Precision measurement of $^{210}$Bi $\beta$-spectrum

I. E. Alekseev, S. V. Bakhlanov, A. V. Derbin, I. S. Drachnev, I. M. Kotina, I. S. Lomskaya, V. N. Muratova, N. V. Niyazova, D. A. Semenov, M. V. Trushin, and E. V. Unzhakov

1 V.G. Khlopin Radium Institute, St. Petersburg 194021, Russia
2 Petersburg Nuclear Physics Institute, Gatchina 188350, Russia
National Research Center “Kurchatov Institute”

The precision measurement of the $\beta$-spectrum shape for $^{210}$Bi (historically RaE) have been performed with a spectrometer based on semiconductor Si(Li) detector. This first forbidden non-unique transition has the transition form-factor strongly deviated from unity and knowledge of its spectrum would play an important role in low-background physics in presence of $^{210}$Pb background. The measured transition form-factor could be approximated as $S(W) = 1 + (-0.4363 \pm 0.0037)W + (0.0523 \pm 0.0010)W^2$, that is in good agreement with previous studies and has significantly increased parameter precision.

I. INTRODUCTION

Precision measurements of the $\beta$-spectra are currently very important in neutron and nuclear $\beta$-decay studies, as a means of searching for the effects beyond the Standard Model (SM) in the low energy region [1, 2]. Accurate studies of nuclear $\beta$-decays have been exploited for many years in various applications of fundamental physics problems, predominately in neutrino physics. In this paper we present the results of the measurement of $\beta$-spectrum of $^{210}$Bi with spectrometer based on Si(Li)-detectors [3, 4]. The problems of the $\beta$-decay of $^{210}$Bi (historically Radium E, RaE) such as strong deviation from the allowed energy distribution and prolonged lifetime has been investigated widely starting from 1930-th in many experimental and theoretical investigations [5–7]. The situation was clarified after the assumptions that the ground state of $^{210}$Bi is the combination of some wave functions [8] and calculations of nuclear matrix elements for $\beta$-decay on the basis of the finite Fermi systems theory [9]. Later spectral measurements of $^{210}$Bi $\beta$-decay were performed in works [8, 10–12] that used magnetic lens and solid state $\beta$-spectrometers.

Isotope $^{210}$Bi is one of the natural radioactive elements of the $^{238}$U decay chain. As a product of the radioactive gas $^{222}$Rn and the subsequent long-lived $^{210}$Pb, the isotope $^{210}$Bi is present inside or on the surface of almost all structural materials. At present the precise knowledge of $^{210}$Bi $\beta$-spectrum is very important for background modeling of modern low-background studies of neutrino physics, dark matter and others rare processes. In particular, the shape of $^{210}$Bi $\beta$-spectrum is very similar to the spectrum of recoil electrons originated from scattering of solar CNO-neutrinos [13] and extraction of the CNO signal requires the shape of $\beta$-spectrum to be determined with the sufficient accuracy.

II. EXPERIMENTAL SETUP

Magnetic [14, 15] and electrostatic [16, 17] $\beta$-spectrometers possess the superior energy resolu-
cessed with a standard CR-3RC analogue shaper and digitized with a 14-bit ADC. The energy resolution determined for 59.6 keV γ-line of $^{241}$Am turned out to be FWHM = 900 eV for the full width at half maximum.

In order to determine the main characteristics of the spectrometer, we used a $^{207}$Bi source, providing γ- and X-rays, conversion and Auger electrons. The $^{207}$Bi source with an activity of $10^4$ Bq was placed inside the vacuum cryostat at a distance of 14 mm from the Si(Li) detector surface. The $^{207}$Bi spectrum, measured with the Si(Li) detector, is shown in Fig. 1 for the interval (0.01 – 2.0) MeV.

Three of the most intense $^{207}$Bi γ-lines have energies of 569.7 keV, 1063.7 keV and 1770.2 keV and are emitted with probabilities of 0.977, 0.745 and 0.069 per single $^{207}$Bi decay, respectively [22, 23]. The corresponding peaks of the conversion electrons from K, L and M shells are clearly visible in the spectrum in Fig. 1. The electron energy resolution determined via 480 keV line is FWHM = 1.8 keV. Energy calibration performed using Pb Kα1 X-ray and γ-line with energies of 74.97 keV and 569.70 keV, correspondingly, predicts the position of 975.66 keV conversion electrons peak with an accuracy better than 0.3 keV.

