Soft X-ray transient light curves as standard candles: exponential versus linear decays

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ABSTRACT

A recent paper by King & Ritter (1998: hereafter KR) proposed that the light curves of soft X–ray transients (SXTs) are dominated by the effect of irradiation of the accretion disc by the central X–rays. This prevents the onset of the cooling wave which would otherwise return the disc to the quiescent state, and so prolongs the outbursts beyond those in dwarf nova discs. KR show that the decay of the resulting X–ray light curve should be exponential or linear depending on whether or not the observed peak X–ray luminosity is sufficient to ionize the outer edge of the accretion disc. Here we examine the observed X–ray decays, and show that they are exponential or linear according as the peak luminosity is greater or smaller than the critical value defined by KR, strongly suggesting that the light curves are indeed irradiation–dominated. We show further that the occurrence of an exponential or linear decay tends to favour the same type of decay in subsequent outbursts, so that systems usually show only one or the other type. We use the equations of KR and the observed X–ray light curve to determine the size $R_h$ of the hot disc at the peak of the outburst. For exponential decays, $R_h$ is found to be comparable to the circularization radius, as expected since the disc consists entirely of material transferred from the secondary since the previous outburst. Further, $R_h$ is directly proportional to the time at which one sees the secondary maximum ($t_s$), as expected if $t_s$ is the viscous timescale of the irradiated disc. This implies that the orders of magnitude of the viscosity parameter $\alpha$ and disc aspect ratio $H/R$ are such that $\alpha(H/R) \sim 0.01$. Observation of a secondary maximum calibrates the peak luminosity and gives the distance ($D_{\text{kpc}}$) to the source as

$$D_{\text{kpc}} = 4.3 \times 10^{-5} t_s^{3/2} \eta^{1/2} f^{1/2} F_p^{-1/2} \tau_d^{-1/2}$$

where $F_p$ is the peak flux, $\tau_d$ is the $\epsilon$–folding time of the decay in days, $\eta$ is the radiation efficiency parameter and $f$ is the ratio of the disc mass at the start of the outburst to the maximum possible.

Key words: accretion, accretion discs – instabilities – stars: X–rays: stars.

1 INTRODUCTION

The soft X-ray transients (SXTs) are a subclass of the low–mass X–ray binaries in which a low–mass star transfers material to a neutron star or black hole. They undergo large outbursts, reaching X–ray luminosities of order the Eddington limit (see Tanaka & Lewin 1995 and Tanaka & Shibazaki 1996 for reviews). The outbursts have similarities to those of dwarf novae, but with important differences, one being that the timescales are much longer; a dwarf nova outburst lasts a few days and typically recurs every few weeks, whereas in SXTs the corresponding timescales are months and years. It is generally accepted that the dwarf nova outburst results from a thermal–viscous instability in the accretion disc (see Cannizzo 1993 for a review). To some extent the models used for dwarf novae can reproduce the correct timescales for the SXT outburst, but only by reducing the viscosity parameter by a factor of 10–100 and choosing a particular functional form for it (e.g. Cannizzo, Chen & Livio, 1995).

Although there are similarities between the discs during outburst in dwarf novae and SXTs, an important observed difference is that the discs in the SXTs are heavily irradiated by the central X–ray source (van Paradijs & McClintock 1994; Shahbaz & Kuulkers 1998). Recently King & Ritter (1998) have pointed out that irradiation prolongs the entire SXT outburst cycle, forcing SXT light curves to have long
exponential or linear tails, because the outburst can only shut off through the viscous decay of the central accretion, rather than via a cooling front as is the case in dwarf novae. (For counterarguments see Cannizzo 1998.) In this paper we apply some of the key features of the KR irradiation model to the X-ray light curves of SXTs.

2 OUTBURSTS OF IRRADIATED DISCS

KR show that the light curves of SXTs can be explained by a disc instability model, modified to take account of irradiation by the central X-ray source during the outburst. Irradiation prevents the disc returning to the cool state until the central accretion rate is sharply reduced. If irradiation is strong enough to ionize all of the disc out to its edge, KR show that the X-ray light curve will be a roughly exponential decay, with most of the disc mass being accreted on a viscous timescale. The recurrence time to the next outburst will be long, as the disc has to be rebuilt by mass transfer from the companion star. If instead the X-rays are too weak to ionize the whole disc, KR show that the X-ray light curve should be a roughly linear decline, returning to quiescence after a viscous timescale. In this case much of the disc mass is not accreted, allowing the next outburst to begin after a shorter recurrence time than if there had been an exponential X-ray decay.

