Critical behavior of Josephson-junction arrays at $f = 1/2$

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Abstract

The critical behavior of frustrated Josephson-junction arrays at $f = 1/2$ flux quantum per plaquette is considered. Results from Monte Carlo simulations and transfer matrix computations support the identification of the critical behavior of the square and triangular classical arrays and the one-dimensional quantum ladder with the universality class of the XY-Ising model. In the quantum ladder, the transition can happen either as a simultaneous ordering of the $Z_2$ and $U(1)$ order parameters or in two separate stages, depending on the ratio between interchain and intrachain Josephson couplings. For the classical arrays, weak random plaquette disorder acts like a random field and positional disorder as random bonds on the $Z_2$ variables. Increasing positional disorder decouples the $Z_2$ and $U(1)$ variables leading to the same critical behavior as for integer $f$. 
1 Introduction

For an ordered two-dimensional array of Josephson junctions at rational values of the flux quantum per plaquette \( f = \Phi/\Phi_0 \), where \( \Phi \) is the magnetic flux through a plaquette and \( \Phi_0 \) the flux quantum, the ground state is a pinned vortex lattice commensurate with the underlying periodic pinning potential leading to discrete symmetries in addition to the \( U(1) \) symmetry of the superconducting order parameter \( \sigma \). When quantum fluctuations due to charging effects can be ignored, yielding the so-called classical array, the critical properties can be described by a frustrated \( XY \) spin model in two dimensions with frustration parameter \( f \). Inclusion of charging effects adds an extra dimension, the time direction, but one-dimensional arrays in the form of ladders can still be described by two-dimensional classical spin models. Of particular interest is the case when \( f = 1/2 \) which has been intensively studied both experimentally and theoretically because of the interplay between continuous \( U(1) \) and discrete \( Z_2 \) order parameters. The \( Z_2 \) Ising symmetry can be associated with an antiferromagnetic arrangement of plaquette chiralities, \( \chi = \pm 1 \), measuring the directions of the circulating currents in each plaquette \( 1 \). \( 2 \). \( 3 \).

It is quite natural to expect for a Josephson-junction array at \( f = 1/2 \), that critical behavior resulting from the interplay between vortices and Ising degrees of freedom, can be described by some form of coupled \( XY \) and Ising models. Such models, containing both continuous \( U(1) \) and discrete \( Z_2 \) symmetries, can give rise to interesting and sometimes unusual critical behavior \( 4 \). Recently, there has been a lot of interest in these systems, not only in relation to Josephson-junction arrays but also in surface phase transitions where the interplay between roughening and reconstruction of the surface is important \( 5 \). \( 6 \). \( 7 \). A minimal model which is general enough and consistent with the basic symmetries can be defined by the \( XY \)-Ising model \( 1 \) \( 8 \). \( 9 \).

\[
\frac{H}{kT} = -\sum_{<ij>} [(A + B\sigma_i\sigma_j) \cos(\theta_i - \theta_j) + C\sigma_i\sigma_j]
\]

where \( \sigma = \pm 1 \) is an Ising spin and \( \theta = [0, 2\pi) \) is the phase of a two-component unit vector \( \langle XY \rangle \) spin. The Ising and \( XY \) variables are coupled by their energy densities in the same way as two independent systems of Ising spins are coupled in the Ashkin-Teller model. The model can also be regarded as the infinite coupling limit, \( h \to \infty \), of two \( XY \) models \( 1 \) \( 10 \). \( 11 \) \( 12 \) coupled by a term of the form \( -h \cos(\theta_1 - \theta_2) \). In the \( AB \)-plane, the model defined by Eq. (1) has a rich phase diagram that depends strongly on the value of \( C \). The model with \( A \neq B \) is relevant for an anisotropic frustrated \( XY \) model \( 13 \) \( 12 \) \( 14 \) which can, in principle, be physically realized as a modulated Josephson-junction array at \( f = 1/2 \), where the Josephson coupling in every other column of junctions is different by a constant factor from the others.

