Fraction interpretation of pre-service mathematics teachers

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Abstract. Fractions are a mathematical concepts applied many fields. Therefore correct interpretation of fraction is an important thing. This research investigated the interpretations of fractions by pre-service mathematics teachers. We interviewed 35 pre-service mathematics teachers to interpret 4/5. The results showed that only 19 participants in the study could interpret fractions as part of whole, quotient, and ratio. There was no participant who were able to interpreted fraction as operators and measures. These results indicated that although pre-service mathematics teachers often use fractions in mathematics topic since elementary school to university, their interpretation of fractions is still incomplete.

1. Introduction

Fraction is one of the mathematical concepts that is widely used in various fields, not only in mathematics but also in other fields of knowledge. Subjects matters which related to fraction are learned since elementary to high school. Nevertheless, fraction is still considered a difficult subject by students. This also happens to students in Japan and China who in fact have a good conceptual understanding [1]. Conceptual understanding of fraction is certainly different from the ability to manipulate algorithms [2]. We cannot assume a person's understanding is good just because he can run the algorithms or mention the definition of fraction [2]. Even though, conceptual knowledge and algorithmic procedure are two different things. This difference can be seen from the questions that discuss procedural and conceptual about the equivalence of fraction following.

![Figure 1. Equivalence tasks examining the notion of one or a whole [3]](image)

From these two questions, we can see that algorithmic procedural abilities differ from conceptual understanding. The results of the study using the two questions indicate that many students can answer
procedural questions, but cannot answer conceptual questions [3]. All this time, learning about fractions taught more algorithmic procedures. Meanwhile, conceptual knowledge about fractions has never been obtained by students in many countries [1].

Generally, fraction have five main interpretation, including fraction as part of whole, fraction as quotient, fraction as ratio, and fraction as operators, and fraction as measures [4, 5, 6]. According to Kieren (1976), the part of whole personality of fractions permeated the aforementioned four sub constructs. Table 1 explains each interpretation.

**Table 1. Interpretation of fractions**

| Interpretation   | Commentary                                                                                                                                 |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Part of whole    | In this interpretation, a unit is partitioned into equivalent pieces (e.g., eighths, sixths, or halves) and fraction can be represented as numbers of these pieces (e.g., three eighths, five sixths, one half). The partition do not have to be equall in shape and size (i.e., congruent), but they must be equivalent in some attribute such as area, volume, or number [4]. In this context, the numerator of the fraction must be less or equal to the denominator [7]. |
| Quotient         | Fraction can interpret the result of division [4, 6, 7]. For example, we want to share four chocolate among 2 peoples. Therefore, each people receives 4/2 or 2 cakes. In this perception, numerator of fraction can be smaller, equal to or bigger than the denominator [7]. |
| Ratio            | Fraction can be interpret as a comparison of two quantities [4, 6]. The ratio is not only be able to compare measure of the same type, but also different type [6]. |
| Operators        | In this interpretation, we think of fraction as function. Fraction act as mappings, taking some set or region, and mapping it onto another set or region. More simply put, the operator notion of fraction is about shrinking and enlarging, contracting and expanding, enlarging and reducing, or multiplying and dividing. For example, 3/4 of is an operator which instructs to multiply by 3 and divide the result by 4 [6]. |
| Measures         | The heart of this interpretation is the idea of a fraction as a length on the number line created by partitioning units into subunits. A unit of measure can always be partitioned into smaller and smaller subunits [4]. A fraction measures the distance of a certain point on the number line from zero [6]. |

Interpretation of fraction related to operation of fraction. Behr, et. al (1983) proposed theory which linking the each interpretation to basic of fractions, fraction equivalence, and problem solving (Figure 2) [5, 7]. Based on the figure, it is important for student construct their knowledge about the all interpretation of fraction before learn about operation of fraction. Students’ activities in the learning about operation of fraction should not only be procedural but also conceptual. The teacher must give students the opportunity to build conceptual knowledge.

In order to develop students' conceptual knowledge appropriately, a teacher must have good conceptual understanding. Pre-service mathematics teachers certainly have a long experience of learning fractions. They certainly can perform algorithmic procedures about fractions such as adding, subtracting, multiplying, and dividing fractions. However, this ability cannot describe the conceptual understanding of pre-service mathematics teachers about fractions.
2. Methods
To investigate the ability of pre-service mathematics teachers in fraction interpretation, we conducted exploratory research involving 35 preservice teachers who were bachelor students of mathematics education at one of the universities in Bengkulu, Indonesia. We ask them to interpret fraction 4/5. The students can interpret fractions in the various models which they think of. The student’s interpretation were classified based on the five interpretation of fractions.

3. Result and Discussion
Table 2 summarized results of bachelor students’ interpretation about fraction 4/5. From 35 students, 5 students gave 2 interpretations, 14 students gave 1 interpretation, and 16 students did not give the correct interpretation.

| Interpretation | Numbers of students |
|---------------|---------------------|
| Part of wholes | 12                  |
| Quotient      | 9                   |
| Ratio         | 3                   |
| Operators     | 0                   |
| Measures      | 0                   |

Most students interpret the fractions as part of the whole. Most students use images to interpret fractions as part of whole, and other students describe by sentences. Some students interpret fractions as quotient by giving relevant example. Of the 35 students, only 3 students interpreted fractions as ratio

It can be seen that part of whole and quotient are the best known interpretation. It was assumed that at the beginning of fraction learning in elementary school, the teacher interprets fractions as part of the whole or quotient. Meanwhile, other interpretations is only viewed as a case in fractions or other mathematical topics that apply fractions, such as topic about scale or ratio. In learning mathematics at school, there is no emphasis that fractions can be interpreted as ratios, operators, and measures. The results also show that more than 40% of students do not have conceptual understanding of fractions. This indicates that so far their success in the fraction topic has only been procedural. If this condition is not corrected then there is a tendency that preservice teachers will teach fractions only procedurally. This situation cause instruction that is not meaningful continually. Therefore, bachelor students must develop conceptual knowledge about fractions completely.
In addition, the fraction interpretation model given by students is relatively same. This shows that there are limitations to the model of fraction interpretation. For example in interpreting quotient, students only mention examples of situations related to fractions as quotient. Even though to be more real, students can use images. This model of interpretation is important so that when they become teachers they can present a variety of interesting models of interpretation and clarify the concept of fractions.

4. Conclusion
It was concluded that the students lack of ability to interpret the fraction. All students can not interpret five means of fraction completely. Each student just can give two interpretation. Generally, the model interpreted by student are equal.

5. References
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