Computer Vision and Metrics Learning for Hypothesis Testing: An Application of Q-Q Plot for Normality Test

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Abstract

This paper proposes a new procedure to construct test statistics for hypothesis testing by computer vision and metrics learning. The application highlighted in this paper is applying computer vision on Q-Q plot to construct a new test statistic for normality test. Traditionally, there are two families of approaches for verifying the probability distribution of a random variable. Researchers either subjectively assess the Q-Q plot or objectively use a mathematical formula, such as Kolmogorov-Smirnov test, to formally conduct a normality test. Graphical assessment by human beings is not rigorous whereas normality test statistics may not be accurate enough when the uniformly most powerful test does not exist. It may take tens of years for statistician to develop a new and more powerful test statistic. The first step of the proposed method is to apply computer vision techniques, such as pre-trained ResNet, to convert a Q-Q plot into a numerical vector. Next step is to apply metric learning to find an appropriate distance function between a Q-Q plot and the centroid of all Q-Q plots under the null hypothesis, which assumes the target variable is normally distributed. This distance metric is the new test statistic for normality test. Our experimentation results show that the machine-learning-based test statistics can outperform traditional normality tests in all cases, particularly when the sample size is small. This study provides convincing evidence that the proposed method could objectively create a powerful test statistic based on Q-Q plots and this method could be modified to construct many more powerful test statistics for other applications in the future.

1. Introduction

Normality assumption is one of the most widely imposed assumptions in many statistical procedures such as t-tests, linear regression analysis, and Analysis of Variance (ANOVA) (Razali et al., 2011). If the assumption of normality is violated, interpretation and inference may not be reliable. Therefore, it is crucial to verify this assumption before conducting further statistical analysis. Traditionally, two common ways to examine the normality assumption are graphical exploration by Q-Q plot (quantile-quantile plot) and formal normality tests.

Q-Q plot is a widely used and effective visualization tool for assessing the empirical probability distribution of a random variable, against any hypothesized theoretical distribution. Q-Q plot compares two probability distributions by plotting theoretical quantile (horizontal axis) against empirical quantile (vertical axis). For example, Q-Q plot is commonly used in linear regression analysis to examine whether regression residuals are normally distributed. If the residuals are normally distributed, the Q-Q plot will exhibit a pattern similar to a 45-degree line. Figure 1(a) illustrates this case. Figure 1(b) illustrates the Q-Q plot of a random variable drawn from Laplace distribution. Throughout this paper, Q-Q plots of a variable drawn from normal distribution are referred to as null hypothesis plots and are abbreviated as H0 plots. On the contrary, plots of random variables drawn from non-normal distribution are referred to as alternative hypothesis plots and are abbreviated as H1 plots.

Although the graphical method can serve as an effective tool
in checking normality, results are still error-prone in that human beings may not be able to make correct and consistent assessment, particularly when the target distribution for hypothesis testing is very similar to a normal distribution. One motivation of this study is to explore whether this kind of human task can be automated by machine learning methods.

If computer vision can automate this visualization task, the other research objective is to investigate whether the new AI-based method can outperform existing normality tests developed by statisticians. In other words, can AI be used to construct more powerful test statistics? There are a compelling number of normality tests available in the literature. The most common normality tests provided in statistical software packages include Kolmogorov-Smirnov (KS) test, Anderson-Darling (AD) test, Shapiro-Wilk (SW) test and Jarque-Bera (JB) test. Explanations of these four methods are deferred to Section 2.1. It is well-known in the statistics literature that the uniformly most powerful statistical test may not exist when the alternative hypothesis is multi-dimensional. In other words, there exist room for improvement in statistical power of existing normality tests because there are numerous alternative distributions.

