CP Violation in $D^0 - \bar{D}^0$ Oscillations: 
General Considerations and Applications 
to the Littlest Higgs Model with T-Parity

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Abstract

The observed $D^0 - \bar{D}^0$ oscillations provide a new stage in our search for New Physics in heavy flavour dynamics. The theoretical verdict on the observed values of $x_D$ and $y_D$ remains ambiguous: while they could be totally generated by Standard Model dynamics, they could also contain a sizable or even leading contribution from New Physics. Those oscillations are likely to enhance the observability of CP violation as clear manifestations of New Physics. We present general formulae for $D^0 - \bar{D}^0$ oscillations, concentrating on the case of negligible direct CP violation. In particular we derive a general formula for the time-dependent mixing-induced CP asymmetry in decays to a CP eigenstate and its correlation with the semileptonic CP asymmetry $\mathcal{A}_{SL}(D^0)$ in $D^0(t) \rightarrow \ell \nu K$. We apply our formalism to the Littlest Higgs model with T-parity, using the time-dependent CP asymmetry in $D \rightarrow K_S \phi$ as an example. We find observable effects at a level well beyond anything possible with CKM dynamics. Comparisons with CP violation in the $K$ and $B$ systems offer an excellent test of this scenario and reveal the specific pattern of flavour and CP violation in the $D^0 - \bar{D}^0$ system predicted by this model. We discuss a number of charm decays that could potentially offer an insight in the dynamics of CP violation in $D$ decays. We also apply our formalism to $B_s - \bar{B}_s$ mixing.
1 Introduction

To obtain a “natural” solution to the Standard Model’s (SM) gauge hierarchy problem it has been conjectured that dynamics beyond it has to enter around the TeV scale. This problem has been further deepened by the fact that the SM has passed the test provided by electroweak parameters even on the level of quantum corrections. Little Higgs Models [1, 2] represent an intriguing response to this challenge. Rather than solving the gauge hierarchy problem they
‘delay the day of reckoning’ to a higher scale. They provide scenarios where New Physics (NP) quanta can be produced at the LHC without creating conflicts with electroweak constraints; at the same time they introduce many fewer additional parameters than SUSY or Extra Dimension scenarios.

In order to avoid stringent electroweak precision constraints, one subclass of those models introduces a discrete symmetry called T-parity [3, 4], under which the new particles are odd and can therefore contribute only at the loop level. As a consequence a set of six T-odd ‘mirror’ quarks needs to be introduced that are organised into three families [5]. While constraints from flavour dynamics are not part of their motivation, this latter class of models is not of the Minimal Flavour Violation (MFV) [6–10] variety. They allow to construct connections between findings in high $p_t$ and flavour dynamics that have the potential to be of practical use due to the relative paucity of their new parameters: ten in the quark flavour sector, among them three CP-violating phases.

If one had observed $D^0 - ar{D}^0$ oscillations with $x_D > 1\% \gg y_D$, one would have had a strong prima facie case for NP enhancing $\Delta M_D$. Such a scenario has probably been ruled out now. The theoretical interpretation of the recent seminal discovery of $D^0 - \bar{D}^0$ oscillations with $x_D \sim y_D \sim (0.5 - 1)\%$ [11–14] remains ambiguous [15–18]: the observed size of $\Delta M_D$ and $\Delta \Gamma_D$ might completely be due to SM dynamics — or $\Delta M_D$ could still contain sizable or even leading NP contributions. A breakthrough in our theoretical control over these quantities is required for resolving this issue on theoretical grounds. Barring that there are two possible interpretations of the present situation: (i) It is beyond our computational abilities to evaluate $\Delta M_D$ and $\Delta \Gamma_D$ with sufficient accuracy. (ii) It represents one example of nature being mischievous with us: $\Delta \Gamma_D$ is anomalously enhanced due to a violation of local quark-hadron duality; $\Delta M_D$ on the other hand is enhanced over the value expected in the SM due to the intervention of NP. There is no way that interpretation (ii) could be validated by theoretical arguments; yet we argue there is a straightforward course of action as outlined below.

A priori LHT models could have generated considerably larger values of $\Delta M_D$ than observed; yet in that case the accompanying weak phase in $\mathcal{L}(\Delta C = 2)$ had to be quite small due to constraints from $K_L \to \pi^+\pi^-$ decays [19]. A forteriori they could generate the observed value or a significant fraction of it. The new feature now is that the $K_L$ constraints are diluted to a degree that large phases can emerge in $\mathcal{L}(\Delta C = 2)$. Their most striking experimental signature would be the observation of time dependent CP asymmetries already for Cabibbo allowed final states like $D^0 \to \phi K_S$ in qualitative — albeit not quantitative — analogy to $B_d \to \psi K_S$.

The remainder of this paper will be organised as follows. In Sect. 2 we briefly recapitulate the basic features of Little Higgs models and describe the relevant ingredients of the Littlest Higgs model with T-parity (LHT) needed for our analysis. Here we discuss in explicit terms the connection between $D$ and $K$ physics that is very transparent in the model in question. Then in Sect. 3, we review the theoretical framework of $D^0 - \bar{D}^0$ oscillations and discuss the possible LHT contributions, which we compare with the experimental evidence. Sect. 4 is dedicated to a model-independent discussion of the effect of indirect CP violation on $D$ decays, where we derive in particular a correlation between the semileptonic asymmetry $a_{SL}$ and the time-dependent asymmetry in $D \to K_S \phi$. In Sect. 5 we apply the formalism of Sect. 4 to the LHT
model and show the possible effects of LHT dynamics in CP violation in $D^0 - \bar{D}^0$ oscillations. In Sect. 6 we consider simultaneously the impact of LHT dynamics on CP violation in $D^0 - \bar{D}^0$ oscillations, rare CP violating $K_L$ decays and CP violation in $B_s - \bar{B}_s$ mixing, measured by the CP asymmetry $S_{\psi\phi}$. A brief summary and outlook is given in Sect. 7. Finally using the formalism of Sect. 4 Appendix A rederives the correlation between the CP asymmetry $S_{\psi\phi}$ and the semileptonic asymmetry $A_{SL}^s$ relevant for $B_s - \bar{B}_s$ mixing. Appendix B collects the input parameters used in our analysis.

2 Little Higgs Basics

2.1 Generalities

The Little Higgs class [1,2] comprises a large variety of New Physics models in which the Higgs boson appears as a pseudo-Goldstone boson of a spontaneously broken global symmetry. Gauge and Yukawa couplings break the global symmetry explicitly; however, Little Higgs models are constructed such that every single coupling preserves enough of the global symmetry to keep the Higgs boson massless. Only when more than one coupling is non-vanishing, the symmetry is broken completely and radiative corrections to the Higgs potential arise, being however at most logarithmically divergent at the one-loop level.

In order to achieve the cancellation of quadratically divergent contributions from the top quark, the electroweak gauge bosons and the Higgs itself, a common feature of all Little Higgs models is a set of new heavy weak gauge bosons, scalars and a top partner $T$ at the TeV scale. In spite of the large variety of existing models on the market, this common feature leads in many cases to similar phenomenological implications. Most phenomenological analyses therefore restrict their attention to the Littlest Higgs (LH) model [20]. This latter model, based on an $SU(5) \rightarrow SO(5)$ global symmetry breaking pattern at a scale $f \sim 1$ TeV, introduces in addition to the SM gauge and matter fields the heavy gauge bosons $W^\pm_H$, $Z_H$ and $A_H$, the heavy top partner $T$ and a scalar triplet $\Phi$. In the remainder of this paper, we will restrict ourselves to this economical realization of the Little Higgs concept. Reviews can be found in [21,22].

2.2 The Littlest Higgs with T-Parity

When studying electroweak precision observables, it turns out that an additional discrete symmetry, called T-parity [3,4], is needed in order to allow for the new particles below the 1 TeV scale. Under this symmetry, the SM particles and the heavy top partner $T_+$ are even, while $W_H^\pm$, $Z_H$, $A_H$ and $\Phi$ are odd. A consistent implementation of T-parity requires also the introduction of mirror fermions — one for each quark and lepton species — that are odd under T-parity [5]. In this manner the Littlest Higgs model with T-parity (LHT) is born.

While the Littlest Higgs model without T-parity belonged to the MFV class of models, implying generally small effects in flavour violating observables [23,24], the mirror fermions in the LHT model introduce new sources of flavour and CP violation [25,26]. Potentially large
deviations from the SM and MFV predictions in flavour changing neutral current processes can thus appear \cite{19,27–32}. A brief review of these analyses can be found in \cite{33}.

