Typical skyrmions versus bimerons: a long-distance competition in ferromagnetic racetracks

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During the last years, topologically protected collective modes of the magnetization have called much attention. Among these, skyrmions and merons have been the object of intense study. In particular, topological skyrmions are objects with an integer skyrmion number $Q$ while merons have a half-integer skyrmion charge $q$. In this work, we consider a $Q = 1$ skyrmion, composed by a meron and an antimeron (bimeron), displacing in a ferromagnetic racetrack, disputing a long-distance competition with its more famous counterpart, the typical $Q = 1$ cylindrically symmetrical skyrmion. Both types of topological structures induce a Magnus force and then are subject to the Hall effect. The influence of the Dzyaloshinskii-Moriya interaction (DMI) present in certain materials and able to induces DMI-skyrmions is also analyzed. Our main aim is to compare the motions (induced by a spin-polarized current) of these objects along with their own specific racetracks. We also investigate some favorable factors which are able to give breath to the competitors, impelling them to remain in the race for longer distances before their annihilation at the racetrack lateral border. An interesting result is that the DMI-skyrmion loses this hypothetical race due to its larger rigidity.

INTRODUCTION

Skyrmions [1] are topologically protected states that have been introduced in the framework of the two-dimensional ($2d$) Heisenberg model (HM) by Belavin and Polyakov [2]. The ($2d$) HM is defined by the Hamiltonian $H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$, where $J > 0$ is the ferromagnetic coupling constant, the sum is over nearest-neighbor spins and the spin field $\vec{S}(\vec{r})$ obeys the constraint $\vec{S}(\vec{r}) = \vec{S}_x(\vec{r}) + \vec{S}_y(\vec{r}) + \vec{S}_z(\vec{r}) = \vec{S}$, with $\vec{S}$ being a constant. Topologically, skyrmions correspond to the mapping of the spin-space sphere $(\sum_{\text{int}}^2)$ onto the continuum plane $\vec{r} = (x, y)$ (physical space $(\sum_{\text{phy}}^2)$). Consequently, they are characterized by a skyrmion integer number $Q = \pm 1, \pm 2, \ldots$, and have finite energy $E_0 = 4\pi JS^2 |Q|$, independent of the skyrmion size $R$ since the continuum limit of the Heisenberg model is scale-invariant.

Considering the mapping $(\sum_{\text{int}}^2) \rightarrow (\sum_{\text{phy}}^2)$, the Belavin-Polyakov skyrmion configurations can have essentially two faces as seen by different perspectives, which depend on the boundary conditions (or stereographic projection). For $\vec{S}(\vec{r}) \rightarrow (0, 0, \pm S)$ as $\vec{r} \rightarrow \infty$, one gets the $|Q|$ core configuration (type $I$-skyrmion) while for $\vec{S}(\vec{r}) \rightarrow (\pm S, 0, 0)$ as $\vec{r} \rightarrow \infty$, one gets the $2|Q|$ core configuration (type $II$-skyrmion). For the same $Q$, both skyrmions (type $I$ and type $II$) have the same energy. Therefore, we mean that the core occupies a small localized region in which $S_x^0 + S_y^0 = 0$ and consequently, $S_z = \pm S$. However, depending on parameters like small anisotropies, external magnetic fields, or others that favor out-of-plane or in-plane spins, structures similar to type $I$ or type $II$ skyrmions, respectively, could be excited in a system.

In our study we consider $Q = \pm 1$ skyrmions since they are energetically favorable. Because type $I$-skyrmions exhibit great potential to be used in storage and processing-information technologies, much attention has been dedicated to study such a spin texture [3] [4]. However, for those applications, some intrinsic difficulties in generating and guiding them along a nanostripe need to be overcome. For instance, to use them in spintronic applications, the main barrier is the inability to move skyrmions straight along applied currents. Indeed, it is well known that type $I$-skyrmions suffer the effect of the Magnus force, which leads to the skyrmion Hall effect. Amongst some theoretical propositions to suppress the skyrmion Hall effect, there are possibilities of engineering magnetic materials [5], the formation of coupled skyrmions displacing in bilayer compounds [6–8] and spin-current driven skyrmion dynamics [9]. Based on the above, it should be relevant to see what occurs with the dynamics of topological structures with different shapes along pathways to get more insights to prevail over intrinsic technological difficulties.

