EFFECT OF NON-LOCAL THERMODYNAMIC EQUILIBRIUM MODEL ATMOSPHERES ON PHOTOMETRIC AMPLITUDES AND PHASES OF EARLY B-TYPE PULSATING STARS

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ABSTRACT

Amplitudes and phases of the light variation of a pulsating star in various photometric passbands contain information about the geometry of observed modes. Because oscillation spectra of early B-type main-sequence stars do not exhibit regular patterns, these observables are very often the only ones from which mode identification can be derived. Moreover, these data can yield valuable constraints on mean stellar parameters, subphotospheric convection, microphysics, and atmospheres. We study all possible sources of inaccuracy in theoretical values of the photometric observables, i.e., amplitude ratios and phase differences, of early B-type main-sequence pulsators. Here, we discuss the effects of parameters coming from both model atmospheres and linear nonadiabatic theory of stellar pulsation. In particular, we evaluate for the first time the effect of the departure from the local thermodynamic equilibrium (LTE) approximation. To this end, for non-LTE model atmospheres, we compute tables with the passband fluxes, flux derivatives over effective temperature and gravity, as well as the nonlinear limb-darkening coefficients in 12 passbands most often used. We make these tables publicly available at the Wrocław HELAS Web site.

Key words: stars: abundances – stars: atmospheres – stars: early-type – stars: oscillations

1. INTRODUCTION

Data on the light and line profile variations of a pulsating star bring information on frequencies of excited modes and their geometry. The latter property is particularly important if oscillation spectra are sparse and lack equidistant patterns. This is the case for main-sequence pulsators of early B spectral type, i.e., δ Scuti variables. The photometric amplitudes and phases can also yield constraints on mean stellar parameters, subphotospheric convection in the case of cooler pulsators, like δ Scuti stars (Daszyńska-Daszkiewicz et al. 2003), or opacity data in the case of B-type pulsators (Daszyńska-Daszkiewicz et al. 2005).

Two of the most popular tools to identify a pulsation mode are the amplitude ratios and phase differences in various photometric passbands. In the zero-rotation approximation, these observables depend on the mode degree, ℓ, but are independent of the azimuthal order, m, the intrinsic mode amplitude, ε, and the inclination angle, i.

The semi-analytical expression for the bolometric light variation was formulated by Dziembowski (1977). Then, Balona & Stobie (1979) and Stamford & Watson (1981) expanded it for the light variation in the photometric passbands. They showed that modes with different values of ℓ are located in separate parts of the amplitude ratio versus phase difference diagrams. Subsequently, this method has been applied to various types of pulsating stars by Watson (1988). Cugier et al. (1994) improved the method by including nonadiabatic effects in calculations for the β Cephei stars. Effects of rotation on photometric observables were studied by Daszyńska-Daszkiewicz et al. (2002) for close frequency modes and by Townsend (2003) and Daszyńska-Daszkiewicz et al. (2007) for long-period g-modes. To compute values of the photometric amplitudes and phases we need two kinds of input. The first one comes from stellar model atmospheres and the second one from computations of stellar pulsation. Both data contain various sources of uncertainties.

The goal of this paper is to examine for the first time an influence of non-local thermodynamic equilibrium (NLTE) effects on theoretical values of the photometric amplitude ratios and phase differences for early B-type main-sequence pulsators. We also check the effects of metallicity and microturbulent velocity in the atmosphere. Subsequently, we compare these effects with uncertainties coming from the linear nonadiabatic theory of stellar pulsation. As an example, we consider main-sequence models with a mass of 10 M⊙ and low degree modes with ℓ = 0, 1, 2. Here, we neglect all effects of rotation on pulsation.

The structure of this paper is as follows. In Section 2, we recall the linear formula for the pulsation complex amplitude in a photometric band. Section 3 contains a description of NLTE model atmospheres and the results of our computations of the band fluxes, corresponding flux derivatives over effective temperature and gravity, and the nonlinear limb-darkening coefficients (LDCs) in 12 photometric passbands: uvbyUBVRIJK. Tables with these data can be downloaded from the Wrocław HELAS Web site (http://helas.astro.uni.wroc.pl). Moreover, we study the effects of temperature, gravity, NLTE, atmospheric metallicity, and microturbulent velocity on the above-mentioned quantities. How these atmospheric uncertainties translate into the pulsation photometric observables, i.e., amplitude ratios and phase differences, is discussed in Section 4. Inaccuracies in the photometric observables coming from linear nonadiabatic theory of stellar pulsation are presented in Section 5. Conclusions and remarks end the paper.

2. LIGHT VARIATION DUE TO STELLAR PULSATION

Stellar pulsations cause changes in the temperature, normal to the surface element, and pressure. If pulsations are linear and all effects of rotation on pulsation can be ignored, then the total amplitude of the light variation in the passband x can be written in the following complex form (Daszyńska-Daszkiewicz et al. 2002):

\[ A_x(i) = -1.086εY^m_x(i, 0)b^x_i(D^1_{1,1}f + D^2_{2,2} + D^3_{3,3}), \]  

(1)

where ε is the intrinsic mode amplitude, \( Y^m_x \) is the spherical harmonic, and i is the inclination angle. The amplitude is given by \( \text{abs}(A_x(i)) \) and phase by \( \text{arg}(A_x(i)) \). The \( D^i_{q,r} \cdot f \) product
stands for temperature changes, where

\[ D_{1,\ell}^i = \frac{1}{4} \frac{\partial \log (F_x | b_{\ell}^i)}{\partial \log T_{\text{eff}}} . \]  

