Shear Capacity of Reinforced Concrete Beams with Square Cross Section Subjected to Biaxial Bending

R Thamrin\textsuperscript{1}, S Haris\textsuperscript{1}, E Dedi\textsuperscript{1} and E Dalmantias\textsuperscript{1}

\textsuperscript{1} Civil Engineering Department, Engineering Faculty, Universitas Andalas, Padang, Indonesia
Corresponding author: rendythamrin@eng.unand.ac.id

Abstract. This paper presents the test result of an experimental study on the shear strength of reinforced concrete beams subjected to biaxial loads. Six simply supported reinforced concrete beams with square cross section were used. Three beams rotated at 22.5 degrees and three beams rotated at 45 degrees to the applied load were tested until failure. All beams were subjected to four-point bending test. The test variable in this study was ratio of longitudinal reinforcement. The values of flexural crack load, diagonal crack load and the ultimate load were observed during the test as well as the midpoint deflection of the beams. Existing empirical equation for shear strength of concrete presented in design codes was used and then compared to the values obtained from the test. Comparison between test results and theoretical shear capacity show that all of equations conservatively estimated the occurrence of shear failure with the values of the test results 25 to 55\% higher than the theoretical values depending on the ratio of longitudinal reinforcement. It was also observed that ratio of longitudinal reinforcement influences the angle of diagonal shear crack.

1. Introduction
Shear failure needs to be avoided in the design of reinforced concrete structures to prevent sudden collapse without warning. To avoid collapse due to shear forces, the structure must be carefully designed based on the load acting on it. The direction of the actual load acting on a building structure during an earthquake or similar natural event will be unknown. The direction of the load may be perpendicular to lateral or vertical axes of the cross section but could also be at a certain angle to lateral and vertical axes as illustrated in figure 1(b).

(a) Load perpendicular to lateral axis
(b) Load that has an angle with lateral and vertical axes

Figure 1. The possible direction of load acting on a reinforced concrete member.
Up until now, research carried out on the shear strength of reinforced concrete beams has focused on uniaxial bending, the load only acting in one direction of the lateral axis of the cross-section of the structural elements as shown in figure 1(a). Information on the effect of biaxial shear forces acting with a certain angle to lateral and vertical axes of cross-section in structural elements is limited even though biaxial bending forces in building structures caused by various combinations of loading direction are common [1,2]. Most reinforced concrete codes for building structures do not yet provide design guidelines and formulas for calculating shear force capacity for this biaxial load condition. The behavior of biaxial shear forces acting on structural elements needs to be investigated to determine the effect on the shear strength of reinforced concrete structural elements covering columns and beams under the load of biaxial shear force.

Based on this fact, this study focuses on shear strength provided by concrete of reinforced concrete beams without stirrups loaded with biaxial loads. A numerical analysis was also conducted to obtain the full response of flexural behavior of the rotated cross section.

2. Experimental study
Tests were carried out on six reinforced concrete beams with square cross sections. The simply supported beam was used and loaded monotonically with four point bending. In order to produce a biaxial shear load, lateral axis of cross section was rotated at 22.5 and 45 degrees as shown in figure 2. Figure 3(a) shows the detail of test setup and equipments used while figure 3(b) shows the beam rotated at 45 degree after testing.

![Figure 2. Loading on the tested beam showing rotated cross sections at various selected positions.](image-url)
As shown in figure 2(a), the distance between supports was 2000 mm, the shear span length 800 mm, the distance between two point loads 400 mm, and the beam end length beyond the support 150 mm. The beam had a square cross section with dimensions of 222 mm x 222 mm as illustrated in figure 2(c). The concrete used had characteristic compressive strength of 25.1 MPa. The test variable in this study was the ratio of longitudinal reinforcement. The longitudinal reinforcement had diameters of 13 mm, 16 mm, or 19 mm with yield strength of 410 MPa, 390 MPa, and 370 MPa, respectively. Table 1 shows the beam data and details of the reinforcement. All beams presented in this report were designed without stirrups. The value of load was measured using a load cell placed on the top of the spreader beam as shown in figure 3(a). In order to obtain deflections, three displacement transducers were positioned at loading points and at midspan of the beam as shown in figure 3(a). The load and displacement that occurred during the test were recorded continuously every 5 seconds with the help of data acquisition system.

(a) Rotation angle 22.5 degree. (b) Rotation angle 45 degree.

**Figure 3.** The beams (a) before and (b) after test.

| Beams Identification | f^c' (MPa) | b (mm) | H (mm) | d (mm) | a (mm) | α | φ_b (mm) | N |
|-----------------------|------------|--------|--------|--------|--------|---|----------|---|
| G2-01-22.5             | 185        | 22.5   | 13     | 8      |        |   |          |   |
| G2-02-22.5             | 184        | 22.5   | 16     | 8      |        |   |          |   |
| G2-03-22.5             | 183        | 22.5   | 19     | 8      |        |   |          |   |
| G2-01-45               | 25.1       | 222    | 800    | 45.0   | 13     | 8 |          |   |
| G2-02-45               | 184        | 45.0   | 16     | 8      |        |   |          |   |
| G2-03-45               | 183        | 45.0   | 19     | 8      |        |   |          |   |

Note: b, d, and H are the dimension before the cross section is rotated.

3. Numerical analysis

In this study, theoretical concrete shear strength for each of the beams was calculated using the empirical equation suggested by ACI 318M-14 [3]:

\[ V_c = 0.172\lambda \sqrt{f_c'}bd \]  \hspace{1cm} (1)

where \( f_c' \) is the compressive strength of concrete, \( b \) is the width of cross section and \( d \) is the effective depth of cross section.

The numerical flexural capacity of the beam was obtained using moment-curvature analysis of reinforced concrete cross section [4,5,6,7]. In this method, the beam cross-section is divided into a number of reinforcement and concrete layers. Each layer is associated with the corresponding stress-strain law. The first step in this method is to obtain strains for each layer with the assumed curvature.
value. The stress values for each layer are obtained by using the strains values obtained from the previous step. The compressive stress-strain law of concrete is obtained from another research report [8] and the tensile stress-strain law of concrete is linear up to the maximum value. A bilinear stress-strain law was applied for the steel reinforcement. Each of the load steps had to satisfy the equilibrium condition of the internal forces, $F_i$, as expressed in Equation 2. This was solved using an iterative procedure. After this step, the moment-curvature relationship could then be obtained. The moment-curvature analysis of beam cross section is conducted with the help of a computer program.

$$\sum F_{si} + \sum F_{ci} \approx 0$$ (2)

4. Test results and discussion
The results obtained from the test and the numerical predictions for all beams are summarized in Table 2. In this study, due to the absent of stirrups, all beams failed in shear as expected. Typical crack patterns of the tested beams at failure are shown in figure 4 and figure 5. It was observed from the test that the first crack occurred in the zone of constant moment when the applied load reached an average of 8.3. After that, as the load increased, the flexural crack extend to the shear span region which then developed to become diagonal shear cracks at an average load of 41.5 kN. It is revealed from figure 4 and figure 5 that diagonal shear cracks occurred in the shear span region for all the beams tested in this study.

![Figure 4](image1)

**Figure 4.** Crack patterns of the beams with rotation angle 22.5 degree.
Figure 5. Crack patterns of the beams with rotation angle 45 degree.

Figure 6. Shear force vs. deflection of the tested beams.
Immediately after diagonal cracking, all the tested beams failed in shear. As listed in Table 1 and shown in Figure 6, the measured maximum loads were considerably higher (by 25 to 55%) than the shear strength provided by concrete, $V_c$, computed using the ACI 318M-14 code. This means that the shear strength design formula provided by ACI 318M-14 is conservative and safe to use for designing structural members that may be subjected to biaxial shear force.

Figure 4 and Figure 5 show that the angle of diagonal crack tended to be quite different for each beam. It can be seen that the slope of diagonal crack decreased as the ratio of longitudinal reinforcement increases. This indicates that the ratio of tensile longitudinal reinforcement affected the slope of diagonal crack. This phenomenon has been also reported in previous studies [5].

