Wess-Zumino term for AdS superstring

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Abstract

We examine a bilinear form Wess-Zumino term for a superstring in anti-de Sitter (AdS) spaces. This is composed of two parts; a bilinear term in superinvariant currents and a total derivative bilinear term which is required for the pseudo-superinvariance of the Wess-Zumino term. The covariant supercharge commutator containing a string charge is also obtained.

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1 Introduction

Superstring actions in anti-de Sitter spaces have been studied in the Green-Schwarz formalism \[1, 2, 3, 4, 5\] motivated by the AdS/CFT correspondence \[6\]. This type of superstring action contains a Wess-Zumino term \[7\] which is required by the \(\kappa\)-symmetry to match the number of dynamical degrees of freedom for bosons and fermions \[8\]. The conventional description of the Wess-Zumino term is used in \[1, 2\] while an alternative description of the Wess-Zumino term, written in a bilinear form of super invariant currents, has been proposed for the AdS superstring theories \[3, 5\] and for an AdS superstring toy model \[4\]. The Wess-Zumino term for a superstring in the \(AdS_5 \times S^5\) space is given in \[1\]

\[
S_{WZ,\text{conventional}} = \int d^3 \sigma \, H_{[3]} = \int d^2 \sigma \, B \quad , \quad H_{[3]} = -i \bar{L} \tau_3 L ,
\]  

(1.1)

while the bilinear form Wess-Zumino term can be given as \[3, 10\]

\[
S_{WZ,[2]} = \int d^2 \sigma \, B_{[2]} \quad , \quad B_{[2]} = \bar{L} \tau_1 L
\]  

(1.2)

where \(L\) and \(L\) are superinvariant spinor and vector currents respectively \[1\]. Carrying out the integral of the Wess-Zumino term in (1.1) is complicated for AdS cases, so equations of motion and symmetry generators are hardly obtained except in the light-cone gauge \[11\]. On the other hand the bilinear form Wess-Zumino term as (1.2) is practical for such computations as demonstrated in \[12\].

The difference between \(B\) of (1.1) and \(B_{[2]}\) of (1.2) is the supersymmetry property. \(B\) is not superinvariant but pseudo-superinvariant, while \(B_{[2]}\) is manifestly superinvariant. The pseudo-superinvariance is necessary to give a topological charge in the superalgebra as explained in \[13, 14\], and topological charges classify Wess-Zumino terms \[15\]. A superstring action should contain a pseudo-superinvariant Wess-Zumino term producing a correct topological charge, a string charge.

Now we propose a pseudo-superinvariant two form as a Wess-Zumino term for an AdS superstring

\[
\tilde{B} = B_{[2]} - d \bar{\theta} \tau_1 d \theta,
\]  

(1.3)

which will coincide with the conventional two form \(B\) in (1.1). The second term in (1.3) is a leading term of \(B_{[2]}\) under the flat limit, and it is never obtained from integral of the three form \(\int_0^1 dt (H_{[3]}|_{\theta \to t \theta})\) in (1.1). So this term should be subtracted in order to be \(\tilde{B} = B\). Especially for the computation of topological terms this subtracted term plays an essential role. For computations of local quantities such as equations of motion and local symmetries, this term does not contribute because it is a surface term of an action. In section 2 in order to show \(B = \tilde{B}\) we examine the following criteria \[1\]:

(a) producing the correct three form gauge field strength, \(H_{[3]}\)

\(^1\) The charge conjugation matrices are \(C\) and \(C'\) which are both anti-symmetric in the notation \[1\]. We denote \(\bar{L}_{\alpha\beta} \tau_1 L = L^\beta \gamma_\alpha C_{\beta\alpha} C'_{\gamma\alpha'}\) and \(L = L^a \gamma_a + i L^a \gamma_a\).
(b) containing the local \( \kappa \) invariance

(c) reducing the correct “flat limit” for the IIB superstring action

instead of direct computation by performing integral of (1.1). The conventional description \( B \) gives its flat limit straightforwardly [1], while naive flat limit makes \( B_2 \) to be a trivial total derivative term. In another word \( B \) in the flat limit is differentiated to be an element of the non-trivial class of the Chevalley-Eilenberg cohomology for the supertranslation group, while \( B_2 \) in the flat limit is differentiated to be zero. We will show the correct flat limit of the Wess-Zumino term \( \tilde{B} \) and suggest the corresponding group contraction which maintains the nondegeneracy of the group metric as discussed in [19].

By using the concrete expression of \( \tilde{B} \) in (1.3), we obtain commutators of the super-AdS\(_5\times S^5\) symmetry charges containing a topological string charge in section 3. Topological charges in curved backgrounds for D-branes appearing in the supersymmetry commutator have been calculated by using BPS equations [16] and by using Noether method in the static gauge in the lower order of \( \theta \) expansion [17, 18]. The static gauge is not applicable for a fundamental superstring, since its zero mode is a superparticle moving along a null geodesics in the AdS space [12] as well as in the flat space. The super-AdS\(_5\times S^5\) algebra in the light-cone gauge has been examined in [11] using \( \theta \) the static gauge in the lower order of \( \theta \) expansion [17, 18]. The static gauge is not applicable for a fundamental superstring, since its zero mode is a superparticle moving along a null geodesics in the AdS space [12] as well as in the flat space. The super-AdS\(_5\times S^5\) algebra in the light-cone gauge has been examined in [11] using \( \theta \) where a string charge does not show up in the light-cone formalism. In this paper we construct global symmetry charges by Neother method in canonical language with neither any approximation nor any gauge fixing. A topological string charge in the AdS background is also obtained.

