Soft elementary compact in soft elementary topology

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Abstract

The notion of soft topology was introduced very recently, built up on soft elementary intersection and union. In this paper, Based on this approach, we introduce the notion of soft elementary compact sets and spaces. Also, we investigate their properties. To that end we prove the soft elementary version of Baire theorem.

2010 AMS Classification: 54A05, 54A10, 54D30, 06D72.

Keywords: Soft set, elementary operations, soft e-topology, e-quasi-compact, e-compact, e-locally compact, e-continuous functions.

1 Introduction

The theory of soft set is a new mathematical tool dealing with uncertainties, it was introduced by Molodtsov [16] in 1999. This theory has several applications in economy, medical sciences, social sciences, engineering, see [6, 15].

After Molodtsov, Maji et al. defined some operators for soft sets [14]. Later; those notions were improved in [1, 19, 20].

In [18], Shabir and Naz defined the soft topology, based on the intersection and union of soft sets as in [19]. This notion of soft topology is used in [2, 3, 5, 11, 12, 17, 21]. Soft concepts of element, interior, closure, separation were also defined in [18].

Hazra et al. [13] gave two approaches of soft topology, and deduce some properties and results.

The notion of soft element defined by Das and Samanta [10], is compatible with the definition of soft subsets, and different from that one defined in [18]. It is used to define soft real number, soft complex and soft metric spaces, see [8, 9, 10]. From this notion of
soft element, Das and Samanta introduced other operations such that elementary union, intersection, and complement.

The compactness was investigated by Aygunoglu and Aygun [3], Bayramov and Gunduz [4], and Zorlutuna et al. [21], which are based at the definition given by Shabir in [18].

In [7], Chiney and Samanta introduced a new definition of soft topology based on the elementary intersection, elementary union, elementary complement. This soft topology is different from these defined in [18, 13]. The necessary soft topological concept tools needed for continuity are defined (soft interior element and set, limiting soft, soft closure set, soft neighborhood, soft base, soft continuous function and soft separation axioms).

In this paper, we adopt this new definition, and for spare a probably paradox, we denote by soft elementary topology (or soft e-topology) for the soft topology which is defined in [7].

The paper comprised five sections. In section 2, we introduce some well-known results in soft set theory, which are needed in the paper. In section 3, we introduce the definition of soft elementary sub-topology, and some other properties and results. In the section 4, we state the definition of soft elementary quasi-compact space, soft elementary compact set and their properties. The last section is devoted to the proof of the soft version of Baire theorem.

## 2 Preliminaries

This section contains definitions and properties of soft sets, and some related notions that will be needed throughout this paper. Let X be an initial universe set, E be the set of parameters, and A be a nonempty subset of E.

**Definition 2.1.** [16] A pair \((F, A)\) is called a soft set over \(X\), if and only if \(F\) is a mapping of \(A\) into \(\mathcal{P}(X)\).

**Definition 2.2.** [1, 14, 19] Let \((F, A)\) and \((G, A)\) be two soft sets over \(X\).

i) \((F, A)\) is called a soft subset of \((G, A)\), (i.e. \((F, A) \subseteq (G, A)\)), if \(F(\alpha)\) is a subset of \(G(\alpha)\), for all \(\alpha \in A\);

ii) \((F, A)\) and \((G, A)\) are called soft equal if \((F, A)\) is a soft subset of \((G, A)\) and \((G, A)\) is a soft subset of \((F, A)\);

iii) the complement or relative complement of a soft set \((F, A)\) is denoted by \((F, A)^C\) and is defined by \((F, A)^C = (F^C, A)\), where \(F^C(\alpha) = C_{X}^{F(\alpha)}\), for all \(\alpha \in A\).

iv) union of \((F, A)\) and \((G, A)\) is the soft set \((H, A)\), defined by \(H(\alpha) = F(\alpha) \cup G(\alpha)\), for all \(\alpha \in A\), and denoted by \((F, A) \cup (G, A)\);

v) intersection of \((F, A)\) and \((G, A)\) is the soft set \((H, A)\), defined by \(H(\alpha) = F(\alpha) \cap G(\alpha)\), for all \(\alpha \in A\), and denoted by \((F, A) \cap (G, A)\).
Example 2.1. Let $X = \{x, y, z\}, A = \{\alpha, \beta\}, (F, A)\text{ and } (G, A)$ such that $F(\alpha) = \{x, y\}, F(\beta) = \{x, z\}, G(\alpha) = \{y, z\}, G(\beta) = \{x\}$. Then; $(F, A) \cap (G, A) = (H, A)$ such that $H(\alpha) = X, H(\beta) = \{x, z\}, (F, A) \cap (G, A) = (J, A)$ such that $J(\alpha) = \{y\}, H(\beta) = \{x\}, (F, A)^c = (F^C, A)$ such that $F^c(\alpha) = \{z\}, F^C(\beta) = \{y\}$.

Definition 2.3. [13] Let $Y$ be a subset of $X$. We denote by $(\bar{Y}, A)$ the soft set $(F, A)$ such that $F(\alpha) = Y$, for all $\alpha \in A$. If $Y = \emptyset, (F, A)$ is called null soft, set denoted by $(\tilde{\Phi}, A)$. If $Y = X, (F, A)$ is called absolute soft set, denoted by $(\tilde{X}, A)$. $S(\tilde{X})$ is the collection of $(\tilde{\Phi}, A)$, and the soft sets $(F, A)$ such that $(F, A)(\alpha) \neq \emptyset$, for all $\alpha \in A$.

It is obvious that if $Y \neq \emptyset$, then $(\bar{Y}, A) \in S(\tilde{X})$, and $(\bar{Y}, A) \neq (\tilde{\Phi}, A)$. We denote by $S(\bar{Y})$ the collection of soft subsets $(F, A)$ of $(\bar{Y}, A)$ such that: $(F, A) = (\tilde{\Phi}, A)$ or $(F, A)(\alpha) \neq \emptyset$ for all $\alpha \in A$.

Remark 2.1. Let $Y$ be a nonempty subset of $X$, and $(F, A) \in S(\tilde{X})$. If $(F, A) \cap (\bar{Y}, A) \in S(\tilde{X})$ then $(F, A) \cap (\bar{Y}, A) \in S(\bar{Y})$. Indeed, $(F, A) \cap (\bar{Y}, A) \subset (\bar{Y}, A)$, and $(F, A) = (\tilde{\Phi}, A)$ or $F(\alpha) \neq \emptyset$ for all $\alpha \in A$. Hence $(F, A) \cap (\bar{Y}, A) \in S(\bar{Y})$.

Definition 2.4. [9]

i) A soft element of $(\tilde{X}, A)$ is a function $\bar{x}$, defined on $A$ to the set $X$. A soft element $\bar{x}$ of $(\tilde{X}, A)$ is said to belong to a soft set $(F, A)$ over $X$, which is denoted by $\bar{x} \in (F, A)$, if $\bar{x}(\alpha) \in F(\alpha)$, for all $\alpha \in A$. Therefore, if $(F, A)$ is such that $F(\alpha) \neq \emptyset$, for all $\alpha \in A$, we have $F(\alpha) = \{\bar{x}(\alpha) : \bar{x} \in (F, A)\}$, for all $\alpha \in A$;

ii) the collection of all soft elements of a soft set $(F, A)$ is denoted by $SE(F, A)$;

iii) for a collection $B$ of soft elements of $(\tilde{X}, A)$, we denote by $SS(B)$ the soft set $(F, A)$ such that: $F(\alpha) = \{\bar{x}(\alpha) : \bar{x} \in B\}$.

