Shape optimization of shell structure by using Genetic Algorithm (GA) method

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Abstract. Shell structure has been popular for various application in building, aerospace, oil and gas, ship and automobile industry. This type of element is light weight and can be provide great stiffness for large structure. Furthermore, it is very useful for building architecture and appearance. However, currently, construction is not only concerning structural strength but also focusing on building cost and its impact to environment. Finding the balance between structural strength, cost and aesthetic appearance is essentials this nowadays. Hence, construction framework to find the leeway between those parameters is greatly important. This paper will discuss the framework for shape optimization of shell element by using Genetic Algorithm (GA) method. The shape of structure will be developed from Non-Uniform Rationale Bi-Spline (NURBS). Finite element analysis based on Reissner-Mindlin plate theory is utilized to find the element stress within the structure. Genetic algorithm method will evaluate and perform search operation inside the result domain with constraints of structural weight and element stress. The framework then was implemented in numerical example to find its effectiveness on obtaining the most optimum shape of shell structure. Based on the numerical result, it is clear that the proposed framework is able to reach the convergence to the optimum structure without violating the given constraints.

1. Introduction

Nowadays, structure are mostly modelled and analysed by Finite Element Method (FEM). This is because Finite Element Method (FEM) can be easily integrated in the computer language program which makes structure analysis by using computer becoming significantly efficient. Among many element in FEM, shell element has got great interest from engineer and researcher due to rapid development of computational resources. Huge interest in shell element makes shell element becoming popular to be applied in many fields from large scale structure such as skyscraper, ship, aircraft, to industrial product such as car and bicycle.

Shell structure has advantages against various types of loadings. Shell structure elements can provide high capacity, so it can be used on structures with large spans with relatively less material usage than other types of structural elements [1]. Its capacity depends on the shape of shell curve which will influence the stiffness of shell element. Previously, in office of residential building, the shape of shell structure usually decided by the architect without concerning its strength and stiffness. The loadings will be beared by structural elements i.e. columns, beams and walls whilst the shell element is installed as façade and architectural elements. However, nowadays, new wave of design
philosophy has emerged. The design must not only provide strength and stiffness, but also it must be environmentally friendly and efficient in cost and materials use. Hence, the element that previously considered as architectural element, now must also utilized as structural element which will resist the external loadings.

Previous researchers developed optimization method that can optimize the shell element. The research on optimization of shell structures based on stress and displacement constraints by using triangular element with thickness as design variable was performed [2]. Synthesis algorithm of DESAP1 program was coupled with FEA package to optimize the shell structure.

Structural optimization is used in preliminary design phase to find the guideline for detailed design process. The development of Computer Aided Design (CAD) for design makes optimization method must consider CAD in the process [3]. In regards to shell structure optimization, the optimization model can use the advantage of modelling curved and meshing of shell structure by using CAD system.

This research aims to investigate the effectiveness of Genetic Algorithm method to minimize the required materials to withstand specific loadings. Furthermore, this research tries to couple Genetic Algorithm (GA) as the optimization method with NURBS curve generation based on CAD system. The goal of this research is to investigate the framework for shell element optimization that can deal with 3D shell problem with minimal design variable for the sake of less computational resource. Genetic algorithm was selected for optimization method because non-linear relation between shape of shell and stress. Genetic algorithm has already well known as metaheuristic method that does not need the linear relation between design variable and result domain.

2. Methodology

2.1. Free-form surface by Non Uniform Rational Bi-Spline (NURBS)

NURBS curve was developed back in 1960s by French car factory Citroen. It is defined by a set of control points, knot vector, degree and weights. In this research the order “k” Bi-spline developed in 2D space as depicted in Figure 1(a) which can be described as follow,

\[ N_{i,j}(u) = \begin{cases} 
1 & u_i \leq u \leq u_{i+1} \\
0 & \text{else} 
\end{cases} \]

\[ N_{i,j}(u) = \frac{u - u_i}{u_{i+j} - u_i} N_{i,j}(u) + \frac{u_{i+j} - u}{u_{i+j+1} - u_{i+1}} N_{i+1,j}(u) \]  

(1)

The basic functions of a NURBS curve can be defined by the following function,

\[ R_{i,p}(u) = \frac{\sum_{j=0}^{n} N_{i,j}(u)w_j}{\sum_{j=0}^{n} N_{j,p}(u)w_j} \]  