(2)

Here, \( D_{1,\ell} \) is the flux in the passband \( x \) and \( f \) is the nonadiabatic complex parameter describing the magnitude of the radiative flux perturbation to the radial displacement at the photosphere level

\[ \delta F_{\text{bol}} \rightarrow \delta F_{\text{bol}} = \text{Re} \left[ \epsilon f Y'_{\mu}(\theta, \varphi)e^{-i\omega t} \right] . \]  

(3)

The geometrical term, \( D_{2,\ell} \), is given by

\[ D_{2,\ell} = (2 + \ell)(1 - \ell) \]  

(4)

and the pressure term, \( D_{3,\ell} \), by

\[ D_{3,\ell} = \left( 2 + \frac{\alpha^2 R^3}{GM} \right) \frac{\partial \log (F_x | b_{\ell}^i)}{\partial \log g} . \]  

(5)

Here, \( b_{\ell}^i \) is the disk averaging factor defined by

\[ b_{\ell}^i = \int_0^1 h_\lambda(\mu) \mu P_\ell(\mu) d\mu, \]  

(6)

where \( h_\lambda(\mu) \) is the limb-darkening law, \( P_\ell \) is the Legendre polynomial, and \( \mu \) is the cosine of the angle between a line of sight and the emergent intensity. Remaining parameters have their usual meanings.

In the above expressions, we can distinguish two kinds of input parameters needed to compute theoretical values of the photometric amplitudes and phases. The first input is derived from models of stellar atmospheres and these are the flux derivatives over effective temperature and gravity (Equations (2) and (5)), as well as limb darkening and its derivatives (Equations (2), (5), and (6)). The second input comes from the nonadiabatic theory of stellar pulsation and this is the \( f \)-parameter (Equations (1) and (3)).

In this paper, we used the Warsaw–New Jersey evolutionary code and the linear nonadiabatic pulsation code of Dziembowski (1977). We considered opacity tables from the OPAL (Iglesias & Rogers 1996) and OP (Seaton 2005) projects, and two determinations of the solar chemical composition: GN93 by Grevesse & Noels (1993) and AGSS09, a recent one by Asplund et al. (2009). As for models of stellar atmospheres, we considered Kurucz models (Kurucz 2004) and TLUSTY models (Lanz & Hubeny 2007).

3. NLTE LINE-BLANKETED MODEL ATMOSPHERES

The most widely used models of stellar atmospheres are line-blanketed, plane-parallel, hydrostatic models of Kurucz (2004) computed within an approximation of local thermodynamic equilibrium (LTE). However, in the case of atmospheres of early B-type stars, effects of the departure from LTE and a proper treatment of line opacity become important. Moreover, an escalation of the quality of spectrophotometric observations calls for a need for high-resolution and accurate models of stellar atmospheres.

A grid of NLTE model atmospheres was computed by Lanz & Hubeny (2003, 2007). These are metal line-blanketed, plane-parallel, hydrostatic model atmospheres of O-type stars (OSTAR2002) and of early B-type stars (BSTAR2006), respectively. In computations, they adopted a solar chemical mixture by Grevesse & Sauval (1998), helium to hydrogen abundance of He/H = 0.1 by number, and two values of the microturbulent velocity, \( \xi_t = 2 \) and \( 10 \) km s\(^{-1} \). Lanz & Hubeny (2003, 2007) showed that in the case of OB stars, neglecting NLTE effects causes differences not only in spectral lines but also in the continuum levels. In the near-ultraviolet (the Balmer continuum), the LTE fluxes are about 10% higher than the NLTE counterparts. In turn, in the far- and extreme-ultraviolet (the Lyman continuum), the LTE fluxes are lower than the NLTE ones. For more details, see Lanz & Hubeny (2003, 2007).

The grid of the OSTAR2002 models were computed for 12 values of effective temperatures appropriate to O-type stars, i.e., between 27,500 and 55,000 K with a step of 2500 K, seven values of surface gravities from log \( g = 3.0 \) to 4.75 with a step of 0.25 dex, and one microturbulent velocity, \( \xi_t = 10 \) km s\(^{-1} \). Moreover, 10 values of the atmospheric metallicity were considered, \((Z/Z_\odot)_{\text{atm}} = 2, 1, 0.5, 0.2, 0.1, 1/30, 1/50, 1/100, 1/1000, 0.0,\) where \( Z \) is the metal abundance by mass and \( Z_\odot \) is the solar value.

The grid of the BSTAR2006 models contains 16 values of effective temperatures between 15,000 and 30,000 K and a step of 1000 K, 13 values of surface gravities in the range from log \( g = 1.75 \) to 4.75 dex, and a 0.25 dex step. In this case, six values of metallicity were considered,\((Z/Z_\odot)_{\text{atm}} = 2, 1, 0.5, 0.2, 0.1, 0.0,\) and two values of the microturbulent velocities, \( \xi_t = 2 \) km s\(^{-1} \) and \( \xi_t = 10 \) km s\(^{-1} \). Models with \( \xi_t = 10 \) km s\(^{-1} \) were computed for two sets of chemical mixture but only for B-type supergiants (log \( g \leq 3.0 \)). The first mixture was the same as for \( \xi_t = 2 \) km s\(^{-1} \), i.e., the solar composition, and the second one was enriched in helium and nitrogen, and depleted in carbon.

