Low Magnetic Field Regime of a Gate-Defined Quantum Point Contact in High-Mobility Graphene

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Gate-defined quantum point contacts (QPC’s) are widely used on semiconductor heterostructures to control electronic transport at the nanoscale in two-dimensional electron gases. In graphene, however, tailoring gate-defined QPC’s remains challenging due to the Dirac band structure that inevitably induces gapless, conductive npn junctions beneath gate electrodes. For narrow split-gate geometry, conductance quantization can only develop in the graphene quantum Hall regime when the broken symmetry states open a gap between electron and hole states. Yet, the transition between the low-field and high-field quantum Hall regimes in the split-gate geometry remains unexplored.

Here, we report on the evolution of the coherent electronic transport through a gate-defined QPC in a high-mobility graphene device from ballistic transport to quantum Hall regime upon increasing the magnetic field. At low field, the conductance exhibits Fabry-Pérot resonances resulting from the npn cavities formed beneath the top-gated regions, with evidence of the graphene’s Berry phase by magnetic steering of electron trajectories. Our findings show that, above a critical field \( B^* \) corresponding to the cyclotron radius equal to the npn cavity length, Fabry-Pérot resonances vanish and snake trajectories are guided through the constriction, marking the emergence of the QPC contribution to the conductance. At higher field, transport is done by quantum Hall edge channels, with the characteristic signature of current equilibration in a QPC geometry.

Controlling electron transport with gate-defined constrictions is key in many quantum coherent experiments. In two-dimensional electron gases formed in semiconductor heterostructures, constrictions are commonly made with nano-patterned split-gate electrodes that draw a short and narrow channel of conduction acting as a quantum point contact (QPC) [1]. Such a local electrostatic gating enables fine tuning of the electron transmission through the QPC with the ensuing quantization of the conductance, both in the ballistic [2, 3] and in the quantum Hall regimes under strong magnetic field [4, 5].

In graphene, engineering such a gate-defined QPC has proven difficult due to the absence of band gap for the Dirac electrons [6], which prevents the formation of insulating depleted regions below gate electrodes [7, 10]. Depleting an electron (hole)-doped graphene indeed leads to a hole (electron)-doped region beneath gate electrodes. At zero magnetic field, the resulting npn junction is partially transparent to charge carriers, thereby hampering the realization of gate-defined QPCs in graphene. Still, other approaches based on etched constrictions [11, 12] or gate-defined long one-dimensional channels [13] have evidenced conductance quantization at zero magnetic field.

Recently, some of us demonstrated that, in the quantum Hall regime of high-mobility graphene, the gap that opens at the charge neutrality point between electron and hole-type broken symmetry states enables to operate a gate-defined QPC with fine-tuning of the edge-channels transmission [10]. This result, obtained with hBN-encapsulated graphene in van der Waals heterostructures, opens the way to more elaborated quantum devices based on local top or bottom gating, towards exploring quantum coherent phenomena in graphene and new correlated fractional quantum Hall states [14–15] with quantum Hall interferometry [19, 20], non-equilibrium measurements [21], shot noise experiments [22, 23] or electron quantum optics [24].

However, the impact of the split-gate geometry on the ballistic transport at zero magnetic field, and the transition to the high-field quantum Hall regime where transport is dominated by the QPC constriction, remain to be investigated. Understanding the behavior over the whole magnetic field range is necessary for designing future of QPC-based graphene devices.

In this work, we show how the transport evolves from a low-field regime dominated by Fabry-Pérot (FP) interference [25, 34] under the top gates, into a high-field regime dominated by the constriction. At low field, the analysis of the FP interference together with self-consistent electrostatic simulations enable to assess the cavity length and estimate the potential profile of the pn junctions and the QPC constriction. Upon raising the magnetic field, the magnetic steering of ballistic trajectories modifies the interference pattern and reveals the graphene Berry phase. At a critical field \( B^* \), we identify a transition from Fabry-Pérot conductance oscillations to transport through the constriction via the guiding of ballistic snake trajectories and, eventually, quantum Hall edge channels.

