ERROR PROPAGATION IN QCD FITS

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The parton momentum density distributions of the proton were obtained from a
NLO QCD analysis of HERA and fixed target structure function data. The result-
ing parton distribution set includes the full information on errors and correlations.

1 Introduction

Standard sets of parton densities are widely used to calculate hard scat-
tering cross sections in hadron-hadron and lepton-hadron collisions. However,
none of these sets give the errors on the parton densities which tend to domi-
nate the uncertainties on the predicted cross sections.

To make a parton distribution set available which includes the full infor-
mation on errors and correlations we have performed a NLO QCD analysis
of HERA and fixed target structure function data. In this contribution we
describe how the experimental errors were propagated in the QCD fit.

2 Error propagation

In the fit, the correlated experimental systematic errors were incorporated in
the model prediction of the structure functions:

\[ F_i(p, s) = F_i^{QCD}(p) \left(1 + \sum_\lambda s_\lambda \Delta_{i\lambda}^{\text{syst}}\right) \]  

where \( F_i^{QCD}(p) \) is the QCD prediction and \( \Delta_{i\lambda}^{\text{syst}} \) is the relative systematic error
on data point \( i \) stemming from source \( \lambda \). The fitted parameters \( p \) describe the
parton densities at the input scale \( Q_0^2 \) and \( s \) denotes the set of systematic
parameters. Assuming that the \( s_\lambda \) are uncorrelated and Gaussian distributed
with zero mean and unit variance, the \( \chi^2 \) was defined in the usual way and
two Hessian matrices \( M \) and \( C \) were evaluated at the minimum \( \chi^2 \):

\[ M_{\lambda\mu} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial p_\lambda \partial p_\mu}, \quad C_{\lambda\mu} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial s_\lambda \partial s_\mu}. \]  

Available from \( \text{http://www.nikhef.nl/user/h24/qcdnum} \)
The statistical and systematic covariance matrices were calculated from
\[ V_{\text{stat}} = M^{-1}, \quad V_{\text{syst}} = M^{-1} \langle C \rangle^T M^{-1}. \]

The error on any function \( F \) of the parameters \( p \) is, to first order, given by
\[ (\Delta F)^2 = \sum_\lambda \sum_\mu \frac{\partial F}{\partial p_\lambda} V_{\lambda \mu} \frac{\partial F}{\partial p_\mu} \]
where \( V \) is the statistical, the systematic, or if the total error is to be calculated, the sum of both covariance matrices.

Three additional sources of error were considered in the analysis: (i) Errors due to the uncertainties on the input parameters (\( \alpha_s \) etc.). The error bands are defined as the envelope of the results from the central fit and two additional fits where each input parameter was lowered or raised by the error; (ii) An ‘analysis’ error band is defined as the envelope of the central fit and 10 alternative fits where, for instance, the cuts on the data were varied; (iii) The renormalization and factorization scale uncertainties, obtained from fits where both scales were independently varied in the range \( Q^2/2 < \mu^2 < 2Q^2 \).

In Fig. 1 we show the parton densities obtained from this analysis (left hand plot). The error bands correspond to the quadratic sum of all errors except the scale uncertainties. The relative error contributions to the gluon density and the singlet quark density are shown in the right hand plot of
Fig. 1. It is seen that the analysis error band is small. For the gluon density the remaining contributions to the error are roughly equal in size whereas for the quarks it turns out that the scale uncertainty is the largest source of error.

3 Parton distribution set

Stored on a computer readable file are the statistical and systematic covariance matrices, the parton densities \( f_i \), the derivatives \( \frac{\partial f_i}{\partial p_\lambda} \) (both from the central fit), the results from the systematic fits (where the input parameters or scales were changed) and the analysis error band. The kinematic range covered by the parton densities is \( 9 \times 10^{-4} < x < 1 \) and \( 1 < Q^2 < 9 \times 10^4 \) GeV\(^2\).

A computer program gives fast access to these results and provides tools, which make use of Eq. (4), to propagate the statistical and systematic errors to any function \( F \) of the parton densities. As an input to the program the user should provide a calculation of the derivatives \( \frac{\partial F}{\partial p_\lambda} \). Let us take as an example a hadron-hadron cross section which can be written as a convolution of the parton densities and a hard scattering cross section,

\[
\sigma = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij}. \tag{5}
\]

To calculate the error on \( \sigma \) it is sufficient to provide a function which computes the derivatives

\[
\frac{\partial \sigma}{\partial p_\lambda} = \sum_{ij} \left[ \frac{\partial f_i}{\partial p_\lambda} \otimes f_j + f_i \otimes \frac{\partial f_j}{\partial p_\lambda} \right] \otimes \hat{\sigma}_{ij}. \tag{6}
\]

This calculation is straightforward since the \( f_i \) and the \( \frac{\partial f_i}{\partial p_\lambda} \) are available from the input file.

Finally, we remark that the errors from a QCD fit are not determined by the experimental errors alone but also depend, and maybe quite strongly, on the assumptions made in the analysis, in particular on the parameterizations chosen for the parton densities at the input scale \( Q^2_0 \).

References

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