Cancellation of pinching singularities in out-of-equilibrium thermal field theory

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Long standing problem in out-of-equilibrium thermal field theories are pinching singularities. We find that the expressions suspect of pinching require loop particles to be on the mass shell. This fact, with the help of threshold effect and similar effect due to spin, leads to the elimination of pinching in single self-energy insertion approximation to propagator in all propagators appearing in QED and QCD under very mild restrictions on particle densities.

This, together with the cancellation of collinear singularities, allows the extraction of useful physical information contained in the imaginary parts of the two loop diagrams.

In some cases of interest ($\pi - \rho$ interaction, electro-weak interaction, decay of Higgs particle, ...) none of the mentioned mechanisms works and one has to resort to the resummed Schwinger-Dyson series. These cases are more sensitive to the limitations related to the finite time range.

Contrary to the equilibrium case\textsuperscript{1,2} where pinch, collinear, and infrared problems have been successfully controlled, out of equilibrium theory\textsuperscript{1,2} has suffered from them to these days. However, progress has been made in this field, too.

Weldon\textsuperscript{1} has observed that the out of equilibrium pinch singularity does not cancel; hence it spoils analyticity and causality. The problem gets worse with more than one self-energy insertions.

Bedaque has argued that in out of equilibrium theory the time extension should be finite. Thus, the time integration limits from $-\infty$ to $+\infty$, which are responsible for the appearance of pinches, have to be abandoned as unphysical. Similar argument, referring to the Fermi’s ”golden rule” is given by Greiner and Leupold\textsuperscript{1}.

Le Bellac and Mabilat\textsuperscript{1} have shown that pinching singularity gives a contribution of order $g^2 \delta n$, where $\delta n$ is a deviation from equilibrium. They have also found that collinear singularities cancel in scalar theory, and in QCD using physical gauges\textsuperscript{1,2}, but not in the case of covariant gauges. Niégawa\textsuperscript{1} has found that the pinch-like term contains a divergent part that cancels collinear singularities in the covariant gauge.

Altherr and Seibert have found that in massive $g^2 \phi^3$ theory pinch singularity does not occur owing to the kinematical constraint\textsuperscript{1,2}.

Altherr has suggested a regularization method in which the propagator is modified by the width $\gamma$ which is an arbitrary function of momentum to be calculated in a self-consistent way. In $g^2 \phi^4$ theory, for small deviations from equilibrium, $\gamma$ was found to be just the usual equilibrium damping rate\textsuperscript{3}.

This recipe has been justified in the resummed Schwinger-Dyson series in various problems with pinching\textsuperscript{1,2,4}.

Baier, Dirks, and Redlich\textsuperscript{1} have calculated the $\pi - \rho$ self-energy contribution to the pion propagator, regulating pinch contributions by the damping rate. In subsequent papers with Schiff\textsuperscript{1,5,6} they have calculated the quark propagator within the HTL approximation\textsuperscript{1,2}; in the resummed Schwinger-Dyson series, the pinch is naturally regulated by $\text{Im} \Sigma_R$.

Carrington, Defu, and Thoma\textsuperscript{1} have found that no pinch singularities appear in the HTL approximation to the resummed photon propagator.

Niégawa\textsuperscript{1} has introduced the notion of renormalized particle-number density. He has found that, in the appropriately redefined calculation scheme, the amplitudes and reaction rates are free from pinch singularities.

By pinching singularity we understand the contour passing between two infinitely close poles:

$$\int \frac{dx}{(x + \imath \epsilon)(x - \imath \epsilon)}. \quad (I.1)$$
When \( \epsilon \) tends to zero, the integration path is "pinched" between the two poles, and the expression is ill-defined. Integration gives an \( \epsilon^{-1} \) contribution plus regular terms. Decomposition of \((x \pm i \epsilon)^{-1}\) into 
\[ PP (1/x) \mp i \pi \delta(x), \]
gives the related ill-defined \( \delta^2 \) expression.

The following expression, which is similar to (I.2), corresponds to the resummed Schwinger-Dyson series:

\[
\int dx \frac{\omega(x)}{(x - \Sigma_R(x) + i \epsilon)(x - \Sigma_R^*(x) - i \epsilon)}. \quad (I.2)
\]

where \( \omega(x) \) and \( \tilde{\omega}(x) \) (which appears in (I.3)) are, respectively, proportional to \( \Omega(x) \) and \( \bar{\Omega}(x) \) where \( \Omega(x) \), \( \Sigma_R(x) \), and \( \bar{\Omega}(x) \) are the components of the self-energy matrix.

In expression (I.3), pinching is absent if \( Im \Sigma_R(x_o) \neq 0 \) at a value of \( x_o \) satisfying \( x_o - Re \Sigma_R(x_o) = 0. \)

The expression corresponding to the single self-energy insertion approximation to the propagator is similar to (I.2):

\[
\int dx \frac{\tilde{\omega}(x)}{(x + i \epsilon)(x - i \epsilon)}. \quad (I.3)
\]

One can rewrite the integral as

\[
\int dx \frac{1}{2} \left( \frac{1}{x + i \epsilon} + \frac{1}{x - i \epsilon} \right) \frac{\tilde{\omega}(x)}{x}. \quad (I.4)
\]

If it happens that

\[
\lim_{x \to 0} \frac{\tilde{\omega}(x)}{x} = K < \infty, \quad (I.5)
\]

then the integral (I.4) decomposes into two pieces that, although possibly divergent, do not suffer from pinching.

There are two cases in which the function \( \tilde{\omega}(x) \) is even identically zero in the vicinity of the \( x = 0 \) point: in thermal equilibrium, because of detailed balance relations; in massive \( g^2 \phi^4 \) theory out of equilibrium, owing to the mass shell condition (4). The latter mechanism also works in out of equilibrium QED if a small photon mass \( m_{\gamma} \) is introduced. However, this elimination of pinching can be misleading: the domain of \( x, \) where \( \tilde{\omega}(x) = 0, \) shrinks to a point as \( m_{\gamma} \to 0. \) We shall show that the elimination of pinching also occurs in the \( m_{\gamma} = 0 \) case.

In this paper (5), we identify two mechanisms leading to relation (I.3). They are based on the observation that in the pinch-like contribution loop particles have to be on mass shell.

