Noncommutative gravity and the relevance of the $\theta$-constant deformation

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Abstract – The breaking of diffeomorphism invariance in the Moyal-Weyl ($\theta$-constant) noncommutative (NC) space-time is a well-known and a long-standing problem. It makes the construction of NC gravity models and interpretation of their results very difficult. In order to solve this problem in this letter we construct a NC gravity action based on the NC $SO(2,3) \hat{\star}$ gauge group and the Seiberg-Witten expansion. The NC equations of motion show that the noncommutativity plays the role of a source for the curvature and/or torsion. Finally, we calculate the NC corrections to the Minkowski space-time and show that in the presence of noncommutativity the Minkowski space-time becomes curved, but remains torsion-free. More importantly, we show that the coordinate system we are using is given by the Fermi normal coordinates; the NC deformation is constant in this particular reference system. The breaking of diffeomorphism invariance is understood as a consequence of working in a preferred reference system. In an arbitrary reference system, the NC deformation is obtained by an appropriate coordinate transformation.

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NC gravity, general. – Formulated at the beginning of the XX century, general relativity and quantum mechanics continue to describe remarkably well physics at different scales: the cosmological scale (from scales of planets to the scale of Universe) and the scale of atoms, respectively. Predictions of both theories have been confirmed by many experiments, the most recent being the detection of gravitation waves [1]. However, a theory that could unite these two theories and provide a description of gravity at quantum scales is still missing. There are different proposals in the literature: string theory, loop quantum gravity, noncommutative (NC) geometry, dynamical triangulation, . . . . None of these proposals is complete, mostly due to the lack of experimental tests.

NC gravity theories rely on the notion of noncommutative space-time and/or noncommutative geometry. NC geometry provides a generalization of the description of space-time by smooth manifolds. Introducing NC relations between coordinates naturally leads to uncertainty relations between coordinates [2] and introduces a “foamy, grainy” structure instead of the usual smooth manifolds. In this way one hopes to solve the problem of divergences in quantum field theories and the problem of singularities in general relativity.

During the last fifteen years, different models of NC gravity theories were considered. We just mention a few different approaches: NC gravity via the twist approach [3], emergent NC gravity via the matrix models [4], fuzzy space gravity models and DFR models [5] and NC gravity via the NC gauge theory and the Seiberg-Witten (SW) map [6,7].

In this letter, we study one particular model of NC gravity as a NC gauge theory. We work with the Moyal-Weyl (canonical, $\theta$-constant) noncommutativity. The main advantage of this NC space-time is that, due to the constant noncommutativity, it is relatively easy to study various physics problems. On the other hand, by introducing a constant NC parameter we explicitly break the diffeomorphism symmetry. This is a well-known problem and there are different attempts to deal with it [8]. The $\theta$-constant deformation is naturally defined for an inertial observer. Therefore, it is not possible to apply the $\theta$-constant deformation for GR solutions in arbitrary coordinates. Here we present a solution of this problem by uniting methods of field theory and gravity. We construct a NC gravity model based on the NC gauge theory approach. The gauge group is chosen to be the $SO(2,3)$, group. Calculating NC gravity equations of motion, we show that noncommutativity plays the role of a source for the curvature and torsion. That is, given a flat/torsion-free space-time, noncommutativity induces nonzero curvature/torsion on this space-time. This result is not completely new, it was...
also discussed in [9] in a different approach to NC gravity. Especially, starting from the Minkowski space-time as a solution of the commutative vacuum Einstein equations, the corrections induced by our NC gravity model lead to a space-time with a constant scalar curvature, (A)dS-like space-time.

As we mentioned earlier, our model breaks the commutative diffeomorphism symmetry. This symmetry breaking can be understood in terms of the preferred coordinate system we are using. Looking more carefully, we find that this particular coordinate system is given by the Fermi normal coordinates (FNC). These are coordinates of an inertial observer moving along a geodesic in curved space-time. In particular, this is the observer who measures the constant noncommutativity. We also suggest that FNC represent the preferred coordinate system in which one should study NC gravity effects.

