Field-Dependent Surface Resistance for Superconducting Niobium Accelerating Cavities – Condensed Overview of Weak Superconducting Defect Model

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Abstract—Small (compared to coherence length) weak superconducting defects when located at the surface, combined with the proximity and percolation effects, are claimed responsible for various observations with superconducting radio frequency (RF) accelerating cavities with nonconstant Q-value, such as “Q-slope” and “Q-drop,” the role of temperature, the result of “nitrogen doping” and its relation to the free path, and the influence of the static external magnetic field. The Ginzburg–Landau equations are used to confirm the results.

Index Terms—Cavity resonators, niobium, particle accelerators, superconducting thin films, surface impedance.

I. INTRODUCTION

This publication is motivated by former studies on the superconducting (SC) radio frequency (RF) system for the large electron-positron collider (LEP) and large hadron collider (LHC) storage rings at CERN. There was a long discussion about the possible choice of technologies, either to build the SC RF cavities from niobium sheet or from copper with a thin niobium coating (Nb/Cu) on top. For LEP, RF cavities made from both technologies were installed, although the bulk of RF cavities was made from Nb/Cu, whereas for the LHC Nb/Cu cavities were used. This decision in favor of the Nb/Cu technology was justified by the lower performance requirements for LEP and LHC in terms of maximum gradient. The Nb/Cu cavities showed a stronger increase of the RF losses with the accelerating gradient than the niobium sheet cavities. Therefore, the latter ones found their application in fairly all recent linear accelerators, such as energy recovery and recirculating machines, etc., as well as in studies on future SC linear colliders (ILC). The reason for this is that, for cost reasons, the overall length of a linear accelerator must be kept as short as possible, since particles pass through the device only once or a few times, whereas in storage rings the accelerating gap is continuously traversed.

However, Nb/Cu technology showed several advantages over niobium sheet technology: lower sensitivity to the ambient magnetic field, better tolerance to lossey defects due to the greater thermal stability of the thin niobium layer supported by copper with high thermal conductivity, and last but not least, lower manufacturing costs. Consequently, there was a strong motivation to understand why the RF losses of the Nb/Cu cavities increased faster with the acceleration gradient than those of the niobium sheet cavities. After a long period of research, this drawback was mitigated but not completely eliminated [1], [2].

This article is an attempt to better understand the problem of increased losses in niobium sheet cavities with emphasis to Nb/Cu cavities in particular, but also in niobium sheet cavities. In the published literature several explanations have been published, [3] being the most recent [4]. These are, for example, suppression of the energy gap or the modification of the density of states with increasing RF field. These explanations are not contested but assumed to be negligibly small compared with the model described in this article.

To expand on this argument, reference should be made to a measurement of an Nb accelerating cavity where a very high Q-value (7·1010 at 1.6 K) was relatively constant up to a maximum field strength of about 190 mT [5]. The Q-value at low field and 1.3 K was 1·1011. The authors assume that this cavity had an almost defect-free surface as a result of a number of fortunate circumstances during preparation. Conversely, a field-dependent Q-value as being caused by defects is presented in this article complementing the intrinsic factors as mentioned before.

The following arguments and conclusions are not new in themselves, but have been published in various places (except Section VII). Some of them have more the character of an assertion of a speculative nature, although substantiated. On the other hand, these claims gain weight when identical reasoning from other observations summarized in this article is considered.

The results can be traced back to a common explanation, namely small (compared with the coherence length) weak SC defects when located at the current carrying surface of the SC cavity, in conjunction with the proximity and percolation effects [6], [7].

In this work, the arguments and observations concerning this explanation are summarized one by one. Only the most important results published so far are given here; for a deeper
insight into the physics, the original papers should be consulted. Some associated figures illustrate the arguments.

Table IV (Appendix) lists additional information on the symbols used in this article.

II. SURFACE ENERGY BALANCE

There is a gain in diamagnetic (surface) energy at the expense of the loss of condensation energy, when a sufficiently small weak SC defect on the surface transforms into a normal conducting (NC) defect under the action of the RF magnetic field B. This effect is illustrated by the commonly plotted quality factor Q as a function of magnetic field B, as shown in Fig. 1, based on a closed-form equation, (1.c).

