On the Impact of Random Residual Calibration Error on the Gibbs ILC CMB Estimates over Large Angular Scales

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Abstract

Residual errors in calibration coefficients corresponding to observed cosmic microwave background (CMB) maps are an important issue when estimating a pure CMB signal. These errors in the input-foreground-contaminated CMB maps, if not properly taken into account in a component separation method, may lead to bias in the cleaned CMB map and estimated CMB angular power spectrum. But the inability to exactly determine the calibration coefficients corresponding to each observed CMB map from a multifrequency CMB experiment makes it very difficult to incorporate their exact and actual values during the component separation method. Hence, the effect of any random and residual calibration error in the cleaned CMB map and its angular power spectrum of a component separation problem can only be understood by performing detailed Monte Carlo simulations. In this paper, we investigate the impact of using input-observed CMB maps with random calibration errors on the posterior density of a cleaned CMB map and theoretical CMB angular power spectrum over large angular scales of the sky following the Gibbs Internal-Linear-Combination (ILC) method. By performing detailed Monte Carlo simulations of WMAP and Planck temperature anisotropy observations, including their estimate on calibration errors, we show that the best-fit map corresponding to the posterior maximum is minimally biased in the Gibbs ILC method by a CMB normalization bias and residual foreground bias. The residual calibration-induced error in the best-fit power spectrum causes an overall 6% increase of the net error when added in quadrature with the cosmic-variance-induced error.

Unified Astronomy Thesaurus concepts: Cosmic microwave background radiation (322); Gibbs Sampler (1891); Posterior distribution (1926)

1. Introduction

In the era of precision cosmology, for cosmic microwave background (CMB) temperature anisotropy and over large angular scales, it is no longer the sensitivity of the detectors but the presence of astrophysical foregrounds and instrumental systematics that hinder the measurement of a pure CMB signal. An accurate CMB signal is essential for better understanding the geometry (Ade et al. 2016a), and composition of the universe (Goldstein et al. 2003) and renders stringent constraints on cosmological parameters (Hinshaw et al. 2013; Ade et al. 2016c; Aghanim et al. 2020a). A comprehensive study of the residual systematic errors on top of the already challenging task of foreground removal from CMB maps is very important for a component separation method. The effect of such residual systematics becomes even more significant with many planned next-generation CMB missions (Kogut et al. 2011, 2016; André et al. 2014; Di Valentinno et al. 2018; Sutin et al. 2018; Hanany et al. 2019) designed to detect the signature of very weak primordial gravitational waves, a unique prediction of the initial inflationary epoch of the universe.

Apart from the presence of foreground contamination in the observed maps due to emissions by various astrophysical sources, the presence of residual and random calibration errors poses a difficult challenge when estimating a pure signal. The source of these residual uncertainties in the calibration factors arises due to statistical and systematic uncertainties during the calibration of the frequency maps. The statistical uncertainties include instrumental noise, while systematic uncertainties are due to effects and assumptions made during the calibration procedure that are not completely well understood (Aghanim et al. 2020a; Akrami et al. 2020). Although these residual calibration errors may be small in a CMB experiment, their presence implies that it is impossible to obtain exact values of the calibration coefficients corresponding to the observed CMB map of each detector. The Planck consortium, for example, after using advanced photometric calibration techniques like spin-synchronous modulation of the CMB orbital dipole (Adam et al. 2016a; Ade et al. 2016b; Akrami et al. 2020) for Low-Frequency Instrument (LFI) and using models of planetary atmospheric emissions and time-variable CMB orbital dipole (Adam et al. 2016b; Aghanim et al. 2020a) for High-Frequency Instrument (HFI) maps, has constrained the calibration uncertainties in the Planck full-sky surveys. Similarly, WMAP used the dipole modulation of the CMB signal due to the observatory’s motion around the Sun (Hinshaw et al. 2003; Jarosik et al. 2011; Bennett et al. 2013) as a means to calibrate its maps. But it is impossible to determine the exact value of the calibration coefficient corresponding to each map. In Table 1, we provide the maximum estimate of the residual calibration factors corresponding to the WMAP and Planck frequency channels. Not accounting for such calibration errors in the observed maps during the foreground minimization procedure leads to bias when estimating a cleaned CMB signal (Dick et al. 2010). It is therefore natural to ask what would be the impact of using such improperly calibrated foreground-contaminated CMB maps as inputs to a foreground minimization algorithm on the final cleaned CMB map and its angular power spectrum? In this article, we focus on the Gibbs Internal-Linear-Combination (ILC) CMB reconstruction method proposed by Sudevan & Saha (2020), which possesses various interesting properties as far as in terms of CMB reconstruction by removing...
foregrounds, as described briefly in the following discussions and in references mentioned therein.

