Research Article

Equivalent Circuit Analysis of Linear Phase-Shifting Transformer with End Effect

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The linear phase-shifting transformers (LPSTs) are a new type of transformers with a structure similar to a linear motor that can be used in multiplex technology. A reasonable equivalent circuit is the premise of control research. Based on one-dimensional electromagnetic field analysis of the LPST, we reference the theory of linear motor and propose an equivalent circuit model of the LPST. The LPSTs, which are a phase-shifting transformers based on linear motor structure, are affected by end effects. The end effects affect the mutual inductance and secondary resistance of the LPST, which is modified by four correction coefficients. This paper calculates the four correction coefficients, inductance, and resistance of the LPST and then proposes a single-phase T-type equivalent circuit model considering end effects. Using the analytical model, the output voltages under the three working conditions are calculated and analyzed, and the accuracy is verified by comparison with results obtained by the finite element method. Finally, the accuracy of the analytical method is further verified by experiments under two working conditions, which indicates that the equivalent model is credible and useful for control research of multi-inverter system based on the LPST.

1. Introduction

Multiple superposed technology can effectively reduce voltage harmonics, reduce pollution to the power grid, improve output voltage waves, and improve output performance. Phase-shifting transformers are essential devices of multiphase equipment, so it is of great significance to design and research them. Linear phase-shifting transformers (LPSTs) are a new type of transformers that can be used in multi-rectifier and multi-inverter systems [1]. In addition to realizing the traditional phase-shifting functions of conventional transformers, the LPSTs offer numerous advantages, such as the capability of realizing arbitrary phase angle shifts, easy adjustment of the magnetic field in the air gap, suitability for high-power settings, and relatively simple structure, which makes LPSTs easy to modularize and expand.

A multi-inverter system based on the LPST is proposed in this paper, which combines multi-overlapping technology with LPSTs. The performances of the LPST directly affect the stability of the whole system in multi-inverter system, so the research of ontology characteristics and control methods must be paid attention to. Because the LPST is similar to linear motor in structure and principle and is also affected by end effect, the characteristic analysis of the LPST is more complicated than that of circular transformer [2], which is mainly based on the following reasons: (1) the magnetic circuit is disconnected, and the mutual inductances of three-phase windings are asymmetric; (2) the end effects cause the distortion of air gap magnetic field; (3) the distribution of the windings is asymmetric [3–5].

In order to research the adjustable multi-inverter system with low switching frequency and good dynamic performance, it is very significant to propose an equivalent circuit of the LPST. However, few domestic and overseas scholars propose a reasonable equivalent circuit to study the characteristics of the LPSTs. In addition, because its structure is not exactly the same as linear motor and rotary motor, there is no equivalent circuit model that can be directly used for the LPST. In order to derive the equivalent circuit of the LPST, based on its structural characteristics (the air gap is very small, and both the secondary side and the primary side
are fixed), some research about linear motors and rotary motors can be referred to. In the procedures of the field analytical theories, the quasi-1D and 2D methods are a practical way to analyze the characteristics of the linear induction motor (LIM), which can consider both the skin effect and end effect [6]. Nonaka and Yoshida [7] proposed a series of accurate models based on the space harmonic method considering the end effects and obtained more accurate calculated results. Lv et al. [8, 9] proposed various transverse m.m.f. models in the end-region. Compared with the space harmonic method, the results calculated by these models were more accurate and were validated by the experimental measurements. Bolton [10] proposed a model considering that the LIM worked with an asymmetric secondary sheet. However, an equivalent circuit for the LPSTs is difficult to derive by the sophisticated space harmonic method. Lipo and Nondahl [11] proposed an equivalent circuit based on the winding function method, and the air gap flux in the LIM was divided into various components. Despite the model being able to analyze the steady-state, transient-state, and dynamic characteristics of the LPSTs, the winding function theory is achieved by some approximate hypotheses neglecting end effects. Kim and Kwon [12] proposed an equivalent model for the LIM developed by the finite element method (FEM) considering skin effect and magnetic saturation. The circuitual approach was greatly useful for both the analysis and synthesis of the LIMs, and its performance could easily be obtained from the steady-state equivalent circuit with the end effects. However, it is a challenge to put the model by FEM into the LPST design and control. Gieras et al. [13] proposed the linear induction motor (LIM) analysis model based on the consideration of the end effect, skin effect, and eddy current of the secondary plate. Kang and Nam [14] proposed the $dq$-axis model neglecting the half-filled slot and the saturation of magnetic circuit and only considering the influence of the secondary eddy current on the air gap flux linkage the transverse end effect. The above two models can meet the control requirements to a certain extent, but the algorithm is very rough. Nozaki [15] proposed the equivalent model of motor by considering longitudinal end effect and skin effect and analyzed the magnetic flux density traveling wave of LIM by Fourier’s pole expansion method and finite element method (FEM). This model has a wide range of applications, but the whole operation and programming process is very complex, so many equations need to be solved iteratively. It is difficult to derive the equivalent circuit of the LPSTs by the above methods. Zare-Bazghaleh et al. [16–18] proposed an equivalent circuit basically derived from the T-model circuit, and its parameters were obtained from field theories.

