Calibration of GRB Luminosity Relations with Cosmography

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For the use of Gamma-Ray Bursts (GRBs) to probe cosmology in a cosmology-independent way, a new method has been proposed to obtain luminosity distances of GRBs by interpolating directly from the Hubble diagram of SNe Ia, and then calibrating GRB relations at high redshift. In this paper, following the basic assumption in the interpolation method that objects at the same redshift should have the same luminosity distance, we propose another approach to calibrate GRB luminosity relations with cosmographic fitting directly from SN Ia data. In cosmography, there is a well-known fitting formula which can reflect the Hubble relation between luminosity distance and redshift with cosmographic parameters which can be fitted from observation data. Using the Cosmographic fitting results from the Union set of SNe Ia, we calibrate five GRB relations using GRB sample at $z \leq 1.4$ and deduce distance moduli of GRBs at $1.4 < z \leq 6.6$ by generalizing above calibrated relations at high redshift. Finally, we constrain the dark energy parameterization models of the Chevallier-Polarski-Linder (CPL) model, the Jassal-Bagla-Padmanabhan (JBP) model and the Alam model with GRB data at high redshift, as well as with the Cosmic Microwave Background radiation (CMB) and the baryonic acoustic oscillation (BAO) observations, and we find the $\Lambda$CDM model is consistent with the current data in 1-$\sigma$ confidence region.

Keywords: Gamma rays: bursts; Cosmography

1. Introduction

Since an intrinsic relation between the peak luminosity and the shape of the light curve of SNe Ia has been found, SNe Ia has now been taken as near-ideal standard candles for measuring the geometry and dynamics of the universe. However, the maximum redshift of the SNe Ia which we can currently use is only about 1.7. On the other hand, the redshift of the last scattering surface of the cosmic microwave
background (CMB) is at $z = 1091.3$.\cite{2}

Recently, Gamma-Ray Bursts (GRBs) were proposed to be a complementary probe to SNe Ia and CMB to explore the early universe. As the most intense explosions observed in the universe so far, GRBs are likely to occur in high-redshift range up to at least $z = 8.2$.\cite{31} Moreover, there are several luminosity relations of GRBs between the spectral and temporal properties which have been extensive discussed, such as the isotropic energy ($E_{\text{iso}}$) - peak spectral energy ($E_{\text{peak}}$) relation,\cite{5} the luminosity ($L$) - spectral lag ($\tau_{\text{lag}}$) relation,\cite{15} the $L$ - variability ($V$) relation,\cite{7,8} the $L$ - $E_{\text{peak}}$ relation,\cite{9,10} the $L$ - minimum rise time ($\tau_{\text{RT}}$) relation,\cite{11} and the collimation-corrected energy ($E_{\gamma}$) - $E_{\text{peak}}$ relation,\cite{12} as well as several multiple relations such as the $E_{\text{iso}}$ - $E_{\text{peak}}$ - $t_{b}$ relation,\cite{13} where $t_{b}$ is the break time of the optical afterglow light curves; the $L$ - $E_{\text{peak}}$ - $T_{0.45}$ relation,\cite{23} where $T_{0.45}$ is the rest-frame “high-signal” timescale; and the $L$ - $E_{\text{peak}}$ - $\tau_{\text{lag}}$ (or $\tau_{\text{RT}}$) relation.\cite{15}

Many authors have made use of GRB luminosity indicators as standard candles at very high redshift beyond SNe Ia redshift range for cosmological research.\cite{16,18,17,19,12,20,21,23,24,13,26,27,28,30,31,32,33,34,35,36,37,38,39} Due to the lack of the sample at low redshift which are cosmology independent, to calibrate the empirical GRB relations, one usually needs to assume a particular cosmological model with certain model parameters as a priori. As a result, the so-called circularity problem could prevent the direct use of GRBs for cosmology.\cite{29} Many of works treat the circularity problem with statistical approach which carried out a simultaneous fit of the parameters in the calibration curves and the cosmology.\cite{16,22,41,42} However, the circularity problem can not be circumvented completely by means of the statistical approaches for an input cosmological model is still required in doing the joint fitting.