The data are obtained for a fine grain niobium monocal cavity at 1300 MHz and 2 K [8]. The continuous lines represent the results from fitting the data with the Ginzburg–Landau parameter \( \kappa = 1.5, 1.3, \) and 0.9 (from bottom to top) [6].

The plot shows the commonly observed dependence of the Q-value on the magnetic field B, usually referred to as “Q-slope” and “Q-drop.” From the Q-value the surface resistance \( R_s \) can be extracted by \( R_s = G/Q \), where G is a constant that depends on the RF field pattern alone. \( R_s \), in turn, is considered to be the sum of the residual surface resistance \( R_{res} \), which is RF-field and temperature independent, and all other contributions, such as a temperature-dependent term often referred to as the “BCS surface resistance” \( R_{BCS} \), and a field-dependent term \( R_{s,f} \)

\[
R = R_{res} + R_{BCS} (\omega, T) + R_{s,f} (B, \omega, T)
\]

with

\[
R_{BCS} (\omega, T) = \frac{1}{2} \cdot \mu_0^2 \omega^2 \kappa^2 |s_{Nb}| \cdot \ln \left( \frac{\Delta}{\hbar \omega} \right) e^{-\Delta/T} \]

(1.a)

\[
R_{s,f} (B, \omega, T) = R_{s,f,b} (B) \cdot R_{s,f,dt} (\omega, T)
\]

(1.b)

\[
R_{s,f,b} (B) = (-\kappa^{-2}) \cdot \left\{ 1 + \frac{\ln \left( 1 - \kappa^2 \left( \frac{B}{B_c} \right)^2 \right)}{\kappa^2 \left( \frac{B}{B_c} \right)^2} \right\}
\]

(1.c)

\( R_{s,f,dt} \) turns out to be quite close to \( R_{BCS} \) [6]. The abrupt drop of the Q-value at increased magnetic field (e.g., \( \sim 0.125 \) T for \( \kappa = 1.5 \)) is also explained, when \( \kappa^2 (B/B_c)^2 \) approaches 1. A more detailed derivation of (1.c) can be found in the Appendix.

A distinction must be made between the situation when the defect is located at the surface and the situation when it is located in the volume but still within a distance given by the penetration depth and thus exposed to the RF current. When the defect is embedded in the volume, the current flows around it on both sides when it becomes NC. In other words, a loop-like microscopic magnetic field is created with the net result that the magnetic induction in the superconductor does not change: the diamagnetic energy remains unchanged, the energy balance does not become negative, and therefore there is no gain in the energy balance. This is the reason why the NC volume under the influence of the magnetic field B can grow only on the surface and not inside the superconductor. Thus, a precondition for growth is the presence of a weak SC defect at the surface.

III. ROLE OF TEMPERATURE

The outermost surface of the RF cavity is assumed to be an inhomogeneous mixture consisting of Nb and NbO, representative of other impurities. Then there are areas with an intimate neighborhood of Nb and NbO (called composite) and other areas with less or no content of NbO (called Nb matrix).

First, in the composites plus matrix, the proximity effect between the strong superconductor (S, Nb) and the weak superconductor (N, NbO) will act. Let the volume fraction of S be \( v_s = v_{s}/(v_s + v_N) \). The weak superconductor (NbO) by itself has a critical temperature \( T_c = 1.34 \) K, but the neighborhood of the strong superconductor increases its critical temperature \( T_{CNS} \) according to \( x \), as prescribed by the Cooper limit approximation, which is used here.

Second, at a sufficiently high temperature, some regions of matrix plus composites become normal conducting, while other regions remain superconducting. Then the question arises to what extent the composites together with the matrix fragment into normal conducting and superconducting sites. In other words, the question is as to what proportion of \( x \), as a function of temperature, sufficiently small normal conducting regions will emerge that will serve as expanding defects as described in the previous section. This is a problem of percolation.

According to the Cooper limit approximation, the relation of the critical temperature \( T_{CNS} \) of the composite versus the volume ratio \( x \) starts at the critical temperature \( T_c = 1.34 \) K for the NbO solely, when the concentration of the S component is zero. The temperature difference \( T_{CNS} - T_{c,NbO} \) follows a quasilinear relation with the concentration \( x \) [6], Fig. 2.