Definition 2.5. [9] For any two soft sets $(F, A), (G, A) \in S(\tilde{X})$. Then

i) elementary union of $(F, A)$ and $(G, A)$ is denoted by $(F, A) \cup (G, A)$, and defined by $(F, A) \cup (G, A) = SS(SE(F, A) \cup SE(G, A))$;

ii) elementary intersection of $(F, A)$ and $(G, A)$ is denoted by $(F, A) \cap (G, A)$, and defined by $(F, A) \cap (G, A) = SS(SE(F, A) \cap SE(G, A))$;

iii) elementary complement of $(F, A)$ is denoted by $(F, A)^C$, and defined by $(F, A)^C = SS(B)$, where $B = \{\bar{x} \in (\tilde{X}, A) : \bar{x} \in (F, A)^C\}$.

Example 2.2. Let $X = \{x, y, z\}, A = \{\alpha, \beta\}$. Then; $(\tilde{X}, A) = SS(\{\bar{x_1}, \bar{x_2}, \bar{x_3}, \bar{x_4}, \bar{x_5}, \bar{x_6}, \bar{x_7}, \bar{x_8}, \bar{x_9}\})$, where: $\bar{x_1}(\alpha) = \bar{x_1}(\beta) = x, \bar{x_2}(\alpha) = \bar{x_2}(\beta) = y, \bar{x_3}(\alpha) = \bar{x_3}(\beta) = z, \bar{x_4}(\alpha) = x, \bar{x_4}(\beta) = y, \bar{x_5}(\alpha) = \bar{x_5}(\beta) = z, \bar{x_6}(\alpha) = \bar{x_6}(\beta) = x, \bar{x_7}(\alpha) = \bar{x_7}(\beta) = y, \bar{x_8}(\alpha) = z, \bar{x_8}(\beta) = x, \bar{x_9}(\alpha) = z, \bar{x_9}(\beta) = y$. Let $(F, A), (G, A) \in S(\tilde{X})$ such that $F(\alpha) = \{x, z\}, F(\beta) = \{y, z\}, G(\alpha) = \{x, y\}, G(\beta) = \{x\}$. Then, $(F, A) = SS(\{\bar{x_3}, \bar{x_4}, \bar{x_5}, \bar{x_9}\})$,
\( (G, A) = SS(\{\bar{x}_1, \bar{x}_6\}) \). Hence, \((F, A) \cup (G, A) = SS(\{\bar{x}_1, \bar{x}_3, \bar{x}_4, \bar{x}_5, \bar{x}_6, \bar{x}_9\}) \), \((F, A) \cap (G, A) = (\Phi, A) \). We have \((F, A) \cup (G, A) = (F, A) \cap (G, A) = (\bar{X}, \bar{A}) \), but \((F, A) \cup (G, A) \neq (F, A) \cap (G, A) \) since \((F, A) \cap (G, A) = (H, \alpha) \), such that \(H(\alpha) = \{x\}, H(\beta) = \emptyset \). Now, \((F, A)^C = (F, A)^C = SS(\{\bar{x}_1, \bar{x}_2, \bar{x}_6, \bar{x}_8\}) \).

**Notation:** Let \(Y \) be a nonempty subset of \(X \) and let \((Z, A) \in S(\bar{Y}) \). We denote by \((Z, A)^C \) the soft set \((W, A) \) over \(Y \), where \(W(\alpha) = Y \setminus Z(\alpha) \) for all \(\alpha \in A \). Also, we denote by \((Z, A)^C \) the soft set of the soft elements \(x \) such that \(x \in (Z, A)^C \).

**Definition 2.6.** \([7]\) Let \(\tau \) be a collection of soft sets of \(S(\bar{X}) \). Then \(\tau \) is called a soft e-topology on \((\bar{X}, \tau, A) \) if the following conditions hold

i) \((\Phi, A) \) and \((\bar{X}, A) \) belong to \(\tau \);

ii) the elementary union of any number of soft sets in \(\tau \) belongs to \(\tau \);

iii) the elementary intersection of two soft sets in \(\tau \) belongs to \(\tau \).

The triplet \((\bar{X}, \tau, A) \) is called a soft e-topological space. A member of \(\tau \) is called soft e-open sets in \((\bar{X}, \tau, A) \).

**Definition 2.7.** \([7]\) A soft set \((F, A) \in S(\bar{X}) \) is called a soft e-closed set in \((\bar{X}, \tau, A) \) if its relative complement \((F, A)^C \) belongs to \(S(\bar{X}) \) and \((F, A)^C \) belongs to \(\tau \).

**Proposition 2.1.** \([7]\)

i) \((\Phi, A) \) and \((\bar{X}, A) \) are soft e-closed soft sets in \((\bar{X}, \tau, A) \),

ii) arbitrary elementary intersection of soft e-closed sets is soft e-closed.

**Remark 2.2.** \([7]\) In general, the elementary union of two soft e-closed sets is not soft e-closed.

**Definition 2.8.** \([7]\) Let \((\bar{X}, \tau, A) \) be a soft e-topological space and \((F, A) \in S(\bar{X}) \). Then the soft e-closure of \((F, A) \), denoted by \(\bar{F, A} \) is defined as the elementary intersection of all soft e-closed super sets of \((F, A) \).

**Definition 2.9.** \([7]\) Let \((\bar{X}, \tau, A) \) be a soft e-topological space. A soft element \(\bar{x} \in (\bar{X}, A) \) is called a limiting soft element of a soft set \((F, A) \in S(\bar{X}) \), if for all \((G, A) \in \tau \), and for any \(\alpha \in A \), \(\bar{x}(\alpha) \in G(\alpha) \). This implies that \(F(\alpha) \cap [G(\alpha) \setminus \bar{x}(\alpha)] \neq \emptyset \).

**Definition 2.10.** \([7]\) Let \((\bar{X}, \tau, A) \) be a soft e-topological space and \((F, A) \in S(\bar{X}) \). A soft element \(\ldots \) is called an interior soft element of \((F, A) \), if there exists \((G, A) \in \tau \) such that \(\bar{x}(\alpha) \in (\bar{X}, A) \). The interior of a soft set \((F, A) \) denoted by \(\text{Int}(F, A) \), is defined by \(\text{Int}(F, A) = \{\bar{x}(\alpha) : \bar{x}(\alpha) \in (\bar{X}, A) \} \).

is called soft interior of \((F, A) \) and denoted by \((\bar{F, A}) \).
Definition 2.11. [7] Let $(\tilde{X}, \tau, A)$ be a soft e-topological space. Then $(\tilde{\Phi}, A) \neq (F, A) \in S(\tilde{X})$ is a soft neighborhood (soft nbd, for short) of the soft element $\tilde{x}$ if there exists a soft set $(G, A) \in \tau$, such that $\tilde{x} \in (G, A) \subset (F, A)$.

