Better Astrometric Deblending of Gravitational Microlensing Events by Using the Difference Image Analysis Method

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ABSTRACT

Due to the choice of very dense star fields for a higher event rate, the current microlensing searches suffer from large uncertainties caused by blending effect. To measure light variations of microlensing events free from the effect of blending, a newly developed method of Differential Image Analysis (DIA) was proposed for microlensing searches. However, even with the light variation curve obtained by using the DIA method, dramatic reduction of the uncertainty in the determined Einstein time scale is not expected due to the difficulty in determining the baseline flux of a source star.

However, we show in this paper that if the blending effect is investigated by detecting the shift of a source star image centroid, the DIA method will allow one to detect the blending effect with a significantly enhanced efficiency compared to the efficiency of the current method based on PSF photometry (PSF method). This is because for a given event the centroid shift measurable by using the DIA method, $\delta \theta_{c,\text{DIA}}$, is always larger than the centroid shift measurable by using the PSF method, $\delta \theta_{c,\text{PSF}}$. We find that the ratio $\delta \theta_{c,\text{DIA}}/\delta \theta_{c,\text{PSF}}$ rapidly increases with increasing fraction of blended light. In addition, for events affected by the same fraction of blended light, the ratio $\delta \theta_{c,\text{DIA}}/\delta \theta_{c,\text{DIA}}$ is larger for the event with a lower amplification. Therefore, centroid shift measurements by using the DIA method will be an efficient method to detect the blending effect especially of highly blended events, for which the uncertainties in the determined Einstein time scale are large, as well as of low amplification events, for which the current method is highly inefficient.
1. Introduction

Searches for Galactic dark matter by detecting flux variations of source stars caused by gravitational microlensing have been and are being carried out by several groups (MACHO: Alcock et al. 1993; EROS: Aubourg et al. 1993; OGLE: Udalski et al. 1993; DUO: Alard & Guibert 1997). To increase the event rate, these searches are being conducted towards very dense star fields such as the Galactic bulge and the Magellanic Clouds. While searches towards these dense star fields result in an increased event rate, it also implies that the observed light curves are affected by the unwanted flux from unresolved nearby stars: blending effect.

The light curve of a microlensing event with an isolated source star is represented by

$$ F = A_0 F_0, \quad (1) $$

where \( F_0 \) is the unlensed flux of the source star (baseline flux). The gravitational amplification is related to the lens-source separation \( u \) normalized by the angular Einstein ring radius by

$$ A_0 = \frac{u^2 + 2}{u \sqrt{u^2 + 4}}; \quad u = \left[ \beta_0^2 + \left( \frac{t - t_0}{t_{E,0}} \right)^2 \right]^{1/2}, \quad (2) $$

where the lensing parameters \( \beta_0, t_0, \) and \( t_{E,0} \) represent the lens-source impact parameter, the time of maximum amplification, and the Einstein ring radius crossing time scale (Einstein time scale), respectively. These lensing parameters are determined by fitting theoretical light curves to the observed one. One can obtain information about the lens mass \( M \) because the Einstein time scale is proportional to the square root of the lens mass, i.e. \( t_{E,0} \propto M^{1/2} \).

When an event is affected by blended light, on the other hand, its light curve differs from that of the unblended event by

$$ F_{\text{PSF}} = A_0 F_0 + B, \quad (3) $$

where \( B \) represents the flux from blended stars. Then, to fit the observed light curve of a blended event, one should include \( B \) as an additional parameter in addition to the three fitting parameters \((\beta_0, t_0, \text{ and } t_{E,0})\) of an unblended event. As a result, the uncertainties in the determined Einstein time scale and the corresponding lens mass for a blended event are significantly larger compared to those for an unblended event (Di Stefano & Esin 1995; Woźniak & Paczyński 1997; Han 1997; Alard 1997).

To resolve the blending problem, a newly developed technique to detect and measure light variations caused by gravitational microlensing was proposed by Tomaney & Crotts (1996), Alard & Lupton (1998), and Alard (1999). This so-called Difference Image Analysis (DIA) method measures the variation of source star flux by subtracting observed images from a normalized reference image, i.e.

$$ F_{\text{DIA}} = F_{\text{obs}} - F_{\text{ref}} = F_0 (A_0 - 1), \quad (4) $$
where $F_{\text{obs}} = A_0 F_0 + B$ and $F_{\text{ref}} = F_0 + B$ represent the source star fluxes measured from the image obtained during the progress of the event and from the reference image, respectively. Since not only the baseline flux of the lensed source star but also the flux from blended stars are subtracted by the DIA method, the light variation measured from the subtracted image is free from the effect of blending. Since photometric precision is improved by removing the blended light, the DIA method was adopted by the MACHO group and actually applied to microlensing searches (Alcock et al. 1999a, 1999b).