In this paper we give attention to type $II$-skyrmion textures also called bimerons [10]. These objects are not cylindrically symmetric [11] [12] and may have also important consequences in quantum magnetism. For instance, considering 2d-antiferromagnets with general spin $S$ and the case $Q = 1$, the merons [13] forming a double core skyrmion [14] are “spin-$S$ spinons” [15] [16], which appear as essential objects in the seek for two-dimensional quantum spin liquid [17] states of spin-half ($S = 1/2$). On the other hand, another kind of bimeron structures may also be found in thin chiral magnetic films [18] induced by nonmagnetic impurities [19], as well as stabilized in confined geometries [20].

The main goal of this paper is to analyze the trajectories
of both types of skyrmions described above in ferromagnetic racetracks. In principle, it is shown that if we consider a massless model to describe the dynamics of a bimeron, its trajectory and velocity along a nanotrack would be the same as that predicted for type I-skyrmions. Nevertheless, due to its non-cylindrical symmetry, the displacement of bimerons mass-center induces an effective mass which is different from the mass of its type I-skyrmion counterpart. Therefore, it should move in a straight line for longer/shorter distances. Thus, by means of analytical calculations and micromagnetic simulations, we study type II-skyrmions focusing on their sensitivity to the Magnus force. The results are compared with the trajectories obtained for type I-skyrmions. Here we have to distinguish two categories of type I-skyrmions, which depends on the specific materials they can reside: type I-skyrmions living in ferromagnetic materials with Dzyaloshinskii-Moriya interaction, described by a coupling constant $D$, added to the Heisenberg Hamiltonian $H$ and genuine type I-skyrmions which subsist in ferromagnets without Dzyaloshinskii-Moriya interaction (DMI). Although they have very similar shapes, the small and basic contrasts between them may lead to different dynamics. For instance, when the DMI is present, the skyrmion has a more rigid structure and its size (controlled by the ratio $D/J$) remains practically constant during movement. For racetrack materials with DMI, hereafter, the skyrmions will be called DMI-skyrmions while the name I-skyrmions, will be held for the natural counterpart of type II-skyrmions.

**THEORETICAL MODEL**

Type II-skyrmions or bimerons have two centers in which a meron and an antimeron are positioned. A meron with a winding number $\eta = \pm 1$ and core polarization $P = \pm 1$ has a half-integer topological charge $q = \eta P/2$ (the meron wraps only half of the sphere). Therefore, a pair constituted by a meron ($\eta = 1$) and an antimeron ($\eta = -1$) with the same polarization (for example $P = 1$) has opposite skyrmion numbers adding to zero ($Q = 0$) and thus, such a pair belongs to the same topological sector as uniform ground states. This object would be then topologically unstable since it can be deformed continuously into a ground state with zero skyrmion number. On the other hand, if a pair has a meron and an antimeron with antiparallel core polarizations, these half-integer structures would have equal skyrmion numbers adding to a total of $+1$ or $-1$, belonging to a nontrivial topological sector and thus cannot be deformed continuously into a ground state. It is exactly what occurs with bimerons, which are characterized by a topological invariant (the skyrmion number), defined as

$$Q = \frac{1}{8\pi} \int d^2 \vec{x} \epsilon_{ijk} \epsilon_{aBd} n_{\alpha} \partial_{\beta} n_{\gamma} \partial_{i} n_{\gamma},$$

where $\hat{n}(\vec{x}) = \vec{S}(\vec{x})/|\vec{S}(\vec{x})|$ is the unit vector parallel to the local magnetization $\vec{S}(\vec{x})$.

