Grey box identification and adaptive control in a water level system

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Abstract. We study the application of Adaptive Control to estimate external disturbances in a Water Level System model, taking advantage of a constructed nonlinear grey box identification that allows to exploit its dynamics. The external disturbances will be specially related to valve manipulations considering limited sensing information using only water height sensors. Our methodology is twofold: (1) Using a grey box model to identify the plant under an operating point and, (2) using adaptive control to react against changes in the system or perturbations on the valve and still lead to a convergent and asymptotically stable controller.

1. Introduction

Water Level Systems are a common equipment in many industries, although PLC with built-in classical controllers are enough for many applications \cite{1}, sometimes system disturbances are mild and therefore intelligent controllers need to be used. When dealing with uncertainties it is common to use a variable structure controller or adaptive control scheme, while controllers like Sliding Mode Control are robust, they generate large control efforts in comparison to adaptive controllers. An adaptive algorithm can be used to tune gains of the controller \cite{2} or to construct a learning law to compensate for the unknown parameters in the plant. Many approaches use ARMA or other black box models to identify a plant; however, using those approaches make a designer diverge from his or her physical interpretation and intuition of the model. If a parameter changes, it is not trivial to map it to the generated model. We propose to use a Grey Box Identification algorithm in order to preserve the integrity of the model and its relationship with the parameters. Once the identification of this plant with a defined structure is realized, varying parameters can be recognized due to unmodeled dynamics or physical changes in the system introduced externally. For instance, in a water level system, the hydraulic resistance is characterized by the slope of the pressure change with respect to the mass flow rate, which renders a nonlinear relationship that changes depending on its operating point \cite{3}. To estimate the parameters to aid in the control action, we propose to use a Lyapunov based adaptive controller \cite{4}. Adaptive schemes have been proposed for water level systems, although several of them do not exploit the problem structure \cite{5,6} and sometimes do not use a Lyapunov verification of stability. We provide a method based on Lyapunov stability, which exploits the structure of the water level dynamics to guarantee asymptotic convergence of the system response to the reference point.

2. Water Level System

A plant corresponding to a two-tank system is shown in figure 1(a), this water level system is well known \cite{3} and can be modelled as two tanks connected through water pipes as shown in figure 1(b). Tank 1 serves as a water supplier for tank 2 and it is refilled through a pump before starting operation.
Figure 1. (a) Physical Water Level System implementation. (b) Model of the two-tank water level system

The tanks have certain water heights \((h_1, h_2)\) in the cylindrical reservoirs with areas \(A_1\) and \(A_2\). An electric pump generates the initial conditions of the tank 1. A solenoid valve regulated by a voltage \(V(t)\) controls the input flow \(q_1(t)\) of the tank 2; the output flow \(q_2(t)\) of tank 2 can be affected by the manual valve 2 that is considered as a system perturbation. The measured signals are the water level \(h_1\) and \(h_2\) of both tanks and the water flow \(q_2(t)\).

2.1. System Modelling

Using Bernoulli’s law, a relationship between any two reference sections in the water can be expressed according to:

\[
g z_1 + \frac{p_1}{\rho} + \frac{v_1^2}{2} = g z_2 + \frac{p_2}{\rho} + \frac{v_2^2}{2}
\]

Where, \(v_k\) is the fluid speed in the considered section \(k\), \(\rho\) is the fluid density, \(p_k\) is the pressure along the flow in the section \(k\), \(g\) is the gravity and \(z_k\) is the height of the water column with respect to the reference for \(k \in \{1, 2\}\)

The subscripts indicates the top and bottom levels respectively. The relationships between the flow rates \(q_1\), \(q_2\) and the water levels \(h_1\), \(h_2\) are as follows.

\[
q_1 = c_1 \sqrt{2 g h_1} = k_1 \sqrt{h_1(t)} u(t) \quad (2)
\]

\[
q_2 = c_2 \sqrt{2 g h_2} = k_2 \sqrt{h_2(t)} a
\]