(2)

where \( w_i \) is weight. For control point \( \{P_i\} \), the NURBS function become,
Having obtained the 2D NURBS curve, the skin of shell element is obtained from 3D rotation of one of Cartesian axis. This technique follows the nature process of making surface from 2D to 3D. In this research, the NURBS is determined in x-z axis and the surface obtained from that NURBS is obtained by 3D rotation of this 2D NURBS relative to z axis. The new coordinate of surface based on this rotation can be obtained from the following 3D transformation function,

\[
3D\text{Rot}_\theta(\hat{z}) = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Due to limitation of computational resource, this research only investigated 5 rotation degrees which are 0\(^{\circ}\), 22.5\(^{\circ}\), 45\(^{\circ}\), 67.5\(^{\circ}\), and 90\(^{\circ}\). This process is depicted in Figure 1(b) below,

**Figure 1.** (a) NURBS Generate from specified control point. (b) Skinning process by 3D rotation of 2D NURBS. (c) Optimization process by changing control point as design variable.

### 2.2. Finite Element Analysis (FEM)

Common theory for analysing shell structure is Reissner-Mindlin Plate theory [4]. Reissner-Mindlin plate theory is excellent for describing thick shell behaviour. In Reissner-Mindlin plate theory, there are 5 degree of freedom (DOF) i.e. rotation on the x and y axes (\(\theta_x\) and \(\theta_y\)) and translation on the x, y and z axis as shown in figure 2. [5].

**Figure 2.** Shell element based on 4 Node quadrilateral elements.
Based on the basic assumptions of the Reissner-Mindlin plate theory, node displacement is written as,

\[ u = z \cdot \theta_x(x, y) \quad v = z \cdot \theta_y(x, y) \quad w = z(x, y) \]  

(5)

So the normal strain that occurs on DOF in each node can be mathematically written as follows,

\[
\varepsilon_{xx} = \frac{\partial \theta_x}{\partial x} \quad \varepsilon_{yy} = \frac{\partial \theta_y}{\partial y} \quad \varepsilon_{zz} = \frac{\partial \theta_z}{\partial z} + \frac{\partial \theta_y}{\partial x}
\]  

(6)

and the shear strain that occurs is,

\[
\gamma_{xy} = z \cdot \left( \frac{\partial \theta_x}{\partial y} - \frac{\partial \theta_y}{\partial x} \right)
\]

(7)

\[
\gamma_{xz} = \frac{\partial w}{\partial x} + \theta_x
\]

(8)

\[
\gamma_{yz} = \frac{\partial w}{\partial y} + \theta_y
\]

(9)

The bending and shear stresses that occur based on the strain formula are,

\[
\sigma_f = D_f \varepsilon_f
\]

\[
\sigma_c = D_c \varepsilon_c
\]  

(10)

Assuming that the plate element is a homogeneous and isotropic material, then,

\[
D_f = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}
\]

(11)

and,

\[
D_c = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}
\]

\[
G = \frac{E}{2(1+\nu)}
\]  

(12)

Shape function for four node shell plate elements is derived as follows,

\[
N_1 = \frac{1}{4}(1-\xi)(1-\eta)
\]

\[
N_2 = \frac{1}{4}(1+\xi)(1-\eta)
\]  

(13)

\[
N_3 = \frac{1}{4}(1-\xi)(1+\eta)
\]

\[
N_4 = \frac{1}{4}(1+\xi)(1+\eta)
\]

(14)
\[ N_3 = \frac{1}{4}(1 + \xi)(1 + \eta) \]  
\[ N_4 = \frac{1}{4}(1 - \xi)(1 + \eta) \]  

where \( \xi \) and \( \eta \) are the shape coefficients of the shell plate elements. By using these shape functions, the contribution of the surface load on the \( F \) loading matrix can be assembled based on the following mathematical formula,

\[ F = \begin{bmatrix} f_i & m_{xi} & m_{yi} \end{bmatrix} \]  

since the assumption that there are no moments that occur on node, then

\[ m_{xi} = m_{yi} = 0 \]  

and \( f_i \) mathematically calculated,

\[ f_i = \int_{-1}^{1} \int_{-1}^{1} N_i q |J| d\xi d\eta \]  

Where \( i = 1,2,3,4 \) and \( N_i \) is shape function of \( i \)-th node, \( |J| \) is the determinant of the Jacobian matrix, and \( q \) is the uniform surface load. The Jacobian matrix \([J]\) is determined as follows,

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\
\frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial}{\partial \xi} \sum_{i=1}^{4} N_i x_i & \frac{\partial}{\partial \xi} \sum_{i=1}^{4} N_i y_i \\
\frac{\partial}{\partial \eta} \sum_{i=1}^{4} N_i x_i & \frac{\partial}{\partial \eta} \sum_{i=1}^{4} N_i y_i
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta}
\end{bmatrix}
\begin{bmatrix}
x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4
\end{bmatrix}
\]  

