Analysis of the Shuttle-Based Storage and Retrieval System

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This work was supported in part by the Science, Technology and Innovation Commission of Shenzhen Municipality under Grant JCYJ20190807094803721, in part by the Fundamental Research Funds for Shandong University under Grant 2018JC035, and in part by the National Natural Science Foundation of China under Grant 61973192.

ABSTRACT The shuttle-based storage and retrieval system (SBS/RS) is composed of a shuttle sub-system that is responsible for horizontal movements and a lift sub-system that is responsible for vertical movements. As the combination of the two sub-systems yields high flexibility, low operating cost and large storage capacity, SBS/RS is becoming more and more popular, but also raises managerial issue of how to coordinate the shuttle and the lift. Based on its operational processes, this study models the tier-to-tier SBS/RS system as a semi-open queuing network (SOQN). By removing or adding the synchronization nodes, the SOQN is further transformed into two different closed queuing networks (CQNs), and applies the approximate mean value analysis (AMVA) algorithm to solve the model and estimate its performance. The system performance is measured by the utilization of shuttles, the utilization of lift, and the task cycle time, under various design configurations. Simulation is carried out to validate the effectiveness of the analytical model and algorithm. Compared with the simulation results, the established semi-open queuing network can accurately estimate task cycle time for different rack configurations. The proposed solution method can help to identify the optimal number of shuttles and guide the design of the SBS/RS system.

INDEX TERMS Automated warehouses, shuttle-based storage and retrieval systems, semi-open queuing network, analytical and numerical modeling, optimization, performance analysis.

I. INTRODUCTION

The shuttle-based storage and retrieval system (SBS/RS) is a new type of automated warehousing systems, which is mainly applied to stock keeping units (SKUs) with high usage frequency. According to whether shuttles can perform cross-tier tasks with the help of a lift, two configurations can be defined: SBS/RS with tier-to-tier shuttles and that with tier-captive shuttles. Multiple studies have done for SBS/RS with tier-captive shuttles (e.g., Ekren et al. [1]; Ning [2]; Tappia et al. [5]). Despite operational flexibility, if one shuttle breaks down, for the tier-captive system, no SKUs of the tier with the failed shuttle can be stored or retrieved. In the tier-to-tier SBS/RS, lifts serve the vertical movement, while shuttles serve the horizontal movement of SKUs. Each shuttle can reach any tier of the storage rack with the help of a lift. Thus, the system has fewer shuttles than tiers. The tier-to-tier system is more robust compared with the tier-captive SBS/RS. For the tier-to-tier system, even if one shuttle does not work, other shuttles can be dispatched to perform storage or retrieval tasks. Due to its higher reliability this paper focuses on the tier-to-tier SBS/RS. Because it is difficult to modify rack configurations of the tier-to-tier SBS/RS, the current and future needs should be fully considered during system designs, requiring the evaluation of the performance of a variety of rack configurations. The aim of this paper is to establish such an analytical model for the tier-to-tiers SBS/RS and to provide guidance for warehousing system designers. The analytical model has been applied to a power instrument maintenance parts warehouse. As a spare parts warehouse, the throughput of the warehouse is low, and the idle rate of equipment in the application of tier-captive SBS/RS is high, resulting in a waste of resources, the warehouse is suitable for tier-to-tier SBS/RS.
The rest part of the article is organized as follows. Section two reviews the relevant literature. Section three mainly describes the structure of the system and its operational processes. The tier-to-tier SBS/RS is modeled as a semi-open queuing network (SOQN) and the approximate mean value analysis (AMVA) algorithm is used to solve it in Section four. Section five compares the simulation results with the results obtained from the SOQN for validation. Besides, the optimal number of shuttles is found under various conditions, such as rack configuration, task arrival rate, and velocity of the shuttle. Section six concludes the paper with a summary of major findings and the direction of further research.

