Robustness of Charge-Qubit Cluster States
to Double Quantum Point Contact Measurement

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We theoretically investigate the robustness of cluster states in charge qubit system based on quantum dot (QD) and double quantum point contact (DQPC). Trap state is modeled by an island structure in DQPC and represents a dynamical fluctuation. We found that the dynamical fluctuations affect the cluster states more than static fluctuation caused by QD size fluctuation. © 2010 The Japan Society of Applied Physics

1. Introduction

One-way quantum computing (1,2) is an important approach for quantum computation based on a series of one-qubit measurements starting from a cluster state of a qubit array. Cluster states are highly-entangled states involving all qubits and are typically generated from an Ising-like Hamiltonian, starting from an initial product state \( |\Psi_0\rangle = |0\rangle \otimes |\cdots\rangle \rightarrow \prod_{i=1}^{N} |+\rangle \), where \( |\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2} \). Here, \( |0\rangle \) and \( |1\rangle \) are the two states of the \( i \)-th qubit in an \( N \)-qubit system. In ref. 3, we showed that cluster states in charge qubits (4–17) can be created by applying a single gate bias pulse, right after preparing the initial product state (one-step generation method), and are more robust against nonuniformities among qubits than decoherence-free (DF) states (18,19) under a noise environment generated by a quantum point contact (QPC) detector, which is a sensitive detector of electric charge distribution (20,21). However, trap sites are often unavoidable in solid-state qubits owing to their small fabrication size and several experiments show that trap states significantly affect electric transport properties of nanoscale devices (22–25). Here, we model the trap site as the island (discrete energy state) between double QPCs (DQPC) and investigate robustness of cluster states in charge qubits measured by the DQPC detector (Fig. 1). The charge qubits are based on quantum dots (QD), in which the position of the excess charge in a qubit affects the QPC current electrically, resulting in detection of charged state. Owing to this additional external degrees of freedom, this setup is considered to be a harsher and more realistic condition for qubits than that of ref. 3. Here, we calculate a time-dependent fidelity of four charge qubits with DQPC by solving density matrix (DM) equations.

This article is organized as follows. In §2, formulation of our model is presented. Section 3 is devoted to the numerical calculations for the cluster state and DF state. The conclusion is given in §4. The detailed coefficients used in the equations in the text are shown in Appendix.

2. Density Matrix Equations

Here we show the formulation of the density matrix equations for Fig. 1. The Hamiltonian for the combined qubits and the QPC for Fig. 1 is written as \( H = H_{\text{qb}} + H_{\text{qpc}} + H_{\text{int}} \). \( H_{\text{qb}} \) is a Hamiltonian for an array of charge qubits with nearest-neighbor interactions, expressed by \( (3,7,8) \)

\[
H_{\text{qb}} = \sum_{i} (\Omega_i \sigma_i^x + \epsilon_i \sigma_i^z) + \sum_{i<j} J_{ij} \sigma_i \sigma_j.
\]

where \( \sigma_i^x \) and \( \sigma_i^z \) are Pauli matrices for the \( i \)-th qubit expressed by \( \sigma_0 = |0\rangle \langle 0| + |1\rangle \langle 1| \) and \( \sigma_z = |0\rangle \langle 0| - |1\rangle \langle 1| \). \( 0 \) and \( 1 \) refer to the two single-qubit states in which the excess charge is localized in the upper and lower dot, respectively. \( \Omega_i \) is the inter-QD tunnel coupling between two coupled QDs. \( \epsilon_i \) is the charging energy and corresponds to the energy difference between \( |0\rangle \) and \( |1\rangle \) controlled by gate voltage. The coupling constants \( J_{ij} \) are derived from the capacitance couplings (20).

The Hamiltonian for the two serially coupled QPCs \( H_{\text{qpc}} \) is described by

\[
H_{\text{qpc}} = \sum_{a=L,R} \sum_{s=\downarrow,\uparrow} \left[ E_{i_s} c_{i_s \downarrow}^\dagger c_{i_s \uparrow} + V_{i_s} (c_{i_s \downarrow}^\dagger d_s + d_s^\dagger c_{i_s \uparrow}) \right]
\]

\[ + \sum_{a=L,R} E_b d_b^\dagger d_s + U d_b^\dagger d_b^\dagger d_s^\dagger d_s^\dagger, \tag{2} \]

where \( c_{i_s} \) (\( c_{i_s} \)) is the annihilation operator of an electron in the \( i_s \)-th \((i_s\)-th) level \( \left[ i_s(i_s) = 1, 2, 3, \ldots \right] \) of the left (right) electrode, \( d_s \) is the electron annihilation operator of the island between the QPCs, \( E_{i_s} (E_{i_s}) \) is the energy level of electrons in the left (right) electrode, and \( E_{i_s} \) is that of the island. Here, we assume only one energy level on the island between the two QPCs, with spin degeneracy. \( V_{i_s} (V_{i_s}) \) is the tunneling strength of electrons between the left (right) electrode state \( i_s \) \((i_s) \) and the island state. \( U \) is the on-site Coulomb energy of double occupancy in the island.

