Mechanism for time reparametrization symmetry breaking in spinor canonical gravity coupled to long-range spinor particles

Eiji Konishi*
Graduate School of Human and Environment Studies, Kyoto University, Kyoto 606-8501, Japan
(Dated: May 19, 2017)

We propose a mechanism for time reparametrization symmetry breaking in canonical gravity. We consider a model of spinor gravity, based on Sen’s reformulation of canonical gravity as a spin system, with one additional long-range self-interacting massive spinor particle that is coupled to spinor gravity. The symmetry breaking is identified with the origin of the quantum mechanical non-unitary evolution. Our theory uses an idea from the Penrose argument for space-time uncertainties in its interpretation of quantum mechanics.

I. INTRODUCTION

Since the advent of the Copenhagen interpretation of quantum mechanics[1], there have been many distinct proposals from quantum physicists with various backgrounds to resolve the problems of measurement and the origin of the quantum state reduction that generates non-unitary time evolution of state vectors[2–13].

In this paper, as a new approach, we address a connection between time itself and non-unitary time evolution in quantum mechanics by extending canonical gravity theory. Our mathematical model is based on the reformulation of canonical gravity as a spin system by Amitabha Sen[17] and in our interpretation of quantum mechanics we use the notion of the Penrose uncertainties assigned to superposed space-times[13], which we relate to the quantum superposition of originally definite (i.e., with no uncertainty) particle positions or of originally definite particle momenta in a coherent second-quantized system. Here, the second-quantized treatment of particles is needed for this interpretation to avoid the issue of the indistinguishability of identical particles whose positions and momenta are originally definite.

Before we explain our strategy, let us first give an overview of the time concept in canonical gravity. In this paper, we consider an SL(2, C) spinor field variable \( \lambda_\alpha \), with four field degrees of freedom, on \( M \) and restrict it to \( \Sigma_{t_0} \). We call this the space spinor. We denote by \( t^\mu \) the field of unit vectors on \( M \) that is normal to the family of \( \Sigma_{t_0} \) in the \( 3 + 1 \) decomposition. We restrict \( t^\mu \) to \( \Sigma_{t_0} \) and then represent the covariant four-vector \( t_\mu = g_{\mu \nu}t^\nu \), at each point \( p \) in \( \Sigma_{t_0} \), by a natural Hermitian isomorphism \( t_{\alpha' \alpha} \) between the complex conjugate space, \( W \), of the complex two-dimensional vector space \( W' \) of primed space spinors of arbitrary given kind at \( p \) and the dual space, \( V^* \), of the vector space \( V \) of unprimed space spinors of one kind:

\[
\lambda_\alpha^1 = -\sqrt{2}t_{\alpha' \alpha}l^1_{\alpha' \alpha}.
\]

Here, we lower and raise space spinor indices by contraction with a non-degenerate symplectic form \( \epsilon_{\alpha \beta} \), with

\[ H_{ADM} = \int_{\Sigma_{t_0}} (Nh_{\text{grav}} - 2N_a h^a_{\text{grav}})dS , \]

where

\[
\begin{align*}
  h_{\text{grav}} &= -R - \pi^{ab} \pi_{ab} + \pi^2, \\
  h^a_{\text{grav}} &= D_a(\pi^{ab} - \pi q^{ab}) ,
\end{align*}
\]

both of which must vanish in the absence of a cosmological constant by including the contributions from other sectors as constraints with Lagrange multipliers \( N \) and \( N^a \), respectively, in the totally constrained system with scalar curvature \( R \) and covariant derivative \( D_a \) on \( (\Sigma_{t_0}, q_{ab}) \) and \( \pi = \pi^{mn} q_{mn} \). Intuitive interpretations of Eqs. (1) and (2) are as follows: (i) when we regard \( \Sigma_{t_0} \) as the embedding of a space manifold \( S \) into spacetime \( M \), denoted by \( \epsilon = \epsilon_{t_0}(x) \), the lapse function \( N \) and the shift vector \( N^a \) are the components of the deformation of the embedding \( \partial e/\partial t_0 \) perpendicular and parallel to \( \Sigma_{t_0} \), respectively[13]: (ii) due to their role as Lagrange multipliers for the \( 3 + 1 \) decomposition in the totally constrained system, the lapse function \( N \) and the shift vector \( N^a \) are the time-dependent variables of time reparametrization and the space diffeomorphism of \( S \), respectively; and (iii) the arbitrariness of the four variables \( N \) and \( N^a \) in the space-time metric \( g_{\mu \nu} \) reflects the arbitrariness of choice of space-time coordinate system.

Next, following Sen, we recast the lapse function and the shift vector in terms of a spin system. As in Ref. [14], we consider an \( SL(2, C) \) spinor field variable \( \lambda_\alpha \), with four field degrees of freedom, on \( M \) and restrict it to \( \Sigma_{t_0} \). We call this the space spinor. We denote by \( t^\mu \) the field of unit vectors on \( M \) that is normal to the family of \( \Sigma_{t_0} \) in the \( 3 + 1 \) decomposition. We restrict \( t^\mu \) to \( \Sigma_{t_0} \) and then represent the covariant four-vector \( t_\mu = g_{\mu \nu}t^\nu \), at each point \( p \) in \( \Sigma_{t_0} \), by a natural Hermitian isomorphism \( t_{\alpha' \alpha} \) between the complex conjugate space, \( W \), of the complex two-dimensional vector space \( W' \) of primed space spinors of arbitrarily given kind at \( p \) and the dual space, \( V^* \), of the vector space \( V \) of unprimed space spinors of one kind:

\[
\lambda_\alpha^1 = -\sqrt{2}\lambda'_{\alpha' \alpha}l_{\alpha' \alpha}.
\]

