Feynman’s Branes and Feynman’s Oscillators

Y. S. Kim
Department of Physics, University of Maryland,
College Park, Maryland 20742, U.S.A.

Marilyn E. Noz
Department of Radiology, New York University,
New York, New York 10016, U.S.A.

Abstract

Based on Feynman’s lifetime efforts on quantum mechanics and relativity, it is concluded that the basic difference between field theory and string theory is that field theory is based on running waves while string theory should deal with standing waves in the Lorentz-covariant regime.

At the 1970 spring meeting of the American Physical Society held in Washington, DC, R. P. Feynman stunned the audience by proposing harmonic oscillators for relativistic bound states, instead of Feynman diagrams. His talk was later published in the paper of Feynman, Kislinger, and Ravndal [Phys. Rev. D, Vol. 3, 2706 (1971)]. These authors noted that the hadron mass spectra can be predicted by the degeneracy of the three-dimensional harmonic oscillators. In so doing, they started with the Lorentz-invariant differential equation for the harmonic oscillator, and obtained Lorentz invariant solutions. However, their solutions are not normalizable in the time-separation variable and cannot carry probability interpretation. It is pointed out that there are solutions normalizable in the time-separation variable within the framework of Wigner’s little-group representation of the Poincaré group. These solutions are not invariant but covariant under Lorentz transformations. These solutions give a covariant bound-state model which gives the quark model and the parton model as two different limiting cases, in the low- and high-speed limits respectively.

1 electronic address: yskim@physics.umd.edu
2 electronic address: noz@nucmed.med.nyu.edu
1 Introduction

Isaac Newton discovered the inverse-square law between two point particles, such as the sun and earth. It took him twenty years to formulate the gravity law for non-point particles such as the sun and earth. He had to invent a new mathematics to solve the problem of extended particles. We all know what mathematics Newton had to invent.

Ninety-nine years ago, Einstein formulated relativistic dynamics for point particles. Since then, particles became entities in quantum world. Yet, Einstein’s energy-momentum relation prevails for all those relativistic particles. The question then is the internal variables. In quantum world, particles have intrinsic spins. Are those spins consistent with Einstein’s Lorentz covariance? This question was raised by Wigner in his 1939 paper on representations of the Poincaré group [1].

Table 1: Massive and massless particles in one package. Wigner’s little group unifies the internal space-time symmetries for massive and massless particles. It is a great challenge for us to find another unification: the unification of the quark and parton pictures in high-energy physics.

| Massive, Slow | COVARIANCE | Massless, Fast |
|---------------|------------|---------------|
| Energy-Momentum | $E = p^2/2m$ | Einstein’s $E = [p^2 + m^2]^{1/2}$ | $E = p$ |
| Internal Space-time Symmetry | $S_3$ | Wigner’s Little Group | $S_3$ |
| $S_1, S_2$ | Gauge Trans. | |
| Relativistic Extended Particles | Quark Model | One Covariant Theory | Parton Model |

Table 1 summarizes the covariant picture of the present particle world. The second row of this table indicates that the spin symmetry of slow particles and the helicity-gauge symmetry of massless particles are two limiting cases of one covariant entity called Wigner’s little group. This issue has been extensively discussed in the
Let us then concentrate on the third row of Table 1. After Einstein formulated his special relativity, a pressing problem was to see whether his relativistic dynamics can be extended to rigid bodies as in the case of Newton’s sun and earth and their rotations. As far as we know, there are no satisfactory solutions to this problem. However, according to quantum mechanics, these extended objects are wave packets or standing waves. It might to easier to deal with waves in Einstein’s relativistic world.

As we all know, it is a trivial matter to write down plane waves in a covariant manner. The expression is Lorentz-invariant. Indeed, quantum field theory is possible because the plane waves take an invariant form. Plane waves are running waves.

As for localized objects, like the sun or earth, we have to consider standing waves. Standing waves are superposition of running waves in opposite directions. Do those superpositions remain invariant under Lorentz boosts? This is the question we wish to exploit from the 1971 paper of Feynman, Kislinger and Ravandal.

