Trade-off of Tactical Technical Indicators Based on Improved Branch and Bound Method

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Abstract. In view of the relative lack of research on the method of determining the Tactical Technical Indicators requirements through the typical indexes trade-off model in the domestic combat technology index demonstration, based on the non-convex quadratic programming problem widely used in engineering design and economies of scale, a new linear relaxation is proposed with the help of the characteristics of the quadratic function. The method uses the new ultra-rectangular reduction and pruning techniques according to the differential median theorem, and uses the branch and bound method to solve, which provides a quantified and efficient method for determining the technical parameters of the weapon equipment design.

1. Introduction

The development of Weapon Systems is a complex project. In order to ensure the development work goes smoothly, the preliminary demonstration work is very important. The demonstration of Tactical Technical Indicators is the basis and key of the preliminary demonstration work. The overall performance of the Weapon System is determined by the technical indicators and technical realization. Whether the indicator requirements are reasonable will directly affect the overall performance of the Weapon system.

Demonstration of Tactical Technical Indicators is an analysis activity that clarifies the main Tactical Technical Indicators requirements of Weapons System according to capability requirements and technical feasibility based on the weapon combat scenario. It is an important input to the overall requirements for Weapon System development. Due to the complex coupling relationship between Tactical Technical Indicators and the huge space for a plan composed of multiple Tactical Technical Indicators, it is difficult to quickly demonstrate Tactical Technical Indicators that meet both capability requirements and technical feasibility. At present, it mainly relies on the experience of the model chief, supplemented by the simulation verification method, which has strong uncertainty in the completion time and quality of the demonstration. Aiming at the problem of non-convex quadratic programming that often appears in the demonstration of Tactical Technical Indicators, this paper proposes new relaxation techniques and pruning methods, improves the branch and bound method, and provides calculations Method for Tactical and Technical Indexes under quadratic non-convex conditions, which can effectively improve the efficiency of combat skill index demonstration.
2. Research Status
Currently, the demonstration of Tactical Technical Indicators of Weapon System generally adopts Cost-Effectiveness model and Simulation Technology methods. Foreign research on the demonstration of combat technical indicators is mainly based on the CAIV method, that is, "cost as an independent variable". The CAIV method takes cost and performance as important design goals, establishes the system effectiveness model and cost model, and determines the value range of performance parameters, and finally integrates the effectiveness model and cost model to establish a Tactical Technical Indicators trade-off model to find the optimal the indicator requirements.

Many domestic scholars have conducted research on the issue of Tactical Technical Indicators demonstration. For example, Qizhong Zhou, Wei Song have studied the method of using mission planning technology to demonstrate the Tactical Technical Indicators of airborne missile weapons in the literature [1]. The main idea is to determine the indicator system firstly, and then analyse whether the main functions of the Weapon meet the performance requirements through mission planning technology. If the performance meets the requirements, establish a technical risk assessment model to evaluate the risk. Through the continuous cycle of the above process, continue to modify the indicator system until the performance requirements are met and the risk is reduced to an acceptable range. In literature [2], Jingtao Wang and Jun Hai based on data visualization, established a multi-dimensional radar chart of the performance of military transport aircraft from different levels, then compared and analysed Tactical Technical Indicators, and finally proposed the tactical indicators of China’s future military transport aircraft based on the design characteristics of foreign Tactical Technical Indicators demand. According to the structural characteristics and working principles of naval guns and missiles, Juncheng Chen and Shiyuan Sun etc. comprehensively balance military requirements and key Tactical Technical Indicators in the literature [3], establish a simulation model of the outer ballistics of the artillery shells for simulation analysis, and finally give the range index. Yepin Qu, Huiming Jin, etc. proposed an indicator demonstration method based on the simulation of the operation process of the carrier-based aircraft in the literature [4], and established the mathematics trade-off model based on the indicators of ship-aircraft adaptability, environmental adaptability, and comprehensive support capabilities. then iteratively optimize the index according to various constraints used by the ship. On the basis of CAIV technology, Fang Liu, Jianyin Zhao, etc. added the analysis of development risk in literature [5], and proposed a Effectiveness-Cost-Risk indicator trade-off model, and a genetic algorithm was used to solve the problem using MATLAB to obtain a set of approximate best solution.