In this paper, we propose a new procedure to construct test statistics by applying computer vision and metrics learning on Q-Q plots. The proposed approach can be considered as a Monte-Carlo approach because the training data consists of many Q-Q plots based on simulated data drawn from a standard normal distribution. The first step is to apply image classification models, such as ResNet and VGG model, to convert each Q-Q plot into a numerical vector. The next step is to identify the centroid of all H0 plots. The centroid can be conceptually considered as the most representative vector of all H0 plots. The last step and is also the key step of the new method is to explore which image similarity function (distance metric) can produce a test statistic with the best statistical power for normality test. Given the centroid and any H0 plot, we can calculate a distance metric by different existing methods. The 95% percentile of the distance metric of all H0 plots is the cutoff value for hypothesis testing. In other words, this distance metric serves the purpose of a test statistic. If the distance score of a new plot is smaller than the 95% cutoff, we classify the new plot as a H0 plot whereas if the score exceeds the 95% cutoff, we classify a new plot as H1 plot because it is not similar enough to the centroid, measured by the distance metric. We comprehensively explore various methods in each one of the three steps in our experimentation.

To evaluate the performance of this new method relative to traditional statistical tests, we simulate datasets with fourteen types of non-normal distributions, including t distribution, uniform distribution, beta distribution, Laplace distribution, gamma distribution and chi-square distribution. Our experimentation results show that our proposed approach indeed outperforms all four traditional normality tests in all fourteen cases, especially when the sample size is small.

Our results may contribute to the literature in the following ways. First, our method demonstrates a completely different avenue for constructing test statistics with superior performance, at least for normality test. The test statistic itself can improve the research quality in many applications due to the importance of normality assumption. Second, our method could be generalized to construct test statistics for other problems. Third, our results could also inspire statisticians to derive new formulas for constructing new test statistics after inspecting the properties of the test statistics constructed from machine learning methods. Last but not the least, this study provides a preliminary evidence that human visualization for analytics can be automated by machine learning methods.

2. Related Works

Our research objective is to develop a new method that outperforms existing non-AI based statistical tests. In this section, we will first review four famous traditional normality tests used for comparison in the experimentation section. Next, we will briefly review the literature for learning similarity metrics between two images by deep learning or by traditional image processing techniques.

2.1. Traditional Normality Tests

In statistics, researchers have developed many normality tests. The famous methods can be categorized into two families. The first family of tests are derived based on the comparison between the empirical distribution function estimated using the data and the theoretical (normal) distribution. The second family of tests are based on the sample moments of the hypothesized distribution.

2.1.1. Kolmogorov-Smirnov Test

Kolmogorov and Smirnov introduced the first empirical distribution based test for normality (Kolmogorov, 1933; Smirnov, 1948). This test is a non-parametric test, which is based on the largest vertical difference between the hypothesized and empirical distribution. Given n ordered data points, \( x_1 < x_2 < \ldots < x_n \), the KS test statistic is defined as,

\[
T_{KS} = \sup_x |F^*(x) - F_n(x)|
\]

where ‘sup’ stands for supremum. \( F^*(x) \) is the hypothesized cumulative distribution function whereas \( F_n(x) \) is the empirical cumulative distribution function estimated based on the random sample. In KS test for normality, \( F^*(x) \) is a normal distribution with known mean, \( \mu \), and known
standard deviation, \( \sigma \).

### 2.1.2. ANDERSON-DARLING TEST

Another commonly used test based on empirical distribution is Anderson–Darling test (Anderson & Darling, 1952). The AD test makes use of the specific hypothesized distribution and gives more weight to the tails of the distribution. Given \( n \) ordered data points, \( x_1 < x_2 < \ldots < x_n \), the statistic for this test is defined as,

\[
T_{AD} = n \int_{-\infty}^{\infty} [F_n(x) - F^*(x)]^2 \psi(F^*(x)) dF^*(x) \tag{2}
\]

where \( \psi \) is a non-negative weight function which can be computed by, \( \psi = [F^*(x)(1 - F^*(x))]^{-1} \).

### 2.1.3. SHAPIRO-WILK TEST

With respect to tests by moments, one of the most famous tests was developed by Shapiro and Wilk (Shapiro & Wilk, 1965). The SW test attempts to detect departures from normality due to either skewness or kurtosis, or both (Althouse et al., 1998). Given \( n \) ordered data points, \( x_1 < x_2 < \ldots < x_n \), the original Shapiro-Wilk test statistic is defined as,

\[
T_{SW} = \frac{\sum_{i=1}^{n} a_i x_i^2}{\left( \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{1/2}} \tag{3}
\]

where the weights \( a_i = (a_1, \ldots, a_n) = \left( \frac{m^T V^{-1} V^{-1} m}{m^T V^{1/2} m} \right)^{1/2} \), and \( m = (m_1, \ldots, m_n)^T \) are the expected values of the order statistics of standard normal distribution and \( V \) is the covariance matrix of those order statistics.

