Mathematical Modelling and Parameter Optimization of Pulsating Heat Pipes

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Abstract

Proper heat transfer management is important to key electronic components in microelectronic applications. Pulsating heat pipes (PHP) can be an efficient solution to such heat transfer problems. However, mathematical modelling of a PHP system is still very challenging, due to the complexity and multiphysics nature of the system. In this work, we present a simplified, two-phase heat transfer model, and our analysis shows that it can make good predictions about startup characteristics. Furthermore, by considering parameter estimation as a nonlinear constrained optimization problem, we have used the firefly algorithm to find parameter estimates efficiently. We have also demonstrated that it is possible to obtain good estimates of key parameters using very limited experimental data.

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1 Introduction

A pulsating heat pipe is essentially a small pipe filled with both liquid and vapour and the internal diameter of the heat pipe is at the capillary scale [1, 2]. The liquids in the pipe can form segments or plugs, between vapour segments. When encountering heat, part of the liquid may evaporate and absorb some heat, thus causing a differential pressure and driving the movement of the plugs. When vapour bubbles meet a cold region, some of the vapour may condensate, and thus releasing some heat. The loop can be open or closed, depending on the type of applications and design. This continuous loop and process will form an efficient cooling system if designed and managed properly for a given task. Therefore, such systems have been applied to many applications in heat exchanger, space applications and electronics, and they can potentially have even wider applications [43]. On the other hand, the emergence of nanotechnology and the steady increase of the density of the large-scale integrated circuits have attracted strong interests in modelling heat transfer at very small scales, and the heat management of microdevices has become increasingly important for next generation electronics and miniaturization.

Both loop heat pipes (LHP) and pulsating heat pipes (PHP) may provide a promising solution to such challenging problems, and thus have attracted renewed attention in recent years [1, 4, 7, 12, 13, 24, 25, 42]. In many microelectronic applications, conventional solutions to heat management problems often use fans, heat exchangers and even water cooling. For examples, the fan-driven air circulation system in desktop computers and many laptops have many drawbacks such as bulky sizes and potential failure of mechanical, moving parts. In contrast, loop heat pipes have no mechanical driving system, and heat circulation is carried out through the pipes, and thus such LHP systems can be very robust and long-lasting. In addition, miniaturization and high-performance heat pipes systems are being developed [11, 15, 18, 39, 26, 43]. Simulation tools and multiphase models have been investigated [17, 16]. All these studies suggested that LHP systems can have some advantages over traditional cooling systems.

A PHP system may often look seemingly simple; however, its working mechanisms are relatively complex, as such systems involve multiphysics processes such as thermo-hydrodynamics, two-phase flow, capillary actions, phase changes and others. Therefore, many challenging issues still remain unsatisfactorily modelled. There are quite a few attempts in the literature to model a PHP system with various degrees of approximation and success.

In this paper, we will use a mathematical model based on one of the best models [24, 25], and will carry out some mathematical analysis and highlight the key issues in the state of the art models. In this paper, we intend to achieve two goals: to present a simplified mathematical model which can reproduce most characteristics of known physics, and to provide a framework for estimating key parameters from a limited number of measurements. The rest of the paper is organized as follows: we first discuss briefly design optimization and metaheuristic algorithms such as firefly algorithm, we then outline the main multiphysics processes in the mathematical models. We then solve the simplified model numerically and compared with experimental data drawn from the literature. After such theoretical analysis, we then form the parameter estimation as an optimization problem and solve it using the efficient firefly algorithm for inversely estimating key parameters in a PHP system. Finally, we highlight the key issues and discuss possible directions for further research.
2 Design Optimization of Heat Pipes

The proper design of pulsating heat pipes is important, so that the heat in the system of concern can be transferred most efficiently. This also helps to produce designs that use the least amount of materials and thus cost much less, while lasting longer without the deterioration in performance. Such design tasks are very challenging, practical designs tend to be empirical and improvements tend to be incremental. In order to produce better design options, we have to use efficient design tools for solving such complex design optimization problems. Therefore, metaheuristic algorithms are often needed to deal with such problems.

