On a criterion of incipient motion and entrainment into suspension of a particle from cuttings bed in shear flow of non-Newtonian fluid

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Abstract. Solids transport is a major issue in high angle wells. Bed-load forms by sediment while transport and accompanied by intermittent contact with stream-bed by rolling, sliding and bouncing. The study presents the results of a numerical simulation of a laminar steady-state flow around a particle at rest and in free motion in a shear flow of Herschel–Bulkley fluid. The simulation was performed using the OpenFOAM open-source CFD package. A criterion for particle incipient motion and entrainment into suspension from cuttings bed (Shields criteria) based on forces and torques balance is discussed. Deflection of the fluid parameters from the ones of Newtonian fluid leads to decreasing of the drag and lift forces and the hydrodynamic moment. Thus, the critical shear stress (Shields parameter) for the considered non-Newtonian fluid must be greater than the one for a Newtonian fluid.

1. Introduction
Moving of spherical particles along a plane in a shear flow or fluid at rest is a common phenomenon in nature as well as in technical solutions. It involves river drifts and drilling mud transporting cuttings out of a well. Both processes can be described using this simplified model, which requires determination of hydrodynamic forces and moments affecting a particle near the wall in a viscous fluid.

The flows heterogeneity has a significant effect on the hydrodynamic processes around a moving particle, such as the drag force. At the same time, it creates a force perpendicular to the flow known as lift force. Its value is usually small if compared to the drag force, but it still plays an important role both in sedimentation and sediment spreading. When a particle approaches a wall, the lift force increases dramatically and becomes comparable to the drag force \([1]\). It occurs due to three main mechanisms such as a shear flow, sphere rotation, and presence of the wall \([2]\).

One of the first publications devoted to flow around a sphere near the wall was written by Bretherton \([3]\) & Goldman et al. \([4, 5]\). Both studies relied on the Stokes approximation. Goldman et al. obtained an analytical solution for the forces and moment affecting a spherical particle near the wall. The authors considered two cases: a sphere at rest in a moving shear flow and a moving rotating sphere in a fluid flow at rest. For the last case, it was shown that the drag force and moment grew logarithmically as the distance to wall reduced. Due to the Stokes approximation, the lift force value was equal to zero. Krishnan and Leighton \([2]\) demonstrated...
that at small Reynolds numbers, the forces affecting a rotating sphere moving along the wall could be presented as a sum of six components. The first three are the forces caused by the shear flow, sphere motion, and its rotation. The remaining three are the forces produced by the interaction of the components listed above. This theory, including the lift-off criterion, was experimentally confirmed by King and Leighton [6]. Accounting for the particle roughness allowed avoiding a logarithmic singularity when compared against the theoretical results.

However, the previously mentioned studies concerned small Reynolds numbers. If moderate Reynolds numbers are considered, the problem of particle and bubble motions at a finite distance along the wall in a fluid at rest was investigated by Takemura [7] and Zeng et al. [8] respectively. Zeng et al. [9] studied the forces and moment affecting a sphere in a shear flow near the wall up to the instant the sphere touching the wall.

Lee and Balachandar [10] developed correlation ratios to describe both drag and lift forces at moderate Reynolds numbers for a sphere moving along a plane. They based the correlations on the theoretical study by Krishnan and Leighton [2], summing the six components, as well as on the results by Zeng [8, 9] for a particle moving along the wall.

In one of the most recent publications Lee and Balachandar [11] generalized the results obtained in the previous studies devoted to the forces affecting a spherical particle, and modeled the free steady motion of a particle on a plane in a shear flow. The steady flow in this study was obtained through modeling of an unsteady flow under the assumption of no friction force between the sphere and the plane while the drag force and moment affecting the particle were equal to zero. Apart from the lift force, the authors obtained the particles free-motion velocity, its rotation rate and the time its motion becomes steady. In their further publications [12] the authors concentrated on a spherical particle moving along a rough surface in a turbulent flow. Thus, for Newtonian fluid at moderate Reynolds numbers, the problem of a particle moving on a plane or near to it has been studied sufficiently.