### 2.1.4. JARQUE-BERA TEST

Jarque and Bera proposed one test using the sample standardized third and forth moments (Jarque & Bera, 1980). The JB test performs well for distributions with long tail, while its statistical power is poor for distributions with short tails (Thadewald & Bünning, 2007). The JB statistic is based on sample skewness \( \sqrt{\hat{m}_3} \) and kurtosis \( \hat{b}_2 \) and is given as

\[
T_{JB} = n \left[ \left( \sqrt{\hat{m}_3} \right)^2 \frac{6}{b_1^2} + \left( \hat{b}_2 - 3 \right)^2 \frac{24}{b_2} \right] \tag{4}
\]

### 2.2. Image Classification by Deep Learning

Convolutional neural networks are designed to recognize visual patterns directly from pixel images. In recent years, deep convolutional neural networks have led to breakthroughs for image classification. LeCun et al. proposed the first convolutional neural network LeNet-5 with only two convolutional layers and less than one million parameters to classify handwritten digits (LeCun et al., 1989). Later, Krizhevsky et al. won the ImageNet competition using a convolutional neural network named AlexNet with 60 million parameters (Krizhevsky et al., 2012). More recently, a succeeding winning network architecture is VGG-Net, which consists of 16-19 layers (Simonyan & Zisserman, 2014). This neural network exhibit a simple yet effective strategy of constructing very deep networks: stacking building blocks of the same shape (Xie et al., 2017). Later, this strategy is inherited by Residual learning network named ResNet (He et al., 2016). Unlike VGG-Net that learns an underlying mapping \( H(x) \) fitted by stacked layers, ResNet explicitly approximates a residual function \( F(x) := H(x) - x \). This residual network is easier to optimize and can surpass the 100-layer barrier.

### 2.3. Similarity Metric Learning

Learning a similarity metric between two images is an important research topic with wide range of applications such as image search. Many prior studies have proposed various algorithms to learn such metrics.

One pioneering algorithm was proposed by Xing et al. (2003) that aims at finding “optimal” metrics for clustering algorithms with additional information about relative similarity. Given relative similarity between pairs of points in \( \mathbb{R}^n \), the authors attempt to learn a distance metric from the Mahalanobis distance family, such that points belonging to different clusters can be identified more effectively. More specifically, the distance metric is of the form \( d(x, y) = ||x - y||_A = \sqrt{(x - y)^T A (x - y)} \) whereas \( A \) are weights for optimization. The relative similarity is characterized by subsets of observations \( S \) and \( D \), respectively. \( S \) is a set of pairs of points known to be similar, and \( D \) is a set of dissimilar pairs. The optimization problem is given by

\[
\min_A \sum_{(x_i, x_j) \in S} ||x_i - x_j||_A^2 \tag{5}
\]

subject to

\[
\sum_{(x_i, x_j) \in D} ||x_i - x_j||_A \geq 1, \tag{6}
\]

\[
A \succeq 0. \tag{7}
\]

This convex optimization problem can be efficiently solved by the combination of gradient ascent and iterative projection algorithm over \( A \).

Online algorithm for scalable image similarity (OASIS) model is designed to learn a similarity metric for a large image data set with relative similarity scores (Chechik et al., 2010). OASIS is also the main metrics learning algorithm modified and employed in our paper.

