High Scale Type-II Seesaw, Dominant $W_L - W_L$ Channel
Double Beta Decay within Cosmological Bound and Verifiable
LFV Decays in SU(5)

M. K. Parida†, Rajesh Satpathy *
Centre of Excellence in Theoretical and Mathematical Sciences
Siksha ‘O’ Anusandhan Deemed to be University (SOADU), Khandagiri Square,
Bhubaneswar 751030, India
September 19, 2018

Abstract

Very recently a novel implementation of type-II seesaw mechanism for neutrino mass
has been proposed in SU(5) grand unified theory with a number of desirable new physical
phenomena beyond the standard model. Introducing heavy right-handed neutrinos and
extra fermion singlets, in this work we show how the type-I seeaw cancellation mechanism
works in this SU(5) framework. Besides predicting verifiable LFV decays, we further show
that the model predicts dominant double beta decay with normal hierarchy or inverted
hierarchy of active light neutrino masses in concordance with cosmological bound. In
addition, a novel mechanism for heavy right-handed neutrino mass generation independent
of type-II seesaw predicted mass hierarchy is suggested in this work.

†email:minaparida@soa.ac.in
*email:rajesh.rajesh.satpathy@gmail.com

1 Introduction

Renormalisable standard model (SM) predicts neutrinos to be massless whereas oscillation
experiments prove them to be massive [1–5]. All the generational mixings have been found to
be much larger than the corresponding quark mixings. Theoretically [6–13] neutrino masses
are predicted through various seesaw mechanisms [14–23, 25–40]. In a minimal left-right
symmetric [41–43] grand unified theory (GUT) like $SO(10)$ [44] where the parity (P) violation
in weak interaction is explained along with fermion masses [45–49], a number of these seesaw
mechanisms can be naturally embedded while unifying the three forces of the SM [50]. More
recently precision gauge coupling unification has been successfully implemented in direct
symmetry breaking of $SO(18) \rightarrow SM$ which may have high potential for new physics [51].

The $SO(10)$ model that predicts the most popular canonical seesaw as well as the type-II
seesaw has also the potential to explain baryon asymmetry of the universe via leptogenesis
through heavy RH neutrino [52] or Higgs triplet decays [53]. But because of underlying
quark lepton symmetry [41], the type-I seesaw scale as well as RHν masses are so large that
the model predicts negligible lepton flavor violating (LFV) decays like $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, $
$\tau \rightarrow e\gamma$, and $\mu \rightarrow eee$. Similarly direct mediation of large mass of scalar triplet required
for type-II seesaw gives negligible contribution to lepton number violating (LNV) and lepton
feature violating (LFV) decays. Ever since the proposal of left-right symmetry, extensive investigations continue in search of experimentally observable double beta decay \[54-56\] in the \(W_R-W_R\) channel \[57,58\]. Adding new dimension to such lepton number violating (LNV) process, the like-sign dilepton production has been suggested as a possible means of detection of \(W_R\)-boson at accelerator energies \[59\], particularly the LHC \[60\]. However, no such signals of TeV scale \(W_R\) have been detected so far. Even if \(W_R\) mass and seesaw scales are large and inaccessible for direct verification, neutrinoless double beta decay \((0\nu \beta \beta)\) in the \(W_L-W_L\) channel \[20,61-66\] is predicted close to observable limit with \(\tau_{\beta\beta} \geq 10^{25}\) yrs provided light neutrino masses predicted by such high-scale seesaw mechanisms are of quasidegenerate (QD) type with masses \(m_\nu \geq \mathcal{O}(0.2)\) eV \[54\]. But as noted by the recent Planck data such QD type masses violate the cosmological bound \[67\]

\[
\Sigma_\nu \equiv \sum_{i=1}^{3} \hat{m}_i < 0.23 \text{ (eV)}. \tag{1}
\]

The fact that such QD type \(\nu\) masses violate the cosmological bound may be unravelling another basic fundamental reason why detection of double beta decay continues to elude experimental observation for several decades. On the other hand if neutrinos have smaller NH or IH type masses there is no hope for detection of these LNV events in near future with RH\(\nu\) extended SM. In other words predicting observable double beta decay in the \(W_L-W_L\) channel with left-handed helicities of both the beta particles has been a formidable problem confronting theoretical and experimental physicists. However, it has been shown that in case of dynamical seesaw mechanism generating Dirac neutrinos the seesaw scale is accessible for direct experimental verification \[68\].

The path breaking discovery of inverse seesaw \[26-32\] with one extra singlet fermion per generation not only opened up the neutrino mass generation mechanism for direct experimental tests, but also lifted up lepton flavor violating (LFV) decays \[69\] from the abyssal depth of experimental inaccessibility of negligible branching ratios \((Br.(l_\alpha \rightarrow l_\beta \gamma) \sim 10^{-50})\) to the illuminating salvation of profound observability \((Br.\simeq 10^{-8}-10^{-16})\) \[70\] which has been discussed extensively \[71-73\]. Despite inverse seesaw, observable double beta decay in the \(W_L-W_L\) channel and the non-QD type neutrino masses remained mutually exclusive until both the RH neutrinos and singlet fermions \((S_i)\) were brought into the arena of LFV and LNV conundrum through the much needed extension of the Higgs sector. The King-Kang \[74\] mechanism cancelled out the ruling supremacy of canonical seesaw which was profoundly exploited in SO(10) models with the introduction of both the SO(10) Higgs representations \(16_H\) and \(126_H^*\) \[13,58,75,79\] with successful prediction of observable double beta decay in the \(W_L-W_L\) channel \[20,61\]. Very interestingly, even though high scale type-II seesaw can govern light neutrino masses of any hierarchy, possibility of observable LFV and double beta decay prediction in the \(W_L-W_L\) channel irrespective of light neutrino mass hierarchies has been realized at least theoretically \[13,79\].

The purpose of this work is to point out that there are new interesting physics realisations with suitable extensions of a non-SUSY SU(5) GUT model proposed recently \[80\] where type-II seesaw, precision coupling unification, verifiable proton decay, scalar dark matter and vacuum stability have been already predicted. However with naturally large type-II seesaw scale \(> 10^{9.2}\) GeV observable double beta decay accessible to ongoing experiments \[54,56\] is possible in this model too with QD type neutrinos only of common mass with \(|m_0| \geq 0.2\) eV like many other high scale seesaw models as noted above. In this work we make additional prediction that dominant double beta decay in the \(W_L-W_L\) channel can be realised with NH
or IH type hierarchy consistent with much lighter neutrino masses $|m_i| < < 0.2$ eV. Thus this realization is consistent with cosmological bound of eq. [1]. Although such possibilities were realized earlier in SO(10) with TeV scale $W_R$ or $Z'$ bosons as noted above, in SU(5) without any expectation of verifiable left-right symmetry, we have shown here for the first time that the dominant double beta decay is mediated by a sterile neutrino (Majorana fermion singlet) of $\mathcal{O}(1)$ GeV mass of first generation. The model further predicts LFV decay branching ratios only $4 - 5$ orders smaller than the current experimental limits. Additional interesting part of the present work is the first suggestion of a new mechanism for heavy RH$\nu$ mass generation that permits these masses to have hierarchies independent of conventional type-II seesaw prediction. Some applications of such masses are briefly noted. Thus the highlights of the present model are

- First implementation of the mechanism for cancellation of type-I seesaw leading to the dominance of type-II seesaw in SU(5).
- Prediction of verifiable LFV decays only $4 - 5$ orders smaller than the current experimental limits.
- Prediction of dominant double beta decay in the $W_L - W_L$ channel close to the current experimental limits for light neutrino masses of NH or IH type in concordance with cosmological bound.
- Suggestion of a new mechanism for right-handed neutrino mass generation independent of type-II predicted mass hierarchy.
- Precision gauge coupling unification with verifiable proton decay which is the same as discussed in [80].

This paper is organised in the following manner. In Sec. 2 we briefly review the SU(5) model along with gauge coupling unification and predictions of the intermediate scales. In Sec. 3 we discuss how type-I seesaw formula for active neutrinos cancels out giving rise to dominance of type-II seesaw and prediction of another type-I seesaw formula for sterile neutrino mass. Fit to neutrino oscillation data is discussed in Sec. 4. In Sec. 5 we present our suggestion for a new mechanism of RH$\nu$ mass generation. Predictions on LFV branching ratios is discussed in Sec. 6. Lifetime prediction for double beta decay is presented in Sec. 7. In Sec. 8 we discuss the results of this work and state our conclusion.

## 2 A NON-SUPERSYMMETRIC SU(5) MODEL

### 2.1 Extension of SU(5)

As noted in ref. [81] the inclusion of the scalar $\kappa(3, 0, 8) \subset 75_H$ with mass $M_\kappa = 10^{9.23}$GeV in the extended non-SUSY SU(5) achieves precision gauge coupling unification. Then type-II seesaw ansatz for neutrino mass is realised by inserting the entire Higgs multiplet $15_H \subset SU(5)$ containing the LH Higgs triplet $\Delta_L(3, -1, 1)$ at the same scale $M_{15_H} = 10^{12}$GeV.

\[
\kappa(3, 0, 8) \subset 75_H, M_\kappa = 10^{9.2}$GeV,
\Delta_L(3, -1, 1) \subset 15_H, M_{15_H} = 10^{12}$GeV,
\chi_s(1, 0, 1), M_{\chi_s} \sim \mathcal{O}(1)$TeV.
\]
The scalar singlet $\chi_S(1, 0, 1)$ has played the crucial interesting role of stabilising the SM scalar potential. The interaction of $15_H$ at any scale $> 10^{9.23}$ GeV in this model maintains precision coupling unification. In the present model we extend the model further by the inclusion of the following fermions and scalars. Being singlets under the SM gauge group they do not affect precision mixing unification of ref. [80].

