Zero-bias anomaly in one-dimensional tunneling contacts

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Abstract

We study the Coulomb interaction effects on the tunneling conductance of a contact constructed of two parallel quantum wires. The following contacts are considered: two clean identical quantum wires, two disordered identical quantum wires, and asymmetric contact of one clean and another disordered quantum wires. We show that the low-voltage anomaly of the tunneling conductance is less singular than the low-energy anomaly of the one-particle density of states.

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1. Introduction

Transport of interacting electrons in quantum wires was considered in Ref. [1] for the Wigner crystal, and in Refs. [2] and [3] for one and multi-channel Luttinger liquid. The approach incorporating the Landauer formalism with the description of the electron-electron interaction as interaction of an electron with fluctuating electromagnetic field created by the other electrons was developed in Refs [4]. The results of these works were applied to quantum wires with constriction or with an impurity. Low and high potential barriers were considered.

In the present paper we consider a different situation, the tunneling transport between two parallel quantum wires. Zero-bias anomaly in the one-particle density of states of disordered electron systems including the quasi-one-dimensional case is well known [5]. Zero-bias anomaly of the tunneling conductance of disordered two-dimensional contacts were studied in Refs. [6] and [7] applying the tunneling action method (quasiclassical approach). Alternative approach developed in Refs. [8] and [9], based on the tunneling hamiltonian and the linear response method, naturally includes the interlayer and intra-layer electron-electron interaction. This method makes a direct connection between collective excitation in a two-layer electron system and the tunneling anomaly and it allows to consider on equal footing disordered systems where collective excitations are diffusion modes and clean systems where collective excitations are gapless two-dimensional plasmons. The asymmetric clean-disordered contact was considered in Ref. [9]. In the present paper we extend the analysis of Refs. [8] and [9] for one dimension.

Plan of the paper is the following. We start with the first order Coulomb electron-electron interaction correction to one-particle density of states and compare it with the result obtained earlier by the bosonization technique [10]. Then we study the tunneling conductance of symmetric clean or disordered quantum wires and compare the anomaly in the tunneling conductance with corresponding anomalies of the one-particle density of states. Finally we consider an asymmetric contact of one clean and another disordered quantum wires.
2. Density of states of one-dimensional interacting electron systems

The one-particle electron density of states is defined as

\[ \nu(\epsilon) = -\frac{2}{\pi} \int \frac{dp}{2\pi} \text{Im}[G^R(P)], \]  

(1)

where \( G^R(P) \) is the retarded electron Green’s functions. Without the electron-electron interaction the electron Green’s function is

\[ G^R_0(P) = [G^A_0(P)]^* = \frac{1}{\epsilon - \xi_p + i/2\tau}, \quad \xi_p = \frac{p^2 - p_F^2}{2m}, \]  

(2)

where \( P = (p, \epsilon) \), and \( \tau \) is the elastic electron-impurity relaxation time, in this chapter we assume \( 1/\tau = 0 \) The correction to electron density of states in one dimension due to the Coulomb interaction is

\[ \delta \nu(\epsilon) = -\frac{2}{\pi} \text{Im} \int \frac{dp}{2\pi} [G^R(P)]^2 \Sigma^R_{\epsilon - \epsilon}(P), \]  

(3)

where the electron self-energy is

\[ \Sigma^R_{\epsilon - \epsilon}(P) = \int \frac{dQ}{(2\pi)^2} \left[ \text{Im}[G^A_0(P + Q)] V^A(Q) \tanh\left(\frac{\epsilon + \omega}{2T}\right) \right. \]
\[ + G^R(P + Q) \text{Im} V^A(Q) \coth\left(\frac{\omega}{2T}\right) \], \]  

(4)

where \( Q = (q, \omega) \).

The screened one-dimensional Coulomb potential is

\[ V^A(Q) = \frac{V_0(q)}{1 - V_0(q) P^A(Q)}. \]  

(5)

The nonscreened Coulomb potential \( V_0(q) \) is

\[ V_0(q) = \frac{2e^2}{\epsilon_0} \ln\left(\frac{1}{qa}\right), \quad qa << 1, \]  

(6)

where \( a \) is a width of a wire and \( \epsilon_0 \) is the static dielectric constant, which we will absorb into \( e^2 \). The polarization operator in the long-wave-length limit, \( \omega >> qv_F \), is
\[ P^A(Q) = \frac{2v_F q^2}{\pi (\omega - i0)^2}. \] (7)

which lead to one-dimensional plasmons \( \omega_q^2 = (2/\pi)V_0(q)v_F q^2 \).