The first mechanism is effective in out of equilibrium QED: in the pinch-like contribution to the electron propagator, phase space vanishes linearly as \( x \to 0. \) In the pinch-like contribution to the photon propagator, the domain of integration is shifted to infinity as \( x \to 0. \) For distributions disappearing rapidly enough at large energies, the contribution again vanishes linearly in the \( x \to 0 \) limit. This mechanism is also valid in QCD in the cases with massive quarks.

In out of equilibrium massless QCD, phase space does not vanish, but there is an alternative mechanism: the spinor/tensor structure in all cases leads to relation (I.3).

In a few cases, none of the mentioned mechanisms works and one has to sum the Schwinger-Dyson series. This is the case of the \( \pi - \rho \) loop in the \( \pi \) self-energy.

Even in the limit of zero pion mass, \( \omega(x) \) vanishes only as \( |x|^{1/2} \) and relation (I.5) is not fulfilled. A similar problem appears in electroweak interactions involving decays of \( Z \) and \( W \) bosons, decay of Higgs particles, etc. Another important case is massless \( g^2 \phi^3 \) theory. In contrast to massless QCD, massless \( g^2 \phi^3 \) theory contains no spin factor to provide a \( q^2 \) factor necessary to obtain (I.3).

The densities are restricted only mildly: they should be cut off at high energies, at least as \( |k_o|^{-3-\delta} \), in order to obtain a finite total particle density; for nonzero \( k_o \), they should be finite; for \( k_o \) near zero, they should not diverge more rapidly than \( |k_o|^{-1} \), the electron (positron) distribution should have a finite derivative.

Furthermore, we were unable to eliminate pinches related to the double, triple, etc., self-energy insertion contributions to the propagator.

The resummed Schwinger-Dyson series must be free from pinching as "for any system which moves towards thermal equilibrium and thus behaves dissipatively, the full propagator must have some finite width" (14).
The propagator satisfies the important condition

$$0 = D_{11} - D_{12} - D_{21} + D_{22}. \quad \text{(II.4)}$$

To obtain the corresponding relations for fermions, we only need to make the substitution

$$\sinh^2 \theta(k_o) \to -\sin^2 \bar{\theta}(k_o). \quad \text{(II.5)}$$

In the case of equilibrium, we have

$$\sinh^2 \theta(k_o) = n_B(k_o) = \frac{1}{\exp \beta |k_o| - 1}, \quad \text{(II.6)}$$

and similarly for fermions. Out of equilibrium, $n_B(k_o)$ and $n_F(k_o)$ will be some given functions of $k_o$.

We transform to the Keldysh components

$$D^R_k = -D^x_R(k) = D_R(-k), \quad \text{(II.7)}$$

$$D^A_k = -D^x_A(k) = D_A(-k), \quad \text{(II.8)}$$

$$D^K_k = D^{11}_k + D^{22}_k = h(k_o)(D_R - D_A) = 2\pi \delta(k^2 - m^2)(1 + 2 \sinh^2 \theta),$$

$$h(k_o) = -\epsilon(k_o)(1 + 2 \sin^2 \bar{\theta}). \quad \text{(II.9)}$$

Again for fermions

$$D^K_k = 2\pi \delta(k^2 - m^2)(1 - 2 \sin^2 \bar{\theta}). \quad \text{(II.10)}$$

The proper self-energy satisfies the condition

$$0 = \Sigma^R_{11} + \Sigma^R_{12} + \Sigma^R_{21} + \Sigma^R_{22}. \quad \text{(II.11)}$$

It is also transformed into the Keldysh form:

$$\Sigma^R_{11} = -(\Sigma^R_{11} + \Sigma^R_{21}), \quad \Sigma^R_{12} = \Sigma^R_{22},$$

$$\Omega = \Sigma^R_{11} + \Sigma^R_{22}. \quad \text{(II.12)}$$

The "cutting rules" (refs. 24, 27; see also ref. 28 for application of the rules out of equilibrium) will convince us that only on-shell loop-particle momenta contribute to $Im \Sigma^R$ and $\Omega$.

The Schwinger-Dyson equation

$$G = G + iG\Sigma G, \quad \text{(II.14)}$$

can be written in terms of Keldysh components as

$$G^R = G_R + iG_R\Sigma R G^R, \quad \text{(II.15)}$$

$$G^K = G_K$$

$$+ i(G_A\Omega G_R + G_K \Sigma_R G^R + G_A \Sigma_A G_K). \quad \text{(II.16)}$$

By expanding (II.13), we deduce the contribution from the single self-energy insertion to be of the form

$$G^R_R \approx G_R + iG_R\Sigma_R G^R, \quad \text{(II.17)}$$

which is evidently well defined, and the Keldysh component suspected for pinching:

$$G^K_K \approx G_K$$

$$+ iG_A\Omega G^R + iG_K\Sigma_R G^R + iG_A \Sigma_A G_K. \quad \text{(II.18)}$$

The equation for $G^R_R$ is simple and the solution is straightforward:

$$G^R_R = \frac{1}{G^{-1}_R - i\Sigma^R} = -G^A_A. \quad \text{(II.19)}$$

To calculate $G^K_K$, we can use the solution (II.19):

$$G^K_K = G_A \left(h(q_o)(G^{-1}_A - G^{-1}_R) + i\Omega\right) G^R_R. \quad \text{(II.20)}$$

The first term in (II.20) is not always zero, but it does not contain pinching singularities! The second term in (II.20) is potentially ill-defined (or pinch-like). The pinch-like contribution appears only in this equation; thus it is the key to the whole problem of pinch singularities. In the one-loop approximation, it requires loop particles to be on mass shell.

We start with (II.18). After substituting (II.19) into (II.18), we obtain the regular term plus the pinch-like contribution:

$$G^K_K \approx G^R_K + G^K_K,$$

$$G^R_K = h(q_o)$$

$$G^K_K = iG_A\Omega G^R_R, \quad \Omega = \Omega - h(q_o)(\Sigma^R - \Sigma_A). \quad \text{(II.23)}$$

For equilibrium densities, we have $\Sigma^R_{21} = e^{-\beta q_o} \Sigma_{12}$, and expression (II.23) vanishes identically.

