Numerical study on Longitudinal control of Cessna 172 Skyhawk aircraft by Tail arm length

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Abstract. This paper covers a new method of longitudinally controlling an aircraft by changing the horizontal tail arm length, without the conventional control surface called 'elevator'. The effects of changing horizontal tail arm length from its steady level trimmed condition are determined numerically for the chosen Cessna aircraft. From the numerical calculations, the possibilities of implementing this type of control in the chosen Cessna aircraft are analyzed. At last, some recommendations are suggested for the effective longitudinal control of the Cessna aircraft by changing horizontal tail arm length. Further, it is also suggested that employing both elevator control and tail arm length control together will make an aircraft to meet any kind of longitudinal controllability and stability requirement at any instant. This optimal aircraft configuration can be achieved by proposing a novel configuration and morphing solutions.

1. Introduction
From the origin of aircraft, stability and control have been a subject of concern that must be considered when designing an aircraft[1], [2]. Similarly for a rocket/missile, or any other type of vehicle. It apparently depicts that without stability and control, a flying vehicle cannot be constructed[3]. The study of stability indicates that whether the vehicle will be easy, difficult or impossible to fly. It has two separate disciplines, namely, static stability and dynamic stability. And, the study of controllability ensures that whether the aircraft will be able to perform the desired flight maneuvers. Stability, as well as control, are identified based on three rotational degrees of freedom namely longitudinal, directional and lateral. Traditionally, an aircraft has been controlled by the primary control surfaces like elevator, rudder, and aileron. Here, the elevator is used for longitudinal control and rudder and aileron are used for directional and lateral control since they are coupled motion.

Currently, there are a lot of research works going on in the field of aircraft’s stability and control in subsonic, supersonic and hypersonic speeds. These researches focus on the requirement and required improvement in static stability[4]–[8], dynamic stability [9]–[11] and control[12].

Generally, stability and control characteristics of an aircraft can be analyzed by analytical (theoretical) methods and experimental methods. Now, Analytical techniques are getting highly advanced with developments in computer modeling. for example, (i)[13] used MATLAB for mathematical modelling for Nonlinear six-degree-of-freedom flight, (ii) [14] used SDSA (Simulation and Dynamic Stability
Analysis) application for analysing the dynamic characteristics of the aircraft just in the conceptual design stage, (iii) [8] used CFD (ANSYS Fluent) with dynamic mesh for the prediction of the longitudinal dynamic of an UAV. However, these methods have to be correlated with experiments before they can be used in confidence[4], [15]–[17]. [18]discusses the considerations in the determination of stability and control derivatives and dynamic characteristics from flight data. A combination of these two methods is commonly used in today’s applications. Whereas, [19]used an integrated (computational and experimental) approach to estimate the X-31 aircraft’s stability & controllability.

Literature verifies that continuous studies have been performed and concentrated on the area of stability and controllability. However, all the studies focused only on the traditional control surfaces. None of the researchers did a research on the other way of controlling an aircraft. This paper focuses on the analyzing the possibilities of longitudinally controlling the aircraft by changing the tail arm length of horizontal stabilizer without elevator. Moreover, this paper covers only the numerical study on the effects of horizontal tail arm length on the static longitudinal stability and controllability of the selected aircraft.

2. Static longitudinal stability
In general, to have the ability of returning to longitudinal trim condition (static longitudinal stability), an aircraft must satisfy the following two conditions[20];

- When AOA increases due to disturbance, the whole aircraft’s center of pressure must move rearward from its center of gravity (cg) and
- When AOA decreases due to disturbance, the whole aircraft’s center of pressure must move forward from its ‘cg’.