- Three right handed neutrino singlets $N_i (i = 1, 2, 3)$, one for each generation, with masses to be fixed by this model phenomenology.
- Three left-handed Majorana fermion singlets $S_i (i = 1, 2, 3)$, one for each generation, similar to those introduced in case of inverse seesaw mechanism [26–32].
- A Higgs scalar singlet $\xi_S(1, 0, 1)$ to generate $S - N$ mixings through its VEV and to ensure vacuum stability of the scalar potential.

2.2 Coupling Unification, GUT Scale, and Proton Lifetime

As already discussed [80, 81] using renormalisation group equations for gauge couplings and the set of Higgs scalars of eq(2), precision unification has been achieved with the PDG values of input parameters [82–84] on $\sin^2 \theta_W(M_Z), \alpha_S(M_Z)$ resulting in the following mass scales and the GUT fine-structure constant $\alpha_G$

\[
M_U = 10^{15.23} \text{GeV}, \\
M_\kappa = 10^{9.23} \text{GeV}, \\
M_{\Delta L} = M_{15_H} = 10^{12} \text{GeV}, \\
\frac{1}{\alpha_G} = 37.765. 
\] (3)

Using threshold effects due to superheavy Higgs scalars [85–92], proton lifetime prediction for $p \rightarrow e^+\pi^0$ turns out to be in the experimentally accessible range [93]

\[
\tau_p(p \rightarrow e^+\pi^0) = (1.01 \times 10^{34\pm0.44} - (5.5 \times 10^{35\pm0.44}) \text{yrs.} 
\] (4)

Extensive investigations with number of SU(5) GUT extensions have been carried out with proton lifetime predictions consistent with experimental limits [94–96]. But implementation of type-II seesaw dominance due to type-I seesaw cancellation resulting in dominant LFV and LNV decays as discussed below are new in the context of non-SUSY SU(5).

3 CANCELLATION OF TYPE-I AND DOMINANCE OF TYPE-II SEESAW

Due to the introduction of heavy RH$\nu$s in the present model which were absent in [80], it may be natural to presume apriori that besides type-II seesaw, type-I seesaw may also contribute substantially to light neutrino masses and mixings. But it has been noted that there is a natural mechanism to cancel out type-I seesaw contribution while maintaining dominance of inverse seesaw [58–74, 78] or type-II seesaw or even linear seesaw [13, 79] as the case may be. Briefly we discuss below how this mechanism operates in the present extended model resulting in type-II seesaw dominance even in the presence of heavy RH$\nu$s.

4
The SM invariant Yukawa Lagrangian of the model is
\[
\mathcal{L}_{\text{Yuk}} = Y_{\ell} \bar{\psi}_L \psi_R \Phi + f \bar{\psi}_L \psi L \Delta_L + y \chi S \phi N C S \chi s (1/2) M_N N C N + h.c.
\] (5)

Using the VEVs of the Higgs fields and denoting \( M = y_\chi < \chi >, M_D = Y < \Phi >, \) a 9x9 neutral-fermion mass matrix has been obtained which upon block diagonalisation yields 3x3 mass matrices for each of the light neutrino (\( \nu_\alpha \)), the right handed neutrino (\( N_\alpha \)), and the sterile neutrino (\( S_\alpha \)) \[75,78,79\]. The block diagonalisation of 9x9 neutral fermion mass matrix was earlier presented in useful format in ref. \[33\] which have been effectively utilised to study the type-I seesaw cancellation mechanism in SO(10) models.

In this model the left-handed triplet \( \Delta_L \) and RH neutrinos \( M_N \) being much heavier than the other mass scales with \( M_{\Delta_L} \gg M_N \gg M_D \) are at first integrated out from the Lagrangian leading to
\[
-\mathcal{L}_{\text{eff}} = \left( m^H_{\nu} + M_D \frac{1}{M_N} M_D^T \right) \nu^T \nu_{\beta} + \left( M_D \frac{1}{M_N} M_D^T \right)_{\alpha m} (\bar{\nu}_\alpha S_m + \bar{S}_m \nu_{\alpha})
+ \left( M \frac{1}{M_N} M_D^T \right)_{mn} S_m^T S_n,
\] (6)

which, in the \( (\nu, S) \) basis, gives the 6x6 mass matrix
\[
\mathcal{M}_{\text{eff}} = \begin{bmatrix} M_D M_N^{-1} M_D^T & m^H_{\nu} M_D M_N^{-1} M_D^T \\ M M_N^{-1} M_D^T & M M_N^{-1} M_D^T \end{bmatrix},
\] (7)

while the 3x3 heavy RH neutrino mass matrix \( M_N \) is the other part of the full 9x9 neutrino mass matrix. This 9x9 mass matrix \( \mathcal{M}_{\text{BD}} \) which results from the first step of block diagonalization procedure as discussed above and in the appendix is
\[
\mathcal{W}_1^\dagger \mathcal{M}_{\nu} \mathcal{W}_2^\ast = \mathcal{M}_{\text{BD}} = \begin{bmatrix} M_{\nu} & 0 & 0 \\ 0 & M_S & 0 \\ 0 & 0 & M_N \end{bmatrix},
\] (8)

Defining
\[
X = M_D M_N^{-1}, \quad Y = M M_N^{-1}, \quad Z = M_D M_N^{-1}.
\] (9)

The transformation matrix \( \mathcal{W}_1 \) has been derived as shown in eqn. \[10\] \[75,79\]
\[
\mathcal{W}_1 = \begin{bmatrix} 1 & -\frac{1}{2} Y Z^\dagger & -\frac{1}{2} Z Y^\dagger & Z \\ -\frac{1}{2} Y Z^\dagger & 1 & -\frac{1}{2} Y Y^\dagger & Y \\ -Z^\dagger & -Y^\dagger & 1 & -\frac{1}{2} (Z^\dagger Z + Y^\dagger Y) \end{bmatrix}
\] (10)

After the second step of block diagonalization, the type-I seesaw contribution cancels out and gives in the \( (\nu, S, N) \) basis
\[
\mathcal{W}_2^\dagger \mathcal{M}_{\text{BD}} \mathcal{W}_2^\ast = \mathcal{M}_{\text{BD}} = \begin{bmatrix} m_{\nu} & 0 & 0 \\ 0 & m_S & 0 \\ 0 & 0 & m_N \end{bmatrix},
\] (11)

5
where $W_2$ has been derived in eqn. [12] [75] [79]. We have used the bare mass of $S_i$ and VEV of $\chi_L(2,-1/2,1)$ to be vanishing i.e $\mu_S = 0, <\chi_L> = 0$ to get the form suitable for this model building

$$W_2 = \begin{pmatrix} 1 - \frac{1}{2}XX^\dagger & X & 0 \\ -X^\dagger & 1 - \frac{1}{2}X^\dagger X & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

In eq. (11), the three $3 \times 3$ matrices are

$$m_\nu = m_\nu^I = f v_L \tag{13}$$

$$m_S = -M M_N^{-1} M^T \tag{14}$$

$$m_N = M_N, \tag{15}$$

the first of these being the well known type-II seesaw formula and the second is the emergence of the corresponding type-I seesaw formula for sterile fermion mass. The third of the above equations represents the heavy RH $\nu$ mass matrix.

In the third step, $m_\nu, m_S, m_N$ are further diagonalized by the respective unitary matrices to give their corresponding eigenvalues

$$U^\dagger_\nu m_\nu U_\nu = \hat{m}_\nu = \text{diag} (m_1, m_2, m_3),$$

$$U^\dagger_S m_S U_S = \hat{m}_S = \text{diag} (m_{S_1}, m_{S_2}, m_{S_3}),$$

$$U^\dagger_N m_N U_N = \hat{m}_N = \text{diag} (M_{N_1}, M_{N_2}, M_{N_3}). \tag{16}$$

The complete mixing matrix [33,75] diagonalizing the above $9 \times 9$ neutrino mass matrix given in (??) turns out to be

$$V = \begin{pmatrix} \nu^\nu_{\alpha i} & \nu^S_{\alpha j} & \nu^N_{\alpha k} \\ \nu^S_{\beta i} & \nu^S_{\beta j} & \nu^S_{\beta k} \\ \nu^N_{\gamma i} & \nu^S_{\gamma j} & \nu^N_{\gamma k} \end{pmatrix} \tag{17}$$

$$= \begin{pmatrix} (1 - \frac{1}{2}XX^\dagger) U_\nu & (X - \frac{1}{2}ZY^\dagger) U_S & Z U_N \\ -X^\dagger U_\nu & (1 - \frac{1}{2}(X^\dagger X + YY^\dagger)) U_S & (Y^\dagger Y) U_N \\ 0 & -Y^\dagger U_S & (1 - \frac{1}{2}X^\dagger Z) U_N \end{pmatrix}, \tag{18}$$

as shown in the appendix. In eqn. (18) $X = M_D M^{-1}, Y = M M_N^{-1}$ and $Z = M_D M_N^{-1}$.

The mass of the singlet fermion is acquired through a type-I seesaw mechanism

$$m_S = -M \frac{1}{M_N} M^T \tag{19}$$

where $M$ is the $N - S$ mixing mass term in the Yukawa Lagrangian eq. (??).

