Brain dynamics through spectral-structural neuronal networks

Maricel Agop $^{1,†}$, Alina Gavrilut $^{1,*}$, Gabriel Crumpei$^{2,†}$, Mitică Craus$^{3,†}$, and Vlad Birlescu $^{4,†}$

$^{1,†}$ Department of Physics, Gheorghe Asachi Technical University of Iaşi, Bd. D. Mangeron, no.67, Iaşi, 700050, Romania, m.agop@yahoo.com
$^{1,*}$ Faculty of Mathematics, Al. I. Cuza University, Carol I Bd. 11, Iaşi, 700506, Romania, gavrilut@uaic.ro
$^{2,†}$ Psychiatry, Psychotherapy and Counseling Center Iaşi, Iaşi, 700115, Romania, crumpei.gabriel@yahoo.com
$^{3,†}$ Technical University Gheorghe Asachi of Iaşi, Romania, Faculty of Automatic Control and Computer Engineering, Department of Computer Science and Engineering 53 A Mangeron Blvd., Iaşi, RO-700050, Romania
E-mails: craus@cs.tuiasi.ro, v.birlescu@cs.tuiasi.ro

Abstract. Starting from the morphological-functional assumption of the fractal brain, a mathematical model is given by activating brain's non-differentiable dynamics through the determinism-nondeterminism inference of the responsible mechanisms. The postulation of a scale covariance principle in Schrödinger's type representation of the brain geodesics implies the spectral functionality of the brain dynamics through mechanisms of tunelling, percolation etc., while in the hydrodynamical type representation, it implies their structural functionality through mechanisms of wave shock, solitons type etc. For external constraints proportional with the states density, the fluctuations of the brain stationary dynamics activate both the spectral neuronal networks and the structural ones through a mapping principle of two distinct classes of cnoidal oscillation modes. The spectral-structural compatibility of the neuronal networks generates the communication codes of algebraic type, while the same compatibility on the solitonic component induces a strange topology (the direct product of the spectral topology and the structural one) that is responsible of the quadruple law (for instance, the nucleotide base from the human DNA structure). Implications in the elucidation of some neuropsychological mechanisms (memory’s location and functioning, dementia etc.) are also presented.

Keyword: communication languages; codes; brain; coherence; neuronal network.

1 Introduction

Many phenomena with complex patterns and structures are widely observed in brain. These phenomena are some manifestations of a multidisciplinary paradigm called emergence or complexity. They share a common unifying principle of dynamic arrays, namely, interconnections of a sufficiently large number
of simple dynamic units can exhibit extremely complex and self-organizing behaviors.

There are many diverse methods of analysis of the dynamics of complex systems, particularly of those addressed to brain. However, due to the fact that they use only differentiability in the study of systems, all of them involve sophisticated and sometimes ambiguous models (Kandel brain’s standard models [1]). In our opinion, a way of analysis necessary in the dynamics of complex systems, especially those related to brain, must respect the recent results related to the harmony between morphology and functioning of a system. Thus, if we only concentrate to brain, from structural point of view, the nature is abundant in examples.

The standard approach, by extrapolating neuron performing way to the description of the performance of the whole brain (Kandel [1]), did not produce expected results, because the neuron is part of a network, and from the complex systems’ point of view, the properties of the constitutive elements can be recaptured in the properties of the whole system only as emergent.

On the other hand, the tackling from a quantum perspective, as proposed by different authors (Atmanspacher [2,3]), did not allow for specifying of some functional models of the brain, because at every scale the emergent phenomenon generates new properties which can be recaptured as emergent in the scales that follow.

Consequently, the performance of brain as an assembly, from the psychic life point of view, seems to be best approached from the perspective of complex systems, with their potential components, nondifferential and noncausal from the structural point of view, between which there is a chaotic part structured via attractors, and highlighted within a phase space.

From the very beginning we point out that, both from brain structural (morphological) and from functional (processing) points of view, examples are brought in order to substantiate the idea set forth as the starting one, namely that the brain dynamics at every level are dictated by the brain functional-structural coherence.

For example, studying the branching pattern of dendritic trees of retina neurons, Caserta et al. [4] identify by box counting, fractal shapes with a fractal dimension of approximately 1.7, which can be explained by a diffusion limited aggregation model (Witten and Sander [5]). A fractal structure was observed by Kniffki et al. [6] for the branching dendrite patterns of thalamic neurons in Golgi impregnated specimens. Moreover, as one can easily observe, the entire neuronal network has a fractal structure.

The fractality is also manifest when the functionality of the brain is considered. Such a statement is based on the idea that a great body of laws governing such a functionality at any scale of resolution, prove to be reducible to a power-type law. As a matter of fact, power laws are to be found both in the functionality and in the structure of brain, which substantiates the idea to suggested in the present work, namely that only the structural-functional unity of the brain can lead to a sound explanation of the complex phenomena met at this level (see Werner [7,8]).
The recording of the brain functions highlighted an electric and magnetic activity correlated with certain brain functions. This electromagnetic activity is spectral in nature, and as such assumes a spectral functionality and processing. One can thereby conclude that the morphology and the fractal structure of the brain should be duplicated by a fractal functioning and processing (de Valois & de Valois [9,10] – processing of the visual image, Békési [11] – processing of proprioceptive sensoriality).

Also, Lowen and Teich [12,13] suggested that the fractal action potential patterning in auditory nerve may be related to fractal activity in the ion channels of the sensory organs feeding into the auditory nerve: that is, the hair cells in the cochlea.

For example, the fractal activity at the site of neural impulse transmission at the neuromuscular junction. The muscular fibers contract nonlinearly due to fractal type mechanisms, which allows for the nonlinear adaptation of muscular contraction to the environmental necessities (sharp transients from rest to maximum function, functional reserve for continuous effort via nonlinear training in time of the muscle fibers).

Our paper sets out to tackling the old problem of brain-mind duality, whereby the description of brain can be approached according to the laws of physics and chemistry, while the mind cannot. We are thus allowed conclude that the two aspects of the duality mind-brain actually embody a structural and functional unity that can be modeled physically and mathematically, and can be analyzed according the the modern scientific paradigms. The psychic life is thus represented by a complex dynamics of exchange between neural and spectral networks.

