The properties of magnitude ranking function of trapezoidal fuzzy numbers

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Abstract. There are many ranking function of a fuzzy number. Yager, Maleki, Adamo, centre of maxima, centre of gravity, and soon. This paper discusses about the ranking function for trapezoidal fuzzy number that is called magnitude method. We get some properties related to the arithmetic operation. The other result says that this magnitude method has ordered properties.

1. Introduction
The fuzzy number ranking function plays an important role of the decision making process in many applications. Several strategies have been proposed for ranking fuzzy number. It is more than twenty fuzzy ranking function definitions have been proposed since 1976. In 1976, [1] introduces a method using the concept of maximizing sets to sort fuzzy numbers. He defines the fuzzy number ranking method only considers the right side membership function. Refer to [2], they use the maximization set to order the fuzzy numbers. In 1980, [3] has proposed the concept of α-level as the basis of α-preference terms. In [4], 1982, Yager has proposed four indexes which could be used for ordering fuzzy quantities at closed intervals [0,1]. Specifically, he defines a ranking function for the trapezium fuzzy number, that is denoted by $\hat{A} = (a^L, a^U, a, \beta)$. It is called Yager’s ranking function for trapezoidal fuzzy number. Based on these definitions, [4] have developed the application of this ranking function to create a new method to solve the linear fuzzy programming problems, which the coefficient of the objective function is trapezoidal fuzzy numbers. This method is called fuzzy simplex method. Maleki, in [5], has also defined the ranking function of trapezoidal fuzzy numbers, which is different from Yager’s ranking function. Based on this Maleki’s definition, a fuzzy simplex method has been developed to solve a trapezoidal fuzzy number linear programming problem. There is a difference between Yager’s and Maleki’s ranking function. Yager’s fuzzy ranking function is constrained for certain cases. As an example in cases when there are more than one optimal solutions. This method has not been able to detect the completion of more than one solution and it produced a different optimal solution. At present, many researchers have developed methods for comparing and defining fuzzy number ranking functions [6,7,8, 9, 10, 11].

The aims of this paper to discuss about the other ranking function of trapezoidal fuzzy number. This function is called magnitude that is defined by [11]. This paper is organized as follow: Section 1, we introduce about many definitions of fuzzy ranking function for fuzzy numbers. We refer these definitions that are defined by several researchers. Section 2 contains many definitions and notations which will be used for this research. Section 3 explain about the results of this research. We get some results, i.e. the
properties of magnitude, the arithmetical properties and the ordering properties. In section 4, give the conclusion of this research. Section 5 presents an acknowledgment for research funder.

2. Preliminaries
In this section, we give some definitions that will be used for this research, related to fuzzy sets, the notations of the components of a fuzzy number, etc. Let \( X \) is a set which has element \( x \). A fuzzy set \( A \) of \( X \) is defined as a set of ordered pair \( A = \{(x, \mu_A(x)) | x \in X \} \), with \( \mu_A(x) \) is a membership function of this fuzzy set. The membership function maps from \( X \) into the closed interval \([0,1]\). Support of a fuzzy set \( A \) is a set of all \( x \in X \) with \( \mu_A(x) > 0 \). Core of a fuzzy set \( A \) is a set of all \( x \in X \) which \( \mu_A(x) = 1 \). A fuzzy set is called normal if the Core is not an empty set.

The crisp set that is sawn the degree of the membership is a certain number, is called \( \alpha - \text{cut} \). It is defined as follow.

**Definition 2.1.** ([4], [5], [6], [7]) An \( \alpha \)-cut (level subset - \( \alpha \)) is a fuzzy set \( A \) is a crisp set that defined as \( A_\alpha = \{x \in X | \mu_A(x) \geq \alpha \} \). A strong \( \alpha \)-cut (strong level subset-\( \alpha \)) is defined as \( A_\alpha^\sharp = \{x \in X | \mu_A(x) > \alpha \} \).

Related to crisp sets, there are sets that are convex. Related to this, analogically defined fuzzy sets are convex as follows.