For simplifying the algebra and lowering the computation load of vector controller in practical engineering, Duncan [19] put forward an intelligible equivalent circuit for LIMs by using the rotary-motor model as a basis through inheriting merely the T-type circuit without any field theories. The rotary-motor model was modified to account for the so-called “end effect” and was used to predict output thrust, vertical forces, and couples. Further improvements [20, 21] were mostly based on Duncan’s model. Xu et al. [22, 23] optimized the previous model by using the relation of equal field-circuit complex power and presented a novel equivalent circuit considering the end effects. The results calculated by the equivalent circuit were validated by the experimental measurements. The solution formulas of fictitious primary symmetrical phase potential, secondary current, secondary resistance, secondary leakage reactance, and excitation reactance were derived.

The special structure of the LPST means that its performance is different from those of above-mentioned motors. Considering the structural characteristics of the LPSTs, based on the analytical model (AM) of linear motors, the AM of the LPSTs is established in this paper. The correction coefficients related to the end effects are introduced to modify the results of the AM. Finally, the inverter system based on the AM is proposed. The accuracy of the proposed AM is verified by comparison of the results obtained with those obtained by the FEM.

The research focuses on the modeling and optimization analysis of the mathematical model of the system in this paper. It is organized as follows. The basic working principle of the LPST inverter system is described in Section 2 in detail. In Section 3, the derivations on how to get four correction coefficients related to the end effects are indicated by the relation of equal field-circuit complex power. The magnetic density of the air gap is analyzed. Based on 1D flux density equations, the per-phase T-model equivalent circuit of the LPST is derived. Then, two axis equivalent circuits are deduced based on the per-phase equations so as to study the LPST dynamic performance. The simulation and experimental verifications of the multi-inverter system based on the LPST are given in Section 4 and Section 5, respectively. Finally, this paper is summarized in Section 6.

2. Basic Working Principle of the LPSTs Inverter System

The working principle of the LPSTs is similar to that of LIMs. Figure 1 indicates a schematic diagram of the structure of the LPSTs. The length of the primary core is equal to that of the secondary core of the LPSTs. Both the primary side and secondary side are fixed. The primary and secondary cores are embedded with $3N$ phase windings. The flexible selection of the phase-shifting angle can be realized by changing the value of $N$. There is an adjustable air gap between the iron cores. Therefore, the LPSTs can be equivalent to a linear motor with a slip rate of 1, which can be widely used in multi-inverter systems [24].

In order to better illustrate the basic principles, an LPST used in multi-inverter is selected as an example. The primary side of the LPST is embedded with 4 sets of 3-phase windings. The phase difference between the adjacent windings is 15°. The secondary side of the transformer is embedded with a group of 3-phase windings, which are connected by star mode. The four sets of three-phase six-step waves output by the inverter circuit, which are different from each other by 120 electrical angles, are connected to the 12-phase winding of the primary side of the LPSTs. The traveling wave magnetic field is generated in the air gap, and a set
of three-phase alternating currents which are similar to sinusoidal waves is induced from the windings of the secondary side [25]. Figure 2 describes the spatial phase relation of the primary 12-phase winding. The multi-inverter system based on the LPSTs is shown in Figure 3.

