Clustering features in decay processes

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Abstract. Decay widths in emission processes are described within the stationary scattering theory. We use a pocket-like potential between emitted fragments, centered on the nuclear surface. This picture predicts a linear dependence between the logarithm of the reduced width and fragmentation potential. It turns out that this law is satisfied by proton emission, alpha-decay and heavy cluster emission processes. In particular, we show that, by using surface Gaussian-like components in single particle orbitals mocking four body correlations, it becomes possible to enhance both electromagnetic and α transition probabilities up to the experimental values in $^{212}\text{Po}$.

1. Introduction
The simplest Gamow approach of the α-decay process assumes a preformed α-particle moving around the core and penetrating the Coulomb barrier [1]. Later on, the cluster was built microscopically from two proton and two neutron single particle configurations [2, 3]. Anyway, the decay widths were by several orders of magnitude smaller than the experimental values. In the late 1970’s it was shown that the inclusion of many configurations increases the value of the absolute decay width by more than four orders of magnitude [4]. Yet, the calculated widths were two orders of magnitude too small. Only a theory where a combined shell- and cluster-model configurations were considered could reproduce the absolute decay width [5, 6]. This may justify the so-called extreme cluster models, where the α-particle moves around the core [7, 8, 9]. This picture can be extended to the emission of heavier clusters, like C, O, Ne, Mg and Si [10].

In this presentation we will explain all emission processes within a simple approach, namely a pocket-like potential surrounded by the Coulomb barrier. This picture is able to provide a simple analytical relation, connecting the reduced width to the fragmentation potential. It is valid for any kind of decay process induced by the strong interaction, except fission, and it is the next natural step beyond the Gamow picture. We will also show that a clustering component should be added to the single particle shell model orbitals in order to simultaneously describe α decay width and electromagnetic transitions in $^{212}\text{Po}$.

2. Cluster description of decay processes
For the process $P(aren) \rightarrow D(augh) + C(lu)$ one has a linear dependence between the logarithm of the half life and Coulomb parameter

$$\chi = \frac{2Z_DZ_Ce^2}{\hbar v},$$

(1)
defined in terms of fragment electric charges and their asymptotic relative velocity $v$. This relation is called Geiger-Nuttal law [11] and it is valid for the cluster emission including $\alpha$-decay, as it is shown in Fig. 1.

![Figure 1. Logarithm of the half life versus the Coulomb parameter (1) for various cluster emission processes.](image)

It can be explained within the stationary scattering theory [12]. For spherical emitters the decay width, which is inverse proportional with respect to the half life, can be written in terms of the so-called scattering amplitude $N_0$ as follows [12]

$$\Gamma \equiv \frac{\hbar \ln 2}{T} = \hbar v |N_0|^2 = \hbar v \left| \frac{R f_0^{(int)}(R)}{G_0(\chi, R)} \right|^2.$$  \hspace{1cm} (2)

Notice that the above ratio does not depend on the cluster-daughter radius $R$, because both internal $f_0^{(int)}(R)$ and external Coulomb $G_0(\chi, R)/R$ wave functions should satisfy the same Schrödinger equation. Traditionally the decay width is rewritten

$$\Gamma = 2 P_0 \gamma^2,$$  \hspace{1cm} (3)

in terms of the penetrability and reduced width squared

$$P_0 = \frac{\kappa R}{[G_0(R)]^2}, \quad \gamma^2 = \frac{\hbar^2}{2\mu R^2} |f_0^{(int)}(R)|^2,$$  \hspace{1cm} (4)

where $\kappa$ is the momentum and $\mu$ the reduced cluster-daughter mass. The monopole irregular Coulomb function $G_0(\chi, R)$, defining the penetrability, depends exponentially on the Coulomb parameter, explaining in this way the Geiger-Nuttall law.

