Research on interpolation of NURBS curve based on fractional power

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Abstract. According to the problem of discontinuous of jerk and low rate of federate change in S-shape federate control method, a jerk-limited federate control scheme which reduces the shock to machine tool in interpolating is presented. The constraints are analyzed from the angle of bow height error and acceleration. The simulation result shows that this method can improve the feed speed faster, realize the smooth transition of acceleration, and reduce NURBS Flexible impact on machine tool during curve interpolation.

1. Introduction
Non-Uniform Rational B-Spline (NURBS) is the only mathematical model for describing free-form curves and surfaces specified by the international organization in Standard for Exchange of Product Model Data (STEP). Whether it has the function of NURBS curve interpolation is one of the important indexes to measure a CNC system. In recent years, a hot issue about NURBS interpolation is how to integrate the control of feed speed into interpolation calculation [1]. There are many traditional speed control methods. Based on the description of velocity time curve, common models are linear, exponential, and S-type [2]. The linear speed control method is simple and the calculation is small.

Compared with linear control, exponential speed control has the advantages of good smoothness and high motion accuracy. However, the two schemes mentioned above have the disadvantages of discontinuous acceleration, impact on machine tools and low machining quality. Pan Haihong [3] et al. Discussed the S-type speed control strategy. This control strategy realizes the smooth transition of acceleration, but there is the problem of discontinuous acceleration. Zhao Guoyong [4] described the speed change with piecewise cubic polynomial, and designed the speed control strategy. This algorithm realizes the continuity of acceleration time curve. However, the acceleration changes slowly, which makes it take a long time for the speed to rise to the maximum value, which affects the rapidity of interpolation. In this paper, a speed control strategy based on fractional power polynomials is proposed, which is based on the continuity, smoothness and maximum limit of acceleration. The acceleration can be rapidly increased to the maximum value, and the transition is continuous and smooth.

2. Direct interpolation of NURBS curve
Direct interpolation of NURBS curve is to subdivide the whole NURBS into several interpolation periods. In each interpolation period, the tool is controlled to move on three coordinates, so as to approach the NURBS curve directly. At present, the commonly used method is segmentation.
according to time. On the basis of determining the tool speed at different positions, the speed $v_i$ of each interpolation point is determined according to the curvature change of the machining path. Then, the interpolation step $\Delta S$ is calculated according to the principle of constant step time. For the NURBS curve $p(u)$ represented by parameter $u_i$, the parameter value $u_{i+1}$ of the next point can be determined according to step $\Delta S$. The coordinates $X$, $Y$, $Z$ and their offsets $\Delta X$, $\Delta Y$, $\Delta Z$ of this point can be finally obtained by $u_{i+1}$. In the above process, because the inverse function of NURBS curve is difficult to calculate, Taylor expansion approximation algorithm is generally used to obtain the relationship between step size and parameters.

3. Speed control based on fractional power polynomial

The traditional speed control method adopts the seven-segment method. The seven segment method divides the acceleration process into three stages: acceleration process, uniform acceleration process and deceleration acceleration process. In these three processes, the acceleration is $j_{\text{max}}$, $0$, $-j_{\text{max}}$. The deceleration process is similar to the acceleration process. Based on the seven segment method, the acceleration and deceleration processes are further divided into seven segments. The specific dynamic model of the acceleration phase is as follows, and the deceleration part is similar to it. According to the general situation, the initial value of acceleration and acceleration is 0, and the initial value of velocity is $v_i$.

$$j(t) = \begin{cases} 
\frac{1}{k} t^2 & (0 \leq t < t_1) \\
\text{j}_{\text{max}} & (t_1 \leq t < t_2) \\
k(t_3 - t)^2 & (t_2 \leq t < t_3) \\
0 & (t_3 \leq t < t_4) \\
-k(t_4 - t)^2 & (t_4 \leq t < t_5) \\
-k\text{j}_{\text{max}} & (t_5 \leq t < t_6) \\
k(t_7 - t)^2 & (t_6 \leq t < t_7)
\end{cases} \quad (1)$$

$$a(t) = \begin{cases} 
\frac{2}{3} k t^2 & (0 \leq t < t_1) \\
\text{a}_1 + \text{j}_{\text{max}} (t - t_1) & (t_1 \leq t < t_2) \\
\text{a}_2 + \frac{2}{3} k(t_3 - t)^2 & (t_2 \leq t < t_3) \\
\text{a}_3 & (t_3 \leq t < t_4) \\
\text{a}_4 - \frac{2}{3} k(t_4 - t)^2 & (t_4 \leq t < t_5) \\
\text{a}_5 - \text{j}_{\text{max}} (t - t_5) & (t_5 \leq t < t_6) \\
\text{a}_6 - \frac{2}{3} k(t_7 - t)^2 & (t_6 \leq t < t_7)
\end{cases} \quad (2)$$

Where, $\text{a}_1 = \frac{2}{3} k t_1^2$; $\text{a}_2 = \text{a}_1 + \text{j}_{\text{max}} (t_2 - t_1)$; $\text{a}_3 = \text{a}_2 + \frac{2}{3} t_3$; $\text{a}_4 = \text{a}_3 - \frac{2}{3} k(t_5 - t_4)^2$; $\text{a}_5 = \text{a}_4 - \frac{2}{3} k(t_6 - t_5)^2$; $\text{a}_6 = \text{a}_5 - \text{j}_{\text{max}} (t_6 - t_5)$.  

