Casimir interaction of two dielectric half spaces with Chern-Simons boundary layers

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Abstract

A diffraction problem for a flat Chern-Simons layer at plane boundary of a dielectric half space is solved. The Casimir energy of two dielectric half spaces with Chern-Simons layers at plane-parallel boundaries separated by a vacuum slit is derived. Crossing from the repulsive to the attractive Casimir force is analyzed for two Au and two Si half spaces with boundary Chern-Simons layers. Boundary quantum Hall layers in external magnetic field lead to Casimir repulsion at nanoscales. We discuss features that make systems with boundary quantum Hall layers unique for force measurements and search of long-range interactions beyond electromagnetism.

1 Introduction

The Casimir effect [1] is a quantum interaction effect between macroscopic objects [2] - [14]. Chern-Simons terms [15] were intensively studied in quantum field theory [16] - [19]. The Casimir interaction of two flat Chern-Simons layers in vacuum was considered in [20, 21]. The Casimir-Polder potential of a neutral anisotropic atom in the presence of a flat Chern-Simons layer was found in [22], charge-parity violating effects due to Chern-Simons layer interacting with an atom were considered in [23].

The force between two Chern-Simons layers in vacuum is either attractive or repulsive at all distances depending on values of constants defining Chern-Simons layers [20, 21]. The situation changes if there are Chern-Simons layers at plane-parallel boundaries of dielectric half spaces separated by a vacuum slit of the width $L$, this problem is considered in the present paper. For coinciding Chern-Simons layers at plane-parallel boundaries we demonstrate there can be a minimum of the Casimir energy and, as a result, Casimir repulsion at nanoscales.

It is natural to perform the Casimir force measurements with quantum Hall layers as Chern-Simons boundary layers. Suppose one puts the system in a constant external magnetic field perpendicular to the layers. The constant $a$ of the Chern-Simons action is uniquely defined by the plateau of the quantum Hall effect in terms of QED fine structure

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coupling $\alpha$ and an integer or a fractional number $n$ characterizing this plateau as $a = \alpha n$. The plateau of the quantum Hall effect is defined by the magnitude of an external magnetic field. Without external electric field parallel to the quantum Hall layer there are no spatial components of the Hall current in the boundary layer, it is reasonable to perform Casimir force measurements in this regime.

The Casimir force between two half spaces with Chern-Simons boundary layers can be evaluated from the first principles and with high precision: constants of Chern-Simons layers are defined by the plateaus of the quantum Hall effect, frequency dependent dielectric permittivities of the underlying semispaces can be extracted from the tabulated optical data [25]. The total force in the Casimir experiments typically consists of two parts: the Casimir and the electrostatic contributions [26, 27]. The difference in the theoretical and experimental determination of the force may be a signal of long-range forces beyond electromagnetism measured in experiment. Long-range interactions beyond electromagnetism arise due to exchange of light scalar particles predicted in many extensions of the Standard Model leading to Yukawa-type potentials [28], Yukawa-type potentials in the extra-dimensional theories with a low-energy compactification scale [29, 30], power-type potentials [31, 32], axion exchange potential [33]. In the interaction range above a few micrometers the strongest constraints on potentials of long-range interactions follow from Cavendish type experiments [34, 35, 36, 37], at shorter separations the strongest constraints on Yukawa-type potentials and axion-to-nucleon coupling constants were obtained from various Casimir effect experiments [12, 26, 27, 38].

To sum up, there are several features that make systems with quantum Hall surface layers unique for force measurements: stabilization of force measurements at short separations due to Casimir repulsion, quick decrease of the Casimir force in system with Chern-Simons layers in comparison to the Lifshitz force for two dielectric half spaces [3] at separations approaching the minimum of the Casimir energy, precise evaluation of the Casimir force from the first principles of quantum field theory, possibility of changing plateaus in the quantum Hall effect regime by external magnetic field resulting in a change of the force at a given separation. Geometry of two half spaces with plane-parallel boundaries separated by a vacuum slit $L$ is the simplest geometry where novel features due to Chern-Simons boundary layers occur.

