The students’ mathematical argumentation in geometry

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Abstract. The main objective of this research is to analyze the student's mathematical argumentation when dealing with geometry. The method is used qualitative method with grounded theory to know how the students provide an explanation or an answer against claims so that the quality of the vernacular students will be drawn up with clear from how students compose a series of arguments. The results showed that there were still many students basically experiencing constraints in argumentation, but the quality of the reasoning appears to be a variation of the argument appeared, include: inductive, algebra, visual and perceptual. In addition, the starting point of the students composes a series of arguments generally starts from claims that arise in a matter. Proof of claim further builds upon the relationship between the characteristics of data with mathematical objects that appear in the acquired mathematical knowledge from previous students. Relationship spelled out in a series of statements and reasons which support the claims through the fourth argument.

1. Introduction
The reasoning is the ability that is always inherent in any mathematical activity. In curriculum documents \cite{1, 2}, it is said that reasoning is one of the fundamental mathematical capabilities. Its implications not only on the basic capabilities in understanding mathematics \cite{3}, but remains a strategic issue in the development of mathematical learning challenges in the future.

The reasoning is one of the very complex mathematical abilities. This complexity makes reasoning as an ability that is not easy to achieve. Some studies report a constraint faced by students when exposed to reasoning \cite{4, 5, 6}. These studies present the constraints in relation to the quality of mathematical reasoning in which students generally give an explanation of the difficulty in using the model or mathematical statement as well as making a statement that supports or refuting arguments \cite{5}.

More recently, a study of the quality of mathematical reasoning is much in touch with the argumentation \cite{7}. Argumentation is seen as the study of effective reasoning \cite{8}, and becomes a fundamental tool that helps students improve the competency of reasoning \cite{9}. The study generally uses on qualitative approaches \cite{7, 10}. The study of argumentation has many researchers with extremely variable themes, among them: the structure of general argument with Toulmin Argumentation Pattern (TAP) \cite{11, 12}, the taxonomy of proof scheme \cite{13, 14}; representation \cite{15}; and verification \cite{7, 15}.

TAP \cite{11} is seen as the starting point for the development of the study of argumentation. According to Toulmin \cite{11}, the validity of logical is not the main thing in an argument, but the orientation of the more
important is the content and the structure of semantic matching. The Toulmin analysis is called informal logic [10], which later inspired many researchers to come up with the term informal argumentation [16]. Some other researchers who distinguish the term formal and informal argumentation [17, 18] in which the first term refers to mathematical proof [18, 19]. Other researchers agreed on the difference between proof and argument [20, 21] and placing proof as a series of stages/logic chains organizing process called cognitive unity [20]. In this study, the term proof is distinguished with the argument, that is, the more degrading aspects of semantic rather than syntactic [22].

This paper only wants to reveal an aspect of the study of argumentation with regard to the representation of a mathematical argument emerges from the students of the junior high school. Study on these aspects is viewed essentially given the students of the junior high school lies on the transition of concrete thinking to formal. Overview of aspects of the representation of this argument will give valuable illumination to the development of learning or certain approaches are needed to improve students' mathematical argumentation. Selected topics in this study were the topic of geometry. The study of the argument on this topic has many experts earlier especially deal with a taxonomy of the proof scheme [9, 14, 23]. Study of van Hiele [23] including the most phenomenal by creating a taxonomy of geometric, include visual, descriptive/analytical, deductive, informal, formal, deductive rigor. Nurhasanah, Sabandar, and Kusumah [24] adopted the van Hiele geometry to investigate the emerge of mathematical abstraction to the study of the geometry of the students. Although not mentioned explicitly about argumentation, an investigation into the mathematical abstraction of the variations that occur in students with regards to the student experience in drawing up arguments. The third researcher this does not compile the taxonomy as well as the van Hiele, but a variation of how students are doing abstraction is the main goal to be achieved in the research. In this way, it has been done before by Liu [15]. Study Liu [15] focusing on variations of answers students in solving geometry problems with 4 gave rise to a form of argumentation, namely: inductive, algebra, visual and perceptual.