![Figure 1. Typical static longitudinal stability graph](image)

The above two conditions can be represented in terms of pitching moment coefficient about the center of gravity \( C_{m, cg} \) with respect to the absolute AOA \( \alpha_a \) as shown in figure1[21]. From the figure1, it is observed that the derivative \( \frac{\partial C_{m, cg}}{\partial \alpha_a} \) must be negative for the static longitudinal stability. In order to fix the derivative value (static longitudinal stability/longitudinal stiffness), the contribution of various parts (wing-body and horizontal tail) of the aircraft has been related.
Figure 2. Wing-body and tail contribution to $C_{m,cg}$[22]

Figure 3. Tail contribution[22]

From the figure 2 and 3, the equation of total pitching moment coefficient about ‘cg’ in terms of lift coefficient is given by

$$C_{m,cg} = C_{m,ac,w} + C_{L,w} \left[ h_{cg} - h_{ac} \right] - V_H C_{L,t}$$

(1)

And in terms of AOA

$$C_{m,cg} = C_{m,ac,w} + (a \times \alpha) \left[ h_{cg} - h_{ac} \right] - V_H a_i \alpha_i$$

(2)

$$C_{m,cg} = C_{m,ac,w} + (a \times \alpha) \left[ h_{cg} - h_{ac} \right] - V_H a_i \left[ \alpha_i - \left( \alpha_o + \frac{\partial \epsilon}{\partial \alpha} \right) \right]$$

(3)

$$C_{m,cg} = C_{m,ac,w} + (a \times \alpha) \left[ h_{cg} - h_{ac} \right] - V_H a_i \left[ \alpha_i - \left( \alpha_o + \frac{\partial \epsilon}{\partial \alpha} \right) \right] + V_H a_i \left( i_i + \epsilon_o \right)$$

(4)

$$C_{m,cg} = C_{m,ac,w} + (a \times \alpha) \left[ h_{cg} - h_{ac} \right] - V_H a \left[ 1 - \frac{\partial \epsilon}{\partial \alpha} \right] + V_H a_i \left( i_i + \epsilon_o \right)$$

(5)

Where Tail volume coefficient $V_H = \frac{l_t \times S_t}{S \times C}$

When $\alpha_o$ is zero, the $C_{m,cg}$ can be termed as zero-lift pitching moment coefficient $C_{m,o}$
The slope of the \( C_{m,cg} \) curve is obtained by differentiating equation (5) with respect to \( \alpha_a \)

\[
\frac{\partial C_{m, cg}}{\partial \alpha_a} = a \left[ h_{cg} - h_{np} - V_H \times \frac{a}{a} \times \left( 1 - \frac{\partial \epsilon}{\partial \alpha_a} \right) \right]
\]  

(7)

The equation (7) clearly shows the powerful influences of the location of the ‘cg’ \( (h_{cg}) \) and the tail volume ratio \( V_H \) in determining the pitch stability of an aircraft. It also makes us to establish a certain viewpoint in the design of an airplane.

\[
\frac{\partial C_{m, cg}}{\partial \alpha_a} = a \left[ h_{cg} - h_{np} \right] = -a \left[ h_{np} - h_{cg} \right]
\]  

(8)

\[
\frac{\partial C_{m, cg}}{\partial \alpha_a} = -a \text{ [Static Margin]}
\]  

(9)

The distance \( (h_{np} - h_{cg}) \) is referred as static margin which is a straight measure of static longitudinal stability. For static longitudinal stability, this static margin must be positive. Moreover, the larger static margin leads to have more longitudinal stability for the airplane.

Therefore,

\[
C_{m, cg} = C_{m, o} + \left( \frac{\partial C_{m, cg}}{\partial \alpha_a} \right) \times \alpha_a
\]

These equations (1) to (9) are derived based on the following assumptions [23]:

- \( \cos \alpha_{wb} \approx 1 \) and \( \sin \alpha_{wb} \approx \alpha_{wb} \),
- Center of gravity is assumed very close to the zero lift line,
- \( Z_i \ll l_i \),
- \( D_z \ll l_i \),
- The angle \( (\alpha_{wb} - \epsilon) \) is small,
- \( \sin (\alpha_{wb} - \epsilon) \approx 0 \) and \( \cos (\alpha_{wb} - \epsilon) \approx 1 \).
- $M_{ac,t}$ is small in magnitude, and
- $\alpha_{wb} \rightarrow \alpha$ (Zero lift line of wing-body and Zero lift line of aircraft are almost same), $a_{wb} \rightarrow a$ and $C_{L,wb} \approx C_{L}$.

