Non-leptonic beauty baryon decays and $CP$-asymmetries based on $SU(3)$-Flavor

analysis

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We consider hadronic weak decays of beauty-baryons into charmless baryons and pseudoscalar mesons in a general framework based on $SU(3)$ decomposition of the decay amplitudes. The advantage of the approach lies in the ability to perform an $SU(3)$ analysis of these decays without any particular set of dynamical assumptions while accounting for the effects of an arbitrarily broken $SU(3)$ flavor symmetry. Dictated by the symmetries of the effective Hamiltonian that allow us to relate or neglect reduced $SU(3)$ amplitudes, we derive several sum rule relations between amplitudes and relations between $CP$ asymmetries in these decays and identify those that hold even if $SU(3)$ is broken.

I. INTRODUCTION

LHCb is poised to collect a large data set of two-body weak decays of beauty-baryons [1,3] into charmless baryons and pseudoscalar mesons paving the way to a better understanding of heavy baryon decays. Significant progress has been made in the theoretical understanding of beauty meson decays [4,53] spurred by the experimental advances at flavor factories Belle and Babar [54,55] as well as in LHCb [1,2,10,12]. The general framework of $SU(3)$ analysis in beauty mesons as well as charm meson decays [13,53] into two pseudoscalars ($PP$), pseudoscalar-vector boson ($PV$), and two vector mesons ($VV$) has yielded several amplitude sum-rules and relationships between $CP$ asymmetries for various decay modes. While attempts have been made to analyze such decays for beauty-baryons, a comprehensive analysis is so far missing in the literature. In this paper, we consider the hadronic beauty-baryon decays into an octet or singlet of light baryons and a pseudoscalar meson based on the $SU(3)$ decomposition of the decay amplitudes approach pioneered for $B$-meson decays by Grinstein and Lebed [12]. In contrast to the methodology employed in [54,55] for bottom and charm hadron decays, our approach [12] facilitates an $SU(3)$ decomposition of the decays in terms of $SU(3)$-reduced amplitudes without any particular set of assumptions about the underlying dynamics.

The number of independent $SU(3)$-reduced amplitudes for any given initial and final state is exactly calculable and relations between decay amplitudes emerge naturally once the set of independent $SU(3)$-reduced amplitudes is smaller than the total number of possible decays. The counting of independent $SU(3)$ reduced amplitudes draws on the choice of the effective Hamiltonian, which in the most general case, indicate 44 independent reduced $SU(3)$ amplitudes equaling the number of all possible $\Delta S = -1$ and $\Delta S = 0$ processes. In practice, the dimension-6 effective Hamiltonian that mediates such hadronic decays of bottom baryons predict only 10 independent reduced $SU(3)$ amplitudes. One can therefore obtain amplitude relations between the decay modes that can be derived explicitly. Moreover, a systematic study of the $SU(3)$-breaking effects at the level of decay amplitudes, order by order expanded in the $SU(3)$ breaking parameter, is required to identify those amplitude relations that survive the $SU(3)$ breaking effects. Starting with the symmetries of the effective Hamiltonian, we relate or neglect reduced $SU(3)$ amplitudes to derive several sum rules relations between amplitudes and relations between $CP$ asymmetries while indicating more general relations that continue to hold when the $SU(3)$ symmetry is no longer exact. This study is crucial for a detailed analysis of the $CP$ asymmetry measurements in bottom baryons decays at the CDF and LHCb in recent times [56,53].

The approach to decompose the decay amplitudes in terms of reduced $SU(3)$ amplitudes is presented in Sec.[II]. The relation between the $SU(3)$ Clebsch-Gordon (CG) coefficients in terms of the isoscalar factors and the $SU(2)$ CG coefficients is outlined in Appendix[A]. The results are summarized in Appendix[B] and[C]. In Sec.[III] we perform the $SU(3)$ decomposition of unbroken effective hadronic weak decay Hamiltonian. The relations between the amplitudes for beauty baryon decays into octets of light baryons and pseudoscalar mesons are derived in Sec.[IV]. The effects of $SU(3)$ breaking on account of $s$-quark mass are considered in Sec.[IVA]. The corresponding relations between $CP$ asymmetries are derived in Sec.[V]. We finally conclude in Sec.[VI].
II. APPLICATION OF SU(3) TO DECAY AMPLITUDES

The SU(3) decomposition of physical amplitudes describing a decay process involves writing it in terms of reduced matrix elements of explicit SU(3) operators with appropriate coefficients. The procedure is a straightforward application of Wigner-Eckart theorem for the group SU(3) where the reduced matrix elements are all possible SU(3) invariants with Clebsch-Gordon (CG) coefficients connecting the basis involving physical states to the group theoretic basis.

The most general Hamiltonian \( \mathcal{H} \) which connects the initial and final states via the matrix elements \( \langle f | \mathcal{H} | i \rangle \), consists of exactly those representations \( R \) appearing in \( f \otimes \bar{i} \), where the labels \( i \) and \( f \) denote both physical states and SU(3) representations. It is important to note that in addition to the usual SU(3) CG coefficients that arise from coupling \( f \otimes \bar{i} \), the most general effective Hamiltonian \( (\mathcal{H}) \) itself involves unknown coefficients appearing in front of every SU(3) representation. A priori these coefficients are all independent of each other which get determined once a particular form of effective Hamiltonian is assumed. The states of SU(3) representations are uniquely distinguished when in addition to the \( I_3 \) and \( Y \) values, the isospin Casimir \( I^2 \) is also specified. The full reduced SU(3) amplitude is thus described by \( \langle f | R_f | i \rangle \). The expression of the amplitudes in terms of reduced SU(3) amplitudes is concisely given as,

\[
A(i \rightarrow f_0 f_m) = (-1)^{f_3 + \frac{1}{2} + s} \sum_{\{f, R\}} C_{A,B,C}^{f, R} \tilde{f} (Y_b^-, I_b^-, I_b^*) (Y_m^-, I_m^-, I_m^*) \tilde{f} (Y_f^+, I_f^+, I_f^*) \langle f | R_f | i \rangle
\]

where, \( C_{A,B,C} \) are the SU(2) Clebsch-Gordon coefficients and

\[
\left( \begin{array}{ccc} \mathbf{R}_a & \mathbf{R}_b & \mathbf{R}_c \\ (Y^a, I^a, I_a^*) & (Y^b, I^b, I_b^*) & (Y^c, I^c, I_c^*) \end{array} \right)
\]

are the SU(3) isoscalar coefficients obtained by coupling the representations \( \mathbf{R}_a \otimes \mathbf{R}_b \rightarrow \mathbf{R}_c \). \( T \) is the triality of a SU(3) representation\(^1\) that ensures the reality of the phase appearing in Eq. (1). The symmetry properties of the SU(3) isoscalar factor and its role in obtaining the SU(3) CG coefficients\(^2\) is outlined in Appendix A.

