Research Article

New Statistical Approaches for Modeling the COVID-19 Data Set: A Case Study in the Medical Sector

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Statistical distributions have great applicability for modeling data in almost every applied sector. Among the available classical distributions, the inverse Weibull distribution has received considerable attention. In the practice of distribution theory, numerous methods have been studied and suggested/introduced to increase the flexibility level of the traditional probability distributions. In this paper, we implement different distribution methods to obtain five new different versions of the inverse Weibull model. The new modifications of the inverse Weibull model are called the logarithm transformed-inverse Weibull, a flexible reduced logarithmic-inverse Weibull, the weighted TX-inverse Weibull, a new generalized-inverse Weibull, and the alpha power transformed extended-inverse Weibull distributions. To illustrate the flexibility and applicability of the new modifications of the inverse Weibull model, a biomedical data set is analyzed. The data set consists of 108 observations and represents the mortality rate of the COVID-19-infected patients. The practical application shows that the new generalized-inverse Weibull is the best modification of the inverse Weibull distribution.

1. Introduction

In the practice of distribution theory, one of the important tasks is to devise an efficient statistical model for real phenomena of nature. Generally, the statistical distributions are implemented to analyze real-life situations that are uncertain and endangered. For example, the probability distributions are frequently applied to analyze data in (i) engineering and related sectors [1], (ii) healthcare engineering [2], (iii) the economic and financial sector [3], (iv) hydrology [4], (v) education [5], (vi) metrology [6], (vii) biological sector [7], and (viii) sports [8].

Due to the applicability of the probability distributions in applied areas/sectors, numerous approaches (probability models) have been proposed and studied. For example, Afify et al. [9] proposed the MOPG-Weibull distribution for analyzing the engineering data set. For further studies related to the engineering sector (i.e., data modeling in the engineering-related area), we refer to studies by Almarashi et al. [10] and Strzelecki [11].

Klakattawi [12] implemented a new extended Weibull (NE-Weibull) model for statistical analysis of the data sets related to cancer patients. For more studies related to the biomedical/healthcare data sets (i.e., data modeling in the
biomedical-related area), we refer to studies by Ahmad et al. [13]; Plana et al. [14]; Xin et al. [15]; and Martínez et al. [16].

Tung et al. [17] proposed the arcsine-Weibull (ASin-Weibull) distribution for analyzing data sets in the business and financial sectors. For more studies related to the financial data sets (i.e., data modeling in the financial-related area), we refer to studies by Zhao et al. [18]; Alfarro et al. [19]; Abubakar and Sabri [20]; and Rana et al. [21].

Bakouch et al. [22] implemented the Gumbel model for analyzing the hydrology data set. Singh et al. [23] provided the assessment of groundwater quality data in Nigeria. Hassan et al. [24] implemented the truncated power Lomax (TP-Lomax) distribution for analyzing the flood data set. For other studies related to the hydrology data sets, we refer to studies by Karahacane et al. [25]; Dodangeh et al. [26]; and Tegegne et al. [27].

Among the above fields (engineering, education, hydrology, and healthcare sectors), statistical distributions are frequently implemented to analyze the biomedical data sets. Since December 2019, researchers have proposed and implemented new probability models for analyzing and predicting the COVID-19 events (Baaleanu et al. [28]; Özköse and Yavuz, M. (2022), Khan et al. [29]; Lella and Pja [30]; Mohan et al. [31]; and Singh et al. [32]).

Maurya et al. [33] proposed a new method called the logarithm transformed (LT) family for introducing flexible probability distributions. Let X has the LT family, if its DF (distribution function) R(x; ψ) is

\[ R(x; ψ) = 1 - \frac{\log[2 - M(x; ψ)]}{\log 2}, \]  

where \( x \in \mathbb{R} \) and \( M(x; ψ) \) is a baseline DF.

