First-order vortices in a gauged $CP(2)$ model with a Chern-Simons term

V. Almeida$^1$, R. Casana$^1$ and E. da Hora$^2$.

$^1$Departamento de Física, Universidade Federal do Maranhão, 65080-805, São Luís, Maranhão, Brazil.

$^2$Coordenadoria Interdisciplinar de Ciência e Tecnologia, Universidade Federal do Maranhão, 65080-805, São Luís, Maranhão, Brazil.

We consider a gauged $CP(2)$ theory in the presence of the Chern-Simons action, focusing our attention on those time-independent solutions possessing radial symmetry. In this context, we develop a coherent first-order framework via the Bogomol’nyi prescription, from which we obtain the corresponding energy lower-bound and the first-order equations the model supports. We use these expressions to introduce effective BPS scenarios, solving the resulting first-order equations by means of the finite-difference scheme, this way attaining genuine field solutions engendering topological configurations. We depict the new profiles, commenting on the main properties they engender.

PACS numbers: 11.10.Kk, 11.10.Lm, 11.27.+d

I. INTRODUCTION

In the context of classical theories, solitons are described as those time-independent solutions arising within highly nonlinear models [1]. In this sense, vortices are radially symmetric solutions coming from planar scenarios in the presence of a gauge field.

Moreover, under very special circumstances, solitons can also be obtained via a set of first-order differential equations (instead of the second-order Euler-Lagrange ones), the resulting solutions minimizing the energy of the effective system [2].

In this sense, first-order vortices were firstly studied in the context of the simplest Maxwell-Higgs electrodynamics [3]. Furthermore, these solutions were verified to occur within the Chern-Simons-Higgs scenario too [4]. Also, first-order vortices were recently considered in connection with nonstandard models [5], the resulting solutions being used as an attempt to explain some cosmological issues [6].

In such a context, it is especially interesting to consider the existence of well-behaved time-independent vortices arising from a $CP(N − 1)$ scenario in the presence of a gauge field, mainly due to the close phenomenological relation between such theory and the four-dimensional Yang-Mills-Higgs one [7].

In a recent investigation, radially symmetric solutions arising from a planar $CP(2)$ theory endowed by the Maxwell term were considered, the author clarifying the way these structures and correlated results depend on the parameters of the model [8]. In that work, however, the vortex configurations were obtained by solving the second-order Euler-Lagrange equations directly (the resulting solutions therefore not saturating the Bogomol’nyi bound).

In the sequel, some of us introduced the first-order vortices inherent to the aforementioned Maxwell $CP(2)$ theory, defining the energy lower-bound and the corresponding first-order equations [9]. In that work, the self-dual profiles were constructed numerically by means of the finite-difference scheme, the resulting structures presenting the typical topological shape.

Moreover, some of us have also studied first-order vortices within a Maxwell $CP(2)$ model in the presence of a nontrivial dielectric function. The point to be raised here is that such function can be used to change the vacuum manifold of the effective theory, from which we have used such freedom to generate self-dual vortices engendering a nontopological profile, the resulting Bogomol’nyi bound being not quantized anymore [10].

We now go a little bit further by investigating a rather natural extension of the aforementioned works, i.e. the search for the first-order planar solitons arising from a $CP(2)$ theory in the presence of the Chern-Simons action.

In order to introduce our results, the present manuscript is organized as follows: in the next Section II, we define the gauged $CP(4)N − 1$ theory and some conveniences inherent to it, focusing our attention on those time-independent solitons possessing radial symmetry. We then develop a coherent first-order framework by minimizing the effective energy according the Bogomol’nyi prescription, this way obtaining general first-order equations and the corresponding energy lower-bound, such construction being only possible due to a differential constraint involving the potential engendering self-duality. In the Section III, we solve the first-order expressions in order to find genuine BPS solutions saturating the Bogomol’nyi bound. We solve the corresponding first-order equations by means of the finite-difference algorithm, from which we depict the numerical solutions, whilst commenting the main properties they engender. We end our work in the Section IV, presenting our final considerations and the perspectives regarding future studies.

