Be born slow or die fast: spin evolution of neutron stars with alignment or counteralignment

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ABSTRACT
We revisit the case of magneto-rotational evolution of neutron stars (NSs) with accounting for changes of the angle $\chi$ between spin and magnetic axes. This element of the evolution of NSs is very important for age estimates and population modeling, but usually it is neglected. In the framework of two models of energy losses we demonstrate that unless NSs are born with the inclination angle $\chi_0$ very close (within $\sim 1^\circ$) to the position of maximal losses, pulsars with short initial periods quickly reach regions of small energy losses (aligned rotators in the case of magneto-dipole losses and orthogonal rotators – in the case of current losses) without significant spin-down. This means that either most of known NSs should be born with relatively long periods close to presently observed values, or the initial inclination angles should be large (close to $90^\circ$) for the magnetodipole model and very small (close to $0^\circ$) for the current losses model, or that both models cannot be applied for the whole evolutionary track of a typical NS. In particular, magnetar candidates should be born with periods close to the observed ones within the scope of both models of energy losses. We discuss how these considerations can influence population synthesis models and age estimates for different types of isolated NSs. Focusing on the model of current losses we illustrate our conclusion with evolutionary track reconstructions on the $P - \sin \chi$ plane. We conclude, that most probably both existing models of energy losses – magneto-dipole and longitudinal current – require serious modifications, on the other hand, estimates obtained under the standard assumption of $\sin \chi = 1 = \text{const}$ does not have solid theoretical ground and should be taken with care.

Key words: stars: neutron — stars: evolution — pulsars: general

1 INTRODUCTION

Radio pulsars have been discovered nearly 40 years ago (Hewish at al.\textsuperscript{1968}), and very quickly they were recognized as magnetized rotating neutron stars spinning down and dissipating their rotational energy. Despite of all the progress in the theory of magneto-rotational evolution of isolated neutron stars, several important questions still remain unclear. The uncertainty concerning the evolution of the angle $\chi$ between the magnetic axis and the spin axis of an isolated neutron star is one of them, although the angle $\chi$ significantly affects the rate of energy release for all mechanisms of radio pulsars emission. At that, among the basic parameters, which determine astrophysical manifestations of radio pulsar, this one is the least popular, and its evolution is usually neglected.

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We discuss evolution and estimate ages of isolated neutron stars taking into account changes of the angle between the magnetic and the spin axes. We consider the magnetodipole model and the model of pulsar radio emission based on current losses in the magnetosphere of a neutron star, focusing on the latter. It should be emphasized here that the evolution of an inclination angle is a key parameter for both models. Unfortunately, many authors either completely disregard the evolution of an inclination angle or at best consider its result to be identical to the magnetic field decay (see, for example, Regimbau & de Freitas Pacheco\textsuperscript{2001}; Faucher-Giguère & Kaspi\textsuperscript{2006}, and references therein).

In the following section of this paper we consider the magnetodipole and current losses models of energy release of radio pulsars. There the basic information about the models themselves and their main contradictions with observations are listed. The expression for the characteristic ages of isolated neutron stars with the account of evolution of an inclination angle is presented. As a result, the ages of
isolated neutron stars estimated by the obtained formulas for different initial parameters (spin periods and inclination angles) are listed in Tables 1 and 2. In the third section we reconstruct possible evolutionary tracks of XDIN RX J0720.4-3125 and AXPE2229+586 varying parameters and assuming the model based on current losses and neglecting the possibility that magnetic field decay is important. Everywhere in the fourth sections we give a discussion. In particular, as the account for changes of χ mainly influence population synthesis models and age estimates, we discuss these topics. Finally, we present our conclusions.

2 THE ROLE OF THE EVOLUTION OF INCLINATION ANGLE IN THE AGE ESTIMATES ON THE ACTIVE RADIO PULSAR STAGE

In this section we estimate duration of active radio pulsar stage for isolated neutron stars taking into account the evolution of the angle between the magnetic and the spin axes. We consider two models of energy release mechanism: the magnetodipole model and current losses model. Everywhere in the paper we neglect the possibility that field decays, and that there are any non-electromagnetic effects like those described, for example, by Macy (1974); Menou et al (2001); Wasserman (2002); Ruderman (2006).

The primary point of our calculations is that the ratio f(χ)/P, where P is spin period, and f(χ) = cos χ for the magnetodipole model and f(χ) = sin χ for the current losses model, is invariant during the evolution of a neutron star on the radio pulsar stage. Therefore, in order to estimate the characteristic age of an isolated neutron star in scope of one of the models we do not need to know the initial inclination angle. The inclination angle and the spin period of a neutron star at the present time and its initial spin period are enough to define the initial inclination angle for each model of energy release. The validity and implications of the invariants are discussed throughout this paper.

2.1 Magnetodipole model

Energy losses of an isolated radio pulsar due to radiation can be associated with two different scenarios. The first one is based on magnetic dipole braking, and we discuss it in this subsection. In this case, assuming a perfect dipole rotating in vacuum, the energy losses can be estimated by the so-called magnetodipole formula (Pacini 1967):

\[ W_{md} = -J\Omega\dot{\Omega} = \frac{1}{6} \frac{B_0^2 Q^4 R_0^6}{c^2} \sin^2 \chi. \]  

(1)

Here \( J \approx 2/5MR^2 \) is a moment of inertia of a neutron star, \( \chi \) is the angle between the spin and the magnetic axes, and \( \Omega = 2\pi/P \) is the angular velocity of a neutron star. The magnetic field is defined as \( B_0 = 2\mu/R^3 \), i.e. this is the field on a pole. The magnetodipole losses for ordinary pulsars are approximately \( 10^{31} - 10^{34} \) erg s\(^{-1}\). A simple estimate of a lifetime of a normal radio pulsar with a spin period \( P = 1 \) s and period derivative \( \dot{P} = 10^{-15} \) s s\(^{-1}\) gives us \( \tau_{ch} = P/2\dot{P} \approx 10^7 \) yrs. One can see that this result is only a rough approximation; for more accurate estimate of the age of a radio pulsar the evolution of the angle between the magnetic and the spin axes should be taken into account.

According to (1), the slow-down rate of a radio pulsar for the magnetodipole model can be defined as follows:

\[ \dot{P} = 0.24 \times 10^{-15} B_0^2 P^{-1} \sin^2 \chi. \]  

(2)

In the scope of the magnetodipole model the radiation-reaction torque is perpendicular to the magnetodipole moment of a neutron star. Therefore, the component of angular velocity \( \Omega \cos \chi \) parallel to the magnetodipole moment (and so perpendicular to the deceleration moment) remains constant Davis & Goldstein (1970), Michel (1991), i.e. the ratio

\[ I_{md} = \frac{\cos \chi}{P} \]  

is invariant during the evolution of a neutron star at the active radio pulsar stage (since birth of a neutron star and up to the moment when it crosses the death line). Thus, the angle between the spin and the magnetic axes during the evolution tends to \( 0^\circ \). Davis & Goldstein (1970) emphasize that this model has the following implication: if radio pulsars are born as fast rotators and are decelerated only by dipole radiation to spin periods close to one second then the initial inclination angles must be very close to \( 90^\circ \).

