DILATONIC SPHALERONS

AND

NON-ABELIAN BLACK HOLES

GEORGE LAVRELASHVILI

Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
Föhringer Ring 6, 80805 Munich, Germany

and

DIETER MAISON

Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
Föhringer Ring 6, 80805 Munich, Germany

ABSTRACT

We discuss properties and interpretation of recently found globally regular and black hole solutions of a Einstein-Yang-Mills-dilaton theory.

1Talk given at the 15th annual MRST meeting on High Energy Physics, Syracuse University, NY, May 14-15, 1993. To be published in the Proceedings
2On leave of absence from Tbilisi Mathematical Institute, 380093 Tbilisi, Georgia
1. Introduction

Soon after Bartnik and McKinnon’s discovery of smooth, static solutions of Einstein-Yang-Mills (EYM) theory [1], black hole solutions of the same system where found [2]. Stability analysis showed that all those solutions are unstable in linear perturbation theory [3]. It was understood that the Bartnik-McKinnon solutions are a kind of gravitational sphalerons [4]. All these solutions are not known in closed form and were obtained by numerical integration. In the meantime mathematically rigorous existence proof for all these solutions were given [5],[6],[7].

On the other hand, various theoretical considerations (e.g. String theory, Kaluza-Klein theories, cosmological models) suggest to supplement the gravitational field by a massless scalar companion, a so-called ‘dilaton’ - a neutral scalar field with conformal coupling to matter. So, it is a natural step to extend the EYM theory to a Einstein-Yang-Mills-dilaton (EYMD) theory. This extended theory was recently studied in [8]-[12].

In the present paper we will discuss properties and interpretation of globally regular and black hole solutions of such a EYMD theory.

We will consider a EYMD theory defined by the action

\[
S = \frac{1}{4\pi} \int \left( -\frac{1}{4G} R + \frac{1}{2} (\partial \varphi)^2 - \frac{e^{2\kappa \varphi}}{4g^2} F^2 \right) \sqrt{-g} d^4 x
\]  

(1)

where \( \kappa \) resp. \( g \) denote the dilatonic resp. gauge coupling constant and \( G \) is Newton’s constant.

This theory depends on a dimensionless parameter \( \gamma = \frac{\kappa}{\sqrt{G}} \). In the limit \( \gamma \to 0 \) one gets the EYM theory studied in [1],[2], whereas for \( \gamma \to \infty \) one obtains the YM-dilaton theory in flat space studied in [8],[9]. The value \( \gamma = 1 \) corresponds to a model obtained from heterotic string theory [14]. It was observed [10] that a very special situation occurs for this value of \( \gamma \). We found strong indications that the lowest lying regular solution may be obtained in closed form.

The rest part of this paper is organized as follows. In the following chapter we will introduce the static spherically symmetric ansatz and derive the corresponding field equations. In chapter 3 we will discuss the asymptotic behavior of solution in the vicinity of singular points. In chapters 4 and 5 we shall present the results of our numerical calculations, indicating that for any value of the dilaton coupling constant \( \gamma \) the EYMD system has:

1. A discrete family of globally regular (sphaleronic type) solutions
2. A discrete family of black hole solutions.

Chapter 6 contains concluding remarks.

Our presentation here is based and closely follows Ref’s [8],[10].

2. Ansatz and Field Equations

We are interested in static, spherically symmetric solutions of EYMD theory defined by the action Eq. (1). A convenient parametrization for the metric turns out to
be
\[ ds^2 = A^2(r)\mu(r)dt^2 - \frac{dr^2}{\mu(r)} - r^2d\Omega^2, \quad (2) \]
where \( d\Omega^2 = d\theta^2 + \sin^2(\theta)d\phi^2 \) is the line element of the unit sphere.

For the \( SU(2) \) YM potential we make the usual ('magnetic') spherically symmetric ansatz
\[ W_0^a = 0, \quad W_i^a = \epsilon_{aik} \frac{x^k}{r^2} (W(r) - 1). \quad (3) \]

Substituting this ansatz into the action we obtain the reduced action
\[ S_{\text{red}} = -\int A \left[ \frac{1}{2G}(\mu + r\mu' - 1) + \frac{r^2}{2}\mu\phi'^2 + \frac{e^{2\gamma\varphi}}{g^2} \left( \mu W'^2 + \frac{(1 - W^2)^2}{2r^2} \right) \right] dr, \quad (4) \]
where a prime denotes \( \frac{d}{dr} \).

