Sensorless Control of Electric Motors with Kalman Filters: Applications to Robotic and Industrial Systems

Regular Paper

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Received 26 Nov 2011; Accepted 02 Dec 2011

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Abstract

The paper studies sensorless control for DC and induction motors, using Kalman Filtering techniques. First the case of a DC motor is considered and Kalman Filter-based control is implemented. Next the nonlinear model of a field-oriented induction motor is examined and the motor’s angular velocity is estimated by an Extended Kalman Filter which processes measurements of the rotor’s angle. Sensorless control of the induction motor is again implemented through feedback of the estimated state vector. Additionally, a state estimation-based control loop is implemented using the Unscented Kalman Filter. Moreover, state estimation-based control is developed for the induction motor model using a nonlinear flatness-based controller and the state estimation that is provided by the Extended Kalman Filter. Unlike field oriented control, in the latter approach there is no assumption about decoupling between the rotor speed dynamics and the magnetic flux dynamics. The efficiency of the Kalman Filter-based control schemes, for both the DC and induction motor models, is evaluated through simulation experiments.

Key words

DC motors, induction motors, Kalman Filter, Extended Kalman Filter, Unscented Kalman Filter, field-oriented control, flatness-based control, sensorless control.

1. Introduction

During the last years, there has been significant effort in improving the performance of electric motors. DC (direct current) motors are widely used in industrial systems, such as robotic manipulators, because their control is relatively simple and they are reliable for a wide range of operating conditions. DC motors are usually modelled as linear systems and then linear control approaches are implemented. Additionally, controllers for nonlinear DC motor models have been developed. On the other hand, the applications of induction motors (IM) are mainly concerned with motion transmission systems. Induction motors have been the most widely used machines in fixed-speed applications for reasons of cost, size, weight, reliability, ruggedness, simplicity, efficiency, and ease of manufacture. With the field-oriented method, the dynamic
behavior of the induction motor is rather similar to that of a separately excited DC motor [Rigatos 2009a].

The possibility to reduce the number of sensors involved in the control of electric motors has been a subject of systematic research during the last years [Holtz 2002], [Hilairet et al. 2009], [Borsje et al. 2005]. As a result, state estimation-based control has become an active research area in the field of electric machines and power electronics. In the control of robotic manipulators, which is actually control of the DC motors that rotate the robot’s joints, the angle of each joint is usually measured with the use of an optical encoder. High performance robot control systems also call for velocity and acceleration information from the joints. However, to reduce equipment cost and to simplify installation and maintenance, tachometers and accelerometers are not always used in the robotic control loops. In other cases the information from tachometer and accelerometer measurements is replaced by the fusion of measurements from encoders and external sensors, such as cameras, laser trackers, etc. Thus, one should be able to deduce the velocity and acceleration of the robot’s joints from angular position measurements, using state estimators [Rigatos 2009b], [Rigatos 2008],[Rigatos 2009c]. Furthermore, speed estimation methods have been of great interest among induction motor control researchers. Elimination of the speed sensors has the advantages of lower cost, ruggedness as well as increased reliability. Again to reconstruct the state vector of the induction motor, from a sequence of angular position measurements and the associated voltage input measurements a state observer has to be used.

The Kalman Filter is a special kind of observer that provides optimal filtering of process and measurement noises if the covariances of these noises are known. For linear systems subject to Gaussian measurement or process noise the Kalman Filter is the optimal state estimator, since it results in minimization of the trace of the estimation error’s covariance matrix [Kamen & Su 1999], [Basseville & Nikiforov 1993], [Xiong 2008], [Rigatos & Tzafestas 2007a]. For nonlinear systems, subject to Gaussian noise one can use the generalization of the Kalman Filter as formulated in terms of the Extended Kalman Filter (EKF). The Extended Kalman Filter is based on a linearization of the systems’ dynamics using a first order Taylor expansion [Rigatos & Zhang 2001]. Although EKF is efficient in several estimation problems, it may be subject to cumulative errors due to the local linearization assumption and this may affect the accuracy of the state estimation or even risk the stability of the observer-based control loop.

To overcome the EKF flaws, the Unscented Kalman Filter can be also considered. UKF is a derivative-free state estimation method of high accuracy. The state distribution in UKF is approximated by a Gaussian random variable, which is represented using a minimal set of suitably chosen weighted sample points. These sigma points are propagated through the true nonlinear system, thus generating the posterior sigma-point set, and the posterior statistics are calculated. The sample points progressively converge to the true mean and covariance of the Gaussian random variable [Rigatos 2009d], [Rigatos 2009e]. The Unscented Kalman Filter has been proposed as a suitable approach for overcoming certain drawbacks of the Extended Kalman Filter, such as (i) the need to compute Jacobians and (ii) the cumulative linearization errors which are due to truncation of higher order terms in the associated Taylor series expansion of the motor’s dynamical model. The use of the Unscented Kalman Filter for state estimation and control of nonlinear electric motor models is a relatively new and promising topic. In [Akin et al. 2003] and [Akin et al. 2006] the Unscented Kalman filter (UKF) has been applied to state observation in field oriented control of an induction motor. The UKF enables to estimate rotor speed and dq-axis flux of an induction motor through the processing of only the stator currents and voltages. In [Kumar et al. 2011] the Unscented Kalman Filter (UKF) has been compared to the Extended Kalman Filter for the state estimation of a three-phase induction motor. In [Karami et al. 2010] the Unscented Kalman Filter is applied to state estimation for fault diagnosis of induction motors. Comparison between the estimated and the real output measurements enables the detection of failures in the motor’s components. Finally, in [Akin et al. 2004] the Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF) are analyzed and compared both experimentally and theoretically in the problem of non-linear state estimation for field-oriented sensorless control of AC drives.

The current paper studies sensorless control for DC and induction motors using Kalman Filtering techniques. The structure of the paper is as follows: In Section 2 the dynamic model of the DC motor is explained and control for the nonlinear DC motor model is analyzed. In Section 3, the field-oriented induction motor model is analyzed. In Section 4, flatness-based control for the complete (sixth-order) induction motor model is analyzed. In Section 5 Kalman Filtering for the DC motor model is introduced. Moreover, the Extended Kalman Filter is proposed to estimate the state vector of the nonlinear electric motor using a limited number of sensors, and control of the induction motor is again implemented through feedback of the estimated state vector. In Section 6, the Unscented Kalman Filter is used to estimate the induction motor’s state vector and subsequently a state estimation-based control loop is implemented. In Section 7 the efficiency of the above mentioned Kalman Filter-based control schemes, for both the DC and induction motor models, is tested through simulation experiments. Finally, in Section 8 concluding remarks are given.

2. DC motor modelling and control

2.1 DC motor model

The linear model of the DC motor shown in Fig. 1 is described by the set of equations

\[ L \dot{I} = -k_e \omega - RI + V \]
\[ J \dot{\omega} = k_e I - k_d \omega - \Gamma_d \]

(1)
with the following notations $L$ : armature inductance, $I$ : armature current, $k_r$ : motor electrical constant, $R$ : armature resistance, $V$ : input voltage, taken as control input, $J$ : motor inertia, $\omega$ : rotor rotation speed, $k_d$ : mechanical damping constant, $\Gamma_d$ : disturbance torque. Assuming $\Gamma_d = 0$ and denoting the state vector as $[x_1, x_2, x_3]^T = [\theta, \dot{\theta}, \dot{\theta}]^T$, a linear model of the DC motor is obtained:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -\frac{k_d}{J} \omega & -\frac{k_d}{J} \omega
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
\frac{1}{J} u
\end{pmatrix} V
\] (2)

