Lepton Flavor Violation in Supersymmetric SO(10) Grand Unified Models

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Abstract

The study for lepton flavor violation combined with the neutrino oscillation may provide more information about the lepton flavor structure of the grand unified theory. In this paper, we study two lepton flavor violation processes, $\tau \rightarrow \mu \gamma$ and $Z \rightarrow \tau \mu$, in the context of supersymmetric SO(10) grand unified models. We find the two processes are both of phenomenological interest. In particular the latter may be important in some supersymmetric parameter space where the former is suppressed. Thus, Z–decay may offer another chance for looking for lepton flavor violation.

I. INTRODUCTION

Super-Kamiokande data on the atmospheric neutrino anomaly presents a strong evidence for the existence of neutrino oscillations. The anomaly can be explained by $\nu_\mu - \nu_\tau$ oscillation
with $\delta m_{23}^2 = (1 - 8) \times 10^{-3} eV^2$ and a large mixing angle $\sin^2(2\theta_{\mu\tau}) = 0.8 - 1$. In addition, the long standing solar neutrino deficit can also be interpreted as another type of neutrino oscillations. Assuming that the LSND anomaly will finally disappear, all these observations about neutrinos can be accommodated in a model with three very light active left-handed neutrinos. Many such models have been proposed to explain the measured neutrino parameters since Super-Kamiokande first published the data. Among them a natural explanation of the neutrino masses is provided by grand unified models which interpret the very light neutrino masses compared with the quarks and charged leptons by the see-saw mechanism. Especially in SO(10) grand unified models, in which the right-handed (RH) neutrinos have masses of the order about unification scale and the lepton masses and the quark masses are related, correct neutrino masses can be obtained. The measured neutrino masses are even regarded as a new evidence supporting the grand unification idea. As been argued in Ref. a value for $m(\nu_\tau) \sim \frac{1}{20} eV$ falls into a natural range predicted by a grand unified model based on either a string-unified G(224) model or a SO(10) grand unified model. Furthermore, grand unified theory (GUT) has the advantage that it relates the neutrino problem, its masses and mixings, with charged lepton and quark masses and mixings into a large fermion flavor problem and thus gives more definite predictions.

Conversely, the parameters measured in the neutrino sector can provide a window on the grand unified theory study and they also give important feedback on the problems of quarks and charged lepton masses as they are all related in the GUT. A direct inference of neutrinos being massive is the existence of a Kabayashi-Maskawa like matrix for the lepton sector. However, the processes violating lepton flavors due to this matrix is too small to be observed because of the tiny neutrino masses. In the supersymmetric GUTs the high energy lepton flavor violation (LFV) interactions may leave trace in the mass matrices of scalar partners of leptons by renormalization effects, which drives low energy LFV processes since the mass matrix of the charged leptons and that of sleptons can not be diagonalized.
simultaneously. Therefore, the study of the LFV process may provide important information on the flavor structure of the supersymmetric GUTs. Various LFV processes have been studied in different theoretical frameworks by a number of authors [10–14].

There are two purposes of this work. One is to study the LFV processes in the context of supersymmetric SO(10) grand unified models with a “lopsided” texture for the mass matrices of the down quark and charged lepton which has been advocated by a number of authors to accommodate the large $\nu_\mu - \nu_\tau$ mixing and small $V_{cb}$ [15,16,8]. If the experimental sensitivity on $\tau \rightarrow \mu \gamma$ can reach down to $1 \times 10^{-9}$ [10], not only this process can be observed but it can even be used to discriminate different fermion textures in the grand unified models. Another purpose of this work is to present the analytic formulas for the branching ratio of the process $Z \rightarrow \tau \mu$ in the minimal supersymmetric standard model (MSSM) and the numerical study in the class of models considered by us. This calculation seems absent in the literature. This calculation is triggered by the Giga-Z option of the Tesla project which may expect the $10^9$ $Z$ bosons at resonance [17]. The upper limit for the branching ratios could be improved down to $BR(Z \rightarrow \mu^\pm \tau^\mp) < f \times 2.2 \times 10^{-8}$ with $f = 0.2 \sim 1$. We find this process is quite interesting phenomenologically since in some supersymmetric parameter space $\tau \rightarrow \mu \gamma$ being suppressed, this process can be important however.

The paper is arranged as follows. In sec II we discuss the origin of the LFV interactions in a supersymmetric grand unified model and the Renormalization Group Equations (RGE). In sec III we present the formulas for the branching ratios of the two processes. In Sec IV we briefly introduce a grand unified model proposed in Ref [13] that we used in our calculations. The numerical results are presented in Sec V and we summarize and give conclusions in Sec VI.
II. RENORMALIZATION GROUP EQUATIONS AND LOW ENERGY
SUPERSYMMETRIC SPECTRUM

A. Origin of Lepton Flavor Violation

If the SM is extended with massive and non-degenerate neutrinos, LFV processes may be
induced. However, such processes are highly suppressed due to the smallness of the neutrino
masses. The branching ratio is proportional to $\delta m^2_\nu / M^2_W$ which is hopeless to be observed
\[\text{[18]}\]. When supersymmetry enters the theory the scene changes completely. The LFV may
also be induced through the generation mixing of the supersymmetric soft breaking terms
of the lepton sector. However, arbitrary mixing of these soft terms in the MSSM are not
predictive. In our calculations we adopt the supergravity mediated supersymmetry breaking
mechanism to produce universal soft terms at the GUT scale $M_{GUT}$, because non–universal
soft terms at $M_{GUT}$ may produce too large low energy LFV observable effects \[\text{[10][11]}\]. The
tree level universal soft terms may induce non-diagonal terms at low energy by radiative
corrections including LFV interactions at high scale. Our procedure includes calculating the
low energy supersymmetric soft terms which is not generation universal now by integrating
the RGEs and then calculate the LFV branching ratios induced by these non-universal soft
terms.

