Helicity transport and dynamo sustainment for helical plasma states

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Abstract

Plasma states dominated by single helical modes are often observed in the Reverse Field Pinch (RFP) plasma confinement devices. In this paper the properties of these states are studied on the basis of a relaxation model that assumes the existence of several topological invariants related to the dominant mode. It is hypothesized that the value of the first invariant in this chain, is determined by the existence of a plasma dynamo mechanism that transport helicity. This hypothesis enables us to determine the steady state properties of the plasma equilibrium and some other interesting physical consequences. Further, by considering the properties of the transfer of helicity from the mesoscale (fluctuations) to the macroscale (equilibrium), through the dynamo field, a nonlinear dynamical model can be constructed, that evolves in time to a steady state with a non-vanishing dynamo field, when helicity is injected in the system, as in the case of the ohmic sustained RFP, while the dynamo oscillates initially, but is damped later in time, for vanishing helicity input.

Magnetic helicity conservation is an important plasma property in ideal magneto-hydro-dynamics, describing the invariance of the degree of twist and linkage of the magnetic field lines [1]. In presence of even a small amount of dissipation, it becomes an approximate invariant. In fact in a resistive plasma, reconnection can produce magnetic field topological rearrangements. Even in this case magnetic helicity can play a crucial role in determining the properties of large scale fields in astrophysics and/or in laboratory plasmas, since it is generally better conserved than energy during reconnection and therefore can determine the plasma relaxation pattern toward minimum magnetic energy states, as was first observed by Woltjer [2] for astrophysical plasmas and by Taylor [3] in interpreting the results obtained in a pinch machine. Moreover, the role played by the topological invariants is considered to be very important for the origin and sustainment of large scale magnetic fields in several natural systems, as galaxies, stars and planets. In fact, it is believed that in most, maybe in all of these cases, a MHD dynamo mechanism is needed to produce and/or amplify the magnetic field. Therefore the study of the relationship between the helicity related invariants and the dynamo amplification and sustainment of the magnetic field, is of fundamental importance for a deeper understanding of different natural phenomena, covering a large variety of space and time scales. While helical symmetric turbulence is generally associated with the dynamo process, in principle, also a single large helical mode could amplify an existing magnetic field. Examples of the presence of large scale helical fields are found in planets [1], in solar eruptions [5] and in active galactic nuclei [6] and also in RFP experiments, where, under certain conditions, a single dominant helical mode emerges [7] over a generally much wider fluctuations spectrum. In a series of recent papers [8–10] this experimental observation has been related to the existence, beside the Taylor’s total (volume integrated) magnetic helicity, of a second invariant, that takes into account the pitch of the dominant mode, as first proposed in [11]. The axi-symmetric relaxed states obtained through the minimization of the energy subject to both invariants have been compared with RFP experimental data and a good agreement has been found. However, it is well known that, according to the Cowling anti-dynamo theorem [12] an axi-symmetric magnetic field can not be sustained through a self-generated axi-symmetric current. Therefore, starting from the axi-symmetric magnetic field, as obtained from the relaxation model, determining the dominant helical mode by using a perturbative approach, an approximate helical equilibrium has been also constructed [9], but in this way,
it was not possible to exactly satisfy the constraint imposed by the Ohm’s law. Additionally, according to the experimental observations, the dominant mode is generally oscillating in time, hence a pure equilibrium approach is likely not the most appropriate one and a different explanation is likely needed to describe the physical mechanisms behind the observed time behavior. In this paper we attempt to overcome the above limitations. We start hypothesizing the existence of a dynamo field generated by a dominant helical mode which is, in principle, able to maintain the system in an axi-symmetric equilibrium also satisfying the Ohm’s law. It is shown that the axi-symmetric projection of the dynamo helical field can be directly derived from the knowledge of the relaxed field profiles. Then we assume that this dynamo electric field contributes to the transport of helicity and in this way determines directly the second helicity invariant, mentioned above, so furnishing a physical explanation for its existence. By doing that we are able to determine several other interesting characteristics of the axi-symmetric ohmic equilibrium state, including the value of the applied external toroidal electric field, necessary to sustain it in time. In a second part of the paper we consider a coupled dynamical time evolution of the dynamo field and magnetic helicity showing that the solutions of this system are compatible with the existence of a steady state with a non-zero dynamo, when helicity is injected, while in absence of an input term, the dynamo field tends to become very small, after a relatively long period, of the order of the resistive diffusion time, of oscillations. The paper is organized as follows: in section 1 the relaxation model is briefly summarized; in section 2 the steady state dynamo electric field is derived and the properties of the equilibrium states are discussed; in section 3 the dynamo time dependent model is presented; finally a summary of the main results and conclusions are given.

