Pseudomoduli Dark Matter and Quiver Gauge Theories

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Abstract: We investigate supersymmetric models for dark matter which is represented by pseudomoduli in weakly coupled hidden sectors. We propose a scheme to add a dark matter sector to quiver gauge theories with metastable supersymmetry breaking. We discuss the embedding of such scheme in string theory and we describe the dark matter sector in terms of D7 flavour branes. We explore the phenomenology in various regions of the parameters.

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1. Introduction

Cosmological observations have established the existence of dark matter which is not composed by any of the Standard Model (SM) particles, and with a relic abundance of the order of \( \Omega h^2 \sim 0.1 \) (for reviews see [1, 2, 3, 4] and reference therein).

A stable particle in thermal equilibrium with the SM in the early universe is usually referred as cold dark matter (DM). As long as the universe expands, the DM ceases to annihilate efficiently and freezes out, leaving a relic abundance

\[
\Omega h^2 \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{\alpha_{DM}}{M_{DM}^2}
\]  

(1.1)

where \( h \) is the Hubble parameter in units of 100 km/sec per Mpc, \( \langle \sigma v \rangle \) is the annihilation cross section, \( \alpha_{DM} \) is the characteristic coupling and \( M_{DM} \) is the mass of the dark matter particle. For a massive particle with weak interaction \( (g_{DM} \sim 1) \), i.e. a WIMP, the required relic abundance is obtained for \( M_{DM} \sim 1 \) TeV. The TeV scale that naturally appears suggests some relations between DM and electroweak symmetry breaking (EWSB), and has been dubbed as the WIMP miracle.

A standard explanation for the origin of the electroweak scale is supersymmetry breaking (for a phenomenological introduction to supersymmetry and references see [5]). The idea that supersymmetry and its breaking should account also for the origin of the DM has prompted an extensive investigation. Much effort has been focused on the mechanism of gravity mediation, where the lightest neutralino of the MSSM should be a stable, viable
DM candidate. A phenomenological drawback for models of gravity mediation is that they do not address the susy flavour problem.

The flavour problem is solved in the gauge mediation scenario [7, 8], because the SM gauge interactions are flavour blind. In gauge mediated models supersymmetry is broken at low energies and the gravitino is the LSP. However, the gravitino is typically too light (with mass in the range of eV to Gev) [9, 10] to be a viable cold DM candidate. Other alternatives for susy DM have been and are under inspection (see for instance [11, 12, 13]).

In gauge mediated theories particles in the hidden sectors can be alternative DM candidates. In such models, supersymmetry breaking in the hidden sector occurs dynamically, via strong dynamics effects. The DM can then be identified with the mesons and the baryons of the strongly coupled hidden sector [14].

On the other hand, one can explore the possibility of realizing dark matter in weakly coupled hidden sector with spontaneous supersymmetry breaking [15, 16]. These models are relevant since they can arise as the low energy description of UV free gauge theories, as shown by [17].

Such weakly coupled models typically break supersymmetry in metastable vacua, with a spectrum of elementary particles. The scalar potential often presents tree level flat directions, which are lifted by one loop quantum corrections. These pseudomoduli fields have weak scale interactions and their one loop mass is of the order of the supersymmetry breaking scale, i.e the TeV scale naturally arises. If they are stable against decay because of a discrete $Z_2$ symmetry, they represent viable cold dark matter candidates. This possibility has been explored in O’Raifeartaigh like models in [15] and also analyzed in [16].

In this paper we shall study models of pseudomoduli as dark matter and wish to answer some questions concerning the relative UV completion. This is a non trivial problem, with severe constraints [18, 19] for consistent embedding in a UV complete theory. We shall propose a quiver model for which a stringy origin can be reached. We embed models with pseudomoduli DM in appropriate quiver gauge theories which arise from $D$-branes at CY singularities. The starting point is a quiver theory, inherited and/or interpreted as an IR Seiberg dual, which provides the hidden sector of dynamical breaking of susy. The next step is the addition of a dark matter sector, represented by an extra node in the quiver connected through matter interactions to the hidden sector. The addition of $D7$ flavour branes in the CY can be put in correspondence with the dark matter sector in the quiver.

The structure of the paper is the following. In section 2 we propose the general strategy to embed pseudomoduli dark matter in quiver gauge theories, we comment on the phenomenological constraints and also on the string theory realization. In section 3 we build a concrete example; we couple pseudomoduli DM to the KOO model [20], and we discuss the related phenomenology. In section 4 we provide for a UV origin to the model in terms of a step of Seiberg duality: the UV model is obtained by deforming an $L^{131}$ non isolated singularity and by adding flavour $D7$ branes. A conclusion follows. In appendix A we review a basic cosmological bound on the supersymmetry breaking scale. In appendix B we review the procedure of flavoring with $D7$ branes and we discuss its relation with the DM sector. In appendix C we provide the details of the one loop computations for the pseudomoduli masses.
As we were finishing this paper, we were informed of [21] which study leptophilic dark matter [22] in quiver gauge theories.

2. Pseudomoduli DM in quiver gauge theories

As anticipated, quiver gauge theories are useful settings for the study of the stringy origin of pseudomoduli dark matter. In the quiver, we distinguish between a supersymmetry breaking sector and a DM sector. The first one is thought as an ISS like model: it is characterized by a metastable vacuum, a weakly coupled $SU(N)$ gauge symmetry and an $SU(F)$ flavour symmetry. We also require $R$-symmetry to be broken in this sector. We parametrize this sector with a spurionic chiral field $X$, an $SU(N)$ singlet, which acquires a vev and an $F$-component, i.e. $X = M + \theta^2 F$, and its fermionic component is the Goldstino. The chiral field $X$ couples to two chiral fields $Q_{12}$ and $Q_{21}$, respectively in the fundamental and antifundamental representation of the gauge group $SU(N)$, and in the antifundamental and fundamental of the flavour symmetry $SU(F)$. A three-linear coupling $XQ_{12}Q_{21}$ induces a mass splitting between the bosonic and fermionic components of the fields $Q_{12}$ and $Q_{21}$, which are the messengers of susy breaking. This sector is depicted in the rectangular region in the figure [1].