More recently, Liang et al. proposed a new method to calibrate GRB luminosity relations in a cosmological model-independent way.\cite{43} The motivation of this calibration method is that objects at the same redshift should have the same luminosity distance in any cosmology. Thus the luminosity distance of a GRB at a given redshift can be obtained by interpolating directly from the Hubble diagram of SNe Ia, therefore GRB relations can be calibrated without assuming a particular cosmological model and the Hubble diagram of GRBs has been constructed. Following this cosmology-independent GRB calibration directly from SNe Ia, the derived GRB Hubble diagram can be used to constrain cosmological models at high redshift avoiding circularity problem\cite{11,15,16,17,44,45,46,47,48,49,50,51} Capozziello & Izzo firstly used two GRB relations calibrated with the so-called Liang method to derive the related cosmography parameters which related to the derivatives of the scale factor.\cite{15} Liang et al. combined the updated distance moduli of GRBs obtained by the interpolating method with the joint data to find the contribution of GRBs to the joint cosmological constraints in the confidence regions of cosmological parameters, and reconstructed the acceleration history of the universe with the distance moduli of SNe Ia and GRBs.\cite{50} On the other hand, besides the interpolation method, the
luminosity distance of a GRB can also be obtained directly from SNe Ia data by other mathematical approach. Liang & Zhang has proposed another approach to calibrate GRB relations by using an iterative procedure which is a non-parametric method in a model independent manner to reconstruct the luminosity distance at any redshift from SNe Ia. Similar to the interpolation method, Cardone et al. constructed an updated GRBs Hubble diagram calibrated by local regression from SNe Ia. Kodama et al. has proposed that the $L - E_{\text{peak}}$ relation can be calibrated with one empirical formula fitted from the luminosity distance of SNe Ia. However, according to the formula fitting approach, various possible formula can be fitted from the SNe Ia data which could give different calibration results of GRB relations. As the cosmological constraints from GRBs are sensitive to GRBs calibration results, and the fitting procedure depends seriously on the choice of the formula, the reliability of this method should be tested carefully. In other words, we should find one certain formula which is totally independent of any cosmological models and could accurately evaluate the Hubble relation.

In Cosmography, there is a well-known formula reflecting the Hubble relation between luminosity distance and redshift which can be extracted directly from basic cosmological principles and observation data, with cosmography parameters (the deceleration, jerk and snap parameters: $q$, $j$, and $s$) which are only related to the derivatives of the scale factor without any priori assumption on the underlying cosmological model. Recently, several authors have already used the cosmographic parameters fitting from SNe Ia and/or GRBs dataset to constrain cosmological parameters. If viewing this point from another angle, the cosmographic formula can be considered as a perfect fitting function to calibrate the GRB relations using SNe Ia data, as long as we take the same assumption that objects at the same redshift should have the same luminosity distance in any cosmology. In this paper, instead of the interpolation method using in Ref. we propose another new approach to calibrate GRB luminosity relations with cosmographic fitting from SNe Ia data. The structure of this paper is arranged as follows. In section 2 we give a brief review of the cosmographic Hubble relation between luminosity distance and redshift. In section 3, we calibrate five GRB luminosity relations with cosmographic fitting results from SNe Ia data. In section 4, we construct the Hubble diagram of GRBs obtained by using the cosmographic methods and constrain the dark energy parameterization models of the Chevallier-Polarski-Linder (CPL) model, the Jassal-Bagla-Padmanabhan (JBP) model, and the Alam model with GRB data at high redshift, as well as with the Cosmic Microwave Background radiation (CMB) and the baryonic acoustic oscillation (BAO) observations. Conclusions and discussions are given in section 5.
2. Cosmographic Hubble relation between luminosity distance and redshift

A completely new cosmology branch – cosmography, in which framework cosmology is pure kinematics and completely independent of the underlying dynamics governing the evolution of the universe has been introduced since 1970s. The only assumption is the basic symmetry principles (the cosmological principle) that the universe can be described by the Friedmann-Robertson-Walker metric. By means of a Taylor series expansion of the luminosity distance, Visser gave a formulation that the luminosity distance can be expressed as a power series in the redshift up to the forth order:

\[ d_L(z) = cH_0^{-1} \left\{ z + \frac{1}{2} (1 - q_0)z^2 - \frac{1}{6} \left( 1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2a^2(t_0)} \right) z^3 + \right. \\
\left. + \frac{1}{24} \left[ 2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0j_0 + s_0 + \frac{2kc^2(1 + 3q_0)}{H_0^2a^2(t_0)} \right] z^4 + \ldots \right\} \] (1)

where the coefficients of the expansion are the so-called cosmographic parameters, Hubble parameters \( H \equiv \dot{a}(t)/a(t) \), deceleration parameters \( q \equiv -H^2\ddot{a}(t)/a(t) \), jerk parameters \( j \equiv H^{-3}\dot{a}^{(3)}(t)/a(t) \), and snap parameters \( s \equiv H^{-4}\dot{a}^{(4)}(t)/a(t) \), which related to the scale factor \( a(t) \) and its higher order derivatives (the subscript “0” indicates the present value of the parameters). Obviously pure cosmography by itself will not predict anything about the scale factor \( a(t) \) and we have to turn to the observational data such as SNe Ia to infer the history of the scale factor \( a(t) \) and some important information about expanding history of our universe.

