Spin misalignment of black hole binaries from young star clusters: comparison to GWTC-2 gravitational wave data

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ABSTRACT

Recent studies indicate that the progenitors of merging black hole (BH) binaries from young star clusters can undergo a common envelope phase just like isolated binaries. If the stars emerge from the common envelope as naked cores, tidal interactions can efficiently synchronize their spins before they collapse into BHs. Contrary to the isolated case, these binary BHs can also undergo dynamical interactions with other BHs in the cluster before merging. The interactions can tilt the binary orbital plane, leading to spin-orbit misalignment. We estimate the spin properties of merging binary BHs undergoing this scenario by combining up-to-date binary population synthesis and accurate few-body simulations. We show that post-common envelope binary BHs are likely to undergo only a single encounter, due to the high binary recoil velocity and short coalescence times. Adopting conservative limits on the binary-single encounter rates, we obtain a local BH merger rate density of \( \sim 6.6 \text{ yr}^{-1} \text{ Gpc}^{-3} \). Assuming low (\( \lesssim 0.2 \)) natal BH spins, this scenario can reproduce the distributions of effective spin \( \chi_{\text{eff}} \) and precession parameters inferred from GWTC-2, including the negative \( \chi_{\text{eff}} \) and the peak at \( \chi_p \sim 0.2 \).

Key words: black hole physics – methods: numerical – gravitational waves – binaries: general

1 INTRODUCTION

The origins of gravitational wave (GW) events from merging black hole (BH) binaries is still disputed. The numerous formation pathways that have been proposed so far can be organized into two broad categories: dynamically interacting binaries in dense environments and isolated binaries evolving due to stellar and binary evolution in the field.

The scenarios belonging to the first category include binaries dynamically assembled in globular clusters (GCs, e.g. Sigurdsson & Phinney 1995; Downing et al. 2010; Tanikawa 2013; Bae et al. 2014; Leigh et al. 2014b; Rodriguez et al. 2016; Askar et al. 2017; Samsing et al. 2018; Arca-Sedda & Gualandris 2018; Hong et al. 2018) and other dense stellar systems (open, young and nuclear star clusters, e.g. Portegies Zwart & McMillan 2000; Ziosi et al. 2014; Fujii et al. 2017; Banerjee et al. 2017b, Petrichov & Antonini 2017; Leigh et al. 2018; Rodriguez et al. 2018; Banerjee 2018; Rastello et al. 2018; Di Carlo et al. 2019; Hong et al. 2020; Mapelli et al. 2020; Banerjee 2021), mergers mediated by dynamical interactions with a supermassive BH or an AGN disk (e.g. Antonini & Perets 2012; Antonini & Rasio 2016; Leigh et al. 2016b; VanLandingham et al. 2016; Stone et al. 2016; Bartos et al. 2017; Hamers et al. 2018; McKernan et al. 2018; Hoang et al. 2018; Yang et al. 2019; Trani et al. 2019; McKernan et al. 2020; Tagawa et al. 2020; Arca Sedda 2020), and mergers in multiple stellar systems (e.g. Antonini et al. 2014; Silsbee & Tremaine 2017; Antonini et al. 2017; Rodríguez & Antonini 2018; Hamers & Safarzadeh 2020; Martinez et al. 2020; Leigh et al. 2020).

Conversely, in the field scenario, a primordial stellar binary becomes a merging binary BH via stellar interactions, either via common envelope evolution in Population I/II (e.g. Tutukov & Yungelson 1973; Bethe & Brown 1998; Dominik et al. 2012, 2013; Belczynski et al. 2002, 2008, 2010, 2016; Mennekens & Vanbeveren 2014; Stevenson et al. 2017; Spera et al. 2015, 2019; Kruckow et al. 2018; Eldridge & Stanway 2016; Eldridge et al. 2019; Mapelli & Giacobbo 2018; Mapelli et al. 2017, 2019; Neijssel et al. 2019; Vigna-Gómez et al. 2020; Banerjee 2021) and Population III stars (e.g. Kinugawa et al. 2014; Inayoshi et al. 2017; Kinugawa et al. 2016; Tanikawa et al. 2020a,b), or via chemically homogeneous evolution (e.g. de Mink & Mandel 2016; Mandel & de Mink 2016; Marchant et al. 2016; Riley et al. 2020; du Buisson et al. 2020).

