Influence of Long-Range Coulomb Interactions on the Metal-Insulator Transition in One-Dimensional Strongly Correlated Electron Systems

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Abstract

The influence of long-range Coulomb interactions on the properties of one-dimensional (1D) strongly correlated electron systems in vicinity of the metal-insulator phase transition is considered. It is shown that unscreened repulsive Coulomb forces lead to the formation of a 1D Wigner crystal in the metallic phase and to the transformation of the square-root singularity of the compressibility (characterizing the commensurate-incommensurate transition) to a logarithmic singularity. The properties of the insulating (Mott) phase depend on the character of the short-wavelength screening of the Coulomb forces. For a sufficiently short screening length the characteristics of the charge excitations in the insulating phase are totally determined by the Coulomb interaction and these quasiparticles can be described as quasiclassical Coulomb solitons.
The metal-insulator transition induced by strong correlations in an electron system is a problem of permanent theoretical interest. In recent years, this problem became particularly important in connection with the discovery of high-temperature superconductivity and subsequent attempts to develop a consistent theory of this phenomenon [1].

Much progress has been achieved in one-dimensional models where exact results or well-developed non-perturbative methods are available, making it possible to obtain an analytical description of the dynamics of the Mott-Hubbard transition in a wide range of bare parameters of the system. For example, using the Bethe-ansatz solution of the 1D Hubbard model, an analytical expression has been recently obtained for the charge stiffness of a finite-size system close to the transition as a function of the on-site repulsion $U$ [2].

It is well known that, at half filling (one electron per site), the 1D repulsive Hubbard model describes a Mott insulator. In this case, the charge excitations have a gap in their spectrum and at weak Hubbard interaction, $U \ll t$ ($t$ being the bandwidth), can be regarded as topological solitons (kinks) of the quantum sine-Gordon model with coupling constant $\beta^2 = 8\pi$ (see e.g. [3]). At finite deviations from half filling the system is in a metallic phase, but the charge carriers (“holons”) keep the memory of the Mott phase; they can still be considered as solitons with a characteristic size proportional to the correlation length $\xi_0$ in the insulating phase. At low densities, $n\xi_0 \ll 1$, the holons interact weakly and can be described as free massive spinless fermions [2,3].

In this paper we study the effects of a long-range Coulomb interaction on the metal-insulator transition in strongly correlated one-dimensional electron systems. We shall assume that the Mott phase for the charge excitations is described in terms of the sine-Gordon model, while the transition to the metallic phase is related to creation of solitons at the chemical potential exceeding the charge gap. Near the phase transition, interactions between the solitons can be neglected, so that in the absence of Coulomb correlations the properties of the system near the transition are completely determined by Fermi statistics of quantum solitons. In particular, the compressibility of the system has a trivial (in fermionic language) square-root singularity typical for the commensurate-incommensurate transition.
Notice that, within a classical description, a system of kinks with a finite density forms a soliton lattice at zero temperature (see, e.g. [7]). However, the interaction between the solitons is short ranged and, at densities \( n\xi_0 \ll 1 \), exponentially small. Quantum fluctuations, representing the Goldstone mode of the soliton lattice, destroy the periodic structure. Therefore, irrespective of the interaction constant, the transition to the Mott phase always occurs from a disordered phase, the latter described in terms of free spinless fermions. When the long-range Coulomb interaction is taken into account, the above described qualitative picture is modified. The unscreened repulsive Coulomb forces between the solitons lead to the formation of a Wigner crystal [8]. Strictly speaking, in a 1D infinite chain of charges the \( 1/r \)-interaction does not fall off fast enough to remove the infrared divergence in the density-density correlation function. However, the \( 4k_F \) oscillating part of the correlator decays slower than any power [9], thus making it reasonable to speak of a nearly ordered state.