2.1 The critical luminosity

KR give expressions for the critical central accretion rates required to ionize the disc out to a given radius $R_h$. There are two rates, depending on whether the central source is point-like or disc-like. In the second case irradiation is weaker by a typical factor $H/R$, since a disc-like source is foreshortened by this factor as seen by matter at the disc surface. As a result the critical accretion rate (and thus luminosity) is higher by a factor $\sim R/H$ than in the point-like case. The critical rates (14) and (15) of KR are equivalent to critical luminosities

\[ L_{\text{crit}}^{(\text{point-like})} = 3.7 \times 10^{36} R_h^2 \text{ ergs s}^{-1} \]  
\[ L_{\text{crit}}^{(\text{disc-like})} = 1.7 \times 10^{37} R_{T}^2 \text{ ergs s}^{-1} \]  

(1) (2)

Here $R_{T}$ is the ionized disc radius $R_h$ in units of $10^{11}$ cm. The maximum possible value of the outer disc radius, and hence the radius $R_h$ of the ionized region, is given by the tidal radius $R_T$ of the disc (we use 80 per cent of the Roche lobe of the primary in the applications in this paper, but it should be noted that the fraction can vary between 70 and 90 per cent). If the accretor is a neutron star it must behave as a point-like source, as its surface will be heated by accretion. If the accretor is a black hole it is possible for it to behave as a disc-like accretor (the relevant formulae were labelled ‘NS’, ‘BH’ in KR). Indeed this effect probably accounts for the very strong tendency of black-hole systems with low-mass companion stars to be transient (King, Kolb & Szuszkiewicz, 1997): the mean mass transfer rate $-\dot{M}_2$ lies below $L_{\text{crit}}^{(\text{disc-like})}$ for all black-hole systems, almost irrespective of the nature of the companion star, so that a persistent state with a stably irradiated disc is impossible for them. However, during outbursts, all transients develop a hard power-law X-ray component which is very probably a central X-ray corona, in addition to the softer blackbody-like component which probably comes from the disc. Thus it is likely that even black-hole accretors become point-like X-ray sources during SXT outbursts, making $L_{\text{crit}}^{(\text{point-like})}$ given by (1) the correct formula to use during the outburst. This is the procedure adopted in this paper (see Fig. 1). Note that the use of $L_{\text{crit}}^{(\text{point-like})}$ during outbursts and $L_{\text{crit}}^{(\text{disc-like})}$ for the mean behaviour is consistent, provided that the power-law coronal component becomes weaker than the blackbody component once the central accretion rate $\dot{M}_2$ declines below the mean mass transfer rate $-\dot{M}_2$ near the end of an outburst.

Equipped with the appropriate value of $L_{\text{crit}}$, we can now predict the form of the light curve decay. If the observed peak X-ray luminosity is less than the value of $L_{\text{crit}}$ corresponding to its outer radius, the outer edge of the disc is not strongly irradiated and the decay will be linear. If however the observed luminosity is greater than this critical luminosity, it is strong enough to ionize the outer parts of the disc and the decay will initially be exponential. Both kinds of decay are observed.

2.2 Linear decays

In a linear decay the X-ray luminosity obeys

\[ L_X = \eta c^2 \left( \frac{3 \nu}{B_1} \right)^{1/2} \left[ \frac{M_h^{1/2}(0) - \left( \frac{3 \nu}{B_1} \right)^{1/2}}{t} \right]. \]  

(3)

(using eqn 23 of KR), where the decay started at time $t = 0$. Here $\nu$ is the kinematic viscosity near the edge of the heating front, $M_h(0)$ is the heated mass at the start of the outburst (peak luminosity), and

\[ B_1 = 4 \times 10^5 \text{ (ergs)} \]  

Thus the peak value $L_p$ of $L_X$ is given by setting $t = 0$, and gives

\[ M_h(0) = \frac{B_1 L_p^2}{3c \eta^2} \]  

(5)

Defining $t_{1/2}$ as the time for $L_X$ to fall by a factor 2 we get

\[ \nu = \frac{B_1 L_p}{6 \eta t_{1/2}} \]  

(6)

Combining these two relations and measuring $t_{1/2}$ in days we find

\[ M_h(0) = \frac{2L_p t_{1/2}}{\eta c^2} = 1.9 \times 10^{-10} L_p \eta^{-1} t_{1/2} \text{ g} \]  

(7)

and

\[ \nu = 2.14 \times 10^{-27} B_1 L_p \eta^{-1} t_{1/2}^{-1} \text{ cm}^2 \text{ s}^{-1} \]  