Flavour mixing in the mirror sector is conveniently described by two unitary mixing matrices $V_{Hu}$ and $V_{Hd}$, parameterising the mirror quark couplings to the SM up- and down-type quarks \cite{25,26}, respectively. With the $V_{Hd}$ matrix being parameterised in terms of three mixing angles and three complex phases as suggested in \cite{26}, the $V_{Hu}$ matrix, relevant for $D^0 - \bar{D}^0$ oscillations is given by

$$V_{Hu} = V_{Hd} V_{\text{CKM}}^\dagger.$$ \hspace{1cm} (2.1)

As the CKM mixing angles are experimentally found to be small and therefore $V_{\text{CKM}} \simeq 1$, we have $V_{Hu} \simeq V_{Hd}$. This relation will turn out to be important in what follows in order to understand the close connection of CP violation in the $K$ and $D$ meson systems. This issue is discussed in more detail in the next section (see also \cite{19,34}).

For an extensive description of the LHT model we refer the reader to \cite{28}, where also a complete set of Feynman rules has been derived. We note that an error in the coupling of the $Z$ boson to the mirror fermions has been pointed out in \cite{35}, see also \cite{32,36}. This has however no impact on the present analysis, as the coupling in question does not appear in the one-loop diagrams contributing to $\Delta F = 2$ processes.

### 2.3 Connection between $D$ and $K$ Physics

In \cite{34} the connection between $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing has been discussed within the framework of approximately $SU(2)_L$-invariant New Physics. Due to the connection between up- and down-type quarks in the SM through the CKM matrix, in this scenario the New Physics contributions to $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing are not independent of each other. This observation has been used in \cite{34} to derive lower bounds on the New Physics scale in various New Physics scenarios, emerging if the experimental constraints on $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing are applied to only the $(V - A) \otimes (V - A)$ contribution. One should keep in mind however that in models in which new operators contribute to $\Delta F = 2$ processes the power of this approach is limited, as the various contributions interplay with each other and dilute the correlation in question.

On the other hand the situation is promising in New Physics models with only SM operators, such as the LHT model. In fact this model provides possibly the best example of the physics discussed in \cite{34}. While the following discussion has been triggered by the analysis of $\Delta C = 2$ processes in the LHT model and uses the notations and conventions of \cite{19,25,28}, it applies as well to all other New Physics scenarios with only SM operators. Similar to the LHT model, the flavour mixing matrices $V_{Hu}$ and $V_{Hd}$ parameterise the misalignment between the New Physics and the SM up- and down-type quarks, respectively, that are related via (2.1). $D^0 - \bar{D}^0$ oscillations are then governed by the combinations $(i = 1, 2, 3)$

$$\xi_i^{(D)} = V_{Hu}^{iu} V_{Hd}^{* \dagger},$$ \hspace{1cm} (2.2)

while for $K$, $B_d$ and $B_s$ physics

$$\xi_i^{(K)} = V_{Hd}^{id} V_{Hd}^{* \dagger}, \hspace{0.5cm} \xi_i^{(d)} = V_{Hd}^{ib} V_{Hd}^{* \dagger}, \hspace{0.5cm} \xi_i^{(s)} = V_{Hd}^{ib} V_{Hd}^{* \dagger},$$ \hspace{1cm} (2.3)
respectively, are relevant. By making use of (2.1), we can now express $\xi^{(D)}_i$ through combinations of $V_{Hd}$ and CKM elements. Using the Wolfenstein parameterisation for $V_{CKM}$ and expanding in powers of $\lambda$, we find

$$
\xi^{(D)}_i = \xi^{(K)}_i + \lambda \left( |V_{Hd}^{is}|^2 - |V_{Hd}^{id}|^2 \right) + \lambda^2 \left( A \xi^{(d)}_i - 2 \text{Re}(\xi^{(K)}_i) \right) + \lambda^3 \left( \frac{1}{2} |V_{Hd}^{id}|^2 + A \xi^{(s)}_i \right) + O(\lambda^4). 
$$

The following comments are in order:

- At leading order $\xi^{(D)}_i = \xi^{(K)}_i$, i.e. $D$ and $K$ physics are governed by the same New Physics flavour structure. We note that the complex conjugation arises, as $|D^0\rangle = |\bar{uc}\rangle$ while $|K^0\rangle = |\bar{sd}\rangle$, and it will give rise to a sign difference in CP violating effects in the $D$ and $K$ systems.

- The correction to linear order in $\lambda$ is real, irrespective of the precise structure of $V_{Hd}$. However, as $\Delta C = 2$ CP violation is governed by

$$
\text{Im}(\xi^{(D)}_i) = 2 \text{Re}(\xi^{(D)}_i) \text{Im}(\xi^{(D)}_i),
$$

corrections to the one-to-one correspondence between $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixings will appear already at $O(\lambda)$ in both CP conserving and violating observables. On the other hand the $\Delta C = 1$ effective Hamiltonian is governed by a single power of $\xi^{(D)}_i$, so that direct CP violation in rare $D$ and $K$ decays will be much more strongly correlated and deviations from the one-to-one correspondence will arise only at $O(\lambda^2)$.

- The order $O(\lambda^2)$ correction can be complex, provided that $\text{Im}(\xi^{(d)}_i) \neq 0$, i.e. that there are new CP violating effects in the $B_d$ system.

- At $O(\lambda^3)$ a complex correction arises due to the CP violation in the CKM matrix, given by $i\eta$. This correction is non-vanishing also in the limit of a real, i.e. CP conserving $V_{Hd}$, and vanishes only if $\xi^{(s)}_i = 0$.

3 $D^0 - \bar{D}^0$ Oscillations

3.1 Theoretical Framework

The time evolution of neutral $D$ mesons is generally described by the Schrödinger equation

$$
\frac{i \partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \begin{pmatrix} M^{D}_{11} - \frac{i}{2} \Gamma^{D}_{11} & M^{D}_{12} - \frac{i}{2} \Gamma^{D}_{12} \\ M^{D*}_{12} & M^{D*}_{11} - \frac{i}{2} \Gamma^{D*}_{11} \end{pmatrix} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}.
$$

In the presence of flavour violation

$$
M^{D}_{12} \neq 0, \quad \Gamma^{D}_{12} \neq 0,
$$

5
and the mass eigenstates can be written as

\begin{align}
|D_1\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}} (p|D^0\rangle + q|\bar{D}^0\rangle), \\
|D_2\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}} (p|D^0\rangle - q|\bar{D}^0\rangle),
\end{align}

where

\[
\frac{q}{p} = \sqrt{\frac{M_{D_{12}}^2 - \frac{1}{2} \Gamma_{D_{12}}^2}{M_{D_{12}}^2 - \frac{1}{2} \Gamma_{D_{12}}^2}},
\]

and we choose the CP phase convention

\[CP|D^0\rangle = +|\bar{D}^0\rangle.\]

\[D^0 - \bar{D}^0\] oscillations can then be characterised by the normalised mass and width differences

\[
x_D \equiv \frac{\Delta M_D}{\Gamma}, \quad y_D \equiv \frac{\Delta \Gamma_D}{2\Gamma}, \quad \Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2),
\]

with

\[
\Delta M_D = M_1 - M_2 = 2 \text{Re} \left[ \frac{q}{p} \left( M_{D_{12}} - \frac{i}{2} \Gamma_{D_{12}} \right) \right],
\]

\[
\Delta \Gamma_D = \Gamma_1 - \Gamma_2 = -4 \text{Im} \left[ \frac{q}{p} \left( M_{D_{12}} - \frac{i}{2} \Gamma_{D_{12}} \right) \right],
\]

The attentive reader will note that our definitions in eqs. (3.8), (3.9) follow the PDG conventions [37], also adopted by the HFAG collaboration [14]; for neutral kaons they lead to \(\Delta M_K \cdot \Delta \Gamma_K < 0\). Note that if \(|\Gamma_{D_{12}}|^2 \ll |M_{D_{12}}|^2\), as appropriate for \(B^0\) mesons, one would recover the familiar expressions \(\Delta M \approx 2|M_{D_{12}}|, \Delta \Gamma \ll \Delta M\).

While \(\Delta M_D\) and \(\Delta \Gamma_D\) tell us nothing about CP symmetry, the ratio \(q/p\) and the relative phase between \(M_{D_{12}}^2\) and \(\Gamma_{D_{12}}\),

\[
\varphi_{12} = \frac{1}{2} \arg \left( \frac{M_{D_{12}}^2}{\Gamma_{D_{12}}} \right),
\]

expresses the CP impurity in the two mass eigenstates through \(|q/p| \neq 1\) and/or \(2\varphi_{12} \neq \{0, \pm \pi\}\). We note that while the phases of \(M_{D_{12}}^2\) and \(\Gamma_{D_{12}}^2\) depend on the phase conventions chosen, \(\varphi_{12}\) is phase convention independent and consequently an observable.

The world averages based on data from BaBar, Belle and CDF read [14]

\[
x_D = 0.0100^{+0.0024}_{-0.0026}, \quad y_D = 0.0076^{+0.0017}_{-0.0018}, \quad \frac{x_D^2 + y_D^2}{2} \leq (1.3 \pm 2.7) \cdot 10^{-4}
\]

\[
\left| \frac{q}{p} \right| = 0.86^{+0.17}_{-0.15}
\]
In the limit of (approximate) CP symmetry $x_D, y_D > 0$ implies the CP even state to be slightly heavier and shorter lived than the CP odd one (unlike for neutral kaons).

