Controlled formation and reflection of a bright solitary matter-wave

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Bright solitons are non-dispersive wave solutions, arising in a diverse range of nonlinear, one-dimensional systems, including atomic Bose–Einstein condensates with attractive interactions. In reality, cold-atom experiments can only approach the idealized one-dimensional limit necessary for the realization of true solitons. Nevertheless, it remains possible to create bright solitary waves, the three-dimensional analogue of solitons, which maintain many of the key properties of their one-dimensional counterparts. Such solitary waves offer many potential applications and provide a rich testing ground for theoretical treatments of many-body quantum systems. Here we report the controlled formation of a bright solitary matter-wave from a Bose–Einstein condensate of 85Rb, which is observed to propagate over a distance of ~1.1 mm in 150 ms with no observable dispersion. We demonstrate the reflection of a solitary wave from a repulsive Gaussian barrier and contrast this to the case of a repulsive condensate, in both cases finding excellent agreement with theoretical simulations using the three-dimensional Gross–Pitaevskii equation.
Solitons are non-dispersive wave solutions that arise in a diverse range of nonlinear systems, stabilised by a focussing or defocussing nonlinearity. First observed in shallow water solitons have subsequently been studied in many other fields including nonlinear optics, biophysics, astrophysics, plasma and particle physics. They are characterized by well localized wavepackets in one-dimension (1D) that maintain their initial shape and amplitude for all time, even following collisions with other solitons. Bose–Einstein condensates (BECs) formed from dilute atomic gases support bright soliton solutions in 1D for attractive interatomic interactions (focussing nonlinearity), manifesting themselves as localized humps in the field amplitude. In contrast, dark solitons appear as localized reductions in an otherwise uniform field amplitude, preserved by a defocussing nonlinearity (repulsive interactions). The control with which these systems can be manipulated, combined with the unique properties of matter-wave solitons, leads to a rich testing ground for theoretical descriptions of quantum many-body systems.

BECs are commonly described by a mean-field treatment leading to the well-known Gross–Pitaevskii equation (GPE) in which the atomic interactions are described by a nonlinear term proportional to the s-wave scattering length $a_s$ and the condensate density. In the 1D, homogeneous limit the GPE takes the form of a nonlinear Schrödinger equation that supports a spectrum of exact soliton solutions. Experiments approach this mathematically ideal scenario by confining the condensate in an elongated, prolate trap typically with tight radial confinement. However, this quasi-1D geometry is usually accompanied by the presence of weak axial harmonic trapping, which removes the integrability of the system and prevents the appearance of true solitons. Nevertheless, solitary wave solutions remain that retain many similarities to the classical soliton solutions, such as propagation without dispersion. The formation of bright solitary matter-waves, the three-dimensional (3D) analogue to the bright soliton solutions of the 1D nonlinear Schrödinger equation, from BECs allows one to explore an array of potential applications including novel interferometric devices using narrow Gaussian potentials as beam splitters, the study of short-range atom-surface potentials and the realization of Schrödinger-cat states.

Previously, bright solitary matter-waves have been realised in three separate experiments using $^7$Li (refs 16,17) and $^{85}$Rb (ref. 18). In each case, a Feshbach resonance was used to switch the interactions from repulsive ($a_s > 0$) to attractive ($a_s < 0$) in order to form solitary waves out of the collapse instability. In two of these experiments, multiple wavepackets were created, allowing the study of the dynamics during collisions in the trap. The observation of solitary waves raises many interesting questions regarding the relationship between the mathematical ideal and the experimental reality. It is unclear how soliton-like the solitary waves created in experiments with finite radial trapping and harmonic axial confinement are. An answer to this question needs to be established before potential applications utilizing solitary waves can be realised. At a more fundamental level it remains to be tested whether or not the GPE treatment fully describes the solitary waves created in experiments. Solitary waves realised experimentally typically contain $\lesssim 1,000$ atoms, placing them well outside of the thermodynamic limit and potentially outside the reach of the mean-field description. Several theoretical studies of bright solitary waves beyond the mean-field description have now been performed, either including effects of quantum noise using the truncated Wigner method or using approximate analytic and numerical methods to simulate the full many-body problem. These generate results potentially in conflict with the behaviour predicted by the GPE treatment.

