Research Article

Radar Circular Data Analysis Using a New Watson’s Goodness of Test under Complexity

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Watson’s test is applied to test either the given angular data follows the specified distribution or not. The existing Watson’s test under classical statistics is applied when there is no uncertainty and indeterminacy in sample size or angular data. Under indeterminacy, the existing Watson’s test cannot be applied for testing purposes. Neutrosophic statistics is an alternative to classical statistics for this kind of situation. The Watson’s test under neutrosophic statistics is proposed in this paper. The test statistic of Watson’s test is introduced first. The operational procedure of the proposed Watson’s test is discussed with the help of radar data. From the data analysis and simulation study, it is concluded the proposed Watson’s test is efficient than the existing Watson’s test.

1. Introduction

The data obtained from devices or systems follows any statistical distribution. For efficient prediction and estimation, the decision-makers are interested to investigate which statistical distribution is suitable for the data. [1] suggested, “when we assume that our data follows a specific distribution, we take a serious risk. If our assumption is wrong, then the results obtained may invalid.” Therefore, the goodness of fit tests is performed for testing either the data follows the specified theoretical distribution or not. These tests are performed using cumulative distribution function or probability density function of the theoretical distribution. [2–10] worked on the goodness of fit tests with applications using various types of data sets.

Usually, the statistical tests are applied to the data obtained from the linear scale. In practice, the data obtained from metrology, space, radars, and ecology are circular observations that are measured in radian or degree. [11] mentioned, “Circular data need special treatment in data analysis: consider that an angle of 355° is much nearer to an angle of 5° than it is to an angle of 330°, and so simple arithmetic mean for example can be quite misleading.” The circular tests are applied for testing the randomness of circular data. [11–22] contributed in designing statistical tests for circular data.

The presence of uncertainty and indeterminacy in the sample size, parameters, and the observations lead to apply the fuzzy-based tests for testing purpose. The fuzzy-based tests are quite flexible and widely applied under the uncertain environment in a variety of fields. [23, 24–29] presented various tests using the fuzzy approach.

[30] introduced neutrosophic logic as a generalization of fuzzy logic. The efficiency of neutrosophic over fuzzy logic and interval-based analysis can be read in [31]. The applications of neutrosophic can be seen [32–36]. The classical statistics cannot be applied when vague, uncertain, and indeterminate observations are presented in the data. To overcome the issue, [37] used neutrosophic logic to introduce neutrosophic statistics. More information of neutrosophic statistics can be seen on the websites https://archive.org/details/neutrosophic-statistics?tab=about and https://archive.org/details/neutrosophic-statistics?tab=collection.

The neutrosophic statistics is more informative than the
classical statistics. [38–44] worked on neutrosophic statistics with applications. Recently, [45] introduced a neutrosophic test for circular data.

The existing Watson’s test under classical statistics cannot be applied when uncertainty is presented. By exploring the literature and best of our knowledge, no work on Watson’s test under neutrosophic statistics is done so far. In this paper, we will introduce Watson’s test under neutrosophic statistics. The proposed Watson’s test is mainly aimed at testing either the given circular data follows the given theoretical distribution. The application of the proposed Watson’s test will be given with the help of data measured from radars. It is expected that the proposed Watson’s test will be efficient, flexible, and informative than the existing Watson’s test.

2. Neutrosophic Watson’s \( U_{2n}^2 \) Test

The existing Watson’s \( U_n^2 \) test under classical statistics is applied to test either the given random sample of angular values is fitted to the specified distribution. The existing Watson’s \( U_n^2 \) test is applied when all circular observations in the data are determined, exact, and uncertain. In the case when the circular observations in the data are uncertain, imprecise, and are in intervals, the existing Watson’s \( U_n^2 \) test under classical statistics cannot be applied for fitting the given distribution. In this section, neutrosophic Watson’s \( U_{2n}^2 \) test will be introduced to fit neutrosophic angular values to the given specified distribution. The methodology of the proposed Watson’s \( U_{2n}^2 \) test by following [14] is discussed as follows.

Suppose that \( \Phi_{1n}, \Phi_{2n}, \Phi_{3n}, \ldots, \Phi_{nn} \) be a random sample of neutrosophic angular values of size \( nn \). The neutrosophic forms of angular values and neutrosophic sample size are given as follows: \( \Phi\_N = \Phi_{1n} + \Phi_{2n}, \Phi_{3n}, \ldots, \Phi_{nn} \) and \( nn = n_1 + n_2 + n_3, \ldots, n_n \), respectively. Note that \( \Phi_{1n}, \Phi_{2n}, \Phi_{3n}, \ldots, \Phi_{nn} \) denote the determined and indeterminate parts, respectively, and \( I_{n_1} = I_{n_2}, I_{n_3}, \ldots, I_{n_n} \) are the associated measure of uncertainty. To implement the proposed test, the first step is to arrange neutrosophic angular data \( \Phi_{1n}, \Phi_{2n}, \Phi_{3n}, \ldots, \Phi_{nn} \) in ascending order \( \Phi_{1n} \leq \Phi_{2n} \leq \cdots \leq \Phi_{nn} \) with respect to midvalues of each interval of \( \Phi_N \). Let \( FN(\Phi_N) \) be a neutrosophic cumulative distribution function (ncdf) of the given theoretical distribution.

