Lucky and Proper Lucky Labeling of Quadrilateral Snake Graphs

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Abstract. The labeling is said to be lucky labeling of the graph if the vertices of the graph are labeled by natural number with satisfying the condition that sum of labels over the adjacent of the vertices in the graph are not equal and if vertices are isolated vertex then the sum of the vertex is zero. The least natural number which labelled the graph is the lucky number. The Lucky Number of graph $G$ is denoted by $\eta(G)$. The labeling defined as proper labeling if the vertices of the graph are labeled by natural number with fulfilling the condition that label of adjacent vertices is not the same. The labeling is defined as proper lucky labeling if labeling is proper and also lucky. The proper lucky number of graph $G$ is denoted by $\eta_p(G)$. Here we obtain a lucky number and proper lucky number for family of quadrilateral snake graph such as quadrilateral snake graph, double quadrilateral snake graph, alternate quadrilateral snake graph and double alternate quadrilateral snake graph.

1. Introduction
In graph theory, graph labeling has a wide area. It has a well-built connection among numbers and the structure of the graph. It was introduced by Rosa in 1967 [10]. Labeling is a function to mark the number to a graph vertex or edge or both with the specific condition. Gallian [4] gave a dynamic survey of graph labeling. The proper labeling was initiated by Karonsik (2004) [7] which mean naming adjacent vertices by a different natural number. Lucky labeling for 3 colourable graphs was studied by Ahadi (2012) [1]. Lucky labeling is compared with proper vertex coloring by Akbari (2013) [2]. The graph which fulfill the lucky labeling conditions are called lucky graph. The graph which fulfill the proper lucky labeling conditions are called proper lucky graph. Proper lucky labeling has a very close association with graph colouring.

Vidya computed that alternate quadrilateral snake graphs are product cordial in 2014 [13]. Lavanya had detected that quadrilateral snake graphs are graceful in 2017 [11]. Prajapati computed that quadrilateral snake graph is SD-Prime Cordial in 2019 [9]. Jadav computed snake graphs are strongly-graph in 2016 [6]. Labeling application is computer networking, clustering, image segmentation etc., it also applicable in astronomy, circuit designing, image process, cryptography and information security. The objective of this paper is to study the lucky number and proper lucky number for quadrilateral snake graph, double
quadrilateral snake graph, alternate quadrilateral snake graph and double alternate quadrilateral snake graph.

2. Preliminaries

2.1 Quadrilateral snake graph

A quadrilateral snake graph \( QS_n \) is obtained from \( 3n - 2 \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( 2n - 2 \) vertices and the edge set \( E(QS_n) = \{u_iu_{i+1}; 1 \leq i \leq n\} \cup \{v_{2i-1}v_{2i}; 1 \leq i \leq n\} \cup \{u_iv_{2i-1}; 1 \leq i \leq n\} \) refer figure 1 [5,12].

2.2 Alternate quadrilateral snake graph

An alternate quadrilateral snake graph \( AQS_n \) is consist of a \( 2n \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( n \) vertices and the edge set \( E(AQS_n) = \{u_iu_{i+1}; 1 \leq i \leq n\} \cup \{v_{2i-1}v_{2i}; 1 \leq i \leq n\} \cup \{u_tv_{2i-1}; 1 \leq i \leq n\} \) refer figure 2 [5,12].

2.3 Double quadrilateral snake graph

A double quadrilateral snake graph \( DQS_n \) is consist of a \( 5n - 2 \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( 2n - 2 \) vertices, \( w_i \) have \( 2n - 2 \) vertices and the edge set \( E(DQS_n) = E(QS_n) \cup \{w_{2i-1}w_{2i}; 1 \leq i \leq n\} \cup \{u_iw_{2i-1}; 1 \leq i \leq n\} \cup \{u_{i+1}w_{2i}; 1 \leq i \leq n\} \) refer figure 3 [5,12].
2.4 Double alternate quadrilateral snake graph
An alternate double quadrilateral snake graph $DAQS_n$ is consist of a $3n$ vertices where $n > 1$, $u_i$ have $n$ vertices, $v_i$ have $n$ vertices, $w_i$ have $n$ vertices and the edge set $E(DAQS_n) = E(QS_n) \cup \{w_{2i-1}w_{2i}: 1 \leq i \leq n\} \cup \{u_{2i-1}w_{2i}: 1 \leq i \leq n\} \cup \{u_{2i-1}w_{2i-1}: 1 \leq i \leq n\}$ refer figure 4 [5,12].

