NEUTRINO EMISSIVITY OF DENSE STARS

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Abstract

The neutrino emissivity of compact stars is investigated in this work. We consider stars consisting of nuclear as well as quark matter for this purpose. Different models are used to calculate the composition of nuclear and quark matter and the neutrino emissivity. Depending on the model under consideration, the neutrino emissivity of nuclear as well as quark matter varies over a wide range. We find that for nuclear matter, the direct URCA processes are allowed for most of the relativistic models without and with strange baryons, whereas for the nonrelativistic models this shows a strong dependence on the type of nuclear interaction employed. When the direct URCA processes are allowed, the neutrino emissivity of hadronic matter is larger than that of the quark matter by several orders of magnitude. We also find that the neutrino emissivity departs from $T^6$ behavior when the temperature is larger than the difference in the Fermi momenta of the particles, participating in the neutrino-producing reactions.

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I. INTRODUCTION

The neutrino emissivity of compact stars, such as neutron stars, has been a subject of interest for quite some time. When collapsed stellar core is formed after the supernova explosion, it initially cools by emitting radiation till the temperature falls to about $10^9$K or $\sim$0.1 MeV. Below this temperature, cooling by radiation emission is not efficient and the star is expected to cool by emitting neutrinos. The neutrino emission occurs by the so-called URCA processes ($n \rightarrow p + e^- + \nu_e$ and $p + e^- \rightarrow n + \nu_e$) and the energy-momentum conservation requires that the proton Fermi momentum ($p_F^p$) should be larger than $0.5 \times p_F^n$. This implies that the proton fraction in the neutron star must be larger than $1/9$ \cite{1}. Initially, it was thought that such high proton fraction will not be present in neutron stars and one has to find some other mechanism of neutrino emission. One of the ways of getting around this problem is to consider the so-called modified URCA processes \cite{2}. In these processes, the neutrino emission occurs in the presence of another particle (a nucleon) which helps in satisfying the energy-momentum conservation. However, the neutrino emissivity due to the modified URCA processes is extremely small \cite{3}-\cite{4} to explain the cooling rates of these stars. Later, it was observed that, if the star contains pion \cite{5} or kaon condensate \cite{6} or if the star consists of quark matter \cite{7}-\cite{12}, the direct URCA processes are allowed and the neutrino emissivity is substantially larger ($\sim$ order of magnitude) than that due to modified URCA processes. Recently, some authors have found that, for certain models of the nuclear matter \cite{13}, the proton fraction could be larger than the critical value above which the direct URCA processes are allowed. In such a situation, the neutrino emissivity of the star could be large and one can explain the cooling rates in terms of the standard neutron star models. This explanation, however, depends on the assumption of the nuclear interactions. In particular, it depends on the nature of the three-body force and its isospin dependence \cite{14}.

The purpose of the present work is to calculate the neutrino emissivity of nuclear and quark matter using different models. For nuclear matter, we use nonlinear Walecka model,
derivative scalar coupling model, chiral $\sigma$ model and nonrelativistic models, whereas, for the quark matter, we use the MIT bag model and the chiral color dielectric (CCD) model. One of the reasons behind this calculation is to determine the nuclear equations of state in which the direct URCA processes are allowed. This is of some importance, since the neutrino emissivity due to direct URCA processes dominates when these are allowed. We also want to explore the dependence of the neutrino emissivity on different quark models.

In our earlier calculations of neutrino emissivity of the quark matter [17], we found that, for certain values of Fermi momenta, the approximate neutrino emissivity formula of Iwamoto [8] fails. In that work [17], we were able to determine an empirical formula which is able to reproduce the calculated neutrino emissivity over a wide range of quark matter densities and temperatures. In the present calculation, we want to investigate, if a similar situation exists for nuclear matter also.

The results of our calculation can be summarised as follows. We find that for all the relativistic models considered here, the direct URCA processes are forbidden if the nuclear density is below a certain value. In other words, for all the models, the proton fraction is not high enough at lower nuclear densities. The neutrino emissivity, for all the models without strange baryons, falls rapidly at higher densities (more than 5 times nuclear matter densities). But when the strange baryons are included, the neutrino emissivity due to direct URCA process at higher densities varies very slowly with baryon densities. Thus, the appearance of strange baryons depletes the neutron fraction and the direct URCA processes are allowed, even though the proton fraction is not large. We find that the reactions involving strange baryons give a significant contribution to the neutrino emissivity. As for the quark matter, we find a large dependence of the neutrino emissivity on the models used, to calculate quark matter equation of state. However, when the direct URCA processes in the nuclear matter are allowed, the neutrino emissivity of the nuclear matter is larger than that of the quark matter at corresponding nuclear density. We also find that the calculated neutrino emissivity departs from $T^6$ [8,13] behaviour for a certain range of Fermi momenta of the constituents. This happens for nuclear as well as for quark matter. Furthermore, we are able to fit the
calculated neutrino emissivity with a simple universal formula. This clearly implies that the
departure from $T^6$ behavior of the neutrino emissivity is kinematical in origin.

