Strategic delegation in a sequential model with multiple stages

Paraskevas V. Lekeas∗ Giorgos Stamatopoulos†

Abstract
We analyze strategic delegation in a Stackelberg model with an arbitrary number, \( n \), of firms. We show that \( n-1 \) firms delegate their production decisions and only one firm (the one whose manager is the first mover) does not. The later a manager commits to a quantity, the higher his incentive rate. Letting \( u_i^* \) denote the equilibrium payoff of the firm whose manager commits in the \( i \)-th stage, we show that \( u_n^* > u_{n-1}^* > \cdots > u_2^* > u_1^* \). We also compare the delegation outcome of our game with that of a corresponding Cournot oligopoly and show that managers who commit late (early) are given higher (lower) incentive rates than managers in the Cournot market.

Keywords: Sequential competition; late-movers’ advantage; delegation

1 Introduction
The Stackelberg model of market competition is a benchmark model of industrial economics. In this model, firms select their market strategies (quantities or prices) sequentially. One of the most important issues in this framework focuses on the relation between timing of commitment and relative profitability of firms. For the case of two players, Gal-Or (1985) showed that if reaction functions are downwards-sloping then the first-mover earns a higher payoff than his opponent. On the other hand, in the case of upwards-sloping reaction functions the advantage is with the second-mover. Further studies showed that this result is not robust to

∗Department of Applied Mathematics, University of Crete, Heraklion, Crete, Greece; email: plekeas@tem.uoc.gr

†Department of Economics, University of Crete, 74100 Rethymno, Crete, Greece; email: gstamato@econ.soc.uoc.gr

1The result of Gal-Or (1985) is obtained in a set-up which includes the Stackelberg duopoly as a special case.
variations of the model. Gal-Or (1987) studied a Stackelberg duopoly where firms compete under private information about market demand. In this model the first-mover might earn a lower profit than his opponent, as he produces a relatively low quantity in order to send a signal for low demand. Liu (2005) analyzed a model where only the first-mover has incomplete information about the demand and showed that in some cases the first-mover loses the advantage.

For the case of \( n \geq 2 \) symmetric firms, Boyer and Moreaux (1986) and Anderson and Engers (1992) showed that the \( i \)-th mover obtains a higher profit than the \( i+1 \)-th mover, for \( i = 1, 2, \ldots, n-1 \). Pal and Sarkar (2001) analyzed a model with \( n \geq 2 \) cost-asymmetric firms under the assumption that the later a firm commits to a quantity, the lower its marginal cost. They showed that if cost differentials are sufficiently low, the firm that moves in stage \( i \) obtains a higher payoff than its successor \( i+1 \); otherwise, the ranking of profits is reversed.

Recently, an integration of the Stackelberg model with the theory of endogenous objectives of oligopolistic firms has taken place. The latter theory was launched with the works of Fershtman and Judd (1985), Vickers (1985) and Sklivas (1987). These works endogenized the objective functions of firms in a context of management/ownership separation by postulating that firms maximize a combination of revenue and profit or quantity and profit. This framework was applied by Kopel and Loffler (2008) to a Stackelberg duopoly with homogeneous commodities (which give rise to downwards sloping reaction functions). Their paper analyzed the impact of delegation on the structure of first versus second mover advantage. The authors showed that only the second mover delegates the production decision to a manager. As a result, the second mover produces a higher quantity than the first mover and earns higher profit.

The current paper analyzes strategic delegation in a Stackelberg model with an arbitrary number of firms. It assumes a fixed order of play and perfect observability of choices at each stage. Our work is an extension of the strategic delegation setup presented in Kopel and Loffler (2008). Our aim is to determine the relations among: (i) the timing of commitment to quantities; (ii) the equilibrium delegation decisions and (iii) the relative performance of firms. Moreover, we are interested in comparing the equilibrium of the sequential market with that of a corresponding Cournot market.

The main results of the paper are as follows: First, we show that all firms delegate their production decision to managers except for the firm whose manager is the first to commit to a quantity. Moreover, the equilibrium incentive rate is an increasing function of the order of commitment. Namely, the later a manager selects a quantity, the higher his incentive rate he is given. More importantly, letting \( u^*_n \) denote the equilibrium payoff of the firm whose manager commits in stage \( i \), we show that \( u^*_n > u^*_{n-1} > \cdots > u^*_2 > u^*_1 \). This ordering of profits is due
to the result that the managers who commit at late stages choose relatively high quantities (as they are given relatively high incentive rates).

