Experimental verification of model-free active vibration control approach using virtually controlled object

Heisei Yonezawa, Itsuro Kajiwara and Ansei Yonezawa

Abstract
The purpose of this study is to develop a simple and practical controller design method without modeling controlled objects. In this technique, modeling of the controlled object is not necessary and a controller is designed with an actuator model, which includes a single-degree-of-freedom virtual structure inserted between the actuator and the controlled object. The parameters of the virtual structure are determined so that indirect active vibration suppression is effectively achieved by considering the frequency transfer function from the vibration response of the controlled object to that of the virtual structure. Since the actuator model, which includes a virtually controlled object, is a simple low-order system, a controller with high control performance can be designed by traditional model-based optimal control theory. In this research, a mixed $H_2/H_\infty$ controller is designed considering both control performance and robust stability. The effectiveness of the proposed method is validated experimentally. The robustness of the controller is demonstrated by applying the same controller to various structures.

Keywords
Model-free control, active vibration control, virtually controlled object, mixed $H_2/H_\infty$ control, inertial mass actuator

1. Introduction
To improve performance while reducing weight, vibrations of structures in mechanical systems must be suppressed. Many systems use active vibration control to reduce vibrations. Although it has a high control performance, designing the controller requires modeling of the controlled object, and the model must be updated each time the controlled object is altered. Hence, active vibration control is not only a burden on designers but also has high development costs. Furthermore, the stability and control performance of model-based control systems largely depend on model accuracy. Modeling errors of the actual system degrade vibration control performance.

Many studies have reported model-free active vibration control systems without using a model of a controlled object to design the control system. One study proposed adaptive model-free control for posture stabilization of flexible spacecraft (Wei et al., 2017). Control systems for nonlinear and time-variant systems often involve real-time calculations for the optimization. Several unique methods have been proposed, for example a driving evaluation by a test driver was used to construct a semi-active control system (Swevers et al., 2007). An acoustic controller was designed by focusing on the frequency response from the disturbance (Meurers et al., 2003). A real-time tuning method using the simultaneous perturbation stochastic approximation has been proposed (Kajiwara et al., 2018).

Introducing neural networks (NNs) into a control system is an effective approach to construct a model-free control system. Many techniques using NNs have been applied to active vibration control (Abdeljaber et al., 2016; Madan, 2005; Yao et al., 2014; Yildirim, 2004; Yin et al., 2018; Yousefi et al., 2008). For example, NNs were used in a vibration control system composed of a simple building structure and an active mass-damper (Yang et al., 2006) and to obtain an inverse model of a magnetorheological elastomer combined with linear quadratic regulator (Gu et al., 2017). For tension control of a wire cable, a proportional–integral NN was proposed (Zhang et al., 2017).
et al., 2017a). However, NN-based approaches generally require a large amount of training data to be learned in advance. Moreover, the system requires a lot of time to learn and optimize the desired control action.

In fuzzy control, several control rules are defined based on empirically obtained knowledge and the controlled object’s characteristics. These rules are used as control logic via fuzzy inference. Accordingly, this approach can realize a model-free design. In particular, many studies related to vibration control have used fuzzy inference (Bui et al., 2017; Edalath et al., 2012; Malhis et al., 2005; Song et al., 2015; Thenozhi et al., 2015). However, no systematic approach is available to design the appropriate control rules and membership functions, which are important processes in fuzzy control. Although design methods combined with other optimization algorithms, which have also been proposed for these processes (Chao and Lai, 2003; Marinaki et al., 2010; Zhang and Gan, 2004), they increase the number of procedures and parameters required for the optimization. Therefore, these methods are not easy from the viewpoints of calculation efficiency and reliability in the whole design process.

Sliding mode control (SMC) has been used for model-free vibration control systems due to its robustness against the uncertainty of the controlled objects. For example, the effectiveness of the sigmoid function for appropriate smoothing was verified by experimentally investigating a rotary inverted pendulum (Yigit, 2017). Regarding a smartly structured cantilever beam, the primary and secondary resonance peaks were selectively suppressed (Parameswaran et al., 2015). In positioning control for active suspension systems, a model-free design was realized by estimating the dynamics using the time-delay estimation method and error compensation using SMC (Wang et al., 2019). Some SMC approaches have been combined with NNs (Lee et al., 2014). However, chattering due to the structure, which adversely affects the control system, is a serious problem in SMC. Although efforts such as smoothing functions have been made to reduce this effect, the controlled frequency bands have been investigated only on the low-frequency side, which has less chattering. If the applied conditions change, the control system parameters must be tuned by trial and error to reduce chattering. This is a burdensome process for designers.