Strain energy formula for Reissner-Mindlin plate theory is derived as follow,

\[ U = \frac{1}{2} \left( \delta^{\varepsilon} \right)^T \int_V \varepsilon^2 B_f^T D_f B_f dV \delta^{\varepsilon} + \frac{\alpha}{2} \left( \delta^{\varepsilon} \right)^T \int_V B_e^T D_e B_e dV \delta^{\varepsilon} \]  

with,

\[ B_f = \begin{bmatrix}
0 & \frac{\partial N_1}{\partial x} & 0 & \ldots & 0 & \frac{\partial N_4}{\partial x} & 0 \\
0 & \frac{\partial N_1}{\partial y} & 0 & \ldots & 0 & \frac{\partial N_4}{\partial y} & 0 \\
0 & \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \ldots & 0 & \frac{\partial N_4}{\partial x} & \frac{\partial N_4}{\partial y}
\end{bmatrix} \]  

\[ B_e = \begin{bmatrix}
\frac{\partial N_1}{\partial x} & N_1 & 0 & \ldots & 0 & N_4 & 0 \\
\frac{\partial N_1}{\partial y} & 0 & N_1 & \ldots & 0 & N_4 & 0
\end{bmatrix} \]
\[
\delta^e = \begin{bmatrix}
w_1 & \theta_{x1} & \theta_{y1} & \ldots & w_4 & \theta_{x4} & \theta_{y4}
\end{bmatrix}
\]  
(24)

By assuming the above strain energy matrix, the stiffness matrices for shell element can be obtained by,

\[
K^e = \frac{t^3}{12} \int_{-1}^{1} \int_{-1}^{1} B^T_D B_f J d\eta d\xi + \alpha \int_{-1}^{1} \int_{-1}^{1} B^T f B_c J d\eta d\xi
\]

(25)

where \( t \) is thickness of shell element. The first phase of that stiffness formula describes contribution of element stiffness on bending mechanism while the second phase is representing shear stiffness.

### 2.3. Genetic Algorithm (GA)

Genetic algorithm mimics the theory of “Survival of the Fittest” introduced by Charles Darwin. In this theory, the organism that not fit against the constraint will perished and only the fit organism will survive. Furthermore, the organism in new generation will be fitter than the previous generation due to cross over and mutation. Hence, in Genetic Algorithm (GA), there are three main steps in optimization process which are the natural selection, cross over and mutation.

In this research, classic Genetic Algorithm (GA) method [6]–[8] which represent design variables as string of bit is applied. The design variable in this occasion is the coordinate of control point (\( P \)) that will shape the NURBS curve. 50 organisms were created for the first generation. The optimization algorithm will select 30 organisms to be parents through natural selection process called Roulette Wheel. This technique allows low fitness organism to join the natural selection process although it also has lower chance to be selected as parent. The optimization model is defined as follow,

\[
\begin{cases}
\text{Min} & 2 \cdot \pi \cdot L_{\text{NURBS}} \cdot t \cdot \rho_{\text{steel}} \\
\sigma_{\text{VM}} & \leq \sigma_{\text{all}} \\
\sigma_{\text{all}} & = \frac{\sigma_{\text{VM}}}{SF}
\end{cases}
\]

(26)

where \( \sigma_{\text{VM}} \) is Von-Mises stress and \( L_{\text{NURBS}} \) is length of NURBS curve. Meanwhile, penalty function for the optimization is mathematically written as follow,

\[
Pen = W_{\text{str}}^2 + r_1 \left( g(s) \right)^2 \quad \text{and} \quad c = \frac{R}{Pen}
\]

(27)

Crossover is swapping the chromosome of parents to create new chromosome of offspring that inherit the characteristic of parent. The crossover and natural selection connect the characteristic of old and new generations and are responsible for passing the acquired fitness from the old to the new population of chromosomes. The main function of crossover is to provide combination of solution in the result domain. The role of mutation is essentially different from crossover. It is utilized to create diversity in the population that has been through crossover process, which may also be viewed as introducing new information into the entire population. Mutation helps the optimization process to avoid premature convergence due to lack of diversity during first generation creation. In this formulation, the probabilities of mutation and crossover are constant during evolution which means that the searched space is always sampled in the same way. The probability of crossover (\( P_{\text{co}} \)) dan probability of mutation (\( P_m \)) is set 0.95 and 0.05 respectively. The flowchart of genetic algorithm is presented in figure 3.
3. Numerical example

The optimization technique applied to solve the pre-design problem on roof of oil storage tank with diameter 39.010 mm made from carbon steel A283 Gr. C with yield strength 200 MPa, Elastic (Young’s) Modulus 190.000 MPa and Poisson’s Ratio 0.29. Based on API 650 point 3.6.2.1, the design should consider 2.5 Safety Factor (SF). The standard thickness of steel is 8 mm. The shell structure subjects to concentrated load of 1 Ton, self-weight and Live Load of 122 kg/m². Density of material was assumed 78.61 kN/m³. Roof structure should be optimized to minimize the weight of structure whilst also have Von-Mises stress lower than the allowable stress.

