Generalized second law of thermodynamics with corrected entropy in tachyon cosmology

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Abstract This work is to study the generalized second law (GSL) of thermodynamics in tachyon cosmology where the tachyon field is coupled to the matter Lagrangian while the boundary of universe is assumed to be a dynamical apparent horizon. The two logarithmic and power law corrected entropy on the apparent horizon is also discussed and the conditions for validity of GSL in both scenarios are investigated by using observational data of Sne Ia. In comparison to other research works, since the model is constrained by observational data, the conditions obtained for the dimensionless constant parameter, \( \alpha \) in both logarithmic and power law entropy correction of GSL are (physically) meaningful and realistic. The model also predicts an accelerating universe with no phantom crossing in the past or future.

Keywords Tachyon cosmology · Thermodynamics · GSL · Entropy corrected · Cosmic acceleration

1 Introduction

Observations confirm that about two third of the universe content is filled with a component dubbed as dark energy (DE) which causes cosmic acceleration (Jarosik et al. 2011). The cosmological constant is the simplest candidate for DE to fit the observational data. However, the measured expansion rate of the universe sets its scale being of order of \( 10^{-12} \) GeV; suffers from fine-tuning when compared with the Planck scale (\( 10^{18} \) GeV) (Shaw and Barrow 2011). Alternatively, there are a number of DE models which exploit scalar fields or some other exotic fields like phantom fields with negative energy (Caldwell 2002; Piao and Zhou 2003). Noting that, such scalar fields are usually so light (order of \( 10^{-33} \) eV) that need an extremely fine tuning. It is also claimed in the literature, that cosmic evolution of some kinds of these fields contradicts with the solar system tests (Nojiri and Odintsov 2004). It is most often the case that such fields interact with matter, (i) directly due to a Lagrangian coupling, (ii) indirectly through a coupling to the Ricci scalar or as the result of quantum loop corrections (Damour et al. 1990; Carroll 1998; Carroll et al. 1992; Biswas et al. 2006). Recently, a new model has been proposed in which a tachyon scalar field non-minimally couples to matter Lagrangian. The validity of this model in many cosmological scenarios has also been investigated for example in Farajollahi (2012, 2011a, 2011b, 2011c, 2011d) and Farajollahi and Salehi (2011).

On the other hand, inspired by the black hole physics, there is a deep connection between gravity and thermodynamics. An evidence for this connection in general relativity (GR) can be shown in deriving Einstein field equations in Rindler spacetime by using the relation between entropy and the horizon area as well as first law of thermodynamics. In addition, the validity of the generalized second law of thermodynamics (GSL) has been under extensive consideration by many researchers (Brustein 2000; Izquierdo and Pavon 2006; Gong et al. 2007a; Horvat 2007). According to GSL, the entropy of the fluid inside the hori-
zon in addition to the entropy associated with the apparent horizon is a nondecreasing function of time. As a result, one can obtain Friedmann equations by applying the Clasius relation to the apparent horizon of Friedmann-Robertson-Walker (FRW) universe (Cai and Kim 2005). However, it should be noted that definition of entropy would rather be modified in order to include quantum effects motivated from the loop quantum gravity (Rovelli 1996; Ashtekar et al. 1998). The modification can be taken as a logarithmic or power-law correction to entropy and manifests itself in derived field equations, such as modified Newtonian gravity, modified Friedmann equations (Sheykhi and Hendi 2011; Sheykhi 2010; Liu et al. 2011), and entropic corrections to Coulomb’s law (Sheykhi and Hendi 2012).

In this article, the GSL briefly explained in FRW cosmological model in Sect. 2. We then study the model in both logarithmic and power-law corrected entropy in Sect. 3. In Sect. 4, the tachyon cosmology in the presence of a scalar field non-minimally coupled to matter Lagrangian in the action is considered. The model is constrained with the observational data of SNe Ia. In Sect. 5, the GSL is discussed by considering the logarithmic and power law corrections to the entropy by using the best-fit parameters.

