Static alignment of inertial navigation systems using an adaptive multiple fading factors Kalman filter

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Kalman filter is not an optimal estimation method for the systems without any exact model. Therefore, a multiple fading factors matrix has been used as a multiplier for the covariance matrices in these systems. In this paper, a novel method, named adaptive multiple fading factor Kalman filter, is proposed for the systems without initial alignment of strap down inertial navigation systems. By applying this algorithm to different channels of Kalman filter, different coefficients of fading factors are computed. The simulation results show a noticeable increment in alignment’s precision and alignment’s speed, and a noticeable decrement in sensitivity to unknown noises.

Keywords: strap down; initial alignment; inertial navigation systems; multiple fading factor filter

1. Introduction

Inertial navigation system (INS) is an autonomous system in small intervals of time which has high precision and good coverage. Although, a strap down inertial navigation system (SINS) is the main hardware of the INS, it does not have the problems of a traditional INS. Most importantly, the SINS has some other advantages such as reducing the total system’s complexity, costs and power consumption.

Recently, these systems have grown rapidly in all fields of application. Nowadays, SINS is widely used for positioning and navigating in aircrafts, ships, vehicles, etc. (Dzhashitov et al., 2014; Silson, 2011). Furthermore, SINS is used as a powerful means in constant tracking of a position, direction and velocity of any moving object without referring to any external reference (Wu & Pan, 2013; Yin, Sun, & Wang, 2013).

Notwithstanding, SINS has some drawbacks. One of its main fallibilities is its initial alignment error that has a negative effect on the system velocity and position error. Initial alignment in SINS is defined as measuring coordinates of transformation matrix from body frame to navigation frame and aligning misaligned angles to zero. More precisely, SINS’s initial alignment contains two stages, coarse alignment and the precise alignment, each of which has its own purposes (Jiong, Lei, Rong, & Jianyu, 2011).

The purpose of the coarse alignment is estimation of transferring matrix coordinates from body frame to navigation frame by taking the best advantages of the vector of the Earth’s gravity $g$ and measuring the value of the Earth’s rotation rate $\omega_{eq}$. In contrast, the purpose of precise alignment is computation of the small misaligned angles between the navigation frame and body frame accurately through processing those data obtained from various sensors. In this section, the constant drift of inertial sensors is omitted and the accuracy of alignment is enhanced.

However, precise alignment complicates system’s mechanism design and decreases system’s accuracy and reliability. Initial alignment in SINS should be done with a lot of care and attention at the shortest possible time. Moreover, there are some consequences which make both alignments really complicated and time consuming.

For initial alignment in SINSs, Kalman filter has a high applicability when the system’s model is accurate and the system’s noise is white Gaussian and uncorrelated. For decades, random estimation techniques, especially Kalman filtering (KF) and its more developed forms, extended Kalman filter (EKF) and unscented Kalman filter (UKF), have been widely used in INSs (Grewal, Henderson, & Miyasako, 1991; Ladetto, 2000; Wang & Chen, 2010). Besides, Kalman filter makes it possible to combine measurements of nearly most navigation system’s sensors and reach more precise estimation of position and velocity.

