Exploring vibration control strategies for a footbridge with time-varying modal parameters

José M. Soria, Iván M. Díaz, Emiliano Pereira, Jaime H. García-Palacios, Xidong Wang

1 Universidad Politécnica de Madrid, ETS Ingenieros de Caminos Canales y Puertos, ES 28040, Madrid, Spain.
2 Universidad de Alcalá, Escuela Politécnica Superior, ES 28805, Alcalá de Henares, Madrid, Spain.

*E-mail: ivan.munoz@upm.es

Abstract. This paper explores different vibration control strategies for the cancellation of human-induced vibration of a structure with time-varying modal parameters. The motivation of this study is an urban stress-ribbon footbridge (Pedro Gómez Bosque, Valladolid, Spain) that, after a whole-year monitoring, it has been obtained that the natural frequency of a vibration mode at approximately 1.8 Hz (within the normal range of walking) changes up to 20%, mainly due to temperature variations. Thus, this paper takes the annual modal parameter estimates (aprox. 14000 estimations) of this mode and designs three control strategies: a) a tuned mass damper (TMD) tuned to the aforementioned mode using its most-repeated modal properties, b) a semi-active TMD with an on-off control law for the TMD damping, and c) an active mass damper designed using the well-known velocity feedback control strategy with a saturation nonlinearity. Illustrative results have been reported from this preliminary study.

1. Introduction

The current trend towards lighter and slender structures has resulted in structures with less inherent damping and lower natural frequencies than in the past, which are more susceptible to excitation by human users. Examples of notable vibrations under human-induced excitations have been reported in footbridges, office buildings, shopping malls and sport stadia, amongst others structures, mainly affecting their serviceability.

Within the possible solutions to overcome vibration serviceability problems in footbridges, the inclusion of control devices to the structure seems to be the easiest way of improving the vibration performance. Among passive control devices available for implementation, tuned mass damper (TMD) based strategies are usually adopted for footbridges [1–3]. Passive control strategies are easy to design and do not require external power. However, they have relatively poor performance for low-level vibration and they exhibit a lack of performance due to the off-tuning issue. Both are well-known problems exhibited by TMDs when they have to cope with low-amplitude human-induced vibrations in pedestrian structures with time-varying modal parameters. The problem
of detuning and the subsequent loss of the efficiency of the TMDs is the main concern in their development. An adequate control strategy would be able to correct the vibration absorber parameters and bring it to the proper tuning during its operation [4]. Under these circumstances, passive mass dampers may be upgraded to semi-active or fully-active mass dampers.

In the last decades, semiactive control systems have been studied intensively since they combine the best features of both passive (stability, robustness) and active (adaptability) control systems using a low quantity of energy to operate (as compared to fully-active systems) [5, 6]. However, they still need a feedback control scheme as active systems. The control law may continuously change the TMD parameters or may be based on ON/OFF control strategies [7]. The latter option, which might be not as effective as the former, is easier to implement in practice. Semiactive TMD (STMD) with an ON/OFF variable damping is considered here. The simplified version of the phase control law for the TMD damping (realizable in practice using a magnetorheological (MR) fluid damper) obtained in [4], has been adopted in this work since the variables to be measured are more feasible and then, its practical implementation is also more feasible.

An in-service steel-plated stress-ribbon footbridge previously analyzed in [8] is considered in this work as a slender structure with “lively” vibration behavior. The influence of the environmental factors on modal parameter estimation was thereof extensively studied [8]. It was found that this structure has a vibration mode at approximately 1.8 Hz (within the normal range of walking) which is the main one into the dynamic response of the structure. Through a peered analysis of a whole-year monitoring, it was obtained that this mode changes up to 20% during the year with both seasonal and daily trends and that these changes were mainly explained by temperature variation. Thus, this paper explores advanced control techniques able to deal with a frequency-changing vibration mode and assesses their performance as compared with classical TMDs. More precisely, the performance of active mass damper (AMD) based on the use of a commercial electrodynamics shaker and STMD based on the use of a MR damper is studied. Velocity feedback control is used for the AMD control law and an on-off control law is used to activate/deactivate the MR damper of the STMD.

