Symmetry Protected Topological Order and Spin Susceptibility in Superfluid $^3$He-B

Takeshi Mizushima, Masatoshi Sato, and Kazushige Machida

$^1$Department of Physics, Okayama University, Okayama 700-8530, Japan
$^2$The Institute for Solid State Physics, The University of Tokyo, Chiba, 277-8581, Japan

(Dated: May 10, 2014)

We here demonstrate that the superfluid $^3$He-B under a magnetic field in a particular direction stays topological due to a discrete symmetry, that is, in a symmetry protected topological order. Due to the symmetry protected topological order, helical surface Majorana fermions in the B phase remain gapless and their Ising spin character persists. We unveil that the competition between the Zeeman magnetic field and dipole interaction involves anomalous quantum phase transition where topological phase transition takes place together with spontaneous breaking of symmetry. Based on the quasiclassical theory, we illustrate that the phase transition is accompanied by anisotropic quantum criticality of spin susceptibilities on the surface, which is detectable in NMR experiments.

PACS numbers: 67.60.H-, 03.65.Vf, 74.20.Rp, 67.30.er

Introduction.— Superfluid $^3$He-B is one of the most concrete examples of time-reversal invariant topological superfluids (TSFs) $^1$ $^2$, where the ground state wave function supports a nontrivial bulk topological invariant in three spatial dimensions $^3$ $^8$. As a consequence of the bulk-edge correspondence, helical Majorana fermions live on its specular surface, and their self conjugate property gives rise to an Ising-like anisotropy of spin susceptibility $^9$ $^13$. Recently the surface Majorana cone has been detected in experiments $^17$.

Since the topological superfluidity in $^3$He-B, which is categorized to class DIII $^3$, is ensured by time-reversal invariance, it is sometimes stated that any time-reversal breaking such as a finite magnetic field immediately wipes out the topological nature. Indeed, in the presence of a strong magnetic field, the Majorana Ising spin ceases to exist $^18$. However, as argued in the Letter, a more careful consideration on the basis of microscopic calculations and symmetry of the system points to a different conclusion. It is worth mentioning that the robustness of a topological phase transition for a topological insulator against the time-reversal breaking is proposed in an extended Kane-Mele model $^19$.

In this Letter, we show that $^3$He-B under a magnetic field in a particular direction stays topological as a symmetry protected topological order. In spite of the time-reversal breaking due to the magnetic field, the topological property is retained by a hidden $Z_2$ symmetry that is obtained by a combination of time-reversal and an SO(3)$_L \times$SO(3)$_S \times$U(1) symmetry $^22$. The gap function is $\Delta(\mathbf{k}, r) = i\sigma \cdot d(\mathbf{k}, r) \sigma_y$, where $d_\mu = d_{\mu \nu} \hat{k}_\nu$ and,

$$d_{\mu \nu}(r) = e^{i \rho} R_{\mu \nu}(\hat{n}, \varphi) \Delta_\nu(r). \quad (1)$$

The broken symmetry SO(3)$_L \times$SO(3)$_S$, the relative rotation between spin and orbital spaces, is described by $R_{\mu \eta}(\hat{n}, \varphi)$ with the rotation axis $\hat{n}$ and the angle $\varphi$.

The phase transition is described by a pair of an order parameter $\ell_z$ and a topological number $w$, which are defined in Fig. 1(b) and in the text below Eq. (4). Conventionally, topological phase transitions are accompanied by creation or destruction of gapless surface states and gap closing of bulk quasi-particle spectra. While the anomalous quantum phase transition in the above involves the destruction of gapless surface states, it does not show the bulk gap closing. Instead, there appears a long range order due to symmetry breaking. Moreover, it takes place between two conceptional different quantum orders: At a critical magnetic field, the system undergoes a transition from topologically ordered state ($w \neq 0$) to conventionally ordered one ($\ell_z \neq 0$).