2.5 Lucky labeling
Let $f: V(G) \rightarrow \mathbb{N}$ be a labeling of the vertices of a graph $G$ by a natural number. Let $S(v)$ indicate the sum of labels over the neighbours of the vertex $v$ in $G$. If $v$ is an isolated vertex of $G$ we put $S(v) = 0$. A labeling $f$ is lucky if $S(u) \neq S(v)$ for an adjacent vertex $u$ and $v$ [3,8].

2.6 Proper lucky labeling
A labeling $f$ is proper lucky labeling, if $u$ and $v$ are adjacent in $G$ then $f(u) \neq f(v) \& S(u) \neq S(v)$. The proper Lucky number of $G$ is denoted by $\eta_p(G)$, is the least natural number $k$ such that $G$ has proper lucky labeling with $\{1, 2, \ldots, k\}$ as the set of labels [3,8].

3. Main results
3.1 Theorem
The quadrilateral snake $QS_n$ with $n > 1$ admits lucky labeling with lucky number $\eta(QS_n) = 2$.

Proof:
Let \( f: V(QS_n) \to \{1,2\} \) quadrilateral snake graph with \( 3n - 2 \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( 2n - 2 \) vertices and the edge set \( E(QS_n) = \{u_i u_{i+1}: 1 \leq i \leq n\} \cup \{v_{2i-1} v_{2i}: 1 \leq i \leq n\} \cup \{u_i v_{2i-1}: 1 \leq i \leq n\} \cup \{u_{i+1} v_{2i}: 1 \leq i \leq n\} \), be defined by
\[
\begin{align*}
  f(u_i) &= \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases} \\
  f(v_i) &= 1
\end{align*}
\]
Here we obtain the sum of neighbour vertices as,
\[
s(u_i) = \begin{cases} 2 & i = 1, i = \text{odd } n \\ 3 & i = \text{even } n \\ 4 & \text{odd } i > 1, i < n \\ 6 & \text{even } i < n \end{cases}
\]
\[
s(v_i) = \begin{cases} 2 & i = 4k - 1, i = 4k - 2 \\ 3 & i = 4k, i = 4k - 3 , k \in \mathbb{N} \end{cases}
\]
Therefore sum of adjacent vertices are not same. So, quadrilateral snake \( QS_n \) with \( n > 1 \) admits lucky labeling with lucky number \( \eta(QS_n) = 2 \).

![Figure 5: Lucky quadrilateral snake graph \( QS_7 \)](image)

3.2 **Illustration:** The lucky number 2 of quadrilateral snake graph \( \eta(QS_7) = 2 \), is shown in figure 5.

3.3 **Theorem**
The double quadrilateral snake \( DQS_n \) with \( n > 1 \) admits lucky labeling with lucky number \( \eta(DQS_n) = 2 \).

**Proof:**
Given double quadrilateral snake graph with \( 5n - 2 \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( 2n - 2 \) vertices, \( w_i \) have \( 2n - 2 \) vertices and the edge set \( E(DQS_n) = E(QS_n) \cup \{w_{2i-1} w_{2i}: 1 \leq i \leq n\} \cup \{u_{i+1} w_{2i}: 1 \leq i \leq n\} \),

Case (i): \( n = \text{even} \)

Let \( f: V(DQS_n) \to \{1,2\} \) for double quadrilateral snake graph be defined by
\[
\begin{align*}
  f(u_i) &= \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases} \\
  f(v_i) &= \begin{cases} 1 & i \neq 2 \\ 2 & i = 2 \end{cases} \\
  f(w_i) &= \begin{cases} 1 & i \neq 2 \\ 2 & i = 2 \end{cases}
\end{align*}
\]
Here we obtain the sum of neighbour vertices as,
\[
s(u_1) = 3 \\
\begin{cases} 6 & \text{odd } i > 1 \\ 8 & \text{even } i < n, i > 2 \\ 10 & i = 2 \end{cases}
\]
Therefore sum of adjacent vertices are not same. So, double quadrilateral snake $DQS_n$ with $n > 1$ admits lucky labeling for even $n$ with lucky number $\eta(DQS_n) = 2$.