In order to avoid problems with the convergence of the series for the highest redshift objects as well as to control properly the approximation induced by truncations of the expansions, Cattoen & Visser pointed out that it is useful to recast \( d_L \) as a function of an improved parameter \( y = z/(1 + z) \) and constrained the cosmographic parameters using SNe Ia data. In such a way, being \( z \in (0, \infty) \) mapped into \( y \in (0, 1) \), the luminosity distance at the fourth order in the \( y \)-parameter becomes:

\[ d_L(y) = \frac{c}{H_0} \left\{ y - \frac{1}{2} (q_0 - 3)y^2 + \frac{1}{6} \left[ 12 - 5q_0 + 3q_0^2 - (j_0 + \Omega_0) \right] y^3 + \frac{1}{24} \left[ 60 - 7j_0 - 105q_0 - 32q_0j_0 + 10q_0\dot{\Omega}_0 + 21q_0^2 - 15q_0^3 + s_0 \right] y^4 + \mathcal{O}(y^5) \right\} , \] (2)

where \( \Omega_0 = 1 + kc^2/H_0^2a^2(t_0) \) is the total energy density. For the flat universe, \( \Omega_0 = 1 \). The luminosity distance as the logarithmic Hubble relations can be expressed as:

\[ \ln \left[ \frac{d_L(y)}{\text{Mpc}} \right] = \ln y + \ln \left[ \frac{c}{H_0} \right] - \frac{1}{2} (q_0 - 3)y + \frac{1}{24} \left[ 21 - 4(j_0 + \Omega_0) + q_0(9q_0 - 2) \right] y^2 \\
+ \frac{1}{24} \left[ 15 + 4\Omega_0(q_0 - 1) + j_0(8q_0 - 1) - 5q_0 + 2q_0^2 - 10q_0^3 + s_0 \right] y^3 + \mathcal{O}(y^4), \] (3)

therefore the distance modulus can be given by

\[ \mu(y) = 25 + \frac{5}{\ln 10} \ln \left[ \frac{d_L(y)}{\text{Mpc}} \right]. \] (4)
Recently, Vitagliano et al. fitted two different truncations (Cosmography I: without the third order term \((y^3)\); Cosmography II: with the third order term \((y^3)\)) of the above expansion with the SNe Ia and GRB datasets. With a flat universe \((\Omega_0 = 1)\) prior, the fitting results showed that the Union dataset gave more stringent constraints on the parameters. For Cosmography I, the cosmographic fitting results with the Union dataset are
\[
q_0 = -0.58 \pm 0.24, \quad j_0 + \Omega_0 = 0.91 \pm 2.21,
\]
and for Cosmography II, the cosmographic fitting results with the Union dataset are
\[
q_0 = -0.50 \pm 0.55, \quad j_0 + \Omega_0 = -0.26 \pm 9.0, \quad s_0 = -4.13 \pm 129.79.
\]

It is noted that the Union dataset of 307 SNe Ia didn’t include the 90 SNe Ia data from CfA3 due to their extremely low redshift \((z < 0.1)\), which would not affect the calibrated results for GRB luminosity relations at \(z \geq 0.17\). With the above cosmographic fitting results from the Union dataset, the cosmographic Hubble relation can be considered as a perfect function to deduce the distance moduli of GRBs directly from SNe Ia data. In the next section, we will deduce the distance moduli of GRBs and then calibrate the GRB luminosity relations with Cosmography I and II respectively.

