We discuss a scenario consisting of an effective 4D theory containing fundamental and composite fields. The strong dynamics sector responsible for the compositeness is assumed to be of extra dimensional origin. In the 4D effective theory the SM fermion and gauge fields are taken as fundamental fields. The scalar sector of the theory resembles a bosonic topcolor in the sense there are two scalar Higgs fields, a composite scalar field and a fundamental gauge-Higgs unification scalar. A detailed analysis of the scalar spectrum is presented in order to explore the parameter space consistent with experiment. It is found that, under the model assumptions, the acceptable parameter space is quite constrained. As a part of our phenomenological
study of the model, we evaluate the branching ratio of the lightest Higgs boson and find that our model predicts a large FCNC mode $h \rightarrow tc$, which can be as large as $O(10^{-3})$. Similarly, a large BR for the top FCNC decay is obtained, namely $B.R. (t \rightarrow c + H) \simeq 10^{-4}$.

PACS numbers:
I. INTRODUCTION

The possible existence of extra dimensions has enabled the creation of several physical scenarios beyond the Standard Model (SM) \[1\]. All of these scenarios contain ideas and mechanisms that attempt to solve or to explain some of the most fundamental problems in particle physics, i.e. the problem of electroweak symmetry breaking (EWSB) and the problem of flavor.

It is clear by now that if extra dimensions do indeed exist, and if they exist at the "right" scale, there is a large amount of physical phenomena that will show up when the window is reached. Determining the specific scenario will be a challenging task. In fact, as it turns out, depending on which particular problem one is trying to solve, whether it is a general problem or one generated within the same model being proposed, there are many different possible solutions and explanations, most of which are cleverly created to evade experimental exclusion. This will of course change dramatically when new experimental results come out, however it is not clear from today’s perspective how cleanly will we be able to differentiate among the vast number of specific models.

Motivated by this situation and by the fact that there clearly are interesting general results and mechanisms that can be drawn from considering the possibility of extra dimensions, we take the following approach: In order to study EWSB one can explore a 4D effective theory of an unspecified extra dimensional theory taking into account the effect of those general results. Following this idea one should consider an effective theory with the possibility of having i) a fundamental scalar field whose extra dimensional origin is associated to a Gauge-Higgs unification scenario \[2, 3, 4, 5, 6\] and ii) a composite scalar field also of extra dimensional origin \[7, 8\]. We choose EWSB as a first step and intend to do a similar analysis with flavor in future work.

This idea is similar in spirit to that of \[9\] and more recently \[10\], where it is assumed that there is an extra dimensional strong theory that generates heavy composite states and that at low energies the SM fields are a possible combination of fundamental and composite fields (except for the Higgs and \(t_R\) which are required to be purely composite fields). There is a sector associated to the fundamental fields, one associated to the composite fields and one corresponding to the possible mixing.

In the present proposal we consider an effective SU(2)×U(1) 4D theory with a funda-
mental scalar and a composite scalar. In generality the gauge bosons can be an admixture of fundamental and composite fields as well.

II. SCALAR SECTOR

As discussed in the introduction we consider an effective theory with two scalar fields $H_E$ and $H_C$. $H_E$ is a fundamental scalar associated with components of a higher dimensional gauge boson field as in a Gauge-Higgs unification scenario. This has the virtue that its quartic self coupling is related to the gauge coupling. $H_C$ on the other hand is a composite scalar with an extra dimensional origin as well. In this case one expects the size of the mass squared term in the potential to be of the same order of the heavy composite states masses (assumed to be of O(TeV) in the present work). As in [10] we consider the following sizes for the couplings:

$$1 \sim \lambda < \lambda_c << 4\pi ,$$

where $\lambda$ denotes a fundamental coupling and $\lambda_c$ denotes the composite field coupling.