Delegation in a Cournot model leads to an equilibrium where all firms end-up with a lower payoff compared to the case of non-delegation. This is not true though for the Stackelberg model: Firms whose managers decide on quantities after a threshold stage prefer the delegation regime over nondelegation. Nonetheless, we show that if the number of firms is \( n \geq 3 \), each firm in the Stackelberg market earns a lower payoff than a Cournot firm.

The rest of the paper is organized as follows. Section 2 describes the model and section 3 presents the results. Section 4 concludes.

## 2 The framework

Consider an \( n \)-firm sequential oligopoly. Firms face the inverse demand function \( P = \max\{a - Q, 0\} \), where \( P \) is the market price and \( Q \) is the total market quantity given by \( Q = q_1 + q_2 + \ldots + q_n \), where \( q_i \) is the quantity of firm \( i = 1, 2, \ldots, n \). The production technology of firm \( i \) is represented by the cost function \( C(q_i) = cq_i \), \( i = 1, 2, \ldots, n \). Firms are characterized by ownership-management separation. The task of firm \( i \)'s manager is to select a quantity by maximizing an objective function delegated to him by the owners of the firm. We assume that this objective function is a combination of profit and quantity (Vickers 1985):

\[
T_i = (P - c)q_i + a_i q_i, \quad a_i \geq 0, \quad i = 1, 2, \ldots, n
\]

where \( a_i \) is manager \( i \)'s incentive rate. The time structure of the interaction among firms and managers is as follows. In stage 0, the firms' owners decide simultaneously on the incentive rates of their managers. In particular, firm \( i \)'s owners choose \( a_i \) so as to maximize the profit function

\[
u_i = (a - Q - c)q_i, \quad i = 1, 2, \ldots, n
\]

These choices are made publicly known. Then play becomes sequential. In stage 1 the manager of firm 1 selects (and commits to) a quantity for his firm. His choice is observed by all other players. In stage 2, firm 2's manager selects a quantity, which is observed by all other players. The process continues this way in stages \( 3, 4, \ldots, n - 1, n \).

We denote the above interaction by \( G_S \). In the next section we identify the sub-game perfect Nash equilibrium (SPNE) outcome of this game.

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4The results of this paper do not change if we assume that the objective function of each firm is a convex combination of profit and revenue (Fershtman and Judd 1985, Sklivas 1987).
3 Results

3.1 Quantity stages

Working backwards, we first analyze the quantity competition stages of $G_s$. We first note that, in essence, managers choose quantities as if their firms face asymmetric marginal costs given by $(c_1, c_2, \cdots, c_n) = (c - a_1, c - a_2, \cdots, c - a_n)$. Thus depending on the 0-stage choices of $(a_1, a_2, \cdots, a_n)$ and the resulting asymmetries we can have, a priori, some managers selecting zero quantities. We will show though that any configuration with one or more managers selecting zero quantities cannot be part of a SPNE outcome of $G_S$.

We begin by describing the managers’ reaction functions. It will be useful for our analysis to define not only the standard reaction function but also the auxiliary concept of step $k$ reaction function where $k$ is a positive integer) on which we elaborate below.

Let $Q_i = q_1 + q_2 + \cdots + q_{i-2} + q_{i-1}$. Consider first stage $n$. We will denote by $f^1_n(q_1, \ldots, q_{n-1})$ the step 1 reaction function or simply the reaction function of manager $n$, defined by

$$f^1_n(q_1, \cdots, q_{n-1}) = \arg \max_{q_n \geq 0} T_n(q_1, \cdots, q_{n})$$

where $T_n(q_1, \cdots, q_{n}) = (a - Q - c + a_n)q_n$. For the moment we do not discuss the positiveness or not of the reaction functions; we will turn to this (critical) issue later on in the analysis. Moving to stage $n - 1$, the (step 1) reaction function of manager $n - 1$ is

$$f^1_{n-1}(q_1, \cdots, q_{n-2}) = \arg \max_{q_{n-1} \geq 0} T_{n-1}(q_1, \cdots, q_{n-1})$$

where

$$T_{n-1}(q_1, \cdots, q_{n-1}) = (a - Q^{n-1} - q_{n-1} - f^1_n - c + a_{n-1})q_{n-1}$$

