Estimation of Near-Bottom Suspension-Carrying Flow around a Pipeline

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Abstract. In a model format, the force effect of a stratified ideal fluid flow on streamlined elements of engineering structures is investigated. Integral representations of wave drag and lift force are obtained. Numerical calculations are performed for the real marine medium. The conditions under which significant changes in hydrodynamic reactions to streamlined elements of technical structures occur are revealed.

1. Formulation and solution of the model problem

The underwater obstacles streamlined by a stratified sea current are subject to viscous drag. In addition, hydrodynamic reactions arise due to the generation of internal waves at jumps in the density of the medium. Near the bottom, this effect can be amplified by bottom-weighted (turbid) streams arising when micro- and fine-grained particles of bottom soil are mixed in them. The velocities of such flows can exceed 1 m/s, and the thicknesses – several tens of meters. At the same time, the effective density of water in the streams may be more than ten percent more than the density of the surrounding water, which causes a more sharp drop in density at the flow boundaries, leading, in turn, to increased stratification of the medium [1] and a corresponding intensification of internal waves.

Thus, the underwater elements of engineering structures (in particular, transport pipelines) streamlined by turbid flows can additionally experience quite significant power effects associated with the generation of internal waves. Therefore, the corresponding problem is considered below in the model version: a dipole is considered as an obstacle in the flow; it simulates a transverse flow past a cylinder (as an element of a pipeline).

To estimate the force effects on the obstacle being flown, we consider a two-layer flow of an ideal fluid, bounded below by a horizontal bottom, that flows stationary around a cylinder with a cross-section of radius $R$. The cross section of the pipeline is modeled by a point dipole with a moment $m = 2\pi VR^2$, where $V$ is the incoming flow velocity. The thickness of the upper layer is $H_1$, the lower – $H_2$ and the densities are $\rho_1$ and $\rho_2$ respectively ($\rho_1 < \rho_2$). The origin of coordinates is placed on the undisturbed boundary between the fluid layers, the $x$ axis is directed along this boundary, the $y$ axis – vertically upwards.

The complex velocity of a flow disturbed by a dipole is sought in accordance with [2-4]. Further, within the framework of the linear theory, two model problems are considered: the dipole is located below the density jump, i.e. at the point $(0, -h)$, and above the jump (at the point $(0, h)$). To calculate the hydrodynamic forces applied to the cylinder under consideration, we represent the complex-
conjugate velocity in each of the layers in the form \( \mu_j = V + U_j, \ j = 1,2 \). Then the mathematical problem of finding the velocity perturbations \( U_j \) introduced into the flow by a dipole localized in the lower layer is formulated as follows: it is required to find analytical functions \( U_1(z) \) and \( U_2(z) \) satisfying the boundary conditions

\[
\text{Im} \left[ i \frac{dU_1}{dz} - \nu U_1 \right] = 0 \text{ at } y = H_1, \tag{1}
\]

\[
\delta \text{Im} \left[ i \frac{dU_2}{dz} - \nu U_2 \right] = \text{Im} \left[ i \frac{dU_2}{dz} - \nu U_2 \right] \text{ at } y = 0, \tag{2}
\]

\[
\text{Im} U_1 = \text{Im} U_2 = 0 \text{ at } y = H_2. \tag{3}
\]

Here, \( \delta = \rho_1/\rho_2, z = x + iy, \nu = g / V^2, \ g \) is acceleration of gravity; the function \( U_j(z) \) is regular in the band \(-\infty < x < +\infty, 0 < y < H_1, \) and \( U_j(z) \) – in the band \(-\infty < x < +\infty, -H_2 < y < 0 \) everywhere except for the point \( z = -ih \), at which it has a pole of the second order: \( U_2(z) \rightarrow -m/2\pi(z+ih)^2, \ z \rightarrow -ih \).

