Radiative decay of $\Upsilon(nS)$ into $S$-wave sbottomonium

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Abstract

A calculation is presented of the radiative decay of the $\Upsilon(nS)$ into a bound state of bottom squarks. Predictions are provided of the branching fraction as a function of the masses of the bottom squark and the gluino. Branching fractions as large as several times $10^{-4}$ are obtained for supersymmetric particle masses in the range suggested by the analysis of bottom-quark production cross sections. Data are shown that limit the range of allowed masses. Forthcoming high-statistics data from the CLEO Collaboration offer possibilities of discovery or significant new bounds on the existence and masses of supersymmetric particles.

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A long-standing puzzle in high-energy strong interactions is the fact that the rate for bottom quark ($b\bar{b}$) production at the Fermilab Tevatron collider is two to three times greater than the theoretical prediction from quantum chromodynamics (QCD) [1]. The next-to-leading order QCD contributions are large, and a combination of further higher-order effects in production and/or fragmentation may eventually reduce the discrepancy [2]. An alternative explanation is offered in Ref. [3] in the context of physics beyond the standard model. There, it is argued that a solution may be provided by the existence of a light bottom squark $\tilde{b}$ and a light gluino $\tilde{g}$, with masses in the ranges $2 \text{ GeV} < m_{\tilde{b}} < 5.5 \text{ GeV}$ and $12 \text{ GeV} < m_{\tilde{g}} < 16 \text{ GeV}$. The $\tilde{g}$ and the $\tilde{b}$ are the spin-1/2 and spin-0 supersymmetric partners of the gluon ($g$) and bottom quark ($b$). The masses of all other supersymmetric particles are assumed to be arbitrarily heavy, i.e., of order the electroweak scale or greater [3]. While this scenario is not among the more popular schemes for supersymmetry breaking, the hypothesis of a light bottom squark is not inconsistent with direct experimental searches and indirect constraints from other observables [4, 5, 6, 7, 8]. Therefore, it is essential either to confirm or to refute this proposal by examining its implications for additional processes. In this Letter, we show that high-statistics data being accumulated and analyzed now at the Cornell Electron Storage Ring (CESR) facility [9] could provide definitive confirmation of the proposal of Ref. [3] or severely constrain the allowed parameter space [10].

In the proposal of Ref. [3], it is possible that the bottom squark is relatively stable and, hence, bound states of a bottom squark and bottom antiquark ($s$bottomonium) could exist. These bound states could be produced in radiative decays of bottomonium states, such as $\Upsilon \rightarrow \tilde{S}\gamma$, where $\tilde{S}$ is the $S$-wave bound state of a $\tilde{b}\tilde{b}^*$ pair. Alternatively, the $\tilde{b}$ could decay promptly via $R$-parity and baryon-number violation [3, 7], and no bound state would be formed.

In this Letter, we compute the rate for an $\Upsilon(nS)$ to decay radiatively into an $S$-wave $\tilde{b}\tilde{b}^*$ bound state. We show that, provided that a bound state is formed, the resonance search by the CUSB Collaboration [11] already increases the allowed lower bounds on $m_{\tilde{b}}$ and $m_{\tilde{g}}$. Discovery of the bound states may be possible with the high-statistics 2002 CLEO-c data set, or a larger range of bottom-squark and gluino masses may be disfavored.

The mass eigenstates of the bottom squarks, $b_1$ and $b_2$, are mixtures of $b_L$ and $b_R$, the
FIG. 1: Feynman diagrams for the process $b\bar{b} \rightarrow \tilde{b}\tilde{b}^*\gamma$.

supersymmetry partners the left-handed (L) and right-handed (R) bottom quarks:

$$|	ilde{b}_1\rangle = \sin\theta_{\tilde{b}}|\tilde{b}_L\rangle + \cos\theta_{\tilde{b}}|\tilde{b}_R\rangle,$$

$$|	ilde{b}_2\rangle = \cos\theta_{\tilde{b}}|\tilde{b}_L\rangle - \sin\theta_{\tilde{b}}|\tilde{b}_R\rangle. \quad (1a)$$

$$|	ilde{b}_2\rangle = \cos\theta_{\tilde{b}}|\tilde{b}_L\rangle - \sin\theta_{\tilde{b}}|\tilde{b}_R\rangle. \quad (1b)$$

