Abstract

We discuss some common misconceptions in Unruh effect [1] and Unruh radiation for the cases of linear and circular uniform acceleration of a charged particle or detector moving in a quantum field. We point to the need to go beyond Unruh effect and develop a new theoretical framework for treating the stochastic dynamics of particles interacting with quantum fields under more general nonequilibrium conditions. This framework has been established in recent years using the influence functional formalism [2, 3, 4] and applied to relativistically moving charged particles [5, 6, 7]. Only with nonequilibrium concepts and methodology applied to particle-field interaction can one grasp the full complexity of the problems of beam physics under more realistic conditions, from electrons and heavy ions to coherent atoms.

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1 Introduction and Summary

In this talk we would like to address two sets of issues, one related to Unruh effect, the other related to moving charges in a quantum field, with the hope of clarifying some misconceptions related to these problems. Unruh effect attests that a detector (made of an oscillator, atom, electron, or particle states of a quantum field) moving with a uniform proper acceleration of magnitude \( a \) sees the vacuum state of a quantum field as a thermal bath with temperature 

\[ T_U = \frac{\hbar a}{2\pi \hbar c k_B}. \]

This effect may be understood purely as a kinematic aspect of ordinary quantum field theory and does not require the notion of horizon, despite the connection with the black hole Hawking effect [8]. It is important to recognize that the Unruh effect is a manifestation of thermal noise in the detector, not radiation from the detector. We explain this point below. The first set of issues of interest are:

1) Is there radiation emitted from a uniformly accelerated detector? This is the title of the other talk by BLH, contained in a summary paper by Hu and Raval in this volume [10]. The simple answer is NO, when the detector has reached a steady state. There is emitted radiation in nonequilibrium conditions associated with transients or nonuniform accelerated motion (though the time for a uniformly accelerating charge to equilibrate may be quite long). One example of nonequilibrium conditions is finite time acceleration. This problem was treated with the influence functional method by Raval, Hu and Koks [3]. The other example of nonequilibrium (though stationary) condition is the case...
of circular motion, to which one can ask the question:

2) Is there a circular Unruh effect \[11\] ? The strict answer is NO, in the sense that the detector undergoing circular motion will NOT detect a thermal bath, and hence there is strictly speaking no associated Unruh temperature. Laboratory (e.g., storage ring) conditions may allow a range of parameters (radius versus angular acceleration) such that a near-equilibrium condition exists, in which case and only in such cases can one use the concept of effective temperature, such as was proposed by Unruh \[12\]. Under general conditions, the moving particle/detector will register a colored noise, (which turns white in linear uniform acceleration), and acquire a stochastic component in its trajectory and other degrees of freedom.

For treating these general cases, one needs to invoke statistical field theory applied to the nonequilibrium dynamics of moving charges or detectors in a quantum field. This is the subject matter of the Ph.D. theses of Alpan Raval and Philip Johnson. A partial summary of the latter work, specifically on the derivation of the Abraham-Lorentz-Dirac (ALD) equation \[13\] and its stochastic counterpart, the ALD-Langevin equation, is contained in our other paper in this volume. To facilitate our discussion of this class of problems, including the “circular Unruh effect”, we need to develop some basic concepts such as backreaction, fluctuations, dissipation and decoherence, and understand the demarcation of quantum, stochastic and semiclassical regimes. For this we bring in the second set of issues:

3) Are radiation reaction (RR) and vacuum fluctuations (VF) related by a fluctuation-dissipation relation (FDR)? The answer is NO, not directly. Is there a FDR at work? YES. But it relates vacuum fluctuations to quantum dissipation distinguished as the quantum backreaction which is over and above the classical radiation reaction. It balances the stochastic component in the particle trajectory so that the noise-averaged mean trajectory follows a semi-classical equation of motion.

4) Are runaway solutions and preacceleration necessary evils of ALD equation? NO, if one adopts the correct conceptual framework and methodology. Key to the resolution of these puzzles is the concept of decoherent history and emergent classical behavior from quantum systems. Vacuum fluctuations not only bring about quantum dissipation, it is also a source for decoherence in the quantum system. Decoherence legitimatizes a classical description such as particle trajectories. We will discuss the gist of these issues in the following sections. Full details can be found in the original papers.

