The research on notch filtering and spectral filling for RFI suppression

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Abstract. Radio frequency interference (RFI) suppression has become an essential step in synthetic aperture radar (SAR) imaging processing. In this paper, a spectral filling filtering method for RFI suppression referred to notch filter is derived and analysed. The derivation results show the reason for the "residual terms" after suppression processing in frequency domain and the influence of the "residual terms". And a conclusion is drawn: the essence of interference suppression based on spectral filling filter is the dynamic change process of "residual terms". Since they cannot be completely eliminated in theory, they will affect the image quality after interference suppression, but the effect of spectral filling method is better than that of the notch filter method. The conclusion is verified by the simulation experiment.

1. Introduction
As an active microwave imaging equipment, synthetic aperture radar (SAR) plays a huge role in earth observation and other aspects [1-2]. At the same time, more and more radio frequency interference (RFI) in space may be received by the SAR, which will adversely affect the subsequent normal imaging processing and target recognition. Therefore, the research of RFI suppression technology has increasingly become an important topic in SAR field.

Since RFI is a peak in the frequency domain, it is a simple and effective method to construct a notch filter at the interference frequency and remove it from the radar mixed echoes [3-4]. Some scholars have also studied the impact of the notch filter [5]. Since this method obviously loses the useful echoes at the interference frequency point, some other improved methods and processing flows have also been further studied [6-7]. The essence of these improved methods is the design of the spectral filling filter, but no further analyses of spectral filling filter operation were presented in these researches.

In this paper, the spectral filling filter is theoretically analysed. Firstly, the notch filter model is further extended and the generalized spectral filling filter is designed. Secondly, the results of filling filter based on point target echoes and further pulse compression are derived and analysed. Through these results, the reason and impact of the "residual terms" after suppression processing are pointed out. In addition, it is further concluded that the essence of RFI suppression based on spectral filling filter is the dynamic change process of "residual term". The "residual terms" are the reason that affect image quality after processing owing to they cannot be completely eliminated in theory, but the effect of spectral filling method is better than that of the notch filter method. Finally, the simulation experiment results also verify the analysis and conclusion.
2. Modelling and Theoretical Analysis

2.1. Modelling

Ideally, the one-dimensional range part of SAR point target received signal affected by a single RFI can be expressed as:

\[ x(t) = s_r(t) + j(t) \]  \hspace{1cm} (1)

where \( s_r(t) \) and \( j(t) \) represent ideal point target part and interference part respectively, and \( s_r(t) \) is:

\[ s_r(t) = \sigma_0 \cdot s(t - \tau) = \sigma_0 \cdot \text{rect}\left[\frac{t - \tau}{T}\right] e^{j k_r (t - \tau)} k_r = B/T \]  \hspace{1cm} (2)

where \( B \) is the bandwidth, \( T \) is the transmitted pulse width, \( k_r \) is the frequency modulation rate, \( \tau \) is the time delay, \( \sigma_0 \) is a constant coefficient. The spectrum obtained by Fourier transform of \( x(t) \) can be expressed as follows:

\[ X(f) = S_r(f) + J(f) = \sigma_0 \cdot S(f) e^{-j 2 \pi f f} + f / B \sigma \cdot \text{rect}\left(f / B\right) e^{-j 2 \pi f f} + J(f) \]  \hspace{1cm} (3)

where \( \sigma \) is also a constant coefficient. It is assumed that the stopband center frequency and width of the filter set for a single RFI are \( f_1 \) and \( B_1 \) respectively. According to reference [5], the frequency response of the notch filter is expressed as:

\[ H(f) = 1 - \text{rect}\left(\frac{f - f_1}{B_1}\right) \]  \hspace{1cm} (4)

According to the above equation, when the latter term is further changed, the frequency response of the filling filter is:

\[ H(f) = 1 - k \cdot \text{rect}\left(\frac{f - f_1}{B_1}\right) \]  \hspace{1cm} (5)

where \( k \) is attenuation factor in the range of \([0,1]\). Figure 1 and equation (6) show the flow of frequency domain filtering and \( k \) calculation respectively.

![Figure 1. The flow of frequency domain filtering.](image)

If the carrier frequency and bandwidth of RFI are estimated accurately, the following relationship exists:

\[ X_f \cdot (1-k) = \hat{w} = k \left( X_f - \hat{w} \right) / X_f \]  \hspace{1cm} (6)
Thus, equation (7) can be simplified as:

\[ Y_i(f) = \sigma \left[ \text{rect}\left( \frac{f}{B_i} \right) - k \cdot \text{rect}\left( \frac{f-f_1}{B_i} \right) \right] e^{-j\pi f f_1/k} e^{-j2\pi f \tau} + (1-k) \cdot J(f) \] (9)

\( y_t(t) \) can be obtained by inverse Fourier transform of \( Y_i(f) \):

\[ y_t(t) = \frac{\sigma \omega_s(t-\tau) - k \sigma \omega_s \cdot \text{rect}\left( \frac{(t-\tau)/T'}{2} \right) e^{\frac{j\pi k (t-\tau)^2}{3}} + (1-k) \cdot j(t)}{1} \] (10)

where \( T' = B_i/k \). And due to the narrow band characteristics of RFI, \( B_i \) and \( T' \) are both small.

