Observation potential for \( \eta_b \) at the Tevatron

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We calculate the cross section for \( \eta_b \) production at the Tevatron at next-to-leading order in the strong coupling and find that more than two millions of \( \eta_b \)'s are expected per inverse picobarn of integrated luminosity. We discuss the decay modes into charmed states and suggest that the decays into \( D^* D^{(*)} \) mesons might be the most promising channels to observe the \( \eta_b \) in Run II.

Considering the rich phenomenology of the \( \Upsilon \) states, it is quite surprising that spin singlet \( b\bar{b} \) states, including the \( 1S_0 \) ground state, have not been observed yet.

Fine and hyperfine splittings of the quarkonia spectra were calculated using phenomenological models for the heavy-quark potential [1,2]. Recent progress both in lattice and in perturbative QCD has allowed the achievement of comparable precisions [3,4]. Various approaches have been successfully adopted to describe the charmonium system and are believed to be even more reliable for the heavier \( b\bar{b} \) states, where relativistic effects are less important. Recent determinations lead to a mass splitting between the \( \Upsilon(1S) \) and the \( \eta_b(1S) \) in the 40–60 MeV range [3,4,5,6,7].

Searches of the \( \eta_b \) have been pursued in various experiments. In \( e^- e^+ \) collisions, cross sections for producing spin singlet states are generally small and the signal is rate-limited. This is compensated by a clean environment, which allows for searches in the inclusive decay modes. Following an original suggestion by Godfrey and Rosner [8], the CLEO collaboration has looked for the \( \eta_b \) in the hindered M1 decay of the \( \Upsilon(3S) \) and found no signal [9]. At LEP II, the ALEPH collaboration analysed the \( \gamma \gamma \) interactions data, basing their analysis on the QCD prediction of the \( \Gamma_{\gamma \gamma} \) partial width [8,10]: no evidence was found in the four- and six-charged-particle decay modes [11].

The situation is exactly reversed in hadron collisions, where the production rates can be large, with millions of events produced at the Tevatron per inverse picobarn of integrated luminosity, yet the intense hadronic activity makes any inclusive analysis unfeasible. In this case it is necessary to identify decay modes that have triggerable signatures and allow for full invariant mass reconstruction of the decaying state. Such exclusive modes are believed to have very small branching ratios. Braaten et al. [12] have suggested that the \( \eta_b \) at the Tevatron could be observed via its decay into \( J/\psi J/\psi \), with the subsequent leptonic decay of the \( J/\psi \)'s; experimental efforts have started in this direction [13]. This signature exploits the upgraded abilities of the CDF detector of triggering on soft muons.

The purpose of this note is twofold. First, we provide the prediction for the inclusive cross section of \( \eta_b \) production at the Tevatron, based on the (NR)QCD calculation at the next-to-leading accuracy in the strong coupling [14]. We find that the cross section is about four times larger than the previous available estimate [12]. Second, stemming from an estimate of the corresponding branching ratio, we suggest a new analysis based on the decay of the \( \eta_b \to D^* D^{(*)} \) meson pairs. Thanks to secondary vertex trigger capabilities of CDF and to the high resolution achievable in the invariant mass reconstruction (\( \sim 20 \) MeV), this could turn out to be the most promising search channel at the Tevatron.

According to the NRQCD factorization approach [12], the inclusive cross section for the \( \eta_b \) in \( p\bar{p} \) collisions can be written as:

\[
\sigma(p\bar{p} \to \eta_b + X) = \sum_{i,j} \int d^4x_1 d^4x_2 f_{i/p} f_{j/\bar{p}} \hat{\sigma}(ij \to \eta_b),
\]

where

\[
\hat{\sigma}(ij \to \eta_b) = \sum_n C_n^{ij} \langle 0 | \mathcal{O}_n^{\eta_b} | 0 \rangle.
\]

The short-distance coefficients \( C_n^{ij} \), calculable in perturbative QCD, describe the production of a quark–antiquark pair state with quantum number \( n \), while the \( \langle 0 | \mathcal{O}_n^{\eta_b} | 0 \rangle \) are the non-perturbative matrix elements that describe the subsequent hadronization of the \( b\bar{b} \) pair into the physical \( \eta_b \) state. These matrix elements can be expanded in powers of \( v^2 \simeq 0.1 \), the
Fig. 1(a), is known parameters (up to corrections of $g_{\text{et}}$ contribution [27]. Hence, given that the non-$\eta$ spin-symmetry and vacuum saturation approximations have been calculated in Refs. [14, 16]. These potential models [17] and lattice calculations [18] fall to 10%. For the non-perturbative matrix element we include virtual corrections to the $2\rightarrow 2$ processes, such as $gg \rightarrow c\bar{c}J/\psi J/\psi'$, Fig. 1(b), and real $2\rightarrow 2$ processes, such as $gg \rightarrow b\bar{b}$. Compared to the leading contribution, the $\eta_b$ production at LO (a), and virtual (b) and real (c) contributions at NLO.

