Conductance oscillation due to the geometrical resonance in FNS double junctions

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We theoretically analyzed the Andreev reflection in ferromagnetic metal (FM) / nonmagnetic metal (NM) / superconductor double junctions with special attention to the electron interference effect in the nonmagnetic metal layer. We showed that the conductance oscillates as a function of the bias voltage due to the geometrical resonance. We found that the exchange field and therefore the spin polarization of the ferromagnetic metal can be determined from the period of the conductance oscillation, which is proportional to the square-root of the exchange field.

Recently much attention has been focused on the Andreev reflection (AR) in ferromagnetic metal (FM) / superconductor (SC) contacts since the spin polarization of conduction electrons is measured through the suppression of the conductance below the superconducting gap. This method is called point contact Andreev reflection (PCAR) spectroscopy.

On the other hand, the quasiparticle (QP) interference in nonmagnetic metal (NM) / SC junctions has been extensively studied in the past. As shown in Refs. 12,13,14,15,16,17,18, the interference of QPs in the SC layer brings about the oscillation of the density of states against the QP energy, which is called a Thomas oscillation. The interference in the NM layer also brings about the oscillation of the density of states in the NM layer12,13, which is known as the deGennes-Saint-James bound state or the McMillan-Rowell oscillation. Nesher and Koren measured the dynamic resistance of YBa$_2$Cu$_3$O$_6$.6 / YBa$_2$Cu$_2$.55Fe$_{0}$.45O$_y$ / YBa$_2$Cu$_3$O$_{6.6}$ junctions and determined the renormalized Fermi velocity of QPs in the YBa$_2$Cu$_2$.55Fe$_{0}$.45O$_y$ layer from the period of the McMillan-Rowell oscillation16.

In this paper, we theoretically analyze the Andreev reflection in a FM/NM/SC double junction system with special attention to the electron interference effect in the NM layer. Following the work of Blondé, Tinkham, and Klapwijk (BTK)20, we solve the Bogoliubov-de Gennes (BdG)20 equations and calculate the conductance. We show that the conductance due to the Andreev reflection oscillates as a function of the bias voltage because of the geometrical resonance predicted by deGennes and Saint-James. We obtain the analytical expression of the probability of the Andreev reflection under the Andreev approximation and find that the period of the conductance oscillation is proportional to the square-root of the exchange field. Therefore, we can determine the exchange field and therefore the spin polarization of the FM layer from the period of the conductance oscillation.

The system we consider is comprised of the FM/NM/SC double junctions shown in Fig. 1(a). The current flows along the $x$-axis, and the interfaces between FM/NM and NM/SC are located at $x = 0$ and $x = d$, respectively. The system is described by the following BdG equation:

$$
\begin{pmatrix}
H_0 - h(x)\sigma & \Delta(x) \\
\Delta^*(x) & -H_0 - h(x)\sigma
\end{pmatrix}
\begin{pmatrix}
f_{\sigma}(r) \\
g_{\sigma}(r)
\end{pmatrix}
= E
\begin{pmatrix}
f_{\sigma}(r) \\
g_{\sigma}(r)
\end{pmatrix},
$$

where $H_0 \equiv -(\hbar^2/2m)^2 + V(x) - \mu_F$ is the single particle Hamiltonian, $E$ is the QP energy measured from the Fermi energy $\mu_F$, $V(x)$ is the interfacial barrier21, and $\sigma = (+(-)$ represents the up-(down)-spin band. The exchange field function $h(x)$ is given by $h(x) = h_0[1 - \Theta(x)]$ where $h_0$ represents the exchange field in the FM layer and $\Theta(x)$ is the Heaviside step function. We employed the two-band Stoner model for the FM layer for simplicity. The superconducting gap function is expressed as $\Delta(x) = \Delta_0\Theta(x - d)$, where $\Delta_0$ represents the superconducting gap in the SC layer. We assume that the system

![FIG. 1: (a) Schematic diagram of a FM/NM/SC double junction. An NM with a thickness of $d$ is sandwiched by FM and SC layers. (b) Schematic diagrams of energy vs. momentum of the FM/NM/SC double junction for a spin-up incident electron are shown. The open circles denote holes, the filled circles electrons, and the arrows point in the direction of the group velocity. The incident electron with up-spin is denoted by 0, along with the resulting scattering processes: Andreev reflection (1), normal reflection (2) at the FM/NM interface, transmission to the NM (3, 4) and reflection at the NM/SC interface (5, 6), transmission as a electron-like quasi-particle to the SC (7) and that as a hole-like quasi-particle (8). (c) Schematic diagrams of energy vs. momentum in the FM layer for a spin-down incident electron are shown.](cond-mat.nesc-hall)
has translational symmetry in the transverse (y and z) direction, and therefore the wave vector parallel to the interface \( k_\parallel \equiv (k_y, k_z) \) is a conserved quantity.

