Abstract
This paper proposes a vibration control method of an automotive drive system with backlash to maintain stability and control performance under the control period constraint due to an engine’s characteristics. Reducing the vibrations of the automotive drive system remains a challenge when improving the riding comfort and driving performance of automobiles. In particular, a vibration control method must be developed to compensate for the backlash of differential gears because this element degrades the vibration control performance. Furthermore, engines used as actuators have a constraint in which control cycles are made longer due to restrictions of the input update. The roughly updated cycles adversely affect not only the high vibration control performance but also the stability. In this study, we validate the control system for an automotive drive system with backlash by considering the input update limitation. First, a basic experimental device, which abstracts actual vehicles to focus on the influence due to backlash while reflecting only the basic structure of an automotive drive system, is created. Then to cope with the control cycle constraint, sampled-data $H_2$ control is applied. The servo system is constructed by applying an approximate integrator and frequency shaping. As an approach to compensate for backlash, we propose a simple and practical control mode switching technique. Finally, the effectiveness of the control system is verified experimentally. The results are compared to the control results with those obtained by the traditional discrete approximation.

Keywords: Automotive drive system, Backlash, Control period constraint, Sampled-data control, Servo system, Nonlinear control

1. Introduction

To improve the comfort of automobiles, the vibrations of the automotive drive system must be reduced. In particular, when the engine torque changes suddenly, the vehicle body vibrates due to torsional mode oscillations of the drive system, which includes components from the engine to the tire. Additionally, the automotive drive system includes backlash as a structural nonlinear element in differential gears, which degrades the vibration control performance. Specifically, the shock torque, which is generated when the gear runs freely and collides with backlash, increases the vibration amplitude. Consequently, for automotive drive systems, it is necessary to develop a vibration control method to suppress the adverse influence due to backlash.

Many studies have investigated the control of machines, including backlash. A neural network was applied to a positioning system (Seidl et al., 1995). A nonlinear adaptive control law was proposed for a manipulator with gears (Yang and Fu, 1996). Moreover, a favorable robustness of the fractional order PID controller and the descriptive function method for backlash systems were examined (Barbosa et al., 2007; Barbosa and Tenreiro Machado, 2017). For
nonlinear systems with a dead band such as a tower crane system, the $H_{\infty}$ control using the adaptive fuzzy method was proposed (Wu et al., 2017). To model the influence of backlash as a disturbance, linear $H_{\infty}$ control issues were used (Acho et al., 2013). Other $H_{\infty}$ control approaches were applied to the discontinuous characteristics of two inertial servo systems (Ponce et al., 2016). Additionally, adaptive stabilization to a gyrostabilization system was studied as a control system with an input dead zone band (Aghababa and Aghababa, 2014).

Previous studies have reported the control of an automotive drive system, including backlash via modeling. In particular, many outcomes are related to model predictive control (MPC) and state estimations (Formentini et al., 2017; Balau and Lazar, 2011; Lagerberg and Egardt, 2007; Rostalski et al., 2007; Baumann et al., 2006; Herceg et al., 2009). These are broadly considered effective techniques. Furthermore, tuning the design parameter of a control system to use a simplified engine model was studied (Berriri and Chevrel, 2008). A nonlinear observer and its application to a model-based pole assignment were also studied (Fietzek and Rinderknecht, 2013).

In actual vehicles, an engine control unit (ECU) calculates the value of the proper torque. Then, the vibrations of the automotive drive system are controlled by the actual torque generated from the engine. That is, an actuator to realize control input is the engine in the vibration control of the automotive drive system. Another problem with vibration control of a drive system is that an engine used as an actuator has restrictions on the control period. That is, the torque in the engine can only be updated at the moment when an explosion occurs in the combustion chamber and the crankshaft rotates by a predetermined angle. The essential problem of this constraint is that stability and good control performance are lost upon extending the control period due to the engine combustion mechanism. This is a serious problem for control when an engine is used as an actuator, and it cannot be solved by simply improving the signal processing of control systems. However, vibration control of automotive drive systems considering the control cycle constraint due to such mechanical characteristics of engines has yet to be studied.

