EFFECTIVE CHIRAL THEORY OF LARGE $N_C$

$QCD$ OF MESONS

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Abstract

An effective chiral theory of pseudoscalars, vectors, and axial-vectors is presented. In this theory mesons are coupled to quarks. At quark level mesons have no kinetic terms which are generated by quark loops. The theory has both explicit chiral symmetry (in the limit, $m_q \rightarrow 0$) and dynamical chiral symmetry breaking. Large $N_C$ expansion is a natural result. Tree diagrams are at leading order and meson loops are at higher orders. In this theory octet pseudoscalars are Goldstone bosons. Masses and

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decay widths of the mesons agree with data. VMD, Wess-Zumino-Witten anomaly, and OZI rule are obtained. Besides three current quark masses there are other two parameters: a universal coupling constant and a cutoff. At low energy the theory goes back to ChPT. The phenomenology of the theory is successful.
1 Introduction

QCD is the theory of strong interactions. The nonperturbative QCD remains unsolved. Before QCD created there were already a lot of successful studies on meson physics: Vector Meson Dominance (VMD), Current algebra, PCAC, Sum rules, Goldstone theorem and Gell-Mann, Oaks, and Renner formula of pion mass, ABJ and WZW anomaly etc..

In order to solve nonperturbative QCD many models and effective theories have been proposed, for instance, Quark models from which we learned that there is a constituent quark mass, Nonlinear $\sigma$ model, NJL model with four fermion interactions, QCD sum rule, Instanton induced model, Chiral perturbation theory (ChPT), many more.

Mesons are bound states of quarks and gluons. Bound state is a nonperturbative problem. It is heuristic to revisit $QED$. Hydrogen atom is a bound state of proton and electron. We divide $QED$ into two parts: nonperturbative and perturbative $QED$.

\[ \mathcal{L}_{QED} = \mathcal{L}_{\text{nonperturbative}QED} + \mathcal{L}_{\text{perturbative}QED}. \]  

(1)

The interaction in $\mathcal{L}_{\text{nonperturbative}QED}$ is mainly coulomb interaction, while in perturbative part is mainly transverse photon. Boglubov has proposed a rigorous method to do the separation of $QED$. We use $\mathcal{L}_{\text{nonperturbative}QED}$ to get bound state solution, while use the $\mathcal{L}_{\text{perturbative}QED}$ to do radiative correction.

Of course, $QCD$ is different from $QED$. Gluon fields are nonabelian fields. However,
$QCD$ is still divided into nonperturbative and perturbative parts.

\[ \mathcal{L}_{QCD} = \mathcal{L}_{\text{nonperturbativeQCD}} + \mathcal{L}_{\text{perturbativeQCD}}. \]

In $\mathcal{L}_{\text{nonperturbativeQCD}}$ soft gluons and instantons play dominant roles, while it is well known that $\mathcal{L}_{\text{perturbativeQCD}}$ is dominant by hard gluons. Unlike $QED$ we do not know the explicit expression of $\mathcal{L}_{\text{nonperturbativeQCD}}$. There are different approaches to study $\mathcal{L}_{\text{nonperturbativeQCD}}$. Effective theory is one of them.

In $QCD$ mesons are bound states of quarks and gluons. Meson physics at low energy is nonperturbative. We have proposed a chiral theory of mesons [1], which is an effective theory of nonperturbative part of $QCD$, to treat meson problems. The perturbative part of $QCD$ is used to calculate the radiative correction by hard gluons.

In this talk we focus on the effective theory of mesons.

## 2 Effective large $N_C$ QCD of mesons

The strategy of constructing the effective theory of nonperturbative $QCD$ of mesons is following.

1. Mesons are coupled to quarks.

   In $QCD$ meson is a pole of four point Green function of quarks. A Green function of many quark pairs can be separated into product of corresponding four point Green
functions of quark pairs and kernel. Mesons are poles of the four point Green functions. At the poles mesons are coupled to quarks (kernel). Based on this picture we try to construct an effective Lagrangian to calculate this part of the Green function.

2. From $SU(3)_L \times SU(3)_R$ current algebra we have learned that vector and axial-vector mesons are coupled to vector and axial-vector currents of quarks respectively.

3. We use the scheme of nonlinear $\sigma$ model to introduce pseudoscalars to the theory.

4. The theory has explicit chiral symmetry in the limit of $m_q \to 0$, where $m_q$ is the current quark mass. Therefore, in the chiral limit pseudoscalars are massless and vector and axial-vector mesons have the same masses.

5. QCD has dynamical chiral symmetry breaking. The dynamical chiral symmetry breaking should be introduced to the theory.