Rescaling \( \varphi \to \varphi/\sqrt{G} \), \( r \to \frac{\sqrt{G}}{g}r \) and \( S \to g/S\sqrt{G} \) removes the dependence on \( G \) and \( g \) from \( S \), hence we may put \( G = g = 1 \) without restriction. The only remaining parameter is the dimensionless ratio \( \gamma = \kappa/\sqrt{G} \), which may be also written as the ratio of masses \( \gamma = \frac{M_{\text{Pl}}}{M_D} \) with \( M_{\text{Pl}} = \frac{1}{\sqrt{G}} \) and \( M_D = \frac{1}{\kappa} \). The resulting field equations are
\[
(Ae^{2\gamma\varphi}\mu W')' = Ae^{2\gamma\varphi} \frac{W(W^2 - 1)}{r^2}, \\
(A\mu r^2\varphi')' = 2\gamma Ae^{2\gamma\varphi} \left( \mu W'^2 + \frac{(1 - W^2)^2}{2r^2} \right), \\
\mu' = \frac{1}{r} \left( 1 - \mu - r^2\mu\varphi'^2 - 2e^{2\gamma\varphi} \left( \mu W'^2 + \frac{(1 - W^2)^2}{2r^2} \right) \right), \\
A^{-1}A' = \frac{2e^{2\gamma\varphi}W'^2}{r} + r\varphi'^2. \quad (5) \]

A few remarks are in order.

Eq. (5) are invariant under a shift \( \varphi \to \varphi + \varphi_0 \) accompanied by a simultaneous rescaling \( r \to re^{\gamma\varphi_0} \), hence solutions regular at infinity can always be normalized to \( \varphi(\infty) = 0 \).

In the flat space limit the system of Eq. (5) reduces to
\[
W'' = \frac{W(W^2 - 1)}{r^2} - 2\gamma\varphi'W', \\
(r^2\varphi')' = 2\gamma e^{2\gamma\varphi} \left( W'^2 + \frac{(1 - W^2)^2}{2r^2} \right). \quad (6) \]

For \( \gamma = 0 \) the dilaton field decouples and one obtains the YM theory governed by the equation
\[ W'' = \frac{W(W^2 - 1)}{r^2}. \]  

Introducing the 'time' co-ordinate \( \tau = \ln r \) we obtain the simple equation

\[ \dot{W} = W(W^2 - 1) + \dot{W}. \]  

where a dot denotes \( \frac{d}{d\tau} \). This equation has a mechanical analogue; it describes the motion of a 'particle' in the potential \( V(W) = -(W^2 - 1)^2/4 \) with a negative friction due to the \( \dot{W} \) term. Globally regular solutions correspond to motions interpolating between local maxima of the potential. Due to the friction such motions do not exist, and hence there are no regular, static solutions of the pure YM theory, in accordance with a known result \[15\].

There are some known exact solutions of Eq.\((2)\). Their YM fields are imbeddings of abelian gauge fields, i.e. we are actually dealing with solutions of the Einstein-Maxwell-dilaton theory. All these solutions describe black holes; no globally regular solutions of this type exist \[16\].

The simplest case is the Schwarzschild solution with trivial YM- and dilaton fields, given by

\[ W \equiv 1, \quad \varphi \equiv 0, \quad A(r) = 1, \quad \mu(r) = 1 - \frac{2M}{r}. \]  

There is an abelian (magnetically charged) Reissner-Nordstrøm type solution with nontrivial dilaton field obtained in \[17\] for special value of \( \gamma = \sqrt{3} \), generalized to arbitrary \( \gamma \) in \[18\].

Of particular interest is the so called 'extremal' limit of this solution. In terms of a radial co-ordinate \( \rho \) related to \( r \) through \( dr = r^2 \sqrt{\mu} e^\varphi \rho^{-2} d\rho \) this 'extremal' solution is given by expression:

\[ ds^2 = e^{2\varphi} dt^2 - e^{-2\varphi} (d\rho^2 + \rho^2 d\Omega^2), \]  

with

\[ e^{2\varphi} = \left(1 + \frac{\sqrt{\gamma^2 + 1}}{\rho}\right)^{\gamma^2+1} \]  

and \( W = 0 \).  