The paper continues in Section 2 with the description of the structure and its monitoring system. Section 3 explains the design of the three vibration control strategies explored in this work and discusses on the results. Finally, some conclusions are drawn and suggestions for future work are given.

2. The stress-ribbon bridge

2.1. Structure description and its monitoring

Pedro Gómez Bosque footbridge, sited in Valladolid (Spain), is a slender and lightweight structure that creates a pedestrian link over the Pisuerga River between a sport complex and the city centre (see Figure 1). This bridge, built in 2011, is a singular stress-ribbon footbridge of 85 m span born by a pre-tensioned catenary-shape steel band (of only 30 mm thick) and precast concrete slabs lying on the band [9].

A structural vibration monitoring system was devised in order to continuously estimate the modal parameters of the structure and to assess their changes under varying environmental conditions. Therefore, apart from the accelerometers needed to perform a modal analysis, sensors for the wind and environmental temperature conditions were installed. The monitoring system comprises 18 triaxial accelerometers, 9 at each side of the deck, a temperature sensor and an anemometer with a vane (see Figure 1). Wires and acceleration sensors were installed inside the handrail so the structure aesthetic was not modified in any way. The monitoring system
2.2. Tracking of vibration modes

Tests of 20 min. and 200 Hz as sampling frequency were considered for the tracking following the method described in [8]. The natural frequency estimates for the more lingering modes over 1-year of continuous dynamic monitoring were obtained. Up to nine vibration modes below 4 Hz were tracked. Figure 2 shows the tracked natural frequency estimates, in which some occasional stops due to technical problems are observed. The covariance-driven SSI method (SSI-cov) was used for the operational modal analysis estimation [11, 12]. Table 1 shows the following statistics of the estimation: mean, standard deviation, absolute percentage variation and their repeatability (the success ratio is included between brackets). Note that the fourth mode at 1.79 Hz (with a damping ratio of only 0.42%) corresponds to the highest success ratio (for a particular mode, the ratio of successful estimates to the total number of estimates). That is, this is the most-repeated mode since it is the most important in the structure service response under human-induced vibrations.

2.3. Structure model

Usually, to assess the vibration serviceability, resonant conditions are assumed and the Single-Degree-of-Freedom (SDOF) approach can be adopted from desing guidelines [13]. This approach is adopted here as a first study of the problem. The fourth mode of Table 1 is the critical mode for the vibration serviceability assessment, so this mode is selected for the model of the structure. A seasonal and daily trends with temperature were identified for this mode [8]. Figure 3a shows the frequency estimates versus temperature. For this mode, increasing temperature leads to an increase into the ribbon sag producing a reduction of band tension and leading to a decrease in the natural frequency [14]. Regarding damping ratios, no clear visual dependencies with any external factor was found (see Figure 3b). Figure 4 shows the distribution of the natural frequency and damping ratio estimates for the whole year.

The SDOF model corresponding to mode 4 is considered for the structure model. The
Figure 2. Tracked natural frequency estimates for the whole year.

Figure 3. Estimates, frequency (a) and damping (b), versus temperature for mode 4.
Table 1. Summary of identified natural frequencies and damping ratios for one year monitoring and their statistics: mean frequency ($\bar{f}$), mean damping ($\bar{\zeta}$), standard deviation (Std) and the corresponding variation ($\nu$).