In the present paper, the brain’s dynamics through spectral-structural neuronal networks are analyzed. Thus, by the determinism-nondeterminism inference in brain dynamics, we quit either the classical determinism [14,15], or the quantum nondeterminism [14,16]. Moreover, the fractal type brain [4,12,13,17,18,19,20] is both morphologically and functionally specified by activating the non-differentiable type brain dynamics [21-24].

This paper is organized as follows: in Section 2, the mathematical model is given, the spectral functionality of the brain dynamics is presented, the structural functionality of the brain dynamics is provided and the communication codes generation are explained through the coherence of the spectral-structural functionality. In Section 3, some possible implications of the mathematical model in the decipher of some neuropsychological mechanisms are presented.

With our approach we finally aim at the solution of the old problem of the duality brain-mind. Our physical-mathematical model offers the vision that both the mind and the brain do form a functional, describable through the dynamics existing between the spectral and neural networks lying at the foundation of the psychic life.
2 Results

First, we present the mathematical model, which explains the structure and the functionality of the brain. Moreover, its own space (the one generated by the brain) is structurally, a fractal in the most general sense given by Mandelbrot [24]. In such space, the only possible functionalities (which are compatible with the brain structure) are achieved on continuous but non-differentiable curves [21-28].

Brain’s structural-functional compatibility (structural-functional duality) as a source of the cerebral dynamics at any scale is thus imposed.

Accepting the structural-functional duality of the brain, the trajectory of motion realized on the structural component must be identified with an element from the functional part. If, according to Caserta et al. [29], we admit that the unharmonic oscillations of the neurofibrils would be the source of the functional part of the brain, then the curve describing the motion of a neurofibrile is a continuous nondifferentiable curve (see Werner [7,8]). So this motion takes place in a fractal space, the one generated by the fractal structure of the brain, and thus it can be identified with the geodesic of the associated fractal space. At yet another scale, the neuron can be identified with its corresponding geodesic. More generally, the wave is identified with the corpuscle, the motion of the corpuscle in the field of its associated wave being obviously a continuous nondifferentiable curve (fractal curve), whence the idea of geodesic. We shall detail these considerations in what follows:

By “brain dynamics” we understand the application between structural component (“space” variables) and functional component (“time” variables). Due to the fact that the dynamics reflects different levels of application, the time gets in through scale resolution and is denoted by $\delta t$. We reserve the notation $dt$ for the usual time as in the hamiltonian dynamics; $\delta t$ will be defined through a special substitution principle.

Then the following consequences of the brain dynamics emerge, which will be explained in detail through the present paper:

i) Any continuous but non-differentiable curve of the brain dynamics (brain non-differentiable curve) is explicitly scale resolution dependent $\delta t$, i.e., its length tends to infinity when $\delta t$ tends to zero;

We mention that, mathematically speaking, a curve is nondifferentiable if it satisfies the Lebesgue theorem, i.e. its length becomes infinite when the scale resolution goes to zero (Mandelbrot [24]). Consequently, in the limit, a curve is as zig-zagged as one can imagine. Thus it exhibits the property of self-similarity in every one of its points, which can be translated into a property of holography (every part reflects the whole). Doesn’t this happen in the brain? Of course. This is why, from the smallest scale to the greatest-sized ones, the brain is a whole! [2,3,30];

ii) The physics of the brain phenomena is related to the behaviour of a set of functions during the zoom operation of the scale resolution $\delta t$. Then,
through the substitution principle, $\delta t$ will be identified with $dt$, i.e., $\delta t \equiv dt$ and, consequently, it will be considered as an independent variable;

The fractal variables are dynamical variables, depending both on space coordinates and time, as well as on the resolution scale. Then, a difference should be made for instance between the variables describing the dynamics at the nanoscale (induced by neurofibrils), the dynamics at the synapse level, nerve impulse transmission at the dendrite level (for details see Kniffki et al [6]). It is the global structural-functional coherence of our brain, which, in our opinion, determines a permanent interdependence between these variables;

iii) The brain dynamics is described through fractal variables, i.e., functions depending both on the space-time coordinates and the scale resolution since the differential time reflection invariance of any dynamical variable is broken. Then, in any point of the non-differentiable curve, two derivatives of the variable field $Q(t, dt)$ can be defined:

\[
\begin{align*}
\frac{d_+ Q(t, dt)}{dt} &= \lim_{\Delta t \to 0^+} \frac{Q(t + \Delta t, \Delta t) - Q(t, \Delta t)}{\Delta t} \quad (1) \\
\frac{d_- Q(t, dt)}{dt} &= \lim_{\Delta t \to 0^-} \frac{Q(t, \Delta t) - Q(t - \Delta t, \Delta t)}{\Delta t}.
\end{align*}
\]

The sign + corresponds to the forward process, while the sign − corresponds to the backwards one;

ii) The differential of the spatial coordinate field $dX^i(t, dt)$ is expressed as the sum of the two differentials, one of them being not scale resolution independent (differential part $d_+ x^i(t)$) and the other one being scale resolution dependent (fractal part $d_\pm \xi^i(t)$), i.e.,

\[
d_\pm X^i(t, dt) = d_\pm x^i(t) + d_\pm \xi^i(t, dt);
\]

v) The non-differentiable part of the brain spatial coordinate field satisfies the fractal equation:

\[
d_\pm \xi^i(t, dt) = \lambda_\pm^i (dt)^{1/D_F},
\]

where $\lambda_\pm^i$ are constant coefficients through which the fractalisation type is specified and $D_F$ defines the fractal dimension of the brain non-differentiable curve. Let us note that any definition (Kolmogorov or Hausdorff-Besicovici fractal dimensions [21-24]) is acceptable for $D_F$, but once a certain definition is admitted, it should be used until the end of the analyzed brain dynamics. Moreover, it should be considered constant.