This study investigates the vibration control of an automotive drive system with backlash considering the control cycle constraint explicitly. First, a basic experimental device, which abstracts actual vehicles to focus on the influence due to backlash while reflecting only the basic structure of an automotive drive system, is created. Next, we propose an approach to apply the sampled-data control theory to an actuator with extended control cycles like the engine discussed in this research. If traditional controllers with discrete approximations are used as a feedback system with longer control periods, the closed-loop system may become unstable due to the influence of the approximation error. On the other hand, the sampled-data controller can explicitly evaluate and optimize the responses between sample points, allowing stability and control performance to be maintained even if the control period constraint exists. The sampled-data $H_2$ controller can be designed as a servo system by applying an approximate integrator and frequency shaping. Furthermore, a simple and practical control mode switching algorithm is proposed to compensate for the adverse effect of backlash due to nonlinearity. Finally, the effectiveness of the control system is verified experimentally by comparing and evaluating the results with those obtained by traditional digital controllers with discrete approximations to those obtained by the sampled-data controller in the control cycle constraint.

2. Basic experimental device
2.1 Model of controlled object

Figure 1(a) shows the automotive drive system. This is a rotary drive system, which transmits the torque from the engine to the tires. In an automotive drive system, an engine flywheel is coupled to the differential gears via a clutch and a transmission. Further, the gear of the output side is coupled to a tire and the vehicle body via a driveshaft.

In this study, a basic model, which abstracts actual vehicles to focus on the influence due to backlash while reflecting only the basic structure of an automotive drive system, is considered as the controlled object (Fig. 1(b)). Although the original automotive drive system is a rotary model to transmit torque, this verification uses a translational model to transfer force in order to simplify the experimental device. This is a three degrees of freedom system, in which an actuator $M_E$, a gear mass $m_G$, and a vehicle body $M_B$ are connected by stiffness and damping. The torsional stiffness and damping of the gear and driveshaft correspond to $(K_G, C_G)$ and $(K_D, C_D)$, respectively. Figure 1(c) shows a dynamical behavior of backlash studied in this research. In the Fig. 1(b), backlash is reproduced as the dead zone which an actuator $M_E$ runs freely in the dead zone, and a collision generating a shock force occurs. The vehicle body $M_B$ is connected by the spring $K_C$ and damper $C_C$ at the most downstream point of the system. In this study, positioning control while suppressing vibrations due to backlash is performed using the vehicle body displacement $X_B$ as an observed output.
2.2 Experimental device

Figure 2 shows the experimental device. A linear motor is used as an actuator, and a control input is the thrust driving the slider mass. On the other hand, the vehicle body mass connected to the fixed ends via coil springs vibrates on a linear guide installed in parallel with the motor. The gear mass is at an intermediate point between the guide and motor. In addition, the stiffness of the gear and drive shaft is reproduced using leaf springs. A gap is set to produce a dead-zone due to backlash by creating a space between the leaf springs on both sides of the gear mass. The backlash length of the device is empirically determined by experiments so that the adverse effects (amplitude increase and overshoot in control) appear with vehicle body vibrations in the experiments. That is, the backlash is set larger than that of an actual vehicle to clearly evaluate the improvement by control even in the experimental device. The damping characteristics ($C_C$, $C_D$, $C_G$, $C_{cl}$ in Fig. 1(b)) of the device were approximately identified. Effects by structural damping, attenuation of combining parts, and friction were approximated by using the single degree of freedom method as the rough values of the damping ratio for each part of the device. Then, the value of each damping coefficient was adjusted so that the model’s response fits the experimental result, which is the frequency response from the entire device. In addition, we prepared the damping material by attaching rubber and a viscoelastic sheet to the leaf springs. Table 1 shows the device specifications.

In this study, the experimental device is aimed at focusing on backlash and its influence on the vehicle body in an original actual vehicle. Therefore, the experimental device was constructed to purposely abstract the structure of an actual vehicle. In other words, the device does not completely reproduce an actual vehicle and its driving conditions. Specifically, in order to make it easy to evaluate the adverse effect due to backlash and the improvement effect by control, we constructed the experimental device for basic study, in which the clutch mechanism, gear ratio, flexibility...
of the tire, and friction of the pavement surface are omitted from an actual vehicle. However, the background of this research is based on the problem in an actual automotive drive system. Therefore, the experimental device reflects only the basic components in a real automotive drive system such as the inertia (mass) of the vehicle body, gear part, and the actuator, and rigidity/damping corresponding to the drive shaft and gear shaft. Moreover, each of these parameters was qualitatively determined as a unique (its own) value of the experimental device so as to clearly (more prominently) reproduce the influence of backlash in an actual vehicle even in the device.

