FINAL MASSES OF GIANT PLANETS. II. JUPITER FORMATION IN A GAS-DEPLETED DISK

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ABSTRACT

First, we study the final masses of giant planets growing in protoplanetary disks through capture of disk gas, by employing empirical formulae for the gas capture rate and a shallow disk gap model, which are both based on hydrodynamic simulations. We find that, for planets less massive than 10 Jupiter masses, their growth rates are mainly controlled by the gas supply through the global disk accretion, and the gap opening does not limit the accretion. The insufficient gas supply compared with the rapid gas capture causes a depletion of the gas surface density even at the outside the gap, which can create an inner hole in the disk. Second, our findings are applied to the formation of our solar system. For the formation of Jupiter, a very low-mass gas disk of several Jupiter masses is required at the beginning of its gas capture because of the continual capture. Such a low-mass gas disk with sufficient solid material can be formed through viscous evolution from a compact disk of initial size ~10 au. By viscous evolution with a moderate viscosity of \( \alpha \sim 10^{-3} \), most of the disk gas accretes onto the Sun and a widely spread low-mass gas disk remains when the solid core of Jupiter starts gas capture at \( t \sim 10^7 \) yr. A very low-mass gas disk also provides a plausible path where type I and II planetary migrations are both suppressed significantly. In particular, the type II migration of Jupiter-size planets becomes inefficient because of the additional gas depletion due to the rapid gas capture by such planets.

Key words: planets and satellites: formation – protoplanetary disks

1. INTRODUCTION

A leading hypothesis of giant planet formation is the core instability model (Mizuno 1980; Bodenheimer & Pollack 1986; Pollack et al. 1996; Ikoma et al. 2000; Hubickyj et al. 2005). In a protoplanetary disk, a solid protoplanet attracts the disk gas and has a proto-atmosphere. When the solid core is around 10 Earth masses, the atmospheric mass becomes comparable to the core mass, and the atmosphere becomes gravitationally unstable, which triggers dynamical collapse of the atmosphere to become a giant planet. Since the gas accretion of the atmosphere onto the core occurs in an unstable and runaway manner, the growth continues as long as gas exists around the planet.

The gas-accretion growth of a giant planet is expected to be terminated when the planet’s own strong gravity creates a gap, which is a low-density annular region along the planet’s orbit, when the planet becomes massive. Two well-known gap-opening conditions have been widely used: the thermal condition and the viscous condition (Lin & Papaloizou 1993; Ida & Lin 2004). The thermal condition is a condition whereby the (specific) gravitational energy at a distance of the disk scale height \( \sim G M_p h / c^2 \), where \( G \) is the gravitational constant, \( M_p \) is the planet’s mass, \( h \) is the disk scale height, and \( c \) is the sound speed of disk gas. The viscous condition is where the planetary gravitational torque exerted on the gas disk is stronger than the viscous torque of the disk due to Keplerian shear motion (Lin & Papaloizou 1986). Since the thermal condition usually requires a larger planetary mass for gap opening than the viscous condition and the required planetary mass is consistent with Jupiter, the final masses of the giant planets have been thought to be determined by the thermal condition. Some hydrodynamic simulations show that the planet can indeed create a deep gap (e.g., Bryden et al. 1999; Crida et al. 2006; Dobbs-Dixon et al. 2007), which would support the two gap-opening conditions and suggest the termination of growth. Some simulations, however, show that the planet creates a shallower gap (e.g., Kley 1999; Lubow et al. 1999), which would suggest that the planet’s growth continues even after the two conditions are met. This means that there is no clear consensus on the gap opening conditions.

Tanigawa & Ikoma (2007, hereafter TI07) have constructed an analytic model for the rate of gas accretion onto a planet embedded in a disk gap as a function of the planetary mass, viscosity, scale height, and unperturbed surface density. By using this, they systematically studied the long-term growth and the final masses of gas giant planets. To calculate the accretion rate, TI07 derived an analytic formula for the surface density distribution in the gap region, where gravitational perturbation by the planet is significant. In addition to the gap formula that considers the balance between the viscous torque and the planetary gravitational torque (e.g., Lubow & D’Angelo 2006), TI07 also included the gap shallowing effect due to the Rayleigh stability condition that inhibits too steep a radial gradient of surface density. The shallowing effect supplies a non-negligible amount of gas into the bottom of the gap, which enables the giant planet to keep on growing even after the gap opens. At the same time, TI07 also proposed that the rate of gas accretion onto the planet can be limited by the rate of disk viscous accretion. An insufficient gas supply by disk accretion inevitably limits the rate of gas accretion onto the planet even if the planet is capable of capturing the ambient gas at a higher rate. Such a limited rate of accretion onto a planet was also used in population synthesis calculations (e.g., Mordasini et al. 2009, 2012b). As a result, TI07 gave much larger final masses (\( \gtrsim 10 \) Jupiter mass at 5 au) than the
Recent hydrodynamic simulations have yielded an empirical formula for surface density at the bottom of the gap (Duffell & MacFadyen 2013; Fung et al. 2014). This formula indicates that the gap is much shallower than the traditional prediction, and is even shallower than the analytic estimate given by TI07, which includes the shallowing effect due to the Rayleigh stability condition. Kanagawa et al. (2015a) analytically derived this shallow gap formula by including the effect of density wave propagation in the disk. Such a shallow gap model maintains high accretion rates onto planets and gives much larger final masses of giant planets than the prediction by TI07, leading to the possibility that Jupiter and Saturn formed in a much lighter disk than the MMSN model.

The rapid accretion onto a planet due to a shallow gap also causes a depletion of the disk gas over a wider radial region, in addition to the narrow gap. This gas depletion may alter the type II planetary migration as well as the planet’s growth. Lubow & D’Angelo (2006) examined this depletion mechanism due to the gas accretion onto a planet by using their semi-analytical model and hydrodynamic simulations. However, their estimations might suffer from large errors since the calculation time of their hydrodynamic simulations is less than 1/10 of the characteristic viscous evolution time. It would be valuable to examine the gas depletion due to the gas accretion onto a planet by using an updated formula for the accretion rate with the shallow gap model.

In this study, we update the growth model of giant planets proposed by TI07 by adopting the empirical shallow gap model, and demonstrate that the termination of giant planet growth by the gap opening is much harder than expected in the traditional prediction. From this result on the growth rate, we propose that a gas-depleted disk is suitable for the formation of Jupiter-sized planets. We also estimate the gas depletion due to the rapid gas accretion onto the planet using the updated formula for the accretion rate, which enables us to quantitatively discuss the inner hole and its effect on type II planetary migration. We first describe the formulation of our model in Section 2. We next show examples of the evolution of gas-capturing growth and the final mass of the giant planets in Section 3. We discuss a plausible path for formation of Jupiter in Section 4. The type II migration is also discussed there. Our results are summarized in Section 5.

2. FORMULATION

2.1. Disk Model

We consider a globally evolving protoplanetary disk. The protoplanetary disk has scale height $h = c/\Omega$, where $c$ and $\Omega$ are the sound speed and the Keplerian angular velocity around the central star, respectively. We set the temperature distribution so that $T/r = 10^{-1.5}$ (r/1 au)$^{1/2}$, where $r$ is the distance from the star. This corresponds to the temperature profile $T \approx 280$ K (r/1 au)$^{1/2}$ for solar-type stars. We use the $\alpha$-model for disk viscosity: $\nu = \alpha c h$, where $\alpha$ is a non-dimensional parameter and independent of the radius $r$ and time (Shakura & Sunyaev 1973). For the above temperature profile, $\nu$ is proportional to $r$. We adopt a self-similar solution for global evolution of the protoplanetary disk (Lynden-Bell & Pringle 1974; Hartmann et al. 1998). The surface density of the solution is given by

$$\Sigma_{\text{un}}(r, t) = \Sigma_{\text{ini}}(r, t)\exp\left(\frac{-t}{\tau_{\text{dep}}}\right),$$

and the disk mass is written as a function of time: $M_{\text{d,ini}}r^{1/2} \exp(-t/\tau_{\text{dep}})$. The additional exponential decay would correspond to some other mechanisms for disk dissipation, such as photoevaporation by ultraviolet radiation from the central star or disk wind (see discussion in Section 5). We use $\Sigma_{\text{un}}$ as the unperturbed disk surface density in this paper.

