Ultra-intense laser pulse propagation in plasmas: from classic hole-boring to incomplete hole-boring with relativistic transparency

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Abstract. Relativistic laser pulse propagation into homogeneous plasmas has been investigated as a function of plasma density. At first, the propagation features are compared systematically between relativistic transparency (RT) and hole-boring (HB). Paramountly, a considerably broad intermediate regime, namely the incomplete HB regime, has been found between the RT regime and the HB regime for an extremely intense circularly polarized (CP) pulse. In this regime HB proceeds in collaboration with RT, resulting in a much faster propagation speed and a higher cut-off energy of fast ions than in the classic HB regime. Similarly to the classic HB regime, formulae are presented to model the laser propagation and the ion acceleration according to the modified momentum flux balance in this incomplete HB regime. The simulations give the density boundary between this incomplete HB regime and the classic HB regime for CP pulses, which is crucial for estimating the maximum mean ion energy and the maximum conversion efficiency that can be achieved by the classic HB acceleration at a given laser intensity. For linear polarization (LP) the propagation mechanism apparently undergoes a transition in time between these two regimes. A detailed comparison between LP and circular polarization is made for these phenomena.

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1. Introduction

As the interaction of a relativistically intense laser pulse with matter is the basis of phenomena such as fast ignition of controlled fusion pellets [1–4] and laser particle acceleration [5, 6], it has attracted increasing attention from both the laser and the plasma communities during the last few years [7]. At the intensity $I \geq 10^{18} \text{Wcm}^{-2}$, laser pulse propagation into the overdense plasma is accomplished by relativistic transparency (RT) [8–10] or and by hole-boring (HB) [11–19]. Reinforcement of these two phenomena may happen by beam self-focusing [20, 21] and filamentation [22]. RT is entirely based on the fast electron response, i.e. their energy conservation [9, 10], but in the HB regime the dynamics occurs on the ion time scale and hence is governed by the conservation of ion momentum. The electrostatic field induced by the ponderomotive force becomes sufficiently strong to drive a collisionless shock into the plasma and to accelerate the ions to high velocity [13–19, 23]. So far, the two different mechanisms have been studied at different densities, but very little attention has been paid to the transition between these two regimes [9, 24]. For instance, the plasma density at which the transition happens is still unknown even in the one-dimensional (1D) case.

In this paper, we will investigate systematically the density dependence of laser pulse propagation, shedding light on the density at which the transition of propagation mechanisms occurs. In a mildly intense circularly polarized (CP) pulse, it is characterized by a step-like transition of the pulse front propagation speed and other features accompanying the pulse propagation, whereas in an extremely intense CP pulse, the existence of a broad intermediate regime, i.e. the incomplete HB regime, will be demonstrated and explained. Laser pulse propagation and ion acceleration in this incomplete HB regime will be compared in detail with that in the classical HB regime. Most important, it is found that at the same laser intensity a much higher cut-off energy of fast ions can be achieved in this incomplete HB regime than in the classic HB regime. A modified momentum flux balance will be introduced to model pulse propagation and ion acceleration in this incomplete HB regime. In order to evaluate the maximum mean ion energy and the maximum conversion efficiency from the classic HB acceleration at a given CP laser intensity, we determine the density boundary between the incomplete HB regime and the classic HB regime in CP pulses with the help of simulations. Finally, pulse propagation in linear polarization (LP) is studied and compared with propagation in CP.
2. Theories in two different regimes

Theoretical models of laser propagation and ion acceleration in the HB regime have been well established over the last decades on the basis of the momentum flux balance \([13–18]\). Assuming that the relativistic laser interacts with a very steep overdense plasma, the momentum flux balance in the frame comoving with the laser front at speed \(v_b\) can be written as \([17, 18]\)

\[
\frac{2I}{c} \left( 1 - \beta_b^2 \right) = 2\gamma_b^2 m_i n_i v_b^2 = \frac{2A}{Z} \gamma_b^2 m_p n_{e0} v_b^2, \tag{1}
\]

\(\beta_b = v_b/c, \ \gamma_b = (1 - \beta_b^2)^{-1/2}, \ A = m_i/m_p, \ m_i \) is the ion mass, \(m_p\) the proton mass and \(Z\) the ionic charge state. It is convenient to introduce the dimensionless parameter

\[
\Pi = \sqrt{\frac{I}{m_i n_i c^3}} = a \sqrt{\frac{Z m_e n_c}{A 2 m_p n_{e0}}}, \tag{2}
\]