Where \(c\) is the discharge coefficient of the valve [2], we assume a model where the input voltage directly affects the flow passing from tank 1 to tank 2. Also, \(k_1\) and \(k_2\) are gains that are defined to obtain a desired restriction ([0 - 1]) for \(a(t)\) and \(u(t)\), the perturbation and the input signal respectively. Hence, the relation between the flow rates \(q_1\), \(q_2\) and the dynamic system are as follows.

\[
\frac{d}{dt} h_1(t) = -\frac{q_1}{A_1} \quad (4)
\]

\[
\frac{d}{dt} h_2(t) = \frac{q_1}{A_2} - \frac{q_2}{A_2} \quad (5)
\]

By combining (2), (3) and (4), (5), the mathematical model of the system is expressed as follows.
\[
\frac{dh_1(t)}{dt} = -\frac{k_1 \sqrt{h_1(t)} u(t)}{A_1} \tag{6}
\]

\[
\frac{dh_2(t)}{dt} = \frac{k_1 \sqrt{h_1(t)} u(t)}{A_2} - \frac{k_2 \sqrt{h_2(t)} a}{A_2} \tag{7}
\]

Where \( u(t) \) is a mapped function for the input voltage \( V(t) \) which operates in the range \([v_0 - V_{max}]\).

The relation between \( u(t) \) and \( v(t) \) is as follows.

\[
u(t) = \frac{V(t) - v_0}{V_{max} - v_0} \tag{8}\]

In this work, the constant parameters \((k_1, k_2, v_0)\) are estimated using the Grey System Identification, through a non-linear model (6), (7). Also \( a(t) \) is a mapped function for the aperture of the valve (figure 1(b)), that is unknown and must be estimated by the algorithm in order to drive the system to the required set-point.

2.2. System setup

The water level system has a central processing unit based on Arduino Due with a 32-bit Atmel SAM3X8E ARM Cortex-M3 microcontroller. In this work, pulse width modulation (PWM) signal is converted to DC voltage that is used as an input of the solenoid valve. This valve provides control action on the system and operates in the range 5.7 - 10 VDC.

As shown in figure 1(b), two sensors are used in the system. A sensor Allen Bradley (871TM-B5N18-H2) measures the water level in tank 2 and an ultrasonic sensor (HC-SR04) is used to measures the water level for tank 1. Additionally, we use a water flow sensor (YF-S201) located in series in the output pipe of tank 2. This sensor is used to verify the estimation process.

3. Nonlinear grey box identification

Linear systems models can describe real models quite well, even when many systems are in fact, non-linear. The reason is that some non-linear systems can be approximated by linear models when the inputs and outputs are within certain ranges. However, sometimes-linear models are unable to capture the system behaviour, especially when operating in a broad range of operating points where non-linear phenomena occur.

If we understand the physics of a system and we can express it as a series, of Ordinary Differential Equations (ODE) we can represent the model with states and known and unknown parameters in the continuous time domain (or discrete time difference equations). With this representation we can use a grey box nonlinear identification. Grey box modelling is useful when you know the relation between variables, constraints or explicit equations that represent the dynamics of the system.

In our case, we use a grey box model in continuous time, for data acquired experimentally using the MATLAB software.

3.1. Nonlinear physical models

Non-linear state space models in continuous time are described by:

\[
\dot{x}(t) = f(x(t), u(t), \theta) \\
y(t) = h(x(t), u(t), \theta) + e(t) \tag{9}
\]

Where \( e(t) \) is white noise, \( x(t) \) the states of the system, \( u(t) \) is the input signal, \( y(t) \) is the system response and \( \theta \) the unknown parameters to be fitted. In a similar manner we define the estimated values \((\hat{x}, \hat{y}, \hat{\theta})\):

\[
\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), \theta)
\]
\[ y(t|\theta) = h(\hat{x}(t), u(t), \theta) + e(t) \]  

(10)

The objective of the identification is minimizing the prediction error using the cost function.

\[ J(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} (y(t_i) - \hat{y}(t_i|\theta))^2 \]  

(11)

In MATLAB Identification Toolbox, non-linear state space models (idnlsgrey) are defined and estimated using the dynamic structure as a MATLAB function. The two-tank system model (6,7) is then coded in the function nltank listing below.

