Initialisation of a hybrid AC/DC power system for harmonic stability analysis using a power flow formulation

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Abstract: This study presents the initialisation of a hybrid alternating current/direct current (AC/DC) power system to determine the operation point around which the system is linearised as a starting point for an electromagnetic or harmonic stability analysis. To this end, each AC or DC power system component is modelled in the Fourier transform domain and further adjusted to a simplified representation compatible with a power flow formulation. The study also presents how voltage source converter (VSC) high-voltage DC-based systems can be analysed when the different VSC controls are applied. The result of the power flow solution is then used for the power converters’ initialisation.

1 Introduction

Our power system more and more takes the form of a hybrid alternating current/direct current (AC/DC) network. This is caused by increased penetration of new installations of high-voltage DC (HVDC), in addition to a rise in converter-interfaced renewable energy sources (e.g. large wind farms). These changes have introduced the need for more accurate modelling and simulation of the entire network. As the name suggests, the resulting hybrid network consists of both the AC and DC systems interfaced with power-electronic converters. These devices take the form of line commutated converters (LCCs) or the currently prevailing voltage source converters (VSCs). For HVDC applications, the voltage source technology can either be a simple two-level or three-level topology or the more recently developed modular multilevel converter (MMC) [1–3].

The increased deployment of power electronics in the transmission system has given rise to new types of stability problems. These are commonly referred to as ‘electromagnetic stability’ [4] or ‘harmonic stability’. A detailed description of the frequency-dependency of traditional power system components along with a detailed model of the converter's circuit equations and control loop layout is needed to analyse the problem. At present, electromagnetic or harmonic stability analyses are typically carried out by considering either the AC or DC side [5] and by focusing on relatively small surrounding areas in the AC power system [6, 7]. Different approaches can be used to perform such a stability assessment, but the impedance-based method is amongst the most commonly applied ones [6–8]. However, to enable an assessment of the impact of AC characteristics on the DC side stability and vice versa, it is necessary to consider the entire network in the analysis and not limit the analysis to one subsystem or a smaller part thereof, as it is commonly done. Since the actual stability analysis relies on Fourier transforms, a first step in the analysis is to determine the operating point of the entire hybrid AC/DC system to perform a linearisation of the non-linear system components (e.g. the converters and their control loops).

The introduction of power electronics in the system complicates the computation of operating states of the power system in general. Over the past few decades, there has been a constant need for conceiving new power flow (PF) models and problem formulations that account for hybrid AC/DC systems; first for LCC HVDC [9] and later for VSC HVDC. As an example, the MATLAB-based tool Matpower [10] was improved by adding converters, their overall control capabilities, as well as a DC power system analysis in [11]. Similarly, optimal PF (OPF) formulations started to include hybrid AC/DC systems. For example, in [12], the hybrid AC/DC system description from [11] was incorporated into an OPF description for AC systems in the Julia programming language [13]. In addition, alternative formulations have been presented in the literature including shunt conductances within the HVDC grids [14].

The initialisation of standard AC systems for dynamic simulations using a PF has been the subject of study in the past [15, 16] and is at present available in commercial software packages. On the contrary, the new PF and OPF algorithms for hybrid AC/DC systems have typically been targeted towards operational studies and contingency analyses. Consequently, their integration with other power system simulation tools has not received much attention before.

The presence of power converters as coupling elements between both AC and DC systems complicates the formulation of accurate initialisation procedures for a hybrid AC/DC network. Using a PF formulation for this purpose implies not only extending existing AC system PF formulations to take into account the peculiarities of DC systems. Indeed, an accurate initialisation requires that steady-state characteristics of the converter and its controls be accurately mapped to equivalent power-flow representations [17]. As a result, accurate initialisation routines are only starting to see the light of day as of recently. This is particularly true when the more complex MMC topology is concerned. For example, the design of an accurate initialisation procedure for an MMC was proposed in the work [18]. However, the presented algorithm focuses on determining the steady-state operating point of the MMC, while its interaction with the rest of the AC and DC power systems is left out of the study. An iterative AC/DC PF is used in [19] for estimating the MMC's operating point together with the system state to perform transient studies. Consequently, a steady-state phasor network model is used for the AC system [20]. The DC side network dynamics are considered, but only a simple equivalent π -section model composed of constant lumped R, L, and C parameters is used for the cable. Moreover, the approach does not consider meshed complex AC/DC networks.

This paper aims at filling this gap by introducing a detailed PF-based initialisation of complex hybrid AC/DC power systems to allow large-system electromagnetic or harmonic stability assessments using complex frequency-dependent component models. The power system contains both HVDC and AC subsystems, interconnected using state-of-the-art MMCs. The
operating point analysis starts from defining equivalent PF models of frequency-dependent passive components (such as cables and overhead lines (OHLs)) together with a simplified equivalent steady-state model of the MMC incorporating the converter's outer controlling loops. After solving the hybrid AC/DC PF, the steady-state operating conditions of the network are used as a starting point for determining the operating conditions of each individual power electronic converter, by initialising inner and outer controlling loops. The developed procedure is described in the flow chart from Fig. 1. The practical implementation of this procedure ensures a short simulation time because (i) the system's equations are decoupled, (ii) complex component models are reduced to fit a PF formulation, and (iii) it is implemented in Julia, a C-related programming language. By making abstraction of inner converter control details, while solving the operating conditions of the network, the method also provides high flexibility in defining the converter's inner and outer controlling loops.