## 2 AdS superstring action

We begin with an action for the AdS superstring with the Wess-Zumino term (1.3) given by

\[
S = \int d^2\sigma \mathcal{L} = \int d^2\sigma (\mathcal{L}_0 + \mathcal{L}_{WZ})
\]

\[
\mathcal{L}_0 = -T \sqrt{-g} g^{ij} (L^a_i L_{a,j} + L^a_i L_{a',j})
\]

\[
\mathcal{L}_{WZ} = \pm \frac{1}{4} T \epsilon^{ij} \left( \tilde{L}^I_i (\tau_I)_I L^I_j - \partial_I \bar{\theta}^I (\tau_I)_I \partial_j \theta^I \right).
\]

The notation is the same as the one used by Metsaev and Tseytlin in [1, 2]. The left-

For AdS\(_5\times S^5\) case \((a = 0, 1, \ldots, 4, \ a' = 5, \ldots, 9)\) and \((\alpha = 1, \ldots, 4, \ \alpha' = 1, \ldots, 4, \ I = 1, 2)\), the super-AdS algebra is given by

\[
[P_a, P_b] = J_{ab}, \quad [P_{a'}, P_{b'}] = -J_{a'b'}
\]

\[
[P_a, J_{bc}] = \eta_{ac} P_b - \eta_{bc} P_a,
\]

\[
[J_{ab}, J_{cd}] = \eta_{ac} J_{bd} + 3 \text{ terms},
\]

\[
[Q_I, P_a] = \frac{1}{2} Q_I \gamma_a \epsilon_{IJ}, \quad [Q_I, P_{a'}] = -\frac{1}{2} Q_I \gamma_{a'} \epsilon_{IJ},
\]

\[
[Q_I, J_{ab}] = -\frac{1}{2} Q_I \gamma_{ab}, \quad [Q_I, J_{a'b'}] = -\frac{1}{2} Q_I \gamma_{a'b'}
\]

\[
\{Q_{\alpha a}, Q_{\beta b} \} = \delta_{a'b'} \left[ -2 i C'_{\alpha' \beta'} (C^a)_{\alpha a} P_a + 2 C_{\alpha \beta} (C')^{a} (C^a)_{\beta a'} P_{a'} \right]
\]

\[
+ \epsilon_{IJ} \left[ C'_{\alpha' \beta'} (C')^{a} (C^a)_{\beta a'} J_{ab} - C_{\alpha \beta} (C'_{\gamma})^{a} (C^a)_{\beta' a'} J_{a'b'} \right].
\]

\[2\]
invariant Cartan one-forms of a coset

\[ \text{SU}(2, 2|4)/[\text{SO}(4, 1) \times \text{SO}(5)] \ni G = G(x, \theta) = e^{xP} e^{\theta Q} \]

are defined by

\[
G^{-1}dG = L^a P_a + L^{a'} P_{a'} + \frac{1}{2} L^{ab} J_{ab} + \frac{1}{2} L^{a'b'} J_{a'b'} + L^{\alpha\alpha'} Q_{\alpha\alpha'}
\]

\[= dz^M L^A_M T_A = d\sigma^i L^A_i T_A \quad (2.2)\]

with \(T_A = \{P_a, P_{a'}, J_{ab}, J_{a'b'}, Q_{\alpha\alpha'}\}\), \(z^M = \{x^m, \theta^\mu\}\) and \(\sigma^i = \{\sigma^0, \sigma^1 = \sigma\}\), and they are given by

\[
\begin{align*}
L^a &= e^a + i\bar{\theta}\gamma^a \left(\frac{\sin(\Psi)}{\Psi/2}\right)^2 D\theta, \quad L^{a'} = e^{a'} - \bar{\theta}\gamma^{a'} \left(\frac{\sin(\Psi)}{\Psi/2}\right)^2 D\theta \\
L^{ab} &= \omega^{ab} - \bar{\theta}\gamma^{ab} \epsilon \left(\frac{\sin(\Psi)}{\Psi/2}\right)^2 D\theta, \quad L^{a'b'} = \omega^{a'b'} + \bar{\theta}\gamma^{a'b'} \epsilon \left(\frac{\sin(\Psi)}{\Psi/2}\right)^2 D\theta \\
L^\alpha &= \sin(\Psi) \frac{\psi}{\Psi} D\theta, \\
e^a &= dx^a + \left(\frac{\sin x}{x} - 1\right) dx^b \mathcal{Y}^a_b, \quad e^{a'} = dx^{a'} + \left(\frac{\sin x'}{x'} - 1\right) dx^{b'} \mathcal{Y}^{a'}_{b'} \\
\omega^{ab} &= \frac{1}{2} \left(\frac{\sinh(\frac{x}{2})}{x/2}\right)^2 dx[a, x^b], \quad \omega^{a'b'} = -\frac{1}{2} \left(\frac{\sinh(\frac{x'}{2})}{x'/2}\right)^2 dx[a, x^b']
\end{align*}
\]

where \([ab] = ab - ba\) and

\[
D\theta = \left[dt - \frac{i}{2} \epsilon(\gamma^a e_a + i\gamma^{a'} e_{a'}) + \frac{1}{4} \epsilon(\gamma^{ab} \omega_{ab} + \gamma^{a'b'} \omega_{a'b'})\right] \theta
\]

\[
(\psi^2)^{\alpha\alpha' I}_{\beta\beta' J} = (\epsilon \gamma^\alpha \theta^\alpha I)_{\beta\beta' J} - (\epsilon \gamma^{a'} \theta^a' I)_{\beta\beta' J} - \frac{1}{2} (\gamma^{ab} \epsilon \omega_{ab})_{\beta\beta' J} + \frac{1}{2} (\gamma^{a'b'} \epsilon \omega_{a'b'})_{\beta\beta' J}
\]

\[
x = \sqrt{x^2} = \sqrt{x^a x_a}, \quad x' = \sqrt{x'^2} = \sqrt{x'^a x_{a'}}
\]

\[
\mathcal{Y}^a_b = \delta^a_b - \frac{x_a x^b}{x^2}, \quad \mathcal{Y}^{a'}_{b'} = \delta^{a'}_{b'} - \frac{x_{a'} x^{b'}}{x'^2} \quad (2.4)
\]