We take $\tilde{b}_1$ to be the eigenstate of lighter mass, and we drop the subscript 1 in the remainder of this paper. The mixing angle $\theta_{\tilde{b}}$ is constrained by the requirement that the coupling of a $\tilde{b}\tilde{b}^*$ pair to the $Z$ boson be sufficiently small to be compatible with data [4]. At lowest order (tree-level), this requirement implies that $\sin^2 \theta_{\tilde{b}} \approx 1/6$ [12].

We calculate the decay rate $\Gamma(\Upsilon \rightarrow \tilde{S}\gamma)$ in the framework of the nonrelativistic QCD (NRQCD) factorization formalism [13]. First, we compute the amplitude in full QCD for the process $b(p) + \bar{b}(p) \rightarrow \tilde{b}(q) + \tilde{b}^*(q) + \gamma(k)$. Typical Feynman diagrams are shown in Fig. 1. The indices $i$, $j$, $k$, and $l$ label the colors of the incident $b$ and $\bar{b}$ and the final $\tilde{b}$ and $\tilde{b}^*$, respectively. The color index of the exchanged gluino is $a$. The Feynman rules [10] depend on the mixing angle $\theta_{\tilde{b}}$.

We carry out the computation in the $b\bar{b}$ rest frame and choose the radiation gauge. Then, the photon-$\tilde{b}\tilde{b}^*$ vertex vanishes, since it is proportional to $\epsilon^* \cdot (2q + k) = \epsilon^* \cdot (2p)$, where $\epsilon$ is the polarization of the photon. Therefore, we need to compute only the two diagrams of the type in Fig. 1(a), in which the photon attaches to the $b$ or the $\bar{b}$. We make use of the

1 In the first paper of Ref. [12], it is argued that one-loop contributions may render the light $\tilde{g}$ and light $\tilde{b}$ scenario inconsistent with data at the $2\sigma$ level, unless the mass of the heavier $\tilde{b}_2$ is less than about 125 GeV. Making somewhat different assumptions, the authors of the second paper obtain a $5\sigma$ bound of 180 GeV and the author of the third paper obtains a $3\sigma$ bound of more than 200 GeV. The possibility that the mass of the $\tilde{b}_2$ could be as low as 100 GeV or so is not excluded by data. Analysis of data from $e^+e^-$ interactions at LEP, for example, would require first a detailed modeling of the decay modes of the $\tilde{b}_2$.

2 One can adapt the NRQCD formalism for spin-1/2 quarks to the case of spin-0 squarks by dropping the spin-dependent interactions and replacing Pauli fields with scalar fields.
Here, we use plane-wave states for the quarks, squarks, and photon, without any factors 1

Squaring the matrix element and averaging over the polarizations of the Υ, we obtain

spin-triplet projector \[14\]

\[
\sum_{\lambda_1,\lambda_2} u(p, \lambda_1) \bar{v}(-p, \lambda_2) (\frac{1}{2} \lambda_1 \frac{1}{2} \lambda_2 | 1 \epsilon_T) = -\frac{1}{\sqrt{2}} \phi_T (p - m_b),
\]

(2)

where \(\epsilon_T\) is the polarization of the \(b\bar{b}\) system, the Dirac spinors \(u(p, s_1)\) and \(v(p, s_2)\) have the relativistic normalization \(\bar{u}(p, s)u(p, s) = \bar{v}(p, s)v(p, s) = 2E_b(p)\), and \(E_i(p) = \sqrt{p_i^2 + m_b^2}\). Projecting onto color-singlet \(b\bar{b}\) and \(\bar{b}b^*\) states, which leads to the color factor \((\delta_{ij} \delta_{kl}/N_c)T^a_k T^a_l = \frac{4}{3}\), we find that the amplitude in full QCD is

\[
\mathcal{M} = \frac{16\sqrt{2}}{3m_\bar{b}} g_s^4 e^2 c_\bar{b}^2 \sin \theta_b \cos \theta_b \epsilon \cdot \epsilon_T.
\]

