Optimal control of dissipation for the example of the spin–boson model

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Abstract. – The interaction of a quantum system with a bath, usually referred to as dissipation, can be controlled if one can establish quantum interference between the system–bath interaction and a coupling of the system to an external control field. This is demonstrated for the example of the spin-boson model in the strong coupling limit for the system–bath interaction. It is shown that driving and trapping of the spin system leads to an optimum control problem which is nonlinear in the external control field. Using an indirect optimization strategy introducing a Lagrange-type adjoint state, we show that the spin system can be trapped in otherwise unstable quantum states and that it can be driven from a given initial state to a specified target state with high fidelity.

Introduction. – Over the last a few years, a number of interesting schemes have been proposed to eliminate the undesirable effects of decoherence in open quantum systems, including decoherence free subspaces [1, 2], quantum error correction codes [3–5], quantum feedback [6] and mechanisms based on unitary ”bang-bang” pulses and their generalization, quantum dynamical decoupling [7, 8]. The key ingredient of dynamical decoupling is the continuous disturbance of the system, which suppresses the system-environment interaction. It has been shown that, in the bang-bang control schemes, the decoherence of the system is effectively suppressed if the pulse rate is much higher than the decoherence rate due to the system-environment interaction [7].

The starting point of the decoupling techniques is the observation that even though one does not have access to the large number of uncontrollable degrees of freedom of the environment, it is still possible to interfere with its dynamics by inducing interactions into the sub-system which drive it so fast that the environment cannot follow [8]. Alternatively, if one can establish a suitable coupling to the system by means of an external control, one can establish quantum interference with the system–bath. In a simple minded model for a dissipative quantum system, where the interference between the system–bath and system–control interaction is ignored or is irrelevant only limited control can be achieved [9]. The situation changes dramatically when interference between the system–environment and system–control
interaction can be used to control the effective system–environment coupling [7, 8, 10–19]. The degree and nature of quantum interference constructive or destructive can be controlled by adjustment of the control field, known as coherent control.

In this paper we apply the concept of coherent control to steer a dissipative quantum system to the spin boson model, in which a quantum two-level system (qubit) is modelled by a spin, the environmental heat bath by quantum oscillators and the spin subjected to external control field is coupled to each bath oscillator independently [19–21]. Achieving decoherence control for this model is formulated using optimal control which is mathematically a problem of functional optimizations under constraints in form of differential equations [9, 22]. The two-level system coupled to a bath provides an adequate model of such diverse phenomena as electron transfer reaction [17], electron–phonon interaction in point defects [23] and quantum dots [24], interacting many–body systems [25], magnetic molecules [26] and bath assisted cooling of spins [12].

**Bloch-Redfield formalism.** Consider a physical system $S$ embedded in a dissipative environment $B$, also referred to as the heat bath, and interacting with a time-dependent classical external field i.e., the “control”. The total Hamiltonian $H_{\text{tot}} = H_S(t) + H_B + H_{\text{int}}$ is the sum of the system Hamiltonian $H_S(t)$, the bath Hamiltonian $H_B$ and their interaction $H_{\text{int}}$, which is responsible for decoherence. Note that the operator $H_S(t)$ contains a time-dependent external field to control the quantum evolution of the system. We suppose that the system-environment interaction Hamiltonian is bilinear $H_{\text{int}} = \sum_\alpha A_\alpha \otimes B_\alpha$ where $A_\alpha$ and $B_\alpha$ are Hermitian operators of the system and the environment, respectively.

The basic assumptions underlying the derivation of the equation of motion for the reduced density matrix $\rho(t) = \text{Tr}_B \{\rho_{\text{tot}}(t)\}$, are that (i) the initial factorization ansatz; we assume that at time $t = 0$ the bath $B$ is in thermal equilibrium and uncorrelated with the system $S$ ($\rho_{\text{tot}}(0) = \rho(0) \otimes \rho_B$, Feynman-Vernon approximation), (ii) weak system-bath interaction limit in which the second-order perturbation theory is applicable ($\rho_{\text{tot}}(t) = \rho(t) \otimes \rho_B + \mathcal{O}(H_{\text{int}})$, Born approximation) (iii) the relaxation time $\tau_B$ of the heat bath is much shorter than the time scale $\tau_R$ over which the state of the system varies appreciably ($\tau_B \ll \tau_R$, justifying the Markov approximation). From the Liouville-von Neumann equation $i\hbar \dot{\rho}_{\text{tot}} = [H_{\text{tot}}, \rho_{\text{tot}}]$ for the total density operator and after performing the above mentioned approximations, one obtains the master equation for the reduced density matrix in Bloch-Redfield form

