New physics in inclusive semileptonic $B$ decays including nonperturbative corrections

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Abstract

In this work we study the effects of New Physics (NP) operators on the inclusive $\bar{B} \to X_c \tau^- \bar{\nu}_\tau$ decay including power ($O(1/m_b^2)$) corrections in the NP operators. In analogy with $R(D^{(*)})$ observables, we study the observable $R(X_c) = \frac{\mathcal{B}(\bar{B} \to X_c \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to X_c \ell^- \bar{\nu}_\ell)}$. We present some numerical results for $R(X_c)$ and compare the results for this observable with and without power corrections in the NP contributions.

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I. INTRODUCTION

Flavor anomalies have attracted a lot of attentions recently. Specially, the anomalies in the measurements of the $B \to D^{(*)}$ transitions are interesting since they are confirmed by many experiments and have persisted for a long time. The measured quantities are the ratios of branching fractions of the semileptonic decays defined by $R(D^{(*)}) = \mathcal{B}(B \to D^{(*)} \tau^- \bar{\nu}_\tau)/\mathcal{B}(B \to D^{(*)} \ell^- \bar{\nu}_\ell)$, where $\ell = e, \mu$ [1–7]. These anomalies are rather robust since most of the experimental and theoretical uncertainties cancel in this ratio. They are interesting as these interactions happen at tree level and if approved, we need a large contribution from new physics (NP) to alleviate these deviations from theoretical predictions. There has been many studies of these anomalies in various NP models (see e.g. [8–14] and references there). Generically, these observables can be considered as tests of the lepton universality, so the assumed NP responsible for these anomalies should couple to leptons non-universally. Since the mass of the $\tau$ lepton is much larger than $\mu$ and $e$, and in view of the lepton flavor non-universality, we usually assume that NP only couples to the $\tau$ lepton [15–17], so it is present only in the $B \to X_c \tau^- \bar{\nu}_\tau$ decay. Here we follow the same approach and consider NP only in the third generation.

The SM predictions for $R(D)$ and $R(D^*)$ are,

\[
R(D)_{SM} = 0.298 \pm 0.003, \\
R(D^*)_{SM} = 0.255 \pm 0.004.
\]  (1)

There are lattice QCD predictions for the ratio $R(D)_{SM}$ in the Standard Model [18–20] that are in good agreement with one another,

\[
R(D)_{SM} = 0.299 \pm 0.011 \quad [\text{FNAL/MILC}], \\
R(D)_{SM} = 0.300 \pm 0.008 \quad [\text{HPQCD}].
\]  (2)  (3)

To calculate the SM predictions for $R(D)$ in Eq. (1), we use the results of [21] where experimental and lattice results are combined to obtain this value. There are also recent analyses of SM predictions of $R(D^*)$ [22–24], we use the results of [24] to calculate the value for $R(D^*)_{SM}$ in Eq. (1).

The averages of $R(D)$ and $R(D^*)$ measurements evaluated by the Heavy-Flavor Averaging Group are [25],

\[
R(D)_{exp} = 0.407 \pm 0.039 \pm 0.024, \\
R(D^*)_{exp} = 0.306 \pm 0.013 \pm 0.007.
\]  (4)

These values exceed the SM predictions by more than $3\sigma$ [25].

In view of these anomalies, it is logical to probe possible new physics effects in other decay modes which are connected to the $R(D^{(*)})$ anomalies via the same parton level transitions. An example of this kind of decay mode is the inclusive
\[ \bar{B} \to X_c \tau^\nu \bar{\nu}_\tau \] decay. In a recent work [26], we studied effects of different NP Dirac structures on the inclusive decay \( \bar{B} \to X_c \ell^- \bar{\nu}_\ell \). There, the NP contributions were considered at leading order. In this work we add the nonperturbative \( 1/m_b \) corrections to these NP Dirac structures and provide some numerical results for the effects of these corrections compared to the case when NP is added at parton level only. In [27] the inclusive \( B \) decay is studied in the two higgs doublet model where a particular combination of the scalar and pseudoscalar couplings appear as NP contributions. In [28], nonperturbative corrections of order \( \mathcal{O}(1/m_b^2) \) in the tensor currents are calculated. Here we present the results of these corrections for the scalar, pseudoscalar, vector and tensor contributions, including all the interference terms. This will help in a more precise study of the inclusive \( \bar{B} \to X_c \tau^\nu \bar{\nu}_\tau \) decay mode in the presence of NP.