We proceed as follows. In Sec.2 we solve a diffraction problem for Chern-Simons layer at the boundary of dielectric half space characterized by a permittivity $\varepsilon(\omega)$, reflection and transmission coefficients for diffraction of an electromagnetic plane wave are derived. In Sec.3 we apply scattering formalism [11, 39 - 45] to derive formulas for the Casimir energy of two dielectric half spaces with Chern-Simons layers at plane-parallel boundaries separated by a vacuum slit $L$. We study Casimir forces for Au and Si half spaces with Chern-Simons boundary layers, crossing from the repulsive to the attractive Casimir force is analyzed.

We use units $\hbar = c = 1$. 
2 Diffraction problem

The action with Chern-Simons layer at $z = 0$ has the form:

$$S = \frac{a}{2} \int \varepsilon^{\nu\rho\sigma} A_\nu F_{\rho\sigma} dt dx dy$$

(1)

with the current $J_\nu = a \varepsilon^{\nu\rho\sigma} F_{\rho\sigma}$ and vector-potential $A_\nu$. Equations of electromagnetic field in the presence of Chern-Simons action (1) can be written as follows:

$$\partial_\mu F^{\mu\nu} + a \varepsilon^{\nu\rho\sigma} F_{\rho\sigma} \delta(z) = 0.$$  

(2)

Consider a flat Chern-Simons layer put at $z = 0$ on a dielectric half space $z < 0$ characterized by a frequency dependent dielectric permittivity $\varepsilon(\omega)$, the magnetic permeability $\mu = 1$. Boundary conditions on the components of the electromagnetic field can be written as follows [46]:

$$E_z|_{z=0^+} - \varepsilon(\omega) E_z|_{z=0^-} = -2a H_z|_{z=0},$$  

(3)

$$H_x|_{z=0^+} - H_x|_{z=0^-} = 2a E_x|_{z=0},$$  

(4)

$$H_y|_{z=0^+} - H_y|_{z=0^-} = 2a E_y|_{z=0}.$$  

(5)

Consider TE (s-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at $z = 0$ on a dielectric half space ($z < 0$) defined by a dielectric permittivity $\varepsilon(\omega)$ (the factor $\exp(i\omega t + ik_y y)$ is dropped for simplicity of notations):

$$E_x = \exp(-ik_z z) + r_s \exp(ik_z z), z > 0$$  

(6)

$$E_x = t_s \exp(-ik_z^{(2)} z), z < 0$$  

(7)

$$H_x = r_s \exp(ik_z z), z > 0$$  

(8)

$$H_x = t_s \exp(-ik_z^{(2)} z), z < 0.$$  

(9)

Here $k_z = \sqrt{\omega^2 - k_y^2}$, $k_z^{(2)} = \sqrt{\varepsilon(\omega) \omega^2 - k_y^2}$.

From the condition (4) it follows

$$r_s - t_s = 2a t_s.$$  

(10)

From $E_x|_{z=0^+} = E_x|_{z=0^-}$ we obtain

$$1 + r_s = t_s.$$  

(11)

From the condition $E_y|_{z=0^+} = E_y|_{z=0^-}$ and Maxwell equation $E_y = -\frac{1}{i\omega \varepsilon(\omega)} \partial_z H_x$ it follows that

$$r_s k_z = -\frac{k_z^{(2)}}{\varepsilon(\omega)} t_s.$$  

(12)

From the condition (5) and Maxwell equation $H_y = \frac{1}{i\omega} \partial_z E_x$ we get

$$k_z (-1 + r_s) + k_z^{(2)} t_s = 2a \frac{k_z^{(2)}}{\varepsilon(\omega)} t_s.$$  

(13)
Solving equations (10)-(13) we find reflection and transmission coefficients for TE plane wave:

\[ r_s = \frac{r_f - a^2T}{1 + a^2T}, \quad t_s = \frac{t_f}{1 + a^2T}, \]

\[ r_{s\rightarrow p} = \frac{aT}{1 + a^2T}, \quad t_{s\rightarrow p} = -\frac{aT}{1 + a^2T} \varepsilon(\omega)k_z, \]

(14)

where

\[ T = \frac{4k_zk_z^{(2)}}{(k_z + k_z^{(2)})\varepsilon(\omega)k_z + k_z^{(2)}} \]

(15)

and

\[ r_s^f = \frac{k_z - k_z^{(2)}}{k_z + k_z^{(2)}}, \quad t_s^f = \frac{2k_z}{k_z + k_z^{(2)}}, \]

(16)

are TE Fresnel coefficients for diffraction on a flat dielectric semispace.

