Confinement, Chiral Symmetry and Hadrons

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Abstract
The Polyakov loop is the appropriate deconfinement order parameter for Yang-Mills theories without quarks or with quarks in the adjoint representation of the gauge group. However it is not a physical state of the theory so the information regarding the center group symmetry must be transferred to the physical states. We briefly review how this transfer of information takes place. When adding quarks the center group is no longer a symmetry for matter in the fundamental representation. This feature allows us to explain why color deconfines when chiral symmetry is restored in hot gauge theories with massless quarks. For quarks in the adjoint representation we show that while deconfinement and the chiral transition do not coincide, entanglement between them is still present. Finally, we discuss also the chemical potential driven phase transition.

Recently in [1, 2, 3] we have analyzed the problem of how, and to what extent the information encoded in the order parameter of a generic theory is transferred to the non-order parameter fields. This is a fundamental problem since in nature most fields are non-order parameter fields. This problem is especially relevant in QCD and QCD-like theories since there is no physical observable for deconfinement which is directly linked to the order parameter field. At nonzero temperature the $Z_{N_c}$ center of $SU(N_c)$ is a relevant global symmetry [5], and it is possible to construct the Polyakov loop $\ell$ operator. This object is charged with respect to the center $Z_{N_c}$ of the $SU(N_c)$ gauge group [5], under which it transforms as $\ell \rightarrow z\ell$ with $z \in Z_{N_c}$. A relevant feature of the Polyakov loop is that its expectation value vanishes in the low temperature regime, and is non-zero in the high temperature phase. The Polyakov loop is thus a suitable order parameter for the Yang-Mills temperature driven phase transition [5].

The question is: How can the information about the Yang-Mills phase transition encoded in the $Z_{N_c}$ global symmetry be communicated to the hadronic states of the theory. In [1, 2], using an effective Lagrangian approach, we have explicitly shown that is always possible on symmetry grounds to couple $\ell$ to the hadronic operators. Effective Lagrangians play a relevant role. In [4], for example, a number of fundamental properties for QCD were derived. Of course, at a more elementary level the picture is that the gauge invariant operators of the theory carrying center group charges couple to hadronic states. If not protected by symmetries only an unnatural act would require these relevant couplings to vanish. Such a picture is also confirmed within theoretical models [6]. We have also showed [2] that spatial correlators of the non-critical field are dominated at the phase transition by the critical behavior of the order parameter field. In this way the Polyakov loop field imprints the knowledge of the phase transition on the non order parameter spatial correlators (e.g. the glueball field). Our results are applicable to any phase transition and as such are universal.

The picture changes considerably when quarks are added to the theory. If fermions are in the fundamental and pseudoreal representations for $N_c = 3$ and $N_c = 2$, respectively, the corresponding $Z_3$ or $Z_2$ center of the group is never a good symmetry. For massless quarks the order parameter is the chiral condensate which characterizes the chiral phase transition. For $N_c = 3$ and two massless quark flavors at finite temperature and zero baryon density, the chiral phase transition is in the same universality class

\begin{footnote}
Speaker at the workshop.
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as the three dimensional $O(4)$ spin model \cite{7}, becoming a smooth crossover as small quark masses are accounted for \cite{8}. For $N_c = 2$ the relevant universality class is that of $O(6)$ both for the fundamental and adjoint representations \cite{9}. Even if the discrete symmetry is broken, one can still construct the Polyakov loop and study the temperature dependence of its properties on the lattice. One still observes a rise of the Polyakov loop from low to high temperatures and naturally, although improperly, one speaks of deconfining phase transition \cite{10}. For fermions in the adjoint representation the center of the group remains a symmetry of the theory, and thus besides the chiral condensate, also the Polyakov loop is an order parameter.

