ETTORE MAJORANA AND HIS HERITAGE
SEVENTY YEARS LATER

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Abstract. The physicists working in several areas of research know quite well the name of Ettore Majorana, since it is currently associated to fundamental concepts like *Majorana neutrinos* in particle physics and cosmology or *Majorana fermions* in condensed matter physics. But, probably, very few is known about other substantial contributions of that ingenious scholar, and even less about his personal background. For non specialists, instead, the name of Ettore Majorana is usually intimately related to the fact that he disappeared rather mysteriously on March 26, 1938, just seventy years ago, and was never seen again.

The life and the work of this Italian scientist is the object of the present review, which will also offer a summary of the main results achieved in recent times by the historical and scientific researches on his work.

1. Introduction

Some time after joining the Fermi group, Majorana already had such an erudition and reached such a high level of comprehension of physics that he was able to discuss with Fermi about scientific problems. Fermi himself held him to be the greatest theoretical physicist of our time. He often was astounded [...] I remember exactly these words that Fermi spoke: ‘Once a problem has already been posed, no one in the world is able of solving it better than Majorana’.

Ettore Majorana’s fame rests on testimonies like this one by Bruno Pontecorvo, a younger colleague of Majorana at the Institute of Physics in Rome directed by Enrico Fermi. It can be promptly recognized just by borrowing Fermi’s own words, given in 1937 in the occasion of a meeting of the board in charge of the competition for a new full professorship in theoretical physics in Italy.

Without listing his works, all of which are highly notable both for their originality of the methods as well as for the importance of the results achieved, we limit ourselves to the following:

In modern nuclear theories, the contribution made by this researcher to the introduction of the forces called “Majorana forces” is universally recognized as the one, among the most fundamental, that allows us to understand theoretically nuclear stability. The
work of Majorana today serves as a basis for the most important research in this field.

In atomic physics, the merit of having resolved some of the most intricate questions on the structure of spectra through simple and elegant considerations of symmetry is due to Majorana.

Lastly, he devised a brilliant method that deals with positive and negative electron in a symmetric way, eventually eliminating the necessity to rely on the extremely artificial and unsatisfactory hypothesis of an infinitely large electrical charge diffused in space, a question that had been tackled by many other scholars without success.\footnote{This and many other interesting documents may be found (unfortunately only in Italian) in the comprehensive book in Ref. \cite{1}.}

With this justification, the board, chaired by Fermi, proposed to apply for the second time a special bill passed few years early in order to give a chair to the Nobel Prize Guglielmo Marconi, and suggested to the Minister of National Education “to appoint Majorana as full professor of Theoretical Physics at some University of the Italian kingdom, for high and well-deserved repute, independently of the competition rules”. The Minister accepted the proposal: evidently, such a “reputation” was sufficiently established on the basis of just few (nine) papers published by the Italian scientist.

Unfortunately enough, the University of Naples hosted his talent for three months only, until the end of March 1938, when Majorana gave his last lesson.

As recalled by one of his students, Gilda Senatore\footnote{See what reported in Ref. \cite{3} and references therein.} on Friday March 25, the day after his 21st lesson, “differently than what he usually did [when no lecture on theoretical physics was scheduled], Majorana came to the Institute and stayed there for few minutes. From the corridor leading to the small room where I was writing, he called me by name: ‘Miss Senatore...’; he didn’t enter in the room but remained in the corridor; I reached him and he gave me a closed folder telling: ‘here’s some papers, some notes, keep them... we will talk about later’; afterward he went away and, turned back, said again ‘we will talk about later’. The day after, the director of the Institute of Physics at the University of Naples, Antonio Carrelli, received a mysterious telegram from Palermo and, then, a letter by Majorana, where he wrote that he had abandoned his initial suicidal intentions and decided to return to Naples. However, the following Monday no news from Majorana reached Carrelli who, worried by these circumstances, called his friend Enrico Fermi in Rome, who immediately realized the seriousness of the situation: Majorana had disappeared. At the time, Fermi was working in his laboratory with the young physicist Giuseppe Cocconi. In order to give him an idea of the serious loss for the community of physicists caused by Majorana’s disappearance, Fermi told Cocconi: “You see, in the world there are various categories of scientists: there are people of a secondary or third level standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science. Then, there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no-one else in the world had”.

\[\text{Ref.} \cite{1} \text{;} \text{Ref.} \cite{3} \text{;} \text{Ref.} \]
2. The family background and the first meeting with Fermi

Ettore Majorana was born on August 5, 1906 in Catania, Sicily (Italy), to Fabio Majorana and Dorina Corso. The fourth of five sons, he had a rich scientific, technological and political heritage: three of his uncles were chancellors of the University of Catania and members of the Italian parliament, while another, Quirino Majorana, was a renowned experimental physicist and once president of the Italian Physical Society. Ettore’s father himself was an engineer founding the first telephone company in Sicily and later on a chief inspector of the Ministry of Communications.

Fabio Majorana took care of the education of his son in the first years of his life, until the family moved to Rome in 1921. Ettore left school in 1923 at the age of 17 and joined the Faculty of Engineering at the University of Rome, where he became good friend of Giovanni Gentile Jr and future Nobel laureate Emilio Segrè.

After the advent of Fermi to the chair of Theoretical Physics at the University of Rome in 1926, a group of young people started to form under his guide, supported by Orso Mario Corbino, the director of the Institute of Physics in Rome and an influential politician, whose plan of supporting a rapid development of physics in Italy[4] led him to hire Franco Rasetti as his assistant and later on, in spring of 1927, to entice the most brilliant students of the Faculty of Engineering to move into physics studies. Segrè and his friend Edoardo Amaldi rose to the challenge, joining Fermi and Rasetti’s group and telling them of Ettore’s exceptional gifts. After some encouragement from Segrè and Amaldi, Majorana eventually decided to meet Fermi in the autumn of the same year. The two beautiful minds immediately started talking about the statistical model of atoms that Fermi was working on, later to be known as the Thomas-Fermi model, which describes the energy of an atom in terms of the density of its surrounding electrons. The model involves a complicated non-linear differential equation. The analytical solution of the equation was unknown at that epoch, but Fermi had managed to obtain a numerical table of approximate values for it. Majorana carefully followed Fermi reasoning, asked few questions and left the Institute. The day after he returned to Fermi’s office and asked for a closer look at the numerical table, so that he could compare it with a similar table he worked up the night before. Once he checked the agreement between the two results, Majorana said that Fermi’s table was correct and left with no further comment.