The case of $\eta_b$ production is particularly simple. Compared to the leading contribution, the $1S_0^{(1)}(b\bar{b})$, the color octet terms, $(1S_0^{[8]}3S_1^{[8]}1P_1^{[8]})$, are all suppressed by $v^4$, while the corresponding short distance coefficients start at least at $\alpha_s^2$ as the singlet contribution [27]. Hence, given that the non-perturbative matrix elements for the singlet production are extracted from $\Upsilon$ decays, there are no unknown parameters (up to corrections of $\mathcal{O}(v^4)$) entering the estimate for $\eta_b$ production.

The leading-order cross section for $gg \rightarrow \eta_b$, Fig. 1(a), is

$$\sigma(gg \rightarrow \eta_b) = \frac{\pi^2}{36m_b^2s} \delta(1 - \frac{4m_b^2}{s})(0|\mathcal{O}_1^{1S_0}(1S_0)|0). \quad (3)$$

Next-to-leading order corrections in the strong coupling have been calculated in Refs. [14, 16]. These include virtual corrections to the $2 \rightarrow 1$ process, Fig. 1(b), and real $2 \rightarrow 2$ processes, such as $gg \rightarrow \eta_b g, gg \rightarrow \eta_b q$. Fig. 1(c). The result for the total cross section in $pp$ collisions at 1.96 TeV of center-of-mass energy is

$$\sigma(pp \rightarrow \eta_b + X) = 2.5 \pm 0.3 \mu_b, \quad (4)$$

where we have adopted CTEQ5M1 parton densities, the corresponding two-loop evolution for $\alpha_S(\mu)$ and $m_b = 4.75$ GeV. The quoted uncertainty has been estimated by summing (in quadrature) the errors coming from various sources. The first is associated to the choice of the renormalization and factorization scales. Varying them independently in the range $m_b < \mu_R, \mu_F < 4m_b$ gives an uncertainty of 10%. For the non-perturbative matrix element we have used the determination from $\Upsilon$ leptonic decay, $\langle \Upsilon|\mathcal{O}_1^{3S_1}|\Upsilon \rangle = 3.5 \pm 0.3$ GeV$^3$, which can be related to the $\eta_b$ production matrix element by using spin-symmetry and vacuum saturation approximation, $\langle 0|\mathcal{O}_1^{1S_0}|0 \rangle = \langle \Upsilon|\mathcal{O}_1^{3S_1}|\Upsilon \rangle$, up to $\mathcal{O}(v^4)$ corrections. Other determinations coming from potential models [17] and lattice calculations [18] fall in the range of the quoted error. Finally, we also included the effect of an uncertainty on the bottom (pole) mass of $\pm 50$ MeV on the cross section. The strong correlation to the non-perturbative matrix element extraction has been exploited to reduce the uncertainty from this source. The result, quoted in Eq. (4), accounts for the “direct” contribution and does not include feed-downs from higher-mass states, such as the $h_b$.

For the experimental analysis it is important to know the distribution of $\eta_b$ at small $p_T$ values. This cannot be described accurately by a fixed-order calculation. In this region of the phase space it is necessary to resum higher-order corrections involving soft-gluon radiation. To this aim, we use PYTHIA [19], matched with the exact matrix elements for $ij \rightarrow \eta_b k$ describing the high-$p_T$ tail, as outlined in Ref. [20]. This procedure, which has been shown to work well for the analogous process $gg \rightarrow H$, has the additional virtue that it can be directly used for simulation in experiments. The results are shown in Fig. 2 where the differential distribution in $p_T$ for the $\eta_b$ is shown (upper curve). Hadronization and initial $k_T$ effects are not included. The normalization of the inclusive $\eta_b$ distributions is obtained from the NLO result, Eq. (4).

As already stated above, the cross section for the $\eta_b$ in the central region is about four times larger than was estimated in Ref. [12] by rescaling the $\Upsilon$ cross section at high transverse momentum. This procedure is expected to provide an underestimate, since it assumes that the $p_T$ spectra for $\Upsilon$ and $\eta_b$ have a similar shape at small $p_T$. In fact $\Upsilon$ color-singlet production proceeds at LO through $gg \rightarrow \Upsilon g$ and therefore vanishes at $p_T \sim 0$, where the largest part of the $\eta_b$’s are produced.

