Abstract

We investigated the bubble collisions during the first order phase transitions. Numerical results indicate that within the certain range of parameters the collision of two bubbles leads to formation of separate relatively long-lived quasilumps - configurations filled with scalar field oscillating around the true vacuum state. Energy is perfectly localized, and density is slightly pulsating around its maximum. This process is accompanied by radiation of scalar waves.

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PACS 04.70.Bw; 04.20.Dw

1 Introduction

Analysis of physical processes predicted in the early Universe on the basis of particle theory, is the important way to study physical conditions in the early Universe and physical mechanisms underlying those conditions. As a result of such an analysis, the existence of hypothetical relics of early Universe, such as primordial black holes, topologically stable or metastable solitons etc, have been predicted. Confrontation of predicted effects with observational data provides certain conclusions concerning both cosmological evolution and particle physics models.

First order phase transitions as predicted by unified theories can occur at several periods of cosmological evolution. They are predicted on the basis of a wide class of models of particle symmetry breaking. They are also considered as the final stage of inflation in a wide class of inflationary models. Detailed study of nonlinear configurations arising at the first order phase transitions and
their dynamics is helpful not only for cosmology - nonlinear dynamics of field theories describes a wide range of phenomena occurring in laboratory physics.

The transition with symmetry breaking consists in decay of a metastable phase by nucleation of bubbles of new phase \( \text{[3]} \). A nucleated bubble is a true vacuum fluctuation large enough to evolve classically. The most likely fluctuation is a spherical bubble nucleated at rest with a certain critical size determined by microphysical processes \( \text{[3]} \). Coleman \( \text{[3]} \) calculated the bubble nucleation rate in flat space and at zero temperature using the euclidean path-integral formulation of a scalar field theory. The nucleation rate in this case is proportional to \( e^{-S_E} \) where \( S_E \) is the euclidean action and solution to the euclidean equation of motion for minimal action is the \( O(4) \) symmetric "bounce" solution.

In the very early Universe phase transitions most likely occur at a finite temperature due to the fact that the form of a scalar field potential becomes temperature-dependent when quantum corrections are taken into account \( \text{[5]} \). Generalization of Coleman results to the case of nonzero temperature is based on the remarkable fact that quantum statistics at nonzero temperature is formally equivalent to quantum field theory in the euclidean space, which is periodic in time coordinate with the period \( T^{-1} \). As a result, most likely fluctuations appear to be not \( O(4) \) symmetric spherical bubbles but \( O(3) \) symmetric (with respect to spatial coordinates) cylindric configurations with certain critical size slightly different from \( O(4) \) symmetric case \( \text{[4, 5]} \).

For bubble created with a size smaller than the critical one, it could seem that the gain in volume energy cannot compensate for the loss in surface energy and such the bubbles would have to quickly shrink to nothing. However, detailed analysis discovered that even in this case effects of nonlinearity lead to nontrivial dynamics. The evolution of subcritical bubbles - unstable spherically symmetric solutions of nonlinear Klein-Gordon equation - was, firstly, studied numerically by Bogolubsky and Makhankov \( \text{[6]} \). Using a quasiplanar initial configuration for the bubbles, they found that for a certain range of initial radii, the bubble, after radiating most of its initial energy, settled into long-lived (as compared with characteristic time-scale) stage and only then disappeared by quick radiating their remaining energy. Those configurations called "pulsons" were later rediscovered and revised by Gleiser who found that their most characteristic feature is not pulsating mechanism for radiating the initial energy, but the rapid oscillations of the amplitude of a scalar field during long-lived pseudo-stable regime when almost no energy was radiated away and radial pulsations were rather small \( \text{[7]} \). It was shown \( \text{[7, 8]} \) that those configurations called "oscillons" exist for symmetric and asymmetric double-well potentials, are stable against small radial perturbations, and have lifetimes "far exceeding naive expectations" \( \text{[8]} \).

Although it is well known, that three-dimensional nontrivial configurations of a scalar field are unstable, they can be relevant for systems with short dynamical time-scales. Detailed study of unstable but long-lived configurations can clarify dynamics of nonlinearities in field theories and their role in a wide
class of phenomena ranging from nonlinear optics to phase transitions both in the Universe and in the laboratory. 