We then add the DM sector, characterized by a $U(1)_d$ gauge symmetry and two pairs of bifundamental fields (see figure [1]). These fields interact with the messengers through

\[ F_{12} = \text{link fields} \]

\[ X_{12} = \text{spurionic chiral field} \]

\[ Q_{12} = \text{fundamental} \]

\[ Q_{21} = \text{antifundamental} \]
the superpotential

\[ W_{DM} = Q_{12} Q_{2d} Q_{d1} + Q_{21} Q_{1d} Q_{d2} + m_{d2} Q_{2d} Q_{d2} \]  \hspace{1cm} (2.1)

where the traces are understood. This new sector does not change the solution of the equations of motion for the hidden sector, and the metastable vacuum remains a tree level minimum of the scalar potential.

In the DM sector, the fields \( Q_{2d} \) and \( Q_{d2} \) have a tree level mass and are stabilized at zero vev. They are called link fields.

The other chiral multiplets \( Q_{1d} \) and \( Q_{d1} \) are massless at tree level. In particular, their scalar components are tree level flat directions not associated with any broken global symmetry, i.e. pseudomoduli. Both the fermionic and bosonic components of these multiplets \( Q_{1d} \) and \( Q_{d1} \) can get a mass at one loop. Since supersymmetry is broken, we expect scalars and fermions to get different masses. Typically, as discussed in [13, 14], the scalar masses are higher than the fermionic masses. This implies that in our setting the fermions \( \psi_{Q_{1d}} \) and \( \psi_{Q_{d1}} \) have lower masses than their scalar partners. These fermions are the cold DM candidates.

**Brief and comments**

The previous scheme can generate viable dark matter candidates provided the hidden sector fulfills some basic requirements.

The mechanism that we consider for communicating the supersymmetry breaking to the visible sector (MSSM) is gauge mediation [4, 8]. This is a standard technique in susy breaking quiver gauge theories [23, 24, 25]. Here we shall concentrate on the direct gauge mediation scheme.

Direct gauge mediation is realized by embedding the MSSM gauge group in some subgroup of the hidden sector. Different choices lead to charged or to uncharged dark matter under the MSSM gauge group. Dark matter is charged if we identify the MSSM gauge group with \( SU(F) \), whereas it is uncharged if we embed the MSSM gauge group in \( SU(N) \). Alternatively, we can give mass to the other pair of chiral fields adding \( m_{1d} Q_{1d} Q_{d1} \) to the superpotential (2.1), setting \( m_{2d} = 0 \). This exchanges the role of dark matter and link fields as well as that one of charged and uncharged dark matter. In this paper we discuss the case of uncharged dark matter, which is less constrained by experiments. In this case the massive link fields are charged under the MSSM gauge group and the \( U(1) \) gauge group. The dark matter is charged under the \( U(1) \) gauge group and a hidden sector flavour group. The interaction between the MSSM gauge group and the dark matter is obtained via kinetic mixing [24]. Recently this mechanism has been investigated in [22, 13].

In models of gauge mediation \( R \)-symmetry has to be broken for the gaugino to acquire non trivial mass. The models analyzed in [15] were required to respect a spontaneously broken \( R \)-symmetry that prevents tree level mass terms and extra couplings for the pseudomoduli dark matter candidates. We also admit the explicit breaking of \( R \)-symmetry in the hidden sector. Indeed in our case the UV stringy origin uniquely settles the structure of the interaction.
There should be a discrete $Z_2$ symmetry that forbids dark matter to decay. In our case this symmetry follows automatically from the non-chiral structure of the DM sector and from its interaction superpotential \[^{[24]}\]. Furthermore the pseudomoduli dark matter $Q_{2d}$ and $Q_{d2}$ have to be stabilized at one loop at the origin of the pseudomoduli space such that this $Z_2$ symmetry is unbroken.

As discussed in \[^{[15, 16]}\], we expect the one loop masses of the DM fermions to be smaller than the masses of their scalar superpartners. This high difference between scalar and fermion masses implies that the decay of the $Q_{2d}$ and $Q_{d2}$ scalars does not affect the dark matter relic density. This property of the one loop masses for the scalars and the fermions has to be checked in any model of pseudomoduli DM.

We require to have a TeV scale dark matter mass, and also a not too heavy superpartner spectrum in the MSSM. Hence we demand the parameter

$$R \equiv \frac{M_{DM}}{m_\lambda} \tag{2.2}$$

to be of order 1, where we estimate the soft mass scale with respect to the gaugino mass $m_\lambda$. The dominant contribution to the dark matter cross section comes from its annihilation into dark photons

$$\langle \sigma v \rangle \simeq \frac{\pi \alpha_d}{M_{DM}^2} \tag{2.3}$$

where $\alpha_d$ is the coupling of the $U(1)_d$ gauge group and $M_{DM}$ is the dark matter mass. For $g_d \sim 1$ and $M_{DM} \sim O$(TeV) this cross section secures a satisfactory relic abundance, and it avoids dark matter overabundance \[^{[15, 16]}\]. Note that for this annihilation to be efficient the dark gauge boson mass needs to be lower than the dark matter mass \[^{[27, 13]}\]. We will not address the naturalness of this new scale here.

The realization we provided of pseudomoduli DM in gauge theories is rather generic. Many models of supersymmetry breaking in metastable vacua with explicitly broken $R$-symmetry have been realized \[^{[28, 29, 30, 31, 32, 33, 34, 35, 36]}\]. The procedure for adding a DM sector just explained can be applied to all these models.

**The stringy origin of the DM sector**

Metastable supersymmetry breaking is common to gauge theories arising from D3-branes at CY singularities. In this framework the DM sector corresponds to D7-flavour branes. Indeed, by properly adding D7-branes, we obtain the field content and the interaction superpotential of the DM sector. In the appendix \[^{[3]}\] we discuss the realization of DM sector as D7-branes. This suggests a connection between pseudomoduli DM and D7-branes at CY singularities.

**3. Coupling DM to the KOO model**

In this section we give a concrete example of the strategy presented above. We consider as the supersymmetry breaking sector the KOO model \[^{[20]}\]. The low energy description of this theory is a three node quiver gauge theory $U(N_1) \times U(N_2) \times U(N_3)$, where

$$N_1 = N_2 = N \quad N_3 = N + M \tag{3.1}$$
We choose $N < M$ such that the $U(N_2)$ gauge group is infrared free. The other gauge groups are considered as very weakly coupled at low energy. The superpotential for this model is

$$W_{	ext{KOO}} = -h \mu_1^2 X_{11} - h \mu_3^2 X_{33} + h m_{13} X_{11} X_{33} + h q \cdot X \cdot \tilde{q}$$

(3.2)

where

$$q = \begin{pmatrix} q_{21} & q_{32} \end{pmatrix} \quad \tilde{q} = \begin{pmatrix} q_{12} \\ q_{23} \end{pmatrix} \quad X = \begin{pmatrix} X_{11} & X_{31} \\ X_{31} & X_{33} \end{pmatrix}$$

(3.3)

This model breaks supersymmetry in a long living metastable vacuum and does not have an $R$-symmetry. The absence of $R$-symmetry implies that direct gauge mediation is a viable mechanism to transmit supersymmetry breaking to the superpartners in the MSSM [20].