The amplitude is written with specific attention to the order in which the representations are coupled, the final state representations are coupled via \( \bar{f}_0 \otimes f_m \rightarrow \tilde{f} \), where the product \( (f) \) is then coupled through the conjugate of the initial representation or equivalently \( f \otimes \bar{i} \rightarrow \mathcal{H} \). This ensures that all possible SU(3) representations are indeed generated in case of the most general effective Hamiltonian. Given a form of effective Hamiltonian (\( \mathcal{H}_{\text{eff}} \)), it can be SU(3) decomposed,

\[
\mathcal{H}_{\text{eff}} = \sum_{\{Y, I, I_3\}} \mathcal{F}_{R}^{(Y, I, I_3)} R_f \mathbf{R}_1,
\]

where \( \mathcal{F}_{R}^{(Y, I, I_3)} \) depends on the SU(3) CG coefficients appearing in front of the SU(3) representations \( (\mathbf{R}_1) \). Moreover \( \mathcal{F}_{R}^{(Y, I, I_3)} \) also contains additional factors entering Eq. (3) in form of Wilson coefficients and CKM elements. It is also important to note that by knowing the dynamical coefficients for different isospin values in a given SU(3) representation, one can drop the isospin Casimir label \( (I) \) and express the Wigner-Eckart reduced matrix element \( \langle f | R_f | i \rangle \), in its usual form, independent of the isospin \( I \) label. By completeness of SU(3) CG coefficients up to a phase factor,

\[
\langle f | R_f | i \rangle = \mathcal{F}_{R}^{(Y, I, I_3)} \frac{\dim f \cdot \dim \mathbf{R}}{\dim \mathcal{H}} \text{(dynamical Coeff. of } \mathcal{H})
\]

Alternatively, one can directly start with the given form of effective Hamiltonian in Eq. (3) and perform an SU(3) decomposition of the decay amplitude;

\[
T((m, n)) = (m - n) \mod 3
\]

\(^1\) For an SU(3)-representation with \( m \) and \( n \) fundamental and anti-fundamental indices, the triality of the representation is given by
\[ A(i \to f_b f_m) = \sum_{Y_H,i,i_3} \mathcal{F}^{(Y,i,i_3)} \sum_{Y_M} \left( C_{i_3}^{T_H_i T_M_Y} \right) \left( f_b \left( Y^{h,i_3}_b, i_i \right), f_m \left( Y^{m,i_3}_m, i_i \right) \right) \]

\[ f \left( Y^{f,i_3}_f, i_i \right) \left( Y^{f,i_3}_f, i_i \right) \left( Y^{f,i_3}_f, i_i \right) \left( Y^{f,i_3}_f, i_i \right) \]

The case of our interest, namely, \( B_b(3) \to B(8) P(1) \), where \( B_b \), the initial anti-triplet \( (\bar{3}) \) beauty-baryon undergoes a charmless decay into an octet baryon \((B)\) and an octet pseudoscalar meson \((P)\), is described by a Hamiltonian with \( \Delta Q = 0 \) and \( \Delta S \) (equivalent to \( \Delta I_1 \) and \( \Delta Y \) representation). The possible decays can be divided into two sub classes, namely the \( \Delta S = 0 \) and \( \Delta S = -1 \) transitions. The allowed final state \( SU(3) \) representations \((\Gamma)\) are: \( 1, 8, 10, 10, 27 \). There are 22 physical process possible for \( \Delta S = 0 \) and another 22 for \( \Delta S = -1 \). In Appendix \( \text{B} \) and Appendix \( \text{C} \) respectively each of these decay modes are decomposed in terms of the \( SU(3) \) reduced amplitudes that add up to 44. Since the physical \( q \) and \( \bar{q} \) mesons are admixtures of octet \( \eta_8 \) and singlet \( \eta_1 \) mesons, a study of \( B_b(3) \to B(8) P(1) \) is also necessary. Therefore there one has to take into account 8 (4 each for \( \Delta S = -1 \) and \( \Delta S = 0 \)) additional independent \( SU(3) \) amplitudes which are also described in Appendix \( \text{B} \) and Appendix \( \text{C} \).

\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[ \lambda_u^{(s)} \left( C_1^{(u)} - Q_1^{(c)} \right) + C_2^{(u)} - Q_2^{(c)} \right] - \lambda_u^{(s)} \left( C_i Q_i^{(c)} - \lambda_i^{(s)} \right) + \lambda_u^{(d)} \left( C_1^{(u)} - Q_1^{(c)} + C_2^{(u)} - Q_2^{(c)} \right) - \lambda_u^{(d)} \left( C_i Q_i^{(c)} - \lambda_i^{(d)} \right) \]

\[ = \lambda_u^{(s)} \left( C_1^{(u)} - Q_1^{(c)} \right) + C_2^{(u)} - Q_2^{(c)} \]

\[ + \lambda_u^{(d)} \left( C_1^{(u)} - Q_1^{(c)} + C_2^{(u)} - Q_2^{(c)} \right) - \lambda_i^{(d)} \left( C_i Q_i^{(c)} - \lambda_i^{(d)} \right) \]

\[ = \lambda_u^{(s)} \left( C_1^{(u)} - Q_1^{(c)} \right) + C_2^{(u)} - Q_2^{(c)} \]

\[ + \lambda_u^{(d)} \left( C_1^{(u)} - Q_1^{(c)} + C_2^{(u)} - Q_2^{(c)} \right) - \lambda_i^{(d)} \left( C_i Q_i^{(c)} - \lambda_i^{(d)} \right) \]

We emphasize that this way of counting accounts for a complete set of reduced amplitudes, regardless of the specific form of interaction Hamiltonian. In particular, this decomposition holds even if the \( SU(3) \) symmetry is arbitrarily broken and there is no physical reason to organize particles in \( SU(3) \) multiplets. At this point, every process is independent and to find relations among them requires assuming a specific form of the interaction Hamiltonian.

