A note on nonlinear electrodynamics

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Abstract – We explore the physical consequences of a new nonlinear electrodynamics, for which the electric field of a point-like charge is finite at the origin, as in the well-known Born-Infeld electrodynamics. However, contrary to the latter, in this new electrodynamics the phenomena of birefringence and dichroism take place in the presence of external magnetic fields. Subsequently we study the interaction energy, within the framework of the gauge-invariant but path-dependent variables formalism. Interestingly enough, the static potential profile contains a linear potential leading to the confinement of static charges.

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Introduction. – One of the most startling predictions of Quantum Electrodynamics (QED) is the light-by-light scattering in vacuum and its physical consequences such as vacuum birefringence and vacuum dichroism [1–6]. Very recently, the ATLAS Collaboration has reported on the direct detection of the light-by-light scattering in LHC Pb-Pb collisions with a 4.4 σ level of confidence [7]. Actually, this proposal to look for light-by-light scattering in ultra-peripheral heavy-ion collisions at the LHC was pioneered in ref. [8]. More recently, inspired by these results, a bound on the β-parameter in the nonlinear Born-Infeld electrodynamics was obtained [9].

On the other hand, in recent times nonlinear electrodynamics have been the object of intensive investigations in the context of black hole physics [10,11]. The interest in studying these electrodynamics is mainly due to the possibility of constructing exact (regular) black hole solutions. In addition, nonlinear electrodynamics has also attracted interest in order to explain the Rindler acceleration as a nonlinear electromagnetic effect [12].

In previous works [13–15], we studied different models of (3 + 1)-D nonlinear electrodynamics in vacuum, and showed that for generalized Born-Infeld, and logarithmic electrodynamics the field energy of a point-like charge is finite. It should be further noted that generalized Born-Infeld, exponential, logarithmic and massive Euler-Heisenberg–like electrodynamics display the vacuum birefringence phenomenon.

Motivated by these observations and given the ongoing experiments related to light-by-light scattering, it should be interesting to acquire a better understanding of what might be the observational signatures presented by vacuum electromagnetic nonlinearities. Hence, our purpose here is to consider a new nonlinear electrodynamics and investigate aspects of birefringence and dichroism, as well as the computation of the static potential along the lines of [13–15], which is an alternative to the Wilson loop approach.

It should be further noted that in the papers of refs. [13–15], we have considered electrodynamical models with other types of nonlinearities. Their intrinsic properties have been discussed. Like the present model, they also exhibit the property of vacuum birefringence and electric charge confinement; however, contrary to the other three previous cases we have investigated, the present model displays an additional property, namely, vacuum dichroism. And, indeed, one of the motivations to write down the model under consideration was to get a model with a less usual property: vacuum dichroism.

Our work is organized as follows: in the next section, we describe this new nonlinear electrodynamics and study aspects of birefringence and dichroism. In the third section we calculate the interaction energy for a fermion-antifermion pair. Interestingly enough, the static potential profile contains a linear term, leading to the confinement...
of static charges. Finally, some concluding remarks are made in the last section.

In our conventions the signature of the metric is \((+1, -1, -1, -1)\).

**The model under consideration.** – In this section, we begin our analysis with a brief description of the model under consideration. This will provide the theoretical setup for our work and fix the notation. In this case the corresponding model is governed by the Lagrangian density:

\[
\mathcal{L} = -\lambda^2 \sqrt{-\mathcal{F}} \left[ \frac{1}{\sqrt{2\lambda + \sqrt{-\mathcal{F}}}} \right],
\]

where \(\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\), and \(\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda}\) is the dual electromagnetic field strength tensor. The constant \(\lambda\) has \((mass)^2\) dimension in natural units. Let us also mention here that, in a purely electric case, the \(\lambda\) constant plays the role of a uniform background electric field as it shall become clear in what follows.