1. Relaxed states with two helicity invariants

As discussed in more details in [9], by following [11], the Taylor’s relaxation theory [3] for cases with a dominant helical mode can be modified by adding a second global invariant. In particular the two invariants:

\[ K_0 = \frac{1}{2} \int_V A \cdot B \, dV \]

which is the Taylor’s original total volume integrated helicity, and:

\[ K_1 = \frac{1}{2} \int_V \chi \cdot A \cdot B \, dV \]

that represents a ‘weighted’ total helicity, are shown to be both well conserved quantities in low collisional plasmas [11]. The volume of integration is supposed to be bounded by a perfectly conducting wall. The weight in \( K_1 \) is the helical flux function: \( \chi = q_N \Psi - \Phi \) where \( q_N = \frac{m}{n} \) is the pitch of the dominant mode (\( m \) and \( n \) being respectively the poloidal and toroidal mode numbers), while \( \Psi, \Phi \) are the poloidal and toroidal fluxes respectively. Assuming a boundary conditions with \( \lambda(a) = 0 \) (where \( \lambda \cdot J/B^2 \) is the current component parallel to the magnetic field) the relaxed state is found to be:

\[ J = \sum_d \frac{\lambda_d (d + 2)}{2} \chi^d \, B, \]

where the \( \lambda_d \)'s are eigenvalues of the system (see below), while \( d \) is an arbitrary null or positive integer (introduced to preserve the gauge invariance of the system, see [8] for more details). At difference with the Taylor’s theory that predicts \( J = \lambda B \), with \( \lambda = \text{const.} \), the \( \lambda \) profile varies with radius according to equation (3). Note also that the mode helicity (pitch), which is a free parameter, is influencing the final states since it enters the definition of the function \( \chi \). Keeping only the first two terms (i.e. \( d = 0, 1 \)) [8], and assuming a cylindrical geometry, equation (3) can be rewritten as a two point boundary value differential problem in four equations for the magnetic field components and for the magnetic fluxes as:

\[
\frac{d y_1}{dr} = -\lambda_0 y_2(r)(1 + 2l_w(y_3 - y_4)) \\
\frac{d y_2}{dr} = -\frac{y_2(r)}{r} + \lambda_0 y_1(r)(1 + 2l_w(y_3 - y_4)) \\
\frac{d y_3}{dr} = \left( \frac{R_a}{n} \right) y_2(r) \\
\frac{d y_4}{dr} = r y_1(r),
\]

where \( y_1 \) and \( y_2 \) are the axial and the poloidal components of the magnetic field respectively and \( y_3, y_4 \) are the poloidal and toroidal fluxes, \( \lambda_0 \) is the eigenvalue defining the characteristics of the solution, \( R_a \) is the aspect ratio \( R/a \) (\( R \) being the cylinder length and \( a \) the cylinder radius). In the following we assume the poloidal mode
number \( m \) to be 1, in agreement with the experiments. The parameter \( l_{m} \) has been introduced to allow the \( \lambda \) profile to assume values different from 0 at the wall (with \( 0 < l_{m} < 1 \) \( \lambda \) reverses sign at the wall radius, while for \( l_{m} > 1 \) it is positive) [8]. The two point boundary conditions for solving equation (4) are set at \( r = 0 \) and \( r = 1 \) (the radius is normalized to the small cylinder radius \( a \)) as follows:
\[
\gamma_{r}(0) = \Delta r/2, \quad \gamma_{r}(1) = 0, \quad \gamma_{\phi}(0) = 0, \quad \gamma_{\phi}(1) = 0.5, \quad \text{where} \quad \gamma_{r} \quad \text{is normalized to the wall axial flux.}
\]
Examples of the solutions of equations (4) can be found in [8, 10] and in the next section of this paper.

