EQUILIBRIUM, ISOSCALING AND NUCLEAR ISOTOPE THERMOMETRY RELATED TO 1 GEV PROTON INDUCED REACTIONS

M.N. Andronenko, L.N. Andronenko, W. Neubert† and D.M. Seliverstov
† Institut für Kern- und Hadronenphysik, FZ Rossendorf, 01314 Dresden, Germany

Introduction

Experimental data, related to the decay modes of systems produced in 1 GeV proton-nucleus interactions, have been analysed by methods probed in nucleus-nucleus collisions. The actual questions are addressed to the next topics: attainment of equilibrium in fragmenting systems [1], isoscaling [2] and nuclear isotope thermometry [3, 4]. With this intention we have investigated:

- reactions with cumulative production of Light Charged Particles (LCP)
- emission of Intermediate Mass Fragments (IMF) in fragmentation processes
- production of residual nuclei in spallation reactions

The present paper is a review of recent publications [5, 6, 7] based on experimental results from [8, 9, 10, 11]. Especially, the earlier and precise data [9] are suited for a comprehensive analysis. Fragment production in proton collisions with light target nuclei [10] allows to test the above mentioned questions including small nuclear systems.

Review of experimental data

The data taken into consideration were obtained in several experimental projects performed at the external proton beam of the PNPI synchrocyclotron in Gatchina.

- One experiment carried out with a lens spectrometer combined with TOF measurements was aimed to the production of LCPs detected at backward angles Θ_{lab}=109° and 156° [8]:

\[ p(1\text{GeV})+({}^6\text{Li,}{}^7\text{Li,}{}^9\text{Be,}{}^{12}\text{C,}{}^{27}\text{Al,}{}^{58}\text{Ni,}{}^{197}\text{Au,}{}^{208}\text{Pb}) \rightarrow \text{LCP}({}^1\text{H,}{}^2\text{H,}{}^3\text{H})+X. \]

- The second data set [9, 10] which was analysed involves isotopically separated IMF’s. The measurements were performed with a setup consisting of a magnetic lens spectrometer equipped with ΔE-E telescopes at Θ_{lab}=60° and 120° with respect to the beam axis. Fragment production in light targets was investigated with two TOF-E spectrometers including PPAC’s and Bragg ionization chambers installed at Θ_{lab}=30° and 126°. The following data were obtained in these experiments:

  - differential cross sections at forward and backward angles:

\[ p(1\text{GeV})+({}^{9}\text{Be,}{}^{12}\text{C,}{}^{58}\text{Ni,}{}^{197}\text{Au,}{}^{238}\text{U}) \rightarrow \text{IMF}(Z \geq 2)+X. \]

  - differential cross sections at Θ_{lab}=60°:

\[ p(1\text{GeV})+({}^{48}\text{Ti,}{}^{58}\text{Ni,}{}^{64}\text{Sn,}{}^{112}\text{Sn,}{}^{124}\text{Sn}) \rightarrow \text{IMF}(Z \geq 2)+X. \]

- The yields of spallation products at E_p=1 GeV incident energy were measured with a high-resolution gamma spectrometer [11]. These copious data allowed us to extend the analysis toward heavier isotopes up to residual nuclei with masses close to the target mass A_T considering the reactions:

\[ p(1\text{GeV})+({}^{51}\text{V,}{}^{54,56}\text{Fe,}{}^{58,60,62,64}\text{Ni,}{}^{70,76}\text{Ge,}{}^{185}\text{Rb,}{}^{109}\text{Ag,}{}^{133}\text{Cs}) \rightarrow \text{Residue}+X. \]
Probe the equilibration in fragmentation and spallation reactions

One important question for the understanding of the nuclear disintegration is the search for signals of equilibration of the emitting source. One access is related to the isotopic yield ratios of the reaction products \[5\]. In the grand canonical approach, the yield ratio \( R \) of two different isotopes \((Z, N_1)\) and \((Z, N_2)\) emitted from one source is given by:

\[
R = \left( \frac{N_2 + Z}{N_1 + Z} \right)^{3/2} \cdot \exp \left( -\frac{\varepsilon_2 - \varepsilon_1}{T} \right) \cdot \exp \left( \frac{\mu_n (N_2 - N_1)}{T} \right)
\]

(1)

where \( T \) is the equilibrium temperature, \( \varepsilon \) is the mass excess and \( \mu_n \) is the chemical potential. The chemical potential of the neutrons is assumed to be a linear function of the neutron-to-proton ratio of the combined system of the target\((Z_t, N_t)\) + projectile\((Z_p, N_p)\). Then for proton-induced reactions, the yield ratio of two isotopes with \( N_1 \) and \( N_2 = N_1 + \Delta N \) produced at the same temperature \( T \) can be expressed by

\[
R \propto \exp(c_o \cdot \Delta N \cdot N_t/(Z_t + 1))
\]

(2)

According to (2), in case of equilibrium the ratios lie in a semi-log plot \( R \) versus \( N_t/(Z_t + 1) \) on a straight line as one can see in Figs. 1 and 2. This is valid for both the fragmentation and the spallation reaction products provided that \( \Delta N \) is kept fixed.

