Synthetic spin-orbit coupling in ultracold $\Lambda$-type atoms

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We consider the simulation of non-abelian gauge potentials in ultracold atom systems with atom-field interaction in the $\Lambda$ configuration where two internal states of an atom are coupled to a third common one with a detuning. We find the simulated non-abelian gauge potentials can have the same structures as those simulated in the tripod configuration if we parameterize Rabi frequencies properly, which means we can design spin-orbit coupling simulation schemes based on those proposed in the tripod configuration. We show the simulated spin-orbit coupling in the $\Lambda$ configuration can only be of a form similar to $p_x \sigma_y$ even when the Rabi frequencies are not much smaller than the detuning.

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I. INTRODUCTION

Many interesting quantum phenomena have been found in condensed matter physics when electrons have a spin-orbit coupling, such as the spin Hall effect and the topological insulator [1, 2]. Ultracold atom systems are now regarded as simulation platforms to study condensed matter physics [3], therefore it is important to realize spin-orbit couplings in these systems. It has been shown theoretically that abelian and non-abelian gauge potentials can be simulated for ultracold atoms via their interaction with laser fields, and non-abelian gauge potentials can be used to generate spin-orbit couplings [4–6]. A general spin-orbit coupling for ultracold atoms has not yet been realized experimentally, however some experimental progress towards this direction have been made [7–13].

Theoretical schemes to simulate spin-orbit couplings for ultracold atoms are usually proposed with atom-field interaction in the so-called tripod configuration, where two dark states are used to form the effective spin space [14–20]. These schemes have a drawback that the two dark states are not the lowest energy dressed states, hence atom-atom interactions can induce collisional decay. Some authors also consider the simulation of spin-orbit coupling with atom-field interaction in the $\Lambda$ configuration, where two lowest energy dressed states are used to form the effective spin space [21, 22]. Recently several experiments have realized the special spin-orbit coupling $p_x \sigma_y$ for ultracold atoms via Raman process [11, 13], which is a scheme of $\Lambda$ configuration with Rabi frequencies much smaller than the detuning. Some other kinds of methods are also proposed to simulate spin-orbit couplings in ultracold atom systems [23–25].

Although many interesting features of ultracold atoms have been theoretically found when they have a general spin-orbit coupling such as the Rashba and Dresselhaus spin-orbit couplings, currently we can only experimentally achieve the special spin-orbit coupling $p_x \sigma_y$ for ultracold atoms via Raman process. Since Raman process is a scheme of $\Lambda$ configuration with Rabi frequencies much smaller than the detuning, it is natural to ask whether more general spin-orbit couplings can be simulated in $\Lambda$ configuration when Rabi frequencies are not much smaller than the detuning. We find the answer is NO at least in our concerned $\Lambda$ configuration.

The structure of the paper is as follows. We first give an analytical expression of the simulated non-abelian gauge potentials in our concerned $\Lambda$ configuration, which can help us to design spin-orbit coupling simulation schemes based on those proposed in the tripod configuration. We then consider a spin-orbit coupling simulation scheme where two plane waves are used in the $\Lambda$ configuration. We find the simulated spin-orbit coupling can only be of a form similar to $p_x \sigma_y$ due to the non-degeneracy of the two lowest energy dressed states. We also analyze how the relative magnitude of the two lasers affect the simulated spin-orbit coupling Hamiltonian in this scheme.

II. NON-ABELIAN GAUGE POTENTIAL SIMULATION IN $\Lambda$ CONFIGURATION

A general theory on the simulation of non-abelian gauge potentials for ultracold atoms is presented in Ref. [5]. Here we focus on atom-field interaction in the $\Lambda$ configuration. As shown in FIG. 1, suppose two internal states $|1\rangle$ and $|2\rangle$ of an atom are coupled to a third common one $|3\rangle$ via laser fields with a detuning. The atom Hamiltonian will be

$$H = \frac{\hat{p}^2}{2m} + \hat{H}_0 + V,$$

where $\hat{H}_0$ is the atom-field interaction and $V$ is the possible external trapping potential. In the interaction picture,

$$\hat{H}_0 = \hbar \Delta |3\rangle \langle 3| + \hbar \Omega_1 |1\rangle \langle 3| + \Omega_2 |2\rangle \langle 3| + H.c.,$$

where $\Delta$ is the detuning that is assumed to be positive, $\Omega_1$ and $\Omega_2$ are Rabi frequencies. We parameterize two Rabi frequencies as

$$\Omega_1 = \frac{\Delta}{2} \tan 2\theta \cos \phi e^{iS_1}, \quad \Omega_2 = \frac{\Delta}{2} \tan 2\theta \sin \phi e^{iS_2},$$

