Crest Shape Effect on the Performance of Rectangular Side Weirs

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Abstract

To study the effect of crest shape on the performance of rectangular side weirs, one hundred and four side weir models were tested, eight of them were for sharp crest and ninety six for rounded crest. For each set of rounded crest models, the radius of crest was changed four times and for each radius, the height of the side weir was varied four times. A total of nine hundred thirty four tests were conducted on these models. For all models of different crest shapes, it was found that the average energy difference between two ends of the side weir is very small and less than one percent which can be ignored. The variation of discharge coefficient ($C_M$) with upstream Froude number ($F_{r1}$) was found to be linear. The correlation between ($C_M$) and ratio of upstream water depth to weir height ($h/p$) was found to be linear. The variation of ($C_M$) with the ratio of upstream head above crest to crest radius ($h/r$) was also found linear. A simple power empirical expression was obtained for the combined effect of ($F_{r1}$) and ($h/p$) on ($C_M$) for free flow over sharp crest side weirs with high correlation coefficient. Three simple power empirical expressions were obtained for the variation of ($C_M$) with ($F_{r1}$), ($h/p$) and ($h/r$) for the free flow over inside semicircular, outside semicircular and circular crest shapes with high correlation coefficients. From the hydraulic performance, it was found that small weir heights gave higher performance for all crest shapes. The highest performance of inside semicircular crest was found for crest radius ($r$) = 3.15cm, the highest performance of outside semicircular crest was found for ($r$) = 3.75cm, while, the highest performance of circular crest was found for ($r$) = 2.5cm and ($r/p$) = 0.25.

Keywords: Crest Shape, Performance, Side Weirs.

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1. Introduction

A side weir is an overflow weir framed in the side of a channel. The major function of a side weir in open channel is to either split flow or discharge excess flow into a side channel when the water level in the main channel exceed a specified limit. The flow over a side weir is a case of spatially varied flow with decreasing discharge. Although various geometric and hydraulic shapes of side weirs and different channel cross sections in which side weirs are placed have been used by hydraulic engineers, the most common side weir is still the simple sharp crest weir installed in a rectangular main channel.

Due to the multi-functions of side weirs, their hydraulic behavior has attracted the attention of many investigators. De Marchi (1934) was one of the earliest investigators who gave equations for the flow over side weirs on the basis of a constant specific energy along the side weir. Collinge (1957) studied experimentally the determination of water surface profiles, examination of application of de Marchi's theory, investigation of the effects of velocity of the main channel and movement of bed load on the side weir discharge coefficients. Subramanya and Awathey (1972) and Ranga Raju et al. (1979) concluded that the de Marchi equation can be used and they presented different equations for calculations of discharge coefficient in terms of upstream Froude number for subcritical flow conditions. Cheong (1991) proposed an equation for discharge coefficient in terms of upstream Froude number for prismatic trapezoidal main channels for subcritical flow conditions and he proposed that his equation can be applied to rectangular channels. The effect of side weir characteristics on discharge coefficient was studied by Swamee et al. (1994) by presenting an expression for the estimation of discharge coefficient in terms of side weir height and upstream water depth. Singh et al. (1994) and Jalili and Borghesi (1996) obtained equations for the calculation of de Marchi discharge coefficient in terms of both upstream Froude number and ratio of weir height to upstream water depth. Borghesi et al. (1999) studied the combined effects of upstream Froude number, ratio of weir height to upstream water depth and ratio of side weir length to main channel width on the de Marchi discharge coefficient. Al-hamdany (2008) studied the characteristics of semicircular rectangular side weirs in a trapezoidal main channel proposing an equation for the discharge coefficient in terms of weir height, upstream water depth, length of side weir and width of main channel. Honar and Keshavarzi (2008) concluded that side weirs with rounded edge entrance discharge 10% more flow rate than squared edge entrance. Uyumaz and Muslu (1985) and Kumar and Pathak (1987) have also used the de Marchi constant specific energy principle to determine the coefficient of discharge for side weirs in triangular and circular channels.

From the above literature review, it is clear that no effort has been put into the subject of changing crest shape. The present investigation has been taken up as a contribution toward a better understanding of the hydraulic and geometric influence of flow in the main channel on the free flow over side weirs of different crest shapes. Rounding crest of side weirs is expected to increase the performance of the side weir and consequently increase the value of discharge coefficient.

2. Theoretical Considerations

De Marchi (1934) proposed the following equation for the estimation of discharge coefficient \( C_M \) on the basis of constant specific energy along the side weir (see Fig. (1)):

\[
C_M = \frac{3}{2} \frac{B}{L} \left[ \Phi_2 - \Phi_1 \right]
\]  

(1)
where,
\[
\Phi_1 = \frac{2E_1 - 3p}{E_1 - p} \sqrt{\frac{E_1 - y_1}{y_1 - p}} - 3\sin^{-1}\sqrt{\frac{E_1 - y_1}{E_1 - p}}
\]
\[
\Phi_2 = \frac{2E_2 - 3p}{E_2 - p} \sqrt{\frac{E_2 - y_2}{y_2 - p}} - 3\sin^{-1}\sqrt{\frac{E_2 - y_2}{E_2 - p}}
\]

\(B = \) main channel width,
\(E_1, E_2 = \) specific energy heads at upstream and downstream ends of the side weir, respectively,
\(y_1, y_2 = \) water depths at upstream and downstream ends of the side weir, respectively,
\(L = \) length of the side weir, and
\(p = \) side weir height.