The global disk accretion rate of the self-similar solution with the additional exponential decay at an orbital radius $r$ is given by

$$M_{\text{d,global}}(r, t) = \frac{M_{\text{d,ini}}}{2\tau_{\text{ss}}}\left(1 - \frac{r}{\tilde{r}_{\text{ss}}R_o/2}\right)^{-3/2}\exp\left(-\frac{r}{\tilde{r}_{\text{ss}}R_o}\right)\exp\left(-\frac{t}{\tau_{\text{dep}}}\right),$$

where the factor $\exp(-t/\tau_{\text{dep}})$ is due to the additional exponential decay.

We put the initial total mass of the protoplanetary disk as

$$M_{\text{d,ini}} = 1.1 \times 10^{-1} f_{\text{2.5 au}} \left(\frac{R_o}{200 \text{ au}}\right) M_\odot,$$

where $f_{\text{2.5 au}}$ is a parameter and $M_\odot$ is the mass of the Sun. When $f_{\text{2.5 au}} = 1$, the initial total disk mass of Equation (6) makes the initial unperturbed surface density $\Sigma_{\text{un}}(5 \text{ au}, t = 0)$ equal to that of the MMSN model at 5 au (i.e., $1.7 \times 10^{-5} M_\odot/ \text{AU}^2 = 1.4 \times 10^3 \text{ kg m}^{-2}$, Hayashi et al. 1985).

It has been reported that photoevaporation by far-ultraviolet (FUV) radiation from a central star can considerably accelerate the dispersal of the circumstellar disk (e.g., Gorti et al. 2009). Photoevaporation by FUV mainly removes the gas in the outer disk at a distance $\gtrsim 100$ au and decreases the disk mass exponentially with time at a relatively early stage ($\sim 10^8$ yr). However, the rate of mass loss by this mechanism is still...
uncertain to about an order of magnitude (e.g., Alexander et al. 2014). In this paper, therefore, we do not include the effect of photoevaporation by FUV on the disk evolution for simplicity.

2.2. A New Simple Model for Gas Accretion onto a Planet

We consider a protoplanet embedded in the evolving protoplanetary disk. The protoplanet with mass $M_p$ is rotating around the central star at a distance $r_p$ from it. The planet starts dynamical gas capture, i.e., after gravitational instability of the proto-atmosphere around a solid core of about 10 Earth masses (Mizuno 1980; Bodenheimer & Pollack 1986; Pollack et al. 1996; Ikoma et al. 2000; Hubickyj et al. 2005).

We introduce a formula for the rate of gas accretion onto the protoplanet from the protoplanetary disk as an explicit function of parameters, which enables us to obtain the time evolution of the planet’s mass and eventually the final mass. We basically follow the method of TI07, but we have improved some points, so we will re-summarize it below.

If sufficient gas is supplied toward the planet’s orbit by the disk accretion, the rate of accretion onto the giant planet is determined by the hydrodynamics of the gas accretion flow onto the planet and is denoted by $M_{p,\text{hydro}}$. This accretion rate is given by the product of the two quantities:

$$M_{p,\text{hydro}} = D \Sigma_{\text{acc}}, \quad (7)$$

where $D$ is the accretion area of the protoplanetary disk per unit time, and $\Sigma_{\text{acc}}$ is the surface density at the accretion channel in the protoplanetary disk. Tanigawa & Watanabe (2002, hereafter TW02) performed two-dimensional hydrodynamic simulations of the accretion flow onto a planet and derived an empirical formula for the accretion rate. According to their result, $D$ is given by

$$D = 0.29 \left( \frac{h_p}{r_p} \right)^{-2} \left( \frac{M_p}{M_\oplus} \right)^{4/3} r_p^2 \Omega_p, \quad (8)$$

where $M_p$ and $M_\oplus$ are masses of the planet and central star, respectively, and $h_p$ and $\Omega_p$ are the scale height and Keplerian angular velocity at the planet’s location, respectively. In previous low-resolution simulations, accretion onto the planet is often described by depleting the disk surface density within the planet’s Hill radius with an assumed accretion timescale parameter $f$ (e.g., Zhu et al. 2011). Equation (8) can be utilized to determine the free parameter $f$. A useful recipe for the parameter $f$ will be presented in Appendix C. Note that, strictly speaking, the product form of $D$ and $\Sigma_{\text{acc}}$ in Equation (7) is valid when the equation of state of the gas around the Hill sphere is isothermal because $D$ depends on temperature, which can depend on gas density especially when density is high, as the referee pointed out. However, Machida et al. (2010) showed that the non-isothermality hardly affect the accretion rate; we thus judged that this formula is good enough to estimate the accretion rate in this paper.

TI07 gave the formula for the surface density $\Sigma_{\text{acc}}$ purely in a theoretical manner, including the Rayleigh stability condition. This condition prevents an unrealistically steep surface density gradient and the resultant too-deep gap, which is a consequence of the simple assumption of the balance between viscous torque and gravitational torque by the planet. However, recent hydrodynamic simulations showed that the gap is even shallower than the prediction by TI07 (Duffell & MacFadyen 2013; Fung et al. 2014), which is also supported by theoretical considerations (Fung et al. 2014; Kanagawa et al. 2015a, 2015b). In this study, we thus use an empirical formula for the gas surface density at the bottom of the gap obtained by these studies:

$$\Sigma_{\text{acc}}(r) = \frac{1}{1 + 0.034 K} \Sigma_{\text{un}}(r, \tau), \quad (9)$$

where

$$K = \left( \frac{h_p}{r_p} \right)^{-5} \left( \frac{M_p}{M_\oplus} \right)^2 \alpha^{-1}. \quad (10)$$

In Equation (9), we assumed that the accretion band is located within the bottom of the gap. If the accretion band were located at the edge of the gap with a higher surface density, the accretion rate given by Equations (7) and (9) would be an underestimate. We set $M_{p,\text{ini}} = 3.24 \times 10^{-5} M_\oplus$ for the initial mass of the protoplanet and $R_o = 200$ au.

Figure 1 shows the accretion rate $M_{p,\text{hydro}}$ as a function of the planet’s mass. In the case of a low-mass planet where the parameter $K$ is much less than 1/0.034 ($\sim$30), there is no gas depletion due to the gap, so $\Sigma_{\text{acc}}$ can be simply replaced by $\Sigma_{\text{un}}$ (see Equation (9)). The accretion rate in this regime is

$$M_{p,\text{hydro}} = M_{p,\text{no gap}}$$

$$= 0.29 \left( \frac{h_p}{r_p} \right)^{-2} \left( \frac{M_p}{M_\oplus} \right)^{4/3} \Sigma_{\text{un}} r_p^2 \Omega_p$$

for $K < 1/0.034$. \quad (11)
In the high-mass case where $K > 1/0.034$, on the other hand, $\Sigma_{\text{acc}}$ is reduced to $\Sigma_{\text{un}}/0.034K$ due to the gap opening, and the accretion rate can be written as

$$M_{p,\text{hydro}} = M_{p,\text{gap}}$$

$$= 8.5 \left( \frac{h_p}{r_p} \right) \left( \frac{M_p}{M_0} \right)^{-2/3} \Sigma_{\text{un}} \nu_p$$

for $K \gg 1/0.034$.\(^\text{(12)}\)

Equation (12) shows that the accretion rate $M_{p,\text{hydro}}$ decreases gradually ($\propto M_p^{-2/3}$) after the gap opens.

In Figure 1, we also plotted the accretion rates obtained by the previous hydrodynamic simulations to check the validity of our simple model. D’Angelo et al. (2003) examined the gas accretion rate onto a planet embedded in a protoplanetary disk by performing three-dimensional global hydrodynamic simulations for various planetary masses. We find that our model reproduces their results well where $M_p < M_J$, but their result is smaller than ours by a factor of 2 at $M_p = M_J$, and may indicate that the accretion rate declines more rapidly than our analytical formula where $M_p > M_J$. On the other hand, the accretion rate obtained by Machida et al. (2010), which is also plotted in the figure, does not show the rapid decrease.\(^6\) The accretion rate used in T107 is also plotted. T107’s accretion rate declines rapidly with increasing mass because of its deeper gap model. But their model deviates more from the numerical results than our model, at least in the plotted range. We will briefly discuss the model of the accretion rate in Section 5. The global disk accretion rate $\dot{M}_{\text{d,global}}$ is also shown as a reference.

