EFFECTIVE LAGRANGIAN FOR $B$ and $D$
SEMILEPTONIC DECAYS

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ABSTRACT

An effective lagrangian incorporating both chiral and heavy quark symmetries and describing the low momentum interactions of heavy mesons and light scalar and vector resonances is presented. Using some available data and assuming nearest pole dominance for the form factors we can calculate, at the leading order in chiral and heavy quark expansion, semileptonic $B$ and $D$ decays into light mesons.

1 Introduction

The combined use of $SU(3)\times SU(3)$ chiral symmetry and Heavy Quark Effective Theory (HQET) allows to describe the low-momentum interactions of hadrons containing a single heavy quark with light mesons, like $\pi$, $K$ and $\eta$ \cite{1}. An effective lagrangian incorporating both symmetries has been build and a number of interesting applications has been made \cite{2}.

Here we will concern ourselves to semileptonic decays of $B$ and $D$ mesons into light hadrons \cite{3}. By light hadrons we mean the pseudoscalar octet $\pi$, $K$, $\eta$ and the vector resonances $\rho$, $K^*$, $\omega$ and $\phi$ belonging to the low lying $1^-$ nonet. These has been introduced in \cite{1} through the hidden gauge symmetry approach. We will also take into account the low-lying positive parity heavy meson states, which contribute to the semileptonic amplitudes of $B$ and $D$ through pole diagrams. We will write down a chiral effective lagrangian describing the interactions of all these states among each other: in order to be predictive we will be forced to discard higher derivative terms acting on the light fields, and therefore the calculations will be reliable when the emitted light meson has low momentum, i.e. when $q^2$, the momentum transfer to the lepton pair, is close to his maximal value $q^2_{\text{max}}$. The extrapolation down to $q^2 = 0$ needs an extra assumption on the $q^2$-dependence of the hadronic form factors; we will take a simple pole behaviour. We will neglect moreover $1/M_Q$ corrections to the leading lagrangian.

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In the numerical analysis we will fix the arbitrary coupling constants introduced by the effective description using the experimental data on $D \rightarrow \pi \ell \nu$ and $D \rightarrow K^* \ell \nu$ and predict the branching ratios for the chiral and flavour related decays.

# 2 The heavy-light chiral lagrangian

To be self-contained and to establish the notations we shall start by reviewing the description of heavy mesons and light mesons by effective field operators and of their effective chiral lagrangian. Negative parity heavy $Qq_a$ mesons are represented by fields described by a $4 \times 4$ Dirac matrix

$$H_a = \frac{1 + \gamma}{2} [P_{a\mu}^\gamma \gamma^\mu - P_a \gamma_5] \quad (2.1)$$

$$\bar{H}_a = \gamma_0 H_a^\dagger \gamma_0 \quad (2.2)$$

Here $v$ is the heavy meson velocity, $a = 1, 2, 3$ (for $u, d$ and $s$ respectively), $P_{a\mu}^\gamma$ and $P_a$ are annihilation operators normalized as follows

$$\langle 0 | P_a | Qq_a(0^-) \rangle = \sqrt{M_H} \quad (2.3)$$

$$\langle 0 | P_{a*}^\gamma | Qq_a(1^-) \rangle = \epsilon^\mu \sqrt{M_H} \quad (2.4)$$

with $\nu^\mu P_{a*}^\gamma = 0$ and $M_H = M_P = M_{P*}$, the supposedly degenerate meson masses. Also $\bar{\nu}H = -H\bar{\nu} = H$, $\bar{\nu}H = -\bar{\nu}\bar{H} = \bar{H}$. The pseudoscalar light mesons are described by

$$\xi = \exp \frac{iM}{f_\pi} \quad (2.5)$$

where

$$M = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta
\end{pmatrix} \quad (2.6)$$

and $f_\pi = 132 MeV$. Under the chiral symmetry the fields transform as follows

$$\xi \rightarrow g_L \xi U^\dagger = U \xi g_R^\dagger \quad (2.7)$$

$$\Sigma \rightarrow g_L \Sigma g_R^\dagger \quad (2.8)$$

$$H \rightarrow HU^\dagger \quad (2.9)$$

$$\bar{H} \rightarrow U \bar{H} \quad (2.10)$$

where $\Sigma = \xi^2$, $g_L$, $g_R$ are global $SU(3)$ transformations and $U$ is a function of $x$, of the fields and of $g_L$, $g_R$.

