RAPIDITY DISTRIBUTIONS OF JETS PRODUCED IN HEAVY ION COLLISIONS AT LHC

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Abstract

We discuss the rapidity distribution of produced jets in heavy-ion collisions at LHC. The process allows one to determine to a good accuracy the value of the impact parameter of the nuclear collision in each single inelastic event. The knowledge of the geometry is a powerful tool for a detailed analysis of the process, making it possible to test the various different elements which, in accordance with present theoretical ideas, take part to the production mechanism.

PACS numbers: 24.85.+p, 25.75.-q.

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1 Introduction
Within a few years it will be possible to observe at LHC the first events with
heavy ions colliding at energies of several TeV in the nucleon-nucleon c.m.
system. The interest in the process is justified by the expectation that a new
phase of matter, namely quark-gluon plasma, could appear as a result of the
large energies - high densities which will be reached accelerating heavy ions at
LHC energies [1]. On the other hand the large number of fragments, resulting
from such large structures, as say Pb nuclei, interacting with so large energy
in the c.m. system, makes the analysis of the events rather difficult and
the search of unambiguous signatures puzzling. The identification of a few
features in the event which can be easily understood and whose dynamics
is, on the contrary, rather transparent would be in this respect an important
handle. The production of jets is, in our opinion, a process which has good
possibilities to be understood in a rather satisfactory way, also at such an
extreme regime as in the case of heavy ion collisions at LHC [2]. In fact, in
the present paper, we discuss the mechanism of production of large $k_t$ jets and
of minijets in heavy ion collisions at LHC energies. The main observation is
that one has the possibility of disentangling the geometrical features and to
obtain as a consequence distributions with universal characteristics. When
geometry is decoupled one has in fact the possibility of constructing various
distributions of jets, produced at different rapidities, with different cut-off
values etc., which are related one to another in a simple way, that is easily
worked out in perturbation theory. This universality property, on the other
hand, represents a regularity feature which is easily tested experimentally and
which is rather independent on the details of the actual theoretical approach.

2 Discussion
The inclusive cross section to produce jets at large $k_t$ is expressed, in the
QCD-parton model, by the factorized form
\[
\frac{d\sigma_J}{dx_A dx_B d^2k_t} = G(x_A)G(x_B) \frac{d\hat{\sigma}}{d^2k_t} \quad (1)
\]
where $G(x)$ is the parton distribution, depending on the fractional momentum $x$ and on the scale $k_t$, and $\frac{d\hat{\sigma}}{d^2k_t}$ is the partonic cross section. By introducing a lower cutoff in the transverse momentum of the produced jets $k_t^c$ one defines the integrated cross section:
\[
\sigma_J(k_t^c) = \int_{x_A x_B > 4(k_t^c)^2} d\sigma_J \int dx_A dx_B d^2k_t \quad (2)
\]
The lower limit for the integration in $x_A$ and $x_B$ is obtained by the condition $sx_Ax_B > 4(k_t^2)$. The differential cross section as a function of $x_A$ (or as a function of $x_B$) results from the corresponding integration on $x_B$ (or on $x_A$). For $x_A$ one has to integrate $x_B$ with the limits $1 > x_B > 4(k_t^2)/sx_A$. The expression of the differential cross section as a function of $x_A$ is therefore written as

$$\frac{d\sigma_J}{dx_A} = G(x_A) \times \int_{x_B > 4(k_t^2)/sx_A} G(x_B) \frac{d\sigma_J}{d^2k_t} dx_B d^2k_t \quad (3)$$

The relation corresponds to a flux of projectile partons, at a given value of momentum fraction $x_A$, interacting with the target. The integral represents the target scattering centers which are seen by a projectile parton with momentum fraction $x_A$. One may notice, by looking at the integration limits of the integral on $x_B$, that when $x_A$ is large the number of scattering centers is also large, while at small $x_A$ the number of scattering centres drops to zero. The size of each scattering centre is set by the value of the partonic cross section, and, although it depends on the value of the cut-off $k_t^c$, for sensible values of $k_t^c$, i.e. for values of the order of a few GeV, the size of the scattering centres is very small as compared to the hadronic dimension.