**Definition 2.2.** ([4], [5], [6], [7]) A fuzzy set \( A \) of \( X \) is called convex if for every \( x,y \in X \) and \( \delta \in [0,1] \) then \( \mu_A(\delta x + (1-\delta)y) \geq \min\{\mu_A(x),\mu_A(y)\} \).

There are many properties of fuzzy sets related to their \( \alpha \)-cuts. For example, A fuzzy set is a fuzzy semigroup if and only if all of the \( \alpha \)-cut that is not empty set is a semigroup. Its must be remembered that \( \alpha \)-cut is a crisp set. The following lemma tells about the equivalence between the convex fuzzy sets and their \( \alpha \)-cut.

**Lemma 2.3.** ([4], [5], [6], [7]) A fuzzy set \( A \) of \( X \) convex if and only if all of the \( \alpha \)-cut that is not empty set is convex.

There are many normal fuzzy sets. On the other hand, there are many convex fuzzy sets. We will collect the fuzzy sets that are normal and convex. This sets is called fuzzy numbers. The definition of fuzzy number is given in the following definition.

**Definition 2.4.** ([4], [5],[6], [7]) A fuzzy set is called a fuzzy number if normal and convex.

The special fuzzy number which has trapezoidal graph is called a trapezoidal fuzzy number. The following definition give this definition including its mapping rule.

**Definition 2.5.** ([4],[5]) A fuzzy number \( \tilde{A} = (a_L, a_U, \alpha, \beta) \) is called a trapezoidal fuzzy number if it is fulfill the following mapping.

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-(a_L-\alpha)}{\alpha}, & a_L - \alpha \leq x \leq a_L \\
1, & a_L \leq x \leq a_U \\
\frac{(a_U+\beta)-x}{\beta}, & a_U \leq x \leq a_U + \beta \\
0, & \text{others}
\end{cases}
\]

The graphic of the trapezoidal fuzzy number give in the Figure 1.
According to the Figure 1, the support of \( \tilde{A} = (a_l, a_u, \alpha_1, \alpha_2) \) is the closed interval \([a_l - \alpha, a_u + \beta]\). The arithmetical operations of the trapezoidal fuzzy number are defined as follow, [4, 5]. Let \( \tilde{A} = (a_l, a_u, \alpha, \beta) \) and \( \tilde{B} = (b_l, b_u, \gamma, \delta) \) be trapezoidal fuzzy numbers and \( r \in \mathbb{R} \), hence we have:

i. For \( r > 0 \), \( r\tilde{A} = (ra_l, r a_u, r\alpha, r\beta) \)

ii. For \( r < 0 \), \( r\tilde{A} = (ra_u, r a_l, -r\beta, -r\alpha) \)

iii. \( \tilde{A} + \tilde{B} = (a_l + b_l, a_u + b_u, \alpha + \gamma, \beta + \delta) \)

iv. \( \tilde{A} - \tilde{B} = (a_l - b_l, a_u - b_u, \alpha + \delta, \beta + \gamma) \)

2.1. Ranking function of fuzzy number

We refer the definition of fuzzy number to [4], [5]. A fuzzy number \( \tilde{A} = (a_l, a_u, \alpha, \beta) \) denote a trapezoidal fuzzy number. Let \( F(\mathbb{R}) \) be a set of all fuzzy numbers. A ranking function is a mapping \( \mathcal{R}: F(\mathbb{R}) \to \mathbb{R} \), from a set of fuzzy number into real number. For two arbitrary trapezoidal fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) are defined ordering as follow.

i. \( \tilde{A} \preceq \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) \leq \mathcal{R}(\tilde{B}) \),

ii. \( \tilde{A} \succeq \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) \geq \mathcal{R}(\tilde{B}) \),

iii. \( \tilde{A} \approx \tilde{B} \) if and only if \( \mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{B}) \).