We now add the dark matter sector. As already explained we add a new gauge group $U(1)_d$ with bifundamental fields connected with the groups $U(N_2)$ and $U(N_3)$. The resulting quiver is depicted in figure 2. The new fields interact with the supersymmetry breaking sector fields via the superpotential

$$W_{\text{DM}} = h q_{23} q_{3d} q_{d2} + h q_{32} q_{2d} q_{d3} + h m_2 q_{2d} q_{d2}$$

(3.4)

The metastable supersymmetry breaking vacuum of the KOO model is not destabilized by the deformation [34]. Hence the vacuum expectation values of the fields are

$$q = \begin{pmatrix} \mu_1 1_N & 0 \end{pmatrix} \quad \tilde{q} = \begin{pmatrix} \mu_1 1_N \\ 0 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 0 \\ 0 & \chi \end{pmatrix}$$

(3.5)

$$q_{3d} = Y \quad q_{d3} = \tilde{Y} \quad q_{2d} = 0 \quad q_{d2} = 0$$

(3.6)

The vev of the fields $q_{12}$ and $q_{21}$ break the groups $U(N_1) \times U(N_2)$ to the diagonal subgroup $U(N)^{1-2}$. The fields $\chi, Y, \tilde{Y}$ are pseudomoduli. Their scalar components get masses via one loop corrections. The Goldstino is a mixture of the fermionic component of $\chi$ and of $X_{11}$ [36, 37], and it is eaten by the gravitino in the super Higgs mechanism [38]. The gravitino is typically too light to be a viable dark matter candidate in this gauge mediation scenario. The fermionic components of the fields $Y, \tilde{Y}$ are then natural dark matter candidates. We shall perform a quantitative analysis of the spectrum to justify this scenario.
As a standard procedure we expand the superpotential around the vacuum

\[
\begin{align*}
q &= \left( \mu_1 \mathbf{1}_N + \sigma_1 \Phi_1 \right) \\
q &= \left( \mu_2 \mathbf{1}_N + \sigma_2 \Phi_2 \right) \\
X &= \left( \sigma_3 \Phi_3 \right)
\end{align*}
\]

and then study the quantum infrared model which is a generalized O’Raifeartaigh model

\[
W = h\chi \Phi_1 \Phi_2 - h\mu_3^2 \chi + h\mu_1(\Phi_1 \Phi_4 + \mu_1 \Phi_2 \Phi_3) + h m_{13} \Phi_3 \Phi_4 \\
+ h Y \Phi_1 \Phi_6 + h \tilde{Y} \Phi_2 \Phi_5 + h m_2 \Phi_5 \Phi_6
\]

(3.9)

The pseudomodulus \( \chi \) get one loop corrections only from the first line in (3.9), which is the same microscopic superpotential as in [20]. The second line in (3.9) is the dark matter sector. The massive fields \( \Phi_5 \) and \( \Phi_6 \) are the link fields. The scalars and the fermions of the fields \( Y \) and \( \tilde{Y} \) get both one loop masses. In the appendix C we give the analytical calculations of these masses. Here we discuss the phenomenology of the model.

**Phenomenology**

The stability of the metastable supersymmetry breaking vacuum requires

\[
\mu_1 \gg \mu_3 \quad \mu_1 > m_{13}
\]

(3.10)

where \( \mu_1 \) is the messenger mass scale, \( \mu_3 \) is the supersymmetry breaking scale, and \( m_{13} \) is the \( R \)-symmetry breaking mass. As already explained, supersymmetry breaking is communicated to the MSSM via direct gauge mediation. The GUT \( SU(5) \) gauge group can be embedded both in \( U(N_3) \) or in the diagonal subgroup \( U(N)_1-2 \). In the first case the pseudomoduli DM is charged under the GUT group. We will not investigate this possibility. We choose the second case, leading to uncharged DM. The gaugino mass is [20]

\[
m_\lambda = \frac{g^2}{16\pi^2} (N + M) \frac{h\mu_3^2 m_{13}}{\mu_1^2} + O \left( \frac{m_{13}^2}{\mu_1^2} \right)
\]

(3.11)

The scalar masses are of the same order provided that \( m_{13} \sim \mu_3 / \sqrt{N + M} \).

We now discuss the quantum aspects of the dark matter sector. The scalar component of the chiral fields \( Y, \tilde{Y} \) acquire positive squared masses and are stabilized at the origin of the moduli space. This preserves the \( Z_2 \) discrete symmetry that makes the DM stable. The DM, i.e. the fermionic component of the field \( Y, \tilde{Y} \), also acquire one loop masses as well.

A detailed computation of the 1-loop scalar and fermionic masses for the components of the chiral fields \( Y, \tilde{Y} \), at all order in the supersymmetry breaking scale, is carried out in the appendix C. Here we report the analytic result at the combined third order in the adimensional parameters \( m_{13} / \mu_1, m_2 / \mu_1 \) and \( \chi / \mu_1 \), and at first order in the supersymmetry breaking scale \( \mu_1^2 / \mu^2 \). The fermion mass results

\[
m_{\psi_Y \psi_{\tilde{Y}}} = (N + M) \frac{h^2}{16\pi^2} \frac{\mu_3^2 m_2}{\mu_1^2} \left( 1 + \frac{2 m_{13}^2}{3 \mu_1^2} - \frac{1}{3} \frac{\chi^2}{\mu_1^2} \right) + \ldots
\]

(3.12)
The scalar diagonal and off diagonal masses are respectively

\[ m_{\tilde{Y}Y}^2 = m_{\tilde{Y}Y}^2 = (N + M) \frac{h^2}{32\pi^2} \frac{\mu_3^4}{\mu_1^4} (1 - \frac{m_2^2}{\mu_1^2} - 2 \frac{\chi^2}{\mu_1^2}) + \ldots \]  

(3.13)

\[ m_{YY}^2 = -(N + M) \frac{h^2}{16\pi^2} \frac{\mu_3^2 m_2 \chi}{\mu_1^4} + \ldots \]  

(3.14)

The eigenvalues of the scalar masses can be obtained diagonalizing the resulting mass matrix. The off diagonal components are subleading and hence the main contribution to the eigenvalues comes from the diagonal masses (3.13). In the expressions above we should insert the vev of \( \chi \) as a function of the other parameters, which is found by minimizing the effective potential.