Let

\[
V_N = F_N(\Phi_N), i = 1, 2, 3, \ldots, n_n. \tag{1}
\]

The neutrosophic average of \( V_N \) is computed as follows:

\[
\bar{V}_N = \frac{\sum_{i=1}^{n_n} V_N^n}{n_n}; n_n = [n_1, n_2, \ldots], \bar{V}_N \in [V_L, V_U]. \tag{2}
\]

The test statistic of the proposed Watson’s \( U_{2n}^2 \) is given by

\[
U_{2n}^2 = \sum_{i=1}^{n_n} V_N^n_N - \sum_{i=1}^{n_n} \left( \frac{C_{2n} V_N^n}{n_n} \right) + n_n \left[ \frac{1}{3} - \left( \frac{V_N - 1}{2} \right)^2 \right]; n_n \in [n_1, n_2], \bar{V}_N \in [V_L, V_U]. \tag{3}
\]

where \( C_{2n} = 2^2 - 1 \).

The statistic \( U_{2n}^2 \) is computed in neutrosophic form by

\[
U_{2n}^2 = U_{2n}^2 + U_{2n}^2; I_{2n} = [I_{2n}, I_{2n}]. \tag{4}
\]

In the given neutrosophic form, the statistic \( U_{2n}^2 \) presents the test statistic under classical statistics. The value of statistic \( U_{2n}^2 \) shows the indeterminate value under uncertainty and \( I_{2n} \) is a measure of uncertainty associated with \( U_{2n}^2 \). The proposed statistic reduces to \( U_{2n}^2 \) when \( I_{2n} = 0 \).

3. Application

The application of the proposed Watson’s \( U_{2n}^2 \) test is given with the aid of angle data obtained from the radar. The decision-makers are interested to investigate either the radar data obtained from the radar systems follow the given theoretical distribution or not. For testing the null hypothesis, \( H_0 \): radar data follows the given theoretical distribution vs. \( H_1 \): the radar data does not follow the given theoretical distribution. For testing this hypothesis, the decision-maker is uncertain about the sample size with the measure of uncertainty \( I_{n_1} = 0.13 \) with \( n_1 = 13 \). The neutrosophic form of sample size is \( n_N = 13 + 15I_{n_1}; I_{n_1} \in (0, 0.13) \). The following radar data is obtained from [45].

\[
n_1 = 13, \tag{5}
\]

\[
\Phi_1 = 2500, \Phi_2 = 275 \circ, \Phi_3 = 285 \circ, \Phi_4 = 2850, \Phi_5 = 290 \circ, \Phi_6 = 290 \circ, \Phi_7 = 295 \circ, \Phi_8 = 300 \circ, \Phi_9 = 305 \circ, \Phi_{10} = 310 \circ, \Phi_{11} = 315 \circ, \Phi_{12} = 320 \circ, \Phi_{13} = 330 \circ, \Phi_{14} = 330 \circ, \Phi_{15} = 5 \circ.
\]
Table 1: Effect of measure of indeterminacy on \( n_N \) and \( U_{Nn}^2 \).

| \( I_{n_N} = I_{U_{Nn}^2} \) | \( n_N \) | \( U_{Nn}^2 \) |
|---|---|---|
| 0 | [13,13] | [0.7221,0.7221] |
| 0.001 | [13,13] | [0.7221,0.7228] |
| 0.005 | [13,13] | [0.7221,0.7257] |
| 0.010 | [13,13] | [0.7221,0.7294] |
| 0.10 | [13,15] | [0.7221,0.7950] |
| 0.20 | [13,16] | [0.7221,0.8678] |
| 0.30 | [13,18] | [0.7221,0.9407] |
| 0.40 | [13,19] | [0.7221,1.0136] |
| 0.50 | [13,21] | [0.7221,1.0865] |
| 0.60 | [13,22] | [0.7221,1.1593] |
| 0.70 | [13,24] | [0.7221,1.2322] |
| 0.80 | [13,25] | [0.7221,1.3051] |
| 0.90 | [13,27] | [0.7221,1.3779] |
| 1.00 | [13,28] | [0.7221,1.4508] |

The proposed test for the given data is implemented as follows:

\[
U_{Nn}^2 = \sum_{i=1}^{n_N} V_{iN}^2 - \sum_{i=1}^{n_N} \left( \frac{C_{iN} V_{iN}}{n_N} \right) + n_N \left[ \frac{1}{3,3} - \left( \bar{V}_N - \frac{1}{\sqrt{2,2}} \right) \right]^2 = [0.7221,0.7287].
\]

(6)

**Step 1.** Arrange the angle data in ascending order and assign number \( i \).

**Step 2.** Compute the values of \( V_i = \Phi_{2,3}/360 \) and \( \bar{V}_N = [0.8226,0.775] \).

**Step 3.** Generate the values of \( V_{iN}^2 \) and \( C_{iN} \). The values of \( \sum_{i=1}^{n_N} V_{iN}^2 = [8.83,9.67] \).