The general solutions of the BdG equation (11) in the FM (NM) layer are given by

\[
\Psi_{\pm k_{\FM(NM),\sigma}}(r) = \begin{cases} 
\frac{1}{0} & e^{\pm ik_{\FM(NM),\sigma}x}S_{k_\parallel}(r_\parallel)
\end{cases},
\]

(2)

where \( S_{k_\parallel}(r_\parallel) \) represents the eigen function in the transverse direction in the \( k_\parallel \) channel and \( k_{\FM(NM),\sigma} \) is the x component of the wave number of an electron (hole) with \( \sigma \)-spin defined as \( k_{\FM,\sigma}^\pm = \frac{\sqrt{2m}}{\hbar} \sqrt{\mu_F \pm E \pm \sigma \hbar \omega - E_\parallel} \) and \( k_{\NM}^\pm = \frac{\sqrt{2m}}{\hbar} \sqrt{\mu_F \pm E - E_\parallel} \), where \( E_\parallel = \frac{k_\parallel^2}{2m} \). In the SC layer, we have

\[
\begin{align*}
\Psi_{\pm k_{\SC}^+}(r) &= \begin{cases} 
(u_0/v_0) & e^{\pm ik_{\SC}^+x}S_{k_\parallel}(r_\parallel)
\end{cases}, \\
\Psi_{\pm k_{\SC}^-}(r) &= \begin{cases} 
(v_0/u_0) & e^{\pm ik_{\SC}^-x}S_{k_\parallel}(r_\parallel)
\end{cases},
\end{align*}
\]

(4)

where \( u_0 \) and \( v_0 \) are the coherence factors expressed as \( u_0^2 = 1 - v_0^2 = \frac{1}{2}\left[1 + \frac{\sqrt{\mu_F - E_\parallel}}{\sqrt{\mu_F + E_\parallel}}\right] \), and \( k_{\SC}^\pm \) is the x component of the wave number of an electron (hole-)like QP defined as \( k_{\SC}^\pm = \frac{\sqrt{2m}}{\hbar} \sqrt{\sqrt{\mu_F - E_\parallel}^2 + \Delta^2 - E_\parallel} \).

The wave function of the FM/NM/SC double junction is given by the linear combination of the above general solutions. Let us consider the scattering of an electron in the \( k_\parallel \) channel with \( \sigma \)-spin injected into the NM from the FM, the eight processes shown in Fig.1(b) are active. Therefore, the wave function in the FM layer \( (x < 0) \) takes the form

\[
\Psi_{\sigma,k_\parallel}(r) = \begin{cases} 
\frac{1}{0} & e^{ik_{\FM,\sigma}x} + a_{\sigma,k_\parallel} \frac{0}{1} e^{ik_{\FM,\sigma}x} \\
+ b_{\sigma,k_\parallel} \frac{1}{0} e^{-ik_{\FM,\sigma}x}
\end{cases},
\]

(6)

In the NM layer \( (0 \leq x < d) \), we have

\[
\Psi_{\sigma,k_\parallel}(r) = \begin{cases} 
\frac{1}{0} & e^{i\kappa_{\NM}^+x} + \beta_{\sigma,k_\parallel} \frac{0}{1} e^{-i\kappa_{\NM}^-x} \\
+ \xi_{\sigma,k_\parallel} \frac{1}{0} e^{-i\kappa_{\NM}^-x} + \chi_{\sigma,k_\parallel} \frac{0}{1} e^{i\kappa_{\NM}^+x}
\end{cases}S_{k_\parallel}(r_\parallel),
\]

(7)

and in the SC layer \( (x \geq d) \)

\[
\Psi_{\sigma,k_\parallel}(r) = \begin{cases} 
\frac{1}{0} & e^{i\kappa_{\SC}^+x} \\
+ d_{\sigma,k_\parallel} \frac{0}{1} e^{-i\kappa_{\SC}^-x}
\end{cases}S_{k_\parallel}(r_\parallel).
\]

(8)