where the dimensionless laser amplitude \(a\) is related to the laser intensity \(I\) by \(I\lambda^2 (\text{W cm}^{-2} \mu\text{m}^2) = 1.37 \times 10^{18} a^2\), with the critical density \(n_c = m_e e_0 \omega^2/e^2\), the laser wavelength \(\lambda\) and frequency \(\omega\), the electron charge \(e\) and mass \(m_e\), the ion density \(n_i\) and the electron density \(n_{e0}\). From equations (1) and (2), one obtains the HB velocity \(v_b\), the mean longitudinal ion momentum \(p_{x,i}\), the mean ion energy \(\varepsilon_i\) and the conversion efficiency of acceleration \(\chi_i\) as follows:

\[
v_b/c = \frac{\Pi}{1 + \Pi}, \tag{3}
\]

\[
p_{x,i}/m_i c = \frac{2\Pi(1 + \Pi)}{1 + 2\Pi}, \tag{4}
\]

\[
\varepsilon_i/m_e c^2 = \frac{2\Pi^2}{1 + 2\Pi}, \tag{5}
\]

\[
\chi_i = \frac{2\Pi}{1 + 2\Pi}. \tag{6}
\]

In contrast, in the RT regime the pulse propagation is found to be governed by the energy balance \([9, 10, 25]\). Compared to the group velocity from dispersion, the propagation velocity \(v_i\) is found to be inhibited significantly \([10]\). At ultra-relativistic intensities \((a \geq 10)\), \(v_i\) can be well estimated by \([10]\)

\[
v_i = \exp \left( -\frac{n_p}{n_{cr}} \right) v_b = \exp \left( -\frac{n_p}{n_{cr}} \right) \left( 1 - \frac{n_{e0}}{n_{cr}} \right)^{1/2} c, \tag{7}
\]

where \(n_p\) is the height of the density peak formed before the laser front, \(n_{cr} = (1 + \Theta a^2)^{1/2} n_c\) is the relativistic critical density with the correction factor \(\Theta\) \([10]\). At ultra-relativistic intensities \((a \geq 10)\), \(n_p \simeq 4n_{e0}\) and \(\Theta \simeq 0.48\) satisfy well for CP pulses, while \(n_p \simeq 2n_{e0}\) and \(\Theta \simeq 0.79\) for LP pulses \([10]\).

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Figure 1. (a) Laser front versus time for different electron densities $n_{e0}$ from PIC simulations. Position of laser front $x_f$ is selected as the point where the local incident laser intensity $I(x_f) = I/4$. (b) Propagation velocity averaged over $50 \leq t/\tau \leq 150$ from PIC simulation $v_{\text{pic}}$ (red star), group velocity $v_g$ (green dotted), RT velocity $v_t$ (blue solid), and HB velocity $v_b$ (magenta dashed) as functions of plasma density. The laser is a CP pulse with $a = 10$. The ions are protons.

3. Particle-in-cell simulation results

In the following, we perform 1D particle-in-cell (PIC) simulations to study the laser propagation in both the HB and the RT regimes. The employed code is the same as that used in previous investigations [10, 26, 27]. We consider a laser pulse with $\lambda = 1 \, \mu\text{m}$ and $a$ in the range from 10 to 200. The pulse increases to a constant intensity after a $\sin^2(\pi t/10\tau)$ temporal profile in the first five cycles with $\tau = 2\pi/\omega$. The simulation box length is $200\lambda$ with 100 cells per wavelength. The initial cold plasma extends from $x = 20\lambda$ to $190\lambda$ with a uniform $n_{e0}$ ranging from $0.1n_c$ to $200n_c$. Each particle species is represented by 1000 macroparticles per cell. Protons, deuterons or tritons are used in different simulations in order to study the effect of the ratio $m_i/q_i$. The difference between circular polarization and linear polarization is also investigated.

3.1. Differences between hole-boring (HB) and relativistic transparency (RT)

In figure 1(a), we show the propagation of a CP laser front in uniform plasmas of different densities. From the slope of each curve it can be seen that the propagation velocity is roughly constant at a given density. It should be noted that a conspicuous jump is observed between the two curves corresponding to the closely sequential densities, i.e. $n_{e0}/n_c = 2.5$ and 2.6. This jump in propagation velocity can be observed immediately in figure 1(b), which shows that the averaged propagation velocity over $50 \leq t/\tau \leq 150$ jumps from $0.244c$ to $0.093c$ as the density increases slightly from $n_{e0}/n_c = 2.5$ to 2.6. From equations (3) and (7), it was predicted that the HB velocity $v_b$ will meet the RT velocity $v_t$ at $n_x$ which is always lower than the relativistic critical density $n_{cr}$. This suggests that the transition of the dominant propagation mechanism occurs at a density lower than $n_{cr}$. In fact, $v_{\text{pic}}$ from PIC simulation jumps from $v_t$ to $v_b$ at a definitely lower density $n_b$. For a CP pulse with $a = 10$, $n_x$ at the crossing point of $v_b$ and $v_t$ is about $4.2n_c$. But $n_b$, where $v_{\text{pic}}$ meets $v_b$, is only about $2.6n_c \simeq 0.619n_x$ as shown in figure 1(b).

