String Clustering and J/ψ Suppression in Nuclear Collisions

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Abstract:

We study the clustering of colour strings in the transverse plane of nucleus-nucleus collisions and argue that deconfinement sets in when the string density within a cluster reaches a critical value. We show that this implies a minimal cluster size at the onset of deconfinement, which in turn results in discontinuous behaviour for J/ψ suppression.

Statistical quantum chromodynamics predicts that strongly interacting matter will become deconfined at high temperatures and/or densities. The aim of high energy nuclear collisions is to produce and study this new state of matter, the quark-gluon plasma, in the laboratory. One prerequisite for such studies is an understanding of the onset of deconfinement. In the most general sense, the deconfinement of quarks and gluons means that these partons are no longer constrained to the distributions by which they are governed within hadrons. Such constraints are presumably removed once enough nucleon-nucleon interactions overlap in space and time, i.e., for a sufficiently high collision density. A partitioning of the coloured partons into colour-neutral subsets is then no longer meaningful; instead, there emerge colour-conducting clusters of much larger than hadronic size.

Cluster formation is the central topic of percolation theory, and hence deconfinement and percolation appear to be closely related. The aim of the present note is to pursue this relation in more detail, in particular for the finite-size environment encountered in nucleus-nucleus interactions, and to apply the results to recent data on J/ψ suppression in Pb−Pb interactions.

Percolation theory in its conventional continuum form investigates the formation of clusters of different sizes, if N extended objects (‘discs’ or ‘spheres’ in two or three space dimensions, respectively) are randomly distributed in a given spatial region, allowing overlap. For reasons to be explained shortly, we consider here the two-dimensional case, taking the spatial region to be a circle $A = \pi R^2$ of radius $R$ and the discs as circles $a = \pi r^2$ of radii $r$ (see Fig. 1). The basic question is how the average geometric size $A_{cl}$ of a cluster of connected discs varies with the overall disc density $N/A$; it is convenient here to use the dimensionless density $n = a(N/A)$. The percolation point $n_p$ is defined to be
that density \( n \) at which in the limit \( R \to \infty \) infinite clusters appear for the first (when \( n \leq n_p \)) or last time (when \( n \geq n_p \)); in two dimensions, it is found to be \( n_p = 1.175 \), in accord with numerical results \([8]\). Approaching \( n_p \) from below, the average cluster size thus diverges,

\[
A_{cl}(n) \sim \left( 1 - \frac{n}{n_p} \right)^{-\tau_1}, \quad n < n_p;
\]

while coming from above, the ratio of average cluster size to overall area vanishes, with

\[
\frac{A_{cl}(n)}{A} \sim \left( 1 - \frac{n_p}{n} \right)^{\tau_2}, \quad n > n_p;
\]

\( \tau_1 \) and \( \tau_2 \) denote the relevant critical exponents. In all previous work on percolation in strong interaction thermodynamics \([2]-[4]\), it was assumed that deconfinement sets in at the percolation point of some specific extended objects. In general, this conjecture does not seem tenable; in the Ising model, for example, random percolation and thermal critical behaviour (spontaneous magnetization) do not coincide \([6, 9]\), and pure \( SU(N) \) gauge theory is in the same universality class as \( Z_N \) spin systems \([10]\). The reason for the difference lies in the different concept of cluster in the two cases, which tends to force thermal critical behaviour to higher densities than required for percolation. Hence a redefinition of percolation clusters is needed to make the two phenomena coincide; on the lattice, suitably defined site-bond clusters are shown to do this \([9]\). The generalization of such considerations to the percolation of extended objects is not clear, however, so that deconfinement could presumably set in at \( n_p \) or at some higher density; for further discussion, see \([11]\). Here we therefore leave the value of the deconfinement threshold \( n_c \) open.

