We study the implications of asymmetric dark matter on neutron stars. We construct a "mixed neutron star" model composed of ordinary baryons and of asymmetric dark matter baryons. We derive the general relativistic structure equations for each specie, the equation for the mass within a given radius, and the redshift as function of radius. We present one specific numerical model as an illustrative example. In this example, the mass of the dark neutron equals half that of the ordinary neutron. The main results are: a total mass of 3.74$M_{\odot}$, a total mass within the neutron-sphere equaling 1.56$M_{\odot}$, the neutrons mass is 1.34$M_{\odot}$, the star radius is 31.9 km, the neutron-sphere radius is 11.1 km, and the redshifts from the neutron-sphere and from the star surface are 0.72, 0.25, respectively. We comment briefly on possible astrophysical implications.

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1. Introduction

1.1. Background

Cold dark matter which is favored by most astrophysical and cosmological observations can be realized in symmetric and (or) asymmetric scenarios. In the first class of models, dark matter is made of stable $X$ particles and an equal amount of $\bar{X}$ antiparticles of mass $m_X$. In the early universe, these were in thermal equilibrium and their residual abundance $\Omega_X$ is fixed at the "freeze-out" value when the rate of the Hubble expansion overcomes that of $\bar{X} - X$ annihilation rate. A prototypical example which have been most extensively studied is SUSY (super-symmetry). Unfortunately, the searches for SUSY partners in the new large hadron collider (LHC) have failed to detect them. More specifically, insofar as SUSY dark matter models are concerned searches for electrons, positrons or photons in clumped dark matter

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in and around our Galaxy, and for energetic neutrinos resulting from annihilations of Xs do not provide solid "indirect" evidence for dark matter. Moreover, the ongoing direct underground searches put very strong bounds on the scattering crosssections of massive Xs on nuclei.

Additional constraints are related to accretion of galactic SUSY WIMPS onto the sun. The increased density of the captured WIMPS, accelerates the rate of particle-antiparticle annihilation. The resulting photons and electrons are trapped in the sun but the resulting ultra high energy neutrinos are not. Data from the ICE-CUBE Cerenkov radiation detector near the south pole, severely constrain such models \[1, 2\]. In the case of asymmetric dark matter, there will be no such annihilation in the sun. Moreover, once the fraction of dark matter particles in the sun exceeds the ratio of $\sigma_{Xn}/\sigma_{XX}$, scattering on the already captured X particles in the sun dominates over scattering on the baryons.

Finally, the observed number of satellite mini-halos around the milky way galaxy is two orders of magnitude smaller than predicted within the symmetric dark matter framework \[3, 4\].

Consequently there has been, in the very recent years, a renewed interest in asymmetric dark matter (ADM) which just like ordinary baryonic matter, is charge non-symmetric with say only the dark baryon (or generally only the particle) excess remains after the annihilation of most antiparticles. While there is no single clear-cut explanation for the ordinary baryonic asymmetry, the required dark matter density is readily achieved if the mass ratio of the X particles and baryons $m_X/m_b$ is tuned inversely with the corresponding ratio of asymmetries. An early example of such a model has been proposed in \[5\]. For recent studies of asymmetric dark matter see \[6, 7\] and references therein.

1.2. Cosmological and astrophysical considerations

The idea of asymmetric dark matter should reconcile with astrophysical and cosmological data. An obvious constraint is imposed by big-bang nucleosynthesis, which implies that the mirror neutrinos and photons do not contribute to the rate of the cosmic expansion at that era. Another constraint is that the mirror large scale structure-formation should precede the recombination of ordinary matter, in order to serve as seeds gravitational potential wells for the latter. In the model described in \[6\] the mirror neutrinos and the mirror photons are massive enough for these constraints to be satisfied. We note that even if the mirror neutrinos and photons are massless the above two constraints can be satisfied provided that the dark cosmic-temperature is lower than the ordinary one.