The lagrangian describing the fields $H$ and $\xi$ and their interactions, under the hypothesis of chiral and spin-flavour symmetry and at the lowest order in light mesons derivatives is

$$\mathcal{L}_0 = \frac{f_\pi^2}{8} \langle \partial^\mu \Sigma \partial_\mu \Sigma \rangle + i \langle H_b \nu^\mu D_{\mu ba} \bar{H}_a \rangle + i g \langle H_b \gamma_\mu \gamma_5 A_{ba}^\mu \bar{H}_a \rangle \quad (2.11)$$
where \( < \ldots > \) means the trace, and
\[
D_{\mu ba} = \delta_{ba} \partial_{\mu} + \mathcal{V}_{\mu ba} = \delta_{ba} \partial_{\mu} + \frac{1}{2} \left( \xi^\dagger \partial_{\mu} \xi + \xi \partial_{\mu} \xi^\dagger \right)_{ba} \tag{2.12}
\]
\[
\mathcal{A}_{\mu ba} = \frac{1}{2} \left( \xi^\dagger \partial_{\mu} \xi - \xi \partial_{\mu} \xi^\dagger \right)_{ba} \tag{2.13}
\]
Besides chiral symmetry, which is obvious, since, under chiral transformations,
\[
D_{\mu} \bar{H} \to U D_{\mu} \bar{H} \\
\mathcal{A}_{\mu} \to U \mathcal{A}_{\mu} U^\dagger \tag{2.14}
\]
the lagrangian (2.11) possesses the heavy quark spin symmetry \( SU(2)_v \), which acts as
\[
H_a \to \hat{S} H_a \tag{2.15}
\]
\[
\bar{H}_a \to \bar{H}_a \hat{S}^\dagger \tag{2.16}
\]
with \( \hat{S} \hat{S}^\dagger = 1 \) and \([\hat{S}, \hat{S}] = 0\), and a heavy quark flavour symmetry arising from the absence of terms containing \( m_Q \).

Explicit symmetry breaking terms can also be introduced, by adding to \( \mathcal{L}_0 \) the extra piece (at the lowest order in \( m_q \) and \( 1/m_Q \)):
\[
\mathcal{L}_1 = \lambda_0 < m_q \Sigma + m_q \Sigma^\dagger > + \lambda_1 < \bar{H}_a H_b (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ba} > + \lambda_2 < m_Q \sigma_{\mu\nu} H_a \sigma_{\mu\nu} > \tag{2.17}
\]
The last term in the previous equation induces a mass difference between the states \( P \) and \( P^* \) contained in the field \( H \), such that
\[
M_P = M_H \\
M_{P^*} = M_H + \delta m_H \tag{2.18}
\]
The preceding construction can be found for instance in the paper by Wise [2], and we have used the same notations.

The vector meson resonances belonging to the low-lying \( SU(3) \) octet can be introduced by using the hidden gauge symmetry approach [3] (for a different approach see [4]). The new lagrangian containing these particles, to be added to \( \mathcal{L}_0 + \mathcal{L}_1 \), is as follows:
\[
\mathcal{L}_2 = - \frac{f_\pi^2}{2} a < (\mathcal{V}_\mu - \rho_\mu)^2 > + \frac{1}{2 g_\pi^2} < F_{\mu\nu}(\rho) F^{\mu\nu}(\rho) > + \frac{i A}{2} < H_b \partial^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} > + \frac{i A}{2} < H_b \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} > \tag{2.19}
\]
where \( F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu] \), and \( \rho_\mu \) is defined as
\[
\rho_\mu = \frac{1}{\sqrt{2}} \partial_\mu \sqrt{2} \rho_\mu \tag{2.20}
\]
\(\hat{\rho}\) is a hermitian \(3 \times 3\) matrix containing the light vector mesons \(\rho^{0,\pm}, K^*, \omega\) and \(\phi\). \(g_V\), \(\beta\) and \(a\) are coupling constants; by imposing the two KSRF relations one obtains

\[
a = 2 \quad g_V \approx 5.8
\] (2.21)

For our subsequent analysis of the heavy mesons semileptonic decays we shall have to introduce the low-lying positive parity \(Q \bar{q}_a\) heavy meson states. For \(p\) waves \((l = 1)\), the heavy quark effective theory predicts two distinct multiplets, one containing a 0\(^+\) and a 1\(^+\) degenerate states, the other one comprising a 1\(^+\) and a 2\(^+\) state [6], [7]. At the leading order only the first multiplet, characterized by a total angular momentum of the light degrees of freedom \(s_l = 1/2\) contributes to the semileptonic form factors; in matrix notation it is described by [3]

\[
S_a = \frac{1 + \gamma^\mu}{2} \left[ D^\mu_1 \gamma_5 \gamma_5 - D_0 \right]
\] (2.22)

