Multi-Agent Coverage in Urban Environments

Shivang Patel¹, Senthil Hariharan¹, Pranav Dhulipala¹
Ming C Lin¹,², Dinesh Manocha¹,²,³, Huan Xu¹,⁴,⁵ and Michael Otte¹,²,⁴

Abstract—We study multi-agent coverage algorithms for autonomous monitoring and patrol in urban environments. We consider scenarios in which a team of flying agents uses downward facing cameras (or similar sensors) to observe the environment outside of buildings at street-level. Buildings are considered obstacles that impede movement, and cameras are assumed to be ineffective above a maximum altitude. We study multi-agent urban coverage problems related to this scenario, including: (1) static multi-agent urban coverage, in which agents are expected to observe the environment from static locations, and (2) dynamic multi-agent urban coverage where agents move continuously through the environment. We experimentally evaluate six different multi-agent coverage methods, including: three types of ergodic coverage (that avoid buildings in different ways), lawn-mower sweep, voronoi region based control, and a naive grid method. We evaluate all algorithms with respect to four performance metrics (percent coverage, revisit count, revisit time, and the integral of area viewed over time), across four types of urban environments [low density, high density] × [short buildings, tall buildings], and for team sizes ranging from 2 to 25 agents. We believe this is the first extensive comparison of these methods in an urban setting. Our results highlight how the relative performance of static and dynamic methods changes based on the ratio of team size to search area, as well the relative effects that different characteristics of urban environments (tall, short, dense, sparse, mixed) have on each algorithm.

Index Terms—Multi-Agent, Coverage, Lawn-mower, Ergodic, Boustrophedon, Voronoi, Urban Environment

I. INTRODUCTION

Multi-agent coverage requires a team of agents to collectively perform a region-based mission, such as sweeping, where the mission objective considers the measure of the sub-region in which the task is performed. Depending on the mission, the coverage region may be defined as a subset of space, time, or space × time. Examples include: vacuuming a building, streaming scientific measurements from the field, and monitoring a secured area for intruders.

In this paper we consider multi-agent coverage problems of surveillance and patrol in urban environments. We assume a team of identical UAV agents, such that each agent is equipped with a downward facing camera. The team is tasked with visually monitoring the subset of the environment that is outside of buildings at ground level.

Authors are with: ¹Maryland Robotics Center, ²The Department of Computer Science, ³The Department of Electrical and Computer Engineering, ⁴The Department of Aerospace Engineering, and ⁵The Institute for Systems Research, University of Maryland, College Park. spatel143@umd.edu, {lin, dm}@cs.edu, {mumu, otte}@umd.edu

This work was supported by DARPA cooperative agreement HR0011820028 as part of DARPA OFFSET, ARO grant W911NF-19-1-0069

Three dimensional Urban environments present unique challenges for coverage using small UAVs. This is true even in the special case of ground level coverage. Buildings are obstacles to navigation that agents must fly over or around. Increasing an agent’s altitude is assumed to have two effects on sensing ability. First, raising the height of the downward-facing sensor cone increases the size of the sensor’s ground-level footprint. Second, the increased footprint size causes loss of ground-level detail because larger patches of ground-level area are mapped to image pixels.

We assume that due to loss of ground level detail the agents’ camera sensors are not useful above a predefined maximum altitude. We also assume that this maximum altitude is below the height of the tallest building in the

Multi-Agent Coverage Algorithms We Experimentally Evaluate In Urban Environments

Lawnmower Sweep | Ergodic Sweep | Non-Uniform Ergodic
--- | --- | ---

Ergodic Obstacle Avoid | Voronoi | Rectangular

Fig. 1. Each algorithm is depicted in three panels where: the top panel represents the start, center panel depicts a time midway through the algorithm’s execution, and the bottom panel shows the ending state and paths taken. Paths over buildings are red. In the Voronoi and Rectangular methods, the regions are colored based on their corresponding robot.
environment (otherwise the navigational constraints of buildings are trivially ignored by having all agents fly above all buildings all the time). We are interested in problems where agents must temporarily alter their paths to fly around or over buildings, yet where flying over a building prevents an agent’s sensor from contributing to the monitoring task.

In static multi-agent coverage agents move to a set of mutually advantageous locations, and then remain at the same locations for the duration of the mission. In dynamic multi-agent coverage, agents move continuously throughout the environment so that every point in the environment is intermittently observed. Many missions exist that are inherently static or dynamic in nature. Perimeter monitoring [1] requires that any point that is monitored must be monitored continuously (static coverage). Autonomous vacuuming [2] requires that every point in the environment be swept as frequently as resources allow (dynamic coverage). However, such an obvious choice may not exist for other coverage problems, and the relative applicability of a static or dynamic solution may depend on the problem instance.

Both static and dynamic coverage can be used for urban monitoring and patrol. Their relative usefulness will depend on the relationship between: the total area that must be monitored, the number of agents available, and the sensor footprint of each agent. When the size of the environment is relatively large with respect to [team size \( \times \) sensor footprint] we are forced to choose between monitoring a small subset of the environment continuously or a larger subset of the environment intermittently.

While deterministic sweep algorithms are useful for many coverage applications (painting, vacuuming), they are less suited to adversarial surveillance tasks. Knowledge of how a deterministic path is generated can enable an adversary to evade detection. This problem can be mitigated by making paths (and thus revisit times) non-deterministic. However, a simple random walk is inefficient because the time to cover the environment is relatively large [3]. Ergodic sweep balances randomness with coverage efficiency to achieve practically useful revisit properties [4].

