Semileptonic B decays into even parity charmed mesons

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By using a constituent quark model we compute the form factors relevant to semileptonic transitions of B mesons into low-lying p-wave charmed mesons. We evaluate the $q^2$ dependence of these form factors and compare them with other model calculations. The Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$ are also obtained in the heavy quark limit of our results.

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I. INTRODUCTION

Recently, BaBar Collaboration has discovered a narrow state with $J^P = 0^+$ with a mass of 2317 MeV, $D_{s0}^{*}(2317)$ [1]. The existence of a second narrow resonance, $D_{sJ}(2460)$ with $J^P = 1^+$, was confirmed by CLEO [2]. Both states have been confirmed by BELLE [3]. Soon after the discovery, another set of charmed mesons, $D_0^{*0}(2308)$ and $D_0^{*+}(2427)$ which have the same quantum numbers $J^P = (0^+, 1^+)$ as $D_{sJ}$ has been discovered by BELLE [3]. Before their discovery, quark model and lattice calculations predicted that the masses of these states, in particular $D_{s0}^{*}(2317)$ and $D_{s1}^{*}(2460)$, would be significantly higher than observed [3,4]. Moreover, these states were predicted to be broad due to the fact that they can decay into $D K$ and $D^* K$, respectively. Experimentally, the masses of $D_{s0}^{*}(2317)$ and $D_{s1}^{*}(2460)$ are below the $D K$ and $D^* K$ thresholds and hence they are very narrow. These facts inspired a lot of theorists to explain the puzzle [7].

In this paper we will focus our attention on the weak semileptonic transitions of B mesons into lower lying p-wave charmed mesons ($D^{**}$). These transitions were studied, within a quark model approach, for the first time in [8] and, more recently, in [9] where the authors take into account the symmetries of QCD for heavy quarks [10], already used in [11]. The light-front covariant model [12] was adopted to study the same subject in [13]. The relevant form factors were also evaluated, in the framework of QCD Sum Rules [14], in [15].

Here we employ a simple constituent quark model [16, 17] to evaluate semileptonic form factors of B mesons into p-wave charmed mesons. The plan of the paper is the following. In the next section we describe our quark model; the third section is devoted to introduce and evaluate the s-wave to p-wave form factors. Our way to fix the free parameters of the model and the resulting form factors are discussed in section four, while in section five the heavy quark limit of the form factors are computed and compared with Heavy Quark Effective Theory predictions; the $\tau_{1/2}$, and $\tau_{3/2}$ are also evaluated. In the last section we show and discuss our numerical results.

II. A CONSTITUENT QUARK MODEL

In our model [16, 17] any heavy meson $H(Q\bar{q})$, with $Q \in \{b, c\}$ and $q \in \{u, d, s\}$, is described by the matrix

$$H = \frac{1}{\sqrt{3}} \psi_H(k) \left[ \frac{q_1^0 + m_Q}{2m_Q} \Gamma \frac{q_2^0 + m_q}{2m_q} \right],$$

where $m_Q$ ($m_q$) stands for the heavy (light) quark mass; $q_1^0$, $q_2^0$ are their 4-momenta (cfr Fig.1). $\psi_H(k)$ indicates the meson’s wave function which is fixed by using a phenomenological approach. The meson constituent quarks vertexes, $\Gamma$ in Eq. (1), are fixed by using the correct transformation properties under C, P and to enforce the relation

$$< H | H > \equiv \text{Tr}\{(-\gamma_0 H^\dagger \gamma_0) \ H\} = \int \frac{d^3k}{(2\pi)^3} |\psi_H(k)|^2 = 2 \ M_H .$$

For the odd parity, s-wave heavy mesons $J^P = (0^-, 1^-)$, $\Gamma$ is given by

$$\Gamma_{0^-} = -i\gamma_5 \sqrt{\frac{2m_Qm_q}{m_H^2 - (m_Q - m_q)^2}},$$

$$\Gamma_{1^-}(\varepsilon, q_1, q_2) = \varepsilon\mu \left[ \gamma^\mu - \frac{q_1^\mu - q_2^\mu}{m_H + m_Q + m_q} \right] \sqrt{\frac{2m_Qm_q}{m_H^2 - (m_Q - m_q)^2}},$$
where $\epsilon$ is the polarization 4-vector of the (vector) meson $H$. The vertexes of the lower lying even parity heavy mesons, instead, are given by the matrices

$$\Gamma_{0h} = \Gamma_{0+} = -i \sqrt{\frac{2m_Qm_q}{m_H^2 - (m_Q + m_q)^2}},$$

$$\Gamma_{1P_1} (\epsilon, q_1, q_2) = \epsilon_\mu \left[ \gamma\!\!\!\!/ \left( q_1^\mu - q_2^\mu \right) \frac{m_H}{m_H^2 - (m_Q - m_q)^2} \right] \gamma_5 \sqrt{\frac{6m_Qm_q}{m_H^2 - (m_Q + m_q)^2}},$$

$$\Gamma_{1P_1} (\epsilon, q_1, q_2) = \epsilon_\mu \left[ \left( q_1^\mu - q_2^\mu \right) \frac{m_H}{m_H^2 - (m_Q - m_q)^2} \right] \gamma_5 \sqrt{\frac{6m_Qm_q}{m_H^2 - (m_Q + m_q)^2}}.$$