While there is close to universal consensus that $D^0 - \bar{D}^0$ oscillations have been observed — $(x_D, y_D) \neq (0, 0)$ — considerable uncertainty exists concerning the relative and absolute sizes of $x_D$ and $y_D$. In what follows we will use the experimental $1\sigma$ ranges for $x_D$ and $y_D$ from (3.11).

No sign of CP violation has been observed yet: the value of $|q/p|$ is fully consistent with unity, and a time integrated CP asymmetry in $D^0 \rightarrow K^+ K^-$ or $\pi^+ \pi^-$ is bounded by about $(0.5 - 1)\%$.

Yet the following should be kept in mind: (i) The experimental uncertainty on $|q/p|$ is still quite large. (ii) $D^0 - \bar{D}^0$ oscillation can induce a time integrated CP asymmetry $\simeq x_D \cdot \sin^2\varphi_f$ or $y_D \cdot \sin^2\varphi_f$ as described in more detail in Sect.4; with $x_D$ and $y_D$ bounded by about 0.01, such an asymmetry can hardly exceed 1%; i.e. we have just entered a regime where one can realistically hope for an effect to emerge.

A few technical remarks are in order to set the basics for our subsequent discussions. The phases of neither $M_{12}$ nor $\Gamma_{12}$ are observable per se, since they depend on the phase convention adopted for $\bar{D}^0$; their relative phase $\varphi_{12}$ however is independent of that convention and represents an observable. The CKM matrix provides a very convenient phase convention for $M_{12}^D$, which we will adopt. In the SM $M_{12}^D$ as well as $\Gamma_{12}^D$ are real to a very good approximation ($\sim O(10^{-3})$); however this still leaves their signs to be decided. While the authors of [18] argue that in the SM $(\Gamma_{12}^D)^{\text{SM}}$ likely carries a relative minus sign with respect to $(M_{12}^D)^{\text{SM}}$, the data on $x_D$ and $y_D$, assuming no NP contribution, imply

\[
(M_{12}^D)^{\text{SM}} \sim 0.012 \text{ps}^{-1}, \quad (\Gamma_{12}^D)^{\text{SM}} \sim 0.018 \text{ps}^{-1},
\]

i.e. a relative plus sign between dispersive and absorptive part of the off-diagonal mixing element. In what follows we will therefore not make any assumption on the signs of $(M_{12}^D)^{\text{SM}}$ and $(\Gamma_{12}^D)^{\text{SM}}$.

### 3.2 LHT Contributions

The leading LHT contribution to $\mathcal{L}(\Delta C = 2)$ is given by the standard $(V - A) \otimes (V - A)$ operator with its Wilson coefficient modified by the exchanges of the mirror quarks and heavy gauge bosons $W_H, Z_H$ and $A_H$ in the relevant box diagrams. While the heavy $T_+ \text{ quark cannot contribute directly to the box diagrams in question, its mixing with the standard up-type quarks can generate tree-level flavour changing } Z \text{ couplings in the up quark sector, leading to a non-vanishing contribution to } D^0 - \bar{D}^0 \text{ mixing } [38]. \text{ However, if the heavy } T_+ \text{ state is quasi-aligned with the SM top quark, as required in order not to spoil the Little Higgs mechanism of collective symmetry breaking, these tree level contributions are found to be smaller by several orders of magnitude than the SM short distance contributions and therefore fully negligible.}

Explicit expressions can be found in [19] and will not be repeated here. We only recall certain properties of these formulae that are relevant for our work:

- The LHT contribution depends on seven new real parameters and three complex phases that are constrained to some extent by the data on FCNC processes in $K$ and $B$ systems,
in particular by the observed CP violation in $K_L \to \pi\pi$ decays through the relations (2.1) and (2.4) and by electroweak precision tests. The hadronic uncertainties in this contribution originate in the matrix element of the relevant $\Delta C = 2$ operator between $D^0$ and $\bar{D}^0$ states. This matrix element is parameterised by the $D$ meson decay constant $F_D$ and the parameter $\hat{B}_D$. It should be emphasised that these two parameters are known from lattice calculations with much higher precision than the analogous quantities in $B_d$ and $B_s$ systems. Therefore in view of other uncertainties in our analysis, primarily related to long distance contributions discussed next, it is justified to set $\hat{B}_D$ and $F_D$ to their central values.

- The remaining two contributions are the SM box contribution and the genuine long distance contribution connected with low energy QCD dynamics. Whatever the nature and strengths of the SM contributions, they have nothing to do with the physics of the LHT model and are always present; we will denote the sum of these two contributions to the off-diagonal element of the $D^0 - \bar{D}^0$ mixing matrix simply by $(M_{12}^D)_{\text{SM}}$. It is real to an excellent approximation \cite{15,16}.

In summary, at present we have two experimental constraints of rather moderate rigour — $\Delta M_D$ and $\Delta \Gamma_D$ — and some order of magnitude estimates for the SM contributions to $M_{12}^D$ and $\Gamma_{12}^D$; furthermore we can count on the complex phases of the latter to be so small that they can be ignored at present. Lastly we have only constraints on the LHT parameters. For all these reasons we can provide only more or less typical scenarios, which we construct in the following way.

We find sets of LHT parameters consistent with the data outside charm dynamics and then compute $M_{12}^D$ from them. To this end we fix the New Physics scale to $f = 1$ TeV, implying masses of the heavy gauge bosons

$$M_{W_H, Z_H} = g f \sim 650 \text{ GeV}, \quad M_{A_H} = \frac{g' f}{\sqrt{5}} \sim 160 \text{ GeV}. \quad \text{(3.14)}$$

While the heavy gauge boson masses are fixed by the choice of $f$, the mirror quark masses $m_{i_H}$ depend on additional free Yukawa coupling parameters $\kappa_i$. Therefore we vary the mirror quark masses over the range

$$300 \text{ GeV} \leq m_{i_H} \leq 1000 \text{ GeV}. \quad \text{(3.15)}$$

Note that the masses of up and down mirror quarks in the same doublet are approximately equal. The parameter $x_L$, describing the mixing between the top quark and the heavy $T_+ \equiv T^0$ is fixed to $x_L = 0.5$ in our analysis. While it does not enter $D^0 - \bar{D}^0$ mixing directly, it is relevant for the constraints from $K^0 - \bar{K}^0$ and $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing.

Setting

$$M_{12}^D = (M_{12}^D)_{\text{SM}} + (M_{12}^D)_{\text{LHT}}, \quad \text{(3.16)}$$

$$\Gamma_{12}^D = (\Gamma_{12}^D)_{\text{SM}}, \quad \text{(3.17)}$$

we then ask what real values are required for $(M_{12}^D)_{\text{SM}}$ and $\Gamma_{12}^D = (\Gamma_{12}^D)_{\text{SM}}$ to reproduce a size for $\Delta M_D$ and $\Delta \Gamma_D$ that is compatible with the data.
Figure 1: \((\Gamma_{12}^{D})_{\text{SM}}\) as a function of \((M_{12}^{D})_{\text{SM}}\). The red (darker grey) and green (lighter grey) points correspond to the two solutions when solving (3.8), (3.9) for the poorly known SM contribution. In this and in all subsequent plots, the thick black points correspond to the SM case, i.e. the LHT contribution has been set to zero and the SM contribution alone reproduces the experimental (central) values of \(x_D\) and \(y_D\).

The result of this procedure is shown in Fig. 1, where we show \((\Gamma_{12}^{D})_{\text{SM}}\) as a function of \((M_{12}^{D})_{\text{SM}}\). As there are generally two solutions for the SM contribution, we determine both and show them as red (darker grey) and green (lighter grey) points, respectively, in this and all further figures. We observe that while for essentially all LHT parameter points the \((\Gamma_{12}^{D})_{\text{SM}}\) is consistent with theoretical estimates, for some points a very large negative \((M_{12}^{D})_{\text{SM}}\) is needed. We have verified explicitly that those points do not coincide with the most spectacular effects discussed below and in the \(K\) and \(B\) physics observables discussed in [27–29,32]. As an example we show in Fig. 2 \(|q/p|\) as a function of \((M_{12}^{D})_{\text{SM}}\), with the deviation of \(|q/p|\) from unity measuring the size of CP violating effects in \(D^0 - \bar{D}^0\) oscillations.