The paper is organised as follows. In Section II, we briefly describe the models used in the
calculation of nuclear and quark equations of state. In Section III, the neutrino emissivity
formulae are presented. Finally, the results are discussed in Section IV.

II. THE MODELS

A. Nuclear Models

Four different models, the nonlinear Walecka model \[18,19\], derivative scalar coupling
model \[20,27\], chiral $\sigma$ model \[21\] and nonrelativistic model \[14\] have been used to calculate
the equation of state of the neutron matter. Of these, the first two have been extensively
used in nuclear structure calculations \[22,20\] and have been able to reproduce the properties
of nuclei over a wide range of the periodic table. This probably ensures that one has correct
nuclear equation of state near the nuclear matter density. The chiral $\sigma$ model has been used
as another model for the nuclear equation of state. This model has been used to calculate
neutron star properties \[21\]. One important fact about this model is that nonlinear terms
can give rise to the three body forces, which is important in the equation of state at high
densities. For all these relativistic models, the nuclear equation of state is calculated by
adopting the mean field ansatz. This is in contrast to the nonrelativistic models, where
actual interaction between the constituents are considered.

The Nonlinear Walecka Model: (NW) The Lagrangian density of the nonlinear
Walecka model \[19\] is given by,

$$
\mathcal{L}(x) = \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i + g_{3i} \sigma + g_{\omega i} \omega_\mu \gamma^\mu - g_{\rho i} \rho_\mu \gamma^\mu T_a) \psi_i - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu}
+ \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \rho_a^\mu \rho^a_\mu + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu
- \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} c (g_{\sigma N} \sigma)^4 + \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i
$$

(1)
The Lagrangian in eq. (1) above includes nucleons, Λ and Σ⁻ hyperons (denoted by subscript $i$), electrons and muons (denoted by subscript $l$) and $\sigma$, $\omega$ and $\rho$ mesons (given by $\sigma$, $\omega^\mu$ and $\rho^{a\mu}$ respectively). The Lagrangian includes cubic and quartic self-interactions of the $\sigma$ field. The meson fields interact with baryons through linear coupling and the coupling constants are different for nonstrange and strange baryons.

In the presence of baryons, the mesons develop nonzero vacuum expectation values ($\bar{\sigma}$, $\bar{\omega}$ and $\bar{\rho}^3$ respectively). Assuming that the baryon densities are uniform, one finds that the time components of $\bar{\omega}$ and $\bar{\rho}^3$, in addition to $\bar{\sigma}$, are nonzero. One can then define effective masses ($\bar{m}_i$) and chemical potentials ($\bar{\mu}_i$) for the baryons as,

$$\bar{m}_i = m_i - g_{\sigma i} \bar{\sigma}$$  \hspace{1cm} (2)

and

$$\bar{\mu}_i = \mu_i - g_{\omega i} \bar{\omega} - I_3 g_{\rho N} \bar{\rho}^3,$$  \hspace{1cm} (3)

where $I_3$ is the value of the z-component of the isospin of baryon $i$. The Fermi momenta ($k_i$) and number densities ($n_i$) of the baryons are given by $k_i = \sqrt{\mu_i^2 - \bar{m}_i^2}$ and $n_i = \frac{k_i^3}{3\pi^2}$. For leptons, the Fermi momenta and number densities are given by $k_l = \sqrt{\mu_l^2 - m_l^2}$ and $n_l = \frac{k_l^3}{3\pi^2}$.

The parameters of the NW model are meson-baryon coupling constants, meson masses and the coefficients of the cubic and quartic self-interactions of $\sigma$ meson ( $b$ and $c$ respectively). The $\omega$ and $\rho$ meson masses have been chosen to be their physical masses. Of the rest of the parameters, the nucleon-meson coupling constants ( $\frac{g_{\sigma N}}{m_\sigma}$, $\frac{g_{\omega N}}{m_\omega}$ and $\frac{g_{\rho N}}{m_\rho}$ respectively ) and the coefficients of cubic and quartic terms of the $\sigma$ meson self interaction ( $b$ and $c$ respectively) are determined by fitting the nuclear matter properties ( the binding energy/nucleon ($-16MeV$) and baryon density (0.15$fm^{-3}$), symmetry energy coefficient (32.5$MeV$), Landau mass (0.83$m_N$) and nuclear incompressibility (250 – 300$MeV$) ). The coupling constants of the hyperon-meson interactions and are not well known. These cannot be determined from nuclear matter properties, since the nuclear matter does not contain
hyperons. Furthermore, properties of hypernuclei do not fix these parameters in a unique way. In the literature, a number of choices have been made. These are (a) same as the nucleon-meson coupling constants (Universal coupling) \[24,25\], (b) $\sqrt{2/3}$ times the nucleon-meson coupling constants \[24,25\], (c) $1/3$ times the nucleon-meson coupling constants \[18\] and (d) $2/3$, $2/3$ and $1$ times the nucleon-meson coupling for $\sigma$, $\omega$ and $\rho$ mesons respectively \[19\].