Gate-defined graphene QPC devices are made of an exfoliated graphene monolayer encapsulated between two hexagonal boron nitride (hBN) flakes via the van-der-Waals pick-up technique [34]. In this study, we focus on a graphene device encapsulated between 18 nm/56 nm.
thick top/bottom hBN flakes sitting on a SiO$_2$/Si++ substrate that serves as a back-gate electrode. Suitable etching allows for one-dimensional edge contacts to be deposited on the edges of the hBN/graphene/hBN stack [34]. Top-gate electrodes are patterned onto the top hBN flake in a split-gate geometry [10], with a separation of 200 nm and a gate width of 300 nm measured by SEM micrograph. Six ohmic contacts enable measurements of Hall and longitudinal resistances $R_H$ and $R_L$ in 4-terminal configuration, as shown schematically in Fig. 1a. All measurements are performed at a temperature of 4.2 K.

We begin our study by the device response to gate voltages and the signature of ballistic transport at zero magnetic field. Figure 1b shows the back-gate voltage dependence of the longitudinal resistance $R_L$ that exhibits a sharp resistance peak corresponding to the Dirac point of the graphene flake. From the Drude conductance, we extract a Hall mobility of 85000 cm$^2$V$^{-1}$s$^{-1}$, which corresponds to a mean free path of 1 $\mu$m at a charge carrier density $n \approx 1 \times 10^{12}$ cm$^{-2}$. This value of the mean free path indicates that transport is ballistic across a distance far superior to the width of the top-gated region. The longitudinal resistance $R_L$ versus back-gate voltage $V_{bg}$ and top-gate voltage $V_{tg}$ is shown in Fig. 1d. The charge neutrality point of the bulk graphene is apparent as the horizontal line independent of $V_{tg}$ at $V_{CNP}^{bg} = 0.5$ V, indicating very little intrinsic doping. The diagonal ridge corresponds to a second peak in resistance, signaling the charge neutrality point of the graphene top-gated region, which is under the electrostatic influence of both the back-gate and top-gate. From its slope, we extract a top-gate capacitance $C_{tg} = 65$ nF/cm$^2$, knowing the back-gate capacitance $C_{bg} = 10$ nF/cm$^2$ from Hall measurements. These two resistance ridges define four regions of different polarities (nn’n, np’n, pp’p, pn’p). This map at zero magnetic field shows no indication of the presence of the constriction (such as conductance quantization) due to the small size of the constriction as compared to the much wider regions below the top gates which are always conducting even in the np’n and pn’p bipolar regimes [8–10].

Inspecting the $V_{tg}$-dependence of the resistance shown in Fig. 2a for a fixed back-gate voltage $V_{bg} = 5$ V, we see that prominent pseudo-periodic oscillations of the longitudinal resistance emerge in the bipolar regime with a pseudo-period $\Delta V_{tg}$ that increases on more negative $V_{tg}$ values. The derivative $dR_L/dV_{tg}$ of the entire resistance map displayed in Fig. 2d reveals their presence in the two bipolar regimes and shows their dispersion with $V_{bg}$ and $V_{tg}$.

Such resistance oscillations are similar to that observed...
FIG. 3. (a) Derivative of the longitudinal resistance $dR_L/dV_{tg}$ as a function of the magnetic field $B$ and $V_{tg}$, at $V_{bg} = 10$ V. Fabry-Pérot oscillations are bent in the presence of magnetic field, and undergo a phase shift at $B = B_0$. (b) Line-cuts at 0 and 0.1 T, as indicated by arrows on the right axis in (a). (c) Same data as in (a) over a larger range of $V_{tg}$ and $B$. The Fabry-Pérot oscillations are only visible at low field in the bipolar regime. Dashed lines on the top are guide for the eye for the Shubnikov-de-Haas oscillations from the constricted area. (d-f) Schematic of the evolution of the ballistic electron trajectories within the upper top-gated region upon increasing magnetic field. (f), At zero field, trajectories are straight. (e), Above $B_0$, trajectories form loops, picking-up the graphene Berry phase. (d), Above $B^\ast$, electrons form closed orbits and no longer reach the opposite side of the junction. Snake orbits appear along the pn interface, guiding the electronic trajectories through the constriction.

