Dynamic modelling of ship using gaussian processes and sg filter

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Abstract. Dynamic modeling of surface ships is a prerequisite for intelligent navigation and ship motion control. It is easy to ignore the nonlinear and strong coupling of ship dynamics by using traditional mechanism modeling. Moreover, the obtained parameter errors will be large due to the influence of noise and multicollinearity, it is difficult to establish a high-precision ship dynamic model. This study use a data-driven nonparametric bayesian model based on gaussian process regression on dynamic modelling of ship, it can capture the strong nonlinearity and motion coupling in ship motion, and deal with the presence of uncertainty and noise. SG filter is used to smooth the data to reduce the influence of noise on modeling. The results indicate that the gaussian processes regression and the SG filter can reproduce the ship's motion well, which is beneficial to the further development of ship control design

1. Introduction
Dynamic modeling of surface ships is a prerequisite for intelligent navigation and ship motion control. The goal of ship dynamic modeling is to establish a mathematical model that can produce the best fit between the measured motion response of the ship and the output state of the model given the same motion state and control force. The more accurate the ship’s control force input and ship motion state output mapping, the more stable the ship controller, which is more can better improve the ship's motion simulation and control.

Ship dynamics modeling is mainly divided into parametric modeling based on mechanics and non-parametric modeling without model. In parametric modeling. In parametric modeling, Methods such as least square (LS), ridge regression (RR) and extended Kalman filter (EKF) have been used in ship dynamic modeling. Luo applied the LSSVR [1] method based on linear kernel to the parameter identification of the Abkowitz model. However, these methods have some inherent disadvantages, such as the obtained parameter errors will be large due to the influence of noise and multicollinearity [2]. The most reliable method is to use Planar Motion Mechanics (PMM) for model testing to obtain experimental data for identification. Xu proposed an Optimal Truncated LS [3] and Optimal Truncated LS-SVM [4] method to reduce the uncertainty of the estimated parameters [5]. However, this optimization method is a biased estimation, which reduces the accuracy of hydrodynamic coefficients and thus reduces the variance of parameters. It is easy to ignore the nonlinear and strong coupling of ship dynamics by using traditional mechanism modeling which is difficult to establish a high-precision ship dynamic model.

In recent years, machine learning methods such as support vector machines and kernel ridge regression have been used in ship motion modeling, and have achieved encouraging results in ship
motion prediction. The hydrodynamic coefficients in the Abkowitz model are identified by using $\epsilon$-support vector egression ($\epsilon$-SVR) [6]. A novel nonparametric identification modeling method based on the locally weighted learning (LWL) [7] is proposed for ship maneuvering system, it provides the ship maneuvering mathematical model with high accuracy for ship-handling simulator and marine control engineering. Wang proposed generalized ellipsoidal basis function fuzzy neural networks [8] to identify the motion dynamics of a large tanker. However, there are certain difficulties in determining the parameters of the ANN network structure. In addition, it is easy to overfit when the data is noisy. Symbolic regression and kernel ridge regression [9] are used for black-box marine system identification of a scale ship and have obtained encouraging results. With the development of machine learning technology, kernel methods such as Gaussian Process Regression are used for nonlinear mapping of model input and output. Rong [10] proposed a probabilistic trajectory prediction model to predict the trajectory uncertainty in real-time.

In this paper, A data-driven nonparametric bayesian model based on gaussian process regression was used to describe accurately the relationship between the motion state and control force of ship without assumptions on the mathematical model of ship. A three-degree-of-freedom (surge, sway and yaw rate) probabilistic motion black-box model is established, it can capture strongly nonlinear effects and obtain the predicted results consist of a mean and variance value. SG filter is used to smooth the data to reduce the influence of noise on modeling. The results indicate that the gaussian processes regression and the SG filter can reproduce the ship's motion well, which is beneficial to the further development of ship control design.

2. Materials and Methods

2.1. Ship dynamics model

Dynamic mathematical models are usually obtained by the application of Newtonian and Lagrangian mechanics. There are many types of ship motion models based on mechanical modelling, mainly including separate models, overall models and response models. These ship models lead to a complex system of coupled equations defined by a series of hydrodynamic parameters. The parameters are the representation of added masses, hydrodynamics damping, restoring moment coefficient and constants related directly with control forces such as propellers and rudders.

