Mathematical modelling of heating features of a cylindrical surface under plasma deposition

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Abstract. Mathematical model for temperature determination in a system ‘coating – base’ under plasma deposition of a powder material on the internal cylindrical surface has been developed. Mathematical model takes into account transformation of Gauss heat source, relative motion velocity of a plasmatron, and an angle of a plasma jet relative to the deposition surface. Effect of process variables of deposition on the process thermal stress and quality of plasma coatings has been analyzed.

1. Introduction
Technology of plasma gas-thermal deposition of coatings is widely adopted in manufacturing of various mechanical engineering products. In this technology, solid body formation is a result of sequential layer deposition of powder materials heated up to the certain temperature under thermomechanical influence. Plasma gas-thermal method has come into common use because of its versatility and possibility of developing protecting and functional coatings of various materials on the product surfaces [1-13].

2. Statement of a mathematical model of the process of cylindrical surface heating under plasma deposition
During deposition a plasmatron moves with a certain constant speed $V$ in relation to the surface of the deposited product which, in its turn, revolves at a constant rate $\omega$ to provide even deposition of a coating. Angle of inclination of the plasmatron to the surface $\gamma$ is also remains constant. The exhausting jet containing melted particles from the plasmatron heats the deposited cylindrical surface up to the certain temperature. To get high quality coating it is necessary to provide heating temperature of a product throughout the coating deposition process within the range from $T_{\text{min}}$ to $T_{\text{max}}$ applying some manufacturing methods.

For the specific case, and considering that $\frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}}} \ll 1$ the complex heat conduction equation for the system ‘coating – substrate’ can be represented by two separate heat conduction equations in cylindrical coordinates for the substrate material ($i = 2$) and for the coating material ($i = 1$) with constant values of thermos-physic parameters that equal average values of corresponding parameters within the given narrow temperature range:
\[
\frac{1}{a_i^2} \frac{\partial T_i}{\partial t} = \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r} \frac{\partial T_i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_i}{\partial \varphi^2} + \frac{\partial^2 T_i}{\partial z^2},
\]

where \( T_i \) – material temperature at the point of time \( t \); \( a_i \) - temperature diffusivity coefficient of the material; \( r, z, \varphi \) - cylindrical coordinates.

Heat conduction equations (1) for the substrate and for the coating are supplemented with initial conditions:

\[
T_i(r, z, \varphi, 0) = T_0,
\]

where \( T_0 \) – temperature at initial time point.

Conjugating boundary condition in the contact zone of the substrate and the coating:

\[
\lambda_1 \frac{\partial T_1}{\partial r} = \lambda_2 \frac{\partial T_2}{\partial r}, \quad T_i = T_{2\text{npu}} \quad r = R,
\]

where \( \lambda_i \) - thermal conductivity coefficient; \( R \) – radius of interface between the substrate and the coating.

Boundary condition at the external surface describes heat exchange of the substrate with environment:

\[
\lambda_2 \frac{\partial T_z}{\partial r} = \alpha_2 \left[T_{c\text{npu}} - T_2 \right]_{r=R},
\]

where \( T_c \) – the environment temperature; \( \alpha_2 \) - heat exchange coefficient with the environment.

Boundary condition at the surface affected by the plasma jet and taking part in coating formation meets Newton’s boundary condition. However, according to the experimental research [3] convective heat transfer conditions can be neglected. As a result, boundary conditions include only assignment of the jet towards the surface that is they are transformed into Neumann’s boundary condition:

\[
\lambda_1 \frac{\partial T_1}{\partial r} = -q(z, \varphi, t),
\]

where \( q \) - heat flow density.