II. LITERATURE REVIEW

Although the tier-to-tier SBS/RS has been applied in some logistics distribution centers, there has been very little analytical research of the system. Azadeh et al. [3] divided the autonomous vehicle storage and retrieval system (AVS/RS) into three categories: Horizontal, Vertical, and Diagonal systems. SBS/RS is developed for small transactions carried in small container shipments and is designed to accommodate a wider range of products and shorter response times than autonomous vehicle storage and retrieval system (AVS/RS) [4]. On the other hand, SBS/RS is designed to aisle captive for high transaction throughput [4]. Thus SBS/RS with higher flexibility, lower operating cost and larger storage capacity is regarded an extension of AVS/RS [5]. Considering the energy consumption and amount of energy regeneration for a transaction, Ekren [6], [7] analyzed the performance of the SBS/RS and finds that there is a tradeoff between energy consumption and cycle time performance measures. Thomas et al. [8] proposed a simulation model to determine the throughput of SBS/RS with the application of storage policies, including class-based storage, sequencing of retrieval requests and warehouse reorganization. Some relevant references of AVS/RS are also included in this literature review. Malmborg [9] designed a conceptual approach to analyze the influence of shelf configurations, storage strategy and the number of vehicles on performance of the AVS/RS. Malmborg [10] proposed a state equation model for predicting the proportion of dual command (DC) cycles in AVS/RS with interleaving. D’Antonio et al. [11] used the analytical model to calculate the cycle time and its standard deviation of deep-lane AVS/RS. On this basis, the research team began to use different queuing network models to study the AVS/RS. Kuo et al. [12] used a nested queuing network model to analyze the system performance such as the vehicle utilization and the lift utilization and modeled the service process of vehicles as an M/G/V and the service process of the lift as a G/G/L queue. Based on random storage and DC transactions, Fukunari and Malmborg [13] developed an approximate cycle time model for the AVS/RS. This model also improved earlier models by scaling efficiently for large problems. The model also had an ability to compare the performance of AS/RS and AVS/RS designs. In addition, they also found AVS/RS had more advantages than AS/RS in capital investment. Fukunari and Malmborg [14] pointed out the shortcomings of the state equation model and the nested queuing network, and established a closed queuing network (CQN) of the AVS/RS. Zhang et al. [15] proposed that outbound/inbound transactions can be thought as customers and vehicle-lift pairs as parallel servers to analyze the system performance. By changing the combination of shuttles and lifts and arrival rate, Ekren et al. [16] built a simulation model to find the key factors for the performance of AVS/RS. Marchet et al. [17] researched the AVS/RS through simulation. They found that the number of lanes and tiers are key parameters of the system performance. Manzini et al. [18] developed an analytic model to determine the travelled distance and time for single-command and dual-command cycles given alternative layout configurations. In addition, given the throughput constraint, an appropriate ratio of length/width can reduce capital investment. Heragu et al. [19] established two open queuing networks (OQN) of AVS/RS and AS/RS, and the manufacturing system performance analyzer (MPA) was applied to estimate key performance of the system. Taking the acceleration and deceleration into account, Marchet et al. [20] modeled tier-captive AVS/RS as an OQN and decomposed the original open network into multiple M/G/1 queues by analyzing the lifts and shuttles respectively. Zou et al. [21] estimated key performance of tier-captive AVS/RS and modeled the system as a fork-join queuing network. Under a certain number of tiers, the parallel operation strategy of shuttles and lift is better than the previous sequential operation strategy. Because there is still a time for the lift to send the shuttle to the destination tier in the tier-to-tier system, the tier-captive model of Zou et al. [21] cannot adapt to the tier-to-tier system. Epp et al. [22] modeled tier-captive AVS/RS as a discrete-time OQN to estimate the transaction cycle time. Wang et al. [23] discussed the retrieval process of a multi-tier shuttle warehousing system and built a two-stage OQN. The storage assignment method was studied according to shuttle waiting time and lift idle time obtained from the OQN. Wu et al. [24] modeled tier-captive SBS/RS as an OQN to find the minimum cost configurations. Michael [25]–[27] modeled tier-captive SBS/RS as a space discrete continuous time OQN to assess the performance.

Compared with the OQN, the semi-open queuing network (SOQN) can model more realistic scenarios where a task may have to wait for a resource or vice versa. Roy et al. [28] considered a single tier of AVS/RS and modeled it as a multi-class SOQN and proposed a decomposition method to evaluate system performance. Roy et al. [29] further studied a single tier of AVS/RS about cross-aisle location and dwell-point policies. The results indicated that cross-aisle placed at the end of aisles and shuttles dwelt at load/unload point were optimal policies. However, Roy et al. only considered one tier and could not calculate the performance of the entire system. Ekren et al. [30] modeled the AVS/RS as a single-class, multiple-server, SOQN and solved the SOQN using an approximate method. Ekren et al. [31] modeled the built
tier-to-tier system as an SOQN and used the Matrix-geometric solution method to solve the model to obtain its key performance measures. Tappia et al. [5] built SOQN models to study the performance of single-tier and multi-tier shuttle-based compact systems. A live-cube compact storage system was studied by Zaerpour et al. [32]. They divided the system into several situations and used closed-form formulas to estimate the important performance measures of the system, such as, order response time. They found that the two-class-based system was faster than the random storage strategy to complete all orders. Roy et al. [33] develop SOQN models to evaluate congestion effects in processing storage and retrieval transactions in tier-to-tier AVS/RS. Güller and Hegmanns [34] create a detailed simulation model and to evaluate the performance of a mini-load multi-shuttle order picking system. Ning et al. [2] developed a simulation model that can be auto-remodeled for different rack configurations.

Ekren et al. [1] evaluated the performance of the SBS/RS in terms of lifts and storage/retrieval devices and cycle times of storage/retrieval transaction. Cao and Zhang [35] studied the scheduling problem about the SBS/RS for storing and picking-up compound operations. Ha and Chae [36] suggested three systems and analyzed the effects of free balance in an SBS/RS. They found that the tier-to-tier SBS/RS can operate more productively than the other two control systems. And then Azadeh et al. [37] modeled the tier-to-tier SBS/RS as an CQN. In addition, Ha and Chae [38] found that the bottleneck of the system is the ability of the lift and proposed a decision model to determine the number of shuttles with Bay 0 (on the first tier) as the dwell point. However, compared with the travel time model [39], the queuing model in this paper is more adaptable to the change of parameters. Tappia et al. [39] modeled the remote OP system as a SOQN, and analyzed the factors affecting order picking.

