}} (1+z)^{3/2}. \tag{14}$$

The same expression for the opacity is also valid in the case of optically thin (to Ly$\alpha$ scattering) discrete clouds as long as $n_{\text{HI}}$ is replaced with the average neutral density of individual clouds times their volume filling factor.
In an expanding Universe homogeneously filled with neutral hydrogen, the above equations apply to all parts of the source spectrum to the blue of Lyα. An absorption trough should then be detected in the level of the rest-frame UV continuum of the quasar; this is the so-called “Gunn-Peterson effect”. Between the discrete absorption lines of the Lyα forest clouds, quasar spectra do not show a pronounced Gunn-Peterson absorption trough. The current upper limit at \( z_e \approx 5 \) is \( \tau_{\text{GP}} < 0.1 \) in the region of minimum opacity, implying from equation (14) a neutral fraction of \( \frac{n_{\text{HI}}}{\bar{n}_\text{H}} < 10^{-6} h \). Even if 99% of all the cosmic baryons fragment at these epochs into structures that can be identified with quasar absorption systems, with only 1% remaining in a smoothly distributed component, the implication is a diffuse IGM which is ionized to better than 1 part in \( 10^4 \).

In modern interpretations of the IGM, it is difficult to use the Gunn-Peterson effect to quantify the amount of ionizing radiation that is necessary to keep the neutral hydrogen absorption below the detection limits. This is because, in hierarchical clustering scenarios for the formation of cosmic structures (the Cold Dark Matter model being the most studied example), the accumulation of matter in overdense regions under the influence of gravity reduces the optical depth for Lyα scattering considerably below the average in most of the volume of the Universe, and regions of minimum opacity occur in the most underdense areas (expanding ‘cosmic minivoids’).

A CLUMPY IGM

Owing to the non-linear collapse of cosmic structures, the IGM is well known to be highly inhomogeneous. The discrete gaseous systems detected in absorption in the spectra of high-redshift quasars blueward of the Lyα emission line are assigned different names based on the appearance of their absorption features (see Figure 1).

The term “Lyα forest” is used to denote the plethora of narrow absorption lines whose measured equivalent widths imply \( \text{H} \text{I} \) column densities ranging from \( 10^{16} \text{ cm}^{-2} \) down to \( 10^{12} \text{ cm}^{-2} \). These systems, observed to evolve rapidly with redshift between \( 2 < z < 4 \), have traditionally been interpreted as intergalactic gas clouds associated with the era of baryonic infall and galaxy formation, photoionized (to less than a neutral atom in \( 10^4 \)) and photoheated (to temperatures close to 20,000 K) by an ultraviolet background close to the one inferred from the integrated contribution from quasars. Recent spectra at high resolution and high signal-to-noise obtained with the \textit{Keck} telescope have shown that most Lyα forest clouds at \( z \sim 3 \) down to the detection limit of the data have undergone some chemical enrichment, as evidenced by weak, but measurable C IV lines. The typical inferred metallicities range from 0.3% to 1% of solar values, subject to uncertainties of photoionization models. Clearly, these metals were produced in stars that formed in a denser environment; the metal-enriched
gas was then expelled from the regions of star formation into the IGM.

An intervening absorber at redshift $z$ having a neutral hydrogen column density exceeding $2 \times 10^{17} \text{cm}^{-2}$ is optically thick to photons having energy greater than 13.6 eV, and produces a discontinuity at the hydrogen Lyman limit, i.e. at an observed wavelength of 912 $(1+z)$Å. These scarcer “Lyman-limit systems” (LLS) are associated with the extended gaseous haloes of bright galaxies near the line-of-sight, and have metallicities which appear to be similar to that of Lyα forest clouds.