![Fig. (1): Definition sketch of subcritical flow over rectangular side weir.](image)

A general relationship for the variables affecting the de Marchi discharge coefficient \((C_M)\) for free flow over a side weir with different shapes of crest can be expressed as:

\[
C_M = f_1 \left( p, L, r, \rho, g, y_1, V_1, B \right)
\]

(2)

where, \(r = \) crest radius in case of rounded crest, \(\rho = \) mass density of fluid,
\(g = \) acceleration due to gravity, and \(V_1 = \) upstream velocity. Using Buckingham Pi-theorem and after certain permissible manipulation Eq. (2) becomes:

\[
C_M = f_2 \left( Fr_1, \frac{h_1}{p}, \frac{h_1}{r}, \frac{L}{y_1}, \frac{B}{y_1} \right)
\]

(3)

where, \(Fr_1 = \) Froude number at upstream end of the side weir = \(V_1 / \sqrt{gy_1} \).
The discharge coefficient \( C_M \) is assumed to be independent of weir length \( L \) and width of the main channel \( B \) is kept constant during the present investigation, therefore, Eq. (3) can be reduced to the form:

\[
C_M = f_s \left( Fr_i, \frac{h_i}{p} \frac{h_i}{r} \right)
\] (4)

For sharp crest side weirs \( r=0 \), the term \( \left( h_i / r \right) \) of Eq. (4) is ignored and Eq. (4) may be reduced to the following form:

\[
C_M = f_s \left( Fr_i, \frac{h_i}{p} \right)
\] (5)

### 3. Experimental Setup

The experimental work of this study was carried out at Hydraulic Laboratory of Water Resources Department of Engineering College, University of Duhok. A comprehensive set of tests on eight models for different ranges of variables such as discharge, side weir length and side weir height has been conducted for sharp crest side weir. Also, the experimental program included the testing of ninety six more models to study the performance of rounded crest shapes, thirty two models for testing of inside semicircular crest, thirty two models for testing of outside semicircular crest and another thirty two models for testing of circular crest (see Fig. (2)). In each set of models the radius of crest was changed four times \( r=2 \text{cm}, 2.5 \text{cm}, 3.15 \text{cm} \text{ and } 3.75 \text{cm} \) and for each radius the height of the side weir was varied four times \( p=10 \text{cm}, 15 \text{cm}, 20 \text{cm}, \text{ and } 25 \text{cm} \). All tests were conducted in a horizontal glass walled rectangular tilting flume of 5m length, 0.3m width and 0.45m depth. Water was circulated through the channel by an electrically driven centrifugal pump providing a total flow of 30 l/sec. The flume was divided into two passage channels one for the main channel of width 0.174 m and the other for the side channel of width 0.12m. The division was made via a Perspex sheet as shown in Fig. (3). At the end of the main channel a normal rectangular thin plate weir was used. This weir was fixed at a distance 2.2 m downstream the end of the side weir in order to measure the discharge in the main channel \( Q_2 \). The total flow in the two channels was measured from a calibrated digital control console \( Q_1 \). The flow passing over the side weir \( Q_w \) was measured as \( Q_w=Q_1-Q_2 \). The longitudinal slope of the flume was set to zero (horizontal) for all tests. Each model was set horizontal and fixed at a distance 2.6 m from upstream end of the flume. Different flow rates were allowed in the flume starting from very low flow rate controlled via a manually operated valve located at the delivery pipe. The flow rate was gradually increased taking eight to ten runs on each model. For each run the corresponding heads above crest of the side weir, starting from upstream end towards the downstream end at variable intervals aligned on the center line of the main channel, were measured using a point gauge. While the corresponding flow rate in the main channel was calculated from the flow equation of the normal rectangular weir. The total flow rate was measured from the digital control console. A total of nine hundred thirty four experiments were conducted during the experimental program of the present investigation. Details of the experimental program are shown in Table (1).
Table (1): Details of the experimental program

| Model No. | Crest shape     | Crest radius \( r \) (cm) | Side weir height \( p \) (cm) | Side weir length \( L \) (cm) | Run No.       |
|-----------|-----------------|---------------------------|-----------------------------|-----------------------------|--------------|
| 1-4       | Sharp           | 0                         | 10,15,20,25                 | 20                          | 1-34         |
| 5-8       | Sharp           | 0                         | 10,15,20,25                 | 30                          | 35-70        |
| 9-24      | Inside semicircular | 2.2,5,3.15,3.75           | 10,15,20,25                 | 20                          | 71-214       |
| 25-40     | Inside semicircular | 2.2,5,3.15,3.75           | 10,15,20,25                 | 30                          | 215-358      |
| 41-56     | Outside semicircular | 2.2,5,3.15,3.75          | 10,15,20,25                 | 20                          | 359-502      |
| 57-72     | Outside semicircular | 2.2,5,3.15,3.75          | 10,15,20,25                 | 30                          | 503-646      |
| 73-88     | circular        | 2.2,5,3.15,3.75           | 10,15,20,25                 | 20                          | 647-790      |
| 89-104    | circular        | 2.2,5,3.15,3.75           | 10,15,20,25                 | 30                          | 791-934      |

(a) Sharp crest shape  (b) Inside semicircular crest shape
(c) Outside semicircular crest shape  (d) Circular crest shape

Fig. (2): Different shapes of side weir crest.

