Latest supernovae constraints on $f(R)$ cosmologies

J. Santos$^{1,*}$, J. S. Alcaniz$^{2,3,†}$, F. C. Carvalho$^{4,‡}$, and N. Pires$^{1,§}$

$^1$Universidade Federal do Rio Grande do Norte, Departamento de Física, 59072-970 Natal - RN, Brasil
$^2$Observatório Nacional, 20921-400 Rio de Janeiro - RJ, Brasil
$^3$Instituto Nacional de Pesquisas Espaciais/CRN, 59076-740 Natal - RN, Brasil and
$^4$Instituto Nacional de Pesquisas Espaciais, 12227-010, Sào José dos Campos - SP, Brasil

(Dated: August 29, 2008)

A class of modified gravity, known as $f(R)$-gravity, has presently been applied to Cosmology as a realistic alternative to dark energy. In this paper we use the most recent Type-Ia Supernova (SNe Ia) data, the so-called Union sample of 307 SNe Ia, to place bounds on a theory of the form $f(R) = R - \beta/R^n$ within the Palatini approach. Given the complementarity of SNe Ia data with other cosmological observables, a joint analysis with measurements of baryon acoustic oscillation peak and estimates of the CMB shift parameter is also performed. We show that, for the allowed intervals of $n$, $\Omega_{\text{mo}}$, and $\beta$, models based on $f(R) = R - \beta/R^n$ gravity in the Palatini approach can produce the sequence of radiation-dominated, matter-dominated, and accelerating periods without need of dark energy.

PACS numbers:

I. INTRODUCTION

One of the key problems at the interface between fundamental physics and cosmology is to understand the physical mechanism behind the late-time acceleration of the Universe. In principle, this phenomenon may be the result of unknown physical processes involving either modifications of gravitation theory or the existence of new fields in high energy physics. Although the latter route is most commonly used, which gives rise to the idea of a dark energy component (see, e.g., [1]), following the former, at least two other attractive approaches to this problem can be explored. The first one is related to the possible existence of extra dimensions, an idea that links cosmic acceleration with the hierarchy problem in high energy physics, and gives rise to the so-called brane-world cosmology [2]. The second one, known as $f(R)$ gravity, examine the possibility of modifying Einstein’s general relativity (GR) by adding terms proportional to powers of the Ricci scalar $R$ to the Einstein-Hilbert Lagrangian [3]. The cosmological interest in $f(R)$ gravity comes from the fact that these theories can exhibit naturally an accelerating expansion without introducing dark energy. However, the freedom in the choice of different functional forms of $f(R)$ gives rise to the problem of how to constrain on theoretical and/or observational grounds, the many possible $f(R)$ gravity theories. Much efforts within the realm, mainly from a theoretical viewpoint, have been developed so far [4] (see also Refs. [3] for recent reviews), while only recently observational constraints from several cosmological data sets have been explored for testing the viability of these theories [4, 5, 8, 9, 10, 11].

An important aspect worth emphasizing concerns the two different variational approaches that may be followed when one works with $f(R)$ gravity theories, namely, the metric and the Palatini formalisms (see, e.g., [3]). In the metric formalism the connections are assumed to be the Christoffel symbols and variation of the action is taken with respect to the metric, whereas in the Palatini variational approach the metric and the affine connections are treated as independent fields and the variation is taken with respect to both. In fact, these approaches are equivalents only in the context of GR, that is, in the case of linear Hilbert action; for a general $f(R)$ term in the action they give different equations of motion.

In the present paper we will restrict ourselves to the Palatini formalism for gravitation and will focus on its application to the flat Friedmann-Robertson-Walker (FRW) cosmological model. We will derive constraints on the two parameters $n$ and $\beta$ of the $f(R) = R - \beta/R^n$ gravity theory from the most recent compilations of type Ia Supernovae (SNe Ia) observations, which includes the recent large samples from Supernova Legacy Survey (SNLS), the ESSENCE Survey, distant SNe Ia observed with HST, and others, giving a sample of 307 SNe Ia events [12]. We also combine the SNe Ia data with information from the baryon acoustic oscillation (BAO) [13] and the CMB shift parameter [14] in order to improve the SNe Ia bounds on the free parameters of the theory.

