Probing Modified Gravity by Combining Supernovae and Galaxy Cluster Surveys

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Possible explanations of the observed accelerated expansion of the Universe are the introduction of a dark energy component or the modifications of gravity at large distances. A particular difference between these scenarios is the dynamics of the growth of structures. The redshift distribution of galaxy clusters will probe this growth of structures with large precision. Here we will investigate how proposed galaxy cluster surveys will allow one to distinguish the modified gravity scenarios from dark energy models. We find that cluster counts can distinguish the Dvali-Gabadadze-Porrati model from a dark energy model, which has the same background evolution, as long as the amplitude of the primordial power spectrum is constrained by a CMB experiment like Planck. In order to achieve this, only a couple of hundred clusters in bins of width $\Delta z = 0.1$ are required. This should be easily achievable with forthcoming Sunyaev-Zel’dovich cluster counts, such as the South Pole Telescope in conjunction with the Dark Energy Survey.

I. INTRODUCTION

During the course of the last ten years evidence for an accelerated expansion of the Universe has been mounting. Initially driven by Type Ia Supernovae (SNe) observations by the Supernovae Cosmology Project (SCP) and the High-z Supernovae Search Team [1–3], the combination of large scale structure surveys like the 2 degree field (2dF) and the Sloan Digital Sky Survey (SDSS) [4, 5], cosmic microwave background (CMB) [6, 7] and x-ray observations of clusters of galaxies [8, 9] confirmed these findings. In recent years even larger samples of Type Ia SNe [10–12] indicate strongly that the expansion of the Universe is speeding up. While the observations are getting better and better, theoretical models of accelerated expansions are still in its infancy. The most natural candidate seems to be a cosmological constant, however this comes with a drawback of fine tuning of the initial conditions to unnatural values.

In general there are three possible approaches to obtain accelerated expansion of the Universe. Firstly there could be an additional component in the energy-momentum tensor, which dominates the late time dynamics of the Universe and leads to accelerated expansion. This component requires negative pressure and an example of such a fluid could be a nearly homogeneous scalar field prevailing in the Universe [13–17]. The second possibility, is that Einstein’s equations of gravity require modification on large scales [18–22]. The third possible explanation is that the Universe is inhomogeneous on large scale, although it is hard to explain the boundary conditions for such a scenario in a natural way [23]. In this article we concentrate on the first two possibilities.

Numerous dark energy models have been suggested in the past, where canonical scalar fields with various potentials are only a sub-class [13–17]. In all these models the background evolution can be described by the equation of state, i.e. the relation between the pressure and density of the dark energy fluid. The standard dark energy models also have a standard evolution of the growth of structures. Theories of modified gravity are only emerging as possible explanations for the observed acceleration. One of the most prominent models is one, which is motivated by brane cosmology involving extra dimensions and hence lead to an effectively modified Friedman equation on the 3+1 dimensional brane [18, 21]. Another possibility is to have inverse curvature terms in the Lagrangian of gravity. The additional terms become relevant at the late time evolution of the Universe and can lead to accelerated expansion and hence explain the Supernovae data [19, 20, 24]. When modifying gravity, one has to be very careful so as not to violate high precision tests of gravity in the local Universe, like the Solar System, and also not to introduce unphysical features in the theory, like negative energy eigenstates, namely ghosts, or superluminal modes [21, 25–32]. For the rest of this article we will concentrate on the Dvali-Gabadadze-Porrati (DGP) model as an example for a modified gravity model. The main reason for this is that in this model the dynamics of linear perturbations has been worked out [33–35]. Although, some care is required on large scales [34, 36], this is not important for the cluster count test, which probes power spectra on sub-horizon scales.

II. DARK ENERGY VERSUS DGP COSMOLOGY

For the analysis presented here we take a DGP model as our fiducial model. The DGP model is an example of a brane world model, where our four dimensional Universe resides in a five dimensional space on a brane [18]. The novelty about this model, compared to other brane world models, is that the flat extra dimension is allowed to be large [21]. The changes of the cosmological evolution on the four dimensional brane, compared to standard gravity, are caused by ‘leakage’ of gravitational degrees of freedom off the brane. Cosmological solutions allow empty Universes with accelerated expansion [37, 38]. The resulting modified Friedman equation for the simplest of
these models is given by [21]

$$H_{\text{DGP}}^2 \frac{H_{\text{DGP}}}{r_c} = \frac{8\pi G}{3} \rho,$$  \hspace{1cm} \text{(1)}

where $H_{\text{DGP}}$ is the Hubble parameter in the DGP model, $G$ the four dimensional gravitational constant and $\rho$ the energy density of the constituents of the Universe. For the analysis in this article we focus on the late time evolution and set overall density $\rho$ equal to the matter density $\rho_m$. Gravity on the brane is affected by 5-dimensional effects above the crossover scale $r_c$. Self-acceleration of the Universe takes place today if $r_c \sim H_0^{-1}$. For a flat Universe, the parameter $r_c$ is related to the matter contents $\Omega_m$, by [21]

$$\frac{1}{H_0 r_c} = 1 - \Omega_m,$$  \hspace{1cm} \text{(2)}

which ensures that one obtains $H_0$ as the expansion rate of the Universe today.