We experimentally evaluate six multi-agent coverage methods in simulation, including: three versions of Multi-agent ergodic coverage (differing in how obstacles are addressed), lawn-mower sweep, a voronoi-based static method, and a naive grid-based method. All six methods are experimentally evaluated with respect to four performance metrics, across four types of urban environments (two different building heights, and two different building densities), and for team sizes ranging from 2 to 25 agents.

This paper is organised as follows: Section II contains related work. Preliminaries, including nomenclature and a formal problem definition, appear in Section III. The algorithms we compare are described in Section IV. Experiments appear in section V and a discussion of result appears in Section VI. Section VII concludes the paper.

II. RELATED WORK

We assume a broad definition of the term ‘coverage,’ and use it to indicate the set of problems including coverage, surveillance, and environmental monitoring. Coverage methods are often divided into the two categories of dynamic coverage and static coverage. In static coverage agents achieve stationary positions that enable the environment to be continuously monitored [5, 6, 7, 8]. In dynamic coverage agents move continuously through the environment so that all points are visited [9, 10, 11, 4, 12]. Depending on the scenario, points may be visited once (painting, vacuuming), or multiple times (patrol). We now survey static coverage, dynamic coverage, and other related problems, respectively.

A. Dynamic Coverage

Lawn mower sweep or Boustrophedon Coverage uses a simple back and forth motion. Assuming a sweep of positive radius, this can be used to cover any obstacle-free convex environment of finite area [9]. Polygonal obstacles can be addressed by partitioning the free space into a finite number of convex regions, and sweeping each region separately [10]. In [11] a multi-agent team is divided into two groups of agents, one performing exploration and the other coverage. Most lawn mower sweep implementations assume a 2D environment such that regions surrounded by obstacles are topologically separated from the rest of the environment. In the 3D urban scenarios we consider we project a lawn mower sweeping path down onto the environment and then stitch together discontinuities in elevation. In other words, agents can adjust their elevation along each lawn mower path to fly up and over buildings. This enables agents to observe topologically separated ground-level regions.

Alternative methods of generating a deterministic coverage path include: Zelinski’s algorithm, which orders the sweep based on the levels sets of a wave-front expansion [13], and ideas motivated by space filling curves [14].

Ergodic sweep is applicable to adversarial coverage problems, such as patrol [4], and uses nondeterministic control laws to balance search efficiency and unpredictability (since, in an adversarial scenario, the practice of revisiting points on a set schedule can be exploited by the adversary). Ergodic sweep control laws are often designed as a function of a target coverage distribution and the current coverage distribution. Biasing coverage to obstacle-free [4] reduces obstacle collisions, but does not eliminate collisions. A method for explicit obstacle avoidance within Ergodic coverage is presented in [12], which combines ergodic coverage control laws with phsiocomemetic vector fields to “repel” agents away from obstacles. Three of the six algorithms that we compare are Ergodic coverage variants that, respectively: (1) ignore obstacles during coverage planning and then fly up and over them, (2) biases search away from obstacles and then fly up and over them if necessary, (3) avoid obstacle by flying around them without a change in elevation.
B. Static Coverage

Static coverage algorithms partition the search space into regions such that there is a mapping from robots to regions, and each robot can view its region from a stationary position. Methods differ in how the space is divided, and how the robot-to-region mapping is calculated.

Centralized methods [5], [6], [15] compute the search space division on a single agent or server, and then send the solution to all agents at runtime or a priori. A cellular decomposition of the environment followed by the calculation of a multi-robot spanning forest is used by [6], and each robot is assigned one tree in the forest. A segmentation technique is used in [15], where each robot is assigned a unique segment. Work in [16] partitions the environment using the weighted K-Means clustering algorithm. Centralization has the disadvantage of introducing single points of failure.

In contrast, decentralized and/or distributed process do not have a single point-of-failure. In many strategies robots start at their initial locations and then tend toward a multi-robot configuration with desirable coverage properties over time. Iterative Voronoi-based approaches achieve such a distributed control strategy [17], [18], [19]. At the start of the mission, robot positions are used as the generator points of a Voronoi decomposition of the search space, then each robot moves a small distance toward the center of the Voronoi cell that its generated. The process is then repeated during each time step, and with the result that agents tend to space themselves evenly throughout the environment. One advantage of this method is that each agent need only communicate with its Voronoi neighbors. The basic Voronoi method is one of the six methods we compare in this paper.

C. Other Problems Related to Coverage

Information gathering [20], [21] is closely related to coverage, the main difference being that the amount of information that can be gathered at a particular location is non-uniform and changes as a function of each sensor measurement.

In target search [22], [23], [24], [25] the goal is to locate one or more stationary or moving targets. Target search problem variants that share similarities with the coverage problem include probability based [26], [27], information based [28], [29], [30], and game theoretic formulations [31].

Exploration is similar to coverage in that both problems are concerned with visiting all points in the environment [32]. However, coverage problems often require repeated visits to each location, while exploration is completed once each location has been visited once.

In previous work [33] we present an collision avoidance algorithm for a swarm of UAVs performing an urban coverage task. The method in [33] focuses on agent-to-agent and agent-to-obstacle local collision avoidance. The method in [33] can be used as a post-processing step for all of the algorithms considered in the current paper.

III. PRELIMINARIES

In Section III-A we define our nomenclature and assumptions, and in Section III-B we formally define the problems of static and dynamic multi-agent urban coverage.