Consider now a central high energy collision of two identical heavy nuclei, during which essentially all nucleons undergo several interactions. These occur in such rapid
succession that the nucleons cannot ‘recover’ between successive interactions. Each individual nucleon-nucleon collision establishes a colour flux tube or string between the collision partners; the additive quark model [12] suggests that at present (SPS) energies it connects two triplet colour charges. The transverse radius $r$ of such strings is expected to have the form [13, 14]

$$
r^2 = \frac{1}{\sigma} \ln \left( \frac{L}{L_0} \right),
$$

where $L$ is the separation between the colour charges and $L_0 \lesssim 0.1 - 0.3$ fm the ‘string formation’ length [15, 16]; $\sigma \simeq 0.16$ GeV$^2$ denotes the string tension. Extensive lattice QCD studies find that in the range $0.5 < L < 2$ fm, $r \simeq 0.2 - 0.3$ fm and varies at most weakly [14, 16]. In a dense environment, colour screening will moreover constrain the possible values of $L$, so that here a radius of the mentioned size appears appropriate.

For nuclear collisions in their early stage, we thus obtain a spaghetti-like structure of intertwined QCD strings, and a cut in the transverse plane results in the picture shown in Fig. 1, giving a distribution of transverse string areas over a circle of nuclear area. We want to study what happens when these overlap more and more to form connected spatial regions of increasing size in the transverse plane; hence the appropriate framework is the two-dimensional case of percolation theory introduced above.

We shall now first study how the cluster density in the transverse plane varies as function of the density of discs, then how the geometric cluster size is related to the cluster density. It is useful to define the dimensionless cluster density

$$
n_{cl} = \left( \frac{N_{cl}}{A_{cl}} \right) a \geq 1,
$$

where $N_{cl}$ denotes the average number of discs making up the cluster of size $A_{cl}$; by definition, $n_{cl} = 1$ for an isolated disc. Similarly, we define dimensionless cluster size measures

$$
s_{cl} = \frac{A_{cl}}{a} \geq 1
$$

and

$$
S_{cl} = \frac{A_{cl}}{A} \leq 1;
$$

these are normalized such that $s_{cl} = 1$ for an isolated disc and $S_{cl} = 1$ when the cluster covers the whole basis area $A$. To simplify notation, we shall from now on always mean the average over many configurations when we speak of quantities such as density, cluster density or cluster size.

We shall see shortly that the cluster density $n_{cl}$ increases smoothly with the overall density $n$. Since also the cluster size $s_{cl}$ grows with $n$, clusters of a certain density will always have a certain size. Hence at the onset of deconfinement, at some $n_{cl} = n^c_{cl}$, clusters will have an intrinsic size, $s^c_{cl} = s_{cl}(n^c_{cl})$. Deconfinement therefore begins in regions which cannot be arbitrarily small; the new phase makes its first appearance in an area of finite size. This is the main result of our note and leads to interesting consequences for the pattern of $J/\psi$ suppression by deconfinement. We now turn to cluster formation in detail.

To consider a specific case, we set $R/r = 20$, corresponding to $R = 5$ fm and $r = 0.25$ fm. The area of the discs thus is $1/400$ of the ‘nuclear’ cross section $A = \pi R^2$ over which

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1The first study of this kind was carried out for a specific phenomenological string model [4].
they are distributed; we shall return later to the role of $R/r$ and its specific value. To
determine average cluster quantities, we calculate the size of the cluster containing the
ingredient of $A$ (the shaded area in Fig. 1) in a given configuration and then average over many
configurations [3]. In Fig. 2a we see that the cluster density $n_{cl}$ increases monotonically
with the overall density $n$, starting from unity for $n \to 0$.

The crucial result of percolation theory is that the size $s_{cl}$ of the cluster, in contrast to
its density, shows a dramatic variation with $n$. In particular, for $R/r \to \infty$, $s_{cl}$ diverges
at the percolation point $n_p = 1.175$, i.e., when the overall density is somewhat above one
disc per disc area; $S_{cl}$ vanishes there. Of course there is a considerable overlap of discs,
and at $n_p$, the ratio of the area covered by discs (in clusters of all sizes) to the overall area
becomes $1 - \exp[-1.12] \approx 0.67$; it converges to unity only for $n \to \infty$. For finite $R/r$, this
critical behaviour will be modified by finite size effects, and for the value $R/r = 20$ chosen
above, we obtain the cluster size behaviour shown in Fig. 2b. As expected, it shows the
strongest variation around the percolation point; for sufficiently large systems it is here
governed by Eq. (1) for $n$ below and Eq. (2) above $n_p$.

