A minimal reaction-diffusion neural model generates *C. elegans* undulation

Anshul Singhvi\textsuperscript{1,2}, Harold M. Hastings\textsuperscript{1}, Jenny Magnes\textsuperscript{3}, and Susannah G. Zhang\textsuperscript{3,4}

\textsuperscript{1}Bard College at Simon’s Rock
\textsuperscript{2}Columbia University
\textsuperscript{3}Vassar College
\textsuperscript{4}University of Georgia

October 22, 2021

Abstract

The small (1 mm) nematode *Caenorhabditis elegans* (Corsi [1], wormbook.org) has become widely used as a model organism; in particular the *C. elegans* connectome has been completely mapped, and *C. elegans* locomotion has been widely studied. We describe a minimal reaction-diffusion model for the locomotion of *C. elegans*, using as a framework a simplified, stylized “descending pathway” of neurons as central pattern generator (CPG) (Xu et al., Proceedings of the National Academy of Sciences 115, 2018 [2]). Finally, we realize a model of the required oscillations and coupling with a network of coupled Keener (IEEE Transactions on Systems, Man, and Cybernetics SMC-13, 1983 [3]) analog neurons. Note that Olivares et al. (BioRxiv 710566, 2020 [4]) present a likely more realistic model more distributed CPG. We use the simpler simulation to show that a small network of FitzHugh-Nagumo neurons (one of the simplest neuronal models) can generate key features of *C. elegans* undulation, and thus locomotion, yielding a minimal, biomimetic model as a building block for further exploring *C. elegans* locomotion.

1 Introduction

The small (1 mm) nematode *Caenorhabditis elegans* (*C. elegans*) has become a widely used model organism (cf. http://www.wormbook.org [1]), and has been among the most studied biological models of neuronal development and locomotion [1, 5]. The *C. elegans* connectome has been completely mapped [6] and, as described below, its locomotion has been widely studied (c.f. Corsi [1] and Gjorgjieva et al. [7]). There are a variety of neuronal models which can generate such undulation, “When crawling on a solid surface, the nematode *C. elegans* moves forward by propagating sinusoidal dorso-ventral retrograde contraction waves. A uniform propagating wave leads to motion that undulates about a straight line.” [8]. A different type of locomotion, often called swimming, occurs when nematodes are submerged in a liquid medium. The nematodes “switch” between these two gaits, by changing the dynamics of the central pattern generator (CPG).

The purpose of this paper is to describe a minimal, biomimetic, reaction-diffusion model for the *C. elegans* central pattern generator (CPG) [2, 9]. We use simulation methods to show that a small network of FitzHugh [10]-Nagumo et al. [11] neurons, see also [12] (one
of the simplest neuronal models), based on a skeleton model of the *C. elegans* CPG, can reproduce key features of *C. elegans* undulation [2] [13] [14], and thus locomotion.

Finally, we describe an analog electronic implementation of our model through solving a modified version of the FitzHugh-Nagumo neuron [10], based upon an analog circuit originally proposed by Keener [3]. This circuit solves the Keener differential equations, and we adjusted it to allow diffusive coupling between neurons. We constructed a small network with these “neuro-nimetic” circuits, and showed that their behaviour replicates FitzHugh-Nagumo simulated behaviour.

There are many other CPG models; for example, Olivares *et al.* [4] proposes a distributed network of self-oscillating systems of neurons, instead of a structured chain like our proposed CPG. Here we describe a minimal working model, rather than striving for fuller realism, in order to explore the fundamental components of a small CPG. We aimed for a simple, “minimal” model to enable exploration of defects or other changes in the CPG in an efficient, reproducible and explainable way. Our model can thus be considered as intermediate between the early “neuro-mechanical” model of Yuk *et al.* [15] and more realistic models such as [4], a role similar to that of Adamatzky *et al.* [16] biochemical computation.

2 The model central pattern generator

Recall that central pattern generator is a small neural circuit which generates and regulates the movement of complex organisms. This structure is present in different forms in many animals, and regulates many types of periodic motion. Changes in gait are driven by changes in the dynamics of the CPG, c.f. [17]. The CPG circuit topology is constant; the mode of locomotion depends on the sequence in which the neurons fire.

