An Unsupervised Deep-Learning Method for Fingerprint Classification: the CCAE Network and the Hybrid Clustering Strategy

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Abstract: The fingerprint classification is an important and effective method to quicken the process and improve the accuracy in the fingerprint matching process. Conventional supervised methods need a large amount of pre-labeled data and thus consume immense human resources. In this paper, we propose a new and efficient unsupervised deep learning method that can extract fingerprint features and classify fingerprint patterns automatically. In this approach, a new model named constraint convolutional auto-encoder (CCAE) is used to extract fingerprint features and a hybrid clustering strategy is applied to obtain the final clusters. A set of experiments in the NIST-DB4 dataset shows that the proposed unsupervised method exhibits the efficient performance on fingerprint classification. For example, the CCAE achieves an accuracy of 97.3% on only 1000 unlabeled fingerprints in the NIST-DB4. The code is available at Github.

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1 Introduction

Human fingerprint carries biometric information and is the most significant and traditional biometric identification technology [1–3]. Due to fingerprints’ uniqueness and invariance [4, 5], human fingerprint identification or matching technique are applied to the matching of the prints not only in criminal evidences [6–8] but also in authentication, customs transit, and public transportation systems [9–12]. Thus, fingerprint matching or fingerprint identification technology develops rapidly [13, 14].

However, the fingerprint data is very large and messy [15]. For example, one has to make comparison between the suspect’s fingerprint and the huge amount of candidates’ fingerprints to
collect criminal evidences [6–8]. According to different conditions of the crime scene, fingerprints are usually incomplete and deformed [16]. This lead to tremendous difficulties in the fingerprint matching process. The fingerprint classification is an important and effective method to shorten the time and improve the accuracy in the fingerprint matching process [17–19]. For example, the pre-classification of fingerprint patterns can reduce the number of candidate fingerprints in the matching stage [20, 21].

Fingerprint patterns are basically divided into three categories and they are arch prints, loop prints, and whorl prints [22]. Researchers pay attention to the algorithms of fingerprint classification since last century [23–25]. Certain algorithms were developed to process fingerprint images [26–28]. For example, in order to distinguish the patterns of fingerprints better, features such as the core point, the bifurcation, and the ridge ending are defined [29, 30]. Frequency analysis based on transforms such as Fourier transform is used to process fingerprint images [31, 32]. The graph theory methods, such as the orientation flow [33–35] and the singular point [36, 37], are also introduced.

Recently, the neural network methods are applied in the fingerprint classification as an universal algorithm [38–40]. Conventional supervised neural network methods report high matching accuracy [32, 38, 41–43]. For example, Jeon et al. [41] reported an accuracy of 98.3% using VGGNet [44]. CaffeNet (a variant model of AlexNet [45]) reported by Peralta et al. [38] has an accuracy of 90.7%. Wu et al. [43] designed FCTP-Net whose accuracy was 92.9%. Moreover, the neural network structures with domain knowledge [42] were designed. Meanwhile, interpretable neural networks were preferred [46, 47]. For example, Yao TANG et al. [42] proposed a model with domain knowledge including orientation-field and singular-point. Furthermore, we collect the performance of existing supervised learning fingerprint classification models, as shown in Table. 1. Form Table. 1, The fingerprint classification task based on supervised learning has achieved high accuracy. We evaluate our model of unsupervised learning against the model with the highest accuracy.

| Works          | Methods                          | Testing sets (size)      | Tag number | Accuracy(%) |
|----------------|----------------------------------|--------------------------|------------|-------------|
| Sen Wang et al. [48] | Directional field + kmeans        | NIST 14 dataset (1000)   | 4          | 89.5        |
| Ruxin et al. [21]   | SAE + softmax regression         | NIST-DB4 (2000)          | 4          | 91.4        |
| Wang-Su et al. [41] | VGGNet                           | FVC 2000, 2002, FVC2004 (100) | 5          | 98.3        |
| Daniel et al. [38]  | Variant CaffeNet                  | NIST-DB4 (1650)          | 5          | 90.7        |
| Fan et al. [43]     | FCTP-Net                         | NIST DB4, DB9, DB10 (2000) | 4          | 92.9        |
| Wen et al. [40]     | Singularity ROI + CNN             | NIST-DB4 (4000)          | 5          | 93.0        |

The key to construct an efficient fingerprint classification algorithm is extracting useful features that are invariant and carry the information of fingerprint patterns [29]. To capture the features of fingerprint patterns manually is difficult [38, 43]. Conventional supervised neural network methods need a large amount of pre-labeled data and thus consume immense human resources.