### 2 GSL in FRW cosmology

In the FRW cosmology, by assuming spatially flat Robertson-Walker (RW) metric, the Friedmann equations are

\[
3H^2 = 8\pi\rho \quad (1)
\]

\[
2\dot{H} + 3H^2 = -8\pi P \quad (2)
\]

where \(\rho\) and \(P\) are density and pressure parameters respectively where an equation of state parameter defined as \(P = \rho w\). In the following, two assumptions are made: (i) an entropy is associated with the horizon in addition to the entropy of the universe inside the horizon; (ii) according to the local equilibrium hypothesis, there is no spontaneous exchange of energy between the horizon and the fluid inside. GSL implies that in an expanding universe, the entropy of the viscous DE, dark matter and radiation inside the horizon together with the entropy associated with the horizon do not decrease with time. In general, there are two approaches to validate GSL on horizon: (i) by applying the first law of thermodynamics and obtain entropy relation on the horizon (Cai and Kim 2005; Bousso 2005), i.e.,

\[
T_h dS_h = -dE_h = 4\pi H R_h^3 K^{\mu}K_{\mu} dt
\]

\[
= 4\pi (\rho_{eff} + P_{eff}) H R_h^3 dt, \quad (3)
\]

where the index \(h\) stands for the horizon and \(K^{\mu} = (1, -Hr, 0, 0)\) is the (approximate) Killing vector (the generator of the horizon), or the future directed ingoing null vector field (Gong et al. 2007b); and (ii) in field equations, by employing the horizon entropy and temperature relation on the horizon,

\[
S_h = \pi R_h^2 \quad (4)
\]

\[
T_h = \frac{1}{2\pi R_h} \quad (5)
\]

Note that the two approaches are equivalent on the apparent horizon. Furthermore, recent observational data from type Ia Supernovae suggests that in an accelerating universe the enclosing surface would be the apparent horizon rather than event horizon (Zhou et al. 2007). Therefore, the universe is assumed to be enclosed by a dynamical apparent horizon with radius \(R_h = H^{-1}\), while the rate of change on entropy on the horizon is

\[
\dot{S}_h = 2\pi R_h \dot{R}_h \quad (6)
\]

or simply

\[
\dot{S}_h = 3\pi (1 + w) R_h \quad (7)
\]

On the other hand, applying the fundamental thermodynamics relation to the fluid inside the horizon (Wang et al. 2006) gives

\[
TdS_{in} = PdV + dE_{in} \quad (8)
\]

where \(S_{in}\) is the entropy within the apparent horizon, \(E_{in} = \rho_{eff} V\) is the internal energy, and \(V = \frac{4}{3} \pi R_h^3\). In the absence of energy exchange between outside and inside of the apparent horizon, thermal equilibrium implies that \(T = T_h\). Therefore, in the case of thermal equilibrium, relation (8) leads to

\[
\dot{S}_{in} = \frac{3}{2} \pi (1 + w)(1 + 3w) R_h \quad (9)
\]

Table 1 summarizes the rate of change of \(\dot{S}_h\) and \(\dot{S}_{in}\) in terms of equation of state parameter.

We observe that in case of universe acceleration (phantom line), the rate of change of entropy inside the apparent horizon and on the apparent horizon is a decreasing function of time. On the other hand, the rate of change of total entropy, \(S_t\), which is entropy of the horizon plus the entropy within the horizon is

\[
\dot{S}_t = \frac{9}{2} \pi (1 + w)^2 R_h \quad (10)
\]

which, regardless of the value of apparent horizon radius, it is obviously a nondecreasing function of time. The relations (7), (9) and (10) can then be re-parameterized as

\[
\frac{dS_x}{dz} = -\frac{R_h}{1 + z} \dot{S}_x, \quad (11)
\]
Table 1  The sign of internal and apparent horizon entropy $\dot{S}_h$ and $\dot{S}_h$ with respect to EoS parameter $w$

| $w$   | $-1$ | $0$ | $1/3$ | $-1/3$ |
|-------|------|-----|-------|--------|
| $\dot{S}_{in}$ | $+$  | $0$  | $-$   | $0$    |
| $\dot{S}_h$     | $-$  | $0$  | $+$   | $+$    |

where $z$ stands for redshift and $X$ for the indices “$i$”, “$h$” and “$t$” respectively. As it is apparent from (11), in an ever-expanding universe, $dS_X/dz \leq 0$ whenever $\dot{S}_X \geq 0$.

3 Corrections to entropy

The horizon entropy is proportional to the area of the horizon, i.e. $S = A/4$ where $A = 4\pi R_h^2$. However, modification of the theory, due to the motivation from loop quantum gravity, leads to a correction to the above relation. For instance, in $f(R)$ gravity, the modified entropy is $f(R)A$ (Wald 1993). Quantum corrections to the semi-classical entropy law, on the other hand, have been often introduced by logarithmic and power-law terms. In the following, contributions from these kinds of corrections will be discussed.