2 Quantum, Stochastic, Semi-classical and Classical

2.1 Quantum Open System

A closed quantum system can be partitioned into several subsystems according to the relevant physical scales. If one is interested in the details of one such subsystem, call it the distinguished system, and decides to ignore certain details of the other subsystems, comprising the environment, the distinguished system is thereby rendered an open-system. The overall effect of the coarse-grained environment on the open-system can be captured by the influence functional technique of Feynman and Vernon, or the closely related closed-time-path effective action method of Schwinger and Keldysh \[14\]. These are initial value formulations. For the model of particle-field interactions under study, this approach yields an exact, nonlocal, coarse-grained effective action (CGEA) for the particle motion \[15\]. The CGEA may be used to treat the nonequilibrium quantum dynamics of interacting particles. However, only when the particle trajectories become largely well-defined (with some degree of stochasticity caused by noise) as a result of decoherence due to interactions with the field can the CGEA be meaningfully tran-
scribed into a stochastic effective action, describing stochastic particle motion. In this program of investigation we take a microscopic view, using quantum field theory as the tool to give a first-principles derivation of moving particle interacting with a quantum field from an open-systems perspective.

2.2 Fluctuation-Dissipation Relations

A consequence of coarse-graining the (quantum field) environment is the appearance of noise which is instrumental to the decoherence of the system and the emergence of a classical particle picture. At the semiclassical level, where a classical particle is treated self-consistently with backreaction from the quantum field, an equation of motion for the mean coordinates of the particle trajectory is obtained. This is identical in form to the classical equation in the case of linearly coupled theories. Backreaction of radiation emitted by the particle on the particle itself is called radiation reaction. (For the special case of uniform acceleration it is equal to zero, due to a balance between the acceleration field and the radiation field [16].) Radiation reaction (RR) is often regarded as balanced by vacuum fluctuations (VF) via a fluctuation dissipation relation (FDR). This is a misconception: RR exists already at the classical level, whereas VF is of quantum nature. There is nonetheless a FDR at work balancing quantum dissipation (the part which is over and above the classical radiation reaction) and vacuum fluctuations. But it first appears only at the stochastic level, when self-consistent backreaction of the fluctuations in the quantum field is included in our consideration. Fluctuations in the quantum field is also responsible for a stochastic component in the particle trajectory (beyond the mean). Their balance is embodied in a set of generalized fluctuation-dissipation relations.

2.3 Decoherent Histories, Preacceleration and Runaway Solutions

Not only can coarse-graining of the environment lead to dissipation in the system dynamics, it is also responsible for the decoherence and emergence of classicality in the system, such as the appearance of a classical trajectory. When the environment is a quantum field and the system decoheres, then quantum fluctuations can act effectively as a classical stochastic noise [17, 18].

The view that semiclassical solutions arise as decoherent histories [19] also suggests a new way to look at the radiation-reaction problem for charged particles. The classical equations of motion with backreaction are the Abraham-Lorentz-Dirac (ALD) equations. The solutions to the ALD equations have prompted a long history of controversy due to such puzzling features as pre-accelerations, runaways, and the need for higher-derivative initial data [20]. It has long been felt that the resolution of these problems must lie in the progenitory quantum theory. But this still leaves open the question of when, if ever, the ALD equation appropriately characterizes the classical limit of particle backreaction; how the classical limit emerges; and what imprints the correlations of the quantum field environment leave. Further questions pertinent to the classical behavior arising from the quantum realm, in the context of a moving charge in a quantum field, include whether the decoherent histories are 1) solutions to the ALD equation, 2) unique and runaway free, and 3) causal (no pre-acceleration). In [7] we show how these puzzles and pathologies, both technical and conceptual, are resolved in the context of the initial value quantum open system approach, and that quantum corrected ALD equations satisfying these criteria describe the semiclassical limit.

3 Radiation Reaction and Vacuum Fluctuations

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3.1 Classical Radiation and Radiation Reaction

Uniformly accelerated charges classically radiate according to the Larmor formula, but experience vanishing RR \([16]\). There is an existing belief that the extra work done on the charge against RR must be the direct source of radiant energy, but this static viewpoint is inappropriate. Fields are dynamical objects and have complex interactions with particles. For example, the acceleration field has been shown to do work on charges (and visa versa) and therefore one can not expect a detailed balance between particle and radiation energy alone since that would require a “freezing” out of the near and intermediate field degrees of freedom in a way incompatible with locality and causality.

3.2 Quantum Radiation and Vacuum Fluctuations

Let us now examine the quantum properties of this system. Our result based on self-consistent backreaction says that the stochastic equations when averaged over the noise distribution (noise-average) gives the (mean-field) semiclassical form. In the uniform acceleration case with linear coupling the expectation value of the field (quantum mean) is exactly the same as the classical value where the particle/detector is treated as a “classical” source, though the mean particle trajectory must be self-consistently determined as we have emphasized. At the stochastic level, the particle detector does fluctuate in its worldline, and other degrees of freedom. How does this stochastic component affect the field? As shown by Ravel, Hu and Anglin \([2]\) (for an alternative derivation, see \([14]\)), fluctuations in a detector modify the near field correlations – a polarization cloud is found around the detector trajectory. The same is true for stochastic particle motion in the linearized regime. This quantum effect of modified field correlations adds on to the average classical field value (the two-point function is different from the free field value).