### 4 TYPE-II SEESAW FIT TO OSCILLATION DATA

#### 4.1 Neutrino Mass Matrix from Oscillation Data

Using diagonalisation of neutrino mass matrix ($m_\nu$) by the PMNS matrix $U_{PMNS}$

$$m_\nu = U_{PMNS} \text{diag}(m_1, m_2, m_3) U_{PMNS}^T, \tag{20}$$
where \( m_i(i = 1, 2, 3) \) denote the mass eigenvalues. For neutrino mixings we use the abbreviated cyclic notations \( t_i = \sin \theta_{jk}, \ c_i = \cos \theta_{jk} \) where \( i, j, k \) are cyclic permutations of generational numbers 1, 2, 3. In the standard parametrisation we \([82-84]\)

\[
U_{\text{PMNS}} = \begin{pmatrix}
    c_3 c_2 & t_3 c_2 & t_2 e^{-i \delta_D} \\
    -t_3 c_1 - c_3 s_1 t_2 e^{i \delta_D} & c_3 c_1 - t_3 s_1 t_2 e^{i \delta_D} & t_1 c_2 \\
    t_3 s_1 - c_3 c_1 s_2 e^{i \delta_D} & -c_3 s_1 t_3 c_1 s_2 e^{i \delta_D} & c_1 c_2
\end{pmatrix} \text{diag}(e^{i \alpha_M}, e^{i \beta_M}, 1) \tag{21}
\]

where \( \delta_D \) is the Dirac CP phase and \((\alpha_M, \beta_M)\) are Majorana phases.

Here we present numerical analyses within 3\( \sigma \) limit of the neutrino oscillation data in the type-II seesaw framework \([80]\). As we do not have any experimental information about Majorana phases, they are determined by means of random sampling: i.e from the set of randomly generated values, each confined within the maximum allowed limit of 2\( \pi \) only one set of values for \((\alpha_M, \beta_M)\) is chosen. Very recent analysis of the oscillation data has determined the 3\( \sigma \) and 1\( \sigma \) limits of Dirac CP phase \( \delta_D \) \([1]\). The best fit values of \( \delta_D \) in the normally ordered (NO) and invertedly ordered (IO) cases are near 1.2\( \pi \) and 1.5\( \pi \), respectively, which we utilise for the sake of simplicity.

Global fit to the oscillation data \([1]\) is summarised below including respective parameter uncertainties at 3\( \sigma \) level

\[
\begin{align*}
\theta_{12}^\circ &= 34.5 \pm 3.25, \quad \theta_{23}^\circ (\text{NO}) = 41.0 \pm 7.25, \\
\theta_{23}^\circ (\text{IO}) &= 50.5 \pm 7.25, \quad \theta_{13}^\circ (\text{NO}) = 8.44 \pm 0.5, \\
\theta_{13}^\circ (\text{IO}) &= 8.44 \pm 0.5, \quad \delta_D/\pi (\text{NO}) = 1.40 \pm 1.0, \\
\delta_D/\pi (\text{IO}) &= 1.44 \pm 1.0, \\
|\Delta m_{21}^2| &= (7.56 \pm 0.545) \times 10^{-5} \text{eV}^2, \\
|\Delta m_{31}^2| (\text{NO}) &= (2.55 \pm 0.12) \times 10^{-3} \text{eV}^2, \\
|\Delta m_{31}^2| (\text{IO}) &= (2.49 \pm 0.12) \times 10^{-3} \text{eV}^2. \tag{22}
\end{align*}
\]

We denote the cosmologically constrained parameter, the sum of the three active neutrino masses,

\[
\Sigma_\nu = \sum_{i=1}^{3} \hat{m}_i \tag{23}
\]

whose cosmologically determined upper bound has been given in eq.\([1]\). For normally hierarchical (NH), inverted hierarchical (IH), and quasi-degenerate (QD) patterns, the experimental data on mass squared differences have been fitted by the following light neutrino mass eigen values with the respective values of the cosmological parameter \( \Sigma_\nu \)

\[
\begin{align*}
\hat{m}_\nu &= (0.00127, 0.008838, 0.04978) \text{ eV (NH)} \\
\Sigma_\nu &= 0.059888 \text{ eV}, \\
\hat{m}_\nu &= (0.04901, 0.04978, 0.00127) \text{ eV (IH)} \\
\Sigma_\nu &= 0.059888 \text{ eV}, \\
\hat{m}_\nu &= (0.2056, 0.2058, 0.2) \text{ eV (QD)}, \\
\Sigma_\nu &= 0.6114 \text{ eV}. \tag{24}
\end{align*}
\]
The last line clearly shows violation of cosmological bound in the QD case. Using oscillation data and best fit values of the mixings we have also determined the PMNS mixing matrix numerically

\[ U_{PMNS} = \begin{pmatrix}
0.816 & 0.56 & -0.0199 - 0.0142i \\
-0.354 - 0.0495i & 0.675 - 0.0346i & 0.650 \\
0.450 - 0.0568i & -0.485 - 0.0395i & 0.75 \\
\end{pmatrix}. \] (25)

4.2 Determination of Majorana Yukawa Coupling Matrix

Now inverting the relation \( \hat{m}_\nu = U_{PMNS}^\dagger M_\nu U_{PMNS}^* \) where \( \hat{m}_\nu \) is the diagonalised neutrino mass matrix, we determine \( M_\nu \) for three different cases and further determine the corresponding values of the \( f \) matrix using \( f = M_\nu/v_L \) where we use the predicted value of \( v_L = 0.1 \) eV.

**NH**

\[ f = \begin{pmatrix}
0.117 + 0.022i & -0.124 - 0.003i & 0.144 + 0.025i \\
-0.124 - 0.003i & 0.158 - 0.014i & -0.141 + 0.017i \\
0.144 + 0.025i & -0.141 + 0.017i & 0.313 - 0.00029i \\
\end{pmatrix}. \] (26)

**IH**

\[ f = \begin{pmatrix}
0.390 - 0.017i & 0.099 + 0.01i & -0.16 + 0.05i \\
0.099 + 0.01i & 0.379 + 0.02i & 0.176 + 0.036i \\
-0.16 + 0.05i & 0.176 + 0.036i & 0.21 - 0.011i \\
\end{pmatrix}. \] (27)

**QD**

\[ f = \begin{pmatrix}
2.02 + 0.02i & 0.0011 + 0.02i & -0.019 + 0.3i \\
0.0011 + 0.02i & 2.034 + 0.017i & 0.021 + 0.21i \\
-0.019 + 0.3i & 0.021 + 0.21i & 1.99 - 0.04i \\
\end{pmatrix}. \] (28)

Randomly chosen Majorana phases \[ \alpha_M = 74.84^\circ, \beta_M = 112.85^\circ \] and the central value of the Dirac phase \( \delta_D = 218^\circ \) have been used in this analysis. Using the well known definition of the Jarlskog invariant

\[ J_{CP} = -t_3c_3t_2c_2c_1s_1 \sin \delta_D, \] (29)

and keeping \( \delta_D \) at its best fit values we have estimated the predicted allowed ranges of the CP-Violating parameter in both cases.

\[ J_{CP} = 0.0175 - 0.0212 \text{ (NH)} \]

\[ J_{CP} = 0.0302 - 0.0365 \text{ (IH)} \] (30)

where the variables have been permitted to acquire values within their respective 3\sigma ranges of the oscillation data. Besides these there are non-unitarity contributions which have been discussed extensively in the literature.
4.3 Scaling Transformation of Solutions

In general there could be type-II seesaw models characterizing different seesaw scales and induced VEVs matching the given set of neutrino oscillation data represented by the same neutrino mass matrix. For two such models

\[ m_\nu = f_1 v_1^{(1)} = f_2 v_2^{(2)}, \]

Then the \( f \) matrix in one case is determined up to good approximation in terms of the other from the knowledge of the two seesaw scales

\[ f^{(1)} \simeq f^{(2)} \frac{M_{\Delta(1)}}{M_{\Delta(2)}}. \]

At \( M_{\Delta(1)} = 10^{12} \text{ GeV} \) our solutions are the same as in [80]. In view of this scaling relation, we can determine the values of the Majorana Yukawa matrix in the present case from the estimations of [80]. For example, if we choose \( M_{\Delta(1)} = 10^{10} \text{ GeV} \) in the present case compared to \( M_{\Delta(2)} = 10^{12} \text{ GeV} \) in [80], we rescale the solutions of [80] by a factor \( 10^{-2} \) to derive solutions in the present case. Thus graphical representations of solutions are similar to those of ref. [80] for \( M_{\Delta(2)} = 10^{12} \text{ GeV} \) which we do not repeat here. The values of magnitudes of \( f_{ij} \) at any new scale are obtained by rescaling them by the appropriate scaling factor while the phase angles remain the same as in [80].

4.4 Dirac Neutrino Mass Matrix

The Dirac neutrino mass matrix \( M_D \) plays crucial role in predicting LFV and LNV decays. In certain SO(10) models [45, 46, 79, 99] this is usually determined by fitting the charged fermion masses at the GUT scale and equating it with the upquark mass matrix. The fact that \( M_D^{0} \simeq M_u^{0} \) at the GUT scale follows from the underlying quark lepton symmetry [41] of SO(10). In SU(5) itself, however, there is no such symmetry to dictate the structure of \( M_D \) in terms of quark matrices. Also in this SU(5) model we do not attempt any charged fermion mass fit at the GUT scale or above it. Since the Dirac neutrino mass matrix is not predicted by the SU(5) symmetry itself, for the sake of simplicity and to derive maximal effects on LFV and LNV decays, we assume \( M_D^{0} \) to be equal to the up-quark mas matrix \( M_u^{0} \) at the GUT scale. Noting that \( N \) is SU(5) singlet fermion, in the context of relevant Yukawa interaction Lagrangian

\[ -\mathcal{L}_{\text{Yuk}} = [Y_N \bar{F}_5.1_F.5H + Y_u 10_F.10F.5H + \ldots] + h.c., \]

this assumption is equivalent to alignment of the two Yukawa couplings

\[ Y_N \simeq Y_u. \]

This alignment is naturally predicted in SO(10) or SO(18) [51], but in the present SU(5) case it is assumed.