In order to remove the foregrounds from CMB observations performed by various satellite missions there exist various (foreground) model-dependent and model-independent methods. Since CMB and different astrophysical components have different emission laws, a component separation method can utilize these differences to separate the CMB from foregrounds. An important CMB reconstruction method is the ILC method (Tegmark & Efstathiou 1996; Bennett et al. 2003; Tegmark et al. 2003; Saha et al. 2006), where in order to obtain a cleaned CMB signal, it is not necessary to explicitly model the individual foreground component’s physical morphology in the form of templates at some reference frequencies or in the form of corresponding frequency spectra. The method is based on the assumption that the frequency spectra of the foregrounds are different from the frequency spectrum of the CMB, which is assumed to be blackbody in nature (Mather et al. 1994; Fixsen et al. 1996). In the ILC method, a cleaned CMB map is obtained by linearly combining multifrequency observed foreground-contaminated CMB maps using some amplitude terms known as weight factors. These weights follow the constraint that their sum should be unity and can be estimated analytically by performing a constrained minimization of the variance of the cleaned CMB map.

In recent years the ILC method has been investigated extensively (Eriksen et al. 2004b; Hinshaw et al. 2007; Saha 2011; Saha & Aluri 2016; Sudevan et al. 2017). A global ILC method in pixel space was proposed by Sudevan & Saha (2018) that takes into account the prior information of the CMB covariance matrix under the assumption that detector noise can be ignored over the large angular scales of the sky. Sudevan & Saha (2020) proposed a method to estimate the CMB posterior density and CMB theoretical angular power spectrum given the observed data over the large angular scales of the sky in a (foreground) model-independent manner using the ILC method discussed in Sudevan & Saha (2018) implemented in harmonic space. They provided the best-fit estimates of both the CMB map and theoretical angular power spectrum along with their confidence interval regions, and estimated the CMB posterior without having to explicitly model the foreground components. The theoretical power spectrum results and its error estimates can directly be integrated into the cosmological parameter estimation process.

We organize subsequent sections of this article as follows. In Section 2, we review the basic idea of the Gibbs ILC method. In Section 3 we discuss how the calibration errors affect the cleaned CMB map. We describe our Monte Carlo simulations to study the effect of calibration errors in Section 4 and show the simulation results in Section 5. In Section 6 we discuss our results and conclude.

Table 1

| Frequency Map | K1 30 GHz | Ka1 44 GHz | V 70 GHz | W 100 GHz | 143 GHz | 217 GHz | 353 GHz |
|---------------|----------|------------|----------|-----------|---------|---------|---------|
| Calibration error, $\sigma_r$, (%) | 0.2 | 0.17 | 0.2 | 0.12 | 0.2 | 0.2 | 0.08 | 0.021 | 0.028 | 0.024 |

