High energy cosmic rays experiments inspired by noncommutative quantum field theory

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Phenomenological analysis of the covariant \( \theta \)-exact noncommutative (NC) gauge field theory (GFT), inspired by high energy cosmic rays experiments, is performed in the framework of the inelastic neutrino-nucleon scatterings, plasmon and \( Z \)-boson decays into neutrino pair, the Big Bang Nucleosynthesis (BBN) and the Reheating Phase After Inflation (RPAI), respectively. Next we have have found neutrino two-point function and shows a closed form decoupling of the hard ultraviolet (UV) divergent term from softened ultraviolet/infrared (UV/IR) mixing term and from the finite terms as well. For a certain choice of the noncommutative parameter \( \theta \) which preserves unitarity, problematic UV divergent and UV/IR mixing terms vanish. Non-perturbative modifications of the neutrino dispersion relations are asymptotically independent of the scale of noncommutativity in both the low and high energy limits and may allow superluminal propagation.

INTRODUCTION

String theory indicated that noncommutative gauge field theory (NCQFT) could be one of its low-energy effective theories [1]. Studies on noncommutative particle phenomenology [2, 3] was motivated to find possible experimental signatures and/or predict/estimate bounds on space-time noncommutativity from collider physics experimental data: for example from the Standard Model (SM) invisible part of \( Z \rightarrow \nu \nu \) decays, and more important from the ultra high energy (UHE) processes occurring in the framework of the cosmic-ray neutrino physics. Constraint on the scale of the NCGFT, \( \Lambda_{\text{NC}} \), is possible due to a direct coupling of neutrinos to photons.

Significant progress has been obtained in the so-called Seiberg-Witten (SW) maps [1] and enveloping algebra based models where one could deform commutative gauge theories with arbitrary gauge group and representation [4,10]. In our construction the noncommutative fields are obtained via SW maps from the original commutative fields. It is commutative instead of the noncommutative gauge symmetry that is preserved as the fundamental symmetry of the theory. The constraints on the \( U_\chi(1) \) charges, stated as “no-go theorem” [11], are also rescinded in our approach [12], and the noncommutative extensions of particle physics covariant SM (NCSM) and the noncommutative grand unified theories (NCGUT) models [10, 12, 18] were constructed. These allow a minimal deformation with no new particle content and with the sacrifice that interactions include infinitely many terms defined through recursion over the NC parameter \( \theta^{\mu\nu} \); in practice cut-off at certain \( \theta \)-order.

In a simple model of NC spacetime local coordinates \( x^\mu \) are promoted to hermitian operators \( \hat{x}^\mu \) satisfying space-time NC and implying uncertainty relations

\[
[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \rightarrow |\Delta x^\mu \Delta x^\nu| \geq \frac{1}{2} |\theta^{\mu\nu}|, \tag{1}
\]

where \( \theta^{\mu\nu} \) is real, antisymmetric matrix. The Moyal-Weyl \( \ast \)-product, relevant for the case of a constant \( \theta^{\mu\nu} \), is defined as follows:

\[
(f \ast g)(x) = e^{\frac{i\hbar}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}} f(x) g(y) \bigg|_{y \rightarrow x}. \tag{2}
\]

The operator commutation relation (11) is then realized by the so-called \( \ast \)-commutator

\[
[\hat{x}^\mu, \hat{x}^\nu] = [x^\mu, x^\nu] = i\theta^{\mu\nu}. \tag{3}
\]

The perturbative quantization of noncommutative field theories was first proposed in a pioneering paper by Filk [19]. Other famous examples are the running of the coupling constant of NC QED [20] and the UV/IR mixing [21, 22]. Later well behaving one-loop quantum corrections to noncommutative scalar \( \phi^4 \) theories [23, 24] and the NC QED [20] have been found. Also the SW expanded NCSM [11, 12, 13, 15, 17] at first order in \( \theta \), albeit breaking Lorentz symmetry is anomaly free [27, 28], and has well-behaved one-loop quantum corrections [20–22, 23, 37]. However, despite of some significant progress in the models [23, 37], a better understanding of various models quantum loop corrections still remains in general a challenging open question. This fact is particularly true for the models constructed by using SW map expansion in the NC parameter \( \theta \), [5, 11, 10, 38, 39]. Resulting models are very useful as effective field theories including their one-loop quantum properties [27, 37] and relevant phenomenology [40–47].

Discussions on the C,P,T, and CP properties of the noncommutative interactions are given in [41], and in particular in [10]. For example, fixing \( \theta \) spontaneously breaks C, P, and/or CP discrete symmetries [16]. A breaking of C symmetry occurs in \( Z \rightarrow \gamma \gamma \) process. One common approximation in those existing works is that only the vertices linear in terms of the NC parameter \( \theta \) were used.

Recently, \( \theta \)-exact SW map and enveloping algebra based theoretical models were constructed in the framework of covariant noncommutative quantum gauge field
theory and applied in loop computation and to the phenomenology, as well.

At \( \theta \)-order there are two important interactions that are suppressed and/or forbidden in the SM, the triple neutral gauge boson, and the tree level coupling of neutrinos with photons, respectively. Here an expansion and cut-off in powers of the NC parameters \( \theta^{\mu\nu} \) corresponds to an expansion in momenta and restrict the range of validity to energies well below the NC scale \( \Lambda_{NC} \). Usually, this is no problem for experimental predictions because the lower bound on the NC parameters \( \theta^{\mu\nu} = e^{\mu\nu} / \Lambda_{NC}^2 \) (the coefficients \( e^{\mu\nu} \) running between zero and one) runs higher than typical momenta involved in a particular process. However, there are exotic processes in the early universe as well as those involving ultra high energy cosmic rays, in which the typical energy involved is higher than the current experimental bound on the NC scale \( \Lambda_{NC} \). Thus, the previous \( \theta \)-cut-off approximate results are inapplicable. To cure the cut-off approximation, we are using \( \theta \)-exact expressions, inspired by exact formulas for the SW map, and expand in powers of gauge fields, as we did in [53]. In \( \theta \)-exact models we have studied the UV/IR mixing, the neutrino propagation, and also some NC photon-neutrino phenomenology, respectively. Due to the presence of the UV/IR mixing the \( \theta \)-exact model is not perturbatively renormalizable, thus the relations of quantum corrections to the observables are not entirely clear.