Models of ship dynamics can be expressed by the coupling of control input $\tau$ and system states.

$$f(s, \dot{s}, \dot{\tau}) = \ddot{s}$$

$s$ represents the position and attitude of the ship

$\dot{s}$ represents the speed of ship

$\ddot{s}$ represents the acceleration of the ship

$\tau$ represents the active control of the ship

The inertial force, damping force and restoring force generated by motion can be expressed by $\ddot{s}, \dot{s}$ and $s$ respectively. $\tau$ is usually expressed by $T$ generated by the thruster and rudder $\delta$.

When only considering the 3 DoF models (the surge, sway and yaw), The restoring force of ship can be ignored. So the model can be simplified as:

$$f(\dot{s}, \dot{\tau}) = \ddot{s}$$

The continuous-time 3 DoF model has the following form in differential equations:

$$f_{uk}(u_k, v_k, r_k, \dot{r}_k, \dot{\delta}_k) = \ddot{u}_k$$

Usually, the acceleration measurement value is very inaccurate, and the speed difference will also amplify the noise and cause the acceleration to be unreliable. We can perform integral conversion on equation (3) to get:

$$u_{k+1} = \int_{t}^{t+\Delta t} f_{uk}(u_k, v_k, r_k, \dot{r}_k, \dot{\delta}_k)dt + u_k$$

When $\Delta t$ is regarded as constant value, it can be repressed:
According to the current speed of the ship \( (u_k, v_k, r_k, T_k, \delta_k) \) and the command control (thruster and rudder) input \( (T_k, \delta_k) \) at time step \( k \), it can get the speed of the ship \( (u_{k+1}, v_{k+1}, r_{k+1}) \) at time step \( k+1 \).

### 2.2. Gaussian Processes

Define a training set of \( N \) observations as:

- input data : \( X = \{ x_1, x_2, \ldots, x_N \} \)
- output data : \( f = \{ f_1, f_2, \ldots, f_N \} \)

Gaussian Process (GP) [11] is a set of random variables. Any finite number of random variables in the set satisfy the joint normal distribution.

\[
\begin{bmatrix}
  f_1 \\
  \vdots \\
  f_N 
\end{bmatrix} 
\sim \mathcal{N}
\left(
\begin{bmatrix}
  m(x_1) \\
  \vdots \\
  m(x_N)
\end{bmatrix}, 
\begin{bmatrix}
  k(x_1, x_1) & \cdots & k(x_1, x_N) \\
  \vdots & \ddots & \vdots \\
  k(x_N, x_1) & \cdots & k(x_N, x_N)
\end{bmatrix}
\right)
\]

(8)

Gaussian Process can be conveniently specified by its mean function and the covariance function.

\[
m(x) = \mathbb{E}[f(x)]
\]

(9)

\[
k(x, x') = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))]
\]

(10)

In many cases, the measured value of the system is noisy,

\[
y = f(x) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma_y^2 \mathbf{I})
\]

(11)

Then we generate a random Gaussian vector with this covariance matrix,

\[
p(f \mid X) = \mathcal{N}(m(X), k(X, X))
\]

(12)

With these assumptions in place, the likelihood function is,

\[
p(y \mid f, X) = \prod_{i=1}^{N} \mathcal{N}(y_i; f_i, \sigma_y^2)
\]

(13)

Then, combining Eq. (12) and Eq. (13), we can obtain the observations values \( f^* \) at a whole set of test points \( X^* \) according to the joint Gaussian prior distribution.

\[
\begin{bmatrix}
  f \\
  f^*
\end{bmatrix} 
\sim \mathcal{N}
\left(
\begin{bmatrix}
  m(X) \\
  m(X^*)
\end{bmatrix}, 
\begin{bmatrix}
  K(X, X) + \sigma_y^2 \mathbf{I}_N & K(X, X^*) \\
  K(X^*, X) & K(X^*, X^*)
\end{bmatrix}
\right)
\]

(14)

Which leads to the Gaussian Process predictive equations,

\[
p(f \mid X^*, y, X) = p(f \mid y) = \mathcal{N}(\hat{m}, \hat{K})
\]