Based on the inputs mentioned above, for two flavors the effective Lagrangian of pseudoscalar, vector, axial-vector mesons has been constructed as\[1\]

$$
\mathcal{L} = \bar{\psi}(x)(i \gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a \gamma_5 - mu(x))\psi(x) - \bar{\psi}(x)M\psi(x)
+ \frac{1}{2}m_0^2(\rho_{i\mu} \rho_{i\mu} + \omega_{i\mu} \omega_{i\mu} + a_{i\mu} \gamma_5 a_{i\mu} + f_{i\mu} f_{i\mu})
$$

(3)

where $a_{i\mu} = \tau_i a_{\mu} + f_{\mu}$, $v_{i\mu} = \tau_i \rho_{\mu} + \omega_{\mu}$, and $u = exp\{i \gamma_5 (\tau_i \pi_i + \eta)\}$, and $M$ is the current quark mass matrix. Since mesons are bound states solutions of QCD they are not independent
degrees of freedom. Therefore, there are no kinetic terms for meson fields. The kinetic terms of meson fields are generated from quark loops. A similar Lagrangian can be constructed for three flavors. According to QED and the Standard Model photon, W and Z fields can be incorporated into this Lagrangian.

This Lagrangian has following features

1. Obviously, in the limit $m_q \to 0$ this Lagrangian has chiral symmetry.

2. In Eq.(3) there is a parameter $m$, the constituent quark mass, which is related to quark condensate

$$
<\bar{\psi}\psi> = \frac{i}{(2\pi)^4} Tr \int d^4p \frac{\gamma \cdot p - m\hat{u}}{p^2 - m^2} = -\frac{3m^3N_C}{4\pi^2} \left\{ \frac{\Lambda^2}{m^2} - \log\left(\frac{\Lambda^2}{m^2} + 1\right) \right\}, \quad (4)
$$

where $\Lambda$ is the cutoff of this effective theory. A cutoff is necessary for an effective theory. It should be the match point between nonperturbative and perturbative QCD. The theory has dynamical chiral symmetry breaking.

3. The effective Lagrangian of mesons is obtained by integrate out quark fields in Eq.(3) or by calculating quark loop diagrams. Using the method of path integral, the effective Lagrangian of mesons is obtained.

$$
\mathcal{L}_E^M = \mathcal{L}_{RE} + \mathcal{L}_{IM},
$$
\[ \mathcal{L}_{RE} = \frac{1}{2} \log \det (D^\dagger D), \quad \mathcal{L}_{IM} = \frac{1}{2} \log \det (D/D^\dagger) \] (5)

where

\[ D = \gamma \cdot \partial - i\gamma \cdot \nu - i\gamma \cdot a\gamma_5 + mu. \] (6)

\[ D^\dagger = -\gamma \cdot \partial + i\gamma \cdot \nu - i\gamma \cdot a\gamma_5 + m\hat{u}, \quad \hat{u} = \exp(-i)\gamma_5(\tau_i \pi_i + \eta). \] (7)

In \( \mathcal{L}_{RE} \) there are even numbers of \( \gamma_5 \), while in \( \mathcal{L}_{IM} \) the number of \( \gamma_5 \) is odd. Therefore, \( \mathcal{L}_{IM} \) is the anomalous action.

The kinetic terms of the vector, axial-vector, and pseudoscalar fields are generated by quark loops and the physical meson fields are defined

\[ \pi \rightarrow \frac{2}{f_\pi} \pi, \quad \eta \rightarrow \frac{2}{f_\eta} \eta, \quad \rho \rightarrow \frac{1}{g} \rho, \quad \omega \rightarrow \frac{1}{g} \omega, \] (8)

\( f_\pi \) is defined by normalizing the kinetic terms of pseudoscalar fields and the universal coupling constant, \( g \), is to normalizing the kinetic terms of vector fields.

\[ f_\pi^2 = F^2 (1 - \frac{2c}{g}), \quad c = \frac{f_\pi^2}{2gm_\rho^2}, \quad g^2 = \frac{F^2}{6m^2}, \] (9)

where

\[ \frac{F^2}{16} = \frac{N_C}{(4\pi)^2m^2} \int d^4p \frac{1}{(p^2 + m^2)^2}. \]

There are mixing between axial-vectors and corresponding pseudoscalars. The substitution

\[ a^i_\mu \rightarrow \frac{1}{g} \left( 1 - \frac{1}{2\pi^2g^2} \right) \frac{i}{2} a^i_\mu - \frac{c}{g} \partial_\mu \pi^i. \] (10)
is used to erase the mixing $a^i_\mu \partial_\mu \pi^i$.

