Bayesian WIMP detection with the Cherenkov Telescope Array

Abhi Mangipudi, Eric Thrane and Csaba Balazs

School of Physics and Astronomy, Monash University, Melbourne VIC 3800, Australia

E-mail: abhi.mangipudi@monash.edu, eric.thrane@monash.edu, csaba.balazs@monash.edu

Abstract. Over the past decades Bayesian methods have become increasingly popular in astronomy and physics as stochastic samplers have enabled efficient investigation of high-dimensional likelihood surfaces. In this work we develop a hierarchical Bayesian inference framework to detect the presence of dark matter annihilation events in data from the Cherenkov Telescope Array (CTA). Cosmic rays are weighted based on their measured sky position $\hat{\Omega}_m$ and energy $E_m$ in order to derive a posterior distribution for the dark matter’s velocity averaged cross section $\langle \sigma v \rangle$. The dark matter signal model and the astrophysical background model are cast as prior distributions for $(\hat{\Omega}_m, E_m)$. The shape of these prior distributions can be fixed based on first-principle models; or one may adopt flexible priors to include theoretical uncertainty, for example, in the dark matter annihilation spectrum or the astrophysical distribution of sky location. We demonstrate the utility of this formalism using simulated data with a Galactic Centre signal from scalar singlet dark-matter model. The sensitivity according to our method is comparable to previous estimates of the CTA sensitivity.
1 Introduction

The Cherenkov Telescope Array (CTA) is well equipped to detect dark-matter annihilation or decay events, probing parameters of the weakly interacting massive particle (WIMP) paradigm [1]. There is a reasonable chance that dark matter belongs to the WIMP class [2], and is a thermal relic of the early Universe [3–10]. The cold dark matter paradigm has proven to be successful in reproducing most observations down to galactic scales with some open questions remaining at smaller scales [11]. With collider and direct detection searches yet to yield universally accepted, statistically significant signals \(^1\), recent years have seen increasing interest in indirect detection methods. With plans of observing the Galactic Centre and some dwarf spheroidal satellite galaxies, CTA will be sensitive to the WIMP thermal cross-section for a dark matter mass in the range \(\sim 200 \text{ GeV} \) to 20 TeV [1].

Indirect detection experiments like CTA can determine \(\langle \sigma v \rangle\) — the velocity-weighted average annihilation cross section — by measuring the flux of gamma rays originating from dark matter annihilation. For dark matter that is a thermal relic of the early Universe with mass of the electroweak scale, the expected value of \(\langle \sigma v \rangle\) is approximately [13],

\[
\langle \sigma v \rangle \approx 3 \times 10^{-26} \text{ cm}^3\text{s}^{-1}.
\]

(1.1)

Here, \(v\) is the relative velocity of the colliding dark matter particles, \(\sigma\) is the total annihilation cross section and the angled brackets denote an ensemble average. The velocity-weighted annihilation cross-section is related to the differential gamma-ray flux from dark matter annihilation:

\[
\frac{d\Phi}{dE} = \frac{1}{4\pi} J \frac{\langle \sigma v \rangle}{2S \chi m_{\chi}^2} \sum f B_f \frac{dN_f}{dE}.
\]

(1.2)

\(^1\)While the purported detection of dark matter particles by DAMA/LIBRA is statistically significant [12], this claim is controversial.
Here, $d\Phi/dE$ is the gamma-ray flux from dark-matter annihilation. The variable $J$ is the so-called $J$-factor,

$$J = \int d\hat{n} \int_{\text{LOS}} ds \rho_\chi^2(r(s, \hat{n})),$$

(1.3)

which depends on the dark-matter density $\rho_\chi(r(s, \hat{n}))$ along the line of sight (LOS). The integral is over the unit vector $\hat{n}$ and $s$ is the distance along the LOS. When observing the Galactic Centre the dark matter density is typically assumed to follow the Einasto profile [14]:

$$\rho_\chi(r) = \rho_0 \exp\left\{-2 \left[\left(\frac{r}{r_s}\right) - \frac{1}{\alpha}\right]\right\}.$$

(1.4)

Here $r_s$ and $\rho_0$ are the radius and density, respectively, at which the logarithmic slope of the density is $-2$, and $\alpha$ is a parameter describing the degree of curvature of the profile [15]. We take $r_s = 20$ kpc, $\alpha = 0.17$ [14]. The parameter $\rho_0$ is fixed such that the local dark matter density is $\rho_\chi(r_\odot) = 0.39$ GeV cm$^{-3}$, where $r_\odot = 8.5$ kpc [16].