Few model-free active vibration control systems without the complicated design process, which puts a burden on designers, have been proposed. From the perspective of control performance and implementation, it is required to adopt a model-based optimal control theory and design a controller systematically with less calculation load. Herein, a model-free vibration control method based on the concept of a virtual structure is proposed as one such technique. The proposed method enables model-free design by inserting a virtual controlled object (hereafter virtual object) between the actuator and the actual controlled structure (hereafter actual object). Specifically, setting appropriate parameters of the virtual object defined as a single-degree-of-freedom (SDOF) system in consideration of the frequency transfer function makes the vibration responses of the virtual object equal to those of the actual object. Because a controller designed without using parameters in the actual object indirectly suppresses its vibrations, a system robust against changes of the actual object can be realized. This technique does not require the complex design processes described above. Since the model of the actuator with a virtual object is a simple low-order system, there are fewer design parameters and a smaller calculation (implementation) load. In addition to being a simple practical method, it uses the same design process as traditional model-based optimal control after introducing the virtual object. Hence, the proposed approach can easily realize high control performance.

The rest of the article is organized as follows. First, the model-free control system with a virtual object and a state equation to design a controller are demonstrated. Second, the feasibility of indirect vibration control is shown using the frequency transfer function from the vibration of the actual object to that of the virtual object. Finally, the proposed method is validated by experiments with an emphasis on controller robustness using different controlled objects.

2. Model-free control system

2.1. Actuator

Figure 1 shows the actuator used for active vibration control. It is an inertial mass-type electromagnetic actuator installed on the surface of the target structure. Specifically, a movable mass composed of coils vibrates vertically along the central axis and applies vertical excitation forces to the contact surface that suppresses the object’s vibration. The value of the excitation force is proportional to that of the current in the actuator circuit depending on the command value from the controller. From this mechanical principle, the actuator can be modeled as a SDOF system.

Figure 1. Inertial mass-type electromagnetic actuator modeled as single-degree-of-freedom.
2.2. State equation including a virtual object

Herein, model-free active vibration control is realized by introducing a virtual object into the control system. Figure 2(a) shows the model of an actual system, where \( m, \ k, \) and \( c \) represent the mass, stiffness, and damping of the vibration system, respectively. The subscripts 0 and 1 indicate parameters of the actuator and actual object, respectively. The structure of the actual object is an arbitrary form and is not specifically defined. The purpose of the control is to suppress vibrations of displacement \( x_1 \) of the actual object via the control input \( u \) generated from the actuator as described for the SDOF system shown in the previous section. For this system composed of the actuator and the actual object, several approaches, including the NNs (Yang et al., 2006), the fuzzy control (Edalath et al., 2012), the SMC (Yiğit, 2017), and the adaptive control, which are shown in Section 1, can be candidates for designing a control system without using the model. However, all of them involve the complicated design processes with a lot of time and preparations, and their many parameter tunings by trial-and-error efforts are quite burdensome. In addition, the calculation loads when the control systems are implemented-off-line and work on-line are heavy. For example in SMC (Yiğit, 2017), the chattering due to its controller structure makes the parameter tuning with a trial-and-error effort more difficult when realizing a model-free design.

We used an approach that fundamentally differs from the other traditional methods to realize a simpler and easier design than the previous model-free control systems shown in Section 1. A new point (our originality) of the model-free method proposed in this study is the idea of introducing the virtual object into the control system. Herein, model-free active vibration control is realized by introducing a virtual object into the control system. That is, the virtual object is regarded as a controlled object when designing a controller, and vibration control of the virtual object displacement \( x_v \) is aimed at indirectly suppressing the vibrations of \( x_1 \). This controller is designed based on the traditional model-based control theory for the two-degree-of-freedom system composed of the actuator and the virtual object.