Bocharov and Kushnir [13] investigated the forces affecting a spherical particle near the wall in a shear flow of a power-law fluid. It was demonstrated that at a distance to the wall exceeding three diameters of the sphere and the Reynolds numbers less than 50 (determined from flow velocities opposite the particles), the lift force comprised less than 5% of the drag force. In case, the distance to the wall was less than one diameter, the lift force, trying to push the particle away from the wall, increased as the distance got smaller at constant Reynolds numbers and reached 25-30% of the drag force.

The current publication presents the results of parametric study of the motion of a spherical particle moving along a plane with and without slip in a shear flow of the Herschel–Bulkley fluid.

2. Problem statement

The study considered the steady rolling of a spherical particle with the diameter d and the velocity $v_p$ moving along a plane the (Figure 1) in a shear flow of non-Newtonian (Herschel–Bulkley) fluid. It was assumed that the sphere moved in the same direction as the flow. There was no friction between the sphere and the wall. The origin of Cartesian coordinates matched the spheres center, where the OX axis correlated with the flow, the OY one- with the planes normal, and $\omega$ denoted the rotational rate of the particle along the OZ axis. The fluid flow was described by a steady system of the Navier-Stokes equations:

$$\rho u \cdot \nabla u = -\nabla p + \nabla \cdot \tau, \nabla u = 0,$$

where the viscous stress tensor $\tau$ depended on the local shear rate $\dot{\gamma}$:

$$\tau = \begin{cases} \frac{k + n}{\gamma} \dot{\gamma} & \text{at } \dot{\gamma} > 0 \\ 0 & \text{at } \dot{\gamma} = 0 \end{cases},$$

thus the effective viscosity was $\mu = \begin{cases} \frac{k + n}{\gamma} & \text{at } \dot{\gamma} > 0 \\ \infty & \text{at } \dot{\gamma} = 0 \end{cases}$. 

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Here $\dot{\gamma} = 0.5 \left( \nabla u + \nabla u^T \right)$ denoted the strain velocity tensor, $-\text{the second invariant of the strain velocity tensor of shear rate}$, $n$ - the power index, $\tau_0$ - the yield stress of fluid. The boundary conditions are:

$$u = \begin{cases} r \times \omega - v_p e_x & \text{at } r = d/2 \\ G (y + d/2) e_x & \text{at } x \to -\infty \end{cases}$$

where $r$ denoted a radius vector from the center of the sphere, $\omega = (0, 0, -\omega)$ - the vector of rotational particle velocity, $G$ - the flow shear rate. The characteristic length was the diameter $d$, and the characteristic velocity - the face velocity $v = Gd/2$. Hence, the flow could be characterized by the following set of dimensionless parameters such as $Re_s = G^2 d^2 \rho / (2(kG^n + \tau_0))$ that denoted shear -Reynolds number with the characteristic Herschel-Bulkley viscosity or $ResPL = G^2 - nd^2 \rho / 2k$ - with the characteristic power-law viscosity, $V_p = v_p / v$ that is the spheres dimensionless velocity, $n$ - the power index, $Bn = ?0 / kGn$ - the Bingham number, and $\Omega = \omega / G$ - the spheres dimensionless rotational rate.

In the calculations, the Reynolds number varied in the range $0.1 \leq Re_s \leq 200$. Two cases were considered: particle at rest ($\nabla u = 0$, $\Omega = 0$) and free motion when the terminal velocity $V_{pss}$ and the terminal rotating rate $\Omega_{pss}$ were found iteratively to satisfy the equalities $F_d = 0$, $M = 0$. The power index $n$ varied in the range $0.4 \leq n \leq 1$ and the Bingham number $0 \leq Bn \leq 10$. The drag force $F_d$, the lift force $F_L$ and the hydrodynamic torque $M$ were calculated by integration of the friction and pressure over the sphere surface area $S$.

For forces and torque analyze dimensionless parameters were used: $C_d = 4F_d / \pi \rho G^2 d^2$ to denote the drag force coefficient, $Y = C_d Re_s / 24$ the normalized drag force coefficient, $C_L = 4F_L / \pi \rho G^2 d^2$ the lift force coefficient, $C_M = 8M / \pi \rho G^2 d^3$ the hydrodynamic torque coefficient.