The details of large scale structure formation will depend on the specific
model for the asymmetrical dark matter. Generally, being self interacting one may expect it to form mirror structures on all astrophysical scales. Also, asymmetric dark matter will no longer provide collisionless dark halos as in the symmetric cold dark matter case, so that the flatness of galactic rotation curves will have to be readdressed in this new context.

2. Composite "mixed neutron star"

2.1. Motivation and scope of present work

The mirror dark matter can form gravitationally bound structures. In particular they can form "dark neutron stars". The latter can accrete ordinary matter and form composite neutron stars. One may contemplate additional formation scenarios for the composite neutron stars. In this paper, we focus on the structure of such a compact object and put aside issues related to and constraints following from formation scenarios, and the broader questions regarding the mirror astrophysics and cosmology. This is an interim report on a work in progress [8].

Such objects can have masses larger than those of ordinary neutron stars, while otherwise having very similar observational signatures. The recent discovery of a $2M_\odot$ binary radio pulsar [9], already severely constrains nuclear matter equations of state. A future observation of a neutron star with a mass exceeding $3M_\odot$ would be very difficult to reconcile with an ordinary neutron star but will pose no problem for a mixed neutron star.

The energy momentum tensor is taken to be that of two ideal fluids

\[ T^{\mu\nu} = (\rho_1 + p_1)u^\mu_1 u^\nu_1 - p_1 g^{\mu\nu} + (\rho_2 + p_2)u^\mu_2 u^\nu_2 - p_2 g^{\mu\nu} \]  

Since any inter-specie interaction must be weaker than the intra-specie interaction by a factor $\geq 10^{12}$, each energy momentum tensor is separately conserved. In addition, the ordinary and dark baryon numbers are separately conserved.

2.2. Structure equations

We look for a spherically symmetric static solution of a 2-fluid "mixed neutron star". The line element squared of a spherically symmetric static metric can be written in the Schwarzschild coordinates $(t, r, \theta, \phi)$ as

\[ ds^2 = e^{2\Phi(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 \left( d\theta^2 + \sin^2(\theta) d\phi^2 \right) \]  

The Einstein field equations that determine the metric, together with the separate covariant conservation laws can be shown [8] to lead to the following structure equations:
\[ e^{-2\lambda(r)} = \left(1 - \frac{2Gm(r)}{c^2 r}\right) \] (3)

where \( m(r) \) is the mass enclosed within \( r \) given by

\[ m(r) = \int_0^r 4\pi \left(\rho_1(r') + \rho_2(r')\right) r'^2 dr' \] (4)

For each of the species there exists a hydrostatic equilibrium equation:

\[ \frac{dp_1(r)}{dr} = -G\left(\rho_1(r) + p_1(r)\right) \frac{m(r) + 4\pi r^3 \left(p_1(r) + p_2(r)\right)}{r \left(r - 2Gc^{-2}m(r)\right)} \] (5)

\[ \frac{dp_2(r)}{dr} = -G\left(\rho_2(r) + p_2(r)\right) \frac{m(r) + 4\pi r^3 \left(p_1(r) + p_2(r)\right)}{r \left(r - 2Gc^{-2}m(r)\right)} \] (6)

and \( \phi(r) \) satisfies:

\[ \frac{d\phi(r)}{dr} = -\left(\rho_1(r) + p_1(r)\right)^{-1} \frac{dp_1(r)}{dr} = -\left(\rho_2(r) + p_2(r)\right)^{-1} \frac{dp_2(r)}{dr} \] (7)

The equations imply that each fluid satisfies its own hydrostatic equilibrium equation which is of the form of a modified TOV equation \cite{10, 11}. The two fluids are coupled through \( m(r) \) and through the total pressure \( p_1(r) + p_2(r) \).

