Proton structure functions and quark orbital motion

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Covariant version of the quark-parton model is studied. Dependence of the structure functions on the 3D quark intrinsic motion is discussed. The important role of the quark orbital momentum, which is a particular case of intrinsic motion, appears as a direct consequence of the covariant description. Effect of orbital motion is substantial especially for polarized structure functions. At the same time, the procedure for obtaining the quark momentum distribution from the structure functions is suggested.

PACS numbers: 13.60.-r, 13.88.+e, 14.65.-q

1. INTRODUCTION

The nucleon structure functions are basic tool for understanding the nucleon internal structure in the language of QCD. And at the same time, the measuring and analysis of the structure functions represent the important experimental test of this theory. Unpolarized nucleon structure functions are known with high accuracy in very broad kinematical region, but in recent years also some precision measurements on the polarized structure functions have been completed [1, 2, 3, 4, 5, 6, 7]. For present status of the nucleon spin structure see e.g. [8] and citations therein. The more formal aspects of the nucleon structure functions are explained in [9]. In fact only the complete set of the four electromagnetic unpolarized and polarized structure functions \( F_1, F_2, g_1 \) and \( g_2 \) can give a consistent picture of the nucleon. However, this picture is usually drawn in terms of the distribution functions, which are connected with the structure functions by some model-dependent way. Distribution functions are not directly accessible from the experiment and model, which is normally applied for their extraction from the structure functions is the well known quark-parton model (QPM). Application of this model for analysis and interpretation of the unpolarized data does not create any contradiction. On the other hand, the situation is much less clear in the case of spin functions \( g_1 \) and \( g_2 \).

In our previous study [11, 12] we have suggested, that a reasonable explanation of the experimentally measured spin functions \( g_1, g_2 \) is possible in terms of a generalized covariant QPM, in which the quark intrinsic motion (i.e. 3D motion with respect to the nucleon rest frame) is consistently taken into account. Therefore the quark transversal momentum appears in this approach on the same level as the longitudinal one. The quarks are represented by the free Dirac spinors, which allows to obtain exact and covariant solution for relations between the quark momentum distribution functions and the structure functions accessible from experiment. In this way the model (in its present leading order version) contains no dynamics but only “exact” kinematics of quarks, so it can be effective tool for analysis and interpretation of the experimental data on structure functions, particularly for separating effects of the dynamics (QCD) from effects of the kinematics. This point of view is well supported by our previous results:

a) In the cited papers we showed, that the model simply implies the well known sum rules (Wanzura-Wilczek, Efremov-Leader-Teryaev, Burkhardt-Cottingham) for the spin functions \( g_1, g_2 \).

b) Simultaneously, we showed that the same set of assumptions implies rather substantial dependence of the first moment \( \Gamma_1 \) of the function \( g_1 \) on the kinematical effects.

c) Further, we showed that the model allows to calculate the functions \( g_1, g_2 \) from the unpolarized valence quark distributions and the result is quite compatible with the experimental data.

d) In the paper [14] we showed that the model allows to relate the transversity distribution to some other structure functions.

These results cannot be obtained from the standard versions of the QPM (naive or the QCD improved), which are currently used for the analysis of experimental data on structure functions. The reason is, that the standard QPM is based on the simplified and non-covariant kinematics in the infinite momentum frame (IMF), which does not allow to properly take into account the quark intrinsic or orbital motion.

* Prepared for the 17th International Spin Physics Symposium, SPIN2006, Kyoto, Japan, Oct. 2.-7., 2006.
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The subject of our previous study was the question: What is the dependence of the structure functions on quark intrinsic motion? The aim of the present paper is a discussion of related problems:

1. How to extract information about the quark intrinsic motion from the experimentally measured structure functions?

2. What is the role of the quark orbital momentum, which is a particular case of intrinsic motion?

The paper is organized as follows. In the first part of Sec. 2 the basic formulas, which follow from the generalized QPM, are presented. Resulting general covariant relations are compared with their limiting case, which is represented by the standard formulation of the QPM in the IMF. In the next part of the section the relations for calculation of 3D quark momentum distributions from the structure functions are derived. The quark momentum distributions are obtained from the experimentally measured structure functions \( F_2 \) and \( g_1 \) and it is shown how their combination allows to calculate the momentum distributions of the positively and negatively polarized quarks. The particular form of the quark intrinsic motion is the orbital momentum. In Sec. 3 the role of the quark orbital momentum in covariant description is discussed and it is shown, why its contribution to the total quark angular momentum can be quite substantial. It is demonstrated, that the orbital motion is an inseparable part of the covariant approach. The problem of quark orbital momentum in the context of nucleon spin was recognized and studied also in many previous papers, see e.g. [11, 12] and references therein. The last section is devoted to a short summary and conclusion.

