Diagnostics of plasma electron density and collision frequency of fluorescent lamp using microwave transmission diagnostics

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Abstract. According to the propagation characteristics of the microwave in the plasma, the collision frequency and electron density in the plasma can be simultaneously obtained by measuring the attenuation and phase shift. A new method is proposed to diagnose the plasma in the T8 and T4 fluorescent lamps. According to the experimental results, the electron density of the T8 and T4 fluorescent lamps range from \(0.8 \times 10^{15}/\text{cm}^3\) to \(5.3 \times 10^{15}/\text{cm}^3\) and \(1.4 \times 10^{15}/\text{cm}^3\) to \(8.1 \times 10^{15}/\text{cm}^3\) respectively. Besides, the collision frequency of T8 and T4 are \((2 - 7) \times 10^{18}\text{Hz}\) and \((2.9 - 7.4) \times 10^{18}\text{Hz}\) respectively. This is basically consistent with the diagnostics results of the two-probe diagnostic method. Moreover, the S-parameters simulation results using the measured plasma parameters based on the Drude’s model are also consistent with the experimental results.

1. Introduction

Plasma technology has been widely used in many fields, and its importance has become increasingly prominent [1-2]. The development of plasma technology is based on a thorough understanding of plasma characteristics [3]. Among the parameters characterizing the physical properties of plasmas, electron density is a very important parameter. The current diagnostic methods are mainly focused on the electron density in the plasma, which can be divided into two categories: passive diagnostics and active diagnostics. The former refers to the measurement method of the plasma parameters by measuring the electromagnetic waves or particles emitted by the plasma itself, such as the electrostatic probe method [4-5], the magnetic probe method [6]. The latter refers to the diagnostics of plasma parameters by measuring changes, such as changes in amplitude and phase of the scattered wave or movement of interference fringes, caused by externally input electromagnetic waves, particle beams, such as laser scattering, spectroscopy, microwave diagnostics [7-8].

The electrostatic probe method can simultaneously diagnose the electron density and temperature of the plasma and has spatial resolution. However, this method cannot directly diagnose the electron collision frequency in the plasma. When it is applied to low-temperature industrial plasma diagnostics, it is easy to deposit a layer of insulating material on the surface of the probe, which seriously affects the volt-ampere characteristics of the probe. In the high-temperature fusion plasma, due to the bombardment of the probe surface by the high-speed charged particles, the probe sputters some impurity particles, thereby contaminating the analyte. The microwave diagnostic technology has been rapidly developed because of its merits, such as the measurement is not affected by the plasma, it does not interfere with the measured plasma, the requirements for the microwave source are not high, and the
plasma parameters can be obtained more accurate. The microwave diagnostics method uses electromagnetic waves as a probe beam to enter the plasma, and uses the principle of interaction between the microwave and the plasma to detect the plasma parameters. Microwave interferometry [9] and microwave reflection [10] are currently the more common method. The microwave interferometry obtains the average of the electron density distribution of the plasma along the propagation path by measuring the phase change produced by the propagation of electromagnetic waves in the plasma path. Therefore, the spatial resolution of the microwave interferometer is limited.

The microwave reflection method uses the cut-off characteristic of electromagnetic waves propagating in the plasma to measure the phase change of the reflected wave, and the plasma electron density distribution on the critical surface can be obtained. The microwave transmission diagnostic technique [11] is based on the change of the amplitude and phase of the electromagnetic wave through the plasma, and can simultaneously obtain the electron density and the electron collision frequency information.

In this paper, a new plasma diagnostic method based on microwave transmission was proposed to simultaneously diagnose plasma electron density and collision frequency. In addition, by comparing with the results mentioned in other literature and simulation results, the accuracy of the method is verified. To investigate the state of the plasma in a fluorescent lamp, a Network Analyzer was used to measure the attenuation and phase shift of the microwave as it passes through the T8 or T4 fluorescent lamps. These measurements are related to the real and imaginary parts of the plasma refractive index in the Appleton equation, which makes it possible to obtain electron density and collision frequency.