The boundary condition (1) describes the constancy of pressure on the free surface, (2) – the continuity of pressure when passing through the interface of the layers, (3) – the absence of fluid flow through this surface, (4) is the no-flow condition on the bottom.

The complex-conjugate hydrodynamic reaction of a cylinder \( F^* = X - iY \) is calculated using the Chaplygin formula [5]. Here, \( X \) is the wave drag, \( Y \) is the lift force of the dipole.

When a dipole is localized under a density jump, we have

\[
F^* = \frac{i\rho m^2}{2\pi} \int_{-\infty}^{+\infty} k(C(k)e^{ih} - D(k)e^{-ih})dk - \frac{\rho m^2}{2} \sum_{j=1}^{s} \text{res} \left\{ k(C(k)e^{ih} + D(k)e^{-ih}) \right\}, \tag{5}
\]

\[
C(k) = \frac{k\{-[1-\delta](k^2 - \nu^2)\sin k(H_1 + h) + [1 + \delta]k^2 + (1-\delta)\nu^2 \}\sin k(H_1 + h) - 2k\nu \sin k(H_1 + h)\}}{2\sin kH_1 \sin kH_2 e^{ih} \{k^2 + [\delta k^2 + (1-\delta)\nu^2 \]\sin kH_1 \sin kH_2 - k\nu (\sin kH_1 + \sin kH_2)\}}, \tag{6}
\]

\[
D(k) = \frac{k \sin k(H_2 - h)[\{1+\delta\}k^2+2k\nu+(1-\delta)\nu^2\]e^{-ih} - (1-\delta)(k^2-\nu^2)e^{ih}\}}{2\sin kH_1 \sin kH_2 \{k^2 + [\delta k^2 + (1-\delta)\nu^2 \]\sin kH_1 \sin kH_2 - k\nu (\sin kH_1 + \sin kH_2)\}}. \tag{6}
\]

The integral in (5) is meant in the sense of the Cauchy principal value, and the residues are taken over all \( s \) poles \( k_j \) of the function \( k(C(k)e^{ih} + D(k)e^{-ih}) \) located on the positive real axis. From (6) it is clear that these poles are the positive roots of the equation

\[
k^2 + [\delta k^2 + (1-\delta)\nu^2 \]\th kH_1 \th kH_2 - k\nu (\th kH_1 + \th kH_2) = 0. \tag{7}
\]

In addition, it is obvious that the points \( k = k_j \) (and only they) are singular for the integrand in the first term of (5) (that is, the poles located on the integration contour).

The analysis carried out in [2,3] showed that equation (7) has two positive roots when the following condition is satisfied

\[
V < V_{cr}^{\text{int}} = \sqrt{\frac{gH_1(\beta + 1 - \sqrt{(\beta + 1)^2 - 4\epsilon\beta})}{2}}
\]

and a single positive root if
\[ V_{cr}^{\text{int}} < V < V_{cr}^{\text{sur}} = \sqrt{gH_1(\beta + 1 + \sqrt{(\beta + 1)^2 - 4\epsilon \beta})}; \]

When \( V > V_{cr}^{\text{sur}} \) there are no positive solutions. Here, \( \epsilon = 1 - \delta = (\rho_2 - \rho_1) / \rho_2 \) – relative density difference between the layers, \( \beta = H_2 / H_1 \). From the physical point of view, the critical velocities \( V_{cr}^{\text{int}} \) and \( V_{cr}^{\text{sur}} \) mean the maximum flow velocity at which waves are formed in the flow behind the streamlined cylinder, due to the presence of a layer of density jump and a free surface respectively (i.e. internal and surface waves).