3.1 Logarithmic corrections

Logarithmic corrections, arises from loop quantum gravity due to the thermal equilibrium and quantum fluctuations (Miessner 2004; Ghosh and Mitra 2004; Chatterjee and Majumder 2004; Banerjee and Modak 2009; Modak 2009) and as a result, entropy on apparent horizon becomes,

$$S_h = \frac{A}{4} + \alpha \ln \frac{A}{4}$$

(12)

where $\alpha$ is a dimensionless constant of order unity that its value is still matter of debate. Following the same arguments in previous section, one finds the entropy on apparent horizon and also total entropy as

$$\dot{S}_h = 3(1 + w)\left(\pi R_h + \frac{\alpha}{R_h}\right)$$

(13)

and

$$\dot{S}_t = \frac{9}{2}\pi (1 + w)^2 R_h + 3\alpha (1 + w) R_h^{-1}.$$  

(14)

According to (14), the GSL is satisfied (i.e. $\dot{S}_t \geq 0$) if

$$\alpha \geq \beta$$  

where $\beta \equiv -\frac{3}{2}\pi (1 + w) R_h^{2}$.

3.2 Power-law corrections

When dealing with the entanglement of quantum fields in and out the horizon, power-law corrections appear, as for example (Radicella and Pavon 2010; Das et al. 2008)

$$S_h = \frac{A}{4} (1 - K_\alpha A^{\frac{1}{2}})$$

(16)

where

$$K_\alpha = \frac{\alpha (4\pi)^{\frac{2}{3}} - 1}{(4 - \alpha) r_c^{2 - \alpha}}$$

(17)

and $r_c$ is the crossover scale. The second term in (16), as a power-law correction to the entropy, has been raised from the entanglement of the wave-functions of a scalar field between its ground state and an exited state. The higher the excitation state the more significant the correction term. It is worth noting that in (17), the correction term tends to zero as the semi-classical limit (large area) is retrieved and consequently the conventional entropy is recovered. The rates of change of the horizon entropy and total entropy are respectively

$$\dot{S}_h = 3\pi (1 + w)\left(1 - K_\alpha (4\pi R_h^2)^{1-(\frac{2}{3})}\right) R_h$$

(18)

and

$$\dot{S}_t = \frac{9}{2}\pi (1 + w)^2 R_h - \frac{3}{2}\pi (1 + w)\alpha R_h^{3-\alpha} r_c^{2-\alpha}$$

(19)

Therefore, the GSL will be valid if

$$\alpha \left(\frac{r_c}{R_h}\right)^\alpha \leq 3(1 + w)\left(\frac{r_c}{R_h}\right)^2.$$  

(20)

Following Radicella and Pavon (2010), and Karimi et al. (2011), by replacing $r_c$ with $R_{h0}$, the above constraint becomes

$$\alpha \left(\frac{R_{h0}}{R_h}\right)^\alpha \leq 3(1 + w)\left(\frac{R_{h0}}{R_h}\right)^2.$$  

(21)

In the next section, these two corrected version of entropy will be employed to study GSL in the tachyon cosmology in the presence of a non-minimal coupling term to matter.

4 Tachyon cosmology

The tachyon cosmology with a non-minimal coupling to matter is given by the action:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi} - V(\phi)\right) \sqrt{1 + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}$$
where $R$ is Ricci scalar and $V(\phi)$ denotes the tachyon potential. Unlike the usual Einstein-Hilbert action, the matter Lagrangian $\mathcal{L}_m$ is modified as $f(\phi)\mathcal{L}_m$, where $f(\phi)$ is an analytic function of $\phi$ setting a non-minimal coupling between matter and scalar field. We assume that the matter field, filled the universe, is cold dark matter. We also assume a spatially flat FRW metric where the tachyon scalar field now is a function of only cosmic time. The field equations are

$$
3H^2 = 8\pi \left( \rho_m f(\phi) + \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \right) .
$$

(23)

$$
2\dot{H} + 3H^2 = -8\pi (-V(\phi)\sqrt{1-\dot{\phi}^2})
$$

(24)

and

$$
\ddot{\phi} + (1-\dot{\phi}^2) \left( 3H \dot{\phi} + \frac{V'(\phi)}{V(\phi)} \right) = \frac{f'(\phi)}{4V(\phi)} (1-\dot{\phi}^2)^{3/2} \rho_m
$$

(25)

where prime denotes differentiation with respect to the tachyon field $\phi$.