The mass of this solution is given by

\[ M = 1/\sqrt{\gamma^2 + 1}. \]  

For later use we determine also the metric coefficient \( \mu \), which turns out to be given by

\[ \mu = \left(1 + \frac{\gamma^2+1}{\sqrt{\gamma^2+1} \rho}\right)^2 \]  

For \( \rho \to 0 \) the function \( \mu(r) \) tends to

\[ \mu_0 = \gamma^4/(\gamma^2 + 1)^2. \]
The fact that this value differs from 1 means that the point \( r = 0 \) is not a regular origin.

3. Asymptotic behavior of solutions

The field equations Eq. (3) have singular points at \( r = 0 \) and \( r = \infty \) as well as for points where \( \mu(r) \) vanishes. Regularity at \( r = 0 \) of a configuration requires \( \mu(r) = 1 + O(r^2) \), \( W(r) = \pm 1 + O(r^2) \), \( \varphi(r) = \varphi(0) + O(r^2) \) and \( A(r) = A(0) + O(r^2) \). Since \( W \) and \( -W \) are gauge equivalent we may choose \( W(0) = 1 \). Similarly we can assume \( A(0) = 1 \) since a rescaling of \( A \) corresponds to a trivial rescaling of the time coordinate. Inserting a power series expansion into Eq. (5) one finds

\[
W(r) = 1 - br^2 + O(r^4),
\]
\[
\varphi(r) = \varphi_0 + 2\gamma e^{2\gamma \varphi_0} b^2 r^2 + O(r^4),
\]
\[
\mu(r) = 1 - 4b^2 e^{2\gamma \varphi_0} r^2 + O(r^4),
\]
\[
A(r) = 1 + 4b^2 e^{2\gamma \varphi_0} r^2 + O(r^4),
\]
where \( b \) and \( \varphi_0 \) are arbitrary parameters.

Similarly assuming a power series expansion in \( \frac{1}{r} \) at \( r = \infty \) for asymptotically flat solutions one finds \( \lim_{r \to \infty} W(r) = \{ \pm 1, 0 \} \). It turns out that \( W(\infty) = 0 \) cannot occur for globally regular solutions, therefore we concentrate on the remaining cases. One finds

\[
W(r) = \pm(1 - \frac{c}{r} + O(\frac{1}{r^2})),
\]
\[
\varphi(r) = \varphi_\infty - \frac{d}{r} + O(\frac{1}{r^4}),
\]
\[
\mu(r) = 1 - \frac{2M}{r} + O(\frac{1}{r^2}),
\]
\[
A(r) = A_\infty(1 - \frac{d^2}{2r^2} + O(\frac{1}{r^4})),
\]
where again \( c, d, M, \varphi_\infty \) and \( A_\infty \) are arbitrary parameters.

Turning to singular points \( r_h \), where \( \mu(r) \) vanishes, we find that solutions of Eq. (3) stay regular at such a point, if

\[
W(r_h + \rho) = W_h + W'_h \rho + O(\rho^2),
\]
\[
\varphi(r_h + \rho) = \varphi_h + \varphi'_h \rho + O(\rho^2),
\]
\[
\mu(r_h + \rho) = \mu'_h \rho + O(\rho^2),
\]
with
\[ W'_{h} = \frac{W_{h}(W_{h}^{2} - 1)}{\mu'_{h}r_{h}^{2}}, \]

\[ \varphi'_{h} = \frac{2\gamma e^{2\gamma\varphi_{h}}(W_{h}^{2} - 1)^{2}}{2\mu'_{h}r_{h}^{4}}, \]

\[ \mu'_{h} = \frac{1}{r_{h}}\left(1 - \frac{e^{2\gamma\varphi_{h}}(W_{h}^{2} - 1)^{2}}{r_{h}^{2}}\right). \]  

(18)

For a given value of \( r_{h} \) there are the adjustable parameters \( W_{h} \) and \( \varphi_{h} \) analogous to the parameters \( b \) and \( \varphi_{0} \) at \( r = 0 \). If \( r_{h}, W_{h} \) and \( \varphi_{h} \) are chosen such that \( \mu'_{h} > 0 \) the surface \( r = r_{h} \) describes a regular event horizon, hence asymptotically flat solutions with this behavior represent black holes.