2. Ohm’s law and dynamo electric field

In a conducting plasma the Ohm’s law can be written, in single fluid approximation, as:
\[
\vec{E} + \vec{\nu} \wedge \vec{B} = \eta \vec{j},
\]  
(5)

where \( \nu \) is the fluid/plasma velocity, \( \vec{B} \) the magnetic field, \( \vec{j} \) the current density, \( \eta \) the fluid/plasma resistivity and \( \vec{E} \) is the electric field. By decomposing the magnetic field in a mean \( \vec{B}_{0} \) (axi-symmetric) and in a fluctuating (non axi-symmetric) component (the velocity is assumed here to have only a non axi-symmetric component) and projecting equation (5) along \( \vec{B}_{0} \), the parallel component of the Ohm’s law can be obtained as:
\[
\vec{E}_{\parallel} \cdot \vec{B}_{0} + (\delta \nu \wedge \delta \vec{B})_{\parallel} \cdot \vec{B}_{0} = \eta \vec{E}_{\parallel} \cdot \vec{B}_{0},
\]  
(6)

where \( \delta \nu \) and \( \delta \vec{B} \) are the fluctuating (helical) components of the fields. The averaged (\( \langle \quad \rangle \)) fluctuating term is interpreted as the dynamo electric field. Note also that, for helical perturbations, the average could be taken over the angular dependence (i.e. the poloidal and toroidal angles) of the fields. A classical textbook decomposition for the dynamo term is [13]
\[
\vec{E}_{d} = \langle \delta \nu \wedge \delta \vec{B} \rangle_{0} = \alpha \vec{B}_{0} - \beta \nabla \wedge \vec{B}_{0},
\]  
(7)

where the averaged fluctuating part is expressed in terms of the mean field components. With the decomposition of equation (7), (6) can be rewritten as:
\[
E_{\parallel} B_{\parallel0} + \alpha B_{0} = (\eta + \beta)\int_{0}^{l} \nabla \cdot \vec{E}_{\parallel} \cdot \vec{B}_{0}.
\]  
(8)

The \( \beta \) term in equations (7) and (8) plays the role of an extra dissipation in the system and by assuming it to be of the same order of \( \eta \) its contribution consists eventually in producing an anomalous (enhanced) value of the plasma resistivity. Equation (8), once the fields and the resistivity are given can be solved for \( \alpha \), determining in this way the dynamo field compatible with the Ohm’s law. Note however that equation (8) also depends on the applied electric field, \( E_{\parallel0} \). In fact, in steady state and for a plasma with a fixed axial flux (i.e. \( E_{\parallel0} = 0 \)) the electric field has only a constant axial component \( E_{\parallel0} \), which should also be determined. By introducing the hypothesis that the \( \alpha \)-dynamo term would be responsible for the transport of helicity from the fluctuating to the mean fields and in particular by assuming that the value of the \( K_{i} \) invariant, introduced in section 2, could be directly determined by this transport of helicity from the helical to mean field components, gives us a physical constraint that can be used to determine \( E_{\parallel0} \). The procedure is as follows: first the magnetic fields are calculated from equations (4) and inserted in equation (8) assuming a reasonable plasma resistivity (for example a Spitzer profile with parabolic temperature) and a given \( E_{\parallel0} \) (an initial arbitrary guess). In this way the \( \alpha \) profile can be calculated. Then, the quantity:
\[
\Delta(E_{\parallel0}) = \frac{4}{3} K_{i} - \int_{0}^{l} r \alpha(r, E_{\parallel0}) B_{\parallel}^{2} dr = \int_{0}^{l} r\left[\left(q_{r} y_{r} - y_{\phi} \right) B_{\parallel}^{2} / \lambda(r) - \alpha(r, E_{\parallel0}) B_{\parallel}^{2}\right] dr
\]  
(9)