In this work, we report the controlled formation of bright solitary matter-waves from a $^{85}$Rb BEC. The experimental geometry is such that the velocity of the wavepackets can be precisely controlled, a key factor in facilitating the future exploration of solitary wave interactions and collisions. In addition, we observe and model the controlled reflection of solitary waves from a broad Gaussian potential barrier, demonstrating their particle-like nature. These results pave the way for new experimental studies of bright solitary matter-wave dynamics to elucidate the wealth of existing theoretical work.

**Results**

**Controlled expansion of a tunable BEC.** $^{85}$Rb is a prime candidate for solitary wave experiments owing to the existence of a broad Feshbach resonance at $\sim 155$ G in collisions between atoms in the $F = 2$, $m_F = 2$ state. We use this resonance to form a stable, repulsively interacting condensate in a crossed optical dipole trap, shown in Fig. 1a. The condensate is then loaded into a quasi-1D waveguide, better suited geometrically to the observation of solitary waves. At the point of release into the waveguide, the magnetic bias field controlling the atomic scattering length is jumped to a new value (see Fig. 1b). As the BEC propagates along the waveguide, the value of $a_s$ determines the rate of expansion of the condensate in the axial direction. We probe this expansion by measuring the condensate size (using destructive absorption imaging) as a function of time for different values of $a_s$ as shown in Fig. 1c. Fitting the experimental data we can extract an expansion rate for the BEC, dependent on $a_s$ and $N$. This is shown in Fig. 1d, along with a 3D GPE simulation of the expansion (the solid line). At $a_s = -11 \, a_0$ and $N = 2,000$ we see the expansion rate of the BEC becomes consistent with zero. This lack of dispersion with time indicates the formation of a bright solitary matter-wave.

Figure 2 shows the propagation of this solitary wave, contrasted to that of a repulsively interacting BEC. As the repulsive wavepacket propagates the axial expansion causes a significant drop in optical depth not seen for the solitary wave. We observe the solitary wave propagating over a distance of 1.1 mm in a time of $\sim 150$ ms with very little distortion.

**Reflection from a broad and repulsive Gaussian barrier.** To probe the stability of the solitary wave we investigate reflection of the wavepacket from a repulsive Gaussian barrier with a $1/a^2$ radius of 130 $\mu$m, shown in Fig. 3a. Figure 3b,c show the position of the solitary wave as a function of time in the presence of a 760 kN barrier potential. In this case, the barrier height is greater than the kinetic energy of the solitary wave and the wavepacket is cleanly reflected.

Using a barrier much wider than the solitary wave size the atomic centre-of-mass coordinate behaves classically, with the solitary wave acting as a single particle ‘rolling up a potential hill’. By varying the height of the potential barrier it is possible to select whether the solitary wave is reflected or allowed to travel over the barrier. The position of the solitary wave after 150 ms is shown in Fig. 3d as a function of barrier height. The solid line is a theoretical trajectory, calculated using Newtonian mechanics with no free parameters, and shows excellent agreement with the data.

In Fig. 3e we compare the effect of reflection from the barrier for a solitary wave and a repulsive BEC and contrast the change in width to the case of a repulsive BEC propagating along the waveguide in the absence of the barrier. The solid lines are the theoretical predictions for the condensate widths. We find that in the parameter regime of the experiment a 1D treatment is insufficient, and so we determine the theoretical widths using a 3D (cylindrically symmetric) GPE. This observation is consistent...
with other recent theoretical studies. As expected, the solitary wave is robust against collisions with a repulsive Gaussian barrier and following the reflection maintains its shape, continuing to propagate without dispersion. In the absence of the barrier, the repulsive BEC expands steadily in time. (We attribute the disagreement between experiment and theory at longer times to a small thermal component making the measurement of the condensate width less accurate.) In the barrier reflection case, an oscillation in the condensate width is induced as a result of the larger spatial extent of the repulsive BEC causing it to be strongly compressed as it is reflected from the barrier. Such contrast in the behaviour of the repulsive BEC and the solitary wave reflection lends weight to previous theoretical prediction regarding the superior characteristics of solitary waves for observing quantum reflection from surfaces.

