Jet Physics at Two-Loop Accuracy

T. Gehrmann

Institut für Theoretische Physik, RWTH Aachen, D-52056 Aachen, Germany

Current phenomenological studies of jet observables at colliders are clearly limited by the theoretical uncertainties inherent in the next-to-leading order QCD description. We discuss the recent progress made towards the calculation of QCD corrections to jet observables at the next-to-next-to-leading order in QCD and highlight future perspectives and yet open issues.

1. Introduction

Jet production observables are among the most sensitive probes of QCD at high energy colliders, where they are used for example to determine the strong coupling constant. At present, the interpretation of jet production data within perturbative QCD is restricted to next-to-leading order (NLO) calculations, with theoretical uncertainties considerably larger than current experimental errors. Going beyond NLO calculations offers moreover a more accurate matching of theoretical and experimental jet definitions and a more detailed modelling of the hadronic final state [1]. The extension of jet calculations to NNLO requires three ingredients: the two-loop corrections to multi-leg amplitudes, the single unresolved limits of one-loop amplitudes and the double unresolved limits of tree amplitudes. Finally, all these contributions have to be combined together into a program for the numerical computation of jet observables from NNLO parton level cross sections. In this talk, I review recent progress made on these subjects as well as the currently open issues.

2. Virtual Two-Loop Corrections

Within dimensional regularization, the large number of different integrals appearing in multi-loop calculations can be reduced to a small number of so-called master integrals by using integration-by-parts (IBP) identities [2, 3]. These identities exploit the fact that the integral over the total derivative of any of the loop momenta vanishes in dimensional regularization.

For integrals involving more than two external legs, another class of identities exists due to Lorentz invariance. These Lorentz invariance identities [4] rely on the fact that an infinitesimal Lorentz transformation commutes with the loop integrations, thus relating different integrals. The common origin of IBP and LI identities is the Poincare invariance of loop integrals within dimensional regularization, as was pointed out by J.J. van der Bij at this conference. Using integration-by-parts and Lorentz invariance identities, all two-loop Feynman amplitudes for $2 \rightarrow 2$ scattering or $1 \rightarrow 3$ decay processes can be expressed as linear combinations of a small number of master integrals, which have to be computed by some different method. Explicit reduction formulae for on-shell two-loop four-point integrals were derived in [3]. Computer algorithms for the automatic reduction of all two-loop four-point integrals were described in [3, 4].

The master integrals relevant to $2 \rightarrow 2$ scattering or $1 \rightarrow 3$ decay processes are massless, scalar two-loop four-point functions with all legs on-shell or a single leg off-shell. Several techniques for the computation of those functions have been proposed in the literature, such as the application of a Mellin-Barnes transformation to all propagators, which was used successfully to compute the on-shell planar double box integral [3, 4], the on-shell non-planar double box integral [3] and two double box integrals with one leg off-shell [3].
Figure 1. Computer Algebra for analytic evaluation of two-loop matrix elements.

Most recently, the same method was used to derive the on-shell planar double box integral with one internal mass scale [11] as well as the high energy limit of the on-shell planar triple box integral [12].

A method for the analytic computation of master integrals avoiding the explicit integration over the loop momenta is to derive differential equations in internal propagator masses or in external momenta for the master integral, and to solve these with appropriate boundary conditions. The computation of master integrals from differential equations proceeds as follows [4]. Carrying out the derivative with respect to an external invariant on the master integral of a given topology, one obtains a linear combination of a number of more complicated integrals, which can however be reduced to the master integral itself plus simpler integrals by applying the reduction methods discussed above. As a result, one obtains an inhomogeneous linear first order differential equation in each invariant for the master integral. The inhomogeneous term in these differential equations contains only topologies simpler than the topology under consideration, which are considered to be known if working in a bottom-up approach. The master integral is then obtained by matching the general solution of its differential equation to an appropriate boundary condition.

Using the differential equation technique, one of the on-shell planar double box integrals [13] as well as the full set of planar and non-planar off-shell double box integrals [14] were derived. The computer algebra structures applied in the computation of the master integrals from the differential equations are displayed in the right hand column of Figure 1. The differential equation approach has recently been extended to phase space
integrals [15].