is evaluated. In equation (9) the near force-free nature of the magnetic fields obtained from equation (4) has been used to express the vector potential \( \vec{A} \) as \( \vec{B} / \lambda \). By finding the value of \( E_{\parallel0} \), for which \( \Delta \) expressed by equation (9), vanishes, the corresponding \( \alpha \) profile can be determined. Equation (9) has been derived starting from the helicity invariant:
\[
\lambda_{0} K_{0} + \lambda K_{1} = \text{const.}
\]  
(10)

using the fact that [11], \( \lambda_{i} = \frac{2 \lambda_{i}}{\Phi_{p}} \) and that (within our normalization) \( \Phi_{p} = 0.5 \), equation (10) can be rewritten as:
\[
\lambda_{0} \left( K_{0} + \frac{4}{3} K_{i} \right) = \text{const.}
\]  
(11)

By deriving this equation in time and assuming that the dynamo term contribution in \( \frac{\partial \vec{E}_{d}}{\partial t} \) balances the \( \frac{\partial \vec{K}}{\partial t} \) term (our main hypothesis) we obtain:
\[
\frac{4}{3} \frac{\partial K_{i}}{\partial t} - \int E_{d} \cdot B_{0} dV = 0
\]  
(12)
and by integrating this last equation in time (assuming an impulsive-like dynamo action), equation (9) can be recovered with a suitable normalization of the axial electric field, incorporating the dynamo time scale, $\tau_{\text{dyn}}$, and accommodating the dimensions. With the above assumption the existence of the $K_1$ invariant is therefore strictly linked to the existence of the dynamo mechanism, which is then responsible for the transport of helicity from the dominant helical mode to the axi-symmetric mean magnetic field. In figure 1 an example of the magnetic field profiles obtained for a typical RFP are shown. The values of $F$ and $\Theta$, $-0.09$ and $1.59$ respectively, are in the range of the experimental measurements (for more details and definitions see [10]). In the normalized units in which the system of equations (4) is expressed, this case has: $E_{\text{zo}} = 35$, $K_0 = 2.34$, $K_1 = -0.84$ and $\Delta$ (given by equation (9)) is approximately zero, implying that $K_1$ is perfectly balanced by the helicity transport through the dynamo electric field, shown in figure 1(b).

While transporting (but not dissipating) helicity, the dynamo field is instead, dissipating energy, as can be checked by calculating the second of the two following volume integrals:

$$W_f = \int_V \eta J^2 \, dV, \quad W_{\text{dyn}} = \int_V E_{\text{d}} \cdot J \, dV.$$

It is also interesting to calculate the ratio between the energy dissipated by the dynamo ($W_{\text{dyn}}$) and that dissipated ohmically ($W_f$). For the case considered in the figure we found that this ratio is around 40%. This value is also consistent with previous RFP calculations based on helicity and energy balances [14]. In figure 2(a), for the same parameters of figure 1, the changes of $\alpha$ and of the dissipation ratio (in%) as a function of the applied electric field $E_{\text{zo}}$ is given. The dynamo electric field is mainly affected in the core region by the $E_{\text{zo}}$ changes and its dissipation can also be made very low in comparison with the ohmic one (see figure 2(b)). By constraining $\Delta$ to be zero the corresponding value of the applied axial electric field and dynamo dissipation is unambiguously determined.