Discussion
There is currently much theoretical interest in the scattering of solitary waves from narrow potential barriers where, if the barrier width is on the order of the solitary wave width, quantum effects are observable. At high kinetic energy, soliton splitting is energetically allowed at narrow repulsive barriers. The effect of quantum tunnelling means the barrier can act as a beam splitter,
Figure 3 | Reflection from a repulsive Gaussian barrier. (a) Potential in the axial direction along the waveguide in the presence of the repulsive barrier. (Inset, upper: combined waveguide and Gaussian barrier potential. Lower: experimental setup.) (b) False colour images of a solitary wave reflecting from the barrier. The white line shows the location of the barrier centre. (c) Horizontal position, relative to the crossed dipole trap (XDT), of a solitary wave propagating in the waveguide in the absence (red) and presence (black) of the repulsive barrier. (d) The position of a solitary wave after 150 ms propagation time as a function of the barrier height. Red (black) points correspond to the solitary wave travelling over (being reflected from) the barrier. Solid lines in (c,d): theoretical trajectory calculated using a classical particle model with no free parameters. (e) Condensate width following reflection from the barrier. In the absence of a barrier, a repulsive BEC \( (a_0 = 58 a_0, N = 3.5 \times 10^5) \) will expand as it propagates (red). With the barrier in place, an oscillation in the condensate width is set up following the strong compression of the condensate at the barrier due to the shape of the potential (black). A solitary wave \( (a_0 = -11 a_0, N = 2.0 \times 10^5) \) undergoing the same collision emerges unaltered (blue). Solid lines are the theoretical condensate widths calculated by solving the 3D (cylindrically symmetric) GPE.

dividing the soliton into two parts\(^{13,22}\). These multiple wavepackets can then be used to investigate the phase dependence of binary collisions\(^{22,23}\) and the behaviour of collisions of two solitary waves on a barrier\(^{22,24}\) and would provide a solid first step towards the realization of a bright solitary wave interferometer. In the limit of low kinetic energy, a mean-field GPE treatment of the problem begins to break down\(^{26}\) and quantum behaviour, (described in the 1D limit by the Lieb-Liniger Hamiltonian\(^{27}\)), becomes more significant. Here, splitting of the soliton is energetically forbidden and it becomes possible to create Schrödinger-cat states\(^{5,15}\).

The use of a narrow potential to controllably split a solitary wave presents an opportunity to investigate one of the key open questions arising from previous work; what governs the dynamics and stability of multiple solitary waves existing in the same trap? The long-lived nature of the solitary waves and their apparent stability during binary collisions has been the subject of a wealth of theoretical work\(^{3,26–31}\). Within the framework of the GPE, the observed stability of soliton collisions can only be explained by imposing a relative phase \( \phi = \pi \) between neighbouring solitary waves. The inclusion of quantum noise\(^3\) or accounting for many-body effects\(^4\) both result in effectively repulsive interactions between solitary waves, irrespective of initial phase. Interestingly, incoherent, fragmented objects are also predicted to form in the many-body formalism\(^1\). Further experimental studies are undoubtedly required to address the role of the relative phase in solitary wave collisions and to test the different theoretical descriptions of quantum many-body systems.

