Review Article

Towards the identification of new physics through quark flavour violating processes

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Abstract
We outline a systematic strategy that should help in this decade to identify new physics (NP) beyond the standard model (SM) by means of quark flavour violating processes, and thereby extend the picture of short distance physics down to scales as short as $10^{-20}$ m and even shorter distance scales corresponding to energies of 100 TeV. Rather than using all of the possible flavour-violating observables that will be measured in the coming years at the LHC, SuperKEKB and in Kaon physics dedicated experiments at CERN, J-PARC and Fermilab, we concentrate on those observables that are theoretically clean and very sensitive to NP. Assuming that the data on the selected observables will be very precise, we stress the importance of correlations between these observables as well as of future precise calculations of non-perturbative parameters by means of lattice QCD simulations with dynamical fermions. Our strategy consists of twelve steps, which we will discuss in detail while illustrating the possible outcomes with the help of the SM, models with constrained minimal flavour violation (CMFV), MFV at large and models with tree-level flavour changing neutral currents mediated by neutral gauge bosons and scalars. We will also briefly summarize the status of a number of concrete models. We propose DNA charts that exhibit correlations between flavour observables in different NP scenarios. Models with new left-handed and/or right-handed currents and non-MFV interactions can be distinguished transparently in this manner. We emphasize the important role of the stringent CMFV relations between various observables as standard candles of flavour physics. The pattern of deviations from these relations may help in identifying the correct NP scenario. The success of this program will be very much facilitated through direct signals of NP at the LHC, even if the LHC will not be able to probe the physics at scales shorter than $4 \times 10^{-20}$ m. We also emphasize the importance of lepton flavour violation, electric dipole moments, and $(g-2)_{\mu}$ in these studies.

Keywords: flavour, particle physics, quark flavour violation

(Some figures may appear in colour only in the online journal)

1. Overture

The main goal of elementary particle physics is to search for fundamental laws at very short distance (SD) scales. From the Heisenberg uncertainty principle we know that to test scales of order $10^{-18}$ m we need an energy of approximately 200 GeV. Therefore, by using the LHC we will be able to probe distances as short as $4 \times 10^{-20}$ m. Unfortunately, it will take some time before we can reach a higher resolution using high energy processes. On the other hand, flavour-violating and CP-violating processes are very strongly suppressed and are governed by quantum fluctuations, which allows us to test energy scales as high as 200 TeV, corresponding to SDs in the ballpark of $10^{-21}$ m. Even shorter distance scales can...
be tested, albeit indirectly, in this manner. Consequently, frontiers in testing ultrashort distance scales belong to flavour physics or, more concretely, to very rare processes, such as particle–antiparticle mixing, rare decays of mesons, CP violation and lepton flavour violation (LFV). Electric dipole moments (EDMs) and \((g - 2)_\mu\) also belong to these frontiers, even if they are flavour conserving. While such tests are not limited by the available energy, they are limited by the available precision. The latter has to be very high because the standard model (SM) has until now been very successful and finding departures from its predictions in the quark sector has become a real challenge. This precision applies both to experiments and theoretical calculations. Among the latter higher order perturbative QCD calculations are important and calculations of non-perturbative parameters by means of QCD lattice simulations. They both play prominent roles in the search for new physics (NP) at very SD scales. In particular, calculations of non-perturbative parameters by means of QCD lattice simulations with dynamical fermions play prominent roles in the search for new physics (NP) at very SD scales.

Over the last two decades, flavour physics has developed into a very broad field of study. In addition to the \(K\), \(D\) and \(B\) decays, and \(K^0 - \bar{K}^0\) and \(B^0_d - \bar{B}^0_d\) mixings that were with us for quite some time, \(B^0_s - \bar{B}^0_s\) mixing, \(B_s\) decays, and \(D^0 - \bar{D}^0\) mixing belong these days to the standard repertoire of any flavour workshop. Similarly, LFV has gained in importance after the discovery of neutrino oscillations and related non-vanishing neutrino masses, even if LFV is basically unmeasurable within the SM enriched with tiny neutrino masses. The recent precise measurement of the parameter \(\theta_{13}\), resulting in a much higher value than expected by many theorists, has enhanced the importance of this field. Simultaneously, new ideas for the explanation of the quark and lepton mass spectra and the related weak mixings, summarized by the CKM [1, 2] and PMNS [3, 4] matrices, have been significantly developed over the last two decades. Moreover, the analyses of EDMs, of the \((g - 2)_\mu\) anomaly, and of flavour changing neutral current (FCNC) processes in top quark decays have intensified in recent years in view of the related experimental progress that is expected to take place in this decade.

The correlations between all these observables and the interplay of flavour physics with direct searches for NP and electroweak precision studies will hopefully tell us one day what the proper extension of the SM is. In writing this paper we have been guided by the impressive success of the CKM picture of flavour changing interactions [1, 2], accompanied by the Glashow, Iliopoulos and Maiani (GIM) mechanism [5] and also by several tensions between the flavour data and the SM that are possibly the first signals of NP. Fortunately, there is still a lot of room for NP contributions, particularly in rare decays of mesons and charged leptons, in CP-violating transitions, and in EDMs of leptons, of the neutron and of other particles. There are also a multitude of models that attempt to explain the existing tensions and predict what experimentalists should find in this decade.

The main goal of this paper is to take another look at this fascinating field. However, we should strongly emphasize that we do not intend to present here a review of flavour physics. Comprehensive reviews, written by a number of flavour experts, are already present on the market [6–8] and, moreover, extensive studies of the physics at future flavour machines and other visions can be found in [9–30].

Even if this overture follows closely that in [12] and some of the goals listed there will be encountered below, our presentation is more explicit and is meant to be a strategy that we hope we can execute systematically in the coming years. Undoubtedly, several ideas presented below have already appeared in the literature, including those present in our papers. But the collection of these ideas in one place, an analysis of the various correlations between them and, in particular, new proposals and observations will hopefully facilitate the monitoring of the coming advances of our experimental colleagues who are searching for the footprints of NP directly at the LHC, and indirectly through flavour- and CP-violating processes and other rare processes in this decade.

However, in contrast to [12], we will not confine our discussion to scales explored by the ATLAS and CMS but will also consider much shorter distance scales.

Our paper is organized as follows. In section 2 we set the scene for our strategy of stressing the importance of the correlations between observables. In section 3 we briefly summarize the theoretical framework for weak decays and briefly present a number of the simplest models that will be used to illustrate our ideas. In particular, these are the models with minimal flavour violation (MFV), and models with tree-level FCNCs, that are mediated by neutral gauge bosons and scalars that transparently exhibit non-MFV interactions and the effects of right-handed (RH) currents. In section 4, as a preparation for the subsequent main section of our paper, we present a classification of the various correlations between various processes that depend on the NP scenario that is being considered.

Section 5, a very long section, is devoted to the presentation of our strategy, which consists of 12 steps and, except for step 12, involves only quark flavour physics. In the course of this presentation we will frequently refer to models of section 3, illustrating our ideas by means of these models. In section 6, we collect the lessons gained in section 5 and propose DNA charts with the goal of transparently exhibiting the correlations between various observables that are characteristic for a given NP scenario. Finally, we briefly review a number of concrete extensions of the SM, investigating how they face the most recent LHCb data. In section 7 we close this report with a shopping list for this decade.

2. Strategy

2.1. Setting the scene

In order to illustrate the basic spirit of our strategy for the identification of NP through flavour-violating processes, we recall here a few deviations from SM expectations that could be signs of NP at work but which require further investigations. For non-experts, the appearance of several observables not familiar to them already at the start could be some challenge. However, various definitions of observables, such as \(\varepsilon_K\) and \(S_{\phi K_s}\), that are related to \(\Delta F = 2\) transitions, can be found in
section 5.3; that is, in step 3 of our strategy for the search for NP. It is also a fact that many observables discussed in this review were at the basis of the construction of the SM and already appear in the textbooks [31, 32], so that the general strategy outlined here should not be difficult to follow. While at first sight the experts could in principle skip this section, we would like to ask them not to do so because our strategy for the identification of NP through quark flavour violating processes differs significantly from other strategies found in the literature.

We begin by recalling a visible tension between the CP-violating observables \( \varepsilon_K \) and \( S_{\bar{K}K} \), within the SM first emphasized in [33, 34]. The nature of this tension depends sensitively on the value of the CKM element \( |V_{ub}| \), for which the exclusive semileptonic decays imply a significantly lower value than the inclusive ones. While the latter problem will hopefully be solved in the coming years, it is instructive to presently consider two scenarios for \( |V_{ub}| \):

- **Exclusive (small)** \( |V_{ub}| \) scenario 1. \( |\varepsilon_K| \) is smaller than its experimental determination, while \( S_{\bar{K}K} \) is close to its central experimental value.
- **Inclusive (large)** \( |V_{ub}| \) scenario 2. \( \varepsilon_K \) is consistent with its experimental determination, while \( S_{\bar{K}K} \) is significantly higher than its experimental value.

The actual size of the discrepancies will be considered in step 3 of our strategy but the message is clear: depending on which scenario is considered, we need either constructive NP contributions to \( |\varepsilon_K| \) (scenario 1) or destructive NP contributions to \( S_{\bar{K}K} \) (scenario 2). However, this NP should not spoil the agreement with the data for \( S_{\bar{K}K} \) (scenario 1) and for \( |\varepsilon_K| \) (scenario 2).

In view of the fact that the theoretical precision on \( S_{\bar{K}K} \) is significantly larger than in the case of \( \varepsilon_K \), one may wonder whether removing the 1–2\( \sigma \) anomaly in \( \varepsilon_K \) by generating a 2–3\( \sigma \) anomaly in \( S_{\bar{K}K} \) is a reasonable strategy. However, we will proceed in this manner because this will teach us how different NP scenarios deal with this problematic. In order to resolve this puzzle we definitely need not only a precise determination of \( |V_{ub}| \) that is not polluted by NP but also precise values of non-perturbative parameters relevant for the SM predictions in this case.

Until 2012 there was another significant tension between SM branching ratio for \( B^+ \rightarrow \tau^+ \nu_\tau \) and the data, with the experimental value being by a factor of two larger than the theory. This would favour the large \( |V_{ub}| \) scenario. However, presently, after the data from BELLE this discrepancy is practically absent, as discussed in step 5 of our strategy. Yet, the agreement of the SM with the data still depends on the chosen value of \( |V_{ub}| \), which enters this branching ratio quadratically. In turn, the kind of NP that would improve the agreement of the theory with the data depends on the chosen value of \( |V_{ub}| \). Other modest tensions between the SM and the data will be discussed as we proceed.

Models with many new parameters can successfully face both scenarios for \( |V_{ub}| \) by removing the deviations from the data for certain ranges of their parameters but, as we will see below, in simpler models often only one scenario can be admitted because only in that scenario for \( |V_{ub}| \) does a given model have a chance to fit \( \varepsilon_K \) and \( S_{\bar{K}K} \) simultaneously. For instance, as we will see in the course of our presentation, models with constrained minimal flavour violation (CMFV) select scenario 1, while the 2HDM with MFV and flavor-blind phases (FBPs), 2HDM_{MFV}, favours scenario 2 for \( |V_{ub}| \). What is interesting is that the future precise determination of \( |V_{ub}| \) through tree-level decays will be able to distinguish between these two NP scenarios. We will see that there are other models that can be distinguished in this simple manner.

Clearly, in order to get the full picture, many more observables have to be considered. For instance in table 4, which can be found in step 3, we illustrate the SM predictions for additional observables, in particular the mass differences \( \Delta M_i \) and \( \Delta M_d \) in the \( B_s,d \rightarrow \bar{B}_s,d \) systems. What is striking in this table is that, with the present lattice input in table 1, the predicted central values of \( \Delta M_s \) and \( \Delta M_d \) are both in good agreement with the latter when hadronic uncertainties are taken into account. In particular, the central value of the ratio \( \Delta M_s/\Delta M_d \) is very close to the data. These results depend strongly on the lattice input and in the case of \( \Delta M_d \) on the value of \( \gamma \). Therefore, to get a better insight, both lattice input and the tree-level determination of \( \gamma \) have to improve. Moreover, the situation changes with time. While one year ago the lattice input was such that models providing 10\% suppression of both \( \Delta M_s \) and \( \Delta M_d \) were favoured, this is no longer the case, as can be seen in table 4.

However, for the purpose of presenting our strategy, it will be useful to keep the old central values from lattice that are consistent within 1\( \sigma \) with the present ones but imply certain deviations from SM expectations. This will allow us to illustrate how NP can remove these deviations. In doing this we will keep in mind that the pattern of deviations from SM expectations could be modified in the future. This is particularly the case for the observables that still suffer from non-perturbative uncertainties, such as \( \Delta M_s,d \). It could turn out that suppressions (enhancements) of some observables required in our examples from NP will be modified to enhancements (suppressions) in the future and it will be of interest to see whether a given model could cope with such changes. Bearing this in mind will lead us eventually in section 6 to a proposal of DNA charts, primarily with the goal of transparently exhibiting the pattern of enhancements and suppressions of flavour observables in a given NP scenario, as well as the correlations between them. Of course, this pattern will also include situations in which no modifications in a given observable relative to the SM will take place.

### 2.2. Towards a new SM in 12 steps

Our strategy involves 12 steps, which we present in detail in section 5. These steps involve a number of decays and transitions, as shown in figure 1, and can be properly adjusted in case the pattern of deviations from the SM will be modified.

For the time being, assuming that the present tensions will be strengthened with time, when the data improves, the specific questions that arise are:

- Which model is capable of removing the \( \varepsilon_K \)-\( S_{\bar{K}K} \) tension and simultaneously providing modifications in \( B^+ \rightarrow \tau^+ \nu_\tau \) and \( \Delta M_{s,d} \) if they are required?
• What are the predictions of this model for:

\[ S_{\psi KS}, \quad B_{s,d} \to \mu^+\mu^- , \quad B \to K^+\ell^+\ell^-, \quad B \to X_s\ell^+\ell^- , \quad B \to X_s\bar{\nu}, \quad B \to K^\ast\nu\bar{\nu}, \quad B \to K\nu\bar{\nu}, \]

and how are these predictions correlated with \( S_{\psi KS} \) and \( \varepsilon_K \)?

The comparison of the processes and observables listed here with those appearing in figure 1 does not imply that the ones missing in (1)–(3), like LFV and EDMs, are less important. But, since we discuss these topics in our review only in general terms, they will in fact remain under the shadow of the processes listed above.

2.3. Correlations between observables

In order to reach our goal, we need a strategy for uncovering the NP that are responsible for the observed anomalies. Possible anomalies will hopefully be found in the future. One line of attack chosen by several authors are model independent studies of the Wilson coefficients, with the goal of finding out how much room for NP contributions is still left in each coefficient. In this context the correlations between various Wilson coefficients are studied. While such study is certainly useful and gives some insight into the room left for NP, one should keep in mind that the Wilson coefficients are scale and renormalization scheme dependent and the correlations between them generally depend on the scale at which they are evaluated as well as on the renormalization scheme used.

Therefore, it is our strong believe that searching for correlations between the measured observables is more powerful. Extensive studies of the correlations between various observables in the concrete models very clearly illustrate the power of this strategy. Quite often, only the qualitative behaviour of these correlations is sufficient to eliminate the model as a solution to observed anomalies or to select models as candidates for a new SM. A detailed review of such explicit studies can be found in [17, 20]. These studies have allowed us to construct various classifications of NP contributions in the form of ‘DNA’ tables [35] and 

flavour codes [17], they have also provided some insight into the physics behind resulting correlations in specific models [36]. Detailed analyses in this spirit have been subsequently performed in [37, 38]. With improved data all these results will be increasingly useful.

In the present paper we will take a slightly different route. Instead of investigating explicit models we will illustrate the search for a new SM using very simple models, while being aware of the fact that in more complicated models certain patterns of flavour violations and correlations between various observables could be washed out and become less transparent. This strategy has been used by us in our most recent papers [39–47]. In this context, a prominent role will be played by new tree-level contributions to FCNC processes mediated either by heavy neutral gauge bosons or neutral heavy scalars. These contributions are governed, in particular, by the couplings \( \Delta_{ij}^{L,R}(Z) \) and \( \Delta_{ij}^{L,R}(H^0) \) for gauge bosons and scalars to quarks, respectively. Here, \((i, j)\) denote quark flavours. As we will see, in addition to a general form of these couplings, it will be instructive to consider the following four scenarios for them while keeping the pair \((i, j)\) fixed:

1. Left-handed scenario (LHS) with complex \( \Delta_{L}^{bq} \neq 0 \) and \( \Delta_{R}^{bq} = 0 \),
2. Right-handed scenario (RHS) with complex \( \Delta_{R}^{bq} \neq 0 \) and \( \Delta_{L}^{bq} = 0 \),
3. Left–right symmetric scenario (LRS) with complex \( \Delta_{L}^{bq} = \Delta_{R}^{bq} \neq 0 \),
The field of weak decays is based on effective Hamiltonians, with the generic form given as follows:

\[ \mathcal{H}_{\text{eff}} = \kappa \sum_i C_i(\mu) Q_i + h.c. \]  

(4)

Here, \( Q_i \) are local operators and \( C_i(\mu) \) their Wilson coefficients, which can be evaluated in RG improved perturbation theory. Details of the calculations of these coefficients and the related technology including QCD corrections at the next-to-leading order (NLO) and next-to-next-to-leading order (NNLO) level can be found in [50–52].

The overall factor \( \kappa \) can be chosen at will in accordance with the overall normalization of Wilson coefficients and operators. Sometimes it is useful to set \( \kappa \) to its value in the SM but this is not always the case, as we will see below. The scale \( \mu \) can be the low energy scale \( \mu_L \) at which actual lattice calculations are performed or any other scale, particularly the matching scale \( \mu_m \), which is the border line between a given full and corresponding effective theory.

The matrix elements of the effective Hamiltonian are directly related to decay amplitudes and can be written generally as follows:

\[ \langle \mathcal{H}_{\text{eff}} \rangle = \kappa \sum_i C_i(\mu_L) \langle Q_i(\mu_L) \rangle \]  

(5)

or

\[ \langle \mathcal{H}_{\text{eff}} \rangle = \kappa \sum_i C_i(\mu_m) \langle Q_i(\mu_m) \rangle. \]  

(6)

These two expressions are equal to each other and the Wilson coefficients in them are connected through

\[ \tilde{C}(\mu_L) = \tilde{U}(\mu_L, \mu_m) C(\mu_m), \]  

(7)

where \( \tilde{U} \) is the RG evolution matrix and \( \tilde{C} \) is a column vector. Which of the formulations is more useful depends on the process and model considered.

Now, the Wilson coefficients depend directly on the couplings present in the fundamental theory. In our paper the quark–gauge boson and quark–scalar couplings will play a prominent role and it is useful to introduce a general notation for them so that they can be used in the context of any model considered.

Quite generally, we can consider the basic interactions of charged gauge bosons \( W^+ \), charged scalars \( H^+ \), neutral gauge bosons \( Z' \) and neutral scalars \( H^0 \) with quarks that are shown as vertices in figures 2 and 3. These gauge bosons are all colourless but this notation could be easily extended to colourless gauge bosons and scalars. They can also be extended to heavy quarks interacting with SM quarks and to interactions of bosons with leptons. It should be emphasized that all of the fields in these vertices are in the mass eigenstate basis. In the course of our presentation we will give the expressions for various coefficients in terms of these couplings.

In figures 2 and 3 the couplings \( \Delta_{L,R} \) are \( 3 \times 3 \) complex matrices in the flavour space with \( i, j \) denoting different quark flavours. In the case of charged boson exchanges, the first flavon index in figure 2 denotes an up-type quark and the second a down-type quark.

In models in which FCNC processes take place first at one-loop level, it is useful to work with (6) and express \( C_i(\mu_m) \) in terms of a set of gauge independent master functions, which result from calculations of penguin and box diagrams and which govern the FCNC processes. In particular, this is the case for those models in which the operator structure is the same as in the SM. We will discuss these models soon.

On the other hand, in models in which new operators with RH currents, and scalar and pseudoscalar currents are

4. Left–right asymmetric scenario (ALRS) with complex 
\( \Delta_{L,R}^\mu \neq 0 \),

with analogous scenarios for the pair \((s, d)\). These ideas can also be extended to the charged gauge boson \((W^+)\) and charged Higgs \((H^+)\) exchanges. We will see that these simple scenarios will give us a profound insight into the flavour structure of the models in which NP is dominated by left-handed (LH) currents or RH currents, or LH and RH currents of approximately the same size.

The idea of looking at such NP scenarios is not new and has been particularly motivated by a detailed study of supersymmetric flavour models with NP dominated by LH currents, RH currents or equal amount of LH and RH currents [35]. Moreover, it has been found in several studies of non-supersymmetric frameworks, such as the LHT model [48] or Randall–Sundrum scenario with custodial protection (RSc) [49], that models with the dominance of LH or RH currents exhibit quite different patterns of flavour violation. Our simple models will demonstrate this in a transparent manner.

There is another point that we would like to make. In several papers predictions for various observables in given extensions of the SM are made using presently available loop processes to determine CKM parameters. As we will emphasize in step 1 below, in our view this is not the optimal time to proceed in this manner. The last few years have shown that such predictions have rather a short lifetime. It appears to us that it is more useful at this stage to develop transparent formulae that will allow us to monitor the future events in flavour physics in the SM and its extensions when the experimental data improve and the uncertainties in lattice calculations decrease.

Our strategy will also be complementary to analyses in which allow for fits using sophisticated computer machinery are made. We will start with a subset of observables that have simple theoretical structure while ignoring first constraints from more complicated observables. In subsequent steps we will gradually include more observables in our analysis, which necessarily will modify our insights gained in the first steps; thereby, teaching us something. Only in section 6 we will look at all observables simultaneously, and the grand view of simple models and the grand view of more complicated models should hopefully allow us to efficiently monitor flavour events in this decade.

With this general strategy in mind we can now enter the details, while first briefly recalling the theoretical framework for weak decays.

3. Theoretical framework

3.1. Preliminaries

The field of weak decays is based on effective Hamiltonians, with the generic form given as follows:

\[ \mathcal{H}_{\text{eff}} = \kappa \sum_i C_i(\mu) Q_i + h.c. \]  

Here, \( Q_i \) are local operators and \( C_i(\mu) \) their Wilson coefficients, which can be evaluated in RG improved perturbation theory. Details of the calculations of these coefficients and the related technology including QCD corrections at the next-to-leading order (NLO) and next-to-next-to-leading order (NNLO) level can be found in [50–52].

The overall factor \( \kappa \) can be chosen at will in accordance with the overall normalization of Wilson coefficients and operators. Sometimes it is useful to set \( \kappa \) to its value in the SM but this is not always the case, as we will see below. The scale \( \mu \) can be the low energy scale \( \mu_L \) at which actual lattice calculations are performed or any other scale, particularly the matching scale \( \mu_m \), which is the border line between a given full and corresponding effective theory.

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These two expressions are equal to each other and the Wilson coefficients in them are connected through

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Now, the Wilson coefficients depend directly on the couplings present in the fundamental theory. In our paper the quark–gauge boson and quark–scalar couplings will play a prominent role and it is useful to introduce a general notation for them so that they can be used in the context of any model considered.

Quite generally, we can consider the basic interactions of charged gauge bosons \( W^+ \), charged scalars \( H^+ \), neutral gauge bosons \( Z' \) and neutral scalars \( H^0 \) with quarks that are shown as vertices in figures 2 and 3. These gauge bosons are all colourless but this notation could be easily extended to colourless gauge bosons and scalars. They can also be extended to heavy quarks interacting with SM quarks and to interactions of bosons with leptons. It should be emphasized that all of the fields in these vertices are in the mass eigenstate basis. In the course of our presentation we will give the expressions for various coefficients in terms of these couplings.

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In models in which FCNC processes take place first at one-loop level, it is useful to work with (6) and express \( C_i(\mu_m) \) in terms of a set of gauge independent master functions, which result from calculations of penguin and box diagrams and which govern the FCNC processes. In particular, this is the case for those models in which the operator structure is the same as in the SM. We will discuss these models soon.

On the other hand, in models in which new operators with RH currents, and scalar and pseudoscalar currents are
where the variable $v$ collects the parameters of a given model. It is often useful to keep the CKM factors outside these functions. Then, in models with MFV without FBPs these functions are real valued and universal with respect to different meson systems, implying various stringent correlations between various decays and related observables. In models with MFV, and flavor-blind CPV phases and genuine non-MFV frameworks, these functions become complex valued and the universality between various meson systems is violated, implying corrections to correlations present in models with MFV but with no FBPs.

Generally, several master functions contribute to a given decay, although decays exist which depend only on a single function. We have the following correspondence between the most interesting FCNC processes and the master functions in the MFV models, in which the operator structure is the same as in the SM:

\[
\begin{align*}
K^0 - K^0 &- \text{mixing (}\varepsilon_K) & S(\nu) \\
B^0_{d,s} - B^+_{d,s} &- \text{mixing (}\Delta M_{d,s}) & S(\nu) \\
K &\to \pi\nu\bar{\nu}, \ B &\to X_{d,s}, \text{\nu} & X(\nu) \\
K_L &\to \mu\bar{\mu}, \ B_{d,s} &\to \ell\bar{\ell} & Y(\nu) \\
K_L &\to \pi^0e^+e^- & Y(\nu), Z(\nu), E(\nu) \\
\varepsilon', \text{Nonleptonic } \Delta B = 1, \ \Delta S = 1 &\to X(\nu), Y(\nu), Z(\nu), E(\nu) \\
B &\to X_{d,s} & \to D'(\nu), E'(\nu) \\
B &\to X_{d,s} & \to Y(\nu), Z(\nu), E(\nu), D'(\nu), E'(\nu).
\end{align*}
\]

This table means that the observables, such as branching ratios, mass differences $\Delta M_{d,s}$ in $B^0_{d,s} - B^+_{d,s}$-mixing and the CP violation parameters $\varepsilon$ and $\varepsilon'$, can all be to a very good approximation entirely expressed in terms of the corresponding master functions and the relevant CKM factors.

3.3. CFMV relations as standard candles of flavour physics

The implications of this framework are so stringent that it appears to us to consider them as standard candles of flavour physics. Even if some of these relations will appear again in the context of our presentation, it is useful here to collect the most important relations in one place. A review of these relations is given in [54]. Since NP effects in FCNC processes appear smaller than anticipated in the past, the importance of these relations increased in 2013.

We have:

1. $S_{\psi K_S}$ and $S_{\psi\phi}$ are as in the SM and are, therefore, given by

\[
S_{\psi K_S} = \sin(2\beta) , \quad S_{\psi\phi} = \sin(2|\beta_\ell|), \quad (9)
\]

where $\beta$ and $\beta_\ell$ are defined in (28).

2. While $\Delta M_d$ and $\Delta M_s$ can differ from the SM values their ratio is as in the SM

\[
\left( \frac{\Delta M_d}{\Delta M_s} \right)_{\text{CFMV}} = \left( \frac{\Delta M_d}{\Delta M_s} \right)_{\text{SM}} . \quad (10)
\]

### Detailed expositions of phenomenological consequences of BFPs

This is possibly the simplest class of BSM scenarios. It is defined pragmatically as follows [53]:

- The only source of flavour and CP violation is the CKM matrix. This implies that the only CP-violating phase is that of the KM phase and that CP-violating FBPs are assumed to be absent.
- The only relevant operators in the effective Hamiltonian below the electroweak scale are those present within the SM.

Detailed expositions of phenomenological consequences of this NP scenario has been given in [54, 55] and recently in [20].

In CMFV models it is useful to work with (6) and express $C_\mu (\mu_{ab})$ in terms of a set of gauge independent master functions, which result from calculations of penguin and box diagrams and which govern the FCNC processes. One has then seven one-loop functions that are denoted by

\[
S(\nu), X(\nu), Y(\nu), Z(\nu), E(\nu), D'(\nu), E'(\nu), \quad (8)
\]

The only relevant operators in the effective Hamiltonian

\[
\begin{align*}
K^0 - K^0 &- \text{mixing (}\varepsilon_K) & S(\nu) \\
B^0_{d,s} - B^+_{d,s} &- \text{mixing (}\Delta M_{d,s}) & S(\nu) \\
K &\to \pi\nu\bar{\nu}, \ B &\to X_{d,s}, \text{\nu} & X(\nu) \\
K_L &\to \mu\bar{\mu}, \ B_{d,s} &\to \ell\bar{\ell} & Y(\nu) \\
K_L &\to \pi^0e^+e^- & Y(\nu), Z(\nu), E(\nu) \\
\varepsilon', \text{Nonleptonic } \Delta B = 1, \ \Delta S = 1 &\to X(\nu), Y(\nu), Z(\nu), E(\nu) \\
B &\to X_{d,s} & \to D'(\nu), E'(\nu) \\
B &\to X_{d,s} & \to Y(\nu), Z(\nu), E(\nu), D'(\nu), E'(\nu).
\end{align*}
\]
Moreover, this ratio is given entirely in terms of CKM parameters and non-perturbative parameter $\xi$:

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_s}}{m_{B_d}} \frac{1}{\xi^2} \frac{V_{td}}{V_{ts}}^2 r(\Delta M), \quad \xi^2 = \frac{\hat{B}_i}{\hat{B}_j} \frac{F_{B_i}}{F_{B_j}} \frac{F_{\Delta M_s}}{F_{\Delta M_d}} \quad (11)$$

where we have introduced the quantity $r(\Delta M)$, that is equal unity in models with CMFV. It parametrizes the deviations from these relations found in several models, as discussed below.

3. These two properties allow the construction of the universal unitarity triangle (UUT) of models of CMFV that uses as inputs the measured values of $S_{\psi K}$ and $\Delta M_s/\Delta M_d$ [53].

4. The flavour universality of $S(\nu)$ allows us to derive universal expressions for $S_{\phi K}$, and the angle $\gamma$ in the UUT that depend only on $|V_{ud}|$, $|V_{cb}|$, known from tree-level decays, and non-perturbative parameters entering the evaluation of $\epsilon_K$ and $\Delta M_{d,s}$ [55, 58, 59], which are valid for all CMFV models. We will present an update of these formulae in step 3 of our strategy. Therefore, once the data on $|V_{ud}|$, $|V_{cb}|$, $\epsilon_K$ and $\Delta M_{d,s}$ are taken into account, one is able in this framework to predict not only $S_{\phi K}$ but also $|V_{ts}|$.

5. For fixed CKM parameters determined in tree-level decays, $|\epsilon_K|$, $\Delta M_s$ and $\Delta M_d$, if modified, can only be enhanced relative to SM predictions [60]. Moreover this happens in a correlated manner [59].

6. Two other interesting universal relations in models with CMFV are

$$\frac{B(B \to X_d \nu \bar{\nu})}{B(B \to X_s \nu \bar{\nu})} = \frac{|V_{td}|^2}{|V_{ts}|^2} r(\nu \bar{\nu}) \quad (12)$$

$$\frac{B(\bar{B}_s \to \mu^+ \mu^-)}{B(\bar{B}_d \to \mu^+ \mu^-)} = \frac{\tau(\bar{B}_d) m_{B_s} F_{B_\Delta M_s}}{\tau(\bar{B}_s) m_{B_d} F_{B_\Delta M_d}} \frac{|V_{td}|^2}{|V_{ts}|^2} r(\mu^+ \mu^-), \quad (13)$$

where we have again introduced the quantities $r(\nu \bar{\nu})$ and $r(\mu^+ \mu^-)$, which are all equal unity in CMFV models.

7. Eliminating $|V_{ud}|/|V_{ts}|$ from (11) and (13) allows us to obtain another universal relation within the CMFV models [61]

$$\frac{B(\bar{B}_s \to \mu^+ \mu^-)}{B(\bar{B}_d \to \mu^+ \mu^-)} = \frac{\bar{B}_s \tau(\bar{B}_s) \Delta M_s r_s}{\bar{B}_d \tau(\bar{B}_d) \Delta M_d r_d} = \frac{r(\Delta M)}{r(\Delta M)} \quad (14)$$

that does not involve $F_{B_\Delta M}$ and CKM parameters and which, consequently, contains smaller hadronic and parametric uncertainties than the formulae considered above. This involves only measurable quantities except for the ratio $\bar{B}_s/\bar{B}_d$ that is now already known from lattice calculations with impressive accuracy of $\pm 2$–3% [62], this precision should be even improved. Therefore, the relation (14) should allow a precision test of CMFV, even if the branching ratios $B(\bar{B}_d \to \mu^+ \mu^-)$ would turn out to deviate from SM predictions by 10–20%.

8. All amplitudes for FCNC processes within the CMFV framework can be expressed in terms of seven real and universal master loop functions listed in (8). The implications of this property are numerous correlations between various observables, which are discussed more explicitly in section 4.

3.4. MFV at Large

In the more general case of MFV, the formulation with the help of global symmetries present in the limit of vanishing Yukawa couplings as formulated in [63] is elegant and useful. See also [64] for a similar formulation that goes beyond the MFV. Other profound discussions of various aspects of MFV can be found in [65–70]. An excellent compact formulation of MFV as effective theory has been given by Gino Isidori [71]. We also recommend the reviews in [72, 73], where phenomenological aspects of MFV are summarized.

In short, the hypothesis of MFV amounts to assuming that the Yukawas are the only sources of the breakdown of flavour and CP violation. The phenomenological implications of the MFV hypothesis formulated in this more granular manner than the CMFV formulation given above can be found to be modelled independently by using an effective field theory (EFT) approach [63]. In this framework the SM Lagrangian is supplemented by all of the higher dimension operators that are consistent with the MFV hypothesis, built using the Yukawa couplings as spurion fields. The NP effects in this framework are then parametrized in terms of a few flavour-blind free parameters and SM Yukawa couplings, which are solely responsible for flavour violation as well as CP violation if these flavour-blind parameters are chosen as real quantities, as done in [63]. This approach naturally suppresses FCNC processes to the level observed experimentally, even in the presence of new particles with masses of a few hundreds GeV. It also implies specific correlations between various observables, which are not as stringent as in the CMFV but which are still very powerful.

Yet, it should be stressed that the MFV symmetry principle in itself does not forbid the presence of flavour-blind CP-violating sources [65, 67–70, 74–78], which effectively make the flavour blind free parameters complex quantities to have FBPs. These phases can in turn enhance the EDMs of various particles and atoms, and in the interplay with the CKM matrix they can also have a profound impact on flavour-violating observables, particularly the CP-violating ones. In the context of the so-called aligned 2HDM model, such effects have also been emphasized in [79].

The introduction of flavour-blind CPV phases compatible with the MFV symmetry principle turns out to be a very interesting set-up [65, 67, 69, 70, 77]. In particular, as noted in [69], a large new phase in $B_0^{\tau - \bar{B}_0^\tau}$ mixing could in principle be obtained in the MFV framework if additional FBPs are present. This idea cannot be realized in the ordinary MSSM with MFV, as shown in [35, 80]. The difficulty of realizing this scenario in the MSSM is due to the suppression in the MSSM of effective operators with several Yukawa insertions. Sizable couplings for these operators are necessary, both to
have an effective large CP-violating phase in $B^0_{\mu} - \bar{B}^0_{\mu}$ mixing and, at the same time, to evade bounds from other observables, such as $B_s \to \mu^+ \mu^-$ and $B \to X_s \gamma$. However, it could be realized in different underlying models, such as: the up-lifted MSSM (as pointed out in [81]), in the so-called beyond-MSSM scenarios [82, 83], and in the 2HDM with MFV and FBPs, the so-called 2HDM$_{MFV}$ [84], to which we will return at various places in this writing. An excellent review of 2HDMs at large can be found in [85].

As we will see in step 3 of our strategy, the present data from the LHCb shows that the new phases in $B^0_{\mu} - \bar{B}^0_{\mu}$ mixing, if present, must be rather small. Consequently, the role of FBPs in describing data has also decreased significantly relative to the one that they played in the studies summarized above. However, the full assessment of the importance of these phases will only be possible when the CP violation in $B^0_{\mu} - \bar{B}^0_{\mu}$ mixing will be precisely measured and when the bounds on the EDMs improve.

3.5. Simplest models with non-MFV sources

In models with new sources of flavour and CP violation, in which the operator structure is not modified, the formulation of FCNC processes in terms of seven one-loop functions is also useful. But when the CKM factors are the only ones kept explicit as overall factors, these functions become complex and are different for different meson systems. We then have $(i = K, d, s)$:

$$S_i \equiv |S_i| e^{i\theta^S_i}, \quad X_i \equiv |X_i| e^{i\theta^X_i}, \quad Y_i \equiv |Y_i| e^{i\theta^Y_i},$$

$$Z_i \equiv |Z_i| e^{i\theta^Z_i}, \quad E_i \equiv |E_i| e^{i\theta^E_i}, \quad D_i' \equiv |D_i'| e^{i\theta^D_i'}, \quad E_i' \equiv |E_i'| e^{i\theta^E_i'}.$$  \hfill (15)

Since the property of the universality of these functions is now lost, the usual CMFV relations between $K, B_s$ and $B_d$ systems listed above can be violated while the parameters $r(k)$ introduced in the context of our discussion of CMFV models are generally different from unity and can be complex. A known example of this is the littlest Higgs model with T-parity (LHT) [48].

3.6. The $U(2)^3$ models

Probably the simplest models with new sources of flavour violation are models in which the $U(3)^3$ symmetry of MFV models is reduced to $U(2)^3$ symmetry [86–92]. As pointed out in [39], a number of properties of CMFV models remains in this class of model; in particular, the relation (14) is still valid. On the other hand, there are profound differences due to the presence of new CP phases, which we will discuss in the course of our presentation.

3.7. Tree-level gauge boson and scalar exchanges

In a number of BSM scenarios NP can already enter at tree level, both in charged current processes and in FCNC processes.

In the case of charged current processes, examples of this include the RH $W^{\pm} \ell$ bosons in left–right symmetric models and charged Higgs ($H^{\pm}$) particles in models with an extended scalar sector, such as two Higgs doublet models and supersymmetric models. New operators are present in these models, the simplest example being $(V + A) \times (V + A)$ operators originating in the exchange of $W^{\pm}$ gauge bosons in the left–right symmetric models. $(V - A) \times (V + A)$ operators also contribute in these models. These operators generate in turn through QCD corrections $(S \pm P) \times (S \pm P)$ operators, which are also present in models with $H^\pm$ particles. In the latter models, $(S \pm P) \times (S \pm P)$ operators are also present. Needless to say, all of these statements also apply to neutral gauge bosons and scalars mediating $\Delta F = 1$ transitions. It should also be stressed that anomalous RH couplings of SM gauge bosons $W^{\pm}$ to quarks can be generated through mixing with heavy vectorial fermions.