![Fig. 1. Isotopic yield ratios in \( p(1 \text{ GeV})+A \) fragmentation reactions for target nuclei from \(^9\text{Be}\) to \(^{238}\text{U}\). The lines are exponential fits to the experimental points (open circles), the ratios for light targets are denoted by black dots](image1)

![Fig. 2. Yield ratios \( R \) of \(^{43-48}\text{Sc}\) isotopes obtained in spallation reactions induced by 1 GeV protons with the target nuclei from \(^{51}\text{V}\) to \(^{133}\text{Cs}\). The same notation as in Fig. 1 is used](image2)

The slope \( K(\Delta N) = c_o \cdot \Delta N \) is proportional to the difference of the neutron numbers \( \Delta N \) and fulfils the condition

\[
K(\Delta N)_{\Delta N=1,2,3,4,...} = 1 : 2 : 3 : 4 : ...
\]

(3)

The confirmation of relation (3) for two classes of reactions is demonstrated in Fig. 3. The fact that the ratios related to light nuclei (see black dots in Fig. 1) don’t coincide with the
fit lines responsible for heavy nuclei may be explained by differences in $N/Z$ of the combined system and the actual ratio $N/Z$ of the fragmenting system. Note that in the case of light disintegrating nuclei the values of $N/Z$ are very sensitive to changes of the proton and neutron constituents by one unit. The slope of $R(N_t/(Z_t + 1))$ dependency for light targets are similar to one for heavy targets.

This observation may be an indication in favour of a similar fragment production mechanism in light nuclei, i.e. there is no reason to refuse the statistical treatment completely.

Summarizing, we found for fragmentation and deep inelastic reactions (spallation) at 1 GeV incident proton energy a consistent behavior which may be attributed to attained equilibration characterized by an effective temperature. The exponential slopes of the isotopic yield ratios for fixed difference of the neutron numbers $\Delta N=1,2,3,...$ were found to be nearly independent of the masses of both the fragments and the emitting system.

**Isotopic scaling and its dependence on the isospin of the emitting source**

Supposing that equilibrium is attained, the grand canonical expression for the yield ratio of specific isotopes with $N, Z$ from two different emitting systems can be expanded to first order in $N$ and $Z$ [2] :

$$Y_2/Y_1 = C \cdot \exp(\alpha N + \beta Z)$$

where $\alpha = \Delta \mu_n/T, \beta = \Delta \mu_p/T$ and $\Delta \mu_n = \mu_{n2} - \mu_{n1}$, $\Delta \mu_p = \mu_{p2} - \mu_{p1}$ are the differences of the neutron (proton) chemical potentials for the two emitting systems and $T$ is the equilibrium temperature. In such a case the scaled isotopic ratios

$$S(N) = Y_2/Y_1 \cdot \exp(-\beta Z)$$

lie along a straight line on a semi-log plot $S(N)$ as function of $N$. This three-parametric isotopic scaling (called isoscaling) has been demonstrated in the restricted range $0 \leq N \leq 11$ for multifragmentation, strongly damped binary collisions and evaporation [2].

We have established the validity of such scaling behavior also for proton induced reactions in the GeV energy region [3]. The isotopic yield ratios for fragmentation products ($2 \leq Z \leq 5$) and spallation residues ($11 \leq Z \leq 28$) obtained from numerous targets, irradiated with 1 GeV and 12 GeV protons, were analysed. Many combinations of systems for both reaction types were considered. Figure 4 demonstrates the scaling behavior in the range of neutron numbers $20 \leq N \leq 33$ for proton induced spallation for one chosen combination of two emitting systems. A complete picture of combinations of yield ratios available for spallation and fragmentation products is shown in Figs. 5 and 6. Obviously, the parameters $\alpha$ and $\beta$ are different for various system combinations $Y_2/Y_1$.  

![Fig. 3.](image-url)
The coefficients $\alpha$ and $\beta$ were found to be dependent on the isoscaling factor

$$\Delta \xi = N_2/(Z_2 + 1) - N_1/(Z_1 + 1)$$

(6)

Here, $N_1$, $Z_1$ and $N_2$, $Z_2$ are the neutron and proton numbers of the source '1' and source '2', respectively. For simplicity, we assume that the nucleonic composition of the emitting source is nearly the same as in the system p+target.