The total number of events announced by Ligo-Virgo-KAGRA (hereafter LVK) collaboration amounts to 50: 11 from the O1/O2 observing runs (LIGO Scientific Collaboration & Virgo Collaboration 2019), and 39 from the first half of the O3 observing run (The LIGO Scientific Collaboration & the Virgo Collaboration 2020a). While we are currently far from being able to discriminate the formation pathways of individual events, as the number of detections increases it will be possible to infer the origin of GW events in a statistical sense, i.e. from the ensemble properties of the entire population (e.g.

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Fishbach et al. 2018; Bouffanais et al. 2019; Zevin et al. 2020). In this work we focus on one of the most informative properties of GW progenitors: the spin parameter.

Recently, an interplay between the two main formation scenarios has been proposed. Kumamoto et al. (2019) showed that, in young star clusters, binary BHs formed via common envelope evolution of dynamically assembled main sequence binaries. These systems contribute to the merger rate more than dynamically assembled binary BHs. These binaries undergo a common envelope phase, like in the isolated channel, but they also undergo dynamical interactions before and after collapsing to BHs. The common envelope phase might be even triggered by such dynamical interactions. Similar evolutionary pathways were also found by Di Carlo et al. (2020).

Binaries undergoing this channel experience a common envelope phase, which dramatically decreases the binary separation. Consequently, tidal interactions in the post-common envelope phase can efficiently spin up the naked stellar cores (Kushnir et al. 2016; Piran & Piran 2020), aligning their spin vectors with the orbital angular momentum. However, this alignment might be lost during a subsequent dynamical encounter in the cluster.

In this work we investigate the effective spin distributions of merging binary BHs from this hybrid path. Specifically, we consider post-common envelope binary BHs that undergo a single dynamical encounter in stellar clusters. In Section 2 we discuss our numerical setup and initial assumptions. Section 3 presents the outcome properties of the three-body encounters, while in Section 4 we discuss the encounter rates in cluster environments. In Section 5 we calculate the merger properties of post-encounter binaries and compare them with the recent GWTC-2 data, including the local merger rate density (Section 5.1), the effective spin distributions per stellar metallicity (Figure 5) and the differential merger rate density at the current epoch (Figure 6).

10 M⊙ to 150 M⊙. The secondary-to-primary mass ratios are drawn from a flat distribution between 0 and 1, imposing a minimum mass of 10 M⊙ for the secondary star. The binary semi-major axes have a flat distribution in logarithmic scale from 1 R⊙ to 10³ R⊙, and their eccentricities follow a thermal distribution. The initial pericenter distances are set to be large enough to avoid the onset of Roche-lobe overflow.

We evolve 3 sets of different metallicities: 1, 0.1, and 0.01 Z⊙, running 10⁵ realizations for each set. The stellar wind models and binary interaction model are the same as those in Tanikawa et al. (2020a, see also Giacobbo et al. 2018). We do not take into account BH natal kicks. Since there is no mechanism which can tilt BH spins from the orbital planes, the BH spins will be parallel to the orbital planes due to tidal synchronization (Hut 1981).

We select those binaries which are likely to have spinning BHs. Specifically, we select only those binaries whose separation is small enough to allow the tidal spin-up of the stars, after the binary exits the common envelope phase as a double Wolf-Rayet star. As shown by (Kushnir et al. 2016, see also Safarzadeh & Hotokezaka 2020), the constraint on semimajor axis translates into a constraint in coalescence time after the secondary turns into a BH. Therefore, we select only those binaries that have a GW merging timescale of < 200 Gyr (before turning into BHs) and we exclude those binaries whose GW timescales are too small (< 50 Myr) to allow for encounters with other stars before their mergers.

The outer orbit is determined in the following way. The velocity at infinity v∞ of the outer orbit is drawn from a Maxwellian distribution with σ∞ dispersion. We consider two main environments in which the three-body encounters take place: open clusters (OCS), which have low velocity dispersions, and massive clusters with high velocity dispersions, such as globular and young massive star clusters. We set σ∞ to 1 km/s and 20 km/s for OC and GC simulations, respectively. Note that this represents the 3D velocity dispersion, rather than the commonly reported line-of-sight velocity dispersion, which is a factor of 3 smaller (Illingworth 1976; Harris 1996; Portegies Zwart et al. 2010). The distribution of the pericenter distance of the outer orbit, pout = aout (1 − eout) is drawn from a distribution uniform in 2pout between 0 and 2aout. This is chosen so as to minimize the number of flyby interactions, since numerical experiments have shown that the cross section for resonant encounters drops for (Hut & Bahcall 1983; Hut 1983, 1993).

