Modeling of Capillary Force-Induced Mold Deformation for Sub-10 nm UV Nanoimprint Lithography

Jingxuan Cai\textsuperscript{1} and Wen-Di Li\textsuperscript{1}\

\textsuperscript{1} Department of Mechanical Engineering, The University of Hong Kong, Pokfulam, Hong Kong, China.

Corresponding author: Wen-Di Li (liwd@hku.hk)

\textbf{KEYWORDS}: hydrodynamic model, UV nanoimprint lithography, stress distribution, finite-element method
Abstract

A model has been developed to predict the dynamic behavior of the filling process and to simulate the capillary force-induced defamation of nanostructures on the imprint mold during ultraviolet nanoimprint lithography (UV-NIL) at sub-10 nm resolution. The dynamic behavior of resist liquid filling under different conditions was studied using hydrodynamics, and the capillary force-induced deformation of mold structures was modeled using beam bending mechanics for both wetting and non-wetting mold structures. A critical aspect ratio of mold structures were proposed and calculated based on the modeling results. The theoretical and simulated results provide possible guide for optimization of imprint mold for high-fidelity UV-NIL at sub-10 nm resolution.
1. Introduction

Nanoimprint lithography (NIL) is a promising technology that enables cost-effective and high-throughput fabrication for micro- and nanopatterns. One of the most advanced NIL techniques under intensive study is the ultraviolet NIL (UV-NIL). During UV-NIL process, a UV-curable resist is spin-coated or dispensed on a substrate in the atmospheric environment. Then the resist is covered by a mold with nanopatterns and the resist liquid will fill the nanocavities due to capillary force. The resist is then cured under ultraviolet exposure afterward, leaving solidified nanoimprinted patterns on the substrate. Fabrications of nanostructures with high transfer fidelity using the UV-NIL have been widely reported, and the patterning resolution could achieve at 10 nm when a helium ion beam lithography (HIBL) patterned hydrogen silsesquioxane (HSQ) mold was used.\(^1\)\(^-\)\(^3\) However, pattern deformation and structure collapse arose as essential issues when advancing the UV-NIL toward sub-10 nm resolution.\(^1\) Therefore, the general understanding of the filling process and the deformation of mold structures during the UV-NIL is highly demanded for the optimization of the UV-NIL process and design of HSQ mold structures for the use in sub-10 nm resolution. Concurrently, numerous efforts have been made to investigate the filling process\(^4\)\(^-\)\(^6\) and bubble formation and dissolution process\(^7\)\(^-\)\(^9\) for the UV-NIL using hydromechanics simulations, as well as molecular dynamics studies.\(^10\) However, with the further reduce of structure dimensions, the structures of the imprint mold would suffer from deformations due to the dramatically increased capillary force\(^11\) and thus limits the further application of UV-NIL. Although the resist deformation for NIL has been investigated using solid mechanics\(^12\) and molecular dynamics methods,\(^13\) and the mold deformation for thermal NIL has been studied,\(^14\) to our best knowledge, the general model of the capillary force induced pattern deformation on the mold for UV-NIL towards sub-10 nm resolution has not been reported yet.
Herein, we have proposed a theoretical model, which incorporates both the hydrodynamics and solid mechanics, to simulate the stress distribution in the model structures during the UV-NIL process through modeling the dynamic filling process of the resist flow for both wetting and non-wetting mold materials and compare the theoretical modeling with the results obtained from a numerical simulation using the finite-element method (FEM). Moreover, the capillary force-induced deformation of mold structures in the sub-10 nm UV-NIL was theoretically investigated. Based on the analysis, the critical aspect ratio of mold structures applicable was proposed and calculated for typical mold made of materials with different yield strengths.