### 2.1 Three form \(H_{[3]}\)

The Cartan one-forms satisfy the following Maurer-Cartan (MC) equations

\[
\begin{align*}
dL^I &= e^I J \left(\frac{1}{2} L^a \gamma_a L^I - \frac{1}{2} L^{a'} \gamma_{a'} L^I - \frac{1}{2} (L^{ab} \gamma_{ab} L^I + L^{a'b'} \gamma_{a'b'} L^I)\right) \\
dL^a &= i L^a \gamma^I L^I - L^b \mathcal{Y}^a_b, \quad dL^{a'} = -L^{a'} \gamma^I L^I - L^{b'} \mathcal{Y}^{a'}_{b'} \\
dL^{ab} &= -L^{ab} L^b - L^{a'} \mathcal{Y}^b_b - L^{a'b'} \epsilon_{IJ} L^I \\
dL^{a'b'} &= L^{a'} L^{b'} - L^{c'a'} L^c - L^{a'b'} \epsilon_{IJ} L^I
\end{align*} \quad (2.5)
\]
The first condition \((a)\) is confirmed by taking an exterior derivative of \(\bar{B}\) in (1.3) using the first MC equation and symmetric property of indices
\[
d\bar{B} = d\bar{L}^I(T_1)_{IJ}L^J + \bar{L}^J(T_1)_{IJ}dL^I - d(\bar{\theta}T_1d\theta)
\]
\[
= 2\bar{L}^K(T_1)_{KI}\{\epsilon^I_J\left(\frac{i}{2}\bar{L}^a\gamma_aL^J - \frac{1}{2}\bar{L}^{a'}\gamma_{a'}L^J\right) - \frac{1}{4}(\bar{L}^{ab}\gamma_{ab}L^J + \bar{L}^{a'b'}\gamma_{a'b'}L^J)\}
\]
\[
= -i\bar{L}^K(T_3)_{KI}L^I = H[3].
\]
The result (2.6) is the expected closed three form, \(dH[3] = 0\).

### 2.2 \(\kappa\)-invariance

In order to confirm that the second condition \((b)\), the \(\kappa\)-invariance, restricts the coefficient of the Wess-Zumino term, we set its coefficient to be \(b\) as
\[
S_{WZ} = b \int d^2\sigma T\bar{B}
\]
where the surface term does not contribute. In order to consider arbitrary variations \(\delta z^M\), it is useful to introduce
\[
\delta L^a_i \equiv \delta z^M_iL^a_M, \quad \delta L^{ab} \equiv \delta z^M_iL^{ab}_M, \quad \delta L^\alpha \equiv \delta z^M_iL^\alpha_M.
\]
The important property of the \(\kappa\)-transformation is given by
\[
\delta\kappa L^\alpha = 2 \bar{L}^K(T_3)_{KI}L^I \equiv H[3].
\]
For a superstring in the general type IIB background the \(\kappa\)-variations of Cartan one-forms are given by
\[
\delta\kappa L^\alpha = D(\triangle\kappa L^\alpha) - \frac{1}{4}\triangle\kappa L^{ab}(\gamma_{ab}L^\alpha) - \frac{1}{4}\triangle\kappa L^{a'b'}(\gamma_{a'b'}L^\alpha)
\]
\[
\delta\kappa L^a_i = 2i(\triangle\kappa L^a_i)L^b(\triangle\kappa L^b) + L^{b'}(\triangle\kappa L^b) + L^b(\triangle\kappa L^b),
\]
\[
\delta\kappa L^{a'}_i = -2(\triangle\kappa L^{a'}_i)L^{b'}(\triangle\kappa L^{b'})
\]
with a covariant derivative \(D\)
\[
D(\triangle\kappa L^I) \equiv d(\triangle\kappa L^I) - \frac{i}{2}\epsilon^I_J\bar{L}^J(\triangle\kappa L^J) + \frac{1}{4}(\bar{L}^{ab}\gamma_{ab}L^I + \bar{L}^{a'b'}\gamma_{a'b'}L^I)(\triangle\kappa L^\alpha).
\]
The \(\kappa\) variation of \(L_0\) is written as
\[
\delta\kappa L_0 = -T\frac{1}{2}\sqrt{-GG^{ij}}\delta\kappa G_{ij}, \quad G_{ij} = L^a_iL_{a,j} + L^{a'}_iL_{a',j}
\]
\[
= -2Ti\overline{\triangle\kappa L}(\sqrt{-GG^{ij}}\bar{L}_{ij})L_i.
\]
On the other hand the $\kappa$-variation of the Wess-Zumino term (2.7) is given by
\[
\delta_\kappa \mathcal{L}_{WZ} = 2ibT \Delta_\kappa L (\tau_3 \epsilon^{ij} \mathcal{L}^i) L_i .
\] (2.12)

The factor of the last expression in (2.11) is related to the one in (2.12) as
\[
-\sqrt{-G} G^{ij} \mathcal{L}^j = \Gamma(1)(\tau_3 \epsilon^{ij} \mathcal{L}^j)
\] (2.13)

where
\[
\Gamma(1) = (\frac{1}{\sqrt{-G}} \frac{1}{2} \tau_3 \mathcal{L})
\] (2.14)
satisfying
\[
(\Gamma(1))^2 = 1, \quad \text{tr} \Gamma(1) = 0.
\] (2.15)

The $\kappa$-variation of the total action becomes
\[
\delta_\kappa (\mathcal{L}_0 + \mathcal{L}_{WZ}) = 2TbT \Delta_\kappa L (\Gamma(1) + b)(\tau_3 \epsilon^{ij} \mathcal{L}^j) L_i ,
\] (2.16)

and the $\kappa$ parameter must satisfy
\[
(\Gamma(1) \pm 1) \kappa = 0, \quad \text{for } b = \pm 1 ,
\] (2.17)

where we used the fact $(\Gamma(1) \pm 1)(\Delta_\kappa L) = 2\mathcal{L}(\Gamma(1) \pm 1)\kappa$.