The lower limit for gravity at a given effective temperature was determined approximately by the Eddington limit. The TLUSTY code used by Hubeny & Lanz (1995) becomes unstable near this limit.

3.1. The Passband Fluxes and Their Derivatives

We computed the fluxes, \( F_x \), for the BSTAR2006 NLTE models in 12 commonly used photometric passbands, i.e., in the Strömgren (\( uvby \)) and Johnson–Cousins–Glass (\( UBVRIJHK \)) systems, according to the formula

\[ F_x = \frac{\int_{\lambda_1}^{\lambda_2} F(\lambda) S(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} S(\lambda) d\lambda} , \]  

(7)

where \( S(\lambda) \) is the response function of the passband \( x \), adopted from the Asiago Database on Photometric Systems (Moro & Munari 2000). The integral in Equation (7) is computed in the wavelength range from \( \lambda_1 \) to \( \lambda_2 \), where \( S(\lambda) \) has nonzero values.

We considered the whole grid of effective temperature, \( T_{\text{eff}} \), gravity, log \( g \), metallicity, \((Z/Z_\odot)_{\text{atm}}\), and microturbulent velocity, \( \xi_t \), for the BSTAR2006 models. Table 1 summarizes our results. The file names are coded by values of \((Z/Z_\odot)_{\text{atm}}\) and \( \xi_t \). We have adopted the same designations as the Lanz & Hubeny (2007) loads, i.e., BC, BG, BL, BS, BT, and BZ denote \((Z/Z_\odot)_{\text{atm}} = 2, 1, 0.5, 0.2, 0.1, 0,\) respectively, whereas v2 and v10 correspond to \( \xi_t = 2 \) km s\(^{-1} \) and 10 km s\(^{-1} \), respectively. The index CN marks a model enriched in helium and nitrogen, and depleted in carbon. Each file contains the following columns: line number, effective temperature, \( T_{\text{eff}} \), log g, gravity, \((Z/Z_\odot)_{\text{atm}}\), microturbulent velocity, \( \xi_t \), and the logarithmic flux, log \( F_x \), in the \( uvbyUBVRIJHK \) passbands.
The most important atmospheric parameters in the expression for the brightness variation of a pulsating star are the flux derivatives over effective temperature, $T_{\text{eff}}$, and gravity, $\log g$. In a given photometric passband $x$, they are defined as

$$\alpha_T = \frac{\partial \log F_x}{\partial \log T_{\text{eff}}} \quad \text{and} \quad \alpha_g = \frac{\partial \log F_x}{\partial \log g}.$$ 

Similar to the passband fluxes, the derivatives were computed in the whole range of parameters of the BSTAR2006 models. Names of the derivative tables are given in Table 1. Each file contains the following columns: line number, effective temperature, $T_{\text{eff}}$, logarithm of the surface gravity, $\log g$, metallicity, $(Z/Z_{\odot})_{\text{atm}}$, microturbulent velocity, $\xi_t$, and the flux derivatives over effective temperature, $\alpha_T$, and gravity, $\alpha_g$, in the $uvbyUBVRIJHK$ passbands.

The values of these flux derivatives depend not only on effective temperature and gravity but also on the metallicity, microturbulent velocity, and the departure from LTE in the star’s atmosphere. In Figure 1, we show the NLTE flux derivatives as a function of temperature for two Strömgren passbands $u\gamma$ and three values of gravity, $\log g = 3.5, 4.0, 4.5$. In the left panel we plot the temperature derivative, $\alpha_T$, and in the right one the gravity derivative, $\alpha_g$. We assumed the solar metallicity $(Z/Z_{\odot})_{\text{atm}} = 1$ and $\xi_t = 2$ km s$^{-1}$, respectively.

The wavelength dependence of the flux derivatives is presented in Figure 2. We considered the central wavelengths, $\lambda_c$, of the $UBVRIJHK$ passbands and a model with $T_{\text{eff}} = 20,000$ K and $\log g = 4.0$. The steepest derivatives are in the ultraviolet filter and then the absolute values of $\alpha_T$ and $\alpha_g$ decrease quite rapidly with $\lambda_c$. This is because the early B-type stars emit most of their energy in the UV wavelength range. In Figure 2, we also compare the NLTE derivatives with the LTE ones. The largest difference can be seen again in the $U$ passband.

The effect of NLTE on the flux derivatives as a function of effective temperature for the Strömgren $u\gamma$ passbands is presented in Figure 3. In general, differences between the LTE and NLTE derivatives are relatively small and they increase with effective temperature, especially in the case of $\alpha_g$ in the $u$ passband. In the case of both model atmospheres, the values of $\alpha_T$ in the $u$ passband change only slightly with temperature.

![Figure 1](image1.png)  
Figure 1. NLTE flux derivatives over $\log T_{\text{eff}}$ (the left panel) and $\log g$ (the right panel) as a function of temperature for the Strömgren $u\gamma$ passbands. Three values of $\log g = 3.5, 4.0, 4.5$ were considered. The values of the atmospheric metallicity and microturbulent velocity were assumed as $(Z/Z_{\odot})_{\text{atm}} = 1$ and $\xi_t = 2$ km s$^{-1}$, respectively.