At low densities \( na_B \ll 1 \) (\( a_B = \hbar^2/e^2m_s \) and \( m_s \) being the Bohr radius and soliton mass, respectively) the Wigner lattice with period \( a \equiv n^{-1} \) is not destroyed at exponentially large distances \( L \leq a \exp(const/na_B) \). Therefore, one can expect, in finite-size (mesoscopic) samples, the ”holons” to form a Wigner crystal, so that the transition to the insulating phase takes place from the Wigner crystal phase. We show that, in this case, the square-root singularity of the compressibility is changed to a logarithmic singularity.

The holon density dependence of the behavior of the Wigner lattice is determined by the screening of the Coulomb interaction both at small and large distances. In quasi-one-dimensional systems, the ultraviolet cutoff is determined, as a rule, by the characteristic width \( \lambda \) of a ”one-mode” channel. Long-distance screening can occur when the one-dimensional chain of length \( L \) is situated close to a large metallic electrode, at distance \( D \ll L \). In this case the behavior of the compressibility \( \kappa = \partial n/\partial \mu \) in the weak-screening (\( nD \geq 1 \)) and strong-screening (\( nD \ll 1 \)) regimes turns out to be qualitatively different. At weak screening, the dependence \( \kappa = \kappa(n) \) does not differ from that in the case of an unscreened Coulomb interaction (with the obvious change \( L \to D \)): \( \kappa(n) \sim 1/\ln(nD) \). On lowering the density, one passes to the strong screening regime \( nD \ll 1 \), where the above
logarithmic behavior is replaced by a power-law: $\kappa(n) \sim (Dn)^{-2}$. In the limit of arbitrarily small densities, this dependence would lead to the exponent $2/3$ in the dependence of the compressibility on the chemical potential near the threshold: $\kappa(\mu) \sim (\mu - \mu_c)^{-2/3}$. However, in the vicinity of the phase transition, quantum fluctuations melt the Wigner lattice and recover the universal square-root singularity $\kappa(\mu) \sim (\mu - \mu_c)^{-1/2}$, characterizing the commensurate-incommensurate transition [4]. The presence of a large metallic electrode allows the range of quantum fluctuations to be changed. On the other hand, it also leads to an additive renormalization of the critical value of $\mu$, $\mu_c = \Delta - e^2/4D$, due to effects caused by image forces.

The magnitude of the gap $\Delta$ in the spectrum of charged excitations in the insulating phase crucially depends on the character of the short-wavelength screening of the Coulomb interaction. This can be easily understood for weak Hubbard interaction $U \ll t$, when the charge excitations are quantum solitons with characteristic correlation length $\xi_0 \sim \hbar c_s/\Delta$ ($c_s$ is the velocity of the charge excitations in the Mott phase). If $(\xi_0/\lambda)\alpha_s \leq 1$ ($\alpha_s = e^2/\hbar c_s$), the long-range Coulomb interaction weakly affects characteristics of the insulating phase, and the charge gap is determined by the well-known formula of Lieb and Wu [10].

When the opposite inequality is realized, $(\xi_0/\lambda)\alpha_s \gg 1$, the Coulomb forces significantly renormalize (increase) the Mott-Hubbard gap. It is physically evident that in this case the charge excitations — solitons — become heavier ($\Delta_C \gg \Delta$) due to the strong electrostatic energy, and “fragile” ($\xi_0 \gg \xi_c$) due to the unscreened Coulomb repulsion of charges “inside” the soliton. It seems reasonable to call these excitations Coulomb solitons, since their behavior in the conduction band at $\mu > \Delta_C$ differs from that described above. Namely, at low densities, $n\xi_s \ll 1$, the Coulomb solitons still condense into a Wigner crystal and display the logarithmic threshold singularity of the compressibility. However, at densities $n\xi_s \geq 1$, but still much lower than $a_B^{-1}$, the Wigner lattice is transformed to a sine-Gordon soliton lattice. The compressibility saturates at values $\kappa \sim (\Delta_C\xi_s)^{-1}$, characteristic for a charge-density wave.