With an unauthentic model of the system and unavailable exact statistical data of the system’s noise, relying on the Kalman filter’s estimations may lead to inaccurate results. Sometimes, it may lead to the diversion of Kalman filter’s estimations from the true state. Therefore, to prevent these problems, a vast amount of research has been done and more efficient models of KF have been proposed such as adaptive KF algorithm (Wang & Chen, 2010),...
of the innovative sequence. Descriptively, the fading algorithm of Kalman filter is such an adaptive algorithm which wipes previous data out by exponent $\alpha$, a method that limits the memory of the Kalman filter (Fagin, 1964; Sorenson & Sacks, 1971). In Ydstie and Co (1985), an adjustable algorithm for fading factor has been suggested, wherein fading factors are determined based on ‘memory length’. A fast fading occurs when the system model is not accurate and slow fading is used for accurate models. Few years later, an optimal fading factors KF algorithm had been offered, in which an exponential weight changing approach was used to balance the model errors and unknown drifts (Gao, Yang, Cui, & Zhang, 2006; Özbek & Alive, 1998; Xia, Rao, Ying, & Shen, 1994; Xu, Qin, & Peng, 2004). Zhang, Jin, and Tian (2003) show the use of adaptive filtering techniques to develop the speed of the dynamic alignment of a micro-electro-mechanical system inertial measurement unit (MEMS IMU) with a real-time kinematic global positioning system (RTK GPS) for a nautical function. In Mohamed and Schwarz (1999), real-time adaptive algorithms are used for GPS information processing. To prevent the covariance matrix of estimate error from being asymmetric, an enhanced algorithm has been suggested, which reaches use of the filter residual. Along with statistical progression of the filter residuals using a chi-square test, the fading factors have been calculated distinctly to increase the predicted variance modules of the state vector (Geng & Wang, 2008; Shi, Miao, & Ni, 2010). In Kim, Jee, Park, and Lee (2009), the stability of the adaptive fading EKF has been studied. Furthermore, an adaptive UKF with several fading factors based on gain adjustment has been presented and examined on the estimation system approach of a Pico-satellite by simulation in Söken and Hajiyev (2009). Also, a new strong tracking square root Cubature Kalman Filter with suboptimal multiple fading factors is used, which can adjust the structure parameters of the filter and improve the performance of target state tracking in Li, Zhu, and Zhang (2014). In Gao et al. (2010a), two types of fading factor Kalman filter algorithms are suggested by computing unbiased estimations of the innovative sequence.

In this paper, an adaptive multiple fading factors Kalman filter is suggested that exploits the two types of fading factors algorithms mentioned in Gao et al. (2010a). The proposed algorithm calculates a fading factor for each channel separately. Additionally, by applying an averaging method, another fading factor is calculated for all channels. Then, the computed fading factor for each channel will be compared with the mentioned fading factor obtained from applying the averaging method. Consequently, the bigger fading factor will be chosen and multiplied by an estimation covariance matrix. By using this strategy, the bigger fading factor values are obtained. This will result in a fast fading since the system model is not accurate.

This paper is organized as follows. Section 2 contains a brief introduction of the aligning error models in SINS. In Section 3, our adaptive multiple fading factors Kalman filter is proposed. Simulation results are presented in Section 4. And consequently, in Section 5, the total overview of the present paper and its conclusions will be manifested.

2. SINS error model for alignment

2.1. Coordinate frames

Studying coordinate frames related to the initial alignment of SINS can be described as follows (Qin, 2006):

- Inertial frame (I frame): its origin is in the centre of earth; Page equator $\z$-axis, the axis of $x$ in Page Equator hidden. Direction is arbitrarily to be chosen, $y$-axis complementary to the right system.
- Navigation frame (n frame): it is a local geographic coordinate frame; $x$-axis towards the East, $y$-axis towards the north, $z$-axis perpendicular to the local level, the local positions.
- Body frame (b frame): origin is of body centre; $x$-axis is along the transverse axis, $y$-axis is the longitudinal axis, $z$-axis perpendicular to the longitudinal symmetry page.
- Earth fixed frame (e frame): its origin is in the centre. $z$-axis coincides with the axis of the earth’s polar axis, while the other two are fixed between the two tropics. Detailed information is available in Gao, Miao, and Shen (2010c) and Qin (2006).

2.2. SINS error model

Assuming that the navigation system is fixed to the ground, to study the behaviour of an INS, a suitable error model must be chosen. Using a linear error model for this situation is quite a good approximation. In the SINS error model, the state vector consists of 12 state variables, for example, 3 velocity errors, 3 attitude angle errors, 3 accelerometer biases and 3 gyroscope drifts. Navigation mode equations are presented as follows:

$$\dot{X} = AX + Gw,$$

where

$$X = [\delta V_E \delta V_N \delta V_u \phi_E \phi_N \phi_u V_x V_y V_z \varepsilon_x \varepsilon_y \varepsilon_z]^T,$$

$$w = [w_{ax} w_{ay} w_{az} w_{gx} w_{gy} w_{gz}]^T,$$

$$A = \begin{bmatrix}
A_1 & A_2 & C_p & 0_{3 \times 3} \\
0_{3 \times 3} & A_3 & 0_{3 \times 3} & -C_p' \\
0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix},$$