A. Nomenclature and Assumptions

A team of \( n \) robots is the set \( R = \{ r_1, \ldots, r_n \} \), where \( r_i \) denotes the \( i \)-th robot. The 3D workspace in which the robots operate is \( \chi^3 \). The obstacle space \( \chi^3_{\text{obs}} \subset \chi^3 \) is the subset of \( \chi^3 \) containing obstacles, while the free space \( \chi^3_{\text{free}} \subset \chi^3 \) is the subset of \( \chi^3 \) that does not contain obstacles. \( \chi^3_{\text{obs}} \cap \chi^3_{\text{free}} = \emptyset \) and \( \chi^3_{\text{obs}} \cup \chi^3_{\text{free}} = \chi^3 \). We assume that the ground plane of the workspace can be approximated by 2D Euclidean space \( \mathbb{R}^2 \). Let the projections of \( \chi^3 \) and \( \chi^3_{\text{obs}} \) directly down onto \( \mathbb{R}^2 \) be denoted \( \chi \) and \( \chi_{\text{obs}} \), respectively. The free space at ground level is defined \( \chi_{\text{free}} = \chi \setminus \chi_{\text{obs}} \) such that \( \chi_{\text{obs}} \cap \chi_{\text{free}} = \emptyset \) and \( \chi_{\text{obs}} \cup \chi_{\text{free}} = \chi \). We assume an urban environment such that \( \chi^3_{\text{obs}} \) contains buildings. The area of desired coverage is defined to be all ground-level terrain (and not the sides or tops of buildings), and denoted \( \chi_{\text{search}} = \chi_{\text{free}} \).

Each robot \( r_i \) is assumed to be a point, and moves along a path in \( \chi^3_{\text{free}} \) and observes \( \chi_{\text{free}} \) using a downward facing camera sensors. We assume a continuous time model starting at \( t = 0 \). The interval of time from the beginning of the mission until \( t = t_{\text{max}} \) is \([0, t_{\text{max}}]\). A valid robot path from time \( t = 0 \) to time \( t = t_{\text{max}} \) is a continuous function \( \rho_i : [0, t_{\text{max}}] \rightarrow \chi^3_{\text{free}} \). For problems involving continuous or continual coverage, we often make the tacit assumption that \( t_{\text{max}} \rightarrow \infty \). In other words, that \( \rho_i \) continues to be well defined as search time increases without bound. (This contrasts with the typical path planning problem, in which paths have well defined start and end points and can thus be represented as maps from the unit interval \([0, 1]\)).

For convince, we abuse our notation and (also) let path \( \rho_i \) denote the geometric set of points along the curve in \( \chi^3_{\text{free}} \) that is traced out by robot \( r_i \) over time, such that \( \rho_i \subset \chi^3_{\text{free}} \). The multipath \( \psi \) is the set of all robots’ paths, \( \psi \) is notationally overloaded in the same way as \( \rho_i \). As a function, \( \psi : [0, t_{\text{max}}] \rightarrow (\chi^3_{\text{free}})^n \), where \( \chi^3_{\text{free}} \) is a copy of the 3-dimensional free space associated with robot \( r_i \) and \( (\chi^3_{\text{free}})^n = \chi^3_{\text{free},1} \times \cdots \times \chi^3_{\text{free},n} \) is the product space of all robot’s free spaces. In the geometric sense \( \psi = \bigcup_{i=1}^n \{ r_i \} \).

The space containing all multipaths is denoted \( \Psi = \bigcup_{i=1}^n \{ r_i \} \).

We assume robots have identical downward facing camera sensors such that when \( r_i \) flies at altitude \( h \) the projection of \( r_i \)’s field-of-view down onto \( \chi_{\text{search}} \) is a disc \( B_h \) of radius \( f(h) \). We assume there exists an optimal altitude \( \hat{h} \) for the camera sensor to be used. For example, a sensor will be used at the highest altitude for which it remains a reliable sensor (increasing altitude increases \( B_h \), which is advantageous, but may degrade sensor reliability, which is disadvantageous). We drop the subscript when the sensor is used at the optimal altitude, \( B = B_{\hat{h}} \). Thus, while observing \( \chi_{\text{free}} \) agents are assumed to fly at altitude \( \hat{h} \) in \( \chi^3_{\text{free}} \). Agents may increase their altitude \( h > \hat{h} \) to fly over obstacles. We assume sensors cannot be used to observe \( \chi_{\text{free}} \) (the ground-plane) during maneuver at increased altitude \( h > \hat{h} \) in particular, during maneuvers up and over buildings. Let \( \rho_i \subset \rho_i \) be the subset of \( \rho_i \) containing all points at which robot \( r_i \)’s camera sensor
is functional, i.e., for which \( r_i \) is at altitude \( \tilde{h} = \tilde{h} \). Similarly, let \( \tilde{\psi} = \bigcup_{t=1}^{n} \{ \tilde{\rho} \} \). The area swept by the team is then:

\[
\chi_{\text{sweep}} = \left( \tilde{\psi} \oplus B \right) \cap \chi_{\text{search}}
\]

where ‘\( \oplus \)’ denotes the Minkowski sum. The intersection with \( \chi_{\text{search}} \) is included in Equation 1 so that \( \chi_{\text{sweep}} \subset \chi_{\text{search}} \) by construction.