However, even with the DIA method dramatic reduction of the uncertainties in the determined Einstein time scales of gravitational microlensing events will be difficult. This is because the DIA method, by its nature, has difficulties in measuring the baseline flux $F_0$ of a source star. Unless the blended light fraction of the source star flux measured in the reference image, and thus the baseline flux $F_0 = F_{\text{ref}} - B$, is determined by some other means, one still has to include $B$ as an additional fitting parameter.\(^1\) Therefore, detecting the blending effect and estimating the blended light fraction in the observed source star flux is still an important issue to be resolved (see more discussion in §2).

There have been several methods proposed for the detection of the blending effect. For a high amplification event, one can determine the unblended baseline flux of the source star from the shape of the light curve obtained by using the DIA method itself (Han 2000). In addition, if the color difference between the lensed and blended stars is large, the effect of blending can be detected by measuring the color changes during the event (Buchalter, Kamionkowski, & Rich 1996). One can also identify the lensed source among blended stars by using high resolution images obtained from the *Hubble Space Telescope* (HST) observations (Han 1997). In addition, Han & Kim (1999) showed that the effect of blending can be detected from the astrometric observations of an event by using high resolution interferometers such as the *Space Interferometry Mission* (SIM). However, these methods either have limited applicability only for several special cases of microlensing events or impractical due to the requirement of using highly-demanding instrument for space observations.

A much more practical method for the detection of the blending effect that can be applicable for general microlensing events is provided by measuring the linear shift of the source star image centroid towards the lensed source star (hereafter centroid shift) during gravitational amplification (Alard, Mao, & Guibert 1995; Alard 1996; Goldberg 1998, see more detail in §3). Goldberg & Woźniak (1997) actually applied this method to the OGLE-1 database and demonstrated the efficiency of this method by detecting centroid shifts greater than $0''.2$ for nearly half of the total tested events (seven out of 15 events). However, even with this method the blending effect for an important fraction of blended events, especially for low amplification events, cannot be detected due to their small

\(^1\)Since higher photometric precision is expected by using the DIA method, the uncertainties of determined lens parameters will be smaller than those of lens parameters determined by using the current method based on PSF photometry. Han (2000) showed that for $\sim 30\%$ of high amplification events, one can determine $F_0$ with uncertainties less than 50\%. 
centroid shifts (Han, Jeong, & Kim 1998).

In this paper, we show that if the blending effect is investigated by detecting the shift of a source star image centroid, the DIA method will allow one to detect the blending effect with a significantly enhanced efficiency compared to the efficiency of the current method based on PSF photometry (PSF method). This is because for a given event the centroid shift measurable by using the DIA method, \( \delta \theta_{c,DIA} \), is always larger than the centroid shift measurable by using the PSF method, \( \delta \theta_{c,PSF} \). We find that the ratio \( \delta \theta_{c,DIA} / \delta \theta_{c,PSF} \) rapidly increases with increasing fraction of blended light. In addition, for events affected by the same fraction of blended light, the ratio \( \delta \theta_{c,DIA} / \delta \theta_{c,PSF} \) is larger for the event with a lower amplification. Therefore, centroid shift measurements by using the DIA method will be an efficient method to detect the blending effect especially of highly blended events, for which the uncertainties in the determined Einstein time scale are large, as well as of low amplification events, for which the current method is highly inefficient.

2. Degeneracy Problem

Even with the blending-free flux variations of a gravitational microlensing event measured by using the DIA method, it will be difficult to know whether the event is affected by the blending effect or not. This because the best-fit light curve obtained under the wrong assumption that the event is not affected by the blending effect matches well with the observed light curve.

The relations between the best-fit lensing parameters of a microlensing event resulting from the wrong determination of its source star baseline flux and their corresponding true values are provided by the analytic equations derived by Han (1999). If a blended event is misunderstood as an unblended event, the baseline flux of the source star is overestimated by \( F_0 + B \), causing mis-normalization of the amplification curve. Then the best-fit impact parameter \( \beta \) determined from the mis-normalized amplification curve differs from the true value \( \beta_0 \) by

\[
\beta = \left[ 2(1 - A_p^{-2})^{-1/2} - 2 \right]^{1/2}; \quad A_p = \frac{A_{p,0} + \eta}{1 + \eta},
\]

where \( A_{p,0} = (\beta_0^2 + 2)/(\beta_0 \sqrt{\beta_0^2 + 4}) \) and \( A_p \) represent the peak amplifications of the true and the mis-normalized amplification curves and \( \eta = \Delta F_0 / F_0 = B / F_0 \) is the fractional deviation of the mis-determined baseline flux. Mis-normalization of the amplification curve makes the best-fit Einstein time scale also differ from the true value by

\[
t_E = t_{E,0} \left( \frac{\beta_{th,0}^2 - \beta_0^2}{\beta_{th}^2 - \beta_0^2} \right)^{1/2}.
\]

