Abstract
The indications in favor of the existence of light sterile neutrinos at the eV scale found in short-baseline neutrino oscillation experiments is reviewed. The future perspectives of short-baseline neutrino oscillation experiments and the connections with $\beta$-decay measurements of the neutrino masses and with neutrinoless double-$\beta$ decay experiments are discussed.

Keywords: neutrino, sterile, oscillations, mass, mixing

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1. Introduction

The 2015 Nobel Prizes in Physics is a great acknowledgment of the fundamental importance of the model-independent discoveries of neutrino oscillations in the Super-Kamiokande atmospheric neutrino experiment [1] and in the SNO solar neutrino experiment [2]. These discoveries, which proved that neutrinos are massive and mixed particles, led to the standard three-neutrino mixing paradigm ($3\nu$), in which the three active neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ are superpositions of three massive neutrinos $\nu_1$, $\nu_2$, $\nu_3$ with respective masses $m_1$, $m_2$, $m_3$ (see Ref. [3]). There are two independent squared-mass differences, the small solar $\Delta m^2_{\text{SOL}} \approx 7.5 \times 10^{-5}$ eV$^2$ and the larger atmospheric $\Delta m^2_{\text{ATM}} \approx 2.3 \times 10^{-3}$ eV$^2$, which can be interpreted as $\Delta m^2_{\text{SOL}} = \Delta m^2_{21}$ and $\Delta m^2_{\text{ATM}} = |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$, with $\Delta m^2_{ij} = m_i^2 - m_j^2$ (see Refs. [4,5]).

The completeness of the $3\nu$ mixing paradigm has been challenged by the following indications in favor of short-baseline neutrino oscillations, which require the existence of at least one additional squared-mass difference, $\Delta m^2_{\text{SBL}} \gg \Delta m^2_{\text{ATM}}$ (see the review in Ref. [7]):

1. The reactor antineutrino anomaly [8], which is an about 2.8$\sigma$ deficit of the rate of $\bar{\nu}_e$ observed in several short-baseline reactor neutrino experiments in comparison with that expected from the calculation of the reactor neutrino fluxes [9,10].
2. The Gallium neutrino anomaly [11–15], consisting in a short-baseline disappearance of $\nu_e$ measured in the Gallium radioactive source experiments GALLEX [16] and SAGE [17] with a statistical significance of about 2.9$\sigma$.
3. The LSND experiment, in which a signal of short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations has been observed with a statistical significance of about 3.8$\sigma$ [18,19].

The additional squared-mass difference $\Delta m^2_{\text{SBL}}$ requires the existence of at least one massive neutrino $\nu_4$ in addition to the three standard massive neutrinos $\nu_1$, $\nu_2$, $\nu_3$. Since from the LEP measurement of the invisible width of the $Z$ boson we know that there are only three active neutrinos (see Ref. [14]), in the flavor basis the additional massive neutrinos correspond to sterile neutrinos [20], which do not have standard weak interactions.

Sterile neutrinos are singlets of the Standard Model gauge symmetries which can couple to the active neutrinos through the Lagrangian mass term. In practice there are bounds on the active-sterile mixing, but there is no bound on the number of sterile neutrinos and on their mass scales. Therefore the existence of sterile neutrinos is investigated at different mass scales. This review is devoted to the discussion of sterile neutrinos at the eV scale, which can explain the indications in favor of short-baseline neutrino oscillations listed above. However, there are other very interesting possibilities which are under study: very light sterile neutrinos at a mass scale smaller than 0.1 eV, which could affect the oscillations of solar [21,23] and reactor [24,30] neutrinos; sterile neutrinos at the keV scale, which could constitute warm dark matter according to the Neutrino Minimal Standard Model (νMSM) [31,35] (see also the reviews in Refs. [36–39]); sterile neutrinos at the MeV scale [40–43]; sterile neutrinos at the electroweak scale [44,45] or above it [45,46], whose effects may be seen at LHC and other high-energy colliders. Let us also note that there are several interesting models with sterile neutrinos at

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different mass scales \[\{47,63\].

The possible existence of sterile neutrinos is very interesting, because they are new particles which could give us precious information on the physics beyond the Standard Model (see Refs. \[64,63\]). The existence of light sterile neutrinos is also very important for astrophysics (see Ref. \[66\]) and cosmology (see Refs. \[7,67–70\]).