3. The Calibration of the Luminosity Relations of Gamma-Ray Bursts

We adopt the 69 GRBs provided in Ref. [30] as our sample for calibrating the GRB luminosity/energy relations. We first deduce the distance moduli of GRBs at \(z \leq 1.4\) within our sample. Then using these deduced distance moduli and the redshifts of corresponding GRBs, we calibrate five GRB luminosity/energy relations i.e., the \(\tau_{\text{lag}} - L\) relation, the \(V - L\) relation, the \(L - E_{\text{peak}}\) relation, the \(E_{\gamma} - E_{\text{peak}}\) relation and the \(\tau_{\text{RT}} - L\) relation. These luminosity relations of GRBs can be generally written in the form
\[
\log y = a + b \log x, \quad (5)
\]
where \(a\) and \(b\) are the intercept and slope of the relation respectively, in addition we introduce \(c\) as the linear correlation coefficient of the relation which will be calculated together with \(a\) and \(b\) below; \(y\) is the luminosity \((L/\text{ergs}^{-1}\) or energy \(E_{\gamma}/\text{erg})\); \(x\) is the GRB parameters measured in the rest frame, e.g., \(\tau_{\text{lag}}(1 + z)^{-1}/(0.1 \text{ s})\), \(V(1 + z)/0.02\), \(E_{\text{peak}}(1 + z)/(300 \text{ keV})\), \(\tau_{\text{RT}}(1 + z)^{-1}/(0.1 \text{ s})\), for the corresponding relations above. For the \(x\) values, we adopt the data from Ref. [30] for the \(y\) values, we drive them with the adjusted luminosity distance of GRBs calculated with cosmographic fitting method (Cosmography I and II). The isotropic luminosity of a burst is calculated by
\[
L = 4\pi d_L^2 P_{\text{bolo}}, \quad (6)
\]
where \( P_{\text{bolo}} \) is the bolometric flux of gamma-rays in the burst and \( d_L \) is the luminosity distance of the burst. The isotropic energy released from a burst is given by

\[
E_{\text{iso}} = 4\pi d_L^2 S_{\text{bolo}} (1 + z)^{-1},
\]

where \( S_{\text{bolo}} \) is the bolometric fluence of gamma-rays in the burst at redshift \( z \). The total collimation-corrected energy is then calculated by

\[
E_{\gamma} = F_{\text{beam}} E_{\text{iso}},
\]

where the beaming factor, \( F_{\text{beam}} = (1 - \cos \theta_{\text{jet}}) \); and the value of the jet opening angle \( \theta_{\text{jet}} \) is related to the jet break time \( (t_b) \) and the isotropic energy for an Earth-facing jet, \( E_{\gamma,\text{iso,52}} = E_{\gamma,\text{iso}} / 10^{52}\text{erg} \). When calculating \( E_{\gamma,\text{iso}} \), we also use the cosmographic fitting method from SNe Ia to avoid the circularity problem.

We determined the values of the intercept \( (a) \) and the slope \( (b) \) with their 1-\( \sigma \) uncertainties with the same regression method (the bisector of the two ordinary least-squares) used in Ref. 30, 43. The calibrate results for Cosmography I and II are summarized in Table 1. From Table 1, we find that the calibration results obtained using two cosmographic fitting (Cosmography I and II) are fully consistent with each other. We also find that results obtained by the cosmographic methods differ only slightly from, but still fully consistent with those calibrated by using the interpolation method and the iterative procedure with the same GRB sample (see details in there 43, 44).

### Table 1

Calibration results (for \( a=\)intercept, \( b=\)slope, \( c=\)correlation coefficient) with their 1-\( \sigma \) uncertainties, for the five GRB luminosity/energy relations within the sample at \( z \leq 1.4 \), using two cosmographic fitting results (Cosmography I and II) directly from SNe Ia data.

| Relation          | Cosmography I |   | Cosmography II |   |
|-------------------|---------------|---|----------------|---|
| \( \tau_{\text{lag}}-L \) | \( \tau_{\text{lag}} \) | \( L \) | \( E_{\text{peak}} \) | \( E_{\gamma}-E_{\text{peak}} \) |
| \( \tau_{\text{lag}} \) | 52.13±0.10 | -1.10±0.13 | -0.89 | 52.21±0.11 | -1.13±0.14 | -0.88 |
| \( L \) | 52.47±0.13 | 2.02±0.27 | 0.64 | 52.54±0.13 | 2.06±0.27 | 0.65 |
| \( E_{\text{peak}} \) | 52.14±0.09 | 1.64±0.10 | 0.89 | 52.21±0.09 | 1.67±0.10 | 0.89 |
| \( E_{\gamma}-E_{\text{peak}} \) | 50.83±0.06 | 1.87±0.11 | 0.95 | 50.88±0.07 | 1.93±0.10 | 0.95 |

