MAGNETOOPTICAL EFFECTS IN QUANTUM WELLS IRRADIATED WITH LIGHT PULSES

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The method of detection and investigation of the magnetopolaron effect in the semiconductor quantum wells (QW) in a strong magnetic field, based on pulse light irradiation and measuring the reflected and transmitted pulses, has been proposed. It has been shown that a beating amplitude on the frequencies, corresponding to the magnetopolaron energy level splitting, depends strongly on the exciting pulse width. The existence of the time points of the total reflection and total transparency has been predicted. The high orders of the perturbation theory on electron-electromagnetic field interaction have been taken into account.

Time resolved scattering (TRS) investigations of the excitons in the semiconductor bulk crystals and QWs have been discussed in the current literature. The most interesting results are due to the discrete energy levels and pulse irradiation of the physical subjects. It is well known also that a pair of the energy levels close to each other results into a new effect: The sinusoidal beatings on the frequency $\Delta E$, corresponding to the energy distance $\Delta E$ between the energy levels, appear in the reflected pulses.

In this paper we investigate theoretically TRS from a semiconductor QW in a strong magnetic field (SMF). When a SMF is directed perpendicularly to the QW surface the discrete energy levels of the electron excitations exist there. These discrete energy levels are characterized with the Landau and size-quantization quantum numbers of electrons and holes. In an ideal situation the Landau levels (LL) are equidistant. It is clear that one cannot restrict the consideration to some pair of the LLs and has to take into account either one LL or a large number of the LLs. The last variant has been used in Ref. [4] where the ladder-type structure of the reflected and transmitted pulses has been predicted for the strongly non-symmetrical exciting light pulse. The ladder-type structure is characterized with the period $2\pi/\omega$, where $\omega = |e|H/\mu c$ is the cyclotron frequency, $\mu = \mu_e \mu_h / (\mu_e + \mu_h), m_e (m_h)$ is the electron (hole) effective mass, $H$ is the magnetic field intensity. In the case of arbitrary exciting pulses the beating structure with the frequency $\omega$ depends on the pulse form, but generally speaking is not a sinusoidal one. In the case of the symmetrical exciting pulse, the duration of which is much longer than the period $2\pi/\omega$, the beatings on the frequency $\omega$ have a very small amplitude. Then one can restrict the consideration by the only LL, with which the exciting pulse is in the resonance.

In Ref. [4] the shape of the reflected and transmitted pulses is determined in the vicinity of the resonance of the exciting pulse carrier frequency $\omega$ with the only energy level in the QW. Maybe this level is either excitonic level at $H = 0$, or a discrete energy level in the SMF. It has been shown that under condition

$$\gamma_r \geq \gamma, \quad (1)$$

where $\gamma_r (\gamma)$ is the inverse radiative (non-radiative) lifetime of the electronic excitation, the strong change of the transmitted and reflected pulses comparatively with the shape of the exciting pulse has to happen.

The above-mentioned regards to the case when the interaction of the electronic excitation with other excitations in the QW (in particular with phonons) does not essentially influence the energy spectrum of the electronic excitations. The role of the electron (hole) - LO phonon interaction grows sharply at the resonant values of $H$ when the magnetopolaron effect appears [5], i.e. electron (hole) energy level splitting happens.

The polaron state formation takes place in both 3D and 2D semiconductor systems. In the 2D case the splitting value is about $\alpha^{1/2} h \omega_{LO}$ [6], where $\alpha$ is the Fröhlich electron-LO phonon coupling constant. The usual, combined and weak magnetopolars exist there. The resonant magnetic field values for the usual magnetopolars are determined as

$$\omega_{LO} = j \Omega_{e(h)}, \quad j = 1, 2, 3, \ldots \quad (2)$$

where $\omega_{LO}$ is the LO phonon frequency, $\Omega_{e(h)} = |e| H/m_{e(h)c}$ is the electron (hole) cyclotron frequency. The fractional $j$ correspond to the weak polarons.

The values $\Delta E$ of the polaron splittings for the weak polarons are smaller than those for the usual polarons: They are of higher order on $\alpha$ than $\alpha^{1/2}$. The values $\gamma_{ra}$ and $\gamma_{rb}$ for both upper ($a$) and lower ($b$) energy levels












have been obtained in Ref. [3] as well as the estimations of $\gamma_a$ and $\gamma_b$.

