Finite element analysis of damage accumulation for structure under impact

Wang Hongyan and Hao Guixiang
School of Mechanical and Vehicle Engineering, Beijing Institute of Technology, Beijing, 100081, China.
Department of Mechanical Engineering, Academy of Armored Force Engineering, Beijing, 100072, China.
E-mail: guixiang_hao@sina.com

Abstract. Responses of structure under impact load are different from quasi-static process and fatigue. Especially when the impact load is cyclic loading and unloading, damage of structure is different form that of structure under single continuous load. Random impact load usually has high peak value, thus material in structure may contain plastic strain, and damage evolution is non-linear. For cyclic loading and unloading, elastic recovery process should be considered. These characteristics make it difficult to compute damage in FEA. In the paper, we use FEM and damage theories to compute cumulative damage of certain metal structure under cyclic loading and unloading, and statistic method is used in load setting. A series of experiments were carried out to verify the simulated results. It is proved that FEM can be used to acquire cumulative damage of structure under cyclic loading and unloading, and the results may be referenced in engineering design.

1. Introduction
The research of structure under impact is mainly concentrate on stress, strain distribution. Less research considers the mechanical property change of structure after impact. Based on FEM and damage mechanics, we focus on the structure damage of certain equipment after airdrop, and assess the influence of impact.
The impact velocity of the airborne equipment would be decreased by parachute-airbag system. Still, there is a large impact at the end of landing process. According to the inspection of airborne equipment after airdrop landing impacts, there are many mechanical faults due to the airdrop impacts. For the reliability and maintainability of airborne equipment, the responses especially damage of the equipment at landing process should be studied. For the high cost of experiments, large numbers of airdrop equipments can not be carried out widely. Computer simulation is economical, flexible and repeatable, and airdrop conditions also can be controlled easily in simulation.

2. Lemitre’s damage theory

2.1. Damage variables
The mechanical properties (elastic, viscoelasticity, plasticity, viscoplasticity et al.) of undamaged material are described perfectly in classical solid mechanics theory. The degradation of material or
structure is a sequential process. So it is incorrect to describe behaviours of damaged material by the constitutive equations of the undamaged material. The damage theory objects to describe the damage mechanism, and predict the rest life by setting up the constitutive model of the damaged material. Damage theory relates to solid-state physics, strength of materials and continuum theory. The results coming from damage theory not only reflect the change of microstructure, but also represent the alteration of the macro mechanical property. And the parameters which used to compute damage must be measurable. The shortage of fracture mechanics would be overcome to a certain extent\[1\]. When considering the damage of materials after impact, it is in the first place to choose the suitable variable to measure the damage. The microstructure and some physical properties of material changed after impact, and all these changes represent material degradation. Some variables can be chosen to assess damage, such as the amount, length, area and volume of microstructure voids. And elastic coefficient, yield stress, strength of extension, extensibility or density could be selected, too.

2.2. Strain equivalence principle
In order to measure damage, Lemite brought forward the strain equivalence principle. According to the principle, the damaged material’s strain arose from real stress is equivalent to the undamaged material’s strain resulted from effective stress. Thus, the constitutive equation of a damaged material can be deduced from that of an undamaged material.

\[
\varepsilon = \frac{\sigma}{E} = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1-D)E} \quad (1)
\]

\[
\sigma = E(1-D)\varepsilon \quad (2)
\]

Equation (2) is the constitutive relation of damaged material in 1D problems, and it can be changed into \( \varepsilon = \sigma / \tilde{E} \).

If the elastic modulus in a damaged material is defined by \( \tilde{E} = E(1-D) \), we obtain

\[
D = 1 - \frac{\tilde{E}}{E} \quad (3)
\]

The derivative of equation (2) is

\[
\frac{d\sigma}{d\varepsilon} = \frac{dE}{d\varepsilon}(1-D)\varepsilon + E(1-D) - E\varepsilon \frac{dD}{d\varepsilon} \quad (4)
\]

Unload when the load increases to certain level. The damage process is irreversible, so the damage is constant during unloading process. Thus \( \frac{dD}{d\varepsilon} = 0 \). \( E \) is the elastic modulus of undamaged material, which is constant. So equation (4) can be simplified to obtain

\[
D = 1 - \frac{1}{E} \frac{d\sigma}{d\varepsilon} \quad (5)
\]

Elastic modulus of damaged material \( \tilde{E} \) is the slope of unloading curve. It is called unloading elastic modulus. The damage-plastic strain curve can be obtained by measuring unloading elastic modulus (Figure 1). The material of specimen is 99.99% copper, \( D_c \) is the critical value of damage, \( \varepsilon_{pD} \) is the plastic strain when damage stars. \( \varepsilon_{pR} \) is the plastic strain when the specimen ruptures\[2\].

![Figure 1. Plastic damage variation for 99.99% copper\[2\].](image-url)
2.3. Lemitre’s damage model[3]

The damage of metal material caused by large plastic deformation is called ductile damage. The large deformation corresponds to a plastic strain of 0.2 to 2.

