Heavy Quarkonium Production at Low $P_{\perp}$ in NRQCD with Soft Gluon Resummation

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Abstract

We extend the non-relativistic QCD (NRQCD) prediction for the production of heavy quarkonium with low transverse momentum in hadronic collisions by taking into account effects from all order soft gluon resummation. Following the Collins-Soper-Sterman formalism, we resum the most singular terms in the partonic subprocesses. The theoretical predictions of $J/\psi$ and $\Upsilon$ productions are compared to the experimental data from the fixed target experiments (E866) and the collider experiments (RHIC, Tevatron, LHC). The associated non-perturbative Sudakov form factor for the gluon distributions is found to be different from the previous assumption of rescaling the quark form factor by the ratio of color factors. This conclusion should be further checked by future experiments on Higgs boson and/or di-photon production in $pp$ collisions. We also comment on the implication of our results on determining the color-octet matrix elements associated with the $J/\psi$ and $\Upsilon$ productions in the NRQCD factorization formalism.
I. INTRODUCTION

Heavy quarkonium production has been an important topic in strong interaction physics where one can apply perturbative QCD calculations and systematically classify the associated non-perturbative physics in heavy quark systems [1]. An effective theory, called non-relativistic QCD (NRQCD) [2], has been developed to study the productions and decays of heavy quarkonia, and especially their productions with large transverse momentum in hadronic collisions, up to the next-to-leading order (NLO) in QCD interactions. These calculations demonstrated that the so-called color-octet mechanism is important to understand the \( J/\psi \) and \( \Upsilon \) productions at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC) [3–9].

However, how large of the associated color-octet matrix elements remains a question. This becomes more important in light of recent studies of the heavy quarkonium polarization in the collider experiments, where two separate NLO calculations yield very different conclusions [3]. The main difference lies on the size of the non-perturbative color-octet matrix elements. In heavy quarkonium production, the differential cross section is written into a factorized form,

\[
d\sigma = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X)\langle \phi^H[n]\rangle.
\]

(1)

Here \( \hat{\sigma}(Q\bar{Q}[n] + X) \) is the cross section at the parton level, which represents the production of a pair of heavy quark in a fixed color, spin and orbital angular momentum state \( n \) and can be calculated perturbatively. The long distant matrix element (LDME) \( \langle \phi^H[n]\rangle \) describes the transition of the heavy quark pair in the configuration of \( Q\bar{Q}[n] \) into the final state heavy quarkonium. The LDMEs are process-independent, which can be extracted from experimental data or calculated from non-perturbative method, such as the potential model for predicting heavy quarkonium states [10]. In addition, the LDMEs are organized in terms of the velocity \( v \) expansion in the NRQCD framework. A fixed order perturbative calculation is performed in orders of both the strong coupling constant \( \alpha_s \) and the power of the velocity for the associated LDMEs. For example, for \( J/\psi \) production in \( pp \) collisions, the differential cross sections depend on the following three color-octet matrix elements:

\[
\langle O^{J/\psi[3S_1]} \rangle, \langle O^{J/\psi[1S_0]} \rangle, \langle O^{J/\psi[3P_J]} \rangle,
\]

(2)

which are at the same order in the velocity expansion. M. Butenschoen and B. A. Kniehl used a fixed-order calculation to extract the values of these three matrix elements by fitting to several sets of transverse momentum \( (P_{\perp}) \) distributions of \( J/\psi \) produced in hadron-hadron collisions [6, 7]. In their fits, the experimental data points in small and mediate \( P_{\perp} \) region are all included. They found that the extracted values of these three LDMEs strongly depend on the selection of the low \( P_{\perp} \) data sets, by imposing a lower limit on the \( P_{\perp} \) of the data included in the fits, cf. Tables II, III and IV of Ref. [7]. On the other hand, K. T. Chao et al demonstrated that \( J/\psi \) production in high \( P_{\perp} \) region is only sensitive to two linear combinations of these three matrix elements [3]. In brief, the above three LDMEs cannot be precisely determined from comparing the current experimental data with a fixed-order perturbative calculation in the framework of NRQCD. Moreover, the values of these matrix elements extracted in [6, 7] seem to be too large when compared to those extracted from the inclusive cross section of \( J/\psi \) production in \( e^+e^- \) annihilation [1, 6, 7].
In this paper, we will extend the NRQCD prediction for heavy quarkonium production to low transverse momentum region with a resummation calculation. This will provide additional information on the color-octet matrix elements. Furthermore, comparing the differential cross section spectrum at low $P_\perp$ and high $P_\perp$ could shed light on the underlying mechanisms for heavy quarkonium production, and may also improve our understanding of the heavy quarkonium polarization. Some of the recent calculations for the production of heavy quarkonia with very high $P_\perp$ could be found in Refs. [11, 12]. The low transverse momentum heavy quarkonium production has its own interest in applying the perturbative QCD factorization formalism. In the low $P_\perp$ region ($\Lambda_{QCD} \ll P_\perp \ll M$), there are large logarithms of the type $\alpha_s^n \ln^{2m-1}(M^2/P_\perp^2)$ in high-order perturbative calculations. Therefore, to obtain a reliable perturbative prediction, we have to resum these large logarithms. In this work we follow the Collins-Soper-Sterman (CSS) formalism [13]. We will derive the relevant resummation coefficients, by comparing the expansion of the resummation formula with the fixed order perturbative calculations within the NRQCD formalism. In particular, the most singular contributions from the color-octet $^1S^0(8)$ and $^3P_J(8)$ channels will be resummed through the CSS formalism. Earlier work on the soft gluon resummation for heavy quarkonium production in low $P_\perp$ region has been performed in Ref. [14] in a model similar to the color-evaporation model for heavy quarkonium production. In this paper, we will calculate the resummation in the NRQCD framework.