Here, we lower and raise space spinor indices by contraction with a non-degenerate symplectic form \( \epsilon_{\alpha \beta} \), with

\[ H_{ADM} = \int_{\Sigma_{t_0}} (Nh_{\text{grav}} - 2N_a h^a_{\text{grav}})dS , \]

where

\[
\begin{align*}
  h_{\text{grav}} &= -R - \pi^{ab} \pi_{ab} + \pi^2, \\
  h^a_{\text{grav}} &= D_a(\pi^{ab} - \pi q^{ab}) ,
\end{align*}
\]
which we equip $V$, having a skew pair of space spinor indices. From this isomorphism, $t^{a\alpha'}$, we obtain a natural Hermitian positive-definite inner product $(\lambda, \eta) = \lambda^{\alpha} \eta_{\alpha}$, between $\lambda_\alpha$ and $\eta_\alpha$ in $V$ at a point $p$, as an additional structure on $V$. The group that preserves the structure of $(V,(\cdot,\cdot),c_{\alpha\beta})$ is $SU(2)$. So, space spinors obtained from elements of $V$ by varying $p$ over $\Sigma_{t_0}$ are in fact $SU(2)$ spinors on $\Sigma_{t_0}$. Due to this feature of space spinors, we can relate them to the geometry of $\Sigma_{t_0}$ with the induced negative-definite metric $g_{\mu\nu} = g_{\mu\nu} - t_{\mu} t_{\nu}$ such that $g_{\mu\nu} t^{\nu} = 0$ holds. For instance, we can express spatial tensors on $\Sigma_{t_0}$ that vanish by contraction with $t^{\mu}$ or $t_{\mu}$ in terms of space spinors by replacing each spatial tensor index by a symmetric pair of space spinor indices: for example, we write $T^{\alpha}$ as $T^{(\alpha\beta)}$ such that $T^{(\alpha\beta)} = -T^{(\beta\alpha)}$ holds for the $SU(2)$ adjoint $\dagger$. Sen modeled the lapse function $N$ and the shift vector $N^a$ as two kinds of squares of a dimensionless space spinor $\lambda_\alpha$ [10]:

$$N = \frac{1}{\sqrt{2}} \lambda^{\alpha} \lambda_{\alpha}, \quad N^{(\alpha\beta)} = c \lambda^{(\alpha} \lambda^{\beta)}.$$  

As a result, time lapses and the causal structure of the world are encoded in the Sen spinor field $\lambda_\alpha$.

An original aspect of our approach is that we extend Sen's spin model by introducing one additional long-range self-interacting massive spinor particle that is physically coupled with Sen's spinor by a spin-exchange interaction. We call this new spinor particle the theta particle and hypothesize it to be cold dark matter. In the present model, we assume that the role of the theta particle is such that it only interacts with time lapses and the causal structure of the world, that is, the temporal part of the space-time metric, via its coupling to Sen’s spinor and has no direct interaction with the standard model elementary particles except for the origin of its mass.

Our strategy for deriving non-unitary time evolution (including event readings) in the world, as measured by its proper time, of a spatially macroscopically coherent quantum matter system $\Psi$ by solving the Schrödinger equation written using time increments is to show that fluctuations of the time increment around its trial mean are induced by a spontaneous time reparametrization symmetry (TRpS) breaking in this extended canonical gravity [11], where the theta particle plays the role of creating the TRp invariant potential in the total system of theta particles and Sen’s spins in the coherence domain of $\Psi$. In canonical classical gravity, TRpS is manifest in its total Hamiltonian. In canonical quantum gravity, as seen from the Wheeler-DeWitt equation of the wave function of the Universe, there is no Newtonian concept of external time, and only an internal clock exists [20–22]. This situation is caused by the fact that, besides canonical gravity being a totally constrained system, the object of its dynamics is the Universe itself.

The structure of this paper is as follows. In Sec. II, we present the Hamiltonian of theta particles based on Sen’s reformulation of canonical gravity as a spin system.

In Secs. III and IV, we describe the mechanism of TRpS breaking and the consequent measurement processes of quantum systems in our model. In Sec. V, we summarize our results and discuss some aspects of our model.

II. THETA PARTICLE HAMILTONIAN

In this paper, our interest lies in the spatial weak-field case

$$(q_{ab} + \delta_{ab})^2 \approx 0, \quad \pi_{ab}^2 \approx 0.$$  

In this section, we introduce the spin $1/2$ fermionic theta particle field on $\Sigma_{t_0}$ and extend canonical gravity. We do this in a way that its four-dimensional relativistic spin-variable space matches that of the Sen spinor field: it is the composite of its own Sen’s spinor (we denote this spinor by $\Lambda_\alpha$) and a scalar particle $\Phi$ that carries the mass. We hypothesize theta particles to be cold dark matter and, to simplify the model setting, we deal with only theta particles with velocities that are very low relative to the speed of light. Then, the theta particle field is approximately described by a non-relativistic Pauli $SU(2)$ two-component spinor field, $\Theta_\alpha$, on $\Sigma_{t_0}$ with dimensions $[L^{-3/2}]$.

In our model, two kinds of short-range orbital interactions are assumed. Namely, though $\lambda_\alpha$ has no orbital self-interaction, we assume that the background $\Lambda_\alpha$ of $\Theta_\alpha$ has a two-body orbital self-interaction and that the triple $\lambda_\alpha - \Lambda_\alpha - \Phi$ has a three-body orbital interaction.