For large deformation, we have

$$ D = -Y \frac{\dot{\varepsilon}}{s_0 (1 - D)^{\alpha_0}} $$

(6)

Introduce Von Mises yield condition

$$ \dot{D} = \frac{\sigma_y^2 s_i}{2E \sigma_0} \frac{\dot{p}}{(1 - D)^{\alpha_0}} $$

(7)

According to the experiment results, damage has a linear relationship with plastic strain rate, so we assume \( \alpha_0 = 0 \). Under proportional loading, triaxial stress ratio \( \sigma_H / \sigma_{eq} \) is a constant. So does \( s_i \). \( p_D \) is the threshold strain value of damage. When \( p \leq p_D \), \( D = 0 \). Integrate equation (7), we obtain

$$ D = \frac{\sigma_y^2 s_i}{2E \sigma_0} (p - p_D) $$

(8)

Assuming \( p_R \) is the plastic strain in time of rupture, \( D_R \) can be acquired

$$ D_R = \frac{\sigma_y^2 s_i}{2E \sigma_0} (p_R - p_D) $$

(9)

From equation (8) and (9), the damage is given by

$$ D = D_R \frac{p - p_D}{p_R - p_D} $$

(10)

\( p_D \) and \( p_R \) have linear relationships with triaxial stress ratio \( \sigma_H / \sigma_{eq} \). We assume that these relationships are similar, thus \( p_D / p_R \) is independent of triaxial stress ratio.

$$ p_D / p_R = \varepsilon_D / \varepsilon_R $$

(11)

\( \varepsilon_D \) and \( \varepsilon_R \) are threshold strain value when damage starts and material ruptures.

Damage can be deduced from equation (10) and (11),

$$ D = D_R \left( \frac{p \varepsilon - \varepsilon_D}{\varepsilon_R - \varepsilon_D p} \right) $$

(12)

Under an uniaxial loads, \( \sigma_H / \sigma_{eq} = \frac{1}{3}, s_i = 1 \). We assume that \( p_D = 0 \) when damage occur and \( p_R = \varepsilon_R \) in time of rupture (elastic strain is ignored). From equation (9), we have \( p_R = \varepsilon_R = \frac{2E \sigma_0 D_R}{\sigma_y^2} \).

Under triaxial loads, we obtain \( p_R \)

$$ p_R = \frac{\varepsilon_R}{s_i} $$

(13)

Therefore equation (11) can be transformed into

$$ D = D_R \frac{p \varepsilon R - \varepsilon_D}{\varepsilon_R - \varepsilon_D} $$

(14)

Where;

\( D \) is the damage value

\( D_R \) is the damage value in time of rupture

\( p \) represents the cumulative plastic strain

\( \varepsilon_D \) is the threshold strain value of damage
$\varepsilon_R$ is the strain value when rupture starts
$s_t$ reflects the influence of triaxial stress ratio, therefore it is called triaxial stress factor.

Under uniaxial stress state, $\sigma_H = \frac{1}{3} \sigma$, $\sigma_{eq} = \sigma$, $s_t = 1$.

$$s_t = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^3$$

In above equations, three parameters, which are $\varepsilon_D$, $\varepsilon_R$ and $D_R$, should be given. They can be calculated from experiments’ data of $D$ and $\varepsilon$.

3. FEA model of equipment

It is very important to establish the finite element model properly for a complex structure. The model not only affects the analysis process, but also influences the reliability of the results.

Modelling of the hull and analysis were completed in HyperMesh. The original structure of hull was simplified when modelling the finite element model. For the most parts of hull are plates, the plate structures were modelled with shell elements.

The finite element model was obtained by discretizing the geometry model. The total amount of nodes and elements are 114028 and 107089; the detailed statements of the type of elements are shown in table 1. The material is aluminium alloy, its mechanical parameters are: $\rho = 2.7 \times 10^3 \text{kg/mm}^3$, $E = 70 \times 10^3 \text{MPa}$ and $\nu = 0.3$.

| Number | Bar2 | Tria3 | Quad4 | Penta6 | Hex8 |
|--------|------|-------|-------|--------|------|
| Total Element Number | 117089 |
| Node Number | 114028 |

4. Simulation of equipment’s airdrop landing

Nowadays, there are two methods to study the impact problems at landing; they are coupling method and uncoupling method [5,6].

(1) For the coupling method, the airbags and airborne equipment are combined and studied together. They are related with consistency condition of the contact, the responses of airbags and airborne vehicle. The forces between them are obtained by solving the equations of consistency conditions.

(2) For the decoupling method, the responses and damage of the equipment are concerned, not of the airbags and ground. Solving the dynamic responses of the equipment separately by applying loads, which are between the airbags and the equipment. Therefore, the dynamic response results can be obtained when the loads between airbags and equipment are simplified reasonably.
The decoupling method used in this paper makes it efficient to obtain the responses of the airborne equipment. In the method, the impact load is force calculated from acceleration data of real experiments. Decoupling method can reduce solving difficulty and dimension of the question. Also, it can avoid dealing with the complexity of coupling. The method can simulate the damage of structure under repeated airdrops more conveniently and more effectively. In general, it provides a proper technical method of damage assessment. The disadvantage of this method is that the load must be acquired first and the accuracy of load will affect the dynamic response results of the equipment.

