Optimal Precoding for Multiuser MIMO Systems With Phase Quantization and PSK Modulation via Branch-and-Bound

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Abstract—MIMO systems are considered as most promising for wireless communications. However, with an increasing number of radio front ends the corresponding energy consumption and costs become an issue, which can be relieved by the utilization of low-resolution quantizers. In this study we propose an optimal precoding algorithm constrained to constant envelope signals and phase quantization that maximizes the minimum distance to the decision threshold at the receivers using a branch-and-bound strategy. The proposed algorithm is superior to the existing methods in terms of bit error rate. Numerical results show that the proposed approach has significantly lower complexity than exhaustive search.

Index Terms—Precoding, low-resolution quantization, MIMO systems, branch-and-bound methods.

I. INTRODUCTION

The increasing growth of data transmission generates a great demand for the development of high performance communication systems. One challenge in the wireless communications area is the minimization of the energy consumption without major bit error rate performance compromise.

With this in mind, systems with low-resolution quantizers are promising, knowing that the energy consumption of data converters scales exponentially with the resolution in amplitude 1.

Several strategies for precoding with low-resolution quantizers exist. Linear approaches such as the Zero-forcing method (ZF) 2 and MMSE 3 have a low complexity but suffer from error floor in the bit error rate. Therefore, nonlinear precoders have been designed with different design criteria.

A conventional design criterion is the MSE which is considered in the branch-and-bound (B&B) algorithm in 4. Another widely used design criterion in given by the maximization of the minimum distance to the decision threshold (Max-Min DDT) 5, 6, 7, 8, which is promising in combination with hard detection. In 7 an optimal precoding algorithm was presented for the Max-Min DDT and 1-bit quantization at transmitter and receiver (QPSK). In 8 a suboptimal algorithm is developed for the Max-Min DDT criterion and 2q-PSK symbols at each transmit antenna for QAM and PSK modulation schemes.

In the present study, we generalize the work of 7 for phase quantizers with arbitrary number of phases at the transmit antennas and PSK modulation. This extension should be considered as non trivial because in the case of PSK, each symbol cannot be decomposed in independent real and imaginary part as done in the 1-bit case. The proposed precoder is optimal in terms of the Max-Min DDT criterion, obtained by using a sophisticated branch-and-bound strategy. The initial step of the proposed method implies the solution of the relaxed problem subsequently rounded to the feasible set and then a tree search based algorithm is devised.

The paper is organized as follows: Section II describes the system model, whereas Section III establishes the precoder’s objectives, explains the criterion and exposes the problem formulation. In Section IV the proposed precoding algorithm is described. Section V presents and discusses numerical results, while Section VI gives the conclusions.

Regarding the notation, note that real and imaginary part operators are also applied to vectors and matrices, e.g., Re \{x\} = [Re \{x_1\}, \ldots, Re \{x_M\}]^T.

II. SYSTEM MODEL

In this study, a single cell MU-MIMO downlink with full channel state information at the base station (BS) is considered, as illustrated in Fig. 1. On the BS there are M transmit antennas that serve K single antenna users. The data symbol for the ith user \(s_i\) is a \(\alpha_s\)-PSK symbol taken from the set \(S\) described by

\[
S = \left\{ s : s = e^{j \frac{\pi (2s + 1)}{2\alpha_s}}, \text{for } i = 1, \ldots, \alpha_s \right\} .
\]  

(1)

The stacked vector with data symbols for the \(K\) users is denoted by \(s = [s_1, \ldots, s_K]^T\). The vector \(x\) is the input for...
Fig. 2: Decision regions for a 8-PSK data case

Fig. 3: Rotated coordinate system

III. PRECODING TASK

This section establishes the objectives of the precoder, presents the used design criterion and exposes the problem formulation. The criterion for the precoder design is the maximization of the minimum distance to the decision threshold or equivalently the maximization of the safety margin at the detectors. With this aim, is to find the vector \( x \) which yields the smallest distance to the decision threshold is maximized. By expressing the corresponding problem in the epigraph form \([9]\), the problem has a linear objective function, linear constraints and a discrete feasible set, which then is a non-convex problem that has a NP hard solution by applying exhaustive search.

This study relies on the distance to the decision threshold \( \epsilon \) for hard detection of PSK symbols and the description of the objective is equivalent to the one presented in \([8]\). Note that for the special case of QPSK modulation the objective is also equivalent to the objective utilized in \([7]\).