The original CSS resummation (also called the transverse momentum resummation) is derived for low transverse momentum Drell-Yan lepton pair (vector Boson) production, and has been applied to Higgs boson and di-photon productions in hadron-hadron collisions and semi-inclusive hadron production in deep-inelastic scattering (DIS) processes. However, there have been few studies to extend the CSS resummation to more complicated processes, such as the di-jet production in hadronic collisions. Because the final state in these hard processes carries color, the soft gluon radiation and resummation might be very different from that of Drell-Yan lepton pair production in Ref. [13]. For example, using the threshold resummation formalism to describe the di-jet production in hadron collisions, it was found that the relevant Sudakov form factor has to be modified into a complicated matrix form [15, 16]. Although the kinematics of the transverse momentum resummation is different from that of the threshold resummation, similar matrix form may exist. (See, for example, a recent calculation in the soft-collinear-effective-theory [17].) Interestingly, when we apply these analyses to the heavy quarkonium production in NRQCD, we find that for heavy quarkonium production at low transverse momentum, the matrix form will be simplified, and can be written as a single exponential form factor in the CSS resummation formalism. The reason is as follows. There is only one color-configuration, in either color-singlet or color-octet state, for heavy quarkonium production in hadron-hadron collisions, and there is no mixing between them. As a result, the soft gluon resummation for the color-singlet channel will have the exact the same form as that presented in Ref. [13]. For the color-octet channel, the most important leading double logarithmic terms are found to be the same as those for the color-singlet channel. On the other hand, for the sub-leading-logarithmic terms, there is an additional term in the color-octet channel, as to be explicitly shown in the one-loop calculation of the differential cross section. Following the same arguments made in Refs. [13, 17] that QCD resummation calculation may be performed for more complicated hard processes (e.g., dijet production) in hadron collisions, we examine in this work the effect of QCD resummation to the production of heavy quarkonium in hadron collisions. It will be interesting to check the validity of the resummation formalism employed here beyond the
NLO, such as at two-loop order for heavy quarkonium production in NRQCD framework.

An important ingredient of the resummation calculation, for predicting low transverse momentum distribution of heavy quarkonium produced via gluon-gluon fusion processes, is the determination of the needed non-perturbative Sudakov form factor. This form factor was previously assumed to be related to that of the quark fusion processes by the ratio of color factors $C_A/C_F = 9/4$ [18]. The latter was determined through a global analysis of Drell-Yan (lepton) pair productions in hadron collisions [19–22]. Our results will show this assumption does not work for heavy quarkonium production. Future experimental data on top quark pair and Higgs boson productions shall provide additional information on determining the non-perturbative Sudakov form factors for gluon-gluon fusion processes.

We would like to emphasize that the transverse momentum resummation for colored final state hard processes, including heavy quarkonium production in the color-octet channel and heavy quark pair production in general, are far less developed as compared to those for the production of color neutral particles, such as Drell-Yan pair and Higgs boson. A call for caution is needed when comparing the form factors determined from this calculation to those from Drell-Yan processes. Also, we have to keep it in mind that there has been no general proof of factorization for hardronic hard processes (such as dijet and heavy quark pair productions) at low transverse momentum in hadron collisions. We note, however, an interesting development on this issue [23], which may provide a support for such factorization.

The paper is organized as follows. In Sec. II, we derive the low $P_\perp$ behavior of heavy quarkonium production in NRQCD by taking the limit of $P_\perp \ll M$. At one-loop order, the perturbative corrections are shown to contain large logarithms. We resum these large logarithms to all orders in Sec. III, following the CSS formalism. The relevant coefficients are obtained at the next-to-leading-logarithmic (NLL) level. The numerical evaluations are carried out in Sec. IV, where we extract the non-perturbative Sudakov form factor for the gluon-gluon fusion processes and the values of the associated color-octet matrix elements in the NRQCD framework. We conclude our paper in Sec. V.

II. LOW $P_\perp$ BEHAVIOR OF FIXED ORDER CALCULATIONS

By applying the NRQCD factorization formalism, heavy quarkonium production in hadron-hadron collisions arises from the partonic processes,

\[ a + b \rightarrow [Q\bar{Q}] + X , \]  

(3)

where $a$ and $b$ stand for the partons from the incoming nucleons. In high energy collisions, it is dominantly produced via the gluon-gluon fusion subprocess. At leading order of $v$ and $\alpha_s$, $(Q\bar{Q})$ pairs are produced in the $1S_0$ or $3P_J$ configuration via gluon-gluon fusion. They can be produced in either color-singlet or color-octet states, i.e., $gg \rightarrow Q\bar{Q}[1S_0^{1,8}]$ or $Q\bar{Q}[3P_J^{1,8}]$. Finite $P_\perp$ arises from the real gluon emission processes. As to be shown later, low $P_\perp$ heavy quarkonia are dominantly produced via the $1S_0^{1,8}$ and $3P_J^{1,8}$ channels. This is because in these channels the initial state gluon radiation contributes to a singular power behavior $1/P_\perp^2$ in the low $P_\perp$ region, whereas all other channels are power suppressed by $P_\perp/M$ in the limit of $P_\perp \ll M$. For example, the $2 \rightarrow 2$ subprocesses for $J/\psi$ and $\Upsilon$ productions include the
following channels,

\[ gg \to [Q\bar{Q}^{J/\psi,T}]_1 + g , \]  
\[ qq \to [Q\bar{Q}^{J/\psi,T}]_2 + q , \]  
\[ q\bar{q} \to [Q\bar{Q}^{J/\psi,T}]_2 + g , \]

where \([Q\bar{Q}^{J/\psi,T}]_1\) can be in \(1S^0, \, 3S^1, \) and \(3P^1\) configurations, while \([Q\bar{Q}^{J/\psi,T}]_2\) can be in \(1S^0, \, 3S^1, \) and \(3P^1\). The differential cross sections for these channels have been previously calculated in Refs. [24, 25, 27, 28]. The low \(P_\perp\) behavior of these cross sections can be obtained by proper expansion in powers of \(P_\perp/M\), where \(P_\perp\) and \(M\) are transverse momentum and mass of the heavy quarkonium, respectively. In this expansion, we only keep the leading order contribution, and neglect all higher order terms of \(P_\perp/M\). In the limit \(P_\perp \to 0\), only some of these cross sections can reproduce the lowest order terms that are proportional to \(1/P_\perp^2\). This case appears when the respective \(2 \to 1\) partonic subprocess, \(ab \to [Q\bar{Q}^{J/\psi,T}]\), exist. Hence, in high energy hadron-hadron collisions, the production cross section of heavy quarkonium can be approximated as follows:

\[
\sigma(J/\psi \text{ or } \Upsilon) = \sigma(gg \to 1S^0_0) + \sigma(gg \to 3S^0_1) + \sigma(gg \to 3P^1_0)
+ \sigma(gg \to \chi_2(1P^1_2)) \cdot \text{Br}(\chi_2(1P^1_2) \to J/\psi \text{ or } \Upsilon + \gamma),
\]

where we have ignored the contribution from \(q\bar{q}\) initial state, \(q\bar{q} \to 3S^0_1\), for its relatively small parton density in high energy collisions. Furthermore, the production channel \(gg \to \chi_0(3P^1_0)\) is not included in our calculation, for its small decay branch ratio (Br) into \(J/\Psi\) or \(\Upsilon\).