In this paper, beyond the supervised method, we propose an unsupervised classification approach of deep learning. The method can extract fingerprint features and classify fingerprint patterns automatically. In this approach, a new convolutional auto-encoder structure named constraint
convolutional auto-encoder (CCAЕ) is used to extract fingerprint features. At the same time, a hybrid clustering strategy is used to obtain the final clusters. A set of experiments in NIST-DB4 dataset shows that, the proposed unsupervised method exhibits the efficient performance on fingerprint classification. It shows that the proposed method, compared with the supervised method, not only need no pre-labeled fingerprint images but also performs better.

This paper is organized as follows. In Sec. 2, we introduce the main method, including the data preprocessing, the CCAЕ neural network model structure, and the hybrid clustering strategy respectively. In Sec. 3, we give the result of the main method using on the NIST-DB4 fingerprint database. Details such as the setting of the experimental environment, the specific configuration of the experimental model, and the performance evaluation of each models are given. Conclusions and discussions are given in Sec. 4. Another details are given in the appendix.

2 Database and the main method

In this section, we introduce the database and the main method.

2.1 Database and preprocessing

The NIST special database 4 (NIST-DB4) [49] is a pre-labeled fingerprint data set. The NIST-DB4 contains 4000 8-bit gray scale fingerprint images (2000 pairs) stored in PNG format. The image size is 512×512 PPI. In this work, instead of using the whole images from the NIST-DB4, we only uses 1000 fingerprint images that are randomly selected from the NIST-DB4.

Some images in NIST-DB4 are affected by noises, interference, and uneven image contrast, as shown in Fig. 1. In this section, we apply successive operations on the raw fingerprint images in order to obtain images with higher quality. The preprocessing includes cropped, denoising, and binarization of the core area of the fingerprint image.

Figure 1. Examples of images in NIST-DB4 that are affected by noises, interference, and uneven image saturation.
Cropping the core area of the fingerprint image. We manually crop out the core area of the fingerprint images. The cropped process will be replaced by neural network method in the following research. The original image is cropping from $512 \times 512$ PPI to $200 \times 200$ PPI, as shown in Fig. 2. The cropped image will be resized to $256 \times 256$ PPI before sending into the $CCAE$ network.

Denoising and adaptive binarization. The Gaussian denoising and the contrast equalization are applied to remove high frequency noise and adjust the image contrast automatically. Then, adaptive binarization processing is used to obtain images with higher quality, as shown in Fig. 2.

![Pre-process: cropping, denoising, and adaptive binarization.](image)

**Figure 2.** The pre-process: cropping, denoising, and adaptive binarization. The blue area is the central point of the fingerprint

2.2 The model

In this section, we introduce a new convolutional auto-encoder structure named the constraint convolutional auto-encoder (CCAE). The $CCAE$ is a variant of the auto-encoder (AE) model. A brief review of the AE, including the convolutional auto-encoder (CAE), and the variational auto-encoder (VAE), is given in the appendix. In constructing the loss function, we use two methods to build the loss function of the $CCAE$. The $CCAE$ with these two loss functions are named $CCAE_a$ and $CCAE_b$ respectively. The $CCAE$, both $CCAE_a$ and $CCAE_b$, has been proved to be an useful tool to extract fingerprint features. For the sake of convenience, the $CCAE$ represent the $CCAE_a$ and the $CCAE_b$ models.

2.2.1 The structure

Instead of using pooling and upsampling layers, the $CCAE$ uses convolutional and deconvolution layers only. Moreover, the convolution layer with big kernel-sizes, such as $15 \times 15$ and $30 \times 30$, is applied. The structure of the $CCAE$ and the parameter setting are shown in Fig. 3.