2. Method
The research was carried out on the even semester on 2016/2017. The research approach is qualitative use the method of grounded theory. Research subjects are assigned 33 students in the class VIII in one of the junior high school in Tangerang, Indonesia. The main data obtained from test results on the topics of geometry, especially cube and block. The test results are analyzed through three stages of grounded theory, namely: open coding, selective coding, and theoretical coding [25]. At the first stage, test results were analyzed through two steps: (1) identification of the structure of mathematical argumentation, and (2) grouping the arguments based on the representation of argumentation that appears. The arguments are then grouped by similarity of the characteristics and the representation of argumentation in the form of ARG 1, ARG 2 and so on. ARG 9 is categorized as irrelevant argumentation, while ARG 10 is categorized as arguing that simply does not show up or no categorized. At the second stage, this is done by considering the sub-categories that appear to define a core category. The measures undertaken are (1) doing analysis of category or subcategory that appears by specifying the dominant symptoms from each of the categories (subcategories), (2) assigns a category to the core by means of linking between categories that have been defined in the previous step, and (3) reviewing deepening against each category core that has been established through the interview against the selected sample theoretically [26]. And the last stage, theory or conjecture is built upon synchronization and triangulation of the data. The measures taken are (1) conducting analysis and synchronization of data obtained in the previous stage, (2) perform a triangulation of data through analysis of the work students and in-depth interview against the respondent elected, (3) devise theories (conjectures based on the results of the analysis), synchronization and triangulation.

The following are the instruments used in a mathematical argument test students.

Koko claims that the areas of the diagonal fields of a cube are equal.
How about your opinion about the revelation of Koko? Make an argument in favor of denying the Koko opinion!
Firman claims that two blocks of the equal volume have a different surface area if the length, width, and height of each block are different.

How about your opinion about the question of Firman? Make an argument in favor of denying the Firman opinion!

3. Result and discussion

3.1. Open coding

The results of studies generally describe the existence of variations in the mathematical argument. This is reflected from the arguments that appear on the stage of open coding, as shown in table 1.

Table 1. The Variation of Student's Mathematical Argumentation

| Argumentation | Question 1 |   | Question 2 |   |
|---------------|------------|---|------------|---|
|               | N   | Percentage | N  | Percentage |
| ARG 1         | 5   | 15         | 22 | 67         |
| ARG 2         | 4   | 12         | 2  | 6          |
| ARG 3         | 3   | 9          | 3  | 9          |
| ARG 4         | 2   | 6          | 0  | 0          |
| ARG 5         | 3   | 9          | 0  | 0          |
| ARG 9         | 6   | 18         | 2  | 6          |
| ARG 10        | 10  | 30         | 4  | 12         |

In table 1, the percentage of the arguments on the question 1 for category irrelevant argumentation (ARG 9) and argumentation which do not appear (ARG 10) goes far beyond other arguments. This shows that students are still experiencing constraints to produce argumentation. Instead, on the question 2, the percentage of ARG 1 is high compared with most other arguments. This gives you an idea that the trend of mathematical argumentation that appears on the question 2 is ARG 1. Likewise, although the arguments in the question 1 dominating in ARG 9 and ARG 10, the argument for the category appears, still the highest argument 1 compared to the other arguments.

3.2. Selective coding

This step begins by identifying the arguments appear appropriate representation. On the argumentation 1, students use some examples to support claims are filed. On the question 1, students must first define a few cubes with a certain size; for example, 4cm. Students then calculate the length of the field diagonals of cube ($4\sqrt{2}$cm), then the area of the diagonals field of the cube ($16\sqrt{2}$cm$^2$). Students who understand the properties of the cube, it is generally claimed that areas of the diagonal fields that opposite sides of 4cm are equal, namely $16\sqrt{2}$cm$^2$. As the basis of the conclusions drawn, the students claim that area of the diagonal fields will always be the same on other cubes which has a rib 6cm ($36\sqrt{2}$cm$^2$), 8cm ($64\sqrt{2}$ cm$^2$), ... and so on. On the question 2, students choose a pair of the block that has the same volume but the area of the same, for example, the pair of the block with a size 12cm x 2cm x 2cm and 4cm x 3cm x 4cm. Students claim that the two blocks in the same volume (48cm$^3$) but the area of the different (104cm$^2$ and 90cm$^2$). Students also have tried a couple of other blocks and the block pairs are claiming that volume is the same, but the size of the corresponding ribs are different, the breadth is definitely different.

On the argumentation 2, students use symbolic representation to assert that claim in each reserved was accepted or not. Students generally bring up any variables to determine a specific value from the element of the unknown material in question. On the question 1, students establish specific variables (s, r, or x) to represent the value of the ribs on any cube. These variables are then used to identify the elements of the diagonal field of any cube last until the conclusion of that vast symbolic field of diagonal fields of a cube is equal. On the question 2, the students set two blocks with different sizes.
The students then claimed that both the block in the same volume and must be pointed out that they are not equal. At this stage, no one can demonstrate that the claims involved proved to be right. Of course, students can use authentication with contra-example, but no one is able to do so.