For a supersymmetric SO(10) grand unified model, the structure below $M_{GUT}$ where the
grand unification has been spontaneously broken is the same as the MSSM supplemented
with MSSM singlet RH neutrino superfields. The superpotential of the lepton sector is now
\[ W = f^{ij}_\nu \hat{H}_2 \hat{L}_i \hat{N}_j + f^{ij}_l \hat{H}_1 \hat{L}_i \hat{E}_j + \frac{1}{2} M^{ij} \hat{N}_i \hat{N}_j + \mu \hat{H}_1 \hat{H}_2 \quad (2.1) \]
where $f_\nu$ and $f_l$ are the Yukawa coupling matrices, $M$ is the RH neutrino mass matrix. $i,$
j are the generation indices. Anti-symmetric tensor $\epsilon^{ab}$ is implicit to contract the SU(2)
doublets with \( \epsilon^{12} = -1 \). In general, \( f_\nu \) and \( f_l \) cannot be diagonalized simultaneously, which is the origin of LFV interactions. Diagonalize \( f_\nu \) and \( f_l \) by bi-unitary rotations,

\[
\begin{align*}
  f^\delta_l &= U_{L,R}^\dagger f_l U_{R} \\
  f^\delta_\nu &= V_{L}^\dagger f_\nu V_{R}
\end{align*}
\]  

(2.2)

where \( U_{L,R}, V_{L,R} \) are all unitary matrices. Then define

\[
V_D = U_L^\dagger V_L
\]

(2.3)

which is analog to the KM matrix \( V_{KM} \) in the quark sector. \( V_D \) is crucial for LFV processes. The RH neutrino masses are not much lower in order compared to \( M_{GUT} \) in SO(10) grand unified models. After the RH neutrinos are decoupled and \( H_2 \) gets VEV \( v_2 \) of weak scale we get three light left–handed (LH) Majorana neutrinos with mass matrix \( m_\nu = -(f_\nu v_2) M^{-1}(f_\nu v_2)^T \) by see–saw mechanism. Suppose \( m_\nu m_\nu^\dagger \) is diagonalized by \( V_L' \), then the matrix

\[
V^{MNS} = U_L^\dagger V_L'
\]

(2.4)

determines the neutrino oscillation parameters. In the published grand unified models which emphasize the neutrino oscillations the large \( \nu_\mu - \nu_\tau \) mixing angle in \( V^{MNS} \) is mainly coming from \( U_L^\dagger \). Thus, on the one hand, we may expect large rates for LFV processes due to large \( \mu - \tau \) mixing in \( U_L^\dagger \), which at the same time causes large \( \nu_\mu - \nu_\tau \) mixing in \( V^{MNS} \) for neutrino oscillations, and, on the other hand, difference between \( V_D \) and \( V^{MNS} \) may be found by neutrino oscillations and LFV processes. If so, important information about GUT structure may be derived then.

The soft breaking terms for the lepton sector is \[\text{\cite{19}}\],

\[
\mathcal{L}_{soft} = -m^2_{H_1} H_1^\dagger H_1 - m^2_{H_2} H_2^\dagger H_2 - (m^2_{L})^{ij} \tilde{L}_i^\dagger \tilde{L}_j - (m^2_{R})^{ij} \tilde{R}_i^\dagger \tilde{R}_j - (m^2_{\nu})^{ij} \tilde{\nu}_i^\dagger \tilde{\nu}_j
\]
\[
\begin{aligned}
&+ \left( B_{\mu} H_1 H_2 + \frac{1}{2} B M^{ij} \tilde{\nu}_i^\ast \tilde{\nu}_j^\ast + (A_E f_\nu)^{ij} H_1 \tilde{L}_i \tilde{R}_j \\
&+ (A_\nu f_\nu)^{ij} H_2 \tilde{L}_i \tilde{\nu}_j + h.c. \right) 
\end{aligned}
\] (2.5)

where \(i, j\) are generations indices. At \(M_{\text{GUT}}\) we assume the universal conditions,

\[
\begin{align*}
    m^2_{H_1} &= m^2_{H_2} = m^2_0 \\
    m^2_\tilde{L} &= m^2_\tilde{R} = m^2_\tilde{\nu} = m^2_0 \\
    A_E &= A_\nu = A_0
\end{align*}
\] (2.6, 2.7, 2.8)

Fig. 1 gives the explanation of the occurrence of low energy LFV processes. The non-diagonal Yukawa couplings induce \(\tilde{\nu}_\mu - \tilde{\nu}_\tau\) mixing through loop effects. This high energy process can be running down by integrating the RGEs to low energy. Thus the study of LFV processes induced by non-universal soft terms can actually reveal high energy fermion flavor structure. In the basis where the \(f_i\) is diagonal we can get the non-diagonal scalar mass in the first order of approximation as

\[
\begin{aligned}
    (\delta \tilde{m}^2)_{23} &\approx \frac{1}{8\pi^2} f_\nu f_\nu^\dagger (3 + a^2) m^2_0 \log \frac{M_{\text{GUT}}}{M_R} \\
    &\approx \frac{1}{8\pi^2} (V_D)_{23} (V_D)_{33} \cdot f^2_{\nu_3} (3 + a^2) m^2_0 \log \frac{M_{\text{GUT}}}{M_R}
\end{aligned}
\] (2.9)

where in the diagonalized Yukawa matrix \(f^\dagger_\nu\) only the (3,3) element \(f_{\nu_3}\) is kept. \(a\) is the universal parameter \(A_0 = am_0\) and \(M_R\) is the scale where the RH neutrinos are decoupled.

**B. RGEs running**

The RGEs used by us are given in the Appendix A. We have paid much effort to keep the RGEs for the soft terms of lepton sector, which is relevant to our calculations, as complete
as possible. Specially speaking, both the diagonal and non-diagonal terms are kept in the RGEs. However, only diagonal terms of the scalar quark soft terms are kept because these non-diagonal terms are small and not relevant to our calculations. For the Yukawa sector, only the (3,3) elements of the diagonalized Yukawa matrices $f_t$, $f_b$, $f_\tau$ and $f_{\nu_3}$ are kept. The evolution of the mixing matrix $V_D$ from $M_{GUT}$ to $M_R$ is ignored.

The integration procedure consists of iterative runnings of the RGEs from the GUT scale to the low energy scale $M_Z$ and back for every set of inputs of $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ and the sign of $\mu$, which are the universal scalar masses, gaugino masses, scalar trilinear parameter, and the standard vacuum expectation values ratio of the two Higgs fields and the Higgsino mass parameter, until the low energy gauge couplings and the Yukawa couplings are all correct within a given range. The parameters $\mu$ and $B$ are given by low energy spontaneous breaking conditions \cite{19,20}.

The RGEs at Appendix A is given in the basis where $f_\nu$ is diagonal. The RH neutrinos are decoupled at $M_R$. Below $M_R$ the basis is rotated to the basis where $f_l$ is diagonal. The MSSM RGEs are obtained by simply dropping the terms including $f_\nu$ and setting $V_D$ equal to 1. The basis rotation leads to

$$
(m_2^L)_{\text{below}} = V_D (m_2^L)_{\text{up}} V_D^T \\
(A_E)_{\text{below}} = V_D (A_E) V_D^T .
$$

Below $M_{SUSY} = 250 GeV$ the SM beta functions are used \cite{21}. Threshold effects are taking into account by decoupling the corresponding particles at its running mass $Q = m(Q)$.