The methodology described in this paper is implemented in the Julia programming language, which offers a wide range of functionalities and fast computation. It includes the definition of each component's PF equivalent to fit the PF formulation from [13], using the package PowerModelsACDC.jl. The MMC is implemented based on the detailed model proposed in [21] since it is the most prevailing technology for VSC HVDC applications.

The paper is organised as follows. Section 2 reviews the basics for a multiport network implementation using ABCD and Y parameters and discusses the conversions between them. The section thereby explains the conversion between the internal modelling method used to construct equivalent grid impedances for the eventual stability assessment as in [22] and the format needed for the PF analysis. Section 3 describes the PF formulation with the corresponding models for the most commonly used power system components. The proper operation of the proposed PF initialisation algorithm is demonstrated in Section 4 on a detailed hybrid AC/DC power system consisting of a multi-terminal HVDC-based power system and small-scale adjacent AC networks. Section 5 presents the concluding remarks.

1.1 Notation

Uppercase bold identifiers are used for matrices, e.g. \( X \), and lowercase bold identifiers are used for vectors, e.g. \( x \). Matrix \( I_n \) presents the \( n \times n \) identity matrix. The following definition is used for a compact set of equations:

\[
J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.
\]

For any matrix \( X \), the notation \( X(i, j) \) presents an element of the matrix at the location \( (i, j) \). The notation \( X(i, j; k:l) \) is used for a submatrix with \( (j-i) \times (l-k) \) elements formed from a portion of matrix \( X \).

The imaginary unit is \( j = \sqrt{-1} \). The Fourier transform's variable \( s \) presents \( s = j \omega \), for an angular frequency \( \omega \). \( \Re \{ x \} \) and \( \Im \{ x \} \) present real and imaginary part of the complex number \( x \in \mathbb{C} \).

To transform three-phase voltages and currents from the stationary \( abc \)-frame to the rotating \( dqz \)-frame, the Park's transformation is used

\[
P_{ao}(t) = \frac{2}{3} \begin{bmatrix} \cos (\omega t) & \cos (\omega t - \frac{2\pi}{3}) & \cos (\omega t - \frac{4\pi}{3}) \\ \sin (\omega t) & \sin (\omega t - \frac{2\pi}{3}) & \sin (\omega t - \frac{4\pi}{3}) \end{bmatrix}.
\]

with \( \omega_t \) the angular frequency. The inverse Park's transformation is given as

\[
P_{ao}(t) = \frac{3}{2} P_{ao}^T(t) + \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.
\]

2 Power system components

For the system description at the basis of the electromagnetic stability assessment, each power system component is considered as a multiport network that can be described in the Fourier domain using \( ABCD \), \( Y \), \( Z \), \( H \), or \( S \) parameters. In this work, we have chosen to employ the definition using \( ABCD \) and \( Y \) parameters since they preserve the physical dimension of the component's voltages and currents.

Assuming that power system components have the same number of input and output nodes—which is correct for all analysed three-phase AC and DC components as well as for both monopolar and bipolar DC power network configurations—each component can be represented using \( ABCD \) parameters as

\[
\begin{bmatrix} V_p \\ I_p \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} V_s \\ I_s \end{bmatrix}.
\]

In (3), the current and voltage vectors at the component input (or primary nodes) have superscript ‘\( p \)’, while the output currents and voltages have superscript ‘\( s \)’. Matrices \( A, B, C, \) and \( D \) represent the interconnection between input and output variables (currents and voltages) specific for the power system component. If the number of input/output nodes of the component is \( n \), then these matrices have sizes \( n \times n \).

Similarly, the \( Y \) parameters for the same component can be defined as

\[
\begin{bmatrix} I_p \\ V_p \end{bmatrix} = Y \times \begin{bmatrix} I_s \\ V_s \end{bmatrix}
\]

and

\[
Y = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix}.
\]

The \( Y \) parameters in the previous form can be obtained from \( ABCD \) parameters as follows [23]:

\[
\begin{bmatrix} I_p \\ V_p \end{bmatrix} = Y \times \begin{bmatrix} I_s \\ V_s \end{bmatrix}.
\]
Similarly, the \( ABCD \) parameters can be determined from \( Y \) parameters in a unified manner as

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
  -Y_{ss} & -Y_{sp}^{-1} \\
  Y_{ps} - Y_{pp}^{-1} Y_{ss} & -Y_{pp}^{-1}
\end{pmatrix}.
\]

(6)

3 PF definition

To perform an electromagnetic or harmonic stability analysis, non-linear descriptions of the components, such as power converters, must be linearized around an operating point calculated for the entire network. This operating point is determined in the initialisation phase by solving the PF equations representing the combined AC/DC system.

To do so, the network is initialised using the OPF tool from [12] implemented as a Julia package, which can be found in the PowerModelsACDC.jl repository [13]. The package relies on the PF models developed for MatACDC [11], which extends the Matpower [10] AC power system models with DC system representations and with power converters. The mentioned PowerModelsACDC.jl library contains models for AC and DC power systems description provided in Table 1.

As a result, the constructed power system is divided into AC systems, DC systems, and converters. It contains AC and DC branches and buses, converters, generators, loads, and shunts. Storage elements can be represented as equivalent injections for the particular operation point. Components included for the electromagnetic stability assessment thus need to be represented using their equivalent models for the PF analysis to obtain the overall system description in terms of system voltages and currents. It should be noted that all models in this paper are developed under the assumption that the power system is balanced.