Next, control for a nonlinear DC motor model will be presented. Usually the DC-motor model is considered to be linear by neglecting the effect of armature reaction or by assuming that the compensating windings remove this effect. Introducing the armature reaction leads to a nonlinear system. In that case the dynamic model of the DC-motor model can be written as an affine in-the-input system [Horng 1999]:

\[
\dot{x} = f(x) + g(x)u
\] (3)

with $\dot{x}$ denoting the derivative of the motor’s state vector, $x = [x_1, x_2, x_3]^T = [\theta, \dot{\theta}, \dot{\theta}]$ ($\theta$ is the position of the motor, $\dot{\theta}$ is the angular velocity of the motor and $i_a$ is the armature current). Functions $f(x)$ and $g(x)$ are vector field functions defined as:

\[
f(x) = \begin{pmatrix}
x_2 \\
x_3 x_2 + k_3 x_2 x_3 + k_4 T_1 \\
k_5 x_2 + k_6 x_2 x_3 + k_7 x_2
\end{pmatrix},
g(x) = \begin{pmatrix}
0 \\
0 \\
k_8
\end{pmatrix}
\] (4)

where $k_1 = -F/J$, $k_2 = A/J$, $k_3 = B/J$, $k_4 = -1/J$, $k_5 = -A/L$, $k_6 = -B/L$, $k_7 = -R/L$, $k_8 = -1/L$, $R$ and $L$ are the armature resistance and induction respectively, and $J$ is the rotor’s inertia, while $F$ is the friction. Now, considering $k_4 T_1$ as disturbance, the state-space equation of the DC motor can be rewritten as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= k_1 x_2 + k_2 x_3 + k_3 x_2^2 + k_4 T_1 \\
\dot{x}_3 &= k_5 x_2 + k_6 x_2 x_3 + k_7 x_2 + k_8 u
\end{align*}
\] (5)

where $T_1$ is the load torque and $u$ is the terminal voltage. From the second row of Eq. (5) one obtains,

\[
\begin{align*}
\dot{x}_2 &= k_1 x_2 + k_2 x_3 + 2k_3 x_2 x_3 \Rightarrow \\
\dot{x}_2 &= k_1 x_2 + k_2 x_3 + 2k_3 k_5 x_2 x_3 + 2k_3 k_6 x_2 x_3 + 2k_3 k_7 x_2 + 2k_3 k_8 x_2 u
\end{align*}
\] (6)

Thus the input-output relation can be written as

\[
x_2 = f(x) + g(x)u
\] (7)

where, $f(x) = k_1 x_2 + k_2 x_3 + 2k_3 k_5 x_2 x_3 + 2k_3 k_6 x_2 x_3 + 2k_3 k_7 x_2$, and $g(x) = 2k_3 k_8 x_3$. Taking $k_1 = 0$ and considering the torque $T_1$ as external disturbance, the nonlinear DC motor model of Eq. (5) becomes

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= k_2 x_3 + k_3 x_2^2 \\
\dot{x}_3 &= k_5 x_2 + k_6 x_2 x_3 + k_7 x_2 + k_8 u
\end{align*}
\] (8)

Then, setting the output to be $y = x_1$ one can see that all state variables $x_i, i = 1, 2, 3$ and the control input $u$ can be expressed as functions of this output and its derivatives. Indeed it holds

\[
\begin{align*}
x_1 &= y \\
x_2 &= y \\
x_3 &= \frac{-k_3 \sqrt{k_3^2 + 4k_1 k_8}}{2k_3} \\
u &= \frac{1}{k_8} \{y^{(3)} - f(x)\}
\end{align*}
\] (9)

The aforementioned system of Eq. (8) can be written in the Brunovsky (canonical) form:
where \( w = \bar{f}(x, t) + \bar{g}(x, t)u \). Using a control input as in Eq. (15) it is possible to make the electric motor’s angle track any desirable setpoint.

2.2 Control of the nonlinear DC motor model

The design of the nonlinear DC motor controller is according to the flatness-based control theory. The main principles of flatness-based control are as follows [Martin & Rouchon 1996], [Rudolph 2003], [Flies & Mounier 1999], [Villagra et al. 2007]. Flatness-based control can be applied to finite dimensional system of the form

\[
\begin{bmatrix}
   y_1 \\
   y_2 \\
   y_3
\end{bmatrix}
= \begin{bmatrix}
   0 & 1 & 0 \\
   0 & 0 & 1 \\
   0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
   y_1 \\
   y_2 \\
   y_3
\end{bmatrix}
+ \begin{bmatrix}
   0 \\
   0 \\
   1
\end{bmatrix}
\begin{bmatrix}
   v
\end{bmatrix}
\tag{10}
\]

For the motor model of Eq. (3) and Eq. (4) an appropriate control law that satisfies the aforementioned flatness properties is

\[
u = \frac{1}{\bar{g}(x, t)} [x_d^{(n)} - \bar{f}(x, t) - K^T e + u_c] \tag{15}\]

with \( e = x - x_d \), \( e^T = [e_1, e_2, \ldots, e^{(n-1)}] \), \( K = [k_1, k_2, \ldots, k_1] \), such that the polynomial \( e^{(n)} + k_1 e^{(n-1)} + \cdots + k_1 e \) is Hurwitz. This controller makes the system’s output (angle \( \theta \) of the motor) follow a given reference signal \( x_d \).

3. Control of the field-oriented induction motor model

3.1 Field-oriented induction motor model

Control for induction motors is also studied. With the field-oriented method, the induction motor dynamics is rather similar to that of a separately excited DC motor [Wai and Chang 2001], [Nounou & Rehman 2007], [Wai & Chang 2004]. A decoupled relationship is obtained after a proper selection of state coordinates and thus, the rotor speed is asymptotically decoupled from the rotor flux, while the speed can be controlled only by the torque current as shown in [Bodson et al. 1994], [Marino et al. 1991], [Leonard 1985], [Wai & Chang 2003], [Lin et al. 2000].

To derive the dynamic model of an induction motor the three-phase variables are first transformed to two-phase ones. This two-phase system can be described in the stator-coordinates frame \( a-b \), and the associated voltages are denoted as \( v_a \) and \( v_b \), while the currents of the stator are \( i_a \) and \( i_b \), and the components of the rotor’s magnetic flux are \( \psi_a \) and \( \psi_b \). Then, the rotating angle of the rotor with respect to the stator is denoted by \( \delta \). Next, the rotating reference frame \( d-q \) on rotor, is defined (Fig. 2).

The state vector of the motor is defined as
\[ x = [\theta, \omega, \psi_r, i_a, i_b] \] and the dynamic model of the induction motor is written as [Hornig 1999]:

\[
x = \dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m
\]

\[
\dot{x} = f(x) + g_a(x)v_a + g_b(x)v_b \tag{17}
\]

with

\[
f(x) = \begin{pmatrix}
x_2 \\
\mu_1 (x_3 x_b - x_4 x_5) - \frac{T_e}{J} \\
-a_1 x_3 - n_p a_2 x_4 + a_1 M x_5 \\
n_p a_2 x_3 - a_1 x_4 + a_1 M x_6 \\
a_1 b_1 x_3 + n_p a_2 b_1 x_4 - \gamma x_5 \\
-a_p b_1 x_3 + a_1 b_1 x_4 - \gamma_1 x_6
\end{pmatrix} \tag{18}
\]
The control signal in the coordinates system \( \alpha \) – \( \beta \) and of the magnetic fluxes of the rotor (\( \psi_{\alpha r} \) and \( \psi_{\beta r} \)) to the reference frame \( d – q \) which rotates together with the rotor (Fig. 2). In the \( d – q \) frame there will be only one non-zero component of the magnetic flux \( \psi_{d r} \), while the component of the flux along the \( q \) axis equals 0. The new control inputs of the system are considered to be \( v_{\alpha r} \), \( v_{\beta r} \), and are associated to the \( d – q \) frame voltages \( v_d \) and \( v_q \), respectively. The control inputs \( v_{\alpha r} \), \( v_{\beta r} \) are connected to \( v_{\alpha s} \), \( v_{\beta s} \) of Eq. (17), according to the relation

\[
\begin{pmatrix}
v_{\alpha s} \\
v_{\beta s}
\end{pmatrix} = \phi(\psi) \begin{pmatrix} \psi_{\alpha r} \\
\psi_{\beta r} \end{pmatrix}^{-1} \begin{pmatrix} v_d \\
v_q \end{pmatrix}
\]  
(20)

where \( \psi = \psi_{d r} \) and \( ||\psi|| = \sqrt{\psi_{\alpha r}^2 + \psi_{\beta r}^2} \). Next, the following nonlinear feedback control law is defined

\[
\begin{pmatrix} v_{\alpha d} \\
v_{\beta q}
\end{pmatrix} = \sigma L_a \begin{pmatrix}
-n_p a \psi_{\beta r} - \frac{a M_i r^2}{\psi_{\beta r}} - a b \psi_{\alpha r} + v_d \\
n_p a \psi_{\alpha r} + b n_p \omega \psi_{\beta r} + \frac{a M_i r^2}{\psi_{\beta r}} + v_q
\end{pmatrix}
\]  
(21)