We have used the above mentioned choices for meson-strange baryon couplings to investigate the neutrino emissivity.

**Derivative Scalar Coupling Model (DSC model):** In this case, the Lagrangian is given by \[20\]

$$L(x) = \sum_i \bar{\psi}_i (i \gamma^\mu \partial_\mu - m_i + g_{\omega_i} \omega^\mu_i \gamma^\mu - g_{\rho_i} \rho^a_\mu \gamma^\mu T_a) \psi_i - \frac{1}{4} \omega^{\mu \nu} \omega_{\mu \nu}$$

$$+ \frac{1}{2} m_\sigma^2 \omega^\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \rho_{\mu \nu} \rho^\mu_\nu + \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^a_\mu$$

$$+ \sum_i g_\sigma \bar{\psi}_i \sigma \psi_i (1 + g_\sigma \sigma/M) + \sum_l \bar{\psi}_l (i \gamma^\mu \partial_\mu - m_l) \psi_l$$

(4)

where $\omega_\mu$ represents the vector meson field, $m_\sigma$ and $m_\omega$ are the masses of the scalar and vector fields, and $F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$. The summation indices $i$ and $l$ stand for fermions and leptons respectively. The DSC Lagrangian differs from the original $\sigma$ - $\omega$ model \[26\] in the baryon-$\sigma$ meson coupling term ($= g_\sigma \bar{\psi} \psi \sigma$ for the $\sigma$ - $\omega$ model), but it has also two parameters: $g_\sigma$ and $g_\omega$ like the original model \[26\].

From the Lagrangian (4), the following definition of (density dependent) effective nucleon mass ($m^*$) is suggested:

$$m^* = M/(1 + g_\sigma \sigma/M)$$

(5)

**Chiral Sigma Model (CS model):** This model includes $\sigma$, $\omega$ and $\pi$ fields. In addition, $\rho$ meson is included in the Lagrangian to reproduce the symmetry energy of the nuclear matter correctly. The Lagrangian for an SU(2) × SU(2) chiral sigma model that includes (dynamically) an isoscalar vector field ($\omega_\mu$) is
\[
\mathcal{L}(x) = \frac{1}{2}(\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4}(\vec{\pi} \cdot \vec{\pi} + \sigma^2 - x_o^2)^2
\]

\[-\frac{1}{4}F_{\mu \nu}F^{\mu \nu} + \frac{1}{2}g_\omega(\sigma^2 + \vec{\rho}^2)\omega_\mu \omega^\mu + g_\sigma \bar{\psi}(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\pi})\psi + \bar{\psi}(i\gamma_\mu \partial^\mu - m_\sigma)\psi - \frac{1}{4}G_{\mu \nu}G^{\mu \nu} + \frac{1}{2}m_\rho^2 \vec{\rho} \cdot \vec{\rho} + \sum_l \bar{\psi}_l(i\gamma_\mu \partial^\mu - m_l)\psi_l (6)\]

where \( F_{\mu \nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \psi \) is the nucleon isospin doublet, \( \vec{\pi} \) is the pseudoscalar pion field and \( \sigma \) is the scalar field. The vector field \( \omega_\mu \) couples to the conserved baryonic current \( j_\mu = \bar{\psi} \gamma_\mu \psi \). The expectation value \( < j_o > \) is identifiable as the nucleon number density.

The interactions of the scalar and the pseudoscalar mesons with the vector boson generates a mass for the latter spontaneously by the Higgs mechanism. The masses for the nucleon, the scalar meson and the vector meson are respectively given by

\[
M = g_\sigma x_o \\
m_\sigma = \sqrt{2\lambda} x_o \\
m_\omega = g_\omega x_o. (7)
\]

where \( x_o \) is the vacuum expectation value of sigma field.

With \( x = (\sigma^2 + \vec{\rho}^2)^{1/2} \), the the effective mass of the nucleon is \( M^* \equiv yM \), where \( y = x/x_o \). For both DSC and CS model, the chemical potential of the baryon can be related to the fermi momentum through a similar relation as eq.(3). The nucleon- meson coupling constant are determined by fitting the nuclear matter properties, binding energy (-16.3 MeV), saturation density (0.153 \( fm^{-3} \)) and symmetry energy coefficient (32.5 MeV). The incompressibility in the above two models are 225 MeV and 700 MeV respectively.

**Nonrelativistic models:** Here we have used the models as proposed by Wiringa et al. by combining different two- nucleon and three- nucleon potentials. In particular three different choices have been considered.