Fig. (3): General plan of the flume and the location of the side weir.
4. Analysis of Results

4.1 De Marchi Discharge Coefficient of Side Weir ($C_M$)

In order to use de Marchi equation for the calculation of discharge coefficients for all models, the assumption of constant energy for subcritical flow has to be checked. Fig. (4) shows the average energy difference in the channel between the ends of the side weir (i.e., $E_1$ and $E_2$). This difference was found to be very small and less than (1%) which can be ignored. This is an evidence to use de Marchi discharge coefficient equation with no fear of uncertainty or lack of accuracy. Borghei et. al. (1999) showed that the energy difference in the channel between the two ends of the side weir is (3.7%), El-Khashab and Smith (1976) estimated a (5%) difference for subcritical flow, whereas Ranga Raju et al. (1979) obtained a mean value of (2%). Thus the assumption of constant energy is acceptable.

![Graph showing specific energy at two ends of the side weir](image)

Fig. (4): Specific energy at two ends of the side weir.

4.2 Variation of ($C_M$) with Upstream Froude Number ($Fr_1$)

The variation of discharge coefficient ($C_M$) with upstream Froude number ($Fr_1$) is studied for all side weir crest shapes, different heights of side weir and two side weir lengths (20 cm and 30 cm). Values of Froude number $\left( Fr_1 = \frac{V_1}{\sqrt{gy_1}} \right)$ were calculated for depths of flow upstream the side weir ($y_1$) for all cases. The obtained results can be defined by simple linear equations of the form:

$$C_M = a_1 + b_1 Fr_1$$  \hspace{1cm} (6)

Values of constants $a_1$ and $b_1$ and the corresponding values of correlation coefficient ($R$) were obtained for all cases and tabulated in Table (2). Sample of plots for the variation of ($C_M$) with ($Fr_1$) are shown in Figs (5), (6), (7) and (8) for sharp, inside semicircular, outside semicircular and circular crest side weirs respectively with height ($p$) =25 cm and crest radius ($r$) = 3.75 cm.
Fig. (5): Variation of $C_M$ with $Fr_1$ for sharp crest with $p = 25$ cm.

Fig. (6): Variation of $C_M$ with $Fr_1$ for inside semicircular crest with $r = 3.75$ cm and $p = 25$ cm.

Fig. (7): Variation of $C_M$ with $Fr_1$ for outside semicircular crest with $r = 3.75$ cm and $p = 25$ cm.

Fig. (8): Variation of $C_M$ with $Fr_1$ for circular crest with $r = 3.75$ cm and $p = 25$ cm.

From the above figures and Table (2), one may observe that the value of $C_M$ decreases with the increase of ($Fr_1$) values. Similar patterns of linear equations were obtained by Ranga Raju et. al. (1979), Singh et. al. (1995) and Pinheiro and Silva (1999) for sharp crested side weirs.

Table (2): Values of constants ($a_1$) and ($b_1$) and correlation coefficient (R) of Eq. (6) for different crest shapes and different heights of side weir.

| Crest shape         | Sharp       | Inside semicircular with radius=2 cm |
|---------------------|-------------|--------------------------------------|
|                     | $a_1$      | $b_1$      | $R$     | $a_1$      | $b_1$      | $R$     |
| $p$ (cm)            |            |            |         |            |            |         |
| 10                  | 0.7463     | -0.9413    | 0.9799  | 0.7335     | -0.7719    | 0.9497  |
| 15                  | 0.7483     | -0.9983    | 0.8806  | 0.7513     | -0.8006    | 0.9392  |
| 20                  | 0.7223     | -1.0349    | 0.8606  | 0.7739     | -0.9975    | 0.9117  |
| 25                  | 0.6484     | -0.8611    | 0.8500  | 0.6804     | -0.6874    | 0.8800  |
| Crest shape | Inside semicircular with radius=2.5 cm | Inside semicircular with radius=3.15 cm |
|-------------|-------------------------------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7319  | -0.7553 | 0.9614 | 0.7587 | -0.8534 | 0.9434 |
| 15          | 0.6799  | -0.5881 | 0.8857 | 0.6616 | -0.5444 | 0.8574 |
| 20          | 0.6976  | -0.7098 | 0.8359 | 0.6859 | -0.6344 | 0.8840 |
| 25          | 0.6508  | -0.6360 | 0.8018 | 0.6615 | -0.7152 | 0.6785 |