As already discussed in [16, 17], the 4-momentum conservation in the meson-constituent quarks vertexes can be obtained defining a heavy running quark mass. For details we address the reader to the references [16, 17]. Here, for the sake of utility, we recall all the remaining rules of our model for the evaluation of the hadronic matrix elements of weak currents:

a) for each quark loop with 4-momentum $k$ we have

$$\int \frac{q^3k}{(2\pi)^3},$$

a colour factor of 3 and a trace over Dirac matrices;

b) for the weak hadronic current, $\bar{q}_2 \Gamma^\mu q_1$, one puts the factor

$$\sqrt{\frac{m_1}{E_1}} \sqrt{\frac{m_2}{E_2}} \Gamma^\mu,$$

where with $\Gamma^\mu$ we indicate a combination of Dirac matrices.

### III. FORM FACTORS

In this section we evaluate the form factors parameterizing the $0^- \to (0^-, 1^-)$ and $0^- \to (0^+, 3P_1, 1P_1)$ weak transitions. The decomposition of these matrix elements of weak currents in terms of form factors are the following

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1 We don’t consider in this paper the tensor mesons.
The calculation of the form factors in Eqs. (10) and (11) for the case of $B \to D(D^*)$ transitions has been done in Ref. [17]. However, for the sake of utility, the analytical expressions are reported in appendix A.

One of the main results of this paper is the calculation of the form factors appearing in Eqs. (12), (13) and (14). By way of example, in the following we describe the calculation of the matrix element $<H_0^+(p')|A_µ|H_0^-(p)>$ and give the expressions of the form factors $F_{±}$. In appendix B we collect the expressions for $G(0)$, $F(0)$ and $A^{(i)}_{±}$. Note that all the calculations are done in the frame where $q_0^β = (E_3, \vec{k} − \vec{q})$, $q_1^µ = (E_1, \vec{k})$, $q_2^µ = (E_2, −\vec{k})$.

Let us start considering the $0^- \to 0^+$ transition,

$$<H_0^+(p')|\bar{Q}_3γµγ5Q_1|H_0^-(p)> = - \int_D \frac{d^3k}{(2π)^3} ψ_0^+(k)ψ_0^−(k) \sqrt{\frac{m_1m_3}{E_1E_3}} \text{Tr} \left[ -\frac{q_2^β + m_2^2}{2m_2} (γ_0Γ_{0+}(q_3, q_2)γ_0) \frac{q_3^β + m_3^2}{2m_3} γ_µγ_5 \frac{q_1^β + m_1}{2m_1} (Γ_{0−}) - \frac{q_2^β + m_2^2}{2m_2} \right]$$  (15)

where $D$ is the integration domain (see Refs. [12, 17]) defined by

$$\text{Max}(0, k−) ≤ k ≤ \text{Min}(K_M, K+) \quad \text{Max}(−1, f(k, |\vec{q}|)) ≤ \cos(θ) ≤ +1 \quad 0 ≤ ϕ ≤ 2π$$  (16)

with

$$k_± = \sqrt{m_F^2 + |\vec{q}|^2 ± m_F^2} \sqrt{m_F^2 + |\vec{q}|^2}$$  (17)

$$f(k, |\vec{q}|) = \frac{2\sqrt{m_F^2 + |\vec{q}|^2} \sqrt{k^2 + m_2^2} - (m_2^2 + m_2^2)}{2k|\vec{q}|}$$  (18)

$ϕ$ and $θ$ are the azimuthal and the polar angles respectively for the tri-momentum $\vec{k}$. $K_M = (m_1^2 - m_2^2)/(2m_1)$ and $m_F (m_F)$ is the mass of the initial (final) meson: in Eq. (15), $m_F = m_0^−$. We choose the $z$−axis along the direction of $\vec{q}$, the (tri-)momentum of the W boson (cfr Fig. 11).

The analytical expressions for the form factors can be obtained by comparing Eq. (15) with Eq. (12)

$$F_+(q^2) = \int \frac{k^2 dk dcosθ ψ_1(k)ψ_1^∗(k)}{8π^2 m_1 \sqrt{-(d_{12}^2 + m_1^2)} (m_F^2 − s_{23})} E_1E_3 \left[ d_{12}s_{23}E_2 - m_F ((d_{13} − 2m_2) m_2 + m_1 E_2) + \frac{k}{|\vec{q}|} \cos(θ) (-m_F^2 m_1 + d_{12}s_{23} (m_1 − E_F) + m_1^2 E_F) \right]$$  (19)

$$F_−(q^2) = \int \frac{k^2 dk dcosθ ψ_1(k)ψ_1^∗(k)}{8π^2 m_1 \sqrt{-(d_{12}^2 + m_1^2)} (m_F^2 − s_{23})} E_1E_3 \left[ m_2m_1s_{13} + (m_1^2 + d_{12}s_{23}) E_2 - \frac{k}{|\vec{q}|} \cos(θ) (m_F^2 m_1 + d_{12}s_{23} (m_1 + E_F) + m_1^2 E_F) \right]$$  (20)

where $(d_{ij} = m_i − m_j$ and $s_{ij} = m_i + m_j$).

