Evaluating the V-band Photometric Metallicity with Fundamental Mode RR Lyrae in the Kepler Field

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Abstract

The aim of this work is to evaluate the performance of photometric metallicity [Fe/H], determined based on V-band light curves photometrically transformed from the gr-band light curves. We tested this by using a set of homogeneous samples of fundamental mode RR Lyrae located in the Kepler field. It was found that the color term is necessary in such photometric transformation. We demonstrated that when including the color term the determined photometric [Fe/H] are in good agreement with the spectroscopic [Fe/H], either based on the calibrated or the transformed V-band light curves. We also tested the impact of Blazhko RR Lyrae in determining the photometric [Fe/H], and found that Blazhko RR Lyrae can give consistent photometric [Fe/H]. Finally, we derived independent gVr-band [Fe/H]–ϕ31–P relations (where ϕ31 and P are the Fourier parameter and pulsation period, respectively) using our light curves. The V-band relation is in good agreement with the most recent determination given in the literature.

Unified Astronomy Thesaurus concepts: RR Lyrae variable stars (1410); Metallicity (1031); Light curves (918)

Supporting material: machine-readable table

1. Introduction

One of the important observed quantities, besides the pulsation periods P, for RR Lyrae is metallicity, commonly denoted as [Fe/H]. This is because the V-band absolute magnitude (M_V) for RR Lyrae is correlated with [Fe/H]. Also, [Fe/H] is one of the independent parameters in the period–luminosity–metallicity (PLZ) relations (especially in the infrared filters). For exemplary reviews on the M_V–[Fe/H] relation or PLZ relations, see Sandage & Tammann (2006), Beaton et al. (2018), Bhardwaj (2020), and references therein.

The best way to measure [Fe/H] is via spectroscopic observations. Errors on the spectroscopically measured [Fe/H] fall in the range of ~0.1 to ~0.3 dex (e.g., see Nemec et al. 2013); in some cases these errors can even reach to ~0.01 dex level (e.g., SX For in Crestani et al. 2021). However, spectroscopic observations for RR Lyrae can be expensive or time consuming. Alternatively, [Fe/H] can be estimated based on the stellar systems (e.g., in globular clusters or dwarf galaxies) or environment (e.g., in the Galactic halo) that the targeted RR Lyrae belong to. Another way to estimate [Fe/H] is using light-curve properties of RR Lyrae, such as amplitudes (e.g., see Alcock et al. 2000; Sandage 2004; Fabrizio et al. 2021) or Fourier parameter ϕ31.

Since the seminal paper of Jurcsik & Kovacs (1996), who derived the [Fe/H]–ϕ31–P relation in the V band for ab-type (fundamental mode) RR Lyrae (hereafter RRab), a number of publications have derived a similar relation (some with additional parameters in such relation) in other filters, as well as for the c-type (first-overtone) RR Lyrae. These works include Sandage (2004) and Morgan et al. (2007) in the V band; Smolec (2005) and Dékány et al. (2021) in the I band; Watkins et al. (2009), Sesar et al. (2010), and Oluseyi et al. (2012) in the Sloan Digital Sky Survey (SDSS) g and/or r band; Nemec et al. (2011) and Nemec et al. (2013) in the Kepler Kp band; Ngeow et al. (2016) in the R_PTF band; Iorio & Belokurov (2021) in the Gaia G band; Mullen et al. (2021) in the WISE W1 and W2 bands; and Wu et al. (2006) for unfiltered or white-light observations. The rms errors from these empirical relations vary from ~0.1 dex (Nemec et al. 2013) to ~0.5 dex (in the WISE band; Mullen et al. 2021).