Returning to Eq. 1.2, the mass of the dark matter particle is $m_\chi$, and $S_\chi = 1(2)$ if this particle is (not) its own anti-particle. The sum is over annihilation states $f$, $B_f$ is the annihilation fraction while $dN_f/dE$ is the photon energy spectrum. In factorising the $J$-factor from the terms in parentheses we assume that the annihilation rate is uncorrelated with $v(r)$ [17].

It is instructive to pause and discuss the components of Eq. 1.2. The left-hand side, the differential gamma-ray flux, is what experiments like CTA measure by looking for excess photons beyond what is expected from the astrophysical background. If we assume a specific model for dark-matter annihilation, most of the terms on the right-hand side can be calculated from first principles. The one right-hand-side term that we cannot uniquely calculate from first principles is $\langle \sigma v \rangle$. Thus, by measuring the differential flux, and assuming a specific dark-matter model, we obtain an estimate for $\langle \sigma v \rangle$. The main mission of indirect detection experiments is to measure $m_\chi$ and $\langle \sigma v \rangle$.

A number of previous publications have explored how CTA will be able to measure $\langle \sigma v \rangle$; see, for example, [15, 17–22]. Ref. [17] uses the on/off approach to demarcate two separate regions of interest—an ‘on’ region in which the signal is expected to dominate over background, and an ‘off’ region in which the background dominates over signal. Ref. [15] utilises a Poisson joint-likelihood method, where the comparisons between dark matter and background fluxes are performed in different energy and spatial bins. Ref. [18] uses a likelihood ratio test statistic, which uses the log likelihood ratio comparing the signal hypothesis to the “total” hypothesis to quantify the statistical significance of the signal, which can in turn be used to estimate $\langle \sigma v \rangle$.

In this work we develop a Bayesian formalism for the measurement of dark matter annihilation in CTA data. In doing so, we aim to leverage Bayesian tools used widely in astronomy such as stochastic samplers, which can be used to efficiently explore high-dimensional likelihood surfaces. A Bayesian formulation is also useful pedagogically, for example, to explicitly formulate assumptions as priors. Our signal model (from dark matter annihilation) and our background model (of gamma rays from astrophysical processes) are framed in terms of prior distributions for sky location $\Omega$ and photon energy $E$. We construct a posterior distribution for the velocity-averaged cross section $\langle \sigma v \rangle$. A dark matter signal is detected when this posterior excludes $\langle \sigma v \rangle = 0$ at high credibility, e.g., 99.99994% for a “five-sigma”
detection. Systematic uncertainty, for example, in the shape of our signal and background
distributions, can be modeled with hyper-parameters; see, e.g., [23]. Marginalizing over these
hyper-parameters, one obtains a new posterior distribution for $\langle \sigma v \rangle$, which is (appropriately)
broadened by systematic uncertainty.

The remainder of this paper is organised as follows. In Section 2, we present the statisti-
cal formalism underpinning our analysis. We describe the prior distributions that characterise
our signal and background models; we describe the likelihood function characterising CTA
measurements of cosmic rays. In Section 3, we demonstrate the formalism using simulated
data. We provide concluding thoughts in Section 4.

2 Formalism

2.1 Overview and basics

In this section we describe a Bayesian formalism to measure the properties of dark-matter
annihilation in CTA data. We consider a dataset consisting of $N$ gamma-ray events, each
indexed by an event number $1 \leq i \leq N$. The dataset $\mathbf{d}$, which describes these $N$ events,
consists of elements $d_i$. Each datum $d_i$ consists of the measured sky location $\hat{\Omega}_m$ and the
measured energy $E_m$ of each cosmic ray:

$$d^i = \{\hat{\Omega}^i_m E^i_m\}. \quad (2.1)$$

Here, the hat signals that $\hat{\Omega}_m$ is a unit vector.

2.2 Likelihood

Each event $i$ is characterized by a likelihood function $\mathcal{L}(d^i|\hat{\Omega}^i, E^i)$. This likelihood is a point
spread function: it relates the measured sky location and energy (denoted with a subscript $m$) to the
true sky location and energy (written without subscripts). It can be factored into
separate components for energy and sky location

$$\mathcal{L}(d^i|\hat{\Omega}^i, E^i) = \mathcal{L}(E^i_m|E^i) \mathcal{L}(\hat{\Omega}^i_m|\hat{\Omega}^i, E^i_m). \quad (2.2)$$

The sky location likelihood is conditional on the measured energy since high-energy cosmic
rays are better localised than low-energy ones. We use the publicly available prod3b-v1 instrument response function (IRF) library, which is optimised for the detection of a point-like source at a $20^\circ$ zenith angle. Note that the Galactic Centre is mostly visible from the southern site [17]. Our likelihood function uses the ctools [24] functions test_sim_edisp for energy and test_sim_psf [26] for sky location. These likelihood functions are shown in Fig. 1a and 1b respectively.