Let us consider first the influence of long-range Coulomb forces on the properties of the
metallic phase near the transition to the insulating state. In what follows, we shall assume that the transition is of the Mott-Hubbard type, although our conclusions remain valid for any one-dimensional charged system near the commensurate-incommensurate transition.

Using the standard approach (see e.g. [11], and also [12]), one can treat the charge excitations of the half-filled, weak-coupling \((U \ll t)\) Hubbard model as topological quantum solitons of the related sine-Gordon model (at \(\beta^2 = 8\pi\)), characterized by correlation length \(\xi_0 = \hbar c_s/\Delta\) (here \(2\Delta\) is the Mott-Hubbard gap, and \(c_s \simeq 2t + U/2\pi\) is the velocity of charged excitations). The long-range repulsive forces will be taken into account by adding the screened (at short distances \(\Delta x \ll \lambda\)) Coulomb interaction energy to the total energy of the system. The screening length \(\lambda\) is determined, for example, by transverse dimensions of the confinement potential which forms the one-dimensional chain.

Let \(\lambda \gg \xi_0\). Then it is reasonable to assume (the exact criterium will be given below) that the Coulomb interaction does not affect the intrinsic characteristics of the solitons (i.e. the Mott phase) but changes the character of interaction between them at large distances, \(\Delta x \geq \lambda\). Therefore, assuming the solitons to be point-like objects, we shall study their dynamics in the conduction band at low densities, \(n \ll a_B^{-1}\). At densities \(n \leq a_B^{-1}\) the Coulomb interaction leads to the formation of a soliton Wigner lattice with period \(a \equiv n^{-1}\) (\(a_B = (\hbar c_s/e^2)\xi_0\) is the Bohr radius for charged particles with mass \(m_s = \Delta/c_s^2\)). This ordered state is expected to be stable in chains of mesoscopically large length and therefore can significantly affect the properties of the metallic phase [13]. We shall study the influence of Wigner crystallization on the characteristics of the transition to the insulating (Mott) phase.

Consider a situation, typical for “mesoscopic” experiments, when a massive metallic electrode is placed at the distance \(D\) (\(\lambda \ll D \ll L\)) from the chain. The electrode leads to screening of the Coulomb interaction at large distances. In this case, the classical (electrostatic) energy of the lattice, with the electrode screening effect taken into account, equals [13]

\[\text{...}\]
\[ E_W = \frac{e^2 n^2}{2} \left[ \sum_{i \neq 0} \frac{1}{|i|} - \sum_{i = -\infty}^{\infty} \frac{1}{\sqrt{i^2 + (2D/a)^2}} \right] = e^2 n^2 \left[ \ln(nD) + C - 2 \sum_{k=1}^{\infty} K_0(4\pi knD) \right], \]  

where \( C \) is the Euler constant, and \( K_0(x) \) is the Macdonald function. The asymptotic behavior of the expression (1) in the limits of high and low densities are

\[ E_W \approx e^2 n^2 \ln(nD), \quad nD \geq 1 \]  
\[ E_W \approx n \left( -\frac{e^2}{4D} \right) + 2\zeta(3)(eD)^2 n^4, \quad nD \ll 1. \]  

Quantum corrections to the energy of the Wigner crystal (1) caused by zero-point fluctuations

\[ \Delta E_W = \frac{\hbar}{2} \int_0^{\pi/a} A \, dk, \quad s^2 = \frac{n}{m_s} \frac{d^2 E_W}{dn^2} \]

are small, \( \Delta E_W \ll E_W \) (\( s \) is the sound velocity for the Wigner crystal). This is true at all densities \( n \ll a_B^{-1} \) in the weak screening regime, and within the interval \( a_B D^{-2} \ll n \ll a_B^{-1} \) at strong screening \( (nD \ll 1) \) as well. As could be expected, a strongly screened Coulomb interaction at low densities \( n \leq a_B D^{-2} \) cannot stabilize the Wigner lattice which is already melted at zero temperature by zero-point fluctuations. In the absence of the macroscopic electrode, the regime Eq.(2) is realized, where \( D \) is replaced by the longitudinal length of the chain \( L \).