In the $SO(3)$ vector spin model, in which all variables are treated as their field amounts, we define the direction of the theta spin by

$$f^{(\alpha\beta)} = \Theta^{(\alpha} \Theta^{\beta)}$$

and require that it physically couples to the Sen spin $\lambda_\alpha$ by a strong coupling ($-J_c > 0$):

$$H_c \approx -J_c \int_{\Sigma_{t_0}} \tau_a N_{a,phys} dS,$$

where $N_{a,phys}$ is the part that is spatial-diffeomorphism gauge independent (i.e., the physical part) of the shift vector $N^a$:

$$N^a = N_{a,phys}^a + N_{a,rest}^a,$$

where both $N_{a,phys}^a$ and $N_{a,rest}^a$ give a spatial displacement of $\Sigma_{t_0}$ in $M$ accompanying the time evolution of $\Sigma_{t_0}$ via their respective couplings to momenta, but the displacement due to $N_{a,rest}^a$ is attributed to the choice of spatial diffeomorphism gauge. $H_c$ is invariant under TRp at the level of canonical equations, since the combination $H_c dt_0$ is invariant under TRp. $H_c$ is regarded as an effective interaction, that is, a spin-exchange interaction between

...
two kinds of Sen spin ($\lambda_\alpha$ and $\Lambda_\alpha$) with common two-dimensional non-relativistic spin eigenspaces. A spin-exchange interaction is the reduced form of an orbital interaction that is obtained as the quantum mechanical average of the orbital interaction by integrating out orbital degrees of freedom[23].

Next, we model the Hamiltonian of the theta particle field $\Theta_\alpha$ itself as that of a spin model with a long-range ferromagnetic interaction via a massless wave of a continuous succession of spin-exchange interactions, which propagates in the background $\Lambda_\alpha$ of $\Theta_\alpha$, 

$$H_{\text{spin}} = \int_{\Sigma_t} (N h_{\text{spin}}^a - 2 N_a h_{\text{spin}}^a) dS ,$$ (12)

where in the Newtonian limit,

$$h_{\text{spin}} \approx \frac{i \hbar}{2m} \partial_\alpha \pi_\alpha \partial^\alpha \Theta_\beta \delta^\alpha\beta - J \int_{\Sigma_0} \frac{1}{|x - x'|} \tau_\alpha(x) \tau_\beta(x') dS' ,$$ (13)

$$h_{\text{spin}}^a \approx -\pi_\alpha \partial^\alpha \Theta_\beta \delta^\alpha\beta$$ (14)

for a theta particle field $\Theta_\alpha$ with mass $m$, canonical momentum $\pi_\alpha = i\hbar \Theta_\alpha$, and a positive-valued long-range decay factor $-J/|x - x'|$, with a constant $-J > 0$, that generates an inverse square law force.

In the form presented here, the system $H_{\text{spin}}$ manifests a global SU(2) spin rotation symmetry. This global symmetry is gauged by the background $\Lambda_\alpha$ of $\Theta_\alpha$ that does not explicitly appear in $h_{\text{spin}}$.

In our model, we also consider the matter sector $\Psi$. The chain of interactions and correlations between $\Theta$, $\lambda$ and $\Psi$ is schematically shown in the diagram

![Diagram](image)

In our model, while (b) is a short-range interaction, (a) is a long-range self-interaction. The process (c) defines the lapse function $N$ and the shift vector $N^a$, and the right-directed arrow of (d) arises from time-dependent processes in the world for $\Psi$ with its proper time including the ADM time lapse. The correlation (e) is indirect and causally a one-way process.

### III. SYMMETRY BREAKING

This paper’s main statement is that, in our model, TRpS can be broken in time-dependent processes in the world of a spatially macroscopically coherent quantum matter system $\Psi$ (assuming that it is, at least typically, above the micrometer scale) and this TRpS breaking leads to non-unitary time evolution in this world, as measured by its proper time.

Throughout this paper, the term coherence is used with the meaning of quantum coherence of superposed de Broglie waves and superposed photons[26, 27].

In a quantum coherent state realized in a spatial domain, there is an overwhelming majority of quanta with a specific mode (i.e., a momentum eigenstate). Then, due to the position-momentum uncertainty relation, in this state, the three position uncertainties are realized with the dimensions of this domain.

Now, we assume a second-quantized matter system $\Psi$ of $n$ identical particles, with the coherence of $\Psi$ limited by macroscopic lengths $\ell^a (a = 1, 2, 3)$ at a time $t_0$. Then, the existence of the macroscopically, in the quantum mechanical sense, ferromagnetically ordered distribution of $n$ physical Sen spins over the time-dependent coherence domain of $\Psi$ in $\Sigma_t$ at time $t_0$ follows. [In general, the $n$ physical Sen spins stochastically accompany a second-quantized $n$-body system $\Psi$ comprising arbitrary kinds of particles, with the same existence probability as that of $\Psi$, while the position uncertainty of $\Psi$ is kept.]

The key fact for showing this existence is that the dimensions $\ell^a$ of the coherence domain are smaller than the spatial stochastic means of the Penrose position uncertainties $\Delta x^a_{\psi}(x)$, in the identification between the spacetimes having the complex-valued probability amplitudes of $\Psi$ under superposition. These uncertainties are taken to be the root mean square error (for its weight, see below) in spatial identification of the originally definite positions of particles (taking into account the Pauli exclusion principle for identical fermions) at a point $x^a$ in the time slice $\Sigma_{t_0}$:

$$\ell^a \lesssim \Delta x^a_{\psi}, \ |n_a \Delta x^a| = |\langle N^a_{\text{phys}}, t_0 \rangle| ,$$ (16)

for a field of unit vectors $n_a^a(x)$, the root mean squares $\Delta x^a_{\psi}$ weighted by the one-particle probability density $\phi(x)$ of $\Psi$ in the particle picture as in $\Delta x^a_{\psi}$ itself[26, 27], the vector modulus of the expectation values $\langle N^a_{\text{phys}, t_0} \rangle$, and the TRp invariant Mandelstam-Tamm time uncertainty[28, 29]

$$\sigma \equiv N \Delta t_0 ,$$ (17)

which is constant over $\Sigma_{t_0}$ and characteristic of the rate of change of the system $(\Sigma_{t_0}, q_{ab})$ itself at the time $t_0$—below this critical time scale $\sigma$, space-like hypersurfaces $\Sigma_{t_0}$ that are selected from superposition by the position measurements of particles are practically identical for different measurement times $t_0$[29]. Thus, we obtain

$$|\langle N^a_{\text{phys}} \rangle| = \frac{|n_a \Delta x^a|}{\sigma} N \equiv \kappa N .$$ (18)