The couplings of these states to the light pseudoscalars and vector mesons are

\[
\mathcal{L}' = i f^\mu < S_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{H}_a > + i \zeta < \bar{S}_a H_b \gamma_\mu (V^\mu - \rho^\mu)_{ba} >
\] (2.23)

\[
+ i \mu < \bar{S}_a H_b \sigma^{\lambda\nu} F_{\lambda\nu}(\rho)_{ba} > + h.c.
\]

We shall see in the following that some information on the coupling constants \(g, \mu, \lambda\) and \(\zeta\) can be obtained by the analysis of the semileptonic decays.

### 3 Weak currents

At the lowest order in derivatives of the pseudoscalar couplings and in the symmetry limit, weak interactions between light pseudoscalars and a heavy meson are described by the weak current [3]:

\[
L^\mu_a = \frac{i \alpha}{2} < \gamma^\mu (1 - \gamma_5) H_b \xi^\dagger_{ba} >
\] (3.1)

where \(\alpha\) is related to the pseudoscalar heavy meson decay constant \(f_H\), defined by

\[
< 0 | \bar{\psi}_a \gamma_\mu \gamma_5 Q | P_b (p) > = ip^\mu f_H \delta_{ab}
\] (3.2)

as follows:

\[
\alpha = f_H \sqrt{M_H}
\] (3.3)

We can in a similar way introduce the current describing the weak interactions between pseudoscalar Goldstone bosons and the positive parity \(S\) fields:

\[
\hat{L}^\mu_a = \frac{i \hat{\alpha}}{2} < \gamma^\mu (1 - \gamma_5) S_b \xi^\dagger_{ba} >
\] (3.4)

and the current by which the \(H\) fields interact with the light vector mesons:

\[
L^\mu_{1a} = \alpha_1 < \gamma_5 H_b (\rho^\mu - V^\mu)_{bc} \xi^\dagger_{ca} >
\] (3.5)

All these currents transform under the chiral group similarly to the quark current \(\bar{\psi} \gamma^\mu (1 - \gamma_5) Q\), i.e. as \((3_L, 1_R)\).
4 Semileptonic decays

Let us first consider the decay

\[ P \rightarrow \Pi \ell \nu \]  

where \( P \) is a heavy meson (\( B \) or \( D \)) and \( \Pi \) a light pseudoscalar meson. The hadronic matrix element can be written in terms of the form factors \( F_0, F_1 \) as follows

\[
\langle \Pi(p')|V^\mu|P(p)\rangle = \left[ (p + p')^\mu + \frac{M_{\Pi}^2 - M_H^2}{q^2} q^\mu \right] F_1(q^2) - \frac{M_{\Pi}^2 - M_H^2}{q^2} q^\mu F_0(q^2)
\]

where \( q^\mu = (p - p')^\mu \), \( F_0(0) = F_1(0) \) and \( M_H = M_P \). The form factors \( F_0 \) and \( F_1 \) take contributions, in a dispersion relation, from the \( 0^+ \) and \( 1^- \) meson states respectively.

We notice here that, by working at the leading order in \( 1/m_\ell \), the possible parametrizations of the weak current matrix element are not all equivalent. Computed in the heavy meson effective theory, the matrix element of eq. (4.2) reads:

\[
\langle \Pi(p')|V^\mu|P(p)\rangle = A v^\mu + B p'^\mu
\]

with \( A \) and \( B \) both scaling as \( \sqrt{M_H} \) at \( q^2 = q_{\text{max}}^2 = (M_H - M_{\Pi})^2 \) (where the theory should provide for a better approximation). The factor \( \sqrt{M_H} \) which gives rise to this scaling behaviour comes just from the wave function normalization of the \( P \) operator, and no other explicit factor \( M_H \) appears in the heavy meson effective field theory. If one introduces the usual form factors \( f_+ \) and \( f_- \) through the following decomposition:

\[
\langle \Pi(p')|V^\mu|P(p)\rangle = f_+(p + p')^\mu + f_-(p - p')^\mu
\]

one has the relations:

\[
f_+ = \frac{1}{2} \left( \frac{A}{M_H} + B \right), \quad f_- = \frac{1}{2} \left( \frac{A}{M_H} - B \right)
\]