In our opinion, the functionality of the cerebral processes implies dynamics on geodesics having various fractal dimensions. Precisely, for $D_F = 2$, quantum type functionalities are generated (percolation in living neural networks, tunneling in neurofibrils etc.). For $D_F < 2$, correlative type functionalities are generated, while for $D_F > 2$, non-correlative type ones can be found (limited or unlimited diffusions-branching pattern of dendritic trees of retina neurons by a diffusion limited aggregation [4-6]).
vi) The differential time reflection invariance of any dynamical variable is recovered by combining the derivates \(d_+/dt\) and \(d_-/dt\) in the non-differentiable operator
\[
\hat{d} \equiv \frac{1}{2}(\frac{d_+ + d_-}{dt}) - i\frac{1}{2}(\frac{d_+ - d_-}{dt}).
\]

This is a natural result of the in complex through differentiability [31,32] procedure. Applying now the non-differentiable operator to the brain spatial coordinate field yields to the brain complex velocity field
\[
\hat{V}^i = \frac{dX^i}{dt} = V^i_D - V^i_F,
\]
with
\[
V^i_D = \frac{1}{2}(v^i_+ + v^i_-), \quad V^i_F = \frac{1}{2}(v^i_+ - v^i_-), \quad v^i_+ = \frac{d_+ x^i + d_+ \xi^i}{dt}, \quad v^i_- = \frac{d_- x^i + d_- \xi^i}{dt}.
\]

The real part \(V^i_D\) is differentiable and scale resolution independent (differentiable velocity field), while the imaginary one \(V^i_F\) is non-differentiable and scale resolution dependent (fractal velocity field);

vii) In the absence of any external constraint there can be found an infinite number of non-differentiable curves (brain geodesics) relating any pair of its points and this is true at all scales. Then, in the brain fractal space, the neuron is substituted with the brain geodesics themselves so that any external constraint (electroencephalogram, functional MRI etc.) is interpreted as a selection of geodesics by the measuring device. The infinity of brain geodesics in the bundle, their non-differentiability and the two values of the derivative imply a generalized statistical fluid like description (brain non-differentiable fluid). Then the average values of the brain fluid variables must be considered in the previously mentioned sense, so the average of \(d_\pm X^i\) is
\[
<d_\pm X^i> \equiv d_\pm x^i
\]
with
\[
<d_\pm \xi^i> = 0;
\]

viii) The brain dynamics can be described through a covariant derivative, whose explicit form is obtained as follows.

Let us now consider that the non-differentiable curves are immersed in a 3-dimensional space (the brain dynamics consciousness is 3-dimensional) and that \(X^i\) are the spatial coordinate field of a point on the non-differentiable curve. We also consider a variable field \(Q(X^i,t)\) and the following Taylor expansion up to the second order
\[
d_\pm Q(X^i,t) = \partial_i Q dt + \partial_t Q d_\pm X^i + \frac{1}{2} \partial_i \partial_k Q d_\pm X^i d_\pm X^k.
\]
These relations are valid in any point of the space and more for the points $X^i$ on the non-differentiable curve which we have selected in (9).

From here, forward and backward value of (9) become

$$< d_\pm Q > = < \partial_t Q dt > + < \partial_i Q d_\pm X^i > + \frac{1}{2} < \partial_t \partial_k Q d_\pm X^i d_\pm X^k > .$$  \hspace{1cm} (10)

We supose that the average value of variable field $Q$ and its derivatives coincide with themselves and the differentials $d_\pm X^i$ and $dt$ are independent. Therefore, the average of their products coincides with the product of averages. Consequently, (10) becomes

$$d_\pm Q = \partial_t Q dt + \partial_i Q < d_\pm X^i > + \frac{1}{2} \partial_t \partial_k Q < d_\pm X^i d_\pm X^k > .$$  \hspace{1cm} (11)

Even the average value of $d_\pm \xi^i$ is null, for the higher order of $d_\pm \xi^i$, the situation can still be different. Let us focus on the averages $< d_\pm \xi^i d_\pm \xi^l >$.

Using (3) we can write

$$< d_\pm \xi^i d_\pm \xi^l > = \pm \lambda_\pm^i \lambda_\pm^l (dt)^{(2/D_F)-1} dt,$$  \hspace{1cm} (12)

where we accepted that the sign $+$ corresponds to $dt > 0$ and the the sign $-$ corresponds to $dt < 0$.

Then (11) takes the form

$$d_\pm Q = \partial_t Q dt + \partial_i Q < d_\pm X^i > + \frac{1}{2} \partial_t \partial_k Q \lambda_\pm^i \lambda_\pm^l (dt)^{(2/D_F)-1} \partial_i \partial_l Q.$$

(13)

If we divide by $dt$ and neglect the terms that contain differential factors (for details see the method from [21-28]) we obtain:

$$\frac{d_\pm Q}{dt} = \partial_t Q + \nu^i \partial_i Q \pm \frac{1}{2} \lambda_\pm^i \lambda_\pm^l (dt)^{(2/D_F)-1} \partial_i \partial_l Q.$$  \hspace{1cm} (14)

These relations also allow us to define the operators

$$\frac{d_\pm}{dt} = \partial_t + \nu^i \partial_i \pm \frac{1}{2} \lambda_\pm^i \lambda_\pm^l (dt)^{(2/D_F)-1} \partial_i \partial_l.$$  \hspace{1cm} (15)

Under these circumstances, taking into account (4), (5) and (15) let us calculate $\hat{d}/dt$. It results

$$\frac{\hat{d}Q}{dt} = \partial_t Q + \hat{\nu}^i \partial_i Q + \frac{1}{4} (dt)^{(2/D_F)-1} D^{lk} \partial_l \partial_k Q,$$

(16)

where

$$D^{lk} = d^{lk} - i d^{lk}$$

$$d^{lk} = \lambda_+^l \lambda_+^k - \lambda_-^l \lambda_-^k, D^{lk} = \lambda_+^l \lambda_+^k + \lambda_-^l \lambda_-^k.$$  \hspace{1cm} (17)
The relation (16) also allows us to define the covariant derivative
\[
\frac{\hat{d}}{dt} = \partial_t + \hat{V}^i \partial_i + \frac{1}{4} (dt)^{(2/D_F)-1} D^{lk} \hat{\partial}_l \hat{\partial}_k.
\]  
(18)

Now, we shall refer to what we call brain’s geodesic.