The control signal in the coordinates system \( a – b \) is

\[
\begin{pmatrix} v_{\alpha s} \\
v_{\beta s}
\end{pmatrix} = ||\psi|| \sigma L_a \begin{pmatrix} \psi_{\alpha r} \\
\psi_{\beta r} \end{pmatrix}^{-1} \begin{pmatrix}
-n_p a \psi_{\beta r} - \frac{a M_i r^2}{\psi_{\beta r}} - a b \psi_{\alpha r} + v_d \\
n_p a \psi_{\alpha r} + b n_p \omega \psi_{\beta r} + \frac{a M_i r^2}{\psi_{\beta r}} + v_q
\end{pmatrix}^{-1} \begin{pmatrix} v_d \\
v_q \end{pmatrix}
\]  
(22)

Substituting Eq. (22) into Eq. (17) one obtains

\[
\frac{d}{dt} \omega = \mu \psi_{r d} v_{d} - \frac{T_L}{J}
\]  
(23)

\[
\frac{d}{dt} i_q = -\gamma i_q + v_q
\]  
(24)

\[
\frac{d}{dt} \psi_{d r} = -a \psi_{r d} + a M_i \psi_{d r}
\]  
(25)

\[
\frac{d}{dt} i_d = -\gamma i_d + v_d
\]  
(26)

\[
\frac{d}{dt} \rho = n_p \omega + \alpha M_i \frac{v_d}{\psi_{d r}}
\]  
(27)

The system of Eq. (23) to Eq. (27) consists of two linear subsystems, where the first one has as output the magnetic flux \( \psi_{r d} \) and the second has as output the rotation speed \( \omega \), i.e.

\[
\frac{d}{dt} \psi_{d r} = -a \psi_{r d} + a M_i \psi_{d r}
\]  
(28)

\[
\frac{d}{dt} i_d = -\gamma i_d + v_d
\]  
(29)

\[
\frac{d}{dt} \omega = \mu \psi_{r d} v_{d} - \frac{T_L}{J}
\]  
(30)

\[
\frac{d}{dt} i_q = -\gamma i_q + v_q
\]  
(31)

If \( \psi_{r d} \rightarrow \psi_{r d}^f \), i.e. the transient phenomena for \( \psi_{r d} \) have been eliminated and therefore \( \psi_{r d} \) has converged to a steady state value then Eq. (30) is decoupled \( \psi_{r d} \), then the two subsystems described by Eq. (28)-(29) and Eq. (30)-(31) are decoupled.

The subsystem that is described by Eq. (28) and Eq. (29) is linear and has as control input \( v_d \), and can be controlled using methods of linear control. For instance the following PI controller has been proposed for the control of the magnetic flux [Marino et al. 1991], [Leonard 1985]

\[
v_d(t) = -k_{d1}(\psi_{r d} - \psi_{r d}^f) - k_{d2} \int_0^t (\psi_{r d}(\tau) - \psi_{r d}^f(\tau))d\tau
\]  
(32)
If Eq. (32) is applied to the subsystem that is described by Eq. (28) and Eq. (29), then one can succeed \( \psi_{rd}(t) \rightarrow \psi_{rd}^{\text{ref}}(t) \). If \( \psi_{rd}(t) \) is not precisely measured using Hall sensors then it can be reconstructed using some kind of observer.

Now the subsystem that consists of Eq. (30) and Eq. (31), are now rewritten in the form of Eq. (33) to Eq. (37). This system of lower order is also flat with \( \psi \) system behavior is described by Eq. (33), Eq. (34) and Eq. (35). For the outer speed and flux control design the inner current control loop can be designed using only Eq. (37). The flat outputs for the voltage-fed induction motor are the angle of the rotor position \( \theta \) and variable \( \rho \), where \( \rho \) has been defined as the rotor flux angle. According to [Dannehl & Fuchs 2006], if the stator current dynamics are much faster than the speed and flux dynamics a faster inner current control loop can be designed using only Eq. (35) and Eq. (36) and assuming the speed and flux as constants. For the outer speed and flux control design the stator currents are treated as new control inputs and the system behavior is described by Eq. (33), Eq. (34) and Eq. (37). This system of lower order is also flat with \( \psi_{rd} \) and \( \theta \) as flat outputs.

It can be shown that all state variables of the induction motor can be written as functions of the flat outputs and their derivatives. Moreover, using Eq. (35) and Eq. (36) a controller that satisfies the flatness properties (and thus it can be also expressed as a function of the flat outputs and their derivatives) is defined as follows:

\[
v_{sd} = \sigma L_s \left( \frac{di_{sd}}{dt} + \gamma i_{sd}^* - \alpha \beta \psi_{rd} - n_p \omega i_{sq} - \frac{\alpha M_i \omega^2}{\psi_{rd}} + v_d \right)
\]

\[
v_{sq} = \sigma L_s \left( \frac{di_{sq}}{dt} + \gamma i_{sq}^* + \beta \psi_{rd} + n_p \omega i_{sd} + \frac{\alpha M_i \omega i_{sd}}{\psi_{rd}} + \gamma \right)
\]

where \( i_{sd}^* \) and \( i_{sq}^* \) denote current setpoints. Substituting Eq. (38) and Eq. (39) into Eq. (35) and Eq. (36) one obtains the dynamics of the current tracking errors.

\[
\frac{d\Delta i_s}{dT} = -\gamma \Delta i_s + v_d
\]

\[
\frac{d\Delta i_q}{dT} = -\gamma \Delta i_q + v_q
\]

Tracking of the reference setpoint can be also succeeded for the rotor’s speed and flux through the application of the control law of Eq. (38) and Eq. (39) to Eq. (33) and Eq. (37). The control inputs are chosen as

\[
i_{sd} = \frac{1}{M_i} \left( \frac{d\psi_{rd}^*}{dt} + \alpha \psi_{rd}^* + i_d \right)
\]

\[
i_{sq} = \frac{1}{M_i} \left( \frac{d\psi_{rd}^*}{dt} + \alpha \psi_{rd}^* + i_q \right)
\]

Denoting \( \Delta \psi_{rd} = \psi_{rd} - \psi_{rd}^* \) and \( \Delta \omega = \omega - \omega^* \) the tracking error dynamics are given by

\[
\frac{d\Delta \psi_{rd}}{dT} = -\alpha \Delta \psi_{rd} + i_d
\]

\[
\frac{d\Delta \omega}{dT} = -\frac{T}{T} + i_q
\]

The convergence of the tracking error to zero can be assured through the application of the following feedback control laws:

\[
i_d = -\alpha_1 \Delta \psi_{rd}
\]

\[
i_q = \frac{T}{T} - \alpha_2 \Delta \omega
\]
5. Kalman and Extended Kalman Filtering for linear electric motor models

5.1 The discrete-time Kalman Filter for the Linear Electric Motors

For the DC motor models described in Section 2 the Kalman Filter is an efficient state estimator. Kalman Filtering can be applied to discrete-time state models of the form

\[ x(k + 1) = \Phi(k)x(k) + L(k)u(k) + w(k) \]
\[ z(k) = Cx(k) + v(k) \]  
\[ (50) \]

where the state \( x(k) \) is a \( m \)-vector, \( w(k) \) is a \( m \)-element process noise vector and \( \Phi \) is an \( m \times m \) real matrix. Moreover, the output measurement \( z(k) \) is a \( p \)-vector, \( C \) is an \( p \times m \)-matrix of real numbers, and \( v(k) \) is the measurement noise. It is assumed that the process noise \( w(k) \) and the measurement noise \( v(k) \) are uncorrelated. Now, the problem of interest is to estimate the state \( x(k) \) of the DC motor based on output measurements. This can be done using the Kalman Filter equations

**Measurement update:** acquire \( z(k) \) and compute

\[ K(k) = P^-(k)C^T[CPC^T + R]^{-1} \]
\[ \hat{\epsilon}(k) = \hat{x}^- (k) - K(k)[z(k) - C\hat{x}^- (k)] \]
\[ P(k) = P^-(k) - K(k)CP^-(k) \]  
\[ (51) \]
\[ (52) \]
\[ (53) \]

**Time update:** compute

\[ P^-(k + 1) = \Phi(k)P(k)\Phi^T(k) + Q(k) \]
\[ \hat{x}^- (k + 1) = \Phi(k)\hat{x}(k) + L(k)u(k) \]  
\[ (54) \]
\[ (55) \]

where \( \hat{x}^- (k) \) is the estimation of the state vector using measurements up to time instant \( k - 1 \), \( \hat{x}(k) \) is the estimation of the state vector using measurements up to time instant \( k \), \( P \) is the covariance matrix of the estimation error, \( R \) is the measurement noise covariance matrix and \( Q \) is the process noise covariance matrix.