It would seem consistent at this point to throw away the terms proportional to \( A \), obtaining

\[
\langle \Pi(p')|V^\mu|P(p)\rangle \simeq B p'^\mu
\]

which however does not reproduce the original expression of the matrix element. This is a clear contradiction since the two terms on the right hand side of eq. (4.3) scale in the same fashion. On the other hand, by making use of the decomposition of eq. (4.2) and working at the leading order we find:

\[
F_1 = \frac{B}{2}, \quad F_0 = \frac{1}{M_H} (A + BM_{\Pi})
\]

which, inserted back in the eq. (4.2), fully reproduces the matrix element given in eq. (4.3). The previous example shows that one must be very careful in the definition of the form factors when working at the leading order in \( 1/m_\ell \) in the heavy meson effective field theory.
Using the previous lagrangians (2.11), (2.23) and the currents (3.1), (3.4) we obtain, at the leading order in $1/m_Q$ and at $q^2 = q_{\text{max}}^2$, the following results

\[ F_1(q_{\text{max}}^2) = \frac{g M_H f_H}{2f_\pi(v \cdot k - \delta m_H)} \]  

\[ F_0(q_{\text{max}}^2) = \frac{f'' H_M}{\sqrt{M_H f_\pi(v \cdot k - \delta m_S)}} - \frac{f_H}{f_\pi} \]  

The r.h.s. in (4.8) and the first term in (4.9) arise from polar diagrams. Finally $k^\mu$ is the residual momentum related to the physical momenta by $k^\mu = q^\mu - M_{\tilde{H}} v^\mu$ (and $p^\mu = M_H v^\mu$).

A similar analysis can be performed for the semileptonic decay process

\[ P \rightarrow \Pi^* \ell \nu \]  

of a heavy pseudoscalar meson $P$ into a light vector $\Pi^*$ particle. The current matrix element is expressed as follows

\[ < \Pi^*(\epsilon, p') | (V^\mu - A^\mu) | P(p) > = \frac{2V(q^2)}{M_H + M_{\Pi^*}} \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p_\alpha p_\beta \]

\[ + i(M_H + M_{\Pi^*}) \left[ \epsilon_\mu^* - \frac{\epsilon_\mu^* \cdot q}{q^2 - q_\mu} A_1(q^2) \right] \]

\[ - i\frac{\epsilon_\mu^* \cdot q}{(M_H + M_{\Pi^*})} \left[ (p + p')_\mu - \frac{M_{\tilde{H}}^2 - M_{\Pi^*}^2}{q^2} q_\mu \right] A_2(q^2) \]

\[ + i \epsilon_\mu^* \cdot q \frac{2M_{\Pi^*}}{q^2} q_\mu A_0(q^2) \]  

where

\[ A_0(0) = \frac{M_{\Pi^*} - M_H}{2M_{\Pi^*}} A_2(0) + \frac{M_{\Pi^*} + M_H}{2M_{\Pi^*}} A_1(0) \]  

Notice that the tensor structures given in square brackets of eq. (4.11) have vanishing divergence and are constant in the limit of infinite $M_H$. Such a decomposition satisfies the same properties discussed above for the form factors $F_0$ and $F_1$. In a dispersion relation the form factor $V(q^2)$ takes contribution from $1^-$ particles, $A_0(q^2)$ from $0^-$ particles and $A_j(q^2)$ $(j = 1, 2)$ from $1^+$ states.

Using the lagrangians (2.19) and (2.23) and the currents (3.1), (3.4) and (3.5) we get at $q^2 = q_{\text{max}}^2$ and at leading order in $1/m_Q$ the results

\[ V(q_{\text{max}}^2) = -\frac{g_V}{\sqrt{2}} \lambda f_H \frac{M_H + M_{\Pi^*}}{v \cdot k - \delta m_H} \]  

\[ A_1(q_{\text{max}}^2) = -\frac{2g_V}{\sqrt{2}} \left[ \frac{\alpha_1 \sqrt{M_H}}{M_H + M_{\Pi^*}} + \frac{\hat{\alpha} \sqrt{M_H}}{M_H + M_{\Pi^*}} \frac{\zeta/2 - \mu M_{\Pi^*}}{v \cdot k - \delta m_S} \right] \]  