In this section, the fundamentals of the components used in the multipower system are described. Only the most common components are considered, being impedance, transformer, transmission line, cable, AC and DC sources/loads, and, given the focus on the electromagnetic instability assessment, the MMC converter. Components are presented using their physical equations, as well as by their model descriptions using \( ABCD \) parameters in the Fourier transform domain.

3.1 AC and DC branches

AC and DC branches typically represent three-phase AC and DC connections between AC and DC buses, respectively. Branches are grouped inside different AC or DC grids when several AC or DC networks are present. AC branches are typically defined in PF routines using their circuit parameters evaluated at 50 Hz, as described in [10], whereas DC branches can generally be reduced to their series resistance at DC [11].

The model of the AC branch as provided in [10] is depicted in Fig. 2. The Julia package PowerModelsACDC.jl [13] supports also a resistive shunt admittance which can also be unsymmetrical with respect to the sending and receiving side. In our formulation, we use a symmetrical resistive and capacitive shunts \((g_c/2) + j(h/2)\). The full expression for the AC admittance parameters is given by the equation:

\[
y_{ac} = \begin{pmatrix}
y_1 & g_c/2 & j h/2 \frac{1}{s^2} - \frac{y_s}{s} \frac{1}{s - j p_{diss}} \\
y_s & y_1 & g_c/2 + j h/2 \frac{1}{s^2} \frac{1}{s - j p_{diss}}
\end{pmatrix}.
\]

It should be noted that the AC network is considered as balanced for the PF solution, and thus, all the components have diagonal matrix models, with equal elements on the matrix diagonal.

Table 1 Definitions of the PF implementation

| Identifier | meaning |
|------------|---------|
| AC node | connection between the three-phase components and AC side of the power converter |
| AC branch | representation of any three-phase series element, such as transmission line, transformer, and impedance |
| load | without component representation, defined as PQ element |
| shunt | three-phase shunt component |
| generator | synchronous generator, wind farm, and grid equivalent |
| DC node | connection between monopolar or bipolar DC components, and with DC side of the power converter |
| DC branch | equivalent DC representation of any series element, such as transmission lines, resistances, etc. |
| DC converter | a connection between a pair of AC and DC nodes using a lossy power electronic converter model |

![Fig. 2 Matpower AC branch model [10]](http://example.com/fig2)

![Fig. 3 MatACDC DC branch model [11]](http://example.com/fig3)

(a) For monopolar DC configuration, (b) For symmetric monopolar DC branch configuration

The DC branch is modelled with its equivalent series resistance [11], as depicted in Fig. 3.

For the initialisation using a PF calculation, a number of components need to be modelled either as AC or DC branches. Their models are described in detail in this subsection.

3.1.1 Impedance: Impedance can be formed between \( n \) input nodes and \( n \) output nodes. The impedance is defined with the \( n \times n \) matrix \( Z \), where

\[
Z = \begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{n1} & Z_{n2} & \cdots & Z_{nn}
\end{bmatrix}
\]

and each component \( Z_{ij} \) represents the impedance between input node \( i \in \{1, \ldots, n\} \) and output node \( j \in \{1, \ldots, n\} \). The AC and DC parameters of the impedance from matrix (8) are given by

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{bmatrix}
I & Z \\
0 & I
\end{bmatrix}.
\]

In the case of a DC resistance, all matrices are of size \( 1 \times 1 \), while three-phase impedances are of size \( 3 \times 3 \). The corresponding DC branch model that is used for this component’s representation is given as a Thévenin equivalent series impedance \( r = \Re[Z] \). The AC branch is modelled according to

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3.1.2 Transformer: The single-phase equivalent of a transformer is depicted in Fig. 4, which presents an accurate model for the transformer operating below the MHz range, as recommended in [24]. It relies on the model as detailed in [4, 25] with the ABCD parametric description

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = Y_t \times (Z_w^k \parallel N \times Z_t^k \parallel Z_s) \times Y_v
\]  

where

\[
\begin{align*}
Y_t &= \begin{bmatrix} 1 & 0 \\ \frac{1}{3}C_t & 1 \end{bmatrix}, \\
Z_w^k &= \begin{bmatrix} 1 & sL_p + R_p \\ 0 & 1 \end{bmatrix}, \\
Y_m &= \begin{bmatrix} 1 & 0 \\ \frac{1}{sL_m + 1} & \frac{1}{R_m} \end{bmatrix}, \\
Z_s &= \begin{bmatrix} 1 & \frac{1}{sL_s} \parallel \frac{1}{R_s} \\ 0 & 1 \end{bmatrix}
\end{align*}
\]

and

\[
N = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}
\]

for \( n \) being the turn ratio.

The single-phase realisation can be extended into the YY and ΔY three-phase configuration [4].

Three-phase transformers are accounted for either in a YY or ΔY configuration, where each of the three single-phase transformers is represented by its equivalent from Fig. 4.