For the actuator and the virtual object, the equations of motion are obtained from Newton’s second law as

\[
m_0 \ddot{x}_0 + c_0 (\dot{x}_0 - \dot{x}_v) + k_0 (x_0 - x_v) = u \tag{1}
\]

\[
m_1 \ddot{x}_v + c_1 (\dot{x}_v - \dot{x}_1) + k_1 (x_v - x_1) + c_0 (\dot{x}_v - \dot{x}_0) + k_0 (x_v - x_0) = -u \tag{2}
\]

Next, equations (1) and (2) are Laplace transformed where all the initial conditions are zero. Then, these equations are written as

\[
m_0 s^2 X_0(s) + c_0 s (X_0(s) - X_v(s)) + k_0 (X_0(s) - X_1(s)) = U(s) \tag{3}
\]

\[
m_1 s^2 X_v(s) + c_1 s (X_v(s) - X_1(s)) + k_1 (X_v(s) - X_0(s)) + c_0 s (X_v(s) - X_0(s)) + k_0 (X_v(s) - X_0(s)) = -U(s) \tag{4}
\]

Here, \( s \) is the Laplace operator, and the functions after the transformation are expressed by capital letters. From equations (3) and (4), we derive transfer function \( T_{xv1}(s) \) from displacement \( x_1 \) of the actual object to displacement \( x_v \) of the virtual object as

\[
T_{xv1}(s) = \frac{X_v(s)}{X_1(s)} = \frac{(c_1 s + k_1) (m_0 s^2 + c_0 s + k_0)}{m_1 s^2 + (c_1 + c_0) s + (k_1 + k_0)} - \frac{(m_1 s^2 + c_0 s + k_0)}{m_0 s^2 + (c_0 + k_0) s + (1 + k_0/k_1)} \tag{5}
\]

The parameters of the virtual object are adjusted as design variables. Regarding equation (5), if the stiffness of the virtual object is designed as \( k_v \rightarrow \infty \), and the mass \( m_v \) becomes zero, the value of the transfer function \( T_{xv1} \) converges to 1. In this condition, displacement \( x_v \) and displacement \( x_1 \) are equal. Hence, the vibrations of the actual object can be indirectly controlled by suppressing the vibrations of the virtual object. However, \( k_v \) cannot be designed to be an infinite variable because the controller is obtained via a numerical calculation. Moreover, the value of
$m_v$ cannot be zero because this creates an uncontrollable system without a mass point to give the control input. Therefore, we aimed to establish the required transfer characteristic approximately

$$T_{v_y1} \approx 1$$  \hspace{1cm} (6)

Designing $k_v$ as a sufficiently large finite value, $m_v$ as a very small finite value, and $c_v$ as zero satisfies the transfer characteristic of equation (6), which allows indirect vibration control to work effectively. Table 1 shows the control system parameters.

In equations (1) and (2), vibrations of the actual object are regarded as disturbances in the following form

$$w = k_v x_1 + c_v \dot{x}_1$$  \hspace{1cm} (7)

From this, the state equation of the 2DOF system is derived as

$$\dot{x}_{va} = A_{va} x_{va} + B_{va1} w + B_{va2} u$$  \hspace{1cm} (8)

Here, each coefficient matrix and state vector are expressed as

$$A_{va} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_v + k_m}{m_v} & \frac{k_m}{m_v} & -\frac{c_v + c_m}{m_v} & \frac{c_m}{m_v} \\ \frac{k_m}{m_0} & -\frac{k_v}{m_0} & \frac{c_m}{m_0} & -\frac{c_v}{m_0} \end{bmatrix},$$  \hspace{1cm} (10)

$$B_{va1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad B_{va2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This state equation does not include parameters of the actual object. Consequently, a model-free control system can be realized by designing a controller using equation (8), in which the virtual object is used as a controlled object. The designed controller can indirectly suppress vibrations of the real structure since the transfer characteristic (6) is established, thanks to the design of the virtual object model. This independence of the actual object parameters gives robustness with respect to the characteristic variations of the actual objects.

NN approaches such as (Yang et al., 2006) involve a lot of time and heavy calculation loads to implement a model-free controller because experimentally system identification of an unknown plant and repeated training of the controller must be performed.

However, in the presented technique, the advantage that the virtual object introduced for model-free design is a simple SDOF system reduces the number of tuning parameters, the time required to implement the controller, and the calculation loads. This is because only the two design parameters, $m_v$ and $k_v$, are necessary to determine the virtual object, and the controller obtained from equation (10) is a low-order state equation (order: 4).

In the proposed technique, closed-loop control is performed by feedback of the vibration of the virtual object as the observed output. However, this vibration does not exist in the real system. Therefore, vibration $x_1$ of the actual object, which is almost equal to $x_i$ from equation (6), is used as the observed output. When a multi-degree-of-freedom (MDOF) system is used as the actual object, $x_1$ at the location where the actuator is installed should be measured by a sensor.

