Some remarks on a nongeometrical interpretation of gravity and the flatness problem

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Abstract

In a nongeometrical interpretation of gravity, the metric \( g_{\mu\nu}(x) = \eta_{\mu\nu} + \Phi_{\mu\nu}(x) \) is interpreted as an effective metric, whereas \( \Phi_{\mu\nu}(x) \) is interpreted as a fundamental gravitational field, propagated in spacetime which is actually flat. Some advantages and disadvantages of such an interpretation are discussed. The main advantage is a natural resolution of the flatness problem.

Keywords: nongeometrical interpretation of gravity, effective metric, flatness problem

1 Introduction

It seems that a gravitational theory based on a scalar or a vector field in a flat Minkowski space cannot describe known experimental data \([1], [2]\). On the other hand, the phenomenological success of Einstein’s theory of gravity suggests that gravity should be described completely, or at least partially, by a symmetric second-rank tensor field. In general, a symmetric second-rank tensor field contains components of spin-0, spin-1 and spin-2 \([3]\). There are many theories of gravity based on a symmetric second-rank tensor field \([4], [5]\). However, if we require that a symmetric second-rank tensor \( \Phi_{\mu\nu} \) describes a massless spin-2 field in a flat Minkowski space with metric \( \eta_{\mu\nu} \) and satisfies a second-order differential equation in which \( \Phi_{\mu\nu} \) is consistently coupled to itself and to other fields, then the most general such equation can be written in the form of the Einstein equation (with a cosmological term) \([6]-[12]\), where the “effective metric” is given by

\[
g_{\mu\nu}(x) = \eta_{\mu\nu} + \Phi_{\mu\nu}(x) \ .
\]
The Einstein equation, when written in terms of $\Phi_{\mu\nu}$ and $\eta_{\mu\nu}$, possesses an infinite number of terms. On the other hand, this equation looks much simpler when it is written in terms of $g_{\mu\nu}$. This suggests, but in no way proves, that $g_{\mu\nu}$, and not $\Phi_{\mu\nu}$, is a fundamental field. Such an interpretation leads to the standard geometrical interpretation of gravity. However, such an interpretation makes gravity very different from other fields, because other fields describe some dynamics for which spacetime serves as a background, while gravity describes the dynamics of spacetime itself. This may be one of the obstacles to formulate a consistent theory of quantum gravity.

The aim of this paper is to investigate a nongeometrical interpretation (NGI) of gravity, in which $\Phi_{\mu\nu}(x)$ is a fundamental gravitational field propagated in a flat Minkowski spacetime with the metric $\eta_{\mu\nu}$, while $g_{\mu\nu}(x)$ has the role of the effective metric only. Some aspects of such an interpretation have already been discussed [13]. In this paper we reconsider some conclusions drawn in [13] and stress some novel conclusions. We find that such an interpretation is not only consistent, but also leads to several advantages with respect to the standard interpretation. In particular, it leads to a natural resolution of the flatness problem. We also comment on some disadvantages of such an interpretation.

2 Global topology and cosmology in the NGI

It has recently been suggested [14] that gravity, as a dynamical theory of the metric tensor $g_{\mu\nu}(x)$, should not be interpreted as a dynamical theory of the space-time topology. The topology should be rather fixed by an independent axiom, while the Einstein (or some other) equation determines only the metric tensor on a fixed manifold. For the Cauchy problem to be well posed, it is necessary that the topology is of the form $\Sigma \times \mathbb{R}$. The most natural choice is $\mathbb{R}^D$ as a global topology, which admits a flat metric $\eta_{\mu\nu}$. Thus the NGI of gravity, which we consider in this paper, supports this nontopological interpretation, because in the NGI it is manifest that the topology is fixed by the background spacetime with a flat metric $\eta_{\mu\nu}$.

The nongeometrical (or nontopological) interpretation may seem to be inconsistent on global level, because it starts with a global $\mathbb{R}^D$ topology of spacetime, while the Einstein equation, which determines $\Phi_{\mu\nu}$ and $g_{\mu\nu}$, possesses solutions for the metric $g_{\mu\nu}$ which correspond to a different topology.

However, this problem is resolved in the Cauchy-problem approach. For example, if the space has $\mathbb{R}^3$ topology on the “initial” Cauchy surface, then it has the same topology at all other instants. Quite generally, if the Cauchy problem is well posed, then the space topology cannot change during the time evolution [14]. The fact that the topology of time in the Friedman universes is not $\mathbb{R}$, but a connected submanifold of $\mathbb{R}$ which is singular on its end(s), can be interpreted merely as a sign of nonapplicability of the Einstein equation for high-energy densities.