Let us show that the values $\Delta E$ of the polaron splittings can be measured in the experiments with the pulse irradiation of a QW in a SMF. We suppose that a light pulse incidents on the single QW from the left perpendicular to the QW surface. The pulse electric field intensity is

$$E_0(z, t) = E_0 e^{-i\omega (t-z n/c)} \{\Theta(t - z n/c)e^{-i\gamma(t-z n/c)/2} + [1 - \Theta(t - z n/c)]e^{i\gamma(t-z n/c)/2}\} + c.c., \quad (3)$$

where $E_0$ is the real amplitude, $e_l$ is the polarization vector, $\omega_l$ is the pulse carrier frequency, $n$ is the refraction index out the QW, $\gamma$ determines the pulse duration, $\Theta(x)$ is the Haeveriside step function. After the Fourier transformation we obtain

$$E_0(z, t) = E_0 e_l \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z n/c)} D_0(\omega) + c.c., \quad (4)$$

$$D_0(\omega) = \frac{i}{2\pi} [(\omega - \omega_l + i\gamma_l/2)^{-1} - (\omega - \omega_l - i\gamma_l/2)^{-1}]. \quad (5)$$

When $\gamma_l \to 0$ the pulse (3) transits into a monochromatic wave. Let us suppose that the incident wave has a circular polarization. We imply that both waves of the circular polarization correspond to the excitations of two types of the EHP, the energies of which are equal. Let us consider a QW, the width of which is much smaller than the light wave length value $c/\omega n$. Then the electric fields on the left (on the right) of the QW are determined by the expressions [3]

$$E_{left(right)}(z, t) = E_0(z, t) + \Delta E_{left(right)}, \quad (6)$$

$$\Delta E_{left(right)}(z, t) = E_0 e_l \times \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z n/c)} D(\omega) + c.c., \quad (7)$$

where the upper (lower) sign corresponds to the index $<left>$ ($<right>$),

$$D(\omega) = -4\pi \chi(\omega) D_0(\omega)/(1 + 4\pi \chi(\omega)), \quad (8)$$

$$\chi(\omega) = (i/4\pi) \sum_{\rho} (\gamma_{rp}/2) [(\omega - \omega_{\rho} + i\gamma_{\rho}/2)^{-1} + (\omega + \omega_{\rho} + i\gamma_{\rho}/2)^{-1}], \quad (9)$$

where $\rho$ is the index of the excited state, $\hbar \omega_{\rho}$ is the excitation energy measured from the ground state energy. Applying Eq. (8) means that the theory is constructed with taking into account the highest orders of the perturbation theory on the electron-EM field interaction [3]. The second term in the square brackets of Eq. (9) is non-resonant one and omitted below.

We consider below the case of two excited energy levels when the index $\rho$ takes two values: $\rho = 1$ and $\rho = 2$. These levels are for instance the lower and upper magnetopolaron energy levels. With the help of Eqs. (6)-(9) we obtain

$$\Delta E_{left}(z, t) = -i E_0 e_l \times \left\{ \Theta(s) \left[ e^{-i\omega_l s + i\gamma_{\rho}/2\left(1 + (\gamma_{\rho} + G_1)/2\right)^{-1}} \left( \frac{\gamma_{11}/2}{\omega_l - \Omega_1 + i(G_1 - \gamma_l)/2} + \frac{\gamma_{12}/2}{\omega_l - \Omega_2 + i(G_2 - \gamma_l)/2} \right) \right] \right\}, \quad (10)$$

where $s = t - z n/c$. The expression for $\Delta E_{right}(z, t)$ differs from Eq. (10) by the substitution $p = t - z n/c$ instead of $s$. The designations are used

$$(\Omega - iG/2)_{1(2)} = 1/2 \left[ \omega_1 + \omega_2 \pm \sqrt{(\omega_1 - \omega_2)^2 - \gamma_{11}\gamma_{22}} \right];$$

$$\tilde{\omega}_1(2) = \omega_1(2) - i\Gamma_{1(2)}/2; \quad \Gamma_{1(2)} = \gamma_{11(2)} + \gamma_{12(2)};$$

$$\tilde{\gamma}_{1(2)} = \gamma_{11(2)} + \Delta \gamma, \quad \tilde{\gamma}_{21(2)} = \gamma_{21(2)} - \Delta \gamma;$$

$$\Delta \gamma = \gamma_{11}[\Omega_2 - \omega_2 - i(G_2 - \gamma_2)/2] + \gamma_{21}[\Omega_1 - \omega_1 - i(G_1 - \gamma_1)/2] \times [\Omega_1 - \Omega_2 + i(G_2 - G_1)/2]^{-1}. \quad (11)$$