It can be observed that the recursion of the Kalman Filter given by Eq. (51) and Eq. (54) is also applicable to the nonlinear DC motor model of Eq. (3) and Eq. (4) if the latter system is written in the form of Eq. (10) and the measured output is taken to be \( y = x_1 \).

5.2 Extended Kalman Filter for nonlinear electric motor models

The following nonlinear time-invariant model is now considered [Rigatos & Tzafestas 2007a]:

\[ x(k + 1) = \phi(x(k)) + L(k)u(k) + w(k) \]
\[ z(k) = \gamma(x(k)) + v(k) \]  
\[ (56) \]

where \( x \in \mathbb{R}^{m \times 1} \) is the system’s state vector, \( w \in \mathbb{R}^{m \times 1} \) is the system’s output vector, while \( v \) and \( \gamma(x) \) are uncorrelated, Gaussian zero-mean noise processes with covariance matrices \( Q(k) \) and \( R \) respectively. The operators \( \phi(x) \) and \( \gamma(x) \) are

\[ \phi(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_m(x)]^T, \quad \gamma(x) = [\gamma_1(x), \gamma_2(x), \cdots, \gamma_p(x)]^T \]  

respectively. It is assumed that \( \phi \) and \( \gamma \) are sufficiently smooth in \( x \) so that each one has a valid series Taylor expansion. Following a linearization procedure, \( \phi \) is expanded into Taylor series about \( \hat{x} \):

\[ \phi(x(k)) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + \cdots \]  
\[ (57) \]

where \( J_\phi(x) \) is the \( m \times m \) Jacobian of \( \phi \) calculated at \( \hat{x}(k) \):

\[ J_\phi(x) = \frac{\partial \phi}{\partial x} |_{x = \hat{x}(k)} = \begin{pmatrix} \frac{\partial \phi_1}{\partial x_1} & \cdots & \frac{\partial \phi_m}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \phi_1}{\partial x_m} & \cdots & \frac{\partial \phi_m}{\partial x_m} \end{pmatrix} \]  
\[ (58) \]

Likewise, \( \gamma \) is expanded about \( \hat{x}^- (k) \):

\[ \gamma(x(k)) = \gamma(\hat{x}^- (k)) + J_\gamma(\hat{x}^- (k))[x(k) - \hat{x}^- (k)] + \cdots \]  
\[ (59) \]

where \( \hat{x}^- (k) \) is the estimation of the state vector \( x(k) \) before measurement at the \( k \)-th instant to be received and \( \hat{x}(k) \) is the updated estimate of the state vector after measurement at the \( k \)-th instant has been received. The \( p \times m \) Jacobian \( J_\gamma(x) \) is

\[ J_\gamma(x) = \frac{\partial \gamma}{\partial x} |_{x = \hat{x}^- (k)} = \begin{pmatrix} \frac{\partial \gamma_1}{\partial x_1} & \cdots & \frac{\partial \gamma_1}{\partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial \gamma_p}{\partial x_1} & \cdots & \frac{\partial \gamma_p}{\partial x_m} \end{pmatrix} \]  
\[ (60) \]

The resulting expressions create first order approximations of \( \phi \) and \( \gamma \). Thus the linearized version of the plant is obtained:

\[ x(k + 1) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + w(k) \]
\[ z(k) = \gamma(\hat{x}^- (k)) + J_\gamma(\hat{x}^- (k))[x(k) - \hat{x}^- (k)] + v(k) \]  
\[ (61) \]

Now, the Extended Kalman Filter (EKF) recursion is as follows: First the time update is considered: by \( \hat{x}(k) \) the estimation of the state vector at instant \( k \) is denoted. Given initial conditions \( \hat{x}^- (0) \) and \( P^- (0) \) the recursion proceeds as:

- **Measurement update.** Acquire \( z(k) \) and compute:

\[ K(k) = P^- (k)J_\gamma(\hat{x}^- (k))J_\phi^T(\hat{x}^- (k))P(k) + R(k)^{-1} \]
\[ \hat{x}(k) = \hat{x}^- (k) + K(k)[z(k) - \gamma(\hat{x}^- (k))] \]
\[ P(k) = P^- (k) - K(k)J_\gamma(\hat{x}^- (k))P^- (k) \]  
\[ (62) \]

- **Time update. Compute:**

\[ P^-(k + 1) = J_\phi(\hat{x}(k))P(k)J_\phi^T(\hat{x}(k)) + Q(k) \]
\[ \hat{x}^- (k + 1) = \phi(\hat{x}(k)) + L(k)u(k) \]  
\[ (63) \]

The schematic diagram of the EKF loop is given in Fig. 3. The Extended Kalman Filter is applicable to nonlinear electric motor models, such as the induction motor described in Sections 3 and 4.
6. Unscented Kalman Filter for nonlinear electric motor models

The Unscented Kalman Filter (UKF) is a nonlinear state estimator of high accuracy, as analyzed in [Julier et al. 2000], [Julier & Uhlmann]. The state distribution is represented again by a Gaussian Random Variable but is now specified using a minimal set of deterministically chosen weighted sample points. The basic sigma-point approach can be described as follows:

1. A set of weighted samples (sigma-points) are deterministically calculated using the mean and square-root decomposition of the covariance matrix of the prior random variable. As a minimal requirement the sigma-point set must completely capture the first and second order moments of the prior random variable.

Higher order moments can be captured at the cost of using more sigma-points.

2. The sigma-points are propagated through the true nonlinear function using functional evaluations alone, i.e. no analytical derivatives are used, in order to generate a posterior sigma-point set.

3. The posterior statistics are calculated (approximated) using tractable functions of the propagated sigma-points and weights. Typically, these take on the form of a simple weighted sample mean and covariance calculations of the posterior sigma points.

The sigma-point approach results in posterior approximations that are accurate to the third order for Gaussian inputs for all nonlinearities. For non-Gaussian
inputs, approximations are accurate to at least the second-order, with the accuracy of third and higher-order moments determined by the specific choice of weights and scaling factors. Furthermore, unlike EKF no analytical Jacobians of the system equations need to be calculated as in the case for the EKF. The concept of UKF for approximating the distribution of a system’s state is given in Fig. 4 [van der Merwe et al. 2004]. It can be observed that comparing to EKF, the UKF (sigma-point) approach succeeds improved estimation of the state vector’s mean value and covariance (only 5 points are needed to approximate sufficiently the 2D distribution).

The UKF is a special case of Sigma-Point Kalman Filters. The UKF is a discrete time filter which uses the unscented transform for approximating solutions to the filtering problem of the form

\[ x_k = f(x_{k-1}, u_{k-1}) + q_{k-1} \]
\[ y_k = h(x_k) + r_k \]

where \( x_k \in \mathbb{R}^n \) is the state, \( y_k \in \mathbb{R}^m \) is the measurement, \( q_{k-1} \sim N(0, Q_{k-1}) \), and \( r_k \in \mathbb{R}^m \) is a Gaussian measurement noise \( r_k \sim N(0, R_k) \). The mean and covariance of the initial state are \( m_0 \) and \( P_0 \), respectively. Some basic operations performed in the UKF algorithm (Unscented Transform) are summarized as follows:

1) Denoting the state vector mean as \( \hat{x} \), a set of \( 2n + 1 \) sigma points is taken from the columns of the \( n \times n \) matrix \( \sqrt{(\lambda + \lambda I)P_{zz}} \) as follows:

\[ x^i = \hat{x} + \sqrt{(\lambda + \lambda I)P_{zz}} \] \( i = 1, \cdots, n \)
\[ x^i = \hat{x} - \sqrt{(\lambda + \lambda I)P_{zz}} \] \( i = n + 1, \cdots, 2n \)

and the associate weights are computed:

\[ W_0 = \frac{\lambda}{(\lambda + \lambda I)} \] \( i = 1, \cdots, n \)
\[ W_i = \frac{1}{2(\lambda + \lambda I)} \] \( i = n + 1, \cdots, 2n \)

Matrix \( P_{xx} \) is the covariance matrix of the state \( x \) and index \( i \) denotes its \( i \)-th column. The parameter \( \lambda \) is a scaling parameter.