In this review, we consider 3+1 \[71,74\] and 3+2 \[75,79\], neutrino mixing schemes in which there are one or two additional massive neutrinos at the eV scale\[88\] and by the experimental bound on neutrinoless double-\[88\] and by the experimental bound on neutrinoless double-\[88\] decay (assuming that massive neutrinos are Majorana particles; see Ref. \[89\]).

In the 3+1 scheme, the effective probability of \[\bar{\nu}_e \rightarrow \bar{\nu}_\mu\] transitions in short-baseline experiments has the two-neutrino-like form \[72\]

\[
P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu} = \delta_{\alpha\beta} - 4|U_{\alpha4}|^2 \left( \delta_{\alpha\beta} - |U_{\beta4}|^2 \right) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right),
\]

where \(U\) is the mixing matrix, \(L\) is the source-detector distance, \(E\) is the neutrino energy and \(\Delta m^2_{31} = m^2_3 - m^2_1 \approx 1 \text{eV}^2\). The electron and muon neutrino and antineutrino appearance and disappearance in short-baseline experiments depend on \(|U_{e4}|^2\) and \(|U_{\mu4}|^2\), which determine the amplitude \(\sin^2 2\theta_{\alpha\mu} = 4|U_{\alpha4}|^2|U_{\beta4}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E}\right)\) of \(\nu_\mu \rightarrow \nu_\tau\) transitions, the amplitude \(\sin^2 2\theta_{\alpha\mu} = 4|U_{\alpha4}|^2 \left(1 - |U_{\mu4}|^2\right)\) of \(\nu_\mu \rightarrow \nu_e\) disappearance, and the amplitude \(\sin^2 2\theta_{\mu\tau} = 4|U_{\mu4}|^2 \left(1 - |U_{\tau4}|^2\right)\) of \(\nu_\tau \rightarrow \nu_\mu\) disappearance.

Since the oscillation probabilities of neutrinos and antineutrinos are related by a complex conjugation of the elements of the mixing matrix (see Ref. \[3\]), the effective probabilities of short-baseline \(\nu_\mu \rightarrow \nu_\tau\) and \(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau\) transitions are equal. Hence, the 3+1 scheme cannot explain a possible CP-violating difference of \(\nu_\mu \rightarrow \nu_\tau\) and \(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau\) transitions in short-baseline experiments. In order to allow this possibility, one must consider schemes with more than one sterile neutrino. In the 3+2 scheme there are four additional effective mixing parameters in short-baseline experiments: \(\Delta m^2_{51} \geq \Delta m^2_{41}, |U_{e4}|^2, |U_{\mu4}|^2\) and \(\eta = \arg \left[ U_{e4}^\ast U_{\mu4} U_{\tau5} U_{\mu5} \right]\) (see Refs. \[17,90\]). Since the complex phase \(\eta\) appears with different signs in the effective 3+2 probabilities of short-baseline \(\nu_\mu \rightarrow \nu_e\) and \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) transitions, it can generate measurable CP violations.

2. Global fits of short-baseline data

Several analyses of short-baseline neutrino oscillation data have been done after the discovery of the LSND anomaly in the middle 90’s \[71,73,75,77,78,81,91–104\]. The interest in short-baseline neutrino oscillations was renewed after the discovery in 2006 of the Gallium neutrino anomaly \[111–114\] and especially after the discovery in 2011 of the reactor antineutrino anomaly \[7,8,15,79,83,85,87,92,112,119\].

Here we review the results of the global fit of short-baseline neutrino oscillation data presented in Ref. \[7\], in which the data of the following three groups of experiments have been considered:

(A) The \(\nu_\mu \rightarrow \nu_e\) appearance data of the LSND \[19\], MiniBooNE \[121\], BNL-ET76 \[122\], KARMEN \[123\], NOMAD \[124\], ICARUS \[125\] and OPERA \[126\] experiments\[1\].