For the isotropic square and triangular classical arrays \( 1 \) and for the ladder quantum array \( 13 \) the relevant subspace of model (1) is the symmetric one, \( A = B \),

\[
\frac{H}{kT} = -\sum_{<ij>} [A(1 + \sigma_i\sigma_j) \cos(\theta_i - \theta_j) + C\sigma_i\sigma_j].
\]

This case has a special symmetry since \( XY \) variables are not coupled across an Ising domain wall where \( \sigma_i\sigma_j + 1 = 0 \). In the original frustrated \( XY \) model, the phases across the chiral domain walls are actually coupled in the ground state \( 1 \). This coupling and additional terms allowed by symmetry have always been assumed to be irrelevant at criticality in the \( XY \)-Ising model description. Recently, Knops et al \( 16 \) have addressed this question using transfer matrix calculations and provided further justification for the relation between the \( XY \)-Ising model and the frustrated \( XY \) model on a square lattice by showing that at the transition the phase coupling across domain walls vanishes for increasing system sizes.

The phase diagram of the \( XY \)-Ising model given by Eq. (2) as inferred by Migdal-Kadanoff renormalization \( 10 \) and Monte Carlo simulations \( 4 \). \( 9 \) consists of three branches which meet at multi-critical point \( P \) as indicated in Fig. 1. One of the branches, \( PT \), corresponds to single transitions with simultaneous loss of \( XY \) and Ising order, and the other two to separate Kosterlitz-Thouless and Ising transitions. The line of single transitions becomes first order at the tricritical point \( T \). The critical behavior of a Josephson-junction array at \( f = 1/2 \) as a function of temperature corresponds to a particular cut through this phase diagram and the single or double character of the transition depends on the relative location of this path to the branch point \( P \). In Monte Carlo simulations \( 4 \), the critical line \( PT \) in the phase diagram...
appeared to be non-universal as the critical exponents associated with the $Z_2$ order parameter were found to vary systematically along this line. More recent work based on transfer matrix calculations \[17\] found no clear evidence for variation of these exponents and suggested that the apparent non-universality may be due to slow convergence and strong corrections to scaling. However, it is clear that the values of the critical exponents estimated by finite-size scaling of large systems are inconsistent with pure Ising critical behavior, $\nu = 1, \eta = 1/4$, as can be seen from Table 1.

Table 1: Critical exponents $\nu$ and $\eta$ for the $Z_2$ order parameter and central charge $c$ of the XY-Ising model on the critical line $PT$ (Fig. 1) at $A = 1$, $C = -0.2885$. Results are from Monte Carlo simulations (MC) and Monte Carlo transfer matrix calculations (MCTM). The value of $c$ is the estimate for the largest system without extrapolation.

|       | MC     | MCTM   |
|-------|--------|--------|
| $\nu$ | Ref. 1 | 0.84   |
| $\eta$|        | 0.31   |
| $c$   | Ref. 17| 0.79   |
|       |        | 0.40   | $\sim 1.60$ |

The central charge $c$, which provides additional information on the nature of this critical line of simultaneous Ising and XY transitions was also evaluated by transfer matrix calculations on infinite strips of widths as large as 30 lattice spacings \[17\]. A value, $c = 1 + 1/2$, would be expected if the critical behavior is the result of a superposition of critical Ising ($c = 1/2$) and Gaussian ($c = 1$) models \[18\]. The numerical estimate for $c$ however is usually higher than this value, as indicated in Table 1 but effective values of $c$ obtained for increasing system sizes decrease significantly. Extrapolation assuming power-law corrections \[17\] yield results not inconsistent with $c = 3/2$. These results suggest that the critical line $PT$ is controlled by a non-trivial fixed point with new critical exponents. In this case the phase transition in the Josephson-junction array system at $f = 1/2$ would be in a new universality class, if this system indeed corresponds to a cut through the XY-Ising model that intersects the critical line of simultaneous XY and Ising transitions.

