THE ZERO-POINT OF THE CLUSTER-CLUSTER CORRELATION FUNCTION:
A KEY TEST OF COSMOLOGICAL POWER SPECTRA

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ABSTRACT

We propose the zero-point of the cluster-cluster correlation function as a sensitive test for the shape of the power spectrum of initial fluctuations. With a wealth of new available redshifts for rich clusters the correlation function is measured to higher accuracy than has previously been done. In particular it is now possible to go beyond the power law description to measure the point at which the correlation function becomes zero. Four independent measurements of the zero-point of the rich galaxy cluster correlation function indicate that the zero point, \( r_0 \), should be in the range \((40-60)h^{-1}\text{Mpc}\). The large value of \( r_0 \) at which the zero-point occurs rules out conventional CDM models independently of the assumed amplitude. The most severe constraints are imposed on the CDM models with the cosmological constant. Models with \( \Omega < 0.25 \) should be rejected because they predict too large \( r_0 \). If the age of the Universe is assumed to be larger than 15 Gyr, models with either \( \Omega < 0.5 \) or \( h > 0.55 \) are rejected.

We present the results of numerical simulations of clusters in an \( \Omega = 1 \) cosmological model with a mixture of cold plus hot dark matter (CHDM). We have identified clusters in the simulations and calculated the cluster-cluster correlation function. The function we determined for the simulated clusters has a zero-point, \( r_0 = 55h^{-1}\text{Mpc} \) that accurately matches the zero point of the observed function. In addition the shape of the function on all available scales (up to \( 100h^{-1}\text{Mpc} \)) is reproduced for both the Abell and the APM clusters.

Subject headings: cosmology: theory — dark matter — large scale structure of the universe — galaxy clusters
1. INTRODUCTION

The clustering properties of rich clusters of galaxies can provide a powerful constraint for theories of galaxy formation. In particular the cluster spatial two-point correlation function provides a useful quantitative measure of clustering for comparison with theories. The first determination of the cluster correlation function for a complete redshift sample gave

\[ \xi_{cc} = (r/r_{cc})^{-1.8} \]

where \( r_{cc} = 25h^{-1}\text{Mpc} \) (Bahcall and Soneira, 1983, Klypin and Kopylov, 1983). The correlation amplitude is much larger than that of galaxies \( r_{gg} \sim 5h^{-1}\text{Mpc} \) (Davis and Peebles, 1983).

Since the first work on the cluster correlation function there has been controversy concerning the true value of the correlation function amplitude (i.e. \( r_{cc} \)). Sutherland (1988) and Sutherland and Efstathiou (1991) have claimed that the selection of Abell clusters is non-uniform on the sky with a tendency to find pairs of clusters close in projection but at very different redshifts. These authors claim a lower correlation than that found by Bahcall and Soneira (1983); \( r_{cc} = 14h^{-1}\text{Mpc} \). Other studies made on subsamples which not suffer from the biases claimed by Sutherland for the whole Abell catalog found an intermediate value for the correlation length \( r_{cc} = 20h^{-1}\text{Mpc} \). These studies include cD clusters (West and van den Bergh, 1990) and X-ray clusters (Lahav et al. 1989). Most recently the studies of Peacock and West (1992) and Postman et al. (1992), using samples of 427 and 351 clusters respectively, find a correlation length close to \( 20h^{-1}\text{Mpc} \).