We realise this matrix \( M_D \) using renormalisation group equations for fermion masses and gauge couplings and their numerical solutions [77] starting from the PDG values [82–84] of fermion masses at the electroweak scale. Following the bottom-up approach and using the down quark diagonal basis, the quark masses and the CKM mixings are extrapolated from low
energies using renormalisation group (RG) equations \[97,98\]. After assuming the approximate equality \( M_D^0 \approx M_u^0 \) at the GUT scale where \( M_u^0 \) is the up-quark mass matrix, the top-down approach is exploited to run down this mass matrix \( M_D^0 \) using RG equations \[97\]. Then the value of \( M_D \) near \( 1 \sim 10 \) TeV scale turns out to be

\[
M_D \simeq \begin{pmatrix}
0.014 & 0.04 - 0.01i & 0.109 - 0.3i \\
0.04 + 0.01i & 0.35 & 2.6 + 0.0007i \\
0.1 + 0.3i & 2.6 - 0.0007i & 79.20 \\
\end{pmatrix} \text{GeV.}
\]

As already noted above, although on the basis of SU(5) symmetry alone there may not be any reason for the rigorous validity of eq.(35), in what follows we study the implications of this assumed value of \( M_D \) to examine maximum possible impact on LFV and LNV decays discussed in Sec.6 and Sec.7. Another reason is that the present assumption on \( M_D \) may be justified in direct SO(10) breaking to the SM which we plan to pursue in a future work \[100\].

5 RIGHT-HANDED NEUTRINO MASS IN SU(5) vs. SO(10)

5.1 RH\( \nu \) Mass in SO(10)

The fermions responsible for type-I and type-II seesaw are the LH leptonic doublets and the RH fermionic singlets of three generations. In SO(10) the left handed lepton doublet \((\nu, l)^T\) and the right-handed neutrino \(N\) are in the same representation \(16_F\).

\[
(\nu, l)^T \subset 16_F,
\]

\[
N \subset 16_F.
\]

In SO(10) GUT the Higgs representation \(126_H^\dagger\) contains both the left-handed and the right-handed triplets carrying \( B - L = -2 \),

\[
126_H^\dagger = \Delta_L(3, 1, -2, 1) + \Delta_R(1, 3, -2, 1) + ....
\]

where the quantum numbers are under the left-right symmetry group \(SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C\). The common Yukawa coupling \(f_{10}\) in the Yukawa term

\[
-\mathcal{L}_{\text{Yuk10}} = f_{10}16_F16_F126_H^\dagger,
\]

generates the dilepton-Higgs triplet interactios both in the left-handed and right-handed sectors giving rise to type-I and type-II seesaw mechanisms. The RH neutrino mass is generated through the VEV of the neutral component of the of \(\Delta_R\)

\[
M_N = f_{10}\langle \Delta_R^0 \rangle.
\]

The type-II seesaw contribution to light neutrino mass is

\[
\mathcal{M}_\nu = f_{10}v_L
\]

where \(v_L\) is the corresponding induced VEV of \(\Delta_L\)

\[
v_L = \lambda_{10}\frac{\langle \Delta_R^0 \rangle v_{ew}^2}{M_{\Delta_L}^2}.
\]
Here $\lambda_{10}$ is the quartic coupling in the part of the scalar potential

$$V_{10} = \lambda_{10} \Delta_L^\dagger \Delta_R \Phi \Phi \subset \lambda_{10} 126_H^\dagger 126_H 10_H 10_H.$$  \hfill (42)

Thus with type-II seesaw dominance, the predicted heavy RH neutrino masses in SO(10) follow the same hierarchical pattern as the active light neutrino masses

$$M_{N_1} : M_{N_2} : M_{N_3} :: m_1 : m_2 : m_3.$$  \hfill (43)

### 5.2 RHν Mass in SU(5)

Feynman diagram for type-II seesaw mechanism in the present model is shown in Fig.1. In contrast to SO(10) where the LH leptonic doublet and the RH $\nu$ are in the single representation $16_F$, in SU(5) they are in different fermionic representations

$$(\nu, l)^T \subset 5_F,$$

$$N \subset 1_F.$$  \hfill (44)

In SU(5), while the dilepton Higgs interaction is given by

$$-\mathcal{L}_{Yukl} = f \bar{5}_F \bar{5}_F 15_H,$$  \hfill (45)

the RH neutrino mass is generated through

$$-\mathcal{L}_{YukNN} = (1/2) f_N N N \sigma_S + h.c.$$  \hfill (46)

The fact that $N$ is a singlet under SU(5) forces $\sigma_S$ to be a singlet too. Further this singlet $\sigma_S$ must carry $B-L=-2$ as its VEV generates the heavy Majorana mass

$$M_N = f_N \langle \sigma_S \rangle.$$  \hfill (47)

In sharp contrast to SO(10) where the LH triplet $\Delta_L$ and the RH triplet $\Delta_R$ scalars contained in the same representation $126_H^\dagger$ generate the type-II seesaw and $M_N$ the situation in
SU(5) is different. Since LH triplet $\Delta_L (3, -1, 2)$ mediating type-II seesaw belongs to Higgs representation $15_H \subset SU(5)$ and $\sigma_S$ belongs to a completely different representation which is a singlet $1 \subset SU(5)$, the two relevant Majorana type couplings in general may not be equal

$$f_N \neq f.$$  

(48)

Also this assertion is further strengthened if we do not assume SU(5) to be a remnant of SO(10).

Then the RH neutrino mass hierarchy can be decoupled from the type-II seesaw prediction. It is interesting to note that in SU(5)

$$v_L = \frac{\mu_\Delta v_{\text{ew}}^2}{M_\Delta^2}$$  

(49)

where $\mu_\Delta$ is the trilinear coupling in the potential term

$$V_{\text{tri}} = \mu_\Delta \Delta_L \Phi \Phi + h.c.$$  

(50)

The VEV of this singlet $\sigma_S$ can explain the dynamical origin of this trilinear coupling through its VEV $v_\sigma = \langle \sigma_S \rangle$

$$\mu_\Delta = \lambda v_\sigma,$$  

(51)

where $\lambda$ is the quartic coupling in the potential term

$$V_{\text{ql}} = \lambda \sigma_S \Delta_L \Phi \Phi + h.c.$$  

(52)

$\subset$ $\lambda \sigma_S 15_H 5_H 5_H + h.c$  

(53)

where the second line represents the SU(5) origin. For GUT-scale $U(1)_{B-L}$ symmetry breaking driving VEV $v_\sigma \simeq M_{\text{GUT}}$ in the absence of any intermediate symmetry, it is possible to ensure $\mu_\Delta \simeq M_{\Delta_L}$ for

$$\lambda \simeq \frac{M_{\Delta_L}}{M_{\text{GUT}}},$$  

(54)

Thus the SU(5) model gives similar explanation for quartic coupling as in direct breaking of SO(10). But the predicted hierarchy of RH$\nu$ masses may not in general follow the same hierarchical pattern as given by SO(10) as shown in eq.(43). This is precisely because the eq.(43) follows because the same diagonalisation matrix $U_{\text{PMNS}}$ diagonalises the LH and the RH neutrino mass matrices which is further rooted in the fact that same Majorana coupling $f_{10}$ that generates the type-II seesaw mass term also generates $M_N$. But because of the general possibility $f_N \neq f$, the RH$\nu$s may acquire a completely different pattern depending upon the value of $f_\ell$. Unlike SO(10), these masses are also allowed to be quite different from the type-II seesaw scale.

Even if the value of $v_\sigma$ may be needed to be near $M_{\Delta_L}$, the value of $M_N$ is allowed to be considerably lower by finetuning the value of $f_N$. Our LFV and LNV decay phenomenology as discussed below may need $M_N = 1 - 10$ TeV which is realizable using this new technique in SU(5). In contrast SO(10) needs $U(1)_R \times U(1)_{B-L}$ or $SU(2)_R \times U(1)_{B-L}$ gauge symmetry and hence new gauge bosons near the TeV scale to generate such RH$\nu$ masses which should be detected at LHC [58, 79]. Thus a new mechanism for RH$\nu$ mass emerges here by noting the coupling $f_N \neq f$ which has the potential to generate RH$\nu$ masses in the range $100 - 10^{15}$
GeV. Thus the RHν mass predictions in the two GUTs in the presence of type-II seesaw dominance are

**Type-II Seesaw Dominated SO(10):**

\[ M_{N_i} \simeq \frac{m_i M^2_{\Delta L}}{v_{ew}^2}. \]  

**Type-II Seesaw Dominated SU(5):**

\[ M_{N_i} = [O(10)\text{GeV} - O(M_{\Delta L})]. \]

Here \( m_i, i = 1, 2, 3 \) represents the three mass eigen values of light neutrinos. It is to be noted that \( m_i \) is absent in the RHS of eq.(56) in the SU(5) case.