In the regime $n_{e0} < n_b$, the laser propagates at a speed $v_t$ that is predicted well by equation (7),

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Figure 2. Laser propagation at (a) \(n_{e0} = 2.5n_c\) in the RT regime and (b) \(n_{e0} = 2.6n_c\) in the HB regime. \(n_e\) and \(\gamma_e\) are the cycle-averaged electron density and the Lorentz factor, respectively. \(E_{in} = [(E_y + B_z)^2 + (E_z - B_y)^2]/4\) and \(E_{re} = [(E_y - B_z)^2 + (E_z + B_y)^2]/4\) are the energy density of the incident wave and the reflected wave, respectively. Snapshots are taken at \(t = 100\tau\). Other parameters are as in figure 1.

which is higher than \(v_b\) but lower than the group velocity from the dispersion relation [9, 10, 25, 28, 29]. In the regime \(n_{e0} > n_b\), the propagation velocity is in perfect agreement with \(v_b\) as predicted in [17, 18]. As a result, in this case we can estimate the propagation velocity in the whole density regime as

\[
v_{\text{prop}} = \begin{cases} 
v_t, & \text{for } n_{e0} < n_b, \\
v_b, & \text{for } n_{e0} > n_b, \end{cases}
\]

and thus one can define the regimes, \(n_{e0} < n_b\) and \(n_{e0} > n_b\), as the RT regime and the HB regime, respectively.

It has previously been predicted by an analytical stationary solution [30] and verified by the simulation [24] that for a CP pulse the transition of the propagation mechanism will happen at a density lower than the relativistic critical density \(n_c\). This earlier transition is attributed to the formation of a thick density peak in front of a CP pulse due to its strong quasi-constant ponderomotive force, which significantly increases the threshold of laser intensity for RT. However, owing to the enhancement of the electron compression with mobile ions [31], the RT regime usually ends at an even lower density than predicted by the stationary solution with fixed ions. For instance, it ends at \(n_b = 2.6n_c\) with mobile ions for a CP pulse with \(a = 10\) as shown in figure 1(b), while the predicted value by the stationary solution is about \(n_b = 4.5n_c\).

Some propagation features in these two density regimes are compared in figure 2. At \(n_{e0} = 2.5n_c\) in the RT regime, the density behind the laser front is roughly equal to the initial value but the Lorentz factor has increased to a value higher than \(n_e/n_c\). It suggests that to some extent the propagation in this regime is similar to that in a classically underdense plasma except for the lower propagation velocity. However, in the HB regime it is found that the plasma is pushed forward as a piston and there are almost no particles remaining behind the laser front as shown in figure 2(b) at \(n_{e0} = 2.6n_c\). Therefore, one can conclude that in the RT regime the laser propagates into the plasma denser than \(n_c\) in a tricky way, i.e. increasing the Lorentz factor of electrons; while in the HB regime it propagates into the overdense plasma in a somewhat ‘violent’ way, i.e. repelling away both electrons and ions on its path [18]. As a result, in the RT regime the laser pulse interacts with a substantially long plasma and strong coupling occurs between the laser and the plasma, which is illustrated by the obviously modulated reflected
Figure 3. $p_x-x$ phase spaces of electrons (a) and (b) and protons (c) and (d) at $t = 100\tau$ from PIC simulations. The left pictures are for $n_{e0} = 2.5n_c$ in the RT regime, while the right pictures for $n_{e0} = 2.6n_c$ are in the HB regime. The distribution functions are on logarithmic scales. Other parameters are the same as in figure 1.

wave shown in figure 2(a), whereas in the HB regime the interaction of the pulse with plasma mainly occurs at the interface of the laser front and the density peak. Consequently, a linear piston model can usually describe HB perfectly.

Owing to essentially different dynamics between RT and HB, the produced high-energy electrons and ions present significantly different features, as shown in figure 3. In the RT regime, the electrons behind the laser front are highly heated due to their strong coupling with the laser, but only a small number of fast ions are generated in this regime. In addition the majority of these fast ions propagate at a speed lower than the laser propagation velocity ($\simeq 0.244c$) (see figure 3(c)). On the other hand, in the HB regime there are only a few electrons remaining behind the laser front, and these residual electrons behave as if they were in the vacuum (see the fine structure in the electron phase space in figure 3(b)). It should be noted that in this regime a large number of fast ions is generated in the vicinity of the interface between the laser and the plasma piston, and the mean longitudinal ion momentum is in good agreement with the value $0.187m_i c$ predicted by equation (4) as shown in figure 3(d). This HB ion acceleration mechanism provides an approach to producing high-energy ions for a wide variety of applications [2–4, 6, 11, 12, 19, 21, 23].