On the argument 3, students build their arguments on the picture to support the proposed claim. On the question 1, students set up a cube. Students then determine two areas of diagonal fields and claim that the wide diagonal both fields are equal. Student outlines one of the broad fields of the diagonal equation. With the properties of the cube is known by previous, students then get the broad field of the other diagonal. This way is done to get the other diagonal fields so that the obtained conclusions that the sixth field of diagonal cube breadth equal. On the question 2, students create an illustration picture of two blocks that have length, width and height are different. A measure of length, width, and height of each block are the length of the line segments representing length, width, and height of each rib block. All students in this category are indeed could not propose further conjecture, so the outcome of this argument is not up to the expected conclusion.

On the argument 4, students base their arguments on the context is known. On the question 1, student imagined a pair of cuboid objects. Both cubes are congruent. If each cube sliced through one of the diagonal fields, then by altering their respective pieces of the cube, a new cube shape is obtained which remains congruent. Furthermore, students can also do other slices on the diagonal and keep generating more new cubes are congruent. The arguments 4 including arguments that at least show-up and even in the reserved question 2 do not appear at all. Basically, students need a lot of experience to interact with the real world that presents contextual problems so arguments 4 can appear in the imagination of the mind of the students.

On the argumentation 5, students base their arguments on the connections between mathematical objects based on the characteristics of the objects that have been previously understood. On the question,1 students begin making arguments by mentioning cube properties. For example, a cube has the properties, among others: the six sides congruent squares, mutually perpendicular sides, ribs-congruent opposite sides and perpendicular to each other. Student directly claims that the area of the diagonal fields of the cube is equal because the sides of the cube are congruent.

3.3. Theoretical coding
From some of the argumentation that appears that students have different ways of making explanations supporting against argumentation. On the argumentation 1, students use some special cases to check the truth of the statement asked. The conclusion is drawn when each case demonstrates the truth of the statement asked. This way is an indication of inductive argumentation. On the argumentation 2, students expressed any case through symbolic representation. Because the case took any, then the truth of the statement that is submitted is a foregone conclusion. This way is an indication of the algebraic argument. On the argument 3, students must first define an image as a starting point for investigation. Students then create math sentences based on the facts contained in the image. Conclusions are drawn when the facts demonstrate the truth of the statement. Because the proof of the truth of the statement is sourced from the image, then this way is an indication of visual argumentation. On the argument 4, the students started the idea of proof from a context that is known to object that resemble the existing context on the statement. Conclusions are drawn based on the perception that the truth of the statement appears in the context of the imagined or manipulated. This way is an indication of perceptual argumentation. On the argumentation 5, students base on characteristics of mathematical objects that exist in the statement. Although this argument brings up new arguments, all students in this group are never completely on the expected conclusions. Basically, new students bring up the initial idea to create an interconnection between the properties owned this object with the statement that will be proven. Thus students are using characteristics of mathematical objects on a statement that would then be applied by means of inductive argumentation, visual, perceptual and algebra. The results of the following interview give an explicit regarding the next student actions based on arguments 5.
Teacher : How are the diagonal field of this and that?
Student 5 : Equal, Sir?
Teacher : Why?
Student 5 : Similarly rectangular? This line and it's is equal, the properties of the cube.
Teacher : How do I calculate the areas?
Student 5 : (Confused), but the length of the rib is not known, Sir? For Example, I take 4 cm.
Teacher : Student 11 mentions that the length of field diagonal of the cube is equal, what that means?
Student 11 : I don't know, but, for example, in this classroom? (While point to the roof of the class). If I pull the line from there to the end here just the same with the draw a line from the tip of next door here to there.
Teacher : Next?
Student 11 : Lines on the wall on your right and left, right Sir?
Teacher : Why?
Student 11 : Properties of the cube.
Teacher : How about Student 24?
Student 24 : From this picture, I know Sir, but not sure?
Teacher : Why?
Student 24 : Since the cube sides of a square, then the diagonal field is equal. Right, Sir?

The results of the interview give an overview that students use the characteristics of mathematical objects arise in planning to make a statement in making an explanation of the claim lodged. The claim is a phenomenon that appears on every student's argumentation in which claims are used as a starting point to justify the statement in question. Referring to this claim, students then create an explanation so that the data obtained can come to the statement of claim filed. This data is connected to students' prior knowledge gained so that students utilize the characteristics of mathematical objects that appear to elaborate on claims based on several cases (inductive), a symbolic representation (algebra), images (Visual), and imagination (perceptual).