Expression (II.23) is the only one suspected of pinch singularities at the single self-energy insertion level. The function $\Omega$ in (II.23) belongs to the type of functions characterized by the fact that both loop particles have to be on mass shell. It is analyzed in detail in Secs. III and IV (for threshold effect) and in Sec. V (for spin effect). With the help of this analysis we show that relation (II.23) transforms into

$$G^K_K = -i\frac{K(q^2, q_o)}{2}$$
are related to the limits
\[ q \to m^2 \pm 0 \]
where \( K(q^2, q_o) = \Omega/(q^2 - m^2) \) multiplied by spinor/tensor factors included in the definition of \( G_{R,A} \). The finiteness of the limit
\[ \lim_{q^2 \to m^2 \pm 0} K(q^2, q_o) = K_\mp(q_o) < \infty, \]
is important for cancellation of pinches. The index \( \mp \) indicates that the limiting value \( m^2 \) is approached from either below or above, and these two values are generally different. To isolate the potentially divergent terms, we express the function \( K(q^2, q_o) \) in terms of functions that are symmetric \( (K_1(q^2, q_o)) \) and antisymmetric \( (K_2(q^2, q_o)) \) around the value \( q^2 = m^2 \):
\[ K(q^2, q_o) = K_1(q^2, q_o) \]
\( \pm \epsilon(q^2 - m^2)K_2(q^2, q_o). \]
These functions are given by
\[ K_{1,2}(q^2, q_o) = \frac{1}{2}(K(q^2, q_o) \pm K(2m^2 - q^2, q_o)). \]
Locally (around the value \( q^2 = m^2 \)), these functions are related to the limits \( K_\pm(q_o) \) by
\[ K_{1,2}(q^2, q_o) = \frac{1}{2}(K_+(q_o) \pm K_-(q_o)). \]
As a consequence, the right-hand side of expression \( (II.24) \) behaves locally as
\[ G_{Kp}(q^2, q_o) \]
\[ \approx -\frac{i}{2}(K_+(q_o) + \epsilon(q^2 - m^2)K_2(q_o)) \]
\[ \frac{1}{q^2 - m^2 + 2i\epsilon q_o} + \frac{1}{q^2 - m^2 - 2i\epsilon q_o}, \]
and the term proportional to \( K_2 \) is capable of producing logarithmic singularity.

**III. Threshold Factor**

In this section we analyze the phase space of the loop integral with both loop particles on mass shell. Special care is devoted to the behavior of this integral near thresholds. The expressions are written for all particles being bosons, and spins are not specified; change to fermions is elementary.

Now, starting from \( (II.12) \) to \( (II.13) \), we calculate \( \Omega \) and \( Im\Sigma_R \):
\[ \Omega = 2iIm\Sigma_{11} = 2\frac{iq^2}{2} \int d\mu N_\Omega(k_o, k_o - q_o)F, \]
where
\[ d\mu = \frac{d^4k}{(2\pi)^4}4\pi^2\delta(k^2 - m_D^2)\delta((k - q)^2 - m_S^2), \]
and
\[ N_\Omega(k_o, k_o - q_o) = -0.5\epsilon(k_o(k_o - q_o)) \]
\[ +0.5 + \sinh^2 \theta_D(k_o)(0.5 + \sinh^2 \theta_S(k_o - q_o)), \]
\[ Im\Sigma_R = \frac{q^2}{2} \int d\mu N_R(k_o, k_o - q_o)F, \]
and
\[ N_R(k_o, k_o - q_o) \]
\[ = \sinh^2 \theta_D(k_o)\epsilon(k_o - q_o) \]
\[ + \sinh^2 \theta_S(k_o - q_o)\epsilon(-k_o) \]
\[ +\Theta(-k_o)\Theta(k_o - q_o) - \Theta(k_o)\Theta(q_o - k_o). \]
\( F \) is the factor dependent on spin and internal degrees of freedom.

It is useful to define \( N_{\Omega}(k_o, k_o - q_o) \) as
\[ N_{\Omega} = N_\Omega - h(q_o)N_R. \]
After integrating over \( \delta \)'s, one obtains
\[ d\mu = \frac{1}{4|q|} \frac{|k_o|dk_o}{|\vec{k}|} d\phi \Theta(1 - z_o^2), \]
and expressions for \( \Omega \) and \( Im\Sigma_R \) take general form
\[ I = \int d\mu N(k_o, k_o - q_o)F(q, k_o, |\vec{k}|, q\vec{k}), \]
where \( |\vec{k}| = (k_o^2 - m_D^2)^{1/2}, \)
\[ q\vec{k} = |q||\vec{k}|z_o, \]
\[ z_o = \frac{q^2 + \vec{k}^2 - (q - \vec{k})^2}{2|\vec{k}||q|}. \]
\( \phi(0, 2\pi) \) is the angle between vector \( \vec{k}_T \) and \( x \) axes.

Let us start with the \( q^2 > 0 \) case. Solution of \( \Theta(1 - z_o^2) \) gives the integration limits
\[ k_{o,1} = \frac{1}{2q^2} (q_o (q^2 + m_D^2 - m_S^2)) \]

\[ \mp \frac{1}{2q^2} |q| ((q^2 - q_{dtr}^2)(q^2 - q_{-tr}^2))^{1/2}, \quad (III.11) \]

\[ q_{\pm tr} = |m_D \pm m_S|. \quad (III.12) \]

Assume now that \( q_{tr} \neq 0 \). In this case, the threshold, the limits shrink to the value

\[ k_{o tr} = \frac{q_o (q_{tr}^2 + m_D^2 - m_S^2)}{2q_{tr}}. \quad (III.13) \]

We define the coefficient \( c_1 \) by

\[ c_1 = \frac{1}{4|q|} \int d\phi N(k_{otr}, k_{otr} - q_o)F. \quad (III.14) \]

Now the expression \((II.8)\) can be approximated by

\[ \mathcal{I} \approx c_1 (|\vec{k}|_2 - |\vec{k}|_1) \]

\[ \approx c_1 (\Theta(q^2 - q_{+tr}^2) + \Theta(-q^2 + q_{-tr}^2)) \]

\[ \frac{q_o ((q^2 - q_{+tr}^2)(q^2 - q_{-tr}^2))^{1/2}}{q^2}. \quad (III.15) \]

Relation \((III.13)\) is the key to further discussion of the threshold effect.