3. Mathematical Modeling with End Effects

3.1. Single-Phase T-Equivalent Circuit End Effect. Both the primary windings and the secondary windings are treated as 12 phases, and the single-phase T-equivalent circuit is obtained. Then, a set of three-phase output voltage can be synthesized by matrix transformation of the 12-phase output voltage of the secondary side. The LPSTs are affected by the end effect that exists in linear motors due to the openings at both ends of the physical structure. Therefore, the influence of the end effect must be considered in the modeling.

For the convenience of the modeling, the following assumptions are made [26]: (1) the magnetic potential produced by the currents in the primary windings is replaced by the surface current layer, and only its fundamental component is considered; (2) the influence of the cogging effect is considered by the air gap coefficient; (3) the various field quantities vary with time according to the sinusoidal wave.

3.2. Mathematical Analysis of Longitudinal End Effect. According to the physical structure of the LPST, the two-dimensional AM of the LPST with the longitudinal end effect can be obtained. After simplification, it can be shown as Figure 4.

The equivalent current layer on the primary side of the LPST can be expressed as follows:

$$J_1 = J_1 \exp[j(\omega_0 t - kx)],$$

$$J_1 = \sqrt{2m_1} W_1 k_{w_1} I_1.$$

Here, $J_1$ is the amplitude of primary traveling wave current layer, $m_1$ is the number of phases in the primary winding, $k = \pi/\tau$, $\omega_0$ is the angular velocity of the primary current, $\rho$ is the polar pairs of the primary side, $k_{w_1}$ is the primary winding coefficient, $W_1$ is the number of coil turns per phase on the primary side, and $I_1$ is the effective value of primary phase current.

Along the path of the rectangle in Figure 4, this yields the following equation:

$$\frac{\partial B_{3y}}{\mu_0} = J_1 + J_2,$$

where $J_2$ is the complex form of the equivalent traveling wave current layer in region 2, $B_{3y}$ is the component of the magnetic flux density in region 3 on the y-coordinate, and $ge$ is equivalent air gap.

Since the current contains only z-component, the vector magnetic potential also contains only z-component. This yields the following equation:

$$\frac{\partial A_{3z}}{\sigma_e} = E_{3z}.$$

Substituting (4) into Maxwell’s equations yields the following equation:

$$J_z = -\sigma_e \frac{\partial A_{3z}}{\sigma_e},$$

where $\sigma_e$ is the surface conductivity of the secondary conductor and conforms to the formula $\sigma_e = \sigma d$. The primary current changes as the function $(\exp(j\omega t))$ of time yields the following equation:

$$A_{3z} = A_{3z}(x, t) = A_e(x) \exp(j\omega t).$$

Substituting (4), (5), and (6), into (2) yields the following equation:

$$\frac{\partial A_{3z}}{\mu_0} d^2 A_{3z} - j\omega_0 \sigma_e A_{3z} = -J_1 \exp(-jkx).$$
The general solution of (7) is obtained as follows:

\[ A_z = c_z \exp(-jkx) + c_1 \exp\left[-\left(\frac{1}{\alpha_1} + j\frac{\pi}{\tau_e}\right)x\right] + c_1' \exp\left(\frac{1}{\alpha_1} + j\frac{\pi}{\tau_e}\right)x, \]  

(8)

\[ A_{3z} = c_z \exp[j(\omega_c t - kx)] + c_1 \exp\left(-\frac{x}{\alpha_1}\right)\exp\left[j\left(\omega_c t - \frac{\pi}{\tau_e}x\right)\right] + c_1' \exp\left(\frac{x}{\alpha_1}\right)\exp\left[j\left(\omega_c t + \frac{\pi}{\tau_e}x\right)\right]. \]  

(9)

Here, \( c_1c_1' \) is the undetermined coefficient, \( \alpha_1 = 1/\sqrt{2}\omega_c \sigma_e \), and \( \tau_e = 2\sqrt{2}\pi g_e/\mu_0 \).

