Dark matter annihilation from intermediate-mass black holes: Contribution to the extragalactic gamma-ray background

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We investigate contributions to the extragalactic gamma-ray background (EGB) due to neutralino dark matter (DM) pair-annihilation into photons, from DM density enhancements (minispikes) surrounding intermediate-mass black holes (IMBHs). We focus on two IMBH formation scenarios; our conservative scenario where IMBHs are remnants of Population-III stars, and our optimistic scenario where IMBHs are formed in protogalactic disks. In both scenarios, their formation in pregalactic halos at high redshift lead to the formation of minispikes that are bright sources of gamma-ray photons. Taking into account minispike depletion processes, we only sum contributions from a cosmological distribution of IMBHs with maintained minispikes. Our conservative scenario (BH mass $10^3M_\odot$ with a $r^{-3/2}$ minispike) predicts gamma-ray fluxes that are an order larger than the equivalent flux, using the same DM parameters (mass 100 GeV and annihilation cross section $3 \times 10^{-26}$ cm$^3$ s$^{-1}$), from the host halo without IMBH minispires. Our optimistic scenario (BH mass $10^5M_\odot$ with a $r^{-7/3}$ minispike) predicts fluxes that are three orders larger, that can reach current EGB observations taken by EGRET (DM parameters as above). This fact may serve interesting consequences for constraining DM parameters and elucidating the true nature of IMBHs. Additionally, we determine the spectra of DM annihilation into monochromatic gamma rays, and show that its flux can be within observational range of GLAST, providing a potential “smoking-gun” signature of DM.

I. INTRODUCTION

Despite compelling indirect evidence, from galactic to cosmological scales, the fundamental nature of the dominant nonbaryonic component in the matter density of the Universe (dark matter, hereafter DM) remains unknown. Intriguingly, extended models of particle physics independently provide us with a host of particle candidates for this as yet unknown matter, of which the most popular is the supersymmetric neutralino (see reviews [1–3] for details). Upgrades of underground direct detectors looking for scattering of DM particles from nuclei, together with future neutrino, antimatter, and gamma-ray detectors looking for products of DM annihilation, will dramatically enhance our chances of understanding the true nature of DM. In particular, the forthcoming launch of the Gamma Ray Large Area Space Telescope (GLAST) [4] and numerous ground based Atmospheric Cerenkov Telescopes make indirect gamma-ray search especially promising.

Since the DM annihilation rate scales as the DM density squared, there is great advantage in observing areas where the DM density is believed to be high. The galactic center (GC) is the immediate choice, and indeed strong gamma-ray emission has been observed and its nature and origin have been investigated by many researchers [5–13]. However, the DM density in the GC is highly uncertain, making accurate predictions difficult. For example, DM enhancements called “spikes” can form during the formation of a central supermassive-BH (SMBH) [14], but it can also be depleted by various processes by varying degrees [15–18]. In addition, nearby astrophysical (non-DM) gamma-ray sources make a potential DM detection impossible for all but a narrow range of DM parameters [19]. Intermediate-mass black holes (IMBHs, see e.g. [20]) provide an alternative source that may work positively for DM detection. Bertone et al. [21] recently investigated the possibility of detecting a “smoking-gun” gamma-ray signature of DM using IMBHs in the Milky Way as point sources. They showed that IMBH formation increases the DM density in its vicinity to produce a “minispike,” and also that DM enhancement depletion processes are generally less significant for IMBHs due to their roughly spherical distribution about the GC. They conclude that under optimistic circumstances, the Energetic Gamma Ray Experimental Telescope (EGRET) may have already seen a few of the IMBH minispires as unidentified sources.

Another avenue of indirect DM search is via the extragalactic gamma-ray background (hereafter EGB) measured over a wide energy range [22,23]. The origin of this background is currently unknown, and it has been speculated that DM annihilation gamma rays from cosmo-

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logical distributions of DM contribute to some degree [24–31]. Since the DM annihilation cross section is so small, a consideration of DM enhancements is crucial for meaningful gamma-ray flux predictions. A popular DM enhancement is those at the centers of galactic DM halos. However, Ando [29] has recently shown that they are strongly constrained by observations of our galaxy. The author assumes universality of galactic DM halo profiles, and shows that DM annihilation cannot significantly contribute to the EGB without exceeding gamma-ray observations from our GC [29]. The author also points out that this constraint could be loosened when one takes DM substructures into account.

In this paper we argue IMBHs minispikes as a substructure in the DM halo, that can lead to enhancements that do not conflict with current observations of our galaxy. We determine contributions to the EGB from IMBH minispikes by summing gamma-ray fluxes from all redshifts. IMBH minispikes are not expected to greatly suffer from depletion processes, but we do take into account BH-BH mergers, which are known to strongly deplete minispikes and do occur in IMBHs. We also consider a conservative case ($10^3 M_\odot$ BHs of Population-III origin [32] with a $r^{-3/2}$ minispike) and an optimistic case ($\sim 10^5 M_\odot$ BHs formed in the centers of protogalactic disks [33] with a $r^{-7/3}$ minispike). Our result is that contributions to the EGB are increased by 1–3 orders in magnitude. In particular, our optimistic case predicts fluxes that can reach current EGB observations. As this has interesting implications for constraining DM parameters and IMBH scenarios, we critically assess uncertainties in our calculation. We then determine the flux of DM annihilation into line gamma rays, and show that under optimistic conditions, it is observable by GLAST. This provides a potential smoking-gun signature of DM.

