Heat transfer characteristics in channel with square rib at different Mach numbers

Heng Li, Duo Wang, Ting Yu and Hongyi Xu

Department of Aeronautics and Astronautics, Fudan University, Shanghai 200433, PR China

1 E-mail: Hongyi_xu@fudan.edu.cn

Abstract. The detailed flow structures and closely-related heat transfer characteristics were investigated along the wall of a cooling channel with rib tabulator by computation. The two-dimensional Navier-Stokes equations were solved using the density-based algorithm and strongly-coupled with the heat-transfer equation. Three typical Reynolds numbers defined by rib height and Mach numbers were set at 500, 750, 1000 and at 0.2, 0.3, 0.4, respectively. Results show that, As the Mach numbers changing from 0.2 to 0.4, the local Nusselt numbers increase and the average heat transfer effects are enhanced. Based on the instantaneous field of vortices evolvement, the vortexes are first generated near the top surface of the rib by the strong shear effects. Then the vortexes are shed and intensified gradually downstream. Particularly, an important phenomenon is, for the first time, discovered that the high Nusselt number has a consistent correlation with the positive and negative sign alteration of the shear layer on the wall, which satisfactorily explains the mechanisms of heat transfer enhancement due to the flow.

1. Introduction

To improve the heat transfer efficiency in a cooling channel of turbine blade in modern aero-engine, some wall structures, such as ribs or fins, were popularly used to enhance the heat transfer efficiency. In the past, a number of experimental studies were conducted. Ali et al. [1, 2] investigated the flow and heat transfer characteristics of channel wall attached by a rib at various Reynolds numbers. As reported based on experiments, large recirculation bubble was observed in the downstream of rib. Besides, a secondary eddy was found in the leeward corner [1]. Recently, Fouladi et al. [3] experimentally investigated the flow and convective heat transfer on a flat plate with a square rib at four typical Reynolds numbers. Comparing with the smooth plate, heat transfer was significantly enhanced in the ribbed plate.

Heat transfer and flow in a single-rib mounted channel were investigated by direct numerical simulation by Miura et al. [4, 5]. They found that, the mean Nusselt number on the ribbed wall was about 1.3 times larger than the one on a smooth wall. Single rib mounted on the wall was concluded as a useful technique for heat-transfer enhancement.

Due to the fact that most of the previous simulations were based on an incompressible model, the present works consider the compressibility by using the density-based algorithm and strongly coupled the model with the heat-transfer equation, which is more accurate to reflect the density variations in real flows. In fact, the temperature differences are remarkable between the cooling fluid and the hot turbine blade surface, and therefore the density variations of cooling gas are significant near the hot
wall even if the Mach number is not very high. Hence, it is more valid to accurately resolve the density and temperature fields with the current density-based compressible algorithm. Usually, in incompressible studies, Reynolds number is chosen as the independent variable. In compressible flow, the heat transfer performances induced by Reynolds number and Mach number are similar, because the Reynolds number increase with the increase of Ma number. Therefore, to distinguish with incompressible flow, we choose Ma number as the independent variable in this paper.

Particularly, the latest vortex identification method [6] is applied to present works to analyze the phenomena of vortical flow structures and their effects on the heat transfer. The flow configuration of cooling channel with a rib tabulator is considered and the Navier-Stokes equations are solved in its two dimensional form. Detailed thermal flow fields are investigated and some strongly-coupled flow and heat transfer phenomena are analyzed.

2. Computational method and solution conditions

2.1. Problem descriptions

Figure 1 shows the computational domain and the Cartesian coordinate system. The channel dimensions are \( L=42 \, \text{e} \) and \( H=10 \, \text{e} \). A square rib with a distance \( L_r=11 \, \text{e} \) is set at the lower wall of channel. The temperature at no-slip wall is kept as 1.5 and the incoming flow temperature is 1.0. To accurately resolve the boundary layers near the walls, the numerical grids are refined towards the channel and rib walls.