The symmetry protected topological order is closely associated with the order parameter manifold of $^3$He-B. Ignoring the dipole interaction and a Zeeman field, the bulk $^3$He-B spontaneously reduces the symmetry SO(3)$_L \times$SO(3)$_S \times$U(1) to SO(3)$_L + S$ $^22$. The gap function is $\Delta(\mathbf{k}, r) = i\sigma \cdot d(\mathbf{k}, r) \sigma_y$, where $d_\mu = d_{\mu \nu} \hat{k}_\nu$ and,

$$d_{\mu \nu}(r) = e^{i \rho} R_{\mu \nu}(\hat{n}, \varphi) \Delta_\nu(r). \quad (1)$$

The broken symmetry SO(3)$_L \times$SO(3)$_S$, the relative rotation between spin and orbital spaces, is described by $R_{\mu \eta}(\hat{n}, \varphi)$ with the rotation axis $\hat{n}$ and the angle $\varphi$.

The phase transition is described by a pair of an order parameter $\ell_z$ and a topological number $w$, which are defined in Fig. 1(b) and in the text below Eq. (4). Conventionally, topological phase transitions are accompanied by creation or destruction of gapless surface states and gap closing of bulk quasi-particle spectra. While the anomalous quantum phase transition in the above involves the destruction of gapless surface states, it does not show the bulk gap closing. Instead, there appears a long range order due to symmetry breaking. Moreover, it takes place between two conceptional different quantum orders: At a critical magnetic field, the system undergoes a transition from topologically ordered state ($w \neq 0$) to conventionally ordered one ($\ell_z \neq 0$).

The symmetry protected topological order is closely associated with the order parameter manifold of $^3$He-B. Ignoring the dipole interaction and a Zeeman field, the bulk $^3$He-B spontaneously reduces the symmetry SO(3)$_L \times$SO(3)$_S \times$U(1) to SO(3)$_L + S$ $^22$. The gap function is $\Delta(\mathbf{k}, r) = i\sigma \cdot d(\mathbf{k}, r) \sigma_y$, where $d_\mu = d_{\mu \nu} \hat{k}_\nu$ and,

$$d_{\mu \nu}(r) = e^{i \rho} R_{\mu \nu}(\hat{n}, \varphi) \Delta_\nu(r). \quad (1)$$

The broken symmetry SO(3)$_L \times$SO(3)$_S$, the relative rotation between spin and orbital spaces, is described by $R_{\mu \eta}(\hat{n}, \varphi)$ with the rotation axis $\hat{n}$ and the angle $\varphi$.

The phase transition is described by a pair of an order parameter $\ell_z$ and a topological number $w$, which are defined in Fig. 1(b) and in the text below Eq. (4). Conventionally, topological phase transitions are accompanied by creation or destruction of gapless surface states and gap closing of bulk quasi-particle spectra. While the anomalous quantum phase transition in the above involves the destruction of gapless surface states, it does not show the bulk gap closing. Instead, there appears a long range order due to symmetry breaking. Moreover, it takes place between two conceptional different quantum orders: At a critical magnetic field, the system undergoes a transition from topologically ordered state ($w \neq 0$) to conventionally ordered one ($\ell_z \neq 0$).

The symmetry protected topological order is closely associated with the order parameter manifold of $^3$He-B. Ignoring the dipole interaction and a Zeeman field, the bulk $^3$He-B spontaneously reduces the symmetry SO(3)$_L \times$SO(3)$_S \times$U(1) to SO(3)$_L + S$ $^22$. The gap function is $\Delta(\mathbf{k}, r) = i\sigma \cdot d(\mathbf{k}, r) \sigma_y$, where $d_\mu = d_{\mu \nu} \hat{k}_\nu$ and,

$$d_{\mu \nu}(r) = e^{i \rho} R_{\mu \nu}(\hat{n}, \varphi) \Delta_\nu(r). \quad (1)$$

The broken symmetry SO(3)$_L \times$SO(3)$_S$, the relative rotation between spin and orbital spaces, is described by $R_{\mu \eta}(\hat{n}, \varphi)$ with the rotation axis $\hat{n}$ and the angle $\varphi$.  