Let us consider the principle of scale covariance (the physics laws are invariant we respect to scale transformations) and postulate that the passage from the differentiable mathematical model to the non-differentiable mathematical model can be implemented by replacing the standard time derivative \(d/dt\) by the non-differentiable operator \(\hat{d}/dt\). Thus, this operator plays the role of the covariant derivative, namely, it is used to write the fundamental equations of brain dynamics under the same form as in the classical (differentiable) case. In these conditions, applying the operator (18) to the complex velocity field (5), the brain geodesics in the presence of an external constraint given by the scalar potential \(U\), have the following form:
\[
\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \hat{\partial}_l \hat{V}^i + \frac{1}{4} (dt)^{(2/D_F)-1} D^{lk} \hat{\partial}_l \hat{\partial}_k \hat{V}^i = -\hat{\partial}^i U.
\]  
(19)

This means that the local acceleration \(\partial_t \hat{V}^i\), the convection \(\hat{V}^l \hat{\partial}_l \hat{V}^i\), the dissipation \(D^{lk} \hat{\partial}_l \hat{\partial}_k \hat{V}^i\) and the forces induced by the external constraints \(\hat{\partial}^i U\) make their balance in any point of the brain non-differentiable curve. Moreover, the presence of the complex coefficient of viscosity type \(\frac{1}{4} (dt)^{(2/D_F)-1} D^{lk}\) specifies that the neuronal medium is a rheological medium, so it has memory, as a datum, by his own structure.

If the fractalisation is achieved by Markov type stochastic processes \([21,22,24]\), then
\[
\lambda_+^l \lambda_+^l = \lambda_-^l \lambda_-^l = 2\lambda \delta^{il},
\]  
(20)

where \(\delta^{il}\) is Kronecker’s symbol with the property
\[
\delta^{il} = \begin{cases} 
1, & i = l \\
0, & i \neq l.
\end{cases}
\]

In these conditions, the equation of brain geodesics takes the simple form
\[
\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \hat{\partial}_l \hat{V}^i - i\lambda (dt)^{(2/D_F)-1} \hat{\partial}^l \hat{\partial}_l \hat{V}^i = -\hat{\partial}^i U
\]  
(21)
or more, by separating the motions on differential and fractal scale resolutions,
\[
\frac{\hat{d}V^i_D}{dt} = \partial_t V^i_D + V^l_D \hat{\partial}_l V^i_D - [V^l_F - \lambda (dt)^{(2/D_F)-1} \hat{\partial}^l] \hat{\partial} V^i_F = -\partial^i U
\]  
(22)

\[
\frac{\hat{d}V^i_F}{dt} = \partial_t V^i_F + V^l_D \hat{\partial}_l V^i_F + [V^l_F - \lambda (dt)^{(2/D_F)-1} \hat{\partial}^l] \hat{\partial} V^i_D = 0.
\]

In what follows, we discuss the spectral functionality through the brain’s geodesics of Schrödinger type.
For irrotational motions
\[ \varepsilon_{ikl} \partial^k \hat{V}^l = 0, \] 
where \( \varepsilon_{ikl} \) is the Lévy-Civita pseudo-tensor. We choose \( \hat{V}^i \) in the form which makes this definition an identity
\[ \hat{V}^i = -2i\lambda (dt)^{(2/D_F)-1} \partial^i \ln \Psi, \] 
where for the moment \( \ln \Psi \) defines the scalar potential of the complex velocity field.

Substituting (24) in (21) we obtain
\[ \frac{d\hat{V}^i}{dt} = -2i\lambda (dt)^{(2/D_F)-1} \{ \partial_t \partial^i \ln \Psi - i[2\lambda (dt)^{(2/D_F)-1}(\partial^l \ln \Psi \partial^i) \cdot \partial^i \ln \Psi](25) + \lambda (dt)^{(2/D_F)-1} \partial^i \partial_t \partial^i \ln \Psi \} = -\partial^i U. \]

Using the identities
\[ \partial^t \partial_t \ln \Psi + \partial_i \ln \Psi \partial^i \ln \Psi = \frac{\partial_t \partial^i \Psi}{\Psi} \] 
\[ \partial^i \left( \frac{\partial^t \partial_t \Psi}{\Psi} \right) = 2(\partial^i \ln \Psi \partial_t) \partial^i \ln \Psi + \partial^t \partial_t \partial^i \ln \Psi \]
the equation (25) becomes
\[ \frac{d\hat{V}^i}{dt} = -2i\lambda (dt)^{(2/D_F)-1} \partial^i \{ \partial_t \ln \Psi - 2i\lambda (dt)^{(2/D_F)-1} \frac{\partial^t \partial^i \Psi}{\Psi} \} = -\partial^i U. \] 

This equation can be integrated up to an arbitrary phase factor, which may be set to zero by a suitable choice of phase of \( \Psi \) and this yields:
\[ \lambda^2 (dt)^{(4/D_F)-2} \partial^t \partial_t \Psi + i\lambda (dt)^{(2/D_F)-1} \partial_t \Psi - \frac{U}{2} \Psi = 0. \] 

The relation (28) is a Schrödinger type equation ("brain geodesics of Schrödinger type") and it implies the following:

i) According to [33], \( \Psi \) is a wave function and it has a direct physical signification only through \( |\Psi|^2 \) as probability density (probability density);

ii) The unpredictable character of the brain dynamics is specified through the wave properties of the neuronal medium (or, neuronal network). In this way, brain’s spectral type functionality is provided;

iii) The mechanisms that are responsible of brain’s spectral functionality are of quantum type only when they are extrapolated for various scale resolutions (such as tunneling, percolation, entanglement states - see Werner [7] etc.).