(8)

Thus given the peak X-ray luminosity and the time taken for the flux to drop by a factor 2 we can determine the ionized disc mass at the start of the outburst and the disc viscosity. Since for a linear decay $L_X$ lies below the critical value which would ionize the whole disc (by definition), the radius of the ionized disc at the start of the outburst is given by

\[ R_h(0)^2 = \frac{B_1 L_p}{\eta c^2} \]  

(9)
A linear decay in an XRT implies that the ratio $L_X/(\text{outer disc radius})^2$ is smaller than a critical value. This can happen either because (a) the central X-rays are weaker than usual, or (b) the disc is very extended, which in turn requires the tidal radius $R_T$ to allow a large disc radius. Case (a) is seen in Aql X-1 (see section 4.1); the 19-hr period means a relatively small disc, but the X-ray luminosity only reaches peak values $L_\nu < 10^{36}$ erg s$^{-1}$. Case (b) must apply in sufficiently long-period binaries: the disc cannot be smaller than the circularization radius, which increases with orbital period, while the X-ray luminosity cannot greatly exceed the Eddington limit. An example is GRS 1744–63, $(P = 11.8$ days; the longest period of any SXT) which is linear right from the start of the observed outburst decline $(\dot{M}_c = 1.5 \times 10^{-8}$ $M_\odot$ yr$^{-1}$ cf $\dot{M}_{\text{crit}}$ (point – like) $= 2.2 \times 10^{-8}$ $M_\odot$ yr$^{-1}$).

### 2.3 Exponential decays

For systems in which the peak X-ray luminosity is strong enough to keep the whole disc in a hot state, KR show that the X-rays decrease exponentially, i.e.

$$L_X = \eta c^2 R_h \nu f \exp(-t/\tau),$$

(10)

where $\rho \approx 3 \times 10^{-8}$ g cm$^{-3}$, $f$ is the ratio of the disc mass at the start of the outburst to the maximum possible, and

$$\tau = \frac{R_h^2}{3 \nu^2},$$

(11)

We can fit the form of (10) at $t=0$ to the observed X-ray light curve, to give the normalisation of the light curve and the decay time allows one to determine $\nu$ and $R_h$. From (10) at $t=0$ we get

$$L_\nu = \rho c^2 R_h \nu f,$$

(12)

and combining this equation with (11) gives

$$\nu = 1.68 \times 10^{-11} L_\nu^{2/3} \eta^{-2/3} f^{-2/3} \tau_d^{-1/3} \text{ cm}^2 \text{ s}^{-1},$$

(13)

and

$$R_h = 2.09 \times 10^{-3} L_\nu^{1/3} \eta^{-1/3} f^{-1/3} \tau_d^{1/3} \text{ cm},$$

(14)

where $\tau_d$ is $\tau$ measured in days.

Equation (11) is equivalent to the statement that the hot disc mass $M_h$ decays exponentially on the viscous timescale $\tau$, i.e.

$$L_X = \eta c^2 \frac{M_h}{\tau},$$

(cf eqn 4 of KR), so the total heated mass at the start of the outburst is

$$M_h(0) = \frac{L_\nu \tau}{\eta c^2} = 1.62 \times 10^{-16} L_\nu \eta^{-1} \tau_d \text{ g}$$

(16)

Note that this equation is very similar to equation (7), i.e. the mass of the hot disc in the linear case. Combining the first form of this equation with equations (12) and (11) shows that the heated mass at the start of the outburst is

$$M_h(0) = \frac{\rho f R_h^3}{3}$$

(17)

The maximum possible value of this mass is given by assuming that the disc has the critical density everywhere ($f = 1$), and that its outer radius has the largest possible size, i.e. $R_h = R_T$. This gives

$$M_{\text{max}} = \frac{\rho R_T^3}{3},$$

(18)

where $\rho \approx 3.0 \times 10^{-8}$ g cm$^{-3}$ and $R_T$ the tidal radius. Since the latter quantity is a function of binary parameters, this value can be compared with the observational estimate (16).

