Study on nonlinear bifurcation characteristics of multistage planetary gear transmission for wind power increasing gearbox

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Abstract: In order to study the nonlinear dynamic characteristics of multistage gear transmission system of wind power increasing gearbox, a three degree of freedom dynamic model of high speed gear pair of wind power increasing box is established. The nonlinear bifurcation characteristics of the gear pair in high-speed level of the wind turbine gearbox are analyzed by numerical analysis method. The research shows that the high speed gear pair of the multistage planetary transmission system of the wind power increasing box shows significant dynamic nonlinearity in the process of dynamic meshing. By controlling the value of internal and external excitation parameters, the influence of the nonlinear bifurcation response of the system is discussed, which can provide a useful reference for the parameter selection and nonlinear dynamic behavior analysis of the gear system.

1. Introduction
A great amount of considerable growth and interest has been received by renewable energy sources over the last years. Wind energy is regarded one of the most magnetic solutions to electric power production's problem, while the economic benefits that is produced by wind turbines could be further enhanced by diminishing construction costs. Gear transmission systems were studied by linear vibration theories, irrespective of nonlinear factors like backlash, time-varying stiffness of teeth. With the increasing requirements of excellent speed, hard transmission, lightweight in modern geared systems. It demands more precise assessment of the dynamic tooth burden and vibration reaction that is related to geared systems' nonlinear aspects. It was realized that geared systems' vibration must be taken into account in a nonlinear framework.

A lot of research efforts have been given in the literature that concerns gear transmission's nonlinear characteristics [1-4]. Yong [5] proved a gear-bearing that couples model of gear trains that are based on a nonlinear dynamics model with two degrees of freedom coupling planetary gear trains' dynamics model. Yoon [6] researched vibro-impacts of a transmission in vehicle systems arise usually at unloaded gear pairs' location, and dissolved the problems by designing multi-staged clutch dampers efficiently. Zhou [7] presented an 8-DOF lumped parameter dynamic model in view of the coupled lateral-torsional vibration and researched the coupled nonlinear characteristics of the spur gear. Li [8] established gear transmission system's 6-DOF dynamic model and studied relative displacement's bifurcation characteristic caused by damping and rotating. Zhu [9] widened the harmonic balance approach to research the nonlinear response of a planetary gear sets. The gear sets' nonlinear properties are researched, and nonlinearities' influence on the frequency reaction characteristic has been researched.

In this paper, the multistage planetary gear transmission system of wind power increasing box is taken as the research object. The nonlinear dynamic model of high speed meshing gear pair is established.
Numerical analysis method is used to solve the dynamic equations of the system. The influence of internal and external excitation parameters on the nonlinear dynamic characteristics of the system is studied, which provides a useful reference for the parameter selection and nonlinear dynamic behavior analysis of the wind power gearbox.

2. Multistage planetary gear transmission mechanism of high power wind power increasing box

The transmission system of large power wind turbine gearbox is usually multistage planetary gear drive. Figure 1 is a two stage planetary gear train with a high speed parallel shaft transmission mechanism. The main components of the transmission mechanism for low level internal gear 1, low-speed planetary gear 2, low-speed planetary gear 3, low level sun gear 4, medium level internal gear 5, medium speed planetary plane 6, medium level sun gear 7, medium speed planetary gear 8, high-speed pinion shaft 9, high speed large gear 10.

Figure 1 multistage planetary gear transmission diagram of high power wind power increasing box

3. Nonlinear dynamic model of multi-stage high speed gear of high power wind power increasing box

Multistage transmission of high-power wind power increasing box is a typical speed increasing drive. Compared with the input stage, the nonlinearity of the high speed gear transmission is significantly enhanced, and the analysis of its nonlinear characteristics can provide a useful reference for the dynamic characteristics of the multi-stage transmission of the high-power wind power increasing box.

The lumped mass method is used to establish the nonlinear dynamic model of the high speed gear in the transmission system of the wind power increasing box, as shown in Figure 2.

Figure 2 nonlinear dynamic model of high speed gear of transmission system of wind power increasing box

The model considers the support clearance of the bearing, the backlash and transmission error of the gear pair, and assumes that the meshing gears are straight cylindrical gears. The main parameter $m_i, I_i, R_i, T_i, \theta_i, \bar{x}_i, F_i (i = d, p)$ of the model respectively represents the translational mass, the moment of inertia, the radius of the base circle, the load torque, the torsional vibration displacement, the lateral
vibration displacement and the radial preload. The parameter \( c_i, k_i, f_i(x_i) (i = d, p, r) \) respectively represents the support damping coefficient and meshing damping coefficient, support stiffness and meshing stiffness, bearing clearance function and backlash function are respectively the main and passive gears.