![Figure 2: The cluster density $n_{cl}$ (a) and the cluster size $s_{cl}(S_{cl})$ (b), as functions of the overall density $n$, for $R=5$ fm, $r=0.25$ fm and a uniform random distribution.](image)
Let us now try to ‘translate’ $n$ and $n_{\text{cl}}$ into variables relevant for deconfinement in nuclear collisions. The number of strings is determined by the number of nucleon-nucleon collisions; this can be determined experimentally by measuring the rate of Drell-Yan dilepton production. Nucleon-nucleon collisions and strings play a basic role in the early stages of the nucleus-nucleus interaction; the modification of the parton distributions associated to the onset of deconfinement can occur through successive nucleon-nucleon interactions, if the time between the collisions is too short for the restoration of a physical nucleon \cite{18}. Eventually the strings fuse into ‘wounded’ nucleons which lead to the observed soft hadron production, and on this level the number of times a nucleon was hit by others no longer matters. In a Glauber-based description \cite{18}, both the average (transverse) collision density and the corresponding density of wounded nucleons can be calculated for nucleus-nucleus collisions at fixed impact parameter $b$. Through convolution with the correlation between the number of wounded nucleons at fixed $b$ and the energy $E_T$ of the measured secondary hadrons, it then becomes possible to determine all quantities of interest in terms of experimental observables. Fig. 2a thus effectively shows the collision density in the average cluster as function of the overall collision density; the latter can then be correlated to the density of wounded nucleons and thus to $E_T$.

As already noted, deconfinement is expected to set in when there is enough ‘internetting’ between interacting nucleons \cite{18}, i.e., when the collision density within a cluster becomes sufficiently high; we denote this value by $n_{\text{cl}}^c$. It is reached at a certain value $n_c = n(n_{\text{cl}}^c)$ of the overall density, which can, but does not need to be the percolation point (see Fig. 2a). By Fig. 2b, this identifies the corresponding critical cluster size $s_{\text{cl}}^c = s_{\text{cl}}(n_{\text{cl}}^c)$. In other words, requiring a critical collision density automatically forces the cluster to have a certain minimum size. This result clarifies the question of how much medium is needed before one can speak about a new phase: systems of geometric size $s_{\text{cl}} < s_{\text{cl}}^c$ will on the average not have a string density sufficient for deconfinement.

The onset of deconfinement in nuclear collisions is thus governed by two parameters: the deconfinement string density $n_{\text{cl}}^c$ (or some equivalent quantity) and the transverse string size $r$; with $n_{\text{cl}}^c$ and $r$ fixed, we can then determine the size $s_{\text{cl}}^c$ of the bubbles of deconfined medium present at the deconfinement threshold. The size of the nuclear transverse area $\pi R^2$ is given by nuclear geometry. For a realistic description, we use Wood-Saxon nuclear distributions \cite{17} and a Glauber-based formalism to calculate the distribution of collisions in the transverse plane as function of the impact parameter in $A-B$ collisions \cite{18}. The distribution of the string cross sections in the transverse plane is thus no longer completely random; it is weighted with the mentioned nucleon-nucleon collision density.

In the remainder of this note, we shall apply our considerations to the study of $J/\psi$ suppression \cite{19} in nuclear collisions as observed in $S-U$ and $Pb-Pb$ data \cite{5}. For this purpose, we will choose $r$ in the range 0.2 - 0.3 fm and fix $n_{\text{cl}}^c$ such that central $S-U$ collisions are just below the deconfinement threshold \cite{21}. We can then address two questions:

- How is the deconfinement threshold thus defined related to the percolation point?
- What is the effect of the finite cluster size on the deconfinement pattern?

As answer to the first question, in previous work \cite{4} the coincidence of deconfinement and percolation had been assumed, $n(n_{\text{cl}}^c) = n_p$. On the second question, a recent study
had already shown that a minimal bubble size for the deconfined medium leads to an abrupt onset of $J/\psi$ suppression as function of the collision centrality. The critical bubble size was there obtained through superheating in a first order phase transition. Such a picture is, however, not as easily adapted to the environment produced in nuclear collisions as the present cluster formation approach, which moreover does not contain the bubble size as an open parameter.

