Electromagnetic fields from the extended Kharzeev-McLerran-Warringa model in relativistic heavy-ion collisions

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Based on the Kharzeev-McLerran-Warringa (KMW) model that estimates the strong electromagnetic (EM) fields generated in relativistic heavy-ion collisions, we generalize the formulas of EM fields in the vacuum by incorporating the longitudinal position dependence, the generalized charge density distribution and retardation correction. We further generalize the formulas of EM fields in the QGP medium by incorporating a constant Ohm electric conductivity. Using the extended KMW model, we observe a slower time evolution and a more reasonable impact parameter dependence of the magnetic field strength than those from the original KMW model in the vacuum. The constant electric conductivity from lattice QCD data helps further prolong the time evolution of magnetic field, so that the magnetic field strength at the thermal freeze-out time matches the required magnetic field strength for the explanation of the observed difference in global polarizations of Λ and ¯Λ hyperons in Au+Au collisions at RHIC. These generalized formulations in the extended KMW model could be potentially useful for many EM fields relevant studies in relativistic heavy-ion collisions at lower colliding energies and for various species of colliding nuclei.

I. INTRODUCTION

The study of strongly interacting matter and its properties in the presence of strong electromagnetic (EM) fields has been a hot topic for more than a decade. The ultra-relativistic heavy-ion collisions provide a unique way for creating and exploring the strongly interacting matter at extremely high temperature and high energy density. The QCD vacuum, topologically non-trivial gauge field configuration with non-zero winding number $Q_w$, of deconfined QGP in the presence of such a strong magnetic field $B$ will induce a non-conserved axial current $j^a_\mu$ as well as a vector current $j_\mu = \sum_f q_f \langle \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f \rangle$, which is respectively called the “charge separation effect” (CSE) and the “chiral magnetic effect” (CME) [2, 42, 50].

Since the axial current $j^a_\mu$ requires a charge imbalanced $C$-odd environment while the vector current $j_\mu$ requires a chirality imbalanced $P$-odd environment, an asymmetry between the amount of positive and negative charges along the direction of magnetic field $B$ in heavy-ion collisions is expected. Experimental observations of CME can be regarded as direct evidences of topologically non-trivial gluon field configurations, and furthermore can be interpreted as indications of event-by-event local $P$ and $CP$ violation of QCD at quantum level [2, 52]. Besides the CME and CSE, it is well known that the strong magnetic field could also influence many QCD processes, e.g., the induction of chiral symmetry breaking, the influences on chiral condensation, and the modification of in-medium particle mass. As an important consequence, the QCD phase diagram may be dynamically modified by such a strong magnetic field, which are respectively called the "charge separation effect" (CSE) and the "chiral magnetic effect" (CME) [2, 6, 42–50].

$eB$}

The constant electric conductivity from lattice QCD data helps further prolong the time evolution of magnetic field, so that the magnetic field strength at the thermal freeze-out time matches the required magnetic field strength for the explanation of the observed difference in global polarizations of Λ and ¯Λ hyperons in Au+Au collisions at RHIC. These generalized formulations in the extended KMW model could be potentially useful for many EM fields relevant studies in relativistic heavy-ion collisions at lower colliding energies and for various species of colliding nuclei.
magnetic field \[60, 61\], e.g. color-superconducting phases at very high baryon densities could also be strongly affected by the strong magnetic field \[62\]. Coupling with some anomaly processes, many interesting effects \[48\], e.g. the formation of \(\pi^0\)-domain walls \[66\], could also be generated by such a strong magnetic field.

Due to the fact that the generated magnetic field \(B\) in heavy-ion collisions can not be directly event-by-event measured, it is an enormous challenge to measure the magnetic field \(B\) induced chiral anomalous effects in current experimental measurements. In heavy-ion collisions, the magnetic field \(B\) is generated along the direction preferentially perpendicular to the reaction plane, hence experimental measurements are usually conducted using the two-particle correlators \(\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle\) firstly proposed by Voloshin \[67\], where \(\alpha\) and \(\beta\) denote the electric charge sign of particles \(\alpha\) and \(\beta\), \(\phi_\alpha\) and \(\phi_\beta\) are their azimuthal angles respectively, and \(\Psi_{RP}\) is the azimuthal angle of the constructed reaction plane for a given event, and \(\langle \cdots \rangle\) denotes the average over all particle pairs and then over all events. Therefore, the same-sign (SS) and opposite-sign (OS) correlators can be respectively written as \(\gamma_{ss} \equiv (\gamma_{++} + \gamma_{--})/2\) and \(\gamma_{os} \equiv (\gamma_{+-} + \gamma_{-+})/2\). Based on \[5,67\], the magnetic field driven CME is expected to contribute to a negative \(\gamma_{ss}\) but a positive \(\gamma_{os}\). The STAR Collaboration \[68,73\] and ALICE Collaboration \[74\] have independently measured the \(\gamma_{ss}\) and \(\gamma_{os}\) correlators, which indeed show the expected features of CME. However, there exit some ambiguities \[75,83\] in the interpretation of the experimental data due to the large background contaminations, potentially arising from the elliptic-flow \(v_2\) driven background contributions, such as the transverse momentum conservation (TMC) \[78,82,83\], local charge conservation (LCC) \[79–81\]. Hence, a dedicated run of the \(^{96}\)Zr \(+\) \(^{96}\)Zr and \(^{96}\)Ru \(+\) \(^{96}\)Ru isobar colliding systems at RHIC has been proposed \[5\], which is expected to yield unambiguous evidence for the CME signal by varying the signal but with the \(v_2\)-driven backgrounds roughly fixed \[5,83,89\].

In this paper, we will review some generic features of an analytical model for the estimation of EM fields generated in relativistic heavy-ion collisions from the original work initiated by Kharzeev, McLerran and Warringa \[2\], which we refer to as the original KMW model. On the ground of it, we formulate our generalizations of the estimated EM fields. We first start from the generalization of charge distributions used in heavy-ion collisions, based on which we point out that the formulas of EM fields in the original KMW model can be properly extended by incorporating the longitudinal position dependence through the generalized relativistic three-dimensional charge distribution models, e.g., three-parameter Fermi model, for both spherically symmetric colliding nuclei and axially deformed ones. Also, we make the retardation correction (RC) to the estimated EM fields contributed by participants, such that our formulation of the estimated EM fields embedded with both generalized charge distributions and retardation correction (RC) can be potentially applied to lower energy region, such as the current beam-energy-scan (BES) program at RHIC, the under planning FAIR, NICA and J-PARC programs. It is because the Lorentz contraction is not so large that the “pancake-shaped disk” approximation used in the original KMW model \[2\] is no longer valid in these energy regions. Besides, we further extend the formulas of EM fields to incorporate the medium feedback effects according to the Faraday’s induction law, where a constant Ohm electric conductivity \(\sigma\) is properly and analytically embedded. Finally, we make some numerical evaluations of the generalized EM fields formulas in the extended KMW model for the detailed comparisons of time evolution, impact parameter \(b\) dependence as well as the prediction of possible lifetime \(t_B\) of the estimated magnetic field \(B(t, r)\) in comparison with those from the original KMW model.

This paper is organized as follows. We present detailed formulations of the estimated EM fields in the extended KMW model for heavy-ion collisions in Sec. \[11\] which consists of three subparts. We first present in Sec. \[11A\] a formal generalization of the charge distributions from the widely used three-parameter Fermi model in which the relativistic Lorentz contraction effects on the geometries of both spherically formed and axially deformed colliding nuclei are taken into account. We then in Sec. \[11B\] make a generalization of EM fields in the vacuum from the widely used Liénard-Wiechert equations, and naturally generalize the EM fields in the vacuum from the original KMW model with the generalized charge distributions and retardation correction. We further extend the EM fields with medium feedback effects by incorporating a constant Ohm electric conductivity \(\sigma\) into the extend KMW model for high-energy heavy-ion collisions in Sec. \[11C\]. Some evaluations and comparisons about the time evolution, centrality (impact parameter \(b\)) dependence, as well as the prediction of possible lifetime of the estimated magnetic field from the extended KMW model in comparison with those from the original KMW model are presented in Sec. \[11\]. We finally summarize the main processes of such generalization along with the conclusions and discussions in Sec. \[14\].

The notation we use in this paper is the rationalized Lorentz-Heaviside units within the natural units, with \(\hbar = c = 1\) and \(\mu_0 = \epsilon_0 = 1\).

II. ELECTROMAGNETIC FIELDS FROM EXTENDED KMW MODEL

Before the detailed discussion, let us first emphasize that the physical situations of heavy-ion collisions consist of event-by-event \(P\) and \(CP\) violation processes due to the effects of topological charge fluctuation with non-trivial QCD gauge field configurations (characterized by the topological invariant, the winding number \(Q_w\)) in the vicinity of the
deconfined QGP phase \cite{2}. This kind of $P$ and $CP$ violation processes can only locally happen under some special and even extreme conditions in QCD, such as in the instantaneous deconfined QGP phase with chirality imbalance at extremely high temperature $T \sim \Lambda_{\text{QCD}}$ in the presence of a strong magnetic field $eB \sim m_{\pi}^2$, which will equivalently generate event-by-event locally non-vanishing $\theta$ angle for the effects of so called “$\theta$-vacuum” \cite{90–92}. In other words, the fluctuations of local $P$ and $CP$ violated metastable domains of the QCD $\theta$ vacuum on the event-by-event basis are intrinsically connected to the amount of charges $Q$ separated by the simultaneous magnetic field $B$ for the non-trivial gluon field configurations with $Q_w \neq 0$. It means that the expectation value of the amount of separated charges $Q$ is proportional to the strength of magnetic field $|B|$, therefore the charge asymmetry fluctuations $\Delta_2^{\pm}$ will directly proportional to $|B|^2$ \cite{2}. It is therefore of crucial importance to quantify the strength of EM fields, especially the time evolution and the possible lifetime $t_B$ of the magnetic field $B(t, r)$ in relativistic heavy-ion collisions \cite{93}.

