Estimation of ordinary kriging parameters for determining characteristics and distribution of groundwater layer in Tondo area, Mantikulore district, Palu

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Abstract. Tondo area as a research location is one of Palu area. The topography of this area besides sloping and interspersed with hilly areas is also a barren and critical groundwater area. Geologically, the rock structure that comprises this area consists of Molasa Celebes deposits namely clay, sand and gravel. So far, clean water in the Tondo area has been supplied by the Palu City Regional Drinking Water Company (PDAM) from surface water source in Kawatuna area. There is also a PDAM Donggala that supplies clean water to this area which is sourced from groundwater. However, the water source is quite deep at around 150 m besides that the depth of groundwater position varies based on the results of groundwater drilling in this region and groundwater cannot always be found. If these results are followed up with Ordinary Kriging research using a Semivariogram to find groundwater, the results are expected to be better and more accurate. Based on the research results obtained that: The best model is the Spherical model with the smallest MSE value of 429.5016 and the resulting parameter values are nuggets = 15.70937, sill = 28.46195, and range = 3343.121. The deepest depth is 23.07785 m.

1. Introduction

Tondo area as a research area is one of Palu area. This area is in the administrative area of Mantikulore district. The topography of this area besides sloping and interspersed with hilly areas is also a barren and critical groundwater area. In urban planning, this area is expected to become the development area of Palu in the future. However, to support this development, there are a number of issues that are tondo areas known as barren and critical groundwater. By Sudradjat, A. (1981) geologically, the rock structure that makes up this area consists of Molasa Celebes deposits namely clay, sand and gravel.

Based on such natural conditions, it is very difficult to obtain groundwater near the surface, which is possible only in deep groundwater [4]. So far, clean water in Tondo area has been supplied by the Palu City Regional Drinking Water Company (PDAM) from surface water sources in the Kawatuna area. In addition, there is also Donggala PDAM which supplies clean water to this area which is sourced from groundwater. However, the water source is quite deep at around 150 m besides that the depth of groundwater position varies based on the results of the drilling [7].

Several previous studies, groundwater search by the geoelectric method in the Tondo area showed that the groundwater surface was lacking in potential, only two point locations gave an indication of groundwater level, the rest did not provide groundwater information [4]. If these results are followed up
with Ordinary Kriging Semivariogram method research for groundwater search which was successfully carried out in the Bandung Basin, it is expected that the results will be better and more accurate. And can improve the results of previous studies [6]. The application of the Ordinary Kriging method for estimating oil and groundwater content has been widely used elsewhere such as [5]. To solve the above problems can be done with semivariogram calculation to get the sill and range which are then applied to the Ordinary Kriging Parameter determination technique. Determination of this parameter can be done through statistical calculations. By knowing the values of the Ordinary Kriging parameters, the determination of the depth of the groundwater layer can be calculated and mapped. Based on the description above, the problem can be formulated as follows: How to search for groundwater in the area of Tondo in order to provide better and accurate results, How to provide information on the depth map of groundwater sources in the area of Tondo. How to determine ordinary Kriging parameters that will be used in determining the presence of groundwater layers in the Tondo area of Mantikulore sub-district.

The Purpose of this Research To determine groundwater search techniques that are better and more efficient and accurate. Determine information on semivariogram parameters used for the Kriging Organization, Make a Depth Map of groundwater sources in the Tondo area.

2. Materials and Methods

2.1 Spatial Data
Spatial data is a measurement of location information. This data is presented in the form of the geographical position of objects, locations and relationships with other objects, which use coordinate points and extents, in the form of discrete or continuous data.

For example, $s \in \mathbb{R}^d$ is the location of data in the d-dimensional Euclid space and the reserve data in the well expressed by $Z(s)$ then $s$ varies with the index set $D \subseteq \mathbb{R}^d$. Spatial process $\{Z(s) : s \in D\}$ satisfies second-order stationary [3], if:

1. $E[Z(s)] = m, \forall s \in D$
2. Covariances between $s$ and $s + h$ depend on vector which is separate $h$ and independent on location $s$.

$$\text{Cov} \{Z(s), Z(s+h)\} = E[Z(s)Z(s+h)] - m = C(h)$$

where $C(h)$ called covariance which’s satisfies $\text{Var}(Z(s)) = C(0)$ for all $s \in D$.