The mass of the third body is drawn from the stellar population synthesis code ssfr, in accordance with the nss simulations used for the inner binaries. It is well known that massive stars and binaries sink to the cores of star clusters faster than lighter stars, due to mass segregation and energy equipartition. (e.g. Giersz & Heggie 1996; Gürkan et al. 2004; Khalisi et al. 2007; Goswami et al. 2012; Tanikawa et al. 2012, 2013; Trani et al. 2014; Fujii & Zwart 2014; Spera et al. 2016; Webb & Vesperini 2016; Pavelk 2020). We mimic this effect by pairing the most massive binaries to the most massive single body, as shown in Figure 1. We combine 3 choices of stellar metallicity with 2 choices of velocity dispersion, for a total of 6 sets of simulations. Each set comprises of 10⁵ realizations, enough to sample the initial parameter space. Table 1 summarizes the initial conditions for each set.

The initial binary-single distance is set to 1000au, which is large enough so that the initial binary is unperturbed by the single. The simulations are run until either the three-body interaction is complete (with an outbound binary-single on a hyperbolic orbit), or there is a merger.
Spin misalignment in black hole binaries

3 ENCOUNTER OUTCOMES

A three-body encounter can be represented as a succession of strong chaotic interactions, wherein all three-bodies exchange energy, and long, regular excursions in which the single is ejected from the binary and forms a temporary hierarchical bound system (e.g. Stone & Leigh 2019). Eventually, assuming all point particles, the single will achieve a velocity above the escape velocity, and the encounter will end with an unbound binary-single. In the context of three-body interactions in the cores of stellar clusters, this process usually leads the binary semimajor axis to shrink, and it is therefore known as binary hardening (Heggie 1975).

After the encounter is concluded, the orientation of the outgoing binary might be different than the initial one. This will cause spin-angle misalignment if initially the BH spins were aligned with the orbit. We measure the tilt angle $\Delta \cos \iota$ between the initial binary plane and the final one, and check if one of the binary members was exchanged with the single BH. Hereafter, we refer to binaries that underwent an exchange as "exchanged binaries", while the binaries that did not are termed "original binaries".

Figure 2 shows the distributions of $\cos \Delta \iota$ for all of our sets. The distributions are essentially the same regardless of the initial velocity dispersion, indicating that encounters in OCs and GCs will lead to very similar tilt angles. This is because the initial binaries are very tight, and the single BH very massive, so that regardless of $\sigma_{\infty}$ we are in the regime of hard binary scatterings (Heggie 1975; Hut 1983; Hut 1984). The tilt angle distribution of original binaries is strongly peaked at $\Delta \cos \iota = 1$, indicating that most original binaries retain their initial orientation. The peak at $\cos \Delta \iota = 1$ becomes more pronounced with increasing metallicity: about 15%, 18% and 34% of the binaries get misaligned by less than 15° at $Z = 0.01, 0.1$ and $1 Z_\odot$, respectively. The tilt angle distribution of exchanged binaries is moderately flat, except at $Z = 1 Z_\odot$ where it becomes more strongly peaked at $\Delta \cos \iota = 0^\circ$.

In Figure 3 we show the distributions of binary recoil velocity for all our sets. As for the $\cos \Delta \iota$ distribution, there is little dispersion between OC and GC cases. However, the escape velocity from OCs is much lower than for GCs. As a conservative choice, we set the cluster escape velocity to $v_{\text{escape}} = 4 \sigma_{\infty}$, which amounts to 80 km/s and 4 km/s for the GC and OC sets, respectively. This results in the majority (>97%) of binaries escaping in the OC sets, while only about 40% of the binaries escape in the GC sets.

The kick distribution shifts towards lower velocities at higher metallicity. This is a consequence of the lower BH masses at high metallicity, which makes three-body interactions less energetic and...
it is unlikely that they undergo a second encounter even if they do not escape from the cluster (see Section 4). For all metallicities, the initial distribution peaks at ≈100 Myr and ranges from 40 Myr to 10 Gyr. This is the result of our selection of initial binaries that have been spun-up by tidal interactions.

The peak coalescence time in the final distribution for all binaries becomes slightly shorter, at ≈10 Myr after the encounter. At $Z = Z_\odot$, this peak is dominated by exchanged binaries: most of the original binaries have a much shorter coalescence time, peaked at ≈1 Myr. At higher metallicity, the discrepancy in coalescence time between exchanged and original binaries lessens.