Let us first analyze the $\alpha$-decay process. The binding energy per nucleon for an $\alpha$-particle is much larger with respect to their neighbours. Due to a smaller nuclear density at surface, an
\(\alpha\)-cluster structure is energetically more favourable in this region [13]. Therefore clustering is a surface effect and it can be simulated by assuming that the \(\alpha\)-particle moves in a pocket-like potential [12]

\[
V(r) = \hbar \omega \frac{\beta (R - R_0)^2}{2} + v_0, \quad R \leq R_B \\
= \frac{Z_D Z e^2}{R} \equiv V_C(R), \quad R > R_B
\]

(5)

where \(R_0\) is the surface pocket radius, as it is shown in Fig. 2.

\[V_s(R) = Z D Z e^2 / R
\]

\[V_{frag}(R) = V_C(R) - Q = Z D Z e^2 / R - \frac{\chi}{\rho} - 1.
\]

(7)

In terms of the Coulomb parameter (1) and reduced radius \(\rho = \kappa R\). Notice that the radial equation of the shifted ho potential is similar to the equation of the one-dimensional oscillator and therefore the wave function of the ground state is given by

\[f_0^{(int)}(R) = A_0 e^{-\beta(R - R_0)^2 / 2}.
\]

(8)

Figure 2. The interaction potential between the cluster and daughter nucleus.
By using Eq. (6) one obtains the following relation [14]

\[
\log_{10} \gamma^2(R_B) = -\frac{\log_{10} e^2}{\hbar \omega} V_{\text{frag}}(R_B) + \log_{10} \frac{\hbar^2 A^3_2}{2e\mu R_B}.
\]

(9)

The slope of this dependence has a negative value and it is connected with the shape of the interaction potential (ho energy \( \hbar \omega \)), while the free term gives information about the amplitude of the cluster wave function. Our calculation has shown that the linear relation (9), but with different coefficients, remains valid in the most general case of the double folding plus repulsive interaction between fragments, used in Refs. [15, 16].

We first analyzed Eq. (9) in \( \alpha \)-decays connecting ground states of even-even nuclei. In Fig. 3 the data are divided into five regions of even-even \( \alpha \) emitters as follows

1) \( Z < 82, \ 50 < N < 82 \) \( \text{Fig. 3 (c), stars} \);  
2) \( Z < 82, \ 82 < N < 126 \) \( \text{Fig. 3 (a), crosses} \);  
3) \( Z > 82, \ 82 < N < 126 \) \( \text{Fig. 3 (b), circles} \);  
4) \( Z > 82, \ 126 < N < 152 \) \( \text{Fig. 3 (c), squares} \);  
5) \( Z > 82, \ N > 152 \) \( \text{Fig. 3 (d), triangles} \).

In calculations it was used the value of the touching radius, i.e.

\[
R_B = 1.2(A_D^{1/3} + A_C^{1/3}).
\]

(11)

Notice that the regions 1-4 contain rather long isotopic chains, while in the last region 5 one has not more than two isotopes/chain. This is the reason why, except for the last region 5, the reduced width decreases with respect to the fragmentation potential, according to the theoretical
Figure 4. Logarithm of the reduced width squared versus the fragmentation potential for even-even (a), even-odd (b), odd-even (c) and odd-odd (d) α-particle emitters.

prediction given by (9). Eq. (9) is also fulfilled for α-decays from even-odd, odd-even and odd-odd emitters, as it is shown in Fig. 4.

In Fig. 5 we plotted the difference between the logarithm of the reduced width squared and fitting lines in Fig. 4. One sees that for most decays this law gives a reasonable description within a factor of three.

Figure 5. The difference between the logarithm of the reduced width squared and the fitting lines in Fig. 4 for even-even (a), even-odd (b), odd-even (c) and odd-odd (d) α-particle emitters.
The linear dependence between $\log_{10} \gamma^2$ and the fragmentation potential (9) remains valid for any kind of cluster emission. This fact is nicely confirmed by heavy cluster emission processes in Fig. 6 (a). Here it is also plotted a similar dependence for $\alpha$-decays corresponding to the same heavy cluster emitters. The straight line is the linear fit for cluster emission processes, except $\alpha$-decays

$$\log_{10} \gamma^2 = -0.586(V_C - Q) + 15.399$$

(12)

Figure 6. (a) Logarithm of the reduced width squared versus the fragmentation potential (7). Different symbols correspond to various cluster decays. The straight line is the linear fit (12) for cluster emission processes, except $\alpha$-decay. (b) Cluster mass number versus the fragmentation potential.