We obtain this also for higher dimension \((D=6, \text{ for example})\).

Owing to \((III.12)\) and \((III.13)\), the function \( \mathcal{I}(q^2, m_D^2, m_S^2) \) has the following properties important for cancellation of pinches.

It vanishes between the thresholds, i.e., the domain \((m_D - m_S)^2 < q^2 < (m_D + m_S)^2 \) is forbidden \((\mathcal{I} = 0)\). If it happens that the bare mass \( m^2 \) belongs to this domain, the single self-energy insertion will be free of pinching. In this case, multiple (double, triple, etc.) self-energy insertions will also be free of pinching.

Massive \( \lambda \phi^4 \) theory \[ \] is a good example of this case.

It is (in principle) different from zero in the allowed domain \( q^2 < (m_D - m_S)^2 \) and \( (m_D + m_S)^2 < q^2 \). In this case, one cannot get rid of pinching. This situation appears in the \( \pi - \rho \) interaction \[ \].

The behavior at the boundaries (i.e., in the allowed region near the threshold) depends on the masses \( m_D \) and \( m_S \) and there are a few possibilities.

If both masses are nonzero and different \((0 \neq m_D \neq m_S)\), then there are two thresholds and \( \mathcal{I} \) behaves as \( (q^2 - q_{dtr}^2)^{1/2} \) in the allowed region near the threshold \( q_{\pm tr}^2 \). For \( m^2 = q_{tr}^2 \), the power 1/2 is not large enough to suppress pinching.

If one of the masses is zero \((m_D \neq 0, m_S = 0 \text{ or } m_D = 0, m_S \neq 0)\), then \((III.13)\) gives that the thresholds are identical (i.e., the forbidden domain shrinks to zero) and one obtains the \((q^2 - m_D^2)^3 \) behavior near \( m_D^2 \). This case \((m^2 = m_D^2)\) is promising. The elimination of pinching in the electron propagator, considered in Sec.IV, is one of important examples.

If the masses are equal but different from zero \((m_D = m_S \neq 0)\), then there are two thresholds with different behavior. The function \( \mathcal{I} \) behaves as \( (q^2 - q_{dtr}^2)^{1/2} \) in the allowed region near the threshold \( q_{tr}^2 = 4m_D^2 \), and this behavior cannot eliminate pinching in the supposed case \( m^2 = 4m_D^2 \).

However, at the other threshold, namely at \( q_{-tr}^2 = 0 \), the physical region is determined by \( q^2 < 0 \) and the above discussion does not apply. In fact, the integration limits \((III.11)\) are valid, but the region between \( k_{o 1} \) and \( k_{o 2} \) is now excluded from integration. One has to integrate over the domain \((\infty, k_{o 1}) \cup (k_{o 2}, +\infty) \). This leads to the limitation in the high-energy behavior of the density functions.

An important example of such behavior, elimination of pinching in the photon propagator \((m_\gamma)\), is discussed in Sec.IV.

If both masses vanish \((m_D = m_S = 0)\), the thresholds coincide, there is no forbidden region and no threshold behavior. The behavior depends on the spin of the particles involved. For scalars, the leading term in the expansion of \( \mathcal{I} \) does not vanish. Pinching is not eliminated.

The case of vanishing masses \((m_D = m_S = 0)\) for particles with spin exhibits a peculiar behavior. In all studied examples (see Sec.V for details), \( \mathcal{I} \) behaves as \( q^2 \to 0 \), which promises the elimination of pinching.

\section*{IV. PINCH SINGULARITIES IN QED}

\textbf{A. Pinch Singularities in the Electron Propagator}

In this subsection we apply the results of preceding section to cancel the pinching singularity appearing in a single self-energy insertion approximation to the electron propagator. To do so, we have to substitute \( m_D = m, m_S = 0 \), \( \sinh^2 \Theta_D(k_o) \to -n_e(k_o), \sinh^2 \Theta_S(k_o - q_o) \to n_r(k_o - q_o), \) and \( h(k_o) = -\epsilon(k_o)(1 - 2n_e(k_o)) \), where \( n_e \) and \( n_r \) are given non-equilibrium distributions of electrons and photons in relations \((III.3), (III.5), (III.6),\), and \((III.9)\). The thresholds are now identical \((q_{\pm tr}^2 = m^2)\), and the integration limits satisfy

\[ |\vec{k}|_2 - |\vec{k}|_1 = \frac{q_o}{q}(q^2 - m^2)). \quad (IV.1) \]

At threshold the limits shrink to the value \( k_{o tr} = q_o, |\vec{k}| tr = |\vec{q}| \).

Then, with the help of \((III.14)\), we define
\[ K(q^2, q_o) = \frac{(q + m)\bar{\Omega}(q + m)}{(q^2 - m^2)} \]
\[ \approx \frac{1}{16\pi^2|q|^2(q^2 - m^2)} \int d\phi N_{\Omega}(k_{otr}, k_{otr} - q_o) \]
\[ (q + m)\bar{\Omega}(q + m)(|\vec{k}|_1 - |\vec{k}|_1). \quad \text{(IV.2)} \]

For \( q^2 \neq 0 \), we can decompose the vector \( k \) as
\[ k = \frac{(k, q)}{q^2} q + \frac{(k, \bar{q})}{q^2} \bar{q} + k_T \]
\[ = (q - q_o) - \frac{m^2 + m^2 + q^2}{2q^2} + \frac{k_o}{|q|} \bar{q} + k_T, \quad \text{(IV.3)} \]

where, in the heat-bath frame we have
\[ q = (q_o, 0, 0, |\vec{q}|), \quad \bar{q} = (|\vec{q}|, 0, 0, q_o), \]
\[ q\bar{q} = 0, \quad \bar{q}^2 = -q^2. \quad \text{(IV.4)} \]