To the fourth order in covariant derivatives in Minkowsky space the lagrangian takes the following form

$$L_{RE} = \frac{N_c}{(4\pi)^2} m^2 D^2(2 - D^2/2) Tr D_\mu U D^\mu U^\dagger$$

$$- \frac{1}{3} \frac{N_c}{(4\pi)^2} \frac{D^2(2 - D^2/2)}{4} \{2\omega_\mu \omega^\mu + Tr \rho_\mu \rho^\mu + 2f_\mu f^\mu + Tr a_\mu a^\mu \}$$

$$+ i \frac{N_c}{2(4\pi)^2} Tr \{ D^\dagger_\mu U D_\nu U^\dagger + D_\mu U^\dagger D_\nu U \} \rho^{\nu\mu}$$

$$+ i \frac{N_c}{2(4\pi)^2} Tr \{ D^\dagger_\mu U - D_\mu U^\dagger \} a^{\nu\mu}$$

$$+ \frac{N_c}{6(4\pi)^2} Tr D_\mu D_\nu U D^\mu D^\nu U^\dagger$$

$$- \frac{N_c}{12(4\pi)^2} Tr \{ D^\dagger_\mu U D^\mu U D^\nu U^\dagger + D^\dagger_\mu U D^\mu U D^\nu U^\dagger D^\nu U - D_\mu U D_\nu U^\dagger D^\mu U^\dagger D^\nu U \}$$

$$+ \frac{1}{2} m^2_0 (\omega_\mu \omega^\mu + \rho^{\dagger}_\mu \rho_\mu + a^{\dagger}_\mu a_\mu + f_\mu f^\mu ), \quad (11)$$

where

$$D_\mu U = \partial_\mu U - i [\rho_\mu, U] + i \{a_\mu, U\},$$

$$D_\mu U^\dagger = \partial_\mu U^\dagger - i [\rho_\mu, U^\dagger] - i \{a_\mu, U^\dagger\},$$

$$\omega_\mu = \partial_\mu \omega^\nu - \partial_\nu \omega^\mu,$$

$$f_\mu = \partial_\mu f^\nu - \partial_\nu f_\mu,$$

$$\rho_\mu = \partial_\mu \rho^\nu - \partial_\nu \rho_\mu - i [\rho_\mu, \rho^\nu] - i [a_\mu, a^\nu],$$

$$a_\mu = \partial_\mu a^\nu - \partial_\nu a_\mu - i [a_\mu, a^\nu] - i [\rho_\mu, a^\nu],$$

$$8$$
\begin{align*}
D_\nu D_\mu U &= \partial_\nu (D_\mu U) - i[\rho_\nu, D_\mu U] + i\{a_\nu, D_\mu U\}, \\
D_\nu D_\mu U^\dagger &= \partial_\nu (D_\mu U^\dagger) - i[\rho_\nu, D_\mu U^\dagger] - i\{a_\nu, D_\mu U^\dagger\}.
\end{align*}

All the vertices of mesons are derived from Eq.(11).

4. Besides three current quark masses, there are other two parameters in this effective theory: cutoff $\Lambda$ and $g$. Input $f_\pi$, the decay rate of $\rho \to ee^+$, $\Lambda$ and $g$ are determined, $\Lambda \sim 1.8 GeV$ and $g = 0.395$. three current quark masses are determined by input $m_\pi^2$, $m_{K^\pm}^2$.

5. $N_C$ comes from the quark loop. We have

$$f_\pi^2 \sim O(N_C), \quad g \sim O(\sqrt{N_C}).$$

All the meson fields are order of $O(\sqrt{N_C})$ and all the meson vertices are $O(N_C)$. Therefore, the diagrams at tree level are $O(N_C)$ and loop diagrams are at higher order in $N_C$ expansion. 't Hooft and Witten have proposed using $N_C$ expansion to study $QCD$. In this theory $N_C$ expansion is a natural result. $N_C$ expansion is used to do physical calculations. The cutoff is about 1.8GeV. Therefore, only low lying mesons contribute to the loop diagrams. Namely, the loop diagrams are calculable. This is the reason why this effective theory is named as effective chiral large $N_C$ $QCD$ of mesons.

Now we can use the vertices derived from Eq.(11) to study meson physics.
3 Symmetry breaking and Masses of mesons

In field theory meson masses are always associated with symmetry breaking. $\pi$, $\rho$, and $a_1$ are all made of u and d quarks

Why pion is very light?

Why $\rho$ is much heavier than pion?

Why $a_1$ is heavier than $\rho$?

How to understand Weinberg’s second sum role $m_a^2 = 2m_\rho^2$?