The reason for this is that at low metallicity, the original binaries have a shorter semimajor axis than the exchanged ones. Since at low metallicity the initial single star can be twice more massive than the binary (see Figure 1), the binary needs to harden more in order to eject the single. On the other hand, if the initial single ejects one of the lower-mass binary components, the binary can maintain a larger separation.

Another way to phrase it is that the outcome distribution of binary binding energies $E_b = Gm_1m_2/a$ of a three-body encounter is the same whether the final binary is the original or exchanged. Therefore, at the same $E_b$, a higher mass product $m_1m_2$ translates into a larger semimajor axis $a$, and vice versa (Valtonen & Karttunen 2005). However, at short coalescence times the distribution has a similar trend for both exchanged and original binaries.

4 ENCOUNTER RATE ESTIMATES

The merger rate density for these kinds of events depends linearly on the encounter rate between the compact binary BHs and single BHs. Here we estimate and discuss the encounter rate for the three-body encounters considered in this work. The encounter rate of binary-single encounters $\Gamma_{\text{enc}}^{2+1}$ can be expressed as (Leigh & Sills 2011):

$$\Gamma_{\text{enc}}^{2+1} = N_{\text{sin}}n_{\text{bin}} \frac{3\pi Gm_{\text{tot}}R_{\text{enc}}}{\sigma_{\text{co}}} = f_{\text{bin}}f_{\text{sin}} N_{\text{sin}} \frac{3\pi Gm_{\text{tot}}R_{\text{enc}}}{\sigma_{\text{co}}} \tag{1}$$

where $n$ is the stellar number density, $N$ is the number of stars, $\sigma_{\text{co}}$ is the velocity dispersion, $m_{\text{tot}} = m_{\text{bin}} + m_3$ is the mass of the binary plus single, $f_{\text{bin}}$ and $f_{\text{sin}}$ are the binary and single fractions, and $R_{\text{enc}}$ is the distance below which the binary-single can undergo a chaotic three-body encounter, which we conservatively set to $2r_{\text{inf}}$. Here we have assumed that the cross section is dominated by gravitational focusing, so that the Safronov number $\Theta = Gm_{\text{tot}}/(R_{\text{enc}}\sigma_{\text{co}}^2) \gg 1$ (Binney 2008). This latter condition is always satisfied given the compactness of the binary.

The velocity dispersion depends on cluster size and mass via the virial relation $\sigma_{\text{co}}^2 = 0.45GM_{\text{cl}}/r_h$, where $M_{\text{cl}}$ is the cluster mass and $r_h$ is the half-mass radius.
The fraction of compact binary BHs \( f_{\text{CBBH}} \) is estimated from our binary population simulations as the ratio between the final selection of binaries and the initial binary number. This fraction depends on the metallicity, and amounts to \( 7.6 \times 10^{-5}, 1.02 \times 10^{-3} \) and \( 2.17 \times 10^{-3} \) for \( Z = 1, 0.1 \) and 0.01 \( Z_\odot \), respectively. This is likely an underestimate of the number of compact binary BHs, because it considers only those formed from isolated evolution. The common envelope phase that leads to such compact binaries may be triggered by dynamical interactions, therefore increasing the fraction \( f_{\text{CBBH}} \) (Kumamoto et al. 2019; Di Carlo et al. 2019). The fraction of single black holes is instead \( f_{\text{BH}} = 0.028 \), consistent with an evolving population of stars that follows a Kroupa (2001) mass function between 0.8 and 150 solar masses.

Finally, we can obtain the encounter rate averaged over the star cluster mass function, which follows a power-law of \( \beta = -2 \), as observed in massive clusters in the Galactic disk and starburst galaxies (Portegies Zwart et al. 2010):

\[
\Gamma_{\text{enc}}^{2+1} = A_{\text{cl}} \int \frac{f_{\text{CBBH}}}{10^{-3}} \frac{f_{\text{BH}}}{0.02} \frac{dM_{\text{cl}}}{M_{\text{cl}}} \tag{3}
\]

where \( A_{\text{cl}} \) is the normalization factor so that the cluster mass function normalizes to 1. The averaged encounter rate is then

\[
\Gamma_{\text{enc}}^{2+1} \approx 10^{-3} \text{Myr}^{-1} \left( \frac{f_{\text{CBBH}}}{10^{-3}} \right) \left( \frac{f_{\text{BH}}}{0.02} \right) \tag{4}
\]

Given the short lives of OCs (~300 Myr, Portegies Zwart et al. 2010), it seems that OCs might only experience a few such binary-single encounters, if any. This low encounter rate justifies our assumption of considering only the effect of a single encounter.