From experiment [6], the temperature-dependent part \( R_{s,f,dt} \) of the field-dependent surface resistance increases abruptly at \( T^* = 2 \) K (Fig. 3), which indicates percolation behavior.

Indeed, from Fig. 2, the corresponding volume ratio \( x^* = 0.03 \) is known as a “void percolation threshold” for “continuum percolation” for a distribution of overlapping spheres (N) with equal radius and voids (S) in between [9].

There, \( x^* \) is interpreted as a percolation threshold, and \( T^* \) as the corresponding “percolation temperature.” They depend on
IV. NITROGEN DOPING

Doping with nitrogen may improve (or sometimes reduce) the Q-value with respect to the field \(10\). A binary uniform metal mixture of “dirty” niobium enriched with dissolved nitrogen, as a “weak” superconductor, is subject to the proximity effect produced by the neighboring high-quality niobium metal as a “strong” superconductor. Because of this proximity effect, the weak superconductor has a lower critical field \(B^c\) and lower critical temperature than the strong superconductor. For a very small RF field \(B\), the Q-value remains constant until \(B^c\) (about 10–20 mT), at which its outermost surface candidates become \(nc\). As \(B\) continues to increase, the NC zone (volume fraction \(f_N\)) penetrates deeper into the surface until the field \(B^c\) (saturation field about 80–90 mT) from where on the weak superconductors are fully NC and the Q-value remains constant. The weak SC zones with electrical conductivity \(s_{\text{Nb}}\) are gradually replaced by those with averaged electrical conductivity \(s_m\). Since the RF field \(B\) at the surface decreases exponentially within the depth \(x\) of the superconductor, \(B(x) = B \cdot e^{-x/\lambda}\), the RF field at \(x\), \(B(x)\), is in turn determined by the logarithm of \(B\), \(x = \lambda \cdot \ln(B/B(x))\), \(\lambda\) being the penetration depth. Thus, the defect layer ranges from \(x = 0\) to \(d_N = \lambda \cdot \ln(B^c/B^c)\). The function \(f(B)\) describes the fraction of the defect layer as a function of the RF field with the range of values between 0 and 1 \([11]\)

\[
f_v(B) = \frac{\ln(B/B^c)}{\ln(B^c/B^c)}.
\]

Hence, the surface resistance decreases (or increases), depending on \(c = s_m/s_{\text{Nb}}\), according to the following formula:

\[
R_s = R_{BCS} \cdot [1 - f_v(B) + c \cdot f_v(B)].
\]

The full amplitude of the magnetic field-dependent part determines the constant \(c\) (all symbols are explained in Table IV).

It is interesting to note that the characteristic increase or decrease of the Q-value at low fields is also observed in undoped cavities, indicating a rather general phenomenon.

To analyze the data, one turns to a model by Landauer \([12]\), who studied current flow in binary mixtures of media of different electrical conductivities. In this effective medium approximation model, the global electrical conductivity \(s_m\) is

\[
s_m = (3x_1 - 1) s_1 + (3x_2 - 1) s_2 + \sqrt{[(3x_1 - 1) s_1 + (3x_2 - 1) s_2]^2 + 8s_1 s_2}
\]

where \(x_1\) stands for the fraction of the total volume occupied by material 1 (weak superconductor), \(x_2\) stands for the fraction of the total volume occupied by material 2 (strong superconductor),...
and $s_1$ and $s_2$ are the respective electrical conductivities, and $s_m$ is the electrical conductivity of the composite.

In this case, the weak superconductor has a typical electrical conductivity of dirty niobium $s_1$, from about $10^6$ to $10^7$ $(\Omega m)^{-1}$. On the other hand, the strong superconductor is pure niobium, of which the electrical conductivity is set to be purely imaginary, $s_2 = i/(\mu_0 \cdot \lambda^2 \cdot \omega)$. Inserting $s_1$ and $s_2$ in (4) results in a curve for $s_m$, the real part of which culminates in a maximum at $x_1 = 2/3$ (Fig. 5). This indicates a percolation path inside the composite. The similarity of this maximum to a (frequency-dependent) bandpass curve of a RF circuit is not accidental; because at the point $x_1 = 2/3$, the real and imaginary parts of $s_m$ are identical, just as in a resonance. This property reflects the assumption of the Landauer model that charges are deposited at defects in a homogeneous medium, much like an LC circuit.