Taking $\tan \beta = 2 \sim 10$, we show the numerical results as follows

$$
\delta m_L^2 = \begin{pmatrix}
0 & (0.92 \sim 2.87) \times 10^{-3} & (-0.77 \sim -2.06) \times 10^{-3} \\
(0.92 \sim 2.87) \times 10^{-3} & 0 & (0.97 \sim 3.0) \times 10^{-2} \\
(-0.77 \sim -2.06) \times 10^{-3} & (0.97 \sim 3.0) \times 10^{-2} & 0
\end{pmatrix} (3 + a^2) m_0^2 \tag{2.11}
$$
where smaller \( \tan \beta \) may give larger contribution.

C. Low energy supersymmetric spectrum

At low energy the supersymmetric particle masses and mixing angles are obtained by diagonalizing the corresponding chargino, neutralino, scalar neutrino and scalar lepton mass matrices numerically. The slepton mass matrix is given by a \( 6 \times 6 \) matrix as

\[
A_E = \begin{pmatrix}
0.7 & (2.42 \sim 8.12) \times 10^{-4} & (-2.41 \sim -5.88) \times 10^{-4} \\
(2.42 \sim 8.12) \times 10^{-4} & 0.7 & (2.5 \sim 8.67) \times 10^{-3} \\
(-2.41 \sim -5.88) \times 10^{-4} & (2.5 \sim 8.67) \times 10^{-3} & 0.7
\end{pmatrix} A_0
\]  

(2.12)

The \( m^2_{\tilde{\nu}} \) is diagonal since only \( f_l \) enters its RGE. The full sneutrino mass matrix has a \( 12 \times 12 \) structure. However, according to \([10,11]\) if we only keep the first order of these terms perturbed by RH neutrino masses we can have a very simple structure, which is relevant to generation mixing,

\[
m^2_{\tilde{\nu}} = m^2_{\tilde{\nu}_L} + \frac{1}{2} M_Z^2 \cos 2\beta
\]  

(2.15)

The mass matrices of chargino and neutrino are standard and given at Appendix B.
III. ANALYTIC FORMULAS

Fig. 2 gives the one loop diagrams relevant to the process $\tau \rightarrow \mu \gamma$. The amplitude of this process can be written in the general form

$$M = e m_i \bar{u}_j(p_2) i \sigma_{\mu \nu} p_3 \nu (A_L P_L + A_R P_R) u_i(p_1) e^\mu(p_3) ,$$  \hspace{1cm} (3.1)

where $P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)$ are the chirality projection operators. $i, j$ represent initial and final lepton flavor. The most convenient way to calculate $A_L$ and $A_R$ is to pick up the one loop momentum integral contributions which are proportional to $\bar{u}_j(p_2) P_{L,R} u_i(p_1) 2 p_1 \cdot \epsilon$ respectively. The neutralino exchanging contribution is

$$A_L^{(n)} = \frac{1}{32\pi^2} \left( \frac{e}{\sqrt{2} \cos \theta} \right)^2 \frac{1}{m_{\tilde{\nu}}^2} \left[ B_{j\alpha a} B^{i\alpha a} \frac{1 - 6k + 3k^2 + 2k^3 - 6k^2 \log k}{6(1 - k)^4} + \frac{m_{\chi^0_a} B_{j\alpha a}}{m_i} A_{i\alpha a} \frac{1 - k^2 + 2k \log k}{(1 - k)^3} \right] ,$$

$$A_R^{(n)} = A_L^{(n)} \text{ } (B \leftrightarrow A) ,$$

where $k = m_{\chi^0_a}^2 / m_{\tilde{\nu}}^2$. $A$ and $B$ are the lepton–slepton–neutralino coupling vertices given in Appendix B. The corresponding contribution coming from exchanging charginos are

$$A_L^{(c)} = - \frac{g_2^2}{32\pi^2} Z_{i\nu}^a Z_{i\nu} \frac{1}{m_{\tilde{\nu}}^2} \left[ Z_{2a}^{-} Z_{2a}^{-} \frac{m_j m_j}{2 M_W^2 \cos^2 \beta} \frac{2 + 3k - 6k^2 + k^3 + 6k \log k}{6(1 - k)^4} + \frac{m_{\chi^0_a}}{\sqrt{2} M_W \cos \beta} Z_{1a}^+ Z_{2a}^+ \frac{m_j}{m_i} \frac{3 - 4k + k^2 + 2 \log k}{(1 - k)^3} \right] ,$$

$$A_R^{(c)} = - \frac{g_2^2}{32\pi^2} Z_{i\nu}^a Z_{i\nu} \frac{1}{m_{\tilde{\nu}}^2} \left[ Z_{1a}^+ Z_{1a}^+ \frac{2 + 3k - 6k^2 + k^3 + 6k \log k}{6(1 - k)^4} + \frac{m_{\chi^0_a}}{\sqrt{2} M_W \cos \beta} Z_{1a}^+ Z_{2a}^+ \frac{3 - 4k + k^2 + 2 \log k}{(1 - k)^3} \right] ,$$

where $k = m_{\chi^0_a}^2 / m_{\tilde{\nu}}^2$. Mixing matrices $Z_{ij}$, $Z^+$ and $Z^-$ are given in Appendix B.

The branching ratio is given by
\[ BR(\tau \rightarrow \mu \gamma) = \frac{\alpha_{em}}{4\pi} m_\tau^5 (|A_L|^2 + |A_R|^2)/\Gamma_\tau, \tag{3.6} \]

where \( \Gamma_\tau = 2.265 \times 10^{-12} \text{GeV}. \)

Fig. 3 gives the diagrams contributing to the \( Z \rightarrow \tau\mu \) process. Neglecting masses of the final fermions the amplitude is given as

\[ M = \sum_i a_i M_i, \tag{3.7} \]

\[ M_{1,2} = \bar{u}(p_2)\gamma^\mu P_L R v(p_1)\epsilon_\mu, \tag{3.8} \]

respectively. The corresponding analytic expressions of \( a_i \) are given in appendix C.

The branching ratio for \( Z \rightarrow \tau\mu \) is given by

\[ \text{Br}(Z \rightarrow \mu\tau) = \frac{\Gamma(Z \rightarrow \tau^\pm\mu^{\mp})}{\Gamma_Z}, \]

where \( \Gamma_Z = 2.49 \text{GeV} \) and

\[ \Gamma(Z \rightarrow \mu\tau) = \frac{1}{48\pi M_Z} \cdot \sum |M|^2 \]

\[ = \frac{1}{48\pi M_Z} \cdot \left[ (|a_1|^2 + |a_2|^2) \cdot 2M_Z^2 + (|a_3|^2 + |a_4|^2) \cdot \frac{M_Z^4}{4} \right], \tag{3.9} \]

where \( \Sigma \) represents the sum of \( \mu, \tau \) spins and \( Z \) polarizations.