On the other hand, the rate of binary-binary encounters in OCs dominates over that of binary-single encounters due to the abundance of wide stellar binaries (e.g. Leigh & Sills 2011; Leigh & Geller 2013; Geller & Leigh 2015). While we limit ourselves to simulating binary-single encounters, we do not expect the outcome of binary-binary encounters to be statistically different. The reason is that wide binaries in OCs have typically a semimajor axis much larger than 10 au, which is \( 10^2 \) larger then the hard binaries of our sample (Raghavan et al. 2010). Such hard-soft binary encounters tend to quickly eject one of the wide binary members, and subsequently continue the evolution as a three-body encounter.

The binary-binary encounter rate is \( \Gamma_{\text{enc}}^{2+2} \) and can be calculated as

\[
\Gamma_{\text{enc}}^{2+2} = \frac{8 \pi G M_{\text{tot}} R_{\text{enc}}}{\sigma_{\infty}} \tag{5}
\]

Assuming the typical size of wide binaries, \( (R_{\text{enc}} \equiv a_{\text{WB}} = 30 \text{ au}, Raghavan et al. 2010), f_{\text{bin},a} \equiv f_{\text{CBBH}} = 10^{-3}, \) and a wide binary fraction of \( f_{\text{bin},b} \equiv f_{\text{WB}} = 0.5, \) Equation 5 leads to a much higher encounter rate of \( \Gamma_{\text{enc}}^{2+2} \approx 40 \text{ Myr}^{-1}. \) However, only close passages between the compact binary BH with a binary member can lead to a meaningful encounter: given its compactness, the binary BH can simply pass through the wide binary without really interacting as a 2-body object. Therefore, to estimate the binary BH-wide binary encounter rate we set \( R_{\text{enc}} \equiv 2 a_{\text{WB}}, \) and double the rate to take into account that each wide binary is composed of two objects. With this, the 2+2 encounter rate becomes \( \Gamma_{\text{enc}}^{2+2} \approx 0.2 \text{ Myr}^{-1}. \) In this assumptions we ignore that some of these encounters may end with the wide binary companion bound to the compact binary, forming a stable hierarchical triple. This triple would be too wide to affect the evolution of the compact binary via the von Zeipel-Kozai-Lidov mechanism.
(von Zeipel 1910; Lidov 1962; Kozai 1962), and would likely be disrupted via a subsequent interaction on a very short timescale.

Note that only encounters with wide binary BHs, rather than stellar wide binaries, can lead to a desirable outcome, that is, tilting of the orbital plane of the binary BH. Most encounters with stellar objects will involve low-mass main sequence stars, because by the time the primordial binary has become a compact binary BH, massive stars have already collapsed into BHs. Such low-mass stars will have a limited impact on the more massive binary BH. To significantly affect the binary BH, the velocity kick from the passing star needs to be comparable to the orbital velocity of the binary BH, \( v_{\text{bin}} = \sqrt{Gm_{\text{BH}}/R_{\text{BH}}} \). From conservation of linear momentum, this requires the star to approach one of the binary members by at least \(-a_{\text{bin}}(m_{\text{star}} + m_{\text{BH}})/m_{\text{bin}}\). This distance is about 0.019 au for a 1 M\(_{\odot}\) star encountering a 40 M\(_{\odot}\) binary BH; this distance is dangerously close to the stellar tidal disruption radius \( R_{\text{star}}(m_{\text{star}}/m_{\text{BH}})^{1/3} \sim 0.012\) au. Hence most of the encounters between compact binary BHs and wide stellar binaries will result in either little impact on the binary BH or stellar tidal disruptions. We can then correct the binary-binary encounter rate by taking into account only encounters with wide binary BHs, whose fraction we can estimate as \( f_{\text{BH}}/f_{\text{WB}} \approx 0.014\). In this approximation we have neglected stellar binary interactions, which are unlikely to occur in wide binaries. The total encounter rate is therefore \( \Gamma_{\text{enc}} = \Gamma_{\text{enc}}^2 + \Gamma_{\text{enc}}^3 \approx 6 \times 10^{-3} \) Myr\(^{-1}\).

This model assumes that most of the encounters will occur in the core, which may not be always correct. Barrera et al. (2020) find that the total integrated (i.e., over the entire volume of the cluster) encounter rate is underestimated by a factor of ~5 when compared to the core rate, as confirmed via N-body simulations. Barrera et al. (2020) found that of order 50% of all interactions occur in the core, with a non-negligible additional contribution coming from interactions occurring just outside the core, and then drifting into the core due to mass segregation. Hence, the correction factor from the integrated rate calculation can simply be multiplied by the total core rate, in order to obtain a total rate for the entire cluster.