Our main interest is the scalar sector and will for the moment assume all the gauge bosons to be fundamental fields. The Lagrangian to be discussed is

$$\mathcal{L}_{\text{eff}} = |D_\mu H_E|^2 + |D_\mu H_C|^2 - V(H_E, H_C) ,$$

with

$$V(H_E, H_C) = -\mu_e^2 |H_E|^2 - \mu_c^2 |H_C|^2 - \kappa^2 (H_E^\dagger H_C + h.c.) + \frac{g^4}{2} |H_E^\dagger H_E|^2 + \lambda_c |H_C^\dagger H_C|^2 .$$

As usually done in general Gauge-Higgs unification scenarios, the coefficient $\mu_e$ is assumed to be radiatively generated. As discussed above, $|\mu_c|$ is expected to be of of O(TeV), i.e. the mass scale of composite fields in general. $\kappa$ is a free parameter that characterizes the amount of mixing and has mass dimension 1. Note that similar mixing parameters can appear when more terms mixing $H_E$ and $H_C$ are incorporated into the potential. In the present work we assume that the dominant contribution is the one coming from the $\kappa$ term in Eq. (3) and so we are assuming all other mixings to be small and negligible. One interesting feature of this Lagrangian is the form of the quartic coupling for the fundamental scalar. Since we are relating this scalar to the gauge structure its quartic coupling is given in terms of the SU(2)
gauge coupling $g$ as in the usual Gauge-Higgs unification scenarios. Finally $1 < \lambda_c << 4\pi$ as stated before.

Note that EWSB can be triggered by the vacuum expectation values (vevs) of both $H_C$ and $H_E$ if $\mu_c^2$ and/or $\mu_e^2$ are positive. In order to explore the parameters we can determine the values of $\mu_c^2$ and $\mu_e^2$ in terms of $\kappa$, $\lambda_c$, $g$ and the (vevs) of the scalar fields. Denoting the vevs of $H_E$ and $H_C$ by $v_e/\sqrt{2}$ and $v_c/\sqrt{2}$ respectively, defining $\tan \xi \equiv v_c/v_e$ and minimizing the potential, one obtains the following expressions:

$$
\mu_e^2 = -\kappa^2 \tan \xi + \frac{g^4}{2} v^2 \cos^2 \xi,
\mu_c^2 = -\kappa^2 \cot \xi + \lambda_c v^2 \sin^2 \xi .
$$

(4)

Using these expressions one can determine regions of parameter space in $|\kappa|$ and say $\tan \xi$ using $\mu_c \sim \text{TeV}$ and $1 < \lambda_c << 4\pi$ where $\mu_e$ is positive or negative and determine the viability of the scenario.

From the Lagrangian Eq.(2) we determine that the scalar, pseudoscalar and charged scalar masses are

$$
M_S^2 = \kappa^2 \begin{pmatrix}
\tan \xi + \frac{g^4}{2} v^2 \cos^2 \xi / \kappa^2 & -1 \\
-1 & \cot \xi + 2\lambda_c v^2 \sin^2 \xi / \kappa^2
\end{pmatrix},
$$
$$
M_P^2 = M_+^2 = \kappa^2 \begin{pmatrix}
\tan \xi & -1 \\
-1 & \cot \xi
\end{pmatrix},
$$

where we have defined

$$
H_E = \begin{pmatrix}
\phi_e^+ \\
\phi_c + v_c + iA_c / \sqrt{2}
\end{pmatrix},
H_C = \begin{pmatrix}
\phi_c^+ \\
\phi_c + v_c + iA_c / \sqrt{2}
\end{pmatrix} .
$$

(5)

From these expressions we obtain the physical states in the usual way by diagonalizing the mass matrices. In the scalar sector we then obtain the lightest ($h^0$) and heavy ($H^0$) states by performing a rotation in the $\phi_c - \phi_e$ space parametrized by an angle $\alpha = \alpha(\lambda_c, \xi, |\kappa|)$. Denoting the elements of $M_S$ by $m_{ij}$, the neutral CP-even masses are

$$
m^2_{H,h} = \frac{1}{2} \left( m_{11} + m_{22} \pm \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2} \right) ,
$$

(6)

and
\[ \tan 2\alpha = \frac{2m_{12}}{m_{11} - m_{22}}. \] (7)

The expression for the physical pseudoscalar mass is simple and given by

\[ m_A^2 = \kappa^2 (\cot \xi + \tan \xi). \] (8)

In the left column of Fig. (1) we show the pseudoscalar mass for different choices of \( \tan \xi \) as a function of \(|\kappa|\). Note that all possibilities lie above the \( \tan \xi = 1 \) curve and that given the form of \( m_A^2 \), there is a \( \xi \to 1/\xi \) correspondence.