Liu et al. [34] introduced a useful method, namely, a FRL-X (flexible reduced logarithmic-X) family for obtaining the modified versions of the existing distributions. Let X has the FRL-X distributions, if its DF R(x; ψ, β) is

\[ R(x; ψ, β) = 1 - \frac{\log[1 - βM(x; ψ) + β]}{\log(β + 1)}, \]  

where \( β \in \mathbb{R}^+ \) is an additional parameter.

Ahmad et al. [35] proposed another new class of probability distributions, called the weighted T-X (WT-X) family of distributions. The DF R(x; ψ) of the WT-X distributions is

\[ R(x; ψ) = 1 - \frac{1 - M(x; ψ)}{e^{M(x; ψ)}}, \]  

with PDF r(x; ψ) given by

\[ r(x; ψ) = \frac{m(x; ψ)}{e^{M(x; ψ)}}, \]  

where \( m(x; ψ) = dl/dxM(x; ψ) \).

Wang et al. [36] studied a NG-X (new generalized-X) family with DF R(x; ψ, θ), provided by

\[ R(x; ψ, θ) = 1 - e^{-M(x; ψ)[1 - M(x; ψ)]^θ}, \]

where \( θ \in \mathbb{R}^+ \) is the additional parameter.

Bo et al. [37] proposed another useful method, namely, the APTEX-X (alpha power-transformed extended-X) family of distributions. The DF R(x; ψ, α₁) of the APTEX-X family is

\[ R(x; ψ, α₁) = \alpha₁ \left(\frac{1 - (1 - M(x; ψ))^{e^{M(x; ψ)}}}{α₁ - 1} - 1\right), \]

where \( α₁ \neq 1, α₁ \in \mathbb{R}^+ \) is an additional parameter.

In the next section, we obtain different modifications of the inverse Weibull (IW) distribution by implementing the approaches defined in Eqs. (1)–(6). For every new modified form of the IW model, the plots of the PDF are also obtained.

2. Some New Modifications of the Inverse Weibull Distribution

This section offers some new different extensions of the IW distribution by incorporating the well-known approaches described in Section 1. Consider the DF \( M(x; ψ), \) PDF \( m(x; ψ), \) SF (survival function) \( S(x; ψ), \) HF (hazard function) \( h(x; ψ), \) and cumulative HF \( H(x; ψ) \) of the IW distribution (with parameters \( α \in \mathbb{R}^+ \) and \( ψ \in \mathbb{R}^+ \)) given by

\[ M(x; ψ) = e^{-ψ/x^α}, \]

\[ m(x; ψ) = \frac{αψ/x^{α+1}}{x^{α+1}} e^{-(ψ/x^α)}, \]

\[ S(x; ψ) = 1 - e^{-(ψ/x^α)}, \]

\[ h(x; ψ) = \frac{αψ/x^{α+1}}{e^{-(ψ/x^α)}(1 - e^{-(ψ/x^α)})}, \]

\[ H(x; ψ) = -\log\left(1 - e^{-(ψ/x^α)}\right), \]

respectively, where \( ψ = (α, ψ). \)

2.1. The Logarithm Transformed-Inverse Weibull Distribution. Here, we implement the LT family approach (see (1)) to introduce a new version of the IW model. The new version of the IW model is called the logarithmic transformed-inverse Weibull (LT-IW) distribution. The DF of the LT-IW distribution is obtained by using (7) in (1). Let X has the LT-IW model, if its DF is expressed by

\[ R(x; ψ) = 1 - \frac{\log\left[2 - e^{-(ψ/x^α)}\right]}{\log 2}, x \in \mathbb{R}^+, α, δ \in \mathbb{R}^+. \]

Associating to Eq. (9), the PDF \( r(x; ψ), \) SF \( R(x; ψ), \) and HF \( h(x; ψ) \) of the LT-IW model are given by
Complexity

\[ r(x; \psi) = \frac{(\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{(\log 2) [2 - e^{-(\psi/x)}]} \]

\[ R(x; \psi) = \log \left[ 2 - e^{-(\psi/x)} \right], \]

\[ h(x; \psi) = \frac{(\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{(\log 2) [2 - e^{-(\psi/x)}] [2 - e^{-(\psi/x)}]} \]

respectively.

