Multi-criteria fuzzy model for system technical condition estimation at the life cycle stages

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Abstract. The article describes the tasks of in-depth estimation of the systems technical condition based on fuzzy ideas about the behavior of parameters within the tolerance limits. The proposed approach allows to more accurately control the influence of influencing factors at different stages of the life cycle of systems compared with the known methods. The approach is based on the application of six local criteria and a comprehensive criterion for assessing the level of system working capacity. The criteria are based on time series, linguistic variables, membership functions, and the proposed fuzzy classifier algorithm. Taking into account the peculiarities of the dynamics of changes in parameters in the tolerance field provides new opportunities for managing the life cycle processes of systems. The value of the evaluation of the level of working capacity is defined as the left most maximum of a fuzzy set for the corresponding output linguistic variable or the smallest of the modal values if the fuzzy set has several modal values. The quantitative estimation of working capacity is complemented by a qualitative estimation, expressed in the form of a linguistic description of the level of working capacity and degree of evaluation confidence in the result of recognition, understandable to the end user and convenient in making management decisions and developing recommendations at the stages of the life cycle.

1. Introduction

Continuous complication of technical systems, increasing requirements for their reliability and increasing responsibility for their functions make the development and improvement of scientific, methodological and technical support for monitoring the technical condition of responsible and expensive systems particularly relevant [1-6]. The classical method and technical means of the pre-limit control are not focused on the estimation of the operational states of products having a different level of functioning quality in accordance with a specific implementation of the parameter vector. These circumstances do not allow timely detection of latent defects at the early stages of their development and facts of poor quality adjustment, which appear as an anomalous change in parameter values within the limits of tolerance fields in time and proximity of parameter values to the limits of tolerance fields. The aim of the work is to develop a multi-criteria fuzzy model for assessing the
technical condition of systems during the organization of the tolerance control processes at all stages of the life cycle (LC), allowing to increase the reliability of the control results taking into account uncertainties. The model is based on a fuzzy classification of the model input parameters [7], determined on the basis of measured values of parameters at the current time and accumulated data in the form of parameter value vectors characterizing the system. The proposed approach to assessing the technical condition of the system expands the possibilities of the pre-limit control by introducing an additional process using seven fuzzy classifiers, taking into account the proximity of the parameter values to the limits of the tolerance fields and the dynamics of their change. The study applied the theories and methods for analyzing time series, fuzzy sets, fuzzy logic, decision making [1-8] and designing intelligent information systems.

2. Features of the estimation of the systems technical condition at different stages of the life cycle

The process approach used in describing the stages of the LC of systems involves the conversion of process inputs into outputs. The enlarged processes corresponding to the stages of the system LC are considered in the article, for definiteness: design and development (S1), production (S2), adjustment and debugging (S3), testing (S4), exploitation (S5). From the point of view of assessing the technical condition at these stages, various indicators are used, which determine the features of technology, resources and limitations. For S1, the inputs are the research results and technical requirements, and the outputs are the design, technological and software documentation. The criteria for the technical condition of the system being created at this stage determine compliance with the input requirements of S1 and the implementation requirements for S2. This is right for the subsequent stages. The parametric description of the technical condition criteria should reflect the continuity for the organization of the end-to-end digital LC with this approach, but can be expressed in different terms and achieved under various conditions at different stages. Table 1 lists examples of technical condition criteria that are not pretending to be complete and typical methods and means of solving problems at stages of LC under digitalization process.

| Stage                          | Technical condition criteria                        | Possible problems                                                                 | Methods and means applied                  |
|--------------------------------|-----------------------------------------------------|-----------------------------------------------------------------------------------|--------------------------------------------|
| design and development (S1)    | Technical level and quality of documentation        | Insufficient quality of materials and components, CAD level                        | Introduction of DFM analysis              |
| production (S2)                | Minimizing non-compliances (defects)                | Insufficient quality control                                                     | Introduction of 2D and 3D inspections and M2M |
| adjustment and debugging (S3)  | Reliability of the control                          | Incomplete knowledge of the technical condition                                   | Automation and control completeness        |
| tests (S4)                     | Level of working capacity in the conditions of external influences | Incomplete knowledge of the technical condition                                   | Improvement of test programs and methods   |
| exploitation (S5)              | Level of workability                                | Incomplete knowledge of the technical state                                      | Additional research                       |