Then the step 2 reaction function of manager $n$ is derived by $f^1_n$ when $q_{n-1}$ is replaced by $f^1_{n-1}$, i.e.,

$$f^2_n(q_1, \cdots, q_{n-2}) = f^1_n|_{q_{n-1}=f^1_{n-1}}$$

Moving on to stage $n - 2$, the step 1 reaction function of manager $n - 2$ is defined by

$$f^1_{n-2}(q_1, \cdots, q_{n-3}) = \arg \max_{q_{n-2} \geq 0} T_{n-2}(q_1, \cdots, q_{n-2})$$

Whenever there is no confusion, we will drop the variables $q_1, q_2$, etc., from the definitions of the various reaction functions. For notational simplicity, the definitions do not include the incentive rates.
consider a vector \( \tilde{a} \). We argue that case (i) cannot be part of any SPNE outcome. To this end, the concavity of \( f \) implies that:

These conditions in turn imply that:

Recall that

The above description will be useful in order to examine what type of quantity configurations can support an SPNE outcome of \( G_s \). To this end, consider the generic stage \( i \). Using our description, manager \( i \) selects \( q_i \) in order to maximize the function

Notice that

Recall that \( f_i^1(q_1, \ldots, q_{i-1}) \) denotes manager \( i \)'s (step 1) reaction function. Then the concavity of \( T_i \) in \( q_i \) (which can be easily established) implies that if

then \( f_i^1(q_1, \ldots, q_{i-1}) = 0 \) whereas if

then \( f_i^1(q_1, \ldots, q_{i-1}) > 0 \).

These conditions in turn imply that:

(i) if \( Q^i + \sum_{k=i+1}^n f_k^{k-i}(q_1, \ldots, 0) \geq a - c + a_i \) then \( f_i^1(q_1, \ldots, q_{i-1}) = 0 \) and

(ii) if \( Q^i + \sum_{k=i+1}^n f_k^{k-i}(q_1, \ldots, 0) < a - c + a_i \) then \( f_i^1(q_1, \ldots, q_{i-1}) > 0 \).

We argue that case (i) cannot be part of any SPNE outcome. To this end, consider a vector \( (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \) of 0-stage choices. Assume that these choices

\({}^6\)To be consistent, when dealing with \( T_1 \) we need to set \( Q^1 = 0 \).
are such that all managers select positive quantities except for one, say manager $i$. Let $(\tilde{q}_1, \ldots, \tilde{q}_{i-1}, 0, \tilde{q}_{i+1}, \ldots, \tilde{q}_n)$ denote this market outcome. Given that $\tilde{q}_k = f_k^{-i}(\tilde{q}_1, \ldots, \tilde{q}_{i-1}, 0)$, $k = i + 1, i + 2, \ldots, n$, and since we are in case (i), we have $\sum_{j \neq i} \tilde{q}_j \geq a - c + \tilde{a}_i$. But then the profit of any firm $j, j \neq i$, in stage 0 is

$$u_j = (a - \tilde{q}_1 - \cdots - \tilde{q}_n - c)\tilde{q}_j \leq (a - (a - c + \tilde{a}_i) - c)\tilde{q}_j = -\tilde{a}_i\tilde{q}_j \leq 0$$

To put it differently, a configuration of the form $(\tilde{q}_1, \ldots, \tilde{q}_{i-1}, 0, \tilde{q}_{i+1}, \ldots, \tilde{q}_n)$ cannot support an SPNE outcome as such a configuration would make the market price fall below the marginal cost. To see this, notice that the conditions $\sum_{j \neq i} \tilde{q}_j \geq a - c + \tilde{a}_i$ and $\tilde{a}_i \geq 0$ imply that $a - \sum_{j \neq i} \tilde{q}_j \leq c$. A similar argument holds for outcomes under which more than one managers select zero quantities.

Hence in what follows we can focus our attention on the case where all firms produce positive quantities.