The differential equation of nonlinear vibration of high speed gear of transmission system of wind power increasing box:

\[
\begin{bmatrix}
    m_d & 0 & 0 \\
    0 & m_p & 0 \\
    -m_e & m_e & m_e
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_d(T) \\
    \ddot{x}_p(T) \\
    \ddot{x}_e(T)
\end{bmatrix}
+
\begin{bmatrix}
    c_d & 0 & 0 \\
    0 & c_p & -c_r \\
    0 & 0 & c_r
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_d(T) \\
    \dot{x}_p(T) \\
    \dot{x}_e(T)
\end{bmatrix}
+
\begin{bmatrix}
    k_d & 0 & k_r \\
    0 & k_p & -k_r \\
    0 & 0 & k_r
\end{bmatrix}
\begin{bmatrix}
    x_d(T) \\
    x_p(T) \\
    x_e(T)
\end{bmatrix}
=
\begin{bmatrix}
    -F_d \\
    F_p \\
    \bar{F}_{R_0} - m_e \ddot{\bar{e}}(T) + \bar{F}_e(T)
\end{bmatrix}
\tag{1}
\]

Where \( m_d \) - translational mass of driving gear, \( m_p \) - translation quality of moving gear, \( m_e \) - equivalent mass, \( I_d \) - the inertia of the driving gear, \( I_p \) - moment of inertia of driven gear, \( R_d \) - radius of base circle of driving gear, \( R_p \) - radius of base circle of driven gear, \( c_d \) - support damping coefficient of driving gear, \( k_d \) - support stiffness coefficient of driving gear, \( k_p \) - coefficient of support stiffness of driven gear, \( c_r \) - meshing damping coefficient, \( k_r \) - meshing stiffness coefficient, \( f_d(x_d) \) - backlash nonlinear function of active gear, \( f_p(x_p) \) - nonlinear function of clearance of driven gear, \( f_r(x_e) \) - nonlinear function of meshing clearance, \( \bar{F}_d \) - radial preload of driving gear, \( \bar{F}_p \) - radial preload of passive gears, \( \bar{e}(T) \) - static transmission error of meshing gear pair, \( \bar{F}_{R_0} \) - the meshing force caused by the average component of the exciting torque, \( \bar{F}_R \) - tooth meshing force caused by variable component of exciting torque.

In order to facilitate the study, the meshing force of the gear teeth caused by the average component of the static transmission error \( \bar{e}(T) \) and the excitation torque of the meshing gear pair \( \bar{F}_{R_0}(T) \) is assumed to be the harmonic function of the single frequency, and the formula (1) is treated by dimensionless method, and solved by numerical method.

4. Influence of internal excitation parameters on nonlinear bifurcation characteristics of the multi-stage high speed gear

The change of the excitation parameters directly affect and restrict the dynamic characteristics of the system. In order to analyze the actual conditions and facilitate the discussion, the internal and external incentives of the system are taken into account. Considering the influence of the internal excitation frequency on the dynamic characteristics of the system, it is assumed that the external excitation parameters and internal excitation parameters of the system are fixed values. Under different internal excitation frequencies, the dynamic characteristics of the system are obtained by numerical simulation, and the system exhibits abundant vibration characteristics.

In order to study the influence of the system parameters on the motion state of the system, the parameter can be changed under the condition of keeping the other parameters unchanged, and the response characteristics of the system with the change of the parameters are investigated. The range of the amplitude parameter is \([0.0, 7.0]\), and the initial condition of the differential equation is zero. Calculate the values of parameters one by one. When each differential equation is solved, the equation
is first entered into a stable state, then a small time interval is calculated, and the maximum value of the interval is obtained. The maximum value is calculated by multiple iterations, and the corresponding maximum value is displayed to obtain the bifurcation diagram of the system, as shown in figure 7.