From energy spectra of electrons and ions in figure 4, it is confirmed that in the RT regime the laser energy is mainly deposited on hot electrons. For $n_{e0} = 2.5n_c$, 35.4% of laser energy has transferred into hot electrons but only 10.6% into fast ions. Instead, in the HB mode, owing to its close affinity to shock heating, ions absorb energy more efficiently than electrons; for $n_{e0} = 2.6n_c$, 17.5% of laser energy has transferred into fast ions but only 2.6% into hot electrons.
Figure 4. Energy spectra of accelerated electrons (left) and protons (right) in the RT regime with \(n_{e0} = 2.5n_c\) (blue dashed) and in the HB regime with \(n_{e0} = 2.6n_c\) (red solid) at \(t = 100\tau\) from PIC simulations. Other parameters are as in figure 1.

In the energy spectrum these fast ions are distributed around the mean ion energy estimated by equation (5). The energy spread of these quasi-monoenergetic fast ions can be further narrowed for practical applications by increasing the plasma density or decreasing the laser intensity [17], at the expense of their energy. From what has been observed above, we can conclude that the electrons play a dominant role in the RT regime, whereas such a role is assumed by the ions in the HB regime. As a principle, the parameter governing electron dynamics is energy rather than momentum, whereas for ion dynamics the reverse is true. Thus the laser propagation appears to be governed by energy balance in the RT regime and by momentum flux balance in the HB regime.

3.2. Incomplete HB regime at ultra-high laser intensity

In the above section, we have shown the existence of a distinct boundary between the RT and the HB regimes, and compared the differences in laser propagation features and plasma response with a CP laser pulse at the intensity \(a = 10\). However, at a higher laser intensity \(a = 100\), we find that the transition from the RT regime to the HB regime occurs less directly as shown in figure 5. Since HB becomes very efficient at such an intensity, the HB velocity will meet the RT velocity at a relatively lower ratio \(n_x/n_{cr}\), as indicated by equations (3) and (7). Hence, the achievable maximum HB velocity is significantly increased and the gap between the HB velocity and the RT velocity becomes very narrow. In this case, the density \(n_b\), where the propagation velocity completely converts into the HB velocity, is about 0.609\(n_c\); but no jump happens in the propagation velocity. In contrast, there is a considerably broad transition region ranging from \(n_{e0} = 8n_c\) to \(12n_c\), wherein the propagation velocity gently varies from the RT velocity to the HB velocity. Figure 5 shows in detail some propagation features of this extremely intense laser pulse into plasma with initial uniform density \(n_{e0} = 10n_c\). In this intermediate regime, the laser pulse penetrates into the relativistically transparent plasma, where most of the electrons have got high Lorentz factors. However, obvious differences can be found in the density profile when compared with the typical case of RT as shown in figure 2(a). On the one hand, the averaged density \(n_t\) behind the laser front is much lower than the initial density \(n_{e0}\). On the other hand, there is a considerably thick region with a density higher than \(n_{e0}\) before the laser front. To some degree, both aspects illuminate that the plasma is compressed by the laser ponderomotive force like in the HB regime. In addition, the existence of a density peak behind the laser front, which is even higher than the initial density, indicates that RT is still quite effective in making the
pulse transmit through the compressed layer partially. As a result, an incomplete hole has been bored by the collaboration with RT in this density regime, which we define as the incomplete HB regime. In comparison with a sharp boundary in the classic HB, such an incomplete HB is characterized by a ‘distributed’ ponderomotive piston in the density phase space, and it can be recognized from the density peak(s) behind the laser front.

If we know the averaged electron density \( n_e \) remaining behind the laser front, then the laser propagation and the corresponding ion acceleration in this incomplete HB regime can be formulated according to the momentum flux balance as in the classic HB regime. Ignoring the longitudinal momentum of the particles behind, the modified momentum flux balance in the frame comoving with laser front in the incomplete HB regime is given by

\[
\frac{2I}{c} \left( 1 - \beta_b \right) \left( 1 + \beta_b \right) = \frac{2A}{Z} v_b^2 m_p (n_{e0} - n_i) v_b^2.
\]