in single or double pn-junction devices [25–33] and provide direct signature of quantum interference of ballistic electron trajectories bouncing between the two pn junctions of the top-gated region. As illustrated in Fig. 2, the two electrodes of the split-gate form a constriction but also Fabry-Pérot cavities beneath their long section with parallel edges. There, the two parallel and partially transmitting pn junctions induce multiple interference for ballistic electrons and thus FP resonances in the transmission of the resulting np’n (pn’p) cavity [35–38]. After averaging over all incidence angles, these sharp resonances translate into smooth resistance oscillations such as those visible in Fig. 2.

Interestingly, despite the fact that the resonant cavity is cut in two parts due to the split-gate geometry, FP oscillations are observed with a significant visibility in the entire area of the bipolar regime under the top gates (Fig. 2). At low back-gate voltage, far from the diagonal ridge, the constriction is also in the bipolar regime, but forms a narrow non-resonant cavity with non-parallel pn interfaces, and the resonances average out to zero. At higher back-gate voltage, close to the diagonal ridge, the constriction is in the unipolar regime and cannot produce interference, but the measured resistance pattern is almost unperturbed. This indicates that the constriction region has a virtually negligible contribution to the total conductance.

Quantitative analysis of the FP oscillations enables to extract an effective cavity length $L_{eff}$ between the two pn junctions, which depends on the applied top-gate and back-gate voltages. The pseudo-period in charge carrier density is given by $\Delta n_{tg} = 2\sqrt{\pi n_{tg}}/L_{eff}$, where $n_{tg}$ is the charge carrier density beneath the top-gate [25, 29]. At $V_{bg} = 10$ V, one can calculate the cavity length by estimating the pseudo-period $\Delta n_{tg}$ over several oscillations. By averaging the pseudo-period between $V_{tg} = -5$ V and $-1.5$ V, we find $L_{eff} \simeq 380$ nm, which is slightly larger than the width of the top-gate electrodes. Self-consistent electrostatic simulations (see Supplementary Information) have been performed to calculate the potential profile across the npn junction (Fig. 1f), and a cavity length of 366 nm has been obtained for $V_{bg} = 10$ V and $V_{tg} = -3$ V, which is close to the value extracted from the data at the same gate voltages.

We furthermore complement this analysis by numerical simulations of the transmission through the npn junctions. We calculate the conductance by averaging the angular dependence of the transmission obtained within the WKB approximation [37] at zero magnetic field. The electrostatic potential profile is modeled by the product of two hyperbolic tangent functions, separated by a distance $d$, with a characteristic transition half-width $w$, determined by self-consistent simulations (see Supplementary Information). The resulting gate-voltage de-
dependence of the transmission shown in Fig. 2 reproduces qualitatively the experimental pattern of resistance oscillations. The curvature of the interference fringes at low back-gate voltage corresponds to a significant increase of the cavity length, due to a weaker screening of the top-gate voltage by the low carrier density in the bulk region.