Figure 3 shows the vehicle body displacement and motor thrust in the experiments. This experiment simulates a sudden change in the engine torque and vehicle body vibrations due to a collision with backlash in actual vehicles. The thrust of the lower graph gives a step input that rises at 2 s, which corresponds to the sudden change in the engine torque. This is the driving force generated from the current in the electric circuit of the motor. This force acts on the motor slider. However, the force shown in Fig. 3 is the value calculated using the current command value. In other words, this force is not measured by a force sensor. But, because the electrical time constant of the motor is very small in this study, the response of the current is sufficiently fast. Therefore, the influence of the delay can be ignored, and we can consider that the force in Fig. 3 acts on the motor slider. The vehicle body displacement of the upper graph shows different responses depending on the change of backlash length. From this, the deterioration of vibration amplitude due to the increase of shock force (backlash length) can be confirmed. Originally, the large transient vibration occurs due to a sudden change in the driving force (black line with no backlash in Fig. 3). Furthermore, this vibration amplitude increases due to the effect of backlash. For example, this vibration deterioration is the amplitude difference between the black line (backlash = 0 mm) and the red line (backlash = 20 mm) in Fig. 3. In general, the tendency where the amplitude increase due to backlash is relatively more dominant was confirmed in experiments where the step input decreases. In addition, from the viewpoint of vibration control, a linear controller can completely suppress the vibrations due to only the step input in the black line. However, the contribution of backlash cannot be compensated by the traditional control method. This point will be discussed in more detail in ‘6.4 Discussion’ from the experiment result in Fig. 11. Consequently, it is verified that the experimental device is available to evaluate the impact due to backlash and the control system.

Fig. 2 Experimental device. This is a translational system where some elements of an actual vehicle are omitted to focus on the influence of backlash. A linear motor is used as an actuator. Its motor slider corresponds to $M_E$ in Fig. 1(b). The purpose of the experiment is to control the vibrations of a vehicle body on the linear guide. The vehicle body is $M_B$ in Fig. 1(b). The stiffness and damping of the gear ($K_G$, $C_G$) and driveshaft ($K_D$, $C_D$) are reproduced by the plate springs and attached rubbers as damping material, respectively. The gap is set to produce a dead-zone due to backlash by creating a space between the two leaf springs $K_G$ in Fig. 1(b).
Table 1  Parameters of the experimental device. Each one was qualitatively determined as a unique value of the device with the aim of prominently reproducing the influence of backlash in an actual vehicle even in the device. Each damping coefficient is experimentally identified as the effect by the damping material, etc.

| $M_B$ | $m_G$ | $K_G$ | $S_w$ |
|-------|-------|-------|-------|
| 0.232 kg | 0.039 kg | 0.31 N/m | 20.12 N/m |

2.3 State equation and nonlinear parameter

The experimental device is modeled for the control system design. From Newton’s second law, the equations of motion are written as

$$\dot{x}_B = \frac{1}{M_B} \{ K_D (x_B - X_B) + C_D (\dot{x}_B - \dot{X}_B) - e_{rr} K_c X_B - Oe r - C_c \dot{X}_B + F_r \},$$

(1)

$$\dot{x}_G = \frac{1}{m_G} \{ S_w K_G (X_E - x_G) + OK G + S w C_G (\dot{x}_G - \dot{x}_G) \}.$$  

(2)

$$\dot{x}_E = \frac{1}{M_E} \{ u_{LM} - C_{cl} \dot{X}_E - S w C_G (\dot{x}_E - \dot{x}_G) - S w K_G (X_E - x_G) - OK G \}.$$  

(3)

Then the state equation and the output equation of the system are derived as

$$\dot{x}_p = A_p x_p + B_{p1} w_p + B_{p2} u_p,$$

$$y = C_p x_p + D_{p1} w_p + D_{p2} u_p.$$  

(4)

The state variables and each coefficient matrix are described as

$$A_p = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\frac{K_D}{m_G} & \frac{M_B}{m_G} & 0 & -\frac{(C_D + C_C)}{M_B} & \frac{C_D}{M_B} & 0 \\
\frac{-(K_D + e_{rr} K_c)}{m_G} & \frac{K_D}{M_B} & 0 & -\frac{C_D}{M_B} & \frac{C_D}{M_B} & 0 \\
\frac{S w K_G + K_D}{m_G} & \frac{S w K_G}{M_G} & \frac{C_D}{m_G} & \frac{S w C_G + C_D}{M_B} & \frac{S w C_G}{M_B} & \frac{S w C_G}{M_B} \\
0 & \frac{S w K_G}{M_B} & -\frac{S w K_G}{M_B} & 0 & \frac{S w C_G}{M_B} & \frac{S w C_G}{M_B} \\
0 & S w K_G & 0 & M_E & S w C_G & \frac{S w C_G + C_c}{M_E} \\
\end{bmatrix},$$