2. STRUCTURE FUNCTIONS AND INTRINSIC QUARK MOTION

In our previous study \([11, 12]\) of the proton structure functions we showed, how these functions depend on the intrinsic motion of quarks. The quarks in the suggested model are represented by the free fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry \( G_k^\pm (p_0) d^3p \), where \( p_0 = \sqrt{m^2 + p^2} \) and symbol \( k \) represents the quark and antiquark flavors. These distributions measure the probability to find a quark of given flavor in the state

\[
u (p, \lambda n) = \frac{1}{\sqrt{N}} \left( \frac{\phi_{\lambda n}}{p_0 + m} \right) ; \quad \frac{1}{2} n \sigma \phi_{\lambda n} = \lambda \phi_{\lambda n}, \quad N = \frac{2 p_0}{p_0 + m}, \tag{1}\]

where \( m \) and \( p \) are the quark mass and momentum, \( \lambda = \pm 1/2, \phi_{\lambda n}^\dagger \phi_{\lambda n} = 1 \) and \( n \) coincides with the direction of proton polarization. The distributions with the corresponding quark (and antiquark) charges \( c_k \) allow to define the generic functions \( G \) and \( \Delta G^1 \),

\[
G(p_0) = \sum_k c_k^2 G_k(p_0), \quad G_k(p_0) \equiv G_k^+(p_0) + G_k^-(p_0), \tag{2}
\]

\[
\Delta G(p_0) = \sum_k c_k^2 \Delta G_k(p_0), \quad \Delta G_k(p_0) \equiv G_k^+(p_0) - G_k^-(p_0), \tag{3}
\]

from which the structure functions can be obtained. If \( q \) is momentum of the photon absorbed by the proton of the momentum \( P \) and mass \( M \), in which the phase space of quarks is controlled by the distributions \( G_k^\pm (p_0) d^3p \), then there are the following representations of corresponding structure functions.

A. Manifestly covariant representation
i) unpolarized structure functions:

\[
F_1(x) = \frac{M}{2} \left( A + \frac{B}{\gamma} \right), \quad F_2(x) = \frac{P q}{2 M \gamma} \left( A + \frac{3B}{\gamma} \right), \tag{4}\]

where

\[
A = \frac{1}{P q} \int G \left( \frac{P p}{M} \right) [pq - m^2] \delta \left( \frac{pq}{P q} - x \right) d^3p \tag{5}\]

\[\]

1 In the papers [11, 12] we used different notation for the distributions defined by Eqs. (2) and (3): \( G_k^\pm \), \( \Delta G_k \) and \( \Delta G \) were denoted as \( h_{k \pm}, \Delta h_k \) and \( H \). Apart of that we assumed for simplicity that only three (valence) quarks contribute to the sums (2) and (3). In present paper we assume contribution of all the quarks and antiquarks, but apparently general form of the relations like [10] - [12] is independent of chosen set of quarks.
\[ B = \frac{1}{Pq} \int G \left( \frac{pP}{M} \right) \left[ \left( \frac{Pp}{M} \right)^2 + \frac{(Pp)(Pq) - pq}{2} \right] \delta \left( \frac{pq}{Pq} - x \right) \frac{d^3p}{p_0} \]  

and

\[ \gamma = 1 - \left( \frac{Pq}{Mq} \right)^2. \]  

The functions \( F_1 = MW_1 \) and \( F_2 = (Pq/M)W_2 \) follow from the tensor equation

\[ \left(-g_{\alpha\beta} + \frac{q_{\alpha}q_{\beta}}{q^2}\right)W_1 + \left(P_\alpha - \frac{Pq}{q^2}q_\alpha\right)\left(P_\beta - \frac{Pq}{q^2}q_\beta\right)W_2 = \frac{1}{M^2} \int G \left( \frac{pP}{M} \right) \left[ 2p_\alpha p_\beta + p_\alpha q_\beta + q_\alpha p_\beta - g_{\alpha\beta}pq \right] \delta \left( (p + q)^2 - m^2 \right) \frac{d^3p}{p_0}. \]  

After modification of the delta function term

\[ \delta \left( (p + q)^2 - m^2 \right) = \delta \left( 2pq + q^2 \right) = \delta \left( 2Pq \left( \frac{pq}{Pq} - \frac{Q^2}{2Pq} \right) \right) = \frac{1}{2Pq} \delta \left( \frac{pq}{Pq} - x \right) ; \quad q^2 = -Q^2, \quad x = \frac{Q^2}{2Pq}. \]

The dependence on the Bjorken \( x \) is introduced. Then contracting with the tensors \( g_{\alpha\beta} \) and \( P_\alpha P_\beta \) gives the set of two equations, which determine the functions \( F_1, F_2 \) in accordance with Eqs. \((4)-(7)\).