In this work we present NCSM extended neutrino gauge bosons actions to all orders of \( \theta \) and study quantum properties: Neutrino two point function. Finally we discuss the decay width \( \Gamma(Z \to \nu\nu) \) as functions of the NC scale \( \Lambda_{NC} \) for light-like noncommutativity which are allowed by unitarity condition.

**UHE COSMIC RAY MOTIVATION**

Direct coupling of gauge bosons to neutral and “chiral” fermion particles, via \( \gamma \)-commutator in the NC background, which plays the role of an external field in the theory, allow us to estimate a constraint on the scale of the noncommutative gauge field theory, \( \Lambda_{NC} \), arising from ultra-high energy cosmic ray experiments involving \( \nu \)-nucleon inelastic cross section, see i.e. Fig. 1.

The observation of ultra-high energy (UHE) \( \nu \)'s from extraterrestrial sources would open a new window to look to the cosmos, as such \( \nu \)'s may easily escape very dense material backgrounds around local astrophysical objects, giving thereby information on regions that are otherwise hidden to any other means of exploration. In addition, \( \nu \)'s are not deflected on their way to the earth by various magnetic fields, pointing thus back to the direction of distant UHE cosmic-ray source candidates. This could

![FIG. 1: Diagrams contributing to \( \nu N \to \nu + X \) processes.](image1)

![FIG. 2: \( \nu N \to \nu + \text{anything} \) cross sections vs. \( \Lambda_{NC} \) for \( E_\nu = 10^{10} \text{ GeV} \) (thick lines) and \( E_\nu = 10^{13} \text{ GeV} \) (thin lines). FKRT and PJ lines are the upper bounds on the \( \nu \)-nucleon inelastic cross section, denoting different estimates for the cosmogenic \( \nu \)-flux. SM denotes the SM total (charged current plus neutral current) \( \nu \)-nucleon inelastic cross section. The vertical lines denote the intersections of our curves with the RICE results.](image2)

![FIG. 3: The intersections of our curves with the RICE results (cf. Fig.2) as a function of the fraction of Fe nuclei in the UHE cosmic rays. The terminal point on each curve represents the highest fraction of Fe nuclei above which no useful information on \( \Lambda_{NC} \) can be inferred with our method.](image3)
also help resolving the underlying acceleration in astrophysical sources.

In the energy spectrum of UHE cosmic rays at $\sim 4 \times 10^{19}$ eV the GZK-structure has been observed recently with high statistical accuracy [60]. Thus the flux of the so-called cosmogenic $\nu$’s, arising from photopion production on the cosmic microwave background $\gamma\nu_{\text{CMB}} \rightarrow \Delta^{\ast} \rightarrow N\pi$ and subsequent pion decay, is now guaranteed to exist. Possible ranges for the size of the flux of cosmogenic $\nu$’s can be obtained from separate analysis of the data from various large-scale observatories [61, 62].

Note that there is the uncertainty in the flux of cosmogenic $\nu$’s regarding the chemical composition of UHE cosmic rays (for details see [62]). Using the upper bound on the $\nu N$ cross section derived from the RICE Collaboration search results [63] at $E_\nu = 10^{11}$ GeV ($4 \times 10^{-3}$ mb for the FKRT $\nu$-flux [61]), one can infer from $\theta$-truncated model on the NC scale $\Lambda_{\text{NC}}$ to be greater than 455 TeV, a really strong bound. Here we have $\theta_{\mu\nu} = e_{\mu\nu}/\Lambda_{\text{NC}}^2$, such that the matrix elements of $c$ are of order one. One should however be careful and suspect this result as it has been obtained from the conjecture that the $\theta$-expansion stays well-defined in the kinematical region of interest. Although a heuristic criterion for the validity of the perturbative $\theta$-expansion, $\sqrt{s}/\Lambda_{\text{NC}} \lesssim 1$, with $s = 2E_\nu M_N$, would underpin our result on $\Lambda_{\text{NC}}$, a more thorough inspection on the kinematics of the process does reveal a more stronger energy dependence $E_\nu^{1/3}/\Lambda_{\text{NC}} \lesssim 1$. In spite of an additional phase-space suppression for small $x$’s in the $\theta^2$-contribution [40] of the cross section relative to the $\theta$-contribution, we find an unacceptably large ratio $\sigma(\theta^2)/\sigma(\theta) \approx 10^4$, at $\Lambda_{\text{NC}} = 455$ TeV. Hence, the bound on $\Lambda_{\text{NC}}$ obtained this way is incorrect, and our last resort is to modify the model adequately to include the full-$\theta$ resummation, thereby allowing us to compute nonperturbatively in $\theta$.

Total cross section, as a function of the NC scale at fixed $E_\nu = 10^{10}$ GeV and $E_\nu = 10^{11}$ GeV, together with the upper bounds depending on the actual size of the cosmogenic $\nu$-flux (FKRT [61] and PJ [62]) as well as the total SM cross sections at these energies, are depicted in our Figure 2. In order to maximize the NC $\theta$-exact effect we choose $c_{01} - c_{13} = c_{02} - c_{23} = c_{03} = 1$.