The reason why we take the root mean square $\Delta x^a_{\psi}$ of $\Delta x^a_{\psi}(x)$ in Eq.(16) is that just after the state reduction, the particle position stochastically takes a definite value $x^a$ in the full sense with the event probability $\phi(x)$ and
this value \( x^a \) needs to match the originally definite position value where, as the root mean square error in the spatial identification of this originally definite position, the uncertainties \( \Delta x^a(x) \) are defined. In Eq. (10), \( n^a \) is the direction of the group velocity \( \langle N^a_{\text{phys}} \rangle \) of the spatial displacement of \( \Sigma_{t_0} \) in \( M \) under superposition. This direction of the group velocity is arbitrarily chosen, due to the \( SU(2) \) gauge symmetry, and needs to be definite within the coherence domain. Here, we note three points about the above statements. First, \( \Delta x^a(x) \) can be realized at a specific point \( x^a \) in \( \Sigma_{t_0} \) only stochastically with event probability \( \varphi(x) \). Second, the time-energy uncertainty relation is not applied to the Mandelstam-Tamm time uncertainty \( \sigma \) because \( \sigma \) is not defined through Hamiltonian mechanics but is just the dispersion of time increment trials given at a time \( t_0 \). Third, the Pauli exclusion principle is not applied to \( \lambda_a \) because \( \lambda_a \) is dimensionless and thus has no orbital degree of freedom.

Now, when we define the magnetization of a system of uniformly distributed spins by the vector modulus of (weighted) arithmetic means of the spin vector components over this system, Eq. (16) shows approximately that the \( SU(2) \) gauge symmetry is fully governed by the Vlasov equation (i.e., the collisionless equilibrium) from the collisionless equilibrium, and its kinetic multipliers: inverse temperature \( \beta \) and chemical potential \( \mu \) for the total energy and chemical potential \( \mu \) for the total mass conservation. This was discovered and called the double Lynden-Bell distribution in Ref. [32], and has, in particular, two coexisting chemical potentials. (The evaporated masses, that is, the masses that have completely escaped from the main cluster are excluded from the local system.) Here, due to the Pauli exclusion principle, which is applicable because of the fermionic nature of \( \Theta_{\alpha} \) that has orbital degrees of freedom, and the alignment of the initial theta spins, we consider an initial phase space \( (\mu\text{-space}) \) distribution of the water-bag type (i.e., with a single non-zero fine-grained level). The ground state of the theta particle system is

\[
\Delta \tau^{a} = \frac{\eta^{(i)}}{\exp(\beta^{(i)}(\varepsilon - \mu^{(i)})) + 1}, \quad i = 1, 2
\]  

(20)

for the Heaviside step function \( \theta(\varepsilon) \) and the Fermi energy \( \varepsilon_F \) (i.e., the chemical potential at zero temperature) of the Lynden-Bell distribution at zero temperature. Here, two remarks about the collisionless equilibrium are made. First, in the collisionless equilibrium of a long-range interacting system, as a result of phase mixing, the variable of the fine-grained Vlasov distribution is the one-particle energy only. The second remark concerns the persistence of this equilibrium. A general property of long-range interacting systems means that, in the thermodynamic limit (i.e., the limit that the particle number \( \nu \) tends to infinity while fixing the energy per particle \( E/\nu \)), the lifetime of the collisionless equilibrium diverges, the theta particle system never approaches the Boltzmann-Gibbs equilibrium (i.e., the collisional equilibrium) from the collisionless equilibrium, and its kinetics is fully governed by the Vlasov equation (i.e., the collisionless Boltzmann equation) when we discuss nonequilibrium properties. This thermodynamic limit can be realized in a quantum mechanically macroscopic scale coherence domain if two assumptions hold. First, the theta particle is light (so, high-density) cold dark matter—like the axion, which is an ultralight candidate for cold dark matter—owing to the low amount of energy exchanged with other theta particles in incoherence domains that do not align nor gather theta particles and occupy the overwhelming majority of space. Second, the ferromagnetic coupling constants \( J_c \) and \( J \) are strong enough to attract and confine such a large number of theta particles to the coherence domain that belongs to an exceptionally minor fraction of space.

In the theta particle ground state of the system, \( H_c \) can be written as

\[
V_c = -J_c \int_{\Sigma_{t_0}} \tau_{\alpha}(x)N_{\text{phys},t_0}^{\alpha}(x)\,dS \quad (22)
\]

\[
\approx J_c\kappa \sum_{i=1,2} \nu \int_{\Sigma_{t_0}} \rho_{\alpha}^{(i)}(x)N_{t_0}^{\alpha}(x)\,dS, \quad (23)
\]

where \( \kappa \) is TRp invariant, \( \nu \) is the theta particle number.
of the system, and \( \rho_0^{(i)} \) is a coarse-grained position distribution of \( f_0^{(i)} \) whose form is determined by the Fermi energy \( \varepsilon_F \) and the form of the self-consistent mean-field potential of the second term of Eq. (13). Here, \( V_c \) is the TRp invariant potential for time lapse \( N \) at the level of canonical equations. The main property of this potential is that \( V_c dt_0 \) gives distinct degeneracies, with respect to TRp, for two different time lapse densities \( \rho_0^{(i)} N \) for \( i = 1, 2 \). Owing to their differing TRp degeneracies, these two time lapse densities define different proper times \( t^{(i)} \) of \( f_0^{(i)} \) and corresponding time lapses \( N^{(i)} \). This property of \( V_c \) is confirmed by noting that, in the particle description, \( V_c \) is written as