\( ii) \) polarized structure functions:

As follows from \[11\] the corresponding spin functions in covariant form read

\[ g_1 = Pq \left( G_S - \frac{Pq}{qS} G_P \right), \quad g_2 = \left( \frac{Pq}{qS} \right)^2 G_P, \]  

where \( S \) is the proton spin polarization vector and the functions \( G_P, G_S \) are defined as

\[ G_P = \frac{m}{2Pq} \int \Delta G \left( \frac{pP}{M} \right) \frac{pS}{pP + mM} \left[ 1 + \frac{1}{mM} \left( pp - \frac{pu}{qu} Pq \right) \right] \delta \left( \frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}, \]  

\[ G_S = \frac{m}{2Pq} \int \Delta G \left( \frac{pP}{M} \right) \left[ 1 + \frac{pS}{pP + mM} \frac{M}{m} \left( pS - \frac{pu}{qu} qS \right) \right] \delta \left( \frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}; \]

\[ u = q + (qS)S - \frac{(Pq)}{M^2} P. \]

\( B. \) Rest frame representation for \( Q^2 \gg 4M^2x^2 \)

As follows from the Appendix in \[11\], if \( Q^2 \gg 4M^2x^2 \) and the above integrals are expressed in terms of the proton rest frame variables, then one can substitute

\[ \frac{pq}{Pq} \rightarrow \frac{p_0 + p_1}{M} \]

and the structure functions are simplified as:

\[ F_1(x) = \frac{Mx}{2} \int G(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \]  

\[ F_2(x) = Mx^2 \int G(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \]  

\[ g_1(x) = \frac{1}{2} \int \Delta G(p_0) \left( m + p_1 + \frac{p_1^2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \]  

\[ g_2(x) = -\frac{1}{2} \int \Delta G(p_0) \left( p_1 + \frac{p_1^2 - p_2^2}{2(p_0 + m)} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \]
where the $p_1$ and $p_T$ are longitudinal and transversal quark momentum components.

**C. Standard IMF representation**

The usual formulation of the QPM gives the known relations between the structure and distribution functions $^{9}$:

\[ F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x), \quad F_2(x) = x \sum_q e_q^2 q(x), \]  
\[ g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x), \quad g_2(x) = 0, \]  
\[ q(x) = q^+(x) + q^-(x), \quad \Delta q(x) = q^+(x) - q^-(x). \]  

where

In the Appendix A we have proved that these relations represent the particular, limiting case of the covariant relations $^{10}$. The three versions of the relations between the structure functions and the quark distributions can be compared:

a) If we skip the function $g_2$ in the version $C$, then the relations (17) and (18) practically represent identity between the structure functions and quark distributions. Such simple relations are valid only for the IMF approach based on the approximation (4), which means that the quark intrinsic motion is suppressed. In more general versions $A$ and $B$, where the intrinsic motion is allowed, the relations are more complex. The intrinsic motion strongly modifies also the $g_2$. In the version $C$ there is $g_2(x) = 0$, but $g_2(x) \neq 0$ in the $A$ and $B$.

b) The version $B$ allows to easily calculate the (substantial) dependence of the first moment $\Gamma_1$ on the rate of intrinsic motion. A more detailed discussion follows in the next section. The same approach implies that functions $g_1$ and $g_2$ for massless quarks satisfy the relation equivalent to the Wanzura-Wilczek term and obey some well known sum rules, that is shown in $^{11}$.

c) The functions $F_1$ and $F_2$ exactly satisfy the Callan-Gross relation $F_2(x)/F_1(x) = 2x$ in the versions $B$ and $C$, but this relation is satisfied only approximately in the $A$: $F_2(x)/F_1(x) \approx 2x + O(4M^2 x^2/Q^2)$.

The task which was solved in different approximations above can be formulated: How to obtain the structure functions $F_1$, $F_2$ and $g_1$, $g_2$ from the probabilistic distributions $G$ and $\Delta G$ defined by Eqs. (2) and (3)? In the next we will study the inverse task, the aim is to find out a rule for obtaining the distribution functions $G$ and $\Delta G$ from the structure functions. In the present paper we consider the functions $F_2$ and $g_1$ represented by Eqs. (14) and (15).

As follows from the Appendix A in $^{12}$, the function

\[ V_n(x) = \int K(p_0) \left( \frac{p_0}{M} \right)^n \delta \left( \frac{p_0 + p_1}{M} - x \right) d^3p \]  

satisfies

\[ V'_n(x)_{x\pm} = \mp 2\pi MK(\xi) \xi^2 \frac{\sqrt{\xi^2 - m^2}}{M} \left( \frac{\xi}{M} \right)^n; \quad x_{\pm} = \frac{\xi \pm \sqrt{\xi^2 - m^2}}{M}. \]  

In this section we consider only the case $m \to 0$, then

\[ V'_n(x)_{x} = -2\pi MK(\xi) \xi^2 \left( \frac{\xi}{M} \right)^n; \quad x = \frac{2\xi}{M}. \]  

As we shall see below, with the use of this relation one can obtain the probabilistic distributions $G$ and $\Delta G$ from the experimentally measured structure functions.

Let us remark that in present stage the QCD evolution is not included into the model. However, this fact does not represent any restriction for the present purpose - to obtain information about distributions of quarks at some $Q^2$ from the structure functions measured at the same $Q^2$. Distribution of the gluons is another part of the proton picture. But since our present discussion is directed to the relation between the structure functions and corresponding (quark) distributions at given scale, the gluon distribution is left aside.