To settle the issue of the reliability of Abell’s cluster catalog automated cluster surveys have been made. Two recent projects are the Automated Plate Measuring Facility (APM) survey (Dalton et al. 1992) and the Edinburgh-Durham Cluster Catalog (Nichol et al. 1992). The correlation functions for clusters found in the catalogs have slopes similar to previous ones and amplitudes (\( \sim 15h^{-1}\text{Mpc} \)) lower than previously determined for the
On the theoretical front one can use models for the growth of structure in the universe by gravitational instability to make predictions concerning the amplitude and shape of the cluster correlation function. These predictions will vary depending on the initial fluctuation spectrum which in turn is determined by the physics of the dark matter particles. Bahcall and Cen (1992) have found that standard biased cold dark matter models ($\Omega = 1$) are inconsistent with the observed cluster mass function and cluster correlation function for any bias parameter. A low density, low-bias ($\Omega \sim 0.25$, $b \sim 1$) CDM model with or without a cosmological constant appears to be consistent with both the cluster correlation length, $r_{cc}$, and the cluster mass function. Jing et al., (1993) have calculated two point correlation functions of clusters in a set of CHDM models. They found that a hybrid model of the universe which contains (in mass) $\sim 30\%$ HDM, $\sim 70\%$ CDM and $\geq 1\%$ baryons could provide a reasonable fit to the observed two point correlation function of Abell clusters. Note however that they were mainly concerned with $\xi_{cc}(r)$ for $r < 50h^{-1}$Mpc and did not actually simulated the hot component. Olivier et al. (1993) have compared the cluster correlation function with the CDM and Textures models. They find that for $40 \text{ Mpc} \leq r \leq 80$ Mpc the observed correlation function is larger than that predicted by these two theories. Holtzman and Primack (1993) using the peaks formalism for gaussian density fields have investigated the prediction for the cluster correlation function in high and low $\Omega$ CDM universes and in the C+HDM model. They find that the data are best fit by the CHDM model. Bartlett and Silk (1993) have reached the same conclusion by making analytical model predictions for the cluster X-ray temperature function, based on the Press-Schechter approximation.

Now that cluster catalogs containing redshifts for several hundred clusters have become
available it is possible to determine the scale at which the cluster correlation function has a value of zero (we refer to this as the $\xi_{cc}$ zero-point). We argue in this paper that this is a key observation for comparison with models. We know of four independent determinations of the correlation function zero-point (Scaramella et al. 1993, Dalton et al. 1992, Postman et al. 1992 and Postman 1993, Peacock and West 1992).

2. COSMOLOGICAL MODELS

We study the CHDM model, which assumes that the Universe has the critical density and the Hubble parameter is $H = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $h = 0.5$ and the cosmological constant is zero. The CHDM model has the following parameters. The mass density of the Universe in the form of neutrino is $\Omega_\nu = 0.30$, density of baryons is $\Omega_b = 0.1$. We use analytical approximations (c.f. eq.(1) of Klypin et al., 1993) for the “cold” and “hot” spectra. The amplitude of fluctuations is normalized so that our realization is drawn from an ensemble producing the quadrupole in the angular fluctuations of the cosmic microwave background at the $17\mu K$ level measured by COBE. This gives the rms fluctuations of mass in a sphere of $8h^{-1}\text{Mpc}$ radius, $\sigma_8 = 0.667$ and the rms velocity of the “cold” dark matter relative to the rest frame $\sigma_v = 750 \text{ km s}^{-1}$.

3. N-BODY SIMULATIONS

Numerical simulations are done using a standard Particle-Mesh (PM) code (Hockney & Eastwood 1981, Kates, Kotok, & Klypin 1991). Two runs with different realizations of the initial spectrum have been done. Both runs are simulated using a $256^3$ mesh for the gravitational force resolution. Each simulation has $128^3$ “cold” particles and two additional sets of $128^3$ particles in each set to represent “hot” neutrinos. We use the same prescription to simulate random thermal velocities of “hot” particles as Klypin et al. (1993).
“Hot” particles are generated in pairs, particles of each pair having random “thermal” velocities of exactly equal magnitude but pointing in opposite directions. The directions of these “thermal” velocities are random. The magnitudes of the velocities are drawn from relativistic Fermi-Dirac statistics. The particles have different relative masses: each “cold” particle has a relative mass 0.7 and each “hot” particle has the mass $0.3/2 = 0.15$.