Spatial process $\{Z(s) : s \in D\}$ stationary intrinsic, if:

1. $E[Z(s+h) - Z(s)] = 0$
2. For all vector $h$, additional of $\{Z(s+h) - Z(s)\}$ has infinity variances which’s not depend on location $s$.

$$\text{Var}(s+h) - Z(s)) = 2 \gamma(h)$$

Where $h$ is vector on $\mathbb{R}^d$ and $\gamma(h)$ called semivariogram.

2.2 Experimental Semivariogram

Semivariogram is a statistical tool that is needed to estimate spatial data, because if there are two spatial values that are located close together, it will be relatively the same value compared to two spatial values that are located far apart.

The variogram is formulated as follows [3]:

$$2\gamma(h) = E[Z(s) - Z(s+h)]^2$$

To estimate spatial data, a device is used to describe, model, and calculate spatial correlations between random
variables $Z(s)$ and $Z(s+h)$, called semivariograms, which are half the value of the variogram. Semivariogram can be defined as follows [8]:

$$\gamma(h) = \frac{1}{2} E[Z(s) - Z(s+h)]^2$$

(4)

Experimental variogram is the estimated value obtained from sampling in the field. Experimental variogram is made based on the value of spatial correlation between two variables separated by a certain distance ($h$). The experimental semivariogram is formulated as follows:

$$\gamma(h) = \frac{1}{2N(h)} \sum[Z(s_i) - Z(s_i+h)]^2$$

(5)

where:

$\gamma(h)$ is the experimental semivariogram at lag distance $h$

$N(h)$ is the number of data pairs at lag distance $h$

There are three parameters used to describe the experimental semivariogram to calculate the parameters that will be used in the calculation of the model semivariogram. Some parameters used to look up values in the semivariogram model are as follows:

a) Nugget Effect ($C_0$)

   Nugget effect is an approach to semivariogram values at distances around zero.

b) Sill ($C_0 + C$)

   Sill is when the semivariogram value tends to reach a stable value

c) Range ($a$)

   Range is the distance when the semivariogram reaches the sill value.

2.3 Semivariogram Model

Calculation of experimental semivariogram will produce parameters that will be used in estimating Ordinary kriging parameters. However, the results of the experimental semivariogram calculation cannot be directly used in the estimation but must be modeled first. The semivariogram models are as follows:

1. Spherical Model

$$\gamma(h) = \begin{cases} C_0 + C \left[ \frac{3h}{2a} - 0.5 \left( \frac{h}{a} \right)^3 \right] & \text{for } h \leq a \\ C_0 + C & \text{for } h > a \end{cases}$$

(6)

where:

$h$ = distance of sample location

$C_0 + C$ = sill, the semivariogram value for distances when magnitude is constant

$a$ = range, the distance when the semivariogram value reaches sill

2. Exponential Model

$$\gamma(h) = C_0 + C \left[ 1 - \exp \left( -\frac{3h}{a} \right) \right]$$

(7)

3. Model Gaussian

$$\gamma(h) = C_0 + C \left[ 1 - \exp \left( -\frac{3h^2}{a^2} \right) \right]$$

(8)

Semivariogram model above in the form of images in figure 1 [2].
2.4 Ordinary Kriging

Ordinary Kriging (OK) is a method that BLUE (Best Linear Unlimited Estimator), which is a good linear unbiased estimator. OK is linear because the estimator is influenced by a linear combination of data, non-biased and good because it aims to get the mean error (mR) equal to zero, and minimize the variance of the error ($\sigma^2_R$). To estimate the value of an arbitrary point, for example $S_0$, namely the kriging estimator $\hat{Z}(S_0)$ is a linear combination of available data values such as: $Z(S_0), Z(S_0), \ldots, Z(S_n)$ or can be written as follows [1].

$$\hat{Z}(S_0) = \sum_{i=1}^{n} W_i Z(S_i) \ldots W_i \in \mathbb{R}$$  \hspace{1cm} (9)

where:
- $Z(S_0)$ fitted value at location is not sampled
- $w_i$ weighted coefficients $Z(s_i)$, with $\sum_{i=1}^{n} W_i = 1$
- $Z(s_i)$ value at location is sampled
- $n$ number of sample

