Mathematical model of wedge-shaped sliding support with low melting metal coated guide for partially filled working gap

M A Mukutadze, A N Opatskikh, A V Morozova and N S Zadorozhnaya

Rostov State Transport University, 2, Rostovskogo Strelkovogo Polka Narodnogo Opolcheniya sq., Rostov-on-Don, 344038, Russia

e-mail: opatskih@yandex.ru.

Abstract. The reported study presents an asymptotic series solution to a system of differential equations in powers of a small parameter $K$ depending on melting and a mechanical energy dissipation rate. Further, a self-similar solution has been found, as a consequence, a profile of velocities, pressure, principal operational characteristics, load capability, and friction force have been determined for a partially grease-filled working gap.

1. Introduction
An increase in loads designed for friction units of new machines and mechanisms is a key factor of their efficiency, total running time, and reliability [1-10]; it depends by far on the structure and quality of bearing joints. A serious drawback all currently-available calculation methods of sliding bearings suffer from [11-20] is that they hardly take into account the partial grease filling (a pre-failure condition).

Therefore, the development of a mathematical model for the case of partially grease-filled working gap and a guide surface coated with a light-melting metal alloy represents an advanced field of the theoretical tribology research, whose findings might be of a high importance.

2. Problem Statement
The study is focused on a wedge-shaped sliding support to consist of a slider block and a guide. A space between a slider block and a guide is supposed to be partially grease-filled; a slide block is stable, and a guide coated with a light-melting metal alloy goes around axis $Oy$ at a velocity of $u'$. 

3. Input equations and boundary conditions
In the Cartesian coordinates $x'o'y'$ (Fig. 1) a linear path equation of a sliding block and melted surface is written as follow:

$$y' = h_0 + x'tg\alpha, \quad y' = -\Phi(x),$$ (1)

where $h_0$ – thickness of a grease film in the initial cross section;
$\Phi_0$ – thickness of the melted layer in the initial cross section;
$\alpha$ – inclination angle of a sliding block linear path to axis $ox'$;
$\Phi(x)$ – function characterizing a melt thickness of the guide surface.
Input base equations are a non-dimensional equation of a viscous uncompressible liquid in approximation of “a thin layer”, a continuity equation, and an equation to determine a melted guide edge in terms of mechanical energy dissipation rate. In Cartesian coordinates this system of equations with boundary conditions is written as follow:

$$\frac{\partial v}{\partial y} = 0, \quad \frac{\partial^2 v}{\partial y^2} = \frac{dP}{dx}, \quad \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad \frac{d\Phi(x)}{dx} = K \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial v}{\partial y} \right)^2 dy$$

$$v = 0, \ u = 0 \quad \text{for} \ y = h(x) = 1 + \eta x, \ \nu = -1, \ u = 0 \quad \text{for} \ y = -\Phi(x),$$

$$\Phi = -\frac{h^*}{l} \quad \text{for} \ x = 0, \ P(x_l) = P(x_r) = 0, \ \Phi(0) = 0$$

where $u$, $v$ – vector components of a grease medium velocity;

$P$ – hydrodynamic pressure in a greasing layer;

$K = \frac{2\mu u^*}{h_l L'}$ – parameter to depend on melting and mechanical energy dissipation rate;

$L'$ – specific heat for a volume unit;

$x_l$ and $x_r$ – angular coordinates of free surface ends in the grease, respectively.

The relation between non-dimensional and dimensional values complies with formulae:

$$x' = lx, \quad y' = h_0 y, \quad v_x = u v, \quad v_y = u \frac{h_0}{l} u, \quad P' = P P', \quad P'' = \frac{\mu u l}{h_0^2}$$

The boundary conditions for $u$ and $v$ on the contour line $y = -\Phi(x)$ are formulated as follow:

$$v(0 - H(x)) = v(0) - \left. \left( \frac{\partial v}{\partial y} \right) \right|_{y=0} \cdot H(x) - \left( \frac{\partial^2 v}{\partial y^2} \right)_{y=0} \cdot H^2(x) + ... = -1$$

$$u(0 - H(x)) = u(0) - \left. \left( \frac{\partial u}{\partial y} \right) \right|_{y=0} \cdot H(x) - \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} \cdot H^2(x) + ... = 0$$

An analytical solution to a system of differential equations (2) with regard to boundary conditions (3) and (5) is found in series in powers of a small parameter $K$:

$$v(x, y) = v_0(x, y) + K v_1(x, y) + K^2 v_2(x, y) + ...$$

$$u(x, y) = u_0(x, y) + K u_1(x, y) + K^2 u_2(x, y) + ...$$

$$P(x) = P_0(x) + K P_1(x) + K^2 P_2(x) + ...$$

$$\Phi(x) = 0 - K \Phi_1(x) - K^2 \Phi_2(x) - K^3 \Phi_3(x) - ...$$
Substituting (6) into the system of differential equations (2) and taking into account boundary conditions (3), following equations are obtained with boundary conditions:

- for zero approximation:
  \[
  \frac{\partial^2 v_0}{\partial y^2} = \frac{dP_0}{dx}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0
  \]
  \(v_0 = 0, u_0 = 0\) for \(y = 1 + \eta x\), \(v_0 = -1, u_0 = 0\) for \(y = 0\), \(P_0(x) = P_0(x_2) = 0\), \(\Phi_0 = \frac{P_0}{l}\)

- for first approximation:
  \[
  \frac{\partial^2 v_1}{\partial y^2} = \frac{dP_0}{dx}, \quad \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0, \quad \frac{d\Phi_1(x)}{dx} = K \int_0^h \left(\frac{\partial v_0}{\partial y}\right)^2 dy, \quad u_i = 0, v_i = 0 \quad \text{при} h(x) = 1 + \eta x,
  \]
  \(v_1(x) = \left(\frac{\partial v_0}{\partial y}\right)_{y=0}, \quad u_1(x) = \left(\frac{\partial u_0}{\partial y}\right)_{y=0}\) \(\Phi_1(x), P_1(x) = P_1(x_2) = 0\)

A solution to the problem in zero approximation is sought as follows:

\[

\psi_0(x,y) = \frac{\partial \psi_0(x,y)}{\partial y} + V_0(x,y), \quad u_0(x,y) = -\frac{\partial \psi_0(x,y)}{\partial x} + V_0(x,y),
\]

\[

\psi_0(x,y) = \tilde{\psi}_0(\xi), \quad \xi = \frac{y}{h(x)}, \quad V_0(x,y) = \tilde{v}_0(\xi); \quad V_0(x,y) = \tilde{u}_0(\xi) \cdot h'(x)
\]

Substituting (11) into the system of differential equations (7) with regard to boundary conditions (8), a system of differential equations is obtained:

\[

\psi''_0(\xi) = \frac{1}{h(x)} \frac{1}{h(x)} + \tilde{c}_1^2 \quad \tilde{c}_1^2 \quad \tilde{c}_2^2
\]

and boundary conditions:

\[

\tilde{\psi}'(0) = 0, \tilde{\psi}'(1) = 0, \tilde{u}_0(1) = 0, \tilde{u}_0(0) = 0, \quad \tilde{v}_0(0) = -1, \int_0^1 \tilde{v}_0(\xi)d\xi = 0, P_0(x_1) = P_0(x_2) = 0
\]

Taking integral of the equation (12) in terms of (13), the result is:

\[

\psi_0(\xi) = \frac{1}{2} \left(\xi^2 - \xi\right), \quad \tilde{v}_0(\xi) = \tilde{c}_1 \frac{\xi^2}{2} + \left(1 - \frac{\tilde{c}_1}{2}\right) \xi - 1, \quad \tilde{c}_1 = -6
\]

Following the formula \(P_0(x_1) = P_0(x_2) = 0\) within the accuracy of the second order of smallness \(o(\eta^2)\) for \(\tilde{c}_2\), it is obtained:

\[

\tilde{c}_2 = 6 \left(1 + \frac{\eta}{2} \left(x_2 - x_1\right)\right)
\]

in terms of (15) for \(P_0\) it is written:

\[

P_0 = 3\eta \left(\left(x - x_1\right)^2 - (x - x_1)(x_2 - x_1)\right)
\]

To determine \(\Phi_1(x)\) in terms of (14), an equation is formulated:

\[

\frac{d\Phi_1(x)}{dx} = h(x) \left(\frac{\tilde{\psi}_0'(\xi)}{h'(x)} + \frac{\tilde{v}_0(\xi)}{h(x)}\right)^2 d\xi
\]

Taking integral of the equation (17), the result is:

\[

\Phi_1(x) = \int_0^x \frac{\Delta_1dx}{h'(x)} + \int_0^x \frac{\Delta_2dx}{h(x)}
\]
where \( \Delta_1 = \frac{1}{12} \left( (\bar{v}^2(\xi)) - \bar{v}_0^2(\xi) \right) \) and \( \Delta_2 = \frac{1}{2} \left( 2\bar{v}_0(\xi) \cdot \bar{v}_0'(\xi) - \bar{v}_0^2(\xi) \right) \) for \( \bar{v}_0(\xi) \in C \).