A synthesis of existing work is provided in TABLE 1 to position this study. Unlike other articles, this paper contributes an SOQN model for the tier-to-tier SBS/RS and uses the AMVA algorithm to solve the model, which can help designers understand the relationship between various configuration parameters and make optimal choices.

Although many scholars have studied the AVS/RS and modeled the system as an SOQN, the solution to the SOQN is too complex for practical applications. Besides, the theoretical research on how to design the SBS/RS is limited. Due to their different operational processes, an SOQN developed for the AVS/RS is not applicable to the SBS/RS. In this paper, the operational processes of shuttles and the lift are divided into multiple stages, and each stage is considered a node. The SOQN is built by analyzing the relationship between nodes. The AMVA algorithm is used to solve the model, which can quickly calculate the performance of the SBS/RS under different rack configurations.

III. SYSTEM DESCRIPTION

Figure 1 illustrates the tier-to-tier SBS/RS studied in this paper. Shuttles are responsible for the SKU retrievals at each tier. A lift is placed at the end of each storage aisle and responsible for vertical movements. The retrieval transactions arrive and end at the I/O locations located at the bottom tier [6]. The bottom tier is the shuttle buffer, where idle shuttles stay after completing a task and wait for a new task. Placing idle shuttles in the first-tier stems from three reasons. Firstly, due to the fact that retrieval operations are more important than storage tasks in e-commerce warehouse, our dwell policy facilitates the retrieval operations at the very beginning. Secondly, it reduces the total number of shuttle movements, in particular when facing a series of outbound tasks. Thirdly, the inspection and maintenance work may be much easier when all shuttles dwell at the bottom. With the help of the lift, a shuttle can access any tier. The lift and shuttles can move simultaneously. This paper studies the performance of the whole system by studying only one aisle due to the similarity among aisles.

A. MAIN ASSUMPTIONS AND NOTATIONS

In order to analyze the system performance through an SOQN, the following assumptions are made.

1. Only consider retrieval tasks.
2. Arrivals follow a Poisson process.
3. The random storage policy is chosen.
4. The tier-to-tier SBS/RS operate on basis of DC cycle.
5. The blockage of the shuttles in the aisle is ignored.
6. Only study one aisle.
7. The bottom tier is the shuttle buffer.
8. The I/O point is located at the first tier of each aisle.
9. The lifts and shuttles conform to the first-come-first-serve principle.

We only consider retrieval tasks [2] [23], because the efficiency of retrieval operations is key to e-commerce business. Retrieval tasks arrive at the system following a Poisson process. The Poisson process assumption is widely used in all relevant studies (e.g., [1], [2] [5]). To save storage space, the random storage policy is chosen, following Heragu [40]. Besides, we will ignore the blockage of the shuttles in the aisle [28], [29] to reduce the complexity of the model. Both shuttles and the lift conform to the first-come-first-serve principle. The lift remains at its position where it finishes a task and becomes idle (i.e., no outstanding requests for the lift). Because we only study retrieval tasks, shuttles will dwell at the first tier of the storage rack.

The notations are defined for the SOQN modeling in TABLE 2.

B. SYSTEM PROCESSES ANALYSIS

Based on the above assumptions, the flowchart for a retrieval task in the tier-to-tier SBS/RS is shown in Figure. 2.

Based on above assumptions and flowchart, we divide a retrieval task into six processes and calculate the average service time of each process for modeling the overall task. The six processes are also illustrated in Figure. 4.

Process 1: The lift associated to the retrieval task moves from its current position to the first tier, and loads an empty shuttle. The SBS/RS is responsible for both inbound and
TABLE 1. Overview of the literature.