Here we outline the problem that most of the authors disregard as they do not take into account the evolution of the angle between the magnetic axis and the spin axis. It is often accepted, especially in population synthesis models, that radio pulsars are born as fast rotators (\( \dot{P}_0 \sim 10 - 30 \) ms). As there is no information about the initial distribution of radio pulsars in the inclination angle (Tauris & Manchester 1995), most of the authors assume that the initial inclination angle to be equiprobable, or just ignore this question. Eq. (3) defines the evolutionary track of a newborn radio pulsar with given \( P_0 \) and \( \chi_0 \) till it reaches the death line, which for the magnetodipole model is generally approximated by the following simple equation (see, for example, Bhattacharya et al. (1992)):

\[ \frac{B}{P^2} = 0.34 \times 10^{12} \text{ G s}^{-2}. \]  

(4)

It should be noted here that the death line approximation (4) is valid for a neutron star with a perfect dipole magnetic field and with the radius \( R = 10^6 \) cm, the moment of inertia \( J = 10^{45} \) g cm\(^2\), and the inclination angle \( \sin \chi \sim 1 \), i.e. \( B \) is calculated out of \( P \) under these assumptions. The lack of dependence of the death line on the angle \( \chi \) should be considered as an important disadvantage of the usual approach in the context of the magnetic dipole braking model. In the model of current losses such dependence is taken into account (see below). Accordingly, the accuracy of age estimates is often lower for the magnetodipole model than for the model based on current losses.

As a result of (1), for the period \( P_{dl} \), at which neutron star is leaving the radio pulsar stage (the death line crossing), we have:

\[ P_{dl} = 1.7 B_{12}^{0.5} \text{ s}, \]  

(5)

here \( B_{12} = B/10^{12} \) G.

Thus, if

\[ P_{dl} > \frac{P_0}{\cos \chi_0}, \]  

(6)

an isolated neutron star becomes an aligned rotator before it can reach the death line and leave the stage of radio emission. As one can see, this way of evolution should be typical.
for most of the radio pulsars with short initial spin periods, except for those with the initial angles between the magnetic and the spin axes very close to 90°. For example, let us consider an evolutionary track of the neutron star with the initial spin period $P_0 = 20$ ms, initial angle between the spin and magnetic axes $\chi_0 = 45^\circ$ and magnetic field $B_0 = 10^{12}$ G. Eq. (3) allows us to estimate the period at which the neutron star becomes an aligned rotator: $P_{al} \approx 28$ ms.

As a result, for the magnetodipole model we conclude the following: if the invariant $I_3$ is valid, and radio pulsars are born as fast rotators, and their initial angle distribution is equiprobable, then most of radio pulsars should become aligned rotators long before they pass the death line. Thus, we should expect the distribution in observable angle between the magnetic and the spin axes of active radio pulsars to have a peak close to 0°. Otherwise, an essential part of radio pulsars should initially have long spin periods (Beskin, Gurevich & Istomin 1993).

Further, we can estimate a lifetime of an isolated neutron star as an active radio pulsar:

$$\tau_{md}[s] = \int_{P_0}^{P_{al}} \frac{dP}{2.4 \times 10^{-16} B_{12}^2 P^{-3} (1 - \cos^2 \chi_0(P^2)/(P_{al}^2))},$$

where $P_0$ and $\chi_0$ are the initial spin period and the angle between the spin and the magnetic axes of a radio pulsar, respectively, and $P_{al}$ is a period at which a radio emission turn-off occurs (a pulsar crosses the death line). If an inclination angle $\chi$ reaches 0° before a pulsar crosses the death line, the period of the transition to aligned rotators $P_{al}$ should be used instead of $P_{al}$.

Substituting Eq. (3) into (7) and integrating, we obtain:

$$\tau_{md} = 2 \times 10^{15} B_{12}^{-2} \left( \frac{P_0}{\cos \chi_0} \right)^2 \ln \left( \frac{1 - \cos^{-2} \chi_0}{1 - \cos^{-2} \chi_0} \right) \text{s.} \quad (8)$$

Therefore, assuming the initial spin period $P_0 = 20$ ms, initial angle between the spin and the magnetic axes $\chi_0 = 89.6^\circ$ and magnetic field $B_0 = 10^{12}$ G, from Eq. (8) we obtain an estimate of the age of an isolated neutron star at the moment it crosses the death line as $\tau_{md} = 226$ Myr. If we assume $\chi_0 = 45^\circ$, the age of the radio pulsar which enters the stage of aligned rotators can be estimated as $\tau_{al} = 0.16$ Myr. Table 1 contains the estimates of duration of the radio pulsar stage for different parameters (such as magnetic fields, initial spin periods and initial inclination angles of radio pulsars).

It should be noted here that the age of a neutron star estimated within the scope of magnetodipole model by (8) significantly depends on the death line. For example, if the coefficient in (8) is twice less, then the lifetime of a typical Crab-like pulsar at the radio pulsar stage estimated by (8) is reduced by at least the factor of 4. At that, the smaller inclination angles $\chi$ at which the pulsar crosses the death line gives the larger decrease in the estimated lifetimes with the coefficient reduction in (8). As the spin-down rate of a neutron star decreases near the death line (due to the factor $\sin^2 \chi$ in Eq. (2), the longest part of a lifetime at the radio pulsar stage is the period when a neutron star stays near the death line. Thus, as we do not take into account the inclination angle $\chi$ for the death line in the scope of the magnetodipole model Eq. (8) only roughly approximates the age of a neutron star. It is most likely that the more accurate expression for the death line can reduce the age estimates by a factor of a few.

### 2.2 Longitudinal current losses model

In this subsection we estimate ages of radio pulsars assuming that the energy losses of an isolated neutron star are associated with longitudinal currents in the pulsar magnetosphere. In our estimations we take into account the evolution of an angle between the magnetic and the spin axes of a neutron star. The energy losses of an isolated neutron star estimated by magnetodipole formula are of the same order of magnitude as the so-called current losses – the energy release determined by ponderomotive forces of longitudinal electric currents, i.e. the currents flowing along the open magnetic field lines and closing on the surface of the star (Beskin et al. 1993).

A simple age estimate of a normal radio pulsar for the model based on current losses is the same as for the magnetodipole model: $\tau_{ch} = P/2P \approx 10^7$ yrs. A more accurate estimate of a radio pulsar lifetime for the current losses model requires to consider evolution of an angle $\chi$ between the spin and the magnetic axes of a neutron star. The deceleration of neutron star rotation in the scope of this model is caused by the surface currents that flow across the magnetic field lines (Beskin et al. 1993). Therefore, the ponderomotive Ampere force arises, and the direction of the appropriate deceleration torque is opposite to the magnetodipole moment. Thus, the component of the angular velocity of a neutron star $\Omega \sin \chi$ perpendicular to the magnetodipole moment (and so perpendicular to the deceleration moment) remains constant, and the ratio

$$I_3 = \frac{\sin \chi}{\Omega} \quad (9)$$

is invariant during the stage of normal radio pulsar. As we can see, in the scope of current losses model the angle $\chi$ between the magnetic and the spin axes evolves to 90°. A time of evolution of an inclination angle is of the same order of magnitude as the slow-down rate of a neutron star.