Even if the future data confirm that UHE cosmic rays are composed mainly of Fe nuclei, as indicated by the PAO data, then still valuable information on $\Lambda_{\text{NC}}$ can be obtained with our method, as seen in Fig 3. Here we see the intersections of our curves with the RICE results (cf. Fig 2) as a function of the fraction $\alpha$ of Fe nuclei in the UHE cosmic rays. On top of results, presented in Figs 2 and 3 we also have the NC scale given as a function of the plasmon frequency, from the plasmon decay into neutrino pairs $\gamma_{\text{pl}} \rightarrow \nu\bar{\nu}$ (Fig 4), and as a function of the $T_{\text{dec}}$ from BBN (Fig 5), respectively. All results depicted in Figs 2, 3, 4, 5 shows convergent behavior. In our opinion those were the strong signs to continue research towards quantum properties and phenomenology of such $\theta$-exact noncommutative gauge field theory model.

### CONSISTENCY OF THE SW MAP AND ENVELOPING ALGEBRA APPROACH TO NCGFT

The choice of gauge group appears to be severely restricted in a noncommutative setting [1]: The star commutator of two Lie algebra valued gauge fields will involve the anti-commutator as well as the commutator of the Lie algebra generators. The algebra still closes for Hermitian matrices, but it is for instance not possible to impose the trace to be zero. This observation can be interpreted in two ways:

(a) The choice of gauge group is restricted to U(N) in the fundamental, anti-fundamental or adjoint represen-
(b) the gauge fields are valued in the enveloping algebra of a Lie algebra and then any (unitary) representation is possible.

The case (a) applies also to the U(1) case and imposes severe restrictions on the allowed charges; it has been studied carefully and has led to “theorems”\(^\text{[64, 65]}\). The second case avoids both the gauge group and the U(1) gauge parameter, but needs to address the potential problem of too many degrees of freedom, since all coefficient functions of the monomials in the generators could a priori be physical fields. The solution to this problem is that the coefficient fields are not all independent. They are rather functions of the correct number of ordinary gauge fields via Seiberg-Witten maps and their generalizations.

The situation is reminiscent of the construction of superfields and supersymmetric actions in terms of ordinary fields in supersymmetry. This method, referred to as Seiberg-Witten map or enveloping algebra approach avoids both the gauge group and the U(1) charge invariance. Thus the classical charge $q$ is split into left and right charges $q = q^L - q^R$, as we have seen above.

**COVARIANT $\theta$-EXACT U(1) MODEL**

We start with the following SW type of NC U(1) gauge model:

$$ S = \int \left[ -\frac{1}{4} F_{\mu\nu} \ast F^{\mu\nu} + i \bar{\Psi} \partial \Psi \right]. $$

with the NC definitions of the nonabelian field strength and the covariant derivative, respectively:

$$ F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - i [A_{\mu} \ast A_{\nu}], $$

$$ D_{\mu} \Psi = \partial_{\mu} \Psi - i [A_{\mu} \ast \Psi]. $$

All noncommutative fields in this action ($A_{\mu}, \Psi$) are images under (hybrid) Seiberg-Witten maps of the corresponding commutative fields ($a_{\mu}, \psi$). Here we shall interpret the NC fields as valued in the enveloping algebra of the underlying gauge group. This naturally corresponds to an expansion in powers of the gauge field $a_{\mu}$ and hence in powers of the coupling constant $\epsilon$. At each order in $a_{\mu}$ we shall determine $\theta$-exact expressions.

In the next step we expand the action in terms of the commutative gauge parameter $\lambda$ and fields $a_{\mu}$ and $\psi$ using the SW map solution\(^\text{[48]}\) up to the $\mathcal{O}(a^3)$ order:

$$ \Lambda = \lambda - \frac{1}{2} \theta^{ij} a_i \ast_2 \partial_j \lambda, $$

$$ A_{\mu} = a_{\mu} - \frac{1}{2} \theta^{\alpha\beta} a_\alpha \ast_2 (\partial_\mu a_\beta + f_{\mu\beta}), $$

$$ \Psi = \psi - \theta^{\alpha\beta} a_\beta \ast_2 \partial_\alpha \psi $$

$$ + \frac{1}{2} \theta^{\mu\nu} \theta^{\alpha\beta} \left[ a_\alpha \ast_2 (\partial_\mu a_\nu + f_{\mu\nu}) \right] \ast_2 \partial_\beta \psi $$

$$ + 2 a_\mu \ast_2 (\partial_\mu (a_\nu \ast_2 \partial_\nu \psi)) - a_\mu \ast_2 (\partial_\mu a_\nu \ast_2 \partial_\nu \psi) $$

$$ - \left( a_\mu \partial_\nu \psi (\partial_\nu a_\sigma + f_{\nu\sigma}) - \partial_\mu \partial_\nu \psi a_\sigma \ast_2 \partial_\sigma \right) a_\sigma, $$

with $\Lambda$ being the NC gauge parameter and $f_{\mu\nu}$ is the abelian commutative field strength $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. 

$$ \hat{L}_{\text{gauge}} = -\frac{1}{4g^2} \text{tr} \left( \hat{F}_{\mu\nu} \ast \hat{F}^{\mu\nu} + \hat{F}^R_{\mu\nu} \ast \hat{F}^{\mu\nu} \right), $$

with $g := e \sqrt{\text{tr}(Q^L)^2 + \text{tr}(Q^R)^2}$. In\(^\text{[12]}\) we have employed this construction on deformed Yukawa couplings. Namely, in the Yukawa terms, a star product deformation would prevent the charge summation. The hybrid SW map\(^\text{[10, 69]}\) is introduced to recover gauge invariance.