A strong check on all these computations of master integrals is given by the completely numerical calculations of [16], which are based on an iterated sector decomposition to isolate the infrared pole structure. The methods of [16] were applied to confirm all of the above-mentioned calculations.

A third approach, which avoids the reduction to master integrals, has been presented in [17]. In this approach, all integrals appearing in the two-loop amplitudes are related to higher transcendental functions, which can be expanded in terms of nested harmonic sums.

The two-loop four-point functions with all legs on-shell can be expressed in terms of Nielsen’s polylogarithms [18, 19]. In contrast, the closed analytic expressions for two-loop four-point functions with one leg off-shell contain two new classes of functions: harmonic polylogarithms [20] and two-dimensional harmonic polylogarithms (2dHPL’s) [21]. Accurate numerical implementations for these functions [21] are available.

2.1. 2 → 2 Processes with all legs on-shell

With the explicit solutions of the integration-by-parts and Lorentz-invariance identities for on-shell two-loop four-point functions and the corresponding master integrals, all necessary ingredients for the computation of two-loop corrections to 2 → 2 processes with all legs on-shell are now available. The generic structure of such a calculation is outlined in the left hand column of Figure 1. In fact, only half a year elapsed between the completion of the full set of master integrals and the calculation of the two-loop QED corrections to Bhabha-scattering [22]. Subsequently, results were obtained for the two-loop QCD corrections to all parton-parton scattering processes [23]. For gluon-gluon scattering, the two-loop helicity amplitudes have also been derived [24]. Moreover, two-loop corrections were derived to processes involving two partons and two real photons [25]. Since the gluon fusion into photons has a vanishing tree level amplitude, these results form part of the NLO corrections to photon pair production [26], yielding a sizable correction.

Finally, light-by-light scattering in two-loop QED and QCD was considered in [27]. The results for the two-loop QED matrix element for Bhabha scattering [22] were used in [28] to extract the single logarithmic contributions to the Bhabha scattering cross section.

2.2. 2 → 2 Processes with one off-shell leg

Using the two-loop master integrals with one off-shell leg [14], the two-loop QCD matrix element for e⁺e⁻ → 3 jets [29] and the corresponding helicity amplitudes [30] were computed following the reduction procedure depicted in Figure 4. The infrared pole structure of these results agrees with the prediction [31] obtained from an infrared factorization formula.

An independent confirmation of part of these results was performed in [32], where two of the seven colour factors (corresponding to the terms proportional to n_f) of the two-loop helicity amplitudes for e⁺e⁻ → 3 jets were derived using the nested sum method of [17]. Processes related to e⁺e⁻ → 3 jets by crossing symmetry are (2 + 1)-jet production in deep inelastic ep scattering and vector-boson-plus-jet production at hadron colliders. The analytic continuation of the e⁺e⁻ → 3 jets two-loop helicity amplitudes to the kinematic regions relevant for these scattering processes has been derived in [33].

3. Real Corrections

Besides the two-loop virtual corrections, a NNLO calculation of jet observables has to include the contributions from single unresolved (soft or collinear) real radiation from one-loop processes as well as from double unresolved real radiation at tree level. Only after summing all these contributions (and including terms from the renormalization of parton distributions for processes with partons in the initial state), do the divergent terms cancel among one another. The factorization properties of both the one-loop, one-unresolved-parton contribution [34] and the tree-level, two-unresolved-parton contributions [35] have been studied, but a systematic procedure for isolating the infrared singularities has so far been
Figure 2. Structure of NNLO parton level Monte Carlo programme.

established only for the one-loop, one-unresolved-parton processes. Although this is still an open and highly non-trivial issue, significant progress is anticipated in the near future.

4. Numerical Implementation

Prior to their implementation into a numerical program, it is needed to analytically extract the infrared pole terms from the one-loop, one-unresolved-parton and two-unresolved-parton contributions. At NLO, two types of methods have been used very successfully in the past. The phase space slicing method [36] divides up the final state phase space into resolved and unresolved regions; the infrared subtraction terms are then integrated only over the unresolved regions. This method avoids overcounting of singular contributions, the required integrals over restricted phase space regions can however be very involved beyond NLO. The subtraction method [37] integrates the subtraction terms over the full phase space. The construction of the subtraction term requires in this case great care to avoid overcounting problems. In general, algorithms based on subtraction are more efficient numerically.