Note that there should be at least one order of magnitude between the lowest eigenvalue of the scalar mass matrix and the fermion mass, otherwise the DM relic abundance is affected by the decay of the scalars \( Y \) and \( \tilde{Y} \). In the appendix A we review how this constraint gives a bound on the scale of suSy breaking. In figure 3 we plot the ratio of fermion and the lowest scalar mass as a function of the parameters of the model: it shows that there are two order of magnitude between the two masses. This constraints the suSy breaking scale to \( \mu_3 < 10^5 \) TeV (see appendix A).

As already explained, we require the DM to be of the same order of the soft masses (\( \sim 1 \) TeV). We estimate the parameter \( R \) (2.2) as

\[ R = \frac{m_{\psi\psi}}{m_\lambda} \sim \frac{hm_2}{g^2 m_{13}} + \ldots \]  

(3.15)
and we plot it in figure 4. Figure 4 shows that we can find a range in the parameter space where the ratio $R$ is of order 1. Notice that the enhancement factor $(N + M)$ cancels in the ratio $R$. This is a specific feature of the model, since the messenger fields and the dark matter are charged under the same global symmetry $U(N_3)$.

As usual in theories with direct gauge mediation, this model can suffer from a Landau pole problem. As pointed out in [37], requiring perturbative unification of the couplings below the Landau pole forces the messenger scale and the supersymmetry breaking scale to be large. This results in a too large mass for the gravitino, outside the cosmological bound worked out in [39]. We leave a detailed analysis of the issue of gauge coupling unification for future studies. One can solve this problem by looking at a different UV completion for the KOO model, for example via a cascading gauge theory. In the next section we show how the same low energy theory arises from a system of $D3$ and $D7$ branes probing a CY singularity through a Seiberg duality.

Alternatively, one can choose a different embedding for the SM gauge group into the flavour group of the supersymmetry breaking sector. This embedding of $SU(5)_{GUT}$ into $U(N_3)$ has been investigated in [37], and it has been shown to be compatible with a gravitino mass which is consistent with the cosmological bound of [39]. For this embedding the pseudomoduli $Y$ and $\hat{Y}$ are charged under the standard model gauge group. If we want to realize DM uncharged under the SM gauge group, we have to exchange the role of the dark matter and of the link fields. This is done by setting to zero the mass term in (3.4), and adding a new one $h m_3 q_3 d q_3$. The phenomenology of the low energy theory and the ratio $R$ are unchanged.
4. Pseudomoduli DM from $D$-brane at CY singularities

String theory provides a natural embedding for pseudomoduli DM. In fact, deformations of non isolated singularities lead to metastable supersymmetry breaking in the gauge theory living on $D3$-branes probing the singularities $[10, 11]$. It has also been shown $[11]$ that the addition of flavour $D7$ branes can break supersymmetry in metastable vacua. Here we use both these effects to build a quiver gauge theory. We show that the resulting model is a concrete realization of the setup for pseudomoduli DM in quiver gauge theories proposed in section 2.

We consider the quiver gauge theory arising from $D3$ branes at the $L^{131}$ singularity. It can be described by the curve in $C^4$

$$xy^3 = wz$$

and we deform it as

$$x(y + \xi)y(y - \xi) = wz$$

We choose the rank of the four gauge group as

$$N_0 = 0 \quad N_1 = 1 \quad N_2 = M \quad N_3 = N$$

and the resulting superpotential is

$$W_0 = X_{11}Q_{21}Q_{12} + \lambda Q_{12}Q_{23}Q_{32}Q_{21} + m_{23}Q_{23}Q_{32}$$

where the mass is related to the parameter in the geometry (4.2) as $m_{23} = \xi \lambda$. We add $N_4 = M - 1$ $D7$-flavour branes associated with the fields $Q_{23}$ and $Q_{32}$ (see appendix $[13]$). This introduces new fields that interacts via the superpotential

$$W_{D7} = \rho Q_{23}Q_{34}Q_{42} + \rho Q_{32}Q_{24}Q_{43} + m_{34}Q_{34}Q_{43} + m_{24}Q_{24}Q_{42}$$

In figure $[13]$ we give a pictorial quiver representation of the model, with complete superpotential

$$W = W_0 + W_{D7}$$

We work in the range $M > 2N$. In this window the node 2 has the strongest coupling and it is in the magnetic free window. The low energy description can be obtained via Seiberg duality. The dual low energy theory has the superpotential

$$W = Tr\begin{pmatrix} q_{21} & q_{23} & q_{24} \\ M_{11} & M_{13} & M_{14} \\ M_{31} & M_{33} & M_{34} \\ M_{41} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} q_{12} \\ q_{32} \\ q_{42} \end{pmatrix} + m_{11}M_{11}X_{11} + m_{13}M_{13}M_{31}$$

$$+ m_{Q34}Q_{34}Q_{43} + m_{MQ34}M_{34}Q_{43} + m_{MQ34}Q_{34}M_{43} - \mu_3^2M_{33} - \mu_4^2M_{44}$$

where the new scales are

$$m_{11} = \Lambda_2, \quad m_{13} = \lambda\Lambda_2^2, \quad m_{Q34} = m_{34}, \quad m_{MQ34} = \rho\Lambda_2, \quad \mu_3^2 = m_{23}\Lambda_2, \quad \mu_4^2 = m_{24}\Lambda_2$$
We work in the regime where $m_{11}$ and $m_{Q34}$ are larger than the other scales of the theory. We can then integrate out the fields $X_{11}, M_{11}, Q_{34}$ and $Q_{43}$, and obtain the superpotential

$$ W = q_{21}M_{13}q_{32} + q_{21}M_{14}q_{42} + q_{23}M_{31}q_{12} + q_{24}M_{41}q_{12} + q_{23}M_{33}q_{32} + q_{23}M_{34}q_{42} + q_{24}M_{43}q_{32} + q_{24}M_{44}q_{42} + m_{13}M_{13}M_{31} + m_{34}M_{34}M_{43} - \mu_3^2M_{33} - \mu_4^2M_{44} $$  \hspace{1cm} (4.9)

where we define $m_{34} = m_{MQ34}^2/m_{Q34}$. Figure 6 is a pictorial quiver representation of the magnetic model. The ranks of the groups in the magnetic theory are

$$ N_1 = 1, \quad \tilde{N}_2 = N_3 = N, \quad N_4 = M - 1 $$  \hspace{1cm} (4.10)