### 2.3 Flat limit

Now we will examine its “flat limit”, the third condition (c). Under the scaling $x \rightarrow (1/R)x$ and $\theta \rightarrow (1/\sqrt{R})\theta$, the Cartan one-form for $Q$’s (2.3) are expanded in a power series of $R$
\[
L^I = \sum_{r=\text{half integer}} \frac{1}{R^r} L^I_r,
\] (2.18)

and especially $L_{1/2}$ and $L_{3/2}$
\[
L^I_{1/2} = d\theta
\] (2.19)
\[
L^I_{3/2} = [ - \frac{i}{2}(dx^a \gamma_a + i dx^{a'} \gamma_{a'}) \epsilon^{IJ} \theta^J + \frac{1}{6} \epsilon^{IJ} (\gamma^{a'} \theta^J \bar{\theta}^K \gamma_a d\theta^K + \gamma^a \theta^J \bar{\theta}^K \gamma_{a'} d\theta^K)
+ \frac{1}{12} (\gamma^a \theta^J \bar{\theta}^K \gamma_{a'} \epsilon^{KL} d\theta^L - \gamma^{a'} \theta^J \bar{\theta}^K \gamma_a \epsilon^{KL} d\theta^L) ]
\] (2.20)

are necessary for examining its flat limit. Corresponding to the expansion of $L$’s, the two form Wess-Zumino term $B_{[2]}$ is also expanded. Its leading term becomes total derivative
\[
\frac{1}{R} \bar{L}^I_{1/2}(\tau_1)_{IJ} L^J_{1/2} = \frac{1}{R} d\bar{\theta}^I(\tau_1)^{IJ} d\theta^J = \frac{1}{R} d(\bar{\theta}^I \tau_1^{IJ} d\theta^J) ,
\] (2.21)
which is subtracted in our $\tilde{B}$. The next to leading term becomes the flat space Wess-Zumino term

$$\frac{1}{R^2}(L_{1/2}^I(\tau_1)_{IJ}L_{3/2}^J + L_{3/2}^I(\tau_1)_{IJ}L_{1/2}^J)$$

$$= \frac{1}{R^2} \left[ i \bar{d} \tilde{\theta} \tau_3 (dx^a \gamma_a + i dx^a \gamma_a) \theta \right.$$  

$$+ \frac{1}{3} \left\{ (d \bar{\theta} \gamma^a \tau_3 \theta) (\bar{\theta} \gamma_a d \theta) \right.$$

$$\left. + \frac{1}{6} \left\{ (d \bar{\theta} \gamma^{ab} \tau_1 \theta) (\bar{\theta} \gamma_{ab} (i \tau_2) d \theta) \right. \right.$$

$$\left. \right) \right] . \quad (2.22)$$

The cyclic identity of this space is equal to the Jacobi identity of three $Q$’s,

$$I_{\alpha \beta \gamma} + I_{\beta \gamma \alpha} + I_{\gamma \alpha \beta} = 0 \quad (2.23)$$

$$I_{\alpha \beta \gamma} = I_{1\alpha \beta \gamma} + I_{2\alpha \beta \gamma}$$

$$I_{1\alpha \beta \gamma}(\phi) = -\delta_{IJ} \left\{ C'_{\alpha \beta'}(C'_{\gamma \alpha})_{\alpha \beta}(\bar{\phi}K_{\gamma a})_{\gamma'} - C_{\alpha \beta}(C'_{\gamma \alpha})(\bar{\phi}K_{\gamma a})_{\gamma'} \right\} \epsilon_{LK}$$

$$I_{2\alpha \beta \gamma}(\phi) = \frac{1}{2} \epsilon_{IJ} \left\{ C_{\alpha \beta'}(C_{\gamma ab})_{\alpha \beta}(\bar{\phi}K_{\gamma a})_{\gamma'} - C_{\alpha \beta}(C'_{\gamma \alpha})(\bar{\phi}K_{\gamma a})_{\gamma'} \right\}$$

where $\alpha$ runs $\alpha \alpha'$ and $\phi$ is an arbitrary spinor. The second and third terms of the two-form (2.22) are expressed in terms of $I$ as

$$\int \frac{1}{R^2} \frac{1}{3} \left\{ I_{\alpha \beta \gamma}(\tau_1 d \theta) \theta^\alpha d \theta^\beta d \gamma \right.$$  

$$\left. \right\} \right] . \quad (2.24)$$

where a partial integration is performed. On the other hand the terms are also expressed in terms of $I_1$ and $I_2$ as

$$2^{\text{nd}} + 3^{\text{rd}} \text{terms of (2.22)} = \frac{1}{R^2} \frac{1}{3} \left\{ I_1(\tau_1 \theta) \theta^\alpha d \theta^\beta d \gamma - I_2(\tau_1 \theta) \theta^\alpha d \theta^\beta d \gamma \right\} . \quad (2.25)$$

The cyclic identity (2.23) multiplied with one $\theta$ and two $d \theta$'s gives following formula

$$I_{2\alpha \beta \gamma}(2 \theta^\alpha d \theta^\beta d \gamma + d \theta^\alpha d \theta^\beta d \gamma) = -2I_{1\alpha \beta \gamma} \theta^\alpha d \theta^\beta d \gamma . \quad (2.26)$$