In section II we briefly describe the inclusive \( B \) decay process and present the results of our calculations, in section III we present the numerical results and in IV we finish the note with a short conclusion.

**II. INCLUSIVE B DECAY**

The inclusive semileptonic \( B \) decay rate can be calculated systematically by expansion in terms of perturbative and nonperturbative corrections. The leading terms in this expansion reproduce the free quark decay rate while higher order terms are written as double expansions in terms of short distance perturbative effect which is an expansion in \( \alpha_s \), and long distance nonperturbative effect which is an expansion in \( \Lambda_{QCD}/m_b \).

Nonperturbative corrections are calculated in the context of operator product expansion (OPE) and heavy quark effective theory (HQET). The techniques to calculate these corrections are known well (see e.g. [29–35]). The expansion is basically written in terms of operators with increasing dimensions where the higher dimension operators are suppressed by powers of \( 1/m_b \). A convenient method to calculate these corrections to arbitrary order in \( 1/m_b \), is presented in [36]. In this note, we extend the SM results by adding the scalar, pseudo-scalar, vector and tensor currents as NP effects. We consider the effective Hamiltonian,

\[
\mathcal{H}_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} \left\{ \left[ \bar{c} \gamma_\mu (1 - \gamma_5) b + g_L \bar{c} \gamma_\mu (1 - \gamma_5) b + g_R \bar{c} \gamma_\mu (1 + \gamma_5) b \right] \bar{\tau} \gamma^\mu (1 - \gamma_5) \nu_\tau + \left[ g_S \bar{c} b + g_P \bar{c} \gamma_5 b \right] \bar{\tau} (1 - \gamma_5) \nu_\tau + \left[ g_T \bar{c} \sigma^{\mu\nu} (1 - \gamma_5) b \right] \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau + \text{h.c.} \right\},
\]

where \( G_F \) is the Fermi constant and \( V_{cb} \) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. When \( g_S = g_P = g_L = g_R = g_T = 0 \), the above equation produces the SM effective Hamiltonian.

To calculate the differential decay rate for \( \bar{B} \to X_c \tau^\nu \bar{\nu}_\tau \), we use the optical theorem to find the imaginary part of the time ordered products of the charged
interference terms, in the following, with subscripts that correspond to contributions of SM, NP and \( 2 \).

The explicit expression of the three-fold decay distribution in terms of the invariant kinematic variable \( q \) can find the \( q \) and the spin interaction energy of the quark in the hadron, respectively. After integrating over the energies of the charged lepton and the corresponding neutrino in the rest frame of the \( B \) meson, the explicit expression of the three-fold decay distribution in terms of the invariant quantities, is provided in the appendix.

The leading order result is the free quark decay distribution and the first non-perturbative correction appears at order \( \Lambda_{QCD}^2/m_b^2 \). This correction is proportional to two hadronic parameters \( \lambda_1 \) and \( \lambda_2 \) which correspond to the kinetic energy and the spin interaction energy of the \( b \) quark in the hadron, respectively.

After integrating over the energies of the charged lepton and the neutrino, we can find the \( q^2 \) distribution as,

\[
\frac{d\Gamma}{dq^2} = N(q^2) \left[ \left| \frac{(1 + g_L)}{2} + \frac{|g_R|^2}{2} \right|_{SM} + Re\left(g_R^*(1 + g_L)\right) \left| \frac{d\Gamma}{dq^2} \right|_{LR} + g_s^2 \left| \frac{d\Gamma}{dq^2} \right|_{S} \right. \\
+ Re\left(g_s^*(1 + g_L + g_R)) \left| \frac{d\Gamma}{dq^2} \right|_{SLR} + |g_P|^2 \left| \frac{d\Gamma}{dq^2} \right|_{P} + Re\left(g_P^*(1 + g_L - g_R)) \left| \frac{d\Gamma}{dq^2} \right|_{PLR} \right. \\
+ \left| g_T \right|^2 \left| \frac{d\Gamma}{dq^2} \right|_{T} + Re\left((1 + g_L)g_T^* \right) \left| \frac{d\Gamma}{dq^2} \right|_{LT} + Re\left(g_Rg_T^* \right) \left| \frac{d\Gamma}{dq^2} \right|_{RT} \right],
\]