To sum up, the domestic Tactical Technical Indicators demonstration method is to qualitatively give the demonstration process. Currently, it is mainly to establish an index trade-off model based on the characteristics of specific equipment and conduct multiple simulation tests to give the optimal index requirements [1-5]. Although the basic ideas of Tactical Technical Indicators demonstrations are Cost-Effectiveness models [6], simulation technology can also accurately determine the optimal value in the trade-off model, but the simulation requires multiple input of different Tactical Technical Indicators requirements to select the optimal solution, which reduces the efficiency of indicator demonstration. Domestic research on how to determine the requirements of Tactical Technical Indicators according to the trade-off model is still weak. In order to make up for the shortcomings of the simulation technology, we based on the existing research and focuses on a typical trade-off model--non-convex quadratic programming which is often encountered in the demonstration, proposed a new solution method to improve the efficiency of Weapon System development demonstration.

3. New branch and bound method to solve non-convex quadratic programming problem
Branch and Bound is the most commonly used method for solving non-convex programming problems. Its main idea is to decompose (branch) the feasible region of the planning problem successively, and calculate the upper and lower bounds of the objective function for the solution set in each constraint subset, and obtain the upper and lower bound sequence of the optimal value (delimitation) , In each branch, delete (prune) the subset whose optimal value exceeds the target value of the known feasible solution set. When the upper and lower bounds are equal or the difference between the two meets the
requirements, the iteration terminates and the global optimal solution is obtained. Otherwise, the iteration continues until the end.

In the optimal programming, a programming problem in which one of the objective function and the constraint function is non-convex is called a non-convex programming problem. The optimization program whose objective function is a non-convex quadratic function is called a non-convex quadratic programming problem. The non-convex programming problem is the most common planning problem in the actual optimization problem. The non-convex quadratic programming problem has a wide range of applications in the social production economy such as engineering design and optimization control. Therefore, We uses non-convex quadratic programming as example, given improved branch and bound method to gain index optimal value.

We adopts a non-convex quadratic programming model in which the constraint function is also a quadratic function. The specific form is as follows:

$$\begin{align*}
\min & \quad x^T Q^0 x + (d^0)^T x \\
\text{s.t.} & \quad \begin{cases} x \in H = \{x^T Q^i x + (d^i)^T x \leq b_i \quad i = 1, 2, \ldots, m \} \\
 x \in J = \{x \in \mathbb{R}^n; \partial \leq x \leq \beta \} \end{cases}
\end{align*}$$ (1)

$$Q^i$$ are real symmetric matrices of order $$n$$, $$Q^i = (q^i_{jk})_{n \times n}$$, $$i=0, 1, 2, \ldots, m$$; $$d^i \in \mathbb{R}^n$$, $$b_i \in \mathbb{R}$$, $$i=1, 2, \ldots, m$$; $$l = (l_1, l_2, \ldots, l_n)^T$$, $$u = (u_1, u_2, \ldots, u_n)^T$$.

The non-convex quadratic programming problem is an NP-hard problem. Since the objective function and constraint function are non-convex, there may be multiple local optimal solutions when solving the problem, and it is difficult to determine the global optimal solution, which making the solution more difficult. Although there have been many researches on non-convex quadratic programming algorithms in recent years, most of them use relaxation techniques and branch and bound method frameworks for solving, but there is still much room for improvement in the accuracy and efficiency of the problem[7-8]. On the basis of these studies, we constructs a new branch and bound method. First, the NQP problem is transformed into an equivalent quadratic programming problem (ENQP). In order to improve the efficiency of the solution, a new linear relaxation technique is used. The objective function and the constraint function are linearly relaxed to obtain the linear relaxation programming problem, and the original objective function limit is determined by solving the linear relaxation programming. Finally, the hyperrectangular reduction technique is used to solve the problem using the branch and bound algorithm. Finally it proved that the linear relaxation technique and pruning method used in this paper can greatly increase the convergence speed and improve the operating efficiency.