Concerning FCNC processes, tree-level transitions are present in any model in which GIM mechanism is absent in some sectors of a given model. This is the case in numerous $Z'$ models, gauged flavour models with new very heavy neutral gauge bosons, and left–right symmetric models with heavy neutral scalars. They can also be generated at one loop in models having GIM at the fundamental level and MFV, of which two-Higgs doublet models with and without supersymmetry are the best known examples. Tree-level $Z^0$ and SM neutral Higgs $H^0$ contributions to $\Delta F = 2$ processes are also possible in models containing vectorial heavy fermions that mix with the standard chiral quarks. This is also the case of models in which $Z^0$ and SM neutral Higgs $H^0$ mix with new heavy gauge bosons and scalars in the process of electroweak symmetry breaking. Recently, two very detailed analyses of FCNCs within models with tree-level gauge boson and neutral scalar and pseudoscalar exchanges have been performed in [40, 44], we will include the highlights from these two papers in our discussion.

In the previous section we defined in figures 2 and 3 the basic interactions of charged gauge bosons $W^{\pm}$, charged scalars $H^+$, with quarks. In the flavour precision era, QCD corrections to tree-level exchanges have also to be taken into account. They depend on whether a gauge boson or scalar is exchanged and, of course, on the process considered. Fortunately, the NLO matching conditions for tree-level neutral gauge bosons $Z$ and neutral scalars $H^0$ exchanges have been recently calculated in [93, 94]. By combining them with previously calculated two-loop anomalous dimensions of four-quark operators, it is possible to perform complete NLO RG analysis in this case.

Finally, we would like to make a general comment on the expressions for various observables in this class of models, which we will encounter below. They are very general and apply also to models in which the FCNC processes enter first at the one-loop level. Indeed, they contain a very general operator structure and general new flavour-violating and CP-violating interactions. However, having a simpler coupling structure than in the case of models in which NP is dominated by loop contributions allows us to have an analytic look at various correlations between various observables, as we will see below.
4. Classifying correlations between various observables

As we have seen in the preceding sections, in the SM and in models with CMFV the observables measured in the processes shown in figure 1 depend on the selected number of basic universal functions that are the same for $K$ and $B_{s,d}$ decays. In particular, $\Delta F = 2$ processes depend only on the function $S(v)$, while the most important rare $K$ and $B_{s,d}$ decays depend on three universal functions: $X(v), Y(v)$ and $Z(v)$.

Consequently, a number of correlations exist between various observables, not only within the $K$ and $B$ systems but also between $K$ and $B$ systems. In particular, the latter correlations are very interesting because they are characteristic for this class of model. A review of these correlations is given in [54]. These correlations are violated in several extensions of the SM, either through the presence of a new source of flavour violation or the presence of new operators. However, given that the SM constitutes the main bulk of most branching ratios, the CMFV correlations can be considered as standard candles of flavour physics with the help of which new sources of flavour violation or effects of new operators could be identified. It is for the latter reason that we prefer to use CMFV correlations as standard flavour candles and not those present in MFV at large, even though models with MFV and one Higgs doublet give the same results.

In [49] a classification of correlations following from CMFV has been presented. In the following, we will somewhat modify this classification so that it fits better to our presentation in the next section, which will consider a number of models in contrast to [49] where only the RSc model has been studied.

We distinguish the following classes of correlations in CMFV models:

Class 1. Correlations implied by the universality of the real function $X$. They involve rare $K$ and $B$ decays with $v\bar{v}$ in the final state. These are:

\[ K^+ \rightarrow \pi^+ v\bar{v}, \quad K_L \rightarrow \pi^0 v\bar{v}, \quad B \rightarrow X_{s,d} v\bar{v}, \]

\[ B \rightarrow K^+(K)v\bar{v}. \]  

(17)

Class 2. Correlations implied by the universality of the real function $Y$. They involve rare $K$ and $B$ decays with $\mu^+\mu^-$ in the final state. These are:

\[ B_{s,d} \rightarrow \mu^+\mu^-, \quad K_L \rightarrow \mu^+\mu^-, \quad K_L \rightarrow 0^+\mu^+\mu^-, \]

\[ K_L \rightarrow 0^+e^+e^- . \]  

(18)

Class 3. In models with CMFV NP contributions enter the functions $X$ and $Y$ approximately in the same manner because, at least in the Feynman gauge, they come dominantly from $Z^0$-penguin diagrams. This implies correlations between rare decays with $\mu^+\mu^-$ and $v\bar{v}$ in the final state. It should be emphasized that this is a separate class because NP can generally have a different impact on decays with $v\bar{v}$ and $\mu^+\mu^-$ in the final state. This class simply involves the decays of class 1 and class 2.

\footnote{We do not include in this list a known model independent correlation between the asymmetries $S_{\phi K}$ and $A_{\phi K}$ [95] that has to be satisfied basically in any extension of the SM.}

Class 4. Here we group correlations between $\Delta F = 2$ and $\Delta F = 1$ transitions in which the one-loop functions $S$ and $(X, Y)$, respectively, cancel out and the correlations follow from the fact that the CKM parameters extracted from tree-level decays are universal. One known correlation of this type involves [96, 97]

\[ K^+ \rightarrow \pi^+ v\bar{v}, \quad K_L \rightarrow \pi^0 v\bar{v} \] and $S_{\phi K}$.  

(19)

another one involves [61]

\[ B_{s,d} \rightarrow \mu^+\mu^- \] and $\Delta M_{s,d}$.  

(20)

As we will see in section 5, some of these correlations, in particular those between $K$ and $B$ decays, are strongly violated in certain models while others are approximately satisfied. Clearly, the full picture is only obtained by looking simultaneously at patterns of violations of the correlations in question in a given NP scenario.

At later stages of our presentation in section 5 we will study correlations in models with tree-level FCNCs mediated by neutral gauge bosons and scalars that go beyond the CMFV framework. In these models the multi-correlations between various observables in a given meson system are predicted. It is useful to group these processes in the following three classes:

Class 5. $e_K, E \rightarrow \pi^+ v\bar{v}, \quad K_L \rightarrow \pi^0 v\bar{v}, \quad K_L \rightarrow \mu^+\mu^-, \quad K_L \rightarrow 0^+ e^+ e^- , \quad e'/e$.  

(21)

Class 6. $\Delta M_{s,d}, S_{\phi K}, B_{s,d} \rightarrow \mu^+\mu^-, S_{\mu \mu}^d$.  

(22)

where the CP-violating asymmetry $S_{\mu \mu}^d$ can only be obtained from time-dependent rate of $B_{s,d} \rightarrow \mu^+\mu^-$ and will remain in the realm of theory for the foreseeable future.

Class 7. $\Delta M_{s}, S_{\phi K}, B_{s} \rightarrow \mu^+\mu^-, S_{\mu \mu}^s, B \rightarrow K v\bar{v}, \quad B \rightarrow K^+ v\bar{v}, \quad B \rightarrow X_{s} v\bar{v}$,  

(23)

where the measurement of $S_{\mu \mu}^s$ will require heroic efforts from experimentalists but apparently is not totally hopeless.

Class 8. $B \rightarrow X_{s} \gamma, \quad B \rightarrow K^+ \gamma, \quad B^+ \rightarrow \tau^+ \nu_{\tau}$.  

(24)

in which new charged gauge bosons and heavy scalars can play a significant role. The first two differ from previous decays because they are governed by dipole operators.

Class 9. $B \rightarrow K \mu^+\mu^-, B \rightarrow K^+ \mu^+\mu^-, B \rightarrow X_{s} \mu^+\mu^-$.  

(25)

to which several operators contribute and for which multitude of observables can be defined. Moreover, in the case of FCNCs mediated by tree-level neutral gauge boson exchanges there are interesting correlations between these observables and those of class 7.

Class 10. Correlations between $K$ and $D$ observables.

Class 11. Correlations between quark flavour violation, LFV, EDMs and $(g - 2)_{\mu}$.
5. Searching for NP in 12 Steps

5.1. Step 1: the CKM matrix from tree-level decays

Given that the SM already represents the dominant part in very many flavour observables, it is crucial to determine the CKM parameters as precisely as possible, independently of NP contributions. Here the tree-level decays governed by $W^{±}$ exchanges play a prominent role. The charged current decays could be affected by heavy charged new gauge boson exchanges and heavy charged Higgs boson exchanges that could contribute directly to tree-level decays. In addition, non-standard $W^{±}$ couplings could be generated through mixing of $W^{±}$ with the new heavy gauge bosons in the process of electroweak symmetry breaking. Moreover, the mixing of heavy fermions, both sequential (such as the case of the fourth generation) or vectorial (which are present in various NP scenarios), could make the CKM matrix non-unitary and not allow us to use the well-known unitarity relations of this matrix. This mixing would also generate non-standard $W^{±}$ couplings to SM quarks.

The non-observation of any convincing NP signals at the LHC until now gives some hints that the masses of new charged particles are shifted above the 500 GeV scale. Therefore, NP effects in charged current decays are likely to be at most at the level of a few per cent. While effects of this sort could play a role one day, in the first step it is a good strategy to assume that tree-level charged current decays are fully dominated by $W^{±}$ exchanges with SM couplings and, consequently, by the CKM matrix.

The goal of this first step is then a very precise determination of

$$|V_{us}| \simeq s_{12}, \quad |V_{ub}| \simeq s_{13}, \quad |V_{cb}| \simeq s_{23}, \quad \gamma = \delta,$$  

(26)

where on the rhs we give the measured quantities and on the lhs the determined parameters of the CKM matrix given in the standard parametrization:

$$\hat{V}_{CKM} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -s_{23} c_{12} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix}.$$

(27)

The phase $\gamma$ is one of the angles of the unitarity triangle that is shown figure 4. We emphasize that the relations in (26) are excellent approximations. Indeed, $c_{13}$ and $c_{23}$ are very close to unity. The parameters $\hat{\rho}$ and $\hat{\eta}$ are the generalized Wolfenstein parameters [58, 98]. Extensive analyses of the unitarity triangle have been performed for many years by CKMfitter [99] and UTfit [100] collaborations, and have recently been performed by the SCAN-method collaboration [101].

Under the assumption made above, this determination would give us the values of the elements of the CKM matrix without NP pollution. From the present perspective, the most important elements are the determinations of $|V_{ub}|$ and $\gamma$ because, as seen in table 1, they are presently not as well known as $|V_{cb}|$ and $|V_{us}|$. In this table we give other, most recent values of the relevant parameters, to which we will return in the course of our review.

Looking at table 1 we are able to make the following observations:

- The element $|V_{us}|$ is already well measured.
- The accuracy of the determination of $|V_{ub}|$ is quite good but the discrepancy between the inclusive and exclusive determinations is disturbing [116, 117], with the exclusive ones being visibly smaller [118]. We quote here also the average value provided by PDG. It should be recalled that the knowledge of this CKM matrix element is very important for rare decays and CP violation in the K-meson system. Indeed, $\varepsilon_K, B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ are all roughly proportional to $|V_{ub}|^4$ and even a respectable accuracy of 2% in $|V_{ub}|$ translates into 8% parametric uncertainty in these observables. This is in particular disturbing for $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ because these branching ratios are practically independent of any theoretical uncertainties.
- Future $B$-facilities accompanied by improved theory should be able to determine $|V_{cb}|$ with a precision of 1–2%.
- The case of $|V_{ub}|$ is more disturbing, with central values from inclusive determinations being by roughly 25% higher than the corresponding value resulting from exclusive semi-leptonic decays. We will see below that, depending on which of these values is assumed, different conclusions on the properties of NP responsible for certain anomalies seen in the data will be reached. Again, future $B$-facilities accompanied by improved theory should be able to determine $|V_{ub}|$ with a precision of 1–2%.
- Finally, the only physical CP phase in the CKM matrix, $\gamma$, is still poorly known from tree-level decays. But LHCb should be able to determine this angle with an error of a few degrees, which would be a great achievement. Further improvements could come from SuperKEKB.

At this point we would like to mention that we reserve the term tree-level determination of $\gamma$ for its extraction from pure tree-level decays, such as $B \rightarrow D K$. But it is known that $\gamma$ can also be determined from the asymmetries $S_{\pi\pi}$ and $S_{\phi K_S}$, because in this determination the NP phase in $B_d^0 - B_d^0$ mixing discussed in step 3 cancels out.

The importance of precise determinations of $|V_{cb}|$, $|V_{ub}|$, and $\gamma$ should not be underestimated. Table 3 and figure 2
in [119] showing SM predictions for various combinations of $|V_{ub}|$ and $|V_{cb}|$ demonstrate this very clearly. Therefore, the consequences of reaching our first goal would be profound. Indeed, precisely determining the four parameters of the CKM matrix without influence from NP will allow us to reconstruct all of its elements. In turn, they could be used efficiently in the calculation of the SM predictions for all decays and, in particular, FCNC processes, both CP-conserving and CP-violating. Moreover, this would allow to calculate not only an important element $|V_{cb}|$ but also its phase $-\beta$, with $\beta$ denoting another, very important, angle of the unitarity triangle in figure 4.

In order to be prepared for these developments, we collect here the most important formulae related to the unitarity triangle and CKM matrix. The phases of $V_{td}$ and $V_{ts}$ are defined by

$$V_{td} = |V_{td}|e^{-i\delta}, \quad V_{ts} = -|V_{ts}|e^{-i\delta}. \quad (28)$$

Next, the lengths $C_A$ and $B_A$ in the unitarity triangle are given, respectively, by

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\theta}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} |V_{ub}|, \quad (29)$$

$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1 - \bar{\theta})^2 + \bar{\eta}^2} = \frac{1}{\lambda} |V_{td}| \quad (30)$$

An important very accurate relation is

$$\sin 2\beta = \frac{2 \bar{\eta}(1 - \bar{\theta})}{R_t^2}. \quad (31)$$

We also note that the knowledge of $(R_b, \gamma)$ from tree-level decays gives

$$|V_{id}| = |V_{ai}||V_{cb}|R_t, \quad R_t = \sqrt{1 + R_b^2 - 2R_b \cos \gamma}, \quad \cot \beta = \frac{1 - R_b \cos \gamma}{R_b \sin \gamma}. \quad (32)$$

Similarly, the knowledge of $(R_t, \beta)$ allows us to determine $(R_b, \gamma)$ through

$$R_b = \sqrt{1 + R_t^2 - 2R_t \cos \beta}, \quad \cot \gamma = \frac{1 - R_t \cos \beta}{R_t \sin \beta} \quad (33)$$

and, consequently, with known $\lambda = |V_{ai}|$ and $|V_{cb}|$ one finds $|V_{td}|$ by means of (29). Similarly, $V_{ts}$ can be calculated. $|V_{ts}|$ is slightly below $|V_{cb}|$ but in the flavour precision era it is better to calculate its value numerically by using the standard parametrization. Then, one also finds that the value of $\beta_t$ is tiny: $\beta_t \approx -\gamma$.

There is still another powerful route to the determination of the Unitarity Triangle. As pointed out in [120], in addition to the determination of UT without any NP pollution through the determination of $(R_b, \gamma)$, in models with CMFV and MFV in which NP is absent in $S_{\psi K_S}$, the determination can proceed through $(\beta, \gamma)$; then,

$$R_t = \frac{\sin \gamma}{\sin(\beta + \gamma)}, \quad R_b = \frac{\sin \beta}{\sin(\beta + \gamma)}. \quad (34)$$

In fact as demonstrated in [120] $(R_b, \gamma)$ and $(\beta, \gamma)$ are the two most powerful ways to determine UT in the sense that the accuracy on these two pairs does not have to be very high in order to determine $(\bar{\theta}, \bar{\eta})$ with good precision. But, as we have seen, $|V_{ub}|$ is not very well known and, even if there are hopes to determine it within few % in the second half of this
decade, it is more probable that the $\gamma$ from tree-level decays will be known with this accuracy first and the $\beta, \gamma$ strategy will be leading in getting ($\tilde{q}, \tilde{q}$) within CMFV and MFV models.

The values of $|V_{ud}|$ and $|V_{ts}|$ are crucial for the predictions of various rare decays but, particularly, for the mass differences $\Delta M_d$ and $\Delta M_s$ and the phases $\beta$ and $\beta$, for the corresponding mixing induced CP-asymmetries $S_{\phi K}$, and $S_{\phi\phi}$, which are defined within the SM in (52). Also, the CP-violating parameter $\varepsilon_K$ depends crucially on $V_{td}$ and $V_{ts}$.

Before making some statements about the present status of the first five super stars of flavour physics

$$\Delta M_d, \Delta M_s, S_{\phi K}, S_{\phi\phi}, \varepsilon_K \text{ (35)}$$

within the SM, we have to make the second very important step.

5.2. Step 2: improved lattice calculations of hadronic parameters

Precise knowledge of the meson decay constants $F_{Ru}$, $F_{Rd}$, $F_{R}$, and of various non-perturbative parameters $B_i$ related to hadronic matrix elements of SM operators and operators found in the extensions of the SM is very important. Indeed, this would allow us in conjunction with step 1 to perform precise calculations of $\Delta M_d$, $\Delta M_s$, $\varepsilon_K$, $B(B_{d,s} \rightarrow \mu^+\mu^-)$, $B(B^+ \rightarrow \tau^+\nu_{\tau})$ and of other observables in the SM. We could then directly see whether or not the SM is capable of describing these observables. The recent unquenched lattice calculations allow for optimism and in fact a very significant progress in the calculation of $B_K$, that is relevant for $\varepsilon_K$, has recently been made. In addition, the weak decay constants $F_{Ru}$, $F_{Rd}$, and $F_{R}$, and some non-perturbative $B_i$ parameters are much better known than they were a few years ago.

In table 1 we collect the most relevant non-perturbative parameters for $\Delta F = 2$ observables that we extracted from the most recent FLAG average [104]. It should be remarked that these values are consistent with those presented in [105, 107] but generally have larger errors since FLAG prefers to be conservative. In particular, in the latter two papers one finds:

$$F_{Ru} \sqrt{B_{Ru}} = 279(13) \text{ MeV}, \quad F_{Rd} \sqrt{B_{Rd}} = 226(13) \text{ MeV}, \quad \xi = 1.237(32), \quad (36)$$

$$F_{R} = 225(3) \text{ MeV}, \quad F_{K} = 188(4) \text{ MeV}, \quad (37)$$

which contain smaller errors than quoted in [104].

We should also mention the recent results from the Twisted Mass Collaboration [62]

$$\sqrt{B_{Ru} F_{Ru}} = 262(10) \text{ MeV}, \quad \sqrt{B_{Rd} F_{Rd}} = 216(10) \text{ MeV}, \quad (38)$$

which are not yet included in the FLAG average but having smaller errors are consistent with the latter.

Evidently, there has been a lot of progress in determining all of these relevant parameters but one would like to decrease the errors further and it appears that this should be possible in the coming years. Selected reviews about the status and prospects can be found in [62, 121–125].

5.3. Step 3: $\Delta F = 2$ observables

5.3.1. Contributing operators. In order to describe these processes in generality we begin by listing the operators that can contribute to $\Delta F = 2$ observables in any extension of the SM. Specifying to the $K^0 - \bar{K}^0$ system, the full basis is given as follows [93, 126]:

$$Q_1^{VLL} = (\bar{s}_y \gamma\mu P_L \bar{d})(\bar{s}_y\gamma\mu P_L d), \quad (39a)$$

$$Q_1^{VRR} = (\bar{s}_y \gamma\mu P_R \bar{d})(\bar{s}_y\gamma\mu P_R d), \quad (39b)$$

$$Q_1^{LR} = (\bar{s}_y \gamma\mu P_L \bar{d})(\bar{s}_y\gamma\mu P_R d), \quad (39c)$$

$$Q_2^{SLL} = (\bar{s}_\mu P_L \bar{d})(\bar{s}_\mu P_R d), \quad (40a)$$

$$Q_2^{SRR} = (\bar{s}_\mu P_R \bar{d})(\bar{s}_\mu P_R d), \quad (40b)$$

$$Q_2^{SL} = (\bar{s}_\mu P_L \bar{d})(\bar{s}_\mu P_L d), \quad (40c)$$

$$Q_2^{SR} = (\bar{s}_\mu P_R \bar{d})(\bar{s}_\mu P_R d), \quad (40d)$$

where $P_{R,L} = (1 \pm \gamma_5)/2$ and we suppressed colour indices as they are summed up in each factor. For instance, $\bar{s}_y \gamma\mu P_L d$ stands for $\bar{s}_y\gamma\mu P_L d_1$ and similarly for other factors. For $B^0_q - \bar{B}^0_q$ mixing our conventions for operators are:

$$Q_1^{VLL} = (\bar{b}_y \gamma\mu P_L q)(\bar{b}_y\gamma\mu P_L q), \quad (41a)$$

$$Q_1^{VRR} = (\bar{b}_y \gamma\mu P_R q)(\bar{b}_y\gamma\mu P_R q), \quad (41b)$$

$$Q_1^{LR} = (\bar{b}_y \gamma\mu P_L q)(\bar{b}_y\gamma\mu P_R q), \quad (41c)$$

$$Q_2^{SLL} = (\bar{b}_\mu P_L q)(\bar{b}_\mu P_R q), \quad (42a)$$

$$Q_2^{SRR} = (\bar{b}_\mu P_R q)(\bar{b}_\mu P_R q), \quad (42b)$$

$$Q_2^{SL} = (\bar{b}_\mu P_L q)(\bar{b}_\mu P_L q), \quad (42c)$$

$$Q_2^{SR} = (\bar{b}_\mu P_R q)(\bar{b}_\mu P_R q), \quad (42d)$$

As already mentioned in step 2, the main theoretical uncertainties in $\Delta F = 2$ transitions reside in the hadronic matrix elements of the contributing operators. These matrix elements are usually evaluated by lattice QCD at scales corresponding roughly to the scale of decaying hadron, although in the case of $K$ meson decays, in order to improve the matching with the Wilson coefficients, the lattice calculations are performed these days at scales of $\mu \approx 2 \text{ GeV}$. However, for the study of NP contributions it is useful, starting from their values at these low scales, to evaluate them at scales where NP is at work. This can be done by means of RG methods. The corresponding analytic formulae to achieve this goal can be found in [126].

The most recent values of the matrix elements of the operators at a high scale $\mu = 1 \text{ TeV}$ are given in table 2. The matrix elements of operators with L replaced by R are equal to those given in this table. The values in table 2 correspond to the MS-NDR scheme and they are based on lattice calculations in [127, 128] for $K^0 - \bar{K}^0$ system and in [129] for $B^0_{d,s} - \bar{B}^0_{d,s}$ systems. For the $K^0 - \bar{K}^0$ system we have simply used the average of the results in [127, 128] that are consistent with each
Table 2. Hadronic matrix elements $\langle Q_i^a(\mu_H) \rangle$ in units of $\text{GeV}^3$ at $\mu_H = 1\text{ TeV}$. Reproduced with permission from [44]. Copyright 2013 SISSA.

| $Q_i^{LR}(\mu_H)$ | $Q_i^{LR}(\mu_H)$ | $Q_i^{QLL}(\mu_H)$ | $Q_i^{QSL}(\mu_H)$ |
|-------------------|-------------------|-------------------|-------------------|
| $K^0 - \bar{K}^0$ | $-0.14$ | $0.22$ | $-0.074$ | $-0.128$ |
| $B_s^0 - \bar{B}_s^0$ | $-0.12$ | $0.34$ | $-0.11$ | $-0.22$ |
| $B_s^0 - \bar{B}_s^0$ | $-0.37$ | $0.51$ | $-0.17$ | $-0.33$ |

Table 3. Hadronic matrix elements $\langle Q_i^a(\mu) \rangle$ in units of $\text{GeV}^3$ at $m_t(m_t)$. Reproduced with permission from [44]. Copyright 2013 SISSA.

| $Q_i^{LR}(\mu)$ | $Q_i^{LR}(\mu)$ | $Q_i^{QLL}(\mu)$ | $Q_i^{QSL}(\mu)$ |
|-----------------|-----------------|-----------------|-----------------|
| $K^0 - \bar{K}^0$ | $-0.11$ | $0.18$ | $-0.064$ | $-0.107$ |
| $B_s^0 - \bar{B}_s^0$ | $-0.21$ | $0.27$ | $-0.095$ | $-0.191$ |
| $B_s^0 - \bar{B}_s^0$ | $-0.30$ | $0.40$ | $-0.14$ | $-0.29$ |

other3. Since the values of the relevant $B_i$ parameters in these papers have been evaluated at $\mu = 3\text{ GeV}$ and $4.2\text{ GeV}$, respectively, we have used the formulae in [126] to obtain the values of the matrix elements in question at $\mu_H$. For simplicity we choose this scale to be $\mu_H$ but any scale of order $O(1)$ would give the same results for the physical quantities up to NNLO QCD corrections that are negligible at these high scales. The renormalization scheme dependence of the matrix elements is canceled by the one of the Wilson coefficients, as discussed below.

In the case of tree-level SM $Z$ and SM Higgs exchanges we evaluate the matrix elements at $m_t(m_t)$ since the inclusion of NLO QCD corrections allows us to choose any scale of $O(M_H)$ without changing the physical results. The values of hadronic matrix elements at $m_t(m_t)$ in the MS-NDR scheme are given in table 3.

The Wilson coefficients of these operators depend on the SD properties of a given theory. They can be directly expressed in terms of the couplings $\Delta^{ij}R(\mu/Z)\Delta^{ij}R(\mu)H_0$ in the case of tree-level gauge boson and scalar exchanges. In models with the GIM mechanism at work they are given in terms of loop functions. Then, couplings $\Delta^{ij}_{L,R}(W^*)^2$ and $\Delta^{ij}_{L,R}(H^*)^2$ enter the game.

5.3.2. SM results. In the SM only the operator $Q_1^{QLL}$ contributes to each meson system. With the information gained in steps 1 and 2 at hand we are ready to calculate the SM values for the five super stars in (35). To this end, we recall the formulae for $\Delta M_{d,s}, S_{\psi K_s}, S_{\psi \phi}$, and $\varepsilon_K$.

Defining

$$\lambda_{\tau_i}^{(K)} = V_{\tau_i}^* V_{\tau_d}, \quad \lambda_{\tau_i}^{(d)} = V_{\tau_b}^* V_{\tau_d}, \quad \lambda_{\tau_i}^{(s)} = V_{\tau_b}^* V_{\tau_s},$$

we have first

$$\Delta M_s = \frac{G_F^2}{12\pi^2} M_W^2 m_{\bar{B}_s} |\lambda_s^{(s)}|^2 F_{\bar{B}_s}^2 \bar{B}_s m_{\bar{B}_s} S_0(x_s),$$

$$\Delta M_d = \frac{G_F^2}{12\pi^2} M_W^2 m_{\bar{B}_s} |\lambda_d^{(s)}|^2 F_{\bar{B}_s}^2 \bar{B}_s m_{\bar{B}_s} S_0(x_s).$$

which results from ($\phi = d, s$)

$$\langle M_{(t)}^a \rangle_{\text{SM}} = \frac{G_F^2}{12\pi^2} F_{\bar{B}_s}^2 \bar{B}_s m_{\bar{B}_s} M_W^2 \left[ \lambda_{\tau_i}^{(q)} \right]^2 \eta_B S_0(x_i)$$

and

$$\Delta M_q = 2 |M_q|^2.$$  

(46)

Here, $x_i = m_i^2/M_W^2$, $\eta_B = 0.55$ is a QCD factor and

$$S_0(x_i) = \frac{4 x_i - 11 x_i^2 + x_i^3}{2(1 - x_i)^2} - \frac{3 x_i^2 \log x_i}{2(1 - x_i)^3}$$

$$= 2.31 \left[ \frac{m_{\bar{B}_s}(m_t)}{163\text{ GeV}} \right]^{1.52}.$$  

(47)

We then find three useful formulae (|$V_{tb}$| = 1)

$$\Delta M_s = 17.7/\text{ps} \cdot \left[ \frac{1}{267\text{ MeV}} \right]^{2} \frac{S_0(x_s)}{2.31}$$

$$\times \left[ \frac{|V_{ts}|}{0.0402} \right]^2 \frac{\eta_B}{0.55}.$$  

(49)

$$\Delta M_d = 0.51/\text{ps} \cdot \left[ \frac{1}{218\text{ MeV}} \right]^{2} \frac{S_0(x_s)}{2.31}$$

$$\times \left[ \frac{|V_{td}|}{8.5 \times 10^{-3}} \right]^2 \frac{\eta_B}{0.55}.$$  

(50)

and

$$R_{\Delta M_s} = \frac{\Delta M_d}{\Delta M_s} = \frac{m_{\bar{B}_s} V_{\bar{B}_s}^* \bar{B}_s F_{\bar{B}_s}^2}{m_{\bar{B}_s} V_{\bar{B}_s}^* \bar{B}_s F_{\bar{B}_s}^2} \frac{|V_{td}|}{|V_{ts}|} = \frac{m_{\bar{B}_s} 1}{|V_{td}|} \frac{|V_{td}|}{|V_{ts}|}.$$  

(51)

The mixing induced CP asymmetries are given within the SM simply by

$$S_{\psi K_s} = \sin(2\beta), \quad S_{\psi \phi} = \sin(2|\beta|).$$  

(52)

They are the coefficients of $\sin(\Delta M_{d,t})$ and $\sin(\Delta M_{d,t})$ in the time dependent asymmetries in $B^0 \to \psi K_s$ and $B^0 \to \psi \phi$, respectively.

For the CP-violating parameter $\varepsilon_K$ we have

$$\varepsilon_K = \frac{\kappa \epsilon \epsilon^*}{\sqrt{2 \Delta (M_K)_{\text{exp}}} \left( \sin \left( M_{(t)} \right) \right)_{\text{SM}}}.$$  

(53)

where $\varphi_\epsilon = (43.51 \pm 0.05)^\circ$ and $\kappa = 0.94 \pm 0.02$ [34, 113] takes into account that $\varphi_\epsilon \neq \frac{\pi}{2}$ and includes long distance effects in $\Delta (\Gamma_{(t)})$ and $\Delta (M_{(t)})$. Moreover,

$$\langle M_{(t)}^a \rangle_{\text{SM}} = \frac{G_F^2}{12\pi^2} F_{\bar{B}_s}^2 \bar{B}_s m_{\bar{B}_s} M_W^2$$

$$\times \left[ \lambda_{\tau_i}^{(s)} S_0(x_i) + \lambda_{\tau_i}^{(s)} S_0(x_i) + 2 \lambda_{\tau_i} \eta_s S_0(x_i, x_i) \right],$$  

(54)

where $\eta_s$ are QCD factors given in table 1 and $S_0(x_i, x_i)$ can be found in [131].

In table 4 we summarize the results for $|\varepsilon_K|$, $B(R^* \to \tau\nu\bar{\tau})$, $\Delta M_{d,t}$, (sin $2\beta$)$_{\text{true}}$, $\Delta M_{d,t}$, $|V_{td}|$, and $|V_{ts}|$ obtained from (32), setting

$$|V_{td}| = 0.2252, \quad |V_{ts}| = 0.0409, \quad \gamma = 68^\circ.$$  

(55)
and choosing five values for $|V_{ub}|$. Two of them correspond to the two scenarios defined in section 2. The value of $\gamma$ is close to its most recent value from $B \to DK$ decays obtained by LHCb using 3 fb$^{-1}$ and neglecting $D^0 - \bar{D}^0$ mixing [132]

$$\gamma = (67.2 \pm 12)^\circ \quad \text{(LHCb)} \quad (56)$$

and to the extraction from U-spin analysis of $B_s \to K^+ K^-$ and $B_d \to \pi^+ \pi^-$ decays ($\gamma = (68.2 \pm 7.1)^\circ$) [133]. In [134] both $B \to DK$ and $B \to D\tau\nu$ decays are used and, furthermore, $D^0 - \bar{D}^0$ mixing is fully included and the combination of results gives a best-fit value $\gamma = 72.6^\circ$ and the confidence interval $\gamma \in [55.4, 82.3]^\circ$ at 68% CL. We do not show the uncertainties in SM predictions but just quote a rough estimate of them:

$$|s_K| : \pm 11\% \quad B(B^+ \to \tau^+\nu_\tau) : \pm 15\% \quad \Delta M_{s,d}: \pm 10\% \quad S_{\psi K_s} : \pm 3.0\% \quad (57)$$

In order to show the importance of precise values of the non-perturbative parameters, we show the results for present central values of $F_B \sqrt{\hat{B}_K}$ and $F_{B_s} \sqrt{\hat{B}_{B_s}}$ in table 1 (I) and for the older values in (36) indicated by (II).

We observe that while $\Delta M_{s,d}$, $|V_{td}|$ and $|V_{ts}|$, practically do not depend on $|V_{ub}|$, this is not the case for the remaining observables, although the $|V_{ub}|$ dependence in $S_{\psi K_s}$ is very weak. Clearly, the data shows that it is difficult to fit simultaneously $s_K$ and $S_{\psi K_s}$ within the SM but the character of the NP that could cure these tensions depends on the choice of $|V_{ub}|$. On the other hand, the agreement of the SM with the data on $\Delta M_s$ and $\Delta M_d$ is very good. In particular, for the set (I) we find that

$$\left( \frac{\Delta M_s}{\Delta M_d} \right)_{\text{SM}} = 34.1 \pm 3.0 \quad \exp : 34.7 \pm 0.3 \quad (58)$$

is in excellent agreement with the data.

We learn the following lessons to be remembered when we start investigating models beyond the SM:

**Lesson 1.** We learn that in the case of exclusive determination of $|V_{ub}|$ any NP model that pretends to be able to remove or soften the observed departures from the data should simultaneously:

- Enhance $|s_K|$ by roughly 20% without significantly affecting the result for $S_{\psi K_s}$.
- Suppress slightly $\Delta M_s$ and $\Delta M_d$ without significantly affecting their ratio in the case of the set (II). This suppression is not required if set (I) is used.

**Lesson 2.** We learn that in the case of inclusive determination of $|V_{ub}|$ any NP model that pretends to be able to remove or soften the observed departures from the data should simultaneously:

- Suppress $S_{\psi K_s}$ by roughly 20% without significantly affecting the result for $|s_K|$.
- Suppress slightly $\Delta M_s$ and $\Delta M_d$ without significantly affecting their ratio in the case of the set (II). This suppression is not required if set (I) is used.

Clearly, $|V_{ub}|$ could have an intermediate value but we find that a more transparent picture emerges for these two values.

**Lesson 3.** The next lesson comes from HQAG [112]:

$$S_{\psi} = -(0.04^{+0.10}_{-0.13}), \quad S_{\psi}^{\text{SM}} = 0.038 \pm 0.005, \quad (59)$$

where we have shown also SM prediction and the experimental error on $S_{\psi}$ has been obtained by adding the statistical and systematic errors in quadrature. Indeed, it looks like the SM still survived another test: mixing induced CP violation in $B_s$ decays is significantly lower than in $B_d$ decays as was already expected in the SM for the last 25 years. However, from the present perspective, $S_{\psi}$ could still be found in the range

$$-0.20 \leq S_{\psi} \leq 0.20 \quad (60)$$

and finding it to be negative would be a clear signal of NP. Moreover, finding it above 0.1 would also be a signal of NP but this would not be as pronounced as the negative value. The question then arises of whether this NP is somehow correlated with the one related to the anomalies identified above. We will return to this issue in the course of our presentation.

**Lesson 4.** The final lesson comes from the recent analysis in [47], where the values $|V_{ub}| = (42.4(9)) \times 10^{-3}$ [116] and $|V_{cb}| = (3.6 \pm 0.3) \times 10^{-3}$ [103] have been used. For these values there is an acceptable simultaneous agreement of the SM with both $S_{\psi K_s}$ and $s_K$ but then

$$\Delta M_s = 18.8 \text{ ps}^{-1}, \quad \Delta M_d = 0.530 \text{ ps}^{-1}, \quad (61)$$

is slightly above the data.
This discussion shows how important the determinations of the CKM and non-perturbative parameters are if we want to identify NP indirectly through flavour-violating processes. We will return to this point below and refer to [46, 47], where extensive numerical analysis of this issue has been presented in the context of models with tree-level FCNC transitions.

5.3.3. Going beyond the SM. In view of NP contributions that are required to remove the anomalies just discussed, we have to generalize the formulae of the SM. First, for $M_{12}^i$, $M_{12}^{i,\text{SM}}$, and $M_{12}^{i,\text{NP}}$, which govern the analysis of $\Delta F = 2$ transitions in any extension of the SM, we have

$$M_{12}^i = (M_{12}^i)_{\text{SM}} + (M_{12}^i)_{\text{NP}} \quad (i = K, d, s),$$

with $(M_{12}^i)_{\text{SM}}$ given in (46) and (54).

For the mass differences in the $B^0_{d,s} - \bar{B}^0_{d,s}$ systems we then have

$$\Delta M_q = 2 |(M_{12}^q)_{\text{SM}} + (M_{12}^q)_{\text{NP}}| \quad (q = d, s).$$

Now

$$M_{12}^q = (M_{12}^q)_{\text{SM}} + (M_{12}^q)_{\text{NP}} = (M_{12}^q)_{\text{SM}} C_B e^{2i\beta_s},$$

where

$$(M_{12}^q)_{\text{SM}} = |(M_{12}^q)_{\text{SM}}| e^{i\beta_s}, \quad (M_{12}^q)_{\text{SM}} = |(M_{12}^q)_{\text{SM}}| e^{i\beta_s}.$$}

The phases $\beta$ and $\beta_s$ are defined in (28), and one has approximately $\beta \approx (22 \pm 3)^\circ$ and $\beta_s \approx -1^\circ$ with precise values depending on $|V_{ub}|$. We then find

$$\Delta M_q = (\Delta M_q)_{\text{SM}} C_B,$$

and

$$S_{\psi K} = \sin(2\beta + 2\psi_B), \quad S_{\psi \phi} = \sin(2|\beta|) - 2|\psi_B|.$$}

Thus, in the presence of non-vanishing $\psi_B$ and $\psi_B$, these two asymmetries do not measure $\beta$ and $\beta_s$, but $|\beta| + |\psi_B|$ and $|\beta| - |\psi_B|$, respectively.

It should be remarked that the experimental results are usually given for the phase

$$\phi_s = 2\beta_s + \phi^{\text{NP}},$$

so that

$$S_{\psi \phi} = -\sin(\phi_s), \quad 2\psi_B = \phi^{\text{NP}}.$$}

In particular, the minus sign in this equation should be remembered when comparing our results with those quoted by the LHCb.