### 3. Dynamo and helicity coupled model

As mentioned in the introduction usually the RFP states where a single helical (SH) mode is dominant are observed not to be stationary in time [7]. Since the helical states are never pure in experiments, that is the spectrum of fluctuating MHD modes contains several harmonics (although generally at lower amplitudes) these time oscillatory behavior could be interpreted as the result of the nonlinear interaction between the dominant mode and the sub-dominant ones [15, 16]. Here we propose a different model in which the conservation of
helicity, its transfer from the dominant helical mode to the mean field (as described in section 3) and the presence of a fluctuating dynamo field become important ingredients, while the interaction with other modes is neglected. Our starting point is the model described in [17] that, we think, can also be applied in the case of the RFP single helical plasmas. The main hypothesis in [17] is that, whenever an $\alpha$-dynamo is present in a system, there would be a transport of helicity from a dominant fluctuating helical field with an inverse scale length $k_2$ to the equilibrium inverse scale length $k_1$, with $k_2 > k_1$. This hypothesis seems to match our present case, in fact we have verified in section 3 that: (i) an $\alpha$-dynamo is able to sustain the RFP at finite values of the applied electric field, (ii) that a transport of helicity from the helical field to the mean field can better explain a relaxation model, which favorably compares with experiments. All the other hypotheses made in [17] to derive their dynamical model seems also to be fulfilled in our case: the quasi force-free nature of the mean fields, the low dissipation, the near ideal wall boundary conditions (needed to eliminate a surface term in the helicity balance equation). However we modify the system derived in [17] in two aspects. First we take into account the input of helicity, which is realized in the RFP through the application of the toroidal loop voltage ($V_z$). The product $2V_z\Phi$ (where $\Phi$ is the toroidal magnetic flux) quantifies this helicity input. Second, in agreement with the steady state model of the previous section, we hypothesize that the helicity transported by the dynamo balances exactly the invariant $K_1$, therefore we can eliminate one of the unknowns of [17], by setting $h_2 = c_1 h_1$, where $c_1$ is a given constant, whose approximate value can be inferred by the steady state calculations of section 2. With these changes the model derived in [17] can be rewritten as:

$$\frac{\partial h_1}{\partial \tau} = c_1 - 2Qh_1^{1/2}\left(\frac{k_2}{k_1}\right)^{1/2} - 2h_1\left(\frac{k_2}{k_1}\right)^2/R_M$$

$$\frac{\partial Q}{\partial \tau} = -\left(\frac{k_2}{k_1}\right)^{1/2}h_1^{1/2}(1 + c_1h_1)/3 + \left(\frac{k_2}{k_1}\right)^{1/2}h_1^{1/2}\chi_o(1 + c_1h_1) - Q(1 + P_M)/R_M,$$

where $h_1$ is the volume integrated helicity (normalized to $k_2/v_2^2$), $(c_1 h_1)$ is the helicity transported from the mesoscale with mode number $k_2$ to the macroscale with mode number $k_1$, $(k_2)$ can be taken of the order of the cylinder minor radius, $a$) by the parallel dynamo electric field, $Q = -E_{dyn}/v_2^2 k_2 R_M = v_2/\nu k_2$ and $P_M = \nu/\eta$ are the magnetic Reynold’s and Prandtl numbers ($\nu$ viscosity and $\eta$ resistivity), while $v_2$ is the typical velocity at the mesoscale $k_2$, $\chi_0 = \beta/v_2^2$ is a constant related to the dynamo $\beta$ effect (see equation (7)). The time in equation (14) is a normalized time $\tau = tv_2 k_2$. The input of helicity is determined by the constant $c_2$. The
nonlinear first order differential system of equation (14) is solved as an initial value problem assigning values to $h_1$ and $Q$ at $\tau = 0$ Note that, in the following, it was set, $k_1 = 1$ and $k_2 = 2$, two length scales closer than those considered in [17]. In fact the length scale of a typical helical mode in RFP can be calculated as:

$$k_2 = \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{R}\right)^2}.$$  

(15)