In Fig. 3 we show the real and imaginary part of \(M_{12}^{D}\), as defined in (3.16). Again the red (darker grey) and green (slightly lighter grey) points fulfil all existing \(K\) and \(B\) physics constraints and correspond to the two solutions for \(((M_{12}^{D})_{\text{SM}},(\Gamma_{12}^{D})_{\text{SM}})\), while for the light blue (grey) triangular points the constraint from \(\varepsilon_K\) has been omitted. We observe that even in the latter case, a strong correlation between \(\text{Re}(M_{12}^{D})\) and \(\text{Im}(M_{12}^{D})\) appears. This is due to the experimental constraints on \(x_D\) and \(y_D\) which enter by solving (3.7) for \(((M_{12}^{D})_{\text{SM}},(\Gamma_{12}^{D})_{\text{SM}})\). We note that very large values of \(\text{Im} M_{12}^{D}\) (Note the vastly different scales on the two axes!) generally have to be compensated by an unnaturally large \((\Gamma_{12}^{D})_{\text{SM}}\) in order to agree with the data. The additional constraint from \(\varepsilon_K \propto \text{Im} M_{12}^{K}\) results in the allowed red (darker grey) and green (slightly lighter grey) areas in the figure. We observe that points with very large \(\text{Im} M_{12}^{D}\) are now excluded, due to the correlation between \(K\) and \(D\) physics discussed analytically in Sect. 2.3. On the other hand we observe that almost the entire range of CP-violating phases is allowed, although phases close to \(\pm 90^\circ\), or
Figure 2: $|q/p|$ as a function of $(M_{12}^D)_{SM}$. The red (darker grey) and green (lighter grey) points correspond to the two solutions for $((M_{12}^D)_{SM}, (\Gamma_{12}^D)_{SM})$.

Figure 3: Im($M_{12}^D$) as a function of Re($M_{12}^D$). The red (darker grey) and green (slightly lighter grey) points fulfil all existing $K$ and $B$ physics constraints and correspond to the two solutions for $((M_{12}^D)_{SM}, (\Gamma_{12}^D)_{SM})$, while for the light blue (grey) triangular points the constraint from $\varepsilon_K$ has been omitted.
equivalently $\varphi_{12} = \pm 45^\circ$, appear to be unlikely. It should be emphasised that independently of what fraction of the observed $\Delta M_D$ is attributed to the SM contribution, a non-negligible phase $\varphi_{12}$ can only come from NP, in our case from LHT contributions. This makes it very clear that an observation of large mixing induced CP asymmetries in $D$ decays which are governed by the phase $\varphi_{12}$, would be a clear signal of NP.

Next in Fig. 4 we show the correlation between $\text{Im}(M_{12}^K)$ and $\text{Im}(M_{12}^D)$. We find a certain correlation between these two quantities, but as expected from the discussion in Section 2.3, this correlation is not a strict one due to the $\mathcal{O}(\lambda)$ corrections to $\xi_i^{(D)}$ relative to $\xi_i^{(K)}$. Consistent with our previous results, we see also from this figure that including the experimental constraint from $\varepsilon_K$ excludes very large values for $\text{Im}(M_{12}^D)$.

4 CP Asymmetries in $D$ Decays

From the first time they entered the stage of fundamental physics through the discovery of $K_L \rightarrow \pi^+\pi^-$, CP studies have demonstrated their power to reveal subtle dynamical features. We have good reason to expect that they will again reveal the intervention of New Physics. One should keep two facts in mind: (i) Baryogenesis requires dynamics beyond the SM CP violation. (ii) With the SM providing one amplitude, a CP asymmetry can be linear in a New Physics amplitude thus exhibiting a relatively high sensitivity to the latter.
4.1 General Formalism

The time evolution of initially pure $D^0$ and $\bar{D}^0$ states, respectively, can be obtained from solving (3.1) and is given by

$$|D^0(t)\rangle = f_+(t)|D^0\rangle - \frac{q}{p} f_-(t)|\bar{D}^0\rangle,$$

$$|\bar{D}^0(t)\rangle = -\frac{q}{p} f_-(t)|D^0\rangle + f_+(t)|\bar{D}^0\rangle,$$

where

$$f_+(t) = e^{-i\bar{M}t}e^{-\Gamma t/2}\cos Qt,$$

$$f_-(t) = ie^{-i\bar{M}t}e^{-\Gamma t/2}\sin Qt,$$

with $q/p$ given in (3.5), $\bar{M} = (M_1 + M_2)/2$ and

$$Q = \sqrt{(M_{12}^2 - \frac{i}{2} \Gamma_{12}) (M_{12}^* - \frac{i}{2} \Gamma_{12}^*)} = \frac{1}{2} (\Delta M_D - \frac{i}{2} \Delta \Gamma_D).$$

From (4.1), (4.2) we find for the time-dependent decay rates of $D^0(t)$, $\bar{D}^0(t)$ to a final state $f$:

$$\Gamma(D^0(t) \to f) = |T(D^0 \to f)|^2 e^{-\Gamma t} \left[ \frac{1}{2} (1 + |\lambda_f|^2) \cosh \frac{\Delta \Gamma_D t}{2} \right. \right.$$

$$\left. + \frac{1}{2} (1 - |\lambda_f|^2) \cos \Delta M_D t - \sinh \frac{\Delta \Gamma_D t}{2} \Re \lambda_f + \sin \Delta M_D t \Im \lambda_f \right],$$

$$\Gamma(\bar{D}^0(t) \to f) = |T(\bar{D}^0 \to f)|^2 e^{-\Gamma t} \left[ \frac{1}{2} \left( 1 + \left| \frac{1}{\lambda_f} \right|^2 \right) \cosh \frac{\Delta \Gamma_D t}{2} \right. \right.$$

$$\left. + \frac{1}{2} \left( 1 - \left| \frac{1}{\lambda_f} \right|^2 \right) \cos \Delta M_D t - \sinh \frac{\Delta \Gamma_D t}{2} \Re \frac{1}{\lambda_f} + \sin \Delta M_D t \Im \frac{1}{\lambda_f} \right],$$

where we dropped the overall phase space factors and defined

$$\lambda_f = \frac{q}{p} \frac{T(D^0 \to f)}{T(\bar{D}^0 \to f)}.$$

These general formulae agree with those of Dunietz and Rosner [39] after their definitions of $\Delta M$ and $\Delta \Gamma$ are adjusted to ours in (3.8) and (3.9).

From these results, one can easily obtain the CP asymmetries

$$\frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)}$$

where $f$ is a CP eigenstate

$$CP|f\rangle = \eta_f |f\rangle, \quad \eta_f = \pm 1.$$

We find

$$\frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)} = \frac{F(-)}{F(+)} e^{\frac{\Delta \Gamma_D t}{2}} + \cos \Delta M_D t,$$
where we have introduced the function
\[
F(\pm) = \frac{1}{2} \left( \frac{|q|}{p} \pm \frac{|p|}{q} \right)^2 \left( \cosh \frac{\Delta \Gamma_D t}{2} - \cos \Delta M_D t \right) \\
- \left[ \left( |\lambda_f| \pm \frac{1}{|\lambda_f|} \right) \cos 2\varphi_f \sinh \frac{\Delta \Gamma_D t}{2} - \left( |\lambda_f| \mp \frac{1}{|\lambda_f|} \right) \sin 2\varphi_f \sin \Delta M_D t \right],
\]
and
\[
\varphi_f = \frac{1}{2} \arg (\lambda_f).
\]
We emphasize that the phase $\varphi_f$ is phase convention independent as it depends only on the relative phase between $q/p$ and $T(\bar{D}^0 \to f)/T(D^0 \to f)$ in (4.8). The expression (4.11) with $F(\pm)$ given above generalizes the well-known formula from the $B$ system to include the effects of $\Delta \Gamma$ and $|q/p| \neq 1$.

For practical purposes, as $x_D, y_D \ll 1$ it is sufficient to consider the CP asymmetry in the limit of small $t$. Then (4.11) reduces to
\[
\frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)} \\
= - \left[ y_D \left( |\lambda_f| - \frac{1}{|\lambda_f|} \right) \cos 2\varphi_f - x_D \left( |\lambda_f| + \frac{1}{|\lambda_f|} \right) \sin 2\varphi_f \right] \frac{t}{2 \tau_D},
\]
where $\tau_D = 1/\Gamma$.

In the case of a non-negligible CP phase $\xi_f$ in the decay amplitude $T(D^0 \to f)$, but $|T(D^0 \to f)| = |T(\bar{D}^0 \to f)|$, $\lambda_f$ simplifies to
\[
\lambda_f = \eta_f \frac{q}{p} e^{-i2\xi_f}, \quad |\lambda_f| = \left| \frac{q}{p} \right|.
\]
Moreover, if in the adopted phase convention (like CKM convention) the phase $\xi_f$ is negligible as assumed in what follows, we have
\[
\lambda_f = \eta_f \frac{q}{p} = \eta_f \left| \frac{q}{p} \right| e^{i2\tilde{\varphi}}.
\]
We then find
\[
\frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)} = S_f \frac{t}{2 \tau_D},
\]
where we defined in analogy with the $B$ system
\[
S_f \approx -\eta_f \left[ y_D \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \cos 2\tilde{\varphi} - x_D \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) \sin 2\tilde{\varphi} \right].
\]
Note that in the $B$ system $y \ll x$ and $|q/p| \approx 1$, so that the above result simplifies considerably in case of the CP asymmetries $S_{\psi K_S}$ and $S_{\psi\phi}$ in the $B_d$ and $B_s$ systems, respectively.