- Argonne \( v_{14} \) (AV14) and Urbana VII (UVII) three nucleon potential,
- Urbana \( v_{14} \) (UV14) two nucleon potential and UVII,
• UV14 and three nucleon interaction (TNI) model of Lagaris and Pandharipande [15].

AV14 and UV14 have identical structure and can be written as a sum of 14 operator components. The main difference between these two models come from the strength of short range tensor force. The UV14 does not have short range tensor components which results in a weak tensor force that vanishes at the origin. The AV14 tensor force is finite at the origin and at intermediate distance looks like Paris potential [16]. The three nucleon potential potential UVII combines a long range two pion exchange part and an intermediate range repulsive part. Out of the three different combinations, \( AV14 + UVII \), \( UV14 + UVII \) and \( UV14 + TNI \) considered, only the second combination produce enough proton fraction to have non-zero neutrino emissivity.

In order to have some idea on the expected behaviour of different models, let us study the proton fractions for different parameters as well as different models. In fig. 1, we have plotted the proton fraction for nonlinear Walecka model with different hyperon couplings and incompressibility. It is evident from this figure that, for all the parameter sets, the required proton fraction is attained over a certain density (\( \sim 2\rho_0 \)), \( \rho_0 \) is the nuclear matter density). A change in incompressibility from 300 MeV to 350 MeV does not give much change in the proton fraction. On the other hand, the variation of hyperon couplings give a large change in the proton fraction. This indicates that the effect of hyperon couplings on neutrino emissivity will be much larger than that of the incompressibilities.

Fig.2. gives a comparison of proton fraction between different models. It shows a strong density dependence of the proton fraction on models. Below a certain density (\( \sim 0.2 fm^{-3} \)), none of these models satisfy the limit and neutrino emissivity will be zero. We also find that nonlinear Walecka model without hyperons yield larger proton fraction at higher density (\( > 0.8 fm^{-3} \)). Also, with hyperons required proton fraction is attained at earlier baryon densities than without hyperons. However, the nonrelativistic model yield lowest proton fraction throughout the density range.

The proton fraction also depends on the symmetry energy coefficients. We have varied
the symmetry energy coefficient from 28-38 MeV for DSC and CS models. With increase in symmetry energy, the required proton fraction for neutrino emissivity in direct URCA process, is attained at lower baryon density.

**B. Quark Models**

We have used CCD model and bag model \[17\] to study the neutrino emissivity. The colour dielectric model is based on the idea of Nielson and Patkos \[28\]. In this model, one generates the confinement of quarks and gluons dynamically through the interaction of these fields with scalar field. Here, we have used chiral extension of this model to study the quark matter neutrino emissivity. The CCD model has already been used to study static properties of baryons \[29\], properties of quark matter at finite density and temperature \[30,31\], hadron-quark phase transition and properties of dense stars \[19,32\]. In CCD quark matter, we assume that meson (\(\phi\)) expectation value is zero i.e. \(<\phi> = 0\) where as \(<\phi^2 > \neq 0\). So, the Lagrangian is rewritten in terms of \(<\phi^2>\). The quark mass becomes density dependent. In this model, with increase in density quark mass decreases and drops to about \(1/4\)th of its initial value around density \(2\, fm^{-3}\). This density is defined as the critical density for chiral transition in our model. There are five parameters in CCD model; bag parameter \(B\), scalar field potential parameter \(\alpha\), \(u\) and \(d\) quark mass, strange quark mass and strong coupling constant \(\alpha_s\). These input parameters are obtained by fitting the baryonic masses. In the present paper, we have discussed the results for the parameter set: \(B^{1/4} = 152.1\, MeV\), \(m_{q(u,d)} = 91.6\, MeV\), \(m_{q(s)} = 294.9\, MeV\), \(\alpha = 36\) and strong coupling constant \(\alpha_s = 0.08\). In the bag model, the neutrino emissivity is calculated with \(m_u = m_d = 0\), \(m_s = 150\, MeV\) and \(\alpha_s = 0.08\). For both CCD and bag model, we consider interaction upto first order in \(\alpha_s\).

**III. EMISSIVITY FORMULAE**

In general, the Lagrangian density for URCA processes in the current current interaction form is written as \[33\],
\[ \mathcal{L}(x) = \frac{G_F}{\sqrt{2}} l_\mu(x) I^\mu(x) + H.C \]  

(8)

where the weak coupling constant \( G_F = 1.435 \times 10^{-49} \text{erg cm}^3 \) and \( l_\mu \) and \( I^\mu \) are the leptonic and hadronic weak currents respectively.

\[ l_\mu(x) = \bar{e} \gamma_\mu (1 - \gamma_5) \nu_e + \bar{\mu} \gamma_\mu (1 - \gamma_5) \nu_\mu + \ldots \ldots + h.c. \]  

(9)

\[ I_\mu(x) = \bar{\psi}_1 \gamma_\mu (A - B \gamma_5) \psi_2 + h.c. \]  