Recently, the V-band [Fe/H]–ϕ31–P relation was updated from two works. Martinez-Vazquez et al. (2016) updated the Jurcsik & Kovacs (1996) relation by using seven globular clusters and eight field RR Lyrae with high-resolution spectroscopic metallicity. Furthermore, Mullen et al. (2021) rederived the V-band [Fe/H]–ϕ31–P relation based on a sample of ~10^5 field RRab with spectroscopic determined [Fe/H]. It is foreseen that the Mullen et al. (2021) V-band relation will be widely applied in various studies on using ab-type RR Lyrae as distance tracers. On the other hand, the SDSS-like (ugriz) filters are becoming more popular in major synoptic sky surveys, including (but not limited to) the Pan-STARRS1 (Chambers et al. 2016), the Zwicky Transit Facility (ZTF; Bellm et al. 2019; Graham et al. 2019), the SkyMapper Southern Survey (Onken et al. 2019), the Dark Energy Survey (Dark Energy Survey Collaboration et al. 2016), the HyperSuprime-Cam Subaru Strategic Program (Aihara et al. 2018), and the Vera C. Rubin Observatory Legacy Survey of Space and Time (LSST; Ivezić et al. 2019). This implies that in order to apply the Mullen et al. (2021) V-band relation, photometric transformations need to be applied to the gr-band data from these surveys to the V band. In principle, such transformations could add extra uncertainties to the final estimated [Fe/H].

Therefore, the goal of this work is to evaluate the performance and accuracy of such photometric transformations in the derivation of photometric [Fe/H], using the V-band [Fe/H]–ϕ31–P relation. Instead of relying on inhomogeneous data taken from the literature, we intended to obtain homogeneous light-curve data using the same telescope and
CCD camera on the same set of RR Lyrae, such that a differential comparison can be made. We selected 30 RRab stars located in the Kepler field (taken from Table 7 in Nemec et al. 2013), which possess homogeneous spectroscopic [Fe/H] measured from high-resolution spectra. Section 2 describes the time-series observations of these RR Lyrae. Photometry and photometric calibration of our light-curve data are presented in Section 3. These light curves were then used to derive their corresponding Fourier parameter $\phi_3$ as mentioned in Section 4. Performance on the photometric [Fe/H] from using the V-band light curves and the transformed light curves are tested in Section 5. We have also derived a set of [Fe/H]−$\phi_3$−$P$ relations based on our light curves in Section 6, followed by discussions and conclusions in Section 7. We note that the transmission curve for the V-band filter lies in between the g- and r-band filters; hence, we only observed our targeted RR Lyrae in these filters.

2. Observations and Image Reduction

Time-series observations of the 30 targeted RR Lyrae in the Kepler field were carried out using the 0.41 m SLT telescope located at Lulin Observatory. This telescope is an f/8.4 Ritchey–Chrétien telescope and is equipped with an Andor iKon-L936 CCD camera, providing a pixel scale of 0.79 pixel. Queue observations were executed, via commercial software MaxIm DL and ACP Observatory Control Software, from 2019 June 18 to 2021 November 24 (weather permitting) in gVr filters. Depending on the brightness of the targeted RR Lyrae, exposure time varied between 2 to 300 s in all filters. After removing problematic images (due to bad seeing or weather, tracking problems, etc.), the number of gVr sequence ranged from $\sim$120 to $\sim$144 for all of the 30 RR Lyrae. Subroutines in IRAF (version 2.16) were used to reduce these images, including bias and dark subtractions, as well as flat-fieldings. Astrometric calibration on the reduced images were done using the astrometry.net software suite. (Lang et al. 2010)

3. Photometric Calibration

For each of our targeted RR Lyrae, we constructed a reference catalog by merging the Pan-STARRS1 Data Release 1 (DR1) photometric data (Chambers et al. 2016; Flewelling et al. 2020) and the $UBV$ photometric catalog published in Everett et al. (2012, hereafter the $UBV$ catalog). A search area with a size of $27'\times27'$ centered at each targeted RR Lyrae was adopted to query the Pan-STARRS1 DR1 photometric data. We applied a number of selection criteria to select only the nonvarying stellar sources in the merged reference catalogs. Further details of the adopted selection criteria were given in Appendix A. These merged reference catalogs were then crossmatched to the catalogs generated from the SExtractor (version 2.25.0; Bertin & Arnouts 1996) on all reduced images. The popular MAG_AUTO implemented in SExtractor was adopted for measuring the instrumental magnitudes. Hence, for each image we have a catalog containing both the gr-band and BV-band photometry from the Pan-STARRS1 and the $UBV$ catalog, respectively, for the reference stars, as well as their instrumental magnitudes.