(B) The following \(\nu_e\) disappearance data: 1) the data of the Bugey-4 \[129\], ROVNO91 \[130\], Bugey-3 \[131\], Gosgen \[132\], ILL \[133\], Krasnoyarsk \[134\], Rosno88 \[135\], SRF \[136\], Chooz \[137\], Palo Verde \[138\], Double Chooz \[139\], and Daya Bay \[140\] reactor antineutrino experiments with the new theoretical fluxes \[8,10,141\]; 2) the data of the GALLEX \[16\] and SAGE \[17\] Gallium radioactive source experiments with the statistical method discussed in Ref. \[14\], considering the recent \(^{115}\text{Ga}/(\text{He},\text{H})^{116}\text{Ge}\) cross section measurement in Ref. \[142\]; 3) the solar neutrino constraint on \(\sin^2 2\theta_{\mu\tau}\) \[15,143,146\]; 4) the KARMEN \[147\] and LSND \[148\] \(\nu_e + ^{12}\text{C} \rightarrow ^{12}\text{He} + e^-\) scattering data \[113\], with the method discussed in Ref. \[116\].

(C) The constraints on \(\nu_\mu\) disappearance obtained from the data of the CDHSw experiment \[149\], from the analysis \[77\] of the data of atmospheric neutrino oscillation experiments\[2\], from the analysis \[115,155\] of the MINOS neutral-current data \[156\] and from the analysis of the SciBooNE-MiniBooNE data \[157\] and antineutrino \[158\] data.

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\[1\] In the literature one can also find studies of the 3+3 \[77,80\], 3+1+1 \[81,82\], and 1+3+3 \[83,84\] schemes.

\[2\] The correct but more complicated analysis of the ICARUS and OPERA data presented in Ref. \[129\] (see also Ref. \[130\]) have not been considered because it would not change significantly the results of the global fits.

\[3\] The analysis of the IceCube data \[150,151\], which could give a marginal contribution, have not been considered because it is too complicated and subject to large uncertainties.
|                | 3+1  | 3+1  | 3+1  | 3+1  | 3+2  | 3+2  |
|----------------|------|------|------|------|------|------|
|                | TotGLO | PrGLO | noMB | noLSND | TotGLO | PrGLO |
| $\chi^2_{\text{min}}$ | 306.0 | 276.3 | 251.2 | 291.3 | 299.6 | 271.1 |
| NDF$_{\text{GLO}}$ | 268   | 262   | 230   | 264   | 264   | 258   |
| GoF$_{\text{GLO}}$ | 5%    | 26%   | 16%   | 12%   | 7%    | 28%   |
| $\chi^2_{\text{min}}$ | 98.9  | 77.0  | 50.9  | 91.8  | 86.0  | 69.6  |
| NDF$_{\text{PG}}$ | 194.4 | 194.4 | 194.4 | 194.4 | 192.9 | 192.9 |
| GoF$_{\text{PG}}$ | 13.0  | 5.3   | 6.2   | 5.3   | 20.7  | 8.6   |
| $\Delta\chi^2_{\text{NO}}$ | 0.1%  | 7%    | 5%    | 7%    | 0.04% | 7%    |
| NDF$_{\text{NO}}$ | 49.2  | 47.7  | 48.1  | 11.4  | 55.7  | 52.9  |
| $n\sigma_{\text{NO}}$ | 6.4$\sigma$ | 6.3$\sigma$ | 6.4$\sigma$ | 2.6$\sigma$ | 6.1$\sigma$ | 5.9$\sigma$ |

Table 1: Results of the global fit of short-baseline data taking into account all MiniBooNE data (TotGLO), only the MiniBooNE data above 475 MeV (PrGLO), without MiniBooNE data (noMB) and without LSND data (noLSND) in the 3+1 and 3+2 schemes. The first three lines give the minimum $\chi^2 (\chi^2_{\text{min}})$, the number of degrees of freedom (NDF$_{\text{GLO}}$) and the goodness-of-fit (GoF$_{\text{GLO}}$) of the global fit (GLO). The following five lines give the quantities relevant for the appearance-disappearance (APP-DIS) parameter goodness-of-fit (PG) [120]. The last three lines give the difference $\Delta\chi^2_{\text{NO}}$ between the $\chi^2$ without short-baseline oscillations and ($\chi^2_{\text{min}}$)$_{\text{GLO}}$, the corresponding difference of number of degrees of freedom (NDF$_{\text{NO}}$) and the resulting number of $\sigma$’s ($n\sigma_{\text{NO}}$) for which the absence of oscillations is disfavored.