The coefficients \( a_{\sigma,k_\parallel}, b_{\sigma,k_\parallel}, c_{\sigma,k_\parallel}, d_{\sigma,k_\parallel}, \alpha_{\sigma,k_\parallel}, \beta_{\sigma,k_\parallel}, \xi_{\sigma,k_\parallel}, \) and \( \chi_{\sigma,k_\parallel} \) are determined by matching the wave function at the boundary of the contact \( x = 0 \) and \( d \). Following the BTK theory, the probabilities of the AR and the normal reflection are given by

\[
A_{\sigma,k_\parallel}(E) = (k_{\FM,\sigma}/k_{\FM,\sigma})a_{\sigma,k_\parallel}^* b_{\sigma,k_\parallel} + B_{\sigma,k_\parallel}(E) = b_{\sigma,k_\parallel}^* c_{\sigma,k_\parallel},
\]

respectively. Since we assume that the temperature is zero, the conductance at bias voltage \( V \) is given by

\[
G = \frac{e^2}{\hbar} \sum_{\sigma,k_\parallel} A_{\sigma,k_\parallel}(eV).
\]

(9)

Let us first consider the most idealistic case where the interfacial scattering potential, \( V(x) \), is assumed to be zero. This simplification enables us to obtain the analytical expression of \( A_{\sigma,k_\parallel}(E) \) under the Andreev approximation.

\[
A_{\sigma,k_\parallel}(E) \approx \frac{4(1 - \zeta)\sqrt{1 - \eta^2}}{2[(1 - \zeta)^2 + (1 - \zeta)(1 - \zeta)^2 - \eta^2] - \eta^2[\epsilon^2 + (1 - 2\epsilon^2)\cos^2((k_{\NM}^+ - k_{\NM}^-)d) - \epsilon\sqrt{1 - \epsilon^2}\sin(2(k_{\NM}^+ - k_{\NM}^-)d)]},
\]

(10)

where we introduced the normalized parameters \( \eta = \hbar_0/\mu_F \), \( \zeta = E_F/\mu_F \), and \( \epsilon = E/\Delta_0 \). We note that Eq. (10) contains trigonometric functions in the denominator. For the FM/SC junction, i.e., \( d = 0 \), the trigonometric functions become constant and Eq. (10) reproduces the de Jong and Beenakker results of the zero-bias conductance. For the FM/NM/SC junctions with \( d \neq 0 \), the trigonometric functions in Eq. (10) give rise to the oscillation of the conductance against the bias voltage. The origin of the conductance oscillation is the interference of electrons in the NM layer. In the FM/NM/SC double junctions, the injected electron propagates across the NM layer to the interface as an electron (3 in Fig.1) and is scattered into a hole (6 in Fig.1) by the superconduct-
The value of the exchange field is taken to be Eq. (10) are plotted by lines and circles, respectively. The exact numerical results and the value of Eq. (10) are plotted by lines and circles, respectively. The value of the exchange field is taken to be $h = 0.3, 0.6, 0.9 \mu_F$ from top to bottom. (b) Same plot for $d=10 \mu m$.

The superconducting gap can pair an excited electron with an electron inside the Fermi sea, leaving a hole excitation. The hole propagates back across the NM layer; however, it cannot interfere with the original electron. This interference produces an electron in the NM layer. It can interfere with the original electron (3 in Fig. 1) by the superconducting gap, and propagate as a hole excitation. The hole propagates back across the NM layer; however, it cannot interfere with the original electron.

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In order to analyze the interference effect on the conductance oscillation, we consider the AR probability $A_{\sigma,k_\parallel}(E)$ of the one-dimensional system; i.e., only the transverse channel with $k_\parallel = 0$ is considered. In Fig. 2 (a), 2 (b) we plot the probability $A_{\sigma,k_\parallel}(eV)$ of the FM/NM/SC double junctions with $d=1 \mu m$ and $10 \mu m$, respectively, as a function of the bias voltage. Since Eq. (10) is an even function of normalized value of the exchange field $\eta$, $A_{\sigma,k_\parallel}(eV)$ is independent of the spin direction $\sigma$; i.e., $A_{\uparrow,k_\parallel}(eV) = A_{\downarrow,k_\parallel}(eV)$. The Fermi energy and the superconducting gap are assumed to be $E_F = 3.8 eV$ ($k_F = 1.0 \text{Å}^{-1}$) and $\Delta_0 = 1.5 \text{meV}$, respectively. The exact numerical results and the approximate values of Eq. (10) are plotted by lines and circles, respectively. The value of the exchange field is taken to be $h = 0.3, 0.6, 0.9 \mu_F$ from top to bottom.