In calculating the term proportional to \((1 - a)\), where \( a \) is the gauge parameter, we have to use the trick
\[ ((k - q)^2 \pm \imath \epsilon)^{-2} \approx \lim_{m_o \to 0} \left[ \frac{\partial}{\partial m_o^2}((k - q)^2 \pm \imath \epsilon) - m_o^2 \right]^{-1}. \quad \text{(IV.5)} \]

Finally, we obtain the "sandwiched" trace factor \( \bar{\Omega} \) calculated with loop particles on mass shell:
\[ (q^2 - m^2) \left( -\frac{q^2 - m^2}{q^2} \bar{q} \right) + \frac{q_o q^2 + m^2}{q^2|q|} + 2\frac{k_o}{|q|} \bar{q} + 2k_T \]
\[ - (1 - a) \left( \frac{q^2 - m^2}{2q^2} - (q - \bar{q} + \frac{q_o}{|q|}\bar{q}) \right). \quad \text{(IV.6)} \]

Now we can study the limit
\[ K(q_o) = \lim_{q^2 \to m^2} K(q^2, m^2, q_o) \]
\[ = (q + m) \frac{q_o}{2|q||m^2} N_{\Omega}(k_{otr}, k_{otr} - q_o). \quad \text{(IV.7)} \]

It is easy to find that \( K(q_o) \) is finite provided that \( m^2 \neq 0 \) and \( N_{\Omega}(q_o, 0) < \infty \). The last condition is easy to investigate using the limiting procedure:
\[ N_{\Omega}(q_o, 0) = \lim_{k_o \to q_o} N_{\Omega}(k_{otr}, k_o - q_o) \]
\[ = \lim_{k_o \to q_o} 2n_e(k_o - q_o)(n_e(q_o) - n_e(k_o)) \]
\[ + \lim_{k_o \to q_o} (n_e(q_o) - n_e(k_o) - \epsilon(q_o)\epsilon(k_o - q_o) n_e(k_o) - n_e(q_o)) \]
\[ \lim_{k_o \to q_o} (n_e(q_o) + n_e(k_o) - 2n_e(q_o)n_e(k_o)). \quad \text{(IV.8)} \]

The integration limits imply that the limit \( k_o \to q_o \) is taken from below for \( q^2 > m^2 \), and from above for \( q^2 < m^2 \). The two limits lead to different values of \( N_{\Omega}(q_o, 0) \). This leads to the discontinuity of \( K(q^2, m^2, q_o) \) at the point \( q^2 = m^2 \).

Only the first term in (IV.8) can give rise to problems. We rewrite it as \( \lim_{k_o \to 0} (2k_o\Theta(k_o) \frac{\partial n_e(k_o + q_o)}{\partial q_o}) \). As relation (IV.7) should be valid at any \( q_o \) we find two conditions:
\[ \lim_{k_o \to 0} k_o n_e(k_o) < \infty, \quad \text{(IV.9)} \]
\[ \left| \frac{\partial n_e(q_o)}{\partial q_o} \right| < \infty. \quad \text{(IV.10)} \]

Under the very reasonable conditions (IV.9) and (IV.10) the electron propagator is free from pinching.

It is worth observing that \( K(q_o) \) is gauge independent, at least within the class of covariant gauges.

B. Pinch Singularities in the Photon Propagator

To consider the pinching singularity appearing in a single self-energy insertion approximation to the photon propagator, we have to make the substitutions \( m_D = m = m_S \), \( \sin^2 \Theta_D(k_o) \to -n_e(k_o) \), \( \sin^2 \Theta_S(k_o - q_o) \to -n_e(k_o - q_o) \), and \( h(k_o) = -\epsilon(k_o)(1 + 2n_e(k_o)) \). There are two thresholds, but only \( q^2_{1, tr} = 0 \) and the domain where \( q^2 < 0 \) are relevant to a massless photon. The integration limits are given by the same expressions (III.11), but now we have to integrate over the domain \((-\infty, k_{o,1}](k_{o,2}, +\infty) \). As \( q^2 \to 0 \), we find \( (k_{o,1} \to -\infty) \) and \( (k_{o,2} \to +\infty) \). The integration domain is still infinite but is shifted toward \( \pm \infty \) where one expects that the particle distribution vanishes:

\[ K_{\mu\nu}(q^2, q^o) = \left( g_{\mu\rho} - (1 - a) \frac{q_{\mu}q_{\rho}}{q^2 - 2iq_o\epsilon} \right) \]
\[ - \bar{\Omega}_{\mu\rho} \left( g_{\sigma\nu} - (1 - a) \frac{q_{\sigma}q_{\nu}}{q^2 + 2iq_o\epsilon} \right) \]
\[ = \frac{1}{16\pi^2|q|^2} \int_{-\infty}^{k_{o,1}} \left( \int_{k_{o,2}}^{k_{o,1}} \frac{k_o dk_o}{|k|} \int d\phi \right) \]
\[ N_{\Omega}(k_o, k_o - q_o) \left( g_{\mu\rho} - (1 - a) \frac{q_{\mu}q_{\rho}}{q^2 - 2iq_o\epsilon} \right) \]
\[ F_{\rho\sigma} \left( g_{\sigma\nu} - (1 - a) \frac{q_{\sigma}q_{\nu}}{q^2 + 2iq_o\epsilon} \right). \quad \text{(IV.11)} \]
To calculate $F_{\mu\nu}$ for the $\epsilon - \bar{\epsilon}$ loop, we parameterize the loop momentum $k$ by introducing an intermediary variable $l$ perpendicular to $q$. $m$ is the mass of loop particles:

$$k = \alpha q + l, \quad q.l = \alpha, \quad k^2 = (k - q)^2 = m^2,$$

$$l^2 = m^2 - \alpha^2 q^2, \quad \alpha = \frac{k^2 + q^2 - (k - q)^2}{2q^2}. \quad \text{(IV.12)}$$

After all possible singular denominators are canceled, one can set $\alpha = 1/2$.

$$F_{\mu\nu} = -Tr(\bar{\epsilon} + m)\gamma^{\mu}(k + q + m)\gamma^{\nu}$$

$$= \left(4m^2q^2 - q^4 A^{\mu\nu}(q)\right) + q^2 \left(4k_0(q_0 - q_o) - 4m^2 - q^2\right) A^{\mu\nu}(q)$$

$$+ \left(-8\left(k_0 - \frac{q_o}{2}\right)^2 + 2q^2\right) B^{\mu\nu}(q) \right), \quad \text{(IV.13)}$$