Let \( \tilde{A}(x) \) be a membership function, for every \( x \in \mathbb{R} \). This function has properties as follow, [11]:

i. \( \tilde{A}(x) \) is upper semi-continuous

ii. \( \tilde{A}(x) = 0 \), for \( x \notin [a_l - \alpha, a_u + \beta] \)

iii. There are real numbers \( a^l, a^u \) such that \( a_l - \alpha \leq a^l \leq a_u \leq a^u + \beta \) and:

a. \( \tilde{A}(x) \) is increasing in the interval \( a_l - \alpha \leq x \leq a^l \)

b. \( \tilde{A}(x) \) is monotone decreasing in interval \( a_u \leq x \leq a^u + \beta \)

c. \( \tilde{A}(x) = 1 \), \( a^l \leq x \leq a^u \)

Furthermore, \( \tilde{A}_L \) denotes a function from \( a_l - \alpha \leq x \leq a_l \) into the closed interval \([0,1]\). This function is called left membership function. The function \( \tilde{A}_U \) is a function from interval \( a_u \leq x \leq a^u + \beta \) into the closed interval \([0,1]\). It is called right membership function.

The parametric form of a fuzzy number \( \tilde{A} \) is an ordered pair \( (\tilde{A}_L, \tilde{A}_U) \) from the functions \( \tilde{A}_L(r), \tilde{A}_U(r) \) with \( 0 \leq r \leq 1 \), \( \tilde{A}_L(r) = a_l - \alpha + ar \) and \( \tilde{A}_U(r) = a^u + \beta + br \), \( 0 \leq r \leq 1 \).

3. Results: arithmetical operations and ordering properties

This section tell about the results of this research related to the magnitude ranking function for trapezoidal fuzzy number. For arithmetical operation, i.e. addition and scalar multiplication of fuzzy numbers have the following properties.

Let \( \tilde{A} = (a_l, a_u, \alpha, \beta) \), \( \tilde{B} = (b_l, b_u, \alpha', \beta') \) be arbitrary fuzzy numbers.

\[
\begin{align*}
(a_l - \alpha) & \quad (a_u - \alpha) & \quad (a_l - \alpha) + (a_u - \alpha)
\end{align*}
\]

**Proof.**

\[
\begin{align*}
\tilde{A} + \tilde{B} &= (a_l + b_l, a_u + b_u, \alpha + \gamma, \beta + \delta) \\
(a_l + b_l)(r) &= (a_l + b_l) - (a + \alpha' + (\alpha + \alpha')r) \\
&= (a_l - \alpha + \alpha r) + (b_l - \alpha' + \alpha' r) \\
&= (\tilde{A}_L(r) + (\tilde{B}_L(r)) \\
(b_l - \alpha) & \quad (b_u - \alpha) & \quad (b_l - \alpha) + (b_u - \alpha)
\end{align*}
\]
Proof.
\[ \bar{A} - \bar{B} = (a^L, a^U, \alpha, \beta) + (-b^U, -b^L, \beta', \alpha') \]
\[ = (a^L - b^U, a^U - b^L, \alpha + \beta', \beta + \alpha') \]
\[ (\bar{A} - \bar{B})_L(r) = (a^L - b^U) - (\alpha + \beta') + (\alpha + \beta')r \]
\[ = (a^L - \alpha + \alpha r) + (-b^U - \beta' + \beta' r) \]
\[ = (\bar{A})_L(r) + (-\bar{B})_L(r) = (\bar{A})_L(r) - (\bar{B})_L(r) \]

c. \[ (k\bar{A})_L(r) = k\bar{A}_L(r) \]

Proof.
For case \( k \geq 0 \), we obtain:
\[ k\bar{A} = (ka^L, ka^U, k\alpha, k\beta) \]
\[ (k\bar{A})_L(r) = ka^L - k\alpha + k\alpha r = k(\alpha^L - \alpha + \alpha r) = k(r) \]

For case \( k < 0 \), we have:
\[ k\bar{A} = (ka^U, ka^L, -k\beta, -k\alpha) \]
\[ (k\bar{A})_L(r) = ka^U - k\beta + k\beta r = k(\alpha^U - \beta + \beta r) = k\bar{A}_L(r) \]

d. \[ (\bar{A} + \bar{B})_U(r) = (\bar{A})_U(r) + (\bar{B})_U(r) \]