The expressions for the remaining form factors in Eqs. (10)−(14) are collected in appendix A and B.
FIG. 2: The $B \rightarrow D^* \ell \nu$ spectrum. The dashed (dotted) line corresponds to the exponential (gaussian) vertex function. The data are taken from Ref. [21].

TABLE I: The two best fit sets of values of the free parameters are obtained for the exponential (exp.) and gaussian (gauss.) vertex function.

| Parameter | fitted values (exp.) | fitted values (gauss.) |
|-----------|-----------------------|------------------------|
| $m_q$     | 23 MeV                | 34 MeV                 |
| $\omega_B$| 101 MeV               | 108 MeV                |
| $\omega_D = \omega_{D^*} = \omega_{D^{**}}$ | 51 MeV | 186 MeV |
| $|V_{cb}|$ | 0.041 | 0.043 |

IV. FIXING THE FREE PARAMETERS

The numerical evaluation of the form factors given in Section III requires to specify the expression for the vertex functions and the values of the free parameters of the model. For the vertex functions we adopt two possible forms, the gaussian-type, extensively used in literature (see for example [18])

$$
\psi_H(k) = 4 \sqrt{M_H} \sqrt{\frac{\sqrt{\pi^3}}{\omega_H^3}} \exp \left\{ -\frac{k^2}{2 \omega_H^2} \right\},
$$

(21)

and the exponential one

$$
\psi_H(k) = 4\pi \sqrt{\frac{M_H}{\omega_H^3}} \exp \left\{ -\frac{k}{\omega_H} \right\},
$$

(22)

which is able to fit the results of a relativistic quark model regarding the shape of the meson wave-functions [19]. In our approach $\omega_H$ is a free parameter which should be fixed by comparing a set of experimental data with the predictions of the model. In this paper we choose to fix the free parameters by a fit to the experimental data on the $Br(B \rightarrow D \ell \nu)$ [20] and on the spectrum of $B \rightarrow D^* \ell \nu$ process [21].

The quality of the agreement between fitted spectrum and the corresponding experimental data may be assessed by looking at the Figure 2. It should be also observed the very small differences between the $B \rightarrow D^* \ell \nu$ spectrum using the vertex functions in Eqs. (21)-(22). Regarding the $B \rightarrow D \ell \nu$ branching ratio, we obtain 2.00 (2.01) % for the exponential (gaussian) vertex function to be compared to the experimental value: $Br(B \rightarrow D \ell \nu) = 2.15 \pm 0.22 (2.12 \pm 0.20)\%$ for the charged (neutral) $B$ meson. At this stage the two different form of the vertex functions agree equally well with the experimental data. However, differences emerge when single form factors are considered (cfr for example Table III).
V. Heavy Quark Limit

In this section we perform the heavy quark limit for the form factors obtained in the previous sections. Before to do this we briefly remind the implications of the HQET on the heavy meson spectrum. In the quark model, mesons are conventionally classified according to the eigenvalues of the observables $J$, $L$ and $S$: any state is labelled with the symbol $2S+1L_J$. So, if we consider the lower lying even parity mesons ($L = 1$), the scalar and the tensor mesons correspond to $^3P_0$ and $^3P_2$ states, respectively. Moreover, there are two states with $J = 1$: the $^3P_1$ and $^3P_1$, they can mix each other if the constituent quark masses are different as in the case of charmed mesons. For heavy mesons the decoupling of the spin of the constituent heavy quark, $s_Q$, suggests to use a different set of observables: the total angular momentum of the light constituent, $j_q (= s_q + L)$, the orbital momentum of the light degree of freedoms respect to the heavy quark, $L$, the total angular momentum $J (= j_q + s_Q)$, any state is labelled with $L^j_J$. In this representation the scalar and the tensor mesons are labelled with $P_0^{3/2}$ and $P_2^{3/2}$, respectively. The axial mesons are classified as $P_1^{3/2}$ and $P_1^{1/2}$ and are related to the $^3P_1$ and $^3P_2$ states by the 11

\[
|P_1^{3/2}\rangle = +\sqrt{\frac{2}{3}} |P_1\rangle + \sqrt{\frac{1}{3}} |P_2\rangle ,
\]

\[
|P_1^{1/2}\rangle = +\sqrt{\frac{1}{3}} |P_1\rangle - \sqrt{\frac{2}{3}} |P_2\rangle .
\]

The scaling laws of the HQET concern the $0^- \rightarrow (P_0^{1/2}, P_1^{1/2}, P_1^{3/2})$ form factors, they are defined combining the form factors in Eqs. (12), (13), (14) and the relations in Eqs. (23) and (24). For example $G^{1/2}(q^2) \equiv \sqrt{1/3} G(q^2) - \sqrt{2/3} G'(q^2)$.