Now, we refer to the structural functionality through brain geodesics of hydrodynamic type.
If $\Psi = \sqrt{\rho} \exp(iS)$ with $\sqrt{\rho}$ the amplitude and $S$ the phase of $\Psi$, the complex velocity field (5) takes the explicit form

$$\hat{V}^i = 2i\lambda(dt)^{(2/D_F)-1}\partial^i \ln \Psi$$
$$V_D^i = 2i\lambda(dt)^{(2/D_F)-1}\partial^i S$$
$$V_F^i = 2i\lambda(dt)^{(2/D_F)-1}\partial^i \ln \rho. \quad (29)$$

Substituting (29) into (19) and separating the real and imaginary parts, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of $\Psi$, we obtain:

$$\partial_t V_D^i + (V_D^j \partial_l) V_D^l = -\partial^i(Q + U) \quad (30)$$
$$\partial_t \rho + \partial^i (\rho V_D^i) = 0 \quad (31)$$

with $Q$ the specific non-differentiable potential

$$Q = -2\lambda(dt)^{(4/D_F)-2}\frac{\partial^i \partial_j \sqrt{\rho}}{\sqrt{\rho}} = -\frac{V_D^i V_D^l}{2} - \lambda(dt)^{(2/D_F)-1}\partial_i V_F^i. \quad (32)$$

Equation (30) represents the specific momentum conservation law, while equation (31) represents the states density conservation law. Equations (30) and (31) define the fractal hydrodynamic model and imply the following:

i) Any neuronal cell is in a permanent interaction with a fractal medium, identified with the neuronal network through the specific non-differentiable potential (32). The physics fractal medium is prone to computational properties [34];

ii) The neuronal network can be identified with a fractal fluid (non-differentiable fluid), whose dynamics is described by the fractal hydrodynamical model;

iii) The fractal velocity field $V_F^i$ does not represent actual motion, but contributes to the transfer of the specific momentum and to the brain energy focus, thus conferring spectral representability to brain functioning through what we call neuronal network. This may be seen clearly from the absence of $V_F^i$ from the states density conservation law and from its role in the variational principle [21,22];

iv) Any interpretation of $Q$ should take cognizance of the self nature of the specific momentum transfer. While the brain energy is stored in the form of the mass motion and potential energy, some is available elsewhere and only the total is conserved. It is the conservation of the energy and the specific momentum that ensures reversibility and the existence of eigenstates, but denies a Lévy type motion [35] of brain interaction with an external medium;

v) The predictable character of the brain activity is specified through the corpuscular properties of the neuronal network. Thus, the brain’s corpuscular type functionality is provided;
vi) The mechanisms that are responsible of brain’s structural functionality are of a hydrodynamical type, but when they are extrapolated for various scale resolutions (shock waves, solitons [36] etc.):

vii) The specific non-differentiable potential coordinates the transitory functionality (the spectral-structural functionality).

Next, we shall present several considerations about the generation of the communication languages.

Both functionalities (either the one which is responsible of the brain activity unpredictable character, or the other one which is responsible of the brain activity predictable character) act simultaneously. By their interconditioning there result either brain coherence or brain incoherence. Indeed, let us admit that both in the brain representation of Schrödinger type and in the brain representation of hydrodynamical fractal type, the external constraint is proportional with the states density, i.e., 

$$U = 2a|\Psi|^2 = 2a\rho,$$

with $a = \text{const.}$ Then for the statonary case ($\partial_t \Psi = 0$ and $\partial_t \rho = 0$, $V_D = \partial^4 S = 0$) the equation (28) becomes:

$$\lambda^2 (dt)^{(4/D_F)-2} \partial^4 \partial_t \Psi + E_\Psi \Psi - a|\Psi|^2 \Psi = 0,$$

(33)

while the equations (30) – (32) get by integration the form:

$$Q + U = -2\lambda^2 (dt)^{(4/D_F)-2} \frac{\partial^4 \sqrt{\rho}}{\sqrt{\rho}} + 2a\rho = 2E_\rho$$

(34)

with $E_\Psi$ and $E_\rho$ constants having specific energies significances.

By the substitutions:

$$\frac{(E_\Psi)^{1/2}}{\lambda k_\Psi}(dt)^{1-(2/D_F)}(k_\Psi x + k_\Psi y + k_\Psi z) = \xi_\Psi$$

$$\frac{(E_\rho)^{1/2}}{\lambda k_\rho}(dt)^{1-(2/D_F)}(k_\rho x + k_\rho y + k_\rho z) = \xi_\rho$$

$$\Psi = \left(\frac{E_\Psi}{a}\right)^{1/2} f, \sqrt{\rho} = \left(\frac{E_\rho}{a}\right)^{1/2} h$$

(35)

(33) and (34) reduce to the equations of Ginzburg-Landau type [36]:

$$\frac{d^2 f}{d\xi_\Psi^2} = f^3 - f$$

(36)

for spectral functionality, respectively

$$\frac{d^2 h}{d\xi_\rho^2} = h^3 - h,$$

(37)

for structural functionality, where $k_\Psi$ and $k_\rho$ are the brain wave vectors.
Using the methodology from [25], these previous equations admit either the infinite energy solutions

\[ |\Psi|^2 = \frac{2s^2_\Psi}{1 + s^2_\Psi} sn^2 \left( \frac{\xi_\Psi - \xi_{\Psi_0}}{\sqrt{2}} ; s_\Psi \right) \] (38)

\[ \rho = \frac{2s^2_\rho}{1 + s^2_\rho} sn^2 \left( \frac{\xi_\rho - \xi_{\rho_0}}{\sqrt{2}} ; s_\rho \right) \] (39)

where \( sn \) are Jacobi’s elliptic functions of modules \( s_\Psi \) and \( s_\rho \) [37]

\[ s_\Psi = \frac{1 - (1 - 2c_\Psi)^{1/2}}{1 + (1 - 2c_\Psi)^{1/2}} \quad s_\rho = \frac{1 - (1 - 2c_\rho)^{1/2}}{1 + (1 - 2c_\rho)^{1/2}} \] (40)

with \( \xi_{\Psi_0}, \xi_{\rho_0}, c_\Psi, c_\rho \) integration constants, or the finite energy solutions (kink solutions [38])

\[ |\Psi|^2 = \tanh \left( \frac{\xi_\Psi - \xi_{\Psi_0}}{\sqrt{2}} \right) \]

\[ \rho = \tanh \left( \frac{\xi_\rho - \xi_{\rho_0}}{\sqrt{2}} \right) \]

obtained through the degeneration of the elliptic functions \( sn \) in the modules \( s_\Psi \) and \( s_\rho \), i.e.,

\[ s_\Psi \rightarrow 1 \quad \text{for} \quad c_\Psi \rightarrow 1/2 \] (41)

\[ s_\rho \rightarrow 1 \quad \text{for} \quad c_\rho \rightarrow 1/2. \]