IV. A SO(10) GRAND UNIFIED MODEL

To give definite predictions we will work within a specific model. Before we turn to introduce this model it is worth noting that our calculations are not very model sensitive. That is the reason why we concentrate on the 2–3 generation mixing. The mixing between the first two generations may be very sensitive to different models and we will discuss the processes separately. From Fig.1, we know that \( \tau - \mu \) mixing is mainly determined by the factor \( \delta m_{23}^2/m_0^2 \), which, from Eq. (2.9), depends only on the parameters \((V_D)_{23} \cdot (V_D)_{33}, \ldots\).
$f_{\nu_3}$, $M_{GUT}$ and $M_R$. $M_{GUT} \approx 2 \times 10^{16}$GeV is required by any supersymmetric grand unified model. In the context of SO(10) grand unified models $f_{\nu_3}$ is related to the top Yukawa coupling $f_t$ at the GUT scale. In addition, with the SuperK data of $m_{\nu_\tau} \approx \frac{1}{20}$eV $M_R \approx 2 \times 10^{14}$GeV is roughly determined by the see–saw mechanism. Thus the only model sensitive parameter in fermion textures is $(V_D)_{23}$ ( $(V_D)_{33}$ is determined by unitary condition of the $V_D$ matrix ). In most published SO(10) models the large $\nu_\mu - \nu_\tau$ mixing is not produced dominantly by $M_R$ mass matrix and thus produce large $\mu - \tau$ mixing, for example, we have $(V_D)_{23} \cdot (V_D)_{33} \sim (0.5 - 0.3)$ corresponding to $\theta_{23} \sim (20^\circ - 70^\circ)$. Thus our discussions on these branching ratios are useful for estimating the predictions for such a class of unified models, although they may be different in details.

We did our calculations within the model given by Albright et al [15]. The model gives excellent predictions of quark and lepton masses and naturally explains the largeness of $\nu_\mu - \nu_\tau$ mixing and the smallness of $V_{cb}$. At $M_{GUT}$, after the SO(10) breaking to the MSSM, the fermion mass textures are given by

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & \epsilon/3 & 1 \end{pmatrix} M_U , \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon/3 \\ \delta' e^{i\phi} \sigma + \epsilon/3 & 1 \end{pmatrix} M_D , \quad (4.1)$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} M_U , \quad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon \\ \delta' e^{i\phi} -\epsilon & 1 \end{pmatrix} M_D , \quad (4.2)$$

where $U$, $D$, $N$ and $L$ are the up quark, down quark, neutrino and lepton mass matrices respectively. The most remarkable feature of the textures is the lopsided parameter $\sigma \sim 1$ at $L$ and $D$. According to the SU(5) relation $D = L^T$, the large $\nu_\mu - \nu_\tau$ mixing due to the $\sigma$ term in $L$ is related to the right–handed down quark mixing, which has no observable physical effects. The smallness of quark mixing $V_{cb}$ is determined by the parameter $\epsilon \sim 0.1,$
which translates to the right-handed lepton mixing.

The model predicts the lepton sector 2-3 mixing with $\theta_{23} \sim 63^\circ$, which leads to the only model structure sensitive quantity for our processes $(V_D)_{23} \cdot (V_D)_{33} \sim 0.4$.

Taking all the fermion masses and $V_{KM}$ elements at $M_Z$ given in Ref. [22] as inputs, we calculate the corresponding values at $M_{GUT}$ with several values of $\tan \beta$ by integrating the RGEs and fit the parameters in Eq. (4.1) and (4.2). We find these parameters are not sensitive to the supersymmetric parameters, except that $M_U$ becomes larger as taking small $\tan \beta$. To keep the predicted neutrino masses in the correct range we take $M_R = 5 \times 10^{14} GeV$ when $\tan \beta = 2$ and $M_R = 2 \times 10^{14} GeV$ when $\tan \beta = 5, 10$ in our later calculations.

V. NUMERICAL RESULT AND DISCUSSION

The relevant parameters on predicting the branching ratios are the universal supersymmetry soft breaking parameters, $m_0, m_{\frac{1}{2}}, A_0$, $\tan \beta$ and the lepton sector mixing matrix $V_D$ defined in Eq. (2.3). We will present dependence of the branching ratio of the processes $\tau \rightarrow \mu \gamma$ and $Z \rightarrow \tau \mu$ on the soft parameters in the model of [13]. In our calculations the soft parameters are constrained by various theoretical and experimental considerations [23], such as the LSP of the model should be a neutralino and masses of all supersymmetric particles be beyond present mass limits and so on.

In Figs. 4-7, we plot the branching ratio of $\tau \rightarrow \mu \gamma$ as functions of $m_0$ for several sets of other soft parameters. Two experimental bounds are plotted in every figure, which correspond to the present experimental limit $BR(\tau \rightarrow \mu \gamma) < 1.1 \times 10^{-6}$ [24] and the expected sensitivity of future experiments $BR(\tau \rightarrow \mu \gamma) < 1.0 \times 10^{-9}$ [11]. The general trend of $BR(\tau \rightarrow \mu \gamma)$ is decreasing dramatically as $m_0$ increases. In Fig. 4, the branching ratio is plotted for $m_{\frac{1}{2}} = 100 GeV$, $\tan \beta = 5$ and $A_0 = 2m_0$, $m_0$, 0, taking $\mu$ either positive
or negative. That branching ratio decreases with $a = A_0/m_0$ is easily understood due to the Eq. (1). It is interesting to note that although this process can not be observed at present, it will be detectable in the future experiments in a large part of the soft parameter space.

Fig. 5 presents the branching ratio for $m_{\pm} = 100, 200, 400 GeV$ and $A_0 = m_0$, $\tan \beta = 5$. We can find that the branching ratio is also very sensitive to the parameter $m_{\pm}$. It will decrease quickly with increasing $m_{\pm}$. If $m_{\pm} > 400 GeV$, this process will be non-observable as displayed in the figure. In Fig. 6, we plot the branching ratio for $m_{\pm} = 100 GeV$, $\tan \beta = 10$ and $A_0 = \pm 2m_0$, $\pm m_0$, 0. We find that the sign of $A_0$ is not very important in the order of magnitude estimate. However, the sign of $A_0$ is still relevant to the precise predictions. Fig. 7 plots the branching ratio at different $\tan \beta$. The branching ratio increases as $\tan \beta$ becomes large. It is explained in Ref. [12] that the dominant term of the amplitude for $\tau \rightarrow \mu\gamma$ is proportional to $\tan \beta$. We note that at $\tan \beta = 2$ the sign of $\mu$ can be significant. The branching ratio when $\mu$ is negative is much smaller than that when $\mu$ is positive.