However, the interesting question is whether the NGI is consistent if the Einstein equation is not treated as a Cauchy problem and singularities are not treated as pathologies of the model. In [14] it was concluded that the NGI of gravity was not appropriate for cosmological problems. Contrary to this conclusion, we argue that the application of the NGI of gravity to cosmological problems is actually the main advantage of this interpretation with respect
to the conventional interpretation, because the NGI predicts that the effective metric $g_{\mu\nu}$ of a homogeneous and isotropic universe is flat, in agreement with observation. In the conventional approach, the assumption that the Universe is homogeneous and isotropic leads to the Robertson-Walker metric

$$\begin{align*}
\text{ds}^2 &= dt^2 - R^2(t) \frac{dx^2 + dy^2 + dz^2}{[1 + (k/4)(x^2 + y^2 + z^2)]^2}.
\end{align*}\quad (2)
$$

If $k = 0$, this corresponds to a flat universe. The observed flatness cannot be explained in the conventional approach. However, in the NGI, (2) is interpreted as an effective metric, whereas the fundamental quantity is the gravitational field $\Phi_{\mu\nu}$. The nonvanishing components of $\Phi_{\mu\nu}$ in (2) are

$$\Phi_{ij}(x) = \left\{1 - \frac{R^2(t)}{[1 + (k/4)(x^2 + y^2 + z^2)]^2}\right\} \delta_{ij},\quad i, j = 1, 2, 3.\quad (3)
$$

Now the assumption that the Universe is homogeneous and isotropic means that $\Phi_{\mu\nu}$ does not depend on $x, y, z$, which leads to the conclusion that the relation $k = 0$ must be satisfied.

3 The question of local consistency of the NGI

The fact that the NGI leads to a natural resolution of the flatness problem suggests that the NGI could be the right interpretation. Thus, it is worthwhile to further explore the consistency of such an interpretation.

Let us start with the motion of a particle in a gravitational field. If we neglect the contribution of the particle to the gravitational field $\Phi_{\mu\nu}(x)$, then the action of the particle with a mass $m$ can be chosen to be 

$$\begin{align*}
S &= m \left[ -\frac{1}{2} \int d\tau \dot{x}^\mu \dot{x}^\nu \eta_{\mu\nu} - \kappa \int d\tau h_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \right],
\end{align*}\quad (4)
$$

where $\tau$ is the proper time of the particle, $\dot{x}^\mu = dx^\mu/d\tau$, $h_{\mu\nu}(x)$ is a redefined gravitational field, $2\kappa h_{\mu\nu}(x) \equiv \Phi_{\mu\nu}(x)$, and $\kappa$ is a coupling constant. The value of $\kappa$ is determined by the definition of $h_{\mu\nu}(x)$. For example, $h_{\mu\nu}(x)$ can be defined such that, in the weak-field limit, $h_{00}(x)$ is equal to Newton’s gravitational potential. The action (4) also can be written as

$$\begin{align*}
S &= m \left[ -\frac{1}{2} \int d\tau g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \right],
\end{align*}\quad (5)
$$

which is the conventional form of the action of the particle in the gravitational field. Both forms of the action lead to the same equations of motion which determine the trajectory $x^\mu(\tau)$. In the conventional geometrical interpretation, this trajectory is interpreted as a motion along a geodesic, which is not the case for the NGI.

In (4) and (5) it was stated that $\tau$ is the proper time, but the proper time was not defined. For (4) one could naively take the definition $d\tau^2 = \eta_{\mu\nu} dx^\mu dx^\nu$. On the other hand, in (5) the proper time is defined as $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$, which leads to results which are in agreement with observations. We require that (4) is equivalent to (5), so in (4) we must take

$$\begin{align*}
d\tau^2 &= [\eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)] dx^\mu dx^\nu.
\end{align*}\quad (6)\phantom{\text{very long space here}}$$
It is interesting to note that the existence of the geometrical interpretation is in no way the property of the symmetric second-rank tensor field only. For example, as noted in [13], the interaction of a particle with a scalar field $\phi(x)$ can be described by the interaction part of the action $S_I = -mk \int dt \phi(x) \dot{x}^\mu \dot{x}^\nu \eta_{\mu \nu}$, which leads to the action of the form (4), where the effective metric is $g_{\mu \nu}(x) = \eta_{\mu \nu}(1 + 2\kappa\phi(x))$.