The outlined approach is part of the concept of building tools for the design and production of cyber-physical systems. A possible approach that goes beyond the article is reduced to building interfaces between individual software packages. Mathematical models converting will eliminate or facilitate the development of such interfaces.

3. List and mathematical models of criteria and subcriteria

The article proposes an expanded and refined set of criteria. Compared with [9 ... 11], the list is supplemented with dynamic criteria. Some models are given in [11]. Since the parameters to be
classified have a different range of possible values, a single segment of the real axis [0,1] is chosen as the carrier of linguistic variables, and the parameters themselves are normalized. Below is the definition and types of models of six local criteria, based on the convolution of which further obtained the seventh complex criterion - the level of working capacity.

3.1. Workability margin by the parameter tolerance
The initial data for determining the numerical value of $x_1$ are: $x_i$ is the current value of the parameter, $x_{\text{min}}$ and $x_{\text{max}}$ are the parameter values that define the lower and upper limits of the tolerance field, and the value of the Spearman's rank correlation coefficient $R_S$ [11], which characterizes the process of directional changes in the time series levels. The parameter $x_1$, which determines the working capacity, allows to estimate the distance of the current value from the boundary of the tolerance field, taking into account the dynamics tendency.

3.2. Steadiness tendency of dynamics of the parameter changes
The input parameter of the 5-level classifier is $x_2$. The numerical value is calculated based on the time series of the parameter values and the $R_S$ coefficient using the formulas [11]. The numerical value $x_2$ from the interval [0,1] characterizes the steadiness of the directional change of the parameter values and the steadiness of the parameter values. A violation of a strictly ranked sequence of parameter values indicates an incomplete steadiness of their directional change, and a deviation from a tendency in one direction or the other direction indicates an incomplete steadiness of parameter values.

3.3. The value of the progressive change of the parameter
The input parameter of the 5-level classifier is $x_3$. The calculation of the numerical value is given in subsection 3.4.

3.4. Value of reverse parameter change
The input parameter of the 5-level classifier is $x_4$. It is proposed to use the following algorithm using production rules to calculate the numerical values of $x_3$ and $x_4$:

1. The absence of a change in the value of the parameter characterizing the current and previous measurement is checked.
   - Rule 1: If "$x_i = x_{i-1}$", then "$x_3 = 0$ and $x_4 = 0$". Otherwise, "Run Rule 2"
   - Rule 2: If "(Rs < 0 and $t < 0$) or (Rs > 0 and $t > 0$)"), then "$\Delta x_3 = x_i - x_{i-1}$ and $\Delta x_4 = 0$". Otherwise, "$\Delta x_3 = x_i - x_{i-1}"$
   - Rule 3: If "(Rs < 0 and $t < 0$) or (Rs > 0 and $t > 0$)"), then "$\Delta x_3 = x_i - x_{i-1}$ and $\Delta x_4 = 0$". Otherwise, "$\Delta x_3 = x_i - x_{i-1}"

2. The equation of a straight line passing through two points having coordinates is $(x_i, t_i)$ and $(x_{i-1}, t_{i-1})$. To do this, use the formula:

$$
\frac{t - t_{i-1}}{t_i - t_{i-1}} = \frac{x - x_{i-1}}{x_i - x_{i-1}}.
$$

The resulting linear equation with two variables $x$ and $t$ is converted to the form:

$$
x = f(t) = kt + b,
$$

where $x$ is a parameter, $t$ is time, $k$ is the angular coefficient of a linear function, $b$ is a free term. Next is the derivative of this function, where $\alpha$ is the angle of inclination of the straight line to the axis $t$. For positive $k$, this angle is acute and $x$ increases on the entire numerical axis $t$ (the graph goes “from top to bottom”), and for negative $k$, it grows blunt and $x$ decreases on the whole numerical axis $t$ (the graph goes “from bottom to top”).