Where, the state vector \( x \) represents \((h_1, h_2)\) and the vector input represents \((V(t), a(t))\), at (8). The next arguments are known parameters \((A_1, A_2, V_{max})\) and unknown parameters \((k_1, k_2, v_0)\). To identify the order and the parameters of the model, a nonlinear grey box model object must be created following the script below.

In the script shown, sysdim=[ny,nu,nx] defines the number of outputs, inputs and states of the model, respectively. More advanced alternatives are also available, including by naming each parameter and defining its minimum and maximum value, [7]. Finally, when running the estimation procedure with (pem), k1_est, k2_est, and v0_est are the resulting estimated parameters.

The basic workflow of the identification consists in the construction of a model for the dynamics of the system (nltank), setting the known and unknown variables (nlp) and then the acquisition of experimental data, which will be used to estimate (pem) the actual model values.

4. Adaptive Control

Given the model of the Water Level System in (6), (7), we can set the output \( \hat{h}_2 = h_2 - h_2^r \) as our objective function to drive to zero. In order to construct a reduced order system, we need to investigate the relative degree of the system corresponding to the desired output \( \hat{h}_2 \).

\[ \dot{\hat{h}}_2 = \hat{h}_2 = -\frac{k_2 \sqrt{h_2}}{A_2} a + \frac{k_1 \sqrt{h_1}}{A_2} u \]  

(12)

It can be observed that the output is relative degree one \((\rho = 1)\) since \( \dot{\hat{h}}_2 \) contains the control signal \( u \).

4.1. Adaptive surface

The construction of the surface \( s \) can be formulated similar to the case of sliding mode control [4,8], considering that it can have the structure in (13), for which we propose \( s = \lambda + \hat{h}_2 + \hat{h}_2 \) as a sliding surface. Note that \( s = 0 \) implies \( h_2 \to 0 \) when \( \lambda \to 0 \).

\[ s = (\lambda + \frac{d}{dt} u)^{\rho-1} \]  

(13)
4.2. Stability
In order to guarantee stability, we will propose a Lyapunov-like function that uses the vector of unknown parameters, \( p \).

\[
V = \frac{1}{2} s^2 + \frac{1}{2} \tilde{p}^T P^{-1} \tilde{p}
\]  
(14)

Where \( \tilde{p} \) is the estimated parameter vector and \( \tilde{p} = \hat{p} - p \) is the estimation error. Therefore, \( P^{-1} \) is a symmetric positive definite function. By the Barbalat lemma, if \( V(t, x) \) is a lower bounded continuously uniform scalar function, with \( \dot{V} \) negative semidefinite along \( \dot{x} = f(t, x) \), then \( \dot{V}(t, x) \to 0 \) as \( t \to 0 \).

\[
\dot{V} = s \dot{s} + \tilde{p}^T P^{-1} \tilde{p}
\]  
(15)

\[
\dot{V} = s\left(-\frac{k_2\sqrt{h_2}}{A_2} a + \frac{k_1\sqrt{h_1}}{A_2} u\right) + \tilde{p}^T P^{-1} \tilde{p}
\]  
(16)

\[
\dot{V} = s\left(-\frac{k_1\sqrt{h_1}}{A_2} a + u\right) + \tilde{p}^T P^{-1} \tilde{p}
\]  
(17)

By the physics governing the system, it is straightforward to note that \((k_1, h_1, h_2)\) are always positive.