Equation (9) is related to the sinusoidal magnetic dense traveling wave, the incoming magnetic dense traveling wave,
and the outgoing magnetic dense traveling wave. The third term $c_1' = 0$ is generally ignored, and other coefficients can be obtained by boundary conditions. The results are shown as follows:

\begin{equation}
  c_s = \frac{\mu_0 j_1}{k' g_e (1 + jG)}.
\end{equation}

Here, $G$ is the quality factor.

\begin{equation}
  c_1 = \frac{-j k c_s}{(1/a_1 + j\pi/\tau_e)}.
\end{equation}

In combination with equations (4), (5), (6), and (9), the following equations can be obtained:

\begin{equation}
  B_{3y} = j k c_s \exp[j(\omega t - k x)] + \left(\frac{1}{a_1} + \frac{\pi}{\tau_e}\right) c_1 \exp\left(\frac{x}{a_1}\right) \exp\left[j\left(\omega t - \frac{\pi}{\tau_e} x\right)\right],
\end{equation}

\begin{equation}
  B_{3y} = B_s \left\{ \exp[j(\omega t - k x + \delta_s)] - \exp\left(-\frac{x}{a_1}\right) \exp[j(\omega t - k x)] \right\}.
\end{equation}

Here, $B_s = j k c_s \exp(-j\delta_s) = G J_1 / \delta_s \sqrt{1 + G^2}$.

\begin{equation}
  E_{3z} = -j \omega e \left\{ c_s \exp[j(\omega t - k x)] + c_1 \exp\left(-\frac{x}{a_1}\right) \exp[j\left(\omega t - \frac{\pi}{\tau_e} x\right)] \right\},
\end{equation}

\begin{equation}
  j_2 = -j \sigma_s \omega e \left\{ c_s \exp[j(\omega t - k x)] + c_1 \exp\left(-\frac{x}{a_1}\right) \exp[j\left(\omega t - \frac{\pi}{\tau_e} x\right)] \right\}.
\end{equation}

According to Poynting’s theorem, considering that the resistance loss, electric field energy, and magnetic field energy in the current layer are all 0, the total complex power is equal to the complex power per unit length in the $z$-direction multiplied by the length in the $z$-direction. The total complex power transmitted from the primary windings to the secondary windings and the air gap is shown as follows:

\begin{equation}
  S_{23} = \int_{-a_1}^{a_1} \int_0^{\rho_2} 0.5 [-j_1^* E_{3z}] dx dz
\end{equation}

\begin{equation}
  = J_1 B_{wa} a_1 V_s \left[ 2 \rho r \cos \delta_s - N_L \left\{ a_1^{-1} \exp\left(-\frac{2 \rho r}{a_1}\right) \sin(\delta_s - \beta + 2 S_L \rho r) \right. \right.
\end{equation}

\begin{equation}
  + S_L \exp\left(-\frac{2 \rho r}{a_1}\right) \cos(\delta_s - \beta + S_L \rho r) - a_1^{-1} \sin(\delta_s - \beta) - S_L \cos(\delta_s - \beta) \left. \right\}
\end{equation}

\begin{equation}
  + j J_1 B_{wa} a_1 V_s \left[ 2 \rho r \sin \delta_s - N_L \left\{ a_1^{-1} \exp\left(-\frac{2 \rho r}{a_1}\right) \cos(\delta_s - \beta + 2 S_L \rho r) \right. \right.
\end{equation}

\begin{equation}
  + S_L \exp\left(-\frac{2 \rho r}{a_1}\right) \sin(\delta_s - \beta + S_L \rho r) + a_1^{-1} \cos(\delta_s - \beta) - S_L \sin(\delta_s - \beta) \left. \right\} = P_2 + j Q_3.
\end{equation}
Here,
\[
\delta_s = \tan^{-1}\left(\frac{1}{G}\right),
\]
\[
\beta = \tan^{-1}\left(\frac{\pi a_1}{\tau_c}\right),
\]
\[
S_L = k - \frac{\pi}{\tau_c},
\]
\[
M_L = \left(a_1^{-1}\right)^2 + S_L^2,
\]
\[
N_L = \frac{a_1 \pi \tau_c}{M_L \tau_c \sqrt{\tau_c^2 + (\pi a_1)^2}}.
\]

(15)

\(P_2\) is the active power transferred to the secondary and \(Q_3\) is the reactive power transferred to the air gap. Then, the effective value of the primary phase current can be calculated as follows:
\[
I_1 = \frac{pr I_1}{\sqrt{2} m_1 W_1 k_{w1}}.
\]

(16)

In (16), \(m_1\) is the number of phases, \(W_1\) is the number of serial turns per phase of the primary winding, and \(k_{w1}\) is the coefficient of the primary winding.

