THERMAL PROPERTIES OF TWO-DIMENSIONAL ADVECTION-DOMINATED ACCRETION FLOW

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ABSTRACT

We study the thermal structure of the widely adopted two-dimensional advection-dominated accretion flow (ADAF) of Narayan & Yi. The critical radius for a given mass accretion rate, outside of which the optically thin hot solutions do not exist in the equatorial plane, agrees with those of one-dimensional studies. However, we find that, even within the critical radius, there always exists a conical region of the flow, around the pole, which cannot maintain the assumed high electron temperature, regardless of the mass accretion rate, in the absence of radiative heating. This could lead to a torus-like advection inflow shape since, in general, the ions too will cool down. We also find that Compton preheating is generally important and, if the radiative efficiency, defined as the luminosity output divided by the mass accretion rate times the velocity of light squared, is above $\sim 4 \times 10^{-3}$, the polar region of the flow is preheated above the virial temperature by Compton heating, which may result in time-dependent behavior or outflow while accretion continues in the equatorial plane. Thus, under most relevant circumstances, ADAF solutions may be expected to be accompanied by polar outflow winds. While preheating instabilities exist in ADAF, as for spherical flows, the former are to some extent protected by their characteristicistically higher densities and higher cooling rates, which reduce their susceptibility to Compton driven overheating.

Subject headings: accretion, accretion disks — black hole physics — radiation mechanisms: nonthermal — radiative transfer

1. INTRODUCTION

Depending on the angular momentum that it carries and how the angular momentum is dissipated, gas accretes onto compact objects in various ways. If the gas contains very little angular momentum or bulk flow, the flow becomes spherical (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952; Loeb & Laor 1992; Foglizzo & Ruffert 1997; Nio, Matsuda, & Fukue 1998). If it has enough angular momentum and if the angular momentum is efficiently removed by some mechanism, the flow flattens to a disk shape (Pringle & Rees 1972; Shakura & Sunyaev 1973). Until recently, only the extremes of these two types of solutions have been studied (see Chakrabarti 1996a, 1996b for a history and a unified scheme of accretion solutions and Park & Ostriker 1998 for review on general accretion flow).

Spherical accretion has the merit of being simple and therefore can be accurately calculated. It generally has an infall velocity close to the free-fall value, and most of the gas energy generated is lost into the hole in the case of accretion onto black holes, making the radiation efficiency of the accretion, $e \equiv L/Mc^2$, where $L$ is the luminosity output and $M$ is the mass accretion rate, a self-consistently calculable quantity depending primarily on the entropy at large radii and the accretion rate. The efficiency can be very low or significantly high (Shapiro 1973a, 1973b; Mészáros 1975; Park & Ostriker 1989; Park 1990a, 1990b). In the opposite limiting case, accretion proceeds in the form of a thin disk when the gas has enough angular momentum and cools efficiently. It has a fixed and high radiation efficiency, $e \sim 0.1$, and does not emit high-energy photons owing to the low temperature of the gas. Although the thin-disk accretion model has been successfully applied to many astronomical sources (see Pringle 1981 and Frank, King, & Raine 1992 for reviews), its radiation spectrum is incompatible with certain kinds of celestial sources believed to be powered by accretion.

There are also other types of accretion disk solutions beyond the cool thin disk. Shapiro, Lightman, & Eardley (1976) showed that geometrically and optically thin, high-temperature accretion disks can exist in which ions and electrons are weakly coupled and have different temperatures. Although this solution was more successful in explaining high-energy sources like Cyg X-1, it proved to be unstable on a thermal timescale (Pringle 1976; Piran 1978; Park 1995). Seminal works on geometrically thick accretion disks appeared a few years later (Jaroszynski, Abramowicz, & Paczyński 1980; Paczyński & Wiita 1980). Abramowicz et al. (1988) further combined these works with an "$\alpha$-viscosity" model to find a so called slim-disk solution. It is a geometrically and optically thick accretion disk solution with a significant fraction of the gas energy being transported via advection, but treated in one-dimensional framework. These slim-disk solutions exist only when the mass accretion rate is near or above the Eddington mass accretion rate defined as