With the modified dimensionless parameter

\[
\Pi' = a \sqrt{\frac{Z m_e}{A 2m_p} \frac{n_e}{n_{e0} - n_i}},
\]

then similarly to the classic HB, the incomplete HB velocity \( v_b' \), the mean ion longitudinal momentum \( p_{x,i}' \), and the mean accelerated ion energy \( \epsilon_i' \) in the incomplete HB regime can be immediately obtained as follows:

\[
v_b' = \frac{\Pi'}{1 + \Pi'}, \quad \frac{p_{x,i}'}{m_i c} = \frac{2\Pi'(1 + \Pi')}{1 + 2\Pi'}, \quad \frac{\epsilon_i'}{m_i c^2} = \frac{2\Pi'^2}{1 + 2\Pi'}.
\]
However, in general it is difficult to analytically determine the average residual density \( n_r \) and the density boundary \( n_b \) between this incomplete HB regime and the classic HB regime. In the next section, we will show the dependence of \( n_b \) and \( n_r \) on the laser intensity and plasma density, and fitted formulae will be given for estimating them roughly with the help of simulations.

In the case of figure 5, the electron density remaining behind is about \( n_r \approx 4.7n_c \) on average, and thus equation (11) results in \( v'_b \approx 0.42c \). As shown in figure 5, the propagation velocity \( v_{\text{pic}} \) from simulation is about 0.43c, which is in quantitative agreement with \( v'_b \) estimated by the incomplete HB model. Later we will show that the mean fast ion momentum \( p'_{x,i} \) and energy \( \varepsilon'_i \) from simulation are also in good agreement with the predicted values by our incomplete HB model. However, in order to calculate the conversion efficiency \( \chi'_i \) correctly, one must keep in mind that only the fraction \((n_{e0} - n_t)/n_{e0}\) of ions is accelerated to fast ions by the incomplete HB, and consequently one obtains

\[
\chi'_i = \frac{n_{e0} - n_t}{n_{e0}} \frac{2\Pi'_i}{1 + 2\Pi'_i}.
\]

Furthermore, this \( \chi'_i \) is only the conversion efficiency of the laser energy into fast ions; the conversion efficiency into total ion kinetic energy will be higher due to the efficient heating in the RT region behind the laser front. For instance, the conversion efficiency into total ion kinetic energy from simulation is about 47.6% in the case of figure 5, which is much higher than the predicted conversion efficiency into fast ions \( \chi'_i \approx 29.5\% \).

In figure 6, the phase space of ions in the incomplete HB regime is compared with that in the classic complete HB regime. It is found that the momenta of accelerated fast ions in the incomplete HB regime as shown in figure 6(a) are distributed around \( p'_{x,i} \approx 1.04m_i c \) as predicted by equation (12). However, the distribution presents itself more chaotic than in the classic HB regime, as figure 6(b) shows. Furthermore, in the incomplete HB there is no fine ribcage-like structure [18], which is a representative structure of the ion phase-space in HB as shown in figure 6(b). In previous investigations, this structure has already been found in the classic HB regime [16] and has been attributed to the regular oscillation of the electric field in ion acceleration [17].

As shown in figure 7(a), the longitudinal electric field \( E_x \) looks like a delta-function centered at the laser front in the HB acceleration, so quasi-monoenergetic fast ions can
be primarily accelerated in this narrow region around the laser front [17]. However, in the incomplete HB acceleration, a few electric field spikes appear, not only around the laser front but also behind the laser front. In addition, these spikes are much broader than the spike in the classic HB acceleration. As a result, the cut-off energy of fast ions in the incomplete HB acceleration is much higher than that in the classic HB acceleration as indicated by the integrated area below the electric fields. However, these ions appear to be distributed into many energy groups according to their initial positions. In figure 7(b), the time evolution of the magnitude of the primary electric field spike around the laser front is shown. In the classic HB acceleration, the electric field magnitude oscillates in a ‘saw-tooth’ form, which can be applied to explain the fine ribcage-like structure developed in the ion phase space as shown in figure 6(b) [17]. While in the incomplete HB acceleration, there is almost no regularity in the oscillation of the electric field, consequently chaos is developed in the ion phase space as shown in figure 6(a).

The broad spread of fast ions over the energy spectrum in the incomplete HB acceleration is another result of the unpredictability of the electric field in time and space. As shown in figure 8, at $n_{e0} = 20n_c$ in the classic HB regime, fast ions are distributed around the mean
ion energy 147 MeV with a high quality of being monoenergetic as predicted. At \( n_{e0} = 10n_c \) in the incomplete HB regime, fast ions are distributed around the estimated mean ion energy \( \varepsilon'_i = 417 \text{ MeV} \) by equation (13) too, but there is a large quantity of ions in each energy interval from 200 MeV to a sharp cut-off at 600 MeV. This indicates that the achievable maximum ion energy has been significantly increased in the incomplete HB acceleration at the expense of a quality loss in monoenergetics [18, 24]. This will greatly limit the application of such fast ion beams in practice. However, it is interesting to note that the energy spectrum is sharply cut at the end and here the number of ions is still considerably large. So this incomplete HB ion acceleration mechanism may find some special applications by using the highest energetic part of accelerated fast ions. As well as the classic HB acceleration, the areal density of fast ions from this acceleration is only limited by the length of laser pulse. So it offers the opportunity to generate much more energetic ions at a lower intensity than the one demanded in the classic HB acceleration.