Next, the parameter $\varepsilon_K$ is given by

$$\varepsilon_K = \frac{\kappa_K e^{i\epsilon}}{\sqrt{2}(\Delta M_K)_{\text{exp}}} \left[ |(M_{12}^K)_{\text{SM}}| + |(M_{12}^K)_{\text{NP}}| \right].$$

Finally, the ratio in (51) can be modified

$$R_{\Delta M_s} = \frac{\Delta M_\beta}{\Delta M_s} = \frac{m_\beta s}{m_B s} \left| \frac{\Delta M_s}{\Delta M_s} \right| r(\Delta M),$$

where the departure of $r(\Delta M)$ from unity signals non-MFV sources at work. In this review we only rarely consider $\Delta M_K$ because it is subject to large hadronic uncertainties. Moreover, generally $\varepsilon_K$ gives a stronger constraint on NP.

We will now investigate which of the models introduced in section 3 could remove the anomalies just discussed, depending on whether an exclusive or inclusive value of $|V_{ub}|$ has been chosen by nature and which models are put under significant pressure in both cases. In the latter case, the hope is that the final value for $|V_{ub}|$ will be some average of inclusive and exclusive determinations; that is, in the ballpark of $|V_{ub}| = 3.7 \times 10^{-3}$. If this turns out not to be the case then the latter models are either close to being ruled out or are incomplete, requiring new sources of flavour and/or CP violation in order to agree with the data. As we will soon see, the simplest models considered by us have a sufficiently low number of parameters so that concrete answers about their ability to remove the anomalies in question can be given, particularly when subsequent steps will be considered.

5.3.4. Constrained minimal flavour violation. The flavour structure in this class of model implies that the mixing induced CP asymmetries $S_{\psi K}$, and $S_{\psi \phi}$ are not modified with respect to the SM and the expressions in (52) still apply.

This structure also implies the flavour universality of loop functions contributing to various processes. In the case of the $\Delta F = 2$ processes considered here, this means that in this class of models NP can only modify the loop function $S_0(x_t)$ to some real valued function $S(v)$ which is shown to be of the form

$$S_0(x_t) \leq S(v).$$

This simply implies that $|\varepsilon_K|$, $\Delta M_d$ and $\Delta M_s$, can only be enhanced in this class of model. Moreover, this happens in a correlated manner. A correlation between $|\varepsilon_K|$, $\Delta M_d$, and $S_{\psi K}$, within the SM has been pointed out in [58, 59] and generalized to all models with CMFV in [55]. This correlation follows from the universality of $S(v)$ and the fact that in all of the CMFV models considered, only the term in $\varepsilon_K$ involving $(V_{us}^* V_{td})^2$ is affected visibly by NP, with the remaining terms described by the SM.

Here, we want to look at this correlation from a different point of view. In fact, by eliminating the one-loop function $S(v)$ in $\varepsilon_K$ in favour of $\Delta M_d$ and using also $\Delta M_s$, one can find universal expressions for $S_{\psi K}$, and the angle $\gamma$ in the UUT that depend only on $|V_{us}|$, $|V_{cb}|$, known from tree-level decays, and non-perturbative parameters entering the evaluation of $\varepsilon_K$ and $\Delta M_{d,s}$. These are valid for all of the CMFV models. Therefore, once the data on $|V_{us}|$, $|V_{cb}|$, $\varepsilon_K$ and $\Delta M_{d,s}$ are taken into account, one is able in this framework to predict not only $S_{\psi K}$, $S_{\psi \phi}$ and $\gamma$, but also $|V_{ub}|$.

Explicitly, we first find

$$S_{\psi K} = \sin 2\beta = \frac{1}{b \Delta M_d} \left[ |\varepsilon_K| + B_K - a \right].$$

where

$$\Delta M_d = \left| \frac{m_\beta s}{m_B s} \right| R_{\Delta M_s} r(\Delta M),$$

and

$$\Delta M_s = \frac{m_\beta s}{m_B s} \left| \frac{\Delta M_s}{\Delta M_s} \right| r(\Delta M).$$

with

$$\Delta M_d = \frac{m_\beta s}{m_B s} \left| \frac{\Delta M_s}{\Delta M_s} \right| r(\Delta M),$$

and

$$\Delta M_s = \frac{m_\beta s}{m_B s} \left| \frac{\Delta M_s}{\Delta M_s} \right| r(\Delta M).$$
where 
\[ a = r_t R_t \sin \beta [\eta_{\ell \ell} S_0(x, x) - \eta_{cc} x_c], \]
\[ b = \frac{r_s}{\eta_B} \frac{1}{2 r_d |V_{us}|^2 F_{B_d}^2 B_{B_s}}, \]  
with
\[ r_e = \kappa_e |V_{us}|^2 \frac{G_1^2 F_0^2 m_K M_{B_s}^2}{6 \sqrt{2} \pi^2 \Delta M_K}, \]
\[ r_d = \frac{G_1^2}{6 \pi^2} M_{B_s}^2 m_{Bs}. \]

The following remarks should be made:

- The second term \( \phi \) in the parenthesis in (73) is roughly by a factor of 4\-5 smaller than the first term. This depends on \( \beta \) through \( \sin \beta \) and \( \lambda = |V_{us}| \)

\[ R_t = \eta_R \left( \frac{\Delta M_d}{\Delta M_f} \right)^{\frac{m_{B_s}}{m_{B_t}}} \]
\[ \eta_R = 1 - |V_{us}|^2 \left( \frac{\Delta M_d}{\Delta M_f} \right)^{\frac{m_{B_s}}{m_{B_t}}} \cos \beta + \frac{\lambda^2}{2} + O(\lambda^4), \]

but this dependence is very weak and 0.34 \( \leq \alpha \leq 0.41 \) in the full range of parameters considered.

- The ratio of \( \eta_{\ell \ell}/\eta_B \) is independent of NP.

- With \( R_t \) and \( \beta \) determined in this manner one can calculate \( \gamma \) and \( |V_{ub}| \) by means of (29) and (33).

- The element \( |V_{ub}| \) appears only as square in these expressions and not as \( |V_{ub}|^4 \) in \( \epsilon_K \), which improves the accuracy of the determination.

We should emphasize that in this determination the experimental input \( \Delta M_{s,f} \) and \( \epsilon_K \) is very precise. \( |V_{us}| \) is very well known and \( |V_{cb}| \) is better known than \( |V_{ub}| \) from tree-level decays.

By setting the experimental values of \( \Delta M_{s,d}, \epsilon_K \) and \( |V_{cb}| \), as well as central values of the non-perturbative parameters in table 1 into (73), we find
\[ S_{\epsilon_K} = 0.81 (0.87) \Rightarrow \beta = 27 (30^\circ) , \quad R_t = 0.92 (0.92) \]  
(77)
and thus
\[ R_b = 0.46 (0.50) , \quad |V_{ub}| = 0.0043 (0.0047) , \quad \gamma = 67.2 (66.4^\circ) , \]  
(78)
where the values in parentheses correspond to the input in (36). This demonstrates sensitivity to the non-perturbative parameters.

While a sophisticated analysis including all of the uncertainties would somewhat wash out these results, the message from this exercise is clear. The fact that \( S_{\epsilon-K} \) is much larger than the data requires the presence of new CP-violating phases, although with the most recent lattice input these phases can be smaller. This exercise is equivalent to the one performed in [33], where \( \epsilon_K \) has been set to its experimental value but \( \sin 2\beta \) was free. On the other hand, by setting \( S_{\epsilon-K} \) to its experimental value but keeping \( \epsilon_K \) free, as done in [34], one finds that \( |\epsilon_K| \) is significantly below the data. Yet, this difficulty can be resolved in CMFV models by increasing the value of \( S(v) \). While, the latter approach is clearly legitimate, it hides possible problems of CMFV since it assumes that this NP scenario can describe the data on \( \Delta M_{s,d} \) and \( \epsilon_K \) simultaneously, which as we will now show is not really the case.

Indeed, with respect to the anomalies discussed above, we note that

- CMFV models favour the exclusive determination of \( |V_{ub}| \) because only then are they capable of reproducing the experimental value of \( S_{\epsilon-K} \).

- \( |\epsilon_K| \) can be naturally enhanced by increasing the value of \( S(v) \), thereby, solving the \( |\epsilon_K| \)-\( S_{\epsilon-K} \) tension.

- \( \Delta M_{s,d} \) are enhanced simultaneously with the ratio \( \Delta M_s/\Delta M_d \) unchanged with respect to the SM (\( r(\Delta M) = 1 \)). While the latter property is certainly good news, the enhancements of \( \Delta M_s \) and \( \Delta M_d \) are clearly problematic. Therefore, the present values of hadronic matrix elements imply new tensions, namely the \( |\epsilon_K| \)-\( \Delta M_{s,d} \) tensions pointed out in [20, 135].

In figure 5 we plot \( \Delta M_s \) and \( \Delta M_d \) as functions of \( |\epsilon_K| \).

In obtaining this plot we have simply varied the master one-loop \( \Delta F = 2 \) function \( S \) while keeping the CKM parameters and other input parameters fixed. The value of \( S \) at which central experimental value of \( |\epsilon_K| \) is reproduced turns out to be \( S = 2.9 \), which is to be compared with \( S_{SM} = 2.31 \). At this value the central values of \( \Delta M_{s,d} \) read
\[ \Delta M_d = 0.64 (6) \text{ ps}^{-1} \quad (0.69 (6) \text{ ps}^{-1}) , \]
\[ \Delta M_s = 21.7 (2.1) \text{ ps}^{-1} \quad (23.9 (2.1) \text{ ps}^{-1}) . \]

They both differ from experimental values by 3\( \sigma \). However, the error on \( |\epsilon_K| \) coming dominantly from the error of \( |V_{ub}| \) and the error of the QCD factor \( \eta_{\ell \ell} \) in the charm contribution [114] is disturbing. Clearly this plot gives only some indication for possible difficulties of the CMFV models and we need...
Table 5. CMFV predictions for various quantities as functions of $S(\nu)$ and $\gamma$. The four elements of the CKM matrix are in units of $10^{-3}$, $F_{B_S} \sqrt{B_{B_S}}$ and $F_{B_d} \sqrt{B_{B_d}}$ in units of MeV and $B(B^+ \to \tau^+\nu)$ in units of $10^{-6}$. Reproduced with permission from [45] under a CC BY 4.0 license.

| $S(\nu)$ | $\gamma$ | $|V_{cb}|$ | $|V_{ub}|$ | $|V_{td}|$ | $|V_{ts}|$ | $F_{B_S} \sqrt{B_{B_S}}$ | $F_{B_d} \sqrt{B_{B_d}}$ | $\xi$ | $B(B^+ \to \tau^+\nu)$ |
|-------------------|--------|---------|---------|---------|---------|----------------|----------------|--------|----------------|
| 2.31              | 63°    | 43.6    | 3.69    | 8.79    | 42.8    | 252.7          | 210.0          | 1.204  | 0.822          |
| 2.5               | 63°    | 42.8    | 3.63    | 8.64    | 42.1    | 247.1          | 205.3          | 1.204  | 0.794          |
| 2.7               | 63°    | 42.1    | 3.56    | 8.49    | 41.4    | 241.8          | 200.9          | 1.204  | 0.768          |
| 2.31              | 67°    | 42.9    | 3.62    | 8.90    | 42.1    | 256.8          | 207.2          | 1.240  | 0.791          |
| 2.5               | 67°    | 42.2    | 3.56    | 8.75    | 41.4    | 251.1          | 202.6          | 1.240  | 0.765          |
| 2.7               | 67°    | 41.5    | 3.50    | 8.61    | 40.7    | 245.7          | 198.3          | 1.240  | 0.739          |
| 2.31              | 71°    | 42.3    | 3.57    | 9.02    | 41.5    | 260.8          | 204.5          | 1.276  | 0.770          |
| 2.5               | 71°    | 41.6    | 3.51    | 8.87    | 40.8    | 255.1          | 200.0          | 1.276  | 0.744          |
| 2.7               | 71°    | 40.9    | 3.45    | 8.72    | 40.1    | 249.6          | 195.7          | 1.276  | 0.719          |

a significant decrease of theoretical errors in order to see how solid this result is.

In summary, we observe that simultaneous good agreement for $\varepsilon_K$ and $\Delta M_{K,d}$ with the data is difficult to achieve in this NP scenario. It also implies that to improve the agreement with the data we need at least one of the following four ingredients:

- **Modification of the values of** $|V_{cb}|$, $F_{B_S} \sqrt{B_{B_S}}$, $F_{B_d} \sqrt{B_{B_d}}$. \(80\)
- **New CP phases**, flavour violating and/or flavour blind,
- **New flavour-violating contributions beyond the CKM matrix**, and
- **New local operators**, which could originate in tree-level heavy gauge boson or scalar exchanges: they could also be generated at one-loop level.

The first possibility has been addressed in [45], where the experimental values of $\Delta M_{K,d}$, $\varepsilon_K$, $|V_{us}|$ and $S_{\phi K_S}$ have been used as input and $\tilde{B}_K$ has been set to 0.75, which is in perfect agreement with the lattice results and the large $N$ approach [136–139]. Subsequently, the parameters in \(80\) have been calculated as functions of $S(\nu)$ and $\gamma$ in order to see whether there is any hope for removing all of the tensions in CMFV simultaneously in case in the future more precise determinations of $F_{B_S} \sqrt{B_{B_S}}$, $F_{B_d} \sqrt{B_{B_d}}$ and $|V_{cb}|$ would result in different values than the present ones. The results of [45] are summarized in table 5 and further details can be found in that paper.

Finally, we note that the most recent values of $F_{B_S} \sqrt{B_{B_S}}$ and $F_{B_d} \sqrt{B_{B_d}}$ have significantly softened the problems of the CMFV in question, even if an enhanced value of $|V_{cb}|$ is still required. For instance, in accordance with lesson 4 above, if one would ignore the present exclusive determination of $|V_{cb}|$ and use the most recent inclusive determination [116]

$$|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3}.$$ \(81\)

CMFV would be in a much better shape and the SM-like values for $S(\nu)$ would also be favoured. We are looking forward to the improved lattice calculations and improved determinations of $|V_{cb}|$ in order to see whether CMFV will survive the flavour precision tests.

5.3.5. 2HDM with MFV and FBPs (2HDM\textsubscript{MFV}). In view of our discussion above, this model [84] has in principle a better chance to simultaneously remove the anomalies in question than CMFV models but as we will soon see it approaches this problem in a different manner. The basic new features in 2HDM\textsubscript{MFV} relative to CMFV are:

- The presence of FBPs in this MFV framework through their interplay with the standard CKM flavour violation modifies the usual characteristic relations of the CMFV framework. In particular, the mixing induced CP asymmetries $S_{\phi K_S}$ and $S_{\phi \phi}$ take the form known from non-MFV frameworks, such as LHT, RSc and SM4 as given in (67).
- The FBPs in the 2HDM\textsubscript{MFV} can appear both in Yukawa interactions and in the Higgs potential. While in [84] only the case of FBPs in Yukawa interactions has been considered, in [140] these considerations have been extended to include also the FBPs in the Higgs potential. The two flavour-blind CPV mechanisms can be distinguished through the correlation between $S_{\phi K_S}$ and $S_{\phi \phi}$, which is strikingly different if only one of them is relevant. In fact, the relation between generated new phases are very different in each case:

$$\varphi_{B_S} = \frac{m_q}{m_S} \varphi_{B_S} \quad \text{and} \quad \varphi_{B_d} = \varphi_{B_S}.$$ \(82\)

for FBPs in Yukawa couplings and Higgs potential, respectively.
- New local operators are generated through the contributions of tree-level heavy Higgs exchanges, which also implies a modified structure of flavour violation relative to CMFV. Sizable FBPs, which are necessary to explain possible sizable non-standard CPV effects in $B_s$ mixing, could, in principle, be forbidden by the upper bounds on EDMs of the neutron and the atoms. This question has been addressed in [140] and it has been shown that, even for $S_{\phi \phi} = O(1)$, this model still satisfied these bounds.

It is not our goal to describe the phenomenology of this model here in detail because such details can be found in
Moreover, a review appeared in [73]. In contrast, we want to emphasize that the model addresses the anomalies in question in a manner which differs profoundly from CMFV and, thus, a distinction between these two models can already be made on the basis of the data on $\Delta F = 2$ processes.

Indeed, in this model new contributions to $\varepsilon_K$ originating in tree-level neutral Higgs exchanges are being suppressed by small quark masses $m_{q,d}$. Consequently, the correct value of $\varepsilon_K$ can only be obtained by choosing a sufficiently large value of $\sin 2\beta$, which corresponds to the large (inclusive) $|V_{ub}|$ scenario. If the formula (52) is used then this in turn implies, as seen in table 4, a value of $S_{\phi K_s}$ which is much larger than the data. However, in this model the interplay of the CKM phase with the FBPs in Yukawa couplings and Higgs potential generates non-vanishing new phases $\psi_B$, and the formulae in (67) instead of (52) should be used. The new phases can suppress $S_{\phi K_s}$, simultaneously uniquely enhancing the asymmetry $S_{\phi \phi}$.

Now, while the rate of the suppression of $S_{\phi K_s}$ for a given $S_{\phi \phi}$ is much stronger if significant FBPs in the Higgs potential rather than in Yukawa couplings are at work, both mechanism share a very important property:

- The necessary suppression of $S_{\phi K_s}$ necessarily and uniquely implies the enhancement of $S_{\phi \phi}$ so that this asymmetry is larger than in the SM and consequently has positive sign. Finding eventually $S_{\phi \phi}$ at the LHC to be negative would be a real problem for the 2HDM$_{\text{MFV}}$.

Now, $\varepsilon_K$ can only be made consistent in this model by properly choosing $\gamma$ and, in particular, a $|V_{ub}|$ that has to be sufficiently large. The question then arises of whether $S_{\phi K_s}$, $S_{\phi \phi}$ and $\Delta M_{d, t}$ can simultaneously be made consistent with the data. We find then [141]:

- The removal of the $\varepsilon_K - S_{\phi K_s}$ anomaly, which proceeds through the negative phase $\psi_B$, is only possible with the help of FBPs in the Higgs potential. This is achieved in the case of the full dominance of the $Q_{1/2}^{\text{LL}}$ operators as far as CP-violating contributions are concerned. If these operators also dominate the CP-conserving contributions then two important properties follow:

$$\psi_{B_1} = \psi_{B_2}, \quad C_{B_1} = C_{B_2}. \quad (83)$$

The second of the equalities implies

$$\left( \frac{\Delta M_s}{\Delta M_d} \right)_{\text{2HDM}_{\text{MFV}}} = \left( \frac{\Delta M_s}{\Delta M_d} \right)_{\text{SM}}. \quad (84)$$

This relation is known from models with CMFV but there $C_{B_1} = C_{B_2} \geq 1$. In 2HDM$_{\text{MFV}}$, $C_{B_1} = C_{B_2} \leq 1$ is also possible. Moreover, the CMFV correlation between $\varepsilon_K$ and $\Delta M_{d, t}$ is absent, and $\Delta M_{d, t}$ can be both suppressed and enhanced if necessary.

- A significant contribution of the operators $Q_{1/2}^{\text{LR}}$ is unwanted because it spoils the relation (84), having a much larger effect on $\Delta M_s$ than $\Delta M_d$. But since this contribution uniquely suppresses $\Delta M_d$ below its SM value, it could turn out relevant one day if the lattice results for hadronic matrix are changed. This contribution cannot help in solving $\varepsilon_K - S_{\phi K_s}$ anomaly because its effect on the phase $\psi_{B_1}$ is very small.

Thus, at first sight, at the qualitative level this model provides a better description of $\Delta F = 2$ data than the SM and models with CMFV. Yet, there is a possible difficulty here. As shown in figure 6 the size of $\psi_B$, that is necessary to obtain simultaneously good agreement with the data on $\varepsilon_K$ and $S_{\phi K_s}$ implies, in turn, $S_{\phi \phi} \geq 0.15$, which is $2\sigma$ away from the LHCb central value in (59).

In summary, 2HDM$_{\text{MFV}}$ is from the point of view of $\Delta F = 2$ observables in a reasonable shape. Yet, finding in the future that nature chooses a negative value of $S_{\phi \phi}$ and/or (exclusive) value of $|V_{ub}|$ would practically rule out 2HDM$_{\text{MFV}}$. In addition, a decrease of the experimental error on $S_{\phi \phi}$ without the change of its central value would be problematic for this model.

We are looking forward to improved experimental data and improved lattice calculations to find out whether this simple model can satisfactorily describe the data on $\Delta F = 2$ observables.

5.3.6. Tree-level gauge boson exchanges. We will next investigate what a neutral gauge boson tree-level exchange can contribute to this discussion. For the neutral gauge boson $Z$ contribution, as shown in figure 7, one has generally [40, 93]

$$M_{12}^{(b)} = \frac{\langle Z \rangle^2}{2 M_Z^2} \frac{C_{1}^{\text{LL}}(\mu_Z)}{C_{1}^{\text{VLL}}(\mu_Z)} \langle Q_{1}^{\text{VLL}}(\mu_Z) \rangle$$

$$+ \frac{\langle Z \rangle^2}{2 M_Z^2} \frac{C_{1}^{\text{VRR}}(\mu_Z)}{C_{1}^{\text{VLL}}(\mu_Z)} \frac{C_{1}^{\text{L}}(\mu_Z)}{C_{1}^{\text{L}}(\mu_Z)} \frac{C_{1}^{\text{R}}(\mu_Z)}{C_{1}^{\text{R}}(\mu_Z)} \frac{C_{1}^{\text{L}}(\mu_Z)}{C_{1}^{\text{L}}(\mu_Z)} \frac{C_{1}^{\text{R}}(\mu_Z)}{C_{1}^{\text{R}}(\mu_Z)}$$

where including NLO QCD corrections [93]

$$C_{1}^{\text{VLL}}(\mu_Z) = C_{1}^{\text{VLL}}(\mu_Z) = 1 + \frac{\alpha_s}{4 \pi} \left( -2 \log \frac{M_Z^2}{\mu_Z^2} + \frac{11}{3} \right). \quad (86)$$

$$C_{1}^{\text{L}}(\mu_Z) = 1 + \frac{\alpha_s}{4 \pi} \left( - \log \frac{M_Z^2}{\mu_Z^2} - \frac{1}{6} \right). \quad (87)$$
The outcome for the phenomenology depends on whether Δ_{L} and Δ_{R} are of comparable size or if one of them is dominant and whether they are real or complex quantities. Moreover, these properties can be different for different meson systems. Evidently, we have in mind here the scenarios LHS, RHS, LRS and ALRS of section 2. Moreover, one has to distinguish between scenario 1 (S1) and scenario 2 (S2) for |V_{ub}|, so that generally one deals with LHS1, LHS2 and similarly for RHS, LRS and ALRS.

As expected with these new contributions, without any particular structure of the Δ_{L,R} couplings all tensions within the SM in the ΔF = 2 transitions can be removed in many ways and it will be important to investigate in the next steps which of them are also consistent with other constraints and which ones remove simultaneously other tensions, which are already present or will be generated when the data and lattice results improve in the future.

In concrete BSM models the couplings Δ_{L,R} corresponding to different meson systems, could be related to each other because they may depend on the same fundamental parameters of an underlying theory. For instance, in the minimal 3–3–1 model, which was analysed recently in [41, 47], the flavour-violating couplings Δ^{d}_{L}(Z), Δ^{d}_{R}(Z) and Δ^{b}_{L}(Z) depend on two mixing angles and two complex phases, instead of six parameters, which implies correlations between observables in different meson systems (see also section 6.5.1).

A very detailed analysis of B^{0}_{s} → B^{0}_{d} and K^{0} → 0 systems has been presented in [40], setting the CKM parameters as in (55) and all the other inputs at the central values in table 1, except that in [40] the input in (36) has been used. Since the latter values are consistent with the present ones, in order to take partially hadronic and experimental uncertainties into account we will still present here the results of [40]. Moreover, as in the latter paper, we require that values of observables in question satisfy the following constraints:

\[16.9/ps \leq \Delta M_{s} \leq 18.7/ps, \quad -0.20 \leq s_{\rho K} \leq 0.2,\]  
\[0.48/ps \leq \Delta M_{d} \leq 0.53/ps, \quad 0.64 \leq S_{\rho K} \leq 0.72,\]  
\[0.75 \leq \Delta M_{K} \leq 1.25,\]  
\[2.0 \times 10^{-3} \leq |s_{K}| \leq 2.5 \times 10^{-3}.\]

The larger uncertainty for s_{K} than ΔM_{s,d} signals its strong |V_{cb}| dependence. ΔM_{K} has even larger uncertainty because of potential long distance uncertainties. When using the constraint from S_{\rho K} and S_{\rho K}, we take into account that only mixing phases close to their SM value are allowed by the data, thereby removing some discrete ambiguities.

By parametrizing the different flavour-violating couplings of Z to quarks as follows

\[\Delta^{d}_{L}(Z) = -\bar{s}_{3} e^{-i\delta_{2}}, \quad \Delta^{d}_{R}(Z) = \bar{s}_{1} e^{-i\delta_{1}}, \quad \Delta^{d}_{L}(Z) = -\bar{s}_{12} e^{-i\delta_{2}}, \quad \Delta^{d}_{L}(Z) = -\bar{s}_{12} e^{-i\delta_{2}},\]  

which is possible to find the allowed regions in the spaces (s_{ij}, δ_{ij}) used to describe Z' effects in each system. The minus sign is introduced to cancel that in V_{ts}.

In the case of B^{0}_{s} → B^{0}_{d} system, the result of this search for M_{Z'} = 1 TeV and LHS1 scenario is shown in figure 8. The red regions correspond to the allowed ranges for ΔM_{s},
while the blue ones correspond to the ranges for $S_{\psi\phi}$. The overlap between red and blue regions (light blue and purple) identifies the ones we were looking for. We observe that the requirement of suppression of $\Delta M_s$ implies $\delta_{23} \neq 0$. Since this system is immune to the value of $|V_{ub}|$, the same results are obtained for LHS2.

We note that for each oasis with a given $\delta_{23}$ there is another oasis with $\delta_{23}$ shifted by 180° but the range for $\delta_{23}$ is unchanged. This discrete ambiguity results from the fact that $\Delta M_s$ and $S_{\psi\phi}$ are governed by $2\delta_{23}$. This ambiguity can be resolved by other observables, as discussed in the next steps. The colour coding for the allowed oases, blue and purple for oasis with small and large $\delta_{23}$, respectively, will be useful in this context.

The corresponding oases for $B^0_d-\bar{B}^0_d$ and $K^0-\bar{K}^0$ systems are shown in figures 9 and 10, respectively. We note that now the results depend on whether LHS1 or LHS2 is considered. Moreover, in accordance with the quality of the constraints in (93)–(95), the allowed oases in the $B^0_d-\bar{B}^0_d$ system are smaller than in the $B^0_s-\bar{B}^0_s$ system, while they are larger in the $K^0-\bar{K}^0$ system. The colour coding for allowed oases in these figures will be useful to monitor the following steps in which rare decays will be discussed and the distinction between the two allowed oases in each case will be possible.

In [40] also the allowed oases in scenarios RHS, LRS and ALRS have been considered. We summarize here the main results and refer for details to this paper:

- In the case of RHS scenarios the oases in the space of parameters related to RH currents are precisely the same as those just discussed for LHS scenarios, except that the parameters $\tilde{\delta}_{ij}$ and $\delta_{ij}$ parametrize now RH and not LH currents. Yet, as we will see in the next steps, in the case of $\Delta F = 1$ observables some distinction between LH and RH currents will be possible.

- In the LRS scenarios NP contributions to $\Delta F = 2$ observables are dominated by new LR operators, whose contributions are enhanced through RG effects relative to LL and RR operators and in the case of $\delta_{ij}$ also through chirally enhanced hadronic matrix elements. Consequently, the oases will differ from the previous ones and, typically, the corresponding $\tilde{\delta}_{ij}$ will be smaller in order to obtain agreement with the data. The results can be found in figures 13–15 of [40]. In order to understand these plots, one should recall that the matrix element of the dominant $O_{1,2}^{LR}$ operator has the sign opposite to SM operators. Therefore, in the case of $B^0_s,d-\bar{B}^0_s,d$ systems this operator naturally suppresses $\Delta M_s$ and $\Delta M_d$, with the phase $\delta_{23}$ and $\delta_{13}$ shifted down by roughly 90° relative to the LHS scenarios. We illustrate this in figure 11 for LRS1 scenario. These plots should be compared with the one in figure 8 and in the left panel of figure 9, respectively.

- The allowed oases in ALR scenarios have the same phase structure as in LHS scenarios because the contributions of the dominant LR operators have the same sign as SM contributions. Only the allowed values of $\tilde{\delta}_{ij}$ are smaller because of larger hadronic matrix elements than in the LHS case.

The implications of these results for rare decays will be presented in the next steps.

5.3.7. Tree-level scalar exchanges. We next turn our attention to tree-level heavy scalar exchanges in $\Delta F = 2$ transitions (see figure 12). Here, one finds [44, 93]

\[
\left( \frac{M_{12}}{M_H} \right)_H = \frac{\left( \frac{\Delta_{1d}}{M_H} \right)^2}{2M_H^2} \times \left[ C_{1}^{\text{SL}}(\mu_H) \langle Q_1^{\text{SL}}(\mu_H) \rangle + C_{2}^{\text{SL}}(\mu_H) \langle Q_2^{\text{SL}}(\mu_H) \rangle \right] - \frac{\left( \frac{\Delta_{1d}}{M_H} \right)^2}{2M_H^2} \left[ C_{1}^{\text{SR}}(\mu_H) \langle Q_1^{\text{SR}}(\mu_H) \rangle \right] + C_{2}^{\text{SR}}(\mu_H) \langle Q_2^{\text{SR}}(\mu_H) \rangle - \frac{\Delta_{ld}^{\text{d}}(H) \Delta_{ld}^{\text{d}}(H)}{M_H^2} \times \left[ C_{1}^{\text{LR}}(\mu_H) \langle Q_1^{\text{LR}}(\mu_H) \rangle + C_{2}^{\text{LR}}(\mu_H) \langle Q_2^{\text{LR}}(\mu_H) \rangle \right],
\]

(97)

where including NLO QCD corrections [93]

\[
C_{1}^{\text{SL}}(\mu) = C_{1}^{\text{SR}}(\mu) = 1 + \frac{\alpha_s}{4\pi} \left[ -3 \log \left( \frac{M_H^2}{\mu^2} \right) + \frac{9}{2} \right],
\]

(98)

\[
C_{2}^{\text{SL}}(\mu) = C_{2}^{\text{SR}}(\mu) = \frac{\alpha_s}{4\pi} \left[ -\frac{1}{12} \log \left( \frac{M_H^2}{\mu^2} \right) + \frac{1}{8} \right],
\]

(99)

\[
C_{1}^{\text{LR}}(\mu) = -3 \frac{\alpha_s}{4\pi},
\]

(100)

\[
C_{2}^{\text{LR}}(\mu) = 1 - \frac{\alpha_s}{4\pi}.
\]

(101)

Note that the scalar contributions to $C_{1,2}^{LR}$ differ from those from gauge bosons. The relevant matrix elements can again be found in tables 2 and 3 for tree-level heavy scalar and SM Higgs contributions. In the latter case $M_H = M_h$ with $h$ standing for the SM Higgs.
For our qualitative discussion it is sufficient to set the Wilson coefficients to the LO values. Then,

$$\langle M_{12}^* \rangle_H = -\left( \frac{(\Delta^{sd}_L(H))^2}{2M_H^2} + \frac{(\Delta^{sd}_R(H))^2}{2M_H^2} \right) \langle Q^{SLL}_1(\mu_H) \rangle$$

with analogous expressions for other meson systems. Now, as seen in table 2 model independently

$$\langle Q^{SLL}_1(\mu_H) \rangle < 0, \quad \langle Q^{SLL}_2(\mu_H) \rangle > 0,$$

$$|\langle Q^{LR}_2(\mu_H) \rangle| \gg |\langle Q^{VLL}_1(\mu_H) \rangle|,$$

which has an impact on the signs and size of the couplings $\Delta_{L,R}(H)$ if these contributions should remove the anomalies in the data.

Interestingly the signs of $\langle Q^a_i \rangle$ that are relevant in gauge boson and scalar cases are such that at the end it is not possible to distinguish these two cases on the basis of the signs of the couplings alone. On the other hand, $\langle Q^{SLL}_2 \rangle$ are absent in the case of gauge boson exchanges and $\Delta_{L,R}(Z')$ and $\Delta_{L,R}(H)$ are generally different from each other, so that some distinction will be possible when other decays will be taken into account in later steps. Otherwise, the qualitative comments made in the context of tree-level gauge boson exchanges can be repeated in this case.

Indeed, as analysed recently in [44] the phase structure of the allowed oases is identical to the one of the gauge boson case. As seen in the plots presented in this paper, only the values of $\tilde{s}_{ij}$ change.
5.3.8. Implications of $U(2)^3$ symmetry. Possibly the simplest solution to the problems of various models with MFV is to reduce the flavour symmetry $U(3)^3$ to $U(2)^3$ \cite{86–92}. As pointed out in \cite{39}, in this case NP effects in $\varepsilon_K$ and $\Delta M_{s,d}$ are not correlated with each other, so that the enhancement of $\varepsilon_K$ and suppression of $\Delta M_{s,d}$ can be achieved if necessary, in principle, for the values of $|V_{cb}|$, $F_{B_s}\sqrt{B_{B_s}}$ and $F_{B_d}\sqrt{B_{B_d}}$ in table 1 or (36).

In particular,

- NP effects in $\varepsilon_K$ are of CMFV type and $\varepsilon_K$ can only be enhanced. However, because of the reduced flavour symmetry from $U(3)^3$ to $U(2)^3$, there is no correlation between $\varepsilon_K$ and $\Delta M_{s,d}$, which was problematic for CMFV models.

- In a $B_{d,s}^0 - \bar{B}_{d,s}^0$ system, the ratio $\Delta M_d/\Delta M_s$ is equal to the one in the SM and in good agreement with the data. But in view of new CP-violating phases $\phi_{B_s}$ and $\phi_{B_d}$, even in the presence of only SM operators, $\Delta M_{s,d}$ can be suppressed. However, the $U(2)^3$ symmetry implies $\phi_{B_s} = \phi_{B_d}$ and consequently a triple $S_{\phi K_s} - S_{\phi K_d} - |V_{ub}|$ correlation, which constitutes an important test of this NP scenario \cite{39}. We show this correlation in figure 13 for $\gamma$ between 58° and 78°. Note that this correlation is independent of the values of $F_{B_s}\sqrt{B_{B_s}}$ and $F_{B_d}\sqrt{B_{B_d}}$.

- As seen in this figure, the important advantage of $U(2)^3$ models over $2HDM_{\text{MFV}}$ is that in the case of $S_{\phi K}$ being very small, or even having an opposite sign to the SM prediction, this framework can survive with a concrete prediction for $|V_{ub}|$.

It is of interest to see how the parameter space in tree-level gauge boson or scalar $\Delta F = 2$ transitions is further constrained when the flavour $U(2)^3$ symmetry is imposed on the $Z'$ or $H$ quark couplings. Indeed, now the observables in $B_d$ and $B_s$ systems are correlated with each other due to the relations:

$$\frac{\tilde{s}_{13}}{|V_{td}|} = \frac{\tilde{s}_{23}}{|V_{ts}|}, \quad \delta_{13} - \delta_{23} = \beta - \beta_s.$$  \hfill (104)

Thus, once the allowed oases in the $B_d$ system are fixed, the oases in $B_s$ system are determined. Moreover, all of the

Figure 11. Ranges for $\Delta M_s$ and $S_{\phi K_s}$ (left) and $\Delta M_d$ and $S_{\phi K_d}$ (right) for $M_{Z'} = 1$ TeV in LRS1 satisfying the bounds in equations (93) and (94). Reproduced with permission from [40]. Copyright 2013 SISSA.

Figure 12. Tree-level flavour-changing $A^0, H^0, h$ contribution to $B_d^0 \to \bar{B}_d^0$ and $B_s^0 \to \bar{B}_s^0$ mixing.

Figure 13. $S_{\phi K_s}$ versus $S_{\phi K_d}$ in models with $U(2)^3$ symmetry for different values of $|V_{ub}|$ and $\gamma \in [58^\circ, 78^\circ]$. From top to bottom: $|V_{ub}| = 0.0046$ (blue), 0.0043 (red), 0.0040 (green), 0.0037 (yellow), 0.0034 (cyan), 0.0031 (magenta), 0.0028 (purple). Light/dark grey: experimental 1σ/2σ region. Reproduced with permission from [39]. Copyright 2013 SISSA.
observables in both systems are described by only one real positive parameter and one phase, such as $(\delta_{23}, \delta_{32})$.

The impact of $U(2) \Gamma$ symmetry on tree-level FCNCs due to gauge boson and scalar exchanges has been analysed in [40] and [44], respectively. Once again, the phase structure in both cases is the same. Figure 14 results from the combination of figures 8 and 9 using the $U(2) \Gamma$ symmetry relations in (104). We observe that, in particular, the $(\delta_{23}, \delta_{32})$ cases are significantly reduced. Moreover, the fact that the results in the $B_d$ system depend on whether LHS1 or LHS2 is considered is now transferred through the relations in (104) into the $B_s$ system. This is clearly seen in figure 14 where the final oases (cyan) in LHS2 are smaller than in LHS1 (magenta) due to the required shift of $\hat{s}_{0,k_2}$. The corresponding results in the scalar case can be found in figure 15 of [44]. It will be interesting to see what the impact of the $U(2) \Gamma$ symmetry on rare decays will be in the next steps.

5.4.1. Preliminaries. We now move to consider two superstars of rare $B$ decays: the decays $B_{s,d} \to \mu^+\mu^-$. We will also discuss $B_{s,d} \to \tau^+\tau^-$, which could become superstars in the future. The particular interest in $B_{s,d} \to \mu^+\mu^-$ is related to the fact that in the SM their branching ratios are not only loop and GIM suppressed, as are other rare decays in the SM. Since the final state is purely leptonic and the initial state is a pseudoscalar, the decays in question are strongly helicity suppressed in view of the smallness of $m_\mu$ and, equally importantly, they do not receive photon-mediated one-loop contributions. Given that all of these properties can be violated beyond the SM, these two decays are particularly suited for searching for NP while being in addition theoretically very clean.