### 2.3. Frequency transfer property to realize indirect vibration control

Since the model of the virtual object cannot realize $k_v \to \infty$ and $m_v \to 0$, the transfer characteristic of equation (6) is established in a limited frequency band. The virtual object must be designed to satisfy equation (6) in a controlled frequency band where vibration suppressions are required for resonance peaks at natural frequencies. The characteristic equation of the transfer function (5) is given by

$$R(s) = \left\{ m_s s^3 + (c_v + c_m) s + (k_v + k_m) \right\} \times \left\{ m_0 s^2 + c_0 s + k_0 \right\} - \left( c_0 s + k_0 \right)^2$$

$$= m_s m_0 s^4 + (m_0 c_v + m_0 c_m + m_s c_0) s^3$$

$$+ (m_0 k_v + m_0 k_0 + c_v c_0 + m_v k_0) s^2$$

$$+ (c_v k_0 + c_0 k_v) s + k_v k_0 = 0$$  \hspace{1cm} (11)

This is also the characteristic equation of a well-known 2DOF vibration system with $x_1 = 0$ in Figure 2(b). That is, the two resonance peaks of the frequency response $|T_{v_y1}(j\omega)|$ occur near the damped natural frequencies of the vibration system described by equations (1) and (2) with $x_1 = 0$. In addition, the system can be regarded as an undamped 2DOF vibration system because the damping ratios of the actuator and the virtual object are very small. Consequently, the

| Properties | Value | Unit |
|------------|-------|------|
| $m_0$      | 0.2013 | kg   |
| $k_0$      | 3518  | N/m |
| $c_0$      | 1.186 | N/s/m |
| $m_v$      | $10^{-5}$ | kg |
| $k_v$      | $7.0 \times 10^5$ | N/m |
| $c_v$      | 0.0   | N/s/m |
The undamped natural angular frequencies $\Omega_1$ and $\Omega_2$ ($\Omega_2 > \Omega_1$) are obtained by solving the eigenvalue problem of equation (12).

$$\Omega_1(m_v, k_v) = \frac{2k_v k_0}{(m_v k_0 + m_0 k_v + m_0 k_0)} + \sqrt{(m_v k_0 + m_0 k_v + m_0 k_0)^2 - 4m_v m_0 k_v k_0}$$

$$\Omega_2(m_v, k_v) = \frac{2k_v k_0}{(m_v k_0 + m_0 k_v + m_0 k_0)} - \sqrt{(m_v k_0 + m_0 k_v + m_0 k_0)^2 - 4m_v m_0 k_v k_0}$$

On the other hand, when the lower and upper limit frequencies defining the vibration control band are $\Omega_1^{cont}$ and $\Omega_2^{cont}$ ($\Omega_2^{cont} - \Omega_1^{cont} \geq k_0/\omega_0$), respectively, the constraint conditions for designing the parameters $(m_v, k_v)$ of the virtual object are written as

$$\begin{cases} \Omega_1(m_v, k_v) < \Omega_1^{cont} \\ \Omega_2(m_v, k_v) > \Omega_2^{cont} \end{cases}$$

The specific constraints can be determined as the following inequality, which should be satisfied to design $(m_v, k_v)$, by rearranging equation (14).

$$\begin{cases} \left( \frac{(\Omega_k^{cont})^2}{k_0} - \frac{1}{m_0} \right) k_v + (\Omega_k^{cont})^2 \geq \frac{1}{m_v} \\ \geq \left( \frac{(\Omega_k^{cont})^2}{k_0} - \frac{1}{m_0} \right) (\Omega_k^{cont})^2, \ (k = 1, 2) \end{cases}$$

Regarding $T_{xv1}(s)$ in equation (5), fluctuations in the gain and the phase characteristic occur near the natural frequencies and above $\Omega_2$. On the other hand, the gain value is almost 1, with a negligible phase delay in the frequency band between $\Omega_1^{cont}$ and $\Omega_2^{cont}$. Setting $k_v$ to be a sufficiently large finite value and $m_v$ to be a small finite value so as to satisfy equation (15) realizes model-free vibration suppression for resonance peaks in the predetermined controlled frequency band. Practically, the parameters are adjusted using the Bode diagram of $T_{xv1}(j\omega)$ in advance.