One may be concerned about mass transfer in the circumplanetary disk because the mass transfer may affect the rate of accretion onto the planet (Szulagyi et al. 2014), and the transfer mechanisms are indeed discussed actively (Martin & Lubow 2011; Rivier et al. 2012; Fuji et al. 2014; Keith & Wardle 2014). However, what we simulate is the evolution of mass, and gas that once accretes inside 0.1 Hill radius cannot escape from the Hill sphere (Tanigawa et al. 2012). Thus the mass and angular momentum transfer mechanisms would not affect our result.

We also consider the case where the gas supply by viscous disk accretion is insufficient. In such a case, the gas accretion onto the planet is regulated by the global disk accretion rate $\dot{M}_{\text{d,global}}(r_p)$ rather than $M_{\text{p,hydro}}$. We need to take this effect into account since the gap opening cannot significantly slow down the gas accretion onto the planet. Furthermore, at an early stage of the gas capture by the planet, an additional treatment is required for the realistic gas supply to the planet’s orbit. At the early stage, a substantial amount of gas still exists near the planet’s orbit. The gas supply from nearby is regulated by local disk diffusion rather than global disk accretion. The disk accretion rate due to the local diffusion is given by

$$\dot{M}_{\text{d,local}} = \sqrt{12 \pi} r_p \Sigma_{\text{un}}(r_p) \frac{\nu_p}{\sqrt{t - t_{\exp}}},$$

(13)

(see Appendix A). In practice, the gas supply would be approximately given by the larger of $\dot{M}_{\text{d,global}}$ and $\dot{M}_{\text{d,local}}$. In our model, therefore, by including the gas supply to the planet’s orbit, we give the gas accretion rate onto the planet, $\dot{M}_p$, as

$$\dot{M}_p = \min(\dot{M}_{\text{p,hydro}}, \max(\dot{M}_{\text{d,global}}, \dot{M}_{\text{d,local}})).$$

(14)

Using this model for the rate of gas accretion onto the planet, we can easily simulate evolution of the planet’s mass (or gas accretion rate) for a given set of disk parameters. To do that, we need only to numerically integrate the ordinary differential equation because the integrand is an explicit function of the disk parameters. The final mass of a planet is simply obtained as

$$\dot{M}_{\text{p,final}}(r_p, \alpha, h_p, M_{\text{d,ini}}, R_o) = \int_0^\infty \dot{M}_p(r_p, \alpha, h_p, M_{\text{d,ini}}, R_o, t) \, dt.$$

(15)

Note that Fung et al. (2014) derive a more elaborate fitting formula by two-dimensional hydrodynamic simulations, which focus on cases for planets more massive than that of Duffell & MacFadyen (2013). But the difference between the two formulae is much smaller than that between Duffell & MacFadyen (2013) and T107, so we use the above equation.
3. RESULTS

3.1. Examples of Time Evolution

Figure 2 plots an example of the time evolution of the rate of gas accretion onto a proto-giant planet located at 5 au and that of planetary mass. The parameters are set to be \( \alpha = 3.2 \times 10^{-3} \), \( f_{\Sigma,5\ au} = 1 \), \( \tau_{\text{dep}} = 10^7 \) yr, \( R_o = 200 \) au, \( r_p = 5 \) au. This example illustrates that the evolution can be divided into four phases:

1. Phase 1: after the onset of dynamical gas accretion, the gas accretion rate is regulated by the hydrodynamic accretion flow without a gap, \( M_{p,\text{no-gap}} \). In this case, there is abundant gas near the planet and nothing to limit the accretion flow.

2. Phase 2: the gas supply from the nearby part of the planet’s orbit, \( M_{d,\text{local}} \), limits the accretion rate onto the planet because it is lower than \( M_{p,\text{hydro}} \) and higher than the global disk accretion rate \( M_{d,\text{global}} \).

3. Phase 3: this is also the case where the gas supply toward the planet’s orbit limits the accretion rate, but the rate is given by \( M_{d,\text{global}} \).

4. Phase 4: this is again the case when the gas supply is regulated by the hydrodynamic accretion flow, but with a deep gap: \( M_{p,\text{gap}} \).

In this case, the final mass of the giant planet is as great as 30\( M_J \), which is about 1/3 of the initial disk mass (see Equation (6)). The parameter set in this case is not very special (neither heavy nor highly viscous), but still results in the formation of a massive planet. This is because a gap does not significantly suppress gas accretion onto the planet.

The next example shown in Figure 3 is a case that produces a Jupiter-size planet. This case adopts a lower viscosity of \( \alpha = 10^{-3} \) and a much lighter disk mass of \( f_{\Sigma,5\ au} = 1/20 \).

Because of the lower surface density, the accretion rate in phase 1 is lower and, as a result, the accretion rate and planetary mass do not significantly increase until the end of phase 1. In phases 2 and 3, the sequence of the phase transition is basically the same as in the previous example, but the absolute values of accretion rates are reduced because of the lower surface density and lower viscosity. Accretion rates in phases 2 and 3 are proportional to \( \Sigma \alpha^{1/2} \) and \( \Sigma \alpha \); respectively; thus the accretion rates in this case are reduced by factors of 36 and 60, respectively, in comparison with the case of Figure 2. As a result of these low accretion rates, the mass of the planet does not significantly increase, and the planet is not able to open a deep gap, which leads to no emergence of phase 4. When \( r_p < \frac{r_o R_o}{2} \), we obtain from Equations (5) and (12) that

\[
\frac{M_{p,\text{gap}}}{M_{d,\text{global}}} \sim 0.90 \times \left( \frac{M_p}{M_*} \right)^{-2/3} \left( \frac{h_p}{r_p} \right).
\]

The emergence of phase 4 requires \( M_{p,\text{gap}} < M_{d,\text{global}} \), which gives

\[
M_p > 1.0 \times 10^{-2} \left( \frac{h_p/r_p}{0.05} \right)^{3/2} M_* \text{.}
\]

which explains the presence or absence of phase 4 in the two cases. Thus, up to \( \sim 10M_j \), the reduction in the accretion rate caused by the gap is not effective. Even in phase 4 of Figure 2, \( M_p \) is not so small compared with \( M_{d,\text{global}} \) because the ratio \( M_{p,\text{gap}}/M_{d,\text{global}} \) depends weakly on \( M_p \) as in Equation (16).

3.2. Final Masses

Figure 4 plots the final mass of a gas-capturing planet as a function of orbital radius of the planets when \( f_{\Sigma,5\ au} = 1 \), \( \alpha = 10^{-3} \), \( R_{\text{out}} = 200 \) au, \( \tau_{\text{dep}} = 10^7 \) yr. The solid line shows the final mass obtained in our model. We find that the final mass is 10–20 Jupiter masses over most of the area and has only a
slight radial dependence. Up to \( \sim 10M_J \), a giant planet grows mostly in phase 3 and the growth rate is regulated by the global disk accretion rate \( M_{\text{d,global}} \) at all radii. Thus the growth rate of giant planets is independent of their radial location. Even in phase 4 where \( M_p \gtrsim 10M_J \), the growth rate is not much smaller than \( M_{\text{d,global}} \). We also plot the final mass in the case of TI07 for comparison. In that case, \( \Sigma_{\text{acc}} \) uses a formula of TI07, while Equation (9) is used in this paper. The final mass with TI07’s formula becomes larger than Jupiter’s mass for most of the region. This is mainly because TI07 considers the violation of the Rayleigh condition for a steep radial density gradient, which limits the gradient, tends to fill the gap, and promotes gas-capturing growth as a result. In this case, the final mass of a planet increases with its orbital radius \( r_p \). This is because gap opening is easier in the inner region in TI07. Thus the difference in final mass between the two cases originates from formulae for the gap depth. We note that we assume a unique value of \( M_{\text{d,ini}} \) in Figure 4 (and Figures 5 and 6 as well), although, in a specific system, a planet with a larger orbital radius usually means a smaller \( M_{\text{d,ini}} \) because a planet with a larger orbital radius, which grows more slowly in general, tends to begin gas accretion later. A smaller \( M_{\text{d,ini}} \) makes the final mass smaller, thus a planet with a larger orbital radius would acquire a smaller final mass.