The parameter \(\lambda\) must be positive and it is a sort a cut-off for the electric and magnetic fields, \(\lambda \gg |E|\) and \(|B|\). Also, we must stress that our effective model only applies for electromagnetic fields such that \(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (E^2 - B^2) \geq 0\).

The mass scale, \(M\), fixed by the \(\lambda\)-parameter \((M = \sqrt{\lambda})\), characterizes a regime where the nonlinearity of the electromagnetic interaction becomes relevant. We can therefore associate \(\lambda^{-1/2}\) to the length scale that appears in brane scenarios, where the particles can be localized in lower-dimensional branes which are separated from each other by a distance in the \(\lambda^{-1/2}\)-scale. We believe \(\gtrsim 1\) TeV. This lower bound is based on the results of the recent paper quoted in ref. [9], where the authors show that the recent measurement of the light-by-light scattering by the ATLAS Collaboration is compatible with the \(M\) mass scale \(\gtrsim 1\) TeV.

Accordingly the field equations read

\[
\partial_\mu \left[ \lambda^2 \frac{F^{\mu\nu}}{2\sqrt{2\lambda + \sqrt{-\mathcal{F}}}} \right] = 0,
\]

while the Bianchi identity is given by

\[
\partial_\mu \tilde{F}^{\mu\nu} = 0.
\]

It should be further noted that Gauss’ law reduces to

\[
\nabla \cdot \left( \frac{\lambda^3}{\sqrt{E^2 - B^2} (2\lambda + \sqrt{E^2 - B^2})} E \right) = 0,
\]

From eq. (4) it follows that, for an external point-like charge sitting at the origin, the \(D\)-field lies along the radial direction and is given by \(D = \frac{Q}{r}\), where \(Q = \frac{e}{4\pi}\). It is also important to observe that for a point-like charge, \(e\), at the origin, the electrostatic field is given by

\[
|E| = -2\lambda + \left( \frac{\lambda^3}{Q} \right)^{1/2} r.
\]

Before we proceed further, we shall pause to ascertain that from eq. (1) we recover the Coulomb law. To do this, we consider the \(\lambda \gg \sqrt{-\mathcal{F}}\) regime in eq. (1), that is

\[
\mathcal{L} = \frac{1}{2} (\nabla E) - \frac{\lambda}{\sqrt{2\lambda + \sqrt{-\mathcal{F}}}},
\]

From this expression it follows that the field equations reduce to

\[
\partial_\mu \left( 1 - \frac{\lambda}{\sqrt{2\lambda + \sqrt{-\mathcal{F}}}} \right) F^{\mu\nu} = 0.
\]

It may now easily be verified that for a point-like charge, \(e\), at the origin, the electrostatic field becomes

\[
|E| = \frac{Q}{r^2} + \lambda.
\]

We thus find that expression (1) is compatible with Coulomb law in the \(\lambda \gg \sqrt{-\mathcal{F}}\) regime.

In order to write the dynamical equations into a more compact and convenient form, we shall introduce the vectors \(D = \partial \mathcal{L}/\partial E\) and \(H = -\partial \mathcal{L}/\partial B\), in analogy to the electric displacement and magnetic field strength. We then have

\[
D = \frac{2\lambda^3}{\sqrt{E^2 - B^2} (2\lambda + \sqrt{E^2 - B^2})} E,
\]

and

\[
H = \frac{2\lambda^3}{\sqrt{E^2 - B^2} (2\lambda + \sqrt{E^2 - B^2})} B.
\]

With this, we can write the corresponding equations of motion as

\[
\nabla \cdot D = 0, \quad \frac{\partial D}{\partial t} - \nabla \times H = 0,
\]

and

\[
\nabla \cdot B = 0, \quad \frac{\partial B}{\partial t} + \nabla \times E = 0.
\]