1. To the leading order in quark mass expansion, the masses of the octet pseudoscalar mesons are derived

\[
m_\pi^2 = -\frac{2}{f_\pi^2}(m_u + m_d) < 0|\bar{\psi}\psi|0 >, \\
m_{K^+}^2 = -\frac{2}{f_\pi^2}(m_u + m_s) < 0|\bar{\psi}\psi|0 >, \\
m_{K^0}^2 = -\frac{2}{f_\pi^2}(m_d + m_s) < 0|\bar{\psi}\psi|0 >, \\
m_\eta^2 = -\frac{2}{3f_\pi^2}(m_u + m_d + 4m_s) < 0|\bar{\psi}\psi|0 >. \tag{12}
\]

pion mass originates in explicit chiral symmetry breaking by current quark masses

On the other hand, it is interesting to point out that there are two diagrams from the
two vertices
\[-i m \bar{\psi} \tau^i \gamma_5 \psi \pi^i - \frac{1}{2} m \bar{\psi} \psi \pi^2\] (13)
contribute to pion mass. The later is a tadpole. There is destructive interference between these two diagrams. The cancellation leads to the mass formula above. In the limit \(m_q \to 0, m_\pi = 0\). Goldstone theorem is satisfied.

2. \(m_\rho\)

In this theory we have
\[m_\rho^2 = m_\omega^2 = \frac{1}{g^2} m_0^2.\] (14)
From Eq.(3) we can see that current algebra is satisfied by this theory. The KSFR sum rule
\[g_\rho = \frac{1}{2} f_{\rho\pi\pi} f_\pi^2\] (15)
can be taken as the equation to determine \(m_\rho\). we have
\[f_{\rho\pi\pi} = \frac{2}{g}, \quad g_\rho = \frac{1}{2} g m_\rho^2.\]
Substituting them into the KSFR sum rule, we obtain
\[m_\rho^2 = 2 \frac{f_\pi^2}{g} = 6 m^2.\] (16)
Therefore, \(m_\rho\) is resulted in dynamical chiral symmetry breaking for the masses of \(K^*\) and \(\phi\) strange quark mass corrections should be taken into account.
In the original Lagrangian because of chiral symmetry both $\rho$ and $a_1$ have the same mass. $\rho$ is coupled to vector quark current and $a_1$ couples to axial-vector current of quarks

$$\bar{\psi} T^i \gamma_\mu \gamma_5 \psi a^i_\mu.$$ 

When we calculate the vacuum polarization diagram, unlike the vector coupling, an additional mass term is generated and chiral symmetry is broken by the axial-vector coupling. We name this symmetry breaking as \textbf{axial-vector symmetry breaking}. We obtain

$$\left(1 - \frac{1}{2\pi^2 g^2}\right)m_{a_1}^2 = \frac{F^2}{g^2} + m_\rho^2 = 2m_\rho^2. \quad (17)$$

The left hand is the result of Weinberg's 2\textsuperscript{nd} sum rule. Here we have a new factor. In deriving the 2\textsuperscript{nd} sum rule Weinberg made an assumption about the high energy behavior of the propagator. The factor on the left hand side of Eq.(17) is originated in the high energy behavior of the propagator of $a_1$ field. This formula fits the data better.

$$m_{a_1} = 1.2\text{GeV} \quad m_{a_1} = 1.09\text{GeV}(\text{Weinberg}) \quad m_{a_1} = 1.23 \pm 0.04\text{GeV}(\text{data})$$

There are similar formulas for $f_1(1280), K_1$ and $f_1(1510)$

$$\left(1 - \frac{1}{2\pi^2 g^2}\right)m_{f_1(1280)}^2 = \frac{F^2}{g^2} + m_\omega^2. \quad (18)$$
\( (1 - \frac{1}{2\pi^2g^2})m_{K_1(1400)}^2 = \frac{F^2}{g^2} + m_{K^*}^2 \) \quad (19)

\( (1 - \frac{1}{2\pi^2g^2})m_{f_1(1510)}^2 = \frac{F^2}{g^2} + m_{\phi}^2 \) \quad (20)

These results agree with data well.

Therefore, there are three symmetry breaking in this theory: explicit chiral symmetry breaking, dynamical chiral symmetry breaking, and axial-vector symmetry breaking.

In the SM intermediate bosons couple to both vector and axial-vector currents of fermions.

There are axial-vector symmetry breaking which lead to

1.

\( m_{\rho}^2 = \frac{1}{2}g^2m_t^2, \quad m_{a}^2 = \frac{1}{2}(g^2 + g^{'2})m_t^2, [9] \)

2. Two charges and one neutral spin-0 particles exist, whose masses are \( \sim 10^{14}\)Gev and they are unphysical. Unitarity of the SM is broken at \( 10^{14}\) GeV[10].

By the way, Weinberg’s first sum rule

\( \frac{g_{\rho}^2}{m_{\rho}^2} - \frac{g_{a}^2}{m_{a}^2} = \frac{1}{4}f_{\pi}^2 \), \quad (21)

is satisfied analytically.