\[
V_c \approx J_c \kappa \sum_{i=1,2} \sum_{x(i) \in \rho_0^{(i)}} N_{t_0}(x^{(i)}) \, , \tag{24}
\]

where time lapses \( N_{t_0}(x^{(i)}) \) correspond to the time lapse densities \( \rho_0^{(i)} N \) in Eq. (23). Here, we note that this coarse-graining is applied not to the proper time \( t \) but to the energy \( V_c \), so \( V_c \) is necessary and \( J_c \) needs to be non-zero. Due to this main property of \( V_c \), the ground state one-(\( \Theta, \lambda_{\text{phys}} \))-pair distribution function of the total spin system is in the form

\[
f_0(x, N_x, p) = \sum_{i=1,2} f_0^{(i)}(x, N_x, p) \, . \tag{25}
\]

This expression means that for \( i = 1, 2 \),

\[
f_0^{(i)}(x, p) = \langle f_0^{(i)}((x, N_x), p)) \rangle_{N_x} \, , \tag{26}
\]

\[
\rho_0 N^{(i)} = \rho_0^{(i)} N / f_0^{(i)} = \rho_0 \langle N_0^{(i)} \rangle \, , \tag{27}
\]

where \( f_0^{(i)}(x, p) \) is the \( i \)-th position-momentum Lynden-Bell distribution function of theta particles with the fixed spin part at zero temperature, \( r^{(i)} \) is the fraction of theta particles in \( \rho f_0^{(i)} \) relative to \( \nu \), and \( \langle \cdot \rangle_X \) and \( \langle \cdot \rangle^{(i)} \) represent the integrating out of \( X \) and the average using \( \rho_0^{(i)}(N_x) = f_0^{(i)}((x, N_x), p))/\rho_0^{(i)}(x) \) as the weight, respectively. In Eq. (26), we apply the definition of a distribution function in statistical mechanics as the weight in the averages of variables to the theta particle system. In Eq. (27), for \( i = 1, 2 \), the first and the second equalities define, respectively, \( N^{(i)} \) and the \( N_x \)-dependence of \( f_0^{(i)} \).

Now, we show that TRpS on the lapse function \( N \) does not hold due to its inconsistency with the theta particle part of \( f_0 \): TRpS is preserved for \( r^{(1)} N^{(1)} + r^{(2)} N^{(2)} \), which is equivalent to the TRp invariance of \( V_c \), but non-zero \( N^{(1)} - N^{(2)} \) breaks TRpS spontaneously. The crucial point here is that, due to the superposition form of the distribution \( f_0 \) and the singleness of its non-zero fine-grained level, for every theta or physical Sen spin, it is in principle impossible to specify to which component \( f_0^{(i)} \) this spin belongs, in the phase space region of overlap between \( f_0^{(1)} \) and \( f_0^{(2)} \). Due to this point, in the ground state \( f_0 \), TRp, that is, a replacement of the clock in the theta particle system—\( t_0 \rightarrow t'_0 = F(t_0) \)—is required to be applied for both proper times \( t^{(i)} \) for \( i = 1, 2 \) by a single correspondence rule (i.e., a single time lapse) between clock and proper time due to the singleness of \( f_0 \). Thus, one of the two proper times becomes an obvious constraint on TRp for the other proper time, which breaks TRpS on the lapse function \( N \) spontaneously in a collisionless equilibrium theta particle system.

Once TRpS breaks spontaneously, the lapse function in Eq. (24) decomposes into

\[
N = \langle N \rangle_0 + \tilde{N} \tag{28}
\]

for ground state expectation value \( \langle N \rangle_0 \) and zero-mean fluctuation \( \tilde{N} \). Thus, the proper time \( t \) also decomposes into

\[
t = \langle t \rangle_0 + \tilde{t} \tag{29}
\]

for ground state expectation value \( \langle t \rangle_0 \) and zero-mean fluctuation \( \tilde{t} \). This decomposition in Eq. (29) arises because, according to the ADM spatial coordinate transformation \( dx^a \rightarrow dx'^a = dx^a + N^a dt_0 \) of the space-time \( (M, g_{\mu\nu}) \) with the signature \((+, -, -, -)\), we set the proper time of the local theta particle system, which is, in the present case, also that of the matter system \( \Psi \), as

\[
dt = N dt_0 \tag{30}
\]

by employing an \textit{a priori} and reversibly flowing time \( t_0 \) that is used in the ADM decomposition just as a reparametrizable parameter \( t_0 \).

Consequently, the trials of time increment \( \delta t \) fluctuate around the trial mean with a non-zero relative variance. This relative variance sets the lower-bound time scale for TRpS breaking. Specifically, below this critical time scale, time-dependent processes of the Universe described by using proper time flows \( t \) having the ground state expectation value \( \langle t \rangle_0 \) are not gauge invariant (i.e., not distinguishable) with respect to the gauge transformation \( t_0 \rightarrow t_0 + \tilde{t} \). So, we identify this relative variance with \( \sigma \) in Eq. (17), both of which are the dispersion of time increment trials and the critical time scale for the distinguishability of \( \Sigma_{t_0} \), in the presence of a group velocity of the spatial displacement of \( \Sigma_{t_0} \) in \( M \) under superposition. Here, it is consistent to assume that the behavior of the time increment fluctuation is governed by a certain statistical law, which can be set independently from the model in Sec. II: in particular, its relative variance \( \sigma \) is to be treated as a physical constant of nature (Milburn postulated that such a \( \sigma \) is Planck time \( \sqrt{\hbar} \)).

