Monitoring voltage collapse margin with synchrophasors across transmission corridors with multiple lines and multiple contingencies

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Abstract—We use synchrophasor measurements of the complex voltage and current at both ends of multiple transmission lines to monitor the online voltage collapse margin. A new reduction is used to reduce the multiple transmission lines to a single line equivalent and determine how to combine the synchrophasor measurements. Generator reactive power limits can be accommodated. The results show that this methodology can capture the effect of multiple contingencies inside the transmission corridors, giving awareness to the operators about the severity of contingencies with respect to voltage stability.

Index Terms—Area angle, area voltage, contingency analysis, maximum loadability, phasor measurement units, power system security, smart grid, Thévenin equivalent, voltage stability.

I. INTRODUCTION

As the load is concentrated in some areas, and abundant generation is generally distant from the load, some areas export or import bulk power through transmission corridors. Contingencies and large transfers of power through the corridors increase the risk of voltage collapse and blackout. For these reasons, it is useful to monitor the margin to voltage collapse from measurements, so that the operator can take prompt action to restore the margin if it becomes too small.

Of course, voltage collapse under n-1 contingency can be assessed based on power flow analysis and the state estimator [1]-[20], but there is scope for quickly monitoring multiple outages based on synchrophasor measurements. Multiple outages are prone to occur during bad weather or cyber-physical attacks. Nowadays, for reasons of cost and computational time, multiple contingencies cannot be feasibly evaluated pre-contingency in a systematic way using power flow or the state estimator [21].

In this paper, we address online operational advice about the voltage margin for multiple contingencies in transmission corridors that are connecting areas. Namely, we propose a complementary methodology for evaluating online the contingencies that are not covered under pre-contingency n-1 analysis, giving awareness and recommending control center actions to remedy the problems of voltage stability.

Since the late nineties, researchers have made vigorous efforts to develop methods based on synchrophasor measurements to detect voltage stability problems in real time [22]. However, these approaches are based on a corridor with a single line, and there are difficulties in applying the methods to corridors with multiple transmission lines. Previously in [23] we addressed this difficulty by proposing a way to combine synchrophasor measurements using the area angle, that approximately reduces a transmission corridor with multiple lines to an equivalent single line, so that the known methods for measuring voltage stability margin for a single line could be applied. This reduction using the area angle does not yet accommodate generator reactive power limits, and our initial experience is that the different reduction used in this paper can give a more accurate estimate of the margin. We continue to explore ways to handle the problem of multiple corridors and lines in this paper because it is a key barrier to applying synchrophasor measurements to avoid voltage collapse in practical power transmission systems that often have multiple corridors joining generation and load areas.

Our new methodology reduces multiple lines of transmission corridors to a single line using synchrophasor measurements of complex power and current at each end of each line in the transmission corridor. The multiple transmission corridors are reduced to a single line while preserving the complex power and currents entering and leaving the corridors, and this reduction shows how the synchrophasor measurements should be combined. Then the known methods for measuring voltage stability margin for a single line can be applied to the combined synchrophasor measurements. This paper presents the complex power reduction and demonstrates how the combined synchrophasor measurements can be used to measure the voltage collapse margin online under unusual increments of load, line outages, or generation outages. We note that a reduction methodology preserving current and complex power was also previously used in the context of reduced dynamic models for oscillations in [24].

The paper is structured as follows. Section II reviews
previous work. Section III shows how to combine the synchrophasor measurements and reduce the system to a single line including reactive limits. Results for the WSCC 9-bus test system are presented in Section IV, and Section V concludes the paper.

II. PREVIOUS WORK

A. Contingency analysis

This subsection reviews some of the extensive previous work on off-line n-1 contingency screening for voltage collapse, see [1]-[20], and references therein.

Reference [2] describes the first procedure for finding the buses with potential voltage stability problems under contingencies, which is based on power flow and the modifications in reactive power consumption of the remaining system.

An efficient methodology for evaluating the change of the voltage collapse margin under contingencies is indicated in [9]. This method is based on the current operating point and the pattern of load increase determined by the load forecast. The bifurcation and the load margin are computed and the load margin sensitivity to line outages is evaluated. Reference [12] improves this sensitivity method by using a continuation power flow decreasing the admittance of the outaged line. A new index is proposed that includes the maximum load flow of the remaining lines and the sensitivities of the load margin under line outages. The method is further improved in [14] by using a two parameter continuation power flow, where one parameter is the load increase pattern and the other parameter controls the line outage. In addition, this method includes reactive limits of the generators giving more realistic and accurate results.