In discussing standing waves, it is common to start with a hard-wall potential, but mathematically it is more comfortable to use harmonic oscillators. Indeed, in their paper of 1971, Feynman et al. start with a Lorentz-invariant harmonic oscillator equation. This equation has many different solutions satisfying different boundary conditions. The solution they use is not normalizable in time-separation variable, and cannot be given any physical interpretation.

On the other hand, it is possible to fix up their mathematics. Their Lorentz-invariant differential equation has a normalizable solution which can form a representation space for Wigner’s little group for massive particles. We shall discuss this solution in more detail in this report.

Then, there is another question. Quantum field theory gives a calculational tool called the S-matrix. In the early 1960s, there was a movement in the physics community to start with the S-matrix, instead of complicated field theory, in spite of its field theoretic origin. This proposal was based on analytic properties of the S-matrix, and on the premise that the physics can be formulated in terms of singularities in the complex plane.

While the S-matrix deals primarily with scattering states, bound states can be found from the poles in the negative-energy region. This is indeed the case for nonrelativistic potential scattering. Therefore there was a movement to understand bound-state problems using analytic properties of the S-matrix.

In order to see what happened in this approach, we shall discuss in Sec. 2 a concrete example of the neutron-proton mass difference which once showed a promise but which did not work out. This case is known as the Dashen-Frautschi fiasco in the physics community. We shall show that this fiasco was caused by the confusion about running and standing waves in quantum mechanics. In Sec 3, we point out there are running
waves and standing waves in quantum mechanics. While it is trivial to Lorentz-boost running waves, it requires covariance of boundary conditions to understand fully standing waves. In Sec. 4 we construct the covariant harmonic oscillator wave functions. These wave functions can be Lorentz-boosted, but they depend on the time-separation variable. It is shown in Sec. 5 the quark and parton models are two different manifestation of the same covariant entity. The most controversial aspect of Feynman’s parton picture is that the partons interact incoherently with external signals.

2 Dashen-Frautschi Fiasco

On April 29, at the 1965 spring meeting of the American Physical Society in Washington, Freeman J. Dyson of the Institute of Advanced Study (Princeton) presented an invited talk entitled "Old and New Fashions in Field Theory," and the content of his talk was published in the June issue of the Physic Today, on page 21-24 (1965). This paper contains the following paragraph.

The first of these two achievements is the explanation of the mass difference between neutron and proton by Roger Dashen, working at the time as a graduate student under the supervision of Steve Frautschi. The neutron-proton mass difference has for thirty years been believed to be electromagnetic in origin, and it offers a splendid experimental test of any theory which tries to cover the borderline between electromagnetic and strong interactions. However, no convincing theory of the mass-difference had appeared before 1964. In this connection I exclude as unconvincing all theories, like the early theory of Feynman and Speisman, which use one arbitrary cut-off parameter to fit one experimental number. Dashen for the first time made an honest calculation without arbitrary parameters and got the right answer. His method is a beautiful marriage between old-fashioned electrodynamics and modern bootstrap techniques. He writes down the equations expressing the fact that the neutron can be considered to be a bound state of a proton with a negative pi meson, and the proton a bound state of a neutron with a positive pi meson, according to the bootstrap method. Then into these equations he puts electromagnetic perturbations, the interaction of a photon with both nucleon and pi meson, according to the Feynman rules. The calculation of the resulting mass difference is neither long nor hard to understand, and in my opinion, it will become a classic in the history of physics.

Dyson was talking about the paper by R. F. Dashen and S. C. Frautschi published in the Physical Review [5]. They use the S-matrix formalism for bound states. In
their paper, Dashen andFrautschiuse the S-matrix method to calculate a perturbed energy level. Of course, they use approximations because they are dealing with strong interactions. There are however “good” approximations and “bad” approximations.

If we translate what they did into the language of the Schrödinger picture, they are using the following approximation for the bound-state energy shift \[6\]
\[
\delta E = \left( \phi_{good}, \delta V \phi_{bad} \right),
\]
where
\[
\phi_{good} \sim e^{-br},
\]
\[
\phi_{bad} \sim e^{br},
\]
as illustrated in Fig. 1.