Let \( \tilde{\Psi} = \bigcup_{t} \{ \tilde{\psi} \} \) be the space containing all \( \tilde{\psi} \). Let \( \psi_{\text{obs}} \) be an indicator function that returns 1 or 0 based on whether or not, at time \( t \), a point \( x \in \chi_{\text{search}} \) is observed by at least one robot in the team. \( \psi_{\text{obs}} : \chi_{\text{search}} \times [0, t_{\text{max}}] \times \tilde{\Psi} \rightarrow \{ 1, 0 \} \), and so \( \psi_{\text{obs}}(x, t, \tilde{\psi}) = 1 \iff x \in \psi(t) \oplus B \) and \( \psi_{\text{obs}}(x, t, \tilde{\psi}) = 0 \iff x \not\in \psi(t) \oplus B \).

Given a particular \( \tilde{\psi} \), the function \( g_{\tilde{\psi}, \text{max}} \) is a map from the search space \( \chi_{\text{search}} \) to the time-duration domain, \( \rho_t : \chi_{\text{search}} \rightarrow [0, t_{\text{max}}] \) that measures the cumulative time that point \( x \in \chi_{\text{search}} \) is observed by at least one robot, \( g_{\tilde{\psi}, \text{max}}(x) = \int_{0}^{t_{\text{max}}} \psi_{\text{obs}}(x, t, \tilde{\psi}) \, dt \), where the integral is Lebesgue. Dynamic coverage algorithms can be characterized by a requirement that points in \( \chi_{\text{search}} \) be visited infinitely often as \( t_{\text{max}} \rightarrow \infty \). Given continuous vehicles paths, this is \( \lim_{t_{\text{max}} \rightarrow \infty} g_{\tilde{\psi}, \text{max}}(x) = \infty \) for all \( x \in \chi_{\text{search}} \).

In static search we seek to maximize the number of points continually visible to at least one robot. For search times \( t_{\text{max}} \) large enough that the complicating effects of the startup phase can be ignored (in which agents move to their optimal static search locations), this is equivalent to finding \( \arg \max_{\psi} \int_{\chi_{\text{search}}} g_{\psi, \text{max}}(x) \, d\chi_{\text{search}} \)

i.e., by rotating the starting positions \( \tilde{\rho} \) of the length around the cycle for agent \( i \) (lines 11). The obstacle avoiding multi-path of the team is given by the n-tupal containing the single agent paths (line 6).

At run-time, each agent follows its own version of the cycle. Agents that do not start the mission at the beginning of their cycles head directly to the nearest point along the cycle at which they will be on schedule. Each agent \( i \) repeatedly moves around its cycle \( \tilde{\rho}_i \).

B. Multi-Agent Urban Ergodic Coverage

Non-determinism is useful in scenarios in which an adversary attempts to avoid detection. In Ergodic coverage the agent/team follows non-deterministic trajectories such that the relative time spent in each non-zero measure region of the environment can be prescribed by a user. The desired properties only hold almost surely in the limit as time approaches infinity. Thus, practical performance can be expected to improve with mission duration.

Ergodic coverage is most easily described—and implemented—as an evolutionary process that generates a path. The subroutine \( \text{singleErgodic}(f_{\chi}) \), described in Algorithm 2, computes the ergodic path for a single agent, assuming a user defined coverage distribution \( f_{\chi}(x) \) over \( \chi \) is desired. The ‘\( \cdot \)’ symbol denotes path concatenation.

The subroutine \( \text{nextStep}(f_{\chi}, M_k, C_k) \) (on line 4 of Algorithm 2) implements the control laws of ergodic coverage for a single agent without explicit obstacle avoidance. This is calculated \( B_{\chi}(t) = \sum R \lambda_k S_k \nabla f_{\chi}(x_j(t)) \) which is further normalized and constrained by velocity \( u_{\text{max}} \).

Where \( \lambda_k \) is constant and \( S_k(t) \) is difference between the current distribution and target distribution, given by \( S_k(t) := C_k(t) - M_k(t) \). \( \nabla f_{\chi}(x_j(t)) \) is gradient of the Fourier basis function which is given by

\[
\nabla f_{\chi}(x_j(t)) = \frac{1}{h_{\chi}} \left[ -k_1 \sin(k_1 x_1) \cos(k_2 x_2), -k_2 \cos(k_1 x_1) \sin(k_2 x_2) \right]
\]

and the Fourier basis function is given by \( f_{\chi}(x) = \frac{1}{\pi} \cos(k_1 x_1) \cos(k_2 x_2) \). The Fourier coefficient \( C_k(t) \) is calculated \( C_k(t) = \sum_{j=1}^{\chi} f_{\chi}(x_j(t)) \) and Fourier coefficient of target distribution is \( M_k(t) := N t_{\mu_i} \). Where \( k_1 = \frac{k_1}{L_1}, k_2 = \frac{k_2}{L_2} \) and \( \mu_i = \langle \mu, f_k \rangle \) where \( \langle \cdot, \cdot \rangle \) is an inner product. \( h_{\chi} = \left( \int_{0}^{L_1} \int_{0}^{L_2} \cos^2(k_1 x_1) \cos^2(k_2 x_2) \right)^{1/2} \).
This feedback law implementing obstacle avoidance would be a repulsive feedback law (Algorithm 2), which is calculated:

Algorithm 2 singleErgodic(f_{\text{free}})