Now a few comments on the interpretation of various components of $g_{\mu \nu}$. For example, if $g_{00}$ depends on $x$, in the conventional interpretation this is interpreted as a phenomenon that the lapse of time depends on $x$. In the NGI, it is interpreted that the effect of gravity is such that all kinds of matter (massive and massless) move slower or faster, depending on $x$. Because of the equivalence principle (the coupling constant $\kappa$ in (4) is the same for all kinds of particles), the motion of all kinds of matter is changed in the same way, namely, in such a way as if the metric of the time itself depended on $x$. Similarly, if $g_{ij}$ depends on $x$, in the NGI it is interpreted that the effect of gravity is such that all kinds of matter are contracted or elongated in the same way, depending on $x$. More details on this aspect of the NGI can be found in [13].

In the NGI, the actual distances are given by $\eta_{\mu \nu}$ instead of by $g_{\mu \nu}$. For example, the actual time distance is given by $dt$ instead of by $\sqrt{g_{00}}dt$. Similarly, the actual space distance in the $x^1$-direction is given by $dx^1$ instead of by $\sqrt{g_{11}}dx^1$. Consequently, the actual velocity of light $dx/dt$ (with $ds^2 = 0$) is no longer a constant. However, as stressed in [13], these actual distances are unobservable. Only the effective metric $g_{\mu \nu}$ can be measured. This is one of the unpleasant features of the NGI, but this does not make it inconsistent.

However, there is even a more serious problem of the NGI. These actual distances are not only unobservable, but they are not uniquely defined, because of the invariance with respect to general coordinate transformations of the Einstein equation. The NGI makes sense only if some coordinate condition is fixed. If we can somehow find the right coordinate condition, then we can also define the actual distances. However, it is difficult to find this, because all coordinate conditions lead to the same observable effects, at least in classical physics.

However, it is possible that, in quantum gravity, different coordinate conditions are not equivalent. Moreover, some alternative classical theories of gravity do not possess the invariance with respect to general coordinate transformations (see, for example, [17]). All this suggests that, perhaps, there is a possibility, at least in principle, of identifying the right coordinate condition experimentally. At present, we can only guess what that might be, using some simplicity and symmetry arguments. If we require that this condition should be expressed in terms of $\eta_{\mu \nu}$ and $\Phi_{\mu \nu}$, and that this should not violate Lorentz covariance, then the simplest choice is the harmonic condition

$$D^\mu \Phi_{\mu \nu} = 0,$$

(7)

where $D^\mu$ is the covariant derivative with respect to a flat metric (i.e., a metric which can be transformed to $\eta_{\mu \nu}$ by a coordinate transformation). This condition is preferred by many authors [8], [13]. The metric (4) does not satisfy this condition, but one can easily transform (4) into coordinates for which this condition is satisfied, and conclude in the same way that $k = 0$. One can also see that (3) for $k = 0$ already satisfies (7).
4 Conclusion

The NGI of gravity is consistent and leads to a natural resolution of the flatness problem. The flatness problem can also be resolved by the inflationary model, which predicts that today the Universe should be very close to be flat, even if it was not so flat in early stages of its evolution. On the other hand, the NGI predicts that in a homogeneous and isotropic universe, the exact flatness must be observed in all stages of its evolution. Both predictions are in agreement with present observational data.

The gravitational field $\Phi_{\mu\nu}(x)$ does not differ much from other fields, because it is a field propagated in a nondynamical flat spacetime. The consistency of the NGI requires that some coordinate condition should be fixed, so the resulting theory is no longer covariant with respect to general coordinate transformations. However, the Einstein equation written in terms of $\eta_{\mu\nu}$ and $\Phi_{\mu\nu}$, and supplemented by (7), is Lorentz covariant.

The disadvantages of the NGI are the following: The actual metric $\eta_{\mu\nu}$ is unobservable, only the effective metric $g_{\mu\nu}$ can be measured, at least if the equivalence principle is exact. The Einstein equation seems very complicated when written in terms of $\Phi_{\mu\nu}(x)$ and $\eta_{\mu\nu}$. The action for a particle in a gravitational field, given by (4) and (6), in the NGI also seems more complicated than in the conventional, geometrical interpretation.

However, if some of the alternative theories of gravity is more appropriate than the theory based on the Einstein equation, it is possible that the equivalence principle is not exact and that the correct equation of motion is not so complicated when written in terms of $\eta_{\mu\nu}$, $\Phi_{\mu\nu}(x)$, and possibly some additional dynamical fields.

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