MATLAB is used in applying the algorithm. The sub-routine in the algorithm resemble the generation of NURBS curve, skinning of structure from 2D to 3D surface, finite element analysis based on Reissner-Mindlin plate theory and fitness evaluation based on objective function and constraint. The program stored Von-Mises stress of each structure that represent organism, fitness and design variable. The iteration will not stop until the convergence criteria have been reached. In this occasion, the convergence criteria are:

**Figure 3. Flow chart of Genetic Algorithm**
GA parameter applied in the design can be described by Table 1.

| Parameter description                     | Value   |
|-------------------------------------------|---------|
| Probability of crossover ($P_{co}$)      | 0.95    |
| Probability of mutation ($P_{m}$)        | 0.05    |
| Coefficient of penalty function ($r_1$)  | $1e(-10)$ |
| Constant of penalty function ($R$)        | $1e(4)$ |

4. Result and discussion

NURBS curve that represent the shape of oil storage roof is constructed by $N$ and $R$ from equation (1) and (2) which also called blend function and basic function. The numerical value of basic function in this research is depicted in the figure 4.

The iteration was performed in 20 generations with 30 organisms in each generation. Having completed the iterative process, the convergence criteria as described in equation (27) was reached. The fittest organism in iteration is described in Table 2.

| Organism Nos. | $x_1$ | $x_2$ | $x_3$ | Weight | Fitness | Max Von-Mises Stress |
|---------------|-------|-------|-------|--------|---------|---------------------|
| 1             | 1.18  | 2.52  | 3.94  | 79.5525 | 1.5778  | 78.118              |
| 2             | 1.26  | 2.20  | 3.94  | 79.5936 | 1.5765  | 77.936              |
| 3             | 1.22  | 2.16  | 3.90  | 79.5982 | 1.5764  | 70.886              |

Table 2 shows that changing shape of NURBS influence the Von-Moses stress and fitness of the organism. Furthermore, by comparing the result from the previous generation, the weight of structure
is getting smaller by the increase of generation as depicted by Figure 5, meanwhile the fitness of structure is increasing until reach its peak at 20th generation. It seems that more linear NURBS curve will produce lower stress. Based on the fittest individu on 20th iteration, the shape of NURBS curve can be illustrated in Figure 6.

![Figure 5](image1.png)  
**Figure 5.** Performance of algorithm to improve fitness (a) and reduce the weight (b)

![Figure 6](image2.png)  
**Figure 6.** NURBS shape of the fittest organism in the 20th iteration

During optimization, the control points of NURBS curve are highly sensitive to each other so they should be optimized simultaneously. The optimization gives a satisfactory result. However, a suitable shape is not enough for the design of a shell. When there is a limit on the height of shell, the shell
cannot form a desirable arch and then the design of shell will be governed by the bending stresses in the shell rather than membrane stresses.

5. Conclusion
In this contribution, this research outlined the framework of shape optimization by using Genetic Algorithm (GA) method that utilizes the NURBS curve. The key part of this framework is to develop a NURBS curve, skinning the shell element based on NURBS, determine the Von-Mises stress based on Finite Element Analysis, and select the fittest structure based on natural selection process. The implementation of framework in numerical problem shows that the framework performs well in determining the optimum solution because it can lead the optimization process to reach convergence. Furthermore, it was found that the optimum structure obtained from algorithm has fulfill the objective function to reduce the weight whilst also has Von-Mises stress that below the allowable stress level. In the end, NURBS technique is useful in controlling the shape of shell structure. Parametric design method under NURBS technique provides easy way in modeling the shells. Further research in this field is widely open. Adding more constraints in the optimization such as, buckling and stability issues can be very interesting subject to be explored. Utilizing Iso-geometric Analysis on FEM sub-routine also deserves attention.

6. Acknowledgment
This research was funded and supported by research scheme of Internal Research Fund of LPPM Universitas Islam Riau 2018.

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