The MiniBooNE data require a special treatment, because they show an anomalous excess in the low-energy bins [121] [159] which, as explained later, induces a tension in the global analysis of the data of short-baseline neutrino oscillation experiments [115] [116]. Hence, we will discuss two types of global fits: “total” (TotGLO) and “pragmatic” (PrGLO). In the total fits all the data listed above of short-baseline neutrino oscillation experiments are taken into account. In the pragmatic fits [85] the anomalous low-energy bins of the MiniBooNE experiment [121] [159] are omitted.

Table 1 summarizes the statistical results obtained from global fits of the data above in the 3+1 and 3+2 schemes. Besides the total and pragmatic fits there is also a 3+1-noMB fit without MiniBooNE data and a 3+1-noLSND fit without LSND data which are explained below.

From Tab. 1 one can see that in all fits which include the LSND data the absence of short-baseline oscillations is nominally disfavored by about 6$\sigma$, because the improvement of the $\chi^2$ with short-baseine oscillations is much larger than the number of oscillation parameters.

In both the 3+1 and 3+2 schemes the goodness-of-fit in the total analysis is significantly worse than that in the pragmatic analysis and the appearance-disappearance parameter goodness-of-fit is much worse. This result confirms the fact that the MiniBooNE low-energy anomaly is incompatible with neutrino oscillations, because it would require a small value of $\Delta m^2_{31}$ and a large value of $\sin^2 2\theta_{\mu\tau}$ [115] [116], which are excluded by the data of other experiments (see Ref. [85] for further details). Note that the appearance-disappearance tension in the 3+2-TotGLO fit is even worse than that in the 3+1-TotGLO fit, since the $\Delta\chi^2_{\text{PG}}$ is so much larger that it cannot be compensated by the additional degrees of freedom. Therefore, we think that it is very likely that the MiniBooNE low-energy anomaly has an explanation which is different from neutrino oscillations. The cause of the MiniBooNE low-energy excess of $\nu_e$-like events is going to be investigated in the MicroBooNE experiment at Fermilab [163], which is a large Liquid Argon Time Projection Chamber (LArTPC) in which electrons and photons can be distinguished.

In the following we adopt the “pragmatic approach” advocated in Ref. [85] which considers the PrGLO fits, without the anomalous MiniBooNE low-energy bins, as more reliable than the TotGLO fits, which include the anomalous MiniBooNE low-energy bins.

The 3+2 mixing scheme was considered to be interesting in 2010 when the MiniBooNE neutrino [159] and antineutrino [164] data showed a CP-violating tension, but this tension almost disappeared in the final MiniBooNE data [121]. In fact, from Tab. 1 one can see that there is little improvement of the 3+2-PrGLO fit with respect to the 3+1-PrGLO fit, in spite of the appearance-disappearance tension in the 3+2-TotGLO fit, which is disfavored by the additional degrees of freedom.

4 One could fit the three anomalous MiniBooNE low-energy bins in a 3+2 scheme [160] by considering the appearance data without the ICARUS [125] and OPERA [126] constraints, but the required large transition probability is excluded by the disappearance data.

5 This behavior has been explained in Ref. [160]. It was found also in the analysis presented in Ref. [87].

6 There is however the possibility that at least some part of the MiniBooNE low-energy anomaly may be explained by taking into account nuclear effects in the energy reconstruction [161,162].

In the MiniBooNE mineral-oil Cherenkov detector $\nu_e$-induced events cannot be distinguished from $\nu_\mu$-induced events which produce only a visible photon (for example neutral-current $n^0$ production in which only one of the two decay photons is visible).
of the four additional parameters and the additional possibility of CP violation. Moreover, the p-value obtained by restricting the 3+2 scheme to 3+1 disfavors the 3+1 scheme only at 1.1σ. Therefore, we think that considering the larger complexity of the 3+2 scheme is not justified by the data and in the following we consider only the 3+1 mixing scheme.

Figure 1 shows the allowed regions in the $\sin^2 2\theta_{ee} - \Delta m^2_{41}$, $\sin^2 2\theta_{\mu\mu} - \Delta m^2_{41}$ and $\sin^2 2\theta_{ee} - \Delta m^2_{41}$ planes obtained in the pragmatic 3+1 global fit PrGLO of short-baseline neutrino oscillation data compared with the 3σ allowed regions obtained from $\nu_e \rightarrow \nu_\mu$ short-baseline appearance data (APP) and the 3σ constraints obtained from $\nu_e$ short-baseline disappearance data ($\nu_e$ DIS), $\nu_\mu$ short-baseline disappearance data ($\nu_\mu$ DIS) and the combined short-baseline disappearance data (DIS). The best-fit points of the global (PrGLO) and APP fits are indicated by crosses.