2.3 Signal model

Gamma rays from dark matter annihilation are distinguishable from the astrophysical back-
ground based on their tendency to cluster near the gravitational potential well in the Galactic
Centre. The energy spectrum from dark matter annihilation is also expected to differ from

2The “raw” data produced from CTA is more complicated than we describe here, consisting of CCD images and arrival times. Our data is the result of pre-processing to convert the raw data into estimates of the sky location and energy of each event.

3Specifically, we use the South\_z20\_50h IRF file.
the astrophysical spectrum, though, the precise shape depends on the details of the unknown dark matter annihilation physics. Our signal model therefore consists of two parts: a description of the angular distribution and the energy spectrum.

The signal prior for the energy of event $i$ is denoted

$$\pi(E^i|S).$$

The $S$ signifies that this prior is for the signal hypothesis (that the cosmic ray was created from dark-matter annihilation), which we contrast below with the background hypothesis (that the cosmic ray was created through astrophysical acceleration) denoted $B$. For illustrative purposes, we will assume a specific dark matter scenario: the scalar singlet model. The energy spectrum for this model $\pi(E|S)$ is shown as the black curve in Fig. 2a. Comparing the signal distribution to the background distribution in Fig. 2b (discussed in greater detail below), we see that the signal appears as an excess (above the astrophysical background) around the assumed dark matter mass, in this case, about 2 TeV.

The scalar singlet model is one of the simplest WIMP models; the model adds one massive real scalar field $S$ to the standard model [27–29]. The only renormalizable interaction terms between the singlet and the standard model permitted in the Lagrangian are of the form $S^2H^2$. This leads to a range of possible phenomenological consequences including: thermal production in the early Universe and present-day annihilation signals [30–32], direct detection and $h \rightarrow SS$ decays [33]. The renormalizable terms involving $S$, permitted by the $Z_2$, gauge, and Lorentz symmetries are,

$$\mathcal{L} = \frac{1}{2} \mu_S S^2 + \frac{1}{2} \lambda_h S^2 |H|^2 + \frac{1}{4} \lambda_S S^4 + \frac{1}{2} \partial \mu S \partial \mu S.$$ (2.4)
The pre-factors are the bare $S$ mass, the Higgs-portal coupling, the $S$ quartic self-coupling, and the last term is the $S$ kinetic term. If the singlet does not obtain a vacuum expectation value, the model only has three free parameters: $\mu_S^2$, $\lambda_{Sh}$ and $\lambda_S$. In the following section we present an example detection and non-detection plot (Fig. 4) for the following choice of parameters: $\lambda_{Sh} = 1$, $\lambda_S = 1$, and $m_S = 2\,\text{TeV}$. These parameters are chosen based on constraints upon the scalar singlet model in [7]. The branching fractions and standard model final states that go into producing the energy spectrum are:

\begin{align*}
B_f(W^+W^-) &= 0.50, \tag{2.5} \\
B_f(Z^0Z^0) &= 0.25, \tag{2.6} \\
B_f(hh) &= 0.25, \tag{2.7}
\end{align*}

which have been calculated using micrOMEGAs [34], a code for the calculation of various dark-matter properties.

We have also assumed simplified models in which the dark matter particle primarily annihilates to specific standard model final states (Fig. 2a): $W^+W^-$ (blue), $\tau^+\tau^-$ (red) and $b\bar{b}$ (green). While scalar singlet dark matter particles mostly annihilate into a $W^+W^-$ final state, they have considerable annihilation fraction to final states that are rarely considered in simplified models. This will diminish our ability to measure $\langle \sigma v \rangle$ since some dark matter particles may annihilate in such a way that does not produce detectable cosmic rays. Varying the mass of the dark matter particle shifts the signal prior $\pi(E|S)$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Prior distributions for energy in our Galaxy. \textit{Left}: example prior distributions for the signal hypothesis $\pi(E|S)$ calculated using the Poor Particle Physicist Cookbook (PPPC) [35]. In this case, the dark matter mass is $m_\chi = 2\,\text{TeV}$. We assume a scalar singlet model (black) for which we show sample detection and non-detection plots in Section 3. We also assume a simplified model in which the dark matter particle annihilates to specific standard model final states: $W^+W^-$ (blue), $\tau^+\tau^-$ (red) and $b\bar{b}$ (green). \textit{Right}: the prior distribution (black) for the background hypothesis $\pi(E|B)$ using \texttt{ctools}. The black curve includes contributions from misidentified charged cosmic rays (green) and interstellar emission (red) (for which we choose the \textsc{Pass-8 Fermi} model).
\end{figure}