As already mentioned there is no true Wigner crystallization in an infinite chain. Nonetheless, in mesoscopic-size systems, Coulomb correlations support almost perfect long-range order [9], making it possible to consider Wigner crystallization in the strict sense. Note that in the absence of impurities producing pinning of the Wigner crystal, the soliton lattice can freely slide along the one-dimensional channel. Therefore, in the ideal system, the response of a Wigner soliton-crystal to a low-frequency electric field is identical to that of a Fermi gas of solitons. (For instance, the charge stiffnesses are identical [14]). On the other hand, it is clear that the compressibilities of the two systems are different, so that, upon
crystallization of solitons, the nature of the singularity at the transition to the insulating phase on changing the chemical potential is changed.

For the Fermi gas of solitons (with the rest energy $\Delta$ and mass $m_s = \Delta/c_s^2$), the thermodynamic potential of the system $\Omega$, calculated in the low-density limit, when the solitons can be treated as noninteracting particles, equals

$$\Omega_F(n) = (\Delta - \mu) n + \frac{\pi^2 c_s^2}{6m_s} n^3.$$  \hfill (5)

Equation (5) leads immediately to the square-root singularity in the equilibrium density of holons near the phase transition to the insulating phase [5,6]:

$$n(\mu) = \begin{cases} 0, & \mu < \Delta \\ \frac{2m_s}{\pi c_s} \sqrt{\mu - \Delta}, & \mu \geq \Delta. \end{cases}$$  \hfill (6-7)

When Coulomb correlations are taken into account, the kinetic energy of the holons given by the last term in (5) should be changed by the electrostatic energy of the Wigner lattice (1).

In the absence of the screening provided by the metallic electrode, the energy is given by Eq.(2), and instead of Eq.(5) we have for the thermodynamic potential

$$\Omega_W(n) = (\Delta - \mu) n + e^2 n^2 \ln(nL).$$  \hfill (8)

Comparing (8) with (5) and (7), we find that the square-root singularity of the compressibility is replaced by a logarithmic singularity:

$$\kappa^{-1}(\mu) \simeq 2e^2 \ln \frac{(\mu - \Delta)L}{e^2}.$$  \hfill (9)

We have dropped irrelevant numerical factors in the argument of the logarithm. It is assumed in Eq.(9) that the deviation $\Delta\mu \equiv \mu - \Delta$ is much larger than the Coulomb energy $e^2/L$. One can easily check that, in this regime, the maximum length of the chain should not exceed

$$L_m \sim a \exp(const/na_B) \gg a.$$  For a larger length, $L \geq L_m$, the ordered structure would be destroyed.
The presence of a massive metallic electrode makes the situation more complex. In the weak screening regime (2), similar to (9), we have a logarithmic dependence of the compressibility on the density

$$\kappa^{-1}(n) = 2e^2 \ln(enD), \quad nD \geq 1. \quad (10)$$

On further decreasing the density, this dependence is replaced by a power-law, corresponding to the strong-screening regime:

$$\kappa^{-1}(n) = 24\zeta(3)(enD)^2. \quad (11)$$

According to Eq.(11), the dependence of the equilibrium density on chemical potential is given by

$$n(\mu) = \begin{cases} 0, & \mu < \Delta^* \\ \frac{1}{2\lambda} \left[ \frac{\mu - \Delta^*}{\zeta(3)(e^2/D)} \right]^{1/3}, & \mu \geq \Delta^*, \end{cases} \quad (12)$$

$$n(\mu) = \frac{1}{2\lambda} \left[ \frac{\mu - \Delta^*}{\zeta(3)(e^2/D)} \right]^{1/3}, \quad \mu \geq \Delta^*, \quad (13)$$