| Crest shape | Inside semicircular with radius=3.75 cm | Outside semicircular with radius=2 cm |
|-------------|-------------------------------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7180  | -0.7557 | 0.9946 | 0.7574 | -0.8201 | 0.9650 |
| 15          | 0.6437  | -0.4946 | 0.7873 | 0.7115 | -0.7831 | 0.8179 |
| 20          | 0.6749  | -0.6568 | 0.8105 | 0.7253 | -0.9159 | 0.8496 |
| 25          | 0.6720  | -0.8144 | 0.7945 | 0.6929 | -1.1264 | 0.8000 |

| Crest shape | Outside semicircular with radius=2.5 cm | Outside semicircular with radius=3.15 cm |
|-------------|-------------------------------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7413  | -0.7219 | 0.9677 | 0.7605 | -0.7974 | 0.9703 |
| 15          | 0.7608  | -0.9621 | 0.8021 | 0.7130 | -0.8190 | 0.8694 |
| 20          | 0.6950  | -0.9297 | 0.8506 | 0.7038 | -0.7905 | 0.8898 |
| 25          | 0.6614  | -1.0134 | 0.8416 | 0.6414 | -0.8360 | 0.8336 |

| Crest shape | Outside semicircular with radius=3.75 cm | Circular with radius=2 cm |
|-------------|-------------------------------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7715  | -0.8191 | 0.9785 | 0.7542 | -0.7347 | 0.9697 |
| 15          | 0.6993  | -0.7900 | 0.9285 | 0.6970 | -0.5363 | 0.8785 |
| 20          | 0.7058  | -0.9980 | 0.8430 | 0.7282 | -0.8135 | 0.7915 |
| 25          | 0.6605  | -0.8237 | 0.7253 | 0.7055 | -0.6966 | 0.7895 |

| Crest shape | Circular with radius=2.5 cm | Circular with radius=3.15 cm |
|-------------|-------------------------------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7570  | -0.7725 | 0.9517 | 0.7336 | -0.7281 | 0.9414 |
| 15          | 0.6877  | -0.4932 | 0.8656 | 0.6696 | -0.4516 | 0.8656 |
| 20          | 0.6760  | -0.5477 | 0.8283 | 0.6448 | -0.4212 | 0.7301 |
| 25          | 0.6703  | -0.5426 | 0.7284 | 0.6661 | -0.5342 | 0.7301 |

| Crest shape | Circular with radius=3.75 cm |
|-------------|-------------------------------------|
| \( p \) (cm) | \( a_i \) | \( b_i \) | \( R \) |
| 10          | 0.7094  | -0.6885 | 0.9231 |
| 15          | 0.6566  | -0.4632 | 0.8554 |
| 20          | 0.6535  | -0.4849 | 0.8235 |
| 25          | 0.6537  | -0.5996 | 0.7552 |
4.3 **Variation of \( (C_M) \) with Relative Depth ( \( h_i / p \))**

The variation of \( (C_M) \) with the ratio of upstream water depth above side weir crest to weir height (\( h_i / p \)) was studied for different crest shapes, different side weir heights and two side weir lengths. The best fit relation between \( (C_M) \) and \( (h_i / p) \) was found to be linear of the following general form:

\[
C_M = a_2 + b_2 \frac{h_i}{p}
\]  

(7)

Values of constants \( (a_2) \) and \( (b_2) \) and corresponding values of correlation coefficient \( (R) \) are tabulated in Table (3). Sample of plots for the variation of \( (C_M) \) with \( (h_i / p) \) are shown in Figs (9), (10), (11) and (12) for sharp, inside semicircular, outside semicircular and circular crest side weirs respectively with height \( (p) =25 \text{ cm} \) and crest radius \( (r) = 3.75 \text{ cm} \). These figures and Table (3), show that the values \( (C_M) \) decrease with the increase of \( (h_i / p) \) values. Similar linear relationships were obtained by Singh et al. (1994) and Pinheiro and Silva (1999) for sharp crest side weirs under subcritical flow conditions.

| Fig. (9): Variation of \( C_M \) with \( h_i / p \) for sharp crest with \( p =25 \text{ cm} \). | Fig. (10): Variation of \( C_M \) with \( h_i / p \) for inside semicircular crest with \( r = 3.75 \text{ cm} \) and \( p = 25 \text{ cm} \). |
| --- | --- |
| ![Graph](image1.png) | ![Graph](image2.png) |

| Fig. (11): Variation of \( C_M \) with \( h_i / p \) for outside semicircular crest with \( r = 3.75 \text{ cm} \) and \( p = 25 \text{ cm} \). | Fig. (12): Variation of \( C_M \) with \( h_i / p \) for circular crest with \( r = 3.75 \text{ cm} \) and \( p = 25 \text{ cm} \). |
| --- | --- |
| ![Graph](image3.png) | ![Graph](image4.png) |
Table (3): Values of constants \((a_2)\) and \((b_2)\) and correlation coefficient \((R)\) of Eq. (7) for different crest shapes and different heights of the side