1: $M_k, C_k \leftarrow$ initErgParams(f_{\text{free}})
2: $\rho_{(0)} \leftarrow \emptyset$
3: for $t = 1 \ldots t_{\text{max}}$
4: $\rho_{(t)\leftarrow \rho_{(t-1)} \cup \text{nextStep}(f_{\text{free}}, M_k, C_k)$
5: $C_{(t)} \leftarrow$ calcVectorField($\rho_{(t-1)} \cup \text{obs}$)
6: $C_k \leftarrow$ updateCurrentDist($C_{(t)}, \rho_{(t)}$

Algorithm 3 singleErgAvoidObs(f_{\text{free}}, \text{obs})

1: $M_k, C_k \leftarrow$ initErgParams(f_{\text{free}})
2: $\rho_{(0)} \leftarrow \emptyset$
3: for $t = 1 \ldots t_{\text{max}}$
4: $\rho_{(t)\leftarrow \rho_{(t-1)} \cup \text{nextStep}(f_{\text{free}}, M_k, C_k)$
5: $\rho_{(t)} \leftarrow$ calcVectorField($\rho_{(t-1)} \cup \text{obs}$)
6: $C_{(t)} \leftarrow$ updateCurrentDist($C_{(t)}, \rho_{(t)}$

Algorithm 4 Multi-Agent Ergodic

1: for $i = 1 \ldots n$
2: $\rho_i \leftarrow \text{singleErgodic}(f_{\text{free}})$
3: $\tilde{\rho}_i \leftarrow \text{flyOverBldgs}(\rho_i, f_{\text{free}})$
4: $\psi \leftarrow (\rho_1, \ldots, \rho_n)$

Algorithm 5 Biased Multi-Agent Ergodic

1: for $i = 1 \ldots n$
2: $\rho_i \leftarrow \text{singleErgodic}(f_{\text{free}})$
3: $\tilde{\rho}_i \leftarrow \text{flyOverBldgs}(\rho_i, f_{\text{free}})$
4: $\psi \leftarrow (\rho_1, \ldots, \rho_n)$

Algorithm 6 Obstacle Avoiding Multi-Agent Ergodic

1: for $i = 1 \ldots n$
2: $\rho_i \leftarrow \text{singleErgAvoidObs}(f_{\text{free}}, \text{obs})$
3: $\psi \leftarrow (\rho_1, \ldots, \rho_n)$

Algorithm 7 Multi-Agent Voronoi Cover

1: $x_0 \leftarrow \text{projectOnto}(R, \chi)$
2: $\{\rho_0, \ldots, \rho_n\} \leftarrow \{x_0, \ldots, x_{0,n}\}$
3: for $i = 1 \ldots n$
4: $\psi \leftarrow \text{voroniPart}(x_{i-1}, \chi)$
5: for $i = 1 \ldots n$
6: $x_{i} \leftarrow \text{centroid}(C_i)$
7: $\rho_i = \rho_{(t)} \circ x_{i}$
8: $x_{i} \leftarrow \{x_{i,1}, \ldots, x_{i,n}\}$
9: for $i = 1 \ldots n$
10: $\rho_i = \text{flyOverBldgs}(\rho_i, f_{\text{free}}, \text{obs})$
11: $\psi \leftarrow (\rho_0, \ldots, \rho_n)$

Algorithm 8 Multi-Agent Grid Cover

1: $x_0 \leftarrow \text{projectOnto}(R, \chi)$
2: $\{\rho_0, \ldots, \rho_n\} \leftarrow \{x_0, \ldots, x_{0,n}\}$
3: $\{C_1, \ldots, C_n\} \leftarrow \text{gridPartition}(x_0, \chi)$
4: for $i = 1 \ldots n$
5: $x_{i} = \text{centroid}(C_i)$
6: $\rho_i = \rho_{(t)} \circ x_{i}$
7: $\rho_i = \text{flyOverBldgs}(\rho_i, f_{\text{free}}, \text{obs})$
8: $\psi \leftarrow (\rho_0, \ldots, \rho_n)$

The subroutine singleErgAvoidObs(\chi, \text{obs})

in Algorithm 3 computes an ergodic path for a single agent that explicitly avoids obstacles by using an obstacle repulsive feedback law (line 5), which is calculated:

This feedback law implementing obstacle avoidance would be governed by $V_j(t):= -\alpha V_j(t) + (1 - \alpha) F_j^3(r_j)$ where $V_j(t):= -\beta \frac{B_j(t)}{||B_j(t)||}$ and $F_j^3(r_j)$ is a repulsive vector field. This is further normalized and constrained by velocity $u_{\text{max}}$ as $u_j(t):= -u_{\text{max}} \frac{V_j(t)}{||V_j(t)||}$. The parameter $\alpha \in [0, 1]$ is a bump factor taking value 1 if the robot is far from the obstacle and reducing to 0 when approaching an obstacle (in the vector field). There are many ways one can define $\alpha$, in this paper we have defined it linearly.

Let the probability distribution functions $f_{\chi, \text{obs}}(x)$ and $f_{f_{\text{free}}}(x)$ respectively define uniform random distribution over the entire space (ignoring obstacles) and over the free space (biasing movement away from obstacles).