Similar to the conventional setup of colliding system for heavy-ion collisions, we choose the $x$ axis along the impact parameter $b$, and $z$ axis along the beam direction of projectile such that the $x-z$ plane is exactly the reaction plane (RP). The $y$ axis is then chosen to be perpendicular to the reaction plane, as illustrated in Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{colliding_system.png}
\caption{(Color online) Illustration of the initial geometry of the colliding system projected on the transverse plane $(z = 0)$ at an impact parameter $b$ for non-central high-energy nucleus-nucleus collisions. The centers of the projectile (denoted as P) and target (denoted as T) nucleus are respectively located at $(b/2, 0, 0)$ and $(-b/2, 0, 0)$ at $t = 0$ when the two colliding nuclei are completely overlapping with each other, moving parallel or anti-parallel to the beam $z$ direction.}
\end{figure}

A. Generalization of Charge Distributions for Heavy-ion Collisions

Since in symmetric heavy-ion collisions such as gold-gold collisions at top RHIC energy with the center of mass energy $\sqrt{s} = 200$ GeV per nucleon pair along the $z$ direction, the Lorentz contraction factor $\gamma = \sqrt{s}/(2m_N) \simeq 106.5$ (where $m_N$ is the average nucleon mass), which corresponds to the beam rapidity $Y_0 = \cosh^{-1}(\gamma) \simeq 5.36$. In this case, the two gold nuclei will be Lorentz contracted to be less than 1% of their original size in the $z$ direction, because of which the original KMW model \cite{2} approximates the two colliding nuclei as “pancake-shape disk”. It is further assumed in this model that the charge distribution can be treated as uniformly distributed within the two colliding nuclei and further be limited only onto the transverse plane with a two-dimensional surface number density. Therefore in this paper, we will first formulate our generalization of the charge distributions, and hence formulate the estimation of generated EM fields in the extended KMW model.

It has been widely acknowledged that final state anisotropies of emitted charged particles in heavy-ion collisions are very sensitive to the initial conditions, such as the experimentally measured final state charge particle elliptic flow $v_2$ is actually very sensitive to the initial eccentricity $\varepsilon_2$ \cite{94}. Hence, the initial conditions may dominantly cause the contamination of the experimentally measured CME signal. Therefore, a better quantification of the initial geometry will be of crucial importance, especially for the study of CME in heavy-ion collisions since the initial geometry of charge distributions may significantly modify not only the centrality dependence but also the time evolution of the generated magnetic field $B(t, r)$.

Besides, due to relativistic motion of colliding nuclei, the overall size of each nucleus will be Lorentz contracted by a factor of $\gamma$ along the beam $z$ direction, such that a spherically symmetric nucleus will become an ellipsoidal nucleus with the volume shrunken by a factor of $\gamma$ while the charge density magnified by a factor of $\gamma$ simultaneously.
Hence, we first start from generalizing the charge distributions into a relativistic three-dimensional form with the corresponding standard parameters obtained from the high-energy electron-scattering measurements [95]. Moreover, what the original high-energy electron-scattering experiments measure is actually the charge spatial distribution in the so-called “Breit Frame” [96] rather that the nuclear density profile with relativistic motion. Hence, we note that the originally inferred charge distribution parameters from electron-scattering experiments should therefore be properly modified [94] so as to be unambiguously implanted into the widely used Monte-Carlo simulations of the initial conditions for the finite-number nucleons sampling of each nucleus [97].

For non-central high-energy nucleus-nucleus (A-A) collisions at center of mass energy \( \sqrt{s} = 2\gamma m_N \) per nucleon pair with the same setup of colliding system as illustrated in Fig. 1, the charge distribution used in the original KMW model can be generalized from the widely used charge distribution models listed in [95], i.e. the widely accepted and properly modified [94] so as to be unambiguously implanted into the widely used Monte-Carlo simulations of the finite-number nucleons sampling of each nucleus [97].

where \( \rho_{0} \) is the spherical monopole charge radius, \( Y_{m}^{(0)}(\theta, \phi) \) denotes the spherical harmonics, with \( \theta \) being the polar angle with respect to the symmetry axis of each nucleus, and \( \phi \) being the corresponding azimuthal angle in each nucleus, and they are defined as \( \phi = \sin^{-1}(y'/r'_\perp) \) and \( \theta = \cos^{-1}(z'/r') \). The diffusion depth of the nuclear surface \( d \) can roughly be related to the skin thickness \( t \) by \( t = 4 \ln(3)d \simeq 4.4d \), where \( t \) equals the distance in which the charge density falls from 90\% to 10\% of its central value \( \rho_{0} \).

Notice that the \( \omega \) in Eq. (1) is an additional parameter [93], introduced to characterize the charge central depression or elevation phenomena of some nuclide such as \( ^{40}Ca \), relative to the traditional two-parameter Fermi model (2pF) with parameter \( c \) (which is in analogue to \( R' \) in 3pF model) and \( d \). Only in 3pF model with \( \omega = 0 \) that the half density radius \( R_{1/2} = R'(\theta, \phi) \), which is the distance from the center of the nucleus to the point at which the charge density equals half its central value \( \rho_{0} \). \( \rho_{0} \) is usually regarded as the charge number normalization constant, which can be obtained from the following charge number normalization condition

\[
Z = \int_{-R_{\perp}}^{+R_{\perp}} dx' \int_{-R_{\perp}}^{+R_{\perp}} dy' \int_{-R_{\perp}/\gamma}^{+R_{\perp}/\gamma} dz' \rho(r').
\]

(4)

where \( R_{\perp} \) is the radius of each nucleus. One should note that the \( \gamma \) factor in front of \( \rho_{0} \) in Eq. (1) is due to the fact that since the volume of each colliding nucleus will be Lorentz contracted by a factor of \( \gamma \) in the \( z \) direction, in accordance to which the charge number density will be simultaneously enlarged by a factor of \( \gamma \) at the same time.

The function \( \Theta(r') \) is introduced so as to constrain that the charge density is just within the ellipsoidal colliding nucleus, and it can further be used to separate the spectators from the participants when restricting it just within the transverse plane, namely \( \Theta(r') \rightarrow \tilde{\Theta}(r'_\perp) \), which are separately defined as

\[
\Theta(r') = \theta \left[ R_{A}^{2} - r_{\perp}'^{2} - (\gamma z')^{2} \right],
\]

\[
\tilde{\Theta}(r'_\perp) = \theta \left[ R_{A}^{2} - r_{\perp}'^{2} \right].
\]

Here we note that \( \Theta(r') \) and \( \tilde{\Theta}(r'_\perp) \) are similar to the function \( \theta(r'_\perp) \) introduced in the original KMW model [2]. The function \( \theta(x) \) in Eq. (5) is the standard unit step function, which reads

\[
\theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0. \end{cases}
\]

(6)
For a spherically symmetric nucleus (β₂ = β₄ = 0), i.e. the ¹⁹⁷Au nucleus, R' is actually independent of polar and azimuthal angle distributions and is just the monopole charge radius, namely R' = R₀ (similar to the parameter c in 2pF model), as a consequence of which charge number density ρ(r') = ρ(r')₀ is only a function of r'(γ). In the relativistic situation under this condition, the relativistic three-dimensional charge number density ρ(r') according to Eq. (1) reads as

$$\rho(r') = \gamma \rho_0 \frac{1 + \omega(r'/R_0)^2}{1 + \exp [(r' - R_0)/d]} \Theta(r').$$  \hspace{1cm} (7)

However, for a axially deformed nucleus, e.g. the ⁹⁶Zr nucleus, its shape may deviate from the ideal sphere and the charge distribution depends on the polar and azimuthal angle distributions, for which Eq. (3) is to some extent introduced so as to account for such kind of charge distribution inside the 3pF model in Eq. (1). In the characterization of deformations in Eq. (3), β₂Y²₀ and β₄Y⁴₀₄ characterize the angular variations of R'(θ, φ) of quadrupole and hexadecapole deformations respectively, where β₂ and β₄ are the corresponding quadrupole and hexadecapole deformation parameters, and Y²₀(θ) and Y⁴₀(θ) are the corresponding spherical harmonics.

### B. Electromagnetic Fields from Extended KMW Model in the Vacuum

Since we have generalized the charge number density in a relativistic three-dimensional form in Eq. (1) and Eq. (7), let us now first quote a simple situation where EM fields are generated by a pair of two constantly but oppositely moving point-like charged particles of the same electric charge Ze (where e is the elementary electric charge, e = |e|) with rapidity Yₙ = ±Y (Y > 0), which corresponds to the velocity vₙ = (0, 0, tanh Yₙ), parallel or anti-parallel to the z direction. We now set that at t = 0 the two point-like charged particles are located at rₙ(t = 0) = (x'ₙ, y'ₙ, z'ₙ). Here the index n = ± respectively represents the charged particles moving in the positive and negative z-directions. Also, it is worth to note that since in the three-dimensional situation in heavy-ion collisions for a charge point rₙ within the two colliding nuclei at time t = 0 when the two colliding nuclei are usually assumed to be completely overlapping with each other, the z coordinate z'ₙ for this charge point in general will be non-zero. One can start from using the Liénard-Wiechert potentials generated by this two point-like charge particles to evaluate the EM fields for a field point r = (x, y, z) at observation time t, which gives rise to the following widely used L-W equations of EM fields 3, 5, 8, 11, 15, 16, 20, 22, 25, 32

$$\epsilon E(t, r) = \alpha EM \sum_{n=\pm} Z_n \frac{R_n (1 - v_n^2)}{\left(R_n^2 - [v_n \times R_n]^2\right)^{3/2}},$$