Consider TM (\(p\)-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at \(z = 0\) on a dielectric half space \((z < 0)\) defined by a frequency dependent dielectric permittivity \(\varepsilon(\omega)\) :

\[ H_x = \exp(-ik_z z) + r_p \exp(ik_z z), \quad z > 0 \]

(17)

\[ H_x = t_p \exp(-ik_z^{(2)} z), \quad z < 0 \]

(18)

\[ E_x = r_{p\rightarrow s} \exp(ik_z z), \quad z > 0 \]

(19)

\[ E_x = t_{p\rightarrow s} \exp(-ik_z^{(2)} z), \quad z < 0 \]

(20)

From the condition \(E_x|_{z=0^+} = E_x|_{z=0^-}\) it follows

\[ r_{p\rightarrow s} = t_{p\rightarrow s}. \]

(21)

From the condition \(E_y|_{z=0^+} = E_y|_{z=0^-}\) and equation \(E_y = -\frac{1}{i\omega\varepsilon(\omega)} \partial_z H_x\) we get

\[ k_z(1 - r_p) = \frac{k_z^{(2)}}{\varepsilon(\omega)}t_p. \]

(22)

From (11)

\[ 1 + r_p - t_p = 2ar_{p\rightarrow s}. \]

(23)

From the condition (5) and Maxwell equations \(H_y = \frac{1}{i\omega} \partial_z E_x, \quad E_y = -\frac{1}{i\omega\varepsilon(\omega)} \partial_z H_x\) we obtain

\[ k_z r_{p\rightarrow s} + k_z^{(2)} t_{p\rightarrow s} = 2a t_p \frac{k_z^{(2)}}{\varepsilon(\omega)}. \]

(24)

Solving equations (21)-(24) we find reflection and transmission coefficients for TM plane wave:

\[ r_p = \frac{r_f + a^2T}{1 + a^2T}, \quad t_p = \frac{t_f}{1 + a^2T}, \quad r_{p\rightarrow s} = t_{p\rightarrow s} = \frac{aT}{1 + a^2T}, \]

(25)

where

\[ r_p^f = \frac{\varepsilon(\omega)k_z - k_z^{(2)}}{\varepsilon(\omega)k_z + k_z^{(2)}}, \quad t_p^f = \frac{2\varepsilon(\omega)k_z}{\varepsilon(\omega)k_z + k_z^{(2)}} \]

(26)

are TM Fresnel coefficients for diffraction on a flat dielectric semispace.
3 Casimir interaction

Consider two half spaces \((z \leq 0 \text{ and } z \geq L)\) characterized in their volume by a frequency dependent permittivity \(\varepsilon(\omega)\). Chern-Simons terms with constants \(a_1, a_2\) are located at plane-parallel boundaries \(z = 0\) and \(z = L\) of dielectric half spaces. There is a vacuum slit \(0 < z < L\) between two half spaces.

The reflection matrix \(R_{\text{down}}(a_1) = R(a_1)\) from the \(z \leq 0\) half space is defined by:

\[
R(a_1) = \begin{pmatrix}
  r_s & r_{p-s} \\
  r_{s+p} & r_p
\end{pmatrix} = \frac{1}{1 + a_1^2 T} \begin{pmatrix}
  r_s^f - a_1^2 T & a_1 T \\
  a_1^2 T & r_p^f + a_1^2 T
\end{pmatrix}.
\] (27)

The reflection matrix from the \(z \geq L\) half space is defined after euclidean rotation by

\[
R_{\text{up}}(a_2) = SR(-a_2)S,
\] (28)

where

\[
S = \begin{pmatrix}
  e^{-L\sqrt{\omega^2 + k_x^2 + k_y^2}} & 0 \\
  0 & e^{-L\sqrt{\omega^2 + k_x^2 + k_y^2}}
\end{pmatrix}
\] (29)

is a matrix due to a change of the coordinate system \(x_1 = x, y_1 = -y, z_1 = -z + L\) (see for example [21, 45]).