The signal prior for the sky location of event $i$ is denoted

$$\pi(\hat{\Omega}_i|S).$$

We take our signal prior to be the Einasto profile, which is defined above in Eq. 1.4. In Fig. 3a, we show the signal prior plotted as a function of Galactic longitude and latitude. Comparing
the signal distribution to the background distribution in Fig. 3b (discussed in greater detail below), we see that the signal appears as an excess (above the astrophysical background) near the Galactic Centre. Some of the most common dark matter density profiles are the Einasto, the NFW (named for the authors, Navarro, Frenk, White) [14], and the Burkert [36]. The Einasto and NFW profiles peak sharply while the Burkert profile levels off [15]. The precise shape of the dark matter distribution near the Galactic Centre is the subject of active research (see, e.g., [37]). Systematic uncertainty in the shape of the dark matter profile leads to uncertainty in the $J$-factors, which propagates to systematic uncertainty in $\langle \sigma v \rangle$ [38]. This is particularly true for Galactic Center searches [39]. The Einasto profile is most consistent with the observed distribution of dark matter [17]. However, the dark matter density is relatively poorly constrained and there remain discrepancies between observation and prediction of the dark matter distribution in dwarf galaxies [40]. Therefore, while we adopt the Einasto profile here for illustrative purposes, we recommend marginalizing over uncertainty in the dark matter profile.

Figure 3. Prior distributions for sky location around the Galactic Centre. Left: sky location prior for the signal hypothesis $\pi(\Omega_i|S)$, the Einasto profile plotted in Galactic latitude and longitude $p_\chi(l,b)$. Right: sky location prior for the background hypothesis $\pi(\Omega_i|B)$. A Gaussian mixture model with ten Gaussians, fitted to 525 hours of simulated data of the astrophysical background obtained using ctools.

2.4 Background model

Gamma rays are produced in the Galaxy from the decay of relativistic particles accelerated through shocks. When crossing a shock front, these high-energy particles gain energy and scatter off approaching scattering centers. The energy spectrum from shock acceleration follows a power law $dN/dE \propto E^{-n_\gamma}$ where $n_\gamma \gtrsim 2$ [17, 41]. The astrophysical background prior for the energy of event $i$ is denoted

$$\pi(E^i|B).$$

(2.9)

For the Galactic Centre target this distribution is shown in Fig. 2b. The shape of the spectra appropriately follows the power law described above. Recent observations of the Galactic Centre indicate an excess of cosmic rays, with dark matter annihilation being a
potential explanation [42]. The origin of cosmic rays with energies up to \(10^{15}\) eV are commonly attributed to supernova remnants [43]. However, some other sources have been proposed such as Sagittarius A* and other TeV-bright objects near the galactic centre [44, 45], and unresolved millisecond pulsars [46] all of which are able to accelerate cosmic rays to very high energies. Additionally, two large gamma ray structures known as “Fermi Bubbles” extend above and below the Galactic Centre [47], though, these bubbles (likely sourced by Sagittarius A*) contribute only a small fraction (\(\ll 1\%\)) of the astrophysical gamma-ray flux.

The background prior for sky location of event \(i\) is denoted

\[ \pi(\hat{\Omega}_i|\mathcal{B}). \] (2.10)

In Fig. 3b we show the background prior plotted as a function of Galactic longitude and latitude. The shape of the distribution is determined by the interstellar emission which extends along the Galactic plane. Gamma rays along the Galactic plane are predominantly produced through two processes: cosmic ray interactions with the interstellar medium gas (primarily through the neutral pion channel), and cosmic rays up-scattering off the interstellar radiation field and/or the cosmic microwave background photons to gamma ray energies (inverse Compton scattering) [17].

