Comparison of Signal Processing Algorithms for High-Resolution FMCW Terahertz Thickness Measurements

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Abstract. When using the frequency-modulated continuous-wave (FMCW) approach for layer thickness determination of dielectrics, such as multilayer pipe walls, the resolution is usually limited by the bandwidth. For thinner layers, we compare the measurement signal with potential model signal outputs. Since our approach results in high computation efforts by using brute force optimization, spectral estimation may present an alternative for high resolution distance and hence thickness determination. In this contribution, we compare results of a parametric algorithm (modified covariance method) and a subspace method (multiple signal classification, MUSIC) with the results of our approach.

1. Introduction
In contrast to ultrasound and X-ray techniques, millimeter wave and terahertz systems for thickness measurements require neither contact nor shielding. We presented an FMCW-based approach for inspecting multilayers [1] with thicknesses of roughly one hundred micrometer [2] up to several centimeters. To exceed the Fourier Transform (FT)-based resolution limit, we use a model-based signal processing technique comparing simulated signals with the measurement data resulting from the sample-under-test with respect to the correlation coefficient. By utilizing brute force maximization, this method requires high computational effort since numerous signal models have to be calculated. Spectral estimation techniques are promising for multi-target distance [3, 4, 5] and thickness measurements [6]. Distances and thicknesses below the resolution limit can be separated as well but may lead to inaccurate results [4, 5, 6] depending on the model order and signal-to-noise ratio (SNR) [3]. In this contribution, we analyze the potential resolution of a parametric algorithm (modified covariance method) and a noise subspace method (MUSIC) for thickness evaluation. In the second section, measurement setup and preprocessing are introduced. Section 3 describes our correlation-based signal processing method, presents the estimation approaches and analyzes the model order. Simulated and measured results are shown in section 4, which is followed by a summary.

2. Thickness Measurement Setup
Our setup, describes in [1], is based on a pre-distorted voltage-controlled oscillator (VCO) in connection with an active multiplier chain, to perform linearly frequency-modulated radiation operating within the
W band. The received reflection is multiplied with the 6th harmonic of the VCO, which results in a sum of characteristic difference frequency oscillations from the different boundary surfaces:

\[ x[n] = \text{Re}\left( \sum A_i \exp \left( j2\pi \frac{B}{T} \frac{n}{f_s} + j2\pi \tau_i \right) \right) \]

with bandwidth \( B = 38.56 \) GHz, duration \( T = 170 \) μs, characteristic delay \( \tau_i \) of each reflection, start frequency of the modulation \( F_1 = 71.16 \) GHz, imaginary unit \( j \), sampling frequency \( f_s = 10 \) MHz, and digital time \( n \in \{0,1,\ldots,N-1\} \), \( N = 1700 \). The difference between corresponding \( \tau_i \) results in the thicknesses of the layers considering the refractive indices \( \eta \) of the layer. The amplitudes \( A_i \) depend on the absorption of the propagated layers and the refractive indices according to Fresnel’s equations. The Rayleigh depth resolution limit is 3.9 mm in air. The total number of reflections is infinite due to multiple reflections as depicted in Figure 1, but the sum can be limited to a finite amount of summands \( I \) due to negligible amplitudes. Figure 1 (right) depicts the preprocessing procedure: To remove spurious reflections by the surroundings, a bandpass (bp) filter is used. Moreover, it prepares the signal for the following Hilbert transform \( \mathcal{H} \), which adds an imaginary part to it to create an analytic representation. The following 2-term calibration [7] is based on an empty room (0%) and a conducting plane measurement (100%). It compensates for reflections caused by the focusing unit and defines the phase center for the measurements (calibration position). Hence, the position of the conduction plate equals 0 after calibration.

![Figure 1](image)

**Figure 1.** (left) Main (solid) and multiple (dashed) reflections, incident normal, tilted for schematic (right) diagram of preprocessing including generation of analytic representation and 2-term calibration

3. Signal Processing Algorithms

Our correlation-based approach compares the measured signal with signal models using potential thicknesses within a predefined interval and an appropriate small step size [1]. With small refractive index differences and rather thicker layers only the main reflections are considered. For other cases, the modified transfer matrix method [8] is an effective variant to calculate all multiple reflections. The calibrated signal is compared with the calculated signals of the corresponding models using the correlation coefficient. The thickness candidates resulting in the highest correlation coefficients correspond to the best thicknesses estimates.

In contrast, spectral estimation algorithms do not require a priori intervals but the model order. Numerous algorithms are described in [9, 10] and are applied in [3, 4, 5, 6]. Promising variants are parametric approaches and subspace methods. Within this contribution, one algorithm of each variant is compared with our correlation approach: We apply the modified covariance method and the MUSIC algorithm, which are part of the MATLAB signal processing toolbox [11], as estimation algorithms.