Finally we introduce the semileptonic asymmetry
\[
a_{SL}(D^0) \equiv \frac{\Gamma(D^0(t) \to \ell^- \bar{\nu}_K K^{+(s)}) - \Gamma(\bar{D}^0 \to \ell^+ \nu K^{-}(s))}{\Gamma(D^0(t) \to \ell^- \bar{\nu}_K K^{+(s)}) + \Gamma(\bar{D}^0 \to \ell^+ \nu K^{-}(s))} = \frac{|q|^4 - |p|^4}{|q|^4 + |p|^4} \approx 2 \left( \left| \frac{q}{p} \right| - 1 \right)
\]
which represents CP violation in $\mathcal{L}(\Delta C = 2)$. In writing the last expression, we assumed that $|q/p| - 1$ is much smaller than unity.
4.2 Correlations

Having all these formulae at hand we can derive two interesting correlations. Following the presentation in [40], we find

\[ \sin^2 2\tilde{\phi} = \frac{x_D^2(1 - |q/p|^2)^2}{x_D^2(1 - |q/p|^2)^2 + y_D^2(1 + |q/p|^2)^2}, \]  

(4.20)

where in the phase conventions adopted in (3.6) \( \tilde{\phi} = 1/2 \arg(q/p) \). In the limit \(|q/p| - 1| \ll 1\), \(x_D \sim y_D\) (4.20) reduces to [40]

\[ \sin 2\tilde{\phi} = \frac{x_D}{y_D} \left( 1 - \left| \frac{q}{p} \right| \right). \]  

(4.21)

Using then (4.18) and (4.19) we find for \(\xi_f = 0\)

\[ S_f = -\eta_f \frac{x_D^2 + y_D^2}{y_D} a_{SL}(D^0), \]  

(4.22)

A similar correlation is familiar from the \(B_s\) system [40–42] and we recall it in Appendix A using the formulation presented above.

The violation of the relation (4.22) in future experiments would imply the presence of direct CP violation at work [40]. In the presence of a significant phase \(\xi_f\) we find

\[ S_f = -\eta_f \left[ \cos 2\xi_f \frac{x_D^2 + y_D^2}{y_D} a_{SL}(D^0) + 2x_D \sin 2\xi_f \right]. \]  

(4.23)

A similar comment applies to the correlation in \(B_s\) physics that we discuss in Appendix A.

5 LHT Results

5.1 SM Expectations

It is generally understood that CKM dynamics can generate direct CP violation in Cabibbo suppressed modes. For — in the Wolfenstein parameterisation of the CKM matrix — \(V_{cs}\) contains a weak phase of order \(\lambda^4\) and on the Cabibbo suppressed level there can be two different, yet coherent amplitudes. The same weak phase can also induce CP violation in \(\mathcal{L}(\Delta C = 2)\) [15–18]. Yet because these effects are largely shaped by long distance dynamics, we cannot go beyond saying that while SM dynamics generate CP violation in some charm transitions, it should happen below the 0.1 % level. Since it seems unlikely that the experimental uncertainties can be suppressed below that regime in the near future, we will ignore in our subsequent discussion SM CP violating effects.

5.2 LHT Scenarios

Our findings above strongly suggest that LHT dynamics presumably generate CP asymmetries in many different channels and Cabibbo levels, both of the indirect and direct variety. Exploring
this rich experimental landscape in a comprehensive way will be left to a future paper. Here we will focus on the simplest case, namely on indirect CP violation entering through $L(\Delta C = 2)$. Its impact on decay rates can be expressed through two types of observables:

$$\left| \frac{q}{p} \right| - 1 ,$$

which describes CP violation in $D^0 - \bar{D}^0$ oscillations and

$$\text{Im} \lambda_f ,$$

reflecting the interplay between CP violation in the oscillations and the transition to a final state $f$. These two types of observables can be probed by analysing the time evolution of $D^0 \to K_S K^+ K^-, K^+ K^-, \pi^+ \pi^-, K_S \pi^+ \pi^-, \ell \nu K^{(*)}$. As stated above, in this paper we will assume that New Physics does not affect the direct decay amplitudes for those transitions in any appreciable way.

From the definition of $q/p$ one easily obtains

$$\left| \frac{q}{p} \right|^4 = 1 + \left| \frac{\Gamma_{12}^D}{2M_{12}^D} \right|^2 \frac{\Gamma_{12}^D}{2M_{12}^D} \sin 2\varphi_{12}$$

$$1 + \left| \frac{\Gamma_{12}^D}{2M_{12}^D} \right|^2 - \frac{\Gamma_{12}^D}{2M_{12}^D} \sin 2\varphi_{12} ,$$

(5.26)

where $\varphi_{12}$ has been defined in (3.10). With $M_{12}^D = (M_{12}^D)_{\text{SM}} + (M_{12}^D)_{\text{LHT}}$ and $\Gamma_{12}^D = (\Gamma_{12}^D)_{\text{SM}}$ as determined above we can evaluate $|q/p|$. Likewise for $\text{Im} \lambda_f$, provided the phase $2 \xi_f$ from the ratio of decay amplitudes can be neglected. Below we go through a typical list of transitions, where these CP observables can be probed.

### 5.2.1 $D^0 \to \ell \nu K^{(*)}$

Because of the SM selection rule, ‘wrong’-sign leptons — $D^0 \to \ell^- \bar{\nu} K^{(*)}$, $\bar{D}^0 \to \ell^+ \nu K^{(*)}$ — are theoretically the cleanest signature for oscillations. Having such wrong sign leptons one can search for a difference in them, expressed through $a_{\text{SL}}(D^0)$ in (4.19), which represents CP violation in $L(\Delta C = 2)$. While the rate of wrong sign lepton production oscillates with time, this CP asymmetry does not.

In Fig. 5 we show $a_{\text{SL}}(D^0)$ as a function of $|q/p|$. We observe that almost any value for $a_{\text{SL}}(D^0)$ can be generated by LHT models, but the existing measurements for $|q/p|$ constrain the possible range for $a_{\text{SL}}(D^0)$. The important point to note here is the following: We know already from the data that the production probability of wrong-sign leptons is very low as expressed by $\frac{x_2^D + y_2^D}{2}$; yet this still allows for a sizable or even large CP asymmetry there: in LHT models one could get numbers as large as

$$-0.8 \lesssim a_{\text{SL}}(D^0) \lesssim +0.3$$

(5.27)

restricted only by the measured bounds on $|q/p|$ (the above range corresponds to the current experimental $2\sigma$ range in (3.12)).
Figure 5: $a_{\text{SL}}(D^0)$ as a function of $|q/p|$. The red (darker grey) and green (lighter grey) points correspond to the two solutions for $\left( (M_{12}^D)_{\text{SM}}, (\Gamma_{12}^D)_{\text{SM}} \right)$. The dark and light grey bands correspond to the experimental $1\sigma$ and $2\sigma$ ranges for $|q/p|$, as given in (3.12).

This finding is relevant even when one cannot measure $a_{\text{SL}}(D^0)$ directly as is the case in hadronic collisions. For CP asymmetries in nonleptonic $D^0$ transitions depend, as we will discuss below, on the same quantity $|q/p|$ which underlies $a_{\text{SL}}(D^0)$, so that stringent correlations between the various asymmetries exist (see Sect. 4).

5.2.2 $D^0 \to K_S \phi, K_S K^+ K^-, K_S \pi^+ \pi^-$

As already indicated above we feel quite safe in ignoring direct CP violation for Cabibbo favoured modes. The theoretically simplest channels would be $D^0 \to K_S \pi^0, K_S \eta, K_S \eta'$ — alas experimentally they are anything like simple. In a hadronic environment they seem to be close to impossible. The next best mode is

$$D^0 \to K_S \phi \to K_S [K^+ K^-]_\phi,$$  

(5.28)

which (apart from a doubly Cabibbo suppressed transition $D^0 \to K^0 \phi$) is given by a single isospin amplitude. The strong phase thus drops out from the ratio $T(D^0 \to K_S \phi)/T(D^0 \to K_S \phi)$, while their SM weak phase can be ignored at first. Therefore we can use (4.18) with $\eta_{K_S \phi} = -1$, in qualitative analogy to $B_d \to \psi K_S$. The effect will be much smaller of course, since the oscillations proceed much more slowly and a priori one cannot ignore the impact of $y_D \neq 0$ and $|q/p| \neq 1$. Furthermore the experimental signature of $\phi$ is not nearly as striking as that of $\psi$. One has to extract it from the $K_S K^+ K^-$ final state and distinguish it from final states like $K_S f^0$. The latter is particularly important, since the CP parities of $K_S f^0$ and $K_S \phi$ are opposite. Therefore these final states would have to exhibit CP asymmetries of equal size, yet opposite sign. Ultimately one has to

\footnote{The KM weak phase in $T(D^0 \to K_S \phi)/T(D^0 \to K_S \phi)$ and $q/p$ actually cancel to good accuracy.}
and can perform a CP analysis of the full Dalitz plot for $K_S K^+ K^-$; describing it is beyond the scope of this paper.

