Modeling Systems Based on the Assessment of Weak Symmetry Breaking in Reconstructed Attractors

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Abstract

The paper proposes to use the research method for chaotic processes based on the identification of weak symmetry breaking of the restored attractor. It is shown how the calculations results can be used to identify systems. An algorithm for creating finite-difference models has been developed, including: calculating by numerical methods the necessary conditions for the existence of chaos, reconstructing an attractor in a time series, searching for symmetric attractor fragments under conditions of weak symmetry breaking, determining the form of nonlinearities, and parametric identification. The result of the algorithm is a system of finite-difference equations in the state space. Criteria for assessing symmetry breaking based on estimates of the divergence of fragments of phase trajectories are introduced. The results of modeling systems with chaotic dynamics are presented.

Keywords: attractor, chaotic processes, symmetry

1. Introduction

Currently, chaotic signals are used in various fields of science and technology — in metallurgy [1], mechanical engineering [2], tasks of automation of technological processes [3, 4], for modeling networks [5, 6] and many others. This indicates the great interdisciplinary significance of scientific results in the field of the theory of chaotic dynamics. In many practical applications, it is convenient for researchers to act on the assumption of the chaotic nature of the observed process. The attractor is easy to recover; there are many numerical methods for estimating dimensions. Methods for the approximate solution of the inverse problem of dynamics are developed, i.e. restoration of a given type of differential equations from experimental data.

However, the assumption of chaotic dynamics is often based only on estimates of the necessary conditions for the existence of chaos, such as an estimate of the senior Lyapunov exponent. The paper presents an algorithm for detecting weak symmetry breaking for the analysis of the chaotic nature of the reconstructed phase space [7, 8]. It is shown how the results of calculations can be used to identify systems.

For the study, a time series \{y_0, y_1, ..., y_k, y_{k+1}, ...\} is used, while it is assumed that the time series is generated by the system, which discretized form has the following form:

\[
x = f(x_k, x_0),
\]

\[
y_k = g(x_k),
\]

where \(x\) — n-dimensional point in the state space; \(y_k\) — observed one-dimensional process; \(k\) — discrete time (number); \(f, g\) — vector function.

Using discrete models allows operating with data obtained as a result of the experiment, and accumulate no errors associated with the transition to continuous models, however, this complicates the qualitative analysis of processes.
As is known, a phase trajectory is a trajectory depicting how the state of the dynamical system $x$ changes with time $t$. For discrete systems, states are connected by lines, in accordance with the sequence of samples $k = 1, 2, \ldots$. The area to which all possible trajectories of the systems evolve (converge) is called an attractor. For discrete systems, as well as for continuous, the attractor is a point, for oscillatory systems — closed trajectories (cycles). For chaotic systems, there is an attractor called strange, in which case the trajectories are contracted, but not to a point, a curve, a torus, but to a certain subset of the phase space.

Currently, there are different points of view on the number and nature of scenarios of transition to chaos:

- Feigenbaum scenario (through a cascade of period doubling bifurcations);
- Afraimovich-Shilnikov scenario (Ruelle-Takens model) (through the destruction of a two-dimensional torus or through the destruction of a closed invariant curve);
- Pomeau-Manneville scenario (alternating in time of almost regular oscillations with intervals of chaotic behavior).

The study of scenarios for transition to chaos plays an important role in practice, since it makes it possible in some cases to predict the possibility of a chaotic behavior of a dynamic system by the control parameters change. When developing the method, the second type of scenario was used, which allows the use of methods of qualitative nonlinear dynamics.

The attractor is an invariant characteristic of the system, i.e. it is stored during transform actions. Dynamic systems for which the $n$-dimensional phase volume decreases are called dissipative; if the phase volume is preserved, then such systems are called conservative. Conservative systems always have at least one conservation law. Conservation laws are determined by symmetries allowed by the system.

The problem considered in the paper consists in constructing a model of the form (1) from the observed time series.

### 2. Modeling systems with chaotic dynamics based on the assessment of weak symmetry breaking in a reconstructed attractor

A method for modeling systems with chaotic dynamics was developed. This method allows finding a solution to the inverse problem of dynamics on a minimal invariant manifold in the class of affine systems.

The essence of the method is as follows. The initial data is the time series generated (presumably) by a system with chaotic dynamics.