Initial positions and velocities of particles were set using the Zel’dovich approximation. The displacement vector was simulated directly. Phases of fluctuations were exactly the same for “hot” and “cold” particles. When generating velocities of “hot” particles, the “thermal” component, as described above, was added to the velocity produced by the Zel’dovich approximation.

The size of the computational box for the simulations is 400Mpc ($h = 0.5$). The simulations were started at redshift $z = 7$ and were run to redshift zero with a constant step $\Delta a = 0.01$ in the expansion parameter $a$. The smallest resolved comoving scale in these simulations is $0.78h^{-1}\text{Mpc}$ and the mass of a “cold” particle was $7.33 \times 10^{11} h^{-1} M_\odot$.

4. IDENTIFICATION OF GALAXY CLUSTERS IN THE SIMULATIONS

Observationally real galaxy clusters are found as local concentrations within some radius. For the Abell clusters the radius is $1.5h^{-1}\text{Mpc}$ and for APM clusters it is $0.75h^{-1}\text{Mpc}$. We mimic this procedure for our simulations by finding all local maxima of the total (“cold” plus “hot”) density above density threshold $\delta \rho/\rho > 100$. The density field is produced on the $256^3$ grid, which corresponds to the spacial resolution $0.78h^{-1}\text{Mpc}$. The effective smoothing scale, corresponding to this mesh is smaller than that of real galaxy clusters. As a result, the number of selected maxima is much larger than the expected number of galaxy clusters for this volume (about 100). At $z = 0$ the number of maxima was 1059 in one simulation and 987 in another. Since the scale is not very far from the radius of real
clusters, we use the density maxima as a starting point for the search of the clusters. A sphere of cluster radius is placed at each density maximum and the centrum of mass of the sphere is found. Then the centre of the sphere is displaced to the centrum of mass and the procedure is repeated. As the result the spheres move in space in the process of searching for maximum of the number of dark matter particles inside the sphere radius. After five iteration the process converges and positions of spheres stop to change. For one of the simulations at $z = 0$ we made 10 iterations and found that there was no difference from results obtained after 5 iterations. Some of the spheres (about 10 per cent) find the same clusters. In order to remove the duplicates, pairs of clusters with distance between centers less than $0.75h^{-1}\text{Mpc}$ are found and only one more massive cluster in a pair is retained for the subsequent analysis. We still have more candidates than the expected number of rich galaxy clusters in our volume because most of our “clusters” are less massive than Abell or APM clusters.

5. CLUSTER PROPERTIES IN THE CHDM MODEL

For clusters found in the APM survey Dalton et al. (1992) give $n = 2.4 \times 10^{-5}h^3\text{Mpc}^{-3}$ for the richness class $\mathcal{R} > 20$. This would mean 195 clusters in our volume. By imposing the mass threshold $M > 1.05 \times 10^{14}h^{-1}\text{M}_\odot$ on our APM-style “clusters”, we find 203 clusters in one simulation and 206 in another, which is close to the expected number of APM clusters. The RMS 1D peculiar velocity of the clusters is $\sigma_v = 418$ km s$^{-1}$. This is compatible with the estimate given by Dalton et al. (1992): they formally estimate $\sigma_v = 700$ km s$^{-1}$, but measuring errors account for about half of the dispersion. This brings the estimate of real velocities to the level of 500 km s$^{-1}$, with a large uncertainty. However, not all of the rms velocity 418 km s$^{-1}$ in the model take part in producing anisotropy of $\xi_{cc}(\sigma, \pi)$, which was the method used by Dalton et al. to estimate to cluster velocities. Most of it is due to very
long waves. As a rough estimate, we should remove from the estimate the velocities of a sphere of radius $40h^{-1}\text{Mpc}$, because the anisotropy of $\xi_{cc}(\sigma, \pi)$ was found well within the radius. This reduces the velocity to the level $\sigma_v = (300 - 350) \text{ km s}^{-1}$. It is not clear if this estimate contradicts results of Dalton et al. because the measuring errors are too close to the estimated $\sigma_v$. Dalton et al. also give the number of clusters with richness $\mathcal{R} > 35$. If one assumes that the mass of a cluster is proportional to its richness, than the CHDM model produces a factor of two smaller of APM clusters of richness $\mathcal{R} > 35$: there are 45 and 46 clusters in our simulations while the observed number is 94. The difference could be due to numerical effects. If more massive APM-style cluster is also slightly thicker and its diameter is larger than 2 cells, we underestimate its mass.