By considering a rotation by \( \arg \{ s_i^* \} = -\phi_{s_i} \) of the coordinate system the symbol of interest is placed on the real axis, as shown in Fig.3. This is done by multiplying both the interest symbol \( s_i \) and the noiseless received signal \( z_i \) by \( e^{-j\phi_{s_i}} = s_i^* \) which reads

\[
\tilde{s}_i = s_i s_i^* = 1, \quad w_i = z_i s_i^*.
\]

The distance of the rotated symbol \( w_i \) to the rotated decision threshold is then expressed as

\[
\epsilon_i = \text{Re} \{ w_i \} \sin \theta - |\text{Im} \{ w_i \}| \cos \theta,
\]

as shown in detail in \([8]\). Since the considered rotation included also the decision thresholds the distance expression in (6) holds also for \( z_i \). The minimum of all \( \epsilon_i \), for \( i = 1, \ldots, M \) is defined as \( \epsilon \), which serves as the objective of the precoding design. The algorithm task is to construct the transmit vector \( x \) that maximizes \( \epsilon \).

Based on a stacked vector notation for \( w_i \), namely \( w = \text{diag}(s^*)Hx \), the equivalent minimization problem reads

\[
[x_{\text{opt}}, \epsilon_{\text{opt}}] = \arg \min_{x \in \mathbb{C}^M, \epsilon} -\epsilon \quad \text{s.t.}
\]

\[
\text{Re} \{ Hs^*x \} \sin \theta - |\text{Im} \{ Hs^*x \}| \cos \theta \geq \epsilon_{12K},
\]

where \( H_{s^*} = \text{diag}(s^*)H \).

IV. PROPOSED BRANCH-AND-BOUND PRECODER

In this section we introduce the proposed precoder and derive the bounding steps for the algorithm. It is divided
into three parts, the description of the mapped version of the Minimum Distance to Decision Threshold Precoder (MDDT-Mapped), a general introduction of branch-and-bound precoding strategy and the description of the MDDT branch-and-bound algorithm.

A. MDDT-Mapped Precoder

One approach for finding a feasible solution of (7) is to solve a relaxed version of the original problem followed by a mapping process to ensure that the precoding vector is in the feasible set of the discrete problem. The relaxation is brought by replacing the set $X^M$ by its convex hull, which then establishes convexity of the considered problem. The corresponding relaxed problem is an LP and reads

$$\begin{align*}
\{x_{lb}, \varepsilon_{lb}\} &= \arg\min_{x, \varepsilon} -\varepsilon \\
\text{s.t.} & \quad \Re\{H_{s^*}x\} \sin \theta - |\Im\{H_{s^*}x\}| \cos \theta \geq \varepsilon 1_{2K} \\
& \quad \Re\{x_m e^{j\phi_i}\} \leq \frac{\cos \left(\frac{\pi}{\alpha_x}\right)}{\sqrt{M}}, \text{ for } m = 1, \ldots, M \\
& \quad \phi_i = \frac{2\pi i}{\alpha_x}, \text{ for } i = 1, \ldots, \alpha_x,
\end{align*}$$

which is basically presented before in (8). Note that unlike the algorithm in (8), where $\alpha_x$ is restricted to integer powers of 2, the problem formulation (8) from above supports $\alpha_x$ to be any integer value. Subsequently the continuous solution $x_{lb}$ is quantized to the point in $X^M$ with the shortest Euclidean distance.

The optimal value of (8) is always a lower bound to the optimal value of the original problem (7). Mapping to the feasible set yields a valid solution $x_{ub}$ and the corresponding value for $-\varepsilon$ provides an upper bound on the optimal value of the original problem (7).

B. Introduction of the Branch-and-Bound method

This part of the algorithm is a tree search problem, where a breadth first search is employed. For constructing the tree we consider that from each node $\alpha_x$ branches go out and that the tree consists of $M$ levels.

For the construction of the discrete precoding vector we consider a constrained minimization of a precoding objective function $f(x, s)$, which could be the negative minimum distance to decision threshold, given by

$$x_{opt} = \arg\min_{x} f(x, s) \quad \text{s.t.} \quad x \in X^M. \quad (9)$$

A lower bound on $f(x_{opt}, s)$ can be obtained by relaxing this problem, e.g., as described in (8). An upper bound on $f(x_{opt}, s)$ can be found by mapping the solution of the relaxed version to $X^M$ and evaluating $f(\cdot)$ accordingly. The upper bound on the optimal value is termed $\bar{f}$.