For the bubbles formed with the radii large enough (overcritical bubbles) it is classically energetically favorable to grow. The newly formed bubble of true vacuum is separated from the surrounding false vacuum region by the wall at rest. Immediately after nucleation, the wall starts to accelerate outwards absorbing energy stored in false vacuum region and converting difference of false and true vacuum energy density into kinetic energy of the wall. That way a bubble spreads off converting false vacuum into the true one. This process continues up to the collision with a spherical wall of another bubble.

In the first-order phase transitions at the end of inflation the collision of bubbles is considered as the leading mechanism of reheating by converting the wall energy into radiation. However situation with two bubbles appears much more complicated. Even nucleation of two bubbles is not yet studied in the literature in general. Only in the case when bubbles are widely separated at the time of nucleation and thus can be treated as noninteracting (at the stage of nucleation) the generalization of a single bubble solution is straightforward.

Two bubble collisions were studied in detail by Hawking, Moss, and Stewart and then by Watkins and Widrow, in elegant approach using symmetry of the problem in zero temperature case. For zero temperature bubbles produced by quantum tunneling, initial state is $O(4)$ symmetric, as well as euclidean equation of motion, in natural assumption that a scalar field $\phi$ is invariant under 4-dimensional Euclidean rotations. In analytical continuation to Minkowski space this becomes $O(3,1)$ symmetry. For two bubbles, the line joining their centers is the preferred axis and solution to the euclidean equation of motion is found in the class $O(3)$ ($O(2,1)$ as continued to the Minkowski space) solutions and field configuration arising in collision belongs to the class of $O(3)$ symmetric solutions.

The aim of present paper is investigation of two-bubble collision in the case of finite temperature. We are interested not in reheating by two bubble collisions but in evolution of two bubble configuration during and after collision. The similar subject was discussed several years ago. As was noted by Hawking, Moss and Stewart and confirmed by Watkins and Widrow, collision of two domain walls does not lead to immediate conversion of the wall energy into a burst of radiation. Two walls reflect off one another and move apart creating a new region of false vacuum between them. Our aim was to investigate an evolution of this new false vacuum region to look if it can form a separated object. We connected with such a possibility the hope of formation of metastable relics of the first order phase transitions such as primordial black holes or selfgravitating particle-like structures with de Sitter-like cores.

To our surprise, a false vacuum configuration evolved into a compact quasilump filled with an oscillating scalar field. The fundamental difference of this object from an oscillon is that it arises dynamically as the result of bubble collisions (which increases probability of its production) and that it is made up from an
oscillating scalar field at the background of true vacuum. We call it quasilump, since it does not satisfy all requirements for lumps as defined by S. Coleman: "Non-singular, non-dissipative solutions of finite energy, lumps of energy holding themselves together by their own self-interactions" [14]. In our numerical simulations we observe non-singular configurations of self-interacting scalar field with asymmetric potential, perfectly localized, but we cannot say that they are non-dissipative, although they are rather long-lived as compared with the characteristic scale for the first order phase transitions.

Our paper is organized as follows. In Sect. 2 we present basic equations and initial configuration. In Sect. 3 we give qualitative analysis of the process of collision to find an optimal range of parameters for which concentration of false vacuum energy (which is later transformed into the energy of oscillations) is maximal. In Sect 4 we present the results of numerical simulations. Sect 5 contains summary and discussion.

2 Basic equations

To study mechanism of formation and evolution of false vacuum regions, we shall consider most favorable regime for their appearance which corresponds to high nucleation rate \( NH^4 \gg 1 \) where \( N \) is the nucleation rate per unit four-volume and \( H \) is the Hubble parameter[10]. We also neglect gravity effects on the process of bubble formation and growth which means that we consider bubbles with the initial size much less than cosmological horizon, \( R(0)H \ll 1 \) [3, 9].

We consider real scalar field \( \phi \) with the Lagrangian

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi).
\]

(1)

This is the effective Lagrangian for a large number of more complex models of Universe involving the first order phase transitions (see [6] for more details). In the "thin wall" approximation, \( \epsilon/\lambda << 1 \), some analytical results are known [6], and we will work in the frame of this approximation.