2. Model and simulation aspect

The model discussed here has been applied to a system in which two parallel beams of width $w$ are separated by a space of gap $d$, the depth of the beams is $L$. The models for the wetting and non-wetting imprinting process are schematically demonstrated in Figure 1a and 1b, respectively. In a typical UV-NIL process with a wetting mold, when the mold approaches the substrate, the capillary force pulls the resist up to the features fabricated on the template. The fluidic flow of the resist can be described as a Newtonian fluidic. For simplification, the mold is considered to be stationary, and the whole system is assumed to be in the standard atmospheric pressure. The elastic deformation of beam structures before collapse during UV-NIL is neglected because the high brittleness of typical mold material.
Figure 1. Schematic of the model for the (a) wetting and (b) non-wetting molds. $T$ is the thickness of the resist, $H$ and $w$ are the height and width of the beams, respectively, and $d$ is the width of the gap. $p_r$ is the pressure in the resist, $p_i$ is the pressure at the air-resist interface, and the air pressure $p_0$ is considered to be consistent during resist filling process. $\theta$ is the contact angle between the mold and the resist. (c) Schematic of the calculation of maximum capillary force induced stress across a beam of the mold. The maximum stress in the beam is located at the corners. (d) Schematic of model structures for FEM simulation. The simulation consisted of two steps, a two-phase flow (TPF) model, and a solid mechanics model. In the TPF model, the $x$- and $y$-direction of the resist are the inlets, and the top of the channel is the outlet. The walls are wetting walls with contact
angles of 30°. In the solid mechanics model, the mold is considered as an elastic material, and the pressure difference obtained in the previous step is applied to the walls of beams.

Since the thickness of the resist layer is much smaller than the lateral dimensions of the substrate and the Reynolds number of the resist flow is low, the kinematics of the resist flow are satisfied with the simplified Navier-Stokes equation with lubrication approximation and continuity equation:\(^\text{15}\)

\[
\frac{\partial p_r}{\partial x} = \mu \frac{\partial^2 u_x}{\partial z^2} \tag{1}
\]

\[
\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \tag{2}
\]

Where \(p_r\) is the pressure in the resist, \(u_x\) and \(u_z\) are the velocities in and perpendicular to the substrate, \(\mu\) is the viscosity of the resist.

With the boundary conditions (BCs),

\[u_x(z = 0) = 0\]

\[u_x(z = H) = 0\]

\[u_z(z = 0) = 0\]

\[u_z(z = H) = 0\]

Solving Equation (1) and (2) using the above BCs,

\[p_r = \text{const.} \tag{3}\]
The pressure difference across the liquid-air interface can be estimated by Young-Laplace equations:\textsuperscript{16-18}

\[ p_r - p_0 = \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \]  \hspace{1cm} (4)

where \( \gamma \) is the surface tension of the resist, and \( r_1 \) and \( r_2 \) are the principal radii of curvature.

while

\[ r_1 = \frac{T}{1 - \cos \theta_i} \] \hspace{1cm} (5)

\[ r_2 = \infty \] \hspace{1cm} (6)

then

\[ p_r = p_0 + \frac{\gamma (1 - \cos \theta_i)}{T} \] \hspace{1cm} (7)

As the deformation of mold structures before collapse is considered to be neglected, the capillary rise process of resist liquid can be derived from a Poiseille flow by the Lucas-Washburn equation:\textsuperscript{19-25}

\[ H(t) = \left( \frac{\gamma d \cos \theta}{2 \mu} \right)^{1/2} \sqrt{t} \] \hspace{1cm} (8)

Consequently, the pressure difference drives the resist liquid flow can be derived from the Darcy-Weisbach equation:\textsuperscript{26}

\[ p_r - p_i = f_p \cdot \frac{h}{2d} \cdot \frac{\rho u^2}{2} \] \hspace{1cm} (9)
The Darcy friction factor is defined as

\[ f_D = \frac{32}{Re} \]  

(10)

where \( Re \) is the Reynolds number of the resist flow,

\[ Re = \frac{\rho D \omega}{\mu} \]  

(11)

For a flow between two plates, \( D=2d \), the Equation (11) reduces to,

\[ Re = \frac{2\rho d \omega}{\mu} \]  

(12)

where,

\[ \nu = \frac{dh}{dt} = \frac{1}{2} \left( \frac{c d \cos \theta}{2\mu} \right)^{1/2} \frac{1}{\sqrt{I}} \]  

(13)

Beams of the mold are considered as elastic material in evaluation of the capillary force-induced deformation, and the yield strength is used as the collapse criterion. Capillary force induced stress is distributed inside the beams corresponding to the pressure difference across the beam.