If we consider $d$ fixed entries of $x$, the precoding vector becomes $x = [x_1^T, x_2^T]^T$, with $x_1 \in X^d$. Then a sub problem can be formulated with

$$x_{2, lb} = \arg\min_{x_2} f(x_2, x_1, s) \quad (10)$$

$$\text{s.t.} \quad \Re\{x_m e^{j\phi_i}\} \leq \frac{\cos \left(\frac{\pi}{\alpha_x}\right)}{\sqrt{M}}, \text{ for } m = 1, \ldots, M - d$$

$$\phi_i = \frac{2\pi i}{\alpha_x}, \text{ for } i = 1, \ldots, \alpha_x.$$ 

If the optimal value of (10) is larger (worse) than a known upper bound $\bar{f}$ on the solution of (9), then all member in the discrete solution set which include vector $x_1$ can be excluded from the search.

C. MDDT Branch-and-Bound algorithm derivation

In this section a branch-and-bound algorithm is proposed which solves (9) by considering the problem in (8) for the initialization and sub problems as given by (10) for computing lower bounds. In order to formulate a real valued problem matrix $H_r$ and vector $x_r$ are defined as follows

$$x_r = \begin{bmatrix}
\Re\{x_1\} \\
\Im\{x_1\} \\
\Re\{x_2\} \\
\Im\{x_2\} \\
\vdots \\
\Re\{x_M\} \\
\Im\{x_M\}
\end{bmatrix}, \quad H_r = \begin{bmatrix}
\Gamma_1 \cdots \Gamma_{1M} \\
\Lambda_1 \cdots \Lambda_{1M} \\
\vdots \\
\Psi_1 \cdots \Psi_{1M} \\
\Delta_1 \cdots \Delta_{1M}
\end{bmatrix}, \quad (11)$$

with

$$\Gamma = \Im\{H_{s^*}\} \cos(\theta) - \Re\{H_{s^*}\} \sin(\theta)$$

$$\Lambda = \Re\{H_{s^*}\} \cos(\theta) + \Im\{H_{s^*}\} \sin(\theta)$$

$$\Psi = -\Im\{H_{s^*}\} \cos(\theta) - \Re\{H_{s^*}\} \sin(\theta)$$

$$\Delta = \Im\{H_{s^*}\} \sin(\theta) - \Re\{H_{s^*}\} \cos(\theta).$$

With the real valued description, the variable vector of the optimization problem can be denoted by $v = [\varepsilon, x_r^T]^T$, such that the discrete optimization problem reads as

$$v_{opt} = \arg\min_v a^Tv$$

$$\text{s.t.} \quad Av \leq 0_{2K},$$

$$\{v_{2m} + jv_{2m+1}\} \in X, \quad \text{for } m = 1, \ldots, M,$$

with

$$a = [-1, 0_{2M}^T]^T, \quad A = [1_{2K}, H_r].$$

Replacing the the discrete solution set by its convex hull yields the relaxed problem given by

$$v_{lb} = \arg\min_v a^Tv$$

$$\text{s.t.} \quad Uv \leq p, \quad (14)$$

with

$$U = [A^T, R^T]^T, \quad R = [0_{M\alpha_x}, R'],$$

$$R' = [(I_M \otimes \beta_1)^T, (I_M \otimes \beta_2)^T, \ldots, (I_M \otimes \beta_{\alpha_x})^T]^T.$$
\[ \beta_i = \cos \phi_i, \sin \phi_i \quad \theta = \left[ \begin{array}{c} 0_{2K} \cos \left( \frac{\pi \alpha_i}{\sqrt{M}} \right) \end{array} \right]^T. \]

In the branch-and-bound method sub problems are solved due to \( v = [e, x_{r_1}, x_{r_2}]^T \), where \( x_{r_1} \) is a fixed vector of length \( 2d \), which belongs to the discrete set according to \( v_{12m} + jv_{12m+1} \in X \), for \( m = 1, \ldots, d \).

The matrix \( U \) can be expressed with the following structure \( U = [U_1, U_2, U_3] \), where \( U_1 \) contains \( 2d \) columns of \( U \) and \( U_1 \) is the first column of \( U \). With this, the matrix \( \tilde{U} = [u_1, U_2] \) and the vector \( \tilde{v} = [e, x_{r_2}]^T \) are composed. Using \( \tilde{U} \) and \( \tilde{v} \) the sub problem for the lower-bounding step can be expressed as

\[
\tilde{v}_{lb} = \arg \min_{\tilde{v}} \tilde{a}^T \tilde{v} \quad \text{s.t.} \quad \tilde{U} \tilde{v} \leq b,
\]

with \( \tilde{a} = [-1, 0_{2(M-2d)}]^T \) and \( b = p - U_1 x_{r_1} \). Solving \([15]\) provides an upper bound on the optimal value of the discrete problem with the condition on \( x_{r_1} \). In case the lower bound conditioned on \( x_{r_1} \) is higher than any upper bound on the original problem \( x_{r_1} \) cannot be part of the solution and every member of the discrete solution set which includes \( x_{r_1} \) can be excluded from the search. The steps of the method are detailed in Algorithm 1.