The remaining finite terms must then be combined into a numerical program implementing the experimental definition of jet observables and event-shape variables. The sketch of such a programme to compute $e^+e^- \rightarrow 3j$ at NNLO is given in Figure 2.

Programs to compute processes with initial state hadrons involve the additional complication of initial state singularities, which have to
be absorbed into the NNLO parton distributions. Lacking the full expressions for the splitting functions at this order, these are not yet available at present, work on them is however well advanced [38].

A first calculation involving the features of NNLO jet calculations was presented for the case of photon-plus-one-jet final states in electron-positron annihilation in [39], thus demonstrating the feasibility of this type of calculations. A prerequisite for such a numerical program computing $n$ jet final states is a stable and efficient next-to-leading order programme for the processes yielding $n + 1$ jet final states. For the processes of highest phenomenological interest, these are already available: $e^+e^- \rightarrow 4j$ [10], $ep \rightarrow (3 + 1)j$ [11], $pp \rightarrow 3j$ [12], $pp \rightarrow V + 2j$ [13].

5. Conclusions and Outlook

Considerable progress has been made in the last two years (in fact since the last “Loops and Legs”- and “RADCOR”-conferences) towards the computation of jet observables at NNLO in QCD. In particular, new methods have been developed for the calculation of two-loop virtual corrections to four-point scattering amplitudes. As a result, the two-loop virtual corrections relevant to all phenomenologically important processes in QCD and QED are now known.

These form however only part of the full calculation required at NNLO accuracy, which also has to take into account contributions from one-loop single-unresolved radiation and tree-level double unresolved radiation processes. While appropriate subtraction terms for these processes have been known for quite some time, their analytic integration (required for the cancellation of infrared poles in physical jet observables) is still an unsolved problem. Once this obstacle has been overcome, the remaining finite parts can be implemented into a numerical programme to compute NNLO jet observables.

Acknowledgement

I wish to thank Ettore Remiddi, Nigel Glover, Lee Garland and Thanos Koukoutsakis for a pleasant and fruitful collaboration on the topics discussed in this talk.