The often-observed Q-increase (sometimes Q-decrease) at very small fields in niobium bulk cavities is supposed to have the same origin caused by the contamination surface layer which is thinner than that of the nitrogen-doped one. This assertion might replace the explanation for the $Q$ increase at low field (due to latent heat) given in [6].

Data as obtained at 2 K and different frequencies [13] could be reproduced making use of (1) and (3) in [11] (Fig. 6, continuous lines). Note that these curves are reproduced by only two fit parameters, provided that $x_1 = 2/3$ and the field strength dependence starts at $E_{acc} = 5$ MV/m as in Fig. 6 (this corresponds to $B^* = 20$ mT).

V. AMBIENT MAGNETIC FIELD

The Q-value depends on the ambient static magnetic field $B_{ext}$, being trapped during cooldown into nc fluxons, as observed in SC cavities of all kinds, although mitigated by fast cooling methods [14], [15]. In the following, data on Nb/Cu cavities are analyzed [16]. According to the experimental data of [16], the affected surface resistance $R_{fl}$ can be parameterized as follows:

$$R_{fl} = (R_{fl}^0 + R_{fl}^1 \cdot B) \cdot B_{ext}. \quad (5)$$

The first contribution can be described as follows [17]:

$$R_{fl}^0 = c_{eff} \cdot (\omega \mu_0)^{3/2} \cdot (2s_{Nb})^{1/2} \cdot \lambda^2 \cdot \frac{1}{B_{c2}}. \quad (6)$$

and is shown in Fig. 7.

The correction factor $c_{eff}$ in (6) (62.5%) takes into account the ratio of the effective magnetic flux component perpendicular to the cavity surface with regard to the overall magnetic flux across the cavity silhouette.

The second contribution is

$$R_{fl}^1 = \frac{1}{s_{Nb} \cdot \lambda \cdot B \cdot B_{c2}} \quad (7)$$

for which a distinction as to the mean free path $l$ has to be made.

For $l \gg \lambda$, the replacements typical of the anomalous skin effect, $s_{Nb} \rightarrow s_{eff}$ and $\lambda \rightarrow \delta_{eff}$

$$s_{eff} = \left(\frac{2}{\mu_0 \omega}\right)^{1/3} \left(\frac{\alpha s_{Nb}}{l}\right)^{2/3}$$

$$\delta_{eff} = \left(\frac{2l}{\alpha \mu_0 s_{Nb} \omega}\right)^{1/3}$$
Fig. 8. Frequency dependence of the fluxon sensitivities $R_{fl}^0$ and $R_{fl}^1$ (full dots: bulk niobium; open dots: niobium film). Their dependence on the frequency suggests $\omega^{3/2}$ and $\omega^{2/3}$, respectively (the open squares fall out of the data collection and represent niobium film on oxidized copper cavities, known to have lower values $R_{fl}^0$ and $R_{fl}^1$).

$R_{fl}^1 \sim 1/\lambda_{rel} - 1$, $l \ll \lambda$. (8)

Eq. (8) shows the typical dependence on frequency of the anomalous skin effect and thus confirms the data in Fig. 8.

For $l \ll \lambda$, one obtains

$$R_{fl}^1 = \frac{1}{ncL} \cdot \frac{1}{Bc^2} \sim \lambda_{rel}^2 - 1, \ l \ll \lambda. \quad (9)$$

The results from the relations (8) and (9) are combined and depicted in Fig. 9 [16].

The preceding analysis shows that the observed RF losses in thin film cavities by the ambient magnetic field by trapping can be best described by the following:

1) they originate from fluxons with a local critical temperature around $T_c = 4.5$ K and a reduced electron density ($\sim 17\%$), compared with standard niobium;
2) they are localized inside and in the close vicinity of these fluxons;
3) they are created by the moving fluxons and the local Hall field directed perpendicular to the current-carrying surface;
4) they follow the anomalous skin effect (for mean free paths larger than the penetration depth) due to the ineffectiveness concept of the shielding current along the fluxons.

VI. AMBIENT MAGNETIC FIELD UNDER N-DOPING

Nitrogen-doped cavities may respond to an ambient magnetic field by showing a maximum of the trapped fluxon sensitivity at a characteristic mean free path, Fig. 10 [18]. Evidently, the model as explained in Section IV must not be applied here, because the fluxons are fully nc independently of the RF magnetic field amplitude.