Beskin et al. (1993) developed a theory in which energy losses are associated with longitudinal currents in the neutron star magnetosphere. In this approach there is no unique formula for $P$ for all phases of a radio pulsar evolution, also different assumptions about particle escape can be made. For the escape model based on the proposal by Ruderman & Sutherland (1974) (hindered escape) and for pulsars not very close to the death line, the following equation is derived:

$$\dot{P} = 10^{-15} B_{12}^{10/7} P_{14}^{11/4} \cos^{3/2} \chi. \quad (10)$$

Note, that the exact value of the coefficient in this equation is not well defined. In this paper we use the value written above. So, in the scope of the current losses model, we can write the following expression for the time that an isolated neutron star lives as a radio pulsar:

$$\tau_{cl}[s] = \int_{P_0}^{P_{al}} \frac{dP}{10^{-15} B_{12}^{10/7} P_{14}^{11/4} (1 - \sin^2 \chi_0(P^2)/(P_{al}^2))^{0.75}},$$

where $\chi_0$ and $\chi_0$ are the initial spin period and the initial angle between the spin and the magnetic axes of a radio pulsar, respectively, and $P_{al}$ is a period at which neutron star
Table 1. Age estimates for the magnetodipole model. $P_0$ and $\chi_0$ are initial parameters of a neutron star, $P_{dl}$ and $\tau_{md}$ are the spin period and the estimated age of a neutron star on the death line, $P_{al}$ and $\tau_{al}$ are the spin period and the estimated age of a neutron star at the moment of transition to the aligned rotators.

| $B$, G | $P_0$, s | $\chi_0$, deg. | $P_{dl}$, s | $\tau_{md}$, Myr | $P_{al}$, s | $\tau_{al}$, Myr |
|--------|-----------|-----------------|-------------|-----------------|-------------|-----------------|
| $10^{12}$ | 0.02 | 30 | - | - | 0.023 | 0.1 |
| $10^{13}$ | 0.02 | 30 | - | - | 0.023 | $10^{-3}$ |
| $10^{14}$ | 0.02 | 30 | - | - | 0.023 | $10^{-5}$ |
| $10^{12}$ | 0.02 | 45 | - | - | 0.028 | 0.2 |
| $10^{13}$ | 0.02 | 45 | - | - | 0.028 | $2 \times 10^{-3}$ |
| $10^{14}$ | 0.02 | 45 | - | - | 0.028 | $2 \times 10^{-5}$ |
| $10^{12}$ | 0.02 | 60 | - | - | 0.04 | 3.6 |
| $10^{13}$ | 0.02 | 60 | - | - | 0.04 | 0.04 |
| $10^{14}$ | 0.02 | 60 | - | - | 0.04 | $4 \times 10^{-4}$ |
| $10^{12}$ | 0.02 | 89.6 | 1.7 | 226 | - | - |
| $10^{13}$ | 0.02 | 89.7 | 5.4 | 46 | - | - |
| $10^{14}$ | 0.02 | 89.9 | 17 | 2.7 | - | - |
| $10^{12}$ | 0.5 | 30 | - | - | 0.577 | 112 |
| $10^{13}$ | 0.5 | 30 | - | - | 0.577 | 1.1 |
| $10^{14}$ | 0.5 | 30 | - | - | 0.577 | 0.01 |
| $10^{12}$ | 0.5 | 45 | - | - | 0.707 | 235 |
| $10^{13}$ | 0.5 | 45 | - | - | 0.707 | 2.4 |
| $10^{14}$ | 0.5 | 45 | - | - | 0.707 | 0.02 |
| $10^{12}$ | 0.5 | 60 | - | - | 1 | 2222 |
| $10^{13}$ | 0.5 | 60 | - | - | 1 | 22 |
| $10^{14}$ | 0.5 | 60 | - | - | 1 | 2.2 |
| $10^{12}$ | 0.5 | 89.6 | 1.7 | 167 | - | - |
| $10^{13}$ | 0.5 | 89.7 | 5.4 | 18 | - | - |
| $10^{14}$ | 0.5 | 89.9 | 17 | 1.8 | - | - |

Table 2. Age estimates for the model based on longitudinal current losses. $P_0$ and $\chi_0$ are initial parameters of a neutron star, $P_{dl}$ and $\tau_{cl}$ are the spin period and the estimated age of a neutron star on the death line, $P_{ort}$ and $\tau_{ort}$ are the spin period and the estimated age of a neutron star at the moment of crossing the boundary of the region of orthogonal rotators.

| $B$, G | $P_0$, s | $\chi_0$, deg. | $P_{dl}$, s | $\tau_{cl}$, Myr | $P_{ort}$, s | $\tau_{ort}$, Myr |
|--------|-----------|-----------------|-------------|-----------------|-------------|-----------------|
| $10^{12}$ | 0.02 | 0.4 | 0.98 | 34 | - | - |
| $10^{13}$ | 0.02 | 0.3 | 2.95 | 4.2 | - | - |
| $10^{14}$ | 0.02 | 0.1 | 9.51 | 0.49 | - | - |
| $10^{12}$ | 0.02 | 30 | - | - | 0.04 | 2.5 |
| $10^{13}$ | 0.02 | 30 | - | - | 0.04 | 0.09 |
| $10^{14}$ | 0.02 | 30 | - | - | 0.04 | 0.004 |
| $10^{12}$ | 0.02 | 45 | - | - | 0.028 | 1.4 |
| $10^{13}$ | 0.02 | 45 | - | - | 0.028 | 0.05 |
| $10^{14}$ | 0.02 | 45 | - | - | 0.028 | 0.002 |
| $10^{12}$ | 0.02 | 60 | - | - | 0.023 | 0.8 |
| $10^{13}$ | 0.02 | 60 | - | - | 0.023 | 0.03 |
| $10^{14}$ | 0.02 | 60 | - | - | 0.023 | 0.001 |
| $10^{12}$ | 0.5 | 0.4 | 1 | 16 | - | - |
| $10^{13}$ | 0.5 | 0.3 | 3.35 | 3.2 | - | - |
| $10^{14}$ | 0.5 | 0.1 | 11.24 | 0.42 | - | - |
| $10^{12}$ | 0.5 | 30 | 0.84 | 18 | - | - |
| $10^{13}$ | 0.5 | 30 | - | - | 1.00 | 2.2 |
| $10^{14}$ | 0.5 | 30 | - | - | 1.00 | 0.08 |
| $10^{12}$ | 0.5 | 45 | 0.68 | 17 | - | - |
| $10^{13}$ | 0.5 | 45 | - | - | 0.707 | 1.3 |
| $10^{14}$ | 0.5 | 45 | - | - | 0.707 | 0.05 |
| $10^{12}$ | 0.5 | 60 | 0.571 | 13 | - | - |
| $10^{13}$ | 0.5 | 60 | - | - | 0.577 | 0.8 |
| $10^{14}$ | 0.5 | 60 | - | - | 0.577 | 0.03 |
leaving the radio pulsar stage. To find \( P_{\text{tr}} \) let us consider two possible subsequent steps of the evolution of an isolated neutron star. At first, as a radio pulsar spins down, it may cross the death line and pass to the region of the so-called extinct radio pulsars. These neutron stars have already ceased to radiate in the radio band, but electrodynamic processes in their magnetospheres still play a crucial role. In this case the critical period \( P_{\text{crit}} \) is defined by the death line of radio pulsars:

\[
P_{\text{crit}} = B_{12}^{9/15} (\cos \chi)^{0.3} \, \text{s}. 
\]