| Mode | Frequency | Damping | Repeatability |
|------|-----------|---------|---------------|
|      | $f$ (Hz)  | Std (Hz) | $\zeta$ (%)   | Std (%)  | $\nu$ (%) | Std (%)  | $\nu$ (%) | Repeatability |
| 1    | 1.0482    | 0.0152   | 0.3665        | 0.1710   | 147.89    | 9667     | (44.7%)    |
| 2    | 1.4145    | 0.0107   | 0.3381        | 0.1513   | 110.74    | 10619    | (49.1%)    |
| 3    | 1.5440    | 0.0181   | 0.6498        | 0.2357   | 133.62    | 9886     | (45.7%)    |
| 4    | 1.7937    | 0.0291   | 0.4192        | 0.1502   | 221.88    | 13817    | (63.8%)    |
| 5    | 1.8594    | 0.0168   | 0.5718        | 0.1605   | 234.74    | 9936     | (45.9%)    |
| 6    | 2.3117    | 0.0425   | 0.3753        | 0.1474   | 128.54    | 8746     | (40.4%)    |
| 7    | 3.3821    | 0.0549   | 0.3868        | 0.1191   | 103.96    | 12210    | (56.4%)    |
| 8    | 3.5512    | 0.0524   | 0.7226        | 0.1884   | 157.48    | 9237     | (42.7%)    |
| 9    | 3.9610    | 0.0624   | 0.3853        | 0.1185   | 230.82    | 10183    | (57.8%)    |

Figure 4. Distribution density. The line shows the normal distribution with the same mean and standard deviation as the original distribution density.

structure mode will be denoted as mode 1, since it is the only mode considered for the primary structure. Thus, the Transfer Function (TF) between the structure acceleration and the force is as follows:

$$G(s) = \frac{\alpha_1 s^2}{s^2 + 2 \zeta_1 \omega_1 s + \omega_1^2},$$

in which $s = j \omega$, $\omega = 2\pi f$ being the circular frequency (rad/s) and $f$ the natural frequency (Hz), $\alpha_1 = 1/m_1$ the inverse of the modal mass, $\omega_1$ is the circular frequency and $\zeta_1$ is the damping ratio. A modal mass of $m_1 = 50,000$ kg was estimated from a Finite Element study [15] and $\omega_1$ and $\zeta_1$ are those estimate for the whole-year. Figure 5 shows the 13817 models considered for the structure. A model with natural frequency of 1.82 Hz and damping ratio of 0.37% has
been adopted as nominal model using the most-repeated modal properties (see Figure 4). It can be seen in Figure 4a that the frequency does not fit to a normal distribution, but a pattern of behavior is sensed by the correlation with temperature.

3. Vibration control strategies

3.1. Loading cases

The force used to evaluate the vibration control strategies has been a chirp input whose frequency increases at a linear range with time, that is

\[ F(t) = F_0 \cdot \sin(2\pi f(t)), \]

with \( f(t) \in (1.4, 2.2) \) Hz, \( F_0 = \{400, 1500\} \) N and \( t \in [0, 300] \) s. \hspace{1cm} (2)

The frequency range has been chosen in order to excite with sufficient frequency margin all the possible natural frequencies (see Figures 4 and 5). The two different force values (400,1500) N are representative of walking and running force dynamic load factors, respectively. The final time of 300 s was chosen so that steady state behavior at each frequency was reached.

3.2. Passive control: Tuned Mass Damper

A TMD consist of a secondary mass (also called moving or inertial mass) attached to the structure (main mass) by means of springs and dampers. The TMD mass is fixed as a fraction of the modal mass (mass ratio denoted as \( \mu \)) of the targeted vibration mode; the stiffness of the springs is
selected to obtain the optimum TMD frequency, and the viscous dampers ensure the operation of the TMD in a range of frequencies around the tuning frequency. Energy is dissipated due to the energy transferring from structure to TMD. However, they have relatively poor performance for low-level vibration and they exhibit a lack of performance due to the off-tuning issue. The detuning of TMDs can be due to uncertainties into the modal parameters of the structure, as it is the case studied here.

Figure 6 shows the mechanical model of a TMD which is composed of an inertial mass $m_2$ attached to a primary system by means of a spring of constant $k_2$ and a viscous damper of constant $c_2$. The primary system is the structure to be controlled and is composed of a mass $m_1$, a spring of constant $k_1$ and a viscous damper of constant $c_1$.