Thus the classical charge $q$ is split into left and right charges $q = q^L - q^R$, as we have seen above.

$$ \delta \hat{\Phi} = i \hat{\Lambda}^L \ast \hat{\Phi} - i \hat{\Phi} \ast \hat{\Lambda}^R. $$

Using the associativity of the star product one can easily verify the formal consistency relation

$$ [\delta_{\hat{\Lambda}}, \delta_{\hat{\Sigma}}] \hat{\Phi} = [i \hat{\Lambda}^L \ast i \hat{\Sigma}^L] \hat{\Phi} - \hat{\Phi} \ast [i \hat{\Lambda}^R \ast i \hat{\Sigma}^R]. $$

Therefore the noncommutative gauge transformations $\hat{\Lambda}^{L/R}$ can be constructed from the classical fields and parameters $A_{\mu}^{L/R} = a_{\mu}(x) Q_{L/R}^{L/R}$ and $\lambda(x) Q_{L/R}^{L/R}$ with $Q_{L/R}^{L/R} = \text{diag}(q_{1L/R}^{L/R}, q_{2L/R}^{L/R}, q_{3L/R}^{L/R})$ and $q_{i} = \delta_{i}^{L} - \delta_{i}^{R}$ by so-called hybrid Seiberg-Witten maps\(^\text{[10, 69]}\). The hybrid covariant derivative is given by $D_{\mu} \hat{\Phi} = \partial_{\mu} \hat{\Phi} - i \hat{A}_{\mu} \ast \hat{\Phi} + i \hat{\Phi} \ast \hat{A}_{\mu}$. 

$$ \hat{A}_{\mu} \hat{\Phi} = \partial_{\mu} \hat{\Phi} - i \hat{A}_{\mu} \ast \hat{\Phi} + i \hat{\Phi} \ast \hat{A}_{\mu}. $$

Thanks to\(^\text{[6]}\) the left and right NC gauge fields $\hat{A}_{\mu}^{L/R}$ are constructed from $\hat{A}_{\mu}^{L/R}$ only, respectively. The gauge field action could be written as

$$ \hat{L}_{\text{gauge}} = -\frac{1}{4g^2} \text{tr} \left( \hat{F}_{\mu\nu} \ast \hat{F}^{\mu\nu} + \hat{F}^R_{\mu\nu} \ast \hat{F}^{\mu\nu} \right). $$

The case (a) applies also to the U(1) case and imposes severe restrictions on the allowed charges; it has been studied carefully and has led to “theorems”\(^\text{[64, 65]}\).
The generalized Mojal-Weyl star products \( \star_2 \) and \( \star_3 \), appearing in [54], are defined, respectively, as

\[
f(x) \star_2 g(x) = \left[ f(x) \ast g(x) \right] = \frac{\sin \theta/3 \partial \theta/3}{\theta/3} f(x_1)g(x_2) \bigg|_{x_1 = x_2 = x},
\]

(10)

\[
(f(x)g(x)h(x))_{\star_3} = \left( \frac{\sin \theta/2 \partial \theta/2}{\theta/2} \left( \sin \frac{\partial \theta(\partial_2 + \partial_3)}{2} \right) \right)
+ \{1 \leftrightarrow 2\} f(x_1)g(x_2)h(x_3) \bigg|_{x_1 = x},
\]

(11)

where \( \star \) is associative but noncommutative, while \( \star_2 \) and \( \star_3 \) are both commutative but nonassociative.

The resulting expansion defines \( \theta \)-exact neutrino-photon \( U_\nu \) actions, for a gauge and a matter sectors respectively. Pure gauge field (3-photon) action reads:

\[
S_g = \int i \partial_\mu a_\nu \ast [a^\mu \ast a^\nu] + \frac{1}{2} \partial_\mu (\theta^{\mu \nu} a_\rho \ast_2 (\partial_\rho a_\nu + f_{\nu \rho}) \ast f^{\rho \nu}) \ast f^{\mu \nu}.
\]

(12)

The photon-fermion action up to 2-photon 2-neutrino fields can be derived by using the first order gauge field and the second order neutrino field expansions,

\[
S_f = \int \left( \bar{\psi} + (\theta^{\mu \nu} \partial_\mu \psi \cdot \star_2 a_j) \right) \gamma^\mu [a_\mu \ast \psi] + i (\theta^{\mu \nu} \partial_\mu \psi \ast_2 a_j) \partial_\nu \psi - i \bar{\psi} \ast \theta \partial_\nu a_\rho \ast_2 (\partial_\rho a_\nu + f_{\nu \rho})
+ \bar{\psi} \gamma^\mu \left[ \frac{1}{2} \theta^{\mu \nu} a_\rho \ast_2 (\partial_\rho a_\nu + f_{\nu \rho}) \ast \psi \right] + i (\theta^{\mu \nu} \partial_\mu \psi \ast_2 a_j) \theta_\rho a_\rho \ast \partial_\nu \psi
+ \frac{i}{2} \theta^{\mu \nu} g^{kl} \left[ \theta a_k \ast_2 (\partial_\rho a_\nu + f_{\nu \rho}) \ast_2 (\partial_\rho \psi) \right] \gamma^\mu \partial_\nu \psi + 2a_i \ast_2 (\partial_\nu a_i \ast_2 \partial_\nu \psi) - a_i \ast_2 (\partial_\nu a_i \ast_2 \partial_\nu \psi)
+ (a_i \partial_\nu \psi \partial_\nu a_i + f_{\nu \rho}) - \partial_\nu \partial_\rho \psi a_i a_i \ast_3 \gamma^\mu \partial_\nu \psi
+ \frac{i}{2} \theta^{\mu \nu} g^{kl} \bar{\psi} \ast_2 \left[ \theta a_k \ast_2 (\partial_\rho a_i + f_{\rho i}) \ast_2 (\partial_\nu a_i) \ast_2 \partial_\nu \psi \right] + 2a_i \ast_2 (\partial_\nu a_i \ast_2 \partial_\nu \psi) - a_i \ast_2 (\partial_\nu a_i \ast_2 \partial_\nu \psi)
+ (a_i \partial_\nu \psi (\partial_\nu a_i + f_{\nu \rho}) - \partial_\nu \partial_\rho \psi a_i a_i \ast_3 \gamma^\mu \partial_\nu \psi)
\]

(13)

Note that actions for gauge and matter fields obtained above, [12] and [13] respectively, are nonlocal objects due to the presence of the star products: \( \ast, \ast_2 \) and \( \ast_3 \).