and $Gw$ is the process noise with covariance matrix $Q$.
where \(\omega_e\) is the Earth rotation rate, \(g\) is the local acceleration gravity and \(L\) is the local latitude. \(\nabla_x\), \(\nabla_y\) and \(\nabla_z\) are the biases of the three accelerometers, \(\varepsilon_x, \varepsilon_y\) and \(\varepsilon_z\) are the drifts of the three gyros and \(\delta v_x, \delta v_y\) and \(\delta v_z\) are the east, north and vertical velocity errors, respectively. \(w\) is the noise vector of the system, \(C^n_b\) is the transformation matrix from the body frame to the navigation frame and \(\phi_E, \phi_N\) and \(\phi_A\) are the east, north and azimuth misalignment angles, respectively. Note that the velocity error is considered as the output and the output equation will be represented as follows:

\[
   z = HX + v, \tag{2}
\]

where \(H = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}\) and \(v = \begin{bmatrix} v_E & v_N & v_o \end{bmatrix}\) are the output noise vector.

3. Adaptive several fading factors Kalman filter

3.1. Development of single fading factor

Discrete linear system state and output equations will be represented as follows:

\[
   x = \phi_{k-1}x_{k-1} + \Gamma_{k-1}w_{k-1}, \tag{3}
\]

\[
   z_k = H_kx_k + v_k. \tag{4}
\]

In this system, \(x_k\) is the state vector at time instant \(k\), \(\phi_{k-1}\) is a state transition matrix, \(\Gamma_{k-1}\) is a system perturbation matrix, \(z_k\) is the output at time \(k\), \(H_k\) is the output matrix, \(w_{k-1}\) is the process noise vector and finally, \(v_k\) is the output noise vector.

When the system noise and the measurement noise are uncorrelated and white Gaussian, the Kalman filter equations will be presented as follows:

\[
   \hat{x}^-_k = \phi_{k-1}\hat{x}^+_k, \tag{5}
\]

\[
   \hat{x}^+_k = \hat{x}^-_k + k_k(z_k - H_k\hat{x}^-_k), \tag{6}
\]

\[
   k_k = p^-_kH_k^T(H_kp^-_kH_k^T + R_k)^{-1}, \tag{7}
\]

\[
   p^-_k = \phi_{k-1}p^+_k\phi_{k-1}^T + \Gamma_{k-1}Q_k\Gamma_{k-1}^T, \tag{8}
\]

\[
   p^+_k = (I - k_kH_k)p^-_k(I - k_kH_k)^T + k_kR_kk_k^T. \tag{9}
\]

In these equations, \(\hat{x}^-_k\) is the propagation state vector, \(\hat{x}^+_k\) is the estimated state vector, \(p^-_k\) is the gain matrix, \(\hat{x}^+_k\) is the estimated state vector, \(p^+_k\) is the covariance matrix for \(\hat{x}^+_k\), \(Q_k\) is the system noise covariance matrix, \(R_k\) represents the measurement noise covariance matrix and \(I\) manifests the identity matrix.

When the system model is accurate, and noise characteristics are under Gaussian conditions, white and known accurately, Kalman filter is an optimal estimator. However, satisfying the aforementioned conditions is usually very difficult in the actual system. As mentioned earlier, Kalman filter performance highly depends on the previous measurements and also system model accuracy. If the measurements are inaccurate, Kalman filter estimations might be inaccurate or even incorrect. To eliminate these problems, the \(\lambda_k\) factor is used to omit exponential fading of previous data, which is a technique to limit the memory of the Kalman filter (Fagin, 1964; Sorenson & Sacks, 1971). The equations of a single fading factor Kalman filter are similar to the standard Kalman filter. The only difference is the time propagation equation of the error covariance that is expressed as follows:

\[
   p_k^- = \lambda_k\phi_k^-1p_k^-\phi_k^-1^T + \Gamma_k^-1Q_k^-1\Gamma_k^-1^T, \tag{10}
\]

where \(\lambda_k > 1\) is the fading factor.