Although reflection and splitting experiments show the potential to settle the theoretical debate over the solitary wave formation and dynamics, the ability to probe such narrow and hence rapidly varying potentials using these wavepackets also lends itself to an obvious application in precision measurement. Atoms close to a surface are subject to the short-range Casimir–Polder and van der Waals potentials, which can be measured using the classical and quantum reflection of bright solitary matter-waves\(^{14}\). Our apparatus includes a super-polished Dove prism for such studies, see Fig. 1a. Further in the future, the ability to deliver and manipulate ultracold atoms near to a solid surface may open up new routes to probe short-range corrections to gravity\(^{34}\) due to exotic forces beyond the Standard Model.

Methods
Production of a tunable BEC. We create a BEC with tunable atomic interactions using the method described in ref. 35. A magnetic Feshbach resonance is used to tune both the elastic and inelastic scattering properties of the atomic sample to
achieve efficient evaporation. Importantly, the resonance at 155 G in collisions between 85Rb atoms in the axial direction. The maximum velocity is given by

\[ v_{\text{max}} = \frac{k}{m} A_{\text{L}} \]

and hence scattering length, to be changed independently of the trapping configuration as shown in Fig. 1a. The term ‘levitated’ refers to the use of an additional 1,064 nm laser beam, focused to a waist of 117 μm and intersecting the crossed trap at 45° to each beam. This enters the glass science cell through the back surface of an anti-reflection coated fused silica Dove prism (to be later used for the study of atom-surface interactions).

The position of the magnetic field zero in the horizontal plane at an angle of 30° (27, 25) Hz. This trap is ill-suited to the observation of bright solitary matter-waves and thus we transfer the condensate into a more quasi-1D waveguide created by an additional 1,064 nm laser beam, focussed to a waist of 117 μm and intersecting the crossed trap at 45° to each beam. This enters the glass cell through the back surface of an anti-reflection coated fused silica Dove prism (to be later used for the study of atom-surface interactions).

To load the condensate into the waveguide the scattering length is ramped close to \( a_s = 0 \) in 50 ms thus reducing the condensate size and creating a BEC approximately in the harmonic oscillator ground state of the crossed trap. The BEC is then held for 10 ms to allow the magnetic field to stabilize before simultaneously switching the waveguide beam on, the crossed beams off and jumping the quadrupole gradient in the vertical direction from \( B_z = 21.5 \text{ G cm}^{-1} \) to 26 G cm\(^{-1}\). Although it is advantageous in terms of the evaporation to be under levitated during the condensation phase, we must increase the gradient correspondingly to transfer the atoms. This ensures a truer levitation of the atoms in the waveguide trap, thus maximizing the trap depth of the beam. In addition, the presence of the quadrupole gradient provides much of the, albeit weak, axial trapping along the beam, \( \omega_{\text{ax}} = 1/2 \sqrt{\mu_0 B_0^2/m} \approx 2 \pi \times 1 \text{ Hz} \) (ref. 36). Here, \( \mu_0 \) is the magnetic moment of atoms with mass \( m \) and \( B_0 \) is the magnetic bias field. The waveguide beam itself contributes \(< 0.1 \text{ Hz} \) to the axial trapping, hence the magnetic confinement dominates in this direction. At a beam power of 0.17 W, the waveguide and quadrupole potential produce a trap of \( \omega_{\text{ax}} = 2 \pi \times (1, 27, 27) \text{ Hz} \). Here, the radial trap frequency \( (\omega_{\text{r}}) \) approximately matches that of the crossed beam trap at the point of condensation.

Propagation in the waveguide. A small offset (2.6 mm) between the crossed dipole trap, that is, the waveguide loading position, and the quadrupole centre means that once loaded into the waveguide, the BEC propagates freely towards the magnetic field minimum along the direction of the waveguide, undergoing harmonic motion. As the BEC propagates its rate of expansion in the axial direction is determined by the scattering length. Although strictly speaking the expansion is nonlinear over the full-range of times measured, a linear approximation is valid over the range 10 ms < t < 100 ms from which we can extract a ‘rate’. The position of the magnetic field zero in the axial direction of the waveguide can be displaced by an amount determined by the scattering length close to the zero crossing of \( \omega_{\text{ax}} \).