II. PALATINI APPROACH

The action that defines an $f(R)$ gravity is given by

$$ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m , \quad (1) $$

where $\kappa^2 = 8\pi G$, $g$ is the determinant of the metric tensor and $S_m$ is the standard action for the matter fields. Treating the metric and the connection as completely indepen-
decreased fields, variation of this action gives the field equations

\[ f' R_{(\mu\nu)} - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \]

where \( T_{\mu\nu} \) is the matter energy-momentum tensor which, for a perfect-fluid, is given by \( T_{\mu\nu} = (\rho_m + p_m) u_\mu u_\nu + p_m g_{\mu\nu} \), where \( \rho_m \) is the energy density, \( p_m \) is the fluid pressure and \( u_\mu \) is the fluid four-velocity. Here, we adopt the notation \( f' = df/dR \), \( f'' = d^2 f/dR^2 \) and so on. In (2) \( R_{\mu\nu} \) is given in the usual way in terms of the independent connection \( \Gamma^\nu_{\mu\rho} \), and its derivatives, which is related with the Christoffel symbol \( \{\nu_{\mu\rho}\} \) of the metric \( g_{\mu\nu} \) by

\[ \Gamma^\nu_{\mu\rho} = \{\nu_{\mu\rho}\} + \frac{1}{2f} (\delta^\nu_\rho \partial_\mu + \delta^\nu_\mu \partial_\rho - g_{\nu\rho} g^{\sigma\tau} \partial_\sigma) f' \]

and \( R = g^{\mu\nu} R_{\mu\nu} \).

We assume a homogeneous and isotropic FRW universe whose metric is \( g_{\mu\nu} = diag(-1, a^2, a^2, a^2) \), where \( a(t) \) is the cosmological scale factor. The generalized Friedmann equation can be written in terms of redshift parameter \( z = a_0/a - 1 \) and the density parameter \( \Omega_{\text{mo}} \equiv \kappa \rho_{\text{mo}}/(3H_0^2) \) as (see Ref. \[7, 10\] for details)

\[ \frac{H^2}{H_0^2} = \frac{3\Omega_{\text{mo}}(1 + z)^3 + f/H_0^2}{6f'\xi^2}, \]

where

\[ \xi = 1 + \frac{9}{2} \frac{f''}{f'} \frac{H_0^2 \Omega_{\text{mo}}(1 + z)^3}{R f'' - f'} \]

and \( \rho_{\text{mo}} \) is the matter density today. The trace of Eq. (2) gives another relation

\[ R f' - 2f = -3H_0^2 \Omega_{\text{mo}}(1 + z)^3, \]

and, as can be easily checked, for the Einstein-Hilbert Lagrangean \( f = R \) Eq. (4) reduces to the known form of Friedmann equation.

By assuming a functional form of the type \( f(R) = R - \beta/R^n \), one may easily show that Eq. (4) evaluated at \( z = 0 \) imposes the following relation among \( n, \Omega_{\text{mo}} \) and \( \beta \)

\[ \beta = \frac{R_0^{n+1}}{n} \left( 1 - \frac{3\Omega_{\text{mo}}H_0^2}{R_0} \right), \]

where \( R_0 \), the value of the Ricci scalar today, is determined from the algebraic equation resulting from equating (4) and (3) for \( z = 0 \). Hence, specifying the values of two of these parameters the third is automatically fixed. In other words, in the Palatini approach, the two parameter of \( f(R) = R - \beta/R^n \) can be thought as the pair \( (n, \beta) \) or \( (n, \Omega_{\text{mo}}) \).

### III. OBSERVATIONAL ANALYSES

Since the very first results showing direct evidence for a present cosmic acceleration (using a small number of SNe Ia events) \[13\], the number and quality of SNe Ia data available for cosmological studies have increased considerably due to several observational programs. The most up to date set of SNe Ia has been compiled by Kowal-\textit{ski et al.} \[12\] and includes recent large samples from SNLS \[16\] and ESSENCE \[17\] surveys, older data sets and the recently extended data set of distant supernovae observed with HST. The total compilation, the so-called \textit{Union} sample, amounts to 414 SNe Ia events, which was reduced to 307 data points after selection cuts.