Since the poloidal mode number for the single helical states is $m = 1$ while the aspect ratio $R/a$ is typically in the range 3–4, and the corresponding toroidal dominant mode has $n = 5–7$, it means that the normalized wave vector $(k_2, a)$ is approximately 2. For zero helicity input, as discussed in [17] and as we also verified, the solutions of equation (14) are initially oscillatory while later they converge to a steady state for the the helicity $h_1$, while $Q$ (the dynamo parallel electric field) is strongly quenched. An example is given in figure 3. The values of $R_M(1000)$ and $P_M(1)$ could be reasonable for RFP s at relatively high temperature (300–500 eV) assuming a mode velocity scale $v_2$ of the order of few $10^3$ m s$^{-1}$ [18]. As shown in [17] it is observed that the dynamo electric field tends to become very small (see figure 3) (note that the amplitude of the dynamo becomes of the order of $1/R_M$ asymptotically), while no dependence of the solutions from the choice of $\chi_0$ (or $\beta$) was found. In our case, although the asymptotic values of the variables are not influenced, the initial oscillatory phase is affected by the extra-dissipation introduced by the $\beta$ dynamo term, as shown in figure 4. The time behavior shows that the frequency of the oscillations increases with decreasing dynamo $\alpha$-effect (i.e. decreasing turbulent dissipation), as it could be expected. Instead, the solutions of the system (14) with a finite helicity injection term ($c_2$ not zero) show that the initial oscillations and the final value of $Q$ depend on the amount of helicity injected in the system, as seen in figure 5. Therefore, in presence of a robust helicity injection, the system can achieve a steady state where an asymptotically non-vanishing dynamo field, consistently with the assumption made in section 2, survives, instead if the helicity input becomes small ($c_2 < 0.01$), the behavior shown in figures 3 and 4 is recovered, the frequency of the oscillations is almost unchanged and the dynamo is asymptotically damped.

4. Discussion and conclusions

In this paper we have proposed a mechanism for the transfer of helicity from a plasma dominant helical mode of a given pitch to the axi-symmetric equilibrium, assuming that the amount of this transfer, through the help of a dynamo electric field, would exactly balance the $K_1$ topological invariant [8, 9, 11]. This hypothesis allows us: to construct a steady state solution, to deduce the profile of the dynamo parallel to $B$ and also, to calculate the applied axial electric field needed to obtain the requested amount of helicity transfer. The construction of an
equilibrium in presence of a dynamo field has been shown to be also important in order to avoid the problem of obtaining an ohmic equilibrium i.e. an equilibrium satisfying the Ohm’s law. In fact in the absence of this term a purely helically deformed plasma could hardly satisfy the constraint imposed by the single fluid electron momentum balance [9]. A dynamo electric field generated by a single dominant mode has been estimated recently in [19], where the radial profile of the dynamo electric potential, as deduced by equilibrium reconstructions, was estimated (see figure 8 of [19]), with a very similar shape to the case with 40% dissipation (and with $\Delta = 0$) shown in figure 2(b) of this paper. In [20, 21] it was assumed that the $\alpha$-dynamo contributed to the transport of total helicity from the fluctuations to the macroscale. This leads to a hyper-resistive dynamo term. Our assumption that the dynamo field balances only the $K_1$ invariant, instead of representing an input for the total helicity $K_o$, circumvents the constraints imposed by a hyper-resistive $\alpha$-dynamo and, as we have shown, is able to predict realistic magnetic field profiles. In the construction of the dynamical model, similar to that proposed in [17], which coupled the two helicities related to the mesoscale (the helical mode) and the macroscale (the axi-symmetric equilibrium) to the dynamo electric field, the results of the steady state dynamo theory developed in section 2 were taken into account. The solution of a relatively simple zero-dimensional nonlinear model has shown that, in presence of a finite dissipation, a time oscillatory phase for all the dynamical variables is obtained followed by a saturation of the helicities to constant values and a strong damping of the dynamo field, for zero helicity input. The total time duration of the oscillatory phase has been shown to be of the order of the magnetic Reynold number (in normalized time units). Moreover, the frequency of these oscillations depends on the turbulent-dissipation introduced by the dynamo, through the $\alpha$ effect. These results imply that: (i) the oscillatory phase can not be maintained indefinitely and (ii) that a dynamo field becomes of the order of $1/R_M$ in presence of a finite dissipation. However these properties of the solutions change drastically when an input of helicity is considered. In this case, the oscillations are quickly damped and the dynamo saturates asymptotically to a non-vanishing value, and a steady state with a non zero dynamo field is reached. Considering the typical values for the helicity injection in RFPs it seems that one of the cases plotted in figure 5 should, at least qualitatively, describe the experiments. However, oscillations in time of the dominant single helical mode has been experimentally detected [7, 22]. Therefore it is tempting to interpret this behavior as a result of a dynamical evolution similar to that described in this paper and therefore as the result of the relationship between dynamo and helicity transport. Obviously it would be very important to understand in which regime are the RFPs operating. Unfortunately all the experimental data are still obtained within limited time windows and the experiments are likely far from being operated in an asymptotical relevant (in the sense described in section 3) regime. Beside the SH scenario, RFPs operate commonly also in the so called Multi-Helical (MH) regime, where many MHD modes are simultaneously excited [7, 25, 26]. Since the experimental operational spaces in SH and