Interestingly, lattice results \cite{10} indicate that for ordinary QCD with quarks in the fundamental representation, chiral symmetry breaking and confinement (i.e. a decrease of the Polyakov loop) occur at the same critical temperature. Lattice simulations also indicate that these two transitions do not happen simultaneously when the quarks are in the adjoint representation. Despite the attempts to explain these behaviors \cite{11}, the underlying reasons are still unknown. Recently, using an interesting model containing only hadrons this issue has also been addressed for ordinary QCD in \cite{12}.

In \cite{3} we have proposed a solution to this puzzle based on the general approach established in \cite{2}, envisioned first in \cite{1}. Two general features introduced in \cite{2} are essential: There exists a relevant trilinear interaction between the light order parameter and the heavy non-order parameter field, singlet under the symmetries of the order parameter field. This allows for an efficient transfer of information from the order parameter to the fields that are singlets with respect to the symmetry of the theory. As a result, the non-critical fields have infrared dominated spatial correlators. The second feature, also due to the existence of such an interaction, is that the finite expectation value of the order parameter field in the symmetry broken phase induces a variation in the expectation value for the singlet field, whose value generally is non-vanishing in the unbroken phase.

1. Fundamental Representation

Here we study the behavior of the Polyakov loop by treating it as a heavy field that is a singlet under chiral symmetry transformations. We take the underlying theory to be two colors and two flavors in the fundamental representation. The degrees of freedom in the chiral sector of the effective theory are $2N_f - N_f - 1$ Goldstone fields $\pi^a$ and a scalar field $\sigma$. For $N_f = 2$ the potential is \cite{13}:

$$V_{\text{ch}}[\sigma, \pi^a] = \frac{m^2}{2} \text{Tr} \left[ M^\dagger M \right] + \lambda_1 \text{Tr} \left[ M^\dagger M \right]^2 + \frac{\lambda_2}{4} \text{Tr} \left[ M^\dagger M M^\dagger M \right]$$

(1)

with $2M = \sigma + i 2\sqrt{2} \pi^a X^a$, $a = 1, \ldots, 5$ and $X^a \in \mathcal{A}(SU(4)) - \mathcal{A}(Sp(4))$. $X^a$ are the generators provided explicitly in equation (A.5) and (A.6) of \cite{13}. The Polyakov loop potential in the absence of the $\mathbb{Z}_2$ symmetry is

$$V_{\chi}[\chi] = g_0 \chi + \frac{m_0^2}{2} \chi^2 + \frac{g_3}{3} \chi^3 + \frac{g_4}{4} \chi^4.$$  

(2)

The field $\chi$ represents the Polyakov loop itself. To complete the effective theory we introduce interaction terms allowed by the chiral symmetry

$$V_{\text{int}}[\chi, \sigma, \pi^a] = \left( g_1 \chi + g_2 \chi^2 \right) \text{Tr} \left[ M^\dagger M \right] = \left( g_1 \chi + g_2 \chi^2 \right) (\sigma^2 + \pi^a \pi^a).$$

(3)

In the phase with $T < T_{\text{cr}}$, where chiral symmetry is spontaneously broken, $\sigma$ acquires a nonzero expectation value, which in turn induces a modification also for $\langle \chi \rangle$. The usual choice for vacuum alinement is in the $\sigma$ direction, i.e. $\langle \pi \rangle = 0$. The extremum of the linearized potential is at

$$\langle \sigma \rangle^2 \simeq - \frac{m_\sigma^2}{\lambda}, \quad m_\sigma^2 \simeq m^2 + 2g_1 \langle \chi \rangle, \quad \text{and} \quad \langle \chi \rangle \simeq \chi_0 - \frac{g_1}{m_\chi^2} (\sigma)^2, \quad \chi_0 \simeq - \frac{g_0}{m_\chi^2},$$