3. **Cessna 172 skyhawk**

Since Cessna 172 Skyhawk aircraft’s design data are freely available in internet, books and other sources, it has been chosen for this study purpose. It is a most popular single-engine trainer aircraft which is favorite for student pilots. It has excellent flight performance, better stall characteristics, and low landing speed. Figure 5 shows the three views diagram of the Cessna 172 Skyhawk aircraft.

![Figure 5. Three views diagram of Cessna aircraft](image)

For numerical calculations of stability, the specifications of Cessna 172 Skyhawk aircraft are referred from Wikipedia and other online sources. The specifications are given below.
3.1 General characteristics [24]
- Crew : one
- Capacity : three passengers
- Length : 8.28 m
- Wingspan : 11 m
- Height : 2.72 m
- Wing area : 16.2 m²
- Aspect ratio : 7.32
- Empty weight : 767 kg
- Gross weight : 1,111 kg
- Fuel capacity : 212 litres

3.2 Performance characteristics [24]
- Cruise speed : 226 km/h
- Stall speed : 87 km/h (power off, flaps down)
- Never exceed speed : 302 km/h (IAS)
- Range : 1,289 km with 45 minute reserve, 55% power, at 12,000 ft
- Service ceiling : 4,100 m
- Rate of climb : 3.66 m/s
- Wing loading : 68.6 kg/m²

3.3 Design characteristics [24]
- Wing Aerofoil [25] : NACA 2412
- Wing’s lift curve slope (α) : 5.143
- Tails’s lift curve slope (at) : 5
- Wing surface area (S) : 16.2 m²
- Mean aerodynamic chord (C) of wing : 1.47 m
- Tail surface area (Sₜ) : 2.83 m²
- Downwash variation with respect to αₜ(δε/δα)[26] : 0.25
- Wing leading edge distance from the nose : 1.934 m
- Wing aerodynamic center from leading edge : 0.3675 m
- hₜₘₐₜ[26] : 0.25
- Clₘₐₓ[27] : 1.3

4. Static longitudinal stability – numerical analysis
Static longitudinal stability of the aircraft can be determined by knowing the ‘cg’ location and contribution of various aircraft parts. Since mass of the aircraft is not constant during flying time, ‘cg’ of the aircraft will not occupy a fixed location. Hence, static longitudinal stability also varies based on the ‘cg’ location during flying. So, it is important to find the extreme locations of the ‘cg’ in its mission.

4.1 Possible position of the ‘cg’
In an aircraft, the masses should be distributed such that it has a defined ‘cg’ position, which is critical. Also, the mass distribution should be such that on certain situations where some components may be consumed or even removed, its ‘cg’ movement should be in a controllable manner so that is not compromised. Table 1 shows the possible extreme position of ‘cg’ during the mission phases of Cessna 172 aircraft.
Table 1. Center of gravity location details of Cessna

| Particulars                          | Most forward position (i.e, with all the crew and full tank of fuel) | Most rearward position (i.e, without all the crew and fuel) |
|--------------------------------------|---------------------------------------------------------------------|-------------------------------------------------------------|
| Centre of gravity position from nose | 2.53563 m                                                           | 2.56053 m                                                   |
| Centre of gravity position from leading edge of the wing | 0.60163 m                                                       | 0.62653 m                                                   |

4.2 Determination of stability parameters

The stability, control, and handling qualities of the aircraft are analyzed by using tail volume coefficient and position of the ‘cg’ as some percentage of wing chord length.

From equation (7)

$$\frac{\partial C_{m,cg}}{\partial \alpha} = a \left[ h_{cg} - h_{ac} - V_{H} \times \frac{a}{\alpha} \times \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right]$$

The thrust has three effects, namely the direct moment of the thrust, the propeller or inlet normal force due to turning of the air, and the propeller wash or jet induced flows upon the aircraft parts. These power or thrust effects on static longitudinal stability are excluded in the above equation.