(12)

Using Eq. (10), we can find an equation which shows how the period of transition of a neutron star to the region of extinct radio pulsars depends on the initial parameters:

\[
P_{\text{tr}} = B_{12}^{9/15} (1 - \sin^2 \chi_0) \frac{P_0^2}{P_0^{1.5}}. 
\]

(13)

If the model based on the current losses is valid, the period derivative (10) reduces during the evolution of a neutron star due to the factor \( \cos^{3/2} \chi \). So, the longest part of the lifetime of an active radio pulsar is the time it spends near the death line (unless a radio pulsar passes to the stage of orthogonal rotators before it can reach the death line - this way of evolution we discuss below). Thus, the age estimate (11) significantly depends on the accuracy of the expression (10) near the death line. Unfortunately, as it was noted above, expression (10) adequately defines the period derivative only for the radio pulsars far from the death line. Thus, the age estimates for the current losses model are very approximate and the more accurate expression for the spin-down rate of a neutron star near the death line can alter the results obtained in (11) by a factor of a few.

As it is noted in the previous paragraph, a NS instead of crossing the death line can follow a different path. An alternative evolutionary track leads an object directly to the stage of orthogonal rotators (Beskin et al. 1993; Beskin & Nokhrina 2004). Here a short explanation should be done. If an inclination angle of a radio pulsar is close enough to \( 90^\circ \), so that

\[
|\pi/2 - \chi| < R_0/R, 
\]

then particles with opposite charge signs can flow out of the polar cap surface. Here \( R_0 = R (\Omega R/c)^{0.5} \) is the polar cap radius. A group of radio pulsars for which the Goldreich-Julian charge density changes the sign on the polar cap surface is called orthogonal rotators. These pulsars are characterized by fast (\( P \sim 0.3 \text{s} \)) or superfast (\( P < 0.1 \text{s} \)) rotation, reduced spin-down rate (\( \dot{P} \sim 10^{-17} - 10^{-19} \text{ s}^{-1} \)), weak radio emission (\( L_{\text{rad}} \sim 10^{35} - 10^{37} \text{ erg s}^{-1} \)), and the existence of an interpulse. The energy losses for this class of objects are reduced by the factor \( \Omega R/c \) in comparison with the normal radio pulsars and are virtually independent of the angle between the magnetic and the spin axes, which is anyway very close to 90 degrees. Some candidates to this class are listed in (Beskin & Nokhrina 2004). They have periods from \( \sim 100 \) up to \( \sim 500 \text{ msec} \). Not all radio pulsar with small \( P \) should be orthogonal rotators. It was noted by Lorimer et al. (2004) that isolated pulsars with small period derivatives, \( \sim 10^{-18} \), and shorter periods about few dozens of milliseconds can be "disrupted recycled pulsars", but objects with longer spin periods require a different explanation which is probably related to emission mechanism.

The boundary of the stage of orthogonal rotators is given by (14):

\[
\cos \chi = (2\pi R/(cP))^{1/2} = 1.45 \times 10^{-2} P^{-0.5}, 
\]

(15)

so it is easy to find the following implicit equation for the period of the transition to this stage:

\[
P_{\text{ort}} = \frac{P_0}{\sin \chi_0} \left( 1 - 2.1 \times 10^{-4} P_{\text{ort}}^{-1} \right)^{0.5}. 
\]

(16)

The death line of radio pulsars (12) and the boundary of the orthogonal rotators stage (13) intersect at the point

\[
P_{\text{int}} \approx 0.35B_{12}^{46} \text{ s}, \sin \chi_{\text{int}} \approx (1 - 6 \times 10^{-4} P_{12}^{46})^{-0.5} \text{ in the } P - \sin \chi \text{ diagram. Therefore, if}
\]

\[
\sin \chi_0/P_0 > \frac{(1 - 6 \times 10^{-4} P_{12}^{46})^{0.5}}{0.35B_{12}^{46}}, 
\]

(17)

a neutron star passes through the death line to the region of extinct radio pulsars. In this case, the period of transition is determined by the death line (12), so that \( P_{\text{tr}} = P_{\text{crit}} \). Thus, assuming the initial spin period \( P_0 \approx 20 \text{ ms} \), the initial angle between the magnetic and the spin axes \( \chi_0 = 1^\circ \), and the magnetic field \( B = 10^{12} \text{ G} \), from Eq. (11) we can estimate the age of a radio pulsar at the point of the transition to the region of extinct radio pulsars as \( \approx 36 \text{ Myr} \). The spin period of a neutron star at the point of the transition is \( P_{\text{int}} \approx 0.88 \text{ s} \), and the inclination angle is \( \chi_{\text{int}} \approx 50^\circ \). For the same spin period and standard period derivative \( \dot{P} = 10^{-15} \text{ ss}^{-1} \), the characteristic age estimate \( \tau_{\text{ch}} = P/2\dot{P} \) results in 14 Myr. As we can see, taking into account the evolution of the angle between the spin and the magnetic axes within the current losses model significantly increases the time that a radio pulsar needs to slow down from the spin period \( P_1 \) to the larger spin period \( P_2 \) (both \( P_1 \) and \( P_2 \) are supposed to belong to the region of active radio pulsars).

Otherwise, if the initial parameters of a pulsar do not meet the above condition (17), the transition to the stage of orthogonal rotators takes place, so that \( P_{\text{tr}} = P_{\text{ort}} \). Note, that the transition to the orthogonal rotator stage should be typical for most of the radio pulsars with comparatively short initial spin periods \( P_0 \sim 10 \text{ ms} \) (the ratio \( \sin \chi/P \) is initially very large for these radio pulsars provided that the angle \( \chi \) is not too close to \( 0^\circ \)). Thus, for the model based on current losses the majority of neutron stars pass to the orthogonal rotator stage before the transition to the region of extinct radio pulsars. Assuming the same initial period \( P_0 \approx 20 \text{ ms} \) and \( \chi_0 = 45^\circ \), we can estimate the age of a neutron star passing from the stage of radio pulsars to the orthogonal rotator stage as follows: \( \tau_{\text{ort}} \approx 1.44 \text{ Myr} \) (assuming \( B = 10^{12} \text{ G} \)), \( \tau_{\text{ort}} \approx 54000 \text{ yrs} \) (\( B = 10^{13} \text{ G} \)), and \( \tau_{\text{ort}} \approx 2000 \text{ yrs} \) (\( B = 10^{14} \text{ G} \)). The spin period at which a radio pulsar crosses the boundary of the orthogonal rotator stage for all three magnetic field strengths is the same \( P_{\text{ort}} \approx 28.2 \text{ ms} \). As one can see, it requires about 1 Myr for an active radio pulsar with comparatively large initial angle \( \chi \) and standard magnetic field to increase its period by only 8 ms. Table 2 contains the age estimates for different parameters (such as magnetic fields, initial spin periods and initial inclination angles of radio pulsars).