The primary air gap electromotive force is assumed to be \(E_m(s)\), and according to the equal relationship of the complex power, the following equation can be obtained:
\[
-m_1 I_s E_m^* (s) = P_2 + jQ_3.
\]

(17)

Substituting (14) and (16) into (17), \(E_m^* (s)\) can be obtained as follows:
\[
E_m^* (s) = \frac{-m_1 I_s \left(m_1 W_1 k_{w1} a_1 \sqrt{2} \sigma_p \tau \right) \left[2 \rho \tau \cos \delta_s - N_L \left[\alpha_1^{-1} \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \sin(\delta_s - \beta + 2 S_L \rho \tau) \right] \right] + S_L \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \cos(\delta_s - \beta + S_L \rho \tau) - \left[\alpha_1^{-1} \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \cos(\delta_s - \beta + 2 S_L \rho \tau) \right] + \sqrt{2} m_1 W_1 k_{w1} a_1 \sqrt{2} \sigma_p \tau \left[2 \rho \tau \sin \delta_s - N_L \left[-\alpha_1^{-1} \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \cos(\delta_s - \beta + 2 S_L \rho \tau) \right] + S_L \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \sin(\delta_s - \beta + 2 S_L \rho \tau) + \left[\alpha_1^{-1} \exp\left(-\frac{2 \rho \tau}{\alpha_1}\right) \cos(\delta_s - \beta + S_L \rho \tau) \right] \right}.\]

(18)

It is further deduced that when only the longitudinal end effect is considered, the secondary resistance and excitation reactance of each phase reduced to the primary can be obtained as follows:
\[
R_s (s) = \frac{m_1^2 E_m^* (s)^2}{P_2}
\]
\[
= \frac{2 a_1 m_1 (W_1 k_{w1})^2}{\sigma_c \rho \tau} \frac{G}{\rho \tau \sqrt{1 + G^2}} \frac{C_1^2 + C_2^2}{C_1^2 + C_2^2}.
\]

(19)

\[
X_m (s) = \frac{m_1^2 E_m^* (s)^2}{Q_3}
\]
\[
= \frac{2 a_1 m_1 (W_1 k_{w1})^2}{g_c \rho \tau} \frac{G}{\rho \tau \sqrt{1 + G^2}} \frac{C_1^2 + C_2^2}{C_1^2 + C_2^2}.
\]

(20)
The coefficient expressions in (19) and (20) are shown as follows:

\[
C_1 = 2pr \cos \delta_i - N_i \left[ \alpha_i^1 \exp \left( \frac{2pr}{\alpha_i} \right) \sin (\delta_i - \beta + 2S_i pr) + S_i \exp \left( \frac{2pr}{\alpha_i} \right) \cos (\delta_i - \beta + 2S_i pr) - \alpha_i^1 \sin (\delta_i - \beta) - S_i \cos (\delta_i - \beta) \right],
\]
\[
C_2 = 2pr \sin \delta_i - N_i \left[ -\alpha_i^1 \exp \left( \frac{2pr}{\alpha_i} \right) \cos (\delta_i - \beta + 2S_i pr) + S_i \exp \left( \frac{2pr}{\alpha_i} \right) \sin (\delta_i - \beta + 2S_i pr) + \alpha_i^1 \cos (\delta_i - \beta) - S_i \sin (\delta_i - \beta) \right].
\]

The expressions of the secondary resistance and excitation reactance of each phase of the LPST are shown as follows:

\[
r_r = \frac{4a_1 m_1 (W_i R_{wl})^2}{\pi g_e p},
\]
\[
K_r^*(s) = 2pr \sqrt{1 + G^2} \frac{C_1^2 + C_1^2}{C_1},
\]
\[
X_m = K_r^*(s),
\]
\[
K_x^*(s) = 2pr \sqrt{1 + G^2} \frac{C_1^2 + C_1^2}{C_2},
\]