In order to compare the evolution of Universe in the DGP world with the one of a dark energy model, we note that the Hubble parameter in standard gravity at late times is given by

$$H_{\text{std}}^2 = \frac{8\pi G}{3} \left\{ \rho_m a^{-3} + \rho_\text{de,0} \exp \left[ -3 \int_{a_0}^{a} \frac{1 + w(a')}{a'} da' \right] \right\},$$  \hspace{1cm} \text{(3)}

with $w(a)$ the equation of state factor of the dark energy component. By identifying the Hubble parameters in Eqn. 1 and 3 we obtain $w(a)$ which mimics the background evolution of the DGP model

$$w(a) = -1 + \frac{\Omega_m a^{-3}}{(r_c H_0)^{-1} + 2\eta},$$  \hspace{1cm} \text{(4)}

with $\eta = \sqrt{\Omega_m a^{-3} + 1/(2r_c H_0)^2}$. This tends to $w(a = 1) = -1/(1 + \Omega_m)$ today and to $w(a = 0) = -1/2$ at early times, if we include the condition in Eqn. (2). In Fig. 1 we show the mimicking equation of state factor (dot-dashed line), if we set $\Omega_m = 0.3$. Given that a $\Lambda$CDM model ($w = -1$, dashed line) fits current observations very well, this model might be already under some strain from observations. The behaviour of the mimic model at late and early times also allows to obtain the coefficients of a popular parameterization of the equation of state factor, which is given by $w(a) = w_0 + w_a (1 - a)$ [39, 40]. For the case we discuss here the coefficients are: $w_0 = -0.77$ and $w_a = 0.27$. The equation of state factor for this parameterization is shown by the triple dot-dashed line in Fig. 1.

We will now address the question how a purely geometrical probe, like the Supernovae magnitude - redshift relation, can constrain the DGP model. For this we calculate the model predictions for the magnitudes, which are given by $m \propto \log d_L$ and the luminosity distance $d_L = (1 + z) \int_0^z dz' / H(z')$. In Fig. 2 we plot the magnitude difference to the fiducial DGP model with $H_0 = 72$km/s/Mpc and $\Omega_m = 0.3$. From this we obtain with Eqn. 2 a fiducial value for $r_c H_0 \approx 1.43$. The shaded region shows the magnitude difference if we vary $r_c H_0$ between 1.38 and 1.48. Note that for these values the Hubble parameter today is still well within the errorbars of its measured value of $H_0 = (72 \pm 8)$km/s/Mpc [41].
We also include mock data points from a SuperNovae Acceleration Probe (SNAP)-like survey [42]. We assume a distribution of 2000 Type Ia SNe between $z = 0.3$ and $z = 1.7$ with a magnitude error of $\sigma_m = 0.15$ for an individual SNe [43]. We plot the statistical error bars in redshift bins of $\Delta z = 0.1$. A SNAP-like experiment will not be able to distinguish DGP models with a geometric probe, like the SNe magnitude-redshift relation, be spot on top of the fiducial solid line in the plot. Nevertheless, it is evident from the plot, that SNAP could distinguish the DGP models from a $\Lambda$CDM model (dashed line). Of course the dark energy model with an equation of state given by Eqn. (4), would be spot on top of the fiducial solid line in the plot. A pure geometric probe, like the SNe magnitude-redshift relation, can not distinguish a modified gravity model from an arbitrary dark energy model. However, a SNAP-like survey is likely to probe the geometry and hence the background evolution very well.

So far we have concentrated on SNe as a cosmological probe and established that SNe alone cannot distinguish dark energy from the DGP model. However, recent studies have shown that weak lensing, baryon oscillations or observations of the integrated Sachs-Wolfe effect have potential to distinguish dark energy from modified gravity [33, 44–47]. The ability of these probes to distinguish the DGP model from dark energy is driven by the different dynamics of the growth of structure. We will investigate if counts of galaxy clusters will be different in modified gravity scenarios [50], the dominant contribution is the different dynamics of the growth of structures. We will investigate if counts of galaxy clusters can distinguish the DGP model from the mimick dark energy model. This is similar to the earlier works, which concentrate on weak lensing, baryon oscillations and integrated Sachs-Wolfe effect observations [33, 44–47].