FIG. 1: (Color online) (a) Schematic phase diagram of $^3$He-B under a parallel magnetic field, where $H^*$ involves the topological phase transition with spontaneous symmetry breaking. (b) Relation between $\ell_z$ and the orientation of $d(0, 0, \ell_z)$ for an arbitrary ($\hat{n}, \varphi$) at the surface.
The state (SABS) is given by

\[ E\mu = \frac{1}{2} \mu_{\text{He}} \delta(\sigma_{\mu}, -\sigma_{\mu}) \],

where \( \mu \) is the mass of He atoms. The pair potential for \(^3\text{He}\) atoms is given by \( U_{12}(r) = \frac{k_{\text{B}}^2}{2M} \delta_{\mu\nu} \Theta(r_1 - r_2) \) with \( M, E_F = k_{\text{B}}^2 / 2M \), and \( \mu_{\text{He}} \) is time-reversal with complex conjugation. In this case, the flipped magnetic field by time-reversal \( T \) is recovered by the \( \pi \)-rotation in the \( xy \)-plane. Therefore, the microscopic Hamiltonian of \(^3\text{He}\) atoms is invariant under the discrete symmetry given by \( TU(\pi) \). We notice here that \( \hat{\ell}_z \) mentioned above is transformed nontrivially as \( \hat{\ell}_z \rightarrow -\hat{\ell}_z \) under this symmetry. Therefore \( \hat{\ell}_z \) is an order parameter of the discrete symmetry, and it should be zero unless the discrete symmetry is spontaneously broken.

Remarkably, one can introduce a topological invariant if the discrete symmetry is not spontaneously broken. In that case, the BdG Hamiltonian (2) in the momentum space is manifestly invariant under the discrete symmetry, \( \mathcal{H}(k_x, k_y, -k_z) = TU(\pi) \mathcal{H}(k_x, k_y, k_z)U^{-1}(\pi) T^{-1} \), where \( T = i\sigma_z K \) is time-reversal with complex conjugate operator \( K \) and \( U(\pi) = i\sigma_z \tau_z \) is the \( \pi \)-rotation. Therefore, combining it with the particle-hole symmetry of the BdG Hamiltonian, \( \mathcal{H}(k) | \mathcal{C} | = -\mathcal{H}(-k) \) with \( \mathcal{C} = \mathcal{U} K \), one obtains the relation \( \mathcal{H}(k_x, k_y, k_z) \mathcal{C}^{-1} = -\mathcal{H}(-k_x, -k_y, k_z) \). On the \( k_z \) axis, this reduces to the so-called chiral symmetry

\[ \{ \mathcal{H}, \mathcal{H}(0, 0, k_z) \} = 0. \]

Thus, following Refs. [20, 21], one can introduce the following one-dimensional (1D) winding number \( w = -\frac{1}{4\pi} \int_{-\infty}^{\infty} dk_z \text{tr} \left[ \mathcal{H}^{-1} \partial_{k_z} \mathcal{H} \right] \) which are evaluated as \( w = 2 \) for \( \mu_{\text{He}} < E_F \) (\( \Delta_{\perp} > 0 \)). Therefore, the system is topologically non-trivial, and the bulk-edge correspondence implies that the SABS satisfies \( E(k_1) = 0 \) at \( k_1 = 0 \) even in the presence of the magnetic field. It should be noted here that one needs the discrete symmetry specific to this system in order to define \( w \). Therefore a topological phase realized here is a symmetry protected topological order [20, 21].
Majorana Ising Spin.— In the absence of the magnetic field, it has been known that helical Majorana fermions in 3He-B have Ising-like spin density \( \hat{\tau} \). Now we show that the symmetry protected topological order discussed above retains the Ising spin character of Majorana fermions.

To see this, we use a general symmetry property of the SABS. Let us consider the low energy limit where only the zero energy SABSs at \( \mathbf{k} || 0 \) contribute. According to the index theorem of Ref. [27], Eq. (4) infers that the zero energy SABSs are eigenstates of \( \Gamma \), and the relation \( w = n_- - n_+ \) holds on the surface of superfluid 3He-B, where \( n_\pm \) is the number of the zero energy SABSs with the eigenvalue \( \Gamma = \pm 1 \). In the present case, \( w = n_- = 2 \) and \( n_+ = 0 \), and thus the SABS \( \varphi_{k_\parallel=0} \) satisfies \( \Gamma \varphi_{k_\parallel=0} = -\varphi_{k_\parallel=0} \) (\( a = 1, 2 \)). Using the particle-hole symmetry, one can also put the relation \( \tau_a \varphi_{k\parallel=0} = \varphi_{k\parallel=0} \) at the same time. From these two relations, \( \varphi_{k\parallel=0} \) has a generic form as \( \varphi_{k\parallel=0} = [\varphi_{e}^a \varphi_{e}^a] \) with a function \( \varphi_e \). Ignoring non-zero energy modes, the quantized field \( \Psi = [\psi_1, \psi_1^\dagger, \psi_2, \psi_2^\dagger]^T \) is expanded as \( \Psi(z) = \sum_{a=1,2} \varphi_{k=0}^a(z \psi_e^a) \) with real \( \psi_e \), and from the general form of \( \varphi_{k\parallel=0} \), one obtains \( \hat{\psi}_e = -\hat{\psi}_e^\dagger \), which is a general consequence of our symmetry protected topological order.