Proof.
\[ \bar{A} + \bar{B} = (a^L + b^L, a^U + b^U, \alpha + \alpha', \beta + \beta') \]
\[ (\bar{A} + \bar{B})_U(r) = (a^U + b^U) + (\beta + \beta') - (\beta + \beta') r \]
\[ = (a^U + \beta - \beta r) + (b^U + \beta' - \beta' r) = (\bar{A})_U(r) + (\bar{B})_U(r) \]

e. \[ (\bar{A} - \bar{B})_U(r) = (\bar{A})_U(r) - (\bar{B})_U(r) = (\bar{A})_U(r) + (-\bar{B})_U(r) \]

Proof.
\[ \bar{A} - \bar{B} = (a^L, a^U, \alpha, \beta) + (-b^U, -b^L, \beta', \alpha') \]
\[ = (a^L - b^U, a^U - b^L, \alpha + \beta', \beta + \alpha') \]
\[ (\bar{A} - \bar{B})_U(r) = (a^U - b^L) + (\beta + \alpha') - (\beta + \alpha') r \]
\[ = (a^U + \beta - \beta r) + (-b^L + \alpha' - \alpha' r) \]
\[ = (\bar{A})_U(r) + (-\bar{B})_U(r) = (\bar{A})_U(r) - (\bar{B})_U(r) \]

f. \[ (k\bar{A})_U(r) = k\bar{A}_U(r) \]

Proof.
For case \( k \geq 0 \), then we have:
\[ k\bar{A} = (ka^L, ka^U, k\alpha, k\beta) \]
\[ (k\bar{A})_U(r) = ka^L + k\beta - k\beta r = k(\alpha^U + \beta - \beta r) = k\bar{A}_U(r) \]

For case \( k < 0 \), it applies:
\[ k\bar{A} = (ka^U, ka^L, -k\beta, -k\alpha) \]
\[(k\tilde{A})_y(r) = ka^l - k\alpha + k\alpha r = k(a^l - \alpha + \alpha r) = k\tilde{A}_y(r)\]

For a trapezoidal fuzzy number \(\tilde{A} = (a^l, a^u, \alpha, \beta)\), we defined \(\tilde{A}_L(r) = a^l - \alpha + \alpha r\) and \(\tilde{A}_U(r) = a^u + \beta - \beta r\), with \(0 \leq r \leq 1\). Based on these definition, [11] defined a function from a set of trapezoidal fuzzy number into real number, i.e.

\[\mathcal{M}(\tilde{A}) = \frac{1}{2}\left(\int_0^1 (\tilde{A}_L(r) + \tilde{A}_U(r) + a^l + a^u)h(r) \, dr\right)\]

with \(h(r)\) is a non negative function, increasing function on the closed interval, \(h(0) = 0\), \(h(1) = 1\) and \(\int_0^1 h(r) = \frac{1}{2}\). There are many functions fulfill these properties. One of them is \(h(r) = r\).

To describe the definition of the magnitude of trapezoidal fuzzy numbers, we give the following some examples.

**Example 3.1.** Let \(\tilde{A} = (2,5,1,2)\) and \(\tilde{B} = (0,4,2,3)\). Find the \(\mathcal{M}(\tilde{A}), \mathcal{M}(\tilde{B}), \mathcal{M}(\tilde{A} + \tilde{B}), \mathcal{M}(\tilde{A} - \tilde{B}), \mathcal{M}(3\tilde{A})\).

**Answer.**

i. \(\tilde{A}_L(r) = a^l - \alpha + \alpha r = 2 - 1 + r = 1 + r\), \(\tilde{A}_U(r) = a^u + \beta - \beta r = 5 + 2 - 2r = 7 - 2r\)

\[\mathcal{M}(\tilde{A}) = \frac{1}{2}\left(\int_0^1 (\tilde{A}_L(r) + \tilde{A}_U(r) + a^l + a^u)h(r) \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 ((1 + r) + (7 - 2r) + 2 + 5)r \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 (15 - r)r \, dr\right) = \frac{1}{2}\left(\int_0^1 (15r - r^2) \, dr\right) = \frac{1}{2}\left[\frac{15}{2}r^2 - \frac{1}{3}r^3\right]_0^1 = \frac{1}{2}\left(\frac{15}{2} - \frac{1}{3}\right) = \frac{1}{2}\left(\frac{25}{6}\right) = \frac{25}{12}\]

