Mathematical analysis of the dynamic behavior three motorcycle disc brakes

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Abstract. The disc brake system is currently used in the motorcycles due to the efficiency in the braking process. In this way, the main objective of this work was to determine mathematically dynamic calculations of the braking system of three different brake discs of different cylinder capacity and, thus, analyze the behavior for optimal operation under different conditions. The geometries of these disc brakes were drawn with the aid of SolidWorks design software. The results obtained show that it was possible to validate the correct functioning of the braking system at different operating conditions using mathematical calculations; also, when the systems that have higher cylinder capacity guarantee better braking distance for a given time and speed. These systems work in optimal conditions of real operation, always guaranteeing high levels of safety and operation compared to other types of geometries.

1. Introduction
Nowadays, the motorcycles used in both wheels disc brake due to the efficiency in the braking process. For a long time, these vehicles used drum brakes, which do not dissipate quickly; the temperature generated because of friction between the disc and bands; also, the maintenance and change of elements were very complicated. On the other hand, the most efficient brake disc is in the high-displacement motorcycles that are expensive, but the cost-benefit is better concerning his performance [1].

The braking process consists of a deceleration phase of the movement of the vehicle (motorcycle); during this process, the heat by friction is generated should be dissipated optimally and quickly to avoid the accumulation of heat that can lead to vaporization of the brake fluid, thermo-elastic deformation at the contact surface, degradation and different types of failures on the surface of the braking track (contact surface). Besides, this heat must be dissipated quickly to the medium to avoid the reduction of the coefficient of friction between the brake and the brake pad [2,3]. Together, the discs suffer mechanical stress all the time, so that wear and deterioration of the structure are generated, to which inadequate maintenance is added, and this the component failure occurs swiftly [4,5]. In other investigations, an analysis was carried out to identify the physical and mechanical characteristics (such as the type of material, geometry, thermal properties, friction, and wear), with the purpose to compare the data obtained with those found in the bibliography, to determine the service life of the brakes under the
conditions to which it is subjected [6,7]. These data are essential for the validation of the correct functioning of braking systems with the support of specific design software.

Melnikov, et al. [8] presented a new method for improving the efficiency of the braking control in motorcycles. The results showed that the negative values of the lateral forces using mathematic methods determinate the lateral sliding of the wheels in contact with the road surface, and thus the loss of vehicle stability; the algorithm is useful due to is able to generate a signal when this phenomenon can occur during the operation of the vehicle. Pires et al. [9] studied the disc brake squeal phenomenon in motorcycles, due to be an indication of possible failures in the braking system. For this research, the authors using the finite element analysis to understanding the properties of the materials in contact under different operational parameters, and therefore evaluate the effect of the young modulus during the friction process on braking; all of this is influenced by several conditions that affect a squeal. For this research, the dynamic analysis of the brake disc for the motorcycles was carried out to obtain an understanding of the influence of the geometries of the brakes with the aid of the mathematical equations; these geometries were drawing in the design software Solidworks, and thus obtain a general visualization of the behavior of this type of system and can propose new geometries with the aid of finite element analysis, due to is important in the automotive industry.

2. Methodology

Practical mathematical Equations were used to estimate the dynamic behavior of three-disc brakes of motorcycles. Generalities during the functioning dynamic of the motorcycles were selected from a specific book, the most representative characteristics of instability generated by disc brake are capsize and heating mainly, note thus the importance of the analysis and evaluation using mathematics estimations and finite element analysis for improving this important system in the automotive industry [10]. On the other hand, Figure 1 shows the methodology applied to the mathematical analysis theories for the brake systems of the motorcycles following the gray frame bottom.

For the carried out the calculations, were made theoretical assumptions about the dynamic behavior in motorcycles according to disc brake parameters currently used in different types of motorcycles. The results were compared with bibliographic sources where they report some conditions of the tests performed, to obtain a broader view of the behavior of braking systems [11-13]. The above helps to improve the mechanical properties of the material of experimentally from for the conditions to which they are exposed and thus contribute to sustainable development based on the waste and safety of the braking system. In the analysis developed for the three-disc brakes, the saturation limit was not used due to the ideal braking conditions assumed for the mathematical calculations, and therefore was assumed a maximum speed of 80 km/h and an environment with a temperature of 25 °C. Due to the above, the mechanical, thermal, and physical properties of the discs were obtained from the catalogs and the reference of Cengel [14].
Figure 2 shown the geometrical views for the three-disc brake used with the aid of design software Solidworks, such as Suzuki/Best-125cc, AKT/CPI-135cc, and Suzuki/GS-150cc, which are commercial in use for motorcycles.