Now following Ref. [4, 10], one can show that the last relation, \( \hat{\psi}_e = -\hat{\psi}_e^\dagger \), yields the Ising character of the SABSs: It is shown that among the local density operator and the spin density operators, which are given by \( \rho = \frac{1}{2}[\hat{\psi}_e^\dagger \hat{\psi}_e - \hat{\psi}_e \hat{\psi}_e^\dagger] \) and \( S_\mu = \frac{1}{4}[\hat{\psi}_e^\dagger (\sigma_\mu)_{ab} \hat{\psi}_e - \hat{\psi}_e (\sigma_\mu)^{ab}_{cd} \hat{\psi}_e^\dagger] \), respectively, only \( S_z \) is nonzero while the other components are identically zero. So, in the low energy limit, the SABSs do not contribute to the local density fluctuation, and its local spin density is Ising-like. Here note that we only use a general property of the chiral symmetry, thus the Ising character is a direct consequence of our symmetry protected topological order.

Quasiclassical Eilenberger theory.— Let us now microscopically determine \( \ell_z \) and \( H^* \). For this purpose, we here utilize the quasiclassical Eilenberger theory, which provides a quantitative theory for superfluid 3He at low pressures [28]. This is based on the quasiclassical Green’s functions \( \tilde{g} \) with the Matsubara frequency \( \omega_n = (2n + 1)\pi T \) (\( n \in \mathbb{Z} \)) and the 2x2 unit matrix \( \sigma_0 \)

\[
\tilde{g} = \left[ \begin{array}{cc} \sigma_0 g_{00} + i \sigma_\mu g_{0\mu} & i \sigma_\mu g_{0\mu} + i \sigma_\mu g_{0\mu} \\ i \sigma_\mu g_{\mu0} + i \sigma_\mu g_{\mu0} & \sigma_0 g_{\mu\mu} + \sigma_0 g_{\mu\mu} \end{array} \right].
\]

The evolution is governed by the Eilenberger equation \( \hat{\omega}_n \hat{S} = \hat{S} \) with the self-energies, \( \Sigma_k \).

where \( \nu_\mu \) denotes the Fermi liquid corrections obtained as \( \nu_\mu(k, r) = \sum_\ell A_\ell^\mu(P_\ell(k, r))g_\ell(k, r; i\omega_n)\). The Eilenberger and gap equations with Eq. (1) provide self-consistent equations for \( \nu \) under a fixed \( \vec{\xi} \).

