The solution of the optimization problem of small energy complexes using linear programming methods

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Abstract. Linear programming methods were used for solving the optimization problem of schemes and operation modes of distributed generation energy complexes. Applicability conditions of simplex method, applied to energy complexes, including installations of renewable energy (solar, wind), diesel-generators and energy storage, considered. The analysis of decomposition algorithms for various schemes of energy complexes was made. The results of optimization calculations for energy complexes, operated autonomously and as a part of distribution grid, are presented.

1. Introduction
In recent decades distributed generation plays increasingly significant role in energy balance of many countries [1], with Russia among them [2]. The requirements of energy saving and energy efficiency, environmental constraints force them to use more complicated schemes of small energy systems: cogeneration and trigeneration schemes, schemes with heat and electric accumulators, hybrid circuits, including installations of renewable energy. Variable loads and vast variety of generating and accumulating facilities make the problem of choosing optimal configuration of energy complex hard to solve without adequate mathematical models, effective numerical methods and modern calculation programs.

Recent years a large number of publications, devoted to the problems of optimization and control of generating plants and energy distribution grids, appeared. They consider variable systems, including hybrid energy complexes, whose composition, along with traditional hydrocarbon fuel installations, include installation of renewable energy and energy storage [3–7]. As a rule, such researches examine specific schemes of energy complexes, and optimization procedure in most cases is reduced to simple options search.

Papers [8, 9] have shown that under certain simplifications optimization problem can be reduced to a linear programming problem. The simplest scheme of the energy complex in the composition of gas-piston cogeneration plant, gas boiler and heat accumulator, was considered.

This work is a further development of approaches and methods proposed in [8, 9]. The conditions of applicability of the simplex method for systems that include installation of renewable energy (photovoltaic, wind), diesel generators and electric storage, have been considered. The results of decomposition algorithms analyze are presented. The basic principles of design of small power complexes, including those that are part of local networks, are presented.
2. Formulation of the problem

The problem of finding optimal structure and operation modes of an energy complex is generally a problem of multi-parameter optimization. The essence of optimization is to find the extremum of the objective function, the form of which is determined by the chosen optimization criterion (in most cases—the cost of energy). Such function in general may contain nonlinear coefficients.

The linear programming methods were found very effective for solving optimization problems for systems, characterized by a large number of variables [10]. The linear programming problem is formulated as follows: find a vector of variables, delivering an extremum of the linear objective function under constraints, given in form of linear equations or inequalities. The problem, formalized by a system of balance equations for energy flows with corresponding linearized models of components of the energy complex, is reduced to a linear programming problem.

One of the most effective methods for the numerical solution of linear programming problems is the simplex method [11]. Simplex method implements a rational bust of acceptable solutions in the form of finite iterative process that improves the objective function value at each step, and allows to find an optimal solution in a finite number of steps, or to establish that there is no optimal solution. To find the minimum of a linear functional using the simplex method, you must construct a mathematical model “generation-grid-consumer”—a system of equations and constraints (the direction of energy flows, non-negativity of accumulators’ energy, restrictions on their capacity, etc), reflecting the physical essence of the problem. It is necessary to ensure the condition of linearity of mathematical model, constraints and objective function.

When solving multi-parameter optimization problems, it is important to find a compromise between detalization level of mathematical models, time intervals, within which the load can be considered constant, and reasonable calculation time. The necessity of taking into account daily, weekly and seasonal fluctuations of consumer’s loads and climate circumstances, results in high dimension of the problem. In [9] it is shown that for energy complex, consisting of 4 facilities with CHP among them, and working as a part of distribution grid, with a rate period of 1 year (8760 hours) and hourly load changes, the dimension of the optimization problem (the dimension of the augmented matrix) will be \( 7 \times 10^9 \).

This problem can be partially solved by using the methods of decomposition of the problem [11]. There are three possible approaches that differ in conditions of applicability: architectural, algorithmic and model.