The low-energy part of the $^{207}$Bi spectrum was used for evaluation of the thickness of the non-sensitive layer on the surface of Si(Li) detector. This area contains a set of peaks corresponding to Pb X-rays from K and L series and Auger electrons. The observed position of 56.94 keV Auger peak ($e_{K,L1,L2}$) appeared to be at 56.22 keV. Taking into account the gold coating thickness of 500 Å, the measured 59 keV electron energy loss is 720 eV that corresponds to 4700 Å of the non-sensitive layer.

The planar source of $^{210}$Bi was prepared with the method of thermal oxidation [24]. The polished stainless steel foil with diameter of 24 mm and thickness of 11 μm was used as substrate for $^{210}$Bi source. A water-alcohol solution that contained $^{210}$Bi atoms was deposited onto the oxidized surface of the foil. The solution was air-dried, which was followed by a short-term diffusion annealing during 3 minutes at 300° C for implanting the radionuclide into the oxidized surface layer of the substrate. The technology allows to reach negligibly small source thickness suppressing effects related with attenuation and scattering of the electrons in the source material. Since these effects are difficult to simulate due to complications of the source geometry reconstruction the produced source decreases systematic uncertainties of the measurement.

III. THE RESULTS OF MEASUREMENTS

The natural radioactivity of the $^{238}$U and $^{232}$Th families, along with the long-lived $^{40}$K isotope, are the main sources of the background for neutrino physics and dark matter searches at energies below 3 – 5 MeV. The main decay modes and half-life $T_{1/2}$ values of daughter nuclei following after a long-lived lead isotope $^{210}$Pb are as follows:

$$^{210}\text{Pb}(\beta, 22.3 \text{ y}) \rightarrow ^{210}\text{Bi}(\beta, 5.0 \text{ d}) \rightarrow ^{210}\text{Po}(\alpha, 138 \text{ d})$$

(1)

The end-point energies of the beta spectrum of $^{210}$Pb and $^{210}$Bi are 63.5 keV and 1162 keV, respectively, the energy of α-particles of $^{210}$Po is 5.407 MeV [22, 23]. Since the source of $^{210}$Pb was specially prepared for this experiment and was free of other Pb isotopes, equilibrium in the chain (1) was not reached yet at the time of measurement.

![FIG. 1. The spectrum of $^{207}$Bi source measured with the Si(Li) detector in energy range of (0.01 – 2.0) MeV. The inset shows the electron peaks corresponding to internal conversion from K, L, M and N shells of the 570 keV nuclear level.](image)

![FIG. 2. Low energy part of $^{210}$Pb → $^{210}$Bi spectra measured with Si(Li)-detector. The inset shows the decay scheme of $^{210}$Pb.](image)
The Fig. 2 shows the low-energy region of the measured spectrum determined mainly by $^{210}$Pb decays. Since the coefficient of internal electron conversion due to the discharge of 46.5 keV nuclear level is large $e/\gamma \approx 20$ [22], the electron peaks corresponding to conversion from $L$, $M$ and $N$ shells are clearly visible in the spectrum. The electron energy resolution of Si(Li) detector determined for 30 keV electron conversion line is FWHM = 1.0 keV and the low energy detection threshold is about 5 keV. The spectrum also shows the peaks of characteristic 10.8 keV $\beta$, 5.9 keV $\gamma$ quanta of $^{210}$Pb and 5.4 keV $\alpha$-peak due to $^{210}$Po, 3.8 keV $\gamma$-quanta of $^{210}$Po. The $^{210}$Pb end-point energy $E_{0}$ is 1162 keV measured with high accuracy in other experiments [22, 23]. The background level near the end-point energy of $^{210}$Bi beta spectrum was 0.18 counts/h/keV and that determined mainly by Compton scattering of 1.46 MeV $\gamma$-quanta of $^{40}$K passing through the passive shielding.

The whole spectrum in the energy range of (0.05 – 5.5) MeV is shown in the Fig. 3. The energy resolution of 5407 keV $^{210}$Pb slightly asymmetric $\alpha$-peak is FWHM = 26 keV. The background level near the end-point energy of $^{210}$Bi beta spectrum was 0.18 counts/h/keV and that determined mainly by Compton scattering of 1.46 MeV $\gamma$-quanta of $^{40}$K passing through the passive shielding.