![Figure 1. Computational domain and coordinate system.](image)

2.2. Computational method and parameters

The non-dimensionalized Navier-Stokes equations for compressible fluid are written in Cartesian coordinate as follows:

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{\partial \mathbf{F}_L}{\partial x} + \frac{\partial \mathbf{G}_N}{\partial y},
\]

where

\[
\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E_i \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E_i + p) \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(E_i + p) \end{bmatrix},
\]

\[
\mathbf{F}_N = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yy} \\ u\tau_{xx} + v\tau_{yy} + \frac{\partial T}{\partial x} \end{bmatrix}, \quad \mathbf{G}_N = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ u\tau_{yx} + v\tau_{yy} + k\frac{\partial T}{\partial y} \end{bmatrix}.
\]
\[ \tau_{xx} = -\frac{2}{3} \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \frac{\partial u}{\partial x} \]  
\[ \tau_{xy} = -\frac{2}{3} \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \frac{\partial v}{\partial y} \]  
\[ \tau_{yy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad \tau_{yz} = \tau_{xy}. \]

where, \( \rho \) denotes density, \( u \) and \( v \) are the velocity components in \( x \) and \( y \) Cartesian directions, respectively. \( p \) and \( T \) represent pressure and temperature. \( \mu \) and \( k \) are molecular viscosity coefficient and thermal conductivity, respectively. \( E_i \) is the total energy defined as \( \rho(c_v T + (u^2 + v^2)/2) \) where \( c_v \) is the specific heat at constant volume.

The two-dimensional compressible Navier-Stokes equations were solved using the density-based algorithm and strongly-coupled with the heat-transfer equation. The spatial derivatives in convective terms were discretized with the third-order MUSCL scheme [7, 8], and the spatial derivatives in viscous term were solved with forth-order centered finite-difference scheme. The third-order TVD Runge–Kutta scheme [9] was used in time advancement. The computational codes were developed based on the above methods.

The Reynolds number is defined as \( Re = \rho_e U \sqrt{\mu_e} \), where \( \rho_e \), \( U_e \) and \( \mu_e \) are the density, streamwise velocity and molecular viscosity coefficient at inlet. The local Nusselt number is defined as \( Nu = -H / (T_w - T_0) \partial T / \partial n \), where \( T_w \) is the wall temperature, \( T_0 \) is the bulk mean temperature, and \( \partial T / \partial n \) is the normal temperature gradient on the wall.

To study the Mach number effects on the heat transfer, three typical Mach numbers, \( Ma = 0.2, 0.3 \) and 0.4, were used here. The non-dimensional time step \( \Delta t \) was given at \( 2 \times 10^{-4} \) to satisfy the Courant–Friedrich–Ley (CFL) condition. The turbulence statistics were accumulated up to \( 1 \times 10^6 \) time steps.

### Table 1. Computational parameters.

| Case | Re  | Ma  | Pr   | Grid     | \( \Delta y^+ \) |
|------|-----|-----|------|----------|-----------------|
| 1    | 500 | 0.2 | 0.71 | 428 \times 262 | 0.35           |
| 2    | 750 | 0.3 | 0.71 | 428 \times 262 | 0.37           |
| 3    | 1000| 0.4 | 0.71 | 428 \times 262 | 0.39           |

The grid resolutions are measured by \( \Delta y^+ \), which is defined as \( \Delta y^+ = \Delta y u_c / \nu \), where \( \Delta y \) is the distance of the first point away from the wall, and \( \nu \) is the kinematic viscosity coefficient. The local friction velocity \( u_c \) is defined as \( u_c = \sqrt{\tau_w / \rho_w} \), where \( \rho_w \) is fluid density nearest the wall, \( \tau_w \) is wall shear stress defined as \( \tau_w = \mu_e (\partial u / \partial y)_{wall} \), where \( \mu_e \) and \( (\partial u / \partial y)_{wall} \) are molecular viscosity coefficient and normal velocity gradient nearest the wall. The skin friction Reynolds number is \( Re_c = u_e \delta / \nu \) where \( \delta \) is the boundary-layer thickness. The grid resolutions, based on the local friction velocity \( u_c \), meet the condition of a maximum of \( \Delta y^+ < 0.4 \) along the \( X \) direction (Table 1).

### 2.3. Boundary condition

For the no-slip wall, a ghost-cell immersed boundary method was used here [10, 11] with the Dirichlet boundary condition was applied and the fluid velocity on the wall was set equal to zero. On the wall, the normal pressure gradient was zero and then the density could be obtained by the perfect-gas state equation. For the outlet, the backpressure was set to the environmental pressure and all other variables were based on the zero gradient condition in streamwise direction, namely, the Neumann boundary.
condition. For the inlet, the fully-developed channel turbulence data [12] was extracted as present inlet velocity conditions.