### Table 1

| Fluxes          | Derivatives | Limb-darkening Coefficients | Range of $T_{\text{eff}}$ (K) | Range of $\log g$ (dex) | $(Z/Z_{\odot})_{\text{atm}}$ | $\xi_t$ (km s$^{-1}$) | Mixture |
|-----------------|-------------|-----------------------------|-----------------------------|--------------------------|-----------------------------|---------------------|---------|
| flux_BCv2       | der_BCv2    | LDC_BCv2                    | 15000–30000                 | 1.75–4.75                | 2.0                         | 2                   | GS98    |
| flux_BCv10      | der_BCv10   | LDC_BCv10                   | 15000–30000                 | 1.75–3.00                | 2.0                         | 10                  | GS98    |
| flux_BCv10CN    | der_BCv10CN | LDC_BCv10CN                 | 15000–30000                 | 1.75–3.00                | 2.0                         | 10                  | CN      |
| flux_BGv2       | der_BGv2    | LDC_BGv2                    | 15000–30000                 | 1.75–4.75                | 1.0                         | 2                   | GS98    |
| flux_BGv10      | der_BGv10   | LDC_BGv10                   | 15000–30000                 | 1.75–3.00                | 1.0                         | 10                  | GS98    |
| flux_BGv10CN    | der_BGv10CN | LDC_BGv10CN                 | 15000–30000                 | 1.75–3.00                | 1.0                         | 10                  | CN      |
| flux_BLv2       | der_BLv2    | LDC_BLv2                    | 15000–30000                 | 1.75–4.75                | 0.5                         | 2                   | GS98    |
| flux_BLv10      | der_BLv10   | LDC_BLv10                   | 15000–30000                 | 1.75–3.00                | 0.5                         | 10                  | GS98    |
| flux_BLv10CN    | der_BLv10CN | LDC_BLv10CN                 | 15000–30000                 | 1.75–3.00                | 0.5                         | 10                  | CN      |
| flux_BSv2       | der_BSv2    | LDC_BSv2                    | 15000–30000                 | 1.75–4.75                | 0.2                         | 2                   | GS98    |
| flux_BSv10      | der_BSv10   | LDC_BSv10                   | 15000–30000                 | 1.75–3.00                | 0.2                         | 10                  | GS98    |
| flux_BSv10CN    | der_BSv10CN | LDC_BSv10CN                 | 15000–30000                 | 1.75–3.00                | 0.2                         | 10                  | CN      |
| flux_BTv2       | der_BTv2    | LDC_BTv2                    | 15000–30000                 | 1.75–4.75                | 0.1                         | 1                   | GS98    |
| flux_BTv10      | der_BTv10   | LDC_BTv10                   | 15000–30000                 | 1.75–3.00                | 0.1                         | 10                  | GS98    |
| flux_BTv10CN    | der_BTv10CN | LDC_BTv10CN                 | 15000–30000                 | 1.75–3.00                | 0.1                         | 10                  | CN      |
| flux_BZv2       | der_BZv2    | LDC_BZv2                    | 15000–30000                 | 1.75–4.75                | 0.0                         | 2                   | GS98    |
| flux_BZv10      | der_BZv10   | LDC_BZv10                   | 15000–30000                 | 1.75–3.00                | 0.0                         | 10                  | GS98    |

Note. GS98: chemical mixture by Grevesse & Sauval (1998); CN: chemical mixture enriched in helium and nitrogen, and depleted in carbon.
whereas in the $y$ passband they are larger for higher $T_{\text{eff}}$. The gravity derivatives get steeper with the higher temperature, except $\alpha_{u}^{\gamma}(T_{\text{eff}})$ for NLTE models, which is a decreasing function up to $T_{\text{eff}} \approx 24,000$ K where it then increases.

The influence of the atmospheric metallicity, $(Z/Z_{\odot})_{\text{atm}}$, and microturbulent velocity, $\xi_{t}$, on the flux derivatives is comparable to NLTE effects, as has been already discussed by Szewczuk & Daszyńska-Daszkiewicz (2010).

### 3.2. Limb Darkening

Knowledge of the distribution of the specific intensity over the stellar disk is crucial in many fields of astrophysics. In the case of pulsating stars, the limb-darkening function and its derivatives over effective temperature and gravity are the second atmospheric data needed to compute theoretical values of the photometric amplitudes and phases.

The use of analytical formulae for the specific intensity distribution saves the computation time enormously. The first limb-darkening law was proposed by Milne (1921) in a linear form. With the development of models of stellar atmospheres, it turned out that this approximation has a poor accuracy. Thereafter, Klinglesmith & Sobieski (1970) published a logarithmic law for early-type stars. A quadratic formula for the limb-darkening law was proposed by Manduca at al. (1977) and Wade & Ruciński (1985). For stars hotter than 8500 K, Diáz-Cordovés & Giménez (1992) suggested a square-root law.

However, all these above formulae were not adequate for all types of stars. A more general law was proposed by Claret (2000) in the following nonlinear form:

$$
\frac{I(\lambda, \mu)}{I(\lambda, 1)} = 1 - \sum_{k=1}^{4} a_{k}^{\gamma} (1 - \mu^{k}),
$$

where $I(\lambda, \mu)$ is a specific intensity at the wavelength $\lambda$, $\mu$ is the cosine of the angle between a line of sight and the emergent intensity, and $I(\lambda, 1)$ is the value at the center of the stellar disk. LDCs, $a_{k}^{\gamma}$, are determined to reproduce the model intensity distribution and to conserve the flux with a high accuracy in the whole range of effective temperatures and gravities.