We look for the metastable vacuum states. We find

$$ M_{13} = M_{31} = 0, \quad M_{34} = M_{43}^T = 0, \quad M_{14} = Y, \quad M_{41} = \tilde{Y} $$  \hspace{1cm} (4.11)

$$ M_{33} = 0, \quad M_{44} = \chi, \quad q_{12} = q_{21} = 0, \quad q_{23} = q_{32} = \mu_3, \quad q_{24}^T = q_{42} = 0 $$

The one loop correction for the pseudomoduli are calculated after expanding the fields of the theory around their expectation value. Nevertheless some of their fluctuations give a
supersymmetric contribution at one loop. The only relevant expansions are

\[ q_{42} = \phi_1 \quad q_{24} = \phi_2 \quad q_{12} = \phi_5 \quad q_{21} = \phi_6 \quad M_{13} = \phi_7 \quad M_{31} = \phi_8 \]  
\[ M_{14} = Y \quad M_{41} = \tilde{Y} \quad M_{34} = \phi_4 \quad M_{43} = \phi_3 \quad M_{44} = \chi \]  

(4.12)

Expanding around the vacuum we find the following structure for the effective superpotential involving the pseudomoduli \( \chi \), \( Y \) and \( \tilde{Y} \)

\[ W_{\text{eff}} = W(\chi) + W(Y, \tilde{Y}) \]  

(4.13)

where

\[ W(\chi) = \chi \phi_1 \phi_2 - \mu_4^2 \chi + \mu_3 (\phi_1 \phi_4 + \phi_2 \phi_3) + m_{34} \phi_3 \phi_4 \]  

(4.14)

and the superpotential for the other pseudomoduli is

\[ W(Y, \tilde{Y}) = Y \phi_1 \phi_6 + \tilde{Y} \phi_2 \phi_5 + \mu_3 (\phi_6 \phi_7 + \phi_5 \phi_8) + m_{13} \phi_7 \phi_8 \]  

(4.15)

This superpotential reduces to (3.9) in the limit \( m_{13} > \mu_3 \). Indeed in this limit we can integrate out supersymmetrically the fields \( \phi_7 \phi_8 \) and obtain an effective mass term for the fields \( \mu_3^2 / m_{13} \phi_5 \phi_6 \). Hence in this limit the phenomenology of the model is the same as in section 3. However, also the case with \( m_{13} < \mu_3 \) is phenomenologically viable, with the parameter \( R \) of order 1. These features are manifest in figure 7 where we plot the parameter \( R \) as a function of the ratios \( m_{13} / \mu_3 \) and \( m_{34} / \mu_3 \). In appendix C we perform the explicit computation for the 1-loop DM fermion mass.

5. Conclusions

In this article we proposed a scheme to realize pseudomodulidark matter in quiver gauge theories with weakly coupled supersymmetry breaking at la ISS. In section 2 we distinguished a metastable supersymmetry breaking sector and a dark matter sector. The former communicates the breaking of supersymmetry to the MSSM through gauge interactions. This mechanism requires \( R \)-symmetry to be broken, spontaneously or explicitly. The dark matter sector contains two pseudomoduli fields and it is coupled to the supersymmetry breaking sector through superpotential terms. This coupling induces, at one loop, supersymmetry breaking masses for the pseudomoduli and their fermionic partners in the dark matter sector. This provides the mechanism to generate a TeV scale mass for the fermions which are the cold dark matter candidates.

We considered only the case of uncharged dark matter with respect to the gauge interactions of the standard model. There is a coupling of the \( U(1)_d \) gauge group, under which DM is charged, to the \( U(1)_Y \) of the standard model via the kinetic mixing. Cases in which the dark matter is charged under a non abelian gauge group are left for future works.

We showed that when the supersymmetry breaking sector is described by \( D3 \) branes probing CY singularity, the dark matter sector corresponds to the addition of \( D7 \) flavour branes.
In section 3 we realized the proposed scenario by coupling a pseudomoduli DM sector to the KOO model, and 1TeV mass dark matter is rather natural. In section 4 we studied a model with D3 and D7 branes wrapped over a deformed CY $L^{131}$ singularity. We have argued that this model reduces, in the infrared, to the KOO model plus DM.

Many extensions of our proposal can be studied. One can find other models by coupling the DM sector to gauge theories different from KOO. For example one can couple DM to models with $R$-symmetry breaking in metastable vacua $[28, 29, 30, 31, 32, 33, 34, 35, 36]$. Another interesting issue is the investigation at large of the UV completion. The generation of the DM sector inside a quiver gauge theory can be a result of a step of Seiberg duality, as we showed in the deformed and flavored $L^{131}$ theory.

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A. Cosmological bounds

Pseudomoduli DM arise from a tree level massless chiral multiplet $Y = (\phi_Y, \psi_Y)$. It includes a complex scalar and a fermion. Both fields acquire masses at one loop. In the model we studied there is a hierarchy among these masses. The scalar mass is typically one or two order of magnitude larger than the fermion mass. There is a $Z_2$ discrete symmetry under which $Y$ is charged. This prevents the field $\psi_Y$ to decay. If it has weakly coupled interactions and mass at the TeV scale, it is a viable DM candidate.