The photometric calibration was completed using the following set of equations (e.g., see Masci et al. 2019):

$$g_{\text{PS1}} - g_{\text{instr}} = ZP_g + C_g(g_{\text{PS1}} - r_{\text{PS1}}),$$  

$$r_{\text{PS1}} - r_{\text{instr}} = ZP_r + C_r(g_{\text{PS1}} - r_{\text{PS1}}),$$

$$V - V_{\text{inst}} = ZP_V + C_V(B_{\text{EHK}} - V_{\text{EHK}}),$$

where $m_{\text{PS1}}$ or EHK are magnitudes from published catalogs (either Pan-STARRS1 or $UBV$ catalog), and $m_{\text{inst}}$ are instrumental magnitudes. An iterative 2σ-clipping linear regression, implemented in astropy, was used to fit these equations to determine the $ZP_m$ and $C_m$ coefficients.

Since our SLT observations did not include the $B$ filter, we employed the photometric transformations given in Tonry et al. (2012) to calibrate the $(B-V)$ colors. We adopted the linear transformation between Johnson and the Pan-STARRS1 photometric system from Table 6 of Tonry et al. (2012): $B - g_{\text{PS1}} = 0.213 + 0.587(g_{\text{PS1}} - r_{\text{PS1}})$ and $V - r_{\text{PS1}} = 0.006 + 0.474(g_{\text{PS1}} - r_{\text{PS1}})$. Then, the color transformation is found to be

$$(B - V) = 0.207 + 1.113(g_{\text{PS1}} - r_{\text{PS1}}).$$

Finally, we can transform the calibrated $g_{\text{PS1}}$ magnitudes to the V-band magnitude via the Tonry et al. (2012) transformation. The transformed V-band magnitudes are denoted as $VT$:

$$VT = r_{\text{PS1}} + 0.006 + 0.474(g_{\text{PS1}} - r_{\text{PS1}}).$$

Applying Equations (1)–(5) to calibrate the instrumental magnitudes requires the $(g_{\text{PS1}} - r_{\text{PS1}})$ colors of the targeted stars to be known. In the case of our SLT observations with a sequence of gVr exposures within a sequence is always less than 30 minutes (with a median of 6.2 minutes), and hence we assume the photometry obtained from the near-simultaneous gr-band observations is equivalent to the $(g-r)$ color at the time of observations. Combining Equations (1) and (2) the instrumental colors can be calibrated to the Pan-STARRS1 photometric system via the following equation:

$$(g_{\text{PS1}} - r_{\text{PS1}}) = \frac{ZP_g - ZP_r + (g_{\text{instr}} - r_{\text{instr}})}{1 - C_g + C_r}.$$  

The calibrated $(g_{\text{PS1}} - r_{\text{PS1}})$ colors can be applied back to Equations (1)–(5) to calibrate the gVr- and VT-band photometry. An example of the calibrated grV-band light curves and the transformed VT-band light curve is shown in Figure 1. All of the calibrated grV-band light curves are provided in Table 1. Photometric errors given in Table 1 and shown in Figure 1 include the errors from the instrumental magnitudes and the propagated errors from the calibration. Typical errors on $ZP_g,r,V$ are $\sim$0.006 mag, $\sim$0.004 mag, and $\sim$0.013 mag, respectively. Similarly, the typical errors on $C_g,r,V$ are $\sim$0.012 mag, $\sim$0.007 mag, and $\sim$0.018 mag, respectively.
4. Fourier Parameters $\phi_{31}$

The $V$-band photometric [Fe/H] given in Mullen et al. (2021) is

$$[	ext{Fe/H}]_V = -1.22[\pm 0.01] - 7.60[\pm 0.24](P - 0.58)$$
$$+ 1.42[\pm 0.05](\phi_{31} - 5.25),$$

(7)

with an rms of 0.41 dex. Precise $P$ for our targeted RR Lyrae are available from Nemec et al. (2013) based on the Kepler observations. Hence, we only need to determine the Fourier parameter $\phi_{31}$ from our calibrated $V$-band light curves to obtain the [Fe/H]$_V$.