It is worth reminding that this cross-section estimate is rather simplistic because it neglects the role of global stellar cluster processes, such as core collapse, dynamical friction and mass segregation, which tend to increase the frequency of encounters. The encounter rates outlined here are to be intended as a lower limit estimate. In other words, an enhanced rate of three-body encounters are an unavoidable consequence of the gravothermal instability of self-gravitating systems, and can accelerate the disruption of star clusters (Leigh et al. 2014a, 2016a).

5 MERGERS PROPERTIES

5.1 Local merger rate

To calculate the local merger rate density, we adopt the same approach of Kumamoto et al. (2020). Particularly, we use their equation (16) to express the local merger rate density:

\[
\Gamma_{\text{loc}} = 9 \times 10^{-4} \frac{d\Phi(Z, t_{\text{loc}})}{dZ} \int_{t_{\text{enc}}} dD(Z, t_{\text{gw}} = t_{\text{loc}}) \frac{d\Psi(Z, t_{\text{loc}})}{dZ} \]  

(6)

where \( D(Z, t_{\text{gw}}) \) is the merger rate of binary BHs originated from one cluster, \( t_{\text{loc}} \) is the lookback time, and \( \Psi(Z, t_{\text{loc}}) \) is the comoving formation rate density of stars.

We calculate \( D(Z, t_{\text{gw}}) \) from the simulations, and include \( \Psi(Z, t_{\text{loc}}) \) as estimated by Chruslinska & Nelemans (2019). We express \( D(Z, t_{\text{gw}}) \) as the product of the binary encounter rate \( \Gamma_{\text{enc}}(Z) \), and the density distribution of delay times \( B(Z, t_{\text{gw}}) \):

\[
D(Z, t_{\text{gw}}) = \Gamma_{\text{enc}}(Z) B(Z, t_{\text{gw}}) \]  

(7)

Here the encounter rate depends on the metallicity of the parent cluster through the compact binary BH fraction \( f_{\text{CBBH}} \), as estimated as in Section 4 (\( \Gamma_{\text{enc}} \approx 0.0005, 0.0066 \) and 0.0143 Myr\(^{-1}\) for \( Z = 1, 0.1 \) and 0.001 Z\(_{\odot}\), respectively). The density distribution of delay times \( P(Z, t_{\text{gw}}) \) is obtained from the OC three-body simulations.

We consider three metallicity ranges: \( Z \approx 0.00632–0.00632, 0.000001–0.0000632 \) for the simulations at 1, 0.1, and 0.01 Z\(_{\odot}\), respectively (i.e. equally spaced in logarithmic scale). Ultimately, the expression for the local merger rate reads as:

\[
\Gamma_{\text{loc}} = \frac{9 \times 10^{-4}}{4 M_{\odot} \ln 10} \int_{t_{\text{enc}}} \int_{Z=1,2,3} D(Z, t_{\text{gw}} = t_{\text{loc}}) \Psi(Z, t_{\text{loc}}) dZ \]  

(8)

where the summation is over the three ranges of metallicities, corresponding to the simulations at \( Z = 1, 0.1 \) and 0.01 Z\(_{\odot}\), and \( \Psi(Z, t_{\text{loc}}) \) is the star formation rate density \( d\Phi(Z, t_{\text{loc}})/dZ \) integrated over the three ranges (see equations 31–35 from Kumamoto et al. 2020).

We obtain a local merger rate of \( \Gamma_{\text{loc}} \approx 6.6 \) yr\(^{-1}\) Gpc\(^{-3}\).

Despite the higher abundance of compact binary BHs at \( Z = 0.01 Z_{\odot} (f_{\text{CBBH}} \approx 2 \times 10^{-3}) \), their contribution to the local merger rate is only \( \approx 2 \) yr\(^{-1}\) Gpc\(^{-3}\). The main reason is that most low-metallicity BHs are born at high redshift (see fig. 6 Chruslinska & Nelemans 2019), but they have a very short delay time (Figure 4). Hence, the contribution to our estimated merger rate comes also from solar and moderately sub-solar metallicity binary BHs.

While our estimated merger rate lies at the lower limit inferred by GWTC-2 (The LIGO Scientific Collaboration & the Virgo Collaboration 2020b), it only applies to the subset of BH-BH mergers undergoing the pathway considered here. Our estimate indicates that such binaries may be already contributing to the current detection sample, and that they will likely emerge from the data after a few hundred detections.