The data were obtained in 634 hours of data-taking in short 1-hour series used for stability control. To determine the energy calibration $E = a + bN$ (where $E$ is a Si(Li) visible energy and $N$ is an ADC channel number), the position of 46.5 keV $\gamma$-peak and the value of $^{210}$Bi end-point energy $E_{0} = 1162$ keV measured with high accuracy in other experiments [22, 23] were used. During the fitting of the spectrum, the calibration slope $b$ equal to the analyzer channel width was free, while the value of the parameter $a$ was fixed by 46.5 keV peak position. The differences of fitting parameters for the all 1-hour runs are in agreement with their statistical uncertainties. The total number of registered $^{210}$Bi decays was $1.0 \times 10^{8}$.

IV. DATA ANALYSIS

The energy distribution $S(W)$ of $\beta$-particles emitted in $\beta$-decay process could be expressed as

$$S(W) = PW(W - W_{0})^{2} \times F(W, Z) \times C(W),$$

where $P$ and $T$ are the electron momentum and energy, $W = T/mc^{2} + 1$ is full electron energy, $W_{0} = T_{0}/mc^{2} + 1$ is $\beta$-spectrum end-point energy, $F(W, Z)$ is the electron Fermi function that takes into account electromagnetic interaction of the outgoing electron with the atom and $C(W)$ is the transition nuclear form-factor that considers the effects of internal nuclear interactions.

The Fermi function $F(W, Z)$ is historically derived in approximation of a point-like nuclei without consideration of the atomic shells [22] that means that comparison with experiments using this model needs application of the same approximation, while the $\beta$-spectrum for practical applications would need a more profound calculation of the Fermi function that was performed according to [26, 28].

The transition investigated in this work is of forbidden type that means that the initial and final momenta can not be explained by a single nucleon decay in the mean nuclear field and the nuclear form-factor $C(W)$ is expected to deviate from unity and is the main subject of the measurement. Since the shape factor of first forbidden non-unique transition with such parity-momentum relations can be expressed with sufficient accuracy by a second degree polynomial, we choose the $C(W)$ parametrization as in [18]:

$$C(W) = 1 + C_{1}W + C_{2}W^{2}$$

with generic values of parameters $C_{1}$ and $C_{2}$ that were defined through maximum likelihood fit with $\chi^{2}$ likelihood function.

The final model of the experimental spectrum expresses as:

$$N(E) = \int_{E/mc^{2}+1}^{W_{0}} S(W) \times R(W, E) dW,$$

where $R(W, E)$ is the spectrometer normalized response function obtained with Monte-Carlo simulation of electrons with energy $W$ exiting the source with uniform distribution within the source and uniform distribution of their momenta directions.

Since the setup in use has the classical “target-detector” geometry, it is quite important to take into account the detector response function that would contain a long low-energy tail caused by fraction of electrons backscattered from the detector as well as by bremsstrahlung exit from the detector crystal. The Si(Li) detector has i-region thickness exceeding the stopping range of an electron with endpoint energy of 1162 keV and thus the geometry of irradiated regions of the...
setup is quite well established. This allows to account for the detector energy response through a precise simulation with the Monte-Carlo method. We used Geant4.10.04 simulation package [29] with the standard G4EmStandardPhysics option4 package of electromagnetic interactions.

The package choice was mainly motivated by the Single Scattering model for electrons, that is the most promising among standard ones according to [30, 31]. The simulation was including modelling of the detector entrance windows, collimator and holders according to the physical setup geometry.

The fit range has the lower bound that comes from presence of $^{210}$Pb in the source that covers the low-energy region. Considering that the nuclear form-factor depends only upon momenta of the electron and neutrino one should not expect sudden behavior in the lower tenth of the energy spectrum so this lower bound should not be important for the form-factor establishment.

The fit with canonical Fermi function $F_0(W, Z)$ was performed in the energy range $120 - 1175$ keV with flat background approximation. The Fermi function was calculated according to [27] as:

$$F_0(W, Z) = 4(2pR(A))^{2(\gamma - 1)} e^\gamma \eta (1 - \gamma^2)$$

where $Y = \alpha Z W / p$ and $\gamma = \sqrt{1 - \alpha^2 Z^2}$, $\alpha$ is the fine-structure constant and $R$ is the nuclear radius defined as $R = 0.0029 \times A^{1/3} + 0.0063 \times A^{2/3} - 0.017 \times A^{-1}$.