\[ A_2(q_{\text{max}}^2) = \frac{\mu g_V}{\sqrt{2}} \sqrt{M_H} v \cdot k - \delta m_S \]
\[ A_0(q_{\text{max}}^2) = - \frac{g_V}{2\sqrt{2}} \frac{\beta f_H M_H}{M_{\Pi^*} \left( v \cdot k - \delta m' \right)} + \frac{g_V \sqrt{M_H}}{\sqrt{2} M_{\Pi^*}} \alpha_1 \]  
(4.16)

where \( \delta m' \) arise from the chiral breaking terms of Eq.(2.17). The first term in (4.14) and the last one in (4.16) arises from the direct coupling between the heavy meson \( H \) and the \( 1^- \) light resonances of Eq.(3.5) and the other ones from polar diagrams.

5 Numerical analysis

The results (4.8),(4.9) and (4.13)-(4.16) are obtained in the chiral limit and for \( m_Q \to \infty \); therefore they should apply (with non-leading corrections) to the decays \( B \to \pi \ell \nu_\ell \) or \( B \to \rho \ell \nu_\ell \). Unfortunately, for those decays there are not sufficient experimental results that could be used to determine the various coupling constants appearing in the final formulae.

On the other hand, for \( D \) decays the experimental information is much more detailed and we could tentatively try to use it to fix the constants as well as to make predictions on the other decays which have not been measured yet.

In order to make contact with the experimental data, we have to know the behaviour of the form factors with \( q^2 \). Except for the direct terms in (4.9), (4.14) and (4.16), all the contributions we have collected arise from polar diagrams, which suggests a simple pole behaviour. Such a behaviour is compatible with the experimental data on \( D \to K \ell \nu \) \[10\]. Theoretically QCD sum rules \[11\] seem to indicate that the axial form factors \( A_1 \) and \( A_2 \) do not show a polar dependence, in contrast with lattice results \[12\]. As in the phenomenological analysis of \( D \) semileptonic decays we will assume for the form factors \( F_1(q^2) \), \( V(q^2) \), \( A_1(q^2) \) and \( A_2(q^2) \) (the form factors \( F_0(q^2) \) and \( A_0(q^2) \) are not easily accessible to measurement since they appear in the width multiplied by the lepton mass) the pole behaviour

\[ F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m^2}} \]  
(5.1)

For the pole masses we use the inputs in Table 1 \[13\] that also agree with the masses fitted by the experimental analyses of \( D \) decays \[14\].

| State | \( J^P \) | \( 0^- \) | \( 1^- \) | \( 0^+ \) | \( 1^+ \) |
|------|-------|-------|-------|-------|-------|
| \( dc \) | \( \bar{d}c \) | 1.87 | 2.01 | 2.47 | 2.42 |
| \( sc \) | \( \bar{s}c \) | 1.97 | 2.11 | 2.60 | 2.53 |
| \( \bar{u}b \) | \( \bar{u}b \) | 5.27 | 5.32 | 5.99 | 5.71 |

Table 1: Pole masses for different states. Units are GeV.

For the \( D \to \pi \) semileptonic decay one thus gets, from (4.8) and (5.1):

\[ F_1(0) = - \frac{g \alpha}{2 f_\pi} \sqrt{M_D} \left( M_{D^*} + M_D - M_\pi \right) \]  
(5.2)
Experimentally one has \(|F_1(0)| = 0.79 \pm 0.20\), which implies

\[|g\alpha| = 0.16 \pm 0.04 \text{ GeV}^3/2 \] (5.3)

Only the value of this product matters for the determination of the form factors of related processes. To extract the value of \(|g|\) we fix \(\alpha\) using the QCD sum rules result for \(f_B(\alpha = f_B\sqrt{M_B})\), \(\alpha \approx 0.40\) \([10]\), obtaining \(|g| \approx 0.4 \pm 0.1\) (we neglect the theoretical uncertainties on \(\alpha\)). This result is in agreement with the result obtained by an analysis of radiative \(D^*\) decays: \(|g| = 0.58 \pm 0.41\) (for \(m_c = 1700\) MeV) \([17]\).