4.1. Operating mode
Landing conditions of airborne equipment are: pavement ground, landing velocity of 8m/s. Some airbags are non-serviceable, impact acceleration was 250%~320% of common landing condition, and the max impact acceleration was less than 20g. The experiment’ impact acceleration curve of bottom can be simplified to a saw-tooth curve (Figure 3) [6].

![Figure 3. Simplified acceleration-time curve.](image)

4.2. Dynamic loads
Based on the simplified impact acceleration curve, the dynamic loads are obtained. The impact between airbags, ground, assemblies and the equipment structure are simulated. The assemblies are engine, gear box, oil box and so on. Considering the gravity of assemblies, add the acceleration by gravity acceleration. The values of dynamic loads are obtained. [6]

\[ F_d = 9_{ai} \left[ \left( a_i + g \right) / g \right] \]

(16)

In above equation: \( F_d \) is static load, \( a_i \) is the acceleration for every step. The dynamic load curves are shown in Figure 4. P1~P10 are loads which different assemblies exert on the structure of equipment.

![Figure 4. Curves of dynamic load.](image)

4.3. Analysis process
When we carry out the simulation for the first time, the strain and damage data are written into a state file. For the second simulation, we introduce the strain and damage as an initial state. Therefore, the damage accumulation of structure can be considered. The detailed steps are given:

(1) Carry out simulation of impact, and write down state files.
(2) Initialize the model of equipment with state files which have been written down before.

4.4. Simulation results

(1) The max Von Mises stress is on the bottom of right vertical shaft, and the value is 321.3MPa. Figure 5 shows the stress contour when max stress occurs. The stress time history curve is shown in Figure 6. We can see that there is about residual stress of 80MPa after impact.

(2) The point of max plastic strain is in the same position. The max plastic strain is 0.416 % (Figure 7). Also, we have the stress-strain curve of the max stress element (Figure 8).

Figure 5. Max Von Mises stress Position.

Figure 6. Stress-time curve of the max Von Mises stress.
5. Structure damage evaluation

Using state files to initialize the model of equipment, we get the responses of structure after six times of impacts.

(1) The stress time history of max stress point is shown in Figure 9.

(2) The detailed plastic strain and damage value are given in Table 2. According to equation (13), we get three parameters’ values: $\varepsilon_D = 0.02$, $\varepsilon_R = 0.6$ and $D_R = 0.22$ [2]. Therefore, the damage values can be calculated.

From the results above, we can get the conclusions:

- When the equipment is under impacts of airdrop landing, the structure’s dangerous point is in the bottom of right vertical shaft.
- Stress values in most part of equipment are far from yield point. The design of equipment is safe enough to endure the airdrop landing impact. Therefore, we could find a lightweight design to reduce the amount of material used in the structure.
- In the 6 times of impacts’ simulation, all the max stress points are in the same position. The max stress values change a little, but the plastic strain values increase remarkably.
Table 2. Max stress and plastic strain under accumulated impact

| Times of Impact | Element     | Max Stress | Plastic Strain | Change | Damage D |
|-----------------|-------------|------------|----------------|--------|----------|
| 1               | E2184088    | 321.3MPa   | 0.416%         | ———   | 0.168    |
| 2               | E2184088    | 324.4MPa   | 0.454%         | 9.1%   | 0.184    |
| 3               | E2184088    | 325.4MPa   | 0.508%         | 12%    | 0.207    |
| 4               | E2184088    | 329.7MPa   | 0.519%         | 2.2%   | 0.212    |
| 5               | E2184088    | 330.3MPa   | 0.535%         | 3.1%   | 0.218    |
| 6               | E2184088    | 331.5MPa   | 0.554%         | 3.6%   | 0.227    |

6. Conclusions
In the paper, we set up models of airdrop equipment and airbag system based on FEM. In succession, the landing process of airdrop equipment was simulated, and the responses of equipment were analyzed. Especially, we studied the cumulative damage of equipment structure under six times of airdrop landing based on Lemitre’s theory. The results could be referenced in engineering design of the equipment.

References
[1] Yu Tian-qing, Qian Ji-cheng. Damage Theory and Its Application[M]. Beijing: National Defense Industry Pr-ess,1993.(in Chinese)
[2] Lemaitre J. Evoluation of dissipation and damage in metals, submitted to dynamic loading. Proc. I. C. M.1, Kyoto Japan, 1971.
[3] Lemaitre J. Formulation and identification of damage, kinetic constitutive equations. CISM Course on damage mechanics, UDINE, 1986.
[4] LI Chu-lin, ZHANG Sheng-lan et al. Analysis and ap-plication by HyperWorks[M]. Beijing: China Machine Press,2008.(in Chinese)
[5] Liu Fu. Bird Impact Dynamic Response Analysis of Aircraft Windshield[D]. Nanjing University of Aeronautics and Astronautics,2006.(in Chinese)
[6] DU Zhi-qi, SHAO Peng-li. Dynamic Finite Element Simulation of the Aluminum Alloy Hull at Landing[J].Acta Armamentarii,2009,30(1):1-4.(in Chinese)