To compare our results with the results obtained by Hawking, Moss and Stewart, and by Watkins and Widrow, we choose asymmetric double-well potential of the same form

\[
V(\phi) = \frac{1}{8} \lambda (\phi^2 - \phi_0^2)^2 + \epsilon \phi_0^3 (\phi + \phi_0).
\]

(2)

At \( T = 0 \) the parameters \( \lambda, \phi_0 \) and \( \epsilon \) are specified by the particle model. At nonzero temperature they are influenced by temperature corrections. In the case of high nucleation rate, a first order phase transition is a quick process and we can consider parameters as \( \lambda \approx \lambda(T_c), \phi_0 \approx \phi_0(T_c), \epsilon \approx \epsilon(T_c) \), i.e. being constant during the phase transition at the temperature \( T = T_c \).
The potential \([2]\) has two minima at different values of field \(\phi\). False vacuum (metastable) state is characterized by the field \(\phi = \phi_0(1 - \epsilon/\lambda - 3/2(\epsilon/\lambda)^2)\), whereas the global minimum of the potential \(V(\phi)\) represents the true vacuum state \(\phi = -\phi_0(1 - \epsilon/\lambda + 3/2(\epsilon/\lambda)^2)\).

In our analysis we assume that both mechanisms of the false vacuum decay could take place - tunneling, that is creation of \(O(4)\) symmetrical bubbles, and formation of \(O(3)\) symmetrical bubbles due to temperature fluctuation. Evidently, if the temperature is small enough, the tunneling mechanism of the false vacuum decay dominate. On the contrary, at large temperatures the decay is realized by the nucleation and growth of the \(O(3)\) symmetrical bubbles.

Consider conditions of dominance of the false vacuum decay due to temperature effects. The temperature decay probability was found in \([4, 5]\):

\[
P_{\text{Temp}} \propto e^{-S_3/T},
\]

where \(T\) is the temperature of a phase transition and \(S_3\) is three-dimensional action for \(O(3)\) symmetrical bubble. The probability of the vacuum decay due to tunneling, is given by

\[
P_{\text{tun}} \propto e^{-S_4}
\]

where \(S_4\) is the action for \(O(4)\) symmetrical bubble. The temperature decay dominates if \(S_3/T < S_4\).

The straightforward calculations of the actions \(S_3\) and \(S_4\) give for our potential the condition for the dominance of the temperature decay (the term proportional to \((\epsilon/\lambda)^2\) was omitted):

\[
T > \frac{32}{27\pi} \frac{\epsilon}{\lambda}
\]

in the units \(m_\phi = 1\).

Consider the equation of motion of the scalar field in spherical coordinates

\[
\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial r^2} - \frac{2}{r} \frac{\partial \phi}{\partial r} = -V'(\phi),
\]

Neglecting terms of order of \(O((\epsilon/\lambda)^2)\), we obtain the well known one-dimensional equation

\[
\frac{d^2 \phi}{dt^2} - \frac{d^2 \phi}{dr^2} = -V'(\phi) \mid_{\epsilon=0}.
\]

The properties of this equation have been extensively discussed in the literature since 1975 \([12]\). The fundamental time independent solution is defined by

\[
r = \int_0^\phi \frac{d\phi}{\sqrt{2V(\phi)}}
\]
It can be easily checked by straight substitution, that for the theory defined by potential (2), the solution is represented in the form

$$\phi = \phi_0 \left( \tanh \frac{m}{2} (r - R(t)) - \epsilon / \lambda \right), \quad R(t) = vt + R_0,$$

where $\gamma = 1 / \sqrt{1 - v^2}$, $v < 1$, $m = \sqrt{\lambda} \phi_0$ and $R_0 = 2\lambda / (3\epsilon m)$ is critical radius of the nucleated bubble.

The initial field configuration can be defined at the moment of the bubble formation $t = 0$ with velocity $v = 0$. But it takes too much computer time and memory to obtain the results of the collision, because the kinetic energy of the walls of the colliding bubbles should be large enough to produce false vacuum bag (FVB) and hence the initial distance between the centers of colliding bubbles should be large comparing with critical radius $R_0$ as well. So, we have to use the initial configuration with already moving walls.