### 2.1. Momentum distribution from structure function $F_2$

In an accordance with the definition (20) in which the distribution $K(p_0)$ is substituted by the $G(p_0)$, the structure function $^{14}$ can be written in the form
Then, with the use of the relation (22) one gets
\[ G(p) = -\frac{1}{\pi M^2} \left( \frac{F_2(x)}{x^2} \right)' = \frac{1}{\pi M^2 x^2} \left( \frac{2F_2(x)}{x} - F'_2(x) \right); \quad x = \frac{2p}{M}, \quad p = \sqrt{p^2} = p_0. \] (24)
Probability distribution \( G \) measures number of quarks in the element \( dp \). Since \( d^3p = 4\pi p^2 dp \), the distribution measuring the number of quarks in the element \( dp/M \) reads
\[ 4\pi p^2 MG(p) = -x^2 \left( \frac{F_2(x)}{x^2} \right)' = \frac{2F_2(x)}{x} - F'_2(x); \quad x = \frac{2p}{M}. \] (25)
Let us note, the maximum value of quark momentum is \( p_{\text{max}} = M/2 \), which is a consequence of the kinematics in the proton rest frame, where the single quark momentum must be compensated by the momentum of the other partons.

Another quantity, which can be obtained, is the distribution of the quark transversal momentum. Obviously the integral
\[ \frac{dN}{dp_T^2} = \int G(p) \delta \left( p_T^2 + p_T^2 - p_T^2 \right) d^3p, \] (26)
which measures the number of quarks in the element \( dp_T^2 \), can be modified as
\[ \frac{dN}{dp_T^2} = 2\pi \int_0^{\sqrt{p_{\text{max}}^2}} G \left( \sqrt{p_T^2 + p_T^2} \right) dp_1. \] (27)
It follows, that the distribution measuring number of quarks in the element \( dp_T/M \) reads:
\[ P(p_T) = M \frac{dN}{dp_T} = 4\pi p_T M \int_0^{\sqrt{p_{\text{max}}^2}} G \left( \sqrt{p_T^2 + p_T^2} \right) dp_1. \] (28)
Let us point out that the distribution \( P(p_T) \), equally as the distributions \( G \) and \( \Delta G \), represents the combination of the distributions related to different quark and antiquark flavors - like in Eqs. (2) or (3). With the use of Eq. (24) one gets distributions
\[ P(p_T) = \frac{4\pi}{M^2} \int_0^{\sqrt{p_{\text{max}}^2}} \frac{1}{x^2} \left( \frac{2F_2(x)}{x} - F'_2(x) \right) dp_1; \quad x = 2\sqrt{p_T^2 + p_T^2}/M. \] (29)
In Fig. 1 the structure function \( F_2 \) together with the corresponding distributions calculated with the use of relations (25) and (29) are displayed. For the proton structure function the phenomenological fit performed in the Appendix of the paper [3] was used. Using the Eq. (24) one can calculate the mean value
\[ \langle p \rangle = \frac{\int pG(p)d^3p}{\int G(p)d^3p} = \frac{M}{2} \frac{\int_0^1 x^3 \left( \frac{F_2(x)}{x^2} \right)' dx}{\int_0^1 x^2 \left( \frac{F_2(x)}{x^2} \right)' dx}, \] (30)
but since the extrapolation of the structure function for \( x \to 0 \) gives in the denominator divergent integral, it follows that \( \langle p \rangle \to 0 \). Nontrivial value can be obtained with the integration cutoff \( x > x_{\text{min}} \).

2.2. Momentum distribution from structure function \( g_1 \)
Now, we shall determine the distribution \( \Delta G \) defined in Eq. (3) from the spin function \( g_1 \) - similarly as distribution \( G \) was obtained from the function \( F_2 \). In the paper [12], Eq. (44), we proved that
\[ g_1(x) = V_0(x) - \int_x^1 \left( \frac{4x^2}{y^3} - \frac{x}{y^2} \right) V_0(y) dy, \] (31)
where the function $V_0$ is defined by Eq. (20) for $n = 0$ and $K(p) = ∆G(p)$. In the Appendix 13 it is shown, that the last relation can be modified to:

$$V_{-1}(x) = \frac{2}{x} \left( g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right).$$  \hspace{1cm} (32)

Then, in accordance with Eq. (22), we obtain

$$V_{-1}'(x) = -\pi M^3 ∆G(p); \hspace{1cm} x = \frac{2p}{M},$$  \hspace{1cm} (33)

so the last two relations imply

$$∆G(p) = \frac{2}{\pi M^3 x^2} \left( 3g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy - xg_1'(x) \right); \hspace{1cm} x = \frac{2p}{M}.$$  \hspace{1cm} (34)

Obviously this distribution together with the distribution (24) allows to obtain the polarized distributions $G^\pm$ as

$$G^\pm(p) = \sum_q e_q^2 G^\pm_q(p) = \frac{1}{2} \left(G(p) ± ∆G(p)\right).$$  \hspace{1cm} (35)

These distributions can be with the use of Eqs. (24) and (34) obtained from experimental data on the $F_2$ and $g_1$. The result is displayed in the left part of Fig. 2, for which the function $g_1$ was parameterized by the fit of the world data 7 at $Q^2 = 4GeV^2$.