Our simple model *C. elegans* central pattern generator has two principal components. The first is the head oscillator. As described by Gjorgjieva *et al.* [7], the head oscillator consists of two “head neurons” with mutually inhibitory coupling. Oscillations are generated when this coupling destabilizes an excitable steady state. Here two FitzHugh-Nagumo neurons, with oscillatory dynamics, stabilized 180° out of phase by mutually inhibitory coupling. This provides a pair of out-of-phase stimuli which propagate through a descending pathway of pairs of coupled, excitable, dorsal and ventral neurons. These follow the body of the worm, and are linked to motor neurons and muscles. The head oscillator drives the descending pathway, and the pathway is kept in sync by mutual inhibitory coupling between neurons.

We use 12 pairs of such neurons, as in *C. elegans* coupled by model gap junctions. This coupling yields phase lags as we descend the pathway from head to tail, enabling the head oscillator to drive traveling waves of excitation along the body of the nematode. See Figure 1 on the following page for descending pathway of Xu *et al.* [2] and our simplified model; the latter depicted as a graph, wherein neurons are nodes, and the arrows between them represent connections.

3 The FitzHugh-Nagumo Neuronal Model

As described above, we sought to use the simplest relevant neuronal model. The classical Hodgkin-Huxley[21] model of squid neurons has led to a variety of simpler conduction models, including the Morris-Lecar[22] and FitzHugh [10]-Nagumo *et al.* [11] (FHN)
The FHN model consists of two dynamical variables; a fast activator variable $v$ corresponding to the (rescaled) membrane potential, and a slow inhibitor variable $w$ corresponding to a generalized gating variable.

\begin{align}
\frac{dv}{dt} &= f(v) - w + I_{ext} \\
\frac{dw}{dt} &= \varepsilon(v - \gamma w + \alpha) \\
f(v) &= v - \frac{v^3}{3}
\end{align}

The parameter $I_{ext}$ is an external driving current, and is used here to model the effect of gap junctions and synapses upon membrane potential. The parameter $\alpha$ determines the vertical position of the $w$-nullcline and thus controls excitability. Action potentials can also be generated by a current injection corresponding to $I_{ext}$. Finally, the parameter
\( \varepsilon \) determines ratio between the time scales of the fast activator variable \( v \) and the slow inhibitor variable \( w \); \( \gamma \) determines the effect of the growth of the slow gate variable \( w \). We used the following standard parameter values: \( \varepsilon = 0.08, \gamma = 0.8 \) [12]. FitzHugh-Nagumo neurons can display either (self-)oscillatory or excitable dynamics; given a sufficiently large stimulus (here a pulse of injected current \( I \)), an excitatory neuron generates an action potential. An oscillatory neuron generates a regular sequence of action potentials. In comparison to the standard value \( \alpha = 0.7 \) [12], we adjusted the parameter \( \alpha \) to control neuronal dynamics, with values larger than the oscillatory-excitable boundary \( \alpha_0 \approx 0.467 \) generating excitable dynamics from a stable steady state, and smaller values generating oscillatory dynamics as the steady state is destabilized.

Moreover, \( f(v) \) can be any function which retains the appropriate dynamics, in that it has the same general shape as the cubic \( f(v) = \frac{v^3}{3} - v \). For example, \( f(v) \) could be replaced by the cubic-like I-V curve of the tunnel diode in the Nagumo et al. [11] circuit, or even the piecewise linear approximation generated in the Keener [3] circuit used in our implementation.

Xu et al. [2] described a simplified two-variable model for \textit{C. elegans} neurons, consisting of a fast, cubic-like activator variable (see the \( v \)-nullcline in Figure 2 on the next page) and a slow, non-linear inhibitor variable (see the \( n \)-nullcline). The Xu et al. [2] neuronal model can thus be interpreted as a FitzHugh-Nagumo type model neuron.

