How to convince others?
Monte Carlo simulations of the Sznajd model

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Abstract

In the Sznajd model of 2000, a pair of neighbouring agents on a square lattice convinces its six neighbours of the pair opinion if and only if the two agents of the pair share the same opinion. It differs from other consensus models of sociophysics (Deffuant et al., Hegselmann and Krause) by having integer opinions like $\pm 1$ instead of continuous opinions. The basic results and the progress since the last review are summarized here.

1 Introduction

The application of cellular automata, Ising models and other tools of (computational or statistical) physics has a long tradition [1, 2, 3, 4, 5, 6]. Of course, thinking human beings are not enthusiastic about being treated like a randomly flipping magnetic moment, since they form their opinions by complicated cognitive processes. But to see general properties of mass psychology, such simple approximations may be realistic enough. Similarly, conceiving a child is a very private affair; nevertheless from average birth rates one can predict reasonably well how many babies will be born next year in a large country. Whether I smoke, drink or eat steaks, has influence on my health and on my time of death; nevertheless, average mortality rates are published in many countries and were already studied three centuries ago by comet researcher Halley.

More recently, starting perhaps with [7] (see [8] for recent simulations and further literature), the “consensus” literature tried to find out when (in a computer simulation) a complete consensus from initially diverging opinions emerges. Deffuant et al [9] (henceforth denoted as D, where earlier papers on this model are cited) and Hegselmann and Krause (HK) [10] had opinions on a continuous scale between $-1$ and 1, while the Sznajd model [11] (for a review see [12]) mostly used the binary choice $\pm 1$ for opinions.

The HK model and most other “voter” models [13] assume that every agent is influenced by its neighbours or by all other agents and takes, for example, the opinion of the majority of them, or of a weighted average, as its own next opinion. The Sznajd model, on the other hand, assumes that every agent tries to influence its neighbours, without caring about what they think first. Thus in the Sznajd model the information flows outward to the neighbourhood, as in infection or rumour.
Figure 1: Can the European Union ever agree on a joint foreign policy? The final distribution of opinions in the model of Deffuant et al is shown for little tolerance against different opinions (+, fragmentation), intermediate tolerance (×, two opposing camps) and high tolerance (*, consensus). Note that some extremist opinions always remain.
spreading, while in most other models the information flows inward from the neighbourhood. The D model is in between: Two people who exchange opinions move closer together in their opinions. The models of HK and D assume that only people whose opinions are already close to each other can influence each other: bounded confidence. The results of these two models are similar but the D model is faster to simulate, up to 450 million agents by the present author, Fig.1. The main result is that a complete consensus is reached if the interval of opinions over which people influence each other is large enough, while for small such intervals at the end several distinct opinions survive. For the D model this can also be seen from approximate analytical treatment \textsuperscript{[14]}. The Sznajd model, with only the opinions $\pm 1$ allowed, always leads to a complete consensus, and this remains true if $Q > 2$ different opinions are allowed and all opinions can influence each other. With bounded confidence in the sense that opinion $q$ can influence only the neighbouring opinions $q \pm 1$, the results are similar to those of D and HK. Full consensus for $Q$ up to three but not for larger $Q$. However, the Sznajd model takes into account the well-known psychological and political fact that “united we stand, divided we fall”; only groups of people having the same opinion, not divided groups, can influence their neighbours.

In contrast to the other consensus models, the Sznajd model as published thus far deals only with communication between neighbours, not between everybody. It is a “word-of-mouth” model.

2 Models

To see if a consensus emerges out of initially different opinions, all three models here start with a random initial distribution of opinions $S_i$, $i = 1, 2, \ldots N$ of $N$ “people”, where $S_i$ is a real number (between 0 and 1) in the D \textsuperscript{[9]} and HK \textsuperscript{[10]} models, while it is an integer in the Galam \textsuperscript{[15]} and the Sznajd model \textsuperscript{[11]}. Only the basic D and HK versions are reviewed here. Fortran programs are listed in \textsuperscript{[12]} or given in the appendix.

2.1 Deffuant et al

In the D model \textsuperscript{[9]}, at every Monte Carlo step a randomly selected pair $i, k$ checks if the opinions $S_i$ and $S_k$ differ by less than a fixed parameter $\epsilon$. If no, nothing happens; if yes, both opinions move closer to each other by an amount $\mu|S_i - S_k|$. In Fig.1, the weight $\mu$ was taken as 0.3, and $\epsilon = 0.05, 0.25$ and 0.40 for low, intermediate and high tolerance of dissent.
2.2 Hegselmann-Krause

Also in the HK model [10], the opinions vary between 0 and 1. At each Monte Carlo step, one randomly selected \( i \) takes the average opinion of all other opinions \( S_k \) which differ less than \( \epsilon \) from \( S_i \). Because of this large sum, the simulation for large numbers \( N \) of people is much slower than in the D model; nevertheless the final results are quite similar, Fig. 2 of [16].

