VAQEM: A Variational Approach to Quantum Error Mitigation

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Summary: A variational approach to quantum error mitigation

• **Background:** VQAs are considered suitable to the NISQ era, but machine fidelity is still too low for real world applicability.

• **Goal:** Apply error mitigation in an optimal manner to VQAs for max fidelity – but this is challenging as device and circuit complexity increase.

• **Proposal:**
  • Integrate EM techniques into VQA’s framework of iterative parameter tuning: enabling a feedback-based approach towards optimal EM for the application / device.
  • Targets two idle-time EM methods: insertion of dynamical decoupling sequences and single-qubit gate scheduling.

• **Result:** Improves the quality of the VQA measured objective by 3x on average.
Variational Quantum Algorithms

Classical Optimizer: Tuning gate rotations

Tuning gate rotations on Ansatz circuits with different measurements

Objective Function (Minimize)

Minimize:

\[ H = \{ \text{XIIIII} + \text{IXIIII} + \text{IIXIII} + \text{IIIXII} \]
\[ + \text{IIIIXI} + \text{IIIIIX} + \text{ZZIIII} + \text{IZZIII} \]
\[ + \text{IIIZZI} + \text{IIIIZZ} + \text{ZIIIIZ} \} \]

Objective Function

Gate angle parameters

Potential energy curve of a molecule

\[ H_2 \]

Variational principle: the energy of any trial wave-function is greater than or equal to the exact ground state energy

\[
\frac{\langle \Psi(\vec{\theta}) | H | \Psi(\vec{\theta}) \rangle}{\langle \Psi(\vec{\theta}) | \Psi(\vec{\theta}) \rangle} \geq E_G
\]

https://qiskit.org/learn/intro-qc-qh/
VQA Fidelity in the NISQ era

Energy ($E$) = $Tr[C_\rho \mathcal{H}]$

= $\langle \phi | \mathcal{H} | \phi \rangle$ (pure state)

- $Tr[C_\rho \mathcal{H}]$ for typical, noisy VQE
- $Tr[C_\rho \mathcal{H}]$ for VAQEM
- Noise-free optimization surface

Noisy outcomes

Region beyond lower bound
Impact of Error Mitigation

Energy \( (E) = \text{Tr} [\mathcal{H}_\rho] \)
\[= \langle \phi | \mathcal{H} | \phi \rangle \text{ (pure state)} \]

- \( \text{Tr} [\mathcal{H}_\rho] \) for typical, noisy VQE
- \( \text{Tr} [\mathcal{H}_\rho] \) for VAQEM
- Noise-free optimization surface

Different EM configurations

Parameter Space

Noisy outcomes

Error-mitigated outcomes

Region beyond lower bound
Compiling for machines with limited connectivity leads to increased depth and long critical paths

As application sizes increases, path lengths become longer and more diverse leading to more slack

Decoherence in idle window

Kaitlin N. Smith, Gokul Ravi, Prakash Murali, Jonathan Baker, Nathan Earnest, Ali Javadi-Abhari, Frederic T. Chong. TimeStitch: Error Mitigation in Quantum Computers through Instruction Scheduling. 2021
Idle Window Signal Refocusing: 1Q gate scheduling

Spin Echo Correction: Details in the paper!

optimal position?

Kaitlin N. Smith, Gokul Ravi, Prakash Murali, Jonathan Baker, Nathan Earnest, Ali Javadi-Abhari, Frederic T. Chong. 

TimeStitch: Error Mitigation in Quantum Computers through Instruction Scheduling. 2021
Idle Window Signal Refocusing: Dynamic decoupling

Spin Echo Correction: Details in the paper!

optimal gate types / number / spacing?
1) Imperfect knowledge of stimuli and their effects makes theory driven EM heuristics less effective.
Optimizing EM: practical challenges

2) Micro-analyzing stimuli effects for every EM instance is not scalable.
3) Stimuli-agnostic outcome driven approaches are not always possible since outcomes are often unknown and usually not of highest probability.

* Ensemble of Diverse Mappings MICRO2019
VAQEM: Tuning EM features in the VQA setting

Design details in the paper!
Minimize Objective (H)

VAQEM: Tuning EM features in the VQA setting

Energy (E) = \langle \phi | H | \phi \rangle = \text{Tr}[\mathcal{H}_p]

\text{Only *quantum* EM – details in the paper!}
VQE benefits from VAQEM I

| Bench     | 6q/f/2r | 6q/c/2r | 4q/c/6r | 4q/f/6r | 6q/c/4r | Li+ | H2 |
|-----------|---------|---------|---------|---------|---------|-----|----|
| Depth     | 54      | 31      | 57      | 101     | 55      | 90  | 61 |
| # Win     | 42      | 24      | 22      | 34      | 30      | 45  | 26 |

**VAQEM: GS**

**VAQEM: XY**

**VAQEM: XX**

**VAQEM: GS+XY**
More in the paper:
Conclusion: A variational approach to quantum error mitigation

Future Directions:

• Variationally tune more features of current EM techniques
• Integrate more EM techniques into the VAQEM framework
• Explore tunable optimizations outside of error mitigation
Thank you!
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VAQEM: arXiv:2112.05821
TimeStitch: arXiv:2105.01760
Backup
VQE benefits from VAQEM I

| Bench            | 6q/f/2r | 6q/c/2r | 4q/c/6r | 4q/f/6r | 6q/c/4r | Li+ | H2 |
|------------------|---------|---------|---------|---------|---------|-----|----|
| Depth            | 54      | 31      | 57      | 101     | 55      | 90  | 61 |
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![Bar chart showing VQE energy relative to baseline for different benchmarks and VAQEM configurations.](chart.png)

- VAQEM: GS
- XY
- VAQEM: XY
- XX
- VAQEM: XX
- VAQEM: GS+XY

**Key:**
- HW_TFIM_6q_f_2r
- HW_TFIM_6q_c_2r
- HW_TFIM_4q_c_6r
- HW_TFIM_4q_f_6r
- HW_TFIM_6q_c_4r
- HW_Li+
- UCCSD_H2
- Geo Mean
EM inspired by spin echo correction
**Ideal Flow**

- **Qiskit Runtime**
  - Circuit w/ angle + EM parameters
  - Optimal Classical Tuner
  - Best suited Quantum Machine

**Feasible Flow**

- **Simulation**
  - Circuit w/ angle parameters
  - Optimal Classical Tuner
  - Noise-free Computation Model

- **Qiskit Runtime**
  - Circuit w/ angle parameters
  - Available Classical Tuner
  - Available Quantum Machine

- **In-house independent Window EM Tuner**
  - Opt. angle circuit w/ EM parameters
  - Per-window optimal EM search
  - Available Quantum Machine

**OR**
Tunable Error Mitigation Scope

Proof in the paper!

* Quantum EM techniques *
VAQEM Tuning Overheads
| Bench | 6q/f/2r | 6q/c/2r | 4q/c/6r | 4q/f/6r | 6q/c/4r | Li+ | H2 |
|-------|---------|---------|---------|---------|---------|-----|----|
| Depth | 54      | 31      | 57      | 101     | 55      | 90  | 61 |
| # Win | 42      | 24      | 22      | 34      | 30      | 45  | 26 |
VQA Fidelity in the NISQ era

* Classical Optimizers for Noisy Intermediate-Scale Quantum Devices QCE 2020
