Thermodynamic modeling of continuous flow liquid helium thermostat

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Abstract. Since the discovery of cryogenic superconductivity, liquid helium-based cryogenic superconductivity has been successfully applied in many disciplines. And continuous flow cryostats have great importance for the efficient and stable operation of the entire cryogenic system, which can provide a cryogenic working environment for large scale cryogenic applications. In this paper, a thermodynamic model of the liquid helium thermostat is developed based on the principle of continuous flow helium cryostat and the thermodynamic processes between liquid helium and gas helium, using mass conservation, energy conservation and pressure-volume-temperature relationships. The research work can provide theoretical guidance for the stability analysis of continuous flow cryostats, and also has practical value in large-scale cryogenic applications.

1. Introduction

With the study of gas liquefaction and the physical properties of materials at low temperatures, low-temperature and superconducting technologies have begun to develop, and they have been widely used in high-energy physics accelerators, magnetically confined nuclear fusion, and low-temperature superconductivity \cite{1}. The continuous flow helium cryostat is one of the key equipment for large-scale superconducting applications. Its performance must meet the requirements of low-temperature users. It will have a direct impact on the low-temperature physics experiment and may even determine the success or failure of the entire experiment. In order to ensure the low temperature of helium The normal operation of the thermostat, the design and maintenance of the low temperature environment required by the helium cryostat and the related control of the liquid helium and gas helium valve box are particularly important \cite{2}. Therefore, it is necessary to analyze and study the mechanism of liquid helium boiling and gas helium emission inside the liquid helium thermostat, so as to provide a theoretical basis for the control of liquid helium and gas helium valves.

At present, there has been a certain literature research on the thermodynamic analysis of cryostats. Wang Bin \cite{3} and others have carried out related thermodynamic research work with the goal of low-temperature propellant liquid hydrogen storage tank pressure control, and established the fluid flow and gas flow in the storage tank. The mathematical model of the liquid phase transition process compares the pressure rise and fall characteristics of the tank air pillow under the two different operating modes of mixing and exhaust, and also analyzes the temperature change law of the liquid in the tank. Zhou
Zhenjun [4] and others built a pressure control test platform for liquid nitrogen thermodynamic exhaust technology, carried out a low temperature storage tank pressure control test in mixed mode and parallel mode, and studied the changes of air pillow pressure and liquid phase temperature in different pressure control intervals law. Han Ruixiong [5] et al. studied and analyzed the source of the cryostat heat load on the basis of the CM4 cryostat.

In summary, the current research objectives for the thermodynamics of low-temperature devices such as liquid hydrogen and liquid nitrogen, but the key to the thermodynamics of liquid low-temperature heating devices, and the most suitable for the heat transfer and optimization design of the liquid helium low-temperature heating device, the thermostat of the liquid helium cryogenic device Study the change between the two phases of helium inside. In order to further improve the performance indicators of the cryogenic system and better serve the cryogenic superconductive equipment, on the basis of the above research, this article sets out to study more accurate mathematics based on the thermophysical properties of liquid helium and the working principle of the continuous flow cryostat Model, based on the conservation of liquid and gas mass, energy conservation, and pressure-temperature-volume relations, a dynamic mathematical model of the gas-liquid two-phase equilibrium inside the liquid helium thermostat was established. It can lay a theoretical foundation for follow-up related research.

2. Liquid helium thermostat structure model

Due to the complexity of the actual thermostat, this article abstracts and simplifies it, as shown in Figure 1. The model is mainly composed of an inner cylinder, a superconducting cavity, an outer cylinder, a vacuum insulation layer, a radiation screen, and liquid nitrogen. Cold screen composition. In order to reduce the influence of external heat leakage, the designed inner cylinder heat insulation part adopts the combined heat insulation of VD-MLI air-filled cooling screen. The air cooling screen consumes liquid nitrogen cooling to reduce the heat leakage of the cryostat. The uninsulated part of the inner cylinder passes through The convective heat exchange of the environment heats the helium temperature flowing out of the inner cylinder to prevent frosting on the surface of the flange cover, and at the same time, to reduce the radiation heat leakage of the flange cover, a multi-layer radiation screen is set between it and the liquid surface , Which greatly weakens the radiation leakage heat [6].