The YY configuration is derived from (10), such that

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \text{diag} \{A\}_{3 \times 3} & \text{diag} \{B\}_{3 \times 3} \\ \text{diag} \{C\}_{3 \times 3} & \text{diag} \{D\}_{3 \times 3} \end{bmatrix}.
\]

The ΔY configuration is more complex and it is modelled using the following equations. The inner primary and secondary parameters of the transformer (i.e. all the components except the parasitic capacitances and the load impedance) are given by

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = Z_{w}^n \times Y_{t} \times N \times \begin{bmatrix} 1 & sL_p + R_p \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{sL_m + 1} & \frac{1}{R_m} \end{bmatrix} \times \begin{bmatrix} \frac{1}{sL_s} \parallel \frac{1}{R_s} \\ 0 & 1 \end{bmatrix}
\]

(11)

The ΔY configuration transforms to the delta side \( v_p^{b,c} \) to the wye side voltages \( v_y^{b,c} \), as \( v_y^{b,c} = \sqrt{3} v_p^{b,c} \), while the currents are related as

\[
v_p^{b,c} = \begin{bmatrix} 1 & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 1 & 0 \\ 0 & -\frac{1}{\sqrt{3}} & 1 \end{bmatrix} \times \sqrt{3} v_y^{b,c}.
\]

Using the previous voltage/current relations and ABCD representation of the inner transfer function in (11), the inner impedance can be obtained as

\[
Z_{\text{inner}} = \begin{bmatrix} \text{diag} \{A\}_{3 \times 3} & \text{diag} \{B\}_{3 \times 3} \\ \text{diag} \{C\}_{3 \times 3} & \text{diag} \{D\}_{3 \times 3} \end{bmatrix}.
\]

(12)

The transformer is now represented using ABCD parameters as

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = (Y_{\text{tun}} \times Z_{\text{inner}} \parallel Z_{\text{stray}}) \times Y_{\text{tun}}.
\]

(13)

Since the transformer model cannot easily be represented as the model in Fig. 2, \( Y \) parameters are extracted from the ABCD parameters as described in (3).

Under the assumption of a balanced system, the submatrices \( Y\{1,3,1,3\}, Y\{1,3,4,6\}, Y\{4,6,1,3\}, \) and \( Y\{4,6,4,6\} \) are diagonal for AC networks and three-phase transformers. Thus, it is sufficient to use a single diagonal value from each submatrix. Then, \( Y\{1,1\} = (y_a + (g_a/2) + (j(b_a/2))(1/\Gamma)) \) and \( Y\{4,4\} = (y_a + (g_a/2) + (j(b_a/2))) \). For the PF model following expressions are derived:

\[
\begin{align*}
\tau &= \frac{\sqrt{Y\{4,4\}}}{\sqrt{Y\{1,1\}}}, \\
\theta_{\text{shift}} &= 0, \\
y_a &= -Y\{1,4\} e^{\text{j} \theta_{\text{shift}}}, \\
y_a &= Y\{4,4\} - y_a, \\
r_a &= \Re \{\frac{1}{y_a}\}, \\
k_a &= \Im \{\frac{1}{y_a}\}, \\
g_c &= \Re \{k_a\}, \\
b_c &= \Im \{k_a\}.
\end{align*}
\]

3.1.3 Transmission line: Transmission lines can take the form of cables, OHLs and mixed OHL-cable systems. Since cross-bonded cables [25] and transposed OHLs do shift the resonance spectrum and hence the electromagnetic stability assessment, they are considered here as well. All transmission lines have frequency-dependent distributed parameter models.

Under the assumption of a balanced system, the submatrices \( Y\{1,3,1,3\}, Y\{1,3,4,6\}, Y\{4,6,1,3\}, \) and \( Y\{4,6,4,6\} \) are diagonal for AC networks and three-phase transformers. Thus, it is sufficient to use a single diagonal value from each submatrix. Then, \( Y\{1,1\} = (y_a + (g_a/2) + (j(b_a/2))(1/\Gamma)) \) and \( Y\{4,4\} = (y_a + (g_a/2) + (j(b_a/2))) \). For the PF model following expressions are derived:

\[
\begin{align*}
\tau &= \frac{\sqrt{Y\{4,4\}}}{\sqrt{Y\{1,1\}}}, \\
\theta_{\text{shift}} &= 0, \\
y_a &= -Y\{1,4\} e^{\text{j} \theta_{\text{shift}}}, \\
y_a &= Y\{4,4\} - y_a, \\
r_a &= \Re \{\frac{1}{y_a}\}, \\
k_a &= \Im \{\frac{1}{y_a}\}, \\
g_c &= \Re \{k_a\}, \\
b_c &= \Im \{k_a\}.
\end{align*}
\]

3.1.3 Transmission line: Transmission lines can take the form of cables, OHLs and mixed OHL-cable systems. Since cross-bonded cables [25] and transposed OHLs do shift the resonance spectrum and hence the electromagnetic stability assessment, they are considered here as well. All transmission lines have ABCD model parameters, which can be estimated as in [26, 27] using known values for the series impedance \( Z^s \) and the shunt admittance \( Y^s \) per unit length as

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh (\Gamma l) & \sinh (\Gamma l) \sqrt{Y^s} \sinh (\Gamma l) \\ \cosh (\Gamma l) \sqrt{Z^s} \cosh (\Gamma l) \end{bmatrix}
\]

(14)

where \( \Gamma = \sqrt{Z^s Y^s}, \) \( Z = Z^{-1}, \) and \( l \) is the line length.

The series impedance \( Z \) and shunt admittance \( Y \) are derived differently depending on the actual OHL or a cable realisation. For the electromagnetic stability assessment, the models are derived as frequency-dependent distributed parameter models.