Equation for the heat flow density in the cylindrical coordinate system is the following:

\[
q_r = \frac{N}{2\pi\sigma_z\sigma_\varphi} \exp\left\{ -\frac{1}{2} \left[ \frac{z-Vt}{\sigma_z} \right]^2 + \left[ \frac{R(\varphi-\omega t)}{\sigma_\varphi} \right]^2 \right\};
\]

\[
\sigma_z = \frac{\sigma_0}{\cos \gamma} \left( 1 + \frac{z-Vt}{l} \sin \gamma \right);
\]

\[
\sigma_\varphi = \sigma_0 \left( 1 + \frac{z-Vt}{l} \sin \gamma \right),
\]

where \( V \) – linear velocity along the axis \( z \); \( \omega \) – frequency of the cylinder rotation; \( \gamma \) – plasma jet angle in relation to the surface.

3. Mathematical modelling of a cylindrical surface heating process under plasma deposition

In general case mathematical model of temperature calculation in the system ‘coating-substrate’ consists of heat conduction equations (1), initial condition (2), and boundary conditions (3-5). Taking into account little thickness of the system it is possible to simplify mathematical model due to using of expanding temperature into a series according to smallness parameter that is characterized by the system thickness. In this case, possibility arises to reduce the system of two differential equations in partial derivatives to the single equation. Solution is simplified due to using of an integral heat conduction equation. To get
this equation, let us consider heat balance for the cylinder with the coating (figure 1). Using well-known methods, the equation for heat flows through the edges of the cylindrical element \( AA'BB'CC'D'D' \) (figure 1) is the following:

\[
\begin{align*}
q_{AA'D'D'} &= \int_{R_1}^{R} \left[ -\lambda(r) \left( \frac{\partial T}{\partial r} \right) \right] dr , \\
q_{BB'C'C'} &= \int_{R_1}^{R} \left[ -\lambda(r) \left( \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] dr , \\
q_{BB'AA'} &= \int_{R_1}^{R} \left[ -\lambda(r) \frac{1}{r} \frac{\partial T}{\partial \varphi} \right] dz , \\
q_{CC'D'D'} &= \int_{R_1}^{R} \left[ -\lambda(r) \frac{1}{r} \frac{\partial T}{\partial \varphi} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \right] dz dr , \\
q_{ABCD} &= \int_{R_1}^{R} \left[ q(z, \varphi, t) \right] R_2 d\varphi dz , \\
q_{RCDC} &= \int_{R_1}^{R} \left[ c(r) \rho(r) \frac{\partial T}{\partial t} \right] dz dr .
\end{align*}
\]

where

\[
\lambda(r) = \begin{cases} 
\lambda_1, & r \in [R_1, R] \\
\lambda_2, & r \in [R, R_2]
\end{cases}
\]

\[
C(r) = \begin{cases} 
C_1, & r \in [R_1, R] \smallskip
C_2, & r \in [R, R_2]
\end{cases}
\]

\[
\rho(r) = \begin{cases} 
\rho_1, & r \in [R_1, R] \smallskip
\rho_2, & r \in [R, R_2]
\end{cases}
\]

Heat supply to the cylinder with the coating results in enthalpy change of this element by the value

\[
\int_{R_1}^{R} \left[ c(r) \rho(r) \frac{\partial T}{\partial t} \right] dz dr .
\]
$$\int_{\tilde{R}_1}^{R_2} \left[ \lambda (r) \frac{\partial^2 T}{\partial z^2} + \lambda (r) \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \right] r dr + q(z, \varphi, t) R_1 +$$

$$+ \alpha_2 \left( -T(R_2, z, \varphi, t) + T_2 \right) R_2 = \int_{\tilde{R}_1}^{R_2} c(r) \rho (r) \frac{\partial T}{\partial z} r dr.$$  \hspace{1cm} (14)

To use derived integral equation in case of two – layer system ‘coating – product’, we assume temperature field taking into account conjugating boundary condition as power series on \( \xi=r-R \) accurate to the first-order term of smallness.

$$T(\xi, z, \varphi, t) = T_0 + \left\{ \begin{array}{l}
(1-\beta_1 \xi) T_{10} - \beta_1 \xi (T_0 - T_2), \xi \in [-h_1, 0] \\
(1-\beta_2 \xi \frac{\lambda_1}{\lambda_2}) T_{10} - \beta_2 \xi \frac{\lambda_1}{\lambda_2} (T_0 - T_2), \xi \in [0, h_2] 
\end{array} \right.$$

where \( \beta_1 = \frac{\alpha_2}{\lambda_1} \cdot \frac{1}{1+Bi} ; \beta_2 = \frac{\alpha_2 h_1}{\lambda_1} \).