(10)

where \( h.c. \) stands for hermitian conjugate. For quarks \( A = B = 1 \) and for baryons the value of \( A \) and \( B \) depends on the specific nature of the particle. Using the above Lagrangian one can calculate the neutrino emissivity \( \epsilon \),

\[ \epsilon = g \int \left\{ \Pi_{i=1}^3 \frac{d^3p_i}{(2\pi)3} \right\} E_\nu W_{fi} F(p_1, p_2, p_e) \]  

(11)

where \( i = 1, 2, e, \nu, E_\nu \) is the energy of the neutrino and \( g \) is the degeneracy factor. The transition rate is

\[ W_{fi} = \frac{(2\pi)^4 \delta^4(P_{in} - P_{fn}) |M|^2}{\Pi_i 2E_i} \]  

(12)

where \( P_{in} \) is the sum of the initial momenta, \( P_{fn} \) is the sum of the final momenta, \( E_i \) is the energy of the \( i \)-th particle and \( |M|^2 \) is the squared invariant amplitude averaged over initial spins and summed over final spins. The symbol \( F(p_1, p_2, p_e) \) for the reactions \( 1 \rightarrow 2 + e^- + \bar{\nu}_e \) and \( 2 + e^- \rightarrow 1 + \nu_e \) are

\[ F_d(p_1, p_2, p_e) = n(p_1)(1 - n(p_2))(1 - n(p_e)) \]  

(13)

and

\[ F_r(p_1, p_2, p_e) = (1 - n(p_1))n(p_2)n(p_e) \]  

(14)

respectively, where

\[ n(p) = \frac{1}{1 + e^{(E - \mu)/T}}. \]  

(15)
A. Hadronic matter

In hadronic matter, several weak decays may contribute to the neutrino emission as given below [34].

\[ n \rightarrow p + e^- + \bar{\nu}_e; \quad p + e^- \rightarrow n + \nu_e \]

\[ A = \cos \theta_c; \quad B = (F + D) \cos \theta_c \] (16)

\[ \Lambda \rightarrow p + e^- + \bar{\nu}_e; \quad p + e^- \rightarrow \Lambda + \nu_e \]

\[ A = -3 \sin \theta_c / \sqrt{6}; \quad B = -\frac{(3F + D)}{\sqrt{6}} \sin \theta_c \] (17)

\[ \Sigma \rightarrow n + e^- + \bar{\nu}_e; \quad n + e^- \rightarrow \Sigma + \nu_e \]

\[ A = -\sin \theta_c; \quad B = -(F - D) \sin \theta_c \] (18)

\[ \Sigma \rightarrow \Lambda + e^- + \bar{\nu}_e; \quad \Lambda + e^- \rightarrow \Sigma + \nu_e \]

\[ A = 0; \quad B = \sqrt{\frac{2}{3}} D \cos \theta_c \] (19)

where \( F = 0.427 \) and \( D = 0.823 \). The angle \( \theta_c \) is the Cabbibo angle and \( \cos \theta_c = 0.948 \). The \( |M|^2 \) for the above processes is given by

\[ |M|^2 = \frac{1}{2} \left[ 64(A^2 + B^2)\{(p_2.p_e)(p_1.p_{\nu}) + (p_2.p_{\nu})(p_1.p_e)\} \right. \]
\[ + 64AB\{(p_2.p_e)(p_1.p_{\nu}) - 2(p_2.p_{\nu})(p_1.p_e)\} \]
\[ \left. + 64(A^2 - B^2)m_1m_2(p_{e.p_{\nu}}) \right] \] (20)

Using the above matrix element, one can calculate the neutrino emissivity for both direct and reverse processes.

\[ \epsilon_{(1 \rightarrow 2+e^-+\bar{\nu}_e)} = \left[ \int p_2p_e p_{\nu}^2 p_1 dp_2 dp_e dp_{\nu} dp_1 \right. \]
\[ \cdot \left\{ C_1 \times \int_{\min\{p_2+p_{\nu},p_1+p_{\nu}\}}^{\max\{p_2-p_{\nu},p_1-p_{\nu}\}} dP \left( 1 - \frac{p_2^2 + p_e^2 - P^2}{2E_1E_{\nu}} \right) \right. \]
\[ \cdot \left( 1 + \frac{p_2^2 + p_e^2 - P^2}{2E_2E_{\nu}} \right) \]
\[ + \left. C_2 \times \int_{\min\{p_1+p_{\nu},p_2+p_{\nu}\}}^{\max\{p_1-p_{\nu},p_2-p_{\nu}\}} dP \left( 1 + \frac{p_2^2 + p_e^2 - P^2}{2E_2E_{\nu}} \right) \right. \]
\[ \cdot \left( 1 - \frac{p_1^2 + p_e^2 - P^2}{2E_1E_{\nu}} \right) \]
\[ - \left. C_3m_1m_2 \times \int p_2^2 p_e^3 p_{\nu}^3 p_1 dp_2 dp_e dp_{\nu} \right] \delta(E_2 + E_e + E_{\nu} - E_1)F_d(p_1,p_2,p_e) \] (21)
\[ \epsilon(2\nu^- \rightarrow 1\nu_e) = \int p_2 p_1 p_{1\nu}^2 p_{2\nu} d\nu d\nu d\nu d\nu \]