A transmission line (OHL, cable, cross-bonded cable, or mixed OHL-cable) is represented using its nominal \( \pi \)-model depicted in Fig. 5, where
\[ Z(j\omega) = Y_c \sinh(T\tau), \quad Y(j\omega) = Y_c \tanh(T\tau). \] (15)

For the DC lines and cables, the shunt admittance is not considered, while the branch resistance is equal to \( r = \Re \{ Z(0) \} \). In the symmetrical monopolar configuration, the transmission line (usually underground cable) is represented with two resistances \( r \). This model can easily be extended to employ shunt conductance as in [14]. For the balanced AC transmission line, the impedance and admittance matrices are diagonal such that \( Z(j\omega) = Z(j\omega)(1,1) \) and \( Y(j\omega) = Y(j\omega)(1,1) \) can be chosen. Then, the AC branch PF model is given by
\[
\begin{align*}
\tau &= 0, \\
r_c &= \Re \{ Z(j\omega) \}, \\
x_c &= \Im \{ Z(j\omega) \}, \\
g_c &= \Re \{ Y(j\omega) \}, \\
b_c &= \Im \{ Y(j\omega) \}.
\end{align*}
\] (16)

### 3.2 Shunt components

Shunt reactors and capacitors are defined with their admittance value as \( y_s = g_s + jb_s \) [10]. Shunt elements are modelled according to [25].

### 3.3 Generators

Models of the generators are represented with the desired values for the voltage magnitude \( V_n \) and phase \( \theta \), and with an estimation of the active power \( P \) and reactive power \( Q \) injections. In this work, they are used for modelling both grid equivalents and AC power sources (hence a simplified representation of synchronous machines), implying a high degree of idealisation. Generators are thus defined as reference buses. Grid equivalents are thus represented in the PF formulation as generators behind complex impedances, using the AC branch model introduced previously.

### 3.4 Power electronic converters

A power electronic converter is modelled in accordance with [11, 28], including its phase reactor, potentially a filter, and a transformer. To match it with the MMC model, only the reactor is considered as part of the converter model.

Internal losses of the converter for the PF formulation are calculated in the form of \( P_{loss} = a + b I_c + c I^2 \). The phase reactor, however, contains a series resistance and reactance as \( z_{pr} = R_c + jX_c \), where \( X_c = \omega L_c \).

Depending on the actual realisation of the converter's controls, the parameters of the converter can control DC voltage, the converter active power or use a DC side voltage-power droop. The model of the power converter with the corresponding default PF directions is depicted in Fig. 6, where the AC node and parameters are presented in black and DC node is presented in blue.

### 3.5 MMC power converter

As mentioned before, the focus of this work is the integration of the MMCs inside a hybrid AC/DC power system. The model incorporated relies on the work presented in [21, 29] and is depicted in Fig. 7, where each submodule is represented as a half-bridge module. The switches inside submodules are considered as ideal in this model and thus, losses of the converter are given only with the constant \( c = (R_{aux}/2) \).

Submodules are represented with their averaged equivalent, and thus, the following equations for voltage and current can be written for the upper and lower arms:
\[
\begin{align*}
v^U_{\text{eq}} &= m^U_{I} v^U_{\text{eq}}, \\
\dot{i}^U_{\text{eq}} &= m^U_{I} \dot{i}^U_{\text{eq}} + m^U_{\Sigma} \dot{\Delta},
\end{align*}
\] (17)

\[ m^U_{I} = m^U_{I}^1 + m^U_{I}^2, \]

where \( m^U_{I} \) are the corresponding upper and lower arm insertion indices.

The converter model is developed by following the methodology reported in [21, 29, 30]. Using the \( \Sigma - \Delta \)

\[ Y_a = \frac{V_a}{I_a}, \quad Y_b = \frac{V_b}{I_b}, \quad Y_c = \frac{V_c}{I_c}. \]

The MMC is described using differential equations
\[
\dot{i}_{\text{eq}}^U = \frac{v_{\text{eq}}^U - (a L_c I_c + R_c) \dot{i}_{\text{eq}}^U - v_{\text{eq}}^D}{L_{\text{eq}}}.
\] (18a)
\[ \zeta_{\text{ref}} = -\frac{v_{\text{ref}}}{I_{\text{ref}}} + \left( R_{\text{arm}}I_{\text{ref}} - 2oL_{\text{arm}}J_{\text{ref}} \right) \frac{I_{\text{ref}}}{I_{\text{ref}}}. \] (18b)

\[ \dot{v} = \frac{v_{\text{dc}}}{2L_{\text{arm}}} - \frac{v_{\text{dc}}^2}{L_{\text{arm}}}, \] (18c)

\[ v_{\text{ref}} = \frac{N}{2C} \frac{\Delta q_{\text{dc}}}{\Delta q_{\text{dc}}} - \alpha_{J} \dot{j}_{\text{dc}}. \] (18d)

\[ v_{\text{ref}} = \frac{N}{2C} \frac{\Delta q_{\text{dc}}}{\Delta q_{\text{dc}}} - \alpha_{J} \dot{j}_{\text{dc}}. \] (18e)

\[ v_{\text{ref}} = \frac{N}{2C} \frac{\Delta q_{\text{dc}}}{\Delta q_{\text{dc}}} + 2 \alpha_{J} \cdot \dot{j}_{\text{dc}}. \] (18f)

\[ v_{\text{dc}} = \frac{1}{C_{\text{dc}}}(v_{\text{dc}} - 3I_{\text{dc}}^2). \] (18g)

where

\[ i_{\text{dc}} = P_{\text{up}} + \frac{P_{\text{up}}(\dot{i}_{\text{dc}})}{2}, \]