Specifically, OASIS algorithm uses triplets of training images \( x_i, x_i^+, x_i^- \in X \), where \( r(x_i, x_i^+) > r(x_i, x_i^-) \). In
other words, training involves selected positive and negative images relative to the focal query image. The objective function is a hinge loss function with a margin at 1.

\[ L_W = \sum_{(x_i, x_i^+, x_i^-) \in X} l_W(x_i, x_i^+, x_i^-). \]  

\[ l_W(x_i, x_i^+, x_i^-) = \max\{0, 1 - S_W(x_i, x_i^+) + S_W(x_i, x_i^-)\}. \]

The similarity metric has the form of

\[ S_W(x_i, x_j) = x_i^T W x_j. \]

In order to be able to learn from a large data set, passive-aggressive algorithm is applied iteratively over triplets until convergence to optimal weights \( W \). Please refer to the original paper for the details of the iteration steps.

2.4. Traditional Image Similarity Metrics

In image quality assessment (IQA) literature, one stream of studies utilize image similarity measures to assess the image quality. Specifically, they calculate the similarity between the distorted image with the reference image to determine the image quality. PSNR is one of the most extensively used measures due to its simplicity. However, PSNR cannot capture the human perception of image fidelity and quality (Wang & Bovik, 2009). Therefore, some similarity measures following top-down strategy have been proposed (Lin & Kuo, 2011; Wang et al., 2004). Specifically, those measures first extract different image features to calculate a similarity map and then summarize the values of the similarity map into a single similarity score. For example, the classic structural similarity index (SSIM) employ the luminance, contrast and structural information to construct a similarity map. SSIM then uses averaging pooling to calculate the final similarity score (Wang et al., 2004). Feature similarity index (FSIM) uses phase congruency and gradient magnitude features to constitute the similarity map (Zhang et al., 2011). Visual Saliency Induced Index (VSI) uses saliency-based features and gradient magnitude for image evaluation (Zhang et al., 2014). Gradient Magnitude Similarity Deviation (GMSD) employs global variation of gradients to construct the image quality maps (Xue et al., 2014). Mean deviation similarity index (MDSI) combines gradient similarity, chromaticity similarity, and deviation pooling to enhance the image evaluation (Nafchi et al., 2016). Structural contrast-quality index (SCQI) utilizes Structural Contrast Index to capture local visual quality perception in terms of different image texture features (Bae & Kim, 2016).

3. Methods

The formula of traditional statistical tests can be considered as a function that maps one row of data \( X \) with sample size \( N \) to a real number, \( T : \mathbb{R}^N \rightarrow \mathbb{R} \). Statistics researchers need to impose mathematical assumptions to derive the probability distribution of \( T(X) \) under null hypothesis and a range of values of \( T(X) \) for rejecting the null hypothesis with confidence level at 95%. All four similarity tests described in Section 2.1 are examples of this kind of function.

In this study, we propose a new approach to construct normality test statistics based on computer vision and metrics learning. The first step is to construct a Q-Q plot. Using Q-Q plot could holistically capture richer information than the information used in the traditional statistical tests. For example, when compared with KS or AD tests, Q-Q plot displays very similar information in that it visualizes the distance between an empirical distribution and theoretical distribution. Q-Q plot can also indirectly visualize the dispersion and moments of the sample data, which are the key variables used in deriving the formula of SW test and JB test. The new method relies on deep learning to convert Q-Q plot to a numerical vector that potentially captures all those information for constructing a more powerful test statistic. Mathematically, each row of sample data is transformed into a vector \( V(X) \) that represents an image, the Q-Q plot. This study explores how computer vision could objectively assess and utilize this kind of visual information. Traditionally, this kind of information was subjectively assessed by human beings.

Next, we rely on existing image metrics learning methods to construct a real-value distance function (dis-similarity score) between two images, \( S(V(X_i), V(X_j)) \). Given this distance function, for any data for hypothesis testing, we can calculate \( S(V(X_i), C) \), where \( C \) is defined as the centroid of all \( H_0 \) plots. The construction of centroid will be discussed in detail later in this section. Loosely speaking, by simulating many Q-Q plots of normally distributed data, we can find out a cutoff value \( S_{95\%} \) that 95% of the \( H_0 \) plots have a score of \( S(V(X_i), C) \) lower than the cutoff value. This cutoff value will be used for future hypothesis testing to reject the null hypothesis under which \( X_i \) is drawn from a normal distribution. In contrast, in traditional test, researchers impose distributional assumptions on key variables for deriving the probability distribution of test statistics whereas the new method employs simulation and metrics learning to derive the distribution of a new test statistic.