The approach that we present in this paper differs from and is complementary to [1]-[20] in using post contingency measurements online rather than pre contingency calculations based on the state estimator. Our measurement-based approach is less accurate than state estimator methods, but can work independently of the state estimator, is fast, and handles multiple contingencies more easily.

B. Measuring voltage collapse margin with synchrophasors

For monitoring voltage stability online, many researchers have made vigorous efforts to apply synchrophasor measurements [22]. However, the initial approaches were based on a corridor with a single line, and there are difficulties in directly applying the methods to corridors with multiple transmission lines. It can be noted that applying the single line methods by increasing one load while the other loads remain constant is an obviously unrealistic system stress.

Due to these problems, some researchers recognized the importance of developing an online voltage stability tool for a system with multiple transmission lines [23], [26], [27]. However, those approaches require strong assumptions, such as known admittance between the loads or generators, making it impossible to capture changes on the transmission corridor, or assuming known complex voltage in the generator. These assumptions can generate inaccurate results, especially during multiple contingencies. In order to help solve these problems, our previous work in [23] proposed an initial way to combine synchrophasor measurements based on area angle that approximately reduces a corridor with multiple lines to an equivalent single line, giving a more justifiable and accurate indication of the margin to voltage collapse.

One general problem with the aforementioned methodologies is the reactive power limits of the generators. When a generator bus that is considered as PV changes to PQ, the maximum transfer of power through the transmission corridor is reduced substantially. Some previous approaches for including reactive limits that are different than our methodology for including reactive limits are in [22], [28], [29].

III. REDUCTION OF TRANSMISSION CORRIDOR WITH MULTIPLE LINES TO A SINGLE LINE SYSTEM USING COMPLEX POWERS

The reduction to a single line equivalent is done for a transmission corridor with n inputs (generators at bus g1 to gn) and n outputs (loads at buses ℓ1 to ℓn), that will be reduced to an equivalent system with one input and one output, see Fig. 1.

The transmission corridor includes all the lines that are connecting the generator area with the load area. The generation area could have loads, but it has a net injection of power, and in the same way the load area could have generators but it is...
a net load. The reduction of the corridor to an approximately equivalent single line enables the application of synchrophasor monitoring.

The complex currents and voltages \( I_{g1}, \ldots, I_{gn}, I_{f1}, \ldots, I_{ft}; V_{g1}, \ldots, V_{gn}, V_{f1}, \ldots, V_{ft} \), are obtained from the PMUs at all the buses that bound the transmission corridor. Then the complex power is obtained from the measured complex currents and voltages:

\[
S_{gi} = V_{gi}^* I_{gi}, \quad S_{fi} = V_{fi}^* I_{fi}. \tag{1}
\]

The complete system will be reduced to a single line equivalent system while preserving the complex powers entering and leaving the corridors. In other words, all the power that is entering (leaving) the transmission corridor is equal to all the power that is entering (leaving) the equivalent system.

\[
S_g = \sum_{i=1}^{n} S_{gi}, \quad S_f = \sum_{i=1}^{n} S_{fi}. \tag{2}
\]

Similarly, the complex current entering (leaving) the transmission corridor is equal to the complex power entering (leaving) the equivalent system:

\[
I_g = \sum_{i=1}^{n} I_{gi}, \quad I_f = \sum_{i=1}^{n} I_{fi}. \tag{3}
\]

Based the complex powers and complex currents of the transmission corridor and its equivalent, the voltages of the equivalent system are

\[
V_g = \frac{S_g}{I_g^*}, \quad V_f = \frac{S_f}{I_f^*}. \tag{4}
\]

Then the voltage across, and admittance of the equivalent are:

\[
V_{gi} = \frac{S_{gi}}{I_{gi}}, \quad V_{fi} = V_{gi} I_{gi}. \tag{5}
\]

A benefit of this new reduction is that all the loads can change independently, making the model more realistic. In addition, we do not need to assume any admittance as known, which is very useful for online application, and for tracking the changes of the system such as contingencies.