This maximum can be explained by the fact that weak SC defects have a different surface resistance than the rest of the niobium surface and therefore build up a space charge when current flows through them. Thus, they act like a capacitor in an alternating field. Together with the inductance formed by the superconductor, they thus form an LC resonant circuit. A lumped-circuit model is used to determine the associated resonant frequency, which, with the help of a fit routine, leads to new results about the properties of the weak SC defects. Compared with standard niobium, they exhibit lower critical temperature and electron density, indicating dirty and/or disordered niobium with many dislocations or dissolved oxygen near the solubility limit.

The fluxon sensitivity $R_{fl}^0$ can be described by [17]

$$R_{fl}^0 = c_{eff} \cdot \frac{R}{1 + \left(\frac{\omega}{\omega_0} - \frac{\omega}{\omega} \right) Q^2} \cdot \frac{1}{Bc^2}. \quad (10)$$

Fig. 9. Trapped fluxon sensitivity $R_{fl}^1$ versus the square of the relative penetration depth $\lambda_{rel}^2 = (\lambda/\lambda_L)^2$. The dashed line is the combination of the two contributions, (8) and (9), as and is superimposed on the data from the work in [16].

Fig. 10. Trapped fluxon sensitivity $R_{fl}^0$ for N-doped niobium cavities at 1.3 GHz (full dots adopted from the work in [18]). The superimposed dashed line (red) results from (10).
The corresponding fit parameters for Fig. 10 are listed in Table I, the percentage of effective trapped magnetic flux is estimated to $c_{eff} = 62.5\%$ [16], and the other quantities are explained elsewhere [[17], Table I and Appendix Table IV].

The fit parameters are of the same order of magnitude (within $<\pm 30\%$) as those in [17], except $s_{Nb}$ (within $<\pm 80\%$), and indicate thus the error margin.

### VII. ROLE OF MEAN FREE PATH UNDER N-DOPING

The applicability of the model of [11] was also investigated for variable mean free paths $l$ [19]. A contradiction with the experimental data was found. For this reason, the model was discarded as inapplicable [20]. In the following, it will be shown why this criticism is unjustified and must therefore be rejected.

The cause of the discrepancy is a linear relationship, mentioned in [11], between $l$ and the RRR value. This relation is valid for $l$ that is larger than the coherence length $\xi$. For small $l \approx \xi$ and below, this linear relation breaks down. However, the authors of [20] applied this relation also for small $l$, which leads to inconsistencies with the data as shown in Fig. 11.

To resolve this apparent contradiction, the data of Fig. 11 were analyzed by applying the methods described in Section IV; details to be looked up elsewhere [21]. The two parameters $c$ and $s_{Nb}$, which occur in (1.a) and (3), are now determined by means of fitting routines, each for the data of a certain mean free path. The other parameters $B^*$ and $B^*_c$ are obtained by inspection. Thus, $s_m$ is also determined $s_m = c \cdot s_{Nb}$. From the reasonable assumption that the surface resistance is dominated by $s_m$ at the percolation maximum (e.g., Fig. 5), the electrical conductivity $s_1$ (or the electrical resistivity $\rho_1$) of the composite’s weak constituent is found for that specific $s_m$ (Table II, last two columns).

As a consistence check, the RRR value associated with $s_1$ is used to determine the nitrogen concentration. DeSorbo finds for 0.23 (0.33, 1.64) at. % nitrogen interstitially dissolved in niobium a low-temperature electrical resistivity of 1.7 (1.9, 1.8) $\mu\Omega\text{cm}$ [22]. Padamsee gives an RRR value of 3900 for 1 wt. ppm nitrogen [23]. These numbers result in an electrical conductivity $s_1$ (or the electrical resistivity $\rho_1$), Table III, last column. The two numbers for $\rho_1$ in Tables II and III are identical within the error margins, which means that there is no contradiction in the method chosen.

### VIII. GINZBURG–LANDAU ANALYSIS AS CONSISTENCY CHECK

The breakdown magnetic fields were studied related to the proximity between a “weak” superconductor (N), being nc if standing alone, and a strong superconductor (S) [24].