The upper (lower) sign in Eq. (11) corresponds to 1(2). The values $\Omega_{1(2)}$ and $G_{1(2)}$ are real by their definition. At $\gamma_l = 0$ the expression Eq. (10) transits into the formula for the induced field at a monochromatic irradiation [3]. At $\gamma_{12} = 0$ we obtain from Eq. (10)

$$\Delta E_{left}(z, t) = -i E_0 e_l (\gamma_{11}/2) \times \left\{ \Theta(s) \left[ e^{-i\omega_l s + i\gamma_{\rho}/2\left(1 + (\gamma_{\rho} + G_1)/2\right)^{-1}} \left( \frac{\gamma_{11}/2}{\omega_l - \Omega_1 + i(G_1 - \gamma_l)/2} + \frac{\gamma_{12}/2}{\omega_l - \Omega_2 + i(G_2 - \gamma_l)/2} \right) \right] \right\}.$$
which corresponds to the case of the single excited energy level. Comparing Eq. (10) and Eq. (12) one finds that the fields from two levels not only add, but the renormalization of the frequencies \(\omega_1(2), \Delta \omega \) and the factors \(\gamma_{r1}(2)\) has happened. They are substituted by \(\Omega_1(2), G_1(2)\) and \(\gamma_{r1}(2)\), respectively. In the case of two merging levels, i.e. under condition \(\gamma_{r1} = \gamma_{r2} = \gamma_2, \omega_1 = \omega_2\), we obtain from Eq. (10) an expression of type of Eq. (12) in which the value \(\gamma_{r1}\) has to be substituted by \(2\gamma_{r1}\). It means that in the case of the twice degenerated level, the twofold value of the inverse radiative lifetime figures in all the formulae.

In Figs. 1-5 the modulus \(\mathcal{P}\) and \(\mathcal{T}\) of the exciting and transiting pulses as the functions of \(p = t - zn/c\), the module \(\mathcal{R}\) of the reflected pulse flux as function of \(s = t + zn/c\) and the absorption which, is defined at \(z = 0\) as

\[
\mathcal{A} = \mathcal{P} - \mathcal{T} - \mathcal{R},
\]

are represented. The modulus of the energy fluxes are represented in units \((e^2/2\pi n)E_0^2\).

It follows from Eq. (10) that the results for the energy fluxes are dependent on the parameters \(\omega_l - \omega_1\) and \(\gamma_l\), characterizing the exciting pulse, and the parameters \(\Delta \omega = \omega_1 - \omega_2, \gamma_{r1}, \gamma_{r2}, \gamma_1, \gamma_2\), characterizing the system with two excited levels. It follows from Eq. (10) that there are resonances on the frequencies \(\omega_l = \Omega_1, \omega_l = \Omega_2\) and beatings on the three frequencies \(\omega_l - \Omega_1, \omega_l - \Omega_2, \Omega_1 - \Omega_2\) exist.

In the case

\[
\omega_l = \Omega_1, \quad \gamma_{r1} = \gamma_{r2} = \gamma_{r}, \quad \gamma_1 = \gamma_2 = \gamma
\]

the results for the fluxes are represented in Figs. 1-5. Thus, only parameters \(\gamma_l, \Delta \omega, \gamma_{r}, \gamma\) are variable ones. Obviously, under conditions Eq. (14) the beatings are possible only at the frequency \(\Delta \Omega = \Omega_1 - \Omega_2\), if the value \(\Delta \Omega \neq 0\), which does not always fulfilled as we will see below. Figs. 1, 2 correspond to the parameters

\[
\Delta \omega = 0.00665, \quad \gamma_r = 0.00005, \quad \gamma = 0.0005, \quad \gamma_l = 0.0005 \text{ (Fig.1)}, \quad \gamma_l = 0.005 \text{ (Fig.2)}.
\]

We use the arbitrary units because the results for the fluxes depend on the parameter interrelations only. If the eVs are used then \(\Delta \omega\) from Eq. (15) correspond \[8,9\] to the usual polaron at \(j = 1\) (see Eq. (2)) at the QW width \(d = 300\AA\) for GaAs. The relation \(\gamma/\gamma_r = 10\) is chosen arbitrarily. \(\Omega_1 \simeq \omega_1\) and \(\Omega_2 \simeq \omega_2\) when the parameters of Eq. (15) are used.