Example 2.3. Let $X = \{a, b, c, d\}$, $A = \{\alpha, \beta\}$, and $\tau = \{(\tilde{\Phi}, A), (\tilde{X}, A), (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$, such that $F_1(\alpha) = \{a\}, F_1(\beta) = \{b\}, F_2(\alpha) = \{b, c\}, F_2(\beta) = \{c, d\}$, $F_3(\alpha) = \{a, b, c\}, F_3(\beta) = \{b, c, d\}, F_4(\alpha) = X, F_4(\beta) = \{b, c, d\}$. Then; $(\tilde{X}, \tau, A)$ is a soft e-topological space, and the collection of e-closed sets is $\{\tilde{\Phi}, A, (\tilde{X}, A), (F_1, A)^c, (F_2, A)^c, (F_3, A)^c\}$. $(F_4, A)^c$ is not a soft e-closed set, since $(F_4, A)^c \in S(\tilde{X})$.

Now, let $(F, A), (G, A)$ be such that $F(\alpha) = F(\beta) = \{c\}, G(\alpha) = \{a, b\}, G(\beta) = \{b\}$.

Then; $(F, A)^c = (F_1, A)^c, (G, A) = (F_1, A)$, and $(G, A)$ is a soft nbd of $(\tilde{x})$ such that $\tilde{x}(\alpha) = a, \tilde{x}(\beta) = b$.

Definition 2.12. [7] Let $(\tilde{X}, \tau, A)$ be a soft e-topological space. Let $\tilde{x}, \tilde{y} \in (\tilde{X}, A)$ such that $\tilde{x}(\alpha) \neq \tilde{y}(\alpha)$, for all $\alpha \in A$. Then, if there exist $(F, A), (G, A) \subseteq \tau$, such that $\tilde{x} \in (F, A), \tilde{y} \in (G, A)$ and $(F, A) \cap (G, A) = (\tilde{\Phi}, A)$, then $(\tilde{X}, \tau, A)$ is called a soft e-T$_2$ space, or soft e-Hausdorff space.

Definition 2.13. [7] A soft e-topological space $(\tilde{X}, \tau, A)$ is called a soft e-normal space if for any soft closed set $(F, A)$ and for any soft element $\tilde{x}$, such that $\tilde{x}(\alpha) \notin (F, A)(\alpha)$, for all $\alpha \in A$, there exist $(G, A), (H, A) \subseteq \tau$, such that $(F, A) \subset (G, A), \tilde{x} \in (H, A)$ and $(F, A) \cap (G, A) = (\tilde{\Phi}, A)$.

Definition 2.14. [7] A soft e-topological space $(\tilde{X}, \tau, A)$ is called a soft e-regular space if for any two soft closed sets $(F, A)$ and $(G, A)$, such that $(F, A) \cap (G, A) = (\tilde{\Phi}, A)$, there exist $(U, A), (V, A) \subseteq \tau$, such that $(F, A) \subset (U, A), (G, A) \subset (V, A)$ and $(U, A) \cap (V, A) = (\tilde{\Phi}, A)$.

Definition 2.15. [7] Let $X, Y$ be two non-empty sets and $\{f_\alpha : X \to Y, \alpha \in A\}$ be a collection of functions. Then a function $f : SE(\tilde{X}) \to SE(\tilde{Y})$ defined by $[f(\tilde{x})](\alpha) = f_\alpha(x(\alpha))$, for all $\alpha \in A$ is called a soft function.

Definition 2.16. [7] Let $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function. Then

i) the image of a soft set $(F, A)$ over $X$ under the soft function $f$, denoted by $f[(F, A)]$, is defined by $f[(F, A)] = SS\{f(SE(F, A))\}$, i.e. $f[(F, A)](\alpha) = f_\alpha(F(\alpha))$, for all $\alpha \in A$.

ii) the inverse image of a soft set $(G, A)$ over $Y$ under the soft function $f$, denoted by $f^{-1}[(G, A)]$, is defined by $f^{-1}[(G, A)] = SS\{f^{-1}(SE(G, A))\}$, i.e. $f^{-1}[(G, A)](\alpha) = f^{-1}_\alpha(G(\alpha))$, for all $\alpha \in A$.

Definition 2.17. [7] Let $(\tilde{X}, \tau, A), (\tilde{Y}, \sigma, A)$ be two soft e-topological spaces and $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function. $f : (\tilde{X}, \tau, A) \to (\tilde{Y}, \sigma, A)$ is called soft e-continuous at $\tilde{x}_0 \in (\tilde{X}, A)$, if for every $(V, A) \in \sigma$ such that $f(\tilde{x}_0) \in (V, A)$, there exists
(U, A) ∈ τ such that \( \tilde{x}_0 \in (U, A) \) and \( f(U, A) \subseteq (V, A) \). \( f \) is called soft e-continuous on \((X, \tau, A)\), if it is soft e-continuous at each soft element \( \tilde{x}_0 \in (X, A) \).

**Proposition 2.2.** Let \((X, \tau, A), (Y, \sigma, A)\) be two soft e-topological spaces and \( f : SE(X) \to SE(Y) \) be a soft function. Then, \( f \) is soft e-continuous on \((X, \tau, A)\), if and only if for any soft e-open set \((U, A) ∈ S(Y)\) in \((Y, \sigma, A)\), \( f^{-1}(U, A) \) is a soft e-open set in \((X, \tau, A)\).

### 3 Soft elementary topology

In this section, based on the definition of soft elementary topology, we introduce the notion of soft e-topological space, and we investigate their properties. First, we start with the following key result.

**Theorem 3.1.** Let \((X, \tau, A)\) be a soft e-topological space such that for all \((O_1, A), (O_2, A) ∈ \tau\) we have, \((O_1, A) \cap (O_2, A) ∈ S(X)\). Let \( Y \) be a nonempty subset of \( X \) such that for all \((O, A) ∈ \tau\) we have \((O, A) \cap (Y, A) ∈ S(X)\). Then, it holds that the collection \( \tau_Y = \{(O_Y, A) = (O, A) \cap (Y, A), (O, A) ∈ \tau\} \) define a soft e-topology for \((Y, A)\).

**Proof.** Let \((X, \tau, A)\) be a soft e-topological space and let \( Y \) be a nonempty subset of \( X \). We show that \( \tau_Y \) satisfies the conditions of soft e-topology. On the one hand, since \( (\Phi, A) = (\Phi, A) \cap (Y, A) \) and \((\tilde{Y}, A) = (\tilde{X}, A) \cap (Y, A)\). Then \((\Phi, A)\) and \((\tilde{Y}, A)\) belong to \( \tau_Y \). On the other hand, suppose that \( \{(O_Y^i, A) = (O^i, A) \cap (Y, A), i ∈ I\} \) is a family of soft sets in \( \tau_Y \). Then \( \{(O^i, A), i ∈ I\} \) is a family of soft sets in \( \tau \), and \( (O, A) = \bigcup_{i ∈ I} (O^i, A) ∈ \tau \),

it follows that
\[
\bigcup_{i ∈ I} (O_Y^i, A) = \bigcup_{i ∈ I} [(O^i, A) \cap (Y, A)] = \bigcup_{i ∈ I} [(O^i, A)] \cap (Y, A)\]

Finally, let \((O_Y^1, A) = (O^1, A) \cap (Y, A), (O_Y^2, A) = (O^2, A) \cap (Y, A)\) be two soft sets in \( \tau_Y \). Then, \((O^1, A), (O^2, A)\) are two soft sets in \( \tau \), and \((O, A) = (O^1, A) \cap (O^2, A) ∈ \tau \), it follows that
\[
(O_Y^1, A) \cap (O_Y^2, A) = [(O^1, A) \cap (Y, A)] \cap [(O^2, A) \cap (Y, A)] = [(O^1, A) \cap (Y, A)] \cap [(O^2, A) \cap (Y, A)].\]

If \( [(O^1, A) \cap (Y, A)] \cap [(O^2, A) \cap (Y, A)] = (\Phi, A) \) then \( (O_Y^1, A) \cap (O_Y^2, A) ∈ \tau_Y \), else we obtain
\[(O_1^Y, A) \cap (O_2^Y, A) = [(O_1^1, A) \cap (Y, A)] \cap [(O_2^2, A) \cap (Y, A)] = [(O_1^1, A) \cap (O_2^2, A)] \cap (Y, A) = (O, A) \cap (Y, A) \in \tau_Y.\]

Thus, the collection \(\tau_Y\) define a soft topology for \((\hat{Y}, A)\).