Figure 1 shows the number density of Abell clusters in the CHDM model as a function of redshift (full curves and filled circles). The Press-Schechter approximation does not give a good fit in this case. If it is fitted to the mass function at $z = 0$, $\delta_c = 1.50$, it underestimates by a factor of 3 the number of clusters with mass larger than $2 \times 10^{14}h^{-1}\text{Mpc}$ at $z = 0.5$. The dashed curves show the PS results for $\delta_c = 1.45$.

The full curve in Figure 2 shows the mass function of Abell clusters in the CHDM model. In order to give a rough estimate of the errors we also show the mass function in the two simulations separately as markers. The dot-dashed curve is for the mass function obtained by the Press-Schechter approximation with the parameter $\delta_c = 1.50$. The dashed curve shows the mass function of Abell clusters estimated by Bahcall & Cen (1993). The errors associated with their results are probably larger than formal statistical uncertainties given by Bahcall & Cen (1993). However, the fit given by Bahcall & Cen is not very far from the results obtained in our simulations. The upper dashed curve provides a very accurate fit to the mass function in the CHDM model. It is the same fit as found by
Bahcall & Cen, but the mass scale is higher by the factor 1.6. Thus, the mass function of Abell clusters in the CHDM is well approximated by

$$n(> M) = 4 \times 10^{-5} (M/M_*)^{-1} \exp(-M/M_*) h^3 \text{Mpc}^{-3}, \quad M_* = 2.9 \times 10^4 h^{-1} M_\odot.$$  

There are some possible reasons for the discrepancy with results of Bahcall & Cen. One of them is due to the fact that a large fraction of cluster mass is gained at relatively large radii from the center of the cluster center. Thus, the total mass significantly depends on such assumptions as the density profile or sphericity. (Note also that “significant” in this context means 30–50 percent). Note that an independent study by Biviano et al., (1993) yields data which lie slightly above our model curve as opposed to Bahcall & Cen data which lie below it. Note also that uncertainties in the estimates for the total mass of the Coma cluster are sufficient to account for this factor. For example, for the mass of the Coma cluster, White et al. (1993) give $1.1 \cdot 10^{15} h^{-1} M_\odot$ as compared with $0.65 \cdot 10^{15} h^{-1} M_\odot$ assumed by Bahcall & Cen. It seems that the systematic errors dominate the statistical ones in estimating the observed cluster mass function.

Another way to characterize the mass distribution of clusters is to show the x-ray temperature function of the clusters, $dn/dT$, which is the number density of clusters with given hot gas temperature. The temperature of the gas in clusters is less sensitive to details of the density distribution at peripheral regions of galaxy clusters. It does change with the distance to the cluster center, but to the first approximation the gas could be considered isothermal. Though the gas temperature is possibly better defined observationally, it is more difficult to make an estimate of the temperature in simulations like ours without additional assumptions. We suppose that the gas temperature is proportional to the velocity dispersion of the dark matter in a cluster: $T \propto v^2$. The relation can be normalized to produce “right” temperature for the Coma cluster. If we take $v_{\text{coma}} = 1010 \text{ km s}^{-1}$ (e.g.}
Zabludoff et al. 1990) and $T_{\text{coma}} = 8.2\text{keV}$ (Henry & Arnaud 1991) then $T(\text{keV}) = (v/v_*)^2$, $v_* = 350\text{kms}^{-1}$. This is the relation, which we adopted for our analysis. The scale $v_*$ is uncertain at best within 20 percent. In Figure 3 we present $dn/dT$ for galaxy clusters in the CHDM model. The full curve shows the results for the model with the gas temperature being estimated using the rms velocity found for the dark matter within radius $1.5h^{-1}\text{Mpc}$. Triangles in the plot indicate the temperature distribution function of Abell clusters obtained by Henry & Arnaud (1991) and the results of Edge et al. (1990) are shown as squares. Bartlett & Silk (1993) used the Press-Schechter approximation ($\delta_c = 1.68$) and the relation $T \propto M^{2/3}$ to estimate the temperature distribution function. They predict significantly more high temperature clusters for the CHDM model than the present study but the difference could be explained by a scaling down the temperature estimated by Bartlett & Silk by a factor $T/1.4$, which is not that large taking into account the uncertainties in both the approximation for the number of clusters and in the $T$ - $M$ relation.