The above value of the slope $-\log_{10} e^2/\hbar \omega$ in Eq. (9) leads to $\hbar \omega \approx 1.5$ MeV, with the same order of magnitude as in the $\alpha$-decay case.

Let us mention that a relation expressing the spectroscopic factor (proportional to the reduced width) for cluster emission processes was derived in Ref. [17]

$$S = S_{\alpha}^{(A_C-1)/3}$$

(13)

where $A_C$ is the mass of the emitted cluster and $S_{\alpha} \sim 10^{-2}$. As can be seen from figure 6 (b), between $A_C$ and $V_{\text{frag}}$ there exists a linear dependence and therefore the above scaling law can be easily understood in terms of the fragmentation potential.

Concerning the reduced widths of proton emitters, in Refs. [18, 19] it was pointed out the correlation between the reduced width and the quadrupole deformation. This fact can be seen in Fig. 7 (a), where the region with $Z < 68$ corresponds to $\beta > 0.1$ (open circles), while the other one with $Z > 68$ to $\beta < 0.1$ (dark circles). The two linear fits have obviously different slopes. Notice that the two dark circles with the smallest reduced widths correspond to the heaviest emitters with $Z > 80$.

At the same time one sees from Fig. 7 (b) that the same data are clustered into two regions, which can be directly related to the fragmentation potential (7). Here, the two linear fits in
terms of the fragmentation potential, corresponding to the two regions of charge numbers, have roughly the same slopes, but different values in origin. Thus, the two different lines seen in proton emission systematics [18] can be directly connected with similar lines in Fig. 7 (b). They correspond to different orders of magnitude of the fragmentation potential, giving different orders to wave functions and therefore to reduced widths.

3. Clustering features in $^{212}$Po

The internal function $f_0^{(int)}(R)$ in Eq. (2) is called $\alpha$-particle formation amplitude. For distances beyond the geometrical touching configuration, where the Pauli principle becomes less important, this quantity can be estimated by the following overlap integral [12]

$$f_0^{(int)}(R) = \langle \Psi_P | \Psi_D | \Psi_\alpha \rangle ,$$

in terms of parent, daughter and $\alpha$-particle wave functions, respectively. In order to properly describe clustering properties, one has to consider at the single particle level two components of the wave function [20]

$$\psi_l(r) = N_l^{SM} \psi_l^{SM}(r) + N_l^{clas} \psi_l^{clas}(r) .$$

The shell model part has a standard spherical ho representation

$$\psi_l^{SM}(r) = \sum_n b_n R_{nl}^{(\beta)}(r) ,$$

where $\beta = M_N \omega / \hbar$ is the ho parameter and the principal quantum number has the values $N = 2n + l \leq 6$. 

Figure 7. (a) Logarithm of the reduced width squared for proton emission versus the quadrupole deformation. By open circles are given proton emitters with $Z < 68$, while by dark circles those with $Z > 68$. The two regression lines fit the corresponding data. (b) Logarithm of the reduced width squared versus the fragmentation potential (7). The symbols are the same as in (a).
For the cluster part one uses the wave function of the shifted oscillator (8) expanded in terms of ho components

$$\psi_{l}^{(clus)}(r) = e^{-\frac{\beta c(r-r_0)^2}{2}} = \sum_n c_n R_{nl}^{(\beta)}(r),$$

(17)

where $N$ is spread around a larger value $N \sim 8 - 10$, as can be seen in Fig. 8.

Figure 8. (a) Gaussian distribution centered on the nuclear surface (solid line) and various expansion ho terms (dot-dashed lines). (b) Expansion coefficients of a Gaussian centered on the nuclear surface given by Eq. (17).

Notice that in Ref. [6] the ho parameter of the cluster part has a smaller value, in order to reduce the dimension of the single particle basis (15). Here, all coefficients are found by a diagonalization procedure of the Woods-Saxon mean field.