The final mass can be estimated with a simple equation. Since the gap opening does not significantly affect \( M_p \) up to \( \sim 10M_J \), the global disk accretion rate \( M_{\text{d,global}} \) determines the final mass in most cases. Thus, using \( M_{\text{d,global}} \) for a planet at \( r_p \ll R_o \), the final mass is approximated by

\[
M_{p,\text{final,p3}} \sim \int_0^{\infty} M_{\text{d,global}} |t_\tau \leq R_o| dt \\
= M_{\text{d,ini}} \left[ 1 - \left( \frac{\tau_{\text{dep}}}{\tau_{\text{ss}}} + 1 \right)^{-1/2} \right] \\
= M_{\text{d,ini}} - M_{\text{d,ss}}(\tau_{\text{dep}}).
\]

This means that all the mass lost from the disk is captured by the planet. In the case where \( \tau_{\text{dep}} \ll \tau_{\text{ss}} \), the final mass of Equation (18) is approximately given by \( (\tau_{\text{dep}}/2\tau_{\text{ss}})M_{\text{d,ini}} \). When \( r_p \) is small, the deviation from Equation (18) becomes larger. This is because the accretion state is switched from phase 3 to phase 4 before the end of the growth. In this case, the final mass is roughly estimated as

\[
M_{p,\text{final,p4}} \sim M_{p,\text{final,p3}} \frac{M_{p,\text{gap}}(M_{p,\text{final,p3}})}{M_{\text{d,global}}}
\]

Note that \( M_{\text{d,ini}} \) is the disk mass at the time when the planet starts its gas capture and can be much smaller than the mass when the disk is formed. The timescale of global viscous evolution \( \tau_{\text{ss}} \) in our fiducial case is

\[
\tau_{\text{ss}} = \frac{R_o^2}{3\nu_o} = 1.1 \times 10^7 \left( \frac{\alpha}{10^{-2}} \right) \left( \frac{h_{\text{AU}} / 1 \text{ au}}{10^{-1.5}} \right)^2 \left( \frac{R_o}{200 \text{ au}} \right) \text{ yr}.
\]

Figure 5 plots final masses for various initial surface densities (or disk masses). The five curves, which correspond to five values of \( f_{5:5 \text{ au}} \), show that final mass is proportional to \( f_{5:5 \text{ au}} \) in general. This is simply because the growth rate in phase 3, \( M_{\text{d,global}} \), which is proportional to surface density, mainly determines the final mass (see Equation (18)), and the gap effect is not significant. The final masses shown in this figure are close to possible maximum masses. In the case of Figure 5, \( \tau_{\text{ss}} \approx \tau_{\text{dep}} \), so

\[
M_{\text{d,ini}} - M_{\text{d,ss}}(\tau_{\text{dep}}) = M_{\text{d,ini}} (1 - 1/\sqrt{2}),
\]

which means that all the disk gas accreting inward is captured by the planet on its way toward the central star and the gap has little effect on suppressing the gas capture. The final mass in the case of \( f_{5:5 \text{ au}} = 1 \) would be about 20\( M_J \) around 5 \text{ au}. For the formation of Jupiter-mass planets, the gas disk should therefore be much less massive than the MMSN disk at the onset of their gas capture.

Figure 6 plots \( M_{p,\text{final}} \) as a function of \( r_p \) in the cases with 10 times larger and smaller values of one of the three parameters: \( \alpha \), \( \tau_{\text{dep}} \), and \( f_{5:5 \text{ au}} \). We can see that the final mass increases with both \( \alpha \) and \( \tau_{\text{dep}} \) and depends only on the product \( \alpha \tau_{\text{dep}} \) over most of the range. For example, the degeneracy occurs at
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\[ r_p \ll 10 \text{ au} \] in the cases with \((\alpha, \tau_{\text{dep}}) = (10^{-3}, 10^8)\) and \((10^{-2}, 10^7)\) or the cases of \((10^{-3}, 10^9)\) and \((10^{-4}, 10^7)\). This is because the final mass is a function of \(\tau_{\text{dep}}/\tau_{\text{ss}}\) which is proportional to \(\alpha \tau_{\text{dep}}\) (see Equation (18)). However, this dependence is weaker than that of \(f \Sigma_{\text{ss}}\) because the dependence of final mass on \(\tau_{\text{dep}}/\tau_{\text{ss}}\) is weaker than linear (see Equation (18)), while that on \(f \Sigma_{\text{ss}}\) is basically linear. Note that final masses for a pair of degenerate cases \((\alpha \tau_{\text{dep}} = 10^{-3} \text{ or } 10^{-5})\) are split at \(r_p \sim R_o\).

This is because most gas accretion is completed by \(M_d,\text{local}\) (i.e., phase 2), which is proportional to \(\alpha^{1/2}\), not like \(M_d,\text{global} \propto \alpha^1\). This situation is realized in the case when \(r_p \sim R_o\) and \(\tau_{\text{dep}} < \tau_{\text{ss}}\).

### 3.3. Gas Depletion Due to the Accretion onto the Planet

In phase 3, the disk gas is further reduced, in addition to the effect of the gap produced by the planetary torque. In this phase, the gas supplied by global disk accretion is insufficient for the rapid gas capture, which causes an additional depletion of the gas surface density even outside the narrow gap region. In this phase, the accretion rate, \(DS_{\text{acc}}\), cannot be larger than \(M_d,\text{global}\). This indicates that \(\Sigma_{\text{acc}}\) should be depleted because \(D\) is independent of the surface density (see Equation (7)). The additional depletion factor due to the gas capture, \(f'\), is obtained from the balance of the mass fluxes (i.e., mass conservation). Including this depletion factor, the disk surface density outside the gap is given by \(f' \Sigma_{\text{acc}}(r_p)\), and thus the hydrodynamic capture rate should be evaluated as \(f' M_{p,\text{hydro}}\) instead of \(M_{p,\text{hydro}}\). Assuming quasi-steady flow in the disk, we obtain an equation for the mass flux balance, \(f' M_{p,\text{hydro}} = M_d,\text{global}\). Hence the additional depletion factor is given by

\[
f' = \frac{M_{d,\text{global}}}{M_{p,\text{hydro}}} \approx 1.1 \left( \frac{M_p}{M_s} \right)^{2/3} \left( \frac{h_p}{r_p} \right)^{-1},
\]

where we used Equation (16). From Equation (21), the additional depletion factor is unity for \(M_p = 0.01M_s\) and 0.2 for \(M_p = M_f\) in a disk with \(h_p/r_p = 0.05\). Note that this additional effect of gas depletion (or enhancement) does not exist in phase 4 because of a sufficient supply from global disk accretion.

Lubow & D’Angelo (2006) also examined the gas depletion due to the accretion onto the planet and derived the radial distribution of the surface density by considering a steady viscous accretion disk with a mass sink by the planet. From this accurate surface density distribution, the additional depletion factor is given by

\[
f' = \frac{\dot{M}_{\text{d,global}}}{M_{p,\text{hydro}} + \dot{M}_{\text{d,global}}}.\]

This agrees with Equation (21) when \(M_{p,\text{hydro}} \gg \dot{M}_{\text{d,global}}\).

Since a detailed derivation was not given in their paper, we present the derivation of the surface density distribution in Appendix B. In Lubow & D’Angelo (2006), the ratio \(M_{p,\text{hydro}}/\dot{M}_{\text{d,global}}\) is called the accretion efficiency. They estimated the accretion efficiency from their two-dimensional hydrodynamic simulations. In Figure 7, we plot their results and our model (i.e., Equation (16)). The differences between their model and ours are within a factor of two. Since the calculation time of their hydrodynamic simulations is less than 1/10 of the characteristic viscous evolution time, their values tends to be larger than those in the steady states. Thus we expect that the difference would become smaller if the calculation time were longer. Further investigation with long-term hydrodynamic simulations is necessary for checking our model.