4. The Hubble diagram of gamma-ray bursts

With the cosmographic fitting results (Cosmography I or II), the moduli for the 27 GRBs at \( z \leq 1.4 \) can be directly obtained from SNe data, therefore we calibrate GRB luminosity relations in a completely cosmology-independent way. Furthermore, if assuming that GRB luminosity relations do not evolve with redshift, we are able to obtain the luminosity \( (L) \) or energy \( (E_{\gamma}) \) of each burst at high redshift \( (z > 1.4) \) by utilizing the calibrated results from Cosmography. Consequently, the corresponding luminosity distance \( (d_L) \) can be derived from Eq.(12) ~ Eq.(14) and
the corresponding distance modulus can be calculated as \( \mu = 5 \log(d/L\text{Mpc}) + 25 \). The uncertainty of the value of the luminosity or energy deduced from a GRB relation is

\[
\sigma_{\log L}^2 = \sigma_a^2 + \sigma_b^2 + (0.4343\sigma_x^2 x^2 + \sigma_{\text{sys}}^2, \tag{9}
\]

where \( \sigma_a \), \( \sigma_b \) and \( \sigma_x \) are 1-\( \sigma \) uncertainty of the intercept, the slope and the GRB measurable parameters, and \( \sigma_{\text{sys}} \) is the systematic error in the fitting that accounts for the extra scatter of the luminosity relations.\(^{30}\) Note that the uncertainty of modulus for each luminosity indicator depends on whether \( P_{\text{bolo}} \) or \( S_{\text{bolo}} \) is used:

\[
\sigma_{\mu} = [(2.5\sigma_{\log L})^2 + (1.086\sigma_{P_{\text{bolo}}}/P_{\text{bolo}})^2]^{1/2}, \tag{10}
\]

or

\[
\sigma_{\mu} = [(2.5\sigma_{\log E_\gamma})^2 + (1.086\sigma_{S_{\text{bolo}}}/S_{\text{bolo}})^2 + (1.086\sigma_{F_{\text{beam}}}/F_{\text{beam}})^2]^{1/2}. \tag{11}
\]

For five luminosity indicators, each burst will have up to five estimated distance moduli, we hence use the same method in Ref.\(^{30}\) to obtain the best estimated \( \mu \) for each GRB which is the weighted average of all available distance moduli:

\[
\mu = \left( \sum_{i} \frac{\mu_i}{\sigma_{\mu_i}^2} \right) / \left( \sum_{i} \sigma_{\mu_i}^{-2} \right), \tag{12}
\]

with its uncertainty \( \sigma_{\mu} = \left( \sum_{i} \sigma_{\mu_i}^{-2} \right)^{-1/2} \), where the summations run from 1 to 5 over the five relations used in Ref.\(^{30}\) with available data. Until now we have ultimately obtained the 42 GRB moduli at \( z > 1.4 \) by utilizing the five relations calibrated
with the sample at \( z \leq 1.4 \) using the fitting method. We have plotted the Hubble diagram of 307 SNe Ia and the 69 GRBs obtained using the cosmographic methods in Fig 1.

Constraints from the GRB data can be obtained consequently by fitting the distance moduli \( \mu(z) \). The \( \chi^2 \) value of the observed distance moduli can be calculated by

\[
\chi^2_{\mu} = \sum_{i=1}^{N} \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma_{\mu,i}^2},
\]

where \( \mu_{\text{obs}}(z_i) \) is the observed distance modulus for the GRBs at redshift \( z_i \) with its error \( \sigma_{\mu,i} \); \( \mu(z_i) \) is the theoretical value of distance modulus from a dark energy model. As mentioned above, high-redshift GRBs are rare database for constraining cosmological parameters. Since those 42 GRBs’ moduli (1.4 < \( z \) ≤ 6.6) calculated above are completely cosmological model independent, we thus utilize this dataset together with CMB and BAO observations to constrain specified cosmological models.

For the CMB observation, we choose one shift parameter \( R \) to limit the model parameters. In a flat universe, it can be expressed as

\[
R = \Omega_M^{1/2} \int_0^{z_{ls}} \frac{dz}{E(z)},
\]

where the last scattering redshift \( z_{ls} = 1091.3 \) from the 7-year WMAP results, and the observational value \( R = 1.725 \pm 0.018 \). The \( \chi^2_{\text{CMB}} \) value is

\[
\chi^2_{\text{CMB}} = \frac{(R - 1.725)^2}{0.018^2}.
\]

For the BAO observation, the size of baryon acoustic oscillation peak can be used to constrain the cosmological parameters. This peak can be denoted by a parameter \( A \), which can be expressed as

\[
A = \Omega_M^{1/2} E(z_{\text{BAO}})^{-1/3} \left[ \frac{1}{z_{\text{BAO}}} \int_0^{z_{\text{BAO}}} \frac{dz}{E(z)} \right]^{2/3},
\]

where \( z_{\text{BAO}} = 0.35 \). The observational value is \( A = 0.469(n_s/0.98)^{-0.35} \pm 0.017 \). The \( \chi^2_{\text{BAO}} \) value is

\[
\chi^2_{\text{BAO}} = \frac{[A - 0.469(n_s/0.98)^{-0.35}]^2}{0.017^2}.
\]

Here we combine these two probes with the GRBs dataset above by multiplying the likelihood functions. The total \( \chi^2 \) value is

\[
\chi^2_{\text{total}} = \chi^2_{\text{GRB}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}.
\]
In the following, we will show the constraining progress and results for three dark energy parametrization models: CPL model, JBP model and Alam model.