$$\epsilon B(t, r) = \alpha EM \sum_{n=\pm} Z_n \frac{v_n \times R_n (1 - v_n^2)}{\left(R_n^2 - [v_n \times R_n]^2\right)^{3/2}},$$  \hspace{1cm} (8)

where Rₙ = r - rₙ(t) are the relative positions of the field point r to the source points rₙ at the same observation time t, rₙ(t) = (x'ₙ(t), y'ₙ(t), z'ₙ(t)) + vₙ(t) are the source positions of the two point-like charge particles at time t, and αEM is the EM fine-structure constant, defined as αEM = e²/4π ≈ 1/137. One should note that the retarded effects due to fields propagation have already been incorporated into above equations. For later use convenience, above Eq. (8) can be directly rewritten, in terms of rapidity Y, in the following form,

$$\epsilon E(t, r) = Z \alpha EM \sum_{n=\pm} \frac{\cosh(Yₙ) \cdot Rₙ}{\left((x - x'_n)^2 + (y - y'_n)^2 + |t \sinh Yₙ - (z - z'_n) \cosh Yₙ|\right)^{3/2}},$$

$$\epsilon B(t, r) = Z \alpha EM \sum_{n=\pm} \frac{\sinh(Yₙ) \cdot e_z \times Rₙ}{\left((x - x'_n)^2 + (y - y'_n)^2 + |t \sinh Yₙ - (z - z'_n) \cosh Yₙ|\right)^{3/2}},$$  \hspace{1cm} (9)

which in nature are well consistent with the derivation in 2 for the magnetic field generated by a point-like charge when reducing z'ₙ → 0 in above Eq. (7). In contrary, z'ₙ ≠ 0 is intended for the three-dimensional form of charge distribution such as Eq. (1) or (7) where the z coordinate of a point-like charge inside the colliding nuclei is not necessarily limited only within the transverse plane, hence we can generalize the “pancake shaped disk” approximation as well as the two-dimensional surface number densities used in the original KMW model 2.

Now one can estimate the strength of EM fields generated in heavy-ion collisions. For a field point r = (x, y, z) = (rₗ, z) at observation time t, the EM fields generated by two identical colliding nuclei with charge Ze and beam
rapidity $\pm Y_0 \ (Y_0 > 0)$ parallel or anti-parallel to the $z$ direction can be estimated by combining the generalized relativistic three-dimensional charge number density $\rho(r')$ in Eq. (1) or Eq. (7) with the EM fields generated by a pair of two point-like charges in Eq. (8) or Eq. (9), and then performing the integration of charge number densities over the corresponding coordinate space within the two colliding nuclei. Nevertheless, the situation will be a little different from the point-like charge case since colliding two heavy ions will separate the charged bulk matter into spectators (denote as S) and participants (denote as P). The participants in general will slow-down to a certain extent due to the baryon-junction stopping effect [100], while the spectators in general are assumed to be moving with the same beam rapidity $Y_S = \pm Y_0$ and do not scatter at all. Therefore, we follow the method proposed in [2] by likewise splitting the contributions to the EM fields by participants can therefore be estimated by inserting Eq. (12) into Eq. (9), and then combining with Eq. (1), which follow as

$$E = E_S^+ + E_S^- + E_p^+ + E_p^-,$$

$$B = B_S^+ + B_S^- + B_p^+ + B_p^-.$$  

For a charge position vector $r' = (x', y', z') = (r'_L, z')$ with the same setup of colliding system as in Fig. 1 the charge number density $\rho(r')$ in Eq. (1) or Eq. (7) as well as the corresponding $\Theta(r')$ and $\tilde{\Theta}(r'_L)$ functions should be accordingly shifted by a half impact parameter vector $b$, namely $\rho_\pm(r') = \rho(r' \mp b/2)$, $\Theta_\pm(r'_L) = \Theta(r'_L \mp b/2)$ and $\tilde{\Theta}_\pm(r'_L) = \tilde{\Theta}(r'_L \mp b/2)$. The contributions to the EM fields by spectators can be elaborated into the following form

$$eE_S^\pm(t, r) = \alpha_{EM} \cosh(Y_0) \int_{V_{\pm}} d^3r' \frac{\rho_\pm(r')}{(r_L - r'_L)^2 + t \sinh(Y_0) - (z' - z) \cosh(Y_0)^2}^{3/2},$$

$$eB_S^\pm(t, r) = \alpha_{EM} \sinh(Y_0) \int_{V_{\pm}} d^3r' \frac{\rho_\pm(r')}{(r_L - r'_L)^2 + t \sinh(Y_0) - (z' - z) \cosh(Y_0)^2}^{3/2},$$  

where $V_{\pm}$ donate the ellipsoidal volumes occupied by the two colliding nuclei, which can be related to the integration domains in Eq. (4) for charge number normalization condition.

For EM fields contributed by participants, we at present do not consider contributions by newly created particles either as [2] because the numbers of newly produced positively and negatively charged particles are nearly equal and their expansion is estimated to be almost spherical. Such an assumption should be reasonable, especially for peripheral collisions. Therefore one only needs to take into account the contributions of baryon-junction stopping bulk matter which are initially there. The normalized rapidity $Y$ distribution of the baryon-junction stopping bulk matter of projectile (+) and target (−) can be empirically estimated [2, 17, 27, 100] as

$$f_\pm(Y) = \frac{\alpha_y}{2 \sinh(\alpha_y Y)} e^{\pm \alpha_y Y}, \quad -Y_0 \leq Y \leq Y_0.$$  

Here we note that the parameter $\alpha_y$ can be estimated as $\alpha_y \approx 0$, where $\alpha_0^B$ is the Regge trajectory intercept, defined as $\alpha_0^B(0) \approx 2\alpha_B(0) - 1 + 3(1 + \alpha_R(0)) \approx 1/2$ in Regge theory [98–100], in which $\alpha_B(0)$ is the Regge intercept while $\alpha_R(0) \approx 0.5$ is the Reggeon intercept. Although it has been mentioned early in [99] that a somewhat lower baryon intercept $\alpha_B(0)$ should be used if nucleon exchange effectively dominates, which will somehow give rise to a relatively smaller $\alpha_y$. Recently, the ALICE Collaboration has reported the data at different center of mass energies $\sqrt{s}$ for the anti-baryons to baryons ratio on baryon stopping [101], which seems to support that $\alpha_y \approx 0.5$ is experimentally reasonable. Therefore we subjectively to use $\alpha_y \approx 0.5$ as the principal value in this paper, and also check that varying $\alpha_y$ between 0.48-0.50 does not largely affect the final results for the time evolution of the magnetic field strength.

The contributions to EM fields by participants can therefore be estimated by inserting Eq. (12) into Eq. (9), and then combining with Eq. (1), which follow as

$$eE_P^\pm(t, r) = \alpha_{EM} \int_{V_{\pm}} d^3r' \int_{-Y_0}^{Y_0} dY \frac{\Psi_\pm(Y) \cosh Y \cdot \rho_\pm(r') \tilde{\Theta}_\pm(r'_L) R_\pm}{(r_L - r'_L)^2 + [t \sinh(Y) - (z' - z) \cosh(Y)^2]^{3/2}},$$

$$eB_P^\pm(t, r) = \alpha_{EM} \int_{V_{\pm}} d^3r' \int_{-Y_0}^{Y_0} dY \frac{\Psi_\pm(Y) \sinh Y \cdot \rho_\pm(r') \tilde{\Theta}_\pm(r'_L) e_z \times R_\pm}{(r_L - r'_L)^2 + [t \sinh(Y) - (z' - z) \cosh(Y)^2]^{3/2}},$$  

where the refined functions $\Psi_\pm(Y)$ for baryon-junction stopping effect are introduced to account for whether the retardation time $t_{ret}$ of a participating charge at observation time $t$ is ahead of the collision time $t_c$ or simply after
We then obtain the following partial differential wave equations for EM fields,

$$\Psi_\pm(Y) = \theta(t - t_c) f_\pm(Y) + \theta(t_c - t_\text{ret}) \delta(Y \mp Y_0).$$

(14)

Here $t_\text{ret}$ is obtained by solving the retardation relation $t_\text{ret} = t - \| \mathbf{r} - \mathbf{r}' - t_\text{ret} \cdot \mathbf{e}_z \tanh Y \|$ for each rapidity $Y$ within the normalized rapidity integral. We refer to this kind of treatment of participants in Eqs. (13)-(14) as retardation correction (RC) in this paper. For the case without retardation correction, the functions $\Psi_\pm(Y)$ are simply previously used normalized rapidity distribution functions $f_\pm(Y)$ in [27].

The expressions for the estimated EM fields in Eqs. (11) and (13) are in general consistent with the derivation in the original KMW model [2]. One distinct difference is that the depth in the z direction is now taken into account in the extended KMW model rather than the infinitely thin depth (zero z-depth) in the original KMW model, which actually extends the dimensions of the charge density integration. The other distinct difference is that the retardation correction (RC) from the perspective of field propagation for the treatment of participating charges as in Eq. (14) through $\Psi_\pm(Y)$ rather that $f_\pm(Y)$ in [27] is now explicitly considered. Hence, we think that our generalization in extended KMW model may give rise to some observable differences for the total estimated EM fields since the retardation due to field propagation is most relevant to the z coordinate. Again, we note that the charge number densities $\rho_\pm(r')$ are not longer the uniformly distributed surface densities, which however are generalized into relativistic 3pF model forms for the incorporation of Lorentz contraction effect and the effect due to deformations with the standard parameters obtained from high-energy electron-scattering measurements [95]. Therefore, Eqs. (11) and (13) will be very easily and appropriately applied to lower energy regions and various colliding systems, where the “pancake-shaped disk” approximation used in the original KMW model [2] is no longer valid.