| Study               | System Type | Configuration | Transaction Types            | Research Objective(s)                      | Technical Approach |
|---------------------|-------------|---------------|------------------------------|--------------------------------------------|-------------------|
| Fukunari (2009)     | AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Closed queuing network |
| Heragu et al. (2011)| AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Open queuing network |
| Marchet et al. (2012)| AVS/RS     | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Open queuing network |
| Roy et al. (2012)   | AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Semi-open queuing networks |
| Roy et al. (2015a)  | AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Semi-open queuing networks |
| Zou et al. (2016)   | AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Fork-join queuing network |
| Manzini (2016)      | AVS/RS      | Tier captive  | Storage and Retrieval       | Lane depth; travel time                    | Simulation         |
| Epp et al. (2017)   | AVS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Open queening network |
| D’Antonio et al. (2018)| AVS/RS  | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Mathematical       |
| Fukunari (2008)     | AVS/RS      | Tier-to-tier  | Storage and Retrieval       | Cycle time model                           | Simulation         |
| Ekren et al. (2010) | AVS/RS      | Tier-to-tier  | Storage and Retrieval       | Estimate the performances                  | Simulation         |
| Marchet et al. (2013)| AVS/RS     | Tier-to-tier  | Storage and Retrieval       | Estimate the performances                  | Simulation         |
| Ekren et al. (2013) | AVS/RS      | Tier-to-tier  | Storage and Retrieval       | Estimate the performances                  | Semi-open queuing networks |
| Ekren et al. (2014) | AVS/RS      | Tier-to-tier  | Storage and Retrieval       | Matrix-geometric solution                  | Semi-open queuing networks |
| Roy et al. (2015b)  | AVS/RS      | Tier-to-tier  | Storage and Retrieval       | Evaluate congestion effects                | Semi-open queuing networks |
| Ekren et al. (2015) | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Simulation         |
| Wang et al. (2016)  | SBS/RS      | Tier captive  | Retrieval                   | Estimate the performances                  | Open queuing network |
| Ning (2016)         | SBS/RS      | Tier captive  | Retrieval                   | Estimate the performances                  | Simulation         |
| Cao (2017)          | SBS/RS      | Tier captive  | Storage and Retrieval       | Task scheduling optimization               | Genetic algorithm  |
| Tappia et al. (2017)| SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Semi-open queuing networks |
| Ekren et al. (2018) | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Analytical         |
| Tappia et al. (2019)| SBS/RS      | Tier captive  | Storage and Retrieval       | Remote OP System                           | Semi-open queuing networks |
| Michael (2019)      | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Time continuous queuing model |
| Michael (2020)      | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Time continuous queuing model |
| Michael (2020)      | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Time continuous queuing model |
| Ekren et al. (2020) | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Simulation         |
| Wu et al. (2020)    | SBS/RS      | Tier captive  | Storage and Retrieval       | Estimate the performances                  | Open queuing network |
| Güller (2014)       | SBS/RS      | Tier-to-tier  | Storage and Retrieval       | Estimate the performances                  | Simulation         |
| Ha and Chae (2018)  | SBS/RS      | Tier-to-tier  | Storage and Retrieval       | Free balancing                             | Simulation         |
| Azadeh et al. (2018)| SBS/RS      | Tier-to-tier  | Storage and Retrieval       | Estimate the performances                  | Closed queuing network |
| Ha and Chae (2019)  | SBS/RS      | Tier-to-tier  | Storage and Retrieval       | Determine the number of shuttles           | Simulation         |

The vertical moving distance for the lift from its dwell tier $x$ to the first tier is

$$H_1(x) = (x - 1)h.$$  \hfill (2)

Taking the acceleration/deceleration and velocity-time relationship of the vehicles travelling [20] into account, the moving time of the lift is

$$\tau_1(x) = \begin{cases} 
\frac{2v_L}{a_L} + \frac{H_1(x) - \frac{v_L^2}{2a_L}}{v_L}, & H_1(x) > \frac{v_L^2}{a_L} \\
2\sqrt{\left(\frac{H_1(x)}{a_L}\right)^2}, & \text{otherwise}
\end{cases}.$$  \hfill (3)

The average service time of Process 1 is

$$ES_1 = \sum_{x=1}^{T} P_1(x) \tau_1(x) + t_L.$$  \hfill (4)

Process 2: The lift with an empty shuttle moves from the first tier to the tier associated with the retrieval task. The lift is released for the next request after it unloads the shuttle. The probability that the tier associated with the retrieval task is $y$ is uniform among all retrieval tiers and is $\frac{1}{T-1}, y = 2, \ldots, T$. The vertical moving distance for the lift from the first tier to tier $y$ is

$$H_2(y) = (y - 1)h.$$  \hfill (5)
TABLE 2. Notation of the tier-to-tier SBS/RS.

| Notation | Parameter | Unit |
|----------|-----------|------|
| $\lambda$ | Arrival rate of retrieval tasks | tasks/hour |
| $T$ | Number of tiers | tier |
| $C$ | Number of lanes on each side of an aisle | lane |
| $w$ | Width of a single storage location | m |
| $x$ | Tier where the lift is located | tier |
| $y$ | Tier associated with the retrieval task | tier |
| $c$ | The retrieval lane | lane |
| $\pi$ | Tier where the lift is located | tier |
| $h$ | Height of a single storage location | m |
| $v_L$ | Maximum horizontal velocity of a shuttle | m/s |
| $a_L$ | Maximum vertical velocity of the lift | m/s |
| $a_{\uparrow}$ | Horizontal acceleration/deceleration of a shuttle | m/s² |
| $a_{\downarrow}$ | Vertical acceleration/deceleration of the lift | m/s² |
| $\tau_2$ | Time for a shuttle to load or unload an SKU | s |
| $\rho_L$ | Lift utilization | % |
| $TCT$ | Task cycle time | s |
| $P_i(x)$ | The probability that the lift is at tier $x$ | — |
| $H_i(x)$ | The vertical moving distance for the lift, $i = 1, 2, 4, 5$ | m |
| $\tau_i(x)$ | The moving time of the lift or shuttle, $i = 1, 2, 4, 5$ | s |
| $ES_i$ | The average service time of Process $i$, $i = 1, 2, 4, 5$ | s |