$$\dot{\rho}_{ij} = -\frac{i}{\hbar} \sum_{kl} (H_{S,ik}(t) \delta_{lj} - \delta_{ik} H_{S,lj}(t)) \rho_{kl} - \sum_{kl} R_{ijkl}(t) \rho_{kl},$$

where the first term represents the unitary part of the dynamics generated by the system Hamiltonian $H_S(t)$ and the second term accounts for dissipative effects of the coupling of the system to the environment. The Redfield relaxation tensor $R_{ijkl}(t)$ is given by [27]

$$R_{ijkl}(t) = \delta_{ij} \sum_r \Gamma_{irrk}^+(t) + \delta_{ik} \sum_r \Gamma_{lrrj}^-(t) - \Gamma_{ijkl}^+(t) - \Gamma_{ijkl}^-(t),$$

where the time-dependent rates $\Gamma_{ijkl}^\pm(t)$ are evaluated through the following expressions:

$$\Gamma_{ijkl}^+(t) = \frac{1}{\hbar^2} \int_0^t d\tau \sum_{\alpha,\beta} \langle B_\alpha(\tau) B_\beta(0) \rangle_B A_{\alpha,ij} \sum_{m,n} U_{S,im}(t, t - \tau) A_{\beta,mn} U_{S,jn}^*(t, t - \tau),$$

$$\Gamma_{ijkl}^-(t) = \frac{1}{\hbar^2} \int_0^t d\tau \sum_{\alpha,\beta} \langle B_\beta(0) B_\alpha(\tau) \rangle_B \sum_{m,n} U_{S,im}(t, t - \tau) A_{\beta,mn} U_{S,jn}^*(t, t - \tau) A_{\alpha,ik}. $$
Fig. 1 – Population transfer from the ground state \( z(0) = 1 \) to the maximally mixed state \( z(t_f) = 0 \). (a) shows the control field vs. time. (b) and (c), respectively, show \( \Gamma_{zz} \) and \( \Gamma_0 \) vs. time. (d) shows the relative population \( \langle \sigma_z \rangle \) vs. time. \( w_f = 1, w_r = 0, \alpha = 0.2, \epsilon_0 = \Delta, \omega_c = 20\Delta, \beta = 1/\hbar \Delta \) and \( t_f = 100/\Delta \). The guess parameters chosen are \((A, \Omega, \phi) = (\Delta, \Delta/10, \pi/3)\) and those computed read \((A_{\text{opt}}, \Omega_{\text{opt}}, \phi_{\text{opt}}) = (5.838 \Delta, 0.211 \Delta, 28.173 \text{ rd})\). \( \Delta \) is an arbitrary unit of frequency.

Here \( U_S(t,t') = T \left\{ \exp \left[ -i / \hbar \int_{t'}^{t} \, d\tau \, H_S(\tau) \right] \right\} \) is the propagator of the coherent system dynamics while \( \langle B_\alpha(t)B_\beta(0) \rangle = Tr_B \{ \rho_B \} \) is the environment correlation function with \( \rho_B = \exp(-\beta H_B)/Z \), the canonical ensemble of the bath at the inverse temperature \( \beta = 1/k_B T \).

Because of the applied control field, the transition rates defined by \( W_{jj}(t) = \Gamma_{jj}^{+} \Gamma_{jj}^{+}(t) + \Gamma_{jj}^{-} \Gamma_{jj}^{-}(t) \), in the secular approximation which we suppose also valid in the driven case, become time dependent [15,16]. The field influence on both the unitary and dissipative contributions to the time evolution of the physical system makes possible an external control of dissipation. In particular, the correlation between the control field and the dissipation leads to the destruction of the detailed balance \( \lim_{t \to \infty} W_{ij}(t)/W_{ji}(t) \neq \exp(-\beta E_i)/\exp(-\beta E_j) \) where \( E_i \) are the energy eigenvalues of the undriven physical system. So, the steady state can be far from equilibrium in the driven case. The influence of the control field on the relaxation tensor via \( U_S(t,t') \) is a direct consequence of quantum interference between the system-bath interaction and the coupling of the system to the external field.