(4)
where \( \lambda = \lambda_1 + \lambda_2 \). Here \( m_2^2 \) is the full coefficient of the \( \sigma^2 \) term in the tree-level Lagrangian which, due to the coupling between \( \chi \) and \( \sigma \), also depends on \( \langle \chi \rangle \). Spontaneous chiral symmetry breaking appears for \( m_2^2 < 0 \). In this regime the positive mass squared of the \( \sigma \) is \( M_\sigma^2 = 2\lambda \langle \sigma^2 \rangle \). The formulae in (4) hold near the phase transition where \( \langle \sigma \rangle \) is small. We have ordered the couplings such that \( g_0/m_\chi \) and \( g_1/m_\chi \) are both much greater than \( g_2 \) and \( g_3/m_\chi \). Near the critical temperature the mass of the order parameter field is assumed to posses the generic behavior \( m_\chi^2 \sim (T - T_c)^x \). Equation (4) shows that for \( g_1 > 0 \) and \( g_0 < 0 \) the expectation value of \( \chi \) behaves oppositely to that of \( \sigma \). As the chiral condensate starts to decrease towards chiral symmetry restoration, the expectation value of the Polyakov loop starts to increase, signaling the onset of deconfinement. This is illustrated in the left panel of figure

When applying the analysis presented in [2], the general behavior of the spatial two-point correlator of the Polyakov loop can be obtained. Near the transition point, in the broken phase, the \( \chi \) two-point function is dominated by the infrared divergent \( \sigma \)-loop. This is so, because the \( \pi^a \) Goldstone fields couple only derivatively to \( \chi \), and thus decouple. We find a drop in the screening mass of the Polyakov loop at the phase transition. When approaching the transition from the unbroken phase the Goldstone fields do not decouple, but follow the \( \sigma \), resulting again in the drop of the screening mass of the Polyakov loop close to the phase transition. We consider the variation of the \( \chi \) mass near the phase transition with respect to the tree level mass \( m_\chi \) defined above the chiral phase transition. We define \( \Delta m_\chi^2(T) = m_\chi^2(T) - m_\chi^2 \), where \( m_\chi \) is the mass at a given temperature near the critical point. Using a large \( N \) framework motivated resummation [2] we deduce:

\[
\Delta m_\chi^2(T) = -\frac{2g_1^2(1 + N_\pi)}{8\pi m_\sigma + (1 + N_\pi)3\lambda} \quad T > T_{c\sigma} \quad \text{and} \quad \Delta m_\chi^2(T) = -\frac{2g_1^2}{8\pi M_\sigma + 3\lambda} \quad T < T_{c\sigma}.
\]

From the above equations one finds that the screening mass of the Polyakov loop is continuous and finite at \( T_{c\sigma} \), and \( \Delta m_\chi^2(T_{c\sigma}) = -2g_1^2/(3\lambda) \), independent of \( N_\pi \), the number of pions. This analysis is not restricted to the chiral/deconfining phase transition. The entanglement between the order parameter (the chiral condensate) and the non-order parameter field (the Polyakov loop) is universal.

2. Adjoint Representation

As a second application, consider two color QCD with two massless Dirac quark flavors in the adjoint representation. Here the global symmetry is \( SU(2N_f) \) which breaks via a bilinear quark condensate to \( O(2N_f) \). The number of Goldstone bosons is \( 2N_f^2 + N_f - 1 \). We take \( N_f = 2 \). There are two exact order parameter fields: the chiral \( \sigma \) field and the Polyakov loop \( \chi \). Since the relevant interaction term \( g_1\chi\sigma^2 \) is now forbidden, one might expect no efficient information transfer between them. This naive statement is supported by lattice data [10]. While respecting general expectations the following analysis suggests the presence of a new and more elaborated structure which lattice data can clarify in the near future.