III. \( SU(3) \) DECOMPOSITION OF UNBROKEN EFFECTIVE HAMILTONIAN

The lowest order effective Hamiltonian \[90-92\] for charmless \( b \)-baryon decays consists \( \Delta S = -1 \) and \( \Delta S = 0 \). Each part is composed from the operators \( Q_1, \ldots, Q_{10} \). The complete Hamiltonian can be written as:

\[ Q_5^{(s)} = (\overline{3}_L^s \gamma^\mu b_L^L) \sum_{q=u,d,s} (\overline{7}_R^s \gamma^\nu q_R^L) \]

\[ Q_6^{(s)} = (\overline{3}_L^s \gamma^\mu b_L^L) \sum_{q=u,d,s} (\overline{7}_R^s \gamma^\nu q_R^L) \]

Out of the four “EWP” (i.e. “Electroweak Penguins”) Operators: \( Q_7, \ldots, Q_{10}, Q_7 \) and \( Q_8 \):

\[ Q_7^{(s)} = \frac{3}{2} (\overline{3}_L^s \gamma^\mu b_L^L) \sum_{q=u,d,s} e_q (\overline{7}_R^s \gamma^\nu q_R^L) \]

\[ Q_8^{(s)} = \frac{3}{2} (\overline{3}_L^s \gamma^\mu b_L^L) \sum_{q=u,d,s} e_q (\overline{7}_R^s \gamma^\nu q_R^L) \]

are typically ignored in hadronic decays because of the smallness of \( C_7 \) and \( C_8 \) with respect to the other Wilson Coefficients.
The remaining “EWP” operators are:

\[
Q^{(s)}_9 = \frac{3}{2}(\overline{s}_L \gamma^\mu b^L) \sum_{q=u,d,s} c_q (\overline{t}_L \gamma_\mu q^L),
\]

\[
Q^{(s)}_{10} = \frac{3}{2}(\overline{t}_L \gamma^\mu b^L) \sum_{q=u,d,s} c_q (\overline{q}_L \gamma_\mu q^L). \tag{10}
\]

\[H_{\text{eff}}\] is a linear combinations of four quark operators of the form \((\overline{q}, b)(\overline{b}q)\). These operators transform as \(3 \otimes 3 \otimes \overline{3} \otimes \overline{3}\) under \(SU(3)\)-flavor and can be decomposed into sums of irreducible operators corresponding to irreducible \(SU(3)\) representations: \(15, \overline{10}, 3^{(6)}, 3^{(8)}\), where the superscript index: ‘6’ \((\overline{3})\) indicates the origin of 3 out of the two possible representations arising from the tensor product of \(q_1\) and \(q_2\). The \(SU(3)\) triplet representation of quarks \(q_1\) and its conjugate denoting the anti-quarks \((\overline{q})\) consist of the flavor states:

\[
q_i = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \overline{q} = \begin{pmatrix} \overline{u} \\ -\overline{d} \\ \overline{s} \end{pmatrix} \tag{11}
\]

According to the sign convention chosen in Eq. (11), the meson wavefunctions are given as,

\[
K^+ = u\overline{u}, \quad K^- = -s\overline{u}, \quad K^0 = d\overline{s}, \quad K^{*0} = sd
\]

\[\pi^+ = u\overline{d}, \quad \pi^- = -d\overline{u}, \quad \pi^0 = \frac{1}{\sqrt{2}}(d\overline{d} - u\overline{u}) \]

\[\eta_8 = -\frac{1}{2\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s}), \quad \eta_1 = -\frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s})\]

The physical mesons \(\eta, \eta'\) are related to the \(\eta_8\) and \(\eta_1\) through the \(SO(2)\) rotation,

\[
\left( \begin{array}{c} \eta' \\ \eta \end{array} \right) = \left( \begin{array}{cc} -\cos \theta & \sin \theta \\ -\sin \theta & -\cos \theta \end{array} \right) \left( \begin{array}{c} \eta_8 \\ \eta_1 \end{array} \right) \tag{12}\]

where the definition of \(\theta\) is consistent with the overall notation for the meson wavefunctions as well as agreeing with the phenomenologically determined value of \(\theta\). In the following table the four quark operators, which appear in \(H_{\text{eff}}\), are decomposed using \(SU(3)\) Clebsch-Gordan tables. It is worthwhile to note that in the Hamiltonian operators appear as \(\overline{q}_1 \overline{q}_2 q_3\) whereas in Table I they are expressed conveniently as \(q_1 q_2 \overline{q}_3\). With the help of Table I the effective Hamiltonian can be expressed in terms of operators having definite \(SU(3)\) transformation properties. In particular, the tree, gluonic and electroweak penguin part of the effective Hamiltonian consist of \(21\).

![Table I. Operator Decomposition](image)

\[
\frac{\sqrt{2}H_T}{4G_F} = \left\{ \begin{array}{l}
\lambda^u_6 \left[ \frac{(C_1 + C_2)}{2} \left( -15_1 - \frac{1}{\sqrt{2}}15_0 - \frac{1}{\sqrt{2}}3^{(6)}_1 \right) + \frac{(C_4 - C_2)}{2} (6_1 + 3_0^{(3)}) \right]
\lambda^d_6 \left[ \frac{(C_1 + C_2)}{2} \left( -\frac{2}{\sqrt{3}}15_{3/2} - \frac{1}{\sqrt{2}}15_{1/2} - \frac{1}{\sqrt{2}}3^{(6)}_{3/2} \right) + \frac{(C_4 - C_2)}{2} (6_{3/2} + 3_0^{(3)}) \right]
\end{array} \right\}, \tag{13}
\]

\[
\frac{\sqrt{2}H_G}{4G_F} = \left\{ \begin{array}{l}
-\lambda^u_8 \left[ \sqrt{2}(C_3 + C_4)3^{(6)}_0 + (C_3 - C_4)3^{(3)}_0 \right] - \lambda^d_8 \left[ \sqrt{2}(C_3 + C_4)3^{(6)}_{1/2} + (C_3 - C_4)3^{(3)}_{1/2} \right]
\lambda^u_8 \left[ -\sqrt{2}(C_5 + C_6)3^{(6)}_0 + (C_5 - C_6)3^{(3)}_0 \right] - \lambda^d_8 \left[ -\sqrt{2}(C_5 + C_6)3^{(6)}_{1/2} + (C_5 - C_6)3^{(3)}_{1/2} \right]
\end{array} \right\}, \tag{14}
\]
responding to the reduced matrix elements is a consequence of the vanishing dynamical coefficients contributing to the Hamiltonian. In addition, the absence of the sums extend over all the corresponding contributions between reduced matrix elements regardless of the initial and final states.

\[
\frac{\sqrt{2}H_{\text{EW}}}{4G_F} = \left\{ -\lambda_t^d \left[ \frac{(C_9 + C_{10})}{2} \left( \frac{3}{2} \begin{array}{c} 15 \end{array} \right) - \frac{3}{2 \sqrt{2}} \begin{array}{c} 15 \end{array} + \frac{1}{2 \sqrt{2}} \begin{array}{c} 3(6) \end{array} \right] + \frac{(C_9 - C_{10})}{2} \left( \frac{3}{2} \begin{array}{c} 6 \end{array} + \frac{1}{2 \sqrt{2}} \begin{array}{c} 3(6) \end{array} \right) \right\}. \quad (15)
\]

It is clear from Table [1] that higher SU(3) representations like 24, 42 and 15' are absent in the unbroken Hamiltonian.