In Fig. 6 we show the correlation between $S_{D\to K_S\phi}$ and $a_{SL}(D^0)$. While we can see that a priori LHT dynamics could generate values for $S_{D\to K_S\phi}$ as large as ±0.05, the experimental constraint on $|q/p|$ in (3.12) and consequently on $a_{SL}(D^0)$ in (5.27) implies an allowed range

$$-0.02 \lesssim S_{D\to K_S\phi} \lesssim +0.01,$$

(5.29)
due to the strong correlation between the two CP asymmetries. We observe that for realistic values of $a_{SL}(D^0)$, as given in (5.27), the strict correlation between these two CP asymmetries is linear to an excellent approximation, with the gradient given by $(x_D^2 + y_D^2)/x_D \sim 0.02$, as derived analytically in (4.22). As already discussed in Section 4.2, the violation of the correlation in question would signal the presence of direct CP violation in the $D^0 \to K_S\phi$ decay.

Another suitable channel on the Cabibbo allowed level is $D^0 \to K_S\pi^+ \pi^-$. One starts with resonant final states $D^0 \to K_S \rho^0$ etc. and then proceeds to a Dalitz plot analysis. There is an additional complication though: in general one has to deal with more than one isospin amplitude. Therefore we leave a detailed study of this channel in the LHT model for future work.

### 5.2.3 $D^0 \to K^+ \pi^-$

Neither CKM nor, it seems, LHT dynamics can generate direct CP violation for this doubly Cabibbo suppressed mode. Any CP asymmetry in this mode has to be of the indirect variety [43, 44] involving oscillations (unless there is still another source of CP violation [45]). Its sensitivity to oscillation effects is actually enhanced, since the direct decay amplitude is con-
siderably reduced by $\sim \lambda^2$. Accordingly it already figures prominently in the data base for $D^0$ oscillations.

5.2.4 $D^0 \to K^+K^-, \pi^+\pi^-$

LHT dynamics can generate direct CP violation here due to Penguin diagrams. Evaluating their impact in a reliable way remains a task to be done. Both transitions can exhibit time dependent CP violation driven by the oscillation phase $2\phi_{12}$.

6 Impact of LHT Dynamics on $K$ and $B$ Decays

While Little Higgs models follow only one among many routes towards New Physics, LHT dynamics create non-trivial connections between what might emerge in high $p_t$ collisions at the LHC and flavour dynamics in principle — SUSY models do that as well —, but also in practice due to its relative paucity in additional model parameters. Here we have discussed its impact on the transitions of neutral $D$ mesons. Yet it creates intriguing effects also in kaon and $B$ decays as described in detail in [25,27–32].

In Fig. 7 we plot the CP asymmetry in $B_s \to \psi\phi (S_{\psi\phi})$ against $|q/p|$ in the $D^0 - \bar{D}^0$ system. In the LHT model, $S_{\psi\phi}$ can easily reach values between $-0.2$ and $+0.3$, i.e. considerably larger than its SM prediction being Cabibbo suppressed [46], larger values (up to $+0.6$) are possible for some points in parameter space [27,31,32]. We observe a cross-like structure in the plot, meaning that while either $S_{\psi\phi}$ or $|q/p|$ in the $D^0 - \bar{D}^0$ system can deviate significantly from their SM predictions 0.04 and 1, respectively, it is unlikely to observe large deviations from the SM values in both quantities simultaneously. Therefore if the present hints for a large non-SM value of $S_{\psi\phi}$ [47–49] will be confirmed by more accurate data, LHT dynamics will probably not lead to large CP violating effects in $D^0 - \bar{D}^0$ oscillations, albeit visible effects are still possible. On the other hand, if eventually $S_{\psi\phi}$ will turn out to be SM-like, the road towards spectacular LHT effects in $D^0 - \bar{D}^0$ CP violation will be open.

Fig. 8 shows $Br(K_L \to \pi^0\nu\bar{\nu})$ plotted against $|q/p|$ in the $D^0 - \bar{D}^0$ system. In order to study the impact of the value of $S_{\psi\phi}$ in this correlation, we show it, in the left panel of Fig. 8, for SM-like values of $S_{\psi\phi}$ ($< 0.05$) and, in the right panel of Fig. 8, for larger values of $S_{\psi\phi}$ ($\geq 0.1$). $Br(K_L \to \pi^0\nu\bar{\nu})$ can reach values up to $1.5 \cdot 10^{-10}$ (more than four times the SM expectation) [32]. For larger values of $S_{\psi\phi}$ (right hand side of the plot) very large enhancements of $Br(K_L \to \pi^0\nu\bar{\nu})$ are not observed\(^2\); this shows that simultaneous large LHT effects in CP violating $K$ and $B$ decays are unlikely [28,31,32]\(^3\). On the other hand, from the distribution of points on the left hand side we can see that large effects in $Br(K_L \to \pi^0\nu\bar{\nu})$ and in $|q/p|$ in the $D^0 - \bar{D}^0$ system do not exclude each other, on the contrary: In contrast to the cross-like structure in 7, we now observe an hourglass-like distribution of points, i.e. for points with large

\(^2\)It should be noted, however, that the right hand side of Fig. 8 contains considerably less parameter points than the left hand side.

\(^3\)The situation is in fact analogous to the one observed in Fig. 7 and shows that simultaneous large effects in $B$ and in $K$ or $D$ decays are generally unlikely in the LHT model.
Figure 7: $S_{\psi\phi}$ as a function of $|q/p|$. The red (darker grey) and green (lighter grey) points correspond to the two solutions for $((M_{D_{12}}^{SM}, (\Gamma_{D_{12}}^{SM})$.

Figure 8: $Br(K_L \to \pi^0 \nu\bar{\nu})$ as a function of $|q/p|$, showing only points which predict $S_{\psi\phi} < 0.05$ (left) and $S_{\psi\phi} \geq 0.1$ (right). The red (darker grey) and green (lighter grey) points correspond to the two solutions for $((M_{D_{12}}^{SM}, (\Gamma_{D_{12}}^{SM})$.

$Br(K_L \to \pi^0 \nu\bar{\nu})$ extreme effects in $|q/p|$ up to $\approx 0.5$ or $\approx 2$ are much more likely than in the case of SM-like $Br(K_L \to \pi^0 \nu\bar{\nu})$. This shows the correlation between CP violation in the $K$ system and in the $D$ system which was already apparent in Figs. 3 and 4.

## 7 Summary and Outlook

While the observed values of $\Delta M_D$ and $\Delta \Gamma_D$ might be generated by SM dynamics alone, $\Delta M_D$ could receive significant or even dominant contributions from New Physics. It would take a breakthrough in our control of nonperturbative effects to arrive at accurate SM predictions for $\Delta M_D$ and $\Delta \Gamma_D$. A more pragmatic approach to the interpretative conundrum posed by the observation of $D^0 - \bar{D}^0$ oscillations is to pursue a dedicated and comprehensive program
of CP studies in $D^0$ transitions in particular. Oscillations can obviously reveal CP violation residing in $\mathcal{L}(\Delta C = 2)$; in addition they can provide access to direct CP violation that otherwise would remain unobservable, namely in doubly Cabibbo suppressed modes.

The fact that baryogenesis requires the intervention of New Physics with CP violation represents a generic motivation for the aforementioned program of CP studies. Here we have provided a much more specific one, namely one based on LHT models. Let us repeat: the construction of these models was guided by considerations based on electroweak rather than flavour dynamics — yet they can have non-trivial consequences for the latter. Due to the paucity of their new parameters — at least relative to their ‘competitors’ — they can create connections between the parameters describing the on-shell behaviour of their new quanta and CP asymmetries in $K$, $B$ and $D$ decays that might be of practical use. What we have shown here is that LHT dynamics can generate sizable CP asymmetries in $D^0$ decays as expressed through $\text{Im} \lambda_f \neq 0$ and $|q/p| \neq 1$. The latter implies among other things that while $D^0 - \bar{D}^0$ oscillations produce only very few ‘wrong-sign’ leptons, those might exhibit a sizable CP asymmetry. More generally both of these portals for CP violation can be studied in nonleptonic channels like $D^0 \to K_S K^+ K^-$, $K^+ K^-$, $\pi^+ \pi^-$ and $K^+ \pi^-$. In summary the main analytic results of our paper can be found in the equations (2.4), (4.11), (4.14), (4.18), (4.20), (4.22), (4.23) and (A.1). The corresponding phenomenological implications are discussed in Sections 5 and 6. More specifically in the present paper:

- We have presented a general formula (4.11) for the mixing induced time dependent CP asymmetry for decays into a CP eigenstate. Compared with the analogous formulae known from the $B$ system, (4.11) includes the effects of $\Delta \Gamma \neq 0$ and $|q/p| \neq 1$.