For projection operators $A, B, C$ and $D$ see \textbf{VII.3}. Now we obtain

$$K_{\mu\nu}(q^2, q_o) = \frac{1}{16\pi^2|q|^2}$$

$$\left(\int_{-\infty}^{k_1} + \int_{k_2}^{\infty} k_0 dk_0 \frac{|k_0|}{|k|} \right) d\phi N_1(k_0, k_0 - q_o)$$

$$\left(4m^2q_0^2 A^{\mu\nu}(q)\right) + q^2 \left(4k_0(q_0 - q_o) - 4m^2 - q^2\right) A^{\mu\nu}(q)$$

$$+ \left(-8\left(k_0 - \frac{q_o}{2}\right)^2 + 2q^2\right) B^{\mu\nu}(q) \right). \quad \text{(IV.14)}$$

In the integration over $k_0$ the terms proportional to $(k_0 q^2)^n$ dominate and $\lim_{q^2 \to 0} |K_{\mu\nu}(q^2, q_o)| < \infty$ if

$$\left|\int_{-\infty}^{k_1} + \int_{k_2}^{\infty} k_0 dk_0 \frac{|k_0|}{|k|} \right| < \infty$$

$$\left(\alpha + 3k_0 q^2\right) \int d\phi N_1(k_0, k_0 - q_o) < \infty. \quad \text{(IV.15)}$$

Here $N_1(k_0, k_0 - q_o)$ is given by

$$N_1(k_0, k_0 - q_o) = -2n_\epsilon(k_0 - q_o)$$

$$-n_\gamma(q_0) - n_\epsilon(k_0) - n_\gamma(q_0) - n_\epsilon(k_0)$$

$$-\epsilon(q_0)\epsilon(k_0 - q_o)(-n_\gamma(q_0) + n_\epsilon(k_0)$$

$$+ 2n_\gamma(q_o)n_\epsilon(k_0)). \quad \text{(IV.16)}$$

Assuming that the distributions obey the inverse-power law at large energies $n_\epsilon(k_0) \propto |k_0|^{-\delta\gamma}$ and $n_\gamma(k_0) \propto |k_0|^{-\delta\epsilon}$, we find that the terms linear in densities dominate. Thus, for $n = 0, 1$, one finds

$$\frac{-1}{q^2} \left(\int_{-\infty}^{k_0-1} + \int_{k_0}^{\infty} \right) \left|k_0|dk_0|k_0|^{2n+\delta}(-q^2)^n \right.$$

$$\propto (\delta - 1 - 2n)^{-1} (|q| m)^{1+2n-\delta}(-q^2)^{\delta/2}. \quad \text{(IV.17)}$$

It follows that (\textbf{IV.15}) is finite (in fact, it vanishes) if $\delta, \delta_\epsilon > 3$. Similar analysis for electron propagator at $q^2 < 0$ (thus outside of our analysis of pinch singularities) leads to $\delta_\gamma > 3$. This is exactly the condition

$$\int d^3 k n_{\epsilon, \gamma, \epsilon}(k_0) < \infty. \quad \text{(IV.18)}$$

Thus the pinching singularity is canceled in the photon propagator under the condition that the electron and positron distributions should be such that the total number of particles is finite.

Also, in the photon propagator, the quantity $\lim_{q^2 \to 0} K_{\mu\nu}(q^2, q_o)$ does not depend on the gauge parameter.

Expression (\textbf{IV.17}) is not valid for $m = 0$.

\section{Pinch Singularities in Massless QCD}

In this section we consider the case of massless QCD. One should observe that the massless quarks and gluons are an idealisation eventually appropriate at the lowest order. In the nonequilibrium HTL resummation scheme both quarks and gluons acquire dynamical mass $E_4$. Pinching singularities, related to massive quarks, are eliminated by the methods used in the preceding section.

Attention is turned to the spin degrees of freedom, i.e., to the function $F$ of the integrand in (\textbf{III.1}) to (\textbf{III.8}). In the calculation of $F$ it has been anticipated that the loop particles have to be on mass shell. In this case, $F$ provides an extra $q^2$ factor which suffices for the elimination of pinching singularities.

The integration limits are now

$$k_{o1,2} = \frac{1}{2} (q_o \mp |q|). \quad \text{(V.1)}$$

The difference $|\vec{k}| - |\vec{k}|$ is finite and there is no threshold effect.

It is worth observing that for $q^2 > 0$, we have to integrate between $k_{o1}$ and $k_{o2}$, whereas for $q^2 < 0$, the integration domain is $(-\infty, k_{o1}) \cup (k_{o2}, +\infty)$. This leads to two limits, $\lim_{q^2 \to \pm 0} K(q^2, q_o) = K_{\pm}(q_o)$, in all cases of massless QCD.
By inspection of the final results (V.3), (V.4), and (V.5), we find that the case \( q^2 < 0 \) requires integrability of the function \( k^2 N_q(k_o, k_o - q_o) \) leading to the condition \( (IV.13) \) on the quark, gluon, and ghost distribution functions.

The function \( K_{\mu\nu}(q^2, q_o) \) related to the gluon propagator is the sum of the contributions from various loops, where the terms in the sum are defined as

\[
K_{\mu\nu}(q^2, q_o) = (g_{\mu\nu} - (1 - a)D_{R_{\mu\nu}})
\]

\[
\frac{\tilde{Q}_{\sigma\nu}}{q^2}(g_{\sigma\nu} - (1 - a)D_{A_{\sigma\nu}}).
\]

(V.2)

The tensor \( F \) related to the massless quark-antiquark contribution to the gluon self-energy is

\[
F^\mu_\bar{\nu} = -\frac{\delta_{ab}}{6}Tr\tilde{k}_\gamma^\mu(k - \tilde{h})\gamma^\nu
\]

\[
= \frac{\delta_{ab}}{6} \left( \frac{q^2}{q^4} \left( 4k_o(k_o - q_o) - q^2 \right) A^{\mu\nu}(q)
\right)
\]

\[+ (8k_o - q_o)^2 B^{\mu\nu}(q) + O(\tilde{k}_T) \right) .
\]

(V.3)

As \( F_{\mu\nu} \) contains only \( A \) and \( B \) projectors, the result does not depend on the gauge parameter.