Next, we introduce the notion of sub-e-topological space, based on the above theorem.

**Definition 3.1.** Let \((X, \tau, A)\) be a soft e-topological space such that for all \((O_1, A)\), \((O_2, A) \in \tau\) we have, \((O_1, A) \cap (O_2, A) \in S(X)\). Let \(Y\) be a nonempty subset of \(X\) such that for all \((O, A) \in \tau\), \((O, A) \cap (Y, A) \in S(X)\), and \(\tau_Y = \{(O_Y, A) = (O, A) \cap (Y, A) \in \tau\}\). The triplet \((\hat{Y}, \tau_Y, A)\) is called a soft sub-e-topological space of \((X, \tau, A)\) and \(\tau_Y\) is called soft sub-e-topology of \(\tau\). The members of \(\tau_Y\) are called soft eY-open sets in \((\hat{Y}, \tau_Y, A)\).

**Definition 3.2.** Let \((X, \tau, A)\) be a soft e-topological space such that for all \((O_1, A)\), \((O_2, A) \in \tau\) we have, \((O_1, A) \cap (O_2, A) \in S(X)\), \((\hat{Y}, \tau_Y, A)\) be a soft sub-e-topological space of \((X, \tau, A)\), and \((Z, A) \in S(\hat{Y})\). The soft set \((Z, A)\) is called a soft eY-closed set in \((\hat{Y}, \tau_Y, A)\), if \((Z, A)^C_Y \in S(\hat{Y})\) and \((Z, A)^C_Y \in \tau_Y\).

**Example 3.1.** Let \(X = \{a, b, c, d\}, A = \{a, \beta\}\), and \(\tau = \{\{\hat{\Phi}, A\}, \{\hat{X}, A\}, \{F, A\}, \{G, A\}, \{H, A\}\}\), such that \(\Phi = \{a\}, F = \{c, d\}, G = \{c, d\}, G = \{a\}, H = \{a, c, d\}\). Then, \((\hat{X}, \tau, A)\) is a soft e-topological space. Let \(Y = \{a, c\}\), then \(\tau_Y = \{(\hat{\Phi}, A), (\hat{Y}, A), (F_Y, A), (G_Y, A)\}\), is a soft topology at \((\hat{Y}, A)\), where \(F_Y = \{a\}, F_Y = \{c\}, G_Y = \{c\}, G_Y = \{a\}\).

In the following proposition, we characterize the soft eY-closed sets in a sub-e-topological space \((\hat{Y}, \tau_Y, A)\).

**Proposition 3.1.** Let \((\hat{X}, \tau, A)\) be a soft e-topological space such that for all \((O_1, A)\), \((O_2, A) \in \tau\) we have, \((O_1, A) \cap (O_2, A) \in S(\hat{X})\), and \((\hat{Y}, \tau_Y, A)\) be a soft sub-e-topological space of \((\hat{X}, \tau, A)\). If the soft set \((Z, A)\) is a soft eY-closed set in \((\hat{Y}, \tau_Y, A)\), then there exists a soft e-closed set \((F, A)\) in \((\hat{X}, \tau, A)\), such that \((Z, A) = (F, A) \cap (Y, A)\).

**Proof.** Let \((Z, A)\) be a soft eY-closed set in \((\hat{Y}, \tau_Y, A)\). There are two cases to be considered

**Case 1** If \((Z, A) = (\hat{\Phi}, A)\), then \((F, A) = (\hat{\Phi}, A)\).

**Case 2** If \((Z, A) \neq (\hat{\Phi}, A)\), then there exists \((O, A) \in \tau\) such that \((Z, A)^C_Y = (Z, A)^C_Y = (O, A) \cap (Y, A)\). Hence, for all \(\alpha \in A\), we have \(Y \setminus Z(\alpha) = Y \cap O(\alpha)\). Then, it follows that \(Z(\alpha) = O(\alpha) \cap Y\), for all \(\alpha \in A\). Since, \((Z, A) \neq (\hat{\Phi}, A)\), we get \((O, A)^C \in S(\hat{X})\), \((O^C(\alpha) \neq \emptyset\) for all \(\alpha \in A\) and \((Z, A)^C_Y = (O^C(\alpha) \cap Y\). Putting \((F, A) = (O, A)^C\), thus \((Z, A)^C_Y = (F, A) \cap (Y, A)\).

**Proposition 3.2.** Let \((\hat{X}, \tau, A)\) be a soft e-topological space, \((F, A)\) a soft subset of \((\hat{X}, A)\), and \(\hat{x} \in (\hat{X}, A)\). If \(\hat{x}\) is a soft limiting element of \((F, A)\), for all \((G, A) \in \tau\); \(\hat{x} \in (G, A)\) implies that there exists \(\hat{y} \in S(E(\hat{X}))\) such that \(\hat{y} \neq \hat{x}\) and \(\hat{y} \in (F, A) \cap (G, A)\).
Let $(\tilde{X},\tau,A)$ be a soft topological space, $(F,A)$ a soft subset of $(\tilde{X},A)$, and $\tilde{x}$ is a soft limiting element of $(F,A)$. Then, for any $(G,A) \in \tau$ and for any $\alpha \in A$, $\tilde{x}(\alpha) \in G(\alpha)$ implies that $F(\alpha) \cap [G(\alpha) \setminus \{\tilde{x}(\alpha)\}] \neq \emptyset$. Hence, there exists $a_\alpha \in X$ such that $\tilde{x}(\alpha) \neq F(\alpha) \cap G(\alpha)$. Let $\tilde{y} \in (\tilde{X},A)$ such that $\tilde{y}(\alpha) = a_\alpha$ for all $\alpha \in A$, then $\tilde{y} \neq \tilde{x}$ and $\tilde{y} \in (F,A) \oplus (G,A)$.

In the following proposition, we show that the sub-e-topological space of an e-Hausdorff space is an e-Hausdorff space.

**Proposition 3.3.** Let $(\tilde{X},\tau,A)$ be a soft e-topological space such that for all $(O_1,A),(O_2,A) \in \tau$ we have, $(O_1,A) \cap (O_2,A) \in S(\tilde{X})$, and $(\tilde{Y},\tau_Y,A)$ be a soft sub-e-topological space of $(\tilde{X},\tau,A)$. If $(\tilde{X},\tau,A)$ is a soft e-Hausdorff space then $(\tilde{Y},\tau_Y,A)$ is a soft e-Hausdorff space.