4 Normal strong Decays

The vertices are found from the Lagrangian of mesons. The amplitudes of decays are calculated in the chiral limit, at the tree level, and up to the fourth order in derivatives.
4.1 \( V \rightarrow PP \) decays

Taking \( \rho \rightarrow \pi \pi \) as an example.

\[
\mathcal{L}_{\rho\pi\pi} = f_{\rho\pi\pi} \epsilon_{ijk} \rho_i^\mu \pi_j^\nu \partial_{\mu} \pi_k,
\]

\[
f_{\rho\pi\pi} = \frac{2}{g} \left\{ 1 + \frac{m_\rho^2}{2\pi^2 f_\pi^2} \left[ \left( 1 - \frac{2c}{g} \right)^2 - 4\pi^2 c^2 \right] \right\}.
\]

(22)

There is a form factor \( f_{\rho\pi\pi} \).

\[
\Gamma_\rho = 143.2 \text{MeV}, \ \text{exp.} = 150 \text{MeV};
\]

We obtain

\[
\Gamma(K^* \rightarrow K\pi) = 44.9 \text{MeV}, \ \text{exp.} = (49.8 \pm .8) \text{MeV};
\]

\[
\Gamma(\phi \rightarrow KK) = 3.54 \text{MeV}; \ \text{exp.} = 3.69(1 \pm 0.028) \text{MeV}.
\]

The form factor \( f_{\rho\pi\pi}^2(q^2) \) contributes 34\%, 46\%, and 61\% to the three decays respectively.

4.2 \( A \rightarrow VP \) decays

\[
\mathcal{L}_{a_1 \rightarrow \rho \pi} = \epsilon_{ijk} \{ Aa_i^\mu \rho_j^\mu \pi_k + Ba_i^\mu \rho_j^\nu \partial_{\mu} \pi_k \},
\]

\[
A = \frac{2}{f_\pi} (1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} \left\{ \frac{F^2}{g^2} + \frac{m_a^2}{2\pi^2 g^2} - \frac{2c}{g} + \frac{3}{4\pi^2 g^2} (1 - \frac{2c}{g})(m_a^2 - m_\rho^2) \right\}
\]

\[
B = -\frac{2}{f_\pi} (1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} \frac{1}{2\pi^2 g^2} (1 - \frac{2c}{g}).
\]

(23)
The width of the decay is calculated to be 326MeV which is comparable with data (about
400MeV). There are s-wave and d-wave in this decay

\[ \frac{d}{s} = -0.1. \]  \hspace{1cm} (24)

The experimental value is \(-0.11 \pm 0.02\). We also obtain

\[ \Gamma(K_1(1400) \rightarrow K^*\pi) = 126MeV, \hspace{0.5cm} exp. = 163.6(1 \pm 0.14)MeV, \]

\[ \Gamma(f_1(1510) \rightarrow K^*K) = 22MeV, \hspace{0.5cm} exp. = 35 \pm 15MeV, \]

Amplitude A plays dominant role, in which there is cancellation between the two terms of
A. In 60’s current algebra has difficulty to get the decay width of \(a_1\) meson. This effective
theory provides an explanation. In this theory in unphysical region the current algebra result
is satisfied. However, because of the cancellation in the amplitude A the physical region is
far away from unphysical.

The decay \(\eta' \rightarrow \eta\pi\pi\) is calculated too.

\[ \Gamma(\eta' \rightarrow \eta\pi^+\pi^-) = 85.7keV, exp. = 87.8 \pm 0.12keV, \]

\[ \Gamma(\eta' \rightarrow \eta\pi^0\pi^0) = 48.6keV, exp. = 41.8 \pm 0.11keV. \]
5 Vector meson dominance

The interactions between photon and mesons are derived as

$$\frac{e}{f_{\rho}}\{\frac{1}{2} F_{\mu\nu}(\partial_\mu \rho_\nu^0 - \partial_\nu \rho_\mu^0) + A_{\mu}^{\rho} j_{\mu}^{0}\}.$$  \hspace{1cm} (25)

$$\frac{e}{f_{\omega}}\{\frac{1}{2} F_{\mu\nu}(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) + A_{\mu}^{\omega} j_{\mu}^{\omega}\},$$

$$\frac{e}{f_{\phi}}\{\frac{1}{2} F_{\mu\nu}(\partial_\mu \phi_\nu - \partial_\nu \phi_\mu) + A_{\mu}^{\phi} j_{\mu}^{\phi}\},$$ \hspace{1cm} (26)

where

$$\frac{1}{f_{\rho}} = \frac{1}{2} g, \quad \frac{1}{f_{\omega}} = \frac{1}{6} g, \quad \frac{1}{f_{\phi}} = -\frac{1}{3\sqrt{2}} g$$ \hspace{1cm} (27)

These are the expressions of VMD proposed by Sakurai. $\rho \to ee^+$ is used to determine $g = 0.395$.