$f_{\chi, \text{obs}}(x) = \begin{cases} \frac{1}{\chi^3} & \text{if } x \in \chi \\ 0 & \text{if } x \not\in \chi \end{cases}$

$f_{f_{\text{free}}}(x) = \begin{cases} \frac{1}{||x||} & \text{if } x \in f_{\text{free}} \\ 0 & \text{if } x \not\in f_{\text{free}} \end{cases}$

We compare three versions of multi-agent ergodic sweep for that differ based on how urban obstacles are handled:

1) **Multi-Agent Urban Ergodic Sweep** is a simple repairing strategy that calculates a path assuming no obstacles exists (Algorithm 4). This method handles obstacle avoidance in the same way as the multi-agent lawn mower sweep algorithm described in the previous section. $\tilde{\rho}_i$ is created from $\rho_i$ by having agent $i$ rise in elevation to fly over a building and descend back to the sweep altitude afterward (Algorithm 4 line 5). This biases $\rho_i$ away from obstacles, but does not eliminate the need to occasionally fly up and over a buildings. $\tilde{\rho}_i$ is created from $\rho_i$ by having agent $i$ fly over buildings when necessary (Algorithm 4 line 5).

2) **Multi-Agent Urban Biased Ergodic Sweep** is similar, except that we define the area of desired uniform coverage to be $f_{\text{free}}$ instead of $\chi_{\text{vacant}}$ (Algorithm 5 line 2). This biases $\rho_i$ away from obstacles, but does not eliminate the need to occasionally fly up and over buildings. $\tilde{\rho}_i$ is created from $\rho_i$ by having agent $i$ fly over buildings when necessary (Algorithm 5 line 3).

3) **Multi-Agent Obstacle Avoiding Urban Ergodic Sweep** forces each $\rho_i$ to avoid obstacles using the vector field algorithm (Algorithm 6). In other words, by using singleErgAvoidObs(f_{\text{free}}, \text{obs}) to prevent $\rho_i$ from intersecting $\text{obs}$.

In our experiments, all agents fly at the optimal sweep elevation such that isolated internal courtyards are not visited.

C. Voronoi Urban Coverage Algorithm

Given a set of $n$ generating points (we use the set $x = \{x_1, \ldots, x_n\}$ containing robot projections onto $\chi$) a Voronoi space partitioning of $\chi$ creates mutually disjoint set of cells by assigning each point $x \in \chi$ to a cell $C_i$ associated with the closest generating point (Algorithm 7 line 4), $C_i = \{x \mid ||x - x_i|| < ||x - x_{j\neq i}||\}$ The voronoi partitioning of $\chi$, i.e., instead of all of $\mathbb{R}^2$, is found by placing reflected versions of points across each boundary of $\chi$, and then truncating the resulting extended Voronoi diagram to the footprint of $\chi$. When $\chi$ is a rectangle this requires $5n$ points.

Voronoi coverage methods work by having each robot move a small distance toward the centroid of its current voronoi cell, recalculate new voronoi cells (Algorithm 7 lines 4-7), and then repeat (lines 10). Over time, this causes robots to greedily space themselves away from their neighbors.

D. Grid-Based Urban Coverage Algorithm

We also implement a naive grid-based algorithm for static coverage. This method uses rectangles, instead of Voronoi regions, to divide the space among agents (Algorithm 8). While the Voronoi region algorithm is the result of a distributed control process, this rectangular division of the space
can be calculated a priori. Each agent simply moves to the center of its assigned rectangular region.

V. EXPERIMENTS

In this section we compare the six multi-agent coverage methods (see Tables I and II) across a variety of environments (see Table III), for teams of size 1 to 25 agents. Environments differ based on building height, building density, and footprint size. Each combination of Environment, Algorithm, and team size is repeated over three random trials.

Experiments are run in the Ubuntu Linux operating system using Robot Operating System (ROS), Gazebo, and the pix4 control package. UA Vs measure $1 \times 1 \times 0.3$ meters.

We empirically evaluate performance with respect to four metrics (described below), by randomly sampling a large number of points in the environment and tracking statistics in a disc surrounding each point. Statistical results for a particular trial are obtained by integrating over these points.

1) **Percent Coverage**: The percentage of the map (point regions) that has been swept at least once.

2) **Visits Count**: The total number of visits a point’s region has been visited.

3) **Revisit Time**: The time duration between successive visits to a point’s region.

4) **Time spent**: The cumulative time that any agent is within a point’s region. We unpausage when any agent enters point’s region and pause the counter when the agent leaves that region.

Agent starting locations are chosen randomly for all methods except for Lawn Mower Sweep. For Lawn Mower Sweep, the initial coordinates of the agents are randomly chosen along the lawn mower path (this causes the lawn mower algorithm to have a slight start-up advantage over other methods because it eliminates the startup phase in which agent travel to their equally spaced positions along the cycle). Experimental results appear in Figures 2-4. We discuss our results in the next section.

VI. RESULTS

In tall environments, Algorithms that consider urban obstacles tend to have better performance than those that do not. This makes sense because more time is required to fly up and over buildings than around them. Biasing movement toward the free space (but still flying over them when necessary) outperforms explicit obstacle avoidance in short environments. Building height effects become more pronounced as team size increases. Increasing building density tends to amplify the differences between the different methods.

The Ergodic methods revisit a particular point less often, on average, than lawn mower sweep. In practice, there are adversarial situations in which the benefits of non-deterministic revisit times are preferred, even if this causes an increase in expected revisit time. The obstacle avoiding Ergodic has lower revisit times than free space biased ergodic, which has lower revisit times than normal Ergodic.

Our experiments provide insights into the trade-off between algorithms that are designed for static and dynamic cases. The percentage of the environment that is covered by the Voronoi and grid methods appears to have a linear relationship with agent number, and an approximately linear function of time for lawn mower sweep and the Ergodic methods (until a maximum value is reached).