The right column of Fig. (1) shows the corresponding results for the lightest scalar mass. Note that for each case the constraint \( m_h \geq 114.4 \, \text{GeV} \) (horizontal line in the plots) sets the scale for \(|\kappa|\). For example for \( \tan \xi = 1 \), \(|\kappa| \geq 118.4 \, \text{GeV} \).

In order to see the explicit \( \alpha \) dependence of these results, we plot it as a function of \(|\kappa|\) for the same values of \( \tan \xi \) in Fig. (2).

### III. YUKAWA SECTOR OF THE MODEL

In this section we explore some salient aspects of the Yukawa sector of the model presented above. For the elementary sector we consider the types of Yukawa couplings that it could have keeping in mind that it comes from a Gauge-Higgs unification scenario. We assume that this elementary sector couples predominantly to the third family, as in some particular 5D or 6D scenarios based on the gauge group \( \text{SU}(3)_w \) where the SM can be embedded [11, 12, 13].

For the composite sector, we use the effective Lagrangian approach working with a scenario where the Flavor scale \( (\Lambda_F) \) is assumed to be low, i.e. below the compositeness scale \( (\Lambda_c) \). Then, depending on the scale, we have

i) For \( E > \Lambda_c > \Lambda_F \): A theory with elementary scalars, but no Higgs and no Yukawas.

ii) For \( \Lambda_c > E > \Lambda_F \): A Higgs, but no Yukawa operators.

iii) For \( \Lambda_c, \Lambda_F > E \): Higgs and Yukawa operators: For instance by using the Froggatt-Nielsen mechanism we can write

\[ \mathcal{L}_{yc} = \lambda^{u}_{ij} \left[ \frac{\langle S \rangle}{\Lambda_f} \right]^{n_{ij}} \bar{Q}_i \tilde{H}_C u_j + \lambda^{d}_{ij} \left[ \frac{\langle S \rangle}{\Lambda_f} \right]^{n_{ij}} \bar{Q}_i H_C d_j + \text{h.c.} \] (9)
FIG. 1: Right plots: Pseudoscalar mass as a function of $|\kappa|$ for different choices of $\tan \xi$. Left plots: The lightest scalar ($h^0$) mass as a function of $|\kappa|$ for the same choices of $\tan \xi$ and for $\lambda_c = 1.4$. The horizontal line corresponds to $m_h = 114.4$ GeV.
Thus the Yukawa Lagrangian of our model is:

\[
\mathcal{L}_Y = \left[ Y_{ij}^u \bar{Q}_i L_i \tilde{H}^u R_j + Y_{ij}^d \bar{Q}_i L_i \tilde{H}^d R_j \right] \\
+ \left[ \eta_t \bar{Q}_i L_3 \tilde{H}^E R_3 + \eta_b \bar{Q}_i L_3 \tilde{H}^E d_3 \right] + h.c. \tag{10}
\]

where the first term in brackets is the contribution from the composite Higgs, the second one is due to the elementary Higgs which only contributes to the third family (with similar expressions for leptons).

After spontaneous symmetry breaking (SSB), one can derive the quark mass matrices from Eqs. \((10)\) namely,

\[
[M^u]_{ij} = \frac{1}{\sqrt{2}} \left( v_c Y_{ij}^u + v_c \eta_t \delta_{3i} \delta_{3j} \right), \tag{11}
\]

\[
[M^d]_{ij} = \frac{1}{\sqrt{2}} \left( v_c Y_{ij}^d + v_c \eta_b \delta_{3i} \delta_{3j} \right). \tag{12}
\]

We now assume that the Yukawa composite matrices $Y^u$ and $Y^d$ have the four-Hermitic-
texture form \[14\]. The quark mass has the same form and it is given by:

$$M_q = \begin{pmatrix} 0 & D_q & 0 \\ D_q & C_q & B_q \\ 0 & B_q & A_q \end{pmatrix} \quad (q = u, d),$$

where \(A_q = v_c Y_{3q}^q + v_\eta \eta_{3q}\). Taking \(M^u\) and \(M^d\) real, and following the analysis in \[14\], we diagonalize them using the matrix \(O\) in the following way:

$$\bar{M}^q = O^T q M^q O_q.$$  \hspace{1cm} (13)