6 Form factors

For a long time pion form factor is expressed as a $\rho$ pole. However, The radius from $\rho$ pole is less than the data by about 10%. Comparing with data, the $\rho$ pole form factor decreases slower in time like region and faster in space like region.

We have studied the form factors of pion and kaon[3]. The pion form factor is expressed as

$$|F_{\pi}(q^2)|^2 = f_{\rho\pi\pi}(q^2)\frac{m_{\rho}^4 + q^2\Gamma_{\rho}^2(q^2)}{(q^2 - m_{\rho}^2)^2 + q^2\Gamma_{\rho}^2(q^2)}$$ \hspace{1cm} (28)
The radius of charged pion is found to be

\[
< r^2 >_\pi = (0.395 + 0.057) \text{fm}^2 = 0.452 \text{fm}^2.
\] (29)

The contribution of the intrinsic form factor, \( f_{\rho\pi\pi} \), is about 13% of the total value. The experimental data is \((0.439 \pm 0.03) \text{fm}^2\).

The decay rate of \( \tau \to \pi\pi\nu \) is dominated by pion form factor, which agrees well with CLEO. We obtain \( B_{\tau^+ \to \pi^+\pi^-\nu} = 22.3\% \). The experimental data is \((25.32 \pm 0.15)\% \).

In the same way, kaon form factors are obtained. The radii of kaons are calculated

\[
< r^2 >_{K^+} = 0.38 \text{fm}^2. \quad < r^2 >_{\text{exp} K^+} = 0.34 \pm 0.05 \text{fm}^2.
\]

The decay rate of \( \tau^- \to K^0 K^- \nu \) is dominated by pion form factor too. We obtain \( B = 1.78 \times 10^{-3} \). The data is \((1.59 \pm 0.24) \times 10^{-3} \).

The form factors of pion and charged kaon agree with data in both timelike and spacelike regions below 1.5GeV.

The form factors of \( K_{l3} \) are dominated by a \( K^* \) pole and the intrinsic form factor \( f_{\rho\pi\pi} \). Theory agrees with data.

The three form factors of \( \pi \to e\gamma\nu \) and \( K \to e\gamma\nu \) are computed. Theory agrees well with data. PCAC is satisfied analytically.
7 $\pi\pi$ and $\pi K$

The amplitudes of $\pi - \pi$ scattering are calculated[1]. For the channel of p-wave and $I = 1$ there is $\rho$ resonance. Theory agrees with data well. For the channel of s-wave and $I = 0$ a $0^{++}$ state is needed. This state could be introduced to the theory.

For $\pi - K$ scattering the $K^*$ resonance plays a role. The comparison between theory and data is shown in Ref.[4].

8 Axial-vector current

The axial-vector part of the weak interactions between $W$ boson($A^i_\mu$) and mesons is derived as

$$L^A = -\frac{g_W}{4\cos\theta_C} \frac{1}{f_a} \left\{ -\frac{1}{2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial_\mu a^i_\nu - \partial_\nu a^i_\mu) + A^{\mu\nu} j^{W}_{\mu} \right\}$$

$$-\frac{g_W}{4\cos\theta_C} \Delta m^2 f_a A^i_\mu a^{i\mu} - \frac{g_W}{4\cos\theta_C} f_\pi A^{\mu}_{\mu} \partial^\mu \pi^i,$$

where

$$f_a = g^{-1}(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}, \quad \Delta m^2 = 6m^2 g^2.$$

Because of the axial-vector symmetry breaking there are two more terms than VMD in the axial-vector current. Three terms together makes PCAC satisfied.
9 weak decays

In weak interaction of mesons there are both vector and axial-vector currents. In this theory both the vector and the axial-vector currents are bosonized. CVC and PCAC are satisfied.

In $\tau$ mesonic decays besides vector and axial-vector currents many meson vertices are involved. All these three parts are fixed in this theory. Systematic studies of $\tau$ mesonic decays have been done without any adjustable parameter\cite{5}. Theory agrees well with data.

Kaon weak decays, $K_{\ell2}$\cite{6}, $K_{\ell3}$\cite{1}, $K_{\ell4}$\cite{7}, and $K \rightarrow e\nu\gamma$\cite{5} have been studied and theory agrees with data.

$\Delta I = \frac{1}{2}$ rule in $K \rightarrow 2\pi$ stands there for almost half century. Theoretical understanding is still lacking. $K \rightarrow 2\pi$ are very complicated processes. At order of $O(N_C)$ there are tree diagram, one loop, and two loop diagrams. All these diagrams are calculable. The good news is that there are diagrams which contribute to $I = 1$ amplitude only.