(1) Chevallier-Polarski-Linder (CPL) model\cite{59,58}, in which dark energy with a parametrization EoS:

$$w(z) = w_0 + w_a \frac{z}{1+z}.$$  \hfill (19)

The corresponding luminosity distance for a flat universe is

$$d_L = cH_0^{-1}(1+z) \int_0^z \frac{dz}{E(z)},$$  \hfill (20)

![Figure 2. Contours of likelihood in the \((w_0, w_a)\) plane in the CPL dark energy model for a flat universe, from CMB and BAO observations together with the 42 GRBs data \((z > 1.4)\) obtained by the cosmographic method. The red line is for Cosmography I, and the red plus sign denotes the best-fit values \((w_0 = -0.79, w_a = -0.36)\). The black line is for Cosmography II, and the black plus sign denotes best-fit values \((w_0 = -0.91, w_a = -0.06)\). The contours correspond to 1 and 2-\(\sigma\) confidence regions.]

| CPL Model | JBP Model | Alam Model |
|-----------|-----------|------------|
| Cosmography I | Cosmography II | Cosmography I | Cosmography II | Cosmography I | Cosmography II |
| \(w_0(A_1)\) | \(-0.79_{-0.34}^{+0.32}\) | \(-0.91_{-0.32}^{+0.32}\) | \(-0.74_{-0.51}^{+0.49}\) | \(-1.37_{-0.51}^{+0.53}\) | \(-0.60_{-0.97}^{+0.97}\) |
| \(w_a(A_2)\) | \(-0.36_{-0.16}^{+0.15}\) | \(-0.06_{-0.06}^{+0.06}\) | \(-0.93_{-3.17}^{+3.17}\) | \(2.02_{-3.16}^{+3.16}\) | \(-0.11_{-0.26}^{+0.26}\) |
| \(\Omega_M\) | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| \(H_0\) | 80 | 75 | 80 | 79 | 79 |
| \(\chi^2_{\text{min}}\) | 28.03 | 26.91 | 28.02 | 26.91 | 28.03 | 26.90 |
where
\[ E(z) = \left[ (1 + z)^3 \Omega_M + (1 - \Omega_M)(1 + z)^3(1 + w_0 + w_a) e^{-3w_a z/(1+z)} \right]^{-1/2}. \] (21)

We set \( H_0 \) and \( \Omega_M \) as free parameters and their prior values are 64km s\(^{-1}\)Mpc\(^{-1} \leq H_0 \leq 80\)km s\(^{-1}\)Mpc\(^{-1} \) and 0.23 \leq \Omega_M \leq 0.31 respectively. We also assume this prior in the following analysis. For cosmography I, we find \( \chi^2_{\text{min}} = 28.02 \) at \( \{H_0, \Omega_M\} = \{80\)km s\(^{-1}\)Mpc\(^{-1}, 0.29\)\}, the best fitting values are \( w_0 = -0.74_{-0.53}^{+0.53} \) and \( w_a = -0.93_{-3.53}^{+3.17} \) (1\σ). For cosmography II, we find \( \chi^2_{\text{min}} = 26.91 \) at \( \{H_0, \Omega_M\} = \{79\)km s\(^{-1}\)Mpc\(^{-1}, 0.28\)\}, the best fitting values are \( w_0 = -0.91_{-0.32}^{+0.38} \) and \( w_a = -0.06_{-1.66}^{+1.06} \) (1\σ). Fig. 2 shows the constraints on \( w_0 \) and \( w_a \) parameters for the CPL model. We present the best-fit value of \( w_0 \) and \( w_a \) with 1\σ uncertainties, as well as the best-fit values of \( \Omega_M, H_0, \) and \( \chi^2_{\text{min}} \) for the CPL model in Table 2.