When there is an unpredictable disturbance in the system, reaching the optimal performance using a fixed exponential fading factor is usually difficult. Therefore, a variable fading factor algorithm has been suggested. Fading factors are verified based on ‘memory length’ (Ydstie & Co., 1985). If the obtained data have an inappropriate fit with the system model, a fast fading will happen. Otherwise, a slow fading will happen if we have an appropriate fit. An optimal fading factor KF algorithm has been proposed in Xia et al. (1994). That can be expressed as follows:

\[
   \text{ALGORITHM 1}
\]

\[
   A_k = \lambda_kH_k\phi_k^-1p_k^-\phi_k^-1^TH_k^-T,
\]

\[
   M_k = H_k\phi_k^-1p_k^-\phi_k^-1^TH_k^-T,
\]

\[
   B_k = H_k\Gamma_k^-1Q_k^-1\Gamma_k^-1^TH_k^-T + R_k,
\]

\[
   N_k = \hat{\delta}_{v_k} - B_k,
\]

\[
   J_k = \phi_k^-1p_k^-\phi_k^-1^T,
\]

where \(\hat{\delta}_{v_k}\) is the estimation of the innovation sequence covariance and optimal fading factor is obtained as

\[
   \lambda_k = \max\{1, \text{tr}(N_kM_k^{-1})/m\}.
\]

In this case, \(m\) is a dimension of the output vector.

In this algorithm, only a single fading factor is multiplied in the estimation error covariance matrix, which can change at any time step. This method may not give optimal performance of filters and in some situations, it leads to the divergence of the filter.
As it was discussed, in most cases, the single fading factor is not sufficient and has different rates for different channels. When matrix $p_k$ is not asymmetric, the time propagation equation of error covariance is offered as follows:

$$p_k^- = \Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} + \Gamma_k Q_k^{-1} \Gamma_k^T.$$  \hspace{1cm} (11)

In this equation, $\Lambda_k$ is a diagonal fading factor matrix. Here, the second algorithm which in this paper is called Algorithm 2 will be described. Assuming that the output matrix satisfies the following equation:

$$H_k = [s_{m \times m} 0_{m \times (n-m)}]_{m \times n},$$ \hspace{1cm} (12)

where $s_{m \times m} = \text{diag}(s_1, s_2, \ldots, s_m)$, $m < n$.

In this case, Algorithm 2 can be described as

$$A_k = H_k \Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} H_k^T,$$

$$B_k = H_k \Gamma_k^{-1} Q_k^{-1} \Gamma_k H_k^T + R_k,$$

$$N_k = \hat{\delta}_{vi} - B_k,$$

$$J_k = \phi_{k-1} p_{k-1} \phi_{k-1}^T.$$  

Now, the optimal fading factors can be extracted as $\Lambda_k = (\lambda_1, \lambda_2, \ldots, \lambda_m, 1, 1, \ldots, 1)$, and $\lambda_i$ can be calculated as follows:

$$\lambda_i = \max \left\{ 1, \frac{\hat{\delta}_{vi}^2 - b_{ii}}{\delta_{ji}^2 1_{ii}} \right\}, \hspace{0.5cm} \hat{\delta}_{vi}^2 > b_{ii}.$$  

$$\lambda_i = 1, \hspace{0.5cm} \hat{\delta}_{vi}^2 < b_{ii}$$

In this algorithm, only $\lambda_1, \lambda_2, \ldots, \lambda_m$ can be adaptively estimated, and other matrix components cannot be estimated which are determined by the dimension of the output.

### 3.2. The proposed adaptive multiple fading factors Kalman filter

In this section, an adaptive multiple fading factors algorithm is presented. As mentioned, Algorithm 1 cannot save the filter optimums completely, and sometimes ends in the divergence of the filter. Furthermore, Algorithm 2 cannot provide the optimal solution. It is because of its method of calculating fading factors $(\lambda_1, \lambda_2, \ldots, \lambda_m)$, which in some cases results in $\lambda_k = 1$. Here, an algorithm will be provided, which based on simulation results has more efficient results than the mentioned algorithms.