IV. AMPLITUDE RELATIONS

From the tree and Electroweak part of the Hamiltonian one can project out the coefficients corresponding to the 15 part of the Hamiltonian and write down the following relations between reduced matrix elements regardless of the initial and final states.

\[
\frac{(f \parallel 15_0 \parallel i)}{(f \parallel 15_1 \parallel i)} = \frac{1}{\sqrt{2}}, \quad \frac{(f \parallel 15_\downarrow \parallel i)}{(f \parallel 15_\uparrow \parallel i)} = \frac{1}{2 \sqrt{2}}, \quad \frac{\lambda_t^d (f \parallel 15_0 \parallel i)_{\text{EW}}}{\lambda_u^d (f \parallel 15_\downarrow \parallel i)_{\text{EW}}} = \sqrt{3}, \quad \frac{\lambda_u^d (f \parallel 15_0 \parallel i)_{\text{EW}}}{\lambda_u^d (f \parallel 15_\downarrow \parallel i)_{\text{EW}}} = \sqrt{3} \quad (16)
\]

In case of several different operator structures contributing to the Hamiltonian as is the case in Eq. (13), the relations between reduced matrix elements are expressed in the following way.

\[
\frac{(f \parallel R_i \parallel i)}{(f \parallel R_{\overline{i}} \parallel i)} = \sum_j C_i C_j \sum_m C_m C_{m'}, \quad (17)
\]

where the \(C_i^{(')}\) are the coefficients of the different components of the Hamiltonian and \(C_j\)'s are the CG coefficients and the sums extend over all the corresponding contributions to the Hamiltonian. In addition, the absence of some of the SU(3) representations in the Hamiltonian is a consequence of the vanishing dynamical coefficients corresponding to the reduced matrix elements \((f \parallel 42 \parallel i), (f \parallel 24 \parallel i)\) and \((f \parallel 15 \parallel i)\), regardless of the \(I\) value and initial and final states. The \(\Delta S = -1\) and \(\Delta S = 0\) decay amplitudes and the reduced SU(3) elements are expressed as column matrices \(A\) and \(\mathcal{R}\) respectively and related by the matrix equation

\[
A = TR, \quad (18)
\]

where \(T\) is the coefficient matrix related to the tree part of the Hamiltonian described in Eq. (13). The rank of matrix \(T\) is lower than the total number of decay modes suggesting that not all of the reduced SU(3) matrix elements are independent. The number of actually independent reduced SU(3) matrix elements are equal to the rank of matrix \(T\). The number of amplitude relations can now be estimated unambiguously which is the difference between the total number decay modes and rank of \(T\).

Similar exercise is performed for the penguins where the coefficient matrix can be rewritten where the entries are nothing but products of Wilson Coefficients (\(C_3\)) and Clebsch-Gordon coefficients. Of course, the number of independent rows remain unchanged and the matrix equations take the form,

\[
\mathcal{A}_T = T \mathcal{R}, \quad \mathcal{A}_P = P \mathcal{R} \quad (20)
\]

for both the \(S\)-wave and the \(P\)-wave part. At this point, it is important to recall the penguin part of the Hamiltonian described in Eqs. (13) and (15). In case of gluonic penguins, 6 and 15 of SU(3) are absent which result in a smaller set of independent reduced SU(3) matrix elements. This implies additional amplitude relations between decay modes, some of which are broken once the Electroweak penguins are taken into account in the unbroken Hamiltonian. We include Electroweak penguins that have parts transforming as 3, 5 and 15 of SU(3) and retain all the reduced SU(3) matrix elements. As a result, the amplitude relations derived hold for the gluonic penguin part as well as the Electroweak penguin part of the unbroken Hamiltonian. We begin with identifying the identical rows of the \(T\) and \(P\) matrices which readily gives the simplest amplitude relations for the tree part

\[
T(\Lambda_b^0 \rightarrow \Sigma^- K^+) = T(\Xi_b^0 \rightarrow \Sigma^- \pi^+), \quad (21)
\]  
\[
T(\Lambda_b^0 \rightarrow p^- \pi^+) = T(\Xi_b^0 \rightarrow \Sigma^+ K^-), \quad (22)
\]  
\[
T(\Xi_b^0 \rightarrow n K^-) = T(\Xi_b^0 \rightarrow \Xi^0 \pi^-), \quad (23)
\]  
\[
T(\Xi_b^0 \rightarrow \Xi^+ K^0) = T(\Sigma_b^0 \rightarrow \Sigma^+ \pi^-), \quad (24)
\]  
\[
T(\Xi_b^0 \rightarrow \Sigma^- K^0) = T(\Lambda_b^0 \rightarrow \Sigma^- \pi^+), \quad (25)
\]  
\[
T(\Xi_b^0 \rightarrow \Sigma^+ K^-) = T(\Lambda_b^0 \rightarrow p^- K^-), \quad (26)
\]  
\[
T(\Xi_b^0 \rightarrow \Sigma^- K^0) = T(\Lambda_b^0 \rightarrow \Xi^0 K^0), \quad (28)
\]
\[ T(\Xi_b^0 \to p^+K^-) = T(\Lambda_b^0 \to \Sigma^+\pi^-), \quad (29) \]
\[ T(\Xi_b^0 \to \Xi^0K^0) = T(\Lambda_b^0 \to nK^0), \quad (30) \]
and the same set of relations for the penguin part,
\[ \mathcal{P}(\Lambda_b^0 \to \Sigma^-\pi^+) = \mathcal{P}(\Xi_b^0 \to \Xi^-\pi^+), \quad (31) \]
\[ \mathcal{P}(\Lambda_b^0 \to p^+\pi^-) = \mathcal{P}(\Xi_b^0 \to \Sigma^+K^-), \quad (32) \]
\[ \mathcal{P}(\Xi_b^- \to nK^-) = \mathcal{P}(\Xi_b^- \to \Xi^0\pi^-), \quad (33) \]
\[ \mathcal{P}(\Xi_b^- \to \Xi^-K^-) = \mathcal{P}(\Xi_b^- \to \Sigma^-K^0), \quad (34) \]
\[ \mathcal{P}(\Xi_b^0 \to \Xi^-K^0) = \mathcal{P}(\Xi_b^- \to \Xi^0\pi^-), \quad (35) \]
\[ \mathcal{P}(\Xi_b^0 \to \Sigma^-\pi^+) = \mathcal{P}(\Lambda_b^0 \to \Xi^-\pi^+), \quad (36) \]
\[ \mathcal{P}(\Xi_b^0 \to \Sigma^+\pi^-) = \mathcal{P}(\Lambda_b^0 \to \Xi^+K^-), \quad (37) \]
\[ \mathcal{P}(\Xi_b^0 \to p^+K^-) = \mathcal{P}(\Lambda_b^0 \to \Xi^0K^0), \quad (38) \]
\[ \mathcal{P}(\Xi_b^0 \to p^-\pi^+) = \mathcal{P}(\Lambda_b^0 \to \Sigma^-\pi^+), \quad (39) \]
\[ \mathcal{P}(\Xi_b^0 \to \Xi^0K^0) = \mathcal{P}(\Lambda_b^0 \to \Xi^0K^0). \quad (40) \]