(3)

We use plane-wave states for the quarks, squarks, and photon, without any factors 1/(2E). Squaring the matrix element and averaging over the polarizations of the Υ, we obtain

\[
|\mathcal{M}|^2 = \frac{1024}{27m_\bar{b}^2} g_s^4 e^2 c_\bar{b}^2 \sin^2 \theta_b \cos^2 \theta_b.
\]

(4)

We match the squared matrix element in Eq. (4) onto NRQCD for the \(b\bar{b}\) system in the \(b\bar{b}\) rest frame and onto NRQCD for the \(\bar{b}b^*\) system in the \(\bar{b}b^*\) rest frame using, respectively,

\[
|\mathcal{M}|^2 = F_1(\bar{b}b) |O_1(\bar{b}b)|^2 ,
\]

(5)

\[
|\mathcal{M}|^2 = \bar{F}_1(\bar{b}b) |O_1(\bar{b}b)|^2 .
\]

(6)

Here, \(F_1\) and \(\bar{F}_1\) are short-distance coefficients, \(O_1(\bar{b}b)\) is a four-quark operator, and \(\bar{O}_1(\bar{b}b)\) is a four-quark operator:

\[
O_1(\bar{b}b) = \psi^i \sigma^k \chi^i \alpha^k \psi,
\]

(7)

\[
\bar{O}_1(\bar{b}b) = \bar{\chi}^* \psi \sum_X [\bar{b}b^* + X] \langle \bar{b}b^* + X | \bar{\chi}^* \psi
\]

(8)

The Pauli field \(\psi\) annihilates a \(b\) quark, the Pauli field \(\chi\) creates a \(b\) antiquark, the scalar field \(\bar{\psi}\) annihilates a \(b\) squark, the scalar field \(\bar{\chi}\) creates a \(b\) squark, and the sum is over all intermediate states \(X\). The short-distance coefficient \(F_1(\bar{b}b)\) for the \(\bar{b}b\) operator contains \(\langle 0 | \bar{O}_1(\bar{b}b) | 0 \rangle\), and the short-distance coefficient for the \(\bar{b}b^*\) operator \(\bar{F}_1(\bar{b}b)\) contains \(\langle \bar{b}b | O_1(\bar{b}b) | \bar{b}b \rangle\). In the Born approximation, the matrix elements are

\[
\langle \bar{b}b | O_1(\bar{b}b) | \bar{b}b \rangle = 2(2E_b)^2 N_c,
\]

(9)

\[
\langle 0 | \bar{O}_1(\bar{b}b) | 0 \rangle = (2\bar{E}_b)^2 N_c,
\]

(10)
where $E_{\tilde{b}}$ is the energy of $\tilde{b}(\tilde{b}^*)$ in the $\tilde{b}\tilde{b}^*$ rest frame. The factors $2E_b$ and $2E_{\tilde{b}}$ appear because the free-particle states are normalized to $2E$ particles per unit volume. The factors $N_c$ come from the color traces, and the additional factor 2 in the $\tilde{b}\tilde{b}$ case comes from the spin trace.

From this matching process, we deduce that

$$|\mathcal{M}|^2 = \frac{512}{243m_3^2(2E_b)^2(2\tilde{E}_{\tilde{b}})^2}g_s^4e^2e_b^2\sin^2\theta_b\cos^2\theta_b\langle\tilde{b}\tilde{b}(|O_1(3S)|\tilde{b}\tilde{b})\langle0|\tilde{O}_1\tilde{b}\tilde{b}(1S_0)|0\rangle. \tag{11}$$