With the use of the relation (14) one can formally calculate the partial structure functions corresponding to the subsets of positively and negatively polarized quarks:

$$F^+_2(x) = Mx^2 \int G^\pm(p)δ \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}.$$  \hspace{1cm} (36)

Apparently it holds

$$F_2(x) = F^+_2(x) + F^-_2(x)$$  \hspace{1cm} (37)

and one can define also

$$∆F_2(x) = F^+_2(x) - F^-_2(x)$$  \hspace{1cm} (38)
FIG. 2: Probability distributions $\Delta G, G, G^+$ and $G^-$ are represented by solid, dashed, dash-and-dot and dotted lines (left) and corresponding structure functions $\Delta F_2, F_2, F_2^+$ and $F_2^-$ (right).

or equivalently

$$\Delta F_2(x) = M x^2 \int \Delta G(p) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \quad (39)$$

This equality can be written as

$$\Delta F_2(x) = x^2 V_{-1}(x), \quad (40)$$

which after inserting from Eq. (32) gives

$$\Delta F_2(x) = 2x \left( g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right). \quad (41)$$

The structure functions $F_2, \Delta F_2, F_2^\pm$ are shown in the right part of Fig. 2 and one can observe:

i) Shape of the function $\Delta F_2$ implies, that contributions of oppositely polarized quarks in Eq. (38) are canceled out in the region of low $x$. In fact the shape is similar to that of the function $F_{2\text{val}}$ corresponding to the valence quarks. This suggests that a dominant spin contribution comes from the valence region.

ii) The distributions $\Delta G, G$ and $G^+$ are very close together in the region of higher momenta and simultaneously $G^-$ is close to zero in the same region. And the same holds for corresponding structure functions $\Delta F_2, F_2$ and $F_2^\pm$ in the higher $x$ region. In an accordance with the definitions (2),(3) it follows, that

$$G_+^q(p) \simeq G_q(p), \quad G_-^q(p) \simeq 0. \quad (42)$$

In the other words, polarization of quarks or partons with higher intrinsic energy (and/or higher $x$) coincides with the proton polarization.

The mean value of the distribution $\Delta G$ can be estimated as

$$\langle p \rangle = \frac{\int p \Delta G(p) d^3p}{\int \Delta G(p) d^3p} = \frac{M}{2} \langle x \rangle; \quad \langle x \rangle = \frac{\int_0^1 x g_1(x) dx}{\int_0^1 g_1(x) dx}. \quad (43)$$

The proof of this relation is done in the Appendix C. The numerical calculation with the $g_1$ fit gives $\langle p \rangle = 0.113 GeV/c$. However, interpretation of this average momentum should be done with some care since $\Delta G_0$ can be negative, in principle. Average momentum $\langle p \rangle$ allows to calculate the mean transversal momentum; $\langle p_T \rangle = \pi/4 \cdot \langle p \rangle$. 

3. INTRINSIC QUARK MOTION AND ORBITAL MOMENTUM

The rule of quantum mechanics says, that angular momentum consists of the orbital and spin part \( j = l + s \) and that in the relativistic case the \( l \) and \( s \) are not conserved separately, but only the total angular momentum \( j \) is conserved. This simple fact was in the context of quarks inside the nucleon pointed out in \[26\]. It means, that only \( j^2, j_z \) are well-defined quantum numbers and corresponding states of the particle with spin \( 1/2 \) are represented by the bispinor spherical waves \[27\]

\[
\psi_{kj;jz} (p) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \left( \frac{i^{-l} \sqrt{p_0 + m \Omega_{kj;jz} (\omega)}}{i^{-l} \sqrt{p_0 - m \Omega_{kj;jz} (\omega)}} \right),
\]

where \( \omega = p/p, l = j \pm \frac{1}{2}, \lambda = 2j - l \) (\( l \) defines the parity) and

\[
\Omega_{j,l,jz} (\omega) = \begin{cases} 
\sqrt{\omega^2 + 4j(j+1)} Y_{l,jz-l/2} (\omega); & l = j - \frac{1}{2} \\
\sqrt{\omega^2 + 4j(j+1)} Y_{l,jz+l/2} (\omega); & l = j + \frac{1}{2}.
\end{cases}
\]

States are normalized as:

\[
\int \psi_{kj;jz}^*(p) \psi_{kj;jz} (p) d^3p = \delta(k-k')\delta_{jj} \delta_{l'l} \delta_{jz,jz}'.
\]