Case (ii): $n = \text{odd}$

Let $f: V(DQS_n) \to \{1, 2\}$ for double quadrilateral snake graph be defined by

$$f(u_i) = \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i \neq n - 1, i \neq 2 \\ 2 & i = n - 1, i = 2 \end{cases}$$

$$f(w_i) = \begin{cases} 1 & i \neq n - 1, i \neq 2 \\ 2 & i = n - 1, i = 2 \end{cases}$$

Here we obtain the sum of neighbor vertices as,

$$s(u_i) = \begin{cases} 3 & \text{odd } i > 1, i < n \\ 6 & \text{even } i < n - 1, i > 2 \\ 10 & i = n - 1, i = 2 \end{cases}$$

$$s(v_i) = \begin{cases} 3 & i = 4k, i = 4k - 3, i \neq n, i \neq 1, k \in \mathbb{N} \\ 4 & i = n, i = 1 \end{cases}$$

$$s(w_i) = \begin{cases} 3 & i = 4k, i = 4k - 3, i \neq n, i \neq 1, k \in \mathbb{N} \\ 4 & i = n, i = 1 \end{cases}$$

Therefore sum of adjacent vertices are not same. So, double quadrilateral snake $DQS_n$ with $n > 1$ admits lucky labeling for odd $n$ with lucky number $\eta(DQS_n) = 2$. Hence, the double quadrilateral Snake $DQS_n$ with $n > 2$ admits lucky labeling with lucky number $\eta(DQS_n) = 2$.

![Figure 6: Lucky double quadrilateral snake graph $DQS_7$](image)

3.4 Illustration: The lucky number 2 of double quadrilateral snake graph $\eta(DQS_7) = 2$ is shown in figure 6.

3.5 Theorem

The alternate quadrilateral snake $AQS_n$ with $n > 1$ admits lucky labeling with lucky number $\eta(AQS_n) = 2$. 
Proof:
Let $f: V(AQS_n) \to \{1,2\}$ for alternate quadrilateral snake graph with $2n$ vertices where $n > 1$, $u_i$ have $n$ vertices, $v_i$ have $n$ vertices and the edge set $E(AQS_n) = \{u_iu_{i+1}: 1 \leq i \leq n\} \cup \{v_{2i}v_{2i+1}: 1 \leq i \leq n\} \cup \{u_{2i}v_{2i}: 1 \leq i \leq n\}$, be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

Here we obtain the sum of neighbour vertices as,

$$s(u_1) = 4$$

$$s(u_n) = 2$$

$$s(u_i) = \begin{cases} 3 & \text{even } i, i < n \\ 6 & \text{odd } i, i > 1 \end{cases}$$

$$s(v_i) = \begin{cases} 2 & \text{odd } i \\ 4 & \text{even } i \end{cases}$$

Therefore sum of adjacent vertices are not same. So, alternate quadrilateral snake $AQS_n$ with $n > 1$ admits lucky labeling with lucky number $\eta(AQS_n) = 2$.

**Figure 7:** Lucky alternate quadrilateral snake graph $AQS_8$

3.6 Illustration: The lucky number 2 of alternate quadrilateral snake graph $\eta(AQS_8)$ is shown in figure 7.

3.7 Theorem
The double alternate quadrilateral snake $DAQS_n$ with $n > 1$ admits lucky labeling with lucky number $\eta(DAQS_n) = 2$.