![Figure 2. Geometrical view for the brake discs (a) Best-115, (b) CPI-135, and (c) GS-150.](image)

The austenitic AISI 309 and AISI 310 stainless steel have excellent mechanical and thermal properties due to its composition of chromium and nickel, which provide the highest corrosion resistance under wet and dry mediums, also are hard to wear by his significant hardness of ~2.0 GPa. These alloys may exhibit low ductility at room temperature due to the precipitation of brittle particles that affect the braking process. The increase of the heat on the surface of these materials can affect the microstructure and, thus, the mechanical behavior during the frictional contact generated during the braking process, and therefore the decrement in the mechanical and chemical properties could appear affecting the safety of braking and driver [12].

The braking forces have many variables, such as the speed of the motorcycle, the acceleration, the coefficient of friction of the pavement and the pad, the mass of the motorcycle and the pilot, the braking distance, among other factors. The Equations (1), Equation (2), Equation (3) and Equation (4) were used for calculating the acceleration, braking time, braking force, and blockage on the front wheel, respectively, such as presented following:

\[
a = \frac{V_f^2 - V_i^2}{2 \times D},
\]

\[
t = \frac{2 \times D}{V_f^2 + V_i^2},
\]

\[
F = (M_m + M_p) \times a,
\]

\[
(M_m + M_p) \times a < P_{front \, wheel} \times \mu.
\]

When \( V_f \) is the final speed of the motorcycle in (m/s), \( V_i \) is the initial speed of the motorcycle in (m/s), \( D \) is the braking distance, \( M_m \) is the motorcycle mass, \( M_p \) is the driver mass, \( a \) is the acceleration, \( \mu \) is the pavement coefficient, and \( P_{front \, wheel} \) is the pressure on the front wheel. If the coefficient of friction between the tire and the ground is less than required, the front wheel will lock and begin to slide on the pavement. To find the weight on the front wheel, it is necessary, to sum up, moments at the point where the front wheel makes contact with the ground and find the reaction with the ground (\( R_b \)) on the rear wheel, using the following expression: \( P_{front \, wheel} = (M_m \times g) - R_b \).
3. Results and discussions

The first values/data that must be obtained for the theoretical/mathematical assumptions are based on the braking forces in the motorcycle given the mass (weight in running order), and the approximate mass of the driver, as shown in Table 1; note that, when more high is the cylinder capacity of the motorcycle, these properties are higher, and thus, the dynamic behavior relative with the geometry should be better [11].

| Calculation                  | Suzuki Best | AKT CPI | Suzuki GS |
|------------------------------|-------------|---------|-----------|
| Motorcycle mass ($M_m$)      | 96.7 kg     | 98 kg   | 134 kg    |
| Pilot mass ($P_m$)           | 70 kg       | 70 kg   | 70 kg     |
| Gravity (g)                  | 9.81 m/s²   | 9.81 m/s² | 9.81 m/s² |
| Total mass ($T_m$)           | 166.7 kg    | 168 kg  | 204 kg    |

Then, it is necessary to calculate the distance from the center of gravity of the motorcycle to the front axis, which is where the disc is situated [11]. For this, it is necessary to sum up the moments in the front wheel and calculate the weight of the rear wheel, which is approximately 10 kg for the Suzuki/Best and AKT/CPI, and 12 kg for the Suzuki/GS. In this way, Table 2 summarizes the data for the developed calculations; note that the distances of gravity centers front-wheel reference point are in mm.