The wavefunction \[44\] is simplified for \( j = j_z = 1/2 \) and \( l = 0 \). Taking into account that

\[
Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = i \sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_{11} = -i \sqrt{\frac{3}{8\pi}} \sin \theta \exp (i\varphi),
\]

one gets:

\[
\psi_{kj;jz} (p) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}} \left( \begin{array}{c}
\sqrt{p_0 + m} \left( \begin{array}{c} 1 \\
0 \end{array} \right) \\
-\sqrt{p_0 - m} \left( \begin{array}{c} \cos \theta \\
\sin \theta \exp (i\varphi) \end{array} \right)
\end{array} \right).
\]

Let us note, that \( j = 1/2 \) is the minimum angular momentum for the particle with spin \( 1/2 \). If one consider the quark state as a superposition

\[
\Psi (p) = \int a_k \psi_{kj;jz} (p) dk; \quad \int a_k^* a_k dk = 1
\]

then its average spin contribution to the total angular momentum reads:

\[
\langle s \rangle = \int \Psi^\dagger (p) \Sigma_z \Psi (p) d^3p; \quad \Sigma_z = \frac{1}{2} \left( \begin{array}{c}
\sigma_z \\
\sigma_z
\end{array} \right).
\]

After inserting from Eqs. \[46\], \[47\] into \[48\] one gets

\[
\langle s \rangle = \int a_k^* a_p \left( \frac{p_0 + m}{16\pi p^2 p_0} \right) d^3p = \frac{1}{2} \int a_k^* a_p \left( \frac{1}{3} + \frac{2m}{3p_0} \right) dp.
\]

Since \( j = 1/2 \), the last relation implies for the quark orbital momentum:

\[
\langle l \rangle = \frac{1}{3} \int a_k^* a_p \left( 1 - \frac{m}{p_0} \right) dp.
\]

It means that for quarks in the state \( j = j_z = 1/2 \) there are the extreme scenarios:
i) Massive and static quarks \( (p_0 = m) \), which implies \( \langle s \rangle = j = 1/2 \) and \( \langle l \rangle = 0 \). This is evident, since without kinetic energy no orbital momentum can be generated.

ii) Massless quarks \( (m \ll p_0) \), which implies \( \langle s \rangle = 1/6 \) and \( \langle l \rangle = 1/3 \).

Generally, for \( p_0 \geq m \), one gets \( 1/3 \leq \langle s \rangle / j \leq 1 \). In other words, for the states with \( p_0 > m \) part of the total angular momentum \( j = 1/2 \) is necessarily generated by the orbital momentum. This is a consequence of quantum mechanics, and not a consequence of the particular model. For instance, if one assumes the quark effective mass of the order thousandths and momentum of the order of tenth of GeV, then the second scenario is clearly preferred.

Further, the mean kinetic energy corresponding to the superposition (47) reads

\[
\langle E_{\text{kin}} \rangle = \int a^*_p a_p E_{\text{kin}} dp; \quad E_{\text{kin}} = p_0 - m
\]

and at the same time the Eq. (50) can be rewritten as

\[
\langle l \rangle = \frac{1}{3} \int a^*_p a_p \frac{E_{\text{kin}}}{p_0} dp.
\]

It is evident, that for fixed \( j = 1/2 \) both the quantities are in the proton rest frame almost equivalent: more kinetic energy generates more orbital momentum and vice versa.

Further, the average spin part \( \langle s \rangle \) of the total angular momentum \( j = 1/2 \) related to single quark according to Eq. (49) can be compared with the integral

\[
\Gamma_1 = \int_0^1 g_1(x) dx,
\]

which measures total quark spin contribution to the proton spin. For the \( g_1 \) from Eq. (15) this integral reads

\[
\Gamma_1 = \frac{1}{2} \int \Delta G(p_0) \left( \frac{1}{3} + \frac{2m}{3p_0} \right) d^3p.
\]

Dependence of both the integrals (49) and (51) on intrinsic motion is controlled by the same term \( (1/3 + 2m/3p_0) \), which in both the cases has origin in the covariant kinematics of the particle with \( s = 1/2 \). In fact, the procedures for calculation of these integrals are based on the two different representations of the solutions of Dirac equation: the plane waves (1) and spherical waves (46). It is apparent that for the scenario of massless quarks \( (m \ll p_0) \), due to necessary presence of the orbital motion, both the integrals \( \Gamma_1 \) and \( \langle s \rangle \) will be roughly three times less, than for the scenario of massive and static quarks \( (m \simeq p_0) \). We discussed this effect in the context of experimental data in [13].