where \( N(q^2) = \frac{G_F^2|\langle V_{ub}\rangle|^2m_b^5(1-m_b^2/q^2)^2}{96\pi^3\sqrt{\lambda(1,2,\rho^2)}} \) and \( \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \). The various terms on the right hand side of the above equation are presented in the following, with subscripts that correspond to contributions of SM, NP and interference terms,

\[
\left| \frac{d\Gamma}{dq^2} \right|_{SM} = \left(1 + \frac{\lambda_1}{2m_b^2}\right) \lambda(1,\rho^2,\rho^2) \left\{ [(1 - \rho)^2 + \hat{q}^2(1 + \rho) - 2(\hat{q}^2)^2] \right\}
\]

\[
\left| \frac{d\Gamma}{dq^2} \right|_{NP} = \left(1 + \frac{\lambda_2}{m_b^2}\right) \lambda(1,\rho^2,\rho^2) \left\{ [(1 - \rho)^2 + \hat{q}^2(1 + \rho) - 2(\hat{q}^2)^2] \right\}
\]

\[
\left| \frac{d\Gamma}{dq^2} \right|_{IF} = \left(1 + \frac{\lambda_{12}}{m_b^2}\right) \lambda(1,\rho^2,\rho^2) \left\{ [(1 - \rho)^2 + \hat{q}^2(1 + \rho) - 2(\hat{q}^2)^2] \right\}
\]

\[
\left| \frac{d\Gamma}{dq^2} \right|_{SM-NP} = \left(1 + \frac{\lambda_{12}}{2m_b^2}\right) \lambda(1,\rho^2,\rho^2) \left\{ [(1 - \rho)^2 + \hat{q}^2(1 + \rho) - 2(\hat{q}^2)^2] \right\}
\]
\[
\frac{d\Gamma}{dq^2}\bigg|_{LR} = -12\sqrt{\hat{q}}\hat{q}^2\left(1 + \frac{\lambda_1}{2m_b^2}\right)\lambda(1, \hat{q}^2, \rho^2) + 4\sqrt{\rho}\frac{3\lambda_2}{2m_b^2}\left\{[2(1 - \rho)^3 - 3\hat{q}^2(1 - \rho)^2 + 12(\hat{q}^2)^3(1 + \rho) - 7(\hat{q}^2)^3]\right\},
\]

\[
\frac{d\Gamma}{dq^2}\bigg|_{S} = \frac{3\hat{q}^2}{4}\left((1 + \sqrt{\rho})^2 - \hat{q}^2]\left(1 + \frac{\lambda_1}{2m_b^2}\right)\lambda(1, \hat{q}^2, \rho^2) + 3\frac{\lambda_2}{2m_b^2}\left((1 - \sqrt{\rho})^2(1 + 6\sqrt{\rho} + 5\rho) - 2\hat{q}^2(1 - 2\sqrt{\rho} + 5\rho) + 5(\hat{q}^2)^2]\right\},
\]

\[
\frac{d\Gamma}{dq^2}\bigg|_{SLR} = \frac{3\hat{m}_\tau}{2}(1 - \sqrt{\rho})(1 + \sqrt{\rho})^2 - \hat{q}^2]\left(1 + \frac{\lambda_1}{2m_b^2}\right)\lambda(1, \hat{q}^2, \rho^2) + 3\frac{\lambda_2}{2m_b^2}\left((1 + \sqrt{\rho})^2(1 - 6\sqrt{\rho} + 5\rho) - 2\hat{q}^2(1 + 2\sqrt{\rho} + 5\rho) + 5(\hat{q}^2)^2]\right\},
\]