Proof:
Let $f: V(DAQS_n) \to \{1,2\}$ for alternate quadrilateral snake graph with $3n$ vertices where $n > 1$, $u_i$ have $n$ vertices, $v_i$ have $n$ vertices, $w_i$ have $n$ vertices and the edge set $E(DAQS_n) = E(AQS_n) \cup \{w_{2i}w_{2i+1}: 1 \leq i \leq n\} \cup \{u_{2i}w_{2i+1}: 1 \leq i \leq n\} \cup \{u_{2i-1}w_{2i-1}: 1 \leq i \leq n\}$, be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(w_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

Here we obtain the sum of neighbour vertices as,

$$s(u_1) = 6$$

$$s(u_n) = 3$$

$$s(u_i) = \begin{cases} 5 & \text{even } i, i < n \\ 8 & \text{odd } i, i > 1 \end{cases}$$

$$s(v_i) = \begin{cases} 2 & \text{odd } i \\ 4 & \text{even } i \end{cases}$$
Therefore sum of adjacent vertices are not same. So, double alternate quadrilateral snake $DAQS_n$ with $n > 1$ admits lucky labeling with lucky number $\eta(DAQS_n) = 2$.

3.8 Illustration: The lucky number 2 of double alternate quadrilateral snake graph $\eta(DAQS_B) = 2$. is shown in figure 8.

3.9 Theorem

The quadrilateral snake $QS_n$ with $n > 1$ admits proper lucky labeling with proper lucky number $\eta_p(QS_n) = 3$.

Proof:

Let $f: V(QS_n) \rightarrow \{1,2,3\}$ for quadrilateral snake graph with $3n - 2$ vertices where $n > 1$, $u_i$ have $n$ vertices, $v_i$ have $2n - 2$ vertices and the edge set $E(QS_n) = \{u_iu_{i+1}: 1 \leq i \leq n\} \cup \{v_{2i-1}v_{2i}: 1 \leq i \leq n\} \cup \{u_{i+1}v_{2i}: 1 \leq i \leq n\}$, be defined by

$$f(u_i) = \begin{cases} 1 & \text{odd } i \\ 2 & \text{even } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i = 4k - 1, i = 4k - 2 \\ 2 & i = 4k, i = 4k - 3 \\ 3 & k \in \mathbb{N} \end{cases}$$

Here we obtain the sum of neighbour vertices as,

$$s(u_i) = \begin{cases} 5 & \text{even } n \\ 10 & \text{odd } i < n \end{cases}$$

$$s(u_n) = \begin{cases} 10 & \text{odd } i > 1, i < n \\ 2 & i = 4k, i = 4k - 3 \end{cases}$$

$$s(v_i) = \begin{cases} 2 & i = 4k - 1, i = 4k - 2 \\ 5 & k \in \mathbb{N} \end{cases}$$

Therefore sum of adjacent vertices and adjacent labeling are not same. So, quadrilateral snake $QS_n$ with $n > 1$ admits proper lucky labeling with proper lucky number $\eta_p(QS_n) = 3$. 

![Figure 8: Lucky double alternate quadrilateral snake graph $DAQS_B$](image)

![Figure 9: Proper lucky quadrilateral snake graph $QS_n$](image)
3.10 Illustration: The proper lucky number 3 of quadrilateral snake graph \( \eta_p(QS_7) = 3 \) is shown in figure 9.

3.11 Theorem

The double quadrilateral snake \( DQS_n \) with \( n > 1 \), for odd \( n \) it admits proper lucky labeling with proper lucky number \( \eta_p(DQS_n) = 2 \).

Proof:
Let \( f: V(DQS_n) \to \{1,2\} \) for odd \( n \) double quadrilateral snake graph with \( 5n - 2 \) vertices where \( n > 1 \), \( u_i \) have \( n \) vertices, \( v_i \) have \( 2n - 2 \) vertices, \( w_i \) have \( 2n - 2 \) vertices and the edge set \( E(DQS_n) = E(QS_n) \cup \{w_{2i-1}w_{2i}: 1 \leq i \leq n\} \cup \{u_{i+1}w_{2i}: 1 \leq i \leq n\} \cup \{u_{i+1}w_{2i}: 1 \leq i \leq n\} \), be defined by

\[
\begin{align*}
    f(u_i) &= \begin{cases} 
    1 & \text{even } i \\
    2 & \text{odd } i 
    \end{cases} \\
    f(v_i) &= \begin{cases} 
    1 & i = 4k, i = 4k - 3 \\
    2 & i = 4k - 1, i = 4k - 2 
    \end{cases} \quad k \in \mathbb{N} \\
    f(w_i) &= \begin{cases} 
    1 & i = 4k, i = 4k - 3 \\
    2 & i = 4k - 1, i = 4k - 2 
    \end{cases} \quad k \in \mathbb{N}
\end{align*}
\]