* The necessary conditions for the existence of chaos are calculated by numerical methods — the largest Lyapunov exponent (for chaotic dynamics it must be greater than zero), etc.
* The attractor is restored by the Eckardt method (delay method).
* If the assumption of chaotic dynamics is confirmed, then research continues.
* Based on the analysis of the reconstructed attractor, the presence of symmetries is checked (in conditions of weak symmetry breaking) [9].
* Using the accepted symmetries, the form of equations in the minimal invariant manifold is constructed using the Hausdorff-Lee formula [10].
* The structure of equations is parametrically identified.
* The qualitative discrepancy between the dynamics of the model and the initial series is estimated.

The output is a model in the form of finite difference equations. An algorithm for searching for symmetric sections of reconstructed trajectories under conditions of weak symmetry breaking was developed [10]. It should be noted that in this formulation, the problem of finding almost symmetric fragments is NP-complete.

### 3. Chaotic signal modeling and generation

It was shown in [11] that robust chaos can occur in piecewise-smooth systems. This result facilitates the task of modeling systems with chaotic dynamics. Attempts to carry out parametric identification of a model with arbitrarily selected types of nonlinearities, as a rule, do not find a solution at all. If a solution is found (which is rather an accident than a natural phenomenon), then the solutions will be naturally sensitive to minor changes in parameters. Indeed, in many physical phenomena, chaos occurs with a slight change in the parameter, however, in technical applications, the use of a chaotic regime usually requires a search for less sensitive ones, i.e. robust models. This requires the very essence of experiments with technical systems, as well as errors in calculations and measurements. In addition, for the problems of control synthesis, it is efficient to use the chaotic properties of systems, but provided that the chaotic regime does not change the qualitative behavior in a certain range of parameters.

It is proposed to use the model in the following form:

$$
\begin{align*}
    x_{k+1} &= A x_k + B \Psi u_k, \\
    y_k &= C x_k.
\end{align*}
$$

Here, the nonlinear function $\Psi$ is determined from the found symmetric properties of the phase trajectory according to the Hausdorff-Lee formula; $u_k$ is a discretized piecewise continuous function of time (for example, piecewise linear).

This type of model representation allows the use of parametric identification methods, for example, the least squares method implemented in the System Identification
Toolbox in MATLAB. As parameters of the ident function, the investigated \{y_k\} (output) and the generated piecewise continuous series \{ \Psi \Phi_k \} (input) are used. The output of the function is the matrix \( A, B, C \) of the found dimensions. Note that the dimension of the state space obtained as a result of identification of the system is always equal to the numerical estimates of the initially reconstructed attractor.

Here, chaos is generated as a periodic change in the linear trajectory, preventing the dynamics from arriving at a stable or periodic trajectory. Of course, not in all applications such models seem convenient, but for control systems the use of the form \((2)\) allows the use of well-known methods of synthesis and parameter estimates. In addition, for engineering tasks, ensuring the security of information systems where chaotic signals are widely used, this method allows generating robust chaos with desired properties (type of attractor, degree of symmetry breaking, etc.).

As an example, we present the type of models obtained for the cooling aluminum ingots in the aviation industry:

\[
\begin{bmatrix}
0.998 & 0.006 & -0.009 & 0.007 & -0.001 & -0.007 \\
0.008 & 0.913 & 0.238 & 0.142 & 0.091 & -0.215 \\
0.006 & 0.132 & 0.091 & -0.928 & -0.439 & -0.038 \\
-8.12 \times 10^{-5} & 0.008 & -0.177 & 0.356 & -0.956 & 0.097 \\
-0.001 & -0.06 & 0.803 & 0.203 & -0.145 & 0.139 \\
0.002 & -0.006 & 0.006 & -0.003 & 0.010 & 0.661 \\
\end{bmatrix}
\]

\[
\Psi = \exp(100)\begin{bmatrix}
-7.24 \times 10^{-5} & -0.087 & 0.209 & -0.074 & -0.440 & -0.061 \\
129.8 & -144.2 & 18.43 & -23.21 & -3.233 & -4.911 \\
582.1 & 4,995 & 2,230 & -1.596 & 2.587 & 4.100 \\
\end{bmatrix}
\]

The attractor is shown in Fig. 1. A comparison of the simulated output process with real data is shown in Fig. 2.

![Figure 1. Reconstructed Attractor.](image1)

![Figure 2. Comparison of real data with the model.](image2)

4. Conclusions

A method for creating mathematical models of chaotic processes is developed. The obtained models can be used to build robust chaos, as well as to generate chaos with specified qualitative properties.

The use of dynamic models of chaotic processes makes it possible to provide predictive proactive management for the dynamic allocation of resources in complex infrastructures. An important task is the choice of the shape of the model. Using the proposed form will allow us to obtain computationally reliable models that, on the one hand, reflect the necessary qualitative properties of nonlinear systems, and on the other, are not sensitive to insignificant changes in parameters.

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