Since in any SPNE outcome all managers produce positive quantities, we can use the results of Pal and Sarkar (2001) who computed the equilibrium quantities in an $n$-stage Stackelberg with cost-asymmetric firms (but without delegation) under the assumption that all firms are active. By adjusting their analysis to ours, the manager of firm $i$ chooses the quantity

$$q_i^* = (P^* - c + a_i)2^{n-i}, \quad i = 1, 2, \ldots, n$$

where

$$P^* = \frac{a}{2^n} + \sum_{i=1}^{n} \frac{c - a_i}{2^i}$$

is the market price.

### 3.2 Delegation stage

Armed with the above, we can move to stage 0 (the delegation stage). Let $(a_1, a_2, \ldots, a_n) = (a_i, a_{-i})$. Using (1), the payoff of firm $i$ in stage 0 is

$$u_i(a_i, a_{-i}) = 2^{n-i}(\frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c)(\frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c + a_i)$$

The maximization problem facing firm $i$'s owners is $\max_{a_i} u_i(a_i, a_{-i}), i = 1, 2, \ldots, n$. Define $D_i = 2^{i+1}/(\sigma(i) - 1)$, $\sigma(i) = (2^{i+1} - 2)/(2^i - 2)$ and $h(n) = -2 + 2n + 2^{2-n}$.

**Lemma 1.** Consider the delegation stage of $G_S$.

(i) The equilibrium incentive rates are

$$a_1^* = 0, \quad a_i^* = D_i\frac{a - c}{2^n h(n)} > 0, \quad i = 2, 3, \ldots, n$$
The inequalities \( a_1^* > a_{n-1}^* > \cdots > a_2^* > a_1^* \) hold.

**Proof.** Appears in the Appendix.

By Lemma 1 all firms except for 1, delegate in equilibrium. Moreover, the later a manager commits to a quantity, the higher his incentive rate. To give some intuition behind this result, we first note that there is a negative relation between any \( a_i \) and the market price. For example, consider firms \( i \) and \( i+j \) with \( j > 0 \). The corresponding effects of \( a_i \) and \( a_{i+j} \) on the market prices are

\[
\frac{\partial P^*}{\partial a_i} = -\frac{1}{2^i} < \frac{\partial P^*}{\partial a_{i+j}} = -\frac{1}{2^{i+j}}
\]

To comprehend why the above inequality holds, let us go back to the managers’ (step 1) reaction functions: the rate \( a_i \) appears in the step 1 reaction functions \( q_1, \ldots, q_{i-1}, q_i \) whereas the rate \( a_{i+j} \) appears in the step 1 reaction of more terms, i.e. \( q_1, \ldots, q_i, \ldots, q_{i+j-1}, q_{i+j} \). Furthermore: (i) the relation between \( a_i \) and any of \( q_1, \ldots, q_i, \ldots, q_{i+j-1}, q_{i+j} \) is negative and so is the relation between \( a_{i+j} \) and any of \( q_1, \ldots, q_i, \ldots, q_{i+j-1} \); (ii) the market price depends negatively on quantities. Points (i) and (ii) explain why \( a_{i+j} \) has a smaller negative impact on price than \( a_i \) has. As a result, the owners of firm \( i+j \) have incentive to make their manager more aggressive than firm \( i \)'s owners.

Using Lemma 1, market price, individual and total market quantities are given respectively by

\[
P_S^* = \frac{a}{2^n-1h(n)} + \frac{c(1 + n2^n - 2^{-n})}{2^n-1h(n)} \tag{2}
\]

\[
q_{iS}^* = (2 - 2^{1-i}) \frac{a - c}{h(n)}, \quad i = 1, 2, \ldots, n \tag{3}
\]

\[
Q_S^* = (a - c)[1 - 2^{-n} + \frac{2n - 4 + 2^{2-n}}{2nh(n)}] \tag{4}
\]

Let \( u_i^* \) denote the equilibrium profit of firm \( i, i = 1, 2, \ldots, n \), in \( G_S \). Our next result ranks these profits.

**Proposition 1.** The inequalities \( u_n^* > u_{n-1}^* > \cdots > u_2^* > u_1^* \) hold in \( G_S \).

**Proof.** Since firms face the same price and they are cost-symmetric, \( u_{i+1}^* > u_i^* \) if and only if \( q_{i+1}^* > q_i^* \), \( i = 1, 2, \ldots, n-1 \), which holds by inspection of (3).