In the SM, and in several of its extensions, $B(B_s \to \mu^+\mu^-)$ is found in the ballpark of $(2-6) \times 10^{-9}$. As several model studies show, this is the case of models in which these decays proceed through $Z$-penguin diagrams and tree-level neutral gauge boson exchanges. Larger values can be obtained in the presence of neutral heavy scalar and pseudoscalar exchanges in 2HDM models and Supersymmetry. Here, these decays are governed by scalar and pseudoscalar penguins when the value of $\tan\beta$ is large. In certain models, contributions from tree-level scalars and pseudoscalars can already arise at the fundamental level. Therefore, the discovery of $B(B_s \to \mu^+\mu^-)$ at $O(10^{-6})$ would be a clear signal of NP, possibly related to such scalar and pseudoscalar exchanges [37]. Unfortunately, as we will see below, the most recent data from LHCb and CMS tells us that the nature does not allow us to make a clear distinction between scalar, pseudoscalar and gauge boson contributions, at least on the basis of the $B(B_s \to \mu^+\mu^-$) alone. Either other observables related to the time-dependent rate of this decay have to be studied [43] or/and correlations with other observables have to be investigated. We will see explicit examples of this below. We refer also to [142, 143], where various virtues of these decays have been reviewed.

In order to discuss these issues we have to present the fundamental effective Hamiltonian relevant for these decays and other $b \to s\ell^+\ell^-$ transitions, like $B \to K^*\ell^+\ell^-$, $B \to K\ell^+\ell^-$ and $B \to X_s\ell^+\ell^-$, which we will consider in step 7.

5.4.2. Basic formulae. There are different conventions for operators [144–146] relevant for $b \to s\ell^+\ell^-$ transitions and one has to be careful when using them along with the expressions for the branching ratios present in the literature. The effective Hamiltonian used here and in several recent

![Figure 14. Ranges for $\Delta M_s$ (red region), $S_{\ell\phi}$ (blue region), $\Delta M_d$ (green region) and $S_{\phi K_2}$ (yellow region) for $M_{\mu} = 1$ TeV in LHS1 (left) and LHS2 (right) in the $U(2) \Gamma$ limit satisfying the bounds in equations (93) and (94). The overlap region of LHS1 (LHS2) is shown in magenta (cyan). Reproduced with permission from [40]. Copyright 2013 SISSA.](image-url)
papers is given as follows:

\[
\begin{align*}
\mathcal{H}_{\text{eff}}(b & \to s \ell \bar{\ell}) = \mathcal{H}_{\text{eff}}(b \to s \gamma) - \frac{4G_F}{\sqrt{2}} \frac{a}{4\pi} V_{ts}^* V_{tb} \\
& \times \sum_{i=9,10,5,\mu} [C_i(\mu) Q_i(\mu) + C_i'(\mu) Q'_i(\mu)]
\end{align*}
\]  

(105)

where

\[
\begin{align*}
Q_9 &= (\bar{s} \gamma_t P_t b)(\bar{\ell} \gamma^\mu \ell), \\
Q'_9 &= (\bar{s} \gamma_t P_t b)(\bar{\ell} \gamma^\mu \ell), \\
Q_{10} &= (\bar{s} \gamma_t P_t b)(\bar{\ell} \gamma^\mu \gamma_5 \ell), \\
Q'_{10} &= (\bar{s} \gamma_t P_t b)(\bar{\ell} \gamma^\mu \gamma_5 \ell).
\end{align*}
\]

(106a)

Here \( \mathcal{H}_{\text{eff}}(b \to s \gamma) \) stands for the effective Hamiltonian for the \( b \to s \gamma \) transition that involves the dipole operators (step 6). While we do not explicitly show the four-quark operators in (105), they are very important for the decays considered in this step, particularly as far as QCD and electroweak corrections are concerned.

One should note the difference of ordering of flavours relative to \( \Delta F = 2 \) operators considered in the previous step. This will play a role as we discuss below (for example the relations of the couplings in (159) are useful when comparing \( \Delta F = 1 \) and \( \Delta F = 2 \) transitions). We neglect the effects proportional to \( m_t \), but keep \( m_t \) and \( m_d \) different from zero when they are shown explicitly. Analogous operators govern the \( b \to d \ell^\pm \ell^- \) transitions, particularly the \( B_d \to \mu^+ \mu^- \) decay.

Concentrating first on \( B_s \to \mu^+ \mu^- \), there are three observables that can be used to search for NP in these decays, these are

\[
\begin{align*}
\mathcal{B}(B_s \to \mu^+ \mu^-), \quad A_{\Delta s}^{\mu \mu}, \quad S_{\mu \mu}^{s}.
\end{align*}
\]

(107)

Here, \( \mathcal{B}(B_s \to \mu^+ \mu^-) \) is the usual branching ratio, which includes \( \Delta \Gamma_s \) effects pointed out in [147–149]. Following [43], we will denote this branching ratio with a \( \text{bar} \), while the one without these effects does not have it. These two branching ratios are related through [147–149]

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = r(y_s) \mathcal{B}(B_s \to \mu^+ \mu^-),
\]

(108)

where

\[
r(y_s) \equiv \frac{1 - y_s^2}{1 + A_{\Delta s}^{\mu \mu} y_s}.
\]

(109)

with [112]

\[
y_s = \frac{\Delta \Gamma_s}{2} = 0.062 \pm 0.009.
\]

(110)

The observables \( A_{\Delta s}^{\mu \mu} \) and \( S_{\mu \mu}^{s} \) can only be measured through time-dependent studies and appear in the time-dependent rate asymmetry as follows:

\[
\begin{align*}
\Gamma(B_s^0(t) \to \mu^+ \mu^-) - \Gamma(B_s^0(t) \to \mu^+ \mu^-) \\
&= \sinh(y_s t / \tau_{B_s}) + A_{\Delta s}^{\mu \mu} \sinh(y_s t / \tau_{B_s}).
\end{align*}
\]

(111)

\( A_{\Delta s}^{\mu \mu} \) can be extracted from the untagged data sample, namely from the measurement of the effective lifetime, for which no distinction is made between initially present \( B_s^0 \) or \( B_s^0 \) mesons. If tagging information is included, requiring the distinction between initially present \( B_s^0 \) or \( B_s^0 \) mesons, a CP-violating asymmetry \( S_{\mu \mu}^{s} \) can also be measured. Presently, only \( \mathcal{B}(B_s \to \mu^+ \mu^-) \) is known experimentally but once \( A_{\Delta s}^{\mu \mu} \) will be extracted from time-dependent measurements we will also be able to obtain \( \mathcal{B}(B_s \to \mu^+ \mu^-) \) directly from the experiment. As emphasized and demonstrated in [43] \( A_{\Delta s}^{\mu \mu} \) and \( S_{\mu \mu}^{s} \) provide additional information about possible NP, which cannot be obtained on the basis of the branching ratio alone. In order to present the results for the trio in (107) in various models, we have to express these observables in terms of the Wilson coefficients in the effective Hamiltonian in (105).

To this end one first introduces

\[
\begin{align*}
P &= \frac{C_{10} - C_{10}^{SM}}{C_{10}^{SM}} + \frac{m^2_{bs}}{m^2_{bs}} \frac{m_b}{m_s} \frac{C_{10} - C_{10}^{SM}}{C_{10}^{SM}} \equiv \left| P \right| \left| \epsilon^{\mu \mu \mu} \right|, \\
S &= \sqrt{1 - \frac{4m^2_{bs}}{m^2_{bs}} \frac{m_b}{m_s} \frac{C_{S} - C_{S}^{SM}}{C_{S}^{SM}}} \equiv \left| S \right| \left| \epsilon^{\mu \mu \mu} \right|,
\end{align*}
\]

(112)

(113)

which carries the full information about the dynamics in the decay. However, due to effects from \( B_s^0 - B_s^0 \) mixing, represented here by \( y_s \), the new phase \( \phi_{Bs} \), in \( B_s^0 - B_s^0 \) mixing will also enter the expressions that are given below.

One then finds three fundamental formulae [43, 149, 150]

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = \left[ 1 + A_{\Delta s}^{\mu \mu} y_s \right] \times \left( \left| P \right|^2 + \left| S \right|^2 \right)
\]

(114)

\[
A_{\Delta s}^{\mu \mu} = \frac{\left| P \right|^2 \cos(2\phi_{P} - 2\phi_{B_s}) - \left| S \right|^2 \cos(2\phi_{S} - 2\phi_{B_s})}{\left| P \right|^2 + \left| S \right|^2},
\]

(115)

\[S_{\mu \mu}^{s} = \frac{\left| P \right|^2 \sin(2\phi_{P} - 2\phi_{B_s}) - \left| S \right|^2 \sin(2\phi_{S} - 2\phi_{B_s})}{\left| P \right|^2 + \left| S \right|^2}.
\]

(116)

where

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = \frac{1}{1 - y_s} \mathcal{B}(B_s \to \mu^+ \mu^-)_{SM}.
\]

(117)

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = \frac{\tau_{B_s} C^2_F}{4\pi} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \times \frac{F_{B_s}^2 m^2_{bs} m_{B_s}}{m^2_{bs}} \left| V_{ts}^* V_{tb} \right|^2 \eta^{B_s}_{\text{eff}} Y_{0}(x_s)^2
\]

(118)

with \( \eta^{B_s}_{\text{eff}} \) and \( Y_{0}(x_s) \) given below.

It follows that in any model the branching ratio without \( \Delta \Gamma_s \) effect is related to the corresponding SM branching ratio through

\[
\mathcal{B}(B_s \to \mu^+ \mu^-) = \mathcal{B}(B_s \to \mu^+ \mu^-)_{SM} \left( \left| P \right|^2 + \left| S \right|^2 \right),
\]

(119)

which is obtained from (114) by setting \( y_s = 0 \).
Finally, all of the formulae given above can be used for $B_d \rightarrow \mu^+\mu^-$ with $s$ replaced by $d$ and $y_d \approx 0$, so that in this case there is no distinction between $B(B_d \rightarrow \mu^+\mu^-)$ and $\theta \equiv \theta(B_s \rightarrow \mu^+\mu^-)$. Still, the CP asymmetry $S^{d,\mu}_{\mu\mu}$ can be considered, although measuring it would be a heroic effort.

These formulae are very general and can be used to study these observables model independently, using as variables

$$|P|, \quad \varphi_P, \quad |S|, \quad \varphi_S.$$  \hspace{1cm} \hspace{1cm} (120)

Such an analysis has been performed in [43]. The classification of popular NP in various scenarios characterized by the vanishing or non-vanishing values of the variables in (120) and of the new phase $\varphi_P$ in $B^0_\mu - \bar{B}^0_\mu$ mixing should help in monitoring the improved data in the future. While some of the results of this paper, and also of related analysis of tree-level gauge boson and scalar contributions in [44], will be presented below, we have already collected in table 6 the properties of the selected models that have been discussed in these two papers with respect to the basic phenomenological parameters listed in (120) and the classes defined in [43] that they belong to.

After this general introduction, we will discuss the results in the SM and its simplest extensions.

5.4.3. SM results and the data. In the SM, $B_{s,d} \rightarrow \mu^+\mu^-$ are governed by $Z$-penguin diagrams and $F = 1$ box diagrams, which depend on the top-quark mass. The internal charm contribution can be safely neglected.

The only relevant Wilson coefficients in the SM are $C_9$ and $C_{10}$, as given by

$$\begin{align*}
\sin^2 \theta_W C^\text{SM}_9 &= \sin^2 \theta_W P^\text{NDR}_0 \\
+ \eta_{\text{eff}} Y_0(x_t) - 4 \sin^2 \theta_W Z_0(x_t), \hspace{1cm} \text{(121)}
\end{align*}$$

$$\sin^2 \theta_W C^\text{SM}_{10} = -\eta_{\text{eff}} Y_0(x_t) \hspace{1cm} \text{(122)}$$

with all the entries given in [40, 46], except for $\eta_{\text{eff}}$, which is discussed below. With $m_t \ll m_b$ we have $C^\text{SM}_9 = C^\text{SM}_{10} = 0$.

Here, $Y_0(x_t)$ and $Z_0(x_t)$ are SM one-loop functions given by

$$\begin{align*}
Y_0(x_t) &= \frac{x_t}{8} \left( x_t - 4 \frac{3 x_t \log x_t}{x_t - 1} \right), \hspace{1cm} \text{(123)}
\end{align*}$$

$$\begin{align*}
Z_0(x) &= -\frac{1}{9} \log x + \frac{18 x^4 - 163 x^3 + 259 x^2 - 108 x}{144 (x - 1)^4} \\
+ \frac{32 x^3 - 38 x^2 - 15 x + 18}{72 (x - 1)^4} \hspace{1cm} \log x. \hspace{1cm} \text{(124)}
\end{align*}$$

We then have

$$C^\text{SM}_9 \approx 4.1, \hspace{1cm} C^\text{SM}_{10} \approx -4.1. \hspace{1cm} \text{(125)}$$

The coefficient $\eta_{\text{eff}}$ was until recently denoted by $\eta_Y$ and included only NLO QCD corrections. For $m_t = m_t(m_t)$ one had $\eta_Y = 1.012$ [151, 152].

Over several years, electroweak corrections to the branching ratios have been calculated [153–156] but they were incomplete, implying dependence on renormalization scheme used for electroweak parameters, as analysed in detail in [157]. Recently complete NLO electroweak corrections [158] and QCD corrections up to NNLO [159] have been calculated. The inclusion of these new higher order corrections that were until now missing has significantly reduced the various scale uncertainties, so that non-parametric uncertainties in both branching ratios are below 2%.

The calculations performed in [158, 159] are very involved and in analogy to the QCD factors, like $\eta_B$ and $\eta_{1-3}$ in $\Delta F = 2$ processes, we find it useful to include all QCD and electroweak corrections into $\eta_{\text{eff}}$ introduced in (122), which without these corrections would be equal to unity. Inspecting the analytic formulae in [160] one then finds [47]

$$\eta_{\text{eff}} = 0.9882 \pm 0.0024. \hspace{1cm} \text{(126)}$$

The small departure of $\eta_{\text{eff}}$ from unity was already anticipated in [156, 157] but only the calculations in [158–160] could put these expectations and conjectures on a firm footing. Indeed, in order to end up with such a simple result it was crucial to perform such involved calculations because these small corrections are only valid for particular definitions of the top-quark mass and of other electroweak parameters involved. In particular, one has to use in $Y_0(x_t)$ the MS-renormalized top-quark mass $m_t(m_t)$ with respect to QCD but on-shell with respect to electroweak interactions. This means $m_t(m_t) = 163.5 \text{GeV}$ as calculated in [160]. Moreover, in using (126) to calculate observables, such as branching ratios, it is important to have the same normalization of effective Hamiltonian as in the latter paper, where this normalization is expressed in terms of $G_F$ and $M_W$ only. Needless to say, one can also directly use the formulae in [160].

In the present review we follow the normalization of effective Hamiltonian in [51], which uses $G_F$, $\alpha(M_Z)$ and $\sin^2 \theta_W$. In order to be consistent with the calculation in [160], our $\eta_{\text{eff}} = 0.991$ and $m_t(m_t)$ is unchanged [47]. Interestingly, in the case of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$, the analogue of $\eta_{\text{eff}}$, which this time multiplies $X_0(\nu)$, is also found to be within 1% from unity [161]. It should be remarked that presently it is only in the case of the $B_{s,d} \rightarrow \mu^+\mu^-$ decays discussed here, and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ decays considered in step 8, that one has to take such a
care about the definition of $m_t$ with respect to electroweak corrections because in most cases such corrections are not known or hadronic uncertainties are too large, so that the value $m_t(m_t) = 163.0$ GeV in table 1 used by us otherwise can easily be defended.

In view of still significant parametric uncertainties, it is useful to show the dependence of the branching ratios on various input parameters involved. Such formulae have been already presented in [43, 157] and have been recently updated by the authors of [158] and [159], who find [160] to be rather close to $3$ with

$$
\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.65 \pm 0.06) \times 10^{-9}
\times \left( \frac{m_t(m_t)}{163.5 \text{ GeV}} \right)^{3.02} \left( \frac{\alpha_s(M_Z)}{0.1184} \right)^{0.032} R_s,
$$

where

$$
R_s = \left( \frac{F_{B_s}}{227.7 \text{ MeV}} \right)^2 \left( \frac{\tau_{B_s}}{1.516 \text{ ps}} \right) \left( \frac{V_{ts}^*V_{ts}}{0.0415} \right)^2
$$

we have also shown the most recent average of the results from LHCb and CMS [162–164]. The agreement of the SM prediction with the data for $B_s \rightarrow \mu^+\mu^-$ in (132) is remarkable, although the rather large experimental error still allows for sizable NP contributions. In $B_d \rightarrow \mu^+\mu^-$ much bigger room for NP contributions is left.

We close our discussion of the SM with the correlations of $\mathcal{B}(B_q \rightarrow \mu^+\mu^-)$ and $\Delta M_{s,d}$ that are free from $F_{B_q}$ and the $|V_{tq}|$ dependence [61]

$$
\mathcal{B}(B_q \rightarrow \mu^+\mu^-) = C \frac{\tau_{B_q}}{S_0(x_q)} \frac{\langle \eta_B \rangle}{\Delta M_q},
$$

with

$$
C = 6\pi \frac{1}{\eta_B} \frac{\alpha}{4\pi \sin^2 \theta_W} \frac{m_t^2}{M_W^2} = 4.291 \times 10^{-10},
$$

where $\hat{B}_q$, known from step 2, enters linearly as opposed to having a quadratic dependence on $F_{B_q}$.

The results for branching ratios obtained in this manner have presently comparable errors to those obtained by direct calculations of branching ratios, their values are close to the ones quoted above. Of interest are also the relations (13) and (14) with $r(\mu^+\mu^-) = 1$ and $r = 1$, which will hopefully be tested one day.

Let us next see what the simple models introduced in section 3 can tell us about these decays.

5.4.4. CMFV. In this class of models there are no new CP-violating phases and no new operators. Therefore, all of the formulae of the SM given until now remain valid, except for the following changes:

- The master functions $S_0(x_t)$ and $Y_0(x_t)$ are replaced by new functions $S(v)$ and $Y(v)$, respectively. Here, $v$ denotes all parameters present in a given CMFV model, that is, coupling and masses of new particles, including those of the SM.

- QCD corrections to $B_{s,d} \rightarrow \mu^+\mu^-$, represented by $\eta_B$, are expected in this class of models to be small and this is also expected for electroweak corrections. On the other hand, $\eta_B$ could be visibly different in these models if the masses of particles involved are larger than $1$ TeV. Yet, related to relatively small anomalous dimension of the $(V-A) \times (V-A)$ operator, this change is much smaller than in the case of LR operators encountered in more complicated models. Therefore, in view of the new parameters present in $S(v)$, it is a good idea to first use just the SM value for $\eta_B$.

A more precise treatment would be to make the following replacement:

$$
S_0(x_t) \rightarrow S_0(x_t) + \frac{\eta_B^{\text{NP}}}{\eta_B^{\text{SM}}} \Delta S_0(v),
$$

where $\eta_B^{\text{SM}}$ equals $\eta_B$ in previous expressions and $\Delta S_0(v)$ is the modification of the loop functions by NP contributions. The new $\eta_B^{\text{NP}}$ can easily be calculated in the LO if the NP scale is known. Then, the sign of the anomalous dimension of the operator $Q_{\text{VLL}}^\mu$ implies $\eta_B^{\text{NP}} \leq \eta_B^{\text{SM}}$ for NP scales larger than the electroweak scale.
Figure 15. $B(B_d \rightarrow \mu^+\mu^-)$ versus $B(B_s \rightarrow \mu^+\mu^-)$ in models with CMFV. SM is represented by the light grey area with black dot. Dark grey region combined 1σ range $B(B_s \rightarrow \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$ and $B(B_d \rightarrow \mu^+\mu^-) = (3.6^{+1.4}_{-1.6}) \times 10^{-10}$.

The branching ratios for $B_{s,d} \rightarrow \mu^+\mu^-$ will now be modified with respect to the SM but, as seen in figure 15, due to relations in (13) and (14) with $r(\mu^+\mu^-) = 1$ and $r = 1$, a strong correlation between these two branching ratios is predicted. In figure 15 we have included $\Delta \Gamma_s, \Delta \Gamma_d$ effects in $B(B_s \rightarrow \mu^+\mu^-)$.

The calculations simplify considerably if CKM factors are fixed in step 1. Then, independently of $q$ we simply have

$$\frac{B(B_q \rightarrow \mu^+\mu^-)}{B(B_q \rightarrow \mu^+\mu^-)_{\text{SM}}} = \left( \frac{Y(v)}{Y_0(x_t)} \right)^2$$

and consequently

$$\frac{B(B_s \rightarrow \mu^+\mu^-)}{B(B_d \rightarrow \mu^+\mu^-)}_{\text{CMFV}} = \frac{B(B_s \rightarrow \mu^+\mu^-)}{B(B_d \rightarrow \mu^+\mu^-)}_{\text{SM}} \approx 34.4 \pm 3.6,$$

where we have used the SM values in (132) and (133). Using (14) with $r = 1$ we would find $33.9 \pm 0.8$. Using (138) together with the measurement of $B(B_s \rightarrow \mu^+\mu^-)$ in (132) implies in turn in the context of these models

$$B(B_d \rightarrow \mu^+\mu^-) = (0.84 \pm 0.19) \times 10^{-10}$$

(CMFV),

which is well below the data in (133). This could then be an indication for new sources of flavour violation. In fact, as seen in figure 15 the present data differ from CMFV correlation between these two branching ratios by roughly 2σ but we have to wait for new improved data in order to claim NP at work. Still, it will be interesting to see what kind of NP could bring the theory close to the present experimental central values for the branching ratios in this figure.

5.4.5. 2HDM$_{\text{SMFV}}$. In 2HDM$_{\text{SMFV}}$ scalar and pseudoscalar penguin diagrams generate new scalar and pseudoscalar operators that can even dominate the decays $B_{s,d} \rightarrow \mu^+\mu^-$ at sufficiently high value of $\tan \beta$. However, due to recent LHCb and CMS results, such large enhancements are no longer possible for $B_s \rightarrow \mu^+\mu^-$ and within this model the same applies to $B_d \rightarrow \mu^+\mu^-$. Indeed, within an excellent approximation we have then, similarly to (138), [140]

$$\frac{B(B_s \rightarrow \mu^+\mu^-)}{B(B_d \rightarrow \mu^+\mu^-)}_{\text{2HDM}_{\text{SMFV}}} = \frac{B(B_s \rightarrow \mu^+\mu^-)}{B(B_d \rightarrow \mu^+\mu^-)}_{\text{SM}}.$$

Combined with (84) we can then conclude that also (14) with $r = 1$ is well satisfied in this model. However, while the ratios in (84) and (140) are the same in 2HDM$_{\text{SMFV}}$ and the SM, the individual $\Delta M_{s,d}$ and $B(B_{s,d} \rightarrow \mu^+\mu^-)$ can differ in these models. Still, the range for $B(B_{s,d} \rightarrow \mu^+\mu^-)$ in (139) also applies and constitutes an important test of this model.

Finally, in the limit $C_9 = -C_{10}$ the lower bounds on the two branching ratios can be derived [43, 165]:

$$B(B_q \rightarrow \mu^+\mu^-)_{\text{2HDM}_{\text{SMFV}}} \geq \frac{1}{2}(1 - \eta_y)B(B_q \rightarrow \mu^+\mu^-)_{\text{SM}},$$

which are also valid in the MSSM [166].

5.4.6. Tree-Level gauge boson exchange. We will next consider the contributions of a tree-level gauge boson exchange to the Wilson coefficients of the operators involved (see figure 16). By including the SM contributions, one has [40]

$$\sin^2 \theta_{Y_q} C_9 = [\eta_y Y_0(x_t) - 4 \sin^2 \theta_W Z_0(x_t)]$$

+ $\frac{1}{8\pi} \frac{1}{M_{Z}^2} \Delta_{V}^{\mu} (Z') \Delta_{V}^{\mu} (Z') \frac{V_{tb}^* V_{tb}}{V_{ts} V_{tb}}$, (142)

$$\sin^2 \theta_{Y_q} C_{10} = -\eta_y Y_0(x_t) - \frac{1}{8\pi} \frac{1}{M_{Z}^2} \Delta_{V}^{\mu} (Z') \Delta_{V}^{\mu} (Z') \frac{V_{tb}^* V_{tb}}{V_{ts} V_{tb}}.$$ (143)

$$\sin^2 \theta_{Y_q} C_{9}' = -\frac{1}{8\pi} \frac{1}{M_{Z}^2} \Delta_{V}^{\mu} (Z') \Delta_{V}^{\mu} (Z') \frac{V_{tb}^* V_{tb}}{V_{ts} V_{tb}}.$$

where we have defined

$$\Delta_{V}^{\mu} (Z') = \Delta_{R}^{\mu} (Z') + \Delta_{L}^{\mu} (Z'),$$

$$\Delta_{A}^{\mu} (Z') = \Delta_{R}^{\mu} (Z') - \Delta_{L}^{\mu} (Z').$$

In order to simplify the presentation we still work with $\eta_y$ and $Y_0(x_t)$, which should be replaced by $Y_{\text{eff}}$ in (122) if the future precision of the experimental data will require it.

The vector Wilson coefficients $C_9, C_{10}$ do not contribute to decays in question but they will enter step 7, where the
decays $B \to X_i \ell^+ \ell^-$ and $B \to K^*(K) \ell^+ \ell^-$ are considered. Assuming that the CKM parameters have been determined independently of NP and are universal, we then find

$$\frac{\mathcal{B}(B_q \to \mu^+ \mu^-)}{\mathcal{B}(B_q \to \mu^+ \mu^-)_{\text{SM}}} = \left| \frac{\mathcal{Y}_A(v)(x_i)}{\eta_i Y_0(x_i)} \right|^2,$$

where

$$\mathcal{Y}_A(v) = \frac{1}{V_{tb} V_{tq}^*} M_{Z}^{2} g_{5\text{SM}} \left[ \alpha_{d}^{(\mu)}(Z') - \alpha_{s}^{(\mu)}(Z') \right]$$

is generally complex and moreover different for $B_q \to \mu^+ \mu^-$ and $B_q \to \mu^+ \mu^-$ implying a violation of the CMFV correlation shown in figure 15. Still, the correlation between $\mathcal{B}(B_q \to \mu^+ \mu^-)_{\text{SM}}$ and $M_{q\mu}$, when all these observables are calculated directly, could offer a useful test of the model.

The correlations between the following observables have been investigated in [40]:

$$\Delta M_J, \quad S_{\Psi \phi}, \quad \mathcal{B}(B_s \to \mu^+ \mu^-), \quad S_{\mu \mu}$$

in the $B_s$-system and

$$\Delta M_d, \quad S_{\Psi K_s}, \quad \mathcal{B}(B_d \to \mu^+ \mu^-), \quad S_{d \mu}$$

in $B_d$ system. To this end

$$\alpha_{d}^{(\mu)}(Z') = 0.5$$

has been chosen, to be compared with its SM value $\Delta_{\mu}^{(\mu)}(Z') = 0.372$.

Note that for fixed $\Delta_{\mu}^{(\mu)}(Z')$ the observables in (149) depend only on two complex variables: $\Delta_{L,R}^{(\mu)}(Z')$ and in fact in the LHS, RHS, LRS and ALR scenarios only on $\delta_{33}$ and $\delta_{31}$. Similarly the observables in (150) depend only on two complex variables: $\Delta_{d}^{(\mu)}(Z')$ and in the LHS, RHS, LRS and ALR scenarios only on $\delta_{13}$ and $\delta_{13}$. Since these parameters have been already constrained in step 3, definite correlations between the observables within each set in (149) and (150) follow. Once the $U(2)$ symmetry is imposed, correlations between the sets in (149) and (150) are found. It will be interesting to investigate the impact on these correlations from the $b \to s \ell^+ \ell^-$ and $b \to s \ell^+ \ell^-$ transitions we consider in steps 7 and 9, respectively.

It will be useful to present a numerical analysis of these correlations together with the ones resulting from tree-level scalar exchanges. Therefore we will now turn our attention to the latter exchanges.

5.4.7. Tree-level scalar and pseudoscalar exchanges. A very detailed analysis of tree-level scalar and pseudoscalar tree-level contributions as shown in figure 17 to decays in question has been performed in [44]. In this case, SM Wilson coefficients remain unchanged but the Wilson coefficients of scalar and pseudoscalar operators become non-zero and are given at $\mu = M_{H}$, as follows:

$$m_{b}(M_{H}) \sin^{2} \theta_{W} C_{S} = \frac{1}{2 g_{\text{SM}}} \frac{1}{M_{H}^{2}} \left[ \Delta_{b}^{(H)}(H) \Delta_{S}^{(H)}(H) \right],$$

$$m_{b}(M_{H}) \sin^{2} \theta_{W} C_{P} = \frac{1}{2 g_{\text{SM}}} \frac{1}{M_{H}^{2}} \left[ \Delta_{b}^{(H)}(H) \Delta_{P}^{(H)}(H) \right].$$

where

$$\Delta_{S}^{(H)}(H) = \Delta_{R}^{(H)}(H) + \Delta_{L}^{(H)}(H),$$

$$\Delta_{P}^{(H)}(H) = \Delta_{R}^{(H)}(H) - \Delta_{L}^{(H)}(H).$$

Here, $H$ stands for a scalar or pseudoscalar but if the mass eigenstates have a given CP parity then it is useful to distinguish between a scalar ($H^{0}$) and a pseudoscalar ($A^{0}$). Then,

$$\Delta_{S}^{(A^{0})}(H^{0}) = 0, \quad \Delta_{P}^{(A^{0})}(H^{0}) = 0$$

and only $\Delta_{S}^{(H^{0})}$ and $\Delta_{P}^{(A^{0})}$ can be non-vanishing. This is not a general property, and in fact in the presence of CP-violating effects scalar and pseudoscalars can have both couplings. For simplicity, as in [44], we will assume (157) to be true.

The crucial property of these couplings following from the hermicity of the Hamiltonian is that $\Delta_{\mu}^{(\mu)}$ is real and $\Delta_{P}^{(\mu)}$ is purely imaginary. Therefore, it is useful to work with

$$\Delta_{P}^{(A^{0})}(A^{0}) = i \Delta_{P}^{(A^{0})}(A^{0}),$$

where $\Delta_{P}^{(A^{0})}(A^{0})$ is real.

It should be emphasized that in terms of the couplings used in the analysis of $B_{d} \to \bar{B}_{d}^{0}$ mixing we have generally

$$\Delta_{R}^{(b)}(H) = [\Delta_{L}^{(b)}(H)]^{*}, \quad \Delta_{L}^{(b)}(H) = [\Delta_{R}^{(b)}(H)]^{*},$$

which should be kept in mind when studying the correlations between $\Delta F = 1$ and $\Delta F = 2$ transitions.

Concerning the values of the $\Delta_{P}^{(\mu)}$ and $\Delta_{S}^{(\mu)}$, we will set as in [44]

$$\Delta_{P}^{(\mu)}(H) = \pm 0.020 \frac{m_{b}(M_{H})}{m_{b}(m_{b})}, \quad \Delta_{S}^{(\mu)}(H) = 0.040 \frac{m_{b}(M_{H})}{m_{b}(m_{b})}.$$
In the general case, the blue and purple allowed regions correspond to oases with small and large $s_{23}$, respectively.

In the $U(2)^3$ symmetry case, the allowed regions are shown in magenta and cyan for LHS1 and LHS2, respectively, because in this case even in the $B_s$ system there is dependence on $|V_{ub}|$ scenario. These regions are subregions of the general blue or purple regions, so that they cover some parts of them.

With the latter factor being 0.61 for $M_H = 1$ TeV. We show this factor explicitly to indicate how the correct scale for $m_b$ affects the allowed range for the lepton couplings. These values assure significant NP effects in $B_s \rightarrow \mu^+\mu^-$ while being consistent with all of the known data.

### 5.4.8. Comparison of tree-level $Z'$, pseudoscalar and scalar exchanges

In Figure 18 we show the correlation between $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ and $S_{\phi\phi}$ for $Z'$ (left panel) and $A^0$ (right panel). The corresponding plots for the correlation between $S_{\mu\mu}$ and $A_{\mu\mu}$ and $S_{\phi\phi}$ are shown in Figures 19 and 20. In Figure 21 we show the corresponding results for the scalar $H^0$.

The colour coding is as follows:

- In the general case, blue and purple allowed regions correspond to oases with small and large $s_{23}$, respectively.
- In the $U(2)^3$ symmetry case, the allowed regions are shown in magenta and cyan for LHS1 and LHS2, respectively, because in this case even in the $B_s$ system there is dependence on $|V_{ub}|$ scenario. These regions are subregions of the general blue or purple regions, so that they cover some parts of them.
- The green points in the $Z'$ case indicate the region that is compatible with constraints from $b \rightarrow s\ell^+\ell^-$ transitions. In the scalar and pseudoscalar case, the whole oases are compatible with $b \rightarrow s\ell^+\ell^-$ (see also section 5.7.3).

We observe several striking differences between the results for $Z'$, $A^0$ and $H^0$, which allow us to distinguish these scenarios from each other:

- In the $A^0$ case, the asymmetry $S_{\mu^+\mu^-}$ can be zero while this is not the case for $Z'$ where the requirement of suppression of $\Delta M_s$ directly translates into $S_{\mu^+\mu^-}$ being non-zero. Consequently, in the $Z'$ case the sign of $S_{\mu^+\mu^-}$ can used to identify the right oasis. This is not possible in the case of $A^0$.
- On the other hand, we observe that in the $A^0$ case the measurement of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ uniquely chooses the right oasis. The enhancement of this branching ratio relatively to the SM chooses the blue oasis while suppression chooses the purple one. Present data from LHCb and CMS favour the purple oasis. This distinction is not possible in the $Z'$ case. The maximal enhancements and suppressions are comparable in both cases but finding...
Figure 20. $A^{\lambda}_{\Delta 1}$ versus $S_{\psi \phi}$ in LHS1 and for $Z'$ (left) and pseudoscalar $A^0$ case (right) both for 1 TeV. The magenta region corresponds to the $U(2)^3$ limit for LHS1 and the cyan region for LHS2. The green points in the $Z'$ case indicate the regions that are compatible with $b \rightarrow s \tau^+ \tau^-$ constraints. In the $A^0$ case $b \rightarrow s \tau^+ \tau^-$ does not give additional constraints. Red point: SM central value. Reproduced with permission from [44]. Copyright 2013 SISSA.

Figure 21. $S_{\psi \phi}$ versus $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$, $S'_{\mu^+ \mu^-}$ versus $S_{\psi \phi}$ and $A^{\lambda}_{\Delta 1}$ versus $S_{\psi \phi}$ for scalar $H^0$ case with $M_H = 1$ TeV in LHS1. The two oases (blue and purple) overlap. The magenta region corresponds to the $U(2)^3$ limit for LHS1 and the cyan region for LHS2. Red point: SM central value. Reproduced with permission from [44]. Copyright 2013 SISSA.

$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ close to SM value would require in the $A^0$ case either larger $M_H$ or smaller muon coupling.

- Concerning the $H^0$ case, the absence of the interference with the SM contribution implies that $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ can only be enhanced in this scenario and this result is independent of the oasis considered. Thus, finding this branching ratio below its SM value would favour the other two scenarios over the scalar scenario. The present data from LHCb and CMS indicate that this indeed could be the case. But the enhancement is not as pronounced as in the pseudoscalar case because in the absence of the interference with the SM contribution the correction to the branching ratio is governed here by the square of the muon coupling and is not linearly proportional to it, as in the pseudoscalar case. Therefore, excluding this scenario requires a significant reduction of the experimental errors.

- In addition, CP asymmetries in the $H^0$ case differ from $Z'$ and $A^0$ cases. Similar to the branching ratio, there is no dependence on the oasis considered but, more importantly, $S'_{\mu^+ \mu^-}$ can only increase with increasing $S_{\psi \phi}$. 

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The correlation between \( A_{\lambda}/\Delta_1/Gamma_1 \) and \( S_{\psi\phi} \) has the same structure for \( Z' \), \( A_0 \) and \( H_0 \) cases. We observe that for \( M_H = 1 \) TeV, even for \( S_{\psi\phi} \) significantly different from zero, \( A_{\lambda}/\Delta_1/Gamma_1 \) does not differ significantly from unity in \( A_0 \) and \( H_0 \) scenarios. Larger effects for the same mass are found in the \( Z' \) case.

In figure 22 we summarize our results in the \( B_s \) system for tree-level \( Z' \), \( H_0 \) and \( A_0 \) exchanges, where we also vary the lepton couplings in a wider range: \(|\Delta_{1\mu\mu}^{CP}(H)| \in [0.012, 0.024]\) and \( \Delta_{1\mu\mu}^{CP}(Z') \in [0.3, 0.7] \). As explained in [44], the striking differences between the \( A_0 \)-scenario and \( Z' \)-scenario can be traced back to the difference between the phase of the NP correction to the quantity \( P \), defined in (112), in these two NP scenarios. Since the oasis structure as far as the phase \( \delta_{23} \) is concerned is the same in both scenarios, the difference enters through the muon couplings, which are imaginary in the case of \( A_0 \)-scenario but real in the case of \( Z' \). In addition, by taking into account the sign difference between \( Z' \) and pseudoscalar propagator in the \( b \rightarrow s\mu^+\mu^- \) amplitude, which is now not compensated by a hadronic matrix element, one finds that

\[
P(Z') = 1 + r_{Z'} e^{i\delta_{23}}, \quad P(A_0) = 1 + r_{A_0} e^{i\delta_{23}} \tag{161}
\]

with

\[
 r_{Z'} \approx r_{A_0}, \quad \delta_{Z'} = \delta_{23} - \beta_s, \quad \delta_{A_0} = \delta_{Z'} - \frac{\pi}{2}. \tag{162}
\]

Therefore, with \( \delta_{23} \) of figure 8 the phase \( \delta_{Z'} \) is around 90° and 270° for the blue and purple oasis, respectively. Correspondingly, \( \delta_{A_0} \) is around 0° and 180°. This difference in the phases is at the origin of the differences listed above. In particular, we understand now why the CP asymmetry \( S_{\mu\mu} \) can vanish in the \( A_0 \) case, while it was always different from zero in the \( Z' \)-case. What is interesting is that this difference is only related to the different particle exchanged: gauge boson and pseudoscalar. We summarize the ranges of \( \delta_{Z'} \) and \( \delta_{A_0} \) in table 7.

By proceeding in an analogous manner for the scalar case we arrive at an important relation:

\[
\phi_S = \delta_{Z'} - \pi, \tag{163}
\]

where the shift is related to the sign difference in the \( Z' \) and scalar propagators. But, as seen in (114)–(116), the three observables given there all depend on \( 2\phi_S \), implying that this shift is irrelevant from the point of view of these quantities. Given that different oases correspond to phases shifted by \( \pi \), this also explains why in the scalar case the results in different oases are the same. That the branching ratio can only be enhanced follows only from the absence of the interference with the SM contributions. In order to understand the signs in \( S_{\mu\mu} \), one should note the minus sign in front of sine in
the corresponding formula. The rest follows from (163) and table 7.