In “damped Lyα systems’ the H $\text{i}$ column is so large ($N_{\text{HI}} \gtrsim 10^{20} \text{cm}^{-2}$, comparable to the interstellar surface density of spiral galaxies today) that the radiation damping wings of the Lyα line profile become detectable. While relatively rare, damped systems account for most of the neutral hydrogen seen at high redshifts. The typical metallicities are about 10% of solar, and do not evolve significantly over a redshift interval $0.5 < z < 4$ during which most of today’s stars were actually formed.

Except at the highest column densities, discrete absorbers are inferred to be strongly photoionized. From quasar absorption studies we also know that neutral hydrogen accounts for only a small fraction, $\sim 10\%$, of the nucleosynthetic baryons at early epochs.

**DISTRIBUTION OF COLUMN DENSITIES AND EVOLUTION**

The bivariate distribution $f(N_{\text{HI}}, z)$ of H $\text{i}$ column densities and redshifts is defined by the probability $dP$ that a line-of-sight intersects a cloud with column density $N_{\text{HI}}$ in the range $dN_{\text{HI}}$, at redshift $z$ in the range $dz$,

$$dP = f(N_{\text{HI}}, z)dN_{\text{HI}}dz.$$  \hspace{1cm} (15)

As a function of column, a single power-law with slope $-1.5$ appears to provide at high redshift a surprisingly good description over 9 decades in $N_{\text{HI}}$, i.e. from $10^{12}$ to $10^{21} \text{cm}^{-2}$. It is a reasonable approximation to use for the distribution of absorbers along the line-of-sight:

$$f(N_{\text{HI}}, z) = A N_{\text{HI}}^{-1.5}(1+z)^{\gamma}. \hspace{1cm} (16)$$

Lyα forest clouds and Lyman-limit systems appear to evolve at slightly different rates, with $\gamma = 1.5 \pm 0.4$ for the LLS and $\gamma = 2.8 \pm 0.7$ for the forest lines. Let us assume, for simplicity, a single redshift exponent, $\gamma = 2$, for the entire range in column densities. In the power-law model (16) the number $N$ of absorbers with columns greater than $N_{\text{HI}}$ per unit increment of redshift is

$$\frac{dN}{dz} = \int_{N_{\text{HI}}}^{\infty} f(N'_{\text{HI}}, z)dN'_{\text{HI}} = 2AN_{\text{HI}}^{-0.5}(1+z)^2. \hspace{1cm} (17)$$
Fig. 1.— High resolution ($\lambda/\Delta\lambda = 37,000$) spectrum of the $z_e = 3.50$ quasar Q1159+123, taken with the Keck High Resolution Spectrograph (exposure time 8 h). The data are taken from Songaila (1998). The Ly$\alpha$ forest is clearly seen in absorption blueward of the atomic hydrogen Ly$\alpha$ emission line from the quasar (the broad peak at 5470 Å), and is produced by resonant Ly$\alpha$ scattering in gas clouds along the line-of-sight between us and the quasar. The spectrum shows a Lyman-limit system just shortward of 4150 Å, i.e. close to the quasar emission redshift. Most of the features between the Ly$\alpha$ and C IV emission (the other broad peak just below 7000 Å) are C IV intergalactic absorption lines.
A normalization value of $A = 4.0 \times 10^7$ produces then $\sim 3$ LLS per unit redshift at $z = 3$, and, at the same epoch, $\sim 150$ forest lines above $N_{\text{HI}} = 10^{13.8} \text{ cm}^{-2}$, in reasonable agreement with the observations.

If absorbers at a given surface density are conserved, with fixed comoving space number density $n = n_0(1 + z)^3$ and geometric cross-section $\Sigma$, then the intersection probability per unit redshift interval is

$$\frac{dP}{dz} = \Sigma n \frac{d\ell}{dz} = \Sigma n_0(1 + z)^3 \frac{d\ell}{dz}. \quad (18)$$

If the Universe is cosmologically flat, the expansion rate at early epochs is close to the Einstein-de Sitter limit, and the redshift distribution for conserved clouds is predicted to be

$$\frac{dP}{dz} \propto (1 + z)^3 \frac{d\ell}{dz} \propto (1 + z)^{1/2}. \quad (19)$$