2.3 Galam

The Galam model [15] is not really a consensus model since dissenters are ignored, not convinced. \( N = 4^n \) people are divided into \( N/4 \) groups of four each, each group determining by majority vote which of two possible opinions the single delegate of that group will support. Four such delegates again select one representative by majority vote, four such representatives select one council member, and so on, until after \( n \) such steps of majority hierarchies one opinion represents the whole community. In case of a 2:2 tie, the status quo is preserved, i.e. the government wins over the opposition. Even if in the initial random distribution of opinions, the opposition has a sizeable majority, at the end the minority government wins, also in the case of more realistic Monte Carlo simulations or modified models [17].

2.4 Panic

Some sort of consensus is also reached if all people in a room on fire run to one of two exits, leaving the other exit unused. This panic is the limiting case of a simulation [18] using molecular dynamics techniques, where in general each person follows partly the majority direction and partly his/her own judgement.

2.5 Sznajd

The Sznajd people usually sit on lattice sites, and a pair of two neighbours \( i, k \) having the same opinion \( S_i = S_k \) convinces all its neighbours of this opinion \( S_i \). Instead of a pair, also a single site, or a plaquette of four agreeing neighbours has been simulated [19, 21] to convince all neighbours.

3 Basic Sznajd Results

The basic Sznajd model with random sequential updating always leads to a consensus, even if more than two opinions are allowed or for higher dimensions. If initially half of the opinions are +1 and the other half −1, then at the end half of the samples will have \( S_i = +1 \) for all \( i \), and the remaining half have \( S_i = −1 \) everywhere. A phase transition is often observed as a function
of the initial concentration $p$ of up spins $S_i = 1$: For $p < 1/2$ all samples end up with $S_i = -1$, and for $p > 1/2$ they all end up in the other fixed point $S_i = +1$, for large enough lattices. This phase transition at $p_c = 1/2$ does not exist in one dimension \cite{11} or when a single site (instead of a pair or plaquette) on the square lattice \cite{10} already convinces its neighbours. Pictures and cluster analysis of the domain formation process \cite{20, 12} show strong similarity with Ising models. The time needed to reach a complete consensus fluctuates widely and (in the cases were a phase transition is found) does not follow a Gaussian or log-normal distribution. If convincing happens only with a certain probability, then no complete consensus is found \cite{11, 21}. A Hamiltonian-like description seems possible (only ?) in one dimension \cite{22}. The number of people who never changed their opinion first decays with a power of time, and then stays at a small but finite value \cite{23}, quite different from Ising models. See \cite{24} for an economic application.

4 Sznajd Modifications

Switching from the square to the triangular or a diluted lattice does not change the qualitative results \cite{25, 26}. Elgazzar \cite{27} and Schulze \cite{28} left the word-of-mouth limit of nearest-neighbour interactions and looked at longer ranges of interaction, using a “small world” network \cite{27} or a power-law force \cite{28}. If the probability to convince others decays with a power of the distance, the phase transition remains in the usual case (when a pair is needed to convince neighbours), but no phase transition appears in the simpler case of a single site being able to convince \cite{28}. In contrast, the Ising model may show a transition for power-law decay of interactions even if for nearest neighbours in one dimension no transition occurs.

Schulze also simulated a “ghost site” connected to all normal sites on the square lattice \cite{29}; this ghost site convinces each normal site of the ghost opinion with a small probability. In marketing, this probability corresponds to the influence of advertising, e.g. through TV commercials. The larger the lattice is the smaller is the amount of advertising needed to convince the whole market.

A different subject is “frustration”: What should I do if my neighbours to the left tell me to vote for +1, and those to the right tell me to vote for $-1$ ? For the usual random sequential updating I follow first the opinion of which I am convinced first, and later I follow the one of which I am convinced later: no problem. But if simultaneous updating is used, then I am frustrated, I do not know whom to follow, and thus do not change my opinion. In this case, a consensus is difficult in small lattices and impossible in big ones \cite{30}. (Our introduction is mostly taken from there.) These difficulties are partially reduced if I am less obedient to authority and in case of conflicting advice follow my own opinion, defined as the majority opinion in my own past voting record \cite{31}. The blocking effect of frustration is also removed by a small amount of noise \cite{31}, when people with a low probability do not follow the above rule. Then after sufficiently long time
a nearly complete consensus is found.