With such a large number of events expected, it is interesting to consider in detail the rare exclusive decays that might give triggerable signatures. It is easy to show that direct decays into photons or lepton pairs give either too small branching ratios or very difficult experimental signatures (such as $\eta_b \rightarrow \gamma \gamma$ whose branching ratio is $\mathcal{O}(10^{-5})$). In Ref. [12], the branching ratio $\text{Br}(\eta_b \rightarrow J/\psi J/\psi')$ was estimated through scaling from the analogous $\eta_c \rightarrow \phi \phi$, finding a value compatible with $7 \times 10^{-4}$ with $1\%$ uncertainty. This is probably an overestimate. As a simple upper bound, let us consider the inclusive decay rate of the $\eta_b$ into four-charm states

$$\Gamma(\eta_b \rightarrow J/\psi J/\psi') < \Gamma(\eta_b \rightarrow c\bar{c}c\bar{c}). \quad (5)$$

The inclusive rate can be calculated at leading order by considering the four Feynman diagrams, such as

![Feynman diagrams](image-url)
FIG. 2: Differential cross sections for $\eta_b$ production at the Tevatron ($p\overline{p}$ at 1.96 TeV). The upper curve is normalized to the NLO total rate and describes the $p^2$ distribution for the $\eta_b$. The lower curve is the corresponding distribution for the $D$ mesons coming from the $\eta_b$ decays, after requiring that they both have $|\eta(D)| < 1$ (no branching ratio included). After the acceptance cut, the rate drops to 15% of the total cross section.

the one shown in Fig. 3(b). The result is

$$\text{Br}(\eta_b \rightarrow c\bar{c}c\bar{c}) = 1.8^{+2.3}_{-0.8} \times 10^{-5}. \quad (6)$$

where $m_c = 1.45$ GeV, $m_b = 4.75$ GeV, $\alpha_S(2m_b) = 0.182$ and the NLO expression for the inclusive total width have been used. The amplitudes have been calculated analytically while the integration over the phase space has been performed numerically. The uncertainty has been estimated by varying the renormalization scale between $m_b < \mu_R < 4m_b$ and the masses in the ±50 MeV range. The four-charm branching ratio is very sensitive to the value of the charm mass, which dominates its uncertainty. The above result shows that the inclusive rate is already smaller than the lower bound obtained in Ref. [22]; further suppression is expected mainly because many other decay modes to charmed mesons (or other charmonium states) should contribute to the saturation of the inclusive rate.

Given the result in Eq. (6), a comment on the reliability of the estimate based on the scaling from $\text{Br}(\eta_c \rightarrow \phi\phi)$ is in order. To this aim we recall that the decay of a scalar $Q\overline{Q}$ meson into two vector states is suppressed in perturbative QCD [22].

A non-trivial check of this selection rule is that the branching ratio $\text{Br}(\eta_b \rightarrow J/\psi J/\psi)$ is exactly zero when calculated at LO in the NRQCD double expansion in $\alpha_S$ and $v^2$. For the same reason one would expect the rate of $\eta_c \rightarrow \phi\phi$ to be suppressed, in contradiction with the measured value of about 1%. This entails that some other (non-perturbative) mechanism is responsible for this decay process [22]. Rescaling by $(m_c/m_b)^4$ the branching ratio of $\eta_c \rightarrow \phi\phi$ to obtain the branching ratio of $\eta_b \rightarrow J/\psi J/\psi$ amounts to rescaling by the same factor also the effect of non-perturbative or higher-order contributions that are likely to be crucial in determining the $\eta_c$ decay, but less and less important as we pass from the $\eta_c$ to the $\eta_b$ system.

From the phenomenological point of view, Eq. (6) implies $\text{Br}(\eta_b \rightarrow \mu^+\mu^-\mu^+\mu^-) < 6 \times 10^{-8}$, which makes a search in this channel very hard. As an alternative, we propose to consider the decays into charmed mesons, $\eta_b \rightarrow D^* D^{(*)}$. A computation of the exclusive decay rates is missing and does not seem to be feasible within the framework of quark models or QCD sum rules. Probably, there is room to face such a problem using lattice techniques. Here we assume that the exclusive decays into $D^* D^{(*)}$ dominate the inclusive rate into charm:

$$\Gamma(\eta_b \rightarrow D^* D^{(*)}) \lesssim \Gamma(\eta_b \rightarrow c\bar{c} + X). \quad (7)$$