**Commutative model.** We start from a commutative gravity models based on the \( SO(2,3) \) gauge symmetry. Models of this type appeared in the 1970s [10] and were used in the construction of supergravity theories. The gauge field and the field strength tensor are defined as

\[
\omega_{\mu} = \frac{1}{2} \omega_{\mu}^{AB} M_{AB} = \frac{1}{4} \omega_{a}^{ab} \kappa_{a}, \quad (1)
\]

\[
F_{\mu\nu} = \frac{1}{2} F_{\mu}^{AB} M_{AB} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu} - i[\omega_{\mu}, \omega_{\nu}],
\]

\[
F_{\mu}^{AB} M_{AB} = \partial_{\mu} \omega_{\nu}^{a} - \partial_{\nu} \omega_{\mu}^{a} - i[\omega_{\mu}, \omega_{\nu}],
\]

\[
(2)
\]

with \( \omega_{\mu}^{a} = \omega_{a}^{5} \) and \( T_{\mu}^{a} = I F_{\mu}^{a5} \). The generators \( M_{AB} \) close the \( SO(2,3) \) algebra,

\[
[M_{AB}, M_{CD}] = i(\eta_{AD} M_{BC} + \eta_{BC} M_{AD} - \eta_{AC} M_{BD} - \eta_{BD} M_{AC}),
\]

\[
(3)
\]

The quantities in (2) are given by

\[
R_{\mu}^{ab} = \partial_{\mu} \omega_{\nu}^{a} - \partial_{\nu} \omega_{\mu}^{a} + \omega_{a}^{b} \quad \eta_{\mu \nu} \omega_{\nu}^{a} - \omega_{a}^{b} \omega_{\mu}^{b} - \omega_{a}^{b} \omega_{\mu}^{b} \gamma_{a},
\]

\[
T_{\mu}^{a} = \nabla_{\mu} \epsilon_{a}^{6} - \nabla_{\nu} \epsilon_{a}^{6},
\]

\[
\gamma_{a} = \partial_{\mu} \epsilon_{a}^{6} + \omega_{a}^{b} \epsilon_{b}^{6}.
\]

More details about the \( SO(2,3) \) algebra and the calculations that follow can be found in [11]. The commutative action that we are going to generalize to the NC setting is a sum of three terms:

\[
S = c_{1} S_{1} + c_{2} S_{2} + c_{3} S_{3}
\]

\[
S_{1} = \frac{i}{16 \pi G_{N}} \int d^{4}x e^{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma},
\]

\[
(4)
\]

\[
S_{2} = \frac{1}{16 \pi G_{N}} \int d^{4}x e^{\mu \nu \rho \sigma} F_{\mu \nu} D_{\rho} \omega_{\sigma}, \quad (i \gamma_{a} \gamma_{5} \gamma_{5})
\]

\[
(5)
\]

\[
S_{3} = \frac{-i}{12 \pi G_{N} l} \int d^{4}x e^{\mu \nu \rho \sigma} D_{\mu} \phi D_{\rho} \phi D_{\nu} \phi D_{\sigma} \phi,
\]

\[
(6)
\]

with an additional scalar field \( \phi = \Phi_{A} \Gamma_{A}, \Gamma_{A} = (i \gamma_{a} \gamma_{5} \gamma_{5}) \) (see footnote 1) transforming in the adjoint representation of \( SO(2,3) \) and \( D_{\mu} \phi = \partial_{\mu} \phi - i [\omega_{\mu}, \phi] \). The field \( \phi \) is used to break the \( SO(2,3) \) gauge symmetry of the action to the \( SO(1,3) \) (local Lorentz) gauge symmetry. The action (4) was introduced by Stelle and West and also analyzed by Townsend and by MacDowell and Mansouri in their papers [10]. Actions (5) and (6) were introduced by Wilczek in [12] as different possibilities to write \( SO(2,3) \) gauge-invariant models.

