I critically examine the notion of “irreversibility,” and discuss in what sense it applies to the spontaneous creation of particles in external fields. The investigation reveals that particle creation in very strong fields can only be described by a non-Markovian transport theory.

I. WHAT IS THE PROBLEM?

Spontaneous pair creation contributes to thermalisation. A heavy-ion collision appears highly irreversible: while the initial state of the two colliding nuclei is almost pure, particles emerging from the reaction can well be described by a thermal distribution. Some of the mechanisms responsible for this thermalisation are well understood: binary collisions of the microscopic constituents (partons), hadrochemical reactions, or radiation. At very high energies, however, an increasingly important role is played by yet another elementary process: the spontaneous creation of $qar{q}$-pairs in strong, coherent chromoelectric fields (“flux tubes”). These pairs are produced with a certain momentum distribution, associated with which is a non-zero entropy. It seems therefore that not only particles, but also entropy is produced ‘out of the vacuum.’ This poses the following question: Is spontaneous pair creation indeed irreversible? And if so, in what sense?

II. WHAT IS IRREVERSIBILITY?

All investigations of “irreversibility” are based upon a classification of the degrees of freedom. The v.Neumann entropy associated with the full statistical operator of a closed quantum system,

$$S[\rho(t)] := -k \text{tr}(\rho(t) \ln \rho(t)) \quad ,$$

(1)

is constant in time and hence for our purposes useless. Non-trivial statements about irreversible behavior all refer to a time-dependent relevant entropy, which is the entropy associated with the relevant degrees of freedom. A distinction between relevant and irrelevant degrees of freedom is thus essential. Often, such a distinction appears in disguise: as a classification of observed vs. unobserved; observable vs. unobservable; subsystem vs. environment; collective vs. noncollective; slow vs. fast; shape vs. randomizing; or extrinsic vs. intrinsic degrees of freedom.

The relevant entropy is obtained by coarse-graining.

The v.Neumann entropy measures the amount of missing information as to the pure state of the system, if one is given the expectation values of all observables of the system. In contrast, the relevant entropy measures the amount of missing information as to the pure state of the system, if one is given the expectation values of only the relevant observables. Going from the former to the latter involves discarding information about the irrelevant degrees of freedom – a truncation which is reflected in the general inequality

$$S_{\text{rel}}(t) \geq S[\rho(t)] \quad \forall \ t \quad ,$$

(2)

and often referred to as “coarse-graining.”

Example: Gibbs vs. Boltzmann entropy. The full state of a classical gas of $N$ indistinguishable particles is given by a symmetric probability density $W(\pi_1 \ldots \pi_N)$ on the $6N$-dimensional $N$-particle phase space. Associated with this full state is the Gibbs entropy

$$S_G := -k \int \! d^6 \pi_1 \ldots d^6 \pi_N W(\pi_1 \ldots \pi_N) \ln W(\pi_1 \ldots \pi_N)$$

(3)

which, due to Liouville’s theorem, stays constant – just like the v.Neumann entropy. For reasons to be discussed shortly, one generally considers only single-particle observables to be relevant, but many-particle correlations to be irrelevant. All information about the relevant degrees of freedom is then contained in the reduced probability density

$$w(\pi) := \int \! d^6 \pi_2 \ldots d^6 \pi_N W(\pi_1 \ldots \pi_N) \quad ,$$

(4)

defined on the 6-dimensional single-particle phase space. Associated with this reduced state is the Boltzmann entropy

$$S_B := -kN \int \! d^6 \pi w(\pi) \ln w(\pi) \quad .$$

(5)

The fact that information about particle correlations has been discarded is reflected in the inequality

$$S_B \geq S_G \quad .$$

(6)
It is the coarse-grained Boltzmann entropy to which all non-trivial statements refer, including the famous $H$-theorem.

The $H$-theorem relies on a strong separation of time scales. A prediction cannot possibly contain more information than the data on which it is based. Therefore, if the evolution of the relevant degrees of freedom is Markovian, i.e., if their expectation values at time $t + dt$ can be predicted on the basis of their expectation values at time $t$, then the associated relevant entropy can only increase or stay constant, but never decrease. This is the $H$-theorem. Clearly, its validity depends crucially on the Markovian property of the evolution. Consider, as an example, the classical gas: If the gas is dilute, and hence the duration of one individual collision much shorter than the average time that elapses between two subsequent collisions, then collisions may be regarded as statistically independent ("Stoßzahlensatz"); the evolution equation (Boltzmann equation) becomes Markovian; and therefore the $H$-theorem holds. If, on the other hand, the gas is dense, and hence the duration of a collision approximately equal to the time between two collisions, then there may be memory effects; the evolution is no longer Markovian; and the $H$-theorem may be temporarily violated.