When considering large nuclei interacting at LHC energy the number of scattering centres which is effective for producing jets with momentum transfer larger than a few GeV is a rather large number. The scattering centres are distributed in the volume occupied by the target nucleus. One may figure out the interaction process by integrating the hadronic parton distributions with the limits discussed above and by multiplying the result by the atomic mass number. The resulting quantity is the number of scattering centres as a function of the momentum fraction of the projectile $x_A$. One may then distribute the scattering centres uniformly in the volume of the target nucleus and take the projection in the transverse plane. In correspondence of each scattering centre, projected in the transverse plane, one can draw a dot whose size is the parton-parton interaction cross section integrated with $k_t > k_t^c$. The result is shown in fig.1 for collisions with 7TeV in the nucleon-nucleon c.m. system, 5GeV as a value of $k_t^c$ and Pb as a nuclear target. The two figures show the distribution of scattering centres for two different values of $x_A$, $x_A = 10^{-3}$ and $x_A = 10^{-1}$, corresponding to the case of mini-jets produced in the central and in the forward rapidity region. The scale in fig.1 is the fm and the size of the dot representing each scattering centre has been obtained by computing the partonic cross section in Born approximation. In the actual forward rapidity case the number of possible scattering centres which are effective is of the order of 20,000.

Fig.1 points out in a spectacular way one of the most relevant problems
one has to deal with when considering production of jets in high energy nuclear collisions. Namely the large number of elementary interactions which take place in a single inelastic event and the geometrical features which result from the localization of the elementary interactions. The first observation is that when the number of scattering centres is very large, there may be overlaps in a few cases. As a consequence the effective number of scattering centres is smaller with respect to the number that has been estimated simply by multiplying the parton distributions with the atomic mass number. To keep this feature into account one should consider the *gluon-fusion* process \cite{3}, which reduces the parton population accordingly. One can then observe that the most peculiar feature of the overall interaction is represented by the large number of elementary partonic collisions, which take place at different points in the transverse plane, inside the overlap region of the matter distribution of the two interacting nuclei. One may in fact expect that the parton population of the projectile is also distributed in transverse space in a way similar to the distribution of the scattering centres shown in fig.1. The hard, or semi-hard, nuclear collision is then obtained by superposing two pictures similar to those shown in fig.1, one picture representing the distribution of projectile partons and one representing the distribution of target partons. Both projectile and target partons are localized in transverse plane in a region whose size is set by the cut-off $k_c$ and which is of the order of the size of the partonic interaction cross section, as in the case of the dots in fig.1. Projectile and target partons interact when the regions where they are localized in transverse plane overlap. The interactions are then localized in the overlap region of the matter distribution of the two nuclei and the whole process depends strongly on the nuclear impact parameter as a consequence.

There are two qualitatively different kinds of multiple partonic interactions:

- different pairs of partons interact at different points in transverse plane, that one may call *disconnected interactions* \cite{4}, and

- a given projectile parton may overlap at the same time with two or more target partons, that corresponds to a *rescatterings* \cite{5} process. In this respect one may notice that there is no need to be in the regime where parton fusion is an important effect in order to have a sizeable rescattering probability.

The final state produced by a single elementary partonic interaction is represented by two or more scattered partons. The simplest possibility is to have just a Born level process in such a way that one has a one to one correspondence between initial state partons and jets observed in the final state.
Further jets produced in the single hard interaction are then to be regarded as corrections of order \( \alpha_S \) to the lowest order process.