2) Transform each of the sigma points as:

\[ z^i = h(x^i) \] \( i = 0, \cdots, 2n \)

3) Mean and covariance estimates for \( z \) can be computed as:

\[ z = \sum_{i=0}^{2n} W_i z^i \]
\[ P_z = \sum_{i=0}^{2n} W_i (z^i - \bar{z})(z^i - \bar{z})^T \]

4) The cross-covariance of \( x \) and \( z \) is estimated as:

\[ P_{xz} = \sum_{i=0}^{2n} W_i (x^i - \bar{x})(z^i - \bar{z})^T \]

The matrix square root of a positive definite matrix \( P_{xz} \) means a matrix \( A = \sqrt{P_{xz}} \) such that \( P_{xz} = AA^T \) and a possible way for the calculation of this variable is Singular Value Decomposition (SVD) [Rigatos & Zhang 2001]. As in the case of the Extended Kalman Filter, the UKF is a special case of Sigma-Point Kalman Filters. The UKF is a discrete time filter which uses the unscented transform for approximating solutions to the filtering problem of the form.

\[ x_k = f(x_{k-1}, u_{k-1}) + q_{k-1} \]
\[ y_k = h(x_k) + r_k \]

where \( x_k \in \mathbb{R}^n \) is the state, \( y_k \in \mathbb{R}^m \) is the measurement, \( q_{k-1} \sim N(0, Q_{k-1}) \), and \( r_k \in \mathbb{R}^m \) is a Gaussian measurement noise \( r_k \sim N(0, R_k) \). The mean and covariance of the initial state are \( m_0 \) and \( P_0 \), respectively. Some basic operations performed in the UKF algorithm (Unscented Transform) are summarized as follows:

1) Denoting the state vector mean as \( \hat{x} \), a set of \( 2n + 1 \) sigma points is taken from the columns of the \( n \times n \) matrix \( \sqrt{(\lambda + \lambda I)P_{zz}} \) as follows:

\[ x^i = \hat{x} + \sqrt{(\lambda + \lambda I)P_{zz}} \] \( i = 1, \cdots, n \)
\[ x^i = \hat{x} - \sqrt{(\lambda + \lambda I)P_{zz}} \] \( i = n + 1, \cdots, 2n \)

and the associate weights are computed:

\[ W_0 = \frac{\lambda}{(\lambda + \lambda I)} \] \( i = 1, \cdots, n \)
\[ W_i = \frac{1}{2(\lambda + \lambda I)} \] \( i = n + 1, \cdots, 2n \)

Matrix \( P_{xx} \) is the covariance matrix of the state \( x \) and index \( i \) denotes its \( i \)-th column. The parameter \( \lambda \) is a scaling parameter.

2) Transform each of the sigma points as:

\[ z^i = h(x^i) \] \( i = 0, \cdots, 2n \)

3) Mean and covariance estimates for \( z \) can be computed as:

\[ z = \sum_{i=0}^{2n} W_i z^i \]
\[ P_z = \sum_{i=0}^{2n} W_i (z^i - \bar{z})(z^i - \bar{z})^T \]

4) The cross-covariance of \( x \) and \( z \) is estimated as:

\[ P_{xz} = \sum_{i=0}^{2n} W_i (x^i - \bar{x})(z^i - \bar{z})^T \]

The matrix square root of a positive definite matrix \( P_{xz} \) means a matrix \( A = \sqrt{P_{xz}} \) such that \( P_{xz} = AA^T \) and a possible way for the calculation of this variable is Singular Value Decomposition (SVD) [Rigatos & Zhang 2001]. As in the case of the Extended Kalman Filter, the Unscented Kalman Filter also consists of prediction stage (time update) and correction stage (measurement update) [Julier & Uhlmann], [van der Merwe et al. 2004], [Särrkä 2007], [Kandepu et al 2008].

**Time update:** Compute the predicted state mean \( \hat{x}_{k|k} \) and the predicted covariance \( P_{x|x_{k|k-1}} \) as:

\[ \hat{x}_{k|k} = UT(f_{k}, \hat{x}_{k-1}, P_{x|k-1}) \]
\[ P_{x|x_{k|k-1}} = P_{x|k-1} + Q_{k-1} \]

**Measurement update:** Obtain the new output measurement \( z_k \) and compute the predicted mean \( \hat{z}_k \) and covariance of the measurement \( P_{zz_k} \):

\[ \hat{z}_k = UT(h_{k}, \hat{x}_k, P_{x|x_{k|k-1}}) \]
\[ P_{zz_k} = P_{zz} + K_k R_k \]

Then compute the filter gain \( K_k \), the state mean \( \hat{x}_k \) and the covariance \( P_{x|x_{k|k}} \) conditional to the measurement \( y_k \):

\[ K_k = P_{zz_k}P_{x|x_{k|k-1}} \]
\[ \hat{x}_k = \hat{x}_{k|k} + K_k(z_k - \hat{z}_k) \]
\[ P_{x|x_{k|k}} = P_{x|x_{k|k-1}} - K_k P_{zz_k} K_k^T \]

The filter starts from the initial mean \( m_0 \) and covariance \( P_{xx_0} \). The stages of state vector estimation with the use of the particle filtering algorithm are depicted in Fig. 5. The Unscented Kalman Filter can be used for state estimation of nonlinear electric motors, such as the induction motor analyzed in Sections 3 and 4.

### 7. Simulation tests

The concept of the simulation experiments was as follows: For linear electric motor models subject to Gaussian measurement or process noise the Kalman Filter is the optimal state estimator, since it results in minimization of the trace of the estimation error’s covariance matrix. For nonlinear electric motor models, subject to Gaussian noise one can use the Extended Kalman Filter. Finally, to overcome some limitations of the EKF (such as the need to calculate Jacobians and cumulative estimation errors due to approximative linearization), the Unscented Kalman Filter can be used.

#### 7.1 Kalman Filter-based control of the DC motor model

The DC motor of Eq. (3) and Eq. (4) in its linearized form of Eq. (10) is considered. The control law of Eq. (15) is used to make the motor track the desirable trajectory. The state vector \( \dot{x} \) used in the control law is estimated through Kalman Filtering, as described in Eq. (51)-Eq. (54). To generate Gaussian noise Matlab’s function `randn()` was used. The process and measurement noises were considered to beuncorrelated. The process noise covariance matrix was taken to be \( \frac{1}{2} e \langle w(i)w^T(j) \rangle \) with diagonal elements with diagonal elements \( w_{ii} = 10^{-3} \). The covariance matrix of the measurement noise was defined as \( \frac{1}{2} e \langle \eta(i)\eta^T(j) \rangle = R \delta(i - j) \), with diagonal elements \( r_{ii} = 10^{-2} \). The estimation error covariance matrix \( P \in \mathbb{R}^{3 \times 3} \) and the KF gain \( K \in \mathbb{R}^{3 \times 1} \) were used in Eq. (51) and (54).
γ and a sinusoidal setpoint are depicted in Fig. 6, Fig. 7 and which is given by

\[ \theta^* = \frac{\partial \gamma}{\partial \theta} \]

To perform sensorless control only measurements of the rotor’s angle \( \theta \) were used. The sampling period was taken to be \( T_s = 0.01 \text{sec} \). The tracking performance of the Kalman Filter-based control loop, in the case of a see-saw and a sinusoidal setpoint are depicted in Fig. 6, Fig. 7 and Fig. 8, while the associated control input is shown in Fig. 9. The reference setpoint in denoted by the red line, the state vector variables of the motor are denoted by the blue line, while the estimated state vector elements are described by the green line.