What is the underlying physics behind the interplay between the spin and orbital momentum? Actually, speaking about the spin of the particle represented by the state (1), one should take into account:

a) Definite projection of the spin in the direction \( \mathbf{n} \) is well-defined quantum number only for the particle at rest \( (p = 0) \) or for the particle moving in the the direction \( \mathbf{n} \), i.e. \( p/p = \pm \mathbf{n} \). In these cases we have

\[
s = u^\dagger (p, \lambda \mathbf{n}) \mathbf{n} \Sigma u (p, \lambda \mathbf{n}) = \pm 1/2.
\]

b) In other cases, as shown in the Appendix D, only inequality

\[
\langle s \rangle = |u^\dagger (p, \lambda \mathbf{n}) \mathbf{n} \Sigma u (p, \lambda \mathbf{n})| < 1/2
\]

is satisfied. Roughly speaking, the result of measuring of the spin (of a quark) depends on its momentum in the defined reference frame (proton rest frame). This obvious effect acts also in the states, which are represented by the superposition of the plane waves (1) with different momenta \( p \) and resulting in \( \langle p \rangle = 0 \), but \( \langle p^2 \rangle > 0 \). In [11] we showed, that averaging of the spin projection (51) over the spherical momentum distribution gives the result equivalent to (51). The state (17) can be also decomposed into plane waves having spherical momentum distribution and the spin mean value given by Eq. (19). Well-defined quantum numbers \( j = j_z = 1/2 \) imply, that the spin reduction due to increasing intrinsic kinetic energy is compensated by the increasing orbital momentum.

4. SUMMARY AND CONCLUSION

We studied covariant version of the QPM with spherically symmetric distributions of the quark momentum in the proton rest frame. The main results obtained in this paper can be summarized as follows.
Then, after some algebra the structure functions (4) read
\[ p \text{dominant contribution to the proton spin comes from the valence quarks}. \]
\[ G \text{which implies} \]
\[ x \text{(and/or higher longitudinal and transversal distributions are accessible as well. Results of the calculation suggest:} \]
\[ \langle \text{whereas} \]
\[ G \text{are present rather only in the lower energy region.} \]
\[ \text{time the partial structure functions} \]
\[ \text{The role of orbital motion increases with the rate of quark intrinsic motion; for} \]
\[ \text{Since the approximation (A1) implies sharply peaked distribution at} \]
\[ \text{In the paper [12] we proved relation} \]
\[ \text{APPENDIX A: STRUCTURE FUNCTIONS IN THE APPROACH OF INFINITE MOMENTUM FRAME} \]
\[ \text{The necessary condition for obtaining equalities (17) - (18) is the covariant relation} \]
\[ p_{\alpha} = yP_{\alpha}, \quad (A1) \]
\[ \text{which implies} \]
\[ m = yM \quad (A2) \]
\[ \text{and} \quad p = 0 \text{in the proton rest frame and} \quad p_{T} = 0 \text{in the IMF.} \]
\[ \text{For calculation of the integrals} \]
\[ \text{Then, after some algebra the structure functions (1) read} \]
\[ F_{1}(x) = \frac{1}{2} M x \int G(yM) \delta(y - x) \pi dp_{T}^{2} \frac{dy}{y}, \quad F_{2}(x) = M x^{2} \int G(yM) \delta(y - x) \pi dp_{T}^{2} \frac{dy}{y}. \quad (A3) \]
\[ \text{Since the approximation (A1) implies sharply peaked distribution at} \quad p_{T}^{2} \rightarrow 0, \text{one can identify} \]
\[ MG_{q}(yM) \pi dp_{T}^{2} = q(y) \quad (A4) \]
\[ \text{and then the Eqs. (17) and (18) after integrating are equivalent.} \]
\[ \text{In the same way the equalities (10) - (12) can be modified. Taking into account that} \quad pS \rightarrow yPS = 0, \text{one obtain} \]
\[ g_{1}(x) = \frac{m}{2} \int \Delta G(yM) \delta(y - x) \pi dp_{T}^{2} \frac{dy}{y}, \quad g_{2}(x) = 0. \quad (A5) \]
\[ \text{If we put} \]
\[ M \Delta G_{q}(yM) \pi dp_{T}^{2} = \Delta q(y) \quad (A6) \]
\[ \text{and take into account Eq. (A2), then it is obvious, that the Eqs. (18) and (A5) are equivalent.} \]
\[ \text{APPENDIX B: PROOF OF THE RELATION (32)} \]
\[ \text{In the paper [12] we proved relation} \]
\[ \frac{V_{j}(x)}{V_{k}(x)} = \left( \frac{x}{2 \pi} \int_{x_{0}}^{x} \frac{dy}{2\pi} \right)^{j-k}, \quad x_{0} = \frac{m}{M}. \quad (B1) \]

Acknowledgments

I would like to thank Anatoli Efremov and Oleg Teryaev for many useful discussions and valuable comments.
which for \( m \to 0 \) implies

\[
V_0(x) = \frac{1}{2} \left( xV_{-1}(x) + \int_0^x V_{-1}(y) dy \right).
\] (B2)

After inserting \( V_0 \) from this relation to Eq. (31) one gets

\[
g_1(x) = \frac{1}{2} \left( xV_{-1}(x) + \int_0^x V_{-1}(y) dy \right)
- 2x^2 \left( \int_x^1 \frac{V_{-1}(y)}{y^2} dy + \int_x^1 \frac{1}{y^2} \int_y^1 V_{-1}(z) dz dy \right)
+ \frac{1}{2} x \left( \int_x^1 \frac{V_{-1}(y)}{y} dy + \int_x^1 \frac{1}{y^2} \int_y^1 V_{-1}(z) dz dy \right).
\] (B3)