\[ + \left( \alpha_{J} \dot{j}_{\text{dc}} \right) \] (19a)

\[ \dot{\gamma}_{\text{dc}} = p_{\text{up}}(\dot{P}_{\text{dc}}) \frac{m_{\text{dc}}}{m_{\text{dc}}} - \frac{P_{\text{up}}(\dot{i}_{\text{dc}})}{2} + \left( \alpha_{J} \dot{j}_{\text{dc}} \right), \]

\[ \dot{\gamma}_{\text{dc}} = p_{\text{up}}(\dot{P}_{\text{dc}}) \frac{m_{\text{dc}}}{m_{\text{dc}}} - \frac{P_{\text{up}}(\dot{i}_{\text{dc}})}{2} + \left( \alpha_{J} \dot{j}_{\text{dc}} \right), \]

\[ \dot{\gamma}_{\text{dc}} = p_{\text{up}}(\dot{P}_{\text{dc}}) \frac{m_{\text{dc}}}{m_{\text{dc}}} - \frac{P_{\text{up}}(\dot{i}_{\text{dc}})}{2} + \left( \alpha_{J} \dot{j}_{\text{dc}} \right), \]

\[ \dot{\gamma}_{\text{dc}} = p_{\text{up}}(\dot{P}_{\text{dc}}) \frac{m_{\text{dc}}}{m_{\text{dc}}} - \frac{P_{\text{up}}(\dot{i}_{\text{dc}})}{2} + \left( \alpha_{J} \dot{j}_{\text{dc}} \right), \]

\[ \Psi = \left[ \begin{array}{c} \dot{\gamma} \dot{m}^2 + 2 \dot{\gamma} \dot{m} \dot{q} + \dot{\gamma}^2 \dot{m}^2 + 2 \dot{\gamma}^2 \dot{m}^2 + 4 \dot{\gamma}^2 \dot{m}^2 \end{array} \right]. \] (19c)

\[ N \text{ being the number of submodules, } C \text{ the capacitance of the half-bridge submodule, } L_{\text{arm}} \text{ and } R_{\text{arm}} \text{ are the equivalent inductance and resistance of the arm, respectively. The expressions for the equivalent inductance and resistance are } L_{\text{eq}} = \frac{L_{\text{eq}}}{L_{\text{eq}} 2} \text{ and } R_{\text{eq}} = R_{\text{eq}} + (R_{\text{eq}} 2), \text{ respectively. The model incorporates } dq \text{-frames at the angular frequency } 2o \text{ and } 3o \text{ for } \Delta q_{\text{sub}} \text{ and components at the frequencies } o \text{ and } 3o \text{ for } \Delta q_{\text{sub}} \text{ and components in accordance with } [21]. \]

The determined operating point for the converters is given in Table 4. These values are further used as a reference for solving the equations for the converter's equilibrium.

The voltage references in (20) are obtained as an output of the PF problem itself takes 54 ms, while the remaining time goes to solving the non-linear equations for the four converters' steady-state estimation and the complete system initialisation. The PF is solved as an AC/DC PF problem using PowerModelsACDC.jl [13] with Ipopt as a solver. Solving the non-linear equations of the MMC is done using the NLsolve package written in the Julia programming language [33].

The operating points (equilibrium) of the converters are presented in Table 5 for all 13 state variables. By comparing the values from Tables 4 and 5 we can see the great correspondence between them, which demonstrates that the operating point of an MMC can be estimated in two steps: first by solving the PF problem with the desired goals and then by solving the set of converter's non-linear equations for determining the equilibrium for all state variables.

The solution time for this example is 29.141 s, including the formulation of the PF problem, solving it, and determining the operating point of each converter on an Intel(R) Core(TM) i7-7700HQ on 2.80 GHz CPU and with 16 GB RAM. The solution of the PF problem itself takes 54 ms, while the remaining time goes to solving the non-linear equations for the four converters' steady-state estimation and the complete system initialisation. The PF is solved as an AC/DC PF problem using PowerModelsACDC.jl [13] with Ipopt as a solver. Solving the non-linear equations of the MMC is done using the NLsolve package written in the Julia programming language [33].

5 Conclusion

This paper presented the use of a PF formulation to determine the operating point of the hybrid AC/DC power system, as a starting point for an electromagnetic or harmonic stability assessment. The
### Table 2 - Hybrid power system example: MMC parameters

| Parameter                  | Value                       |
|----------------------------|-----------------------------|
| arm losses                 | $L_{\text{arm}} = 30.6 \ \text{mH}$ |
| submodule                  | $N = 200$                    |
| reactor                    | $L_c = 62.9 \ \text{mH}$    |
| $\tilde{i}_d^c$ control   | $\tilde{\zeta}^c = 0.7$    |
| $\tilde{i}_q^c$ control   | $\tilde{\zeta}^q = 0.7$    |
| $W_c^c$ control            | $K_{W_c^c} = 120$           |
| $\nu_b$ control           | $K_{\nu_b} = 0.01$          |
| $P, Q$ control            | $K_{P, Q} = 2.002 \times 10^{-7}$ |
|                           | $K_{I_P, Q} = 1.001 \times 10^{-4}$ |
|                           | $R_{\text{arm}} = 0.6017 \ \Omega$ |
|                           | $C = 4.23 \ \text{mF}$     |
|                           | $R_c = 0.3429 \ \Omega$    |
|                           | $\omega_c^c = 300 \ \text{rad}/\text{s}$ |
|                           | $\omega_c^q = 1000 \ \text{rad}/\text{s}$ |
|                           | $K_{I_1} = 400$             |
|                           | $K_{I_2} = 2$               |
|                           | $K_{I_3} = 1.001 \times 10^{-4}$ |
### Table 3  Hybrid power system example: OHLs' and cables' parameters

| OHL identifier | Length |
|---------------|--------|
| h_{1,2,3,4}   | 200 km |