![Diagram of liquid helium thermostat structure](image-url)
### 3. Analysis of Mathematical Model of Liquid Helium Thermostat

The four conservation equations used in the dynamic mathematical model of liquid helium thermostat gas-liquid two-phase equilibrium are liquid mass conservation, gas mass conservation, energy conservation, and volume conservation. The mass conservation of gas and liquid can be written as:

\[
\frac{dm_L}{dt} = \frac{dm_{in}}{dt} - \frac{dm_{Boil}}{dt} \tag{1}
\]

\[
\frac{dm_G}{dt} = \frac{dm_{Boil}}{dt} - \frac{dm_{out}}{dt} \tag{2}
\]

Where \(\frac{dm_{in}}{dt}\) represents the quality of the injected liquid helium, \(\frac{dm_{Boil}}{dt}\) represents the mass rate of boiling liquid helium, and \(\frac{dm_{out}}{dt}\) represents the mass rate of helium discharged through the pressure.

The sum of energy in the cryostat can be obtained from the sum of the energy of the liquid and the energy of the gas, which are derived from the time respectively:

\[
\frac{de}{dt} = u_L \frac{dm_L}{dt} + m_L \frac{du_L}{dt} + u_G \frac{dm_G}{dt} + m_G \frac{du_G}{dt} \tag{3}
\]

Where \(e\) represents the internal energy of the thermostat, \(u_L\) and \(u_G\) represents the specific internal energy of liquid helium and gas helium respectively. At the same time, the rate of change of energy can also be calculated according to the enthalpy of the input and output of the liquid and gas, which can be obtained by combining the above equations:

\[
\frac{dQ}{dt} = \frac{dm_{in}}{dt} h_L + \frac{dm_{Boil}}{dt} (h_G - h_L) - \frac{dm_{out}}{dt} h_G \tag{4}
\]

Where \(\frac{dQ}{dt}\) represents the external heat transfer to the liquid helium thermostat and the heat leakage of the liquid helium thermostat itself, \(h_L\) and \(h_G\) represents the enthalpy value of liquid and the enthalpy value of gas. Substituting formulas (1) and (2) into (4) respectively, it can be obtained by simplification:

\[
\frac{dQ}{dt} + \frac{dm_{in}}{dt} (h_L - u_L) + \frac{dm_{Boil}}{dt} (h_G - h_L + u_L - u_G) + \frac{dm_{out}}{dt} (u_G - h_G) = (m_L \frac{du_L}{dP} + m_G \frac{du_G}{dP}) \frac{dP}{dt} \tag{5}
\]

Define \(a_1 = \frac{dQ}{dt} + \frac{dm_{in}}{dt} (h_L - u_L) + \frac{dm_{out}}{dt} (u_G - h_G)\) and \(a_2 = (m_L \frac{du_L}{dP} + m_G \frac{du_G}{dP})\).

Therefore, the dynamic mathematical model of the internal pressure of the liquid helium thermostat can be obtained as:

\[
\frac{dP}{dt} = a_1 \frac{dm_{Boil}}{dt} \frac{(h_G - h_L + u_L - u_G)}{a_2} \tag{6}
\]

\(\frac{dm_{Boil}}{dt}\) it can be known from the reference to set it as a constant value\(^{[6]}\). From equation (6), the enthalpy and internal energy of liquid and gas need to be solved.
In order to calculate the internal energy value of liquid and gas, according to NIST data [7], which contains the relationship between the internal energy and pressure of helium along the saturated liquid and vapor curve to fit a curve, the following relationship is found after research:

\[ u = \lambda P^2 + \gamma P + \eta \] (7)