Separating in (5) the real and imaginary parts, we finally get the following expressions for the wave drag and lift force:

\[
X = -2\pi^2 \rho V^2 R^4 \sum_{j=1}^{k} \text{res} \left\{ k(C(k)e^{ikh} + D(k)e^{-ikh}) \right\},
\]

\[
Y = -2\pi \rho V^2 R^4 \int_{0}^{\infty} k(C(k)e^{ikh} - D(k)e^{-ikh})dk.
\]

Here, the calculation of residues is carried out according to the formula [6]

\[
\text{res} \left\{ k(C(k)e^{ikh} + D(k)e^{-ikh}) \right\} = \frac{k(g_1(k)e^{ikh} + g_2(k)e^{-ikh})}{dg_1(k) / dk} \bigg|_{k=k_j},
\]

\[
g_1(k) = -k((\delta - 1)(k^2 - \nu^2) \sin k(H_1 - h) + [(1 + \delta)k^2 + (1 - \delta)\nu^2] \sin k(H_1 + h) - 2\nu \cosh k(H_1 + h))
\]

\[
g_2(k) = \frac{k \cosh k(H_2 - h)[(1 + \delta)k^2 + 2\nu(1 - \delta)\nu^2] e^{ikh} - (1 - \delta)(k^2 - \nu^2)e^{-ikh}}{2 \cosh k H_1 \cosh k H_2},
\]

\[
g_3(k) = k^2 + [\delta k^2 + (1-\delta)\nu^2] \cosh k H_1 \cosh k H_2 - k \nu (\cosh k H_1 + \cosh k H_2).
\]

Further, in the same formulation, we study the problem of determining the hydrodynamic load on a dipole located in the upper layer of the flow (above a density jump). Solving this problem in the way indicated above, we obtain:

\[
F^\ast = \frac{i\rho m^2}{2\pi} \int_{0}^{\infty} k(A(k)e^{ikh} - B(k)e^{-ikh})dk - \frac{\rho m^2}{2} \sum_{j=1}^{k} \text{res} \left\{ k(A(k)e^{ikh} + B(k)e^{-ikh}) \right\}, \tag{8}
\]

\[
A(k) = \frac{k([\delta k + (1-\delta)\nu] \cosh k H_1 - k[k \cosh k(H_1 - h)] - k \cosh k(H_1 + h))}{\cosh k H_1 \left\{ k^2 + [\delta k^2 + (1-\delta)\nu^2] \cosh k H_1 \cosh k H_2 - k \nu (\cosh k H_1 + \cosh k H_2) \right\}},
\]

\[
B(k) = \frac{k(k + \nu) e^{ikh}}{\cosh k H_1 \left\{ k^2 + [\delta k^2 + (1-\delta)\nu^2] \cosh k H_1 \cosh k H_2 - k \cosh k H_1 \cosh k H_2 - k \cosh k H_2 \right\}}.
\]

Hence, we have the following expressions for the wave drag and lift force:

\[
X = -2\pi^2 \rho V^2 R^4 \sum_{j=1}^{k} \text{res} \left\{ k(A(k)e^{-ikh} + B(k)e^{ikh}) \right\},
\]

\[
Y = -2\pi \rho V^2 R^4 \int_{0}^{\infty} k(A(k)e^{-ikh} - B(k)e^{ikh})dk.
\]
The integral in (8), as in (5), is meant in the sense of the Cauchy principal value, $k_j$ are the poles of the function $k(A(k)e^{-kh} + B(k)e^{kh})$ (solutions of equation (7)), and the residues in (8) are calculated similarly to the first problem.

2. Calculation results

Calculations of the hydrodynamic effects on the model cylinder were carried out for the values of the characteristics of the medium corresponding to the actual sea conditions. Thus, the density in the upper layer is $\rho_1 = 1024\text{kg/m}^3$, and the density drops between the upper and lower layers are $\rho_2 / \rho_1 = \{1.01; 1.10\}$, the second value of $\rho_2 / \rho_1$ corresponds to the presence of turbid flow in the lower layer. The total depth of the stream is $H_0 = H_1 + H_2 = 50\text{m}$, the thickness of the upper layer is $H_1 = 40\text{m}$, the radius of the cylinder is $R = 0.71\text{m}$. The dipole was successively localized at distances $h = \{1.5\text{m}, 2\text{m}, 3\text{m}, 4.5\text{m}\}$ from the interface between the layers.