### 3 CONDITIONS IN THE QUIESCENT DISC

We have seen that exponential decays are expected when the whole of the accretion disc can be kept in the hot state by the central X-rays. In this case a substantial fraction of the disc mass is likely to be accreted in the course of the outburst. This in turn suggests a substantial recurrence time before the disc can be rebuilt and produce the next outburst. By contrast, in a linear decay, the outer disc remains cool and is essentially unaffected by the outburst. Since it probably contains a large fraction of the disc mass, viscous evolution can produce the critical density again somewhere within the disc fairly quickly, allowing another outburst. Usually, such repeated linear outbursts can be expected to deplete the disc mass faster than it is refilled by mass transfer. Thus after a series of linear outbursts, the disc may well enter a much longer quiescent state in which the disc is rebuilt. This kind of ‘supercycle’, with a long quiescent state alternating with a rapid series of linear outbursts, is reminiscent of the outburst behaviour actually observed in linear-decline systems such as GRO J1744–28: no outbursts at all were seen by any instrument until the first observed one in 1996, yet this was followed by a further outburst only $\sim 1$ yr later (Kouveliotou & van Paradijs 1998).

A more subtle point concerns the size of the rebuild disc after an exponential decay. Since this disc consists almost entirely of material transferred from the secondary since the last outburst, and there has been negligible accretion on to the central star in this time, its mean specific angular momentum $j$ must be precisely the same as that transferred through the inner Lagrange point $L_1$, i.e.

$$j = (GM_1 R_{\text{circ}})^{1/2},$$

(19)

where $M_1$ is the primary mass and $R_{\text{circ}}$ is by definition the circularization radius, i.e. the radius of the Keplerian circle having the same specific angular momentum as $L_1$: typically $R_{\text{circ}} \sim 0.5 R_T$. [Eqn 19] does not hold for a disc which has undergone accretion on to the central star, as it increases $j$ above $(GM_1 R_{\text{circ}})^{1/2}$ by selectively ridding itself of low-angular-momentum material.] Thus given the surface density $\Sigma(R)$ of a post-explosional disc, the condition (10) fixes its outer edge $R_{\text{out}} = R_h(0)$ when the outburst starts through the requirement

$$2\pi \int_0^{R_{\text{out}}} \frac{\rho(R) R^2 \Sigma R dR}{2\pi \int_0^{R_{\text{out}}} \Sigma R dR} = (GM_1 R_{\text{circ}})^{1/2}.$$  

(20)

With $\Sigma \propto R^a$ we find

$$R_{\text{out}} = \left(\frac{2a + 5}{2a + 4}\right)^2 R_{\text{circ}},$$

(21)
A quiescent disc generally has \( n \approx 1 \), giving \( R_{\text{out}} \approx 1.36R_{\text{cic}} \sim 0.7R_T \). Thus we expect a rather small disc at the beginning of the next outburst after an exponential decay. Once the outburst begins the disc will try to evolve viscously to a more centrally-condensed state \( n < 0 \), and so will tend to spread towards \( R_T \) in order to conserve angular momentum. However this only occurs when the outburst is well advanced, and the values of \( R_b(0) \) we find for exponential decays should be significantly smaller than \( R_T \). Of course for a disc left over after a linear decay we would expect a radius comparable with \( R_T \) in any case. This bimodal behaviour means that each type of outburst tends to produce conditions favourable for the same type of decay in the following outburst. Thus an exponential decay will mean a small disc, which is more easily kept fully irradiated, making an exponential decay likely in the next outburst. Similarly, a linear decay leaves behind a large disc, tending to keep subsequent decays linear also.

### 4 APPLICATION TO THE SXTS

In this section we consider the decline from outburst of a sample of SXTs. The X-ray light curves which most reflect the bolometric luminosity lie in the 0.4–10 keV range (see Chen, Shrader & Livio 1997, hereafter CSL). Therefore we only consider systems with X-ray light curves in this energy range. Systems where only very hard X-ray light curves are available are more difficult to interpret e.g. GRO J0422+32 and GRS 1009–45. Also it should be noted that the value for the observed luminosity depends on the distance to the source. This will introduce some uncertainty, in particular in the poorly studied or faint systems where reliable distances are not available. We further restrict our sample to SXTs with known orbital periods, as this information is required in order to calculate the tidal and circularization radii and thus predict the critical X-ray luminosity needed to keep the outer disc edge ionized. In Table 1 we give the peak X-ray luminosities and decay timescales for the SXTs which have well sampled X-ray light curves and for which the distance and orbital period is known.