In the limit of \(P_\perp \ll M\), the differential cross section of the gluon-gluon scattering process, \(gg \to Q^{[cl]} + g\), can be expressed as

\[
\frac{d\sigma}{dyd^2P_\perp}|_{P_\perp \ll M} = \sigma_0(Q^{[cl]}) \frac{\alpha_s C_A}{2\pi^2} \int f(x)df(x')dx' \frac{1}{P^2_\perp} \left[ \frac{2(1-\xi + \xi^2)}{(1-\xi)} \delta(1-\xi) + \frac{2(1-\xi + \xi^2)}{(1-\xi)} \delta(1-\xi) + \left(2 \ln \frac{M^2}{P^2_\perp} - \delta_{sc}\right) \delta(1-\xi) \delta(1-\xi) \right],
\]

where \(y\) and \(P_\perp\) are rapidity and transverse momentum of heavy quarkonium, respectively. \(\xi_1 = M e^y/x\sqrt{S}, \xi_2 = M e^{-y}/x'\sqrt{S},\) and \(\sqrt{S}\) is the center-of-mass energy of the hadron-hadron collider. \(f(x)\) and \(f(x')\) are parton distribution functions (PDFs). \(Q^{[cl]}\) represents \(1S^0_{1,8}\) or \(3P^1_{1,8}\) state, and \(\delta_{sc} = 1\) or \(0\) for color-octet or singlet channel production. \(\sigma_0\) is proportional to the leading order partonic cross sections, and

\[
\sigma_0(Q^{[3P^1_2]}) = \frac{64 \alpha_s^2}{15} \frac{\pi^3}{M^7} \langle O[3P^1_2] \rangle,
\]
\[
\sigma_0(Q^{[1S^0_0]}) = \frac{5 \alpha_s^2}{12} \frac{\pi^3}{M^5} \langle O[1S^0_0] \rangle,
\]
\[
\sigma_0(Q^{[3P^0_0]}) = \frac{5 \alpha_s^2}{3} \frac{\pi^3}{M^7} \langle O[3P^0_0] \rangle,
\]
\[
\sigma_0(Q^{[3P^1_2]}) = \frac{4 \alpha_s^2}{3} \frac{\pi^3}{M^7} \langle O[3P^0_0] \rangle.
\]

It is interesting to compare the above expression to that for the color-singlet scalar particle (such as the Higgs boson) production. (See, for example, Ref. [29].) We find that for the
color-octet channel, there is an additional term with \( \delta(1 - \xi_1)\delta(1 - \xi_2) \), cf. Eq. (8), which generates sub-leading logarithmic contribution in the low \( P_\perp \) region. It is originated from the interference of initial and final (colored) state soft gluon radiations, which is absent in the production via color-singlet channel.

In order to perform the resummation calculation at the next-to-leading logarithmic (NLL) level, we have to Fourier transform the above expression to the impact parameter space, and include the virtual diagram contributions. The Fourier transformation into the impact parameter space is defined as \( W(b) = \int d^2 P_\perp e^{-i P_\perp \cdot b} \frac{d\sigma}{db d^2 P_\perp} \). In the impact parameter space, the logarithmic term will yield a soft divergence in terms of \( 1/\epsilon \), where \( \epsilon = (4 - D)/2 \) is the dimensional regularization parameter in \( D \)-dimension space. This soft divergence will be cancelled by the virtual diagram contribution. After adding up the contributions from real emission and virtual diagrams, we are left with collinear divergence in term of \( 1/\epsilon \). In our calculation, we adopt the modified minimal subtraction (\( \overline{\text{MS}} \)) scheme to regularize the real emission and virtual diagrams, we are left with collinear divergence in term of \( 1/\epsilon \). In our calculation, we adopt the modified minimal subtraction (\( \overline{\text{MS}} \)) scheme to regularize the remaining collinear divergences. Finally, the differential cross sections for describing the production of heavy quarkonia in the low \( P_\perp \) region, via \( gg \rightarrow ^1S_0, ^3P_J \) channels, can be expressed in the impact parameter space, within the NRQCD formalism, as

\[
W^{(1)}(b, M^2) = \sigma_0(Q^{[c]}) \frac{\alpha_s C_A}{\pi} \int dxdx' f(x) f(x') \left\{ \xi_1 \mathcal{P}_{gg}(\xi_1) \delta(1 - \xi_2) \left( -\frac{1}{\epsilon} + \ln \frac{4e^{-2\gamma_E}}{\mu^2 b^2} \right) 
+ (\xi_1 \rightarrow \xi_2) + \delta(1 - \xi_1)\delta(1 - \xi_2) \left[ (b_0 + \frac{1}{2} \delta_{c8}) \ln \frac{b^2 M^2}{4} e^{2\gamma_E} - \frac{1}{2} \ln^2 \left( \frac{M^2 b^2}{4} e^{2\gamma_E} \right) - \frac{\pi^2}{6} + \frac{B^{[c]}_{Q}}{C_A} \right] \right\},
\]

where \( b_0 = \left( \frac{1}{6} C_A - \frac{2}{3} T_F n_f \right)/N_c \), with \( C_A = N_c = 3 \), \( T_F = 1/2 \), and \( n_f \) is the number of light quark flavors. The expressions for \( B^{[c]}_{Q} \) can be found in Ref. [23], and \( f(x) \) represents gluon PDF. The gluon splitting function is defined as \( \mathcal{P}_{gg}(x) = \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \delta(x-1) \ln \frac{b_0}{2} \).