Using the Voronoi and Grid methods, the average time between visits is 0, i.e., instantaneous—at least, to those points that are ever visited. This reflects the fact that methods designed for static coverage have agents move to and remain at static locations. The lawn-mower and Ergodic methods have decreasing average revisit times with more agents, as well as increasing mean and standard deviations of revisit times as time increases. The increasing mean revisit time is partially an artifact of the fact that we initialize revisit times to 0 (since re-visit implies at least two visits, and so a point is essentially not tracked, with respect to this metric, until it is visited a second time). The increasing standard deviation appears to asymptote toward a constant value for the lawn mower algorithms, but continues grow over the timescales that we evaluate for the stochastic algorithms. Similar results are reflected in the total visit count to each revisited point (lawn mower and Ergodic continue to increase, while Voronoi and grid method taper off).

VII. SUMMARY AND CONCLUSION

The main contribution of this work is an empirical evaluation of six multi-agent coverage algorithm in urban environments. Urban environments present interesting challenges for coverage algorithms because buildings and other structures are obstacles to navigation. Such obstacles are especially relevant when the coverage sensor cannot be used above a maximum altitude and that altitude is below the tallest building in the environment.

We evaluate two static and four dynamic coverage algorithms. Three of the dynamic coverage algorithms are relevant to adversarial scenarios and use ergodicity to create random revisit times to each location in the environment. These differ in their approach to obstacle avoidance. Biasing ergodic motion away from obstacles (and then greedily flying over buildings in the rare cases it is necessary), appears to work well in environments with short buildings. On the other
High Density Tall Buildings

At 15000 Time Steps

**Coverage**

Area Covered (%)

Number of Agents

Ave. Duration Between Visits

Duration (s)

Ave. Time Near Each Point

Cumulative Time (s)

For Teams of 10 Agents

Coverage

Area Covered (%)

Number of Agents

Ave. Duration Between Visits

Duration (s)

Ave. Time Near Each Point

Cumulative Time (s)

Fig. 2. Performance of different size teams after 15000 time steps (left) and performance for 10 agents over time (right) in environments with tall buildings and high building density (top 4 rows) or low building density (bottom 4 rows) density.

hand, it appears better to maneuver around tall buildings. While this is an intuitive result, previous ergodic methods appearing in the literature consider only 2D coverage scenarios. Our paper investigates how such algorithms perform when obstacles are 3D and the coverage region is defined to be the 2D ground plane.

Our results also confirm the intuition that, for the dynamic methods, mean revisit times appear to scale linearly with agent number. While for the static methods, coverage percentage appear to scale linearly with agent number. We observe that the grid-based method occasionally outperformed the Voronoi method with respect to the time spent in sampled areas, especially for teams with $\leq 15$ agents.

REFERENCES

[1] M. Gupta, M. C. Lin, D. Manocha, H. Xu, and M. Otte, “Monitoring access to user defined areas with multi-agent team in urban environments,” in 2019 International Symposium on Multi-Robot and Multi-Agent Systems (MRS). IEEE, 2019, pp. 56–62.

[2] S. C. Wong, L. Middleton, B. A. MacDonald, and N. Auckland, “Performance metrics for robot coverage tasks,” in Proceedings of Australasian Conference on Robotics and Automation, vol. 27, 2002, p. 29.

[3] U. Feige, “A tight upper bound on the cover time for random walks on graphs,” Random structures and algorithms, vol. 6, no. 1, pp. 51–54, 1995.

[4] G. Mathew and I. Mezić, “Metrics for ergodicity and design of ergodic dynamics for multi-agent systems,” Physica D: Nonlinear Phenomena, vol. 240, no. 4–5, pp. 432–442, 2011.

[5] N. Hazon and G. A. Kaminka, “Redundancy, efficiency and robustness in multi-robot coverage,” in Robotics and Automation, 2005. ICRA 2005. Proceedings 2005 IEEE International Conference on. IEEE, 2005, pp. 735–741.

[6] Y. Gabriely and E. Rimon, “Spanning-tree based coverage of continuous areas by a mobile robot,” Annals of mathematics and artificial intelligence, vol. 31, no. 1-4, pp. 77–98, 2001.

[7] N. Hazon, F. Mieli, and G. A. Kaminka, “Towards robust on-line multi-robot coverage,” in Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on. IEEE, 2006, pp. 1710–1715.
Fig. 3. Performance of different size teams after 15000 time steps (left) and performance for 10 agents over time (right) in environments with short buildings and high building density (top 4 rows) or low building density (bottom 4 rows) density.

[8] D. W. Gage, “Command control for many-robot systems,” Naval Command Control and Ocean Surveillance Center Rdt And E Div San Diego CA, Tech. Rep., 1992.

[9] H. Choset, “Coverage of known spaces: The booustrophedon cellular decomposition,” Autonomous Robots, vol. 9, no. 3, pp. 247–253, 2000.

[10] A. Ntawumenyikizaba, H. H. Viet, and T. Chung, “An online complete coverage algorithm for cleaning robots based on booustrophedon motions and a* search,” in 2012 8th International Conference on Information Science and Digital Content Technology (ICIDT2012), vol. 2. IEEE, 2012, pp. 401–405.

[11] I. Rekleitis, A. P. New, E. S. Rankin, and H. Choset, "Efficient booustrophedon multi-robot coverage: an algorithmic approach," Annals of Mathematics and Artificial Intelligence, vol. 52, no. 2-4, pp. 109–142, 2008.