Consider the system consisting of two dielectric half spaces with Chern-Simons layers at their plane-parallel boundaries separated by a vacuum slit of the width \(L\) at zero temperature. Temperature effects in the Casimir effect can be neglected at distances between half spaces of the order 10 nm we are interested in. The Casimir energy is equal

\[
E(a_1, a_2, L) = \frac{1}{2} \iiint \frac{d\omega dk_x dk_y}{(2\pi)^3} \ln \det(I - R_{\text{up}}(a_2)R_{\text{down}}(a_1)) = \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln \det(I - e^{-2Lr} R(-a_2)R(a_1)).
\] (30)

Consider two Au half spaces separated by a vacuum slit with Chern-Simons boundary layers satisfying the condition \(a \equiv a_1 = a_2\). At large separations the Casimir force is attractive. At short separations there exists a range of parameters \(a\) and distances with a repulsive Casimir force. It is instructive to plot the position of the energy minimum separating regions of the repulsive and the attractive force for three different models of dielectric permittivity: a Drude model \(\varepsilon(\omega) = 1 - \omega_p^2/\omega(\omega + i\gamma)\) with \(\omega_p = 9 eV, \gamma = 0.035 eV\), six-oscillator Drude model [17] and Drude model with full set of Au data [25] taken into account through Kramers-Kronig relations to evaluate \(\varepsilon(i\omega)\). On Fig.1 dependence of the position of the energy minimum \(L_0\) on the parameter \(a\) is shown. The force changes its sign at separations \(L_0 \sim 5 \text{ nm}\). These distances are typical for the nonretarded region of the Casimir interaction between two dielectric/metal half spaces separated by a vacuum slit. To evaluate the Casimir force in the nonretarded limit one should use optical data for frequency dispersion of the dielectric permittivity on the whole frequency axis. This is the reason why the simple Drude model of Au and even the six-oscillator model of Au can not be used for precise calculations of the Casimir forces at these separations. Energy plot corresponding to Chern-Simons parameters \(a_1 = a_2 = 0.565\) (corresponding to a maximum
value \( L_0 = 3.65 \text{ nm} \) on Fig.1 is shown on Fig.2. The change of the force from repulsive to attractive behavior takes place at the distance \( L_0 = 3.65 \text{ nm} \) in this case.

For dielectric permittivity of intrinsic Si we used the model [48]. The change of the force sign for Si corresponding to \( a_1 = a_2 = 0.567 \) takes place at the distance \( L_0 = 6.39 \text{ nm} \) (Fig.3, Fig.4), which is about 2 times larger than in the case of Au.

It is also instructive to plot ratio of the Casimir force with Chern-Simons layers at the boundaries of two half spaces \( F \) to the Lifshitz force \( F_s \) [3] (the force between two dielectric half spaces separated by a distance \( L \)). These ratios for Au and Si are shown on Fig.5 and Fig.6 respectively; transition between repulsive and attractive regimes of the Casimir force due to Chern-Simons boundary layers is clearly seen. Note that ratio of the forces quickly decreases at \( L_0 < L \lesssim 20 \text{ nm} \) for Au and \( L_0 < L \lesssim 30 \text{ nm} \) for Si systems.

Casimir repulsion at short separations is explained as follows. Lifshitz force power law effectively changes from attractive retarded \( L^{-4} \) to attractive nonretarded \( L^{-3} \) behaviour for dielectrics (metals) at distances of the order \( L \sim 10 \text{ nm} \). On the other hand, the force between two Chern-Simons layers in vacuum has \( L^{-4} \) behavior at all separations and thus dominates the total force at separations of the order \( L \lesssim 10 \text{ nm} \). For an interval \( a \in [0, a_0] \), where \( a_0 \approx 1.032502 \), and the condition \( a \equiv a_1 = a_2 \) the Casimir force between two Chern-Simons layers in vacuum is always repulsive. As a result, the sum of the Lifshitz force and the force between two Chern-Simons layers in vacuum effectively leads to Casimir repulsion at short separations.

Chern-Simons constants of the layers at \( z = 0 \) and \( z = L \) will be equal \( (a \equiv a_1 = a_2) \) in a constant external magnetic field perpendicular to the layers if one uses quantum Hall layers at the boundaries of dielectric half spaces. Quantum Hall layers are effectively described by Chern-Simons terms at plateaus of the quantum Hall effect with Chern-Simons constants \( a = \alpha n \), where \( \alpha \) is QED fine structure constant, \( n \) is an integer or a fractional number corresponding to integer or fractional quantum Hall effect. It is reasonable to put external electric field parallel to the layers equal to zero to avoid spatial components of Hall currents in the boundary layers. Quantum Hall layers will lead to Casimir repulsion at nanoscales for \( a \lesssim 1 \) for Au and Si. Essential decrease of the Casimir force due to presence of boundary quantum Hall layers in comparison with Lifshitz force should be seen at separations \( L \lesssim 20 \text{ nm} \) for Au and \( L \lesssim 30 \text{ nm} \) for Si half spaces.