The mass of such a (regular and black hole) solution is given in units of \( M_{Pl}/g = \frac{1}{g\sqrt{G}} \) by

\[ M = \lim_{r \to \infty} \frac{r(1 - \mu(r))}{2}. \]  

(19)

The generic solution with regular boundary conditions of the type Eq. (15) or Eq. (17), Eq. (18) develops a singularity with \( \mu(r_{s}) = 0 \) at some value \( r = r_{s}(b) \) with finite values of \( W(r_{s}), \varphi(r_{s}) \) and diverging \( W'(r_{s}) \). Closer analysis reveals these singularities as coordinate singularities, where the radius \( r \) is stationary. Regular, asymptotically flat resp. black hole solutions avoid this singularity and interpolate smoothly between the described asymptotic behavior at \( r = 0 \) resp. \( r = r_{h} \) and \( r = \infty \). This may be achieved by a suitable choice of the parameter \( b \). In fact, numerical integration of Eq. (3) indicates the existence of discrete families of globally regular resp. black hole solutions for any given \( r_{h} \). The various members of these families are distinguished by the number of zeros of \( W(r) \).

4. Dilatonic Sphalerons

4.1. Flat Space Solution

It is known that pure YM equations in four dimensions have no static solutions because of the repulsive nature of the YM self-interaction. This repulsion can be compensated by the introduction of a Higgs field. This way one obtains e.g. the t’Hooft-Polyakov magnetic monopole and the sphaleron.

The role of the attractive field can be also played by the dilaton. In fact, it was observed (see also [1]) that the introduction of a dilaton field produces a discrete sequence of particle-like solutions of finite energy.

In the next subsection we will discuss in detail this type of solution in presence of gravity.

4.2. Gravitating Dilatonic Sphalerons

Globally regular solutions have to interpolate between the asymptotic behavior Eq. (15) at \( r \to 0 \) and Eq. (16) at \( r \to \infty \). Using a suitably desingularized version of Eq. (3)
at $r = 0$ we have integrated these equations numerically starting from $r = 0$. Due to the invariance of the field equations under a shift of $\varphi$ accompanied by a suitable rescaling of $r$ as already mentioned, it is sufficient to vary the parameter $b$ at the origin. The normalization $\varphi(\infty) = 0$ can be adjusted afterwards, accompanied by a rescaling of the parameter $b$ and of the mass $M$ of the solution.

For small values of $b$ the solution develops a zero of $\mu$ for some $r = r_s(b)$ with $W(r_s) < -1$. On the other hand for large $b$ the singularity occurs before $W(r)$ has a zero. This behavior is completely analogous to the one found for the EYM system [1]. In that case it has been used to prove rigorously the existence of globally regular solutions [5].

For all the considered values of $\gamma$ we found a discrete set \{ $b_N$ \} for which the YM-potential $W_N$ has $N$ zeros and then approaches $(-1)^N$. Since with growing $N$ the precise numerical computation of the solutions becomes increasingly difficult we present only solutions up to $N = 6$.

The numerically determined values of the parameters of the solutions up to $N = 6$ for $\gamma = 1.0$ are collected in Tab 1. The masses $M_N$ given in Tab 1. correspond to the normalization $\varphi(\infty) = 0$, obtained by the shift $\varphi \rightarrow \varphi - \varphi_\infty$ accompanied by a rescaling $M \rightarrow Me^{-\gamma\varphi_\infty}$.

| $N$ | $b$     | $\varphi_\infty$ | $M$         | $\mu_{\text{min}}$ |
|-----|---------|------------------|-------------|------------------|
| 1   | 0.166667| 0.932284         | 0.57695     | 0.5864           |
| 2   | 0.231800| 1.792793         | 0.68481     | 0.3705           |
| 3   | 0.246862| 2.692205         | 0.70344     | 0.2868           |
| 4   | 0.249484| 3.597983         | 0.70651     | 0.2637           |
| 5   | 0.249916| 4.504705         | 0.70702     | 0.2587           |
| 6   | 0.249986| 5.411575         | 0.70709     | 0.2534           |

It is remarkable that for $\gamma = 1$ the parameter $b$ for the lowest lying solution seems to be the rational number $\frac{1}{6}$ (this is valid at least up to 12 digits). This fact together with other regularities which we found suggest that the $N = 1$ solution for $\gamma = 1$ may be obtained in closed form.

The $\gamma$ dependence of the solutions was studied in [10]. We found that the mass in natural units $M_{Pl}/g$ goes to a finite value for $\gamma \rightarrow 0$ and drops to zero like $\frac{1}{\gamma^2}$ for $\gamma \rightarrow \infty$.