[12] H. Salman, E. Ayvali, and H. Choset, “Efficient booustrophedon multi-robot coverage: an algorithmic approach,” Autonomous Robots, vol. 9, no. 3, pp. 247–253, 2000.

[13] A. Zelinsky, R. A. Jarvis, J. Byrne, and S. Yuta, “Planning paths of complete coverage of an unstructured environment by a mobile robot,” in Proceedings of international conference on advanced robotics, vol. 13, 1993, pp. 533–538.

[14] S. V. Spires and S. Y. Goldsmith, “Exhaustive geographic search with mobile robots along space-filling curves,” in Collective robotics. Springer, 1998, pp. 1–12.

[15] K. M. Wurm, C. Stachniss, and W. Burgard, “Coordinated multi-robot exploration using a segmentation of the environment,” in Intelligent Robots and Systems, 2008. IROS 2008. IEEE/RSJ International Conference on. IEEE, 2008, pp. 1160–1165.

[16] A. Solanas and M. A. Garcia, “Coordinated multi-robot exploration through unsupervised clustering of unknown space.” in IROS. Citeseer, 2004, pp. 717–721.

[17] A. Okabe and A. Suzuki, “Locational optimization problems solved through voronoi diagrams,” European Journal of Operational Research, vol. 98, no. 3, pp. 445–456, 1997.

[18] M. Schwager, J. McLurkin, and D. Rus, “Distributed coverage control with sensory feedback for networked robots.” in robotics: science and systems, 2006, pp. 49–56.

[19] A. Breitenmoser, M. Schwager, J.-C. Metzger, R. Siegwart, and D. Rus, “Voronoi coverage of non-convex environments with a group of networked robots,” in Robotics and Automation (ICRA), 2010 IEEE International Conference on. IEEE, 2010, pp. 4982–4989.

[20] B. J. Julian, M. Angermann, M. Schwager, and D. Rus, “Distributed robotic sensor networks: An information-theoretic approach,” The International Journal of Robotics Research, vol. 31, no. 10, pp. 1134–1154, 2012.

[21] A. Khan, E. Yanmaz, and B. Rinner, “Information exchange and
Mixed Density Mixed Height Buildings

At 15000 Time Steps

For Teams of 10 Agents

Fig. 4. Performance of different size teams (left) and performance over time (right) in environments with mixed density and mixed buildings.

decision making in micro aerial vehicle networks for cooperative search,” *Control of Network Systems, IEEE Transactions on*, vol. 2, no. 4, pp. 335–347, 2015.

[22] B. Koopman, “The theory of search. ii. target detection,” *Operations research*, vol. 4, no. 5, pp. 503–531, 1956.

[23] T. H. Chung, G. A. Hollinger, and V. Isler, “Search and pursuit-evasion in mobile robotics,” *Autonomous Robots*, vol. 31, no. 4, pp. 299–316, 2011.

[24] S. Waharte and N. Trigoni, “Supporting search and rescue operations with uavs,” in *Emerging Security Technologies (EST)*, 2010 *International Conference on*, Sept 2010, pp. 142–147.

[25] S. Nayak, S. Yeotikar, E. Carrillo, E. Rudnick-Cohen, M. K. M. Jaffar, R. Patel, S. Azarm, J. Herrmann, H. Xu, and M. W. Otte, “Experimental comparison of decentralized task allocation algorithms under imperfect communication,” *IEEE Robotics and Automation Letters*, vol. to appear, 2020.

[26] T. H. Chung and J. W. Burdick, “Analysis of search decision making using probabilistic search strategies,” *IEEE Transactions on Robotics*, vol. 28, no. 1, pp. 132–144, Feb 2012.

[27] P. Dames and V. Kumar, “Autonomous localization of an unknown number of targets without data association using teams of mobile sensors,” *Automation Science and Engineering, IEEE Transactions on*, vol. 12, no. 3, pp. 850–864, 2015.

[28] P. Dames, M. Schwager, V. Kumar, and D. Rus, “A decentralized control policy for adaptive information gathering in hazardous environments,” in *IEEE Conference on Decision and Control*, Dec 2012, pp. 2807–2813.

[29] G. A. Hollinger, S. Yerramalli, S. Singh, U. Mitra, and G. S. Sukhatme, “Distributed data fusion for multirobot search,” *Transactions on Robotics*, vol. 31, no. 1, pp. 55–66, 2015.

[30] J. Berger and J. Happe, “Co-evolutionary search path planning under constrained information-sharing for a cooperative unmanned aerial vehicle team,” in *Evolutionary Computation (CEC), 2010 IEEE Congress on*, July 2010, pp. 1–8.

[31] M. Otte, M. Kuhlman, and D. Sofge, “Competitive target search with multi-agent teams: symmetric and asymmetric communication constraints,” *Autonomous Robots*, pp. 1–24, 2017.

[32] W. Burgard, M. Moors, D. Fox, R. Simmons, and S. Thrun, “Collaborative multi-robot exploration,” in *Robotics and Automation, 2000. Proceedings. ICRA ’00. IEEE International Conference on*, vol. 1, 2000, pp. 476–481 vol.1.

[33] S. H. Arul, A. J. Sathyamoorthy, S. Patel, M. Otte, H. Xu, M. C. Lin, and D. Manocha, “Lswarm: Efficient collision avoidance for large swarms with coverage constraints in complex urban scenes,” *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 3940–3947, Oct 2019.