Decomposition of the Vector Field of Preferred Directions for Optimization of Five-Axis Machining

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Abstract. A new algorithm to increase the production rate of a five-axis milling machine through improving the coordinates of the to-be-milled points transformed from the workpiece to the machine coordinates is presented and analysed. The method optimizes the cutting path by following a vector field (VF) of optimal directions maximizing the material removal rate (MRR). The algorithm includes grid generation, space filling curves (SFC) and a VF decomposition using rotation invariant complex moments. The case of a radial tool path requires a special treatment called Compact Radial Zigzag (CRZ). To reduce the redundancy, the CRZ is composed of layers with a varying step between the tracks. The combination of the proposed techniques generates tool paths which produce complex shaped Stereolithography (STL) surfaces faster than the conventional methods.

1. Introduction

A five-axis machine is a programmable mechanism fed with the CNC program (G-code) i.e. a sequence of commands composed of three Cartesian coordinates of the tool-tip in the machine coordinate system and two rotation angles required to establish the orientation of the tool (Fig 1). Matching a five-axis toolpath with a VF of preferred directions (VFPD) has been analysed in many publications and is becoming popular in the five-axis machining industry. However, the majority of the algorithms obtain toolpaths by propagating an initial curve inside the parametric region. Optionally, the region is subdivided into clusters characterized by homogeneous VFs. Most of the algorithms have not been verified on industrial formats such as STY, IGES, or STEP.

The VFPD is machine-dependent and is evaluated using the kinematics transformations based on coordinate systems shown in figure 1. The idea to match the tool path and the VFPD based on the maximization of the machining strip was proposed by Chiou and Lee [1]. The first algorithm to cluster the VFPD was introduced in [2]. Some recent modifications are the tensor based approach [3] and segmentation into regions of similar pathlines [4]. A comprehensive survey of these techniques is given in [5] (see also [6],[7]).

When a VFPD-method offsets an initial path and propagates it inside the region, the path may substantially deviate from the VFPD. Then the algorithm re-initializes it. This procedure often creates configurations which does not actually match the desired VFPD. The tool paths often become redundant i.e. the distance between the tracks is considerably smaller than the maximum allowed machining strip. Besides, these algorithms are sensitive to the choice of the initial path which itself is a computationally expensive, NP-hard problem. Finally, as mentioned above, only a few algorithms have been verified on real industrial formats such as STL, IGES or STEP [8].

In this paper a new VFPD-type toolpath generation method based on curvilinear grids [9]-[11] and Adaptive Space Filling Curves (ASFC) [12]-[13] has been proposed. The method has been applied to
the most popular industrial format-STL. The resulting VF-aligned path combines the adaptivity of path-propagating methods with a convenient, regular topology of the conventional patterns such as the zigzag or the spiral. The VF can be obtained using a variety of utility functions such as the machining time, toolpath length (machine independent), kinematic error, etc. In this paper the algorithm constructs a curvilinear path which partly or even entirely aligns with the VF based on the maximum material removal rate MMRR. The reference methods are the standard iso-parametric path (ISO), MasterCam and advanced toolpath generation algorithms of NX11 (former UG) e.g. Helical/Spiral (HS) and Follow Periphery (FP). The cutting experiments have been performed on a milling machine HAAS VF2TR.

Figure 1. Milling machine HAAS VF2TR, National Institute of Metrology of Thailand (a) milling machine with a high speed spindle, (b) the rotational and translational axis of the machine: O1-workpiece coordinate system, O2-coordinate system of the first rotary part (B-axis), O3-coordinate system of the second rotary part (A-axis), O4-coordinate system of the spindle.

The proposed toolpath generation technology consists of the following steps:

— Flatten the surface (parametrization).
— Select a cost function and constraints to evaluate the optimal directions.
— Evaluate the VFPD.
— Cluster the VFPD using complex moment invariants, and a region growing algorithm.
— For each cluster, generate a toolpath aligned with the VF.
— Extend the boundaries of the clusters.
— Define the order of the machining using the shortest path strategy.