Table 2. Stability parameters of Cessna

| Particulars                          | For the Most forward position of ‘cg’ | For the Most rearward position of ‘cg’ |
|--------------------------------------|--------------------------------------|---------------------------------------|
| $h_{cg}$                             | 0.40927                               | 0.426                                 |
| $l_{t}$                              | 4.03083 m                             | 4.03069 m                             |
| $V_{H}$[26]                          | 0.479                                 | 0.47899                               |
| Longitudinal Stiffness               | $\frac{\partial C_{m,cg}}{\partial \alpha}$ | -0.97717                              | -0.89                                 |
| Stability curve slope in degree      | -44.34°                               | -41.67°                               |
| $h_{ap}$                             | 0.07                                  | 0.07                                  |
| Neutral Point from leading edge of the wing | 0.882 m                               | 0.882 m                               |
| Neutral Point from nose              | 2.816 m                               | 2.816 m                               |
| Static Margin[26]                    | 0.19                                  | 0.17305                               |

The calculated static stability curves of Cessna aircraft for the most forward location of ‘cg’ and for the most rearward location of ‘cg’ is shown in figure 6.
The Cessna aircraft is being controlled longitudinally by a primary control surface called elevators. The elevators are usually hinged to the tailplane or horizontal stabilizer. It controls the pitching of an aircraft, and therefore the \( \alpha \) and the lift of the wing by altering the pressure distribution of horizontal stabilizer. The effectiveness of elevators on pitching moment coefficient about ‘cg’ is measured as \( C_{m,e} \).

From the literature, for Cessna aircraft \( C_{m,e} = -1.28 \)

Note: Upward movement of the elevator is assumed as negative and downward movement of the elevator is assumed as positive. The stick force given by a pilot for this type of control is usually very small.

5. **Longitudinal control by tail arm length (without elevator)**

   Apparently, horizontal tail arm length value (\( l_t \)) plays important roles in deciding the longitudinal stability and controllability of an aircraft. It has great influences. So, it is intended to have the longitudinal control by tail arm length. As schematically represented in figure 7, it is proposed to control the pitching of an aircraft by changing the horizontal tail arm length value. This can be achieved by implementing a mechanism to move the horizontal tail. In this paper, the numerical analysis of longitudinal control by ‘\( l_t \)’ has been done for the selected aircraft (Cessna 172 Skyhawk).
Figure 7. Typical diagram of longitudinal control by tail arm length

5.1 For the most forward location of ‘cg’

For the most forward location of the ‘cg’, the effects of tail arm length variation on $C_{m_{o}}$, longitudinal stiffness and its corresponding change in $\alpha_{a}$ are tabulated in the table 3.

Table 3. Details of tail arm length ($l_t$) control for the most forward location of ‘cg’