Following Collins & Richmond [17] and Yanagita et al. [23], we now use generalized diffusion coupling between FHN neurons to model gap junctions and synapses. The effect of gap junctions and synapses upon the membrane potential is modeled by replacing the external current \( I_{\text{ext}} \) of the system, by a generalized diffusion term, \( D \max(\Delta v, 0) \). A positive diffusion coefficient \( D \) is used to simulate a gap junction, or an excitatory synapse and a negative coefficient to simulate an inhibitory synapse ([17]).

This yields a system of diffusion-coupled FitzHugh-Nagumo equations, c.f. Essaki Arumugam & Spano [24]:

\[
\begin{align*}
\frac{dv}{dt} &= f(v) - w + D(\Delta v) \\
\frac{dw}{dt} &= \varepsilon(v - \gamma w + \alpha) \\
f(v) &= v - \frac{v^3}{3}
\end{align*}
\]

where \( \Delta v \) is the rectified difference in voltage between the driving and driven neurons, essentially \( \Delta v = \max(v_{\text{driven}} - v_{\text{driving}}, 0) \). For example, consider the potential of an oscillatory FitzHugh-Nagumo neuron to drive an excitatory FitzHugh-Nagumo neuron. The consequent dynamics depends upon the magnitude of positive diffusion coupling \( D > 0 \): \( D \) must be sufficiently large to generate an action potential in the driven excitatory neuron, which then responds with a time delay corresponding inversely to the magnitude of \( D \).

### 4 Simulation

We start with the network shown in Figure 1 on the preceding page, bottom. We perform simulations in Python, using the standard SciPy ODE solvers, which wrap LSODA,
Figure 2: Dynamics of (A) *C. elegans* neurons adapted from Xu *et al.*, Fig. 5B, Copyright National Academy of Sciences [2] (left) and (B) Fitzhugh-Nagumo neurons [10, 11] (right). We have identified Xu’s *n*-variable (the potassium current in the Hodgkin-Huxley model [21] (with the *w*-variable (slow inhibitor) in the Fitzhugh-Nagumo model. In both models, the nullclines for sodium dynamics (*v*) has a cubic-like shape. We show the *v*-nullcline in two positions: one generating oscillatory dynamics (green); the other excitable (non-oscillatory) dynamics (green). The type of dynamics, oscillatory or excitable, is controlled by coupling of AVB-B gap junctions in Xu’s model. The type of dynamics can be controlled by the injected current *I*, an analog of the above AVB-B coupling or the parameter *α* in the Fitzhugh-Nagumo model. We added a dashed blue line to (A) Xu’s experimentally derived simulation (A) to indicate that the *n*-nullcline is essentially linear in the relevant range of dynamics. It therefore seems reasonable to use the linear *w*-nullcline in the simpler Fitzhugh-Nagumo model. Finally, the thin dashed black line in the graph of Fitzhugh-Nagumo nullclines (A) represents the dynamics of Keener’s [3] implementation of the Nagumo circuit (below). The Keener circuit yields a piecewise linear approximation to Fitzhugh-Nagumo *v* dynamics.

the Livermore Solver for Ordinary Differential Equations. The following block of code illustrates these calculations. Much like Izquierdo & Beer [25] did, we take a segment-based approach to the worm model. Pairs of muscles on either side determine the overall angular displacement per segment. The magnitude of the “contraction” resulting from the smoothing is interpreted as an angular displacement for that segment. We use cubic spline interpolation to smooth the worm body.

```python
# Imports
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
import math

# Constants
# d for dorsal
# v for ventral
# x for FHN membrane potential, namely v in equations (1)-(2)
# x for FHN gate variable, namely w in equations (1)-(2)

# Initial data, some symmetry breaking
```
# dynamics largely independent of initial data
dx0= 1.0
dy0=-0.51
vx0= 1
vy0= -0.49
# more, omitted here