2.4. Model validation

In order to validate present model and code, a classical case of flowing over a fixed circular, was studied. The computational domain was $20D \times 10D$, where $D$ was the cylindrical diameter. The Reynolds number based on $D$ was set at 40 and 1000. The grid number was $600 \times 600$ in $X$ and $Y$ directions and the grid resolution was chosen to satisfy $\Delta X \approx \Delta Y \approx 1/150 D$ near the cylindrical wall.

Figure 2 shows the cylindrical surface pressure coefficient defined as $C_p = (p - p_\infty) / (1/2 \rho U_\infty^2)$. From the comparisons, it can be seen that good agreements were obtained between the present results and the reference data given by Luo et al. [10] and Ghais et al. [13].

![Figure 2](image_url)

Figure 2. Comparison of pressure coefficient $C_p$.

To further validate present codes, the flow and heat transfer were computed in a channel with a square rib. The Reynolds number defined by rib height was set at 500. The Nusselt number was extracted. As shown in Figure 3, present results agree well with the results given by Miura et al. [4, 5].

![Figure 3](image_url)

(a) (b)

Figure 3. Local Nusselt number of $Re = 500$; (a) bottom wall; (b) Nusselt number enhancement. (Please note that the equivalent Reynolds number based on rib height is 455 in the computation given by Miura et al. [4, 5]) (Here $Nu$ and $Nu_0$ are the local Nusselt number on rib-wall and smooth wall).
2.5. Grid dependence study
The simulations were performed on two grid levels in $X$ direction, namely coarse grids ($324 \times 262$) and fine grids ($428 \times 262$). Results show that there are no obvious differences between these two grids. The simulations were also performed on three grid levels in $Y$ direction, namely coarse grids ($324 \times 133$), medium grids ($324 \times 262$) and fine grids ($324 \times 520$). Results show that there are no obvious differences between these three grids. Therefore, as a compromise of the grid resolution and computational work, all present studies were conducted on grids $428 \times 262$.

3. Results

3.1. Heat transfer characteristics at different Mach numbers
As shown in Figure 4(a), it can seen that the $Nu$ increase with the increase of Mach number. The local $Nu$ are larger and the peak value locations are further forward with the increase of $Ma$. These phenomena can be explained from the flow fields (Figure 5 to Figure 7). With the increase of Mach number, the two vortices are more apparent behind the rib. Near the reattachment points, the velocity vectors and temperature gradients vectors are more synergetic. According to the field synergy principle [14], field synergy numbers are larger and the temperature boundary layer are thinner. As a result, the local $Nu$ are larger and the heat transfer effects are more significant.

As shown in Figure 4(b), at low Mach number ($Ma=0.2$), the location of $Nu / Nu_0 > 1$ is near at $X = 5$. At medium Mach number ($Ma=0.3$), the location of $Nu / Nu_0 > 1$ is near at $X = 4$. While, at high Mach number ($Ma=0.4$), the location of $Nu / Nu_0 > 1$ is near at $X = 1.3$. These phenomena can be explained from the flow fields (Figure 5 to Figure 7). With the increase of Mach number, the size of the secondary vortex behind the rib decreases and the reattachment point moves forward. Accordingly, the low heat transfer region decreases and the average heat transfer effects are enhanced. With the increase of Mach number, the Reynolds number increase and the flow stability decrease, which leads the instability occurs easier and the size of the secondary vortex behind the rib decreases.

![Figure 4. Local Nusselt number (a) bottom wall; (b) Nusselt number enhancement.](image)

![Figure 5. Mean temperature contours and stream lines at $Ma=0.2$.](image)
At high Ma of $Ma = 0.4$, there is a smaller high temperature region behind the rib because of the smaller-sized wake as seen in Figure 7. As a result, the smaller regions of low-density fluids are observed (Figure 10). Besides, as shown in Figure 8, Figure 9 and Figure 10, the density gradients near the rib and the wall are larger at high Ma. The phenomena can be explained by the thinner temperature boundary layer near the rib and wall.
3.2. Instantaneous vortexes and shear layers

In order to further understand the convective heat transfer, the vortexes and shear layers are extracted based on the latest vortex identification method, namely Rortex method [6]. The main idea of this method is that, the vorticity vector $\nabla \times \mathbf{v}$ is decomposed into a rotational vector $\mathbf{R}$ which is called the Rortex and a non-rotational vector $\mathbf{S}$ representing the shear effect, namely $\nabla \times \mathbf{v} = \mathbf{R} + \mathbf{S}$.