Talking about voting, the results of Brazilian elections (distribution of number of votes among many candidates) were reproduced quite well if the Sznajd model with many different opinions (instead of only ±1) is put on a Barabási-Albert network [20, 32]. If the Sznajd dynamics is simultaneous to the growth of this network, complete consensus no longer is possible [33]. On the square lattice, if only $Q = 4$ or 5 parties are simulated [34, 33] with bounded confidence, even-odd oscillations as a function of the opinion number may appear at intermediate times, with the effect that the party which was on second place halfway through the convincing process ends up with no votes, just as the fourth-ranked party, while the third-ranked party at the end still has a small number of followers. Also, bounded confidence makes it difficult to reach a consensus for a large number $Q$ of parties [34].

Returning to one dimension, Behera and Schweitzer showed that numerically their Sznajd results cannot be distinguished from a probabilistic voter model with interactions from nearest and next-nearest neighbours [35].

5 Summary

In its first three years, the Sznajd model [11], first rejected by Phys. Rev. Letters, found followers in four continents. Some of its results are Ising like, others are not. More sociological numbers than only Brazilian elections would be nice for comparison.

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6 Appendix: Sznajd Chain and Programs

In one dimension one can see without computer simulations why an initial random distribution of votes (which is not up-down periodic everywhere like an antiferromagnet) results in a complete consensus. After some time large domains of + and − are formed, with domain boundaries like $+++---$. If the rightmost plus pair in this picture is selected, it convinces the leftmost minus to become plus, and the boundary is shifted to the right. If instead the leftmost minus pair is selected, it shifts the boundary to the left. Thus the boundaries undergo a random walk until they annihilate each other or are annihilated at the two sample ends. An intermediate step of this annihilation process are configurations like $+++-++$ or $++--++$. Here the inner mixed region cannot expand since in contrast to Ising models only adjacent pairs convince; but the inner region can shrink to annihilation when the outer pairs are selected to convince their neighbours. Thus all boundaries finally vanish and we arrive at a complete consensus. These arguments apply also to more than two opinions, while simulations show this behaviour also in
Three-dimensional Sznajd model with \(41^3\) sites. Time in Monte Carlo steps per site

Figure 2: Variation of the magnetization, the difference between the two opinions, in ten separate runs of a \(41^3\) Sznajd lattice with neighbors in agreement convincing their 18 neighbours. The two horizontal lines indicate complete consensus. (For larger lattices and higher dimensions blocking cannot be excluded because of the enormous fluctuations seen in this figure.)

higher dimensions, Fig.2, in not too large lattices. Simple programs for D and HK are listed below.
parameter(n=200,max=30,iseed=1,eps=0.4,weight=0.3)
c Deffuant et al consensus (Weisbuch’s C program)
c s(i)=opinion of agent i; n=number of agents
dimension s(n)
print *, n, eps, max, iseed
ibm=2*iseed-1
factor=1.0d0/2147483648.0d0
do 1 i=1,n
   ibm=ibm*16807
   s(i)=iabs(ibm)*factor
1 do 2 iter=1,max
do 3 i=1,n
   ibm=ibm*16807
   j=1+(ibm*factor+0.5)*n
   if(j.le.0.or.j.gt.n) goto 4
   if(abs(s(i)-s(j)).gt.eps) goto 3
   shift=weight*(s(j)-s(i))
   s(i)=s(i)+shift
   s(j)=s(j)-shift
3 print *, iter, s(i)
2 continue
stop
end

parameter(n=200,eps=0.40,max=10,iseed=1)
c Hegselmann-Krause consensus with sequential updating
c s(i) = opinion of agent i, n = number of agents
dimension s(n)
print *, n, eps, max, iseed
ibm=2*iseed-1
factor=1.0d0/2147483648.0d0
do 1 i=1,n
   ibm=ibm*16807
   s(i)=iabs(ibm)*factor
1 do 2 iter=1,max
   do 3 i=1,n
      sum=0.0
neighb=0
   si=s(i)
do 4 j=1,n
   if(abs(s(j)-si).gt.eps) goto 4
   sum=sum+s(j)
   neighb=neighb+1
4  continue
   s(i)=sum/neighb
3   print *, iter, s(i)
2  continue
stop
end

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