The quantities which are needed for the actual description of the nuclear hard interaction process are the (non-perturbative) initial state partonic nuclear distribution and the (perturbative) interaction probability. Because of the multiple partonic interactions, the information which is needed on the initial state is however much more detailed with respect to the nuclear parton distributions, usually considered in inclusive processes. As one realizes looking at fig.1, one needs in fact to know not only the number of partons with given momentum fraction but also their position in transverse plane. In addition, given the number of partons with given momentum fraction and at a given position in transverse plane, one needs to know the parton population at a different transverse point and with a different \( x \) value. The non-perturbative input to the process is in fact represented by the whole nuclear multi-parton distribution \( [6] \), which corresponds to a much more detailed knowledge on the nuclear parton structure than presently available. One needs in fact to know the whole set of all multi-parton correlations, which are independent quantities with respect to the parton distributions usually considered.

It seems however reasonable that when dealing with many partons localized in different space-time regions, as in the case of the parton population of a large nucleus, correlations are not a major effect. When correlations are completely neglected the parton population is described by a Poisson distribution. The probability to have a configuration with \( n \) partons with coordinates \( x_1, b_1, \ldots x_n, b_n \) is therefore given by

\[
\Gamma(x_1, b_1, \ldots x_n, b_n) = \frac{D(x_1, b_1) \cdots D(x_n, b_n)}{n!} e^{-\int D(x, b) \, dx \, db}
\]

In this simplest case the whole distribution is expressed by means of the average number of partons with coordinates \( x, b \), namely \( D(x, b) \). The connection with the usual parton distributions is \( \int D(x, b) \, db = G(x) \), where \( G(x) \) is the nuclear parton distribution usually considered in large \( k_t \) processes. The whole multi-parton distribution is singular in the infrared region and the \( x \)-integration in the normalization factor is done by keeping into account the cutoff \( k_t^c \).

Given a configuration with \( n \) projectile partons and \( m \) target partons, the interaction probability is obtained from the elementary probability of interaction \( p_{ij} \), where \( i \) and \( j \) label a given projectile and a given target parton respectively. Then \( 1 - p_{ij} \) is the probability of no interaction for the pair \( i, j \) and \( \prod_{i,j} (1 - p_{ij}) \) represents the probability of no interaction for the configuration with \( n \) projectile partons and \( m \) target partons. The quantity
which one needs for the cross section is the probability to have at least one interaction \[7\], namely
\[ P(n, m) \equiv 1 - \prod_{i,j}^{n,m}(1 - p_{ij}), \]
which is in fact equal to the sum over all interaction probabilities.

The hard cross section \( \sigma_H \) is obtained by integrating on the impact parameter \( \beta \) the probability to have at least one hard interaction. The probability to have at least one hard interaction in the nuclear collision is the result of the sum over all partonic configurations of the projectile and of the target nucleus, multiplied by the probability to have at least one hard interaction in the given partonic configuration. The hard cross section is therefore expressed as \[8\]
\[ \sigma_H = \int d^2 \beta \sum_{n,m} \Gamma_n \cdot \Gamma_m \cdot P(n, m) \quad (5) \]

The relation above is too complicated to be read off and computed directly, one can obtain a closed analytic form if one takes the drastic attitude of neglecting all parton rescatterings. One may however derive a few simple relations out of Eq.(5). If one works out the average number of parton collisions \( \langle \nu \rangle \) \[8\] the result is the single scattering expression of the perturbative QCD parton model:
\[ \langle \nu \rangle \times \sigma_H = \int_{sx_A x_B > 4(k_c)^2} G(x_A) G(x_B) \frac{d\hat{\sigma}}{d^2 k_t} dx_A dx_B d^2 k_t \quad (6) \]
The relation shows that a rather general non-trivial feature, namely the AGK cancellation, which is expected to hold as a consequence of the AGK cutting rules \[9\] when multiple interactions are present, is implemented explicitly in the actual case.