The double integrals can be reduced by integration by parts with the use of formula

\[
\int_x^1 a(y) \left( \int_y^1 b(z) dz \right) dy = \int_x^1 (A(y) - A(x)) b(y) dy; \quad A'(x) = a(x),
\] (B4)

then the relation (B3) is simplified:

\[
g_1(x) = \frac{1}{2} xV_{-1}(x) - x^2 \int_x^1 \frac{V_{-1}(y)}{y^2} dy.
\] (B5)

In the next step we extract \( V_{-1} \) from this relation. After the substitution \( V(x) = V_{-1}(x)/x \) the relation reads

\[
\frac{g_1(x)}{x^2} = \frac{1}{2} V(x) - \int_x^1 \frac{V(y)}{y} dy,
\] (B6)

which implies the differential equation for \( V(x) \):

\[
\frac{1}{2} V'(x) + \frac{V(x)}{x} = \left( \frac{g_1(x)}{x^2} \right)'.
\] (B7)

The corresponding homogeneous equation

\[
\frac{1}{2} V'(x) + \frac{V(x)}{x} = 0
\] (B8)

gives the solution

\[
V(x) = C \frac{x^2}{x},
\] (B9)

which after inserting to Eq. (B7) gives

\[
C'(x) = 2x^2 \left( \frac{g_1(x)}{x^2} \right)'.
\] (B10)

After integration one easily gets the relation inverse to Eq. (B5):

\[
V_{-1}(x) = \frac{2}{x} \left( g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right),
\] (B11)

which coincides with Eq. (B2).
APPENDIX C: PROOF OF THE RELATION \((43)\)

The relation \((34)\) implies
\[
\int \Delta G(p) d^3 p = \int_0^1 \left( 3g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy - xg_1'(x) \right) dx
\]
and
\[
\int p \Delta G(p) d^3 p = \frac{M}{2} \int_0^1 \left( 3xg_1(x) + 2x \int_x^1 \frac{g_1(y)}{y} dy - x^2 g_1'(x) \right) dx; \quad x = \frac{2p}{M}.
\]

If one denotes
\[
\Gamma_1 = \int_0^1 g_1(x) dx, \quad \Gamma_2 = \int_0^1 x g_1(x) dx,
\]
then integration by parts gives
\[
\int_0^1 \int_x^1 \frac{g_1(y)}{y} dy dx = \Gamma_1, \quad \int_0^1 x g_1'(x) dx = -\Gamma_1
\]
and
\[
\int_0^1 2x \int_x^1 \frac{g_1(y)}{y} dy dx = \Gamma_2, \quad \int_0^1 x^2 g_1'(x) dx = -2\Gamma_2.
\]

Now, one can easily express the ratio
\[
\frac{\int p \Delta G(p) d^3 p}{\int \Delta G(p) d^3 p} = \frac{M}{2} \frac{\Gamma_2}{\Gamma_1},
\]
in this way the relation \((43)\) is proved.

APPENDIX D: PROOF OF THE RELATION \((56)\)

With the use of rule
\[
p \sigma \cdot n \sigma + n \sigma \cdot p \sigma = 2pn
\]
the term in Eq. \((56)\) can be modified as
\[
u^i (p, \lambda n) n \Sigma u (p, \lambda n) = \frac{1}{2N} \phi_{\lambda n}^i \left( n \sigma + \frac{p \sigma \cdot n \sigma \cdot p \sigma}{(p_0 + m)^2} \right) \phi_{\lambda n}
\]
\[
= \frac{1}{2N} \phi_{\lambda n}^i \left( n \sigma + \frac{p \sigma \cdot n \sigma \cdot p \sigma - 2pn}{(p_0 + m)^2} \right) \phi_{\lambda n}
\]
\[
= \frac{1}{2N} \phi_{\lambda n}^i \left( n \sigma \left( 1 - \frac{p^2}{(p_0 + m)^2} \right) + \frac{2pn \cdot p \sigma}{(p_0 + m)^2} \right) \phi_{\lambda n}
\]
\[
= \frac{1}{2p_0} \phi_{\lambda n}^i \left( m \cdot n \sigma + \frac{pn \cdot p \sigma}{p_0 + m} \right) \phi_{\lambda n}.
\]

Since
\[
|\phi_{\lambda n}^i n \sigma \phi_{\lambda n}| = 1, \quad |\phi_{\lambda n}^i p \sigma \phi_{\lambda n}| \leq p, \quad pn = pc \cos \alpha,
\]
\[
\tag{D3}
\]
it follows

\[ |u^\dagger(p, \lambda n) n \Sigma u(p, \lambda n)| \leq \frac{1}{2p_0} \left( m + \frac{p^2}{p_0 + m} \right) = \frac{1}{2}. \]  

(D4)

Obviously

\[ |u^\dagger(p, \lambda n) n \Sigma u(p, \lambda n)| = \frac{1}{2} \]  

(D5)

only for \( p/p = \pm n \) or \( p = 0 \).

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