The symmetry breaking is done by choosing a particular form of the field \( \phi \), namely \( \phi_{a}^{\mu} = 0, \phi^{5} = 1 \). The symmetry breaking can also be spontaneous, for more details see [12]. After the symmetry breaking the resulting commutative action is written as

\[
S = c_{1} S_{1} + c_{2} S_{2} + c_{3} S_{3}
\]

\[
= -\frac{1}{16 \pi G_{N}} \int d^{4}x \left( \frac{1}{16} \epsilon^{\mu \nu \rho \sigma} e_{abcd} R_{\mu \nu}^{ab} R_{\rho \sigma}^{cd} + \sqrt{-g} \left( (c_{1} + c_{2}) R - \frac{6}{\ell^{2}} (c_{1} + 2c_{2} + 3c_{3}) \right) \right),
\]

\[
(7)
\]

with \( \sqrt{-g} = \det e_{\mu}^{a} \) and \( R = R_{\mu \nu}^{ab} e_{\mu}^{a} e_{\nu}^{b} \). The constants \( c_{1}, c_{2} \) and \( c_{3} \) are arbitrary and, as we shall see later, can be determined from additional conditions.

The first term in the action (7) is the topological Gauss-Bonnet term. It does not contribute to the equations of motion and we shall not consider it further. The other two terms are the Einstein-Hilbert term and the cosmological constant term. Note that, depending on the sign of the coefficient \( c_{1} + 2c_{2} + 3c_{3} \), the cosmological constant can be positive, negative or zero. The spin connection \( \omega_{\mu}^{ab} \) and the vierbeins \( e_{\mu}^{a} \) are independent fields in the model. Varying the action (7) with respect to these fields gives two equations of motion. The variation with respect to \( \omega_{\mu}^{ab} \) gives the vanishing torsion equation. This equation enables to express the spin connection in terms of the vierbeins. The variation of the action (7) with respect to \( e_{\mu}^{a} \) gives the Einstein equation, written in terms of the vierbeins. Finally, we conclude that after the symmetry breaking and solving for the spin connection in terms of vierbeins we obtain the usual GR: Einstein equations with an arbitrary cosmological constant.

**Noncommutative model and the low-energy expansion.** We wish to generalize the commutative model (7) to the noncommutative space-time. We work in the canonically deformed \( (\theta\text{-constant noncommutative}) \) space-time and use the approach of the deformation quantization. The noncommutative fields are functions of commuting coordinates while their multiplication is given by the noncommutative (but associative) \( \ast \)-product. In the case of the \( \theta\text{-constant NC space-time, the} \ast \)-product is the Moyal-Weyl product given by

\[
f \ast g = e^{\frac{1}{2} \theta^{\alpha \beta} \frac{\partial f}{\partial \theta^{\alpha}} \frac{\partial g}{\partial \theta^{\beta}}} f(x) g(y) |_{y \rightarrow x}
\]

\[
= f \ast g + i \frac{1}{2} \theta^{\alpha \beta} \left( \partial_{\alpha} f \partial_{\beta} g \right)
\]

\[
- \frac{1}{8} \theta^{\alpha \beta \gamma \delta} \left( \partial_{\alpha} \partial_{\beta} f \partial_{\gamma} \partial_{\delta} g \right) + \ldots .
\]

\[
(8)
\]
The deformation parameter is a constant antisymmetric matrix $\theta^{\alpha\beta}$. The $*$-product reduces to the usual commutative multiplication in the zeroth order in the deformation parameter and has higher-order corrections that lead to $f \ast g \neq g \ast f$.