A quantity of interest is the average number of wounded partons \[8\], namely partons which have undergone at least one hard interaction. Those are in fact the initial state partons that are observed as jets in the final state. In the simplest case, when the elementary parton-parton interaction is represented as a two to two process, all jets observed in the final state are wounded partons. The average number of wounded partons of the projectile nucleus \( A \), with a given momentum fraction \( x_A \) in a nuclear collision with impact parameter \( \beta \), \( W(x_A) \), can be derived from Eq.(5). The resulting expression is:
\[ W(x_A) = \int d^2 b D(x_A, b - \beta) \cdot \left\{ 1 - \exp\left[ - \int D(x_B, b)\hat{\sigma}(x_A, x_B) dx_B \right] \right\} \quad (7) \]
\( \hat{\sigma}(x_A, x_B) \) is the partonic cross section, integrated on the momentum transfer \( k_t \) with the cut off \( k_c \) and \( x_B \) is integrated within the limits \( 4(k_c)^2/sx_A < x_B < 1 \).

One may notice that, while the initial state parton distribution is singular for
small $x$ and $k^t_c$ values, the average number of wounded partons is on the contrary a smooth function in the infrared region. In fact the expression above results from the product of the average number of $A$-partons, $D(x_A, b - \beta)$, and the interaction probability, namely $\{1 - \exp[- \int D(x_B, b) \hat{\sigma}(x_A, x_B) dx_B]\}$.

At small $x_A$ the number of projectile partons is very large, in that limit the interaction probability however goes to zero. The interaction probability in addition has a finite limit, actually the black disk limit, when $x_A \to 1$ or when $k^t_c \to 0$.

A further observation is that, when the interaction probability is small, if the dependence of the average number of partons on $x$ and on $b$ is factorized, the expression for the average number of wounded partons is also factorized in the same variables. More explicitly, if one writes

$$D(x, b) = G(x)\tau(b)$$

where $\tau(b)$ is the nuclear thickness function normalized to one, one obtains, in the limit of small interaction probability,

$$W(x_A) = G(x_A) \int G(x_B) \hat{\sigma}(x_A, x_B) dx_B \times \int d^2b\tau_A(b - \beta)\tau_B(b)$$

The average number of wounded partons $W(x_A)$, in a nuclear collision with impact parameter $\beta$, is therefore proportional in this case to the overlap of the matter distribution of the two interacting nuclei $\int d^2b\tau_A(b - \beta)\tau_B(b)$.

Given the explicit form of the cross section, one may work out all higher moments of the distribution in the number of wounded partons. The resulting expression is more and more involved when higher and higher moments are considered. An important simplification is however obtained, in the distribution of the wounded partons of the projectile, if one makes the hypothesis of neglecting all interactions where different projectile partons hit the same target parton. The set of events of this kind is rather small when compared with the whole set of possible events. When this simplification is made the distribution in the number of wounded partons, at a given value of $\beta$, turns out to be a Poissonian and it is therefore determined completely by its average value. A characteristic feature of the interaction is therefore that the fluctuation of the distribution in the number of wounded partons is less and less important, as compared with the average value, when the average value grows. It is precisely this feature that allows one to disentangle the geometrical aspect of the interaction event by event.

When the distribution of the wounded partons is known one can easily simulate the production of jets in a collision involving heavy nuclei at LHC energy. In fig.2 one may see how the number of produced jets is distributed...
in rapidity in a central Pb – Pb collision with 7TeV energy in the nucleon-nucleon c.m. system. In fig.2a one shows the distribution of jets produced with a transverse momentum larger than 20GeV, in fig.2b the cutoff $k_t^c$ is lowered to 10GeV and in fig.2c $k_t^c = 5GeV$. In the figures the histograms represent the simulated event. The continuous line is the average number of wounded partons, as it results after summing projectile and target wounded partons, as expressed in Eq.(7) using rapidity as a variable. The dashed line is the same average number when using rather the approximate expression in Eq.(9), namely neglecting semi-hard parton rescatterings.