We now turn our attention to the $B_d \to \mu^+\mu^-$ decay. Here, we have to distinguish between LHS1 and LHS2 scenarios. Our colour coding is such that in the general case yellow and green allowed regions correspond to oases with small and large $\delta_{13}$, respectively. We do not show the impact of the imposition of the $U(2)^3$ symmetry because the resulting reduction of the allowed areas amounts typically to 5–10% at most and it is, therefore, more transparent not to show it.

In figures 23 and 24 we show $S_{\rho K_\pi}$ versus $B(B_d \to \mu^+\mu^-)$ and $S_{\mu\mu}$ versus $B(B_d \to \mu^+\mu^-)$ for the $Z'$ scenario. The corresponding plots for the $A^0$ and $H^0$ scenarios are shown in figures 25–28.

In order to understand the differences between these two scenarios of NP, we again look at the phase of the correction to $P$ in (161), which is now given as follows:

$$r_{Z'} \approx r_{A^0}, \quad \delta_{Z'} = \delta_{13} - \beta, \quad \delta_{A^0} = \delta_{Z'} - \frac{\pi}{2}. \quad (164)$$

Note that this time the phase of $V_{ud}$ enters the analysis with $\beta \approx 19^\circ$ and $\beta \approx 25^\circ$ for S1 and S2 scenario of $|V_{ub}|$, respectively. We find then in scenario S2 the phase $\delta_{Z'}$ is around $115^\circ$ and $295^\circ$ for yellow and green oases, respectively. Correspondingly, $\delta_{A^0}$ is around $25^\circ$ and $205^\circ$. We summarize the ranges of $\delta_{Z'}$ and $\delta_{A^0}$ in table 7.

With this insight at hand we can easily understand the plots in question, noting that the enhancements and suppressions of $S_{\rho K_\pi}$ are governed by the cosine of the phase and the signs of $S_{\mu\mu}$ by the corresponding sines. We leave this exercise to the motivated readers and refer to [44] for a detailed description of the plots. What is interesting is that the suppressions or enhancements of certain observables and correlations or anti-correlations between them could tell us one day which of the three NP scenarios, if any, is favoured by nature. In fact, if the present central experimental value for $S_{\mu\mu}$ will be confirmed by more precise measurements tree-level $Z'$, $A^0$ and $H^0$ exchanges will not be able to describe such data alone when the constraints from $\Delta F = 2$ transitions are taken into account.

Finally, let us make a few comments on the impact of the imposition of the $U(2)^3$ symmetry. The main effect is on $B_s \to \mu^+\mu^-$, and we have shown this in all of the above plots. Presently, most interesting in this context is the correlation between $S_{\phi\phi}$ and $B(B_s \to \mu^+\mu^-)$. We observe that already the sign of $S_{\phi\phi}$ will decide whether LHS1 or LHS2 is favoured. Moreover, if $B(B_s \to \mu^+\mu^-)$ will turn out to be suppressed relative to the SM then only one oasis will survive in each scenario. A comparison with future precise value of $|V_{ub}|$ will confirm or rule out this scenario of NP. These correlations are particular examples of the correlations in $U(2)^3$ models, as pointed out in [39]. What is new here is that in a specific model considered by us, the $|V_{ub}| = S_{\phi\phi}$ correlation now also has implications not only for $B(B_s \to \mu^+\mu^-)$ but also for $S_{\mu\mu}$, as seen in other plots. Analogous comments can be made in the case of $A^0$ and $H^0$.

### Table 7

| Oasis | $\delta_{Z'}$ | $\delta_{A^0}$ |
|-------|-------------|---------------|
| $B_s$ (blue) | $50^\circ-130^\circ$ | $-40^\circ-(-40^\circ)$ |
| $B_s$ (purple) | $230^\circ-310^\circ$ | $140^\circ-220^\circ$ |
| $B_s$ (S1) (yellow) | $57^\circ-86^\circ$ | $-33^\circ-(-44^\circ)$ |
| $B_s$ (S1) (green) | $237^\circ-266^\circ$ | $147^\circ-176^\circ$ |
| $B_s$ (S2) (yellow) | $103^\circ-125^\circ$ | $13^\circ-35^\circ$ |
| $B_s$ (S2) (green) | $283^\circ-305^\circ$ | $193^\circ-215^\circ$ |
| $U(2)^3$ (S1) (blue, magenta) | $55^\circ-84^\circ$ | $-35^\circ-(-6^\circ)$ |
| $U(2)^3$ (S1) (purple, magenta) | $235^\circ-264^\circ$ | $145^\circ-174^\circ$ |
| $U(2)^3$ (S2) (blue, cyan) | $101^\circ-121^\circ$ | $11^\circ-31^\circ$ |
| $U(2)^3$ (S2) (purple, cyan) | $291^\circ-301^\circ$ | $201^\circ-211^\circ$ |

5.4.9. Dependence of $\Delta F = 1$ transitions on $M_{Z'}$. The nominal value of $M_{Z'}$ in the plots presented in this review is 1 TeV, except for few cases where higher values are considered. The results for $M_{Z'} = 3$ TeV in 331 models can be found in [41, 47]. Here, following [40, 47], we would like to summarize how our results for $\Delta F = 1$ transitions can be translated into other values of $M_{Z'}$ in case higher values would be required by the LHC and other constraints in a given model. In this translation, the lepton couplings have to be held fixed.

Since presently the constraints on $Z'$ models are dominated by $\Delta F = 2$ transitions, it turns out that for a given allowed size of $\Delta S(B_{s})$, NP effects in the one-loop $\Delta F = 1$ functions are proportional to $1/M_{Z'}$. That these effects are only suppressed like $1/M_{Z'}$ and not like $1/M_{Z'}^2$ is the consequence of the increase with $M_{Z'}$ of the allowed values of the couplings $\Delta_{L,R}(Z')$ extracted from $\Delta F = 2$ observables. When NP effects are significantly smaller than the SM contribution, only interference between SM and NP contributions matters and, consequently, this dependence is transferred to the branching ratios. In summary, by denoting $\Delta \mathcal{O}_{NP}(M_{Z'}^{(i)})$ NP contributions to a given $\Delta F = 1$ observable in $B_s$ and $B_d$ decays at two $(i = 1, 2)$ different values $M_{Z'}^{(i)}$ we have a scaling law

$$\Delta \mathcal{O}_{NP}(M_{Z'}^{(1)}) = \frac{M_{Z'}^{(2)}}{M_{Z'}^{(1)}} \Delta \mathcal{O}_{NP}(M_{Z'}^{(2)}). \quad (165)$$

This scaling law is valid in most of observables in $B_s$ and $B_d$ systems as NP effects are bounded to be small. In the rare $K$ decays, like $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, where NP contributions for sufficiently low values of $M_{Z'}$ could be much larger than the SM contribution, NP modifications of branching ratios will decrease faster than $1/M_{Z'}$ $(1/M_{Z'}^2$ in the limit of full NP dominance), until NP contributions are sufficiently small that the $1/M_{Z'}$ dependence and (165) is once again valid.

Needless to say, when also lepton couplings can be varied in order to compensate for the change of $M_{Z'}$, the scaling law could be modified. In this case, the correlations between NP corrections to various one-loop functions, derived in [40, 47], are helpful in translating our results into those obtained for different $M_{Z'}$ and lepton couplings. We refer in particular to [47], where using the data from LEP-II, CMS and ATLAS the bounds on $M_{Z'}$ in various 331 models with different lepton couplings have been analysed.
5.4.10. Flavour-violating SM Z and SM Higgs boson. Let us next look at the possibility that NP will only be detectable through modified Z and Higgs couplings. Beginning with flavour-violating Z-couplings, they can be generated in the presence of other neutral gauge bosons and or new heavy vectorial fermions with $+2/3$ and $-1/3$ electric charges. RSc is an explicit model of this type [49, 167] (see also [168]). Recently, an extensive analysis of flavour violation in the presence of a vectorial $+2/3$ quark has been presented in [169], where references to the previous literature can be found.

The formalism developed for $Z'$ can be used directly here by setting

$$M_Z = 91.2 \, \text{GeV}, \quad \Delta_L^{\nu}(Z) = \Delta_A^{\mu\nu}(Z) = 0.372, \quad \Delta_V^{\mu\nu}(Z) = -0.028.$$  \tag{166}$$

The implications of these changes are as follows:

- The decrease of the neutral gauge boson mass by an order of magnitude relatively to the nominal value $M_{Z'} = 1 \, \text{TeV}$ used by us decreases the couplings $\tilde{s}_{ij}$ by the same amount.
• Without any impact on the phases $\delta_{ij}$ when the constraints from $\Delta F = 2$ processes are imposed.

• As pointed out in [40], once the parameters $\tilde{s}_{ij}$ are constrained through $\Delta F = 2$ observables the decrease of neutral gauge boson mass enhances NP effects in rare $K$ and $B$ decays. This follows from the structure of tree-level contributions to FCNC processes and is not generally the case when NP contributions are governed by penguin and box diagrams.

• The latter fact implies that already the present experimental bounds on $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(B_{s,d} \to \mu^+\mu^-)$, as well as the data on $B \to X_s\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$ and $B \to K\ell^+\ell^-$ decays, become more powerful than the $\Delta F = 2$ transitions in constraining flavour-violating couplings of $Z$, so that effects in the $\Delta F = 2$ processes cannot be as large as in the $Z'$ case.

The patterns of flavour violation through $Z$ in $B_s$, $B_d$ and $K$ are strikingly different from each other:
In the $B_s$ system, when the constraints from $\Delta M_f$ and $S_{\psi\phi}$ are imposed $B(B_s \to \mu^+\mu^-)$ is always larger than its SM value and is mostly above the data, except in the LRS case where NP contributions vanish. Further constraints follow from $b \to s\ell^+\ell^-$ transitions, so that one has to conclude that it is very difficult to suppress $\Delta M_f$ sufficiently in LHS, LRS and RHS scenarios without violating the constraints from $b \to s\mu^+\mu^-$ transitions. Thus, we expect $B(B_s \to \mu^+\mu^-)$ to be enhanced over the SM value but simultaneously possible tension in $\Delta M_f$ cannot be solved if the relevant parameters are like those in $(36)$. Future lattice calculations will tell us whether this is indeed a problem. Similar conclusions have been reached in $[38, 170]$. Yet, as demonstrated recently in $[46]$, by changing the non-perturbative parameters agreement with both data on $\Delta F = 2$ observables and $B(B_s \to \mu^+\mu^-)$ can be obtained, we will summarize this analysis below.

In the $B_d$ system all $\Delta F = 2$ constraints can be satisfied. We again observe that $B(B_d \to \mu^+\mu^-)$ can be enhanced by almost an order magnitude and this begins to be a problem for certain choices of couplings in view of recent LHCb and CMS data. This is shown in figure 29 for the LHS1 and LHS2 scenarios. Evidently, NP effects are much larger than in the $Z'$ case. We also show the results in ALRS1 and ALRS2 scenarios in which NP effects are smaller than in the LHS1 and LHS2 scenarios. With an improved upper bound on $B(B_d \to \mu^+\mu^-)$, LHS1 and LHS2 scenarios could be put into difficulties, while in ALRS1 and ALRS2 one could easier satisfy this bound. If such a situation really took place and NP effects would be observed in this decay, this would mean that both LH and RH $Z$-couplings in the $B_d$ system would be required but with opposite sign.

As we will see in step 8, the effects of flavour-violating $Z$ couplings in $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ can be, in principle, very large in LHS, RHS and LRS scenarios but they can be bounded by the upper bound on $K_L \to \mu^+\mu^-$, except for the LR scenarios and the case of purely imaginary NP contributions in all these scenarios where this bound is ineffective. We show in step 8 in figure 37 a few examples which demonstrate that even with the latter constraint taken into account flavour violating $Z$ can have impact on rare $K$ decays, which is significantly larger than in the $Z'$ case.

In summary, flavour-violating $Z$ couplings in $B_d \to \mu^+\mu^-$ decay, similarly to $Z'$ couplings in rare $K$ decays discussed in step 8, could turn out to be an important portal to SD scales that cannot be explored by the LHC. However, for the $B_s \to \mu^+\mu^-$ decay this does not seem to be the case any longer.

Concerning the tree-level SM Higgs contributions to FCNCs, one finds that once the constraints on flavour-violating couplings from $\Delta F = 2$ observables are imposed, the smallness of Higgs couplings to muons precludes any measurable effects in $B(B_d \to \mu^+\mu^-)$ and $B(B_s \to \mu^+\mu^-)$ can be only enhanced by at most 8% $[44]$. Still, the presence

![Figure 29](image-url)
of such contributions can remove all possible tensions within the SM in $\Delta F = 2$ transitions without being in conflict with constraints from rare decays.

Similarly to modifications of $Z$ and SM Higgs couplings, couplings of $W^\pm$ could also be modified by NP. There are many papers studying the implications of such modifications for FCNC processes. We refer to the recent detailed analysis in [171], where further references can be found. In particular the constraints on the anomalous $tWb$ interactions turn out to be superior to present direct constraints from top decays and production measurements at Tevatron and the LHC.

5.4.11. Facing the violation of CMFV relation (14). As shown in figure 15, the stringent CMFV relation in (14) appears to be violated by the present data. Even if this violation is still not statistically significant in view of very inaccurate data on $B_d \rightarrow \mu^+\mu^-$, it is of interest to see whether tree-level exchanges of $Z'$ and $Z$ could with a certain choices of quark and lepton couplings reproduce these data while satisfying $\Delta F = 2$ constraints and the constraints from $B_d \rightarrow K^*(K)\mu^+\mu^-$ considered in step 7. As in the numerical analysis presented so far, NP in $\Delta F = 2$ processes was governed by (36) and consequently $C_{B_s} \approx C_{B_d} \approx 0.93$, it is also interesting to see what happens when these values are modified.

Such an analysis has been recently performed in [46] by concentrating on the LHS scenario, which as discussed in step 7 gives a plausible explanation of the $B_d \rightarrow K^*(K)\mu^+\mu^-$ data. Its outcome can be briefly summarized as follows:

- The LHS scenario for $Z'$ or $Z$ FCNC couplings provides a simple model that allows for the violation of the CMFV relation between the branching ratios for $B_{d,s} \rightarrow \mu^+\mu^-$ and $\Delta M_{s,d}$. The plots in figures 30 and 31 for $Z'$ and $Z$, respectively, illustrate this.

- However, to achieve this in the case of $Z'$, the experimental value of $\Delta M_{s}$ must be very close to its SM value ($C_{B_s} = 1.00 \pm 0.01$) and $\Delta M_{s}$ is favoured to be a bit larger than $(\Delta M_{s})_{SM}$ ($C_{B_s} = 1.04 \pm 0.01$). $S_{\phi\phi}$ can still deviate significantly from its SM value.

- In the case of $Z$, both $\Delta M_{s}$ and $S_{\phi\phi}$ must be rather close to their SM values while $\Delta M_{d}$ is favoured to be smaller than $(\Delta M_{d})_{SM}$ ($C_{B_d} = 0.96 \pm 0.01$).

Details on the dependence of the correlation between branching ratios for $B_{d,s} \rightarrow \mu^+\mu^-$ and the CP-asymmetries $S_{\phi\phi}$ and $S_{\phi K^*_s}$ on the values of $C_{B_s}$ and $C_{B_d}$ can be found in [46].
The anatomy of the plots in figures 30 and 31 is also presented there. With the improved data and increased precision of lattice calculations, such plots will be more informative than they are presently.

5.4.12. \( B(B_s \rightarrow \mu^+\mu^-) \) as an electroweak precision test (EWPT). Our review deals predominately with flavour violation. Yet, in particular, NP models the relations between flavour-violating and flavour-conserving couplings exist, so that additional correlations between flavour-violating and flavour-conserving processes are present. Such correlations can involve: on the one hand, LH \( Zb\bar{b} \) and \( Zb\bar{s} \) couplings and, on the other hand, corresponding RH couplings. In particular, it is known that the measured RH \( Zb\bar{b} \) coupling disagrees with its SM value by 3\( \sigma \). In some NP models, the physics responsible for this anomaly can through correlations also have an impact on FCNC processes.

Such a correlation has been pointed out first in [172], and analysed in detail in the context of MFV in [173]. At that time the information on \( Z \rightarrow bb \) couplings was by far superior to that from \( B_s \rightarrow \mu^+\mu^- \), so that the bounds on possible deviations of \( Z \rightarrow bb \) from their SM values implied interesting bounds on FCNC processes, including \( B_s \rightarrow \mu^+\mu^- \). As pointed out recently in [142], the situation is now reversed and the present data on \( B(B_s \rightarrow \mu^+\mu^-) \) already set the dominant constraints on possible modified flavour diagonal \( Z \)-boson couplings. In the case of MFV models, where significant NP effects are expected only in LH \( Z \)-couplings, the present bound derived in [142] from \( B(B_s \rightarrow \mu^+\mu^-) \) is not much stronger than the one derived from \( Z \rightarrow bb \). On the other hand, in generic models with partial compositeness \( B(B_s \rightarrow \mu^+\mu^-) \) sets already now constraint on the RH \( Zb\bar{b} \) coupling that is significantly more stringent than that obtained from \( Z \rightarrow bb \). As a result, in this class of models the present anomaly in the RH \( Zb\bar{b} \) coupling cannot be explained. Needless to say, such constraints on the diagonal \( Zb\bar{b} \) coupling will become even more powerful when the measurement of \( B(B_s \rightarrow \mu^+\mu^-) \) improves so that this decay will offer EWPTs.

5.4.13. \( B_{d,s} \rightarrow \tau^+\tau^- \). The leptonic decays \( B_{d,s} \rightarrow \tau^+\tau^- \) could one day play a significant role in the tests of NP models. In particular, interesting information on the interactions of new particles with the third generation of quarks and leptons could be obtained in this manner. In the SM, the branching ratios in question are enhanced by roughly two orders of magnitude over the corresponding decays to the muon pair:

\[
\frac{B(B_q \rightarrow \tau^+\tau^-)}{B(B_q \rightarrow \mu^+\mu^-)} \approx \sqrt{1 - \frac{4m_{\text{t}}^2}{m_{\text{b}}^2 m_{\text{\mu}}^2}} \approx 210. \quad (167)
\]

Tree-level exchange of a neutral SM Higgs with quark flavour violating couplings could become important, and the same applies to tree-level heavy scalar and pseudoscalar exchanges. Although there are presently no experimental limits on these decays, the interplay with \( \Gamma_{\tau^\pm} \), and the latest measurements of \( \Gamma_{\tau^\pm}/\Gamma_{\tau^0} \) by LHCb would imply the upper bound for branching ratio for \( B_0 \rightarrow \tau^+\tau^- \) of 3% at 90% CL [174, 175]. Due to significant experimental challenges of observing these decays at the LHCb, it is unlikely that we will benefit from them in this decade and, therefore, we will not discuss them further.

5.5. Step 5: \( B^+ \rightarrow \tau^+\nu_\tau \)

5.5.1. Preliminaries. We now look at the tree-level decay \( B^+ \rightarrow \tau^+\nu_{\tau} \), which was the subject of great interest in the previous decade as the data from BaBar [176] and Belle [177] implied a world average in the ballpark of \( \mathcal{B}(B^+ \rightarrow \tau^+\nu_{\tau})_{\exp} = (1.73 \pm 0.35) \times 10^{-4} \), roughly higher than the SM value by a factor of 2. Meanwhile, this situation has changed considerably due to 2012 data from Belle [178], so that the present world average that combines BaBar and Belle data reads [112]

\[
\mathcal{B}(B^+ \rightarrow \tau^+\nu_{\tau})_{\exp} = (1.14 \pm 0.22) \times 10^{-4}, \quad (168)
\]

which is fully consistent with the values quoted in table 4, with some preference for the inclusive values of \( |V_{ub}| \). Yet, the rather large experimental error and parametric uncertainties in the SM prediction still allow, in principle, for sizable NP contributions.

In this context one should recall that one of our working assumptions was the absence of significant NP contributions to decays governed by tree diagrams. Yet, the decay in question could be one of the exceptions because it is governed by the smallest element of the CKM matrix \( |V_{ub}| \) and its branching ratio is rather small for a tree-level decay. We will, therefore, briefly discuss it in the simplest extensions of the SM.

The motivation for this study is the sensitivity of this decay to the new heavy charged gauge bosons and scalars that we did not encounter in the previous steps, where neutral gauge bosons and neutral scalars and pseudoscalars dominated the scene.

5.5.2. SM results. In the SM, \( B^+ \rightarrow \tau^+\nu_{\tau} \) is mediated by the \( W^\pm \) exchange, with the resulting branching ratio given by

\[
\mathcal{B}(B^+ \rightarrow \tau^+\nu_{\tau})_{\SM} = \frac{G_F^2 m_B m_{B^+}}{8\pi} \left(1 - \frac{m_{\tau}^2}{m_B^2} \right)^2 \left(\frac{F_{B^+}}{m_B}\right)^2 |V_{ub}|^2 \tau_{\tau^+}. \quad (169)
\]

Evidently, this result is subject to significant parametric uncertainties induced in (169) by \( F_{B^+} \) and \( |V_{ub}| \). However, recently the error on \( F_{B^+} \) from lattice QCD has decreased significantly, so that the dominant uncertainty comes from \( |V_{ub}| \). Indeed, as seen in table 4, for fixed remaining input parameters, varying \( |V_{ub}| \) in the range shown in this table modifies the branching ratio by roughly a factor of two.

In the literature in order to find the SM prediction for this branching ratio, one eliminates these uncertainties by using \( \Delta M_{ds} \), \( \Delta M_{d}/\Delta M_{s} \) and \( S_{bK_s} \), [35, 179], and taking experimental values for these three quantities. To this end, \( F_{B^+} = F_{B_s} \) is assumed in agreement with lattice values. This strategy has a weak point because the experimental values of \( \Delta M_{ds} \) used in this strategy may not be the ones corresponding to the true value of the SM. However, proceeding in this manner one finds [35]

\[
\mathcal{B}(B^+ \rightarrow \tau^+\nu_{\tau})_{\SM} = (0.80 \pm 0.12) \times 10^{-4}, \quad (170)
\]

with a similar result obtained by the UTfit collaboration [179]. As seen in table 4, this result corresponds to \( |V_{ub}| \) in the
ballpark of $3.6 \times 10^{-3}$ and is fully consistent with the data in (168).

Unfortunately, the full clarification of a possible presence of NP in this decay will have to wait for the data from SuperKEKB. In the meantime, hopefully, the error on $F_{B\tau}$ from lattice QCD will be further reduced and theoretical advances in the determination of $|V_{ub}|$ from tree-level decays will be made, allowing us to make a precise prediction for this decay without using the experimental value for $\Delta M_d$.

It should be emphasized that for a low value of $|V_{ub}|$, the increase of $F_{B\tau}$, while enhancing the branching ratio in question, would also enhance $\Delta M_d$, which in view of our discussion in step 3 is not favoured by the data. On the other hand, the increase of $|V_{ub}|$ while enhancing $B(B^+ \to \tau^+\nu)_{\text{SM}}$ would also enhance $S_{\theta_K}$, shifting it away from the data. This discussion clearly shows that it will be difficult to use this decay for the identification of NP before all of these parameters are known significantly more precisely than is the case now. In fact, the decays $B_{s,d} \to \mu^+\mu^-$ are presently in a much better shape than $B^+ \to \tau^+\nu$ because they are governed by $|V_{ts}|$, which is presently much better known than $|V_{ub}|$.

In view of this uncertain situation, our look at the simplest models providing new contributions to this decay will be rather brief.

5.5.3. CMFV. To our knowledge the $B^+ \to \tau^+\nu\tau$ decay has never been considered in CMFV. Here, we would like to point out that in this class of models the branching ratio for this decay is enhanced (suppressed) for the same (opposite) sign of $\tan \beta$, which is presently much better known than $|V_{ub}|$.

In view of this uncertain situation, our look at the simplest models providing new contributions to this decay will be rather brief.

5.5.4. 2HDM$^{\text{MFV}}$. Interestingly, when the experimental branching ratio was significantly above its SM value, the tension between theory and experiment in the case of $B(B^+ \to \tau^+\nu)$ increased in the presence of a tree-level $H^\pm$ exchange. Indeed, such a contribution interferes destructively with the $W^\pm$ contribution if there are no new sources of CP violation.

This effect was calculated a long time ago by Hou [180] and was in modern times first calculated by Akeroyd and Recksiegel [181], and later by Isidori and Paradisi [182] in the context of the MSSM. The same expression is valid in the 2HDM$^{\text{MFV}}$ framework and is given as follows [183]:

$$B(B^+ \to \tau^+\nu)_{\text{2HDM}^{\text{MFV}}} = B(B^+ \to \tau^+\nu)_{\text{SM}} \times \left[ 1 - \frac{m_B^2}{m_H^2} \left( 1 + \frac{\tan^2 \beta}{1 + \epsilon_i} \right) \right]^2. \quad (172)$$

In the MSSM $\epsilon_i$ are calculable in terms of supersymmetric parameters. In 2HDM$^{\text{MFV}}$ they are just universal parameters that can enter other formulae, implying correlations between various observables. If $\epsilon_i$ are real, positive definite numbers, similar to MSSM, then in this model this branching ratio can be strongly suppressed unless the choice of model parameters is such that the second term in the parenthesis is larger than 2. Such a possibility, which would necessarily imply a light charged Higgs and large $\tan \beta$ values, seems to be very unlikely in view of the constraints from other observables, as stressed in the past in the context of MSSM in [184] and more recently in the context of the 2HDM$^{\text{MFV}}$ in [183].

However, Isidori and Blankenburg point out that in 2HDM$^{\text{MFV}}$, where $\epsilon_0$ and $\epsilon_1$ are complex numbers

$$1 + (\epsilon_0 + \epsilon_1) \tan \beta \leq 0 \quad (173)$$

is possible provided that $\tan \beta$ is large. But then these authors find $B(B \to X_{\tau\nu})$ to be suppressed relative to the SM, which is not favoured by the data. We will discuss this issue in the next step.

Let us stress in this context that the subscript ‘SM’ in (172) could be misleading because what is really meant there is the formula for this decay in the SM. While the SM selects the low (exclusive) value for $|V_{ub}|$, in order to be in agreement with the experimental value of $S_{\theta_K}$, 2HDM$^{\text{MFV}}$ chooses the large (inclusive) value of $|V_{ub}|$ in order to be consistent with experimentally observed value of $\epsilon_K$. The resulting problem with $S_{\theta_K}$ is then solved as discussed in step 3 by new phases in $B_{s,d}^0 - \bar{B}_{s,d}^0$ mixing. But with the inclusive value of $|V_{ub}|$, $B(B^+ \to \tau^+\nu)$ is enhanced and, as seen in table 4, is in agreement with the data that can be obtained.

It appears then that the simplest solution to the possible problem with $B(B^+ \to \tau^+\nu)$ in this model is the absence of relevant charged Higgs contributions to this decay and a sufficiently large value of $|V_{ub}|$.

5.5.5. Tree-level charged gauge boson exchange. Let us write the effective Hamiltonian for the exchange of a charged gauge bosons $W^\pm$ contributing to $B^+ \to \tau^+\nu\tau$, as follows:

$$H_{\text{eff}} = C_L O_L + C_R O_R, \quad (174)$$

where

$$O_L = \langle \bar{b}\gamma_\mu P_L u \rangle \langle \bar{\tau}_\nu\gamma^\mu P_L \tau^- \rangle, \quad (175)$$

$$O_R = \langle \bar{b}\gamma_\mu P_R u \rangle \langle \bar{\tau}_\nu\gamma^\mu P_L \tau^- \rangle$$
and
\[ C_L = C_{L}^{SM} + \frac{\Delta_{L}^{ab}(W^*)\Delta_{L}^{ab}(W^*)}{M_{W^*}^2}, \]
\[ C_R = \frac{\Delta_{R}^{ab}(W^*)\Delta_{R}^{ab}(W^*)}{M_{W^*}^2} \]  
(176)

with \( C_{L}^{SM} \) having the same structure as the correction from \( W^* \) with
\[ \Delta_{L}^{ab} = \frac{g}{\sqrt{2}} V_{ab}, \quad \Delta_{L}^{ab} = \frac{g}{\sqrt{2}} \Delta_{R}^{ab} = 0. \]  
(177)

The couplings \( \Delta_{L,R}^{ab}(W^*) \) could be complex numbers and contain new sources of flavour violation.

5.6.1. SM results. The radiative decays in question, in particular \( B \rightarrow X_s \gamma \), have played an important role in constraining NP in the last two decades because both the experimental data and the theory have been in good shape for some time.

The Hamiltonian in the SM is given as follows:
\[ H_{\text{eff}}(b \rightarrow s \gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts} V_{tb} \left[ C_{7\gamma}(\mu_b) Q_{7\gamma} + C_{5\gamma}(\mu_b) Q_{5\gamma} \right]. \]  
(182)

where \( \mu_b = O(m_b) \). The dipole operators are defined as
\[ Q_{7\gamma} = \frac{e}{16\pi^2} m_b \delta_\alpha \sigma^{\mu\nu} P_\mu b_\nu F_{\mu\nu}, \]
\[ Q_{5\gamma} = \frac{g_5}{16\pi^2} m_b \delta_\alpha \sigma^{\mu\nu} P_\mu T^{\alpha}_{\mu\nu} b_\nu G^{\mu\nu}. \]  
(183)

While we do not show explicitly the four-quark operators in (182), they are very important for decays considered in this step, particularly as far as QCD and electroweak corrections are concerned.

The special role of these decays is that quite generally they are loop generated processes. As such, they are sensitive to NP contributions and in contrast to tree-level FCNCs mediated by neutral gauge bosons and scalars depend on the masses and couplings of new heavy fermions. But of course new heavy gauge bosons and scalars contribute to these decays in many models. At the CKM-suppressed level, tree-level \( b \rightarrow u \gamma s \gamma \) transitions can also contribute but they are small for the photon energy cut-off 1.6 GeV that is usually used [194].

The NNLO QCD calculations of \( B(B \rightarrow X_s \gamma) \), that involve a very important mixing of dipole operators with current-current operators, have been in the last decade at the forefront of perturbative QCD calculations in weak decays. The first outcome of these efforts, which included the dominant NNLO corrections, was already a rather precise prediction within the SM [195]
\[ B(B \rightarrow X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}, \]  
(183)

for \( E_\gamma \geq 1.6 \) GeV. Since then, several new perturbative contributions have been evaluated [194, 196–202]. Most recently, the \( Q_{1,2} - Q_7 \) interference was found in the \( m_t = 0 \) limit [203]. An updated NNLO prediction should be available soon.

4 For a historical account of NLO and NNLO corrections to this decay see [52].
Experimentalists have also made impressive progress in measuring this branching ratio, reaching an accuracy of 6.4% [112]:

$$B(B \to X_s \gamma)_{\text{exp}} = (3.43 \pm 0.22) \times 10^{-4}. \quad (185)$$

One expects that in this decade the SuperKEKB will reach the accuracy of 3%, so that very precise tests of the SM and its extensions will be possible. Comparing the theory with the experiment we observe that the experimental value is a bit higher than the theory, although the experimental and theoretical errors decrease down to 3% without the change in central values, we will be definitely talking about an anomaly and models in which this branching ratio will be enhanced over the SM result will be favoured. Yet, such models also have to satisfy other constraints.

In principle, a very sensitive observable to NP CP-violating effects is the direct CP asymmetry in $b \to s \gamma$, that is, $A_{CP}(b \to s \gamma)$ [204], because the perturbative contributions within the SM amount to only 40.5% [205–207]. Unfortunately, the analysis [208] shows that this asymmetry, similar to other direct CP asymmetries, suffers from hadronic uncertainties originating here in the hadronic component of the photon. These uncertainties lower the predictive power of this observable. Consequently, we do not consider this asymmetry as a superstar of flavour physics and will not include it in our investigations. Similar comments apply to the $B \to X_d \gamma$ decay, although the CP averaged branching ratio could still provide useful results. Yet, we will also leave this decay from our discussion because the remaining observables considered in our paper are evidently more effective in the search for NP from the present perspective.

Comparing the $B \to V \gamma$ decay, we refer first to two fundamental papers that include NLO QCD corrections [209, 210]. While the branching ratios can already offer useful information, even more promising is the time-dependent CP asymmetry in $B \to K^* \gamma$ [211–213]:

$$\Gamma(B^0(t) \to K^{0\gamma}) - \Gamma(B^0 \to K^{0\gamma}(t)) = S_{K^*\gamma} \sin(\Delta M_B t) - C_{K^*\gamma} \cos(\Delta M_B t). \quad (186)$$

In particular, $S_{K^*\gamma}$ offers a very sensitive probe of RH currents. It vanishes for $C_{7\gamma} \to 0$ and, consequently, in the SM, being suppressed by $m_s/m_b$, is very small [213]:

$$S_{K^*\gamma}^\text{SM} = (-2.3 \pm 1.6)\%. \quad (187)$$

A useful and rather accurate expression for $S_{K^*\gamma}$ has been provided in [212]

$$S_{K^*\gamma} \simeq \frac{2}{|C_{\gamma\gamma}|^2 + |C_{7\gamma}|^2} \text{Im} \left( e^{-ib_d} C_{\gamma\gamma} C_{7\gamma}^* \right), \quad (188)$$

with Wilson coefficients evaluated at $\mu = m_b$ and $\sin(\delta_d) = \sin(\delta_{K^*\gamma})$.

On the experimental side, while the present value of $S_{K^*\gamma}$ is rather inaccurate [214–216]

$$S_{K^*\gamma}^\text{exp} = -0.16 \pm 0.22, \quad (189)$$

the prospects for accurate measurements at SuperKEKB are very good [19].

Isospin asymmetries in $B \to V \gamma$ also provide interesting tests of the SM and of NP. A detailed recent analysis with references to earlier papers can be found in [217]. On the experimental side, the isospin asymmetry in $B \to K^* \gamma$ agrees with the SM, while a $2\sigma$ deviation from the SM is found in the case of $B \to \rho \gamma$ [112].

5.6.2. $B \to X_s \gamma$ beyond the SM. Our discussion of NP contributions to this decay will be very brief. The latest review can be found in [218] and a detailed analysis of the impact of anomalous $Wtb$ couplings has been presented in [219], where further references to the earlier literature can be found.

Since the SM agrees well with the data, NP contributions can be at most in the ballpark of 20% at the level of the branching ratio and they should rather be positive than negative. Consequently, this decay will mainly bound the parameters of a given extension of the SM. Here, we only make a few comments.

It is known that $B \to X_s \gamma$ can bound the allowed range of the values of charged Higgs ($H^\pm$) mass and of $\tan\beta$, both in 2HDM and the MSSM. In 2HDM II, the contribution of $H^\pm$ enhances the branching ratio and $M_{H^\pm}$ must be larger than 300 GeV for any value of $\tan\beta$. In the MSSM, this enhancement can be compensated by chargino contributions and the bound is weaker.

As we already stated and discussed in more detail in [218], the fact that the SM prediction is below the data presently favours the models that allow for an enhancement of the branching ratio and disfavours those in which only suppression is possible. Table 1 in [218] is useful in this respect. In particular,

- In 2HDM II, littlest Higgs model without T-parity (LH) and RS $B(B \to X_s \gamma)$ can only be enhanced and in the LIHT the enhancement is favoured.
- In MFV SUSY GUTs [220] and in models with universal extra dimensions it can only be suppressed. In particular, in the latter case lower bound on the compactification scale $1/R$ of 600 GeV can be derived [221–224] in this manner.
- In more complicated models, like MSSM with MFV, general MSSM and left-right models both enhancements and suppressions are possible.

Another important virtue of this decay is its sensitivity to RH currents. In the case of LH currents the chirality flip, necessary for $b \to s \gamma$ to occur, can only proceed through the mass of the initial or the final quark. Consequently, the amplitude is proportional to $m_b$ or $m_s$. In contrast, when RH currents are present, the chirality flip can take place on the internal top quark line, resulting in an enhancement factor $m_t/m_b$ of the NP contribution relatively to the SM one at the level of the amplitude. This is the case of left–right symmetric models in which $B \to X_s \gamma$ has been analysed by many authors in the past [131, 225–234]. In models with heavy fermions ($F$) that couple through RH currents to SM quarks this enhancement, being proportional to $m_F/m_b$, can be very large [135] and the couplings in question must be strongly...
suppressed in order to obtain agreement with the data. This is, for instance, the case of gauge flavour models, which we will briefly describe in section 6. It should be emphasized that the comments on the $m_d/m_b$ and $m_{s}/m_{b}$ enhancements also apply for charged and neutral gauge bosons, as well as for charged and neutral heavy scalars and pseudoscalars.

5.7. Step 7: $B \rightarrow X_i \ell^+ \ell^-$ and $B \rightarrow K^*(K)\ell^+ \ell^-$

5.7.1. Preliminaries. While the branching ratios for $B \rightarrow X_i \ell^+ \ell^-$ and $B \rightarrow K^* (K)\ell^+ \ell^-$ put already significant constraints on NP, the angular observables, CP-conserving ones like the well-known forward–backward asymmetry, and CP-violating ones will definitely be useful for distinguishing various extensions of the SM when the data improve. During the last three years, a number of detailed analyses of various CP averaged symmetries ($S_i$) and CP asymmetries ($A_i$) provided by the angular distributions in the exclusive decay $B \rightarrow K^*(\rightarrow K\pi)\ell^+ \ell^-$ have been performed in [37, 38, 147, 170, 235–242]. In particular, the zeros of some of these observables can be accurately predicted. The pioneering experimental analyses performed at BaBar, Belle and Tevatron [243–245] have already provided interesting results for the best-known forward–backward asymmetry. Yet, the recent data from LHCb [246, 247] surpassed the latter ones in precision, demonstrating that the SM is consistent with the present data on the forward–backward asymmetry. On the other hand, these decays, as we will see below, bring new challenges because the data on $A_i$ and $S_i$ improved last year. Yet, in order to reach clear cut conclusions, further improvement in the data and the reduction of theoretical uncertainties is necessary. Meanwhile, the present data already serves to bound the parameters in several extensions of the SM.

Compared with previous steps, this one is more challenging as far as transparency is concerned. Indeed, the effective Hamiltonian for these decays involves more local operators and corresponding Wilson coefficients, which are generally complex quantities. On the other hand, the numerous symmetries $S_i$ and asymmetries $A_i$ when precisely measured will one day allow a detailed insight into the physics behind the values of the Wilson coefficients in question. In this context, it is important to select those $S_i$ and $A_i$ that are particularly useful for the tests of NP and are not subject to large form factor uncertainties. While significant progress in this direction has already been done in the literature, a more transparent picture will surely emerge once the precision of these angular observables increases with time. The most recent reviews on various optimal strategies for extraction of NP from angular observables can be found in [248, 249]. Details on these strategies can be found in [236–238, 240, 241, 250–254].