We now turn to the magnetic field dependence of the FP oscillations, which enables to reveal additional graphene properties. In particular, FP oscillations can be used to evidence the Berry phase in graphene, through magnetic steering of the ballistic trajectories [25-27, 29, 39, 40]. The magnetic field dependence of the resistance oscillations is displayed in Fig. 3 at a back-gate voltage \( V_{bg} = 10 \) V. Upon increasing \( B \), the oscillations slightly shift towards more negative \( V_{tg} \) and undergo a sudden phase jump at a magnetic field of about 0.05 T (and \(-0.05\) T). This phase jump is clearly evidenced by comparing the line-cuts at 0 and 0.1 T shown in Fig. 3b, and has been observed at several back-gate voltages (data not shown). This phase jump, that has been observed in earlier works using top-gated graphene devices [25-27, 29, 39], originates from closed orbits in momentum space. At low field, cyclotron orbits within the top-gated region do not enclose the origin (see Fig. 3f). Upon increasing magnetic field, trajectories bend further, until the transverse momentum changes sign, causing the orbit in \( k \)-space to enclose the Dirac point (see Fig. 3f). At this particular field, electrons pick up an additional Berry phase of \( \pi \), causing a sudden phase shift of the oscillation pattern.

One can geometrically calculate that this shift occurs for a critical field \( B_0 \approx \hbar k_F e/L_{eff} \), where \( h \) and \( e \) are the reduced Planck constant and electron charge respectively, \( k_F \) is the Fermi wave-vector in the central region and \( \theta \) is the typical incidence angle of the trajectories contributing the most effectively to the FP interference [25, 29]. This typical incidence angle on the barrier emerges from the strong angular dependence of the tunneling transmission through a smooth pn junction (Klein collimation) [30]. It can be calculated as the angle for which the transmission is 1/2, giving \( \theta = \sqrt{\ln(2) eE/\pi \hbar v_F k_F^2} \) where \( E \) is the electric field at the pn junction. Self-consistent simulations for \( V_{bg} = 10 \) V and \( V_{tg} = -3 \) V (\( n = p = 6.2 \times 10^{11} \text{ cm}^{-2} \)) give the value \( E = 3.3 \times 10^5 \text{ V/m} \) (see Supplementary Information), which results in \( \theta = 13^\circ \) and \( B_0 = 0.057 \) T using \( L_{eff} = 380 \) nm. The good agreement with the experimental value of \( B_0 \) confirms the Berry phase origin of the observed phase shift.

Next, we show that on increasing further the magnetic field towards the quantum Hall regime, the contribution of the QPC emerges and becomes predominant in the conductance.

Figure 3e displays the derivative \( dR_L/dV_{tg} \) versus a larger range of \( V_{tg} \) and \( B \), at \( V_{bg} = 10 \) V. The graph can be divided into unipolar \( nn'n \) (right) and bipolar \( np'n \) (left) regimes separated by the Dirac point in the top-gated region located at \( V_{tg} = -1.5 \) V. In the bipolar region, the FP oscillations disperse with magnetic field and eventually disappear at a field of about 0.25 T.

The vanishing of FP oscillations corresponds to the field \( B^* = \hbar k_F /eL_{eff} \) at which the cyclotron radius equals the effective length \( L_{eff} \) of the cavity. At \( V_{bg} = 10 \) V and \( V_{tg} = -3 \) V, we calculate \( B^* \approx 0.24 \) T using \( L_{eff} = 380 \) nm, in excellent agreement with the data. Below \( B^* \), the electron trajectories inside the top-gated region bend under the magnetic field, but still reach the far side of the cavity and are partially transmitted (Fig. 3f).

Above \( B^* \), the cyclotron radius is smaller than the cavity length, and the electron trajectories, which are close to normal incidence due to collimation, do not reach the opposite side of the cavity. Interference no longer occurs, resulting in the fading of the oscillations. Simultaneously, snake orbits appear around the top-gated regions, guiding the electrons towards the constriction (Fig. 3f). The field \( B^* \) therefore marks the onset of electron transport through the constriction. Notice that resistance oscillations characteristic of these peculiar paths are not expected here, since snake trajectories have no possibility in our split-gate geometry to return to the injector side, contrary to devices with a continuous top-gate crossing the entire width [33, 11, 12].