One question raised at this point is how does the performance of firms in \( G_S \) compare with their performance in a sequential market without any delegation.
activities. Let \( \bar{u}_i \) denote the equilibrium profit of the \( i \)-th firm in the latter market. We have the following.

**Corollary 1.** There exists a stage \( i' = i'(n) \) such that \( u^*_i > \bar{u}_i \) if and only if \( i > i'(n) \).

**Proof.** Appears in the Appendix

Therefore, firms whose managers select quantities after the \( i \)’th stage prefer the delegation regime over nondelegation; the opposite holds for the remaining firms. This result is explained by our previous finding that the late-moving managers are relatively aggressive at the expense of the early movers.

### 3.3 Comparison with Cournot competition

In this section we compare the equilibrium outcome of \( G_{S} \) with the outcome of the corresponding Cournot market. In the latter framework, we have a two stage interaction which evolves as follows: in stage 0, firms’ owners choose the incentive rates of their managers. These choices are made publicly known. Then in stage 1, the managers of the \( n \) firms select simultaneously quantities for their firms, using the incentive schemes decided upon in stage 0. Let \( G_{C} \) denote this game (which was first analyzed by Vickers, 1985).

It is known that in the absence of delegation, the Stackelberg market produces a higher total quantity than the Cournot market (Anderson and Engers, 1992). When delegation is introduced then: (i) In \( G_{S} \) not all firms delegate; (ii) in \( G_{C} \) all firms delegate. Hence a direct ranking of the Stackelberg and Cournot total market quantities under certain delegation is not obvious. Corollary 2 below provides this comparison. It also compares incentive rates and profiles across the two frameworks (in what follows, \( Q^*_S, a^*_C, u^*_C \) denote the equilibrium total market quantity, incentive rate and profit respectively under Cournot competition).

**Corollary 2.** Consider the games \( G_{S} \) and \( G_{C} \). The following hold.

(i) For any \( n \geq 2 \), \( Q^*_S > Q^*_C \).

(ii) For any \( n \geq 2 \), \( a^*_n > a^*_C \) and \( a^*_C > a^*_i \), \( i = 1, 2, \ldots, n - 1 \).

(iii) For \( n = 2 \), \( u^*_2 > u^*_C > u^*_1 \); for \( n \geq 3 \), then \( u^*_C > u^*_i \), for all \( i \).

**Proof.** Appears in the Appendix.

Corollary 2 shows an interesting relation: All firms in \( G_{S} \), except for the last mover, choose lower incentive rates than firms in \( G_{C} \). Nonetheless, regarding consumers, the Stackelberg market remains more efficient than the Cournot market as it results to a higher market quantity.

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\(^8\)Due to the symmetry of the model, all firms in the Cournot market choose the same incentive rate and have the same profit in equilibrium.
In the absence of delegation, Anderson and Engers (1992) compared the profitability in the Stackelberg and Cournot models: for \( n = 2 \); the first (second) mover earns a higher (lower) profit than the Cournot duopolists; for \( n \geq 3 \), all Stackelberg firms earn a lower profit than the Cournot firms. When delegation is introduced and \( n = 2 \), the second (first) mover earns a higher (lower) profit than the Cournot duopolists (this case is analyzed by Kopel and Loffler, 2008); for \( n \geq 3 \), all Stackelberg firms earn a lower profit than the Cournot firms (Corollary 2(iii)).

Let us, at this point, recall our assumption that each stage’s choices are perfectly observable. The issue of imperfect observability in strategic games has been analyzed in a series of works. Katz (1991) demonstrated that if delegation choices are not observed by rivals then delegation has no value. Bagwell (1994) showed that observing the rival’s action with noise destroys the impact of first mover’s commitment. Vardy (2004) analyzed a sequential game where observing the first mover’s choice is costly. He showed that being the first mover has no value, no matter how small the observation cost is. Other authors delivered more positive results: Fershtman and Kalai (1997) provided a framework where the value of delegation can be restored, provided there is a positive probability that the delegation contracts are accurately observed. van Damme and Hurkens (1997), Guth et al. (1998) and Maggi (1999) showed that commitment under imperfect observability has an impact on the outcome of the game if one allows for either mixed strategy equilibria (first two papers) or for private information on behalf of the first mover (last paper).