The astrophysical background in our analysis consists of various components. Following [17], in the background we include a contribution from charged cosmic rays and interstellar emission. Due to the relatively low expected flux, we neglect the Fermi Bubbles and point sources. The contribution from charged cosmic rays is the most dominant background component, which is mostly due to misidentified electrons. Ref. [17] uses three different models for the interstellar emission: the Gamma, BASE+GALACTIC RIDGE, and PASS-8 FERMI models. For a detailed description of these models we refer the reader to Ref. [17]. We use the PASS-8 FERMI model for the interstellar emission part of the background, which is data-driven, relying less than other models on theoretical assumptions. These background components have been simulated using ctools for a 525 hour observation period with the publicly available prod3b-v1 IRF library. The resulting energy spectra are shown in Fig. 2b. The simulated data are then fit using a Gaussian mixture model with ten Gaussians. The resulting fit is shown in Fig. 3b.

### 2.5 Marginal likelihoods

Using the prior distributions for our signal and background models, the next step is to calculate marginal likelihoods. For each cosmic ray, we calculate two marginal likelihoods, one for the signal hypothesis

\[ \mathcal{L}(d^i|\mathcal{S}) = \int d\hat{\Omega}_i \int dE^i \mathcal{L}(d^i|\hat{\Omega}_i E^i) \pi(\hat{\Omega}_i, E^i|\mathcal{S}), \] (2.11)

and one for the background hypothesis

\[ \mathcal{L}(d^i|\mathcal{B}) = \int d\hat{\Omega}_i \int dE^i \mathcal{L}(d^i|\hat{\Omega}_i E^i) \pi(\hat{\Omega}_i, E^i|\mathcal{B}). \] (2.12)

\(^4\)Data available at https://github.com/cta-observatory/cta-gps-simulation-paper/tree/master/skymodel/iem

\(^5\)Specifically, we employ the gll_iem_v07 model, available at https://fermi.gsfc.nasa.gov/ssc/data/access/lat/BackgroundModels.html
The marginal likelihoods quantify the support for each hypothesis and the (signal/background) Bayes factor

$$\text{BF}_{SB}^{S,i} = \frac{L(d^i|S)}{L(d^i|B)},$$ (2.13)

describes the relative likelihood that event $i$ is drawn from one hypothesis versus the other. However, many events are required to confidently detect a dark-matter annihilation signal.

There are a number of ways to compute the marginal likelihoods. One option is to represent the likelihoods in Eq. 2.2 using posterior samples. These posterior samples are obtained using an uninformative, fiducial prior $\pi_0$, for example, which is uniform in (the logarithm of) $E$ and isotropic in $\Omega$. For each event $i$, there are $n_i$ fiducial samples, each consisting of an ordered pair:

$$(E^i_k, \hat{\Omega}^i_k).$$ (2.14)

Here, $k$ is the sample number. The fiducial samples may be obtained with a stochastic sampler, e.g., [48–51]. The fiducial samples can be “recycled” (or “reweighted”) to obtain marginal likelihoods [23]

$$L(d^i|S) = Z_0(d^i) \sum_k^{n_i} \frac{\pi(E^i_k, \hat{\Omega}^i_k|S)}{\pi(E^i_k, \hat{\Omega}^i_k|\theta)}.$$ (2.15)

Here, $Z_0$ is the fiducial evidence obtained with the fiducial prior:

$$Z_0(d^i) = \int d\hat{\Omega}^i \int dE^i L(d^i|\hat{\Omega}^i, E^i) \pi(\hat{\Omega}^i, E^i|S).$$ (2.16)

This procedure—which is a special case of importance sampling [52, 53]—is convenient because a single set of fiducial samples can be recycled for different analyses (with different signal and/or background distributions) 6.

### 2.6 Combined likelihood

The next step is to combine our data to construct a single likelihood function for all the data $\vec{d}$. If we allow for each event to be either signal or background, then the single-event likelihood can be written as a mixture model of the signal and background hypotheses:

$$L(d^i|\lambda) = \lambda L(d^i|S) + (1 - \lambda) L(d^i|B).$$ (2.17)

Here, $\lambda \in (0, 1)$ is the probability that event $i$ is drawn from the signal distribution.

Since each measurement is uncorrelated, we can write the combined likelihood as product of single-event likelihoods.

$$L(\vec{d}|\lambda) = \prod_i^N L(d^i|\lambda) = \prod_i^N \lambda L(d^i|S) + (1 - \lambda) L(d^i|B)$$ (2.18)