3.1. Deep-Learning Based Normality Test

The steps of our methods are summarized as follows.

step 0 We simulate training dataset based on standard normal distribution. Next, we produce Q-Q plot following standard procedure in statistics. This step can be considered as data pre-processing step.
step 1 Each one of the simulated Q-Q plot is converted into a numerical vector by a deep learning image classification model. Next, dimension reduction methods could be applied to this output vector.

step 2 The centroid of all numerical vectors obtained from Step 1 is calculated by mean or median of all vectors.

step 3 The distance function between two images is learned. Given the learned distance function, we can calculate the distance between centroid and each one of the training image (H0 plots). We set the cut-off value for hypothesis testing at the 95% percentile of distance metric among all training H0 plots, which is equivalent to setting the confidence level at 95% in traditional statistics.

step 4 For hypothesis testing, given a new Q-Q plot, a distance metric to the centroid will be calculated. If the distance is less than the cutoff value constructed in Step 3, this plot is classified as a H0 plot. Otherwise, this plot is classified as a H1 plot.

We experimented with a number of approaches in Steps 1, 2, and 3. In Step 1, we experimented with several pre-trained models and this study only reports four sets of results. Two sets of results are derived from ResNet-50 (He et al., 2016), and VGG-16 (Simonyan & Zisserman, 2014) because the performance of these two models are better than those of LeNet, AlexNet, and other variants of ResNet and VGG models. Since ResNet and VGG are trained based on ImageNet, which includes many different kinds of photos but not statistical plots, we could further improve the classification performance to construct customized image features that better fit our purpose. Therefore, we also experimented with triplet learning model for image classification to transform one Q-Q plot to a numerical vector based on the method proposed in Wang et al. (2014). To reduce the training time, we replace the convolutional neural network with pre-trained ResNet-50 and VGG-16 within the complicated triplet neural network architecture. Therefore, we can fine-tune the triplet neural network and finally obtain two more models, Triplet_Resnet and Triplet_vgg. As to the dimension reduction in Step 1, we experimented with full singular-value decomposition (SVD) and truncated SVD with a vector of 300 elements. We also experimented with features selection but the performance is worse than SVD. As a result, for each one of the four cases, we will obtain three sets of features: original features without dimension reduction, features with full SVD, and features with truncated SVD.

In Step 2, given the output features from Step 1, we compute the mean or median of the feature vector. This new vector is defined as the “centroid” of vectors of all simulated H0 plots. We also experimented with clustering method but the overall performance is worse than using this simple method. As a consequence, after Step 2, for each one of the 4 models, we have $3 \times 2$ candidate “centroids” (3 types of dimension reduction and 2 averaging methods).

In Step 3, after finding the centroid, we need to calculate the distance between centroid and Q-Q plot. We have experimented with Euclidean distance, Cosine similarity as distance, Manhattan distance, and two customized distance functions as distance metrics. Two customized distance functions, including Bilinear distance (Chechik et al., 2010) and Mahalanobis distance (Davis et al., 2007), are learned by metrics learning methods in Chechik et al. (2010). As a result, we evaluated $3 \times 2 \times 5$ sub-cases for each one of the four deep learning models in our experimentation.

Complexity of training in this new method depends on the complexity of Step 1 because it is relatively time consuming to convert an image into a numerical vector by deep learning, especially when we apply triplet neural network. However, it is linear in the number of H0 plots. The training time in other steps are much shorter and negligible. Step 3 is not time-consuming because metrics learning is applied on a vector after dimension reduction.

3.2. Traditional Image Similarity Measures

Traditional image processing techniques can directly compute a similarity metric by “hand-crafted” image features. As a result, we do not need to convert one image into a numerical vector as in Step 1 in Section 3.1. In step 2, we re-define the centroid by taking average of the pixel values of all H0 plots in the training set. We experimented with other heuristics to find the most representative H0 plot but the performance does not improve much while the speed is much lower. Step 3 is conceptually the same as the counterpart in Section 3.1 except that the distance function used in Section 3.1 is replaced by one traditional image similarity measure. We experimented with seven image similarity measures, respectively. Seven image similarity measures include PSNR, SSIM, SCQI, MDSI, FSIM, GMSD and VSI described in Section 2.4.