Comparing (19), (20) and (22), (23), it is known that the secondary resistance and excitation reactance of the LPSTs are affected by the longitudinal end effect, which can be modified by the coefficients \(K_r^*(s)\) and \(K_x^*(s)\), as shown in the following equations:

\[\frac{\partial^2 B_{ly}}{\partial x^2} + \frac{\partial^2 B_{ly}}{\partial z^2} = -\frac{\mu_0 g_e}{g_e} \frac{\partial B_{ly}}{\partial t} = -\frac{\mu_0 g_e}{g_e} \frac{\partial f_1}{\partial x} \]

Assuming \(B_{ly} = B_{ly}(x, z, t) = B(z) \exp \{j(\omega t - kx)\}\), substituting it into yields the full solution as follows:

\[B_{ly}(x, z, t) = \left( B_c \cosh \alpha z - \frac{\mu_0 f_1}{g_e} R^2 1 \right) \exp \{j(\omega t - kx)\},\]

where \(R^2 = 1 + jG, \alpha^2 = k^2 + j\omega x_0 \sigma_e / \epsilon_e\), and \(B_c\) is the undetermined coefficient, which can be obtained by the boundary conditions and the current continuity theorem.

\[B_c = -j f_1 \frac{\mu_0}{g_e} \frac{1 - R^2}{R^2 \cosh \alpha z} \lambda.
\]

In addition,

\[\lambda = \frac{1}{1 + 1/R \tanh (a_1 \alpha) \tanh k(c_2 - a_1)}\]

Because \(c_2 = a_1\) and \(\lambda = 1\), according to (30) and (31), we can obtain

\[B_{ly}(x, z, t) = -j f_1 \frac{\mu_0}{g_e} \frac{1 - R^2}{R^2 \cosh \alpha z} \exp \{j(\omega t - kx)\}.
\]

The magnetic flux of each pole is supposed to be \(\phi(t)\), which can be expressed as follows:

\[\phi = \int_{a_1}^{a_1} B_{ly}(x, z, t) dz dx\]

The instantaneous value of the excitation potential of each phase of the primary can be obtained as follows:

\[e_m = \int_{a_1}^{a_1} \frac{d}{dt} [\phi] = -W_{kwl} \frac{d}{dt} [\phi] = -\sqrt{2} E_m(s) \exp (j\omega t).
\]

Here,

\[e_m(s) = \frac{4\sqrt{2} \mu_0 W_{kwl} a_1^2 t^2}{\pi g_e} I_1 \left\{ \left[ R^2 + (1 - R^2) \frac{1}{a_i \alpha} \tanh a_i \alpha \right] \right\}.
\]
The power transmitted from the primary to the air gap and the secondary is shown as follows:

\[-\frac{1}{2} m_1 I_s E_m (s) = P_2 + j Q_3\]

\[= \frac{2 \mu_0 a_1 f \rho L_1}{\pi g_e} f_1\]

\[= \left\{ \text{Re} \left( j \left[ R^2 + \left(1 + R^2 \right) \frac{1}{a_1} \tanh a_1 \alpha \right] \right) \right\} + j \left\{ \text{Im} \left( j \left[ R^2 + \left(1 + R^2 \right) \frac{1}{a_1} \tanh a_1 \alpha \right] \right) \right\} \]

Here,

\[T = j \left[ R^2 + \left(1 - R^2 \right) \frac{1}{a_1} \tanh a_1 \alpha \right]. \]

According to the above analysis, the T-type equivalent circuit of LPST with end effects can be obtained, as shown in Figure 6.