Fig. 1 shows the critical luminosity \( (L_{\text{crit}}) \) for point-like central sources (eqn 7) plotted against orbital period for total binary masses \( M = 2M_\odot, M = 10M_\odot \), typical for neutron-star and black-hole systems respectively. In each case there are two lines, corresponding to the fact that the disc radii are typically \( R_T \) and \( 0.7R_{\text{cic}} \) for exponential and linear decays respectively. Accordingly there is a luminosity band in which exponential and linear decays can coexist, even for the same assumed type of central X-ray source. In Fig. 1 we assume that the disc radius is a factor 0.7 smaller in the exponential case compared with the linear one, so that this luminosity band is just a factor 1/0.7^2 ∼ 2. If the peak observed luminosity (in the 0.4–10 keV range) is larger than the relevant value of \( L_{\text{crit}} \), the initial decline of the outburst is expected to be exponential, becoming linear as the luminosity declines through \( L_{\text{crit}} \). Outbursts in which \( L_p \) is less than the relevant \( L_{\text{crit}} \) should be entirely linear (see KR). We also show in Fig. 1 the observed peak X-ray luminosity for those SXT outbursts where we can determine the shape of the decline. The data are taken from CSL and the RXTE database. As expected, the outbursts divide into exponential and linear for peak luminosities above and below the relevant \( L_{\text{crit}} \).

In Table 2 we determine the physical parameters for accretion discs in SXTs with known orbital periods and where the shape of the outburst decline is known.

#### 4.1 Aql X–1 (=4U1908+005)

The (February 1997 and August 1997) outbursts of Aql X–1 both had peak luminosities much less than \( L_{\text{crit}} \) for a neutron star. Thus we expect the decline of these outbursts to be entirely linear; as demonstrated by Fig. 2. However, the 1978 outburst of Aql X–1 (Charles et al. 1980) is expected to be exponential, since it lies above \( L_{\text{crit}} \) for a neutron star. CSL interpret this outburst as having an initial plateau i.e. its initial decay is exponential. Note that this would be consistent with the model of KR. It should also be noted that

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**Table 1. Parameters for the SXTs**

| Source     | Type | Year | \( \log F_p \) (ergs s\(^{-1}\) cm\(^{-2}\)) | D (kpc) | \( \log L_p \) (ergs s\(^{-1}\)) | \( \tau_d \) (days) | \( \frac{P_{\text{orb}}}{P_{\text{orb}}^\text{crit}} \) | \( R_T \) (10\(^{11}\) cm) |
|------------|------|------|---------------------------------|---------|---------------------------------|-------------------|----------------|----------------|
| SAX J1808.4–3658 E | 4/1998 | -8.62 | 4.0 | 36.66 | 8.4 | 2.0 | 0.2 |
| GRO J0422+32 E | 7/1992 | -7.46 | 2.0±0.5 | 37.22 | 40.1 | 5.1 | 0.9 |
| A0620–00 E | 8/1975 | -6.00 | 0.9±0.4 | 37.99 | 26.3 | 7.8 | 1.2 |
| GS 2000+25 E | 5/1988 | -6.71 | 2.0±1.0 | 37.97 | 30.1 | 8.3 | 1.3 |
| GS 1124–683 E | 1/1987 | -6.77 | 4.0±1.0 | 38.51 | 28.3 | 10.4 | 1.3 |
| Cen X–4 E | 5/1979 | -7.11 | 1.2±0.3 | 37.13 | 4.8 | 15.1 | 1.1 |
| Aql X–1 L/E | 5/1978 | -7.50 | 2.3±0.1 | 37.30 | 32.6/35.7 | 19.0 | 1.3 |
| Aql X–1 L | 2/1997 | -8.08 | 2.3±0.1 | 35.72 | 15.0 | 19.0 | 1.3 |
| Aql X–1 L | 8/1997 | -8.28 | 2.3±0.1 | 35.52 | 15.0 | 19.0 | 1.3 |
| 4U 1543–47 E | 3/1971 | -7.24 | 8.0±1.0 | 38.65 | 42.7 | 29.6 | 2.3 |
| GRO J1655–40 L | 6/1997 | -7.40 | 3.2±0.2 | 37.69 | 30.0 | 62.9 | 4.4 |
| GRO J1744–28 L | 1/1997 | -7.72 | 6.5±1.5 | 37.98 | 30.0 | 283.2 | 7.6 |

E and L refer to exponential and linear decays respectively.

For the linear decays \( \tau_d \) is \( t_1/2 \).

The distance to Aql X–1 is taken from Shahbaz et al. (1998).
the X-ray light curve of the 1978 outburst is best fitted with a linear decline (see Fig. 2), rather than the commonly assumed exponential decay. However, the uncertainties in the data are large and so the true form of the decay may be masked.