3.1. Convert ENQP

First use variable substitution to convert the (NQP) problem into (ENQP), let:

$$f_i(x) = x^T Q^i x + (d^i)^T x = x^T (Q^i_1 x, Q^i_2 x, \ldots, Q^i_n x)^T + (d^i_1 x_1 + d^i_2 x_2 + \cdots + d^i_n x_n)$$ (2)

$$Q^i_j$$ represents the jth line of $$Q^i$$, and $$d^i_j$$ represents the jth line of $$d^i$$, $$i=1, 2, \ldots, m$$, $$j=1, 2, \ldots, n$$.

Let:

$$y^i_j = Q^i_j x + d^i_j$$ (3)

Then the objective function and constraint function of NQP can be written as:

$$f_i(x) = \sum_{j=1}^{n} x_j y^i_j$$ (4)
Construct super rectangle of the feasible region J of NQP, $H^0 = [x^0, x^0]$, $\bar{x}^0 = (x_1^0, x_2^0, ..., x_n^0)$, and $\tilde{x}^0 = \min x_i \; x_i \in \{ J \cap H \}$; $\tilde{x}^0 = \max x_i \; x_i \in \{ J \cap H \}$.

Then find the maximum value of $y^j_i$ and minimum value of $y^j_i$:

$$y^j_i = \max y^j_i$$
\[\text{s.t.} \left\{ y^j_i = Q^j_i x + d^j_i \right\} \quad (5)\]

and

$$y^j_i = \min y^j_i$$
\[\text{s.t.} \left\{ y^j_i = Q^j_i x + d^j_i \right\} \quad (6)\]

Let $y^i = (y^i_1, y^i_2, ..., y^i_n)$, $\bar{y}^i = (\bar{y}^i_1, \bar{y}^i_2, ..., \bar{y}^i_n)$, solve the above two optimization problems to get a super rectangle $Y^i = \{ y^i, \bar{y}^i \leq y^i \leq \bar{y}^i \}$.

Then, the NQP problem can be transformed into the ENQP problem on $H^k$, $H^k$ is the sub-superrectangle of the Kth subdivision of $H^0$, $H^k = (H^k)^{n \times 1}$, $H^k = [\bar{x}^k, \tilde{x}^k]$ [9-10]. ENQP is as follows:

$$\min f_0(x, y) = \sum_{j=1}^{n} x_j y^0_j$$
\[\left\{ \begin{array}{l}
 f_i(x, y) = \sum_{j=1}^{n} x_j y^i_j \leq b_i \quad i = 1, 2, ..., m \\
 y^i_j = Q^i_j x + d^i_j, \; y^i_j \in Y^i \\
 i = 1, 2, ..., m, \; j = 1, 2, ..., n \\
 x \in H^k 
\end{array} \right. \quad (7)\]

### 3.2. New linear relaxation method

In order to solve the NQP problem, the objective function and constraint function of ENQP need to be linearly relaxed. In the ENQP question in the previous section,

$$x_j y^j_i = \frac{1}{4} \left[ (x_j + y^i_j)^2 - (x_j - y^i_j)^2 \right]$$

$$\geq \frac{1}{2} \left[ (\bar{y}^i_j + y^i_j) x_j + (\bar{x}_j + x_j) Q^i_j x \right] + \frac{1}{2} (\bar{x}_j + x_j) d^i_j + \frac{1}{4} (\bar{x}_j - y^i_j) (x_j - \bar{y}^i_j)$$

$$\geq \frac{1}{10} (\bar{x}_j + x_j + y^i_j + \bar{y}^i_j)^2$$

$$= \frac{1}{2} \left[ (\bar{y}^i_j + y^i_j + x_j q^j_l + \bar{x}_j q^j_l) x_j + \sum_{j \neq k} (\bar{x}_j + x_j) q^j_k x_k \right] + \frac{1}{2} (\bar{x}_j + x_j) d^i_j$$

$$+ \frac{1}{4} (\bar{x}_j - y^i_j) (x_j - \bar{y}^i_j)$$

So for all $i=0,1,2,..,m$, there are
According to the above formula, the linear relaxation programming (LRP) of ENQP can be obtained:

\[
\begin{align*}
\min l_0(x) &= \sum_{j=1}^{n} l_j^0(x) \\
\text{s.t.} \quad l_i(x) &= \sum_{j=1}^{n} l_j^i(x) \leq b_i \quad i = 1, 2, \ldots, m \\
x &\in H^k
\end{align*}
\]