C. Electromagnetic Fields from Extended KMW Model with Medium Feedback Effects

Up to now, we limit our discussions on the EM fields without considering any medium feedback effects, namely the Ohm electric conductivity $\sigma = \sigma_{\text{Ohm}} = 0$ in above discussion for the extended KMW model. For high-temperature ($\sim \Lambda_{\text{QCD}}$) hot QCD matter, the real QGP is a conducting medium and the electric conductivity $\sigma$ is generally acknowledged to be proportional to the plasma temperature $T$ [102-108], namely $\sigma \propto T$. Since it is known according to the Faraday’s induction law that the electric conductivity $\sigma$ may (to some extent) substantially slow down the decrease of the generated EM fields, and therefore largely prolong the lifetime of EM fields, which is verified to be crucial for the estimation of CME in heavy-ion collisions [93, 109]. We therefore implement a constant electric conductivity $\sigma$ into the following Maxwell’s equations so as to incorporate the medium feedback effects,

$$\nabla \cdot \mathbf{E} = \frac{\rho_{\text{ext}}}{\varepsilon},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B},$$

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \sigma \mathbf{E} + \mathbf{j}_{\text{ext}}.$$  

(15)

We then obtain the following partial differential wave equations for EM fields,

$$(\nabla^2 - \partial_t^2 - \sigma \partial_t) \mathbf{B} = -\nabla \times \mathbf{j}_{\text{ext}},$$

$$((\nabla^2 - \partial_t^2 - \sigma \partial_t) \mathbf{E} = \partial_t \mathbf{j}_{\text{ext}} + \nabla \left( \frac{\rho_{\text{ext}}}{\varepsilon} \right).$$

(16)

The solutions for above EM wave equations have been analytically solved in [23], in which the authors have embedded both electric conductivity $\sigma$ and chiral magnetic conductivity $\sigma_\chi$ for the EM fields generated by a relativistic point-like charge. In the absence of chiral magnetic conductivity $\sigma_\chi$ (since in general, $\sigma_\chi \ll \sigma$), we obtain the following solution in the cylindrical coordinates for the magnetic field generated at a field point $r = (r_\perp, z) = (x, y, z)$ as

$$\begin{pmatrix}
B_r(t, r) \\
B_\phi(t, r) \\
B_z(t, r)
\end{pmatrix} = \frac{Q}{4\pi} \frac{\gamma \nu R_\perp}{\Delta^{3/2}} \left[ 1 + \frac{\gamma |v| \sigma}{2\sqrt{\Delta}} \right] e^A \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix},$$

(17)

where

$$R_\perp = \| r_\perp - r'_\perp \| = \sqrt{(x - x')^2 + (y - y')^2},$$

$$\Delta = \gamma^2 (vt + z'_0 - z)^2 + R_\perp^2,$$

$$A = \frac{\gamma |v| \sigma}{2} \left[ \gamma (vt + z'_0 - z) \right] - \frac{\gamma |v| \sigma}{2} \sqrt{\Delta},$$

(18)
for a point-like charged particle with electric charge \( Q = +ZE \) moving with velocity \( \mathbf{v} = (0, 0, +v) \) along the positive \( z \) direction, which is located at \( \mathbf{r}' = (x', y', z'_0 + vt) \) at observation time \( t \). The radial and longitudinal components of magnetic field \( B_r \) and \( B_z \) are both vanishing when only embedded with the electric conductivity \( \sigma \), namely \( B_r = B_z = 0 \).

The transformations of EM fields \( \mathbf{F} \) (where \( \mathbf{F} = \mathbf{B}, \mathbf{E} \)) in the Cartesian coordinates \((F_x, F_y, F_z)\) with that in the cylindrical coordinates \((F_r, F_\phi, F_z)\) simply read

\[
\mathbf{F} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} F_r \cos \phi - F_\phi \sin \phi \\ F_r \sin \phi + F_\phi \cos \phi \end{pmatrix},
\]

where \( \phi \) is the azimuthal angle (with respect to \( x \)-axis) of the transverse relative position vector, namely \( \mathbf{R}_\perp = \mathbf{r} - \mathbf{r}' \).

Note that since we have \( \mathbf{e}_z \times \mathbf{R} = (y' - y, x - x', 0) \), where \( \mathbf{R} = \mathbf{r} - \mathbf{r}' \) is the relative position vector, the magnetic field \( \mathbf{B} \) in Eq. (17) in the Cartesian coordinates therefore can be rewritten in a more compact form, which reads

\[
\mathbf{B}(t, \mathbf{r}) = \frac{Q}{4\pi} \frac{\gamma v \mathbf{e}_z \times \mathbf{R}}{\Delta^{3/2}} \left[ 1 + \frac{\gamma v|\sigma|}{2} \sqrt{\Delta} \right] e^A,
\]

which shares almost the same form as the L-W form of magnetic field in Eq. (9) except for the factor \((1 + \gamma v|\sigma|\sqrt{\Delta})/2\) due to the inclusion of electric conductivity \( \sigma \) for the QGP medium response. Therefore, Eq. (20) can naturally reproduce the well-known L-W form of magnetic field in the vacuum, i.e., Eq. (8) or Eq. (9), when the electric conductivity \( \sigma \) is vanishing. For a special situation in Eq. (17) when the azimuthal angle \( \phi = 0 \), the \( x \) component of magnetic field \( B_x \) is consequently vanishing and the tangential \( \phi \) component of magnetic field \( B_\phi \) is actually \( B_y \) and along the \( y \) direction, as is usually mentioned in literature [14]. Also, we note that the expression of \( \mathbf{B} \) in Eq. (17) was firstly derived in Ref. [17].

For the electric field \( \mathbf{E} \), the tangential component of electric field \( E_\phi \) can be easily obtained by applying the relation \( E_\phi = -\mathbf{v} B_r \). Meanwhile, the authors in [23] have obtained the following explicit expressions for radial as well as longitudinal components \( E_r \) and \( E_z \) in the leading linear order of \( \sigma \) for high-energy heavy-ion collisions as

\[
\begin{align*}
E_r(t, \mathbf{r}) &= \frac{Q}{4\pi} \left\{ \frac{\gamma v |\sigma|}{2\sqrt{\Delta}} \left[ 1 + \frac{\gamma v|\sigma|}{2} \frac{\sqrt{\Delta}}{3} \right] - \frac{\gamma v}{|\mathbf{R}_\perp|} \left[ 1 + \frac{\gamma v}{\sqrt{\Delta}} \left( t - z - z'_0 \right) \right] e^{-\gamma v|\sigma|(t - (z - z'_0)/v)} \right\} e^A, \\
E_\phi(t, \mathbf{r}) &= 0, \\
E_z(t, \mathbf{r}) &= \frac{Q}{4\pi} \left\{ \frac{\sigma^2 v}{|\mathbf{R}_\perp|^3} e^{-\gamma v|\sigma|(t - (z - z'_0)/v)} \Gamma(0, -A) + \frac{e^A}{\Delta^{3/2}} \left[ \gamma (z - z'_0 - vt) - \frac{v}{|\mathbf{R}_\perp|} A\sqrt{\Delta} - \frac{\gamma v^2}{v^2} \Delta \right] \right\},
\end{align*}
\]

where \( \Gamma(0, -A) = \int_{-A}^{\infty} dt \exp(-t)/t \) is the incomplete gamma function. According to the transformations in Eq. (19), the radial and longitudinal components of electric field \( E_r \) and \( E_z \) are respectively along the \( x \) and \( z \) directions when the azimuthal angle \( \phi = 0 \). Here we note that Eqs. (17) and (21) can naturally reduce to the well-known L-W equations for EM fields in the vacuum when the electric conductivity \( \sigma \) is vanishing (\( \sigma = 0 \)). It has been further proved in [23] that Eqs. (17) and (21) are self-consistent and well satisfy the Maxwell’s equations in Eq. (15). Hence in the following, we will generalize above EM fields for point-like charge in Eqs. (17) and (21), and make them more applicable for high-energy nucleus-nucleus collisions.

We combine the electric conductivity \( \sigma \) embedded EM fields in Eqs. (17) and (21) with the relativistic three-dimensional charge number density in Eq. (1) or Eq. (7), and then apply the decomposition of EM fields as in Eq. (10). In analogue to Eqs. (11) and (13), we finally obtain the following explicit expressions for magnetic field embedded with a constant electric conductivity \( \sigma \). For the contributions of spectators, we obtain

\[
\mathbf{eB}_S(t, \mathbf{r}) = \lim_{Y \rightarrow \pm Y_0} \sigma_{EM} \sinh Y \int_{\mathbb{R}_\pm} d^3r \mathbf{e}_z \times \mathbf{R}_\perp \frac{\rho_\pm (\mathbf{r}')}{\Delta^{3/2}} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \sqrt{\Delta} \right] e^A,
\]

with the definitions in Eq. (18) accordingly rewritten as

\[
\begin{align*}
R_\perp &= |\mathbf{r}_\perp - \mathbf{r}'_\perp| = \sqrt{(x - x')^2 + (y - y')^2}, \\
\Delta &= \left( t \sinh Y - (z - z') \cosh Y \right)^2 + R_\perp^2, \\
A &= \frac{\sigma \sinh Y}{2} \left[ t \sinh Y - (z - z') \cosh Y \right] - \frac{\sigma \sinh |Y|}{2} \sqrt{\Delta},
\end{align*}
\]
transform the expressions for electric field into the Cartesian coordinates after using Eq. (19) as follows,

$$E\pm = \frac{\psi_\perp(Y) \sinh Y \cdot \rho_\perp(r') \hat{\theta}_\perp(r') e_z \times \mathbf{R}_\pm}{\Delta^2} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \sqrt{\Delta} \right] e^A. \quad (24)$$