### III. ALL ORDER RESUMMATION

The presence of large double logarithmic corrections in Eq. (10) is a generic feature in low \( P_\perp \) differential cross section. In order to have a reliable prediction for low \( P_\perp \) heavy quarkonium production, we perform an all order resummation in the CSS formalism. For the color-singlet channel, there is no final state radiation because of its colorless nature. High order soft gluon radiation can only come from initial state, and the CSS resummation will follow the case of Drell-Yan pair production. For the color-octet channel, the final state carries color, and the soft gluon radiation from the color-octet \( (Q \bar{Q}) \) state will contribute an additional soft factor to the Sudakov exponent when factorizing the heavy quarkonium production process in the low transverse momentum region. This feature has been clearly demonstrated in the above fixed order calculation, cf. the factor of \( \delta_{c8} \) in Eqs. (8,10).

Soft gluon resummation for hard processes with colored final state in hadronic processes has been systematically studied in the literature, for example, for the threshold resummation of heavy quark pair production in Ref. [16]. Recently, a transverse momentum resummation has been carried out in the soft-collinear-effective-theory approach [17]. In the following, we will derive the similar formula for the heavy quarkonium production. Since heavy quark pair production and heavy quarkonium production do share similarity in the hard processes, we...
would like to pay close attention to the calculations of Refs. \[16,17\] to check if we can gain some insight for the resummation of soft gluon radiation in heavy quarkonium production, which can be viewed as a special case of heavy quark pair production. In NRQCD, the heavy quark pair is produced at short distance with fixed color and spin configuration, in particular, the heavy quark and antiquark share the same momentum (half of quarkonium momentum) in the non-relativistic limit. From the calculations of Refs. \[16,17\], for heavy quark pair production in pp collisions, it was found that additional soft gluon radiation from the final state particles introduces a matrix form for the soft and hard factors. The hard factor can be calculated from fixed order perturbative diagrams, whereas the effect of soft gluon radiation can be resummed through the anomalous dimensions associated with the soft factor \[16,17\]. Both the hard and soft factors are expanded in the orthogonal color basis in the hard processes. For example, for \(g_\alpha g_\beta \rightarrow Q_i \overline{Q}_j\) process, the following color basis are chosen \[16\]:

\[
c_1 = \delta^{ab} \delta_{ij}, \quad c_2 = if^{abc}T_{ij}^{c}, \quad c_3 = d^{abc}T_{ij}^{c},
\]

(11)

where \(T^c\) is the generator of \(SU(3)\) in the fundamental representation, \(a, b\) and \(i, j\) are initial state gluons and final state quark pair color indices, respectively. Clearly, \(c_1\) corresponds to the color-singlet final state, while \(c_2\) and \(c_3\) to the color-octet final states. The low \(P_\perp\) production of heavy quarkonium in color-octet state is dominated by the \(^1S_0^{(8)}\) and \(^3P_J^{(8)}\) channels, which only couple to the \(c_3\) color basis. In other words, if we follow the calculations of \[16\] and decompose the hard factor into the above three color bases, we will find that at the leading order, only \(H_{33}\) is non-zero. In additional, the one-loop virtual corrections from \[25\] also show that at the NLO, only \(H_{33}\) is non-vanishing. We expect this to be true for higher order calculations as well. The reason is that the production of the color-octet \(^1S_0\) and \(^3P_J\) channels requires the initial state color indices to be symmetric, and hence a non-vanishing contribution can only come from the \(c_3\) color basis. More importantly, the anomalous dimension \(\Gamma\), which governs the soft factor contribution to the resummation calculation, has been calculated up to one-loop order in Ref. \[16\]. Applying those results to the case of color-octet heavy quarkonium production in the NRQCD framework, we find that the anomalous dimension \(\Gamma\) becomes diagonalized, and

\[
\Gamma = \frac{\alpha_s}{\pi} \begin{pmatrix}
0 & 0 & 0 \\
0 & -\frac{C_A}{2} & 0 \\
0 & 0 & \frac{C_A}{2}
\end{pmatrix},
\]

(12)

where we have only kept the real part of the matrix elements, and they correspond to the partonic threshold limit with heavy quark pair produced at rest, cf. Eq.(4.8) of Ref. \[16\]. We note that taking the partonic threshold limit with heavy quark pair produced at rest resembles the dominant kinematics of the heavy quarkonium production in NRQCD. Similar results were also found in the soft-collinear-effective-theory calculations \[17\]. After solving the renormalization group equation, we will obtain an additional soft factor in the CSS resummation formula, as compared to that for colorless particle, such as Higgs boson, production. This conclusion also agrees with our explicit calculations.

The above analysis indicates that there is no complicated structure in the resummation calculation for heavy quarkonium production in either color-singlet or color-octet channel. Therefore, we can directly follow the CSS method to derive its final result. In particular, from Eq. \[10\] we can write down a differential equation with respect to \(\ln M^2\):

\[
\frac{\partial W(b, M^2)}{\partial \ln M^2} = (K + G')W(b, M^2),
\]

(13)
where $K$ and $G'$ are soft and hard evolution kernels, and at one-loop order, we have

$$K + G' = -\frac{\alpha_s C_A}{\pi} \ln \left( \frac{M^2 b^2}{4} e^{2\gamma_E - b_0 - \frac{\delta_{cs}}{2}} \right). \tag{14}$$

The soft part $K$ depends on the scale $1/b^2$ and the renormalization scale $\mu$, while $G'$ depends on the hard scale $M^2$ and $\mu$. Compared to that in Higgs boson production, the only difference is the additional term $\frac{\delta_{cs}}{2}$, which arises from the interference of the initial and final state soft gluon radiation in the color-octet channel and is absent in the color-singlet channel. Both the soft and hard parts $K$ and $G'$ obey the renormalization group equation [13],

$$\frac{\partial K}{\partial \ln \mu} = -\frac{\partial G'}{\partial \ln \mu} = -\gamma_{Kg} = -2\frac{\alpha_s C_A}{\pi}, \tag{15}$$

where $\gamma_{Kg}$ is the cusp anomalous dimension. After solving the renormalization group equations, and evolving from the low energy scale of $\mathcal{O}(1/b)$ to the high energy scale of $\mathcal{O}(M)$, we obtain,

$$W(b, M^2) = e^{-S_{\text{Sud}}(M^2, b, C_1, C_2)} W(b, C_1, C_2), \tag{16}$$

where the Sudakov form factor is

$$S_{\text{Sud}} = \int_{C_1^2/b^2}^{C_2^2 M^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{C_2^2 M^2}{\mu^2} \right) A(C_1, \mu) + B(C_1, C_2, \mu) \right]. \tag{17}$$