3. Numerical Method and Verification

To avoid computational problems, as recommended in [6], a gap equal to $d/100$ between the sphere and the wall, was considered for in all the further studies modeling similar problems. Since we modeled a laminar flow with no symmetry plane ($OXOY$), only half of the flow was considered. The Navier-Stokes equations were solved by shifting the particles reference system and using a stationary computational mesh.

The computational volume was a rectangular 3D parallelepiped, whose surface was isomorphically reflected onto a hemisphere that transferred onto the spherical surface of the particle, forming a structured hexahedral computational grid. The distance from the sphere to the distant boundaries of the computational domain was equal to twenty-five diameters ($25d$) of the sphere. The maximum grid size on the sphere in the radial direction was $\pi d/100$. The size of the computation cell on the spheres surface did not exceed . The cell sizes had been selected
Table 1. Comparison of force characteristics against those in [9].

| $Re_s$ value | Zeng, 2009 [9] present | discrepancy, % |
|--------------|------------------------|---------------|
| 2            | $C_d$ 2.349 · 10       | 2.29          |
| 2            | $C_L$ 2.653             | 3.09          |
| 2            | $C_M$ 8.005             | 2.40          |
| 10           | $C_d$ 6.445             | 0.37          |
| 10           | $C_L$ 1.305             | 0.90          |
| 10           | $C_M$ 1.778             | 0.52          |
| 200          | $C_d$ 1.381             | 2.79          |
| 200          | $C_L$ 3.384 · 10⁻¹      | 4.13          |
| 200          | $C_M$ 1.044 · 10⁻¹      | 2.23          |

to make it possible for the grid to condense by a factor of two in all the directions without significant changes in the forces and moment (< 1%) affecting the sphere.

The numerical solution of the Navie–Stokes equations was found using the OpenFOAM 3.0.1 open-source hydrodynamic package [14], whose solver (simpleFoam) was equipped with the SIMPLE-C algorithm. For discretization of the convective terms, a linear upwind scheme was applied. The coefficient of the lift force, drag force and moment of forces for $V_p = 0$ published by Zeng et al. [9] was used to test the computational mesh and the numerical method. For modeling, Zeng et al. [9] had used NEK5000, an open–source hydrodynamic package based on the spectral method providing high level of computational accuracy. The table below contains the values and their discrepancy, which does not exceed 5%.

4. Results and Discussion

4.1. Forces and Torque acting on a particle

Numerical simulation showed that the growing power law index $n$ led to decreasing of drag force coefficient $Y$ (Figure 2a). This, evidently, occurred because of the apparent viscosity decrease. Quite the reverse, the increasing Bingham number $Bn$ led to increasing of $Y$, because of the apparent viscosity increase.

![Figure 2a](image1)

![Figure 2b](image2)

Figure 2. Drag force and lift force coefficient vs power–law Reynolds number $Re_{sPL}$.

When deriving the coefficient $Y$, we intentionally used the Reynolds number without taking


into account the yield stress. The graph demonstrates that all the coefficient curves show the same asymptote as the Reynolds number increases independent of the Bingham exponent (Figure 2a). This holds true for the Herschel–Bulkley fluid, whose asymptotic inclination depends on the power index \( n \). At the same time, despite the significant dependence of the drag coefficient on the fluid rheology, the lift force coefficient changes insignificantly (Figure 2b).

![Figure 3. Torque coefficient behavior vs Reynolds number \( Re_s \).](image)

It was convenient to consider the moment of hydrodynamic forces as one multiplied by the Reynolds number \( Re_s \). This expression does not change much for a wide range of the Reynolds numbers. Figure 3 shows that as the Reynolds number increases, the moment of forces may reduce significantly, becoming negative for non-Newtonian fluids. In such a way, the moment at the large Reynolds numbers may help the particle stay at rest. A very interesting effect observed here is that the values of \( C_M Re_s \) are constant for both low and high values of the Reynolds numbers.