### 6 LEPTON FLAVOR VIOLATIONS

In the SM extensions there has been extensive investigation of lepton flavor violating phenomena \( l_\alpha \to l_\beta + \gamma \) and other processes like \( \mu \to e e e \) including unitarity violations [71–73]. In the flavor basis we use the standard charged current Lagrangian

\[ \mathcal{L}_{CC} = -\frac{1}{\sqrt{2}} \sum_{\alpha = e, \mu, \tau} \left[ g_2 L_{\alpha L} \gamma_{\mu} W^\mu_{\alpha L} \right] + \text{h.c.} \]  

(57)

In predicting the LFV branching ratios we have used the relevant formulas of [71] while assuming a simplifying diagonal structure for \( M, \)

\[ M = \text{diag.} (M_1, M_2, M_3), \]

(58)

which, in combination with eq.(35), gives the elements of the \( \nu - S \) mixing matrix

\[ V^{(IS)} = \begin{pmatrix} M_{D_{\epsilon 1}}/M_1 & M_{D_{\epsilon 2}}/M_2 & M_{D_{\epsilon 3}}/M_3 \\ M_{D_{\mu 1}}/M_1 & M_{D_{\mu 2}}/M_2 & M_{D_{\mu 3}}/M_3 \\ M_{D_{\tau 1}}/M_1 & M_{D_{\tau 2}}/M_2 & M_{D_{\tau 3}}/M_3 \end{pmatrix}. \]

(59)

The \( S - N \) mixing matrix

\[ V^{(SN)} = \frac{M}{M_N}. \]

(60)

Noting that the physical neutrino flavor state \( \nu_\alpha \) is a mixture of \( \hat{\nu}, \hat{S} \) and \( \hat{N} \)

\[ \nu_\alpha = U_{\alpha i} \hat{\nu}_i + V^{IS}_{\alpha i} \hat{S} + V^{(SN)}_{\alpha i} \hat{N}_i. \]

(61)

where \( U \sim U_{PMNS} \) and the other two mixings violate unitarity. For large \( M_N \gg M \) the third term in the RHS of eq.(61) can be dropped leading to the unitarity violation parameter \( \eta \)

\[ U' \simeq (1 - \eta)U_{PMNS}. \]

(62)

where

\[ \eta_{\alpha \beta} = \frac{1}{2} (X X^\dagger)_{\alpha \beta}, \]

\[ X = \frac{M_D}{M}. \]

(63)
There has been extensive discussion on the constraint imposed on this parameter \[72,73\]. The largest out of these is \( \eta_{\tau\tau} \leq 0.0027 \). Theoretically

\[
\frac{1}{2} \left[ \sum_i \frac{M_{D\tau_i} M_{D\tau_i}^*}{M_i^2} \right] \leq 0.0027.
\]

(64)

In the completely degenerate case of \( S - N \) mixing, \( M_1 = M_2 = M_3 = M \) we get

\[
M \geq 1280\text{GeV}
\]

(65)

The RH neutrinos in the present model being degenerate of masses \( M_{N_i} \gg m_S \) have much less significant contributions than the singlet fermions. The predicted branching ratios being only few to four orders less than the current experimental limits \[70\] are verifiable by ongoing searches,

\[
\begin{align*}
BR(\mu \to e\gamma) &= 5.90 \times 10^{-17}, \\
BR(\tau \to e\gamma) &= 7.80 \times 10^{-16}, \\
BR(\tau \to \mu\gamma) &= 2.39 \times 10^{-12}.
\end{align*}
\]

(66)

For the sake of completeness we present the variation of LFV decay branching ratios as a function of the lightest neutrino mass in Fig. 2.

---

**Figure 2:** Variation of LFV decay branching ratios as a function of the lightest neutrino mass. Colored horizontal lines represent I : \( BR(\mu \to e\gamma) \), II : \( BR(\tau \to e\gamma) \), and III : \( BR(\tau \to \mu\gamma) \).

In this approach the LFV decay rate mediated by the \( W_L \) boson in the loop depends predominantly upon \( N - S \) mixing matrix \( M \) and the Dirac neutrino mass matrix \( M_D \), although subdominantly upon the RH\( \nu \) mass matrix \( M_N \). However in the high scale type-II
seesaw ansatz in this case LFV decay rate is independent of light neutrino masses. This behavior of LFV decay rates are clearly exhibited in Fig.2 where the three branching ratios have maintained constancy with the variation of $m_\nu$.

7  DOMINANT $W_L - W_L$-CHANNEL DOUBLE BETA DECAY WITHIN COSMOLOGICAL BOUND

7.1  Double Beta Decay Mediation by Sterile Neutrinos

As the $W_R$ boson has mass $> 10^{15}$ GeV and the doubly charged Higgs bosons have masses $> 10^{9.2}$ GeV, they have negligible contributions for direct mediations of $0\nu\beta\beta$ process. Feynman diagrams for $0\nu\beta\beta$ decay amplitude due to the exchanges of Majorana fermions $\nu$, $S$, and $N$ are shown in Fig. 3. In Fig.4 we present Feynman diagram for $0\nu\beta\beta$ decay amplitude due to the sterile neutrino exchange where its mass insertion has been explicitly indicated. Mass eigen values of different sterile neutrinos for different sets of $(M_1, M_2, M_3)$ consistent with constraints on unitarity violating parameters $\eta_{\alpha\beta}$ are presented in Table 1. We have used the singlet fermion mass seesaw formula of eq.(15) and $M_{N_1} = M_{N_2} = M_{N_3} = 4000$ GeV. These

Figure 3: Feynman diagrams representing neutrino-less double beta decay due to exchanges of all three types of Majorana fermions $\nu$, $S$, and $N$.

Figure 4: Feynman diagram representing neutrino-less double beta decay amplitude due to exchanges of sterile neutrino $S$ with explicit mass insertion $m_s = -M_{S} \frac{1}{M^{T}}$. 

15
Table 1: Prediction of singlet fermion masses for different values of $(M_1, M_2, M_3)$ where we have used $M_{N_1} = M_{N_2} = M_{N_3} = 4000$ GeV.

| $M$ (GeV)       | $\hat{m}_s$ (GeV) |
|-----------------|--------------------|
| (60, 1200, 1200)| (0.9, 360, 360)    |
| (70, 1200, 1200)| (1.22, 360, 360)   |
| (80, 1200, 1200)| (1.60, 360, 360)   |
| (90, 1200, 1200)| (2.00, 360, 360)   |
| (100, 1200, 1200)| (2.50, 360, 360)   |
| (110, 1200, 1200)| (3.00, 360, 360)   |
| (120, 1200, 1200)| (3.60, 360, 360)   |
| (130, 1200, 1200)| (4.22, 360, 360)   |
| (140, 1200, 1200)| (4.90, 360, 360)   |
| (150, 1200, 1200)| (5.62, 360, 360)   |

solutions are displayed in Fig. 5.

We use normalisations necessary for different contributions \cite{101, 102}, due to exchanges of light-neutrinos, sterile neutrinos, and the heavy RH neutrinos in the $W_L - W_L$ channel. They lead to the inverse half life \cite{75, 78, 79},

$$T_{1/2}^{0\nu} \simeq G_{01} \frac{M_{\nu}^{0\nu}}{m_e} |(M_{ee}^{\nu} + M_{S}^{\nu} + M_{N}^{\nu})|^2,$$

$$= K_{0\nu} |(M_{ee}^{\nu} + M_{S}^{\nu} + M_{N}^{\nu})|^2,$$

$$= K_{0\nu} |\text{M}_{\text{eff}}|^2. \quad (67)$$

Here $G_{01} = 0.6 \times 10^{-14} \text{Yrs}^{-1}$, $M_{\nu}^{0\nu} = 2.58 - 6.64$, and $K_{0\nu} = 1.57 \times 10^{-25} \text{Yrs}^{-1} \text{eV}^{-2}$. In eq. (67) the three effective mass parameters have been defined as

$$M_{ee}^{\nu} = \sum_i (\nu_{ei}^{\nu})^2 m_{\nu_i}, \quad (68)$$

$$M_{S}^{\nu} = \sum_i (\nu_{ei}^{S})^2 \frac{|p|^2}{m_{S_i}}, \quad (69)$$

$$M_{N}^{\nu} = \sum_i (\nu_{ei}^{N})^2 \frac{|p|^2}{M_{N_i}}, \quad (70)$$

with

$$\text{M}_{\text{eff}} = M_{ee}^{\nu} + M_{S}^{\nu} + M_{N}^{\nu}. \quad (71)$$

The quantity $\hat{m}_S$ is the i-th eigen value of the $S-$ fermion mass matrix $m_S$. The magnitude of neutrino virtuality momentum $|p|$ has been estimated to be in the allowed range $|p| = 120 \text{ MeV} - 200 \text{ MeV}$ \cite{101, 102}. The RH $\nu$s being much heavier than the singlet fermions, their contributions have been neglected.
Figure 5: Prediction of singlet fermion mass eigen values as a function of $N - S$ mixing mass parameters $M_i(i = 1, 2, 3)$ for $M_{N_i} = 4 \text{TeV}(i = 1, 2, 3)$. The horizontal red coloured line represents solutions for two other eigen values for $M_2 = M_3 = 1200 \text{ GeV}$.

7.2 Singlet Fermion Assisted Enhanced Double Beta Decay Rate

We use neutrino oscillation data to estimate $M_{ee}$ for NH and IH cases with the values of Dirac phase and Majorana phases as discussed above. We further use the values of $M_i$ from Table 1 and Fig. 5 and the Dirac neutrino mass matrix from eq. (35) to estimate $M_{ee}$ while treating the RH $\nu$ mass at its assumed degenerate value of $M_{N_i} = 4 \text{TeV}(i = 1, 2, 3)$. The variation of effective parameter $m_{ee}$ as a function of lightest neutrino mass is shown in Fig. 6 when $m_{S_1} = 2 \text{ GeV}$.

As noted from the analytic formulas the effective mass parameter in the singlet fermion dominated case being inversely proportional to $m_{S_1}$, it will proportionately decrease for the larger value of the mediating particle mass. This feature has been shown in Fig. 7. We present predictions of double-beta decay half life as a function of the singlet fermion mass in Fig. 8. It is clear that while for $m_{S_1} = 2 \text{ GeV}$ the half life saturates the current experimental limit, for larger masses the half life increases. Neglecting heavy RH $\nu$ contributions but including those due to the lightest sterile neutrino and the IH type light neutrinos our predictions of half life as a function of the lightest sterile neutrino mass is shown in Fig. 9.