We can now give predictions for the processes related to \(D \to \pi \ell \nu\) by heavy quark and chiral symmetries. The results for form factors, widths and branching ratios are presented in Table 2. We stress that the reported values are obtained in the leading order in \(1/M_Q\) expansion; in \([3]\) we reported also the results of a fit obtained introducing mass corrections to the leptonic decay constants ratio \(f_B/f_D = \sqrt{M_D/M_B}\) predicted by the HQET. However, as discussed in \([18]\), a few non leptonic \(B\) decays seem to disagree with this latter solution.

| Decay       | \(F_1(0)\) | \(\Gamma (10^{11} \text{ s}^{-1})\) | BR (%) |
|-------------|-------------|----------------------------------|--------|
| \(D^0 \to \pi^-\) | 0.79        | 0.092                            | 0.39   |
| \(D^+ \to \bar{K}^0\) | 0.67        | 0.66                             | 7.0    |
| \(D^+ \to \eta\) | 0.29        | 0.0056                           | 0.060  |
| \(D_s \to \eta\) | 0.57        | 0.60                             | 2.70   |
| \(D_s \to K^0\) | 0.76        | 0.064                            | 0.29   |
| \(B^0 \to \pi^-\) | 0.53        | 0.0038                           | 0.049  |
| \(B_s \to K\) | 0.52        | 0.0036                           | 0.046  |

The results of Table 2 cannot be fully compared to experiments due to the lack of data. The only available one, apart \(D \to \pi \ell \nu\) used to fix the relevant coupling, is \(BR(D^+ \to \bar{K}^0 \ell \nu) = (5.5 \pm 1.1) \times 10^{-2}\) \([14]\), in agreement with our result \(7.0 \times 10^{-2}\).

Let us now turn to semileptonic decays into vector mesons. The experimental inputs we can use are from \(D \to K^* \ell \nu \ell\) and are as follows:

\[
\begin{align*}
V(0) &= 0.95 \pm 0.20 \\
A_1(0) &= 0.48 \pm 0.05 \\
A_2(0) &= 0.27 \pm 0.11
\end{align*}
\] (5.4)

They are averages between the data from E653 \([19]\) and E691 \([10]\) experiments. The calculated weak couplings at \(q^2 = 0\) are:

\[
V(0) = \frac{g\nu \lambda (M_D + M_{K^*})(M_{D^*_s} + M_D - M_{K^*})}{\sqrt{2} M_{D_s}^2} \frac{\alpha}{\sqrt{M_D}}
\] (5.5)
\[ A_1(0) = -\sqrt{2} g_V \frac{(M_{D_1} + M_D - M_{K^*}) \sqrt{M_D}}{(M_D + M_{K^*}) M_{D_1}^2} \times \left[ \alpha_1(M_{D_1} - M_D + M_{K^*}) - \hat{\alpha} (\frac{\zeta}{2} - \mu M_{K^*}) \right] \]  
(5.6)

\[ A_2(0) = -\frac{g_V \mu (M_D + M_{K^*}) (M_{D_1} + M_D - M_{K^*}) \hat{\alpha}}{\sqrt{2} M_{D_1}^2 \sqrt{M_D}} \]  
(5.7)

Using the experimental inputs (5.4) we obtain

\[ \lambda \alpha = 0.16 \pm 0.03 \text{ GeV}^{1/2} \]  
(5.8)

\[ \hat{\alpha} \mu = -0.06 \pm 0.02 \text{ GeV}^{3/2} \]  
(5.9)

By using the result \( \hat{\alpha} = 0.46 \pm 0.06 \text{ GeV}^{3/2} \) from QCD sum rules [20], one obtains:

\[ \mu = -0.13 \pm 0.05 \]  
(5.10)

For the \( A_1 \) coupling the experimental data do not allow for a separate determination of \( \alpha_1 \) and \( \zeta \). However we notice that the combination:

\[ \alpha_{\text{eff}} = \alpha_1(M_{D_1} - M_D + M_{K^*}) - \hat{\alpha} \left( \frac{\zeta}{2} - \mu M_{K^*} \right) \]  
(5.11)

is almost flavour independent and, at leading order in the \( 1/M_Q \) expansion is scaling invariant. From the \( D \to K^* \) data given in Eq.(5.4) we find:

\[ \alpha_{\text{eff}} = -0.22 \pm 0.02 \text{ GeV}^{3/2} \]  
(5.12)

We can now present the predictions for the class of semileptonic decays related to \( D \to K^* \ell \nu \), always by working strictly at the leading order. Form factors, widths (transverse, longitudinal and total) and branching ratios are given in Table 3. We assume ideal mixing between \( \omega \) and \( \phi \).