For the 1997 outbursts of Aql X–1 the values of $M_\text{b}(0)$ (2.3 \times 10^{23} \text{ g} and 1.5 \times 10^{23} \text{ g} for the February and August outbursts respectively) are much smaller than one would infer from assuming that the disc had almost the critical surface density all the way out to $R_\text{p}$ [5.5 \times 10^{24} \text{ g} and 2.9 \times 10^{24} \text{ g} for the February and August outbursts respectively]; the maximum possible hot disc mass is given by equation (14). This may imply that not much of the mass in the disc is accreted, and so there is sufficient mass left in the disc after the outburst, such that it may be triggered again much sooner than otherwise expected. Alternatively, it may be that the disc simply never reached its maximum mass before the surface density reached its critical value and triggered an outburst. However, the second possibility would not result in an “outside–in” outburst (which requires a high disc mass), as is observed in the 1997 August outburst (Shahbaz et al. 1998). The first possibility seems to be consistent with the short observed recurrence time; Aql X–1 shows quasi–periodic outbursts every 309 and 125 days (Kitamoto, Tsunemi & Miyamoto 1993). If we take the recurrence time to be 0.6 yr and use equation (35) of KR for linear decays, we obtain a mass transfer rate $\sim 2 \times 10^{-10} M_\odot \text{ yr}^{-1}$. Note that the estimate of the mass transfer rate is roughly what is expected for a subgiant filling its Roche lobe in a binary of this period (see King, Kolb & Burderi 1996 and CSL).

4.2 Cen X–4 (=4U1456–32)

For Cen X–4 with an orbital period of 15 hrs we find $L_\text{crit} = 1.6 \times 10^{37} \text{ erg s}^{-1}$ (using eqn. 11). Both the 1969 and 1979 outbursts (Evans et al. 1970; Kaluzienski et al. 1980) had peak luminosities higher than $L_\text{crit}$ for a neutron star, so one expects exponential decays. In Fig. 3 we show exponential fits to the decline of the 1969 and 1979 outbursts of Cen X–4, exactly as expected (see Fig. 1).

4.3 4U1543–47

The 1971 March X-ray outburst light curve of XN 1543–47 is best fitted with an initial exponential decay (see Fig. 3); the final part of the decay may be linear but it is much harder to determine because of the presence of the secondary maxima. $L_\rho (= 4.5 \times 10^{38} \text{ erg s}^{-1})$ is higher than $L_\text{crit}$ (point-like) for a 8M$\odot$ binary (we have used a distance of 8 kpc; see Orosz et al. 1997) and so the compact object is most probably a black hole.

5 THE SECONDARY MAXIMUM; A STANDARD CANDLE

The monotonic decay of X-ray/optical light curves of many of the SXTs are often interrupted by a flux increase of a factor of 2 or more over a timescale $\lesssim 10$ d, followed by the resumption of the normal decay. The secondary maximum has predominantly been seen in the BH SXTs. However, it should be noted that a closer inspection of the X-ray light curves of Cen X–4 (Kaluzienski et al. 1980), Aql X–1 (see Fig. 3) and GRO J1744–28 (Kouveliotou & van Paradijs 1998) also show similar secondary maxima. Note that GRO J1655–40 is the only BH SXT that shows a linear decay (see Fig. 4).

We define the time in days after the outburst peak that the secondary maxima occurs as $t_\text{s}$. As KR point out, these secondary maxima features may be caused by material in the outer disc triggering small outbursts during the linear decay phase. A related idea is simply that the increase in viscosity when irradiation begins is more marked in the outer regions than further in, causing a ‘pulse’ of extra mass to move inwards. In both cases the secondary maximum should appear one irradiated–state viscous time after the initial outburst. In Table 2 we determine $t_\text{s}$ and $R_\text{b}(0)$ for those SXTs in which the secondary maximum has been observed, which we plot in Fig. 5. Note that all data points from exponential decays lie in a straight line, whereas the data from linear decays are systematically higher. This may suggest that there is a different relation for linear decays, which may not be so surprising as one would expect the density wave to behave differently depending on whether or not it started in the outermost part of the original disc. Only more observations of linear decays with secondary maxima will resolve this.