This paper is structured as follows. In Sec. II we introduce IMBHs, starting with their existence, followed by their formation scenarios, and finishing off with a summary of recent numerical studies. Then in Sec. III we develop our calculation frameworks, first for the EGB, followed by DM annihilation, then minispike formation, moving finally on to our IMBH number density fitting. Calculations and results are in Sec. IV, and discussions and conclusions in Sec. V. In all our calculations we adopt the standard flat cosmological constant plus cold DM ($\Lambda$CDM) cosmology, with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\sigma_8 = 0.9$.

II. IMBH: EVIDENCE AND PROPERTIES

A. Evidence for IMBH

Clues for the existence of a class of BHs with masses heavier than stellar BHs but lighter than supermassive black holes (SMBH) have accumulated in recent years. We call these BHs intermediate-mass BHs (IMBHs) and loosely define their mass range as $20 \leq M_{BH}/M_\odot \leq 10^6$.

We briefly discuss the observational evidences and theoretical motivations for their existence.

Observationally, studies of objects known as ultraluminous x-ray sources (ULXs, [34,35]) reveal that they may harbor IMBHs. Although most x-ray sources can be understood as accretion by compact objects such as BHs, there is an upper limit on the luminosity for a given BH mass, known as the Eddington limit. For $20 M_\odot$, commonly accepted as the upper limit of stellar BHs, this limit is $\sim 2.8 \times 10^{39}$ ergs s$^{-1}$. ULXs are observed to exceed this limit. This phenomena can be explained by several mechanisms, including a short-term super-Eddington phase [36], beaming [37], or normal accretion by an IMBH. Although the debate has not been settled, evidence favoring the IMBH mechanism over the other two has accumulated in recent years; these come in various forms, including spectral analysis [20,38], evidence for a low temperature ($\sim 0.1$ keV) black-body component [39], analysis of break frequencies of the power density spectrum [40], and observation of broad Fe lines and quasi-periodic oscillations (QPO) [41]. It seems that at least a fraction of the ULXs, in particular, the most luminous ones, are IMBHs. The BH mass inferred from ULX observations is of the order $\sim 10^5 M_\odot$ (assuming no beaming and accretion efficiency $\sim 10\%$). Also, from their positions in the host galaxies we can deduce an upper limit of $\leq 10^6 M_\odot$, in order not to sink to the galactic center by dynamical friction within a Hubble time [20].

Theoretically, the existence of a population of IMBHs helps explain the origin of SMBHs. There is as yet no definitive SMBH formation scenario, but the discovery of quasars at redshifts $z = 6$ in the Sloan Digital Sky Survey [42–46] suggests that they were already formed at high redshifts. Such an early formation lends itself to scenarios with massive seed-BHs, frequent mergers, and rapid accretion. A hierarchical formation scenario starting from massive seed-BHs also helps explain the tight correlations observed between the SMBH mass and properties of the host galaxy and halo [33]. The natural outcome of this hierarchical scenario is the existence of “wandering” BHs in galactic halos, resulting from seed-BHs that did not successfully merge into a SMBH [47–49], which we call IMBHs. However, despite the theoretical interests, it is difficult to obtain conclusive evidence for their existence. One viable strategy is to search for gravitational waves produced in seed-BH mergers, which may become possible with the launch of the Laser Interferometer Space Antenna (LISA) [48,50].

B. IMBH formation scenarios

In this study, we follow a previous study by Bertone et al. [21] and focus on two seed-BH formation scenarios covering the wide range of possible IMBH masses. In the first scenario, which we refer to as scenario A, the seed-BHs are remnants of the collapse of Population-III (or first
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...generation) stars [32]. These stars are generally massive due to suppression of mass-loss processes, a result of their very low-metal composition. As such, they are also called zero-metallicity or very massive stars (VMS). The fate of VMS have been studied by e.g. [51, 52]: stars with $100 \leq M/M_\odot \leq 250$ encounter the electron-positron pair instability and explode in a giant nuclear-powered explosion, leaving no compact remnant, while heavier stars collapse completely to a BH containing at least half of the stellar mass [52].

We need to know is the mass-function of these BHs. Unfortunately, the mass-function of Population-III stars is not well known, although recent studies indicate a double peaked function that extends up to a few $10^3 M_\odot$ [53]. We base our scenario on the one proposed in [32] and further studied by [21, 47, 49, 54]. Interestingly, if BHs with masses $\gtrsim 10^2 M_\odot$ form at high redshifts in minihalos representing $\sim 3\sigma$ peaks of the smoothed density field, the resulting baryon mass fraction is found to be comparable to those observed for SMBHs in nearby galaxies [32].

As in [21], we conservatively consider BHs of mass $10^2 M_\odot$ forming in minihalos of masses larger than $M_{v,\text{crit}} \sim 10^7 M_\odot$ at formation redshift $z_f \sim 18$.