# vector of x and y values
v=[vx0,vy0,dx0,dy0,vx1,vy1,dx1,dy1,vx2, ...]

# time range for simulations (arbitrary units)
t = np.linspace(0, 2000, 20001)

def FHND(v, t):
    # calculates the derivatives in FHN with diffusion
    vx0, vy0, dx0, dy0, vx1, vy1, dx1, dy1, vx2, ... = v
    # v denotes vector passed to the function FHND
    # FitzHugh-Nagumo parameters, see eqs. (2) and (3).
    # allow for different epsilons
    e0 = 0.08   # epsilon for head neurons
    e1 = 0.08   # epsilon for body neurons
    g = 0.8
    b0 = 0.46
    b1 = 0.47
    # diffusion constants
    Dhead = -0.2
    # Drest= -0.02
    Drest= 0
    Dgap=0.05
    J = 0

    dvdt=[
        # neurons 0 (ventral and dorsal)
        vx0-(vx0**3/3)- vy0+ Dhead*max(dx0-vx0,0) + J,
        e0*(vx0-g*vy0+b0),
        dx0-(dx0**3/3)- dy0+ Dhead*max(vx0-dx0,0) + J,
        e0*(dx0-g*dy0+b0),
        # coupled by 1-way diffusion
        # negative diffusion constant Dhead
        # simulated inhibitory synapse
        # neurons 1 (ventral and dorsal)
        vx1-(vx1**3/3)- vy1+ Drest*max(dx1-vx1,0) + Dgap*max(vx0-vx1,0) + J,
        e1*(vx1-g*vy1+b1),
        # driven by neurons 0 through one-way
        dx1-(dx1**3/3)- dy1+ Drest*max(vx1-dx1,0) + Dgap*max(dx0-dx1,0) + J,
        e1*(dx1-g*dy1+b1),
        # Dgap simulates gap junction in descending chain
        # Drest simulates inhibitory synapse
        # coupling ventral and dorsal neurons 1, ...
        # more, omitted
        return dvdt

Muscle response is simulated by applying Gaussian filters to the positive component of
membrane potentials:

```python
import scipy.ndimage.filters as filt

effective_signal=np.zeros((2001,24))
for neuron_number in range(0,number_of_neurons):
    effective_signal[:,neuron_number]=
        (sol[:,4*neuron_number]+abs(sol[:,4*neuron_number]))/2
    effective_signal[:,neuron_number]+=n
    effective_signal[:,neuron_number]+=filt.gaussian_filter1d(effective_signal[:,neuron_number],40)

# Gaussian filter simulates a combination of
# muscle response to neural stimulus,
# effects of elastic properties of worm and
# also interaction with fluid
```

The worm is then described initially as a sequence of nodes, joined by line segments, with
the angle at each node determined by muscular tension. This is followed by a cubic spline
interpolation. Finally, we generated a simple video by fixing the head of a worm to the
origin; see Figure 3 on the next page.

**Simulation versus experimental results.** Our model generates traveling waves of approx-
imately sinusoidal form, with approximately constant amplitude from head to tail. In
comparison, real *C. elegans* undulations are characterized by approximately sinusoidal
oscillations, but with amplitude decreasing slightly from head to tail, c.f. [2]. In addition,
a search for "markers of chaos" as in Magnes et al. [14] finds that undulations of the simu-
lated worm are too regular (neutrally stable), in contrast to undulations of real nematodes
(with Max \(\text{Re}(\lambda)\) \(\approx 1\text{s}^{-1}\)), likely because we have yet to implement proprioceptive neu-
ronal inputs, c.f. Calhoun et al. [26] and references therein. The neutral stability of our
model may allow it to respond readily but relatively consistently to inputs. Moreover, a
small, negative Max \(\text{Re}(\lambda)\) would typically maintain relatively consistent undulation,
but be readily overcome by somewhat larger inputs or noise, the latter generating random
changes in undulation pattern.

Further comparisons and a more detailed non-linear analysis of real *C. elegans* undulation
will appear in a forthcoming paper [Zhang et al., in preparation].