The model structure for calculating the capillary force induced stress distribution is schematically illustrated in Figure 1c. The second axial momentum of area is

\[ I = \frac{1}{12} L w^3 \]  

(14)

From the elastic beam bending model, the maximum stress across the beam could be estimated by
\[
\sigma_{\text{max}} = 6\left(\frac{2\gamma \cos \theta}{d} - p_r\right)\left(\frac{H}{w}\right)^2
\]  
(15)

For the non-wetting mold, the surface tension prevents resist liquid from filling the cavities. Thus, external pressure is required to overcome the surface tension and push the resist liquid into channels of the mold structures. The model for the non-wetting imprint process is schematically illustrated in Figure 1b. The pressure difference across the liquid-air interface, according to the Young-Laplace equations for the forward contact angle \(\theta\), which is the critical pressure maintaining the hydrostatic force balance, is given by

\[
p_r - p_0 = \frac{2\gamma \cos(\pi - \theta)}{d}
\]  
(16)

When the external pressure exceeds the critical pressure, the resist liquid will fill the channels due to the pressure difference.

Similar to the wetting-wall model, the maximum stress across the beam could be estimated by,

\[
\sigma_{\text{max}} = \frac{6\gamma \cos(\pi - \theta)}{d} \left(\frac{H}{w}\right)^2
\]  
(17)

Finite element method (FEM) simulations are performed using the level-set method and coupled with solid mechanics to validate theoretical modeling. The modeling structure is schematically illustrated in Figure 1d. The integration time step is 0.025 \(\mu\)s and the total simulation time is usually within a few microseconds. The contact angles between the walls and the resist are 30\(^\circ\), and the upper and lower boundaries are fixed. Left and right boundaries are the inlets, and the deformation of the structures is neglected in the simulation process using fluidic dynamics. The simulation of solid mechanics is then implemented to simulate the mechanical behavior of mold
structures by applying the pressure difference obtained in the previous simulation of fluidic
dynamics. The simulations of fluidic dynamics and solid mechanics are conducted using the two-
phase flow (TPF) and solid mechanics modules of COMSOL Multiphysics software,
respectively.

3. Results and discussion

There are several independent variables that must be specified to solve in our model. These
variables include the contact angle of the resist liquid with the mold $\theta_1$ and substrate $\theta_2$, the surface
tension of resist liquid $\gamma$, the thickness of resist film $H$, Young’s modulus $E$ and yield strength $\sigma_Y$
of mold material, and two structure dimensions $w$, $h$ and $d$. The numerical values of these physical
parameters used are listed in Table 1.

Table 1. Numerical values of the physical parameters used in FEM simulation and theoretical
calculation

| Physical parameter                         | Symbol | Value   |
|-------------------------------------------|--------|---------|
| Surface tension (N • m$^{-1}$)            | $\gamma$ | 0.03    |
| Resist viscosity (Pa • s)                 | $\mu$  | 0.01    |
| Resist thickness (nm)                     | $T$    | 400     |
| Ambient pressure (bar)                    | $p_0$  | 1.01    |
| Pressure inside the resist                | $p_r$  |         |
| Pressure at the resist-air interface      | $p_i$  |         |
| Contact angles                            | $\theta_1$, $\theta_2$ | 45°, 30° |
| Density of resist (kg • m$^{-3}$)         | $\rho$ | $1.0 \times 10^3$ |