It is now important to notice that the complicated field problem can be greatly simplified if the above equations are linearized. As is well known, this procedure is justified for the description of a weak electromagnetic wave \((E_p, B_p)\) propagating in the presence of a strong constant external field \((E_0, B_0)\). For computational simplicity our analysis will be developed in the case of a purely magnetic field, that is, when the external electric field \(E_0 = 0\). This then implies that

\[
D = \Gamma E_p,
\]

and

\[
H = \Gamma \left[ B_p - \frac{\lambda}{2\Gamma} (B_0 \cdot B_p) B_0 \right]
\]

with

\[
\Gamma = \frac{1}{2} \left( 1 + \frac{i\lambda}{\sqrt{B_0^2}} \right),
\]

where we have kept only linear terms in \(E_p, B_p\).
\[ \varepsilon_{ij} = \Gamma \delta_{ij}, \]  
and
\[ (\mu^{-1})_{ij} = \Gamma \left( \delta_{ij} - \frac{i\lambda}{2\Gamma(B_0^2)^{3/2}} B_{0i} B_{0j} \right). \]

Next, without restricting generality we take the z-axis as the direction of the magnetic field, \( B_0 = B_0 \hat{e}_3 \), and assuming that the light wave moves along the x-axis. We further make a plane wave decomposition for the fields \( E_p \) and \( B_p \), that is
\[ E_p(x,t) = E e^{-i(wt-kx)}, \quad B_p(x,t) = B e^{-i(wt-kx)}, \]
so that the Maxwell equations become
\[ \left( \frac{k^2}{u^2} - \varepsilon_{22} \mu_{33} \right) E_2 = 0, \]  
and
\[ \left( \frac{k^2}{u^2} - \varepsilon_{33} \mu_{22} \right) E_3 = 0. \]

Here, it is worth to remark that the equations above, (19) and (20), were obtained in the limit \( B_0 \gg B_p \) and \( \lambda \gg |B_0| \).

As a consequence, we have two different situations: First, if \( E \perp B_0 \) (perpendicular polarization), from (20) \( E_3 = 0 \), and from (19) we get \( \frac{k^2}{u^2} = \varepsilon_{22} \mu_{33} \). Hence we see that the dispersion relation of the photon takes the form
\[ n_{\perp} = \sqrt{1 + \frac{i}{2\Gamma(B_0^2)}}. \]

Second, if \( E \parallel B_0 \) (parallel polarization), from (19) \( E_2 = 0 \), and from (20) we get \( \frac{k^2}{u^2} = \varepsilon_{33} \mu_{22} \). In this case, the corresponding dispersion relation becomes
\[ n_{\parallel} = \sqrt{\Gamma}. \]

We then easily verify that there are two optical features for the model under consideration. First, the electromagnetic waves with different polarizations have different velocities or, more precisely, the vacuum birefringence phenomenon is present. The second point is related to the existence of an imaginary part of the index of refraction which gives rise to vacuum dichroism. As is well known, this refers to the absorption of photons in a vacuum depending on photon polarization.

In view of this important result it will be useful to have an \textit{ab initio} explanation of it starting with a generic Lagrangian. For this purpose, we shall calculate the full energy-momentum tensor of the system (photon + external field) to get the energy-momentum balance equation and then to account for the photon energy exchanged with the external field. To this end, we consider a generic Lagrangian
\[ \mathcal{L} = \mathcal{L}(F, G), \]
where \( F \equiv -\frac{i}{2} F^2 = \frac{1}{2}(E^2 - B^2) \) and \( G \equiv -\frac{i}{2} F \cdot \tilde{F} = E \cdot B \). In such a case, the energy-momentum tensor comes out as given below
\[ \Theta^\mu_\kappa = \frac{\partial \mathcal{L}}{\partial F^{\mu\nu}} F_{\nu\kappa} + \frac{\partial \mathcal{L}}{\partial G^{\mu\nu}} G_{\nu\kappa} - \delta^\mu_\kappa \mathcal{L}, \]