(5)

$$B_{p1} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{M_E} & 1 & 0 \\
\end{bmatrix},
B_{p2} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
\frac{1}{M_E} & 1 & 0 \\
\end{bmatrix},
C_p = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$  

(5)

$$D_{p1} = 0, \quad D_{p2} = 0,$$

$$x_p = [ X_B \quad x_G \quad X_E \quad \dot{X}_B \quad \dot{x}_G \quad \dot{X}_E ]^T.$$  

Each external input is given by

$$w_p = \begin{bmatrix}
OK G \\
Oe r \\
Fr \end{bmatrix}, \quad u_p = u_{LM}.$$  

(6)

In this study, nonlinear characteristics such as backlash are expressed by the switching nonlinear parameters included in the state equation. The switching nonlinear parameters due to backlash are $S w$ in $A_p$ and $OK G$ in the external input. Backlash is treated as a dead zone. The relative displacement of $M_E$ and $m_G$ is $\Delta X$, and the
relationship between the nonlinear parameters and the transferred force $F$ is defined as

$$F = Sw \cdot KG \cdot \Delta X + OKG = Sw \cdot KG \cdot (X_E - x_G) + OKG,$$

$$Sw = \begin{cases} 
1, & X_E - x_G > |B| \\
1, & X_E - x_G < -|B|, \\
0, & |X_G - x_G| \leq |B| 
\end{cases}, \quad OKG = \begin{cases} 
-|KG \times |B||, & X_E - x_G > |B| \\
|KG \times |B||, & X_E - x_G < -|B|, \\
0, & |X_E - x_G| \leq |B| 
\end{cases}. \tag{7}$$

If the relative displacement is within the backlash length $|B|$, they are not in contact with each other, and the force is not transmitted. On the other hand, the force can be transmitted when the relative displacement exceeds the backlash length. $Sw$ is a parameter to judge the contact, and $OKG$ is the offset force of $|B|$. There are other nonlinearities by nature in the experimental device by due to its structure. One is the nonlinearity of the coil spring constant $K_C$. The spring force with the displacement $X_B$ is approximated by a cubic equation to express $K_C$ characteristics. The gradient $e_{rr}$ in $A_p$-matrix and offset $Oe_r$ obtained by linearizing it are treated as time-varying parameters. In addition, the dynamic friction existing originally in the linear guide is modeled as Coulomb’s frictional force $Fr$ to be a structural limitation.

3. Control period constraint due to engine characteristics

In this chapter, the control period constraint of an engine (4-cylinder, 4-cycle) is described. Figure 4 shows the relationship between the mechanism to generate engine torque and the control cycles. A series of processes composed of the exhaust, intake, compression, explosion, and expansion is required to generate torque in engines. In other words, the engine torque can only be updated at the moment when the fuel explodes in a cylinder and the crankshaft rotates by 180°. This explosion occurrence interval depending on the engine speed is the actual interval to update the control input when the engine is used as an actuator. The mechanism to update the torque shown in Fig. 4 demonstrates that it is difficult for an actual engine system to achieve a high sample-rate control calculated in engine control units (ECUs). Therefore, longer control periods must be used, resulting in degraded vibration suppression performance.

As mentioned above, the engine has variable intervals to update the control input, which depend on the engine speed. As this is a basic study for the problem, we aim to construct a control system that can suppress the deterioration of the control performance even if the control cycle is extended using a model with fixed intervals to update the input. An approach with the sampled-data control is effective to solve this problem. As shown in the next chapter, the sampled-data control can evaluate the responses between sampling points without an approximation. Consequently, the control performance should be improved by applying the sampled-data controller designed with the abovementioned extended control period.
4. Control system design

4.1 Optimal sampled-data control

We propose a method to apply sampled-data control (Chen and Francis, 1991; Khargonekar and Sivashankar, 1991; Chen and Francis, 1992; Bamieh and Pearson, 1992; Hayakawa et al., 1994) for actuators with limited control cycles such as the engines discussed in this study. Figure 5(a) shows the sampled-data control system, which is a digital control system to control a continuous-time plant \( G(s) \) using a discrete-time controller \( K[z] \). \( G(s) \) and \( K[z] \) comprise a closed-loop system by interposing the ideal sampler \( S_h \) and the zeroth-order holder \( H(\theta) = I \), which are synchronized in a sampling period \( h \).