2 Classical arrays: the frustrated XY model

There have been several numerical studies of the frustrated XY model on triangular and square lattices, but so far no definitive conclusion has been reached as to the nature of the transition. These are the simplest models for a two-dimensional array at $f = 1/2$ when capacitive effects can be neglected \[1\] and the critical behavior describes the superconducting to normal transition in these systems. Different numerical techniques, including finite-size analysis, seem to lead to conflicting results which suggest either a single \[14, 21, 22, 16\] or double transitions \[23, 24, 25, 26\]. It is clear however, that if there is no single transition then there are two transitions which occur at temperatures very close to each other and in a generalized version of the model they could join depending on the effective parameter controlling the XY and Ising excitations. In fact, a Coulomb-gas representation of the frustrated XY model with an additional coupling between nearest-neighbor vortices suggests a mechanism that reconciles the apparently contradictory results: this model has a phase diagram \[27\] with similar structure as in Fig. 1 for the XY-Ising model with the occurrence of double or single transitions depending on this additional coupling. Possibly a Josephson-junction array at $f = 1/2$ corresponds to a cut through the phase diagram of the XY-Ising model close to the branch point $P$. If the critical behavior associated with the $U(1)$ and $Z_2$ symmetries are determined separately, the single or double nature of the transition is likely to remain unresolved on purely numerical grounds. Computing the critical point of a Kosterlitz-Thouless transition even in the simplest case is problematic and requires scaling forms that lead to highly unstable fits. As a consequence, the error bars of the critical point make resolving two critical points impossible in many cases.

If the frustrated XY models and the coupled XY-Ising model are in fact in the same universality class then in order to verify the single nature of the transition it is sufficient to study the $Z_2$ degrees of freedom \[1, 20\]. If the critical exponents are inconsistent with pure Ising values, the transition cannot
correspond to the Ising branch of a double transition as in Fig. 1. This point of view has the advantage of not requiring a precise study of the behavior of the $XY$ variables which for a model of this nature cannot be interpreted using what is known from the standard $XY$ model. The $Z_2$ critical exponents of the $XY$-Ising model on the critical line have been compared with those of the frustrated $XY$ model on square and triangular lattices using the same numerical methods [21]. In particular, the estimate of the thermal exponent, $\nu$, was obtained by finite-size scaling of Monte Carlo data [20] which is insensitive to the estimate of the critical temperature, $T_c$. In most of the other numerical work, the exponent $\nu$ is determined in an indirect way with several fitting parameters while this evaluation involved only a one-parameter fit to the data. The results, $\nu = 0.83$, $\eta = 0.28$ and $\nu = 0.85$, $\eta = 0.31$ for the triangular and square lattice, respectively, are significantly different from the pure Ising values and thus favor a single-transition scenario. They are also in fair agreement with the critical exponents in Table 1 for the $XY$-Ising model on the critical line. These were the first results which clearly indicated that the chiral exponents deviate from pure Ising critical behavior. Independent estimates using finite-size scaling of correlation functions [22], Monte Carlo transfer matrix calculations [21], simulations of the lattice coulomb gas [25] and exact transfer matrix calculations [16] seems to agree on the estimate of $\nu$ and the deviation of the chiral exponents from the pure Ising behavior.

The deviation from the pure Ising exponents for the square lattice has recently been questioned [26] on the basis of a possible failure of the finite-size scaling used in previous estimates. Under the assumption that the transition takes place in two stages very close in temperature, another divergent length scale should be included in the scaling analysis which could be the reason for the non-Ising exponents reported so far. The result for the temperature dependence of the $Z_2$ correlation length [26] appear to be consistent with the pure Ising exponent $\nu = 1$ but we note that this calculation was performed in a temperature range near $T_c$ which is significantly greater than the one where deviations from the Ising exponent have been found [21]. Due to the nearby Ising and $XY$ transitions as in Fig. 1 and crossover effects, a possible non-trivial fixed point on the single line can only be reached if calculations are carried out close enough to the critical line which requires both large systems and temperatures very close to $T_c$. Other calculations of the $Z_2$ correlation length but closer to $T_c$ are consistent with deviations from the pure Ising behavior [24].

Table 2 summarizes recent results obtained for the chiral order parameter of the frustrated $XY$ model.