Feynman rules from above actions, represented in Fig. 6, are given explicitly in [54].

As depicted in Fig. 7 there are four Feynman diagrams contributing to the \( \nu \)-self-energy at one-loop. With the aid of (13), we have verified by explicit calculation that the 4-field tadpole \( (\Sigma_1) \) does vanish. The 3-fields tadpoles \( (\Sigma_3 \text{ and } \Sigma_4) \) can be ruled out by invoking the NC charge conjugation symmetry [16]. Thus only the \( \Sigma_1 \) diagram needs to be evaluated. In spacetime of the di-
mensionality $D$ we obtain

\[
\Sigma_1 = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \left( \sin \frac{\theta p}{2} \right)^2 \frac{1}{q^2 (p + q)^2} \cdot \left[ (q\theta p)^2 (4 - D)(\phi + \phi) + (q\theta p) (\hat{p}(2p^2 + 2p \cdot q) - \phi(2q^2 + 2p \cdot q)) + \left( \phi(q^2 (p^2 + 2p \cdot q) - q^2 (p^2 + 2p \cdot q)) + \phi(\hat{p}(p^2 + 2p \cdot q) - p^2 (q^2 + 2p \cdot q)) \right) \right]
\]

where $\hat{p}^\mu = (\theta p)^\mu = \theta^{\mu\nu} p_\nu$, and in addition $\hat{p}^\mu = (\theta p)^\mu = \theta^{\mu\nu} \theta_{\nu\rho} p^\rho$. To perform computations of those integrals using the dimensional regularization method, we first use the Feynman parametrization on the quadratic denominators, then the Heavy Quark Effective theory (HQET) parametrization \[51\] is used to combine the quadratic and linear denominators. In the next stage we use the Schr"{o}dinger parametrization to turn the denominators into Gaussian integrals. Evaluating the relevant integrals for $D = 4 - \epsilon$ in the limit $\epsilon \to 0$, we obtain the closed form expression for the self-energy

\[
\Sigma_1 = \gamma_5 \left[ p^\mu A + (\theta p)^\mu \left( \frac{p^2}{(\theta p)^2} B \right) \right],
\]

\[
A = -\frac{1}{(4\pi)^2} \left[ p^2 \left( \frac{\text{tr} \theta p - 2(\theta p^2)}{\theta p} \right) A_1 + \left( 1 + p^2 \left( \frac{\text{tr} \theta p - 2(\theta p^2)}{\theta p} \right) A_2 \right) \right],
\]

\[
A_1 = \frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) + \ln(\pi \epsilon \gamma_{\text{E}}) + \sum_{k=1}^{\infty} \frac{(p^2(\theta p)^2/4)^k}{\Gamma(2k + 2)} \left( \ln \frac{p^2(\theta p)^2}{4} + 2\psi_0(2k + 2) \right),
\]

\[
A_2 = -\frac{(4\pi)^2}{2} B = -2
\]

\[
\sum_{k=0}^{\infty} \frac{(p^2(\theta p)^2/4)^{k+1}}{(2k + 1)(2k + 2)\Gamma(2k + 2)} \left( \ln \frac{p^2(\theta p)^2}{4} + 2\psi_0(2k + 2) - \frac{8(k + 1)}{(2k + 1)(2k + 3)} \right),
\]

with $\gamma_{\text{E}} \simeq 0.577216$ being Euler’s constant.

The $1/\epsilon$ UV divergence could in principle be removed by a properly chosen counterterm. However due to the specific momentum-dependent coefficient in front of it, a nonlocal form for it is required.

**UV/IR mixing**

Turning to the UV/IR mixing problem, we recognize a soft UV/IR mixing term represented by a logarithm,

\[
\Sigma_{\text{UV/IR}} = \tilde{\rho} \frac{1}{(4\pi)^2} \left( \ln \frac{1}{\mu^2(\theta p)^2} \right) \left( \frac{\text{tr} \theta p - 2(\theta p)^2}{\theta p} \right).
\]

Instead of dealing with nonlocal counterterms, we take a different route here to cope with various divergences besetting \[15\]. Since $\theta_{0i} \neq 0$ makes a NC theory nonunitary \[58\], we can, without loss of generality, chose $\theta$ to lie in the $(1, 2)$ plane

\[
\theta_{\mu\nu} = \frac{1}{\Lambda_{\text{NC}}} \left(\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).
\]

Automatically, this produces

\[
\frac{\text{tr} \theta p}{(\theta p)^2} + 2(\theta p)^2 = 0, \forall p.
\]

With \[21\], $\Sigma_1$, in terms of Euclidean momenta, receives the following form:

\[
\Sigma_1 = -\frac{1}{(4\pi)^2} \gamma_\mu \left[ p^\mu \left( 1 + \frac{\text{tr} \theta p}{2(\theta p)^2} \right) - 2(\theta p)^2 \frac{p^2}{(\theta p)^2} \right] A_2.
\]

By inspecting \[18\] one can be easily convinced that $A_2$ is free from the $1/\epsilon$ divergence and the UV/IR mixing term, being also well-behaved in the infrared, in the $\theta \to 0$ as well as $\theta p \to 0$ limit. We see, however, that the two terms in \[22\], one being proportional to $\tilde{\rho}$ and the other proportional to $\tilde{\phi}$, are still ill-behaved in the $\theta p \to 0$ limit. If, for the choice \[20\], $P$ denotes the momentum in the $(1, 2)$ plane, then $\theta = \theta P$. For instance, a particle moving inside the NC plane with momentum $P$ along the one axis, has a spatial extension of size $|\theta P|$ along the other. For the choice \[20\], $\theta P \to 0$ corresponds to a zero momentum projection onto the $(1, 2)$ plane. Thus, albeit in our approach the commutative limit ($\theta \to 0$) is smooth at the quantum level, the limit when an extended object (arising due to the fuzziness of space) shrinks to zero, is not. We could surely claim that in our approach the UV/IR mixing problem is considerably softened; on the other hand, we have witnessed how the problem strikes back in an unexpected way. This is, at the same time, the first example where this two limits are not degenerate.