To compute the decay rate of the process $\Upsilon \to \tilde{S}\gamma$, we replace the $\tilde{b}\tilde{b}$ state by the $\Upsilon$ state and replace the $\tilde{b}\tilde{b}^*$ state by the $\tilde{S}$ state in the squared matrix element, multiply by the two-body phase $\Phi_2 = (1/8\pi)[1 - (M_S^2/M_\Upsilon^2)]$, and multiply by $2M_\tilde{S}$, where $M_\tilde{S}$ is the mass of the $\tilde{b}\tilde{b}^*$ bound state, and $M_\Upsilon$ is the $\Upsilon$ mass.\footnote{The factor $2M_\tilde{S}$ appears for the following reason. The $\tilde{S}$ state is normalized to one particle per unit volume in the $\tilde{S}$ rest frame. In order to preserve that normalization in the $\Upsilon$ rest frame, one must multiply by the Lorentz-contraction factor for the volume, namely, $2M_\tilde{S}/(2E_\tilde{S})$, where $E_\tilde{S}$ is the energy of the $\tilde{S}$ in the $\Upsilon$ rest frame. The factor $1/(2E_\tilde{S})$ is absorbed into the conventional definition of the phase space.}

The result is

$$\Gamma(\Upsilon \to \tilde{S}\gamma) = \frac{256\pi^2e_b^2\alpha^2}{243}\sin^2\theta_b\cos^2\theta_b\langle\Upsilon|\langle O_1(3S)|\Upsilon\rangle\rangle\langle0|\tilde{O}_1\tilde{S}(1S_0)|0\rangle(2M_S)(1 - \frac{M_\tilde{S}}{M_\Upsilon^2}). \tag{12}$$

If one considers specific polarizations of the $\Upsilon$, then the decay rate is no longer independent of the angle $\theta$ between the photon and the axis that defines the direction of longitudinal polarization. (In $\Upsilon$ production in $e^+e^-$ annihilation, for example, the polarization is transverse to the beam direction.) In the case of equal population of the two transverse polarization states, $d\Gamma/d(cos\theta) = (3/8)(1 + cos^2\theta)\Gamma$, while in the case of longitudinal polarization, $d\Gamma/d(cos\theta) = (3/4)(1 - cos^2\theta)\Gamma$, where $\Gamma$ is found in Eq. (12).

Using the nonrelativistic approximations $E_b \approx m_b$ and $E_{\tilde{b}} \approx m_\tilde{b}$ in Eq. (12), we obtain the branching fraction

$$\text{Br}(\Upsilon \to \tilde{S}\gamma) = \frac{\Gamma(\Upsilon \to \tilde{S}\gamma)}{\Gamma(\Upsilon \to \mu^+\mu^-)} \times \text{Br}(\Upsilon \to \mu^+\mu^-)_{\text{Exp}}$$

$$= \frac{64\pi\alpha^2}{81\alpha} \sin^2\theta_b\cos^2\theta_b\langle0|\tilde{O}_1\tilde{S}(1S_0)|0\rangle \frac{m_\Upsilon^2m_\tilde{S}^2m_\tilde{b}^2}{M_\Upsilon^2M_\tilde{S}} \times \left(1 - \frac{M_\tilde{S}}{M_\Upsilon^2}\right) \text{Br}(\Upsilon \to \mu^+\mu^-)_{\text{Exp}}. \tag{13}$$

In deriving Eq. (13) we use

$$\Gamma(\Upsilon \to \mu^+\mu^-) = \frac{8\pi\alpha^2}{3} \frac{\langle T|\psi^\dagger\sigma^k\chi|0\rangle \langle0|\chi^\dagger\sigma^k\psi|T\rangle}{M_\Upsilon^2} \tag{14}$$
FIG. 2: Branching fraction for the process $\Upsilon \rightarrow \bar{S}\gamma$ as a function of $M_\bar{S}$. The shaded area is excluded at the 90% confidence level by the $\Upsilon \rightarrow X\gamma$ search of the CUSB Collaboration [11].

and take the vacuum-saturation approximation

$$\langle \Upsilon | O_1(3S_1) | \Upsilon \rangle \approx \langle \Upsilon | \psi^\dagger \sigma^k \chi | 0 \rangle \langle 0 | \chi^\dagger \sigma^k \psi | \Upsilon \rangle,$$

which neglects terms of relative order $v^4$. Here, and throughout this paper, $v$ is the heavy-quark or heavy-squark velocity in the onium rest frame.