There are several triangle relations connecting the $\Delta S = -1$ decays modes:
\[ T(\Lambda_b^0 \to \Sigma^+\pi^-) + T(\Lambda_b^0 \to \Sigma^-\pi^+) + 2T(\Lambda_b^0 \to \Sigma^0\pi^0) = 0, \]
\[ T(\Xi_b^- \to \Xi^-\pi^-) - \sqrt{3}T(\Xi_b^- \to \Xi^-\eta_b) + \sqrt{2}T(\Xi_b^- \to \Xi^-K^0) = 0, \]
\[ T(\Xi_b^- \to \Sigma^0\pi^-) - \sqrt{3}T(\Xi_b^- \to \Lambda^0\pi^-) - \sqrt{2}T(\Xi_b^- \to nK^-) = 0, \]
\[ T(\Xi_b^- \to \Sigma^-\pi^-) + T(\Xi_b^- \to \Xi^-K^-) + T(\Lambda_b^0 \to \Sigma^-\pi^-) = 0, \]
\[ T(\Xi_b^- \to \Sigma^-K^-) - T(\Lambda_b^0 \to \Sigma^-\pi^-) + T(\Lambda_b^0 \to \Sigma^-\pi^-) = 0, \]
\[ T(\Xi_b^- \to \Sigma^-K^-) - T(\Lambda_b^0 \to \Xi^-\pi^-) + T(\Lambda_b^0 \to \Xi^-\pi^-) = 0, \]
\[ T(\Xi_b^- \to \Xi^-K^-) - T(\Xi_b^- \to \Xi^-K^-) + T(\Lambda_b^0 \to p^-\pi^+) + T(\Lambda_b^0 \to p^-\pi^+) = 0. \]
\[ (41) \]

The simplest amplitude relations for the case of $\mathbf{3}_{M_\Lambda} \to \mathbf{8}_S \otimes \mathbf{1}_M$ involving the $SU(3)$ singlet $\eta_1$ are indicated,
\[ T(\Xi_b^0 \to \Xi^0\eta_1) = T(\Lambda_b^0 \to n\eta_1), \]
\[ T(\Xi_b^- \to \Xi^-\eta_1) = T(\Xi_b^- \to \Sigma^-\eta_1), \]
along with triangle relation for $\Delta S = -1$ processes
\[ T(\Lambda_b^0 \to \Lambda\eta_1) - \frac{1}{\sqrt{3}}T(\Lambda_b^0 \to \Sigma^0\eta_1) \]
\[ - \frac{\sqrt{2}}{\sqrt{3}}T(\Xi_b^- \to \Xi^0\eta_1) = 0, \]
and for $\Delta S = 0$ processes,
\[ T(\Lambda_b^0 \to n\eta_1) + \frac{\sqrt{3}}{2}T(\Xi_b^0 \to \Lambda^0\eta_1) \]
\[ - \frac{1}{\sqrt{2}}T(\Xi_b^- \to \Sigma^0\eta_1) = 0, \]
\[ (43) \]

While there is no ground state $SU(3)$ singlet $\Lambda$ baryon, there can be $l = 1$ excited state spin-3/2 $\Lambda_b^{0*}$-baryon, for which one can derive amplitude relations in the case of $\mathbf{3}_{M_\Lambda} \to \mathbf{1}_B \otimes \mathbf{8}_M$:
\[ T(\Xi_b^0 \to \Lambda_b^{0*}K^0) = T(\Lambda_b^0 \to \Lambda_b^{0*}K^0) \]
\[ T(\Xi_b^0 \to \Lambda_b^{0*}\eta_8) = T(\Xi_b^- \to \Lambda_b^{0*}K^-) \]
\[ (44) \]

triangle $\Delta S = -1$ relations:
\[ T(\Lambda_b^0 \to \Lambda_b^{0*}\pi_0) - \frac{1}{\sqrt{3}}T(\Lambda_b^0 \to \Lambda_b^{0*}\eta_8) \]

\[ + \frac{\sqrt{2}}{\sqrt{3}}T(\Xi_b^0 \to \Lambda_b^{0*}K^0) = 0, \]

triangle $\Delta S = 0$ relations:
\[ - \frac{1}{\sqrt{3}}T(\Xi_b^0 \to \Lambda_b^{0*}\eta_8) - T(\Xi_b^0 \to \Lambda_b^{0*}\pi_0) \]
\[ - \frac{\sqrt{2}}{\sqrt{3}}T(\Lambda_b^0 \to \Lambda_b^{0*}K^0) = 0 \]