A least squares linear fit to the data points from exponential decays gives

$$R_\text{b} = 0.128(\pm 0.007)10^{10} t_\text{s} \text{ cm};$$

where the fit has a correlation coefficient of 0.92 and 1–σ errors are given (a constant term in the fit is only significant at the 34 percent confidence level). i.e. this fit is consistent with the physical requirement that it pass through the origin. Thus given a complete X-ray outburst of a SXT then $t_\text{s}$ is determined by the observation of a secondary maximum. This can then be used in eqn. (14) to predict the peak luminosity of the outburst. For an irradiated disc the distance to the source ($D_{\text{kpc}}$) can then be determined given the observed peak X-ray flux $F_\nu$. Thus for exponential decays we obtain

$$D_{\text{kpc}} = 4.3 \times 10^{-5} t_\text{s}^{3/2} \eta^{1/2} f^{1/2} F_\nu^{-1/2} \tau_d^{-1/2}$$

5.1 The viscosity

The gradient of the $R_\text{b}$ versus $t_\text{s}$ fit is related to the rate with which $R_\text{b}$ decreases, i.e. the speed of the density wave producing the secondary maxima. The speed of this wave is given by the radial drift velocity

$$V_\nu = \frac{3\nu}{2R} = \frac{3}{2} \alpha c_s \frac{H}{R}$$

where $\alpha$ is the accretion disc viscosity parameter, $c_s$ is the equatorial–plane sound speed in the hot gas and $H$ is the disc scale height at disc radius $R$. For an irradiated disc the temperature at the edge of the density wave is given by

$$\log T = -4.79 + 0.25 \log L_\nu - 0.5 \log R_\text{b} \text{ K}$$

(van Paradijs 1996) which then gives $c_s$. We find a mean $c_s$ of 1.0 \times 10^6 \text{ cm s}^{-1} (see Table 2), in line with our expectation that the outer region of the disc must be close to but just above the ionization temperature $\sim 6500 – 10,000$ K. From the gradient of eqn. (22) we find $V_\nu = 1.5 \times 10^6 \text{ cm s}^{-1}$, implying that the average aspect ratio and viscosity parameter for SXTs in outburst have orders of magnitude such that...
Table 2. Derived physical parameters for the SXT accretion discs

| Source          | $t_s$ (days) | $\nu$ ($10^{15}$ cm$^{-2}$ s$^{-1}$) | $M_h$ (g) ($10^{23}$) | $R_h$ (cm) ($10^{10}$) | $c_S$ ($10^{6}$ cm s$^{-1}$) | $f^{1/3} R_h(0)/R_T$ (%) | $M_h(0)/M_{max}$ (%) |
|-----------------|--------------|-------------------------------------|-----------------------|------------------------|----------------------------|--------------------------|----------------------|
| SAX J1808.4–3658| E 13         | 0.1                                 | 0.24                  | 1.3                    | 1.2                        | 57                       | 18.4                 |
| GRO J0422+32    | E 38         | 0.2                                 | 8.3                   | 4.4±0.7                | 1.1                        | 50                       | 13                   |
| A0620–00        | E 54         | 0.6                                 | 24.7                  | 6.3±1.1                | 1.2                        | 53                       | 15                   |
| GS 2000+25      | E 75         | 0.5                                 | 27.0                  | 6.5±2.1                | 1.2                        | 51                       | 13                   |
| GS 1124–683     | E 74         | 1.3                                 | 87.9                  | 9.6±1.6                | 1.4                        | 73                       | 40                   |
| Cen X–4         | E 12         | 0.2                                 | 0.4                   | 1.6±0.3                | 1.0                        | 15                       | 0.3                  |
| Aql X–1 5/1978  | L -          | 3.5                                 | 8.3                   | 24.3±2.4               | 1.0                        | 193                      | 4.2                  |
| Aql X–1 5/1978  | E -          | 0.1                                 | 4.6                   | 3.6±0.4                | 1.0                        | 28                       | 2.3                  |
| Aql X–1 2/1997  | L -          | 2.0                                 | 1.0                   | 12.5±1.2               | 0.8                        | 99                       | 0.5                  |
| Aql X–1 8/1997  | L 18         | 1.3                                 | 0.6                   | 9.9±1.0                | 0.8                        | 79                       | 0.3                  |
| 4U 1543–47      | E 15         | 1.4                                 | 183.1                 | 12.2±1.0               | 1.3                        | 53                       | 15                   |
| GRO J1655–40    | L 40         | 9.3                                 | 42.3                  | 46.7±1.9               | 0.8                        | 105                      | 0.5                  |
| GRO J1744–28    | L 52         | 17.1                                | 39.1                  | 53.2±10.6              | 0.8                        | 70                       | 0.1                  |

For GRO J0422+32 we use the UV secondary maxima seen after the BATSE peak (Shrader et al. 1994).

For GRO J1744–28 a secondary maximum is seen in the BATSE data (Kouveliotou & van Paradijs 1998).

\[ \alpha(H/R) \sim 0.01. \]  \hspace{1cm} (25)

A typical high-state value for $\alpha$ is of order 0.1, so this result is consistent with the expected aspect ratio $H/R \sim 0.1$ of the outbursting disc.