**Step 4.** Generate the values of \( \sum_{i=1}^{n_N} (C_{iN} V_{iN}/n_N) = [11.09,12.81] \).

**Step 5.** Finally, compute the values of the proposed test statistic as follows.

Let \( \alpha = 0.05 \), and the tabulated value from [14] is 0.184. By comparing the values of statistic \( U_{Nn}^2 \) with the critical value, the null hypothesis \( H_0 \): radar data follows the given theoretical distribution is rejected. Based on the study, it is concluded that angle data obtained from the radar does not follow the given theoretical distribution.

**4. Comparative Study**

In this section, the efficiency of the proposed Watson’s \( U_{Nn}^2 \) test will be compared with the existing Watson’s \( U_n^2 \) test in terms of flexibility, the measure of uncertainty, and information. As mentioned before, the existing Watson’s \( U_n^2 \) test is a special case of the proposed Watson’s \( U_{Nn}^2 \) test. The proposed Watson’s \( U_{Nn}^2 \) test becomes the existing Watson’s \( U_n^2 \) test when \( I_{U_{Nn}^2} = 0 \). The neutrosophic form of the statistic \( U_{Nn}^2 e(U_{L_n}^2, U_{U_n}^2) \) for the given data is as follows: \( U_{Nn}^2 = 0.7221 + 0.7287 I_{U_{Nn}^2} ; I_{U_{Nn}^2} \in [0,0.0091] \). From the neutrosophic form, it can be seen that the statistic \( U_{Nn}^2 e(U_{L_n}^2, U_{U_n}^2) \) adopts the value in an indeterminate interval. Under uncertainty, the proposed statistic \( U_{Nn}^2 e(U_{L_n}^2, U_{U_n}^2) \) takes the value from 0.7221 to 0.7287. On the other hand, the existing Watson’s \( U_n^2 \) test takes only a single value. Therefore, under indeterminacy, the proposed Watson’s \( U_{Nn}^2 \) test is flexible than the existing Watson’s \( U_n^2 \) test. In addition, the proposed Watson’s \( U_{Nn}^2 \) test gives information about the measure of uncertainty that the existing test cannot provide. For the radar data, the measure of uncertainty associated with Watson’s \( U_n^2 \) test is 0.0091. For testing \( H_0 \): radar data follows the given theoretical distribution vs. \( H_1 \): the radar data does not follow the given theoretical distribution when \( \alpha = 0.05 \), the proposed Watson’s \( U_{Nn}^2 \) test indicates that the probability of accepting \( H_0 \) is 0.95, the probability of committing type-1 error is 0.05, and the probability of in-decision is 0.0091. From the study, it can be seen that the proposed Watson’s \( U_{Nn}^2 \) test is informative than the existing Watson’s \( U_n^2 \) test.

**5. Simulation Study**

A simulation study is performed to see the effect of the indeterminacy parameters on the sample and the proposed Watson’s \( U_{Nn}^2 \) test. The various values of the indeterminacy parameters are considered to see the behavior of sample size and test statistic \( U_{Nn}^2 \). The values of \( n_N \) and \( U_{Nn}^2 \) for various values of \( I_{n_N} = I_{U_{Nn}^2} \) are shown in Table 1. From Table 1, it can be seen that when \( I_m = I_{U_{Nn}^2} \), the values of \( n_N \) and \( U_{Nn}^2 \) increase as the values of \( I_{n_N} \) increase from 0 to 1.00. From this simulation study, it can be observed that the measure of indeterminacy plays a significant role in determined \( n_N \) and \( U_{Nn}^2 \); therefore, the sample size should be selected keeping in mind the measure of indeterminacy.

**6. Concluding Remarks**

The existing Watson’s test under classical statistics was applied when there is no uncertainty and indeterminacy in sample size or angular data. An extension of the existing Watson’s test was presented in the paper. The proposed test can be applied when uncertainty is presented in circular data. From the radar data analysis and simulation study, it is concluded that the proposed outperforms the existing Watson’s test. The proposed test has some limitations that it can be very practical if the software is available to perform it. The test can be applied to test neutrosophic random data. The proposed test for big angular data can be considered as future research.
Data Availability
The data is given in the paper.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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