On this condition we found a linear dependence of $\alpha$ and $\beta$ on $\Delta \xi$:

$$\alpha = \alpha' \cdot \Delta \xi \quad \text{and} \quad \beta = \beta' \cdot \Delta \xi$$

(7)

Figures 7 and 8 show the parameters $\alpha$ and $\beta$ versus $\Delta \xi$ together with the linear fits from which $\alpha'$ and $\beta'$ were obtained.
Using relations (6) and (7) the equation (4) can be corrected for the isoscaling factor $\Delta \xi$ of the two systems under consideration:

$$Y_2/Y_1 = C' \cdot \exp((\alpha' N + \beta' Z) \cdot \Delta \xi)$$  \hspace{1cm} (8)

In the following we use modified scaled isotopic ratios:

$$S'(N) = (\Delta \xi)^{-1} \cdot (\ln(Y_2/Y_1) - \beta Z)$$  \hspace{1cm} (9)

The corresponding results are illustrated in Fig. 9. In this case, the parameters obtained by fits of the relations (7) are taken into account. A pronounced scaling behavior becomes evident for fragmentation and spallation products. Figure 9 shows that all sets of linear dependences presented in Figs. 5 and 6 can be transformed into corresponding uniform dependences with similar slopes. On the other side, the data related to the yields of spallation products obtained with 12 GeV protons form a separate line displayed in the upper part of Fig. 9.

Summarizing the results of this section one can conclude:

- Scaling relationships of isotopic distributions have been observed for both reaction mechanisms, i.e. proton induced fragmentation and spallation
- The influence of the isospin of the emitting systems on the isoscaling parameters $\alpha$ and $\beta$ has been established
- The scaling parameters $\alpha'$ and $\beta'$ vary with the energy of the primary proton beam

An analysis of proton induced fragmentation using double isotopic yield ratios

The double yield ratio $R = R_1/R_2$ of isotopes is related to the nuclear temperature $T_{app}$ as shown by Albergo and coworkers [3]:

$$T_{app} = B / \ln(a \cdot R)$$

where the single isotope ratios are defined by the corresponding yields $Y$

$$R_1 = Y(A_i, Z_i) / Y(A_i + \Delta A, Z_i + \Delta Z)$$
$$R_2 = Y(A_j, Z_j) / Y(A_j + \Delta A, Z_j + \Delta Z)$$

Each combination of four isotopes terms an 'isotope thermometer'. The quantity $B$ is the difference of the binding energies related to the above given ratios. The parameter $a$ includes the spin degeneration factor and mass numbers of the considered isotopes.

This approach became well-known with the pioneering studies in ref. [4] aimed to prove the nuclear phase transition in nuclear matter. We applied this kind of analysis to data available from inclusive measurements at 1 GeV proton interactions with various target nuclei to study...
the mass dependence of $T_{app}$ \[7\]. This method provides the most reliable temperature estimations (so called ‘isotopic temperature’) as far as the excitation energy of the emitting system is restricted to $\simeq 3 - 5 A \cdot MeV$.

Concerning LCP production at 1 GeV proton energy, we assumed that cumulative particles originate from some statistical-like process. Figure 10 shows the results obtained with the LCP thermometer \[(^2\text{H}/^3\text{H})/(^4\text{H}/^5\text{H})\] whereby the yields of the hydrogen isotopes were measured at two angles. The temperature $T_{app}$ is nearly constant at $\Theta_{lab}=109^\circ$ within the range of target masses $6 \leq A_T \leq 208$. The lower part of Fig. 10 shows temperatures which are derived from the differential cross sections of hydrogen isotopes detected at $\Theta=156^\circ$. It should be noted that an increase of $T_{app}$ at $A_T \leq 10$ cannot be excluded. Corrections for the $\Delta$-isobaric state contributions to the differential cross section (as shown in ref. \[8\]) amount to $\leq 20\%$, but it does not change the tendency of increasing $T_{app}$.

Additional examples of temperatures obtained from He and IMF yields are plotted in Fig. 11 as function of $A_T$. This picture confirms the regular behavior of temperatures displayed in Fig. 10. The observed agreement of the studied thermometers implies that possibly each of them is suitable for relative temperature measurements without the hitherto introduced limitation $B \geq 10$ MeV. This condition was introduced in ref.\[2\] to minimize fluctuations due to contributions to the isotopic yields from sequential decays. For a given thermometer the influence of sequential decays seems to be independent on the origin of the excited primordial fragments. This behavior is rather surprising since the target mass numbers (or the volumes of the fragmenting nuclei, respectively) change by a factor of about 25.