**Driven spin boson model.** – The Hamiltonian of the driven spin boson model where the two-level system is bilinearly coupled to an ensemble of harmonic oscillators is given by [19,21]

\[
H = -\hbar \Delta \sigma_x - \hbar \left( \epsilon_0 + \varepsilon(t) \right) \sigma_z + \frac{1}{2} \sum_i \left( \frac{p_i^2}{m_i} + m_i \omega_i^2 x_i^2 \right) + \frac{\sigma_z q_0}{2} \sum_i c_i x_i.
\]  

(5)

where \( \sigma_\alpha \) with \( \alpha = x, y, z \) are Pauli spin matrices; \( \hbar \Delta \) is the tunneling splitting, \( \hbar \varepsilon_0 \) is an energy bias and \( \hbar \varepsilon(t) \) is its modulation by a time-dependent external control field; and the heat bath is represented by a set of harmonic oscillators of mass \( m_i \), angular frequency \( \omega_i \), momentum \( p_i \) and position coordinate \( x_i \). The oscillators are coupled independently to the spin coordinate with strength measured by the set \( \{ c_i \} \) while \( q_0 \) measures the distance between the left and right potential wells. The coupling constants enter in the spectral density function of the environment defined by, \( J(\omega) = \frac{1}{2} \sum_i \frac{c_i}{m_i \omega_i} \delta(\omega - \omega_i) \).

In order to compute the Redfield tensor, it is necessary to determine the propagator of
the coherent system dynamics $U_\xi(t, \tau)$. An analytical expression for $U_\xi(t, \tau)$ is not trivial because the Hamiltonian of the physical system $H_\xi(t) = -\frac{h}{2}\Delta \sigma_x - \frac{h}{2}(\varepsilon_0 + \varepsilon(t)) \sigma_z$ is time-dependent and not diagonal. To get round this difficulty, we transform the Hamiltonian $H$ by the unitary operator, i.e., polaron transformation $U = e^\mathcal{O}$ with $\mathcal{O} = -\frac{i}{2}\sigma_z \Omega$ and $\Omega = \sum_i \Omega_i$ where $\Omega_i = (q_0c_i/h\omega_i^2) \rho_i$ [21]. The transformed Hamiltonian $H' = UHU^{-1}$ takes the following form

$$H' = -\frac{h}{2}(\varepsilon_0 + \varepsilon(t)) \sigma_z + \frac{1}{2} \sum_i \left( \frac{p_i^2}{m_i} + m_i \omega_i^2 x_i^2 \right) - \frac{1}{2} h \Delta \left( \sigma_+ e^{i\Omega} + \sigma_- e^{-i\Omega} \right), \quad (6)$$

where $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$. After invoking the polaron transformation the coherent propagator is trivial and reads $U_\xi(t, t') = \cos[(\varepsilon_0(t - t')/2 + f(t, t'$/2)I + i\sin[(\varepsilon_0(t - t')/2 + f(t, t'/2)\sigma_z$

where the function $f(t, t') = \int_{t'}^t d\varepsilon(\tau)$ captures the effects of the external control field.

**Rate master equation.** - Let us consider the polaron transformed spin-boson Hamiltonian and derive the explicit form for the corresponding master equation for small $\Delta$. Here, the system and the environment operators are $S_1 = h\Delta \sigma_+/2$, $S_2 = S_1^\dagger = h\Delta \sigma_-/2$ and $B_1 = e^{-i\Omega}$, $B_2 = B_1^\dagger = e^{i\Omega}$, respectively. The Bloch-Redfield formalism leads to the following master equation for the diagonal elements of the the reduced density matrix (the populations)

$$\dot{\rho}_{11}(t) = \rho_{22}(t)W_{12}(t) - \rho_{11}(t)W_{21}(t), \quad (7)$$

$$\dot{\rho}_{22}(t) = \rho_{11}(t)W_{21}(t) - \rho_{22}(t)W_{12}(t), \quad (8)$$