In the second scenario, which we refer to as scenario B, BHs form from low angular momentum gas in protogalactic disks at high redshifts, producing a population of seed-BHs with masses $\sim 10^3 M_\odot$. We use the scenario proposed in [33], which we briefly summarize. During the collapse of the first halos, gas cools and a pressure supported disk forms if the halo is massive enough to contain a relatively large fraction of molecular hydrogen (molecular hydrogen is the main coolant, see [55]). Local gravitational instabilities in the disk manifest themselves as an effective viscosity that transfers angular momentum outwards and cause an inflow of gas, in particular, the low angular momentum tail. In halos that are both massive enough to contain enough hydrogen for cooling and do not experience mergers with other halos, the protogalactic disk can evolve uninterrupted until ultimately being terminated by the heating and disruption caused by supernovae of Population-III stars. During this time, mass transfer of order $\sim 10^5 M_\odot$ occurs. The central mass may be briefly pressure supported but will ultimately collapse to a BH due to post-Newtonian instabilities. Since Population-III stars have typical lifetimes of 1–10 Myrs, the halo must be avoid of mergers over many dynamical times. This sets a stringent lower limit on the required halo mass. At $z_f = 12$, this limit is $M_{v,\text{crit}} = 10^8 M_\odot$ (see [33] for an expression). The masses of the BHs that form have a near log-normal distribution with peak mass $M_{\text{bh},0} = 2.3 \times 10^3 M_\odot$ independent of $z_f$, with spread $\sigma_{\text{bh}} = 0.9$ [33].

Scenario B naturally contains many parameters other than $z_f$ (such as fraction of gas cooled, lifetime of Population-III stars, etc.). However, uncertainties in these parameters largely affect $M_{\text{bh},0}$, and as such are ultimately masked by uncertainties in $z_f$, which affects the total number of seed-BHs formed. We therefore treat scenario B formation to be described by one parameter, $z_f$. Now, the epoch of cosmological reionization places a lower limit on $z_f$. This is because the heating of the intergalactic medium and the subsequent ionization of molecular hydrogen (i.e. reionization) terminates further baryon cooling. Without molecular hydrogen, scenario B formation cannot proceed, even in the heaviest halos which satisfy $M_v > M_{v,\text{crit}}$. Therefore, for scenario B, $z_f = z_{\text{re}}$, where $z_{\text{re}}$ is the redshift of reionization (we assume for simplicity that seed-BH formation stops abruptly at reionization).

C. IMBH number density: Results from numerical studies

Now that we have discussed IMBH formation, we will summarize how their number density evolves with time. Since in this work we are only interested in IMBHs with minispikes, from here on we will use “IMBH” to imply “an IMBH with a maintained minispike.” First, although seed-BH formation can lead to minispike formation, various processes destroy it by varying degrees (see Sec. III C), and the strength and number of minispikes decrease with time. BH mergers, i.e. mergering with another IMBH or a SMBH, are the most destructive and we must therefore consider its effects. Before doing so however (in Sec. III D) we summarize basic procedures and results of previous numerical studies.

The basic approach focuses on constructing a statistical sample of halo formation histories (each called a “realization”) in the context of the hierarchical CDM model for structure formation, followed by computing the dynamical evolution of halos and BHs within halos (see [21, 47, 48, 56] for further details). The first step is to consider a virialized halo of mass $M_{v,0}$ at $z = 0$ and construct a merger history, i.e. a list of all the smaller halos that merged together to form the final halo, as well as the redshifts at which the mergers occurred. The next step is to plant BHs in halos that satisfy seed-BH formation conditions. That is, if a halo satisfies $M_v > M_{v,\text{crit}}$ and $z > z_f$, a seed-BH is planted at the earliest time $M_v > M_{v,\text{crit}}$ still holds. This point is labeled $z_{\text{bh}}$. The last step involves evolving the halos and BHs forward to $z = 0$, as described in [48, 56]. During the last step, a BH that comes within a distance $\text{min}(0.01 r_{\text{vir}}, 1 \text{kpc})$, where $r_{\text{vir}}$ is the virial radius of the halo, of another BH is considered a BH-BH “potential pair,” and is decoupled from the simulation. Previous work focused on the gravitational waves produced by the potential pairs’ subsequent mergers [48], but for our purposes we exclude all potential pairs from our EGB calculation.

What necessary results can we obtain from these studies? First, the distribution of BH formation redshifts $z_{\text{bh}}$ is found to be exponential, i.e. using the Milky Way galaxy
with $M_o = 10^{12.7} h^{-1} M_\odot$ and 200 realizations, [48] finds that the number of BHs formed peaks at $z_f$ and decreases exponentially for higher $z_f$ (see their Fig. 2). Second, by following the dynamical evolution of BHs, [21] finds that $N_{bh,A} = 1027 \pm 84$ scenario A BHs and $N_{bh,B} = 101 \pm 22$ scenario B BHs remain unmerged in a Milky Way-sized halo. Errors denote the 1$\sigma$ halo-to-halo scatter (200 realizations were performed). Finally, [48] finds that the number of potential pairs formed is highest at $z_f$ when the BH number density is largest, and decreases as a power-law of $(1 + z)$ with decreasing $z$ (see their Fig. 3).

III. FORMULATIONS

A. Extragalactic gamma-ray background

To calculate contributions to the EGB flux from unresolved cosmological DM sources, we adopt the methodology in Ullio et al. [25], but extend it for our purposes of including IMBH minispike enhancements.