In general, the light curve for a periodic variable star can be fitted with an $n$-order Fourier expansion in the following form (e.g., see Deb & Singh 2009):

$$m(\Phi) = m_0 + \sum_{i=1}^{n} [a_i \cos(2\pi i \Phi) + b_i \sin(2\pi i \Phi)],$$

(8)

where $\Phi = t/P - \text{INT}(t/P)$ is the pulsational phase (between 0 and 1) after folding a time series $t$ with $P$. With trigonometric identities, Equation (8) can be rewritten either as a sine series or a cosine series, i.e., in the form of $m(\Phi) = m_0 + \sum_{i=1}^{n} a_i \sin(2\pi i \Phi + \phi_i)$ or $m(\Phi) = m_0 + \sum_{i=1}^{n} a_i \cos(2\pi i \Phi + \phi_i)$. Following Simon & Lee (1981), the Fourier parameter $\phi_{31}$ is either defined as $\phi_{31}^s = \phi_{31}^c - 3\phi_1^c$ or $\phi_{31}^c = \phi_{31}^s - 3\phi_1^s$, with a conversion of $\phi_{31}^s = \phi_{31}^c - \pi$ (Ngeow et al. 2016). To be consistent with Mullen et al. (2021), we adopted the sine series and $n=5$ for obtaining the Fourier parameter $\phi_{31}^c$. Since the error on $\phi_{31}$ is the same for either the sine series or the cosine series, we calculated the error on $\phi_{31}^c$ using the prescription given in Petersen (1986) and Petersen (1994).

For the 13 Blazhko RR Lyrae, we have also removed the modulated components following a similar procedure as described in Ngeow et al. (2016, in their Section 5.2; also see the references therein), where the modulated (or Blazhko) periods, $P_{\text{BL}}$, were adopted from Nemec et al. (2013). The difference between this work and Ngeow et al. (2016) is we fixed $n=5$, and only vary the $(r, q)$ Fourier orders when fitting the modulated components. The Fourier orders $r$ and $q$ are similar to Equation (8) but for the modulated frequency $f_m = 1/P_{\text{BL}}$, and the combined frequencies $f_0 \pm f_m$ (where $f_0 = 1/P$ is the pulsation frequency, and $k$ is an integer run from 0 to $q$), respectively. The same $(r, q)$ Fourier orders were adopted to fit the gr$V$ and VT light curves for a given Blazhko RR Lyrae. We found that there are four Blazhko RR Lyrae (KIC 5559631, 10789273, 11125706, and 12155928) that did not show Blazhko modulation on their light curves. In contrast, improvements can be seen on the light curves for six Blazhko RR Lyrae (KIC 6183128, 7505345, 7610811, 9001926, 9578833, and 9697825) after removing the Blazhko modulation. For the remaining three Blazhko RR Lyrae (KIC 3864443, 4484128, and 7198959) we cannot effectively remove the Blazhko modulation for a variety of $(r, q)$ combinations. Examples of these three cases are presented in Figure 2. After removing the Blazhko modulation of the 13 Blazhko RR Lyrae (including the three RR Lyrae at which their Blazhko modulations cannot be effectively removed), we redetermined their Fourier parameter $\phi_{31}^c$ from the nonmodulated light curves. As an example, Figure 3 compares the V-band $\phi_{31}^c$ Fourier parameters before and after removing the Blazhko modulations. All of the derived $\phi_{31}^c$ for the 30 RR Lyrae are summarized in Table 2.

5. Testing the Relation

In this section, we test the performance of derived photometric [Fe/H] from the $V$ band and the VT band in several cases, assuming the spectroscopic [Fe/H], [Fe/H]$\text{spec}$ adopted from Nemec et al. (2013), is the “ground truth.” Since the [Fe/H]$\text{spec}$ from Nemec et al. (2013) and the [Fe/H]$_V$ based on Equation (7) are in Carretta et al. (2009) and Crestani et al. (2021) scale, respectively, we added an offset of +0.08 dex (as determined in Mullen et al. 2021) to the [Fe/H]$\text{spec}$ when comparing these two [Fe/H] values.