3.4. Transformer Mathematical Modeling. The LPSTs can be regarded as a linear motor with a slip rate of 1, and its basic structure is similar to that of a linear motor. The voltage equations of the four groups of three-phase windings on the primary side of the LPSTs are shown as follows:

\[U_1 = R_1 I_1 + \frac{d \psi_1}{dt}, \]

\[\psi_1 = L_{11} I_1 + L_{12} I_2. \]

In (41), \( U_1 \) is the primary-side input voltage vector. \( R_1 \) is the primary-side winding resistance matrix. \( I_1 \) is the primary-side current matrix. \( \psi_1 \) is the primary-side flux matrix of the LPSTs. \( L_{11} \) is the primary-side inductance matrix. \( L_{12} \) is the primary-side and the mutual inductance matrix on the secondary side. \( I_2 \) is the current matrix on the secondary side.
\[
U_1 = [U_{s1}, U_{s2}, \ldots, U_{s12}]^T,
\]
\[
R_1 = \text{diag}[r_s, r_s, \ldots, r_s],
\]
\[
I_1 = [I_{s1}, I_{s2}, \ldots, I_{s12}]^T,
\]
\[
L_{11} = \begin{bmatrix}
L_{s1s1} & L_{s1s2} & \cdots & L_{s1s12} \\
L_{s2s1} & L_{s2s2} & \cdots & L_{s2s12} \\
\vdots & \vdots & \ddots & \vdots \\
L_{s12s1} & L_{s12s2} & \cdots & L_{s12s12}
\end{bmatrix} + L_{1s}I_{12s12}, \tag{42}
\]
\[
L_{12} = \begin{bmatrix}
L_{s1r1} & L_{s1r2} & L_{s1r3} \\
L_{s2r1} & L_{s2r2} & L_{s2r3} \\
\vdots & \vdots & \vdots \\
L_{s13r1} & L_{s13r2} & L_{s13r3}
\end{bmatrix}.
\]

Since the primary side and the secondary side of the LPSTs are fixed, the inductance matrix of the primary side and the secondary side does not change with time. Substituting the flux linkage expression into the primary voltage equation yields the following equation:
\[
U_1 = R_1I_1 + L_{11}\frac{dI_1}{dt} + K'_cC_xL_{12}\frac{dI_2}{dt} \tag{43}
\]

The secondary windings of the LPSTs are connected to a three-phase symmetrical load, the load impedance of each phase is \(Z_l = r_l + j\omega L_l\), and the output voltage equation on the secondary side is shown as follows.
\[
\begin{cases} 
K'_cC_xU_2 = K'_cC_xR_2I_2 + \frac{d\psi_2}{dt}, \\
\psi_2 = L_{22}I_2 + K'_cC_xL_{23}I_1.
\end{cases} \tag{44}
\]

R_2 is the secondary-side winding resistance matrix, R_1 is the load resistance matrix, L_1 is the load inductance matrix, L_{21} is the secondary-side and primary-side mutual inductance matrix, and L_{22} is the secondary-side inductance matrix.

Here,
\[
R_2 = \text{diag}[r_r, r_r, r_r],
\]
\[
R_l = \text{diag}[r_l, r_l, r_l],
\]
\[
L_l = \text{diag}[L_l, L_l, L_l],
\]
\[
L_{21} = L_{12}, \tag{45}
\]
\[
L_{22} = \begin{bmatrix}
L_{r1r1} & L_{r1r2} & L_{r1r3} \\
L_{r2r1} & L_{r2r2} & L_{r2r3} \\
L_{r3r1} & L_{r3r2} & L_{r3r3}
\end{bmatrix} + L_{2s}I_{3s3}.
\]

### 4. Simulation Analysis and Comparison

In order to verify the accuracy of the analytical model (AM), the main parameters of the LPST which can be used for multi-rectification are listed in Table 1. There are twelve-phase windings on the primary side, and there are three-phase windings on the secondary side. The FEM is established as shown in Figure 7.

The AM is used to obtain the output voltage waveform and FFT analysis under three different working conditions, as shown in Figures 8–13.

Figures 8–13 show the analysis of three-phase voltage and A-phase FFT of output on the secondary side of the LPST under different load conditions. Table 2 shows the main performance indexes changing rules with load. Analysis shows the following:

1. When there is no load, the inverter output is 24 step waves, and the harmonic content is 9.42%. Under rated load, the output voltage waveform of the inverter system is very close to the sine wave, and the harmonic content of the output voltage is 1.38%, which meets the requirement of less than 5% in the national standard. The harmonic content of the output voltage increases with the decrease of the load and reaches the maximum at no load. Because the winding material is perceptual, when the current flows through the winding, the role of partial voltage have been filtered.