3.1. Modified Covariance Method

Parametric approaches are based on autoregression aiming to predict one time sample \( x[n] \) by \( p \) past samples. The modified covariance method usually results in statistically stable (pseudo) spectrums and a high spectral resolution [9]. The modified covariance function minimizes the forward and backward error, simultaneously:

\[ r_{k,l} = \sum_{n=p}^{N-1} x[n-l]x^*[n-k] + x[n-p+l]x^*[n-p+k], \]

where the operator \((*)\) denotes conjugate complex. Matrix equation
\[
\begin{pmatrix}
  r_x[1,1] & r_x[2,1] & \cdots & r_x[p, 1] \\
  r_x[1,2] & r_x[2,2] & \cdots & r_x[p, 2] \\
  \vdots & \vdots & \ddots & \vdots \\
  r_x[1,p] & r_x[2,p] & \cdots & r_x[p,p]
\end{pmatrix}
\begin{pmatrix}
  a[1] \\
  a[2] \\
  \vdots \\
  a[p]
\end{pmatrix}
= \begin{pmatrix}
  r_x[0,1] \\
  r_x[0,2] \\
  \vdots \\
  r_x[0,p]
\end{pmatrix}
\]

determines the vector \((a[1], a[2], \ldots, a[p])^T\), which equals the denominator coefficients of the pseudo spectrum, whose pole indicate the frequencies [9] conforming to \(\tau_i\).

3.2. Noise Subspace Method

A common noise subspace approach is the MUSIC algorithm with promising results also for measurement [3, 6]. The autocorrelation matrix \(R_x\) with the dimension \(q \times q\) conforming \(q > p\) of a stationary noisy process is assumed to be the sum of the autocorrelation matrix of signal and the one of noise, due to their orthogonality such as for additive white Gaussian noise (AWGN). The \(q - p\) lowest eigenvalues of \(R_x\) indicate the noise variance and their eigenvectors are estimates of the noise subspace basis, which can be used to calculate a filter with poles at the frequencies [9].

3.3. Model Order

For an ideal setup, the model order \(p\) is known from the number of layers: For \(I\) reflections (\(I\) sinusoids) it equals \(p = 2I\) for \(I\) negative and \(I\) positive peaks in the frequency range. Ideally, the Hilbert transform cuts of the negative frequencies and reduces the model order to \(I\). However, due to the restricted measurement interval \(T\) or rather bandwidth \(B\), the peaks result from sinc functions instead of Dirac distributions. By cutting off sidelobes or even parts of the main lobe, additional peaks occur. Moreover, the division of the calibration affects the shape of the spectrum. The influence is visualized by a simulation depicted in Figure 2 (a) and (b). Bandpass, 0 % measurement and multiple reflections are neglected for this purpose. Uncalibrated positions of 60 cm, 60.4 cm and 61.4 cm for 100 % position and two reflectors are used, respectively. Due to calibration, the two reflections occur at 0.4 cm and 1.4 cm. The Fourier transformed signal exhibits small additional (artificial) peaks. The reflectors are not separable for MUSIC for the ideal model order \(p = 2\) and are unaffected (neglecting preprocessing) for model order \(p = 4\). Hence, the preprocessing affects the signal shape and significantly influences the model order. Additional peaks model the artifacts as well as the reflector positions for a minimum value of \(p = 22\) for MUSIC \((p = 38\) for the modified covariance method).

![Figure 2](image)

**Figure 2.** (a), (b): FT of calibrated signal (green) and pseudo spectrums of MUSIC algorithm with different model orders \(p\); from (a) to (b) the x-axis is rescaled, (c) X-ray image of multilayer tube wall section, (d) thickness results by the correlation approach.

4. Simulation and Measurement Results

Figure 2 (c) shows an X-ray image of the cross-section of a tube wall, which was inspected by the setup described in section 2 at the position indicated by the arrow. The thickness of the sample is approximately 1.3 cm. The obtained FT signal, presented in Figure 3 (a), shows that the 4 main peaks from the 4 boundary surfaces are interfering to 2 peaks. For model orders of \(p = 4\) (ideal) and \(p = 8\) (unaffected), the peaks are not resolvable: Fitted model orders of \(p = 850\) for the covariance method and \(p = 400\) for the MUSIC algorithm separate them but result in inaccurate thicknesses. The best fit...
of the correlation approach results in positions which are marked by the vertical lines. This approach enables an accurate thickness evaluation.

Additional equidistant points are measured along the tube. The correlation approach reliably determines the thickness for all positions as depicted in Figure 2 (d). The evaluated positions are used for further investigations on model order and SNR. In [12], the influence of noise to our setup was analyzed, which resulted in an estimated time domain value of $SNR_{TD} = 26$ dB and an indication of AWGN. For these values, the MUSIC algorithm identifies the layers for $p = 200$ but not for $p = 100$. The present measurement results in contrast, may not be as accurate due to a nonlinearity of the frequency modulation or multiple reflections. However, in case of increased $SNR_{TD}$ in Figure 3 (b), lower model orders are sufficient. As expected, higher SNRs tend to increase the accuracy of the distance evaluation as shown in [3] and hence, the accuracy of the estimated thicknesses.

![Figure 3](image-url)

**Figure 3.** (left) FT of measured signal, pseudo spectrum of modified covariance method (pmcov) and MUSIC algorithm (pmusic), (right) results of MUSIC for varying $SNR_{TD}$ and model order

### 5. Summary

Modified covariance and MUSIC methods were utilized for FMCW thickness measurements and compared with a model-based signal correlation approach. For the estimation algorithms, the required model order is significantly influenced by SNR and preprocessing including Hilbert transform. Fitting the model order results in the separation of peaks closer than the Rayleigh resolution limit, but accuracies less than the ones of our correlation approach are observed. The correlation approach instead reliably resolves the layers for different measurement point along the presented samples even for lower SNRs. Our future work will focus on optimization algorithms, in order to reduce the calculation complexity of our correlation approach.

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