IV. MEASUREMENT PROCESSES

A selective quantum measurement consists of two processes: non-selective measurement that makes the state be a mixture of eigenstates, and the subsequent event reading. There have already been many proposals for the
with respect to the expectation values of all observables, \{\hat{A}\}, exists, the time increment fluctuation cannot be described by using \(|\Psi(0)\rangle\) or imposing Eq.\,(36) on \(|\Psi(0)\rangle\) (instead, it can be described by using \(|\Psi(0)\rangle\) and imposing Eq.\,(32) on \(|\Psi(0)\rangle\)) . This is because Eq.\,(35) is the condition for a state change with no dynamical element (i.e., no change of time) and thus it cannot extinguish the quantum coherence. However, just after a non-selective energy measurement of a pure state \(|\Psi\rangle\) (e.g., the time evolution resulting from Eq.\,(35)), viewed as the preparation of the state \(|\Psi(\hat{t})\rangle\) just before the event reading at \(\hat{t}=0\), quantum coherence of \(|\Psi(\hat{t})\rangle\) with respect to \{\hat{A}\} cannot be observed (i.e., that, for \(\hat{A}\), its total interference is less than its uncertainty width). At such a time, for the unaveraged state vector \(|\Psi(0)\rangle\), Eq.\,(36) induces a quantum state reduction that satisfies the Born rule\,[1]: this is the subsequent event reading.

Finally, we make two remarks.

First, the lifetime \(t_{\text{life}}\) of quantum coherence of \(|\Psi\rangle\) with respect to \{\hat{A}\} is equal to the characteristic time of exponentially decaying off-diagonal elements of the averaged density matrix \(|\bar{\Psi}\rangle\langle\bar{\Psi}|\) expressed in the energy eigenbasis. From this, we obtain \(t_{\text{life}}=2\hbar^2/\langle\sigma(E)\rangle^2\).

Second, because the Hamiltonian in quantum mechanics does not have a canonically conjugate operator, von Neumann-type entanglement\,[39] between a given quantum measured system \(\psi\) with measured observable \(\hat{O}\) and the event reading system \(\Psi\) with energy eigenstates is to be by unitary time-evolution of a quantum energy feedback process \(\sum_n c_n|\langle n|E_n\rangle\rightarrow\sum_n c_n|\langle n|E_n\rangle\rangle\), controlled by an external agent, in the open quantum system \(\psi+\Psi\) where the energy reservoir is traced out. This \(\hat{O}\)-to-\(E\) conversion process reflects, and is subsequent to, the non-selective measurement of the observable \(\hat{O}\) in the system \(\psi\).

V. SUMMARY AND DISCUSSION

In our model, we have proposed the scheme shown in Eq.\,(15) consisting of \(\Theta\), \(\lambda\) and \(\Psi\) in the time increment description. Summarizing the contents of the diagram Eq.\,(15) briefly, the superposed form collisionless equilibrium (i.e., double Lynden-Bell) distribution of a local theta particle system \(\Theta\) is dynamically generated from the initial ferromagnetically ordered distribution prepared by the matter system \(\Psi\), provided \(\Psi\) is spatially macroscopically coherent, via the coupling of its vector part with the configuration of the shift vector. This configuration is a ferromagnetically ordered spatial-diffeomorphism that is gauge independent and generated from the spatial quantum coherence structure of \(\Psi\) (refer to Fig.\,1). Then, this local theta particle system \(\Theta\) converts the reversibly flowing time \(t_{\text{f}}\) into an irreversibly flowing (see the argument below) proper time \(t\) with increments fluctuating within the world for the matter system \(\Psi\) by the spontaneous breaking of TRpS due to the

non-selective measurement\,[6-10], but no proposal for the event reading exists. In this section, we show that these two processes accompany TRpS breaking.

In time-dependent processes of the matter system \(\Psi\) with its Hamiltonian \(\hat{H}\), the statistical variance of time increment trials that accompanies TRpS breaking is represented by the non-unitary time evolution of the state vector of the system \(\Psi\) in the Schrödinger picture, as seen in the following way. In the Schrödinger picture, the time-evolution equation of the state vector \(|\Psi(t)\rangle\) of the matter system \(\Psi\) is the Schrödinger equation. We write it in the time increment formalism using the proper time of both the local theta particle system and the matter system \(\Psi\) as

\[
\frac{i\hbar}{\delta t}\frac{\delta |\Psi(t)\rangle}{\delta t} = \hat{H}|\Psi(t)\rangle .
\] (31)

The average of its formal solution over time increment trials is

\[
\langle |\Psi(\mu)\rangle = \int \delta t \varphi(\delta t) \left\{ \exp \left( -i\delta t^\prime \hat{H} \right) |\Psi(0)\rangle \right\} \right. \] (32)

\[
= \exp \left( -i\mu \hat{H} - \frac{\sigma^2 t}{2\hbar^2} \hat{H}^2 \right) |\Psi(0)\rangle ,
\] (33)

when the time increment trials \(\delta t\) are simply assumed to follow a normal distribution \(\varphi(\delta t)\) with mean \(\mu\) and variance \(\sigma^2\). This formula gives the time evolution of the averaged state vector \(|\bar{\Psi}\rangle\) in the history representation

\[
|\bar{\Psi}(t)\rangle = \exp \left( -i\frac{t}{\hbar} \hat{H} - \frac{\sigma^2 t^2}{2\hbar^2} \hat{H}^2 \right) |\Psi(0)\rangle .
\] (34)

For \(|\bar{\Psi}\rangle\), we change \(t\) to \(-t\) in Eq.\,(34). In Eq.\,(34), the first exponential factor gives unitary time evolution. To understand the formal meaning of the second exponential factor, we note the following points. In quantum mechanics, a Hamiltonian \(\hat{H}\) is a Hermitian operator. So, for its real eigenvalues \(\{\lambda\}\), there exists a unique spectral family \(\{d\hat{H}(\lambda)\}\). From the projection property of the spectral components \(d\hat{H}(\lambda)\), it follows that

\[
\hat{H}^2 = \int \lambda^2 d\hat{H}(\lambda) .
\] (35)

Due to this, on average, the second exponential factor in Eq.\,(34) forms a family with the properties of a contraction semi-group with parameter \(t\). This is a dynamical state change based on the Schrödinger equation.

