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Coherence and Decay of Higher Energy Levels of a Superconducting Transmon Qubit

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We present measurements of coherence and successive decay dynamics of higher energy levels of a superconducting transmon qubit. By applying consecutive π-pulses for each sequential transition frequency, we excite the qubit from the ground state up to its fourth excited level and characterize the decay and coherence of each state. We find the decay to proceed mainly sequentially, with relaxation times in excess of 20 μs for all transitions. We also provide a direct measurement of the charge dispersion of these levels by analyzing beating patterns in Ramsey fringes. The results demonstrate the feasibility of using higher levels in transmon qubits for encoding quantum information.

Universal quantum information processing is typically formulated with two-level quantum systems, or qubits [1]. However, extending the dimension of the Hilbert space to a d-level system, or “qudit,” can provide significant computational advantage. In particular, qudits have been shown to reduce resource requirements [2, 3], improve the efficiency of certain quantum cryptanalytic protocols [4–7], simplify the implementation of quantum gates [8, 9], and have been used for simulating multi-dimensional quantum-mechanical systems [10]. The superconducting transmon qubit [11] is a quantum LC-oscillator with the inductor replaced by a Josephson junction [Fig. 1(a)]. The non-linearity of the Josephson inductance renders the oscillator weakly anharmonic, which allows selective addressing of the individual energy transitions and thus makes the device well-suited for investigating multi-level quantum systems. The transmon’s energy potential is shallower than the parabolic potential of an harmonic oscillator, leading to energy levels that become more closely-spaced as energy increases [Fig. 1(b)]. Although leakage to these levels can be a complication when operating the device as a two-level system [12], the existence of higher levels has proven useful for implementing certain quantum gates [13, 14]. Full quantum state tomography of a transmon operated as a three-level qutrit has also been demonstrated [15].

In this work, we investigate the energy decay and the phase coherence of the first five energy levels of a transmon qubit embedded in a three-dimensional cavity [16]. We find the energy decay of the excited states to be predominantly sequential, with non-sequential decay rates suppressed by two orders of magnitude. The suppression is a direct consequence of the parity of the wave functions, in analogy with the orbital selection rules governing transitions in natural atoms. We find that the sequential decay rates scale as i, where i = 1, ..., 4 is the initial excited state, thus confirming the radiation scaling expected for harmonic oscillators [17, 18]. The decay times remain in excess of 20 μs for all states up to i = 4, making them promising resources for quantum information processing applications. In addition, we characterize the quantum phase coherence of the higher levels by performing Ramsey-type interference experiments on each of the allowed transitions, and find strong beating in the resulting interference pattern, due to quasiparticle tunneling. This experimental result provides a direct measurement of the charge dispersion of the different levels [19–24].

Our device is a transmon qubit with a transition frequency f01 = 4.97 GHz for the first excited state, embedded in an aluminum 3D cavity with a bare fundamental mode fc = 11 GHz, and thermally anchored at a base temperature of 15 mK inside a dilution refrigerator. The interactions between the qubit in state |i⟩ and the cavity causes a dispersive shift χi of the cavity resonance to a new frequency fi = fc + χi, which is exploited for the readout of the qubit state [25]. We probe the state by sending coherent readout microwaves of frequency fμw through the resonator at a chosen detuning Δμw = fμw − fc from the bare cavity resonance, and measure the averaged transmission coefficient S21 of the signal over many experiments. Through a heterodyne detection scheme, the voltage amplitude of the transmission signal at fμw is recorded, from which the qubit state occupation is then directly extracted. The resonator transmission takes the form of a Lorentzian peak S21(fμw) = p/|1 + 21Q/[μw − fi]| (see [26]), centered around the qubit state-dependent frequency fi, with magnitude pi representing the state population, and Qf the total quality factor. When the total population p is distributed over several states |i⟩, the transmission becomes S21(fμw) = ∑i S21(fμw).