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connecting applied laser fields, therefore the off-diagonal elements of that Hamiltonian \[5\]

We note that \(\Psi = (\Psi_1, \Psi_2, \Psi_3)^T\) we can find that the column vector of wave functions of the dressed states Eq. (1), we can find that the column vector of wave functions \(\Psi_1, \Psi_2, \Psi_3\) of \(\Psi_3\) are usually has a magnitude of momentum \(\bar{p}\), with \(\bar{p}^2 \ll \hbar^2\), \(|\langle \psi_n | V | \psi_m \rangle | \ll \hbar^2\) and atoms move very slowly (i.e., \(\bar{p}^2 \ll \hbar^2\)). If these conditions are satisfied the wave functions \((\Psi_1, \Psi_2)^T\) will be approximately decoupled from \(\Psi_3\) and evolve under the Hamiltonian \[5\]

\[\hat{H}_{eff} = \left(\frac{\bar{p}^2}{2m} + \bar{V} + \Phi\right).\] (8)

Here \(\bar{A}, \bar{V}\) and \(\Phi\) are \(2 \times 2\) matrices, the elements of \(\bar{A}\) and \(\bar{V}\) are described in Eq. (7) with \(n, m = 1, 2, 3\).

The Hamiltonian \(\hat{H}_{eff}\) in Eq. (8) simulates the movement of a particle with spin-1/2 in gauge potentials, where two lowest energy dressed states \(|\psi_1\rangle\) and \(|\psi_2\rangle\) represent spin up and spin down respectively. Here we emphasize that the \(\theta\) is not required to be small to get \(\hat{H}_{eff}\), i.e., the magnitudes of Rabi frequencies \(\Omega_1\) and \(\Omega_2\) are not required to be much smaller than the detuning \(\Delta\).

We can get an analytical expression of the simulated gauge potentials \(\bar{A}\) and \(\bar{V}\) in \(\hat{H}_{eff}\) by substituting Eq. (4) into Eq. (7). But note that the dressed states \(|\psi_1\rangle, |\psi_2\rangle\) and \(|\psi_3\rangle\) have the same mathematical structures as the two dark states \(|D_1\rangle, |D_2\rangle\) \[5\] and the bright state \(|B\rangle = |D_0\rangle\) \[26\] in the tripod configuration respectively, and in the tripod configuration the simulated gauge potentials can be expressed as \(\bar{A}_{n,m} = i\hbar \langle D_n | \nabla D_m \rangle\) and \(\Phi_{n,m} = \frac{1}{2m} \bar{A}_{n,0} \cdot \bar{A}_{0,m}\) \[26\]. We can conclude immediately that the simulated gauge potentials in our concerned \(\Lambda\) configuration have the same mathematical structures as those simulated in the tripod configuration, i.e., we can obtain an analytical expression for the simulated gauge potentials \(\bar{A}\) and \(\bar{V}\) of \(\hat{H}_{eff}\) in our concerned \(\Lambda\) configuration just through replacing \(S_{13}\) and \(S_{23}\) in Eq. (13) and Eq. (14) of Ref. \[5\] by \(S_1\) and \(S_2\) respectively. Here we write them down for completeness:

\[\hat{A}_{1,1} = \frac{\hbar \cos^2 \phi \nabla S_2 + \sin^2 \phi \nabla S_1}{2m},\] (10)

\[\hat{A}_{1,2} = \frac{\hbar \cos \theta \left[\frac{1}{2} \sin(2\phi) (\nabla S_1 - \nabla S_2) - i \nabla \phi\right]}{2m},\]

\[\hat{A}_{1,3} = \frac{\hbar \cos \theta (\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2)}{2m},\]

\[\Phi_{1,1} = \frac{\hbar^2}{2m} \sin^2 \theta \left[\frac{1}{4} \sin^2(2\phi)(\nabla S_1 - \nabla S_2)^2 + (\nabla \phi)^2\right],\]

\[\Phi_{1,2} = \frac{\hbar^2}{2m} \sin \theta \left[\frac{1}{4} \sin(2\phi)(\nabla S_1 - \nabla S_2) - i \nabla \phi\right] \nabla \phi + \left[\frac{1}{2} \sin(2\phi)(\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2) - i \nabla \theta\right],\]

\[\Phi_{1,3} = \frac{\hbar^2}{2m} \sin \theta \left[\frac{1}{4} \sin^2(2\phi)(\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2)^2 + (\nabla \theta)^2\right].\]

We will give explicit examples of \(S_1\) and \(S_2\) to show how spin-orbit coupling Hamiltonian can be obtained.