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**TABLE I**

| Variable fit parameters | | |
|-------------------------|------------------|------------------|
| $n$ [m$^{-1}$]          | $5.4 \times 10^3$ |
| $s_{Bo}[1/(Dm)^2]$     | $3.3 \times 10^8$ |
| $l_0$ [mm]             | 115              |
| $\xi$ [mm]             | 49               |
| $B_{c0}(0)$ [Gauss]    | $10^6$           |
| $l$ [nm] /RRR          | 3.7              |

n.b.

$\lambda(l) = \lambda_0 [1 + n l^2 / (2 - l)]$

$\xi(l) = 1 / (l + \xi^2)$

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Fig. 11. Surface resistance $R_{BCS}$ versus peak magnetic surface field $B$ for 1.3 GHz niobium sheet cavity at different temperatures (in the same vertical order as indicated in the insert). The dots represent the data measured at Cornell University [19], the lines indicate the fitting results with the parameters as in Table II. The three plots were obtained for three different mean free paths $l = 4.5, 34$, and 213 nm (from a to c).
−20 mT and the saturation field $B_a = \Delta_\lambda (11)$

is $\lambda \approx f (12) \Delta (14)$ and the penetration depth

are solved by $B_x \lambda$ in N and the

vanish. $d \kappa [11] (14)$

where the deviation flattens out to a $\chi \approx f (12) \Delta (14)$ and the saturation field

up to $51 \text{ nm}$, and so was the

and $\lambda = 236 \text{ nm}$ and $\lambda_0 = 51 \text{ nm}$, and so was the

lower critical magnetic field $B_c \approx 24 \text{ mT}$ [11].

To check these numbers, the general Ginzburg–Landau equa-
tions were used. Their range of application is close to the

critical temperature, which is considered to be satisfied for the

weak superconducting defects discussed here. The cubic term

is canceled, which is justified for the small energy gap $\Delta$ in N
[24]. These equations are

$$\frac{1}{\kappa^2} \frac{\partial^2 f}{\partial x^2} - f \left(1 + a^2\right) = 0$$

(11)

$$\frac{\partial^2 a}{\partial x^2} - f^2 a = 0$$

(12)

with the reduced energy gap $f = \Delta / \Delta (x = 0, B = 0)$, the

reduced vector potential $a_0(B) = 2\pi / B \cdot \lambda_0 / (\Phi_0 \cdot K)$ and the coordinate $x$ measured from the N/S interface in units of $\lambda_0$.

The Ansatz for solving the system of coupled nonlinear differen-
tial (11) and (12) is

$$f (x) = \cosh \left[ (d_N - x) \sqrt{1 + a^2 (x; B)} \cdot \kappa \right]$$

$$+ \cosh \left[ d_N \kappa (x) \right]$$

$$- \cosh \left[ d_N \sqrt{1 + a^2 (x; b)} \cdot \kappa (x) \right]$$

(13)

and

$$a (x, B) = a_0 (B) \cdot e^{f(x; x-d_N)}$$

(14)

with $\kappa(x) = \lambda(x) \cdot K$, $d_N$ being the depth of the N-doped layer and $\lambda(x) = \lambda_0 \cdot e^{-K x}$. The two (13) and (14) are solved by mutually inserting one into the other and by adding minute trial corrections $f^\prime$ to $f$. A fairly good solution is found for the RF magnetic field $B$ up to $B^* \approx 20 \text{ mT}$. From here upwards, the right-hand sides of (11) and (12) deviate more and more from

zero up to $B^* \approx 60 \text{ mT}$, where the deviation flattens out to a constant value (Fig. 12).

This behavior is interpreted as reflecting the penetration of magnetic flux into the weak superconductor between its lower critical field $B^*$ and the saturation field $B_c^*$. As a comparison, the previous numbers correspond fairly well with the calculated lower critical field $B^* = 20 \text{ mT}$ and the saturation field $B_c^* = 66.5 \text{ mT}$ [11].

