Note Added to “Proof of the Gauge Independence of the Conformal Anomaly of Bosonic String in the Sense of Kraemmer and Rebhan”

Mitsuo Abe\textsuperscript{a,1} and Noboru Nakanishi\textsuperscript{b,2}

\textsuperscript{a}Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{b}12-20 Asahigaoka-cho, Hirakata 573-0026, Japan

Abstract

A comment is given to the reply of Kraemmer and Rebhan to our paper.

In our previous paper\cite{1}, it has been pointed out that the Kraemmer and Rebhan’s proof of gauge independence of conformal anomaly of bosonic string for various gauge-fixings\cite{2} is wrong, but the gauge independence is proved for the gauge-fixings which reduced to the linearized de Donder gauge in the flat limit of the background metric. After our paper was circulated, Kraemmer and Rebhan have written a reply to it\cite{3}. While they agree to our pointing-out of the noncommutativity between the BRS transformation and $\frac{\delta}{\delta \hat{g}_{\mu\nu}}$, where $\hat{g}_{\mu\nu}$ denotes the classical background metric, they still claim the validity of their proof. So, in order to avoid misunderstanding, we supplement our claim that their proof is wrong.

Their claim seems to be based on their wrong belief that the effective action contains the conformal anomaly \textit{a priori}. One should recognize that the conformal anomaly \textit{cannot} be seen in the effective action itself. In order to obtain the conformal anomaly, one must take $\delta/\delta\hat{g}_{\mu\nu}$. The error committed so far is that this procedure would be made merely to visualize the already existing conformal anomaly. This is \textit{not} the case, however. The truth is that the conformal anomaly term — more precisely speaking, the nonlocal term from which people wish to deduce the prefactor of Polyakov’s induced action — is \textit{produced}

\textsuperscript{1} E-mail: abe@kurims.kyoto-u.ac.jp
\textsuperscript{2} Professor Emeritus of Kyoto University. E-mail: nbr-nakanishi@msn.com
by this procedure. The fact that the anomaly term is produced is more clearly seen in
the FP-ghost number current: The FP-ghost number conservation is not violated at all
in the complete exact solution, but its anomaly is encountered if one considers the two-
point function through \( \delta / \delta \hat{g}_{\mu \nu} \) (see Ref.4).

The fact that the effective action itself does not have any anomaly term independent of \( D \) can be shown by applying the Kraemmer-Rebhan gauge variation procedure
(2.26) until one totally eliminates the contribution from the gauge-fixing plus FP-ghost
Lagrangian.\(^3\) It is quite unreasonable to suppose that this limit is nonexistent because the
anomaly term is a simple sum of the contribution (proportional to \( D \)) from the string La-
grangian and the one (independent of \( D \)) from the gauge-fixing plus FP-ghost one. Since
this procedure is thoroughly carried out without making any disturbance on the BRS ex-
actness, there is no reason to believe the sudden appearance of \(-26\) in the effective ac-
tion in the case in which the gauge-fixing plus FP-ghost Lagrangian is present. Nonzero
anomaly contribution can arise only as a consequence of the violation of BRS exactness
cased by taking \( \delta / \delta \hat{g}_{\mu \nu} \).

Anyway, Kraemmer and Rebhan never demonstrated the existence of the conformal
anomaly proportional to \( D - 26 \) without using \( \delta / \delta \hat{g}_{\mu \nu} \) in the model considered. Thus there
is a big logical gap between the gauge invariance of the effective action itself and that of
the conformal anomaly proportional to \( D - 26 \). If they wish to claim the validity of their
proof, they must directly prove their belief that the conformal anomaly is contained in
the effective action.

References

[1] M. Abe and N. Nakanishi, hep-th/9710131, Preprint RIMS-1161 (Kyoto University).
[2] U. Kraemmer and A. Rebhan, Nucl. Phys. B315 (1989), 717.
[3] U. Kraemmer and A. Rebhan, hep-th/9711075.
[4] M. Abe and N. Nakanishi, hep-th/9710132, Int. J. Mod. Phys. A (1998), to be
published.

\(^3\) It should be noted that there is no reason to reject to consider this limit in their procedure
because no normalization condition is imposed on the gauge-fixing plus FP-ghost Lagrangian in
their proof.