where $\Delta^* = \Delta - e^2/4D$. Substituting (13) into (11), one finds that the logarithmic singularity (9) of the compressibility, taking place for unscreened Coulomb potential, is changed to a power-law $\kappa^{-1} \sim (\mu - \Delta^*)^{2/3}$. However, in the strong screening regime, at densities $n \leq a_B D^{-2}$, the Wigner lattice is destroyed locally by quantum fluctuations. Therefore, Eq.(13) is valid at $\Delta \mu = \mu - \Delta^* \geq (e^2/D)(a_B/D)^3$. There is a region near threshold where the universal square-root singularity, characterizing the commensurate-incommensurate transition, is recovered. Notice that the massive electrode leads to a shift of the critical value of the chemical potential, $\mu_c = \Delta - e^2/4D$, caused by the image forces (see e.g. [15]). This shift is small in the case of weak Coulomb coupling $\alpha_s = e^2/\hbar c_s \ll 1$. In the strong-coupling regime, $\alpha_s \geq 1$, one should also account for strong Coulomb interactions affecting the insulating phase.

For strong Coulomb coupling, $\alpha_s \geq 1$, the characteristic (Bohr) radius $a_B$ turns out to be shorter than the correlation length $\xi_0$, indicating the possibility of strong renormalization of the characteristics of single solitons by Coulomb interaction. From physical considerations
one expects that, due to strong Coulomb interaction, solitons become heavier and extended. In other words, the effective correlation length $\xi_s$, being the size of the “Coulomb” soliton, may substantially exceed $\xi_0$. We shall study this situation assuming that screening is extremely small, $\lambda \leq \xi_0$.

A consistent solution of this problem suggests introducing the long-range Coulomb interaction at the level of the Hubbard model and calculation of the resulting spectrum of the charge excitations in such a system. However, for a half-filled system, repulsive Coulomb effects can only increase the charge gap. For this reason, reduction of the problem to the sine-Gordon model still remains reasonable. First we shall study the effect of Coulomb forces on topological solitons of the quasiclassical sine-Gordon model. Then we shall present arguments explaining the applicability of the obtained results to the holon dynamics in the Hubbard model.

In the presence of long-range Coulomb forces, the Lagrangian of the sine-Gordon model has the form:

$$
\mathcal{L} = \frac{\hbar}{c_s} \left[ \frac{1}{2} \dot{\varphi}^2 - \frac{c_s^2}{2}(\varphi')^2 + \frac{\omega_0^2}{\beta^2} (\cos\beta\varphi - 1) \right] - \frac{e^2}{8\pi^2} \frac{\beta^2}{y} \int_{-\infty}^{\infty} dy \frac{\partial_x \varphi(x,t) \partial_y \varphi(y,t)}{\sqrt{(x-y)^2 + \lambda^2}}.
$$

(14)

We shall be interested in topological solitons (kinks) of the model (14). At $\beta^2 \ll 1$ the model is quasiclassical, and the excitations we are interested in can be found by using trial functions.

A trial function, describing a static topological soliton $\Delta \varphi \equiv \varphi(x = +\infty) - \varphi(x = -\infty) = 2\pi/\beta$ of the model Eq.(14), will be chosen in the form corresponding to the sine-Gordon model:

$$
\varphi_C(x) = \frac{4}{\beta} \arctan \left( \exp \left( \frac{x - x_0}{d^*} \right) \right).
$$

(15)

Here $x_0$ is the center of the soliton, and $d^*$ is a variational parameter which determines the soliton size. Its value is found by minimizing the soliton rest energy:
\[ E(d^*) = \frac{\hbar}{c_s} \int_{-\infty}^{\infty} dx \left[ \frac{c_s^2}{2} (\partial_x \varphi_C)^2 + \frac{\omega_0^2}{\beta^2} (1 - \cos \beta \varphi_C) \right] \]
\[ + \frac{e^2 \beta^2}{8\pi^2} \int \int dx \, dy \, \frac{\partial_x \varphi_C(x) \partial_y \varphi_C(y)}{\sqrt{(x-y)^2 + \lambda^2}}. \]  \tag{16} \]