**Proof.** Let $\tilde{x},\tilde{y} \in (\tilde{Y},A)$ such that $\tilde{x}(\alpha) \neq \tilde{y}(\alpha)$ for all $\alpha \in A$, then $\tilde{x},\tilde{y} \in (\tilde{X},A)$ and since $(\tilde{X},\tau,A)$ is a soft e-Hausdorff space there exist $(F,A),(G,A) \in \tau$ such that $\tilde{x} \in (F,A),\tilde{y} \in (G,A)$ and $(F,A) \cap (G,A) = (\Phi,A)$. 

$\tilde{x} \in (F_Y,A) = (F,A) \cap (\tilde{Y},A), \tilde{y} \in (G_Y,A) = (G,A) \cap (\tilde{Y},A)$ and $(F_Y,A) \cap (G_Y,A) = (\Phi,A)$. Then $(\tilde{Y},\tau_Y,A)$ is a soft e-Hausdorff space.

### 4 Soft e-compact space and soft e-compact set

This section contains basic definitions and properties of soft e-quasi compact space, e-compact spaces, sets. First, we introduce the notion of e-open cover.

**Definition 4.1.** Let $(\tilde{X},\tau,A)$ be a soft e-topological space, $(F,A) \in S(\tilde{X})$, and $\{(O_i,A)\}_{i \in I}$ be a family of soft e-open sets in $(\tilde{X},\tau,A)$.

i) $\{(O_i,A)\}_{i \in I}$ is called a soft e-open cover of $(\tilde{X},A)$ if $(\tilde{X},A) = \underset{i \in I}{\cup} (O_i,A)$.

ii) $\{(O_i,A)\}_{i \in I}$ is called a soft e-open cover of $(F,A)$ if: $(F,A) \subseteq \underset{i \in I}{\cup} (O_i,A)$.

In the following definition, we introduce the notion of a soft e-quasi compact space.

**Definition 4.2.** Let $(\tilde{X},\tau,A)$ be a soft e-topological space. $(\tilde{X},\tau,A)$ is called a soft e-quasi compact space if every soft e-open cover of $(\tilde{X},A)$ has a finite sub-e-cover of $(\tilde{X},A)$.

In the following theorem, we introduce a necessary condition, so that a soft e-topological space has a soft e-quasi compact space.

**Theorem 4.1.** Let $(\tilde{X},\tau,A)$ be a soft e-quasi compact; then for every family $\{(F_i,A)\}_{i \in I}$ of soft e-closes such that $\bigcap_{i \in I} (F_i,A) = (\Phi,A)$, we can extract a finite subfamily $\{(F_i,A)\}_{i \in I_0 \subseteq I}$ such that $\bigcap_{i \in I_0} (F_i,A) = (\Phi,A)$.
Proof. Assume that $(\bar{X}, \tau, A)$ is soft e-quasi compact, and let $\{ (F_i, A) \}_{i \in I}$ be a family of soft e-closed such that $\bigcap_{i \in I} (F_i, A) = (\bar{X}, A)$. Hence, $\{ (F_i, A)^c \}_{i \in I}$ is a family of soft e-opens, and we have $\bigcup_{i \in I} (F_i, A)^c = (\bar{X}, A)$. Since $(\bar{X}, \tau, A)$ is a quasi e-compact, there exists $I_0 \subset I$ such that $\bigcap_{i \in I_0} (F_i, A)^c = (\bar{X}, A)$. Then, $\bigcap_{i \in I_0} (F_i, A) = (\bar{F}, A)$.

Remark 4.1. Since the complementary of a soft e-open set is not a soft e-close set in general, the converse of theorem 4.1 is not true in general. This is shown in the following counter example. Let $X = [1, +\infty], A = [1, +\infty], I = [1, +\infty]$, and $\tau = \{ (\Phi, A) \} \cup \{(O_i, A), i \in I\}$, where: $(O_i, A)(\alpha) = \left[ \frac{1 + i\alpha}{i + \alpha}, +\infty \right]$, for all $\alpha \in A$. First, it is obvious that $(\bar{\Phi}, A) \in \tau$, and $(\bar{X}, A) = (O_1, A) \in \tau$. On other hand, for any collection $I_0 \subset I$ we have: $\bigcup_{i \in I_0} (O_i, A) = (O_{i_0}, A)$, where $i_0 = \min\{i, i \in I_0\}$. Finally, for all $i, j \in I$ such that $i < j$, we have: $(O_i, A) \cap (O_j, A) = (O_j, A)$. Then, $(\bar{X}, \tau, A)$ is a soft e-topological space.

Now, for all $i \in I : (O_i, A)(1) = X$, then $O_i(1) = \Phi$, and $O_i(\alpha) \neq \Phi$ for all $\alpha \neq 1$ i.e. $(O_i, A)^c \not\in \mathcal{S}(X)$ for all $i \in [1, +\infty]$. The collection of soft e-closed is only $\{(\bar{\Phi}, A), (\bar{X}, A)\}$, which is finite, but the family $\{(O_i, A)\}_{i \in [1, +\infty]}$ is a soft e-open cover of $(\bar{X}, A)$, and we can't extract a finite e-open subcover of $(\bar{X}, A)$.

In the following definition, we introduce the notion of a soft e-compact space.

Definition 4.3. Let $(\bar{X}, \tau, A)$ be a soft e-topological space. $(\bar{X}, \tau, A)$ is called a soft e-compact space, if $(\bar{X}, \tau, A)$ is soft quasi e-compact space, and soft e-Hausdorff space.

Now, we introduce a necessary condition, so that a soft e-topological space has a soft e-compact space.

Theorem 4.2. Let $(\bar{X}, \tau, A)$ be a soft e-compact space, and $\{ (F_i, A) \}_{i = 1}^{\infty}$ be a family of decreasing soft e-closed sets, then $\bigcap_{i = 1}^{\infty} (F_i, A) \neq (\bar{\Phi}, A)$.

Proof. Assume that $\bigcap_{i = 1}^{\infty} (F_i, A) = (\bar{\Phi}, A)$, then $\bigcup_{i = 1}^{\infty} (F_i, A)^c = (\bar{X}, A)$. Since $(\bar{X}, A)$ is a soft e-compact, and $\{ (F_i, A)^c \}_{i = 1}^{\infty}$ is a family of soft e-opens sets, we can extract a decreasing finite subfamily $\{ (F_{i_k}, A)^c \}_{k = 1}^{n}$ such that $\bigcup_{k = 1}^{n} (F_{i_k}, A)^c = (\bar{X}, A)$. Then $(F_{i_k}, A) = \bigcap_{k = 1}^{n} (F_{i_k}, A)^c = (\bar{\Phi}, A)$, which is a contradiction.

In the following definition, we introduce the notion of a soft e-compact set.

Definition 4.4. Let $(\bar{X}, \tau, A)$ be a soft e-Hausdorff space, and $(F, A) \in \mathcal{S}(\bar{X})$ such that $(F, A)^c \in \mathcal{S}(\bar{X})$. $(F, A)$ is called a soft e-compact set if all soft e-open cover of $(F, A)$ has a finite sub-e-cover of $(F, A)$.