Let $dN_{\gamma}/dE(E, M_{bh}, z)$ be the differential energy spectrum for the number of gamma-ray photons emitted per unit time from a single IMBH of mass $M_{bh}$ at redshift $z$, and $dn/dM_{bh}(M_{bh}, z)$ the comoving number density of IMBHs of mass $M_{bh}$ at redshift $z$. From these we can determine the number of photons emitted in a proper volume $dV$ say, at redshift $z$ in time interval $dt_0$ and energy range $(E, E + dE)$. Then, assuming isotropic emission, the corresponding number of photons $dN_{\gamma}$ collected by a detector on Earth with an effective area $A$ in the redshifted energy range $(E_0, E_0 + dE_0)$ over a time $dt_0$, is

$$dN_{\gamma} = \int dM_{bh}(1 + z)^3 \frac{dn}{dM_{bh}}(M_{bh}, z) \frac{dN_{\gamma}}{dE} \times (E, M_{bh}, z) e^{-\tau(E_0, z)} \frac{dVdA}{4\pi R_0^2} dE_0 dt_0,$$

where we have used the fact that $dEdt = (1 + z)dE_0(1 + z)^{-1}dt_0 = dE_0 dt_0$. The exponential term is an attenuation factor which accounts for the absorption of gamma rays during propagation to Earth, the factor $(1 + z)^3$ converts from comoving to proper IMBH number density, and $R_0$ comes from the metric of our cosmology as defined in Sec. I. We define $dV$ by radial increment $dr$ and angular increment $d\Omega$ as

$$dV = \frac{(R_0 r)^2 R_0}{(1 + z)^3} dr d\Omega.$$

The main absorption of gamma rays of a few GeV is via pair production on the extragalactic background light emitted by galaxies in the optical and infrared bands. Although at these gamma-ray energies attenuation is almost negligible, we use the form $e^{-c z / \bar{z}_{max}}$, where we approximate $\bar{z}_{max} = 3.3(E_0/10 \text{ GeV})^{-0.8}$, which is a parametrization that reproduces results of [57] with enough accuracy. Substituting $dV$ into Eq. (1) and changing the integration along the line of sight $dr$ to along $dz$, we get for the flux

$$\frac{d\Phi_{\gamma}}{dE_0} = \frac{dN_{\gamma}}{dAdd\Omega dE_0} \times (E_0(1 + z), M_{bh}, z),$$

where $c$ is the speed of light, $H_0$ is the Hubble parameter now, and $h(z)$ is the function $h(z) = \sqrt{\Omega_M(1 + z)^3 + \Omega_k}$.

We use Eq. (2) to compute the final EGB flux. In the following subsections, we will deal with the physical quantities in the expression in detail.

B. Dark matter annihilation gamma rays

In order for DM particles to satisfy cosmological constraints, they are expected to have a small but nonzero annihilation cross section into standard model particles. This ensures they are in chemical equilibrium in the early universe. By constraining their relic density by cosmological observations, one can obtain limits on their annihilation cross section. For DM that is a thermal relic, the required cross section is $3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. However this should be taken as an upper limit, because processes such as coannihilation can allow smaller cross sections (see [1, 3] for details). The DM particle mass is constrained from below by collider experiments and above by theory, giving a commonly accepted range of $m_\chi \sim 50 \text{ GeV} - 10 \text{ TeV}$.

Now, the flux of gamma rays from a single IMBH minispike is quantified by the term $dN_{\gamma}/dE(E, M_{bh}, z)$ in Eq. (2). For gamma rays of DM origin, we can rewrite

$$\frac{dN_{\gamma}}{dE} = \frac{\sigma \nu}{m_\chi} \frac{dN_{\gamma}(E)}{dE} \int_{r_{sp}}^{r_{lim}} n_\chi^2 d^3r$$

where the factor $1/2$ appears due to the fact this is an annihilation of identical particles [25], $\rho(r)$ is the DM density profile around an IMBH ranging from radii $r_{sp}$ (we discuss these in Sec. III C), $\sigma \nu$ is the annihilation cross section times relative velocity, and $dN_{\gamma}(E)/dE$ is the differential gamma-ray yield per annihilation. The latter can be divided into continuum and monochromatic (line) emissions, and can be written as [25]

$$\frac{dN_{\gamma}(E)}{dE} = \sum_{F} b_{\gamma F} n_{\gamma F} \delta(E - m_\chi(1 - M_{bh}^2 / 4m_\chi^2))$$

where the second term refers to the continuum. The continuum photons are produced by annihilation into the full set of tree-level final states $F$ including fermions and gauge or Higgs bosons which generate photons on decay. The
bulk of the photons however are produced in the hadronization and decay of neutral pions (decay mode $\pi^0 \rightarrow 2\gamma$), with a branching ratio of $b_F = 98.8\%$. The monochromatic emission on the other hand is the result of prompt annihilation into two-body states including a photon, a process that is forbidden at tree-level and only allowed in higher order perturbation theory. Although subdominant, these gamma rays have the advantage of producing a smoking-gun signature of DM annihilation, i.e. photons of energy $E = m_\chi$ for the $2\gamma$ final state and $E = m_\chi(1 - M_\gamma^2/4m_\chi^2)$ for the photon and particle $Y$ final state. The parameter $b_{2\gamma}$ is the branching ratio into the respective channels, and $n_{2\gamma}$ is the number of photons emitted per annihilation.

In the current work we consider both the continuum and monochromatic gamma-ray emissions. For the continuum we consider the $\pi^0$ branch, and use a conveniently parameterized form of the rest frame energy distribution per annihilation,

$$dN_{\text{cont}}(E)/dE = (0.42/m_\chi)e^{-x}/(x^{1.5} + 0.00014)$$

where $x = E/m_\chi$. For the monochromatic emission we consider the $\chi\chi \rightarrow 2\gamma$ process, which exists in many supersymmetric models. We use for the branching ratio $b_{2\gamma} = 10^{-3}$.