3. The values of $\Delta x_3$ - the deviation of the parameter in the forward direction (forward) and $\Delta x_4$ - the deviation of the parameter in the opposite direction (reverse) are determined.
4. The numerical values of $x_3$ and $x_4$ are determined by their normalization. In this case, the upper limit of the range of possible values is predetermined by an expert method, and the module $\Delta x_3$ or $\Delta x_4$
is substituted into the formula as the current normalized value. In the event that the modulus of the current value is equal to or greater than the value of the upper limit of the range, the value is equal to the value of the upper limit.

3.5. The rate of change of the parameter

The input parameter of the 5-level classifier is $x_5$. The rate of change of state parameters $v$ characterizes the speed at which the current value of the parameter increases or decreases during the time interval between the current and the previous measurement. The positive (negative) value of $v$ expresses the rate of growth (decrease) of the levels of the series. The value of the input parameter $x_5$ is determined similarly to the values of $x_3$ and $x_4$.

3.6. Acceleration of parameter change

The input parameter of the 5-level classifier is $x_6$. Acceleration of changes in the state parameters $a$ characterizes the rapidity of the change in the velocity $v$, with which the current value of the parameter has increased or decreased during the time interval between the current and the previous measurement. The numerical value is calculated by the formula:

$$a = \frac{v_i - v_{i-1}}{t_i - t_{i-1}},$$

where $v_i$ is the rate of change of the parameter value over the time interval $[t_{i-1}, t_i]$, $v_{i-1}$ is the rate of change of the parameter value over the time interval $[t_{i-2}, t_{i-1}]$. The value of the input parameter $x_6$ is determined similarly to the values of $x_3$ and $x_4$.

In table 2, each criterion is represented by a classified variable, which corresponds to a linguistic variable, term set, and membership function of a fuzzy set. As a term set for each linguistic variable, a set is used $\{EC \text{ -- "excellent condition"}, GC \text{ -- "good condition"}, SC \text{ -- "satisfactory condition"}, DC \text{ -- "dangerous condition"}, PS \text{ -- "pre-failure status"}\}$. Membership functions for terms of linguistic variables in general can be defined analytically by the following expressions:

$$\mu_{EC}(x, a, b) = \begin{cases} 
1, & x \leq a \\
\frac{b-x}{b-a}, & a < x < b \\
0, & b \leq x
\end{cases}$$

$$\mu_{GC}(x, a, b) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & c \leq x
\end{cases}$$

$$\mu_{SC}(x, a, b) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a < x < b \\
1, & b \geq x
\end{cases}$$

where $a, b, c$ – are some numerical parameters that take arbitrary real values and are ordered by the relation: $a \leq b \leq c$. 


Table 2. Fuzzy representation of local and comprehensive criteria.