with time dependent transition rates $W_{12}(t) = \Gamma_{21,12}(t) + \Gamma_{12,21}(t)$ and $W_{21}(t) = \Gamma_{12,21}(t) + \Gamma_{12,21}(t)$. Up to the second order in $\Delta$, the populations decouple from the non-diagonal terms. The master equation for the population difference $z(t) = \langle \sigma_z \rangle_t = \rho_{11}(t) - \rho_{22}(t)$ reads

$$\dot{z}(t) = -\Gamma_{zz}(t)z(t) + \Gamma_0(t), \quad (9)$$

where $\Gamma_{zz}(t) = W_{12}(t) - W_{21}(t)$ and $\Gamma_0(t) = W_{12}(t) + W_{21}(t)$ defined as

$$\Gamma_{zz}(t) = \Delta^2 \int_0^t d\tau e^{-Q_2(\tau)} \cos[f(t, t - \tau)] \cos[Q_1(\tau)], \quad (10)$$

$$\Gamma_0(t) = \Delta^2 \int_0^t d\tau e^{-Q_2(\tau)} \sin[f(t, t - \tau)] \sin[Q_1(\tau)]. \quad (11)$$

The quantities $Q_1(\tau)$ and $Q_2(\tau)$ are the imaginary and the real part, respectively, of the function $\Phi(\tau) = \frac{d^2}{d\omega} \int_0^\infty d\omega (\omega - \Omega) [(1 - \cos \omega \tau) \coth(\omega \beta/2) + i \sin \omega \tau]$, where $e^{-\Phi(\tau)} = \langle e^{i\Omega(\tau)}e^{-i\Omega(0)} \rangle$ is the environment correlation function [28]. In the present work, the spectral density of the environment $J(\omega) = \eta \omega e^{-\omega/\omega_c}$ is assumed to be Ohmic with exponential cutoff. Here $\eta$ is a phenomenological friction coefficient. For an Ohmic bath and at low temperatures regime ($h\omega_c/\beta \gg 1$) one can use the approximation [21]

$$Q_1(\tau) = 2\alpha \arctan(\omega_c \tau), \quad (12)$$

$$Q_2(\tau) = \alpha \ln(1 + \omega_c^2 \tau^2) + 2\alpha \ln \left[ \frac{\sinh(\pi \tau/\beta h)}{\pi \tau/\beta h} \right] \quad (13)$$

where the dimensionless dissipation constant $\alpha = q_0^2 \eta/2\pi h$ have been introduced.
Quantum optimal control problem. – Let time \( t \) be in the interval \([0, t_f]\) for fixed \( t_f \). The evolution of the state variable \( z(t) \) governed by the master equation depends not only on the initial state \( z(0) = z_i \) but also on the time-dependent control variable \( \varepsilon(t) \). The task is now to find a control field that will steer the system from its initial state to a desired final state at specified time \( t_f \). Typically, it is possible to define a cost functional incorporating the objective. The goal of optimal control algorithms is to calculate a control field which can induce a specified system dynamics by minimizing this cost functional. Consider then the following quantum optimal control problem of minimizing the cost functional \( J(p) \) subject to the dynamical constraint, i.e., the master equation for \( z(t) = \langle \sigma_z \rangle_t \)

\[
\begin{cases}
\min J(p) = \frac{w_f}{2} (z(t_f) - z_d)^2 + \frac{w_r}{2t_f} \int_0^{t_f} dt \ (z(t) - z_r(t))^2, \\
\dot{z}(t) = -\Gamma_{zz}(t) z + \Gamma_0(t), \quad t \in [0, t_f], \\
z(0) = z_i.
\end{cases}
\]  

(14)

The first term in the cost functional \( J(p) \) measures the deviation of the final state \( z(t_f) \) from the desired state \( z_d \). During the time interval \([0, t_f]\), there may also exist a desired state trajectory \( z_r(t) \). This objective is incorporated in the second term. Here, it is assumed that \( 0 \leq w_f \leq 1 \) and \( 0 \leq w_r \leq 1 \) with \( w_f + w_r = 1 \). \( J(p) \) is the sum of the so-called final time cost functional and running cost functional. The vector \( p \in \mathbb{R}^{N_p} \) is a set of parameters on which the control field depends, i.e., \( z(t) \equiv \varepsilon(t, p) \). Here, an optimal solution of this problem is characterised by first order optimality conditions in the form of the Pontryagin’s Minimum Principle [22]. These conditions are formulated with the help of a Hamilton function that has the following form in our problem:

\[
\mathcal{H}(z, \lambda, p) = \frac{w_r}{2t_f} (z(t) - z_r(t))^2 + \lambda(t) \left\{ -\Gamma_{zz}(t) z + \Gamma_0(t) \right\}.
\]  