Time-Dependent Filling Process of Imprint Resist
Figure 2a-d show the time-dependent filling process of the resist flow in a 50 nm width, 100 nm height channel of different times. The results clearly suggested that the resist filling process during the UV-NIL is relatively fast (< 0.5 μs). The filling process takes only 0.1 μs to achieve ~90% filling ratio. We also investigated the effect of the gap width on the resist filling behavior. Figure 2e-f display the simulated volume fraction of the resist flow in 100 nm height channels with different gap widths ranging from 10 nm to 50 nm at $t = 0.05 \mu s$. The filling ratio for narrower channel is lower than the wider channel at the same filling time because of the higher viscous resistance. However, the difference is lower than 5 %, indicating that the gap width is not dominant in the filling behavior during the UV-NIL.

Figure 2. (a) Simulated volume fraction of the resist flow in a 100-nm-height channel with a 50-nm-width gap between two beams on the mold during the resist filling process at a different time. (b) The simulated volume fraction of the resist flow in 100-nm-height channels with the gap width range from 10 nm to 50 nm at time $t = 0.05 \mu s$. 
Time-Dependent Stress Distribution in the Mold

The von Mises distribution in the 50-nm-width beams due to the resist flow in a 50 nm width channel at $t = 0.05 \, \mu s$ is plotted in Figure 3a. The maximum stress occurs at the corners, which is in correspondence with previous discussions (Figure 1b). The stress also increases with the filling ratio, due to the increase of the interaction area between the resist flow and mold beams. The relationship between the filling time and the maximum von Mises stress across the beams with different gap widths is summarized in Figure 3b. Apparently, the maximum von Mises stress increases with the filling ratio and reaches the maximum value at ~ 100 % filling ratio, which is also consistent with the theoretical analysis. The relationship of the maximum von Mises stress versus the gap width at ~ 100 % filling is then summarized in Figure 3c. We fitted the FEM simulated data using an inverse proportion model, and the results clearly show that the maximum stress is inversely proportional to the gap width, as predicted in Equation (15).
Figure 3. Time-dependent stress distribution inside the mold. (a) Von Mises stress distribution inside two 50-nm-width, 100-nm-height beams induced by the resist flow in the channel between the beams at \( t = 0.05 \) \( \mu \)s. (b) Max von Mises stress across the beam at different filling stages with beam widths ranging from 10 nm to 50 nm. The heights of the beams were held at 100 nm. (c)
Max von Mises stress inside the beam for different gap widths near 100% filling ratio. The heights of the beam were held at 100 nm. The data is fitted using an inverse proportion model.

**Comparison between FEM Simulated and Theoretically Calculated Results**

As discussed in the previous section, the interaction area of the Laplace pressure increases with the filling time, but the Laplace pressure decreases with the filling time because of the declining velocity of the resist flow. The theoretical calculation according to **Equation (15)** implies that for typical grating structures with the periodicity smaller than 500 nm and aspect ratio smaller than 10, the maximum stress distributed across the beam on the mold reaches its maximum value at nearly 100% filling, and the stress distribution can be obtained from FEM simulations. However, the FEM simulation is time-consuming and not suitable for quick evaluation. Therefore, in addition to the FEM simulation, we have proposed a theoretical model for quick assessment of the capillary force induced stress in the mold structures in nanoscale UV-NIL, based on **Equation (15)** and compared with the FEM simulations for the cross-validation. **Figure 4** shows the maximum von Mises stress in the beams as a function of the height of the beams for 5 nm (**Figure 4a**) and 100 nm (**Figure 4b**) width parallel beams obtained from FEM simulations and theoretical calculations, respectively. The results show that for both cases, the model agrees well with the FEM simulation, which proves that our model is suitable for the evaluation of capillary force induced stress distribution in mold structures in UV-NIL from sub-10 nm to 100 nm resolutions.
Figure 4. The plot of the maximum von Mises stress in two parallel beams as a function of the height of the beams for (a) 5 nm and (b) 100 nm width beams obtained from FEM simulation (black squares) and theoretical calculation (red circles), respectively. The heights of the beams were held at 100 nm.