(2) Recently, Jassal, Bagla & Padmanabhan argued that in CPL model some problems will present at high redshifts, they thus proposed a new parametrization EoS (JBP model)\(^{60}\)
\[ w(z) = w_0 + w_a \left( \frac{z}{1+z} \right)^2, \] (22)
which can model a dark energy component that has the same value at lower and higher redshifts, with rapid variation at low \( z \). The corresponding luminosity distance for a flat universe is the same with Eq.(26), and Eq.(27) becomes
\[ E(z) = \left[ (1 + z)^3 \Omega_M + (1 - \Omega_M)(1 + z)^3(1 + w_0 + w_a) e^{-3w_a z/(1+z)} \right]^{1/2}. \] (23)

For cosmography I, we find \( \chi^2_{\text{min}} = 28.02 \) at \( \{H_0, \Omega_M\} = \{80\)km s\(^{-1}\)Mpc\(^{-1}, 0.29\)\}, the best fitting values are \( w_0 = -0.74_{-0.53}^{+0.53} \) and \( w_a = -0.93_{-3.53}^{+3.17} \) (1\σ). For cosmography II, we find \( \chi^2_{\text{min}} = 26.91 \) at \( \{H_0, \Omega_M\} = \{79\)km s\(^{-1}\)Mpc\(^{-1}, 0.25\)\}, the best fitting values are \( w_0 = -1.37_{-0.51}^{+0.52} \) and \( w_a = 2.02_{-3.48}^{+3.16} \) (1\σ). Fig. 3 shows the constraints on \( w_0 \) and \( w_a \) parameters for the JBP model. We present the best-fit value of \( w_0 \) and \( w_a \) with 1\σ uncertainties, as well as the best-fit values of \( \Omega_M, H_0, \) and \( \chi^2_{\text{min}} \) for the JBP model in Table 2.

(3) The third dark energy parametrization model that we consider is\(^{61}\)
\[ w(z) = \frac{1 + z}{3} \frac{A_1 + 2A_2(1 + z)}{\Omega_{\text{DE}}(z)} - 1, \] (24)
where
\[ \Omega_{\text{DE}}(z) = A_1(1 + z) + A_2(1 + z)^2 + 1 - \Omega_M - A_1 - A_2, \] (25)
thus
\[ E(z) = \left[ (1 + z)^3 \Omega_M + A_1(1 + z) + A_2(1 + z)^2 + 1 - \Omega_M - A_1 - A_2 \right]^{1/2}. \] (26)

For cosmography I, we find \( \chi^2_{\text{min}} = 28.03 \) at \( \{H_0, \Omega_M\} = \{79\)km s\(^{-1}\)Mpc\(^{-1}, 0.29\)\}, the best fitting values are \( A_1 = 0.6_{-0.96}^{+0.96} \) and \( A_2 = -0.11_{-0.26}^{+0.26} \) (1\σ). For
cosmography II, we find $\chi^2_{\text{min}} = 26.90$ at \{${H_0, \Omega_M}$\} = \{78$\text{km s}^{-1}\text{Mpc}^{-1}$, 0.26\}, the best fitting values are $A_1 = -0.39^{+0.9}_{-0.87}$ and $A_2 = 0.1^{+0.24}_{-0.23}$ (1$\sigma$). Fig. 4 shows the constraints on $A_1$ and $A_2$ parameters for the Alam model. We present the best-fit value of $A_1$ and $A_2$ with 1$\sigma$ uncertainties, as well as the best-fit values of $\Omega_M$, $H_0$, and $\chi^2_{\text{min}}$, for the Alam model in Table 2.

We can find that when combining the GRBs dataset obtained by cosmographic fitting method with the CMB and BAO observations, the combined data can give more stringent results on different dark energy parametrization models. It is noted that all these results for above models are consistent with the $\Lambda$CDM model in 1-$\sigma$ confidence region.

5. Summary and Discussion

Due to the lack of the GRB sample at low redshift, there has been a so-called circularity problem which can always be an obstacle for applying GRBs data to constrain cosmological parameters. Based on the basic assumption that objects at the same redshift should have the same luminosity distance, Liang et al. proposed a new method to calibrate GRB relations in a completely cosmology-independent way, namely obtaining the distance modulus of a GRB by interpolating from the Hubble diagram of SNe Ia and then calibrate the GRB relations with these calculated distance moduli. There is a well-known fitting formula in cosmography, which can reflect the Hubble relation between luminosity distance and redshift with cosmographic parameters which can be fitted from SNe Ia. In this work, we propose another approach to calibrate GRB luminosity relations with cosmography fitting.