Relation (V.2) contains only terms proportional to \( q^2 \), and \( \lim_{q^2 \to 0} K_{\mu\nu}(q^2, q_o) \) is finite.

For the ghost-ghost contribution to the gluon self-energy, the tensor \( F \) is given by

\[
F^\mu_\bar{\nu} = -\frac{\delta_{ab} N_c k^{\mu}(k - q)^\nu}
\]

\[= -\delta_{ab} N_c \frac{q^2}{2} \left( \frac{4k_o(k_o - q_o)}{q^2} + q^2 A^{\mu\nu}(q)
\right)
\]

\[- (k_o - q_o)^2 B^{\mu\nu}(q) - \frac{q^2}{4} D^{\mu\nu}(q) + O(\tilde{k}_T) \right) .
\]

(V.4)

The tensor \( F \) for the gluon-gluon contribution to the gluon self-energy is

\[
F^\mu_\bar{\nu} = \frac{\delta_{ab} N_c}{2}
\]

\[
\left( g^{\mu\sigma}(q + k) \gamma^\tau - g^{\tau\sigma}(2k - q) \gamma^\mu + g^{\tau\mu}(k - 2q) \gamma^\sigma \right)
\]

\[
\left( g_{\sigma\rho} - (1 - a) \frac{(k - q)_\sigma (k - q)_\rho}{(k - q)^2} \right) + 2i(k_o - q_o) \epsilon
\]

\[
\left( g^{\nu\rho}(q + k) \gamma^\tau - g^{\tau\rho}(2k - q) \gamma^\nu + g^{\tau\nu}(k - 2q) \gamma^\rho \right)
\]

\[
\left( g_{\tau\eta} - (1 - a) \frac{k_o k_\eta}{k^2 - 2ik_o} \right)
\]

\[= \frac{\delta_{ab} N_c q^2}{2} \left( \frac{1}{q^2} \left( 10(k_o - \frac{q_o}{2})^2 + \frac{3}{2} q^2 \right) A^{\mu\nu}(q)
\right)
\]

\[- (10(k_o - \frac{q_o}{2})^2 + 4q^2) B^{\mu\nu}(q) - \frac{q^2}{2} D^{\mu\nu}(q)
\]

\[- (1 - a) \left( \frac{1}{2} A^{\mu\nu} - B^{\mu\nu} - \frac{q_o}{|q|} C^{\mu\nu} \right)
\]

\[+ (1 - a)^2 \left( - \frac{q^2}{2} A^{\mu\nu} + 2 \frac{q_o}{q^2} B^{\mu\nu} - 2 \frac{q_o}{|q|} C^{\mu\nu}
\]

\[- 2D^{\mu\nu} \right) + O(\tilde{k}_T) \right) .
\]

(V.5)

Expressions (V.3), (V.4), and (V.5) for the ghost-ghost, quark-antiquark, and gluon-ghost contributions to the gluon self-energy contain only terms proportional to \( q^2 \). The function \( K_{\mu\nu}(q^2, q_o) \) approaches the finite value \( K_{\mu\nu}(\pm, q_o) \).

Thus we have shown that the single self-energy contribution to the gluon propagator is free from pinching under the condition \( (IV.18) \).

The \( K \) spinor for the quark-gluon contribution to the massless quark propagator is defined as

\[
\tilde{K}(q^2, q_o) = \tilde{h} \frac{\tilde{Q}_{\sigma\nu}}{q^2} \tilde{h}.
\]

(V.6)

In the self-energy of a massless quark coupled to a gluon the "sandwiched" spin factor \( \tilde{h} F \tilde{h} \) is given by (as the term proportional to \( k_T \) vanishes after integration, we drop it).

\[
\tilde{h} F_{\mu\nu} \tilde{h} = \delta_{ab} \frac{N_c^2 - 1}{2N_c}
\]

\[
\left( g^{\mu\nu} - \frac{(1 - a)(k - q)_\mu (k - q)_\nu}{(k - q)^2 + 2i(k_o - q_o) \epsilon} \right) \tilde{h} \gamma^\mu \gamma^\nu \tilde{h}
\]

\[= \delta_{ab} \frac{N_c^2 - 1}{2N_c} \frac{q^2}{2}
\]

\[
\left( -\tilde{h} \frac{q_o}{|q|} \tilde{h} + 2 \frac{k_o}{|q|} \tilde{h} - \frac{1 - a}{2} \left( -\tilde{h} + \frac{q_o}{|q|} \tilde{h} \right) \right),
\]

(V.7)

which contains the damping factor \( q^2 \).

By inserting (V.7) into (V.6), we obtain (II.25) free from pinches.

To calculate \( K(q_o) \), we need the limit

\[
\lim_{q^2 \to 0} \frac{\tilde{h} F_{\mu\nu} \tilde{h}}{q^2} = \delta_{ab} \frac{N_c^2 - 1}{2N_c} \frac{2(k_o - q_o)}{q_o} \tilde{h}.
\]

(V.8)

From (V.8) we conclude that \( K(q_o) \) does not depend on the gauge parameter.

Omitting details, we observe that pinching is absent from the quark propagator, also in the Coulomb gauge, with the same limit (V.8).

The \( K \) factor for the ghost-gluon self-energy contribution to the ghost propagator is defined as
The $F$ factor for the ghost-gluon contribution is

\[
F_{ghg} = \delta_{ab} N_c k^\alpha q^\nu \left( g_{\mu\nu} - \frac{(1 - a)(k_\mu - q_\mu)(k_\nu - q_\nu)}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon} \right) 
\]

\[
\rightarrow \delta_{ab} N_c \frac{q^2}{2}. \quad (V.10)
\]

The factor $q^2$ ensures the absence of pinch singularity and a well-defined perturbative result.