The scalar component, which is heavier, can decay in a gravitino and a fermion, $\phi_Y \rightarrow G \psi_Y$. This decay can modify the relic density of the DM candidate $\psi_Y$. We thus require that the decay temperature of the scalar $\phi_Y$ is larger than the freeze out temperature of the fermionic DM $\psi_Y$. The scalar decay rate is

$$\Gamma(\phi_Y \rightarrow \psi_Y G) = \frac{m_{\psi_Y}^5}{16\pi f^2} \left(1 - \frac{m_{\psi_Y}^2}{m_{\phi_Y}^2}\right)^4 \approx \frac{m_{\psi_Y}^5}{16\pi f^2}$$

(A.1)

The temperature associated to this decay is $T \sim \Gamma^{1/2}$

$$T_{\phi_Y} = \left(\frac{m_{\psi_Y}}{100 \text{TeV}}\right)^{5/2} \left(\frac{10^4 \text{TeV}}{\sqrt{f}}\right)^2 3 \text{TeV}$$

(A.2)

whereas the freeze out temperature of the fermion is $T_{\text{freeze}} \simeq m_{\psi_Y}/20$. The requirement $T_{\phi_Y} > T_{\text{freeze}}$ translates in a bound on the supersymmetry breaking scale

$$\sqrt{f} \lesssim \left(\frac{m_{\phi_Y}}{100 \text{TeV}}\right)^{5/4} \left(\frac{1 \text{TeV}}{m_{\psi_Y}}\right)^{1/2} 2.5 \cdot 10^5 \text{TeV}$$

(A.3)

B. Flavoring with $D7$ branes

In this paper we have proposed how to realize a dark matter sector in quiver gauge theories. Here we show how this new sector can be described as $D7$-flavour branes [42].

We briefly review the technique introduced in [41] to add $D7$ branes to toric quiver gauge theories and to extract the interaction superpotential; we refer to the original paper for a detailed explanation.

Consider a toric quiver gauge theory realized as $D3$ branes probing a toric CY singularity. The system can be described in terms of a dimer diagram and a useful tool is the Riemann surface in the mirror configuration [43].

By the use of these tools, it turns out that in the quiver we can associate to every bifundamental field a supersymmetric four cycle, which passes through the singular point, on which a $D7$-brane can be wrapped. Call one of this cycle $\Sigma_{ij}$ and label the two gauge groups under which the bifundamental is charged as $U(N_i)$ and $U(N_j)$. When adding
$D7$-branes, one should control the cancellation of RR tadpoles, corresponding, on the field theory side, to an anomaly free theory.

The addition of the $D7$-brane adds new bifundamental fields corresponding to strings stretched between the $D7$ brane and the $D3$ branes. These new degrees of freedom are charged under the gauge groups $U(N_i)$ or $U(N_j)$ and under the $U(1)_A$ symmetry introduced by the $D7$ brane \footnote{If there are $K$ $D7$ branes on the same cycle and with the same Chan-Paton structure the symmetry is $U(K)_A$.}. We show in the figure 8 the resulting quiver.

There is an interaction superpotential term of the type $33 - 37 - 73$ that can be obtained analyzing the disk on the mirror Riemann surface, and it is

$$W_{int} = X_{ij} q_j A q_{Ai} \quad (B.1)$$

If there are $D7$ branes on different four cycles, each of them introduces a couple of bifundamental fields and interaction terms as in (B.1). There are also interaction terms of the type $37_A - 7_A 7_B - 7_B 3$. These terms can lead to masses for the 37 fields if a $7_A 7_B$ field get a non trivial vacuum expectation values. The vev breaks the $U(1)_A \times U(1)_B$ groups associated to the $D7_A$ and $D7_B$ branes to the diagonal subgroup.

For instance consider the addition of $D7$-branes associated to two different bifundamentals $X_{ij}$ and $X_{ji}$ charged under the group $U(N_i)$ and $U(N_j)$. The resulting theory is...
the quiver depicted in figure 9. A vev for the $T_A T_B$ field give raise to the following mass terms in the superpotential, that involves both set of flavours,

$$W_{\text{mass}} = m_1 q_A i q_i B + m_2 q_B j q_j A$$

(B.2)
in addition to the interaction superpotential

$$W_{\text{int}} = X_{ij} q_j A q_i A + X_{ji} q_i B q_B j$$

(B.3)

The mass parameters can be related to geometrical quantities as follows. The mass term corresponds in the geometry to the recombination of two $D_7 A$ and $D_7 B$ branes. The two cycles recombine in one cycle which passes at some distance $\epsilon$ from the singular point. Taking local holomorphic coordinates we can parametrize the two four-cycle as $z_1 = 0$ and $z_2 = 0$. The mass term corresponds to a recombination of the $D_7 A$ and $D_7 B$ brane such that they now wrap the cycle $z_1 z_2 = \epsilon$. The parameter $\epsilon$ is the distance of the four-cycle from the singular point, and it is related to the gauge theory parameters as

$$\epsilon \sim m_1 m_2$$

(B.4)

Then the two mass parameters have to be both turned on.

In the paper we used this technique to add flavours to quiver gauge theories and to extract their interaction superpotential. In section 2 we add $D7$-branes associated to the fields $Q_{12}$ and $Q_{12}$. We introduced only one of the two mass terms of (B.2). This approximation can be considered as a limiting case where there is a large hierarchy among the two masses, and hence we neglect one of the two. This is generally the approximation to adopt when adding a $D7$-brane dark matter sector to a weakly coupled supersymmetry breaking sector, as in section 2 and in section 3.

In section 4 we analyzed a UV complete theory, and there we introduced $D7$-flavour branes associated with the fields $Q_{23}$ and $Q_{32}$ both, with the mass terms of (B.2). The setup of section 2 is then obtained as the low energy description of the model by performing Seiberg duality.

C. An explicit calculation

In this appendix we show a detailed calculation of the one loop masses for pseudomoduli DM. We consider the superpotential

$$W_1 = f X + X \phi_1 \phi_2 + \mu (\phi_1 \phi_3 + \phi_2 \phi_4) + m_1 \phi_3 \phi_4$$

(C.1)

This superpotential represents the supersymmetry breaking sector. Here $R$-symmetry is explicitly broken by the term $m_1 \phi_3 \phi_4$. In the non supersymmetric minimum the fields $\phi_i$ have zero vev. The fields $X$ is a pseudomodulus. The pseudomoduli space is tachyon free and stable if

$$|\mu^2 \pm m_1 X|^2 - f (m_1^2 + \mu^2) > 0$$

(C.2)

The one loop analysis shows that this pseudomodulus is stabilized at $\langle X \rangle \neq 0$ [20, 37].
DM and KOO model

The model of section 3 is recovered by adding to (C.1) the superpotential for the dark matter sector. This is

$$W_2 = m_2 \phi_5 \phi_6 + Y \phi_1 \phi_5 + \tilde{Y} \phi_2 \phi_6$$

The fields $\phi_5$ and $\phi_6$ are stabilized at zero vev in the non supersymmetric vacuum, while the fields $Y$ and $\tilde{Y}$ are pseudomoduli.