The output of the filling filter is used as the input of the matched filter, and the frequency response of the matched filter is assumed to be \( S^*(f) \), where \( (\ast)' \) means conjugate operation. Then the output spectrum \( Y_2(f) \) is:

\[ Y_2(f) = Y_i(f) S^*(f) = \sigma \left[ \text{rect}\left( \frac{f}{B} \right) - k \cdot \text{rect}\left( \frac{f-f_1}{B} \right) \right] e^{-j2\pi f \tau} + (1-k) \cdot J(f) e^{j\pi f^2/k} \] (11)

Since \( J(f) \) is narrow-band, the result of pulse compression obtained by inverse Fourier transform is:

\[ y_2(t) \approx \sigma' \cdot \sin c \left[ \pi B (t-\tau) \right] - \sigma'' \cdot \sin c \left[ \frac{\pi B_i (t-\tau)}{2} \right] e^{-j\pi f (t-\tau)^2} + \sigma''' \cdot \text{rect}\left( (t/T') e^{-j\pi k} \right) \] (12)

where \( \sigma', \sigma'' \) and \( \sigma''' \) are constant coefficients after pulse compression. It is worth noting that the "1" part of equation (10) and equation (12) are the ideal results, and the remaining "2" and "3" parts are undesired results.

2.2. Analysis based on filling filter

The design of the filling filter determines the interference suppression performance after filtering, so the selection of the attenuation factor is very important. The analysis shows the \( k \) value, filtered and pulse compression results in different situations (a) ~ (d):

(a) No RFI in the echo.

When there are only useful target echo signal in the mixed signal, that is \( J(f) = 0 \) and \( X_j = S_j = \hat{\omega} \).

So for the attenuation factor in equation (6), there is a result: \( k = 0 \). At the same time, according to equation (10) and equation (12), since there are \( j(t) = 0 \) and \( k = 0 \), the results after interference suppression and pulse compression only contain "1" part:

\[ y_1(t) = \sigma_0 \omega_s (t-\tau) = x(t) \] (13)

\[ y_2(t) = \sigma' \cdot \sin c \left[ \pi B (t-\tau) \right] \] (14)

It can be seen that the above equations are the results of the ideal point target. It should be noted that the response of the filling filter is 1 in this situation.

(b) There is RFI in the echo and the notch filtering operation is performed.

The frequency domain notch filter is used to filter the mixed signal with interference, and it is unnecessary to estimate the useful signal, so that is \( \hat{\omega} = 0 \) and \( k = 1 \). Similarly, for equation (10) and equation (12), the results contain "1" and "2" parts:

\[ y_1(t) = \sigma_0 \omega_s (t-\tau) - \sigma_0 \cdot \text{rect}\left( (t-\tau)/T' \right) e^{j\pi k (t-\tau)^2} \] (15)

\[ y_2(t) = \sigma' \cdot \sin c \left[ \pi B (t-\tau) \right] - \sigma'' \cdot \sin c \left[ \frac{\pi B_i (t-\tau)}{2} \right] e^{-j\pi f (t-\tau)} \] (16)

(c) There is RFI in the echo and filling filter based on accurate spectrum estimation is carried out.

\[ J(f) \text{rect} \left( \frac{f-f_1}{B_i} \right) \approx J(f) \] (8)
In this case, the interference can be completely eliminated by using the accurate spectrum value, we have the following equation:

\[ Y_1(f) = X(f)H(f) = S(f) + J(f) + 1 - k \cdot \text{rect}\left(\frac{f - f_1}{B_1}\right) \approx S(f) + J(f) - X(f)k = S(f) \]  

(17)

So the result is:

\[ k_0 = \frac{J_f}{X_f} = \frac{(X_f - S_f)}{X_f} = \frac{(X_f - \hat{w})}{X_f} \]  

(18)

where \( \hat{w} = S_f \), and \( J_f \) is the spectrum of RFI. So other results after filtering are the same as situation (a). At the same time, we can get the approximate expression of interference in time domain: the equality of "2" and "3" in equation (10) is listed as the equation and simplified:

\[ (1 - k_0)j(t) = k_0\sigma_0\text{rect}\left(\frac{t - \tau}{T'}\right)e^{j\omega(t - \tau)} \]  

(19)

\[ j(t) = \frac{k_0}{(1 - k_0)}\sigma_0\text{rect}\left(\frac{t - \tau}{T'}\right)e^{j\omega(t - \tau)} = \frac{J_f}{(X_f - \hat{w})}\sigma_0\text{rect}\left(\frac{t - \tau}{T'}\right)e^{j\omega(t - \tau)} \]  

(20)

The above equation can be regarded as the time-domain form of RFI. It can be seen that the signal has a form similar to chirp within the pulse duration \( T' \), which can be used as a basis for establishing an interference model.