Exciting the transmon to a higher level first requires us to measure and analyse Rabi oscillations between adjacent pairs of energy levels, working sequentially up the ladder of states, as depicted in Fig. 1(b). Combined with qubit spectroscopy at each step, this protocol allows us to obtain the successive transition frequencies up to fi−1,i and to accurately calibrate the corresponding π-pulses. Starting with the qubit in the ground state |0⟩, we ap-
FIG. 1. (a) Schematic of the physical transmon qubit (not to scale) housed in the 3D cavity. The cross represents the Josephson junction, situated between the two junction electrodes forming the capacitor. (b) Simulated energy spectrum of the transmon with parameters $E_J/E_C = 58$, where $U[0]$ is the Josephson potential. (c) Rabi oscillations between state $|0\rangle$ and $|1\rangle$ under a Rabi drive tone of varying duration at $f_{01}$. The white dashed line indicates the position of the first $\pi$-pulse at $\tau_{01} = 40\,\text{ns}$. (d) Rabi oscillations between $|1\rangle$ and $|2\rangle$ with $\tau_{12} = 29\,\text{ns}$, obtained by adding a Rabi drive tone at $f_{12}$ after initializing state $|1\rangle$ by applying $\pi_{01}$-pulse. (e) Rabi oscillations on each successive qubit transition up to state $|4\rangle$ using the depopulation readout method. The corresponding excitation pulse sequence and respective depopulation sequence are shown for each Rabi drive. The solid lines are best-fit curves allowing the extraction of the Rabi frequencies $\Omega_{ij}$ method. The corresponding excitation pulse sequence and respective depopulation sequence are shown for each Rabi drive.

When driving Rabi oscillations on the transition $|i\rangle \leftrightarrow |i+1\rangle$ for $i \geq 2$, the readout by the method presented above is not possible in this device, because state $|3\rangle$ does not appear as a conditional shift to the resonator. This is a consequence of the fact that certain states escape the dispersive regime due to their mixing with higher-excited states that have transition frequencies close to the resonator frequency, see simulation in [26]. As a result, we use a modified readout protocol, which does not require measurement pulses at the shifted resonance $f_3$ or $f_4$. After preparing the qubit in state $|i\rangle$ via the upward sequence of $\pi$-pulses $S^i_1 = (\pi_{01}, \pi_{12}, \ldots, \pi_{i-1,i})$, we additionally apply a depopulation sequence $S^i_2 = (\pi_{i,i-1}, \ldots, \pi_{21}, \pi_{10})$ to the qubit immediately before the readout. This maps the population $p_i$ of state $|i\rangle$ onto that of the ground state $|0\rangle$, allowing us to measure $p_i$ by simply probing the resonator at the frequency $f_0$.

By incorporating the depopulation sequence we are able to drive Rabi oscillations of the transmon up to state $|4\rangle$, as shown in Fig. 1(e). The Rabi frequencies $\Omega_{ij}$, extracted via a best-fit curve, are proportional to the matrix elements $\langle i|\hat{n}|j\rangle$ between the states $i$ and $j$, where $\hat{n}$ denotes the number of Cooper pairs transferred between the two junction electrodes forming the capacitor [26]. Consequently, $\Omega_{ij}$ increase as $\langle i|\hat{n}|j\rangle \propto \sqrt{\chi_i}$, as expected from the coupling between the transmon states and the resonator [11]. Having thus obtained all the transition frequencies and $\pi$-pulse calibrations, the qubit can be initialized in any state up to $|4\rangle$ with the sequence $S^4_1$, and we proceed to investigate the decay and phase
coherence of these higher levels.

We start by measuring the dynamics of the state population decay by introducing a varying time delay before the readout process. The calibrated and normalized population evolutions starting from states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are plotted in Fig. 2(a)-(c). The decay from state $|4\rangle$ has also been measured and is presented in [26]. We model the data with a multi-level rate equation describing the evolution of the state population vector $\tilde{p}$, with $\Gamma_{ij}$ representing the decay rate from state $i$ to state $j$: $d\tilde{p}(t)/dt = \Gamma^{T} \cdot \tilde{p}(t)$. The decay rates matrix $\Gamma$ has diagonal elements $(\Gamma)_{jj} = -\sum_{k=0}^{j-1} \Gamma_{jk}$ and off-diagonal elements $\Gamma_{ij}$ for $i \neq j$. The upward rates are considered to be negligible by setting $\Gamma_{ij} = 0$ for all $i < j$, as $k_BT \ll hf_{ij}$ for all $i, j$. Indeed, the quiescent state-$|1\rangle$ population of our transmon is measured to be less than 0.1% [27]. The state occupations of the model are plotted (solid lines) and compared to the experimental data in Fig. 2 for each state, whereby the rates $\Gamma_{ij}$ are used as fitting parameters to extract all the system’s relaxation rates. The fitting was performed iteratively, starting with the decay from $|1\rangle$, where we fit $\Gamma_{10}$ and then fix it for the next decay from $|2\rangle$, where $\Gamma_{21}$ and $\Gamma_{20}$ are determined, and so forth.