**Neutrino dispersion relations**

In order to probe physical consequence of the 1-loop quantum correction, with $\Sigma_{1-\text{loop}}$ from Eq. (3.25) in \[50\], we consider the modified propagator

\[
\frac{1}{\Sigma} = \frac{1}{\tilde{\rho} - \Sigma_{1-\text{loop}}} = \frac{\Sigma}{\Sigma^2}.
\]
We further choose the NC parameter to be \( \Lambda_{NC} \) so that the denominator is finite and can be expressed explicitly:

\[
\Sigma^2 = p^2 \left[ \tilde{A}_2 \left( \frac{p^4}{p_r^4} + 2 \frac{p^2}{p_r^2} + 5 \right) - \tilde{A}_2 \left( 6 + 2 \frac{p^2}{p_r^2} \right) + 1 \right],
\]

where \( p_r \) represents \( r \)-component of the momentum \( p \) in a cylindrical spatial coordinate system and \( \tilde{A}_2 = e^2 A_2/(4\pi)^2 = -B/2 \).

From above one see that \( p^2 = 0 \) defines one set of the dispersion relation, corresponding to the dispersion for the massless neutrino mode, however the denominator \( \Sigma^2 \) has one more coefficient \( \Sigma' \) which could also induce certain zero-points. Since the \( \tilde{A}_2 \) is a function of a single variable \( p^2/p_r^2 \), with \( p^2 = p_0^2 - p_1^2 - p_2^2 - p_3^2 \) and \( p_r^2 = p_1^2 + p_2^2 \), the condition \( \Sigma' = 0 \) can be expressed as a simple algebraic equation

\[
\tilde{A}_2^2 z^2 - 2 \left( \tilde{A}_2 - \tilde{A}_2^2 \right) z + \left( 1 - 6 \tilde{A}_2 + 5 \tilde{A}_2^2 \right) = 0,
\]

of new variables \( z := p^2/p_r^2, \) in which the coefficients are all functions of \( y := p^2/\Lambda_{NC}^2 \).

The two formal solutions of the equation \( \Sigma \)

\[
z = \frac{1}{\tilde{A}_2} \left[ (1 - \tilde{A}_2) \pm 2 \left( \tilde{A}_2 - \tilde{A}_2^2 \right)^{1/2} \right],
\]

are birefringent. The behavior of solutions \( \Sigma \), is next analyzed at two limits \( y \to 0, \) and \( y \to \infty. \)

**The low-energy regime:** \( p^2 p_r^2 \ll \Lambda_{NC}^4 \)

For \( y \ll 1 \) we set \( \tilde{A}_2 \) to its zeroth order value \( e^2/2\pi^2, \)

\[
p^2 \sim \left( \frac{8\pi^2}{e^2} - 1 \right) \pm 2 \left( \frac{8\pi^2}{e^2} - 1 \right)^{1/2} \cdot p_r^2,
\]

obtaining two (approximate) zero points. From the definition of \( p^2 \) and \( p_r^2 \) we see that both solutions are real and positive. Taking into account the higher order (in \( y \)) correction these poles will locate nearby the real axis of the complex \( p_0 \) plane thus correspond to some metastable modes with the above defined dispersion relations. As we can see, the modified dispersion relation \( \Sigma \) does not depend on the noncommutative scale, therefore it introduces a discontinuity in the \( \Lambda_{NC} \to \infty \) limit, which is not unfamiliar in noncommutative theories.

**The high-energy regime:** \( p^2 p_r^2 \gg \Lambda_{NC}^4 \)

At \( y \gg 1 \) we analyze the asymptotic behavior of

\[
\tilde{A}_2 \sim \frac{\pi^2}{8} \sqrt{y} \left( 1 - \frac{16i}{\pi y} e^{-\frac{\pi}{2} \sqrt{y}} \right) + \mathcal{O}(y^{-1}),
\]

from \( \Sigma \), therefore \( \Sigma \) can be reduced to

\[
z \sim -1 \pm 2i \rightarrow p_0^2 \sim p_3^2 \pm 2ip_r^2.
\]

We thus reach two unstable deformed modes besides the usual mode \( p^2 = 0 \) in the high energy regime. Here again the leading order deformed dispersion relation does not depend on the noncommutative scale \( \Lambda_{NC} \).

**The alternative action self-energy**

Using the Feynman rule of the alternative action (2.15) from Ref. [50], which is a consequence of the SW freedom, we find the following contribution to the neutrino self-energy from diagram \( \Sigma_1 \)

\[
\Sigma_{\text{alt}} = \frac{g}{3} \frac{1}{\pi^2} \left( \frac{1}{(\theta p)^2} \right) \left( \text{tr} \theta \theta - \text{tr} \theta \theta \right) \left( \frac{(\theta p)^2}{\Lambda_{NC}^4} + \frac{1}{\Lambda_{NC}^4} \right).
\]

The detailed computation is presented in Appendix B of Ref. [50]. We notice that there are no hard 1/\( \epsilon \) UV divergent and no logarithmic UV/IR mixing terms, and the finite terms like in \( A_1 \) and \( A_2 \) are also absent. Thus the subgraph \( \Sigma_1 \) for the alternative action (2.15) in [50] does not require any counter-term. However, the result \( \Sigma_{\text{alt}} \) does express powerful UV/IR mixing effect, that is in terms of scales terms, the \( \Sigma_{\text{alt}} \) experience the forth-power of the \( \Lambda_{NC}/\text{momentum-scale} \) ratios \( \sim |p|^{-2} |\theta p|^{-2} \) in [20], i.e. we are dealing with the \( \Sigma_{\text{alt}} \sim p \Lambda_{NC}^4/\Lambda_{NC}^4 \) within the ultraviolet and infrared limits for \( \Lambda_{NC} \) and \( p \), respectively.