In the vacuum-saturation approximation, the color-singlet production matrix element $\langle 0 | \tilde{O}_1(1S_0) | 0 \rangle$ may be replaced by the corresponding decay matrix element $\langle \bar{S} | \tilde{O}_1(1S_0) | \bar{S} \rangle$, with uncertainties of relative order $v^4$. Furthermore, the sbottomonium decay matrix element $\langle \bar{S} | \tilde{O}_1(1S_0) | \bar{S} \rangle$ is related to the heavy-quarkonium (HQ) matrix element of the same mass by

$$\langle \bar{S} | \tilde{O}_1(1S_0) | \bar{S} \rangle \approx \langle \text{HQ} | O_1(3S_1) | \text{HQ} \rangle / 2,$$

where we neglect spin-dependent contributions to the HQ matrix element of relative order $v^2$. The expressions on the left and right sides of Eq. (16) are proportional to the squares of the sbottomonium and quarkonium wave functions at the origin, respectively. The value of the HQ matrix element is known at the $\Upsilon$ mass. We estimate its value at smaller quarkonium masses by assuming that it scales as $m_q^{3/2}$. This scaling behavior is approximately that which one obtains from the Martin potential [15], which describes the $J/\psi$ and $\Upsilon$ systems reasonably well. Then we have

$$\langle \bar{S} | \tilde{O}_1(1S_0) | \bar{S} \rangle \approx \left( \frac{m_q}{m_0} \right)^{3/2} \langle \Upsilon | O_1(3S_1) | \Upsilon \rangle / 2.$$
We use a recent lattice measurement of the bottomonium matrix element: \( \langle \Upsilon | \mathcal{O}_1 (^3S_1) | \Upsilon \rangle = 4.10 \pm 0.42 \text{ GeV}^3 \) (Ref. [16]).

In Fig. 2, we plot the branching fraction (13), for several values of \( m_{\tilde{b}} \), as a function of \( M_{\tilde{S}} \approx 2m_{\tilde{b}} \), using the estimate of the sbottomonium matrix element in Eq. (17). Here, and in all further numerical estimates, we take \( \sin^2 \theta_b = 1/6 \), \( m_b = 4.73 \pm 0.20 \) GeV, \( \alpha = 1/137 \), \( \alpha_s = 0.2 \pm 0.02 \), and \( \text{Br}(\Upsilon \rightarrow \mu^+\mu^-)_{\text{Exp}} = 2.48 \pm 0.06\% \) (Ref. [17]). This value for \( \alpha_s \) corresponds approximately to the renormalization scale \( m_b \), which is an upper bound on the momentum transfer in the radiative decay process.

In order to compare our result for the branching fraction with the experimental resolution, it is necessary to know the width of the \( S \)-wave sbottomonium state. The total width into light hadrons is given, in leading order in \( \alpha_s \), by the width into two gluons. This quantity is computed, in leading order in the nonrelativistic expansion, by Nappi [18]:

\[
\Gamma \left( \tilde{S} \rightarrow gg \right) = \frac{4\alpha_s^2}{3M_{\tilde{S}}^2} |R(0)|^2 ,
\]

where \( R(0) \) is the \( \tilde{S} \) radial wave function at the origin. Using Eq. (17) and taking \( M_{\tilde{S}} \approx 2m_{\tilde{b}} \), we have

\[
|R(0)|^2 = \frac{4\pi}{N_c} \langle \tilde{S} | \tilde{\mathcal{O}}_1 (^1S_0) | \tilde{S} \rangle.
\]

The width of the sbottomonium state into light hadrons is less than 10 MeV in the range of parameters proposed in the light-bottom-squark scenario [3]. This width is less than the energy resolution in the CUSB search for monochromatic photon signals [11]. Therefore, we can compare our estimate of the branching fraction \( \text{Br}(\Upsilon \rightarrow \tilde{S}\gamma) \) directly with the CUSB 90% confidence level for the exclusion of \( \Upsilon \rightarrow X\gamma \), which is plotted, along with our estimate, in Fig. 2. We see that, if a bound state is formed, then a part of the range of mass parameters proposed in the light-bottom-squark scenario is disfavored.