$$M_{\text{Edd}} \equiv \frac{L_{\text{Edd}}}{c^2} = 2.19 \times 10^{-9} M_\odot \text{ yr}^{-1}, \quad (1)$$

where $L_{\text{Edd}}$ is the Eddington luminosity and the numerical value is for the pure hydrogen. (In the case of accretion onto neutron stars or thin-disk accretion, the Eddington accretion rate has sometimes been defined as $e^{-1}L_{\text{Edd}}/c^2$ since $e$ is almost a fixed value. However, in the type of accretion in which $e$ is not known a priori, this definition would be confusing and misleading because even when $M \sim M_{\text{Edd}}$, the luminosity from the accretion can be much smaller than...
$L_E$. This choice of definition makes $M_{\text{Edd}}$ in this work 10 times smaller than $M_{\text{Edd}}$ in Narayan & Yi 1995b.) Slim disks have a radiation spectrum similar to that of the thin disks.

A new type of accretion solution that stands somewhere between the spherical and thin-disk accretion had been found by Ichimaru (1977) to explain the bimodal properties of Cygnus X-1. It has recently been rediscovered and extensively studied (Narayan & Yi 1994, 1995a, 1995b, hereafter NY1, NY2, and NY3, respectively, and NY collectively; Abramowicz et al. 1995; see Narayan, Mahadevan, & Quataert 1998 for review and references). These solutions are called “advection-dominated accretion flow” (ADAF) because most of the gas energy is advected with the flow and ultimately into the hole owing to relatively large infall velocity. This type of accretion is possible at sufficiently low mass accretion rate so that radiative cooling is inefficient and ions can be kept near virial temperature. The isodensity contours of self-similar ADAF changes from that of a sphere to a somewhat flattened spheroid, depending on the parameters chosen (NY2), but in general the flow looks spherical or spheroidal rather than disklke. It is in a way a reminiscence of ion-torus model (Rees et al. 1982).

ADAF has a number of distinct and desirable properties. First, like spherical accretion, the radiation efficiency can span a large range and is generally much smaller than $\sim0.1$. Second, the electron temperature is much higher than that in thin-disk accretion and, therefore, can produce high-energy photons. Third, the self-similar form of the solutions makes calculation of various properties very easy (Spruit et al. 1987; NY1). These merits have led to many successful applications of ADAF to various accretion powered sources (Narayan, Kato, & Honma 1997; Chen, Abramowicz, & Lasota 1997; Nakamura et al. 1997; Gammie & Popham 1998; Popham & Gammie 1998; Chakrabarti 1996a) even though the desirable properties depend explicitly on the two-dimensional characteristics of the flow. The two-dimensional nature of ADAF has been explored only in self-similar form (NY2; Xu & Chen 1997) or by numerical simulations of adiabatic, inviscid flow very close to the hole (Molteni, Lanzafame, & Chakrabarti 1994; Ryu et al. 1995; Chen et al. 1997; Igumenshchev & Beloborodov 1997). When treated as a height-integrated disk instead of two-dimensional flow, locally produced photons are assumed to escape the flow without affecting other parts of the flow, which is only true when the flow is truly disklke. Also, parts of the thermal and dynamical structure of the two-dimensional flow can be widely different from those of the averaged disk, under general accretion conditions, because of the flow structure in the vertical direction.

So, in this work, we study the thermal properties of the ADAF based on the two-dimensional self-similar flow solutions of NY2. One important difference between this work and previous studies is the allowance for the preheating of the flow at large radii by photons produced at the inner, hotter part of the flow. When the luminosity is high enough, this can substantially change the dynamics of the ADAF, leading, quite possibly, to some kind of time-dependent behavior or outflow.