![Figure 1: Good and bad wave functions from the S-matrix theory. Bound-state wave functions satisfy the localization condition and are good wave functions. Analytic continuations of plane waves do not satisfy the localization boundary condition, and become bad wave functions at the bound-state energy.](image)

The Schrödinger equation is a second-order differential equation with two solutions. If the energy positive, there are two running-wave solutions. For negative energies, the two solutions take “good” and “bad” forms as indicated in Eq.(2). The good wave function is normalizable and carries probability interpretation. The bad wave function is not normalizable, and cannot be given any physical interpretation. If we demand that this bad wave function disappear, energy-levels become discrete. This is how the bound-state energy levels are quantized.

In the S-matrix formalism, the bound-states appear as poles in the complex energy plane. Those bound-state poles correspond to “good” localized wave functions in the
Schrödinger picture. At all other places, there are unlocalized “bad” wave functions. Dashen and Frautschi overlooked this point when they used approximations in the S-matrix theory, and ended up with the “bad” formula given in Eq. (1).

3 Running Waves and Standing Waves

The Dashen-Frautschi fiasco teaches us an important lesson. There are running waves and standing waves in quantum mechanics. Even though the standing wave is a superposition of running waves, it requires an additional care of boundary conditions. We do not know how to deal with this problem in the S-matrix formalism.

If not impossible, it is very difficult to formulate Lorentz boosts for rigid bodies. On the other hand, it seems to be feasible to boost waves. Indeed, quantum mechanics allows us to look at extended objects as wave packets or standing waves. Thus, we are interested in boosting waves. We should note here also that there are standing and running waves.

Plane waves are running waves. It is trivial to Lorentz-boost the plane wave of the form

\[ \exp \left\{ i (\vec{p} \cdot \vec{x} - p_0 t) \right\}, \]

because the exponent is invariant under Lorentz transformations. However, what would happen when different waves are superposed? Would the spectral function be covariant or invariant? What would happen for standing waves which consist of superposition of waves moving in opposite directions?

While quantum field theory based on Feynman diagrams starts with running waves, quantum mechanics within a localized space-time region deals with standing waves. If string theory is set to solve the problem inside particles, the physics of string theory is necessarily the quantum mechanics of standing waves.

In an attempt to obtain the answers to these questions, we can start with some examples. As usual in quantum mechanics, the first example for standing waves should be a set of harmonic oscillator wave functions. With this point in mind, let us see what Feynman did for harmonic oscillators in the relativistic regime.

4 Can harmonic oscillators be made covariant?

Quantum field theory has been quite successful in terms of perturbation techniques in quantum electrodynamics. However, this formalism is based on the S matrix for scattering problems and useful only for physical processes where a set of free particles becomes another set of free particles after interaction. Quantum field theory does not address the question of localized probability distributions and their covariance under Lorentz transformations. The Schrödinger quantum mechanics of the hydrogen atom
deals with localized probability distribution. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be a time-separation between the two constituent particles.

Before 1964 [7], the hydrogen atom was used for illustrating bound states. These days, we use hadrons which are bound states of quarks. Let us use the simplest hadron consisting of two quarks bound together with an attractive force, and consider their space-time positions $x_a$ and $x_b$, and use the variables

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)/2\sqrt{2}. \quad (4)$$

The four-vector $X$ specifies where the hadron is located in space and time, while the variable $x$ measures the space-time separation between the quarks. According to Einstein, this space-time separation contains a time-like component which actively participates as can be seen from

$$\left( \begin{array}{c} z' \\ t' \end{array} \right) = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \left( \begin{array}{c} z \\ t \end{array} \right), \quad (5)$$

when the hadron is boosted along the $z$ direction. In terms of the light-cone variables defined as [8]

$$u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}, \quad (6)$$

the boost transformation of Eq.(5) takes the form

$$u' = e^\eta u, \quad v' = e^{-\eta} v. \quad (7)$$

The $u$ variable becomes expanded while the $v$ variable becomes contracted, as is illustrated in Fig. 2.

Does this time-separation variable exist when the hadron is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Indeed, this variable belongs to Feynman’s rest of the universe. In this report, we shall see the role of this time-separation variable in the decoherence mechanism.