**PHENOMENOLOGY: \( Z \to \nu \bar{\nu} \) DECAY RATE**

To illustrate another phenomenological effects of our \( \theta \)-exact construction, we present a computation the \( Z \to \nu \bar{\nu} \) decay rate in the \( Z \)-boson rest frame, which is then readily to be compared with the precision Z resonance measurements, where \( Z \) is almost at rest. Since the complete \( Z \nu \nu \) interaction on noncommutative spaces was discussed in details in [12], we shall not repeat it here. We only give the almost complete \( Z \nu \bar{\nu} \) vertex from [12]

\[
\Gamma^\mu(p',p) = i \frac{g}{2 \cos \theta_W} \left( \gamma^\mu + \frac{i}{2} F_\nu \gamma^{\nu} \right) \left( \frac{1}{2} - \gamma_5 \right) \frac{1}{2} \right)
\]

\[
+ \frac{\alpha}{2} \tan \theta_W F_{\mu z} \left( \gamma^\mu - \gamma^\nu \right) \left( \gamma^\nu - \gamma^\mu \right)
\]

\[
\left( \gamma^\nu - \gamma^\mu \right) \left( \gamma^\mu - \gamma^\nu \right) \left( \gamma^\nu - \gamma^\mu \right)
\]

(30)
where $\kappa$ is an arbitrary constant, and

$$ (p'\theta p) F_{\nu}(p', p) = -2i \left( 1 - \exp \left( i \frac{M_Z p \cos \vartheta}{2 \Lambda_{\text{NC}}^2} \right) \right), $$

$$ (p'\theta p) F_{\nu}(p', p) = -2 \sin \left( \frac{M_Z p}{2 \Lambda_{\text{NC}}} \cos \vartheta \right). \quad (31) $$

Note here that due to the equations of motions, for massless on-shell neutrinos the terms $[(\theta'p')\bar{p} - (\theta p')\bar{p}'] (1 - \gamma_5)$ in the vertex $[30]$ do not contribute to the $Z \rightarrow \nu \bar{\nu}$ amplitude. Thus the vertex $[30]$ has the same form as the SM vertex $\gamma^\mu (g_V - g_A \gamma_5)$ $[71]$ $[72]$ with

$$ g_V = 1 - \frac{1}{2} \exp \left( i \frac{M_Z p \cos \vartheta}{2 \Lambda_{\text{NC}}^2} \right) $$

$$ + 2i\kappa \sin \theta_W \sin \left( \frac{M_Z p \cos \vartheta}{2 \Lambda_{\text{NC}}^2} \right), \quad (32) $$

$$ g_A = 1 - \frac{1}{2} \exp \left( i \frac{M_Z p \cos \vartheta}{2 \Lambda_{\text{NC}}^2} \right). \quad (33) $$

The temporary component $\bar{E}_\theta$ of $\theta$ is reduced from equations above since for the $Z$-boson at rest we have

$$ p'\theta p = -M_Z \bar{p} \cdot \bar{E}_\theta = -\frac{M_Z p \cos \vartheta}{\Lambda_{\text{NC}}^2}. \quad (34) $$

with $|\bar{E}_\theta| = 1/\Lambda_{\text{NC}}^2$ and $\vartheta$ the angle between $\bar{p}$ and $\bar{E}_\theta$.

Using $Z\nu\bar{\nu}$ vertex $[30]$, we obtain the following $Z \rightarrow \nu \bar{\nu}$ partial width $[73]$

$$ \Gamma(Z \rightarrow \nu \bar{\nu}) = \Gamma_{\text{SM}}(Z \rightarrow \nu \bar{\nu}) $$

$$ + \frac{\alpha}{3M_Z |E_\theta|} \left[ \kappa \left( 1 - \kappa + \kappa \cos 2\theta_W \right) \sec^2 \theta_W \cos \left( \frac{M_Z^2 |E_\theta|}{4} \right) \right. $$

$$ - 8 \cos^2 2\theta_W \right] \sin \left( \frac{M_Z^2 |E_\theta|}{4} \right) $$

$$ + \frac{\alpha M_Z^4}{12} \left[ -2\kappa^2 + (\kappa(2\kappa - 1) + 2) \sec^2 \theta_W + 2 \cos^2 \theta_W \right], \quad (35) $$

whose NC part vanishes when $\bar{E}_\theta \rightarrow 0$, i.e. for vanishing $\theta$ or space-like noncommutativity, but not light-like $[58]$, $[59]$.

A comparison of the experimental $Z$ decay width $\Gamma_{\text{invis}} = (499.0 \pm 1.5)$ MeV $[74]$ with its SM theoretical counterpart, allows us to set a constraint $\Gamma(Z \rightarrow \nu \bar{\nu}) - \Gamma_{\text{SM}}(Z \rightarrow \nu \bar{\nu}) \lesssim 1$ MeV, from where a bound on the scale of noncommutativity $\Lambda_{\text{NC}} = |\bar{E}_\theta|^{-1/2} \gtrsim 140$ GeV is obtained (see Fig. 8), for the choice $\kappa = 1$.

**DISCUSSION AND CONCLUSIONS**

We have presented the tree level cosmogenic neutrinos ($\nu'$s) scatterings: $\nu N \rightarrow \nu + \text{anything}$ and particle decays: $((\gamma p, Z) \rightarrow \nu \bar{\nu})$ in the covariant $\theta$-exact noncommutative quantum gauge theory based on Seiberg-Witten maps and enveloping algebra formalism.