4.3. Controller construction
A vector of known functions \((Y)\) is introduced, together with the vector of unknown parameters \((p)\).

\[
Y = \frac{\sqrt{h_2}}{k_1\sqrt{h_1}}, \quad p = k_2a
\]  
(18)

In order to drive \( s \) to zero, a controller dependent on \( Y \) and \( p \) is proposed together with a linear action on the surface \( s \):

\[
u = Y\hat{p} - ks
\]  
(19)

\[
\dot{V} = s\frac{k_1\sqrt{h_1}}{A_2} (Y\hat{p} - ks) + \tilde{p}^T P^{-1} \tilde{p}
\]  
(20)

To obtain a negative definite \( \dot{V} = \beta s^2, \beta > 0 \), the rest of the terms will be cancelled through the selection of an adequate \( \hat{p} \). Note that \( \tilde{p} = \hat{p} \).

\[
s\frac{k_1\sqrt{h_1}}{A_2} Y + \tilde{p}^T P^{-1} = 0
\]  
(21)

\[
\hat{p} = \frac{s k_1\sqrt{h_1}}{A_2} p^T Y^T
\]  
(22)

The adaptation law depends only in known parameters and the system states; thus, it can be implemented together with the controller. As the algorithm learns the parameter \( p = k_2a \), the control action improves and drives the surface \( s \) to zero.
5. Results

**Grey Box Identification:** Considering the system (7) defined with \( a = 0 \) we can apply ID Grey to estimate the values of both \( k_1, v_0 \), without worrying about \( k_2 \). Capturing data from our implementation for different values for \( V(t) \), and using the identification algorithm we achieve an accuracy of: 93.6\% and a final prediction error (FPE) of 9.6063e-06. The model correlation is observed in figure 2(a). The resulting parameters obtained through the identification are \( k_1 = 2.18 \times 10^{-4}, v_0 = 5.7009 \). Similarly, using a known value of \( a \), for instance when it is completely open (\( a = 1 \)) the estimation of the parameter \( k_2 \) has an accuracy of: 98.54\% and an FPE: 1.4067e-06. The model correlation is observed in figure 2(b). The resulting value for \( k_2 \) is \( k_2 = 1.7286 \times 10^{-4} \). It should be noted that accuracy of the identified model has a low prediction error (FPE), however, changes on parameters can still affect the plant dynamics. It has been observed during experimentation that the constant \( k_2 \) can also change depending on the operating point of the system (dependent of the presence of laminar or turbulent flow).

![Simulated Response Comparison](image1)

**Figure 2.** Estimation using \( a = 0 \) and \( a = 1 \).

**Setpoint tracking:** A setpoint is defined at \( h_2 = 0.20m \) with initial condition of the system \( h_1 = 0.2149m, h_2 = 0.1187m, \) and \( a_0 = 0.2 \).

Applying the adaptive control algorithm reveals that experimental data and simulation data are very similar in their evolution. Tracking results in the system closed loop system shows that the controller drives the height of tank 2 to the reference value as shown in figure 3(a). Furthermore, it is observed that the unknown parameters converge to their true value. The adaptation law has an overshoot due to the slowly evolution of the level of the tank 2. Although, it has not been formally proved, a rich reference that captures different operating points will provide an asymptotic estimation of parameters \( p \), this behavior is observed by both simulation and experimentation.

![Simulated Response Comparison](image2)

**Figure 3.** Height variation in Tank 2, control action \( u \) and parameter estimation for (a) simulation and experiments and (b) experiments with heavy perturbations.
**External disturbances:** The system is tested against external disturbances when the valve 2 is operated twice during the experiment. The results in figure 3(b) indicates again a great correlation between simulation and experimentation. Also, we observe that asymptotic tracking is achieved even when disturbances are applied. Finally, the parameter $a$ is estimated to its actual value even in the presence of two external disturbances.

6. **Conclusions**

We propose a workflow to work with Water Level Systems with unknown parameters by using a grey box model and an adaptive control scheme that can handle varying parameters and drive the system to the reference values. From our results it is clear that we obtain asymptotically stability of the system even in the presence of external disturbances. The system also has the power of learning the actual value of $a$, important for industrial operators that might be worried about open valves. Also, our controller has a very simple structure that can be easily implemented on micro-controllers as in our setup with Arduino Due with a loop frequency at 1Hz.

7. **References**

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