There are several uncertainties in our calculation. The uncertainty in \( \text{Br}(\Upsilon \rightarrow \mu^+\mu^-) \) is 2.4%. The uncertainty in \( m_b \) is 4.2%. The uncertainties in the value of \( \alpha_s \) and in the lattice computation of the bottomonium matrix element are each about 10%. There is also an uncertainty from the extrapolation of the HQ matrix element from \( M_\Upsilon \) to \( M_{\tilde{S}} \). We estimate it by checking the accuracy of the extrapolation against the phenomenological values of the wave functions at the origin for the \( \Upsilon \) and the \( J/\psi \), as determined from the data for the decay rates into lepton pairs combined with the next-to-leading-order QCD
FIG. 3: The regions of the $m_{\tilde{b}}$-$m_{\tilde{g}}$ parameter space that are excluded at the 90% confidence level by the $\Upsilon \rightarrow X \gamma$ search of the CUSB Collaboration (shaded region) [11]. The solid curve represents the central value of the theoretical calculation, and the dashed curves show the uncertainties on the theoretical values, as described in the text. The strip shows the region $2 < m_{\tilde{b}} < 5.5$ GeV, $12 < m_{\tilde{g}} < 16$ GeV proposed in the light-bottom-squark scenario [3].

expressions for those decay rates. We conclude that the extrapolation error is approximately $31\% \times (M_\Upsilon - M_{\tilde{S}})/(M_\Upsilon - M_{J/\psi})$. There are uncalculated relativistic corrections of the order of the square of the heavy-quark or squark velocity in the onium rest frame. We estimate these to yield an uncertainty in the decay rate of $20\% \times (M_\Upsilon - M_{\tilde{S}})/(M_\Upsilon - M_{J/\psi}) + 10\%$. There are uncertainties from uncalculated corrections of higher order in $\alpha_s$ and from the imprecision in the value of the renormalization scale. We estimate these by varying the scale of $\alpha_s$ from $m_b/2$ to $2m_b$. This procedure yields uncertainties in $\alpha_s$ of $+33\%$ and $-18\%$. As $M_{\tilde{S}}$ approaches $M_\Upsilon$, the momentum transfer in the radiative decay becomes considerably less than $m_b$, and, in this region, our choice of scale probably results in an underestimate of the decay rate. In this same region, we expect to find violations of the NRQCD factorization, which holds only for $M_\Upsilon - M_{\tilde{S}} \gg \Lambda_{\text{QCD}}$.

In Fig. 3 we show the region excluded by the CUSB data. We calculate the uncertainty band on the boundary of the excluded region by adding, in quadrature, the theoretical uncertainties mentioned above. We also plot the values of $m_{\tilde{g}}$ and $m_{\tilde{b}}$ that are suggested in the light-bottom-squark scenario [3]. At $m_{\tilde{g}} = 12$ GeV, provided that the bottom squark lifetime is great enough to permit formation of the $\tilde{b}\tilde{b}^*$ bound state, the mass range $m_{\tilde{b}} < 3.5^{+0.4}_{-0.6}$ GeV is excluded at the 90% confidence level by the CUSB data. At $m_{\tilde{g}} = 16$ GeV,
the central and upper values of the excluded range are $m_{\tilde{b}} = 3.0$ and $3.6$ GeV, but the theoretical uncertainties do not permit us to specify a lower limit at the 90% confidence level. One can probe the region of higher bottom-squark masses by increasing the statistics of the photon sample and by examining decays from bottomonium states of higher mass, such as the $\Upsilon(3S)$ and the $\Upsilon(4S)$. ⁴

Bottom squarks with mass $2$ GeV $< m_{\tilde{b}} < 5.5$ GeV along with gluinos with mass $12$ GeV $< m_{\tilde{g}} < 16$ GeV are proposed in Ref. ³ to explain the larger-than-predicted rate for bottom quark ($b\bar{b}$) production at the Fermilab collider. In this Letter, we show that CUSB data on radiative $\Upsilon$ decays already provide an important additional constraint on the mass ranges. The high-statistics 2002 CLEO data should either permit discovery of squarkonium bound states, confirm the exclusion region of the earlier CUSB data, or further narrow the allowed range of supersymmetry parameter space.