\[
\begin{align*}
    y[k] & := y(kh) = S_h y(t), \quad k = 0, 1, 2, \ldots, \quad (8) \\
    u(kh + \theta) & = H(\theta) u[k] = u[k], \quad 0 \leq \theta < h \\
    \| G_s(G(s), K[z]) \|_2^2 & = \frac{1}{h} \int_0^h \left\{ \int_0^\infty \text{trace} (\phi(t, \mu)\phi^T(t, \mu)) \, dt \right\} \, d\mu. \quad (10)
\end{align*}
\]

Therefore, the system in Fig. 5(a) is a hybrid system. It includes the signals of the continuous-time disturbance \( w(t) \) applied to the generalized plant, the continuous-time controlled variable \( z(t) \) to be evaluated, the continuous-time control input \( u(t) \), the continuous-time observed output \( y(t) \), the discrete-time control input \( u[k] \), and the discrete-time observed output \( y[k] \).

The sampled-data \( H_2 \) controller is designed by focusing on the vibration suppression performance in this study. Mapping from \( w(t) \) to \( z(t) \) is expressed as \( G_s(G(s), K[z]) \) in Fig. 5(a). The discrete-time controller \( K[z] \), which minimizes the \( H_2 \) norm defined as Eq. (10) while stabilizing the closed-loop system in Fig. 5(a), is designed in the sampled-data \( H_2 \) control.

Here, \( \mu \) is an arbitrary time in the first sampling interval \( t \in [0, h) \). In Eq. (10), the square integral average value of the response \( \phi(t, \mu) \) of \( G_s \) via the impulse input applied at \( \mu \) is evaluated as the performance index.
The sampled-data $H_2$ control problem can be solved by converting the hybrid system in Fig. 5(a) into an equivalent discrete-time control system in which the $H_2$ norm minimization problem of the continuous-time performance index is completely preserved. Figure 5(b) shows the equivalent discrete-time feedback system.

The traditional approach for digital controls with discretization loses information between sampling points because it requires a discrete approximation to implement. On the other hand, the sampled-data control is a problem using the performance index based on the continuous-time input/output signals, as shown in Fig. 5(a). Therefore, control, which explicitly considers and optimizes the responses between sampling points, can be achieved. This feature suggests that the sampled-data controller can suppress the deterioration of the control performance even if it is used in a relatively longer control cycle. Additionally, the sampled-data controller can exhibit effective control by using the engine as an actuator. In this research, experiments are conducted in a specific condition where the update timing of the control input is forcibly limited on the actuator side and control periods have to be extended accordingly.

### 4.2 Controller design

Figure 6 shows the generalized plant used to design the sampled-data controller. By regarding the shock force generated by backlash as the disturbance $w(t)$, its effect on the controlled object $P(s)$ can be suppressed. The observed output $y(t)$ is the vehicle body displacement, and the control input is the motor thrust. $z_1$ and $z_2$ are the controlled variables with respect to the vehicle body displacement and the control input, which are evaluated as indicators of the transient response performance and the input restriction, respectively. Furthermore, the frequency weighting functions $W_1(s)$ and $W_2(s)$ corresponding to each controlled variable are introduced into the generalized plant.

As the weighting function $W_2(s)$, a third-order high-pass filter is used to restrict the control input at high frequencies beyond the control frequency range. On the other hand, the vehicle body displacement must follow the target displacement without offsets while suppressing vibrations in the control frequency range, including the natural frequency of the plant. To achieve this, a weighting function, which is defined by multiplying a low-pass filter with the
passband of the control frequency range and an approximate integrator, is a candidate for $W_1(s)$. However, if this weighting function is used as $W_1(s)$, vibration suppression is difficult because the following band near 0 Hz interferes with the vibration frequency band due to the effect of the approximate integrator and the gain in the vibration suppression band decreases. Especially, under the condition where the control periods are extended, this adverse effect is more prominent since the chances of updating the control input are limited. In this study, a frequency weighting function described as Eq. (11) is used to compensate for this effect.

$$M(s) = \frac{s + (2\pi \times \varepsilon_2)}{1.0}, \quad W_1(s) = \frac{2.481 \times 10^5}{s^3 + 125.7s^2 + 7896s + 2.482 \times 10^5} \times \frac{1.0}{s + (2\pi \times \varepsilon_1)}$$

(11)

$$W_1(s) = \overline{W}_1(s) \cdot M(s).$$

(12)

Here, $\varepsilon_1$ is the cut-off frequency to realize an approximate integrator, and $\varepsilon_2$ is set as the frequency just before the vibration control band between the natural frequency of the plant and $\varepsilon_1$. The approximate integrator is shaped flat in the vibration suppression band by introducing the function $M(s)$. Figure 7 shows the gain properties of the weighting functions in the generalized plant. Especially for $W_1(s)$, the integral characteristics are realized around 0 Hz, while the flat passband of the low-pass filter can be maintained in the vibration suppression band (1 Hz – 10 Hz) without decreasing the gain.