6. CORRELATION FUNCTIONS

In Figure 4 the correlation function of Abell clusters of richness $\mathcal{R} \geq 0$ is shown as big circles. We show the data of Postman et al. (1992) scaled to $\Omega = 1$ with the addition of more accurate redshifts (Postman, 1993) and a more accurate estimator for the correlation function: $(\text{DD}-2\text{DR}+\text{RR})/\text{RR}$ (Landy & Szalay 1993). The full curve shows predictions of the CHDM model. For the mass limit we chose $M > 2.5 \times 10^{14}h^{-1}\text{M}_\odot$. This is a compromise between the number density of the Abell clusters and mass of the clusters estimated by Bahcall & Cen. In the simulations there are slightly more clusters above the threshold as compared with the expected number of Abell clusters (148 versus 110) and the threshold mass of the “clusters” is slightly higher than the mass threshold estimated by Bahcall & Cen ($M > 1.8 \times 10^{14}h^{-1}\text{M}_\odot$). The dashed line in the plot shows the power law:
The cluster-cluster correlation function

\[ \xi_{cc}(r) = \left(\frac{r}{20h^{-1}\text{Mpc}}\right)^{-1.8}, \]

which at scales less than \((20 - 30)h^{-1}\text{Mpc}\) gives reasonably good fit for both the observational points and theoretical predictions. At larger radii the correlation function falls below the power law and becomes negative at \(r > 50h^{-1}\text{Mpc}\). The dashed line on the linear part of the Figure presents the correlation function of the dark matter estimated by the linear theory and scaled up by the factor \(b^2 = 6^2\) to match the correlation function at \(40h^{-1}\text{Mpc}\).

In Figure 5 we show the correlation function of APM clusters (circles; Dalton et al. 1992) and predictions from the numerical simulations (the full curve). The mass limit for the APM-style clusters in the model was \(M > 1.05 \times 10^{14}h^{-1}\text{M}_\odot\) and the number of the clusters in the simulation box was 203 and 206 for the two runs. Error bars shown in figures 4 and 5 are computed using poissonian errors \((1 + \xi)/\sqrt{N_{\text{pairs}}}\).