Thus, we show that if the evolution of an inclination angle \( \chi \) is taken into account, the slow-down rate of an active radio pulsar turns out to be several times lower, and a lifetime of a star at that stage turns out to be much longer than...
it is commonly assumed. Moreover, as the angle $\chi$ between the spin and the magnetic axes of a pulsar increases, the slow-down rate of the star reduces by the factor of $(\cos \chi)^m$, where the power $m$ is $\approx 1.5 - 2$. Furthermore, as it is discussed in [Beskin & Nokhrina 2004], a period derivative $P$ of neutron stars at the stage of orthogonal rotators becomes approximately $10^5$ smaller than for the normal radio pulsars, so we should see some weakly radiating old radio pulsars with comparatively small spin periods and with the angles between the magnetic and the spin axes close enough to 90°. After all, if radio pulsars are born as fast rotators and their initial angle distribution is equiprobable, for the model of current losses the most of radio pulsars should become orthogonal rotators long before they can pass through the death line. Thus, as opposed to the magnetodipole model, we can expect the distribution in present day angles between the magnetic and the spin axes of active radio pulsars to have its maximum close to 90°. If the angle distribution has no such peak, we can assume that either the initial inclination angles are very small, or an essential part of observable radio pulsars was born with long spin periods. The latter conclusion is quite probable, and has a long history of discussion in the literature (see, for example, Harding & Lai (2000) and references therein).

3 RECONSTRUCTION OF EVOLUTIONARY TRACKS: CURRENT LOSSES MODEL

As it is shown above, the evolution of the inclination angle of an isolated neutron star significantly affects the estimate of its characteristic age through the influence of this parameter on the spin-down rate and the shape of evolutionary track of a neutron star. Usually, evolutionary tracks of NSs are shown on the $P - P\dot{\chi}$ plot. However, in the case of evolving $\chi$ it is convenient to use the $P - \sin \chi$ diagram. In this section we describe the procedure of reconstruction of evolutionary tracks for the isolated neutron stars assuming that the model based on current losses is valid. Two models of particle acceleration region should be considered: the model, in which the longitudinal electric field near the neutron-star surface hinder the particle escape (Ruderman & Sutherland 1975) and the model, in which the longitudinal electric field does not exist near the neutron-star surface (the model with free particle escape, Arons 1973; Mestel 1981).

The evolutionary track of a neutron star in the $P - \sin \chi$ diagram does not depend on the magnetic field strength directly. At that, the rate at which a neutron star moves along its evolutionary track depends on the magnetic field. Besides, the magnetic field strength affects the death line and the line of the transition to the propeller stage, so it can affect the evolutionary track through the track lengths of a neutron star at different stages of evolution. Further, we want to note that the boundary of the region of orthogonal rotators does not depend on the magnetic field either. At last, we should emphasize that the bias of the track in the region of active radio pulsars is defined solely by the ratio $P_0$ for the current losses model. Therefore, while constructing the evolutionary tracks by the initial parameters (direct way), one should keep in mind that Eq. $P_0$ is very susceptible to the small changes in the initial angle value in the case of the short (millisecond) initial period.

Many of the newly discovered types of isolated neutron stars have long spin periods and may belong to the next evolutionary stages such as extinct radio pulsars or supersonic propellers, or even to orthogonal or aligned rotators. In that case, in order to estimate the age of the star we should reconstruct the whole evolutionary track of a neutron star and consider a neutron star age as a sum of time intervals during which a star lives at each of the stages.

As example, in this section we present reconstructions of evolutionary tracks for two NSs belonging to different classes: the Magnificent seven and anomalous X-ray pulsars. The choice is motivated at first by the fact that for both of them some information about inclinations angles is available. For one of these NSs (RX J0720.4-3125) the angle is small or intermediate, as it appears from modeling it’s X-ray light curve (see below). For another, AXP 1E2259+586, the angle is large, as it is clear from the double peaked X-ray pulse profile (see, for example, Woods & Thompson (2004)). Then both of these NSs has relatively high magnetic fields, which means that the rate of their magneto-rotational evolution is high, too. Finally, poor, but independent, age estimates are available for them. In particular, the characteristic age for RX J0720.4-3125 is $\tau = P/2P\dot{\chi} = 1.9$ Myr (Haberl 2006) is larger than it can be estimated from cooling curves. Cooling age should be $< 1$ Myr (Page et al. 2004). In each of the following plots an independent age estimate is marked by an asterisk. The independent age estimates are taken as 1 Myr for RX J0720.4-3125 (excluding two plots with the characteristic ages less than the independent ones, in which cases the mark is placed at the origin, indicating that initial period and angle should be small) and 0.01 Myr for 1E 2259+586. The latter is based on the associations of the source with supernova remnant CTB 109 (Kaspi et al. 2003).

At first, we consider the evolution of the X-ray dim isolated neutron star RX J0720.4-3125 within the model of current losses, and reconstruct possible evolutionary tracks in $P - \sin \chi$ plane assuming that the angle between spin and magnetic axes at the present time is $\chi_{\text{obs}} = 50°$ (Perez-Azorin et al. 2006). Basing on the present day period $P_{\text{obs}} = 8.39$ s and the angle $\chi_{\text{obs}}$, this object can be either in the region of active radio pulsars or in the region of extinct radio pulsars depending on the magnetic field strength. We consider the reverse track reconstruction, i.e. knowing the parameters (the spin period and the inclination angle) of the neutron star at the present time we reconstruct the evolutionary track. After that we can find, how the initial parameters are related to each other. The initial spin period $P_0 = 20$ ms is used only for the age estimate of the neutron star.

Here we reconstruct the evolutionary tracks of RX J0720.4-3125 assuming the following three values of the magnetic field: $B = 10^{14}$ G, $B = 10^{13.5}$ G, and $B = 10^{13}$ G. Knowing the magnetic field strength, we can define the death line of radio pulsars and the lines of the transitions to the stages of orthogonal rotators and supersonic propellers. After that we can determine if the point with the presently observed spin period $P_{\text{obs}}$ and the presently observed inclination angle $\chi_{\text{obs}}$ is located in the region of active radio pulsars or it falls to the extinct radio pulsar stage. According to the position of this point on $P - \sin \chi$ diagram, we can define the next steps to reconstruct the evolutionary path.
Spin evolution of neutron stars with alignment or counteralignment

\[ \cos \chi_{\text{dl}} = B_{12}^{0.75} \left( \frac{\cos \chi_{\text{obs}}}{P_{\text{obs}}} \right)^{0.41} \approx 0.027B_{12}^{0.75}. \]  

Knowing the coordinates of the point, where the evolutionary track for the extinct radio pulsars as we define for the active radio pulsars (the procedure is the same as it is shown for the example with \( B = 10^{14} \) G).