In Liu’s paper [6], the mathematical definition of Rortex were provided and the validations were conducted based on the late-transition DNS data of semi-flat plate, which well presented the effectiveness of Rortex function in representing the characteristics of vortical flow. Therefore, the present work are based on Rortex to analyze the vortical structures and the its effects on the convective heat transfer on the ribbed wall.

![Figure 11](image)

**Figure 11.** Evolution of Rortex $R$ (a1 to a8) and Shear $S$ (b1 to b8) with a dimensionless time interval of $\Delta t=2$.

There, the typical cases at $Ma=0.2$ are illustrated. After the fields are fully developed, the evolutionary process of rotational vector and non-rotational vector are extracted with dimensionless time interval $\Delta t=2$ as shown in Figure 11.

The magnitude of Rortex variable $R$ represents the vortex strength as shown in Figure 11(a). As the time evolving, several small-scale vortices were generated on the upper side of rib. With a strong shear near the top surface of rib, those small-scale vortices started shedding and evolved into the more intensive large-scale eddies as show in Figure 11. Particularly, the counter-rotating tail vortices
indicated by the red region in Figure 11(a) were observed downstream. It is noteworthy that the counter-rotating eddies were generated, strengthened and then gradually weakened, eventually disappeared downstream.

Besides, the prominent strong shear along the upper edge of the separated region and along the channel wall were observed in Figure 11(b). From a time-sequential frames of the shear variable $S$, it can be seen that the distinctive positive and negative shear regions alternated when evolving downstream along the wall. Specifically, after a distance of $3 \varepsilon$ behind the rib, the strong shear banded regions were alternatively changed, which well explained the phenomenon of strong heat transfer effects.

4. Conclusions

The paper investigated the convective heat transfer on the channel wall attached with a square-shape rib by resolving the two-dimensional Navier-Stokes equations. Three typical Mach numbers were set at 0.2, 0.3, 0.4, respectively. Some important conclusions are summarized as following:

1. As Mach number increasing from 0.2 to 0.4, the secondary vortex size decreases and the location of $Nu/Nu_0=1$ moves forward. As the increase of Mach number, the temperature boundary layer are thinner near the rib and wall in the vicinity of separated region. Accordingly, the local $Nu$ numbers increase and the average heat transfer effects are enhanced.

2. Based on the instantaneous field of vortices evolvement, vortexes appear near the top surface of the rib with the strong shear effects. Then the vortexes shed and intensify gradually downstream. Particularly, an important phenomenon is, for the first time, discovered that the alternate regions of the positive and negative shear layer on the wall are consistent with the locations of high Nusselt number.

Acknowledgments

The research was financially supported by the United Innovation Program of Shanghai Commercial Aircraft Engine with the Grant No. AR908 (the program was founded by Shanghai Municipal Commission of Economy and Information, Shanghai Municipal Education Commission and AECC Commercial Aircraft Engine Co., LTD).

References

[1] Ali M S, Tariq A and Gandhi B K 2012 Proc. ASME Int. Mech. Eng. Congr. Expo.--2012 pp. 1259-71
[2] Ali M S, Tariq A and Gandhi B K 2013 Exp. Fluids 54 pp 1-15
[3] Fouladi F, Henshaw P, Ting S K and Ray S 2017 J. Heat Mass Transf. 104 pp 1202-16
[4] Miura T, Matsubara K and Sakurai A 2010 J Therm Sci Tech-jpn, 5 pp 135-150
[5] Miura T, Matsubara K and Sakurai A 2012 J Therm Sci Tech-jpn, 7 pp 120-134
[6] Liu C Q, Gao Y S, Tian S L and Dong X R 2018 Phys. Fluids 30 p 035103
[7] Leer B V 1979 J. Comput. Phys. 32 101-136
[8] Colella P 1982 Siam Journal on Scientific & Statistical Computing 6 pp 104-117
[9] Shu C W 1988 Siam Journal on Scientific & Statistical Computing 9 pp 1073-84
[10] Luo K, Zhuang Z Y, Fan J R and Haugen N. E. L 2016 J. Heat Mass Transf. 92 pp 708-717
[11] Luo K, Mao C L, Zhuang Z Y, Fan J R and Haugen N. E. L 2017 J. Heat Mass Transf. 104 pp 98-111
[12] Xu H Y 2009 J. Fluid Mech. 621 pp 23-57
[13] Ghias R, Mittal R and Dong H 2007 J. Comput. Phys. 225 pp 528-553
[14] Guo Z Y, Tao W Q and Shah R K 2005 Int. J. Heat Mass Transf. 48 pp 1797-1807