\[
\left\{ C_1 \times \int_{\min\{|p_2 + p_1|, |p_1 + p_{2\nu}|\}}^{\max\{|p_2 + p_{2\nu}|, |p_1 + p_{2\nu}|\}} dP \quad \left(1 + \frac{p_1^2 + p_{2\nu}^2 - P^2}{2E_1 E_\nu}\right) \left(1 + \frac{p_2^2 + p_{1\nu}^2 - P^2}{2E_2 E_\nu}\right) \right. \\
+C_2 \times \int_{\min\{|p_2 + p_{2\nu}|, |p_1 + p_{2\nu}|\}}^{\max\{|p_2 + p_{2\nu}|, |p_1 + p_{2\nu}|\}} dP \quad \left(1 - \frac{p_2^2 + p_{1\nu}^2 - P^2}{2E_2 E_\nu}\right) \left(1 - \frac{p_1^2 + p_{2\nu}^2 - P^2}{2E_1 E_\nu}\right) \right \}

- C_3 m_1 m_2 \times \left[ \frac{p_2^2 p_{1\nu}^3 p_{2\nu}^3 p_{1\nu} d\nu d\nu d\nu}{E_1 E_2 E_\nu} \right] \delta(E_2 + E_\nu - E_1) F_r(p_1, p_2, p_{2\nu}) \quad (22)

where \( C_1 = \frac{G_F^2 (A+B)^2}{2(2\pi)^3} \), \( C_2 = \frac{G_F^2 (A-B)^2}{2(2\pi)^3} \) and \( C_3 = \frac{2G_F^2 (A^2 - B^2)}{(2\pi)^3} \)

Similar decays involving \( \mu^- \), instead of \( \nu^- \), are also possible. Here we have considered only \( \nu^- \) channel, because \( \nu^- \) density will be much more than the \( \mu^- \) and hence it will contribute more. In URCA processes involving only one hyperon, there is a change in strangeness, so the neutrino emission rate which is proportional to \( \sin^2 \theta_c \), is less than one tenth of those for nucleonic URCA processes (\( \propto \cos^2 \theta_c \)). But in the processes involving only hyperons, there is no change in strangeness and hence such reactions are not Cabbibo suppressed.

The nonrelativistic reduction of the matrix element (eq. (20)) can be obtained by neglecting the baryon momenta and replacing baryon energy by corresponding masses. The reduced matrix element becomes,

\[ |M|^2 = \frac{1}{2} \left[ 64 \left\{ (A^2 + 3B^2) E_\nu E_{1\nu} + (A^2 - B^2) (p_{1\nu} p_{2\nu}) \right\} m_1 m_2 \right] \]

The corresponding neutrino emissivity formula \([34]\) can be derived from eq. (11) by performing the phase space integral using Fermi liquid theory,

\[ \epsilon = \frac{457\pi}{10080} G_F C^2 (A^2 + 3B^2) m_1 m_2 \mu_e T^6 \quad (24) \]

### B. Quark matter

In quark matter, the neutrino emission takes place due to following processes.

\[ d \rightarrow u + e^- + \bar{\nu}_e; \quad u + e^- \rightarrow d + \nu_e, \]

\[ s \rightarrow u + e^- + \bar{\nu}_e; \quad u + e^- \rightarrow s + \nu_e. \]
The matrix element for the quark URCA process is

$$|M|^2_{d(s)} = 64\cos^2\theta_C (\sin^2\theta_C) [ (p_2.p_e)(p_{1}\nu)]$$  \hspace{1cm} (27)

The neutrino emissivity for the quark URCA processes are given by

$$\epsilon_{u\rightarrow d \ (u\rightarrow s)}(e^-) = C_{ud}(C_{us}) \int p_u p_e p_d dE_e dE_d \delta(E_u + E_e - E\nu - E_d)$$

$$\times \frac{1}{e^{(E_u - \mu_u)/T} + 1} \frac{1}{e^{(E_e - \mu_e)/T} + 1} \frac{1}{e^{(\mu_d(E_d) - E_d)/T} + 1}$$

$$\times \int_{\text{min}}^{\text{max}} |p_u + p_e| \ |p_d| dE_e$$

$$\epsilon_{d\rightarrow u \ (s\rightarrow u)}(e^-) = C_{du}(C_{su}) \int p_d p_u p_s dE_s dE_u \delta(E_d - E_u - E\nu - E_s)$$