The same set of relations hold for penguin part of the all the above mentioned amplitude relations. Finally, we consider the trivial case of $\mathbf{3}_{M_\Lambda} \to \mathbf{1}_B \otimes \mathbf{1}_M$, where the final state baryon and meson are both $SU(3)$ singlets. The only relevant decay, $\Lambda_b^0 \to \Lambda_b^{0*}\eta_1$, satisfying the $SU(3)$ quantum numbers involve a single reduced $SU(3)$ amplitude matching with the counting of the number of possible independent $SU(3)$ reduced amplitudes. This concludes our discussion of all possible $\mathbf{3}_{M_\Lambda} \to \mathbf{8}_S \otimes \mathbf{8}_M, \mathbf{3}_{M_\Lambda} \to \mathbf{8}_S \otimes \mathbf{1}_M, \mathbf{3}_{M_\Lambda} \to \mathbf{1}_B \otimes \mathbf{8}_M, \mathbf{3}_{M_\Lambda} \to \mathbf{1}_B \otimes \mathbf{1}_M$ decays of $b$-baryons. The most general $SU(3)$ relations can also be obtained in this approach by starting from the $T$ matrix and expressing the dependent rows as a linear combination of the independent ones. We do not list those relations here as they are not particularly illuminating. Nevertheless, in the next section where $SU(3)$ breaking effects are taken into account, we do consider a couple of interesting $SU(3)$ amplitude relations that should hold under some general dynamical assumptions.
A. $SU(3)$ breaking effect

While isospin symmetry holds to a good approximation, the $SU(3)$ symmetry of the light quarks is broken by the mass of the $s$ quark ($m_s$). To incorporate such $SU(3)$ violating effects on decay amplitudes, one can parametrize the breaking of flavor $SU(3)$ by the following interaction $^{43, 44, 52, 03, 05}$,

$$\delta H = \epsilon \bar{q} \gamma_5 \lambda_m q$$

where $\lambda_m$ is the Gell-Mann matrix that contributes to the $SU(3)$-breaking and the breaking parameter $\epsilon$ depends on $m_s$. The $SU(3)$ structure of the unbroken Hamiltonian is modified by this term and to the first order in $m_s$, the broken Hamiltonian is made of the following $SU(3)$ representations $^{43}$,

$$(3 \oplus \overline{6} \oplus 15) \otimes (1 + \epsilon 8) = (3 \oplus \overline{6} \oplus 15)$$

$$+ \epsilon (3, \overline{3}, 15_1 + 15_2 + 15_3$$

$$+ \overline{15}_1 \oplus \overline{15}_2 \oplus 24 \oplus 42),$$

where the subscript $i = 1, 2, 3$ indicates the origin of that representation from $3, \overline{3}, 15$ respectively. The set of reduced $SU(3)$ amplitudes thus gets enlarged and there are less number of relations as a result. The isospin relation,

$$\mathcal{T}(\Lambda_b^0 \to \Sigma^+ \pi^-) + \mathcal{T}(\Lambda_b^0 \to \Sigma^- \pi^+)$$

$$+ 2\mathcal{T}(\Lambda_b^0 \to \Sigma^0 \pi^0) = 0 \quad (46)$$

continues to hold even after including the $SU(3)$ breaking effect to the linear order.

There are other amplitude relations that can be derived on more general grounds. For instance, the isospin symmetry of the unbroken Hamiltonian forbids a $\Delta I = 2$ and $\Delta I = 5/2$ transition. As a consequence, the $SU(3)$-reduced matrix elements $\langle f \parallel R_{I=2} \parallel i \rangle$ and $\langle f \parallel R_{I=5/2} \parallel i \rangle$ must have a vanishing contribution to the decay amplitude for arbitrary initial and final states. Such $SU(3)$ breaking but isospin conserving relations are given below,

$$\frac{\mathcal{T}(\Xi_b^0 \to \Sigma^0 K^0)}{3} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^- K^0)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^0 \to \Xi^0 \pi^0)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^0 \to \Xi^- \pi^0)}{3\sqrt{2}}$$

$$+ \frac{\sqrt{2}\mathcal{T}(\Lambda_b^0 \to \Sigma^0 \pi^0)}{3} + \frac{\mathcal{T}(\Lambda_b^0 \to \Sigma^+ \pi^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Lambda_b^0 \to \Sigma^- \pi^+)}{3\sqrt{2}} + \frac{\mathcal{T}(\Lambda_b^0 \to \Xi^0 \pi^0)}{3\sqrt{2}} + \frac{\mathcal{T}(\Lambda_b^0 \to \Xi^- \pi^0)}{3\sqrt{2}} = 0, \quad (47)$$

$$\frac{\mathcal{T}(\Xi_b^0 \to \Sigma^0 K^0)}{\sqrt{6}} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^+ K^-)}{2\sqrt{3}} + \frac{\mathcal{T}(\Xi_b^0 \to \Sigma^- K^0)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^0 \to \Xi^0 \pi^0)}{\sqrt{6}} - \frac{\mathcal{T}(\Xi_b^0 \to \Xi^- \pi^0)}{\sqrt{6}} = 0. \quad (48)$$

The same set of relations hold for the penguin parts as well.

V. CP RELATIONS

The total decay rate for a two body decay of a spin-1/2 anti-triplet $b$-baryon ($B_b$) to a spin 0 pseudo-scalar ($M$) and a spin 1/2 baryon ($B$) has the following form $^{67, 88, 91, 08}$

$$\Gamma(B_b \to B M) = \frac{|p_B|}{8\pi m_{B_b}^2} \left[ |S|^2 + |P|^2 \right]$$

where $|p_B|$ is the momentum of the final state baryon. Since the decay products can be in any one of the two possible relative angular momentum states, $l = 0$ and $l = 1$, the amplitude can also be decomposed in terms of $S$-wave and $P$-wave parts. Including the phase space corrections the $S$-wave and $P$-wave amplitudes are expressed with kinematic factors factored out $^{92, 84}$ as

$$S = \sqrt{2m_{B_b}(E_B + m_B)} A^S$$

$$P = \sqrt{2m_{B_b}(E_B - m_B)} A^P$$

where $A^S$ and $A^P$ are the the $SU(3)$-reduced amplitudes defined in Eq. (10). The decay rate is then expressed as

$$\Gamma = \frac{|p_B|}{4\pi m_{B_b}} \left[ |A^S|^2 + \left( \frac{|p_B|}{E_B + m_B} \right)^2 |A^P|^2 \right] \quad (50)$$

$A_{CP}$ is defined subsequently as $^{94}$,

$$A_{CP} = \frac{\Gamma(B_b \to B M) - \Gamma(B_b \to \overline{B} \overline{M})}{\Gamma(B_b \to B M) + \Gamma(B_b \to \overline{B} \overline{M})}$$
\[ \bar{\Gamma}(B_b \to B.M) = \frac{1}{2} (\Gamma(B_b \to B.M) + \Gamma(B_b \to \bar{B}.\bar{M})) \]