\[
\frac{d\Gamma}{dq^2}\bigg|_{P} = \frac{3\hat{q}^2}{4}\left((1 - \sqrt{\rho})^2 - \hat{q}^2]\left(1 + \frac{\lambda_1}{2m_b^2}\right)\lambda(1, \hat{q}^2, \rho^2) + 3\frac{\lambda_2}{2m_b^2}\left((1 + \sqrt{\rho})^2(1 - 6\sqrt{\rho} + 5\rho) - 2\hat{q}^2(1 + 2\sqrt{\rho} + 5\rho) + 5(\hat{q}^2)^2]\right\},
\]

\[
\frac{d\Gamma}{dq^2}\bigg|_{PLR} = \frac{3\hat{m}_\tau}{2}(1 + \sqrt{\rho})(1 - \sqrt{\rho})^2 - \hat{q}^2]\left(1 + \frac{\lambda_1}{2m_b^2}\right)\lambda(1, \hat{q}^2, \rho^2) + 3\frac{\lambda_2}{2m_b^2}\left((1 + \sqrt{\rho})^2(1 - 6\sqrt{\rho} + 5\rho) - 2\hat{q}^2(1 + 2\sqrt{\rho} + 5\rho) + 5(\hat{q}^2)^2]\right\},
\]

\[
\frac{d\Gamma}{dq^2}\bigg|_{T} = 8(1 + \frac{2\hat{m}_\tau}{\hat{q}^2})\left(1 + \frac{\lambda_1}{2m_b^2}\right)(2(1 - \rho)^4 - 5q^2(1 - \rho)^2(1 + \rho) + (q^2)^2(3 + 2\rho + 3\rho^2)
\]
\[ + \left( \hat{q}^2 \right)^3(1 + \rho) - \left( \hat{q}^2 \right)^4 \right) + \frac{3\lambda_2}{2m_b^2} \left( 2(-1 + \rho)(3 + 5\rho) + \hat{q}^2(3 + 17\rho + 5\rho^2 - 25\rho^3) \right. \\
+ \left. \left( \hat{q}^2 \right)^2(3 + 14\rho + 15\rho^2) + 5(\hat{q}^2)^3(1 + \rho) - 5(\hat{q}^2)^4 \right), \]

\[
\frac{d\Gamma}{dq^2}_{LT} = 36\hat{m}_\tau \sqrt{\rho} \left[ \left( 1 + \frac{\lambda_1}{2m_b^2} \right) \left( -1 + \rho \right)^3 + \hat{q}^2\left( 1 + 2\rho - 3\rho^2 \right) + \left( \hat{q}^2 \right)^2(1 + 3\rho) - (\hat{q}^2)^3 \right] \\
+ \frac{\lambda_2}{2m_b^2} \left( (1 - \rho)^2(1 + 15\rho) + \hat{q}^2(3 + 10\rho - 45\rho^2) + (\hat{q}^2)^2(19 + 45\rho) - 15(\hat{q}^2)^3 \right),
\]

\[
\frac{d\Gamma}{dq^2}_{RT} = -36\hat{m}_\tau \left[ \left( 1 + \frac{\lambda_1}{2m_b^2} \right) \left( -1 + \rho \right)^3 - \hat{q}^2\left( -3 + 2\rho + \rho^2 \right) - (\hat{q}^2)^2(3 + \rho) + (\hat{q}^2)^3 \right] \\
+ \frac{\lambda_2}{2m_b^2} \left( (1 - \rho)^2(5 + 11\rho) + \hat{q}^2(1 - 18\rho - 15\rho^2) - (\hat{q}^2)^2(13 + 3\rho) + 7(\hat{q}^2)^3 \right). \]

Here we have defined the normalized quantities, \( \hat{q}^2 = q^2/m_b^2 \), \( \rho = m_b^2/m_b^2 \) and \( \hat{m}_\tau = m_\tau/m_b \). Note that there is no scalar-pseudoscalar or (pseudo)scalar-tensor interference terms in the \( q^2 \) distribution. For \( g_S = g_P = g_L = g_R = g_T = 0 \), we reproduce the SM results and for \( g_S = g_P = g_L = g_R = 0 \) we reproduce the results given in [28].