Figure 3 shows the Bode diagram of $T_{xv1}(j\omega)$ for the control system validated in this article. The fluctuations in the gain and phase characteristics cannot establish the desirable condition of equation (6) near the natural frequencies and above $\Omega_2$. However, the required characteristics where the gain value is almost 1 with a negligible phase delay are achieved in the frequency band of $f \in (f_1 \geq 20.6 \text{ Hz}, f_2 \geq 4.03 \times 10^4 \text{ Hz})$, which includes the controlled band.

In fuzzy approaches such as those in Edalath et al. (2012), there is not a clear design policy to determine the membership functions to realize model-free controllers.

On the other hand, because the clear design policy in equation (15) easily determines $m_v$ and $k_v$, only two design parameters actually need to be tuned when using a model-based control theory described later. In addition, since the model-free design focuses on the frequency response $T_{xv1}(s)$ off-line, the calculation loads of the fixed fourth-order controller in real time are smaller.

3. Controller design

In the proposed method, a controller with a high damping performance is easily obtained because the design after introducing the virtual object uses the same process as that of traditional and familiar model-based optimal controllers (Chilali and Gahinet, 1996). This is another practical superiority against the other traditional model-free technologies explained in Section 1.
The $(H_2/H_\infty)$ control theory considering high vibration control performance and robust stability is applied to the plant composed of the virtual object and the actuator to design a controller (Chilali and Gahinet, 1996; Nishidome and Kajiwara, 2003). Figure 4 shows a block diagram of the control system (Yonezawa et al., 2019).

$P(s)$ is the plant, where the actuator including the virtual object is regarded as a controlled object. $K(s)$ is the controller, $w$ is the disturbance, and $u$ is the control input. The observed output $y$ is the virtual object acceleration $\ddot{x}_v$, that is fed back to the controller. $z_2$ is the controlled variable with respect to the virtual object velocity $x_v$, which is evaluated by $H_2$ norm to achieve good vibration suppression over a wide frequency band. To realize robust stability, another controlled variable is $z_x$ with respect to the control input evaluated by the $H_\infty$ norm. Table 2 shows the weighting constants $W_2$ and $W_\infty$ for $z_2$ and $z_x$, respectively. The state equation of the generalized plant $G(s)$ is described as

$$
\begin{align*}
\dot{x}_G &= A_G x_G + B_{G1} w + B_{G2} u \\
z_x &= C_{Gx} x_G + D_{G1x} w + D_{G2x} u \\
z_2 &= C_{G2x} x_G + D_{G12} w + D_{G22} u \\
y &= C_{Gy} x_G + D_{G1y} w + D_{G2y} u
\end{align*}
$$

(16)

For the closed-loop system, the controller is designed to minimize the $H_2$ norm of the transfer function $T_{z2w}(s)$ from $w$ to $z_2$ under the $H_\infty$ norm constraint of the transfer function $T_{z2w}(s)$ from $w$ to $z_x$. Specifically, the optimization problem (18) is solved by the linear matrix inequality approach to obtain the controller. We used MATLAB’s `Control System Toolbox’ and ‘Robust Control Toolbox’ for the calculation.

$$
\begin{align*}
\min \|T_{z2w}(s)\|_2 \\
\text{subject to } \|T_{z2w}(s)\|_\infty < \gamma
\end{align*}
$$

(18)

4. Vibration control experiment

4.1. Configuration of the experimental system and verification condition

The proposed method is verified experimentally. Two different structures are used as controlled objects. Figure 5(a) and (b) shows a cantilever plate (190 mm × 248 mm × 10 mm) and a both-end-supported plate (548 mm × 100 mm × 10 mm), respectively. Both objects are composed of aluminum. A shaker is used to apply the disturbance to each object, and the load cell installed at the excitation point (Figure 5, red dot) measures the excitation force. The actuator (Figure 5, black dot) provides the control input in the $z$-axis direction. The observed output is measured by an accelerometer attached to the backside of the plate at the actuator location. The structures shown in Figure 5(a) and (b) have an optimal location to excite or suppress the
vibrations, which depends on each mode (Gupta et al., 2010). Before conducting control experiments, the influences of the locations on each mode of the structure must be investigated and the optimal actuator location must be determined for the mode to be controlled. This study used the actuator location, which can provide higher damping effects for the vibration modes on the low-frequency side, and the shaker location, which considers space limitations in the experiment. In particular, the actuator should be installed at the location where original and pure vibration control performances of the proposed controller can be clearly evaluated. In addition to the above two structures, control experiments using the cantilever plate shown in Figure 5(a) with a weight of 0.685 kg or 1.37 kg to cause characteristic variations are also carried out to verify the robustness. The purpose of giving two kinds of fluctuations (0.685 kg or 1.37 kg) in the natural frequencies and modes is to investigate the versatility of the same controller for various different structures. A total of four experiments are performed. The controlled frequency band is set from 50 Hz to 1000 Hz.