Here we obtain the sum of neighbour vertices as,

\[
\begin{align*}
    s(u_i) &= 3 \\
    s(v_i) &= \begin{cases} 
    12 & \text{odd } i > 1, i < n \\
    6 & \text{even } i 
    \end{cases} \\
    s(w_i) &= \begin{cases} 
    4 & i = 4k - 1, i = 4k - 2, k \in \mathbb{N} \\
    2 & i = 4k, i = 4k - 3 \\
    2 & i = 4k - 1, i = 4k - 2, k \in \mathbb{N}
    \end{cases}
\end{align*}
\]

Therefore sum of adjacent vertices and adjacent labeling are not same. So, double quadrilateral snake \( DQS_n \) with \( n > 1 \) admits proper lucky labeling for odd \( n \) with proper lucky number \( \eta_p(DQS_n) = 2 \).

3.12 Illustration: The proper lucky number 2 of double quadrilateral snake graph \( \eta_p(DQS_7) = 2 \) is shown in figure 10.

3.13 Theorem

The double quadrilateral snake \( DQS_n \) with \( n > 1 \), for even \( n \) it admits proper lucky labeling with proper lucky number \( \eta_p(DQS_n) = 3 \).

Proof:
Let $f : V(DQS_{n}) \to \{1, 2, 3\}$ for even $n$ double quadrilateral snake graph with $5n - 2$ vertices where $n > 1$, $u_i$ have $n$ vertices, $v_i$ have $2n - 2$ vertices, $w_i$ have $2n - 2$ vertices and the edge set $E(DQS_{n}) = E(QS_{n}) \cup \{w_{2i-1}w_{2i} : 1 \leq i \leq n\} \cup \{u_{i+1}w_{2i} : 1 \leq i \leq n\}$, be defined by

$$f(u_i) = \begin{cases} 
1 & \text{even } i \\
2 & \text{odd } i 
\end{cases}$$

$$f(v_i) = \begin{cases} 
1 & i = 4k, i = 4k - 3, i \neq 2n - 2 \\
2 & i = 4k - 1, i = 4k - 2, i \neq 2n - 2, \ k \in \mathbb{N} \\
3 & i = 2n - 2 
\end{cases}$$

$$f(w_i) = \begin{cases} 
2 & i = 4k - 1, i = 4k - 2, i \neq 2n - 2, \ k \in \mathbb{N} \\
3 & i = 2n - 2 
\end{cases}$$

Here we obtain the sum of neighbour vertices as,

$$s(u_1) = 3$$

$$s(u_n) = 8$$

$$s(u_i) = \begin{cases} 
6 & \text{odd } i > 1 \\
12 & \text{even } i < n 
\end{cases}$$

$$s(v_i) = \begin{cases} 
2 & i = 4k - 1, i = 4k - 2, i \neq 2n - 3 \\
4 & i = 4k, i = 4k - 3, i \neq 2n - 3, \ k \in \mathbb{N} \\
5 & i = 2n - 3 
\end{cases}$$

$$s(w_i) = \begin{cases} 
4 & i = 4k - 1, i = 4k - 2, i \neq 2n - 3, \ k \in \mathbb{N} \\
5 & i = 2n - 3 
\end{cases}$$

Therefore sum of adjacent vertices and labeling are not same. So, double quadrilateral snake $DQS_{n}$ with $n > 1$ admits lucky labeling for even $n$ with proper lucky number $\eta(DQS_{n}) = 3$.

![Figure 11: Proper double quadrilateral snake graph $DQS_{6}$](image.png)

3.14 Illustration: The proper lucky number 3 of double quadrilateral snake graph $\eta_p(DQS_{6}) = 3$. is shown in figure 11.