3. Experimental perspectives

There is an impressive program of many experimental projects which will explore the existence of light sterile neutrinos at the eV scale in the next years (see also the reviews in Refs. [185,191]). It is convenient to divide them in the following categories.

3.1. $\nu_e$ disappearance experiments

The aim of these experiments is to reveal short-baseline oscillations in a robust way by measuring distortions of the neutrino spectrum or variations of the flavor neutrino detection probability as a function of distance. They can be divided in the following subcategories.

Source experiments. These experiments use radioactive sources of $\nu_e$ or $\bar{\nu}_e$ placed near or inside a large detector [192]. Table 2 presents a list of
Accelerator experiments. There have been proposals to use a reactor (for example PROSPECT [179] and CARR [184]) or a movable detector (for example DANSS [177]).

Reactor experiments. These experiments use a reactor \( \bar{\nu}_e \) source with a detector placed at a distance of the order of 10 m. There are several experiments in preparation, as shown by the list in Tab. 4 (see also Ref. [189]). They are planned to have a sufficient energy resolution in order to be sensitive to the distortions in the neutrino spectrum due to the oscillations. Some experiments (for example Stereo [176]) will have a length which may allow to observe the variations of the \( \bar{\nu}_e \) survival probability as a function of distance. Others use will use two detectors at different distances (for example PROSPECT [179] and CARR [184]) or a movable detector (for example DANSS [177]).

In source experiments with monochromatic \( \nu_e \)’s generated by nuclear electron capture (for example SAGE [166] and CrSOX [167]), \( \nu_e \) disappearance can be measured as a function of distance. In source experiments with a continuous \( \bar{\nu}_e \) source with a detector placed at a distance \( \lesssim 1 \) m, \( \nu_e \) survival probability as a function of distance should be able to check unambiguously the indications of short-baseline neutrino oscillations.

Figure 2 shows the sensitivities in the \( \sin^2 2\theta_{\alpha\beta} - \Delta m^2_{\alpha\beta} \) plane of the CeSOX [167-168] source experiment and of the Stereo [176], SoLid [178], DANSS [177] and NEOS [180] reactor experiments in comparison with the region allowed by the pragmatic 3+1 global fit PrGLO. One can see that these experiments should be able to check unambiguously the indications of short-baseline neutrino oscillations.

### Table 2: Main features of new source experiments and their status according to our knowledge.

| Project       | \( P_{\text{in}} \) (MW) | \( M_{\text{target}} \) (tons) | \( L \) (m) | Depth (m.w.e.) | Status       |
|---------------|---------------------------|---------------------------------|-------------|----------------|--------------|
| Nucifer (FRA) | 70                        | 0.8                             | 7           | 13             | operating    |
| Stereo (FRA)  | 57                        | 1.75                            | 9 – 12      | 18             | in preparation |
| DANSS (RUS)   | 3000                      | 0.9                             | 10 – 12     | 50             | in preparation |
| SoLid (BEL)   | 45 – 80                   | 3                               | 6 – 8       | 10             | in preparation |
| PROSPECT (USA) | 85                      | 3, 10                           | 7 – 12, 15 – 19 | few | in preparation |
| NEOS (KOR)   | 16400                     | 1                               | 25          | 10 – 23        | in preparation |
| Neutrino-4 (RUS) | 100               | 1.5                             | 6 – 11      | 10             | proposal     |
| Poseidon (RUS) | 100              | 3                               | 5 – 8       | 15             | proposal     |
| Hanaro (KOR) | 30                        | 0.5                             | 6           | few            | proposal     |
| CARR (CHN)   | 60                        | ~ 1                             | 7, 11       | few            | proposal     |