2.5 Mean Square Error (MSE)

MSE is a method for measuring the accuracy of a model. The MSE value is used to determine the best semivariogram model by comparing the semivariogram value of the model with the smallest MSE value, which will be used to determine the estimation of special data. With the following formula:

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$  \hspace{1cm} (10)

where
- $y_i$ actual value
- $\hat{y}_i$ predicted value
- $n$ number of data

2.6 The Geological Structure of the Palu Valley

The main structure that developed in this region is a graben structure known as the Palu-Koro fault [9]. As a result of these activities the Palu valley was formed and the breakage of granite, schist and genes breakthrough rocks. Other geological structures in the form of selebes molasa deposits are often found in the region of East Palu Tondo.
2.7 Data
In this research used secondary data, where the data needed is wellbore data, location data coordinates. Height from sea level.

2.8 Data Analysis
Data analysis in this study was conducted using the Ordinary Kriging (OK) method with the help of Software R 3.5.2. The steps for data analysis are in the following Mind Flow Chart in figure 2.

![Mind Flow Chart for Research](image)

**Figure 2.** Mind Flow Chart for Research
3. Results and Discussion

3.1. Experimental Semivariogram

An experimental semivariogram is a tool to describe, model, and calculate spatial correlations between random variables $Z(s)$ and $Z(s + h)$, in estimating spatial data. An experimental semivariogram model is shown in figure 3:

![Figure 3. Experimental Semivariogram Model](image)

The experimental semivariogram above (figure 3) was obtained from experimental semivariogram values. So, the resulting parameter values are nugget = 20, sill = 24, and range = 5000.

3.2. Theoretical Semivariogram

Theoretical semivariogram has 3 types of models, namely Spherical, Gaussian, and Exponential models. Each of the three models will be calculated using sill, nugget, and range values based on experimental semivariogram. However, for the estimation calculation, the parameter values will be used according to the fit values of each model. Of the three models, only 1 model will be chosen to be used in the Ordinary Kriging estimation. The selection is based on the smallest Mean Square Error (MSE) value.

3.3. Structural Analysis

Structural analysis is an analysis for matching between experimental semivariogram and theoretical semivariogram with a comparison of the Mean Square Error (MSE) values. Semivariogram model with the smallest MSE value that will be used to make estimates using Ordinary Kriging. In this study the smallest MSE value obtained is the Spherical model of 429.5016. The semivariogram is shown in figure 4.

The parameter values that have been fit with the best model based on structural analysis are Nugget at 15.70937, Sill at 28.46195, and Range = 3343.121. The best model is used in estimating Ordinary Kriging. The first point to be estimated is (805573, 9887090) with a depth of 23,07785 meters. With the same formula in the estimation using Ordinary Kriging $\hat{Z}(S_0) = \sum_{(i=1)} Z(S_i)$, then the assumption of other points will be known with the depth of each point.
Based on structural analysis, the model for semivariogram that will be used in estimating Ordinary Kriging is the Spherical model. The plot for estimating the depth of the wellbore can be seen in the figure 5 below.

In the figure above shows the results of the estimated depth of the wellbore. Where for black indicates that the wellbore has a low depth, and for yellow indicates that the wellbore has the deepest depth based on this map.
3.5. Ordinary Kriging Estimation Contour Map

Contour maps are drawn based on contour lines that are used to provide information or data about the height of a place on the surface of the earth. In this study, contour maps are used to see the depth of the wellbore from the estimation results using Ordinary Kriging. Figure 6 is a contour map of the estimated results of the wellbore depth using the Ordinary Kriging method.

![Figure 6. Drill Well Depth Contour Map](image)

4. Conclusion

Based on the results obtained in this study it can be concluded that the best model in estimating Ordinary Kriging is the Spherical model with the smallest MSE value among other models that is equal to 429.5016. The resulting parameter values are nugget = 15.70937, sill = 28.46195, and range = 3343.121. The deepest depth is 23.07785 meters.

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