Now, some conclusions are obvious:

i) Both the probability density fluctuations

\[ \delta |\Psi|^2 = 1 - \frac{1 + s^2_\Psi}{2s^2_\Psi} |\Psi|^2 = cn^2 \left( \frac{\xi_\Psi - \xi_{\Psi_0}}{\sqrt{2}} ; s_\Psi \right) \] (42)

and the states density ones

\[ \delta \rho = 1 - \frac{1 + s^2_\rho}{2s^2_\rho} \rho = cn^2 \left( \frac{\xi_\rho - \xi_{\rho_0}}{\sqrt{2}} ; s_\rho \right) \] (43)

obtained based on the infinite energy solutions express oneself through the cnoidal oscillations modes of the spatial coordinates fields \( \xi_\Psi \) and \( \xi_\rho \), where \( cn \) are Jacobi’s elliptic functions of modules \( s_\Psi \) and \( s_\rho \) [37];

ii) Generally speaking, as it also results from [36], the cnoidal oscillation modes are equivalent to one-dimensional Toda type lattices of nonlinear oscillators [39,40]. Moreover, according to [14-16], their mapping implies Toda type neuronal networks. That is why, based on the above results, we shall be able to define two Toda type neuronal networks, one of them being specific to the spectral functionality (and which will be called the spectral neuronal network)
and the other one being specific to the structural functionality (and which will be called the structural neuronal network);

iii) Since both functionalities, the spectral one and the structural one, define the same physical object, these imply the identities:

\[ \xi_\Psi \equiv \xi_\rho, \quad E_\Psi \equiv E_\rho, \quad k_\Psi \equiv k_\rho, \quad |\Psi|^2 \equiv \rho. \quad (44) \]

Consequently, the probability density fluctuations are identified with the states density fluctuations, which specifies the brain coherence (or brain compatibility) of the two neuronal networks (the spectral one and the structural one). Such a situation implies mathematically the functionality of the elliptic functions equivalence theorem [37] and, in consequence, it implies the existence of certain algebraic relations among the variables which define brain dynamics on the two neuronal networks, particularly,

\[ \delta |\Psi|^2 = F(\delta \rho). \quad (45) \]

By the algebraic relations, self-structuring, communication codes (languages) of the physical object result;

iv) According to [27,28], the finite energy solutions (see (41) and (42)) can be also obtained by field theories with spontaneous symmetry breaking, which also implies strange topologies [28,41]. In any of these topologies one can always define through the associated topological charges, two distinct stable physical states. Now, according to our results, we shall have on one hand a topology specific to the spectral functionality (called spectral topology) which will define the spectral states through the spectral topological charge and, on the other hand, the topology specific to the structural functionality (called structural topology) which will define the structural states through the structural topological charge. Moreover, due to their physical object status, the two topologies, the spectral one and the structural one act simultaneously, influencing one each other. Practically speaking, we have a unique topology which encompasses both of them in the form of their direct product [41]. This has as a consequence the existence of four distinct stable states. In our opinion, these states could be associated with the nucleotide base from the human DNA structure [42].

3 Discussion

In this section, we talk about possible implications of the mathematical model in the decipher of some neuropsychological mechanisms.

The implementattion of the functional structure of complex systems to psychic life, can explain a series of classical concepts circulated during te last century. Thus the unconscious from psychoanalysis can be associated with the unpredictable, noncausal and potential part from the structure of the complex system, while the conscience, as well as the unconscious behavioral patterns (superego of psychoanalysis) can be associated with the structured, causal and
deterministic part of the complex system. Between the two parts, there exist a permanent dynamics through attractors, describable in the phase space. The chaoticity existing between the two components is absolutely necessary for the functioning of the brain. When it is affected by repetitive cycles (epilepsy crisis) the conscience is blurred.

This new representation on brain’s functioning leads to new conclusions concerning different mental processes that have not been fully understood yet:

i) Consciousness could be the dynamics result between the two networks: the spectral neuronal network and the structural one. For instance, the anesthetic techniques block the structural network. When this structural network becomes again functional, it recovers its dynamics through the multifocal coherence phenomenon with the spectral network (where the memory, the core personality can be found, as detailed in the following). The same thing happens in epileptic crisis, in concussions, electroshocks etc., the structural network being unable to achieve coherent dynamics with the spectral network;

ii) In the structure of the brain (as a physical object), the memory may be located in the spectral neuronal network, whose spectral, and thus fractal character has all the properties that are necessary for the information storage. The memory means coherence achievement among certain structures of the structural neuronal network and the spectral one, where those information have been memorized;

iii) Memory localization could give clues on how personality is structured. Classically, the personality has two components: temperament and character. The temperament is constituted of behavioral and information processing patterns originating from the genetic setting (and which are organized in the structural network). The second component, the character, represents the programs built from the individual relationship with the external medium (education, experiences, cultural environment, analyzers’ setting etc.). It is organized in the spectral network, representing information, behavioral and information processing patterns, that are set in programs resulting from the system and the external medium dynamics. The genetic patterns found in the structural neuronal network give stability and the programs built in the spectral neuronal network are adapted to the environment, in a specific form given by the dynamics with the genetic patterns from the structural network. In this way, personality has stability via some of its components, but it also has specificity and adaptability;

iv) It seems that the potentiality Chomski [43] was talking about, related to every child’s ability to learn the language or the languages to which he is exposed, is related to the spectral neuronal network which gives the memory space, while the structural network represented by Wernicke and Broca center (of speech understanding and speech expression, built by patterns transmitted at the genetic information level) offers the language processing structure;