Here $C_1$ and $C_2$ are two free parameters, at the order of unity. The functions $A$ and $B$ can be expanded perturbatively in powers of $\alpha_s$, with $A = \sum_{i=1}^{\infty} A^{(i)} \left( \frac{\alpha_s}{\pi} \right)^i$ and $B = \sum_{i=1}^{\infty} B^{(i)} \left( \frac{\alpha_s}{\pi} \right)^i$. Furthermore, in Eq. (16),

$$W(b, C_1, C_2) = \sigma_0(Q^{[c]} S M^2) \int_{x_1}^{x_2} \frac{dx}{x} \frac{dx'}{x'} C_{gg} \left( \frac{x}{x'}, b, C_1, C_2, \mu^2 \right) C_{gg} \left( \frac{x'}{x}, b, C_1, C_2, \mu^2 \right) f(x, \mu) f(x', \mu), \tag{18}$$

where $x_1 = M e^{y}/\sqrt{S}$, $x_2 = M e^{-y}/\sqrt{S}$, and the scale $\mu = C_3/b$. After choosing $C_1 = C_3 = 2 e^{-\gamma_E}$ and $C_2 = 1$, we obtain, for the channels $gg \to 1S_0^{[c]} 3P_J^{[c]}$,

$$A^{(1)} = C_A, \quad B^{(1)} = -(b_0 + \frac{1}{2} \delta_{cs}) C_A, \quad C_{gg}^{(1)} = \delta(1-x), \quad C_{gg}^{(2)} = \left( -\frac{\pi^2}{12} C_A + \frac{B_Q^{[c]}}{2} \right) \delta(1-x), \quad C_{gg}^{(1)} = \frac{C_F}{2} x. \tag{19}$$

Together with $A^{(2)} = C_A \left[ \left( \frac{67}{36} - \frac{\pi^2}{12} \right) N_c - \frac{5}{18} N_f \right]$, which is the same as that in the $gg \to H$ process [26], the above result contains all the needed coefficients for performing the resummation calculation at the next-to-leading logarithmic level for the heavy quarkonium production in hadron collisions. We note that both $B^{(1)}$ and $C^{(1)}$ for color-octet channels are different from those for color-singlet channels. The additional contribution in $B^{(1)}$ for
the color-octet channel is exactly the same as that found in Refs. \cite{16,17} for studying the production of heavy quark pair in the threshold resummation formalism.

It is interesting to note that the numerical values of the $C^{(1)}_{ggg}$ coefficients for the color-octet channels are not very different from each other. With the given $B^{[c]}_Q$ values from Ref. \cite{25}, we obtain

$$C^{(1)}_{ggg}(1S^0_0) = 3.16, \quad C^{(1)}_{ggg}(3P^8_0) = 3.76, \quad C^{(1)}_{ggg}(3P^8_2) = 2.80.$$  \hfill (20)

Consequently, the ratio between the color-octet $3P_J$ and $1S_0$ contributions, derived from Eq.(9), can be approximated as

$$\left( \frac{C^{(1)}_{ggg}(3P^8_0)}{\sigma_0(3P^8_0)} + \frac{C^{(1)}_{ggg}(3P^8_2)}{\sigma_0(3P^8_2)} \right) : \left( \frac{C^{(1)}_{ggg}(1S^0_0)}{\sigma_0(1S^0_0)} \right) \approx \frac{7.1}{M_Q^2 \langle \sigma[3P^8_0] \rangle} \approx \frac{7}{M_Q^2 \langle \sigma[1S^0_0] \rangle},$$  \hfill (21)

where $M_Q$ is the heavy quark mass, and $M_Q = M/2$ in NRQCD factorization formalism. Here, we have simplified the two $P$-wave color-octet contributions by applying the heavy quark symmetry \( \langle \sigma[3P^8_2] \rangle = 5 \langle \sigma[3P^8_0] \rangle \). Therefore, at the NLO accuracy, the total contribution from the above three channels are nearly proportional to the linear combination of color octet matrix elements:

$$\sigma(Q^{[8]}) \sim \left( \langle \sigma[1S^0_0] \rangle + \frac{7}{M_Q^2} \langle \sigma[3P^8_0] \rangle \right),$$  \hfill (22)

which is the same as that goes into the leading order cross section calculation. The above combination will enter into the cross section calculation of the low $P_\perp$ heavy quarkonium production. This combination is different from that needed for calculating the high $P_\perp$ distributions \cite{3,4,9}. This difference mainly comes from the fact that the effective gluon-gluon-heavy-quark-pair coupling varies with kinematics. At low $P_\perp$, we can take heavy quark mass limit ($M \gg P_\perp$), where both gluons are almost on-shell. At large $P_\perp$, one of the gluon in the $2 \to 2$ subprocesses must be far off-shell (at order of $P_\perp$), and the gluon-gluon-heavy-quark-pair coupling will be different from that in the low $P_\perp$ region. These changes depend on the configuration of the heavy quark pair and higher order corrections. However, in the low $P_\perp$ region, the soft (and collinear) gluon radiation dominates, and the differential cross section yields a result proportional to the Born level contribution, cf. Eq.(22). This has been confirmed by the exact NLO calculation which includes all the $2 \to 3$ subprocesses \cite{3,4}.

IV. NUMERICAL CALCULATION

In the numerical calculation, the differential cross section can be written as:

$$d\sigma = d\sigma^{\text{resum}} + (d\sigma^{\text{pert}} - d\sigma^{\text{asym}}),$$  \hfill (23)

where $\sigma^{\text{resum}}$ is the resummation of terms which are proportional to $1/P^2_\perp$ in each order of perturbative calculation, and $\sigma^{\text{asym}}$ is constructed to cancel the same terms in the fixed-order cross section $\sigma^{\text{pert}}$. Therefore, the second term $(d\sigma^{\text{pert}} - d\sigma^{\text{asym}})$ is power suppressed by $P_\perp/M$ in low $P_\perp$ region. The above expression is in principle valid in the whole transverse
momentum region. In this paper, we focus on the low $P_\perp$ region, where we can safely neglect the contribution from the second term.