ii. \(\tilde{B}_L(r) = b^l - \gamma + \gamma r = 0 - 2 + 2r = 1 + r\), \(\tilde{B}_U(r) = b^u + \delta - \delta r = 4 + 3 - 3r = 7 - 3r\)

\[\mathcal{M}(\tilde{B}) = \frac{1}{2}\left(\int_0^1 ((-2 + 2r) + (7 - 3r) + 0 + 4)r \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 (9 - r)r \, dr\right) = \frac{1}{2}\left(\int_0^1 9r^2 - \frac{1}{3}r^3 \right)_0^1 = \frac{1}{2}\left(\frac{9}{2} - \frac{1}{3}\right) = \frac{1}{2}\left(\frac{25}{6}\right) = \frac{25}{12}\]

iii. \(\tilde{A} + \tilde{B} = (2,5,1,2) + (0,4,2,3) = (2,9,3,5) = \tilde{C}\)

\(\tilde{C}_L(r) = 2 - 3 + 3r = -1 + 3r\), \(\tilde{C}_U(r) = 9 + 5 - 5r = 14 - 5r\)

\[\mathcal{M}(\tilde{A} + \tilde{B}) = \mathcal{M}(\tilde{C}) = \frac{1}{2}\left(\int_0^1 (\tilde{C}_L(r) + \tilde{C}_U(r) + c^l + c^u)r \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 (-1 + 3r + 14 - 5r + 2 + 9)r \, dr\right) = \frac{1}{2}\left(\int_0^1 (24 - 2r)r \, dr\right)\]

\[= \frac{1}{2}\left(12 - \frac{2}{3}\right) = \frac{1}{2}\left(\frac{35}{3}\right) = \frac{34}{6}\]

Hence, we obtain \(\mathcal{M}(\tilde{C}) = \frac{34}{6}\).

iv. \(\tilde{A} - \tilde{B} = (2,5,1,2) - (0,4,3,2) = (-2,5,4,4) = \tilde{D}\)

\(\tilde{D}_L(r) = -2 - 4 + 4r = -6 + 4r\), \(\tilde{D}_U(r) = 5 + 4 - 4r = 9 - 4r\)

\[\mathcal{M}(\tilde{A} - \tilde{B}) = \mathcal{M}(\tilde{D}) = \frac{1}{2}\left(\int_0^1 (\tilde{D}_L(r) + \tilde{D}_U(r) + d^l + d^u)r \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 ((-6 + 4r) + (9 - 4r) - 2 + 5)r \, dr\right)\]

\[= \frac{1}{2}\left(\int_0^1 6r \, dr\right) = \frac{3}{2}\]

v. \(\tilde{E} = 3\tilde{A} = (6,15,3,6)\)

\(\tilde{E}_L(r) = 6 - 3 + 3r = 3 + 3r\), \(\tilde{E}_U(r) = 15 + 6 - 6r = 21 - 6r\)

\[= \frac{1}{2}\left(\int_0^1 (3 + 3r) + (21 - 6r) + 6 + 15) \, dr\right)\]
\[= \frac{1}{2} \int_0^1 (45 - 3r) r \, dr = \frac{1}{2} \left[ \frac{45}{2} r - r^3 \right]_0^1 = \frac{1}{2} \left( \frac{45}{2} - 1 \right) = \frac{1}{2} \cdot \frac{43}{2} = \frac{43}{4} \]

The value of \( \mathcal{M}(\tilde{A}) \) sign the degree of membership, which can be consider as a ranking function. Related the ordering between trapezoidal fuzzy number and their Magnitude, the Magnitude function is preserve an ordering.