For the electric field $E$ generated by two colliding nuclei, we first notice that $E_\phi$ is vanishing in the point-like charge case, therefore there is no contribution from both spectators and participants for the tangential component electric field $E_\phi$. The cylindrical coordinates are usually not extensively used in heavy-ion collisions, we therefore directly transform the expressions for electric field into the Cartesian coordinates after using Eq. (19) as follows,

$$\begin{align*}
\left( \frac{eE^\pm}{eE^\pm_{x,S}} \right)(t,r) &= \lim_{Y \to \pm Y_0} \alpha_{EM} \int_{V_\pm} d^3r' \rho_\perp(r') \left[ 1 - \hat{\theta}_\parallel(r'_\perp) \right] \left\{ \frac{R_+ \cosh Y}{\Delta^3/2} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \right] \right\} e^A(x - x', y - y'), \\
\left( \frac{eE^\pm}{eE^\pm_{x,p}} \right)(t,r) &= \alpha_{EM} \int_{V_\pm} d^3r' \int_{-Y_0}^{Y_0} dY \, \psi_\pm(Y) \rho_\perp(r') \hat{\theta}_\parallel(r'_\perp) \left\{ \frac{R_+ \cosh Y}{\Delta^3/2} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \right] \right\} e^A(x - x', y - y'),
\end{align*}$$

which are respectively contributions of spectators and participants for the $x$ and $y$ components of electric field $E_x$ and $E_y$. For the $z$ component of electric field $E_z$ contributed by spectators and participants, we have

$$\begin{align*}
\left( \frac{eE^\pm}{eE^\pm_{z,S}} \right)(t,r) &= \lim_{Y \to \pm Y_0} \alpha_{EM} \int_{V_\pm} d^3r' \rho_\perp(r') \left[ 1 - \hat{\theta}_\parallel(r'_\perp) \right] \left\{ \frac{\Delta^2}{\Delta^3/2} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \right] \right\} e^A(x - x', y - y'), \\
\left( \frac{eE^\pm}{eE^\pm_{z,p}} \right)(t,r) &= \alpha_{EM} \int_{V_\pm} d^3r' \int_{-Y_0}^{Y_0} dY \, \psi_\pm(Y) \rho_\perp(r') \hat{\theta}_\parallel(r'_\perp) \left\{ \frac{\Delta^2}{\Delta^3/2} \left[ 1 + \frac{\sigma \sinh |Y|}{2} \right] \right\} e^A(x - x', y - y'),
\end{align*}$$

Here the sign function $\text{sgn}(Y) = Y/|Y|$ denotes the sign of rapidity $Y$. Note that Eqs. (25) quantify the $x$ and $y$ components of EM fields contributed in heavy-ion collisions with a constant electric conductivity $\sigma$ implemented, where we also incorporate the generalized relativistic charge distributions, e.g., the 3pF model in Eqs. (1) or (7), and the retardation correction (RC) through $\psi_\perp(Y)$. On the other hand, we currently in Eqs. (25) do not include the chiral magnetic conductivity $\sigma_\chi$ into the Maxwell’s equations in Eq. (15) since in general $\sigma_\chi \ll \sigma$. Also, the value of electric conductivity $\sigma$ from recent lattice calculations [102, 108] still shows very large uncertainties, which may easily submerge the contributions from chiral magnetic conductivity $\sigma_\chi$. On the other hand, as has shown in [29] that when the chiral magnetic conductivity $\sigma_\chi$ is included, the dominant components such as $B_\phi$, $E_\phi$ and $E_z$ will not be changed at all. Therefore, we hold the point of view that Eqs. (25) can well quantify the dominant contributions of medium feedback effects for the time evolution and centrality (impact parameter $b$) dependence of EM fields generated in heavy-ion collisions.

Here we note that the chiral magnetic conductivity $\sigma_\chi$ in general will be complex due to the spatially anti-symmetric part of the off diagonal photon polarization tensor, as has been formally discussed and evaluated in [44] by using the linear response theory. It has also been shown in [44] that the chiral magnetic conductivity $\sigma_\chi$ has very strong frequency $\omega$ and temperature $T$ dependence, and the induced current $j(t)$ can even change from positive to negative in the time evolution. We therefore currently postpone the inclusion of the chiral magnetic conductivity $\sigma_\chi$ into the extended KMW model, but leave it to our future study.

Before we move forward, let us make some short remarks here. The merits of above generalization for the charge number densities as well as the formulations of estimated EM fields both in the vacuum in Eqs. (10, 14) and in the medium in Eqs. (25) in the extended KMW model can be at least boiled down to two points: one is that it is more realistically akin to the physical situations for charge distributions with relativistic motion, for which both spherical formed and axially deformed charge number densities are explicitly elaborated, e.g., Eqs. (1) or (7). Therefore, the
formulations of estimated EM fields based on the generalized charge number densities can be easily applied to various colliding systems, such as the asymmetric Cu + Au colliding system or the ongoing STAR experiments for isobar colliding systems in which the $^{96}$Zr and $^{96}$Ru nuclei are widely regarded as an axially deformed nucleus [34] [37]. Here we note that a lot of discussions and the related estimations for the EM fields generated in isobar collisions have been presented [30] [32] [36] [86] [88] [89] [110].

Meanwhile, since the charge number density $\rho(r')$ in Eqs. (1) or (7) is formally embedded with relativistic effect due to Lorentz contraction, it will be much more appropriate and adaptable for the situation at very low energy region when the Lorentz contraction effect is not that significantly large, such as at the lower RHIC energy with $\sqrt{s} = 7.7$ GeV where the Lorentz factor $\gamma \sim 4$. As a comparison, the “pancake shape disk” approximation used in [2] will only be a good approximation in high energy region where the Lorentz factor $\gamma$ is substantially large. Therefore the model proposed in this paper will be much more appropriate for applications to the STAR-BES program, and some even lower energy region like the under planning FAIR, NICA and J-PARC programs.

The other point is that since the estimation of generated EM fields are explicitly embedded with medium feedback effects through a constant electric conductivity $\sigma$ and refined baryon-junction stopping effect with proper retardation correction in Eq. (14), the generalized formulas of EM fields from Eqs. (10-14) in the vacuum or Eqs. (22-26) in the medium hence can further be used for many EM fields related studies, such as the CME related charge asymmetry correction in Eq. (14), the generalized formulas of EM fields from Eqs. (10-14) in the vacuum or Eqs. (22-26) in the effects through a constant electric conductivity $\sigma$ [30], CME current $J = \sigma B$ and its related studies for chiral magnetic conductivity $\sigma_\gamma$ [44], in-medium particle mass [63] [69] as well as the QCD phase diagram under strong magnetic field [60] [65], and so on.

Due to the ellipsoidal-type geometry and also the peculiar charge domains occupied by spectators and participants, the numerical integrations of the charge distribution for the estimated EM fields from Eqs. (10-14) or Eqs. (22-26) are actually not very easily and quickly performed. Meanwhile, we notice that the L-W equations of EM fields in the vacuum, i.e. Eq. (8), are widely accepted and used. We therefore propose one possible simplification of the formulas of estimated EM fields in the conducting QGP medium from Eqs. (22-26) by replacing the continuous integration of the charge distribution with the discrete summation of all point-like charges, which turns out to be an alternative solution to do Monte-Carlo numerical simulations of generated EM fields from the L-W equations [3] [5] [8-11] [15] [20-22] [28-33] as follows

$$
e\mathbf{B}^\pm_S(t, r) = \lim_{Y \to \pm y_0} \alpha_{EM} \sum_{N_S} Z_n \frac{\sinh(Y) \cdot \mathbf{e}_z \times \mathbf{R}_\pm}{\Delta^{3/2}} \left[ 1 + \frac{\sigma \sinh |Y| \sqrt{\Delta}}{2} \right] e^A,$$

$$e\mathbf{B}^\pm_P(t, r) = \alpha_{EM} \sum_{N_P} Z_n \int_{-y_0}^{y_0} dY \frac{\Psi_\pm(Y) \sinh Y \cdot \mathbf{e}_z \times \mathbf{R}_\pm}{\Delta^{3/2}} \left[ 1 + \frac{\sigma \sinh |Y| \sqrt{\Delta}}{2} \right] e^A,$$

(27)

for the decomposed magnetic field $\mathbf{B}$ contributed by spectators and participants. For that of the electric field $\mathbf{E}$, we obtain the following expressions for $x$ and $y$ components of the electric field $E_x$ and $E_y$,

$$
eE^\pm_x(t, r) = \lim_{Y \to \pm y_0} \alpha_{EM} \sum_{N_S} Z_n \frac{R_\perp \cosh Y}{\Delta^{3/2}} \left( 1 + \frac{\sigma \sinh |Y| \sqrt{\Delta}}{2} \right) e^A \frac{x - x'}{R_\perp} \left( y - y' \right),$$

$$- \frac{\sigma}{R_\perp \tanh |Y|} \left[ 1 + \frac{\sinh |Y| \left( t - \frac{z - z'}{\tanh Y} \right) \exp \left( -\sigma \left[ t - \frac{z - z'}{\tanh Y} \right] \right) \right] e^A \frac{x - x'}{R_\perp} \left( y - y' \right),$$

$$eE^\pm_y(t, r) = \alpha_{EM} \sum_{N_S} Z_n \int_{-y_0}^{y_0} dY \frac{\Psi_\pm(Y) \cosh Y}{\Delta^{3/2}} \left( 1 + \frac{\sigma \sinh |Y| \sqrt{\Delta}}{2} \right) e^A \frac{y - y'}{R_\perp} \left( y - y' \right),$$

(28)

for $x'$ and $y'$ components. Notice that the $\cosh Y$ and $\sinh Y$ factors in the integrand ensure that $eE_\perp$ is always positive definite, which implies that $eE_\perp$ is a well defined finite function.
and also that for the z component of the electric field $E_z$,

$$
eE_{z,S}^{\pm}(t,r) = \lim_{Y \to Y_0^+} \alpha_{EM} \sum_{n=1}^{N_N} Z_n \left\{ \text{sgn}(Y)\sigma^2 \exp\left( -\sigma \left[ t - \frac{z - z'}{\tanh Y} \right] \right) \right\} \Gamma(0, -A)$$

$$+ \frac{eA}{\Delta^{3/2}} \left\{ (z - z') \cosh Y - t \sinh Y - \text{sgn}(Y)A\Delta - \frac{\sigma \sinh Y}{\tanh^2 Y}\Delta \right\}.$$