### Table 3: Main features of new reactor experiments and their status according to our knowledge.

| Project       | \( P_{\text{in}} \) (MW) | \( M_{\text{target}} \) (tons) | \( L \) (m) | status       |
|---------------|---------------------------|---------------------------------|-------------|--------------|
| SAGE [166]    | \( \nu_e \) \( ^{37}\text{Cr} \) | 0.75                           | \( \lesssim 1 \) | in preparation |
| CeSOX [167-168] | \( \bar{\nu}_e \) \( ^{144}\text{Ce} \) | 1.8 – 3                         | 5 – 12      | in preparation |
| CrSOX [167]   | \( \nu_e \) \( ^{51}\text{Cr} \) | 0.75                           | 5 – 12      | proposal     |
| Daya Bay [169-170] | \( \bar{\nu}_e \) \( ^{144}\text{Ce} \) | 1.8 – 3                         | 1.5 – 8    | proposal     |
| JUNO [171]    | \( \bar{\nu}_e \) \( ^{144}\text{Ce} \) | 1.8 – 3                         | \( \lesssim 32 \) | proposal     |
| LENS [172]    | \( \nu_e, \bar{\nu}_e \) \( ^{51}\text{Cr}, ^{3}\text{He} \) | 0.75, \( \lesssim 3.5 \) | \( \lesssim 3 \) | abandoned    |
| Neutrino-4 (RUS) | \( \nu_e \) \( ^{144}\text{Ce} \) | 1.8 – 3                         | \( \lesssim 6 \) | abandoned    |
| LENA [174]    | \( \nu_e \) \( ^{51}\text{Cr}, ^{37}\text{Ar} \) | 0.75, 0.81                      | \( \lesssim 90 \) | abandoned    |
neutrino flux and cross section which allows to measure the oscillations between the two detectors with small systematic uncertainty.

Figure 2 shows the sensitivities in the \( \sin^2 2\theta_{\mu\tau}, \Delta m_{41}^2 \) plane obtained in the pragmatic 3+1 global fit PrGLO of short-baseline neutrino oscillation data with the sensitivities of the CeSOX [167, 168] source experiment, of the Stereo [176], SoLiD [173], DANSS [177] and NEOS [180] reactor experiments and of the KATRIN [201] \( \beta \)-decay experiment.

3.3. \( \nu_\mu \) disappearance experiments

The accelerator experiments in Tab. 4 (see also the NESSiE proposal in Ref. [209] and the low-energy neutrino factory studies in Refs. [198, 200, 208]) can measure also the short-baseline \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance which is necessarily associated with \( \nu_\mu \rightarrow \nu_e \) oscillations. Let us emphasize that it is important to measure short-baseline \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance for which so far there are only upper limits, whereas the short-baseline \( \nu_e \) and \( \bar{\nu}_e \) disappearance associated with \( \nu_\mu \rightarrow \nu_e \) oscillations is given by the Gallium and reactor anomalies. The consistency of the short-baseline neutrino oscillation scenario with any number of sterile neutrinos requires that also \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance must be observed [165].

Figure 3 shows the sensitivities in the \( \sin^2 2\theta_{\mu\tau}, \Delta m_{41}^2 \) plane obtained in the pragmatic 3+1 global fit PrGLO of short-baseline neutrino oscillation data with the sensitivities of the SBN [202] and nuPRISM [205] accelerator experiments. One can see that also \( \nu_\mu \) disappearance should be observed if the short-baseline neutrino oscillations indicated by the LSND, reactor and Gallium anomalies really exist.

3.4. Neutral-current measurements

In principle, measuring the neutral-current scattering of active neutrinos is the best way to probe their disappearance into sterile states. However, neutral-current measurements are extremely difficult, because
the only observable signal is the recoil of the target particle.

The signal can be enhanced at low neutrino energies by the coherent scattering on nuclei [210, 211] for which the cross section is approximately proportional to the square of the number of neutrons in the nucleus (the proton contribution is suppressed by \(1 - 4 \sin^2 \theta_W \ll 1\), where \(\theta_W\) is the weak mixing angle). This process has not been observed so far, but it is actively searched for [212–216]. In the future it may lead to the direct measurement of active-sterile transitions [217–219].