### IV. Methodology for Evaluating Voltage Collapse Margin Across the Transmission Corridor with Synchrophasors

The methodology that we present in this section captures the effect of the outages in the transmission corridor through the tracking of the voltage collapse margin. To obtain the voltage collapse margin across the transmission corridor, we require a PMU at both ends of the transmission lines that form the corridor. As the transmission corridors are composed of relatively few lines, we estimate that the number of PMUs required is between six and twenty, which is a feasible number of PMUs.

In order to locate the bifurcation point of the system we assume a stable initial operating equilibrium and a slowly varying parameter, which is the increment of the load power varying slowly compared with the dynamics of the system. Under these assumptions, the power system can be modeled by static equations. In addition, to apply the usual synchrophasor monitoring approaches, the model is assumes PV generation buses and PQ load buses.

Generator reactive power limits are handled by sensing when generators reach their reactive power limits and changing that PV bus to a PQ bus in the transmission corridor model. That is, the generator with reactive power limits is modeled as a negative load. The appropriate signals indicating generator reactive power limits can be obtained from standard control center signals or by processing PMU measurements at the generator.

The procedure is as follows:

1) Measure with PMUs the complex voltage and current at both ends of all the transmission lines that connect the generation area with the load area, see Fig. 1.

2) Check the reactive power limit signal of the generation buses. In case that the generator bus reaches its reactive limit then the bus is considered as PQ. For example, in this case we are considering that bus \( g2 \) of the system shown in Fig. 1 reaches its reactive limit.

3) Use the synchrophasor measurements to find the complex power of the generation area and the load area. The generation bus with reactive limits that changed to a PQ bus is treated as a negative load:

\[
S_g = \sum_{i=1}^{n} S_{gi} - S_{g2}, \quad S_f = \sum_{i=1}^{n} S_{fi} + S_{g2}. \tag{6}
\]

4) Combine the complex current into the equivalent single line current:

\[
I_g = \sum_{i=1}^{n} I_{gi} - I_{g2}, \quad I_f = \sum_{i=1}^{n} I_{fi} + I_{g2}. \tag{7}
\]

5) Using the complex power of each area and the current to find the voltages of the reduced system, see (4).

6) Evaluate the voltage stability index from (8).

\[
\text{Index} = \frac{|V_{gi}|}{|V_f|} \times 100. \tag{8}
\]

Index (8) indicates the maximum transfer of load that can be achieved across the transmission corridor that connect the areas under the measured condition of the corridor. Using this index, an alarm can be triggered when a sufficient percentage of the index is exceeded. For example, the alarm could be triggered when the index exceeds 80%.

### V. Results

#### A. Evaluation of voltage collapse margin under multiple contingencies for WSCC 9-bus test system

In this section, we study the WSCC 9-bus test system shown in Fig. 2. We divide this system into two areas, the generation area and the load area. Each area is composed of three buses, and the areas are connected by six lines which form the transmission corridor. This system requires six PMUs to measure the complex voltage and current in all six buses bounding the transmission corridor.
In order to explain better the reduction, we first evaluate the voltage stability margin of the WSCC 9-bus test system without a contingency, see Table I. For this, we measure with the PMUs the complex voltage and currents in all the buses bounding the transmission corridor, and calculate the complex power entering or leaving at each bus. Combining the complex powers and current for the generation and load area, we obtain the equivalent voltage for each area, reducing the transmission corridor to a single line system to which the available methods for voltage stability in radial system can be applied.

Now we simulate n-1, n-2 and n-3 contingencies without shedding load to show the effect of contingencies on the voltage stability margin, see Table II. In real time, when the contingency occurs, the PMU measurements will track the changes in the voltage and current, then we use the methodology for reducing the corridor to a single line system and evaluate the voltage stability margin. This procedure is updating constantly to track the voltage stability margin across the transmission corridor.

The results shown in Table II demonstrate that multiple outages can generate voltage stability problems. For example, under n-1 the highest voltage stability index is 28%, however with an additional outage the voltage stability index can increase drastically to 85%, indicating severe problems.

During real operation, it is desirable to take remedial action promptly tracking the voltage stability margin, covering multiple contingencies in real time. For this, is necessary to define a security limit margin, which the operator should maintain in order to avoid voltage stability problems and blackout. In this way, if under any contingency the limit is violated, the operator should decrease the transfer of power across the corridor or in more severe cases shed load. For example, if we consider the security margin as eighty percent, under contingency number 20, the system operator should take action in order to maintain the security level required, see Table II.

