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Bifurcations in the regularized Ericksen bar model. (English) Zbl 1134.74018
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Summary: We consider the regularized Ericksen model of an elastic bar on elastic foundation on an interval with Dirichlet boundary conditions as a two-parameter bifurcation problem. We explore, using local bifurcation analysis and continuation methods, the structure of bifurcations from double zero eigenvalues. Our results provide evidence in support of S. Müller’s conjecture [Calc. Var. Partial Diff. Equ. 1, No. 2, 169–204 (1993; Zbl 0821.49015)] concerning the symmetry of local minimizers of the associated energy functional and describe in detail the structure of primary branch connections that occur in this problem. We give a reformulation of Müller conjecture and suggest two further conjectures based on the local analysis and numerical observations. We conclude by analysing a “loop” structure that characterizes \((k, 3k)\) bifurcations.

MSC:
74G60 Bifurcation and buckling
74K10 Rods (beams, columns, shafts, arches, rings, etc.)

Keywords:
Lyapunov-Schmidt analysis; continuation methods; Müller conjecture

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References:
[1] Armbruster, D., Dangelmayr, G.: Coupled stationary bifurcations in no-flux boundary value problems. Math. Proc. Camb. Phil. Soc. 101, 167–192 (1987) · Zbl 0633.58011 · doi:10.1017/S0305004100006650
[2] Aston, P.: Scaling laws and bifurcations. In: Roberts, M. Stewart, I. (eds.) Singularity Theory and its Applications, Warwick 1989. Part II. Lecture Notes in Mathematics, vol. 1463, pp. 1–21. Springer-Verlag, Berlin (1991)
[3] Ball, J.M.: Dynamics and minimizing sequences. In: Kirchgässner, K. (ed.) Problems Involving Change of Type. Lecture Notes in Physics, vol. 359, pp. 3–16. Springer-Verlag, Berlin (1990)
[4] Ball, J.M., Holmes, P.J., James, R.D., Pego, R.L., Swart, P.J.: On the dynamics of fine structure. J. Nonlin. Science 1, 17–70 (1991) · Zbl 0791.35030 · doi:10.1007/BF01209147
[5] Doedel, E.J., Champneys, A.R., Fairgrieve, T.F., Kuznetsov, Y.A., Sandstede, B., Wang, X.: AUTO 97: Continuation and Bifurcation Software for ODEs, http://www.maths.surrey.ac.uk/personal/st/B.Sandstede/publications/auto97.pdf
[6] Ericksen, J.: Equilibrium of bars. J. Elasticity 5, 191–202 (1975) · Zbl 0324.73067 · doi:10.1007/BF00126984
[7] Friesecke, G., McLeod, J.B.: Dynamics as a mechanism preventing the formation of finer and finer microstructure. Arch. Rat. Mech. Anal. 133, 199–247 (1996) · Zbl 0920.73345 · doi:10.1007/BF00380893
[8] Friesecke, G., McLeod, J.B.: Dynamic stability of non-minimizing phase mixtures. Proc. Royal Soc. London A 453, 2427–2436 (1997) · Zbl 1020.74002 · doi:10.1098/rspa.1997.0130
[9] Golubitsky, M., Schaeffer, D.G.: Singularities and Groups in Bifurcation Theory. Springer-Verlag, New York (1985) · Zbl 0607.35004
[10] Grinfeld, M., Novick-Cohen, A.: Counting stationary solutions of the Cahn–Hilliard equation by transversality arguments. Proc. Royal Soc. Edinburgh A 125, 351–370 (1995) · Zbl 0828.34007
[11] Grinfeld, M., Novick-Cohen, A.: The viscous Cahn-Hilliard equation: morse decomposition and structure of the global attractor. Trans. AMS 351, 2375–2406 (1999) · Zbl 0927.35045 · doi:10.1090/S0002-9947-99-02445-9
[12] Grinfeld, M., Novick-Cohen, A.: (in preparation)
[13] Healey, T.J., Miller, U.: Two-phase equilibria in the anti-plane shear of an elastic solid with interfacial effect via global bifurcation. Proc. Royal Soc. A 463, 1117–1134 (2007) · Zbl 1132.74033 · doi:10.1098/rspa.2006.1807
[14] Hunt, G.W.: Hidden (a)symmetries of elastic and plastic bifurcation. Appl. Mech. Rev. 36, 1165–1186 (1986) · doi:10.1115/1.3149518
[15] Huo, Y., Müller, I.: Interfacial and inhomogeneity penalties in phase transitions. Continuum Mech. Thermodyn. 15, 395–407 (2001) · Zbl 1068.74590 · doi:10.1007/s00161-003-0124-6
[16] Kalies, W.D., Holmes, P.J.: On a dynamical model for phase transformation in nonlinear elasticity. Fields Inst. Commun. 5, 255–269 (1996) · Zbl 0876.73015
Kamerich, E.: A Guide to Maple. Springer-Verlag, New York (1999) - Zbl 0926.68060

Müller, S.: Singular perturbations as a selection criterion for periodic minimizing sequences. Calc. Var. 1, 169–204 (1993) - Zbl 0821.49015 · doi:10.1007/BF01191616

Swart, P.J., Holmes, P.J.: Energy minimization and the formation of microstructure in dynamic anti-plane shear. Arch. Rat. Mech. Anal. 121, 37–85 (1992) - Zbl 0786.73066 · doi:10.1007/BF00375439

Truskinovsky, L., Zanzotto, G.: Ericksen’s bar revisited: energy wiggles. J. Mech. Phys. Solids 44, 1371–1408 (1996) - doi:10.1016/0022-5096(96)00020-8

Vainchtein, A.: Dynamics of phase transitions and hysteresis in a viscoelastic Ericksen’s bar on an elastic foundation. J. Elasticity 57, 243–280 (1999) - Zbl 1003.74054 · doi:10.1023/A:1007661727193

Vainchtein, A.: Stick-slip interface motion as a singular limit of the viscosity-capillarity model. Math. Mech. Solids 6, 323–341 (2001) - Zbl 1057.74028 · doi:10.1177/108128650100600307

Vainchtein, A., Healey, T.J., Rosakis, P.: Bifurcation and metastability in a new one-dimensional model for martensitic phase transition. Comput. Methods Appl. Mech. Engrg. 170, 407–421 (1999) - Zbl 0949.74049 · doi:10.1016/S0045-7825(98)00205-9

Vainchtein, A., Healey, T., Rosakis, P., Truskinovsky, L.: The role of the spinodal region in one-dimensional martensitic phase transitions. Physica 115D, 29–48 (1998) - Zbl 0962.74530

Vainchtein, A., Rosakis, P.: Hysteresis and stick-slip motion of phase boundaries in dynamic models of phase transitions. J. Nonlinear Sci. 9, 697–719 (1999) - Zbl 0989.74054 · doi:10.1007/s003329900083

Yip, N.K.: Structure of stable solutions of a one-dimensional variational problem. Control, Optim. Calc. Variations 12, 721–751 (2006) - Zbl 1117.49025 · doi:10.1051/cocv:2006019

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