IX. CONCLUSION

In this article, a total of seven examples of measurements or

more general considerations regarding the RF field dependence

of the surface resistance in SC cavities are given, all of which can be traced back to a single explanation: weak SC defects at the surface exposed to the RF field. These, if they stood alone, would be nc at sufficiently low temperature. But by proximity to a strong superconductor like niobium, they also become, though only weakly, SC. Arguments are given that these must be defects at the surface. They can increase their expansion by percolation when, for example, increasing the bath temperature. The phenomena amenable to explanation are quite different. They consist of trapping of magnetic flux during the cooling process, abrupt increase of the surface resistance at a specific temperature, anomalous increase of the surface resistance under the influence of the RF field (Q-slope and Q-drop), decrease (or even increase) of the surface resistance after so-called “N-doping,” dependent on the RF frequency, or decrease of the surface resistance for variable mean free path. An application of the Ginzburg–Landau equations to these phenomena concludes this article.

| I [nm] | c (average) | $s_{\text{NE}}$ [(Ωm)$^{-1}$] | $s_m$ [(Ωm)$^{-1}$] | $s_1$ [(Ωm)$^{-1}$] | $\rho_1 [\mu \Omega \text{cm}]$ |
|-------|------------|-----------------------------|-----------------|----------------|-----------------------------|
| 4.5   | 0.68 ± 0.03 | (3.1 ± 1.4)·10$^8$     | 2.09·10$^8$    | 4.85·10$^6$   | 21 ± 15                   |
| 34    | 0.57 ± 0.05 | (4.9 ± 0.3)·10$^8$     | 2.83·10$^8$    | 7.19·10$^6$   | 14 ± 3                    |
| 213   | 1.00 ± 0.04 | (1.03 ± 0.06)·10$^8$   | 1.03·10$^9$    | 7.02·10$^7$   | 1.4 ± 0.2                 |

$\rho_1$ of the “weak” component$^1$)

The RRR of the “weak” component is defined as $s_1/s_{\text{NE}} (300 \text{ K})$; $s_{\text{NE}} (300 \text{ K}) = 7.6·10^6 [(\Omega \text{m})^{-1}]$; $s_1$ from Table 2.

Fig. 12. Mean chi-square deviation $\chi^2$ as a measure how close the right-hand side of (11) and (12) vanish.
APPENDIX

Remarks With Regard to (1.c)

A weakly superconducting hemispherical surface defect is assumed with a dimension a much smaller than the coherence length $\xi$ and much smaller than the penetration depth $\lambda$. Furthermore, let the current carrying zone be divided into a relatively contaminated region very close to the surface with a coherence length $\xi \approx \lambda$ and a relatively clean region with a coherence length $\xi \approx \lambda$ deeper in the interior.

At the moment of a transition at $B = B^*$ of the defect to the normal state, the loss of condensation energy $\Delta E_c = B_c^2 \cdot V_c / (2 \cdot \mu_0)$ is balanced by the gain of diamagnetic energy $\Delta E_B = B^2 \cdot V_m / (2 \cdot \mu_0)$, where $V_c$ and $V_m$ are the associated volumes and $B_c$ the critical magnetic field of the niobium: $B_c^2 \cdot V_c = B^2 \cdot V_m$. From this, the critical field $B^*$ of the defect is derived as follows:

$$B^* = \sqrt{\frac{V_c}{V_m}} B_c.$$  

With the hemispherical volumes near the surface ($a \ll \xi, \lambda$)

$$V_m \approx \frac{2}{3} \pi \lambda^3 \quad \text{and} \quad V_c \approx \frac{2}{3} \pi \xi^3.$$  

$B^*$ can be quite small at the surface

$$B^* = \left( \frac{\xi}{\lambda} \right)^{3/2} B_c = \frac{B_c}{\kappa^{3/2}}$$

but deeper in the interior will be close to $B_c$ ($\kappa$ is the Ginzburg–Landau parameter).

The equality of the two energies $\Delta E_c$ and $\Delta E_B$ defines the volume of the normal conducting zone $V_c$ as a function of the RF field $B$. The incremental change $\Delta V_c$ then drives the size of the normal conducting zone from a to $\alpha + \Delta a$ and is given by

$$\Delta V_c \approx \frac{2 \cdot B \cdot V_m}{B_c^2} \cdot \Delta B + \frac{B^2}{B_c^2} \cdot \Delta V_m.$$  

With the ratio of the volumes $\Delta V_m / \Delta V_c \approx (\lambda / \xi)^2 \approx \kappa^2$ (Fig. 13), which are still considered small compared with $\lambda$ and $\xi$, so that

$$\Delta V_c \approx \frac{2 \cdot B \cdot V_m}{B_c^2 - B^2 \kappa^2} \cdot \Delta B.$$  