2 The term ‘amplification curve’ represents the changes in the amplification of the source star flux as a function of time.
Here $\beta_{th,0} = 1.0$ and $A_{th,0} = 3/\sqrt{5}$ represent the threshold impact parameter and the corresponding threshold amplification for event detection, respectively. However, due to the mis-normalization of the amplification curve, the actually applied threshold amplification and the corresponding threshold impact parameter differ from $A_{th,0}$ and $\beta_{th,0}$ by

$$A_{th} = A_{th,0}(1 + \eta) - \eta \tag{7}$$

and

$$\beta_{th} = \left[2(1 - A_{th}^{-2})^{-1/2} - 2\right]^{1/2}. \tag{8}$$

Figure 1 shows the degeneracy problem in the light curve measured by using the DIA method. In the figure, the solid curve presents the light variation curve when an example event with lensing parameters $\beta_0 = 0.5$ and $t_{E,0} = 1.0$ is observed by using the DIA method. The event has baseline flux of $F_0 = 0.5$ and it is affected by the blended flux of $B = 0.5$. The dotted curve represents the best-fit light curve obtained by assuming that the event is not affected by the blending effect. The best-fit lensing parameters of the mis-normalized light curve determined by using the relations in equations (5)–(8) are $\beta = 0.756$ and $t_E = 0.742$, respectively. One finds that the two light curve matches very well, implying that it will be not easy to detect the blending effect only from the light variation curve obtained by using the DIA method.

### 3. Centroid Shifts by Using the PSF and DIA Methods

In previous section, we show that detection of the blending effect for general microlensing events will be difficult even with the blending-free light variation curve obtained by using the DIA method. However, we show in this section that if the blending effect of a microlensing event is investigated by detecting the centroid shift of a source star image, the DIA method will allow one to detect the blending effect with a significantly increased efficiency than the current method based on PSF photometry does. This is because for a given event the centroid shift measurable by using the DIA method is always larger than the shift measurable by the PSF method.

For the comparison of the centroid shifts of an event measurable by the DIA and the PSF methods, let us consider a blended source star image within which multiple stars with individual positions and fluxes $x_i$ and $F_{0,i}$ are included. Gravitational amplification occurs only for one of these stars. Since the position of the lensed star usually differs from the centroid of the blended image, the centroid of the source star image is shifted toward the lensed source during the event. If the lensed source star with a baseline flux $F_{0,j}$ is located

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\footnote{For Galactic microlensing event, the typical lens-source separation is of the order of milli-arcsec, while the average separation between stars is $(O)10^{-1}$ arcsec. Therefore, simultaneous lensing for multiple source stars hardly happens.}
at a position \( \mathbf{x}_j \), the amount of this centroid shift, which is measurable by using the current PSF method, is

\[
\delta\theta_{c, \text{PSF}} = D (\langle \mathbf{x} \rangle - \mathbf{x}_j); \quad D = \frac{f (A_0 - 1)}{f (A_0 - 1) + 1},
\]

(9)

where \( f = F_{0,j} / \sum_i F_{0,i} \) represents the fractional flux of the lensed source out of the total flux of the blended stars (including \( F_{0,j} \)) within the integrated seeing disk and \( \langle \mathbf{x} \rangle = \sum_i \mathbf{x}_i F_{0,i} / \sum_i F_{0,i} \) is the position of blended image centroid before the gravitational amplification, i.e. the position of the blended source image centroid on the reference frame (Goldberg 1998). On the other hand, the position of the centroid measured on the subtracted image by using the DIA method is the true position of the lensed source star, i.e. \( \mathbf{x}_j \). Therefore, the centroid shift measurable by using the DIA method is

\[
\delta\theta_{c, \text{DIA}} = \langle \mathbf{x} \rangle - \mathbf{x}_j.
\]

(10)

Then the ratio between the centroid shifts measurable by the two methods is

\[
\frac{\delta\theta_{c, \text{DIA}}}{\delta\theta_{c, \text{PSF}}} = D^{-1}.
\]

Since \( f (A_0 - 1) > 0 \), and thus \( D < 1 \), for a given event the centroid shift measurable by the DIA method is always larger than the shift measurable by the PSF method.