The concept of gauge symmetry can be generalized to the NC setting. We use the enveloping algebra approach and the Seiberg-Witten map [13]. The noncommutative $SO(2,3)$ gauge transformation of the NC gauge field $\hat{\omega}_\mu$ is defined as

$$\delta^\mu \hat{\omega}_\mu = \partial_\mu \hat{\Lambda}_\epsilon + i[\hat{\Lambda}_\epsilon \ast \hat{\omega}_\mu],$$

with the NC gauge parameter $\hat{\Lambda}_\epsilon$. It can be shown that the algebra of NC gauge transformations closes in the enveloping algebra only\(^2\). Since the enveloping algebra is infinite dimensional, it would seem that the NC gauge theory should have infinitely many degrees of freedom. The idea of the Seiberg-Witten map is that the NC gauge transformations are induced by the commutative gauge transformations, $\delta_\epsilon \rightarrow \delta^\epsilon$. Then

$$\hat{\omega}_\mu = \hat{\omega}_\mu (\omega_\mu), \quad \hat{\phi} = \hat{\phi} (\phi, \omega_\mu),$$

that is, the NC fields are functions of the corresponding commutative fields. The method for solving the SW map equations is introduced in [14,15] and the explicit solutions for the $SO(2,3)$ gauge group are given in [16].

The important result of this approach is that there are no new degrees of freedom but new interaction terms appear. The relations between different (new) interaction terms are fixed by the SW map.

The NC generalization of (7) is given by

$$S_{\text{NC}} = c_1 S_{\text{NC}} + c_2 S_{2\text{NC}} + c_3 S_{3\text{NC}},$$

with

$$S_{1\text{NC}} = -\frac{i}{64\pi G_N} \text{Tr} \int d^4x e^{\mu\nu\rho\sigma} F_{\mu\nu} \ast F_{\rho\sigma} \ast \hat{\phi},$$

$$S_{2\text{NC}} = \frac{1}{128\pi G_N} \int d^4x e^{\mu\nu\rho\sigma} \hat{F}_{\mu\nu} \ast \hat{D}_\rho \hat{\phi} \ast \hat{D}_\sigma \hat{\phi} + \text{c.c.},$$

$$S_{3\text{NC}} = -\frac{i}{128\pi G_N} \int d^4x e^{\mu\nu\rho\sigma} D_\mu \hat{\phi} \ast D_\nu \hat{\phi} \ast D_\rho \hat{\phi} \ast D_\sigma \hat{\phi} + \text{c.c.}.$$}

The action is invariant under the NC $SO(2,3)$ gauge symmetry and the SW map guarantees that after the expansion it will be invariant under the commutative $SO(2,3)$ gauge symmetry. Using the SW map solutions for the fields $\hat{F}_{\mu\nu}$ and $\hat{\phi}$ [16] and the $*$-product (8), the action (11) is expanded in orders of the NC parameter $\theta^{\alpha\beta}$. The expansion is done around the commutative vacuum $\phi^5 = l$.

\(^2\)Only in the case of $U(N)$ in the fundamental representation the NC gauge transformations still close in the corresponding Lie algebra.

The second-order correction is the first non-vanishing correction. The calculation is long, some details and the result can be found in [11]. The result, being long and complicated is not so easy to analyze. Nevertheless, it enables to study different sectors of our model: high curvature or low curvature, big, small or vanishing cosmological constant, etc. In this letter we analyze the low-energy sector of the model. Therefore, in the expanded NC gravity action we keep only terms that are of zeroth, first and second-order in the derivatives of the vierbeins $e^a_\mu$. The resulting action is given by

$$S_{\text{NC}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R - \frac{6}{l^2} (1 + c_2 + 2c_3) \right) + \frac{\theta^{\alpha\beta} \theta^{\gamma\delta}}{128\pi G_N} \int d^4x e^{\mu\nu\rho\sigma} (2 + c_2 + 38c_3) R_{\alpha\beta\gamma\delta} + (4 - 12c_2 + 44c_3) R_{\alpha\beta\delta} \delta_{\gamma\delta} - (6 + 22c_2 + 36c_3) g_{\alpha\beta} R_{\gamma\delta} \delta_{\mu\nu} + (5 - \frac{9}{2} c_2 - 7c_3) T_{\alpha\beta} \gamma_{\delta c} + \ldots \right).$$