- Assuming the absence of direct CP violation in the $\Delta C = 1$ decay amplitudes, we have presented an expression for the CP asymmetry $S_f$ (4.18) that generalises the familiar expressions for $S_{\psi K_S}$ and $S_{\psi \phi}$ to include the effects of $\Delta \Gamma \neq 0$ and $|q/p| \neq 1$.

- We have derived a correlation between $S_f$ and $a_{\text{SL}}(D^0)$ (4.22) that depends only on $x_D, y_D$ and $\eta_f$. A similar dependence has recently been pointed out in the case of $B_s - \bar{B}_s$ mixing in [40]. We confirm the latter result apart from a factor of 2.

- We have discussed the correlation between $D$ and $K$ decays in the spirit of the recent analysis in [34], demonstrating that the LHT model exhibits these correlations very transparently. To this end the expression (2.4) turned out to be very useful.

- Analysing in detail the LHT model we have found observable CP violating effects in $D^0 - \bar{D}^0$ oscillations well beyond anything possible with CKM dynamics. The correlation between $S_f$ and $a_{\text{SL}}(D^0)$, illustrated here for $f = K_S \phi$ will serve as a useful test (see Fig. 6) of the LHT dynamics.

- We have identified a clear pattern of flavour violation predicted by the LHT model:
  - While either the CP asymmetry $S_{\psi \phi}$ in $B_s - \bar{B}_s$ mixing or $|q/p|$ in the $D^0 - \bar{D}^0$ system can deviate significantly from their SM predictions, it is unlikely to observe
large deviations from the SM values in both quantities simultaneously. The improved measurements of $S_{\psi\phi}$ at the Tevatron and the LHC in the coming years will therefore have a large impact on the possible size of CP violating effects in $D$ decays within the LHT model.

- The strong correlation between the $K$ and $D$ systems implies that large New Physics effects in $K$ and $D$ decays are possible simultaneously.

- On the other hand simultaneous large effects in $K$ and $B$ decays are unlikely [28, 31, 32]. The latter property has also been pointed out recently in the context of RS models with custodial protection [50, 51].

Finally the analysis presented here can also be viewed as a proof of principle in two ways:

1. Charm decays might reveal the intervention of dynamics that so far has remained hidden.

2. While the CP phenomenology of $K \rightarrow \pi\nu\bar{\nu}$, $B_s \rightarrow \psi\phi$ and of $D^0$ decays has some overlap, it is also fully complementary and its dedicated study thus mandatory: new dynamics that might hardly affect $B_s \rightarrow \psi\phi$ can leave an identifiable footprint in $K \rightarrow \pi\nu\bar{\nu}$ and $D^0$ decays.

Acknowledgements

I.B. thanks the Excellence Cluster of the TUM for the wonderful hospitality afforded to him last year, when this work was begun. A.J.B. would like to thank the theory group of the Physics Department of Cornell University for great hospitality during the final stages of this paper. He would also like to thank Robert Fleischer for useful discussions. This research was supported by the NSF under the grant number PHY-0807959, the Deutsche Forschungsgemeinschaft (DFG) under contract BU 706/2-1, the DFG Cluster of Excellence ‘Origin and Structure of the Universe’ and by the German Bundesministerium für Bildung und Forschung under contract 05HT6WOA.

A CP Violation in $B_s - \bar{B}_s$ Mixing

The general formulae for CP asymmetries discussion in Sect. 4 can be applied to the $B_s$ meson system as well. We recall that in the latter system $|q/p| = 1$ with good accuracy, and in addition $y \ll x$. Therefore the correlation (4.22) simplifies even further in this case, and we find

$$A_{SL}^s = 2 \frac{y_s}{x_s} S_{\psi\phi},$$  \hspace{1cm} (A.1)

which agrees with the findings in [40] apart from the factor 2 and represents an alternative derivation of the correlation found in [41, 42]. Note that we used the convention

$$A_{SL}^s = \frac{\Gamma(B_s(t) \rightarrow \ell^+X) - \Gamma(B_s(t) \rightarrow \ell^-X)}{\Gamma(B_s(t) \rightarrow \ell^+X) + \Gamma(B_s(t) \rightarrow \ell^-X)},$$  \hspace{1cm} (A.2)
and $S_{\psi\phi}$ is the coefficient of $\sin \Delta M_s t$ in

$$\frac{\Gamma(\bar{B}_s(t) \to \psi\phi) - \Gamma(B_s(t) \to \psi\phi)}{\Gamma(\bar{B}_s(t) \to \psi) + \Gamma(B_s(t) \to \psi\phi)}.$$  \hspace{1cm} (A.3)

Further

$$x_s = \frac{m_H - m_L}{\Gamma_s}, \quad y_s = \frac{\Gamma_H - \Gamma_L}{2\Gamma_s},$$

where we stress that our definition of $y_s$ differs by sign from the HFAG one.

**B Numerical input**

We use the following values of the experimental and theoretical quantities as input parameters:

| Quantity                  | Value                  |
|---------------------------|------------------------|
| $\lambda = |V_{us}|$       | 0.226(2)               |
| $|V_{ub}|$                 | 3.8(4) $\cdot 10^{-3}$ |
| $|V_{cb}|$                 | 4.1(1) $\cdot 10^{-2}$ |
| $\gamma$                  | 78(12)$^\circ$         |
| $\Delta M_K$              | 0.5292(9) $\cdot 10^{-2}$ ps$^{-1}$ |
| $|\varepsilon_K|$         | 2.229(12) $\cdot 10^{-3}$ |
| $\Delta M_d$              | 0.507(5) ps$^{-1}$     |
| $\Delta M_s$              | 17.77(12) ps$^{-1}$    |
| $S_{\psi K S}$            | 0.675(26)              |
| $\bar{m}_c$               | 1.27(2) GeV            |
| $\bar{m}_t$               | 162.7(13) GeV          |
| $F_K$                     | 156(1) MeV             |
| $\bar{B}_K$               | 0.75(7)                |
| $\bar{B}_{B_s}$           | 1.22(12)               |
| $\bar{B}_{B_d}$           | 1.22(12)               |
| $F_D$                     | 212 MeV                |
| $\bar{B}_D$               | 1.17                   |
| $G_F$                     | $1.16637 \cdot 10^{-5}$ GeV$^{-2}$ |
| $M_W$                     | 80.398(25) GeV         |
| $\sin^2 \theta_W$        | 0.23122                |
| $m_{K^0}$                 | 497.614 MeV            |
| $m_{B_d}$                 | 5279.5 MeV             |
| $m_{B_s}$                 | 5366.4 MeV             |
| $m_{D_0}$                 | 1864.6 MeV             |
| $\eta_1$                  | 1.43(23)               |
| $\eta_2$                  | 0.577(7)               |
| $\eta_B$                  | 0.55(1)                |
| $\xi$                     | 1.21(4)                |
| $\kappa_L$                | $2.31 \cdot 10^{-10}$  |

**References**

[1] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, *Deconstructing dimensions*, *Phys. Rev. Lett.* **86** (2001) 4757–4761, [hep-th/0104005].

[2] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett.* **B513** (2001) 232–240, [hep-ph/0105239].

[3] H.-C. Cheng and I. Low, *TeV symmetry and the little hierarchy problem*, *JHEP* **09** (2003) 051, [hep-ph/0308199].

[4] H.-C. Cheng and I. Low, *Little hierarchy, little Higgses, and a little symmetry*, *JHEP* **08** (2004) 061, [hep-ph/0405243].
[5] I. Low, *T-parity and the Littlest Higgs*, JHEP 10 (2004) 067, [hep-ph/0409025].

[6] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, Phys. Lett. B500 (2001) 161–167, [hep-ph/0007085].

[7] A. J. Buras, *Minimal flavor violation*, Acta Phys. Polon. B34 (2003) 5615–5668, [hep-ph/0310208].

[8] G. D’Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, *Minimal flavour violation: An effective field theory approach*, Nucl. Phys. B645 (2002) 155–187, [hep-ph/0207036].

[9] L. J. Hall and L. Randall, *Weak scale effective supersymmetry*, Phys. Rev. Lett. 65 (1990) 2939–2942.

[10] R. S. Chivukula and H. Georgi, *Composite Technicolor Standard Model*, Phys. Lett. B188 (1987) 99.

[11] BABAR Collaboration, B. Aubert et. al., *Evidence for D⁰ − D̄⁰ mixing*, Phys. Rev. Lett. 98 (2007) 211802, [hep-ex/0703020].