5.2 Tra X−1 (4U1524–62)

The 1974 November X−ray outburst light curve of Tra X−1 is best fitted with an exponential decay (see Fig. 3). There is a secondary maximum present $\sim 52$ days after the outburst peak; the peak observed flux is $3.0 \times 10^{-9}$ ergs s$^{-1}$ cm$^{-2}$ and $\tau_0 = 57.4$ days. Using equation (25) below we estimate $D_h = 16$ and 12 for the neutron star and black hole cases respectively.

Using the lower limit to the distance we can estimate the absolute magnitude of the secondary star. Given $V \sim 21$ and $A_V = 2.4$ mags (CSL) we find $M_V > +3.2$. The secondary star is either a main sequence star later than F or a subgiant later than K5; the latter seems to be more probable.

5.3 SAX J1808.4–3658

The 1998 April X−ray outburst light curve of the X−ray pulsar/transient SAX J1808.4–3658 is best fitted with an exponential decay (see Fig. 6). There is also a secondary maximum present $\sim 13$ days after the outburst peak. Given the observed peak flux $2.38 \times 10^{-9}$ ergs s$^{-1}$ cm$^{-2}$ (Gilfanov et al. 1998) and $\tau_0 = 8.4$ days, using equation (24) we estimate the distance to be 5.5 kpc, assuming $f=1.0$. However, if $f=0.5$, then the we obtain 3.9 kpc which agrees very well with the distance estimate obtained from assuming that the type 1 X−ray burst was Eddington limited (in’t Zand et al. 1998).

6 CONCLUSIONS

We have shown that the X−ray light curves of soft X−ray transients are divided into exponential or linear according as their peak luminosities lie above or below the critical value which can maintain the outer disc in the hot state. This is as expected in the model of King and Ritter (1998) for transient outbursts. We were also able to estimate the radius and mass of the heated disc region. For exponential decays these are consistent with the idea that the disc is filled to a significant fraction of the maximum possible mass before the outburst is triggered, presumably near its outer edge. This edge is found to be close to the circularization radius, as expected.

For exponential decays the secondary maximum appears to occur one irradiated−state viscous time after the start of the outburst, as suggested by King and Ritter (1998). This result can be used to provide an estimate of the distance to the system.

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Figure 1. The critical luminosity needed to ionize the entire disc. The dotted and solid lines show the critical luminosities for binaries with total mass $M = 2M_\odot, 10M_\odot$, typical for neutron-star and black-hole systems respectively. The critical luminosities are a factor 2 smaller for exponential decays than for linear ones because the disc radii in these two cases are assumed to differ by a factor 0.7, corresponding to the circularization radius and tidal radius respectively. The systems given in Table 1 are plotted here, i.e. the SXTs with known orbital periods and whose outburst decline shape can be determined. In order of increasing orbital period, the SXTs shown are SAX J1808.4–3658, GRO J0422+32, A0620–00, GS2000+25, GS1124–68, Cen X–4, Aql X–1, 4U1543–47, GRO J1655–40 and GRO J1744–28. E and L denote observed exponential and linear decays respectively.
Figure 2. The X-ray light curves of Aql X–1. The top panel shows the 1978 (Ariel 5/ASM) outburst. The bottom panel show the RXTE February and August 1997 outbursts. The February outburst has been shifted vertically by 10 counts/sec for clarity. We show on all the light curves linear fits to the decay.
Figure 3. X-ray light curves of SXTs. Top left: 1971 (Vela 5) X-ray outburst of 4U1543–47; top right: 1969 (Ariel 5/ASM) and 1979 (Vela 5) X-ray outbursts of Cen X–4; bottom left: 1974 (Ariel 5/ASM) X-ray outburst of 4U1524–62; bottom right: 1997 RXTE X-ray outburst of GS1354–64. E and L denote exponential and linear fits respectively. The data were taken from CSL.
Figure 4. The RXTE/ASM 1996 X-ray light curve GRO J1655–40. The secondary maximum can clearly be seen. A linear fit is also shown.
Figure 5. The radius of the heating front at the peak of the outburst ($R_h$) versus the time of the secondary maximum after the peak of the outburst ($t_s$). The stars and crosses are data taken from exponential and linear decays respectively. The left and right ordinate refer to the exponential and linear decay data points respectively. A linear fit to the exponential decays is also shown.
Figure 6. The RXTE/PCA 1998 X-ray light curve SAX J1808.4–3658 (Gilfanov et al. 1998). The secondary maximum can clearly be seen. An exponential fit is also shown.