The moving time of the lift from tier one to tier $y$ is

$$\tau_2(y) = \begin{cases} 
2v_L/a_L + \frac{H_2(y) - \frac{v^2_L}{a_L}}{v_L}, & H_2(y) > \frac{v^2_L}{a_L} \\
2\sqrt{\frac{H_2(y)}{a_L}}, & \text{Otherwise.} 
\end{cases}$$

Thus, the average service time of Process 2 is

$$ES_2 = \frac{\sum_{y=2}^{T} \tau_2(y)}{T - 1} + t_L.$$  \hfill (7)

**Process 3:** The shuttle moves from lane zero to the retrieval lane to obtain the SKU and then moves back to lane zero in the tier. The horizontal moving distance is $D(c) = c \cdot w$ if the load for the retrieval task is assumed at lane $c$. Having the acceleration and deceleration of the shuttle taken into account, the travel time for the shuttle is

$$\tau_3(c) = \begin{cases} 
2(2v_S/a_S + (D(c) - \frac{v_S^2}{a_L})/v_S), & D(c) > \frac{v_S^2}{a_S} \\
2\left(\sqrt{\frac{D(c)}{a_S}}\right), & \text{Otherwise.} 
\end{cases}$$

If $c$ is assumed to be uniformly distributed among the total $C$ lanes, the average service time of Process 3 is

$$ES_3 = \frac{\sum_{c=1}^{C} \tau_3(c)}{C} + t_S.$$  \hfill (9)

**Process 4:** The empty lift moves from its current location $z$ to tier $y$ and loads the shuttle with the SKU. Similar to Process 1, the probability that the lift is at tier $z$ when requested by a loaded shuttle is

$$P_1(x) = \begin{cases} 
1, & z = 1 \\
\frac{1}{2(T - 1)}, & z = 2, 3, \ldots, T. 
\end{cases}$$

The vertical move distance for the lift is

$$H_4(y, z) = |z - y| \cdot h.$$  \hfill (10)

The moving time of the lift is

$$\tau_4(y, z) = \begin{cases} 
2v_L/a_L + \frac{H_4(y, z) - \frac{v^2_L}{a_L}}{v_L}, & H_4(y, z) > \frac{v^2_L}{a_L} \\
2\left(\sqrt{\frac{H_4(y, z)}{a_L}}\right), & \text{Otherwise.} 
\end{cases}$$

Hence, the average service time of Process 4 is

$$ES_4 = \frac{\sum_{z=1}^{T} \sum_{y=2}^{T} P_4(z) \tau_4(y, z)}{T - 1} + t_L.$$  \hfill (12)

**Process 5:** The lift carrying the shuttle with the SKU moves to the first tier and then unloads the shuttle upon arrival. The move distance for the lift is $H_5(y) = (y - 1)h$. The move time for the lift is

$$\tau_5(y) = \begin{cases} 
2v_L/a_L + \frac{H_5(y) - \frac{v^2_L}{a_L}}{v_L}, & H_5(y) > \frac{v^2_L}{a_L} \\
2\left(\sqrt{\frac{H_5(y)}{a_L}}\right), & \text{Otherwise.} 
\end{cases}$$

The shuttle enhances the lift to the retrieval tier. The lift travels from the current tier to the retrieval tier. The lift travel and reads the SKU. The lift is released.
Therefore, the average service time of this process is

$$ES_5 = \frac{\sum_{y=2}^{T} \tau_5(y)}{T - 1} + t_L. \quad (15)$$

Process 6: The shuttle is released after it releases the SKU and its service time is $ES_6 = t_5$.

**IV. QUEUING MODEL AND SOLUTION FOR TIER-TO-TIER SBS/RS**

**A. SEMI-OPEN QUEUING NETWORK**

In this section, the retrieval process of the tier-to-tier SBS/RS is modeled as an SOQN. Various performance metrics of the system are investigated, including the utilization of shuttles, the utilization of lift, and task cycle time. When a retrieval task comes, it is assigned to a shuttle, if available, before entering the queuing network system, so the number of tasks under operations within the queuing network is not greater than the number of shuttles. If no shuttle is available, the task needs to wait in an external queue. A shuttle that has been associated with a task waits for the lift in the queue. A retrieval task will leave the queuing network when it is completed after the six processes and releases the associated shuttle. The released shuttle is assigned to or waits for the next retrieval task.

The SOQN of the system is illustrated in Figure 3. In this network, $\mu_1$ represents the rate by which a shuttle travels from the first lane to the retrieval lane to get an SKU and return to the first lane waiting for the lift. $\mu_2$ is the rate by which a shuttle unloads an SKU at the bottom tier. Once a shuttle is paired with a retrieval task, the shuttle will not be released until the task is completed, but the lift can be released more than once, since a shuttle will request the lift twice during performing a retrieval task. The first request for the lift is to transport the empty shuttle with two stages:

1. The lift travels to the first tier and loads the empty shuttle;
2. The lift with the empty shuttle moves to the retrieval tier and then unloads the shuttle. The two stages are denoted by $w_1$ and $w_2$ in Figure 4, respectively. The second request for the lift is to transport the shuttle carrying the SKU and its movement is reverse to that in the first time, whose two stages are represented by $w_3$ and $w_4$ in Figure 4, respectively.