Our application will be based on the successive suppression scenario \[22\] - \[24\], in which the $\chi$ and the $\psi'$ are dissociated essentially at the deconfinement point, the $J/\psi$ later. This implies that the fraction (about 40 \%) of $J/\psi$’s coming from $\chi$ and $\psi'$ decay will be suppressed when $n_{cl} = n_{cl}^c$, if these states are produced within a deconfined cluster. The directly produced $J/\psi$’s then survive until $n_{cl}^{diss}(J/\psi)$; based on energy density estimates \[22, 23\], this is expected to be considerably larger than $n_{cl}^c$. Note that in our approach also the onset of direct $J/\psi$ suppression will be abrupt, governed by the cluster size at $n_{cl}^{diss}(J/\psi)$. – Since the very loosely bound $\psi'$ can in addition also be dissociated by hadronic comovers \[18\], we use below $n_{cl}^c$ the experimental suppression for the 8 \% of the $J/\psi$ from this state.

We now calculate the distribution of nucleon-nucleon collisions and the corresponding cluster density and cluster size in $S - U$ and $Pb - Pb$ collisions as function of the impact parameter $b$. In the Table, we show the results for $r = 0.20, 0.26$ and 0.30 fm. It is seen that the cluster density for central $(b = 0)$ $S - U$ collisions is reached in $Pb - Pb$ collisions for an impact parameter $8.0 \text{ fm} < b < 8.5 \text{ fm}$. This result is found to be quite independent of $r$, even though the associated cluster densities and sizes show a strong $r$-dependence.

| $b$ (fm) | $r$ (fm) | S-U | Pb-Pb |
|---------|---------|-----|-------|
| $N_{coll}$ | 0 | 0 | 4 | 8 | 8.5 | 10 |
| $A$ (fm$^2$) | 37.6 | 137.9 | 85.6 | 38.8 | 33.5 | 19.3 |
| $N_{coll}/A$ (fm$^{-2}$) | 5.32 | 6.0 | 7.59 | 6.21 | 5.97 | 5.17 |
| $N_{cl}$ | 16.7 | 139.1 | 98.5 | 19.9 | 14.8 | 6.6 |
| $A_{cl}$ (fm$^2$) | 1.47 | 10.9 | 7.81 | 1.70 | 1.29 | 0.62 |
| $n_{cl}/a$ (fm$^{-2}$) | 11.2 | 12.6 | 12.4 | 11.3 | 11.1 | 10.2 |
| $N_{cl}$ | 91.8 | 828.6 | 551.1 | 110.3 | 78.8 | 21.2 |
| $A_{cl}$ (fm$^2$) | 11.9 | 91.5 | 62.5 | 14.1 | 10.3 | 3.00 |
| $n_{cl}/a$ (fm$^{-2}$) | 7.70 | 9.05 | 8.82 | 7.81 | 7.62 | 6.95 |
| $N_{cl}$ | 149.0 | 878.4 | 605.2 | 179.9 | 136.5 | 40.2 |
| $A_{cl}$ (fm$^2$) | 23.7 | 111.0 | 80.0 | 28.2 | 22.0 | 7.16 |
| $n_{cl}/a$ (fm$^{-2}$) | 6.32 | 7.91 | 7.57 | 6.40 | 6.22 | 5.60 |

Table: Collision densities and cluster sizes for $S - U$ and $Pb - Pb$ collisions.

Tuning $r$ in the given range leads to variations in the size of the cluster at the onset of deconfinement, and this in turn determines how abruptly the $J/\psi$ production rate drops at that point. We find best agreement with the most recent data \[25\] for $r = 0.26$ fm. With a total nuclear overlap area of 35 - 40 fm$^2$ for both central $S - U$ collisions and $Pb - Pb$ collisions at $b = 8$ fm, the cluster size at the onset of deconfinement is for this $r$ value less than half the total area; hence the density profile varies little within a
given cluster. The resulting form for the anomalous $J/\psi$ suppression is shown in Fig. 3. As direct consequence of the finite cluster size at the onset of deconfinement we get an abrupt onset of the suppression as function of impact parameter $b$. As function of the measured transverse energy $E_T$, the drop is softened by the inherent $E_T - b$ smearing of the experiment, which is here included in the form described in [18]. The agreement between our model and the data is seen to be quite good.