Next we would like to discuss the behavior of the discrete family of solutions for some fixed value of $\gamma$ parametrized by the number $N$ of zeros of $W(r)$. Already for moderately large $N$ the solutions develop a characteristic behavior. Two different regions can be clearly distinguished. In an inner region the function $\mu_N(r)$ falls to the value $\mu_0 = \gamma^2/(\gamma^2 + 1)^2$ and then stays constant on an interval, whose length increases with $N$. The function $W_N(r)$ approaches a universal form, decreasing first to $W \approx -0.15$ and then oscillating with decaying amplitude around $W = 0$. While
\( \mu_N \) stays more or less constant the function \( \varphi_N \) grows linearly with \( \ln r \). In a second, outer region \( \mu_N \) raises monotonically to \( \mu = 1 \), while \( W_N \) stays still very small. Finally \( W_N(r) \) raises to its asymptotic value \( \pm 1 \).

In the limit \( N \to \infty \) we obtain two different limiting solutions, one describing the interior part and another one the exterior one, differing by an infinite shift of \( \varphi \). The limiting interior solution, which is regular at \( r = 0 \), is normalized to \( \varphi(0) = 0 \). It extends to arbitrarily large values of \( r \). For large \( r \), where \( W(r) \) is very small and \( \mu(r) \) is more or less constant, the solution can be well approximated by an asymptotic form obtained by linearization in \( W \):

\[
\begin{align*}
W_{as} &= r^{-\frac{\gamma^2+1}{2\gamma^2}} \sin \left( \omega \ln r + \delta \right), \\
\varphi_{as} &= \frac{1}{\gamma} \ln r - \frac{1}{2\gamma} \ln(\gamma^2 + 1), \\
\mu_{as} &= \frac{\gamma^4}{(\gamma^2 + 1)^2}, \\
\ln A_{as} &= A_0 + \frac{1}{\gamma^2} \ln r,
\end{align*}
\]

with \( \omega = (1 + 1/\gamma^2) \sqrt{3}/2 \).

Since \( \varphi_N(\infty) \) grows without limit for \( N \to \infty \) the exterior limiting solution can only be obtained by renormalizing \( \varphi \) to \( \varphi(\infty) = 0 \), accompanied by an infinite rescaling of \( r \). It turns out that \( W_N(r) \) tends to zero if the rescaled value of \( r \) is kept fixed as \( N \) tends to infinity. The resulting exterior solution is the 'extremal' abelian solution discussed in chapter 2. The masses \( M_N \) of the regular solutions tend to the limiting value Eq. (12) from below, whereas the minima \( \mu_{min} \) tend to \( \mu_0 \) Eq. (14) from above.

As in [7] one can find asymptotic formulae for the \( b_N \) values and for the mass \( M_N \) of the solutions. The asymptotic formula for \( b_N \) for \( \gamma = 1.0 \) is

\[
b_N \approx \frac{1}{4} - \frac{1}{12} e^{-\frac{\pi}{\sqrt{3}}(N-1)}.
\]

A better approximation for small \( N \) is given by the expression

\[
b_N \approx \frac{1}{4(1 + 1/2 e^{-\frac{\pi}{\sqrt{3}}(N-1)})},
\]

where

\[
b_1 = \frac{1}{6}, \quad \text{and} \quad b_\infty = \frac{1}{4} = 0.25
\]

have been taken into account.

Eq. (21) and Eq. (22) are very precise already for small \( N \).
All the solutions were found \cite{10} to be unstable in linearized perturbation theory, as to be expected from experience with the Bartnik-McKinnon’s solutions \cite{3}.

5. Non-Abelian Black Holes

Black hole solutions have to interpolate between the asymptotic behavior Eq. (17) at \( r = r_h \) and Eq. (16) at \( r \to \infty \).

Our numerical results indicate that there is again a discrete infinite family of such black hole solutions for any given values of \( \gamma \) and \( r_h \). For \( r_h \to 0 \) the functions \( W(r) \) and \( \varphi(r) \) tend to those of the regular solutions. For large values of \( r_h \) the function \( \varphi(r) \) varies very little between \( r_h \) and \( r \to \infty \). Thus these solutions resemble those of the EYM theory without the dilaton. Tab. 2 contains the corresponding parameters and masses. Renormalizing \( \varphi \) to \( \varphi(\infty) = 0 \) clearly involves now also a rescaling of \( r_h \). The values \( r_{h0} \) given in Tab. 2 are those before the rescaling (corresponding to the normalization \( \varphi(r_h) = 0 \)).