2. Formalism

In the Gibbs ILC approach (Sudevan & Saha 2020), we estimate the CMB posterior density $P(S, C|D)$, where $S$ is the true CMB signal, $C_i$ denotes the theoretical CMB angular power spectrum, and $D$ is the given observed CMB data, by drawing samples of $S$ and $C_i$ from the distribution using the Gibbs sampling technique (Geman & Geman 1984; Eriksen et al. 2004a; Larson et al. 2007; Eriksen et al. 2008; Groeneboom 2009). In Gibbs sampling, at the beginning of any Gibbs iteration $i$, a CMB signal $S^{i+1}$ is sampled from the conditional density of $S$, $P_i(S|D, C_i)$, given both observed data $D$ and a theoretical CMB angular power spectrum $C_i^{(i)}$, i.e.,

$$S^{i+1} \leftarrow P_i(S|D, C_i^{(i)}).$$

Using the sampled CMB signal $S^{i+1}$, a theoretical CMB angular power spectrum $C_i^{(i+1)}$ is sampled from the conditional density of $C_i$, $P_2(C_i|D, S)$, given both observed data $D$ and a CMB map, $S^{i+1}$, i.e.,

$$C_i^{(i+1)} \leftarrow P_2(C_i|D, S^{i+1}).$$

At the end of $i$th iteration, there is a new pair of $S^{i+1}$ and $C_i^{(i+1)}$ and all sampled $S$ and $C_i$ from previous $(i - 1)$ iterations. These two steps are repeated a large number of times where at each step $C_i^{(i)}$ in Equation (1) is replaced by $C_i^{(i+1)}$ of Equation (2) and similarly $S^{i+1}$ of Equation (2) is replaced by $S^{i+1}$ from the new Equation (1). Removing some initial samples of $S$ and $C_i$ (the burn-in phase), all other samples in the sequence are statistically equivalent to those sampled from the joint CMB posterior density $P(S, C_i|D)$ rather than those sampled from their individual conditional probability distributions.

Since we intend to reconstruct the joint CMB posterior density in a foreground model-independent manner we sample $S$ at each Gibbs iteration by minimizing the foregrounds present in the observed data using the global ILC method. Let us assume that we have $n$ mean subtracted foreground-contaminated full-sky CMB maps $X_i$, at a frequency $f_i$, with $i = 1, 2, \ldots, n$. Then a CMB estimate $\hat{S}$ of the underlying true CMB signal $S$ is obtained by linearly combining these $n$ input maps, i.e.,

$$\hat{S} = \sum_{i=1}^{n} w_i X_i,$$

where $w_i$ is the weight corresponding to the $i$th frequency channel. To preserve the CMB signal in the cleaned map the weights follow the constraint that the sum of all the weights corresponding to $n$ frequency channels should be unity i.e.,

$$\sum_{i=1}^{n} w_i = 1.$$
Using this condition on weights, we perform a constrained minimization of the CMB covariance weighted variance, \( \sigma^2 = \hat{S}^T C \hat{S} \) where \( C \) is the theoretical CMB covariance matrix (Sudevan & Saha 2018, 2020) and \( \dagger \) represents the Moore–Penrose generalized inverse (Penrose 1955), in order to estimate the weights. The choice of weights that minimizes \( \sigma^2 \) is obtained by following a Lagrange’s multiplier approach,

\[
W = \frac{\hat{A}^\dagger e}{e^\dagger \hat{A} e},
\]

where \( \hat{A}_{ij} = X_i^T C X_j \), \( W \) is an \( (n \times 1) \) weight vector, and \( e \) is the \( n \times 1 \) shape vector of the CMB in thermodynamic temperature units. Typically, if the input CMB maps are calibrated correctly across all frequency channels, the shape vector is then an \( (n \times 1) \) identity column vector, therefore \( \sum_{i=1}^n w_i e_i = 1 \). The cleaned CMB map, \( \hat{S} \), estimated using the global ILC method is given by

\[
\hat{S} = DW = D \hat{A}^\dagger e / e^\dagger \hat{A} e,
\]