$$\times \frac{1}{e^{(E_u - \mu_u)/T} + 1} \frac{1}{e^{(E_s - \mu_s)/T} + 1} \frac{1}{e^{(E_d - \mu_d)/T} + 1}$$

$$\times \int_{\text{min}}^{\text{max}} |p_d + p_s| \ |p_u| dE_u$$

where  \( C_{ud} = C_{du} = \frac{6G_F^2 \sin^2\theta_C}{(2\pi)^5} \),  \( C_{su} = C_{us} = \frac{6G_F^2 \sin^2\theta_C}{(2\pi)^5} \),  \( G_F \) is the weak decay constant and \( \theta_C \) is the Cabibbo angle. Here \( E_i \) and \( \mu_i \) are energy and chemical potential of the \( i \)-th species (\( i = u, d, s, e \)). The single particle energy momentum relation, required to evaluate neutrino emissivity for both hadronic and quark matter, is defined in analogy with the relation between chemical potential and fermi momentum.

**IV. RESULTS AND DISCUSSION**

In the present neutrino emissivity calculation, we have considered only the direct URCA processes for hadron and quark matter. Also, the neutrino emissivity due to direct decay process and its reverse, at chemical equilibrium, are taken to be equal. The neutrino emission from charge neutral hadronic or quark matter at chemical equilibrium, for small temperatures (\( T \leq 1 MeV \)), occur due to those fermions whose momenta lie close to their Fermi surfaces. Therefore, the required kinematical criterion for reaction to occur is that the momentum conservation condition be satisfied for the fermion momenta around the respective Fermi surfaces. For the process \( 1 \rightarrow 2 + e^- + \nu \), the momentum conservation condition can be written as \( p_2^F + p_e^F - p_1^F = \Delta p > 0 \).
In fig. 3, we have plotted the density dependence of neutrino emissivity at $T = 0.5\, MeV$ from different hadronic models which are nonlinear Walecka model, derivative scalar coupling model and chiral sigma model. The quark matter neutrino emissivity using CCD and bag model are also plotted in fig. 3. In CCD model, quarks are massive ($m_{(u,d)} = 125\, MeV$, $m_s = 300\, MeV$). The $u$ and $d$ quarks masses decrease with increase in density due to non zero $<\pi^2>$. On the other hand, $m_s$ remains unchanged as $<K^2>$ and $<\eta^2>$ remain zero in the medium throughout the range of densities considered. We find that the neutrino emissivity in CCD model is higher ($\sim$ order of magnitude) than the bag model, due to the additive effect of quark masses on the neutrino emissivity, the strong interaction coupling constant being same ($\alpha_s$ is equal to 0.08 for both CCD and bag model where $\alpha_s = g_s^2/4\pi$). Fig.3. shows that upto density $1.0\, fm^{-3}$, neutrino emissivity from DSC and CS model matter is higher ($\sim$ factor of 2-4) than the other hadronic models. But beyond that only nonlinear Walecka model with hyperons dominate. Here one thing should be noted that with increase in the baryon density, for all the models, the mean field value of $\rho_0^3$ increases due to increase in proton fractions. As a result, the neutron energy and hence the neutrino energy decreases. After a certain density, neutrino energy becomes negative and the reaction stops. This phenomena is more pronounced in case of models without hyperons at densities greater than $1\, fm^{-3}$. The neutrino emissivity from nonrelativistic model is of the same order as the relativistic models, whereas, the neutrino emissivity due to quark URCA processes are in general lower compared to nucleonic URCA processes.

In fig.4, we have plotted neutrino emissivity from the reaction $n \rightarrow p + e + \bar{\nu}_e$, with temperatures, for two different $\Delta p$. The curves for both exact and approximate results are given. We find that exact In our calculation we find that our exact neutrino emissivity is consistently smaller than the approximate results. In fact, when $\Delta p$ is large compared to the temperature, our numerical result is almost same as the neutrino emissivity obtained using the analytic formula (eq. (24)). On the other hand, for small $\Delta p(\sim T)$, there is a deviation from the analytic result. The similar results were obtained for quark matter system. The deviation of approximate from exact result can be explained in the following ways.
In calculating the approximate formula, the neutrino momentum is neglected in the delta function. As long as $\Delta p$ is much larger than $T$, this approximation gives correct result and neutrino emissivity varies as $T^6$. On the other hand, when $\Delta p$ is small ($\sim T$), an additional power of $T$ may come in the denominator [17] and neutrino emissivity is no longer proportional to $T^6$. The other reason for the difference comes from the factorization of angle and momentum integrals. The momenta may differ from the corresponding Fermi momenta by $T$ in the integral. When $\Delta p \sim T$, there are reasons in momentum space where $\cos \theta_{pn}$ ($\theta_{pn}$ is the angle between proton and neutron) is greater than 1, and the rest of the integrand is not small. Clearly these regions must be excluded from the integration as these values of $\cos \theta_{pn}$ are unphysical. If one does not put this restriction, which happens when one factorizes angle and momentum integrals, the phase space integral will be overestimated.