The one-bubble solution (8) is the approximate solution to exact Eq.(6). It also satisfies the correct boundary conditions at infinity up to the terms of order $(\epsilon / \lambda)^2$ and hence can be chosen as new initial condition at definite moment $t$ or at definite radius of the expanding bubble $R = R(t)$. The only thing that remains to do is to connect the radius $R$ and the velocity $v$.

To find the velocity $v$ in the one-bubble solution (8) at an arbitrary moment $t$ or at definite bubble radius $R(t)$ we note that the energy

$$E = \int \left( \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right) d^3x$$

is conserved if the field $\phi$ is governed by Eq.(6). The substitution of the field $\phi$ in form (8) leads after simple calculations to the expression

$$E \simeq \frac{8\pi}{3} \frac{1}{\lambda} R(t)^2 [\gamma - R(t) \epsilon / \lambda] = \text{Const}$$

(9)

The $\text{Const}$ can be determined at $t = 0$, because we know the values of the parameters at this moment: $\gamma(t = 0) = 1$ and $R(t = 0) = 2\lambda / 3\epsilon$. Substituting it into expression (8), we find the connection between the bubble radius $R = R(t)$ and $\gamma$–factor (or, equivalently, the velocity $v$):

$$\gamma = \frac{R}{\lambda} \frac{\epsilon}{\lambda} + \frac{4}{3\epsilon^2 R^2} \left( \frac{\lambda^2}{2} \right)$$

(10)

Thus, the initial conditions for one bubble of radius $R$ is represented by formula (8) with $\gamma$–factor (10).

3 Bubble collisions - Qualitative analysis

Let us introduce the dimensionless variables $\psi = \phi / \phi_0$, $\lambda^{1/2} \phi_0 t \to t$ and $\lambda^{1/2} \phi_0 r \to r$. The classical equation of motion for the scalar field of Lagrangian (1) has the form

$$\frac{1}{2} \frac{d^2 \psi}{dt^2} + \frac{1}{2} (\nabla \psi)^2 + V(\psi) = 0.$$
\[ \partial_t^2 \psi - \nabla^2 \psi = -\frac{1}{2} \psi (\psi^2 - 1) - \epsilon/\lambda \]  

(11)

The suitable initial two-bubble configuration has in our dimensionless variables the form

\[
\begin{align*}
\psi &= \psi_0 \left\{ \text{th}\left[ \frac{1}{2} (r_+ - R) \right] - \epsilon/\lambda \right\}, z < 0, \\
\psi &= \psi_0 \left\{ \text{th}\left[ \frac{1}{2} (r_- - R) \right] - \epsilon/\lambda \right\}, z > 0, \\
r_{\pm} &= \sqrt{x^2 + y^2 + (z \pm b)^2}, b > R.
\end{align*}
\]  

(12)

To start numerical simulations of bubble collisions, we need a set of simple criteria indicating a proper range of parameters favorable to formation of separated false vacuum regions.

Let us first find the condition at which the region of a false vacuum can be formed as a result of a collision of two relativistic bubbles.

A field configuration in a wall is just transition from a true vacuum inside to a false vacuum outside. While propagating through a false vacuum before collision, bubble absorbs the energy of a surrounding false vacuum and transforms it into a kinetic energy of the wall. The kinetic energy is characterized by the Lorentz factor \( \gamma = 1/\sqrt{1 - v^2} \). To get a region of a false vacuum between bubbles as a result of a collision, energy absorbed by walls from a false vacuum to the moment of a collision, must be sufficient to form a false vacuum state at least at the scale of the wall width. Let us estimate the lower limit for \( \gamma \) at which such minimal region can be formed.