5.2 \( \chi_{\text{eff}} \) and \( \chi_{\text{p}} \) distributions

The information on BH spin during the merger is encoded into two parameters, the effective spin parameter \( \chi_{\text{eff}} \) and the effective precession parameter \( \chi_{\text{p}} \). Both parameters describe the orientation of the spin vector with respect to the binary orbit: the effective spin \( \chi_{\text{eff}} \) relates to the spin component parallel to the orbital angular momentum vector, while the effective precession \( \chi_{\text{p}} \) relates to the orbital precession caused by the in-plane spin component.

Given the dimensionless spin \( \chi \) and the spin obliquity \( \theta \) (i.e. the angle between the spin vector and the orbital angular momentum vector), we can express \( \chi_{\text{eff}} \) and \( \chi_{\text{p}} \) as:

\[
\chi_{\text{eff}} = \frac{m_1 x_1 \cos \theta_1 + m_2 x_2 \cos \theta_2}{m_1 + m_2} = \frac{(m_1 x_1 + m_2 x_2) \cos \Delta \theta}{m_1 + m_2} \]  

(9)

\[
\chi_{\text{p}} = \max \left[ x_1 \sin \theta_1, x_2 \sin \theta_2 \frac{4q + 3}{4 + 3q} \right] \]  

(10)

where the indices 1 and 2 refer to the primary and secondary BHs,
so that \( m_1 > m_2 \), and \( q = m_2/m_1 \leq 1 \) is the mass ratio. The last identity takes into account our initial setup, where both spins are initially aligned and \( \theta_1 = \theta_2 = \Delta i \).

The dimensionless spin of BHs at birth is largely uncertain. Its precise value depends on the interplay between winds and tidal interactions of the progenitor binary, and on the physics of angular momentum transport during the last stages of core collapse. In the following, we test 3 different phenomenological models for the dimensionless spin. In the maximum model, we assume that both BHs in the original binaries are born maximally spinning, i.e. \( \chi_1 = \chi_2 = 1 \). In the uniform model, the spin of the BHs is instead randomly drawn from a uniform distribution between 0 and 1. In the beta model, the spin is sampled from a beta distribution with scale parameters \((\alpha, \beta) = (2.3, 4)\). The beta distribution peaks at about \( \chi \approx 0.2 \) with a long tail at 0.3–0.7. The spin of the initially isolated BH can affect the effective spin distributions of exchanged binaries; it is assumed to be zero in all three models.

Figure 5 shows the cumulative distribution of \( \chi_{\text{eff}} \) and \( \chi_p \) for each simulated set. Because the outcome distributions are the same regardless of the initial velocity dispersion of the three-body encounter, we omit plotting the curves from the GC simulations.

In the maximum model, the \( \chi_{\text{eff}} \) distributions are characterized by a peak at \( \chi_{\text{eff}} \approx 1 \), stronger at high metallicity. This peak is mainly composed of original binaries that experienced only weak encounters with moderate tilting. Hence, the peak at \( \chi_{\text{eff}} \approx 1 \) simply traces the distribution of \( \chi_{\text{eff}} \) before the encounter, which is identical to 1 in the maximum model. The slope between \( \chi_{\text{eff}} \approx -0.5, 0.5 \) is composed of exchanged binaries, wherein the primary BH is the non-spinning, exchanged one. Overall, the slope in the maximum model is too shallow the match the one inferred from the observations.

Going from the maximum to the beta model, the average dimensionless spin of the BHs decreases, and the cumulative distributions of \( \chi_{\text{eff}} \) becomes more peaked at zero. Particularly, the distribution at \( Z = 0.1 \) and 0.01 \( Z_\odot \) for the beta model matches well the inferred distribution from GWTC-2.

The \( \chi_p \) cumulative distribution has a similar trend, with the beta model more peaked at zero, and the maximum model favoring larger \( \chi_p \). Overall, the uniform model matches better the \( \chi_p \) constraints from the GWTC-2 data.

In addition to displaying the distribution of \( \chi_{\text{eff}} \) and \( \chi_p \) for each metallicity, we compute the local merger rate for different bins of \( \chi_{\text{eff}} \) and \( \chi_p \). Each domain range is divided in 25 uniform bins, and the procedure to calculate the local merger rate is repeated for each binary subset. Figure 6 shows the obtained local merger rate as a function of \( \chi_{\text{eff}} \) and \( \chi_p \). The end result can be thought of as a combination of the distributions in Figure 5, weighted by merger rate per metallicity range, plus second order effects from the correlations between effective spin and delay time.