They are stabilized at one loop at the origin of the moduli space, and their scalar components and their fermions components get both a non zero mass. The fermions $\psi_Y$ and $\tilde{\psi}_Y$ get one loop masses, differently from $\psi_X$, because they are not associated with the goldstino. However their masses are generically different from the masses of their bosonic partners, because they feel the effects of supersymmetry breaking.

In this appendix we explicitly calculate the scalar and fermion masses. The calculations are performed by using the approach of [44]. It consists of calculating the mass term for the model with $f$ turned on and repeat the same calculation, but with $f = 0$. The two models, the one with $f \neq 0$ and the one with $f = 0$, have the same interactions, the same field content, but a different spectrum. Since the masses of the fields get corrected only in the non supersymmetric case, the difference between the mass in the non supersymmetric case and the mass in the supersymmetric case coincide with the mass in the non supersymmetric case. Trivially we can write the equation

$$m^{(1)}_{f \neq 0} = m^{(1)}_{f = 0} - m^{(1)}_{f = 0}$$

This trick reduces the number of diagrams necessary to calculate the masses of the pseudomoduli. Indeed the only diagrams that contribute to the mass are the ones depending on $f$ in the non-supersymmetric case.

We first calculate the fermion mass term in the effective Lagrangian of the form

$$\mathcal{L}_{\text{eff}} \supset M_\psi \psi_Y \psi_Y + \text{h.c.}$$

This term arises from the one loop diagrams due to the interactions

$$\mathcal{L} \supset \psi_Y (\psi_5 \phi_1 + \phi_5 \psi_1) + \tilde{\psi}_Y (\psi_6 \phi_2 + \phi_6 \psi_2) + \text{h.c.}$$

The calculation is not immediate, since the mass matrix for the fields $\phi_1$ and $\phi_2$ are not diagonal. We have to diagonalize the bosonic mass matrices for $X \neq 0$ and $Y = \tilde{Y} = 0$. For simplicity we first diagonalize the fermionic squared mass matrix, and then we diagonalize the bosonic one, breaking the holomorphic structure. The eigenvalues of the fermionic mass matrix for the fields $\phi_1, \ldots, \phi_4$ are

$$m^2_F = m_1^2 + X^2 + 2 \mu^2 \pm \sqrt{(m_1^2 - |X|^2)^2 + 4 \mu^2 (m_1^2 + 2 \text{Re}(X) m_1 + |X|^2)}$$

The diagonal combination of the fields in the superpotential are

$$\phi_1 = \cos \tau \rho_1 + \sin \tau \rho_4, \quad \phi_2 = \cos \tau \rho_2 + \sin \tau \rho_3$$
$$\phi_3 = -\sin \tau \rho_2 + \cos \tau \rho_3, \quad \phi_4 = -\sin \tau \rho_1 + \cos \tau \rho_4$$
where

$$\sin^2 \tau = \frac{\mu^2 + m^2 - m_{m}^2}{m^2_{B^*} - m^2_{F^*}}$$  \hspace{1cm} (C.9)$$

and

$$m^2_{\psi(\rho_1)} = m^2_{\psi(\rho_2)} = m^2_{F^*} \quad m^2_{\psi(\rho_3)} = m^2_{\psi(\rho_4)} = m^2_{F^*}$$  \hspace{1cm} (C.10)$$

Since supersymmetry is broken, the complex chiral fields \( \rho_i \) do not have holomorphic masses. It is necessary to find the combinations of these fields that diagonalize the bosonic mass matrix. The eigenvalues of this matrix are

$$m_{B^{2\mu}} = \frac{-\eta f + m_1^2 + X^2 + 2 \mu^2 + \rho \sqrt{(\eta f + m_1^2 - |X|^2)^2 + 4 \mu^2 (m_1^2 + |X|^2 + 2 \text{Re}(X)m_1)}}{2}$$  \hspace{1cm} (C.11)$$

where \( \eta = \pm 1 \) and \( \rho = \pm 1 \). The scalar components that diagonalize the boson mass matrix are

\[
\begin{align*}
\xi_1^A &= -\text{Im}(\rho_1 + \rho_2) \cos \theta + \text{Im}(\rho_3 + \rho_4) \sin \theta, \\
\xi_2^A &= -\text{Re}(\rho_1 - \rho_2) \cos \theta - \text{Re}(\rho_3 - \rho_4) \sin \theta, \\
\xi_1^B &= \text{Im}(\rho_1 + \rho_2) \sin \theta + \text{Im}(\rho_3 + \rho_4) \cos \theta, \\
\xi_2^B &= \text{Re}(\rho_1 - \rho_2) \sin \theta - \text{Re}(\rho_3 - \rho_4) \cos \theta, \\
\xi_1^C &= \text{Im}(\rho_1 - \rho_2) \cos \gamma - \text{Im}(\rho_3 - \rho_4) \sin \gamma, \\
\xi_2^C &= \text{Re}(\rho_1 + \rho_2) \cos \gamma + \text{Re}(\rho_3 + \rho_4) \sin \gamma, \\
\xi_1^D &= -\text{Im}(\rho_1 - \rho_2) \sin \gamma - \text{Im}(\rho_3 - \rho_4) \cos \gamma, \\
\xi_2^D &= -\text{Re}(\rho_1 + \rho_2) \sin \gamma + \text{Re}(\rho_3 + \rho_4) \cos \gamma
\end{align*}
\]

where

$$\sin^2 \theta = \frac{f(m_1^2 - X^2) + (m_f^2 - m_f^2)(m_B^2 + m_B^2 + m_f^2 + m_f^2)}{2(m_B^2 + m_B^2 + m_f^2 + m_f^2)}$$

$$\cos^2 \gamma = \frac{f(m_1^2 - X^2) + (m_f^2 - m_f^2)(m_B^2 + m_B^2 - m_f^2 - m_f^2)}{2(m_B^2 + m_B^2 - m_f^2 - m_f^2)}$$  \hspace{1cm} (C.13)$$

The diagonal masses of these fields are

$$m_A^2 = m_B^{2+} \quad m_B^2 = m_B^{2+} \quad m_C^2 = m_B^{2-} \quad m_D^2 = m_B^{2+}$$  \hspace{1cm} (C.14)$$