(15)
Fig. 3 – Trapping of the spin in unstable excited state, \( z(0) = 1 \). (a) and (b), respectively, show \( \Gamma_{zz} \) and \( \Gamma_0 \) vs. time. (c) shows the relative population \( z = \langle \sigma_z \rangle \) vs. time. (d) shows the cost functional vs. the number of iteration. \( w_f = 0, w_r = 1, \alpha = 0.2, \epsilon_0 = -\Delta, \omega_c = 20\Delta, \beta = 1/\hbar\Delta \) and \( t_f = 50/\Delta \). The guess parameters chosen are \( (A, \Omega, \phi) = (12\Delta, 5\Delta, 0) \) and those computed read \( (A_{\text{opt}}, \Omega_{\text{opt}}, \phi_{\text{opt}}) = (218.196\Delta, 89.995\Delta, -1.117 \text{ rd}) \). \( \Delta \) is an arbitrary unit of frequency. Note that \( \Omega_{\text{opt}} \) is much higher than \( \omega_c \).

Pontryagin’s minimum principle states that a necessary condition for \((z, p)\) to be a solution of the above optimal control problem is the existence of an adjoint state \( \lambda \) such that

\[
\begin{align*}
\dot{z}(t) &= -\frac{\partial H}{\partial \lambda} = -\Gamma_{zz}(t)z + \Gamma_0(t), \\
\dot{\lambda}(t) &= -\frac{\partial H}{\partial z} = -\frac{w_f}{\gamma_f}(z(t_f) - z_d) + \Gamma_{zz}(t)\lambda(t), \\
z(0) &= z_i, \quad \lambda(t_f) = w_r (z(t_f) - z_d), \\
0 &= \frac{\partial H}{\partial p_i}, \quad i = 1\ldots N_p \quad \text{and} \quad t \in [0,t_f].
\end{align*}
\]  

The minimum principle requires the solution of a set of complicated nonlinear algebraic equations, namely the optimality conditions \( \partial H/\partial p_i = 0, i = 1\ldots N_p \) which can only be solved in an iterative manner. The present optimal control problem is not singular because \( \det \left( \frac{\partial^2 H}{\partial p_i \partial p_j} \right) \neq 0, i, j = 1\ldots N_p \), since \( \Gamma_{zz}(t) \) and \( \Gamma_0(t) \) depend nonlinearly on \( \epsilon(t,p) \). The optimality conditions \( \partial H/\partial p_i = 0, i = 1\ldots N \), gives the gradient for the cost functional \( J(p) \) with respect to control parameters \( p_i \)

\[
\frac{\partial J}{\partial p_i} = \int_0^{t_f} \lambda(t) \left[ \frac{\partial \Gamma_{zz}(t)}{\partial p_i} z(t) + \frac{\partial \Gamma_0(t)}{\partial p_i} \right] dt, \quad i = 1\ldots N_p.
\]

In summary, the computation of the gradient towards an optimal solution requires the computation of \( z(t) \) forward in time and the adjoint state \( \lambda(t) \) backward in time.

**Numerical Results.** – In the present work, we write the control field as monochromatic plane wave \( \epsilon(t,p) = \epsilon(t,A,\Omega,\phi) = A \cos(\Omega t + \phi) \). In this case, an analytic expression can be found for the function \( f(t,t') = f(t,t',A,\Omega,\phi) = \int_t^{t'} d\tau \epsilon(\tau,A,\Omega,\phi) \) which greatly reduces the numerical effort for finding a solutions to this optimization problem. The task is then to find a set of three parameters namely the amplitude \( p_1 = A \), the frequency \( p_2 = \Omega \) and the phase \( p_3 = \phi \) such that the cost functional \( J(p) \) defined above is minimal. The optimization
is performed using the gradient method. More precisely, we used the subroutine DMNG of port library [29] implementing the quasi-Newton method as variant of gradient algorithms.