These coefficients were calculated for LTE model atmospheres in many photometric systems and a wide range of effective temperatures, gravities, metallicities, and for several values of the microturbulent velocities (Claret 2000, 2003, 2008; Claret & Hauschildt 2003). A comparison of different numerical methods of computations of LDCs was discussed in detail by Díaz-Cordovés et al. (1995), Claret (2000), Heyrovský (2007), and Claret (2008).

In this paper, we determine nonlinear LDCs for the metal line-blanketed, NLTE, plane-parallel, hydrostatic model atmospheres of Lanz & Hubeny (2007). In the first step, we computed monochromatic specific intensities for 20 equally separated points of $\mu$ in the range of $\langle 0.001, 1 \rangle$ and in the wavelength range of $(2950, 26500)$ Å, for all parameters of the BSTAR2006 atmospheres. All calculations were performed using the SYNSPEC program (Hubeny et al. 1985; Hubeny & Lanz 2000). This code is intended to compute specific intensities and fluxes from the model atmosphere input, which we took from the NLTE...
models described above. Subsequently, for each disk position $\mu$, we integrated intensities over the $uvby$ and $UBVRIJHK$ bands

$$I_x(\mu) = \int_{\mu_a}^{\mu_b} S(\lambda) d\lambda,$$

where $I_x(\mu)$ is the specific intensity in the passband $x$ and $I(\lambda, \mu)$ is the monochromatic specific intensity obtained by SYNOPSPEC. The response function, $S(\lambda)$, was interpolated to the wavelengths of the computed monochromatic specific intensity by the cubic spline method.

The nonlinear LDCs, $a_k^x$, and the center intensity, $I_x(1)$, were determined using the least-squares method by minimizing

$$\chi^2(a_1, a_2, a_3, a_4) = \sum_{i=1}^{20} (I_{\text{LDC}_i} - I_i)^2,$$

where $I_{\text{LDC}_i}$ is the specific intensity computed with the Claret nonlinear law and $I_i$ is the corresponding model intensity at the point $\mu_i$.

Similar to Heyrovský (2007), we evaluated the quality of our fits by the relative residuals

$$\sigma^2 = \frac{\sum_{i=1}^{20} (I_i - I_{\text{LDC}_i})^2}{\sum_{i=1}^{20} I_i^2},$$

whereas the conservation of the flux was controlled by computing the relative flux excess

$$\frac{\Delta F_x}{F_x} = \frac{\int_0^{\mu_1} I_x(\mu) d\mu - \int_0^{\mu_2} I_x(\mu_{\text{LDC}}) d\mu}{\int_0^{\mu_1} I_x(\mu) d\mu}.$$

The limb-darkening law, $h(\mu)$, given in Equation (6), is defined as (Daszyńska-Daszkiewicz et al. 2002)

$$h_x(\mu) = \frac{2\pi L_x(\mu)}{F_x} = \frac{2\pi a_3^x (1 - \mu^{1/2})}{1 - \sum_{k=1}^{4} \frac{k}{k+1} a_k^x}.$$

The names of the LDC files are given Table 1 and they are coded in the same way as the flux and flux derivative tables (cf. Section 3.1). Each file contains the following columns: names of LDCs, $a_k^x$; effective temperature, $T_{\text{eff}}$; gravity, log $g$; metallicity, $(Z/Z_\odot)_{\text{atm}}$; microturbulent velocity, $\xi_\tau$; and values of LDCs in the Strömgren and Johnson–Cousins–Glass photometric passbands, in the order $uvbyUBVRIJHK$.

In Figure 4, we show distributions of the normalized specific intensity in the $UBVRIJHK$ passbands for a model with the following parameters: $T_{\text{eff}} = 20,000$ K, log $g = 4.0$, $\xi_\tau = 2$ km s$^{-1}$, and $(Z/Z_\odot)_{\text{atm}} = 1$. In this figure, we compare the specific intensity computed by means of SYNOPSPEC for 20 equally separated points of $\mu$ (squares) and the fitted limb-darkening function defined by Equation (8) (solid lines). Intensities for passbands $BVI$ were shifted downward by $n \cdot 0.1$, where $n = 1, 2, \ldots, 7$. As we can see, the quality of the limb-darkening fit is very accurate.

To compare the values of the NLTE and LTE intensities, in Figure 5 we plot $I_\nu$ (erg cm$^{-2}$ s$^{-1}$ sr$^{-1}$) as a function of the angle $\mu$ for the Strömgren $uvby$ passbands. The same model as in Figure 4 was considered. The NLTE intensities get lower values, in particular, for the $u$ passband. This is caused by the location of the $u$ filter on the Balmer continuum where the difference in the amount of energy radiated in the LTE and NLTE models is more pronounced than in a region to the right of the Balmer jump, where other filters ($vby$) are defined.

In Figures 6 and 7, we plot the NLTE LDCs, $a_1, a_2, a_3, a_4$, as a function of $T_{\text{eff}}$ in the Strömgren $u$ passband, for different values of gravity, log $g$, and metallicity, $(Z/Z_\odot)_{\text{atm}}$, respectively. Panels of the given figure have the same scale. As we can see, there is a strong dependence of LDC on effective temperature, gravity, and metallicity. The sensitivity on $T_{\text{eff}}$ is stronger for lower values of log $g = 3.5$. The sensitivity to metallicity is similar for all LDCs. The effect of the microturbulent velocity, $\xi_\tau$, not shown here, is comparable to the effect of the metallicity.