Integrating over the electron momentum in Eq. (3) we have

\[ \frac{\delta \nu(\epsilon, T)}{\nu} = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \tanh\left(\frac{\epsilon + \omega}{2T}\right) \text{Im} \int_{-\infty}^{\infty} dq \frac{V_0(q)}{(\omega - i0)^2 - (2/\pi)V_0(q)v_F q^2}. \] (8)

where \( \nu = 1/(\pi v_F) \). Performing momentum integration within the logarithmic accuracy we get,

\[ \frac{\delta \nu(\epsilon, T)}{\nu} = \frac{1}{4} \left( \frac{e^2}{\pi v_F} \right)^{1/2} \int_{-\infty}^{\infty} d\omega \tanh\left(\frac{\epsilon + \omega}{2T}\right) \times [\ln(|\omega|)]^{1/2}, \quad E = \left( \frac{2e^2 v_F}{\pi a^2} \right)^{1/2} \] (9)

The correction to the density of states for \( T << \epsilon \) is

\[ \frac{\delta \nu(\epsilon, T)}{\nu} = -\frac{1}{6} \left( \frac{e^2}{\pi v_F} \right)^{1/2} [\ln(E/\epsilon)]^{3/2}, \] (10)

Equation (10) corresponds to the first term in the expansion of the equation

\[ \nu(\epsilon) = \frac{1}{\pi v_F} \exp\left[ -\frac{1}{6} \left( \frac{e^2}{\pi v_F} \right)^{1/2} [\ln(E/\epsilon)]^{3/2} \right]. \] (11)

obtained earlier by bosonization technique in Ref. [10].

2. Two clean quantum wires

The tunneling current in the linear response theory is given by

\[ j = -e \text{Im} \Pi^R(eV), \] (12)

where \( e \) is the electron charge, \( V \) is the applied voltage, and \( \Pi^R \) is the retarded polarization operator with lower and upper Green’s functions related to different banks, and the tunneling matrix element \( t \) stands as vertices. The momentum does not conserve at vertices. For noninteracting electrons the tunneling conductance of asymmetric contact is

\[ G_0 = \frac{\partial j}{\partial V} = \frac{1}{2} e^2 \pi |t|^2 \nu_1 \nu_2, \] (13)
where $\nu_1$ and $\nu_2$ are the electron density of states in each bank of the tunneling contact. We assume that both banks have the same chemical potentials to ensure that we have zero-bias anomaly, thus $\nu_1 = \nu_2 = \nu$.

Now we discuss the Coulomb interaction in a system of two parallel identical quantum wires. The nonscreened Coulomb potentials within the wire, $V_0$, and between electrons in different wires, $U_0$, are

$$V_0(q) = 2e^2 \ln(1/qa), \quad qa << 1,$$
$$U_0(q) = 2e^2 \ln(1/qd), \quad qd << 1,$$ (14)

where $d$ is the distance between the wires. The screened interlayer $U$ and intra-layer potentials $V$ are described by the equations

$$U = U_0 + V_0 PU + U_0 PV,$$
$$V = V_0 + V_0 PV + U_0 PU,$$ (15)

where $P$ is the polarization operator defined by Eq. (16). It is convenient to present the potentials $V$ and $U$ in the form

$$V = \frac{1}{2} \frac{V_0 + U_0}{1 - P(V_0 + U_0)} + \frac{1}{2} \frac{V_0 - U_0}{1 - P(V_0 - U_0)},$$ (16)

$$U = \frac{1}{2} \frac{V_0 + U_0}{1 - P(V_0 + U_0)} - \frac{1}{2} \frac{V_0 - U_0}{1 - P(V_0 - U_0)}.$$ (17)

In all further calculations small momentum transfers are important, thus we assume $qa << 1$ and $qd << 1$ thus

$$V_0 - U_0 = (1/2)\pi v_F A, \quad V_0^2 - U_0^2 = 2e^2 A\pi v_F \ln(1/qb), b = (ad)^{1/2}, \quad A = \frac{4e^2}{\pi v_F} \ln(d/a).$$ (18)

The contribution from the intra-layer and inter-layer interactions are

$$\delta_{11}G + \delta_{22}G = e^2|t|^2 \text{Im} \int \frac{d\epsilon}{2\pi} \int \frac{d\omega}{2\pi} \frac{\partial S(\epsilon + eV)}{\partial \epsilon} \times S(\epsilon + \omega) \int \frac{dq}{2\pi} V^R(Q) \int \frac{dp'}{2\pi} (G^A - G^R)(p', \epsilon) \times \int \frac{dp}{2\pi} (G^A(P))^2 G^R(P + Q),$$ (19)
\[
\delta_{12}G = e^2|t|^2 \text{Im} \int \frac{d\epsilon}{2\pi} \int \frac{d\omega}{2\pi} \frac{\partial S(\epsilon + eV)}{\partial \epsilon} S(\epsilon + \omega)
\times \int \frac{dq}{2\pi} U^R(Q) \left[ \int \frac{dp}{2\pi} G^A(P) G^R(P + Q) \right]^2,
\] (20)

where \( S(\epsilon) = -\tanh(\epsilon/2T) \).