The fit results are shown on fig. 4. The obtained minimum of $\chi^2/NDF = 1775.3 / 1705$ corresponds to Pearson P-value = 0.12 and form-factor parameters $C_1 = -0.4523 \pm 0.0031$ and $C_2 = 0.0560 \pm 0.0008$. The easily computed values of $F_0(W, Z)$ that together with obtained coefficients $C_1$ and $C_2$ allows to calculate the shape of the $\beta$-spectrum that references to our measurements.

In order to have a fair comparison with results of [8, 11] we performed the fit with the Fermi function $F(W, Z)$ calculated in accordance with formalism presented [28], attempting to improve the precision of the analytical description. In this work $F(E, Z)$ was enhanced by including second and third terms of $pr$-power expansion of electron wave function at small values of $r$ ($F_0(E, Z)$ is obtained by neglecting all but the first term). Also, additional corrections were included, such as the finite size of the nucleus and atomic shell screening into account [32, 33]. The values of $F(E, Z) = F_0(E, Z) \cdot \chi \cdot \eta$ used in the calculation were taken from the Table 14 of [28] for $Z = 83$ and $A = 210$.

The fit range was increased with respect to the improved $F(W, Z)$ that takes into account the nucleus final size and the screening corrections. The same procedure gives the form-factor parameters as $C_1 = -0.4339 \pm 0.0012$ and $C_2 = 0.0513 \pm 0.0004$. These values one can compare with $C_1' = 0.46 \pm 0.01$ and $C_2' = 0.0586 \pm 0.002$ obtained in [8]. The errors of $C_1$, $C_2$ obtained in the present work are more than five times less, however, the parameters $C_1$, $C_2$ and $C_1'$, $C_2'$ are consistent with each other within the 1.5 $\sigma$.

The final fitting procedure was repeated using the classic definition of the Fermi function with several corrections that included atomic shell screening effect [34], finite size distribution of electromagnetic and weak charge inside nucleus [35] and QED radiative corrections [36]. The final $F(E, Z)$ had the following form:

$$F(E, Z) = F_0(E, Z) \times S(E, Z) \times L_0(E, Z) \times C(E, Z) \times G_\beta(E)$$

where $E$ is full electron energy, $Z$ is the charge of a daughter nucleus, $F_0(E, Z)$ is Fermi function, $S(E, Z)$ screening correction, $L_0(E, Z)$ and $C(E, Z)$ are electromagnetic and weak finite size corrections and $G_\beta(E)$ is radiative correction. The results of the final fitting procedure are given in Fig. 5.

Implication of a more precise Fermi function allowed to lower energy threshold down to 100 keV, providing good P-value = 0.13 that is an evidence of better agreement of the corrected beta-spectrum with the experimental data. The minimum of $\chi^2/NDF = 1803.9 / 1738$ corresponds to form-factor parameters $C_1 = -0.4363 \pm 0.0037$ and $C_2 = 0.0523 \pm 0.0010$. These values $C_1$ and $C_2$ are obtained taking into account the most complete knowledge of the interactions emitted electron with atom. One
should note that the parameters $C_1$ and $C_2$ have quite strong correlation in the fit of the experimental data, having the correlation coefficient of 0.94.

As the response function model is based on the simplified interaction models used in the simulation, it is important to estimate the uncertainties concerning its imperfection. Consideration of these uncertainties was performed through analytical modification of the response function as:

$$
\tilde{R}(E, W) = \begin{cases} 
R(E, W) \times (1 + A \ln(BW)), & \text{if } E < T - 5\sigma \\
R(E, W), & \text{if } E > T - 5\sigma 
\end{cases}
$$

where $\sigma$ is the detector resolution at kinetic energy $T$ and $A, B$ are free parameters. Eventually, six parameters were free in the fit: the common normalization coefficient, the slope of the energy calibration, the form factor parameters $C_1$ and $C_2$, and response function parameters $A$ and $B$.

The dependence $A \ln(B\text{W})$ used for the variation of the response function tail approximately corresponds to the uncertainties of the response function for different GEANT4 simulation packages. The response function was renormalized to conserve detection efficiency of the original simulation.