3.3. Boundary between the incomplete HB regime and the classic HB regime

Because of the emergence of an incomplete HB regime, we have shown in the above section that there is no clear boundary between the RT and the HB regimes at an extremely high laser intensity. On the other hand, we can define the density boundary \( n_b \) between the incomplete HB regime and the classic HB regime as the density where the propagation velocity meets the predicted classic HB velocity, and consider the RT as the special case of the incomplete HB in the limit of \( n_r = n_{e0} \). For a CP pulse at intensity \( a = 100 \), such a defined \( n_b \) is about \( 12n_c \) as shown in figure 5. In the regime above this density, we found that not only the propagation velocity but also the mean fast ion energy and the conversion efficiency from simulations are in good quantitative agreement with estimations by equations (3)–(6) from the classic HB model. To our surprise, such a defined density boundary \( n_b \) is approximately located at the golden section point of the cross density \( n_x \), i.e.

\[
n_b \simeq \Phi n_x,
\]

at the whole test intensity regime from \( a = 10 \) to \( a = 200 \), where \( \Phi \simeq 0.618 \) is the reciprocal of the golden ratio. From its definition, the cross density \( n_x \) can be numerically solved from equations (3) and (7) under the condition \( v_b = v_t \). For protons, we find that the following formula:

\[
n_x \simeq (1 + a^2)^{0.314} n_c,
\]

fits well the numerical solution at all test laser intensities as shown in figure 9(a), in which the agreement between the estimated \( n_b \) by equations (15) and (16) and that from PIC simulations is also illustrated.

Furthermore, equations (5) and (6) suggest that at a given laser intensity both the maximum mean ion energy and the maximum conversion efficiency in a classic HB acceleration are achieved at this density boundary \( n_b \). Therefore, by substituting equations (15) and (16) into equations (5) and (6), we can conveniently estimate the maximum mean ion energy \( (\varepsilon_{\text{i, max}}) \) and the maximum conversion efficiency into fast ions \( (\chi_{\text{i, max}}) \) that can be achieved in the classic HB acceleration as follows:

\[
\varepsilon_{\text{i, max}} = \frac{2\Pi_{\text{max}}^2}{1 + 2\Pi_{\text{max}}^2} m_e c^2 = \frac{2\Pi_{\text{max}}^2}{1 + 2\Pi_{\text{max}}^2} \times 938 \text{ MeV},
\]
Figure 9. (a) The cross density $n_x$ where the HB velocity meets the relativistic transparent velocity and the lower density boundary of the HB regime $n_b$ as functions of the incident laser intensity. The discrete ▼ and ★ points are from the numerical solutions and PIC simulations, respectively; the continuous curves are from the fitted formulae. (b) The maximum mean ion energy ($\varepsilon_{i,\text{max}}$) and the maximum conversion efficiency ($\chi_{i,\text{max}}$) that are achievable in the HB acceleration as functions of laser intensity. The ions are protons.

\[ \chi_{i,\text{max}} = \frac{2\Pi_{\text{max}}}{1 + 2\Pi_{\text{max}}}, \]

with $\Pi_{\text{max}} \approx 0.021a(1 + a^2)^{-0.157}$, and the resultant values are shown as functions of the laser intensity in figure 9(b).

As discussed in the above section, the averaged residual density $n_r$ is crucial for modeling the incomplete HB. With the help of PIC simulations, we find that $n_r$ can be fitted well by the following formula:

\[
n_r \approx \begin{cases} 
    n_{e0} \left( \frac{n_b - n_{e0}}{n_b} \right)^{\frac{n_b - 1}{36}}, & \text{for } n_{e0} < n_b, \\
    0, & \text{for } n_{e0} > n_b
\end{cases}
\]

in the whole test laser intensity regime from $a = 10$ to $a = 200$, where $n_b$ is given by equation (15). At mildly relativistic intensity the exponent $(n_b - 1)/36$ in equation (19) is negligible, so $n_r$ decreases sharply to 0 at $n_{e0} = n_b$. However, at ultra-high relativistic intensity this exponent can no longer be neglected, so $n_r$ decreases gently to 0 when $n_{e0}$ increases to $n_b$. Therefore, a considerably broad incomplete HB regime appears at the ultra-high relativistic intensity. The good agreement between this fitted formula and the PIC simulations is illustrated at both laser intensity $a = 10$ and $a = 100$ in figure 10.