This gas depletion would also create an inner hole, which is a depleted region inside a certain radius of a disk (e.g., Williams & Cieza 2011). Here we consider the possibility that the inner holes are formed by planets. We simply assume in Equation (21) that all the gas approaching the planet’s orbit is captured by the planet when \(M_{p,\text{hydro}} > \dot{M}_{d,\text{global}}\) (i.e., in Phase 3), but Equation (22) means that not all the gas is necessarily captured even in such a case. This can be interpreted as follows. Gas capture by a planet reduces surface density in a wide region, which reduces the gas capture rate in turn. When the reduced gas accretion rate is smaller than the rate of global disk viscous accretion, a fraction of gas that is not captured by the planet would need to pass through the planet’s orbit in a steady state. This inward flow creates an inner disk with a lower surface density, which could be the origin of the observed inner holes. The surface density at the inner hole would be given by \(f' \Sigma_{\text{acc}}(r)\), whereas we neglected this small amount of mass loss through the inner hole in phase 3 in this paper. Note that the gas depletion considered here is different from that caused by gap formation, which is created by gravitational torque from the planet and is usually much narrower. The gas depletion considered here is a depletion in addition to that due to gap formation. Furthermore, the gas depletion due to gas capture also affects the type II migration of the planet. This will be discussed in detail later.

![Figure 7. Accretion rate onto a planet normalized by the disk viscous accretion rate. Circles show accretion efficiency \(\epsilon\) in Table 1 of Lubow & D’Angelo (2006), and the solid line shows \(M_{p,\text{gap}}/M_{d,\text{global}}\). As a reference, \(M_{p,\text{no-gap}}/M_{d,\text{global}}\) is also shown by the dashed line.](image-url)
Figure 8. Final masses of giant planets at 5 au as a function of $f_{5,5\text{AU}}$ when $\alpha = 10^{-3}$ and $R_{\text{out}} = 200$ au in the cases of $\tau_{\text{dep}} = 10^7$ yr (red solid) and $10^6$ yr (blue dashed). Since $\alpha$ and $\tau_{\text{dep}}$ are degenerate in most cases, the case with $\tau_{\text{dep}} = 10^7$ yr corresponds to the case with $\tau_{\text{dep}} = 10^6$ and $\alpha = 10^{-2}$, for example. The dotted–dashed line shows $M_{\text{final}}$, which corresponds to the possible maximum mass of the planet, and the dotted line shows $M_{\text{p,ini}}$. Note that a factor $f_{5,5\text{AU}}$ is defined when the dynamical gas accretion begins, not when the protoplanetary disk forms. Note also that the mass of Saturn, which is at 9.5 au, is also plotted just for reference, while this figure is based on the case of 5 au.

4. IMPLICATION FOR THE ORIGIN OF OUR SOLAR SYSTEM

4.1. A Suitable Gas Disk for Jupiter Formation

In this paper, we updated the model for the growth of giant planets by employing the shallow gap model revealed by recent hydrodynamic simulations. The updated model showed that the formation of Jupiter-mass planets requires much less massive gas disks than the MMSN model at the stage of dynamical gas capture by planets. This is because the gap is not so deep as to terminate the gas accretion onto the planet.

Figure 8 plots $M_{\text{p,final}}$ at 5 au as a function of $f_{5,5\text{AU}}$, which is the degree of depletion of the disk gas relative to the MMSN model when the rapid gas capture begins. The final mass is proportional to the degree of depletion, $f_{5,5\text{AU}}$, when the final mass is much larger than the initial core mass and smaller than $10M_J$ (i.e., within phase 3). In the case of $\alpha = 10^{-3}$ and $\tau_{\text{dep}} = 10^7$ yr, a Jupiter-mass planet is formed in a gas disk with $f_{5,5\text{AU}} = 0.04$. The total mass of this gas disk is about 4 $M_J$. Such a very low-mass gas disk also has the advantage that type I and II planetary migrations are both suppressed significantly. In a less viscous case $\alpha = 10^{-4}$ (or equivalently a case of short disk lifetime, $\tau_{\text{dep}} = 10^6$ yr), $f_{5,5\text{AU}} = 0.2$ is suitable. However, such a moderate-mass disk does not slow down the planetary migrations significantly. Hence we adopt the former case as a suitable gas disk for Jupiter formation. If we adopt a higher viscosity $\alpha > 10^{-3}$ or a longer depletion time ($\tau_{\text{dep}} > 10^7$ yr), the suitable disk mass decreases further. As mentioned in Section 3, the final mass depends on the product $\alpha \tau_{\text{dep}}$. Thus, the case of $\alpha = 3 \times 10^{-3}$ and $\tau_{\text{dep}} = 3 \times 10^6$ yr also gives a suitable disk with $f_{5,5\text{AU}} = 0.04$ as well as the former case.

For the case of Saturn, $f_{5,5\text{AU}}$ should be an even lighter disk ($\sim 0.01$ when $\tau_{\text{dep}} = 10^7$ yr and $\alpha = 10^{-3}$) or a later start (shorter duration) of the dynamical gas capture (e.g., $f_{5,5\text{AU}} \sim 0.05$ when $\tau_{\text{dep}} = 10^6$ yr). We note for Figure 8 that we should not assume a single set of values for $\tau_{\text{dep}}$ and $f_{5,5\text{AU}}$ for both Jupiter and Saturn simultaneously because Saturn is thought to have been formed sometime after the formation of Jupiter. In other words, each giant planet in a system has its own $\tau_{\text{dep}}$ and $f_{5,5\text{AU}}$ in general.

We also note that, before the rapid gas capture, there is a slowly contracting gas-capturing phase after the planet’s mass reaches the critical core mass, and that $\tau_{\text{dep}}$ does not include the slowly contracting phase. The slow phase lasts for $\gtrsim 10^6$ yr (Pollack et al. 1996; Ikoma et al. 2000), and the total time for the disk dissipation (from the disk formation) could be short, comparable to the slow phase, so it would not be realistic to consider the case when $\tau_{\text{dep}}$ is much shorter than the duration of the slow phase. If we were to consider the case with $\tau_{\text{dep}} < 10^6$ yr, that would correspond to the case when the disk suddenly dissipates in a timescale less than $10^6$ yr after the long-lasting slowly contracting phase ($\gtrsim 10^6$ yr).

4.2. Viscous Evolution to a low-mass Disk with High Metallicity from a Compact Disk

Jupiter and Saturn have solid cores and also contain a considerable amount of heavy elements in their H/He envelopes. The two giant planets are expected to have solid components of 30–60$M_E$ in total (e.g., Baraffe et al. 2014). This shows that Jupiter and Saturn have much higher metallicity than the solar composition. If we also consider solids in the other planets, the formation of our solar system requires solid materials of $\gtrsim 80M_E$ in total, which is consistent with the amount of solid included within 50 au of the MMSN disk. However, the above low-mass disk with $4M_J$ contains solid components of less than 20$M_E$ if it has the solar composition, in which heavy elements are 1.4% by mass. In order to form Jupiter and Saturn, therefore, the low-mass disk should have very high metallicity, at least at the beginning of the gas capture.

A low-mass disk with a sufficient solid component can be formed through a viscous disk evolution from a compact size described by Equation (1). Consider an initially compact disk with radius of $\sim 10$ au and with the solar composition. The disk mass is $\sim 18M_J$, and it thus contains solid material of $\sim 80M_E$. Because of the compactness, the gas and solid surface densities of this disk are twice those of the MMSN model at 5 au. First, planetesimals are formed in situ and decoupled from the gaseous disk before the disk evolution. Second, the compact gas disk suffers viscous accretion and loses most of its gas within a relatively short time. The gas disk spreads out to $\sim 200$ au and is reduced in mass to $\sim 4M_J$ at $t = 10^7$ yr in the case of $\alpha = 10^{-3}$ (see Equation (20)). Planetary embryos grow during the disk evolution, and finally a sufficiently large solid core causes dynamical collapse of its envelope and starts to capture its surrounding disk gas rapidly. This scenario explains the suitable low-mass disk with a sufficient solid component for Jupiter formation in a natural way. The formation of Saturn would also be reasonable if the onset of dynamical gas capture occurs at an even later time. It can also naturally explain why the metallicity of Jupiter and Saturn is higher than solar by a factor of $\sim 10$. 