### III. NUMERICAL RESULTS

For our numerical calculations in this section we use the \( 1S \) mass scheme [37, 38]. We also include the \( \mathcal{O}(1/m_b^3) \) correction in SM which is derived in [39]. In doing so, one other hadronic parameter \( \rho_1 \) is introduced in calculating the total rate. The input parameters we use are \( m_b = 4.691 \pm 0.037 \text{ GeV} \), \( \lambda_1 = -0.362 \pm 0.067 \text{ GeV}^2 \) and \( \rho_1 = 0.043 \pm 0.048 \text{ GeV}^3 \) taken from the global fit which is presented in [40] and \( \delta m_{bc} = 3.40 \pm 0.02 \text{ GeV} \) and \( \lambda_2 = 0.12 \pm 0.03 \text{ GeV}^2 \) as used in [35].

Besides nonperturbative effects, we include the \( \mathcal{O}(\alpha_s) \) perturbative corrections in SM calculated in [41, 42]. The effects of higher order perturbative corrections are very small in the observables where the ratio of rates are calculated [26, 43], so we include only \( \mathcal{O}(\alpha_s) \) corrections. We find for the ratio of branching ratios in SM,

\[
R(X_c)_{SM} = \frac{B(B \to X_c \ell^+ \nu_\ell)_{SM}}{B(B \to X_c \ell^+ \nu_\ell)_{PS}} = 0.217 \pm 0.006. \]

Adding the NP effects, we can find,

\[
\frac{R(X_c)}{R(X_c)_{SM}} \simeq 1 + 1.147(|g_L|^2 + |g_R|^2 + 2Re(g_L)) + 0.031|g_P|^2 + 0.326|g_S|^2 + 12.643|g_T|^2 \]

\[
- 0.714Re((1 + g_L)g_R^*) + 0.096Re((1 + g_L - g_R)g_P^*) + 0.493Re((1 + g_L + g_R)g_S^*) \\
+ 5.516Re(g_Rg_T^*) - 3.399Re((1 + g_L)g_T^*).
\]

(18)
FIG. 1: The ratio of decay rates $R(X_c)$ when one coupling at a time is present. The dashed red curves correspond to the case when the NP contribution is added at parton level while the solid red curves correspond to the case when power corrections are included in the NP contributions. Green bands are the constraints on the couplings due to $R(D^{(*)})_{\exp}$ within $3\sigma$ and $B_c$ lifetime. The pink band is $R(X_c)_{\exp}$ within $1\sigma$.

Using the ALEPH measurement, $B(b \to X\tau^-\bar{\nu}_\tau)_{\exp} = (2.43 \pm 0.32) \times 10^{-2}$ [44], and the world average for the semileptonic branching ratio into the light lepton [25], $B(B \to X_c\ell^-\bar{\nu}_\ell)_{\exp} = (10.65 \pm 0.16) \times 10^{-2}$, we can find an experimental value for the ratio,

$$R(X_c)_{\exp} = 0.228 \pm 0.030.$$ (19)

In Fig. (1) we present the results for the observable $R(X_c)$ when we turn on one NP coupling at a time. We consider two cases: the first case is when the NP contribution is considered only at parton level(dashed red curves), and the second case is when we add the subleading $1/m_b$ corrections to these NP contributions(solid red curves). The gray and brown bands correspond to the uncertainties of this observable when we vary the values of the parameters within their uncertainties. The green bands are the constraints on the couplings when we consider the measurements of $R(D^{(*)})$ within $3\sigma$. For the $g_P$ coupling, it is well known that the $B_c$ lifetime leads to a tight constraint [45–47]. We use $B(B_c \to \tau^-\bar{\nu}_\tau) \leq 30\%$ as in [48], to include this constraint on the $g_P$ coupling which is included in the green band in the plot. The pink band, is the value of $R(X_c)_{\exp}$ within $1\sigma$.