From Eqs. (11, 12) we can write

$$[\tilde{Y}^q] = \frac{\sqrt{2}}{v_c} [M_q] - \eta_{q3} \cot \xi \tilde{h}^q,$$ \hspace{1cm} (14)

where \([\tilde{Y}^q] = [O^T_q] [Y^q] [O_q]\), and \([\tilde{h}^q] = [O^T_q] [Diag\{0, 0, 1\}] [O_q]\). Note that the second term (proportional to \(\eta_{q3}\)) induces FCNC. Parameterizing \(A_q\) as \(A_q = m_{q3} - \beta_q m_{2q}\) \[15\] and performing the product \([O^T_q] [Diag\{0, 0, 1\}] [O_q]\), the term \([\tilde{h}^q]\) goes like

$$[\tilde{h}^q]_{ij} \sim \frac{\sqrt{m_i m_j}}{m_{q3}}.$$

Finally the interactions of the neutral Higgs bosons \((h^0, H^0, A^0)\) with quark pairs acquire the following form:

$$\mathcal{L}_Y^q = \bar{d}_i \left[ \frac{m_{d_i} \cos \alpha}{v_s} \sin \xi \delta_{ij} - \eta_b \frac{\sqrt{m_{d_i} m_{d_j}} \cos(\alpha + \xi)}{m_b} \sqrt{2} \sin \xi \right] d_j H^0$$

$$+ \bar{d}_i \left[ - \frac{m_{d_i} \sin \alpha}{v_s} \sin \xi \delta_{ij} + \eta_b \frac{\sqrt{m_{d_i} m_{d_j}} \sin(\alpha + \xi)}{m_b} \sqrt{2} \sin \xi \right] d_j h^0$$

$$+ i \bar{d}_i \left[ - \frac{m_{d_i} \cot \xi \delta_{ij}}{v_s} \sin \xi - \eta_b \frac{\sqrt{m_{d_i} m_{d_j}}}{m_b} \frac{1}{\sqrt{2} \sin \xi} \right] \gamma^5 d_j A^0,$$

$$+ \bar{u}_i \left[ \frac{m_{u_i} \cos \alpha}{v_s} \sin \xi \delta_{ij} - \eta_t \frac{\sqrt{m_{u_i} m_{u_j}} \cos(\alpha + \xi)}{m_t} \sqrt{2} \sin \xi \right] u_j H^0$$

$$+ \bar{u}_i \left[ - \frac{m_{u_i} \sin \alpha}{v_s} \sin \xi \delta_{ij} + \eta_t \frac{\sqrt{m_{u_i} m_{u_j}} \sin(\alpha + \xi)}{m_t} \frac{1}{\sqrt{2} \sin \xi} \right] u_j h^0$$

$$+ i \bar{u}_i \left[ \frac{m_{u_i} \cot \xi \delta_{ij}}{v_s} \sin \xi - \eta_t \frac{\sqrt{m_{u_i} m_{u_j}}}{m_t} \frac{1}{\sqrt{2} \sin \xi} \right] \gamma^5 u_j A^0.$$ \hspace{1cm} (16)
IV. HIGGS AND TOP FCNC DECAYS

We now present some generic numerical results in order to show the potential of this model. A complete phenomenological study of this model is underway and will be presented elsewhere. If we go back to the main motivation of this model, recall that we incorporated both composite and fundamental scalars of extra dimensional origin in order to write a two Higgs doublet model in 4D. Furthermore, we assumed, motivated by previous works, that the composite scalar couples to all quarks while the elementary (fundamental) one couples only to the third family. Consider now the following approach: Let’s suppose that naturally one would expect the top quark to have a mass of $O(m_W)$ and that the reason for its heaviness is precisely that it has an extra contribution, in this case from the elementary sector. Following this idea we explore the model in the case where this only happens to the top quark and hence set $\eta_b = 0$ in the following analysis.