The most prominent feature of the data is that the decay proceeds mainly sequentially [28], with the non-sequential decay rates suppressed by two orders of magnitude. The extracted decay times are in excess of 20 $\mu$s for all states up to $|4\rangle$, and are listed in Tab. I. For the sequential rates, we find that the rates scale linearly with state $i$, as plotted in Fig. 2(d): this behavior is consistent with decay processes related to fluctuations of the electric field (like Purcell or dielectric losses), for which we expect the lifetimes to be inversely proportional to $|\langle ii|n\rangle|^2$ (see [26] for numerical calculations of the matrix elements). Furthermore, theoretical relaxation rates between neighboring levels due to quasiparticle tunneling also respect this approximate dependence $\Gamma_{i,i-1} \propto \Gamma_{10}$ [29, 30]. We note that the anharmonicity of this device is sufficiently weak that its decay rates scale as those of Fock states in a harmonic oscillator [17, 18].

![FIG. 2.](image)

Figure 2. (a)-(c) Population decay traces of the qubit states up to $|3\rangle$, obtained by varying the time delay $\Delta t$ before the depopulation sequence. The solid lines are state occupations from the multi-level decay model taking into account all decay channels. (d) The sequential decay rates $\Gamma_{i,i-1}$ (green dots) for increasing energy state $i$, showing the roughly linear dependence (solid line). (e)(f) Zoom of decay curves in panel (b) and (c) respectively, showing the model with all transition rates allowed (solid line) compared to the model with only neighboring transitions allowed (dashed line). The extracted decay times are listed in Tab. I.

| Frequency | $f_{01}$ | $f_{12}$ | $f_{23}$ | $f_{34}$ |
|-----------|---------|---------|---------|---------|
| Exp. $f$ (GHz) | 4.9692 | 4.6944 | 4.3855 | 4.0280 |
| Sim. $f$ (GHz) | 4.9692* | 4.6944* | 4.3874 | 4.0475 |
| Sequ. decay $^{-1}$ | $\Gamma_{10}$ | $\Gamma_{21}$ | $\Gamma_{32}$ | $\Gamma_{43}$ |
| time ($\mu$s) | 84 ± 0.24 | 41 ± 0.21 | 30 ± 0.21 | 22 ± 2 |
| Non-seq. decay $^{-1}$ | $\Gamma_{20}$ | $\Gamma_{31}$ | $\Gamma_{40}$ |
| time ($\mu$s) | 1812 ± 223 | 1314 ± 359 | 2631 ± 694 |
| Dephasing $T_2$ | $T_{2}(01)$ | $T_{2}(12)$ | $T_{2}(23)$ | $T_{2}(34)$ |
| time ($\mu$s) | ±20% | 72 | 32 | 12 | <2 |

TABLE I. Comparison of experimental and simulated values for the transition frequencies $f_{i,i+1}$, the relaxation times $\Gamma_{ij}$ for the sequential and non-sequential rates, the dephasing times $T_2(ij)$ for the superpositions of states $|i\rangle$ and $|j\rangle$, and the dispersive shifts $\chi_i$. The measured charge dispersion splittings $\epsilon_{ij}(n_{\text{f}})$ extracted from Ramsey fringes are compared to the simulated maximum splittings $\epsilon_{ij}^{\text{max}}(n_{\text{f}})$.

To illustrate the effect of the non-sequential rates, we also fitted the data to a model involving only sequential rates (dashed lines in Fig. 2(e)(f)). Although the deviations between the two fits are small, inclusion of the rates $\Gamma_{20}^{-1}$, $\Gamma_{30}^{-1}$ and $\Gamma_{31}^{-1}$ does provide somewhat better matching for the initial increase in ground state population $|0\rangle$ for $t < 70 \mu$s where we would expect the largest impact. From numerical simulations of the qubit-
resonator Hamiltonian, we expect the rates $\Gamma_{20}$ and $\Gamma_{31}$ to be strongly suppressed due to the parity of those states [31], whereas the matrix element $|\langle 3|\hat{n}|0\rangle|^2$ relevant for $\Gamma_{30}$ is about 100 times smaller than $|\langle 1|\hat{n}|0\rangle|^2$ [26]. Quasiparticles also contribute to relaxation rates for non-neighboring levels, and theory [21] predicts that they are suppressed by at least three orders of magnitude, a much stronger suppression than we extract. This suggests that the non-sequential decay rates are dominated by some non-quasiparticle process, such as dielectric loss or coupling to other cavity modes.