References

1. Russell, J. S. Report on waves. In Report of the Fourteenth Meeting of the British Society for the Advancement of Science 311–390 (John Murray, 1845); http://go.nature.com/Ozdafg.

2. Dousse, T. & Peyrard, M. Physics of Solitons (Cambridge University Press, 2006).

3. Dabrowska-Wüster, B., Wüster, S. & Davis, M. J. Dynamical formation and interaction of bright solitary waves and solitons in the collapse of Bose-Einstein condensates with attractive interactions. New J. Phys. 11, 053017 (2009).

4. Streltsov, A. L., Alon, O. E. & Cederbaum, L. S. Formation and dynamics of many-boson frustrated solitons in one-dimensional attractive ultracold gases. Phys. Rev. Lett. 100, 130401 (2008).

5. Weiss, C. & Castin, Y. Creation and detection of a mesoscopic gas in a nonlocal quantum superposition. Phys. Rev. Lett. 102, 010403 (2009).

6. Pethick, C. J. & Smith, H. Bose-Einstein Condensation in Dilute Gases (Cambridge University Press, 2001).

7. Pitashev, L. & Stringari, S. Bose-Einstein Condensation (Clarendon Press, Oxford, 2003).

8. Billam, T. P., Marchant, A. L., Cornish, S. L., Gardiner, S. A. & Parker, N. G. Bright solitary matter waves: formation, stability and interactions. In Progress in Optical Science and Photonics, Vol. 1, Spontaneous Symmetry Breaking. Self-Trapping and Josephson Oscillations. (ed. Malomed, B. A.) (Springer, Berlin, Heidelberg, 2013).

9. Salasnich, L., Parola, A. & Reatto, L. Effective wave equations for the dynamics of cigar-shaped and disk-shaped Bose condensates. Phys. Rev. A 65, 043614 (2002).

10. Malomed, B. A., Mihalache, D., Wise, F. & Torner, L. Spatially periodic optical solitons. J. Opt. B Quantum Semiclass. Opt. 7, S53–S72 (2005).

11. Billam, T. P., Wraithall, S. A. & Gardiner, S. A. Variational determination of approximate bright matter-wave soliton solutions in anisotropic traps. Phys. Rev. A 85, 013627 (2012).

12. Cronin, A. D., Schmiedmayer, J. & Pritchard, D. E. Optics and interferometry with atoms and molecules. Rev. Mod. Phys. 81, 1051–1129 (2009).

13. Dawe, D., Sidong, L., Pollack, S., Dries, D. & Hulet, R. Interactions of bright matter-wave solitons with a barrier potential. In 42nd Annual Meeting of the APS Division of Atomic, Molecular and Optical Physics Vol. 56, OPE.10 (APS, 2011).

14. Cornish, S. L. et al. Quantum reflection of bright matter-wave solitons. Physica D 238, 1299–1305 (2009).

15. Streltsov, A. L., Alon, O. E. & Cederbaum, L. S. Scattering of an attractive Bose-Einstein condensate from a barrier: formation of quantum superposition states. Phys. Rev. A 80, 043616 (2009).

16. Strecke, K. E., Partridge, G. B., Truscott, A. G. & Hulet, R. G. Formation and propagation of matter-wave soliton trains. Nature 417, 150–153 (2002).

17. Khaykovich, L. et al. Formation of a matter-wave bright soliton. Science 296, 1290–1293 (2002).

18. Cornish, S. L., Thompson, S. T. & Wieman, C. E. Formation of bright matter-wave solitons during the collapse of attractive Bose-Einstein condensates. Phys. Rev. Lett. 96, 170401 (2006).

19. Ruprecht, P. A., Holland, M. J., Burnett, K. & Edwards, M. Time-dependent solution of the nonlinear Schrödinger equation for Bose-condensed trapped neutral atoms. Phys. Rev. A 51, 4704–4711 (1995).