### 2. THERMAL PROPERTIES OF ADVECTION-DOMINATED ACCRETION FLOWS

In generic accretion flow, the thermal state of the gas at a given point is determined by the balance between various heating and cooling processes. Gas can be heated by $PdV$ work, viscous dissipation, and the interaction with radiation. It cools mostly by emission of radiation. However, heating and cooling are not always balanced, and the extra energy gain or loss will be stored as internal energy and carried with the flow. This advection of internal energy mostly acts as loss of gas energy at a fixed point in the accretion flow and sometimes is called advective cooling.

The advection-dominated accretion flow solutions of NY refer to a family of solutions in which gas at a given point is heated by viscous dissipation and cooled mostly by the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the advection, with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the
3. COOLING IN TWO-DIMENSIONAL ADAF

The self-similar ADAF is, like the Bondi solution, very attractive because of its simplicity. Most physical quantities are simple power laws of radius and can be readily calculated. However, this also implies that the solution may not be appropriate everywhere for any real accretion flow.

The simplicity of ADAF self-similar solution is based on the simple energy balance equation. The assumption of a constant ratio between the viscous heating, advective cooling, and radiative cooling, \( 1: f: 1 - f \), maintains the self-similarity of the solutions yet puts severe restrictions on cooling processes. Should the ratio be constant in radius, the energy equation would be satisfied at only one point. NY overcome this problem by finding a semi-consistent variable \( f \) for applications to real accretion flows. However, this is only possible when a positive \( f \) can be found: the cooling should be smaller than the heating. Solutions do not exist if positive \( f \) is not possible. Cooling may be so strong that the flow cannot be kept at a high temperature (Rees et al. 1982). This leads to the critical mass accretion \( M_{\text{crit}} \) for a given radius above which the optically thin hot solutions do not exist (Abramowicz et al. 1995; NY3). And for a given \( m_\text{a} \), hot solutions exist only within some critical radius, \( r_{\text{crit}} \).

First, we will study similar issues in two-dimensional ADAF. The viscous heating per unit volume in self-similar ADAF is simply

\[
q_{\text{vis}} = f^{-1} q_{\text{adv}} = \frac{3}{2} \epsilon \frac{\rho |v_r| c_s^2}{r} ,
\]

where \( \rho(r, \theta) \) is the gas density, \( v_r \) is the radial velocity, and \( c_s \) is the isothermal sound speed. The composite formula for atomic cooling and nonrelativistic bremsstrahlung (Stellingwerf & Buff 1982; Nobili, Turolla, & Zampieri 1991) extended to relativistic bremsstrahlung is used for cooling,

\[
C = \sigma_T c \alpha_f m_e c^2 n_i^2 \left\{ [\lambda_{\text{br}}(T_e) + 6.0 \times 10^{-22} \theta_e^{-1/2}]^{-1} + \left( \frac{\theta_e}{4.82 \times 10^{-6}} \right)^{-12} \right\}^{-1} ,
\]

where \( \sigma_T \) is the Thomson cross section, \( \alpha_f \) is the fine-structure constant, \( n_i \) is the number density of ions, and \( \theta_e = k T_e/m_e c^2 \). The relativistic bremsstrahlung rate is (Svensson 1982; Stepney & Guilbert 1983; NY3)

\[
\lambda_{\text{br}} = \left( \frac{n_e}{n_i} \right) \left( \sum Z_i^2 \right) F_{el}(\theta_e) + \left( \frac{n_e}{n_i} \right)^2 F_{ee}(\theta_e) ,
\]

where

\[
F_{el} = 5 \left( \frac{2}{n^3} \right)^{1/2} \theta_e^{1/2} (1 + 1.781 \theta_e^{-1/4}) \quad \text{for } \theta_e < 1 \\
= 9 \frac{\theta_e \ln(1.123 \theta_e + 0.48) + 1.5}{2\pi} \quad \text{for } \theta_e > 1
\]

and \( Z_i \) is the charge of ions. The unknown electron temperature is assumed to be described by the fitting formula \( T_e = T_e(T_e/T_e + T_a) \) with \( T_a \approx 10^8 \) K, and \( (k T_e/m_e c^2) = \theta_e^2 \theta_a^2 (r/r_a)^{-1/4} \), with \( r_a = 2GM/c^2 \), which approximates the electron temperature profile in ADAF (NY3).