![Fig. 3. Same as Fig. 2, but fitting the JBP model. For Cosmography I: the best-fit values ($w_0 = -0.74, w_a = -0.93$); for Cosmography II: the best-fit values ($w_0 = -1.37, w_a = 2.02$).](image-url)
from SNe Ia data. We adopted the fitting results from the Union set of SNe Ia for
the so-called Cosmography I and II\cite{57} and calibrate five GRB relations by this cos-
mographic fitting method. The calibration results obtained using two cosmographic
fitting are fully consistent with each other. Assuming that GRB luminosity relations
do not evolve with redshift, we obtained the distance modulus of the GRB data at
higher redshift $1.4 < z \leq 6.6$. Note that the circularity problem could be completely
avoided if we apply these GRBs data to constrain cosmological parameters. We thus
constrained three parameterization dark energy models of the CPL, JBP and Alam
models by combining the GRB data at high redshift with the CMB and BAO ob-
servations, and the constraint results are all consistent with the $\Lambda$CDM model in
$1$-$\sigma$ confidence region.

Compared to previous cosmology-independent calibration method, we find that
results obtained by the cosmographic method consistent with those calibrated by
using the interpolation method and the iterative procedure from SNe Ia with the
same GRB sample\cite{43,44}; this situation are consistent and similar with the recent
conclusions that the cosmographic Amati relation agrees in the errors with other
cosmology-independent calibrations\cite{69} and that the calibration coefficients and the
intrinsic scatter actually do not depend on the adopted calibration procedure se-
riously, based on the use of a fiducial model, the cosmographic method and the
local regression.\cite{70} It should be noted that the fitting procedure used in Kodama
et al.\cite{53} depends seriously on the choice of the formula. In Cosmography, the cos-
mographic formula is totally independent of any cosmological models and could

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4}
\caption{Same as Fig. 2, but in the $(A_1, A_2)$ plane in the Alam model for a flat universe. For
Cosmography I: the best-fit values ($A_1 = 0.6, A_2 = -0.11$); for Cosmography II: the best-fit
values ($A_1 = -0.39, A_2 = 0.1$).
}
\end{figure}
accurately evaluate the Hubble relation between luminosity distance and redshift, and can be considered as a perfect fitting function to calibrate the GRB relations using SNe Ia data. The reliability of this method should be more reasonable than the fitting procedure which chooses a formula in arbitrary. However, since the distance moduli of GRBs calculated by the fitting method including the cosmographic method are obviously relevant to all of SN Ia data points for calibration, it will be not appropriate for directly combining these two datasets to constrain the cosmological parameters. But like showing in this work, we can combine the GRBs dataset obtained by cosmographic fitting method with the CMB and BAO observations and the combined data can give more stringent results. As to the interpolation method for deducing one individual GRB, only a few SNe Ia close to this GRB were used. In this case, the measurement error for a single SNe Ia’s modulus may influence the final result of calibration. In the cosmographic method, the full information of SNe Ia data is completely used in the calibration. It should be noted that our approach provides no more accurate cosmological parameter constraints than other works such as the simultaneous fit method. However, the primary motivation of this work is not on improvement of the statistical error, rather on avoiding the circularity problem more clearly in logic.

For comparing GRB to SNe Ia, GRBs are almost immune to dust extinction, whereas SNe Ia observations suffer extinction from the interstellar medium. GRBs can extend the Hubble diagram to much higher redshifts beyond SNe Ia data. Different dark energy models may have very different Hubble diagrams at high redshifts. On the other hand, due to the large statistical scatters of the relations and the small dataset compared to SNe Ia, the contribution of GRBs to the cosmological constraints would not be sufficiently significant at present. However, it should be noted that a single GRB at high redshift will provide more information than a single maximal redshift SNe Ia, and the larger scatter of GRB data is somewhat coming from the observational limit of nowadays technological level, which means the GRB data may eventually turn into good data as our observational capability enhance. It has been found that cosmological constraints would improve substantially with more simulated GRBs expected by future observations through Monte Carlo simulations. With more and more GRBs observed from Fermi Gamma-ray Space Telescope with much smaller scatters, and its combination with the increasing Swift data, GRBs could be used as an additional choice to set tighter constraints on cosmological parameters of dark energy models. At that time when GRB data becomes common used data sample to constrain high redshift cosmology, the so called circularity problem for GRB data will be a key problem to solve. Moreover, it is noted that accurately calibrating the GRB relations is also very important for the GRB theory system itself even without considering the cosmology. We have to note stress again that the high-redshift GRB dataset we obtained here is completely cosmology-independent, it will ultimately fill the data crack between SNe Ia and CMB after more GRBs being observed in the future.
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