Experimentally, except the input \( D \to K^* \ell \nu \), the only measured decay is up to now \( D_s^+ \to \phi \ell \nu \); our prediction for the branching ratio, \( 1.3 \times 10^{-2} \), well compares to the experimental measure [13] \( 1.4 \pm 0.5 \times 10^{-2} \). For the decay \( D^+ \to \rho^0 \ell^+ \nu_\ell \) one has the upper limit [15] \( BR < 3.7 \times 10^{-3} \), which is satisfied by our result \( BR(D^+ \to \rho^0 \ell^+ \nu_\ell) = 2.4 \times 10^{-3} \). For the decay \( B^- \to \rho^0 \ell^- \bar{\nu}_\ell \) we obtain \( BR = 0.28 \times 10^{-3} \), compatible with the upper limit of about \( 0.3 \times 10^{-3} \) found by CLEO collaboration [21].

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Table 3: Predictions for semileptonic $D$ and $B$ decays into a vector meson. Partial widths are in units of $10^{11} \text{s}^{-1}$. The branching ratios and the widths for $B$ must be multiplied for $|V_{ub}/0.0045|^2$.

| Decay      | $A_1(0)$ | $A_2(0)$ | $V(0)$ | $\Gamma_L$ | $\Gamma_T$ | $\Gamma$ | BR (%) | $\Gamma_L/\Gamma_T$ |
|------------|----------|----------|--------|------------|------------|----------|--------|------------------|
| $D \rightarrow K^*$ | 0.48     | 0.27     | 0.95   | 0.21       | 0.16       | 0.37     | 1.6    | 1.31             |
| $D \rightarrow \rho^\pm$ | 0.55     | 0.28     | 1.01   | 0.026      | 0.019      | 0.045    | 0.19   | 1.40             |
| $D \rightarrow \omega$ | 0.55     | 0.28     | 1.01   | 0.026      | 0.019      | 0.045    | 0.19   | 1.40             |
| $D_s \rightarrow K^*$ | 0.52     | 0.30     | 1.06   | 0.021      | 0.017      | 0.037    | 0.17   | 1.26             |
| $D_s \rightarrow \phi$ | 0.45     | 0.28     | 0.99   | 0.16       | 0.13       | 0.29     | 1.3    | 1.23             |
| $B \rightarrow \rho^\pm$ | 0.21     | 0.20     | 0.62   | 0.0018     | 0.0024     | 0.0043   | 0.055  | 0.75             |
| $B \rightarrow \omega$ | 0.21     | 0.20     | 0.62   | 0.0018     | 0.0024     | 0.0043   | 0.055  | 0.75             |
| $B_s \rightarrow K^*$ | 0.20     | 0.21     | 0.64   | 0.0015     | 0.0024     | 0.0040   | 0.051  | 0.60             |

It is curious to observe that the leading order results could have been obtained in a model independent way by assigning, in the parametrization of the matrix element, the scaling behaviour of the various form factors. For instance, for the $D \rightarrow K^*$ process we can write:

\[ V \frac{(M_D + M_{K^*})}{\sqrt{M_D}} = \frac{v}{\sqrt{M_D}} \] (5.13)

\[ (M_D + M_{K^*})A_1 = a_1 \sqrt{M_D} \] (5.14)

\[ \frac{A_2}{(M_D + M_{K^*})} = \frac{a_2}{\sqrt{M_D}} \] (5.15)

where $v$, $a_1$ and $a_2$ are constants as $M_D$ grows. This behaviour simply follows from the definitions of $V$, $A_1$ and $A_2$, and from the fact that the matrix element $< K^* | J^\mu | D >$ scales as $\sqrt{M_D}$. The above relations are valid at $q^2 = q^2_{\text{max}} = (M_D - M_{K^*})^2$ and they should be appropriately modified at $q^2 = 0$. To do so we assume a simple polar behaviour for the form factors. Notice that the quantities $v$, $a_1$ and $a_2$ will in general depend on $M_D$, $M_{K^*}$ and the relevant pole mass $M_{\text{pole}}$, with the restriction that they should be constant in the large $M_D$ limit. At $q^2_{\text{max}}$ the polar behaviour provides a factor:

\[ \frac{M_{\text{pole}}^2}{M_{\text{pole}}^2 - (M_D - M_{K^*})} \sim \frac{1}{2} \frac{1}{M_{\text{pole}}^2 - (M_{\text{pole}} - M_D + M_{K^*})} \] (5.16)