This anecdote recalled by Rasetti, Segrè and Amaldi [4, 5] shows that Majorana arrived at a series solution of the Thomas-Fermi equation using a very peculiar method. He first transformed the Thomas-Fermi equation into an Abel equation, with a very original method that can be used for a large class of differential equations; this was probably done in the sake of applying known theorems on the existence and uniqueness of solution of the Abel equation. However, he then transformed again the Thomas-Fermi equation into another first-order differential equation, whose series solution was explicitly given in terms of only one quadrature, and from this method Majorana obtained a table of numerical values as accurate as (at least) that of Fermi.

It is a remarkable achievement of Majorana’s genius the fact that he obtained many results well before several renowned mathematicians and physicists or, as for his solution of the Thomas-Fermi equation (see the historical and scientific
assessment presented in Refs. [6] and [7]), which have not been independently found by anyone else.

3. Researches in physics

Majorana gave fundamental contributions in several different areas of theoretical physics, in part as collaborator of the Fermi group in Rome, but he published only few scientific articles, so that his brilliant activity was not immediately recognized outside the Fermi group. The complete list of the published articles is the following:

(1) G. Gentile and E. Majorana, Sullo sdoppiamento dei termini Roentgen ottici a causa dell’elettrone rotante e sulla intensità delle righe del Cesio, Rend. Acc. Lincei 8, 229 (1928);
(2) E. Majorana, Sulla formazione dello ione molecolare di He, Nuovo Cim. 8, 22 (1931);
(3) E. Majorana, I presunti termini anomali dell’Elio, Nuovo Cim. 8, 78 (1931);
(4) E. Majorana, Reazione pseudopolare fra atomi di Idrogeno, Rendi. Acc. Lincei 13, 58 (1931);
(5) E. Majorana, Teoria dei tripletti $P'$ incompleti, Nuovo Cim. 8, 107 (1931);
(6) E. Majorana, Atomi orientati in campo magnetico variabile, Nuovo Cim. 9, 43 (1932);
(7) E. Majorana, Teoria relativistica di particelle con momento intrinseco arbitrario, Nuovo Cim. 9, 335 (1932);
(8) E. Majorana, Über die Kerntheorie, Z. Phys. 82, 137 (1933); Sulla teoria dei nuclei, Ric. Scientifica 4(1), 559 (1933);
(9) E. Majorana, Teoria simmetrica dell’elettrone e del positrone, Nuovo Cim. 14, 171 (1937);
(10) E. Majorana, Il valore delle leggi statistiche nella fisica e nelle scienze sociali, Scientia 36, 55 (1942), edited by G. Gentile jr. (posthumous).

The largest part of Majorana’s work was left unpublished. We are now left with his master thesis on “the quantum theory of radioactive nuclei”, 5 notebooks (“Volumetti”), 18 booklets (“Quaderni”), 12 folders with spare papers, and the set of the lecture notes on theoretical physics prepared for a class at the University of Naples. Almost all this material is presently at the Domus Galilaeana in Pisa, Italy; the complete set of Volumetti was translated in English and published in Ref. [9], that of the Naples’ lecture notes is (in Italian) in Ref. [10]. A forthcoming publication will appear soon (in English) with a reasoned selection of the most interesting material present in the Quaderni [11].

In the following we give an account of key studies performed by Majorana during his short scientific life, as emerging from recent historical and scientific researches to date.

3.1. Atomic and molecular physics. In 1928, when Majorana started his collaboration (still as a University student in physics) with the Fermi group in Rome, he already shown an outstanding capacity of solving very involved mathematical problems in a very interesting and clear way such as, for example, by obtaining the

\footnote{In this list we do not include the short communication presented by the young student Majorana at the end of 1928, at Fermi’s request, during the 1928 annual meeting of the Italian Physical Society. The published text of that communication [8] was not written by himself; the original work can be found in Ref. [9].}
The semi-analytical series solution of the Thomas-Fermi equation, as mentioned above. The whole work on this topic is contained in some spare sheets, and diligently reported by the author himself in his *Volumetti*\[^9\]. From these it is evident the considerable contribution given by Majorana even in the achievement of the statistical model, anticipating, in many respects, some results reached later by leading specialists. The major finding by Majorana was his solution (or, rather, methods of solutions) of the Thomas-Fermi equation, which remained completely unknown, until recent times \[^7\], to the physicists community, which failed to realize that the non-linear differential equation relevant for atoms and other systems could be solved semi-analytically \[^12\]. An intriguing property in the Majorana derivation of the solution of the Thomas-Fermi equation is that his method can be easily generalized and applied to a large class of particular differential equations. Several generalizations of the Thomas-Fermi method for atoms were proposed as well by Majorana, but they were considered by the physicists community, unaware of Majorana’s unpublished works, only many years later \[^6\]. Indeed, Majorana studied \[^13\] the problem of an atom in a weak external electric field, i.e., atomic polarizability, and obtained an expression for the electric dipole moment for a (neutral or arbitrarily ionized) atom. Furthermore, he also started to consider the application of the statistical method to molecules, rather than single atoms, studying the case of a diatomic molecule with identical nuclei \[^14\]. Finally, Majorana also considered the second approximation for the potential inside the atom, beyond the Thomas-Fermi approximation, with a generalization of the statistical model of neutral atoms to those ionized \(n\) times, including the case \(n = 0\) \[^15\]. This was achieved by focusing the attention on the effective potential acting on the given electron inside an atom, assumed to be generated by a rescaled charge density which takes into account the finite charge of the given electron on which the potential acts. As discussed in Ref. \[^16\], such an approach is basically the same which is currently adopted in the renormalization of physical quantities in modern gauge theories. The specific example considered by Majorana may be regarded as the first application of this modern viewpoint to an atomic problem.