(15)

\[
\hat{m} = m(X^*) + K(X^*, X)(K(X, X) + \sigma_y^2 \mathbf{I}_N)^{-1}(y - m(X))
\]

(16)

\[
\hat{K} = K(X^*, X^*) - K(X^*, X)(K(X, X) + \sigma_y^2 \mathbf{I}_N)^{-1}K(X, X^*)
\]

(17)

The squared exponential (SE) covariance function was used expressed as,

\[
K(x, x') = \sigma_y^2 \exp\left(-\frac{(x - x')^t \Lambda(x - x')}{2}\right)
\]

(18)

where \( \sigma_y^2 \) denotes signal variance and \( \Lambda \) is the length-scale hyperparameters. Typically, the hyperparameters can be learned according to optimization the maximum likelihood.
3. Test Results and Discussions

3.1. Dynamic modelling by Gaussian Processes
A data-driven nonparametric model based on gaussian process regression was used to describe accurately the relationship between motion state and control force of ship. Gaussian process is generally used for multiple input and single output. Therefore, it is necessary to establish three Gaussian process models in the 3 DoF (surge, sway and yaw). The models are shown in Fig. 1.

![Fig.1 The input and output of gussian process model](image)

According to the current speed of the ship \((u_k, v_k, r_k)\) and the command control(thruster and rudder) input \((T_k, \delta_k)\) at time step \(k\), The Gaussian process models can predict the speed of the ship \((u_{k+1}, v_{k+1}, r_{k+1})\) at time step \(k+1\).

3.2. Train data and SG filter
Accurate measurements of the speeds is important for dynamic modelling of surface ship. However the data is usually disturbed by noise, so it is difficult to know the real value. In order to have the real value as a reference to verify whether the model can make correct predictions under noise interference, this study generates training data as a real value through Marine Systems Simulator (MSS), and adds noise to the data to verify the Gaussian process and the SG filtering algorithm.

The core idea of the SG filter[13] method (Savitzky-Golay Filter) is to perform weighted filtering on the data in the window, which is obtained by least squares fitting a given high-order polynomial to the data in the window.

The experimental design is of great significance to the identification and modeling. The steady-state navigation of the ship cannot effectively stimulate the characteristics of the ship's maneuvering motion system. Zigzag maneuvers and the turning test are often used in traditional mechanism modelling. Compared with zigzag maneuvers, the turning test is not a good choice because that the rudder angle will no longer change and the ship's speed status is also stable. It is of lack of persistence of excitation. \(15^\circ/15^\circ\) Zigzag maneuvers is selected to train the model. Using a time step of 2s and total time of 600s, there are a total of 300 data samples for training. The training data will be affected by Gaussian white noise following the method[12].

Ship data is usually disturbed by noise, so it is difficult to know the real value. In order to have the real value as a reference to verify whether the model can make correct predictions under noise interference, this study generates training data as a real value through Marine Systems Simulator (MSS), and adds noise to the data to verify the Gaussian process and the SG filtering algorithm.
3.3. Validation and discussion

The squared exponential (SE) covariance function was adopted for model expressed as Eq. (18), and the hyperparameters can be learned according to optimization the maximum likelihood. The method of cross validation is hold-out validation, where 90% of the data is used for training and 10% is used for validation.

Fig. 3–5 depicts the 15°/15° zigzag maneuvers test results of the speeds in the 3-DOF motions. It showing higher accuracy and reproduce the ship's motion well, the variance can be seen as a confidence level of the predicted mean.
4. Conclusion
In this study, a data-driven nonparametric bayesian model based on gaussian process regression was used to dynamic modelling of ship, it can capture the strong nonlinearity and motion coupling in ship motion, and deal with the presence of uncertainty and noise. SG filter is used to smooth the data to reduce the influence of noise on modeling. The methodology has been validated with 15°/15° Zigzag maneuvers from a coupled 3-DOF model. It showing higher accuracy and reproduce the ship's motion well, which is beneficial to the further development of ship control design.

Forthcoming work will focus on improving the test plan so that the sampled data covers a wide range of ship motion characteristics. Besides, we will introduce the local gaussian process regression methods in the proposed scheme to calculate large-scale experimental data fast enough.

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