The innovative sequence is a part of measurements that includes new information about the state of the system. Indeed, it is a Gaussian white process with zero mean and covariance $H_k p_k^- H_k^T + R_k$. Considering Equation (11), the following result is obtained:

$$\delta_{vi} = H_k (\Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} H_k^T + \Gamma_k Q_k^{-1} \Gamma_k H_k^T + R_k)$$

$$= H_k \Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} H_k^T + H_k \Gamma_k Q_k^{-1} \Gamma_k H_k^T + R_k$$

$$+ R_k = A_k + B_k,$$ \hspace{1cm} (13)

where

$$A_k = H_k \Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} H_k^T,$$

$$B_k = H_k \Gamma_k^{-1} Q_k^{-1} \Gamma_k H_k^T + R_k.$$  

The unbiased estimate of the innovation sequence covariance can be extracted as

$$\hat{\delta}_{vi} = \frac{1}{k-1} \sum_{i=1}^{k} v_i v_i^T,$$ \hspace{1cm} (14)

where $v_i$ is the innovation sequence, therefore

$$\hat{\delta}_{vi} = A_k + B_k.$$ \hspace{1cm} (15)

Generally, the output matrix equation in navigation and positioning applications can be considered as Equation (12). If the output matrix $H_K$ satisfies Equation (12), then we have

$$A_k = H_k \Lambda_k p_{k-1} \phi_{k-1}^T \Lambda_k^{-1} H_k^T = H_k \Lambda_k J_k \Lambda_k^{-1} H_k^T$$

$$= [s_{m \times m} 0_{m \times (n-m)}]_{m \times n} \Lambda_k^{-1} H_k^T$$

$$\times \left[ \begin{array}{c} \Lambda^1_{m \times m} \times 0_{m \times (n-m)} \\
\Lambda^2_{m \times m} \times 0_{m \times (n-m)} \\
\vdots \\
\Lambda^n_{m \times m} \times 0_{m \times (n-m)} \\
0_{m \times (m \times (n-m))} \times \Lambda^1_{m \times m} \\
0_{m \times (m \times (n-m))} \times \Lambda^2_{m \times m} \\
\vdots \\
0_{m \times (m \times (n-m))} \times \Lambda^n_{m \times m} \\
0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))} \\
\end{array} \right]$$

$$= \left[ A^1_{m \times m} \times 0_{m \times (m \times (n-m))} \\
A^2_{m \times m} \times 0_{m \times (m \times (n-m))} \\
\vdots \\
A^n_{m \times m} \times 0_{m \times (m \times (n-m))} \\
0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))} \right]$$

$$\times \left[ s_{m \times m} \times 0_{m \times (n-m)} \\
s_{m \times m} \times 0_{m \times (n-m)} \\
\vdots \\
s_{m \times m} \times 0_{m \times (n-m)} \\
0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))} \right]$$

$$= \left[ B^1_{m \times m} \times 0_{m \times (m \times (n-m))} \\
B^2_{m \times m} \times 0_{m \times (m \times (n-m))} \\
\vdots \\
B^n_{m \times m} \times 0_{m \times (m \times (n-m))} \\
0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))} \right]$$

$$\times \left[ s_{m \times m} \times 0_{m \times (n-m)} \\
s_{m \times m} \times 0_{m \times (n-m)} \\
\vdots \\
s_{m \times m} \times 0_{m \times (n-m)} \\
0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))} \right]$$

$$= \Lambda_{m \times m} \times 0_{m \times (m \times (n-m))} \times 0_{m \times (m \times (n-m))}$$

Therefore, the aforementioned equation can be rewritten as

$$a_{ii} = \lambda_i^2 \delta_{ji}^2$$ \hspace{0.5cm} (16)

where $a_{ii}$ and $j_{ii}$ are the $i$th diagonal elements of matrix $A_K$ and $J_K$, respectively.
By considering the obtained results of the aforementioned equation and according to Equation (15), the following conclusion is achieved:

\[ \hat{\delta}_{v_i} = a_{ii} + b_{ji} \]  
(17)

In this conclusion, \( \hat{\delta}_{v_i} \) is the \( i \)th diagonal element of matrix \( \delta_{v} \) and \( b_{ji} \) is the \( j \)th diagonal element of matrix \( B_k \).

Based on the earlier discussions, the fading factors are calculated using the following equation:

\[ \lambda_i^2 = \frac{\hat{\delta}_{v_i} - b_{ii}}{s_{ij}^2} \quad (i = 1, 2, \ldots, m). \]  
(18)

Here, we will propose a new formula to calculate \( \lambda_k \).