Substituting (15) into (16) and doing the integrals, we easily find that, for a weak coupling \( \alpha_s \ll 1 \), the parameter \( d^* \) coincides with the size of an unperturbed kink, \( d^* = d_0 = c_s/\omega_0 \), \( E(d^*) = E_s = 8\hbar \omega_0/\beta^2 \). For strong coupling, \( \alpha_s \geq 1 \), the unscreened Coulomb interaction leads to multiplicative renormalizations of the size and energy of the soliton:

\[ d^* = d_0 \left( \frac{\alpha_s \beta^2}{2\pi^2} \right)^{1/2} \ln^{1/2} \left[ \frac{d_0}{\lambda} \sqrt{\frac{\alpha_s \beta^2}{2\pi^2}} \right] \gg d_0 \]  \tag{17} \]
\[ E_C = \frac{8\hbar \omega_0}{\beta^2} \left( \frac{\alpha_s \beta^2}{2\pi^2} \right)^{1/2} \ln^{1/2} \left[ \frac{d_0}{\lambda} \sqrt{\frac{\alpha_s \beta^2}{2\pi^2}} \right] \gg E_0. \]  \tag{18} \]

Let us show that expression (18) can be obtained within a consistent scheme, using solutions of the sine-Gordon equations. For a linearized problem, the Coulomb interaction can be taken into account exactly, leading to a modification of the spectrum of small perturbations (“optical phonons”) of the field \( \varphi \) (see, e.g. [9])

\[ \omega^2(k) = \omega_0^2 + k^2 c_s^2 \left[ 1 + \alpha_s \frac{\beta^2}{2\pi^2} K_0(k \lambda) \right]. \]  \tag{19} \]

Since at \( x \geq 1 \) \( K_0(x) \) is exponentially small, the Coulomb interaction mostly affects the long-wavelength dynamics, where the spectrum (19) takes the form

\[ \omega^2(k) = \omega_0^2 + k^2 c_s^2 \frac{\alpha_s \beta^2}{2\pi^2} \ln \frac{1}{k \lambda}, \quad k \lambda \ll 1. \]  \tag{20} \]

Expression (20) differs from the “phonon” spectrum of the unperturbed sine-Gordon equation in that the constant velocity \( c_s \) has been replaced by a momentum dependent effective “velocity” \( c_s(k) \sim c_s \sqrt{\ln(1/|k| \lambda)} \). Analysing the structure of the last term in Eq.(16), one concludes that, when studying the influence of weakly screened \( \lambda \ll d_0 \) Coulomb interaction on the topological soliton of the sine-Gordon model, it is sufficient to change the exact Lagrangian Eq.(14) by an approximate one in which the effects of Coulomb interaction are incorporated in a coordinate dependent velocity
\[ c_s^2 \rightarrow c^2(x-x_0) = c_s^2 \alpha_s \beta^2 \ln \left| \frac{x-x_0}{\lambda} \right|. \] (21)

In Eq.(21) it is assumed that \( |x-x_0| \gg \lambda \). For this reason, it is possible to neglect in all calculations terms containing derivatives of the “velocity”, \( |c'(x)/c(x)| \ll 1 \).

In the framework of such an “adiabatic” perturbation theory, the topological soliton has the standard form:

\[ \varphi_C(x) = \pm \frac{4}{\beta} \arctan[\exp \left( \frac{x-x_0}{d(x)} \right)], \] (22)

where the soliton size, \( d_0 \), is replaced by a coordinate dependent, smooth function \( d(x) \equiv c(x)/\omega_0 \). One can easily check that the energy of the Coulomb soliton (22) exactly coincides with Eq.(18).