3.15 Theorem
The alternate quadrilateral snake $AQS_{n}$ with $n > 1$ admits proper lucky labeling with proper lucky number $\eta_p(AQS_{n}) = 2$.

Proof:
Refer theorem 3.5, since it satisfies the condition the sum of adjacent vertices and adjacent labeling are not same. So, alternate quadrilateral Snake $AQS_{n}$ with $n > 1$ admits proper lucky labeling with proper lucky number $\eta_p(AQS_{n}) = 2$. 
3.16 **Theorem**
The double alternate quadrilateral snake $DAQS_n$ with $n > 1$ admits proper lucky labeling with proper lucky number $\eta_p(DAQS_n) = 2$.

**Proof:**
Refer theorem 3.7, since it satisfies the condition the sum of adjacent vertices and adjacent labeling are not same. So, Double alternate quadrilateral snake graph $DAQS_n$ with $n > 1$ admits proper lucky labeling with lucky number $\eta_p(DAQS_n) = 2$.

3.17 **Corollary**
Lucky number and minimum degree of quadrilateral snake graph, alternate quadrilateral snake graph, double quadrilateral snake graph and double alternate quadrilateral snake graph are equal i.e., $\eta(G) = \delta(G), G \in QS_n, AQS_n, DQS_n, DAQS_n$.

3.19 **Corollary**
Lucky labeling is half of maximum degree of quadrilateral snake graph and double alternate quadrilateral snake graph i.e., $\eta(G) = \frac{1}{2} \Delta(G), G \in QS_n, DAQS_n$.

3.20 **Corollary**
Lucky number is one third of maximum degree of double quadrilateral snake graph i.e., $\eta(DQS_n) = \frac{1}{3} \Delta(DQS_n)$.

3.21 **Corollary**
Lucky number is one less than maximum degree of alternate quadrilateral snake graph i.e., $\eta(AQS_n) = \Delta(AQS_n) - 1$.

3.22 **Corollary**
Proper lucky number of quadrilateral snake graph is one more than minimum degree i.e., $\eta_p(QS_n) = \delta(QS_n) + 1$.

3.23 **Corollary**
Proper lucky number and minimum degree of double quadrilateral snake graph are equal, if $n$ is odd i.e., $\eta_p(DQS_n) = \delta(DQS_n)$, for odd $n$.

3.23 **Corollary**
Proper lucky number is one more than minimum degree of double quadrilateral snake graph, if $n$ is even i.e., $\eta_p(DQS_n) = \delta(DQS_n) + 1$, for even $n$.

3.24 **Corollary**
Proper lucky number and minimum degree of alternate quadrilateral snake graph and double alternate quadrilateral snake graph are equal i.e., $\eta_p(G) = \delta(G), G \in AQS_n, DAQS_n$.

3.27 **Corollary**
Proper lucky number of quadrilateral and alternate quadrilateral snake graph are one less than maximum degree i.e., $\eta_p(G) = \Delta(G) - 1, G \in QS_n, AQS_n$.

3.28 **Corollary**
Proper lucky number of double quadrilateral snake graph is one third of maximum degree if n is odd i.e.,
\[ \eta_p(DQS_n) = \frac{1}{3} \Delta(DQS_n), \text{for odd } n. \]

3.29 Corollary
Proper lucky number of double quadrilateral snake graph is half of maximum degree if n is even i.e.,
\[ \eta_p(DQS_n) = \frac{1}{2} \Delta(DQS_n), \text{for even } n. \]

3.30 Corollary
Proper lucky number of double alternate quadrilateral snake graph is half of the maximum degree i.e.,
\[ \eta_p(DATS_n) = \frac{1}{2} \Delta(DATS_n). \]

4. CONCLUSION
In this paper we compute the lucky number and proper lucky number of the quadrilateral snake graph,
double quadrilateral snake graph, alternate quadrilateral snake graph and double alternate quadrilateral
snake graph. Also we connected it with minimum and maximum degree. Further we take this work to
standard graph and networks.

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