where \( D \) is a set of \( n \) observed CMB maps \( (X_1, X_2, ..., X_n) \). On large angular scales of the sky (for e.g., at a pixel resolution defined by HEALPix \(^5\) pixel resolution parameter \( N_{side} = 16 \) and beam-smoothed by a Gaussian beam of FWHM = 9°) the observed CMB maps have negligible detector noise levels. Therefore, the global ILC weights adjust themselves in such a way that they cancel out the correlated foregrounds across frequency channels while doing so they minimize the bad effects of CMB-foreground-fractional correlation as well, over large angular scales, thereby providing a cleaned CMB map \( \hat{S} \) very close to the true CMB signal, \( S \). The Monte Carlo simulation results presented in Sudevan & Saha (2020) show that the error while reconstructing a cleaned CMB map using the Gibbs ILC method is less than ±0.5 \( \mu \)K outside the galactic region and less than ±4.2 \( \mu \)K inside the galactic plane. Equation (5) shows the relation between global ILC weights, \( W \), and the CMB shape vector, \( e \).

### 3. Bias in the Presence of Residual Calibration Error

If in a CMB experiment, the observed CMB maps are not calibrated correctly, then in the presence of calibration uncertainties \( \delta e_i \) corresponding to the CMB map observed in the frequency channel \( \nu_i \), the elements of the CMB shape vector in thermodynamic temperature units will be modified as \( e_i' = 1 + \delta e_i \), or following a vector notation \( e' = e + \delta e \). If \( \delta e \) were completely known, the weights estimated using the new shape vector \( e' \), while minimizing the foregrounds in these observed maps, would still be subjected to the constraint that \( \sum_{i=1}^n w_i e_i' = 1 \) so that it will not introduce any multiplicative bias in the cleaned CMB amplitude. But in any CMB experiment, it is not possible to obtain the exact numerical values of calibration coefficients corresponding to the observed maps even after using advanced calibration techniques. Therefore, there will always be, however small, some residual calibration uncertainties in each of the observed maps. Hence, it is worthwhile to understand what will happen if we use the Gibbs ILC method on those input maps with some level of calibration uncertainties in each map while (incorrectly) assuming the CMB shape vector to be the unit vector \( e \) in Equation (5) (or in Equation (6)).

In the presence of calibration error the cleaned map following Equation (3) is given by

\[
\hat{S} = \sum_{i=1}^n \left( w_i e_i' S + w_i e_i' \sum_{k=1}^{n_f} f^k_0 \right),
\]

where \( S \) and \( f^k_0 \) respectively represent the true sky CMB signal and foreground template for the foreground component \( k \) at some reference frequency, and \( n_f \) denotes the total number of foreground components. \(^3\) The factor \( f^k_\nu \) represents the \( \nu \)th element of the \( k \)th foreground shape-vector \( f^k \). Defining, \( g^k_i = e'_i f^k_\nu \) we can write Equation (7) following the matrix notation as follows:

\[
\hat{S} = [W^T \cdot e'] S + \left[ W^T \cdot \sum_{k=1}^{n_f} g^k \right] f^0.
\]

Using this equation we note that in the presence of calibration error the foreground shape vectors modifies to \( g^k \) from the initial \( f^k \) without any alteration of the total number of foreground components or the underlying foreground degrees of freedom. Using Equation (8) we can infer the presence of different kinds of bias in the presence of calibration error as discussed below.

#### 3.1. CMB Bias or Normalization Bias

Although in the presence of calibration error \( e' \) enters in Equation (8) while estimating the weights using Equation (5), we assume that there are no calibration errors in the observed maps, i.e., we keep \( e \) as a unit \( n \times 1 \) vector, hence \( [W^T \cdot e'] = 1 \) in Equation (8). This leads to CMB normalization bias in \( \hat{S} \). Depending upon whether \( [W^T \cdot e'] > 1 \) or \( < 1 \) the CMB map will be biased high or low than the sky CMB signal. We note that, even in presence of calibration error, if it so happens \( [W^T \cdot \delta e] \sim 0 \), then \( [W^T \cdot e'] \sim 0 \). Hence, if random \( \delta e \) are such that \( [W^T \cdot \delta e] \sim 0 \) the net normalization bias in \( \hat{S} \) will be close to zero. A larger deviation of \( [W^T \cdot \delta e] \) from 0 will lead to greater normalization bias in the cleaned map.