We first compare the [Fe/H]$_V$ calculated from Equation (7) and [Fe/H]$\text{spec}$ using the calibrated or transformed light curves, and after removing the Blazhko modulation for the Blazhko RR Lyrae, in the top panels of Figure 4. Good agreements between [Fe/H]$_V$ and [Fe/H]$\text{spec}$ can be seen from the top left panel of Figure 4, with

Figure 1. Calibrated gr$V$-band light curves and the transformed VT-band light curve for a non-Blazhko RR Lyrae. The error bars include errors from the instrumental magnitudes and the calibration processes.
\[
\left\langle \frac{[\text{Fe}]/[\text{H}]}{\text{spec}} \right\rangle = -0.08 \text{ dex.}
\]

Similarly, photometric \([\text{Fe}/\text{H}]\) from the transformed VT-band light curves agree well with the \([\text{Fe}/\text{H}]_{\text{spec}}\) with
\[
\left\langle \frac{[\text{Fe}]/[\text{H}]}{\text{VT}} - \frac{[\text{Fe}]/[\text{H}]}{\text{spec}} \right\rangle = -0.01 \text{ dex.}
\]

In both cases, the standard deviations (\(\sigma\)) on the averaged values are
\[
\sigma = 0.23 \text{ dex and } \sigma = 0.24 \text{ dex, respectively, well within the rms of 0.41 dex given in Equation (7).}
\]

Since the Blazhko periods might not be reliably determined, or might not be determined at all, for (possible) Blazhko RR Lyrae found in the synoptic time-series imaging surveys, we tested the V-band photometric \([\text{Fe}/\text{H}]\) if the Blazhko modulations were not removed on these Blazhko RR Lyrae. The middle panels of Figure 4 are similar to the top panels, except that the Blazhko modulations were not removed. For the 13 Blazhko RR Lyrae with Blazhko modulations removed, we found
\[
\left\langle \frac{[\text{Fe}]/[\text{H}]}{\text{V}} - \frac{[\text{Fe}]/[\text{H}]}{\text{spec}} \right\rangle = -0.09 \text{ dex (} \sigma = 0.23 \text{ dex)}
\]

and
\[
\left\langle \frac{[\text{Fe}]/[\text{H}]}{\text{VT}} - \frac{[\text{Fe}]/[\text{H}]}{\text{spec}} \right\rangle = -0.02 \text{ dex (} \sigma = 0.25 \text{ dex).}
\]

In comparison, the averaged values changed to
\[-0.06 \text{ dex (} \sigma = 0.26 \text{ dex) and 0.00 dex (} \sigma = 0.29 \text{ dex) in the V and VT bands, respectively, if the Blazhko modulations were not removed. Nevertheless, these averaged differences (and their } \sigma) \text{ are well within the rms of Equation (7). Therefore, it is possible to include the Blazhko RR Lyrae in estimating the V-band photometric [Fe/H] without removing their Blazhko modulations. A similar conclusion was also found in Ngeow et al. (2016).}
\]

Finally, we consider an extreme case such that the Blazhko modulations were not removed for the Blazhko RR Lyrae, at the same time the color corrections were ignored (see the
bottom panels of Figure 4). That is, assuming \( B - V = 0.0 \) mag and \((g^p - P^b) = 0.0\) mag in Equations (3) and (5) when calibrating the \( V \)– and \( VT \)-band light curves, \( V = \text{\textit{V}}^\text{inst} + Z_P \) and \( VT = \text{\textit{V}}^\text{inst} + Z_P + 0.006\). As a result, the \( VT \)-band light curves are equivalent to \( r \)-band light curves. In this case the averaged difference for the \( V \)-band photometric \([\text{Fe}/\text{H}]\) is \(-0.03\) dex \((\sigma = 0.25\) dex\)), which is still reasonable to apply Equation (7) to estimate the photometric \([\text{Fe}/\text{H}]\). In contrast, the \( VT \)-band photometric \([\text{Fe}/\text{H}]\) displays a large offset from the 1:1 relation in the bottom right panel of Figure 4, with a much larger averaged difference of \(0.28\) dex \((\sigma = 0.28\) dex\)). Even though this value is still within the 0.41 dex rms of Equation (7), it is large enough to induce a bias in the derived photometric \([\text{Fe}/\text{H}]\). When the color corrections were ignored or set to zero, \( \phi_{31} \) values derived from the \( VT \)-band light curves are equivalent to those from the \( r \)-band light curves; hence, Equation (7) should not be used.