7. DISCUSSION

Four recent papers have determined that the cluster correlation function becomes negative on large scales (Postman et al. 1992 and Postman 1993, Peacock and West, 1992, Dalton et al. 1992, Scaramella et al. 1993). The zero point we infer from these four studies is \(r_0 = (50 \pm 10)h^{-1}\text{Mpc}\). This seems to be a robust result as the correlation functions have been computed using different samples and different methods. It is clear from previous studies that the CDM model cannot reproduce the cluster-cluster correlation function (Bahcall and Cen, 1992, Olivier et al. 1993) because it has a zero point of \(r_0 = 33h^{-1}\text{Mpc}\). For the CDM model with a cosmological constant (Kofman et al. 1993) the zero point occurs at \(r_{0,CDM+\Lambda} = 16.5(\Omega h^2)^{-1}\text{Mpc}\). The observed limits on the zero point put severe constraints on this model. The observed zero point lies in the range \(40 - 60h^{-1}\text{Mpc}\) implying the \(\Omega h\) lies in the range 0.27–0.41. Thus models with \(h < 1\) and \(\Omega < 0.25\) are in conflict with the observations. Assuming the age of the universe to be larger than
15 Gyr (to get a benefit from introducing the cosmological constant) rejects all models with $\Omega < 0.5$ or $h > 0.55$. For the tilted CDM model the constraint on the zero point implies that the large scale slope of the power spectrum should be in the range $h = 0.6 - 0.8$. The CDM + $\lambda$ model and low $\Omega$ models claimed by Bahcall and Cen (1992) to fit the data at $r < 25h^{-1}\text{Mpc}$ but do not account for the key observation of the cluster zero point (i.e. their model correlation functions go negative only beyond $r = 100h^{-1}\text{Mpc}$ in conflict with the observations). As shown in figures 4 and 5 the CHDM model provides an excellent fit to the APM and Abell cluster correlation functions over the range $5 < r < 100h^{-1}\text{Mpc}$ for which the function has been determined.

Note that two systematic errors can affect the determination of the zero point. Firstly for a finite sample the correlation function will be negative on large scales. The amplitude of this effect is $-n_c/N$ where $n_c$ is the number of clusters per clump and $N$ is the total number. For the Abell sample $n_c$ is $\sim 2$ and the total number $N$ is $\sim 200$. This effect alone would produce a negative value of $\xi_{cc}$ on large scales of $\sim -0.01$. Note however that observed and predicted negative amplitude is $\sim -0.05$ considerably larger than this effect.

Secondly, the finite box size or observational volume does not have long waves which should have the effect of reducing the zero-point. The amplitude of this effect depends on the spectrum. For the CHDM model and a 400Mpc box we estimate this to be a 10-20% effect. Results of our numerical simulations indicate that small nonlinear effects on 50Mpc scales move the zero point up by almost the same amount, thus basically compensating the effect of the finite box size.

We thus emphasize that it is now necessary to go beyond the power law representation of $\xi(r)$ since this is no longer a good fit to the available data.

We thank Marc Postman for sending us his correlation function via electronic mail.
FIGURE CAPTIONS

Figure 1. Number density of Abell clusters in the CHDM model as a function of redshift (full curves and filled circles). The dashed curves show the PS results for $\delta_c = 1.45$.

Figure 2. The cluster mass function. The solid curve shows the mass function predicted by the CHDM model. The dot-dashed curve is the mass function obtained by the Press-Schechter approximation with the parameter $\delta_c = 1.5$. The short-dashed curve shows the mass function of Abell clusters estimated by Bahcall & Cen (1993). The long-dashed curve provides a very accurate fit to the mass function in the CHDM model. It is the same fit as found by Bahcall & Cen, but the mass scale is higher by the factor 1.6.

Figure 3. The X-ray temperature distribution function. The full curve shows the results for the rms velocity found for the dark matter within radius $1.5h^{-1}$Mpc. Triangles in the plot indicate the temperature distribution function of Abell clusters obtained by Henry & Arnaud (1991) and the results of Edge et al. (1990) are shown as squares.

Figure 4. The Abell cluster correlation function. The correlation function of Abell clusters of richness $R \geq 0$ is shown as big circles (Postman et al. 1992, Postman 1993). The full curve shows predictions of the CHDM model. The dashed line in the plot shows the standard power law correlation function with a correlation length of $20h^{-1}$Mpc. The dashed line on the linear part of the plot presents the correlation function of the dark matter predicted by the linear theory.

Figure 5. The APM cluster correlation function. The correlation function of APM clusters (Dalton et al.) is shown as circles and the full curve shows predictions from the numerical simulations. The mass limit for the APM-style clusters in the model was $M > 1.05 \times 10^{14}h^{-1}M_\odot$ and the number of the clusters in the simulation box was 203.
and 206 for the two runs.
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