In Figs. 8~10, we present the quantitative results of the branching ratio of the decay $Z \rightarrow \tau\mu$. We find in most parameter space this process can not be detected by the Giga-Z option [17]. However, we can still find several interesting features in this process which are different from the $\tau$ radiative decay process. The most remarkable feature about this LFV process is that its branching ratio becomes large at first and then approaches to a constant when $m_0$ is increasing, which gives a sharp contrast with the process $\tau \rightarrow \mu\gamma$. Thus this process may have advantage over the $\tau \rightarrow \mu\gamma$ process in some region of parameter space. The reason for this different behavior between the two processes can be traced back to the different coupling structures between the processes as shown in Eqs. (3.1) and (3.7), (3.8). The magnetic structure of $\tau$ decay determines that its amplitude is inversely proportional to the sfermion masses square, whereas there is a vector current coupling in Eq. (3.7) which
determines the $Z \to \mu \tau$ amplitude’s trend as increasing $m_0$.

Fig. 8 shows the branching ratio as a function of $m_0$ for $m_{\frac{1}{2}} = 100 GeV$, $\tan \beta = 5$ and $A_0 = 2m_0$, $m_0$, 0 with both positive and negative sign of $\mu$. The sign of $\mu$ is quite irrelevant when $m_0 > 500 GeV$. Fig. 9 displays the same function for different $m_{\frac{1}{2}}$ values, 100, 200, 400 GeV, with $A_0 = 2m_0$ and $\tan \beta = 5$. The dependence of $BR(Z \to \tau \mu)$ on $a$ and $m_{\frac{1}{2}}$ is similar to that of $BR(\tau \to \mu \gamma)$. We note again that the sign of $\mu$ is irrelevant when $m_{\frac{1}{2}} > 200 GeV$. Fig. 10 gives the branching ratio for $\tan \beta = 2$, 5, 10 with $A_0 = 2.5m_0$, $m_{\frac{1}{2}} = 150 GeV$. We can see another important feature of this LFV process that it is more favorable for the small $\tan \beta$ value in contrast with the $\tau \to \mu \gamma$ process. At the extreme case, the branching ratio may access $1 \times 10^{-8}$, which may be detectable. The relationship between the branching ratio and $\tan \beta$ is easily understood. For small $\tan \beta$ large non–universal soft mass term $\delta \tilde{m}^2_{3i}$ will be produced due to a large $f_{\nu_3}$ in Eq. (2.9). We note that when $\tan \beta$ becomes smaller, the $BR(Z \to \tau \mu)$ increases quickly.

In summary, according to our calculations we find that the $\tau \to \mu \gamma$ is more feasible than $Z \to \tau \mu$ to reveal charged lepton flavor violation in the context of SO(10) SUSY–GUTs. In most parameter space $\tau \to \mu \gamma$ has a branching ratio that can be detected in the future experiments [10]. The $Z \to \tau \mu$ is hopeful to be detected in a small parameter space. We note the remarkable feature of the $Z$ decay process that its dependence on the supersymmetry soft parameters $\tan \beta$ and $m_0$ is opposite to that of $\tau$ decay. Thus it can supplement the LFV search besides the $\tau \to \mu \gamma$ process.

VI. SUMMARY AND CONCLUSION

In this work, we present the dependence of the branching ratios of two charged lepton flavor violation processes $\tau \to \mu \gamma$ and $Z \to \tau \mu$ on the supersymmetry soft breaking parameters
in the context of SO(10) grand unified models with the "lopsided" texture of mass matrices for charged leptons and down quarks. We expect these processes may be detected in the future experiments. The first process is more hopeful to be observed. The second process may offer useful information about the soft parameters if it is also observed. The different behaviors of the two processes depending on $\tan \beta$ and $m_0$ implies that the simultaneous study of the two processes will be interesting.

We expect the study of charged lepton flavor process may provide another window of high energy physics besides the neutrino oscillation study. The combined study of the LFV processes and neutrino oscillation may feed light on the sector of right-handed neutrinos, which may be necessary in a model to naturally explain the small neutrino masses.

ACKNOWLEDGMENTS

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Appendix A

In this appendix we give the renormalization group equations of the MSSM supplemented with RH neutrinos. The two–loop RGEs of the gauge couplings can be found in many literatures which will not be affected by the presence of the RH neutrinos. We give one–loop RGEs of the Yukawa coupling matrices and the soft terms which are affected by the presence of RH neutrinos. In this appendix we denote the Yukawa couplings of up quark, down quark, lepton and neutrino as $U$, $D$, $E$ and $N$ respectively. Denote the soft terms as $A_U \cdot U = U_A$ and so on. The RGEs of the Yukawa couplings are

$$\frac{dU}{dt} = \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 + 3UU^\dagger + DD^\dagger + Tr[3UU^\dagger + NN^\dagger] \right] U ,$$  \hspace{1cm} (A.1)
\[
\begin{align*}
\frac{dD}{dt} &= \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 + 3DD^\dagger + UU^\dagger + Tr[3DD^\dagger + EE^\dagger] \right] D , \\
\frac{dE}{dt} &= \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 + 3EE^\dagger + NN^\dagger + Tr[3DD^\dagger + EE^\dagger] \right] E , \\
\frac{dN}{dt} &= \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 + 3NN^\dagger + EE^\dagger + Tr[3UU^\dagger + NN^\dagger] \right] N ,
\end{align*}
\]

where \( t = \log Q \), \( c_i = \left( \frac{13}{15}, 3, \frac{16}{3} \right) \), \( c'_i = \left( \frac{7}{15}, 3, \frac{16}{3} \right) \), \( c''_i = \left( \frac{9}{5}, 3, 0 \right) \), \( c'''_i = \left( \frac{3}{5}, 3, 0 \right) \). The RGEs of \( \mu \) and soft parameters of Higgs sector are

\[
\begin{align*}
\frac{d\mu}{dt} &= \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 + Tr[3UU^\dagger + 3DD^\dagger + EE^\dagger + NN^\dagger] \right] \mu , \\
\frac{dB}{dt} &= \frac{2}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i + Tr[3UU_A + 3DD_A + EE_A + NN_A] \right] , \\
\frac{dm_{H_u}^2}{dt} &= \frac{2}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i^2 + 3Tr[U(M_{Q_L}^2 + M_{U_R}^2)U^\dagger + m_{H_u}^2 UU^\dagger + U_A U_A^\dagger] \\
&\quad + Tr[M_{Q_L}^2 NN^\dagger + NM_{\nu}^2 N^\dagger + m_{H_u}^2 NN^\dagger + N_A N_A^\dagger] \right] , \\
\frac{dm_{H_d}^2}{dt} &= \frac{2}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i^2 + 3Tr[D(M_{Q_L}^2 + M_{D_R}^2)D^\dagger + m_{H_d}^2 DD^\dagger + D_A D_A^\dagger] \\
&\quad + Tr[E(M_L^2 + M_{R}^2)E^\dagger + m_{H_d}^2 EE^\dagger + E_A E_A^\dagger] \right] .
\end{align*}
\]

\( M_i \) in the above expressions are the gaugino masses whose RGEs are same as those in the MSSM.