Fig. 7 Gain of frequency weighting functions. $W_2(s)$ restricts the control input at high frequencies beyond the control frequency range. For $W_1(s)$, the integral characteristics are realized around 0 Hz, while the flat passband of the low-pass filter can be maintained in the vibration suppression band by shaping the approximate integrator flat near the natural frequency of the plant. Therefore, it can achieve both vibration suppression and the following target value.

4.3 Feedforward input

Figure 8 depicts the servo system to follow the target value. It is constructed using the sampled-data controller designed in the previous chapter. In this system, the thrust, which multiples the spring constant $K_C$ by the target displacement, is used as the feedforward input to realize the rapid following to the target value.

Fig. 8 Block diagram of the servo system to follow the target value.

The feedforward input is used for rapidly following.

5. Compensation for backlash

In actual vehicles, there is dead time before the driver’s operations are reflected on the vehicle body behavior. In this study, vibrations due to backlash can be improved by processing to eliminate backlash in advance during this dead
time (Yonezawa et al., 2018). Considering the characteristics of actual vehicles, the dead time before the target displacement rise is also set for the experimental device. In the experimental device, the value of this dead time is set as 65 ms. Figure 9 shows the control mode switching algorithm to realize above processing.

Each control mode (I, II, III, IV) in Fig. 9 is described below. \((A_C, B_C, C_C, D_C)\) is a state-space representation of the sampled-data \(H_2\) controller.

(Control Mode I)

\[
e(k) = r(k) - X_B(k),
\]

\[
x_C(k + 1) = A_C x_C(k) + B_C e(k),
\]

\[
u_1(k) = C_C x_C(k) + D_C e(k) + K_C r(k).
\]

(Control Mode II)

\[
e_{pre}(k) = r_{pre}(k) - X_B(k) = r_{pre} - X_B(k),
\]

\[
x_{AW}(k + 1) = A_C x_C(k) + B_C e_{pre}(k),
\]

\[
x_{AW}(k) = z^{-1}[x_{AW}(k + 1)], \quad (x_{AW}(k) = x_C(k)) \quad \text{(16)}
\]

\[
x_C(k + 1) = x_{AW}(k), \quad (\text{Substitute } x_{AW}(k) \text{ for } x_C(k + 1)) \quad \text{(18)}
\]

\[
u_2(k) = C_C x_C(k) + D_C e_{pre}(k) + K_C r_{pre}. \quad \text{(20)}
\]

(Control Mode III)

\[
x_C(k + 1) = A_C x_C(k) + B_C e_{pre}(k), \quad \text{(21)}
\]

\[
u_3(k) = C_C x_C(k) + D_C e_{pre}(k) + K_C r_{pre}. \quad \text{(22)}
\]

(Control Mode IV)

\[
x_{AW}(k + 1) = A_C x_C(k) + B_C e(k), \quad \text{(23)}
\]

\[
x_{AW}(k) = z^{-1}[x_{AW}(k + 1)], \quad \text{(24)}
\]

\[
x_C(k + 1) = x_{AW}(k), \quad \text{(25)}
\]

\[
u_4(k) = C_C x_C(k) + D_C e(k) + K_C r(k). \quad \text{(26)}
\]
Control Mode I does not compensate for backlash. The control error $e(k)$ is calculated using the target value $r(k)$ and the vehicle body displacement $X_B(k)$ along with the feedforward input $u_F = K_C r(k)$.

Control Mode II is a mode, in which the compensation action starts at 65 ms before rising of the target value. This mode achieves gradual coupling upstream and downstream of the system by reducing backlash and preventing the accumulation of the control error. Specifically, the inputs generated by switching the target value to a small positive value $r_{pr}$ reduce backlash. However, only switching the target value results in an accumulation of errors due to an uncontrolled time zone, which causes excessive control inputs to be calculated. To avoid this, anti-windup, which temporarily stops updating the state variable $x_C(k)$ in the controller, is also used. During reducing backlash, anti-windup resets the control errors of the controller at every calculation step. In Eq. (18), $z^{-1} = e^{-hs}$ is a one-cycle $h$ delay operator.