The Eilenberger equation with \( \tilde{g} = -\pi^2 T \Sigma \) is numerically solved with the Riccati parameterization in the system that two specular walls normal to \( \vec{x} \) are situated at \( z = 0 \) and \( z = 20 \xi \). The numerical procedure is same as that in Refs. [14, 38]. We solve the gap equation with \( |\Delta|^2 = \pi T c_{\xi} \omega_0 |\Delta| \) and \( \Delta \omega / \Delta^2 = 2 \times 10^{-4} \) (2x10^-5), where we set the cutoff \( E_c = 20 T c_{\xi} \). Note that since the ratio \( \Delta / \omega^2 \) is associated with the distortion of \( \Delta \) in the thermodynamic limit [32], it is independent of the energy cutoff. The coherence length \( \eta \equiv v_F / T c_{\xi} \) is estimated about 80nm at the zero pressure of 3He-B and \( T c_{\xi} \approx 1mK \) is the critical temperature of the bulk B-phase under \( H = 0 \). In a slab geometry, \( \vec{n} \) may be assumed to be spatially uniform, because the length scale of the spatial variation of \( \vec{n} \) is macroscopically large [22], compared with the thickness of the sample 20\xi \approx 1.6\mu m in typical experiments [34, 35]. Then, the stable configuration of \( \hat{\nu} \) is determined by minimizing the thermodynamic potential \( \Omega(\hat{\nu}) \) whose explicit form is same as that given by Vorontsov and Sauls in Ref. [30]. Note that for the thickness 20\xi, the magnetic field induces the first-order phase transition from B-phase to A- or planar phase at the critical field \( H_{AB} = 0.09 T c_{\xi} / \mu_n \approx 3.6 kG \) [37].
Numerical results. — Since the \( \mathbf{n} \)-vector always points to \( \mathbf{z} \) in the case of the perpendicular field \( \mathbf{H} \parallel \mathbf{z} \), we here consider a parallel field \( \mathbf{H} \parallel \mathbf{x} \). Figures 2a and 2b describe the energy landscape on the unit sphere of \( \mathbf{n} \). Figure 2c depicts the energy gap of the SABS evaluated from Eq. (3). The SABS becomes gapless along the certain trajectory on the sphere of \( \mathbf{n} \), which coincides with the condition of \( \ell_z = 0 \). The stable configuration of \( \mathbf{n} \) is determined as a consequence of the interplay between dipole interaction and Zeeman energy. The former favors the situation where \( \mathbf{n} \) is normal to the surface, namely, \( \ell_z = 0 \), while the condition for minimizing the Zeeman energy (2a), \( \ell_z = 1 \), is same as the condition that opens the maximum energy gap in the SABS. Hence, in the magnetic field lower than the dipolar field \( H_D \approx 30G \sim 0.001\pi T_{c0}/\mu_n \), as displayed in Fig. 2a, \( \mathbf{n} \) points to \( \mathbf{z} \). It is seen in Fig. 2b that for the larger \( H \)'s it tends to tilt from \( \mathbf{z} \) to the direction with \( \ell_z \neq 0 \).

The field dependence of \( \ell_z \) estimated with the stable configuration of \( \langle \mathbf{n}, \varphi \rangle \) is displayed in Fig. 2d. In the limit of the low field, \( \ell_z \) is locked to be \( \ell_z = 0 \), which ensures the existence of surface Majorana fermions. \( \ell_z \) stays zero up to the critical value \( \mu_nH/\pi T_{c0} \approx 0.001 \), which is consistent with the argument that the systems with \( \ell_z = 0 \) has the discrete symmetry. At \( H \gtrsim H^* \), the symmetry protected topological phase with \( \ell_z = 0 \) undergoes a change to non-topological phase with \( \ell_z \neq 0 \).

In Fig. 3a) we plot the field dependence of the local spin susceptibility on the surface, \( \chi_{\mu\nu}(z) = 0 \), defined as \( \chi_{\mu\nu}(z)/\chi_N \equiv M_\mu(z)/M_N \) for a magnetic field \( \mathbf{H} \parallel \mathbf{r}_F \). The local magnetization \( M_\mu(z) \) is estimated as \( M_\mu(r) = M_N[\hat{h}_\mu + \frac{1}{\mu_nH}(g_\mu(\hat{k}, r; i\omega_n))]_{k, \omega_m} \) with that in the normal state \( M_N = \frac{2\gamma^2}{1+\kappa_0^2}N_FH \), where \( N_F \) is the density of states of the normal \( ^3\text{He} \). It is seen from Fig. 3a) with the solid line that for \( \mathbf{H} \parallel \mathbf{z} \), the local spin susceptibility on the surface, \( \chi_{zz}(0) \), is considerably enhanced, compared with \( \chi_{zz}(z=10\xi) \) (the dashed line) \( \hat{\chi} \).