Then we give RGEs of soft mass terms of lepton sector

\[
\begin{align*}
\frac{dM_L^2}{dt} &= \frac{2}{16\pi^2} \left[ -\Sigma c''_i g_i^2 M_i^2 + \frac{1}{2}[NN^\dagger M_L^2 + M_L^2 NN^\dagger] + \frac{1}{2}[EE^\dagger M_L^2 + M_L^2 EE^\dagger] \\
&\quad + EM_{R}^2 E^\dagger + m_{H_d}^2 EE^\dagger + E_A E_A^\dagger + NM_{\nu}^2 N^\dagger + m_{H_u}^2 NN^\dagger + N_A N_A^\dagger \right] , \\
\frac{dM_R^2}{dt} &= \frac{2}{16\pi^2} \left[ -\frac{12}{5} g_1^2 M_1^2 + E^\dagger EM_{R}^2 + M_R^2 E^\dagger E + 2(E^\dagger M_R^2 E + m_{H_d}^2 E^\dagger E + E_A E_A) \right] , \\
\frac{dM_{\nu}^2}{dt} &= \frac{2}{16\pi^2} \left[ N^\dagger NM_{\nu}^2 + M_{\nu}^2 N^\dagger N + 2(N^\dagger M_L^2 N + m_{H_u}^2 N^\dagger N + N_A N_A) \right] .
\end{align*}
\]

The RGEs of trilinear terms of lepton sector are

\[
\frac{dA_E}{dt} = \frac{1}{16\pi^2} \left[ -2\Sigma c''_i g_i^2 M_i + 2Tr(3A_D DD^\dagger + A_E EE^\dagger) + 2A_N NN^\dagger \right]
\]
In the basis where $N$ is diagonal and only keep the third family Yukawa coupling constants $f_t$, $f_b$, $f_{\nu_3}$ and $f_\tau$, then the RGEs are simplified as

\[
\frac{dA_t}{dt} = \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i + 2Tr(3A_tUU^\dagger + A_N NN^\dagger) + 2A_tEE^\dagger \right. \\
\left. + (5NN^\dagger + EE^\dagger)A_N + A_N (N^\dagger - EE^\dagger) \right].
\]

(A.12)

\[
\frac{dA_b}{dt} = \frac{1}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i + 2Tr(3A_bUU^\dagger + A_N NN^\dagger) + 2A_bEE^\dagger \right. \\
\left. + (5NN^\dagger + EE^\dagger)A_N + A_N (N^\dagger - EE^\dagger) \right].
\]

(A.13)
\[
\left( \frac{dA_E}{dt} \right)_{ij} = \frac{1}{16\pi^2} \left[ (-2\Sigma c''_i g_i^2 M_i + 6f_b^2 A_b + 2f_\tau^2 A_\tau)\delta_{ij} + V_{3i}^* V_{3j} f_\tau^2 (5(A_E)_{jj} + (A_E)_{ii}) + f_{\nu_3}^2 (2(A_\nu)_{3i} \delta_{j3} + (A_E)_{3j} \delta_{i3} - (A_E)_{33} \delta_{ij}) \right], \tag{A.27}
\]

\[
\left( \frac{dA_\nu}{dt} \right)_{ij} = \frac{1}{16\pi^2} \left[ (-2\Sigma c''_i g_i^2 M_i + 6f_i^2 A_i + 2f_{\nu_3}^2 A_{\nu_3})\delta_{ij} + V_{3i}^* V_{3j} f_\tau^2 (2(A_E)_{ii} + (A_\nu)_{ii} - (A_\nu)_{ii}) + f_{\nu_3}^2 (5(A_\nu)_{3j} \delta_{i3} + (A_\nu)_{3i} \delta_{ij}) \right]. \tag{A.28}
\]

The matrix \( V \) in the above equations refers to \( V_D \) defined in Eq. (2.3). Below \( M_R \), RH neutrinos are decoupled and the RGEs of the MSSM are used. This can be achieved by setting \( f_{\nu_3} = 0 \) and \( V_{ij} = \delta_{ij} \) in the above expressions.

## Appendix B

In this appendix we list the relevant pieces of the Lagrangian and conventions which we take in our calculations [19]. The Lagrangian pieces are

\[
\mathcal{L}_{\tilde{\nu}\tilde{Z}} = \frac{-ig_2}{2\cos\theta_W} (\tilde{\nu}^{I*} \bar{\tilde{\nu}}' \bar{\nu}' ) Z^\mu, \tag{B.1}
\]

\[
\mathcal{L}_{\tilde{\nu}\tilde{Z}} = \frac{ig_2}{\cos\theta_W} \left( \frac{1}{2} Z_L^{\mu} Z_L^{\mu*} - \sin^2 \theta_W \delta_{ij} \right) (\tilde{i}^I_{ij} \bar{\tilde{i}}^I_{ij} ) Z^\mu, \tag{B.2}
\]

\[
\mathcal{L}_{X^+X^-Z} = \frac{g_2}{2\cos\theta_W} \chi^I_{ij} \gamma^\mu \left[ (Z_{ij}^{N} Z_{ij}^{N} + Z_{ij}^{N} Z_{ij}^{N} ) P_L + \delta_{ij} \cos 2\theta_W ] \chi^0_{ij} Z^\mu \right. \tag{B.3}
\]

\[
\mathcal{L}_{X^0X^0Z} = \frac{g_2}{2\cos\theta_W} \chi^I_{ij} \gamma^\mu \left[ C^{ij} P_L + D^{ij} P_R \right] Z^\mu \chi^I_{ij} \mu \tag{B.4}
\]

\[
\mathcal{L}_{X^+1\tilde{\nu}} = -g_2 \tilde{i}^I_{ij} \left[ Z_{ij}^{N} P_R - \frac{m_{\nu_i}}{\sqrt{2} M_W \cos \beta} Z_{2i} P_L \right] Z_{ij}^{N} \chi^0_{ij} \tilde{\nu}_j + h.c. \tag{B.5}
\]

\[
\mathcal{L}_{X^{0\tilde{\nu}_i}} = \frac{e}{\sqrt{2} \cos \theta_W} \chi^0_{ij} \left[ Z_{ij}^{N} (Z_{ij}^{N} + Z_{ij}^{N} \cot \theta_W) - \cot \theta_W \frac{m_{\nu_i}}{M_W \cos \beta} Z_{ij}^{N} (Z_{ij}^{N} + Z_{ij}^{N} \cot \theta_W) \right] P_L \tilde{\nu}^0_{i} + h.c. \tag{B.6}
\]