Above 0.25 T, a second set of oscillations emerges (Fig. 3f), which disperse with magnetic field and bend toward higher positive top-gate voltages. This dispersion is non-linear, which is particularly visible in the unipolar regime. Such a behavior is at odds with the linear dispersion of Landau levels expected to appear in this range of magnetic field. Given the electronic mobility \( \mu = 85000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \), Landau levels should appear roughly above a field such that \( B = 1/\mu = 0.12 \) T. To understand the origin of the second set of oscillations, we first calculate the number of Shubnikov-de-Haas oscillations expected in the top-gated region at \( V_{bg} = 10 \) V, within the \( V_{tg} \) range from \(-1.5 \) to 5 V. Since the Fermi energy varies from 0 to 121 meV and given the graphene Landau quantization \( \epsilon_N = v_F \sqrt{2eBN} \), \( N \) being the Landau level index, this corresponds to a total of 39 Landau levels at 0.7 T (the maximum field of Fig. 3f). However, only around 10 oscillations can be observed here, such that they cannot originate from the top-gated region.

The origin of these features is revealed by measuring their relative coupling to the gates. Figure 4 presents the dependence of the oscillations with back-gate and top-gate voltages, at different magnetic fields. At \( B = 0.2 \) T (Fig. 4a), the FP oscillations are visible in the bipolar regime. From \( B = 0.6 \) T on, another set of parallel lines appears extending from the unipolar to the bipolar regime, with a smaller slope than that of the Dirac ridge of the top-gated area (Fig. 4c, d). Since Landau levels in the bulk graphene should manifest as horizontal lines independent of the top-gate voltage, and those
FIG. 4. (a-d) Derivative of the longitudinal resistance $dR_L/dV_{bg}$ versus $V_{bg}$ and $V_{tg}$ at $B = 0.2$ T, 0.3 T, 0.6 T, and 1 T. (e) $R_L$ versus $V_{bg}$ and $V_{tg}$ at $B = 1$ T. The pattern is typical of edge-channel transport in the quantum Hall regime through a split-gated device. Dashed lines are guides for the eye showing the limits of regions with different filling factors $\nu_b$ and $\nu_{QPC}$ in the bulk and QPC regions, respectively.

in the top-gated area should be parallel to the diagonal Dirac ridge, these new lines hence correspond to a region with an intermediate capacitive coupling, which is the region located between the two electrodes of the split-gate [10]. From the slope of these new lines, we extract a coupling ratio $C_{QPC}^{tg}/C_{bg}$ = 1.4, a value close to the value 1.2 obtained from a self-consistent electrostatic simulation of the QPC geometry (see Supplementary Information). One can notice small variations in the slope of these lines, which could result from a specific screening of the electrical field in the region of the constriction. Knowing the capacitive coupling of the QPC, the charge carrier density in the QPC can be calculated for any voltages ($V_{bg}$, $V_{tg}$). At $B = 1$ T, for $V_{bg} = 10$ V and $V_{tg}$ varying from $-5$ to 5 V, the Fermi energy in the QPC varies from 50 to 121 meV, which corresponds to a change from 2 to 11 Landau levels. Such a change is fully consistent with the 9 oscillations observed over this range in Fig. 4. Similarly, for the data previously discussed in Fig. 3, at $B = 0.7$ T, we calculate a variation of 9 Landau levels between $V_{tg} = -1.5$ and 5 V, very close to the 10 observed experimentally. This analysis leads us to conclude that the new set of dispersing oscillations in Fig. 3 are nothing but the Shubnikov-de Haas oscillations in the QPC, which emerge above $B^*$, once the cyclotron radius is smaller than the length of the nnp cavity formed by the top gates.