Having performed all necessary transformations by turns, we get values to determine temperature in the system for the whole two – layer cylindrical shell:

$$T(\xi, z, \varphi, t) = T_0 - \beta_1 \xi \frac{\lambda_1}{\lambda_2} (T_0 - T_2) + \left( 1 - \beta_1 \xi \frac{\lambda_1}{\lambda_2} \right).$$

$$\left[ -(T_0 - T_2) \left( 1 - \frac{\exp(-Aa^2t)}{1-\beta_1 \xi \frac{\lambda_1}{\lambda_2}} \right) + q_0 BR^2 Q \left( \frac{z}{R}, \varphi, \frac{a^2t}{R} \right) \right]$$

4. Experimental research of heat flow effect on the cylindrical surface

To determine result adequacy of theoretical assessment of cylindrical surface heating according to developed methods of calculation of experimental research of heat flow effect from the plasma jet scanning internal surface of the cylindrical shell on the integral temperature in the transition zone from the substrate to the coating where maximum temperatures of the product is reached, has been carried out.

Special installation was used to carry out research. The deposited cylinder was installed in a device rotated by the electric motor; thermal resistor was a temperature sensor. Thermal resistor was embedded into the copper rod pressed to the milled cylinder surface from the side opposite to the deposited one, to provide good thermal contact. Change of sensor resistance depending on the temperature resulted to the change of frequency of a frequency modulator. The frequency modulator had autonomous power supply; it was mounted on the rotating panel together with the cylinder. To eliminate thermal effect of a plasma jet on radio circuit elements a thermal shield was used. On exit of the frequency modulator the coil was switched on, its axis coincided with the device shaft axis for deposition. The coil inductively coupled with the coil at the exit was installed at the stationary part of the device for deposition. Frequency-modulated signal from the coil was amplified by a limiting amplifier and sent to the frequency meter, voltages from which were measured and recorded. Sensor calibration was done on the readings of mercury thermometer and platinum-rhodium thermocouple that were submerged together with the sensor into organosilicon liquid.

Heat flow from the plasma jet was firstly evaluated on its bulk mean enthalpy at the nozzle exit section of the plasmatron determined on parameters of arc mode (current strength, arc voltage, plasma-supporting gas consumption) and thermal losses with cooling water. Next, the heat flow was checked...
by measurements directly on the samples. Correlation among these experimental evaluation results of a heat flow was used to plot theoretical curves describing dependence of heating rate of cylindrical samples on the process variables of deposition.

Parameters of distribution of heat flow density in the heat spot in case of orthogonal orientation of the jet relative to the deposited surface were determined according to the dispersion of Gauss distribution that, in turn, was determined from the heat spot diameter.

\[ \sigma = \frac{d_H}{4.89}. \] (17)

Determination of \( d_H \) was done applying both method of calculation evaluation of geometric parameters of the jet and using calorimetry of the jet through the screen with an opening. In the latter case, location of outer boundaries of the jet was determined by means of evaluating the distance from the nozzle exit section of the plasmatron to the cross section having the diameter identical with size of the opening in the screen. In this position, we noticed sharp bend in the character of dependence of the heat capacity that got into the calorimeter, on the distance from the nozzle exit section; this sharp bend was resulted from lateral dimensions excess of the jet over the diameter of the feed opening in the screen.

Variation range of process variables in performed calculation-experimental research was the following: heat capacity of argon-hydrogen plasma jet \( Q: (1...3)\cdot10^5 \text{ Wt/m}^2 \); rotation frequency of the cylinder \( \omega: 0.05...2.65 \text{ rev/s}; \) linear velocity of the plasmatron movement \( V: 0.85...100 \text{ mm/s}; \) product wall thicknesses \( h: 3...5 \text{ mm}; \) coating thicknesses \( 0.1...0.4 \text{ mm}; \) angle of inclination of the plasmatron axis \( 80...30^\circ \). Research was carried out for the material \( \text{Zr}_2\text{O}_3 \).