While it appears from the present perspective that the observables in $B_{s,d} \rightarrow \mu^+ \mu^-$ decays are subject to smaller hadronic uncertainties than the observables considered here, the strength of $B \rightarrow K^* \mu^+ \mu^-$ is not only the presence of several symmetries $S_i$ and asymmetries $A_i$, or other constructions like $A^\gamma_{10}$, $P_l$, $H^I_{+}$ and alike. Indeed, the presence of an additional variable, the invariant mass of the dilepton ($q^2$), is an important virtue of these decays. Studying different observables in different $q^2$ bins can one day, as stressed in particular in [238, 240, 248, 249], not only help to discover NP but also to identify it. The most recent study [255] of the so-called primary observables $P_l$ and $P'_l$ introduced in [241, 248] in the context of the most recent LHCb data [247, 256] illustrates this in explicit terms and we will return to these data and the related analyses [255, 257] below.

The story of the departures of LHCb data from the SM in the decays in question is rather involved but interesting. In particular, the previous indications for a deviation from SM value of the isospin asymmetry in $B \rightarrow K^* \mu^+ \mu^-$ decay have now disappeared [258]. On the other hand, the corresponding asymmetry in $B \rightarrow K \mu^+ \mu^-$ decay disagrees presently with the SM [258]. A recent very detailed analysis of the isospin asymmetries in these decays can be found in [217].

However, as pointed out in [255, 257] and analysed in detail, sizable departures from the SM expectations in some of the observables $P_l$ or $S_l$ are seen in most recent LHCb data [247, 256].

In order to have a closer look at these issues, we need the effective Hamiltonian for these decays, which is given in (105) with the first term given in (182). The stars in these decays are the Wilson coefficients entering these Hamiltonians. The most important are

$$C_{7 \gamma}, \ C_9, \ C_{10}, \ C'_{7 \gamma}, \ C'_{9}, \ C'_{10} \quad (190)$$

where the primed Wilson coefficients correspond to primed operators obtained through the replacement $P_l \leftrightarrow P_R$. The scalar and pseudoscalar coefficients are more constrained by $B_s \rightarrow \mu^+ \mu^-$ decay, we will make few comments on them below.

The values of the coefficients in (190) have been calculated in the SM and in its numerous extensions. Moreover, they have been constrained in model independent analyses in which they have been considered as real or complex parameters. To this end, the data on $B \rightarrow X_i \gamma$, $B \rightarrow K^* \gamma$, $B \rightarrow X_i \ell^+ \ell^-$, $B \rightarrow K^* \ell^+ \ell^-$, $B \rightarrow K \ell^+ \ell^-$ and $B_s \rightarrow \mu^+ \mu^-$ have been used. The fact that these coefficients enter universally in a number of observables allows us to obtain the correlations between their values. We only refer to the selected papers that we have found particularly useful for our studies of NP. These are [37, 38, 239, 257], where model-independent constraints on NP in $b \rightarrow s$ transitions have been updated and generalized. Further references can be found there and in the text above.

It is useful to consider $B \rightarrow X_i \ell^+ \ell^-$ decay and $B \rightarrow K^* \ell^+ \ell^-$ in two different regions of the dilepton invariant mass. The low $q^2$ region with $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$, considered already for a long time, and the high $q^2$ region with $q^2 > 14.4 \text{ GeV}^2$ became very relevant after theoretical progress made in [259]. First, in these regions one is not sensitive to the $c\bar{c}$ resonances. Moreover, while the branching ratios in the high $q^2$ region are mainly sensitive to NP contributions to the Wilson coefficients $C'_{-9}$ and $C'_{10}$, the branching ratio in the low $q^2$ region also depends strongly on $C'_{7 \gamma}$. Therefore, one expects some correlation between NP contributions at low $q^2$ and those in $B \rightarrow X_i \gamma$ decay.

In [37, 38] the NP scenarios without important contributions from scalar operators have been considered. Various
analyses show that once the experimental upper bound on the branching ratio for $B_s \rightarrow \mu^+\mu^-$ has been taken into account, the impact of pseudoscalar operators $O_9^{(P)}$ on $B \rightarrow X_u\ell^+\ell^-$ and $B \rightarrow K^*(K)^\ast \ell^+\ell^-$ is minor. However, as stressed in [237], when lepton mass effects are taken into account there is one observable among the many measured in $B \rightarrow K^*\ell^+\ell^-$ that is sensitive to scalar operators $O_7^{(S)}$. This is interesting because $B_{s,d} \rightarrow \mu^+\mu^-$ decays generally involve both scalar and pseudoscalar operators. In this sense the angular distribution in $B \rightarrow K^*\ell^+\ell^-$ allows us to probe the scalar sector of a theory beyond the SM, in a way that is theoretically clean and complementary to $B_s \rightarrow \mu^+\mu^-$. We refer for more details to [237], in particular to figure 5 of that paper. However, the recent very improved result from LHCb and CMS on $B_s \rightarrow \mu^+\mu^-$ in (132) imposed on this figure precludes this study from the present perspective.

While $B \rightarrow K^*\ell^+\ell^-$ is not as theoretically clean as $B_s \rightarrow \mu^+\mu^-$ because of the presence of form factors, recent advances in lattice calculations [260] give some hopes for improvements. This is also the case of $B \rightarrow K^\ast\ell^+\ell^-$, where progress in lattice calculations of the relevant form factors has been reported in [261, 262].

As stressed in particular in [239], a simultaneous consideration of $B \rightarrow K^*\ell^+\ell^-$ together with $B_s \rightarrow \mu^+\mu^-$ provides useful tests of extensions of the SM. Indeed, while $B_s \rightarrow \mu^+\mu^-$ is sensitive only to the differences $C_7 - C_9'$ and $C_8 - C_9'$, the decay $B \rightarrow K^*\ell^+\ell^-$ is sensitive to their sum $C_7 + C_9'$ and $C_8 + C_9'$. A very extensive model independent study of $C_7(C_9')$ and $C_8(C_9')$ in the context of the data on $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow K^*\ell^+\ell^-$ has been performed in [239]. With improved data a new insight on the importance of scalar and pseudoscalar operators will be possible.

As we have already stated above, the picture resulting from these analyses is very rich and a brief summary of these sometimes numerically challenging analyses is a challenge in itself. In what follows we will limit our discussion to a number of observations referring to the rich literature for details, in particular to [37, 38, 239, 257], because the spirit of these papers fits well to our strategies.

5.7.2. Lessons from recent analyses. The studies of these decays in the SM and its extensions have been the subject of numerous analyses for almost the last twenty years [155, 263–268]. The most recent studies can be found in [37, 235, 238, 240, 255, 257, 269, 270], where references to the older papers can be found. The progress in the recent years is the inclusion in these analyses of the data on angular observables in $B \rightarrow K^*\ell^+\ell^-$. In the simplest case, the allowed ranges in the space of the real or imaginary parts of a pair of Wilson coefficients, or in the complex plane of a single Wilson coefficient are shown. As stressed in [37], the conclusions drawn from such studies are only valid if the chosen Wilson coefficients are indeed the dominant ones in a given NP scenario. In fact, this is approximately the case in a number of models considered in the literature. A few examples are:

- In MFV models with dominance of $Z$ penguins and without new sources of CP violation only the real parts of $C_{7,9}$ and $C_{10}$ are relevant.
- In MSSM with MFV and FBPs [78], in effective SUSY with FBPs [271], and in effective SUSY with a $U(2)^3$ symmetry [86, 87], NP effects in $\Delta B = \Delta S = 1$ processes are dominated by complex contributions to $C_7$ and $C_8$.

The analysis of this type in [37] uses the data on $B \rightarrow K^*\mu^+\mu^-$ at low and high $q^2$, $B \rightarrow X_u\ell^+\ell^-$, $B \rightarrow X_u\gamma$ and $B \rightarrow K^\ast\gamma$. The resulting figure 2 in that paper containing twelve plots depicts the allowed ranges for various pairs of real and/or imaginary parts of chosen Wilson coefficients. While very impressive, such plots are rather difficult to digest at first sight. Yet, the message from this analysis is clear. Already present data can exclude sign-flips of certain coefficients in certain NP scenarios relative to SM values. Such plots will be more informative when the data is improved.

As in many NP models, several Wilson coefficients could be affected by new contributions. For example, the authors of [37] perform probably for the first time a global fit of all Wilson coefficients. In this context, in addition to the general case, they consider specific examples of NP scenarios that are similar in spirit to those introduced in section 2. These are the cases of real LH currents, complex LH currents and complex RH currents. Again, 32 plots resulting from this study shows the complexity of such analyses. With improved data, such plots will be useful for obtaining an insight into the physics involved. Even if some time has passed since this analysis has been published, the following observations from this global analysis remain valid:

- For $C_{7,9}$ and $C_{10}$ there is little room left for constructive interference of real NP contributions with the SM.
- A flipped sign solution with $C_{7,9} \simeq -C_{7,9}^{\text{SM}}$, $C_9 \simeq -C_9^{\text{SM}}$, and $C_{10} \simeq -C_{10}^{\text{SM}}$ is allowed by the data.
- Sizable imaginary parts for all coefficients are still allowed.

A detailed study of CP symmetries and CP asymmetries in concrete BSM scenarios can also be found in [237]. In particular, it has been found that these observables could allow us clear distinction of LHT, general MSSM, and MSSM with flavour-blind phases (FBMSSM), not only from SM predictions but also among these three scenarios.

This picture could be modified by the most recent LHCb data [247, 256] on angular observables in $B_d \rightarrow K^*\mu^+\mu^-$ that show significant departures from SM expectations. Moreover, new data on the observable $F_L$, consistent with LHCb value in [247], have been presented by CMS [272]. These anomalies in $B_d \rightarrow K^*\mu^+\mu^-$ have recently triggered two sophisticated analyses [255, 257] with the goal of understanding the data and to indicate what type of NP could be responsible for these departures from the SM. Both analyses point towards NP contributions in the modified coefficients $C_{7,9}$ and $C_9$, with the following shifts with respect to their SM values:

$$C_{7,9}^{\text{NP}} < 0, \quad C_9^{\text{NP}} < 0. \quad (191)$$

Other possibilities, in particular involving RH currents ($C_9 > 0$), have been discussed in [257]. Subsequently, several other analyses of these data have been presented [46, 47, 273–279]. In particular, a recent comprehensive Bayesian analysis
of the authors of [170, 253] in [275] finds that, although SM works well, if one wants to interpret the data in extensions of the SM then scenarios in which chirality-flipped operators are included work better than those without them. In that case, they find that the main NP effect is still in $C_9$ and in agreement with [257], who find that in the $C_9 - C_9'$ plane the SM point is outside the $2\sigma$ range.

It should be emphasized at this point that these analyses are subject to theoretical uncertainties, which have been investigated at length in [242, 255, 259, 270, 279–281] and it remains to be seen whether the observed anomalies are only the result of statistical fluctuations and/or underestimated error uncertainties. This has been particularly emphasized by the authors of [275], who do not think that it will be possible to convincingly demonstrate the presence of NP in the decays in question without significant improvement in the understanding of $1/m_b$ corrections and reduction of the uncertainties in hadronic form factors.

Assuming that NP is really at work here, we have investigated in [46] whether tree-level $Z'$ and $Z$-exchanges could simultaneously explain the $B_d \rightarrow K^+\mu^+\mu^-$ anomalies and the most recent data on $B_s,d \rightarrow \mu^+\mu^-$. In this context, we have investigated the correlation between these decays and $\Delta F = 2$ observables. The outcome of this rather extensive analysis for $B_{s,d} \rightarrow \mu^+\mu^-$ has been already summarized at the end of step 4. In particular the plots in figures 30 and 31 demonstrate that LHS scenario for $Z'$ or $Z$ FCNC couplings provides a simple model that allows for the violation of the CMFV relation between the branching ratios for $B_{s,d} \rightarrow \mu^+\mu^-$ and $\Delta M_{s,d}$.

As far as the anomalies in $B \rightarrow K^*\mu^+\mu^-$ are concerned,

- $Z'$ with only LH couplings is capable of softening the anomalies in the observables $F_L$ and $S_3$ in a correlated manner as proposed in [255, 257]. However, a better description of the present data is obtained by also including RH contributions with the RH couplings of approximately the same magnitude but with an opposite sign. This is our ALRS scenario. We illustrate this in figure 32. This is in agreement with the findings in [257].

Several analogous correlations can be found in [46]. We should emphasize that if $Z'$ is the only new particle at scales $O(\text{TeV})$ then $C_{7y}$ can be neglected, implying nice correlations, as shown in figure 32.

- The SM $Z$ boson with FCNC couplings to quarks cannot describe the anomalies in $B \rightarrow K^*\mu^+\mu^-$ due to its small vector coupling to muons.

In summary, while the modification of the Wilson coefficient $C_{7y}$ together with $C_9$ could provide the explanation of the data [255, 257], it appears that the most favourite scenario is the one with participation of RH currents [46, 257, 277]

$$C_9 < 0, \quad C_9' > 0, \quad C_9' \approx -C_{9\text{NP}}.$$ (192)

Yet, the case of NP being present only in the coefficient $C_9$ cannot be presently excluded [46, 47, 255, 273–275]. Concerning the dynamics, the favourite physical mechanisms behind these deviations emerging from these studies is the presence of tree-level $Z'$ exchanges. We will summarize the recent results in 331 models [47] in section 6.5.1.

We are looking forward to improved LHCb data in order to see how the story of NP in $B \rightarrow K^* (K)\mu^+\mu^-$ and $B_{s,d} \rightarrow \mu^+\mu^-$ decays evolves with time.

5.7.3. Explicit bounds on Wilson coefficients. In the present review, we have used the results discussed above to constrain the correlations between various observables in models with tree-level neutral gauge boson, and neutral scalar and pseudoscalar exchanges. Such constraints can be found in the plots presented in steps 4 and 9. To this end, in the case of gauge boson exchanges we use the bounds from figures 1 and 2 of [38]. Approximately, these bound can be summarized as follows:

$$-2 \lesssim \Re(C_{10}'^\mu) \lesssim 0, \quad -2.5 \lesssim \Im(C_{10}') \lesssim 2.5,$$ (193a)

$$-0.8 \lesssim \Re(C_{10}^\text{NP}) \lesssim 1.8, \quad -3 \lesssim \Im(C_{10}^\text{NP}) \lesssim 3.$$ (193b)

5 The latest updates [143, 282] show that the recent LHCb measurement of the CP asymmetry $A_{FB}$ [247] leads to a slightly stronger constraint on the imaginary part of $C_{10}^\mu$: $-1.5 \lesssim \Im(C_{10}') \lesssim 1.5$.  

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**Figure 32.** Left: $\langle F_L \rangle$ versus $\langle S_3 \rangle$ in LHS, where the magenta line corresponds to $C_9^\text{NP} = -1.6 \pm 0.3$ and the cyan line to $C_9^\text{NP} = -0.8 \pm 0.3$. Right: the same in ALRS for different values of $C_9^\text{NP}$: $-2$ (blue), $-1$ (red), $0$ (green) and $1$ (yellow). The light and dark grey area corresponds to the experimental range for $\langle F_L \rangle$ with all data and only LHCb+CMS data, taken into account, respectively. The black point and the grey box correspond to the SM predictions. Reproduced from [257] under a CC BY 4.0 license.
Especially, the LHCb data on $B \to K^* \mu^+ \mu^-$ only allows for negative values of the real part of $C_{10}^{(i)}$
\[ \Re (C_{10}^{(i)}) \leq 0 \]  
and this has an impact on our results in the RH and LR scenarios that are presented in steps 4 and 9. However, for the numerical analysis we use the exact bounds that are smaller than these rectangular bounds. For $C_{10}^{(i)}$—relevant for LHS—the latter allow a much larger parameter space whereas for $C_{10}^{(i)}$—relevant for RHS—the approximation above gives very similar results to the exact bounds in our plots. In figures 18, 19, 20, 41 and 42 the green regions in the $Z^*$ case are compatible with the exact bound from [38]. The black points in RHS show the excluded regions where the bound in (194) is violated, which as one can see nearly coincides with the correct bounds (see figures 41 and 42).

Concerning the bounds on the coefficients of scalar operators, we quote here the bounds derived from the analysis in [239]. By adjusting their normalization of Wilson coefficients to ours, the final result of this paper reads:
\[ m_b |C_S^{(i)}| \leq 0.7, \quad m_b |C_P^{(i)}| \leq 1.0, \]  
where the scale in $m_b$ should be the high matching scale. As demonstrated in [44], these bounds do not presently have an impact on the values of these coefficients in scenarios with tree-level scalar and pseudoscalar exchanges.

In summary, this step will definitely bring new insight into SD dynamics during the upgraded analyses of the LHCb and SuperKEKB will also play an important role in these studies.

5.8. Step 8: $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \pi^0 \nu \bar{\nu}$ and $K_L \to \mu^+ \mu^-$

5.8.1. Preliminaries. Among the top highlights of flavour physics in this decade will be the measurements of the branching ratios of two golden modes: $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$. $K^+ \to \pi^+ \nu \bar{\nu}$ is CP conserving while $K_L \to \pi^0 \nu \bar{\nu}$ is governed by CP violation. Both decays are dominated in the SM and many of its extensions by $Z$-penguin diagrams. It is well known that these decays are theoretically very clean and their branching ratios have been calculated within the SM, including NNLO QCD corrections and electroweak corrections [153, 161, 283–285]. Moreover, extensive calculations of isospin breaking effects and non-perturbative effects have been done [286, 287]. Reviews of these two decays can be found in [288–291]. In particular, in [288] the status of NP contributions as of 2008 has been reviewed. A recent short review of NP signatures in Kaon decays can be found in [292].

Assuming that light neutrinos couple only to LH currents, the general SD effective Hamiltonian describing both decays is given as follows:
\[ H_{\text{eff}}(\nu \bar{\nu}) = g_{\text{SM}}^2 V_{td}^* V_{td} [X_L(K)(i\gamma^\mu P_L d) + X_R(K)(i\gamma^\mu P_R d)] \times (i\gamma^\mu P_L v), \]  
where $g_{\text{SM}}$ is defined in (90). We have suppressed the charm contribution that is represented by $P_C(X)$ below.

The resulting branching ratios for the two $K \to \pi \nu \bar{\nu}$ modes can be written generally as
\[ B(K^+ \to \pi^+ \nu \bar{\nu}) = \kappa_s \left[ \left( \frac{\text{Im} X_{\text{eff}}}{\lambda_5^2} \right)^2 + \left( \frac{\text{Re} X_{\text{eff}}}{\lambda_5^2} - P_C(X) \right)^2 \right], \]  
\[ B(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im} X_{\text{eff}}}{\lambda_5^2} \right)^2, \]  
where \[ \kappa_s = (5.36 \pm 0.026) \times 10^{-11}, \]  
\[ \kappa_L = (2.31 \pm 0.01) \times 10^{-10} \]  
and [283–287]
\[ P_C(X) = 0.42 \pm 0.03. \]  
The SD contributions are described by
\[ X_{\text{eff}} = V_{td}^* V_{td} X_L(K) + X_R(K) \equiv V_{td}^* V_{td} X(x_t)(1 + \xi e^{i\theta}). \]  
Here,
\[ X_L^{\text{SM}}(K) = \eta_X X_0(x_t) = 1.464 \pm 0.041, \]  
results within the SM from $Z$-penguin and box diagrams with
\[ X_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + 3x_t - 6 \right. \left( x_t - 1 \right)^2 \ln x_t]. \]  
and $\eta_X = 0.994$ for $m_t(m_t)$.

It should be remarked that with the definitions of electroweak parameters as in table 1, in particular $\sin^2 \theta_W$, the electroweak corrections to $X_L^{\text{SM}}(K)$ are totally negligible [161] and are, therefore, not exhibited here. To this end $m_t(m_t)$, as discussed in the context of the $B_{s,d} \to \mu^+ \mu^-$ decays in step 4, should also be used. That is, for $m_t$ only QCD corrections are MS renormalized, whereas $m_t$ is on-shell as far as electroweak corrections are concerned. See [157, 161] for more details.

In order to describe NP contributions we have introduced the two real parameters $\xi$ and $\theta$ that vanish in the SM. These formulae are in fact very general and apply to all extensions of the SM. The correlation between the two branching ratios generally depends on two variables $\xi$ and $\theta$ [288, 293, 294], and measuring these branching ratios one day will allow us to determine $\xi$ and $\theta$ and compare them with model expectations. We illustrate this in figure 33.

Unfortunately, on the basis of these twodi branching ratios it is not possible to find out how important the contributions of RH currents are because their effects are hidden in a single function $X_{\text{eff}}$. In this sense, the decays governed by $b \to \nu \bar{\nu}$ transitions that we will discuss soon are superior. Indeed, in this case we have three branching ratios to our disposal and one is also sensitive to the direction of the spin of $K^+$.

Experimentally, we have [295]
\[ B(K^+ \to \pi^+ \nu \bar{\nu})_{\exp} = (17.3 \pm 11.5) \times \text{10}^{-11}, \]  
and the 90% CL upper bound [296]
\[ B(K_L \to \pi^0 \nu \bar{\nu})_{\exp} \leq 2.6 \times \text{10}^{-8}. \]
The prospects for improved measurements of $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ are very good. One should stress that a measurement of this branching ratio with an accuracy of 10% will already give us a very important insight into the physics at SD scales. Indeed, the NA62 experiment at CERN aims at this precision and a new experiment at Fermilab (ORKA) should be able to reach an accuracy of 5%, which would be truly fantastic. It will take longer in the case of $K_L \rightarrow \pi^0\nu\bar{\nu}$ but KOTO experiment at J-PARC should provide interesting results in this decade on this branching ratio. It should be emphasized that the combination of these two decays is particularly powerful in testing NP. The extraction of the SD part from the data is subject to considerable uncertainties. The most recent estimate already encountered in $B_{s,d}(123)$. We note the minus sign in front of $X_R$ in (201) that results from the fact that only the $\gamma_\mu\gamma_5$ part contributes.

The SD contributions are described by

$$Y_{\text{SM}}(K) = \eta_Y Y_0(x_t)$$

(210)

already encountered in $B_{s,d} \rightarrow \mu^+\mu^-$ decays and given in (123). We note the minus sign in front of $Y_R$ as opposed to $X_R$ in (201) that results from the fact that only the $\gamma_\mu\gamma_5$ part contributes.

The extraction of the SD part from the data is subject to considerable uncertainties. The most recent estimate gives $B_{\text{SD}}(K_L \rightarrow \mu^+\mu^-) \approx 2.5 \times 10^{-9}$, (211) to be compared with $(0.8 \pm 0.1) \times 10^{-9}$ in the SM [304].
5.8.2. SM results. The branching ratios for $K^+ \to \pi^+ \nu \overline{\nu}$ and $K_L \to \pi^0 \nu \overline{\nu}$ in the SM are given by

$$B(K^+ \to \pi^+ \nu \overline{\nu}) = \kappa_+ \left[ \left( \frac{\text{Im} \lambda_i}{\lambda^2} X(x_i) \right)^2 + \left( \frac{\text{Re} \lambda_i}{\lambda^2} X(x_i) - P_c(X) \right)^2 \right],$$

(212)

and

$$B(K_L \to \pi^0 \nu \overline{\nu}) = \kappa_L \left( \frac{\text{Im} \lambda_i}{\lambda^2} X(x_i) \right)^2.$$

(213)

The important feature of these expressions is that these two decays are described by the same real function $X(x_i)$. The present theoretical uncertainties in $B(K^+ \to \pi^+ \nu \overline{\nu})$ and $B(K_L \to \pi^0 \nu \overline{\nu})$ are at the level of 2–3% and 1–2%, respectively. By calculating the branching ratios for the central values of the parameters in table 4, we find for $|V_{ub}| = 0.0034$

$B(K^+ \to \pi^+ \nu \overline{\nu})_{\text{SM}} = 8.5 \times 10^{-11}$,

$B(K_L \to \pi^0 \nu \overline{\nu})_{\text{SM}} = 2.5 \times 10^{-11},$

(214)

while for $|V_{ub}| = 0.0040$ we find

$B(K^+ \to \pi^+ \nu \overline{\nu})_{\text{SM}} = 8.4 \times 10^{-11}$,

$B(K_L \to \pi^0 \nu \overline{\nu})_{\text{SM}} = 3.4 \times 10^{-11}.$

(215)

We observe that, whereas $B(K^+ \to \pi^+ \nu \overline{\nu})$ is rather insensitive to $|V_{ub}|$, $B(K_L \to \pi^0 \nu \overline{\nu})$ increases with increasing $|V_{ub}|$. The main remaining uncertainty in these branching ratios comes from the $|V_{ub}|^2$ dependence and if the present value from tree-level decays is used then this uncertainty amounts to roughly 10%. As we demonstrated in [45] this uncertainty within the SM can be decreased significantly with the help of $\varepsilon_K$, particularly when the angle $\gamma$ will be known from tree-level decays. Therefore, we expect that when the data from NA62 will be available, the total uncertainties in both branching ratios will be in the ballpark of 5%.

These results should be compared with the experimental values given in (204) and (205). Certainly, there is still significant room left for NP contributions and we will now turn our attention to them in the context of simplest extensions of the SM.

5.8.3. CMFV. In these models $K^+ \to \pi^+ \nu \overline{\nu}$ and $K_L \to \pi^0 \nu \overline{\nu}$ are described by a single real function $X(v)$, implying a strong correlation between the two branching ratios, as emphasized in [97]. We show this correlation in figure 34. Thus, once the branching ratio for $K^+ \to \pi^+ \nu \overline{\nu}$ has been measured with high precision by NA62 and later at Fermilab, we will also precisely know the corresponding branching ratio for $K_L \to \pi^0 \nu \overline{\nu}$ that will be universal for the full class of CMFV models.

5.8.4. 2HDM$_{\text{||}}$. In this class of models the dominant new contribution comes from charged Higgs ($H^\pm$) exchanges in $Z^0$-Penguin diagrams and box diagrams. While an explicit calculation with present input is missing, we do not expect large NP contributions in this scenario.

Figure 34. $B(K_L \to \pi^0 \nu \overline{\nu})$ versus $B(K^+ \to \pi^+ \nu \overline{\nu})$ in CMFV. Red point: SM central value. Grey region: experimental range of $B(K^+ \to \pi^+ \nu \overline{\nu})$. The black line corresponds to the Grossman–Nir (GN) bound.
Figure 35. $B(K_L \to \pi^0\bar{\nu} \nu)$ versus $B(K^+ \to \pi^+\bar{\nu} \nu)$ for $M_{Z'} = 1$ TeV (upper panels, C1: cyan, C3: pink.) and $M_{Z'} = 5$ TeV (cyan), $10$ TeV (blue) and $30$ TeV (purple) (lower panels) in LHS1 (left) and LHS2 (right). Black regions are excluded by the upper bound $B(K_L \to \mu^+\mu^-) \leq 2.5 \times 10^{-9}$. Red point: SM central value. Grey region: experimental range of $B(K^+ \to \pi^+\bar{\nu} \nu)$. The black line corresponds to the GN bound. Reproduced with permission from [40]. Copyright 2013 SISSA.

We will now illustrate this in explicit terms by considering the set $\varepsilon_{K,K^+ \to \pi^+\bar{\nu}, K_L \to \pi^0\bar{\nu}, K_L \to \mu^+\mu^-}$ (218) in the scenarios LHS, RHS and LRS for the $\Delta_{L,R}$ couplings in question.

The inclusion of $K_L \to \mu^+\mu^-$ in this discussion leads to interesting results. Indeed, now

$$Y_\ell(K) = Y(x_i) + \frac{\Delta_{A}^{\mu\bar{\mu}}(Z') \Delta_{s}^{\ell\bar{\nu}}(Z')}{g_{\text{SM}}^2 M_{Z'}^2 V_{ts}^* V_{td}},$$

(219)

$$Y_R(K) = \frac{\Delta_{A}^{\mu\bar{\mu}}(Z') \Delta_{R}^{\ell\bar{\nu}}(Z')}{g_{\text{SM}}^2 M_{Z'}^2 V_{ts}^* V_{td}},$$

(220)

We note that up to the lepton couplings NP corrections are the same as in $X_{L,R}(K)$. However, very importantly, the function $Y_R(K)$ enters with the opposite sign to $X_R(K)$ into the branching ratio for $K_L \to \mu^+\mu^-$, so that effectively one has

$$Y_A(K) = \eta Y_0(x_i) + \frac{[\Delta_{A}^{\mu\bar{\mu}}(Z') \Delta_{s}^{\ell\bar{\nu}}(Z') - \Delta_{R}^{\ell\bar{\nu}}(Z')]}{2 g_{\text{SM}}^2 M_{Z'}^2 V_{ts}^* V_{td}}$$

(221)

$$\equiv |Y_A(K)| e^{i\delta_R},$$

The minus sign in front of $\Delta_{R}^{\ell\bar{\nu}}(Z')$ implies an anti-correlation between $K^+ \to \pi^+\bar{\nu} \nu$ and $K_L \to \mu^+\mu^-$ branching ratios, as noted already within the RSe scenario in [49].

We will now summarize the results obtained in [40], where the leptonic couplings have been chosen to be

$$\Delta_{L}^{\ell\bar{\nu}}(Z') = \Delta_{A}^{\mu\bar{\mu}}(Z') = 0.5,$$

(222)

for which NP contributions to $\varepsilon_K$ vanish. As seen in figure 10, this is only allowed for scenario S2, for which SM agrees well with the data and NP contributions are not required. In this scenario $\tilde{s}_{12}$ can even vanish. In scenario S1, in which NP contributions are required to reproduce the data, $\tilde{s}_{12}$ is bounded from below and $\delta_{12}$ cannot satisfy (223).

In the upper panels of figure 35 we show the correlation between the branching ratios for $K^+ \to \pi^+\bar{\nu} \nu$ and $K_L \to \pi^0\bar{\nu} \nu$ in LHS1 and LHS2 for $M_{Z'} = 1$ TeV [40]. Since only vector currents occur, we get the same result for RHS1 and RHS2. We observe the following pattern of deviations from the SM expectations:

- There are two branches in both scenarios. The difference between LHS1 and LHS2 originates from required NP contributions in LHS1 in order to agree with the data on $\varepsilon_K$, and the fact that in LHS1 there are two oases and only one in LHS2.
The horizontal branch in both plots corresponds to $n = 0, 2$ in (223), for which the NP contribution to $K \to \pi \nu \bar{v}$ is real and vanishes in the case of $K_L \to \pi^0 \nu \bar{v}$.

- The second branch corresponds to $n = 1, 3$ in (223), for which the NP contribution is purely imaginary. This is parallel to the GN bound [306] that is represented by the solid black line.

- The deviations from the SM are significantly larger than in the case of rare $B$ decays. This is a consequence of the weaker constraint from $\Delta S = 2$ processes compared to $\Delta B = 2$ and the fact that rare $K$ decays are more strongly suppressed than rare $B$ decays within the SM. Yet, as seen, the largest values corresponding to black areas are ruled out through the correlation with $K_L \to \mu^+ \mu^-$, as discussed below.

- We observe that even at $M_{Z'} = 10$ TeV both branching ratios can still considerably differ from SM predictions and for $M_{Z'} \lesssim 20$ TeV NP effects in these decays, in particular the $K_L \to \pi^0 \nu \bar{v}$ should be detectable in the flavour precision era.

Of particular interest is the correlation between $B(K^+ \to \pi^+ \nu \bar{v})$ and $B(K_L \to \mu^+ \mu^-)$ that we show in figure 36. In the case of LHS1 scenario, a correlation analogous to this is found in the LHT model [307] but due to fewer free parameters in case of LHS1 scenario, a correlation analogous to this is found.

From figures 35 and 36 we obtain the following results:

- In the case of the dominance of real NP contributions, we find for $M_{Z'} = 1$ TeV

$$B(K^+ \to \pi^+ \nu \bar{v}) \lesssim 16 \times 10^{-11}. \quad (224)$$

In this case, $K_L \to \pi^0 \nu \bar{v}$ is SM-like and $B(K_L \to \mu^+ \mu^-)$ reaches the upper bound in (211).

- In the case of the dominance of imaginary NP contributions, the bound on $B(K_L \to \mu^+ \mu^-)$ is ineffective and both $B(K^+ \to \pi^+ \nu \bar{v})$ and $B(K_L \to \pi^0 \nu \bar{v})$ can be significantly larger than the SM predictions while $B(K^+ \to \pi^+ \nu \bar{v})$ can also be larger than its present experimental central value. We also find that for such large values the branching ratios are strongly correlated.

Inspecting in the LHS2 scenario when the branch parallel to the GN bound leaves the grey region corresponding to the 1$\sigma$ region in (204) we find a rough upper bound

$$B(K_L \to \pi^0 \nu \bar{v}) \lesssim 85 \times 10^{-11}, \quad (225)$$

which is much stronger than the present experimental upper bound in (205).

Finally, in the right panel of figure 36 we show the correlation between $B(K^+ \to \pi^+ \nu \bar{v})$ and $B(K_L \to \mu^+ \mu^-)$ in the RHS1 scenario. Indeed, the correlations in both oases differ from those in LHS1. This feature is already known from different studies, particularly in RSc scenario [49], and originates in the fact that while $K^+ \to \pi^+ \nu \bar{v}$ is sensitive to vector couplings, $K_L \to \mu^+ \mu^-$ is sensitive to the axial-vector couplings. We also note that, in the case of the dominance of imaginary NP contributions corresponding to the horizontal line, $B(K^+ \to \pi^+ \nu \bar{v})$ and $B(K_L \to \pi^0 \nu \bar{v})$ can be large. But otherwise, $B(K^+ \to \pi^+ \nu \bar{v})$ is suppressed with respect to its SM value and $B(K_L \to \pi^0 \nu \bar{v})$ is SM-like.

Finally, we also discuss what happens if we exchange the $Z'$ boson with the $Z^0$ boson with flavour-violating couplings. Except for the LR scenario, and in case of purely imaginary NP contributions, these effects are bounded by $K_L \to \mu^+ \mu^-$. In figure 37 we show our result for LHS2, RHS2 and LRS2 where the effects can be much larger than in the $Z'$ case.

$$\text{5.8.6. Tree-level scalar exchanges.}$$

If the masses of neutrinos are generated by the couplings to scalars then the contributions of these scalars to decays with neutrinos in the final state are definitely negligible. But if the masses of neutrinos are generated by a different mechanism than coupling to scalars, as in the case of the see-saw mechanism, it is not a priori obvious that such couplings in some NP scenarios could be measurable. Our working assumption in the present paper will be that this is not the case. Consequently, NP effects of scalars in $K^+ \to \pi^+ \nu \bar{v}$, $K_L \to \pi^0 \nu \bar{v}$ and $b \to s \nu \bar{v}$...
transitions considered next will be assumed to be negligible in contrast to the $Z'$ models, as we have just seen. As demonstrated in [44], scalar contributions to $K_L \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \pi^0\ell^+\ell^-$, although in principle larger than for $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $b \rightarrow s\nu\bar{\nu}$ transitions, are found to be small and consequently we will not discuss them here.

5.9. Step 9: Rare $B$ decays $B \rightarrow X_s\nu\bar{\nu}$, $B \rightarrow K^+\nu\bar{\nu}$ and $B \rightarrow K\nu\bar{\nu}$

5.9.1. Preliminaries. The rare decays in question are among the important channels in $B$ physics because they allow a transparent study of $Z$ penguin and other electroweak penguin effects in NP scenarios in the absence of dipole operator contributions, and Higgs (scalar) penguin contributions that are often more important than $Z$ contributions in $B \rightarrow K^+\ell^+\ell^-$ and $B \rightarrow \ell^+\ell^-$ decays [308–310]. However, their measurements appear to be even harder than those of the rare $K$ decays that have just been discussed. Yet, SuperKEKB should be able to measure them at a satisfactory level.

The inclusive decay $B \rightarrow X_s\nu\bar{\nu}$ is theoretically as clean as $K \rightarrow \pi\nu\bar{\nu}$ decays but the parametric uncertainties are a bit larger. The two exclusive channels are affected by form factor uncertainties but in the case of $B \rightarrow K^+\nu\bar{\nu}$ [310] and $B \rightarrow K\nu\bar{\nu}$ [311] significant progress was made a few years ago. In the latter paper, this has been achieved by considering simultaneously also $B \rightarrow K\ell^+\ell^-$. Non-perturbative tree-level contributions from $B^+ \rightarrow \tau^+\nu$ to $B^+ \rightarrow K^+\nu\bar{\nu}$ and $B^+ \rightarrow K^{*+}\nu\bar{\nu}$ at the level of roughly $10\%$ have been pointed out [312]. Therefore, the expressions in equations (228)–(230) given below, as well as the SM results in (234), refer only to the SD contributions to these decays. The latter are obtained from the corresponding total rates, subtracting the reducible long-distance effects pointed out in [312].

The general effective Hamiltonian, including also RH current contributions, that is used for the $B \rightarrow (X_s, K, K^*)\nu\bar{\nu}$ decays is given as follows:

$$H_{eff} = g_{SM}^2 V_{tb} V_{ts} \times \left[ X_L(B_s) (\tilde{\sigma}_t P_L b) + X_R(B_s) (\tilde{\sigma}_t P_R b) \right] \times (\bar{v}\gamma_{\mu} P_L v).$$

(226)

It has a very similar structure to the one for $K \rightarrow \pi\nu\bar{\nu}$ decays in (196). In particular,

$$X_L^{SM}(B_s) = X_L^{SM}(K)$$

(227)

with $X_L^{SM}(K)$ given in (202). Moreover, in models with MFV there is a striking correlation between the branching ratios for $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $B \rightarrow X_s\nu\bar{\nu}$ because the same one-loop function $X(v)$ governs the two processes in question [97]. This relation is generally modified in models with non-MFV interactions, particularly RH currents. As we will see below, there are also correlations between $K_L \rightarrow \pi^0\nu\bar{\nu}$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $B \rightarrow K^+\nu\bar{\nu}$ that are useful for the study of various NP scenarios.