Contributing to the above discussion is not a goal (or an ambition) of the current paper. Just to provide some real-world facts we quote from Scalera and Zazzaro (2008):

"... the assumption of contract observability seems in some cases to be quite realistic. When firms compete to hire managers, it is likely that contractual clauses are publicly declared."

Further, the same present the argument that

"... in many countries, at least as regards quoted companies, firms are obliged by regulators to announce manager compensations to the market and this eases their commitment to the contracts signed with managers."\(^9\)

### 4 Conclusions

We analyzed strategic delegation in a Stackelberg model with an arbitrary number of firms. We showed that the later a firm’s manager commits to a quantity, the higher his firm’s profit. Delegation improves the payoff of the late-movers and hurts early-movers. Namely, firms whose managers commit late (early) to a quantity end-up with a higher (lower) payoff compared to the non-delegation regime. This

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\(^9\)Katz (1991) questions the impact of this type of announcements as they refer to the actual payments of the managers and not to the rules that generate these payments.
is different from the case of delegation under Cournot competition, where all firms are hurt by delegation.

Our paper has analyzed a framework with linear demand and cost functions. Introducing a more general framework will allow us to examine the robustness of our results. Further the analysis of a market where incentive contracts are imperfectly observed is of special interest.

Appendix

Proof of Lemma 1. (i) Notice that

$$\frac{\partial u_i(a_i,a_{-i})}{\partial a_i} > 0 \iff \frac{1}{2^n} (a - c - a_i) + \left( \frac{a}{2^n} + \sum_{j=1}^{n} \frac{c - a_j}{2^j} - c \right) (1 - \frac{1}{2^n}) > 0$$

or iff

$$a_i < \left( \frac{a - c}{2^n} - \sum_{j \neq i}^{n} \frac{a_j}{2^j} \right) \left( \frac{2^i (2^i - 2)}{2^{i+1} - 2} \right)$$

Clearly, for $i = 1$, the derivative is negative and hence the equilibrium incentive rate that firm 1 chooses is $a_1^* = 0$. Let now $i \geq 2$. Then the reaction function of firm $i$ is given by

$$a_i = \begin{cases} 0, & \text{if } \sum_{j \neq i} a_j/2^j \geq (a - c)/2^n, \\ \frac{2}{\sigma(i)} [(a - c)/2^n - \sum_{j \neq i} a_j/2^j], & \text{if } \sum_{j \neq i} a_j/2^j < (a - c)/2^n. \end{cases}$$

where $\sigma(i) = (2^{i+1} - 2)/(2^i - 2)$, $i \geq 2$. We first notice that in any equilibrium of the delegation game only firm 1’s owners choose a zero incentive rate; all the remaining firms choose positive incentive rates. To see this, consider an outcome $a_i = 0$ and $a_j > 0, j = 2, \cdots, i - 1, i + 1, \cdots, n, j \neq i$. The market price is

$$P = \frac{a}{2^n} + \sum_{i=1}^{n} \frac{c - a_i}{2^i}.$$ 

Since $a_i = 0$, we have $\sum_{j \neq i} a_j/2^j \geq (a - c)/2^n$. But then it is easy to show the last inequality implies that the corresponding price would fall below the marginal cost $c$. Hence we restrict attention to the case where $\sum_{j \neq i} a_j/2^j < (a - c)/2^n$ for all $i$. Then we have the system

$$\frac{1}{2^2} a_2 + \frac{1}{2^3} a_3 + \cdots + \sigma(i) \frac{1}{2^n} a_i + \cdots + \frac{1}{2^n} a_n = \frac{a - c}{2^n}, \quad i = 2, 3, \ldots, n \quad (5)$$

$$\sigma(i) = \frac{2^{i+1} - 2}{2^i - 2}.$$ 

Using (5) the equations for firms $i$ and 2 we get,

$$a_i = \frac{\sigma(2) - 1}{\sigma(i) - 1} 2^{i-2} a_2 \quad \text{(6)}$$
\[ a_2 = \frac{a - c}{2^{n-2}} (\sigma(2) + \sum_{i=3}^{n} \frac{\sigma(2) - 1}{\sigma(i) - 1})^{-1} \]  \hspace{1cm} (7)

It is straightforward to show that \( \sigma(2) + \sum_{i=3}^{n} \frac{\sigma(2) - 1}{\sigma(i) - 1} = -2 + 2n + 2^{2-n} \equiv h(n) \).