### C. Dark matter enhancement around IMBH

The emergence of a deeper gravitational potential due to the formation of a BH inevitably alters the DM halo in which it is formed. It has been shown that the adiabatic formation of a SMBH results in an enhancement of the nearby DM density, called a “spike” [14]. However, it has also been shown that spike formation depends on initial conditions [15], and even formed, it is depleted by processes such as BH-BH mergers [16], dynamical processes such as gravitational scattering off stars [18], and DM annihilation itself [17]. Fortunately, minispires around IMBHs may not be greatly affected by these problems. First, we have selected IMBHs that survive without experiencing any major mergers. Second, these unmerged IMBHs are not necessarily localized in the galactic center, and null observations of ULXs in our galaxy suggest they reside in satellite halos with no significant stellar component. These imply that dynamical processes are not very significant. Third, a central BH formation is a built-in property of scenario B, predicting a strong minispike. However, we must make clear that the precise likelihood of minispike formation and survival are still uncertain. With these in mind, we treat scenario B as our optimistic case and scenario A our conservative.

The BH in scenario B forms in the center of its host halo [33], and therefore predicts a strong minispike. To compute its shape we need to specify the initial DM profile. If we write the initial inner DM profile as $\rho(r) \propto r^{-7}$, the minispike has the form [21]

$$\rho_{\text{sp}}(r) = \rho(r_{\text{sp}}) (\frac{r}{r_{\text{sp}}})^{-\gamma_{\text{sp}}},$$

where $r_{\text{sp}}$ is the radius of the minispike and $\gamma_{\text{sp}}$ is the gradient, given as

$$\gamma_{\text{sp}} = \frac{9 - 2\gamma}{4 - \gamma}.$$  

Values for $\gamma$ have been proposed by analytic fits of N-body simulations, e.g. $\gamma = 1$ by Navarro, Frenk, and White (NFW) [58], and $\gamma = 1.5$ by Moore et al. [59]. Recent simulations have produced shallower profiles (see [60] and references therein) and as well as somewhere between 1–1.5 [61–63]. In this work we assume a middle value of $\gamma = 1$, but keep in mind that a smaller value will lead to less gamma rays. For reference, our values for halo radii are summarized in Table I.

For scenario A, while some works show that they are formed in the centers of their host halos [65], others show fragmentation may lead to off-center BHs [66]. Although motivations for a central BH are strong, to remain conservative we consider an off-center BH formation. It has been shown that BH formation in a uniform DM background will form a mild $r^{-3/2}$ minispike [21,54,67,68]. Specifically, we consider the minispike studied in [21,54], with $p(r) = \rho_h(r/r_h)^{-3/2}$ where $r_h = 0.045$ pc.

In both scenario A and B minispires, the very DM annihilation we are studying sets a limit on the inner minispike radius. Assuming DM annihilation is the main process by which the inner density decreases, the DM number density $n_x$ obeys the evolution equation $\dot{n}_x(r, t) = -\sigma v n_x^2(r, t)$. Solving this yields the solution

$$n_x(r, t) = \frac{n_x(r, t_f)}{1 + n_x(r, t_f)\sigma v (t - t_f)},$$

where $t - t_f$ is the time elapsed since BH formation. The upper limit is of order $m_\chi/\sigma v (t - t_f)$, and we define $r_{\text{lim}}$ as the radius at which the following holds

$$\rho_{\text{sp}}(r_{\text{lim}}) = \frac{m_\chi}{\sigma v (t - t_f)} \equiv \rho_{\text{lim}},$$

where $m_\chi$ is the DM mass. For common DM parameters, $r_{\text{lim}}$ has grown to $\approx 10^{-3}$ pc by $z = 0$.

### TABLE I. Representative values of halo radii for scenario B, for two formation redshifts. Shown are the halo virial radius $r_{\text{vir}}$, the scale radius $r_s$, $r_{\text{lim}}$, and the inner radius of the minispike $r_{\text{sp}}$. DM parameters are $m_\chi = 100$ GeV and $\sigma v = 3 \times 10^{-26}$ cm$^3$s$^{-1}$.

| Radii, $z_f = 10.9$ [pc] | Radii, $z_f = 12$ [pc] |
|-------------------------|-------------------------|
| $r_{\text{vir}}$        | 1300                     | 1100                      |
| $r_s$                   | 510                      | 480                       |
| $r_h$                   | 34                       | 33                        |
| $r_{\text{sp}}$        | 6.8                      | 6.7                       |
| $r_{\text{lim}}(z = 0)$| $5 \times 10^{-3}$       | $4 \times 10^{-3}$        |


D. Modelling the IMBH number density

Now we parametrize the decreasing IMBH number density, using results of previous numerical studies. To determine the initial (i.e. at $z_f$) IMBH number density, we consider a delta-function seed-BH formation occurring at $z_f$, and plant BHs in halos that satisfy $M_v \approx M_{v,\text{crit}}$. This simplified picture neglects BHs that would have formed earlier. That is, for some halos we plant one BH where in fact there would be two (or more). However, as [48] shows, the number of BHs formed in $z > z_f$ decreases exponentially. This, coupled to the fact that massive halos become increasingly unlikely for higher redshifts, the effect of ignoring them is minimal. We also stress that this picture in no way overestimates EGB contributions, because (i) we have underestimated the initial BH density, and (ii) we have neglected all DM annihilations before $z_f$. Moreover, any underestimation is masked by uncertainties caused by $z_f$.