The TMD has been designed using the Assami and Nishihara [16] formulas based on an $H_\infty$ norm approach for vanishing structural damping ($\zeta_1 \approx 0$). TMD parameters $k_2$ and $c_2$ for a given TMD mass $m_2$ are derived by minimising the $H_\infty$-norm of the TF between the structure acceleration $\ddot{x}_1$ and the force $F(t)$. That is,

$$\eta = \sqrt{\frac{1}{1+\mu}}$$  \hspace{1cm} (3)$$

$$\zeta_2 = \sqrt{\frac{3\mu}{8(1+\mu)}} \sqrt{1 + \frac{27}{32}\mu},$$  \hspace{1cm} (4)

where $\mu = m_2/m_1$ is the mass ratio, which is assumed to be 1\% ($m_2 = 500$ kg), $\eta = f_2/f_1$ is the natural frequency ratio of TMD and primary structure and the stiffness and damping ratio for the TMD are obtained from $k_2 = (2\pi f_2)^2 m_2$ and $c_2 = 2\zeta_2 m_2 (2\pi f_2)$, respectively.
3.3. Semi-active control: Semi-Active Tuned Mass Damper

It is assumed a STMD based on a MR damper, which can be used to update continuously the damping force depending on structure and TMD movement. Phase control for the TMD damping is considered [17]. The adapted version from [17] proposed by Moutinho [4] has been used since this is clearly geared to practical implementation due to the variables needed: the structure acceleration instead of displacement and the TMD mass velocity instead of the relative velocity, as usual [7]. The control law achieves a phase lag between the control force (the force coming from the TMD) and structure displacement close to 90° even in situation of significant detuning.

The control law adopted is of ON/OFF type due to its simplicity. Thus, the adopted control law is defined as follows [4]:

\[
\begin{cases}
\ddot{x}_1 \cdot \dot{x}_2 \leq 0 & \Rightarrow c_2 = c_{\text{min}} \\
\ddot{x}_1 \cdot \dot{x}_2 > 0 & \Rightarrow c_2 = c_{\text{max}}
\end{cases}
\]

in which \( c_{\text{max}} = 50 \cdot c_{\text{min}} \), \( c_{\text{min}} \) is the optimal damping obtained from Equation (4), \( \ddot{x}_1 \) is the structure acceleration (obtain by an accelerometer) and \( \dot{x}_2 \) is the absolute velocity of the TMD mass (easy to be obtained from the integration of an accelerometer signal installed on the TMD mass).

3.4. Active control: Active Mass Damper

A proof-mass actuator generates inertial forces in the structure on which it is placed without the need for a fixed reference. The TF between the inertial force applied to the structure \( F_A(t) \) and the input voltage \( V(t) \) can be closely described as a linear second-order system as follows [18]:

\[
G_A(s) = \frac{K_A s^2}{s^2 + 2 \zeta_A \omega_A s + \omega_A^2},
\]

in which \( K_A \) is the actuator gain, \( \omega_A = 2\pi \cdot 0.9 \text{ rad/s} \) is the natural frequency associated with the suspended moving mass and \( \zeta_A = 0.5 \) is the damping ratio (including electrical and mechanical effects). The natural frequency \( \omega_A \) must be sufficiently below the first natural frequency of the structure \( \omega_1 \) (see Equation (1)), in such a way that the phase distortion introduced by the proof-mass actuator does not affect significantly the efficacy of the AVC system at the structure frequency. Typically, it is recommended that \( \omega_A \) less than half of \( \omega_1 \) [19]. This model corresponds to a commercial shaker (APS Dynamics Model 400 electrodynamic shaker) whose moving mass is of \( m_A = 30.4 \text{ kg} \) and its maximum force is of 400 N. Note that the resonance of the shaker can be adapted to the structure necessities through the use of a local (or inner) feedback control as it was shown in [20].