The number of clusters per redshift bin is given by [51]

$$\frac{dN}{dz} = \Delta \Omega \frac{dV}{dz d\Omega}(z) \int_{M_{\text{lim}}(z)}^\infty \frac{dn}{dM} dM,$$  

where $dV/(dz d\Omega) = [r(z)]^2/H$ is the comoving volume in a flat universe, with $r(z) = \int_0^z H^{-1}(z')dz'$ the coordinate distance and $\Delta \Omega$ is the angular sky coverage of the survey. $M_{\text{lim}}(z)$ is the limiting mass, which will depend in general on the parameters of the survey and cosmology.

The near future will see a few surveys, which will perform a blind search for galaxy clusters and identify them via the so called Sunyaev-Zel’dovich decrement in the CMB radiation [52]. The interesting feature of this effect, is that it is independent of redshift and hence could potentially identify clusters at large redshift. Upcoming surveys are the South Pole Telescope (SPT) [53], Atacama Cosmology Telescope (ACT) [54] and Atacama Pathfinder EXperiment (APEX) [55]. In order to perform number counts of galaxy clusters in redshift bins we also require the redshift. The proposed Dark Energy Survey (DES) [56] is designed to obtain the redshifts of the clusters identified by SPT. SPT will observe 4,000 deg$^2$ of sky with a 1 arcminute beam. In order not to be too SPT specific in the analysis presented here, we do not calculate the limiting mass of the survey from the instrument parameters for SPT. We assume an effective constant mass limit of $M_{\text{lim}} = 1.7 \times 10^{14} h^{-1} M_\odot$, which corresponds roughly to the mass limit of the SPT survey [51]. However the results we find will apply to any kind of galaxy cluster survey. Apart from the cosmological parameters mentioned so far, we also require for the analytical prediction of the cluster counts the normalization and slope of the primordial matter power spectrum. We choose a slope of $n = 1$ and $\sigma_8 = 0.75$ [7]. For this survey and fiducial model we find $\approx 7200$ clusters of galaxies. In Fig. 3 we show the results for counts of clusters in redshift bins of $\Delta z = 0.1$. The solid line is the fiducial DGP model, with randomly generated mock data points including Poisson errors. The faint dotted lines on either side of the fiducial model correspond to $1/(r_c H_0) = 1.38$ and $1/(r_c H_0) = 1.48$. Note that these models are hardly distinguishable from the fiducial model in the number counts. The dot-dashed line is for the mimick dark energy model with equation of state given by Eqn. 4. The difference is, particularly in the peak of the distribution,
well above the 2σ threshold. This is driven by the difference in the growth factor of the DGP model, given by Eqn. 5 and the mimic dark energy model, which is obtained for δσ = 0.03 and the dark shaded on for δσ = 0.01.

FIG. 3: Number of clusters of galaxies above a threshold of $M_{\text{lim}} = 1.7 \times 10^{14} h^{-1} M_\odot$ for the concordance ΛCDM Universe (dashed) and the fiducial DGP model (solid). The thin dashed line is for a model with $w = -0.8$. The dotted lines are for the $r_c = 1.38/H_0$ and $r_c = 1.48/H_0$ models respectively. The dot-dashed line is for a dark energy model, which mimics the background evolution of the DGP scenario. The errorbars assume Poisson errors for each bin with $\Delta z = 0.1$ for a SPT-like experiment. The shaded regions are illustrating the uncertainties on the mimic model from propagating errors on $w_0$, $w_a$, $\Omega_m$ and $\sigma_8$ from SNe and CMB observations. The light shaded region is obtained by assuming $\delta\sigma_8 = 0.03$ and the dark shaded on for $\delta\sigma_8 = 0.01$.

We have shown that the background evolution of the DGP model can be entirely mimicked by a dark energy model with a suitably chosen equation of state, given in Eqn. 4. Due to this fact Type Ia SNe alone can not distinguish modified gravity models from dark energy. However, due to the different time evolution of the linear perturbations, galaxy cluster counts can distinguish the DGP model from a dark energy model with the same background evolution. The difference between these two models is more then at the 2σ level, even for relatively low number of clusters $\approx 7000$. This result is only slightly weakened if we include parameter degeneracies in the mimic model. We have presented a very simplified analysis in the article, but the striking difference between the mimic dark energy and the DGP model, raises hopes that this result also stands up in a more careful analysis, which for example includes uncertainties in the gas physics of clusters in order to establish the mass limit [61].

A further interesting question would be to see whether our findings also hold up for other modified gravity scenarios, like the inverse curvature model [19]. In order to answer this question a perturbative analysis of these models is required, which is not an easy task. It would be very surprising if the standard perturbation result holds for these models. In general we would expect that modified gravity models lead to a modification of the growth of structure. Clusters of galaxies in combination with a geometrical probe, like SNe distances, could provide a uniquely sensitive window for the distinction between general modifications of gravity and dark energy.

IV. CONCLUSION
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