The required calculation is

$$n_{\text{bh}}(z_f) = \int_{M_{v,\text{crit}}}^{\infty} dM_v \frac{dn}{dM_v} (z = z_f), \quad (9)$$

where $n_{\text{bh}}$ is the comoving number density of IMBHs that have not experienced any mergers, and $dn/dM_v$ is the halo mass-function. For the mass-function we use one postulated by Press-Schechter theory [69],

$$\frac{dn}{dM} = \frac{\bar{\rho}_0}{M^2} \nu f(\nu) \frac{d\log \nu}{d\log M}, \quad (10)$$

where $\bar{\rho}_0$ is the comoving matter background density, $\bar{\rho}_0 = \rho_c \Omega_M$, with $\rho_c$ the critical density. The parameter $\nu \equiv \delta_c(z)/\sigma(M)$ is defined as the ratio between the critical linear fractional overdensity required for virialization over $\sigma(M)$ the present rms linear density fluctuation in spheres containing a mean mass $M$. For the multiplicity function $f(\nu)$ we use the ellipsoidal collapse model [70], normalized to results of $N$-body simulations of the Virgo Consortium [71], as was done in [25].

Now, we assume that the number of IMBHs present and the number of potential pairs formed are proportional, and fit a power-law redshift dependency to $n_{\text{bh}}$,

$$n_{\text{bh}}(z) = n_{\text{bh}}(z_f) \left(\frac{1 + z}{1 + z_f}\right)^\beta, \quad (11)$$

where $\beta$ is a free parameter. We can derive representative values for $\beta$ by fitting $n_{\text{bh}}(0)$ to numerical results by [21]. From their value of $N_{\text{bh,A}}(N_{\text{bh,B}})$, we interpolate the number of unmerged IMBHs in other galaxies by their halo masses, and determine $n_{\text{bh}}(0)$ as

$$n_{\text{bh}}(0) = N_{\text{bh}} \int_{M_{v,\text{crit}}}^{\infty} dM_v \left(10^{12} h^{-1} M_\odot\right)^\alpha \frac{dn}{dM_v}(0), \quad (12)$$

where $\alpha$ is some constant which we assume to be 1. This gives $\beta \approx 0.2 (0.3)$ for scenarios A (scenario B). Although in reality we do not expect $\alpha$ to be exactly 1 (e.g. we expect less unmerged IMBHs in older elliptical galaxies), we find that for reasonable deviations from $\alpha = 1$ our estimates of $\beta$ do not appreciably change. Moreover, uncertainties caused by deviations in $\alpha$ are far smaller than those caused by $z_f$.

Lastly, we conclude this section with the parameters of scenarios A and B in Table II.

### IV. CALCULATION AND RESULTS

#### A. Contribution to the EGB

Substituting Eq. (3) into Eq. (2) we find the gamma-ray flux observed at Earth as

$$\frac{d\Phi_\gamma}{dE_0} = \frac{\sigma v}{8\pi H_0 m^2_e} \int_0^{z_f} dz \frac{\Delta^2(z) dN_{\gamma}(E')}{h(z)} \frac{dE'}{dE} e^{-z/z_{\text{max}}}, \quad (13)$$

where $E' = E_{\text{0}}(1 + z)$ and $\Delta^2(z)$ is defined

$$\Delta^2(z) = \frac{1}{\bar{\rho}_0^2} \int dM_{\text{bh}} \frac{dn}{dM_{\text{bh}}}(M_{\text{bh}}, z) \int_{r_{\text{lim}}}^{r_{\text{sp}}} \rho^2(r) d^3r. \quad (14)$$

Note the disappearance of the $(1 + z)^3$ term in Eq. (13) compared to [25]. The reason for this is that while the DM density in the halo is a function of $z$, the density in the minispike around an IMBH is only a function of $z_f$; the $z$ dependence is taken into account instead by $r_{\text{lim}}$.

For scenario B, since the BH mass distribution is near log-normal, and the enhancement due to the minispike scales linearly with the BH mass, we find that with very good accuracy we can approximate all the BH mass to have the peak value $M_{\text{bh,0}}$. Hence, we substitute $dn/dM_{\text{bh}}(M_{\text{bh}}, z) = n_{\text{bh}}(z) \delta(M_{\text{bh}} - M_{\text{bh},0})$, which gives the final form we will use,

$$\Delta^2(z) = \frac{1}{\bar{\rho}_0^2} n_{\text{bh}}(z) \int_{r_{\text{lim}}}^{r_{\text{sp}}} \rho(r)^2 d^3r. \quad (15)$$

#### TABLE II. Scenario A and B parameters, including formation redshift $z_f$, the minimum halo mass required for formation to occur $M_{v,\text{crit}}$, the number density of IMBH $n_{\text{bh}}$ at $z = z_f$ determined from Eq. (9), the number of IMBHs residing in a Milky Way-size galactic halo $N_{\text{bh}}$ [21], $n_{\text{bh}}$ at $z = 0$ derived from Eq. (12), and $\beta$ derived from our fitting.

| Scenario A | Scenario B |
|------------|------------|
| $z_f$      | 18         | $10^{9.25}^{+0.5}_{-0.5}$ |
| $M_{\text{bh}} [M_\odot]$ | $10^2$ | $10^5$ |
| $M_{v,\text{crit}}(z_f) [M_\odot]$ | $4 \times 10^6$ | $10^8$ |
| $n_{\text{bh}}(z_f)$ [Mpc$^{-3}$] | 23 | 2.5 |
| $N_{\text{bh}}$ [Mpc$^{-3}$] | 1027 $\pm$ 84 | 101 $\pm$ 22 |
| $n_{\text{bh}}(0)$ [Mpc$^{-3}$] | 12 | 1.1 |
| $\beta$ | 0.2 | 0.3 |

$z_{\text{min}}$ taken from Wilkinson Microwave Anisotropy Probe (WMAP) 3rd year results [72].
In Fig. 1 we show the redshift dependence of the enhancement function \( \Delta(z)^2/h(z) \) for scenario B, for three values of \( z_{f} = z_{re} \) are plotted, 8, 10, and 12. The strongest enhancement occurs at \( z_{f} \), partly because of the highest IMBH number density, but mainly because of the presence of the sharpest minispike. This displays the merits of considering the EGB, because high redshift enhancements are most easily observed as a diffuse background. On the other hand, we must therefore carefully consider the IMBH number density at \( z_{f} \). However, a major uncertainty in the hierarchical formation scenario of SMBHs is that the “occupation number,” that is, the fraction of galaxies containing a seed-BH at high redshift, is highly model dependent. In other words, contributions to the EGB flux can potentially constrain seed-BH models. To this effect, our calculation for the initial density is a conservative value (see Sec. III D).