The continuum limit of the 2$d$-isotropic ferromagnet described by a Hamiltonian $H$ consists in the famous nonlinear $\sigma$-model, given by $(J/2) \int d^2 \vec{x} (\partial_{i} \vec{S}(\vec{x}))^2$, $v = 1.2$ and the constraint $S^2 = 1$ (without loss of generality, we use an unit spin vector). The explicit static spin configuration of a bimeron can be obtained by using boundary conditions $\vec{S}\rightarrow (1,0,0)$ at $r\rightarrow \infty$. Then, parametrizing the spin vector $\vec{S}(\vec{r})$ by two scalar fields, the polar and azimuthal angles $\theta$ and $\phi$, $\vec{S} = (\cos \theta \cos \phi, \sin \theta \sin \phi, \cos \phi)$, this static solution with $Q = 1$ (energy equal to $4\pi J$), size $R$ (merons separated by a distance $R$) and mass center localized at the origin can be written as

$$\theta_{2c}^{(h)} = \arccos \left( \frac{R c_i}{\rho^2 + R^2/4} \right),$$

$$\phi_{2c}^{(h)} = \arctan \left( \frac{c_i - R/2}{c_j} \right) - \arctan \left( \frac{c_i + R/2}{c_j} \right),$$

where $\rho = \sqrt{\xi_x^2 + \xi_y^2}$, and $(c_i, c_j) = (x, y)$ and $(c_i, c_j) = (y, x)$ for type II-skyrmions with the cores aligned horizontally ($h$-bimeron) and vertically ($v$-bimeron), respectively. If $\xi = \xi$, we obtain a regular rigid bimeron in which the two cores are not deformed. If $\xi \neq \xi$, we obtain a bimeron having an elliptical shape. In Fig. 1, we show the vector field of the above described model for a $v$-bimeron with the cores aligned vertically, and $\xi = \xi = 1$.

Aiming to compare the dynamics of $I$- and $II$-skyrmion, we can describe a $Q = 1$ $I$-skyrmion solution (Fig. 1.b) with characteristic radius $R$, energy equal to $4\pi J$, and placed at $(0,0)$, as

$$\theta_{1c} = \arccos \left( \frac{R^2 - \rho^2}{R^2 + \rho^2} \right), \quad \phi_{1c} = \arctan \left( \frac{y}{x} \right).$$

Micromagnetic simulations are performed to study the stabilization and dynamics of these skyrmion structures. Firstly, we have stabilized the bimeron in a racetrack composed by an isotropic Heisenberg ferromagnetic material (Fig. 1a) at zero temperature by relaxation and using the solutions of the $O(3)$ nonlinear $\sigma$-model given by expressions [2]. The investigated racetrack has a width (distance between the upper and lower lateral borders) equal to $L_y = 80a$ and length $L_x = 300a$, where $a$ is the lattice parameter. The calculations consider periodic boundary conditions along the $x$-direction and open boundary condition along the $y$-direction. The bimeron was stabilized with the following parameters: $J = 1$ and $R = 4a$. Similar parameters are also used for type I-skyrmion (Fig. 1.b). Both tracks in Fig. 1 are organized in parallel to simulate a hypothetical race between the $I$- and $II$-skyrmions. Since we are studying 4 structures ($I$-skyrmion, $DMI$-skyrmion, $h$-bimeron and $v$-bimeron), our imaginary running track is constituted by 4 lanes, each one made by a ferromagnetic material with characteristics able to support its resident competitor.