Let us now discuss the accuracy of our determinations of the Claret LDCs for the BSTAR2006 NLTE models. In Table 2, we give the average and maximum values of the merit functions, i.e., the relative residuals, $\sigma$, and relative flux excess, $\frac{\Delta F_x}{F_x}$. These values were calculated for the entire range of $T_{\text{eff}}$, i.e., (15000, 30000) K, and log $g$, i.e., (1.75, 4.75) dex, assuming the solar metallicity, $(Z/Z_\odot)_{\text{atm}} = 1$, and the microturbulent velocity of $\xi_\tau = 2$ km s$^{-1}$. We used 20 equally spaced points of $\mu$ instead of 17 unequally spaced points as used in the LTE Kurucz models. Moreover, our NLTE values of $I(\mu)$ were computed closer to the stellar limb, i.e., up to $\mu = 0.001,
whereas the lowest angle of \( \mu \) in the LTE models is 0.01. We considered two sets of passbands: \( uvbyUBVRIJHK \) and \( BVRI \). The second set was used to compare our results with those of Heyrovský (2007) who made computations for the LTE Kurucz atmosphere models using 17 and 11 points of \( \mu \). As we can see the quality of our fit is very good. The average value of \( \sigma \) amounts to 0.119% and for the worst fitted profile we got \( \sigma = 0.432\% \). Our average value of \( \sigma \) is smaller than that of Heyrovský’s by a factor of two. When we limited our analysis to the \( BVRI \) passband, as in Heyrovský (2007), the result is even better. In this case, the average and maximum values of \( \sigma \) amount to 0.089% and 0.251%, respectively. This indicates that with the larger number of equally separated points of \( \mu \) in the fitting procedure one reproduces more accurately the model intensities.

Finally, let us check the conservation of the flux. The average value of \( \left| \frac{\Delta F}{F} \right| \) from our fitting for the NLTE models is slightly worse than in Heyrovský (2007) for the LTE model atmospheres,
but is of the same order of magnitude. The maximum values of \(|\Delta F\) are 9.10 \times 10^{-4} for all 12 passbands and 5.28 \times 10^{-4} for BVRI. The corresponding average values are 14.5 \times 10^{-5} and 9.50 \times 10^{-5}, respectively. For larger values of gravities, the fluxes computed with the Claret limb-darkening law are overestimated whereas for smaller values of log g, fluxes are underestimated. This is caused by a steeper slope of the intensity near the limb for lower values of log g. Consequently, the fitted limb darkening for lower gravities is slightly below model intensities.

4. UNCERTAINTIES IN PHOTOMETRIC PULSATIONAL OBSERVABLES FROM MODEL ATMOSPHERES

The atmospheric metallicity, \((Z/Z_\odot)_{\text{atm}}\), microturbulent velocity, \(\xi_t\), and effects of NLTE are the most important factors which can affect the photometric amplitudes and phases of a pulsating star.

In all comparisons, we used a reference model computed with the mass of \(M = 10 \, M_\odot\), the OPAL opacity tables, the AGSS09 mixture, hydrogen abundance of \(X = 0.7\), metal abundance of \(Z = 0.02\), without overshooting from a convective core, \(\Delta \alpha_{\text{op}} = 0.0\), and NLTE-TLUSTY models of stellar atmospheres with metallicity of \((Z/Z_\odot)_{\text{atm}} = 1.0\) and microturbulent velocity of \(\xi_t = 2 \, \text{km} \, \text{s}^{-1}\).

In Figure 8, we compare photometric observables computed with LTE and NLTE model atmospheres at the same values of metallicity \((Z/Z_\odot)_{\text{atm}} = 1\) and microturbulent velocity \(\xi_t = 2 \, \text{km} \, \text{s}^{-1}\), for the 10 \(M_\odot\) model in the course of its main-sequence evolution. Here, to illustrate our results we chose the \(uv\) Strömgren passbands because this photometric system is very often used in observations of pulsating stars. Moreover, the pulsation amplitude of the B-type pulsators is the highest in the u band and the lowest in the y band and this combination is most useful for mode identification. But we have to stress that to make a unique mode identification and to infer some information on parameters of model and theory at least three passbands are needed (e.g., Daszyńska-Daszkiewicz et al. 2003). In the left panel, we show the amplitude ratios, \(A_u/A_y\), and in the right one the corresponding phase differences, \(\phi_u - \phi_y\), as a function of \(T_{\text{eff}}\) for the first three radial modes: \(p_1\), \(p_2\), and \(p_3\). As we can see, the fundamental mode is most sensitive to the NLTE effects. In general, the NLTE values of the amplitude ratio, \(A_u/A_y\), are smaller than the LTE ones for the hotter models and larger for more evolved models.

Subsequently, we checked the NLTE effect for nonradial modes. In Figure 9, we plot the same photometric observables as in Figure 8, but for the \(\ell = 1\) mode considering one pressure mode, \(p_1\), and one high-order gravity mode, \(g_{14}\). Figure 10 illustrates the same but for two \(\ell = 2\) modes: \(p_1\) and \(g_{22}\). In the case of nonradial modes, the amplitude ratio, \(A_u/A_y\), has larger values for NLTE computations. We can also see different behaviors and values of photometric observables for pressure and high-order gravity modes, especially for the \(\ell = 2\) modes.