Predicted lifetimes are seen to decrease with increasing sterile neutrino mass. The sterile neutrino exchange contribution completely dominates over light neutrino exchange contributions for $m_{S_1} = 1.3 - 7 \text{ GeV}$ in case of IH but for $m_{S_1} = 1.5 - 20 \text{ GeV}$ in case of NH. At $m_{S_1} \simeq 1.5 \text{ GeV}$ both types of solutions saturate the current laboratory limits reached by different experimental groups. For double beta decay half-life expectations in standard and non-standard scenarios see ref. [107].
Figure 6: Variation of effective mass parameter as a function of lightest active neutrino mass $m_1$ for $m_{s_1} = 2$ GeV. For comparison, predictions in the standard model supplemented by light neutrino masses of NH type is shown by green dot-dashed curve. For IH pattern of mass hierarchy the standard prediction is shown by red dot-dashed curve.

Figure 7: Same as Fig.6 but for $m_{s_1} = 4.0$ GeV.
Figure 8: Prediction of double beta decay half-life as a function of sterile neutrino mass $m_{s_1}$ GeV (blue shaded curve) where the NH type light neutrino and the sterile neutrino exchange contributions have been included. Effects of much larger masses ($m_{s_2}, m_{s_3}$) $\gg m_{s_1}$ have been neglected. The spread in the curve reflects uncertainty in the virtuality momentum $p = 120 - 190$ MeV. For comparison the standard prediction with NH and IH pattern of light neutrino mass hierarchies are shown by the two respective horizontal lines. The bottom most thick red horizontal line closest to the X-axis represents overlapping experimental bounds from different groups [54, 56].
Figure 9: Same as Fig. 8 but with contributions of IH type light neutrinos combined with lightest sterile neutrinos.

8 SUMMARY, DISCUSSION AND CONCLUSION

A recently proposed scalar extension of minimal non-SUSY SU(5) GUT has been found to realize precision gauge coupling unification, high scale type-II seesaw ansatz for neutrino masses, and prediction of a WIMP scalar DM candidate that also completes vacuum stability of the scalar potential. But the LFV decays are predicted to have negligible rates inaccessible to ongoing searches in foreseeable future. Like wise experimentally verifiable double beta decay rates measurable by different search experiments are possible only for quasi-degenerate neutrino mass spectrum with large common mass scale $|m_0| \geq 0.2$ that violates the recently measured cosmological bound $\sum_i m_i \leq 0.1$ eV. In order to remove these deficits we have extended this model by the addition of three RH $\nu$s, three extra Majorana fermion singlets $S_i (i = 1, 2, 3)$ and a scalar singlet $\xi_S (1, 0, 1)$ that generates $N - S$ mixing mass term through its vacuum expectation value. In the original theory of type-I seesaw cancellation mechanism, although the choice of particles is same as $N_i, S_i$ and $\xi_S (1, 0, 1)$, the neutrino mass is given by double seesaw [74]. Further there is no grand unification of gauge couplings or prediction of proton decay in this model [74], and the scalar potential of the model has vacuum instability. In addition the $N_i$ are not gauged. The model does not predict dominant contributions to double beta decay through this mechanism with NH or IH type neutrino masses. In non-SUSY SO(10) models of unification of three forces, implementing the cancellation of type-I seesaw [58, 75, 78], the TeV scale RH neutrinos are gauged but the neutrino masses are controlled by inverse seesaw. But in [13, 79] the RH $\nu$s are gauged and the neutrino mass formula is linear seesaw or type-II seesaw [13]. In all type-II seesaw dominated SO(10) models, the RH $\nu$ masses have the same hierarchy as the left-handed neutrino masses: $M_{N_1} : M_{N_2} : M_{N_3} :: m_1 : m_2 : m_3$. This happens precisely because the left-handed and the right-handed
dilepton Yukawa interactions originate from the same SO(10) invariant term: $f_{16}F^c_{16}F_{126}$. In SU(5), however, as the LH triplet $\Delta_L(3, -1, 1)$ generating type-II seesaw and the singlet $\sigma_S(1, 0, 1)$ generating RH$\nu$s belong to different scalar representations $15_H \subset SU(5)$ and $1_H \subset SU(5)$, respectively, they can possess different Majorana couplings in their respective Yukawa interactions: $f_{126} \Delta_L^c$ and $f_N \sigma_S N N$. Because of this reason the generated RH$\nu$ masses through $M_N = f_N \sigma_s$ no longer follows the predicted type-II hierarchical pattern. Then the allowed fine tuning $|f_{126}| << |f|$ permits $M_N \sim O(1 - 10)$ TeV RH neutrino mass scale even though, unlike SO(10) models, there are no low mass $W_R$ or $Z'$ bosons at this scale in this SU(5) model. The apprehension of unacceptably large active neutrino mass generation through type-I seesaw mechanism is rendered inoperative through the well established procedure of cancellation mechanism that is also shown to operate profoundly in this SU(5) model. Such RH$\nu$s generating $N - S$ mixing mass $M \approx O(100 - 1000)$ GeV now reproduce the well known results on LFV decay branching ratios only $4 - 5$ orders lower than the current experimental limit as well as the extensively investigated non-unitarity effects. Through the sterile neutrino canonical seesaw formula emerging from this cancellation mechanism (in the presence of $N_i$), $m_S = -M \frac{1}{M_N} M^T$, this mechanism predicts their masses over a wide range of values, $m_{s_1} = O(1 - 100)$ GeV and $m_{s_2}, m_{s_3} \sim O(10 - 1000)$ GeV. The lightest sterile neutrino mass $m_{s_1}$ now predicts dominant double beta decay in the $W_L - W_L$ channel through the $\nu - S$ mixing close to the current experimental limits even though the light neutrino masses are of NH or IH type ($m_i << |0.2| \text{ eV}$) which satisfy the cosmological bound. For larger values of $m_{s_1}$ the predicted decay rate decreases and the sterile neutrino contribution becomes negligible for $m_{s_1} > 50$ GeV. In the limiting case when all the singlet fermion masses have such large values, the double beta decay rates asymptotically approach the respective standard NH or IH type contributions. The new mechanism of RH$\nu$ mass generation also allows the second and the third generation sterile neutrino masses to be quasi-degenerate (QD) near $1 - 10$ TeV scale while keeping $m_{s_1} \sim 1 - 10$ GeV suitable for dominant double beta decay mediation. There is a possibility that such TeV scale QD masses while maintaining observable predictions on LFV decays can effectively generate baryon asymmetry of the universe via resonant leptogenesis [79]. Vacuum stability of the scalar potential can be implemented though the scalar singlet $\xi_S(1, 0, 1)$ following the method of [103][104]. A scalar singlet DM can be easily accommodated as discussed in [80]. Irrespective of scalar DM, the model can also accommodate a Majorana fermion singlet dark matter [105] which can emerge from the additional fermionic representation $24_F \subset SU(5)$.

The predictions of new fermions has an additional advantage over scalars as these masses are protected by leptonic global symmetries [106]. The predictions of such Majorana type sterile neutrinos can be tested by high energy and high luminosity accelerators through their like-sign dilepton production processes [108]. For example at LHC they can mediate the process $pp \rightarrow W_L X \rightarrow l^+l^-jjX$ where the jets could manifest as mesons. It would be quite interesting to examine emergence of such SU(5) theory as a remnant of SO(10) or $E_6$ GUTs.

We conclude that even in the presence of SM as effective gauge theory descending from a suitable SU(5) extension, it is possible to predict experimentally accessible double beta decay rates in the $W_L - W_L$ channel satisfying the cosmological bound on active neutrino masses as well as verifiable LFV decays. The RH$\nu$ masses can be considerably different from those constrained by conventional type-I or type-II seesaw frameworks which are instrumental in predicting interesting physical phenomena even if there are no non-standard heavy gauge bosons anywhere below the GUT scale.
ACKNOWLEDGMENT

M. K. P. thanks the Science and Engineering Research Board, Department of Science and Technology, Government of India for grant of research project SB/S2/HEP-011/2013. R.S. thanks Siksha ‘O’ Anusandhan University for research fellowship.

References

[1] P. F. de Salas (Valencia U., IFIC), D. V. Forero, C. A. Ternes, M. Tortola, J. W. F. Valle, “Status of Neutrino Oscillations 2018: First Hint for Normal Ordering and Improved CP Sensitivity”, e-Print: arXiv:1708.01186v2[hep-ph][INSPIRE].

[2] T. Schwetz, M. Tartola, J. W. F. Valle, “Global neutrino data and recent reactor fluxes: status of three-flavour oscillation parameters”, New J. Phys. 13 (2011) 063004 [arXiv:1103.0734] [INSPIRE].

[3] D. V. Forero, M. Tartola, J. W. F. Valle, “Neutrino oscillations refitted”, Phys. Rev. D 90 (2014) 093006 [arXiv:1405.7540] [INSPIRE].

[4] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A. M. Rotunno, “Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP-violation searches”, Phys. Rev. D 86 (2012) 013012 [arXiv:1205.5254] [INSPIRE].

[5] M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, “Global Analyses of Neutrino Oscillation Experiments”, Nucl. Phys. B 908 (2016) 199 [arXiv:1512.06856] [INSPIRE].