The combined contribution after integrating over electron momentums in the region \( qv_F < |\omega| \) is

\[
\frac{\delta G}{G_0} = \frac{1}{G_0} \sum_{ij} \delta_{ij} G = -\frac{1}{\pi^2} \int d\omega f(\omega, eV) J_c(\omega),
\] (21)

where

\[
J_c(\omega) = \text{Im} \int_0^{\omega/|v_F|} dq V^R(q, \omega) - U^R(q, \omega) \frac{V(q, \omega)}{\omega^2},
\] (22)

\[
f(\omega, eV) = \frac{1}{2} \int d\epsilon \frac{\partial S(\epsilon + eV)}{\partial \epsilon} S(\epsilon + \omega).
\] (23)

We see that the result depends on the effective potential \( V - U \). According to Eqs. (16-18),

\[
V^R - U^R = \frac{V_0 - U_0}{1 - P^R(V_0 - U_0)} = \frac{A\pi v_F \omega^2}{2(\omega + i0)^2 - A v_F q^2}.
\] (24)

If \( A >> 1 \) the pole in Eq. (24) is within the limit of integration of Eq. (22) and

\[
J_c(\omega) = -\frac{\pi^2}{4\omega} A^{1/2}.
\] (25)

Further we consider the case of \( T << eV \), thus

\[
\frac{\delta G}{G_0} = \frac{A^{1/2}}{4} \ln(eV/E).
\] (26)

We see that singularity in the tunneling conductance is weaker than in the one-particle density of states because the effective potential \( V - U \) is less singular than \( V \).

2. Two disordered quantum wires

We consider the tunneling contact of two identical disordered quantum wires. The disordered quantum wire is considered as a quasi-one-dimensional system for voltages satisfying
the condition $eV \ll D/a^2$, see Ref. [5], where $D$ is the diffusion coefficient. The polarization operator in a disordered wire is

$$P^R(q, \omega) = -\nu \frac{Dq^2}{-i\omega + Dq^2}, \quad q\ell \ll 1, \quad \omega\tau \ll 1,$$

(27)

where $\ell = v_F\tau$ is the electron mean free path. Generalizing corresponding equation of Ref. [5] for the quasi-one-dimensional case we get

$$\frac{\delta G}{G_0} = -\frac{1}{2\pi (p_Fa)^2} \int d\omega f(\omega, eV) J_d(\omega),$$

(28)

where

$$J_d(\omega) = \text{Im} \int_0^{1/\ell} dq \frac{V^R(Q) - U^R(Q)}{-i\omega + Dq^2}.$$  

(29)

The effective potential $V - U$ is

$$V^R(Q) - U^R(Q) = 2e^2 \ln(d/a) \frac{-i\omega + Dq^2}{-i\omega + D'q^2}, \quad D' = (1 + A/2)D.$$  

(30)

Therefore

$$J_d(\omega) = -\frac{\pi^2 v_F A}{(2\omega)^{3/2}} \frac{1}{(D')^{1/2} - D^{1/2}}.$$  

(31)

Finally we have

$$\frac{\delta G}{G_0} = -\frac{1}{4} [(1 + A/2)^{1/2} + 1] \left( \frac{1}{2eV\tau} \right)^{1/2},$$

(32)

for

$$\frac{1}{\tau(p_Fa)^4} \ll eV \ll \min \left[ \frac{1}{\tau}, \frac{D}{a^2} \right].$$

(33)

The left inequality guarantees applicability of the perturbation theory for Eq. (32).

Comparing Eq. (32) with the correction to the one-particle density of states in disordered quasi-one-dimensional case [5] we obtain

$$\delta \nu(\epsilon) \sim \frac{1}{\epsilon^{1/2}(p_Fa)^2} (\ln \epsilon)^{1/2},$$

(34)
we again see that the correction to the tunneling conductance is less singular than the density of states.