The interesting feature of these three $b \rightarrow s\nu\bar{\nu}$ transitions, in particular when taken together, is their sensitivity to the RH currents [308, 309] that were recently studied in [310].
Following the analysis of the latter paper, the branching ratios of the $B \to \{X_s, K, \eta\} \nu\bar{\nu}$ modes in the presence of RH currents can be written as follows:

\begin{align}
B(B \to K \nu\bar{\nu}) &= B(B \to K \nu\bar{\nu})_{\text{SM}} \times |1 - 2\eta| \epsilon^2, \\
B(B \to K^* \nu\bar{\nu}) &= B(B \to K^* \nu\bar{\nu})_{\text{SM}} \times |1 + 1.31\eta| \epsilon^2, \\
B(B \to X_s \nu\bar{\nu}) &= B(B \to X_s \nu\bar{\nu})_{\text{SM}} \times |1 + 0.09\eta| \epsilon^2,
\end{align}

where we have introduced the variables
\begin{align}
\epsilon^2 &= \frac{|X_L(B_\ell)|^2 + |X_R(B_\ell)|^2}{|\eta X_0(x_t)|^2}, \\
\eta &= \frac{\text{Re} (X_L(B_\ell) X_R^*(B_\ell))}{|X_L(B_\ell)|^2 + |X_R(B_\ell)|^2},
\end{align}

with $X_{L,R}$ defined in (226).

We observe that the RH currents signaled here by a non-vanishing $\eta$ enter these three branching ratios in a different manner, thereby allowing an efficient search for the signals of these currents. In addition, the average of the $K^*$ longitudinal polarization fraction $F_L$ used in the studies of $B \to K^* \ell^+\ell^-$ is a useful variable because it only depends on $\eta$:

$$
\langle F_L \rangle = 0.54 \frac{1 + 2\eta}{1 + 1.31\eta}.
$$

The experimental bounds [313–315] read
\begin{align}
B(B \to K \nu\bar{\nu}) &< 1.4 \times 10^{-5}, \\
B(B \to K^* \nu\bar{\nu}) &< 8.0 \times 10^{-5}, \\
B(B \to X_s \nu\bar{\nu}) &< 6.4 \times 10^{-4}.
\end{align}

5.9.2. SM results. In the absence of RH currents, $\eta = 0$ and all three decays are fully described by the function $X(x_t)$. The updated predictions for the SM branching ratios are [310–312]
\begin{align}
B(B \to K \nu\bar{\nu})_{\text{SM}} &= (3.64 \pm 0.47) \times 10^{-6}, \\
B(B \to K^* \nu\bar{\nu})_{\text{SM}} &= (7.2 \pm 1.1) \times 10^{-6}, \\
B(B \to X_s \nu\bar{\nu})_{\text{SM}} &= (2.7 \pm 0.2) \times 10^{-5}.
\end{align}

5.9.3. CMFV. In this class of models all of the branching ratios are described as in step 8 by the universal function $X(\nu)$

$$
X_L(B_\ell) = X(\nu), \quad X_R(B_\ell) = 0
$$

and, consequently, they are strongly correlated. However, the correlation between the $K \to \pi \nu\bar{\nu}$ branching ratios and the $B \to s \bar{\nu}\nu$ transitions considered here are most characteristic of this class of models. This correlation is in particular stringent once the CKM parameters have been determined in tree-level decays. We show this in figure 38.

5.9.4. 2HDM$_{\text{MFV}}$. To our knowledge, similarly to the case of $K \to \pi \nu\bar{\nu}$ decays, no detailed analysis of $b \to s \nu\bar{\nu}$ transitions exists in the literature. Yet, because of tiny couplings of scalar particles to neutrinos, such effects could only be relevant at one loop level with charged Higgs contributions at work. We expect these contributions to be small.

5.9.5. Tree-level gauge boson exchanges. By including the SM contribution, in this case the couplings $X_L$ and $X_R$ are given as follows:

\begin{align}
X_L(B_q) &= \eta L X_0(x_t) + \frac{\Delta^{\nu\nu}_{\ell}(Z')}{M_Z^2 S_{\text{SM}}} \frac{\Delta^{\nu\nu}_{\ell}(Z')}{V_{tb} V_{tb}}, \\
X_R(B_q) &= \frac{\Delta^{\nu\nu}_{\ell}(Z')}{M_Z^2 S_{\text{SM}}} \frac{\Delta^{\nu\nu}_{\ell}(Z')}{V_{tb} V_{tb}}.
\end{align}

A detailed analysis of these decays has been performed in [40]. We summarize here the most important results of this analysis.

In figure 39 (left) we show $B(B \to X_s \nu\bar{\nu})$ versus $B(B \to \mu^+\mu^-)$ in LHS1 scenario. This correlation is valid in any oasis due to the assumed equal sign of the leptonic couplings in (222), although, as seen in the plot, the size of NP contribution may depend on the oasis considered. Significant NP effects are still possible and the suppression of $B(B_s \to \mu^+\mu^-)$ below the SM value will also imply the suppression of $B(B \to X_s \nu\bar{\nu})$. If the future data will disagree with this pattern then the rescue could come from the flip of the signs in $\nu\bar{\nu}$ or $\mu^+\mu^-$ couplings,
indicate the regions that are compatible with $b \to s \ell^+ \ell^-$ constraints. In the SM, QCD penguins give positive contributions while the electroweak penguins give negative contributions. In order to obtain useful prediction for $\varepsilon'/\varepsilon$ in the SM, the precision on the corresponding hadronic parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ should be at least 10%. Although recently significant progress has been made in the case of $B_8^{(3/2)}$ that is relevant for electroweak penguin contribution [316], the calculation of $B_6^{(1/2)}$ is even more important. There are some hopes that this parameter could also be known with satisfactory precision in this decade [125, 317].

This would really be good because the calculations of SD contributions to this ratio (Wilson coefficients of QCD and electroweak penguin operators) within the SM have already been known for 20 years at the NLO level [318, 319] and present technology could extend them to the NNLO level if necessary. First steps in this direction have been done in [320, 321].

In the most studied extensions of the SM, the QCD penguin contributions are not modified significantly. On the other hand large NP contributions to electroweak penguins are possible. But they are often correlated with $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ decays so that considering $\varepsilon'/\varepsilon$ and these two decays simultaneously useful constraints on model parameters can be derived, again subject to the uncertainties in $B_6^{(1/2)}$ and $B_8^{(3/2)}$.

The present experimental world average from NA48 [322] and KTeV [323, 324],

$$\varepsilon'/\varepsilon = (16.6 \pm 2.3) \times 10^{-4},$$

could have an important impact on several extensions of the SM discussed if $B_6^{(1/2)}$ and $B_8^{(3/2)}$ were known. An analysis of $\varepsilon'/\varepsilon$ in the LHT model demonstrates this problem in explicit terms [325]. If one uses $B_6^{(1/2)} = B_8^{(3/2)} = 1$ as obtained in the large $N$ approach [139, 326], then $(\varepsilon'/\varepsilon)_{\text{SM}}$ is in the ballpark of the experimental data, although below it and sizable departures of $B(K_L \to \pi^0 \nu \bar{\nu})$ from its SM value are not allowed. $K^+ \to \pi^+ \nu \bar{\nu}$ being CP conserving and consequently not as strongly correlated with $\varepsilon'/\varepsilon$ as $K_L \to \pi^0 \nu \bar{\nu}$ could still be enhanced by 50%. On the other hand, if $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are different from unity and $(\varepsilon'/\varepsilon)_{\text{SM}}$ disagrees with experiment, much more room for enhancements of rare $K$ decay branching.

Figure 39. $B(B \to X_s \nu \bar{\nu})$ versus $B(B_s \to \mu^+ \mu^-)$ (left) and $B(B \to X_s \nu \bar{\nu})$ versus $S_{\phi'}$ (right) in LHS1 for $M_{Z'} = 1$ TeV. The green points indicate the regions that are compatible with $b \to s \ell^+ \ell^-$ constraints. Reproduced with permission from [40]. Copyright 2013 SISSA.

Figure 40. $\varepsilon$ versus $\eta$ for scenario LHS1, RHS1, LRS1 and ALRS1. Reproduced with permission from [40]. Copyright 2013 SISSA.
ratios through NP contributions is available. See also new insight from the recent analysis in [119]. Reviews of $\varepsilon'/\varepsilon$ can be found in [327–331].

5.10.2. Basic formula in the SM. In the SM, ten operators pay tribute to the $\varepsilon'/\varepsilon$, these are

Current–current:

\[ Q_1 = (\bar{s}_u u)_{V-A} \quad Q_2 = (\bar{s}_u u)_{V-A}, \]

\[ Q_3 = (\bar{s}_d d)_{V-A} \quad Q_4 = (\bar{s}_d d)_{V-A}, \]

\[ Q_5 = (\bar{s}_d d)_{V-A} \quad Q_6 = (\bar{s}_d d)_{V-A}, \]

QCD-penguins:

\[ Q_7 = (\bar{s}_u u)_{V-A} \quad Q_8 = (\bar{s}_u u)_{V-A}, \]

\[ Q_9 = (\bar{s}_d d)_{V-A} \quad Q_{10} = (\bar{s}_d d)_{V-A}. \]

Here, $\alpha, \beta$ denote colours and $e_q$ denotes the electric quark charges reflecting the electroweak origin of $Q_7, \ldots, Q_{10}$. Finally, $(\bar{s}_d d)_{V-A} \equiv s_u y_u (1 - y_s) d_u$.

The NLO RG analysis of these operators is rather involved [318, 319] but eventually one can derive an analytic formula in terms of the basic one-loop functions [328]. The most recent version of this formula is given as follows [119]:

\[ \varepsilon'/\varepsilon = a \text{Im} \lambda^K_{ij} \cdot F_\varepsilon(x_i), \]
where $\lambda_{i}^{(K)} = V_{i}D_{i}^{*}$, $a = 0.92 \pm 0.02$ and

$$F_{i}(x_{i}) = P_{0} + P_{x} \phi_{i}(x_{i}) + P_{t} \psi_{i}(x_{i})$$

$$+ P_{x} Z_{i}(x_{i}) + P_{E} E_{i}(x_{i}).$$

(245)

with the first term dominated by QCD-penguin contributions, the next three terms by electroweak penguin contributions and the last term being totally negligible. The one-loop functions $X_{0}$, $Y_{0}$ and $Z_{0}$ can be found in (203), (123) and (124), respectively. The coefficients $P_{i}$ are given in terms of the non-perturbative parameters $R_{6}$ and $R_{8}$ defined in (248) as follows:

$$P_{i} = r_{i}^{(0)} + r_{i}^{(6)} R_{6} + r_{i}^{(8)} R_{8}.$$  

(246)

The coefficients $r_{i}^{(0)}$, $r_{i}^{(6)}$ and $r_{i}^{(8)}$ comprise information on the Wilson-coefficient functions of the $\Delta S = 1$ weak effective Hamiltonian at the NLO and their numerical values can be found in [119]. These numerical values are chosen to satisfy the so-called $\Delta I = 1/2$ rule and emphasize the dominant dependence on the hadronic matrix elements residing in the QCD-penguin operator $Q_{6}$ and the electroweak penguin operator $Q_{8}$. From table 1 in [119] we find that for the central value of $\alpha_{s}(M_{Z}) = 0.1185$ the largest are the coefficients $r_{0}^{(6)}$ and $r_{2}^{(8)}$, representing QCD-penguin and electroweak penguin contributions, respectively:

$$r_{0}^{(6)} = 16.8, \quad r_{2}^{(8)} = -12.6.$$  

(247)

The fact that these coefficients are of similar size but have opposite signs has been a problem since the end of 1980s, when the electroweak penguin contribution increased in importance due to the large top-quark mass [332, 333].

The parameters $R_{6}$ and $R_{8}$ are directly related to the $B$-parameters $B_{6}^{(1/2)}$ and $B_{8}^{(1/2)}$, representing the hadronic matrix elements of $Q_{6}$ and $Q_{8}$, respectively. They are defined as

$$R_{6} \equiv 1.13 B_{6}^{(1/2)} \left[ \frac{114 \text{ MeV}}{m_{s}(m_{c}) + m_{s}(m_{c})} \right]^{2},$$

$$R_{8} \equiv 1.13 B_{8}^{(1/2)} \left[ \frac{114 \text{ MeV}}{m_{s}(m_{c}) + m_{s}(m_{c})} \right]^{2},$$

(248)

where the factor 1.13 signals the decrease of the value of $m_{s}$ since the analysis in [328] has been done.

A detailed analysis of $\varepsilon/\varepsilon'$ is clearly beyond this review and we would like to make only a few statements.

In [328] it has been found that with $R_{8} = 1.0 \pm 0.2$ as obtained at that time from lattice QCD, the data could be reproduced within the SM for $R_{6} = 1.23 \pm 0.16$. While in 2003 this value would correspond to $B_{6}^{(1/2)} = 1.23$, the change in the value of $m_{s}$ would imply $B_{6}^{(1/2)} = 1.05$, which is very close to the large $N$ value. Now, the most recent evaluation of $B_{6}^{(3/2)}$, from lattice QCD [316, 334, 335] finds $B_{6}^{(3/2)} \approx 0.65$ and, thereby, implies that $R_{6} \approx 0.8$.

A very recent analysis of $\varepsilon/\varepsilon'$ in the SM [119] which uses this lattice result finds that for $B_{6}^{(1/2)} = 1.0$ the agreement of the SM with the data is indeed good, although parametric uncertainties, in particular due to $|V_{ub}|$ and $|V_{cb}|$, still allow for sizable NP contributions. Undoubtedly, we need sufficient precision on $B_{6}^{(1/2)}$ and these two CKM parameters in order to have a clear cut picture of $\varepsilon'/\varepsilon$. We are looking forward to the improved values of $|V_{ub}|$, $|V_{cb}|$, $B_{6}^{(1/2)}$ and $B_{8}^{(3/2)}$ and expect that in the second half of this decade $\varepsilon'/\varepsilon$ will again become an important actor in particle physics. The correlations with $K_{L} \rightarrow \pi^{0}v\bar{v}$ and $K^{+} \rightarrow \pi^{+}v\bar{v}$, which were reanalysed recently in [119], should then help us to select favourite NP scenarios, particularly if the experimental branching ratios for these decays will be known with sufficient accuracy.

5.11. Step 11: Charm and Top Systems

5.11.1. Preliminaries. Our review is dominated by mixing and decays in $K$, $B_{d}$ and $B_{s}$ meson systems. In the last two steps we want to emphasize that charm and top physics (this step) as well as LFV, EDMs, and $(g-2)_{e,\mu}$ discussed in the next step play important roles in the search for NP. Our discussion will be very brief but we hope that general statements and the selected references are still useful for non-experts.

5.11.2. Charm. The study of $D$ mesons allows us to explore in a unique manner the physics of up-type quarks in FCNC processes. This involves $D^{0} - \bar{D}^{0}$ mixing, direct and mixing induced CP violation, and rare decays of mesons. An excellent summary of the present experimental and theoretical status as well of the future prospects for this field can be found in chapter 4 of [22]. We cannot add anything new to the information given there, but by not working recently in this field we can provide a number of unbiased statements.

Charm decays have the problem that the intermediate scale of roughly 2 GeV does not allow us, on the one hand, to use methods such chiral perturbation theory or large $N$ that are useful for $K$ physics. On the other hand, methods such as heavy quark effective theories are not as useful here as in the $B_{s,d}$ systems. Fortunately, lattice simulation are mostly done around this scale so that the future of this field will definitely depend on the progress made by lattice QCD.

Due to the presence of down quarks in the loop diagrams governing FCNCs within the SM, the GIM mechanism is very effective, so that the SD part of any SM contribution is strongly suppressed. Consequently, the background to possible NP contributions from this part is significantly smaller than in the case of $K$ and $B_{s,d}$ meson systems. This is particularly the case of the CP violation, which is predicted to be tiny in $D$ meson system. Unfortunately, a large background to NP from hadronic effects makes the study of NP effects in this system very challenging and even the originally large direct CP violation observed by LHCb [336] could not be uniquely attributed to the signs of NP. The recent update shows that the anomaly in question has basically disappeared [337] but NP could still be hidden under hadronic uncertainties.

This situation could improve in the future and a large amount of theoretical work prompted by these initially exciting LHCb results will definitely be very useful when the data is improved. It is impossible to review this work, which is summarized in [22], and we will mention here only few papers that fit very well to the spirit of our review because they discuss correlations between CP violation in charm decays and other observables [338–340]. These correlations, as in the
decays discussed by us in previous steps, depend on the model considered, so they may help us to identify the NP at work. They involve not only observables in the charm system, such as rare decays $D^0 \rightarrow \phi \gamma$ or $D^0 \rightarrow \mu^+ \mu^-$, but also observables measured at high-$p_T$, such as $t\bar{t}$ asymmetries, which is another highlight from the LHC.

In this context one should mention correlations between $D$ and $K$, which could be used to constrain NP effects in $K$ system through those in charm and vice versa [341, 342]. In particular, the universality of CP violation in flavour-changing decay processes elaborated in [342] allows us to predict direct correspondence between NP contributions to the direct CP violation in charm and $K_L \rightarrow \pi \pi$ represented by $\varepsilon'/\varepsilon$. There is no question that charm physics will play a significant role in the search for NP by constraining theoretical models and offering complementary information to the one available from $K$ and $B_{d,s}$ system. Yet, from the present perspective, clear cut conclusions about the presence or absence of relevant NP contributions will be easier to reach by studying observables considered by us in previous steps.

5.11.3. Top quark. The heaviest quark, the top quark, has already played a dominant role in our review. It governs SM contributions to all observables discussed by us. The fact that the SM is doing well indicates that the structure of the CKM matrix with three hierarchical top quark couplings to lighter quarks

$$|V_{td}| \approx 8 \times 10^{-3}, \quad |V_{ts}| \approx 4 \times 10^{-2}, \quad |V_{tb}| \approx 1$$

(249)

combined with the GIM mechanism represents the flavour properties of the top quark well. Yet, as the LHC became a top quark factory, properties of the top can also be studied directly through its production and decay. In the latter case, FCNC processes such as $t \rightarrow c \gamma$ can be investigated. It is also believed that the top quark is closely related to various aspects of electroweak symmetry breaking and the problem of naturalness. Indeed, the top quark having the largest coupling to the Higgs field is the main reason for the severe fine tuning that is necessary to keep the Higgs mass close to the electroweak scale.

For these reasons, we expect that the direct study of top physics, both flavour conserving and flavour violating, will give us a profound insight into SD dynamics, particularly as hadronic uncertainties at such SD scales are much smaller than in decays of mesons. The observation of a large forward backward asymmetry in $t\bar{t}$ production at the Tevatron and the intensive theoretical studies aiming to explain this phenomenon have shown that this type of physics has great potential in constraining various extensions of the SM. Since this material goes beyond the goals of our review, we just wanted to emphasize that this is an important field in the search for NP. A useful collection of articles that deal with top and flavour physics in the LHC era can be found in [343]. A detailed study of flavour sector with up vector-like quarks including correlations among various observables can be found in [169].

5.12. Step 12: LFV, $(g-2)_\mu$, and EDMs

5.12.1. Preliminaries. Our review deals dominantly with quark flavour violating processes. Yet, in the search for NP an important role will also be played by

- neutrino oscillations, neutrinoless double $\beta$ decay;
- charged lepton violation;
- anomalous magnetic moment of the muon $a_\mu = \frac{1}{2}(g-2)_\mu$;
- EDMs of the neutron, atoms and leptons.

In the following we will only very briefly discuss these items. Selected reviews of these topics can be found in [7, 23, 344–346], where many references can be found. The study of correlations between LFV, $(g-2)_\mu$ and EDMs in supersymmetric flavour models and SUSY GUTs can be found in [35, 347–349]. Analogous correlations in models with vector-like leptons have been presented in [350] and general expressions for these observables in terms of Wilson coefficients of dimension-six operators can be found in [351].

Concerning the first item, the observation of neutrino oscillations is a clear signal of physics beyond the SM and so far, together with dark matter and the matter–antimatter asymmetry observed in our Universe, is the only clear sign of NP. In order to accommodate neutrino masses one needs to extend the SM. The most straightforward way to this is to proceed in the same manner as for quark and charged lepton masses and just introduce three RH neutrinos that are singlets under the SM gauge group anyway. A Dirac mass term is then generated via the usual Higgs coupling $\bar{\nu}_i Y_i H v_\nu$. However, there is also the possibility for a Majorana mass term for the RH neutrinos since it is gauge invariant. One would need to introduce or postulate a further symmetry to forbid this term, which is also already an extension of the SM. Furthermore, this Majorana mass term introduces an additional scale $M_R$ and since it is not protected by any symmetry it could be rather high. Then, the seesaw mechanism is at work and can generate light neutrino masses as observed in nature. Another possibility to get neutrino masses without RH neutrinos is the introduction of an additional Higgs-triplet field. Either way, the accommodation of neutrino masses requires an extension of the SM.

In the second and last point from above, the interest in the related observables is based on the fact that they are suppressed within the SM to such a level that any observation of them would clearly signal physics beyond the SM. In this respect, they differ profoundly from all processes discussed by us until now, which suffer from a large background coming from the SM while one needs precise theory and precise experiment to identify NP. Although $a_{e,\mu}$ are both flavour- and CP-conserving, they also offer powerful probes to test NP.

5.12.2. Charged LFV. The discovery of neutrino oscillations has shown that the individual lepton numbers are not conserved. However, no charged lepton flavour violating decays have been observed to date. In the SM enriched by light neutrino masses, lepton flavour violating decays $\ell_i \rightarrow \ell_j \gamma$ occur at unobservable small rates because the transition amplitudes are suppressed by a factor of $(m_{\ell_j}^2 / m_{\ell_i}^2) / M_W^2$. 
On the other hand, in many extensions of the SM, such as supersymmetric models, LHT or the SM with sequential fourth generation (SM4) measurable in this decade branching ratios are particularly predicted when the masses of involved new particles are in the LHC reach. However, it should be stressed that in principle LFV can even be sensitive to energy scales as high as 1000 TeV. For a recent analysis within mini-split supersymmetry see [352].

The most prominent role in the LFV studies is played by the decays
\[ \mu \rightarrow e\gamma, \quad \tau \rightarrow \mu\gamma, \quad \tau \rightarrow e\gamma \] (250)
but also the study of decays
\[ \mu^- \rightarrow e^- e^+ e^-, \quad \tau^- \rightarrow \mu^- \mu^+ \mu^-, \quad \tau^- \rightarrow e^- e^+ e^- \] (251)
while \( \mu \rightarrow e \) conversion in nuclei also offer in conjunction with \( l_i \rightarrow l_j \gamma \) powerful tests of NP.

Since our review is dominated by correlations, let us just mention how a clear cut distinction between supersymmetric models, LHT model and SM4 is possible on the basis of these decays. While it is not possible to distinguish the models, LHT model and SM4 is possible on the basis of mention how a clear cut distinction between supersymmetric models. Other analyses of LFV in the LHT model were also identified. A detailed analysis of LFV in various extensions of the SM is also motivated by the prospect of measurements of LFV processes that will have much higher sensitivity than those presently available. In particular, the MEG experiment at PSI is already testing \( B(\mu \rightarrow e\gamma) \) at the level of \( \mathcal{O}(10^{-13}) \). The current upper bound is [363]
\[ B(\mu \rightarrow e\gamma) \leq 5.7 \times 10^{-13}. \] (252)
This bound puts also some GUT models under pressure, as in for example the model discussed in section 6.5.5. An upgrade for MEG is also already approved [364], where they expect to improve the sensitivity down to \( 6 \times 10^{-14} \) after three years of running and there is an approved proposal at PSI to do \( \mu \rightarrow eee \) [365]. The planned accuracy of SuperKEKB of \( \mathcal{O}(10^{-8}) \) for \( \tau \rightarrow \mu\gamma \) is also of great interest. This decay can also be studied at the LHC.

An improved upper bound on \( \mu \rightarrow e \) conversion in titanium will also be very important. In this context, the dedicated J-PARC experiment PRISM/PRIME [366] should reach the sensitivity of \( \mathcal{O}(10^{-18}) \); that is, an improvement by six orders of magnitude relative to the present upper bound from SINDRUM-II at PSI [367]. Mu2e collaboration will measure \( \mu \rightarrow e \) conversion on aluminum to \( 6 \times 10^{-17} \) at 90% CL around 2020 [368], which is a factor of \( 10^4 \) better than SINDRUM-II. Meanwhile, an improvement of a factor 10 is planned to be reached with Project X at Fermilab [27]. In [369] the model discriminating power of a combined phenomenological analysis of \( \mu \rightarrow e\gamma \) and \( \mu \rightarrow e \) conversion on different nuclei targets is discussed. They found that, in most cases, going from aluminum to titanium is not very model-discriminating. A realistic discrimination among models requires a measure of \( \mathcal{B}(\mu \rightarrow e, T\gamma)/\mathcal{B}(\mu \rightarrow e, A\gamma) \) at the level of 5% or better.

For further detailed review of LFV see [7, 370, 371]. An experimenter’s guide for charged LFV can be found in [346].

5.12.3. Anomalous magnetic moments \((g - 2)_{\mu,e}\)

The anomalous magnetic moment of the muon
\[ a_\mu = \frac{(g - 2)_\mu}{2} \] (253)
provides an excellent test for physics beyond the SM. It can be extracted from the photon-muon vertex function \( \Gamma^\mu(p', p) \)
\[ \tilde{a}(p')\Gamma^\mu(p', p)u(p) = \tilde{a}(p') \left[ \gamma^\mu F_V(q^2) + (p + p')^\mu F_M(q^2) \right] u(p), \] (254)
with
\[ a_\mu = -2m_\mu F_M(0). \] (255)
On the theory side \( a_\mu \) receives four dominant contributions:
\[ a_\mu^{SM} = a_\mu^{QED} + a_\mu^{ew} + a_\mu^{\gamma\gamma} + a_\mu^{\text{NP}}. \] (256)
While the QED [372–375] and electroweak contributions [344, 376] to \( a_\mu^{SM} \) are known very precisely and the light-by-light contribution \( a_\mu^{\gamma\gamma} \) is currently known with an acceptable accuracy [377, 378], the theoretical uncertainty is dominated by the hadronic vacuum polarization. A review of the relevant calculations of all these contributions and related extensive analyses can be found in [344, 379].

According to the most recent analysis in [379], the very precise measurement of \( a_\mu \) by the E821 experiment [380] in Brookhaven differs from its SM prediction by roughly 4.6σ:
\[ a_\mu^{exp} - a_\mu^{SM} = (39.4 \pm 8.5) \times 10^{-10}, \] (257)
where we have added various errors discussed in [379] in quadrature.

Many models beyond the SM have tried to explain this discrepancy, supersymmetric models were especially popular [381–387]. In SUSY the discrepancy could easily be accommodated for relatively light smuon masses and large \( \tan B \). However, so far no light SUSY particles have been discovered. Another approach was followed in [388], where the interplay of \((g - 2)_\mu \) and a soft muon Yukawa coupling that is generated radiatively in the MSSM was studied. With
the increased SUSY mass scale the explanation of \((g - 2)_{\mu}\) anomaly becomes difficult \cite{389}.

Of course, a new experiment would also be desirable. Fortunately, the \((g - 2) - 2\) ring at BNL has been disassembled and is on its way to Fermilab for a run around 2016. The overall error should go down by a factor of 2. Thus, if the central value will remain unchanged then the discrepancy with the SM will increase to more than 8.0\(\sigma\).

The anomalous magnetic moment of the muon \(a_{\mu}\) is more sensitive to lepton flavour conserving NP than \(a_{e}\), and, consequently, the latter was not as popular as \(a_{\mu}\) in the last decade. However, as emphasized in \cite{349}, the fact that \(a_{e}\) is very precisely measured and very precisely calculated within the SM means that it can also be used to probe NP, even if the theory agrees very well with experiment. Indeed, \(a_{e}\) plays a central role in QED since its precise measurement provides the best source of \(\alpha_{em}\), assuming the validity of QED \cite{390}. Conversely, one can use a value of \(\alpha_{em}\) from a less precise measurement and insert it into the theory prediction for \(a_{e}\) to probe NP. The most recent calculation yields \(a_{e} = 1.159652182.79(7.71) \times 10^{-12}\) \cite{391}, where the largest uncertainty comes from the second-best measurement of \(\alpha_{em}\), which is \(\alpha_{em}^{-1} = 137.03599884(91)\) from a rubidium atom experiment \cite{392}. Usually, NP contributions to \(a_{e}\) are small due to the smallness of the electron Yukawa coupling and the suppression of the NP scale. However, multiple flavour changes, resulting effectively in a lepton flavour conserving loop could, be enhanced due to the \(\tau\) Yukawa coupling \cite{349}.

5.12.4. Electric dipole moments. Even though the experimental sensitivities have improved a lot, no EDM of a fundamental particle has so far been observed. Nevertheless, EDM experiments have already put strong limits on NP models. A permanent EDM of a fundamental particle violates both T and P, and thus—assuming CPT symmetry—is another way to measure CP violation. In the SM the only CP-violating phase of the CKM matrix enters quark EDMs first at three loops (two loop EW + one loop QCD), which results in negligibly small SM EDMs. Consequently, EDMs are excellent probes of new CP-violating phases of NP models, especially FBPs, and of strong CP violation.

A recent review about EDMs can be found in \cite{345}, which updates the review in \cite{393}. See also \cite{394}. As discussed in \cite{345}, by naive dimension analysis EDMs probe a NP scale of several TeV. This assumes order one CP-violating phases \(\phi_{CP}\) for the electron EDM that arises at one loop order:

\[
de_{e} \approx m_{e} \frac{\alpha_{e}}{\Lambda^{2}} \sin \phi_{CP} \approx \frac{1}{2} \left(\frac{1 \text{ TeV}}{\Lambda}\right)^{2} \sin \phi_{CP} \times 10^{-13} \text{e fm},
\]

(258)

Recently, the upper bound on \(d_{e}\) has been improved by an order of magnitude with respect to the previous bound in \cite{395} and reads \cite{396}

\[
|d_{e}| \lesssim 8.7 \times 10^{-16} \text{e fm}.
\]

(259)

This implies for the CP-violating phase \(\sin \phi_{CP} \lesssim \frac{1}{16\pi^{2}}\). The implications of this new bound on MFV have been investigated in \cite{397} and other analyses are expected in the near future.

The scale of NP can be even higher for the neutron and \(^{199}\text{Hg}\) EDMs because they are sensitive to the chromo-magnetic EDM that enters with a factor of \(\alpha_{s}\) rather than the fine structure constant \(\alpha_{em}\), pushing the sensitivity closer to 10 TeV. As one can see from (258), the sensitivity to the NP scale goes as \(1/\Lambda^{2}\), whereas in many other cases, such as LFV, the sensitivity goes as \(1/\Lambda^{4}\). Future EDM measurements aim to improve their sensitivity by approximately two orders of magnitude, which will then push the mass scale sensitivity into the 20–100 TeV range.

There are different sources for EDMs. For hadronic EDMs there is the \(\Theta\) term of QCD, which is very much constrained due to the non-observation of permanent EDMs of the \(^{199}\text{Hg}\) atom and neutron. Apart from the \(\Theta\) term, the SM CKM induced EDMs would be far smaller in magnitude than the next generation EDM sensitivities. Consequently, one does not need the same kind of refined hadronic structure computations as one often needs in flavour physics to interpret the EDM results in terms of NP. That being said, the hadronic matrix element problem remains a considerable challenge. At dimension six, one encounters several different operators for the first generation fermions that could give rise to EDMs: pure gauge operators \(\bar{G}G\), four-fermion operators (semi-leptonic and non-leptonic), gauge-Higgs operators \(\psi^\dagger \psi \bar{G}G\), and gauge-Higgs-fermion operators \((\bar{Q}T^AQq)\psi G\).

In experiments one often deals with composite systems and, thus, nuclear physics is important in determining the EDMs of neutral atoms. Nuclear structure can also provide an amplifier of atomic EDMs. In heavier neutral systems there is the shielding of the EDMs of constituents of one charge by those of the other (e.g. protons and electrons). The transmission of the CP violation through a nucleus into an atom must overcome this shielding. Its effectiveness in doing so is expressed by a nuclear Schiff moment. In nuclei with asymmetric shapes these Schiff moments can be enhanced by two or three orders of magnitude. For example, an octupole deformed nucleus, such as \(^{225}\text{Ra}\), gives enhanced nuclear Schiff moments and, thus, an enhanced atomic EDMs in a diamagnetic system.

Flavour diagonal CP-violating phases as needed for electroweak baryogenesis can be strongly constrained by EDMs. In the MSSM, for example, this requires rather heavy first and second generation sfermions but at the same time light electroweak gauginos below one TeV, as well as a subset of the third generation sfermions (see \cite{398} for details). However, as can be deduced from the plots in \cite{399}, the improved bound on \(d_{e}\) in (259) nearly excludes this possibility. While the bino-driven baryogenesis analysed in \cite{400} is still allowed by this new measurement, it further constrains this scenario.

A new and largely unexplored direction for electroweak baryogenesis is the flavour non-diagonal CPV that would enter the \(B\) or \(D\) meson systems \cite{401–403}. Flavour non-diagonal CP violation is far less susceptible to EDM constraints than flavour diagonal phases since it arises at multi-loop order. In the SM, for example, it is a two-loop effect that involves the one-loop CP-violating penguin operator and a hadronic loop with two \(\Delta S = 1\) weak interactions.
Finally, let us quote recent studies of EDMs in 2HDM models with FBPs [140, 404] and supersymmetry [352], where further references to the rich literature can be found.

6. Towards selecting successful models

6.1. Preliminaries

We have seen in previous sections that by considering several theoretically clean observables in the context of various extensions of the SM there is a chance that we could identify new particles and new forces at very SD scales that are outside the reach of the LHC. In fact, this strategy is not new as most of elementary particles of the SM have been predicted to exist on the basis of low energy data well before their discovery6. Moreover, this has been achieved by not only the desire to understand the data but simultaneously with the goal to construct a fundamental theory of elementary matter and elementary interactions that is predictive and consistent with all physics principles we know. Yet, the present situation differs from the days when one started to discover first quarks in the following manner. Based on the time and resources that were required to build the LHC, it is rather unlikely that a machine probing directly 100–200 TeV energy scales or SD scales in the ballpark of a zeptometer (10^−21 m) will exist in the first half of this century. Rather, a machine such as an international linear collider with the energy of 1 TeV will be built in order to study the details of physics up to this energy scale. Therefore, the search for new phenomena below 4 × 10^−20 m, which is beyond the LHC, will be in the hands of flavour physics and very rare processes.

There is no question that the progress in the search for NP at the shortest distance scales will require an intensive collaboration of experimentalists and theorists. In this context, there is the question of whether a top-down or a bottom-up approach will turn out to be more efficient in reaching this goal. While the bottom-up approach using exclusively effective theories with basically arbitrary coefficients of local operators allowed by symmetries of the SM can provide some insight in what is going on, we think that the top-down approach will eventually be more effective in the flavour precision era in identifying NP beyond the LHC’s reach. Yet, needless to say, it would be extremely important to get some directions from direct discoveries of new phenomena at the LHC. This would particularly allow for the correlations between high energy and low energy observables, which is only possible in a top-down approach.

Thus, our basic strategy, as already exemplified on previous pages, is to look at different models and study different patterns of flavour violation in various theories through identification of correlations between various observables. The question then arises of how to do it most efficiently and transparently.

In principle, global fits of various observables in a given theory to the experimental data appears to be most straightforward. The success or failure of a given theory is then decided on the basis of χ² or other statistical measures. This is clearly a legitimate approach and it is used almost exclusively in the literature. Yet, we think that in the first phase of the search for NP a more transparent approach could turn out to be more useful, which we will present next.

6.2. DNA chart

As reviewed in [17, 20], extensive studies of many models have allowed us to construct various classifications of NP contributions in the form of ‘DNA’ tables [35] and flavour codes [17]. The ‘DNA’ tables in [35] aimed to indicate whether, in a given theory, a value of a given observable can differ by a large, moderate or only tiny amount from the prediction of the SM. The flavour codes [17] were more a description of a given model, in terms of the presence or absence of LH or RH currents in it, and the presence or absence of new CP phases, flavour violating and/or flavour conserving.

Certainly, in both cases there is room for improvement. In particular, in the case of the ‘DNA’ tables considered in [35], we now know that in most quark flavour observables NP effects can be at most by a factor of 2 larger than the SM contributions. Exceptions to this includes those cases in which some branching ratios or asymmetries vanish in the SM. But the particular weakness of this approach is the difficulty in depicting the correlations between various observables that could be characteristic for a given theory. Such correlations are much easier to show on a circle and in what follows we would like to formulate this new idea and illustrate it with a few examples.

\begin{align}
\text{Step 1.} & \quad \text{We construct a chart showing different observables, typically a branching ratio for a given decay or an asymmetry, like CP-asymmetries } S_\psi K, \psi K^∗, S_\psi φ, S_\psi φ^∗, \Delta M_{K^∗}, \Delta M_{φ}, \delta_{K^∗} \text{ and } \delta_{φ}. \text{ The important point is to select the optimal set of observables that are simple enough so that definite predictions in a given theory can be made.} \\
\text{Step 2.} & \quad \text{In a given theory we calculate the selected observables and investigate whether a given observable is enhanced or suppressed relative to the SM prediction or is basically unchanged. To this end a measure like two } \sigma \text{ will be required. In the case of asymmetries, we will proceed in the same manner if its sign remains unchanged relative to the one in the SM but otherwise we define the change of its sign from } + \text{ to } − \text{ as a suppression and the change from } − \text{ to } + \text{ as an enhancement. For these three situations we will use the following colour coding:} \\
\text{enforcement} & \quad \text{yellow, } \text{no change} = \text{white} \quad \text{suppression} = \text{black} \quad (260) \\
\text{Step 3.} & \quad \text{It is only seldom that a given observable in a given theory is uniquely suppressed or enhanced but frequently two observables are correlated or uncorrelated; that is, the enhancement of one observable implies uniquely an enhancement (correlation) or suppression (anti-correlation) of} \\
\end{align}
Another observable. It can also happen that no change in the value of a given observable implies no change in another observable. There are of course other possibilities. The idea then is to connect in our DNA chart a given pair of observables that are correlated with each other by a line. The absence of a line means that two given observables are uncorrelated. In order to distinguish the correlation from anti-correlation, we will use the following colour coding for the lines in question: correlation $\leftrightarrow$ blue, anti-correlation $\leftrightarrow$ green.

(261)

We will first make selection of the optimal observables that can be realistically measured in this decade and we will subsequently illustrate the DNA chart in few simple models.