*Heating.* - As a first example, we consider driving the system from a pure state $z(0) = 1$, corresponding to the temperature-zero ground state of the two-level system, into the mixed target state $z(t_f) = 0$. Fig. 1(d) shows that in the absence of the control, the system inevitably relaxes to the thermal equilibrium state

$$z_{st} = \frac{\Gamma_0(\epsilon_0)}{\Gamma_{zz}(\epsilon_0)} = \frac{W_{12}(\epsilon_0) - W_{21}(\epsilon_0)}{W_{12}(\epsilon_0) + W_{21}(\epsilon_0)} = \tanh(\hbar \beta \epsilon_0 / 2). \quad (18)$$

Fig. 1 shows also guess and optimal harmonic control, obtained via the gradient method, the rate $\Gamma_{zz}(t)$ and the inhomogeneous term $\Gamma_0(t)$, as well as the time evolution of the relative population $z(t)$. Owing to the control dependence of $\Gamma_{zz}(t)$ and $\Gamma_0(t)$ which reflects the periodic nature of the control field, Fig. 1(b) and 1(c), the objective posed can be achieved perfectly.

*Cooling.* - As a second example, we consider a spin-flip in which $\langle \sigma_z \rangle = 1$ at t=0 is changed to $\langle \sigma_z \rangle = -1$ at target time $t_f$ with an optimal solution displayed in Fig. 2a. Here, the simplest optimum harmonic solution selected is a low-frequency field which simply changes the level sequence in the two-level system, at the same time, renormalising $\Gamma_{zz}$ and $\Gamma_0$. Approaching target time $t_f$, $\Gamma_{zz}(t)$ approaches $-\Gamma_0(t)$ to provide $z(t_f) = -1$, as shown in Figs. 2(b) and 2(c). In Fig. 2(d) we also show the instantaneous equilibrium value $z_{st}(t) = \tanh(\hbar \beta (\epsilon_0 + \epsilon_{opt}(t))/2]$ which corresponds to the ratio $(W_{12}(t) - W_{21}(t))/(W_{12}(t) + W_{21}(t)) = \Gamma_0(t)/\Gamma_{zz}(t)$ relevant in the adiabatic limit, valid when $|d\epsilon(t)/dt| \ll \Delta^2$ as discussed in the literature [19,26]. In the long-time limit, the prediction from the adiabatic approximation is fulfilled by the numerically selected optimal electric field $\epsilon_{opt}(t)$ in Fig. 2(d). In the context of bath-assisted cooling of spins, our finding has certain analogies with the results of Ref. [12] where a version of the spin-boson model under influence of externally controlled pulses is studied. Similar to our situation, an interference between the external fields and the spin-bath interaction create a mechanism that cool the spins much below the bath temperature. Starting from the initially infinite-temperature spin the authors cool it down to very low temperatures by increasing the spin’s polarisation.

*Trapping.* - As a third example, we consider trapping of the spin system for $\epsilon_0 = -\Delta$ in state $z(t) = 1$ for all time $t \in [0, t_f]$ which now corresponds to the (unstable) excited state of the system. In isolated case, trapping is made possible by dynamic localization using a periodic control field to renormalize the coupling between the two levels from $\Delta$ to $\Delta J_0(A/\Omega)$ for a large frequency of the field; here $J_0(x)$ is the zero order Bessel function [19,31]. Therefore, when $A/\Omega$ is such that $J_0(A/\Omega) = 0$, the two-levels are decoupled and the tunneling between them is suppressed. In the presence of dissipation the trapping is destroyed, the undriven system relaxes to the equilibrium state, and the guess harmonic field with high frequency is not able to trap the system as shown in Fig. 3(c). For an optimum harmonic control field, we find that trapping becomes truly effective when the control frequency exceeds the cut-off frequency which was set as $\omega_c = 20\Delta$. The optimum solution for the control parameters is $(A_{opt}, \Omega_{opt}, \phi_{opt}) = (218.196,\Delta, 89.996,\Delta, -1.119 \text{ rad})$. As shown in Figs. 3(a) and 3(b), this high-frequency control renormalizes $\Gamma_{zz}(t)$ and $\Gamma_0(t)$ to values of the order $10^{-5}\Delta$ such that $z(t) \approx 0$ for all time $t \in [0, t_f]$. This finding is in qualitative agreement with the bang-bang method used to study the independent spin-boson model [7]. As we mentioned in the introduction, this study showed that the control has to be switched at a rate greater than $1/\omega_c$ in order to provide effective decoupling of the physical system from the environment.
Fig. 3(d) shows the cost functional versus the number of iteration. According to this figure, the cost functional is monotonically decreasing and its convergence to zero is reached after 15 iterations.