The ratio \( f \) is calculated from the definition,

\[
1 - f = C q_{\text{vis}} ,
\]

in the \((r, \theta)\) plane. The two-dimensional structure of the flow is basically determined by one parameter \( \epsilon' = \epsilon/f \) (see NY2). Contours of \( f = 0.9 \) (innermost curve), 0.8, 0.7, 0.6, 0.5, 0.3, 0.2, 0.1, and 0.0 (outermost curve) for ADAF with \( m_a = 0.03, \epsilon' = \epsilon/f = 1.0 \), and \( z = 0.1 \) are shown in Figure 1. The heavily shaded region above the contours represents \( f < 0 \), i.e., the region where hot electrons cannot exist owing to cooling. The one-dimensional calculation of NY3 (Fig. 3) shows that \( f = 0.5 \) flow has \( r_{\text{crit}} \approx 10^4 r_a \), which agrees well with \( r_{\text{crit}} \approx 1.1 \times 10^4 r_a \) for \( z = 0 \) in Figure 1.

Note that there is a roughly cone-shaped shaded region around the pole where the hot solution is not possible. For \( z \ll 1 \), this region extends down to \( \sim r_a \). This is mainly due to the slow infall velocity near the pole (NY2), to which the advective cooling is proportional. Since the viscous heating and radiative cooling are proportional to the advective cooling in self-similar ADAF, a small infall velocity near the pole implies a small advective cooling, and hence small viscous heating and radiative cooling. But even atomic plus bremsstrahlung cooling can have a far higher cooling rate.
This will create a funnel around the polar axis (Fig. 1), and in fact, as we shall show in Park & Ostriker (1999), this is the virial value, resulting in the collapse of the polar region. That the ion temperature would consequently drop below the virial temperature is near virial. But the low temperature of the electrons is usually the most important interaction. They can either heat or cool the gas, depending on the spectrum of the radiation and the temperature of the gas. High-energy photons heat the electrons, while low-energy ones cool them. However, the flow at large radius is generally heated because most photons are produced at hotter inner regions, and this is called preheating (Ostriker et al. 1976).

The importance of preheating on the flow can be estimated by comparing the relevant timescales. If $t_{\text{vis}} \ll t_H$, preheating can be safely ignored. However, if there exists a region where $t_H \ll t_{\text{vis}}$, then preheating dominates over the viscous heating or the advective cooling. Since radiative fluxes are, in the lowest approximation, quadratic in the flow rate (Shapiro 1973a, 1973b), whereas advective heating/cooling terms are linear in the flow rate, preheating will become significant for high enough accretion rates. In the spherical case we found that the solutions are significantly altered for the mass accretion rate $m \equiv M/M_{\text{Edd}} \gtrsim 10^{-1.5}$ and the luminosity $L \equiv L/L_{\text{Edd}} \gtrsim 10^{-7.5}$, with no high-temperature solutions possible having $l \gtrsim 0.03$ and $e \gtrsim 3 \times 10^{-4}$ because of the preheating instability (Park 1990a, 1990b). Since the density of the ADAF is higher than that in spherical accretion flow for the same mass accretion rate, owing to the small infall velocity, we expect that this limit will occur for higher values of $e$ and $l$.