Additionally, in Table II we evaluate the accuracy of this approach. We contrast the voltage stability index using the synchrophasor measurements with the exact answers for the stability margin obtained using the well known continuation power flow. These results shown that our method for combining multiple lines using measurements is a reasonable approximation, with an error in the index less than 15%. The error reduces for the more highly stressed cases of interest. For future work, we will analyze and explain the origin of the error.

VI. CONCLUSION

We show how to reduce multiple lines in several transmission corridors to a single line equivalent to which online monitoring of voltage stability with synchrophasors can be applied. The reduction is based on synchrophasor measurements of complex power and current at both ends of the lines, and the reduction shows how to combine the synchrophasor
measurements so that they are effective in monitoring voltage stability.

The approach can give a fast, online indication of voltage stability that can accommodate both multiple contingencies and generator reactive power limits. These capabilities should increase operator situational awareness under emergency conditions, and should be complementary to methods that make pre-contingency calculations from a model based on the state estimator.

The new methodology for analyzing online voltage stability margin for multiple transmission lines and multiple contingencies was tested in the WSCC 9-bus system. Our results suggest that we have found a promising and systematic approach for online monitoring of voltage stability margin for multiple transmission lines and multiple contingencies. Planned future work will generalize the methodology for corridors that include load or generation inside the corridors and analyze the approximations made.

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REFERENCES

[1] K. Nara, K. Tanaka, H. Kodama, R. Shouls, M.-S. Chen, P. Van Olinda, and D. Bertagnolli, “On-line contingency selection algorithm for voltage security analysis,” IEEE Power Engineering Review, vol. PER-5, no. 4, pp. 41–42, April 1985.

[2] G. Ejebe, H. Van Meeteren, and B. Wollenberg, “Fast contingency screening and evaluation for voltage security analysis,” IEEE Trans. Power Systems, vol. 3, no. 4, pp. 1582–1590, Nov 1988.

[3] N. Hadjsaid, M. Benahmed, J. Fandino, J. Sabonnadiere, and G. Nerin, “Fast contingency screening for voltage-reactive considerations in security analysis,” IEEE Trans. Power Systems, vol. 8, no. 1, pp. 144–151, Feb 1993.

[4] G. Ejebe, G. Iraisari, S. Mokhtari, O. Obadina, P. Ristanovic, and J. Tong, “Methods for contingency screening and ranking for voltage stability analysis of power systems,” in Power Industry Computer Application Conference, May 1995, pp. 249–255.

[5] H.-D. Chiang, A. Flueck, K. Shah, and N. Balu, “Cflow: a practical tool for tracing power system steady-state stationary behavior due to load and generation variations,” IEEE Trans. Power Systems, vol. 10, no. 2, pp. 623–634, May 1995.

[6] G. Ejebe, G. Iraisari, S. Mokhtari, O. Obadina, P. Ristanovic, and J. Tong, “Methods for contingency screening and ranking for voltage stability analysis of power systems,” IEEE Trans. Power Systems, vol. 11, no. 1, pp. 330–356, Feb 1996.

[7] H.-D. Chiang, C.-S. Wang, and A. Flueck, “Look-ahead voltage and load margin contingency selection functions for large-scale power systems,” IEEE Trans. Power Systems, vol. 12, no. 1, pp. 173–180, Feb 1997.

[8] S. Repo and P. Jarenautasa, “Contingency analysis for a large number of voltage stability studies,” in International Conference on Electric Power Engineering, PowerTech Budapest, Aug 1999, pp. 34–

[9] R. Greene, I. Dobson, and F. Alvarado, “Contingency ranking for voltage collapse via sensitivities from a single nose curve,” IEEE Trans. Power Systems, vol. 14, no. 1, pp. 232–240, Feb 1999.

[10] A. Flueck and J. Dondeti, “A new continuation power flow tool for investigating the nonlinear effects of transmission branch parameter variations,” IEEE Trans. Power Systems, vol. 15, no. 1, pp. 223–227, Feb 2000.

[11] I. Musirin, T. Khawa, and A. Rahman, “Simulation technique for voltage collapse prediction and contingency ranking in power system,” in Student Conference on Research and Development, 2002, pp. 188–191.