As shown in Figs. 2 (a) and 2 (b), Eq. (10) and therefore the Andreev approximation are valid for all values of the exchange field. According to Eq. (10), the period of the oscillation is determined by the condition that $k^+ - k^- = n\pi$, where $n$ is an integer. Since $k^\pm \simeq k_F (1 \pm E/2\mu_F)$, the period is obtained as

$$\Delta V_{1D} \simeq \frac{\hbar^2 \pi k_F}{2m e d},$$

which is inversely proportional to the thickness of the NM layer, $d$. For the one-dimensional system, the period $\Delta V_{1D}$ is independent of the exchange field of the FM layer as shown in Figs. 2 (a) and 2 (b), and in Eq. (11). However, as we shall show later, the period of the conductance oscillation due to the geometrical resonance depends on the exchange field of the FM layer because the number of $k_\parallel$ channels available for the AR is restricted by the exchange field.

The period of the oscillation of $A_{\sigma,k_\parallel}(eV)$ with finite $k_\parallel$ is given by $\Delta V_{k_\parallel} \simeq \frac{\hbar}{\sqrt{2m e d}} \sqrt{E_|| - E_\parallel}$. From Eq. (9) the conductance is obtained by summing up $A_{\sigma,k_\parallel}(eV)$ for all available $k_\parallel$. Since the spin of the Andreev reflected hole is opposite to that of the incident electron, the maximum value of $k_\parallel$ and therefore $E_\parallel$ is limited by the exchange field $h_0$ as $\max E_\parallel = \mu_F - h_0$. We assume that oscillations of $A_{\sigma,k_\parallel}(eV)$ with different periods cancel out each other and the period of the sum $\sum_{\sigma,k_\parallel} A_{\sigma,k_\parallel}(eV)$ is determined by the shortest period. Thus, the period of the conductance oscillation of the three-dimensional system is obtained as

$$\Delta V_{3D} \simeq \min \Delta V_{k_\parallel} = \frac{h \pi}{\sqrt{2m e d}} \sqrt{h_0}. \quad (12)$$

In Figs. 3 (a), 3 (b), and 3 (c) we plot the conductance of the FM/NM/SC junction $G_{FNS}$, normalized by that of the FM/NM/SC junction, $G_{FNN}$, against the bias voltage. As shown in Fig. 3 (a), the oscillation due to the geometrical resonance does not appear in the conductance-voltage curve if the thickness of the NM layer, $d$, is less than or of the order of nm. The conductance-voltage curve is indistinguishable from that of the FM/SC junction. Hence, we can use the conventional PCAR analysis for a FM film, the surface of which is coated by a thin vacuum.
FIG. 4: (a) The normalized conductance, $G_{\text{FNS}}/G_{\text{FNN}}$ for $d=1\mu m$, $Z=0.2$ is plotted against the bias voltage. (b) Same plot for $d=10\mu m$, $Z=0.2$.

(less than a few nm) NM layer.

Figures 3 (b) and 3 (c) show the conductance-voltage curves for the FM/NM/SC double junctions with $d=1\mu m$ and $d=10\mu m$, respectively. One can see that the period does depend on the exchange field, $h_0$, in the FM layer as well as the thickness of the NM layer. The period is a decreasing function of the exchange field. We can easily confirm that the period is proportional to the square-root dependence of the exchange field by looking at Fig. 3 (d). The exact numerical results (filled circles) agree well with Eq. 12 (dotted line). The results suggest that we can determine the exchange field and therefore the spin polarization of the FM layer from the period of the conductance oscillation.

Next we consider the effect of the interfacial scattering potential at the NM/SC interface. We assume that the interfacial potential is represented by the delta-function potential at the NM/SC interface. We assume that the period of the conductance oscillation is due to the geometrical resonance in the NM layer as well as the thickness of the NM layer. The period of the oscillation is proportional to the square-root of the exchange field.

In summary, we studied the conductance oscillation due to the geometrical resonance in a FM/NM/SC double junction theoretically. We showed that the conductance due to the Andreev reflection oscillates as a function of the bias voltage due to the geometrical resonance. We found that the exchange field and therefore the spin polarization of the FM layer can be determined from the period of the conductance oscillation because the period of the conductance oscillation is proportional to the square-root of the exchange field.

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