v) In the context of new discoveries about mirror neurons, our model concerning psyche’s functionality and structuring could give explanations about
mirror neurons’ functioning and their integration in the psychological functioning in general. So far, experimental data emphasizes only the elements from the structural neuronal network (excited neurons, highlighted by electrodes implanted or brain areas highlighted by fMRI). Accepting the spectral network could explain complex phenomena, concepts, feelings that could not be generated only by the activity of several neurons, but by complex processing that could take place only in the spectral neuronal network. It might even be possible that the neurons excitation is achieved through the spectral network, where the information originates through interpersonal communications spectral vibrational ways. It could be thus explained a series of controversies about mimetic learning, empathy, mind theory, language etc.;

vi) In neuropathology, our model could also generate new conclusions concerning both mental and neuropsychological illness. For instance, in vascular dementia, the blood deficiency affects on one hand the neurons (the structural neuronal network) and on the other hand, it influences the dynamics between the two networks, while in Alzheimer dementia, the dynamics between the two networks is primarily affected, with the impossibility to access the information stored, with the damage of the spatial-temporal orientation, but also of the behavior and even of the entire personality (see the considerations from ii)). Certain somatic trauma cases when the phantom limb sensation manifests itself (Ramachandran [44]) could have an explanation by the model we conceived; the structural network can be inhibited or destroyed by the respective limb or organ, its representation remaining in the spectral neuronal network, generating the painful and contracted phantom limb symptoms and allowing the alleviating and curing through a suggestion and autosuggestion mechanism (the mirror box technique);

vii) The neuroplasticity phenomenon related to brain’s adaptive capacity would be more understandable if, causally, according to our model, the neurons and the neuronal connections development would achieve by the dynamics between the two networks, based on the patterns developed in the spectral side from the reaction with the environment.

4 Conclusion

With our approach we finally aim at the solution of the old problem of the duality brain-mind. Our physical-mathematical model offers the vision that both the mind and the brain do form a functional, describable through the dynamics existing between the spectral and neural networks lying at the foundation of the psychic life.

We made obviously that the brain dynamics can be represented by a Bohmian mechanics (Bohm [30]) exactly the way the regular Schrödinger wave mechanics is represented. Only here there are now hidden parameters. Their place is explicitly filled by neuronal medium.

The main conclusions of the present paper are the following:
i) The morphological-functional assumption of the fractal brain (in the most general sense of this concept described by Mandelbrot [24]) induces a mathematical model by activating brain’s non-differentiable dynamics. Thus, one gives up either on the classical determinism, or on the quantum nondeterminism through the determinism-nondeterminism inference of the responsible mechanisms;

ii) Through the mathematical model, the scale covariance principle induces brain geodesics in the velocity representation, situation in which any neuron from the network is substituted with the brain geodesics themselves and, moreover, any external constraint (electroencephalogram, functional MRI etc.) is interpreted as a selection of the brain geodesics by the measuring device;

iii) Through the Schrödinger type representation of the brain geodesics, the spectral (wave) functionality of the brain dynamics is explicited, which implies specific mechanisms of tunneling type, enteglement states type etc., while through the hydrodynamical representation, the structural (corpuscular) functionality of the brain dynamics is explicited, which induces specific mechanisms of shock waves type, solitons type etc.;

iv) The inference of these two representations sets forth the transitory (spectral-structural) functionality, which is controlled by the specific non-differentiable potential;

v) If the external constraint is proportional with the states density, then the stationary dynamics through their fluctuations activate both the spectral neuronal network and the structural neuronal network. These classes of neuronal networks result by the mapping of certain classes of one-dimensional Toda type networks. Usually, the one-dimensional Toda type networks are associated with certain cnoidal oscillation modes that, in our situation, can be specific either to the spectral character, or to the corpuscular one;

vi) The structural-functional compatibility (coherence) of these two classes generates classes of algebraic type communication codes;

vii) The spectral-structural compatibility of the neuronal networks only on the solitonic component simultaneously activates the functionality of a strange topology (the direct product of the spectral topology and the structural topology), which induces four distinct physical states through the associated topological charges. Thus, the quadruple logic elements are generated. In our opinion, such logic could be associated with the nucleotide base from the human DNA structure;

viii) Possible implications of the mathematical model in the elucidation of some neuropsychological mechanisms (for instance, memory’s location and functioning, brain trauma consequences, dementia etc.) are presented.

Author Contributions
In this paper, Maricel Agop provided the original idea and constructed its framework. Together with Alina Gavriliţ, the detailed calculation was conducted, and they were responsible for drafting and revising the whole paper. The implications and correspondences in neuroscience activity of this model were given by Gabriel Crumpei. The role in communication codes was highlighted by Mitică Craus and Vlad Bîrlescu. All authors have read and approved the final manuscript.

Acknowledgments. The authors are indebted to Nicolae Mazilu for valuable discussions and remarks.

References

[1] E. Kandel, Principles of Neural Science, Fifth Edition, the McGraw-Hill Companies, 2013.

[2] H. Atmanspacher, Quantum approaches to consciousness, The Stanford Encyclopaedia of Philosophy, Spring 2011 Edition.

[3] H. Atmanspacher, W. Fach, A structural-phenomenological typology of mind-matter correlation, Journal of Analytical Psychology 2013, 58, 219-244.

[4] F. Caserta, W.D. Eldred, E. Fernandez, R.E. Hausman, L.R. Stanford, S.V. Bulderev, S. Schwarzer, H.E. Stanley, Determination of fractal dimension of physiologically characterized neurons in two and three dimensions, J. Neurosci. Methods 1995, 56, 133–144.

[5] T.A. Witten Jr., L.M. Sander, Diffusion-limited aggregation, a kinetic critical phenomenon, Phys. Rev. Lett. 1981, 47, 1400-1403.

[6] K.D. Kniffki, M. Pawlak, C. Vahle-Hinz, Scaling behavior of the dendritic branches of thalamic neurons, Fractals, 1993, 1, 171-178.

[7] Werner, G., Perspectives on the Neuroscience of Cognitions and Consciousness, BioSystems, 2007, 87, 82-95.

[8] Werner, G., Consciousness related neural events viewed as brain state space transition, Cogn. Neurodyn. 2009, 3, 83-95.