Consider collision of two spherically \( O(3) \) bubble walls described by the solution (8) with the parameters \( R \) and \( \gamma_{in} \) to the moment of a collision. The leading term in the energy density of a wall, as calculated for the quasiplanar solution (6), is

\[ \rho_w \simeq \frac{\gamma^2}{4} \cosh^4 \left( \gamma (R - vt)/2 \right) \]

. Before a collision, in the solid angle

\[ \Delta \Omega = \frac{\pi r^2}{R^2} \ll 1 \]

, each wall has the energy

\[ E_{in} = \frac{2}{3} \Delta \Omega R^2 \gamma_{in}. \]

After a collision, the walls reflect with a final kinetic energy \( E_{fin} \). If we want bubble wall collision to form, between reflecting walls, a false vacuum region of a radius \( r \) and width \( h \) within the solid angle \( \Delta \Omega \), we must have

\[ E_{in} = E_{fin} = \frac{2}{3} \Delta \Omega R^2 \gamma_{fin} + 2 \frac{\epsilon}{\lambda} V_{fr}, \]
where \( \rho_{\text{vac}} = 2\epsilon/\lambda \) is false vacuum density for the case of the potential \( \Omega \), and \( V_{\text{fvr}} \) is the volume of a false vacuum region within a cone with a solid angle \( \Delta\Omega \) which is given by
\[
V_{\text{fvr}} = \frac{\pi}{6} h(3r^2 + h^2) \approx \frac{1}{2} \Delta\Omega R^2 h
\]
The width of a false vacuum region is of order of a width of a wall to the moment of reflection which is \( 2/\gamma_{\text{fin}} \). For \( \gamma_{\text{fin}} = 1 \), the width is \( h = 2 \). It gives us the constraint for \( \gamma_{\text{in}} \) with which a wall came to the first collision in the form
\[
\gamma_{\text{in}} \geq 1 + \frac{3}{2} \frac{\epsilon}{\lambda} h \geq 1 + \frac{3}{2} \frac{\epsilon}{\lambda}.
\]
Now let us specify the line joining centers of bubbles as \( z \) axis. Let us show that the energy conservation puts constraint on the propagation of a false vacuum region in \( z \) direction.

Consider a slice of a false vacuum region originated from the collision in the element \( \Delta\Omega \) of spherical bubble walls which has radius \( R \) in the moment of collision. The acceleration of the considered element of the wall came from transformation of the energy of surrounding false vacuum into kinetic energy of the wall on the way to its first collision, when the true vacuum bubbles grow from the initial radius \( R(0) \) to the radius \( R \gg R(0) \) in the moment of collision. So, the kinetic energy absorbed by the wall from a false vacuum to the moment of collision, is
\[
E_{\text{kin}} = \frac{2\epsilon}{\lambda} \Delta\Omega \frac{1}{3} (R^3 - R(0)^3) \approx \frac{2\epsilon}{\lambda} \Delta\Omega \frac{1}{3} R^3.
\]
The walls reflect each other in the moment of the first collision and move outwards, creating a false vacuum region between them. Each wall stops when all its kinetic energy has been transformed into the energy of a false vacuum region formed between the walls. In this moment the walls radius is \( R_{\text{max}} \) and a false vacuum, created by each wall, fills a region between the spherical shells \( R_{\text{max}} \) and \( R \). The energy balance gives
\[
\frac{2\epsilon}{\lambda} \Delta\Omega \frac{1}{3} R^3 \approx \frac{2\epsilon}{\lambda} \Delta\Omega \frac{1}{3} (R_{\text{max}}^3 - R^3),
\]
so that
\[
R_{\text{max}} \simeq 2^{1/3} R.
\]
Since we consider overcritical bubbles, the wall surface energy is neglected in this treatment, provided that \( \epsilon R_{\text{max}}/\lambda \gg 1 \) The same result has been obtained for the case of \( O(4) \) symmetric bubbles by HMS [10].

One finds from the equation (14) that after the collision a false vacuum is formed and occupies a region between the outgoing walls, with a maximal size given by a distance between the planes \( z = \pm (2^{1/3} - 1)b \), where \( 2b \) is the initial separation of the centers of true vacuum bubbles.
After the walls stop their outward movement at \( R_{\text{max}} = \pm(2^{1/3} - 1)R \) in the region of walls intersection, the parts of walls in this region reflect off one another and next time they collide at \( \Delta t \sim 2(2^{1/3} - 1)R \) after the first collision. The shortest interval between two subsequent collisions is at \( r = 0 \) when \( R = b \). Using the condition (13), we find from the Equation (10) the minimal \( \gamma \) at which a false vacuum region is formed between the walls after the second collision. Before the second collision at \( r = 0 \), the value of \( \gamma \) in the walls is given by

\[
\gamma_2 = R \frac{\epsilon}{\lambda} \approx (2^{1/3} - 1)b \frac{\epsilon}{\lambda}.
\]

