Fractal Model for Predicting Elemental Sulfur Saturation in the Presence of Natural Fracture

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ABSTRACT: Sour gas reservoirs are an important part of unconventional gas reservoirs, which are widely distributed in the world. However, elemental sulfur deposition, channel plugging, and productivity reduction consequentially occur in the development of high sour gas fields as pressure drops. The accurate prediction of sulfur deposition is a very important work for sour gas reservoirs. In this paper, a fractal model is presented for predicting elemental sulfur saturation in the presence of natural fracture. The model takes into consideration the effects of non-Darcy flow. In addition, the influence parameters such as fractal dimension, fractal index, and non-Darcy flow are studied. The results showed the following: (1) sulfur deposition was overestimated by Hu’s model, and this paper model is more accurate for prediction of sulfur deposition; (2) elemental sulfur deposition decreases with the increase of the fractal dimension, while elemental sulfur deposition increases with the decrease of the fractal index; and (3) non-Darcy flow should be considered because it causes a faster rate of sulfur deposition. This research will provide a basis and reference for predicting elemental sulfur saturation in the presence of natural fracture for sour gas reservoirs.

1. INTRODUCTION

With the depletion of conventional energy and the increasing demand of humans, unconventional energy exploitation has become a global focus.1−3 Sour gas reservoirs are an important part of unconventional gas reservoirs and are widely distributed all over the world.4,5 However, elemental sulfur is often precipitated when the temperature and pressure drop. Elemental sulfur blocks pores, resulting in a decrease in porosity and permeability, and then the production of sour gas wells will be reduced. Sulfur deposition is of great significance for the development of sour gas reservoirs.6

Many scholars have been focusing on sulfur deposition, and the models of sulfur solubility in sour gas were developed. Kuo and Colsmann7 built the model of elemental sulfur in porous media, and the influencing factors of the model are analyzed. The relationship of deposited sulfur and reservoir permeability was proposed by Kuo,8 and an empirical model for porosity and permeability change because of sulfur deposition was established by Adin.9 Chrastil10 proposed a simple correlation to predict the solubility of elemental sulfur. Roberts11 presented a sulfur deposition damage model under steady flow conditions. Based on Roberts’s model, some researchers12−14 presented a sulfur deposition damage model considering non-Darcy flow. Mahmoud et al.15 presented an analytical model of sulfur deposition, which incorporates rock porosity, relative permeability, and rock wettability. Guo et al.14 presented an analytical model of elemental sulfur solubility in a sour gas mixture. The above models have been widely applied to the prediction of sulfur deposition in high sulfur gas reservoirs. However, few focus on sulfur deposition damage in fractured sour gas reservoirs.

Based on Roberts’s model,11 Qin et al.16 improved the model that considers reservoir compaction for the carbonate gas reservoir. The result of the model is verified by the model of Guo et al.14 and has higher accuracy in the calculation of sulfur deposition in the fractured sour gas reservoir. Hands et al.16 studied the effect of natural fractures on sulfur gas reservoirs in the carbonate gas reservoir. Hu et al.17 proposed sulfur deposition of a carbonate sour gas reservoir in the presence of natural fracture. Through the model, factors such as fracture aperture, fracture height, and production rate are analyzed. Hu et al.17 studied the influence of sulfur deposition on fractured horizontal well performance. These studies assume that the fracture width is equal. In fact, the results of some scholars’ research show that natural fractures have fractal characteristics in oil and gas reservoirs.18,19 Therefore, it is
necessary to study and develop a fractal model for sulfur deposition damage in the presence of natural fracture.

In this paper, a fractal model is presented for predicting sulfur saturation in the presence of natural fracture; the model takes into account non-Darcy flow. This model is compared with Hu’s model; the result shows that this model is more accurate. Then, the influence parameters such as fractal dimension, fractal index, and non-Darcy flow are studied. This model can guide the prediction of sulfur deposition in carbonate reservoirs with a high sulfur content.

2. ELEMENTAL SULFUR SOLUBILITY

It is important to predict elemental sulfur critical temperature and pressure in sour gas reservoir development. Based on the associative law and entropy principle, an empirical solubility equation was presented.  

\[
c_r = \rho^k \exp \left( \frac{a}{T} + b \right)
\]  

(1)

According to eq 1, an empirical equation with Brunner and Woll experimental data is obtained:  

\[
c_r = \rho^k \exp \left( \frac{-4666}{T} - 4.5711 \right)
\]  

(2)

Elemental sulfur may precipitate when pressure drops down to critical pressure, given the effects of deposited sulfur on reservoir permeability, an empirical equation about relative permeability and sulfur saturation is provided:  

\[
\ln k = as_i
\]  

(3)

Sulfur saturation (S_i) is defined as the ratio between the deposited sulfur volume and pore volume.