| Crest shape | Sharp | Inside semicircular with radius=2 cm | Inside semicircular with radius=2.5 cm | Inside semicircular with radius=3.15 cm | Inside semicircular with radius=3.75 cm | Outside semicircular with radius=2 cm | Outside semicircular with radius=2.5 cm | Outside semicircular with radius=3.15 cm | Circular with radius=2 cm | Circular with radius=2.5 cm | Circular with radius=3.15 cm |
|-------------|-------|------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| \(p\) (cm) | \(a_2\) | \(b_2\) | \(R\) | \(a_2\) | \(b_2\) | \(R\) | \(a_2\) | \(b_2\) | \(R\) | \(a_2\) | \(b_2\) | \(R\) | \(a_2\) | \(b_2\) | \(R\) | \(a_2\) | \(b_2\) | \(R\) |
| 10          | 0.7259 | -0.4524 | 0.9231 | 0.7451 | -0.4434 | 0.9681 | 0.7848 | -0.4704 | 0.9355 | 0.7501 | -0.4312 | 0.9546 | 0.7848 | -0.4704 | 0.9355 | 0.7501 | -0.4312 | 0.9546 |
| 15          | 0.7881 | -0.6271 | 0.9313 | 0.7774 | -0.5146 | 0.9344 | 0.7218 | -0.3732 | 0.9417 | 0.6927 | -0.3121 | 0.8846 | 0.7218 | -0.3732 | 0.9417 | 0.6927 | -0.3121 | 0.8846 |
| 20          | 0.7629 | -0.6787 | 0.8871 | 0.8108 | -0.6586 | 0.9349 | 0.7076 | -0.4011 | 0.9102 | 0.6787 | -0.3406 | 0.8654 | 0.7076 | -0.4011 | 0.9102 | 0.6787 | -0.3406 | 0.8654 |
| 25          | 0.6856 | -0.5708 | 0.9332 | 0.7067 | -0.4635 | 0.9256 | 0.7063 | -0.4361 | 0.8618 | 0.7022 | -0.4305 | 0.9008 | 0.7063 | -0.4361 | 0.8618 | 0.7022 | -0.4305 | 0.9008 |
4.4 Variation of \((C_M)\) with Relative Depth \((h_1/r)\)

Since crest of the side weir is rounded, thus, the discharge coefficient \((C_M)\) is expected to depend on crest radius \((r)\). In dimensional analysis, it was shown that \((C_M)\) is dependent on the ratio of upstream water depth above side weir crest to the crest radius \((h_1/r)\). The variation of \((C_M)\) with \((h_1/r)\) was studied for different crest shapes, different side weir heights and two side weir lengths. Form the analysis of experimental results, the general relation between \((C_M)\) and \((h_1/r)\) was found to be linear and of the form:

\[
C_M = a + b \frac{h_1}{r} \tag{8}
\]

Values of constants \((a)\) and \((b)\) and corresponding values of correlation coefficient \((R)\) are tabulated in Table (4). Sample of plots are shown in Figs (13), (14) and (15) for inside semicircular, outside semicircular and circular crest side weirs respectively with height \((p) = 25\) cm and crest radius \((r) = 3.75\) cm.

| Crest shape | Circular with radius=3.75 cm |
|-------------|-------------------------------|
|             | \(a\) | \(b\) | \(R\) |
| 10          | 0.7362 | -0.4239 | 0.9574 |
| 15          | 0.6846 | -0.3306 | 0.9052 |
| 20          | 0.6817 | -0.3568 | 0.9014 |
| 25          | 0.7383 | -0.6271 | 0.7314 |
From the above figures and Table (4), it is obvious to realize that values of \((C_M)\) decrease with the increase of \((h_i/r)\) values and the best fitting is a linear relationship with high value of correlation coefficient.

Table (4): Values of constants \((a_3)\) and \((b_3)\) and correlation coefficient \((R)\) of Eq. (8) for different crest shapes and different heights of side weir.

| Crest shape     | Inside semicircular with radius=2 cm | Inside semicircular with radius=2.5 cm |
|-----------------|--------------------------------------|----------------------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7451  | -0.0887 | 0.9681 | 0.7397  | -0.1061 | 0.9564 |
| 15              | 0.7774  | -0.0686 | 0.9344 | 0.7083  | -0.0665 | 0.9180 |
| 20              | 0.8108  | -0.0659 | 0.9349 | 0.7345  | -0.0619 | 0.8982 |
| 25              | 0.7067  | -0.0371 | 0.9256 | 0.6854  | -0.0458 | 0.9156 |

| Crest shape     | Inside semicircular with radius=3.15 cm | Inside semicircular with radius=3.75 cm |
|-----------------|-----------------------------------------|----------------------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7676  | -0.1503 | 0.9372 | 0.7166  | -0.1511 | 0.9124 |
| 15              | 0.6957  | -0.0810 | 0.9333 | 0.6800  | -0.0920 | 0.9018 |
| 20              | 0.7134  | -0.0673 | 0.9087 | 0.6984  | -0.0839 | 0.8787 |
| 25              | 0.7067  | -0.0687 | 0.8038 | 0.7111  | -0.0854 | 0.8767 |

| Crest shape     | Outside semicircular with radius=2 cm | Outside semicircular with radius=2.5 cm |
|-----------------|---------------------------------------|----------------------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7591  | -0.0896 | 0.9291 | 0.7451  | -0.1021 | 0.9412 |
| 15              | 0.7624  | -0.0748 | 0.9377 | 0.8178  | -0.1123 | 0.9127 |
| 20              | 0.7712  | -0.0620 | 0.9214 | 0.7327  | -0.0740 | 0.8881 |
| 25              | 0.7416  | -0.0610 | 0.8722 | 0.6899  | -0.0607 | 0.8426 |