LRP is a convex programming problem, which is easy to solve. An effective lower bound of the optimal value of ENQP can be given by solving the optimal value of LRP. The above relaxation method provides a new method of defining the lower bound of NQP, which speeds up the convergence speed of the calculation.

3.3. Branch and Bound Algorithm

In this section, combining with the differential median theorem, using super-rectangular subdivision and reduction techniques, a branch and bound algorithm for solving the global optimal value of the NQP problem is given.

3.3.1. Super rectangular division. According to section 2.1, the super rectangle of the k-th subdivision is \( H^k = (H^k_j)_{j=1}^n \), \( H^k_j = [x^k_j, \bar{x}^k_j] \), \( H^k \in H^0 \). To divide \( H^k \) into sub-super rectangles \( H^{k+1}_1, H^{k+1}_2 \), take

\[
\alpha = \{ \alpha: x^k_{\alpha} - x^k_{\alpha} = \max (x^k_j - x^k_j), j = 1, 2, \ldots, n \}
\]

Let \( x^k_{\alpha} = \frac{x^k_{\alpha} + x^k_{\alpha}}{2} \), decompose \( H^k \) with \( x^k_{\alpha} \) as the node, and obtain two sub-super rectangles \( H^{k+1}_1, H^{k+1}_2 \), where:

\[
\begin{align*}
H^{k+1}_1 &= [x^{k+1,1}, \bar{x}^{k+1,1}] \\
H^{k+1}_2 &= [x^{k+1,2}, \bar{x}^{k+1,2}] \\
\bar{x}^{k+1,1} &= (x^{k+1,1}, x^{k+1,1}, \ldots, x^{k+1,1}) \\
\bar{x}^{k+1,2} &= (x^{k+1,2}, x^{k+1,2}, \ldots, x^{k+1,2}) \\
\bar{x}^{k+1,1} &= (x^{k+1,1}, x^{k+1,1}, \ldots, x^{k+1,1}) \\
\bar{x}^{k+1,2} &= (x^{k+1,2}, x^{k+1,2}, \ldots, x^{k+1,2})
\end{align*}
\]

3.3.2. Area reduction method. The linear objective function and constraint function \( l^k_i(x) \) in the linear relaxation programming problem LRP can be written as:

\[
l^k_i(x) = \sum_{j=1}^{n} d_{ij} x_j + \beta_i, \quad i = 0, 1, 2, \ldots, m
\]

To construct a hyperrectangular reduction method, let
\[ RL^k_i = \sum_{j=1}^{n} \min \left\{ \partial_{ij}^k x^k_j, \frac{\partial_{ij}^k x^k_j}{\partial_{ij}^k} \right\} + \beta_i \]  
\[ RU^k_i = \sum_{j=1}^{n} \max \left\{ \partial_{ij}^k x^k_j, \frac{\partial_{ij}^k x^k_j}{\partial_{ij}^k} \right\} + \beta_i \]  
i=0, 1, 2, \ldots, m, \text{ and } x^k \in H^k.