According to (2), by the time of observation the neutron star lives as an extinct radio pulsar:

\[ \tau_2 = \int_{P_0}^{P_{\text{obs}}} \frac{dP}{2.4 \times 10^{-18}B_{12}^2 P_{\text{obs}} - \left( 1 - \cos^2 \chi_{\text{obs}} \right) /(P_{\text{obs}}^2)} \].

Finally, for the lifetime of a neutron star as an extinct radio pulsar at the time of observation we have:

\[ \tau_2 = \frac{2 \times 10^{16}B_{12}^2 P_{\text{obs}}^2 \cos^2 \chi_{\text{obs}}}{\ln \left( \frac{P_{\text{obs}}^2 - P_{\text{obs}}^2}{P_0^2 - P_{\text{obs}}^2} \right) \cos^2 \chi_{\text{obs}}} \]  

Thus, the age of an extinct radio pulsar can be estimated as:

\[ \tau = \tau_1 + \tau_2, \]

where \( \tau_1 \) is the time neutron star lives as radio pulsar and \( \tau_2 \) is a lifetime in the region of extinct radio pulsars.

Fig. 2 shows the reconstructed evolutionary track for RX J0720.4-3125 within the model with hindered particle escape from the neutron star surface for magnetic field strengths \( B = 10^{13.5} \) G.

If the model with free particle escape from the neutron star surface is valid then an angle between the spin and the magnetic axes tends to 90° for both active and extinct radio pulsar regions. So, we can simply follow the same procedure for the extinct radio pulsar as we define for the active radio pulsars (see Fig. 4 and the example with \( B = 10^{14} \) G above) and use (14). Assuming the spin period of the star at birth was \( P_0 = 20 \) ms, the age of X-ray dim isolated neutron star RX J0720.4-3125 within the model with free particle escape can be estimated as 11 Myr and 2 Myr for \( B = 10^{15} \) G and \( B = 10^{13.5} \) G, respectively. It should be noted here, that the theory of the energy release associated with the longitudinal currents in the neutron star magnetosphere was generally developed by Beskin et al. (1993) under the assumption of the hindered particle escape (Ruderman & Sutherland 1972). Thus, if particles leave the surface of a neutron star freely (Arons 1974; Mestel 1999), expression (10) is a rough approximation of the period derivative and so the ages estimated under the assumption of the free particle escape can be altered by a factor of a few for the more accurate spin-down rate expression.

In addition, in Figs. 3–5 we present the evolutionary tracks of AXP 1E2259+586 for a set of possible values of magnetic field strength and presently observed angle between the magnetic and the spin axes (see data on AXPs in general and 1E2259+586 in particular in Woods & Thompson (2004)). Finally, in Figs. 6–8 we reconstruct the evolutionary tracks of RX 0720.04-3125, assuming \( \chi_{\text{obs}} = 5° \) (Haberl et al. 2006). The validity of the model with hindered particle escape from the neutron star surface is assumed in these figures.

Figure 1. The reconstructed evolutionary track of XDIN J0720.4-3125. \( B = 10^{14} \) G, \( P_{\text{obs}} = 8.39 \) s, \( \chi_{\text{obs}} = 50° \). Model based on current losses is used. The figure is valid for both hindered and free particle escape. The presently observed spin period and inclination angle correspond to the region of active radio pulsars. If the initial spin period \( P_0 = 20 \) ms, the age estimate is \( \tau \approx 0.4 \) Myr. The “star” marks the initial parameters corresponding to the independent age estimate \( \tau_{\text{ind}} = 0.4 \) Myr.

If we assume the magnetic field \( B = 10^{14} \) G, the present parameters of J0720.4-3125 correspond to the point in the region of active radio pulsars. Recall, that during this stage the ratio \( \sin \chi/P \) is invariant. Therefore, the evolutionary track of RX J0720.4-3125 can be represented by a straight line between the points \( (P_{\text{obs}}; \sin \chi_{\text{obs}}) \) and \( (0;0) \) (see Fig. 1). The point defining the initial spin period \( P_0 \) and the initial inclination angle \( \chi_0 \) is located on this line; thus, if we know the initial spin period, we can unambiguously define the initial angle between the magnetic and the spin axes. If the initial spin period is short enough (\( P_0 = 20 \) ms), the age of RX J0720.4-3125 can be estimated as \( 0.41 \) Myr (11).

For lower magnetic field strengths \( B = 10^{14} \) G, the present parameters of J0720.4-3125 correspond to the point in the region of active radio pulsars. Recall, that during this stage the ratio \( \sin \chi/P \) is invariant. Therefore, the evolutionary track of RX J0720.4-3125 can be represented by a straight line between the points \( (P_{\text{obs}}; \sin \chi_{\text{obs}}) \) and \( (0;0) \) (see Fig. 1). The point defining the initial spin period \( P_0 \) and the initial inclination angle \( \chi_0 \) is located on this line; thus, if we know the initial spin period, we can unambiguously define the initial angle between the magnetic and the spin axes. If the initial spin period is short enough (\( P_0 = 20 \) ms), the age of RX J0720.4-3125 can be estimated as \( 0.41 \) Myr (11).

For lower magnetic field strengths the observed parameters of RX J0720.4-3125 correspond to the region of extinct radio pulsars. In that case, the evolutionary tracks of the isolated neutron star will be different for the following two models of an acceleration region: the model with hindered particle escape (Ruderman & Sutherland 1972) and the model with free particle escape (Arons 1974; Mestel 1999) from the neutron star surface.

Within the model with hindered particle escape plasma fills only the inner region of the pulsar magnetosphere, and a neutron star slows down according to the magnetodipole formula (2), therefore \( \cos \chi/P \) is invariant for extinct radio pulsars. Hence, the track for this XDIN in the region of extinct radio pulsars in \( \sin \chi \) plane can be represented as follows:

\[ \sin \chi = \left( 1 - P^2 (\cos \chi_{\text{obs}}/P_{\text{obs}})^2 \right)^{0.5}. \]  

In order to continue the evolutionary track of the star to the region of active radio pulsars we should find the parameters, at which neutron star passes through the death line of radio pulsars (13). As a result, for RX J0720.4-3125 we have:

\[ P_{\text{dl}} = B_{12}^{0.75} \left( \frac{\cos \chi_{\text{obs}}}{P_{\text{obs}}} \right)^{0.41} \approx 0.35B_{12}^{0.75} \text{ s}. \]
As, most probably, both objects left the region of active radio pulsars, then the track reconstruction shows that initial periods of these objects are long enough.

4 DISCUSSION

In this section we provide a brief discussion on several topics related to our analysis.

We discuss possible modifications of models due to non-electromagnetic effects. Then we discuss the possibility to choose between free and hindered particle escape basing on the data on inclinations angles of extinct pulsars.
Figure 6. The reconstructed evolutionary track of XDIN J0720.4-3125. $B = 10^{14}$ G, $P_{\text{obs}} = 8.39$ s, $X_{\text{obs}} = 5^\circ$. Model based on current losses is used. The figure is valid for both hindered and free particle escape. The presently observed spin period and inclination angle correspond to the region of active radio pulsars. If the initial spin period $P_0 = 20$ ms, the age estimate is $\tau \approx 0.34$ Myr. The asterisk marks the initial parameters correspondent to the independent age estimate $\tau_{\text{ind}} = 0.34$ Myr.