When using the architectural decomposition, the original problem is divided into \( n \) subtasks, each having its own simplex tableau and objective function. The value of objective function for original problem is calculated as a sum of objective functions of its subtasks. The calculation time is reduced due to significant reduction of operations, when determining the pivot element [11]. If an energy complex does not include accumulators and installations of renewable energy, the optimization calculation for each time interval, characterized by constant loads, will not depend from calculations for previous time intervals. In this case any degree of decomposition can be taken, up to the estimated amount of time with a constant load. When solving the problem of determining optimal operating modes of power complex with given power amounts of generating facilities and capacity of accumulators, the permissible degree of decomposition depends on the mode of accumulators’ work.

If the task is to find the optimal power of wind or solar energy installations, the architectural decomposition cannot be applied.

An algorithmic approach to decomposition is significantly more difficult to implement, but has almost no limitations in use. The essence of the approach is as follows: each variable and each equation, relating to a single interval of time, is “marked” by the number of the interval and after determination of pivot column (actually—variable introduced into the basis) the choice of pivot row is done only between rows with the same mark as the introduced into basis element. Similarly to the architectural, algorithmic method reduces the number of operations...
while determining the pivot element, although reduction is less significant—pivot column is still selected from all the columns in the tableau. However, this method can be used regardless of the problem being solved and the composition of energy complex under consideration.

Model approach involves such composition of constraints system (in fact—model of energy complex), which reduces the mutual influence of the complex operation modes in different time intervals. It is applicable, for example, when calculating energy complexes, including accumulators. In this case model decomposition results in additional constraints, indicating regularity of saved energy zeroing. This, on one hand, brings a certain error in the calculation results, on the other—significantly reduces the computation time. Thus it is necessary to find a compromise between the accuracy of calculations and reasonable computation time.

Depending on the purpose the optimization problems of energy distributed power systems can be divided into 4 types (in all cases optimization criterion is a minimum cost of energy supply):

1. The calculation of the optimal operation modes of energy complex of specified composition.
2. Comparative calculations of different scheme solutions of energy complex for a certain consumer for the purpose of choosing the best one.
3. Finding the optimal solution for partial reconfiguration of energy complex (replacement or addition of equipment).
4. Design calculation of the optimal configuration and operation modes of energy complex equipment.

3. Mathematical model
Mathematical model of energy complex must reflect two aspects of its functioning: energy and economic. For ordering a large number of free parameters in the equations of the mathematical model, every power plant in the complex, and each time slot is assigned a number, and the titles of free parameters include two relevant indexes, pointing to facilities and time intervals those parameters refer to. Parameters describing the installed power of the equipment do not have time indexes since they general constraints for all time intervals.

Energy aspect includes the distribution of heat and electric energy flows between facilities and consumer, limitations on the instantaneous power of generating facilities, as well as the amount of energy stored in accumulators at any time (for each time interval). The balance equations for heat and electric energy flows are based on the conditions of complete coverage of consumer’s loads at any time. For any energy complex including m facilities at a j-th time interval the balance equations can be written as follows:

\[ \sum_{i=1}^{m} Q^j_i = Q^j_{ec}, \quad (1) \]
\[ \sum_{i=1}^{m} N^j_i = N^j_{ec}. \quad (2) \]

Here \( Q^j_{ec} \) and \( N^j_{ec} \) are consumer’s heat and electrical loads at j-th time interval; \( Q^j_i \) and \( N^j_i \) are heat and electric power of i-th at the same time.

For heat-generating facilities all values of \( N^j_i \) will be zero, as the values of \( Q^j_i \) for electricity-generating facilities. Since cogeneration facilities usually have strict dependence between heat and electric power, their heat power in corresponding balance equations is expressed by a linear function of electric power, depending on characteristics of cogeneration facility. For each time
interval constraints of instantaneous power of generating facilities are introduced:

\[ Q_i^j = Q_i^{\text{max}} , \] \hspace{1cm} (3)

\[ N_i^j = N_i^{\text{max}} . \] \hspace{1cm} (4)

Where \( Q_i^{\text{max}} \) and \( N_i^{\text{max}} \) are the installed heat and electric power values for \( i \)-th facility.