2. Vector field of optimal tool directions
Consider a part surface \( S(u,v) \). Let \( W_i \) be a CC (cutter contact) point. Define a collection of points given by \( \Omega_{W_i} = \{ W : \text{dist}_s(W,W_i) = l_i \} \), where \( \text{dist}_s \) is the geodesic distance and \( l_i \) is a prescribed step. The image of \( \Omega_{W_i} \) in the machine coordinates is denoted by \( \Omega_{M_i} \). The distance between \( M_i \) and \( M \) is denoted by \( l_{i,M} = l_{i,M}(W) \).

The machining strip corresponding to \( W_i \) is denoted by \( \Gamma_{W_i} \). \( \Omega_{W_i} \) and \( \Omega_{M_i} \) are obtained by the corresponding inverse kinematic transformations [11]. Note that \( \Omega_{W_i} \) is a geodesic circle, whereas \( \Omega_{M_i} \) is represented by an irregular closed curve. The distance on \( S(u,v) \) is not proportional to the distance in the machine coordinates. The machining time \( t \) depends on the translations and rotations of the machine parts necessary to establish the required position and orientation of the tool.

Define the MRR in the direction \( W_i \) by \( R_i(W) = \frac{F l_i(W) w_i(W)}{l_{i,M}} \) [mm\(^2\)/s], where \( F \) is the feed rate.

The machining strip \( W_i \) depends on, the curvature of the part surface, the shape of the tool and the
effective cutting profile. Methods of evaluating the machining strip are presented in [11]. The direction \( W_{1} \rightarrow W_{2} \) becomes optimal, if \( W_{2} = \arg \max_{W_{2} \in \Omega_{2}} R_{d}(W) \) e.g. \( W_{2} \) produces MMRR. Vectors \( W_{1} \rightarrow W_{2} \) evaluated for each surface point and transferred into the parametric coordinates form a VFPD. Denote VFPD by \( V(u,v) \). Unfortunately, a continuous, smooth toolpath maximizing the utility function \( R_{d}(W) \) for every \( W \) on a complex shaped part surface usually is not possible. On the one hand, the tool path has to follow the prescribed VF. On the other hand, the desirable geometric structure of such a path is the zigzag, spiral or the radial pattern. The latter requirement improves the smoothness of the machined part, reduces the possibility (although does not guarantee the avoidance) of gouging and global collisions.

3. Grid generation

We arrange the cutter location (CC) points \( \{(u_{i,j}, v_{i,j})\}, 0 \leq i \leq N_{i}, 0 \leq j \leq N_{j}\) as a curvilinear grid in the parametric coordinates \( u,v \). Mathematically, the CC points define a mapping from the computational region \( \Delta = \{0 \leq \xi \leq N_{\xi}, 0 \leq \eta \leq N_{\eta}\} \) into a domain \( K \) defined in the parametric coordinates. In other words, there exists a pair of functions \( \{(\xi,\eta), (\xi,\eta)\} \) such that the rectangular grid \( \{(i,j)\} \) being fed to \( \{(\xi,\eta), (\xi,\eta)\} \) becomes \( \{(u_{i,j}, v_{i,j})\} \).

According to this model, the required VF is partitioned into \( (a(u,v),b(u,v)) \), creating a dual VF, corresponding to the \( \xi \) and \( \eta \) directions (figure 2):

\[
\begin{align*}
    a(u,v) &= \begin{cases} 
        V(u,v) \in \Omega_{a}, & \text{if } \xi(\xi,\eta), \\
        0, & \text{otherwise,}
    \end{cases} \\
    b(u,v) &= \begin{cases} 
        V(u,v) \in \Omega_{b}, & \text{if } \eta(\xi,\eta), \\
        0, & \text{otherwise,}
    \end{cases}
\end{align*}
\]

where \( \Omega_{a}, \Omega_{b} \) are prescribed subsets of \( V(u,v) \) selected according to a certain criteria.

![Figure 2. The dual VF (a)- a(u,v), (b)- b(u,v)](image)

The grid \( \{(\xi,\eta), (\xi,\eta)\} \) is adapted to the dual VF \( (a(u,v),b(u,v)) \) using a modification of a standard grid generation approach [6]-[9]. The smoothness of the grid is represented by a functional given by

\[
F_{3} = \int \int u_{\xi}^{2} + u_{\eta}^{2} + v_{\xi}^{2} + v_{\eta}^{2} d\xi d\eta,
\]

where subscripts denote the partial derivatives. The numerical solution is based on approximating of the second-order elliptic PDEs and an iterative solution by the Newton-Raphson method.