The $K$ factor for the scalar-photon self-energy contribution to the scalar propagator is defined as

\[
K(q^2, q_o) = \frac{\bar{\Omega}}{q^2}. \quad (V.11)
\]

The $F$ factor for the massless scalar-photon contribution to the scalar self-energy,

\[
F_{\gamma\gamma} = (q + k)^\mu(q + k)^\nu \left( g_{\mu\nu} - \frac{(1 - a)(k_\mu - q_\mu)(k_\nu - q_\nu)}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon} \right) \rightarrow 2q^2, \quad (V.12)
\]

clearly exhibits the $q^2$ damping factor!

VI. CONCLUSION

Studying the out of equilibrium Schwinger-Dyson equation, we have found that ill-defined pinch-like expressions appear exclusively in the Keldysh component ($\mathcal{G}_K$) of the resummed propagator (II.21), or in the single self-energy insertion approximation to it (II.23). This component does not vanish only in the expressions with the Keldysh component (II.13) ($\Omega$ or $\bar{\Omega}$ for the single self-energy approximation) of the self-energy matrix. This then requires that loop particles be on mass shell. This is the crucial point to eliminate pinch singularities.

We have identified two basic mechanisms for the elimination of pinching: the threshold and the spin effects.

For a massive electron and a massless photon (or quark and gluon) it is the threshold effect in the phase space integration that produces, respectively, the critical $q^2 - m^2$ or $q^2$ damping factors.

In the case of a massless quark, ghost, and gluon, this mechanism fails, but the spinor/tensor structure of the self-energy provides an extra $q^2$ damping factor.

We have found that, in QED, the pinching singularities appearing in the single self-energy insertion approximation to the electron and the photon propagators are absent under very reasonable conditions: the distribution function should be finite, exceptionally the photon distribution is allowed to diverge as $k_o^{-1}$ as $k_o \rightarrow 0$; the derivative of the electron distribution should be finite; the total density of electrons should be finite.

For QCD, identical conditions are imposed on the distribution of massive quarks and the distribution of gluons; the distributions of massless quarks and ghosts (observe here that in the covariant gauge, the ghost distribution is not required to be identically zero) should be integrable functions; they are limited by the finiteness of the total density.

In the preceding sections we have shown that all pinch-like expressions appearing in QED and QCD (with massless and massive quarks!) at the single self-energy insertion level do transform into well-defined expressions. Many other theories behave in such a way. However, there are important exceptions: all theories in which lowest-order processes are kinematically allowed do not acquire well-defined expressions at this level. These are electroweak interactions, processes involving Higgs and two light particles, a $\rho$ meson and two $\pi$ mesons, $Z$, $W$, and other heavy particles decaying into a pair of light particles, etc. The second important exception is massless $g^2\phi^3$ theory. This theory, in contrast to massless QCD, contains no spin factors to provide $[\phi^3]$. In these cases, one has to resort to the resummed Schwinger-Dyson series. One can also expect that, in these cases, higher order contributions become more important and provide artificial cutoff which reduces the contribution of pinch-like terms. In ultimate case this points out to the limitations of the method.

The main result of the present paper is the cancellation of pinching singularities at the single self-energy insertion level in QED- and QCD-like theories. This, together with the reported cancellation of collinear singularities, allows the extraction of useful physical information contained in the imaginary parts of the two-loop diagrams. This is not the case with three-loop diagrams, because some of them contain double self-energy insertions. In this case, one again has to resort to the sophistication of resummed propagators.

VII. APPENDIX

We start by defining a heat-bath four-velocity $U_\mu$, normalized to unity, and define the orthogonal projector

\[
\Delta_{\mu\nu} = g_{\mu\nu} - U_\mu U_\nu. \quad (VII.1)
\]

We further define spacelike vectors in the heat-bath frame:

\[
\kappa_\mu = \Delta_{\mu\nu} q^\nu, \quad \kappa_\mu \kappa^\mu = \kappa^2 = -q^2. \quad (VII.2)
\]
There are four independent symmetric tensors (we distinguish retarded from advanced tensors by the usual modification of the \( i\epsilon \) prescription) \( A, B, \) and \( D \) (which are mutually orthogonal projectors), and \( C \):

\[
A_{\mu\nu}(q) = \Delta_{\mu\nu} - \frac{\kappa_{\mu}\kappa_{\nu}}{q^2},
\]

(VII.3)

\[
B_{\mu\nu}(q) = U_{\mu}U_{\nu} + \frac{\kappa_{\mu}\kappa_{\nu}}{q^2} - \frac{q_{\mu}q_{\nu}}{(q^2 + 2iq_{\mu}\epsilon)},
\]

(VII.4)

In addition to the known multiplication properties (for convenience we drop \( q \)-dependence)

\[
AA = A, \quad BR, AB, A = BR, A,
\]

(VII.7)

\[
C_{\mu\nu}(q) = \frac{(\kappa^2)^{1/2}}{U q}
\]

(VII.5)

\[
\left(\frac{(U q)^2}{\kappa^2} U_{\mu}U_{\nu} - \frac{\kappa_{\mu}\kappa_{\nu}}{q^2} + \frac{q_{\mu}q_{\nu}}{q^2(q^2 + 2iq_{\mu}\epsilon)}\right),
\]

(VII.6)

\[
C_{\mu\nu}(q) = \frac{q_{\mu}q_{\nu}}{q^2 + 2iq_{\mu}\epsilon}.
\]

(VII.10)

we need mixed products

\[
B_{\mu\nu}(q) = \frac{1}{2}(B_{\mu\nu} + B_{\nu\mu}),
\]

(VII.11)

\[
C_{\mu\nu}(q) = \frac{1}{2}(C_{\mu\nu} + C_{\nu\mu}),
\]

(VII.12)

\[
D_{\mu\nu}(q) = \frac{1}{2}(D_{\mu\nu} + D_{\nu\mu}),
\]

(VII.13)

\[
(C_{\mu\nu}D_{\mu\nu})_{\mu\nu} = (D_{\mu\nu}C_{\mu\nu})_{\mu\nu}
\]

(VII.14)

\[
= \frac{1}{2}\left(\frac{q_{\mu}q_{\nu}}{q^2 + 2iq_{\mu}\epsilon} + \frac{q_{\mu}q_{\nu}}{q^2 - 2iq_{\mu}\epsilon}\right).
\]

(VII.14)

\[\text{References}\]

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