The fading factors may satisfy the following condition:

\[ \lambda_k = d_k, \]  
(19)

\[ M_k = H_k \phi_k^{-1}p_{k-1}^{+} \phi_k^{-1}H_k^T, \]  

\[ N_k = \hat{\delta}_{v} - B_k, \]  

\[ d_k = \max\{1, \text{abs}(\text{tr}(N_kM_k^{-1}))\}. \]  
(20)

Here, \( d_k > 1 \) is also considered as a fading factor. Considering the sign of square root \( \lambda_i \), the optimal fading factors matrix can be expressed as

\[ \lambda_i = \max \left\{ d_k, \sqrt{\frac{\hat{\delta}_{v_i} - b_{ii}}{s_{ij}^2}}, \hat{\delta}_{v_i} > b_{ii} \right\}. \]  
(21)

Now, the proposed algorithm can be described based on the aforementioned equations as follows. If the output matrix satisfies Equation (12), we have

\[ A_k = H_k \Lambda_k \phi_k^{-1}p_{k-1}^{+} \phi_k^{-1}H_k^T; \]  

\[ B_k = H_k \Gamma_k^{-1}Q_k^{-1} + R_k; \]  

\[ M_k = H_k \phi_k^{-1}p_{k-1}^{+} \phi_k^{-1}H_k^T; \]  

\[ N_k = \hat{\delta}_{v} - B_k; \]  

\[ J_k = \phi_k^{-1}p_{k-1}^{+} \phi_k^{-1}. \]  

In the next phase, the fading factors matrix can be expressed as follows:

\[ \Lambda_k = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, d_k, d_k, \ldots, d_k\}, \]  

where \( \lambda_i, d_i \) can be calculated from Equations (20) and (21).

The calculation of the fading factor for each channel is independent of other channels. In other words, the fading factors matrix is made up of some independent fading factors. As known, the fading factors matrix is used to set the covariance matrix \( p_k^{-} \) in order to adjust the gain matrix \( k_k \). In practice, \( \hat{\delta}_{v_i} \) is estimated as follows:

\[ \hat{\delta}_{v_i} = \frac{1}{N} \sum_{j=0}^{N} v_{k-j} v_{k-j}^T, \]  
(22)

where \( N \) is the window width.

Determination of the window width properly is difficult. With a very low \( N \), some of the information may be lost and the unbiased estimate of the innovation sequence covariance cannot be calculated. On the other hand, if \( N \) is chosen to be very large, the volume of information will be too large, and the short-range characteristics of the innovation sequence covariance are hard to reflect.

When the velocity error is considered as the output, the output matrix of the SINS alignment can be modelled as follows:

\[ H_k = [I_{3 \times 3} \ 0_{3 \times 9}]. \]

And the optimal fading factors matrix can be as follows:

\[ \Lambda_k = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, d_k, d_k, \ldots, d_k\}, \]  

\[ \lambda_i = \begin{cases} \max \left\{ d_k, \sqrt{\frac{\hat{\delta}_{v_i} - b_{ii}}{s_{ij}^2}}, \hat{\delta}_{v_i} > b_{ii} \right\}, \\ d_k, \quad \hat{\delta}_{v_i} < b_{ii} \end{cases} \quad (i = 1, 2, 3). \]

4. Simulation results

In this section, the proposed method has been evaluated based on simulation. Both the constant and drifts of each gyro are selected as 0.02(°)/h; and both the constant and random biases of each accelerometer are selected as \( 1 \times 10^{-4} \) g; the true attitude angles of the system are 0°, 0°, and 0° and the local latitude of SINS place is 39.96°. After the coarse alignment, the horizontal accuracy is 0.1°, and the azimuth accuracy is 0.3°, both of which meet the applicability demand of linear model. In the presence of external interference, to demonstrate the capabilities of the multipurpose fading factors Kalman filter algorithm, we assume the range to be 100–150 and 200–250 s and system noise to be \( Q' = 10Q \) and \( Q'' = 12Q \), respectively. Here, standard Kalman filter, Algorithm 1, Algorithm 2 and proposed algorithm are used in SINS initial alignment individually.