Figure 4. Same case as in figure 3(b) but with (a) $\chi_o = 0.3$ and (b) $\chi_o = 0$. 
in MH are similar, it can be hypothesized that the shape of the dynamo electric field would be also similar in the two cases, although in MH possibly more modes can contribute to it. Since generally the transport properties of the plasma are worst in MH it is, however, expected that the intensity of the dynamo should increase in order to be able to sustain a more dissipative plasma. Likely also the power dissipation by the dynamo should be similar in the two cases, when normalized to the respective ohmic input power. It is clear that the theory derived here, based on the existence of some well conserved invariants, requires the existence of at least some regions containing well defined magnetic surfaces similarly to what was proposed in [27] within a different approach. In Summary: according to the picture presented in this paper, when the RFP dynamics develops a single dominant helical mode, its axi-symmetric magnetic field can be determined as a minimum energy state subjects to topological invariants linked to the prevailing mode helical pitch. The helical perturbation is essential in generating an axi-symmetric dynamo electric field that, in turn, is needed to sustain the mean field and satisfying the constraint imposed by the Ohm’s law. The dynamo field is also responsible for the transport of helicity from the helical mode scale length (mesoscale) to the macroscopic equilibrium scale (macroscale). It has been shown that this model, in steady state, can predict several features that are in agreement with the experimental results obtained in RFPs. It has also been found that the dynamo electric field dissipate almost 40%–50% of the total ohmic power. This is the amount of energy required by the fluctuations themselves to generate the dynamo. Studying, with a simplified dynamical model, the interaction between the helicity transport and the dynamo field, it was shown that, as time evolves, the dynamo is quenched, whenever the helicity input is zero, on the contrary, a perfect steady state can be achieved in presence of a robust helicity input. The so called catastrophic dynamo quenching (CDQ) problem is an open and active research issue in astrophysics [23]. Several dynamo models applied to the Sun, for example, predict a quenching of the dynamo field in time, which has not been really observed. Boundary conditions and/or footprints motion characteristics are very important in determining and possibly mitigating the CDQ [24], through helicity transport or helicity input respectively. A complete solution of this problem is still not available [23]. RFP research could possibly help to solve the mystery of CDQ and reveal not well understood properties of the dynamo effect, since there are, as we discussed here,

![Figure 5. Same quantities as in figure 3 with Q not rescaled versus normalized time, τ. The initial conditions are (b_i = 10^{-4}, Q = 0) with R_M = 1000, P_M = 1, \chi = 0.6, c_1 = -0.4, and k_1 = 1 and k_2 = 2 with c_2 = 0.1 (a), c_2 = 1 (b) and c_2 = 10 (c).](image-url)
several basic physical characteristics in these experiments that have possible similarities to those observed in magnetic fields measured in planets, stars or galaxies.

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