**Effects of Physical Parameters of Resist and Mold Materials**

In principle, a narrower beam can lead to a smaller momentum of inertia, and a higher beam results in a larger moment, which both cause a higher level of stress distribution. **Figure 5a** and **5b** plots the maximum stress as a function of gap width and width of gratings, respectively. The results
evidence that the gap width and height of mold beams are the dominant factors for the stress distribution during the UV-NIL process.

The thickness of the resist, which is considered to be constant in this model, is a minor factor that affects the stress distribution. When advancing the dimension of the patterns on the mold to tens of nanometers scale, the Laplace pressure (several MPa) becomes much larger than the pressure in the thin resist film (several kPa), as long as the thickness of the resist is still greater than the height of the beams on the mold. Therefore, the maximum stress would decrease according to Equation (7) and Equation (9). Figure 5c shows the influence of the thickness of resist layer on the highest stress in mold structures, in the case of two parallel beams with 100 nm width, 100 nm gap width and 200 nm height. The result indicates that the thickness of the resist is a less dominant parameter in the UV-NIL, because when the thickness of resist layer increases from 100 nm to 500 nm, the maximum stress only increases by 15%.
Figure 5. Effects of physical parameters of the resist and mold materials on the maximum stress inside the mold structures, showing the maximum stress versus (a) gap width between the beams, (b) height of the beams, (c) thickness of the resist, (d) surface tension of the resist, and (e) contact angle between the resist and the mold. The heights of the beams were fixed at 200 nm in (c) – (d).

Physical parameters of resist liquid also play an essential role in the imprint process. As discussed in the previous paragraphs, the viscosity of the resist flow affects only the filling process but not the stress distribution in the mold structures. However, the surface tension and contact angle of resist liquid change the maximum stress inside the beams significantly. The relationships of surface tension and contact angle with the maximum stress inside two 100 nm width, 100 nm gap width beams are summerized in Figure 5d and 5e, respectively. The results indicate that the maximum stress increases linearly with the surface tension, due to the increased capillary force and Laplace pressure, and decreases with the contact angle, because lower contact angle leads to better wettability and increases the capillary force accordingly.

**Critical Aspect Ratio of the Mold**

In solid mechanics, the structures collapse when the maximum stress exceeds the yield strength of materials. As illustrated in Figure 1b, the maximum stress occurs at the corner of a beam of the imprint mold during the imprinting process. The stress distributed at the corner near the resist liquid flow is compressive; while the stress at the opposite corner is tensile. Therefore, the mold structure collapses through the unbalanced loading. The collapsing process could be predicted by comparing the maximum stress with the yield strength of the mold material. For different mold materials, the yield strength varies significantly, e.g., 7000 MPa for silicon, 364 MPa for silicon dioxide. For the mold used in sub-10 nm UV-NIL made of crosslinked HSQ using HIBL, we
assume the yield strength of HSQ ranges from 60 – 100 MPa, taking the adhesion of HSQ structures with the substrate into consideration.

**Figure 6a** shows the critical aspect ratio as a function of gap width of a two-parallel-beams imprint mold for different yield strength ranging from 60 MPa to 100 MPa, the result indicates that the highest aspect ratio allowed for transferring the 10 nm period beams to the resist through UV-NIL is only 1.8, even for a mold with yield strength of $\sigma_Y = 100 \text{ MPa}$. Therefore, to improve the critical aspect ratio, materials with higher yield strength is demanded.
Figure 6. (a) Critical aspect ratio of the beams as a function of the period of the wetting imprint mold with different yield strengths of 60 MPa, 80 MPa, and 100 MPa, respectively. (b) Critical imprint pressure as a function of the period of the non-wetting imprint mold. (c) Critical aspect ratio of the beams as a function of the period of the non-wetting imprint mold with different yield strengths of 60 MPa, 80 MPa, and 100 MPa, respectively.
According to Equation (9), a critical imprint pressure exists in the non-wetting UV-NIL system. **Figure 6b** shows the critical imprint pressure as a function of the gap width of the beams on the non-wetting imprint mold in UV-NIL, and the width of the beams is the same as the gap width. In the figure, the critical pressure is calculated for a contact angle of 110°, and the result indicates that critical imprint pressure increases when the gap width of the beams shrinks. For example, for gratings with a sub-10 nm period, the critical imprint pressure exceeds 4 MPa, almost 40 atm.