[9] R.L. de Valois, K.K. de Valois, Spatial vision, Oxford Psychology series no. 14, New York, Oxford University Press, 1988.

[10] R.L. de Valois, K.K. de Valois, A multi-stage color model, Vision Res. 1993, 33, 1053-1065.

[11] von Békési, G., Problems relating psychological and electrophysiological observations in sensory perception, Perspectives in Biology and Medicine 1970, 11, 179-194.
[12] S.B. Lowen, M.C. Teich, Fractal auditory nerve firing patterns may derive from fractal switching in sensory hair cell ion channels, AIP Conf. Proceedings 285, 1993, eds Handel P. H., Chung A. L., editors. American Institute of Physics, 745-748.

[13] S.B. Lowen, M.C. Teich, Fractal based Point Processes, 2005, New York: Wiley.

[14] Slavova, A. Cellular Neural Networks: Dynamics and Modeling. Mathematical Modeling: Theory and Applications, Vol. 16, Springer, 2003.

[15] Karayiannis, N.B.; Venetsanopoulos, A.N. Artificial Neural Networks, Learning Algorithms, Performance Evaluation, and Applications. The Springer International Series in Engineering and Computer Science, Vol. 209, 1993.

[16] Chow, T.W.S. Neural Networks and Computing. Learning Algorithms and Applications (Series in Electrical and Computer Engineering), 2007.

[17] Díaz M.H.; Córdova, F.M.; Cañete, L.; Palominos, F., Cifuentes, F.; Sánchez, C.; Herrera, M. Order and chaos in the brain: fractal time series analysis of the EEG activity during a cognitive problem solving task. Information Technology and Quantitative Management (ITQM 2015), Procedia Computer Science 2015, 55, 1410-1419.

[18] Diaz, H.; Córdova, F. Harmonic Fractals in the Brain: Transient Tuning and Synchronic Coordination in the Quasi-Chaotic Background of Ongoing Neural EEG Activity. Procedia Computer Science, 2013, 17, 403-411.

[19] A. Khodabakhsh, A. Mehran, R. Majid, A.M. Esfandiar, S. Firoozeh, Brain activity of women is more fractal than men, Neuroscience Letters, 2013, 535, 7-11.

[20] A.J. Ibáñez-Molina, S. Iglesias-Parro, Fractal characterization of internally and externally generated conscious experiences, Brain and Cognition, 2014, 87, 69-75.

[21] Nottale, L. Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity. World Scientific: Singapore, Singapore, 1993.

[22] Nottale, L. Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics. Imperial College Press: London, UK, 2011.

[23] Nottale, L. Scale Relativity: A Fractal Matrix for Organization in Nature. Electron. J. Theor. Phys. 2007, 4, 187-274.

[24] Mandelbrot, B. The Fractal Geometry of Nature. W. H. Freeman and Company: New York, NY, USA, 1983.
[25] Timofte, A.; Casian-Botez, I.; Scurtu, D.; Agop, M. System Dynamics Control through the Fractal Potential. *Acta Phys. Pol. A* **2011**, *119*, 304-311.

[26] Agop, M.; Forna, N.; Casian-Botez, I. New Theoretical Approach of the Physical Processes in Nanostructures. *J. Comput. Theor. Nanosci.* **2008**, *5*, 483-489.

[27] Agop, M.; Gavriluț, A.; Crumpei, G.; Doroftei, B. Informational Non-differentiable Entropy and Uncertainty Relations in Complex Systems. *Entropy* **2015**, *16*, 6042-6058.

[28] Agop, M.; Gavriluț, A.; Ștefan, G.; Doroftei, B. Implications of Non-Differentiable Entropy on a Space-Time Manifold. *Entropy* **2015**, *17*, 2184-2197.

[29] Caserta, F., Stanley, H.E., Eldred, W.D., Daccord, G., Hausman, R.E., Nittman, J., Physical Mechanisms Underlying Neurite Outgrowth: A quantitative Analysis of Neuronal Shape, *Physical Review Letters*, **1990**, *64*, 95-98.

[30] Bohm, D., A suggested interpretation of the quantum theory in terms of "hidden" variables, *Phys. Rev.* **1952**, *85*, 166-179.

[31] Cresson J. Scale relativity theory for one dimensional non differentiable manifolds. *Chaos, Solitons and Fractals*, **2002**, *14*, 553-562.

[32] Cresson, J. Scale Calculus and the Schrödinger Equation. *J. Math. Phys.* **2003**, *44*, 4907-4938.

[33] Phillips, A.C. Introduction to Quantum Mechanics; *Wiley: New York*, NY, USA, 2003.

[34] Birlescu, V.S.; Agop, M.; Craus, M. Computational properties of a fractal medium. *Int. J. Quantum Inform.* 12, **2014**, [22 pages] DOI: 10.1142/S0219749914500221.

[35] P. Lévy, Theories de l’Addition Aléatoires, Paris, *Gauthier-Villars*, 1937.

[36] Jackson, E.A. Perspectives on Nonlinear Dynamics. *Cambridge University Press*, Cambridge, 1992.

[37] E.W.F. Armitage J.V. Elliptic functions. *Cambridge University Press*, Cambridge, 2006.

[38] Chaichian, M.; Nelipa, N.F. Introduction to Gauge Field Theories. *Springer-Verlag Berlin Heidelberg*, 1984.

[39] Toda, M. Theory of Nonlinear Lattices. *New York: SpringerVerlag*, 1981.
[40] Toda, M. Nonlinear lattice and soliton theory, *IEEE Trans. CAS*, 1983; 30: 542-554.

[41] Willard, S. (1970). General Topology. *Reading, Mass.: Addison-Wesley Pub. Co.* ISBN 0486434796. Retrieved 2013.

[42] Brown, T.A. Genomes, 2nd edition. *Oxford: Wiley-Liss*, 2002, ISBN-10: 0-471-25046-5.

[43] Chomski, N. Language and the Study of Mind. *Tokyo: Sansyusya Publishing*, 1982.

[44] Ramachandran, V.S. Mirror neurons and imitation learning as the driving force behind the great leap forward in human evolution. *Edge Foundation web site*. Retrieved October 19, 2011.