[6] G. Altarelli, “Neutrinos Today: An Introduction”, in Proceedings: 49th Rencontres de Moriond on Electroweak Interactions and Unified Theories, Thuile, Italy, March 15-22, (2014);

[7] A. Yu. Smirnov, “Theories of Neutrino Masses and Mixings”, Nuovo Cim. C 037, no. 03, 29 (2014) [arXiv:1402.6580] [hep-ph];

[8] R. N. Mohapatra, “From Old Symmetries to New Symmetry Quarks, Leptons, and B-L”, in “50 Years of Quarks” pp 245-263 (World Scientific, 2015);

[9] R. N. Mohapatra, “Neutrino Mass as a Signal for TeV Scale Physics”, Nucl. Phys. B 908 (2016) 423-433;

[10] O. G. Miranda, J. W. F. Valle, “Neutrino Oscillation and Seesaw Origin of Neutrino Masses”, Nucl. Phys. B 908, 436 (2016) [arXiv:1602.00864] [hep-ph];

[11] J. W. F. Valle, “Neutrino Physics from A to Z: Two Lectures at Corfu”, PoS CORFU2016 (2017) 007 arXiv:1705.00872.

[12] G. Senjanovic, “Neutrino mass: From LHC to grand unification”, Riv. Nuovo Cim. 34 (2011) 1-68.

[13] M. K. Parida, B. P. Nayak, “Singlet fermion assisted dominant seesaw with lepton flavor and number violations and leptogenesis”, Adv. High Energy Phys. 2017 (2017) 4023493 [arXiv:1607.07236] [hep-ph].
[14] P. Minkowski, “mu → e gamma at a Rate of One Out of 1-Billion Muon Decays?”, Phys. Lett. B 67, (1977) 421.

[15] T. Yanagida, in Workshop on Unified Theories, KEK Report 79-18, p. 95, 1979.

[16] M. Gell-Mann, P. Ramond and R. Slansky, Supergravity, p. 315, Amsterdam: North Holland, 1979.

[17] S. L. Glashow, 1979 Cargese Summer Institute on Quarks and Leptons, p. 687, New York: Plenum, (1980).

[18] R. N. Mohapatra and G. Senjanovic, “Neutrino Mass and Spontaneous Parity Violation,” Phys. Rev. Lett. 44, (1980) 912.

[19] J. Schechter, J. W. F. Valle, “Neutrino Masses in SU(2) × U(1) Theories”, Phys. Rev. D 22, (1980) 2227.

[20] J. Schechter, J.W. F. Valle, “Neutrinoless double beta decay in SU(2) × U(1)y theories”, Phys. Rev. D 25 (1982) 2951;

[21] M. Magg, C. Wetterich, Phys. Lett. B 94, (1980) 61.

[22] G. Lazaridis, Q. Shafi, C. Wetterich, Nucl. Phys. B 181, (1981) 287.

[23] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, (1981) 165.

[24] E. Ma, U. Sarkar, “Neutrino Masses and Leptogenesis with Heavy Triplets”, Phys. Rev. Lett. 80 (1998) 5716, arXiv:hep-ph/9802445.

[25] R. N. Mohapatra, M. K. Parida, “Type-II seesaw dominance in non-supersymmetric SO(10) grand unification”, Phys. Rev. D 84 (2011) 095021, arXiv:1109.2188.

[26] R. N. Mohapatra, Phys. Rev. Lett. 56 (1986) 561.

[27] R.N. Mohapatra, J. W. F. Valle, Phys. Rev. D 34 (1986) 1642.

[28] D. Wyler, L. Wolfenstein, “Massless neutrinos in left-right symmetric models”, Nucl. Phys. B 218 (1983) 205.

[29] C. H. Albright, “Search for solutions of superstring neutrino mass problem”, Phys. Lett. B 178 (1986) 219;

[30] S. Nandi, U. Sarkar, “A solution to neutrino mass problem in superstring E6 theory”, Phys. Rev. Lett. 56 (1986) 564.

[31] E. Witten, “New issues in manifolds with SU(3) holonomy”, Nucl. Phys. B 268 (1986) 79;

[32] E. Ma, “Lepton number non-conservation in E6 inspired superstring models”, Phys. Lett. B 191 (1987) 287.

[33] W. Grimus, L. Lavoura, JHEP 0011, (2000) 042; arXiv: 0008179 [hep-ph]; P. M. Ferreira, W. Grimus, D. Jurciuonis, L. Lavoura, “Flavor symmetries in a renormalizable SO(10)”, Nucl. Phys. B 906 (2016) 289-320, arXiv:1510.02641 [hep-ph].
[34] R. Foot, H. Lew, X. G. He, G. C. Joshi, Z. Phys. C 44 (1989) 441; B. Bajc, G. Senjanovic, “Seesaw at LHC”, JHEP 0708 (2007) 014 [arXiv:hep-ph/0612092].

[35] E. K. Akhmedov and M. Frigerio, Phys. Rev. Lett. 96 (2006) 061802 [hep-ph/0509299]; E. K. Akhmedov and M. Frigerio, JHEP 0701 (2007) 043 [hep-ph/0609046].

[36] M. K. Parida, B. P. Nayak, R. Satpathy, R. L. Awasthi, “Standard coupling unification in SO(10), hybrid seesaw neutrino mass, leptogenesis, dark matter, and proton lifetime predictions”, JHEP 1704 (2017) 075, [arXiv:1608.03956 [hep-ph]] [INSPIRE].

[37] E. Akhmedov, M. Lindner, E. Schnapka, J. W. F. Valle, “Left-right symmetry breaking in NJL approach”, Phys. Lett. B 368 (1996) 270-280, hep-ph/9507275; M. Malinsky, J. C. Romao, J. W. F. Valle, “Novel supersymmetric SO(10) seesaw mechanism”, Phys. Rev. Lett. 95 (2005) 161801, hep-ph/0506296; S. M. Barr, Phys. Rev. Lett. 92 (2004) 101601, hep-ph/0309152.

[38] E. Ma, Phys. Rev. D 73 (2006) 077031;

[39] M. K. Parida, Phys. Lett. B 704 (2011) 206;

[40] M. K. Parida, Pramana, 79 (2012) 1271.

[41] J. C. Pati, A. Salam, Phys. Rev. D 10 (1974) 275.

[42] R. N. Mohapatra, J. C. Pati, Phys. Rev. D 11, 566, (1975) 2558.

[43] G. Senjanović, R. N. Mohapatra, Phys. Rev. D 12, (1975) 1502; G. Senjanovic, Nucl. Phys. B 153, 334 (1979).

[44] H. Georgi, Particles and Fields, Proceedings of APS Division of Particles and Fields, ed C. Carlson, (AIP, New York, 1975), p.575; H. Fritzsch, P. Minkowski, Ann. Phys. (Berlin) 93, 193 (1975).

[45] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 70, (1993) 2845.

[46] A. S. Joshipura, K. M. Patel, “Fermion masses in SO(10) model”, Phys. Rev. D 83 (2011) 095002, arXiv:1102.5148 [hep-ph].

[47] G. Altarelli, G. Blankenburg,”Different SO(10) paths to fermion masses and mixings”, JHEP 1103 (2011) 133, arXiv:1012.2697 [hep-ph].

[48] S. Bertolini, T. Schwetz, M. Malinsky, “Fermion masses and mixings in SO(10) models and the neutrino challenge to supersymmetric grand unified theories”, Phys. Rev. D 73 115012, [hep-ph/0605006v4].

[49] H. S. Goh, R. N. Mohapatra, and S. Nasri, Phys. Rev. D 70, 075022 (2004).

[50] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. 52, 1072 (1984); D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. D 30, 1052 (1984); D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D 31, 1718 (1985); D. Chang, R. N. Mohapatra, M. K. Parida, Phys. Lett. 142B (1984) 55-58; M. K. Parida, Phys. Lett. B 126 , 220 (1983); M. K. Parida, Phys. Rev. D
[51] M. Reig, J. W. F. Valle, C. A. Vaquera-Arazu, F. Wilczek, “A Model for Comprehensive Unification”, Phys. Lett. B774 (2017) 667-670, [arXiv:1706.03116 [hep-ph]].

[52] S. Fukuyama and T. Yanagida, Phys. Lett. B174 (1986) 45.

[53] T. Hambye, E. Ma, U. Sarkar, “Supersymmetric Triplet Higgs Model of Neutrino Mass and Leptogenesis”, Nucl. Phys. B602 (2001) 23 [hep-ph/0011192].

[54] H.V. Klapdor-Kleingrothaus, A. Dietz, L. Baudis, G. Heusser, I.V. Krivosheina, S. Kolb, B. Majorovits, H. Pas, H. Strecker, V. Alexeev, A. Balysh, A. Bakalyarov, S.T. Belyaev, V.I. Lebedev, S. Zhukov (Kurchatov Institute, Moscow, Russia) Eur.Phys.J. A12, (2001) 147.

[55] C. Arnaboldi et al. [CUORICINO Collaboration], Phys. Rev. C78, (2008) 035502; C. E. Aalseth et al. [ IGEX Collaboration ], Phys. Rev. D65, (2002) 092007.

[56] J. Argyriades et al. [NEMO Collaboration], Phys. Rev. C80, (2009) 032501; I. Abt, M. F. Altmann, A. Bakalyarov, I. Barabanov, C. Bauer, E. Bellotti, S. T. Belyaev, L. B. Bezrukov et al., [hep-ex/0404039];

[57] M. Nemevsk, G. Senjanovic, V. Tello, Phys. Rev. Lett. B110, (2013) 151802.

[58] M. K. Parida, B. Sahoo, “Planck scale induced left-right gauge theory at LHC and experimental tests”, Nucl.Phys. B906 (2016) 77-104 [arXiv:1411.6748 [hep-ph]].

[59] W. Y. Keung, G. Senjanovic, “Majorana neutrinos and the production of right-handed charged gauge bosons”, Phys. Rev. Lett. 50 (1983) 1427.