2. Microwave transmission diagnostic technology
Microwave transmission diagnostic technology enables simultaneous diagnostics of electron density and collision frequency by measuring the attenuation and phase shift of electromagnetic waves through the plasma. The microwave transmission system is shown as Figure 1. The transmitting antenna illuminates the target, and the receiving antenna receives only the transmitted wave passing through the tested target, which is the role of the metal plate.

2.1. Dielectric constant diagnostics by free space method
The free space inversion method was originally proposed by Cullen.A.L. from University of London in 1987 [12]. The test principle is that with the transmitting antenna and the receiving antenna connected with the two ports of the Network Analyzer respectively, and the electromagnetic parameters can be calculated by measuring the scattering parameters.

The basic principle of the free space method is to use the transmitting antenna to transmit the microwave signal to the free space. When the electromagnetic wave encounters the sample to be tested, it will reflect and transmit. The receiving antenna receives both these reflected signals and transmitted signals, and measures the reflection and transmission coefficient. The coefficient, combined with its relationship with the dielectric constant, is used to calculate the dielectric constant.
Assuming that the measured object has a thickness of \(d\). In free space, a plane wave of frequency \(f\) is perpendicularly incident from the free space to the test sample, and the propagation direction propagates forward along the z-axis, as shown in Figure 2. The left and right sides of the sample are air, and the propagation constant of electromagnetic waves in air is \(\beta_0(\beta_0 = \omega \sqrt{\mu_0 \varepsilon_0})\), where \(\omega\) represents the circular frequency of the incident wave, and \(\mu_0\) and \(\varepsilon_0\) are denoted as permeability and dielectric constant in air, respectively. \(\mu_\infty\) and \(\varepsilon_\infty(\varepsilon(\omega) = \varepsilon'(\omega) - j \varepsilon''(\omega))\) are respectively indicated as relative complex permeability and relative complex permittivity of the dielectric material.

![Figure 2. Electromagnetic wave propagation through the sample.](image)

Combined with Maxwell's equations and the definition of S-parameters, the S-parameters can be written as

\[
S_{11} = \frac{(\eta^2-1)(1-e^{-2j\beta d})}{(\eta+1)^2-(\eta-1)^2e^{-2j\beta d}}
\]

and

\[
S_{21} = \frac{4n_\infty e^{-\beta d}}{(\eta+1)^2-(\eta-1)^2e^{-2j\beta d}}.
\]

Where \(\eta_r\) represents the characteristic impedance of the object to be measured, \(\beta\) is the transmission coefficient of the electromagnetic wave in the object to be measured, whose relationship with complex permittivity are

\[
\eta_r = \sqrt{\frac{\mu_r}{\varepsilon_r}},
\]

\[
\beta = \beta_0 \sqrt{\mu_r \varepsilon_r}.
\]

2.2. Calculation of plasma electron density and collision frequency

Supposed that a lossy plasma is isotropic, non-magnetized, with refractive index of \(n = n_r - fn_j\), where [13]

\[
n_r = \frac{\omega_p^2}{\omega^2 - \omega_p^2} + \frac{1}{2} \left( \frac{1}{(1 - \frac{\omega_p^2}{\omega^2 - \omega_p^2})^2} + \frac{v_e^2}{\omega^2 \omega_p^2} \frac{\omega_p^2}{\omega^2 - \omega_p^2} \right)^{1/2},
\]

and

\[
n_j = \left\{ -\frac{1}{2} \left( 1 - \frac{\omega_p^2}{\omega^2 - \omega_p^2} \right) + \frac{1}{2} \left( \frac{1}{(1 - \frac{\omega_p^2}{\omega^2 - \omega_p^2})^2} + \frac{v_e^2}{\omega^2 \omega_p^2} \frac{\omega_p^2}{\omega^2 - \omega_p^2} \right)^{1/2} \right\}^{1/2}.
\]

\(\omega_p\) is the plasma frequency, and \(\omega\) is the incident frequency in radians.