By extrapolating the RHA results to 3+1 dimensions, one may see that these altered field correlations showing up as vacuum polarization drop off faster than \(1/r^2\) and hence are not seen by observers at infinity \([21]\). Since the equivalence of a quantum mean to the classical value holds only under the one-loop, Gaussian approximations, when these conditions are lifted, there may be new effects as yet undiscovered.

Whether there is quantum-corrected radiation from a nonuniformly accelerated charge or detector is therefore what one should focus on here when one asks a question like “Is there emitted radiation in Unruh effect?” Our result obtained with self-consistent backreaction of quantum fluctuations shows that the (noise-averaged) of a decohered particle trajectory obeys the ALD equation, which is known to be consistent with the classical Larmor formula (if one include the nonlocal acceleration field effects, as one must). This applies to any accelerated trajectory, uniform or nonuniform, which implies that there is no additional “extra” average radiation in the semiclassical/stochastic regime beyond the usual classical quantity, even though there are fluctuations (noise) induced in the particle (the Unruh effect in the uniform acceleration case). It has been verified that the presence of detector fluctuations is not inconsistent with the absence of additional radiation.

When quantum decoherence is incomplete, the mean-field equations of motion for both radiation and particle have quantum corrections (an example of this is Schwinger’s synchrotron radiation calculation \([22]\)) which must be included to answer questions beyond the semiclassical or stochastic domain.

3.3 Nonequilibrium quantum dynamics of charges

One major improvement of our approach to the problem of moving charges in a quantum field is the consideration of full backreaction of the quantum field on the particle in the determi-
Dynamical backreaction ensures self-consistency between the particle/detector and the quantum field. The lack thereof is where many of the problems and paradoxes arise. We also find that conceptual issues are easier to consider if we deal with such problems at four distinct levels: quantum, stochastic, semiclassical and classical, as explained earlier. Confusion will arise when one mixes physical processes of one level with another without knowing their interconnections, such as drawing the equivalence between radiation reaction with vacuum fluctuations. Before summarizing our thoughts for processes under nonequilibrium conditions, which cover most cases save a few special yet important ones, such as uniform acceleration, let us remark that these well-known cases are what we would call ‘test field’ or prescribed (trajectory) cases and not self-consistent or backreaction-sensitive. These cases are easier to study because they possess some special symmetry, such as is present for the uniform acceleration case (Rindler spacetime), inertial case (Minkowski), or the eternal black hole case (Killing tensor). They are legitimate only if the backreaction of the field on the particle permits such solutions. Under these special conditions, a detector feels a thermal bath (in the inertial case it is the zero-temperature vacuum).

Let us analyze the physics of nonequilibrium processes at separate levels:

**Classical** level- the decohered self-consistent (mean) solutions for particle and field. If the system is sufficiently coarse-grained and decohered, the particle obeys classical equations of motion, such as the ALD equation from QED. There is no Unruh effect because it is quantum in nature (at the classical level the effect of quantum fluctuations are averaged out).

**Semiclassical** level – defined as a classical system (particles or detectors) interacting with a quantum field. Coarse-graining over quantum field for reduced particle dynamics at one-loop gives back the classical equations of motion for the mean trajectory of the particle. Higher-order quantum corrections arising from nonlinearities modify the mean of the quantum equations of motion for the particle. Quantum corrections may not however show up significantly at the low energy macroscopic description because decoherence tends to suppress these higher-order (e.g., higher-loop) nonlinear quantum effects.

**Stochastic** level - where fluctuations of the quantum field manifest as stochastic noise in the system dynamics. Coarse-graining the field (to some but not the fullest –classical –extent), one obtains a classical stochastic equation for the system (such as the Einstein Langevin equation for semiclassical stochastic gravity or the ALD-Langevin equation for QED). It is possible to encode much of the quantum statistical information of the field and the state of motion of the system in the noise correlator and the two point function of the particle. Thus effects of both quantum (field environment) and kinematic (particle system) nature show up as a stochastic component in the particle trajectory which is self-consistently determined. The stochastic equations of motion have a quantum dissipation term (not classical radiation reaction!) that balances the quantum fluctuations, and is governed by a FDR. The latter is described by the noise kernel, which for general conditions is nonlocal, entailing that the noise in the detector is colored and temperature is no longer a viable concept.