### 7.2 Extended Kalman Filter-based control of the field-oriented induction motor

The nonlinear model of the decoupled field-oriented induction motor, given in Eq. (30) and Eq. (31), was considered. The model’s state variables were taken to be \( x_1 = \theta \) and \( x_2 = \dot{\theta} \). The measurement update of the EKF is given by Eq. (62), while the time update of the EKF is given by Eq. (63). The measured state variable was supposed to be the rotor’s angle \( \theta \). The Jacobian \( J_p(x) \) is the \( 2 \times 2 \) Jacobian of \( \phi \) calculated through the expansion:

\[ \phi(x(k)) = \phi(\hat{x}^-(k)) + J_p[x(k) - \hat{x}^-(k)] + \cdots \]

and which is given by

\[ J_p = \frac{\partial \phi}{\partial x} |_{x = \hat{x}^-} (k) = \begin{pmatrix} \frac{\partial \phi_1}{\partial x_1} & \frac{\partial \phi_1}{\partial x_2} \\ \frac{\partial \phi_2}{\partial x_1} & \frac{\partial \phi_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -mglT \cos(x_1(k)) & 1/T \end{pmatrix} \]

Likewise, \( \gamma \) is expanded about \( \hat{x}^- (k) \)

\[ \gamma(x(k)) = \gamma(\hat{x}^-(k)) + J_p[x(k) - \hat{x}^-(k)] + \cdots \]

where \( \hat{x}^- (k) \) is the estimation of the state vector \( x(k) \) before measurement at the \( k \)-th instant to be received and \( \hat{x}(k) \) is the updated estimation of the state vector after measurement at the \( k \)-th instant has been received. The \( 1 \times 2 \) Jacobian \( J_p(x) \) is

\[ J_p(x) = \begin{pmatrix} \frac{\partial \gamma}{\partial x} |_{x = \hat{x}^-} (k) \end{pmatrix} = \begin{pmatrix} \frac{\partial \gamma_1}{\partial x_1} & \frac{\partial \gamma_1}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \]

(74)

To implement sensorless control for the decoupled field-oriented induction motor model only measurements of the rotor’s angle \( \theta \) were used. Again the tracking performance of the proposed control scheme was tested in the case of a sinusoidal and a see-saw setpoint. As it can be seen in Fig. 10 and Fig. 11 the sensorless controller succeeded asymptotic elimination of the tracking error despite abrupt changes in the reference trajectory, or the existence of process and measurement noises. The control signal of the Extended Kalman Filter-based control loop is depicted in Fig. 12.

### 7.3 Unscented Kalman Filter-based control of the induction motor model

In Unscented Kalman Filter-based control a set of suitably chosen weighted sample points (sigma points) were propagated through the nonlinear system and used to approximate the true value of the system’s state vector and of the state vector’s covariance matrix. The UKF algorithm consisted of two stages, the time update and the measurement update, which are summarized as follows:

The time update of the UKF is

\[ x_{i+1} = \phi(x_i) + L(k-1)u(k-1), \quad i = 0, 1, \cdots, 2n \]

\[ \hat{x}_{k+1} = \sum_{i=0}^{2n} w_i x_i \]

\[ P_{x_{k+1}} = P_{x_{k+1}x_k} + Q_k \]
Kalman Filter can be also used in place of the Extended Kalman Filter. The Unscented Kalman Filter is an efficient approach through Kalman Filtering becomes clear. It can be noticed that the Extended Kalman Filter is an efficient approach for the implementation of state estimation-based control of the sixth-order induction motor model. The Unscented Kalman Filter can be also used in place of the Extended Kalman Filter and in the latter case there will be no need to use the Unscented Kalman Filter for reconstructing its state vector from output measurements.

The simulation experiments of Fig. 13 and 14 show the good tracking performance of the UKF-based control loop, in the case of time varying setpoints (such as see-saw and sinusoidal reference trajectories). Furthermore, the diagrams presented in Fig. 15 show that the control signal, which is applied to the decoupled field-oriented induction motor model, remains smooth.

The approach on flatness-based control of the induction motor that was presented in Section 4 needs knowledge of the electric motor’s state vector \( \mathbf{x} = [\theta, \omega, \psi_{s1}, \psi_{s2}, i_s, i_r, \rho] \). It will be shown that it is possible to implement state estimation for the electric motor using measurements only the rotation angle \( \theta \) and of the stator currents \( i_s \) and \( i_r \). The Extended Kalman Filter can give estimates of the non-measured state vector elements, i.e. of the rotation speed \( \omega \), of the magnetic flux \( \psi_{s1} \) and of the angle \( \rho \) between the flux vectors \( \psi_{s1} \) and \( \psi_{s2} \). Measuring currents \( i_s \) and \( i_r \) and using the estimate of angle \( \rho \), the input measurements \( i_s \) and \( i_r \) can be provided to the Extended Kalman Filter. Thus one has

\[
\begin{pmatrix}
i_s \\
i_r
\end{pmatrix}
= \begin{pmatrix}
\cos(\rho) & \sin(\rho)
\end{pmatrix}
\begin{pmatrix}
i_s \\
i_r
\end{pmatrix}
\tag{75}
\]

The proposed flatness-based control scheme with the use of Extended Kalman Filtering for estimation of the non-measurable parameters of the motor’s state vector is depicted in Fig. 16.

To implement the Extended Kalman Filter in the induction motor’s model that is expressed in the \( dq \) reference frame the following Jacobian matrices are calculated:

\[
J_{\psi} = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & \mu x_5 \\
0 & 0 & 0 \\
0 & n_p x_5 & -\alpha - \frac{a M x_5^2}{x_5} \\
0 & -\beta n_p x_3 - n_p x_4 - \beta n_p x_2 + \frac{a M x_5 x_4}{x_5} & -\gamma \\
0 & n_p x_2 - \frac{a M x_5 x_3}{x_5} & -\gamma - \frac{a M x_5}{x_5}
\end{pmatrix}
\tag{76}
\]

\[
J_\theta = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\tag{77}
\]

Simulation results on flatness-based control of the induction motor when using the Extended Kalman Filter for reconstructing its state vector from output measurements are presented in Fig. 17 to Fig. 22. The real state variable is denoted by the dashed blue line, the estimated state variable is denoted by the dashed green line, while the associated reference setpoint is denoted by the continuous red line. It can be observed that, although using a reduced number of sensors, the proposed state estimation-based control scheme for the induction motor provides accurate tracking of the reference setpoints. Taking into account that several variables of the induction motor state vector (e.g. rotation speed and magnetic flux) are not directly measurable (due to sensors cost and limited reliability, sensors failures and difficulties in sensors installation) the significance of state estimation through Kalman Filtering becomes clear. It can be noticed that the Extended Kalman Filter is an efficient approach for the implementation of state estimation-based control of the sixth-order induction motor model. The Unscented Kalman Filter can be also used in place of the Extended Kalman Filter and in the latter case there will be no need to compute Jacobian matrices.

Remark 1: DC motor control is performed using as control input the armature current or the field voltage. Field oriented induction motor control assumes transformation of the induction motor model from the \( ab \)-reference frame to the \( dq \) reference frame and that the magnetic flux \( \psi_{s1} \) converges to a steady-state value \( \psi_{s1}^{set} \). The field-oriented control for induction motors is implemented through nested control loops. In the outer loop, control of the magnetic flux is performed enabling decoupling between the motor’s speed dynamics and the flux dynamics. This decoupling makes possible to develop controllers of the rotor’s speed/position with methods already applied to DC motors. Finally, flatness-based control for induction motors considers also a nested control loops scheme as depicted in Fig. 16. Transformation to the \( dq \) reference frame is again performed, however this time there is no assumption about decoupling between the rotor speed dynamics and the magnetic flux dynamics. In the inner loop, current control is implemented providing as control inputs to the induction motor the voltages \( v_d \) and \( v_q \). In the outer loop position/speed control and flux control are implemented, providing the current setpoints \( i_d \) and \( i_q \) which become inputs to the inner voltage control loop.
Figure 6. Parameter $x_1$ of the state vector of the DC motor in state estimation with use of the Kalman Filter (a) when tracking a see-saw set-point (b) when tracking a sinusoidal setpoint

Figure 7. Parameter $x_2$ of the state vector of the DC motor in state estimation with use of the Kalman Filter (a) when tracking a see-saw set-point (b) when tracking a sinusoidal setpoint

Figure 8. Parameter $x_3$ of the state vector of the DC motor in estimation was performed with use of the Kalman Filter (a) when tracking a see-saw setpoint (b) when tracking of a sinusoidal setpoint
Figure 9. Control signal of the Kalman Filter-based control loop for the DC motor model (a) when tracking a see-saw setpoint (b) when tracking of a sinusoidal setpoint