The chiral part of the potential is given by [11] with \( 2M = \sigma + i 2\sqrt{2}\pi^a X^a \), \( a = 1, \ldots, 9 \) and \( X^a \in \mathcal{A}(SU(4)) - \mathcal{A}(O(4)) \). \( X^a \) are the generators provided explicitly in equation (A.3) and (A.5) of [13]. The \( \mathbb{Z}_2 \) symmetric potential for the Polyakov loop is

\[
V_\chi[\chi] = \frac{m_\chi^2}{2}\chi^2 + \frac{g_1}{4}\chi^4,
\]

and the only interaction term allowed by symmetries is

\[
V_{\text{int}}[\chi, \sigma, \pi] = g_2\chi^2 \text{Tr} \left[ M^\dagger M \right] = g_2\chi^2(\sigma^2 + \pi^a\pi^a).
\]

Here we review the case in which the chiral phase transition happens first, i.e. \( T_{c\chi} \ll T_{c\sigma} \). For \( T_{c\chi} < T < T_{c\sigma} \) both symmetries are broken, and the expectation values of the two order parameter
fields are linked to each other:

\[
\langle \sigma \rangle^2 = -\frac{1}{\lambda} \left( m^2 + 2 g_2 \langle \chi \rangle^2 \right) \equiv -\frac{m^2_\sigma}{\lambda}, \quad \langle \chi \rangle^2 = -\frac{1}{g_4} \left( m^2_{0\chi} + 2 g_2 \langle \sigma \rangle^2 \right) \equiv -\frac{m^2_\chi}{g_4}.
\]

(7)

The coupling \( g_2 \) is taken to be positive. The expected behavior of \( m^2_\chi \sim (T - T_{c\chi})^{\nu_{\chi}} \) and \( m^2_\sigma \sim (T - T_{c\sigma})^{\nu_{\sigma}} \) near \( T_{c\chi} \) and \( T_{c\sigma} \), respectively, combined with the result of eq. (7), yields in the neighborhood of these two transitions the qualitative situation, illustrated in the right panel of figure 1. On both sides of \( T_{c\chi} \) the relevant interaction term \( g_2 \langle \sigma \rangle \sigma \chi^2 \) emerges, leading to a one-loop contribution to the static two-point function of the \( \sigma \) field \( \propto \langle \sigma \rangle^2/m_\chi \). Near the deconfinement transition \( m_\chi \to 0 \) yielding an infrared sensitive screening mass for \( \sigma \). Similarly, on both sides of \( T_{c\sigma} \) the interaction term \( \langle \chi \rangle \chi \sigma^2 \) is generated, leading to the infrared sensitive contribution \( \propto \langle \chi \rangle^2/m_\sigma \) to the \( \chi \) two-point function. We conclude, that when \( T_{c\chi} \ll T_{c\sigma} \), the two order parameter fields, a priori unrelated, do feel each other near the respective phase transitions. It is important to emphasize that the effective theory works only in the vicinity of the two phase transitions. Interpolation through the intermediate temperature range is shown by dotted lines in the right panel of figure 1. Possible structures here must be determined via first principle lattice calculations.

The infrared sensitivity leads to a drop in the screening masses of each field in the neighborhood of the transition of the other, which becomes critical, namely of the \( \sigma \) field close to \( T_{c\chi} \), and of the \( \chi \) field close to \( T_{c\sigma} \). The resummation procedure outlined in the previous section predicts again a finite drop for the masses:

\[
\Delta m^2_\chi(T_{c\sigma}) = -\frac{8 g_2^2 \langle \chi \rangle^2}{3 \lambda}, \quad \Delta m^2_\sigma(T_{c\chi}) = -\frac{8 g_2^2 \langle \sigma \rangle^2}{3 g_4}.
\]

(8)

We thus predict the existence of substructures near these transitions, when considering fermions in the adjoint representation. Searching for such hidden behaviors in lattice simulations would help to further understand the nature of phase transitions in QCD.

We have shown how deconfinement (i.e. a rise in the Polyakov loop) is a consequence of chiral symmetry restoration in the presence of fermions in the fundamental presentation. In nature quarks have small, but nonzero masses, which makes chiral symmetry only approximate. Nevertheless, the picture presented here still holds: confinement is driven by the dynamics of the chiral transition. The argument can be extended even further: If quark masses were very large then chiral symmetry would be badly broken, and could not be used to characterize the phase transition. But in such a case the \( Z_2 \) symmetry becomes more exact, and by reversing the roles of the protagonists in the previous discussion, we would find that the \( Z_2 \) breaking drives the (approximate) restoration of chiral symmetry. Which of the
underlying symmetries demands and which amends can be determined directly from the critical behavior of the spatial correlators of hadrons or of the Polyakov loop [2].