Then, as the main impact of poor knowledge of $\chi$ evolution on age estimates and population synthesis models, we touch these subjects.

4.1 The role of invariants and modifications

One of highlights of this paper can be summarized as follows. If a neutron star is born rapidly rotating, it reaches the stage of low energy losses with very short spin period, unless the initial inclination angle of a star is very close to the value correspondent to maximal spin down. This conclusion crucially depends on the existence of the invariants $\mathbf{4}$ or $\mathbf{9}$. The existence of the invariants is based on several simplifying assumptions:

- A neutron star is an ideal sphere;
- Magnetic field is due to an ideal dipole;
- Magnetic dipole braking or current losses are the only electro-magnetic processes which slows down the star;
- There are no additional non electro-magnetic processes related to internal structure of a NS or to external agents.

Clearly, all four assumptions can be violated. In such a case most of the results presented above have to be reconsidered taking into account additional processes or/and conditions.

In particular, we want to pay attention to the old paper by [MacV 1974]. This author notes, that if the dominant internal field is poloidal, then due to non-sphericity of a star the alignment process is working. If, oppositely, the toroidal component of the field dominates – then the counteralignment takes place. This arguments are particularly interesting as magnetars, especially SGRs, are supposed to have huge toroidal fields, which are dissipated on the spin-down time scale of these objects and deform the star [Stella et al. 2007].

Another possibility is related to remnant discs [Menou et al. 2001]. They can provide additional spin-down without significant changes in the inclination angle, especially at early stages of evolution. So, when the disc becomes inefficient a pulsar already has a relatively long period, but did not significantly change its inclination angle, yet.

Also, a process of vortex line migration during spin-down can be important [Ruderman 2006]. Effectively, this process results in counteralignment, and so can partly compensate alignment in magneto-dipole model. However, as alignment goes as $\cos \chi \propto P^n$ and vortex line migration leads to counteralignment as $\sin \chi \propto P^n$, $0 < n < 0.5$, compensation is only partial.

Finally, for a rigid and non-spherical neutron star the free mutation process plays significant role regulating the tendency of the magnetic axis to align of counter-align with the spin axis. [Goldreich 1970] showed that in case of magnetic dipole braking model the nutation amplitude decreases if the inclination angle of a neutron star is less than $\approx 55^\circ$ and increases if the inclination angle is larger than $\approx 55^\circ$.

Electromagnetic processes can also lead to small changes in the angle $\chi$. Probably, it can be the case for PSR B1931+24 which shows a peculiar behavior [Kramer et al. 2008], if the explanation is in switching between magneto-dipole and current losses [Beskin & Nokhrin 2006].

4.2 Free vs. hindered particle escape

Potentially, in the case of the longitudinal current losses the distribution in the inclination angle of the neutron stars behind the death line can provide necessary information to choose between free [Arons 1979; Mestel 1999] and hindered [Ruderman & Sutherland 1975] particle escape. The reasoning is simple: if the model of free escape is realized, then (especially among the strongly magnetized neutron stars) most of the neutron stars should have $\chi$ close to $90^\circ$. In the opposite case, there should be no neutron stars very close to $\chi = 90^\circ$, but this does not mean that most of them have $\chi \approx 0^\circ$. However, this easy approach can be done only if neutron stars are significantly evolved, i.e. if their present-day spin periods are much larger than the initial ones. Unfortunately, our analysis presented above shows that for the radio pulsars and magnetars the initial periods cannot be much smaller than the observed ones if either the magnetodipole or current losses model is valid.

Among all highly magnetized NSs the Magnificent seven can have relatively small initial periods in comparison with the present day values, as it comes out after comparison of there age estimates based on the standard assumption and its analogue for current losses with independent estimates. If this is the case, then it is interesting to note, that they do not show overabundance of orthogonal rototators. This can be considered as a (weak) argument in favor of the hindered escape model.

4.3 Age estimates for highly magnetized NSs and their initial spin periods

For the current losses model we compare the standard characteristic ages and the age estimates with the account of the angle evolution for the different types of isolated neutron stars under the specific assumption that all considered isolated neutron stars still remain in the region of active radio pulsars, i.e. they did not pass either through the death
Table 3 presents the results for one individual neutron star from each of the following types: HB-PSRs, SGRs, AXPs, X-ray dim isolated neutron stars and RRATs. For some sources the validity of the above assumption, of course, can be questioned. But for the considered objects this assumption results in the lower age estimate, and that is what we want to obtain.

As there is no exact information available about the angle between the spin and the magnetic axes for almost all of these isolated neutron stars, at first we consider the low estimates of their ages assuming constant inclination angle equal to maximal losses position, $\tau_{\min}$.

To get the lower age estimate for the case when sources are already at the stage of extinct pulsars, $\tau_{\chi_{\max}}$, in the framework of the current losses model, we allow the angle $\chi$ to evolve and find the maximum angle between the magnetic and the spin axes at which the neutron star with the presently observed spin period could still be located in the region of active radio pulsars in the plane $P - \sin \chi$ (see Fig. 7). The fact, that for extinct pulsars in the scope of free particle escape model such an estimate provides the lowest possible age is obvious. For the observed period an extinct pulsar has $\chi > \chi_{\max}$, so for the same initial period the one which is now extinct has larger also initial inclination angle. Then this means, that for all the same evolutionary parameters it always had larger angle, and therefore, smaller $\dot{P}$. To reach the same observed period from the same initial it takes longer for the one which is now extinct. For the hindered particle escape model we made numerical estimates, which demonstrate that even in these case for all considered parameters, which have been chosen according to parameters of the sources in Table 3, our value $\tau_{\chi_{\max}}$ still is a lower age estimate for extinct radio pulsars.

The age of the NS with $\chi = \chi_{\max}$ is calculated according to Eq. (11), where $\chi_0 = \chi_{\max} P_0 / P_{\text{tr}}$. Here $P_{\text{tr}}$ is the observed period.

All objects presented in Table 3 belongs to classes of highly magnetized NSs with long spin periods. For them magneto-rotational evolution goes quicker, so any evolution of $\chi$ on a spin-down time scale should be very pronounced. Even without precise information about inclination angles of highly magnetized NSs, observations allow us to say that they cover a wide range: from nearly orthogonal, to small $\chi$. From Tables 1 and 2 one can see, that NSs with $B$ larger $\sim 10^{13}$ born with short periods rapidly, in less than few tens of thousand years or even faster, become aligned or orthogonal rotators without significant spin-down, unless they were born with $\chi$ extremely close to the critical value of maximal losses. As the latter does not seem very probable for all of these sources (for such long periods and strong fields the proximity to the critical values should be of order of minutes), we can conclude that initial periods of highly magnetized NSs should be close to the presently observed values.

Note that this conclusion is in correspondence with recent investigation of properties of supernova remnants associated with magnetars (Vinc & Kuiper 2006). The authors do not find evidence that these neutron stars were formed from rapidly rotating proton neutron stars. Thus, it is likely that magnetars, as active radio pulsars, can be born with comparatively long periods. Recent calculation (Shibata et al. 2006) support the conclusion that highly magnetized NSs are born slowly rotating. Furthermore, Ferrario & Wickramasinghe (2003, 2006) discussed that the neutron stars with strong magnetic fields tend to be born as slow rotators, so initial spin period of a neutron star may depend on the magnetic field strength. On the other hand, for magnetars non-electromagnetic contributions to the evolution of $\chi$ can be important.