We will pick up a suitable combination of (2.24) and (2.25) in such a way that the second and the third terms of (2.22) are rewritten to include only $I_1$ by using the formula (2.26)

$$2^{\text{nd}} + 3^{\text{rd}} \text{terms of (2.22)}$$

$$= A(2.24) + (1 - A)(2.23)$$

$$= \frac{1}{R^2} \left\{ I_{1\alpha \beta \gamma} d \theta^\alpha d \theta^\beta d \gamma - (1 - 2A)I_{2\alpha \beta \gamma} d \theta^\alpha d \theta^\beta d \gamma - A I_{2\alpha \beta \gamma} d \theta^\alpha d \theta^\beta d \gamma \right\}$$

$$A = 1/4 \frac{1}{R^2} \frac{1}{3} \frac{3}{2} I_1 \theta^\alpha d \theta^\beta d \gamma . \quad (2.27)$$
Collecting (2.27) and other terms in (2.22) leads to the following expression of the Wess-Zumino term
\[ \tilde{B} = \tilde{L}_1 \tilde{L} - d\tilde{\theta}_1 d\theta = \frac{1}{R^2} (2\tilde{L}_{1/2} \tilde{L}_{3/2}) + o\left( \frac{1}{R^3} \right) \]
2\tilde{L}_{1/2} \tilde{L}_{3/2} = [i\bar{d} \tilde{\theta}_3 (dx^\alpha \gamma_a + idx^{\alpha'} \gamma_{a'}) \theta
+ \frac{1}{2} ((\tilde{\theta}_3 \gamma^a d\theta) (\tilde{\theta}_a d\theta) - (\tilde{\theta}_3 \gamma^{a'} d\theta) (\tilde{\theta}_a d\theta))] \]
= [i\bar{d} \tilde{\theta}_3 dx^\alpha \Gamma_\alpha \theta + \frac{1}{2} (\tilde{\theta}_3 \Gamma_\alpha d\theta) (\bar{\theta} \Gamma_\alpha d\theta)]
(2.28)
where \( \hat{a} \) runs both \( a \) and \( a' \). The \( 1/R^2 \)-part is the two-form Wess-Zumino term for a superstring in a flat space. However this \( L_{3/2} \) can not be preserved by the conventional IW contraction where only \( L_{1/2} \) is preserved. In order to define the WZ term consistently even after IW contractions, new limiting procedure is required.

3 Super-AdS charges and string charge

Now we will compute Noether charges for the Super-AdS\(_5\times S^5\) space and their commutators. The total derivative term which is subtracted from the current bilinear term plays an essential role for the global supersymmetry. The pseudo invariance of the Wess-Zumino term (1.3) gives a surface term contribution to the supercharge. Under the global supersymmetry transformation with a parameter \( \varepsilon \), the variation of the Lagrangian (2.1) comes from only the subtracted total derivative term
\[ \delta_\varepsilon \mathcal{L} = \partial_\varepsilon \left( \mp T \partial_\varepsilon \tau_1 \partial_\varepsilon \theta \right) \]
= \( \mp T \partial_\varepsilon \epsilon_{ij} \left( \delta_\varepsilon \tilde{\theta}_1 \partial_\varepsilon \theta + \tilde{\theta}_1 \partial_\varepsilon \delta_\varepsilon \theta \right) \)
\( \equiv \partial_\varepsilon U_\varepsilon^i \). (3.1)
There is no contribution from Cartan one forms \( L^A \), since they are invariant under the global supersymmetry up to the local Lorentz which is cancelled in the Lorentz invariant Lagrangian. The supersymmetry charge is written as
\[ \varepsilon \mathcal{Q} = \int d\sigma \left[ p \delta_\varepsilon x + \zeta \delta_\varepsilon \theta - U_\varepsilon^0 \right] , \quad U_\varepsilon^0 = \mp 2T \tilde{\theta}_1 \partial_\varepsilon (\delta_\varepsilon \theta) \] (3.2)
with \( p \) and \( \zeta \) being canonical conjugates of \( x \) and \( \theta \). The last term gives the topological string charge in the superalgebra.

The symmetry transformation rules \( \delta_\varepsilon x \) and \( \delta_\varepsilon \theta \) are determined as follows. Under the supersymmetry transformation an element of a coset \( G/H \) is transformed as \( G \rightarrow gGh \) with \( g \in G \) and \( h \in H \) respectively. For an infinitesimal global parameter \( \varepsilon \), variational one-form (2.8) is given as
\[ G^{-1} \delta_\varepsilon G = G^{-1} (g - 1) G + (h - 1) = \triangle_\varepsilon L^A T_A \]
= \( \delta_\varepsilon z^M L^A_M T_A \). (3.3)
Therefore once $\triangle^\varepsilon L$’s and $L^{-1}$ are obtained, symmetry transformation rules are determined as

$$\delta_{\varepsilon z}^M = \triangle^\varepsilon L^A (L^{-1})^M_A.$$  \hspace{1cm} (3.4)