2.2.2 The loss function: the constraint on the latent vector

To extract effective features in the fingerprint images, the model need indeed learn some of the morphological features of the fingerprint, but need not learn all the morphological features. Because the entire morphological features of the fingerprint are redundant for the fingerprint classification,
and lead to low classification accuracy. That is, the model should "learn" and at the same time "forget" the features of the fingerprint. In this section, we introduce two methods to build the loss function of the CCAE.

**CCAE_a.** In the CCAE_a, the latent vector parameters $y$ are sampled from a Gaussian distribution with mean-value of 0. The constraint is transformed as an additional term in the loss function, as shown in Fig. 7. That is

$$\text{loss} = \frac{1}{M} \sum_{i=1}^{M} \left( x_i - x_i' \right)^2 + \frac{1}{N} \sum_{j=1}^{N} y_j^2,$$  

where $M$ is the number of fingerprint image, $N$ is the length of latent vector, $x_i'$ is the decoding data.

**CCAE_b.** The CCAE_b model inspired from the work of Aytekin et al.[50], where the loss function reads

$$\text{loss} = \frac{1}{M} \sum_{i=1}^{M} \left( x_i - x_i' \right)^2, \quad x_i' = D\left( \frac{y_j}{\pi \sum_{j=1}^{N} y_j^2} \right)$$

Figure 3. The structure of the CCAE.
where $D$ is the decoding block proceeding. That is the $CCA E$ with Aytekin’s loss function, Eq. (5), is the $CCA E_b$ model.

### 2.3 The hybrid clustering algorithm

In this section, we introduce a new clustering strategy called the hybrid clustering algorithm into the fingerprint clustering. The strategy is first proposed in the previous work "Automatic morphological classification of galaxies: convolutional auto-encoder and bagging based multi-clustering model” that is under publishing. The previous work shows that, a single clustering method groups the subsample from a single perspective of view and thus will lead to miss-classification. By considering the clustering result of various kinds of models, one can group subsamples from a comprehensive view, that is a multi-perspective of view, and thus the clustering result is more reliable.

Here, the encoding data is used to divide the fingerprint data into four categories (arch, whorl, right loop and left loop) by different clustering methods. Different clustering methods make different clustering effects. The hybrid clustering algorithm, bagging algorithm based on multi-clustering model, is used to get the optimal clustering results. It shows that the hybrid clustering algorithm can cluster the patterns of fingerprint better than the traditional single clustering algorithm.

The Hybrid clustering algorithm can improve the accuracy of clustering greatly. The algorithm flow chart is shown in Fig. 4.

**Figure 4.** An illustration of the hybrid clustering strategy. For example, the purple ball (g) ball is classified into different clusters, so it is not well clustered and thus rejected. Blue balls, such as balls d and o, and yellow balls, such as balls a and f, are well clustered by both clustering methods.

Here we use three traditional clustering methods, including the Euclidean distance k-mean algorithm [51], the agglomerative (AGG) algorithm [52], and the balanced iterative reducing and
clustering using hierarchies (BIRCH) algorithm [53]. A brief review of the k-mean, the AGG, and the BIRCH algorithms is given in the appendix.

Although the bagging algorithm based multi-clustering model will result in more rejective data, the accuracy will be improved obviously, which will be shown in the following sections. For the sake of clarity, Fig. 5 gives an overview of the main method.

![Flow chart](image)

**Figure 5.** The flow chart.

### 3 Results

In this section, we provide model evaluation indicators for several models. The highlight here is to transfer the evaluation method with supervised learning to our unsupervised task. All of our models used an Adam [54] optimizer with a learning rate of $3 \times 10^{-4}$ and training 1000 epochs on datasets.
3.1 Environment settings.

The experiment system environment used for learning and testing is set as follows. The operating system is Linux Ubuntu 18.04.5. The hardware consists of an Intel Xeon CPU E5-2690v3 2.60GHz, 62 GB memory, one NVIDIA GeForce GTX 1070Ti GPU, and the Tensorflow 2.4.0 deep learning framework.