**Conclusion.** In summary, we have formulated and solved an optimal control problem for the spin–Boson model to demonstrate feasibility of controlling the effective system–bath interaction by an external control field. An external control not only renormalises the spin Hamiltonian but also the effective coupling between system and environment. We have demonstrated that the effective system-bath interaction can be increased or decreased in controlled fashion. This mechanism can be used to optimize dynamic driving and trapping of the spin system. Results were presented for control of the relative population $z$ of the spin system and a harmonic control field. This work has been extended to a control of all components of the Bloch vector simultaneously, as well as to general control field shapes. The results will be presented elsewhere.

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**REFERENCES**

[1] P. Zanardi and M. Rasetti, Phys. Rev. Lett. 79, 3306 (1997).
[2] D. A. Lidar, I. L. Chuang and K. B. Whaley, Phys. Rev. Lett. 81, 2594 (1999).
[3] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK, 2000).
[4] J. Preskill, Proc. Roy. Soc. Lond. A 454, 385 (1998).
[5] E. Knill, R. Laflamme and L. Viola, Phys. Rev. Lett. 84, 2525 (2000).
[6] J. Wang and H. Wiseman, Phys. Rev. A 64, 063810 (2001).
[7] L. Viola et al., Phys. Rev. A 58, 2733 (1998); Phys. Rev. Lett. 82, 2417 (1999); 83, 4888 (1999).
[8] D. Vitali and P. Tombesi, Phys. Rev. A 59, 4178 (1999); 65, 012305 (2001).
[9] H. Jirari and W. Pötz, Phys. Rev. A 72, 013409 (2005).
[10] L. Hartmann, I. Goychuk, M. Grifoni and P. Hänggi, Phy. Rev. E 61, R4687 (2000).
[11] K. M. Fonseca Romero et al., Chem. Phys. 296, 307 (2004); Phys. Rev. Lett. 95, 140502 (2005).
[12] A. E. Allahverdyan et al., Phys. Rev. Lett. 93, 260404 (2005).
[13] X. Hu and W. Pötz, Phys. Rev. Lett. 82, 3116 (1999).
[14] R. Xu et al., J. Chem. Phys. Lett. 120, 6600 (2004).
[15] O. Kocharovskaya et al., Phys. Rev. A 49, 4928 (1994).
[16] D. H. Schirrmeister and V. May, Chem. Phys. Lett. 297, 383 (1998).
[17] M. Morillo et al., Phys. Rev. B 54, 13962 (1996); Chem. Phys. 212, 157 (1996).
[18] Yu. Dakhnovskii, Phys. Rev. B 49 (1994);
[19] M. Grifoni and P. Hänggi, Phys. Rep 304, 1 (1998).
[20] U. Weiss, Quantum Dissipative Systems, (World Scientific, Singapore 1999).
[21] A. J. Leggett, et al., Rev. Mod. Phys 59, 1 (1987).
[22] A. E. Bryson, Jr. Yu-Chi Ho, Applied Optimal Control (Hemisphere, New York, 1975);
[23] B. Krumhauer, V.M. Axt and T. Kuhn, Phys. Rev. B. 65, 195313 (2002).
[24] U. Hohenester and G. Stadler, Phys. Rev. Lett 92, 196801 (2004).
[25] J. Zhang and W. Pötz, Phys. Rev. B 48, 11583 (1993).
[26] I. Roussochatzaki et al., Phys. Rev. Lett 94, 147204 (2005).
[27] K. Blum, *Density Matrix Theory and Applications*, (Plenum Press, New York, 1996).
[28] G. D. Mahan, Many Particles Physics, (Plenum Press, New York, 1999).
[29] http://www.netlib.org/port, D. M. Gay, *Usage Summary for Selected Optimization Routines*, Computing Science Technical Report No 153, AT&T Bell Laboratories (1990).
[30] F. Grossmann, T. Dittrich, P. Jung, and P. Hänggi, Phys. Rev. Lett 67 516 (1991).
[31] J. Stockburger, Phys. Rev. E 59, R4709 (1999).