Complexity of this method depends on the time for computing similarity score between two plots in Step 2 and similar to Section 3.1, this computation time is linear in the number of H0 plots.

4. Experimentation

Our visualization for the Q-Q plots is very similar to traditional ones, with theoretical quantiles and sample quantiles displayed as the x-axis and y-axis, respectively. Specifically, Q-Q plot is created by qqnorm() command in R with a red line added by abline() command. We also add this red 45-degree line ($y = x$ line) to highlight the possible
deviation from normality because for H1 plots without a red line, some sample points may also form a straight line as displayed in Figure 2 (b)(c)(d). Without the anchoring 45-degree line, both human beings and computer vision may not be able to detect the deviations from perfect normality. Our experimentation results also confirm the benefits of adding the true 45-degree line. Formatting of Q-Q plots more or less may affect the final performance of our method but is left for future exploration due to page limit.

The standard procedure used in statistics to demonstrate the superior performance of a newly proposed test statistic is by drawing statistical power plots. Researchers need to extensively simulate cases under the alternative hypothesis and show that the new method can reject more cases than traditional methods. In other words, the type I error in this problem is always fixed at 5% whereas the type II error will be compared in the statistical power plots. Following Razali et al. (2011) that evaluates the statistical power of normality tests, we simulate 14 cases of non-normal distributions to examine the statistical power of our new method. The detailed specifications of these 14 distributions are provided in Table 1. The first seven cases are symmetric distributions whereas the other seven are asymmetric distributions. One special case is that t distribution converges to normal distribution when the degree-of-freedom approaches infinity and therefore t(50) is a very challenging case for hypothesis testing. As to the sample size, we experimented with different number of observations: \( n = 10, 25, 50, 100, 250, 500, 1000 \). This is consistent with the statistics literature to illustrate the statistical power of a new test when being applied on different sample sizes. Figure 2 provides two examples of Q-Q plots for \( n = 10 \) and 1000, respectively. For constructing a training set, we simulated 7,000 H0 plots and we do not need to simulate H1 plots except for training triplet learning methods and metrics learning methods. In triplet learning and metrics learning, we simulated 7,000 H0 plots and 1,000 H1 plots in each case. This is because triplet and metrics learning methods require three types of input images, namely, the query image, the positive image and the negative image. Query image and positive image are sampled from all H0 plots and the negative image is sampled from all H1 plots in 14 cases. The test set for evaluating statistical power only needs H1 plots and the sample size is 1,000 new H1 plots in each case. It is worthy of noting that our training set is not large and the performance reported is conservative and could be under-estimated.

Table 1. 14 Cases of Non-Normal Distributions

| No | Type | Specification |
|----|------|--------------|
| 1  | t distribution | \( e \sim t(2) \) |
| 2  | t distribution | \( e \sim t(5) \) |
| 3  | t distribution | \( e \sim t(10) \) |
| 4  | t distribution | \( e \sim t(50) \) |
| 5  | uniform distribution | \( e \sim U(0, 1) \) |
| 6  | Laplace distribution | \( e \sim Laplace(0, 1) \) |
| 7  | beta distribution | \( e \sim Beta(2, 2) \) |
| 8  | beta distribution | \( e \sim Beta(6, 2) \) |
| 9  | beta distribution | \( e \sim Beta(3, 2) \) |
| 10 | beta distribution | \( e \sim Beta(2, 1) \) |
| 11 | gamma distribution | \( e \sim Gamma(1, 5) \) |
| 12 | gamma distribution | \( e \sim Gamma(4, 5) \) |
| 13 | chi-square distribution | \( e \sim \chi^2(4) \) |
| 14 | chi-square distribution | \( e \sim \chi^2(20) \) |

Figure 2. Q-Q plots for \( n=10, 1,000 \) under different distributions

5. Results

5.1. Comparison among ML Approaches

The important findings are summarized as follows,
1. Among deep learning methods, ResNet model performs the best.
2. Deep learning methods perform better than traditional image processing approaches.
3. Overall, the best-performing case is using ResNet with truncated SVD, customized Mahalanobis similarity metric, and using median of features as the centroid.