As suggested by equations (3) and (7), the HB velocity is strongly dependent upon the mass to charge ratio of the ions, while the RT velocity is independent of the ion properties. This indicates that the density $n_x$ and hence the lower density boundary $n_b$ of the HB regime will shift for ion species with different mass to charge ratios. As ion species with three representative mass to charge ratios, the propagation velocities in plasmas with pure protons, deuterons and tritons are compared in figure 11. First, we confirm that the propagation velocity in the RT regime is independent of the ion species. From this aspect, it is also demonstrated that the laser propagation is indeed dominated by the energy balance with electrons in RT. However, in the HB regime the propagation velocity is in good quantitative agreement with the HB velocity from equation (3) for each ion species, respectively, and it shows a strong dependence on the ion mass to charge ratio. For each ion species, the velocities predicted by the HB model and the RT model...
Figure 10. The averaged electron density \( n_r \) remaining behind the laser front as a function of the initial plasma density in a CP pulse with intensity (a) \( a = 10 \); (b) \( a = 100 \). The discrete ⋄ points show the values from PIC simulations; the continuous curves are from the fitted formula (19).

Figure 11. RT velocity \( v_t \), group velocity \( v_g \), propagation velocity from PIC simulation \( v_{\text{pic}} \) and HB velocity \( v_{\text{hb}} \) as functions of plasma density. The incident laser is a CP pulse with intensity \( a = 10 \). The propagation velocities and the HB velocities are compared among plasmas with different ion species: protons (marked with ‘P’), deuterons (marked with ‘D’) and tritons (marked with ‘T’).

meet at a different density \( n_x \), which brings up a different lower boundary \( n_b \) of the HB regime. For protons \( n_b/n_x \approx 2.6n_c/4.2n_c \approx 0.619 \), for deuterons \( n_b/n_x \approx 2.9n_c/4.8n_c \approx 0.604 \) and for tritons \( n_b/n_x \approx 3.1n_c/5.1n_c \approx 0.608 \). On the one hand, this illustrates that the RT regime can be extended to a little higher density in the plasma with a bigger ion mass to charge ratio. On the other hand, much to our surprise, each \( n_b \) is still approximately located at the golden section of \( n_x \) for different ion species, i.e. equation (15) is satisfied well for different ion species.

3.4. Effect of laser polarization

Under the assumption of immobile ions, the propagation of an LP pulse is compared with that of a CP pulse in the RT regime [10]. In figure 12(a), we show the propagation of an LP laser front in a uniform plasma with mobile ions. It is found that the LP pulse propagation can be divided into two stages in a wide density range \( 4n_c \leq n_0 \leq 6n_c \). For instance, at density \( n_0 = 4n_c \) the laser front is well predicted by the RT model in the first stage \( t < 100\tau \), but in the second stage...
Figure 12. (a) Laser front of an LP pulse against time for different electron densities \( n_{e0} \) from PIC simulations; the fronts predicted by the RT model and the HB model at \( n_{e0} = 4n_c \) are shown as discrete ▲ and ▼ points, respectively. (b) Propagation velocity \( v_{\text{pic}} \) (⋆) averaged over \( 50 \leq t/\tau \leq 150 \) from PIC simulation. \( x_t, v_g, v_{\text{rt}} \) and \( v_{\text{hb}} \) are as defined in figure 1. The incident laser is an LP pulse with \( a = 10 \). The ions are protons.

For \( t > 100\tau \) it is in good agreement with the predicted value by the HB model. As a result, there is no clear boundary between the RT regime and the HB regime for LP pulse as indicated by figure 12(b); hence it is difficult to analytically predict the propagation of an LP pulse in the whole density regime. By comparing figure 12 with figure 1, we confirm that for LP pulses RT dominates \( (n_{e0} \leq 3n_c) \) or plays an important role \( (n_{e0} \leq 6n_c) \) up to a higher density than that for CP pulses, and the RT velocity for LP pulses is much higher than that for CP pulses.