As the response function model is based on the simplified interaction models used in the simulation, it is important to estimate the uncertainties concerning its imperfection. Consideration of these uncertainties was performed through analytical modification of the response function as:

$$
\tilde{R}(E, W) = \begin{cases} 
R(E, W) \times (1 + A \ln(BW)), & \text{if } E < T - 5\sigma \\
R(E, W), & \text{if } E > T - 5\sigma 
\end{cases}
$$

where $\sigma$ is the detector resolution at kinetic energy $T$ and $A, B$ are free parameters. Eventually, six parameters were free in the fit: the common normalization coefficient, the slope of the energy calibration, the form factor parameters $C_1$ and $C_2$, and response function parameters $A$ and $B$.

The dependence $A \ln(B\text{W})$ used for the variation of the response function tail approximately corresponds to the uncertainties of the response function for different GEANT4 simulation packages. The response function was renormalized to conserve detection efficiency of the original simulation.
dimensional Gaussian distribution that includes the correlation coefficient obtained in the fit. One can see that both of our spectra are consistent with Daniel (1962) and Carles & Malonda (1996) spectra within uncertainties. The current study shows significantly increased precision with respect to the previous studies.

V. CONCLUSIONS

The spectrometer based on the Si(Li) detector was used to precisely measure the \( \beta \)-spectrum of \(^{210}\)Bi nuclei. As a result of the 634 hours measurements with a total number of \( 1.0 \times 10^8 \) of registered electrons it was established that the \( \beta \)-spectrum is described by form-factor \( S(W) = 1 + (−0.4523 \pm 0.0031)W + (0.0560 \pm 0.0008)W^2 \) if the Fermi function is calculated according to formula [5].

for a point-like nucleus. The obtained values of the parameters \( C_1 \) and \( C_2 \) together with [5] can be used for calculation of the electron spectrum of \(^{210}\)Bi.

When the additional above-mentioned corrections to the Fermi function are taken into account, the form-factor parameters are equal \( C_1 = (−0.4363 \pm 0.0037) \) and \( C_2 = (0.0523 \pm 0.0010) \), that can be useful for calculation of specific nuclear matrix elements. The obtained parameters of the form-factor are in agreement with the previous studies and have significantly increased precision.

VI. ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project nos. 19-02-00097 and 20-02-00571).

[1] P. Herczeg. Beta decay beyond the standard model. *Progress in Particle and Nuclear Physics*, 46(2):413–457, 2001. ISSN 0146-6410. doi:10.1016/S0146-6410(01)00149-9.

[2] Jeffrey S. Nico and Michael Snow. Fundamental neutron physics. *Annual Review of Nuclear and Particle Science*, 55(1):27–69, 2005. doi:10.1146/annurev.nucl.55.090704.151611.

[3] I. E. Alexeev, S. V Bakhlanov, N. V. Bazlov, E. A. Chmel, A. V. Derbin, I. S. Drachnev, I. M. Kotina, V. N. Muratova, N. V. Pilipenko, D. A. Semyonov, E. V. Unzhakov, and V. K. Yeremin. Beta-spectrometer with Si detectors for the study of \(^{210}\)Po–\(^{214}\)Po decays. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 890:64–67, 2018. ISSN 0168-9002. doi:10.1016/j.nima.2018.02.031.

[4] N. V. Bazlov, S. V. Bakhlanov, A. V. Derbin, I. S. Drachnev, V. K. Eremin, I. M. Kotina, V. N. Muratova, N. V. Pilipenko, D. A. Semyonov, E. V. Unzhakov, and E. A. Chmel. A beta spectrometer based on silicon detectors. *Instruments and Experimental Techniques*, 61:323–327, 2018. ISSN 0168-9002. doi:10.1134/S0001820X1803017X.

[5] J. S. O’Conor. The beta-ray spectrum of Radium E. *Phys. Rev.*, 52:303–314, Aug 1937. doi:10.1103/PhysRev.52.303.

[6] G. J. Neary and J. D. Cockcroft. The beta-ray spectrum of Radium E. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 175(960):71–87, 1940. doi:10.1098/rspa.1940.0044.

[7] E. J. Konopinski and G. E. Uhlenbeck. On the Fermi theory of \( \beta \)-radioactivity. II. The “forbidden” spectra. *Phys. Rev.*, 60:308–320, Aug 1941. doi:10.1103/PhysRev.60.308.