| Horizontal tail arm length ($l_t$) | $C_{m_{o}}$ | Longitudinal Stiffness $\left( \frac{\partial C_{m_{cog}}}{\partial \alpha_{a}} \right)$ | Angle of attack ($\alpha_{a}$) in degree | Stability curve slope in degree | Remark |
|-----------------------------------|------------|-------------------------------------------------|----------------------------------------|---------------------------------|--------|
| 1.500                             | -0.0022    | 0.1507                                          | 0.84                                   | 8.6                             | Unstable |
| 1.838                             | 0.0074     | 0.0000                                          | -11511.66                              | 0.0                             | Neutral  |
| 1.900                             | 0.0092     | -0.0276                                         | 19.13                                  | -1.6                            | Stable   |
| 2.000                             | 0.0121     | -0.0722                                         | 9.58                                   | -4.1                            | Stable   |
| 2.500                             | 0.0263     | -0.2950                                         | 5.12                                   | -16.4                           | Stable   |
| 3.000                             | 0.0406     | -0.5178                                         | 4.49                                   | -27.4                           | Stable   |
| 3.500                             | 0.0549     | -0.7406                                         | 4.25                                   | -36.5                           | Stable   |
| 4.031                             | 0.0700     | -0.9772                                         | 4.11                                   | -44.4                           | Stable   |
| 4.500                             | 0.0834     | -1.1863                                         | 4.03                                   | -49.9                           | Stable   |
The effects of tail arm length on the AOA are shown in figure 8. And the effects of tail arm length on AOA and stability are shown in figure 9. From the figure 8, 9 and table 3, it is observed that the tail arm length change will not create considerable changes in decreasing the AOA from the steady level trim condition since stability increases greatly. Stability curve is almost perpendicular line after the tail arm length value is 20 m, whereas, decrement in tail arm length leads to an increase in AOA as well as a decrement in stability. In this case, even a slight decrement in tail length creates a huge difference in angle of attack. This also creates stability problems. Stability is neutral for the tail arm length value 1.838 m. below this value makes the aircraft in unstable condition. As a maximum control extent, by this way it can be controlled for the AOA form 3.2 degree to stall AOA theoretically. So it is recommended that not to go for the longitudinal control to decrease the $\alpha_a$ from the steady level trim condition by changing tail arm length. Even to increase the $\alpha_a$ from the steady level trim condition, it brings stability problems.
Figure 8. AOA variation with respect to tail arm length ($l_t$) for the most forward location of ‘cg’ case

![Diagram showing AOA variation with respect to tail arm length](image)

Figure 9. Tail arm length control stability curves for the most forward location of ‘cg’

5.2 For the most rearward location of ‘cg’

For the most rearward location of the ‘cg’, the effects of tail arm length variation on $C_{m_o}$, longitudinal stiffness and its corresponding change in $\alpha_a$ are tabulated in the table 4.

Table 4. Details of tail arm length ($l_t$) control for the most rearward location of ‘cg’

| Horizontal tail arm length ($l_t$) | $C_{m_o}$ | Longitudinal Stiffness ($\frac{\partial C_{m_o}}{\partial \alpha_a}$) | Angle of attack ($\alpha_a$) in degree | Stability curve slope in degree | Remark |
|-----------------------------------|-----------|------------------------------------------------|-----------------------------------|---------------------------------|--------|
| 2.000                             | 0.0121    | 0.0139                                          | -49.80                            | 0.8                             | Unstable |
| 2.031                             | 0.0129    | 0.0001                                          | -10500.62                         | 0.0                             | Neutral  |
| 2.100                             | 0.0149    | -0.0307                                         | 27.87                             | -1.8                            | Stable   |
| 2.200                             | 0.0178    | -0.0752                                         | 13.54                             | -4.3                            | Stable   |
| 2.300                             | 0.0206    | -0.1198                                         | 9.87                              | -6.8                            | Stable   |
| 2.500                             | 0.0263    | -0.2089                                         | 7.22                              | -11.8                           | Stable   |
| 3.000                             | 0.0406    | -0.4318                                         | 5.39                              | -23.4                           | Stable   |
| 3.500                             | 0.0549    | -0.6546                                         | 4.80                              | -33.2                           | Stable   |
| 4.031                             | 0.0700    | -0.8911                                         | 4.50                              | -41.7                           | Stable   |
| 4.500                             | 0.0834    | -1.1002                                         | 4.34                              | -47.8                           | Stable   |
| 5.000                             | 0.0977    | -1.3230                                         | 4.23                              | -52.9                           | Stable   |
| 5.500                             | 0.1119    | -1.5459                                         | 4.15                              | -57.1                           | Stable   |
| 6.000                             | 0.1262    | -1.7687                                         | 4.09                              | -60.5                           | Stable   |
| 6.500                             | 0.1405    | -1.9915                                         | 4.04                              | -63.4                           | Stable   |
| 7.000                             | 0.1547    | -2.2143                                         | 4.01                              | -65.7                           | Stable   |
| 7.500                             | 0.1690    | -2.4371                                         | 3.97                              | -67.7                           | Stable   |
| 8.000                             | 0.1833    | -2.6600                                         | 3.95                              | -69.4                           | Stable   |
The effects of tail arm length on the AOA are shown in figure 10. And the effects of tail arm length on AOA and stability are shown in figure 11. From the figure 10, 11 and table 4, it is observed that the tail arm length change will not create considerable changes in decreasing the AOA from the steady level trim condition since stability increases greatly. Stability curve is almost perpendicular line after the tail arm length value is 20 m. whereas, decrement in tail arm length leads to an increase in AOA as well as a decrement in stability. In this case, even a slight decrement in tail length creates a huge difference in angle of attack. This also creates stability problems. Stability is neutral for the tail arm length value 2.031 m. below this value makes the aircraft in unstable condition. As a maximum control extent, by this way it can be controlled for the AOA from 4 degree to stall AOA theoretically. So it is recommended that not to go for the longitudinal control to decrease the αa from the steady level trim condition by changing tail arm length. Even to increase the αa from the steady level trim condition, it brings stability problems.