The transition of the propagation mechanism for LP pulses at a given density in the range \( 4n_c \leq n_{e0} \leq 6n_c \) can be attributed to the development of a thick high-density peak before the laser front. Since the ponderomotive force of an LP pulse \( f_{\text{p}}^L = -\frac{me}{4} \frac{\partial}{\partial x} a^2(x)[1 + \cos(2\omega t)] \) is oscillating in time, even in a moderately dense plasma the high-density peak cannot always be formed thick enough immediately to prevent RT as in a CP pulse with a stable ponderomotive force \( f_{\text{p}}^C = -\frac{me}{4} \frac{\partial}{\partial x} a^2(x) \). Indeed if the initial plasma density is low enough, RT will be too fast to give electrons and ions time to accumulate abundantly before the laser front, so that RT can dominate the propagation over the whole process. Otherwise, in a moderate dense plasma the electrons and ions will accumulate continually before the laser front to form a relativistically overdense plasma peak; hence the propagation mechanism will gradually convert from RT into HB. In the extreme case of an initial relativistically overdense plasma, RT does not have any chance to happen from the beginning. The propagation of a relativistic pulse in a moderate dense plasma \( n_{e0} = 5n_c \) has been shown in figure 13. It is found that the modulation of the density profile at \( t = 50\tau \) is very weak, which indicates that RT still dominates the pulse propagation until this time, while the strong modulation of density profile is presented at \( t = 150\tau \) and a very thick high-density peak has formed before the laser front, which illustrates that the propagation mechanism has already converted into HB.

With the transition of the propagation mechanism, the ion energy spectrum has also changed in essence for an LP pulse. As shown in figure 14, the ion energy spectrum decreases exponentially at \( t = 50\tau \), because there are almost no accelerated fast ions but only hot ions, owing to heating at this time. Peaks present in the high-energy region in the spectrum at \( t = 150\tau \), which indicates the generation of fast ions by the HB acceleration. However, the HB ion acceleration in an LP pulse is usually much less effective than that in a CP pulse. Therefore,
Figure 13. Snapshots of LP pulse propagation in a uniform plasma with \( n_{e0} = 5n_c \) at (a) \( t = 50\tau \) and (b) \( t = 150\tau \). \( n_e, \gamma_e, E_{in} \) and \( E_{re} \) are as defined in figure 2. The ions are protons, and the laser intensity \( a = 10 \).

Figure 14. Comparison of fast ion energy spectra at \( t = 50\tau \) and \( t = 150\tau \) from the interaction of an LP pulse with a uniform plasma, laser intensity \( a = 10 \) and plasma density \( n_{e0} = 5n_c \). The ion energy spectrum at \( t = 150\tau \) in a CP pulse with the same intensity is also drawn for comparison. The ions are protons.

the height of the dominant quasi-monoenergetic peak in the ion energy spectrum for an LP pulse is much lower than that for a CP pulse at the same intensity as shown in figure 14. In particular, the quality of these fast ions being monoenergetic is also much poorer than that for a CP pulse at the same intensity, although the cut-off of the energy has been greatly increased in the HB acceleration using an LP pulse.

4. Summary and discussion

To summarize, we have studied the relativistic laser pulse propagation in overdense plasmas with different densities in the RT regime, the HB regime and the probable transition regime (i.e. the incomplete HB regime). It is found that in the RT regime electrons dominate the interaction with the laser and the pulse propagation is governed by the energy balance. Conversely, in the HB regime pulse propagation and ion acceleration are governed by the ion momentum flux balance. In particular, a considerably broad transition regime is found between the RT regime and the HB regime at extremely high intensities (e.g. \( a = 100 \)). In this regime, the incomplete HB happens much more efficiently in collaboration with RT. As a result, the propagation velocity and the cut-off energy of fast ions have been significantly increased in comparison with that of the classic HB, which represents a possible way of obtaining higher...
fast ion energies at the same laser intensity than was previously predicted for classic HB acceleration. Introducing the averaged density \( n_r \) remaining behind the laser front into the modified momentum flux balance, the pulse propagation and the corresponding ion acceleration are well formulated by equations (11)–(14). With the help of PIC simulations, it is found that the density boundary between the incomplete HB regime and the classic HB regime in CP pulses is approximately located at the golden section of the cross density \( n_x \) (equation (15)). Combining equation (15) and the HB model equations (5) and (6), the maximum mean ion energy and the maximum conversion efficiency to fast ions can be estimated at any given CP laser intensity in the classic HB acceleration. For LP pulses, there is no clear boundary between the RT regime and the HB regime, because a thick high-density peak may develop before the laser front with time in a wide density range. This density peak will terminate RT, the propagation mechanism converts into HB and ion acceleration begins.

In this paper, we have restricted the study to interactions of laser pulses with thick enough plasma targets in one dimension. In multi-dimensional cases, the filamentation [22] and the relativistic self-focusing [20, 21] may have a strong impact upon the laser propagation, particularly the quality of accelerated fast ions may be affected by the possible transverse instabilities [32]. However, it has been demonstrated that both the quality and the cut-off energy of accelerated ions can be significantly improved by using a thin target, in which the underlying acceleration mechanisms are the Light-Sail for CP pulses [32–36] and the Breakout Afterburner [37, 38] for LP pulses, respectively. In addition, if the laser intensity is much more intense than \( a = 100 \), radiation reaction should be considered [39].

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