In the parameter space of interest, adding the $1/m_b$ corrections to the NP contributions causes a change of $R(X_c)$ that is numerically at percent level. This change is mostly noticeable in the $g_S$ and $g_T$ case where the maximum correction, in the parameter space that is favored by $R(D^{(*)})$, is $\approx 5\%$. 
IV. CONCLUSIONS

Recent measurements of $R(D^{(*)})$ show large deviations from SM predictions and this could be a signal of nonuniversal NP. The quark level transition in this observable is $b \to c\tau^-\bar{\nu}_\tau$ and we can probe this transition in other decay modes. In a recent work [26], we studied the inclusive $\bar{B} \to X_c\tau^-\bar{\nu}_\tau$ decay in view of the anomalies in the $R(D^{(*)})$ measurements. In this work we extended this study by including the effects of $1/m_b$ corrections in the NP Dirac structures. We presented the results of our calculations for the differential decay rate $\frac{d\Gamma}{dq^2}$ as well as the three-fold decay distribution and presented some numerical results of the effects of these power corrections on the observable $R(X_c)$. By constraining the NP parameters by the existing $R(D^{(*)})$ measurements, we presented the favored parameter region by these measurements to illustrate if the power corrections in the NP part are important. We found that, in the parameter range of interest, these corrections are generically at percent level (except for the $g_P$ coupling which is smaller) and the maximum effect of these corrections is in the $g_S$ and $g_T$ part which is $\approx 5\%$. 
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Appendix A: The differential rate

In this appendix we present the three-fold differential rate in terms of invariant quantities. We write the distribution in the presence of all NP couplings in the form,

$$\frac{d^3\Gamma}{dx^3} = \frac{G_F^2|V_{cb}|^2}{8\pi^3} \left\{ |1 + g_L| \frac{d^3\Gamma}{dx^3} \right|_{SM} + |g_R| \frac{d^3\Gamma}{dx^3} \right|_R + |g_S| \frac{d^3\Gamma}{dx^3} \right|_S + |g_P| \frac{d^3\Gamma}{dx^3} \right|_P + |g_T| \frac{d^3\Gamma}{dx^3} \right|_T \right. + Re((1 + g_L)g_R^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{LR} + Re((1 + g_L + g_R)g_S^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{SLR} + Re((1 + g_L - g_R)g_P^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{PLR} + Re((1 + g_L)g_T^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{LT} + Re(g_Rg_T^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{RT} + Re((g_S - g_P)g_T^*) \frac{d^3\Gamma}{dx^3} \left. \right|_{SPT} \right\}, \quad (A1)

where the three independent variables are usually taken to be $dx^3 = dq^2dE_\tau dE_\nu$ or $dx^3 = dq^2dE_\tau dq.v$, $v$ being the four velocity of the $B$ meson. Each contribution to the differential rate can be written as,

$$\frac{d^3\Gamma}{dx^3} \bigg|_A = \frac{1}{\Delta_0} \frac{d^3\Gamma}{dx^3} \bigg|_A^{(1)} + \frac{1}{\Delta_0^2} \frac{d^3\Gamma}{dx^3} \bigg|_A^{(2)} + \frac{1}{\Delta_0^3} \frac{d^3\Gamma}{dx^3} \bigg|_A^{(3)}. \quad (A2)$$

Here we have defined $\Delta_0 = p^2 - m_c^2$ with $p = m_B v - q$. The contributions (A2) to the decay distribution are given by the substitutions [29, 30],

$$\frac{1}{\Delta_0} \rightarrow \delta(p^2 - m_c^2), \quad \frac{1}{\Delta_0^2} \rightarrow -\delta'(p^2 - m_c^2), \quad \frac{1}{\Delta_0^3} \rightarrow \frac{1}{2} \delta''(p^2 - m_c^2). \quad (A3)$$

In the following we present various contributions to this distribution.
The SM contribution is given as,

\[
\frac{d^3\Gamma}{dx^3}_{SM}^{(1)} = \frac{4}{3m_b} \left[ 6m_b p.p.p_\nu \nu + (\lambda_1 + 3\lambda_2)(2p_\tau.p_\nu - 5p_\tau.v_\nu.v) \right]
\]  
(A4)

\[
\frac{d^3\Gamma}{dx^3}_{SM}^{(2)} = \frac{4}{3m_b} \left[ 2(\lambda_1 + 3\lambda_2)(-2p.p_\nu + 5p.v_\nu.v)p.p_\tau + 2m_b \lambda_1 (2p.v_\tau.v - 5p.p_\nu) p_\nu.v + 6m_b \lambda_2 (p.v_\tau.p_\nu - p.p_\tau.p_\nu) \right]
\]  
(A5)

\[
\frac{d^3\Gamma}{dx^3}_{SM}^{(3)} = \frac{32\lambda_1}{3} \left[ p.p - (p.v)^2 \right] p.p.p_\nu \nu .
\]  
(A6)