2. At rated load, the efficiency is 92.3%. In other load conditions, the efficiency (EF) of the inverter system based on the LPST is above 90%, which meets the design requirements. The greater the load, the
Figure 7: Diagram of model structure associated with the finite element method (FEM).

Figure 8: Output three-phase voltage waveform at no load.

Figure 9: Output A-phase voltage harmonic analysis at no load.
greater the current, and the stronger the filtering effect of the LPST.

(3) Three-phase imbalance factor (IF) increases the loss of the transformer and causes zero sequence current to be too large and local metal parts temperature to rise. Under different loads, the IF of the output voltage of the secondary windings of the LPST is much less than 2%, which meets the national standard.

(4) In the harmonic analysis diagram, the A-phase voltage mainly contains 5th and 7th harmonics. The output voltages under three different load conditions are compared with the FEM results, and the amplitude error is extremely small within 2%, which proves the accuracy of the AM.

5. Experiment

According to the proposed multi-inverter system based on LPST above, we have fabricated a prototype of multi-inverter system as shown in Figure 14.

The LPST is on the far right of Figure 14. To modify the wiring method, the terminal of each coil winding on the left side of the LPST is connected to the pillar terminal to facilitate wiring and to modify the wiring method. The control circuit is on the far left.

The results show the following: (1) From Figures 15–18, the harmonic content of A-phase at 50% load is 5.45%, and
Figure 13: Output A-phase voltage harmonic analysis at rated load. The main performance indexes obtained from the analytical model and the FEM are shown in Table 2.

| AM load (%) | THD (%) | Fundamental | IF | EF (%) |
|-------------|---------|-------------|----|--------|
| 100         | 1.38    | 238.7       | 0  | 92.3   |
| 50          | 2.72    | 241.2       | 0  | 92.7   |
| 0           | 9.42    | 243.6       | 0  | —      |

| FEM load (%) | THD (%) | Fundamental | IF (%) | EF (%) |
|--------------|---------|-------------|--------|--------|
| 100          | 1.78    | 240.8       | 0.26   | 91.8   |
| 50           | 3.16    | 243.5       | 0.16   | 92.6   |
| 0            | 9.33    | 245.8       | 0.12   | —      |

Figure 14: Multi-inverter system based on the LPST experimental platform.

Figure 15: Experimental diagram of output A-phase voltage at 50% load.
that of A-phase at rated load is 3.75%, which are larger than the results of the AM on the equal conditions. Due to the processing level, it is difficult for the air gap to be very small, and it is difficult for the primary and secondary sides to be symmetrical, which makes the experimental value slightly larger. (2) From Figures 9, 11, and 13, when the AM and FEM are performed, the A-phase output voltage harmonics are mainly concentrated at 5th and 7th times, while the even harmonics and the multiple harmonics of 3 are very few and can be neglected. As shown in Figures 16 and 18, the 3rd, 5th, and 7th harmonics are more significant, and the even harmonics are also more obvious. This is due to the fact that it is difficult to achieve absolute uniformity of the iron core during the manufacture of the LPST, and the asymmetry of
the three-phase windings is more obvious. At the same time, the LPST is close to saturation, which leads to even harmonics and multiple harmonics of 3.

6. Conclusion

In this paper, the equivalent circuit of the LPSTs was derived by considering the influence of the end effect on the mutual inductance and the secondary-side circuit. The following conclusions can be drawn:

(1) The proposed equivalent circuit of the LPST was derived based on the 1D air gap flux density equation, and four coefficients were got to describe the influence on the mutual inductance and secondary resistance brought by the longitudinal and transversal end effects. These coefficients had clear physical meanings so as to understand the end effects. Hence, it was convenient to study the performance of the LPST.

(2) The FEM was carried out. The results in three different working conditions showed that the error between the AM and the FEM was less than 2%. The accuracy of the AM was verified by comparison with the results obtained by the FEM.

(3) A small prototype was made, and experimental research was carried out. The experimental results were analyzed in detail. The amplitude of the output voltage obtained by AM was higher than the experimental results, which is explained in detail in Section 5. The accuracy of the AM was far verified by comparison with the experimental results.

As a result, we believe that the equivalent circuit model proposed in this paper can be used to analyze the characteristics and control of LPSTs.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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