Example 4.1. Let $X = \mathbb{R}, A = \{\alpha, \beta\}$, and $\tau$ be the collection of soft sets $(O, A) \in \mathcal{S}(\bar{X})$ such that $(O, A) = (\bar{\Phi}, A)$, or for all $x \in (F, A)$, there exist $r_\alpha > 0, r_\beta > 0$ such that $|x(\alpha) - r_\alpha, x(\alpha) + r_\alpha| \subset O(\alpha)$, and $|x(\beta) - r_\beta, x(\beta) + r_\beta| \subset O(\beta)$. It is obvious that $(\bar{X}, \tau, A)$ is a soft e-topological space, and $\tau_\alpha = \{ O(\alpha), (O, A) \in \tau \}, \tau_\beta = \{ O(\beta), (O, A) \in \tau \}$ are...
two crisp topologies of $X$, equivalent to the topology of metric space $(\mathbb{R}, |.|)$. Now, let $\tilde{x}, \tilde{y}$ be two soft elements of $(X, A)$ such that $\tilde{x}(\alpha) \neq \tilde{y}(\alpha), \tilde{x}(\beta) \neq \tilde{y}(\beta)$, and $r_{\alpha} = |\tilde{x}(\alpha) - \tilde{y}(\alpha)|, r_{\beta} = |\tilde{x}(\beta) - \tilde{y}(\beta)|$ then $|\tilde{x}(\alpha) - \frac{r_{\alpha}}{2}, \tilde{x}(\alpha) + \frac{r_{\alpha}}{2} \cap \tilde{y}(\alpha) - \frac{r_{\alpha}}{2}, \tilde{y}(\alpha) + \frac{r_{\alpha}}{2}| = \emptyset$, and $|\tilde{x}(\beta) - \frac{r_{\beta}}{2}, \tilde{x}(\beta) + \frac{r_{\beta}}{2} \cap \tilde{y}(\beta) - \frac{r_{\beta}}{2}, \tilde{y}(\beta) + \frac{r_{\beta}}{2}| = \emptyset$. Hence, there exists $(F, A)$ a soft e-nbd of $\tilde{x}$, $(G, A)$ a soft e-nbd of $\tilde{y}$ such that $(F, A) \cap (G, A) \neq \emptyset$, hence $((F, A), (G, A), (H, A), (X, A))$, such that $F(\alpha) = \{a\}, F(\beta) = \{c, d\}, G(\alpha) = \{c, d\}, G(\beta) = \{a\}, H(\alpha) = \{a, c, d\}, H(\beta) = \{a, c, d\}$, $Y = \{b, c, d\}$. $(\tilde{Y}, A)$ is a soft compact set, since it is a finite soft set, but we can’t introduce a soft e-compact space, hence we can’t extract a soft e-sub-cover of the e-cover open $\{(F_n, A), n \in \mathbb{N}\}|(F_n(\alpha) = F_n(\beta) = [-n, n]|$, but the soft set $([-1, 1], A)$ is a soft e-compact set.

In the following theorem, we show that the sub-e-compact space is a soft e-compact set.

**Theorem 4.3.** Let $(\tilde{X}, \tau, A)$ be a soft e-Hausdorff space. Assume that for all $(O_1, A), (O_2, A) \in \tau$ we have: $(O_1, A) \cap (O_2, A) \in S(\tilde{X})$. Let $Y$ be a nonempty subset of $X$ such that $Y \neq X$, and for all $(O, A) \in \tau$ we have: $(O, A) \cap (Y, A) \in S(\tilde{X})$. If $(Y, A)$ is a soft $e$-compact space, then $(\tilde{Y}, A)$ is a soft e-compact set in $(\tilde{X}, \tau, A)$.

**Proof.** Since $Y \neq \emptyset, Y \neq X$, we obtain $(\tilde{Y}, A), (\tilde{Y}, A)^C \in S(\tilde{X})$. Now, let $\{(O_i, A), i \in I\} \subseteq \tau$ such that $(\tilde{Y}, A)^C \subseteq \bigcup_{i \in I} (O_i, A)$. Then $(\tilde{Y}, A) = (\tilde{Y}, A) \cap (\bigcup_{i \in I} (O_i, A)) = \bigcup_{i \in I} (\tilde{Y}, A) \cap (O_i, A)$. Hence, the family $\{(\tilde{Y}, A) \cap (O_i, A), i \in I\}$ is a soft $e$-open cover of $(\tilde{Y}, A)$, and since $(\tilde{Y}, \tau, A)$ is a soft e-compact space we can extract a finite family $\{(\tilde{Y}, A) \cap (O_i, A), i \in I_0\}$ such that $(\tilde{Y}, A) = \bigcup_{i \in I_0} (\tilde{Y}, A) \cap (O_i, A) = (\tilde{Y}, A) \cap \bigcup_{i \in I_0} (O_i, A)$. Then, $(\tilde{Y}, A) \subseteq \bigcup_{i \in I_0} (O_i, A)$, hence $(\tilde{Y}, A)$ is a soft e-compact set in $(\tilde{X}, \tau, A)$.

**Remark 4.2.** The converse of theorem [4.3] is not true in general. This is shown in the following counter example. Consider the soft e-topological space, which is introduced in example [4.4]. Let $X = \{a, b, c, d\}, A = \{a, \beta\}$, and $\tau = \{\emptyset, (F, A), (G, A), (H, A), (X, A)\}$, such that $F(\alpha) = \{a\}, F(\beta) = \{c, d\}, G(\alpha) = \{c, d\}, G(\beta) = \{a\}, H(\alpha) = \{a, c, d\}, H(\beta) = \{a, c, d\}$. Let $Y = \{b, c, d\}$. $(\tilde{Y}, A)$ is a soft compact set, since it is a finite soft set, but we can’t introduce a soft sub-e-topological space from $(\tilde{Y}, A)$, since $(F, A) \cap (\tilde{Y}, A) \notin S(\tilde{X})$.

In the following two theorems, we show the relationship between soft e-compact set and soft e-closed set.

**Theorem 4.4.** Let $(\tilde{X}, \tau, A)$ be a soft e-Hausdorff such that for all $(O_1, A), (O_2, A) \in \tau$ we have: $(O_1, A) \cap (O_2, A) \in S(\tilde{X})$. Let $(F, A)$ be a soft e-compact set, then $(F, A)$ is a soft e-closed set.

**Proof.** Assume that $(F, A)^C \neq \emptyset, (F, A)$, and let $\tilde{y} \in (F, A)^C$, then for all $\tilde{x} \in (F, A)$ we have $\tilde{x}(\alpha) \neq \tilde{y}(\alpha)$ for all $\alpha \in A$. Since $(\tilde{X}, \tau, A)$ is a soft e-Hausdorff, there exist $(G_x, A), (H_x, A) \in \tau$ such that $\tilde{x}(G_x, A), \tilde{y}(H_x, A)$ and $(G_x, A) \cap (H_x, A) = \emptyset, (F, A)$. In the following two theorems, we show the relationship between soft e-compact set and soft e-closed set.
We have \((F, A) \subseteq \bigcup_{x \in (F, A)} (G_x, A)\), and since \((F, A)\) is a soft e-compact set, there exist \(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n \in (F, A)\) such that \((F, A) \subseteq \bigcup_{i=1}^n (G_{\bar{x}_i}, A) = \bigcup_{i=1}^n (G_i, A)\). Putting \((H, A) = \bigcup_{i=1}^n (H_{\bar{x}_i}, A) \in \tau\), then \(\bar{y} \in (H, A)\) and \((H, A) \cap (G_i, A) = (\bar{H}, A)\) for all \(i = 1, \ldots, n\).