4.2. Control experiment results and discussion

Figures 7–10 show the frequency responses from the disturbance (load cell output) to the measured acceleration (sensor output), for the nominal cantilever plate, the cantilever plate with a weight of 0.685 kg, the cantilever plate with a weight of 1.37 kg, and the both-end-supported plate.
plate, respectively. The same controller is used in all experiments. The blue line and red line denote the response without control and the closed-loop frequency response with control in each graph, respectively. The controller designed in this study aims to reduce only the resonance peaks occurring at the natural frequencies, which may maximum the vibration amplitude and cause instability, for the frequency components of the disturbance. In particular, the damping effects for the resonance peaks on the low-frequency side, which are serious problems for practical uses, are more important.

Compared to the experiment without control, satisfactory vibration control performances are achieved for the cantilever plate (Figures 7–9). Table 3 shows the vibration reduction levels of major resonance peaks. The single model-free controller provides good vibration control characteristics at major peaks below 500 Hz. Moreover, the acceleration responses at other peaks between 500 Hz and 1 kHz do not deteriorate. Especially in the low-frequency band in Figures 8 and 9, the controller maintains vibration control characteristics similar to those with the nominal structure (Figure 7). That is, Figures 7–9 demonstrate that the resonance peaks on the low-frequency side are sufficiently reduced, thanks to the robustness of the same model-free controller, which was easily designed without using the plant model. Compared to the vibration reduction of the cantilever plate, the controller provides a higher control performance at the resonance peak near 224 Hz for the both-end-supported plate (Figure 10 and Table 3). The both-end-supported plate in Figure 10 has only the one clear resonance peak of the first mode at 224 Hz originally in the state of open-loop without control. Because the purpose of

![Figure 8. Frequency responses of the closed-loop system when the controlled object is the cantilever plate with a weight of 0.685 kg.](image)

![Figure 9. Frequency responses of the closed-loop system when the controlled object is the cantilever plate with a weight of 1.37 kg.](image)
control is to reduce only this resonance peak of the first mode, the red line in Figure 10 clearly demonstrates the effectiveness (damping effect) of the controller. All the experiments use the same controller, which is designed using the parameters in Tables 1 and 2, demonstrating the robustness of the proposed method against characteristic fluctuations in the actual object. Fluctuations in the natural frequencies and modes are induced in the target structure (Figures 7–9). In addition, the boundary conditions of the structure in Figure 10 completely differ from those in Figures 7–9. Consequently, the proposed method has two kinds of robustness: one against characteristic changes and one against structural changes. This is because the controller is designed without detailed mathematical models of the actual structures and is less susceptible to the modeling errors.

The actuator location on the plate induces performance variations. The optimal point to apply the control input depends on each mode in the case where the MDOF system with multiple modes is the controlled object (Gupta et al., 2010). Furthermore, detailed analysis of the control characteristics reveals that the reduction in the resonance peak for the both-end-supported plate (Figure 10) is larger than those in the cantilever plates (Figures 7–9). This difference is attributed to the fact that the both-end-supported plate has one major mode below 1 kHz, demonstrating that it can be regarded as the SDOF system and the actuator location is optimal to reduce the first mode vibration. Although the locations of the actuator and shaker affect the experimental results, the results certainly demonstrate the effectiveness of the proposed controller. This is because the excitations by the disturbance are sufficient, and control of the several modes is inherently difficult due to the actuator location even if a model-based controller is applied. Future tasks related to the actuator locations include the development of a model-free method to search for the optimal location and introducing multiple actuators.