Using the solution (22), the standard scheme of quasiclassical quantization can be easily developed (see, e.g. [16]). It can be shown that the one-loop quantum correction to the classical energy of the Coulomb soliton has the same form as in the usual sine-Gordon model, \( \Delta E_s = -\hbar \omega_0 / \pi \). Within the traditional scheme of quasiclassical quantization of solitons, the relative smallness of quantum corrections is provided by the small value of the coupling constant \( \beta^2 \ll 1 \). In our case, solitons can be treated classically due to large Coulomb energy. Therefore, it seems natural to suggest that formula (18) remains valid in the extreme quantum case \( \beta^2 \gg 1 \) (from the point of view of the usual sine-Gordon model).

When the charge sector of the half-filled, weakly repulsive \( (U/t \ll 1) \) Hubbard model is mapped onto the sine-Gordon model, the coupling constant \( \beta^2 \) turns out to be equal \( 8\pi \) [11]. The above analysis allows one to put forward a hypothesis that the long-range Coulomb interaction in the strong-coupling case \( e^2/\hbar c_s \gg 1 \) lead to a multiplicative renormalization of the Mott-Hubbard gap

\[ \Delta \rightarrow \Delta_C = \Delta \sqrt{\frac{e^2}{\hbar c_s}} \ln^{1/2} \left( \frac{\xi_0}{\lambda} \sqrt{\frac{e^2}{\hbar c_s}} \right). \] (23)

Notice that the appearance of fractional powers of a large logarithm is typical for various quantum problems that explicitly incorporate long-range Coulomb forces [13,9,17].
Now we consider Coulomb solitons in the conduction band at $\mu > \Delta_C$. At low densities $n\xi_C \ll 1$, where

$$\xi_C \simeq \xi_0\alpha_s^{1/2} \ln^{1/2} \left( \frac{\xi_0}{\lambda \sqrt{\alpha_s}} \right)$$

(24)

is the characteristic size of the Coulomb soliton, the charges in the conduction band can be considered as point-like. The unscreened Coulomb interaction leads to the formation of a Wigner crystal (see Eqs.(2),(9)). On increasing the density $n \geq \xi_C^{-1} \ll a_B^{-1}$, the solitons start to overlap strongly, forming a sine-Gordon lattice.

The energy density of such a classical lattice can be readily estimated, using well-known periodic solutions of the sine-Gordon model

$$\varphi_p(x) = \frac{1}{\beta} \left\{ \pi + 2am \left[ \frac{x}{d^*} \frac{1}{k(n)} \right] \right\}.$$  

(25)

Here $am(z)$ is the elliptic amplitude, and $k(n)$ is the elliptic modulus fixed by the soliton density

$$2kK(k) = (nd^*)^{-1},$$  

(26)

where $K(k)$ is the complete elliptic integral of the I-order, and $d^*$ is defined in Eq.(17). The energy density of the soliton lattice equals (see also [18])

$$E_p(n) = E_C n K \left\{ E(k) - \frac{1}{2} (1 - k^2) K(k) \right\}$$

(27)

($E(k)$ is the complete elliptic integral of the II-order, $E_C$ being the energy of the Coulomb soliton). Expressions (25)-(27) are valid in our case at densities $nd^* \geq 1$. In the limit of a dense soliton system $nd^* \gg 1$ ($k \to 0$), the soliton lattice smoothly transforms into a charge-density wave

$$E_p(n) \simeq \frac{\pi^2}{4} (\Delta_c d^*) n^2$$

(28)

and the compressibility of the system is no longer dependent on the density: $\kappa^{-1} \sim \Delta_c d^* = \text{const.}$
In conclusion we have shown that long-range Coulomb forces drastically modify the properties of 1D electron systems in vicinity of the metal-insulator phase transition. In the metallic phase a weakly-screened Coulomb interaction leads to the formation of a Wigner crystal of charged quasiparticles and therefore affects the critical behaviour of the system at the transition point. The properties of the insulating phase are also changed if the short wavelength screening length is sufficiently small. In this case the Mott-Hubbard gap is strongly renormalized and the charged excitations in the Mott phase can be described as quasiclassical Coulomb solitons.

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