In ADAF, all physical quantities are products of a radial part, a function of radius $r$ only, with an angular part, a function of spherical polar angle $\theta$ only. So

$$t_{\text{vis}} = \frac{1}{\epsilon} \Omega K^{-1} r^{-1} v^{-1}(\theta) ,$$

where the radial infall velocity is defined as $v_r = r\Omega_K(r)v(\theta)$, with $\Omega_K(r) \equiv (GM/r^3)^{1/2}$.

$$t_H = \frac{(m_e c^2)}{4kT_x} \frac{L_{\text{E}}}{{L_{\text{X}}}^3} \frac{2}{3} c_s^2(r) \frac{r}{c} ,$$

where $m_e$ is the electron mass; $c$ is the speed of light; $T_x$ is the Compton temperature of the radiation defined as the energy-weighted mean of photon energy averaged over the photon spectral number density, $4\pi T_x \equiv \langle \hbar \nu \rangle / \langle \hbar \nu \rangle$; $L_x$ is the luminosity of the Comptonizing radiation; $L_E$ is the Eddington luminosity; $c_s(\theta)\equiv v(\theta)$ is the isothermal sound speed divided by the Keplerian velocity $c_s(\theta) \equiv (p/\rho)^{1/2} \equiv \Gamma \Omega K(\theta)$; $p$ is the total pressure; and $\rho$ is the gas density (Levich & Syunyaev 1971). The inequality $t_H < t_{\text{vis}}$ now reduces to

$$\frac{v(\theta)c_s^2(\theta)}{r} < \frac{2}{3} \frac{4kT_x}{m_e c^2} \left( \frac{L_x}{L_E} \right) .$$

We take the case of the flow $L_x/L_E = 3 \times 10^{-4}$, $T_x = 10^9$ K, and $\epsilon = 0.1$ as an example (NY3). Equation (15) represents the region above the solid curve in Figure 2, in which preheating is the dominant heating process and should not be ignored in the thermal balance equation. In advection-dominated flow, $t_{\text{flow}} \approx t_{\text{adv}} \approx t_{\text{vis}}$ and, therefore, $t_H < t_{\text{flow}}$. The flow is not adiabatic anymore, and dynamics of the flow will be significantly altered.

Similarly, there could be many soft photons, thereby lowering the radiation temperature $T_x \ll T_e$, and the flow would be cooled by Compton scattering. The condition for this is

$$\frac{v(\theta)c_s^2(\theta)}{r} < \frac{2}{3} \frac{4kT_x}{m_e c^2} \left( \frac{L_x}{L_E} \right) ,$$

which corresponds to the region above the dotted curve in Figure 2 for the same flow parameters.

For the flow near the equator, Compton heating or cooling may be ignored within some radius. But above the equatorial plane at large radius or around the pole, Compton heating or cooling can be more important than the viscous heating. It is quite possible that Compton preheating would heat the flow near the pole, which would otherwise cool down, to a high-temperature. Or if there are abundant soft photons, the flow would cool because of the...
Compton cooling. This result is independent of the mass accretion rate and depends only on the Comptonizing luminosity. Esin (1997) similarly found the importance of Compton cooling (called “non-local cooling”) under certain conditions in a careful one-dimensional analysis, whereas Compton heating was found to be generally unimportant. The main reason for this discrepancy is that a different radiation temperature, i.e., spectrum, is assumed and that physical parameters of height-integrated flow can be widely different from those of two-dimensional flow, especially near the polar axis (NY2).

However, a real ADAF could be far more complicated than this. First, the radiation temperature $T_X$ can vary from place to place. It should correctly represent the spectrum of the radiation field at a given position, which is the sum of the radiation transferred from other regions of the flow and that produced locally. Second, the luminosity $L_X$ is also a function of position, which is again related to the density and temperature of the gas. Third, the gas temperature is determined by the amount of Compton heating and therefore by the radiation temperature and the radiation density. Hence, more accurate analysis requires simultaneously solving the energy equations for gas and radiation. This has been done only for spherical accretion flow (Park 1990a, 1990b; Nobili et al. 1991; Mason & Turolla 1992).