3. One clean and another disordered quantum wires

The screened potentials are satisfied the equations

\[
\begin{pmatrix}
V_{11} & U_{12} \\
U_{21} & V_{22}
\end{pmatrix}
= \begin{pmatrix} V_0 & U_0 \\ U_0 & V_0 \end{pmatrix} + \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} V_{11} & U_{12} \\
U_{21} & V_{22} \end{pmatrix}.
\] (35)

We will use the definitions: \(V_{11} = V_1, V_{22} = V_2\), and \(U_{12} = U_2\). The solution of Eq. (35) is

\[
U = \frac{U_0}{D}, \quad V_{1,2} = \frac{V_0 - (V_0^2 - U_0^2)P_{2,1}}{D},
\]

\[
D = (1 - V_0 P_1)(1 - V_0 P_2) - U_0^2 P_1 P_2.
\] (36)

We assume layer 1 is clean and layer 2 is disordered. The polarization operator in the disordered wire, \(P_2\), is defined by Eq. (27) and \(P_1\) by Eq. (7) for \(qv_F << \omega\) and \(P_1 = -\nu\) for \(\omega << qv_F\). The corrections to the tunneling conductance in an asymmetric two-dimensional contact were derived in Ref. [9]. Generalizing them for the one-dimensional case we have

\[
\frac{\delta_{22}}{G_0} = -\frac{1}{2\pi(p_F a)^2} \int d\omega f(\omega, eV) \times \text{Im} \int_0^{1/\ell'} dq \frac{V_R^2(Q)}{(-\omega + Dq^2)^2},
\] (37)

\[
\frac{\delta_{11}}{G_0} = \frac{1}{2\pi(p_F a)^2} \int d\omega f(\omega, eV) \text{Im} \int_0^{\omega/v_F} dq \frac{V_R^1(Q)}{\omega^2},
\] (38)

\[
\frac{\delta_{12}}{G_0} = \frac{1}{\pi^2(p_F a)^2} \int d\omega f(\omega, eV) \times \text{Im} \int_0^{1/\ell'} dq \frac{\Gamma_R(Q)U_R(Q)}{-i\omega + Dq^2}.
\] (39)

where

\[
\Gamma_R(Q) = \frac{1}{\nu} \int \frac{dp}{2\pi} G^A(P) G^R(P + Q) = \frac{i\pi}{\omega - qv_F},
\] (40)

and \(\ell' = \max[\ell, a, d]\). There are two characteristic regions of momentum in Eqs. (37-39), the plasmon region, \(q < |\omega|/v_F < 1/\ell'\), and the diffusion region, \(|\omega|/v_F < q < 1/\ell'\). In the
plasmon region $Dq^2 << |\omega|$ and $\Gamma^R(q, \omega) \approx i\pi/\omega$, thus the combined contribution from all terms in Eqs. (37-39) is

$$\frac{\delta G}{G_0} = \frac{1}{2\pi(p_F a)^2} \int d\omega f(\omega, eV) J(\omega), \quad (41)$$

where

$$J(\omega) = \text{Im} \int_{\omega/\nu_F}^{\omega} \frac{dq}{\omega^2} (V^R_1(Q) + V^R_2(Q) - 2U^R(Q)). \quad (42)$$

In the plasmon region $(qv_F/\omega)^2 >> Dq^2/|\omega|$, thus $|P_1| >> |P_2|$, and the effective potential $V_1 + V_2 - 2U$ takes the form

$$V_1 + V_2 - 2U = \frac{1}{D} \left[2(V_0 - U_0) - (V_0^2 - U_0^2)(P_1 + P_2)\right]$$

$$= \pi v_F A \left[1 - \frac{2e^2 (qv_F)^2}{\omega^2} \ln(1/qa)\right]$$

$$1 - \frac{2e^2 (qv_F)^2}{\omega^2} \ln(1/qa) + A \frac{Dq^2}{i\omega} \ln(1/qa). \quad (43)$$

It is clear from Eqs. (42) and (43) that the important contribution to the tunneling conductance appears only if

$$\ln(1/qa) << A \frac{Dq^2}{|\omega|} \ln(1/qa). \quad (44)$$

Inequality (44) sets the lower limit $q_0$ in the integral of Eq. (42). Within logarithmic accuracy $q_0 = (|\omega|/2AD)^{1/2}$. Provided inequality (44) is satisfied, the singular contribution to the tunneling conductance originates from the following part of the effective potential

$$V_1 + V_2 - 2U = -\frac{2i\pi e^2 \omega (qv_F)^2 \ln(1/qa)}{i\omega^3 + 2A e^2 / \pi v_F Dq^2 (qv_F)^2 \ln(1/qa)}. \quad (45)$$