At $B = 1$ T, the longitudinal resistance displayed in Fig. 4 exhibits plateaus with a parallelogram shape delimited by lines of equal filling factors in the bulk graphene (horizontal lines) and in the QPC (the new diagonal lines discussed above). The horizontal strips of constant bulk filling factor $\nu_b$ and the diagonal strips of constant filling factor in the QPC $\nu_{QPC}$ are indexed in the figure. The observation of resistance plateaus indicates the onset of the quantum Hall regime, with an electron transmission controlled by the QPC. However, at the magnetic field considered here, which is too small for broken symmetry state to develop and to open a gap between electron and holes, the exact value of the resistance within each plateau results from current equilibration between co-propagating edge channels along the pn interfaces [8–10]. In this case, as shown in Ref. [10], the conduction through the system is governed by three filling factors: $\nu_b$, $\nu_{QPC}$, and also $\nu_g$ below the top gates, with different equilibration rules in the unipolar and bipolar regimes of the top gates.

In conclusion, we have studied the evolution with magnetic field of an electrostatically-defined constriction in high-mobility graphene. At low magnetic field, Fabry-
Pérot oscillations within the ballistic top-gated region develop and exhibit at a particular magnetic field a phase shift due to the graphene Berry phase. Upon entering the quantum Hall regime, Shubnikov-de-Haas oscillations corresponding to Landau levels in the constriction take place, marking the onset of the conduction regime dominated by the QPC. We identify the magnetic field $B^*$ at which the transition between the two regimes occurs, that is, when the cyclotron radius becomes smaller than the Fabry-Pérot cavity length. Our findings show that signatures of the split-gate defined constriction can emerge at very-low magnetic field due to the guiding of electron towards the constriction via snake trajectories. Our study provides a complete understanding of the functioning of a split-gate-defined constriction in monolayer graphene from zero-magnetic field up to the quantum Hall regime.

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**A. SELF-CONSISTENT ELECTROSTATIC SIMULATION OF THE GATE-DEFINED GRAPHENE QPC**

![Simulation Domain](image)

**FIG. S1.** (a) Simulation domain of size $2 \times 2 \times 1.3 \, \mu m^3$. The split-gate is 300 nm wide, 100 nm thick, and the gap is 200 nm wide. The SiO$_2$ layer is 280 nm thick. The bottom (top) BN layer is 20 nm (40 nm) thick. The SiO$_2$ and BN dielectric constants are set to $\epsilon_{r, SiO_2} = 3.9$ and $\epsilon_{r, BN} = 4.0$. The top surface in vacuum is set at zero voltage. The back-gate voltage is applied on the bottom surface. The top-gate voltage is applied on all faces of the split-gate volume. The zero-surface-charge boundary condition is applied on the lateral surfaces of the simulation domain. (b) Self-consistent electrostatic energy profile $E = -eV$ across the npn junction, for a continuous top gate, at $V_{bg} = 3 \, V$ and $V_{tg}$ varying from 0 to -5 V. (c) Cavity length defined as the distance between the two zeros of density in (b). The cavity length is larger than the gate size due to the close proximity of the top gate as compared to the back gate. (d) Normalized energy profiles (black curves) identical to those shown in (b) and analytical function (red curve) modeling the ensemble of energy profiles. (e) Self-consistent carrier density in the graphene sheet for $V_{bg} = 3 \, V$ and $V_{tg} = -2.1 \, V$. The position of the zero-density curve (dashed line) indicates that the QPC contains the same type of carriers as the bulk at this gate voltages (electrons corresponds to negative density). The position of the split-gate is represented by the white line. (f) Density in the bulk of the graphene sheet (green dots), below the top gates far from the constriction (blue dots), and at the saddle-point of the constriction (red dots), for $V_{bg} = 3 \, V$ and $V_{tg}$ varying from 0 to -5 V. The blue arrow at $V_{tg} = -0.4 \, V$ indicates the charge neutrality condition below the top gates and the red arrow at $V_{tg} = -2.5 \, V$ indicates the charge neutrality condition in the QPC.