The effect of atmospheric metallicity on photometric observables for the radial modes is shown in Figure 11. Here, we considered \((Z/Z_\odot)_{\text{atm}} = 0.1\) in addition to our standard value of \((Z/Z_\odot)_{\text{atm}} = 1.0\). As we can see, the effect of the atmospheric metallicity on photometric observables is relatively small and comparable to the effect of the assumption of LTE in stellar atmosphere models. The effect of the microturbulent velocity, \(\xi_t\), is of the same order as shown by Szewczuk & Daszyńska-Daszkiewicz (2010).

5. UNCERTAINTIES IN PULSATIONAL PHOTOMETRIC OBSERVABLES FROM PULSATION THEORY

Linear nonadiabatic theory of stellar pulsation provides eigenfrequencies, corresponding eigenfunctions, and information on mode excitation. The value of the flux eigenfunction at the level of the photosphere enters the expression for the light variation and is called the \(f\)-parameter, as has been introduced in Section 2. For a given mode frequency, this parameter depends, in general,
1.5
1.55
1.6
1.65
20 21 22 23 24 25
Au/Ay
Teff [kK]
p1

Figure 9. Same as in Figure 8 but for $p_1$ and $g_{14}$ nonradial modes with $\ell = 1$.

1.5
1.55
1.6
1.65
20 21 22 23 24 25
ϕu−ϕy [rad]
Teff [kK]
p1

Figure 10. Same as in Figure 8 for $p_1$ and $g_{22}$ nonradial modes with $\ell = 2$.

1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
2.2
2.4
2.6
2.8
3
3.2
20 21 22 23 24 25
Au/Ay
Teff [kK]
p1
Z/Z⊙atm=1.0
Z/Z⊙atm=0.1

Figure 11. Effect of metallicity on the amplitude ratios, $A_u/A_y$ (left panel) and phase differences, $\phi_u - \phi_y$ (right panel) for 10 $M_\odot$ main-sequence models as a function of $T_{\text{eff}}$ for three radial modes: $p_1$, $p_2$, and $p_3$. NLTE model atmospheres with the microturbulent velocity of $\xi_t = 2 \text{ km s}^{-1}$ and two values of the atmospheric metallicities, $(Z/Z_\odot)_{\text{atm}} = 1$ and 0.1, were used. Linear nonadiabatic pulsations were computed with $Z = 0.02$, $X = 0.7$, OPAL opacity tables, and AGSS09 chemical mixture.

on mean stellar parameters, chemical composition, opacity data, subphotospheric convection, etc. Moreover, in the case of high-order gravity modes, the $f$-parameter also depends on the mode degree, $\ell$ (e.g., Daszyńska-Daszkiewicz & Walczak 2010). This is why the amplitude ratios and phase differences for the $p$- and $g$-modes of a given degree, $\ell$, behave so differently. This is exemplified most pronouncedly in Figure 10.

Pulsations of B-type stars are driven by the $\kappa$ mechanism operating in the “$Z$-bump” layer. The nonadiabatic complex parameter, $f$, is also defined in this subphotospheric layer, therefore its value strongly depends on metal abundances, chemical mixture, and hence opacities (Daszyńska-Daszkiewicz et al. 2005).

In Figure 12, we show the effects of the internal abundances of hydrogen, $X$, and metals, $Z$, on the photometric amplitude ratio, $A_u/A_y$, and phase differences, $\phi_u - \phi_y$, for the first three radial modes $p_1$, $p_2$, and $p_3$. The same models as in Section 4 were considered. Here, we compare computations obtained with $Z = 0.02$ versus $Z = 0.03$ and $X = 0.7$ versus $X = 0.75$. As we can see, the amplitude ratios computed with $Z = 0.03$ are larger than those obtained with $Z = 0.02$, whereas increasing hydrogen abundance, $X$, decreases the amplitude ratios. The value of $\phi_u - \phi_y$ changes by about 0.1 rad at most. These effects are particularly distinct for the radial fundamental mode, $p_1$. Comparing Figure 12 with Figures 8 and 11, we can also see that the effects of $X$ and $Z$ on photometric observables are much more pronounced than the effects of NLTE and $(Z/Z_\odot)_{\text{atm}}$. Please note that for a given mode degree, $\ell$, the scales of the amplitude ratios and phase differences are the same in all figures. A potential of deriving constraints on metallicity for the $\beta$ Cep star HD129929 has been already pointed out by, e.g., Dupret et al. (2004).
1.6
1.8
2
2.2
2.4
2.6
3
3.2
19 20 21 22 23 24 25
Au/Ay
Teff [kK]
p1
p2
p3

X=0.7 Z=0.02
X=0.75 Z=0.02
X=0.7 Z=0.03

-0.2
-0.1
0
0.1
0.2
0.3
19 20 21 22 23 24 25
ϕ
u − ϕ
y [rad]

Teff [kK]
p1
p2
p3

Figure 12. Effects of the internal abundances of hydrogen, X, and metal, Z, on photometric observables for the three radial modes as a function of Teff for the 10 M⊙ main-sequence models. In the left panel, we plot the amplitude ratios Au/Ay and in the right one the corresponding phase differences ϕu − ϕy. The NLTE models with the atmospheric metallicity of (Z/Z⊙)atm = 1 and microturbulent velocity of ξt = 2 km s⁻¹ were used. Linear nonadiabatic pulsations were computed with OPAL opacity tables and AGSS09 chemical mixture.