In order to accurately link the pressure in the cryostat with the internal energy, for the pressure range of 0.9-1.5 bar, we found that the coefficients in Table 1 were used to fit the internal energy and pressure curve:

| Coefficient | Liquid | Gas |
|-------------|--------|-----|
| \( \lambda \) | 0      | -1.9034 \times 10^{-7} |
| \( \gamma \) | 0.04897 | 0.03977 |
| \( \eta \)  | -5794.82 | 12638.58 |

Knowing the internal energy of thermodynamics, the change of its enthalpy can be defined by the internal energy of thermodynamics:

\[ h = u + \frac{P}{\rho} \] (8)

Where \( P \) indicates stress, \( \rho \) represents the density of gas or liquid. In the above process, the internal energy and value of liquid helium and gas helium and the internal energy and pressure of liquid helium and gas are accurately transmitted.

Finally, we get the dynamic mathematical model of the gas-liquid two-phase equilibrium inside the liquid helium thermostat. The dynamic mathematical models of liquid level and pressure are:

\[
\frac{dL}{dt} = \frac{dm_{\text{in}}}{dt} - \frac{dm_{\text{out}}}{dt} - \frac{P}{\rho_L S}
\] (9)

\[
\frac{dP}{dt} = \frac{a_1}{a_2} \frac{dm_{\text{out}}}{dt} (h_G - h_L + u_L - u_G) + b_2
\] (10)

Where \( \rho_L \) represents the density of liquid helium, \( S \) represents the bottom area of the thermostat.

4. Model verification and simulation

There are two ways to define the inlet and outlet values on the cryostat. One way is to determine what inlet and outlet values are required to cause a specific behaviour in the level and pressure within the vessel. This approach is helpful in validating the model. Another way is to determine the level and pressure responses to a given set of input and output conditions, and determine how these vary in time. This aspect is more applicable to situations where the model is being used to determine how the real system will perform under certain circumstances.

Because the cold GHe return system is involved in determining the mass flow of gas out of the cryostat, simulating the actual control valves is impossible unless a simulation of the GHe return system is combined with this cryostat model. Flow rates into the cryostat are dictated by the LHe supply line, meaning that unless some model for the LHe supply line is attached to the model of the cryostat, the
level controller and valve position cannot be used to indicate the actual flow rates into the vessel. Therefore, for the stand-alone operation of the vessel simulation used in the model validation process, simple discrete proportional integral (PI) controllers of the form.

Figure 2 shows the actual and simulated fill rates for the cryostat, indicates that overall the slopes of the actual and simulated fill level data match fairly well as do the shapes of the plots. Thus the cryostat model can predict the filling of the cryostat with reasonable accuracy. From Figure 3, it is worth noting that the simulated pressure curve matches the actual pressure curve almost exactly.

5. Conclusion
This article takes the continuous flow cryostat of a large cryogenic system as the research object to ensure that the performance indicators of the cryostat meet the requirements of the stability and working accuracy of the cryogenic system. Through in-depth investigation and research on the liquid helium thermostat system, and low temperature Analysis of the working mechanism of the liquid helium thermostatt to realize the thermodynamic analysis of the cryostat and the detailed mathematical analysis of
the liquid level and pressure changes caused by the boiling of liquid helium and the generation of helium to express the time-varying liquid helium thermostat. The working state provides a theoretical basis for the transmission of liquid helium and the discharge of gas helium in the liquid helium thermostat. And use the data obtained through experiments for simulation to predict the behavior of the cryostat with reasonable accuracy LHe level during filling and outlet mass flow gas used for heat load of various cryostats.

Acknowledgments
This article is based on the National Natural Science Foundation of China (51767013), the Natural Science Foundation of Gansu Province (20JR5RA395), and the Youth Science Foundation of Lanzhou Jiaotong University (2019032).

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