| Classified variable | Linguistic variable                        | Therm-set | Functions of fuzzy sets |
|---------------------|--------------------------------------------|-----------|------------------------|
| \( x_1 \)           | \( A1 \) - estimation of the state of the parameter according to the criterion of the workability margin by the parameter tolerance | \( EC_1 \) | \( \mu_2 (x_1; 0.1, 0.3) \) |
|                     |                                             | \( GC_1 \) | \( \mu_2 (x_1; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_1 \) | \( \mu_2 (x_1; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( DC_1 \) | \( \mu_2 (x_1; 0.5, 0.7, 0.9) \) |
|                     |                                             | \( PS_1 \) | \( \mu_2 (x_1; 0.7, 0.9) \) |
| \( x_2 \)           | \( A2 \) - estimation of the state of the parameter according to the stability criterion of the dynamics tendency steadiness | \( EC_2 \) | \( \mu_2 (x_2; 0.1, 0.3) \) |
|                     |                                             | \( GC_2 \) | \( \mu_2 (x_2; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_2 \) | \( \mu_2 (x_2; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( DC_2 \) | \( \mu_2 (x_2; 0.5, 0.7, 0.9) \) |
| \( x_3 \)           | \( A3 \) - estimation of the state of the parameter according to the criterion of the value of the progressive change of the parameter | \( EC_3 \) | \( \mu_2 (x_3; 0.1, 0.3) \) |
|                     |                                             | \( GC_3 \) | \( \mu_2 (x_3; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_3 \) | \( \mu_2 (x_3; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( DC_3 \) | \( \mu_2 (x_3; 0.5, 0.7, 0.9) \) |
| \( x_4 \)           | \( A4 \) - estimation of the state of the parameter according to the criterion of the value of the reverse parameter change | \( EC_4 \) | \( \mu_2 (x_4; 0.1, 0.3) \) |
|                     |                                             | \( GC_4 \) | \( \mu_2 (x_4; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_4 \) | \( \mu_2 (x_4; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( DC_4 \) | \( \mu_2 (x_4; 0.5, 0.7, 0.9) \) |
| \( x_5 \)           | \( A5 \) - estimation of the parameter state by the criterion of the rate of change of the parameter | \( EC_5 \) | \( \mu_2 (x_5; 0.1, 0.3) \) |
|                     |                                             | \( GC_5 \) | \( \mu_2 (x_5; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_5 \) | \( \mu_2 (x_5; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( DC_5 \) | \( \mu_2 (x_5; 0.5, 0.7, 0.9) \) |
| \( x_6 \)           | \( A6 \) - estimation of the state of the parameter according to the criterion of acceleration of the parameter change | \( EC_6 \) | \( \mu_2 (x_6; 0.1, 0.3) \) |
|                     |                                             | \( GC_6 \) | \( \mu_2 (x_6; 0.1, 0.3, 0.5) \) |
|                     |                                             | \( SC_6 \) | \( \mu_2 (x_6; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( DC_6 \) | \( \mu_2 (x_6; 0.5, 0.7, 0.9) \) |
| \( y \)             | \( B \) - estimation of the parameter state by the criterion of working capacity level | \( EC_7 \) | \( \mu_2 (y; 0.7, 0.9) \) |
|                     |                                             | \( SC_7 \) | \( \mu_2 (y; 0.3, 0.5, 0.7) \) |
|                     |                                             | \( GC_7 \) | \( \mu_2 (y; 0.5, 0.7, 0.9) \) |

4. Estimation of the level of working capacity by a complex criterion

To obtain to obtain a numerical estimate, a linguistic description of the level of health and the degree of evaluative confidence in the recognition result, it is proposed to use the following fuzzy classification algorithm based on the fuzzy model of E. Mamdani.

4.1. Input of the initial data

Baseline data are:

- values of the time series \( x^* = \{ x^*_t : t \in T \} \), where \( x^*_t \) - the value of the monitored parameter of the technical system, recorded at time \( t \);
• a set of input linguistic variables \( A = \{ A_i \}_{i=1}^6 \);
• output linguistic variable \( B \);
• basic term-sets of linguistic variables are \( A_i \) and \( B \)
\( T(A) = \{ EC_i, GC_i, SC_i, DC_i, PS_i \}_{i=1}^6 \; T(B) = \{ PS_i, DC_i, SC_i, GC_i, EC_i \} \) with corresponding triangular, z-shaped and s-shaped membership functions;
• base of fuzzy production rules \( P = \{ RULE \}_{j=1}^k \) containing the following formulas and systems of fuzzy production rules:

\[
\{ RULE \}_{j=1}^k : \text{IF } A_i \text{ is } EC_i \text{ and } A_2 \text{ is } EC_2 \text{ and } A_3 \text{ is } EC_3 \text{ and } A_4 \text{ is } EC_4 \text{ and } A_5 \text{ is } EC_5 \text{ and } A_6 \text{ is } EC_6 \text{, THEN } B \text{ is } EC_7;\]