### 3.5. \(\beta\)-decay mass measurements

The most sensitive experiments on the search of the effects of neutrino masses in \(\beta\) decay use the Tritium decay process\(^8\)

\[
^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e. \quad (3.1)
\]

Non-zero neutrino masses distort the measurable spectrum of the emitted electron. It is convenient to consider the Kurie function (see Ref. [3])

\[
K^2(T) = (Q - T_e) \sum_k |U_{ek}|^2 \sqrt{(Q - T_e)^2 - m_k^2} \Theta(Q - T_e - m_k), \quad (3.2)
\]

where \(T_e\) is the electron kinetic energy, \(Q = M_{^3\text{He}} - M_{^3\text{He}} - m_e \approx 18.574\) keV is the \(Q\)-value of the process, and \(\Theta\) is the Heaviside step function. Considering an experiment in which the energy resolution is such that \(m_k \ll Q - T_e\) for the three standard light neutrino masses \((k = 1, 2, 3)\), the Kurie function can be approximated by

\[
K^2(T) \approx (Q - T_e) \sqrt{(Q - T_e)^2 - m_\beta^2} \Theta(Q - T_e - m_\beta) + (Q - T_e) \sum_{k=4}^3 |U_{ek}|^2 \sqrt{(Q - T_e)^2 - m_k^2} \Theta(Q - T_e - m_k), \quad (3.3)
\]

with the effective light neutrino mass \(m_\beta\) given by

\[
m_\beta^2 = \sum_{k=1}^3 |U_{ek}|^2 m_k^2. \quad (3.4)
\]

Hence, \(m_\beta\) causes a distortion of the end-point of the electron kinetic energy spectrum and a heavy non-standard neutrino mass \(m_k\) with \(k \geq 4\) can be measured by observing a kink of the kinetic energy spectrum of the emitted electron at \(Q - m_k\) below the end point [116, 225–229]. Recently, the Mainz [233] and Troitsk [234, 235] collaborations obtained upper bounds for the mixing factor \(|U_{e4}|^2\) for \(m_4^2 \gtrsim 10\) eV\(^2\). In the 3+1 scheme these bounds imply an exclusion curve in the \(\sin^2 2\theta_{\mu\mu} - \Delta m_{41}^2\) plane for \(\Delta m_{41}^2 \gtrsim 10\) eV\(^2\) [119], which is well above the allowed region obtained in the pragmatic 3+1 global fit PrGLO shown in Fig. 1.

The experiment KATRIN [236], which is under construction and is scheduled to start data taking in 2016, will aim to reach a sensitivity of 0.2 eV at 90% C.L. for \(m_\beta\) in five years of running. Some studies have been performed to analyze the sensitivity of the KATRIN experiment to the effects of heavy sterile neutrinos with keV-scale masses [201, 237, 238] and light eV-scale sterile neutrinos [201, 232, 240, 242]. Figure 2 shows the KATRIN sensitivity presented in Ref. [201]. One can see that it covers a significant portion of the PrGLO allowed region. Hence, there is a concrete possibility that KATRIN can observe the effect of \(m_4\) if \(\nu_4\) exists and both \(m_4\) and \(|U_{e4}|^2\) are not too small.

### 3.6. Neutrinoless double-\(\beta\)-decay

The implications of non-standard mainly sterile massive neutrinos at the eV scale for neutrinoless double-\(\beta\)-decay experiments have been studied by several authors [15, 102, 243, 252].

If massive neutrinos are Majorana particles (see the recent reviews in Refs. [89, 252]), in the case of 3+1 mixing the rate of neutrinoless double-\(\beta\)-decay is proportional to the square of the effective Majorana mass

\[
|m_{\beta0}| \equiv |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3 + |U_{e4}|^2 e^{i\alpha_4} m_4. \quad (3.5)
\]
In this expression there are three completely unknown complex phases $\alpha_2, \alpha_3, \alpha_4$ which depend on the Majorana phases in the neutrino mixing matrix. These unknown complex phases can generate cancellations between the different mass contributions. Figure 5 shows the range of allowed values of $|m_{3\beta\beta}|$ as a function of the lightest neutrino mass in the cases of $3\nu$ and $3+1$ mixing with normal and inverted ordering of the three lightest neutrinos [251]. The $3\nu$ mixing parameters are those obtained in Ref. [253] and the sterile neutrino mixing is that obtained in the global pragmatic $3+1$ PrGLO fit of short-baseline neutrino oscillation data discussed in Section 2.