**B. THE SOLUTION PROCEDURE OF SOQN**

The queuing network is decomposed into a closed queuing network and an open queuing network [41], [42], in which the closed queuing network is solved by the approximate mean value analysis algorithm (AMVA). The queuing network is solved by the following three steps.

Step 1: The queuing network is conformed into a closed queuing network (CQN) by removing the synchronization node. There are six nodes in the closed queuing network and the system throughput $TH_1$ can be calculated through the AMVA method.

Step 2: The synchronization node is replaced by a load-dependent exponential node as the second CQN. Similarly, the throughput of the second CQN, $TH_2$, can also be calculated through the AMVA method.

Step 3: The synchronization node is synchronized and the visit of node $i$ equals to the task arrival rate, since the system can be stabilized only if the task arrival rate is smaller than the system’s maximum throughput rate determined in the first step.

**C. THE PROCEDURE OF AMVA**

The main parameters used in AMVA are listed in TABLE 3. For the visiting ratio of each node, we can refer to the method of Bolch et al. [43]. The visiting rate of node $i$ is $\lambda_i = \sum_{j=1}^{N} \lambda_j P_{ji}$. The visiting ratio at movement node $i$ is $v_i = \lambda_i / \lambda$. The calculation process of AMVA method is detailed as follows.

1. Initialization.

\[P_i(0)(0) = 1, \quad i = 1, \ldots, N,\]
\[Q_i(0) = 0, \quad i = 1, \ldots, N,\]
\[L_i(0) = 0, \quad i = 1, \ldots, N,\]
\[EL_i(0) = 0, \quad i = 1, \ldots, N.\]

2. For $i = 1, 2, \ldots, N$,

\[ES_{rem,i} = \frac{C_i - 1}{C_i + 1} ES_i + \frac{2}{C_i + 1} \frac{1}{2ES_i}.\]

3. For $j = 1, 2, \ldots, R$,

\[ET_i(j) = Q_i(j-1)ES_{rem,i} + EL_i(j-1)\frac{ES_i}{C_i} + ES_i.\]
TABLE 3. Parameters used in the AMVA.

| Parameters | Meaning |
|------------|---------|
| $\lambda_i$ | The visiting rate of node $i$ |
| $P_{ji}$ | The probability that a job is transferred to node $j$ immediately from node $i$. |
| $N$ | Number of nodes in the SOQN |
| $R$ | Number of shuttles in the system |
| $ES_{em,i}$ | Mean time until the first departure at movement node $i$ |
| $ET_i$ | Response time of a job at movement node $i$ if there are $j$ shuttles in the queuing network |
| $ES_j$ | Average service time of the servers at movement node $i$ |
| $ES_j^2$ | Second moment of the service time at movement node $i$ |
| $L_i$ | The service time of the lift at node $i$ |
| $L_{0j}$ | Queue length of shuttles (tasks) at movement node $i$ if there are $j$ shuttles in the queuing network |
| $EL_i(j)$ | Queue length of shuttles (tasks) at movement node $i$, excluding the shuttle in service, if there are $j$ shuttles in the queuing network |
| $C_i$ | Number of servers at movement node $i$ |
| $Q(j)$ | Probability of all servers being busy at movement node $i$ if there are $j$ shuttles |
| $P_i(k|j)$ | Probability of $k$ servers being busy at movement node $i$ if there are $j$ shuttles in the queuing network |
| $TH(j)$ | Overall throughput if there are $j$ shuttles in the queuing network |
| $v_i$ | Visiting ratio at movement node $i$ |
| $TH_i$ | The throughput of the system, $i = 1, 2$ |
| $L_f$ | The queue length of shuttles |
| $L_s$ | The sum of the queue length of shuttles at each node except the synchronization node |