3.2 A comparison with the existing supervised methods

In order to show the difference between the existing supervised methods and our model, a table is given to show the performance of various approaches. See Table 2 below.

Table 2. A comparison between the existing supervised methods and the CCAE. our model uses only 1000 randomly selected subsamples from the NIST-DB4 dataset. \( A_{all} \) stands for the all fingerprint type accuracy. R.r is the Reject rate, which is the percentage of the total number of rejections after the hybrid clustering mechanism.

| Method                        | Training sets (size) | Testing sets (size) | \( A_{all}(\%) \) | R.r(\%) |
|-------------------------------|----------------------|---------------------|-------------------|---------|
| Ruxin et al. [21]             | NIST-DB4 (2000)      | NIST-DB4 (2000)     | 91.4              | \( \times \) |
| Fan et al. [43]               | NIST DB4, DB9, DB10 (100000) | NIST DB4, DB9, DB10 (2000) | 92.9              | \( \times \) |
| The CCAE_{a} and the hybrid clustering | None                | NIST-DB4 (1000)     | 96.4              | 5.1     |
| The CCAE_{b} and the hybrid clustering | None                | NIST-DB4 (1000)     | 97.3              | 6.3     |

Tables 1 and 2 shows that the CCAE_{a} and the CCAE_{b} achieve higher accuracy than the supervised method without any pre-labeled training dataset. The number of rejected subsamples in the CCAE_{a} and the CCAE_{b} are only 51 and 63 respectively. In other word, the actually size of testing set is 949 and 937 respectively.

3.3 The effectiveness of the hybrid clustering algorithm

In this section, in order to show the effectiveness of the hybrid clustering algorithm, we make comparison between the result of the single clustering model and the hybrid clustering model, as shown in the Table 3 below.

Table 3. Comparison between the hybrid clustering model and the single clustering model in terms of their performance.

| Model                          | Performance evaluation | \( A_{all}(\%) \) | R.r(\%) |
|--------------------------------|------------------------|-------------------|---------|
| The CCAE_{a-k-means}           |                        | 92.6              | \( \times \) |
| The CCAE_{a-BIRCH}             |                        | 94.9              | \( \times \) |
| The CCAE_{a-AGG}               |                        | 94.9              | \( \times \) |
| The CCAE_{a}                   |                        | 96.4              | 5.1     |
| The CCAE_{b}                   |                        | 97.3              | 6.3     |
Table 3 shows that the accuracy of the hybrid clustering model is at least 1.5% higher than the single clustering model at the cost of only 6.3% rejected subsamples at most. The high-purity clustering result can be used as pre-labeled training dataset for downstream tasks.

3.4 The analysis of the CCAE

In this section, we further analyze the effectiveness of the CCAE. Particularly, we discuss the center-crop operation, the big convolutional kernel, and L2 constraint on the loss function.

3.4.1 The center-crop operation

The strong prior operation is favorable for model learning features. We used t-SNE visualization method to directly show the improvement brought by strong prior operation. Fig. 6 below shows the results based on the t-SNE visualization.

![Figure 6. The effect of prior cropped operation on data compression.](image)

3.4.2 The big convolutional kernel

We use big convolution kernels for convolution operation, whose sizes are 30×30, 15×15 and 9×9. The small kernel size is 3×3. Fig. 6 shows that, Base on the CCAE, the effect of big convolution kernel is critical. Beside, a comparison between the CCAE and Aytekin’s model is given in the appendix.

3.4.3 The constraint on the latent vector

A constraint on the latent vector is a method that designed to "forget" the features of the fingerprint. It shows that the CCAE is good at learning the required morphological features of the fingerprint.
Table 4. Comparison of different model in terms of their performance. $A_{all}$ stands for the accuracy, $P_{average}$ stands for the average precision, and $R_{average}$ stands for the average recall. The detail of the calculation of $A_{all}$ and e.t.c., is given in the appendix. The cropped CAE model is the big convolution kernel CAE model with the cropped input image.