With quarks in the adjoint representation we investigated the physical scenario [10] in chiral symmetry is restored after deconfinement set in, i.e. $T_{c\chi} \ll T_{c
\sigma}$. In this case we have pointed to the existence of an interesting structure, which was hidden until now: There are still two distinct phase transitions, but since the fields are now entangled, the transitions are not independent. This entanglement is shown at the level of expectation values and spatial correlators of the fields. Lattice simulations will play an important role in checking these predictions.

The analysis can be extended for phase transitions driven by a chemical potential. For 2 colors there is a phase transition from a quark-antiquark condensate to a diquark condensate [14]. We hence predict, in two color QCD, that when diquarks form for $\mu = m_\pi$, the Polyakov loop also feels the presence of the phase transition exactly in the same manner as it feels when considering the temperature driven phase transition. Such a situation is supported by recent lattice simulations [15]. The results presented here are not limited to describing the chiral/deconfining phase transition and can readily be used to understand phase transitions sharing similar features.

References
[1] F. Sannino, Phys. Rev. D 66, 034013 (2002) [arXiv:hep-ph/0204174].
[2] A. Mocsy, F. Sannino and K. Tuominen, Phys. Rev. Lett. 91, 092004 (2003) [arXiv:hep-ph/0301229].
A. Mocsy, F. Sannino and K. Tuominen, Induced universal properties and deconfinement, arXiv:hep-ph/0306069.
[3] A. Mocsy, F. Sannino and K. Tuominen, Confinement versus chiral symmetry, arXiv:hep-ph/0308135.
[4] F. Sannino and M. Shifman, Effective Lagrangians for orientifold theories, arXiv:hep-th/0309252.
[5] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B 210, 423 (1982).
[6] P. N. Meisinger and M. C. Ogilvie, Phys. Rev. D 66, 105006 (2002) [arXiv:hep-ph/0206181].
[7] F. Wilczek, Int. J. Mod. Phys. A 7, 3911 (1992) [Erratum-ibid. A 7, 6951 (1992)].
[8] O. Scavenius, A. Mocsy, I. N. Mishustin and D. H. Rischke, Phys. Rev. C 64, 045202 (2001).
[9] S. Holtmann and T. Schulze, arXiv:hep-lat/0305019.
[10] F. Karsch and M. Lutgemeier, Nucl. Phys. B 550, 449 (1999) [arXiv:hep-lat/9812023].
[11] An incomplete list: G. E. Brown, A. D. Jackson, H. A. Bethe and P. M. Pizzochero, Nucl. Phys. A 560, 1035 (1993); S. Digal, E. Laermann and H. Satz, Nucl. Phys. A 702, 159 (2002); K. Fukushima, arXiv:hep-ph/0310121.
[12] N. O. Agasian and S. M. Fedorov, “Hadron resonance gas and nonperturbative QCD vacuum at finite temperature,” arXiv:hep-ph/0310249.
[13] T. Appelquist, P. S. Rodrigues da Silva and F. Sannino, Phys. Rev. D 60, 116007 (1999). J. T. Lenaghan, F. Sannino and K. Splittorff, Phys. Rev. D 65, 054002 (2002). For the Wess-Zumino anomaly terms see: Z. Y. Duan, P. S. Rodrigues da Silva and F. Sannino, Nucl. Phys. B 592, 371 (2001) [arXiv:hep-ph/0001303].
[14] For a review on 2 color QCD see S. Hands, Nucl. Phys. Proc. Suppl. 106, 142 (2002).
[15] B. Alles, M. D’Elia, M. P. Lombardo and M. Pepe, arXiv:hep-lat/0210039.