| Crest shape     | Outside semicircular with radius=3.15 cm | Outside semicircular with radius=3.75 cm |
|-----------------|------------------------------------------|----------------------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7717  | -0.1451 | 0.9738 | 0.7791  | -0.1749 | 0.9645 |
| 15              | 0.7598  | -0.1180 | 0.9568 | 0.7307  | -0.1249 | 0.9493 |
| 20              | 0.7337  | -0.0807 | 0.9099 | 0.7314  | -0.1011 | 0.8664 |
| 25              | 0.6725  | -0.0667 | 0.8729 | 0.7089  | -0.0910 | 0.8608 |

| Crest shape     | Circular with radius=2 cm | Circular with radius=2.5 cm |
|-----------------|----------------------------|----------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7598  | -0.0844 | 0.9449 | 0.7848  | -0.1176 | 0.9355 |
| 15              | 0.7251  | -0.0503 | 0.9161 | 0.7218  | -0.0622 | 0.9417 |
| 20              | 0.7643  | -0.0564 | 0.8340 | 0.7076  | -0.0501 | 0.9102 |
| 25              | 0.7473  | -0.0434 | 0.9122 | 0.7063  | -0.0436 | 0.8618 |

| Crest shape     | Circular with radius=3.15 cm | Circular with radius=3.75 cm |
|-----------------|-------------------------------|-------------------------------|
| **p (cm)**      | **a_3** | **b_3** | **R** | **a_3** | **b_3** | **R** |
| 10              | 0.7501  | -0.1358 | 0.9546 | 0.7299  | -0.1562 | 0.9628 |
| 15              | 0.6927  | -0.0655 | 0.8846 | 0.6846  | -0.0826 | 0.9052 |
| 20              | 0.6787  | -0.0537 | 0.8654 | 0.6817  | -0.0669 | 0.9014 |
| 25              | 0.7022  | -0.0542 | 0.9008 | 0.6850  | -0.0657 | 0.8742 |
4.5 Variation of \( (C_M) \) with \((Fr_i)\) and \((h_i / p)\) for Sharp Crest Side Weirs

The functional relationship for the discharge coefficient \( (C_M) \) of free flow over side weirs of sharp crest can be written as a function of upstream Froude number \( (Fr_i) \) and \((h_i / p)\), (see Eq. (5)). To study the effects of both \((Fr_i)\) and \((h_i / p)\) together on \( (C_M) \), all experimental results of free flow over sharp crest side weirs were used as input data in a regression analysis computer program to obtain an empirical power expression of the following form:

\[
C_M = 0.434 \left( \frac{Fr_i^{0.207}}{h_i^{0.306}} \right) 
\]

with a correlation coefficient =0.837 and a standard deviation =0.055. The relation between values of \( (C_M) \) predicted by Eq. (9) and those observed experimentally is plotted in Fig. (16) showing quite good agreement.

4.6 Variation of \( (C_M) \) with \((Fr_i)\), \((h_i / p)\) and \((h_i / r)\) for Rounded Crest Shape Side Weirs

The functional relationship for the discharge coefficient \( (C_M) \) of free flow over side weirs with different rounded crest shapes may be written as a function of \((Fr_i)\), \((h_i / p)\) and \((h_i / r)\) (as shown in Eq. (4)). All experimental results of free flow over inside semicircular crest shape side weirs were used as input data in the regression analysis program to obtain an empirical power expression of the following form:

\[
C_M = 0.471 \left( \frac{Fr_i^{0.1}}{h_i^{0.314}} \left( \frac{h_i}{r} \right)^{0.021} \right) 
\]

with a correlation coefficient =0.865 and a standard deviation =0.045. Fig. (17) shows the relation between \( (C_M) \) values predicted by Eq. (10) and those observed experimentally showing good agreement.

Fig. (16): Variation of predicted values of \(C_M\) with observed ones for side weirs of sharp crest.

Fig. (17): Variation of predicted values of \(C_M\) with observed ones for side weirs of inside semicircular crest.
All experimental results of free flow over outside semicircular crest shape side weirs were used as input data in the same regression analysis program to obtain an empirical power expression of the following form:

\[
C_M = 0.519 \frac{Fr_1^{0.1}}{h_p^{0.26} h_r^{0.105}}
\]  

(11)

with a correlation coefficient =0.865 and a standard deviation =0.045. The relation between \(C_M\) values predicted by Eq. (11) and those observed experimentally is plotted in Fig. (18) showing good agreement.

All experimental results of free flow over circular crest shape side weirs were used as input data in the same regression program to obtain an empirical power expression of the following form:

\[
C_M = 0.483 \frac{Fr_1^{0.107}}{h_p^{0.313} h_r^{0.011}}
\]

(12)

with a correlation coefficient =0.865 and a standard deviation =0.032. A comparison between \(C_M\) values predicted by Eq. (12) and experimentally observed values is shown in Fig. (19) showing good agreement.