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where, \( v_1 = v_3 + \frac{4}{15}k_1^2 \); 
\( v_2 = v_1 + a_1(t_2 - t_1) + \frac{1}{2}j_{\max}(t_2 - t_1)^2 \); 
\( v_3 = v_2 + \frac{31}{15} \); 
\( v_4 = v_3 + a_{\max}(t_4 - t_3) \); 
\( v_5 = v_4 + a_4(t_5 - t_4) - \frac{4}{15}k(t_5 - t_4)^2 \); 
\( v_6 = v_5 + a_5(t_6 - t_5) - \frac{1}{2}j_{\max}(t_6 - t_5)^2 \).

4. Relevant constraints

4.1. Calculation of radius of curvature

The radius of curvature describes the degree of curvature of the curve. For a NURBS curve, the radius of curvature \( R_i \) at \( u_i \) is shown in Equation (4).

\[
R_i = \frac{|p'(u_i)|^3}{|p''(u_i)|} \tag{4}
\]

From the homogeneous coordinate representation of NURBS curve, we can get:

\[
p(u) = \sum_{i=0}^{n} P_i N_{i,k}(u) \tag{5}
\]

Therefore, we can get:

\[
p'(u) = k \sum_{i=0}^{n} P_i \left[ N_{i,k-1}(u) \frac{N_{i+1,k-1}(u)}{u_{i+k} - u_i - u_{i+k+1} - u_i} \right] \tag{6}
\]

Similarly, \( p''(u) \) can be obtained. Now, the radius of curvature at this point can be calculated by taking it into Equation (4).

4.2. Velocity constraint based on bow height error

If the bow height error between two adjacent points \( u_i \) and \( u_{i+1} \) is \( \varepsilon \), then according to the definition of bow height error, we can get:

\[
\varepsilon = r_i - \sqrt{r_i^2 - \left( \frac{L_i}{2} \right)^2} \tag{7}
\]

Because \( L_i = v_iT \), we can get:
In order to ensure the machining accuracy, under the constraint of bow height error, the speed \( v_i \) at \( v_c \) should meet the following relationship:

\[
v_c = \begin{cases} 
  \frac{v_{\text{max}}}{2} & \frac{\varepsilon(2R_l - \varepsilon)}{T} > \frac{v_{\text{max}}}{2} \\
  2\sqrt{\varepsilon(2R_l - \varepsilon)} / T & \frac{\varepsilon(2R_l - \varepsilon)}{T} \leq \frac{v_{\text{max}}}{2} 
\end{cases}
\]

(9)

4.3. Velocity constraint based on the maximum value of acceleration

In the interpolation process, in addition to the acceleration discontinuity will have a flexible impact on the machine tool, the absolute value of the acceleration will also have an impact on the machine tool. In Literature [5], the relationship between acceleration and feed speed is discussed, and the corresponding formula is derived. For NURBS curve \( p(u) \), the relationship among acceleration \( j \), acceleration \( a \) and velocity \( v \) are shown in Figure 1.

![Figure 1. The relationship among acceleration and velocity in the process of interpolation.](image)

Because the distance between two adjacent interpolation points is very small, the value of \( \theta \) is very small. The interpolation path between adjacent interpolation points \( p_i \) and \( p_{i+1} \) can be approximately circular arc. At the same time \( \sin(\theta/2) \approx \theta/2 \). If the feed speed under the constraint of maximum acceleration \( j_{\text{max}} \) is \( v_j \), it can be seen from Figure 1.

\[
v_j T = 2r_i \sin \left( \frac{\theta}{2} \right) = 2r_i \frac{\theta}{2}
\]

(10)

\[
\Delta a_n = 2a_n \sin \left( \frac{\theta}{2} \right) = \frac{v_j^2}{r_i} \times \frac{v_j T}{r_i}
\]

(11)

From the definition of acceleration, we can get:

\[
j_i = \frac{\Delta a_n}{T} = \frac{v_j^3}{r_i^2}
\]

(12)

Therefore,

\[
v_j = \sqrt[3]{r_i^2 j_{\text{max}}}
\]

(13)

5. Simulation analysis

From Equation (1) - Equation (3), the acceleration time curve, acceleration time curve and speed time curve of the fractional power polynomial speed control strategy (here referred to as \( f \) algorithm) in the acceleration uniform deceleration stage can be obtained. In order to verify its rationality, it is compared with the seven segment method speed control strategy (called \( s \) algorithm here) in Literature
[3] and the cubic polynomial speed control strategy (called t algorithm here) in Literature [4]. Here, $k=1$, $j_{\text{max}}=1 \text{mm/s}^3$, $a_{\text{max}}=2 \text{mm/s}^2$, $v_{\text{max}}=8 \text{mm/s}$, $v_f=0 \text{mm/s}$. The comparison results are shown in Figure 2.

![Comparison of speed control strategies of F algorithm, T algorithm and S algorithm.](image)

**Figure 2.** Comparison of speed control strategies of F algorithm, T algorithm and S algorithm.

It can be seen from the figure that the acceleration and speed of algorithm S change rapidly, but the acceleration has a step jump. T algorithm and F algorithm realize acceleration continuity, but F algorithm changes acceleration and velocity more quickly. It can be seen that the F algorithm proposed in this paper can make the speed reach the maximum value faster under the condition that the acceleration speed is continuous.

### 6. Conclusions

Based on the analysis of existing algorithms, this paper proposes a speed planning method based on fractional power polynomials. The constraints based on acceleration control are analyzed. According to the change of curvature, the calculation method of acceleration and deceleration stage and deceleration point is discussed and the judgment rule is given. The analysis results show that the
algorithm can effectively improve the machining speed and meet the interpolation requirements of NURBS curve with high speed and high precision on the premise of continuous acceleration speed.

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