### 2.3. Solution procedure

Given the two equations of state, and the two central energy densities, the TOV equations \cite{15, 16} are integrated up to \( r = R_1 \) where \( p_1(R_1) = 0 \). Specie 1 is confined within this radius. From this radius on, only TOV2 (equation 6) is integrated until the radius \( R_2 \) where \( p_2(R_2) = 0 \). This is the star radius. Once the solution is obtained, equation (7) is solved with the boundary condition \( \Phi(R_2) = \frac{1}{2} \ln \left(1 - \frac{2G m}{\frac{c^2}{r_{R_2}}}\right) \), with \( m = m(R_2) \) being the mass of the mixed neutron star.

In this way, a two-parameters (the central densities) family of static models is obtained, in contrast with the ordinary neutron star models that form a one-parameter (the central density) family.
3. An illustrative example

To illustrate the model characteristics we present here a generic example. We employ a nuclear matter equation of state that was obtained by fitting observational data of x-ray bursters [12]. Fig. 1 displays the pressure and the baryon number as functions of the energy density. The maximal ordinary neutron star mass for this equation of state is $2.44M_\odot$ and the corresponding radius is 11.7 km.

For the dark baryons we use the same equation of state as for the neutrons with the appropriate scaling, taking into account baryons masses ratio. This choice seems to be the minimal and simplest one.

The ordinary neutrons star masses satisfy $m \simeq m_{\text{pl}}^3 m_b^{-2}$ with $m_{\text{pl}}$ denoting the Planck mass. Therefore, in order to obtain a mixed neutron star with mass larger than ordinary neutron stars, it is required that $m_D < m_b$ where $m_D$ is the dark baryon mass. In this example we chose $m_D = 0.5m_b$, so that the dark equation of state is

$$p_2(\rho_2) = \frac{1}{16} \rho_1(16\rho_2)$$

The mass of a pure dark neutron star will be $\sim 8M_\odot$ and the corresponding radius is $\sim 50km$. It is expected that the mixed neutron star solution would yield a mass, and radius intermediate between those of a neutron star and a pure dark neutron star.

The values of the central energy densities in this example are:

$$\rho_1(0) = 600 \text{ MeV fm}^{-3}, \quad \rho_2(0) = \frac{1300}{16} \text{ MeV fm}^{-3}$$
4. Computation results

The computation results are summarized in Table 1. The first row contains the star total mass, the dark mass, the neutrons mass, and the total mass within the neutron-sphere. The second row specifies the star radius, the neutron-sphere radius, the redshifts for these two radii, and the neutrons binding energy. The detailed r-dependence of the energy densities, the enclosed mass $m(r)$, and $\Phi(r)$ are displayed in figures (2) and (3), respectively.

Table 1. Model Results

| $m$            | $m_{\text{dark}}$ | $m_{\text{neutrons}}$ | $m(R_1)$ |
|----------------|-------------------|------------------------|----------|
| $3.74M_\odot$  | $2.4M_\odot$     | $1.34M_\odot$         | $1.56M_\odot$ |

| $R_2$ | $R_1$ | Redshifts | Neutrons BE |
|-------|-------|-----------|-------------|
| 31.9 km | 11.1 km | $z(R_1)=0.72$, $z(R_2)=0.25$ | 22% |

Fig. 2. energy densities (left) and enclosed mass (right) as function of radius.

5. Discussion

We have demonstrated that a mixed neutron star can, as expected, have a mass higher than ordinary neutron stars. At the same time the physical radius, as probed by ordinary massless and massive particles, is the neutron-sphere radius which is similar to the radius of ordinary neutron stars.

An important question, not addressed here, is that of stability. Since the models form a two-parameters family (the central densities) the question of stability is more complex than in the one parameter family of ordinary
neutron star models. There are a number of quite interesting astrophysical implications with regard to phenomenology of compact x-ray sources. Can some of the stellar mass binary black holes be actually mixed neutron stars? The neutron-sphere redshift is about 50% higher than in the ordinary neutron star case, which may have interesting results for the temperature, radius and luminosity measured by a distant observer. The larger neutrons binding energy would lead to a smaller value of the maximal neutrons mass, compared to an ordinary neutron star.

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