The PDF plots of the LT-IW model are provided in Figure 1. The plots of the LT-IW model in Figure 1 are obtained for \( \alpha = 0.2, \psi = 0.5 \) (red line), \( \alpha = 3.4, \psi = 0.4 \) (green line), \( \alpha = 2.5, \psi = 0.5 \) (black line), and \( \alpha = 2.1, \psi = 1.5 \) (blue line).

2.2. A Flexible Reduced Logarithmic-Inverse Weibull Distribution. Here, we use the FRL-X approach (see (2)) to introduce a novel generalized version of the IW distribution. The new updated form of the IW distribution is called the FRL-IW distribution. The DF of the FRL-IW model is obtained by using Eq. (7) in (2). Let \( X \) has the FRL-IW distribution, if its DF is given by

\[ R(x, \psi, \beta) = 1 - \frac{\log \left[ 1 - \beta e^{-(\psi/x)} + \beta \right]}{\log (\beta + 1)}, \]

\( x \in \mathbb{R}^+, \psi, \beta \in \mathbb{R}^+ \).

Corresponding to Eq. (12), the PDF \( r(x, \psi, \beta) \), SF \( R(x, \psi, \beta) \), and HF \( h(x, \psi, \beta) \) of the FRL-IW model are given by

\[ r(x, \psi, \beta) = \frac{\beta (\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{[\log (1 + \beta)]^{-1}} \]

\[ R(x, \psi, \beta) = \log \left[ 1 + \beta - e^{-(\psi/x)} \right], \]

\[ h(x, \psi, \beta) = \frac{\beta (\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{[\log (1 + \beta)]^{-1} \left[ 1 + \beta - e^{-(\psi/x)} \right]} \]

respectively.

Different plots of \( r(x, \psi, \beta) \) of the FRL-IW distribution are presented in Figure 2. The plots of \( r(x, \psi, \beta) \) in Figure 2 are obtained for \( \alpha = 1.2, \psi = 0.7, \beta = 1.2 \) (red line), \( \alpha = 3.4, \psi = 0.4 \) (green line), \( \alpha = 2.5, \psi = 0.9, \beta = 2.8 \) (black line), and \( \alpha = 3.1, \psi = 0.3, \beta = 0.9 \) (blue line).

2.3. The Weigted TX-Inverse Weibull Distribution. In this section, we apply the WT-X distribution approach to propose a modified version of the IW distribution, called the weighted TX-inverse Weibull (WT-X-IW) distribution. The DF of the WT-X-IW distribution is obtained by using Eq. (7) in (3). Let \( X \) has the WT X-IW model, if its DF is

\[ R(x; \psi) = 1 - \frac{1 - e^{-(\psi/x)}}{e^{-(\psi/x)}}, \ x \in \mathbb{R}^+, \alpha, \psi \in \mathbb{R}^+. \]

In link to (15), the PDF \( r(x; \psi) \), SF \( R(x; \psi) \), and HF \( h(x; \psi) \) of the WT X-IW model are given by

\[ r(x; \psi) = [2 - e^{-(\psi/x)}] \frac{(\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{e^{-(\psi/x)}}, \]

\[ R(x; \psi) = \frac{1 - e^{-(\psi/x)}}{e^{-(\psi/x)}}, \]

\[ h(x; \psi) = [2 - e^{-(\psi/x)}] \frac{(\alpha \psi/x^{\alpha+1}) e^{-(\psi/x)}}{1 - e^{-(\psi/x)}} \]

respectively.
Some possible plots for the PDF of the WTX-IW model are sketched in Figure 3. The plots in Figure 3 are sketched for $\alpha = 1.5, \psi = 1.8$ (red line), $\alpha = 1.2, \psi = 2.7$ (green line), $\alpha = 1.2, \psi = 4.4$ (black line), and $\alpha = 1.4, \psi = 0.8$ (blue line).

### 2.4. A New Generalized-Inverse Weibull Distribution

In this section, we incorporate a NG-X method and introduce another extended form of the IW distribution. The new extended form of the IW model is called the NG-IW (new generalized-inverse Weibull) model. The DF of the NG-IW model is obtained by using Eq. (7) in (5). Let X has the NG-IW model, if its DF is given by

$$ R(x; \vartheta, \psi) = 1 - \left[ 1 - e^{-\left( \psi/x \right)^{\vartheta}} \right]^{\psi}, \quad x \in \mathbb{R}^+, \vartheta, \psi \in \mathbb{R}^+. $$

(18)

In link to (8), the PDF $r(x; \vartheta, \psi)$, SF $R(x; \vartheta, \psi)$, and HF $h(x; \vartheta, \psi)$ of the NG-IW model are, respectively, given by

$$ r(x; \vartheta, \psi) = \frac{\alpha \psi e^{-\left( \psi/x \right)^{\vartheta}}}{x^{\vartheta+1}} \left[ 1 - e^{-\left( \psi/x \right)^{\vartheta}} \right]^{\vartheta-1} \left[ (1 + \vartheta) - e^{-\left( \psi/x \right)^{\vartheta}} \right], $$

$$ R(x; \vartheta, \psi) = \frac{1 - e^{-\left( \psi/x \right)^{\vartheta}}}{e^{-\left( \psi/x \right)^{\vartheta}}}, $$

$$ h(x; \vartheta, \psi) = \frac{\alpha \psi e^{-\left( \psi/x \right)^{\vartheta}}}{x^{\vartheta+1}} \left[ 1 - e^{-\left( \psi/x \right)^{\vartheta}} \right] \left[ (1 + \vartheta) - e^{-\left( \psi/x \right)^{\vartheta}} \right]. $$

(19)

Some possible PDF $r(x; \vartheta, \psi)$ plots of the NG-IW distribution are sketched in Figure 4. The plots of $r(x; \vartheta, \psi)$ in Figure 4 are sketched for $\alpha = 1.5, \psi = 1.8$ (red line), $\alpha = 1.2, \psi = 2.7$ (green line), $\alpha = 3.4, \psi = 4.7, \theta = 0.5$ (green line), $\alpha = 2.7, \psi = 1.2, \theta = 0.8$ (black line), and $\alpha = 3.5, \psi = 6.7, \theta = 0.1$ (blue line).

### 2.5. The Alpha Power-Transformed Extended-Inverse Weibull Distribution

This section offers another new extension/generalization of the IW model called the alpha power transformed extended-inverse Weibull (APTE-IW) model. The DF of the APTE-IW distribution is obtained by using Eq. (7) in (6). Let X has the APTE-IW model, if its DF is

$$ R(x; \alpha_1, \psi) = \frac{(1 - e^{-\left( \psi/x \right)^{\alpha_1}})^{\alpha_2} - 1}{\alpha_1 - 1}, $$

$$ x \in \mathbb{R}^+, \alpha_1 \neq 1, \alpha_1, \alpha_2, \delta \in \mathbb{R}^+. $$

(21)

In link to (21), the PDF $r(x; \alpha_1, \psi)$, SF $R(x; \alpha_1, \psi)$, and HF $h(x; \alpha_1, \psi)$ of the APTE-IW model are given by

$$ r(x; \alpha_1, \psi) = \frac{(\log \alpha) \alpha \psi x^{\alpha_1} e^{-\left( \psi/x \right)^{\alpha_1}}}{\alpha_1 - 1} \left( \frac{1 - e^{-\left( \psi/x \right)^{\alpha_1}}}{\alpha_1 - 1} \right)^{\alpha_2}, $$

$$ R(x; \alpha_1, \psi) = \frac{\alpha_1 - \alpha_2}{\alpha_1} \left( \frac{1 - e^{-\left( \psi/x \right)^{\alpha_1}}}{\alpha_1 - 1} \right)^{\alpha_2}, $$

$$ h(x; \alpha_1, \psi) = \frac{(\log \alpha) \alpha \psi x^{\alpha_1} e^{-\left( \psi/x \right)^{\alpha_1}}}{\alpha_1 - 1} \left( \frac{1 - e^{-\left( \psi/x \right)^{\alpha_1}}}{\alpha_1 - 1} \right)^{\alpha_2}, $$

respectively.

Some possible PDF $r(x; \alpha_1, \psi)$ plots of the APTE-IW distribution are provided in Figure 5. The plots of $r(x; \alpha_1, \psi)$ in Figure 5 are sketched for $\alpha = 1.2, \psi = 2.1, \theta = 2.2$ (red line), $\alpha = 1.5, \psi = 0.8, \theta = 3.2$ (green line), $\alpha = 1.7, \psi = 1.2, \theta = 3.8$ (black line), and $\alpha = 1.8, \psi = 2.1, \theta = 2.1$ (blue line).
3. Data Analysis

Here, we demonstrate the applicability of the updated versions of the IW distribution. All the proposed updated versions of the IW distribution are applied to a data set concerned with the COVID-19 pandemic. These data are recorded between March 4, 2022, and July 20, 2020 [38].

The considered data set has one hundred eight observations and is given by 1.041, 1.205, 1.402, 1.800, 1.815, 1.923, 2.058, 2.065, 2.070, 2.077, 2.326, 2.352, 2.438, 2.500, 2.506, 2.601, 2.926, 2.988, 3.027, 3.029, 3.215, 3.218, 3.219, 3.228, 3.233, 3.257, 3.286, 3.298, 3.327, 3.336, 3.359, 3.395, 3.440, 3.499, 3.537, 3.632, 3.751, 3.778, 3.922, 4.089, 4.120, 4.292, 4.344, 4.424, 4.557, 4.648, 4.661, 4.697, 4.730, 4.909, 4.949, 5.143, 5.242, 5.317, 5.392, 5.406, 5.442, 5.459, 5.854, 5.985, 6.015, 6.105, 6.122, 6.140, 6.182, 6.327, 6.370, 6.412, 6.535, 6.656, 6.697, 6.814, 6.968, 7.151, 7.260, 7.267, 7.486, 7.630, 7.840, 7.854, 7.903, 8.108, 8.325, 8.551, 8.696, 8.813, 8.826, 9.284, 9.391, 9.550, 9.935, 10.035, 10.043, 10.158, 10.383, 10.685, 10.855, 11.665, 12.042, 12.878, 13.220, 14.604, 14.962, and 16.498.

The summary values of the COVID-19 data are given by minimum = 1.041, maximum = 16.498, range = 15.457, mean = 5.822, variance = 10.56173, standard deviation = 3.249882, skewness = 0.9732453, 1st quartile = 3.289, 2nd quartile or median = 5.279, 3rd quartile = 7.594, interquartile range = 4.305, and kurtosis = 3.666136. Furthermore, some summary plots of the data set are presented in Figure 6.

Here, we consider four frequently used analytical measures (statistical tests or statistical procedures) to show which probability distribution better fits the biomedical data. These measures are given by the following:

(i) The AIC:

\[
AIC = 2p - 2\pi(v). \tag{24}
\]

(ii) The BIC:

\[
BIC = p \log(m) - 2\pi(v). \tag{25}
\]

(iii) The CAIC:

\[
CAIC = \frac{2mp}{m - p - 1} - 2\pi(v). \tag{26}
\]

(iv) The HQIC:

\[
HQIC = 2p \log(\log(m)) - 2\pi(v). \tag{27}
\]

In a general sense, the above-mentioned analytical measures are used for comparative analysis. A statistical model that has smaller values of the statistical tests is considered the most suitable model among other competing statistical models.

Table 1 gives the MLEs (\(\hat{\alpha}_{MLE}, \hat{\psi}_{MLE}, \hat{\theta}_{MLE}, \hat{\alpha}_{1,MLE}\)) of the competitive probability models using the COVID-19 data set. The analytical measures for the COVID-19 data using the considered probability models are presented in Table 2.

Based on the reported results in Table 2, it is obvious that the NG-IW model provides the best fit to the biomedical data. For the NG-IW model, the values of the considered test statistics are AIC = 531.7657, CAIC = 532.0010, BIC = 539.7561, and HQIC = 535.0043. Based on the numerical results in Table 2, the second appropriate model is the FRL-IW distribution. For the FRL-IW model, we have AIC = 546.7329, CAIC = 546.9682, BIC = 554.7232, and HQIC = 549.9714. The 3rd best model is the LT-IW distribution. For the LT-IW model, AIC = 549.0155, CAIC = 549.1321, BIC = 554.3424, and HQIC = 551.1746. The 4th best model is the WTX-IW distribution. For the WTX-IW model, AIC = 551.7866, CAIC = 551.9032, BIC = 557.1135, and HQIC = 553.9457. The 5th best model is the APTE-IW distribution. For the APTE-IW model, AIC = 553.3800, CAIC = 553.6043, BIC = 561.5085, and HQIC = 556.6775.

As we have seen that the NG-IW model provides a close fit to the biomedical data. Therefore, we provide the profiles of the log-likelihood function (LLF) of the NG-IW distribution. Based on the \(\hat{\alpha}_{MLE}, \hat{\psi}_{MLE}, \) and \(\hat{\theta}_{MLE},\) the LLF profiles of the NG-IW distribution are obtained in Figure 7. The graphs in Figure 7 confirm the unique values of the \(\hat{\alpha}_{MLE}, \hat{\psi}_{MLE},\) and \(\hat{\theta}_{MLE}.\)

After the numerical illustration of the NG-IW model using the COVID-19 data set (see Table 2), next we show visually that the NG-IW model provides the best fit to the COVID-19 data set. For the visual illustration of the NG-IW model, the plots of the fitted PDF \(r(x; \hat{\theta}, \hat{\psi}),\) DF \(R(x; \hat{\theta}, \hat{\psi}),\) SF \(\hat{R}(x; \hat{\theta}, \hat{\psi}),\) HF \(h(x; \hat{\theta}, \hat{\psi}),\) cumulative HF \(H(x; \hat{\theta}, \hat{\psi}),\) probability-probability (PP), and QQ (quantile-quantile) are obtained in Figure 8. The plots of \(r(x; \hat{\theta}, \hat{\psi}), R(x; \hat{\theta}, \hat{\psi}), \hat{R}(x; \hat{\theta}, \hat{\psi}), h(x; \hat{\theta}, \hat{\psi}),\) and \(H(x; \hat{\theta}, \hat{\psi})\) are obtained using the following expressions:
Figure 6: Some summary plots of the COVID-19 data set.