| Calculation                  | Suzuki Best | AKT CPI | Suzuki GS |
|------------------------------|-------------|---------|-----------|
| Height of the center of gravity of the motorcycle ($h_{cdg_p}$) | 740 mm | 765 mm | 773 mm |
| Distance of center of gravity of driver ($d_{cdg_p}$)   | 791 mm | 700 mm | 869 mm |
| Distance of center of gravity of motorcycle ($d_{cdg_m}$) | 496 mm | 439 mm | 545 mm |
| Height of the center of gravity of driver ($h_{cdg_m}$)   | 550 mm | 569 mm | 574 mm |
| Distance between centers ($d_{axis}$)                      | 1,240 mm | 1,225 mm | 1,340 mm |
| Coefficient of pavement ($\mu$)                            | 0.9 | 0.9 | 0.9 |
| Pill coefficient                                             | 0.45 | 0.45 | 0.45 |
| Rolling radius                                               | 280 mm | 280 mm | 298 mm |
| Diameter of the disc                                         | 220 mm | 220 mm | 275 mm |

Table 3 shows the calculations made for each of the disc brakes using constant data such as speed (80 km/h = 22.22 m/s), for the calculations of the braking distance (D), acceleration of the motorcycle (a), braking time (t), braking force (F), reaction with the ground ($R_0$), the force on the front wheel ($W_f$), and the total force applied by the pistons ($T_p$). All of these calculations act during the braking system operation, which affects the mechanical properties of the material, and thus the safety. The force during braking is negative due to the force applied is the compression on the disc brake. Also, the weight on the front wheel to check if there is a blockage or not, it is was determined on the motorcycle Suzuki/Best; the lock on the front wheel is presented at a braking distance of 185 m. The AKT/CPI motorcycle has a lock on the front wheel at a braking distance of 105 m, the Suzuki GS motorcycle has a lock on the front wheel at a braking distance of 95 m, considering that's motorcycles is going at a speed of 80 km/h and that the rear brake is not performing its function [15]. On the other hand, the conditions for the braking distance calculations were an initial speed of 80 km/h and the final speed of 0 km/h.

Figure 3 shows the braking distances for each of the three-disc brakes. The braking distances for the three discs are influenced by the heating generated by friction; these values vary in a matter of thousandths of a second, in the function of the distances used until the braking system is blocking. To obtain the performance of the braking system from Figure 3(a) is observed, the efficiency in function of the braking distance for each disc brake under ideal conditions of braking. Also, this behavior the braking time varies according to the speed interval influenced by braking time and distance [16].
These braking times are found in 7.6 s and 11.7 s for a braking speed of 1.9 m/s and 2.9 m/s. The opposite is the case with the disc brake of the smallest motorcycle, where the braking times and speeds are between 15.7 s and 18.9 s for a speed of 1.1 m/s and 1.4 m/s. Additionally, the $t_{RG}$ (reaction time of the system) that is around 3 s is not considered because it elapses since the brake pedal is actuated until the required force is reached [10]. In the same way, aerodynamic actions are not taken into account for the calculations due to this parameter is useful when the speed on the motorcycles are higher than 90 km/h. Also, it can be seen that the braking distance is influenced by the motorcycle cylinder, as can be seen in the braking efficiency of the highest cylinder motorcycles of 135 and 150 cubic centimeters, which oscillates between 85 m and 130 m for both cases also related as a simultaneous effect between time and braking speed.

Also, note that from Figure 3(b), when higher, the cylinder capacity of the motorcycles increases the value of each for each disc brake, and the calculations are related to the geometry of the disc. Also, the safety is affected by the short distances of braking, generating a blocking of the system as described above. The braking distance and time are affected by the inclination angle/descent on the road, which increases for each motorcycle when the angle increase. However, aerodynamic characteristics can include such as factor C, that was are not included in the calculations. This factor is affected by the design of the front of the motorcycle, and thus when more area is involved, this value increases, so the braking distance becomes less [16].

The results presented on the three-disc brake are similar to those reported in some bibliographic references because there is little information reported on the disc brakes of motorcycles. On the other hand, the dynamic behavior of motorcycles is similar to that of an automobile, but the differences are the type of tire, power, and load capacity, among others.