(2) $TH(j) = \sum_{i=1}^{j} \left( v_i \cdot ET_i(j) \right)$

(3) For $i = 1, 2, \ldots, N$, and $k = 1, 2, \ldots, \min(C_i - 1, j)$,

$$P_i(k|j) = \frac{ES_i}{k} \cdot v_i \cdot TH(j) \cdot P_i(k-1|j-1);$$

(4) For $i = 1, 2, \ldots, N$, if $j < C_i$, $Q_i(j) = 0$, otherwise,

$$Q_i(j) = \frac{ES_i}{v_i} \cdot TH(j) \cdot Q_i(j-1) + P_i(C_i - 1|j-1);$$

(5) For $i = 1, 2, \ldots, N$,

$$P_i(0|j) = 1 - \sum_{k=1}^{\min(C_i - 1, j)} P_i(k|j) - Q_i(j);$$

(6) For $i = 1, 2, \ldots, N$, if $j < C_i$; $EL_i(j) = 0$, otherwise,

$$EL_i(j) = \frac{ES_i}{v_i} \cdot TH(j) \cdot [EL_i(j-1) + Q_i(j-1)];$$

(7) For $i = 1, 2, \ldots, N$,

$$L_i(j) = TH(j) \cdot ET_i(j)$$

In the second step, the throughput of the system, $TH_2$, and the queue length $L_f$ of shuttles at node $N + 1$ can be obtained through the AMVA method. Thus, the utilization of a shuttle is $\rho_S = 1 - \frac{L_f}{T_C}$. The sum of the queue length of shuttles at each node except the synchronization node is $L_s = \sum_{i=1}^{N} L_i$. Then the task cycle time is $TCT = \frac{L_s}{\rho_L}$. The utilization of lift is $\rho_L = \frac{TH_2}{\sum_{i=1}^{N} (v_i \cdot ES_i')}$, where $ES_i'$ denotes the service time of the lift at node $i$.

TABLE 4. Parameters of the simulation.

| Parameter | Value |
|-----------|-------|
| rack tiers | 11 |
| lanes | 100 |
| shuttles | 3 |
| $\lambda$ | 60, 80 and 100 tasks/hour |
| $v_i$ | 2m/s |
| $v_L$ | 3m/s |
| $a_L$ | 1m/s² |
| $a_L'$ | 1m/s² |
| $w$ | 0.5m |
| $h$ | 0.8m |
| $t_r$ | 4.5s |
| $t_L$ | 3s |

D. SIMULATION VALIDATION

The retrieval process of the tier-to-tier SBS/RS is simulated by MATLAB 2015 to validate the performance obtained from solving the SOQN model. The analysis model established through SOQN can be calculated directly with MATLAB, which takes 0.0625 seconds using a normal laptop. For simulation, the time required is approximately 2 hours per quarter run. For each scenario, 50 replications of a 24-hour simulation are run. The parameters of the simulation are presented in TABLE 4 and all parameters are set based on the actual data of warehouse construction and equipment performance parameters. The 3-dimension parameters conform to the real-world e-commerce warehouse project located in Guangdong Province China, and widely used in other articles [43], [44]. The warehouse is less than 10 meters high and about 70 meters in length. These two values meet the depth and the number of racks settings in the simulation. More, the mechanical parameters of equipment are also representative. In generally, the velocity of the lift ranges between 2 and 5 meters per second with its acceleration varies from 1 to 4 meters per second squared; and the velocity of the shuttle ranges between 1 and 5 meters per second with acceleration varies from 1 to 2 meters per second squared. According to the actual situation of the warehouse, the task arrival rate is between 60 and 100 tasks/hour. The tasks are randomly generated and the input data for simulation are tasks with arrival rate of 60, 80 and 100 tasks/hour respectively. Based on the above simulation environment, the simulated task cycle time is about 50 to 60 seconds, which is in line with the actual situation.
The performance metric, namely the utilization of shuttles, the utilization of lift and the task cycle time ($TCT$) are used to validate the analytical model. The simulation results are shown in Table 5, and the detailed data are in Table 6.

The relative error percentages are calculated by formula $\frac{|\text{Sim} - \text{Ana}|}{\text{Sim}} \times 100\%$, where $\text{Ana}$ and $\text{Sim}$ represent the results from the analytical SOQN model and simulation, respectively. The results are summarized in Figure 5. In these figures, the abscissa represents the percentage of relative error, and the ordinate represents the frequency of the corresponding relative error. The mean relative errors of the utilization of shuttles and the utilization of lift are less than 10%, and when the arrival rate is low, the mean relative error of task cycle time is low. As the arrival rate increases, due to the error of the task waiting queue, the mean relative error of $TCT$ will become larger. Due to the assumption that the blockage of the shuttles in the aisle is ignored, with the increase of the arrival rate, the blockage of the shuttle will increase, so the relative error increases. It can be seen that SBS/RS is suitable for the situation with low arrival rate. However, the error is acceptable because of the complexity of the system.

V. NUMERICAL ANALYSIS

Based on the built SOQN, the system performance is evaluated for various configurations. First, the effect of the number of shuttles, $R$, on $TCT$ is discussed. Figure 6(a) summarizes $TCT$ for each rack configuration (i.e., the number of tier $T = 9, 11,$ and $13$) where the task arrival rate is set 80 tasks/hour. With the similar pattern in the relationship...
between the number tiers and shuttles (as shown in Figure. 6(a), we set the number of tiers, T, at 11 and explore the influence of task arrival rate on the optimal number of shuttles, as shown in Figure. 6(b). According to task arrival rate and TCT in the Figure. 6(b), the following conclusions can be drawn: (1) Task arrival rate has a great influence on TCT. TCT increases significantly in task arrival rate because the shuttle blockage was ignored, especially when the number of shuttles is two. (2) When the task arrival rate \( \lambda \leq 80 \), the TCT of four or more shuttles is basically the same. So, four shuttles are enough. When the rate \( \lambda > 80 \), the TCT of 4 vehicles increases faster, and five shuttles are needed to reduce the TCT. When the number of shuttles is more than five, it has little effect on TCT, and it will increase the waiting time of shuttles, resulting in a waste of resources. It can be seen that when the arrival rate increases to a point, continuing to increase the number of shuttles will not improve TCT, indicating that the lift is the bottleneck of the system. (3) In summary, the threshold number of shuttles is four. When there are fewer than four shuttles, TCT will increase steeply while the number of shuttles is reduced. However, when there are more than four shuttles, adding more shuttles can only improve TCT little.