The switching of the target value ends when the original target value $r(k)$ exceeds $r_{pr}$, and the control error is calculated to return to $r(k)$. Furthermore, anti-windup ends at the moment when backlash is completely eliminated. To judge the backlash elimination, a jerk given by differentiating the vehicle body acceleration can be used. The backlash elimination time at which the jerk increases sharply due to the discontinuously change of transferred thrust and exceeds a threshold value, is determined.

Control Mode III is a mode to stop only anti-windup since backlash has been eliminated. On the other hand, although the switching of the target value ends, since backlash remains, anti-windup is continuously applied to the controller until it is completely eliminated in Control Mode IV.

6. Control experiments
6.1 Verification condition

In this study, we consider the essential problem due to the engine characteristics of actual vehicles as a basic examination and reproduce the condition where control periods are forcibly extended. To investigate this problem, the input update timings are limited by forcibly holding the control input value calculated from the digital controller with a zeroth-order hold at a certain period. This period is given by a multiplying factor of the natural frequency to be controlled for vibration suppression. The natural frequency of the plant is about 3.5 Hz. Specifically, the performance is evaluated when the control frequency is 50 times (175 Hz), 10 times (35 Hz), 7 times (24.5 Hz), and 5 times (17.5 Hz) of the natural frequency. Furthermore, digital controllers with traditional discrete approximations are also applied to compare the control results with those obtained by the sampled-data controller. The traditional approach includes two types of digital controllers obtained by discretizing continuous-time $H_2$ controller with zeroth-order hold (Continuous-Time Controller: CTC) and designing for a discretized plant by the discrete-time $H_2$ control theory (Discrete-Time Controller: DTC).

6.2 Configuration of the experimental system

Figure 10 overviews the configuration of the experimental system. In the experiments, vehicle body displacement is measured with a laser displacement sensor (LDS : KEYENCE, IL-300). In the digital signal processor (DSP : mtt, iBIS, DSP7101A), the control inputs are calculated using the feedback displacement. Then the thrust output from the DSP is amplified by the servo amplifier. According to the signals, the motor slider is driven for positioning control. Furthermore, the displacement and input signals are recorded by a data logger (CATEC INC, Cat System). The input update cycle explained in the previous section is the period to hold the inputs at the output of the DSP.
6.3 Control experiment results

Figures 11–13 depict the control experiment results with the sampled-data $H_2$ controller, the CTC, and the DTC. In each figure, the left graph sets {[(a), (b)] and [(c), (d)] show the time waveforms of vehicle body displacement and control input when the control frequency is 50 times and 10 times the natural frequency, respectively. The right graph sets {[(e), (f)] and [(g), (h)] are the results when the control frequency is 7 times and 5 times the natural frequency, respectively.

Regarding the displacement, the blue line shows the response via the step thrust in Fig. 3 without control, while the green line is the target displacement. The red line shows the control result with a digital controller and the compensation for backlash as described in chapter 5. Especially in Fig. 11, for comparison, the control result without compensation for backlash is shown in the black line.

Regarding the thrust (control inputs), the responses in each color correspond to the displacement.
Fig. 11 Control experiment results with sampled-data controller; (a) Displacement when control frequency is 50 times the natural frequency; (b) Driving force when control frequency is 50 times the natural frequency; (c) Displacement when control frequency is 10 times the natural frequency; (d) Driving force when control frequency is 10 times the natural frequency; (e) Displacement when control frequency is 7 times the natural frequency; (f) Driving force when control frequency is 7 times the natural frequency; (g) Displacement when control frequency is 5 times the natural frequency; (h) Driving force when control frequency is 5 times the natural frequency

Fig. 12 Control experiment results with continuous-time controller; (a) Displacement when control frequency is 50 times the natural frequency; (b) Driving force when control frequency is 50 times the natural frequency; (c) Displacement when control frequency is 10 times the natural frequency; (d) Driving force when control frequency is 10 times the natural frequency; (e) Displacement when control frequency is 7 times the natural frequency; (f) Driving force when control frequency is 7 times the natural frequency; (g) Displacement when control frequency is 5 times the natural frequency; (h) Driving force when control frequency is 5 times the natural frequency
Fig. 13 Control experiment results with discrete-time controller: (a) Displacement when control frequency is 50 times the natural frequency; (b) Driving force when control frequency is 50 times the natural frequency; (c) Displacement when control frequency is 10 times the natural frequency; (d) Driving force when control frequency is 10 times the natural frequency; (e) Displacement when control frequency is 7 times the natural frequency; (f) Driving force when control frequency is 7 times the natural frequency; (g) Displacement when control frequency is 5 times the natural frequency; (h) Driving force when control frequency is 5 times the natural frequency