**Conductors**

- number: $n_c = 3$
- position: $\Delta x_{bc} = 10\,\text{m}$, $\Delta y_{bc} = 30\,\text{m}$, flat
- sag: $d_{sag} = 10\,\text{m}$
- radius: $r_c = 0.015\,\text{m}$
- resistivity: $R_{dc} = 0.063\,\Omega\text{m}$

**Ground wires**

- position: $\Delta x_g = 6.5\,\text{m}$, $\Delta y_g = 7.5\,\text{m}$
- sag: $d_{sag_g} = 10\,\text{m}$
- radius: $r_g = 0.0062\,\text{m}$
- resistivity: $R_{dgc} = 0.92\,\Omega\text{m}$

**Cable**

- cable identifier
  - $c_{1,5}$: 100 km
  - $c_2$: 150 km
  - $c_{3,4}$: 200 km
- number of cables: $n = 2$
- underground positions: $x = -0.5\,\text{m}$, $0.5\,\text{m}$, $y = 1\,\text{m}$
- core (Cu): $r_c = 24.25\,\text{mm}$
- insulator 1 (XLPE): $\Delta r = 17.5\,\text{mm}$
- sheath (Pb): $\Delta r = 4.5\,\text{mm}$
- insulator 2 (XLPE): $\Delta r = 3.5\,\text{mm}$
- armour (steel): $\Delta r = 10.8\,\text{mm}$
- insulator 3 (XLPE): $\Delta r = 5.2\,\text{mm}$

### Table 4  Operating point of the MMCs determined by PF

**MMC 1**

| $P_{ac}$ | $Q_{ac}$ | $P_{dc}$ | $V_{dc}$ | $V_m$ | $\theta$ |
|----------|----------|----------|----------|-------|---------|
| 120.03 MW | 0.00     | -120.01 MW | 640.00 kV | 399.80 kV | -0.05 rad |

**MMC 2**

| $P_{ac}$ | $Q_{ac}$ | $P_{dc}$ | $V_{dc}$ | $V_m$ | $\theta$ |
|----------|----------|----------|----------|-------|---------|
| -60.00 MW | 0.00     | 60.00 MW | 639.98 kV | 405.07 kV | 0.00 rad |

**MMC 3**

| $P_{ac}$ | $Q_{ac}$ | $P_{dc}$ | $V_{dc}$ | $V_m$ | $\theta$ |
|----------|----------|----------|----------|-------|---------|
| -20.00 MW | 0.00     | 20.00 MW | 639.98 kV | 404.00 kV | -0.01 rad |

**MMC 4**

| $P_{ac}$ | $Q_{ac}$ | $P_{dc}$ | $V_{dc}$ | $V_m$ | $\theta$ |
|----------|----------|----------|----------|-------|---------|
| -40.00 MW | 0.00     | 40.00 MW | 639.99 kV | 404.54 kV | -0.01 rad |
simulation is performed in two steps since it couples procedures for the AC/DC power system’s PF and the solution of non-linear differential equations for the operating point of an MMC. In the first step, the PF problem of the combined power system is solved, in which power converters are modelled only with their losses and controlling loops for DC voltage and active and reactive power. The PF solution is used in the second step to set the initial values of the power converters, and then the complete solution for the converter’s internal dynamics and controls at the beginning. At the same time, it provides sufficient accuracy, which is demonstrated by the fact that the full order converter’s equilibrium can easily be determined. The proposed procedure ensures a high degree of flexibility in the definition of the power system as well as for the inner and outer controlling loops of the MMCs.