Would it be possible to use a mean color when transforming the \( gr \)-band light curves to the \( VT \)-band light curves? We tested this scenario by using the mean colors for each RR Lyrae, and repeated the same procedures. In this case \([\text{Fe}/\text{H}]_{\text{VT}} - [\text{Fe}/\text{H}]_{\text{PEC}}) = 0.28\) dex with \( \sigma = 0.29\) dex. This scenario is similar to the previous case for setting the color to zero, as the constant color term can be “absorbed” to the \( Z_{P,m} \); hence, the \( VT \)-band light curves would be similar to the \( r \)-band light curves. To remedy this, we provided a template color curve in Appendix B, such that colors can be estimated at various pulsational phases using the template color curve and an estimation of the mean color, and not assuming a constant or zero color. There are various approaches to estimate the mean color for an ab-type RR Lyrae, such as using the observed light curves, using prior information (e.g., from other surveys or observations), using a period–color relation (e.g., in Ngeow et al., 2022), etc. It is up to the researchers to decide which approach to use, depending on their situations, needs, and goals.

### 6. Deriving the Relations

The Fourier parameter \( \phi_{31} \) presented in Table 2 for all of the 30 ab-type RR Lyrae can be used to derive independent \([\text{Fe}/\text{H}] - \phi_{31} \)–\( P \) relations in the \( gVr \) bands. Following Mullen et al. (2021), we adopted a \([\text{Fe}/\text{H}] - \phi_{31} \)–\( P \) relation in the form of \([\text{Fe}/\text{H}] = a + b(P - P_0) + c(\phi_{31} - \phi_{31}) \), where \( P_0 \) and \( \phi_{31} \) are the mean period and \( \phi_{31} \) in the sample. The zero-point \((a)\), the period coefficient \((b)\), and the \( \phi_{31} \) coefficient \((c)\) were fitted using the orthogonal distance regression (ODR) implemented in the SciPy package (e.g., scipy.odr), at which errors on both \([\text{Fe}/\text{H}]\) and \( \phi_{31} \) were included in the fittings. The mean period for our sample is \( P_0 = 0.55 \) days, and the mean \( \phi_{31} \) values in the \( gVr \) band are 4.97, 5.06, and 5.27 rad,
respectively. To be consistent with Mullen et al. (2021), we adopted $P_0 = 0.58$ days and $\phi_31 = 5.25$ rad, and derived the following relations:

$$\begin{align*}
[\text{Fe/H}]_V &= -1.01[\pm 0.06] - 7.43[\pm 0.80] \\
&\times (P - 0.58) + 1.69 \\
&\times [\pm 0.16](\phi_{31} - 5.25), \text{ rms } = 0.24 \text{ dex}, \quad (9)
\end{align*}$$

$$\begin{align*}
[\text{Fe/H}]/V &= -1.21[\pm 0.05] - 7.67[\pm 0.78] \\
&\times (P - 0.58) + 1.50 \\
&\times [\pm 0.14](\phi_{31} - 5.25), \text{ rms } = 0.24 \text{ dex}, \quad (10)
\end{align*}$$

Figure 4. Comparisons of the photometric [Fe/H], either in the V band (left panels) or VT band (right panels), derived from Equation (7) to the [Fe/H]spec. Errors on photometric [Fe/H] include the rms error and propagated errors on $\phi_31$ from Equation (7), and we assume errors on $P$ are negligible. The top, middle, and bottom panels are the comparisons for three different cases as discussed in the text (see Section 5). The solid lines represent the 1:1 relation and they are not the fits to the data. The dashed lines show the expected $\pm 0.41$ dex rms on the photometric [Fe/H].
If we adopted $P_0$ and $\phi_{31}^0$ from our sample, then the zero-points ($a$) were changed to $-1.26 \pm 0.04$, $-1.26 \pm 0.04$, and $-1.27 \pm 0.05$ in the $gVr$ band, respectively. We emphasize that Equations (9)–(11) are only applicable to ab-type RR Lyrae.

