A METHOD OF EXTRACTING THE MASS OF THE TOP QUARK IN THE DI-LEPTON CHANNEL USING THE DØ DETECTOR

Sarosh N. Fatakia
Boston University
for the DØ collaboration

ABSTRACT
We present a method for extracting the mass of the top quark from the di-lepton decays of top anti-top quark pairs. In this decay channel two neutrinos remain undetected. Extraction of the mass of the top quark by kinematic reconstruction is not possible because the event is under-constrained. We therefore employ a dynamical likelihood method to solve the problem.

1 The di-lepton event topology
In the di-lepton channel top anti-top quark pairs decay via $t \rightarrow Wb$, followed by $W \rightarrow l\nu$. We can identify and measure the 4-momenta of the jets\(^1\) and the charged leptons. The two neutrinos in the event remain undetected. The vector sum of their transverse momenta can be inferred from the observed missing $p_T$. This leaves us with 14 observables \(\{o\}\) out of 18 values \(\{v\}\) needed to describe the six particle final state. In order to constrain the $t\bar{t}$ kinematics, three additional constraints on the invariant masses of final state particle combinations are introduced:
\[ m(l_1^+, \nu_1) = m(l_2^-, \bar{\nu}_2) \equiv m_W, \]
\[ m(b, l_1^+, \nu_1) = m(\bar{b}, l_2^-, \bar{\nu}_2) \]
\(^1\)Both jets are assumed to be $b$-jets
In the end the set of equations are still under-constrained by one equation. Extraction of the mass of the top quark by kinematic reconstruction is therefore not possible.

2 The Analysis

This analysis was developed during Run I\[1\]. Ideally we would like to calculate the probability to measure the 14 observables \( \{o\} \), given the top quark mass \( m_t \).

\[
P(\{o\} | m_t) \propto \int_{\{v\}} f(x) f(\bar{x}) |\mathcal{M}|^2 p(\{o\}|\{v\}) d^{18}\{v\} dx d\bar{x}
\]  

(1)

is the probability density function (pdf) for measuring \( \{o\} \) given the 18 final state parameters \( \{v\} \) for a hypothetical \( m_t \) and for the parton (anti-parton) momentum fraction \( x (\bar{x}) \). Here \( f(x) \) \( f(\bar{x}) \) is the proton (anti-proton) parton distribution function and \( \mathcal{M} \) is the matrix element for the process. A hypothetical value of \( m_t \) is used as the last constraint. We then obtain up to 4 real solutions for the \( \nu \) and \( \bar{\nu} \) momenta\[3\]. There is a two-fold combinatoric ambiguity in pairing the charged leptons and \( b \)-jets. As a result up to eight solutions of the neutrino momenta are possible. Instead of rigorously computing the pdf in Equation 1 event weights\[2\][3] are used to characterize the physics. In this poster we calibrate the effect of this simplification by comparing the measured mass of the top quark obtained from the likelihood fits to simulated events, versus the value used in their generation.

For every event, a weight \( (W_k) \) which is a function of the hypothesized mass of the top quark \( (m_t) \), is derived\[1\][2][3] that corresponds to the \( k^{th} \) neutrino momentum solution. The event weight

\[
W_k(m_t) \propto f(x) f(\bar{x}) p(E'|m_t)p(\bar{E}'|m_t),
\]

(2)

represents the likelihood to observe the event for a given \( m_t \). Here \( p(E'|m_t) \) is the probability density function for the energy of the charged lepton to be \( E' \) in the rest frame of the top quark of mass \( m_t \)[4]. The quantity \( p(\bar{E}'|m_t) \) is the analogue for the anti-top quark. With \( n \) solutions for the neutrino momenta the total event weight \( W \) is defined as:

\[
W = \text{normalization} \sum_{k=1}^{n} W_k.
\]

(3)

The value of \( m_t \) at which the weight curve peaks is used as the mass estimator in the analysis. All simulated events are first filtered using the same criteria as for data events[5]. Figure[1] (left) shows the weight curve for a simulated event with input \( m_t = 175.0 \) GeV. The peak values from many events generated
with the same $m_t$ are binned into histograms to construct templates, one of which is shown in Figure 1(right).

It is not possible to generate MC events with continuously varying input $m_t$. Seven distinct $m_t$ values are used to generate the signal MC template distributions from 120.0 GeV to 230.0 GeV. Templates representing contamination from background processes are also constructed and then added to the signal templates in the proportion of the expected background. Normalized templates represent the likelihood of observing the particular event if the mass of the top quark equals $m_t$.

Sets of eight MC events are used to construct simulated experiments. A binned maximum likelihood fit is performed for each such ensemble using the template distributions. One such fit to ensemble events is illustrated in Figure 2(left). Many different ensembles are constructed, and the exercise is repeated many times to obtain a distribution of maximum likelihood estimates (MLEs) as in Figure 2(center). The mean of this distribution yields the mean fitted estimate of $m_t$. Repeating the same exercise for different input MC $m_t$ yields values which can be used to check the performance of the method. A straight line parameterized as:

$$\text{fitted mass} = p_1 \cdot (\text{input mass} - 175.0 \text{ GeV}) + p_0 \text{ GeV}$$

gives the best fit to the ensemble test results for $p_1 = 1.00\pm0.04$ and $p_0 = 175.8\pm0.08$ GeV. This fit to the set of points, shown in Figure 2(right), is consistent with a straight line of unit slope and 175.0 GeV offset.

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2Eight emu events observed from 142.73 pb$^{-1}$ data.

Figure 1: (left) Example weight distribution from a simulated event generated with $m_t = 175.0$ GeV. (right) Template constructed from peak values obtained from many events each having input $m_t = 175.0$ GeV.
Figure 2: (left) Binned maximum log-likelihood fit of an ensemble of events to the template histograms yield the most likely estimate (MLE) of $m_t$ for that ensemble. (center) A histogram of MLEs derived from 100 different ensembles, all signal events used have an input $m_t = 175.0$ GeV. (right) Calibration of a series of different input $m_t$ values. The 175 GeV input MC value follows from the histogram in the center.

3 Conclusion

The calibration of this method verifies its performance, and indicates absence of bias in the method.

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