## 5 Analog implementation

We show that a diffusion-coupled Keener [3] electronic analog neurons can imitate key fea-
tures of the dynamics of our network of Fitzhugh-Nagumo neurons. Recall that Nagumo
et al. [11] proposed a circuit to simulate a FitzHugh-Nagumo neuron, shown in Figure 4
on page 9. It used a tunnel diode to achieve a cubic-like activation function, and an
inductor to differentiate the potassium current (slow gate variable) Figure 4 on page 9

However, given that tunnel diodes are expensive and rarely available, Keener [3] proposed
a modified Nagumo circuit which used the saturation properties of operational ampli-
fiers ("op-amps") to achieve cubic-like non-linearity in the FHN model. This yields a
piecewise linear approximation to the Fitzhugh-Nagumo \(v\) (sodium, fast activator) null-
clines, as shown in Figure 2 on page 5. The nullclines are sufficiently similar that the
dynamics are effectively the same [3]. Keener also used an op-amp and a capacitor to
simulate the inductor in the original Nagumo circuit. Roughly, differentiation is simulated
Figure 3: A "dashboard view" of our simulated *C. elegans*. (a) Graphical representation of a traveling wave of simulated membrane potentials of ventral neurons, numbered from head to tail. Time and membrane potential units are arbitrary. (b) Successive "images" of the simulated *C. elegans*. Muscle response to neural potentials was simulated using a Gaussian filter to simulate muscle dampening, one dorsal (resp., ventral) muscle per dorsal (resp., ventral) neuron. A cubic spline is used to simulate a "smoothed" worm body between joints as nodes. (c) For comparison, here are shadow images of a real *C. elegans* in a cuvette.

by anti-integration. Later Sanchez-Sinencio & Linares-Barranco [27] proposed a CMOS implementation of the Nagumo circuit.

Finally, diffusion coupling is simulated by a follower, and then a coupling resistor followed by a diode (positive diffusion, excitatory synapse or gap junction) or a follower, inverting amplifier of unit gain, coupling resistor and a diode in series (negative diffusion, inhibitory synapse), c.f. [17, 23] for the role of diffusion, c.f. Essaki Arumugam & Spano [24] for the use of resistors to emulate diffusion. The diffusion constant is inversely proportional to the coupling resistance. Coupling resistors are tuned to obtain desired synchronization or time delays; for example, coupling resistors in the descending chain of neurons are \( \sim 1-3 \) M\( \Omega \), with larger resistors yielding longer delays until eventually the injected current is too small to generate an action potential. Typical experimental results for a pair of neurons in the descending chain coupled by a gap junction are shown in Figure 5 on page 10.
Figure 4: Top: The original circuit proposed by Nagumo et al. [11]. Note the use of inductors for differentiation, and a tunnel diode to supply a cubic-like V-I curve for sodium dynamics in the Fitzhugh-Nagumo model. Bottom: Our circuit, a minor modification of Keener [3], using a single pair of power supplies and fine bias control, is shown in the same layout, to make the similarities and differences more explicit. The cubic-like V-I curve for sodium dynamics is generated by op-amp $U_1$ and associated resistors. In fact this circuit provides a polygonal approximation to Fitzhugh-Nagumo sodium dynamics. The inductor in the Nagumo circuit is simulated by op-amp $U_2$ and associated passive components. Roughly, differentiation is simulated by "anti-integration."

6 Conclusion

We have shown that the undulatory motion of *C. elegans* can be generated using a simple network of Fitzhugh-Nagumo neurons, the network capturing using a structured central pattern generator, and simple, biomimetic neurons. In particular, our simple, "minimal"
Figure 5: Dynamics of analog implementation of coupling from one neuron in our head oscillator to the next corresponding neuron in our descending chain of neurons; see Figure 1 on page 3. (A) Theoretical results, as in Figure 3 on page 8. (B) Experimental results from diffusion coupled Keener neurons. Note similarity with (A). Our experimental period, \( \sim 5\text{ms} \) agrees with that of Keener [3]. The head oscillator and additional downstream couplings are also similarly reproduced by diffusion-coupled Keener neurons.

Our model generates traveling waves of sinusoidal undulations, as in [2]. An initial exploration shows that small changes in parameters do not alter qualitative dynamics; further study is needed to explore the range of possible dynamics.

Our model thus lies in between the "Shape memory alloy-based small crawling robot" mechanical model of Yuk et al. [15] and more realistic (and complex) descriptions of Izquierdo & Beer [25] and Olivares et al. [4], in the spirit of Adamatzky et al. [16] description of the neural system as a computational system. Because of the simplicity and flexibility our our model, we expect that it may prove useful in studying the effects of stimuli, aging and mutations upon the dynamics of *C. elegans* undulation and locomotion.