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6In applied statistics, the fiducial prior (in the denominator of Eq. 2.15) is known as the “proposal distribution” while the new prior (in the numerator of Eq. 2.15) is known as the “target distribution.”
After combining the data for $N$ measurements, the parameter $\lambda$ is interpreted as the proportion of events drawn from the signal hypothesis ($N_S$). Our goal is to infer $\lambda$, which determines the extent to which a dark-matter signal is present in the data. The posterior for $\lambda$ is

$$p(\lambda | \vec{d}) = \frac{\mathcal{L}(\vec{d} | \lambda) \pi(\lambda)}{\mathcal{Z}(\vec{d})},$$

where $\pi(\lambda)$ is our prior on the mixing fraction and

$$\mathcal{Z}(\vec{d}) = \int d\lambda \pi(\lambda) \mathcal{L}(\vec{d} | \lambda),$$

is the Bayesian evidence for the data given the full dataset given our signal+background mixture model. It appears in Eq. 2.19 as a normalization constant, but we describe below how it can be used for model selection. If we take this prior to be uniform $^7$ then the posterior is an order-$N$ polynomial. However, due to the central limit theorem, the posterior for $\lambda$ converges to a Gaussian distribution as $N$ becomes large and the data become informative.

One can claim a dark-matter detection when this posterior rules out $\lambda = 0$ at high credibility. However, it is desirable to convert statements about $\lambda$ to statements about the velocity-averaged cross section $\langle \sigma v \rangle$. There are several steps. First, we relate $\lambda$ to the number of dark-matter annihilation events in the dataset. Since we expect that the number of background events ($N_B$) will be overwhelmingly larger in CTA than the number of signal events, we make the approximation that $N \approx N_B$, yielding

$$\lambda = \frac{N_S}{N} \approx \frac{N_S}{N_B}.$$  

The number of signal events $N_S$ is related to the differential gamma-ray flux from dark-matter annihilation:

$$N_S = T \int dE \frac{d\Phi}{dE}(E) A(E).$$

Here, $T$ is the observation time while $A(E)$ is the energy-dependent collection area of CTA (obtained using ctools [24]). The differential flux $d\Phi/dE$ is linearly related to $\langle \sigma v \rangle$ via Eq. 1.2. Thus, there is a one-to-one mapping between $\lambda$ and $\langle \sigma v \rangle$.

### 3 Demonstration

In this section we demonstrate our formalism using mock data targeting the Galactic Centre. We simulate both astrophysical background and dark-matter annihilation signal assuming a scalar singlet model. We generate 525 hours (21 days) of background data using random draws from the background distributions described in Section 2. The background distribution of sky location is shown in Fig. 3b while the background energy spectra is shown in Fig. 2b. This observation time corresponds to what we might expect to obtain for CTA measurements of the galactic centre [17]. This dataset corresponds to $\approx 10^8$ gamma-ray events. To this dataset we swap in a small number of signal events. The signal distribution of sky location is shown in Fig. 3a while the signal energy spectra is shown in Fig. 2a. We vary the precise

$^7$A more physically motivated prior for $\lambda$ can be obtained by using an $N_S = 0$ Poisson distribution scaled based on the results of previous upper limits on the dark-matter gamma-ray flux.
number of signal events in order to control the statistical significance of the dark-matter annihilation signal.

First, we tune the number of signal events to illustrate what a dark-matter detection looks like in our Bayesian formalism. In Fig. 4a, we plot the posterior distribution for $\langle \sigma v \rangle$ using a dataset for which we can (only just) rule out the null hypothesis $\langle \sigma v \rangle = 0$ with 99% credibility. In this case we set the mass of the dark matter particle to $m_\chi = 2$ TeV. In order to achieve this statistical significance, we employed $7.6 \times 10^4$ signal events out of a total set of $1.08 \times 10^8$ events (0.07%). The vertical red line in Fig. 4a indicates the true value of $\langle \sigma v \rangle$. The posterior is statistically consistent with the true value, though, it does not peak at the true value of $\langle \sigma v \rangle$ due to statistical fluctuations. The shaded region shows the (highest posterior density) 99% credible interval, which covers $(0, 8.64) \times 10^{-27}$ cm$^3$s$^{-1}$.

Next, we set the number of signal events to zero in order to illustrate what a non-detection result looks like. In Fig. 4b, we plot the posterior distribution for $\langle \sigma v \rangle$, again setting $m_\chi = 2$ TeV. In this panel, we cannot rule out $\langle \sigma v \rangle = 0$ with even marginal significance, and so we obtain only an upper limit of $\langle \sigma v \rangle > 6.17 \times 10^{-27}$ cm$^3$s$^{-1}$ (99%) credibility. As the observation time $T$ is increased, the typical upper limit scales like $T^{-1/2}$.