Some of these works is also recognizable in the underlying framework of the first published paper \(^1\) made in collaboration with G. Gentile, where they derived the ionization energy of an electron in the 3\(d\) orbit of gadolinium and uranium, in good agreement with the experimental values. In addition, by applying first-order perturbation theory to the Dirac equation, they also calculated the fine structure splitting of different (X-ray transitions) spectroscopic terms in gadolinium, uranium and caesium.

After that, by the end of 1931 the 25-year-old physicist had published two articles \(^2,4\) on the chemical bonding of molecules and two more papers \(^3,5\) on spectroscopy. In paper \(^2\), by starting to discuss the still unclear experimental result on the band structure in helium emission spectrum, Majorana approached the problem of formation of the molecular ion \(\text{He}_2^+\) in a way similar to the one proposed earlier by Heitler and London \[^17\] for the \(\text{H}_2\) molecule, where the idea of quantum exchange in the explanation of the chemical bond was introduced. This same concept based on the resonance force was also used in paper \(^4\), where he faced the unexplained anomalous \(X\) term observed in the spectrum of the \(\text{H}_2\) molecule, by assuming a pseudopolar binding between two ions in the hydrogen molecule.
The far-reaching interest in the helium spectrum involved also Majorana who, in paper 3, performed some calculations on certain doubly excited levels of helium, by taking into account all helium levels generated by combining two hydrogenoid orbits with principal quantum number $n = 2$ and including the mutual electron repulsion as a perturbation. This paper anticipated results later obtained (1934-1935) by a collaborator of S. Goudsmith on the Auger effect in helium and by some other people [18]. However, the most important article published in 1931, dealing with atomic spectroscopy problems, was paper 5, where the characterization of spectra of different atoms with two electrons in the outer shell was given. A theoretical explanation of the missing lines in the predicted $P$ triplet levels in the absorption spectra of Hg, Cd and Zn was indeed presented by introducing a novel process now known as autoionization and equivalent to the Auger process already known in X-ray emissions. Such a process was independently introduced in the same year by A.G. Shenstone [19], and its important role was later largely recognized in a great variety of atomic and molecular spectra. The puzzle of the missing lines was eventually clarified on the experimental side in 1955 with the observations of W.R.S. Garton and A. Rajaratnam [20], while the correctness of the Majorana’s spectroscopic assignments was proved only in 1970 by W.C. Martin and V. Kaufman [21].

As Edoardo Amaldi has written [5], an in-depth examination of these works leaves one struck by their quality. They reveal both a deep knowledge of the experimental data, even in the most minute detail, and an uncommon (and without equal at that time) ability in using the symmetry properties of the quantum states, resulting in a remarkable simplification of the problems and a brilliant choice of the most suitable method for their quantitative resolution.

These published papers, however, do not exhaust the entire work performed by Majorana on atomic physics, which was the main research topic investigated by the Fermi group in Rome in the years 1928-1933. On one hand, some echo of the work reported in those papers is present in the Quaderni [11], where it is pointed out that, especially in the case of paper 5 on the incomplete $P'$ triplets, some material was not included by the author in the published work. On the other hand, several other problems were considered and solved by Majorana, without reporting the results obtained in published articles, thus being practically unknown to physicists.

Several expressions for the wavefunctions and the different energy levels of two-electron atoms (and, in particular, of helium) were, for example, considered by Majorana, mainly in the framework of a variational method aimed at solving the corresponding Schrödinger equation. Numerical values for the energy terms were reported in large summary tables, and some approximate expressions were also obtained for three-electron atoms (in particular, for lithium) and for alkali, including the effect of polarization forces in hydrogen-like atoms [11].

The problem of the hyperfine structure of the energy spectra of complex atoms was considered in some detail as well, revealing the careful attention of Majorana to the existing literature. A generalization of the Landé formula for the hyperfine splitting to non-Coulomb atomic field was given, along with a relativistic formula for the Rydberg corrections of the hyperfine structures [11]. Such a detailed study by Majorana formed the basis of what discussed by Fermi and Segrè in a well-known paper of 1933 on this issue [22], as acknowledged by those authors themselves.
Some substantial help on difficult theoretical calculations given by Majorana to several people working in Rome, led in some cases to notable results. For example, in 1932, stimulated by Segrè [5], Majorana published his paper on the non-adiabatic spin-flip of atoms in a magnetic field, which was later extended in 1937 by Nobel laureate I.I. Rabi and, more in general, in a celebrated work of 1945 by F. Bloch and Rabi (who explicitly recognized the work by Majorana as seminal for the solution of the problem). It established the theoretical basis for the experimental method used to reverse the spin of neutrons by a radio-frequency field, a method that is still employed today in all polarized-neutron spectrometers. That paper contained also an independent derivation of the well-known Landau-Zener formula (published later in the same year) for non-adiabatic transition probability. It also introduced a novel mathematical tool for representing spherical functions (Majorana sphere), rediscovered only in recent times.

Several problems of molecular physics were also faced by Majorana. He studied in some detail, for example, the helium molecule and considered the general theory of the vibration modes in molecules as well, with particular reference to the C\textsubscript{2}H\textsubscript{2} molecule of acetylene (which presents peculiar geometric properties) [11].

Finally, other notable results concerned the problem of ferromagnetism in the framework of the Heisenberg model with the exchange interaction. The approach used by Majorana in this study, however, is original [11], since it does not follow neither the Heisenberg formulation nor the subsequent van Vleck formulation in terms of spin Hamiltonian. By using statistical arguments, he calculated the magnetization (with respect to the saturation value) of the ferromagnetic system when an external magnetic field is applied, and the spontaneous magnetization. Several examples of ferromagnetic materials, with different geometries, were also reported in his own notebooks.

3.2. Nuclear physics. Majorana was among the firsts who studied nuclear physics in Rome, at least since 1929 when he defended his master thesis on “the quantum theory of radioactive nuclei”. However quite unexpectedly, since the Fermi group was not effectively involved in nuclear physics until 1933, he continued to study such topics for several years, independently of the main research topics of the Fermi group, till his famous theory of nuclear exchange forces published in paper 8 of 1933. The antecedents that led to such a discovery are quite intriguing and we dwell here a bit on these.