We can now evaluate the one loop fermion mass, by using the diagram in figure [10]. From this diagram one computes the function \( I(m_B, m_F) \), which is

$$I(m_B, m_F) = \frac{-m_F}{16\pi^2} \left( \log \frac{A^2}{m_F^2} - \frac{m_B^2}{m_B^2 - m_F^2} \log \frac{m_B^2}{m_F^2} \right)$$  \hspace{1cm} (C.15)$$

and the mass for the fermion is

\[
\begin{align*}
M_{\psi \psi} &= (\cos \theta \cos \tau - \sin \theta \sin \tau)^2 I(m_A, m_2) + (\sin \theta \cos \tau + \cos \theta \sin \tau)^2 I(m_B, m_2) \\
&\quad - (\cos \gamma \cos \tau + \sin \gamma \sin \tau)^2 I(m_C, m_2) - (\sin \gamma \cos \tau - \cos \gamma \sin \tau)^2 I(m_D, m_2)
\end{align*}
\]
Analogously we can evaluate the mass term acquired at one loop by the scalars \(Y\) and \(\tilde{Y}\). The mass term in the Lagrangian is

\[
\mathcal{L} = m^2_{YY^*}|Y|^2 + m^2_{\tilde{Y}Y^*}|	ilde{Y}|^2 + (m_{YY}Y\tilde{Y} + cc)
\]  
(C.16)

The one loop masses acquired by the pseudomodiuli are

\[
m^2_{YY^*} = m^2_{\tilde{Y}Y^*} = \left( (\cos \theta \cos \tau - \sin \theta \sin \tau)^2 (m^2 + X^2) + (\cos \theta \sin \theta + \cos \theta \sin \tau)^2 \mu^2 \right) K(m_A, m_2) + (\cos \theta \to \sin \theta, \sin \theta \to -\cos \theta, m_A \to m_B)
\]
(C.17)

and

\[
m_{YY} = 2mX(\cos \theta \cos \tau - \sin \theta \sin \tau)^2 K(m_A, m_2) + (\cos \theta \to \sin \theta, \sin \theta \to -\cos \theta, m_A \to m_B)
\]
(C.18)

where the functions \(K(m_{B1}, m_{B2})\) and \(J(m_B)\) are associated with the Feynman diagrams of figure 11 and 12. The computation of these diagrams gives

\[
K(m_{B1}, m_{B2}) = \frac{1}{16\pi^2} \left( \frac{\Lambda^2}{m_{B2}^2} - \frac{m_{B1}^2}{m_{B1}^2 - m_{B2}^2} \log \frac{m_{B1}^2}{m_{B2}^2} \right)
\]
(C.19)

\[
J(m_B) = \frac{1}{16\pi^2} \left( \Lambda^2 - m_B^2 \log \frac{\Lambda^2}{m_B^2} \right)
\]
(C.19)

In the \(R\) symmetric limit, \(m_1 \to 0\), the vev of the scalar pseudomodulus \(X\) vanishes. In this limit the masses are

\[
m^2_{YY^*} = m^2_{\tilde{Y}Y^*} = \frac{\mu^2((1 - \nu^2)((1 - \epsilon^2)(\nu^2 - 1 - \epsilon) \log(1 - \epsilon) + (1 + \epsilon^2)(\nu^2 + \epsilon - 1) \log(1 + \epsilon)) - 4\epsilon^2 \nu^4 \log \nu}{32\pi^2(\nu^2 - 1)(\epsilon^2 - (\nu^2 - 1)^2)}
\]  
(C.20)

where we defined \(\epsilon = \frac{f^2}{\mu^2}\) and \(\nu = \frac{m}{\mu}\). At the lowest order in the supersymmetry breaking scale this mass reduces to

\[
m^2_{YY^*} = m^2_{\tilde{Y}Y^*} = \frac{f^2}{32\pi^2\mu^2} M(\nu) = \frac{f^2}{32\pi^2\mu^2} \frac{(1 - 4\nu^2 + 3\nu^4 - 4\nu^4 \log \nu)}{(1 - \nu^2)^3}
\]  
(C.21)

**Figure 10:** The one loop Feynman diagram associated with the function \(I(m_B, m_F)\)
where $M(\nu)$ is a positive function.

**The flavored deformed $L^{131}$ model**

In section [4] we studied a different dark matter sector in addition to the superpotential (C.1). The superpotential for this DM sector is

$$W_3 = Y\phi_1\phi_6 + \tilde{Y}\phi_2\phi_5 + \mu_2(\phi_5\phi_8 + \phi_6\phi_7) + m_2\phi_7\phi_8$$  \hfill (C.22)

In this case the squared mass matrices of the fields $\phi_5, \ldots, \phi_8$ have to be rotated in a diagonal form. This is done by defining the function

$$\sin^2 \alpha = \frac{\mu_2 - \lambda_+}{\lambda_+^2 - \lambda_-^2}$$  \hfill (C.23)

where

$$\lambda_{\pm} = \frac{m_2^2 + 2\mu_2^2 - m\sqrt{m^2 + 4\mu_2^2}}{2}$$  \hfill (C.24)

The diagonal combinations $\rho_1, \ldots, \rho_8$ appearing in $W_3$ are defined by

$$\phi_5 = -\sin \alpha \rho_5 + \cos \alpha \rho_8, \quad \phi_8 = \sin \alpha \rho_5 + \cos \alpha \rho_8$$

$$\phi_6 = -\sin \alpha \rho_6 + \cos \alpha \rho_7, \quad \phi_7 = \cos \alpha \rho_5 + \sin \alpha \rho_8$$  \hfill (C.25)
The fermion mass in this case is

\[
M_{\psi \psi \tilde{\psi}} = (\cos \theta \cos \tau - \sin \theta \sin \tau)^2 (\cos^2 \alpha \cos \mu (m_A, m_f) + \sin^2 \alpha \cos \mu (m_A, m_f)) \\
+ (\sin \theta \cos \tau + \cos \theta \sin \tau)^2 (\cos^2 \beta \cos \mu (m_B, m_f) + \sin^2 \beta \cos \mu (m_B, m_f)) \\
- (\cos \gamma \cos \tau + \sin \gamma \sin \tau)^2 (\cos^2 \beta \cos \mu (m_C, m_f) + \sin^2 \beta \cos \mu (m_C, m_f)) \\
- (\sin \gamma \cos \tau - \cos \gamma \sin \tau)^2 (\cos^2 \beta \cos \mu (m_D, m_f) + \sin^2 \beta \cos \mu (m_D, m_f))
\]  
(C.26)

where

\[
m_{f \pm} = \frac{m_2 \pm \sqrt{m_2^2 + 4\mu_2^2}}{2}  
\]  
(C.27)

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