At first let us calculate $L^{-1}$ in (3.4). Coefficients of Cartan one forms are given from (2.3) as

$$L_m^A = \left( \begin{array}{ccc}
L_m \dot{a} & e_m \dot{\bar{a}} + \Theta_m \mu L_m \dot{\bar{a}} & \omega_m \dot{\bar{a}} + \Theta_m \mu L_m \dot{\bar{a}} & L_m^\alpha = \Theta_m \mu L_m^\alpha \\
L \mu \dot{\bar{a}} & L \mu \dot{\bar{a}} & L \mu \dot{\bar{a}} & L \mu ^\alpha
\end{array} \right)$$  \hspace{1cm} (3.5)

with

$$\begin{align*}
e_m^a &= \delta_m^a + \left( \begin{array}{c}\sinh x \\ \frac{x}{x}
\end{array} \right) \delta_m^a, \quad e_m^{a'} = \delta_m^{a'} + \left( \begin{array}{c}\sin x' \\ \frac{x'}{x'}
\end{array} \right) \delta_m^{a'} \\
\omega_m^{ab} &= \frac{1}{2} \left( \begin{array}{c}\sinh \left( \frac{x}{2} \right) \\ \frac{x}{2}
\end{array} \right) \delta_m^{[ab]}, \quad \omega_m^{a'b'} = -\frac{1}{2} \left( \begin{array}{c}\sinh \left( \frac{x'}{2} \right) \\ \frac{x'}{2}
\end{array} \right) \delta_m^{[a'b']}
\end{align*}$$  \hspace{1cm} (3.6)

Its inverse is defined by $L_m^A(L^{-1})^A_N = \delta_m^N$ and is given by

$$\begin{pmatrix}
(L^{-1})^a_n = (e^{-1})^a_m (L^{-1})^a_\bar{\nu} \\
(L^{-1})^m_{\bar{a} b} & (L^{-1})^a_{\bar{a} b'} \\
(L^{-1})^n_{\bar{a} \nu} & (L^{-1})^\alpha_n
\end{pmatrix}$$  \hspace{1cm} (3.7)

with

$$\begin{align*}
(L^{-1})^a_n &= (e^{-1})^a_n = \delta^a_n + \left( \begin{array}{c}x \\ \sinh x
\end{array} \right) \delta^a_n, \quad (L^{-1})_{ab}^m = 0 \\
(L^{-1})^{a'}_n &= (e^{-1})^{a'}_n = \delta_n^{a'} + \left( \begin{array}{c}x \\ \sin x
\end{array} \right) \delta_n^{a'} \\
(L^{-1})^a_{\bar{a} \nu} &= -i \bar{\theta} \left( \gamma^a (e^{-1})^a_n + i \gamma^a (e^{-1})^{a'}_n \right) \frac{(2\sin(\Psi/2))^2}{\Psi \sin \Psi}, \quad (L^{-1})_{a'b'} = 0 \\
(L^{-1})^\alpha_n &= -\frac{1}{2} (\epsilon \gamma^a \theta)_{\alpha}^\beta \left( \begin{array}{c}2\sin(\Psi/2)^2 \\ \Psi \sin \Psi
\end{array} \right)_{\beta}^\alpha
\end{align*}$$  \hspace{1cm} (3.8)

where this solution is well-defined in a flat limit although an ambiguity exists caused by the rectangular matrix (3.5) and (3.7) which is removed by the local Lorentz degrees of freedom.
Next let us calculate $\Delta_\varepsilon L^A$ for global supersymmetry in (3.4). From the relation of (3.3), i.e. $G^{-1} \delta_\varepsilon G = G^{-1} \varepsilon QG + \frac{i}{2} h_{\bar{a}b} J_{\bar{a}b}$, $\Delta_\varepsilon L^A$'s are obtained as:

$$
\begin{align*}
\Delta_\varepsilon L^a &= -2i\bar{\theta} \gamma^a \sin \frac{\Psi}{2} \bar{\varepsilon}, \quad \Delta_\varepsilon L^{a'} = 2\bar{\theta} \gamma^{a'} \sin \frac{\Psi}{2} \bar{\varepsilon} \\
\Delta_\varepsilon L^{ab} &= 2\bar{\theta} \gamma^{ab} \varepsilon \sin \frac{\Psi}{2} \bar{\varepsilon} + h^{ab}_e, \quad \Delta_\varepsilon L^{a'b'} = -2\bar{\theta} \gamma^{a'b'} \varepsilon \frac{\sin \Psi}{2} \bar{\varepsilon} + h^{a'b'}_e
\end{align*}
$$

(3.9)

with

$$
\begin{align*}
h^{ab} &= -\bar{\theta} \left( \epsilon \gamma^{ab} + i \gamma^{a} x^{b} \right) \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin x / x} \right) \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin \Psi / \Psi} \right)^2 \bar{\varepsilon} \\
h^{a'b'} &= -\bar{\theta} \left( \epsilon \gamma^{a'b'} - \gamma^{a'} x^{b'} \right) \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin x / x} \right) \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin \Psi / \Psi} \right)^2 \bar{\varepsilon}.
\end{align*}
$$

(3.10)

Combining (3.4), (3.8), (3.9) and (3.10) gives supertransformation rules as

$$
\begin{align*}
\delta_\varepsilon x^m &= -i \bar{\theta} \gamma^m \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin \Psi / \Psi} \right)^2 \bar{\varepsilon} \\
\delta_\varepsilon \theta^\mu &= \frac{\Psi}{\sin \Psi} \bar{\varepsilon} \\
&\quad + \frac{1}{4} \left[ \gamma^{ab} \theta \bar{\theta} \left( \epsilon \gamma^{ab} + i \gamma^{c} \omega^{ab}_c \right) - \gamma^{a'b'} \theta \bar{\theta} \left( \epsilon \gamma^{a'b'} + \gamma^{c'} \omega^{a'b'}_c \right) \right] \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin \Psi / \Psi} \right)^2 \bar{\varepsilon}
\end{align*}
$$

(3.11)

with

$$
\begin{align*}
\gamma^m &= \left\{ \begin{array}{ll}
\gamma^a (e^{-1})_a^m, & \text{for } m = 0 \sim 4 \\
i \gamma^{a'} (e^{-1})_{a'}^m, & \text{for } m = 5 \sim 9
\end{array} \right.
\\
\omega^{ab}_c &= (e^{-1})_c^m \omega_m^{ab} = \delta_c^{[a} x^{b]} \frac{1}{2} \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin x / x} \right)^2 \\
\omega^{a'b'}_{c'} &= (e^{-1})_{c'}^m \omega_m^{a'b'} = -\delta_{c'}^{[a'} x^{b']} \frac{1}{2} \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin x / x} \right)^2.
\end{align*}
$$