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7 References
[1] Yang, X., Xue, Y., Wen, P., et al.: ‘Comprehensive understanding of dc-pole-to-pole fault and its protection for modular multilevel converters’, High Volt., 2018, 3 (4), pp. 246–254
[2] Xiang, M., Hu, J., Qiu, Y.: ‘Coordinated control of power loss and capacitor voltage ripple reduction for ac voltage boosted FBMS MMC with second harmonic circulating current injection’, High Volt., 2018, 3 (4), pp. 272–278
[3] Zhao, S., Chen, Y., Peng, L.: ‘Semiconductor loss calculation of dc–dc modular multilevel converter for HVDC interconnections’, High Volt., 2018, 3 (4), pp. 263–271
[4] Bayo Salas, A.: ‘Control interactions in power systems with multiple VSC-HVDC converters’ (KU Leuven, Belgium, 2018)
[5] Ji, K., Tang, G., Yang, J., et al.: ‘Harmonic stability analysis of MMC-based AC system using dc–dc–dc–AC converter’, IEEE J. Emerging Sel. Top. Power Electron., 2020, 2 (2), pp. 1152–1163
[6] Harfors, L., Bongiorno, M., Lundberg, S.: ‘Input-admittance calculation and shaping for controlled voltage-source converters’, IEEE Trans. Ind. Electron., 2007, 54 (6), pp. 3232–3334
[7] Lyu, J., Zhang, X., Cai, X., et al.: ‘Harmonic state-space based small-signal impedance modeling of a modular multilevel converter with consideration of internal harmonic dynamics’, IEEE Trans. Power Electron., 2018, 33 (3), pp. 2134–2148
[8] Agbemambo, A.J., Domínguez García, J.L., Prieto Armijo, E., et al.: ‘Dynamic modeling and interaction analysis of multi-terminal VSC-HVDC grids through an impedance-based approach’, Int. J. Electr. Power Energy Syst., 2019, 113, pp. 874–887
[9] El Marsafawy, M., Mathur, R.: ‘A new, fast technique for load-flow solution of integrated multi-terminal dc/ac systems’, IEEE Trans. Power Appar. Syst., 1980, PAS-99 (1), pp. 246–255
[10] Zimmerman, R.D., Sánchez, C.E.: ‘MATPOWER 6.0 user’s manual’ (PSERC, Temple, AZ, USA, 2016)
[11] Beerten, J.: ‘MATACDC 1.0 user’s manual’ (Department of Electrical Engineering, University of Leuven, Leuven, 2012). Available at https://www.esat.kuleuven.be/electa/teaching/matacdc/MatACDCManual
[12] Engan, H., Dave, J., Van Hertem, D., et al.: ‘Optimal power flow for AC/DC grids: formulation, convex relaxation, linear approximation, and implementation’, IEEE Trans. Power Syst., 2019, 34 (4), pp. 2980–2990
[13] Zhang, H., Jing, L., Wu, X., et al.: ‘Power flow control scheme for multiport power electronics transformers’, High Volt., 2018, 3 (4), pp. 255–262
[14] Stepnov, A., Saad, H., Karagac, U., et al.: ‘Initialization of modular multilevel converter for the simulation of electromagnetic transients’, IEEE Trans. Power Deliv., 2019, 34 (1), pp. 290–300
[15] Zhu, S., Liu, K., Liu, W., et al.: ‘Modeling and initialization of modular multilevel converters based high-voltage dc transmission in power system dynamics simulations’, 2018 IEEE PES Asia-Pacific Power and Energy Engineering Conf. (APPEEC), Kota Kinabalu, Malaysia, 2018, pp. 223–228
[16] Ravikumar, G.: ‘MATTRANS: a MATLAB power system transient stability simulation package’, 2015 IEEE 3rd Int. Conf. Grid Integration of Renewable Energy Sources (GRID), (2016), pp. 1–3
[17] Bergna Diaz, G., Freytes, J., Guillaud, X., et al.: ‘Generalized voltage-based state-space modeling of modular multilevel converters with constant equilibrium in steady state’, IEEE J. Emerging Sel. Top. Power Electron., 2018, 6 (2), pp. 707–725
[18] Lecić, A., Beerten, J.: ‘Generalized multiport representation of power systems using ABCD matrices for harmonic stability analysis’, XXI Power Systems Computation Conf., Porto, Portugal, 2020, pp. 1–8
[19] Reveyrand, T.: ‘Multiport conversions between S, Z, Y, H, ABCD, and T parameters’. 2018 Int. Workshop on Integrated Nonlinear Microwave and Millimetre-wave Circuits (INMMIC), 2018, pp. 1–3
[20] CIGRE JWG C4/B4.38. ‘Network modelling for harmonic studies’. CIGRE, 2019, TB 766
[21] Wu, L.: ‘Impact of HV/UHV underground power cables on resonant grid behavior’ (Eindhoven University of Technology, Netherlands, 2014)
[22] Castellanos, F., Martí, J.: ‘Full frequency-dependent phase-domain transient line model’, IEEE Trans. Power Syst., 1997, 12 (3), pp. 1331–1339
[23] Morched, A., Gustavsen, B., Taribli, M.: ‘A universal model for accurate calculation of electromagnetic transients on overhead lines and underground cables’, IEEE Trans. Power Deliv., 1999, 14 (3), pp. 1032–1038
[24] Beerten, J., Cole, S., Belmans, R.: ‘Generalized steady-state VSC MTDC model for sequential ac/dc power flow algorithms’, IEEE Trans. Power Syst., 2012, 27 (2), pp. 821–829
[25] Bergna Diaz, G., Zonetti, D., Sanchez, S., et al.: ‘PI passivity-based control and performance analysis of MMC multi-terminal HVDC systems’, IEEE J. Emerging Sel. Top. Power Electron., 2019, 7 (4), pp. 2453–2466
[26] Freyste, J.: ‘Small-signal stability analysis of modular multilevel converters and application to MMC multi-terminal dc grids’ (École Centrale de Lille, France, 2017)
[27] Saini, O.C., Beerten, J.: ‘Generalized dynamic phasor modeling of the MMC for small-signal stability analysis’, IEEE Trans. Power Deliv., 2019, 34 (3), pp. 991–1000
[28] Bergna Diaz, G., Zonetti, D., Sanchez, S., et al.: ‘PI passivity-based control and performance analysis of MMC multi-terminal HVDC systems’, IEEE J. Emerging Sel. Top. Power Electron., 2019, 7 (4), pp. 2453–2466
[29] Mogensen, P.K.: ‘NL-Solve repository: Julia non-linear solvers’, 2018. Available at https://github.com/JuliaNL/Solvers/NL.solver.jl

Table 5 MMCs’ equilibrium values

| VDC, V | VCS, V | VCD, V | VDC, V |
|--------|--------|--------|--------|
| 639,996.76 | 639,985.50 | 639,987.79 | 639,985.11 |