In this Section, we will use this SNe Ia sample to place limits on the \( n - \Omega_{\text{mo}} \) (or, equivalently, \( \beta - \Omega_{\text{mo}} \)) parametric space. This analysis, therefore, updates the results of Refs. \[4, 7\]. We also perform a joint analysis involving the \textit{Union} SNe Ia sample and measurements of the baryonic acoustic oscillations (BAO) from SDSS \[13\] and the CMB shift parameter as given by the WMAP team \[14\] to break possible degeneracies in the \( n - \Omega_{\text{mo}} \) plane (for more details on the statistical analyses discussed below we refer the reader to Ref. \[18\]).

#### A. Latest SNe Ia constraints

The predicted distance modulus for a supernova at redshift \( z \), given a set of parameters \( P = (n, \Omega_{\text{mo}}) \), is

\[ \mu_0(z|P) = m - M = 5 \log d_L + 25, \]

FIG. 1: Hubble diagram for 307 SNe Ia from the \textit{Union} sample \[12\]. The curves correspond to the best-fit pairs of \( n \) and \( \Omega_{\text{mo}} \) arising from statistical analyses involving SNe Ia (only) and SNe Ia + BAO + CMB shift parameter. For the sake of comparison the flat \( \Lambda \text{CDM} \) scenario with \( \Omega_{\text{mo}} = 0.26 \) is also shown.
where $m$ and $M$ are, respectively, the apparent and absolute magnitudes, and $d_L$ stands for the luminosity distance (in units of megaparsecs),

$$d_L(z; P) = (1 + z) \int_0^z \frac{dz'}{H(z'; P)},$$

(9)

where $H(z; P)$ is given by Eqs. (4) - (6).

We estimate the best fit to the set of parameters $P$ by using a $\chi^2$ statistics, with

$$\chi^2_{SN} = \sum_{i=1}^{N} \left[ \frac{\mu_i^b(z|P) - \mu_i^{obs}(z)}{\sigma_i} \right]^2,$$

(10)

where $\mu_i^b(z|P)$ is given by Eq. (5), $\mu_i^{obs}(z)$ is the extinction corrected distance modulus for a given SNe Ia at $z_i$, and $\sigma_i$ is the uncertainty in the individual distance moduli. Since we use in our analysis the Union sample (see [12] for details), $N = 307$.

Figure (1) shows the Hubble diagram for the 307 SNe Ia events of the Union sample. The curves stand for the best-fit $f(R)$ models obtained from SNe Ia and SNe Ia + BAO + CMB analysis. For the sake of comparison, the standard $\Lambda$CDM model with $\Omega_{mo} = 0.26$ is also shown. Note that all models seem to be able to reproduce fairly well the SNe Ia measurements. In Fig. (2a) we show the first results of our statistical analyses. Contour plots (68.3%, 95.4% and 99.7% c.l.) in the $n \times \Omega_{mo}$ plane are shown for the $\chi^2$ given by Eq. (10). We clearly see that SNe Ia measurements alone do not tightly constrain the values of $n$ and $\Omega_{mo}$, allowing for a large interval of values for these parameters, with $n$ ranging from -1 to even beyond 1, and $\Omega_{mo}$ consistent with both vacuum solutions ($\Omega_{mo} = 0$), as well with universes with up to 90% of its energy density in the form of non-relativistic matter. The best-fit values for this analysis are $\Omega_{mo} = 0.57$ and $n = 0.99$, with the reduced $\chi^2 = \chi^2_{m}/\nu \simeq 1.01$ ($\nu$ is defined as degrees of freedom).