#### 3.2. Foreground Bias

In the presence of calibration errors, weights are expected to be dependent on \( \delta e \). This may cause weights to deviate from the optimal values that would have otherwise removed foregrounds satisfactorily in the absence of the calibration error. It is interesting, therefore, to ask how much foreground bias may be caused in the cleaned map due to calibration errors? Following an analysis similar to Sudevan & Saha (2020), Saha & Aluri (2016), we obtain

\[
W = \frac{(I - C_i^j) e'}{e'^T (I - C_i^j) e'} \left[ 1 + 2(\delta e)^T \hat{A} e' \right] - \frac{(\delta e)^T \hat{A} e'}{e'^T \hat{A} e'},
\]

\(^3\) In Equation (7) we have assumed the detector noise is negligible, which is the case for WMAP and Planck observations for temperature anisotropy over large angular scales of the sky.
We note that as in Sudevan & Saha (2020) where \( I \) represents the empirical foreground covariance matrix in multipole space in observed data with calibration error. \( C_f \) represents the theoretical CMB angular power spectrum. In zero order of the small calibration error \( \epsilon \), Equation (9) reduces to
\[
W \sim \frac{(I - C_f C_f^T)\epsilon}{\epsilon^T (I - C_f C_f^T)\epsilon}.
\]
(11)

We note that \( C_f C_f^T \) is a projector on the column space of \( C_f \). Now following Sudevan & Saha (2020), if \( n > n_f \) then the null space of \( C_f \) is a nonempty set and \( (I - C_f C_f^T) \) is a projector on the null space. From Equation (11), we see that the weight vector \( W \) (which is actually estimated after incorrectly assuming CMB shape vector to be a unit vector, \( \epsilon \) in Equation (5)) satisfies
\[
W^\dagger g_k \sim 0 \quad \forall \quad k,
\]
(12)
since \( g_k \) lies completely inside the column space of \( C_f \). Since any deviation of \( W^\dagger g_k \) from zero (for any \( k \)) causes foreground residual in the cleaned map, Equation (12) implies that if the residual calibration errors of the input maps are small, there will be only very small residual foreground bias in the foreground-cleaned CMB map even if the weights are estimated assuming no calibration error in the input maps.

4. Methodology

The Planck consortium, using advanced photometric calibration techniques like spin-synchronous modulation of the CMB orbital dipole for LFI, and using models of planetary atmospheric emissions and time-variable CMB orbital dipole for HFI maps, has dramatically brought down the calibration uncertainties in the Planck full-sky surveys. However, it is impossible to determine the exact value of the residual calibration uncertainties corresponding to each map. Therefore, performing detailed Monte Carlo simulations, where we simulate foreground and detector noise-contaminated maps that mimic the real-life observed CMB maps, is the only way to understand the impact of using incorrect calibration coefficients during a CMB reconstruction method.

In the current analysis, we perform 1000 different sets of Monte Carlo simulations of the entire Gibbs ILC procedure for a comprehensive study of the impact of using input frequency maps with varying levels of (residual) calibration errors corresponding to different simulation sets, on the Gibbs ILC results. The calibration errors used in these simulations are consistent with the WMAP (Jarosik et al. 2011; Bennett et al. 2013) and Planck 2018 results (Aghanim et al. 2020a; Akrami et al. 2020). We mention the calibration error levels in Table 1 assuming they represent 1σ error levels.

In these Monte Carlo simulations, in each set, we simulated foregrounds and detector noise-contaminated CMB maps at all WMAP and Planck frequency channels at a pixel resolution \( N_{\text{side}} = 16 \) and beam-smoothed by a Gaussian beam of FWHM 9°. The free–free, synchrotron, and thermal dust emissions at different frequency channels are obtained at \( N_{\text{side}} = 256 \) and at a beam resolution of 1° following the procedure described in Sudevan et al. (2017). These maps are then downgraded to \( N_{\text{side}} = 16 \) and we performed an additional smoothing with a Gaussian beam of FWHM = \( \sqrt{540^2 - 60^2} \) to bring all the foreground maps to 9° beam smoothing. We generated a CMB temperature map using the theoretical CMB power spectrum consistent with cosmological parameters obtained by the Planck collaboration (Ade et al. 2016c) at \( N_{\text{side}} = 16 \) and beam smoothing of 9°. We follow the same procedure given in Sudevan et al. (2017) to generate detector noise maps corresponding to each input map at \( N_{\text{side}} = 16 \) and 9° smoothing, the detector noise levels in accordance with the estimate provided by WMAP and the Planck science team. The final simulated foreground-contaminated maps at different Planck and WMAP frequencies are obtained by linearly combining the CMB, various foregrounds, and the detector noise maps.