[12] Belle Collaboration, M. Staric et. al., *Evidence for D⁰ − D̄⁰ Mixing*, Phys. Rev. Lett. 98 (2007) 211803, [hep-ex/0703036].

[13] BELLE Collaboration, K. Abe et. al., *Measurement of D⁰ − D̄⁰ mixing in D⁰ → K_Sπ⁺π⁻ decays*, Phys. Rev. Lett. 99 (2007) 131803, [arXiv:0704.1000].

[14] Heavy Flavor Averaging Group (HFAG) Collaboration, E. Barberio et. al., *Averages of b-hadron properties at the end of 2006*, arXiv:0704.3575. Updates available on http://www.slac.stanford.edu/xorg/hfag.

[15] I. I. Y. Bigi and N. G. Uraltsev, *D⁰ − D̄⁰ oscillations as a probe of quark-hadron duality*, Nucl. Phys. B592 (2001) 92–106, [hep-ph/0005089].

[16] S. Bianco, F. L. Fabbri, D. Benson, and I. Bigi, *A Cicerone for the Physics of Charm*, Riv. Nuovo Cim. 26N7 (2003) 1–200, [hep-ex/0309021].

[17] A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, *SU(3) breaking and D⁰ − D̄⁰ mixing*, Phys. Rev. D65 (2002) 054034, [hep-ph/0110317].

[18] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, *The D⁰ − D̄⁰ mass difference from a dispersion relation*, Phys. Rev. D69 (2004) 114021, [hep-ph/0402204].

[19] M. Blanke, A. J. Buras, S. Recksiegel, C. Tarantino, and S. Uhlig, *Littlest Higgs Model with T-Parity Confronting the New Data on D⁰ − D̄⁰ Mixing*, Phys. Lett. B657 (2007) 81–86, [hep-ph/0703254].
[20] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson, The Littlest Higgs, JHEP 07 (2002) 034, [hep-ph/0206021].

[21] M. Schmaltz and D. Tucker-Smith, Little Higgs review, Ann. Rev. Nucl. Part. Sci. 55 (2005) 229–270, [hep-ph/0502182].

[22] M. Perelstein, Little Higgs models and their phenomenology, Prog. Part. Nucl. Phys. 58 (2007) 247–291, [hep-ph/0512128].

[23] A. J. Buras, A. Poschenrieder, and S. Uhlig, Particle antiparticle mixing, $\varepsilon_K$ and the unitarity triangle in the Littlest Higgs model, Nucl. Phys. B716 (2005) 173–198, [hep-ph/0410309].

[24] A. J. Buras, A. Poschenrieder, S. Uhlig, and W. A. Bardeen, Rare K and B decays in the Littlest Higgs model without T-parity, JHEP 11 (2006) 062, [hep-ph/0607189].

[25] J. Hubisz, S. J. Lee, and G. Paz, The flavor of a little Higgs with T-parity, JHEP 06 (2006) 041, [hep-ph/0512169].

[26] M. Blanke et. al., Another look at the flavour structure of the Littlest Higgs model with T-parity, Phys. Lett. B646 (2007) 253–257, [hep-ph/0609284].

[27] M. Blanke et. al., Particle antiparticle mixing, $\varepsilon_K$, $\Delta\Gamma_q$, $A^q_{SL}$, $A_{CP}(B_d \to \psi K_S)$, $A_{CP}(B_s \to \psi \phi)$ and $B \to X_s,d\gamma$ in the Littlest Higgs model with T-parity, JHEP 12 (2006) 003, [hep-ph/0605214].

[28] M. Blanke et. al., Rare and CP-violating K and B decays in the Littlest Higgs model with T-parity, JHEP 01 (2007) 066, [hep-ph/0610298].

[29] M. Blanke, A. J. Buras, B. Duling, A. Poschenrieder, and C. Tarantino, Charged Lepton Flavour Violation and $(g-2)_\mu$ in the Littlest Higgs Model with T-Parity: a clear Distinction from Supersymmetry, JHEP 05 (2007) 013, [hep-ph/0702136].

[30] M. Blanke, A. J. Buras, S. Recksiegel, C. Tarantino, and S. Uhlig, Correlations between $\varepsilon'/\varepsilon$ and Rare K Decays in the Littlest Higgs Model with T-Parity, JHEP 06 (2007) 082, [0704.3329].

[31] M. Blanke, A. J. Buras, S. Recksiegel, and C. Tarantino, The Littlest Higgs Model with T-Parity Facing CP-Violation in $B_s - \bar{B}_s$ Mixing, arXiv:0805.4393.

[32] M. Blanke, A. J. Buras, B. Duling, S. Recksiegel, and C. Tarantino, FCNC Processes in the Littlest Higgs Model with T-Parity: a 2009 Look, in preparation.

[33] M. Blanke and A. J. Buras, A guide to flavour changing neutral currents in the Littlest Higgs model with T-parity, Acta Phys. Polon. B38 (2007) 2923, [hep-ph/0703117].

[34] K. Blum, Y. Grossman, Y. Nir, and G. Perez, Combining $K - \bar{K}$ mixing and $D - \bar{D}$ mixing to constrain the flavor structure of new physics, arXiv:0903.2118.
[35] T. Goto, Y. Okada, and Y. Yamamoto, *Ultraviolet divergences of flavor changing amplitudes in the Littlest Higgs model with T-parity*, Phys. Lett. B670 (2009) 378–382, [arXiv:0809.4753].

[36] F. del Aguila, J. I. Illana, and M. D. Jenkins, *Precise limits from lepton flavour violating processes on the Littlest Higgs model with T-parity*, arXiv:0811.2891.

[37] Particle Data Group Collaboration, C. Amsler et. al., *Review of particle physics*, Phys. Lett. B667 (2008) 1. Updates available on [http://pdg.lbl.gov/](http://pdg.lbl.gov/).

[38] J. Y. Lee, *A Vector-like heavy quark in the Littlest Higgs model*, JHEP 12 (2004) 065, [hep-ph/0408362].

[39] I. Dunietz and J. L. Rosner, *Time Dependent CP Violation Effects in $B^0 - \bar{B}^0$ Systems*, Phys. Rev. D34 (1986) 1404.

[40] Y. Grossman, Y. Nir, and G. Perez, *Testing New Indirect CP Violation*, arXiv:0904.0305.

[41] Z. Ligeti, M. Papucci, and G. Perez, *Implications of the measurement of the $B^0_s - \bar{B}^0_s$ mass difference*, Phys. Rev. Lett 97 (2006) 101801, [hep-ph/0604112].

[42] M. Blanke, A. J. Buras, D. Guadagnoli, and C. Tarantino, *Minimal Flavour Violation Waiting for Precise Measurements of $\Delta M_s$, $S_{\psi\phi}$, $A_{SL}^s$, $|V_{ub}|$, $\gamma$ and $B^0_{s,d} \to \mu^+\mu^-$*, JHEP 10 (2006) 003, [hep-ph/0604057].

[43] I. I. Bigi, in: *Proc. XIII ICHEP*, S. C. Loken (ed.), World Scientific (1986) p. 857; SLAC-PUB-4074.

[44] G. Blaylock, A. Seiden, and Y. Nir, *The Role of CP violation in $D^0 - \bar{D}^0$ mixing*, Phys. Lett. B355 (1995) 555–560, [hep-ph/9504306].

[45] G. D’Ambrosio and D.-N. Gao, *The diquark model: New physics effects for charm and kaon decays*, Phys. Lett. B513 (2001) 123–129, [hep-ph/0105078].

[46] I. I. Y. Bigi and A. I. Sanda, *Notes on the Observability of CP Violations in B Decays*, Nucl. Phys. B193 (1981) 85.

[47] CDF Collaboration, T. Aaltonen et. al., *First Flavor-Tagged Determination of Bounds on Mixing- Induced CP Violation in $B^0_s \to J/\psi\phi$ Decays*, Phys. Rev. Lett. 100 (2008) 161802, [arXiv:0712.2397].

[48] D0 Collaboration, V. M. Abazov et. al., *Measurement of $B^0_s$ mixing parameters from the flavor-tagged decay $B^0_s \to J/\psi\phi$*, Phys. Rev. Lett. 101 (2008) 241801, [arXiv:0802.2255].

[49] CDF Collaboration, G. Brooijmans, *Mixing and CP Violation at the Tevatron*, arXiv:0808.0726.
[50] M. Blanke, A. J. Buras, B. Duling, S. Gori, and A. Weiler, $\Delta F = 2$ Observables and Fine-Tuning in a Warped Extra Dimension with Custodial Protection, *JHEP* 03 (2009) 001, [arXiv:0809.1073].

[51] M. Blanke, A. J. Buras, B. Duling, K. Gemmler, and S. Gori, Rare $K$ and $B$ Decays in a Warped Extra Dimension with Custodial Protection, *JHEP* 03 (2009) 108, [arXiv:0812.3803].