Thus, we have
\[ \partial_\mu \Theta^\mu_\kappa = j^\mu F_{\mu\kappa}, \]
where \( j^\mu \) is some external distribution of charges and currents. In our case, we set it to zero. Next, we split \( F_{\mu\nu} \) as the sum of an external background \( F_{\mu\nu}^{\text{ext}} \), and a small fluctuation (the photon field), \( f_{\mu\nu} \), namely, \( F_{\mu\nu} = f_{\mu\nu} + F_{\mu\nu}^{\text{ext}} \). Therefore, eq. (25), up to quadratic terms in the fluctuations, can be expressed as
\[ \partial_\mu (\Theta^{\text{ext}} + \Theta^{\text{ext}}, f + \Theta^{f,f})^\mu_\kappa = j^\mu F_{\mu\kappa} + j^\mu f_{\mu\kappa}. \]
where, in the full energy-momentum tensor, we have separated the part that corresponds to the external field, another piece which mixes the external field and the fluctuation and, finally, the purely photonic contribution. From eq. (26), it can be readily seen that, in the case of external currents are absent and the external background is constant, the exchange of energy and momentum between the photon and the external field is expressed as
\[ \partial_\mu (\Theta^{\text{ext}} + \Theta^{\text{ext}}, f + \Theta^{f,f})^\mu_\kappa = -\partial_\mu (\Theta^{\text{ext}}, f)^\mu_\kappa. \]
It is precisely the right-hand side of eq. (27) that illustrates the photon’s energy absorbed by the background field.

\textbf{Interaction energy.} – With these considerations in mind, we shall now examine the interaction energy between static point-like sources for the model under study. To this end, we will calculate the expectation value of the energy operator \( \mathcal{H} \) in the physical state \( \langle \Phi \rangle \), along the lines of refs. [13–15]. Here it is worth emphasizing that, consistently with the approximation of the previous section (\( \lambda \gg \sqrt{-F} \)), the initial point of our analysis is the Lagrangian density
\[ \mathcal{L} = \left( \frac{1}{2} \right) (-\mathcal{F}) - \frac{\lambda}{2\sqrt{2}} \sqrt{-F}. \]

As we have indicated in [13–15], to handle the exponent \( 1/2 \) in expression (28), we incorporate an auxiliary field \( v \) such that its equation of motion gives back the original theory. Therefore the corresponding Lagrangian density takes the form
\[ \mathcal{L} = -\frac{1}{4} \left( \frac{1}{2} - \frac{2\lambda}{\sqrt{2}} \right) F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{8\sqrt{2}} \frac{1}{v}. \]

With the redefinition \( \frac{1}{v} = \frac{1}{2} - \frac{\lambda}{\sqrt{2}} v \), eq. (29) becomes
\[ \mathcal{L} = -\frac{1}{4} \frac{1}{V} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} \frac{V}{\left( \frac{1}{2} - 1 \right)}. \]
It is worthwhile sketching at this point the canonical quantization of this theory from the Hamiltonian analysis point of view. It may now easily be verified that the canonical momenta are \( \Pi^0 = -\frac{1}{\lambda} F^{0a} \), so one immediately identifies the two primary constraints \( \Pi^0 = 0 \) and \( p \equiv \frac{\partial H}{\partial \dot{\phi}} = 0 \). Furthermore, the momenta are \( \Pi_i = \frac{\partial H}{\partial \dot{v}_i} \). Here \( E_i = F_{i0} \). In such a case, the canonical Hamiltonian reduces to

\[
H_C = \int d^3x \left\{ \Pi_i \partial^i A_0 + \frac{V}{2} \Pi^2 + \frac{1}{2V} B^2 \right\} + \frac{1}{8} \int d^3x \frac{V}{[\sqrt{2}V - 1]}. \tag{31}
\]