In order to express \( CP \) relation among modes we rely on the identity \( \text{Im}(V_{ub} V_{us}^* V_{kb} V_{ks}^*) = -\text{Im}(V_{ub} V_{us}^* V_{kb} V_{kd}^*) = J \), where \( J \) is the well known Jarlskog invariant. Notice, Eqs. (19) and (31) imply that \( A_{CP} \) is the sum \( CP \) violation in the \( S \) and \( P \) waves. We define a quantity \( \Delta_{CP}^a = |A^a|^2 - |A^0|^2 \), for the partial wave \( a \), where \( a = \{ S, P \} \) and \( A^a \) are defined in Eq. (19) with phase-space factors removed from the respective partial waves. By definition,

\[
\Delta_{CP}^a(B_b \to B.M) = -4J \times \text{Im} \left[ A_{CP}^a(B_b \to B.M)A_{CP}^a(B_b \to B.M) \right].
\]

Based on amplitude relations for the tree and penguin parts obtained in Eqs. (21–20) and Eqs. (31–40), the following ten \( \Delta_{CP}^a \) relations are obtained,

\[
\begin{align*}
\Delta_{CP}^a(\Lambda_b^0 \to \Sigma^- K^+) &= -\Delta_{CP}^a(\Xi_b^0 \to \Xi^- \pi^+), \\
\Delta_{CP}^a(\Lambda_b^0 \to p^+ \pi^-) &= -\Delta_{CP}^a(\Xi_b^0 \to \Sigma^+ K^-), \\
\Delta_{CP}^a(\Xi_b^0 \to nK^-) &= -\Delta_{CP}^a(\Xi_b^0 \to \Xi^0 \pi^-), \\
\Delta_{CP}^a(\Xi_b^0 \to nK^-) &= -\Delta_{CP}^a(\Lambda_b^0 \to \Sigma^- K^+), \\
\Delta_{CP}^a(\Xi_b^0 \to \Xi^- K^+) &= -\Delta_{CP}^a(\Xi_b^0 \to \Sigma^+ K^-), \\
\Delta_{CP}^a(\Xi_b^0 \to \Xi^- K^+) &= -\Delta_{CP}^a(\Lambda_b^0 \to \Sigma^- \pi^+), \\
\Delta_{CP}^a(\Xi_b^0 \to nK^-) &= -\Delta_{CP}^a(\Lambda_b^0 \to \Xi^0 K^0), \\
\Delta_{CP}^a(\Xi_b^0 \to \Xi^- K^+) &= -\Delta_{CP}^a(\Lambda_b^0 \to \Sigma^+ \pi^-), \\
\Delta_{CP}^a(\Xi_b^0 \to nK^-) &= -\Delta_{CP}^a(\Lambda_b^0 \to nK^0),
\end{align*}
\]

for both \( a = S \) and \( a = P \). Finally we obtain \( A_{CP} \) relations using,

\[
A_{CP}(B_b \to B.M) = \frac{\tau_{B_b}}{BR(B_b \to B.M)} \Delta_{CP}(B_b \to B.M),
\]

where, \( \tau_{B_b} \) is the lifetime of the beauty-baryon. The relation between \( A_{CP} \) and \( \Delta_{CP} \) is,

\[
\Delta_{CP} = \frac{\left| p_B \right|}{4\pi} \frac{(E_B + m_B)}{m_{B_b}} \left[ \delta_{CP} + \left( \frac{\left| p_B \right|}{E_B + m_B} \right)^2 \delta_{CP} \right] \]

Since, \( \Delta_{CP} \) depends on the masses of the initial and final baryons as well as the final state meson, some approximation is needed to obtain \( A_{CP} \) relations between various modes. Ignoring \( p_B \) and \( m_B \) differences between the various modes, \( CP \) violation relations between various modes can be experimentally verified using the relation,

\[
\frac{A_{CP}(B_b \to B_j M_k)}{A_{CP}(B_b \to B_j M_{\bar{k}})} \approx \frac{\tau_{B_b}}{BR(B_b \to B_j M_k)} \frac{BR(B_b \to B_m M_n)}{\left( E_B + m_B \right)^2},
\]

where \( i, j, k \) and \( l, m, n \) are indices corresponding to the various baryons belonging to the above mentioned \( \delta_{CP} \) relations. There is a further simplification in case \( i = l \), resulting in

\[
\frac{A_{CP}(B_b \to B_j M_k)}{A_{CP}(B_b \to B_m M_{\bar{n}})} \approx \frac{BR(B_b \to B_j M_k)}{BR(B_b \to B_m M_{\bar{j}})} \frac{\left| A^j \right|^2 - |A^0|^2}{\left| A^k \right|^2 + |A^0|^2},
\]

where the uncertainties due to lifetime measurement also cancel out. Alternatively, if the longitudinal polarization of the daughter baryon can be measured from an angular distribution study of the final states, one can estimate the relative strength of the \( P \)-wave contribution in the total decay width. The longitudinal polarization of the daughter baryon is given by,

\[
\alpha = \frac{2 \text{Re}(A^P A^0)}{|A^0|^2 + |A^P|^2 (E_B + m_B)^2}
\]

The \( P \)-wave contribution can now be systematically taken into account resulting in a more reliable prediction for \( A_{CP} \) relations. These relations serve as an important test of flavor \( SU(3) \) symmetry in beauty-baryon non-leptonic decays and one can compare these findings with the analogous decays of bottom mesons to have a better understanding of the \( SU(3) \) flavor symmetry breaking pattern.

**VI. CONCLUSIONS**

We consider a general framework for hadronic beauty-baryon decays into octet or singlet of light baryon and a pseudoscalar meson, based on \( SU(3) \) decomposition of the decay amplitudes. We show that in the most general case, the 44 distinct decay modes require 44 independent reduced \( SU(3) \) amplitudes to describe all possible \( \Delta S = -1 \) and \( \Delta S = 0 \) processes. In practice, the dimension-6 effective Hamiltonian that mediates such non-leptonic decays of bottom baryons predicts only 10 independent reduced \( SU(3) \) amplitudes. As a consequence there must exist relations between the decay amplitudes. We explicitly derive several sum rules relations between decay amplitudes as well as relations between \( CP \) asymmetries. Moreover, we systematically study the \( SU(3) \)-breaking effects in the decay amplitudes at leading order in the \( SU(3) \) breaking parameter. We further identify an amplitude relation that survives even when the \( SU(3) \) flavor symmetry is no longer exact.