Since \((H, A), (G_i, A) \in \tau\), we have \((H, A) \cap (G_i, A) = (H)\) for all \(i = 1, \ldots, n\). Then for all \(\alpha \in A\) we have \(\bigcup_{i=1}^n ([H, A] \cap (G_i, A)) = \bigcup_{i=1}^n ([H, A] \cap (G, A)) = (H, A)\). Hence \((H, A) \cap \bigcup_{i=1}^n (G_i, A) = (H, A) \cap \bigcup_{i=1}^n (G, A) = (\bar{H}, A)\). Then, \((H, A) \subseteq \bigcup_{i=1}^n (G_i, A) \quad \forall (F, A)\), so \((F, A) \subseteq \tau\), and \((F, A)\) is soft e-closed set.

**Theorem 4.5.** Let \((\widetilde{X}, \tau, A)\) be a soft e-Hausdorff. Assume that there exists a soft e-compact set \((K, A)\) such that \((F, A) \subseteq (K, A)\), and let \((F, A)\) be a soft e-closed set, then \((F, A)\) is a soft e-compact set.

**Proof.** Since \((F, A)\) is a soft e-closed set, \((F, A)^C \subseteq S(\widetilde{X})\) and \((F, A)^C \in \tau\). Let \(\{(O_i, A), i \in I\}\) be a soft e-cover open of \((F, A)\), then \((\widetilde{X}, A) = (F, A)^C \cup \bigcup_{i \in I} (O_i, A)\), and \((K, A) \subseteq (F, A)^C \cup \bigcup_{i \in I} (O_i, A)\). Since \((K, A)\) is soft e-compact set, there exists a finite subfamily \(\{(O_i, A), i \in I_0 \subset \{\in I\}\}\) such that \((K, A) \subseteq (F, A)^C \cup \bigcup_{i \in I_0} (O_i, A)\). Then, \((F, A) \subseteq \bigcup_{i \in I_0} (O_i, A)\), hence \((F, A)\) is a soft e-compact set.

The following proposition provide that the soft e-compactness is compatible as the soft elementary union, and elementary intersection (in more conditions).

**Proposition 4.1.** Let \((\widetilde{X}, \tau, A)\) be a soft e-Hausdorff. Then

1. *elementary union of two soft e-compact sets is a soft e-compact set.*

2. *if \((\widetilde{X}, \tau, A)\) is a soft e-compact, and for all \((O_1, A), (O_2, A) \in \tau\) we have: \((O_1, A) \cap (O_2, A) \in S(\widetilde{X})\), then elementary intersection of any soft e-compact sets is a soft e-compact set.*

**Proof.** Let \((\widetilde{X}, \tau, A)\) be a soft e-Hausdorff space.

1. Let \((K_1, A), (K_2, A)\) be two soft e-compact sets, and let \(\{(O_i, A), i \in I\}\) be an e-open cover of \((K_1, A) \cup (K_2, A)\). Then \(\{(O_i, A), i \in I\}\) be an e-open cover of \((K_1, A)\) and \((K_2, A)\). We can extract a finite subcover \(\{(O_i, A), i \in I_1\}\) of \((K_1, A)\) and a finite subcover \(\{(O_i, A), i \in I_2\}\) of \((K_2, A)\). Hence, \(\{(O_i, A), i \in I_1 \cup I_2\}\) is a subcover of \((K_1, A) \cup (K_2, A)\).

2. Assume that \((\widetilde{X}, \tau, A)\) is a soft e-compact and for all \((O_1, A), (O_2, A) \in \tau\) we have: \((O_1, A) \cap (O_2, A) \in S(\widetilde{X})\). Let \(\{(K_i, A), i \in I\}\) be a family of soft e-compact sets, then \(\{(K_i, A), i \in I\}\) be a family of soft e-closed sets. \(\bigcap_{i \in I} (K_i, A)\) is a soft e-closed set and soft subset of any soft e-compact set \((K_i, A)\), hence \(\bigcap_{i \in I} (K_i, A)\) is a soft e-compact set.
Remark 4.3. We can replace the condition \((X, \tau, A)\) be a soft e-compact by the condition: there exists a soft e-compact set \((K, A)\) such that for all \(i \in I\) we have \((K_i, A)\subseteq(K, A)\).

Now, we prove some properties of soft e-compact sets, and spaces.

**Theorem 4.6.** Let \((\tilde{X}, \tau, A)\) be a soft e-Hausdorff, \((K, A)\) be a soft e-compact and \((F, A) \in S(\tilde{X})\) be a soft subset not finite of \((K, A)\). Then \((F, A)\) has a limiting soft element.

**Proof.** Let \((\tilde{X}, \tau, A)\) be a soft e-Hausdorff, \((K, A)\) be a soft e-compact, \((F, A) \in S(\tilde{X})\). Assume that \((F, A)\subseteq(K, A)\) is not finite, and has not a soft limiting element, then by lemma \ref{lemma2} for all \(\tilde{x} \in (\tilde{X}, A)\), there exists \((G_x, A) \in \tau\) such that for all \(\tilde{y} \in (\tilde{X}, A)\), \(\tilde{y} \in (\tilde{F}, A) \cap (G_x, A)\) implies that \(\tilde{y} = \tilde{x}\). The family \(\{(G_x, A), \tilde{x} \in (\tilde{X}, A), i = 1 \ldots n\}\) is a soft e-open cover of \((K, A)\). So, we can extract a finite cover \(\{(G_{x_i}, A), \tilde{x}_i \in (\tilde{X}, A), i = 1 \ldots n\}\) \(\subseteq\{(G_x, A), \tilde{x} \in (\tilde{X}, A), i = 1 \ldots n\}\) there exists a subfamily \(\{\tilde{x}_i \in (\tilde{X}, A), i \in \{1, 2, \ldots, n\}\}\). Since, \((K, A) \subseteq (\cup_{i=1}^{n}(G_i, A)\]\(\subseteq (\tilde{F}, A)\) \(\cup (\cup_{i=1}^{n}(G_i, A))\]. Then, \((F, A)\) is a soft subset of \(SS\{\tilde{x}_i \in (\tilde{F}, A), i = 1 \ldots n\}\), hence \((F, A)\) is finite, which is a contradiction.

**Theorem 4.7.** Let \((\tilde{X}, \tau, A)\) be a soft e-compact, \((\tilde{X}, \tau, A)\) is a soft e-regular space.

**Proof.** Let \((F, A)\) be a soft e-closed and let \(\tilde{y} \in (F, A)^c\). From the proof of theorem \ref{lemma4} there exist \((G, A), (H, A) \in \tau\) such that \((F, A) \subseteq (G, A), \tilde{y} \in (H, A), (G, A) \cap (H, A) = (\tilde{F}, A)\) and \((G, A)\) is a soft e-regular space.