Our parameterization of the two Rabi frequencies not only gives us a convenient way to obtain an analytical expression of the simulated gauge potentials as shown above, but also gives us a way to design spin-orbit simulation schemes in the...
Spin-orbit coupling in the tripod configuration can only be of a form similar to the two lowest energy dressed states in the tripod configuration. This is an example to show the form of the simulated spin-orbit coupling Hamiltonian in this scheme.

As shown in FIG. 2 two internal states \(|1\rangle\) and \(|2\rangle\) of an atom are coupled to a third common one \(|3\rangle\) via two plane waves respectively with the same detuning. The directions of the two waves are in the x-z plane, and the phases of the two Rabi frequencies are assumed to be

\[
S_1 = \vec{k}_1 \cdot \vec{r} = kx \sin \varphi + k z \cos \varphi, \\
S_2 = \vec{k}_2 \cdot \vec{r} = -kx \sin \varphi + k z \cos \varphi,
\]

where \(k\) is the wave vector of the applied plane waves and \(\varphi\) determines the wave directions. Substitute \(S_1\) and \(S_2\) into the expressions of simulated gauge potentials \(\vec{A}\) and \(\Phi\) in Eq. (10) and note that only \(S_1\) and \(S_2\) are position dependent, we get

\[
\frac{A_x}{\hbar k} = -\frac{1}{2} \sin^2 \theta \cos(2\phi) \sin \varphi \sigma_0 + \cos \theta \sin(2\phi) \sin \varphi \sigma_z, \\
\frac{A_z}{\hbar k} = \frac{1}{2} (2 - \sin^2 \theta) \cos(2\phi) \sin \varphi \sigma_z,
\]

(12)

where \(\sigma_x, \sigma_y, \sigma_z\) are three Pauli matrices and \(\sigma_0\) is the 2 × 2 identity matrix. If we set \(\phi = \pi/4\) and substitute \(\vec{A}\) above into \(\hat{H}_{eff}\) then we can get a spin-orbit coupling proportional to

\[
\cos \theta \sin \varphi \sigma_x + \frac{1}{2} \sin^2 \theta \cos \varphi \sigma_z.
\]

(13)

Recall the effective Hamiltonian \(\hat{H}_{eff}\) is simulated under the conditions \((\hbar k)^2 \ll \hbar \Delta\) and \((\hbar k)^2 \ll \hbar \Delta\). Now we show under these conditions some terms in \(\hat{H}_{eff}\) can be further neglected. First we find the potential \(\Phi\) satisfies

\[
|\Phi_{n,m}| \leq \frac{(\hbar k)^2}{2m} \sin^2 \theta \ll \hbar \Delta \frac{\sin^2 \theta}{\cos(2\theta)},
\]

(14)

therefore \(\Phi\) can be ignored compared to \(\hat{V}\). Second we write \(\cos \theta\) in \(A_x\) of Eq. (12) as \(1 - 2 \sin^2 (\theta/2)\) and then substitute \(A_x\) and \(A_z\) into \(\hat{H}_{eff}\), we can get some energy terms proportional to \(\sin^2 \theta\) or \(2 \sin^2(\theta/2)\) [27], which are of magnitude \(max\left[\frac{\bar{\sigma}^2}{2m}, \frac{(\hbar k)^2}{2m}\right]\) and can also be ignored compared to \(\hat{V}\). Therefore in \(\hat{H}_{eff}\) we can safely write \(\Phi = 0\) and

\[
A_x = \hbar k \sin \varphi \sin(2\phi) \sigma_x - \cos(2\phi) \sigma_z, \\
A_z = \hbar k \cos \varphi \sigma_0.
\]

(15)

We note that this result is obtained due to the non-degeneracy of the dressed states \(|e_1\rangle\) and \(|e_2\rangle\), not by assuming \(\theta\) is close to zero. From Eq. (15) we find that only the movement in the \(x\) direction of the atom is coupled to its pseudo spin, and the coupled movement is governed by

\[
\hat{H}_{xs} = \frac{\hat{p}_x^2}{2m} + \hat{V},
\]