Table 2: Recent estimates for the chiral critical exponents $\nu$, $\eta$ and central charge $c$ of the frustrated $XY$ model on a square and triangular(T) lattice. Results considered consistent with $\nu = 1$ are indicated by $\sim 1$

| Ref. | $\nu$ | $\eta$ | $c$ |
|------|-------|-------|-----|
| 19   | $\sim 1$ | 0.40  | 1.66 |
| 20   | 0.82  | 0.31  |     |
| 20   | 0.83  | 0.28  | (T) |
| 20   | $\sim 1$ | 0.26  |     |
| 22   | 0.86  | 0.22  |     |
| 21   | 0.80  | 0.38  | 1.61 |
| 25   | 0.84  | 0.26  |     |
| 16   | 0.77  | 0.28  | 1.55 |
| 26   | $\sim 1$ |     |     |

3 Effects of disorder

In real arrays, disorder is always present due to fabrication processes but it can also be deliberately introduced in order to study its effects [29]. Two kinds of disorder may be present: random plaquette areas and positional disorder [30, 31]. These types of disorder are irrelevant in zero applied magnetic field but become important and have interesting effects at higher fields. Randomness of plaquette areas
is realized when the superconducting grains have random sizes but the magnetic field is sufficiently low to permit a partial Meissner effect in individuals grain so that the fluxes can be regarded as independent random variables. This random-flux type of disorder has been argued to lead to the destruction of superconductivity and to a finite correlation length \( \xi \sim 1/H \), which depends on the field \( H \), but vortex pinning \([32, 33]\) by disorder is likely to dominate the superconducting behavior. If only weak positional disorder is present, phase coherence is possible within a range of temperature \( T^- < T < T^+ \) for sufficiently low fields or disorder \([30, 34]\). For an average value of flux quanta per plaquette, \( f_o \), the maximum field or disorder for which quasi-long-range phase coherence is possible is bounded by \( f_o \Delta \leq 1/\sqrt{32\pi} \) where the probability distribution of the grain positions is \( P(\vec{u}) \propto \exp(-\vec{u}^2/2\Delta^2) \) and \( \vec{u} \) is the displacement of a grain from its average position. The analysis also shows that the behavior of the superconducting transition at \( T^+ \) has the same features as the Kosterlitz-Thouless transition but with a non-universal jump of the helicity modulus. The disorder-induced reentrant phase transition which is predicted to occur at the lower temperature, \( T^- \), does not take into account vortex pinning by disorder. Since the dynamics is expected to be very slow, experimental or numerical study of the this transition is difficult.

Experiments on proximity-coupled arrays with deliberate positional disorder \([39]\) and numerical simulations \([35, 36]\) for integer \( f_o \) appear to be consistent with the existence of a critical field but find no evidence for reentrance. A recent renormalization group treatment \([37]\) reaches the conclusion that the transition to the normal phase is not reentrant but vanishes at the same critical disorder.

For \( f_o = n + 1/2 \), the ordered array has a \( \mathbb{Z}_2 \) symmetry in addition to the \( U(1) \) symmetry. With a Coulomb-gas representation it has been shown that random plaquette areas act like random fields on the \( \mathbb{Z}_2 \) order parameter and positional disorder acts like random bonds \([31, 30]\). Consequently, random plaquette areas should destroy chiral order and phase coherence for any amount of disorder. This result seems to agree with energy balance arguments for the chiral ground state \([38]\). Positional disorder, however, is expected to affect the chiral order less dramatically and have different effects on the \( \mathbb{Z}_2 \) and \( U(1) \) excitations. Increasing the amount of disorder should reduce the \( XY \) transition temperature to below that of the Ising if the \( \mathbb{Z}_2 \) and \( U(1) \) degrees of freedom are treated separately. Eventually, increasing disorder should lead to a double transition scenario where phase coherence is lost within the chiral ordered phase where domain walls and corner charges should have no effect. This leads to the same critical field as for integer \( f_o \). In the experiments \([29]\) on proximity-coupled arrays, the critical field \( f_c(q) \) appears to be independent of \( q \) where \( f_o = p/q \) which is consistent with the proposed scenario \([30]\) if it is realized for all rational values of \( f_o \). Monte Carlo simulations of the frustrated \( XY \) model on a square lattice finds evidence for the splitting into separated \( XY \) and Ising transitions \([28]\) as predicted but no evidence for re-entrance \([39]\).