Related the ordering between trapezoidal fuzzy number and their Magnitude, for arbitrary trapezoidal fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \), we obtain the following properties:

i. \( \mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B}) \) if and only if \( \tilde{A} > \tilde{B} \)

ii. \( \mathcal{M}(\tilde{A}) < \mathcal{M}(\tilde{B}) \) if and only if \( \tilde{A} < \tilde{B} \)

iii. \( \mathcal{M}(\tilde{A}) = \mathcal{M}(\tilde{B}) \) if and only if \( \tilde{A} \approx \tilde{B} \)

iv. \( \mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B}) \) if and only if \( \tilde{A} \geq \tilde{B} \) or \( \tilde{A} \approx \tilde{B} \)

v. \( \mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B}) \) if and only if \( \tilde{A} \geq \tilde{B} \) or \( \tilde{A} \approx \tilde{B} \)

The corollary of the previous properties, the following properties are hold:

i. If \( \inf \sup(\tilde{A}) \geq 0 \) or \( \inf \tilde{A}(r) \geq 0 \) then \( \mathcal{M}(\tilde{A}) \geq 0 \)

ii. If \( \sup \sup(\tilde{A}) \geq 0 \) or \( \sup (\tilde{A}(r)) \geq 0 \) then \( \mathcal{M}(\tilde{A}) \leq 0 \)

iii. For arbitrary trapezoidal fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \), we have \( \mathcal{M}(\tilde{A} + \tilde{B}) = \mathcal{M}(\tilde{A}) + \mathcal{M}(\tilde{B}) \)

**Proof.** Let \( \tilde{A} = (a^L, a^U, \alpha, \beta) \) and \( \tilde{B} = (b^L, b^U, \gamma, \delta) \), so we get

\[
\tilde{C} = \tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \delta)
\]

\[
\tilde{C}(r) = (a^L + b^L) - (\alpha + \gamma) + (\alpha + \gamma) r = (a^L - \alpha + a r) + (b^L - \gamma + \gamma) r = \tilde{A}(r) + \tilde{B}(r)
\]

By the same way, we get \( \tilde{C}(r) = \tilde{A}(r) + \tilde{B}(r) \). As the consequence, we have:

\[
\mathcal{M}(\tilde{C}) = \frac{1}{2} \int_0^1 (\tilde{c}(r)+\tilde{c}(r)+c^L+c^U)r \, dr = \frac{1}{2} \int_0^1 ((\tilde{a}(r)+\tilde{b}(r))+\tilde{a}(r)+\tilde{b}(r) + (a^L+b^L)) r \, dr
\]

\[
= \frac{1}{2} \int_0^1 \left( ((\tilde{a}(r)+\tilde{b}(r))+\tilde{a}(r)+\tilde{b}(r) + (a^L+b^L) r \, dr
\right) + \frac{1}{2} \int_0^1 (\tilde{b}(r)+\tilde{b}(r)+b^L+b^U) r \, dr)
\]

\[
= \mathcal{M}(\tilde{A}) + \mathcal{M}(\tilde{B})
\]

iv. For an arbitrary symmetry trapezoidal fuzzy number, i.e. \( \tilde{A} = (-a^L, a^L, \alpha, \alpha) \), then \( \mathcal{M}(\tilde{A}) = 0 \).

**Proof.** \( \tilde{A}(r) = -a^L + \alpha - a r, \tilde{A}(r) = a^L - \alpha + a r, \)

\[
\mathcal{M}(\tilde{A}) = \frac{1}{2} \int_0^1 ((-a^L + \alpha - a r) + (a^L + \alpha + a r)-a^L, -a^L) r \, dr = \frac{1}{2} \int_0^1 0 \, dr = 0
\]

v. For two arbitrary symmetry trapezoidal fuzzy numbers, i.e. \( \tilde{A} = (a^L, a^U, \alpha, \alpha), \tilde{B} = (a^L, a^U, \beta, \beta), \)

then \( \mathcal{M}(\tilde{A} + \tilde{B}) = \mathcal{M}(\tilde{B}) \)

**Proof.** \( \tilde{A}(r) = a^L + \alpha - a r, \tilde{A}(r) = a^U - \alpha + a r, \)

\[
\mathcal{M}(\tilde{A}) = \frac{1}{2} \int_0^1 ((a^L + \alpha - a r) + (a^U - \alpha + a r)+a^L + a^U) r \, dr = \frac{1}{2} \int_0^1 (a^L + a^U) r \, dr
\]

\[
= \frac{1}{2} \int_0^1 ((a^L + \beta - \beta r) + (a^U - \beta + \beta r+a^L + a^U) r \, dr = \mathcal{M}(\tilde{B})
\]

Let \( \Omega, \Omega' \) be finite subset of the collection of trapezoidal fuzzy numbers. Related to these sets, we get many properties which are declared in the following lemmas.