Now, we consider an unaveraged state change with no dynamical element. For two different time increment trials \(\delta t_1\) and \(\delta t_2\), the equivalence \(\simeq\) of two state vectors up to a phase-factor difference

\[
e^{-\frac{i\delta t_1}{\hbar} \hat{H} |\Psi(0)\rangle} \simeq e^{-\frac{i\delta t_2}{\hbar} \hat{H} |\Psi(0)\rangle} , \ \delta t_1 \neq \delta t_2
\] (36)

holds if and only if the unaveraged state vector \(|\Psi(0)\rangle\) is an energy eigenstate. When quantum coherence of \(|\Psi(0)\rangle\)
existence of the TRp invariant double Lynden-Bell potential Eq. (24) for the time lapse; the time lapse is rescaled in the ground state Eq. (26) in two ways (see Eq. (27)). [On the other hand, spatial diffeomorphism invariance is not broken. The reason why spatial diffeomorphisms behave differently from TRp is that the physical part $H_c$ of the total Hamiltonian of the whole system is spatial-diffeomorphism-gauge independent and is invariant under TRp only.] By solving the Schrödinger equation of the state vector written in the time increment formalism (e.g., Eq. (31)), we have shown that this conversion of time results in non-unitary events, which include gravitational state mixings (that is, non-selective measurements) and event readings, in the world for the matter system $\Psi$, with irreversibility arising from using the time fluctuations $\tilde{t}$ in Eq. (29) but expressed by using the a priori reversible temporal flow $t_0$.

In our interpretation of quantum mechanics, we reverse the roles of particles and space-time in uncertainties of the positions and momenta of particles. Namely, while the original positions $\mathbf{r}^a$ and momenta $\mathbf{p}^a$ of the localizable second-quantized particles in $\Psi$ are simultaneously definite variables, at every time, uncertainties of $\mathbf{r}^a$ and $\mathbf{p}^a$ are owned by space-like hypersurfaces as the uncertainties of the variables $x^a$ and $h_{\text{grav}}^a(x)$, respectively. In the case of positions, we have discussed this ownership by space-like hypersurfaces in Sec. III. In the case of momenta, this ownership by space-like hypersurfaces is realized via the replacement of a particle momentum uncertainty $\Delta p^a_{\text{mat}}$ arising from the field $h_{\text{mat}}^a$ with a space-time momentum uncertainty $\Delta p^a_{\text{grav}}(q)$ at the originally definite particle position $q^a$. This replacement is allowed by the momentum constraint $h_{\text{grav}}^a(q) = 0$ in the whole system, and the space-time momentum uncertainty $\Delta h_{\text{grav}}^a(q)$ at the originally definite particle position $q^a$ is defined by the root mean square error in the identification of the originally definite momentum of a particle in $\Psi$ by using the momentum probability density of this particle as the weight of the root mean square. In our picture, quantum states of space-like hypersurfaces related to their semi-classical background $\Psi$ give quantum eigenstates of the observable particle position or particle momentum under superposition and the observable particle position and particle momentum are mutually exclusively definite canonical variables satisfying the canonical commutation relation. We treat the particle spin separately because it is independent from the spatial motion of the particle.

Next, it is worthwhile to briefly compare our proposal for resolving the measurement problem with a few other proposals that have formal completeness in the description by a state vector. First, the essential difference from the program of quantum decoherence[6–8] is that, while that approach must describe the time-dependent processes considered in the measurement problem statistically using the mixed state of the quantum system with a density matrix, our mechanism for non-unitary time-dependent processes works fundamentally with the state vectors. Second, compared with the continuous spontaneous collapse model in Ref.[9], the advantage of our model is that, while the evolution law of the continuous spontaneous collapse model is modified a priori, ours is not except for introducing a lower-bound for the time scale of TRpS breaking; in particular, our time-evolution equation is not modified from the Schrödinger equation. Finally, a particularly novel feature of our proposal is that, in the mechanism invoking gravity, the main source of its properties is not gravity itself but a particle, that is, the theta particle, which we hypothesize to be cold dark matter. This feature and our gravitational state mixing mechanism distinguish our proposal from the Diési-Penrose proposal for quantum state reduction[12–13].

We close this paper by offering a perspective. In the quantum mechanical regime of the theta particle system, there is pairing instability in the ground state because the theta particle is a fermion and its self-interactions through the aligned spin are attractive[10]. In such a situation, we expect, below a certain critical temperature, the realization of superconductivity-like macroscopically coherent behavior by quantum theta particles evading the Pauli exclusion principle. If our proposed mechanism can be tested experimentally, then investigating this aspect of our model, and in particular determining the scale of the critical temperature, will be a significant issue. It

![FIG. 1: A physical Sen spin configuration in $\Sigma_0$ at a time instance $t_0$ is schematically shown at a quantum mechanical spatial scale. Here, every Sen spin has no momentum and is fixed at a spatial point in $\Sigma_0$. The center boxed red domain and the outer boxed blue domain represent a coherence domain owned by space-like hypersurfaces as the uncertainty domain, respectively. The five thick red aligned rea...](image-url)
must be done at the same time as primary determination of the scales of the two coupling strengths and the mass of theta particles, in connection to the issue of the thermodynamic limit, and examination of the behavior of the time increment fluctuation accompanying TRpS breaking.