Figure 10. AOA variation with respect to tail arm length (lₜ) for the most rearward location of ‘cg’ case
6. Conclusion
In this numerical study, the effects of changing the horizontal tail (without elevators) arm length on the static longitudinal stability of the chosen Cessna 172 Skyhawk are presented. It shows how the AOA and stability curve varies based on the tail arm length value. The study concludes the following two points (i) Increasing AOA by decreasing tail arm length from the steady level trim condition is possible but it also affects the stability greatly, and (ii) Decreasing AOA by increasing tail arm length from the steady level trim condition is not achievable as expected. Hence, it is recommended that employing both elevator control and tail arm length control together will make an aircraft to meet any kind of longitudinal controllability and stability requirement at any instant especially during dynamic oscillations. This embedded aircraft configuration can be attained by proposing aircraft morphing technologies which have been an emerging field of research.

Symbols and abbreviations

cg Center of gravity
ac Aerodynamic center
np Neutral point
AOA Angle of Attack
$C_{m, cg}$ Pitching moment coefficient about ‘cg’ of the aircraft
$\alpha_a$ Absolute AOA of the aircraft
$\alpha_f$ AOA of fuselage
$\alpha_{wb}$ AOA of wing-body
$\alpha_w$ AOA of wing
$\alpha_t$ Absolute AOA of tail
$C_{m, ac, w}$ Moment coefficient of the wing about aerodynamic center
$C_{m, o}$ Zero lift pitching moment
$h_{cg}$ Location of cg from leading edge of the wing in terms of percentage of chord
$h_{ac}$ Location of wing ‘ac’ from the leading edge in terms of percentage of chord
$h_{np}$ Location of ‘np’ from the leading edge of wing in terms of percentage of chord
\[V_\infty\] Free stream velocity
\[L_w\] Lift force produced by the wing
\[D_w\] Drag force produced by the wing
\[L_t\] Lift force produced by the tail
\[D_t\] Drag force produced by the tail
\[W\] Weight of the aircraft
\[C_{L,w}\] Coefficient of lift of wing
\[C_{L,t}\] Coefficient of lift of wing-body
\[C_{L,t}^{\text{max}}\] Coefficient of lift of horizontal tail
\[a\] Lift curve slope of aircraft
\[a_{\text{wb}}\] Lift curve slope of wing-body
\[a_t\] Lift curve slope of horizontal tail
\[i_w\] Wing incidence angle
\[S\] Wing reference surface area
\[C\] Wing chord length (approximately equal to zero lift chord length of wing)
\[M_{\text{ac},w}\] Moment about aerodynamic center of wing
\[M_{\text{ac},t}\] Moment about aerodynamic center of tail
\[\varepsilon\] Downwash produced by the wing
\[V_H\] Horizontal tail volume coefficient
\[i_t\] Tail arm length
\[z_t\] Perpendicular distance between ‘cg’ of the aircraft and ‘ac’ of the tail.
\[S_t\] Tail surface area
\[\delta_e\] Elevator deflection angle

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