**Lemma 3.1.** For every \( \tilde{A} \in \Omega, \tilde{A} \succeq \tilde{A} \)

**Proof.** Trivial.
Lemma 3.2. Let $\tilde{A}, \tilde{B} \in \Omega$. If $\tilde{A} \triangleright \tilde{B}$, $\tilde{B} \triangleright \tilde{A}$, then $\tilde{A} \simeq \tilde{B}$.

Proof. If $\tilde{A} \triangleright \tilde{B}$, then $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$. Based on the previous property, we obtain:

$$\tilde{A} \triangleright \tilde{B} \quad \text{or} \quad \tilde{A} \simeq \tilde{B}$$

(3.1)

Furthermore, analogously with the condition $\tilde{B} \triangleright \tilde{A}$ has consequence $\mathcal{M}(\tilde{B}) \geq \mathcal{M}(\tilde{A})$, so we get

$$\tilde{B} \triangleright \tilde{A} \quad \text{at} \quad \tilde{B} \triangleright \tilde{A}$$

(3.2)

Based on the (3.1) and (3.2), we prove that $\tilde{A} \simeq \tilde{B}$.

Lemma 3.3. If $\tilde{A}, \tilde{B}, \tilde{C} \in \Omega$, $\tilde{A} \triangleright \tilde{B}$, $\tilde{B} \triangleright \tilde{C}$, then $\tilde{A} \triangleright \tilde{C}$

Proof. If $\tilde{A} \triangleright \tilde{B}$, then $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$. It has consequence the following conditions

$$\tilde{A} \triangleright \tilde{B} \quad \text{or} \quad \tilde{A} \simeq \tilde{B}$$

(3.3)

Analogously, if $\tilde{B} \triangleright \tilde{C}$, then $\mathcal{M}(\tilde{B}) \geq \mathcal{M}(\tilde{C})$. Hal ini berarti berlaku jika dipenuhi:

$$\tilde{B} \triangleright \tilde{C} \quad \text{atau} \quad \tilde{B} \simeq \tilde{C}$$

(3.4)

Refer to (3.3) and (3.4) we have conditions $\tilde{A} \triangleright \tilde{C}$ or $\tilde{A} \simeq \tilde{C}$. Then we have $\mathcal{M}(\tilde{B}) \geq \mathcal{M}(\tilde{C})$. It can be conclude that $\tilde{A} \triangleright \tilde{C}$.

Lemma 3.4. Let $\tilde{A}, \tilde{B} \in \Omega$. Then we have:

(i) If $\inf \sup(\tilde{A}) \geq \sup \sup(\tilde{B})$, then $\tilde{A} \triangleright \tilde{B}$

(ii) If $\inf \sup(\tilde{A}) > \sup \sup(\tilde{B})$, then $\tilde{A} > \tilde{B}$

Proof. Omitted

Lemma 3.5. Let $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$. $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ in $\Omega'$ if and only if $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ ini $\Omega$.

Proof. $(\Rightarrow)$ It is known that $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ in $\Omega'$, then $\tilde{A} > \tilde{B}$ in $\Omega'$. It is also known that $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$, so it is fulfill that $\tilde{A} > \tilde{B}$ in $\Omega$. Hence $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ in $\Omega$.

$(\Leftarrow)$ By reversing the proof, we have $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ in $\Omega$, so that $\tilde{A} > \tilde{B}$ in $\Omega$. It is known that $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$, then we have $\tilde{A} > \tilde{B}$ in $\Omega'$. Accordingly, $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ in $\Omega'$.