\[
\{ RULE \}_{k} : \text{IF } A_i \text{ is } PS_i \text{ and } A_2 \text{ is } PS_2 \text{ and } A_3 \text{ is } PS_3 \text{ and } A_4 \text{ is } EC_4 \text{ and } A_5 \text{ is } PS_5 \text{ and } A_6 \text{ is } PS_6 \text{, THEN } B \text{ is } PS_7.\]

(7)

4.2. Calculation of input parameters of fuzzy classifiers
Definition of the set \( X = \{ x_1, x_2, x_3, x_4, x_5, x_6 \} \) of numerical values of the input variables of fuzzy classifiers based on \( x^* = \{ X^* : t \in T \} \) formulas and systems of fuzzy production rules.

4.3. Fuzzification of input variables
Determination of the set \( C = \{ C_{ij} \} \) of values of membership functions for each \( i \)-th of the subconditions of the rule

\[
P = \{ RULE \}_{j=1}^k \text{ base and for all input variables } X = \{ x_1, x_2, x_3, x_4, x_5, x_6 \}.\]

4.4. Aggregation of sub-conditions in \( P = \{ RULE \}_{j=1}^k \)
Definition of a set \( L = \{ \alpha_f \}_{f=1}^\varepsilon \) of numerical values of cut-off levels \( \alpha \) (degree of truth of conditions) according to each of the active rules \( P = \{ RULE \}_{j=1}^k \) of the rule base. The active rules are those in which the degrees of truth of all sub-conditions are non-zero. Since the condition of each \( f \)-th rule consists of six sub-conditions with sequence numbers \( i \), the degree of truth of a complex utterance is determined based on the known values of the degrees of truth of the sub-conditions \( C = \{ C_{ij} \} \) of active rules. The min-conjunction is used as an aggregation method, since in all rules, only a fuzzy conjunction is used as a logical connection for sub-conditions. To determine the result of a fuzzy conjunction, the formula is used:

\[
\alpha_f = \min \{ C_{ij} \}_{j=1}^\varepsilon.\]

(8)

4.5. Activation of the conclusions of each rule from the base of active rules \( P_a = \{ RULE \}_{j=1}^\varepsilon \)
Determining the degree of truth and the membership function of each conclusion for the output linguistic variable \( B \) considered. Production base rules \( P_a = \{ RULE \}_{j=1}^\varepsilon \) do not contain subconclusions and do not contain weighting factors determining the significance of the rule or confidence in the truth
degree of the conclusion obtained according to a separate rule. Therefore, to obtain the result of the activation, it is not necessary to make additional calculations of the degrees of truth of the subcontracts taking into account the weighting factors, but the numerical values of the set can be directly used. The activated ("truncated") membership functions of fuzzy sets of values of the output linguistic variable are determined using the min-activation method using the formula:

$$\mu^*_f(y) = \min\{\alpha_f, \mu_f(y)\}_{f=1}^g,$$

where $\mu_f(y)$ is the term membership function, which is the value of some output variable in the $f$-th production rule.

4.6. Accumulation conclusions of fuzzy production rules
Receiving the total membership function of a fuzzy set of values of the output linguistic variable $B$ by combining the activated membership functions $\mu_1^*(y), \mu_2^*(y), \ldots, \mu_g^*(y)$ of all conclusions. The max-disjunction operation is used as an accumulation method. To determine the result of a fuzzy disjunction, use the formula:

$$\mu_{acc}(y) = \max\{\mu_1(y)\}_{f=1}^g.$$

4.7. Defuzzification Fuzzy Sets
The procedure for determining the quantitative value of the output linguistic variable in the form of a real number. The process of transition from the membership function of the output linguistic variable $\mu_{acc}(y)$ to its precise value $y^*$ is carried out by the method of the left most maximum. The value of the output variable is defined as the modal value of a fuzzy set for the corresponding output linguistic variable or the smallest modal value (leftmost) if the fuzzy set has several modal values. The left most maximum of $y^*$ is calculated by the formula:

$$y^* = \min\{y_m\} = \min\{\arg\max\{\mu_{acc}(y)\}\},$$

where $y_m$ is the modal value of a fuzzy set with $\mu_{acc}(y)$.