One may notice that while the average number grows the fluctuation around the average grows much less rapidly. The effect of the unitarization of the hard parton-nucleus interaction, namely the difference between the continuous and the dashed line, is also shown to be a rather sizable effect when $k_t^c = 5GeV$.

Since the fluctuations are relatively less important when the average numbers are large, one may try to determine the value of the impact parameter by counting, in a given event, all jets produced above some lower cutoff $k_t^c$. In fig.3 we show the distribution in the total number of jets produced in central collision with different cutoff values (20GeV fig.3a, 10GeV fig.3b and 7GeV fig.3c).

In the case of a 7GeV cutoff the fluctuation in the number of produced minijets is 2.5%. The overall number of produced minijets in a central collision with a 7GeV cutoff is however larger than 1500, and the actual possibility of detecting such a large number of minijets is out of reach. A quantity which is likely to be more accessible experimentally could be the overall energy carried by all the produced minijets. Same numbers of produced minijets can carry different amounts of energy. The relative fluctuation in energy is therefore larger with respect to the relative fluctuation in the number of produced minijets. Anyhow with a 7GeV cutoff the fluctuation in energy turns out to be less than 4%. If one lowers the cutoff to 5GeV the fluctuation in the energy carried by all the produced minijets is close to 2.5%. A similar variation in the average energy of produced jets is obtained by varying the value of the impact parameter by an amount sizeably smaller than 1 fm. It seems therefore plausible that the measure of the total energy carried by jets, produced with relatively small momentum transfer, will allow the determination of the value of the impact parameter of the nuclear collision with an uncertainty smaller than 1 fm.

A possible procedure in classifying events with jets in heavy ion collisions could be therefore the following:

In each single event one may measure the total energy of the produced jets as a function of $k_t^c$. Since the distribution is narrower and narrower when $k_t^c$
is smaller and smaller, the energy carried by the jets with relatively small \( k_t^c \) allows one to estimate the value of the impact parameter of the collision. One may then collect all events with the same impact parameter \( \beta \). At each rapidity value and for a given cutoff \( k_t^c \) one has therefore a distribution in the number of observed minijets. One expects that all these distributions are universal, to a large extent. One expects in fact that the distributions are the same at different rapidity values and with different cutoffs, the only difference being in the average values. The dependence of the average values, as a function of rapidity and cutoff, is obtained through pQCD.

As an example one may write the expression for the average number of produced minijets, at a given rapidity \( y \), for all events with the same impact parameter \( \beta \). When \( k_t^c \) is not too small, say it is larger than \( 10^{-15} GeV \), semi-hard rescatterings can be neglected and one obtains:

\[
\frac{dN}{dy} = \int d^2b \tau_A(b - \beta) \tau_B(b) \times \left[ g_A(y) \int_{y' < y} g_B(y') \hat{\sigma}(y - y', k_t^c) dy' + g_B(y) \int_{y < y'} g_A(y') \hat{\sigma}(y' - y, k_t^c) dy' \right] \tag{10}
\]

here \( g(y) = xG(x) \) and rapidities and fractional momenta are connected by the usual 2 \( \to \) 2 kinematics. The dependence on \( y \) and on the cutoff \( k_t^c \), which is described by the term in square parenthesis, is obtained by the usual pQCD parton model inputs. Geometry is factorized and it enters in the first term in Eq.(10) only. All different averages of distributions with the same impact parameter are related one to another according with pQCD as shown in Eq.(10). When the impact parameter is changed the distributions are obtained, from the corresponding ones measured previously, simply by rescaling the averages with the value of the overlap function.