Acknowledgement. We acknowledge assistance from our students Cheris Congo, Miranda Hulsey-Vincent, Rifah Tasnim and Naol Negassa.
7 References

1. Corsi, A. K. A Transparent Window into Biology: A Primer on Caenorhabditis Elegans. WormBook, 1–31. ISSN: 15518507. http://www.wormbook.org/chapters/www_celegansintro/celegansintro.html (2020) (June 18, 2015).

2. Xu, T. et al. Descending Pathway Facilitates Undulatory Wave Propagation in Caenorhabditis Elegans through Gap Junctions. Proceedings of the National Academy of Sciences 115, E4493–E4502. ISSN: 0027-8424, 1091-6490. http://www.pnas.org/lookup/doi/10.1073/pnas.1717022115 (2020) (May 8, 2018).

3. Keener, J. P. Analog Circuitry for the van Der Pol and FitzHugh-Nagumo Equations. IEEE Transactions on Systems, Man, and Cybernetics SMC-13, 1010–1014. ISSN: 0018-9472, 2168-2909. http://ieeexplore.ieee.org/document/6313098/ (2020) (Sept. 1983).

4. Olivares, E., Izquierdo, E. & Beer, R. A neuromechanical model of multiple network rhythmic pattern generators for forward locomotion in C. elegans. BioRxiv, 710566 (2020).

5. Katz, P. S. Evolution of Central Pattern Generators and Rhythmic Behaviours. Philosophical Transactions of the Royal Society B: Biological Sciences 371, 20150057. https://royalsocietypublishing.org/doi/10.1098/rstb.2015.0057 (2020) (Jan. 5, 2016).

6. Jabr, F. The Connectome Debate: Is Mapping the Mind of a Worm Worth It? Scientific American. https://www.scientificamerican.com/article/c-elegans-connectome/ (2020).

7. Gjorgjieva, J., Biron, D. & Haspel, G. Neurobiology of Caenorhabditis Elegans Locomotion: Where Do We Stand? BioScience 64, 476–486. ISSN: 1525-3244, 0006-3568. http://academic.oup.com/bioscience/article/64/6/476/289633/Neurobiology-of-Caenorhabditis-elegans-Locomotion (2020) (June 1, 2014).

8. Kim, D., Park, S., Mahadevan, L. & Shin, J. H. The Shallow Turn of a Worm. Journal of Experimental Biology 214, 1554–1559. ISSN: 0022-0949, 1477-9145. http://jeb.biologists.org/cgi/doi/10.1242/jeb.052092 (2020) (May 1, 2011).

9. Wen, Q. et al. Proprioceptive Coupling within Motor Neurons Drives C. Elegans Forward Locomotion. Neuron 76, 750–761. ISSN: 08966273. https://linkinghub.elsevier.com/retrieve/pii/S0896627312008057 (2020) (Nov. 2012).

10. FitzHugh, R. Mathematical Models of Threshold Phenomena in the Nerve Membrane. The Bulletin of Mathematical Biophysics 17, 257–278. ISSN: 0007-4985, 1522-9602. http://link.springer.com/10.1007/BF02477753 (2020) (Dec. 1955).

11. Nagumo, J., Arimoto, S. & Yoshizawa, S. An Active Pulse Transmission Line Simulating Nerve Axon. Proceedings of the IRE 50, 2061–2070. ISSN: 0096-8390. http://ieeexplore.ieee.org/document/4066548/ (2020) (Oct. 1962).

12. Izhikevich, E. M. & FitzHugh, R. Fitzhugh-nagumo model. Scholarpedia 1, 1349 (2006).

13. Magnes, J., Susman, K. & Eells, R. Quantitative Locomotion Study of Freely Swimming Micro-Organisms Using Laser Diffraction. Journal of Visualized Experiments, 4412. ISSN: 1940-087X. http://www.jove.com/video/4412/quantitative-locomotion-study-freely-swimming-micro-organisms-using (2020) (Oct. 25, 2012).
14. Magnes, J. et al. Chaotic markers in dynamic diffraction. *Applied Optics* **59**, 6642–6647 (2020).