| Table 3. Calculation of braking distance. |
|------------------------------------------|
| D  | a  | t   | F   | $R_b$ | $W_f$ | $T_f$ | $(M_m + M_p) \times a$ | $W_f \times \mu$ | Checking the wheel lock |
|----|----|-----|-----|-------|-------|-------|------------------------|---------------------|------------------------|
| 210| 1.18| 18.90| -196.00| 717.95| 230.68| 1,174.35| 196.00 | 207.61 | No |
| 205| 1.20| 18.45| -200.78| 715.52| 233.10| 1,186.71| 200.78 | 209.79 | No |
| 200| 1.23| 18.00| -205.80| 712.97| 235.65| 1,199.69| 205.80 | 212.09 | No |
| 195| 1.27| 17.55| -211.08| 710.29| 238.33| 1,213.33| 211.08 | 214.50 | No |
| 190| 1.30| 17.10| -216.63| 707.47| 241.15| 1,227.70| 216.63 | 217.04 | No |
| 185| 1.33| 16.65| -222.49| 704.50| 244.13| 1,242.84| 222.49 | 219.72 | Yes |
| 180| 1.37| 16.20| -228.67| 701.36| 247.27| 1,258.81| 228.67 | 222.54 | Yes |
| 175| 1.41| 15.75| -235.20| 698.04| 250.59| 1,275.71| 235.20 | 225.53 | Yes |
| 130| 1.90| 11.70| -319.09| 567.44| 393.94| 2,005.51| 319.09 | 354.55 | No |
| 125| 1.98| 11.25| -331.85| 560.66| 400.72| 2,040.02| 331.85 | 360.65 | No |
| 120| 2.06| 10.80| -345.68| 553.32| 408.06| 2,077.44| 345.68 | 367.26 | No |
| 115| 2.15| 10.35| -360.71| 545.33| 416.05| 2,118.05| 360.71 | 374.44 | No |
| 110| 2.24| 9.90| -377.10| 536.63| 424.75| 2,162.31| 377.10 | 382.28 | No |
| 105| 2.35| 9.45| -395.06| 527.09| 434.29| 2,210.92| 395.06 | 390.86 | Yes |
| 100| 2.47| 9.00| -414.82| 516.60| 444.78| 2,264.35| 414.82 | 400.31 | Yes |
| 95 | 2.60| 8.55| -436.65| 505.00| 456.38| 2,323.37| 436.65 | 410.74 | Yes |
| 120| 2.06| 10.80| -419.75| 778.78| 535.76| 2,322.29| 419.75 | 482.18 | No |
| 115| 2.15| 10.35| -438.00| 770.03| 544.51| 2,360.17| 438.00 | 490.06 | No |
| 110| 2.24| 9.90| -457.91| 760.49| 554.05| 2,401.55| 457.91 | 498.65 | No |
| 105| 2.35| 9.45| -479.72| 750.04| 564.50| 2,446.86| 479.72 | 508.05 | No |
| 100| 2.47| 9.00| -503.70| 738.54| 576.00| 2,496.66| 503.70 | 518.40 | No |
| 95 | 2.60| 8.55| -530.22| 725.83| 588.71| 2,551.75| 530.22 | 529.84 | Yes |
| 90 | 2.74| 8.10| -559.67| 711.72| 602.82| 2,612.93| 559.67 | 542.54 | Yes |
| 85 | 2.90| 7.65| -592.59| 695.94| 618.60| 2,681.30| 592.59 | 556.74 | Yes |
4. Conclusions

The braking system used for the dynamic mathematical calculations is the most used in the motorcycles currently as a function of the cylinder capacity. Note that, perhaps all motorcycles have disc brakes of different geometric on both wheels that aid the dissipation of heat and safety of the system. However, due to the costs, some motorcycles used brake drums almost in the wheel rear due the most absorbed energy is in the lead wheel, but in this last, the heat is no evacuate quickly, affecting the efficiency of the braking. The temperature, environment, and maintenance of the braking systems were not considered in the calculations, due to is a hard part for the comprehension because is involved the microstructure, mechanical and chemical properties of the disc brakes. Thus, it is essential to consider by the drivers to gain the optimal safety of the system.

This research was conducted to study the relationship of the geometry of brake disc towards better braking performance in terms of dynamic behavior during the braking distance and inclination angles on the ideal movement conditions of a motorcycle. From the results, it shows that the higher the cylinder capacity, the braking effect will be more efficient, so the disc brake 3 is considered to have the best dynamic properties for the system according to his geometry.

Finally, the application of the design software for the development of finite element analysis is the high importance due to the considerations of the design and mechanical properties of the materials currently use to the improvement of the different parameters that affect the braking systems. Also, with this tool can corroborate calculus and experimental analysis obtained in this type of research.

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