This factor exhibits a certain flavour dependence, which we may account for by incorporating it in $v$, $a_1$ and $a_2$:

\[ v = \frac{\hat{v}}{(M_{\text{pole}} - M_D + M_{K^*})} \] (5.17)

and similarly for $a_1$, $a_2$. We can assume that $\hat{v}$, $\hat{a}_1$ and $\hat{a}_2$ are approximately flavour independent. In this way we obtain the following expressions

\[ V(0) = \frac{(M_D + M_{K^*})(M_{\text{pole}} + M_D - M_{K^*})}{M_{\text{pole}}^2 \sqrt{M_D}} \hat{v} \] (5.18)
\begin{align*}
A_1(0) &= \frac{(M_{\text{Pole}} + M_D - M_{K^*})\sqrt{M_D}}{(M_D + M_{K^*})M_{\text{Pole}}^2}\hat{a}_1 \\
A_2(0) &= \frac{(M_D + M_{K^*})(M_{\text{Pole}} + M_D - M_{K^*})}{M_{\text{Pole}}^2\sqrt{M_D}}\hat{a}_2
\end{align*}

The constants \( \hat{v}, \hat{a}_1 \) and \( \hat{a}_2 \) are determined by the data for \( D \to K^* \) given in Eq.(5.4).

A comparison with our model gives:

\begin{align*}
\hat{v} &= \frac{g\nu\lambda}{\sqrt{2}} \alpha \\
\hat{a}_1 &= -\sqrt{2}g\nu\alpha_{\text{eff}} \\
\hat{a}_2 &= -\frac{g\nu\mu}{\sqrt{2}}\hat{\alpha}
\end{align*}

therefore the predictions obtained from this scaling argument coincide with those obtained at leading order from an effective lagrangian. The same observation applies to the results of Table 2.

In Table 4 we compare our results with other existing calculations. The comparison is made for the ratios of the form factors at \( q^2 = 0 \) to the corresponding form factors for the \( D \) meson, from which we have fixed our parameters.

Table 4: Comparison among our predictions and other theoretical calculations of the form factors at \( q^2 = 0 \).

| Decay                  | Our result | WSB[22] | AW[23] | Lattice[12] |
|------------------------|------------|---------|--------|-------------|
| \( F_1(D \to K)/F_1(D \to \pi) \) | 0.85       | 1.10    | 2.8    | 1.09        |
| \( F_1(B \to \pi)/F_1(D \to \pi) \) | 0.67       | 0.48    | 0.40   | 0.85        |
| \( A_1(D \to \rho)/A_1(D \to K^*) \) | 1.15       | 0.89    | 0.90   | 0.85        |
| \( A_2(D \to \rho)/A_2(D \to K^*) \) | 1.04       | 0.80    | 0.62   | 0.11        |
| \( V(D \to \rho)/V(D \to K^*) \)  | 1.06       | 0.97    | 1.30   | 0.91        |
| \( A_1(B \to \rho)/A_1(D \to K^*) \) | 0.44       | 0.32    | 0.31   |             |
| \( A_2(B \to \rho)/A_2(D \to K^*) \) | 0.74       | 0.24    | 0.72   |             |
| \( V(B \to \rho)/V(D \to K^*) \)  | 0.65       | 0.26    | 0.71   |             |

6 Conclusions

The leptonic decays of a heavy pseudoscalar meson into a light pseudoscalar or into an octet vector resonance have been studied with an effective lagrangian by including the allowed direct coupling and the lowest contributing poles. Chiral and heavy quark symmetries are incorporated in this lagrangian and allow to calculate, from the data on the \( D \to K \) and \( D \to K^* \), form factors and widths for semileptonic decays of \( B \) and \( D \).
into light mesons. Comparison with other calculations in the literature shows the still uncertain status of the theory in this field, particularly for the $B$.

We expect our formalism to be reliable around $q_{\text{max}}^2$, where the light mesons have low momentum and therefore the derivative expansion should be valid. The extrapolation down to $q^2 = 0$ is made assuming pole dominance for the form factors. The main problem of this approach are the $1/M_Q$ corrections to the leading results; they are probably significant at the charm scale and they could modify the simple scaling behaviour of the form factors. Inclusion of subdominant terms in the lagrangian would lead to a loss of predictivity, due to the introduction of too many unknown effective couplings: therefore in this framework there is no simple way to improve our leading order treatment. Additional new experimental data will test for the validity of the approximations used here and will allow for a more precise determination of the parameters of the leading effective lagrangian.

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