Table 1: The values of the maximum likelihood estimators of the fitted models using the COVID-19 data set.

| Distributions | $\tilde{\alpha}$ | $\tilde{\psi}$ | $\tilde{\beta}$ | $\tilde{\theta}$ | $\tilde{\alpha}_1$ |
|---------------|------------------|-----------------|-----------------|-----------------|------------------|
| LT-IW         | 1.829195         | 8.721454        | —               | —               | —                |
| FRL-IW        | 2.291612         | 8.003594        | 11.582096       | —               | —                |
| WTX-IW        | 1.339224         | 9.164487        | —               | —               | —                |
| NG-IW         | 0.705908         | 8.586715        | —               | 9.655237        | —                |
| APTE-IW       | 0.633391         | 8.119927        | —               | —               | 12.65028         |

Table 2: The values of the analytical measures of the fitted models using the COVID-19 data set.

| Distributions | AIC        | CAIC       | BIC        | HQIC       |
|---------------|------------|------------|------------|------------|
| LT-IW         | 549.0155   | 549.1321   | 554.3424   | 551.1746   |
| FRL-IW        | 546.7329   | 546.9682   | 554.7232   | 549.9714   |
| WTX-IW        | 551.7866   | 551.9032   | 557.1135   | 553.9457   |
| NG-IW         | 531.7657   | 532.0010   | 539.7561   | 535.0043   |
| APTE-IW       | 553.3800   | 553.6043   | 561.5085   | 556.6775   |

Figure 7: The profiles of the log LF of the NG-IW distribution using the COVID-19 data set.
Figure 8: Visual illustration of the NG-IW distribution using the COVID-19 data set.

\begin{align}
    r(x; \hat{\theta}, \hat{\psi}) &= \frac{6.061430 e^{-8.586715/\alpha^{0.705908}}}{\alpha^{1.705908}} \left[ 1 - e^{-8.586715/\alpha^{0.705908}} \right] 8.655237 \\
    &\times \left[ (10.655237) - e^{-8.586715/\alpha^{0.705908}} \right] \\
    &\times e^{-8.586715/\alpha^{0.705908}}, \\
    \\
    R(x; \hat{\theta}, \hat{\psi}) &= 1 - \left[ 1 - e^{-8.586715/\alpha^{0.705908}} \right] 9.655237, \\
    \\
    \bar{R}(x; \hat{\theta}, \hat{\psi}) &= \left[ 1 - e^{-8.586715/\alpha^{0.705908}} \right] 9.655237, \\
    \\
    h(x; \theta, \psi) &= \frac{6.061430 e^{-8.586715/\alpha^{0.705908}}}{\alpha^{1.705908}} \left[ 1 - e^{-8.586715/\alpha^{0.705908}} \right] (10.655237) - e^{-8.586715/\alpha^{0.705908}}, \\
    \\
    H(x; \hat{\theta}, \hat{\psi}) &= -\log \left( \frac{1 - e^{-8.586715/\alpha^{0.705908}}}{e^{-8.586715/\alpha^{0.705908}}} \right) 9.655237,
\end{align}

respectively.
The empirical and fitted plots in Figure 8 reveal that the NG-IW distribution provides a close fit to the COVID-19 data set.

4. Concluding Remarks

In recent times, statistical models have been frequently used to analyze data in applied sectors, such as engineering, hydrology, education, finance, and biomedical sectors. To provide the best description of the phenomena under consideration, a number of statistical models have been introduced and implemented. Among these models, the IW distribution has received considerable attention. Therefore, numerous modifications of the IW distribution have been proposed and applied. In this paper, we introduced five different modifications of the IW distribution for modeling real-life data sets. Finally, the new modified forms of the IW distribution were applied to real-life data taken from the biomedical sector. The practical application showed that the NG-IW distribution was the best candidate model for analyzing the COVID-19 data set.

In the future, we are motivated to implement the LT-IW, FRL-IW, WTX-IW, NG-IW, and APTE-IW models in other applied sectors. Furthermore, the bivariate extensions of the LT-IW, FRL-IW, WTX-IW, NG-IW, and APTE-IW models can also be introduced to deal with the bivariate data sets. Bayesian estimation of the LT-IW, FRL-IW, WTX-IW, NG-IW, and APTE-IW models using different types of censored samples can be discussed [39].

Data Availability

All data are included in the paper.

Conflicts of Interest

The authors declare no conflicts of interest.

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