To extract the heavy quark mass dependence from the expressions of the form factors we follow the same approach used in our previous paper [17]. We introduce the variable $x$, defined by $x = (2\alpha k)/m_F$, in such a way, neglecting the light quark mass respect to the heavy ones, the integration domain, near the zero-recoil point, simplify to 0 $\leq x \leq \pi$ and 0 $\leq \phi \leq 2\pi$. Therefore, if we look at the expressions of $F_{\pm}(q^2)$, Eqs. (19)-(20), near the zero recoil point (i. e. $q^2 \approx q_{max}^2$) we have, neglecting terms of the order of $x^3$,

\[
F_{\pm}(q^2)\big|_{q^2 \approx q_{max}^2} = \int_0^\infty dx \, \psi_l(k(x)) \, \psi_F(k(x)) \, \frac{m_F}{m_F + m_I} \frac{(7 - 3w) \, x^2}{384 \, m_I \, \pi^2 \, \alpha^3} .
\]

For $\alpha \ll 1$ the integration can be easily done giving

\[
F_{\pm}(q^2)\big|_{q^2 \approx q_{max}^2} = \frac{m_F + m_I}{\sqrt{m_F m_I}} \left\{ \begin{array}{c}
G^2 (q^2) \\
F^2 (q^2) \\
A^2 (q^2)
\end{array} \right\} = \frac{1}{\sqrt{3}} \left\{ \begin{array}{c}
G(q^2) \\
F(q^2) \\
A(q^2)
\end{array} \right\} - \frac{2}{3} \left\{ \begin{array}{c}
G'(q^2) \\
F'(q^2) \\
A'(q^2)
\end{array} \right\} \]

Thus, the $\tau_{1/2}$ Isgur-Wise function resulting from our model is given by

\[
\tau_{1/2}(w) = \frac{1}{3} - \frac{1}{4}(w - 1) + \frac{19}{96}(w - 1)^2 + o((w - 1)^3) ,
\]

where we have also written the term $(w - 1)^2$ which was neglected in Eq. (26).

A similar analysis can be performed on the heavy to heavy $0^- \rightarrow P_1^{3/2}$ form factors. Let us start considering the combination

\[
\left\{ \begin{array}{c}
G^{1/2}(q^2) \\
F^{1/2}(q^2) \\
A^{1/2}(q^2)
\end{array} \right\} = \frac{1}{\sqrt{3}} \left\{ \begin{array}{c}
G(q^2) \\
F(q^2) \\
A(q^2)
\end{array} \right\} - \frac{2}{3} \left\{ \begin{array}{c}
G'(q^2) \\
F'(q^2) \\
A'(q^2)
\end{array} \right\}
\]

which defines the $0^- \rightarrow P^{1/2}$ form factors. It is very simple to obtain their scaling laws in the limit of heavy quark masses. Following the above method we obtain

\[
\left\{ \begin{array}{c}
G^{1/2}(q^2) \\
F^{1/2}(q^2) \\
A^{1/2}(q^2)
\end{array} \right\} = N \left\{ \begin{array}{c}
\sqrt{m_F m_I} \\
2 \sqrt{m_F m_I (w - 1)} \\
\pm 1 / \sqrt{m_F m_I}
\end{array} \right\} \frac{1}{3} \left( 1 - \frac{3}{4}(w - 1) \right) + o((w - 1)^2) ,
\]
TABLE II: The Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$ at zero recoil and their slope parameters.

| $\tau_{1/2}(1)$ | $\rho_{1/2}^2$ | $\tau_{3/2}(1)$ | $\rho_{3/2}^2$ | Ref.          |
|-----------------|----------------|-----------------|----------------|--------------|
| 0.33            | 0.75           | 0.83            | 1.29           | This work    |
| 0.34            | 0.76           | 0.59            | 1.09           | [8]          |
| 0.22            | 0.83           | 0.54            | 1.5            | [22]         |
| 0.31            | 1.18           | 0.61            | 1.73           | [23]         |
| 0.13 ± 0.04     | 0.50 ± 0.05    | 0.43 ± 0.09     | 0.90 ± 0.05    | QCDSR [27]   |
| 0.35 ± 0.08     | 2.5 ± 1.0      | –               | –              | QCDSR(NLO) [26] |
| 0.38 ± 0.04     |                | 0.53 ± 0.08     |                | Lattice [24] |

where

\[
N = \begin{cases} 
2\sqrt{2} \left( \frac{\omega_I \omega_F}{\omega_I^2 + \omega_F^2} \right)^{3/2} & \text{gaussian – type} \\
8 \sqrt{\omega_I^3 \omega_F^3} \left( \omega_I + \omega_F \right)^3 & \text{exponential – type} 
\end{cases}
\] (30)

Similarly, we can evaluate the $\tau_{3/2}$ Isgur-Wise function obtaining

\[
\tau_{3/2}(\omega) = \frac{5}{6} - \frac{31}{24} (\omega - 1) + \frac{93}{64} (\omega - 1)^2 + o((\omega - 1)^3).
\] (31)

A comparison between our results and some others coming from quark models, QCD sum rules and Lattice calculations can be done looking at the Table II. The values of the $\tau$ functions at zero recoil point and their slopes are compatible. In particular, it should be observed that our results for $\tau_{1/2}$ are practically the same obtained in the Isgur Scora Grinstein Wise (ISGW) model [8] and QCD Sum Rules findings [26, 27].2 Regarding $\tau_{3/2}$, our result at zero recoil point is slightly larger of the results coming from other models, while the slope is comparable with others.