"Irreversible" is the flow of information from slow to fast degrees of freedom. While the constancy of the v. Neumann entropy shows that complete information about the system is retained in a full microscopic description, the variation of the relevant entropy indicates that the amount of information carried by the relevant degrees of freedom continuously changes. An obvious interpretation is that in the course of the system’s evolution, information about the system is being transferred between relevant and irrelevant degrees of freedom. How exactly the relevant entropy behaves, depends crucially on the choice of the "relevant vs. irrelevant" classification. In general, the flow of information need not have a unique direction; it may occur either from relevant to irrelevant, or from irrelevant to relevant degrees of freedom. As a consequence, the relevant entropy may either increase or decrease. It is only when the relevant degrees of freedom are slow and the irrelevant degrees of freedom are fast that, according to the general $H$-theorem, the information flow is uniquely directed from relevant to irrelevant degrees of freedom. This "leaking" of information into fast degrees of freedom is perceived as irreversible. However, the information is not really lost – it only becomes inaccessible to a certain coarse-grained level of description. In the example of a dilute classical gas, information is being transferred from single-particle observables to many-particle correlations. Since in general these correlations will not be measured, part of the information about the system becomes experimentally inaccessible.

The study of irreversible behavior reduces to a time scale analysis. "Irreversibility" refers to the experimenter’s ability – or the lack thereof – to prepare, control, or monitor certain degrees of freedom; it thus seems to be a purely "anthropomorphic" concept. Nevertheless, irreversible features of the dynamics are not entirely subjective. For an observable to be measurable in practice, it is usually necessary that it varies slowly. In this case the experimentally monitored degrees of freedom constitute some subset of the set of all slow degrees of freedom. Accordingly, the flow of information from observed to unobserved degrees of freedom is intimately tied to the flow of information from slow to fast degrees of freedom. But the latter is an objective property of the dynamics: it is determined by the presence of disparate time scales, and not dependent on any observer. A study of irreversible features of the dynamics should therefore focus on the identification of slow and fast degrees of freedom, and on the analysis of the associated time scales. The choice of the "relevant vs. irrelevant" classification is then no longer as arbitrary and subjective as it may have seemed at first; rather, the proper choice is determined by objective physical criteria – by the time scales.

### III. IS SPONTANEOUS PAIR CREATION IRREVERSIBLE?

The relevant degrees of freedom are the occupation numbers of momentum states. What is commonly being measured in an experiment, and what enters into most transport equations such as the quantum Boltzmann equation, are the momentum distributions $n_{\pm}(\vec{p}, t)$ of the produced particles and antiparticles. This strongly suggests choosing the occupation numbers of momentum states as the relevant degrees of freedom. Associated with this choice is the relevant entropy (for spin-1/2 fermions)

$$S_{rel}(t) := -2k \int_{\vec{p}} \left[ \frac{n_{-}}{2} \ln \frac{n_{-}}{2} + \left(1 - \frac{n_{-}}{2}\right) \ln \left(1 - \frac{n_{-}}{2}\right) \right] \left( n_{-} \leftrightarrow n_{+} \right)$$

(7)

(where $n_{\pm} \equiv n_{\pm}(\vec{p}, t)$). Whether or not the relevant degrees of freedom are sufficiently slow; i.e., whether or not their evolution is Markovian; whether or not the relevant entropy obeys an $H$-theorem; and hence whether or not in this description pair creation appears irreversible – all this must be the subject of a thorough time scale analysis.

I focus on the pair creation process proper. Irreversible information flows occur at two different stages of the evolution: (i) during the pair creation process proper, and (ii) during the subsequent “decoherence.” During the pair creation process proper, information leaks from relevant occupation numbers into correlations and relative phases. Under the influence of subsequent collisions, this phase information is then being transferred further to an unobservable “environment” (or “heat bath”) of high-frequency partons. Such a transfer of phase information to an unobservable environment is often referred to as “decoherence,” and discussed elsewhere in these
Proceedings. Here, I wish to concentrate on stage one of the evolution, the pair creation process proper.

A time scale analysis requires knowledge of the non-Markovian equation of motion. The question to be addressed is the following: Is the initial flow of information from occupation numbers to correlations and relative phases irreversible? In other words, does the relevant entropy obey an $H$-theorem? According to our previous discussion, the answer to this question hinges upon a careful analysis of time scales. It is necessary to derive a generally non-Markovian equation of motion for the occupation numbers; to identify the memory time of this non-Markovian equation as well as the time scale on which the occupation numbers evolve; to compare these, and thus to find a criterion for the validity of the Markovian approximation. If and only if this criterion is satisfied, the $H$-theorem holds. If, on the other hand, the criterion is not satisfied, then the memory time furnishes the typical time scale on which the relevant entropy may temporarily decrease.