(29)

Here the expressions for $R_\perp$, $\Delta$ and $A$ are the same as the definitions in Eq. (23), and $\Psi_{\pm}(Y)$ in Eq. (14). $N_S$ and $N_P$ denote the total numbers of spectators and participants of point-like charges, respectively. Also, we note that a similar Monte-Carlo simulation using above formulas of EM fields for point-like charges in Eqs. (27-29) has firstly been performed in [23], since such an alternative formulation can be more directly obtained from the EM fields of point-like charge in [23] when further embedded with the refined baryon-junction stopping effect with retardation correction as in Eq. (14). If one assumes that each proton at nucleon level in the two colliding nuclei can be treated as point-like charge, as has been extensively assumed in literatures [3, 6, 8-11, 15, 20, 22, 28, 53], then above generalization of EM fields in Eqs. (27-29) for Monte-Carlo simulations in heavy-ion collisions can have at least two merits: one is that the EM fields from Eqs. (27-29) are analytically embedded with a constant electric conductivity $\sigma$ for the incorporation of medium effects rather than the original L-W equations in the vacuum; the other is that the EM fields in Eqs. (27-29) are also implanted with the refined baryon-junction stopping effect with retardation correction as in Eq. (14).

III. RESULTS AND DISCUSSIONS

Since we have explicitly formulated the estimations of generated EM fields for heavy-ion collisions in the vacuum in Eqs. (10-14) or in the medium in Eqs. (22-26), as well as an alternative solution to the currently used Monte-Carlo simulations of point-like charges in the medium in Eqs. (27-29), let us first give some pre-analysis before performing the ab initio integration of charge distribution or event-by-event simulations of generated EM fields at specific space-time points. Due to the mirror and centrosymmetric symmetries of the colliding system (as is illustrated in Fig. 1), the total electric field $E$ will vanish while the total magnetic field $B$ will only remain its $y$ component pointing in the negative $y$ direction at the center point $r = 0$ of the overlapping region, which further results in a charge separation due to the CME. Therefore, we only focus on the $y$ component of magnetic field $B_y$ in this paper.

Let us first declare some abbreviations that we will use in the following. The original KMW model is incorporated with a two-dimensional (2D) surface charge number density, which is abbreviated as 2D KMW model. The extended KMW model is generalized by incorporating the longitudinal position dependence and generalized three-dimensional (3D) volume charge number density, which is likewise abbreviated as 3D KMW model but including two cases: with (w/) and without (w/o) retardation correction (RC). The collision time $t_c$ for simplicity is chosen at fully overlapping time, namely $t_c = 0$. Besides, we do check that even when one considers the collision time $t_c$ totally from the geometry consideration regardless of the causality reason according to the theory of special relativity, namely the collision time $t_c$ can be estimated as $t_c = -t_d$, where $t_d$ is a departure time defined in Eq. (30), our results basically will not be changed due to a much earlier retardation time $t_{ret}$

It should also be noted that the formulation of estimated magnetic field in the original KMW model is actually only in the vacuum without retardation correction. In order to make a better comparison of the two models in the conducting medium, we also show the generalized 2D KMW model embedded with a constant Ohm electric conductivity $\sigma$, which can be easily obtained from the reduction of extended KMW model by removing the longitudinal position dependence and replacing the corresponding charge number density $\rho$. Such a generalization of the formulas of the magnetic field in the conducting medium actually is consistent with that in Refs. [17, 27]. Moreover, since it is mainly the magnetic field strength that has been numerically evaluated in the original KMW model [2], and also the following papers [17, 27]. For a comparison consideration, we only show the results of estimated magnetic field in this paper. The results of estimated electric field can be similarly evaluated from the extended KMW model in the same way as we present here.
A. Time Evolution of the Magnetic Field Strength

Let us now make a sample comparison of the time evolution of the total magnetic field strength estimated from Eq. 10 in the vacuum. In Fig. 2, we show the estimation of time evolution of the total magnetic field strength $eB(t, r)$ in the vacuum at the center point $r = 0$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200$ GeV with different impact parameters $b = 4, 8, 12$ fm. From the inserted plot in the upper panel, one can clearly see that the magnetic field from 3D KMW model with (w/ ) and without (w/o) retardation correction (RC) at $t = 0$ will systematically yield a relatively smaller magnetic field strength (especially for larger impact parameter $b$), compared with that too sharp peak of magnetic field strength around $t \sim 0$ from the original KMW model. It can intuitively be attributed to the more localized and central elevated charge distribution used in the original KMW model. We check that our results at $t = 0$ actually are indeed consistent with the numerical simulations from HIJING model in [10], which will be clearly demonstrated in the next subsection.

![Figure 2: Comparison of the time evolution of the total magnetic field strength in the vacuum from extended KMW model with (w/) and without (w/o) retardation correction (RC) and that from original KMW model at different impact parameters $b = 4, 8, 12$ fm for the center point $r = 0$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200$ GeV. The two dashed horizontal lines indicate the later-time constraints of the magnetic field strength estimated in [93] from the difference between global polarizations of $\Lambda$ and $\bar{\Lambda}$ hyperons in Au+Au collisions at $\sqrt{s} = 200$ GeV in [112] at one standard deviation ($1\sigma$) and three standard deviations ($3\sigma$), respectively.]

However, compared with that “seemly enhanced” magnetic field strength from the extended (3D) KMW model without retardation correction, it is also noticeable that the original KMW model will generally give rise to a weaker magnetic field strength just shortly ($\sim 0.03$ fm/c) after the collision, and the differences in between become rather noticeable at later times for more central (or smaller impact parameter $b$) collisions. Thus the extended KMW model without retardation correction will yield a longer lifetime $t_B$ of the magnetic field than the original KMW model. When incorporated with retardation correction, however, the magnetic field strength from the extended (3D) KMW model will firstly decrease very fast, and then decrease slowly and even grows up to a certain extent for some time range at some impact parameters, e.g., $b = 4, 8$ fm, as clearly shown in the lower panel of Fig. 2. This can be intuitively understood as follows: when incorporated with the retardation correction (RC) in Eq. 14, the baryon-junction stopping effects represented through $f_{\pm}(Y)$ in Eq. 14 are basically ruled out by the $\theta(t_{\text{ret}} - t_c)$ function during the comparison of retardation time $t_{\text{ret}}$ with the collision time $t_c$. The contributions from charged participants are mainly dominated by those with beam rapidity $\pm Y_0$, hence the magnetic field strength at early stage decreases much faster than those without retardation correction; At middle or later stage time evolution, however, the contribution from baryon-junction stopping bulk matter grows dominant since the spectators have fly away. The resulting magnetic field strength contributed by those participants will increase first and then decrease. Hence the resulting total magnetic field strength will show non-monotonic behavior, especially for smaller impact parameter $b$.
due to larger baryon-junction stopping bulk matter. This can be interpreted as a result of the competition between retardation effect of field propagation and baryon-junction stopping effect of participants.

The electric conductivity \( \sigma \) has been calculated by using a first principal lattice QCD approach in the quenched approximation in Ref. [105], according to which we set the electric conductivity \( \sigma = 5.8 \text{ MeV} \) in this paper so as to make estimations of the magnetic field generated in the conducting QGP medium. In Fig. 3, we show the estimations of time evolution of the magnetic field strength \( eB(t, r) \) in the conducting medium with \( \sigma = 5.8 \text{ MeV} \) from the extended KMW model with (w/) and without (w/o) retardation correction (RC) and that from the original KMW model at the center point \( r = 0 \) of the overlapping region in Au+Au collisions at \( \sqrt{s} = 200 \text{ GeV} \) with different impact parameters \( b = 4, 8, 12 \text{ fm} \). One can clearly see that the magnetic field strength in the conducting QGP medium case in Fig. 3 decays much slower than that in Fig. 2 in the vacuum case. There clearly show non-monotonic behaviors of the magnetic field strength at early time stage for different impact parameters in the conducting QGP medium rather than the monotonically decreasing behaviors of magnetic field strength in the vacuum. Besides, compared with the original KMW model (2D), we find the extended KMW model (3D) will in general systematically yield a relatively larger magnetic field strength in the conducting QGP medium except at very early stage around \( t \sim 0 \). Also, we notice that the differences between any two cases become more significant around the non-monotonic peaks \( (t \sim 0.1 \text{ fm}/c) \), especially for smaller impact parameter \( b \).