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4.3. Type I and Type II Migration in Gas-depleted Disks

Here we discuss more in detail type I and type II migration in gas-depleted disks. In particular, we show below that the type II migration of Jupiter-size planets or smaller is inefficient because of the additional gas depletion due to the rapid gas capture described in Section 3.

Because of the problematic rapid type I migration (Tanaka et al. 2002), studies of the synthesis of the planet population prefer a gas-depleted (or high-metallicity) disk or a significant reduction in the type I migration speed (Daïsaka et al. 2006; Ida & Lin 2008a; Mordasini et al. 2012a). In the gas-depleted disk adopted in our scenario, the solid surface density is twice that of the MMSN disk whereas the gas surface density is depleted to 4% at the end of the core-growth stage. This requires the solid-to-gas ratio to be 50 times as large as that of the solar composition. This enhancement is comparable to that required in the population synthesis calculations. Hence our scenario for Jupiter formation is a plausible and natural path that can overcome the type I migration problem. Mordasini et al. (2012c) also included the viscous disk evolution in their population synthesis calculations but they fixed the outer radius of the initial disk to be 30 au. Such an intermediate-size disk takes a longer time to deplete the disk gas enough by disk accretion. More compact initial disks should also be examined in population synthesis calculations.

Next we examine type II migration in the gas-depleted disk. In our model of giant planet formation, we did not include the effect of type II migration. It is worthwhile to estimate the timescale of type II migration for our gas-depleted disk. Recently Duffell et al. (2014) have derived an empirical formula for migration speed in the classical type II regime from their hydrodynamic simulations, and the revised timescale for type II migration is given by

\[ t_{\text{migII,p}} = 0.14 \frac{M_p}{\Sigma_{\text{out}} r_p^2} t_{\text{migII,d}} \]

\[ = 0.089 \frac{M_p}{r_p \Sigma_{\text{out}}} \]  

(23)

where \( \Sigma_{\text{out}} \) is the gas surface density outside the gap and corresponds to \( \Sigma(r_p) \) in Appendix B, and \( t_{\text{migII,d}} = 2 r_p^2 / (3 \dot{r_p}) \). Equation (23) corresponds to the “planet-dominated” case where \( M_p > \Sigma_{\text{out}} r_p^2 \), while \( t_{\text{migII,d}} \) corresponds to the “disk-dominated” case, which is the traditional type II migration. The condition for the planet-dominated case is safely satisfied in the gas-depleted disk we consider. Equation (23) agrees well with the hydrodynamic simulations by Dürmann & Kley (2015) within a factor of \( \sim 2 \) in the “planet-dominated” case and is consistent with analytically derived formulae by Armitage (2007), which are used in population synthesis calculations.

As newly pointed out in this paper, for a giant planet smaller than \( \sim 10 M_J \), the gas surface density outside the gap \( \Sigma_{\text{out}} \) suffers an additional depletion from the unperturbed disk because of the rapid gas capture by that disk. The additional depletion factor \( f' \) is given by Equation (21). It gives 0.2 for Jupiter mass and \( h_p / r_p = 0.05 \). This additional depletion factor is derived from the mass-flux balance in the outer disk (see Section 3.1 and Appendix B). This effect slows down the type II migration and lengthens \( t_{\text{migII,p}} \) by a factor of \( 1/f' \). Also note that the additional depletion decreases the lower limit of \( M_p \) for the “planet-dominated” case. Any previous models of type II migration do not include the effect of this additional gas depletion. In fact, the gas capture rate, \( M_{\text{p,hydro}} \), is not taken into account in population synthesis calculations and thus they cannot evaluate this depletion factor \( f' = M_{\text{p,global}} / M_{\text{p,hydro}} \).

The growth time of giant planets less than \( \sim 10 M_J \) is given by

\[ t_{\text{grow}} = \frac{M_p}{M_p - \frac{M_p}{3 \pi r_p^2 \Sigma_{\text{un}}} h_p} \]  

which is almost equal to \( t_{\text{migII,p}} \) if the unperturbed surface density \( \Sigma_{\text{un}} \) is replaced by \( \Sigma_{\text{out}} \). Hence the ratio \( t_{\text{grow}} / t_{\text{migII,p}} \) is given by

\[ \frac{t_{\text{grow}}}{t_{\text{migII,p}}} = f' \approx 1.2 \times \left( \frac{M_p}{M_*} \right)^{2/3} \left( \frac{h_p}{r_p} \right)^{-1} \]  

(25)

This indicates that planets of Jupiter mass or smaller suffer only a small radial drift by type II migration during their growth. According to our growth model, the growth of giant planets terminates only when the disk is depleted to a negligible mass. Hence our results indicate that type II migration is ineffective for Jupiter-mass planets or smaller. On the other hand, since the previous models neglect this additional gas depletion, the growth time is always comparable to \( t_{\text{migII,p}} \), provided that the growth is controlled by the global disk accretion (Benz et al. 2014).

Figure 9 shows growth–migration curves and the current distribution of extrasolar planets. Filled circles are Jupiter and Saturn.

Figure 9. Growth-migration curves. Solid curves show evolution paths based on our result (Equation (25)), and dashed curves are the cases where the traditional type II migration is used. Small dots show the distribution of extrasolar planets. Filled circles are Jupiter and Saturn.
Jupiters (Mayor et al. 2011; Ida et al. 2013). Hasegawa & Ida (2013) discussed possible mechanisms that slow down the type II migration but did not find any effective slowdown mechanisms. In the above and Figure 9, we showed that type II migration of giant planets smaller than \(\sim 10M_J\) slows down because of the additional gas depletion due to their rapid gas capture. Our slowdown mechanism may resolve the problem of type II migration.

For giant planets larger than \(\sim 10M_J\) (i.e., in phase 4), the gap effect prolongs the growth time compared with Equation (24) and the additional gas depletion does not occur. Then we find that the time ratio is again given by Equation (25) but \(f'\) given by Equation (21) is larger than unity in this case. The timescale of type II migration is shorter than the growth time for \(M_p > 10M_J\). This may explain why extrasolar planets more massive than \(10M_J\) are observed less frequently. A detailed population synthesis calculation would be necessary, including our slowdown mechanism for type II migration.

It may be worthwhile to summarize briefly that the evolution of planetary growth with migration for the “disk-dominated” case corresponds to the traditional type II migration, in contrast to the “planet-dominated” case. The timescale of the “disk-dominated” case \(t_{\text{migII,d}}\) is always shorter than the disk global viscous timescale \(\tau_{ss}\) simply because \(R_o > r_p\). Thus a planet subject to “disk-dominated” type II migration always falls toward the central star on a timescale much shorter than the disk viscous evolution time. Even if there are mechanisms for rapid disk dissipation (such as strong photoevaporation), \(t_{\text{migII,d}}\), which is typically \(\sim 10^5\) yr for 5 au and \(\sim 10^4\) yr for the “cool Jupiter” \((\geq M_J\) at \(\sim 1\) au), is still too short to explain the distribution of cool Jupiters naturally. When the disk becomes as light as the planet, the migration becomes “planet-dominated,” which is slower and thus mitigates the infall problem. However, even if the gas accretion rate is as fast as \(M_{\text{d,global}}\), the migration timescale \(t_{\text{migII,p}}\) is always comparable to the growth timescale, and thus it is still difficult to explain the cool Jupiters (Figure 9). In addition to the planet-dominated migration, gas depletion due to accretion onto the planet further reduces migration whilst maintaining the accretion rate onto the planet (Section 3.3). This would explain the distribution of cool Jupiters found among extrasolar planets. We should emphasize that this argument is not based on a special parameter set but holds for rather general disk parameters such as viscosity or surface density.

5. SUMMARY AND DISCUSSION

We examined the growth rates and the final masses of giant planets embedded in protoplanetary disks through capture of disk gas, by employing an empirical formula for the gas capture rate and a shallow disk gap model, which are both based on hydrodynamic simulations. Our findings are summarized as follows.