![Bifurcation diagram with $F_i$](image1)

![Bifurcation diagram of system with $\psi_i$](image2)

Figure 3 Bifurcation diagram with $F_i$

Figure 4 Bifurcation diagram of system with $\psi_i$

It can be seen from Figure 3 that when the internal excitation amplitude $F_i$ is small, the response of the system is two times periodic response. When the internal excitation amplitude $F_i$ increases, the response of the system is chaotic response. The internal excitation amplitude continues to increase, and the system exhibits two times periodic response. When the internal excitation amplitude increases to about 1.2, the response of the system begins to become periodic response. As the amplitude of the internal excitation continues to increase, the system response has experienced the period doubling response, the chaotic response and the periodic response.

The internal excitation amplitude is kept constant, and the internal excitation phase is changed independently. The bifurcation diagram of the system with the internal excitation phase $\psi_i$ is shown in Figure 4.

In the Figure 4, the bifurcation diagram of the system is given when the internal excitation $\psi_i$ phase varies from 0 to 6.8. It can be seen from the diagram that when the internal excitation phase is small (less than 1.2), the response of the system is chaotic. There is a period doubling bifurcation window in this interval. When the internal excitation phase varies from 1.2 to 1.9, the response is two times of periodic motion. With the increase of the internal excitation phase, the response of the system in the interval [1.9,2.5] becomes quasi periodic motion. As the internal excitation phase continues to increase, the system moves from the periodic motion to the chaotic motion.

5. Influence of external excitation parameters on nonlinear bifurcation characteristics of the system

The obtained parameters are the same as those of the previous ones, while the internal excitation parameters are kept constant. The bifurcation diagram of the system is shown in Figures 5, 6 and 7, respectively, by changing the frequency $\omega_o$, the amplitude $F_o$ and the phase $\psi_o$ of the external excitation individually.
Figure 5 Bifurcation diagram of system with $\omega_o$

In Figure 5, the bifurcation diagram of the system response is given when the frequency of the external excitation varies from 0 to 6. It can be seen from the diagram that when the frequency of the external excitation is small, the response is doubly periodic motion. With the increase of the frequency of the external excitation, the response of the system becomes chaotic. When the frequency of external excitation is between 1.8 and 2.6, the response of the system is periodic motion. When the frequency continues to increase, the system undergoes double periodic motion and quasi periodic motion.

Figure 6 gives a bifurcation diagram when the phase of the external excitation varies from 0 to 6.5. When the phase value of the external excitation is small, the dynamic response of the system has four bifurcations, and the response of the system jumps to quasi periodic response when the value is near 0.5. As the phase of the external excitation increases, the response of the system changes from quasi periodic response to periodic response. When the phase of external excitation is increased to about 2, the response bifurcation of the system is two times periodic response. When the phase of the external excitation increases to about 2.5, the response of the system continues to bifurcate into four times periodic response. When the phase of the external excitation is in the range $[4.1,5.0]$, the system response becomes obvious chaotic motion. When the phase of external excitation is near 5.3, the response of the system continues to bifurcate into eight times of periodic response, and the response of the system continues to bifurcate as the increase continues.

Figure 7 Bifurcation diagram with the amplitude $F_o$

Figure 7 gives a bifurcation diagram when the amplitude of the external excitation varies from 0 to 6.5. It can be seen from Figure 11 that the amplitude of the external excitation significantly affects the response of the system. When the amplitude of the external excitation is small, the dynamic response of the system has five bifurcations, and when the value is near 0.5, the response of the system has six bifurcations, one of which is quasi periodic response. As the amplitude of the external excitation continues to increase, the response of the system is approximately three times of the periodic motion around 0.8. When the amplitude of external excitation is about 1.2, the response bifurcation of the system is five times periodic response. When it increases to about 2.4, the system bifurcation is seven times periodic response. As the amplitude of the external excitation continues to increase, the system response becomes chaotic in the interval $[3.9,5.2]$. When the amplitude of the external excitation is about 5.6, the
response bifurcation of the system is nine times of periodic noise, and the response of the system continues to bifurcate as the amplitude of the external excitation continues to increase.

6. Conclusion
The high speed gear pair of the multistage planetary transmission mechanism of the wind power increasing box shows significant dynamic non-linearity in the transmission meshing process. Internal excitation and external excitation have an important influence on the nonlinear dynamic response, and the parameters of the internal and external excitation will directly affect the nonlinear characteristics of the system. The internal excitation frequency has a great influence on the response of the system, which makes the system exhibit different vibration states. By controlling the value of internal and external excitation parameters, the influence of the nonlinear bifurcation response on the nonlinear bifurcation response of the system is discussed, which can provide a useful reference for the parameter selection and nonlinear dynamic behavior of the gear system.

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