Fig. 1 and Fig. 2 differ from each other sharply. The beatings are almost invisible in Fig. 1. It means that these results correspond to the induced electric field Eq. (12) under condition \(\omega_l = \omega_1\). In Fig. 2 the beatings at the frequency \(\Delta \omega = \omega_1 - \omega_2\) are seen brightly in the reflected pulse. Reflection and absorption are much smaller than unity in Fig. 1, 2, and reflection is much smaller than absorption. These features characterize the case \(\gamma_r < < \gamma\). As for beatings, their appearance depends sharply on the pulse duration \(\gamma_l^{-1}\). Beatings are clearly visible for the short pulses when \(\gamma_l \approx \Delta \omega\). This is clear physically: The frequencies, which are close to \(\omega_1\) and to \(\omega_2\), are essential in the frequency spectrum Eq. (5) of the short pulse.

The former parameters \(\Delta \omega = 0.00665, \gamma_r = 0.00005\), but \(\gamma = 0\) correspond to Fig. 3 and Fig. 4. The parameter \(\gamma_l\) in Fig. 3 and Fig. 4 coincide with \(\gamma_l\) in Fig. 1 and Fig. 2, respectively. The beatings in the reflected and absorbed pulses are expressed only in Fig. 4 when \(\gamma_l \approx \Delta \omega\). In Figs. 3, 4 reflection and absorption are prolonged more than in the Figs. 1, 2, because in the case \(\gamma = 0\) the prolongation is determined by the value \(\gamma_{r1}^{-1}\).

In Figs. 2, 3, 4 there is the time point \(t_0\), where absorption equals to zero and changes its sign. At \(s_0 = t_0\) the reflected flux is equal to the exciting flux and the transmitted flux equals to zero at \(p_0 = t_0\). This is the time point in which the field from the right of the QW equals zero (the total reflection point). An analysis shows that in the case of a single excited level at \(\omega_l = \omega_1\) incidentally the special point exists only at \(t_0 > 0\), which corresponds to the condition \(\gamma_l > \gamma\),

\[
t_0 = \frac{2}{\Gamma_1 - \gamma_l} \ln \frac{2\gamma_{r1}(1)}{\gamma_l}\Gamma_1 \gamma_l). (16)
\]

In Figs. 1, 3 the curves are presented at which the level 2 almost does not practically influence. The condition \(\gamma_l > \gamma\) is satisfied for Fig. 3, but not for Fig. 1. Therefore the point of total reflection is presented in Fig. 3 and it is absent in Fig. 1. In Ref. 4 it has been shown that the total reflection points exist also for other forms of pulses including non-symmetrical ones.

The relations

\[
\mathcal{P} \approx 0, \quad \mathcal{T} \approx \mathcal{R}, \quad \mathcal{A} \approx -2\mathcal{T}
\]

are fulfilled for the times very much larger than \(\gamma_l^{-1}\) in Figs. 2, 3, 4. The sense of them is clear: If \(\gamma_l >> \gamma + \gamma_r\) then the fields created by the exciting pulse are very small at quite large values \(t = zn/c\), and the fluxes of \(\mathcal{T}\) and \(\mathcal{R}\) are determined only by the induced fields \(\Delta E_{left(right)}\). The picture becomes a symmetrical one relative to the plane \(z = 0\) where the QW is placed, the fluxes from the left and right of the plane are equal in absolute values. The absorbed flux is negative and equals to the sum of fluxes going to the left and to the right. That means that the QW gives back the accumulated energy.

In Fig. 5 the parameters
\[ \Delta \omega = 0.00665, \]
\[ \gamma_r = 0.00666, \quad \gamma = 0.0001, \quad \gamma_l = 0.001 \] (17)

are used, i.e. the special case \( \Delta \omega \approx \gamma_r \) is represented. The beating absence is due to \( \Delta \Omega = 0 \), but not to the small influence of one of the levels. Indeed, as it follows from Eqs. (11), (14), \( \Omega_1 = \Omega_2 = (\omega_1 + \omega_2)/2 \) at \( |\Delta \omega| \leq \gamma_r \). Thus, if \( \omega_l = \Omega_1 \), the pulse carrier frequency is situated exactly between the two levels in the case of Fig. 5.