The rate of increase of $f(N_{\text{HI}}, z)$ with $z$ in both the Ly$\alpha$ forest and LLS is considerably faster than this, indicating rapid evolution. The mean proper distance between absorbers along the line-of-sight with columns greater than $N_{\text{HI}}$ is

$$L = \frac{d\ell}{dz} \approx c \frac{N_{\text{HI}}^{1/2}}{H_0 \Omega_M^{1/2} 2A(1 + z)^{4.5}}. \quad (20)$$

For clouds with $N_{\text{HI}} > 10^{14}$ cm$^{-2}$, this amounts to $L \sim 0.7 h^{-1} \Omega_M^{-1/2}$ Mpc at $z = 3$. At the same epoch, the mean proper distance between LLS is $L \sim 30 h^{-1} \Omega_M^{-1/2}$ Mpc.

### INTERGALACTIC CONTINUUM OPACITY

Even if the bulk of the baryons in the Universe are fairly well ionized at all redshifts $z \lesssim 5$, the residual neutral hydrogen still present in the Ly$\alpha$ forest clouds and Lyman-limit systems significantly attenuates the ionizing flux from cosmological distant sources. To quantify the degree of attenuation we have to introduce the concept of an effective continuum optical depth $\tau_{\text{eff}}$ along the line-of-sight to redshift $z$,

$$\langle e^{-\tau} \rangle \equiv e^{-\tau_{\text{eff}}}, \quad (21)$$

where the average is taken over all lines-of-sight. Neglecting absorption due to helium, if we characterize the Ly$\alpha$ forest clouds and LLS as a random distribution of absorbers in column density and redshift space, then the effective continuum optical depth of a clumpy IGM at the observed frequency $\nu_o$ for an observer at redshift $z_o$ is

$$\tau_{\text{eff}}(\nu_o, z_o, z) = \int_{z_o}^{z} dz' \int_{0}^{\infty} dN_{\text{HI}} f(N_{\text{HI}}, z)(1 - e^{-\tau}). \quad (22)$$
where $\tau = N_{\text{HI}} \sigma_H(\nu)$ is the hydrogen Lyman continuum optical depth through an individual cloud at frequency $\nu = \nu_o(1 + z)/(1 + z_o)$. This formula can be easily understood if we consider a situation in which all absorbers have the same optical depth $\tau_0$ independent of redshift, and the mean number of systems along the path is $\Delta N = \int dz dN/dz$. In this case the Poissonian probability of encountering a total optical depth $k\tau_0$ along the line-of-sight (with $k$ integer) is $p(k\tau_0) = e^{-\Delta N}\Delta N^k/(k!)$, and $\langle e^{-\tau} \rangle = e^{-k\tau_0}p(k\tau_0) = \exp[-\Delta N(1 - e^{-\tau_0})]$.

If we extrapolate the $N_{\text{HI}}^{-1.5}$ power-law in equation (16) to very small and large columns, the effective optical depth becomes an analytical function of redshift and wavelength,

$$
\tau_{\text{eff}}(\nu, z_o, z) = \frac{4}{3} \sqrt{\pi \sigma_L} A \left(\frac{\nu_o}{\nu_L}\right)^{-1.5} (1 + z_o)^{1.5} \left[(1 + z)^{1.5} - (1 + z_o)^{1.5}\right].
$$

(23)

Due to the rapid increase with lookback time of the number of absorbers, the mean free path of photons at 912 Å becomes so small beyond a redshift of 2 that the radiation field is largely ‘local’. Expanding equation (23) around $z$, one gets $\tau_{\text{eff}}(\nu_L) \approx 0.36(1 + z)^2 \Delta z$. This means that at $z = 3$, for example, the mean free path for a photon near threshold is only $\Delta z = 0.18$, and sources of ionizing radiation at higher redshifts are severely attenuated.

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