20. Holdaway David, I. H., Weiss, C. & Simon A., Gardiner Quantum theory of propagation of matter-wave soliton trains. New J. Phys. 4, 176 (2002).

21. Cuevas, J., Kevrekidis, P. G., Malomed, B. A., Dyke, P. & Hulet, R. G. Interactions of solitons with an attractive barrier: splitting and recombination in quasi-1d and 3d. Preprint at http://arXiv.org/abs/1301.3958 (2013).

22. Helm, J. L., Billam, T. P. & Gardiner, S. A. Bright matter-wave soliton collisions at narrow barriers. Phys. Rev. A 85, 053621 (2012).
23. Billam, T. P., Cornish, S. L. & Gardiner, S. A. Realizing bright-matter-wave-soliton collisions with controlled relative phase. Phys. Rev. A 83, 041602 (2011).
24. Martin, A. D. & Ruostekoski, J. Quantum dynamics of atomic bright solitons under splitting and recollision, and implications for interferometry. New J. Phys. 14, 043040 (2012).
25. Ernst, T. & Brand, J. Resonant trapping in the transport of a matter-wave soliton through a quantum well. Phys. Rev. A 81, 033614 (2010).
26. Gertjerenken, B., Billam, T. P., Khaykovich, L. & Weiss, C. Scattering bright solitons: quantum versus mean-field behavior. Phys. Rev. A 86, 033608 (2012).
27. Lieb, E. H. & Liniger, W. Exact analysis of an interacting Bose gas. I. the general solution and the ground state. Phys. Rev. 130, 1605–1616 (1963).
28. Al Khawaja, U., Stoof, H. T. C., Hulet, R. G., Strecker, K. E. & Partridge, G. B. Bright soliton trains of trapped Bose-Einstein condensates. Phys. Rev. Lett. 89, 200404 (2002).
29. Parker, N. G., Martin, A. M., Adams, C. S. & Cornish, S. L. Bright solitary waves of trapped atomic Bose-Einstein condensates. Physica D 238, 1456–1461 (2009).
30. Parker, N. G., Martin, A. M., Cornish, S. L. & Adams, C. S. Collisions of bright solitons in Bose-Einstein condensates. J. Phys. B 41, 045303 (2008).
31. Carr, L. D. & Brand, J. Spontaneous soliton formation and modulational instability in Bose-Einstein condensates. Phys. Rev. Lett. 92, 040401 (2004).
32. Gordon, J. P. Interaction forces among solitons in optical fibers. Opt. Lett. 8, 596–598 (1983).
33. Khaykovich, L. & Malomed, B. A. Deviation from one dimensionality in stationary properties and collisional dynamics of matter-wave solitons. Phys. Rev. A 74, 023607 (2006).
34. Dimopoulos, S. & Geraci, A. A. Probing submicron forces by interferometry of Bose-Einstein condensed atoms. Phys. Rev. D 68, 124021 (2003).
35. Marchant, A. L., Händel, S., Hopkins, S. A., Wiles, T. P. & Cornish, S. L. Bose-Einstein condensation of $^{85}$Rb by direct evaporation in an optical dipole trap. Phys. Rev. A 85, 053647 (2012).
36. Lin, Y.-J., Perry, A. R., Compton, R. L., Spielman, I. B. & Porto, J. V. Rapid production of $^{87}$Rb Bose-Einstein condensates in a combined magnetic and optical potential. Phys. Rev. A 79, 063631 (2009).
37. Marchant, A. L., Händel, S., Wiles, T. P., Hopkins, S. A. & Cornish, S. L. Guided transport of ultracold gases of rubidium up to a room-temperature dielectric surface. New J. Phys. 13, 125003 (2011).

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Author contributions
A.L.M. performed the experiments and data analysis. T.P.B. carried out the numerical simulations. T.P.W. and M.M.H.Y. assisted with the development of the apparatus. S.A.G. provided theoretical support. S.L.C. conceived and managed the project. A.L.M., T.P.B. and S.L.C. prepared the manuscript.

Additional information
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