(3.12)

Inserting (3.11) into (3.2), the supersymmetry charge is written as

$$
\begin{align*}
Q_{\alpha\alpha'} &= \int d\sigma \left[ -i \bar{\theta} (\gamma^a x^a + i \gamma^{a'} x^{a'}) \left( \frac{\sin \frac{\Psi}{2} / \frac{\Psi}{2}}{\sin \Psi / \Psi} \right)^2 e^{i\frac{\theta}{2}} \right] \\
&+ \zeta \left\{ 1 + \frac{1}{4} \left( \gamma^{ab} \theta \bar{\theta} (\epsilon \gamma^{ab} + i \gamma^{c} \omega^{ab}_c) - \gamma^{a'b'} \theta \bar{\theta} (\epsilon \gamma^{a'b'} + \gamma^{c'} \omega^{a'b'}_c) \right) \left( \frac{\sin \frac{\Psi}{2}}{\frac{\Psi}{2}} \right)^2 \right\} \frac{1}{\sin \Psi / \Psi} e^{i\frac{\theta}{2}} \\
&\pm 2 T \bar{\theta} \tau_1 \partial_\sigma \left\{ 1 + \frac{1}{4} \left( \gamma^{ab} \theta \bar{\theta} (\epsilon \gamma^{ab} + i \gamma^{c} \omega^{ab}_c) - \gamma^{a'b'} \theta \bar{\theta} (\epsilon \gamma^{a'b'} + \gamma^{c'} \omega^{a'b'}_c) \right) \left( \frac{\sin \frac{\Psi}{2}}{\frac{\Psi}{2}} \right)^2 \right\} \frac{1}{\sin \Psi / \Psi} e^{i\frac{\theta}{2}} \right\}
\end{align*}
$$

(3.13)
The momentum charge and the Lorentz charge are analogously obtained in the appendix and given by

\[ P_a = \int d\sigma \left[ p_a + p_b x_{\hat{c} \theta \hat{a}} + \zeta \gamma_{\hat{c} \hat{b}} \theta \hat{a} \right] , \quad \text{for} \; \hat{a}, \hat{b}, \hat{c} = 0 \sim 4 \text{ or } 5 \sim 9 \]

\[ J_{\hat{a} \hat{b}} = \int d\sigma \left[ p_{[\hat{a} \hat{b}]} + \frac{1}{2} \zeta \gamma_{\hat{a} \hat{b}} \theta \right] \quad . \tag{3.14} \]

These charges of the super-AdS$_5 \times S^5$ space satisfy the following commutators

\[ \{ Q_{\alpha' I}, Q_{\beta' J} \} = -2i\delta_{IJ}(CC' \gamma^\hat{a})_{\alpha' \beta'} \mathcal{P}_a + \epsilon_{IJ}(CC' \gamma^\hat{a})_{\alpha' \beta'} \mathcal{J}_{\hat{a}} \]

\[ -2i(\tau_3)_{IJ}(CC' \gamma^\hat{a})_{\alpha' \beta'} \mathcal{Z}_{\hat{a}} \tag{3.15} \]

\[ [P_a, P_b] = J_{ab} , \quad [P_a', P_{b'}] = -J_{a'b'} \]

\[ [\hat{P}_{\hat{a}}, \mathcal{J}_{\hat{b} \hat{c}}] = \eta_{\hat{a} \hat{b}} \hat{P}_{\hat{c}} - \eta_{\hat{a} \hat{c}} \hat{P}_{\hat{b}} , \quad [\mathcal{J}_{\hat{a} \hat{b}}, \mathcal{J}_{\hat{c} \hat{d}}] = \eta_{\hat{a} \hat{c}} \mathcal{J}_{\hat{b} \hat{d}} + 3 \text{ terms} \tag{3.16} \]

\[ [Q_I, P_a] = \frac{i}{2} Q_I \gamma_a \epsilon_{IJ} , \quad [Q_I, P_{a'}] = -\frac{1}{2} Q_I \gamma_a' \epsilon_{JI} , \quad [Q_I, \mathcal{J}_{\hat{a} \hat{b}}] = -\frac{1}{2} Q_I \gamma_{\hat{a} \hat{b}} \]

up to the local Lorentz generator. The topological term $Z$ is obtained from the surface term as explained in the beginning of this section

\[ \left[ \delta \epsilon (-\int d\sigma \ U^0_\epsilon) - (\epsilon \leftrightarrow \epsilon') \right]_{\theta=0} = \epsilon^\hat{a} (\hat{a} J_{\hat{a} J}) (\pm 2T) \left( CC' e^{-i\epsilon_{IJ}/2} \tau_1 \partial_\theta (e^{i\epsilon_{IJ}/2}) \right)_{\hat{a} \hat{b}} \]

\[ = \epsilon^\hat{a} (\hat{a} J_{\hat{a} J}) (\pm T) (\tau_3) \partial_\theta (iCC' \sinh X / X) \]

\[ = \epsilon^\hat{a} (\hat{a} J_{\hat{a} J}) (\pm T) (CC' \gamma_{\hat{a}} \sinh X / X) \tag{3.17} \]

where $X = \sqrt{x^a x_a + x^{a'} x_{a'}}$ and

\[ Z_{\hat{a}} = \pm T \int d\sigma \partial_\sigma \left( x_{\hat{a}} \sinh X / X \right) . \tag{3.18} \]

For a zero-mode of the string this string charge vanishes and 32 supersymmetries remain. For massive excited states this string charge breaks half supersymmetries as same as BPS states. After rescaling $x \rightarrow x/R$ and taking the flat limit $R \rightarrow \infty$ this reduces into the flat string charge, $T \int d\sigma \partial_\sigma x_{\hat{a}}$.