The \( A = R \) contribution is derived from SM part by the substitutions \( p_\tau \rightarrow p_\nu \) and \( p_\nu \rightarrow p_\tau \),

\[
\frac{d^3\Gamma}{dx^3}_{R}^{(i)} = \frac{d^3\Gamma}{dx^3}_{SM}^{(i)} (p_\tau \leftrightarrow p_\nu) \quad i = 1, 2, 3.
\]  
(A7)

For \( A = S \) we have,

\[
\frac{d^3\Gamma}{dx^3}_{S}^{(1)} = \frac{1}{2m_b^2} \left[ 2m_b^2 (p.v + m_\tau) + (m_\tau + m_\nu) (\lambda_1 + 3\lambda_2) \right] p_\tau.p_\nu
\]  
(A8)

\[
\frac{d^3\Gamma}{dx^3}_{S}^{(2)} = - \frac{(\lambda_1 + 3\lambda_2)}{3m_b} \left[ 3m_b (p.v + m_\tau) - 3m_\tau.p.v + 2p.p - 5(p.v)^2 \right] p_\tau.p_\nu
\]  
(A9)

\[
\frac{d^3\Gamma}{dx^3}_{S}^{(3)} = \frac{4\lambda_1}{3} (p.v + m_\tau) \left[ p.p - (p.v)^2 \right] p_\tau.p_\nu .
\]  
(A10)

while the \( A = P \) case can be derived from \( A = S \) case by the substitution \( m_\tau \rightarrow -m_\tau \),

\[
\frac{d^3\Gamma}{dx^3}_{P}^{(i)} = \frac{d^3\Gamma}{dx^3}_{S}^{(i)} (m_\tau \rightarrow -m_\tau) \quad i = 1, 2, 3.
\]  
(A11)

For \( A = T \) we find,

\[
\frac{d^3\Gamma}{dx^3}_{T}^{(1)} = \frac{16}{3m_b} \left[ 6m_b (2p.p_\nu p_\tau.v + 2p.p_\tau.p_\nu - p.v_\tau.p_\nu) + 5(\lambda_1 + 3\lambda_2)(p_\tau.p_\nu - 4p_\tau.vp_\nu.v) \right]
\]  
(A12)

\[
\frac{d^3\Gamma}{dx^3}_{T}^{(2)} = - \frac{32}{3m_b} \left[ (\lambda_1 + 3\lambda_2)(8p_\tau.p.p_\nu - 2p.p_\tau.p_\nu + 5(p.v)^2 p_\tau.p_\nu - 10p.p_\nu.vp_\tau.v
\] 
\[ - 10p.p_\tau.vp_\nu.v) + 2m_b (5\lambda_1 - 3\lambda_2) (p_\nu.p_\nu.v + p.p_\tau.p_\nu.v) \right]
\]
\[-3m_b(\lambda_1 - \lambda_2)p.v.p.p - 8m_b\lambda_1p.v.p.v.p.v.v\]  \hspace{2cm} (A13)

\[\frac{d^3\Gamma}{dx^3}|_T^{(3)} = -\frac{128\lambda_1}{3}[p.p - (p.v)^2][p.v.p.p - 2p.p.p.v - 2p.p.v.v] \hspace{2cm} (A14)\]

For \( A = LR \),

\[\frac{d^3\Gamma}{dx^3}|_{LR}^{(1)} = -\frac{4m_c}{m_b}(2m_b^2 + \lambda_1 + 3\lambda_2)p.p.p \hspace{2cm} (A15)\]

\[\frac{d^3\Gamma}{dx^3}|_{LR}^{(2)} = \frac{8m_c}{m_b}[-(\lambda_1 + 3\lambda_2)p.v.p.p + m_b(\lambda_1 + \lambda_2)p.p.p - 4m_b\lambda_2p.v.p.v.v] \hspace{2cm} (A16)\]

\[\frac{d^3\Gamma}{dx^3}|_{LR}^{(3)} = -\frac{32m_c\lambda_1}{3}[p.p - (p.v)^2]p.p.p \hspace{2cm} (A17)\]