Relations between the slope of Isgur-Wise function and $\tau$ functions at zero recoil points were derived, in the form of sum rules, by Bjorken [28] and Uraltsev [29]

\[
\rho^2 - \frac{1}{4} = \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2 \sum_n |\tau_{3/2}^{(n)}(1)|^2,
\] (32)

\[
\frac{1}{4} = \sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2,
\] (33)

where $n$ stands for the radial excitations and $\rho^2$ is the slope of the Isgur-Wise function $\xi(\omega)$ which, in our model [17], is

\[
\xi(\omega) = 1 - \frac{11}{12} (\omega - 1) + \frac{77}{96} (\omega - 1)^2 + o((\omega - 1)^3).
\] (34)

Our results for $n = 0$ oversaturate both the sum rules. For the Bjorken sum rule this is due to the small value we obtain for the derivative of the Isgur-Wise function ($\rho^2$) which is in any case compatible with the experimental value $\rho^2 = 0.95 \pm 0.09$ [21]. We plan to study this problem in a separate work. However, a detailed discussion on these sum rules and the findings of quark models can be found in [30].

VI. NUMERICAL RESULTS AND DISCUSSION

All the results discussed in the previous section has been obtained without fixing the free parameters of the model. In this one we use the fitted values of the free parameters in Table I (cfr section IV for discussion) to obtain the

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2 The values and the slopes of $\tau$ functions are obtained fitting the numerical results obtained in Morenas et al. (see Ref [8]) using the ISGW model.
TABLE III: $B \rightarrow D^{**}$ form factors evaluated at $q^2 = 0$ and at $q^2_{\text{max}} = (m_B - m_{D^{**}})^2$ by using the vertex function in Eq. (22). In parentheses the values obtained using the gaussian vertex function (cfr. Eq. (21)).

| Form Factor | This work | Ref. [8] | Ref. [13] |
|-------------|-----------|----------|----------|
| $F_1$       | -0.32 (-0.30) | -0.35 (-0.33) | -0.18 | -0.24 | -0.24 | -0.34 |
| $F_0$       | -0.32 (-0.30) | -0.025 (-0.036) | -0.18 | 0.008 | -0.24 | -0.20 |
| $A_{1/2}^{(1)}$ | -0.25 (-0.23) | -0.31 (-0.28) | -0.18 | -0.39 | -0.075 | -0.083 |
| $A_{2}^{(1)}$ | 0.096 (0.088) | -0.0018 (-0.0029) | 0.070 | -0.002 | 0.073 | 0.071 |
| $V^{(1/2)}$ | 0.69 (0.63) | 0.87 (0.79) | 0.49 | 0.91 | 0.32 | 0.56 |
| $A_{0}^{(3/2)}$ | -0.61 (-0.58) | -0.81 (-0.77) | -0.20 | -0.46 | -0.47 | -0.76 |
| $A_{1/2}^{(3/2)}$ | -0.13 (-0.13) | -0.016 (-0.023) | -0.005 | -0.008 | -0.20 | -0.26 |
| $A_{3/2}^{(3/2)}$ | 0.70 (0.65) | 1.27 (1.19) | 0.33 | 0.72 | 0.25 | 0.47 |
| $V^{(3/2)}$ | -0.81 (-0.77) | -1.09 (-1.03) | -0.44 | -0.71 | -0.61 | -1.24 |

TABLE IV: Parameters of the $B \rightarrow D^{**}$ form factors. The functional $q^2$ dependence is either polar: $F(q^2) = F(0)/(1 - a q^2/m_B^2)$ or linear: $F(q^2) = F(0)(1 + b q^2)$. In parentheses the values obtained using the gaussian vertex function (cfr. Eq. (21)).

| Form Factor | F(0) | $a$ | $b$ (GeV$^{-2}$) |
|-------------|------|----|----------------|
| $F_1$       | -0.32 (0.30) | 0.263 (0.233) |
| $F_0$       | -0.32 (0.30) | -0.109 (-0.103) |
| $A_{1/2}^{(1)}$ | -0.25 (-0.23) | 0.661 (0.626) |
| $A_{2}^{(1)}$ | 0.10 (0.092) | -0.120 (-0.120) |
| $A_{3/2}^{(1)}$ | 0.69 (0.64) | 0.695 (0.680) |
| $V^{(1/2)}$ | 0.67 (0.61) | 0.695 (0.679) |
| $A_{0}^{(3/2)}$ | -0.61 (-0.59) | 0.846 (0.834) |
| $A_{1/2}^{(3/2)}$ | -0.13 (-0.13) | -0.102 (-0.0956) |
| $A_{3/2}^{(3/2)}$ | 0.72 (0.67) | 1.53 (1.54) |
| $V^{(3/2)}$ | -0.81 (-0.77) | 0.872 (0.873) |