We now proceed to investigate the phase coherence of the higher levels by performing a Ramsey-fringe measurement, whereby we obtain the dephasing times $T_2$. A Ramsey experiment on state $|i\rangle$ consists of first applying $\pi$-pulses to bring the transmon to state $i-1$, followed by a $\pi/2$-pulse at frequency $f_{i-1,i}$ to bring it into a superposition of states $|i-1\rangle$ and $|i\rangle$, then allowing a variable free-evolution time $\Delta t$ to pass, and finally applying a second $\pi/2$-pulse before applying the depopulation sequence and performing the readout. The measured Ramsey fringes are shown in Fig. 3 for each state up to $|4\rangle$. The frequency of the $\pi/2$-pulses was purposefully detuned to generate oscillating traces. The power spectral density of the data, obtained via a discrete Fourier transform, reveals two well-defined frequency components for states $|2\rangle$ and $|3\rangle$, and a number of frequencies for state $|4\rangle$. As described in the supplementary material, we fit the Ramsey fringes in Fig. 3(a) to a sum of two damped sinusoids, and the extracted dephasing times $T_2$ are listed in Tab. I.

FIG. 3. (a) Ramsey oscillations experiment on each subsequent energy transition up to state $|4\rangle$. (b) The Power Spectral Density (PSD), obtained from the discrete Fourier transforms of the corresponding Ramsey fringes. The Ramsey pulse sequence (top) corresponds to the fourth-row panel, representing a Ramsey sequence on state $|4\rangle$, with the black $\pi/2$-pulses representing the $\pi/2$-pulses performed at frequency $f_{34}$ to bring the transmon into the superposition state $|3\rangle + |4\rangle)/\sqrt{2}$ before allowing the free evolution time $\Delta t$.

The splitting of the transition frequencies can be understood in terms of quasi-particle tunneling between the two junction electrodes [26]. Despite the large $E_J/\hbar\omega$ ratio, the transmon retains some sensitivity to charge fluctuations, and the charge dispersion approximately grows in an exponentially way with increasing level number [26]. From our full numerical transmon-cavity simulation, we calculate the change in level splitting $\epsilon_{ij}(n_g)$ between levels $i, j$ as a function of the effective offset charge $n_g$ [11], shown in Fig. 4. The maximum change in $\epsilon_{ij}$ due to quasi-particle tunneling is given by $\epsilon_{ij}(n_g = 1/2) - \epsilon_{ij}(n_g = 0)$, as marked by vertical dashed lines, but note that what we measure experimentally is the dispersion between $n_g + 1/2$ and $n_g$ for an unknown value of $n_g$. The measured splittings are compared to the calculated maximum splittings $\epsilon_{ij}^{(\text{max})}$ in Tab. I. State $|1\rangle$ is unresolved, in agreement with the small splitting of 2.5 kHz predicted, whereas we find that the splittings of states $|2\rangle$ and $|3\rangle$ are reasonably well predicted by the simulation. Charge traps between the substrate and the deposited metal film, or the presence of two-level fluctuators in the junction, also lead to charge fluctuations, possibly explaining the additional peaks seen in the spectrum of the state $|4\rangle$. It should be noted that for quantum information purposes, the noise causing the beating in the Ramsey fringes can be refocused with an echo sequence by adding a temporally short $\pi$-pulse (broad frequency spectrum) to the center of the Ramsey sequence [32].

In conclusion, we have demonstrated the preparation and control of the five-lowest states of a transmon qubit in a three-dimensional cavity. We observed predominantly sequential energy relaxation, with non-sequential rates suppressed by two orders of magnitude. In addition,

FIG. 4. (a) Calculated change in transition frequency $\epsilon_{ij}(n_g)$ as a function of the effective offset charge $n_g$, expressing the charge dispersion of each transition. (b) Zoom of the lowest two transitions 0-1 and 1-2.
our direct measurement of the charge dispersion at higher levels agrees well with theory and facilitates further studies of the crucially important dephasing characteristics of quantum circuits. The measured qubit lifetimes in excess of 20 $\mu$s at energy states up to |4⟩ expands the practicability of transmons for quantum information applications and simulations using multi-level systems.

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[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, (Cambridge University Press, 2010).

[2] A. Muthukrishnan and C. R. Stroud, Phys. Rev. A 62, 052309 (2000).

[3] S. S. Bullock, D. P. O’Leary, and G. K. Brennen, Phys. Rev. Lett. 94, 230502 (2005).

[4] H. Bechmann-Pasquinucci and A. Peres, Phys. Rev. Lett. 85, 3313 (2000).

[5] N. J. Cerf, M. Bourennane, A. Karlsson, and N. Gisin, Phys. Rev. Lett. 88, 127902 (2002).