2.1 Metaheuristics

Metaheuristic algorithms such as the firefly algorithm and bat algorithm are often nature-inspired [28, 31], and they are now among the most widely used algorithms for optimization. They have many advantages over conventional algorithms, such as simplicity, flexibility, quick convergence and capability of dealing with a diverse range of optimization problems. There are a few recent reviews which are solely dedicated to metaheuristic algorithms [28, 29, 34]. Metaheuristic algorithms are very diverse, including genetic algorithms, simulated annealing, differential evolution, ant and bee algorithms, particle swarm optimization, harmony search, firefly algorithm, cuckoo search, flower algorithm and others [29, 30, 33, 36].

In the context of heat pipe designs, we have seen a lot of interests in the literature [5, 7]. However, for a given response, to identify the right parameters can be considered as an inverse problem as well as an optimization problem. Only when we understand the right working ranges of key parameters, we can start to design better heat-transfer systems. To our knowledge, this is the first attempt to use metaheuristic algorithms to identify key parameters for given responses. We will use the firefly algorithm to achieve this goal.

2.2 Firefly Algorithm

Firefly algorithm (FA) was first developed by Xin-She Yang in 2008 [28, 30] and is based on the flashing patterns and behavior of fireflies. In essence, FA uses the following three idealized rules:

1. Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.

2. The attractiveness is proportional to the brightness and they both decrease as their distance increases. Thus for any two flashing fireflies, the less brighter one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly in the form of a random walk.

3. The brightness of a firefly is determined by the landscape of the objective function.

As a firefly’s attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the variation of attractiveness $\beta$ with the distance $r$ by

$$\beta = \beta_0 e^{-\gamma r^2},$$

where $\beta_0$ is the attractiveness at $r = 0$. 

3
The movement of a firefly $i$ attracted to another more attractive (brighter) firefly $j$ is determined by
\[ x_{i}^{t+1} = x_{i}^{t} + \beta_0 e^{-\gamma r_{ij}^2} (x_{j}^{t} - x_{i}^{t}) + \alpha \epsilon_{i}^{t}, \quad (2) \]
where the second term is due to the attraction. The third term is randomization with $\alpha$ being the randomization parameter, and $\epsilon_{i}^{t}$ is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time $t$. If $\beta_0 = 0$, it becomes a simple random walk. Furthermore, the randomization $\epsilon_{i}^{t}$ can easily be extended to other distributions such as Lévy flights.

The Lévy flight essentially provides a random walk whose random step size $s$ is drawn from a Lévy distribution
\[ \text{Lévy} \sim s^{-\lambda}, \quad (1 < \lambda \leq 3), \]
which has an infinite variance with an infinite mean. Here the steps essentially form a random walk process with a power-law step-length distribution with a heavy tail. Some of the new solutions should be generated by Lévy walk around the best solution obtained so far; this will speed up the local search. Lévy flights are more efficient than standard random walks [29].

Firefly algorithm has attracted much attention [3, 23, 35]. A discrete version of FA can efficiently solve non-deterministic polynomial-time hard, or NP-hard, scheduling problems [23], while a detailed analysis has demonstrated the efficiency of FA for a wide range of test problems, including multiobjective load dispatch problems [3, 32]. Highly nonlinear and non-convex global optimization problems can be solved using firefly algorithm efficiently [9, 35]. The literature of firefly algorithms has expanded significantly, and Fister et al. provided a comprehensive literature review [8].

3 Mathematical Model for a PHP System

3.1 Governing Equations

Mathematical modelling of two-phase pulsating flow inside a pulsating heat pipe involves many processes, including interfacial mass transfer, capillary force, wall shear stress due to viscous action, contact angles, phase changes such as evaporation and condensation, surface tension, gravity and adiabatic process. All these will involve some constitutive laws and they will be coupled with fundamental laws of the conservation of mass, momentum and energy, and thus resulting in a nonlinear system of highly coupled partial differential equations. Consequently, such a complex model can lead to complex behaviour, including nonlinear oscillations and even chaotic characteristics [14, 19, 20, 24, 27, 41].

A mathematical model can have different levels of complexity, and often a simple model can provide significant insight into the working mechanism of the system and its behaviour if the model is constructed correctly with realistic conditions. In most cases, full mathematical analysis is not possible, we can only focus on some aspects of the model and gain some insight into the system.