![Fig. (18): Variation of predicted values of \(C_M\) with observed ones for side weirs of outside semicircular crest.](image1)

![Fig. (19): Variation of predicted values of \(C_M\) with observed ones for side weirs of circular crest.](image2)

The empirical equation Eq. (9) was obtained for limited values of \((Fr_1)\) between 0.025 and 0.443, while, Eqs. (10), (11) and (12) were obtained for limited values of \((Fr_1)\) between 0.024 and 0.483, \((h_i / p)\) values between 0.08 and 0.8, \((h_i / r)\) values between 0.533 and 4.75 and \((r / p)\) values between 0.08 and 0.375. For ranges outside the above given ranges, the obtained empirical equations may be used with care.
4.7 Hydraulic Performance of the Side Weir with Different Crest Shapes

Recalling Eq. (7) and Table (3) for the variation \( C_M \) with \( (h_i / p) \) and plotting this equation for the range of \( (h_i / p) \) between (0.15) and (0.6) for sharp crest side weirs and for all heights \( (p=10\text{cm}, 15\text{cm}, 20\text{cm}, 25\text{cm}) \) as in Fig. (20), one may observe that weirs of low heights give higher values of \( C_M \), (i.e., higher performance). Plotting Eq. (7) for the range of \( (h_i / p) \) between (0.15) and (0.6) for different rounded crest shapes of the side weir and for all heights of the side weir and any crest radius (say \( r=3.75\text{cm} \)) as shown in Figs. (21) to (23), one may conclude that values of \( C_M \) decrease with the increase of \( (h_i / p) \) values and side weirs of low height \( (p=10\text{cm}) \) give higher values of \( C_M \) and consequently higher performance. This conclusion is similar to that concluded for side weirs of sharp crest. This behavior is explained that for small heads above crest the discharge is small and velocity head of flow is negligibly small, changes in depth due to contraction also are negligibly small and the operating head is the same at every point along the crest. The discharge per unit length of crest is the same everywhere. Thus, for small heads above crest and low weir heights, the weir performance tends to be ideal. As head increases the discharge and the velocity head increase, consequently, a large proportion of the crest operates under a head less than that in the main channel with a corresponding fall in performance and \( C_M \) value. Therefore, for all crest shapes the smaller height \( (p=10\text{cm}) \) performs better than the other heights. To find out which crest shape is performing better than others, the variation of \( C_M \) with \( (r/p) \) for constant values of \( (h_i / p) =0.2 \) and \( (p=10\text{cm}) \) is plotted in Fig. (24). It is interesting to find that the highest performance of side weirs of inside semicircular crest shape was found for \( (r/p=0.315, \text{i.e.,} r=3.15\text{cm}) \) among other radii, while side weirs of outside semicircular crest shape of \( (r/p=0.375, \text{i.e.,} r=3.75\text{cm}) \) gave the highest performance(highest value of \( C_M \)) among other radii, and circular crest side weir of \( (r/p=0.25, \text{i.e.,} r=2.5\text{cm}) \) gave the highest performance among other radii. In addition, it is interesting to observe that side weir of circular crest shape of \( (r/p=0.25) \) gave the best performance over the other shapes and radii, this can be attributed to the continuous curvature of flow lines passing over the circular crest shape with very little head loss compared to other shapes.
Fig. (20): Plot of Eq. (7) for \( h_1/p = 0.15 \) to 0.6 of sharp crest.

Fig. (21): Plot of Eq. (7) for \( h_1/p = 0.15 \) to 0.6 of inside semicircular crest with \( r = 3.75 \) cm.

Fig. (22): Plot of Eq. (7) for \( h_1/p = 0.15 \) to 0.6 of outside semicircular crest with \( r = 3.75 \) cm.

Fig. (23): Plot of Eq. (7) for \( h_1/p = 0.15 \) to 0.6 of circular crest with \( r = 3.75 \) cm.

Fig. (24): Variation of \( C_M \) with \( r/p \) for different crest shapes of \( h_1/p = 0.2 \) and \( P = 10 \) cm
4.8 Comparison of Present Study Results with those of other Investigators

It is so necessary and important to compare the results of the present study with those of other investigators to verify them and to make sure that the results obtained by the present study are logically acceptable. Because most of the previous works were done on the hydraulic characteristics of flow passing over side weirs of sharp crest, thus, the comparison is restricted to the results of sharp crest side weirs only. Equations of de Marchi discharge coefficient obtained by the previous investigators are shown in Table (5). In this table, the ranges of \((F_r)\) and \((L/B)\) are also shown. The summary of the comparison of present study results for different sharp crest heights \((p=10\text{cm}, 15\text{cm}, 20\text{cm} \text{and} 25\text{cm})\) with those of previous investigators is shown in Fig. (25) which prevails that the present study results lie well within lines obtained by other investigators.