In the CSS resummation formalism, the resummation part of total cross section can be written as

$$d\sigma \over d^2P_\perp dy \bigg|_{P_\perp \ll M} = \frac{1}{(2\pi)^2} \int d^2b e^{iP_\perp \cdot \vec{b}} W(b, M, x_1, x_2),$$

(24)

where $W(b, M, x_1, x_2)$ has been extensively discussed in the last section. The $b$ integral contains contribution from the non-perturbative region where $b$ is so large that $\alpha_s(1/b)$ cannot be reliably calculated perturbatively. To model the contribution from the non-perturbative region, we follow the $b_*$-prescription to add a phenomenological non-perturbative form factor [13], and write

$$W(b) = W(b_*) W^{NP}(b),$$

(25)

where $b_*$ is defined as $b_* = b/\sqrt{1 + (b/b_{\text{max}})^2}$. Here, $b_*$ cannot exceed $b_{\text{max}}$, which is equivalent to making a cutoff on the variable $b$ at $b_{\text{max}}$.

Generally, the non-perturbative form factor $W^{NP}$ depends on the flavor of the initial state partons. There have been many studies in the literature to extract $W^{NP}$ associated with the initial state quarks by comparing theoretical predictions with experimental data of Drell-Yan lepton pair production and $Z^0$ and $W$ boson productions in $pp$ ($p\bar{p}$) collisions. (See, for example, Refs. [19–21].) On the other hand, for the gluon initiated processes, there has been no precise determination of the corresponding $W^{NP}(b)$ in the literature. For example, in Ref. [13], di-photon productions via gluon-gluon fusion processes in hadron collisions have been investigated, where the associated $W^{NP}(b)$ factor was assumed to scale with the color-factor $C_A/C_F = 9/4$, with respect to that for the quark initiated processes [22]. This is because at the Tevatron energy, the production rate of di-photon events is dominated by quark initiated subprocesses, so that it is difficult to use that data to extract the non-perturbative factor $W^{NP}(b)$ associated with the gluon initiated subprocesses. In this work, we will take the BLNY parameterization form, a 3-parameter pure Gaussian form, as proposed in Refs. [20] [22],

$$W^{NP}(b) = \exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 \ln \left( 100 x_1 x_2 \right) \right] b^2,$$

(26)

and extract the non-perturbative form factor associated with the gluon initiated processes by performing a global fit to the low $P_\perp$ distributions of heavy quarkonia produced in high energy hadron-hadron collisions. More specifically, we will follow the previous studies and take the following fixed parameters: $Q_0 = 1.6$ GeV and $b_{\text{max}} = 0.5$ GeV$^{-1}$, in addition to the three free parameters $g_1$, $g_2$, and $g_3$. Furthermore, for simplicity, we will assume the same non-perturbative function for calculating both the color-octet and color-single heavy quarkonium state productions. This approximation is justified by that the low $P_\perp$ heavy quarkonia are dominantly produced at high energy colliders via color-octet channel.

As we discussed in the last section, heavy quarkonium (charmonium and bottomonium) production rate depends on the associated color-octet matrix elements in the NRQCD factorization formalism. In particular, we found that the differential cross section in the low $P_\perp$ region is proportional to the combination of the two hadronic matrix elements, as shown in Eq. (22). Hence, we shall treat the very combination of $Q^{[S_0^8]}$ and $Q^{[P_0^8]}$ as one free parameter to be determined by the global fits to the $J/\psi$ and $\Upsilon$ production data, separately. We note that the hadronic matrix elements are different for $J/\psi$ and $\Upsilon$ productions. In our
fits, we include the experimental data on $\Upsilon$ production from fixed target experiment by the E866 Collaboration [30], and the collider experiments on $\Upsilon$ production at the LHC [31-33] and the Tevatron [34], and $J/\psi$ production at the RHIC [35] with $1.2 < |y| < 2.4$. The other color-singlet matrix elements, $\langle O^{J/\psi[3P_2]} \rangle$ and $\langle O^{\Upsilon[3P_2]} \rangle$, are taken from Ref. [3] and Ref. [8] for $J/\psi$ and $\Upsilon$ production, respectively. In total, our fits contain ten free parameters: $g_{1,2,3}$, two linear combinations of color-octet matrix elements for $J/\psi$ and $\Upsilon$ production, respectively, and five normalization parameters $N_{fit}$ which were introduced to account for the normalization uncertainty in each experiment, following the same procedure as done in Ref. [21].

To determine the free parameters $g_{1,2,3}$ and the two linear combination of color-octet matrix elements in the global $\chi^2$ fits, we minimize the $\chi^2$ contribution from the five experimental data sets, as shown in Table I, while allowing the normalization of each experimental data set to float within one standard deviation of the published experimental error of collider luminosity. The result of our fits are shown in Figs. 1, 2 and 3, in which we have plotted the shifted theory prediction (multiplied by $N_{fit}$ for Figs. 1 and 2) with the experimental data for the above five experimental data sets. In addition, we also compare the theory prediction, with the fitted theory parameters, to the $J/\psi$ production at the RHIC with $|y| < 0.35$ (cf. Fig. 2(B)) and to the $\Upsilon$ production at the LHCb (cf. Fig. 3). The fitted parameters are summarized in Table I. The uncertainties quoted for $g_{1,2,3}$ are evaluated with the other parameters fixed at their values given by the best fit. The overall agreement between the theoretical predictions, based on the NRQCD and soft gluon resummation, and the exper-
FIG. 2: $J/\psi$ production at $\sqrt{S} = 7$ TeV with $1.6 < |y| < 2.4$ at the LHC (A), at $\sqrt{S} = 200$ GeV with $|y| < 0.35$ (B) and $1.2 < |y| < 2.4$ (C) for $pp$ collision at the RHIC. The curve C has been multiplied by a factor 1/10 to separate it from the others. The data points are from Refs. [32, 35] for CMS and PHENIX Collaborations, respectively.

The data is very good, considering the energy span of the experiments, ranging from fixed target to high energy collider experiments. In particular, if we compare the experimental data from the Tevatron and the LHC with those from the fixed target experiment, cf. Fig. 1, we find that there is a strong energy dependence for the $P_{\perp}$-spectrum, which is reasonably described by the energy dependence of the non-perturbative form factor in Eq. (26). Similar conclusion holds for the $J/\psi$ production between RHIC and LHC, as shown in Fig. 2. From Figs. 2 and 3, we find that the agreement between theoretical predictions and experimental data for $J/\psi$ production is not as good as that for $\Upsilon$ production. This may be because the mass of $J/\psi$ is not very large (as compared to $\Lambda_{QCD}$) and the theoretical uncertainties from the non-perturbative factor of Eq. (26) become sizable at relative low mass region.