In the energy range of interest, $10^{10}$ to $10^{11}$ GeV, where there is always energy of the system ($E$) larger than the NC scale ($E/\Lambda_{\text{NC}} > 1$), the perturbative expansion in terms of $\Lambda_{\text{NC}}$ retains no longer its meaningful character, thus it is forcing us to resort to those NC field-theoretical frameworks involving the full $\theta$-resummation. Our numerical estimations of the contribution to the processes coming from the photon exchange, pins impeccably down a lower bound on $\Lambda_{\text{NC}}$ to be as high as around up to $O(10^6)$ GeV, depending on the cosmogenic $\nu$-flux.

For above analysis it was necessary to use results of $[12]$ which shows explicitly that the “no-go theorem” $[11]$ is certainly not applicable to our SW-map based $\theta$-exact models of the NCGFT. Namely, it is known to be impossible in noncommutative geometry to directly form tensor products from the NC fields as long as there is no additional underlaying mathematical structure. The SW-map based models do however have an additional underlaying mathematical structure: They can be understood as the deformation quantization of ordinary fiber bundles over a Poisson manifold. With this additional structure, tensor products are possible and survive the quantization procedure $[66]$. However, the authors in $[11]$ failed to directly form tensor products of noncommutative fields. The proof of this failure is given in $[12]$.

Now we first discuss $\theta$-exact computation of the one-loop quantum correction to the $\nu$-propagator. We in particular evaluate the neutrino two-point function, and demonstrate how quantum effects in the $\theta$-exact SW map approach to NCGFT’s, together with a combination of Schwinger, Feynman, and HQET parameterization, reveal a much richer structure yielding the one-loop quantum correction in a closed form.

General expression for the neutrino self-energy $[16]$ contains in $[17]$ both a hard $1/\epsilon$ UV term and the UV/IR mixing term with a logarithmic infrared singu-
larity \(\ln|\theta p|\). Results shows complete decoupling of the UV divergent term from softened UV/IR mixing term and from the finite terms as well. Our deformed dispersion relations at both the low and high energies and at the leading order do not depend on the noncommutative scale \(\Lambda_{NC}\). The low energy dispersion relation \((24)\) is, in principle, capable of generating a direction dependent superluminal velocity, this can be seen clearly from the maximal attainable velocity of the neutrinos

\[
\frac{v_{max}}{c} = \frac{dE}{d|\vec{p}|} \sim \sqrt{1 + (859 \pm 59) \sin^2 \vartheta}, \tag{36}
\]

where \(\vartheta\) is the angle with respect to the direction perpendicular to the NC plane. This gives one more example how such spontaneous \(\theta\)-background breaking of Lorentz symmetry could affect the particle kinematics through quantum corrections, even without divergent behavior like UV/IR mixing. On the other hand one can also see that the magnitude of superluminosity is in general very large in our model as a quantum effect, thus seems contradicting various observations which suggests much smaller values \([73 \ldots 77]\). On the other hand, note that the large superluminal velocity issue may also be reduced/removed by taking into account several considerations and/or properties:

1. Selection of a constant nonzero \(\theta\) background in this paper is due to the computational simplicity. The results will, however, still hold for a NC background that is varying sufficiently slowly with respect to the scale of noncommutativity. There is no physics reason to expect \(\theta\) to be a globally constant background ether. In fact, if the \(\theta\) background is only nonzero in tiny regions (NC bubbles) the effects of the modified dispersion relation will be suppressed macroscopically. Certainly a better understanding of possible sources of NC is needed.

2. We have considered only the purely noncommutative neutrino-photon coupling. However, it has been pointed out that modified neutrino dispersion relation could open decay channels within the commutative standard model framework \([78]\). In our case this would further provide decay channel(s) which can bring superluminal neutrinos to normal ones.

3. Note that the model 1 is not the only allowed deformed model with noncommutative neutrino-photon coupling. And as we have shown for our model 2, there could be no modified dispersion relation(s) for deformation(s) other than 1, therefore it is reasonable to conjecture that Seiberg-Witten map freedom may also serve as one possible remedy to this issue.

4. Our results differs with respect to \([64]\) since in our case both terms are proportional to the spacetime noncommutativity dependent \(\theta\)-ratio (the scale-independent structure!) factor in \((21)\), which arise from the natural non-locality of our actions. Besides the divergent terms, a new spinor structure \((\theta\eta p)\) with finite coefficients emerges in our computation, see \([13] \ldots [15]\). All these structures are proportional to \(p^2\), therefore if appropriate renormalization conditions are imposed, the commutative dispersion relation \(p^2 = 0\) can still hold, as a part of the full set of solutions obtained in \((23)\).

5. Finally, we mention that our approach to UV/IR mixing should not be confused with the one based on a theory with UV completion (\(\Lambda_{UV} < \infty\)), where a theory becomes an effective QFT, and the UV/IR mixing manifests itself via a specific relationship between the UV and the IR cutoffs \([79] \ldots [81]\).

From the same actions \([12] \ldots [13]\), but for three different cosmological laboratories, that is from UHE cosmic ray neutrino scatterings on nuclei \([52]\), from the BBN and from the RPAI \([47]\), we obtain very similar, a quite strong bounds on the NC scale, of the order of \(10^6\) GeV. Note in particular that all results depicted in Figs\([25] \ldots 8\) show closed-convergent forms.

All above summarized properties are previously unknown features of \(\theta\)-exact NC gauge field theory. They appear in the model with the action presented in section 4. The alternative action, and the corresponding \(\nu\)-self-energy \([29]\), has less striking features, but it does have it’s own advantages due to the absence of a hard UV divergences, and the absence of complicated finite terms. The structure in \([29]\) is different (it is NC-scale/energy dependent) with respect to the NC scale-independent structure from \([21]\), as well as to the structure arising from fermion self-energy computation in the case of \(\nu\)-product only expanded theories \([64] \ldots [81]\). However, \([21]\) does possesses powerful UV/IR mixing effect. This is fortunate with regard to the use of low-energy NCQFT as an important window to holography \([57]\) and quantum gravity \([82]\).

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