Finally, the proper number of shuttles related to the maximum velocity of the shuttle is studied. In our experiment, the number of rack tiers is fixed at 13; the task arrival rate is fixed at 80 tasks/hour. Generally, the height of the rack is not high enough for the lift to reach its maximum velocity, so it is not necessary to consider the effect of the maximum velocity of the lift, then we define \( v_L = 3m/s \). Since there is only one lift in each aisle, the efficiency of the system would not be improved with more shuttles beyond a certain value. The variation of the proper value corresponding to the maximum velocity of shuttles is shown in Figure. 7(a), and the related TCT is presented in Figure. 7(b). The TCT corresponding to a specific velocity and proper shuttle number represents the minimum TCT of the system with specific acceleration of the lift, rack configuration and task arrival rate.

According to the analysis, we can choose a proper number of shuttles according to the maximum velocity of the shuttle when designing the system. As shown in Figure. 7(a), for example, when \( a_L = 1m/s^2 \) and \( a_L = 2m/s^2 \) and the velocity of the shuttle is beyond 3\( m/s \), the proper number of shuttles maintains 3, and the respective minimum TCT of the system is shown in Figure. 7(b). When \( a_L = 3m/s^2 \), and the velocity of the shuttle is beyond 2\( m/s \), the proper number of shuttles maintains 3. Figure. 7(b) also indicates that when the acceleration of the lift is beyond 2\( m/s^2 \), the efficiency of the system will not change remarkably when the velocity of the shuttle is more than 3\( m/s \). On the one hand, the lift generally does not reach the maximum velocity, so the acceleration of the lift has a great impact on the efficiency of the tier-to-tier SBS/RS. On the other hand, the shuttle will reach the maximum velocity in most cases, so the maximum velocity of the shuttle has a greater impact on the tier-to-tier SBS/RS.
VI. CONCLUSION

This paper models the tier-to-tier SBS/RS and analyzes for the coordination of the shuttle sub-system and the lift sub-system, in which each shuttle can visit any tier of the rack with the help of a lift. An SOQN is established based on a detailed analysis of the movement processes of shuttles and the lift. By removing or adding the synchronization node, the SOQN is further transformed into two different CQNs, and the AMVA algorithm is used to solve the model and obtain the system performance. Numerical experiments based on a simulation model are used to validate the SOQN-based analytical model. The comparison indicates that the queuing network can calculate the performance of the system with high accuracy. By applying this model, the performance of the tier-to-tier SBS/RS can be estimated in a short time, and the efficiency of the SBS/RS can be optimized by changing the parameters, such as the number of shuttles, storage rack parameters and device performance.

The TCT is affected in many ways, such as the number of rack tiers, the velocity of shuttles, the acceleration of the lift, and the task arrival rate. We found that adding more shuttles beyond a certain number into the system cannot improve its performance. These results show that an effective analytical model is useful during the design phase of the system.
tier-to-tier SBS/RS. When deciding the optimal configuration (e.g., the number of shuttles and lifts), designers must take device speeds and costs into consideration. More shuttles will sure increase capital investment but not always lead to higher efficiency.

Because of the complexity and diversity of SBS/RS, further research is necessary for other types of SBS/RS, such as the tier-captive SBS/RS. In future research it may be necessary to optimize the model to make the presented SOQN network model suitable for SBS/RS with higher arrival rates. In the current setting, the error of our SOQN-based analytical model is acceptable for designing systems with low arrival rate. Moreover, the presented network model may be extended by considering dual command cycles. Studying the blocking problem of shuttles in one aisle is also of great interest. Moreover, we only consider the random storage strategy. However, other storage strategies (such as class-based or dedicated storage) can also be interesting. In addition, if multiple lifts are used, the model may be adapted to a higher task arrival rate.

**APPENDIX**

See Tables 6 and 7.

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| TABLE 7. Abbreviations. |
|------------------------|
| **Abbreviation** | **Meaning** |
| SBS/RS | Shuttle-Based Storage and Retrieval System |
| AVS/RS | Autonomous Vehicle Storage and Retrieval System |
| CQN | Closed Queuing Network |
| QQN | Open Queuing Network |
| SOQN | Semi-Open Queuing Network |
| SKU | Stock Keeping Units |
| AMVA | Approximate Mean Value Analysis |
| DC | Dual Command |
| TCT | Task Cycle Time |
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