The above resummation results are based on the NRQCD factorization formalism for heavy quarkonium production. Hence, the resulting non-perturbative factor for the gluon-gluon initiated low $P_{\perp}$ heavy quarkonium production processes, cf. Eq. (26), may not be the same as that for the gluon-gluon initiated Higgs boson production in hadron-hadron collisions. The former is mainly produced via color-octet channel, based on the NRQCD factorization formalism, while the latter is produced via color-singlet channel. Although the resummed cross section has properly taken into account the different effects of multiple soft gluon radiation for producing a color-octet or singlet state in perturbative calculation, a different non-perturbative factor might be needed to describe the Higgs boson production, as compare to the heavy quarkonium production via color-octet channel. Nevertheless, we
shall compare our result to the assumption made in Ref. [21] for Higgs boson production,

\[\begin{align*}
g_1 &= 0.21 \times (9/4) = 0.47, \\
g_2 &= 0.68 \times (9/4) = 1.53, \\
g_1 \times g_3 &= -0.29, \\
\end{align*}\]

which were scaled by the ratio of color factors involved in the gluon-gluon versus quark-quark fusion processes. From our fit, we find that the absolute values for the two parameters: \(g_2\) which controls the scale dependence (\(\ln Q\) term), and the combination \(g_1 \times g_3\) which controls the energy dependence (\(x_1x_2\) term), are both smaller than those in Eq. (27) It will be interesting to further investigate this important issue and the relevant phenomenological consequences. In particular, our determinations of the above parameters are only based on heavy quarkonium production in \(pp\) collisions, where the NRQCD factorization contains color-octet channel contributions and the associated non-perturbative form factor might be different from that for the color-singlet processes. To further check the scaling proposed in Ref. [21], we need to study the transverse momentum distribution of Higgs boson and/or di-photon production in \(pp\) collisions. Finally, in order to estimate the uncertainties in our fit, we show in Fig. 4 the contour plots of the uncertainties of \(g_1\), \(g_2\), and \(g_1 \times g_3\).

Meanwhile, the differential cross sections in Figs. 1-3 also depend on the color-octet matrix elements. As discussed in the last section, in our calculations, we can only determine the combinations of the color-octet matrix elements in the form of Eq. (22). From the fit,
we find the following results for $J/\psi$ and $\Upsilon$ productions:

\begin{align}
\langle \phi^{J/\psi}[1S_0^8] \rangle &+ \frac{7}{m_c^2} \langle \phi^{J/\psi}[3P_0^8] \rangle = 0.0197 \pm 0.0009 \text{ GeV}^3, \\
\langle \phi^{\Upsilon}[1S_0^8] \rangle &+ \frac{7}{m_b^2} \langle \phi^{\Upsilon}[3P_0^8] \rangle = 0.0321 \pm 0.0014 \text{ GeV}^3,
\end{align}

(28)

respectively. It’s interesting to note that the result of Eq. (28) agrees well with that determined by comparing the total cross section in photoproduction [36] and fixed-target hadroproduction [37] of $J/\psi$, which are 0.02 GeV$^3$ and 0.03 GeV$^3$, respectively. Comparing
TABLE I: The results of the fits at 68%, 90% and 95% CL. Here, $N_{\text{fit}}$ is the fitted normalization factor for each experiment, which is multiplied to the corresponding theoretical prediction to yield the comparison presented in Figs. 1 and 2.

| Parameter | BLNY fit            |
|-----------|---------------------|
| $g_1$ (CL 68%) | 0.03±0.056          |
| $g_2$ (CL 68%) | 0.87±0.065          |
| $g_3 \ast g_1$ (CL 68%) | -0.17±0.011       |
| $g_1$ (CL 90%) | 0.03±0.111          |
| $g_2$ (CL 90%) | 0.87±0.080          |
| $g_3 \ast g_1$ (CL 90%) | -0.17±0.013       |
| $g_1$ (CL 95%) | 0.03±0.113          |
| $g_2$ (CL 95%) | 0.87±0.134          |
| $g_3 \ast g_1$ (CL 95%) | -0.17±0.030       |

E866 ($\Upsilon$) $N_{\text{fit}} = 1.06$ (5 points) $\chi^2 = 7.9$

Tevatron ($\Upsilon$) $N_{\text{fit}} = 0.96$ (5 points) $\chi^2 = 3.0$

CMS($\Upsilon$) $N_{\text{fit}} = 1.05$ (5 points) $\chi^2 = 9.2$

RHIC($J/\psi$) $N_{\text{fit}} = 0.9$ (6 points) (1.2 $< |y| < 2.4$) $\chi^2 = 3.4$

CMS($J/\psi$) $N_{\text{fit}} = 1.11$ (5 points) $\chi^2 = 14.6$

$\chi^2/\text{DOF}$ 38

$\chi^2/\text{DOF}$ 1.46

These results with those extracted from the high $P_\perp$ heavy quarkonium production \[3,9\]:

$$\langle \phi^{[1 S_0]} \rangle + \frac{3.9}{m_c^2} \langle \phi^{[3 P_0^8]} \rangle = 0.074 \pm 0.019 \text{ GeV}^3,$$

$$\langle \phi^{[1 S_0]} \rangle + \frac{0.45}{m_b^2} \langle \phi^{[3 P_0^8]} \rangle = 0.113 \pm 0.020 \text{ GeV}^3,$$

(29)

we find that the color-octet matrix element of $[^3 P_0^8]$ is likely to take a negative value. Studying the production of heavy quarkonium in high $P_\perp$ region also yields the following relation \[3,9\]:

$$\langle \phi^{[1 S_1]} \rangle + \frac{-0.56}{m_c^2} \langle \phi^{[3 P_0^8]} \rangle = 0.0005 \pm 0.00028 \text{ GeV}^3,$$

$$\langle \phi^{[3 S_1]} \rangle + \frac{-0.045}{m_b^2} \langle \phi^{[3 P_0^8]} \rangle = 0.061 \pm 0.012 \text{ GeV}^3.$$  