Similar to Nemec et al. (2013) and Ngeow et al. (2016), we compare the predicted photometric $[\text{Fe}/\text{H}]$ derived from Equations (9), (10), and (11) to the $[\text{Fe}/\text{H}]_{\text{spec}}$ in Figure 5. The Fourier parameters $\phi_{31}$ were adopted from Table 2, including the Blazhko RR Lyrae with Blazhko modulations being removed, when calculating the predicted photometric $[\text{Fe}/\text{H}]$. All of the three equations give $\langle [\text{Fe}/\text{H}]_{\text{spec}} \rangle - [\text{Fe}/\text{H}]_{\text{r}} = 0.0$ dex, with $\sigma_g = 0.24$ dex, $\sigma_{Vr} = 0.23$ dex, and $\sigma_r = 0.29$ dex, respectively. These standard deviations are the same or similar to the rms given in Equations (9)–(11), and no systematic offsets were seen from Figure 5.

Recall that the $[\text{Fe}/\text{H}]_{gr}$, $[\text{Fe}/\text{H}]_v$, and $[\text{Fe}/\text{H}]_{r}$ in Equations (9)–(11) are in the Carretta et al. (2009) scale. If the $[\text{Fe}/\text{H}]$ were converted to the Crestani et al. (2021) scale, same as in Mullen et al. (2021), then the $gVr$-band zero-points ($a$) of the relation became $-0.93 \pm 0.06$, $-1.13 \pm 0.05$, and $-1.46 \pm 0.07$, respectively.

7. Discussions and Conclusions

In this work, we tested the performance of $[\text{Fe}/\text{H}]_v$ from Mullen et al. (2021) using a set of homogeneous ab-type RR Lyrae located in the Kepler field. The most important conclusion from our work is good agreement was found between $[\text{Fe}/\text{H}]_v$ and $[\text{Fe}/\text{H}]_{\text{spec}}$. This conclusion also holds if the $V$-band light curves are photometrically transformed from the $gr$-band light curves, as long as the color terms are properly taken into account when calibrating the photometry. On the other hand, the transformed $V$-band (i.e., the $VT$-band) light curves should not be used to derive $[\text{Fe}/\text{H}]_v$ if the color terms are ignored or set to a constant. Modern synoptic time-series surveys often will not acquire near-simultaneous $g$- and $r$-band photometry\(^5\); hence, estimating the $(g-r)$ colors at certain pulsational phases can be a nontrivial task. In Appendix B, we provide a template color curve such that the $(g-r)$ colors can be estimated at various pulsational phases.

Our sample also contains \(~40\%\) Blazhko RR Lyrae, providing an opportunity to test the inclusion of Blazhko RR Lyrae in deriving $[\text{Fe}/\text{H}]_v$. Our test results suggested Blazhko RR Lyrae can be used even if the Blazhko modulations are not removed. This implies RR Lyrae found in the synoptic time-series surveys (e.g., the LSST in coming years; see Hernitschek & Stassun 2022) can be used to determine the photometric $[\text{Fe}/\text{H}]$ without first identifying if they are Blazhko RR Lyrae or not.

Finally, we derive independent $[\text{Fe}/\text{H}]_{\phi_{31}-P}$ relations, for the ab-type RR Lyrae, in the $gVr$ bands based on our data. Good agreement between the coefficients can be seen in the $V$-band $[\text{Fe}/\text{H}]_{\phi_{31}-P}$ relation derived in Mullen et al. (2021) and in our work, suggesting the $V$-band $[\text{Fe}/\text{H}]_{\phi_{31}-P}$ relation is robust, no matter if it is derived from an “all-sky” sample as done in Mullen et al. (2021, with \(~10^3\) ab-type RR Lyrae) or from a “local” sample of 30 ab-type RR Lyrae located in the Kepler field. Furthermore, the $V$-band light curves are totally independent in these works. Together with the infrared $[\text{Fe}/\text{H}]_{\phi_{31}-P}$ relation presented in Mullen et al. (2021), i.e., $[\text{Fe}/\text{H}]_{\text{WISE}} = -1.47 - 8.33(P - 0.58) + 0.92(\phi_{31} - 1.90)$, we found that both period coefficient ($b$) and $\phi_{31}$ coefficient ($c$) in the $[\text{Fe}/\text{H}]_{\phi_{31}-P}$ relations monotonically decrease when the wavelength is increasing.