Suppose UB is the current upper bound of the NQP problem. If \( RL^0_i > UB \), then the problem NQP has no optimal solution on \( H^k \), delete this \( H^k \) \[10\]-\[12\]. If \( RL^0_i \leq UB \), consider the following two situations:

(a) For \( j=1, 2, \ldots, n \), if \( \partial_{ij}^k x^k_j > 0 \), then the NQP problem has no global optimal solution in \( H_{i,j}^k = (H_{ij}^k)_{n \times 1} \),
\[ \bar{x}_j^k x^k_j \partial_{ij}^k x^k_j = \min \left( \bar{x}_j^k x^k_j, \frac{UB - RL^0_i x^k_j}{\partial_{ij}^k} \right) + \min \left\{ \partial_{ij}^k x^k_j, \frac{\partial_{ij}^k x^k_j}{\partial_{ij}^k} \right\} \]

(b) For \( j=1, 2, \ldots, n \), if \( \partial_{ij}^k x^k_j < 0 \), then NQP problem has no global optimal solution in \( H_{i,j}^k = (H_{ij}^k)_{n \times 1} \),
\[ \bar{x}_j^k x^k_j \partial_{ij}^k x^k_j = \min \left( \bar{x}_j^k x^k_j, \frac{UB - RU^k_i x^k_j}{\partial_{ij}^k} \right) + \max \left\{ \partial_{ij}^k x^k_j, \frac{\partial_{ij}^k x^k_j}{\partial_{ij}^k} \right\} \]

According to the above process, the area can be effectively reduced and the calculation efficiency can be improved.

3.3.3. Branch and Bound Steps. Suppose that \( L(H^k) \) and \( x^k = x(H^k) \) represent the optimal value and optimal solution of the linear relaxation programming problem (LRP) on the subregion \( H^k \) respectively, and \( O \) represents the remaining super rectangle after pruning Subset, \( W \) represents the set of all feasible points. According to the branching and reduction methods given in 2.3.1 and 2.3.2, the specific steps for using the branch and bound to solve the NQP problem are given below:

Step 1: Determine the convergence error \( \delta > 0 \). Solve the LRP \( (H^0) \) problem, get the optimal solution \( x^0 \) and the optimal value \( L(x^0) \) (denoted as \( L^0 \)). If \( x^0 \) is a feasible solution to the NQP problem, the upper bound of the optimal value of NQP is \( UB(0) \), denoted as UB0. If \( UB^0 - L^0 < \delta \) or \( O = \emptyset \), the calculation ends, and \( x^0 \) is a global optimal solution of the NQP problem. Otherwise, go to step 2.

Step 2: Divide the super rectangle \( H^k \) into two sub-super rectangles \( H_{i,1}^k, H_{i,2}^k \) according to the super-rectangle subdivision method in 2.3.1.

Step 3: Use the reduction method of 2.3.2 to reduce \( H_{i,1}^k \) and \( H_{i,2}^k \). The reduced sub-regions still use \( H_{i,1}^k \) and \( H_{i,2}^k \) Represents, \( O = (O \backslash H^k) \cup \{H_{i,1}^k\} \). Then solve the LRP \( (H_{i,1}^k) \), \( i = 1, 2 \) on the two subregions respectively, and calculate the optimal solution \( x_{i,1}^{k+1}, x_{i,2}^{k+1} \) and the optimal value \( L(x_{i,1}^{k+1}), L(x_{i,2}^{k+1}) \), \( W = W \cup \{x_{i,1}^{k+1}, x_{i,2}^{k+1}\} \).

Step 4: Update the upper and lower bounds of the NQP question. Upper bound \( UB^0 = \min \{f(x) : x \in W\} \), lower bound \( L^0 = \min \{L(H_{i,1}^k), H_{i,2}^k \in O\} \).

Step 5: Pruning: \( T = T \backslash \{H^k : L(H^k) > UB^0, H^k \in O\} \).

Finally, let \( k = k + 1 \), and perform step 2.

4. Application example

4.1. Background
The design of Weapon System needs to consider its economy and feasibility while meeting the performance requirements of the system. The following describes how to determine the requirements of
the corresponding Tactical Technical Indicators under the constraints of military transport aircraft efficiency and economic costs.