Essentially, equation (6) enables effective active vibration control in the controlled frequency band without using a plant model. Figure 3 demonstrates that the required characteristics $|T_{\text{ext}}(j\omega)| \approx 1$ (gain) and $\phi = \tan^{-1}(\text{Im}(T_{\text{ext}})/\text{Re}(T_{\text{ext}})) \approx 0$ (phase) are almost obtained from 50 Hz to 1000 Hz, resulting in indirect vibration suppression for the actual objects. Figure 11 shows the time responses of acceleration from the control experiments for the cantilever plates, where Figure 11(a)–(c) corresponds to the controlled objects in Figures 7–9, respectively. The black line denotes the acceleration measured as the observed output, and the magenta dashed line shows the acceleration of the virtual object. Because the virtual object does not exist in the real system, $\ddot{x}_v$ cannot be measured. Therefore, the state vector (9) was estimated using the linear Kalman filter designed for the system of equation (8). The magenta dashed line shows the estimated value of $\ddot{x}_v$. The waveforms of $\ddot{x}_v$ almost match those of the acceleration of the real structures, indicating that the virtual object can be regarded as a controlled object when designing a controller. Even if the characteristics of the cantilever plate change, both accelerations consistently match each other as

![Figure 10. Frequency responses of the closed-loop system when the controlled object is the both-end-supported plate.](image)

| Experimental results (Figures) | Figure 7 | Figure 8 | Figure 9 | Figure 10 |
|-------------------------------|---------|---------|---------|---------|
| Major resonance frequency (Hz) | 114     | 257     | 105     | 190     |
| Amount of resonance peak reduction (dB) | -16.1 | -12.6 | -10.9 | -11.2 |
|                               | -9.6    | -17.2   | -21.2   |         |

Table 3. Vibration suppression performance at major resonance peaks obtained by the same model-free controller.
shown in Figure 11(a)–(c). From Figure 11, it is possible to visually and easily understand that the most important fundamental condition (6) to achieve model-free control is certainly satisfied. In addition, because one of the main points presented in this article is an experimental verification, this confirmation for condition (6) is crucial.

To enhance the reliability of the proposed controller, additional verifications by control simulations were also performed, and those results similar to the above experimental results were obtained. Figures 12 and 13 show the simulation results for the cantilever plate and the both-end-supported plate, respectively, where the color has the same meaning as in Figures 7 and 10. Here, based on the experimental modal analysis (Kajiwara et al., 2018; Zhang et al., 2017b), the plant models in the simulations are obtained by applying system identification (curve fitting) to the frequency responses of the structures shown in Figure 5(a) and (b). The model-free controller is the same as that used in the experiments. The fact that the same model-free controller provides good vibration suppressions for both structures confirms its robustness.
5. Conclusion

This research proposed model-free active vibration control based on the virtual object. The proposed technique realizes indirect vibration control without using plant models by inserting the SDOF virtual object between the actuator and the actual object. In addition, the state equation for controller design was derived, and the parameters of the virtual object were determined by considering the frequency transfer function from the vibration of the actual object to that of the virtual object. Next, the mixed $H_2/H_\infty$ controller was designed by a traditional model-based approach for the derived system. Finally, experiments verified the effectiveness of the proposed controller and confirmed the robustness of the controller by applying the same controller to various structures.

Future research involves three distinct tasks. The first is to derive a more quantitative design policy for the parameters of the virtual object. The second is to apply the present approach in various mechanical systems to declare its versatility. Third, we are planning to experimentally compare the model-free controller by the virtual object with the traditional approaches referenced in Section 1.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Itsuro Kajiwara https://orcid.org/0000-0003-2651-7289

References

Abdeljaber O, Avci O and Inman DJ (2016) Active vibration control of flexible cantilever plates using piezoelectric materials and artificial neural networks. *Journal of Sound and Vibration* 363: 33–53.

Bui HL, Nguyen CH, Bui VB, et al. (2017) Vibration control of uncertain structures with actuator saturation using hedge-algebras-based fuzzy controller. *Journal of Vibration and Control* 23(12): 1984–2002.

Chao PCP and Lai CL (2003) Boundary control of an axially moving string via fuzzy sliding mode control and fuzzy neural network methods. *Journal of Sound and Vibration* 262: 795–813.

Chilali M and Gahinet P (1996). $H_\infty$ design with pole placement constraints: an LMI approach. *IEEE Transactions on Automatic Control* 41(3): 358–367.

Edalath S, Kukreti AR and Cohen K (2012) Enhancement of a tuned mass damper for building structures using fuzzy logic. *Journal of Vibration and Control* 19(21): 1763–1772.

Gu X, Yu Y , Li J, et al. (2017) Semi-active control of magnetorheological elastomer base isolation system utilizing learning-based inverse model. *Journal of Sound and Vibration* 406: 346–362.

Gupta V , Shama M and Thakur N (2010) Optimization criteria for optimal placement of piezoelectric sensor and actuator on a smart structure: a technical review. *Journal of Intelligent Material Systems and Structures* 21(12): 1227–1243.