In Fig. 2 we show contributions to the EGB from DM annihilation into continuous gamma rays (we discuss monochromatic emission later). We use DM parameters \( m_{\chi} = 100 \text{ GeV} \) and \( \sigma v = 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1} \). The values of \( z_{f} \) and \( \beta \) in Table II were used. We do not expect low-redshift IMBHs to be resolved; it has been shown that GLAST will resolve minispires in our galaxy and possibly Andromeda, but not further [21]. Moreover, we find that sources within \( z = 0.01 \) contribute less than a tenth of the total EGB contribution. We therefore show the flux, integrated safely from \( z = 0 \) to \( z_{f} \). We find that minispires increase the gamma-ray flux from DM halos by 1–3 orders. In particular, scenario B can give fluxes that are of the order of current observations. Scenario A fluxes are two orders smaller, but we stress that scenario A is a conservative case; a BH of mass \( 10^{2} M_{\odot} \) with a mild \( r^{-3/2} \) minispike.

\[ \text{FIG. 1. Enhancement factor } \Delta^2 \text{ as a function of redshift, for scenario B IMBH minispikes. Three values of } z_f = z_{re} \text{ are plotted, 8, 10, and 12. The peak enhancement at } z_f \text{ is largely due to the fact that the minispike is sharpest just after it is formed. This demonstrates the effectiveness of the minispike in enhancing contributions to the EGB.} \]

\[ \text{FIG. 2. Contributions to the EGB flux, from scenario A and scenario B IMBH minispikes. Also shown are EGRET data and predictions of our minimal host halo only scenario (i.e. no spikes and minispikes). We see that minispikes increase EGB contributions by 1–3 orders in magnitude. The 1\sigma scatter in } N_{bh,A} \text{ [21] is shown for scenario A. For scenario B, the 1\sigma scatter in } N_{bh,B} \text{ as well as } z_{re} \text{ are shown combined. In all calculations, } z_f \text{ and } \beta \text{ shown in Table II are used, with DM parameters } m_{\chi} = 100 \text{ GeV and } \sigma v = 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}. \text{ Note that scenario A is our conservative case; a BH of mass } 10^{2} M_{\odot} \text{ with a mild } r^{-3/2} \text{ minispike.} \]

B. Constraining DM parameters

Instead of performing a complete scan over DM parameters space, we follow Bertone et al. [21] and consider two discrete cases. In addition to our previously chosen set \( m_{\chi} = 100 \text{ GeV} \) and \( \sigma v = 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1} \), we define a new set \( m_{\chi} = 1 \text{ TeV} \) and \( \sigma v = 10^{-29} \text{ cm}^3 \text{s}^{-1} \). Using...
scenario B with $z_{\text{re}} = 10.9$ and $\beta = 0.3$, we plot in Fig. 4 the predicted EGB flux for both sets.

Now in Fig. 5 we show an advantage, from the point of view of constraining DM parameters, of considering IMBH minispike enhancements. We show contributions to the EGB at an energy of 1 GeV, against the DM annihilation cross section. As before, we use $z_{\text{re}} = 10.9$ and $\beta = 0.3$, and assume for now $m_x = 100$ GeV. The advantage of minispikes is observed in the gradient of the plotted line. Although one would naively expect gamma-ray fluxes to scale as $\sigma v/m_x^2$, the presence of a minispike alters this dependence. This is due to the fact that the dominant term in the minispike enhancement [the integral in Eq. (15)] is given by the expression $\rho_{\text{lim}}^2 \rho_{\text{lim}}^3$, which brings the gamma-ray flux scaling as $(\sigma v)^{2/7} m_x^{9/7}$, for minispikes growing out of a $\gamma = 1$ profile. Physically speaking, a smaller cross section works to maintain a denser minispike, which compensates for the decrease in flux due to the smaller cross section. One can say this weak dependence on DM parameters makes minispikes particularly suited for DM detection.

To determine the potential of the weak dependence on DM parameters, we consider gamma-ray observations by GLAST, which is expected to have more than an order better point source sensitivity than EGRET. With its launch, many gamma-ray sources that could not have been resolved until now will be detected, and taking these into account, a smaller EGB flux is expected. The most widely considered candidate for the dominant EGB contributor is unresolved blazars, i.e. a beamed population of active galactic nuclei (see [73] and references therein), and the fraction of these blazars that can be removed with GLAST depends on their luminosity function. The latest
calculation predicts that the resolvable fraction is around 20% [73]. If the EGB flux is entirely due to blazars, then the EGB will be reduced by the same fraction, and hence the sensitivity to $\sigma \nu$ will be improved by a factor of 2. We should, however, keep in mind that [73] concluded that blazars cannot fully explain the EGB flux, but only 25%–50%. Therefore, the remaining 50%–75% may be due to other astrophysical objects of either known or unknown origin, which may or may not be resolved by GLAST. This indicates that a significant fraction may still be resolved with GLAST, depending on the property of this additional contributor. In an optimistic case, where a fraction of $0.75 + 0.25 \times 0.2 = 0.8$ can be resolved with GLAST, the cross section sensitivity will be around $3 \times 10^{-29}$ cm$^{-2}$ s$^{-1}$, depending on values of $z_f$ and $N_{bh}$ (see error bars in Fig. 5). The sensitivity will also depend on $M_{bh}$; if half $M_{bh}$, the sensitivity decreases to $\sim 10^{-28}$ cm$^{-2}$ s$^{-1}$. These sensitivities are so small that no other experiments can compete in the next decades.