In March 1932 James Chadwick announced the discovery of the neutron, after which Majorana revealed to his friends and colleagues in Rome [5] that he had built a theory of light nuclei based on the quantum concept of exchange forces. Although encouraged by Fermi to go public with his results, Majorana’s hypercritical judgement prevented him from doing so. True to style, his work was not recognized until a few months later when it was independently elaborated by Werner Heisenberg. This fact caused a sensation in the Rome group, and Fermi urged Majorana, successfully, to visit Heisenberg in Leipzig, for a six-month period in 1933, where Majorana’s capacities and results largely impressed Heisenberg himself.

In the Heisenberg model, atomic nuclei were supposed to be composed of protons and neutrons only, without any need of electrons as was previously largely accepted (before the discovery of the neutron, protons and electrons were the only known “elementary” particles). It was also assumed that the leading forces responsible
for nuclear stability were those between neutrons and protons (neglecting neutron-neutron and proton-proton forces), which were deduced to be exchange forces in analogy to what had been proven by Heitler and London for the $\text{H}_2^+$ molecular ion (in this case, an H atom and an H$^+$ ion are held together by the exchange of an electron). According to Heisenberg, the underlying nuclear forces should be interpreted in terms of nucleons exchanging spinless electrons, implicitly assuming that the neutron was practically formed by a proton and an electron. Majorana immediately realized this defect of the Heisenberg’s theory. Indeed, in Majorana’s view, the neutron was pictured as a “neutral proton” [5], as effectively is, and in his model the forces between neutrons and protons were explained in terms of the exchange interaction arising from the quantum effect coming from interchanging the space coordinates of identical nucleons, rather than from interchanging their electric charge as in the Heisenberg model. The direct consequences of the Majorana version of the nuclear model were that the exchange forces considered are independent of nuclear density of the total mass and prevent the collapse of the nucleus even without a repulsive force at short distance. The observed particular stability of the $\alpha$-particle (and not the unobserved one of deuterium nucleus, as predicted by the Heisenberg model) was thus explained by a “saturation phenomenon more or less analogous to valence saturation”. It was especially this saturation of nuclear forces for the $\alpha$-particle that quickly led to the recognition by Heisenberg himself and others of the Majorana model as the most appropriate one. On several occasions, in fact, while discussing the “Heisenberg-Majorana” exchange forces, Heisenberg mentioned his own contribution only marginally, rather emphasizing that of Majorana [27].

This model was certainly the most renowned contribution by Majorana to the contemporary physicists community; however, he did not limit himself to study this particular nuclear physics topic. In the research notes of the Quaderni [11], several pages were devoted to study possible forms of the nucleon potential inside a given nucleus, describing the interaction between neutrons and protons. Although generic nuclei were often considered in the discussion, some particular care was given by Majorana to light nuclei (deuteron, $\alpha$-particles, etc.). In a sense, the researches performed by Majorana on this subject are, at the same time, preliminary studies and generalizations of what had been published by himself in his well-known paper 8, revealing a very rich and peculiar way of reasoning. Note also that, probably before his studies leading to paper 8, a relativistically invariant field theory for nuclei composed of scalar particles was also elaborated by Majorana [28], which described the transitions between different nuclei.

In 1930 Majorana elaborated a dynamical theory of $(\alpha, p)$ reactions on light nuclei [30], whose experimental results were interpreted by Chadwick and G. Gamov [29], describing the energy states in terms of the superposition of a continuous spectrum and a discrete level. He provided a complete theory for the artificial disintegration of nuclei by means of $\alpha$-particles (with and without $\alpha$ absorption) approaching the problem by considering the simplest case with an unstable state of the system formed by a nucleus plus an $\alpha$-particle, which spontaneously decays with the emission of an $\alpha$-particle or a proton. In particular Majorana obtained the explicit expression for the integrated cross section of the nuclear process. The peculiar aspect of Majorana theory [31] was the introduction of quasi-stationary states (the superposition of discrete and continuous states), which were only later
considered by U. Fano [32] in 1935 in a completely different context and then widely applied in the framework of condensed matter physics about 20 years later. In addition to this, other topics were also considered by Majorana [9, 11], and we here only mention the study of the problem of the energy loss of $\beta$-particles passing through a medium, where he deduced the Thomson formula by using classical arguments. This study could be potentially of some interest, for correct historical reconstructions, in relation with the famous theory elaborated by Fermi on the nuclear $\beta$ decay just in 1934.

3.3. Relativistic fields and group theory. Among the papers published by Majorana in 1932, the most important one is certainly paper 7 concerning a relativistic wave mechanics of particles with arbitrary spin, which makes no use of the negative-energy states. Around 1932 it was commonly thought that one could write relativistic quantum equations only in the case of particles with zero or half-integer spin. Majorana had quite a different belief (see several works carried out in the Quaderni [11]), and he began constructing suitable quantum-relativistic equations for higher spin values (one, three-halves, etc.), even devising a general method for writing the equation for a generic spin-value. Still, he did not publish anything, until he discovered that one could write a single equation to cover an infinite series of cases, that is, a whole, infinite family of particles of arbitrary spin: this was finally reported in paper 7. In order to implement his programme with these “infinite components” equations, Majorana invented a technique for the representation of a group several years before Eugene Wigner did. Remarkably, Majorana obtained the simplest infinite-dimensional unitary representations of the Lorentz group that were re-discovered by Wigner in his 1939 and 1948 works [33]. The entire theory was re-invented by Soviet mathematicians (in particular Gelfand and collaborators [34, 35]) in a series of articles starting from 1948 and finally applied by physicists, first to hadronic physics and then to modern string theories, years later. The importance of Majorana’s article was first realized by B.L. van der Waerden [36] but, unfortunately, it remained unnoticed for more than three decades until D. Fradkin, informed by Amaldi, reexamined that pioneering work in light of later developments and clearly explained the relevance of Majorana’s approach and results accomplished many years earlier [37].

Some work behind paper 7 can be found as well in the Quaderni [11]. Here, by starting from the usual Dirac equation for a 4-component spinor, he obtained explicit expressions for the Dirac matrices in the cases of 6-component and 16-component spinors. Interesting enough, at the end of his discussion, Majorana also treated the case of spinors with an odd number of components, namely a 5-component field.