\(\ldots\) include terms of the form: $\theta^\alpha \nabla e \nabla e, \theta^\alpha T \nabla e, \theta^\alpha \nabla T$ and $\theta^\alpha T^2$, where $T$ labels the torsion tensor $T_{\mu\nu}^\alpha$. These terms are of no importance for the specific solution we analyze in this letter. Therefore, we did not write them explicitly. These complete result is written in [11]. It is quite interesting to observe that the $\theta^\alpha R$ term vanishes in the action (12). Thus, the Einstein-Hilbert term is stable under NC corrections.

The action (12) is invariant under the local $SO(1,3)$ symmetry. The diffeomorphism symmetry is broken, as expected. Terms that explicitly break this symmetry are of the form $\theta^{\alpha\beta} \theta^{\gamma\delta} R_{\alpha\beta\gamma\delta}$ and $\nabla_\mu e^a_\nu$. The first type of terms, despite the contracted indices, are not scalars since $\theta^{\alpha\beta}$ is not a tensor but a constant matrix. The second type of terms can be rewritten using the metricity condition as $\nabla_\mu e^a_\nu = \Gamma^\sigma_{\mu\nu} e^a_\sigma$ with the affine connection $\Gamma^\sigma_{\mu\nu}$ explicitly appearing in the action.

To obtain the equations of motion, we have to vary the action (12) with respect to the spin connection $\omega^a_\mu$ and the vierbein $e^a_\mu$. The equations are given by

$$R^\mu_\nu - \frac{1}{2} g^{\mu\nu} R + \Lambda g^{\mu\nu} = \tau^\mu_\nu,$$

$$\tau^\mu_\nu = -\frac{8\pi G_N}{e} e^a_\mu \delta S_{\text{NC}} \frac{\partial S_{\text{NC}}}{\partial e^a_\mu}.$$
\[
T_{ac} c^a - T_{bc} c^b - T_{ab}^\mu = S_{ab}^\mu, \quad (14)
\]

where \( S_{NC}^{(2)} \) is the second-order expansion of the action (12). The explicit expressions for \( \tau^{\mu \nu} \) and \( S_{ab}^\mu \) are given in [11]. Even without explicitly solving these equations, we can make some important comments. Looking at eq. (14) we see that noncommutativity is a source of torsion. Namely, even if we start from a torsion-free solution (in the zeroth order in the deformation parameter), NC corrections will introduce non-zero torsion in the second order in the deformation parameter. The analogous conclusion follows from eq. (13): the flat space-time will become curved due to the presence of noncommutativity. In the last part of the letter, we analyze one flat space-time solution, namely the Minkowski space-time.

**NC corrections to the Minkowski space-time.** Minkowski space-time is a solution to the vacuum Einstein equations without the cosmological constant. Therefore, we first have to assume that \( 1 + c_2 + 2c_3 = 0 \), that is the cosmological constant in not present in the zeroth order. Note that in our previous works [16,17] we were not able to choose the value of the cosmological constant, but to be able to do that we have to first study cosmological solutions and their corrections induced by our NC gravity model.

The right-hand side of eq. (16) is constant. Therefore, the scalar curvature\(^3\) of this solution is given by
\[
R = -\frac{11}{l^6} g^{\alpha \beta} g^{\gamma \delta} \eta_{\alpha \gamma} \eta_{\beta \delta} = \text{const.} \quad (18)
\]

This shows that the noncommutativity induces curvature and the Minkowski space-time becomes (A)dS-like. The sign of the scalar curvature will depend on the particular values of the parameter \( \theta^{\alpha \beta} \). The induced curvature is very small, being quadratic in \( \theta^{\alpha \beta} \) and it will be difficult to measure it. However, qualitatively we showed that noncommutativity is a source of curvature, just like matter or a cosmological constant. It is tempting to try to relate the quantity \( \theta^{\alpha \beta} g^{\gamma \delta} \eta_{\alpha \gamma} \eta_{\beta \delta} / l^6 \) with the actual value of the cosmological constant, but to be able to do that we have to first study cosmological solutions and their corrections induced by our NC gravity model.