[60] B. Bajc, G. Senjanovic, “Seesaw at LHC”, JHEP, 0708 (2007) 014 [arXiv:hep-ph/0612029].

[61] J. W. F. Valle,”Neutrinoless double beta decay with quasidirac neutrinos”, Phys. Rev. D27 (1983) 1672-1674.

[62] J. Schechter, J.W. F. Valle, “Neutrino Decay and Spontaneous Violation of Lepton Number”, Phys. Rev. D25 (1982) 774.
Aguila and J. A. Aguilar-Saavedra and J. de Blas, Acta Phys. Polon. B 40, (2009) 2901 ; arXiv:0910.2720 [hep-ph]; A. van der Schaaf, J. Phys. G 29, (2003) 2755; Y. Kuno, Nucl. Phys. B, Proc. Suppl. 149, (2005) 376.

[74] S. K. Kang and C. S. Kim, Phys. Lett. B 646, 248 (2007)

[75] R. L. Awasthi, M. K. Parida, and S. Patra, J. High Energy Phys. 1308 (2013) 122, arXiv:1302.0672[hep-ph]; M. K. Parida, S. Patra, Phys. Lett B 718 (2013) 1407, arXiv:1211.5000[hep-ph].

[76] S. K. Majee, M. K. Parida, A. Raychaudhuri, Phys. Lett. B 668 (4) (2008) 299-302, arXiv:0807.3959[hep-ph].

[77] M. K. Parida and A. Raychaudhuri, Phys. Rev. D 82, 093017 (2010); arXiv:1007.5085[hep-ph].

[78] M. K. Parida, R. L. Awasthi, P. K. Sahu, JHEP 1501 (2015) 045, arXiv:1401.1412[hep-ph].

[79] B.P. Nayak and M.K. Parida, Eur. Phys. J. C 75, (2015) 5, 183, arXiv:1312.3185[hep-ph].

[80] Biswonath Sahoo, Mainak Chakraborty, M. K. Parida, “Neutrino mass, Coupling Unification, Verifiable Proton Decay, Vacuum Stability and WIMP Dark Matter in SU(5)”, Adv. High Energy Phys. 2018 (2018) 4078657, arXiv:1404.01803[hep-ph].

[81] M.L. Kynshi, M.K. Parida, “Higgs Scalar in the Grand Desert with Observable Proton Lifetime in SU(5) and Small Neutrino Masses in SO(10)”, Phys.Rev. D 47 (1993) 4830 (Rapid Communication).

[82] Particle Data Group, J. Beringer et al., “Review of Particle Physics (RPP)”, Phys. Rev. D 86 (2012) 010001.

[83] K. A. Olive et al. (Particle Data Group), “Review of Particle Physics (RPP)”, Chin. Phys. C 38 (2014) 090001.

[84] C. Patrignani et al (Particle Data Group), “Review of Particle Physics (RPP)”, Chin. Phys. C 40 (2016) no.10, 100001.

[85] S. Weinberg, “Effective Gauge Theories” Phys. Lett. B 91 (1980) 51.

[86] L. Hall, “Grand Unification of Effective Gauge Theories”, Nucl. Phys. B 178 (1981) 75.

[87] B. Ovrut, H. Schnitzer, “Effective Field Theory in Background Field Gauge”, Phys. Lett. B 110 (1982) 139.

[88] M. K. Parida, “Heavy Particle Effects in Grand Unified Theories with Fine-Structure Constant Matching”, Phys. Lett. B 196 (1987) 163.

[89] M. K. Parida, C. C. Hazra, M. K. Parida, C. C. Hazra, “Superheavy Higgs Scalar Effects in Effective Gauge Theories From SO(10) Grand Unification With Low Mass Right-handed Gauge Bosons”, Phys.Rev. D40 (1989) 3074-3085.

[90] R. N. Mohapatra, M. K. Parida, Phys. Rev. D 47 (1993) 264, arXiv:hep-ph/9204234.
[91] P. Langacker, N. Polonsky, Phys. Rev. D 49 (1994) 1454, arXiv:hep-ph/9306235.

[92] Dae-Gyu Lee, R. N. Mohapatra, M. K. Parida, M. Rani, Phys. Rev. D 51 (1995) 229, arXiv:hep-ph/9404238.

[93] Superkamiokande Collaboration, K. Abe, Y. Haga, Y. Hayato, M. Ikeda, K. Iyogi et al., “Search for Proton Decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in 0.31 Megaton Years Exposure of Water Cherenkov Detector”, Phys. Rev. D 95 (2017) no.1 012004 arXiv:1610.03597[hep-ex][INSPIRE].

[94] I. Dorsner, S. Fajfer, A. Greljo, J. F. Kamenik and N. Košnik, “Physics of leptoquarks in precision experiments and at particle colliders”, Phys. Rept. 641, 1 (2016) arXiv:1603.04993 [hep-ph]; I. Dorsner, “A scalar leptoquark in SU(5)”, Phys. Rev. D 86, 055009 (2012) arXiv:1206.5998 [hep-ph]; I. Dorsner, S. Fajfer and I. Mustac, “Light vector-like fermions in a minimal SU(5) setup”, Phys. Rev. D 89, no. 11, 115004 (2014) arXiv:1401.6870 [hep-ph]; M. K. Parida, P. K. Patra, A. K. Mohanty, “Gravity induced large grand unification mass in SU(5) with higher dimensional operator”, Phys. Rev. D 39 (1989) 316-322;

I. Doršner, S. Fajfer and N. Košnik, “Leptoquark mechanism of neutrino masses within the grand unification framework”, Eur. Phys. J. C 77, no. 6, 417 (2017) arXiv:1701.08322 [hep-ph]; I. Dorsner, S. Fajfer and N. Kosnik, “Heavy and light scalar leptoquarks in proton decay”, Phys. Rev. D 86, 015013 (2012) arXiv:1204.0674 [hep-ph]; I. Dorsner and P. Fileviez Perez, “Upper Bound on the Mass of the Type III Seesaw Triplet in an SU(5) Model”, JHEP 0706, 029 (2007) hep-ph/0612216; P. Fileviez Perez, “Supersymmetric Adjoint SU(5)”, Phys. Rev. D 76, 071701 (2007) arXiv:0705.3589 [hep-ph] .

[95] P. Nath, P. Filviez Perez, Phys. Rep. 441 (2007) 191, arXiv:hep-ph/0601023.

[96] P. Langacker, Phys. Rept. 72 , 185 (1981).

[97] C. R. Das and M. K. Parida, “New Formulas and Predictions for Running Fermion Masses in SM, 2HDM, and MSSM” Eur. Phys. J. C 20 (2001) 121, arXiv:0010004[hep-ph]; M. K. Parida, B. Purkayastha, Eur. Phys. J. C 14 (2000) 159, arXiv:9902374[hep-ph]; M. K. Parida, N. N. Singh, Phys. Rev. D 59 (1999) 032002, arXiv:9710328[hep-ph].

[98] D. Meloni, T. Ohlsson, S. Riad, “Renormalization Group Running of Fermion Observables in an Extended Non-Supersymmetric SO(10) Model”, JHEP 03 (2017) 045 arXiv:1612.07973[hep-ph] [INSPIRE].

[99] R. L. Awasthi and M. K. Parida ,Phys.Rev.D 86 , (2012) 093004, e-Print.arXiv:1112.1286[hep-ph].

[100] M. K. Parida, Rajesh Satpathy, Under Preparation.

[101] G. Pantis, F. Simkovic, J. Vergados, and A. Faessler, Phys. Rev. C 53, (1996) 695 ; arXiv:nucl-th/9612036 [nucl-th]; J. Suhonen and O. Civitarese, Phys. Rept. C 300 (1998) 123; J. Kotila and F. Iachello, Phys. Rev. C 85, (2012) 034316 ; arXiv:1209.5722 [nucl-th].
[102] R. N. Mohapatra, Phys. Rev. D 34 (1986) 3457; M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1; F. Simkovic, G. Pantis, J. Vergados, and A. Faessler, Phys. Rev. C 60, (1999) 055502; arXiv:hep-ph/9905509 [hep-ph]; A. Faessler, A. Meroni, S. T. Petcov, F. Simkovic, Nucl. Phys. B 692 (2004) 303; arXiv:hep-ph/0309342.

[103] A. Falkowski, C. Gross, O. Lebedev, “A Second Higgs from Higgs Portal”, JHEP 05 (2015) 057, arXiv:1502.01361 [hep-ph].

[104] B. Sahoo, M. K. Parida, M. Chakraborty, “Matter Parity Violating Dark Matter Decay in Minimal SO(10), Unification, Vacuum Stability and Verifiable Proton Decay”, e-Print: arXiv:1707.01286 [hep-ph].

[105] M. Cirelli, A. Strumia and M. Tamburini, “Cosmology and Astrophysics of Minimal Dark Matter”, Nucl. Phys. B 787, 152 (2007) arXiv:0706.4071 [hep-ph].

[106] G. ’t Hooft, in Proceedings of the 1979 Cargese Summer Institute on Recent Developments in Gauge Theories, edited by G. t Hooft et al. (Plenum Press, New York, 1980).

[107] Shao-Feng Ge, W. Rodejohann, K. Zuber, “Half-life Expectations of Neutrinoless Double Beta Decay in Standard and Non-Standard Scenarios”, Phys. Rev. D 96 (2017) no.5, 055019 arXiv:1707.07904 [hep-ph].

[108] B. P. Nayak, M. K. Parida, “Dilepton Events with Displaced Vertices, Double Beta Decay, and Resonant Leptogenesis with Type-II Seesaw Dominance, TeV Scale Z' and Heavy Neutrinos ”, e-Print arXiv:1509.06192 [hep-ph].