The abbreviations defined in the above expressions will be used in the next Appendix.
The mixing matrices $Z$ in the above expressions are given below. The scalar lepton and scalar neutrino mass matrices are given in Sec II. The corresponding mixing matrices are defined as

$$Z_L^T m^2 Z_L = \text{diag} \left( m^2_{_L i} \right), \; i = 1 \ldots 6$$  \hspace{1cm} (B.7)

$$Z_{
u}^T m^2_{\nu} Z_{\nu} = \text{diag} \left( m^2_{\nu i} \right), \; i = 1, 2, 3$$  \hspace{1cm} (B.8)

The mass matrix of charginos is

$$M_\chi = \begin{bmatrix} m_2 & \sqrt{2} M_W \sin \beta \\
\sqrt{2} M_W \cos \beta & \mu \end{bmatrix}.$$  \hspace{1cm} (B.9)

The mixing matrices $Z^\pm$ is defined as

$$(Z^-)^T M_\chi Z^+ = \text{diag} \left( m_{\chi_1}, m_{\chi_2} \right).$$  \hspace{1cm} (B.10)

The mass matrix for neutralinos is

$$M_{\chi^0} = \begin{bmatrix} m_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\
0 & m_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & -\mu \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & -\mu & 0 \end{bmatrix},$$  \hspace{1cm} (B.11)

which is diagonalized by

$$Z_N^T M_{\chi^0} Z_N = \text{diag} \left( m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0} \right).$$  \hspace{1cm} (B.12)

**Appendix C**

In this appendix we give the analytic expressions of $a_i$ defined in Sec III.
\[ a_1 \text{ is given by} \]
\[
   a_1 = \frac{g_2}{2 \cos \theta_W} \frac{g_2^2}{16 \pi^2} (a_1(a) + a_1(b) + a_1(c) + a_1(d) + 2a_1(e) + 2a_1(f)) , 
\]
\[ \text{(C.1)} \]

where \( a_1(i) \) is coming from the corresponding Feynman diagram. They are given by

\[
a_1(a) = |Z_{1a}^+|^2 Z_{1a}^{i\alpha} Z_{i\nu}^{j\alpha^*} (-2)C_{00} ,
\]
\[ \text{(C.2)} \]

\[
a_1(b) = -\tan^2 \theta_W \left( \frac{1}{2} Z_{L\alpha}^{i\alpha} Z_{L\beta}^{j\beta} - \sin^2 \theta_W \delta^{\alpha\beta} \right) (-2)A^{i\alpha\alpha^*} A^{j\beta\alpha} C_{00} ,
\]
\[ \text{(C.3)} \]

\[
a_1(c) = -Z_{1a}^{j\alpha} Z_{1b}^{j\alpha} Z_{i\nu}^{i\alpha^*} \left[ (Z_{1\nu}^{j\alpha} + \delta^{ab} \cos 2\theta_W) m_{\chi_a^0} m_{\chi_b^-} - (Z_{1\nu}^{j\alpha} + \delta^{ab} \cos 2\theta_W) m_{\tilde{\nu}_{\alpha}}^2 \right] C_0
\]
\[
   - (Z_{1b}^{j\alpha} Z_{1a}^{i\alpha} + \delta^{ab} \cos 2\theta_W)(B_0 - 2C_{00}) \right) ,
\]
\[ \text{(C.4)} \]

\[
a_1(d) = -\frac{1}{2} \tan^2 \theta_W A^{i\alpha\alpha^*} A^{jab} \left[ 2D^{ab} C_{00} + \left( C^{ab} m_{\chi_a^0} m_{\chi_b^0} - D^{ab} m_{\tilde{\nu}_{\alpha}}^2 \right) C_0 - D^{ab} B_0 \right] ,
\]
\[ \text{(C.5)} \]

\[
a_1(e) = -2(0.5 - \sin^2 \theta_W) |Z_{1a}^{j\alpha} Z_{1\nu}^{j\alpha^*} (B_0 + B_1) ,
\]
\[ \text{(C.6)} \]

\[
a_1(f) = -\tan^2 \theta_W (0.5 - \sin^2 \theta_W) A^{i\alpha\alpha^*} A^{j\alpha\alpha} (B_0 + B_1) ,
\]
\[ \text{(C.7)} \]

where \( i, \alpha \) and \( a \) represent the flavors of lepton, slepton or sneutrino and chargino or neutralino respectively. \( a_2 \) is given by

\[
   a_2 = \frac{g_2}{2 \cos \theta_W} \frac{g_2^2}{16 \pi^2} (a_2(b) + a_2(d) + 2a_2(f)) ,
\]
\[ \text{(C.8)} \]

where

\[
a_2(b) = -\tan^2 \theta_W \left( \frac{1}{2} Z_{L\alpha}^{i\alpha} Z_{L\beta}^{j\beta} - \sin^2 \theta_W \delta^{\alpha\beta} \right) (-2)B^{i\alpha\alpha^*} B^{j\alpha\alpha} C_{00} ,
\]
\[ \text{(C.9)} \]

\[
a_2(d) = -\frac{1}{2} \tan \theta_W B^{i\alpha\alpha^*} B^{j\alpha\beta} \left[ 2C^{ab} C_{00} + \left( D^{ab} m_{\chi_a^0} m_{\chi_b^0} - C^{ab} m_{\tilde{\nu}_{\alpha}}^2 \right) C_0 - C^{ab} B_0 \right] ,
\]
\[ \text{(C.10)} \]

\[
a_2(f) = \tan^2 \theta_W \sin^2 \theta_W B^{i\alpha\alpha^*} B^{j\alpha\alpha} (B_0 + B_1) ,
\]
\[ \text{(C.11)} \]

Then we have

\[
a_i = \frac{g_2}{2 \cos \theta_W} \frac{g_2^2}{16 \pi^2} (a_i(b) + a_i(d)), \quad i = 3, 4
\]
\[ \text{(C.12)} \]
and

\[ a_3(b) = 2 \tan^2 \theta_W \left( \frac{1}{2} Z_L^\alpha Z_L^\beta - \sin^2 \theta_W \delta^{\alpha\beta} \right) B^{\alpha\beta} \delta^\alpha_\lambda \delta^\beta_\alpha m_{\chi^\alpha_\lambda} (C_0 + C_1 + C_2) , \quad (C.13) \]

\[ a_3(d) = \tan^2 \theta_W B^{\alpha\beta} \delta^\alpha_\lambda \delta^\beta_\alpha m_{\chi^\alpha_\lambda} (C_0 + C_1 + C_2) , \quad (C.14) \]

\[ a_4(b) = 2 \tan^2 \theta_W \left( \frac{1}{2} Z_L^\alpha Z_L^\beta - \sin^2 \theta_W \delta^{\alpha\beta} \right) A^{\alpha\beta} \delta^\alpha_\lambda \delta^\beta_\alpha m_{\chi^\alpha_\lambda} (C_0 + C_1 + C_2) , \quad (C.15) \]

\[ a_4(d) = \tan^2 \theta_W A^{\alpha\beta} \delta^\alpha_\lambda \delta^\beta_\alpha m_{\chi^\alpha_\lambda} (C_0 + C_1 + C_2) . \quad (C.16) \]