C. Line gamma rays

In the previous subsection we discussed constraining DM parameters using contributions to the EGB from DM annihilation into continuous photons, and highlighted its potential using the future GLAST mission. However, the continuum component lacks a distinguishing signature to separate it from other sources, such as unresolved blazars. Although the spectrum of other sources decreases rapidly at high energies, this is nonetheless a difficulty. Ideally, we would like to identify DM annihilation without these complications. The monochromatic component of DM annihilation provides a means of achieving this. As discussed earlier, the monochromatic emission provides a smoking-gun signature of DM annihilation due to its energy at the DM mass. Here, we present the monochromatic spectrum from the $\chi \bar{\chi} \rightarrow 2 \gamma$ process with a fixed branching ratio of $b_{2\gamma} = 10^{-3}$. Our result is shown in Fig. 6 for two sets of DM parameters, $m_{\chi} = 100$ GeV and $\sigma \nu = 10^{-26}$ cm$^{3}$ s$^{-1}$ (note this is not excluded by the continuum component), and $m_{\chi} = 1$ TeV and $\sigma \nu = 10^{-29}$ cm$^{3}$ s$^{-1}$. Again, we consider scenario B IMBH minispikes with $z_{re} = 10.9$ and $\beta = 0.3$.

The spectral shape of Fig. 6 arises because of distortion due to cosmological redshift and absorption of gamma rays during propagation. The larger peak at $E_0 = m_{\chi}$ is a characteristic of DM annihilation, and if detected provides very convincing evidence for DM. The smaller peak is a characteristic of IMBH minispikes; it is due to the strong enhancement factor at $z_f$, as shown in Fig. 1. If detected, this second peak identifies the presence of a high enhancement factor at high redshift, supporting the case of seed-BHs and minispikes.

In the past, line gamma rays had not been intensively considered, because the EGB spectrum taken by EGRET did not reach up to the required $O(100)$ GeV energies. With GLAST however, the energy window extends up to 300 GeV (and with better energy resolution), and line gamma rays could be a serious candidate of the first DM detection. Furthermore, as shown in Fig. 6, the minispike around IMBHs provide such high EGB line fluxes that it might give better evidence than the continuum flux. The characteristic spectral feature, combined with good energy resolution of GLAST, work quite positively for the detection of line gamma rays, even if gamma rays from other astrophysical sources give considerable contribution at the same energy.

V. DISCUSSION AND CONCLUSIONS

We have studied contributions to the EGB from DM annihilation in minispikes around IMBHs. Our results are plotted in Fig. 2 using DM parameters $m_{\chi} = 100$ GeV and $\sigma \nu = 3 \times 10^{-26}$ cm$^{3}$ s$^{-1}$. We find that a consideration of minispikes increase the contribution by 1–3 orders, so that in optimistic scenarios the predicted gamma-ray flux may reach current EGB values. The EGB can therefore potentially be used to constrain DM parameters and/or IMBH scenarios, particularly when better EGB observations are taken by GLAST. In our work, we considered two IMBH formation scenarios, scenario A being remnants of Population-III stars [32], and scenario B being formed in the centers of protogalactic disks [33]. Scenario A is our conservative case (mass $10^2 M_\odot$ with a mild $r^{-3/2}$ minis-
BH formation are not well known, and are likely to be [21]. However, the collapse and accretion processes during scale to the dynamical time scale at some relevant distance is satisfied when one compares the BH formation time by the collisionless nature of particle DM, and adiabaticity the halo center. These conditions are generally supported center of its host halo, and very cold initial DM orbits near and symmetric BH formation, formation of a BH in the enhancements.

Additionally, such unmerged IMBHs are likely to reside in spikes cannot significantly contribute to the EGB [29]. As a consequence, GLAST may be able to probe down to \( \sigma v \sim 10^{-29} \text{ cm}^3 \text{s}^{-1} \) if we optimistically assume scenario B IMBH minispikes, and that 80% of the EGB is resolvable by GLAST. This is far smaller than any other experiment, and excludes the allowed \( \sigma v \) region due to the standard thermal relic SUSY DM scenario. However, evidence concerning IMBH properties and EGB composition are required before setting such constraints. In this respect, it has recently been shown that the EGB power spectrum can be used to discriminate DM contributions from other sources [31].

All our calculations can be applied to other numerous DM candidates by simply substituting the appropriate differential gamma-ray spectrum per annihilation Eq. (4). A detailed search over particle DM candidates using EGB contributions should become possible with the launch of GLAST, and these should then be cross correlated with constraints from other potential DM signals. Finally, we have used EGB contributions to reveal DM properties, but we stress that the fact that scenario B predicts gamma-ray fluxes that are two orders greater than scenario A may also shed light on IMBH and SMBH models.

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