4. Conclusion
The student’s mathematical argumentation include inductive, algebra, visual and perceptual argumentation. The future of argumentations emerge from the fourth argument is inductive, the conclusion is based on the truth of some cases; algebraic, the conclusion is based on the general case with representation symbolic; visual, the conclusion is based on geometry picture; perceptual, the conclusion is based on a known context. In the justification of a statement, students generally bring up claims as a starting point. Students then elaborate data by connecting on mathematical object characteristics in question. Students use inductive, algebra, visual, and perceptual and to make a series of statements and reasons that support against claims filed.

References
[1] National Council of Teachers Mathematics 2000 Principles and standards for school mathematics (Reston V A : National Council of Teachers of Mathematics)
[2] Sundayana R, Herman T, Dahlan J A, and Prahmana R C I 2017 Using ASSURE learning design to develop students’ mathematical communication ability World Transactions on Engineering and Technology Education 15 245
[3] Ball D and Bass H 2003 Making mathematics reasonable in school In J Kilpatrick, G Martin and D Schifter eds A Research Companion to Principles and Standards for School Mathematics (Reston V A : National Council of Teachers of Mathematics) p 27
[4] Wardhani S and Rumiati 2011 Instrument penilaian Hasil belajar matematika SMP: Belajar dari PISA dan TIMSS (Yogyakarta: P4TK Matematika)
[5] Rizta A, Zulkardi, and Hartono Y 2013 Pengembangan soal penalaran model TIMSS matematika
SMP Jurnal Penelitian dan Evaluasi Pendidikan 17 230

[6] Tanujaya B, Prahmana R C I, and Mumu, J 2017 Mathematics instruction, problems, challenges, and opportunities: A case study in Manokwari regency, Indonesia World Transactions on Engineering and Technology Education 15 287

[7] Bergqvist T 2005 How students verify conjectures: Teachers’ expectations Journal of Mathematics Teacher Education 8 171

[8] Zarebski Toulmin’s model of argument and the “logic” of science discovery Studies in Logic, Grammar and Rhetoric 16 267

[9] Lee T-N 2015 Developing a theoretical framework to assess Taiwanese primary students’ geometric argumentation In M Marshman, V Geiger and A Bennison eds Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australia (Sunshine Coast: MERGA) p 365

[10] Inglis M, Mejia-Ramos J P, and Simpson A 2007 Modelling mathematical argumentation: The importance of qualification Educational Studies Mathematics 66 3

[11] Toulmin S E 1958 The use of argument (Cambridge: Cambridge University Press)

[12] von Aufschnaiter C, Erduran S, Osborne J, and Simon S 2008 Arguing to learn and learning to argue: Case studies of how students’ argumentation relates to their scientific knowledge Journal of Research in Science Teaching 45 101

[13] Pedemonte B 2008 Argumentation and algebraic proof ZDM, The International Journal of Mathematics Education 40 385

[14] Stylianides A J 2007 Proof and proving in school mathematics Journal for Research in Mathematics Education 38 289

[15] Liu Y 2013 Aspects of mathematical arguments that influence eighth-grade students’ judgment of their validity Dissertation (Ohio: The Ohio State University)

[16] Brem S K and Rips L J 2000 Explanation and evidence in informal argument Cognitive Science 24 573

[17] Aberdeen A 2005 The uses of argument in mathematics Argumentation 19 287

[18] Yackel E 2002 What we can learn from analyzing the teacher’s role in collective argumentation Journal of Mathematics Behavior 21 423

[19] Banegas J A 2003 Argumentation in mathematics In An Aberdeen and I J Dove eds The Argument of Mathematics (Netherland: Springer) p 47

[20] van Bendegem J P and van Kerkhove B 2009 Mathematical arguments in context Foundations of Science 14 45

[21] Prahmana R C I and Kusumah Y S 2016 The hypothetical learning trajectory on research in mathematics education using research-based learning Pedagogika 123 42

[22] Larios-Osorio V and Acuña-Soto C 2009 Geometrical proof in the institutional classroom environment In F-L Lin, F-J Hsieh G Hanna, and M de Villiers eds Proceedings of the International Commission on Mathematical Instruction (Taipei: National Taiwan Normal University, Taiwan) p 59

[23] van Hiele P M 1986 Structure and insight: A theory of mathematics education (New York: Academic Press)

[24] Nurhasanah F, Kusumah Y S, and Sabandar J 2017 Concept of triangle: Examples of mathematical abstraction in two different contexts International Journal on Emerging Mathematics Education 1 53

[25] Jones M and Alony I 2011 Building the use of grounded theory in doctoral studies International Journal of Doctoral Studies 6 95

[26] Creswell J W 2010 Research design pendekatan qualitative, quantitative dan mixed A Fawaid (Translator) (Yogyakarta: Pustaka Pelajar)