In contrast, when the parallel field \( \mathbf{H} \parallel \mathbf{x} \) is applied, the magnetization \( M_\mu(z) \) on the surface is sensitive to the orientation of \( \ell \). It is seen in Fig. 3b) with the dashed line that \( M_\mu(z) \) at \( \mu_nH/\pi T_{c0} = 9.2 \times 10^{-4} \) is strongly suppressed in the surface region, where \( \mathbf{n} \parallel \mathbf{z} \), that is \( \ell_z = 0 \), is energetically favored. This implies that the SABS does not contribute to \( M_\mu(z) \) on the surface and is consistent with the property of the Majorana Ising spins.

In the relatively high field \( \mu_nH/\pi T_{c0} = 0.0018 \), however, \( M_\mu(z) \) is enhanced around the surface, while \( M_\mu(z) \) which is perpendicular to \( \mathbf{H} \parallel \mathbf{x} \) emerges on the surface. This emergence of \( M_\mu(z) \) on the surface reflects the stable configuration of \( \langle \mathbf{n}, \varphi \rangle \), where \( \ell_z = 0 \), \( \mathbf{n} \parallel \mathbf{z} \), deviates from zero and is less than unity. As displayed in Figs. 3a) and 3b), the magnetic field within the range of \( 0 < \ell_z < 1 \) significantly induces \( M_\mu(z) \) and \( \hat{\chi}_{xx}(z) \) on the surface, where the SABS opens the finite energy gap and the winding number \( w \) is not defined.

Conclusions. — Here, we have clarified that the interplay of dipole interaction and a magnetic field in \(^3\text{He-B}\) involves a new class of quantum phase transition, that is, the topological phase transition with the spontaneous breaking of the hidden \( Z_2 \) symmetry. Using the quasi-classical theory, we have demonstrated that \(^3\text{He-B}\) stays topological as the symmetry protected topological order and Majorana Ising spins exist unless the \( Z_2 \) symmetry is spontaneously broken at the critical field \( H^* \). The quantum phase transition is accompanied by the anomalous behavior of spin susceptibilities, which is observable through NMR experiments in a slab geometry.

The authors are grateful to S. Higashitani, M. Ichioda, O. Ishikawa, R. Nomura, J. Saunders, R. Shindou, Y. Okuda, and Y. Tsutsumi for fruitful discussions and comments. This work was supported by JSPS.

FIG. 2: (Color online) Energy landscape on the unit sphere of \( \mathbf{n}, \delta\Omega(\mathbf{n}) \), at \( \mu_nH/\pi T_{c0} = 9.2 \times 10^{-4} \) (a) and 0.0061 (b) where we fix \( \varphi/\pi = -0.5537 \) which minimizes the dipole interaction. We also set \( \mathbf{H} \parallel \mathbf{x} \) and \( T/T_{c0} = 0.2 \). The bright (dark) color depicts the higher (lower) energy. The energy gap \( \min \delta\Omega \) is seen in Fig. 2(b) that for the larger \( \mu_nH/\pi T_{c0} \), that is \( \ell_z < 1 \) significantly induces \( M_\mu(z) \) and \( \chi_{xx}(z) \) on the surface, where the SABS opens the finite energy gap and the winding number \( w \) is not defined.

FIG. 3: (Color online) (a) Field dependence of \( \chi_{\mu\nu}(z)/\chi_N \) at \( T = 0.2T_{c0} \). The solid (dashed) lines denote \( \chi_{xx}(0) \) (\( \chi_{xx}(10\xi) \)) for \( \mathbf{H} \parallel \mathbf{z} \) and the symbols correspond to \( \chi_{\mu\nu}(0) \) for \( \mathbf{H} \parallel \mathbf{x} \). (b) \( M_\mu(z) \) for \( \mathbf{H} \parallel \mathbf{z} \) at \( \mu_nH/\pi T_{c0} = 9.2 \times 10^{-4} \) (dashed line) and 0.0018 (solid lines), where \( M_\mu(z) \) at \( \mu_nH/\pi T_{c0} = 9.2 \times 10^{-4} \) are zero. All data are taken with \( \Lambda_D/\Lambda^2 = 2 \times 10^{-4} \).
(No. 2074023303, 2134010303 and 22540383) and the MEXT KAKENHI (No. 22103002 and No. 22103005).