A key relationship concerning the inner diameter of a typical pulsating heat pipe is the range of capillary length, and many design options suggest to use a diameter near the critical diameter [5]
\[ d_c = 2 \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}}, \quad (4) \]
where $\sigma$ is the surface tension (N/m), while $g$ is the acceleration due to gravity, which can be taken as $9.8 \, \text{m/s}^2$. $\rho_l$ and $\rho_v$ are the densities of liquid and vapour, respectively. In the rest of the paper, we will focus on one model which may be claimed as the state-of-the-art, as it is based on the latest models [24, 25], with a simplified model for a system of liquid plugs and vapour bubbles as described in [37].

### 3.1.1 Temperature Evolution

The temperature $T_v$ in a vapour bubble is governed by the energy balance equation

$$m_v c_v \frac{dT_v}{dt} = -h_l f_v (T_v - \tau) \pi (d_i - 2\delta) - r_m h_v \pi (d_i - 2\delta) - p_v \frac{dV}{dt},$$

where

$$r_m = \frac{2\sigma_0}{(2 - \sigma_0)} \frac{1}{\sqrt{2\pi R}} \frac{p_v}{\sqrt{T_v}} - \frac{p_l}{\sqrt{\tau}}.$$  

Here $L$ is the mean plug length, $d_i$ is the inner diameter of the pipe, and $\delta$ is the thickness of the thin liquid film. $h$ corresponds to the heat transfer coefficient and/or enthalpy in different terms. Here $\sigma_0$ is a coefficient and $R$ is the gas constant.

In addition, the volume in the above equation is given by

$$V = \frac{\pi d_i^2}{4} (L + L_v),$$

where the length of the vapour bubble is typically $L_v = 0.02 \, \text{m}$, and $\sigma \approx 1$.

The temperature $\tau$ in the liquid film is typically $L_v = 0.02 \, \text{m}$, and $\sigma \approx 1$.

For a vapour bubble, rate of change in mass is governed by the conservation of mass

$$\frac{dm_v}{dt} = -\pi (d_i - 2\delta) r_m.$$  

As $m_v = \rho_v V$, we have

$$\rho_v \frac{dV}{dt} = \pi (d_i - 2\delta) r_m - \rho_v \pi d_i L_v,$$

where $\rho_v \approx 1 \, \text{Kg/m}^3$ and the transfer rate $r_v = 0$ if $T_w > T_v$.

### 3.1.2 Mass Transfer

The position $x_p$ of the liquid plug is governed by the momentum equation

$$m_p \frac{d^2 x_p}{dt^2} = \frac{\pi}{4} (d_i - 2\delta)^2 (p_{v1} - p_{v2}) - \pi d_i L_p s_w + m_p g,$$

where

$$m_p = \rho_l \frac{\pi d_i^2}{4} (L_0 - x_p).$$

5
with \( L_0 \approx 25d_i \) and \( \rho_l = 1000 \text{ Kg/m}^3 \). Here the term \( p_{v1} - p_{v2} \) is the differential pressure between the two sides of the plug.

In the above equations, we also assume the vapour acts as an ideal gas, and we have

\[
p_v = \frac{m_v RT_v}{V}.
\]

The shear stress \( s_w \) between the wall and the liquid plug is given by an empirical relationship

\[
s_w = \frac{1}{2} C_f \rho_l v_p^2,
\]

where \( C_f \approx 16/Re \) when \( Re = \rho_l v_p d_i / \mu_l \leq 1180 \); otherwise,

\[
C_f \approx 0.078 Re^{-1/4}.
\]

Here, we also used that \( v_p = dx_p/dt \).

The mathematical formulation can include many assumptions and simplifications. When writing down the above equations, we have tried to incorporate most the constitutive relationships such contact angles, capillary pressures and viscous force into the equations directly so that we can have as fewer equations as possible. For details, readers can refer to \[24, 25, 5\]. This way, we can focus on a few key equations such as the motion of a plug and temperature variations, which makes it more convenient for later mathematical analysis.

### 3.2 Typical Parameters

The properties of the fluids and gas can be measured directly, and typical values can be summarized here, which can be relevant to this study. Most of these values have been drawn from earlier studies \[6, 24, 38\].

Typically, we have \( L = 0.18 \text{ m}, d_i = 3.3 \times 10^{-3} \text{ m}, \delta = 2.5 \times 10^{-5} \text{ m} \). The initial temperature is \( T_v \approx \tau = 20^\circ \text{C} \), while the initial wall temperature is \( T_w = 40^\circ \text{C} \). The initial pressure can be taken as \( p_v \approx 5.5816 \times 10^4 \text{ Pa} \). \( h_{f-fw} = 1000 \text{W/m}^2\text{K}, h_v = 10 \text{ W/m}^2\text{K} \).