For each facility only one restriction is chosen, depending on the type of energy produced. For cogeneration facilities an electric power restriction is introduced. The equations that describe accumulators’ operation consider the example of electric accumulators. As the accumulator can both store and deliver energy, and parameters must be non-negative when solving simplex task, the change of energy amount in the accumulator will be described by two parameters at each time interval:

\[ \Delta E_N^j = t^j (N_{\text{acc}+}^j - N_{\text{acc}-}^j) . \] \hspace{1cm} (5)

Where \( E_N^j \) is the change of energy stored in accumulator at \( j \)-th time interval; \( t^j \) is the duration of this time interval; \( N_{\text{acc}+}^j \) and \( N_{\text{acc}-}^j \) are values of power, received and delivered by the accumulator. For each time interval two constraints of amount of stored in accumulator energy are written: firstly, it can not be negative, secondly, it cannot exceed the accumulator’s capacity.

\[ \sum_{k=1}^{j} (N_{\text{acc}+}^k - N_{\text{acc}-}^k) \geq 0, \] \hspace{1cm} (6)

\[ \sum_{k=1}^{j} (N_{\text{acc}+}^k - N_{\text{acc}-}^k) \leq E_N^{\text{max}} . \] \hspace{1cm} (7)

Where \( E_N^{\text{max}} \) is accumulator’s capacity.

The amount of stored energy at each time interval is determined as a sum of energy changes of all previous intervals. The current version of mathematical model does not impose restrictions on the rate of charging or discharging, it does not take into account residual charge. All of the above also applies to the heat accumulator.

The economic efficiency of the energy complex is determined by the costs of energy supply, by the objective function in other words. In general, the objective function includes optimization parameters that characterize the operation of equipment, and optimization parameters that determine the optimal values of the facilities’ installed power (capacity for accumulators). Each parameter type corresponds to a component of energy cost. The component can be either fuel expenses, which are determined by the operation mode of energy complex, or constant expenses, determined by the sum of installed powers and operation costs of facilities. In accordance with the contribution to the objective function, there are three types of facilities:

1. Facilities, which can be optimized for operation, but not the installed power. These are hydrocarbon fuels using facilities with a pronounced dependence of efficiency on relative power: gas turbines, gas piston units, diesel generators. Rated power of such installations is determined by expert assessments on the basis of the consumers’ load charts, category of power supply reliability and other data.

2. Facilities, which can be optimized both for operation and installed power. These include facilities with weak dependence between efficiency and operation mode: gas boilers, electric boilers, heat and electricity accumulators.

3. Facilities, which can be optimized only for constant expenses, but not the operation mode. These include installations of renewable energy: photovoltaic batteries, wind power plants. Their operation mode is determined by weather and can not be controlled by operator.
Assuming that a certain energy complex, including $m$ facilities, has facilities of the first type under numbers from 1 to $a$, facilities of the second type under numbers from $a + 1$ to $b$, and the third type—from $b + 1$ to $m$, we can write for such a complex the objective function in general form:

$$ R = \sum_{i=a+1}^{m} F_i(P_i^{\text{max}}) + \sum_{j=1}^{n} \sum_{i=1}^{b} f_i(P_j^i). $$  \hfill (8)

Where $R$ is the value of objective function; the first summand in the right part is a sum of fuel expenses during rate period; second summand is a sum of constant expenses for those facilities, which can be optimized installed power (capacity); $P_j^i$ and $P_i^{\text{max}}$ are the values of instantaneous power and installed power for $i$-th facility.

Thus it does not matter, what kind of energy a facility produces. In case of cogeneration facilities, the values of $P_j^i$ and $P_i^{\text{max}}$ are equal to those of instantaneous and maximal electric power. The function $f_i(P_j^i)$ determines dependence between instantaneous power and fuel expenses and $F_i(P_i^{\text{max}})$—between installed power and constant expenses for $i$-th facility. In accordance with the terms of the applicability of the simplex method, these dependences must be linear.