The beatings are absent in Fig. 5, but the figure structure differs radically from the structure for the case of two energy levels. In Fig. 5 there is the time point \( t_x \) of the total transparency in which reflection and absorption are equal to 0 and the transmitted flux is equal to the exciting flux. An analysis shows that under conditions Eq. (14) and \( |\Delta \omega| = \gamma_r \),

\[ t_x = \frac{2}{\gamma_l - \Gamma} \ln \frac{(\gamma_l - \gamma)(\Gamma + \gamma_l)^2}{2\gamma(\gamma_l^2 + \gamma_l^2 - \gamma^2)}, \] (18)

and the total transparency point exists in the case \( \gamma_l < \Gamma \), which is fulfilled for Fig. 5. The electric field to the left of the QW equals zero in the time point \( t_x \).

In all the figures, besides Fig. 1, it is seen that the absorbed flux equals zero in the total reflection or total transparency point and changes its sign, i.e. becomes negative one. That means that the electronic system at first accumulates the energy of the created electron-hole pairs and then gives it back. In the case \( \gamma_1 = \gamma_2 = 0 \) all the accumulated energy comes back as radiation, i.e. the integral absorption equals zero. This result is true for the cases of Figs. 3, 4.

In Figs. 1-5 there are only a few examples of the shapes of the reflected, absorbed and transmitted pulses, corresponding to some combinations from the seven parameters. It is worth stressing that the experimental results on the pulse irradiation of the QWs in a magnetic field allow us in principle to detect and investigate the magnetopolaron states. Measuring of time lags of the reflected and transmitted pulses allows us to determine the lifetimes of the polaron states. The special subject of interest is the investigation of the points of the total reflection and total transition, the first appear in the case of short pulses. For the usual polarons the case of Figs. 1, 2 is the most real one. With the help of the pulse irradiation of the QWs one can investigate not only usual, but weak polarons also. The smaller \( \Delta \omega \) for the weak polarons are even preferable because, first, the beating frequency diminishes and, second, one can use more longer pulses, than in the case of usual polarons.

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[1] H. Stolz, Time - Resolved Light Scattering from Excitons, Springer Tracts in Modern Physics, Springer, Berlin, 1994.

[2] J. Shah, Ultrafast Spectroscopy of Semiconductors and Semiconductor Nanostructures, Springer, Berlin, 1996.

[3] I. G. Lang, V. I. Belitsky. Physics Letters A, 245 (1998), 329 - 333.

[4] I. G. Lang, V. I. Belitsky. Solid State Commun. 107, 577 - 582 (1998).

[5] E. J. Johnson, D. M. Larsen. Phys. Rev. Lett. , 16, 655 (1966).

[6] L. I. Korovin, S. T. Pavlov, B. E. Eshpulatov. Sov. Phys. Solid State, 20, 2077 (1978).

[7] H. Fröhlich, Adv. Phys. 3, 325 (1954).

[8] L. I. Korovin, I. G. Lang, S. T. Pavlov, JETP, 88, 105 (1999) (Zh. Eksp. Teor. Fiz., 115, 187 (1999)).

[9] I. G. Lang, L. I. Korovin, A. Contreras Solorio and S. T. Pavlov. Lanl archive preprint, cond-mat/0001248.

[10] L. C. Andreani, F. Tassone, F. Bassani. Solid. State Commun. , 77, 641 (1991).

[11] L. C. Andreani, G. Panzarini, A. V. Kavokin, M. R. Vladimirova. Phys. Rev. B 57, 4670 (1998).

FIG. 1. The modulus of the exciting, transmitted, reflected and absorbed energy fluxes as time functions: \( \omega_l = \Omega_1, \Delta \omega = 0.00665, \gamma_r = 0.00005, \gamma = 0.0005, \gamma_l = 0.0005 \). All the parameters are given in eV.

FIG. 2. Same as Fig 1 for \( \gamma_l = 0.005 \).

FIG. 3. Same as Fig. 1 for \( \gamma = 0, \gamma_l = 0.0005 \).

FIG. 4. Same as Fig 1 for \( \gamma = 0, \gamma_l = 0.005 \).

FIG. 5. Same as Fig. 1 for \( \gamma_r = 0.00666, \gamma = 0.0001, \gamma_l = 0.001 \).