Majorana’s admiration for the work of Hermann Weyl and others on the application of group theory to quantum mechanics was first recalled by Amaldi [5] and, apart from the published paper 7, is well testified by a number of unpublished notes [9, 11]. The Weyl approach, indeed, greatly influenced the entire scientific thought and work of Majorana [39]. This is particularly evident in his notebooks [40] where, for example, he gave a detailed analysis of the relationship between the representations of the Lorentz group and the matrices of the (special) unitary group in two dimensions. In these notes, a strict connection with the Dirac equation was always taken into account, and the explicit form of the transformations of every
bilinear in a given spinor field which is relevant in the Dirac theory of a spinning particle was reported.

The Dirac equation was, indeed, a favorite and long-lasting topic studied by Majorana. In the Quaderni [11], the relativistic equation describing spin-1/2 particles was usually considered in a Lagrangian framework (in general, the canonical formalism was adopted), obtained from a least action principle. After an interesting preliminary study of the problem of the vibrating string, where Majorana obtained a (classical) Dirac-like equation for a two-component field, he then went on to consider a semiclassical relativistic theory for the electron, where the Klein-Gordon equation and the Dirac equation were deduced from a semiclassical Hamilton-Jacobi equation. Later on, the field equations and their properties (Lorentz invariance, issues related to the probabilistic interpretation, and so on) were considered in details, and the quantization of the (free) Dirac field discussed by means of the standard formalism, using annihilation and creation operator formalism. Finally, the electromagnetic interaction was introduced in the Dirac equation and the superposition of the Dirac and Maxwell fields studied in a very peculiar way, obtaining the expression for the quantized Hamiltonian of the interacting system using a normal mode decomposition.

Real (rather than complex) Dirac fields were considered by Majorana in his paper 9 on a “symmetrical theory for electrons and positrons”, where he introduced the so-called (and now well-known to particle physicists) Majorana neutrino hypothesis. This hypothesis was no doubt revolutionary, because it first put forward the possibility that the antimatter partner of a given matter particle could be the particle itself. This was in direct contradiction to what Dirac had successfully assumed in order to solve the problem of negative-energy states in quantum field theory (i.e., the existence of the positron). With amazing farsightedness Majorana suggested that the neutrino, which had just been postulated by Wolfgang Pauli and Fermi to explain puzzling features of radioactive beta decay, could be such a particle. This would make the neutrino unique among the elementary particles and, moreover, enable it to have mass. Today many experiments are still devoted to detect these peculiar properties, which include the phenomenon of neutrino oscillations: we have not yet succeeded to find a definite answer to Majorana’s proposal.

3.4. Quantum electrodynamics. A large part of Majorana’s interests was devoted to several theoretical problems of the rising quantum electrodynamics [9][11], especially after he returned from Leipzig (after 1933). This is also testified by a letter to his uncle Quirino [41], who constantly pressed Majorana for theoretical explanations of his own experiments [42]. However, also in this case, according to his hypercritical judgment Majorana decided not to publish any paper on his studies.

Fortunately we know from his notebooks that he generally considered quantum electrodynamics in a Lagrangian and Hamiltonian framework, with the use of a least action principle. In one of his studies [11], as it is now customary, the electromagnetic field was decomposed in plane wave operators, and its properties studied in a full Lorentz-invariant formalism by employing group-theoretic arguments. Explicit expressions for the quantized Hamiltonian, creation and annihilation operators for the photons, as well as angular momentum operator, were deduced in several different bases, along with the appropriate commutation relations. Apart from the relevance of the scientific results contained in such writings, and achieved
by Majorana well in advance with respect to other scholars, they also testify on the
approach followed by Majorana on dealing with these topics. In fact, once more
being ahead of his time, he adopted the more mathematical and more powerful
methods introduced by Heisenberg, Born, Jordan and Klein, instead of following
the plain (and quite famous) approach by Fermi [43].

This is also the case for a recently retrieved text written by Majorana in French
[44], where Majorana dealt with quite a peculiar topic in quantum electrodynamics.
It is instructive, for this topic, just to quote directly from the Majorana’s paper:

Let us consider a system of \( p \) electrons and put the following as-
sumptions: 1) the interaction between the particles is sufficiently
small allowing to speak about individual quantum states, so that
we may consider that the quantum numbers defining the configura-
tion of the system are good quantum numbers; 2) any electron has
a number \( n > p \) of inner energetic levels, while any other level has
a much greater energy. We deduce that the states of the system
as a whole may be divided into two classes. The first one is com-
posed of those configurations for which all the electrons belong to
one of the inner states. Instead the second one is formed by those
configurations in which at least one electron belongs to a higher
level not included in the \( n \) levels already mentioned. We will also
assume that it is possible, with a sufficiently degree of approxima-
tion, to neglect the interaction between the states of the two classes.
In other words we will neglect the matrix elements of the energy
corresponding to the coupling of different classes, so that we may
consider the motion of the \( p \) particles in the \( n \) inner states, as if only
these states exist. Then, our aim is to translate this problem into
that of the motion of \( n - p \) particles in the same states, such new
particles representing the holes, according to the Pauli principle.

Majorana, thus, by following a track left by Heisenberg, applied the formalism of
field quantization to Dirac’s hole theory, obtaining the general expression for the
QED Hamiltonian in terms of anticommuting holes quantities. We also point out
the peculiar justification for using anticommutators for fermionic variables given by
Majorana; this, indeed, “cannot be justified on general grounds, but only by the
particular form of the Hamiltonian. In fact, we may verify that the equations of
motion are satisfied to the best by these last exchange relations rather than by the
Heisenberg ones”.

In the Quaderni we also find several studies, following an original idea by Op-
penheimer [45], aimed at exploring the possibility to describe the electromagnetic
field by means of the Dirac formalism, as already pointed out in the literature.
Some emphasis was given to consider the properties of the electromagnetic field as
described by a real wavefunction for the photon, a study which was well beyond
the contemporary works of other authors [46].