Kajiwara I, Furuya K and Ishizuka S (2018) Experimental verification of a real-time tuning method of a model-based controller by perturbations to its poles. *Mechanical Systems and Signal Processing* 107: 396–408.

Lee LH, Huang PH, Shih YC, et al. (2014) Parallel neural network combined with sliding mode control in overhead crane control system. *Journal of Vibration and Control* 20(5): 749–760.

Madan A (2005) Vibration control of building structures using self-organizing and self-learning neural networks. *Journal of Sound and Vibration* 287: 759–784.

Figure 13. Frequency responses of the closed-loop system obtained by control simulation for the both-end-supported plate model.
Malhis M, Gaudiller L and Hagopian JD (2005) Fuzzy modal active control of flexible structures. *Journal of Vibration and Control* 11: 67–88.

Marinaki M, Marinakos Y and Stavroulakis GE (2010) Fuzzy control optimized by PSO for vibration suppression of beams. *Control Engineering Practice* 18: 618–629.

Meurers T, Veres SM and Tan ACH (2003) Model-free frequency domain iterative active sound and vibration control. *Control Engineering Practice* 11: 1049–1059.

Nishidome C and Kajiwara I (2003) Motion and vibration control of flexible-link mechanism with smart structure. *JSME International Journal (C)* 46: 565–571.

Parameswaran AP, Ananthakrishnan B and Gangadharan KV (2015) Design and development of a model free robust controller for active control of dominant flexural modes of vibrations in a smart system. *Journal of Sound and Vibration* 355: 1–18.

Song C, Zhou Z, Xie S, et al. (2015) Fuzzy control of a semi-active multiple degree-of-freedom vibration isolation system. *Journal of Vibration and Control* 21(8): 1608–1621.

Swevers J, Lauwerys C, Vandersmissen B, et al. (2007) A model-free control structure for the on-line tuning of the semi-active suspension of a passenger car. *Mechanical Systems and Signal Processing* 21: 1422–1436.

Thenozhi S and Yu W (2015) Active vibration control of building structures using fuzzy proportional-derivative/proportional-integral-derivative control. *Journal of Vibration and Control* 21(12): 2340–2359.

Wang J, Jin F, Zhou L, et al. (2019) Implementation of model-free motion control for active suspension systems. *Mechanical Systems and Signal Processing* 119: 589–602.

Wei C, Luo J, Dai H, et al. (2017) Adaptive model-free constrained control of postcapture flexible spacecraft: a Euler-Lagrange approach. *Journal of Vibration and Control* 24: 4885–4903.

Yang SM, Chen CJ and Huang WL (2006) Structural vibration suppression by a neural-network controller with a mass-damper actuator. *Journal of Vibration and Control* 12(5): 495–508.

Yao J, Jiang G, Gao S, et al. (2014) Particle swarm optimization-based neural network control for an electro-hydraulic servo system. *Journal of Vibration and Control* 20(9): 1369–1377.

Yigit I (2017) Model free sliding mode stabilizing control of a real rotary inverted pendulum. *Journal of Vibration and Control* 23(10): 1645–1662.

Yildirim S (2004) Vibration control of suspension systems using a proposed neural network. *Journal of Sound and Vibration* 277: 1059–1069.

Yin K, Zhao H and Lu L (2018) Functional link artificial neural network filter based on the q-gradient for nonlinear active noise control. *Journal of Sound and Vibration* 435: 205–217.

Yonezawa H, Kajiwara I, Sato S, et al. (2019) Vibration control of automotive drive system with nonlinear gear backlash. *Journal of Dynamic Systems, Measurement, and Control* 141(12): 121002.

Yousefi H, Hirvonen M, Handroos H, et al. (2008) Application of neural network in suppressing mechanical vibration of a permanent magnet linear motor. *Control Engineering Practice* 16: 787–797.

Zhang Q, Wang S, Zhang A, et al. (2017a) Improved PI neural network-based tension control for stranded wire helical springs manufacturing. *Control Engineering Practice* 67: 31–42.

Zhang Y, Hiruta T, Kajiwara I, et al. (2017b) Active vibration suppression of membrane structures and evaluation with a non-contact laser excitation vibration test. *Journal of Vibration and Control* 23(10): 1681–1692.

Zhang QZ and Gan WS (2004) Active noise control using a simplified fuzzy neural network. *Journal of Sound and Vibration* 272: 437–449.