[8] A. Grau Carles and A. Grau Malonda. Precision measurement of the RaE shape factor. *Nuclear Physics A*, 506(1):83–90, 1996. ISSN 0375-9474. doi:10.1016/0375-9474(95)00381-9.

[9] S. A. Fayans and Khodel V. A. Calculations of nuclear matrix elements for beta-decay of RaE. *Physics Letters B*, 31(3):99–102, 1970. ISSN 0370-2693. doi:10.1016/0370-2693(70)90120-4.

[10] E. A. Plassmann and L. M. Langer. Beta spectrum of Radium E. *Phys. Rev.*, 96:1593–1598, Dec 1954. doi:10.1103/PhysRev.96.1593.

[11] H. Daniel. Das \( \beta \)-spektrum des raE. *Nuclear Physics*, 31:293–307, 1962. ISSN 0029-5582. doi:10.1016/0029-5582(62)90745-9.

[12] D. Flothmann, W. Wiesner, R. Löhken, and H. Rebel. \( \beta \)-Spektroskopie mit Halbleiterdetektoren beim Zerfall von \(^{32}\)P, \(^{49}\)Sc, \(^{204}\)Tl und \(^{210}\)Bi. *Zeitschrift für Physik A Hadrons and Nuclei*, 225:164–194, 1969. doi:10.1007/BF01392517.

[13] M. Agostini, K. Altenmüller, S. Appel, V. Atoschenko, Z. Bagdasarian, D. Basilio, G. Bellini, J. Benziger, G. Bonfini, D. Bravo, B. Caccianiga, F. Calaprice, A. Caminata, L. Cappelli, S. Caprioli, M. Carlini, P. Cavalcante, F. Cavanna, A. Chepurnov, K. Choi, L. Collica, D. D’Angelo, S. Davini, A. Derbin, X. F. Ding, A. Di Ludovico, L. Di Noto, I. Drachnev, K. Fomenko, A. Formozov, D. Franco, F. Gabriele, C. Galbiati, M. Gschwender, C. Ghiano, M. Giannamarchi, A. Goretti, M. Gromov, D. Guffanti, T. Houdy, E. Hungerford, Aldo Ianni, Andrea Ianni, A. Jany, D. Jeschke, S. Kumanar, V. Kobychev, G. Korga, T. Lachenmaier, M. Laubenstein, E. Litvinovich, P. Lombardi, L. Ludhova, G. Lukyanenko, L. Lukyanenko, I. Machulin, G. Manuzio, S. Marcocci, J. Maricic, J. Martyn, E. Meroni, M. Meyer, L. Miramonti, M. Misiaszek, V. Muratova, B. Neumair, M. Nieslyon, L. Oberauer, V. Orekhov, F. Ortica, M. Pallavicini, L. Papp, O. Penek, L. Pietrafaccia, N. Pilipenko, A. Pocar, A. Porcelli, G. Raikov, G. Ranucci, A. Razeto, A. Re, M. Redchuk, A. Romani, N. Rossi, S. Rottemaner, S. Schönert, D. Semenov, M. Skorokhvatov, O. Smirnov, A. Sotnikov, L. F. F. Stokes, Y. Suvorov, R. Tartaglia, G. Testera, J. Thurn, E. Unzhakov, F. Villante, A. Visheska, R. B. Vogelaar, F. von Feilitzsch, S. Weinz, M. Woj-
[32] L. A. Sliv. On theory of forbidden beta-transitions. *JETP*, 17(12):1049–1058, 1947.

[33] M. E. Rose. A note on the possible effect of screening in the theory of beta-disintegration. *Phys. Rev.*, 49:727–729, May 1936. doi:10.1103/PhysRev.49.727.

[34] Wolfgang Buhring. The screening correction to the Fermi function of nuclear β-decay and its model dependence. *Nuclear Physics A*, 430(1):1 – 20, 1984. ISSN 0375-9474. doi:https://doi.org/10.1016/0375-9474(84)90190-8.

[35] D.H. Wilkinson. Evaluation of beta-decay: finite mass and size effects. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 290(2):509 – 515, 1990. ISSN 0168-9002. doi:https://doi.org/10.1016/0168-9002(90)90570-V.

[36] A. Sirlin. General properties of the electromagnetic corrections to the beta decay of a physical nucleon. *Phys. Rev.*, 164:1761–1767, Dec 1967. doi:10.1103/PhysRev.164.1767.