![Graph showing J/ψ suppression](image_url)

Figure 3: The anomalous $J/\psi$ suppression given by our model, compared to data [25]; the righthand figure shows the results after division by pre-resonance absorption [18].

Finally we return to the relation of deconfinement and percolation. For an infinite two-dimensional continuum system, percolation occurs at $n = 1.175$. For $r = 0.25 \text{ fm}^2$, we obtain $n_c \simeq 1.2$, for $r = 0.26 \text{ fm}^2$, $n_c \simeq 1.3$; thus the threshold value obtained from central $S-U$ collisions is quite close to the infinite area percolation point, supporting the conjecture of percolation as basis for deconfinement [2]-[4], [11].

In conclusion: the onset of deconfinement for a sufficient overlap of strings appears to provide a conceptually clear picture for the beginning of quark-gluon plasma formation. Its main and model-independent consequence is that a critical collision density implies clusters of a critical size. The resulting abrupt onset of deconfinement is found to agree with recent data on anomalous $J/\psi$ suppression in $Pb-Pb$ collisions.

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References

[1] See e.g., D. Kharzeev and H. Satz, in Quark-Gluon Plasma 2, R. C. Hwa (Ed.), World Scientific, Singapore 1995.

[2] G. Baym, Physica 96A (1979) 131.

[3] T. Çelik, F. Karsch and H. Satz, Phys. Lett. 97B (1980) 128.

[4] N. Armesto et al., Phys. Rev. Lett. 77 (1996) 3736.

[5] L. Ramello (NA50), in Proceedings of Quark Matter '97, Tsukuba/Japan, to appear in Nucl. Phys. A.

[6] For a recent survey, see D. Stauffer and A. Aharony, Introduction to Percolation Theory, Taylor & Francis, London 1992.

[7] For a recent survey, see e.g. M. B. Isichenko, Rev. Mod. Phys. 64 (1992) 961.

[8] U. Alon, A. Drory and I. Balberg, Phys. Rev. A 42 (1990) 4634.

[9] A. Coniglio and W. Klein, J. Phys. A 13 (1980) 2775.

[10] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B 210 [FS6] (1982) 423.

[11] H. Satz, ‘Percolation and Deconfinement’, Bielefeld Preprint BI-TP 98/11, May 1998.

[12] E. M. Levin and L. L. Frankfurt, JETP Lett. 2 (1965) 65;
H. J. Lipkin and F. Scheck, Phys. Rev. Lett. 16 (1966) 71;
H. Satz, Phys. Lett. 25B (1967) 220.

[13] M. Lüscher, G. Münster and P. Weisz, Nucl. Phys. B180 (1981) 1.

[14] For a survey, see E. Laermann, in QCD - 20 Years Later, P. Zerwas and H. Kastrup (Eds.), World Scientific, Singapore 1993.

[15] O. Alvarez, Phys. Rev. D24 (1981) 440.

[16] For recent precision studies, see G. Bali, K. Schilling and C. Schlichter, Phys. Rev. 51 (1995) 5165 and Nucl. Phys. B (Proc. Supp.) 42 (1995) 273.

[17] C. W. deJager, H. deVries and C. deVries, Atomic Data and Nuclear Data Tables 14 (1974) 485.

[18] D. Kharzeev, C. Lourenço, M. Nardi and H. Satz, Z. Phys. C 74 (1997) 307.

[19] T. Matsui and H. Satz, Phys. Lett. 178B (1986) 416.

[20] D. Kharzeev, M. Nardi and H. Satz, hep-ph/9707308, July 1997.

[21] J.-P. Blaizot and J.-Y. Ollitrault, Phys. Rev. Lett. 77 (1996) 1703.

[22] F. Karsch, M. T. Mehr and H. Satz, Z. Phys. 37, (1988) 617.
[23] F. Karsch and H. Satz, Z. Phys. C 51 (1991) 209.

[24] S. Gupta and H. Satz, Phys. Lett. B283 (1992) 439.

[25] A. Romana (NA50), XXXIIIrd Rencontre de Moriond, March 1998, Les Arcs, France.