B. Joint analysis

1. BAO

The acoustic oscillations of baryons in the primordial plasma leave a signature on the correlation function of galaxies as observed by Eisenstein et al. [13]. This signature furnishes a standard rule which can be used to constrain the following quantity:

$$A = D_V \sqrt{\Omega_{mo} H_0^2} \frac{1}{z_*},$$

(11)

where the observed value is $A_{obs} = 0.469 \pm 0.017$, $z_* = 0.35$ is the typical redshift of the SDSS sample and $D_V$ is the dilation scale, defined as $D_V = [D_M(z_0|P)2/z_0/H(z_0|P)]^{1/3}$ with the comoving distance $D_M$ given by $D_M(z_*|P) = \int_0^{z_*} dz'/H(z'|P)$. In Fig. (2b) we show the confidence contours (68.3%, 95.4% and 99.7% c.l.) in the $n - \Omega_{mo}$ plane arising from this measurement of $A$. As expected, since this quantity has been measured at a specific redshift ($z_* = 0.35$), it forms bands on this parametric space, instead of ellipsoids as in the case of SNe Ia data.

2. Shift Parameter

The shift parameter $R$, which determines the whole shift of the CMB angular power spectrum, is given by
FIG. 3: Left: Confidence intervals at 68.3%, 95.4% and 99.73% in the \( n \times \Omega_{mo} \) plane arising from the combined fit involving SNe Ia \( \text{Union} \) sample, BAO and CMB shift parameter. The best-fit values for this analysis is \( n = -0.12 \) and \( \Omega_{mo} = 0.26 \) (\( \beta = 3.45 \)). Right: Effective equation of state as a function of redshift for a \( f(R) = R - \beta R^n \) theory in the Palatini formalism. The parameters values correspond to the best-fit solution of our joint analysis.

\[
R \equiv \sqrt{\Omega_{mo}} \int_0^{z_{ls}} \frac{H_0 dz'}{H(z'; P)},
\]

where the \( z_{ls} = 1089 \) is the redshift of the last scattering surface, and the current estimated value for this quantity is \( R_{obs} = 1.70 \pm 0.03 \) [14]. Note that, to include the CMB shift parameter into the analysis, the equations of motion must be integrated up to the matter/radiation decoupling, \( z \simeq 1089 \). Since radiation is no longer negligible at this redshift, a radiation component with an energy density today of \( \Omega_{\gamma} = 5 \times 10^{-5} \) has been included in our analysis. Figure (2c) shows the constraints on the \( n - \Omega_{mo} \) plane from the current WMAP estimate of \( R \).
1 + z for the best-fit solution of our joint SNe Ia + BAO + CMB analysis. Note that, for this particular combination of parameters, the universe goes through the last three phases of cosmological evolution, i.e., radiation-dominated \((w = 1/3)\), matter-dominated \((w = 0)\) and the late time acceleration phase (in this case with \(w \approx -1\)). Therefore, the arguments of Ref. [20] about the \(w_{eff}\) in the metric approach seem not to apply to the Palatini formalism, at least for the interval of parameters \(n, \Omega_{mo}\) and \(\beta\) given by our statistical analysis. In Table I we summarize the main results of this paper compare them with recent determinations of the parameters \(n, \Omega_{mo}\) and \(\beta\) from independent analyses.

IV. CONCLUSIONS

\(f(R)\)-gravity based cosmology has presently been thought of as a realistic alternative to general relativistic dark energy models. In this paper, we have worked in the context of a \(f(R) = R - \beta R^{-n}\) gravity with equations of motion derived according to the Palatini approach. We have performed consistency checks and tested the observational viability of these scenarios by using the latest sample of SNe Ia data, the so-called Union sample of 307 events. Although the current SNe Ia measurements alone cannot constrain significantly the model parameters \(n, \Omega_{mo}\) and \(\beta\), when combined with information from BAO and CMB shift parameters, the fit leads to very restrictive constraints on the \(n - \Omega_{mo}\) (or, equivalently, \(\beta - \Omega_{mo}\)) parametric space. At 99.7% c.l., e.g., we have found the intervals \(n \in [-0.25, 0.35]\) and \(\Omega_{mo} \in [0.2, 0.31]\) \(([\beta \in [2.3, 7.1]]\). We note that, differently from results in the metric formalism [20], the universe corresponding to the best-fit solution for a combined SNe Ia+BAO+CMB \(\chi^2\) minimization (\(\Omega_{mo} = 0.26\) and \(n = -0.12\)) shows all three last phases of the cosmological evolution: radiation era, matter era and a late time cosmic acceleration.