By using (23) and (25), one can easily arrive at the generalized conservation equation:

$$
\left( \rho_m f(\phi) \right) + 3H \rho_m f(\phi) = -1/4 \rho_m \dot{f}(\phi).
$$

(26)

In comparison with (2) and (2), the effective equation of state parameter is found to be

$$
w_{eff} = \frac{-V(\phi)\sqrt{1-\dot{\phi}^2}}{\rho_m f(\phi)} + \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}.
$$

(27)

In the following we constrain the parameters of model by using the observational data. Without losing the generality we first rewrite the equations by introducing the following new variables,

$$
X = \frac{8\pi \rho_m \dot{f}}{3H^2} , \quad Y = \frac{8\pi V}{3H^2} , \quad Z = \dot{\phi} , \quad U = \frac{1}{H}.
$$

(28)

We assume that $f(\phi) = f_0 \exp(\delta_1 \phi)$ and $V(\phi) = V_0 \times \exp(\delta_2 \phi)$ where $\delta_1$ and $\delta_2$ are constants characterizing the slope of potential $V(\phi)$ and coupling field $f(\phi)$. Cosmological model with potentials in the form of exponential functions have been used in a variety of contexts, such as accelerating expansion cosmological models (Halliwell 1987), cosmological scaling solutions (Yokoyama and Maeda 1988; Ivashchuk et al. 2003), chameleon models (Khoury and Weltman 2004), and models with attractor solutions (Maeda and Fujii 2009; Barreiro et al. 2000; Sen and Sethi 2002; Franca and Rosenfeld 2002). Using new variables, (23)–(26) become

$$
\frac{dX}{dN} = X(1-1/4\delta_1 ZU - 3Y \sqrt{1-Z^2})
$$

(29)

$$
\frac{dY}{dN} = Y(2ZU + 3 - 3Y \sqrt{1-Z^2})
$$

(30)

$$
\frac{dZ}{dN} = \frac{1/4\delta_1 XU(1-Z^2)^{3/2}}{Y} - (1-Z^2)(3Z + \delta_2 U)
$$

(31)

$$
\frac{dU}{dN} = \frac{3}{2} U(1 - Y \sqrt{1 - Z^2})
$$

(32)

where $N = \ln a$ with $a$ being scalar factor. The Friedmann equation (23), in terms of the new dynamical variables becomes

$$
X + \frac{Y}{\sqrt{1-Z^2}} = 1.
$$

(33)

Using the Friedmann constraint, (29)–(32) reduce to

$$
\frac{dX}{dN} = X(-1/4\delta_1 ZU - 3(1-X)(1-Z^2))
$$

(34)

$$
\frac{dZ}{dN} = \frac{1/4\delta_1 XU(1-Z^2)}{1-X} - (1-Z^2)(3Z + \delta_2 U)
$$

(35)

$$
\frac{dU}{dN} = \frac{3}{2} U(1 - (1-X)(1-Z^2)).
$$

(36)

Finally, the effective equation of state parameter, as a function of these new variables, can be rewritten as

$$
w_{eff} = \frac{-Y(1-Z^2)}{X \sqrt{1-Z^2} + Y}.
$$

(37)

To constrain the model parameters with the recent observational data, we use “Union2” sample (Amanullah et al. 2010) consisting of 557 usable SNe Ia data. The $\chi^2$ method, is the method used to best-fit the model for the parameters $\delta_1$ and $\delta_2$, and the initial conditions $X(0)$, $Z(0)$ and $U(0)$. In this case, the $\chi^2$ function is introduced as

$$
\chi^2_{5N}(\delta_1, \delta_2, X(0), Z(0), U(0)) = \sum_{i=1}^{557} \frac{[\mu_i^{the}(z_i) | \delta_1, \delta_2, X(0), Z(0), U(0)) - \mu_i^{obs}]^2}{\sigma_i^2},
$$