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Appendix A
Selection of Reference Stars

We summarize a number of selection criteria to select nonvarying stellar sources from the catalogs merged with Pan-STARRS1 DR1 photometric catalog and the UBV photometric catalog given in Everett et al. (2012). These nonvarying stellar sources will be used to construct a reference star catalog for each of the targeted RR Lyrae. The selection criteria are listed below, some of them were inspired from, or similar to those adopted in, the ZTF Science Data System Explanatory Supplement.6

1. Excluding the targeted RR Lyrae.
2. Separation of the matched sources when crossmatching the Pan-STARRS1 DR1 and the UBV catalogs is smaller than 1″.
3. Sources without measurements in any of the BVgri bands in the merged catalogs were excluded.
4. Sources with number of observations in each of the gri bands is greater than 5.
5. Sources with mean point-spread function (PSF) magnitudes (meanPSF) in the gri band that are in between 14 mag and 20 mag.
6. Sources with a difference between the i-band mean PSF magnitudes and mean Kron magnitudes (meanKron), meanPSFi − meanKroni, that are within the range of −1.5 mag and −0.04 mag.
7. Sources with color, meanPSFg − meanPSFi, that are within the range of −0.5 mag and 3.0 mag.
8. Sources with photometric errors in all of the BVgr bands that are smaller than 0.1 mag.
9. Sources with standard deviations (magStd) in the gri-band meanPSF that are smaller than 0.1 mag.

For the remaining sources after applying the above selection criteria, a third-degree polynomial function was fitted to the gri-band meanPSF and magStd in a two-step process. In the first step, we removed data points that were larger than 1 standard deviation from the best-fit polynomial function. The rest of the data points were refit in the second step of the process (see Figure 6). We only retained sources that have magStd smaller

![Figure 6](https://irsa.ipac.caltech.edu/data/ZTF/docs/ztf_explanatory_supplement.pdf)
than the best-fit polynomial function in the gr band as reference stars for performing the photometric calibration (see Section 3). The number of the selected reference stars varies from \( \sim 160 \) to \( \sim 1120 \) for the 30 targeted RR Lyrae.

Appendix B
Color-curve Template

The near-simultaneous gr-band observations of our targeted RR Lyrae allow the construction of color curves via Equation (6); an example is presented in the left panel of Figure 7. These color curves can be used to construct a template color curve in the \((g-r)\) color. We first removed data points with errors larger than 0.1 mag (they tend to be outliers on the color curves), and then normalized the color curves by subtracting the means and scaled with their amplitudes. Finally, we rephased the color curves using the reference epoch \( t_0 \) adopted from Nemec et al. (2013, their Table 1). The composite color curve is presented in the right panel of Figure 7, and fitted with a low-order Fourier sine series. The best-fit template color curve, shown as red curve in the right panel of Figure 7, is

\[
c(\Phi) = 0.280 \sin(2\pi \Phi + 4.205) + 0.111 \sin(4\pi \Phi + 4.088) + 0.062 \sin(6\pi \Phi + 4.430),
+ 0.027 \sin(8\pi \Phi + 4.808) + 0.011 \sin(10\pi \Phi + 5.138),
\]

(B1)

with an rms of 0.131, where \( c = (g-r) \).

Figure 7. Left panel: comparison of the instrumental \((g_{\text{inst}}^{\text{inst}} - r_{\text{inst}}^{\text{inst}})\) colors (in red open circles) and the calibrated \((g_{\text{PS1}} - r_{\text{PS1}})\) colors (in black filled triangles), using Equation (6), for the same RR Lyrae as presented in Figure 1. Right panel: composite of the normalized \((g-r)\) color curve based on 27 RR Lyrae (excluding KIC 3864443, 4484128, and 7198959; due to larger scatters seen on their color curves); the red curve represents the best-fit template color curve (see the text for details).
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