In Figure 2, we illustrate the centroid shifts measurable by the DIA and the PSF methods for visualization. On the left side, we present the contours (measured at an arbitrary flux level) of two source stars with identical baseline fluxes (the inner two dotted circles with their centers marked by ‘+’) and their integrated images (the outer solid curve centered at ‘x’) before (upper part) and during (lower part) gravitational amplification. Among the two stars within the blended image, the right one is lensed. Between the two left contours of integrated image, we marked the centroid shifts measurable by the PSF (\( \delta\theta_{c, \text{PSF}} \)) and the DIA (\( \delta\theta_{c, \text{DIA}} \)) methods. To better show the centroids shifts, the region enclosed by a dot-dashed line is expanded and presented on the right side. One finds that \( \delta\theta_{c, \text{DIA}} > \delta\theta_{c, \text{PSF}} \).

Then, how much larger is the centroid shift measurable by the DIA method than the shift measurable by the DIA method? The ratio \( \frac{\delta\theta_{c, \text{DIA}}}{\delta\theta_{c, \text{PSF}}} \) depends not only on the blended light fraction \( 1 - f \), but also on the amplification \( A_0 \) of an event. To see these dependencies, we present in Figure 3 the relations between the ratio \( \frac{\delta\theta_{c, \text{DIA}}}{\delta\theta_{c, \text{PSF}}} \) and the fraction of blended light for events with various impact parameters. From the figure, one finds the two following important trends. First, the ratio \( \frac{\delta\theta_{c, \text{DIA}}}{\delta\theta_{c, \text{PSF}}} \) increases rapidly with the increasing fraction of blended light. This implies that compared to the PSF method the DIA method will be able to better detect the blending effect of highly blended events for which the uncertainties in the determined Einstein time scales are large. Second, if events are affected by the same fraction of blended light, the ratio \( \frac{\delta\theta_{c, \text{DIA}}}{\delta\theta_{c, \text{PSF}}} \) is bigger for the event with lower amplification, implying that the blending effect of low amplification events can be better detected by using the DIA method. For events with low amplifications, the expected centroid shifts measurable by the PSF method are very small, making it difficult to detect the blending effect by this method. Therefore, by increasing the detection
efficiency for centroid shifts for low amplification events, the DIA method will allow one to
detect the blending effect for a significant fraction of Galactic microlensing events.

4. Summary

We analyze the centroid shifts of the source star image centroid of gravitational
microlensing events caused by the blending effect. The findings from the comparison of the
centroid shifts measurable by the current method based on PSF photometry and the newly
developed DIA method are summarized as follows.

1. For a given event the centroid shift measurable by using the DIA method is always
larger than the shift measurable by using the PSF method, allowing one to better
detect the blending effect of microlensng events.

2. The ratio between the centroid shifts measurable by using the DIA and the PSF
methods, $\delta \theta_{c,\text{DIA}} / \delta \theta_{c,\text{DIA}}$, rapidly increases as the fraction of the blended light
increases. Therefore, detection of the centroid shifts by using the DIA method is an
efficient method for the detection of the blending effect especially for highly blended
events which cause large uncertainties in the determined Einstein time scales.

3. If the blended light fraction is the same, the ratio $\delta \theta_{c,\text{DIA}} / \delta \theta_{c,\text{DIA}}$ is larger for the
events with a lower amplification. Therefore, the DIA method enables one to better
detect the source star image centroid shifts of low amplification events, for which
detection of their blending effect by using the PSF is very difficult due to their small
expected shifts.

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Figure 1: The degeneracy problem in the light variation curve of a gravitational microlensing event observed by the DIA method. The solid curve represents the light variation curve which is expected when an example event with $\beta_0 = 0.5$, $t_{E,0} = 1.0$, and $F_0 = 0.5$ is observed by using the DIA method. The dot-dashed curve represents the best-fit curve obtained under the assumption that the flux of the source star is not affected by the blending effect, despite that the source star flux in the reference image is influenced by the blended light with an amount $B = 0.5$. 
Figure 2: Illustration of the source star image centroid shifts measurable by using the PSF ($\delta\theta_{c,\text{PSF}}$) and the DIA ($\delta\theta_{c,\text{DIA}}$) methods. On the left side, the contours (measured at an arbitrary flux level) of two source stars with identical baseline fluxes (the inner two dotted circles with their centers marked by ‘+’) and their integrated images (the outer solid curve centered at ‘x’) before (upper part) and during (lower part) gravitational amplification are presented. Among the two stars within the blended image, the right one is lensed. Between the two left contours of integrated image, the centroid shifts measurable by the PSF ($\delta\theta_{c,\text{PSF}}$) and the DIA ($\delta\theta_{c,\text{DIA}}$) methods are marked. To better show the centroids shifts, the region enclosed by a dot-dashed line is expanded and presented on the right side.
Figure 3: The relations between the centroid shift ratio $\delta \theta_{c,DIA}/\delta \theta_{c,PSF}$ and the fraction of blended light $1 - f$ for events with various impact parameters. To better show the relations with $(1 - f) \leq 0.7$, the region is enlarged and presented in a separate box.