The RF power dissipation per square $p$ is proportional to $p \sim V_c \cdot B^2$, such that $\Delta p \sim 2 B V_c \cdot \Delta B + B^2 \cdot \Delta V_c$. The first summand represents the RF power loss associated with the square of the RF field and is therefore of no further interest. Only the second summand describes the RF losses, which increase faster than quadratic with the RF field and will therefore be discussed in more detail here. The power dissipation per square

$$p = \frac{\sigma_n}{4} \omega^2 \lambda^3 B^2$$

leads to

$$\Delta p = \frac{1}{4} \omega^2 \lambda^3 B^2 \Delta \sigma_n$$

and with

$$\frac{\Delta \sigma_n}{\sigma_n} = \frac{\Delta V_c}{V_c}$$

ends up with

$$\Delta p = \frac{\sigma_n}{4} \omega^2 \lambda^3 B^2 \frac{\Delta V_c}{V_c} = \frac{\sigma_n}{2} \omega^2 \lambda^3 \frac{V_m}{V_c} B^3 B^2 \frac{B^3}{B_c^2 - B^2 \kappa^2} \Delta B.$$  

The RF power dissipation $p$ per square for a high-frequency cycle from the low RF field to the high RF field is proportional to the integral

$$p = \frac{\sigma_n}{2} \omega^2 \lambda^3 \frac{V_m}{V_c} \int_{B \to B^*} B^3 \frac{B^3}{B_c^2 - B^2 \kappa^2} dB',$$

where $B^*$ is set close to zero as explained earlier. Finally, after integration [6] and with

$$R_{s,fd} (B, \omega, T) = \frac{2 p}{(B / \mu_0)^2}$$

the field-dependent RF surface resistance $R_{s,fd}$ is obtained as follows:

$$R_{s,fd} (B, \omega, T)$$

$$= \sigma_n (T) \omega^2 \mu_0^2 \lambda^3 \frac{V_m}{V_c} \left( -\kappa^{-2} \right) \left[ 1 + \frac{1 - \kappa^2 \left( \frac{B}{B_c} \right)^2}{\kappa^2 \left( \frac{B}{B_c} \right)^2} \right],$$

the second factor of which corresponds to (1.c).

In a more descriptive way, (1.c) can be developed into an infinite series as follows:

$$R_{s,fd} \sim \frac{1}{2} \left( \frac{B}{B_c} \right)^2 + \frac{\kappa^2}{3} \left( \frac{B}{B_c} \right)^4 + \frac{\kappa^4}{4} \left( \frac{B}{B_c} \right)^6 + \ldots.$$  

Fig. 13. Weak superconducting defect at low RF field (left) and after incremental increase of the RF field (right) above the critical temperature $B^*$ of the defect.
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TABLE IV

| Symbol | Description | Symbol  | Description |
|--------|-------------|---------|-------------|
| a      | Reduced vector potential | k^1     | Effective coherence length |
| B      | Peak rf magnetic surface field | I       | Mean free path |
| B_N    | Critical magnetic field | n       | Density of electrons |
| B_{sat} | Saturation field from N-doping | Q       | Q-value |
| B_{lc}  | Lower critical field for weak superconducting defect | R       | Lumped circuit resistance |
| B_{ln}  | Upper critical magnetic field | R_{Bc}  | “BCS” surface resistance |
| B_{amb} | Ambient magnetic field | R_{amb} | rf field dependent surface resistance |
| c_{eff} | Percentage of effective trapped magnetic flux | R_{sat} | Field-dependent surface resistance as a function of the rf field B |
| c       | Q-slope improvement (or degradation) ratio from N-doping | R_{sat} | Field-dependent surface resistance as a function of the temperature T |
| d_n     | Depth of N-doped layer | R_{D}   | Surface resistance at low field |
| E_{acc} | Accelerating gradient | R_{D}   | Surface resistance from ambient magnetic field B_{amb} |
| f       | Reduced energy gap | R_{0}   | Component of R_{D} |
| f_n     | Volume fraction of “weak” superconductor | R_{D}   | Component of R_{D} |
| G       | Geometry factor | R_{g}   | Surface resistance |

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