To complement and support our analysis of the split-gated graphene device, we carried out self-consistent electrostatic simulations of the carrier density in the graphene plane as a function of the back-gate and top-gate voltages. The geometry of the simulation is represented in Fig. S1(a). The dimensions of the split-gate and the thicknesses of the SiO$_2$ and BN layers are close to that of the measured device. The graphene sheet is modeled by a charge density $\sigma$ linked to the electrostatic potential $V$ by the relation:

$$\sigma = (-\epsilon) \text{sign}(V) \frac{e^2 V^2}{\pi \hbar^2 v_F^2}.$$ 

The mesh grid is computed using Gmsh [http://gmsh.info] and the electrostatic problem is solved self-consistently using a modified version of MaxFEM [http://www.usc.es/en/proyectos/maxfem], an electromagnetic simulation software based on the finite element method.
Figure 1(c) of the article shows the profile of the potential energy $E = -eV$ across the top-gated region (using a continuous top-gate without gap) for a fixed back-gate voltage $V_{bg} = 3$ V and various top-gate voltages $V_{tg} = -0.2, -0.4, -1, -2$ V. A larger set of potential profiles is plotted in Fig. S1(b). Above the threshold of the bipolar regime (npn), the cavity length between the two zeros of density increases quickly to the gate size of 300 nm, and then continues to increase slowly to larger values (Fig. S1(c)).

For comparison with the experimental value of the cavity length $L_{eff} = 380$ nm measured at $V_{bg} = 10$ V and $V_{tg} = -3$ V, we computed the potential profile at these gate voltages and obtained a cavity length of 366 nm, in good agreement with the experimental value. The electric field $E = -dV/dx$ at the pn interface can also be obtained from this potential profile and we obtain $E = 3.3 	imes 10^6$ V/m. This value is used in the article to calculate the collimation angle $\theta$ through the pn interface.

For the transport simulations described in the next section, the potential profile needs to be modeled by an analytical function. We use the exact potential profiles calculated at $V_{bg} = 3$ V and $V_{tg}$ varying between 0 and -5 V (Fig. S1(b)) to obtain an averaged profile (Fig. S1(d)) which is then modeled by the function:

$$f(x) = \frac{1}{4} \left( 1 + \tanh \left( \frac{x + d/2}{w} \right) \right) \left( 1 + \tanh \left( \frac{d/2 - x}{w} \right) \right)$$

This modeled profile is the product of two step functions separated by a distance $d = 380$ nm with a characteristic transition half-width $w = 60$ nm. This value of $d$ is taken as a fixed parameter for all gate voltages during the transport simulations. Note that $d$ is not equal to the cavity length $L_{eff}$, which instead strongly depends on the gate voltages as discussed above.

Finally, the capacitive coupling between the QPC and the gates is obtained from the calculation of the spatial distribution of the carrier density within the graphene sheet (Fig. S1(e)) for the split-gate geometry (Fig. S1(a)). The densities in the bulk, below the top gates, and at the saddle-point of the constriction, are plotted in Fig. S1(f) for a fixed back-gate voltage and various top-gate voltages. For $V_{bg} = 3$ V, the charge neutrality in the QPC is obtained $V_{tg} = -2.5$ V. The capacitance between the top gate and the QPC is thus found to be 1.2 times larger than the back-gate capacitance, in correct agreement with the experimental value of 1.4 obtained from Fig. 4(d) of the article. The small discrepancy might result from the experimental uncertainty on the precise dimensions of the split-gate, and on the thickness and dielectric constant of the BN flakes.

**B. SIMULATION OF THE CONDUCTANCE THROUGH THE GATE-DEFINED NPN JUNCTION**

Numerical simulations of the transmission through the npn junction have been performed to compare with experimental data. For a given incident angle $\theta$ on the left pn interface, the transmission $T$ through the npn junction can be calculated in the WKB approximation [37] at zero magnetic field with the formula:

$$T(\theta) = \left| \frac{t_1 t_2}{1 - r_1 r_2 \exp(i2\phi)} \right|^2$$

The coefficient $t_1 = e^{-\lambda}$ with $\lambda = \int_{x_1}^{x_1'} (k_y^2 - k_F^2)^{1/2} dx$ is the transmission amplitude across the pn interface which forms a tunneling barrier between the turning points $x_1'$ and $x_1$ (Fig. S2(a)). The coefficient $r_1 = (1 - e^{-2\lambda})^{1/2}$ is the reflection amplitude at the pn interface from the inside the npn cavity. The quantity $\phi = \int_{x_1}^{x_2} (k_y^2 - k_F^2)^{1/2} dx$ is the phase accumulated in the central region between the turning points $x_1$ and $x_2$. At zero magnetic field, the two pn interfaces have the same transmission and reflection coefficients $t_1 = t_2$ and $r_1 = r_2$.

The parallel wave vector $k_y(\theta) = k_y^\text{bulk} \sin(\theta)$ is conserved and fixed by the incidence angle $\theta$ (Fig. S2(b)), whereas the total Fermi wave vector $k_F(x) = k_F^\text{bulk} + (k_F^\text{barrier} - k_F^\text{bulk}) \times f(x)$ is a function of the position $x$ across the npn junction. The Fermi wave vectors $k_F^\text{bulk}$ and $k_F^\text{barrier}$ are obtained from the density $n$ within each region by $k_F = \sqrt{\pi n}$. The function $f(x)$, that characterizes both the energy profile $E_F(x)$ and the wave vector profile $k_F(x) = E_F(x)/\hbar v_F$, is modeled by the product of two step functions separated by a fixed distance $d = 380$ nm with a characteristic transition half-width $w = 60$ nm (see details in the previous section).

The angular dependence of the transmission $T(\theta)$ shows a series of resonant peaks corresponding to constructive interference (red curve in Fig. S2(c)). The presence of a tunneling barrier at the pn interface is responsible for the Klein collimation effect with a small range of incident angles being transmitted (blue curve). The resonant peak around $\theta = 13^\circ$ corresponds to half transmission of the pn interface and gives the main contribution to the conductance oscillations.
FIG. S2. (a) Schematics of the npn junction, with a Fermi wave vector in the n region determined by the back-gate only, and a Fermi wave vector in the p region determined by both the back-gate and the top-gate. (b) Due to conservation of the parallel wave vector $k_y$, an electron arriving at the Fermi level with incident angle $\theta$ has to cross a tunneling barrier at the pn interface (region in gray). The classical turning points $x_1'$ and $x_1$ (resp. $x_2$ and $x_2'$) for the first (second) pn interfaces are obtained as the intersections of the potential profile $k_F(x)$ with the constant $\pm k_y$ horizontal lines. (c) Angular dependence of the transmission $T(\theta)$ calculated for $V_{bg} = 3$ V, $V_{tg} = -1$ V, and the potential profile described in the text (red line). The red dashed lines indicated the envelope of the interference peaks ($\phi \equiv 0$ and $\pi$). The blue line is the transmission $|t_1|^2$ of a single pn interface showing the Klein collimation effect.

The conductance $G$ of the npn junction is obtained by averaging the angular dependence of the transmission according to the formula:

$$G = \frac{4e^2}{h} \frac{W k_{F,\text{bulk}}}{\pi} \int_{-\pi/2}^{\pi/2} T(\theta) \frac{d\theta}{\pi}$$

where $W$ is the width of the sample. The choice of a uniform averaging over the incident angle $\theta$ is justified by the fact that all angles are almost equiprobable for our sample geometry (see Fig. 1a in the article), as opposed to experiments on nanoribbons with transverse quantization of the incident wave vector.

The conductance oscillations calculated as a function of the top-gate and back-gate voltages are shown in Fig. 2(d) of the article. They reproduce correctly the experimental pattern of resistance oscillations in Fig. 2(c). Interestingly, the curvature of the interference fringes at low back-gate voltage corresponds to a significant increase of the cavity length, which results from a weaker screening of the top-gate voltage by the low carrier density in the bulk region.