In addition, we have the initial values: \( m_v0 = p_v0 V_0 / (R_v(T_v0 + 273.15)) \) where \( T_v0 = 20^\circ \text{C}, p_v0 = 10^5 \text{ Pa}, R_v = 461 \text{ J/Kg K}, \) and \( m_f0 \approx m_v0 / 10 \).

### 4 Nondimensionalization, Analysis and Simulation

After some straightforward mathematical simplifications, the full mathematical model can be written as the following equations:

\[
\frac{m_v c_v dT_v}{dt} = -h_{f-ve}(T_v - \tau)L\pi(d_i - 2\delta) - r_m h_v L\pi(d_i - 2\delta)
- p_v \frac{\pi}{2} [(d_i - 2\delta)r_m - d_i L_v r_v],
\]

\[
\frac{m_f c_v dT_v}{dt} = -h\tau(T_v - T_w)L\pi d_i + h_{f-ve}(T_v - \tau)L\pi(d_i - 2\delta)
+ r_m h_v L\pi(d_i - 2\delta),
\]

\[
\frac{dm_v}{dt} = -\pi(d_i - 2\delta)r_m,
\]

\[
\frac{m_p d^2x_p}{dt^2} = \frac{\pi}{4} (d_i - 2\delta)^2 (p_{vk} - p_{vk}) - \pi d_i L_p s_w + m_p g.
\]
Though analytically intractable, we can still solve this full model numerically using any suitable numerical methods such as finite difference methods, and see how the system behaves under various conditions with various values of parameters. Preliminary studies exist in the literature [24, 6], and we have tested our system that it can indeed reproduce these results using the typical parameters given in Section 3.2 [37]. For example, the mean temperature in a plug is consistent with the results in [41], while the oscillatory behaviour is similar to the results obtained by others [12, 5, 41, 25, 40].

4.1 Non-dimensionalized Model

For the current purpose of parameter estimation, the system can be considered stationary under appropriate conditions, and thus many terms can be taken as constants. Then, the full mathematical model (16) can be non-dimensionalized and written as

\[
\begin{align*}
\frac{d u}{dt} &= a(T_v - \tau) - \alpha_1 - \alpha_2, \\
\frac{d \tau}{dt} &= b(T_v - \tau) - \epsilon \tau + \alpha_3, \\
\frac{d u}{dt} &= -\Delta, \\
\frac{d^2 x_p}{dt^2} &= \frac{1}{(\beta_1 - \beta_2 x_p)} \left[ \beta - \gamma \left( \frac{dx_p}{dt} \right)^2 \right],
\end{align*}
\]

where

\[
\begin{align*}
a &= \frac{h_1 f_L \pi (d_i - 2\delta)}{c_{vv}}, & \alpha_1 &= \frac{r_m h_L \pi (d_i - 2\delta)}{c_{vv}}, \\
\alpha_2 &= \frac{p_v \pi}{\rho_v c_{vv}} [(d - 2\delta) r_m - d_i L_v r_v], & b &= \frac{-h_1 f_L \pi (d_i - 2\delta)}{m_f c_{vl}}, \\
\epsilon &= \frac{h_1 f_L \pi d_i}{m_f c_{vl}}, & \alpha_3 &= \frac{r_m h_L \pi (d_i - 2\delta)}{m_f c_{vl}}, \\
\Delta &= \frac{\pi (d_i - 2\delta) r_m}{m_{v0}}, & \beta &= \frac{\pi (d_i - 2\delta)^2 (p_v 1 - p_v 2)}{4} + g, \\
\gamma &= \frac{\pi d_i L_p C_v \rho_l}{2}, & \beta_1 &= \frac{\rho L_0 \pi d_i^2}{4}, & \beta_2 &= \frac{\rho_l \pi d_i^2}{4}.
\end{align*}
\]

In the above formulation, \( u \) is the dimensionless form of \( m_v \), that is \( u = m_v / m_{v0} \).