3. ELEMENTAL SULFUR SATURATION PREDICTION MODEL AND SOLUTION

The elemental sulfur deposition prediction model employs the following assumptions for descriptions of the sour gas reservoir flow:

1. The sour gas reservoir is horizontal and homogeneous and has a uniform thickness;
2. Flow through the gas reservoir is radial steady-state;
3. Non-Darcy flow is taken into account in the model;
4. The elemental sulfur is a solid particle;
5. The natural fractures are evenly distributed in all directions.

The sour gas flows from the matrix rock to the natural fracture, and sour gas flows into the wellbore from the natural fracture.

Because fractal theory has been widely used in reservoir characterization and productivity prediction, the fractal model is used to describe the changes of permeability and porosity.  

The relationships between fractal permeability and porosity flow are given as:  

\[
k(r) = k_w \left( \frac{r}{r_w} \right)^{d-\theta-2}
\]  

(4)

\[
\phi(r) = \phi_w \left( \frac{r}{r_w} \right)^{d-2}
\]  

(5)

where r_w is the radius of the wellbore, \( k_w \) is permeability, mD; \( \phi_w \) is porosity; d is the mass fractal dimension; \( \theta \) is the conductivity index; d denotes the fractal dimension representing the dimension of the fractal fracture network embedded in the Euclidean matrix; and \( \theta \) is the connectivity index characterizing the diffusion process.

Based on the turbulent flow equation, the non-Darcy flow equation is obtained as follows:

\[
d_p = \frac{\mu}{k_w} + \beta \mu c^2
\]  

(6)

where \( \beta \) is the turbulence coefficient, m^-1, it can be gained as follows:

\[
\beta = \frac{7.89 \times 10^{10}}{k_e^{1.60}(1 - S_w)^{0.404}}
\]  

(7)

According to assumptions, gas flow is mainly dependent on natural fractures, and the gas production rate can be expressed as the following:

\[
q = q_{sc} B_g = 2\pi r h v
\]  

(8)

where

\[
B_g = \frac{B_{sc} ZT}{Z_{sc} T_{sc} p} = 3.458 \times 10^{-4} \frac{ZT}{p}
\]  

(9)

The production rate is constant, and flow velocity (radius is r and time is dt) is

\[
v = 3.458 \times 10^{-4} \frac{ZT}{p} \frac{q_{sc}}{2\pi h r}
\]  

(10)

Combined with eqs 6 and 10:

\[
\frac{d_p}{d_t} = 1.843 \times 10^{-3} \frac{\mu q}{k_k s_i h r} + 3.397 \times 10^{-18} \frac{\beta \rho c^2}{r h^2}
\]  

(11)

Elemental sulfur saturation in sour gas is changed, which is the reason for pressure dropping in interval (\( d_i \)) at r (sulfur deposition radius). The precipitated sulfur volume can be expressed as:  

\[
d_v = \frac{q(d_i/d_p) d_i}{10^5 \rho_i} = 4.831 \times 10^{-7} \frac{q_{sc}}{d_p} d_i d_t
\]  

(12)

The sour gas reservoir pore volume can be expressed as:

\[
d_v = 2\pi h f(1 - S_w) d_i
\]  

(13)

Thus, elemental sulfur saturation can be calculated as:

\[
d_s = \frac{d_v}{d_i} = 7.693 \times \frac{q(d_i/d_p)}{r h F(1 - S_w) d_t}
\]  

(14)

Combined with eqs 4, 5, 7, 11, and 14:
\[
\frac{dS_s}{dt} = 1.418 \times 10^{-4} \frac{\mu q^2 (d_v/d_p)}{r^2 h^2 \phi g \left( \frac{r}{r_w} \right)^{d-2} (1 - S_{sw}) k k_s \left( \frac{r}{r_w} \right)^{d-\theta-2}} + 2.062 \times 10^{-14} \]

\[
\rho q^3 (d_v/d_p) \frac{r^2 h^2 \phi g \left( \frac{r}{r_w} \right)^{d-2} (1 - S_{sw}) k k_s \left( \frac{r}{r_w} \right)^{d-\theta-2}}{r^2 h^2 \phi g \left( \frac{r}{r_w} \right)^{d-2} (1 - S_{sw}) k k_s \left( \frac{r}{r_w} \right)^{d-\theta-2}} = 1.404
\]

For convenient calculation, we can define \( A \) and \( B \):

\[
A = 1.418 \times 10^{-4} \frac{\mu q^2 (d_v/d_p)}{r^2 h^2 \phi g \left( \frac{r}{r_w} \right)^{d-2} (1 - S_{sw}) k k_s \left( \frac{r}{r_w} \right)^{d-\theta-2}}
\]

\[
B = 2.062 \times 10^{-14} \frac{\rho q^3 (d_v/d_p)}{r^2 h^2 \phi g \left( \frac{r}{r_w} \right)^{d-2} (1 - S_{sw}) k k_s \left( \frac{r}{r_w} \right)^{d-\theta-2}}
\]

\[
\rho = \frac{M \gamma g P}{ZRT} = 3.48 \times 10^{-3} \frac{\gamma g P}{ZT}
\]