Comparison of results of experimental evaluation and calculation results has revealed good convergence (within 12 %) and has confirmed reliability of assumptions, and developed methodology of calculation and analytical evaluation of cylindrical shell heating when the coating is deposited on its internal surface.

5. Results and discussion

Results of theoretical and experimental research of influence of processing factors on the cylinder heating have revealed that the temperature in the system can change within a wide range (Figure 2) from insufficiently high to provide required strength properties of the coating to excessively high resulting in worsening of physical-mechanical characteristics of the main structural material of the product.

As it is seen from calculation results, displayed at Figure 2, increase in heat flow capacity results in proportional increase of heat intensity of the cylinder, and at \( Q=4\cdot10^5 \text{ Wt/m}^2 \) critical temperatures are reached that characterize worsening of design characteristics of the substrate material.

The research has also shown that decreasing of cylinder revolution frequency on deposition results in heat intensity. It appears weakly under changing \( \omega \) from 2.65 to 0.65 rev/s, and it appears strongly under revolution deceleration up to 0.05 rev/s. Attainable temperature level in the latter case greatly exceeds critical values among the researched variants, and stabilization rate of heat exchange in the system ‘plasma jet – coating – cylinder – environment’ essentially increases. It should be noted that in all other researched cases this transition takes 240...320 s.

The most noticeable, in comparison with angular rate of cylinder rotation, effect on the temperature change of the wall is the adjustment of movement speed of the plasmatron.

Movement speed decrease of the heat source from 100 mm/s to 0.85 mm/s results in temperature increase in the system ‘coating – product’ by 2...2.5 times. It is connected with specific power increase of the plasma jet approaching to the product due to increase of action time of the heat source, and consequently, heating of ‘coating – substrate’ system. Thus, deceleration of plasmatron movement speed up to low values значений \( V<0.1 \text{ mm/s} \) can result in overheating of the substrate material (i.e. to reaching critical temperatures). It should be noted, that the temperature in the system ‘coating – substrate’ is increasing during about 6...7 passages of the plasmatron, and then it keeps certain value determined by the heat flow density and the speed of plasmatron movement.
Figure 2. Temperature of cylinder heating depending on source thermal capacity and deposition time at: Q, Wt./m²: 1 - 10⁵; 2 - 2·10⁵; 3 - 4·10⁵; V – 3.35 mm/s; h=4 mm; coating thickness 0.2 mm; \( \omega = 1.35 \text{ rev/s}; \gamma = 40^\circ \).

It can be noted that wall thickness of the deposited cylinder markedly affects heat intensity of the product at values close to \( h=1 \) mm. Increase of cylinder wall thickness diminishes heating by a factor of 1.2 ... 1.5, but in general, this parameter has substantially smaller influence than parameters of jet generation and coating deposition. Similar conclusions could be done relative to coating thickness influence on the heating rate of the cylinder.

Changing axis inclination angle of the plasma jet relative to the deposited surface also sufficiently influences heating rate of the cylinder. Adjustment of the angle \( \gamma \) from 80 to 30° results in reduction of heating rate. This results from increase of deposition spot area along the cylinder generating line. Temperature variation value decreases on the same reason. However, according to influence level on the heating rate the angle of jet inclination should be referred to the group of weakly influencing parameters, thicknesses of cylinder walls and deposited coating are also referred to this group.

6. Conclusion
Mathematical model taking into account features of heating of a cylindrical surface under plasma deposition has been developed. Influence of process variables of deposition such as cylinder rotation frequency, speed of plasmatron movement and inclination angle of the plasma jet on the process thermal stress and quality of plasma coatings has been analyzed. Technological guidelines for the process of deposition on internal cylindrical surface have been developed.

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