The initial state vector is assumed to be

\[ X(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \]

The initial state covariance matrix is set as

\[
p_0 = \text{diag}(0.1 \text{ m/s})^2 \quad (0.1 \text{ m/s})^2 \quad (0.1 \text{ m/s})^2 \quad (1^\circ)^2 \quad (1^\circ)^2 \\
x (1^\circ)^2 \quad (1 \times 10^{-4} \text{ g})^2 \quad (1 \times 10^{-4} \text{ g})^2 \\
\times (0.02(°)/h)^2 \quad (0.02(°)/h)^2 \quad (0.02(°)/h)^2. \]
System noise is set as

\[ Q = \text{diag}((1 \times 10^{-4} \text{g})^2, (1 \times 10^{-4} \text{g})^2, (1 \times 10^{-4} \text{g})^2, \]
\[ \times (0.02(\circ)/h)^2, (0.02(\circ)/h)^2, (0.02(\circ)/h)^2). \]

The measurement noise is also set as \( R = \text{diag}((0.1 \text{m/s})^2, (0.1 \text{m/s})^2, (0.1 \text{m/s})^2) \). When using Algorithms 1 and 2 and also the proposed algorithm, window widths are set as \( N = 100 \).

If the estimation of a state by each of the presented algorithms is convergent, the state will be observable, but if the estimation of a state is not convergent, the state will not be observable. Moreover, if the rank of the observability matrix is equal to the order of the system, the system will be completely observable; on the contrary, if the rank of the observability matrix is less than the order of the system, the difference between the order of the system and the rank of the observability matrix will be the number of the unobservable states. The observability matrix can be written as

\[ O = \begin{bmatrix}
\varphi & \varphi H & \varphi H^2 & \ldots & \varphi H^{11}\end{bmatrix}^T. \]

Since \( \text{Rank}(O) = 9 \), there are only nine observable states, while the other three states are unobservable. Based on the observability analysis of the SINS error model on a stationary base (Gao, Miao, & Ni, 2010b; Gao et al., 2010c; Qin, 2006), observable misalignment angles are essential. The unobservable states combination could be \( \nabla_N, \nabla_E \) and \( \varepsilon_E \) or \( \nabla_E, \varepsilon_E \) and \( \varepsilon_U \). However, the most acceptable choice of three unobservable states is \( \nabla_E, \nabla_N \) and \( \varepsilon_E \) for the initial

![Figure 1. Estimation error of \( \varphi_E \).](image1)

![Figure 2. Estimation error of \( \varphi_U \).](image2)
Figure 3. Estimation error of $\phi_N$.

Figure 4. Estimation of $\nabla_z$.

Figure 5. Estimation of $\varepsilon_y$. 
alignment process (Ali & Ushaq, 2009). Observability of the system is solely characterized by the system model and not by the different filtering algorithms. In other words, the filtering algorithm does not have any influence on observability of the system. Curves of the states $\delta\varphi_U, \delta\varphi_N$ and $\varphi_U$ are shown which are observable according to the results of the observability analysis. The estimation of $\delta\varphi_U, \delta\varphi_N$ and the estimation error of $\delta\varphi_U, \delta\varphi_N$ and $\delta\phi_U$ are shown in Figures 1, 2 and 3, respectively. As it is shown, the speed convergence of estimation error of the proposed algorithm is faster than any other algorithm and also, $\delta\varphi_U$ and $\delta\varphi_N$ are approximations to $\delta\varphi_U$ and $\delta\varphi_N$ are shown in Figures 4 and 5, respectively. From Figures 1 to 5, the following results can be obtained:

- Algorithm 1 is not superior to standard Kalman filter and in the presence of external noise, the standard Kalman filter and Algorithm 1 are not good but Algorithm 2 and the proposed algorithm have good performances.
- As can be seen in the curves, the proposed algorithm has a quicker response than any other algorithm. Besides, it is more stable.
- When the proposed algorithm is applied for initial alignment of SINS, the final estimated accuracy of misalignment angles is much more accurate than other algorithms which prove its authenticity. However, using the standard KF algorithm and Algorithm 1, the final estimation accuracy is determined by the constant drifts of the inertial sensors. The proposed algorithm can still have a perfect estimation effect. The azimuth alignment accuracy can be improved up to 50% intensity.

5. Conclusions

In this paper, a novel algorithm has been proposed for initial alignment in SINS. This algorithm can compute several fading factors based on the innovation sequence. According to the theoretical analysis, the proposed method not only has good convergence properties but also provides accurate estimations. In other words, this algorithm is more efficient than the single fading factor KF algorithm. Furthermore, computer simulation results have manifested that the new algorithm has strong robustness and adaptability.

Disclosure statement

No potential conflict of interest was reported by the authors.

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