In order to show how our results vary with the dark-matter mass, we calculate the Bayesian upper limit (99% credibility) for $\langle \sigma v \rangle$ (assuming null results) for different values of $m_\chi$. By changing $m_\chi$, we change the expected signal distribution—$\pi(E|S)$ (Eq. 2.3)—which, in turn, affects the extent to which signal can be distinguished from background. For each value of $m_\chi$, we calculate the 99% upper limit for the case where no signal events are present in the data. In Fig. 5, we plot these limits on $\langle \sigma v \rangle$ as a function of $m_\chi$. The coloured curves represent different simplified models of dark matter annihilation final state, and the scalar singlet dark matter model.

We reproduced the $W^+W^-$ limit, shown by the dash-dotted line, from Ref. [17]. When comparing the two $W^+W^-$ limits we have to keep in mind that, among various differences, they are extracted by different statistical methods and are based on a different observational

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In order to reduce computation time, we have employed a total set of $1.08 \times 10^5$ events and scaled the upper limit using this well known scaling relation to present the expected upper limits for $10^8$ events.
strategy. Our limit, for example, relies on the full dataset arriving from the Galactic Centre while the one in Ref. [17] is based on an On-Off observing strategy. The latter reduces the number of detected photons from dark matter annihilation and is expected to produce a slightly weaker limit. We account the difference in shape to other differences in the inference such as the different statistical meaning of frequentist confidence and Bayesian credibility intervals.

![Figure 5](image)

**Figure 5.** Projected 99% upper limits on the velocity-averaged cross section \(\langle \sigma v \rangle\) as a function of dark-matter mass \(m_\chi\). The limits are calculated assuming 525 hours of observation of the Galactic Centre. We assume simple models whereby the dark matter particle annihilates completely to \(W^+W^-\) (with electroweak corrections) (blue), \(\tau^+\tau^-\) (red) and \(b\bar{b}\) (green), and the scalar singlet model (black). The dash-dotted \(W^+W^-\) line is reproduced from Ref. [17]. The dashed horizontal line corresponds to the dark matter ‘thermal’ cross-section (Eq. 1.1). The projected upper limits fall below the dashed black line for dark-matter masses of \(\approx 0.2 - 5\) TeV, indicating that CTA has the required sensitivity to detect dark matter in this range.

4 Discussion

Bayesian methods are well known, rigorous statistical tools which are widely used in (particle) astrophysics. Bayesian inference is useful because of its transparency, namely, that it can simply and quantify various assumptions and their effect on the inferred quantities. Due to the popularity of Bayesian methods various numerical tools, such as likelihood calculators or samplers, exist that (future extensions of) our analysis can take advantage of. Because of this, our Bayesian analysis is flexible and expandable, since it is easy to change or add models, assumptions, and uncertainties without altering the statistical framework itself. Additionally, with small extensions, the same framework can be used for multiple purposes such as parameter estimation or model comparison. The analysis presented here, for example, can be extended to take advantage of the full machinery of Bayesian inference. We sketch out some of the most important next steps in order to describe how this formalism can be
used (i) to measure model parameters, (ii) to take into account systematic error, and (iii) for Bayesian model selection.

**Measuring model parameters.**—In the demonstration above, we fix the dark matter mass \( m_\chi \) to just one value at a time. However, in practice, the dark-matter mass should be treated as a free parameter. Some dark-matter models have multiple parameters, as is the case with the scalar singlet model described in Section 2; Eq. 2.4, which describes this model, includes three independent parameters. We denote the list of parameters associated with a model \( \theta \) so that, for example, \( \theta = \{ J, m_\chi, \lambda_{S_h}, \lambda_S \} \). The signal prior is conditional on these parameters.

\[
\pi_S(\hat{\Omega}, E|\theta). \tag{4.1}
\]

The parameter \( \theta \) is sometimes referred to as a *hyper-parameter* [23] since it controls the shape of the distribution of other parameters \( (E, \hat{\Omega}) \). Models like Eq. 4.1, where the prior is conditional on one or more hyper-parameters, are referred to as *hierarchical* models [23]. It is necessary to marginalize over the free parameters in \( \theta \) when calculating the marginal signal likelihood. The generalization of Eq. 2.11 is

\[
\mathcal{L}_S(d^i|\theta) = \int d\hat{\Omega}^i \int dE^i \mathcal{L}(d^i|\hat{\Omega}^i, E^i) \pi_S(\hat{\Omega}^i, E^i|\theta) \pi(\theta). \tag{4.2}
\]