4. Numerical and cutting experiments

This section presents a VF-grid alignment method combined with the ASFC [10], tested against ISO, “Follow Periphery” (FP) and “Helical or Spiral” (HS) routines of NX11. A five-axis toolpath cannot be generated by NX11 directly from the STL format. Therefore, we convert the STL to a solid model, using fit surface functions of NX11. The average Euclidean error is 0.05 mm, which ensures the accuracy of the approximated surface. The HS and FP toolpaths are generated from the solid model. An example of the test surface is shown in figure 3. The blank workpiece is 200x200x50 mm. The curvilinear grid adapted to the dualVF in figure 3 is shown in figure 3(a) and the corresponding tool path in figure 3 (b).

The surfaces produced by the conventional ISO zigzag and by the proposed method are displayed in figure 4 (a) and (b) respectively. Table 1 shows the machining time. The proposed method outperforms
the conventional algorithms for every prescribed scallop height, $h$. For instance, for $h=0.25\text{mm}$ the VF tool path is 5.3 times faster than that generated by MasterCam, by 23% faster than ISO, 7.8 times faster than HS and by 40% faster than FP.

Figure 3. (a) adaptive grid, (b) SFC- tool path

Figure 4. The machining results (a) ISO zigzag (b) the VF method

5. Decomposition of the VF using flusser complex moments

Several algorithms have been proposed to decompose the VF of optimal directions for tool path generation of five-axis machines e.g. [3],[4]. However, most of the algorithms are not applicable to the STL surfaces since the corresponding VF is usually highly irregular and noisy. In this section we present a clustering approach based on complex moments proposed by Flusser [14] to circumvent the above drawbacks. We illustrate the algorithm by radial (star) pattern which requires a special treatment due to the high redundancy near the pole. The basic idea of the CRZ is minimization of the machining time by decomposing the region into several subregions characterized by an increasing stepover between the radial tracks[15]. A video demo of the algorithm is available at: https://drive.google.com/open?id=1OM_z4cAOUqGu2RPAzkZOIBcEnfptdTq7.

The model of the crown of the canine tooth produced by the proposed method is shown in figure 5.

Finally, the machining time for the test examples in figures 3 and 4 is compared with the conventional methods in Tables 1 and 2.

Figure 5. Application of the CRZ to a dental crown, (a) two-layer CRZ, (b) virtual machining (Vericut), (c) workpiece prepared for the fine cut, (d) the result

6. Conclusions

A new method for generation of VF-aligned tool paths for five-axis machining has been presented and analysed. The new idea is numerical generation of a curvilinear tool path adapted to the VF of optimal directions. This is combined with clustering of the VF and generation of local optimal subgrids providing faster machining. By introducing internal radial layers, the proposed algorithms eliminates the redundant parts. The experiments show a substantial decrease in the machining time with regard to preceding methods. In our experiments the proposed method outperforms the conventional algorithms for every prescribed scallop height, moreover, the advantage increases as the scallop height decreases. The CRZ partition makes it possible to set up the maximum machining strip on every circular boundary, reducing
the redundancy, and therefore, reducing the machining time. The process is repeated until the total machining time cannot be improved.

| Method | Scallop height, mm | Scallopl height, mm |
|--------|-------------------|---------------------|
|        | 0.25   | 0.1    | 0.05   |
| NX11 HS| 2:59:36 | 7:10:23 | 14:54:49 |
| FP     | 1:27:47 | 2:18:48 | 3:16:02  |
| ISO zigzag | 0:33:49 | 0:53:10 | 1:15:04 |
| ASFC   | 0:26:43 | 0:41:56 | 0:56:26 |

| Method | Scallop height, mm | Scallopl height, mm |
|--------|-------------------|---------------------|
|        | 1.0    | 0.5    | 0.1    |
| NX11 HS| 04:19. | 04:30 | 08:21 |
| FP     | 04:20 | 06:00. | 09:01 |
| ISO zigzag | 01:32 | 02:09 | 05:16. |
| Contour  | 03:39 | 04:00. | 07:09 |
| Radial  | 01:47 | 02:17. | 02:32. |
| Iso-planar | 01:58 | 02:45. | 03:34 |
| CRZ    | 01:47 | 01:58 | 02:06. |

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