(19)

Seeking for a solution to the equation (18)-(19) within the accuracy of the second order of smallness \( o(\eta^2) \), it is obtained:

\[
\Phi_i(x) = 13x - \frac{25}{2} \eta x^2 + 9\eta(x_i - x)
\]

(20)

Taking into account (20) and boundary conditions (10), it is written as follow:

\[
v_i(x) = \left\{ \begin{array}{ll} 0, & u_i = 0 \text{ for } y = 1 + \eta x, \quad u_i = \left( \frac{\partial u_0}{\partial y} \right)_{y=0} = 0, \\
-\frac{39}{2} \eta x (2x - x_i - x) - \frac{51}{2} \eta x^2 - 9\eta x(x_i - x) + 13x, & P_i(x_i) = P_i(x) = 0.
\end{array} \right.
\]

(21)

A solution to the equation of a viscous uncompressible liquid for the first approximation (9) and boundary conditions (21) brings about:

\[
v_i(x) = \frac{dP_i}{dx} \frac{x^2}{2} - \frac{39}{2} \eta x (2x - x_i - x) - \frac{51}{2} \eta x^2 - 9\eta x(x_i - x) + 13x
\]

(22)

\[-\frac{dP_i}{dx} \cdot 1 + \eta x \cdot y - \frac{39}{2} \eta x (2x - x_i - x) - \frac{51}{2} \eta x^2 - 9\eta x(x_i - x) + 13x
\]

To estimate the hydrodynamic pressure a Leibniz formula vs. parameters is used

\[
\frac{\partial}{\partial x} \int_0^{\mu_1} v_i(x) dy = \frac{\partial}{\partial x} \left[ -\frac{1}{12} \frac{dP}{dx} (1 + \eta x)^3 - \frac{39}{2} \eta x (2x - x_i - x) - \frac{51}{2} \eta x^2 - 9\eta x(x_i - x) + 13x (1 + \eta x) \right] = 0
\]

(23)

Solving an equation (23) for the hydrodynamic equation and boundary conditions (10), it is obtained:

\[
P_i = 12 \left( \frac{39}{4} \eta (x_i^2 - x^2) (x_i + x) - \frac{155}{6} \eta (x_i^3 - x^3) - \frac{9}{2} \eta (x_i^2 - x^2) (x_i - x) + \frac{13}{4} (x_i^2 - x^2) \right)
\]

(24)

4. Results and Discussions

In terms of (7), (8), (16) and (24) for bearing capacity and friction force it is obtained:

\[
w = P \cdot \int_0^l \left( \frac{P_i + KP}{x} \right) dx = \mu \frac{\mu_1^2}{k_0} \left[ 3\eta (x_i - x) + 12(x_i - x_i) \left( \eta \left( \frac{147}{24} x_i^2 + \frac{81}{2} x_i^4 + \frac{589}{24} x_i^2 x_i^2 + \frac{13}{2} x_i^4 \right) + \frac{13}{12} (2x_i^2 - x_i x_i - x_i^2) \right) \right]
\]

(25)

\[
L_m = \mu \int \left[ \frac{\partial v_i}{\partial y} \bigg|_{y=0} + K \frac{\partial v_i}{\partial y} \bigg|_{y=0} \right] dy = \mu \frac{\mu_1^2}{k_0} \left( x_i - x \right) x
\]

\[
 \left[ 2\eta \frac{x_i}{2} - \eta \frac{x_i}{2} - 1 + 6K \left( \frac{613}{24} x_i^4 + \frac{13}{4} x_i^4 - \frac{147}{24} x_i^4 x_i + \frac{39}{4} x_i x_i + \frac{7}{12} x_i - 6x_i + \frac{7}{12} x_i^2 - \frac{32}{24} x_i^3 + \frac{157}{12} x_i x_i - \frac{9}{2} x_i + \frac{155}{4} \right) \right] \eta
\]

\[
- \left[ \frac{26}{12} x_i + \frac{13}{6} x_i x_i + \frac{13}{12} x_i + \frac{13}{2} x_i + \frac{13}{2} x_i \right]
\]
5. Experimental research

Using calculation models developed in the theoretical part, an experimental study is carried out. The most important outcome to emerge from the study is a future application sphere of the designed tribosystem outlined in the form of loading and velocity conditions, as well as principal tribotechnical characteristics.

The experimental study was focused on a sliding bearing with a light-melting metal coating for the case of a partially grease-filled gap (a pre-failure condition).

The experiment findings were used to reveal a period of a hydrodynamic friction mode. A series of experiments has confirmed the validity of developed theoretical calculation models and data on their numerical analysis within the scope of design and performance parameters of tribosystems coated with a light-melting metal, so theoretical and experimental results demonstrate their convergence.

An analysis of the correlations and graphs obtained allows the following conclusions:

1. An expansion of the loaded zone \((x_2 - x_1)\) increases significantly the bearing capacity, whereas the friction falls; this fact is in line with the parameters under consideration, outcomes of other researchers, verifying the efficiency of designed models, as a consequence.
2. If a guide surface is coated with a light-melting metal alloy, the tribotechnical testing carried out by an end-type friction machine has shown the considerable reduction (to 27%) of a wear spot and the longer stability of a greasing film (to 48%) in comparison with a traditional grease.

Conclusion

More specified calculation models of wedge-shaped sliding supports make it possible to change a ratio between its bearing capacity and friction coefficient via varying a light-melting alloy coating. The study has established the acceptable convergence of theoretical and experimental results, supporting therefore theoretical conclusions.

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