The recycling trick described in Eq. 2.15 is useful here since we can use one set of fiducial samples for many different values of \( \theta \). The likelihood of the complete data given signal fraction \( \lambda \) and dark-matter parameters \( \theta \) is

\[
\mathcal{L}(\vec{d}|\lambda, \theta) = \prod_i \left( (1 - \lambda) \mathcal{L}(d^i|B) + \lambda \mathcal{Z}_\nu(d^i) \sum_k \frac{\pi_S(E^i_k, \hat{\Omega}^i_k|\theta)}{\pi(E^i_k, \hat{\Omega}^i_k|\theta)} \right). \tag{4.3}
\]

Programming the likelihood into a stochastic sampler, we can obtain posterior samples for \( \theta \), allowing us to, for example, to measure the dark-matter mass in case of signal detection. The sum over posterior samples can sometimes be evaluated rapidly using graphical processor units (GPUs) [54].

**Taking into account systematic error.**—In our demonstration, we have assumed that the astrophysical background model is perfectly well specified. In reality, the flux of astrophysical cosmic rays is an active area of research, and so our model is subject to uncertainty (discussed in Section 2D). One way to accommodate this systematic uncertainty in our framework is with noise-model hyper-parameters, which we denote by \( \eta \):

\[
\pi_B(\hat{\Omega}|\eta). \tag{4.4}
\]

The power-law slope of the Einasto profile, for example, can be treated as a free parameter. If we choose, we can add this slope to \( \eta \). It is necessary to marginalize over the free parameters in \( \theta \) when calculating the marginal signal likelihood. The generalization of Eq. 2.12 is

\[
\mathcal{L}_B(d^i|\eta) = \int d\hat{\Omega}^i \int dE^i \mathcal{L}(d^i|\hat{\Omega}^i, E^i) \pi_B(\hat{\Omega}^i|\eta). \tag{4.5}
\]

---

9In this discussion, we sometimes denote the signal or background labels \( S \) and \( B \) as subscripts for readability.
As in Eq. 4.3, one may recycle fiducial background samples for computational efficiency.

**Model selection.**—Above, we carried out our demonstration with a single dark-matter model. However, it will likely be useful to perform analyses for multiple dark-matter models in order to see which one best fits the data. In order to do this, one may calculate the Bayesian evidence for the entire dataset (as in Eq. 2.20) given each dark-matter model \((S_1, S_2, \ldots)\):

\[
\mathcal{Z}(\bar{d}|\alpha) = \int d\lambda \pi(\lambda) \mathcal{L}(\bar{d}|\lambda, \alpha).
\]

(4.6)

Here, \(\alpha\) is an index that numbers the different dark-matter models. The Bayes factor comparing models \(\alpha = 1\) and \(\alpha = 2\) is

\[
BF_2^1 = \frac{\mathcal{Z}(\bar{d}|\alpha = 2)}{\mathcal{Z}(\bar{d}|\alpha = 1)}.
\]

(4.7)

A large Bayes factor (for example, \(\ln BF > 8\)) indicates that one dark-matter model is strongly preferred to another [23]. One should compare the Bayes factor to the ratio of maximum likelihood values for each model in order to understand the extent to which the Bayes factor is influenced by the prior distributions of each model:

\[
\frac{\max_{\theta_2} \mathcal{L}(\bar{d}|\theta_2, \alpha = 2)}{\max_{\theta_1} \mathcal{L}(\bar{d}|\theta_1, \alpha = 1)}.
\]

(4.8)

Here, \(\theta_1, \theta_2\) are the parameters for model \(\alpha = 1, 2\) respectively.

Indirect searches for dark matter via gamma rays considerably matured during the last decade. Observations of the Fermi Gamma-ray Space Telescope [55] and those of H.E.S.S. [56], among others, paved the way toward precision measurements of cosmic gamma rays. The capabilities of CTA will surpass those of the earlier gamma ray instruments on the front of energy range, resolution and sensitivity. This opens an exciting new era in the next decade of dark matter indirect detection. The increasingly precise gamma-ray observations have to be matched by increasingly sophisticated inference tools to determine the fundamental physical properties of dark matter particles, such as their mass, spin and interaction strengths. In this spirit, the formalism presented in this work provides one of the potentially useful frameworks for CTA measurements of dark matter.

5 Acknowledgements

We are grateful to Christopher Eckner and Torsten Bringmann for collaboration on the early version of this project based on frequentist inference. We also thank Giacomo Damico, Fabio Iocco, Manuel Meyer, and Manuela Vecchi for valuable comments on the manuscript. ET is supported by the Australian Research Council (ARC) Centre of Excellence CE170100004. The work of CB is supported by the ARC Discovery Project grants DP180102209 and DP210101636.

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