The relationship of the critical aspect ratio for non-wetting molds versus the gap width of the beams is plotted in **Figure 6c**. The results indicate that for gating patterns with a given period, the critical aspect ratio could be improved through hydrophobic treatment of the mold.

### 3. Conclusion

In conclusion, we have established a model to describe the dynamic mechanical behavior in UV-NIL towards sub-10 nm resolution. We have theoretically and numerically investigated the capillary force induced fluidic flow of resist and pattern deformation in UV-NIL. The maximum stress and critical aspect ratio in both wetting and non-wetting molds have been calculated for pattern structures ranging from 5 nm to 200 nm. This study has reported here the key factors affecting the durability of imprint molds and provided guidelines for reducing the pattern deformation on the imprint mold. Based on our study, the critical aspect ratio for grating patterns with a given period is limited by both the mechanical properties of the resist and the mold. High-fidelity replication of nanostructures at sub-10 nm resolution demands proper selection of resist and aspect ratio of the nanostructures on the mold.
References

1. Li, W. D.; Wu, W.; Williams, R. S., Combined Helium Ion Beam and Nanoimprint Lithography Attains 4 Nm Half-Pitch Dense Patterns. *J Vac Sci Technol B* 2012, 30 (6).

2. Shen, Y. M.; Yao, L.; Li, Z. W.; Kou, J. L.; Cui, Y. S.; Bian, J.; Yuan, C. S.; Ge, H. X.; Li, W. D.; Wu, W.; Chen, Y. F., Double Transfer Uv-Curing Nanoimprint Lithography. *NanoTechnology* 2013, 24 (46).

3. Wang, C.; Xia, Q. F.; Li, W. D.; Fu, Z. L.; Morton, K. J.; Chou, S. Y., Fabrication of a 60-Nm-Diameter Perfectly Round Metal-Dot Array over a Large Area on a Plastic Substrate Using Nanoimprint Lithography and Self-Perfection by Liquefaction. *Small* 2010, 6 (11), 1242-1247.

4. Du, J.; Wei, Z.; He, W.; Tang, Y., Effects of Mold Geometries and Initial Resist Thickness on Filling Behavior in Uv-Nanoimprint Lithography. *J Comput Theor Nanos* 2012, 9 (8), 1029-1035.

5. Kim, K.-D.; Kwon, H.-J.; Choi, D.-g.; Jeong, J.-H.; Lee, E.-s., Resist Flow Behavior in Ultraviolet Nanoimprint Lithography as a Function of Contact Angle with Stamp and Substrate. *Jpn J Appl Phys* 2008, 47 (11R), 8648.

6. Yoneda, I.; Nakagawa, Y.; Mikami, S.; Tokue, H.; Ota, T.; Koshiba, T.; Ito, M.; Hashimoto, K.; Nakasugi, T.; Higashiki, T. In *A Study of Filling Process for Uv Nanoimprint Lithography Using a Fluid Simulation*, SPIE Advanced Lithography, International Society for Optics and Photonics: 2009; pp 72712A-72712A-7.

7. Liang, X.; Tan, H.; Fu, Z.; Chou, S. Y., Air Bubble Formation and Dissolution in Dispensing Nanoimprint Lithography. *NanoTechnology* 2007, 18 (2), 025303.