5. PREHEATING LIMIT

If preheating increases above some critical value, it can affect the accretion flow more dramatically than just changing the thermal balance. Ostriker et al. (1976) found that in Bondi-type spherical accretion flow too much preheating changes the dynamics of the flow around the sonic radius and would disrupt the steady flow. This results in various time-dependent behavior in the flow and in the outcoming radiation (Cowie, Ostriker, & Stark 1978; Ciotti & Ostriker 1997).

Our next question is whether there is a similar preheating limit for ADAF. Since ADAF is self-similar, it does not have an accretion radius or outer boundary. So we have to look at the temperature structure in preheated ADAF. We will use simplified thermal balance equation to solve for the temperature for the ADAF with strong preheating luminosity.

In ADAF, the cooling rate is assumed to be a constant fraction of the advective cooling to obtain self-similar forms of solutions. This simplification may not be too bad if the solution is taken as height-integrated (or angle-averaged) form. But in two-dimensional ADAF, the flow time approaches infinity along the polar direction and is an order of magnitude larger than the usual free-fall time along the equatorial direction (NY2). There is no radial advection along the pole, and we expect the Compton heating to be balanced by the radiative cooling. Similarly, we apply the thermal balance equation to all parts of the flow to estimate the preheating effect, which is valid when $t_H < t_{vir}$ (Eq. 2).

The temperature is determined by requiring $H(T_{eq}; r, \delta) = C(T_{eq}; \delta, \delta)$. When atomic line cooling and bremsstrahlung are the dominant processes, $T_*(r)$ has the form that outside some radius $r_*$, $T(r) \approx 10^4$ K and at $r_*$, $T(r)$ jumps to $T_* \approx 2 \times 10^6$ K, at which temperature the bremsstrahlung cooling rate becomes comparable to the peak of the atomic line cooling (Buff & McCray 1974). In this domain there is a classical phase change and the temperature suddenly jumps, because there is no stable equilibrium between $\sim 10^4$ K and $T_*$. If we further assume that the luminosity profile $L_*(r)$ is constant in $r$, the temperature profile inside $r_*$ is simply $T(r) = T_*(r/r_*)^{-1}$ if the gas cools only by nonrelativistic bremsstrahlung.

At the transition radius $r_*$, Compton heating is equal to the peak of the cooling curve by definition (Eq. [8]),

$$\frac{d \epsilon k(T_X - T_{eq})}{m_n c^2} = \frac{L_*(r_*)}{m} \frac{1}{n(\delta)} \left( \frac{r_*}{r} \right)^{-1/2} = 8.8 \times 10^{-5}, \quad (17)$$

where the gas number density is defined as $n(r, \delta) = n(\delta) (M/4\pi c m_n r_*^2) (r/r_*)^{-3/2}$. Since the temperature of the electrons is not too different from $\sim 10^9$ K at the inner part of the flow, where most of the radiation is produced, we take $T_*(r) \sim 10^8$ K $\gg T_*(r)$. This choice of $T_*$ means that the radiation should contain enough hot photons to heat the gas. If there are too many soft photons, e.g., synchrotron photons, $T_X$ could be lower.

The accretion flow would be disrupted, if the flow is heated above the virial temperature $T_{vir}$, defined as $(5/2)kT_{vir} \equiv GM \mu/m$. The flow temperature suddenly jumps from $10^4$ K to $T_*$ at radius $r_*$, and the inflow would stop or reverse (i.e., become outflow) if $T_* > T_{vir}(r_*)$. The condition $T_* > T_{vir}(r_*)$ is equivalent to $r_*>r_* \approx 9.4 \times 10^5 r_*$ (Eq. [17]) since $r_*$ is defined as $T_{vir}(r_*) = T_*$.