As can be seen from Figure 6, the shapes of the load deflection response of all beams demonstrate suddenly drop after the maximum load. However, the slopes of load deflection curves before the maximum load were significantly different for different beams. The difference in these slopes can be attributed to the difference in the ratio of longitudinal reinforcement.

Figure 7 compares numerical analysis and test results. The purpose of this comparison is to show more clearly that no beams reached flexural capacity because they failed due to shear before reaching bending capacity. Even though Figure 7(a) and Figure 7(d) show that the tensile reinforcement yielded before shear failure, however none of the tested beams reached the ultimate flexural strength because none of the beams experienced concrete crushing in the compression zone. It was also obtain from the test that beams with 45 degree angle show slightly higher shear strength compare to that beams with 22.5 degree angle.

### Table 2. Test results and the numerical predictions.

| Beams Identification | $V_c$ (kN) | $V_f$ (kN) | $V_d$ (kN) | $V_m$ (kN) |
|-----------------------|------------|------------|------------|------------|
| G2-01-22.5            | 34.3       | 42.4       | 8.0        | 43.5       |
| G2-02-22.5            | 34.1       | 56.6       | 6.0        | 35.0       |
| G2-03-22.5            | 33.9       | 72.0       | 14.5       | 37.5       |
| G2-01-45              | 34.3       | 46.3       | 6.3        | 42.6       |
| G2-02-45              | 34.1       | 63.0       | 6.0        | 44.1       |
| G2-03-45              | 33.9       | 79.8       | 9.3        | 46.0       |

The 2nd Global Congress on Construction, Material and Structural Engineering
IOP Publishing
IOP Conf. Series: Materials Science and Engineering 713 (2020) 012029
doi:10.1088/1757-899X/713/1/012029
5. Conclusions
Six reinforced concrete beams with square cross sections designed without stirrups were tested until failure to obtain shear capacity on being subjected to a biaxial shear load. The results indicate that the following conclusions can be written:
- The maximum loads obtained from the test were noticeably higher by 25 to 55% than the theoretical shear strength provided by the concrete, $V_c$, computed using the ACI 318M-14 equation. This indicates that structural members subjected to biaxial shear force still can be safely designed using the shear strength design formula provided by ACI 318M-14.
- The longitudinal reinforcement ratio significantly affects the diagonal crack angle.
- Numerical flexural strength obtained using layer element method confirms that none of the beams reached flexural capacity because they failed in shear due to lack of shear strength provided in the shear span zone.

6. References
[1] Mark P 2007 No AccessShear-resistant design of biaxially loaded RC beams Magazine of Concrete Research 59(1) 21-28
[2] Tinini A, Minelli F, Belletti B and Scolari M 2016 Biaxial shear in RC square beams: Experimental, numerical and analytical program Engineering Structures 126 469–480
[3] American Concrete Institute 2014 Building Code Requirements for Structural Concrete and Commentary (United States: Farmington Hills Michigan)
[4] R Park, and T Paulay 1975 Reinforced Concrete Structures (New York: John Wiley)
[5] Thamrin R, Tanjung J, Aryanti R, Nur O F and Devinus A 2016 Journal of Engineering Science and Technology 11(4) 548–62
[6] Thamrin R, Kurniawan R, and Melinda A P 2017 Shear and flexural capacity of reinforced concrete members with circular cross section Procedia Engineering 171 957-964
[7] Thamrin R 2017 Analytical Prediction on Flexural Response of RC Beams Strengthened with Steel Plates Int. Symposium on Civil and Environmental Eng. 2016 ISCEE 2016 (Melaka) vol 103 (Paris: MATEC Web of Conference) 02012
[8] Mander J B, Priestley M J N and Park R 1988 Theoretical Stress-Strain Model for Confined Concrete Journal of Structural Engineering 114(8) 1804-1826

Acknowledgements
The authors would like to acknowledge their thanks to the Engineering Faculty, Andalas University for the financial support through Hibah Publikasi in financial year 2019.