Next, we also notice that by requiring the primary constraint \( \Pi^0 \) to be preserved in time, one obtains the secondary constraint \( \Gamma_1 = \partial_t \Pi^0 = 0 \). Similarly for the constraint \( p \), we get the auxiliary field \( v \) as

\[
V = 2 \left( 1 + \frac{\lambda}{2 \sqrt{\Pi^2}} \right), \tag{32}
\]

which will be used to eliminate \( V \). We observe that to get this last expression we have ignored the magnetic field in eq. (31), because it add nothing to the static potential calculation, as we will show below. According to usual procedure, the corresponding total Hamiltonian that generates the time evolution of the dynamical variables is \( H = H_C + \int d^2x (u_0(x) \Pi_0 + u_1(x) \Gamma_1(x)) \), where \( u_0(x) \) and \( u_1(x) \) are the Lagrange multiplier utilized to implement the constraints. It is a simple matter to verify that \( \dot{A}_0 = [A_0, H] = u_0(x) \), which is an arbitrary function. Since \( \Pi^0 = 0 \) always, neither \( A_0 \) nor \( \Pi^0 \) are of interest in describing the system and may be discarded from the theory. Hence, we can write

\[
H = \int d^3x \left\{ w(x) \partial^i \Pi_i + V \Pi^2 \right\} + \frac{1}{8} \int d^3x \frac{V}{[\sqrt{2}V - 1]}, \tag{33}
\]

where \( w(x) = u_1(x) - A_0(x) \) and \( V \) is given by (32).

We can at this stage impose a gauge condition, so that in conjunction with the constraint \( \Pi^0 = 0 \), it is rendered into a second class set. A particularly convenient choice is

\[
\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^i A_i(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0, \tag{34}
\]

where \( 0 \leq \lambda \leq 1 \) is the parameter describing the space-like straight path \( z^i = \xi^i + \lambda (x - \xi)^i \), and \( \xi \) is a fixed point (reference point). We also recall that there is no essential loss of generality if we restrict our considerations to \( \xi \xi^0 = 0 \). Hence the only nontrivial Dirac bracket for the canonical variables is given by

\[
\{ A_i(x), \Pi^0(y) \}^* = \delta_i^0 \delta^{(3)}(x - y)
\]

\[-\partial^i \int_0^1 d\lambda x^i \delta^{(3)}(\lambda x - y). \tag{35}
\]

We now proceed to compute the interaction energy for the model under consideration. As mentioned above, to do that we need to compute the expectation value of the energy operator \( H \) in the physical state \( | \Phi \rangle \). Following Dirac [16], we write the physical state \( | \Phi \rangle \) as

\[
| \Phi \rangle \equiv | \Psi(y)\Psi(y') \rangle = \bar{\psi}(y) \exp \left( ie \int_{y'}^y dz^i A_i(z) \right) \psi(y') | 0 \rangle, \tag{36}
\]

where \( | 0 \rangle \) is the physical vacuum state and the line integral appearing in the above expression is along a space-like path starting at \( y' \) and ending at \( y \), on a fixed time slice. The above expression clearly shows that each of the states \( | \Phi(\xi) \rangle \) represents a fermion-antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance.

Taking the above Hamiltonian structure into account, we see that

\[
\Pi_\lambda(x) | \Psi(y)\Psi(y') \rangle = \bar{\Psi}(y) \Psi(y') | \Pi_\lambda(x) | 0 \rangle
\]

\[+ \int_{y'}^y \int d^3z \delta^{(3)}(z - x) | \Phi \rangle. \tag{37}
\]

As a consequence of this, by employing (37), (33) and (32), the interaction energy takes the form

\[
\langle H \rangle_{\Phi} = \langle H \rangle_0 + V_1 + V_2, \tag{38}
\]

where \( \langle H \rangle_0 = \langle 0 | H | 0 \rangle \). The \( V_1 \) and \( V_2 \) are given by

\[
V_1 = \int d^3x | \Phi | \Pi_\lambda^2 | \Phi \rangle, \tag{39}
\]

and

\[
V_2 = \lambda \int d^3x | \Phi | \sqrt{\Pi^2} | \Phi \rangle. \tag{40}
\]

At this point we should mention that the reason why we eliminated from the Hamiltonian the magnetic field now becomes clear, that is, the commutator for the magnetic field is zero.