(38)

where $\mu_i^{the}$ and $\mu_i^{obs}$ are the distance modulus parameters obtained from our model and observations, respectively, and $\sigma_i$ is the estimated error of $\mu_i^{obs}$. Noting that the difference between the absolute and the apparent luminosity of a distance object is given by

$$
\mu(z) = 5 \log_{10} D_L(z) - \mu_0
$$

(39)
Table 2  Constraints on model parameters and initial values

| Model parameters | δ₁  | δ₂  | X₀  | U₀  | Z₀  |
|------------------|-----|-----|-----|-----|-----|
| Best-fit values  | −0.90 | −0.50 | 0.14 | 1.43 | 0.38 |

where \( \mu_0 = 5 \log_{10} h - 42.38 \) and \( h = (H_0/100) \text{ km/s/Mpc} \). The luminosity distance quantity, \( D_L(z) \) in (39) is derived as

\[
D_L(z) = (1 + z)H_0 \int_0^z \frac{dz'}{H(z')}. \tag{40}
\]

Table 2, summarizes the results derived by minimizing (38).

5  GSL in tachyon cosmology

In this section, by using best fitted model parameters, the GSL is investigated. In Fig. 1 top and middle panel plots of effective EoS parameter and rate of changes of entropies are shown as a function of redshift. The graph of \( w_{\text{eff}} \) versus the redshift \( z \) shows that \( w_{\text{eff}} > -1 \) (no phantom crossing) in the past and future while the universe is currently in quintessence era.

The EoS parameter shows that the universe begins to accelerate at about \( z \simeq 0.6 \). To probe GSL, plot of \( dS_h/dz, dS_i/dz \) and \( dS_{\text{int}}/dz \) versus redshift (the middle panel) shows that the total and apparent horizon entropies are never positive while the internal entropy become positive only when the universe begin to accelerate. This can be seen more clearly in Fig. 1 bottom panel where the rate of change of entropies are sketched with respect to effective EoS parameter. Noting that at this stage no correction is done to the entropy.

Next we consider the case of logarithmic correction to the entropy. By plotting \( \beta \), the right hand side of (15), in the top panel of Fig. 2, we see that parameter \( \beta \) approaches zero from below. To satisfy GSL, we have the condition \( \alpha \geq \beta \), \( \forall z \) which forces the constant parameter \( \alpha \) to be nonnegative as it is apparent from (15). The bottom panel of Fig. 2 is an evidence for this claim.

Finally, in the case of power-law entropy corrections, we plot \( dS_h/dz \) against redshift for different values of parameter \( \alpha \) in (19). The graphs in Fig. 3 confirm that GSL is satisfied for \( \alpha \leq 0 \). As can be seen in middle and bottom panels, for \( \alpha > 0 \), GSL is violated either in the past or future.

6  Summary and remarks

This paper is proposed to study GSL and its corrected versions in tachyon cosmological model in the presence of a non-minimal coupling to the matter field. The model is best fitted with observational data in order to validate the findings. For the model, the entropy for the apparent horizon,
the fluid inside the universe and total entropy is derived. To include quantum effects motivated from loop quantum gravity, two kinds of corrections to the entropy; logarithmic and power law corrections are also investigated. For the model with no correction to the entropy, the GSL is always satisfied as $\frac{dS}{dz} < 0$ for any redshift $z$. However, in the logarithmic correction case, GSL is satisfied only when the dimensionless constant $\alpha$ is greater than zero. Similarly, in power law entropy correction, GSL is satisfied for $\alpha \leq 0$. Noting that since the constrained model reveals no phantom crossing in the past and future, the rate of change of entropy of apparent horizon, $dS_h/dz$, is always positive both in accelerating and decelerating eras. On the other hand, the model predicts that the universe begins to accelerate at $z \simeq 0.6$, and thus the en-

Fig. 2 Logarithmic correction to entropy. (Top) The dynamics of $\beta$ vs. redshift. (Bottom) The dynamics of $\frac{dS}{dz}$ vs. redshift, for different values of $\alpha$

Fig. 3 Power-law entropy correction. The dynamics of $\frac{dS}{dz}$ vs. redshift for $\alpha < 0$ (top), and for $\alpha > 0$ (middle and bottom)
ropy within the horizon, \(dS_n/dz\), changes from negative in decelerating era to positive in accelerating era. However, to satisfy GSL with entropy corrections, in case of logarithmic correction, the dimensionless constant \(\alpha\) has to be positive for all redshifts whereas, in case of power law correction, \(\alpha\) has to be equal or less than zero for all redshifts. Our model also predicts universe acceleration while no phantom crossing occurs in the past or future. It is noticeable to know that the constraints on dimensionless constant \(\alpha\) in both correction scenarios are physically meaningful as our model fitted with the observational data.

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