(15)

Remember that before the first collision this factor for the walls at \( r = 0 \) is given by the Eq.(10) as

\[
\gamma_1 = b \frac{\epsilon}{\lambda}.
\]

(16)

If we want the false vacuum region be maintained after the second collision of the reflected parts of walls, we must satisfy \( \gamma_2 > \gamma_{\text{in}} \). Then it follows from the Equations (13,15,16), that \( \gamma \) before the first collision must be

\[
\gamma_1 > \gamma_{\text{min}} = \frac{1 + 3\epsilon/\lambda}{(2^{1/3} - 1)}.
\]

(17)

It indicates the favorable range of the \( \gamma \) parameter before the first collision needed for numerical simulation, and also, by (10,16), the favorable range for the parameters \( R \) and \( b \).

In the case \( \gamma_1 \gg \gamma_{\text{min}} \), a false vacuum region undergoes the succession of oscillations - expansions and contractions - along the \( z \) axis in the region confined by

\[
-b(2^{1/3} + 1) < z < b(2^{1/3} - 1).
\]

(18)

Repeating the above reasoning for the subsequent collisions we find easily that in the limit of large \( \gamma \) the period of the \( n \)-th oscillation decreases as \((\sqrt{2n})^{-1}\). This agrees with the HMS result [10] for the \( O(4) \) symmetry case. The reason for such a coincidence can be easily understood. The main difference between the \( O(3) \) and \( O(4) \) cases is in the form of the initial wall configurations, taken in our case as a quasiplanar \( O(3) \) solution. However, in all the above reasoning the internal structure of the walls was not involved which just resulted in similar estimation for the decreasing of period of oscillations.

For large \( \gamma \) we can treat the oscillations of a false vacuum region along the axis \( z \) as the continuous propagation of a spherical wave moving with speed of light (in our units \( c = 1 \)). In the frame with the origin in, say, \( z = -b \), the element \( \Delta \Omega \) of the wall with the angle \( \alpha \) with respect to the \( z \) axis, follows the trajectory \( r = z \tan \alpha \). Assume that to the moment of reflection considered element of the wall has the coordinate \( z = ct \). At the same moment its radial coordinate is \( r = z \tan \alpha \). The region of causal contact along the axis \( r \) satisfies the condition \( r \leq ct \). It follows then that only for the angles \( \alpha < \pi/4 \), the
region of intersection of walls is in causal contact. Therefore the boundary of
the region of causal contact within a false vacuum region is the cone
\[ \alpha = \pi/4. \]  
(19)

It means that further evolution of a false vacuum region confined within the
boundary (19) does occur independently on dynamics of field outside this bound-
ary. So, considered region of a false vacuum is separated in its further causal
and hence dynamical evolution.

Now we can easily estimate the total energy of a separated false vacuum
region. The energy density of a false vacuum is given by \( \rho_{\text{vac}} = 2\epsilon/\lambda \). The
volume of a FVB is the volume of two cones whose height is equal \( b \) and base
area \( \pi b^2 \). So, the mass confined within this region is
\[ M = \frac{4\epsilon \pi b^3}{3\lambda}. \]  
(20)

It is evident that separation occurs at the time of order of \( t_{\text{sep}} = (\sqrt{2} - 1)b \).

4 Numerical Results

In the cylindric coordinates the equation of motion for a scalar field (11) equation
takes the form
\[ \partial_t^2 \psi - \partial_r^2 \psi - \partial_z^2 \psi = -\frac{1}{2} \psi (\psi^2 - 1) - \epsilon/\lambda. \]
The solution to this equation has the axial symmetry and reflection symmetry
with respect to \( z = 0 \) plane. The initial configuration described by solutions
\[ \psi = th[\gamma/2(r_+ - R - vt)] + th[\gamma/2(r_- - R - vt)] - 1 - \epsilon/\lambda \]  
(21)
is shown in Fig.1. The walls already have kinetic energy that is indicated by \( \gamma \)
factor equals 5.