The typical example of a heavy nucleus where the $\alpha$-clustering effects are very important is $^{212}$Po. Here, the structure of low-lying states can be explained in terms of two proton and two neutron orbitals above the double magic inert core $^{208}$Pb [21]. Anyway, by using the standard components of the two protons and two neutrons the decay width is underestimated by two orders of magnitude. It is necessary to consider the second cluster component in Eq. (15), with $N_{l}^{(clus)} = 0.3$, in order to describe the absolute value of the $\alpha$-decay width, as can be seen in Fig. 9 (b) (solid line). In spite of the fact that in figure 9 (a) the difference between the formation probability within the shell model approach (dashed line) and mixed approach (solid line) is rather small, the enhancement of the decay width is about two orders of magnitude. Notice that the same value was obtained in Ref. [5], by using a diagonalisation procedure of the residual interaction.

This picture is also supported by analyzing electromagnetic transitions. The electric transition probability of the multipolarity $\lambda$ is proportional to the reduced matrix element of
the transition operator squared, i.e.

$$B(E\lambda : J \rightarrow J') = \frac{1}{2J + 1} \left| \langle J' | \hat{T}_{\lambda} | J \rangle \right|^2,$$

(18)

where $|J\rangle$ and $|J'\rangle$ denote initial and final states and the transition operator is proportional to spherical harmonics

$$\hat{T}_{\lambda \mu} = r^{\lambda} Y_{\lambda \mu}.$$

(19)

The radial part of the quadrupole transition matrix element is proportional to the principal quantum number

$$\langle n_1 l_1 | n_2 l_2 \rangle \sim N = 2n + l.$$

(20)

It is known that $B(E2)$ values are satisfactory described within the shell model for $^{210}\text{Pb}$, but they are by one order of magnitude less than the experimental values in $^{212}\text{Po}$. They are of the same order of magnitude as those in $^{210}\text{Pb}$ [22]. In order to explain this discrepancy we used the mixed representation (15) with $\mathcal{N}^{(\text{clus})}_l = 0.3$. From Table I one can see a nice agreement with respect to experimental data. The enhancement of the $B(E2)$ value in $^{212}\text{Po}$ is explained by larger principal quantum numbers $N$ involved by the cluster component (15) in transition matrix elements (20). This is a very convincing evidence in favour of the $\alpha$-clustering structure of this nucleus.
Table I

Experimental and theoretical B(E2) values in $^{210}$Po (second and third columns), $^{210}$Pb (fourth and fifth columns) and $^{212}$Po (sixth and last columns) in Weiskopf units.

| $J \rightarrow J'$ | $^{210}$Po $B(E2)_{exp}$ | $^{210}$Pb $B(E2)_{th}$ | $^{210}$Po $B(E2)_{exp}$ | $^{210}$Pb $B(E2)_{th}$ | $^{212}$Po $B(E2)_{exp}$ | $^{212}$Pb $B(E2)_{th}$ |
|------------------|-----------------|----------------|-----------------|----------------|----------------|----------------|
| 2 $\rightarrow$ 0 | 0.56(12)         | 1.4(4)         | 3.9             | —              | 9.2            |
| 4 $\rightarrow$ 2 | 4.6(2)           | 3.2(7)         | 3.5             | —              | 20.8           |
| 6 $\rightarrow$ 4 | 3.0(1)           | 2.2(3)         | 2.4             | 13.5(36)       | 14.4           |
| 8 $\rightarrow$ 6 | 1.18(3)          | 0.62(5)        | 1.0             | 4.60(9)        | 5.8            |

4. Conclusions

Based on a simple model of the cluster-core dynamics, namely a shifted harmonic oscillator potential surrounded by the Coulomb interaction, it was derived an universal analytical relation expressing the logarithm of the reduced width squared as a linear function in terms of the fragmentation potential, defined as the difference between the Coulomb barrier and Q-value. It is fulfilled with a reasonable accuracy by all experimental decay data, describing transitions between ground states. In heavy nuclei this cluster component cannot be build from usual single particle orbitals because protons and neutron lie in different major shells. Thus, an additional gaussian centered on the nuclear surface, predicted by the above mentioned universal law, is necessary to be included in the single particle basis. We described both $\alpha$-decay width and B(E2) value in $^{212}$Po by using a mixed single particle basis, containing shell-model and $\alpha$-cluster components.

Acknowledgments

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