Also in the present form of quantum mechanics, there is an uncertainty relation between the time and energy variables. However, there are no known time-like excitations. Unlike Heisenberg’s uncertainty relation applicable to position and momentum, the time and energy separation variables are $c$-numbers, and we are not allowed to write down the commutation relation between them. Indeed, the time-energy uncertainty relation is a $c$-number uncertainty relation [9], as is illustrated in Fig. 3.

How does this space-time asymmetry fit into the world of covariance [10]. This question was studied in depth by the present authors in the past. The answer is that Wigner’s $O(3)$-like little group is not a Lorentz-invariant symmetry, but is a covariant
symmetry [1]. It has been shown that the time-energy uncertainty applicable to the time-separation variable fits perfectly into the $O(3)$-like symmetry of massive relativistic particles [4].

The c-number time-energy uncertainty relation allows us to write down a time distribution function without excitations [4]. If we use Gaussian forms for both space and time distributions, we can start with the expression

$$\left( \frac{1}{\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2} (z^2 + t^2) \right\}$$

for the ground-state wave function. What do Feynman et al. say about this oscillator wave function?

In their classic 1971 paper [3], Feynman et al. start with the following Lorentz-invariant differential equation.

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x).$$

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. Feynman et al. insist on Lorentz-invariant solutions which are not normalizable. On the other hand, if we insist on normalization, the ground-state wave function takes the form of Eq. (8). It is then

Figure 2: Lorentz boost in the light-cone coordinate system.
possible to construct a representation of the Poincaré group from the solutions of the above differential equation $[4]$. If the system is boosted, the wave function becomes
\[
\psi_\eta(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp \left\{-\frac{1}{2} \left( e^{-2\eta u^2} + e^{2\eta v^2} \right) \right\}.
\] (10)

This wave function becomes Eq.\,(8) if $\eta$ becomes zero. The transition from Eq.\,(8) to Eq.\,(10) is a squeeze transformation. The wave function of Eq.\,(8) is distributed within a circular region in the $uv$ plane, and thus in the $zt$ plane. On the other hand, the wave function of Eq.\,(10) is distributed in an elliptic region with the light-cone axes as the major and minor axes respectively. If $\eta$ becomes very large, the wave function becomes concentrated along one of the light-cone axes. Indeed, the form given in Eq.\,(10) is a Lorentz-squeezed wave function. This squeeze mechanism is illustrated in Fig.\,4.

There are many different solutions of the Lorentz invariant differential equation of Eq.\,(9). The solution given in Eq.\,(10) is not Lorentz invariant but is covariant. It is normalizable in the $t$ variable, as well as in the space-separation variable $z$. How can we extract probability interpretation from this covariant wave function?
Figure 4: Effect of the Lorentz boost on the space-time wave function. The circular space-time distribution at the rest frame becomes Lorentz-squeezed to become an elliptic distribution.

5 Feynman’s Parton Picture

It is a widely accepted view that hadrons are quantum bound states of quarks having localized probability distribution. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [3, 4].

In 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many “partons” whose properties appear to be quite different from those of the quarks [11]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

a. The picture is valid only for hadrons moving with velocity close to that of light.
b. The interaction time between the quarks becomes dilated, and partons behave as free independent particles.

c. The momentum distribution of partons becomes widespread as the hadron moves fast.

d. The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together.

In order to resolve this paradox, let us write down the momentum-energy wave function corresponding to Eq. (10). If the quarks have the four-momenta $p_a$ and $p_b$, we can construct two independent four-momentum variables

$$ P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b), $$

(11)

where $P$ is the total four-momentum and is thus the hadronic four-momentum.

$q$ measures the four-momentum separation between the quarks. Their light-cone variables are

$$ q_u = (q_0 - q_z)/\sqrt{2}, \quad q_v = (q_0 + q_z)/\sqrt{2}. $$

(12)

The resulting momentum-energy wave function is

$$ \phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{ -\frac{1}{2} \left( e^{-2\eta q_u^2} + e^{2\eta q_v^2} \right) \right\}. $$

(13)

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [4, 12, 13].