Military Transport Aircraft refers to aircraft used to transport soldiers, weapons and other military materials. With large load capacity and endurance, it can carry out airlift, airborne, and airdrop to ensure that the ground troops can quickly maneuver from the air. Airlift effectiveness refers to the effect of the transportation system in completing transportation tasks in a given combat environment. According to the literature [9-11], the transportation efficiency index system includes the effects achieved by performing transportation tasks and the resources consumed to achieve these effects. From the perspective of the final manifestation of transportation efficiency, indicators such as task success rate, safety, and transportation volume can be used to give a comprehensive transportation effect. In order to facilitate data collection and modeling, relative values or probability values are used to indicate the completion of each indicator degree.

Suppose the success rate index is represented by \( R \), the safety index is represented by \( S \), and the transportation volume index is represented by \( T \). According to the data inquired, it is assumed that the safety index \( S \) is determined by the availability and credibility of the system itself, the success rate index \( R \) represents the proportion of undamaged materials successfully delivered to the designated area in the total delivered materials, and the transportation volume index \( T \) refers to the carrying capacity of the transport aircraft. Assuming that the comprehensive transport efficiency index \( E \) can be calculated by the product of the three indicators of quality, safety and accuracy, namely:

\[
E = T(S + R)
\]  

(20)

Generally speaking, the higher the transportation efficiency of military transport aircraft, the higher the space, personnel, equipment and economic costs required for transportation. Make \( C \) to represent the cost of military transport aircraft. Given the transport efficiency requirements of military transport aircraft, it is required to minimize the cost while meeting the transport efficiency, and give the requirements for the above three technical indicators.

4.2. Effectiveness and Cost Trade-Off Model

Before establishing the trade-off model between effectiveness and costs, we need to establish the relationship between cost and success rate indicators, safety indicators and transportation volume indicators.

The cost can be determined by the success rate, safety and transportation volume. Because these three indicators have a certain interaction and are positively related to the development cost, the planning problems in the engineering design are generally non-convex quadratic programming. So it is assumed that the development cost \( C \) can be expressed by the following formula:

\[
C = 4S^2 + 3T^2 + 4R^2 + 5ST + 6RT
\]  

(21)

According to the statistical results of the relevant literature, the \( S \) of the system is generally between 0.8 and 1, that is, \( S \in [0.8,1] \); the proportion of undamaged materials successfully delivered to the designated area \( R \in [0.7,1] \); the load capacity of military transport aircraft requires \( T \) to be between 100t and 110t, and the dimensionless treatment of \( T \) makes \( T \in [1,1.1] \).

Assuming that the transport efficiency of military transport aircraft is greater than 0.9, that is:

\[
E = T(S + R) \geq 2.7
\]  

(22)

According to the constraints of the variables, we can get the planning problem as follows:

\[
\begin{align*}
\text{min}\ C &= 4S^2 + 3T^2 + 4R^2 + 5ST + 6RT \\
\text{s. t. } \quad T(S + R) &\geq 2.9 \\
&0.8 \leq S \leq 1 \\
&1 \leq T \leq 1.1 \\
&0.7 \leq R \leq 1
\end{align*}
\]  

(23)

\( NQP: \)
4.3. Model solving

This section solves the non-convex quadratic programming problem in Section 3.2 with the new branch and bound given in Section 2.

**Step1** Perform an equivalent transformation on the NQP problem according to the method introduced in section 2.1 and convert it into an ENQP problem. Here \( Q^0 = \begin{bmatrix} 4 & 0 & 0 \\ 5 & 3 & 0 \\ 0 & 6 & 4 \end{bmatrix}, \quad Q^1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \) while using \( y_j^i = Q_j^i x \) to transform the NQP problem into a linear function, find the super rectangle \( H^0 \) of the NQP constraint set is obtained, and the super rectangle \( Y^0 \) of \( y_j^i = Q_j^i x \) is solved accordingly, and the feasible region of the corresponding ENQP problem is given.

**Step2** According to the linear relaxation method given in 2.2, combine \( d \) with the \( H^0 \) and \( Y^0 \) calculated by Step1, the ENQP is linearly relaxed to obtain a linear relaxation programming (LRP). The programming problem is a simple multivariable linear programming problem with a feasible region of \( H^0 \). The solution method and solution process are very simple and fast. The optimal solution obtained by LRP \( (S^0, T^0, R^0) \) and the optimal value \( L(S^0, T^0, R^0) \) is denoted as \( L_0 \), at this time the upper bound of the optimal value of the original NQP problem \( UB^0 = E(S^0, T^0, R^0) \).