1. Because of the shallow gap revealed by recent hydrodynamic simulations, giant planets do not stop their gas-capturing growth. Only the depletion of the whole gas disk can terminate their growth. For planets less massive than \(10M_J\), their growth rates are mainly controlled by the gas supplied by global disk accretion rather than their gaps. For such a mass range, the final planetary mass is given by Equation (18). For the more massive planets, their growth rates are limited by deep gaps and their final mass is given by Equation (19).

2. For planets less massive than \(10M_J\), the gas supplied to the planets by disk accretion is insufficient. This also causes a depletion of the gas surface density even outside the gap and creates an inner hole in the protoplanetary disk. The additional gas depletion factor is given by Equation (21) (see also Appendix B). Our result suggests that less massive giant planets can create deeper inner holes than more massive ones.

3. Because of the continual growth, the formation of Jupiter requires a very low-mass gas disk of a few or several \(M_J\) at least, at the beginning of its gas capture. This disk is much less massive than the MMSN model, whereas solid material of \(\sim 80\) Earth masses is also necessary for the formation of the planets in our solar system. That is, we need a very low-mass disk with a high metallicity. These requirements can be achieved by the viscous evolution from a compact disk of initial size \(\sim 10\) au with the solar composition. For a disk with a moderate viscosity of \(\alpha \sim 10^{-3}\), most of the disk gas accretes onto the central star and a widely spread low-mass gas disk remains at \(t \sim 10^7\) yr. This scenario can explain the high metallicity in the giant planets of our solar system.

4. A very low-mass gas disk also provides a plausible path where type I and II planetary migrations are both suppressed significantly. In particular, we also showed that the type II migration of Jupiter-size planets is inefficient because of the additional gas depletion due to their rapid gas capture. This slow type II migration is consistent with the radial distribution of observed extrasolar planets in which Jupiter-size planets congregate at \(\sim 1\) au.

In this paper, we proposed the formula that describes the gas accretion rate onto the planet, which combines an empirical formula of gas accretion rate obtained by a local hydrodynamic simulation (TW02) and another empirical formula of gap depth obtained by global hydrodynamic simulation (Duffell & MacFadyen 2013). Although the accretion rate of our model is in agreement with those of hydrodynamic simulations (see Figure 1), our model might have the tendency that the accretion rate decreases more slowly than hydrodynamic simulations for
the higher mass range \((M_p > M_J)\). The smaller accretion rate suggested by hydrodynamic simulations might, however, have difficulty in explaining the distribution of extrasolar planets in the plane of mass versus orbital radius, especially for cool Jupiters \((0.5–5\,\text{au}, M_p > 100M_\text{Earth})\) because the timescale of the gas-capturing growth would be longer than that of the inward type II migration, resulting in the formation of more warm or hot Jupiters. Set against the gap-deepening effect, there is a mechanism whereby the high-mass planets \((M_p \geq 5M_J)\) increase the eccentricity of the gas motion in the gap edge, which promotes gas transport into the gap region, leading to enhancement of the gas accretion rate onto the planets by a factor of two (Kley & Dirksen 2006). This might compensate for the steep decrease in the accretion rate to some degree. At this stage, it seems difficult to accurately model the accretion rate, which is a function of the temperature and viscosity, as well as the planet’s mass. It therefore becomes more important to determine the rate of gas accretion onto the planets and the dependence on disk parameters in detail.

We also proposed the formation of large-scale disk depletion created by gas capture of the planet based on an analytic argument (see Appendix B). This depletion arises also from long-term viscous evolution of the gas disk, which has not been studied so far. To check this, high-resolution hydrodynamic simulations with long-term evolution should be done in future work. Malik et al. (2015) have recently investigated the gap-opening criterion of migrating planets in protoplanetary disks and found that the gap opening is more difficult than the traditional condition, which is based on torque balance (e.g., Lin & Papaloizou 1993; Crida et al. 2006). Other recent hydrodynamic simulations (Duffell & MacFadyen 2013; Fung et al. 2014) also showed that the gap opening is less significant, although there are some differences in assumptions and purposes. Also, gas in the gap region tends to be turbulent due to magneto-rotational instability (Gressel et al. 2013; Keith & Wardle 2015), which would lead to an even shallower gap. Further studies would be necessary to quantify the gap-deepening effect.

In this paper, we considered the rapid gas capture of the proto-Jupiter in a very light disk in which “planet-dominated” type II migration can be applied. Here we discuss a case of a more massive disk than the MMSN in which a planet suffers “disk-dominated” type II migration. Since the growth timescale is shorter than the migration timescale in this case, a Jupiter-sized planet can form within \(\sim 10^4\) yr before severe inward migration (D’Angelo & Lubow 2008). However, if the disk remains massive after that, the planet would grow further and also suffer severe inward migration. In order to leave a Jovian-sized planet, the disk has to dissipate rapidly \(\sim 10^4\) yr just after the planet’s formation. A mechanism for such rapid disk dissipation has not been proposed so far. Therefore the massive disk would not be suitable for Jupiter formation.

In situ formation of hot Jupiters is thought to be difficult in general. This is because (1) it is easier for a planet in the inner region to form a gap than one in the outer region, which means that the planet’s growth stops at a mass much less than that of Jupiter, (2) even if a hot Jupiter forms, it is susceptible to type II migration and falls into the central star, and (3) there is not enough solid material to trigger the dynamical gas capture of the protoplanet. Our model showed that the gap has little effect on suppressing the rate of gas accretion onto planets. In this sense, the first point is no longer a reason to prevent in situ formation. We also showed that type II migration is less effective than previously thought, which would not rule out in situ formation either. Although we make no claim that there is enough solid material to form large solid cores, we should note that the difficulty of in situ formation of hot Jupiters is greatly mitigated.

Our model assumes forced exponential decay with respect to time for the protoplanetary disk, but the times of disk dissipation would impact significantly on disk evolution and thus also on the planet formation scenario described above. Several mechanisms in addition to viscous disk accretion are proposed, such as photoevaporation (Hollenbach et al. 1994; Clarke et al. 2001; Owen et al. 2012), solar wind stripping (Horedt 1978; Matsuyama et al. 2009), solar wind induced accretion to the central star (Elmegreen 1978), and disk wind (Suzuki & Inutsuka 2009, 2014). Among them, photoevaporation is thought to be the dominant mechanism to dissipate disks.

Photoevaporation has been actively studied recently and the role of X-ray and UV irradiation from the central star has been substantially understood. Since, however, the luminosity of X-rays, EUV, and FUV and the time evolution have large uncertainty, the evolution of protoplanetary disks would vary widely. If the rate of disk accretion toward the central star, 

\[
M_{d,\text{global}} = 5 \times 10^{-9} f_{\Sigma, 5 \, \text{au}} \left(\frac{\alpha}{10^{-3}}\right) \left(M_\text{Sun}/1 \, \text{au}\right)^2 M_\odot \, \text{yr}^{-1},
\]

is larger than the mass-loss rate by photoevaporation, the effect of photoevaporation can be neglected. Our scenario suggests that \(f_{\Sigma, 5 \, \text{au}} \sim 0.04\) is a plausible parameter, which gives \(M_{d,\text{global}} = 2 \times 10^{-10} M_\odot \, \text{yr}^{-1}\). According to Owen et al. (2012), the mass-loss rate can be in the range from \(10^{-12}\) to \(10^{-7} M_\odot \, \text{yr}^{-1}\) depending on X-ray luminosity, which means that our scenario is realized when the X-ray luminosity is not strong.

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we obtain the exact solution (e.g., Landau & Lifshitz 1987):
\[
\Sigma(x, t) = \Sigma_{\text{un}} \text{erf} \left( \frac{x}{2\sqrt{3} \nu t} \right),
\]
where \( \Sigma_{\text{un}} \) is a constant.