Once we simulated these input maps, they were then scaled by calibration factors obtained by randomly drawing a unit mean Gaussian random variable \( x_i \), where \( i = 1, ..., n \) (total number of maps), with standard deviation equal to the desired amount of calibration error mentioned in Table 1. This generates a given set of input maps with the randomly chosen calibration errors. For the purpose of Monte Carlo simulations, we simulate a total of 1000 different sets of input maps with random calibration errors.

After simulating the foreground-contaminated maps with different levels of calibration error for different sets, we use the Gibbs ILC algorithm to minimize the foregrounds. While implementing the Gibbs ILC code, we assumed that all the input simulated maps are properly calibrated, i.e., the shape vector, \( \epsilon \), is a unit vector. In the current implementation of the Gibbs ILC procedure, each simulation consists of 10 chains each, with randomly chosen initial points and 5000 Gibbs steps. We reject the first 50 samples (each from cleaned CMB maps and sampled CMB theoretical angular power spectra) in each chain corresponding to the initial burn-in phase. This results in a total of 49,500 samples from each simulation.

5. Results

In this section, we discuss the results obtained after performing detailed Monte Carlo simulations of CMB reconstruction using 1000 different sets of input maps with random calibration errors consistent with WMAP and Planck observations. While implementing the Gibbs ILC method, during the foreground minimization using the global ILC method we do not take into account the presence of calibration errors in the map. We follow the same procedure as outlined in Sudevan & Saha (2020) for calculating the CMB posterior, best-fit CMB map, and best-fit theoretical CMB angular power spectrum from the cleaned CMB map and theoretical angular power spectrum samples generated.

5.1. Cleaned Maps

Using 49,500 sampled maps from a given set of simulations we estimate the best-fit CMB map corresponding to the maximum likelihood pixel values for each pixel. Using best-fit CMB maps from all 1000 simulation sets we estimate a simple mean map. In Figure 1, we show the mean best-fit cleaned CMB map in the top panel. This map agrees very well with the best-fit CMB map when the simulation involved no calibration
bias for the same reason (e.g., see Sections 3.2 and 3.1). The normalization bias arises since in the case of input frequency maps with calibration errors, weights satisfy $W^i e = 1$ instead of $W^i e' = 1$, where $e' = e + \delta e$ and $\delta e$ represent the unknown residual calibration error, to incorrectly normalize the underlying CMB component in the cleaned map. A measure of the normalization bias in any given experiment with a given set of unknown residual calibration error is then $W^i e' - 1$. We show the distribution of CMB normalization bias in percentage level using 1000 sets of CMB reconstruction using the Gibbs ILC method in Figure 2. As seen from this figure the normalization bias is only 0.17\% at the 1\$\sigma$ level in a CMB reconstruction method using Gibbs ILC method. In Figure 3 we show the normalization bias map for the chosen input CMB map of this work corresponding to the 1\$\sigma$ value of $\sum w_i c_i^1 - 1$. For the calibration error levels of WMAP and Planck, the normalization bias is less than 0.2\,\mu K in magnitude.

5.2. Best-fit CMB Angular Power Spectrum

Since exact values of calibration coefficients are unknown in any CMB experiment, estimated values of the best-fit CMB angular power spectrum are different from the actual ones that would have been estimated in a hypothetical case where the calibration coefficients were exactly known. Such differences may result in a bias in the estimated best-fit CMB angular power spectrum apart from causing larger errors due to the random nature of the calibration uncertainties. Using 1000 Monte Carlo simulations of CMB posterior estimations using the Gibbs ILC method, in this section we assess this bias and error in the best-fit CMB angular power spectrum in the presence of calibration uncertainties.