(30)
Combining equations from Eq. (28) to Eq. (30), we deduced:

\begin{align*}
\langle \phi^{J/\psi \, ^1S_0^8} \rangle &= 0.1423 \pm 0.044 \text{GeV}^3, \\
\langle \phi^{J/\psi \, ^3P_0^8}/m_c^2 \rangle &= -0.0175 \pm 0.0064 \text{GeV}^3, \\
\langle \phi^{J/\psi \, ^3S_1^8} \rangle &= -0.0093 \pm 0.0038 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^1S_0^8} \rangle &= 0.119 \pm 0.021 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^3P_0^8}/m_b^2 \rangle &= -0.012 \pm 0.0033 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^3S_1^8} \rangle &= 0.060 \pm 0.012 \text{GeV}^3.
\end{align*}

and

\begin{align*}
\langle \phi^{\Upsilon \, ^1S_0^8} \rangle &= -0.0175 \pm 0.0064 \text{GeV}^3, \\
\langle \phi^{J/\psi \, ^3P_0^8}/m_c^2 \rangle &= -0.0093 \pm 0.0038 \text{GeV}^3, \\
\langle \phi^{J/\psi \, ^3S_1^8} \rangle &= 0.1423 \pm 0.044 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^1S_0^8} \rangle &= 0.119 \pm 0.021 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^3P_0^8}/m_b^2 \rangle &= -0.012 \pm 0.0033 \text{GeV}^3, \\
\langle \phi^{\Upsilon \, ^3S_1^8} \rangle &= 0.060 \pm 0.012 \text{GeV}^3.
\end{align*}

The above result shows that the color-octet matrix elements can be determined to some degree by fitting the three linear combinations of the LDMEs to the transverse momentum distributions of heavy quarkonia \((J/\psi \text{ and } \Upsilon)\) produced in high energy hadron-hadron collisions.

Before closing this section, we note that in the NRQCD factorization formalism, it has been assumed that the momentum of the observed quarkonium is the same as the heavy quark-antiquark pair produced at the short distance, where the soft gluon radiation from the later stage \((i.e., \text{hadronization process})\) was neglected. In principle, the (non-perturbative) soft gluon radiation from the final state long distance process may affect the \(P_\perp\) distribution of heavy quarkonium. It is worthwhile to further investigate its effect and implication in heavy quarkonium production in hadron-hadron collisions. However, that is beyond the scope of the current paper.

V. SUMMARY AND CONCLUSION

In this paper, we combine the NRQCD and soft gluon resummation formalism to calculate the \(P_\perp\) distribution of \(J/\psi\) and \(\Upsilon\) production in hadronic collision in the low \(P_\perp\) region. At high energy colliders, the dominant production mechanism of heavy quarkonium is via gluon-gluon fusion, similar to the production of Higgs boson at the CERN Large Hadron Collider (LHC). Our analytic calculation shows that the CSS resummation formalism can be applied to the \(J/\psi\) and \(\Upsilon\) hadroproduction processes, similar to the Higgs boson hadroproduction. As compared to the color-singlet Higgs boson hadroproduction, the coefficient function \(B^{(1)}\) of the Sudakov factor, in the CSS resummation formalism, has to be modified in order to take into account the interference effect of initial and final state soft gluon radiations, for the color-octet \(J/\psi\) and \(\Upsilon\) hadroproductions, while the coefficients \(A^{(1)}\) and \(A^{(2)}\) remain to be the same for the color-singlet and color-octet cases. In order to numerically evaluate the Sudakov form factor in the large impact parameter \((b)\) region, which is relevant to the low transverse momentum \((P_\perp)\) region, we need to introduce a non-perturbative function \(W^{NP}\) in the CSS resummation formalism. In this work, we use a 3-parameter pure Gaussian form (BLNY form) to parameterize the \(W^{NP}\), and fit these free parameters by five experimental data sets of \(J/\psi\) and \(\Upsilon\) hadroproductions, with a total number of 26 data points. We find that we need to modify the non-perturbative function \(W^{NP}\) previously assumed. The result of our analysis is summarized in Table I. Though the non-perturbative function \(W^{NP}\) extracted from heavy quarkonium data could in principle be applied to studying the distribution of Higgs boson produced via gluon-gluon fusion process, it should be emphasized that the main
contribution of the $J/\psi$ and $\Upsilon$ productions in hadronic collision is from color-octet final state channels, in contrast to the production of the color-single Higgs boson. Hence, they might need different non-perturbative factor $W^{NP}$ to describe their low $P_\perp$ distributions. Nevertheless, it remains useful to compare this newly determined $W^{NP}$ to that currently used for the Higgs boson and di-photon productions in hadronic collisions.

As emphasized in the above discussions, our resummation calculation for the heavy quarkonium production was derived from a NLO result in the RQCD framework. Because of the simple color configuration and non-relativistic nature in this formalism, the soft gluon radiation can be resummed into a simple exponential form. This feature is consistent with the soft gluon radiation in heavy quark pair production previously studied in the partonic threshold limit with heavy quark pair produced at rest. In particular, the matrix form of the Sudakov form factor can be simplified in this limit, which is consistent to our results. This is encouraging for further investigations on the transverse momentum resummation for more complicated hard processes. Of course, we have to keep in mind that the resummation formula may break down at higher orders because there has been not general proof of the factorization for hardronic hard processes (such as dijet, heavy quark pair, and heavy quarkonium productions) in hadron collisions.

We have also shown how to extract the values of the color-octet matrix elements from studying the transverse momentum distribution of $J/\psi$ and $\Upsilon$, in both the low and high $P_\perp$ regions. The result of this analysis is given in Eqs. (31) and (32). These matrix elements are found to be consistent with the inclusive total cross section in photoproduction [36] and fixed-target hadroproduction [37] of $J/\psi$. Further investigations are needed to clarify the underlying mechanisms for heavy quarkonium production in the whole $P_\perp$ range.

We would like to emphasize that the resummation formula in our calculations are based on the extension of the NRQCD factorization in the low transverse momentum region, where the most singular contributions are found to follow the CSS resummation expansion at one-loop order. It will be interesting to check if this holds at higher orders. Meanwhile, we note that the color-single $3_1^{(1)}$ channel does not contain singular contributions at low transverse momentum, because it cannot be produced via a $2 \to 1$ process. Hence, we did not consider the effect of soft gluon resummation for this production channel in the current analysis.

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