(16)

where \(\hat{V}\) is the effective potential due to the applied plane waves and \(\hat{p}_x\) is the momentum operator in the \(x\) direction.
where $\alpha = \frac{1}{2m}\hbar ks\sin\varphi$, $h = \hbar \Delta \sin^{2}\theta \over 2\cos(2\theta)$ and a constant term $c = \frac{(\hbar k)^{2}}{2m}\sin^{2} \varphi - \hbar \Delta \sin^{2}\theta \over 2\cos(2\theta)$ is neglected. If we use $|e_{1}'\rangle = \cos\theta |e_{1}\rangle - \sin\theta |e_{2}\rangle$ and $|e_{2}'\rangle = \sin\theta |e_{1}\rangle + \cos\theta |e_{2}\rangle$ instead of $|e_{1}\rangle$ and $|e_{2}\rangle$ to represent spin up and spin down respectively, then the coupled Hamiltonian will be

$$\hat{H}'_{xs} = \frac{\hbar^{2}}{2m} + 2\alpha\hat{p}_{x}\sigma_{z} + h[\cos(2\theta)\sigma_{x} + \sin(2\theta)\sigma_{z}],$$

where $2\alpha\hat{p}_{x}\sigma_{z}$ represents spin-orbit coupling. Thus we have shown the simulated spin-orbit coupling in our concerned $\Lambda$ configuration can only be of a form similar to $p_{x}\sigma_{y}$ even when the Rabi frequencies are not much smaller than the detuning.

The roles of $h$ and $\phi$ seem clear in $\hat{H}'_{xs}$; one controls the magnitude of the “magnetic field” and the other controls its direction. However, as we change $\phi$, which is determined by the relative magnitude of two lasers, not only $\hat{H}'_{xs}$ but also the spin basis states $|e_{1}'\rangle$ and $|e_{2}'\rangle$ will be changed, and it may be not easy to see the underlying physics. Now we assume $\hbar\Delta$ is so bigger that a small $\theta = \theta_{m}$ can lead to $h\sigma_{z}$ dominating $\hat{H}_{xs}$, then we can study the underlying physics only varying $\theta$ between $-\theta_{m}$ and $\theta_{m}$. Since $\theta$ is small, i.e., the Rabi frequencies are much smaller than the detuning, there is $|e_{2}'\rangle = \cos \varphi e^{-iS_{z}} |1\rangle + \sin \varphi e^{-iS_{z}} |2\rangle$ and our simulation scheme reduces to the Raman process in recent experiment [11]. At this time we get $|e_{1}'\rangle = e^{-iS_{z}} |2\rangle$ and $|e_{1}'\rangle = e^{-iS_{z}} |1\rangle$, which are independent of $\phi$. Experimentally we can study the phase transition of $\hat{H}'_{xs}$ due to the change of $\phi$ and $\theta$ as in Refs. [11,12].

IV. DISCUSSION AND SUMMARY

We have given an example that the simulated spin-orbit coupling in the $\Lambda$ configuration can only be of a form similar to $p_{x}\sigma_{y}$ even when the Rabi frequencies are not much smaller than the detuning. The same conclusion can also be obtained when we consider the spin-orbit coupling simulation schemes in Refs. [21,22]. So if we want to get a more general spin-orbit coupling in our concerned $\Lambda$ configuration, we should find a way to eliminate the Zeeman term in $\hat{H}_{eff}$ due to the non-degeneracy of the two lowest energy dressed states. If we assume the trapping potential $V = V_{1} |1\rangle \langle 1| + V_{2} |2\rangle \langle 2| + V_{3} |3\rangle \langle 3|$ with $V_{1} = V_{2}$ and $V_{3} = V_{1} - E_{2}/\sin^{2}\theta$, then this Zeeman term will be eliminated. However this trapping potential will give a coupling between $\Psi_{2}$ and $\Psi_{3}$ that cannot be ignored. How to effectively eliminate the Zeeman term due to the non-degeneracy of the two lowest energy dressed states is still under investigation.

In summary, we have given an analytical expression of the simulated non-abelian gauge potentials in our concerned $\Lambda$ configuration based on a special parameterization of the two Rabi frequencies. We have shown the simulated spin-orbit coupling in our concerned $\Lambda$ configuration can only be of a form similar to $p_{x}\sigma_{y}$ even when the Rabi frequencies are not much smaller than the detuning.

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