One expects in addition that if one plots the relative rates of events with different numbers of minijets, with \( k_t \) larger that some fixed value \( k_t^c \) and at the same rapidity, one obtains a quantity which tests the overlap function, namely which is sensitive to the actual distribution in space of the nuclear partonic matter. As an example in fig.4a we plot the relative rates of events with jets at \( y = 0 \) and \( k_t^c = 20 GeV \) in the case of a Woods-Saxon distribution in space of the nuclear partonic matter (continuous histogram) and in the case of an hard-sphere distribution (dotted histogram). The difference is much more evident if one plots the ratio of the two histograms (fig.4b).
3 Conclusions

A large number of jets are expected in the case of nuclear collisions at LHC. The global features of the events are understood in terms of the pQCD parton model, once the geometrical features of the interaction are taken into account. The picture of the interaction just described is easily tested: One may first look at central collision events only, by taking a veto trigger in the forward direction. All central events are expected to show a universal behavior in the distributions of jets produced at different rapidity values and with different cutoff values. The jets observed are in fact the wounded partons, namely the initial state partons which have undergone at least one hard interaction. The expectation is that the different distributions in the number of wounded partons, and therefore in the number of observed jets, are close to a Poisson distribution when the impact parameter of the nuclear collision is fixed. At fixed impact parameter all different distributions in the number of observed jets are therefore equal. The only difference is in the average value, which is calculated in the pQCD parton model in a straightforward way.

One can then proceed estimating the impact parameter event by event. That can be done by looking at the total energy carried by the jets produced in each single event at small $k_T^c$. The value of the energy, compared with the corresponding value in the case of a central collisions, gives the estimate of the size of overlap of the two interacting nuclei. Within the simple approach to the nuclear interaction actually described, collecting all events with the same overlap one obtains distributions which have to be the same as those observed in the case of central collisions, after rescaling the averages with the corresponding value of the overlap function.

The simple picture of the interaction just described is easily tested experimentally. The universal behaviour of the distributions in the number of observed jets, just discussed, can be tested in a model independent way, which allows a rather strict and detailed control on the different theoretical elements which enter in the representation of the process. Once the production mechanism is tested in detail, possible deviations from the universal behavior in particular conditions (heavy nuclei, high energy, central collisions) would represent a convincing signal of a different production mechanism taking place.

Acknowledgements

This work was partially supported by the Italian Ministry of University and of Scientific and Technological Research by means of the Fondi per la Ricerca scientifica - Università di Trieste.
**Figure captions**

Fig. 1a. Projection in transverse plane of the scattering centres in a collision of a parton with a \( Pb \) nucleus. The scale is the \( fm \) and the size of the dots corresponds to the parton-parton cross section integrated with the cut off in momentum transfer \( k_t^c = 5GeV \). The momentum fraction \( x_A \) of the projectile parton is \( x_A = 10^{-3} \) and the energy in the nucleon-nucleon c.m. system is \( 7TeV \).

Fig. 1b. Same as in fig. 1a with \( x_A = 10^{-1} \).

Fig. 2a. Rapidity distribution of produced jets in a central \( Pb - Pb \) collision with \( 7TeV \) energy in the nucleon-nucleon c.m. system. The histogram is the simulated event, the continuous line is the average number of wounded partons expected, as from Eq.(7), summing the projectile and target wounded partons and expressing the resulting numbers as a function of rapidity. The dashed line is the same average number when using the approximate expression in Eq.(9), namely after neglecting semi-hard parton rescatterings. \( k_t^c = 20GeV \).

Fig. 2b. Same as in fig. 2a with \( k_t^c = 10GeV \).

Fig. 2c. Same as in fig. 2a with \( k_t^c = 5GeV \).

Fig. 3a. Distribution in the total number of jets produced in central collisions with cutoff \( k_t^c = 20GeV \).

Fig. 3b. Same as in fig. 3a with \( k_t^c = 10GeV \).

Fig. 3c. Same as in fig. 3a with \( k_t^c = 7GeV \).

Fig. 4a. Relative rates of events with jets at \( y = 0 \), \( k_t^c = 20GeV \) in the case of a Woods-Saxon distribution in space of the nuclear partonic matter (continuous histogram) and for an hard-sphere distribution (dotted histogram).

Fig. 4b. Ratio of the two histograms in fig. 4a.
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