Figure 10. Parameter $x_1$ of the state vector of the field-oriented induction motor model in estimation with use of the Extended Kalman Filter (a) when tracking a see-saw set-point (b) when tracking a sinusoidal setpoint

Figure 11. Parameter $x_2$ of the state vector of the field-oriented induction motor model in state estimation with use of the Extended Kalman Filter (a) when tracking a see-saw set-point (b) when tracking a sinusoidal setpoint
Figure 12. Control signal of the Extended Kalman Filter-based control loop for the field-oriented induction motor model (a) when tracking of a see-saw setpoint (b) when tracking of a sinusoidal setpoint

Figure 13. Parameter $x_1$ of the state vector of the field-oriented induction motor model in estimation was performed with use of the Unscented Kalman Filter (a) when tracking a see-saw setpoint (b) when tracking of a sinusoidal setpoint

Figure 14. Parameter $x_2$ of the state vector of the field-oriented induction motor model in estimation was performed with use of the Unscented Kalman Filter (a) when tracking a see-saw setpoint (b) when tracking of a sinusoidal setpoint
Figure 15. Control signal of the Unscented Kalman Filter-based control loop for the field-oriented induction motor model (a) when tracking a see-saw setpoint (b) when tracking of a sinusoidal setpoint

Figure 16. Schematic diagram the proposed flatness-based control scheme with the use of Extended Kalman Filtering

Figure 17. Flatness-based control of the induction motor with the use of Extended Kalman Filtering in case of tracking a seesaw setpoint (a) rotor’s angle θ (b) rotor’s speed ω
Figure 18. Flatness-based control of the induction motor with the use of Extended Kalman Filtering in case of tracking a seesaw setpoint
(a) stator’s current \( i_d \) (b) stator’s current \( i_q \)

Figure 19. Flatness-based control of the induction motor with the use of Extended Kalman Filtering in case of tracking a constant setpoint
(a) rotor’s angle \( \theta \) (b) rotor’s speed \( \omega \)

Figure 20. Flatness-based control of the induction motor with the use of Extended Kalman Filtering in case of tracking a constant setpoint
(a) stator’s current \( i_d \) (b) stator’s current \( i_q \)
Remark 2: To compensate for external disturbances and parameter variations in state estimation-based nonlinear control schemes the following approaches are possible (i) use of adaptive Kalman Filter, (ii) redesign of the Kalman filter in the form of a disturbance observer, (iii) redesign of the Kalman Filter in the form of a high-gain Extended Kalman Filter. Regarding (i), adaptive Kalman Filtering is implemented through the adaptation of the process and measurement noise covariance matrices in the Kalman Filter recursion so as to cope with variable noise levels [Alonge & Ippolito 2010]. Regarding (ii), additive disturbances and parametric changes can be identified with the use of Kalman Filters that operate as disturbance observers (see work on Kalman Filtering and disturbance observers in [Rigatos 2011]). Once the disturbance affecting the nonlinear system becomes known it can be compensated through the introduction of an additional control term in the loop. Thus, it can be assured again that the estimation error $x - \hat{x}$ will be minimal and the performance of the control loop will be satisfactory. There are several results on disturbance observers (e.g. Extended State Observers, Unknown Input Observers or Perturbation Observers) and on their use within a Kalman Filter framework [Miklosovitch et al 2006], [Kwon & Chung 2003]. Such results can be exploited so as to make Kalman Filtering for electric motors as well as the associated state estimation-based control loop be more robust [Rigatos 2011]. Regarding (iii) the high-gain Extended Kalman Filter can provide additional robustness to state estimation under external disturbances and parametric variations [Boizot et al. 2010], [Besançon et al. 2004].

8. Conclusions

The paper has studied sensorless control, for DC and induction motors, using Kalman Filtering techniques. First the case of a DC motor was considered. The Kalman Filter was proposed for estimating the state vector of the electric motor and Kalman Filter-based control was implemented. Next the nonlinear model of a field-oriented induction motor was considered. The motor’s angular velocity was estimated by an Extended Kalman Filter which
used rotor angle measurements, and sensorless control of the induction motor was again implemented through feedback of the estimated state vector. Moreover, a state estimation-based control loop was implemented, using the Unscented Kalman Filter to estimate the induction motor’s state vector. Finally, state estimation-based control was developed for the complete (sixth-order) dynamical model of the induction motor using a nonlinear flatness-based controller and the state estimation provided by the Extended Kalman Filter. Unlike field-oriented control, in the latter approach there is no assumption about decoupling between the rotor’s speed dynamics and the magnetic flux dynamics. The efficiency of the aforementioned Kalman Filter-based control schemes, for both the DC and the induction motor models, was tested through simulation experiments.

The paper has shown that in the design of state estimation-based control for electric motors the following should be taken into account: (i) for linear electric motor models subject to Gaussian measurement or process noise the Kalman Filter is the optimal state estimator, since it results in minimization of the trace of the estimation error’s covariance matrix, (ii) for nonlinear electric motor models, subject to Gaussian noise one can use the generalization of the Kalman Filter as formulated in terms of the Extended Kalman Filter. The EKF appears to be an efficient estimator for the implementation of state estimation-based control schemes. (iii) to overcome certain limitations of the EKF (such as the need to compute Jacobian matrices and the cumulative linearization errors due to approximative linearization of the motor dynamics), Sigma Point Kalman Filters (SPKF), and particularly the Unscented Kalman Filter (UKF) can be used.

The Kalman Filtering approaches examined in this paper have shown that it is possible to reduce the number of sensors involved in the control loops of electric motors and to implement efficient state estimation-based control. Kalman Filter-based control for DC and induction motors can have several applications for the design of industrial and robotic systems of improved performance.

9. References

[Alonge & D’ippolito 2010] Alonge, F. & D’ippolito, A. (2010). Robustness analysis of an Extended Kalman Filter for sensorless control of induction motors 2010 IEEE International Symposium on Industrial Electronics. ISIE 2010, Bari, Italy, July 2010.

[Akin et al. 2003] Akin, B.; Orguner, U. & Ersak, A. (2003). State estimation of induction motor using unscented Kalman filter. Control Applications, 2003. CCA 2003. Proceedings of 2003 IEEE Conference, Instabul, Turkey, 2003.

[Akin et al. 2004] Akin, B.; Orguner, U. & Ersak, A. (2004). A comparative study on Kalman filtering techniques designed for state estimation of industrial AC drive systems. Proceedings of the IEEE International Conference on Mechatronics 2004, ICM’04; Istanbul; June 2004.

[Akin et al. 2006] Akin, B.; Orguner, U. & Ersak, A. (2006). Simple derivative-free nonlinear state observer for sensorless AC drives. IEEE/ASME Transactions on Mechatronics, vol. 11, no. 5, pp. 634-643.

[Bassecriveille & Nikiforov 1993] Bassecriveille, M. & Nikiforov, I. (1993). Detection of abrupt changes: Theory and Applications. Prentice-Hall.

[Boedson et al. 1994] Bodson, M.; Chiasson, J. & Novotnak, R. (1994). High-Performance Induction Motor Control via Input-Output Linearization. IEEE Control Systems Magazine, pp. 24-33.

[Borsje et al. 2005] Borsje, P.; Chan, T.F., Wong, Y.K. & Ho, S.L. (2005). A comparative study of Kalman filtering for sensorless control of a permanent-magnet synchronous motor drive. 2005 IEEE International Conference on Electric Machines and Drives, art. no. 1531434, pp. 815-822, San Antonio, Texas, 2005.

[Dannhe & Fuchs 2006] Dannhe, J. & Fuchs, F.W. (2006). Flatness-based control of an induction machine fed via voltage source inverter - Concept, control design and performance analysis. IECN 2006, IEEE Conference on Industrial Electronics, pp. 5125-5130.

[Delaleau et al. 2001] Delaleau, E.; Louis, J.P. & Ortega, R. (2001). Modelling and Control of Induction Motors. International Journal of Applied Mathematics and Computer Science, vol.11, no.1, pp. 105-129.

[Flies & Mounier 1999] Flies, M. & Mounier, H. (1999). Tracking control and ϕ-freeness of infinite dimensional linear systems. In: G. Picci and D.S. Gilliam Eds.,Dynamical Systems, Control, Coding and Computer Vision, vol. 258, pp. 41-68, Birkhãuser.