10 Anomaly

The Effective Lagrangian $\mathcal{L}_{IM}$ is the anomalous action of the mesons, which contains odd number of $\gamma_5$.

Vertex $\pi\omega\rho$ is anomalous. It is obtained from

$$\frac{1}{g} \omega^\mu < \bar{\psi} \gamma_\mu \psi > = \frac{-i}{(2\pi)^D} \int d^Dp Tr \gamma_\mu s_F(x,p) \omega^\mu. \quad (30)$$
The anomalous part of this vertex is obtained

\[
\frac{1}{g} \omega^\mu < \bar{\psi} \gamma_\mu \psi > = \omega^\mu \partial^2 \omega_\mu + \frac{N_c}{(4\pi)^2} 2 \frac{1}{g} \epsilon^{\mu \nu \alpha \beta} \omega_\mu \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \\
+ \frac{N_c}{(4\pi)^2} 2 \frac{1}{g} \epsilon^{\mu \nu \alpha \beta} \partial_\alpha \omega_\mu \partial_\beta U U^\dagger \{ \frac{i}{g} [\partial_\beta U U^\dagger (\rho_\alpha + a_\alpha) - \partial_\beta U^\dagger U (\rho_\alpha - a_\alpha)] \\
- \frac{2}{g^2} (\rho_\alpha + a_\alpha) U (\rho_\beta - a_\beta) U^\dagger - \frac{2}{g^2} \rho_\alpha a_\beta \}. \tag{31}
\]

This formula is exactly the same as the one obtained by the Syracuse group and by Witten.

\[ L_{\omega \rho \pi} \] is derived

\[
L_{\omega \rho \pi} = -\frac{N_c}{\pi^2 g^2 f_\pi} \epsilon^{\mu \nu \alpha \beta} \partial_\mu \omega_\nu \rho^\alpha_\beta \partial_\alpha \pi^\beta_i; \tag{32}
\]

\[
\Gamma(\omega \to 3\pi) = 7.7MeV, \hspace{1em} exp. = 7.49(1 \pm 0.02)MeV.
\]

\[
\Gamma(\omega \to \gamma \pi) = 724keV.
\]

The experimental value is 717(1 \pm 0.07)keV.

\[
\Gamma(\rho \to \gamma \pi) = 76.2keV.
\]

The experimental data is 68.2(1 \pm 0.12)keV.

Using VMD,

\[
L_{\pi^0 \to \gamma \gamma} = -\frac{\alpha}{\pi f_\pi} \epsilon^{\mu \nu \alpha \beta} \pi^0_\mu \partial_\nu A_\alpha \partial_\alpha A_\beta; \tag{33}
\]

This is ABJ anomaly.
The form factor of $\pi^0 \to \gamma\gamma^*$ is obtained[8]

$$F_\pi(q^2) = f_{\pi\rho\omega}(q^2) \frac{1}{2} \left\{ \frac{-m^2_\rho + i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2 - m^2_\rho + i\sqrt{q^2}\Gamma_\rho(q^2)} + \frac{-m^2_\omega + i\sqrt{q^2}\Gamma_\omega(q^2)}{q^2 - m^2_\omega + i\sqrt{q^2}\Gamma_\omega(q^2)} \right\},$$

$$f_{\pi\rho\omega}(q^2) = 1 + \frac{g^2}{2f_\pi^2} \left( 1 - \frac{2c}{g} \right)^2 q^2. \quad (34)$$

$f_{\pi\rho\omega}$ is the intrinsic form factor of $\pi^0\gamma\gamma^*$. For very low momentum we obtain

$$F_\pi(q^2) = 1 + a \frac{q^2}{m^2_\pi}, \quad (35)$$

$$a = \frac{m^2_\pi}{2} \frac{1}{m^2_\rho} + \frac{1}{m^2_\omega} + \frac{m^2_\pi}{2f_\pi^2} g^2 \left( 1 - \frac{2c}{g} \right)^2. \quad (36)$$

The first term comes from the $\rho$ and $\omega$ poles and the second term comes from the intrinsic form factor of $\pi^0\gamma\gamma^*$, which is from anomaly too.

$$a = 0.0303 + 0.0157 = 0.046.$$  

There are other anomalous processes for $\eta$ and $\eta'$. Theory agrees with the data.

$K^* \to K\pi\pi$ decay originates in anomaly and is calculated

$$\Gamma = 2.65 keV$$

which is below the experimental limit.

11 ChPT

The ChPT is constructed by chiral symmetry. This theory is working when $E < 500 MeV$.

To the fourth order in derivatives there are 10 parameters which are determined by fitting
data. Any effective theory for higher energy should take the ChPT as a low energy limit, namely the 10 parameters should be predicted. Indeed, the effective large $N_C$ QCD of mesons goes back to ChPT($E_im, ho$) and the 10 parameters are predicted[6].