Time evolution of a scalar field in the center of the region of collision \( \psi(t, r = z = 0) \) shown in Fig.2, was calculated for the parameters \( \Gamma = 5; b = 52; R = 50 \).
The qualitative behavior of the field with time has been discussed in previous
section. From the beginning the field changes in the manner discussed in \( [10], [9] \), then it oscillates around true vacuum for a long time and finally, large
secondary fluctuations appear again.

As we shall see below, the energy of oscillations is perfectly localized. This
behaviour does not change with changing the step in calculations. Field config-
urations in different moments of time are shown in Figs.3,4.

The energy density profile calculated from scalar field potential, is shown in
the next series of figures. They demonstrate concentration of field energy in the
center of region of collision. In the fig.5 one can see time dependence of energy
density in the center of the region of collision. Large secondary peak is created due to the coherent field oscillation which are coming from outside.

The density profile at this time is represented in the Fig.6. This localized configuration oscillates for some time and finally is converted into outgoing radiation. Only gravity could prevent this process. Till now we did not consider gravitational effects, but they will be estimated below. Consider the evolution of energy contained in the sphere of certain radius as shown in Fig.7-8.

This pictures show two peaks of energy - the first is due to energy in the moment of collision, the second is the energy of the quasilump which is formed as a result of the collision. It becomes evident by comparing these figures that the energy is strongly localized. Indeed, the energy contained in internal sphere of radius 5 is only in 1.5 times smaller than that contained in external sphere, while the ratio of their volumes is 8. Gravitational radius of the quasilump in our units is

$$ r_g = \left( \frac{m}{m_{pl}} \right)^2 \frac{E}{\lambda} $$

If the first order phase transition happens at the end of inflation, the mass $m$ of inflaton field is rather large and $m_{\phi}/m_{pl} \sim 10^{-5}$. Substituting this value into (22) and the value of energy $E \approx 1000$ obtained from fig.7 one can easily find the condition when gravitational radius is comparable with the size of the quasilump, which can be taken as $r_0 \approx 5$ in our dimensionless units. This condition is satisfied if coupling constant $\lambda \sim 10^{-8}$. For $\lambda < 10^{-8}$ gravitational forces become essential and the probability of black hole formation grows up to unity when $\lambda$ tends to $10^{-8}$.

5 Summary

In this paper we give qualitative arguments supported by numerical simulation for the existence of long-lived fluctuation that arise as a result of a collision of two expanded bubbles.

The two-bubble collision leads, first, to the formation of short-living false vacuum region in the center of collision. Numerical results indicate separation of a false vacuum region at the time $t \sim b$. Then it evolves into rather compact object - quasilump made up of a scalar field oscillating around its true minimum, with lifetime enough to be captured by its gravitational field. At small coupling constants black hole can be produced.

Till now the similar object discussed in literature was oscillon [16]. The main difference between these two objects is as follows. i) Oscillon represent a subcritical bubble of true vacuum inside a false vacuum, that arise due to temperature fluctuations. Our object is the fluctuation of scalar field in the true vacuum background that arise as a result of dynamical process. ii) To be long-lived, oscillon should have rather large initial radius, though less than
critical one, and rather flat initial distribution of scalar field. The evolution of
the oscillon consists of oscillation of field value with almost constant radius of the
field configuration. Our lump is much more compact object with the amplitude
value of scalar field being much larger than that of the field in its potential
minimum. For $\lambda \leq 10^{-8}$ gravitational forces are essential and the probability
of PBH creation is of order unity. iii) Oscillon, being produced in spite of small
probability \[17\], is extremely long lived object with lifetime $10^3 - 10^4 \text{1/m}$, $m$
being the mass of scalar field. The life-time of our lump is of the same order of
magnitude but lumps can be produced with much higher probability, because
they result from collisions of overcritical bubbles whose rate of nucleation is
much bigger than for subcritical bubbles.

6 Acknowledgement

This work was supported by the Polish Committee for Scientific Research through
the Grant 2P03D.017.11. The work of MyuK and SGR was partially performed
in the framework of Section ”Cosmoparticle physics” of Russian State Scientific
Technological Program ”Astronomy. Fundamental Space Research”, with teh
support of Cosmion-ETHZ and Epcos-AMS collaborations.

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