1.5
1.55
1.6
1.65
19 20 21 22 23 24 25
Au/Ay
Teff [kK]
p1

X=0.7 Z=0.02
X=0.75 Z=0.02
X=0.7 Z=0.03

-0.03
-0.02
-0.01
0
0.01
19 20 21 22 23 24 25
ϕ
u − ϕ
y [rad]

Teff [kK]
p1

Figure 13. Same as in Figure 12 but for two ℓ = 1 modes: p1 and g14.

1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
19 20 21 22 23 24 25
Au/Ay
Teff [kK]
p1

X=0.7 Z=0.02
X=0.75 Z=0.02
X=0.7 Z=0.03

-0.4
-0.3
-0.2
-0.1
0
19 20 21 22 23 24 25
ϕ
u − ϕ
y [rad]

Teff [kK]
p1

Figure 14. Same as in Figure 12 but for two ℓ = 2 modes: p1 and g22.

The effects of hydrogen and metallicity abundance for the ℓ = 1, p1, and g14 modes are shown in Figure 13, and for the ℓ = 2, p1, and g22 modes in Figure 14. As one can see, in the case of the nonradial modes these parameters have smaller influence on the photometric observables than in the case of radial modes. The amplitude ratio, A_u/A_y, for the ℓ = 1 mode increases with increasing abundances of both Z and X. This is true also for the ℓ = 2, p1 mode, whereas the amplitude ratio of the ℓ = 2, g22 mode does not behave monotonically. The important result is that in the case of nonradial modes the effects of X and Z are considerably smaller than the NLTE effects (cf. Figures 9 and 10), which is opposite to the radial mode case.

Finally, we evaluated effects of the opacity data and chemical mixture. In Figure 15, we show the photometric observables for the radial modes computed with two sources of the opacity tables and two chemical mixtures. Here, we compared computations obtained with the OPAL versus OP tables and the GN93 versus AGSS09 mixture. Again, the largest effects are for the p1 mode in both the amplitude ratios and phase differences. The amplitude ratios computed with GN93 are smaller than those obtained with AGSS09, because of the relatively higher abundance of iron in AGSS09. Also computations with the OP data give smaller values of A_u/A_y compared to those obtained with the OPAL data. Moreover, the maximum of the OP amplitude ratio is shifted toward cooler effective temperatures. This is connected with the location of the Z-bump layer, which occurs at a slightly higher temperature in the OP data. The use of different opacity data changes the value of ϕ_u − ϕ_y by about 0.2 rad at most. The chemical mixture has a rather minor effect on phase differences. We can also see that these input data affect photometric observables of radial modes far more than the atmospheric parameters (cf. Figures 8 and 11).
Effects of the opacity tables and chemical mixtures for nonradial modes with \( \ell = 1 \) and 2 are shown in Figures 16 and 17, respectively. These data affect the \( \ell = 1 \) modes very subtly and are far less important than the NLTE effects. In the case of the \( \ell = 2 \) modes, effects of opacities and mixture are comparable to the NLTE effects.

As we could see, generally, in the case of radial modes, effects of pulsational parameters on photometric observables are much larger than effects of atmospheric parameters discussed in the previous section. The opposite is true for nonradial modes which are more sensitive to model atmospheres, in particular to the NLTE effects.

6. CONCLUSIONS AND REMARKS

We have presented a comprehensive overview of possible sources of uncertainties in theoretical values of the photometric amplitudes and phases of early B-type pulsators. These data are of particular importance because they serve as tools for mode identification in the case of main-sequence pulsators, and can also yield valuable constraints on mean stellar parameters and physics.

The uncertainties are embedded in stellar model atmospheres and nonadiabatic theory of stellar pulsation. The atmospheric input consists of the flux derivatives over \( T_{\text{eff}} \) and \( \log g \), and limb darkening and its derivatives, in photometric passbands. These quantities are sensitive to the atmospheric metallicity and microturbulent velocity, as well as to the departure from the LTE approximation. From pulsation computations, we get the nonadiabatic complex parameter, \( f \), whose value depends on chemical composition, opacities, and subphotospheric convection, if present. If the \( f \)-parameter can be derived from observations, then we get an extra seismic probe by means of which we can test this input physics.

We began with computations of tables with various quantities, needed to evaluate the light variation in a given photometric...
band, for NLTE-TLUSTY model atmospheres. These tables include data on the passband flux, flux derivatives over effective temperature and gravity, and the nonlinear LDCs in the 12 most popular passbands, uvbyUBVRIJK. All these data are publicly available and can be retrieved from the Wrocław HELAS Web site.

Then, we studied effects of these input parameters on the photometric observables, i.e., amplitude ratios and phase differences, of the 10 $M_\odot$ main-sequence models for the low degree modes with $\ell = 0, 1, 2$. In particular, the effect of NLTE model atmospheres was studied for the first time. This effect is comparable to the influence of atmospheric metallicity and microturbulent velocity. Subsequently, we compared the effect of atmospheric parameters with the effect of pulsational parameters. This comparison showed that the photometric observables of the radial modes are by far more sensitive to the pulsation input. In turn, in the case of nonradial modes the NLTE effect becomes more important, especially in the case of the $\ell = 1$ modes.

A complete seismic model should reproduce not only pulsational frequencies but also the observed values of the photometric amplitude and phases, which can be translated into the empirical values of the $f$-parameter. Our studies show that such a comprehensive approach should take into account all possible inaccuracies.

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