Another unknown contribution by Majorana, which has been brought to light
only recently [28] is nothing but the theory of scalar quantum electrodynamics,
as elaborated several years later (1934) by Pauli and V.F. Weisskopf [47]. The
results obtained by Majorana were well summarized by Gian Carlo Wick [48], who
was aware of them directly from Majorana (although Wick never saw the written
calculations, reported in [11]):
I was admitted as a bystander to an international meeting on nuclear physics, which was organized by the Royal Academy in Rome in the fall of 1931. I was asked by Heitler to act as a sort of interpreter between him and Majorana. He spoke hardly any Italian, and Majorana’s German was a bit weak. So during lunches, Heitler expressed a curiosity in what Majorana was doing; Fermi must have told him how bright he was. So Majorana began telling, in that detached and somewhat ironical tone, which was typical of him, especially when discussing his own work, that he was developing a relativistic theory for charged particles. It was not true, he said, that the Schrödinger equation for a relativistic particle had to have the form indicated by Dirac. It was clear by now, that in a relativistic theory one had to start from a field theory and for this purpose the Klein-Gordon wave-equation was just as legitimate as Dirac’s. If one starts with the first one, and quantizes it, one get a theory with consequences quite similar to Dirac positron theory, with positive and negative charges, the possibility of pair creation etc. You can see what I am driving at: Majorana had the Pauli-Weisskopf scheme all worked out already at that time. Please do not think I am disputing the merit of these authors. Majorana never published this work, he did not seem to take it very seriously, and I don’t think Pauli and Weisskopf ever even heard of it. Heitler probably forgot all about it, and so did I, until I saw the paper by Pauli and Weisskopf.

As well known, the relevance of the Pauli-Weisskopf theory for a scalar field is certainly related to its possible direct applications, i.e. meson theory, but, in addition, it can be considered a milestone in the development of quantum electrodynamics. In fact, it showed unambiguously that the marriage between quantum mechanics and special relativity did not necessarily require a spin 1/2 for the correct interpretation of the formalism, as erroneously believed. In this respect, it is interesting that the formulation of such a theory by Majorana can be dated as early as 1929-1930. The Majorana theory, however, presents also several different theoretical aspects with respect to the Pauli-Weisskopf theory: this is the case, for example, for the use of general sets of plane waves in the expansion of the field variables, or the adoption of four (for matter particles) plus four (for photons) instead of four plus two operators describing the quanta of the appropriate fields.

Other peculiar investigations concerned the possibility to introduce an intrinsic time delay (as a universal constant) in the expressions for the electromagnetic retarded fields, and the study on the modification of the Maxwell equations in presence of magnetic monopoles. Besides the fully theoretical work about quantum electrodynamics, some applications were also considered by Majorana. This is the case of the free electron scattering, where he gave an explicit expression for the transition probability, and the coherent scattering of bound electrons. Several other scattering processes were considered as well in the framework of perturbation theory, by means of the Dirac or the Born methods.

Finally, we also mention an intriguing attempt, made by Majorana as early as in 1928, to find a relation between the fundamental constants \( e, h, c \). The interest for this work is not related to the particular mechanical model used by him (that,
indeed, led to a result $e^2 \approx hc$ far from the truth, as noted by Majorana himself), but rather to the interpretation of the electromagnetic interaction in terms of exchanged particles. According to Majorana, indeed, the electromagnetic field generated by charged particles is, in some sense, quantized, and two electrons interact by exchanging particles each another.

As a first approximation we can describe the situation in terms of a pointlike mass moving with a group velocity equal to the light speed $c$. Let us also make the arbitrary assumption that such point-particle moves periodically between $A$ and $B$ and back. Let us further suppose that it is free of interactions while travelling between $A$ and $B$, whereas in $A$ and in $B$ it inverts its velocity due to the collision with the electrons setting there.

As it is evident, such an interpretation of the quantized electromagnetic field substantially coincides with that introduced more than a decade later by Feynman in quantum electrodynamics, the point-particles exchanged by electrons being assumed to be the photons.

3.5. Other studies. In some notes, probably prepared for a seminar at the University of Naples in 1938 [50], Majorana gave a physical interpretation of quantum mechanics which anticipated of several years the Feynman approach in terms of path integral, independently of the underlying mathematical formulation.

The starting point in Majorana’s paper was to search for a meaningful and clear formulation of the concept of quantum state. This was achieved by considering some sets of ‘solutions that differ for the initial conditions’ which, in the Feynman language of 1948 [51], correspond precisely to the different integration paths: the different initial conditions were always referred to the same initial time, while the determined quantum state corresponded to a fixed end time. Moreover, the crucial point in the Feynman formulation of quantum mechanics, namely that of considering not only the paths corresponding to classical trajectories, but all possible paths joining the initial and final points, was introduced by Majorana after an interesting discussion on the harmonic oscillator, where its quantum energy levels were interpreted in terms of classical oscillations: “we can say that the ground state with energy $E_0 = h\nu/2$ corresponds roughly to all classical oscillations with energy between 0 and $h\nu$, the first excited state with energy $E_1 = 3h\nu/2$ corresponds to the classical solutions with energy between $h\nu$ and $2h\nu$, and so on.” Here we stress also the key role played by the symmetry properties of the physical system in Majorana analysis, a feature which is, again, quite common in all his papers. Furthermore, although no trace can be found of the formalism underlying the Feynman path-integral approach to quantum mechanics in the Majorana manuscript, nevertheless it is very intriguing that the main physical issue, the novel way of interpreting the theory of quanta, were realized well in advance by Majorana. This is particularly impressive if we take into account that, in the known historical path, the interpretation of the formalism has only followed the mathematical development of the formalism itself.

Finally, we also mention paper 10, published posthumous by Majorana’s friend Gentile and dealing with a completely different subject: “the value of statistical laws in physics and social sciences”. As suggested by Gentile, this was probably intended for a sociology journal, and was likely written during a period ranging from
1933 to 1937, when Majorana’s studies extended to cover economics, politics, philosophical problems, etc. This paper presented the point of view of a physicist about the value of statistical laws, in physics and social sciences, to people of different disciplines such as sociology and economics. Of some interest was the observation that statistical laws are investigation tools to be used in economic and social modelling, and have the same epistemological status of irreducible probabilistic laws as quantum mechanics. Then, even in this case, the approach by Majorana was well ahead with respect to that of the vast majority of his contemporaries working in this field.

4. Professor of theoretical physics

As we have seen, Majorana contributed significantly to many theoretical researches which were considered as the frontier topics in the 1930s. However, his own peculiar contribution ranged also on the basics and applications of quantum mechanics, as Majorana’s lectures on theoretical physics clearly demonstrate.