\( H_{\text{int}} \) is the capacitive interaction between the qubits and the QPC, which induces dephasing between different eigenstates of \( \sigma_z \) (18,19). Most importantly, it takes into account the fact that localized charge near the QPC increases the energy of the system electrostatically, thus affecting the tunnel coupling between the left and right electrodes.

1 1
1 2
1 3
1 4
gate

Fig. 1. (Color online) Four qubits that use double QD charge states are capacitively coupled to a double QPC detector with an island. One excess charge is injected into each qubit. Single energy level is assumed in each QD and the island. This figure shows \( |0\rangle \) state.

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\[ H_{\text{int}} = \sum_{\alpha=L,R} \sum_{i=1}^{2} \delta V_{\alpha,i} \sigma_i + \sum_{i=3}^{5} \delta V_{\alpha,i} \sigma_i \]
\[ = \left( \sum_{i=3}^{5} \delta V_{\alpha,i} \sigma_i \right) \left( e_{\alpha,i} d_{\alpha,i} + d_{\alpha,i}^\dagger c_{\alpha,i} \right) \]
where \( \delta V_{\alpha,i} \) is the effective change of the tunneling strength between the electrodes and QPC island. Hereafter, we neglect the spin dependence of \( V_{\alpha,i} \) and \( \delta V_{\alpha,i} \). We assume that the tunneling strength of electrons weakly depends on the energy \( V_{\alpha,i} = V_{\alpha}(E_{\alpha,i}) \) and the Fermi surface \( \mu_{\alpha} \). Then, the qubit states influence the QPC tunneling rates \( \Gamma_L \) and \( \Gamma_R \) by
\[ \Gamma_L^{-1} = \Gamma_1^{-1} + \Gamma_2^{-1} \quad \text{and} \quad \Gamma_R^{-1} = \Gamma_3^{-1} + \Gamma_4^{-1} \]
through \( \Gamma_{1,2} \equiv 2\pi g_{0,0}(\mu_0) |V_{\alpha,1}(\mu_0)|^2 \) and \( \Gamma_{3,4} \equiv 2\pi g_{0,0}(\mu_0 + U)|V_{\alpha,1}(\mu_0 + U)|^2 \), depending on the qubit state \( \sigma = \pm 1 \) (\( V_{\alpha,1}(\mu_0) \) is the density of states of the electrodes \( \alpha = L,R \) and \( \Gamma_i \) is a tunneling rate when the island lies in the \( \sigma \) state). The values of \( \Gamma_{1,2,3,4} \) are determined by the geometrical structure of the system and depend on the distance between qubits and the QPC as \( \Gamma_{i} = \Gamma_{0} \pm \Delta \Gamma_{i} = \Gamma_{0}(1 \pm \xi) \) with the measurement strength \( \xi \equiv \Delta \Gamma/\Gamma_{0} \).

The DM equations of four qubits and the DQPC detector at zero temperature of Fig. I are derived similarly to refs. 27 and 28 by

where \( z_1, z_2 \) indicate qubit states such as 0000, 0001, \ldots, 1111 and, \( \rho^0_{z_1 z_2} \), \( \rho^1_{z_1 z_2} \), and \( \rho^{c_1}_{z_1 z_2} \) are density matrix elements when no electron (a), one electron, and two electrons (c) exist in the QPC island, respectively. \( J_{\alpha} = \sum_{i=1}^{4} e_{\alpha,i} J_{12} + J_{23}, J_{0001} = \sum_{i=1}^{4} e_{\alpha,i} + J_{12} + J_{23}, J_{1111} = - \sum_{i=1}^{4} e_{\alpha,i} + J_{12} + J_{23} \). \( g_{z_1 z_2} \) is introduced for the sake of notational convenience and determined by the relative positions between qubit states (see Appendix and ref. 27). In the present case, there are 768 equations to be solved.