Since the disk mass reduced from the initial condition is equal to the mass sunk into the boundary at \( x = 0 \), we have
\[
\int_0^t F(t')dt' = 2\pi r_p \int_0^{\infty} (\Sigma_{\text{un}} - \Sigma(x, t)) dx
\]
\[
= 2\pi r_p \Sigma_{\text{un}} \frac{2 \sqrt{3} \nu t}{\sqrt{\pi}},
\]
where \( F(t) \) is the mass flux toward \( x = 0 \) from \( x > 0 \), and we use a mathematical formula: \( \int_0^\infty \text{erf}(x) dx = 1/\sqrt{\pi} \). Differentiating Equation (A6) with respect to time yields the mass accretion rate to the boundary \( x = 0 \) as a function of time:
\[
F(t) = 2\pi r_p \Sigma_{\text{un}} \frac{3\nu \nu t}{\pi t}.
\]

**APPENDIX B**

**A SOLUTION OF A STEADY ACCRETION DISK WITH A MASS SINK TO AN EMBEDDED PLANET**

Lubow & D’Angelo (2006) obtained the radial gas distribution of a steady viscous accretion disk with a mass sink by a planet and showed that the mass sink causes gas depletion over a wide region, but did not give a detailed derivation of it. Here, we present the derivation of the solution of the surface density distribution in such a case.

In an accretion disk, the radial angular momentum flux \( F_J \) is given by (Lynden-Bell & Pringle 1974)
\[
F_J = j F_M + 3\pi r^2 \nu \Sigma \Omega,
\]
where \( F_M \) is the radial mass flux and the specific angular momentum \( j \) is given by \( r^2 \Omega \). Keplerian rotation is assumed for \( \Omega \). Thus, for a given (constant) mass flux and angular momentum flux, the surface density of the quasi-steady disk is expressed as
\[
\Sigma(r) = \frac{-F_M + F_J/j(r)}{3\pi \nu (r)}.
\]

We here consider an accretion disk having a mass sink at a rate of \( M_p \) at \( r = r_p \) due to accretion onto the planet. The disk angular momentum sinks into the planet at a rate \( j(r_p) M_p \). Then the mass and angular momentum fluxes are discontinuous at \( r_p \) and given by
\[
F_M = \begin{cases} 
(M_p + M_s) & (r > r_p), \\
-M_s & (r < r_p),
\end{cases}
\]
and
\[
F_J = \begin{cases} 
(j_p M_p + j_s M_s) & (r > r_p), \\
-j_s M_s & (r < r_p),
\end{cases}
\]
\[\text{where } M_s \text{ is the mass accretion rate onto the central star and } r_s \text{ is the radius of the inner disk edge. The specific angular momenta } j_p \text{ and } j_s \text{ are the values at } r_p \text{ and } r_s, \text{ respectively. Note that the sum } M_p + M_s \text{ is equal to the global accretion rate } M_{\text{d,global}}. \text{ The negative fluxes indicate the inward transport of the disk mass and angular momentum. Substituting Equations (36) and (37) into (35), we obtain}
\]
\[
\Sigma(r) = \begin{cases} 
\frac{M_s}{3\pi \nu} \left( 1 - \frac{r_s}{r} \right) + \frac{M_p}{3\pi \nu} \left( 1 - \frac{r_p}{r} \right) & r > r_p, \\
\frac{M_s}{3\pi \nu} \left( 1 - \frac{r_p}{r} \right) & r < r_p,
\end{cases}
\]
\[
\text{The ratio } M_p/M_s \text{ is determined by the accretion formula of Equation (7). From Equation (B5), we obtain}
\]
\[
\Sigma(r_p) = \frac{M_s}{3\pi \nu},
\]
\[
\text{where we omitted the term } \sqrt{r_s/r_p}. \text{ This approximation would be valid for a planet with } r_p \gtrsim 1 \text{ au since } r_s \text{ would be less than 0.1 au. Note that the solution given by Lubow & D’Angelo (2006) does not assume that } r_s \ll r_p. \text{ From Equations (7), (9) and (B6), we obtain}
\]
\[
M_p = \frac{D'}{3\pi \nu} M_s,
\]
\[
\text{where } D' \text{ is defined by}
\]
\[
D' = \frac{1}{0.034K + 1}
\]
\[
\nu_p = \nu(r_p), \text{ and we equate } \Sigma(r_p) \text{ with } \Sigma_{\text{un}} \text{ in Equation (9). Since the ratio } D'/3\pi \nu \text{ is equal to } M_p/\text{M}_{\text{hydro}}/\text{M}_{\text{d,global}} \text{ by definition, the ratio is given by Equation (16):}
\]
\[
\frac{D'}{3\pi \nu} = 0.90 \left( \frac{M_p}{M_s} \right)^{-2/3} \left( \frac{h_p}{r_p} \right)
\]
\[
= 4.5 \left( \frac{M_p}{M_s} \right)^{-2/3} \left( \frac{h_p/r_p}{0.05} \right).
\]
\[
\text{Noting } M_p + M_s = \text{M}_{\text{d,global}} \text{ and using Equation (B7), we obtain the accretion rates as}
\]
\[
M_p = \frac{D'/3\pi \nu}{1 + D'/3\pi \nu} \text{M}_{\text{d,global}},
\]
\[
M_s = \frac{1}{1 + D'/3\pi \nu} \text{M}_{\text{d,global}}.
\]
\[
\text{Substituting Equation (B10) into (B5), we finally obtain the expression for the surface density of the disk with a mass sink}
\]
of the planetary gas capture as
\[
\Sigma(r) = \begin{cases} 
\frac{M_{\text{global}}}{3\pi(1 + D'/3\pi a_p)} \left(1 - \frac{r_p}{\sqrt{r}}\right) & \text{for } r > r_p, \\
\frac{D'}{3\pi a_p} \left(1 - \frac{r_p}{r}\right) & \text{for } r < r_p.
\end{cases}
\]

(44)

From this expression, the additional gas depletion factor due to gas capture is given by \((1 + D'/3\pi a_p)^{-1}\), which is approximately equal to Equation (22) in the text. In Figure 10 we plot the surface density distribution obtained for a typical case. In the vicinity of the planet, the gas, which is an additional gas depletion due to the gravitational torque from the planet, should also exist. Even though the obtained surface density does not show this gap, its effect is included in this formulation through the parameter \(D'\) of Equation (41).

APPENDIX C
RECIPE FOR THE GAS ACCRETION ONTO THE PLANET IN HYDRODYNAMIC SIMULATIONS

Tanigawa & Watanabe (2002) obtained the fitting formula of the gas accretion rate from their local high-resolution hydrodynamic simulations (see Equation (8)). However, it is usually difficult for global hydrodynamic simulations to have such a high resolution near the planet; one has to treat the gas accretion onto the planet in a simplified way. One simple way is to reduce the surface density at a constant rate within a certain radius \(a_{\text{fit}}\):

\[
d\Sigma = -\frac{\Omega_s^2 \Sigma}{f} \quad \text{for } r < a_{\text{fit}},
\]

(45)

where \(f\) is a parameter for the reduction rate of the gas, \(r\) is the distance from the planet, and \(a_{\text{fit}}\) is a parameter for the radius within which the gas surface density is reduced. Using the above equation, we can write the accretion rate onto the planet as

\[
M_p = \int_0^{a_{\text{fit}}} \left| \frac{d\Sigma}{dr} \right| 2\pi rdr = \frac{a_{\text{fit}}^2}{f} \frac{\Sigma r_H^2}{\Omega_p r_H}.
\]

(46)

Comparing Equation (46) with Equations (7) and (8), we have

\[
f = \frac{\pi a_{\text{fit}}^2}{1.26} \left(\frac{h_p}{r_H}\right)\left(\frac{M_p}{M_*}\right)^{-2/3}
\]

(47)

This means that the coefficient \(f\) is not an independent parameter, but given as a function of the disk aspect ratio and the planet’s mass when the normalized radius \(a_{\text{fit}}\) is given. One may want to have two radii within which the gas surface densities are reduced on different timescales (e.g., Kley 1999; Zhu et al. 2011). Even in this case we can derive the same equation with a different coefficient. For example, in the case of Zhu et al. (2011), we have

\[
f = 1.7 \left(\frac{h_p}{r_p}\right)^2 \left(\frac{M_p}{M_*}\right)^{-2/3},
\]

(48)

where they adopt \(a_{\text{fit}} = 0.45\) for the inner radius and \(a_{\text{fit}} = 0.75\) for the outer radius, and the reduction rate within the outer radius is three times slower than that within the inner radius. Note that the above recipe assumes a constant surface density within the radius \(a_{\text{fit}}\) whereas the surface density in a numerical simulation varies with the position, which would not lead to serious error for simulations that have low spatial resolution near the planet.

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