Numerical results of the form factors. First of all, in Table III are collected the values of the form factors for the $B \rightarrow D^{**}$ transitions evaluated at zero recoil point ($q^2_{\text{max}} = (m_B - m_{D^{**}})^2$) and at $q^2 = 0$. We consider the charmed final state with the following masses: $m(D_0^{**}) = 2.40 \text{ GeV}$, $m(D_1^{**}) = 2.43 \text{ GeV}$, $m(D_1) = 2.42 \text{ GeV}$ (Ref. [1]). Note that we are considering, for a better comparison with other calculations, the eikonal form factors (cfr for definitions, for example, [32]). Looking at the Table III we can see that the absolute values of our form factors (at $q^2 = 0$) are larger than the ones in Ref. [8, 13], this naturally implies larger branching ratios in our model. In particular, our predictions on the branching ratios, using the exponential (gaussian) vertex function, are ($\tau_B = 1.536 \times 10^{-12} \text{ s}$) (Ref. [20]).

\[
Br(B^0 \rightarrow D_0^{**} \ell^- \bar{\nu}_\ell) = 2.3(2.1) \times 10^{-3} (|V_{cb}|/0.041)^2,
\]

\[
Br(B^0 \rightarrow D_1^{**} \ell^- \bar{\nu}_\ell) = 2.0(1.6) \times 10^{-3} (|V_{cb}|/0.041)^2,
\]

\[
Br(\bar{B}^0 \rightarrow D_1^{**} \ell^- \bar{\nu}_\ell) = 8.0(7.3) \times 10^{-3} (|V_{cb}|/0.041)^2.
\]

Regarding the $q^2$ dependence of the form factors, we find a very good agreement with numerical results assuming

\[D_1 \text{ and } D_1 \text{ represent, respectively, the two different physical axial-vector charmed meson states. The physical } D_1^{**} \text{ (} D_1 \text{)} \text{ is primarily } F_{1/2}^{(1)} (F_{3/2}^{(1)}). \text{ They differ by a small amount from the mass eigenstates in the heavy quark limit, for a discussion see }[33]. \text{ In this paper we neglect these differences.}\]
the following polar expression

\[ F(q^2) = \frac{F(0)}{1 - a \frac{q^2}{m_N^2}} \]  

(36)

the fitted values of \(a\) can be found in Table IV. It is interesting to observe that the effective pole mass is not far from the mass of the \(B_c\) meson. A different \(q^2\) dependence exhibit the form factors \(F_0\) and \(A_1\); for them we use the form

\[ F(q^2) = F(0) \left(1 + b q^2\right) \]  

(37)

and the values of \(b\) are collected in Table IV.

In conclusion we have obtained in a very simple constituent quark model all the semileptonic form factors relevant to the transition of \(B\) into the low-lying odd and even parity charmed mesons. The free parameters of the model have been fixed by comparing model predictions with the \(B \to D^* \ell \nu\) spectrum and \(B \to D \ell \nu\) branching ratio. Our numerical results are generally larger than the results of other models. However, form factors reproduce the scaling laws dictated by the HQET in the limit of infinitely heavy quark masses.

APPENDIX A: FORM FACTORS \(f_\pm, g, f, a_\pm\)

In this appendix we collect the analytical expressions for the \(0^- \to 0^-\) and \(0^- \to 1^-\) form factors defined in Eqs. (10) and (11), respectively.

\[ f_+(q^2) = \int k^2 dk d\cos \theta \frac{\psi_I(k) \psi_F^*(k)}{8 \pi^2 m_I \sqrt{(d_{23}^2 - m_F^2) (d_{12}^2 - m_I^2) E_1 E_3 (m_F^2 - E_F^2)}} \times \]

\[ \left\{ (d_{12} d_{23} E_2 - m_I (m_2 (-2 m_2 + s_{13}) + m_I E_2)) (-m_F^2 + E_F^2) + k |\vec{q}| \cos(\theta) (d_{12} d_{23} (m_I - E_F) + m_I (-m_F^2 + m_I E_F)) \right\} \]  

(A1)

\[ f_-(q^2) = \int k^2 dk d\cos \theta \frac{\psi_I(k) \psi_F^*(k)}{8 \pi^2 m_I \sqrt{(d_{23}^2 - m_F^2) (d_{12}^2 - m_I^2) E_1 E_3 (m_F^2 - E_F^2)}} \times \]

\[ \left\{ (d_{12} d_{23} E_2 + m_I (d_{13} m_2 + m_I E_2)) (-m_F^2 + E_F^2) - k |\vec{q}| \cos(\theta) (d_{12} d_{23} (m_I + E_F) + m_I (m_F^2 + m_I E_F)) \right\} \]  