After having stabilized the bimeron, adjusting its configuration inside the given racetrack, fourth-order Runge-Kutta method is employed to compute the dynamics of the magnetic
moment, \( \mathbf{S}_i \), by solving the the Landau-Lifshitz-Gilbert (LLG) equation [21, 22],
\[
\frac{\partial \mathbf{S}_i}{\partial t} = -\gamma \mathbf{S}_i \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{S}_i \times \frac{\partial \mathbf{S}_i}{\partial t} \tag{4}
\]
where \( \gamma \) is the gyromagnetic ratio, \( \mathbf{H}_{\text{eff}} = -\frac{1}{\gamma} \frac{\partial \mathbf{H}_0}{\partial \mathbf{S}_i} \) is the net effective magnetic field on each spin, and \( \alpha \) is the Gilbert damping coefficient. The spin-polarized current is introduced by using the Berger spin-transfer torque [23],
\[
\tau_B = p \left( \mathbf{j} \cdot \nabla \right) \mathbf{S}, \tag{5}
\]
and
\[
\tau_{\text{DB}} = p \beta S \times \left( \mathbf{j} \cdot \nabla \right) \mathbf{S}, \tag{6}
\]
where Eq. (5) and Eq. (6) are the adiabatic and non-adiabatic torque, respectively. Here \( p \) is the spin polarization of the electric current density \( \mathbf{j} \), while \( \beta \)-parameter characterizes its relative strength to the Berger’s torque (Eq. (5)).

RESULTS

After stabilizing the skyrmions, we have performed micromagnetic simulations to obtain their mass center position as a function of time for four configurations: i) a DMI-skyrmion; ii) a I-skyrmion; iii) a v-bimeron; and iv) a h-bimeron. Before presenting the main results, we have to say something about the particularities of the above structures. Specifically, different from I- and II-skyrmions, DMI-skyrmions demand extra parameters and factors to be stabilized in a magnetic compound, such as the coupling \( D \) and the presence of an external magnetic field along the direction perpendicular to the magnetic plane. Instead of using the field, we stabilize this kind of structure by a small easy-axis anisotropy \( k_z/J = 0.11 \). In addition, we use \( D/J = 0.26 \) for the Dzyaloshinskii-Moriya coupling constant. These factors convert DMI-skyrmion configurations in rigid structures, much more inflexible than the other skyrmions investigated here. Indeed, DMI-structures are heavier than the other skyrmions and their size does not suffer significant variation during their motions as will be discussed below. This hardness is not expected for I- and II-skyrmions, since they are described only by a Heisenberg Hamiltonian. As a consequence, their sizes may suffer some fluctuations during their motion, mainly when the spin current is initially applied. Further, at first sight, because the II-skyrmion has two merons with opposite winding numbers, one may expect that the meron tends to suffer the Magnus force impelling it to, let’s say, the upper border, while its counterpart antimeron tends to go to the opposite side, i.e., the lower border (see Fig. 1-a). Nevertheless, the type II-skyrmion as a whole has a topological number \( Q = 1 \) and, therefore, it tends to suffer the Magnus force, similar to what happens to I-skyrmion (all that depends on \( q = \eta P/2 \); both merons of the bimeron have positive charge \( q = 1/2 \), moving in the same direction). In other words, the total Magnus force on the structure as a whole is not zero. Therefore, the bimeron mass center moves along the racetrack suffering the skyrmion Hall effect. The ob-
I. Trajectories of process. This makes the skyrmion may rotate slightly around its mass center structure, mainly because the racetrack has a finite size. In-...
Assuming that

\[ M_v = \text{some expression} \]

these assumptions, we obtain that the mass elements of the

\[ \text{mass elements} \]

are given by

\[ M - \xi \approx \text{some expression} \]