In order to understand any possible bias in the best-fit CMB angular power spectrum in the Gibbs ILC method due to residual calibration error, we show in the top panel of Figure 4 the mean best-fit CMB angular power spectrum obtained from 1000 simulations (in green) with calibration errors, along with the best-fit angular power spectrum when the input frequency maps contained no calibration error (in red). The green filled region around the mean best-fit angular power spectrum represents the 1$\sigma$ error levels at different multipoles obtained from 1000 best-fit angular power spectra. The mean best-fit with calibration error matches very well with the zero calibration error case. In the middle panel of this figure, we show any bias in the best-fit angular power spectrum with respect to the zero calibration error case by plotting the difference of mean best-fit CMB angular power spectrum (with calibration error) and the best-fit spectrum without any calibration error. Visually, the differences take comparatively larger and more positive values between $\ell = 2$ and $\ell = 15$, whereas there are somewhat smaller but more negative values between $\ell = 16$ to $\ell = 32$. The average bias in the first multipole range is as small as $\sim 28\,\mu K^2$. Average bias between $\ell = 16$ to $\ell = 32$ is just $-4.7\,\mu K^2$. In the bottom panel of Figure 4, we show the calibration-uncertainty-induced error in the best-fit CMB angular power spectrum by plotting the ratio of standard deviations ($\sigma_{C_\ell}^{\text{cal}}$) of the best-fit spectra in the presence of calibration error and cosmic-variance-induced error ($\sigma_{C_\ell}$). The standard deviation from simulations varies between 8\% (at $\ell = 4$) and a maximum of 22\% at $\ell = 10$ of cosmic-variance-induced error. The calibration-induced mean fractional error between $\ell = 2$ and $\ell = 32$ is just 13\%. The calibration-induced error is expected to combine in quadrature with the cosmic-
method. We see that the mode of the distribution is centered around 1, in percentage level corresponding to the 1000 sets of CMB reconstruction using the Gibbs ILC method. We see that the mode of the distribution is centered around $-0.015\%$, which translates into about $\pm 0.00018 \mu K$ difference in temperature at each pixel at the $1\sigma$ level.

![Figure 2](image1.png)

**Figure 2.** Distribution of CMB normalization bias, $\sum w_i e'_i - 1$, in percentage level corresponding to the 1000 sets of CMB reconstruction using the Gibbs ILC method. We see that the mode of the distribution is centered around $-0.015\%$, which translates into about $\pm 0.00018 \mu K$ difference in temperature at each pixel at the $1\sigma$ level.

![Figure 3](image2.png)

**Figure 3.** Normalization bias in the CMB map corresponding to the $1\sigma$ level of $\sum w_i e'_i - 1$. We see that at $1\sigma$ of normalization bias the maximum change in pixel temperature is of the order of $\pm 0.2 \mu K$.

Considering this mean level of fractional error, the net error becomes $\sqrt{1 + 0.13} = 1.06$ times the cosmic-variance-induced error. This causes a 6% error increase from the cosmic variance prediction between the multipole range $2 \lesssim \ell \lesssim 32$.

Summarizing the simulation results of this section, we conclude that even in the presence of calibration uncertainties in the input foreground and detector noise-contaminated CMB maps, our Gibbs ILC method produces a best-fit cleaned CMB map that has a very minor level of residual foreground contamination bias and an almost negligible CMB normalization bias. For the best-fit power spectrum, calibration-induced bias and error both remain small.