4.2 Simplified Model

Under the assumptions of stationary conditions and \( x_p / L_0 \ll 1 \) (for example at the startup stage), the last equation is essentially decoupled from the other equations. In this case, we have an approximate but simplified model

\[
\begin{align*}
\frac{d \tau}{dt} &= b(T_v - \tau) - \epsilon \tau + \alpha_3, \\
\frac{d^2 x_p}{dt^2} &= \frac{1}{\beta_1} \left[ \beta - \gamma \left( \frac{dx_p}{dt} \right)^2 \right],
\end{align*}
\]

Now the second equation can be rewritten as

\[
\begin{align*}
\frac{d^2 x_p}{dt^2} &= A - B \left( \frac{dx_p}{dt} \right)^2, & A &= \frac{\beta}{\beta_1}, & B &= \frac{\gamma}{\beta_1}.
\end{align*}
\]
For the natural condition $x_p = 0$ at $t = 0$, the above equation has a short-time solution

$$x_p = \frac{1}{B} \ln \left[ \cosh(\sqrt{AB} \ t) \right], \tag{25}$$

which is only valid when $x_p/L_0$ is small and/or $t$ is small. Indeed, it gives some main characteristics of the startup behaviour as shown as the dashed curve in Fig. 1.

Furthermore, the first equation for the liquid plug temperature is also decoupled from the PDE system. We have

$$\frac{d\tau}{dt} = Q_1 - Q_2 \tau, \quad Q_1 = bT_v + \alpha_3, \quad Q_2 = b + \epsilon. \tag{26}$$

For an initial condition $\tau = 0$ at $t = 0$, we have the following solution

$$\tau = Q_1(1 - e^{-Q_2 t}), \tag{27}$$

which provides a similar startup behaviour for small times as that in [41].

The approximation solutions are compared with the full numerical solutions and experimental data points, and they are all shown in Fig. 1. Here the experimental data points were based on [24, 25, 5]. It is worth pointing that the approximation can indeed provide some basic characteristics of the fundamental behaviour of the startup, and the full numerical results can also give a good indication of the final steady-state value.

![Figure 1: Comparison of the full numerical results with approximations and experimental data [25].](image)

This simplified model also confirms that some earlier studies using mainly $x_p$ as the dependent variable can indeed give good insight into the basic characteristics of the complex system. For example, Wong et al. used an equivalent viscous damping system without considering the actual heat transfer system [27], and their main governing equation can be written as

$$\frac{d^2 x}{dt^2} + a \frac{dx}{dt} + b(k + t)x = 0, \tag{28}$$
where \(a, b, k\) are constants. This system has typical features of a viscous damping system. On the other hand, Yuan et al. used a system with primarily a second-order nonlinear ODE for \(x_p\)

\[
\frac{d^2x_p}{dt^2} + \frac{2C}{L} \left(\frac{dx_p}{dt}\right)^2 + \frac{2g}{L} x_p = \frac{(p_{e1} - p_{e2})}{L\rho_l},
\]

(29)

where \(C\) is the friction coefficient. This system can have even richer characteristics of oscillatory behaviours [40].

In comparison with earlier simpler models, our simplified model can produce even richer dynamic features of the system without using much complex mathematical analysis, thus provides a basis for further realistic analysis and simulations.

5 Inverse Parameter Identification and Optimization

The aim of an inverse problem is typically to estimate important parameters of structures and materials, given observed data which are often incomplete. The target is to minimize the differences between observations and predictions, which is in fact an optimization problem. To improve the quality of the estimates, we have to combine a wide range of known information, including any prior knowledge of the structures, available data. To incorporate all useful information and carry out the minimization, we have to deal with a multi-objective optimization problem. In the simplest case, we have to deal with a nonlinear least-squares problem with complex constraints [21, 22].

The constraints for inversion can include the realistic ranges of physical parameters, geometry of the structures, and others. The constraints are typically nonlinear, and can be implicit or even black-box functions. Under various complex constraints, we have to deal with a nonlinear, constrained, global optimization problem. In principle, we can then solve the formulated constrained problem by any efficient optimization techniques [37, 29]. However, as the number of the degrees of freedom in inversion is typically large, data are incomplete, and non-unique solutions or multiple solutions may exist; therefore, metheuristic algorithms are particularly suitable.