Moreover, we notice that the time evolution of the estimated magnetic field strength \( B(t, r) \) in the conducting medium at the later-time stage evolution in Fig. 3 is quiet different from that in the vacuum case in Fig. 2 for the impact parameter \( b \) dependence. This can be intuitively understood as follows: since in the vacuum case there is no medium feedback effect due to the vanishing electric conductivity \( \sigma \), the magnetic field decays quickly and there is only the baryon-junction stopping effect that may to some extent hinder the moving charged bulk matter from flying away from the field point \( r = 0 \). Thus the baryon-junction stopping effect will dominate in the later stage of time evolution especially for smaller impact parameter \( b \) due to larger bulk matter of the participants, the resulting total magnetic field strength therefore becomes larger for smaller impact parameter \( b \) at later time stage. In the conducting medium, however, since the electric conductivity \( \sigma \) is non-negligible, it is the Faraday's induction effect rather than the baryon-junction stopping effect that dominates for the later time evolution. The resulting total magnetic field strength in the conducting medium therefore shows rather distinct impact parameter \( b \) dependence at later time stage from that in the vacuum. We also notice that the original (2D) KMW model and the extended (3D) KMW model can

\[
\begin{align*}
\Delta P_{\mu} (3\sigma) &
\end{align*}
\]

\[
\begin{align*}
\Delta P_{\mu} (1\sigma) &
\end{align*}
\]

\[
\begin{align*}
\sqrt{s} = 200 \text{ GeV}, \bar{r} = 0
\end{align*}
\]

FIG. 3: (Color online) Comparison of the time evolution of the total magnetic field strength in the medium with constant Ohm electric conductivity \( \sigma = 5.8 \text{ MeV} \) from extended KMW model with (w/) and without (w/o) retardation correction (RC) and that from generalized original KMW model at different impact parameters \( b = 4, 8, 12 \text{ fm} \) for the center point \( r = 0 \) of the overlapping region in Au+Au collisions at \( \sqrt{s} = 200 \text{ GeV} \). The two dashed horizontal lines indicate the later-time constraints of the magnetic field strength estimated in [93] from the difference between global polarizations of \( \Lambda \) and \( \bar{\Lambda} \) hyperons in Au+Au collisions at \( \sqrt{s} = 200 \text{ GeV} \) in [112] at one standard deviation (1\( \sigma \)) and three standard deviations (3\( \sigma \)), respectively.
give almost identical results after \( t > 1.0\,\text{fm}/c \), and the difference in between grows little during the time evolution which is quite different from that in the vacuum case.

The “kinks” marked as the points at different impact parameters \( b \) for the extended (3D) KMW model in Fig. 3 are due to the fact that the two colliding nuclei start to separate from each other. The corresponding vertical lines indicate the departure time \( t_d \) when the two colliding nuclei just depart from each other, which can be estimated from the following geometry relation in the \( x-z \) plane (RP) at an impact parameter \( b \) as follows

\[
t_d = t_d(b) = \frac{R_A}{\gamma} \sqrt{1 - \frac{b^2}{2R_A}}.\tag{30}
\]

The original (2D) KMW model due to the same reason actually also have “kinks”, which however are all located at \( t = 0 \) (we do not show in Fig. 3), since the charges in the original KMW model are restricted within the transverse plane with zero depth in the \( z \) direction.

By using the interesting idea proposed in the recent Ref. [93], one can use the experimentally measured polarization data [111, 112] on \( \Lambda \) and \( \bar{\Lambda} \) hyperons to make some constraints on the allowed later-time magnetic field strength, which has been also recently explored in Ref. [113]. It has been estimated in [93] that in the one standard deviation (1σ) limit of the recent measurements of the global polarization of \( \Lambda \) and \( \bar{\Lambda} \) hyperons in Au+Au collisions at \( \sqrt{s} = 200 \,\text{GeV} \) [112], the magnetic field strength at thermal freeze-out can be estimated as

\[
\epsilon|B| = \frac{\epsilon T_s |\Delta P_H|}{2|\mu_{\Lambda}|} < 2.8985 \times 10^{-3} m_{\pi}^2.\tag{31}
\]

where \( T_s \approx 150 \,\text{MeV} \) is the temperature of the emitting source, \( \Delta P_H = P_{\Lambda} - P_{\bar{\Lambda}} \) is the difference in global polarizations of \( \Lambda \) and \( \bar{\Lambda} \) hyperons, and \( \mu_{\Lambda} = -\mu_{\bar{\Lambda}} = -0.613\mu_N \). In the three standard deviations (3σ) limit, the magnetic field strength is estimated as \( \epsilon|B| < 1.8021 \times 10^{-2} m_{\pi}^2 \) [93]. Here we choose \( m_{\pi} \) as the mass of \( \pi^0 \) rather than that of \( \pi^\pm \), which is consistent with the pion mass that we use throughout this paper. Therefore, we can immediately use these possible constraints to compare with the magnetic field strength estimated from the extended KMW model and that from the original KMW model both in the vacuum in Fig. 2 and in the conducting medium in Fig. 3 which could enable us to make some estimations of the possible lifetime \( t_B \) of the generated magnetic field in Au+Au collisions at \( \sqrt{s} = 200 \,\text{GeV} \).

By comparing with the possible constraints shown by two dashed horizontal lines at 1σ and 3σ limits in Fig. 2 and Fig. 3, we notice that the magnetic field strength in the vacuum in Fig. 2, at a possible lifetime \( t_s \approx 5 \,\text{fm}/c \) of the QGP in Au+Au collisions at \( \sqrt{s} = 200 \,\text{GeV} \) [112], is extremely smaller than the 1σ bound of the possible constraint from the polarization data. This means that although we have improved the estimation of time evolution of the magnetic field strength in the vacuum in the extended KMW model, which results in a prolonged lifetime of magnetic field compared with that from the original KMW model, but the strength of the magnetic field at thermal freeze-out time \( t = t_s \) still appears quite inadequate (about two orders of magnitude smaller) for explaining the observed difference between global polarizations of \( \Lambda \) and \( \bar{\Lambda} \) hyperons. Hence, this indicate that the generated QGP in heavy-ion collisions need to maintain the magnetic field stronger for a longer lifetime than that in the vacuum, which further requires that the generated QGP is more like a conducting medium.

In Fig. 3, as our expectation above, we surprisingly notice that the magnetic field strength in the conducting medium with a constant electric conductivity \( \sigma = 5.8 \,\text{MeV} \) at \( 3 < t < 5 \,\text{fm}/c \) almost lies between the 1σ and 3σ bounds. We find that the magnetic field strength at impact parameter \( b = 4-12 \,\text{fm} \) lie between \( \epsilon B \approx (2.7094 - 7.8672) \times 10^{-3} m_{\tau}^2 \) at \( t = t_s \approx 5 \,\text{fm}/c \), which meets the required condition of magnetic field strength for the explanation of observed difference between global polarizations of \( \Lambda \) and \( \bar{\Lambda} \) hyperons. Our results support that if the generated QGP is a conducting medium, the enough strength of magnetic field can be reached at the thermal freeze-out time.

### B. Impact Parameter Dependence of the Magnetic Field Strength

Let us now compare the impact parameter \( b \) dependences of magnetic field strength in the vacuum and in the conducting QGP medium.

In Fig. 4 we show the impact parameter \( b \) dependences of the estimated magnetic field strength in the vacuum from the extended (3D) KMW model and from the original (2D) KMW model with (w/ ) and without (w/o) retardation correction (RC) at the center point \( r = 0 \) of the overlapping region in Au+Au collisions at \( \sqrt{s} = 200 \,\text{GeV} \) at \( t = 0 \). Meanwhile, we also show the results from numerical simulations with HIJING model in [10, 50] and that from a sample analytical approach (denote as LGK model) initially proposed in [16], both of which have been used for the comparison of impact parameter dependence of magnetic field in Ref. [50].
FIG. 4: (Color online) (a) Comparison of impact parameter $b$ dependence of the estimated total magnetic field strength in the vacuum at the center point $r = 0$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200$ GeV at $t = 0$ fm/c from six different approaches. (b) The corresponding ratio (2D/3D) of the total magnetic field strength from 2D and 3D KMW models with (w/) and without (w/o) retardation correction (RC). The dashed vertical line indicates the boundary at $b = 2R_A$, above which the two colliding nuclei will miss each other and there is no contribution from participants, as indicated by the colored bands. The corresponding “kinks” are also marked by the points along the dashed vertical line.

At first glance, one may notice that the magnetic field strength in the vacuum at smaller impact parameter $b$ from the 3D KMW model seems to deviate a lot from the result obtained with HIJING model in [10]. Meanwhile, the LGK model seems to work well and be consistent with the HIJING result. But we should point out that in the evaluation of generated magnetic field with HIJING model in [10], at $t = 0$ when the two colliding nuclei are completely overlapping with each other, all nucleons inside the two colliding nuclei are assigned with the same beam velocity $v_z = 1 - (2m_N/\sqrt{s})^2$, and in the LGK model presented in [50] all the moving charges inside the two colliding nuclei are regarded as spectators and assigned with the beam velocity $\pm v_z$, but they both neglect the baryon-junction stopping effect during the overlapping process. Unlike these two approaches, in the extended KMW model, we have included the baryon-junction stopping effect through the experimentally supported $f_{\pm}(Y)$ in Eq. (12) along with the generalised charge distribution. Therefore, it is not surprising that the simulation with HIJING model and the LGK model in [50] will roughly yield similar and consistent results, and both of them will give slightly larger magnetic field strength, compared with the extended (3D) KMW model without (w/o) retardation correction (RC), especially at smaller impact parameter $b$ due to a larger charged bulk matter of participants which is affected by $f_{\pm}(Y)$. When the retardation correction (RC) is included, however, the extended (3D) KMW model with (w/) retardation correction (RC) give a slightly larger magnetic field strength at larger impact parameter $b$ than the HIJING and LGK models, which can be mainly attributed to the generalised charge distribution for the incorporation of Lorentz contraction effect on the geometries of the two colliding nuclei used in the extended KMW model.

In Fig. 4, one may notice the “kinks” at the boundary $b = 2R_A$ from both original and extended KMW models where the two colliding nuclei begins to miss each other and there is no contribution from participants (which will be demonstrated in the following Fig. 5), compared with the other two models that seem to smoothly across the boundary. The “kinks” are due to the fact that the charge number densities in both 2D and 3D KMW models are accompanied with the step functions $\Theta_{\pm}$ so as to constrain that the charges are limited within the two colliding nuclei (with radius $R_A$ before the Lorentz contraction), i.e. Eq. (1). Here we note that the HIJING model uses an relatively large upper limit of the nucleus radius for the corresponding sampled protons inside, which actually largely exceeds the generally supposed nucleus charge radius $R_c$ estimated from the nucleus root mean square (RMS) charge radius $R_{rms}$ measured in experiments [115] such as $R_A \simeq R_c = \sqrt{5/3}R_{rms}$, while the LGK model used in [50] also employs an relatively large nuclear charge radius $R_c$ in the treatment of effective charge $Z_{eff}$.

