Higher Moments of Net-Proton Multiplicity Distributions*

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Abstract—Higher moments of event-by-event net-proton multiplicity distributions have been applied to search for the QCD critical point. Model results are used to provide a baseline for this search. The measured moment products, $\kappa \sigma^2$ and $S \sigma$ of net-proton distributions, which are directly connected to the thermodynamical baryon number susceptibility ratio in Lattice QCD and Hadron Resonance Gas (HRG) model, are compared to the transport and thermal model results. We argue that a non-monotonic dependence of $\kappa \sigma^2$ and $S \sigma$ as a function of beam energy can be used to search for the QCD critical point.

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1. INTRODUCTION

Lattice QCD calculations predict that a crossover from the hadronic phase to the Quark Gluon Plasma (QGP) phase occurs above a critical temperature with zero baryon chemical potential ($\mu_B$). The corresponding cross-over temperature range has been estimated to be about 170–190 MeV [1]. At large $\mu_B$, QCD-based model calculations indicate that the transition from the hadronic phase to the QGP phase is of first order. The end point of the first-order phase transition line is the QCD Critical Point (CP) [2, 3]. There are large theoretical uncertainties of its location and even its existence is not confirmed [3]. Experimentally, we study the QCD phase diagram by varying the colliding energy in heavy-ion collisions [4]. The possibility of the existence of the CP has motivated our interest to search for it with the RHIC beam energy scan program [5]. By tuning the collision energy from a center-of-mass energy of 200 GeV down to 5 GeV, we will be able to vary the baryon chemical potential from $\mu_B \sim 20$ to $\mu_B$ of about 500 MeV.

The characteristic feature of a CP are a large correlation length ($\xi$) and critical fluctuations. Recently, theoretical calculations have shown that higher moments of multiplicity distributions of conserved quantities, such as net-baryon, net-charge, and net-strangeness, are sensitive to the correlation length $\xi$ [6].

In Lattice QCD calculation with $\mu_B = 0$, higher-order susceptibilities of the baryon number, which can be related to the higher-order moments of the net-baryon multiplicity distributions, show a non-monotonic behavior near $T_c$ [7]. A similar behavior is expected at CP in the finite $\mu_B$ region. Experimentally, it is hard to measure the net-baryon number event-by-event while the net-proton number is measurable. Theoretical calculations show that fluctuations of the net-proton number can be used to infer the net-baryon number fluctuations at the CP [8].

In this paper we will show the centrality and energy dependence of various moments and moment products of net-proton multiplicity distributions from published STAR AuAu data [9] at $\sqrt{s_{NN}} = 19.6, 62.4, 200$ GeV and model calculations.

2. OBSERVABLES

The various moments of the event-by-event multiplicity distributions are defined as: Mean, $M = \langle N \rangle$, Variance, $\sigma^2 = \langle (\Delta N)^2 \rangle$, Skewness, $S = \langle (\Delta N)^3 \rangle / \sigma^3$, and Kurtosis, $\kappa = \langle (\Delta N)^4 \rangle / \sigma^4 - 3$, where $\Delta N = N - \langle N \rangle$. Skewness and Kurtosis are used to characterize the asymmetry and peakedness of the multiplicity distributions, respectively. For gaussian distributions, they are equal to zero. Thus, the Skewness and Kurtosis are ideal probe to demonstrate the non-Gaussian fluctuation feature as expected near the CP. In particular, a sign change of the skewness or kurtosis may be an indication that the system crossed the phase boundary [6, 10].

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3. RESULTS

We have calculated the various moments of net-proton ($\Delta p = N_p - N_\bar{p}$) distributions from transport models (AMPT (ver. 2.11) [11], Hijing (ver. 1.35) [12], UrQMD (ver. 2.3) [13]) and a thermal model (Therminator (ver. 1.0) [14]). By using several models with different physics implemented, we can study the effects of physics correlations and backgrounds which are present in the data and that might modify purely statistical emission patterns, like resonance decays, jet-production (Hijing), coalescence mechanism of particle production (AMPT), thermal particle production (Therminator), and hadronic rescattering (AMPT, UrQMD).

The kinetic coverage of protons and anti-protons used in our analysis is $0.4 < p_T < 0.8$ GeV/$c$ and $|y| < 0.5$. Figure 1 shows the number of participant ($N_{\text{part}}$) dependence of four moments ($M$, $\sigma$, $S$, $\kappa$) extracted from net-proton distributions of AuAu collisions at $\sqrt{s_{NN}} = 200$ GeV for the various models. $M$ and $\sigma$ show a monotonic increase with $N_{\text{part}}$ for all of the models, while $S$ and $\kappa$, which are positive, decrease monotonically with $N_{\text{part}}$, which means the shape of the net-proton distributions become more symmetric as the centrality increases. The dashed lines in Fig. 1 are derived from the independent emission source model [15]. The centrality evolution of the various moments of net-proton distributions in Fig. 1 can be well described by such a model.

Figure 2 shows the $N_{\text{part}}$ dependence of moment products $S\sigma$ and $\kappa\sigma^2$ of net-proton distributions for AuAu collisions of various energies from AMPT String Melting model calculation. In the upper panel, the $S\sigma$ shows almost no centrality dependence for high energy, but it shows a small decreasing trend for low energies. We can also see that the $S\sigma$ has a strong energy dependence decreasing with increasing energy. In the lower panel of Fig. 2, the $\kappa\sigma^2$ shows almost no centrality dependence and the values are around unity for all energies.
In Fig. 3, the energy dependence of moment products $S_\sigma$, $\kappa_\sigma^2$ for most central net-proton distributions from STAR data [9] are compared with the results from various models. We see the data are in good agreement with the Hadron Resonance Gas (HRG) model ($\kappa_B \sigma^2_B = 1$, $S_B \sigma^2_B = \tanh(\mu_B/T)$) [16] and the thermal model (Terminator) results. HIJING, UrQMD, and AMPT default fail to describe $S_\sigma$ and $\kappa_\sigma^2$ simultaneously. For $\kappa_\sigma^2$, the results from various models show no dependence on energy and are close to unity. A large deviation from constant as a function of $N_{\text{part}}$ and collision energy for $\kappa_\sigma^2$ may indicate new physics, such as critical fluctuations [6]. Recent lattice QCD calculations from [17] have shown that $\kappa_\sigma^2$ non-monotonically depends on colliding energy in the neighbourhood of the CP.

4. SUMMARY

Higher moments of the distributions of conserved quantities are predicted to be sensitive to the correlation length at CP and to be related to the susceptibilities computed in Lattice QCD and the HRG model. Various non-CP models (AMPT, HIJING, Terminator, UrQMD, HRG) have been applied to study the non-CP physics background effects on the higher moments of net-proton distributions. The moment products $S_\sigma$, $\kappa_\sigma^2$ of net-proton distributions have almost no dependence on collision centrality and $\kappa_\sigma^2$ is also found to be constant as a function of energy for various models. On the other hand, the high-energy data are in good agreement with the HRG model.

Both the model as well as the data do not show non-monotonic behavior. Thus, they can serve as a baseline of the behavior expected from known physics effects for the RHIC beam energy scan. The presence of a critical point in that region may result in non-Gaussian fluctuations and non-monotonic behavior of the observables studied here as a function of collision energy.

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