The Reimann tensor for the solution (17) can be calculated easily. A very interesting and unexpected observation follows: knowing the components of the Riemann tensor, the components of the metric tensor in (17) can be written as
\[
\begin{align*}
  g_{00} & = 1 - R_{0mn} x^m x^n, \\
  g_{0i} & = -\frac{2}{3} R_{0mn} x^m x^n, \\
  g_{ij} & = \delta_{ij} - \frac{1}{3} R_{mijn} x^m x^n. \quad (19)
\end{align*}
\]

This form of the metric tensor is typical for a special type of coordinates, the Fermi normal coordinates. These coordinates are inertial coordinates of a local observer moving along a geodesic. The time coordinate is just the proper time of the observer moving along the geodesic. The space coordinates \( x^i \) are defined as affine parameters along the geodesics in the hypersurface orthogonal to the actual geodesic of the observer. Unlike the Riemann normal coordinates which can be constructed in a small neighborhood of a point, FNC can be constructed in a small neighborhood of a geodesic, that is inside a small cylinder surrounding the geodesic [18,19]. Along the geodesic these coordinates are inertial, that is
\[
g_{ij} |_{\text{geod.}} = \eta_{ij}, \quad \partial_i g_{ij} |_{\text{geod.}} = 0. \quad (20)
\]

The measurements performed by the local observer moving along the geodesic are described in FNC. Especially, she/he is the one that measures \( \theta^{\alpha \beta} \) to be constant! In any other reference frame (any other coordinate system) observers will measure space-time dependent \( \theta^{\alpha \beta} \).

With this observation we now understand the breaking of diffeomorphism symmetry in the following way: there is a preferred reference system defined by FNC which we label by \( x^\mu \) and the noncommutativity is constant in that

\(^*\)Let us point out that, since the commutative diffeomorphism symmetry is broken by noncommutativity, NC quantities do not transform covariantly under this symmetry. Only the commutative limit of a NC quantity transforms covariantly under the commutative diffeomorphism. This is a common situation in NC field theories.
particular reference system. In an arbitrary reference system with \( y^\alpha \) coordinates the noncommutativity is calculated from \([y^\alpha \cdot y^\beta] \). The \( \cdot \)-product is the Moyal-Weyl \( \cdot \)-product (8) and \( y^\alpha \) are understood as functions of FNC \( x^\mu \). In this way we obtain

\[
[y^\alpha \cdot y^\beta] = i \theta^{\mu \nu} \frac{\partial y^\alpha}{\partial x^\mu} \frac{\partial y^\beta}{\partial x^\nu} - \frac{i}{24} \theta^{\mu \nu \rho \sigma} \theta^{\lambda \delta} \frac{\partial^3 y^\alpha}{\partial x^\delta \partial x^\rho \partial x^\mu} \frac{\partial^3 y^\beta}{\partial x^\lambda \partial x^\sigma \partial x^\nu} + \ldots \quad (21)
\]

Note that the leading term coincides with a tensor transformation law. However, the expression \([y^\alpha \cdot y^\beta]\), being a commutator of two coordinates, is not a tensor.

We conclude that the constant NC deformation is consistent only with the reference system given by FNC. In our future work we plan to investigate other solutions of our NC gravity model, like NC Schwartzschild solution and cosmological solutions. Especially, we are interested in the role of FNC in these solutions and in this way we hope to gain a better understanding of NC gravity.

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