Equation 2 can be deformed as:

\[
\frac{dS_s}{dp} = 4 \left( 3.48 \times 10^{-3} \frac{\gamma g P}{ZT} \right) \exp \left( -4666 \frac{T}{T} - 4.5711 \right)
\]

Combined with eqs 15–17:

\[
\frac{dS_s}{dt} = \frac{A}{k_{lg}} + \frac{B}{k^{1.6}_{lg}}
\]

Combined with eqs 3 and 20:

\[
\frac{\sigma^{1.605}}{Ac^{1.605} + B} dS_s = dt
\]

Integrating eq 22, we can obtain:

\[
t = \int_{0}^{S_s} \frac{\sigma^{1.605}}{Ac^{1.605} + B} dS_s
\]

If non-Darcy flow is not considered, \( B = 0 \) in eq 23, then

\[
t = (e^{\alpha S_s} - 1) / \alpha / A
\]

4. RESULTS AND DISCUSSION

This paper takes a well of a sour gas reservoir in Sichuan basin as an example. The well parameters can be seen in Table 1. Comparison of results between this paper model and Hu’s model is shown in Figure 1. According to this paper model, with a production rate of \( 8.0 \times 10^4 \) m\(^3\)/day, at \( r = 0.8 \) m, sulfur saturation is 0.5 at approximately 998 days while Hu’s model shows the same saturation at approximately 941 days. This is because the fractal characteristics of fractures increase the permeability of fractures, so the sulfur deposition rate calculated in this paper is slower than that in Hu’s model. Because the fractures in fractured gas reservoirs have fractal characteristics, the model established in this paper is more accurate for predicting sulfur deposition. The comparison with the field data also proves the accuracy of this paper model.

Figure 2 shows the relationship between fractal dimension and sulfur saturation when the production rate is \( 8.0 \times 10^4 \) m\(^3\)/day and the radius of sulfur deposition is 0.8 m. We can see that the sulfur saturation of 0.5 is observed at 710 days when the fractal dimension is 0.8, while the same saturation is observed at 1396 days when the fractal dimension is 1.2. It shows that as the fractal dimension increases, the elemental sulfur deposition decreases. This is because the permeability of fractures increases with the increase of fractal dimension, resulting in smaller elemental sulfur saturation.

Table 1. Sour Gas Well Parameters of Sichuan Basin

| basic parameter                  | value  |
|----------------------------------|--------|
| reservoir initial pressure, MPa  | 36     |
| reservoir temperature, K         | 363    |
| gas average viscosity, Pa·s      | 2.52 × 10^{-3} |
| reservoir porosity, %            | 9.8    |
| roughness coefficient, m         | 0.2 × 10^{-6} |
| gas relative density, dimensionless | 0.72  |
| universal gas constant, MPa·m^{-3}/kmol·K | 0.008471 |
| sulfur density, g/cm\(^3\)       | 2.07   |
| sulfur deposition radius, m       | 0.8    |
| gas reservoir radius, m           | 1500   |
| compressibility factor, dimensionless | 0.92  |
| air molecular weight, dimensionless | 28.97 |
| fractal dimension, dimensionless | 0.8    |
| fractal index, dimensionless      | 0      |

Figure 1. Result comparison between the model and field data in a sour gas well of Sichuan basin.
permeability of fractures decreases with the increase of the fractal index, resulting in greater elemental sulfur saturation. Figure 4 shows the comparison of the results obtained from Darcy and non-Darcy flow in predicting sulfur saturation. If non-Darcy flow exists in the development of actual gas reservoirs, the prediction of sulfur deposition in non-Darcy flow is more accurate. As shown in Figure 4, compared with Darcy flow, damage caused by sulfur deposition is more serious. This is because when the development speed of the high sulfur gas reservoir is too fast, the fluid flow in the gas reservoir is non-Darcy flow. However, if the development speed is too low, the economic development cannot be guaranteed. Hence, it is necessary to select the development speed reasonably, so as to minimize the formation damage caused by sulfur deposition and ensure certain economic benefits.

5. CONCLUSIONS
In this paper, we use porosity and permeability functions that reflect the fractal nature of sulfur gas reservoirs. Based on fractal theory, an elemental sulfur saturation model is presented for natural fractures in sour gas reservoirs. The influence parameters such as fractal dimension, fractal index, and non-Darcy flow are studied. The results showed that the following: (1) sulfur deposition was overestimated by Hu’s Model, and this paper model is more accurate for the prediction of sulfur deposition; (2) elemental sulfur deposition decreases with the increase of the fractal dimension, while elemental sulfur deposition increases with the decrease of the fractal index; and (3) non-Darcy flow should be considered because it causes a faster rate of sulfur deposition.

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Notes
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