| Model   | Performance evaluation |
|---------|------------------------|
|         | $A_{all}$(%) | $P_{average}$(%) | $R_{average}$(%) | $F_{1average}$ | R.r(%) |
| The cropped CAE | 95.5 | 94.3 | 95.2 | 0.94 | 5.0 |
| $CCAE_a$ | 96.4 | 95.8 | 96.4 | 0.96 | 5.1 |
| $CCAE_b$ | 97.3 | 97.0 | 97.3 | 0.97 | 6.3 |

It shows that adding Gaussian constraint to the encoding data can improve the performance of clustering with an increase of 1% $\sim$ 2%. Moreover, the $CCAE_b$ model clustering result can be further improved by 1% $\sim$ 2%. From Table. 4, even if the $CCAE_b$ model is highly accurate, the rejection rate is also high. This indicates that different l2 constraint methods are suitable for different scenarios, including high precision rate scenarios or high recall rate scenarios.

4 Conclusions and Discussions

In this paper, a fingerprint classification algorithm which use the unsupervised $CCAE$ to learn the fingerprint features and use the hybrid clustering algorithm to clustering fingerprints by their patterns is presented. To integrally extract fingerprint structure information, we use generative model, rather than discriminative model. The purpose of the discriminant model is to find the high-dimensional representation of the data in the high-dimensional space, and then map it to a low-dimensional space for classification and discrimination. It may contain incomplete structural characteristics of the data. However, the generation model can compress the dimension of the data while the generation module preserves the structural information of the data. This is a more reasonable way of data compression, and can provide higher robustness and interpretability for downstream data matching and data classification.

We have following main contributions in this paper. Instead of using the popular supervised deep learning method, (1) we propose a novel unsupervised model, the $CCAE$, to learn the fingerprint features automatically and (2) apply the hybrid clustering strategy to obtain the final groups. It shows that the proposed method, compared with the supervised method, not only need no pre-labeled fingerprint images but also performs better.

In the future work, different network structure will be considered, such as the residual [56] and the inception [57]. Furthermore, we will migrate the model to various domains, such as palm prints, faces, celestial bodies, medical imaging, and so on.

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6 Appendix

6.1 A brief review of the AE, the CAE, and the VAE

The CCAE is a variation of the convolutional auto-encoder (CAE) and the variational auto-encoder (VAE). In this section, we give a brief review of the auto-encoder (AE), the CAE, and the VAE. Their models are shown in Fig. 7.

AE. The AE is an efficient method to extract important informations from the raw data [58]. By the encoding block and the decoding block, which are composed of multiple fully connected layers, the AE reduces the dimension of the input and at the same time preserves the main information. Usually, the encoding block can be regarded as a nonlinear mapping $f_{\text{encode}}(x)$, that maps the input
data $x$ to the latent vector parameters $y$. For example,

$$y = f_{\text{encode}}(x) = \sigma(Wx + b),$$

(1)

where $\sigma(x)$ is the nonlinear activation function, $W$ and $b$ are parameter and weight of the encoding block. The decoding block, another nonlinear mapping, maps the latent vector parameters $y$ back to the reconstructed data $x'$. By adjusting the weights both in the encoding and decoding blocks, the AE minimizes the differences between the reconstructed data $x'$ and input data $x$.

**CAE.** The convolutional operation is an effective method to extract features of images. For example, the Lenet-5 network [59] and the AlexNet [45] are the variations of the convolutional neural networks, these two networks enhanced the image classification efficiency enormously. In a conventional auto-encoder (CAE), the encoding block of the AE is replaced by a convolutional neural networks, which makes the CAE efficient at compressing the dimension of images.

**VAE.** It has been shown that, applying a constraint on the hidden parameters $y$ helps the AE to learn more effective features [60]. For example, in the VAE, the hidden parameters $y$ are usually sampled from the Gaussian distribution. Aytekin et al.[50] sampled the latent vector parameters from the unit ball space.

**Resize.** To compare with CAE model, we set up the resize model, which adjusts the original $512 \times 512 \times 3$ image to $32 \times 32 \times 2$ directly with Lanczos interpolation algorithm [61, 62]. From the perspective of the clustering results, we prove that there is no difference between the full convolutional auto-encoder and the pure scaling operation. This means that full CAE is not enough to achieve the learning feature effect.