Acknowledgments

The authors are very grateful to Edivaldo Moura Santos for helpful conversations and a critical reading of the manuscript. JS and NP thank financial support from PRONEX (CNPq/FAPERN). FCC acknowledge financial support from FAPESP. JSA’s work is supported by CNPq.

[1] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9 (2000) 373; T. Padmanabhan, Phys. Rep. 380, 235 (2003); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003); J. S. Alcaniz, Braz. J. Phys. 36, 1109 (2006); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1755 (2006);

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B. 485 208 (2000); C. Deffayet et al., Phys. Lett. B, 502, 199 (2001); J. S. Alcaniz, Phys. Rev. D 65, 123514 (2002); D. Jain, A. Dev and J. S. Alcaniz, Phys. Rev. D 66, 083511 (2002) arXiv:astro-ph/0206224; V. Sahni and Y. Shtanov, JCAP 0311, 014 (2003); M. D. Maia et al., Class. Quant. Grav. 22, 1623 (2005).

[3] H. A. Buchdahl, Mont. Not. R. Astr. Soc. 150, 1 (1970); A. A. Starobinsky, Phys. Lett. B 91, 99 (1980); R. Kerner, Gen. Relativ. Gravit. 14, 453 (1982); J. D. Barrow and A. C. Ottewill, J. Phys. A: Math. Gen. 16, 2757 (1983); J. D. Barrow and S. Cotsakis, Phys. Lett. B 214, 515 (1989); B. Li and J. D. Barrow, Phys. Rev. D 75, 084010 (2007).

[4] M. Ferraris, M. Francaviglia and I. Volovich, Class. Quantum Grav. 11, 1505 (1994); A.D. Dolgov and M. Kawasaki, Phys. Lett. B 573, 1 (2003); É. É. Flanagan, Phys. Rev. Lett. 92, 071101 (2004); G. J. Olmo, Phys. Rev. D 72, 083505 (2005); Phys. Rev. D 75, 023511 (2007); Phys. Rev. Lett. 98, 061101 (2007); S. Capozziello, S. Nojiri, S.D. Odintsov and A. Troisi, Phys. Lett. B 639, 135 (2006); A. W. Brookfield, C. van de Bruck and L.M.H. Hall, Phys. Rev. D 74, 064028 (2006); T. Multamäki and I. Vilja, Phys. Rev. D 73, 024018 (2006); N. J. Poplawski, Phys. Rev. D 74, 084032 (2006); A. de la Cruz-Dombriz and A. Dobado, Phys. Rev. D 74, 087501 (2006); L. Amendola, R. Gamoujui, D. Polarski and S. Tsujikawa, Phys. Rev. D 75, 083504 (2007); L. Amendola, D. Polarski and S. Tsujikawa, Int. J. Mod. Phys. D 16, 1555 (2007); W. Hu and I. Sawicki, Phys. Rev. D 76, 064004 (2007); O. Bertolami, C.G. Böhmer, T. Harko and F.S.N. Lobo, Phys. Rev. D 75, 104016 (2007); B. Li, J.D. Barrow and D.F. Mota, Phys. Rev. D 76, 104047 (2007); I. Navarro and K. Van Acoleyen, JCAP 0702, 022 (2007); S. Baghram, M. Farhang and S. Rahvar, Phys. Rev. D 75, 044024 (2007); T. P. Sotiriou, Phys. Lett. B 645, 389 (2007); J. Santos, J.S. Alcaniz, M.J. Rebouças and F.C. Carvalho, Phys. Rev. D 76, 083513 (2007) arXiv:0708.0411 [astro-ph]; S. Fay, S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 76, 063504 (2007); C.G. Böhmer, T. Harko and F.S.N. Lobo, JCAP 0803, 024 (2008); S.A. Appleby and R.A. Battye, JCAP 0805, 019 (2008); E. Barausse, T.P. Sotiriou and J.C. Miller, Class. Quantum Grav. 25, 105008 (2008); G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, L. Sebastiani and S.Zerbini, Phys. Rev. D 77, 046009 (2008); S. Nojiri and S.D. Odintsov, Phys. Rev. D 77, 026007 (2008); T. P. Sotiriou, S. Liberati and V. Faraoni, Int. J. Mod. Phys. D 17, 399 (2008); C. G. Boehmer, T. Harko and F. S. N. Lobo, Astrophys. Phys. 29, 386 (2008); S. Nojiri and S.D. Odintsov, arXiv:0804.3519 [hep-th]; S. Capozziello, M. Francaviglia and S. Mercadante, arXiv:0805.3642 [gr-qc]; T.P. Sotiriou, arXiv:0805.1160 [gr-qc]; C.S.J. Pun, Z. Kovács and T. Harko, arXiv:0806.0679 [gr-qc]; V. Faraoni, arXiv:0806.0766 [gr-qc]; F. S. N. Lobo, arXiv:0807.1640 [gr-qc];