The beta spin model best matches the current observational data. The \( \chi_{\text{eff}} \) distribution is compatible to the observed one, with a peak slightly above 0 and a tail at negative \( \chi_{\text{eff}} \). The \( \chi_p \) distribution has a broad peak at \( \approx 0.2 \), in reasonable agreement with the GWTC-2 data (see fig. 9 of The LIGO Scientific Collaboration & the Virgo Collaboration 2020b). In contrast, the maximum model predicts a peak at \( \chi_{\text{eff}} \approx 1 \), which is not present in the data, while the uniform model strongly favors zero values of \( \chi_p \).

6 CAVEATS

We remind here the assumptions we made along this work. First, we have considered only compact binary BHs undergoing a single three-body encounter. This was justified in Section 4 and Section 3, by noting that the encounter rate of such binaries is small compared to cluster lifetimes and GW coalescence times. Moreover, the binaries get ejected from most stellar clusters due to the high recoil kicks from the encounters, preventing further encounters.

Another assumption of our work is that the BH spins are initially aligned with the orbit. This assumption is reflected in the choice of the initial conditions: all our binary BHs come from post-common envelope evolution, and form a close Wolf-Rayet binary before collapsing into BHs. Our binary sample has a median period less than 1 day, so that tidal forces can efficiently align the spin of the progenitor stars (Kushnir et al. 2016; Hotokezaka & Piran 2017; Piran & Piran 2020). In general, this is not true for longer binary periods and stars that underwent significant mass transfer; however this depends on the tidal efficiency and the angular-momentum transport within the stellar interiors, which are highly uncertain (Stegmann & Antonini 2020).

Finally, we neglected BH natal kicks that might tilt the binary plane and the stellar spins. In our case, the kick magnitude should be >600 km/s to affect the binary angular momentum, which is unlikely (Mandel 2016; Mirabel 2016; Wysocki et al. 2018).

![Figure 5. Cumulative distributions of \( \chi_{\text{eff}} \) (top) and \( \chi_p \) (bottom) for each simulated set. Green lines: \( Z = 1 Z_\odot \), Orange lines: \( Z = 0.1 Z_\odot \), Blue lines: \( Z = 0.01 Z_\odot \). The OC and GC distributions overlap, so we show only the OC ones. The line style denotes the model for the natal dimensionless spin \( \chi \). Dotted lines: maximally spinning BHs (\( \chi_1 = \chi_2 = 1 \)). Dot-dashed line: \( \chi_1, \chi_2 \) drawn from a uniform distribution in (0,1). Solid lines: \( \chi_1, \chi_2 \) sampled from a beta distribution with \( (\alpha, \beta) = (1.3, 4) \). In all three models the isolated BHs are non-spinning \( \chi_3 = 0 \). The gray shaded area shows the 90% confidence interval of the distribution reconstructed from GWTC-2 (default spin model of The LIGO Scientific Collaboration & the Virgo Collaboration 2020b).](image-url)
properties, we consider both low velocity dispersion environments (corresponding to open clusters) and high velocity dispersion environments (corresponding to globular and young massive star clusters). We show that the encounter outcome is the same regardless of the velocity dispersion, due to the compactness of the binaries.

We estimate the local merger rate considering the cosmic star formation rate density at different metallicities, taking into account the delay time of BH mergers and the binary encounter rates. We obtain a local merger rate of $\Gamma_{\text{loc}}^\text{gw} \approx 6.6 \text{yr}^{-1} \text{Gpc}^{-3}$, which shows that this pathway might be contributing to the events detected so far.

We also estimate the differential merger rate for the effective $\chi_{\text{eff}}$ and precession $\chi_p$ spin parameters. Because the value of natal BH spins $\chi$ is subject to numerous uncertainties, we test three different phenomenological models.

Assuming low dimensionless spins $\chi \lesssim 0.2$ in binaries and non-spinning isolated BHs, our scenario can reproduce the distributions of $\chi_p$ and $\chi_{\text{eff}}$ inferred from GWTC-2. In particular, our model can explain the negative $\chi_{\text{eff}}$ inferred the GWTC-2 population and the broad peak at $\chi_p \sim 0.2$ in the precession spin parameter.

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DATA AVAILABILITY

The tsunami code, the initial conditions and the simulation data underlying this article will be shared on reasonable request to the corresponding author.

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