Note that the computation of the optimal precoding vector in each symbol period can correspond to an enormous computational complexity. Nevertheless, the method might be a practical solution for channels with large coherence time, where the finite number of different precoding vectors can be precomputed and stored as suggested in \([10]\).

V. Numerical Results

For comparison of the proposed method with the state-of-the-art algorithms, the uncoded bit error rate is evaluated, where Gray-coding is considered. The considered signal-to-noise ratios (SNR) is defined by \( \text{SNR} = \frac{\text{SINR}}{\alpha_n^2} \), where \( N_0 \) denotes the noise power density.

The numerical computations were made with \( K = 2 \) users, and the number of antennas at the BS is \( M = 6 \) and \( M = 9 \) and 1000 random channel realizations. One conventional configuration is considered with 8-PSK symbols (\( \alpha_x = 8 \), \( \alpha_s = 8 \)). In addition, to demonstrate the flexibility of the proposed framework, a more exotic configuration is considered where \( x_i \) is a 3-PSK symbol using QPSK modulation at the same time (\( \alpha_x = 3 \), \( \alpha_s = 4 \)), which is compatible only with a subset of the existing methods. The corresponding BER performances are illustrated in Fig. 5 and Fig. 4 respectively.

The proposed method is compared with the following methods from the literature: 1. The MSM-Precoder \([8]\), which corresponds to solving an LP with computational complexity in the order of \( O(2M + 1)^{3.5} \), when using interior point methods (IPM); 2. The ZF precoder with constant envelope \([2]\) with \( O(K^2M) \), which precoding vectors are subsequently phase quantized; 3. The CIO precoder implemented via CVX \([11]\), which corresponds to solving a second order cone program with \( O((2M + 1)^{3.5}) \), when using IPM. In addition, the MaxMin DDT precoder with full resolution and per antenna power constrained is considered, which yields a higher optimal value \( \epsilon \) value, because relaxation of the feasible set results in an upper bound of the optimal value of the original problem. As expected the proposed algorithm shows a significantly lower BER than existing suboptimal algorithms, which confirm the aptitude of the Max Min DDT design objective in the context of hard detection.

Note that the proposed algorithm does not yield an error floor which occurs for suboptimal precoding algorithms with phase quantization at the BS.

The proposed branch-and-bound method yields the same solution as the exhaustive search but with a lower average complexity. The complexity of the algorithm heavily depends on finding as early as possible a tight upper bound that permits many exclusions of possible candidates while going down the tree. By using IPM for solving sub problems \([15]\) corresponds to a computational complexity given by \( O(n^{3.5}) \).

Algorithm 1 Proposed B&B Precoding for solving \((7)\)

initialization:
Given the channel \( H \) and transmit symbols \( s \) compute a valid upper bound \( \hat{f} \) on the problem in \((7)\), e.g., by solving \((8)\) followed by a mapping to the closest precoding vector \( x \in X^M \).