4.8. Classification $y^*$
Recognition of the obtained numerical value $y^*$ is performed using a fuzzy five-level classifier of the parameter $y$. The result of the classification is a linguistic description of the level of health and the degree of confidence of the expert in this recognition result. Thus, the proposed fuzzy classifier allows us to obtain an estimate of the level of efficiency of the parameter in quantitative and qualitative form.

5. Conclusions
The proposed approach to assessing the technical condition of the system complements the technology of tolerance control by a fuzzy representation of the behavior of parameters within the tolerance limits. The quantitative estimation of working capacity is complemented by a qualitative estimation, expressed in the form of a linguistic description of the level of working capacity and degree of evaluation confidence in the result of recognition, understandable to the end user and convenient in making management decisions and developing recommendations at the stages of the life cycle. The proposed system of criteria is supplemented, as compared with previous publications [9 ... 11], with criteria that take into account the peculiarities of the dynamics of changes in parameters. The convolution of criteria in the form of fuzzy sets is performed using E. Mamdani's fuzzy algorithm and supplemented with an estimation of the level of parameter working capacity in a qualitative form. The resulting multi-criteria model and algorithms can be applied to different stages of systems LC and allow them to take into account their specifics by clarifying the dependence of criteria and constraints.
The following formal representations will be used to model end-to-end digital LC. The results obtained can be applied in the development of applications of intelligent decision support systems for monitoring the state in the responsible systems production.

References

[1] Huaguang Zhang Derong Liu Fuzzy 2006 Modeling and Fuzzy Control 416
[2] Ramezani S and A Memariani 2011 A fuzzy rule based system for fault diagnosis, using oil analysis results Int. J. Ind. Eng.Prod.Res. 22 91-8
[3] Zimmermann H-J 2000 Practical Applications of Fuzzy Technologies Springer p 667
[4] Amo A, Montero J, Biging G and Cutello V 2004 Fuzzy classification systems European Journal of Operational Research Elsevier 156(2) 495-507
[5] Werro N 2015 Fuzzy Classification of Online Customers Springer p 148
[6] S Stormer N, Werro, and D Risch 2006 Recommending Products by the Mean of a Fazzy Classification In Proc. of the European Conference on Collaborative Electronic Commerce Technology and Research CollECTeR pp 29-37
[7] Zadeh L A 1978 Fuzzy sets as a basis for a theory of possibility Fuzzy Sets and Systems 1 3-28
[8] Fishburn P C 1970 Utility Theory for Decision Making (New York: John Wiley and Sons) p 234
[9] Smirnov V A 2013 Malfunction searching in onboard control systems during acceptance control Informatsionno-upravliaisuchchie sistemy 2 24-8
[10] Korshunov G I, Nazarevich S A and Smirnov V A 2018 Fuzzy classification of technical condition at lifecycle stages of responsible appointment systems Proc. of the II International Scientific and Practical Conference “Fuzzy Technologies in the Industry – FTI 2018” Ulyanovsk Russia October 23-25 2018 CEUR Workshop Proceedings Vol-2258 urn: nbn:de:0074-2258-5 pp 427-37
[11] Korshunov G I, Smirnov V A, Frolova E A and Nazarevich S A. 2019 Fuzzy Models and System Technical Condition Estimation Criteria Proc. Conf. in Fourth International Congress on Information and Communication Technology (ICICT 2019) Venue - Brunel University taking place in London UK during February 25 and 26 2019. www.icict.co.uk (to be published)