The results of the calculation of the wave drag $X$ and lift force $Y$ (per linear meter of cylinder length), as functions of the speed of the incident flow $V$, are shown in Fig. 1–2. Each graph shows 4 curves corresponding to different distances $h$, and, as can be seen in the figures, the increase of $h$ leads to a decrease in the maximum of drag and lift force.

Attention is drawn to the significant effect of the density jump power $\varepsilon$ on changes in the magnitude of the maximum of wave drag and lift force. So, the comparison of graphs in fig. 1-2 and constructed for two different values of the jump power $\varepsilon$, shows that such changes are almost directly proportional to the magnitude of $\varepsilon$. The values of the maxima themselves shift towards an increase in the flow velocity when the cylinder is shifted from the jump layer. At the same time, calculations showed that the values of $X$ and $Y$ practically do not depend on the thicknesses of the flow layers $H_1$ and $H_2$. Thus, the power of the density jump $\varepsilon$ and the distance $h$ from the dipole modeling the cylinder to the density jump are the main parameters of the problem; their variations can lead to significant changes in the hydrodynamic reactions to the obstacle streamlined.

It should be noted that at flow rates greater than the critical speed $V_{cr}^{int}$, the wave drag $X(V)$ experiences a discontinuity of the first kind, sharply decreasing to almost zero. Then, it again reaches significant values only at significantly higher values of $V$ ($V > 4\text{m/s}$), i.e. at stream speeds not characteristic to real sea conditions. At the same time, the value of the critical velocity itself increases with increasing power of the density jump layer $\varepsilon$, which corresponds to the expansion of the range of speeds inside which the cylinder experiences significant wave drag. In this case, the graphs (Fig. 1) are generally shifted toward larger values of $V$.

The graphs of function $Y(V)$ presented in Fig. 2 reflect an important feature in the variability of lift force. Thus, in a relatively narrow range of flow velocity, the lift force abruptly changes its direction to the opposite. The question arises about the possible consequences of such a drastic and reversible change of $Y(V)$. To answer it, one may use the Bernoulli integral, which relates the change in pressure with the change in the velocity of the stream. Since in real sea conditions the flow velocity is non-uniform in space, the pressure field is also non-uniform. And the change in pressure directly affects the change in force effects on the streamlined underwater obstacles. As a result, alternating force effects can lead to the deformation of individual horizontally extended elements of streamlined structures (for example, transport pipelines). This result is new and must be taken into account when designing and constructing underwater structures on the seabed.

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Figure captions

Fig. 1. Wave drag of a dipole as a function of the flow velocity. The dipole is located beneath the density jump (the left pair of graphs), dipole is located above the jump (the right pair); $\rho_2 / \rho_1 = 1.01$ for the top pair of graphs; $\rho_2 / \rho_1 = 1.1$ for the lower pair; curves 1–4 correspond to $h = 1.5$ m, 2 m, 3 m, and 4.5 m, respectively.

Fig. 2. Lift force of a dipole as a function of the flow velocity. The dipole is located beneath the density jump (the left pair of graphs), dipole is located above the jump (the right pair); $\rho_2 / \rho_1 = 1.01$ for the top pair of graphs; $\rho_2 / \rho_1 = 1.1$ for the lower pair; curves 1–4 correspond to $h = 1.5$ m, 2 m, 3 m, and 4.5 m, respectively.
beneath the density jump

\[ \frac{\rho_2}{\rho_1} = 1.01 \]

above the density jump

\[ \frac{\rho_2}{\rho_1} = 1.1 \]

Fig. 1

beneath the density jump

\[ \frac{\rho_2}{\rho_1} = 1.01 \]

above the density jump

\[ \frac{\rho_2}{\rho_1} = 1.1 \]

Fig. 2