For \( A = SLR \),

\[\frac{d^3\Gamma}{dx^3}|_{SLR}^{(1)} = m_\tau \frac{2m_b^2(p.p + \lambda_1 + 3\lambda_2)p.p.m_c.m_c.v.v} \hspace{2cm} (A18)\]

\[\frac{d^3\Gamma}{dx^3}|_{SLR}^{(2)} = \frac{2m_\tau}{2m_b}[(\lambda_1 + 3\lambda_2)(-3p.p.p + 3m_b\lambda_1 + 2m_\tau + 5m_c.p.v.v + 2m_c.p.v)] \hspace{2cm} (A19)\]

\[\frac{d^3\Gamma}{dx^3}|_{SLR}^{(3)} = \frac{8m_\tau\lambda_1}{3}[p.p - (p.v)^2](p.p + \lambda_1 + 3\lambda_2)p.p.v.v \hspace{2cm} (A20)\]

For \( A = PLR \) we have,

\[\frac{d^3\Gamma}{dx^3}|_{PLR}^{(i)} = \frac{d^3\Gamma}{dx^3}|_{SLR}^{(i)} (m_c \rightarrow -m_c) \hspace{2cm} i = 1, 2, 3. \hspace{2cm} (A21)\]

For \( A = LT \),

\[\frac{d^3\Gamma}{dx^3}|_{LT}^{(1)} = -48m_c(m_c.m_c.p.v.v) \hspace{2cm} (A22)\]

\[\frac{d^3\Gamma}{dx^3}|_{LT}^{(2)} = -\frac{16m_\tau m_c}{m_b}[(\lambda_1 + 3\lambda_2)(-2p.p.v + 5p.v.v.v) - 3m_b(\lambda_1 - \lambda_2)p.v.v.v] \hspace{2cm} (A23)\]

\[\frac{d^3\Gamma}{dx^3}|_{LT}^{(3)} = -64m_\tau m_c\lambda_1[p.p - (p.v)^2](p.v.v) \hspace{2cm} (A24)\]
For $A = RT$,

$$\frac{d^3\Gamma}{dx^3} \bigg|_{RT}^{(1)} = \frac{24 m_r}{m_b^2} \left[ 2m_b^2 p.p_\nu + (\lambda_1 + 3\lambda_2)(p.p_\nu - m_b p_\nu.v) \right]$$

(A25)

$$\frac{d^3\Gamma}{dx^3} \bigg|_{RT}^{(2)} = -\frac{16 m_r}{m_b} \left[ -3(\lambda_1 + 3\lambda_2)p.p_\nu.p_v + m_b(5\lambda_1 - \lambda_2)p.p_\nu - 2m_b(\lambda_1 + \lambda_2)p.v p_\nu.v \right]$$

(A26)

$$\frac{d^3\Gamma}{dx^3} \bigg|_{RT}^{(3)} = 64 m_\tau \lambda_1 [p.p - (p.v)^2] (p.p_\nu)$$

(A27)

For $A = SPT$,

$$\frac{d^3\Gamma}{dx^3} \bigg|_{SPT}^{(1)} = 8(p.p_\nu.p_\tau.v - p.p_\tau.p_\nu.v)$$

(A28)

$$\frac{d^3\Gamma}{dx^3} \bigg|_{SPT}^{(2)} = \frac{8}{3m_b} \left[ 5(\lambda_1 + 3\lambda_2)p.v - m_b(5\lambda_1 + 3\lambda_2) \right] (p.p_\nu.p_\tau.v - p.p_\tau.p_\nu.v)$$

(A29)

$$\frac{d^3\Gamma}{dx^3} \bigg|_{SPT}^{(3)} = \frac{32\lambda_1}{3} [p.p - (p.v)^2] (p.p_\nu.p_\tau.v - p.p_\tau.p_\nu.v)$$

(A30)
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