[1] M. Born, Zeitschrift. für. Physik. 38, 803 (1926); N. Bohr, Nature. 121, 580 (1928).
[2] J. von Neumann, Math. Ann. 104, 570 (1931); Mathematische Grundlagen der Quantenmechanik (Springer-Verlag, Berlin, 1932); E. P. Wigner, Am. J. Phys. 31, 6 (1963).
[3] L. de Broglie, J. Phys. Radium. 8, 225 (1927).
[4] D. Bohm, Phys. Rev. B 39, 160 (1952); ibid. 85, 180, (1952).
[5] H. Everett, Rev. Mod. Phys. 29, 454 (1957); J. A. Wheeler, ibid. 29, 453 (1957).
[6] H. D. Zeh, Found. Phys. 1, 69 (1970).
[7] W. H. Zurek, Phys. Rev. D 24, 1516 (1981); ibid. 26, 1862 (1982); Rev. Mod. Phys. 75, 715 (2003).
[8] E. Joos and H. D. Zeh, Z. Phys. B 59, 223 (1985).
[9] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34, 470 (1986); A. Bassi and G. C. Ghirardi, Phys. Rep. 379, 257 (2003).
[10] H. Araki, Prog. Theor. Phys. 64, 719 (1980).
[11] F. Károlyházy, Nuovo Cimento 52, 390 (1966).
[12] L. Diósi, Phys. Lett. A 120, 377 (1987); Phys. Rev. A 40, 1165 (1989).
[13] R. Penrose, Gen. Relativ. Gravit. 28, 581 (1996).
[14] A. Sen, J. Math. Phys. 22, 1781 (1981).
[15] A. Sen, Int. J. Theor. Phys. 21, 1 (1982).
[16] A. Sen, Phys. Lett. B 119, 89 (1982).
[17] P. A. M. Dirac, Proc. Roy. Soc. Lond. A 246, 326 (1958); ibid. 246, 333 (1958); Phys. Rev. 114, 924 (1959).
[18] R. Arnowitt, S. Deser and C. W. Misner, in Gravitation: An Introduction to Current Research, L. Witten (ed.) (Wiley, New York, 1962).
[19] K. Kuchař, J. Math. Phys. 17, 777 (1976).
[20] J. A. Wheeler, in Battelle Rencontres, C. De Witt and J. A. Wheeler (eds.) (Benjamin, New York, 1968).
[21] B. S. De Witt, Phys. Rev. 160, 1113 (1967).
[22] A. Vilenkin, Phys. Rev. D 39, 1116 (1989).
[23] W. G. Unruh and R. M. Wald, Phys. Rev. D 40, 2598 (1989).
[24] C. J. Isham, in Integrable Systems, Quantum Groups, and Quantum Field Theories, L. A. Ibort and M. A. Rodríguez (eds.) (Kluwer, London, 1993).
[25] L. Landau and E. Lifshitz, Quantum Mechanics (Non-Relativistic Theory), Course of Theoretical Physics, Vol. 3 (Butterworth-Heinemann, Oxford, 2000).
[26] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. 21, 400 (1949); A. S. Wightman, ibid. 34, 845 (1962).
[27] Coherent states of photons can be localized without any limitations. For these states, the full localization of the photon number density can be obtained. See I. Bialynicki-Birula and Z. Bialynicka-Birula, Phys. Rev. A 79, 032112 (2009) for details.
[28] L. Mandelstam and I. G. Tamm, J. Phys. (USSR) 9, 249 (1945).
[29] A. Messiah, Quantum Mechanics. Vol I (North-Holland, Amsterdam, 1972).
[30] D. Lynden-Bell, Mon. Not. R. Astron. Soc. 136, 101 (1967).
[31] R. Pakter and Y. Levin, Phys. Rev. Lett. 106, 200603 (2011); Y. Levin, R. Pakter, F. B. Rizzato, T. N. Téles and F. P. C. Benetti, Phys. Rep. 535, 1 (2014).
[32] E. Konishi and M. Sakagami, Phys. Rev. E 91, 032144 (2015); for a comprehensive review, see E. Konishi, Int. J. Mod. Phys. B 30, 1630007 (2016).
[33] Y. Levin, R. Pakter and F. B. Rizzato, Phys. Rev. E 78, 021130 (2008).
[34] A. Campa, T. Dauxois, D. Fanelli and S. Ruffo, Physics of Long-range Interacting Systems (Oxford University Press, Oxford, 2014).
[35] W. Braun and K. Hepp, Comm. Math. Phys. 56, 101 (1977).
[36] L. J. Rosenberg and K. A. van Bibber, Phys. Rep. 325, 1, (2000).
[37] G. J. Milburn, Phys. Rev. A 44, 5401 (1991); New. J. Phys. 8, 96 (2006).
[38] L. Diósi, Braz. J. Phys. 35, 260 (2005).
[39] E. Joos, in Decoherence and the Appearance of a Classical World in Quantum Theory, D. Giulini et al. (eds.) (Springer-Verlag, Berlin, 1996).
[40] L. N. Cooper, Phys. Rev. 104, 1189 (1956).
[41] In this paper, the spontaneous breaking of a gauge symmetry $G$ refers to that of the global symmetry part (i.e., the pre-gauged symmetry) of $G$ at each point in a space-time domain. Thus, the corresponding ground state expectation values are functions of the space-time coordinates.
[42] Based on Sen’s picture of gravity as a spin system in $\Sigma$, we fix the flow of time $t_0$ common to all of the clocks instead of fixing the flow of proper time $t$. Here, we note that if TRpS is unbroken, then each clock (including the Universe itself: $t_0 = t$ and $N = 1$) is equivalent to the other clocks; the inverse of this statement for each clock is also true.