(d) General situation: when RFI exists and there is an error in the estimated spectrum of the filling filter, the value of \( k \), the filtered and the pulse compression results are consistent with the results in 2.1. The approximation of equation (8) is the premise of the above derivation and analysis. In practice, this approximation is not valid due to the error in the interference carrier frequency and bandwidth detection. In this case, the RFI that is existing in the mixed signal cannot be completely eliminated anyway. Through the above analysis, the following conclusions can be drawn:

The first part ("1") of equation (10) and equation (12) represent the ideal result, and the second and third parts ("2" and "3") are both undesired results ("residual terms"). The "residual terms" affect the result of the target pulse compression together: 1) The superimposed effect of them will raise the sidelobes after pulse compression, making PSLR and ISLR worse. 2) At the same time, they have opposite signs. As the value of \( k \) changes, the second part and the third part ("residual term") change continuously, and cancellation can be achieved at a certain accurate value of \( k \). In fact, it is difficult to accurately estimate the value of \( k \), so the influence of interference on the data cannot be completely eliminated. It can only be mitigated by relatively accurate estimation within an acceptable range.

3. Simulation Experiment

Point target experiment and measured data experiment are set up for simulation verification. For the convenience of explanation, the three items in equation (10) and equation (12) are \( \text{part1\_desired} \), \( \text{part2\_fill} \) and \( \text{part3\_fill} \) respectively. Similarly, the results after notch filtering can be expressed by \( \text{part1\_desired} \) and \( \text{part2\_notch} \) respectively. And above, \( \text{part1} \) represents the desired part, and others are "residual terms".

3.1. Point target simulation

| Parameters | Values |
|-----------|--------|
| Bandwidth | 180 MHz |
The parameters setting of point target simulation is shown in the Table 1. In this experiment, one-dimensional range data is processed by: the notch filter method, the filling filter method with $k = 0.6k_0$ and $k = 1.2k_0$, where $k_0$ represents the value of $k$ that has accurately filled. The simulation results are presented in two ways: the filtered spectrum and the signal after pulse compression in time domain.

The original signal spectrum, mixed signal spectrum and filtered spectrum results are shown in figure 2(a) and 2(b). In this two figures, the result of using the notch filter method has the greatest loss to the useful echo signal. And the filling filter method with $k = 0.6k_0$ and $k = 1.2k_0$ represent under-compensation and over-compensation respectively, which may occur in actual processing. Compared with the notch filter method, many signals with the same frequency as the interference are retained by using filling filter method. Figure 3(a) and 3(b) show the results of pulse compression after filtering, and the curves correspond to the desired data, mixed data, notch filtered data, filling filtered data with $k = 0.6k_0$ and $k = 1.2k_0$ respectively. It can be clearly seen from the figures that the sidelobes of the pulse compression result are significantly raised by the interference, and the sidelobes after using notch and filling filter are improved significantly. The corresponding PSLR, ISLR and IRW results are given in Table 2. The data in this table indicate that RFI can significantly raise the sidelobes (resulting in large ISLR) and slightly affect the IRW. In addition, the PSLR and ISLR decrease and the IRW is the same as the desired result after using notch and filling filter method, which indicates that notch or filling filter method does not affect the resolution and filling filter technology is better than the former.

| Parameter       | Value   |
|-----------------|---------|
| Pulse width     | 10 μs   |
| Carrier frequency | 9.6 GHz |
| PRF             | 1666.7 Hz |

Figure 2. The results of spectrum in frequency domain. (a) Original results. (b) Enlarged results

Figure 3. The results of pulse compression. (a) Original results. (b) Enlarged results.
Table 2. Results after pulse compression

|                  | PSLR(dB) | ISLR(dB) | IRW (m) |
|------------------|----------|----------|---------|
| Original data    | -13.2611 | -9.6806  | 1.5469  |
| Mixed data       | -14.7021 | 3.6967   | 1.5938  |
| Notched data     | -13.1952 | -9.5837  | 1.5469  |
| Filled data \(k = 0.6k_u\) | -13.2188 | -9.6274  | 1.5469  |
| Filled data \(k = 1.2k_u\) | -13.2424 | -9.6466  | 1.5469  |