Some remarks are in order:

In order of get the most economical set of Yukawa parameters, we expressed the composite Yukawa in terms of both the particle masses and the elementary Yukawa (see Eq. 14). As a consequence, the fermion-fermion-CP even Higgs ($H^0$ or $h^0$) couplings have the same form for both type of fermions. In the case of the fermion-fermion-CP odd Higgs ($A^0$) couplings the only difference is a sign in Eq. (16).

In order to avoid dangerous FCNC, we need to specify the magnitude of $\eta_{3q}$. To do so, we perform the following analysis: The Cheng-Sher ansatz [18] for the fermion-fermion-Higgs boson coupling is

$$\bar{f}_i f_j \phi^0 \sim c_{ij} \frac{\sqrt{m_{f_i} m_{f_j}}}{v},$$

(17)

and FCNC are kept under control if $|c_{ij}| \sim 10^{-1}$. In our case the fermion-fermion-Higgs boson coupling is found to be

$$\bar{f}_i f_j \phi^0 \sim \eta_{f3} \frac{\sqrt{m_{f_i} m_{f_j}}}{m_{f_3}},$$

(18)

and so $c_{ij} = \eta_{f3} \frac{v}{m_{f_3}} \Rightarrow |\eta_{f3}| = \frac{m_{f_3}}{v} |c_{ij}|$. Then for u-quarks $|\eta_t| \sim 10^{-1}$ and for d-quarks $|\eta_b| \sim 10^{-2}$.

Using the expressions in Eq. (16) we compute the branching ratios (BR) for the lightest scalar $h^0$ decays to $b\bar{b}$, $\tau\bar{\tau}$, $ZZ$, $WW$, $t\bar{t}$ and $t\bar{c}$. Fig. 3 shows the results for $\tan \xi = 1$ (above)
and \( \tan \xi = 10 \) (below). We present plots for three different choices of \( \alpha \) corresponding to values within the desired \( |\kappa| \) range, i.e. \( 15 \text{ GeV} \leq |\kappa| \leq 500 \text{ GeV} \). Note in particular the result for the BR corresponding to the flavor changing decay \( h^0 \rightarrow tc \). We see that for all three cases this BR is larger than the SM prediction by about 10 orders of magnitude [16, 17].

The discussion above deals with Higgs boson decays in our model. However, if it so happens that the Higgs particle is light enough, then it could show up in FCNC top decays, namely \( t \rightarrow c + h \). The evaluation of the corresponding decay branching ratio leads to \( \text{B.R.}(t \rightarrow c + h) \approx 10^{-4} \), (see Fig. (4)) which is in the right range to be detected at the LHC [19].

\[ \text{V. CONCLUSIONS} \]

We have presented a model based on the fact that if there are extra dimensions of a size relevant to particle physics phenomenology, then it is possible to have 4D scalars associated to the extra dimensional physics. As a result, the model assumes the existence of both composite and fundamental scalars whose origin is extra dimensional. Concretely, the model contains two Higgs doublets, one of each kind, where one of the Higgses couples to all fermions while the other couples only to third family fields. In this simplest version of the model all other fields (gauge and fermions) are treated as fundamental.

A detailed study of the scalar spectrum has revealed that this simple model has a rather constrained parameter space consistent with the model assumptions. We have computed the branching ratios for the lightest Higgs decay and found that the one corresponding to the \( h^0 \rightarrow tc \) mode is much larger than the one obtained in the Standard Model (10 orders of magnitude approximately). Lastly we present the branching ratio for the FCNC top decay \( t \rightarrow c + h \) which we find to be of \( \text{O}(10^{-4}) \).

\[ \text{Acknowledgments} \]

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FIG. 3: Branching ratios for $h^0$ decay as a function of $m_h$ for $\eta_b = 0$ and $|\eta_t| = 10^{-1}$. The results are shown for $\tan \xi = 1$ (top) and $\tan \xi = 10$ (bottom) and for three different choices of $\alpha$. These
FIG. 4: Branching ratio for $t \rightarrow c + h$ for different choices of $\alpha$ and $\tan \xi$.

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