Using (6) and (7), the solution for \( a_i, i \geq 2 \), is \( a_i^* = D_i \frac{a - c}{2^n h(n)} \), where \( D_i = \frac{2^{i+1}}{\sigma(i) - 1} \).

(ii) Notice that \( a_{i+1}^* > a_i^* \) if and only if \( D_{i+1} > D_i \) or \( 2^{i+2}/(\sigma(i + 1) - 1) > 2^{i+1}/(\sigma(i) - 1) \), which holds because \( \sigma(i+1) < \sigma(i) \).

**Proof of Corollary 1.** The equilibrium profit of the \( i \)-th mover in \( G_S \) is \( u_i^* = (a - c)^2 (1 - 2^{-i})/[2^{n-2} h(n)^2] \) whereas the profit of the same firm in a market without delegation activities is \( \bar{u}_i = 2^{-i} (a - c)^2 / 2^n \). Then, \( u_i^* > \bar{u}_i \) if and only if \( 2^{2+i} > 4 + [h(n)^2] \). Let \( r(i) = 2^{2+i} \). It is easy to show that \( r(1) < 4 + [h(n)^2] < r(n) \); further, \( r(i) \) is increasing in \( i \). Hence there exists a unique \( i'(n) < n \) such that \( u_i^* > \bar{u}_i \) if and only if \( i > i'(n) \).

**Proof of Corollary 2.** (i) Consider the last stage of \( G_C \). The quantity that the manager of firm \( i \) chooses is

\[ q_{iC}(a) = \max \{(a - n(c - a_i) + \sum_{i \neq j} (c - a_j))/(n+1), 0\} \], \( i = 1, 2, \ldots, n \)

Equilibrium delegation schemes are

\[ a_i^* = a_C^* = \frac{n - 1}{n^2 + 1} (a - c), \quad i = 1, 2, \ldots, n \]

Hence, individual and total market quantities are given respectively by

\[ q_{iC}^* = q_C^* = \frac{n(a - c)}{n^2 + 1}, \quad Q_C^* = \frac{n^2(a - c)}{n^2 + 1}, \quad i = 1, 2, \ldots, n \]

Recall that market quantity in \( G_S \) is

\[ Q_S^* = (a - c)[1 - 2^{-n} + \frac{2n - 4 + 2^{2-n}}{2^nh(n)}] \]

It is then easy to show that \( Q_S^* > Q_C^* \) if and only if \((n-1)2^{1+n} + 2 - 2n^2 > 0\) which holds.

(ii) Notice that \( a_i^* > a_C^* \) iff \( 2^{i+1} > 4 + [(n-1)2^n h(n)]/(n^2 + 1) \). Define the function \( w(i) = 2^{i+1} \) and notice that \( w(n-1) < 4 + [(n-1)2^n h(n)]/(n^2 + 1) < w(n) \). Since
$w(i)$ is strictly increasing in $i$, we conclude that $a_i^* < a_C^*$ for $i = 1, 2, \ldots, n - 1$ and $a_n^* > a_C^*$.

(iii) The equilibrium profit of the $i$-th mover in $G_S$ is $u_i^* = \frac{4(1 - 2^{-i})}{2n[h(n)]^2}(a - c)^2$ whereas the profit of each firm in $G_C$ is $u_C^* = \frac{n}{(n^2 + 1)^2}(a - c)^2$. Notice that $u_i^* > u_C^*$ if and only if $4 - 2^{2-i} > \frac{n^2(h(n))^2}{(n^2 + 1)^2}$. Let $y(n)$ denote the right part of the last inequality. For $n \geq 3$, $y(n) > 4 > 4 - 2^{2-i}$. On the other hand, if $n = 2$, $u_1^* = (a - c)^2/18 < u_C^* = 2(a - c)^2/25 < u_2^* = (a - c)^2/12$.

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\(^{10}\)In the absence of delegation, each Cournot firm earns $(a - c)^2/(n + 1)^2$. Thus the non-delegation regime is preferable by all Cournot firms over the delegation regime.
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