Table (5): Discharge coefficient equations of different investigators

| Source                          | Coefficient Equation                                                                 | Froude Number | L/B  | p (m)             |
|---------------------------------|---------------------------------------------------------------------------------------|---------------|------|-------------------|
| Subramanya and Awasthy(1972)    | \(C_M = 0.864 \frac{1-F_r^2}{2+F_r^2}\)                                               | 0.02-0.85     | 0.2-1| 0-0.6             |
| Nandesamoorthy Thomson (1972)   | \(C_M = 0.432 \frac{2-F_r^2}{1+2F_r^2}\)                                              | -             | -    | 0-0.6             |
| Ranga Raju et al. (1979)        | \(C_M = 0.81 - 0.6F_r\)                                                              | 0.1-0.5       | 0.33-0.5| 0.2-0.5          |
| Hager (1987)                    | \(C_M = 0.7275 \frac{2+F_r^2}{2+3F_r^2}\)                                            | 0-0.87        | 3.33 | 0-0.2             |
| Cheong (1991)                   | \(C_M = 0.45 - 0.22F_r^2\)                                                            | -             | -    | 0                 |
| Singh et. al. (1994)            | \(C_M = 0.99 - 1.26F_r\)                                                             | 0.2-0.4       | 0.4-0.8| -                |
|                                | \(C_M = 0.33 - 0.18F_r + 0.49p/y_1\)                                                 |               |      |                   |
| Jalili and Borghei (1996)       | \(C_M = 0.71 - 0.41F_r - 0.22p/y_1\)                                                 | 0.1-2         | 0.67-2.5| 0.01-0.19        |
| Borghei et. al. (1999)          | \(C_M = 0.55 - 0.47F_r^2\)                                                           | 0.1-0.9       | 0.67-2.33| 0.01-0.19        |
|                                | \(C_M = 0.7 = 0.48F_r + 0.3p/y_1 + 0.06L/B\)                                         |               |      |                   |
| Present study(sharp)            | \(C_M = 0.434F_r^{0.207} \left(\frac{h}{p}\right)^{-0.506}\)                       | 0.025-0.443   | 1.15-1.724| 0.1-0.25         |
Fig. (25): Variation of \( C_M \) with \( Fr_1 \) for the present study results compared to those of previous investigators for sharp crest side weir

5. Conclusions

Within the limits of the experimental data of the present investigation, the following conclusions can be drawn:

1. For all models of different crest shapes, it was found that the average energy difference between the two ends of the side weir is very small and less than one percent which can be ignored and the de Marchi discharge coefficient equation can be used with no fear of uncertainty or lack of accuracy. See Fig. (4)

2. The variation of \( (C_M) \) with \( (Fr_1) \) for all crest shapes studied, different side weir heights and two side weir lengths (L=20 cm and 30 cm) was found to be linear as shown in Eq. (6). It was observed that the value of \( (C_M) \) decreases with the increase of \( (Fr_1) \) values. See Table (2) and Figs (5) to (8).

3. The correlation between \( (C_M) \) and \( (h_1/p) \) was found to be linear for different crest shapes, different weir heights and two side weir lengths as shown in Eq. (7). It was realized that values of \( (C_M) \) decrease with the increase of \( (h_1/p) \) values with high correlation coefficients. See Table (3) and Figs (9) to (12).

4. The variation between \( (C_M) \) and \( (h_1/r) \) was found to be linear with high value of correlation coefficient as shown in Eq. (8). It was found the values of \( (C_M) \) decrease with the increase of \( (h_1/r) \) values for different crest shapes, different weir heights and two side weir lengths. See Table (4) and Figs (13) to (15).

5. The combined effects of both \( (Fr_1) \) and \( (h_1/p) \) on \( (C_M) \) were studied for the free flow over sharp crest side weirs. A simple power empirical expression was obtained with
a correlation coefficient $= 0.837$ and a standard deviation $= 0.055$ as shown in Eq. (9). The predicted values of ($C_M$) were in a good agreement compared with those observed experimentally. See Fig. (16).

6. The combined effects of ($F_{r_1}$), ($h_1 / p$) and ($h_1 / r$) on ($C_M$) were studied for the free flow over rounded crest shape side weirs. Three power empirical expressions were obtained. One was for inside semicircular crest shape as shown in Eq. (10) with a correlation coefficient $= 0.865$ and a standard deviation $= 0.045$. The second expression was for outside semicircular crest shape as shown in Eq. (11) with a correlation coefficient $= 0.865$ and a standard deviation $= 0.045$. While, the third expression was for circular crest shape as shown in Eq. (12) with a correlation coefficient $= 0.865$ and a standard deviation $= 0.032$. The predicted values of ($C_M$) obtained from these expression were in good agreements with those obtained experimentally. See Figs (17), (18) and (19).

7. From the study of hydraulic performance of the different crest shapes, one may observe the followings:

a) For sharp crest, low side weir height ($p=10cm$) gave the highest performance compared with other heights. For side weirs of low heights and low heads, the hydraulic performance tends to be ideal. See Fig. (20).

b) For all crest shapes studied, the smaller side weir height ($p=10cm$) performs better than other heights. The highest performance of inside semicircular crest shape was with crest radius ($r$) $= 3.15cm$. While, side weir of outside semicircular crest shape of radius ($r$) $= 3.75cm$ gave the highest performance among other radii. Circular crest side weir of radius ($r$) $= 2.5cm$ gave the highest performance among other radii. Also, it was found that circular crest shape of ($r/p=0.25$) gave the best performance over other shapes and radii. See Figs (21) to (24).

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