The feedback control scheme considered is shown in Figure 7. In this case, velocity feedback control has been adopted together with a saturation nonlinearity included to avoid actuator overloading [21]. That is, the initial control voltage is clipped up to ±2 V. The compensator is an integrator and a control gain as follows

\[
C(s) = \frac{K_c}{s},
\]

with \( K_c = 1174 \) is the control gain obtained from the root locus method (see Figure 8) for the nominal structure model in such a way that the damping of the closed-loop poles corresponding to the actuator are approximately \( \zeta_A = 0.2 \), which avoid stroke saturation due to low frequency noise. Note that if the value of \( K_c \) is increased, the imparted damping to the structure is higher,
Figure 7. Active control scheme.

Figure 8. Root locus. × open loop poles, ■ closed loop poles.

however, the control loop system is more sensitive to stroke saturation for low frequency noise (see the root locus of Figure 8).

4. Discussion and conclusions
The results are compared in terms of the Maximum Transient Vibration Value (MTVV) calculated from the 1 s running root-mean-squared acceleration for each loading test.

Figures 9 and 10 shows the MTVV obtained for the uncontrolled case and the three control
strategies using the 13817 models for both loading cases. As it is observed, the STMD always improves results as compared to the TMD even for the nominal model for which the TMD was designed, being the results very similar in that case. It is also observed that the AMD considered (moving mass of 30.5 kg, the TMD and STMD mass was of 500 kg) can cope with low-amplitude vibration (chirp of 400 N) very efficiently; however, it is not able to cope with vibrations coming from a stronger force (chirp of 1500 N). The amplitude level of MTVV obtained is directly proportional to the damping of the structure for each case (see Figures 9b and 10b).

Finally, Figure 11 shows the percentage of reduction distribution for the whole year and for both loading cases. Again, the STMD performances better than the TMD: the reduction is higher and the distribution band is narrower so that its behavior is low-dependent on the structure model. The AMD for the 400-N force is totally independent of the structure model and is able to reduce up to 98% of the uncontrolled case. However, for the case of 1500-N force, the AMD is not able to cancel them due to its limitation in the maximum force.

A preliminary study has been carried out in order to explore different control strategies. This study will help in making a decision about the more convenient control technique to be considered for a future implementation. The following conclusions can be extracted:

- The TMD, even though it performs quite well, its performance degrades significantly
Figure 10. Results for 1500-N chirp amplitude.

when model frequencies move away from the nominal model.

• The STMD has been shown to be quite insensitive to its initial tuning and able to cancel effectively the vibration independent of the modal properties of the vibration mode over the whole year.

• The commercial shaker considered as AMD (30 kg moving mass) is extremely effective for low amplitude vibration but it is not able to cope with large amplitude due to extreme loading, since maximum control force delivered by it is not able to cope with large amplitudes due to extreme loading.

Future work will consider the inclusion of several vibration modes of the structure at the control location and the analysis of the vibration control strategies.

5. Acknowledgements

The authors acknowledge the financial support provided by the Ministry of Economy and Competitiveness of the Government of Spain to make this work possible by funding from the REVES-P Research Project, with reference DPI2013-47441. This work is also supported by project SETH of INNPACTO Program, with reference IPT-2012-0703-380000.
Figure 11. Distribution of % of vibration reduction for the three strategies.

References

[1] E. Caetano, Á. Cunha, C. Moutinho, and F. Magalhães. Studies for controlling human-induced vibration of the Pedro e Inês footbridge, Portugal. Part 2: Implementation of tuned mass dampers. *Engineering Structures*, 32(4):1082–1091, 2010.

[2] C. Moutinho, Á. Cunha, and E. Caetano. Analysis and control of vibrations in a stress-ribbon footbridge. *Structural Control and Health Monitoring*, 2010.

[3] C. M. Cassado, I.M. Díaz, J.D. Sebastián, A.V. Poncela, and A. Lorenzana. Implementation of passive and active vibration control on an in-service footbridge. *Structural Control and Health Monitoring*, 20:70–87, 2013.