For the efficiency $\epsilon = l/m = 3 \times 10^{-5}$ flow with $\epsilon = 1.0$ and $T_X = 10^9$ K, $r_*$ as a function of $\delta$ is shown in Figure 3 as a solid curve and $r_*$ is shown as a dotted one. The part of the flow between $r_*$ and $r_*$ with $r_* > r_*$ is overheated and the steady inflow is not possible, whereas the part of the flow with $r_* < r_*$ can accrete normally. The flow in regions A and C of Figure 3 has low temperature $\sim 10^4$ K and is stable. The flow in regions B and D are Compton heated above $T_*$, with the flow in region B being unstable, while that in region D being stable. So it would be possible that the flow accretes along the equatorial plane while there is outflow along the pole due to preheating. Blandford & Begelman (1999) also propose advection-dominated inflow...
outflow via a different mechanism: Part of the conservative flow can have positive energy due to the energy flux transported by the viscous torque (see also NY2).

The critical efficiency \(e_c\) above which this overheating starts to occur in any part of the flow corresponds to \(r_\infty \big| \delta = r_\infty \) since preheating is most effective in the polar direction as a result of lower infall velocity and lower density. Figure 4 shows \(e_c\) determined for \(T_\infty = 10^9\) K as squares. We choose \(T_\infty = 10^9\) K as the representative value.

The electron temperature in ADAF generally flattens at \(2-5 \times 10^9\) K (NY3). So we expect \(T_\infty\) to be in the range \(5 \times 10^8\) to \(5 \times 10^9\) K, depending on the degree of Comptonization of bremsstrahlung photons. However, if there are enough synchrotron photons, \(T_\infty\) can be smaller. Esin (1997) assumes a flat spectrum covering 10 decades in frequencies. This type of spectrum can lower \(T_\infty\) by a factor larger than 10. On the other hand, near critical, \(e \approx e_c\), solutions of Park & Ostriker (1999) show \(T_\infty \approx 6 \times 10^9\) K for photons from bremsstrahlung plus synchrotron emission.

Depending on the value of \(e'\) (see Fig. 4), any flow with efficiency above \(e_c \sim 4 \times 10^{-3}\) will suffer preheating instability at or near the pole and can develop the time-dependent behavior or outflow as in spherical case (Cowie, Ostriker, & Stark 1978). This value of \(e_c \sim 4 \times 10^{-3}\) corresponds to \(l \approx 2 \times 10^{-4}\) and \(\dot{m} \approx 0.05\) in NY3 solutions and is comparable to that in the spherical accretion flow. Since the radiation temperature assumed here is higher than that of the self-consistent spherical flow (Park 1990a), the critical efficiency, which is inversely proportional to the radiation temperature, should be smaller. However, the gas density of ADAF is on average \(30 \sim 100\) times greater for the same total mass accretion rate owing to the smaller infall velocity than that in the spherical accretion flow, which is almost freely falling (Park 1990a; NY2), resulting in similar critical efficiency. For the same radiation temperature, i.e., spectral shape, higher density provides a major advantage for ADAF (in comparison with spherical flow): higher luminosities and efficiencies should be possible.

6. LUMINOSITY FROM ADAF

One of the unique differences between ADAF (or spherical accretion) onto black holes and thin-disk accretion is that its radiation efficiency cannot be assumed a priori but must be determined self consistently. In thin-disk accretion, whatever the energy input to the flow, it is locally radiated away because of the long infall time, and its radiation efficiency is essentially determined by the position of inner edge of the disk. However, accretion flow with significant radial velocity can carry the energy into the hole as well as radiates away. So the out-coming radiation can be either significant or negligible depending on the dynamics and thermal structure of the flow (see Park & Ostriker 1998 for review).