[6] T. Durt, D. Kaszlikowski, J.-L. Chen, and L. Kwek, Phys. Rev. A 69, 032313 (2004).

[7] I. Bregman, D. Aharonov, M. Ben-Or, and H. S. Eisenberg, Phys. Rev. A 77, 050301 (2008).

[8] B. P. Lanyon, M. Barbieri, M. P. Almeida, T. Jennewein, T. C. Ralph, K. J. Resch, G. J. Pryde, J. L. O’Brien, A. Gilchrist, and A. G. White, Nature Phys. 5, 134 (2008).

[9] J. M. Chow, J. M. Gambetta, A. W. Cross, S. T. Merkel, C. Rigetti, and M. Steffen, New J. Phys. 15, 110512 (2013).

[10] M. Neeley, M. Ansmann, R. C. Bialczak, M. Hofheinz, E. Lucero, A. D. O’Connell, D. Sank, H. Wang, J. Wenger, A. N. Cleland, M. R. Geller, and J. M. Martinis, Science 325, 722 (2009).

[11] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 76, 042319 (2007).

[12] J. M. Chow, L. DiCarlo, J. M. Gambetta, F. Motzoi, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 82, 040305 (2010).

[13] L. DiCarlo, J. M. Chow, J. M. Gambetta, L. S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature 460, 240 (2009).

[14] A. A. Abdumalikov, J. M. Fink, K. Juliussion, M. Pechal, S. Berger, A. Wallraff, and S. Filipp, Nature 496, 482 (2013).

[15] R. Bianchetti, S. Filipp, M. Baur, J. M. Fink, C. Lang, L. Steffen, M. Boissonneault, A. Blais, and A. Wallraff, Phys. Rev. Lett. 105, 223601 (2010).

[16] H. Paik, D. I. Schuster, L. S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. Lett. 107, 240501 (2011).

[17] N. Lu, Phys. Rev. A 40, 1707 (1989).

[18] H. Wang, M. Hofheinz, M. Ansmann, R. Bialczak, E. Lucero, M. Neeley, A. D. O’Connell, D. Sank, J. Wenger, A. Cleland, and J. M. Martinis, Phys. Rev. Lett. 101, 240502 (2008).

[19] J. A. Schreier, A. A. Houck, J. Koch, D. I. Schuster, B. R. Johnson, J. M. Chow, J. M. Gambetta, J. Majer, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. B 77, 180502 (2008).

[20] A. A. Houck, J. A. Schreier, B. R. Johnson, J. M. Chow, J. Koch, J. M. Gambetta, D. I. Schuster, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. 101, 080502 (2008).

[21] G. Catelani, S. E. Nigg, S. M. Girvin, R. J. Schoelkopf, and L. I. Glazman, Phys. Rev. B 86, 184514 (2012).

[22] L. Sun, L. DiCarlo, M. D. Reed, G. Catelani, L. S. Bishop, D. I. Schuster, B. R. Johnson, G. A. Yang, L. Frunzio, L. Glazman, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. Lett. 108, 230509 (2012).

[23] C. Wang, Y. Y. Gao, I. M. Pop, U. Vool, C. Axline, T. Brecht, R. W. Heeres, L. Frunzio, M. H. Devoret, G. Catelani, L. I. Glazman, and R. J. Schoelkopf, arXiv:1406.7300 (2014).

[24] D. Ristè, C. C. Bultink, M. J. Tiggelman, R. N. Schouten, K. W. Lehnert, and L. DiCarlo, Nature Comm. 4 (2013).

[25] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. 95, 060501 (2005).

[26] See Supplementary Material at [URL will be inserted by publisher], which includes Ref. [33], for further description of the measurement techniques, data analysis, further data to Fig. 2, and for the full numerical simulation of the transmon and the coupled qubit-cavity Hamiltonian.

[27] XY Jin et al., in preparation.

[28] M. Dykman and M. Krivoglaz, Soviet Physics Reviews 5, 265441 (1984).

[29] G. Catelani, J. Koch, L. Frunzio, R. J. Schoelkopf, M. H. Devoret, and L. I. Glazman, Phys. Rev. Lett. 106, 077002 (2011).

[30] G. Catelani, R. J. Schoelkopf, M. H. Devoret, and L. I. Glazman, Phys. Rev. B 84, 064517 (2011).

[31] P. Deppe, M. Mariantoni, E. Solano, and R. Gross, Nature Phys. 4, 686 (2008).

[32] E. L. Hahn, Phys. Rev. 80, 580 (1950).

[33] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 69, 062320 (2004).