In our earlier section, we have discussed that a strangeness changing reaction is proportional to $\sin^2 \theta_c$. Hence, it would have lower neutrino emissivity compared to the reactions where there is no change in strangeness. In the present calculation, we find that the neutrino emissivity from $n$ to $p$ decay and $\Sigma$ to $\Lambda$ decay are larger ($\sim$ order of magnitude) than the neutrino emissivity from $\Lambda$ to $p$ and $\Sigma$ to $p$ decay, which are strangeness changing reactions. Overall, the neutrino emissivity from the neutron decay is higher than all the other decays.

We have studied the neutrino emissivity from hadronic matter for different parameter sets. It is found that there is only a small variation in emissivity with incompressibility. Also, as mentioned earlier, hyperon couplings are varied in our calculation. With decrease in hyperon couplings, it becomes energetically favourable to convert nucleons into hyperons as hyperons do not feel the predominantly repulsive force. As a result, with decreasing couplings more and more hyperons get populated. This implies that with decrease in hyperon couplings, the neutrino emissivity due to the hyperon decay increases.

In our calculation for neutrino emissivity from hadronic matter, we find that the departure from the usual approximation [17] arises due to large $T/\Delta p$. The similar behaviour have been found for quarks also. So it may be possible to fit the numerically calculated $\epsilon$ ($\epsilon_{exact}$) with a function of the form $\epsilon_{approx}(f(x))^{-1}$, where $x = T/\Delta p$. The function $f(x)$
should be such that for small values of $x$ it should approach unity. In fig.5, we have plotted $\epsilon_{approx.}/\epsilon_{exact}$ against $T/\Delta p$ for different decays with different parameters. We have fitted the above graph with a function $f(x) = 1 + ax + bx^2 + cx^3$, with $a = -2.5$, $b = 100$ and $c = 30$ as obtained in the case of interacting quarks [17]. It is evident that with an overall multiplicative factor $\sim 1.5$, same function can describe both hadronic and quark emissivities. This factor 1.5 is due to the nonrelativistic approximation, which is used to calculate approximate formula of neutrino emissivity in hadronic decays.

V. CONCLUSION

In the present paper, we have studied the neutrino emissivity due to hadronic and quark URCA processes for different models. We have also considered effect of incompressibility and hyperon couplings on neutrino emissivity. It is found that relativistic models, considered here, in general, have nonzero neutrino emissivity and these are higher compared to neutrino emissivity due to quark URCA processes. But the scenario is not so simple in case of nonrelativistic models. In such models, neutrino emissivity is highly sensitive to the nature of three-nucleon interactions. In fact in the present study, we find that only $UV14 + UVII$ gives nonzero emissivity. For relativistic models, neutrino emissivity is more sensitive to the hyperon couplings than the incompressibilities.

Our calculation shows that as in the case of quark matter, approximate formula is not valid for hadronic weak decays when $T/\Delta p$ is large i.e $\Delta p$ is smaller compared to $T$. An alternative formula can be used to calculate the neutrino emissivity in such cases for both hadronic and quark matter.

In conclusion, direct Urca processes in hadronic decays provides an alternate scenario for rapid cooling, without the necessity of phase transition to quark matter phase inside neutron stars.
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FIGURES

FIG. 1. Proton fractions for nonlinear Walecka (NW) model, first three curves are for incompressibility $K = 300 MeV$

FIG. 2. Proton fractions for (a) DSC model, (b) CS model, (c) NW model without hyperons, $K = 300 MeV$, (d) NW model with hyperons, $K = 300 MeV$ and (e) nonrelativistic model UV14+UVII

FIG. 3. Variation of neutrino emissivity in c.g.s. units ($gm/cm^3/sec.$) with density for $T = 0.5 MeV$, (a) DSC model, (b) CS model, (c) NW model without hyperons, $K = 300 MeV$, (d) NW model with hyperons, $K = 350 MeV$, (e) CCD model, (f) bag model and (g) nonrelativistic model UV14+UVII

FIG. 4. Neutron decay emissivity in c.g.s. units ($gm/cm^3/sec.$) for incompressibility ($K$) $300 MeV$; for $n_B = 0.4 fm^{-3}$ and $\Delta p = 54.53$ (a) Our result (b) Approximate result; for $n_B = 1.4 fm^{-3}$ and $\Delta p = 0.50 MeV$ (c) Our result, (d) Approximate result

FIG. 5. Neutrino emissivity ratio is plotted against $x = T/\Delta p(\Delta p = \Delta p)$. The points corresponds to different hadronic decays. The line corresponds to the function $f(x) = 1 + ax + bx^2 + cx^3$. $a = -2.5 , b = 100$ and $c = 30$ as obtained in ref.[17] for quarks.