Time-dependent fidelity \( F(t) \equiv \text{Tr}[\varrho(0)\rho(t)] \) is calculated by tracing over the elements of the reduced DM obtained from eq. (4).

We use these DM equations to derive four qubits, and compare fidelity of a cluster state
\[ |\Psi_{\text{CS}} \rangle = \frac{1}{2} \left( |+1 \rangle |0 \rangle_1 + |+1 \rangle |0 \rangle_2 |+1 \rangle |0 \rangle_3 |+1 \rangle |0 \rangle_4 \right) + \left( |+1 \rangle |0 \rangle_1 |+1 \rangle |0 \rangle_2 |+1 \rangle |0 \rangle_3 |+1 \rangle |0 \rangle_4 \right) \]
with that of a DF state
\[ |\Psi_{\text{DF}} \rangle = \frac{1}{2} \left( |1100 \rangle - |1001 \rangle - |0110 \rangle + |0011 \rangle \right). \]

3. Cluster States in Charge Qubits

In one-way quantum computing, quantum computation is carried out by measuring a series of qubits in cluster states. Cluster states are directly generated if we can prepare an Ising-like Hamiltonian \( H_\text{Ising} = (g/4) \sum_{i,j} (1 - \sigma_i^z)(1 - \sigma_j^z) \), starting from the initial state \( |\Psi_0 \rangle \). A unitary evolution \( U_\text{Ising}(t) = \exp(-i t H_\text{Ising}) \) (we use \( h = 1 \)) at \( g t = (2n t + 1) \pi \) with an integer \( n t \) transforms \( |\Psi_0 \rangle \) into cluster states. In the case of charge qubit in eq. (1), the \( e_{\alpha,i} \) term needs to be switched off during the creation of the cluster state and then switched on when measurements are carried out. To realize this controllability, additional gates are required. However, these additional gates make the qubit system complicated, ending by producing decoherence and crosstalk between qubits themselves and between qubits and the environment. In addition, for some qubit systems, once the qubit array is made, \( \Omega_0 \) and \( J_{ij} \) are fixed, and only \( e_{\alpha,i} \) is controllable via the gate voltage bias (we call these “single-design qubits”).

In ref. 3, we proposed how to effectively generate cluster states in such simple-design qubits (“one-step generation method”). In this method, cluster states are obtained by applying a gate voltage \( e_i \) for the \( i \)-th qubit, expressed as
\[ e_i = e_i^{\text{CS}} = \sqrt{E_i^\text{CS} - \Omega_i}, \]
and then making the state sensitive to the existence of trap sites. Figures 3(a) and 3(b) show the combined effects of the island structure and the nonuniformities among qubits when parameters \( \Omega_0, e_i, \) and \( \Gamma_i \) fluctuate by 10%. The smaller sizes of QDs are preferable for qubits based on QDs. If we assume that a diameter of each QD is
We can see that the difference is slight for various types of nonuniformities when $\eta$ is sufficiently small. We can also see that the fidelity of the larger gate bias case (1) is degraded slightly more than that of the smaller gate bias case (2). Note that this difference is much smaller in cluster states than in DF states, which is a preferable result for one-way computing in solid-state qubits. From Figs. 3 and 4, we can confirm that the cluster state is robust to nonuniformity, whereas the cluster state is weak against local electronic state (trap state), which is similar to the DF state.

5. Conclusions
We investigated the effect of local electronic fluctuations (trap state) in addition to nonuniformities, as a model of a decoherence mechanism in cluster states. We found that the island (trap site) affects the fidelity of cluster states significantly. Because the local electronic state provides dynamical fluctuation whereas the nonuniformities provide static fluctuation, we can say that the cluster states are sensitive to dynamical fluctuations. Experimentally, it is difficult to reduce the number of trap sites. Thus, more detailed understanding of effects of trap states will be required in the future.