Tab 2. Size dependence of the black hole parameters for \( \gamma = 1.0 \).

| \( r_{h0} \) | \( r_h \)  | \( W_h \)  | \( \varphi_{\infty} \) | \( M \)  |
|---------|---------|---------|---------|---------|
| 0.1     | 0.040083| 0.998331| 0.914212| 0.5849  |
| 1.0     | 0.510138| 0.824278| 0.673075| 0.6933  |
| 10.0    | 9.903090| 0.273001| 0.009738| 4.9997  |
| 50.0    | 49.98067| 0.268148| 0.000387| 24.999  |

Again, all these non-abelian black holes were found \cite{10} to be unstable in linearized perturbation theory.

6. Concluding Remarks

We find globally regular and black hole solutions of a EYMD theory. The regular solutions are sphaleron-type configurations - gravitating dilatonic sphalerons. One expects fermionic zero modes and as a result unsuppressed fermion number non-conservation processes in the background of this type of solutions. Due to the high mass of the solutions the only situation where they could play a role is in the Early Universe, but at the moment there seems to be no natural physical scenario where we could make use of these solutions.

It is interesting that the black holes we found have a non-vanishing YM field outside of a horizon. As they differ from the Schwarzschild solution, but do not carry any gauge charge we might consider them as violating the No-Hair Conjecture (stating that black holes are specified by their gauge charges). Yet, being unstable they do not constitute a strong case of such a violation.

The case \( \gamma = 1 \) corresponds to a model obtained from string theory. We find strong indications that the \( N = 1 \) globally regular solution can be obtained in closed
form.

7. Acknowledgements

We are indebted to P. Breitenlohner and P. Forgács for many stimulating discussions.

8. References

1. R. Bartnik and J. McKinnon, *Phys. Rev. Lett.* 61 (1988) 141.
2. M.S. Volkov and D.V. Gal’tsov, *JETP Lett.* 50 (1990) 346; H.P. Künzle and A.K.M. Masood-ul-Alam, *J. Math. Phys.* 31 (1990) 928; P. Bizon, *Phys. Rev. Lett.* 61 (1990) 2844.
3. N. Straumann and Z.H. Zhou, *Phys. Lett.* B243 (1990) 33.
4. M.S. Volkov and D.V. Gal’tsov, *Phys. Lett.* B273 (1991) 255; D. Sudarsky and R.M. Wald, *Phys. Rev.* D46 (1992) 1453.
5. J.A. Smoller, A.G. Wasserman, S.T. Yau and J.B. McLeod, *Comm. Math. Phys.* 143 (1991) 115.
6. J.A. Smoller and A.G. Wasserman, *Comm. Math. Phys.* 151 (1993) 303; J.A. Smoller, A.G. Wasserman and S.T. Yau, *Comm. Math. Phys.* 154 (1993) 377.
7. P. Breitenlohner, P. Forgács and D. Maison, *On Static Spherically Symmetric Solutions of the Einstein-Yang-Mills Equations*, Preprint, MPI-Ph/93-41 (1993).
8. G. Lavrelashvili and D. Maison, *Phys. Lett.* B295 (1992) 67.
9. P. Bizon, *Phys. Rev.* D47 (1993) 1656.
10. G. Lavrelashvili and D. Maison, *Regular and Black Hole Solutions of Einstein-Yang-Mills-Dilaton Theory*, Preprint, MPI-Ph/92-115 (1992) to arrear in *Nucl. Phys.* B (1993).
11. E.E. Donets and D.V. Gal’tsov, *Phys. Lett.* B302 (1993) 411.
12. P. Bizon, *Acta Physica Polonica* B24 (1993) 1209.
13. T. Torii and K. Maeda, *Black Holes with Non-Abelian Hair and their Thermodynamical Properties*, Waseda University preprint, WU-AP/28/93.
14. M.B. Green, J.H. Schwarz and E. Witten *Superstring Theory*, Cambridge U.P., Cambridge, 1987.
15. S. Coleman, in: *New Phenomena in Subnuclear Physics*, ed. A. Zichichi, Plenum, NY, 1975;
   S. Deser, *Phys. Lett.* B64 (1976) 463.
16. P. Breitenlohner, D. Maison and G. Gibbons, *Comm. Math. Phys.* 120 (1988) 295.
17. P. Dobiasch and D. Maison, *Gen. Rel. and Grav.* **14** (1982) 231.
18. G.W. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741.