We predicted all the 10 coefficients by using $\pi\pi$ and $\pi K$ scatterings, masses of pseudoscalar mesons, radius of pion, form factors of $\pi \rightarrow e\gamma\nu$. Up to the second order in current quark masses, the expressions of pseudoscalar mesons are the same as obtained by ChPT.

| $10^3L_1$ | $10^3L_2$ | $10^3L_3$ | $10^3L_4$ | $10^3L_5$ | $10^3L_6$ | $10^3L_7$ | $10^3L_8$ | $10^3L_9$ | $10^3L_{10}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1.0      | 2.0      | -5.16    | 0        | 4.77     | 0        | 0        | -0.079   | 8.3      | -7.1     |

In this theory the quark condensate(12) is obtained

$$<\bar{u}u>=<\bar{d}d>=<\bar{s}s>=-(0.378GeV)^3.$$  \hspace{1cm} (37)

A larger quark condensate leads to greater value of $L_5$. On the other hand, $\frac{f_K}{f_\pi} = 1.19$ is obtained in this paper, which is 2.5% away from the data. Because of the same reason a smaller $L_8$ is obtained in this paper.

Small current quark masses are obtained

$$m_u = 0.91MeV, \quad m_d = 2.15MeV, \quad m_s = 52.3MeV.$$
As a check, the effect of current quark mass in $\pi^0 \rightarrow \gamma\gamma$ is calculated

$$L_{\pi^0 \rightarrow \gamma\gamma} = -\frac{\alpha}{\pi f_\pi} f_q \pi^0 \varepsilon^{\mu\nu\lambda\beta} \partial_\mu A_\nu \partial_\lambda A_\beta,$$

(38)

$$f_q = 1 - \frac{1}{2m} (m_u + m_d) + \frac{g^2}{2 f_\pi^2} (1 - \frac{2c}{g})^2 m_{\pi^0}^2,$$

(39)

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha^2 m_{\pi}^2}{16 \pi^3 f_\pi^2} \{1 - \frac{1}{m} (m_u + m_d) + \frac{g^2}{2 f_\pi^2} (1 - \frac{2c}{g})^2 m_{\pi^0}^2\}^2$$

(40)

$$f_q = 1 + 4.97 \times 10^{-3}.$$  

(41)

The decay width of $\pi^0 \rightarrow \gamma\gamma$ is increased by 1%.

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.89 eV.$$  

(42)

The data is $7.83(1 \pm 0.071)eV$.

### 12 OZI rule

At the tree level $\phi \rightarrow \rho\pi$ if forbidden. Only loop diagrams of mesons contribute to this decay. At tree level, the decay amplitude is $O(N_C)$ and at the loop level of this decay the order is $O(1)$. It is well known that the decay $\phi \rightarrow \rho\pi(3\pi)$ is forbidden by OZI rule. In this theory there is no tree diagram for this decay mode and only loop diagrams of mesons contribute to this decay mode. The order of the amplitude of this decay mode is $O(1)$. Because the cutoff determined is about 1.8GeV, therefore, only low lying meson states contribute to the
loop diagrams of the decay, $\phi \rightarrow \rho \pi(3\pi)$. This decay is calculable. It is a serious test on this effective chiral large $N_C$ theory.

13 Summary

This effective theory is phenomenologically successful. It has most nice features obtained in previous studies. Especially, this theory uses $N_C$ expansion to do concrete physical study. On the other hand, there are more work ahead:

1. $K \rightarrow 2\pi$ decays and $\Delta I = \frac{1}{2}$ rule,

2. CP violation in rare kaon decays,

3. OZI rule suppressed decays $\phi \rightarrow \rho \pi(3\pi)$,

4. Loop corrections, such as in $\rho \rightarrow 2\pi$...et al.,

5. Current quark mass corrections,

6. The match between this theory and perturbative QCD.

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Figure Captions

FIG. 1. Pion form factor in time-like region.

FIG. 2. Pion form factor in space-like region.

FIG. 3. Pion form factor in space-like region.

FIG. 4. Charged kaon form factor in time-like region.

FIG. 5. Charged kaon form factor in space-like region.

FIG. 6. Phase shift $\delta^{1/2}_1$.

FIG. 7. Phase shift $\delta^{1/2}_1$.

FIG. 8. Phase shift $\delta^{3/2}_0$.

FIG. 9. $K^+\pi^-$ p-wave cross section.
FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.
FIG. 5.
FIG. 6.
FIG. 5

FIG. 8.
FIG. 7

$K^+\pi^-$ p-wave cross section (mb)

Energy (GeV)

FIG. 9.