We find that the largest contribution to the decay of the $\eta_b$ into charmed states is given, Fig. 3(a), by

$$\text{Br}(\eta_b \rightarrow c\bar{c}g) = 1.5_{-0.4}^{+0.8} \%, \quad (8)$$

where we used the same input parameters and estimated the uncertainties as in the decay into the four-charm states. It can now be argued that not much suppression is expected in passing from the inclusive to the exclusive decays. For example, decay rates into charmonium states should be of the same size $Y \rightarrow J/\psi + X$, i.e., $\mathcal{O}(10^{-3})$ [24]. Emission of extra pions should also be considered, but some of these contributions would be automatically included in the experimental analysis, e.g., $\eta_b \rightarrow D D^* \rightarrow D D\pi$, as non-resonant diagrams. However, to be conservative, we consider the range $10^{-3} < \text{Br}(\eta_b \rightarrow D^* D^{(*)}) < 10^{-2}$ in the following phenomenological analysis.
There are other decay modes leading to a two-charge final state that have not been included in our estimate of the branching ratio. One is the decay of $\eta_b \rightarrow g^* g^* \rightarrow cc$ through a (box) loop, which proceeds at order $\alpha_s^3$ and is further suppressed by loop factors. Another is the decay $\eta_b \rightarrow g^* \rightarrow cc$, via a $3 P_1^{(8)}$ state, which is only, $\alpha_s^3$ but is is suppressed by the color octet matrix element $\langle \eta_b | O_8^{(8)} S_8 | \eta_b \rangle$. Assuming a scaling $O(V^4)$ for the non-perturbative matrix element, this process gives a non-negligible contribution to the branching ratio, about 5 thousandths. However, this result is affected by a large uncertainty and could be much smaller as suggested by studies [25, 26] about the size of color octet matrix elements in $\Upsilon$ decays.

Results for the $p_T$ distribution of the $D$ mesons from the $\eta_b$ decay are shown in Fig. 2 (lower curve). No branching ratio is included, the difference in rate coming only from the requirement that both $D$ mesons be central, $|y(D)| < 1$. As expected, the $p_T$ distribution peaks just below $M_{\eta_b}/2$. The efficiency for the geometrical acceptance of the detector is found to be about 15%. By adding the requirement that at least one $D$ meson has $p_T > 5$ GeV, one is left with only 4% of the total number of events produced. The above efficiency can be folded with our estimate of the branching ratio leading to $10^{-4}$ $D^* D^{(*)}$ triggerable events expected from the $\eta_b$ decay in 100 pb$^{-1}$ of integrated luminosity at the Tevatron. The final number of reconstructed events will depend on the decay modes of the $D$ and $D^*$ mesons and on the associated experimental efficiencies. We leave this to more detailed experimental studies and only add a few comments. First we note that, according to the arguments outlined above, perturbative QCD predicts the $\eta_b \rightarrow D^* D^{*}$ decay to have a smaller rate with respect to $\eta_b \rightarrow D^* D$. In this case, it is reasonable to expect that different charge assignments, such as $D^* D^0, D^+ D^-, D^{*+} D^-$, will occur with the same probability of $\frac{1}{4}$. Finally, we recall that the cleanest signatures have small branching ratios, $\text{Br}(D^0 \rightarrow K^- \pi^+) = 3.90\%$ and $\text{Br}(D^+ \rightarrow K^0 \pi^+) = 2.77\%$ [24], leading to a factor of about $10^{-3}$ drop in the rate if both $D$ mesons are required to decay through these channels. We foresee that sizeable improvements could be achieved by requiring that just one of two $D$ mesons decays through a very clean signature, providing an efficient trigger.

To summarize, we have presented the NLO QCD prediction for $\eta_b$ production at the Tevatron, including a resummed result for the $p_T^2$ spectrum obtained with a dedicated implementation in PYTHIA. The production rate is large, of the order of a few $\mu$b, and allows for the search of the $\eta_b$ through rare exclusive decays. We argued that the branching ratio into $J/\psi J/\psi$ is probably too small and we suggested to look for the $\eta_b$ through its decay into $D^* D^{(*)}$ mesons. Our results indicate that the $\eta_b$ could be eventually observed during Run II at the Tevatron.

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[27] This is at variance with the $J/\psi$ or $\Upsilon$ production where the $C^{gg}_9$ coefficient is $O(\alpha_3)$ and therefore color-octet contributions can be important.

[28] The decay $\eta_b \rightarrow DD$ is forbidden by parity conservation.