### 6.2 A brief review of the conventional clustering algorithm

**The k-means algorithm: a brief review.** The k-means algorithm[51] divides the data set samples into $k$ cluster classes by different distance formulas. The cluster center is obtained by initializing the mean vector at the beginning. Through the greedy strategy, the distance between the sample and the cluster center is minimized, at the same time, the cluster center is updated. Finally, clustering results can be obtained.

**The AGG and the BIRCH algorithms: a brief review.** The AGG algorithm [52] and the BIRCH algorithm [53] belong to hierarchical clustering algorithm, which divides the data set at different levels into form a tree-like structure. The AGG algorithm is a bottom-up aggregation strategy. Firstly, each sample in the data set is regarded as an initial cluster, and then, in each step of the algorithm running, the two clustering clusters closest to each other are found to merge. The merging process is repeated until reach the preset number of cluster clusters. The BIRCH algorithm uses the clustering feature (CF) tree to perform hierarchical clustering. The algorithm builds a CF tree based on input data first. Then, the clustering algorithm and the outlier processing on leaf nodes are conducted. At the end of clustering, each leaf node becomes a cluster of a sample set.

### 6.3 The structure of Aytekin’s model

The loss function of the $CCAE_b$ model is inspired form the Aytekin’s model. In this section, we make a comparison between the $CCAE$ model and the Aytekin’s model. Fig. 8 gives the result and Table. 5 gives the structure of the Aytekin’s model.
Table 5. The Aytekin’s model and comparison with the CCAE_a model and the CCAE_b model.

| Layer | Method          | Model                  | Output size     |
|-------|-----------------|------------------------|-----------------|
|       | The Aytekin’s model |                        |                 |
| layer1| Conv[5,32,2]    | 128×128×32             |                 |
| layer2| Conv[5,64,2]    | 64×64×64               |                 |
| layer3| Conv[3,128,2]   | 32×32×128              |                 |
| layer4| Dense[2048]     | 2048×1                 |                 |
| layer5| Deconv[3,128,2]| 64×64×128              |                 |
| layer6| Deconv[5,64,2]  | 128×128×64             |                 |
| layer7| Deconv[5,32,2]  | 256×256×32             |                 |
| layer8| Deconv[3,1,1]   | 256×256×1              |                 |

6.4 The calculation of evaluation index

We calculate the overall accuracy of the fingerprints, the average precision, the average recall, and the average F1 score of the model. These parameters respectively represent the overall accuracy of the model, the intra-class clustering accuracy, the inter-class clustering accuracy, and the intra-class and the inter-class accuracy harmonic average. The F1 score is a good indicator of model accuracy. The closer its value goes to 1, the better the model. Their calculation formula is shown as:

\[
Accuracy_{all} = \frac{\sum_{class} TP_{class}}{\sum_{class} Total_{class}},
\]
\[
\text{Precision}_{\text{average}} = \frac{1}{N} \sum_{\text{class}} \frac{TP_{\text{class}}}{FP_{\text{class}} + TP_{\text{class}}},
\]
\[
\text{Recall}_{\text{average}} = \frac{1}{N} \sum_{\text{class}} \frac{TP_{\text{class}}}{TP_{\text{class}} + FN_{\text{class}}},
\]
\[
F_1_{\text{average}} = \frac{1}{N} \sum_{\text{class}} \frac{2 \times \text{Precision}_{\text{class}} \times \text{Recall}_{\text{class}}}{\text{Precision}_{\text{class}} + \text{Recall}_{\text{class}}},
\]

where \text{class} is the fingerprint pattern, it contains arch (arch and tented arch), whorl, left loop, and right loop. \(TP\) is the correct number of clustering categories and \(FP\) is the number of the fingerprint intra-class error clustering. \(FN\) is the number of the fingerprint inter-class error clustering. \(N\) is the number of the fingerprint patterns. The \text{Precision}_{\text{class}} and the \text{Recall}_{\text{class}} are precision and recall within a single class. This approach is a multi-classification problem, so we use the average values to evaluate performance of the model.

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