[5] S. Capozziello and M. Francaviglia, Gen. Relativ. Gravit. 40, 357 (2008); T. P. Sotiriou and V. Faraoni,
[6] M. Amarzguioui, Ø. Elgarøy, D.F. Mota and T. Multamäki, Astron. Astrophys. 454, 707 (2006).
[7] S. Fay and R. Tavakol, Phys. Rev. D 75, 063509 (2007).
[8] M. Fairbairn and S. Rydbeck, JCAP 0712, 005 (2007).
[9] M. S. Movahed, S. Baghram and S. Rahvar, Phys. Rev. D 76, 044008 (2007).
[10] F.C. Carvalho, E.M. Santos, J.S. Alcaniz and J. Santos, JCAP (in press). arXiv:0804.2878 [astro-ph].
[11] T. Koivisto, Phys. Rev. D 76, 043527 (2007).
[12] M. Kowalski et al., arXiv:0804.4142 [astro-ph].
[13] D. J. Eisenstein et al., Astrophys. J., 633, 560 (2005).
[14] D. N. Spergel et al., Astroph. J. Suppl. 148, 175 (2003); D. N. Spergel et al., Astroph. J. Suppl. 170, 377 (2007).
[15] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[16] P. Astier et al., Astron. Astrophys. 447, 31 (2006).
[17] W. M. Wood-Vasey et al., Astrophys. J. 666, 694 (2007).
[18] T. Padmanabhan and T. R. Choudhury, Mon. Not. R. Astron. Soc. 344, 823 (2003); P. T. Silva and O. Bertolami, Astrophys. J. 599, 829 (2003); Z.-H. Zhu and M.-K. Fujimoto, Astrophys. J. 585, 52 (2003); J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, Phys. Rev. D 68, 023510 (2003) arXiv:astro-ph/0303388; S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 70, 043531 (2004); J. S. Alcaniz and N. Pires, Phys. Rev. D 70, 047303 (2004) arXiv:astro-ph/0404146; J. A. S. Lima and J. S. Alcaniz, Astron. Astrophys. 357, 393 (2000) arXiv:astro-ph/0003189; T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. 429, 807 (2005); J. S. Alcaniz and Z.-H. Zhu, Phys. Rev. D 71, 083513 (2005) arXiv:astro-ph/0411604; Z. K. Guo, Z. H. Zhu, J. S. Alcaniz and Y. Z. Zhang, Astrophys. J. 646, 1 (2006) arXiv:astro-ph/0603632; L. Samushia and B. Ratra, Astrophys. J. 650, L5 (2006); M. A. Dantas, J. S. Alcaniz, D. Jain and A. Dev, Astron. Astrophys. 467, 421 (2007) arXiv:astro-ph/0607060; E. M. Barboza and J. S. Alcaniz, Phys. Lett. B (in press). arXiv:0805.1713 [astro-ph].
[19] J. R. Bond, G. Efstathiou, & M. Tegmark, MNRAS, 291, L33 (1997).
[20] L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett. 98, 131302 (2007).