When the hadron is at rest with $\eta = 0$, both wave functions behave like those for the static bound state of quarks. As $\eta$ increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the $z$-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function, as is shown in Fig. 5. The longitudinal momentum distribution becomes wide-spread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from non-relativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a wide-spread distribution.
Figure 5: Lorentz-squeezed space-time and momentum-energy wave functions. As the hadron’s speed approaches that of light, both wave functions become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman’s parton picture.
However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the momentum-energy width increase in the same direction as the hadron is boosted. This is of course an effect of Lorentz covariance. This indeed is the key to the resolution of the quark-parton paradox [4, 12].

After all these qualitative arguments, we are interested in whether Lorentz-boosted bound-state wave functions in the hadronic rest frame could lead to parton distribution functions. If we start with the ground-state Gaussian wave function for the three-quark wave function for the proton, the parton distribution function appears as Gaussian as is indicated in Fig. 6. This Gaussian form is compared with experimental distribution also in Fig. 6.

For large $x$ region, the agreement is excellent, but the agreement is not satisfactory for small values of $x$. In this region, there is a complication called the “sea quarks.” However, good sea-quark physics starts from good valence-quark physics. Figure 6 indicates that the boosted ground-state wave function provides a good valence-quark physics.

**Concluding Remarks**

The present authors have been interested in question of covariant harmonic oscillators since 1973 [10]. We started with the covariant oscillator wave function as a purely phenomenological mathematical instrument. We then noticed that the covari-
The ant oscillator formalism can serve as a representation of the Wigner's little group for massive particles, capable of the fundamental symmetry representation for relativistic particles. This allows us to deal with the c-number time-energy uncertainty relation without excitations. Furthermore, the Lorentz-boosted Gaussian wave function produces a parton distribution in satisfactory agreement with experimental data.

What are then Feynman's contributions to this subject? In addition to the formulation of the parton picture, he suggested the use of harmonic oscillator wave functions to understand bound-state problems in the covariant regime. Then where does the Feynman diagram stand in his scheme? Feynman diagrams start with plane waves which are running waves. Harmonic-oscillator wave functions are standing waves. For standing waves, we have to take care of the covariance of boundary conditions or spectral functions. This is precisely what we are reporting in this report.

It is gratifying to note that there is only one covariant quantum mechanics for both scattering and bound states. For scattering states, we are dealing with asymptotically free waves, and Feynman diagrams start with plane waves. For bound states, we should start with standing waves. The harmonic oscillator wave functions constitute the starting example.

It is our understanding that the purpose of string theory is to understand the physics inside particles. Since particles are localized entities in the space-time region, string theory is necessarily a physics of standing waves if we are to preserve the present form of quantum mechanics. The Lorentz covariance of the standing waves is the major issue in string or brane theory.

References

[1] E. P. Wigner, Ann. Math. 40, 149 (1939).
[2] Y. S. Kim and E. P. Wigner, J. Math. Phys. 31, 55 (1990).
[3] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).
[4] Y. S. Kim and M. E. Noz, Theory and Applications of the Poincaré Group (Reidel, Dordrecht, 1986).
[5] R. F. Dashen and S. C. Frautschi, Phys. Rev. 135, B1190 and B1196 (1964).
[6] Y. S. Kim, Phys. Rev. 142, 1150 (1966).
[7] M. Gell-Mann, Phys. Lett. 13, 598 (1964).
[8] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).
[9] P. A. M. Dirac, Proc. Roy. Soc. (London) A114, 234 and 710 (1927).
[10] Y. S. Kim and M. E. Noz, Phys. Rev. D 8, 3521 (1973).

[11] R. P. Feynman, *The Behavior of Hadron Collisions at Extreme Energies*, in *High Energy Collisions*, Proceedings of the Third International Conference, Stony Brook, New York, edited by C. N. Yang et al., Pages 237-249 (Gordon and Breach, New York, 1969).

[12] Y. S. Kim and M. E. Noz, Phys. Rev. D 15, 335 (1977).

[13] Y. S. Kim, Phys. Rev. Lett. 63, 348 (1989).