15. Yuk, H., Kim, D., Lee, H., Jo, S. & Shin, J. H. Shape memory alloy-based small crawling robots inspired by C. elegans. *Bioinspiration & biomimetics* **6**, 046002 (2011).

16. Adamatzky, A., De Lacy Costello, B. & Shirakawa, T. Universal Computation with Limited Resources: Belousov-Zhabotinsky and Physarum Computers. *International Journal of Bifurcation and Chaos* **18**, 2373–2389. ISSN: 0218-1274, 1793-6551. [https://www.worldscientific.com/doi/abs/10.1142/S0218127408021750](https://www.worldscientific.com/doi/abs/10.1142/S0218127408021750) (Aug. 2008).

17. Collins, J. J. & Richmond, S. A. Hard-Wired Central Pattern Generators for Quadrupedal Locomotion. *Biological Cybernetics* **71**, 375–385. ISSN: 0340-1200, 1432-0770. [http://link.springer.com/10.1007/BF00198915](http://link.springer.com/10.1007/BF00198915) (Sept. 1994).

18. Chen, B. L., Hall, D. H. & Chklovskii, D. B. Wiring optimization can relate neuronal structure and function. *Proceedings of the National Academy of Sciences* **103**, 4723–4728 (2006).

19. Boyle, J. H., Berri, S. & Cohen, N. Gait modulation in *C. elegans*: an integrated neuromechanical model. *Frontiers in computational neuroscience* **6**, 10 (2012).

20. Zhen, M. & Samuel, A. D. *C. elegans* locomotion: small circuits, complex functions. *Current opinion in neurobiology* **33**, 117–126 (2015).

21. Hodgkin, A. L. & Huxley, A. F. A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve. *The Journal of Physiology* **117**, 500–544. ISSN: 0022-3751, 1469-7793. [https://onlinelibrary.wiley.com/doi/abs/10.1113/jphysiol.1952.sp004764](https://onlinelibrary.wiley.com/doi/abs/10.1113/jphysiol.1952.sp004764) (Aug. 28, 1952).

22. Morris, C. & Lecar, H. Voltage Oscillations in the Barnacle Giant Muscle Fiber. *Biophysical Journal* **35**, 193–213. ISSN: 00063495. [https://linkinghub.elsevier.com/retrieve/pii/S0006349581847820](https://linkinghub.elsevier.com/retrieve/pii/S0006349581847820) (July 1981).

23. Yanagita, T., Ichinomiya, T. & Oyama, Y. Pair of Excitable FitzHugh-Nagumo Elements: Synchronization, Multistability, and Chaos. *Physical Review E* **72**, 056218. ISSN: 1539-3755, 1550-2376. [https://link.aps.org/doi/10.1103/PhysRevE.72.056218](https://link.aps.org/doi/10.1103/PhysRevE.72.056218) (Nov. 28, 2005).

24. Essaki Arumugam, E. M. & Spano, M. L. A chimeric path to neuronal synchronization. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **25**, 013107 (2015).

25. Izquierdo, E. J. & Beer, R. D. From Head to Tail: A Neuromechanical Model of Forward Locomotion in *Caenorhabditis Elegans*. *Philosophical Transactions of the Royal Society B: Biological Sciences* **373**, 20170374. ISSN: 0962-8436, 1471-2970. [https://royalsocietypublishing.org/doi/10.1098/rstb.2017.0374](https://royalsocietypublishing.org/doi/10.1098/rstb.2017.0374) (Oct. 19, 2018).

26. Calhoun, A. J., Chalasani, S. H. & Sharpee, T. O. Maximally informative foraging by *Caenorhabditis elegans*. *Elife* **3**, e04220 (2014).

27. Sanchez-Sinencio, E. & Linares-Barranco, B. Circuit implementation of neural FitzHugh-Nagumo equations in *Proceedings of the 32nd Midwest Symposium on Circuits and Systems*, (1989), 244–247.

12