Next, we further analyse each part of the results after pulse compression. It should be pointed out that the calculations of each part of the filling filter result are based on assumption: the echo \(X(f)\) only contains \(S(f)\). Figure 4(a) gives the results of subtracting desired signal from each results after pulse compression, and the curves correspond to RFI, notch filtered result, filling filtered results with \(k = 0.6k_u\) and \(k = 1.2k_u\). The RFI in this figure is broadened after pulse compression, and the influence of the "residual term" can be reduced to around -35dB through using notch filter and filling filter. And the "residual term" of filling filter method is smaller than notch filter method. In fact, \(k\) cannot be accurately estimated (under-compensation and over-compensation), this effect on the sidelobes will always exist. Figure 4(b) and 4(c) show the absolute value curves of part2_fill and part3_fill at \(k = 0.6k_u\) and \(k = 1.2k_u\), respectively. In case \(k = 0.6k_u\) in figure 4(b), the condition is under-compensation and that is \(|\text{part2_fill}| < |\text{part3_fill}|\). And in case \(k = 1.2k_u\) in figure 4(c), the condition is over-compensation and that is \(|\text{part2_fill}| > |\text{part3_fill}|\). The results further show that the filling filtering is a dynamic process, which also verifies the conclusion.

3.2. Measured data simulation

The parameters settings of the measured simulation experiment are shown in the Table 3, and the data come from Radarsat.

Table 3. Parameters of measured simulation

| Parameters       | Values          |
|------------------|-----------------|
| Bandwidth        | 30.111 MHz      |
| Pulse width      | 41.74 \(\mu\)s |
| Carrier frequency| 5.3 GHz         |
| PRF              | 1256.98 Hz      |

The original signal spectrum, mixed signal spectrum and filtered spectrum results are shown in figure 5(a). In this experiment, in order to correspond to under-compensation and over-compensation, set \(k = 0.2k_u\) and \(k = 1.5k_u\). Figure 5(b) shows the result of subtracting the ideal data from the processed data, and the result indicates that the effect of the filling filter is better than that of the notch filter.
Figure 5(c) shows the absolute value curves of \( \text{part2\_fill} \) and \( \text{part3\_fill} \) at \( k = 0.2k_0 \), the condition is under-compensation and that is \( |\text{part2\_fill}| > |\text{part3\_fill}| \). And figure 5(d) is corresponded to \( k = 1.5k_0 \), there is \( |\text{part2\_fill}| < |\text{part3\_fill}| \). The above results are the same as the results of the point target.

Figure 5. The results after pulse compression. (a) The results of pulse compression. (b) The results of subtracting desired signal from each results after pulse compression. (c) The results of filling filter with \( k = 0.2k_0 \). (d) The results of filling filter with \( k = 1.5k_0 \).

Figure 6. The result of imaging. (a) Imaging results polluted by RFI. (b) Imaging results using notch filter method. (c) Imaging results using filling filter method with \( k = 0.2k_0 \). (d) Imaging results using filling filter method with \( k = 1.5k_0 \).
Figure 6 shows the result of imaging using two-dimensional data, where the horizontal is the range and the vertical is the azimuth. Compared with Figure 6(a), after RFI suppression results figure 6(b), 6(c) and 6(d) are obviously clearer. Figure 7 is the schematic diagram of "residual terms" imaging. It should be noted that: in order to show the effect of "residual term" on the image, the image is not normalized with the strong scattering point in ideal image. In this figure, image of the "survival terms" covers the entire scene, which actually raises the sidelobes of the scene targets, although this effect is not obvious in figure 6. Table 4 shows the average values of each "residual terms" images after using three kinds of suppression processing. Through comparison, the results in this table are the same as the point target simulation results, which also verifies the conclusion.

| | part 2 + part 3 | part 2 | part 3 |
|---|---|---|---|
| Notched data | 2.1746 | 2.1746 | -- |
| Filled data (k = 0.2k0) | 1.7397 | 2.1744 | 0.4347 |
| Filled data (k = 1.5k0) | 1.0873 | 2.1735 | 3.2605 |

4. CONCLUSION
In this paper, two types of RFI suppression methods based on spectral operations are focused: notch filter and filling filter methods. Through derivation and analysis, the interference suppression operation based on spectrum filtering is summarized as the selection of attenuation factor k. In addition, the conclusion is given: (a) By setting k reasonably, the performance of spectral filling method is better than notch filter method. (b) The essence of interference suppression based on spectral filling filter is the dynamic change process of "residual terms". (c) Since the "residual terms" cannot be completely eliminated in theory, they will affect the image quality after interference suppression. The simulation results of point targets and measured data also confirm the analysis and conclusions.

The derivation and experiment of this paper are mainly to illustrate the essence of interference suppression based on spectrum operation, and to achieve more ideal interference suppression results: for spectral filling method itself, it is necessary to find a more accurate estimation of attenuation factor k. In addition, some other effective spectral filtering methods should also be further explored. We hope that our work will be helpful to colleagues who are devoted to SAR and RFI suppression research.

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