5.1 Parameter Estimation as an Inverse Problem

Inverse problems in heat transfer and other disciplines tend to find the best parameters of interest so as to minimize the differences between the predicted results and observations. In the simplest case, a linear inverse problem can be written as

\[
u = Kq + w,
\]

(30)

where \(u\) is the observed data with some noise \(w\), \(q\) is the parameter to be estimated, and \(K\) is a linear operator or mapping, often written as a matrix. An optimal solution \(q^*\) should minimize the norm \(\|u - Kq\|\). In case when \(K\) is rectangular, we have the best estimate

\[
q^* = (K^T K)^{-1} K^T u.
\]

(31)

But in most cases, \(K\) is not invertible, thus we have to use other techniques such as regularization. In reality, many inverse problems are nonlinear and data are incomplete. Mathematically speaking, for a domain \(\Omega\) such as a structure with unknown but true parameters \(q^*\) (a vector of multiple parameters), the aim is find the solution vector \(q\) so that
the predicted values $y_i$ (i=1,2, …, n), based on a mathematical model $y = \phi(x,p)$ for all $x \in \Omega$, are close to the observed values $d = (d_1, d_2, ..., d_n)$ as possible.

The above inverse problem can be equivalently written as a generalized least-squares problem [21, 37]

$$\min f = ||d - \phi(x,q)||^2,$$

or

$$\min \sum_i [d_i - \phi(x_i,q)]^2,$$

Obviously, this minimization problem is often subjected to a set of complex constraints. For example, physical parameters must be within certain limits. Other physical and geometrical limits can also be written as nonlinear constraints. In general, this is equivalent to the following general nonlinear constrained optimization problem

$$\min f(x,q,d)$$

subject to

$$h_j(x,q) = 0, \quad (j = 1, ..., J), \quad g_k(x,q) \leq 0, \quad (k = 1, ..., K).$$

where J and K are the numbers of equality and inequality constraints, respectively. Here all the functions $f$, $h_j$, and $g_k$ can be nonlinear functions. This is a nonlinear, global optimization for $q$. In the case when the observations data are incomplete, thus the system is under-determined, some regularization techniques such as Tikhonov regularization are needed. Therefore, the main task now is to find an optimal solution to approximate the true parameter set $q^*$. From the solution point of view, such optimization can be in principle solved using any efficient optimization algorithm. However, as the number of free parameters tends to be very large, and as the problem is nonlinear and possible multimodal, conventional algorithms such as hill-climbing usually do not work well. More sophisticated metaheuristic algorithms have the potential to provide better solution strategies.

Though above inverse problems may be solved if they are well-posed, however, for most inverse problems, there are many challenging issues. First, data are often incomplete, which leads to non-unique solutions; consequently, the solution techniques are often problem-specific, such as Tikhonov regularization. Secondly, inverse problems are highly nonlinear and multimodal, and thus very difficult to solve. In addition, problems are often large-scale with millions of degrees of freedom, and thus requires efficient algorithms and techniques. Finally, many problems are NP-hard, and there is no efficient algorithms of polynomial time exists. In many of these cases, metaheuristic algorithms such as genetic algorithms and firefly algorithm could be the only alternative. In fact, metaheuristic algorithms are increasingly popular and powerful [28, 37]. In the rest of this paper, we will use the firefly algorithm discussed earlier to carry out the parameter estimations for pulsating heat pipes.

5.2 Least-Squares Estimation

Now we try to solve the following problem for parameter estimation: Suppose we measure the response (i.e., the location $x_p$ of a plug) of a pulsating heat pipe, as the results presented in Fig. [1] can we estimate some key parameters using these results through our simplified mathematical model? One way to deal with this problem is to consider it as a nonlinear, least-squares best-fit problem as described earlier in Eq. (33).
Table 1: Least-Squares Parameter Estimation.

| Parameters | Estimates | True values |
|------------|-----------|-------------|
| $L$        | $0.15 \sim 0.24$ | $0.18$ (m) |
| $d_i$      | $0.001 \sim 0.005$ | $0.0033$ (m) |
| $T_v$      | $10 \sim 27$ | $20$ ($^\circ$C) |
| $T_w$      | $20 \sim 49$ | $40$ ($^\circ$C) |
| $p_v$      | $40 \sim 129$ | $100$ (kPa) |

Table 2: Parameter Estimation by Constrained Optimization.