All the coupling vertices \( A, B, C, D \) are defined in the Appendix B. The \( B_{0,1} \) and \( C_{0,1,2,00} \) are the standard two-point and three-point functions with its definition given in the program LoopTools [25]. The arguments of function \( B \) from Fig. (c), (d), (e), (f) are \( (M^2_Z, m^2_{\chi^\alpha_\lambda}, m^2_{\chi^\alpha_\lambda}), (M^2_Z, m^2_{\chi^\alpha_\lambda}, m^2_{\chi^\alpha_\lambda}), (0, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}), (0, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}) \) respectively. The arguments of functions \( C \) from Figs. (a), (b), (c), (d) are \( (0, M^2_Z, 0, m^2_{\chi^\alpha_\lambda}, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}), (0, M^2_Z, 0, m^2_{\chi^\alpha_\lambda}, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}), (0, M^2_Z, 0, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}), (0, M^2_Z, 0, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}, m^2_{\nu_\alpha}) \) respectively, where the external fermion masses have been set to zero.
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FIGURE CAPTIONS

FIG. 1 Feynman diagrams contributing to $\tau \rightarrow \mu \gamma$. The universal scalar neutrino masses become non-degenerate and mixed by the non–diagonal Yukawa couplings. This effect gives low energy scalar neutrino mixing by RGEs running.

FIG. 2 The Feynman diagrams contributing to the process $\tau \rightarrow \mu \gamma$.

FIG. 3 The Feynman diagrams contributing to the process $Z \rightarrow \mu \tau$. The other two self-energy diagrams coming from $\tau$ legs are omitted.

FIG. 4 Branching ratio of $\tau \rightarrow \mu \gamma$ as a function of $m_0$ for $m_{1/2} = 100 GeV$, $\tan \beta = 5$ and $A_0 = 2m_0$, $m_0$, 0. The solid line is for $\mu > 0$ and the dashed line is for $\mu < 0$.

FIG. 5 Branching ratio of $\tau \rightarrow \mu \gamma$ as a function of $m_0$ for $A_0 = m_0$, $\tan \beta = 5$ and $m_{1/2} = 100 GeV$, 200 GeV, 400 GeV. The solid line is for $\mu > 0$ and the dashed line is for $\mu < 0$.

FIG. 6 Branching ratio of $\tau \rightarrow \mu \gamma$ as a function of $m_0$ for $m_{1/2} = 100 GeV$, $\tan \beta = 10$, $\mu > 0$ and $A_0 = \pm 2m_0$, $\pm m_0$, 0. The solid line is for $A_0 > 0$ and the dashed line is for $A_0 < 0$.

FIG. 7 Branching ratio of $\tau \rightarrow \mu \gamma$ as a function of $m_0$ for $A_0 = m_0$, $m_{1/2} = 100 GeV$, and $\tan \beta = 2$, 5, 10. The solid line is for $\mu > 0$ and the dashed line is for $\mu < 0$.

FIG. 8 Branching ratio of $Z \rightarrow \mu \tau$ as a function of $m_0$ for $m_{1/2} = 100 GeV$, $\tan \beta = 5$ and $A_0 = 2m_0$, $m_0$, 0. The solid line is for $\mu > 0$ and the dashed line is for $\mu < 0$.

FIG. 9 Branching ratio of $Z \rightarrow \mu \tau$ as a function of $m_0$ for $A_0 = 2m_0$, $\tan \beta = 5$ and $m_{1/2} = 100 GeV$, 200 GeV, 400 GeV. The solid line is for $\mu > 0$ and the dashed line is for $\mu < 0$. 

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FIG. 10 Branching ratio of $Z \rightarrow \mu\tau$ as a function of $m_0$ for $A_0 = 2.5m_0$, $m_\frac{1}{2} = 150\text{GeV}$, and $\tan \beta = 2, 5, 10$. 
Fig. 1
Fig. 2
Fig. 3
Fig. 4

$m_{1/2} = 100\text{ GeV}, \tan\beta = 5$

Experimental bound

Expected Sensitivity

$\text{BR}(\tau \to \mu \gamma)$

- $A_0 = 2 \cdot m_0$
- $m_0$
- $A_0 = 0$

$\mu > 0$
$\mu < 0$
$A_0 = m_0$, $\tan(\beta) = 5$

- $\mu > 0$
- $\mu < 0$

$\text{BR}(\tau \rightarrow \mu\gamma)$

- $m_{1/2} = 100\,\text{GeV}$
- $m_{1/2} = 200\,\text{GeV}$
- $m_{1/2} = 400\,\text{GeV}$

Experimental bound

Expected Sensitivity
Fig. 6

Experimental bound

$A_0 = \pm 2m_0$

$A_0 = 0$

$m_{1/2}=100\text{GeV}, \tan \beta=10, \mu > 0$

$A_0 > 0$

$A_0 < 0$

Expected Sensitivity
Fig. 7

Experimental bound

$A_0=m_0$, $m_{1/2}=100$GeV

$\tan\beta=10$

$\mu > 0$

$\mu < 0$

$\tan\beta=2$

$\tan\beta=5$

Expected Sensitivity

$BR(\tau \to \mu \gamma)$

$m_0$ (GeV)
Fig. 8

Expected Sensitivity

$\text{BR}(Z \rightarrow \mu\tau)$

- $A_0 = 2m_0$
- $A_0 = m_0$
- $A_0 = 0$

$m_{1/2} = 100\text{GeV}, \tan\beta = 5$

$\mu > 0$
$\mu < 0$

$m_0$ (GeV)
Fig. 9

Expected Sensitivity

$A_0 = 2 \cdot m_0$, $\tan \beta = 5$

- $m_{1/2} = 100 \text{GeV}$
- $m_{1/2} = 200 \text{GeV}$
- $m_{1/2} = 400 \text{GeV}$

$\mu > 0$

$\mu < 0$
Fig. 10

$\tan\beta = 2$

$\tan\beta = 5$

$\tan\beta = 10$

$A_0 = 2.5 \cdot m_0, m_{1/2} = 150 \text{GeV}$

Expected Sensitivity