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Appendix: Coefficients in DM Equations
Here, we show all coefficients used in eq. (4). For legibility, we denote $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ as $|A\rangle$, $|B\rangle$, $|C\rangle$, $|D\rangle$, respectively. Four-qubit states are written by $|AA\rangle$, $|AB\rangle, \ldots, |DD\rangle$. Then, $J_{zi}$ ($z_i = AA, \ldots, DD$) are expressed as

$$
\begin{align*}
J_{AA} &= \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + J_{12} + J_{23} + J_{34}, \\
J_{AC} &= \epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4 + J_{12} + J_{23} - J_{34}, \\
J_{BA} &= \epsilon_1 - \epsilon_2 + \epsilon_3 + \epsilon_4 - J_{12} - J_{23} + J_{34}, \\
J_{BC} &= \epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4 - J_{12} + J_{23} - J_{34}, \\
J_{CA} &= \epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4 - J_{12} + J_{23} + J_{34}, \\
J_{CC} &= \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 - J_{12} - J_{23} + J_{34}, \\
J_{DA} &= -\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + J_{12} - J_{23} + J_{34}, \\
J_{DC} &= -\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4 + J_{12} + J_{23} - J_{34}, \\
J_{DD} &= -\epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4 + J_{12} + J_{23} + J_{34}.
\end{align*}
$$

$g(z_i)$ ($i = 1, \ldots, 4, z_i = AA, \ldots, DD$) are given by

Fig. 2. Time-dependent fidelity of four-qubit states for a cluster state, a DF state, and a product state $|1010\rangle$, in case (i). $\Gamma_0 = J$, $\Gamma_1 = \Gamma_2$, and $\zeta = 0.6$. The "weak" indicates a weak measurement case of $\zeta = 0.2$ and the "strong" indicates a strong measurement case of $\zeta = 0.6$.

Fig. 3. Time-dependent fidelities $F(t)$ of a cluster state and a DF state when there are nonuniformities in qubit parameter. $\Omega = 4J$ and $n_{\Omega} = 1$ (case (1)). $\Gamma_0 = J$, $\Gamma_1 = \Gamma_2$, and $\zeta = 0.6$. (a) Strong measurement case. (b) Weak measurement case. Nonuniformity in the qubit parameters is introduced as $\Omega_i = 2\Omega(1 - \eta_i)/\zeta$, $\epsilon_i = \epsilon + \eta_i J$, and $\Gamma_i^{(i)} = (1 - \eta_i)\Gamma^{(i)}_i$, with $i$ indicating the $i$-th qubit. Here $\eta_1 = 0$ for all qubits besides $\eta_2 = 0.1$ (i), $\eta_2 = 0.1$ (ii), and $\eta_2 = 0.1$ (iii). The fidelities of $|\Psi_{CS}\rangle$ for (i) and (ii) mostly overlap.

Fig. 4. Fidelities of four-qubit cluster state and DF state at $t = 50\Gamma_0^{-1}$ as a function of nonuniformity $\eta$. Parameters are the same as those in Fig. 3.
$g_1(AA) = CA$, $g_2(AA) = BA$, $g_3(AA) = AC$, $g_4(AA) = AB$

$g_1(AB) = CB$, $g_2(AB) = BB$, $g_3(AB) = AD$, $g_4(AB) = AA$

$g_1(AC) = CC$, $g_2(AC) = BC$, $g_3(AC) = AA$, $g_4(AC) = AD$

$g_1(AD) = CD$, $g_2(AD) = BD$, $g_3(AD) = AB$, $g_4(AD) = AC$

$g_1(BA) = DA$, $g_2(BA) = AA$, $g_3(BA) = BC$, $g_4(BA) = BB$

$g_1(DB) = DB$, $g_2(DB) = AB$, $g_3(DB) = BD$, $g_4(DB) = BA$

$g_1(BC) = DC$, $g_2(BC) = AC$, $g_3(BC) = BA$, $g_4(BC) = BD$

$g_1(BD) = DD$, $g_2(BD) = AD$, $g_3(BD) = BB$, $g_4(BD) = BC$

$g_1(CA) = AA$, $g_2(CA) = DA$, $g_3(CA) = CC$, $g_4(CA) = CB$

$g_1(CB) = AB$, $g_2(CB) = DB$, $g_3(CB) = CD$, $g_4(CB) = CA$

$g_1(CC) = AC$, $g_2(CC) = DC$, $g_3(CC) = CA$, $g_4(CC) = CD$

$g_1(CD) = AD$, $g_2(CD) = DD$, $g_3(CD) = CB$, $g_4(CD) = CC$

$g_1(DA) = BA$, $g_2(DA) = CA$, $g_3(DA) = DC$, $g_4(DA) = DB$

$g_1(DB) = BB$, $g_2(DB) = CB$, $g_3(DB) = DD$, $g_4(DB) = DA$

$g_1(DC) = BC$, $g_2(DC) = CC$, $g_3(DC) = DA$, $g_4(DC) = DD$

$g_1(DD) = BD$, $g_2(DD) = CD$, $g_3(DD) = DB$, $g_4(DD) = DC$.

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