Following our earlier procedure [17,18], the static potential turns out to be

\[
V = -\frac{e^2}{4\pi r} + e\lambda r, \tag{41}
\]

after subtracting a self-energy term.

It is of interest also to notice that the electric charge confining regime we are getting out of the model is a feature of the limit \( \lambda \gg \sqrt{-\mathcal{F}} \). This then means that we are in the deep nonlinear phase. At this energy scale, electromagnetic interaction no longer decouples from the non-Abelian electroweak processes. The non-Abelian character of the physics underlying electromagnetic interaction at this scale becomes relevant and we can, thereby, understand why electric charge confinement comes into play. A possible suggestion to test this phenomenon could be through experiments testing the electromagnetic interaction in systems like positronium or quarkonium in a scale above 100 GeV.
Before concluding this subsection it is constructive to briefly examine an alternative derivation of our previous result, which permits us to check the internal consistency of our procedure. In order to illustrate the discussion, we begin by recalling that

\[ V \equiv e(A_0(0) - A_0(L)), \]

where the physical scalar potential is given by

\[ A_0(t, r) = \int_0^1 d\lambda r^i E_i(t, \lambda r). \]  

This equation follows from the vector gauge-invariant field expression

\[ A_\mu(x) \equiv A_\mu(x) + \partial_\mu \left( - \int_\xi^x dz^\nu A_\nu(x) \right), \]

where the line integral is along a space-like path from the point \( \xi \) to \( x \), on a fixed slice time. It should again be stressed here that the gauge-invariant variables (44) commute with the sole first constraint (Gauss law), showing in this way that these fields are physical variables.

It should be noted that Gauss’ law for the present theory reads

\[ \partial_t \Pi^i = J^0, \]

where \( E^i = \Pi^i \) and \( V \) is given by eq. (32). Note that we have included the external current \( J^0 \) to represent the presence of external charges. In such a case, for two opposite charges located at \( y \) and \( y' \), \( J^0(x) = e(\delta^{(3)}(x - y) - \delta^{(3)}(x - y')) \), the electric field reduces to

\[ E(x) = \frac{e}{2\pi} \left( \frac{1}{|x - y|^2} - \frac{1}{|x - y'|^2} \right) \hat{x} + \lambda \hat{x}, \]

where \( \hat{x} = \frac{y - y'}{|y - y'|} \). Making use of this expression, we find that eq. (43) reduces to

\[ A_0(x) = - \frac{e}{2\pi} \left( \frac{1}{|x - y'|} - \frac{1}{|x - y|} + \frac{1}{|x|} - \frac{1}{|y'|} \right) - \lambda |x|, \]

Accordingly, by employing eq. (47) and after subtracting a self-energy term, we recover the potential of eq. (41) for a pair of static point-like opposite charges located at \( y \) and \( y' \), where \( r = |y' - y| \).

Final remarks. – In summary, we have considered a new nonlinear electrodynamics. It was shown that in this new electrodynamics the phenomena of birefringence and dichroism take place in the presence of external magnetic fields. Subsequently, we have studied the interaction energy. To do this, once again we have exploited a key aspect for understanding the physical contents of gauge theories, that is, the correct identification of field degrees of freedom with observable quantities. Interestingly enough, our analysis reveals that the static potential profile contains a linear potential leading to the confinement of static charges. It remains to be worked out how to connect our \( \lambda \)-parameter to the recent measurement of light-by-light scattering \([7]\) and to the PVLAS experiment of light’s nonlinearity effects. We shall be reporting on that in a forthcoming work.

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