## 4 Quantum ladder

A one-dimensional ladder of Josephson junctions, at zero temperature, undergoes a superconductor to insulator transition as a function of charging energy \( E_c \) due to capacitive effects. The transition is strongly affected by the magnetic field \([10, 11]\) through the value of \( f \). As for two-dimensional arrays, the universality class of this transition is a problem of great interest specially in relation to experiments \([12, 13, 11]\) and theoretical predictions of universal properties \([14, 15]\). At \( f = 1/2 \), the effective Hamiltonian \([12, 13, 11]\) describing fluctuations from a commensurate phase of pinned vortices is expected to be in the same universality class of the \( XY \)-Ising model in the parameter space \( A = B \). The location of the cut through the phase diagram in Fig. 8 depends on the ratio \( E_x/E_y \) between the interchain \( E_x \) and intrachain \( E_y \) Josephson couplings of the ladder. Since, the superconducting-to-insulator transition at \( T = 0 \) is to be identified with the loss of phase coherence, this transition in the decoupled region is in the Kosterlitz-Thouless universality class, but in the single transition region it is possibly in a new universality class, the same as for a two-dimensional classical array at finite temperature.

The critical behavior of the quantum ladder at \( f = 1/2 \) has been studied using a Monte Carlo transfer matrix applied to the path-integral representation of the model and a finite-size scaling analysis \([12]\). In this formulation, the one-dimensional quantum model is mapped into a two-dimensional classical model with an extra dimension, the time direction. The parameter \( \alpha = \sqrt{E_y/E_c} \) plays the role of an inverse temperature in the classical model, where \( E_c \) is charging energy. Near the critical point \( \alpha_c \), chiral order
is destroyed by quantum fluctuations with an energy gap vanishing as $|\alpha - \alpha_c|^\nu$. At the critical point, the correlation function decay as a power with an exponent $\eta$. The results for the critical coupling $\alpha_c$ and critical exponents $\nu$ and $\eta$ for two different values of the ratio $E_x/E_y$ are indicated in Table 3. For equal interchain and intrachain couplings, $E_x = E_y$, the results for the critical exponents differ significantly from the pure two-dimensional Ising model and are in fair agreement with the corresponding values at the single line of the XY-Ising model in Table 1. For $E_x/E_y = 3$, the results are in reasonable agreement with pure Ising values. In the phase diagram of the XY-Ising model, this corresponds to a path in the decoupled region where XY and Ising transitions can take place at different points. In fact, for this case, the values obtained for the critical coupling associated with chiral order $\alpha_I = 1.16$, and the one associated with the universal jump in the helicity modulus, $\alpha_{XY} = 1.29$, clearly indicate that there are two transitions. Somewhere in between $E_x/E_y = 1$ and 3 there should be a bifurcation point where a single transition with simultaneous loss of phase coherence and chiral order decouples into separate transitions, the chiral transition being located in the insulating phase.

Table 3: Chiral critical exponents $(\nu, \eta)$, for the quantum ladder [15].

| $E_x/E_y$ | $\nu$   | $\eta$   |
|-----------|---------|---------|
| 1         | 0.81(4) | 0.47(4) |
| 3         | 1.05(6) | 0.27(3) |

5 Conclusions

Numerical results from Monte Carlo simulations and transfer matrix computations support the identification of the critical behavior of Josephson-junctions arrays at $f = 1/2$ flux quantum per plaquette with the universality class of the XY-Ising model. For classical arrays without charging effects, in the form of square and triangular lattices, the results are consistent with a transition corresponding to a cut through the XY-Ising phase diagram intersecting the critical line of simultaneous XY and Ising transitions. Some conflicting results regarding the single or double nature of the transition may be explained as resulting from the proximity of the cut to the branch point in the phase diagram. For the quantum ladder, the ratio between interchain and intrachain Josephson couplings can be used to tune the transition near the branch point resulting in a simultaneous or double transition depending on this ratio. For classical arrays, weak random plaquette disorder acts like a random field and positional disorder as random bonds on the $Z_2$ order parameter. Positional disorder tend to decouple the XY and Ising variables leading to the same critical disorder as for integer $f$.

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Figure 1: Phase diagram of the XY-Ising model obtained by Monte Carlo simulations [4,9]. Solid and dotted lines indicate continuous and first-order transitions, respectively. The precise locations of $P$ and $T$ are uncertain.