(A2)
\[ g(q^2) = \int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{m_2 + k^2\sin^2(\theta) + \frac{d_{12}E_2 + k\cos(\theta)(d_{23}m_I + d_{12}E_F)}{m_I}}{8\pi^2 \sqrt{(d_{23}^2 - m_F^2)} (d_{12}^2 - m_I^2)} \frac{1}{E_1E_3}, \] (A3)

\[ f(q^2) = \int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{1}{8\pi^2 (m_F + s_{23}) \sqrt{(d_{23}^2 - m_F^2)} (d_{12}^2 - m_I^2)} \times \]
\{ -k^2 m_F^2 (d_{12} m_F - m_I E_F) \} + |q|^2 E_2 (d_{12} m_F E_2 + m_I (-m_3^2 + (m_2 + m_F)^2 - 2 E_2 E_F)) + k \cos(\theta) (k \cos(\theta) (d_{12} m_F - m_I E_F) (m_F^2 + 2 E_F^2) + |q| E_F (d_{12} m_F E_2 + m_I (m_3^2 - (m_2 + m_F)^2 + 4 E_2 E_F))) \} \times \]

\[ a_+(q^2) + a_-(q^2) = \int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{1}{4\pi^2 |q|^2 m_I \sqrt{(d_{23}^2 - m_F^2)} (d_{12}^2 - m_I^2)} \frac{1}{E_1E_3E_F} \times \]
\{ -k^2 (m_F^2 (d_{12} m_F - m_I E_F)) - |q|^2 (m_F + s_{23} - m_I E_2) E_F + d_{12} (k^2 m_F (|q|^2 + m_F)^2) - |q|^2 (m_F + s_{13}) E_F + k \cos(\theta) (k \cos(\theta) (-d_{12} m_F + m_I E_F) (|q|^2 + m_F^2 + 2 E_F^2) + |q| E_F (d_{12} m_F E_2 + m_F + s_{13}) E_F + m_I (d_{23} (m_F + s_{23}) - (m_I + 2 E_2) E_F))) \} \times \]

\[ a_+(q^2) - a_-(q^2) = \int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{1}{4\pi^2 |q|^2 m_I \sqrt{(d_{23}^2 - m_F^2)} (d_{12}^2 - m_I^2)} \frac{1}{E_1E_3E_F} \times \]
\{ k^2 \sin^2(\theta) m_F^2 (-q^2 + 2 d_{12} d_{23} + m_F^2 + m_I^2) - 2 (|q|^2 E_2 - k \cos(\theta) E_F) \times \}
\{ (|q|^2 (d_{23}^2 - m_F^2) m_I + 2 d_{12} d_{23} E_2 + (-q^2 + m_F^2 + m_I^2) E_2) - k \cos(\theta) (-q^2 + 2 d_{12} d_{23} + m_F^2 + m_I^2) E_F \} \times \]

\[ \text{APPENDIX B: FORM FACTORS } G^{(q)}, F^{(q)}, A^{(q)}_\pm \]

In this appendix we give the expressions of the form factors appearing in Eq. (14) \((0^- \to 1^P_1)\) transitions. We use the same notations of the previous appendix.

\[ G(q^2) = -\int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{k^2 \sin^2(\theta) m_F}{8\pi^2 (-d_{23}^2 + m_F^2) \sqrt{(-d_{12}^2 + m_I^2)} (m_F^2 - s_{23}^2)} E_1 E_3 \] (B1)

\[ F(q^2) = -\int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{\sqrt{3} k^2 \sin^2(\theta) m_F (-q^2 + 2 d_{12} d_{23} + m_F^2 + m_I^2)}{8\pi^2 (d_{23}^2 - m_F^2) \sqrt{(-d_{12}^2 + m_I^2)} (m_F^2 - s_{23}^2)} E_1 E_3 \] (B2)

\[ A_+(q^2) + A_-(q^2) = -\int k^2dkd\cos\theta \psi_I(k)\psi_F^*(k) \frac{1}{8\pi^2 |q|^2 (d_{23}^2 + m_F^2) m_I \sqrt{d_{12}^2 - m_I^2} (-m_F^2 + s_{23}^2)} E_1 E_3 \times \]
\{ k^2 \sin^2(\theta) m_F^2 (-q^2 + 2 d_{12} d_{23} + m_F^2 + m_I^2) - 2 (|q|^2 E_2 - k \cos(\theta) E_F) \times \}
\{ (|q|^2 (d_{23}^2 - m_F^2) m_I + 2 d_{12} d_{23} E_2 + (-q^2 + m_F^2 + m_I^2) E_2) - k \cos(\theta) (-q^2 + 2 d_{12} d_{23} + m_F^2 + m_I^2) E_F \} \} \times \] (B3)
\[ A_+ (q^2) - A_- (q^2) = - \int \frac{k^2 \, dk \, d \cos(\theta) \, \psi_1 (k) \psi_1^\ast (k) \sqrt{3} \, m_F}{4 \pi^2 |q|^2 (d_{23}^2 - m_F^2) \, m_I (-q^2 + m_F^2 + m_I^2) \sqrt{(d_{12}^2 - m_I^2) (-m_F^2 + s_{23}^2)}} \times \]
\[
\left\{ \begin{array}{l}
2 \, m_I \, E_F \left( |q|^2 \left( d_{12}^2 - m_F^2 \right) E_2 + k^2 \, E_F \left( d_{12} \, d_{23} + m_I \, E_F \right) + \right) \\
k \, \cos(\theta) \left( -3 \, k \, \cos(\theta) \, E_F \left( d_{12} \, d_{23} + m_I \, E_F \right) + \right) \\
|q| \left( m_I^2 \, E_F + 2 \, m_I \, E_F + d_{12} \left( 2 \, d_{23} \, E_2 - d_{12} \, E_F \right) \right) \right\} \tag{B4}
\]