4. Software implementation

The proposed method of schemes and operation modes of energy complexes optimization was realized in Delphi programming environment in the form of a software package Smart Energy Complex. The software package includes three main modules: the interface module, the math module and the module of mathematical modelling.

The interface module includes description of the program’s control elements, procedures of data input and graphical representation of the original data and the results of calculations, functions of saving results of calculations and loading original data from file.

Math module contains a description of the simplex method for solving the problem, as well as the types of data to work with simplex tables, linear equations and variables.

The module contains a description of the mathematical modeling of data types for storing and processing information about the characteristics of power plants and complexes, as well as the procedures for drawing up a system of equations, constraints, objective function and the simplex table based on the data of the energy complex.

The module of mathematical modeling contains description of data types for storing and processing data on energy facilities and energy complexes, and procedures of object function and simplex tableau drafting on basis of these data.

In addition, the program requires a text file that contains the graph of loads and climatic data (wind speed, insolation), in case the energy complex includes installations of renewable energy. Also a file, containing information about all facilities that can be included in the scheme of energy complex, is needed. The program’s possibilities can be expanded by simply adding new facilities data to this file.

5. Results and discussion

As an illustration of proposed approaches, a problem of optimization of autonomous energy complex, providing energy for a dwelling house (several houses) in climatic conditions of Moscow region [12] with total heated area of 7000 square meters, was considered. It was assumed that characteristic period of consumer’s loads changing is one week (consisting of weekdays and weekends), respectively, the problem was divided into weekly block, the energy of accumulators zeroed at end of each week. The year was divided into four specific periods of time: winter, summer, autumn-spring and fall and transitional autumn-spring period without heating load. It was assumed that the heat load of the hot water system presents throughout the year. Typical
The scheme of independent power supply, consisting of piston gas unit and gas boiler, was chosen as a basic scheme for comparison. The original composition of energy complex (before graphs of the relative load of a residential building in the weekdays and weekends were used to simulate the electrical load. The twenty-four-hour day was divided into hourly intervals, within which the energy loads were assumed constant (figure 1).

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optimization procedure) piston gas unit with combined heat production (PGU), gas boiler (GB), electric boiler (EB), photovoltaic cell (PVC), lead-acid battery (LAB) and heat storage (HS). As a result of optimization, electric boiler was excluded from composition of energy complex. Compared to the basic scheme, the energy production expenses (capital and operational) fell by 4.2% and consumption of natural gas—by 16%. The PVC area appeared 78 m$^2$, the capacity of LAB—208 kWh, the volume of heat storage tank—12.4 m$^3$. The optimal power of gas boiler appeared 270 kW. It should be noted that energy stores in spite of the high capital costs, provide even more significant gains in fuel costs. The optimal graph of the equipment operation during the weekday in winter is shown in figure 2.

Estimated structure of energy supply is shown in the diagram (figure 3).

Despite the relatively unfavorable conditions in the Moscow region for solar cells, the percentage of consumer’s load coverage of photovoltaic cells was quite noticeable (figure 3C). The trend of solar cells’ cost reduction, opposed to the growth gas fuel and electricity costs, will further increase the efficiency of the use of renewable energy installations as a part of energy complexes.

6. Conclusion
It is shown that the simplex method, which is well-known and widely used at solving of problems of mathematical programming in the fields of economics, management, planning and logistics, can also be successfully applied to solving optimization problems in the field of small distributed power. Various classes of practical problems were considered and general approach to solving them was formulated. The results of comparative optimization calculations for energy complexes’ schemes, including various generating and accumulating facilities, showed that optimization problems with rate period of 1 year and hour sequence of time intervals with constant loads, with using decomposition, can be solved on PC within reasonable time.

The results can be used in the design of new energy complexes of small power, both standalone and connected with external energy grids, in modernization of existing systems and to determine optimal operation modes for equipment of energy complex.

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