As realized only recently, Majorana revealed a genuine interest in advanced physics teaching starting from 1933, soon after he obtained at the end of 1932 the professorship degree of “libero docente” (analogous to the German privatdozent). In view of this position, he proposed some academic courses at the University of Rome, as testified by the programs of three courses he would have given between 1933 and 1937 [52]. Although Majorana never gave lectures, probably due to the lacking of students, they are particularly interesting and informative due to a very careful choice of the topics he intended to treat in his courses.

The first course (academic year 1933-34) was that of Mathematical Methods of Quantum Mechanics, whose syllabus contained the following topics:

1) Unitary geometry. Linear transformations. Hermitian operators. Unitary transformations. Eigenvalues and eigenvectors. 2) Phase space and the quantum of action. Modifications to classical kinematics. General framework of quantum mechanics. 3) Hamiltonians which are invariant under a transformation group. Transformations as complex quantities. Non compatible systems. Representations of finite or continuous groups. 4) General elements on abstract groups. Representation theorems. The group of spatial rotations. Symmetric groups of permutations and other finite groups. 5) Properties of the systems endowed with spherical symmetry. Orbital and intrinsic momenta. Theory of the rigid rotator. 6) Systems with identical particles. Fermi and Bose-Einstein statistics. Symmetries of the eigenfunctions in the center-of-mass frames. 7) The Lorentz group and the spinor calculus. Applications to the relativistic theory of the elementary particles.

The second course (academic year 1935-36) was instead on Mathematical Methods of Atomic Physics covering the following topics:

Matrix calculus. Phase space and the correspondence principle. Minimal statistical sets or elementary cells. Elements of the quantum dynamics. Statistical theories. General definition of symmetry problems. Representations of groups. Complex atomic spectra. Kinematics of the rigid body. Diatomic and polyatomic molecules.
Relativistic theory of the electron and the foundations of electrodynamics. Hyperfine structures and alternating bands. Elements of nuclear physics.

Finally, the third course (academic year 1936-37) was planned to discuss Quantum Electrodynamics:

- Relativistic theory of the electron. Quantization procedures. Field quantities defined by commutability and anticommutability laws. Their kinematical equivalence with sets with an undetermined number of objects obeying the Bose-Einstein or Fermi statistics. Dynamical equivalence. Quantization of the Maxwell-Dirac equations.
- Study of the relativistic invariance. The positive electron and the symmetry of charges. Several applications of the theory. Radiation and scattering processes. Creation and annihilation of opposite charges. Collisions of fast electrons.

Majorana effectively lectured on theoretical physics only in 1938 when, as recalled above, he obtained a position as a full professor at University of Naples. He started on January 13 till March 24, one day before his disappearance, thus about two months only, but his activity was intense, and his interest for teaching extremely high. For the benefit of his students, he prepared careful notes for his lectures [10]. A recent analysis has shown that Majorana's 1938 course was very innovative for that time [52, 39], and this has been confirmed by the retrieval (on September 2004) of a faithful transcription of the whole set of Majorana's lecture notes (the so-called "Moreno lecture notes") comprising 6 lectures not included in the original collection [3].

The first part of his course on theoretical physics dealt with the phenomenology of the atomic physics and its interpretation in the framework of the old quantum theory of Bohr-Sommerfeld. This part was quite analogous to the course given by Fermi in Rome (1927-28) attended by the student Majorana.

The second part started with the classical radiation theory, reporting explicit solutions of the Maxwell equations, scattering of the solar light and some other applications. It then discussed the theory of relativity: after the presentation of the corresponding phenomenology, a complete discussion of the mathematical formalism required by the theory was given, ending with some applications as the relativistic dynamics of the electron. Finally, a discussion of several effects for the interpretation of quantum mechanics, such as the photoelectric effect, the Thomson scattering, the Compton effects and the Franck-Hertz experiment, were addressed.

The last part of the course, more mathematically oriented, dealt with the Schrödinger and the Heisenberg formulation of quantum mechanics. This part did not follow the Fermi approach, but rather referred to previous personal studies by Majorana [9], also following the original Weyl's book [38] on group theory and quantum mechanics.

Majorana's course(s) thus consisted of a fruitful mixture of an original approach - very similar to that of today's courses on quantum mechanics - and of some consolidated lines of development, which were the clear legacy of the lines Fermi had adopted in his courses. Both the lecture notes for the Naples' course and the programs of the three courses that Majorana submitted in Rome between 1933 and 1936, reveals his forefront approach and his search for new routes in dealing with quantum mechanics.
5. Conclusion

Majorana’s course on theoretical physics was suddenly and unexpectedly interrupted by his mysterious disappearance three months after his appointment at Naples.

On Friday March 25, 1938 Majorana went to the Institute of Physics and handed over the lecture notes and some other papers to one of his students. After that, he returned to his hotel and, after writing farewell letters to his family and to the director of the Institute of Physics, Carrelli, apparently embarked on a ship to Palermo. He reached his destination the following morning, where he lodged for a short time in the Grand Hotel Sole. It was there that he wrote a telegram and a letter to Carrelli pointing out a change of mind about his decisions. On Saturday evening Majorana embarked on a ship from Palermo to Naples. From here onwards, no other reliable information about him are available.

There have been several conjectures about the fate of Majorana, including suicide, a retreat in a monastery and a flight to a foreign country [1]. Understanding the root of such a dramatic decision is perhaps impossible and it could be triggered by personal and familiar reasons, such as Majorana’s peculiar relationship with his extremely possessive mother (especially after the death of his father in 1934), or more elaborated reasons reported in nice literary tales. However, quoting Majorana himself on his approach to physics: “We cannot give to such hypothesis greater likelihood than to some other theoretical presumptions without a too much subjective appraisal”.

Attention should not, however, be shifted far from the outstanding work performed by the Italian scientist, only briefly outlined in this review. We prefer to end with Fermi’s own words, written not long after Majorana’s disappearance [53]:

Able at the same time to develop audacious hypothesis and criticize acutely his work and that of others: very skilled calculating man, a deep-routed mathematician that never loses the very essence of the physical problem behind the veil of numbers and algorithms, Ettore Majorana has at the highest level that rare collection of abilities which form the theoretical physicist of very first-rank. Indeed, in the few years during which his activity has been carried out, until now, he has been able to outclass the attention of scholars from all over the world, who recognized, in his works, the stamp of one of the greatest mind of our times and the promise of further conquests.