**Theorem 4.8.** Let \((\tilde{X}, \tau, A)\) be a soft e-compact, \((\tilde{X}, \tau, A)\) is a soft e-normal space.

**Proof.** Let \((F, A)\) be a soft e-closed set, and let \(\tilde{y} \in (F, A)^c\). From the proof of theorem \ref{lemma6} if \((F_1, A), (F_2, A)\) are two soft e-closed sets such that \((F_1, A) \cap (F_2, A) = (\tilde{F}, A)\), and for all \(\tilde{y} \in (F_2, A)\) there exist \((G_{1y}, A), (G_{2y}, A) \in \tau\) such that \((F, A) \subseteq (G_{1y}, A), \tilde{y} \in (G_{2y}, A), (G_{1y}, A) \cap (G_{2y}, A) = (\tilde{F}, A)\). We can extract a sub-e-cover \(\{(G_{2y}, A)\] \(\subseteq\{(G_{2y}, A)\] of the cover \(\{(G_{2y}, A), \tilde{y} \in (F_2, A)\}\) of \((F_2, A)\). Putting, \((G_1, A) = (G_{1y}, A), (G_2, A) = (G_{2y}, A)\) for all \(i = 1 \ldots n\), \((G_1, A) = (G_{1y}, A), (G_2, A) = (G_{2y}, A)\), hence \((F_1, A) \subseteq (G_1, A), (F_2, A) \subseteq (G_2, A)\) and \((G_1, A) \cap (G_2, A) = (\tilde{F}, A)\). Then, \((\tilde{X}, \tau, A)\) is a soft e-normal space.

This proposition provides that the image of a soft e-compact set by a soft e-continuous function in a soft e-Hausdorff space is a soft e-compact set.

**Proposition 4.2.** Let \((\tilde{X}, \tau, A), (\tilde{Y}, \sigma, A)\) be two soft e-Hausdorff spaces. Let \(f : SE(\tilde{X}) \to SE(\tilde{Y})\) be a soft function and \((K, A)\) be a soft e-compact set of \((\tilde{X}, \tau, A)\). If \(f\) is a soft e-continuous then \(f[(K, A)]\) is a soft e-compact set of \((\tilde{Y}, \sigma, A)\).

**Proof.** Let \(\{(U_i, A) \in \sigma, i \in I\}\) be an open cover of \(f[(K, A)]\), then \(f[(K, A)] \subseteq (\cup_{i \in I}(U_i, A))\) By Proposition 5.4 of \ref{lemma7}, we have \((K, A) \subseteq f^{-1}[(f[(K, A)])] \subseteq f^{-1}[(\cup_{i \in I}(U_i, A)]= \)
\[ \bigcup_{i \in I} f^{-1}[(U_i, A)] \]. Since \( f \) is a soft \( e \)-continuous function, then \( \{ f^{-1}[(U_i, A)], i \in I \} \) is a soft \( e \)-open cover of \((K, A)\), and since \((K, A)\) is a soft \( e \)-compact set we can extract a finite subcover \( \{ f^{-1}[(U_i, A)], i \in I_0 \} \) of \((K, A)\), i.e. \((K, A) \subseteq \bigcup_{i \in I_0} f^{-1}[(U_i, A)]\). Then

\[
 f[(K, A)] \subseteq f\left( \bigcup_{i \in I_0} f^{-1}[(U_i, A)] \right) = \bigcup_{i \in I_0} f(f^{-1}[(U_i, A)]) \subseteq \bigcup_{i \in I_0} (U_i, A). \]

Hence \( f[(K, A)] \) is a soft \( e \)-compact set of \((\tilde{Y}, \sigma, A)\).

\[ \text{Definition 5.3.} \]

Let \((\tilde{X}, \tau, A)\) be a soft \( e \)-compact neighborhood of \((X, \tau, A)\). Then

\[
 f[\tilde{X}] = f(\bigcup_{i \in I_0} \tilde{U}_i) = \bigcup_{i \in I_0} f(\tilde{U}_i) \subseteq \bigcup_{i \in I_0} (U_i, A). \]

In this theorem, we give the soft elementary version of Baire theorem, using the soft elementary intersection.

\[ \text{Theorem 5.1.} \]

Let \((\tilde{X}, \tau, A)\) be a soft locally compact space such that for all \((O_1, A), (O_2, A) \in \tau\) we have: \((O_1, A) \cap (O_2, A) \in S(\tilde{X})\), then \((\tilde{X}, \tau, A)\) is a soft Baire space.

\[ \text{Proof.} \]

Let \((\tilde{X}, \tau, A)\) be a soft \( e \)-locally compact space. Let \( \{ (F_i, A) \}_{i=1}^{\infty} \) be a family of soft \( e \)-closed sets such that \((F_i, A) = (\tilde{F}_i, A)\) for all \( i = 1, 2, \ldots \). We set \( \bigcup_{i=1}^{\infty} (F_i, A) = (F, A)\). To prove that \( (F, A) = (\tilde{F}, A)\) it is enough to prove that for all \( (O, A) \in \tau\)
we have \((O, A) \cap (F, A)^C \neq (\tilde{\Phi}, A)\). Since \((F_1, A) = (\tilde{\Phi}, A)\) we have \(((O, A) \cap (F, A)^C \neq (\tilde{\Phi}, A)\). Since \((O, A), (F_1, A)^C \in \tau, (O, A)^\cap (F_1, A) \in S(\tilde{X})\), there exists a soft element \(\tilde{x}_1 \in (O, A) \cap (F_1, A)^C\), and since \((\tilde{X}, \tau, A)\) is a soft e-locally compact space there exists a soft e-compact set \((K_1, A) \subseteq (O, A) \cap (F, A)^C\) such that \(\tilde{x}_1 \in (K_1, A)\). Next, \((F_2, A) = (\tilde{\Phi}, A)\) and \((K_1, A) \neq (\tilde{\Phi}, A)\), then there exists a soft element \(\tilde{x}_2\) and a soft e-compact set \((K_2, A)\) such that \(\tilde{x}_2 \in ((K_2, A) \cap (F_2, A)^C\). Proche to proche we construct a countable family of e-closed sets \(\{(K_i, A)\}_{i=1}^\infty\) such that \(\tilde{x}_2 \in (K_2, A) \cap (F_2, A)^C\). The family \(\{(K_i, A)\}_{i=1}^\infty\) is a family of decreasing soft subsets of the soft e-compact set \((K_1, A)\). So, \(\{(K_i, A)\}_{i=1}^\infty\) is a family of decreasing soft e-closed subsets of the soft e-compact set \((K_1, A)\), then by theorem 4.2 we have \(\cap_{i=1}^\infty (K_i, A) = (\tilde{\Phi}, A)\). Let \(\tilde{x} \in \cap_{i=1}^\infty (K_i, A) \neq (\tilde{\Phi}, A)\), then \(\tilde{x} \in (K_i, A) \cap (F_i, A)^C\) for all \(i = 1, 2, \ldots\). Hence, \(\tilde{x} \in (F_i, A)^0\) for all \(i = 1, 2, \ldots\). Then, \(\tilde{x} \notin (F, A)^0\), which need to \((O, A) \neq (F, A)^0\), hence \(\cap_{i=1}^\infty (K_i, A) = (\tilde{\Phi}, A)\).

6 Conclusion

In this paper, basing on the approach of Chiney and Samanta [7], we have introduced a definition of soft elementary compact set, and space. We have investigated some properties of the soft elementary compactness, and we have proved the main result, which is the soft elementary version of Baire theorem.

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