Besides, one can clearly notice that, compared with other three 3D models used in Fig. 4 the original (2D) KMW
FIG. 5: (Color online) Comparison of impact parameter $b$ dependence of the magnetic field strength in the vacuum contributed by spectators (S) in the upper panels (a) and (b), and that contributed by participants (P) in the lower panels (c) and (d) at the center point $r = 0$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200$ GeV at $t = 0$ fm/c from original (2D) KMW model (left panels) with that from extended (3D) KMW model (right panels). The dashed vertical lines indicate the boundary at $b = 2R_A$, above which the two colliding nuclei will miss each other and there is no contribution from participants, as indicated by the colored bands. The corresponding “kinks” are also marked by the points along the dashed vertical lines.

The model indeed largely overestimates the magnetic field strength at $t = 0$, as shown in Fig. 2 that the original (2D) KMW model will give too sharp peaks of magnetic field strength around $t \sim 0$ at different impact parameters due to the more localized and central elevated two-dimensional surface charge number density and no longitudinal position dependence of the EM fields. Moreover, we show the ratios (2D/3D) of the total estimated magnetic field strength from the original (2D) KMW model to that from the extended KMW model with (w/ ) and without (w/o) retardation correction in the lower panel of Fig. 4. One can clearly see that the original (2D) KMW will overestimate the magnetic field strength roughly by a factor of $\sim 1.7$ (1.5) at relatively smaller impact parameter $b$ with (without) retardation correction (RC), especially at the boundary $b = 2R_A$ by a factor of $\sim 2.4$ (2.4).

In Fig. 5, we make a detailed comparison of the impact parameter $b$ dependence of the estimated magnetic field strength in the vacuum contributed by spectators (S) and participants (P) from the original (2D) KMW model (left panels) with that from the extended (3D) KMW model (right panels) with (w/ ) and without (w/o) retardation correction (RC). From the upper panels of Fig. 5 one can clearly see that it is mainly the contributions of spectators in the original (2D) KMW model that cause the too shape peak of the estimated magnetic field strength (left), compared with that result from the extended (3D) KMW model (right), which is also consistent with observed too shape peaks in the time evolution in Fig. 4 at $t \sim 0$.

In the lower panels of Fig. 5 we also notice that the magnetic field strength contributed by participants with retardation correction systematically shows strong enhancements relative to that without retardation correction for both 2D and 3D KMW models. This is mainly due to the fact that the baryon-junction stopping effect represented by $f_{\pm}(Y)$ in Eq. (14) is basically ruled out by the $\theta(t_{ret} - t_c)$ function during the comparison between retardation time $t_{ret}$ and collision time $t_c$. The contributions of charged participants actually are all calculated with beam rapidity $\pm Y_0$ instead of the normalized rapidity distribution $f_{\pm}(Y)$. Hence this kind of enhancement is quite significant at $t = 0$, especially for the 2D KMW model due to the more localized two-dimensional surface charge number density. It is quite noticeable that the magnetic field strength contributed by participants in the 3D KMW model for a middle impact parameter $b$, e.g., $b = 8$ fm, is almost one third of that in the 2D KMW model for the case with retardation correction (RC).

Different from the situation in the vacuum in Fig. 4, the centrality dependence of the generated magnetic field strength in the conducting medium should be realistically evaluated at some time after the collision, since the formation of QGP medium certainly needs some time after the collision time $t_c$ for a more realistic consideration. At present, it is still not very clear how long it takes for the formation of QGP medium in heavy-ion collisions, so we estimate it
from the widely used (pre-) equilibration time in hydrodynamical simulations, namely $t = \tau_0 = 0.6 \text{ fm}/c$ \cite{114,116}, with a constant Ohm electric conductivity $\sigma = 5.8 \text{ MeV}$ from extended (3D) KMW model and that from original (2D) KMW model at the center point $r = 0$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200 \text{ GeV}$. (b) The corresponding ratio (2D/3D) of the total magnetic field strength from 2D and 3D KMW models with (w/ ) and without (w/o) retardation correction (RC). The dashed vertical line indicates the boundary at $b = 2R_A$ in Fig. 4, compared to which there is no “kink” at all since the two colliding nuclei have already missed each other before $t = \tau_0$ and the corresponding shaded color bands are also not shown.

Likewise, in Fig. 7 we show a detailed comparison of the impact parameter $b$ dependence of the estimated magnetic field strength in the conducting QGP medium contributed by spectators (S) and participants (P) at $t = \tau_0 = 0.6 \text{ fm}/c$ \cite{114,116} with a constant electric conductivity $\sigma = 5.8 \text{ MeV}$ from the original (2D) KMW model (left panels) with that from the extended (3D) KMW model (right panels) with (w/ ) and without (w/o) retardation correction (RC). In the upper panels of Fig. 7, one can clearly see that the original (2D) KMW model and the extended (3D) KMW model can give almost the same impact parameter $b$ dependent magnetic field strength contributed by spectators, although the result in the extended (3D) KMW model is slightly stronger. In the lower panels of Fig. 7, we find that the retardation correction (RC) can only mildly modulate the impact parameter $b$ dependent magnetic field strength contributed by participants in the conducting medium, which is more noticeably stronger in the extended (3D) KMW model than that in the original (2D) KMW model, similar to the situation of spectators in the upper panels of Fig. 7. These behaviors in the conducting medium, however, are distinctly different from those in the vacuum at $t = 0 \text{ fm}/c$ in Fig. 5, where the original (2D) KMW model will systematically yield a stronger magnetic field strength than the extended (3D) KMW model.
FIG. 7: (Color online) Comparison of impact parameter $b$ dependence of the magnetic field strength in the conducting QGP medium contributed by spectators (S) in the upper panels (a) and (b), and that from participants (P) in the lower panels (c) and (d) at hydrodynamical (pre-) equilibrium time $t = \tau_0 = 0.6 \text{fm}/c$ \cite{114, 116} with a constant Ohm electric conductivity $\sigma = 5.8 \text{ MeV}$ at the center point $r = 0 \text{ fm}$ of the overlapping region in Au+Au collisions at $\sqrt{s} = 200 \text{ GeV}$ from original (2D) KMW model (left panels) with that from extended (3D) KMW model (right panels). The dashed vertical lines indicate the boundary at $b = 2R_A$ in Fig. 4, compared to which there is no “kink” at all since the two colliding nuclei have already missed each other before $t = \tau_0$ and the corresponding shaded color bands are also not shown.

IV. SUMMARY

In summary, based on the Kharzeev-McLerran-Warringa (KMW) model \cite{2} for the estimation of strong EM fields generated in heavy-ion collisions, we make an attempt to generalize the formulas of estimated EM fields in the original KMW model, which eventually turns out to be the above formulations in the extended KMW model.

We first start from generalizing the widely used charge distribution (or charge number density) models by incorporating the Lorentz contraction effect for the ellipsoidal geometry of each colliding nucleus due to relativistic motion, e.g., the three-parameter Fermi (3pF) model, for both spherically symmetric colliding nuclei as well as axially deformed colliding nuclei used in heavy-ion collisions. We then combine the widely used L-W equations of EM fields generated by point-like charges with the generalized relativistic three-dimensional charge number densities. By employing the decomposition method for the contributions of spectators and participants of the two colliding nuclei as that in the original KMW model \cite{2}, we then naturally generalize the formulas of EM fields in the vacuum by incorporating the longitudinal position dependence and retardation correction (RC) for heavy-ion collisions.

Since the medium feedback effects in the high-temperature QCD matter or the conducting QGP medium may substantially modify the time evolution of the generated EM fields according to the Faraday’s induction law, we explicitly and analytically embed a constant electric conductivity $\sigma$ into the Maxwell’s equations for the incorporation of medium effects, and eventually formulate the estimation of generated EM fields with medium feedback effects in the extended KMW model for heavy-ion collisions. For simplicity, we also propose one possible kind of simplification of the formulas of EM fields in the conducting QGP medium, which turns out to be an alternative way to the currently used Monte-Carlo simulations of generated EM fields from widely used L-W equations in heavy-ion collisions, where we incorporate both conducting medium effects through a constant Ohm conductivity $\sigma$ and the refined baryon-junction stopping effect with retardation correction (RC).

Our formulations of the estimated EM fields in the extended KMW model will result in a slower time evolution (or a longer lifetime $t_B$) and a more reasonable impact parameter $b$ dependence of the magnetic field strength in the vacuum. The formulations of estimated EM fields embedded with medium feedback effects through a constant electric conductivity $\sigma = 5.8 \text{ MeV}$ from lattice QCD data further prolong the time evolution of the magnetic field. We surprisingly find that the magnetic field strength at the estimated thermal freeze-out time $t = t_s$ can meet the
required magnetic field strength for the explanation of the observed difference in global polarizations of Λ and ¯Λ hyperons in Au+Au collisions at $\sqrt{s} = 200$ GeV. This supports that if the generated QGP is a conducting QCD medium, an enough strength of magnetic field can be maintained until the thermal freeze-out time.

It should be emphasized that we only applied the generalization of charge distributions with the three-parameter Fermi model (3pF) for Au+Au collisions at $\sqrt{s} = 200$ GeV in this work, which is just a simple case of spheric nucleus, various colliding nuclei such as axially deformed isobaric nuclei (zirconium vs ruthenium experiment at RHIC) can be similarly generalized in the same way as we present here. Also, the extended KMW model can be applied to various colliding energies, especially for the lower energy regions like the ongoing STAR-BES program and the under planning FAIR, NICA and J-PARC programs. Hopefully, these generalized formulations also could be employed for many EM fields relevant studies, such as the charge asymmetry correlators or fluctuations related to the experimental searching for the CME, in-medium particle mass, as well as the QCD phase diagram under strong magnetic field.

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