Here we estimate how much outgoing radiation is produced self consistently by self-similar ADAF. The amount of radiative cooling in ADAF is always assumed to be some fraction \((1-f)\) of the viscous heating. The remaining fraction \(f\) is advected with the flow and not radiated. Hence the radiative luminosity would be \((1-f)\) times the sum of all viscous heating (eq. [7]) over all radii and angles,

\[
L_{\text{rad}} = \frac{3}{2} e'(1-f) \int_{-\infty}^{\infty} \int_0^{\infty} 2\pi d\delta |v_r| c_s^2 r. \tag{18}
\]

Since the parameter \(f\) is not determined a priori for self-similar ADAF, we take \(f = 0\) to estimate the maximum luminosity that can be produced. Substituting physical quantities of NY2 yields the dimensionless maximum luminosity as a function of \(e'\). Since the right-hand side of the equation (18) is proportional to \(\dot{M}\), the maximum efficiency \(e_{\text{max}}\) rather than the maximum luminosity is determined. We assume \(r_\infty = 3r_\ast\), and the values of \(e_{\text{max}}\) for each \(e'\) are shown in Figure 4 as circles. Comparison with the critical efficiency, \(e_c\), shows that the flow with \(e' \geq 0.07\) has a possibility of preheating disruption. Higher \(e'\) flows are more vulnerable to preheating because they have higher infall velocity and pressure and therefore higher viscous heating and radiation output.

The exact value of \(e_{\text{max}}\) may differ somewhat from the values above, because simple thermal balance equation is not always valid. In reality, the local cooling rate can be larger than the viscous heating because of the additional \(Pd\dot{V}\) work. A very good example is the spherical accretion flow without viscous dissipation. The viscous heating is zero, yet the gas is heated by compression due to gravity and radiates (Shapiro 1973a, 1973b; Park 1990a, 1990b). Therefore, \(e_{\text{max}}\) can be higher.

7. SELF-CONSISTENT FLOWS

So far we have discussed the cases where gas near the polar axis or at large radius is cooled in the absence of preheating, or it can be heated too much and disrupted by preheating. However, it is also possible that the flow can be maintained at high temperature by preheating without being disrupted. In spherical accretion flow, there exist two branches of solutions for certain mass accretion rate (Park 1990a, 1990b; see Park & Ostriker 1998 for references): In the lower luminosity branch, gas is cooled down to \(\sim 10^4\) K and thus not much energy is released through radiation. In the other, higher luminosity branch, gas at large radius is Compton heated by hot radiation produced in the inner region, and a higher radiation efficiency is achieved. We do find that this is also true for ADAF. For some mass accre-
tion rates there exist hot solutions self-consistently maintained by Compton preheating, whereas the flow would cool down to thin disk in the absence of preheating (Park & Ostriker 1999).

8. SUMMARY

We have studied the thermal properties of the self-similar, two-dimensional advection-dominated accretion flow (NY2) with special consideration given to the radiative cooling and heating. We find the following:

1. A hot solution is possible only within some critical radius for a given mass accretion rate in the equatorial plane, confirming the one-dimensional analysis (NY3). Also, for any mass accretion rate, a roughly conical region around the pole cannot maintain high-temperature electrons in the absence of radiative heating with the collapsed region shrinking as $\dot{m}$ is reduced. If ions become coupled to the now cold electrons (as seems likely), an empty funnel around the polar axis will be created.

2. Part of the flow at large radii or above the equatorial plane should be affected through Compton heating or cooling by photons produced at smaller radii, if the luminosity is high enough. If the radiation efficiency of the accretion is above $\sim 4 \times 10^{-3}$ and the out-coming radiation has mean photon energy comparable to the electron temperature of the inner region, preheating due to the inverse Compton scattering would overheat the polar region of the flow and may create time-dependent behavior or outflow while accretion still goes on in the equatorial directions. For NY3 solutions, these phenomena should begin to occur for luminosities $L \gtrsim 2 \times 10^{-4}$ and accretion rate $\dot{m} \gtrsim 0.05$ in Eddington units.

3. The role of Compton preheating is quite intriguing in ADAF as in spherical flows. On the one hand it can have the attractive feature of driving polar winds whenever the total luminosity is above some rather low bound. But it may also allow another branch of solutions making the original ADAF solutions more viable for higher mass accretion rates than those for which solutions were valid. The detailed calculation of this new branch of solutions will be given in Park & Ostriker (1999).

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