6.4 Discussion

As the control cycle becomes longer, the tracking and vibration suppression performances deteriorate in the CTC and the DTC. In the results of Figs. 12 and 13, since high control performance can be achieved with a relatively short control period (50 times the natural frequency), the design specification of the controllers, which reflect the generalized plant, is appropriate. Additionally, the performance tendency accompanying the change in control cycles confirms that the control performance degradation is due to the control period constraint because neither the CTC nor the DTC explicitly takes the responses of the plant between the timings updating the control input into account. Since both design processes require discrete approximations, information on the performance index ($H_2$ norm) between input update points is lost. Therefore, the influence of the discrete approximation error increases as the control periods are extended, resulting in a deteriorated vibration control performance. Especially for the CTC, the stability of the closed-loop system is not guaranteed because the discrete approximation is performed only for the controller. As a result, the CTC tends to become more unstable in a control cycle that is five times the natural frequency. On the other hand, the DTC can stabilize the closed-loop system at each input update point, but the control performance deteriorates as the control cycle becomes longer.

All of the digital controllers are designed from the specifications of the servo system with a continuous-time generalized plant. Therefore, from the viewpoint of the discrete approximation error for the continuous-time plant, the cause of the expanded steady-state errors can also be explained by whether the responses between input update points are evaluated or not. The integral characteristics improve the offsets by accumulating the control inputs with errors at each calculation step. However, if control cycles are extended (5-7 times), it is difficult to improve the steady-state error due to the limitation of chances at the accumulate inputs. Furthermore, modeling errors in the experimental device (e.g., the maximum static friction and damping) promote the deterioration of the integral characteristics.

On the other hand, a high control performance can be achieved with the sampled-data $H_2$ controller even when the control period is extended to 5 times the natural frequency. This is because the sampled-data controller can optimize the plant responses between input update points. The digital controller is designed for the equivalent discrete-time system, in which the performance index ($H_2$ norm) between the input update points is preserved without an approximation. Therefore, even if control cycles are significantly extended, the control input to minimize the $H_2$ norm between update

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points is calculated and applied to the plant at each timing updating the input. In addition, the degradation of the integral characteristics is improved by preserving the design specification in the continuous-time generalized plant. The input gain larger than that with the CTC and the DTC is calculated at each update timing to improve the offset even with a small number of input update chances (Fig. 11). Consequently, the effectiveness of the sampled-data controller with the input update interval was confirmed for vibration control of the automotive drive system considering the control period constraint.

Finally, the effect of backlash compensation is discussed from the results in Fig. 11. Without the control mode switching (Fig. 11, black lines), overshooting occurs just after the target value rises due to the adverse effect of backlash. When the target displacement suddenly changes at 2 s, the system is in an uncontrollable state where the thrust is not transferred to the vehicle body due to backlash. Therefore, the vehicle body cannot immediately follow the target displacement, causing control errors to accumulate. Then the controller calculates unnecessarily large control inputs with the accumulation of errors. The excessive input accelerates the upstream side of the system. As a result, a collision occurs when backlash is eliminated, and the shock force is transferred to the vehicle body. On the other hand, in the red lines with the compensation for backlash, a high control performance to suppress overshooting can be achieved. From the control inputs (b), (d), (f), (h): red line), it can be seen that backlash is reduced beforehand during the dead time as described in Chapter 5. Furthermore, excessive control inputs due to the accumulation of errors can be avoided by anti-windup and backlash is gently eliminated. Consequently, the control mode switching technique proposed in this study can suppress the adverse influence due to backlash.

7. Conclusion

In this study, the control period constraint due to an engine’s characteristics was considered to achieve appropriate vibration control of an automotive drive system with backlash. First, the basic experimental device, which abstracts actual vehicles to focus on the influence due to backlash while reflecting only the basic structure of an automotive drive system, was created. Next, a mechanism to generate the engine torque in actual vehicles and its problem with respect to the control input update limitation were demonstrated. Then an approach to apply the sampled-data control, which can optimize responses between sampling points, was proposed to maintain the performance under the control period constraint. The servo sampled-data $H_2$ controller was designed to follow the target value without deterioration of the vibrations. Furthermore, a simple control mode switching technique to compensate for the effect due to backlash was proposed. Finally, the effectiveness of the proposed control approach was validated by experiments and comparison of the control results with those obtained by the traditional discrete approximation.

The future work is to investigate a control under variable control period constraint depending on the engine speed in actual vehicles using the basic experimental device employed in this study. Furthermore, considering practical applications, we are planning to improve the experimental device to be more like a real vehicle in the future.

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