8. Morihara, D.; Nagaoka, Y.; Hiroshima, H.; Hirai, Y., Numerical Study on Bubble Trapping in Uv Nanoimprint Lithography. *J Vac Sci Technol B* 2009, 27 (6), 2866-2868.

9. Hiroshima, H.; Komuro, M., Control of Bubble Defects in Uv Nanoimprint. *Jpn J Appl Phys* 2007, 46 (9S), 6391.

10. Taga, A.; Yasuda, M.; Kawata, H.; Hirai, Y., Impact of Molecular Size on Resist Filling Process in Nanoimprint Lithography: Molecular Dynamics Study. *J Vac Sci Technol B* 2010, 28 (6), C6M68-C6M71.

11. Li, W. D.; Wu, W.; Williams, R. S., Combined Helium Ion Beam and Nanoimprint Lithography Attains 4 Nm Half-Pitch Dense Patterns. *J Vac Sci Technol B* 2012, 30 (6), 06F304-06F304-4.

12. Yoshimoto, K.; Stoykovich, M. P.; Cao, H. B.; de Pablo, J. J.; Nealey, P. F.; Drugan, W. J., A Two-Dimensional Model of the Deformation of Photoresist Structures Using Elastoplastic Polymer Properties. *J Appl Phys* 2004, 96 (4), 1857-1865.

13. Woo, Y. S.; Lee, D. E.; Lee, W. I., Molecular Dynamic Studies on Deformation of Polymer Resist During Thermal Nanoimprint Lithographic Process. *Tribol Lett* 2009, 36 (3), 209-222.

14. Lazzarino, F.; Gourgon, C.; Schiavone, P.; Perret, C., Mold Deformation in Nanoimprint Lithography. *J Vac Sci Technol B* 2004, 22 (6), 3318-3322.

15. Oron, A.; Davis, S. H.; Bankoff, S. G., Long-Scale Evolution of Thin Liquid Films. *Reviews of modern physics* 1997, 69 (3), 931.

16. Young, T., An Essay on the Cohesion of Fluids. *Philosophical Transactions of the Royal Society of London* 1805, 65-87.

17. Laplace, P. S., *Traité De Mécanique Céléste/Par Ps Laplace...; Tome Premier [-Quatrieme].* de l’Imprimerie de Crapelet: 1805; Vol. 4.

18. Gauss, C. F., *Principia Generalia Theoriae Figurarum in Statu Aequilibrii.* Springer: 1877.

19. Lucas, R., Uber Das Zeitgesetz Des Kapillaren Aufstiegs Von Flüssigkeiten. *Colloid & Polymer Science* 1918, 23 (1), 15-22.

20. Washburn, E. W., The Dynamics of Capillary Flow. *Physical review* 1921, 17 (3), 273.

21. Loeb, G. I.; Schrader, M. E., *Modern Approaches to Wettability: Theory and Applications.* Springer Science & Business Media: 2013.

22. Zhmud, B.; Tiber, F.; Hallstensson, K., Dynamics of Capillary Rise. *Journal of Colloid and Interface Science* 2000, 228 (2), 263-269.

23. Kornev, K. G.; Neimark, A. V., Modeling of Spontaneous Penetration of Viscoelastic Fluids and Biofluids into Capillaries. *Journal of colloid and interface science* 2003, 262 (1), 253-262.

24. Martic, G.; Gentner, F.; Seveno, D.; Coulon, D.; De Coninck, J.; Blake, T., A Molecular Dynamics Simulation of Capillary Imbibition. *Langmuir* 2002, 18 (21), 7971-7976.

25. Dimitrov, D. I.; Milchev, A.; Binder, K., Molecular Dynamics Simulations of Capillary Rise Experiments in Nanotubes Coated with Polymer Brushes†. *Langmuir* 2008, 24 (4), 1232-1239.

26. Brown, G. O., The History of the Darcy-Weisbach Equation for Pipe Flow Resistance. *Environmental and Water Resources History* 2002, 38 (7), 34-43.