### 6. Conclusions and Discussions

The levels of calibration uncertainties present in the observed CMB maps have been drastically reduced by following advanced photometric calibration techniques like spin-synchronous modulation of the CMB orbital dipole (Adam et al. 2016a; Ade et al. 2016b; Akrami et al. 2020) using models of planetary atmospheric emissions (Adam et al. 2016b; Aghanim et al. 2020a), etc. But the presence of the residual calibration error (however small it be) in the observed CMB maps may pose a difficult challenge when estimating a pure CMB signal. Here we study the impact of random calibration errors on the CMB map and its angular power spectrum that is obtained using the Gibbs ILC method. Since it is impossible to obtain the exact value of calibration uncertainties corresponding to each individual observed CMB map, we perform detailed Monte Carlo simulations of the Gibbs ILC method with realistic residual calibration errors compatible with WMAP and Planck observations, after simulating realistic foreground and detector noise-contaminated CMB maps over large angular scales of the sky.

Using analytical results we show in Section 3 that residual errors in calibration coefficients lead to two distinct types of bias in the Gibbs ILC method, which is implemented over large angular scales of the sky. The first kind of bias is called the CMB normalization bias, which arises since the empirical weights satisfy $W^T e = 1$ instead of $W^T e' = 1$, where $e'$ represents the true CMB shape vector in the presence of calibration error. The second type of bias is due to residual foreground contamination in the cleaned maps. By estimating the best-fit cleaned CMB maps corresponding to the maximum posterior density with detailed Monte Carlo simulations, in Section 5 the normalization bias is merely $0.17\%$ at a confidence level of $1\sigma$. The residual foreground bias is small as well. Our Monte Carlo simulations show that the residual calibration errors tend to maximally bias only the galactic central region, with a magnitude of $\sim 2 \mu K$ at the $1\sigma$ confidence level. Between $2 \leq \ell \leq 15$ (mean) bias in the best-fit CMB angular power spectrum, when calibration errors are present in the input maps, Gibbs ILC method finds $\sim 28 \mu K^2$ with respect to the ideal case of zero calibration error. This bias decreases with an increase in multipoles and is just $-4.7 \mu K^2$ between $16 \leq \ell \leq 32$. The calibration error widens the error intervals on the best-fit CMB angular spectrum. The average increase of net error level between $2 \leq \ell \leq 32$ is $-6\%$ over the cosmic variance induced error.

Based upon our Monte Carlo simulations we conclude that for an analysis over large angular scales of the sky, even if we use maps with realistic (residual) calibration errors without accounting for them in the Gibbs ILC algorithm (by modifying the CMB shape vector), this leads to a very minor level of bias in the best-fit cleaned CMB map. The bias and error in the best-fit CMB angular power spectrum are both small; however, they
may not completely negligible. It would be important to incorporate such bias and error in the angular power spectrum during cosmological parameter estimation and investigate their role.

Finally, we note an interesting advantage of the Gibbs ILC method with regard to the impact of residual calibration errors on the CMB reconstruction. Since our method does not require modeling the frequency spectrum or any templates for foreground components to reconstruct the CMB products to a good accuracy, our foreground removal is independent of the calibration error, as long as the later is small (e.g., Section 3.2).

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Figure 4. In the top panel, we show the mean best-fit estimated from the 1000 Monte Carlo simulations using the green line. The best-fit theoretical angular power spectrum obtained from simulations with no calibration error is shown using red points and the input CMB angular power spectrum used in all the simulations is shown using yellow points. The 1σ standard deviation region corresponding to the best-fit spectra from the Monte Carlo simulations is shown as a light green shaded region around the mean best-fit angular power spectrum. We see from this plot that the mean best-fit agrees well with both the input CMB and best-fit (from the simulation with no calibration errors) angular power spectrum. Both the input CMB angular power spectrum and best-fit angular power spectrum lie inside the 1σ region of the mean best fit. In the middle panel we show the difference of the mean best-fit angular power spectrum where calibration error was included in the simulations and the best-fit spectrum without any calibration error. The bottom panel compares the standard deviations of the best-fit spectra with calibration error with the cosmic variance. The calibration-induced errors appear to be approximately uniformly distributed over multipoles with respect to the cosmic-variance-induced errors. The yellow line represents the mean fractional error of 13% between the entire multipole range.