[4] C. Moutinho. Testing a simple control law to reduce broadband frequency harmonic vibrations using semi-active tuned mass dampers. *Smart Materials and Structures*, 24(5):055007, 2015.

[5] M. Setareh, J.K. Ritchey, T.M. Murray, J.H. Koo, and M. Ahmadian. Semiactive tuned mass damper for floor vibration control. *Journal of Structural Engineering*, 242(113):242–250, 2007.

[6] F. Weber. Dynamic characteristics of controlled mr-stmds of wolgograd bridge. *Smart Materials and Structures*, 22(095008):16pp, 2013.

[7] J.H. Koo, M. Ahmadian, M. Setareh, and T. Murray. In Search of Suitable Control Methods for Semi-Active Tuned Vibration Absorbers. *Journal of Vibration and Control*, 10(2):163–174, 2004.

[8] J.M. Soria, I.M. Díaz, J.H. García-Palacios, and N. Ibán. Vibration monitoring of a steel-plated stress-ribbon footbridge: uncertainties in the modal estimation. *Journal of Bridge Engineering*, 108(5):1175–1183, 2016.

[9] A.J. Narros. Pasarela Peatonal “Pedro Gómez Bosque” sobre el Río Pisuerga en la Ciudad de Valladolid. Un Nuevo Récord de Longitud en Pasarelas Colgadas de Banda Tesa. *Revista Técnica Cemento-Hormigón*,
947:80–86, 2011. (In Spanish).

[10] J. de Sebastián, A. Escudero, R. Arnaz, I.M. Díaz, A. Poncela, and A. Lorenzana. A low-cost vibration monitoring system for a stress-ribbon footbridge. In 6th ECCOMAS Conference on Smart Structures and Materials, 2013.

[11] P. Van Overschee and B.D. Moor. Subspace Identification for Linear Systems. Boston: Kluwer Academic, 1996.

[12] B. Peeters and G. De Roeck. Reference-Based Stochastic Subspace Identification for Output-Only Modal Analysis. Mechanical Systems and Signal Processing, 13(6):855–878, November 1999.

[13] F. du béton and I.F.S. Concrete. Guidelines for the Design of Footbridges: Guide to Good Practice. Bulletin (fib Fédération internationale du béton). International Federation for Structural Concrete, 2005.

[14] D. Cobo del Arco, A.C. Aparicio, and A.R. Mari. Preliminary design of prestressed concrete stress ribbon bridge. Journal of Bridge Engineering, 6:234–242, August 2001.

[15] J. Castaño, O. Cosido, J. Pereda, M. Cacho-Pérez, and A. Lorenzana. Static, modal and dynamic behaviour of a hanging footbridge: experimental and computational results. In Third International Conference on Mechanical Models in Structural Engineering (CMMoST 2015), 2015.

[16] T. Asami and O. Nishihara. Closed-Form Exact Solution to $H_\infty$ Optimization of Dynamic Vibration Absorbers (Application to Different Transfer Functions and Damping Systems). Journal of Vibration and Acoustics, 125(3):398, 2003.

[17] L. Chung, Y. Lai, and C.W. Yang. Semi-active tuned mass dampers with phase control. Journal of Sound and Vibration, 332(15):1–16, 2013.

[18] A. Preumont. Vibration Control of Active Structures. An Introduction. Kluw Academic Publishers, 2002.

[19] I.M. Diaz and P. Reynolds. Robust saturated control of human-induced floor vibrations via a proof-mass actuator. Smart Materials and Structures, 18(12):125024, 2009.

[20] I.M. Diaz, E. Pereira, M.J. Hudson, and P. Reynolds. Enhancing active vibration control of pedestrian structures using inertial actuators with local feedback control. Engineering Structures, 41:157–166, 2012.

[21] I.M. Diaz and P. Reynolds. On-off nonlinear active control of floor vibrations. Mechanical Systems and Signal Processing, 24(6):1711–1726, 2010.