[Hilairet et al. 2009] Hilairet, M.; Augerb, F. & Berthelot, E. (2009). Speed and rotor flux estimation of induction machines using a two-stage extended Kalman filter. Automatica, Elsevier, vol. 45, pp. 1819-1827.

[Holtz 2002] Holtz, J. (2002). Sensorless control of induction motor drives. Proceedings of the IEEE, vol. 90, no. 8, pp. 1359-1394.

[Horng 1999] Horng, J.H. (1999). Neural adaptive tracking control of a DC motor. Information Sciences, Elsevier, vol. 118, pp. 1-13.

[Julier et al. 2000] Julier, S.; Uhlmann, J. & Durrrant-Whyte, H.F. (2000). A new method for the nonlinear transformations of means and covariances in filters and estimators. IEEE Transactions on Automatic Control, vol.45, no.3, pp. 477-482.

[Julier & Uhlmann 2004] Julier, S. & Uhlmann, J.K. (2004). Unscented Filtering and Nonlinear Estimation. Proceedings of the IEEE, vol.92, pp. 401-422.

[Kamen & Su 1999] Kamen, E.W. and Su, J.K. (1999). Introduction to Optimal Estimation. Springer.

[Kandepu et al. 2008] Kandepu, R.; Foss, B. & Immland, M. (2008). Applying the unscented Kalman filter for nonlinear state estimation. Journal of Process Control, Elsevier, vol. 18, pp. 753-768.

[Karami et al. 2010] Karami, F.; Poshtian, J. & Poshtian, M. (2010). Model-based fault detection in induction Motors. 2010 IEEE International Conference on Control Applications, CCA 2010, Yokohama, Japan, September 2010.

[Kumar et al. 2011] Kumar, S.; Prakash, J. & Kanagasabapathy, P. (2011). A critical evaluation
and experimental verification of Extended Kalman Filter, Unscented Kalman Filter and Neural State Filter for state estimation of three phase induction motor. Applied Soft Computing Journal, Elsevier, vol. 11, no. 3, pp. 3199-3208.

[Kwon & Chung 2003] Kwon, S. & Chung, W.K. (2003). Combined synthesis of state estimator and perturbation observer. Journal of Dynamic Systems Measurement and Control, ASME, vol. 125, pp. 19-26.

[Leonard 1985] Leonard, W. (1985). Control of Electrical Drives. Springer-Verlag.

[Lin et al 2000] Lin, F.J.; Wai, R.J., Lin, C.H. & Liu, D.C. (2000). Decoupled stator-flux-oriented induction motor drive with fuzzy neural network uncertainty observer. IEEE Transactions on Fuzzy Systems, vol. 8, no.5, pp. 193-197.

[Marino & Zafiriou 1991] Marino, R.; Peresada, S. & Valigi, P. (1991). Adaptive Input-Output Linearizing Control of Induction Motors. IEEE Transactions on Automatic Control, vol. 38, no.2, pp. 208-221.

[Martín & Rouche 1996] Martín, P. and Rouche, P. (1996). Two remarks on induction motors. CES’96 IMACS Multiconference, vol. 1, pp. 76-79, Lille, France.

[van der Merwe et al. 2004] van der Merwe, R.; Wan, E.A. & Julier S.I. (2004). Sigma-Point Kalman Filters for nonlinear estimation and sensor-fusion applications to integrated navigation. Proc. of the AIAA Guidance, Navigation and Control Conference, Providence, RI, USA, August, 2004.

[Miklosovitch et al. 2006] Miklosovitch, R.; Radke, A. & Gao, Z. (2006). Discrete implementation and generalization of the Extended State Observer. Proceeding of the American Control Conference, Minneapolis, Minnesota, June 2006.

[Nounou & Rehman 2007] H.N. Nounou and H. Rehman, Application of adaptive fuzzy control to AC machines. Applied Soft Computing, Elsevier, vol. 7, no.3, pp. 899-907, 2007.

[Rigatos & Zhang 2001] Rigatos, G. & Zhang, Q. (2001). Fuzzy Model Validation using the Local Statistical Approach. Publication Interne IRISA No 1417, Rennes, France.

[Rigatos & Tzafestas 2007a] Rigatos, G.G. & Tzafestas S.G. (2007). Extended Kalman Filtering for Fuzzy Modelling and Multi-Sensor Fusion. Mathematical and Computer Modelling of Dynamical Systems, Taylor & Francis, vol. 13, pp. 251-266.

[Rigatos & Tzafestas 2007b] Rigatos, G.G. & Tzafestas, S.G. (2007). $H_\infty$ tracking of uncertain SISO nonlinear systems: An observer-based adaptive fuzzy approach. International Journal of Systems Science, Taylor and Francis, vol. 38, no.6, pp. 459-472.

[Rigatos 2008] Rigatos, G.G. (2008). Particle Filtering for state estimation in industrial robotic systems. IMech Journal of Systems and Control Engineering, vol. 222, no.6, pp. 437-455.

[Rigatos 2009a] Rigatos, G.G. (2009). Adaptive fuzzy control of DC motors using state and output feedback. Electric Power Systems Research, Elsevier, vol. 79, no. 11, pp. 1579-1592.

[Rigatos 2009b] Rigatos, G.G. (2009). Particle and Kalman filtering for state estimation and control of DC motors. ISA Transactions, Elsevier, vol. 48, no. 1, pp. 62-72.

[Rigatos 2009c] Rigatos, G.G. (2009). Particle Filtering for State Estimation in Nonlinear Industrial Systems. IEEE Transactions on Instrumentation and Measurement, vol. 58, no.11, pp. 3885-3901.

[Rigatos 2009d] Rigatos, G.G. (2009). Particle and Kalman filtering for fault diagnosis in DC motors. IEEE VPPC 2009, IEEE VPPC 2009, IEEE 5th Vehicular Power Propulsion Conference, Michigan, USA, Sep. 2009

[Rigatos 2009e] Rigatos, G.G. (2009). Sigma-point Kalman Filters and Particle Filters for integrated navigation of unmanned aerial vehicles. Intl. Workshop on Robotics for Risky Interventions and Environmental Surveillance, RISE 2009, Brussels, Belgium, Jan. 2009.

[Rigatos 2011] Rigatos, G.G. (2011). Modelling and Control for Intelligent Industrial Systems: Adaptive Algorithms in Robotics and Industrial Engineering, Springer.

[Rudolph 2003] Rudolph, J. (2003). Flatness Based Control of Distributed Parameter Systems. Steuerungs- und Regelungstechnik, Shaker Verlag, Aachen, 2003.

[Särkkä 2007] Särkkä, S. (2007). On Unscented Kalman Filtering for state estimation of continuous-time nonlinear systems. IEEE Transactions on Automatic Control, vol.52, no.9, pp.1631-1641.

[Villagra et al 2007] Villagra, J.; d’Andrea-Novel, B., Mounier, H. & Pengov, M. (2007). Flatness-based vehicle steering control strategy with SDRE feedback gains tuned via a sensitivity approach. IEEE Transactions on Control Systems Technology, vol. 15, pp. 554-565.

[Wai & Chang 2001] Wai R.J. & Chang J.M. (2001). Intelligent control of induction servo motor drive via wavelet neural network. Electric Power Systems Research, Elsevier, vol. 61, no.1, pp. 67-76.

[Wai & Chang 2003] Wai R.J. & Chang J.M. (2003). Implementation of Robust Wavelet-Neural-Network Sliding-Mode Control for Induction Servo Motor Drive. IEEE Transactions on Industrial Electronics, vol. 50, no.6, pp. 1317-1334.

[Wai & Chang 2004] Wai R.J. & Chang, H.H. (2004). Backstepping Wavelet Neural Network Control for Indirect Field-Oriented Induction Motor Drive. IEEE Transactions on Neural Networks, vol. 15, no. 2, pp. 367-382.

[Xiong 2008] Xiong, J. (2008). An Introduction to Stochastic Filtering Theory. Oxford University Press.

[Boizot et al. 2010] Boizot, N.; Busvelle, E. & Gauthier, J.-P. (2010). An adaptive high-gain observer for nonlinear systems. Automatica, Elsevier, vol. 46, pp. 1483-1488.

[Besançon et al. 2004] Besançon, G., Zhang, Q. & Hammouri H., High-gain observer based state and parameter estimation in nonlinear systems. IFAC Symposium on Nonlinear Control Systems (NOLCOS), Stuttgart, Germany, 2004.