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[1] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, Nucl. Phys. B 431, 453 (1994); P. Nason et al., in Proc. of the Workshop on Standard Model Physics (and more) at the LHC, Geneva, 1999, pp 231-304.

[2] M. Cacciari and P. Nason, Phys. Rev. Lett. 89, 122003 (2002)

[3] E. L. Berger, B. W. Harris, D. E. Kaplan, Z. Sullivan, T. M. P. Tait, and C. E. M. Wagner, Phys. Rev. Lett. 86, 4231 (2001) [arXiv:hep-ph/0012001].

[4] M. Carena, S. Heinemeyer, C. E. M. Wagner, and G. Weiglein, Phys. Rev. Lett. 86, 4463 (2001).

[5] CLEO Collaboration, V. Savinov et al., Phys. Rev. D 63, 051101 (2001) [arXiv:hep-ex/0010047].

⁴ In order to apply the expression for the branching fraction in Eq. ¹³ to decay from a state other than the $\Upsilon$, one must replace $\text{Br}(\Upsilon \to \mu^+\mu^-)_{\text{Exp}}$ with the rate for the decaying state and replace $m_b$ with one-half the mass of the decaying state.
[6] DELPHI Collaboration, P. Abreu et al., Phys. Lett. B 444, 491 (1998) [arXiv:hep-ex/9811007].

[7] E. L. Berger, arXiv:hep-ph/0201229, Int. J. Mod. Phys. A, in press.

[8] References to an extensive body of recent theoretical papers can be found in E. L. Berger, C. W. Chiang, J. Jiang, T. M. P. Tait, and C. E. M. Wagner, arXiv:hep-ph/0205342, Phys. Rev. D (in press).

[9] CLEO Collaboration, R. A. Briere et al., “CLEO-c and CESR-c: A New Frontier of Weak and Strong Interactions”, CLNS 01/1742 (2001).

[10] If $m_{\tilde{b}} < m_b$, direct decays such as $\Upsilon \rightarrow \tilde{b}\tilde{b}^*$ and $\chi_b \rightarrow \tilde{b}\tilde{b}^*$ could proceed with sufficient rates for observation. See, E. L. Berger and L. Clavelli, Phys. Lett. B 512, 115 (2001) [arXiv:hep-ph/0105147]; E. L. Berger and J. Lee, Phys. Rev. D 65, 114003 (2002) [arXiv:hep-ph/0203092].

[11] CUSB Collaboration, P. Franzini et al., Phys. Rev. D 35, 2883 (1987).

[12] One-loop contributions to the ratio $R_b$ at the $Z$ pole are considered in J. Cao, Z. Xiong, and J. M. Yang, Phys. Rev. Lett. 88, 111802 (2002); G. C. Cho, Phys. Rev. Lett. 89, 091801 (2002); S. W. Baek, Phys. Lett. B 541, 161 (2002).

[13] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995); 55, 5853(E) (1997).

[14] J. H. Kühn, J. Kaplan, and E. G. Safiani, Nucl. Phys. B157, 125 (1979); B. Guberina, J. H. Kühn, R. D. Peccei, and R. Rückl, Nucl. Phys. B174, 317 (1980); E. L. Berger and D. Jones, Phys. Rev. D 23, 1521 (1981).

[15] A. Martin, Phys. Lett. B 93, 338 (1980); 100, 511 (1981).

[16] G. T. Bodwin, D. K. Sinclair, and S. Kim, Phys. Rev. D 65, 054504 (2002) arXiv:hep-lat/0107011.

[17] Particle Data Group, D. E. Groom et al., Eur. Phys. J. C 15, 1 (2000).

[18] C. R. Nappi, Phys. Rev. D 25, 84 (1982).