After processing, the electron collision frequency is

\[
v_e = \frac{2n_e n_j \omega}{1-n_e^2 + n_j^2}.
\]

The plasma frequency is

\[
\omega_p = \omega \left[ (1-n_e^2 + n_j^2) \left( 1 - \frac{4n_e^2 n_j^2}{1-n_e^2 + n_j^2} \right) \right]^{1/2}.
\]
Thus, the plasma density is
\[ n_e = \frac{\omega^2 e_0 m_e}{\varepsilon_r^2} \cdot (1 - n_e^2 + n_j^2) \cdot (1 - \frac{4n_e^2 n_j^2}{1-n_e^2+n_j^2}). \]  
(9)

The relationship between the complex refractive index and the complex permittivity is \( n = \sqrt{\varepsilon_r} \). In this way, according to equations (7) and (9), we can obtain the plasma density and the collision frequency by the refractive index. While the refractive index can also be obtained from equations (5) and (6), when the S21 parameter was measured. So, the plasma electron density and the collision frequency can be obtained by measuring the S21 parameter.

3. Measuring system

Based on the previous analysis, a microwave transmission diagnostic system was designed to diagnose plasma electron density and collision frequency. Figure 3 shows the experimental measurement system. The system mainly consists of ballasts, fluorescent lamps array, horn antennas, a Network Analyzer (Agilent 8719ES), a computer, a microwave darkroom system. The ballast is Philips EBC-218 and connected to 220v/50Hz AC. The fluorescent lamps are T8 or T4 fluorescent lamps, whose length is 600mm and diameter is 25.72mm. The three-layered arrangement of the lamps is shown in Figure 3, considering that it can be better equivalent to a homogeneous media sheet. The linearly polarized horn antenna is operated at 6-18 GHz.

4. Experiment and analysis

4.1. Results of T8 fluorescent lamps

The physical relationship between electron density and the EMF scattering parameters is typically established by the thermodynamic description of the plasmas using Boltzmann’s equation [14]. For the local thermodynamic equilibrium state (LTE), the detailed theory developed in this area is based on the deduction of the Saha-Boltzmann equation for the state population of particles in a plasma, where the electron density must be high enough to meet the criterion of the LTE quantum state condition [15]. Although global thermodynamic equilibrium (GTE) state with macroscopically homogeneous intensive parameters, e.g., the electron temperature, is hard to be generated or identified, the hypothesis of the LTE state of the plasma, crucial to the simplification of the theoretical treatment of the Boltzmann’s model [16], is still widely accepted for many globally non-thermal equilibrium plasma systems. For the Appleton-Lassen’s equation used in microwave diagnostics of plasma frequency and collision frequency, the thermodynamic equilibrium state is not quoted by the authors, typically. For example, in [13], the authors used the Lassen’s relation to diagnosis the electron density for a Plasma lamp with a good agreement of the other diagnostic methodology. However, this is not a direct demonstration of the role of thermal equilibrium state of the plasmas being diagnosed through various refractive index methodology [17]. Consequently, the diagnosed parameters based on the Lassen’s equation is used to calculate the scattering features of the plasma lump model in a time-domain algorithm solution of the
Maxwell’s equation, where the Maxwellian distribution of the electron energy with the LTE condition is served as the underlying hypothesis.

**Figure 4.** Transmission attenuation curve of plasma in T8 lamps.

**Figure 5.** Diagnostic results of plasma in T8 lamps, (a), (b) and (c) represent complex permittivity, the collision frequency, and the electron density, respectively.
The current in the fluorescent lamps is about 60mA. The sweep frequency source has a sweep range of 6-12 GHz and a sweep interval of 15MHz. Firstly, in the absence of power, no plasma is generated in the fluorescent lamp. The amplitude and phase data of the transmitted wave was collected and stored for calibration of the system. Secondly, connecting the ballasts to 220V AC. Finally, by processing the data collected and stored, the electron density and collision frequency of the plasma can be obtained.