From Fig. 5 one can see that the presence of an additional massive neutrino at the eV scale can change dramatically the predictions for the possible range of values of $|m_{3\beta\beta}|$ in Refs. [15, 102, 213, 252]. In the case of a normal $3\nu$ mass hierarchy ($m_1 < m_2 < m_3$) the value of $|m_{3\beta\beta}|$ is dominated by the contribution of $\nu_4$, which implies that $1 \times 10^{-2} \lesssim |m_{3\beta\beta}| \lesssim 7 \times 10^{-2}$ eV. This range of values of $|m_{3\beta\beta}|$ is larger than that predicted by the standard $3\nu$ mixing in the case of a normal hierarchy and similar to that predicted in the case of an inverted hierarchy in the standard $3\nu$ mixing scheme. On the other hand, in the case of an inverted $3\nu$ mass ordering there can be a complete cancellation between the contribution of $\nu_4$ and those of the three standard light neutrinos, leading to the disappearance of the lower limit for $|m_{3\beta\beta}|$ predicted by the standard $3\nu$ mixing scheme.

The next generation of neutrinoless double-beta decay experiments (see Refs. [254, 259]) is planned to explore the range of $|m_{3\beta\beta}|$ between about $1 \times 10^{-2}$ and $5 \times 10^{-2}$ eV predicted by the standard $3\nu$ mixing in the case of an inverted hierarchy. They are not expected to reach the range of $|m_{3\beta\beta}|$ between about $8 \times 10^{-4}$ and $5 \times 10^{-3}$ eV predicted by the standard $3\nu$ mixing in the case of a normal hierarchy. From Fig. 5 it is clear that the predictions are dramatically changed in the $3+1$ neutrino mixing scheme and a positive result in these experiments is guaranteed in the case of a normal mass hierarchy, whereas in the case of an inverted mass hierarchy the allowed range of $|m_{3\beta\beta}|$ goes from zero to about 0.1 eV.

4. Conclusions

The reactor, Gallium and LSND anomalies can be explained by neutrino oscillations if the standard three-neutrino mixing paradigm is extended with the addition of light sterile neutrinos which can give us important information on the new physics beyond the Standard Model.

The global fits of short-baseline neutrino oscillation data in the framework of mixing schemes with one or more sterile neutrinos suffer from a tension between the results of appearance and disappearance short-baseline neutrino oscillation experiments. This tension can be alleviated adopting the “pragmatic approach” advocated in Ref. [85], in which the anomalous MiniBooNE low-energy excess of $\nu_e$-like events is neglected from the global analysis of short-baseline neutrino oscillation data. The cause of the MiniBooNE low-energy excess is going to be investigated in the MicroBooNE experiment at Fermilab [163].
Moreover, the cosmological data indicate a tension between the necessity to have a sterile neutrino mass at the eV scale and the expected full thermalization of the sterile neutrinos through active-sterile oscillations in the early Universe \cite{2012CPPYP7}. Hence, the possible existence of light sterile neutrinos at the eV scale is controversial and needs new reliable experimental checks.

The impressive program of new experiments reviewed in Section \ref{sec:exp} gives us confidence that the question of the existence of the light sterile neutrinos indicated by the reactor, Gallium and LSND anomalies will be answered in a definitive way in the next years.

For neutrino physics, the discovery of the existence of light sterile neutrinos would open a rich field of experimental and theoretical research on the properties of the sterile neutrinos, their mixing with the active neutrinos and their role in neutrino experiments (e.g. in solar \cite{[15][18][20][22][26][27][29][30][31][32]} long-baseline \cite{[26][27][28]} and atmospheric \cite{[73][92][151][153][271][276]} neutrino experiments) in astrophysics (e.g. in supernova neutrino experiments \cite{[255][256][257][258][259][260][261]} and indirect dark matter detection \cite{[284][285][286][287][288]}), high-energy cosmic neutrinos \cite{[285][288]} and, in cosmology, at the eV scale and the expected full thermalization of energy cosmic neutrinos \cite{[284][285][286][287][288]}, and indirect dark matter detection \cite{[285][286][287][288]}, high-energy cosmic neutrinos \cite{[285][288]}, and in cosmology (see Refs. \cite{[7][67][70]}).

Let us finally emphasize that the discovery of the existence of sterile neutrinos would be a major discovery which would have a profound impact not only on neutrino physics, but on our whole view of fundamental physics, because sterile neutrinos are elementary particles beyond the Standard Model. The existence of light sterile neutrinos would prove that there is new physics beyond the Standard Model at low-energies and their properties can give important information on this new physics. Without any doubt, such a discovery would deserve a new Nobel Prize in Physics.

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