4.4 Population synthesis

One of areas in which poor knowledge of the evolution of $\chi$ can have strong impact is population synthesis of binary and isolated NSs. Below we mainly focus on the case of solitary objects, but in many aspects this discussion can be applied to binaries, too.

The inclination angle $\chi$ explicitly appears in the formulas for spin-down rate and energy release in both models (magneto-dipole and current losses). The death line should also depend on this parameter, and in the case of the current losses model this is shown explicitly. Finally, extreme values of this parameter can result in low energy losses. Taken all together, it appears that modeling the evolution of this parameter is at least as important as the evolution of the magnetic field.

Most (if not all) of population synthesis models accept the standard assumption about spin-down, which is Eq.(1) with $\sin \chi = 1 = \text{const}$. The death line for radio pulsars is
Table 3. Age estimates for different classes of isolated neutron stars in the scope of current losses model for the constant inclination angle $\chi = 0^\circ$ and for the inclination angle evolving to the maximum possible value $\chi_{\text{max}}$. We assume that a neutron star is located in the region of active radio pulsars on the $P – \sin \chi$ diagram and the initial spin period is taken to be $P_0 = 20$ ms.

| Name          | Class | $P, s$ (10$^{14}$) | $B, G$ | $\tau_{cl},$ Myr | $\chi_{\text{max}},$ deg | $\tau_{\chi_{\text{max}}},$ Myr |
|---------------|-------|---------------------|--------|------------------|--------------------------|--------------------------|
| J1734-3333    | HB-PSR| 1.17                | 0.05   | 89.2             | 0.1                      |                          |
| 1800-20       | SGR   | 7.5                 | 0.3    | 77.5             | 0.5                      |                          |
| 1E2259+586    | AXPs  | 6.98                | 0.6    | 64.3             | 0.8                      |                          |
| J0720.4-3125  | XDN   | 8.39                | 0.3    | 71.4             | 0.5                      |                          |
| J1819-1458    | RRA   | 4.26                | 0.5    | 83.7             | 0.9                      |                          |

taken in the form of Eq. (4). It is often believed that the uncertainty in the initial values of $\chi$ can be hidden in uncertainties of $B$. In some sense, this is true. But only when we deal with active radio pulsars, and if the inclination angle does not significantly evolve. On later evolutionary stages (propellers, accretors) $\chi$ and $B$ “decouples”. Such “decoupling” means that we become interested in actual values of parameters, not in some particular combination. In particular, to predict properties of isolated accreting NSs it is very important to know if most of them are expected to be aligned rotators (and so, non-pulsating sources), or, oppositely, they tend to be orthogonal rotators showing clear pulsations.

Also, the death line should depend on both $\chi$ and $B$, and at least in the case of current losses the combination of these parameters in the definition of the line is different from the combination in the spin-down rate. The use of the standard assumption, which, as demonstrated above, does not have solid theoretical grounds, makes many results of population synthesis not very trustable. So, if the aim is to produce a kind of a global population synthesis, like the one presented in Popov et al. (2000), including post-PSR stages, then it is necessary to follow the evolution of $\chi$ and $B$ separately. Unfortunately, it has never been done, and there are serious difficulties in fulfilling such a program, as none of theories gives such an evolution in good correspondence with observations of radio pulsars.

5 SUMMARY

We revisited magneto-rotational evolution of NSs with account of the evolution of the angle $\chi$ between spin and magnetic axes. In our consideration we used two models of energy losses: magneto-dipole and longitudinal currents. In general, the results obtained in the framework of the former are not new, but are presented for comparison and illustration. We compare output of our calculations with observations, and with estimates according to the standard assumption widely used for age estimates and in population synthesis. In this assumption the spin-down is calculated due to magneto-dipole model with constant $\chi = 90^\circ$.

Our calculations demonstrate that for initial angles not too close to the critical ones (90$^\circ$ in the case of magneto-dipole losses, and 0$^\circ$ – for current losses) NSs do not become normal extinct pulsars crossing the death line, but appear in the region of low losses (aligned rotators in the case of magneto-dipole model, and orthogonal rotators in the case of current losses). At that, for small initial periods and strong (but still not magnetar) magnetic fields this evolution proceeds very rapidly, in tens or hundreds thousand years. This fact does not correspond with observations. We consider three possibilities to resolve this problem:

- an essential part of radio pulsars are born with comparatively long spin periods, close to the observed ones, or
- the initial angles between the magnetic and the spin axes are close to critical values, or
- both magnetodipole model and the model based on current losses require significant modifications and cannot be directly applied for detailed calculations of NS evolution.

To test the first assumption it is necessary to perform a detailed population synthesis modeling. However, it seems possible, that the results of such computations would not support the hypothesis, as we expect an unobserved over-abundance of old pulsars with angles corresponding to low energy losses. Also, the total birth rate of pulsars should become higher, in contradiction with independent estimates.

The second possibility that initial angles $\chi$ are very close (within 1 degree) to one of the critical values seems too artificial, and does not have a solid physical ground. Clearly, it should be modified.

Most probably, our understanding of NS evolution on the radio pulsar stage in not satisfactory, now, and so both theories cannot be directly applied for detailed computations. Some non-electromagnetic effects can be important, especially during early stages of spin-down. They can be related to the presence of debris discs around NSs, or with mechanical effects related to non-sphericity or to processes in NS interiors. These effects can be different for different types of NSs.

Highly magnetized NSs seem to be most favorable sources to test predictions of theories of magneto-rotational evolution, and to probe initial parameters. The evolution of such stars is the most rapid, for them there are estimates of angles $\chi$ and independent ages estimates. Our calculations (including track reconstructions) show that if such sources as SGRs, AXPs, RRATs, and the Magnificent seven are beyond the death line, then their initial spin periods should be close to the observed ones. This conclusion is valid if one of the discussed theories is working, or, in general, if the inclination angles significantly evolve on the spin-down time scale.

Poor understanding of the details of evolution related to changes of the inclination angle is mostly important for age estimates and for population synthesis studies. Taking in account important role of $\chi$ in both models of energy losses (at least, equal to the role of the magnetic field strength) we can say, that neglect of its evolution is a “skeleton in a cupboard” of population synthesis models. The account of the evolution of $\chi$ can be crucial for radio pulsar stud-
ies, and significantly impact calculations of the later stages (propellers, accretors, etc.). Summarizing, we can say that our understanding of NS magneto-rotational evolution is not enough to make detailed population models and to obtain exact ages. The use of the standard assumption of constant $\chi$ contradicts present day models of radio pulsar energy losses and require serious modifications or basis. Most probably, detailed numerical modeling (Timokhin 2006; Contopoulos & Spitkovsky 2006) can provide the correct answer about details of magneto-rotational evolution, in the future, but, probably, additional inclusion of non-electromagnetic processes can be necessary.

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