In this manuscript, we adopt $\eta^{\mu\nu} = (+−−)$ as the metric signature for the flat spacetime, together with the natural units system, for the sake of simplicity.
II. THE MODEL

We begin our investigation by presenting the Lagrange density defining the gauged \( CP(N-1) \) model in the presence of the Chern-Simons term (with \( \epsilon^{12} = +1 \)), i.e.

\[
\mathcal{L} = \frac{k}{4} \epsilon^\mu\nu A_\alpha F_{\mu\nu} + (P_{ab} D_\mu \phi_b)^* P_{ac} D_\mu \phi_c - V(|\phi|).
\]

Here, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) stands for the electromagnetic field strength tensor, \( D_\mu \phi = \partial_\mu \phi - ig A_\mu Q_{ab} \phi_b \) representing the covariant derivative (\( Q_{ab} \) is a real diagonal charge matrix). Furthermore, \( P_{ab} = \delta_{ab} - h^{-1} \phi_a \phi_b^* \) is a projection operator defined conveniently. In this work, the Greek indexes run over the space-time coordinates, the Latin ones counting the complex fields underlying the \( CP(N-1) \) sector (with \( \phi^*_a \phi_a = h \)).

The Euler-Lagrange equation for the gauge field is given by

\[
\frac{k}{2} \lambda^\mu\nu F_{\mu\nu} = J^\lambda,
\]

where

\[
J^\lambda = ig \left[(P_{ab} Q_{bc} \phi_f)^* P_{ac} D^\lambda \phi_c - (P_{ab} D^\lambda \phi_b)^* P_{ac} Q_{cb} \phi_f \right] \tag{3}
\]

represents the current vector. It is then instructive to write down the Gauss law for time-independent configurations, which reads (here, \( B = F_{21} \) is the magnetic field)

\[
kB = \rho, \tag{4}
\]

with

\[
\frac{\rho}{ig} = (P_{ab} D^\theta \phi_b)^* P_{ac} Q_{cd} \phi_d - P_{ab} D^\theta \phi_b (P_{ac} Q_{cd} \phi_d)^* \tag{5}
\]

and \( D^\theta \phi = -ig Q_{bc} \phi_c \). In this sense, given that \( A^0 = 0 \) does not solve the Gauss law identically, the temporal gauge does not hold anymore, the final structures possessing both electric and magnetic fields.

In this work, we look for radially symmetric solutions inherent to the gauged \( CP(2) \) scenario by using the usual map

\[
A_i = -\frac{1}{g} e^{ij} n_j A(r),
\]

\[
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{pmatrix} = h \begin{pmatrix}
e^{im_1 \theta} \sin(\alpha(r)) \cos(\beta(r)) \\
e^{im_2 \theta} \sin(\alpha(r)) \sin(\beta(r)) \\
e^{im_3 \theta} \cos(\alpha(r))
\end{pmatrix}, \tag{7}
\]

with \( m_1, m_2 \) and \( m_3 \in \mathbb{Z} \) standing for winding numbers, \( e^{ij} \) being the bimdimensional Levi-Civita tensor (with \( e^{12} = +1 \)), \( n^j = (\cos \theta, \sin \theta) \) representing the unit vector. Therefore, regular solutions presenting no divergences are obtained via those profile functions \( \alpha(r) \) and \( A(r) \) satisfying

\[
\alpha(r \to 0) \to 0 \quad \text{and} \quad A(r \to 0) \to 0. \tag{8}
\]

It is already known that, in order to support configurations with nontrivial topology, we must fix \( m_1 = -m_2 = m, m_3 = 0 \) and \( Q = \lambda_3/2 \), with \( \lambda_3 \) = diag\((1, -1, 0)\) (the choice \( m_1 = m_2 = m, m_3 = 0 \) and \( Q = \lambda_3/2 \), with \( \sqrt{3} \lambda_3 \) = diag\((1, 1, -2)\), mimicking the first one), the profile function \( \beta(r) \) then holding for two constant solutions, i.e.

\[
\beta(r) = \beta_1 = \frac{\pi}{4} + \frac{\pi}{2} k \quad \text{or} \quad \beta(r) = \beta_2 = \frac{\pi}{2} k, \tag{9}
\]

with \( k \in \mathbb{Z} \); for additional details, the reader is referred to the Eq. (9) of the Ref. [9] and the discussion therein.

It is important to highlight that, from this point on, our expressions describe the effective scenario defined by the conveniences introduced in the previous paragraph.

We look for genuine first-order solutions saturating an energy lower-bound. In this sense, we proceed the minimization of the overall energy, the starting-point being the energy-momentum tensor related to the effective scenario, i.e.

\[
T_{\lambda \rho} = 2 (P_{ab} D_\lambda \phi_b)^* P_{ac} D_\rho \phi_c - \eta_{\lambda \rho} \mathcal{L}_{\text{ntop}}, \tag{10}
\]

where

\[
\mathcal{L}_{\text{ntop}} = (P_{ab} D_\mu \phi_b)^* P_{ac} D_\rho \phi_c - V(|\phi|) \tag{11}
\]

stands for the nontopological Lagrange density, the energy density reading

\[
\varepsilon = \frac{k^2 B^2}{g^2 h W} + V + h \left[ \frac{\alpha}{\sqrt{W}} \frac{d\alpha}{dr} + \frac{W}{r} \left( \frac{A}{2} - m \right)^2 \right], \tag{12}
\]

where we have introduced the Gauss law (4). Here, we have defined the auxiliary function

\[
W = W(\alpha, \beta) = \sin^2 \alpha (1 - \sin^2 \alpha \cos^2 (2\beta)). \tag{13}
\]

The point to be raised is that, whether the potential is constrained to satisfy

\[
\frac{2k}{g^2 \sqrt{h}} \frac{d}{dr} \sqrt{W} = -h \sqrt{W} \frac{d\alpha}{dr}, \tag{14}
\]

the expression for the energy density can be rewritten according the Bogomol’nyi prescription, therefore giving rise to

\[
\varepsilon = \left( \frac{kB}{g \sqrt{hW}} \mp \sqrt{V} \right)^2 + h \left( \frac{\alpha}{\sqrt{W}} \frac{d\alpha}{dr} + \frac{W}{r} \left( \frac{A}{2} - m \right) \right)^2
\]

\[
\mp \frac{2k}{g^2 \sqrt{h}} \frac{1}{r} \frac{d}{dr} \left( A - 2m \right) \sqrt{\frac{V}{W}}, \tag{15}
\]

where we have used \( B(r) = -A'/gr \) for the magnetic field (prime denoting derivative with respect to \( r \)), the resulting first-order equations standing for

\[
\frac{d\alpha}{dr} = \pm \frac{\sqrt{W}}{r} \left( \frac{A}{2} - m \right), \tag{16}
\]
the solution for $\beta(r)$ being necessarily one of those stated in (9).

The scenario can be summarized as follows: given the potential fulfilling the constraint (14), the model (1) effectively supports radially symmetric solutions satisfying the first-order equations (16) and (17), the final configurations saturating an energy lower-bound given by

$$E_{bps} = 2\pi \int r \varepsilon_{bps} dr = \pm \frac{8\pi mk}{g^2\sqrt{h}} \sqrt{\frac{V_0}{W_0}}, \quad (18)$$

where

$$\varepsilon_{bps} = \mp \frac{2k}{g^2\sqrt{h}} \frac{1}{r} \frac{d}{dr} \left[ (A - 2m) \sqrt{\frac{V}{W}} \right] \quad (19)$$

stands for the energy density of the first-order structures, the upper (lower) sign holding for negative (positive) values of $m$. Here, we have supposed that $(A_\infty - 2m) \sqrt{V_\infty/W_\infty}$ vanishes, with $\sqrt{V_0/W_0}$ being finite. Moreover, we have defined $V_0 \equiv V(r \to 0)$, $W_0 \equiv W(r \to 0)$, $V_\infty \equiv V(r \to \infty)$, $W_\infty \equiv W(r \to \infty)$ and $A_\infty \equiv A(r \to \infty)$.

It is also instructive to calculate the magnetic flux $\Phi_B$ the first-order solutions support. It reads

$$\Phi_B = 2\pi \int r B(r) dr = -\frac{2\pi}{g} A_\infty, \quad (20)$$

where we have used $B(r) = -A'/gr$ again. We demonstrate below that the energy lower-bound (18) can be verified to be proportional to the magnetic flux (20), both quantities being quantized according the winding number $m$, as expected for topological solitons.

### III. First-Order Scenarios and Their Numerical Solutions

We now demonstrate how the first-order framework we have developed generates genuine radially symmetric solitons. Here, in order to present our results, we proceed as follows: firstly, we choose a particular solution for $\beta(r)$ coming from (9), whilst solving the constraint (14) for the potential engendering self-duality. We then use such conveniences to obtain the asymptotic boundary conditions $\alpha(r)$ and $A(r)$ must obey in order to fulfill the finite-energy requirement, i.e. $\varepsilon(r \to \infty) \to 0$, from which we also calculate the energy lower-bound (18) and the magnetic flux (20) explicitly, showing that they are proportional to each other, as expected. Finally, we solve the corresponding first-order equations numerically by means of the finite-difference scheme, whilst commenting on the main properties they engender.

#### A. The $\beta(r) = \beta_1$ Case

We go further into our investigation by choosing

$$\beta(r) = \beta_1 = \frac{\pi}{4} + \frac{\pi}{2} \epsilon, \quad (21)$$

from which one gets $\cos^2(2\beta_1) = 0$, the fundamental constraint being reduced to

$$\frac{2k}{g^2\sqrt{h}} \frac{d}{dr} \left[ \frac{\sqrt{V}}{\sin \alpha} \right] = h \frac{d}{dr} (\cos \alpha), \quad (22)$$

whose solution is

$$V(\alpha) = \frac{g^4}{16k^2} \frac{h^3}{3} \sin^2(2\alpha), \quad (23)$$

i.e. the potential supporting self-duality (here, we have used $C = 0$ for the integration constant).

We now implement (21) and (23) into (12), the resulting expression being

$$\varepsilon(r) = \frac{k^2B^2}{g^2h\sin^2 \alpha} + \frac{g^4}{16k^2} \frac{h^3}{3} \sin^2(2\alpha)$$

$$+ \frac{h}{2} \left[ \left( \frac{d\alpha}{dr} \right)^2 + \sin^2 \alpha \left( \frac{A}{2} - m \right) \right] \sin^2(2\alpha), \quad (24)$$

from which we attain $\varepsilon(r \to \infty) \to 0$ by imposing

$$\alpha(r \to \infty) \to \frac{\pi}{2} \text{ and } A(r \to \infty) \to 2m, \quad (25)$$

standing for the boundary conditions the profile functions obey in the asymptotic limit.

In view of (21), (23) and (25), the energy lower-bound (18) can be verified to be equal to

$$E_{bps} = \pm 4\pi hm, \quad (26)$$

the magnetic flux $\Phi_B$ (20) standing for

$$\Phi_B = -\frac{4\pi}{g} m, \quad (27)$$

from which we get that $E_{bps} = \pm gh \Phi_B$, both $E_{bps}$ and $\Phi_B$ being proportional to each other and quantized according the winding number $m$, as expected. Here, we have used

$$\frac{\sqrt{V_0}}{\sin \alpha_0 \sqrt{1 - \sin^2 \alpha_0 \cos^2(2\beta_1)}} = \frac{g^2\sqrt{h}}{2k}, \quad (28)$$

this way also verifying our previous assumption, see the discussion just after (19).

The first-order equations (16) and (17) can be rewritten as

$$\frac{d\alpha}{dr} = \pm \frac{\sin \alpha}{r} \left( \frac{A}{2} - m \right), \quad (29)$$

$$\frac{1}{r} \frac{dA}{dr} = \mp \frac{g^4}{4k^2} h^2 \sin(2\alpha) \sin \alpha, \quad (30)$$

which must be solved according the boundary conditions (8) and (25).
FIG. 1: Numerical solutions to $\alpha(r)$ coming from (29) and (30) in the presence of (8) and (25). Here, we have fixed $h = k = 1$ and $g = \sqrt{2}$, varying the winding number: $m = 1$ (solid black line), $m = 2$ (dashed blue line) and $m = 3$ (dash-dotted red line).

**B. The $\beta(r) = \beta_2$ case**

We now consider

$$\beta(r) = \beta_2 = \frac{\pi}{2},$$

(31)

via which one gets $\cos^2(2\beta_2) = 1$, the corresponding constraint being

$$\frac{4k}{g^2h} \frac{d}{dr} \left[ \frac{\sqrt{V}}{\sin(2\alpha)} \right] = \frac{h}{4} \frac{d}{dr} \left( \cos(2\alpha) \right),$$

(32)

is solution standing for the self-dual potential, i.e.

$$V(\alpha) = \frac{g^4}{16k^2} \left( \frac{h}{4} \right)^3 \sin^2(4\alpha),$$

(33)

where we have chosen $C = 0$ for the integration constant.

We proceed in the very same way as before, i.e. we use (31) and (33) into (12), from which one gets the general expression

$$\varepsilon(r) = \frac{4k^2B^2}{g^2h\sin^2(2\alpha)} + \frac{g^4}{16k^2} \left( \frac{h}{4} \right)^3 \sin^2(4\alpha)
+ h \left[ \left( \frac{d\alpha}{dr} \right)^2 + \sin^2(2\alpha) \left( A - m \right)^2 \right],$$

(34)

the finite-energy requirement $\varepsilon(r \to \infty) \to 0$ being attained by those profile functions fulfilling

$$\alpha(r \to \infty) \to \frac{\pi}{4} \quad \text{and} \quad A(r \to \infty) \to 2m,$$

(35)

i.e. the boundary conditions in the limit $r \to \infty$.

Now, due to (31), (33) and (35), the energy bound (18) reduces to

$$E_{bps} = \mp \pi hm,$$

(36)

the magnetic flux $\Phi_B$ still being given by the result in (27). Therefore, one gets that $E_{bps} = \pm gh\Phi_B/4$, the lower-bound being proportional to the flux of the magnetic field, both ones being again quantized. Here, we have calculated

$$\frac{\sqrt{V_0}}{\sin \alpha_0 \sqrt{1 - \sin^2 \alpha_0 \cos^2(2\beta_2)}} = \frac{g^2\sqrt{h}}{2k} \frac{h}{4}$$

(37)

in order to verify our previous conjecture.

In this case, the first-order expressions (16) and (17) can be written in the form

$$\frac{d\alpha}{dr} = \pm \frac{\sin(2\alpha)}{2r} \left( A - m \right),$$

(38)

$$\frac{1}{r} \frac{dA}{dr} = \mp \frac{g^4}{4k^2} \left( \frac{h}{4} \right)^2 \sin(4\alpha) \sin(2\alpha),$$

(39)

which must be considered in the presence of the conditions (8) and (35).

It is worthwhile to point out that the equations (38) and (39) can be obtained directly from those in (29) and (30) via the redefinitions $\alpha \to 2\alpha$ and $h \to h/4$, the energy bound and the self-dual potential behaving in a similar way, the magnetic flux remaining the same. Therefore, given that the two first-order scenarios introduced
the conditions (we verify the monotonic manner these fields approach the asymptotic profiles reading $A(r \to \infty) \to 2m$.

The Figure 3 shows the solutions to the magnetic field $B(r)$, the resulting flux being confined on a ring centered at the origin, its radius increasing as the winding number itself increases. It is also interesting to note that the magnetic field vanishes asymptotically, this way fulfilling the finite-energy requirement $\epsilon (r \to \infty) \to 0$.

In the Figure 4, we depict the profiles to the energy density $\epsilon_{bps} (r)$ inherent to the first-order configurations, these solutions also engendering rings centered at $r = 0$, their radii (amplitudes) increasing (decreasing) as $m$ increases. Here, we point out that $\epsilon_{bps} (r = 0)$ vanishes for $m \neq 1$.

In the figures 5 and 6, we present the solutions to the electric potential $A^0(r)$ and to the electric field $E(r)$ inherent to it, respectively, this last one behaving in the same general way the magnetic field does (i.e. yielding well-defined rings), both $E(r = 0)$ and $E(r \to \infty)$ vanishing identically.

We end this Section by studying the Bogomol’nyi limit supporting self-duality. In this sense, we proceed the linearisation of the first-order equations (29) and (30) around the boundary values (8) and (25), for $m > 0$ (lower signs in the first-order expressions), from we get the approximate solutions near the origin

$$\alpha (r) \approx C_0 r^m$$

and

$$A(r) \approx \frac{g^2 h^2 C_0^2}{4k^2 (m + 1)} r^{2(m + 1)},$$

the asymptotic profiles reading

$$\alpha (r) \approx \frac{\pi}{2} - C_\infty e^{-M_\alpha r},$$

and

$$A(r) \approx 2m - \frac{g^2 h}{k} C_\infty r e^{-M_A r},$$

$M_\alpha = M_A = g^2 h/2k$ being the masses of the corresponding bosons (for $h = k = 1$ and $g = \sqrt{2}$, both $M_\alpha$ and $M_A$ equal the unity), the relation $M_\alpha/M_A = 1$ defining the Bogomol’nyi limit. Here, $C_0$ and $C_\infty$ stand for real positive integration constants to be fixed by requiring the correct behavior at $r = 0$ and $r \to \infty$, respectively.
IV. FINAL COMMENTS AND PERSPECTIVES

We have investigated the first-order radially symmetric solutions inherent to the $CP(2)$ model in the presence of the Chern-Simons action, from which we have obtained regular solitons saturating a quantized energy lower-bound.

We have introduced the overall theory and the conventions inherent to it, focusing our attention on those time-independent configurations presenting radial symmetry. In the sequel, we have applied the Bogomol'nyi prescription, rewriting the expression for the effective energy in order to introduce a well-defined lower-bound (i.e. the Bogomol'nyi bound). The point to be raised is that such construction was only possible due to a differential constraint involving the potential supporting self-duality.

We have considered separately the cases defined by the two different solutions the additional profile function $\beta(r)$ supports, this way verifying that these two contexts are phenomenologically equivalent, therefore existing only one effective scenario. We have then solved the corresponding first-order equations numerically by means of the finite-difference algorithm, depicting the resulting profiles we have found this way. We have pointed out the main properties the final configurations engender, also studying the Bogomol'nyi limit explicitly.

We highlight that the results we have presented in this work only hold for the radially symmetric structures defined by the map in (6) and (7), being therefore not possible to ensure that the original model supports first-order solitons outside the radially symmetric proposal, such question lying beyond the scope of this manuscript.

Ideas regarding future investigations include the search for the nontopological first-order solitons coming from (1) and the development of a well-defined self-dual framework inherent to a $CP(2)$ theory in the presence of both the Maxwell and the Chern-Simons terms simultaneously. These issues are currently under consideration, and we hope positive results for an incoming contribution.

Acknowledgments

The authors thank CAPES, CNPq and FAPEMA (Brazilian agencies) for partial financial support.

[1] N. Manton and P. Sutcliffe, Topological Solitons (Cambridge University Press, Cambridge, England, 2004).
[2] E. Bogomol'nyi, Sov. J. Nucl. Phys. 24, 449 (1976). M. Prasad and C. Sommerfield, Phys. Rev. Lett. 35, 760 (1975).
[3] H. B. Nielsen and P. Olesen, Nucl. Phys. B 61, 45 (1973).
[4] R. Jackiw and E. J. Weinberg, Phys. Rev. Lett. 64, 2234 (1990). R. Jackiw, K. Lee and E. J. Weinberg, Phys. Rev. D 42, 3488 (1990).
[5] D. Bazeia, E. da Hora, C. dos Santos and R. Menezes, Phys. Rev. D 81, 125014 (2010); Eur. Phys. J. C 71, 1833 (2011). D. Bazeia, R. Casana, M. M. Ferreira Jr. and E.
da Hora, Europhys. Lett. 109, 21001 (2015). R. Casana, E. da Hora, D. Rubiera-Garcia and C. dos Santos, Eur. Phys. J. C 75, 380 (2015). R. Casana, M. M. Ferreira Jr., E. da Hora and C. Miller, Phys. Lett. B 718, 620 (2012). R. Casana, M. M. Ferreira Jr., E. da Hora and A. B. F. Neves, Eur. Phys. J. C 74, 3064 (2014). R. Casana and G. Lazar, Phys. Rev. D 90, 065007 (2014). R. Casana, C. F. Farias and M. M. Ferreira Jr., Phys. Rev. D 92, 125024 (2015). R. Casana, C. F. Farias, M. M. Ferreira Jr. and G. Lazar, Phys. Rev. D 94, 065036 (2016). L. Sourrouille, Phys. Rev. D 87, 067701 (2013). R. Casana and L. Sourrouille, Mod. Phys. Lett. A 29, 1450124 (2014).

[6] C. Armendariz-Picon, T. Damour and V. Mukhanov, Phys. Lett. B 458, 209 (1999). V. Mukhanov and A. Vikman, J. Cosmol. Astropart. Phys. 02, 004 (2005). A. Sen, J. High Energy Phys. 07, 065 (2002). C. Armendariz-Picon and E. A. Lim, J. Cosmol. Astropart. Phys. 08, 007 (2005). J. Garriga and V. Mukhanov, Phys. Lett. B 458, 219 (1999). R. J. Scherrer, Phys. Rev. Lett. 93, 011301 (2004). A. D. Rendall, Class. Quantum Grav. 23, 1557 (2006).

[7] A. D’Adda, M. Luscher and P. D. Vecchia, Nucl. Phys. B 146, 63 (1978). E. Witten, Nucl. Phys. B 149, 285 (1979). A. M. Polyakov, Phys. Lett. B 59, 79 (1975). M. Shifman and A. Yung, Rev. Mod. Phys. 79, 1139 (2007).

[8] A. Yu. Loginov, Phys. Rev. D 93, 065009 (2016).

[9] R. Casana, M. L. Dias and E. da Hora, Phys. Lett. B 768, 254 (2017).

[10] R. Casana, M. L. Dias and E. da Hora, Nonempty first-order vortices in a gauged $CP(2)$ model with a di-electric function, submitted to the Physical Review D (2017).