### 4.2. Free rolling particle

Lee et al. [11] consider the equilibrium rolling with a slip of a particle moved by a shear flow. The equilibrium condition here is the drag force \( F_d \) and moment equal \( M \) to zero. In our study, a similar problem statement was considered by making changes in the numerical algorithm. Calculating the solution we found the spheres equilibrium (terminal) velocity \( V_{pss} (C_d = 0) \) and rotation rate \( \Omega_{pss} (M = 0) \). In the case of free rotation, the equilibrium velocity \( V_{pss} \) reduces, but it still may exceed one Figure 4a). As for the case without slip, the equilibrium velocity grows monotonically as the Reynolds number, \( n \) and \( Bn \) deviate from Newtonian values.

The particles free rotation rate does not change much in the Newtonian fluid, which is impossible to say about the power–law fluid where the increased limiting stress leads to a reduced dependence between the rotation rate and the Reynolds number (Figure 4b).

The lift force coefficient monotonously decreases as the rheological parameters deviate from Newtonian ones (Figure 5). However, lines for different fluid types are placed not as close as in the case in Figure 2b. For a free rolling particle in a Newtonian fluid, the lift force coefficient deviates by 20% in comparison to the particle at rest (Figure 5b). When fluid properties deviate from the Newtonian values a significant reduction of this ratio is observed. There are two opposite factors that determine the lift force in a shear flow of a Newtonian fluid. Increase in the velocity leads to decrease of the lift force and the rotation leads to its increase. But for a non-Newtonian fluid increase of rotation leads to a decrease of apparent viscosity and to significant losses in the lift force.
4.3. Critical shear stress
Using forces and torque we could formulate a criterion for incipient motion and particle lift-off. Lets consider it for a case with particles of the same size. In Figure 6 you can see the scheme

Figure 6. Sphere on a cuttings bed.
that we used to obtain force and torques balance equations for a spherical particle on a sediment bed. Consider the equilibrium of force moments relative to point A:

\[ M + F_d \delta_v + F_L \delta_h - F_A \delta_h - mg \delta_h \geq 0 \]

Figure 7. The critical Shields parameter vs Figure 8. Relative contribution of different the Reynolds number \( Re_s \) and fluid properties. factors in incipient motion criterion.

For the spheres of the same size \( \alpha \approx 60^\circ \). Now, an equation for the Shields number can be obtained, which is derived as a ratio of the shear stress at the top of the bed to the lift force acting on the particle \( \tau_I = \tilde{\tau}_{wall}/(\rho_f - \rho_p)gd \) and previous parameters \( C_d, C_L, C_M \) and \( Re_s \):

\[ \tau_I = \frac{8}{3\sqrt{Re_s}} \left( C_d + \sqrt{3} C_L + 2C_M \right)^{-1}. \]

Let us consider the contribution of each factor \( (C_d, C_L \text{ and } C_M) \) in as a function of \( \tau_I \). From Figure 8 we can say that for all types of fluids and a wide range of the Reynolds numbers the main input is given by the drag force \( C_d \). The input of the lift force is negligible for small \( Re_s \), but with growing \( Re_s \) the impact of \( C_L \) stands to be compatible with \( C_d \). The torque \( C_M \) decreases as \( Re_s \) increases and its input to the criterion becomes negligible at the moderate Reynolds numbers.

The next phase of particle motion is its lift-off from the sediment bed. Neglecting the interaction with other particles, lets consider the lift force for a single particle while free motion. From Newton’s law, we derive:

\[ F_L - mg \geq 0 \]

In its dimensionless form this inequality produces a criterion for contact loss in terms of Shields parameter \( \tau_L \):

\[ \tau_L = \frac{8}{3C_L Re_s} \]

The effects of suggested criteria are shown on Figure 7. The domain of shear stresses \( \tau \) on the wall bed now divided into three sub-domains: stationary sludge settlings \( (\tau < \tau_I) \), moving sludge settlings \( (\tau_I \leq \tau < \tau_L) \) and stirring-up (roiling) \( (\tau \geq \tau_L) \).
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