Microscopic model: Schwinger mechanism. All essential features of the pair creation process are exhibited by the well-known Schwinger mechanism, the spontaneous creation of $e^+e^-$ pairs in a constant, homogeneous external electric field. It is this simple model which has been considered in much of the literature and which I now want to subject to further analysis. Let $q$ be the electron charge, $\vec{E}$ the external field, $\vec{p}(t) := \vec{p} + q\vec{E}t$ the time-dependent momentum, $m$ the spin component, and $\phi_f$ the dynamical phase accumulated between times $t_i$ and $t_f$. Then in the Heisenberg picture the evolution mixes particle and antiparticle ($a^\dagger$) and field operators, with respective amplitudes $\alpha_{fi}$ and $\beta_{fi}$:

$$\mathcal{U}(t_2, t_1) \left( \begin{array}{c} a^\dagger(\vec{p}(t_1), m) \\ b(-\vec{p}(t_1), -m) \end{array} \right) = \left( \begin{array}{cc} \alpha_{21} & \beta_{21} \\ -\beta_{21} & \alpha_{21} \end{array} \right) \left( \begin{array}{c} e^{\mathcal{i}\phi_{21}} 0 \\ 0 e^{-\mathcal{i}\phi_{21}} \end{array} \right) \left( \begin{array}{c} a^\dagger(\vec{p}(t_2), m) \\ b(-\vec{p}(t_2), -m) \end{array} \right)$$

(8)

Hence on the microscopic level, spontaneous pair creation is described by a time-dependent Bogoliubov transformation.

In the equation of motion, pair creation is accounted for by a non-Markovian source term. The equation of motion for the occupation numbers has the structure of a quantum Boltzmann equation. Aside from the usual acceleration and (possibly) collision terms, it contains an additional source term to account for the spontaneous pair creation. By means of the so-called projection method, this source term may be related to the coefficients of the Bogoliubov transformation:

$$\dot{n}^{\text{non}}(\vec{p}, t) = 4 \text{Re} \int_0^{t-t_0} d\tau \left( \frac{\partial \beta}{\partial t} \right) (\vec{p}, -\vec{p}) e^{-\mathcal{i}2\mathcal{E}(\vec{p}, 0)} \beta(0, 0) \times S(\vec{p} - q\vec{E}t, t - \tau)$$

(9)

Here $t_0$ denotes the initial time at which the external field is switched on; $\beta(t_1, t_1)$ is a shorthand for $\partial \beta(t_2, t_1)/\partial t|_{t_2=t_1}$; and the factor $S \equiv [1 - n_-/2][1 - n_+/2] - (1/4)n_-n_+$ accounts for Pauli blocking as well as the possible annihilation of pairs back into the field. Evidently, the source term involves an integration over the entire history of the system and is therefore non-Markovian.

The memory time combines a quantum mechanical and a classical time scale. Careful analysis reveals that significant contributions to the above source term come only from times $\tau$ which are smaller than the characteristic memory time

$$\tau_{\text{mem}} \sim \frac{h}{\epsilon_\perp} + \frac{\epsilon_\perp}{qE}$$

(10)

(where $\epsilon_\perp$ denotes the transverse energy or mass). The two terms have very different physical origins. (i) The time $h/\epsilon_\perp$ is quantum mechanical. It corresponds to the time-energy uncertainty relation – to the time needed to create a virtual particle-antiparticle pair, and may thus be regarded as the “time between two production attempts.” (ii) The time $\epsilon_\perp/qE$, on the other hand, is classical. It can be interpreted in various ways, depending on the picture employed to visualize the pair creation process. If pair creation is viewed as a tunneling process from the negative to the positive energy continuum, this classical time coincides with the time needed for the wave function to traverse the barrier with the speed of light.

The Markovian approximation is valid only for weak fields. The occupation numbers evolve on a typical time scale set by the inverse production rate $[\dot{n}^{\text{non}}(\vec{p})]^{-1}$. Assuming $p_\parallel = 0$ for simplicity, one obtains as a representative scale

$$\tau_{\text{prod}} \sim \frac{\epsilon_\perp}{qE} \exp \left( \frac{\pi \epsilon_\perp^2}{2h qE} \right)$$

(11)

This production scale is much larger than the memory time, and hence the evolution is Markovian, only if $E \ll m^2/hq$.

First conclusion: The flow of information from occupation numbers to correlations and phases is strictly irreversible only if the field is weak. As long as the field is sufficiently weak, the evolution of occupation numbers is Markovian, the $H$-theorem holds, and hence the relevant entropy increases monotonically. But as soon as $E \sim m^2/hq$, both memory time and production scale attain the same magnitude $\tau \sim h/m$. As a result, the Markovian approximation breaks down, and there may be temporary violations of the $H$-theorem (on the same time scale $\tau \sim h/m$). Indeed, oscillations of the relevant entropy have been observed in numerical simulations.

Second conclusion: Pair creation in very strong fields must be described by a non-Markovian transport theory. Whenever the external field is stronger than the critical value $m^2/hq$, there may be sizeable memory effects which cannot be accounted for in a Markovian transport theory. Particle masses of the order 0.5, 15 or 500 MeV correspond to critical field strengths of the order $10^{-3}$,
1 or $10^3$ MeV/fm, respectively.] This strong-field domain is in fact the only one which is physically relevant, because only there does spontaneous pair creation occur at an appreciable rate. Hence for all practical purposes pair creation must always be described by a non-Markovian transport theory.

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