If besides inequality $\omega \tau << 1$ we assume also that $A\omega \tau << 1$, then we can neglect $i\omega^3$ term in the denominator of Eq. 45. As a result

$$J(\omega) = -\frac{\pi v_F}{\omega^{3/2}} \left(\frac{A}{D}\right)^{1/2}, \quad (46)$$

and the correction to the conductance is

$$\frac{\delta G}{G_0} = \frac{1}{2(p_F a)^2} \left(\frac{3A}{eV\tau}\right)^{1/2} \quad (47),$$
for

\[ \frac{1}{\tau(p_Fa)^4} < eV < \min \left[ \frac{1}{A\tau}, \frac{D}{a^2} \right]. \]  

(48)

Now we consider the diffusion region where only to terms \( \delta_{22}G \) and \( \delta_{12}G \) contribute. In the diffusion region \( \omega \sim Dq^2 \), and \( \omega << qv_F \), thus following Eq. (19) we can put \( P_1 = -\nu \), and present the function \( D(Q) \) in the following form

\[ (-i\omega + Dq^2)D(Q) = (1 + V_0\nu)(-i\omega + D''q^2), \quad \frac{D''}{D} = 1 + \frac{V_0\nu + (V_0^2 - U_0^2)\nu^2}{1 + V_0\nu} \]  

(49)

We start with the correction \( \delta_{22}G \). The momentum integral in Eq. (37)

\[ J_1(\omega) = \text{Im} \int_{|\omega|/v_F}^{1/\ell} dq \frac{V_2^R(Q)}{(-i\omega + Dq^2)^2} \]  

(50)

will be calculated within logarithmic accuracy while the lower limit will be shifted to zero and the upper one to infinity. As a result

\[ J_1(\omega) = \frac{\pi}{\omega^{3/2}(2D)^{1/2}} [1 + C(\omega)], \]  

(51)

where

\[ C^2(\omega) = \frac{2e^2\nu(ln(1/q_1a) + A\ln(1/q_1b))}{1 + 2e^2\nu \ln(1/q_1a)}, \]  

(52)

where \( q_1 = (\omega/D)^{1/2} \). Finally for the voltage satisfying condition of Eq. (33) we have

\[ \frac{\delta_{22}G}{G_0} = -\frac{1}{2(p_Fa)^2}[1 + C(eV)] \left( \frac{1}{2eV\tau} \right)^{1/2}. \]  

(53)

Calculating \( \delta_{12}G \) we take into account that in the diffusion region \( \Gamma^R(q, \omega) = -i\pi/qv_F \), thus

\[ \frac{\delta_{12}G}{G_0} = -\frac{1}{\pi^2v_F} \int d\omega f(\omega, eV)J_2(\omega), \]  

(54)

where

\[ J_2(\omega) = \text{Re} \int_0^{1/\ell} dq \frac{U^R(Q)}{-i\omega + Dq^2}. \]  

(55)
Using Eq. (36) for the potential $U$ we get

$$J_2(\omega) = \text{Re} \int_0^{1/\ell} \frac{dq}{-i\omega + D''q^2} \frac{2\pi e^2 \ln(1/qd)}{1 + 2\pi e^2 \nu \ln(1/q_1d)}$$

$$= \frac{\pi}{4\omega} \frac{1}{D''(q_1)} \frac{2e^2 \ln(1/q_1d)}{1 + 2e^2 \nu \ln(1/q_1d)}$$

(56)

We see that $J_2(\omega)$ is less singular than $J_1(\omega)$ and therefore the correction $\delta_{12}G$ in the diffusion region may be neglected.

4. Conclusions

We considered the following types of tunneling contacts: two clean identical quantum wires, two disordered identical quantum wires, and an asymmetric contact of one clean and another disordered quantum wires. We have shown that in symmetric contacts the zero-bias anomaly of the tunneling conductance due to the Coulomb interaction is weaker than corresponding singularity of the one-particle density of states. This effect is due to partial cancellation of contributions from intra-wire and inter-wire interactions. In an asymmetric clean-disordered contact only the anomaly associated with the disordered wire survived, though slightly modified. We see the zero-bias anomaly as a signature of collective excitations, such as plasmons in clean double-wire systems or diffusion modes in disordered double-wire system. Absence of a plasmon mode in the coupled clean-disordered system lead to absence of corresponding anomaly in the tunneling conductance.

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