Acknowledgments. The interest of the present author in the work by Ettore Majorana was stimulated, many years ago, by Erasmo Recami who, since then, always encouraged him in further studies. His help and support is here gratefully acknowledged. The author is also indebted to Gianpiero Mangano for a very careful reading of the manuscript.

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ETTORE MAJORANA AND HIS HERITAGE SEVENTY YEARS LATER

APPENDIX A. CHRONOLOGY OF THE LIFE AND WORK OF ETTORE MAJORANA

1906
August 5
Ettore Majorana was born in Catania to Fabio Massimo (1875-1934) and Salvatrice (Dorina) Corso (1876-1965), fourth of five sons (Rosina, Salvatore, Luciano, Ettore and Maria).

1921
The Majorana family moved from Sicily to Rome.

1923
July
He completed secondary school (Liceo-Ginnasio Statale “Torquato Tasso” in Rome) at the age of 17.
November 3
He joined the Faculty of Engineering at the University of Rome.

1927
He started to write the “Volumetti”, personal notebooks where he wrote down his own studies and/or researches (the date reported in the first of the five notebooks is March 8, 1927).
Summer
E. Amaldi and E. Segré, students in Engineering, decided to join the Fermi group (the formal passage was registered on November 1927 for Segré and on February 1928 for Amaldi), on advice of the Director of the Institute of Physics in Rome, O.M. Corbino.
Autumn
Segrè encouraged Majorana to meet Fermi; after this meeting, he moved from Engineering to Physics studies (the formal passage was registered on November 19, 1928).

1928
In collaboration with G. Gentile jr. he published his first paper 1, following the spectroscopic researches of the Fermi group (the paper was presented at the Accademia dei Lincei on July 24).
December 29
Still University student, he participated at the XXII General Meeting of the Italian Physical Society (directed by his uncle Quirino), giving a talk on a Ricerca di un’espressione generale delle correzioni di Rydberg, valevole per atomi neutri o ionizzati positivamente (the summary of his talk appeared in Nuovo Cim. 6, XIV (1929) while the original work is in the Volumetto II.)

1929
July 6
He graduated with a master degree in Physics, with a thesis on La teoria quantistica dei nuclei radioattivi (The quantum theory of radioactive nuclei). It is the first time that the subject of nuclear physics appears among the activity of the Rome group.
1931
He published two papers (2,4) on the chemical bond of molecules and two further papers (3,5) on spectroscopic researches.

1932
He published two papers (6,7). In the first one, stimulated by Segrè, he studied for the first time, from a theoretical point of view, the non-adiabatic spin-flip, whose transition probability was independently obtained by Landau and Zener in the same year. In the second paper, again for the first time, he introduced the infinite-dimensional representation of the Lorentz group, anticipating the results obtained by E. Wigner in 1938 and 1948.

March
After the discovery of the neutron by J. Chadwick, Majorana discuss with the members of Fermi group a theory of light nuclei made of only protons and neutrons (without electrons). Although encouraged by Fermi and his group, he decided to not publish his work. A similar result, with some imperfection, is published independently by W. Heisenberg in the following July.

November 12
He got the “Libera Docenza” degree in Theoretical Physics at the University of Rome.

1933

January
Upon suggestion of E. Fermi, he obtained a fellowship from the Italian C.N.R. for visiting the Institute of Theoretical Physics in Leipzig head by Heisenberg.

January 19
In the evening he arrived at Leipzig.

March 3
Encouraged by Heisenberg, he finally sent, to the German journal Zeitschrift für Physik, his paper 8 on nuclear theory, whose main result had been already obtained one year before and improving the Heisenberg theory on nuclear interactions.

March 4
From Leipzig he moved to Copenhagen, where he stayed for about one month at the Institute of Theoretical Physics directed by N. Bohr.

April 15
He left Copenhagen and come back to Rome for the Easter holidays.

May
At the University of Rome he presented the syllabus of a course on Mathematical Methods of Quantum Mechanics, which he never taught.

May 5
He returned to Leipzig to continue his visit.

May 11
Upon request of the Italian C.N.R., he sent to the official journal of this agency the Italian version of the paper 8 published in German.

July
The Majorana family (the mother of Ettore with his sisters Rosina and Maria and his brother Salvatore) visited Ettore in Leipzig.

August 5
He definitively returned to Rome.

1934
His father died; such a tragic event had a strong influence on his following life.

1935
April 30
At the University of Rome he presented the syllabus of a course on Mathematical Methods of Atomic Physics, he never taught.

1936
April 28
At the University of Rome he presented the syllabus and materials for a course on Quantum Electrodynamics, never taught in the following.

1937
He published his last paper 9, where he presented results obtained some years before. It contained the fundamental theory on the Majorana neutrino, which twenty years later has been recognized as a basic result in order to explain the phenomena related to the problem of the neutrino mass.

June
He decided to participate to the national call for a full professorship in Theoretical Physics at the University of Palermo (obtained at the request of E. Segrè). Among the participants we find G. Gentile jr, L. Pincherle, G. Racah, G. Wataghin, G.C. Wick.

October 25
The committee for the competition (headed by Fermi) met for the first time, and immediately stopped its work in order to send a letter to the Minister of Education with the request to give a professorship in Theoretical Physics to Majorana “for high and well deserved repute”, independently of the usual competition rules.

November 2
The Minister of Education accepted the request and appointed Majorana as full professor at the University of Naples (starting from the following November 16).

1938
January 10 or 11
He arrived in Naples to take up the chair of Theoretical Physics.

January 13
At 9.00 he delivered the opening lecture for his course on Theoretical Physics. His family arrived from Rome for the occasion.

January 15
He effectively started his course (which was usually delivered on Tuesday, Thursday and Saturday of each week).

March 12
After his lecture, he visited his family in Rome for the last time.

March 24
He gave his last lecture (N.21).

March 25
In the morning he went to the Institute of Physics of Naples to give a folder with the notes of his lectures to a student of him. After a stop at his Hotel, at 17.00 he left for an unknown destination.