In the next step we converted the above values of $T_{app}$ into temperatures $T$ at the breakup point as far as the correction factors for sequential decay \[13\] were available. The mean correction amounts to $\simeq 5\%$ but does not exceed $15\%$.

The target mass dependence for thermometers including isotopic pairs with $^3\text{He}/^4\text{He}$ is demonstrated in the Fig. 12. Some features of Fig. 12 are worth to discuss.

(i) The temperatures which have been derived from the differential cross sections at $\Theta_{lab}=60^\circ$...
(open circles) are larger in comparison with those obtained from production cross sections. Enhanced temperatures in forward direction and strong variations of such thermometers which involve $^3\text{He}/^4\text{He}$ ratios were also reported for heavy-ion induced reactions.

(ii) We observed some structures in the temperatures obtained from fragment yields at $\Theta_{lab}=60^\circ$ for $^{48}\text{Ti}$, $^{58}\text{Ni}$, $^{64}\text{Ni}$, $^{112}\text{Sn}$ and $^{124}\text{Sn}$ targets (open circles). These data are characterized by the low cut-off in the measured kinetic energy distributions and small corrections for the missing part of the spectra to obtain energy integrated yields. Regardless of the indicated errors, Fig. 12 shows within a limited region of $A_T$ a common systematic trend to higher temperatures with increasing ratio $N_t/Z_t$ of the target. The isotopically separated targets $^{58,64}\text{Ni}$ and $^{112,124}\text{Sn}$ form such groups which look like local fluctuations but more probably they are caused by different $N/Z$ ratios of the fragmenting nuclei.

(iii) If we consider the full range of $A_T$ from Be to U, the opposite tendency dominates: the breakup isotopic temperature decreases weakly with increasing $A_T$. The temperatures which include the ratio $^3\text{He}/^4\text{He}$ show the most pronounced decrease with increasing $A_T$. This behavior seems to be related to the exceptional properties of $^4\text{He}$ which were attributed to a predominant production by evaporation.

![Fig. 12. Isotopic temperatures $T$ as function of target mass number. Open circles: data from measurements at $\Theta_{lab}=60^\circ$. Black dots connected by solid lines are evaluated from isotope production cross sections](image)

On the other side, double ratios involving heavier isotopes ($\text{Li, Be, B}$) provide temperatures which are nearly independent on the target mass within the error bars.

Within the considered range of $A_T$ the average isotopic temperature $\langle T \rangle \simeq 4$ MeV was obtained from measured cross sections (Fig. 12, black dots). It should be noted here that thermometry analysis performed for isotopic yields of ternary charged particles accompanying low energy fission of heavy nuclei resulted in significant lower temperature $\simeq 1$ MeV.

![Fig. 13. Comparison of temperatures $T$ as function of the excitation energy $E^*/A$ derived from 1-GeV proton induced fragmentation (black dots) with IMF production in Au+Au collisions presented in the caloric curve](image)

Figure 13 gives another presentation of the obtained results: the isotopic temperature against the excitation energy. The excitation energy $\langle E^*/A \rangle$ was estimated from earlier measurements of the mass loss and the linear momentum transfer for nuclei emitting IMF’s. The data obtained from 1 GeV proton induced fragmentation show an agreement with those derived from heavy ion collisions. We mention the difference between the caloric curve, which characterizes the temperature evolution of one system as a function of the excitation energy and the black points which are presented in Fig. 13. The latter one’s belong to different excited systems in a wide range of target masses $A_T$. 

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Summarizing, different isotope thermometers employing inclusive data obtained in 1 GeV proton induced fragmentation, involving various target nuclei, were analysed.

- It was found that even thermometers which involve isotope pairs with $B < 10$ MeV provide steady results which may be suitable for relative temperature measurements.
- The estimated average breakup temperature $\langle T \rangle \simeq 4$ MeV, derived from double isotopic yield ratios of the fragmentation products at 1 GeV proton energy, is consistent with the corresponding one characterizing the plateau of the caloric curve, obtained for heavy-ion induced multifragmentation.
- The weak dependence of the isotopic temperatures on the target mass $A_T$ suggests speculations about a comprehensive applicability of thermodynamical concepts in nearly all nuclei involved in this analysis.

Overall conclusion

Fragmentation and spallation reactions induced by 1 GeV protons show features of equilibration. The statistical mode of fragment formation seems to prevail not only in heavy ion reactions below $\simeq 100$ $A$-MeV but also in proton-nucleus collisions in the GeV domain. Isospin degree of freedom plays an important role in the description of residual nucleus and fragment production.

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