Such a deformation can be represented by

\[ \text{profile when they are displacing under the action of a cur-

trajectories of type} \]

\[ \text{initial position of the bimeron affects its route, causing dif-

x}\text{-skyrmions, neglecting terms of the order of} \]

\[ \delta. \] The previous equation reveals that if the

\[ v - \text{bimeron is} \]

\[ \sim \text{some value} \]

\[ \delta \ll 1, \] we can expand the mass elements of the

\[ I\text{-skyrmions, neglecting terms of the order of} \]

\[ \delta^2. \] Under these assumptions, we obtain that the mass elements of the

\[ v\text{-bimeron configuration are} \]

\[ M_d = M_v + \frac{4\pi \delta b}{R^2} \left[ 2\sqrt{A} \left( \frac{1}{A} - 2b^2 + 8Ab^4 \right) - \frac{1}{\sqrt{B}} - \frac{1}{\sqrt{C}} \right], \]

where

\[ A = (R^2 + 4b^2)^{-1}, \quad B = (R - 2b)^{-2}, \] and

\[ C = (R + 2b)^{-2}. \] The previous equation reveals that if the \( v\)-bimeron is

flattened along the \( x\)-axis direction (\( \zeta > \xi \)), its mass increases, while if the \( v\)-meron is flattened along the \( y\)-axis direction (\( \zeta < \xi \)), its mass decreases. The mass elements for the \( h\)-bimeron can be also obtained. However, the equations describing them are cumbersome and will be omitted here. In

\[ \text{Fig. 5} \] we show the behavior of \( M_d \) of the \( h\)-bimeron as a

function of \( \delta \). It can be observed that the mass elements of the

\( h\)-bimeron behave contrary to the \( v\)-bimeron case. That is, for \( \delta < 0 \) the mass increases when compared to the \( M_v \) and for \( \delta > 0 \), the mass decreases. Additionally, the effect of the deformation on the mass is more prominent for \( h\)-bimerons.

From the above discussion, we are now in a position to explain the results obtained from micromagnetic simulations. Indeed, from the mass-center trajectory equation \( y(x) = (D_2 \alpha / g) x \propto M_v x \) (or Eqs. (9)), one can observe that the position of the skyrmion depends on its mass in such a way that the larger the mass, more quickly the skyrmion approaches the lateral border of the racetrack. In principle, the annihilation of the structure at the racetrack lateral border would occur at a smaller \( x\)-position. In this context, because the \( v\)-bimeron mass practically does not change when it deforms, its trajectory should be almost the same as that of the type \( I\)-skyrmion. On the other hand, the \( h\)-bimeron diminishes its mass when it is flattened along the \( y\)-axis direction. Nevertheless, because the changes in the skyrmion mass are more pronounced for \( h\)-bimerons, the trajectory of this structure must have a more pronounced difference as compared with the type \( I\)-skyrmion pathway. Such results agree with the simulations. However, when all structures are near the stripe border \((x \sim x_i, c)\), the deformation along \( x\)-axis direction increases the \( h\)-bimeron mass and it is rapidly destroyed in the stripe border. Because we have considered a model for very small \( \delta \), the trajectories obtained analytically are almost superposed, and then a most complete model should consider larger deformations. In addition, at \( x \sim x_i, c \), the skyrmion-border interaction must also be very important for the skyrmion deformations, changing drastically the skyrmion trajectories as indicated by the simulations.
DISCUSSION AND CONCLUSION

In summary, we have investigated how different skyrmion configurations travel along isotropic ferromagnetic racetracks. Since these skyrmions reside, in general, on different circumstances or materials (for instance, in-plane or out-of-plane boundary conditions dictate their structures), we have considered a race competition among them in which each skyrmion moves in its own appropriated lane. Since all the objects analyzed here experience the Hall skyrmion effect, they inevitably will die after running some distance along the racetrack (striking the lateral border). We show that the trajectories of these skyrmions depend on their mass, in such a way that, small modifications in the mass may result in an additional last breath, making determined skyrmion to live a bit more in the track. Our results show that a bimeron positioned in the v-bimeron mode is the best long-distance runner since it could go through a little more spatial extension before its annihilation at the lateral border of its racetrack. In spite the skyrmion-border interaction is not included, the presented theory gives a useful tool to understand the behavior of these different magnetic textures.

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