REFERENCES

[1] E.W.N. Glover, these proceedings.
[2] G. ’t Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.
[3] F.V. Tkachov, Phys. Lett. 100B (1981) 65; K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. B192 (1981) 159.
[4] T. Gehrmann and E. Remiddi, Nucl. Phys. B580 (2000) 485.
[5] V.A. Smirnov and O.L. Veretin, Nucl. Phys. B566 (2000) 469; C. Anastasiou, E.W.N. Glover and C. Oleari, Nucl. Phys. B575 (2000) 416; B585 (2000) 763(E); C. Anastasiou, T. Gehrmann, C. Oleari, E. Remiddi and J.B. Tausk, Nucl. Phys. B580 (2000) 577.
[6] S. Laporta, Int. J. Mod. Phys. A15 (2000) 5087.
[7] V.A. Smirnov, Phys. Lett. B460 (1999) 397.
[8] C. Anastasiou, J.B. Tausk and M.E. Tejeda-Yeomans, Nucl. Phys. B (Proc. Suppl.) 89 (2000) 262.
[9] J.B. Tausk, Phys. Lett. B469 (1999) 225.
[10] V.A. Smirnov, Phys. Lett. B491 (2000) 130; B500 (2001) 330.
[11] V.A. Smirnov, Phys. Lett. B524 (2002) 129.
[12] V.A. Smirnov, hep-ph/0209193 and these proceedings.
[13] T. Gehrmann and E. Remiddi, Nucl. Phys. B (Proc. Suppl.) 89 (2000) 251.
[14] T. Gehrmann and E. Remiddi, Nucl. Phys. B601 (2001) 248; B601 (2001) 287.
[15] C. Anastasiou and K. Melnikov, hep-ph/0207004 and these proceedings.
[16] T. Binoth and G. Heinrich, Nucl. Phys. B585 (2000) 741 and these proceedings.
[17] S. Moch, P. Uwer and S. Weinzierl, J. Math. Phys. 43 (2002) 3363; S. Weinzierl, Comput. Phys. Commun. 145 (2002) 357.
[18] N. Nielsen, *Der Eulersche Dilogarithmus und seine Verallgemeinerungen*, Nova Acta Leopoldina (Halle) 90 (1909) 123; L. Lewin, *Polylogarithms and Associated Functions* (North Holland, Amsterdam 1981); K.S.
Kölbig, SIAM J. Math. Anal. 17 (1986) 1232.
[19] K.S. Kölbig, J.A. Mignaco and E. Remiddi, BIT 10 (1970) 38.
[20] E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys. A15 (2000) 725.
[21] T. Gehrmann and E. Remiddi, Comput. Phys. Commun. 141 (2001) 296; 144 (2002) 200.
[22] Z. Bern, L. Dixon and A. Ghinculov, Phys. Rev. D63 (2001) 053007.
[23] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, Nucl. Phys. B601 (2001) 318; B601 (2001) 347; B605 (2001) 486; E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, Nucl. Phys. B605 (2001) 467.
[24] Z. Bern, A. De Freitas and L. Dixon, JHEP 0203 (2002) 018.
[25] Z. Bern, A. De Freitas and L. J. Dixon, JHEP 0109 (2001) 037 and these proceedings; C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, Nucl. Phys. B629 (2002) 255.
[26] Z. Bern, L. Dixon and C. Schmidt, hep-ph/0206194 and these proceedings.
[27] Z. Bern, A. De Freitas, L.J. Dixon, A. Ghinculov and H.L. Wong, JHEP 0111 (2001) 031.
[28] E.W.N. Glover, J.B. Tausk and J.J. van der Bij, Phys. Lett. B516 (2001) 33.
[29] L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, Nucl. Phys. B627 (2002) 107.
[30] L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, Nucl. Phys. B642 (2002) 227.
[31] S. Catani, Phys. Lett. B427 (1998) 161; G. Sterman and M.E. Tejeda-Yeomans, hep-ph/0210130.
[32] S. Moch, P. Uwer and S. Weinzierl, hep-ph/0207043 and these proceedings.
[33] T. Gehrmann and E. Remiddi, Nucl. Phys. B640 (2002) 379.
[34] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, Nucl. Phys. B425 (1994) 217; D.A. Kosower, Nucl. Phys. B552 (1999) 319; D.A. Kosower and P. Uwer, Nucl. Phys. B563 (1999) 477; Z. Bern, V. Del Duca and C.R. Schmidt, Phys. Lett. B445 (1998) 168; Z. Bern, V. Del Duca, W.B. Kilgore and C.R. Schmidt, Phys. Rev. D60 (1999) 116001; S. Catani and M. Grazzini, Nucl. Phys. B591 (2000) 435.
[35] J.M. Campbell and E.W.N. Glover, Nucl. Phys. B527 (1998) 264; S. Catani and M. Grazzini, Phys. Lett. B446 (1999) 143; Nucl. Phys. B570 (2000) 287; F.A. Berends and W.T. Giele, Nucl. Phys. B313 (1989) 595.
[36] K. Fabricius, I. Schmitt, G. Kramer and G. Schierholz, Z. Phys. C11 (1981) 315; W.T. Giele and E.W.N. Glover, Phys. Rev. D46 (1992) 1980.
[37] R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B178 (1981) 421; S. Catani and M.H. Seymour, Nucl. Phys. B485 (1997) 291.
[38] S. Moch, J. A. Vermaseren and A. Vogt, hep-ph/0209100 and these proceedings.
[39] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, Phys. Lett. B414 (1997) 354; A. Gehrmann-De Ridder and E.W.N. Glover, Nucl. Phys. B517 (1998) 269.
[40] L.J. Dixon and A. Signer, Phys. Rev. Lett. 78 (1997) 811; Phys. Rev. D56 (1997) 4031; Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. 79 (1997) 3604; J.M. Campbell, M.A. Cullen and E.W.N. Glover, Eur. Phys. J. C9 (1999) 245; S. Weinzierl and D.A. Kosower, Phys. Rev. D60 (1999) 054028.
[41] Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. 87 (2001) 082001.
[42] Z. Nagy, Phys. Rev. Lett. 88 (2002) 122003.
[43] J. Campbell and R.K. Ellis, Phys. Rev. D65 (2002) 113007.