Regarding the form factors in Eq. (13) we have
\[
G' (q^2) = - \sqrt{\frac{3}{2}} \int \frac{k^2 \, dk \, d \cos(\theta) \, \psi_1 (k) \psi_1^\ast (k)}{8 \pi^2 |q|^2 \left( -d_{23}^2 + m_F^2 \right) m_I \sqrt{(d_{12}^2 - m_I^2) (-m_F^2 + s_{23}^2)}} \times \]
\[
\left\{ - \left( |q| \left( d_{23} \left( k^2 + d_{23} \, m_2 \right) - m_2 \, m_F^2 \right) m_I + d_{12} \left( d_{23}^2 - m_F^2 \right) E_2 \right) + \right) \\
k \, \cos(\theta) \left( k \, |q| \, \cos(\theta) \, d_{23} \, m_I + \left( d_{23}^2 - m_F^2 \right) \left( m_I \, s_{23} + d_{12} \, E_F \right) \right) \right\} \tag{B5}
\]
\[
F' (q^2) = - \sqrt{\frac{3}{2}} \int \frac{k^2 \, dk \, d \cos(\theta) \, \psi_1 (k) \psi_1^\ast (k)}{8 \pi^2 \left( -d_{23}^2 + m_F^2 \right) \sqrt{(d_{12}^2 - m_I^2) (-m_F^2 + s_{23}^2)}} \times \]
\[
\left\{ -2 \, k^2 \sin(\theta)^2 \left( d_{12} \, m_F^2 + d_{23} \, m_I \, E_F \right) + \left( d_{23}^2 - m_F^2 \right) \left( -m_I^2 \, s_{23} + m_I^2 \, s_{23} + m_I \, \left( -m_F^2 + s_{23}^2 \right) + \right. \right. \\
- m_2 \left( m_F^2 - m_3 \, s_{23} - 2 \, m_I \, E_F \right) \right\} \tag{B6}
\]
\[
A'_+ (q^2) + A'_- (q^2) = - \sqrt{\frac{3}{2}} \int \frac{k^2 \, dk \, d \cos(\theta) \, \psi_1 (k) \psi_1^\ast (k)}{16 \pi^2 |q|^2 \left( -d_{23}^2 + m_F^2 \right) m_I^2 \sqrt{(d_{12}^2 - m_I^2) (-m_F^2 + s_{23}^2)}} \times \]
\[
\left\{ 4 \left( |q|^2 \, E_2 \left( m_F^2 \, m_I \, s_{23} + 2 \, d_{12} \, m_F^2 \, E_2 + d_{23} \, m_I \left( -d_{23} \, s_{23} \right) + 2 \, E_2 \, E_F \right) \right) - \right) \\
k \, |q| \, \cos(\theta) \, E_F \left( m_F^2 \, m_I \, s_{23} + 4 \, d_{12} \, m_F^2 \, E_2 + d_{23} \, m_I \left( -d_{23} \, s_{23} + 4 \, E_2 \, E_F \right) \right) + \right. \\
k^2 \left( d_{12} \, m_F^2 + d_{23} \, m_I \, E_F \right) \left( -m_F^2 + \cos(\theta)^2 \left( m_F^2 + 2 \, E_F^2 \right) \right) \right\} \tag{B7}
\]
\[
A'_+ (q^2) - A'_- (q^2) = + \sqrt{\frac{3}{2}} \int \frac{k^2 \, dk \, d \cos(\theta) \, \psi_1 (k) \psi_1^\ast (k)}{8 \pi^2 |q|^2 \left( -d_{23}^2 + m_F^2 \right) m_I \, E_F \sqrt{(d_{12}^2 - m_I^2) (-m_F^2 + s_{23}^2)}} \times \]
\[
\left\{ - \left( k^2 \left( d_{12} \, m_F^2 + d_{23} \, m_I \, E_F \right) \left( -m_F^2 + 2 \, E_F^2 \right) \right) + |q|^2 \left( 2 \, d_{23}^2 \, m_2 \, m_I \, E_F + d_{23} \, \left( k^2 \, m_I + \right. \right. \\
2 \left( m_I^2 + m_2 \, m_3 - m_I^2 - m_1 \, s_{23} \right) \, E_2 \right) \, E_F + m_F^2 \left( k^2 \, d_{12} - 2 \, \left( m_F^2 + d_{12} \, E_2 \right) \, E_F \right) + \right. \\
k \, E_F \left( -3 \, k \, \cos(2 \, \theta) \, E_F \left( d_{12} \, m_F^2 + 2 \, m_I \, E_F \right) - \right. \right. \\
2 \, |q| \, \cos(\theta) \left( d_{23}^2 \, m_I \, s_{23} - m_F^2 \left( m_3 \, s_{23} + 2 \, d_{12} \, E_2 \right) + \right. \\
d_{23} \, m_I^2 \, E_F + \left( m_2^2 \, m_3 - m_1 \, \left( m_F^2 + d_{23} \, s_{23} \right) + m_3 \, m_I \left( m_1 + 2 \, E_2 \right) - \right. \right. \\
m_2 \left( m_3^2 - m_F^2 + m_I^2 + 2 \, m_I \, E_2 \right) \, E_F \right) \right\} \tag{B8}
\]

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