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Appendix A: SU(3) isoscalar factors

The isoscalar factors depend on the identity of the representations, and on the hypercharges and isospins of the isomultiplets that are coupled. Let us denote them by the following notation:

\[
\begin{pmatrix}
  r & r' & R \\
  (y, i, i_3) & (y', i', i_3') & (Y, I, I_3)
\end{pmatrix}.
\]  

(A1)

The SU(3) Clebsch-Gordan coefficients are found as products of isoscalar factors and SU(2) Clebsch-Gordan coefficients:

\[
\langle R, Y, I, I_3 | r, y, i, i_3, r', y', i', i_3' \rangle = \begin{pmatrix}
  r & r' & R \\
  (y, i, i_3) & (y', i', i_3') & (Y, I, I_3)
\end{pmatrix} \langle I, I_3 | i, i_3' \rangle.
\]  

(A2)

where \( \langle I, I_3 | i, i_3' \rangle \) are the SU(2) CG coefficients. The order in which the SU(3) representations are coupled is \( r \otimes r' \rightarrow R \). The two symmetry relations involving the SU(3) isoscalar factors are as follows:

A) If the order in which the representations are coupled is reversed (i.e. \( r' \otimes r \rightarrow R \)) then the isoscalar factors pick up a phase factor:

\[
\begin{pmatrix}
  r' & r & R \\
  (y', i', i_3') & (y, i, i_3) & (Y, I, I_3)
\end{pmatrix} = (-1)^{I + I'} \xi(R; r, r') \begin{pmatrix}
  r & r' & R \\
  (y, i, i_3) & (y', i', i_3') & (Y, I, I_3)
\end{pmatrix}.
\]  

(A2)

Here \( \xi(R; r, r') \) is the phase factor that depends only on the identity element of \( r, r' \) and \( R \) and the phase convention chosen for the highest weight state.

B) Conjugation operation on all three representations also give rise to a phase factor:

\[
\begin{pmatrix}
  r & r & R \\
  (y, i, i_3) & (y', i', i_3') & (Y, I, I_3)
\end{pmatrix} = (-1)^{I + I'} \xi(R; r, r') \begin{pmatrix}
  r & r' & R \\
  (y, i, i_3) & (y', i', i_3') & (Y, I, I_3)
\end{pmatrix}.
\]  

(A3)

Similar to the previous case, \( \xi(R; r, r') \) is the phase factor that depends only on the identity element of \( r, r' \) and \( R \) and the phase convention chosen for the highest weight state. As a corollary of Eqs. (A2) and (A3),

\[
\xi(R; r, r') = \xi(R; r, r'),
\]  

(A4)

\[
\xi(R; r', r) = \xi(R; r, r').
\]  

(A5)

Following, we have adopted the Condon-Shortley and de Swart phase convention that requires eigenvalues of the isospin(I) as well as \( V \) spin raising and lowering operators are real and positive. An additional requirement on the highest weight state is Clebsch-Gordan coefficient between these three states be real and positive, i.e.

\[
\langle R, Y_h, I_h, I_{h3} | r, y_h, i_h, i_{h3}, r', y_h', i_h', i_{h3}' \rangle > 0
\]  

These conditions ensure that SU(3) CG coefficients and isoscalar factors are all real.
The decay amplitudes for $\gamma S\gamma S\gamma S \rightarrow S S S$ in the following way:

\[ \text{Appendix B: decomposition of } \Omega \rightarrow \]
The $\Delta S = 0$ decay amplitudes for $\Xi_{B_s} \to 8_B 1_M$ are $SU(3)$ decomposed in the following way:

$$
\begin{pmatrix}
A(\Lambda_b^0 \to n\eta_1) \\
A(\Xi_b^0 \to \Lambda^0\eta_1) \\
A(\Xi_b^0 \to \Sigma^0\eta_1) \\
A(\Xi_b^- \to \Sigma^-\eta_1)
\end{pmatrix} =
\begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\
-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0
\end{pmatrix}
\begin{pmatrix}
8 || 3_{1/2} || 3 \\
8 || 1_{1/2} || 3 \\
8 || 15_{1/2} || 3 \\
8 || 15_{3/2} || 3
\end{pmatrix}
$$

(B1)

The $\Delta S = 0$ decay amplitudes for $\Xi_{B_s} \to 1_B 8_M$ are $SU(3)$ decomposed in the following way:

$$
\begin{pmatrix}
A(\Xi_b^0 \to \Lambda^0n\eta_8) \\
A(\Xi_b^0 \to \Lambda^0\pi_0) \\
A(\Xi_b^- \to \Lambda^0\pi^-) \\
A(\Lambda_b^0 \to \Lambda^0* K^0)
\end{pmatrix} =
\begin{pmatrix}
-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0
\end{pmatrix}
\begin{pmatrix}
8 || 3_{1/2} || 3 \\
8 || 6_{1/2} || 3 \\
8 || 15_{1/2} || 3 \\
8 || 15_{3/2} || 3
\end{pmatrix}
$$

(B2)

Appendix C: $SU(3)$ decomposition of $\Delta S = -1$ processes

The $\Delta S = -1$ decay amplitudes for $\Xi_{B_s} \to 8_B 1_M$ are $SU(3)$ decomposed in the following way:

$$
\begin{pmatrix}
A(\Lambda_b^0 \to \Lambda^0\eta_1) \\
A(\Lambda_b^0 \to \Sigma^0\eta_1) \\
A(\Xi_b^- \to \Sigma^-\eta_1) \\
A(\Xi_b^0 \to \Sigma^0\eta_1)
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{2} & 0 & \frac{1}{2} & 0 \\
0 & \frac{1}{2} & 0 & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
8 || 3_0 || 3 \\
8 || 6_0 || 3 \\
8 || 15_0 || 3 \\
8 || 15_1 || 3
\end{pmatrix}
$$

(C1)

The $\Delta S = -1$ decay amplitudes for $\Xi_{B_s} \to 1_B 8_M$ are $SU(3)$ decomposed in the following way:

$$
\begin{pmatrix}
A(\Lambda_b^0 \to \Lambda^0*\pi^0) \\
A(\Lambda_b^0 \to \Lambda^0*\eta_8) \\
A(\Xi_b^- \to \Lambda^0* K^-) \\
A(\Xi_b^0 \to \Lambda^0* K^0)
\end{pmatrix} =
\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
8 || 3_0 || 3 \\
8 || 6_0 || 3 \\
8 || 15_0 || 3 \\
8 || 15_1 || 3
\end{pmatrix}
$$

(C2)
The decay amplitudes for the \( \Sigma \) and \( \Lambda \) are decomposed in the following way:
