Nonlinear Oscillations of Elastic Curved Plate Carried to the Associated Masses System

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Abstract. Curved plates are used in many spheres of our life. Often curved plates have an articulated support. Often, the operation of such structures is associated with fluctuations as well as with the systems of attached masses. The vibrations of such structures have not been fully studied yet. In the laboratory of building structures of Komsomolsk-on-Amur State Technical University a test bench was prepared for testing curved plates hinged at the edges and carrying an attached mass or a system of attached masses. The attached mass is represented by a sensor fixed to the body of a curved plate carrying an attached mass. Experimental and theoretical data correlate with each other. Minimal discrepancies were obtained with the minimum attached mass and the maximum discrepancies at the maximum investigated attached mass. Theoretical calculations were performed using the known equations of plate oscillations.

1. Introduction
The wide use of plates in construction and machine building is due to the ease of manufacture and efficiency, in terms of strength, geometric shape of the plates. The occurrence of natural oscillations from intense dynamic loads, when operating such structures, does not limit the operation of hinged-supported plates. Phenomena of this kind are often accompanied by an attached mass to the plate or a system of attached masses. Although this circumstance has not been fully studied, plate operation is present. And interest in analyzing the dynamics of thin plates is very high in broad areas of activity. [1-4]

2. Experimental Section
To investigate the effect of the presence of an attached mass or a system of attached masses, a special experiment was conducted in the laboratory of building structures of KnASTU. The prototype, located in a special bench (Figure 1), is equipped with an attached mass, was subjected to an additional system of attached masses, of a different nature, in the presence of natural oscillations.

This work reflects the investigation of the natural oscillations of a thin rectangular plate in a plane that is hinged on both sides. Equations of natural oscillations of the plate are obtained according to the generally accepted theory of plate oscillations, as well as experimental data reflecting the dependence of the effect of the attached mass and the system of attached masses on the natural frequencies of the shell oscillations. Oscillations with moderate amplitudes of natural oscillations were decomposed,
according to the equations [5,6]. A discrete nonlinear model of vibrations of a thin plate supported at
the edges, obtained during research, was investigated using a method of many scales.

The sample is a thin plate, rectangular in plan, made of galvanized steel. The composition of the
plate is given in Table 1. The geometric characteristics of the object: \( L = 890 \text{ mm}, B = 370 \text{ mm}, H = 0.4 \text{ mm}. \) A sample is shown in Figure 1.

![Figure 1. The sample shell.](image)

The sample consists of grade 3 steel, semi-boiling. The shell model is fixed in a steel stand. This
stand has the form of a table made of equal-angled corners L45x3 steel st3sp, especially for this
experiment. Hinge support is realized with the help of glass plates 30x30 mm 4 mm thick, laid in the
form of a corner. The boundary conditions are as close as possible to the real ones. The attached mass
is the accelerometer BC110, located on the sample according to figure 1. Accelerometer BC110
measures the oscillation frequency with maximum accuracy.

![Figure 2. Typical accelerometer BC 110.](image)

The accelerometer BC 110 transmits the readings to the signal amplifier, which, amplifying the
signal, transmits further along the circuit to the analog-to-digital converter, which transmits the signal
further to the personal computer. The thermometer is located closest to the shell, but does not touch it,
transmits the readings on the software "Zet-lab", where in the compartment with the data of the
oscillations the dependence of some parameters on others is reflected, in the real time. Software "Zet-
lab" allows you to display and record fluctuations in the real time. The heating element creates a
temperature gradient. A block diagram of the experimental setup for carrying out the experiment is
shown in Figure. 3.
The experiment is aimed at studying the natural oscillations of a plate in "rest" and revealing the dependencies of forced and natural oscillations on the effect of an attached mass or mass system. Forced oscillations, going into their own oscillations, were set by impact with a test hammer AU03. Also there was a contactless sensor for measuring the vibration of the plate, not shown in the diagram. This sensor is a checker, serves to check and reject erroneous data of the accelerometer BC110.

3. Equations and mathematics

Let us turn to the mathematical model of natural oscillations of a plate, rectangular in plan, hinged-supported from two sides. The process of oscillation of a plane thin homogeneous plate is described by the equation:

$$ u_{tt} = a^2 (u_{xx} + u_{yy}) $$  

The function $u(x,y,t)$ characterizes the deflection of the plate. The initial conditions are described as follows:

$$ u(x,y,0) = \phi(x,y), \quad u_t(x,y,0) = \psi(x,y) $$  

And boundary conditions are

$$ u_y(0,y,t) = 0, u_y(b_1,y,t) = 0, \quad u_x(x,0,t) = 0, u_x(x,b_2,t) = 0 $$

$X^*(x) + \nu X(x) = 0$ - Second-order equation, homogeneous linear differential with constant coefficients. The parameter is meaningful. For the solution, we should consider the case of finding a parameter above zero equal to zero, below zero.

Then, taking into account the boundary conditions, we express:

$$ X'(x) = D_1 \sqrt{\nu} \sin \sqrt{\nu} x + D_2 \sqrt{\nu} \cos \sqrt{\nu} x, \quad X'(0) = \sqrt{\nu} D_2 = 0 \Rightarrow D_2 = 0, \quad \nu \neq 0, \quad X'(b_1) = D_1 \sqrt{\nu} \sin \sqrt{\nu} b_1 = 0 $$

$D_1 \neq 0$, Since Important are non-trivial solutions, $\nu \neq 0$, Consequently
\[
\sin \sqrt{v} b_1 = 0; \\
\sqrt{v} b_1 = \pi n, \; n \in Z; \\
v = \left(\frac{\pi n}{b_1}\right)^2, \; n \in Z
\]  

(5)

From this it follows that only for values of equal \( v_n = v = \left(\frac{\pi n}{b_1}\right)^2 \), there exist a nontrivial solution of the equations, has the form:

\[
X_n(x) = \cos \frac{\pi n}{b_1} x
\]

(6)

Similarly we obtain the solution of the problem:

\[
Y_m(y) = \cos \frac{\pi m}{b_2} y;
\]

(7)

\[
\mu_m = \left(\frac{\pi m}{b_2}\right)^2
\]

Eigenvalues \( \lambda_{n,m} = \left(\frac{\pi n}{b_1}\right)^2 + \left(\frac{\pi m}{b_2}\right)^2 \), Relevant functions

\[
\nu_{n,m} = A_{n,m} \cos \frac{\pi n}{b_1} y \cos \frac{\pi m}{b_2} y
\]

(8)

where \( A_{n,m} \) - some constant factor. We represent it in such a way that the function \( \nu_{n,m} \) was a unit of unity

\[
\int_0^h \nu_{n,m}^2 \, dx \, dy = A_{n,m}^2 \int_0^h \cos^2 \frac{\pi n}{b_1} \, x \, dx \int_0^h \cos^2 \frac{\pi m}{b_2} \, y \, dy = 1
\]

(9)

The integrals are calculated separately:

\[
\int_0^h \cos^2 \frac{\pi n}{b_1} \, x \, dx = \int_0^h \frac{1 + \cos \frac{2\pi n}{b_1} x}{2} \, dx = \frac{1}{2} \int_0^h dx + \frac{1}{2} \int_0^h \cos \frac{2\pi n}{b_1} \, x \, dx = \frac{1}{2} \left[ h + \frac{b_1}{2\pi n} \sin \frac{2\pi n}{b_1} h - \frac{h}{2\pi n} \sin \frac{2\pi n}{b_1} 0 \right] = \frac{1}{2} b_1 + \frac{b_1}{2\pi n} \sin 2\pi n - \frac{b_1}{2\pi n} \sin 0 = \frac{1}{2} b_1;
\]

\[
\int_0^h \cos^2 \frac{\pi m}{b_2} \, y \, dy = \frac{1}{2} b_2.
\]

\[
A_{n,m}^2 \frac{1}{2} b_1 \frac{1}{2} b_2 = 1; \quad A_{n,m} = \frac{4}{b_1 b_2} \quad \Rightarrow \quad A_{n,m} = \frac{4}{\sqrt{b_1 b_2}};
\]

(10)

\[
\nu_{n,m} = \frac{4}{b_1 b_2} \cos \frac{\pi n}{b_1} y \cos \frac{\pi m}{b_2} y
\]

(11)
The number of integer solutions \( n \) and \( m \) of equation

\[
\lambda_{n,m} = \left( \frac{n \pi}{b_1} \right)^2 + \left( \frac{m \pi}{b_2} \right)^2
\]

(12)

Depends on the number of eigenfunctions, belong to \( \lambda_{n,m} \).

The values \( \lambda_{n,m} \) belong to the solution of equation \( T'' + a^2 \lambda T = 0 \):

\[
T_{n,m}(t) = \overline{B_{n,m}} \cos \sqrt{\lambda_{n,m}} at + \overline{B_{n,m}} \sin \sqrt{\lambda_{n,m}} at
\]

(13)

where \( \overline{B_{n,m}} \) and \( \overline{B_{n,m}} \) - arbitrary constants.

Starting from the additional conditions, the initial solution of the problem \( u_{n,m} = a^2 \left( u_{nx} + u_{ny} \right) \) will be found with the help of particular solutions, which, in turn, have:

\[
u_{n,m} (x,y,t) = \overline{B_{n,m}} \cos \sqrt{\lambda_{n,m}} at + \overline{B_{n,m}} \sin \sqrt{\lambda_{n,m}} at
\]

(14)

We calculate the general solution:

\[
u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \overline{B_{n,m}} \cos \sqrt{\lambda_{n,m}} at + \overline{B_{n,m}} \sin \sqrt{\lambda_{n,m}} at
\]

(15)

where \( \nu_{n,m} (x,y) \) is determined by the decision, and the coefficients \( \overline{B_{n,m}} \) and \( \overline{B_{n,m}} \) are equal:

\[
\overline{B_{n,m}} = \frac{1}{\sqrt{b_1 b_2}} \left( \frac{4}{b_1 b_2} \right) \int_{0}^{h} \int_{0}^{b_2} \phi(x,y) \cos \frac{n \pi}{b_1} x \cos \frac{m \pi}{b_2} y \, dx \, dy
\]

(16)

4. Results and Discussion

This equation shows the dependence of the deflection of the plate on the boundary conditions. The boundary conditions reflect the characteristics of the condition of the problem, which include the system of attached masses. The change in the temperature gradient affects the oscillations of the plate.

5. Conclusion

Figure 4 reflects the dependence of the results of calculating the natural frequencies and modes of vibration of a steel rectangular plate in plan view from the attached mass. The oscillation frequencies are in Hz. As a result, it becomes clear that the theoretical data and the calculated data do not coincide. The difference between the theoretical and experimental data at a temperature of \( t = 22 \) °C and with an attached mass of less than 150 g is less than 5%. This fact indicates the accuracy of calculating the natural vibrations of a plate carrying a small attached mass, and the experiment was carried out as accurately as possible. The maximum discrepancy between the theoretical and calculated data occurs with the maximum attached mass used in the study - 250 grams. The difference is 15%. With an attached mass of 50 grams, the difference is 1%. This circumstance indicates the need for additional studies of wafer vibrations involving an attached mass or a system of attached masses.
**Figure 4.** Dependence of the oscillation frequency of the shell on the value of the attached mass.

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