∗ Electronic address: mizushima@mp.okayama-u.ac.jp
† Electronic address: msato@nuap.nagoya-u.ac.jp
Permanent address: Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan

[1] Y. Tanaka, M. Sato, and N. Nagaosa, J. Phys. Soc. Jpn. 81, 011013 (2012).
[2] X.L. Qi and S.C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[3] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B 78, 195125 (2008).
[4] X.L. Qi, T.L. Hughes, S. Raghu, and S.C. Zhang, Phys. Rev. Lett. 102, 187001 (2009).
[5] M. Sato, Phys. Rev. B 79, 214526 (2009).
[6] M. Sato, Phys. Rev. B 81, 220504(R) (2010).
[7] A. Kitaev, AIP Conf. Proc. 1134, 22 (2009).
[8] G. E. Volovik, JETP Lett. 90, 587 (2009).
[9] M. Stone and R. Roy, Phys. Rev. B 69, 184511 (2004).
[10] S.-B. Chung and S.-C. Zhang, Phys. Rev. Lett. 103, 235301 (2009).
[11] Y. Nagato, S. Higashitani, and K. Nagai, J. Phys. Soc. Jpn. 78, 123603 (2009).
[12] G. E. Volovik, JETP Lett. 90, 398 (2009).
[13] R. Shinhou, A. Furusaki, and N. Nagaosa, Phys. Rev. B 82, 180505(R) (2010).
[14] Y. Tsutsuki, T. Mizushima, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. 79, 113601 (2010).
[15] T. Mizushima and K. Machida, J. Low Temp. Phys. 162, 204 (2011).
[16] M. A. Silaev, Phys. Rev. B 84, 144508 (2011).
[17] S. Murakawa, Y. Wada, Y. Tamura, M. Wasai, M. Saitoh, Y. Aoki, R. Nomura, Y. Okuda, Y. Nagato, M. Yamamoto, S. Higashitani, and K. Nagai, J. Phys. Soc. Jpn. 80, 013602 (2011) and references therein.
[18] G. E. Volovik, JETP Lett. 91, 201 (2010).
[19] Z. Ringel and E. Altman, arXiv:1207.0581.
[20] Z.-C. Gu and X.-G. Wen, Phys. Rev. B 80, 155131 (2009).
[21] F. Pollmann, E. Berg, A. M. Turner, and M. Oshikawa, Phys. Rev. B 85, 075125 (2012).
[22] D. Vollhardt and P. Wölfle, The Superfluid Phases of Helium-3 (Taylor and Francis, London, 1990).
[23] W. F. Brinkman, H. Smith, D. D. Osheroff, and E. I. Blount, Phys. Rev. Lett. 33, 624 (1974).
[24] A. I. Ahonen, M. Kruisius, and M. A. Paalainen, J. Low Temp. Phys. 25, 421 (1976).
[25] O. Ishikawa, Y. Sasaki, T. Mizusaki, A. Hirai, and M. Tsubota, J. Low Temp. Phys. 75, 35 (1989).
[26] M. Sato and S. Fujimoto, Phys. Rev. B 79, 094504 (2009).
[27] M. Sato, Y. Tanaka, K. Yada, and T. Yokoyama, Phys. Rev. B 83, 224511 (2011).
[28] J.W. Serene and D. Rainer, Phys. Rep. 101, 221 (1983).
[29] A. J. Leggett, J. Phys. C 6, 3187 (1973); Ann. Phys. (N.Y.) 85, 11 (1974).
[30] L. Tewordt and D. Einzel, Phys. Lett. 56A, 97 (1976).
[31] N. Schopohl, J. Low Temp. Phys. 49, 347 (1982).
[32] R. S. Fishman, Phys. Rev. B 36, 79 (1987).
[33] Y. Tsutsuki, M. Ichioka, and K. Machida, Phys. Rev. B 83, 094510 (2011).
[34] S. Miyawaki, K. Kawasaki, H. Inaba, A. Matsubara, O. Ishikawa, T. Hata, and T. Kodama, Phys. Rev. B 62, 5855 (2000).
[35] R.G. Bennett, L.V. Levitin, A. Casey, B. Cowan, J. Parpia, and J. Saunders, J. Low Temp. Phys. 158, 163 (2010).
[36] A.B. Vorontsov and J.A. Sauls, Phys. Rev. B 68, 064508 (2003).
[37] T. Mizushima, arXiv:1208.1318.