**Step3** Given the convergence error \( \delta = 10^{-5} \), judge whether \( UB^0 - L_0 < \delta \) holds, if yes, the calculation is terminated, at this time \((S^0, T^0, R^0)\) is the global optimal solution. If not, proceed to the fourth step.

**Step4** Divide \( H^0 \) into \( H_1^0, H_2^0 \) according to the super rectangular subdivision method in Section 2.3.1. That is, take the longest side \( a \) of \( H^0 \) and divide \( H^0 \) into \( H_1^0 = [x_1^{1,1}, x_1^{1,1}], H_2^0 = [x_1^{1,2}, x_1^{1,2}] \), where

\[
\begin{align*}
x_1^{1,1} &= (x_1^{1,1}, x_2^{1,1}, \ldots, x_n^{1,1}), \\
x_1^{1,2} &= (x_1^{1,2}, \ldots, x_n^{1,2})
\end{align*}
\]

**Step5** Reduce \( H_1^0 \) and \( H_2^0 \) respectively according to the area reduction method in Section 2.3.2. First calculate \( RL^0 = \sum_{j=1}^3 \min \{ \partial_{0j} x_j^{1,1}, \partial_{0j}^0 x_j^{1,1} \} \), where \( \partial_{0j}^0 \) is calculated based on \( H_1^0 \) and \( y_j^{1,1} \), and the specific calculation formula is shown in 2.3.2. If \( RL^1 > UB^0 \), then delete \( H_1^0 \) which has no optimal solution on \( H_1^0 \), delete \( H_1^0 \), and then find \( \bar{y}_j^{(1,1)}(x_1) \) based on 2.3.2 points, reduce \( H_1^0, H_2^0 \) and exclude the part of the global optimal solution. The area obtained after reduction is still represented by \( H_1^0 \) and \( H_2^0 \), update \( O = (O \backslash H^0) \cup \{ H_1^0, H_2^0 \} \).

**Step6** Solve \( LRP(H_1^0), LRP(H_2^0) \) respectively, calculate the optimal solution \( x_1^{1,1}, x_1^{1,2} \) and the optimal value \( L(x_1^{1,1}), L(x_1^{1,2}) \), update \( W = W \cup \{ x_1^{1,1}, x_1^{1,2} \} \).

**Step7** Re-determine the upper bound \( UB^0 \), and the lower bound \( L^0 = \min \{ L(H^1), H^1 \in T \} \).

**Step8** Delete the branches whose optimal value is not between \( UB^0 \) and \( L^0 \).

Finally, return to Step4 and start to repeat the above operations until the end.

Through the above steps, the optimal solution for minimizing the development cost under the transportation efficiency constraint is obtained as follows:

| Optimal solution | Optimal Value | Number of iterations | Running Time(s) | Accuracy |
|------------------|--------------|----------------------|-----------------|----------|
| (0.9156,1.6745,0.8342) | 30.5958 | 9 | 1.2576 | 10^{-5} |

5. Conclusion

When conducting Weapon System development demonstration, it is necessary to provide Tactical Technical Indicators requirements based on capability requirements and technical feasibility.
requirements. The Branch and Bound optimization algorithm for non-convex quadratic programming proposed in this paper can greatly save and improve the efficiency of demonstration.

In addition to non-convex quadratic programming, the non-convex programming problems encountered in the demonstration of Tactical Technical Indicators include linear ratio sum problems and generalized linear multi-product programming problems. Regarding the direct solution of non-convex optimization, all the problems in non-convex cases have not been completely solved so far. For the larger-scale and higher-dimensional non-convex problems in the Tactical Technical Indicators trade-off model, it is still necessary to study more efficient solutions. The next step is to deal with higher-dimensional non-convex quadratic programming and other types of non-convex programming problems.

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