Table 2 reports the average statistical power (i.e., average rejection rate of H1 plots across different sample size) of each method in 14 cases. Due to page limit, we only report the best case out of 30 trials in Table 2. Specifically, for each model, we compared results from three cases with or without dimension reduction, five distance functions, and two methods to calculate the centroid. An interesting and surprising finding is that the best case out of 30 trials for all 4 methods is the same. The best dimension reduction method is truncated SVD, centroid should be calculated by median, and the best metric is constructed by trained Mahalanobis
5.2. Comparing New Method with Statistical Tests

This section summarizes the comparison results between traditional normality tests and the best case based on ResNet model. For traditional statistical tests, we employ those four tests described in Section 2.1. We can demonstrate that the new method indeed significantly outperforms all 4 traditional normality tests in all 14 cases of alternative hypotheses. The statistical powers of the four traditional normality tests and our method under selected alternative distributions are presented in Figure 3. In these figures, the x-axis is the sample size for normality test and y-axis is the power of the normality test. Statistical power means the rejection rate of H1 plots and the larger value implies the performance of the test statistic is better. Due to page limit, we omit the power plots of $t(5)$ and $Beta(3, 2)$. Plot for $t(5)$ is qualitatively similar to the plot for $t(10)$ while the plot for $Beta(3, 2)$ is qualitatively similar to that of $Beta(6, 2)$.

From the figures, we can reach the following conclusions. First, for all non-normal distributions, our method outperforms all four traditional normality tests. Particularly, the difference of performance is larger when the sample size is smaller, except for $t$-distribution. This finding is intuitive in that traditional methods rely heavily on the larger sample size mathematically whereas in computer vision approach, sample size does not alter much of the shape of a Q-Q plot. Second, for $t$ distribution, when the only parameter, degree of freedom, increases to infinity, $t$ distribution converges to standard normal distribution. For human vision, the probability distribution of a $t(10)$ distribution is already indistinguishable from a standard normal distribution. Our new method can still outperform traditional test on a $t(50)$ distribution across all sample sizes, proving that deep learning can capture subtle differences in Q-Q plots to construct a very powerful statistical test for normality.

6. Conclusion and Future Works

In this study, we propose a new learning method to construct test statistics for normality test based on Q-Q plots. Our methods are built upon the literature about deep-learning based image classification and similarity metrics learning. Our experimentation results show that using ResNet on Q-
Figure 3. (a) Power comparisons for $t(2)$ at 5% significance level. (b) Power comparisons for $t(10)$ at 5% significance level. (c) Power comparisons for $t(50)$ at 5% significance level. (d) Power comparisons for $U(0, 1)$ at 5% significance level. (e) Power comparisons for $Beta(2, 2)$ at 5% significance level. (f) Power comparisons for $Laplace(0, 1)$ at 5% significance level. (g) Power comparisons for $Beta(6, 2)$ at 5% significance level. (h) Power comparisons for $Beta(2, 1)$ at 5% significance level. (i) Power comparisons for $Gamma(1, 5)$ at 5% significance level. (j) Power comparisons for $Gamma(4, 5)$ at 5% significance level. (k) Power comparisons for $\chi^2(4)$ at 5% significance level. (l) Power comparisons for $\chi^2(20)$ at 5% significance level.

Q plots with truncated SVD for dimension reduction and metrics learning based on Mahalanobis distance delivers the best performance in our trials.

This study provides preliminary yet convincing evidence that computer vision on statistical plots can be utilized to construct powerful test statistics. Our method could be generalized and be applied to any distributional test based on Q-Q plot since Q-Q plot can be applied to any distribution. At the same time, it is convincing that better formatting of Q-Q plot or other new type of statistical plot could further improve the performance of the AI-based method for normality test. Furthermore, it seems convincing that our method could be generalized to construct test statistics for other famous hypothesis testing in statistics based on other statistical visualizations, such as residual plots for linear regression analysis or auto-correlation and partial auto-correlation plots for time series analysis.
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