### 6.3. Optimal observables

On the basis of our presentation in the previous sections, we think that one should first have a closer look at the following observables.

$$\begin{align*}
\epsilon_{K}, \quad \Delta M_{s,d}, \quad S_{\phi K_{s}}, \quad S_{\phi \phi}, \\
K^{+} \to \pi^{+}\nu\bar{\nu}, \quad K_{L} \to \pi^{0}\nu\bar{\nu}, \quad e'/e, \\
B_{d,d} \to \mu^{+}\mu^{-}, \quad B \to X_{s}\nu\bar{\nu}, \quad B \to K^{\star}(K)\nu\bar{\nu}, \\
B \to X_{l}Y, \quad B^{*} \to \tau^{+}\nu_{\tau}, \quad B \to K^{\star}(K)\mu^{+}\mu^{-},
\end{align*}$$

where in the latter case we mean theoretically clean angular observables. The remaining observables discussed by us will then serve as constraints on the model and if measured could also be chosen.

### 6.4. Examples of DNA charts

The first DNA chart that one should in principle construct is the one dictated by experiment. This chart will have no correlation lines but will show where the SM disagrees with the data and comparing it with DNA chart specific to a given theory will indicate which theories have survived and which have been excluded. Unfortunately, in view of the significant uncertainties in some of the SM predictions and the rather weak experimental bounds on most interesting branching ratios, such an experimental chart is rather boring at present because it is basically white. However, in the second half of this decade, when LHC and other machines will provide new data and lattice calculations will increase their precision, it will possible to construct such an experimental DNA chart and we should hope that it will not be completely white.

Here we want to present four examples of DNA charts. In figure 43 we show the DNA chart of CMFV while the corresponding chart for $U(2)^3$ models is shown in figure 44. The DNA charts representing models with LH and RH flavour-violating couplings of $Z$ and $Z'$ can be found in figure 45.

The interested reader may check that these charts summarize compactly the correlations that we have discussed in detail at various places in this review. In particular, we observe the following features:

- When going from the DNA chart of CMFV in figure 43 to the one for the $U(2)^3$ models in figure 44, the correlations between $K$ and $B_{s,d}$ systems are broken because the symmetry is reduced from $U(3)^3$ down to $U(2)^3$. The anti-correlation between $S_{\phi \phi}$ and $S_{\phi K_{s}}$ is just the one shown in figure 13.
- As the decays $K^{+} \to \pi^{+}\nu\bar{\nu}$, $K_{L} \to \pi^{0}\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ are only sensitive to the vector quark currents, they do not change when the couplings are changed from LH to RH. On the other hand, the remaining three decays in figure 45 are sensitive to axial-vector couplings, implying an interchanged of enhancements and suppressions when going from $L$ to $R$ and also a change of correlations to anti-correlations between the latter three and the former three decays. Note that the correlation between $B_{s} \to \mu^{+}\mu^{-}$ and $B \to K^{\star}\mu^{+}\mu^{-}$ does not change because both decays

![Figure 43. DNA chart of CMFV models. Yellow means enhancement, black means suppression, and white means no change. Blue arrows $\leftrightarrow$ indicate correlation and green arrows $\leftrightarrow$ indicate anti-correlation.](image)

![Figure 44. DNA chart of $U(2)^3$ models. Yellow means enhancement, black means suppression, and white means no change. Blue arrows $\leftrightarrow$ indicate correlation and green arrows $\leftrightarrow$ indicate anti-correlation.](image)
are sensitive only to axial-vector coupling.

- However, it should be remarked that, in order to obtain the correlations or anti-correlations in LHS and RHS scenarios, it was assumed that the signs of the LH couplings to neutrinos and the axial-vector couplings to muons are the same, which does not have to be the case. If they are opposite then the correlations between the decays with neutrinos and muons in the final state change to anti-correlations, and vice versa.
- On the other hand, due to $SU(2)_L$ symmetry, the LH $Z'$ couplings to muons and neutrinos are equal and this implies the relation

$$\Delta_{L}^{\nu\nu}(Z') = \frac{\Delta_{L}^{\mu\nu}(Z') - \Delta_{A}^{\mu\nu}(Z')}{2}.$$  

(266)

Therefore, once two of these couplings are determined, the third follows uniquely without the freedom mentioned in the previous item.

- In the context of the DNA charts in figure 45, the correlations involving $K_L \rightarrow \pi^0 \nu \bar{\nu}$ apply only if NP contributions carry some CP-phases. If this is not the case, then the branching ratio for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ will remain unchanged. This is evident from our discussion in step 8 and the plots presented there.

In this context, let us summarize the following important properties of the case of tree-level $Z'$ and $Z$ exchanges when both LH and RH quark couplings are present, which in addition are equal to each other (LRS scenario) or differ by sign (ALRS scenario):

- In LRS, NP contributions to $B_{s,d} \rightarrow \mu^+ \mu^-$ vanish but not to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.
- In ALRS, NP contributions to $B_{s,d} \rightarrow \mu^+ \mu^-$ are non-vanishing and this also applies to $B_d \rightarrow K^+ \mu^+ \mu^-$, as seen in the right hand panel of figure 32. On the other hand, they vanish in the case of $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $B_d \rightarrow K \mu^+ \mu^-$.

6.5. Reviewing concrete models

The realization of this strategy in the case of more complicated models is more challenging in view of many parameters involved, which often have to be determined beyond flavour physics. However, we expect that when more data from the LHC and flavour machines around the world becomes available it will be possible to be more concrete, which will also be the case for these more complicated models. Two rather detailed reviews of various patterns of flavour violation in a number of favorite and less favourite extensions of the SM appeared in [17, 20]. In view of the fact that no totally convincing signs of NP in flavour data has been observed since the appearance of the second review, there is no point in presently updating these reviews. Basically, all of these models can fit the present data by adjusting the parameters or increasing the masses of new particles. Therefore, we only make a few remarks on some of these models and indicate in which section of [20] more details on a given model and related references to the original literature can be found.

6.5.1. $331$ model. A concrete example for $Z'$ tree-level FCNC discussed in section 3.7 and at various places in section 5 is a model based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$, the so-called $331$ model, which was originally developed in [405, 406]. There are different versions of the $331$ model, which are characterized by a parameter $\beta$ that determines the particle content. In [41] we consider the $\beta = 1/\sqrt{3}$-model to be called an $\overline{331}$ model. Since only LH currents are flavour violating and the effects in $\epsilon_K$ are rather small, it favours inclusive $|V_{ub}|$ and thus belongs to LHS2. Furthermore, the lepton couplings are no longer arbitrary but come out automatically from the Lagrangian: $\Delta_{L}^{\nu\nu}(Z') = 0.14$ and $\Delta_{A}^{\mu\nu}(Z') = -0.26$ for $\beta = 1/\sqrt{3}$. For the general $Z'$ scenario, we used $\Delta_{L}^{\nu\nu}(Z') = 0.5$ and $\Delta_{A}^{\mu\nu}(Z') = 0.5$.

In the breaking $SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$ to the SM gauge group, a new heavy neutral gauge boson $Z'$ appears that mediates FCNC already at tree level. A nice
theoretical feature is that from the requirement of anomaly cancellation and asymptotic freedom of QCD, it follows that one needs $N = 3$ generations. Anomaly cancellation is only possible if one generation (usually the third is chosen) is treated differently than the other two generations.

Further studies of the 331 model can be found in [407, 408], where the lepton sector was analysed in detail, and in [409–411], where mixing of neutral mesons as well as a number of rare $K$ and $B_{d,s}$ decays have been considered. The decay $b \rightarrow s\gamma$ was considered in [412, 413] and in [414], neutral scalar contributions were also included.

Flavour structure

The 331 model studied in [41] has the following fermion singlets under $SU(3)_L$.

neutral scalar contributions were also included.

| $e, \mu, \tau$ | $\nu_e, \nu_\mu, \nu_\tau$ | $\tau$ |
|---------------|------------------|-------|
| $\nu_e, \nu_\mu, \nu_\tau$ | $\nu_\mu, \nu_\tau$ | $\nu_\tau$ |
| $\mu$ | $\tau$ | $\tau$ |

Additionally, the neutral scalar contributions were also included.

We need the same number of triplets and anti-triplets due to anomaly cancellation. If one takes into account the three colours of the quarks, then we have six triplets and six anti-triplets with this choice. Neutral currents mediated by $Z'$ are affected by the quark mixing because the $Z'$ couplings are generation non-universal. However, only LH quark currents are flavour violating, thus we are left with LHS. Except for the $Z'$ mass, the tree-level FCNCs in $B_{d,s}$ and $K$ meson systems depend effectively on 2 angles and 2 phases $\delta_{23}, \delta_{13}, \delta_{12},$ such that the $B_S$ sector depends only on $\delta_{13}, \delta_1$ and the $B_L$ sector on $\delta_{23}, \delta_2$. Then, in contrast to the general $Z'$ models, as discussed before, the NP parameters in $K$ sector are fixed. In particular, CP violation is governed there by the phase difference $\delta_2 - \delta_1$. In more general $Z'$ models the $K$ sector is decoupled from $B_{d,s}$ sector.

The phenomenology here is more restrictive than that in a general $Z'$ model with LH couplings and it is of interest to investigate how the 331 models with arbitrary $\beta$ face the new data on $B_{d,s} \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow K^*(K)\mu^+ \mu^-$ while taking into account the present constraints from $\Delta F = 2$ observables, low energy precision measurements, LEP-II, and the LHC data. Such an analysis has been performed in [47] and we summarize the main results of this paper where numerous correlations between various flavour observables can be found.

By studying the implications of these models for $\beta = \pm \pi/\sqrt{3}$ with $n = 1, 2, 3$ we find that the case $\beta = -\pi/\sqrt{3}$ leading to Landau singularities for $M_Z \approx 4 \text{ TeV}$ can be ruled out when the present constraints on $Z'$ couplings, in particular from LEP-II, are taken into account. For $n = 1, 2$, interesting results are found for $M_Z < 4 \text{ TeV}$ with largest NP effects for $\beta < 0$ in $B_d \rightarrow K^* \mu^+ \mu^-$ and the ones in $B_{s,d} \rightarrow \mu^+ \mu^-$ for $\beta > 0$. Since $Re(C^N_{9})$ can reach the values $-0.8$ and $-0.4$ for $n = 2$ and $n = 1$, respectively, the $B_d \rightarrow K^* \mu^+ \mu^-$ anomalies can be softened with the size depending on $\Delta M_{r, \Delta M_{r,S}}$ and the CP-asymmetry $S_{\phi, \phi}$. A correlation between $Re(C^N_{9})$ and $\overline{B}(B_d \rightarrow \mu^+ \mu^-)$, identified for $\beta < 0$, implies for negative $Re(C^N_{9})$ uniquely suppression of $\overline{B}(B_d \rightarrow \mu^+ \mu^-)$ relative to its SM value, which is favoured by the data. In turn, $S_{\phi, \phi} < S_{\phi, \phi}$ is also favoured with $S_{\phi, \phi}$, having a dominantly opposite sign to $S_{\phi, \phi}$ and closer to its central experimental value. There is another triple correlation between $Re(C^N_{9})$, $\overline{B}(B_d \rightarrow \mu^+ \mu^-)$ and $N$ of the light CP-even transitions, and $K^+ \rightarrow \pi^0\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ turn out to be small.

We also find that the absence of $B_d \rightarrow K^* \mu^+ \mu^-$ anomalies in the future data and confirmation of the suppression of $\overline{B}(B_d \rightarrow \mu^+ \mu^-)$ relative to its SM value would favour the $331$ model ($\beta = 1/\sqrt{3}$), which is summarised in detail above, and $M_Z \approx 3 \text{ TeV}$. Assuming lepton universality, we find an upper bound $|C^N_{9}| < 1.1(1.4)$ from LEP-II data for all $Z'$ models with only LH flavour-violating couplings to quarks when NP contributions to $\Delta M_4$ at the level of 10% (15%) are allowed.

6.5.2. Littlest Higgs model with T-parity. As stressed in section 3.6 of [20] the LHCb data can be considered as a relief for this model.

- In this model, it was not possible to obtain $S_{\phi, \phi}$ of $O(1)$ and values above 0.3 were rather unlikely. In addition, negative values for $S_{\phi, \phi}$ as opposed to 2HDM$_{\text{SM}}$ are possible in this model.

- Because of new sources of flavour violation originating in the presence of mirror quarks and new mixing matrices, the usual CMFV relations between $K$, $B_d$ and $B_s$ systems are violated. This allows us to remove the $S_{\phi, \phi}$, $S_{\phi, \phi}$ anomaly for both scenarios of $V_{ub}$ and also improve agreement with $\Delta M_{4,5}$. 

- The small value of $S_{\phi, \phi}$ from LHCb still allows for sizable enhancements of $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ and $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$, which would not be possible otherwise.

- On the other hand, rare $B$-decays turn out to be SM-like but some enhancements are still possible. In particular, $B(\bar{B}_s \rightarrow \mu^+ \mu^-)$ can be enhanced by 30% and a significant part of this enhancement comes from the T-even sector. The effects in $B(\bar{B}_d \rightarrow \mu^+ \mu^-)$ can be larger and suppression is also possible.

6.5.3. CP conserving 2HDM II. The authors of [415] made a global fit of the CP conserving 2HDM II with a softly broken $Z_2$ symmetry. Their analysis includes the experimental constraints from LHC on the mass and signal strength of the Higgs resonance at 126 GeV (which is always interpreted as the light CP-even 2HDM Higgs boson $h$), the non-observation of additional Higgs resonances, EWPO and flavour data on $B^0 - \bar{B}^0$ mixing and $B \rightarrow X_s\gamma$. Furthermore, the following theoretical constraints are taken into account: vacuum stability and perturbativity. The authors find that the parameter region with $\beta - \alpha \approx \frac{5}{7}$, where the couplings of the light CP-even
Higgs boson are SM like, is favoured. The allowed range in the tan $\beta - m_{H^+}$ plane is shown in figure 46 (left). The lower bound on $m_{H^+}$ of 322 GeV (400 GeV) at 2$\sigma$ (1$\sigma$) for tan $\beta > 1$ follows from the constraint from $B \rightarrow X_s \gamma$. The allowed mass regions for $H^0/A^0/H^+$ is shown on the right hand side of figure 46. Flavour and EWP observables exclude scenarios with both $m_H$ and $m_A$ below 300 GeV at 2$\sigma$.

Other recent analyses of 2HDM II can be found in [416–419]. In [193] it was even stated that 2HDM-II is ruled out by $B \rightarrow D(D^*)\tau\nu$ data. However, it seems to us that such a statement is premature because the data could change in the future and, moreover, this would also imply that the SM is ruled out because the 2HDM-II contains the SM in its parameter space in the decoupling limit.

6.5.4. Supersymmetric flavour models (SF). None of the supersymmetric particles have been seen so far. However, one of the important predictions of the simplest realization of this scenario, the MSSM with R-parity, is a light Higgs with $m_H \lesssim 130$ GeV. Although the discovery of a Higgs boson at the LHC around 125 GeV could indeed be the first hints for a Higgs of the MSSM, it will take some time to verify it. In any case, MSSM still remains a viable NP scenario at scales $\mathcal{O}(1 \text{ TeV})$, although the absence of the discovery of supersymmetric particles is rather disappointing. Similarly, the SUSY dreams of large $B(B_s \rightarrow \mu^+\mu^-)$ and $S_{\phi\phi}$ have not been realized at LHCb and CMS. However, the data from these experiments listed in (59), (132) and (133) have certainly had an impact on SUSY predictions.

In view of the rather rich structure of various SF models, as analysed in detail in [35] and summarized in section 3.8 of [20], it is not possible to discuss them adequately here. We make only two comments:

- The new data on $B(B_{s,d} \rightarrow \mu^+\mu^-)$ indicate that there is more room for NP contribution dominated by LH currents than RH currents.
- Although the large range of departures from SM expectations found in [35] has been significantly narrowed, there is still significant room for novel SUSY effects present in quark flavour data. Assuming that SUSY particles will be found, the future improved data for $B_{s,d} \rightarrow \mu^+\mu^-$ and $S_{\psi\phi}$, as well as $\gamma$ combined with $|V_{ub}|$, should help in distinguishing between various supersymmetric flavour models.

6.5.5. Supersymmetric SO(10) GUT model. Grand unified theories open the possibility to transfer the neutrino mixing matrix $U_{\text{PMNS}}$ to the quark sector and, therefore, correlate leptonic and hadronic observables. This is accomplished in a controlled way in a concrete SO(10) SUSY GUT, which was proposed by Chang, Masiero and Murayama (CMM model), where the atmospheric neutrino mixing angle induces new $b \rightarrow s$ and $\tau \rightarrow \mu$ transitions [420, 421]. In [422] we have performed a global analysis in the CMM model of several flavour processes containing $\Delta M_s$, $S_{\phi\phi}$, $b \rightarrow s\gamma$ and $\tau \rightarrow \mu\gamma$, including an extensive RG analysis to connect Planck-scale and low-energy parameters. A short summary of this work can also be found in [20, 423, 424].

Here, we want to shortly summarize the basic features of this model. At the Planck scale the flavour symmetry is exact but it is already broken at the SO(10) scale, which manifests itself in the appearance of a non-renormalizable operator in...
the SO(10) superpotential. The SO(10) symmetry is broken down to the SM gauge group via SU(5), and the whole \( 5 \)-plet \( S_i = (d^c_{R_i}, \xi_{Li}, -v_{Li})^T \) and the corresponding supersymmetric partners are then rotated by \( U_{\text{PMNS}} \). While at \( M_{\text{Pl}} \) the soft masses are still universal, we get a large splitting between the masses of the first, second and third down-squark and charge-slepton generation at the electroweak scale due to RG effects of \( \eta_i \). The flavour effects in the CMM model are then mainly determined by the generated mass splitting and the structure of the PMNS matrix.

In [422] we used tribimaximal mixing in \( U_{\text{PMNS}} \). However, the latest data now shows that the reactor neutrino mixing angle \( \theta_{13} \approx 8^\circ \) is indeed non-zero. Consequently, whereas effects in \( K^0 \rightarrow \bar{K}^0 \) mixing, \( B_d^0 \rightarrow B_d^\pm \) mixing and \( \mu \rightarrow e\gamma \) are very small in the original version of the model, this changes when \( \theta_{13} \approx 8^\circ \) is taken into account. Large effects in \( \mu \rightarrow e\gamma \) are now possible. With tribimaximal mixing, large contributions were only predicted in observables connecting the second and third generation. So we focused on \( b \rightarrow s\gamma \), \( \tau \rightarrow \mu\gamma , \Delta M_t \) and \( S_{\phi\phi} \). Concerning \( B \rightarrow \mu^+\mu^- \), effects are small because the CMM model at low energies appears as a special version of the MSSM with small tan \( \beta \), such that this branching ratio stays SM-like. Another observable that needs further investigation is the Higgs mass, which in the CMM model tends to be too small. The analysis of [422] was done prior to the detection of the Higgs boson and there we pointed out the Higgs mass could be up to 120 GeV in the parameter range consistent with flavour observables. However, an updated analysis of the CMM model shows that the two new experimental results, \( \theta_{13} \approx 8^\circ \) and \( M_H = 126 \text{ GeV} \), put the CMM model under pressure [425, 426]. The constraint from \( B(\mu \rightarrow e\gamma) \) (see equation (252)) supersedes those from \( b \rightarrow s \) and \( \tau \rightarrow \mu\gamma \) FCNC processes and requires very heavy sfermion and gaugino masses (\( \approx (8-10) \text{ TeV} \)). It is very difficult to find a range in the parameter space which simultaneously satisfy the Higgs mass constraint and the experimental upper bound on \( B(\mu \rightarrow e\gamma) \). A Higgs mass of \( M_H = 126 \text{ GeV} \) can be accommodated by passing from the MSSM to the NMSSM.

6.5.6. The minimal effective model with right-handed currents: RHMFV. A few years ago there was an interest in taking another general look at the RH currents, which originated in \( K^0 \rightarrow \bar{K}^0 \) mixing, \( B_d^0 \rightarrow B_d^\pm \) mixing and \( \mu \rightarrow e\gamma \) of the CMM model under pressure [425, 426]. The constraint from [422] for details, we would like to summarize the present status of this model:

- In this model, the high inclusive value of \( |V_{ub}| \) is selected by the model as the true value of this element, providing simultaneously the explanation of the smaller \( |V_{ub}| \) found in SM analysis of exclusive decays and the very high value of \( |V_{ub}| \) implied by the previous data for \( B(\tau \rightarrow \mu\nu_\tau) \). The decrease of the latter branching ratio casts some doubts on the explanation of the tension between inclusive and exclusive values of \( |V_{ub}| \) by RH currents but the large experimental error on \( B(B^+ \rightarrow \tau^+\nu_\tau) \) does not yet exclude this idea. It could be that the true value of \( |V_{ub}| \) determined in inclusive decays is somewhere between its present central inclusive and exclusive values, like \( |V_{ub}| = 3.8 \times 10^{-3} \), and that the effect of RH currents is smaller than previously anticipated.

- A value like \( |V_{ub}| = 3.8 \times 10^{-3} \) still implies \( \sin 2\beta \approx 0.74 \), but in this model in the presence of SM-like \( S_{\phi\phi} \) measured by LHCb it is possible due to new phases to achieve agreement with the experimental value of \( S_{\phi\phi} \). However, this would not be possible for \( S_{\phi\phi} = O(1) \), as stressed in [294].

- As far as the decays \( B_{s,d} \rightarrow \mu^+\mu^- \) are concerned, in 2010 the constraint from \( B \rightarrow X_{\mu\mu} \) precluded already \( B(B_{s,d} \rightarrow \mu^+\mu^-) \) to be above \( 1 \times 10^{-8} \). Moreover, NP effects in \( B_{d} \rightarrow \mu^+\mu^- \) have been generally found to be smaller than in \( B_s \rightarrow \mu^+\mu^- \). But the smallness of \( S_{\phi\phi} \) from LHCb has modified the structure of the RH matrix and one should expect that the opposite is true in accordance with the room left for NP in \( B_{d} \rightarrow \mu^+\mu^- \) by the LHCb data, although to be sure a more detailed numerical analysis is required.

There are other interesting consequences of this NP scenario that can be found in [294, 429], even if some of them will be modified due to changes in the structure of the RH matrix. It looks like RHMFV could still remain as a useful framework when more precise experimental data for the observables just mentioned will be available in the second half of this decade.

6.5.7. A Randall–Sundrum model with custodial protection. Models with a warped extra dimension, as first proposed by Randall and Sundrum, provide a geometrical explanation of the hierarchy between the Planck scale and the EW scale [430, 431]. Moreover, when the SM quarks and leptons are allowed to propagate in the fifth dimension (bulk), these models naturally generate the hierarchies in the fermion masses and mixing angles through different localization of the fermions in the bulk.

In order to avoid problems with EWPTs and FCNC processes, the gauge group is generally larger than the SM gauge group [432, 433]:

\[
G_{\text{RSC}} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X
\]
and, similarly to the LHT model, new heavy gauge bosons, KK gauge bosons, are present. Moreover, a special choice of fermion representation protects the LH flavour conserving couplings in order to agree with the data, particularly in the case of $Z \rightarrow b\bar{b}$ [434].

The increased symmetry also provides a custodial protection for LH flavour-violating couplings of $Z$ to down-quarks and to corresponding RH couplings to up-quarks [49, 167, 435]. We will call this model RSc to indicate the custodial protection. Detailed analyses of electroweak precision tests and FCNCs in a RS model, both without and with custodial protection, can also be found in [436–438].

The different placing of fermions in the bulk generates non-universal couplings of fermions to KK gauge bosons and $Z$ and after the rotation to mass eigenstates this induces FCNC transitions at the tree level. As we have discussed, tree-level $Z$ non-universal couplings of fermions to KK gauge bosons and to corresponding RH couplings to up-quarks [49, 167, 435], protection, can also be found in [436–438].

Detailed analyses of electroweak precision tests and FCNCs in a RS model, both without and with custodial protection. Therefore, in spite of some tensions in this NP scenario, the techniques developed in the last decade will certainly play an important role in the phenomenology if a new strong dynamics will show up at the LHC after its upgrade.

6.5.8. Composite Higgs and partial compositeness. This brings us to the following idea, which still has not been ruled out in spite of the discovery of a boson that looks like the Higgs boson of the SM. The severe fine-tuning problem that this model faces can still be avoided if the Higgs boson is a composite object. The question then arises of how, in such a model, the fermion masses can be generated without at the same time violating the stringent bounds of FCNCs. The most popular mechanism to achieve this goal is an old 4D idea, which is known as partial compositeness [448]. In this NP scenario, SM fermions couple linearly to heavy composite fermions with the same quantum numbers. The SM fermion masses are then generated in a seesaw-like manner and the mass eigenstates are superpositions of elementary and composite fields. Light quarks are dominantly elementary while the degree of compositeness is large for the top quark.

This idea for explaining the fermion mass hierarchies by hierarchical composite-elementary mixings, which was already used in the RS scenario that was discussed previously, leads to a suppression of FCNCs, even if the strong sector is completely flavour-anarchic [449–451]. Yet, as we have seen in the 5D setting, even this mechanism is not powerful enough to satisfy the bounds from FCNCs without some degree of fine-tuning for the masses of KK gluons, as represented here by the resonances of the strong sector, in the reach of the LHC [435, 440, 452]. For this reason, various mechanisms have been suggested to further suppress flavour violation. One idea is to impose a flavour symmetry under which the strong sector is invariant and which is only broken by the composite-elementary mixings [88, 453–456]. Alternative solutions include flavour symmetries that are also broken in the strong sector [457–459]. An extension of the (flavour-blind)
global symmetry of the strong sector has also been proposed in [460].

In addition as we have seen in the case of RSc, protection mechanisms have to be invoked to satisfy the electroweak precision tests, in particular related to the $T$ parameter, [432, 433] which requires the extension of the gauge group. In the 4D setting this means that the strong sector should be invariant under the custodial symmetry $SU(2)_L \times SU(2)_R \times U(1)_X$. Moreover, the presence of heavy vectorial composite fermions that mix with the SM fermions and the presence of new heavy vector resonances implies modifications of $Z$ couplings, leading to unacceptable $Z$ coupling to LH $b$ quarks and tree-level FCNCs mediated by $Z$. As already discussed in the context of RS, a particular choice of fermion representation allows us to remove these problems for both $Z \rightarrow bb$ [434] and also FCNCs [49, 167, 435]. In the 4D setting this is equivalent to making the strong sector (approximately) invariant under a discrete symmetry [434].

The important point to be made here, which was also recently emphasized by Straub [461], is that the resulting pattern of FCNCs mediated by $Z$ will generally depend on

- The flavour symmetry imposed on the strong sector admitting also the case of an anarchic strong sector,
- Choice of the fermion representations to satisfy the bounds on $Z$ couplings.

A simple 4D effective framework to study the phenomenology of these different possibilities is given by the two-site approach proposed in [462]. In this framework, one considers only one set of fermion resonances with heavy Dirac masses, as well as spin-1 resonances associated to the global symmetry $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$, which can be considered as new ‘heavy gauge bosons’. This approach can be viewed as a truncation of 5D warped (RS) models, taking into account only the lightest set of KK states. This approximation has already been used in [49, 167, 435] in the context of RSc, as discussed above, and is particularly justified in the case when FCNCs appear already at tree level.

In the language of 4D strongly coupled theories, the RSc scenario discussed previously is custodially protected flavour-anarchic model where the LH quarks couple to a single composite fermion. In such a framework the NP effects in rare $K$ and $B_{s,d}$ decays, as analysed in [49, 167, 435], are fully dominated by RH $Z$-couplings and the pattern of flavour violation with implied correlations is described by the RH DNA chart in figure 45.

However, there are other possibilities [461]. In a custodially protected flavour-anarchic model, where the LH up- and down-type quarks couple to two different composite fermions, rare $K$ and $B_{s,d}$ decays are fully dominated by LH $Z$-couplings. The pattern of flavour violation with implied correlations is summarized by the LH DNA chart in figure 45. Indeed, the results for this scenario in figure 4 in [461] can easily be understood on the basis of the DNA chart in figure 45.

Next, one can consider a model with partial compositeness in which the strong sector possesses $U(2)^c$ flavour symmetry [88, 463], minimally broken by the composite-elementary mixings of RH quarks. In this case, as already discussed at length by us and also seen in figure 4 of [461], the pattern of flavour violation with implied correlations is summarized by the DNA chart in figure 44. A useful set of references to models with partial compositeness can be found in [461].

### 6.5.9. Gauged flavour models.

In these models [234, 464, 465] a MFV-like ansatz is implemented in the context of maximal gauge flavour (MGF) symmetries: in the limit of vanishing Yukawa interactions these gauge symmetries are the largest non-Abelian ones allowed by the Lagrangian of the model. The particle spectrum is enriched by new heavy gauge bosons, carrying neither colour nor electric charges, and exotic fermions to cancel anomalies. Furthermore, the new exotic fermions give rise to the SM fermion masses through a seesaw mechanism, which is in a way similar to how the light LH neutrinos obtain masses by the heavy RH ones.

Even if this approach has some similarities to the usual MFV description, the presence of flavour-violating neutral gauge bosons and exotic fermions introduces modifications of the SM couplings and tends to lead to dangerous contributions to FCNC processes mediated by the new heavy particles.

In [135] a detailed analysis of $\Delta F = 2$ observables and of $\mathcal{B}(B \rightarrow X_s \gamma)$ in the framework of a specific MGF model of Grinstein et al [464], including all relevant contributions, has been presented. The number of parameters in this model is much smaller than in some of the extensions of the SM discussed above and, therefore, it is not obvious that the present tensions on the flavour data can be removed or at least softened. Therefore, it is of interest to summarize the status of this model in light of the discussions of FCNCs in the previous sections. The situation is as follows:

- After imposition of the constraint from $\epsilon_K$, only small deviations from the SM values of $S_{\phi K}$ and $S_{\phi\phi}$ are allowed. Meanwhile, at the time of our analysis in [135] this appeared as a possible problem, this result is fully consistent with present LHCb data. Consequently this model selects the scenario with exclusive (small) value of $|V_{ub}|$.
- The structure of correlations between $\Delta F = 2$ observables is very similar to models with CMFV and is represented by the DNA chart in figure 43. In particular, $|\epsilon_K|$ is enhanced without modifying $S_{\phi K}$, $\Delta M_d$ and $\Delta M_s$ are strongly correlated in this model with $\epsilon_K$ and the enhancement of the latter implies the enhancement of $\Delta M_s$.
- The main problem of this scenario in 2011 was the branching ratio for $B \rightarrow \tau^+ \nu$, which in this model is in the ballpark of $0.7 \times 10^{-4}$. This has softened significantly in view of the 2012 data from Belle.
based on [42, 466], which can be seen as a minimal theory of fermion masses (MTFM). The idea is to explain SM fermion masses and mixings by their dynamical mixing with new heavy vectorlike fermion $F$. Very simplified, the Lagrangian has the following form: $\mathcal{L} \propto m\bar{F}F + M\bar{F}F + \lambda hFF$, where $M$ denotes the heavy mass scale, $m$ characterizes the mixing, and $\lambda$ is a Yukawa coupling. Thus, the light fermions have an admixture of heavy fermions with explicit mass terms.

This mass generation mechanism bears some similarities to the one in models with partial compositeness and gauge flavour models, as just discussed. Since in this model the Higgs couples only to vectorlike heavy fermions but not to chiral fermions of the SM, the SM Yukawas arise solely through mixing. We reduce the number of parameters such that it is still possible to reproduce the SM Yukawa couplings and at the same time flavour violation is suppressed. In this way we can identify the minimal FCNC effects. A central formula is the leading order expression for the SM quark masses

$$ m^X_{ij} = v^X \epsilon^{X}_{ij} \lambda^X, \quad (X = U, D), \quad \epsilon^{Q,U,D}_i = \frac{m^{Q,U,D}_i}{M^{Q,U,D}_i}. $$

In [466] the heavy Yukawa couplings $\lambda^{U,D}$ have been assumed to be anarchical $O(1)$ real numbers, which allowed a first look at the phenomenological implications. In [42] the so-called TUM (trivially unitary model) was studied in more detail. We assumed universality of heavy masses $M^Q = M^U = M^D = M$ and unitary Yukawa matrices. With this, the flavour structure simplified considerably. Furthermore we concentrated on flavour violation in the down sector and thus set $\lambda^U = 1$. After fitting the SM quark masses and the CKM matrix we are left with only four new real parameters and no new phases: $M$, $\epsilon^Q_i$, $s^d_3$, $s^d_2$. The latter two parameters are angles of $\lambda^D$ (the third angle is fixed by the fitting procedure) and from fitting $m_t$ it follows that $0.8 \leq \epsilon^D < 1$.

The new contributions to FCNC processes are dominated by tree-level flavour-violating $Z$ couplings to quarks. The simplest version of the MTFM, the TUM, is capable of describing the known quark mass spectrum and the elements of the CKM matrix favouring $|V_{ub}| \approx 0.0037$. Since there are no new phases in the TUM, $S_{\psi K_S}$ stays SM-like and thus the large inclusive value of $|V_{ub}|$ is disfavored. Although effects in $\epsilon_K$ can in principle be large, the effects are bounded by $B(K_L \to \mu^+\mu^-)_{SD} \leq 2.5 \times 10^{-9}$. For a $|V_{ub}|$ in between exclusive and inclusive value, it is still possible to find regions in the parameter space that satisfy equations (95) and (211) but then the prediction of the model is that $S_{\psi K_S} \approx 0.72$, which is by $2\sigma$ higher than its present experimental central value. In figure 47 (left) we show the correlation $B(K_L \to \mu^+\mu^-)$ versus $|\epsilon_K|$ for $M = 3$ TeV, where only the green points satisfy (211) and (95) simultaneously. In the TUM effects in $B_{s,d}$ the mixings are negligible and the pattern of deviations from SM predictions in rare $B$ decays is CMFV-like, as can be seen on the right-hand side of figure 47. However, $B(B_{s,d} \to \mu^+\mu^-)$ are uniquely enhanced over their SM values. For $M = 3$ TeV, these enhancements amount to at least 35% and can be as large as a factor of two. The enhancements decrease with increasing $M$. However, they remain sufficiently large for $M \leq 5$ TeV to be detected in the flavour precision era. The effects in $K \to \pi\nu\bar{\nu}$ transitions are also enhanced by a similar amount.

At the time when our paper was published there was hope that the enhancement of $B(B_s \to \mu^+\mu^-)$ uniquely predicted by the model would be confirmed by the improved data. As seen on the right-hand side of figure 47, the most recent data from LHCb and CMS do not support this prediction and either the value of $M$ has to be increased or the TUM has to be made less trivial.

7. Summary and shopping list

Our review of strategies for the identification of new physics through quark flavour violating processes is approaching the end. In the spirit of our previous reviews [12, 17, 20], we have addressed the question how in principle one could identify NP with the help of quark flavour violating processes. In contrast to [12, 17], we have concentrated on the simplest extensions of the SM, describing more complicated ones only in the final part of this review. These simple constructions are helpful in identifying certain patterns of flavour violation. In particular, correlations between various observables characteristic for
these scenarios can distinguish between them. These features are exposed compactly by the DNA charts in figures 43–45. Our extensive study of models in which flavour violation is governed by tree-level exchanges of gauge bosons, scalars, and pseudoscalars with different couplings, as exemplified by LHS, RHS, LRS and ALRS scenarios, shows that future measurements can tell us which one is favoured by nature.

However, we are aware of the fact that these simple scenarios are not fully representative for more complicated models in which a collection of several new particles and a number of new parameters can wash out the various correlations that have been identified by us. This is particularly the case of models in which FCNCs appear first at one-loop level and the FCNC amplitudes depend on the masses of exchanged gauge bosons, fermions and scalars and their couplings to SM particles. In CMFV, MFV at large and models of exchanged gauge bosons, fermions and scalars and their loop level and the FCNC amplitudes depend on the masses correlations that have been identified by us. This is particularly a number of new parameters can wash out the various scenarios are not fully representative for more complicated by LHS, RHS, LRS and ALRS scenarios, shows that future and pseudoscalars with different couplings, as exemplified governed by tree-level exchanges of gauge bosons, scalars, Our extensive study of models in which flavour violation is involves only quark flavour observables:

- Precise values of all non-perturbative parameters relevant for \( \Delta F = 2 \) transitions from lattice QCD. This also means hadronic matrix elements of new operators outside the framework of CMFV. In fact, this will be the progress made in the coming years when most of the experiments will sharpen their tools for the second half of this decade.
- Precise determinations of CKM parameters from tree-level decays. This goal will be predominantly addressed by SuperKEKB but in the case of the angle \( \gamma \), LHCb will provide a very important contribution.
- Precise values of \( S_{\phi K_s} \) and \( S_{\phi \phi} \) together with improved understanding of hadronic uncertainties represented by QCD penguins.
- Precise measurements of \( B(B_s \rightarrow \mu^+ \mu^-) \) and \( B(B_d \rightarrow \mu^+ \mu^-) \). It is important that both branching ratios are measured as this together with \( \Delta M_S \) and \( \Delta M_D \), and precise values of \( B_{B_s} \) and \( B_{B_d} \), would provide a powerful test of CMFV. It is evident from our presentation that the observables related to the time-dependent rate would be far enrich these studies.
- Precise measurements of the multitudes of angular observables in \( B \rightarrow K (K^*) \ell^+ \ell^- \) accompanied by improved form factors can still contain important information about NP. In particular, it will be important to clarify the anomalies observed recently by the LHCb experiment, as discussed in step 7 of our strategy.
- Precise measurements of \( B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) and \( B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \). The first messages will come from NA62 and then, hopefully, from J-Parc and ORKA.
- Precise measurements of the branching ratios for the trio \( B \rightarrow X_s \nu \bar{\nu} \), \( B \rightarrow K^+ \nu \bar{\nu} \) and \( B \rightarrow K \nu \bar{\nu} \). These decays are in the hands of SuperKEKB.
- Precise determination of \( B(B^+ \rightarrow \tau^+ \nu \bar{\tau}) \), which is again in the hands of SuperKEKB.
- Precise measurement of \( B(B \rightarrow X_s \gamma) \).
- Precise lattice results for the parameters \( B_{B_s}^{(1/2)} \) and \( B_{B_s}^{(3/2)} \) entering the evaluation of \( \varepsilon'/\varepsilon \).

A special role will be played by charm physics because it allows us to learn more about flavour physics in the up-quark sector. But the future of this field will depend on the progress on reduction of the hadronic uncertainties.

Next, a very important role in the search for NP, as discussed in step 12, will be played by lepton flavour violating decays, EDMs and \( (g-2)_\mu \). But this is another story and we have discussed these topics only very briefly in our review.

Finally, a crucial role in these investigations will be played by theorists, both in the case of inventing new ideas for identifying NP and constructing new extensions of the SM with fewer parameters and, thereby, they will be more predictive.

In any case, this decade is expected to bring a big step forward in the search for new particles and new forces, and we should hope that one day the collaboration of experimentalists and theorists will enable us to get some insight into the Zeptouniverse.

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