4 ‘Circular Unruh Effect’ – Misconceptions

We now apply these ideas to discuss radiation from a particle in circular motion in a quantum field and in particular we address two common sets of misconceptions related to it. (We only present the main points here, see for calculations and further discussions.) These misconceptions arise from unclear distinction between a) linear uniform acceleration and circular motion, b) thermal radiance felt by the detector/charge in uniform acceleration (Unruh ef-
fect) versus emitted radiation (misconjured as Unruh ‘radiation’) sensed by probes afar, and c) emitted radiation of classical and quantum origin.

It has been asserted that Unruh radiation is already observed in storage rings \[11\]. This is the so-called circular Unruh effect. For this discussion we assume that RF fields give the particle average circular (steady state) motion by restoring the energy loss from synchrotron radiation. Questions:

4.1 Is there a circular Unruh effect?

NO. In fact, the circular case displays nonequilibrium (albeit steady state) quantum field statistics that are more general than the linear uniform (thermal) Unruh case. There is a difference between linear acceleration and angular acceleration. Just from dimensional grounds, there is only one parameter in the linear case, the proper acceleration \(a\), but two in the circular case, the angular acceleration \(\alpha\) and the radius of the orbit \(R\). In the linear case, as the velocity of the particle increases to the speed of light, an event horizon forms. In the circular case, the direction of velocity changes but its magnitude remains constant, there is no event horizon. (Invoking Kerr metric to describe circular motion is unnecessary and misleading, as the problem is basically about kinematics in relativistic quantum field theory.)

4.2 Is temperature a viable concept?

NO. To the extent that the existence of an event horizon is the condition for the appearance of an Unruh or Hawking temperature (this is the traditional argument based on global geometry \[26\], the modern one is via kinematic effect, which enables one to consider nonequilibrium conditions \[8\]), one can already see that there is no well-defined Unruh temperature in circular motion. For circular motion one needs to incorporate the effect of a second physical scale other than acceleration (e.g., the radius). If the system is in near-equilibrium conditions, one can introduce an 'effective (frequency dependent) temperature' \[12\].

4.3 Emittance and Vacuum fluctuations

A related point is the emittance (spread) of particle beams, which is commonly understood to result from quantum field-induced fluctuations. One can treat beam emittance without invoking temperature or Unruh effect. For general cases there is no need for temperature to play the intermediary between quantum field and induced beam fluctuations (on this point we concur with Jackson \[27\]).

Beam emittance is indeed the working of kinematic effects (particle motion) on vacuum fluctuations (quantum noise). (For viewing Hawking-Unruh effect in this light see \[8\]). Beams in linear uniform acceleration are expected to show thermal spread (neglecting possible sources of non-thermal noise). Beams in circular motion do not come into thermal equilibrium, though they may achieve a steady state balance between vacuum fluctuations and quantum dissipation. Our prediction is that the detector (a particle with internal degrees of freedom such as an electron with spin) will see colored noise whose correlator is related to the nonthermal electron populations in their two polarization states. This is more general than the Unruh effect as it is under nonequilibrium conditions.

4.4 Isn’t synchrotron radiation Unruh radiation?

No. Synchrotron radiation occurs for classical systems (where there is no \(\hbar\)); or arises in the semiclassical limit of quantum systems where quantum noise has been averaged out. The Unruh effect is thermal radiance in the system arising from quantum fluctuations; it is seen in the stochastic and quantum limit. One argument views synchrotron radiation as the scattering of virtual vacuum fluctuations into real photons by a moving charge. But in the Unruh effect there
is no radiation after the system has equilibrated, yet there are thermal fluctuations in the particle. This highlights the distinction between emitted radiation (synchrotron or Larmor) and thermal radiance felt by the particle/detector (Unruh effect). There is no direct link between the classical limit of radiation and the quantum Unruh effect; but at the stochastic level a FDR relates quantum dissipation and vacuum fluctuations [7].

4.5 Is there emitted quantum radiation from the charge?

At the stochastic level there is nonequilibrium noise in the particle/detector; these fluctuations alter field correlations around the particle trajectory as a polarization effect [3]. At the quantum level one can use the open system approach but coarse-grain the particle, and determine the quantum corrections to radiation. Take note that quantum corrections modifying both the mean-field radiation and noise-average trajectory must be found self-consistently. The result should be compared with Schwinger’s [22] and/or the quasi-classical operator method because discrepancies, if any, will be of considerable interest.

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