Define the first level \( (d=1) \) of the tree by \( G_d := X \)
for \( d = 1 : M - 1 \) do
Partition \( G_d \) in \( x_1, \ldots, x_1 | G_d \)
for \( i = 1 : |G_d| \) do
Express \( x_{1,i} \) with stacked vector notation due to \((11)\) as \( x_{r_1,i} \)
Conditioned on \( x_{r_1,i} \) solve \( \hat{v}_{lb} \) from \((15)\)
Determine \( \epsilon = |\hat{v}_{lb}| \)
Compute the lower bound: \( \text{lb}(x_{1,i}) := -\epsilon \)
Map \( x_{2,i} \) to the discrete solution with the closest Euclidean distance:
\( \tilde{x}_2(x_{2,i}) \in X^{M-d} \)
Using \( \tilde{x}_2 \) find the smallest (negative) distance to the decision threshold \( \text{ub}(x_{1,i}) := \text{max}_k \left[ \left| \text{Im} \left( H_s^T x_{1,i}, \tilde{x}_2 \right) \right| \cos \theta - \text{Re} \left( H_s^T x_{1,i}, \tilde{x}_2 \right) \right] \)
Update the best upper bound with:
\( \hat{f} = \min \left( \hat{f}, \text{ub}(x_{1,i}) \right) \)
end for
Build a reduced set by comparing conditioned lower bounds with the global upper bound \( \hat{f} \)
\( G_{d+1} := G_d \times X \)
end for
Search method for the ultimate level \( d = M \),
Partition \( G_1 \) in \( x_1, \ldots, x_1 | G_1 \)
\( \epsilon(x_{1,i}) := \min_k \left[ \text{Re} \left( H_s^T x_{1,i} \right) \sin \theta - |\text{Im} \left( H_s^T x_{1,i} \right)| \cos \theta \right] \)
The global solution is
\( x_{opt} = \arg \max_{x_{1,i} \in G_1} \epsilon(x_{1,i}) \)
An optimal algorithm for precoding constrained to constant envelope and phase quantization for PSK modulation and hard detection is proposed. The design criterion maximizes the envelope and phase quantization for PSK modulation and hard detection is proposed. The design criterion maximizes the number of sub problems decrease when climbing down the tree. The average number of sub problems is illustrated in Fig. 6. Based on Fig. 6 it turns out that the average number of sub problems is only a small fraction of the number of candidates which are evaluated in the exhaustive search. Taking into account that each candidate evaluation in the exhaustive search corresponds to a complexity of $O(MK)$ justifies the utilization of the proposed branch-and-bound approach, when the optimal precoding vector is desired.

VI. Conclusions

An optimal algorithm for precoding constrained to constant envelope and phase quantization for PSK modulation and hard detection is proposed. The design criterion maximizes the minimum distance to the decision threshold at the receivers. The proposed algorithm outperforms the state-of-art techniques for this class of precoding in terms of BER. Numerical results confirm the efficiency of the proposed branch-and-bound strategy.

References

[1] R. Walden, “Analog-to-digital converter survey and analysis,” IEEE J. Sel. Areas Commun., vol. 17, no. 4, pp. 539–550, Apr. 1999.
[2] S. K. Mohammed and E. G. Larsson, “Per-antenna constant envelope precoding for large multi-user MIMO systems,” IEEE Trans. Commun., vol. 61, no. 3, pp. 1059–1071, March 2013.
[3] A. Mezghani, R. Ghiat, and J. A. Nossek, “Transmit processing with low resolution D/A-converters,” in Proc. of the 16th IEEE Int. Conf. on Electronics, Circuits and Systems - (ICECS 2009), Hammamet,Tunisia, Dec 2009, pp. 683–686.
[4] S. Jacobsson, W. Xu, G. Durisi, and C. Studer, “MSE-optimal 1-bit precoding for multiuser MIMO via branch and bound,” in Proc. of 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Calgary, Alberta, Canada, April 2018, pp. 3589–3593.
[5] L. Landau, S. Krone, and G. P. Fettweis, “Intersymbol-interference design for maximum information rates with 1-bit quantization and oversampling at the receiver,” in Proc. of the Int. ITG Conf. on Systems, Communications and Coding, Munich, Germany, Jan. 2013.
[6] J. Mo and R. W. Heath Jr, “Capacity analysis of one-bit quantized MIMO systems with transmitter channel state information,” IEEE Trans. Signal Process., vol. 63, no. 20, pp. 5498–5512, Oct 2015.
[7] L. T. N. Landau and R. C. de Lamare, “Branch-and-bound precoding for multiuser MIMO systems with 1-bit quantization,” IEEE Wireless Commun. Lett., vol. 6, no. 6, pp. 770–773, Dec 2017.
[8] H. Jedda, A. Mezghani, A. L. Swindlehurst, and J. A. Nossek, “Quantized constant envelope precoding with PSK and QAM signaling,” IEEE Trans. Wireless Commun., vol. 17, no. 12, pp. 8022–8034, Dec 2018.
[9] S. Boyd and L. Vandenberghe, Convex Optimization. New York, NY, USA: Cambridge University Press, 2004.
[10] H. Jedda, J. A. Nossek, and A. Mezghani, “Minimum BER precoding in 1-bit massive MIMO systems,” in Proc. of IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM), Rio de Janeiro, Brazil, July 2016.
[11] P. V. Amadori and C. Masouros, “Constant envelope precoding by interference exploitation in phase shift keying-modulated multiuser transmission,” IEEE Trans. Commun., vol. 16, no. 1, pp. 538–550, Jan 2017.