Corollary 3.6. Let $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$. $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega'$ if and only if $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega$.

Proof. $(\Rightarrow)$ We have $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega'$, hence $\tilde{A} > \tilde{B}$ or $\tilde{A} \simeq \tilde{B}$ di $\Omega'$. It is known that $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$, The consequence is $\tilde{A} > \tilde{B}$ or $\tilde{A} \simeq \tilde{B}$ in $\Omega$. Accordingly, we get $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega$.

$(\Leftarrow)$ By reversing the proof, we have $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega$. The consequence is $\tilde{A} > \tilde{B}$ or $\tilde{A} \simeq \tilde{B}$ in $\Omega$. It is known that $\tilde{A}, \tilde{B} \in \Omega \cap \Omega'$. Accordingly, $\tilde{A} > \tilde{B}$ or $\tilde{A} \simeq \tilde{B}$ in $\Omega'$. Hence, $\mathcal{M}(\tilde{A}) \geq \mathcal{M}(\tilde{B})$ in $\Omega'$.

Lemma 3.7. Let $\tilde{A}, \tilde{B}, \tilde{A} + \tilde{C}, \tilde{B} + \tilde{C}$ are trapezoidal fuzzy numbers. If $\tilde{A} > \tilde{B}$, then $\tilde{A} + \tilde{C} > \tilde{B} + \tilde{C}$.

Proof. It is known that $\tilde{A} > \tilde{B}$. So, we have $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$. Both of sides are added by $\mathcal{M}(\tilde{C})$. We know that $0 \leq \mathcal{M}(\tilde{C}) \leq 1$. Therefor $\mathcal{M}(\tilde{A}) + \mathcal{M}(\tilde{C}) > \mathcal{M}(\tilde{B}) + \mathcal{M}(\tilde{C})$. Refer to the property of the function $\mathcal{M}$, we obtain $\mathcal{M}(\tilde{A} + \tilde{C}) > \mathcal{M}(\tilde{B} + \tilde{C})$. The consequence is $\tilde{A} + \tilde{C} > \tilde{B} + \tilde{C}$.

Corollary 3.8. Let $\tilde{A}, \tilde{B}, \tilde{A} + \tilde{C}, \tilde{B} + \tilde{C}$ are trapezoidal fuzzy numbers. If $\tilde{A} \geq \tilde{B}$, then $\tilde{A} + \tilde{C} \geq \tilde{B} + \tilde{C}$.

Proof. We have $\tilde{A} \geq \tilde{B}$, then $\tilde{A} > \tilde{B}$ or $\tilde{A} \simeq \tilde{B}$. It means that $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$ or $\mathcal{M}(\tilde{A}) = \mathcal{M}(\tilde{B})$. There are two cases. The first case $\mathcal{M}(\tilde{A}) > \mathcal{M}(\tilde{B})$. Based on the Lemma 3.7, it has been proven. The second case is $\mathcal{M}(\tilde{A}) = \mathcal{M}(\tilde{B})$. We can add $\mathcal{M}(\tilde{C})$ on the both of sides. We get $\mathcal{M}(\tilde{A}) + \mathcal{M}(\tilde{C}) = \mathcal{M}(\tilde{B}) + \mathcal{M}(\tilde{C})$, because $0 \leq \mathcal{M}(\tilde{C}) \leq 1$. According to the property of function $\mathcal{M}$, we obtain $\mathcal{M}(\tilde{A} + \tilde{C}) \geq \mathcal{M}(\tilde{B} + \tilde{C})$. Hence, $\tilde{A} + \tilde{C} \approx \tilde{B} + \tilde{C}$. Finally we have proven that $\tilde{A} + \tilde{C} \geq \tilde{B} + \tilde{C}$. 


4. Conclusions
   Based on the previous discussion on the section 3, we can conclude that the Magnitude function forms a fuzzy ranking function of trapezoidal fuzzy number. Beside that, the properties of arithmetical and ordering numbers are fulfilled by this magnitude ranking function. Thus, the magnitude ranking function preserves the arithmetical operation and the ordering numbers.

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