| Parameters | Estimates (mean ± standard deviation) | True values |
|------------|--------------------------------------|-------------|
| $L$        | $0.17 \pm 0.1$ | $0.18$ (m) |
| $d_i$      | $0.0032 \pm 0.0004$ | $0.0033$ (m) |
| $T_v$      | $20.4 \pm 0.9$ | $20$ ($^\circ$C) |
| $T_w$      | $39.2 \pm 1.1$ | $40$ ($^\circ$C) |
| $p_v$      | $98 \pm 8.8$ | $100$ (kPa) |

Let us focus on the key parameters $L$, $T_v$, $T_w$, $d_i$, and $p_v$. When we carry out the estimation using the least squares methods, we can only get crude estimates as shown in Table 1. Here we have used 25 data points to establish estimates for 5 key parameters. The wide variations of these parameters, though near the true values, suggest that there are insufficient conditions for the inverse problem to have unique solutions. This can be attributed to the factors that the measured data points were sparse and incomplete, no constraints were imposed on the ranges of the parameters, and additional constraints might be needed to make the inverse problem well-posed.

5.3 Constrained Optimization by the Firefly Algorithm

In order to get better and unique solutions to the problem, to increase of the number of measurements is not the best choice, as experiments can be expensive and time-consuming. Even with the best and most accurate results, we may still be unable to get unique solutions. We have to make this problem well-posed by imposing enough conditions.

First, we have to impose more stringent bounds/limits. Thus, we apply $L \in [0.15, 0.22]$ m, $d_i \in [0.002, 0.004]$ m, $T_v \in [15, 25]$ $^\circ$C, $T_w \in [35, 45]$ $^\circ$C, and $p_v \in [80, 120]$ kPa. Then, we have to minimize the parameter variations, in addition to the residual sum of squares. Thus, we have

$$\text{Minimize} \quad \sum_{i} \left[ d_i - \phi(x_i, q) \right]^2 + \sum_{k=1}^{5} \sigma_k^2,$$  \hspace{1cm} (36)

where $\sigma_k^2$ is the variance of parameter $k$. Here we have 5 parameters to be estimated. Now we have a constrained optimization problem with imposed simple bounds.

By using the firefly algorithm with $n = 20$, $\gamma = 1$, $\beta = 1$ and 5000 iterations, we can solve the above constrained optimization. We have run the results 40 times so as to obtained meaningful statistics, using the same set of 25 observed data points, and the results are summarized in Table 2.
We can see clearly from the above results that unique parameter estimates can be achieved if sufficient constraints can be imposed realistically, and the good quality estimates can be obtained even for a number of sparse measurements. This implies that it is feasible to estimate key parameters of a complex PHP system from experimental data using the efficient optimization algorithm and the correct mathematical models.

6 Discussions and Conclusions

Though a pulsating heat pipe system can be hugely complex as it involves multiphysics processes, we have shown that it is possible to formulate a simplified mathematical model under one-dimensional configuration, and such a model can still be capable to reproduce the fundamental characteristics of time-dependent startup and motion of liquid plugs in the heat pipe. By using scaling variations and consequently non-dimensionalization, we have analysed the key parameters and processes that control the main heat transfer process. Asymptotic analysis has enabled us to simplify the model further concerning small-time and long-time behavior, so that we can identify the factors that affect the startup characteristics. Consequently, we can use the simplified model to predict and then compare the locations of a plug with experimental data. The good comparison suggests that a simplified model can work very well for a complex PHP system.

Even though we have very good results, however, there are still some significant differences between the full numerical model and experimental data, which can be attributed to unrealistic parameter values, incomplete data, oversimplified approximations and unaccounted experimental settings. Further work can focus on the extension of the current model to 2D or 3D configuration with realistic geometry and boundary conditions.

By treating parameter estimation as an inverse problem and subsequently a nonlinear constrained optimization problem, we have used the efficient metaheuristic algorithms such as firefly algorithm to obtain estimates of some key parameters in a 1D PHP system. We have demonstrated that a least-squares approach is not sufficient to obtain accurate results because the inverse problem was not well-posed, with insufficient constraints. By imposing extra proper constraints on parameter bounds and also minimizing the possible variations as well as minimizing the best-fit errors, we have obtained far more accurate estimates for the same five key parameters, and these parameter estimates are comparable with their true values.

There is no doubt that further improvement concerning the mathematical model and parameter estimation will provide further insight into the actual behaviour of a PHP system, and subsequently allows us to design better and more energy-efficient PHP systems.

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