For \( a_1 = -a_2 \) the force is always attractive, ratios of the Casimir force with Chern-Simons layers at the boundaries of two half spaces \( F \) to the Lifshitz force \( F_s \) are plotted on Fig.7 and Fig.8 for Au and Si respectively. The attractive Casimir force in this case is due to a theorem that opposites obtained by mirror images of each other attract when they are separated by a vacuum slit [49].

### 4 Conclusions

In this paper we present a solution of a diffraction problem for a Chern-Simons plane layer located at the boundary of a dielectric half space characterized by a frequency dependent dielectric permittivity \( \varepsilon(\omega) \). Casimir forces for two dielectric half spaces with Chern-Simons boundary layers separated by a vacuum slit \( L \) have remarkable properties. For equal Chern-Simons terms the Casimir energy has a minimum at \( L = L_0 \) for an interval of constants \( a \) of Chern-Simons terms, the Casimir force for such systems is repulsive at short separations.
and attractive at large separations. We have shown that for Au half spaces with Chern-Simons boundary layers the minimum of the Casimir energy can be achieved at separation $L_0 = 3.65$ nm, for Si half spaces with Chern-Simons boundary layers - at $L_0 = 6.39$ nm. Repulsive behaviour of the Casimir force at short separations should enhance stability of force measurements at $L \sim L_0$. Quick decrease of the Casimir force in comparison to the Lifshitz force at separations approaching the energy minimum is another interesting feature of systems with Chern-Simons boundary layers that can be verified in experiments.

It is natural to consider experimental implementation of the Chern-Simons layers as quantum Hall layers in the presence of constant external magnetic field perpendicular to the layers and zero external electric field parallel to the layers. Knowledge of the Chern-Simons constant $a = \alpha n$ at each plateau of the quantum Hall effect (characterized by an integer or a fractional number $n$) is important for precise evaluation of the Casimir force from the first principles.

Finding the difference between the theoretical force and its experimental values is a natural way to search for long-range interactions beyond electromagnetism. Systems with Chern-Simons boundary layers have unique features that provide novel possibilities in experimental search of long-range interactions beyond electromagnetism via force measurements.

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Figure 1: Position of the minimum of the energy $L_0$ for Chern-Simons layers on Au semispaces, $a = a_1 = a_2$. Results for three models of Au dielectric permittivity are shown.
Figure 2: Energy on a unit surface for Chern-Simons layers on Au semispaces obtained from full set of known optical data for Au. Chern-Simons constant is $a_1 = a_2 = 0.565$, which corresponds to the minimum of energy at $L_0 = 3.65$ nm.
Figure 3: Position of the minimum of the energy $L_0$ for Chern-Simons layers on intrinsic Si semispaces, $a = a_1 = a_2$. 
Figure 4: Energy on a unit surface obtained for Chern-Simons layers on intrinsic Si semispaces. Chern-Simons constant is $a_1 = a_2 = 0.567$, which corresponds to the minimum of the energy at $L_0 = 6.39$ nm.
Figure 5: Ratio of the force $F$ with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force $F_s$ between two Au semispaces separated by a distance $L$. Chern-Simons constants are $a_1 = a_2 = 0.565$. 

$F/F_s$ vs $L$, nm
Figure 6: Ratio of the force $F$ with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force $F_s$ between two intrinsic Si semispaces separated by a distance $L$. Chern-Simons constants are $a_1 = a_2 = 0.567$. 

\[ \frac{F}{F_s} \]
Figure 7: Ratio of the force $F$ with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force $F_s$ between two Au semispaces separated by a distance $L$. Chern-Simons constants are $a_1 = -a_2 = 0.565$. 
Figure 8: Ratio of the force $F$ with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force $F_s$ between two intrinsic Si semispaces separated by a distance $L$. Chern-Simons constants are $a_1 = -a_2 = 0.567$. 