Figure 4 shows the transmission attenuation curve of the plasma in the lamps. In the case of discharge, the microwave absorption in the frequency range of 6-11GHz is weaker than that in 11-12GHz. In order to improve the accuracy of the measurement data post processing scheme, the data of 6-11GHz were used for analysis, and the diagnostics results are shown in Figure 5. The accurate values are shown in Table 1.

Table 1. Numerical results of plasma in T8 lamps.

| Parameter        | Values                        |
|------------------|-------------------------------|
| Complex permittivity | ~1.2 − 0.6i (Related to incident frequency) |
| Plasma frequency | 1.6 × 10^{10} − 4.2 × 10^{10} rad/s |
| Collision frequency | 2 × 10^{10} − 7 × 10^{10} Hz |
| Electron density | 0.8 × 10^{17} − 5.3 × 10^{17} m^{-3} |

According to diagnostic results, the plasma electron density in the T8 fluorescent lamps is roughly between 0.8 × 10^{11}/cm^{3} and 5.3 × 10^{11}/cm^{3}, while the collision frequency is approximately 2 × 10^{10}Hz to 7 × 10^{10}Hz. When using double probe diagnostics [18], the plasma electron density obtained is 10^{11}/cm^{3} to 3 × 10^{12}/cm^{3}, the collision frequency is 10^9 to 10^{11}Hz. Comparing the value of plasma density and collision frequency in the double probe diagnostics, it can be seen that the data measured in this paper is numerically correct. Further, the diagnostic results using the new method show a narrower data range which indicates a higher numerical accuracy.

The S-parameters of the same arrangement of the T8 fluorescent lamps are calculated with FEM method, where the plasma is described based on the Drude’s model. According to the experiment results, the plasma frequency and the collision frequency in the model are set to 4.2 × 10^{10} rad/s and 60GHz, respectively. Comparing it with the experimental results of S21, as shown in Figure 6, we can find that the simulation results of S21 are consistent with the experimental results of S21 within a certain error range, which all have a strong absorption peak at 11-12GHz.

![Figure 6. Comparison of S21 between simulation and experiment in T8 lamps.](image)

Theoretically, the plasma electron density and collision frequency in the lamp are independent of the incident frequency. But as shown in Figure 5, electron density and collision frequency are not a constant, which is caused by systematic errors. The error sources of this method mainly come from the instrument, violation of far-field conditions, multiple reflections in the fluorescent lamps, surface scattering, etc. Multiple reflection effects can be reduced by using the time domain gate technology, but other sources of error are difficult to be eliminated.
4.2. The result of T4 fluorescent lamps

The plasma in the T4 fluorescent lamp was diagnosed in the same way as the T8 lamps. As shown in Figure 7, the data between 6GHz and 16GHz were chosen for analysis. After processing the data, the plasma electron density in the T4 fluorescent lamps is in the range of $1.4 \times 10^{11} / \text{cm}^3$ to $8.1 \times 10^{11} / \text{cm}^3$, and the plasma collision frequency is between $2.9 \times 10^{16} \text{Hz}$ to $7.4 \times 10^{19} \text{Hz}$.

![Figure 7. Transmission attenuation curve of plasma in T4 lamps.](image)

5. Conclusions

The microwave transmission diagnostic method can simultaneously diagnose electron density and collision frequency in plasma. Compared with both the results in the literature and the numerical simulation results based on the Drude’s model, it is shown that the novel frequency-sweep microwave transmission diagnostic method proposed by the authors exhibits a narrower diagnostics data range which indicates a higher numerical accuracy. Besides, it can simultaneously diagnose the electron density and collision frequency of the plasma. Further, it is found that the electron density diagnosed through the method is dependent on the incident frequency. It can be explained by the system error is frequency-dependant for this methodology. The future work is mainly to model and reduce the system errors and further improve diagnostic accuracy.

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