4 Conclusions and discussions

We have shown that the pseudo-superinvariant Wess-Zumino term written in the form of $\tilde{B}$ in (3.3) satisfies the three conditions, (a) correct three form $H_{[3]}$, (b) $\kappa$-invariance and (c) correct flat limit. We have constructed global charges of the super-AdS space.
\(Q, P, J\) and a string charge \(Z\) in (3.18) appeared in the supercharge commutator. We have also mentioned that a generalization of the IW contraction is required to give a correct flat limit of the bilinear Wess-Zumino term, where the next leading term in the limiting procedure is preserved to make the fermionic part of the group metric to be nondegenerate, i.e. not only \(L_{1/2}\) but also \(L_{3/2}\) should be preserved in (2.18) as shown in (2.28). In a flat space the bilinear Wess-Zumino term can not exist as shown in [14], but in AdS spaces the bilinear form Wess-Zumino terms exist. This fact is a reflection of the fact that the super-AdS algebra is nondegenerate. In order to have the bilinear Wess-Zumino term even in a flat space after some group contraction, we need a generalization of the IW-contraction where the scale parameter does not disappear completely and the resultant superalgebra is nondegenerate [10].

Once AdS brane actions are obtained, pp brane actions can be easily obtained by an analogous limiting procedure to the section 2.3 corresponding to the contraction of super-AdS groups [20].

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**A Appendix**

The momentum and Lorentz charges are also obtained analogously to the supersymmetry charges by taking infinitesimal parameters \(y^\tilde{a}\) and \(\lambda^{\tilde{a}b} = \{\lambda^{ab}, \lambda^{a'b'}\}\). The momentum charge is given by

\[
y^\tilde{a}P_{\tilde{a}} = \int d\sigma[p_{m}\delta_{y}x^{m} + \zeta_{\mu}\delta_{y}^{\mu}] .
\]

(A.1)

Translation variation leads to \(\triangle_{y}L^{A}\) in (B.4) as

\[
\begin{align*}
\triangle_{y}L^{a} & = \tilde{y}^{a} - i\tilde{\theta}\gamma^{a}\sin\frac{\Psi}{\Theta}\tilde{Y} \theta , \quad \triangle_{y}L^{a'} = \tilde{y}^{a'} + \tilde{\theta}\gamma^{a'}\sin\frac{\Psi}{\Theta}\tilde{Y} \theta \\
\triangle_{y}L^{ab} & = \tilde{z}^{ab} + \tilde{\theta}\gamma^{ab}\epsilon\sin\frac{\Psi}{\Theta}\tilde{Y} \theta + h_{y}^{ab} , \quad \triangle_{y}L^{a'b'} = \tilde{z}^{a'b'} - \tilde{\theta}\gamma^{a'b'}\epsilon\sin\frac{\Psi}{\Theta}\tilde{Y} \theta + h_{y}^{a'b'} .
\end{align*}
\]

(A.2)

with

\[
\begin{align*}
\tilde{Y} \theta & = \left[-\frac{i}{2}\epsilon(\tilde{y}^{a}\gamma_{a} + i\tilde{y}^{a'}\gamma_{a'}) + \frac{1}{4}(\tilde{z}^{ab}\gamma_{ab} + \tilde{z}^{a'b'}\gamma_{a'b'})\right]\theta \\
\tilde{y}^{a} & = y^{a} + (\cosh x - 1)y^{a}\gamma_{a} , \quad \tilde{y}^{a'} = y^{a'} + (\cosh x' - 1)y^{b'}\gamma_{b'} \\
\tilde{z}^{ab} & = -x^{[a}y^{b]}\sinh x , \quad \tilde{z}^{a'b'} = x^{[a'}y^{b']\sinh x'.}
\end{align*}
\]

(A.3)
The subgroup parameter $h$’s are determined by

$$h^a_y = \delta_y x^m L^a_{m} - \Delta_y L^a_y, \quad h^{a'b'}_y = \delta_y x^m L^{a'b'}_{m} - \Delta_y L^{a'b'}$$

(A.4)

where $\delta_y x^m$ is determined independently on $h$’s because of $(L^{-1})_a^m = 0$. Using the above relations the transformation rules under the translation are obtained analogously to (3.4) by

$$\delta_y x^m = \left\{ \begin{array}{ll}
y^m + (\cosh x - 1)y^m \Upsilon^m_n, & \text{for } m, n = 0 \sim 4 \\
y^m + (\cos x' - 1)y^m \Upsilon^m_m, & \text{for } m, n = 5 \sim 9 \end{array} \right..$$

(A.5)

The Lorentz charge is given by

$$\frac{1}{2} \lambda^a \lambda^b J_{ab} = \int d\sigma[p_m \delta_x x^n + \zeta_\mu \delta_x \theta^\mu]. \quad (A.6)$$

The Lorentz variation leads to $\Delta_y L^A$ in (3.4) obtained as (A.2) where $\tilde{y}^a, \tilde{y}^{a'b'}, \tilde{\gamma}$ and the subscript $y$ are replaced by $\tilde{\lambda}^\hat{a}, \tilde{\lambda}^{\hat{a}'}, \tilde{\gamma}$ and a subscript $\lambda$ with

$$\tilde{\lambda}^\hat{a} = \lambda^a \frac{\sinh x}{x}, \quad \tilde{\lambda}^{\hat{a}'} = \lambda^{a'} \frac{\sin x'}{x'}. \quad \tilde{\lambda}^{a'b'} = \lambda^{a'b'} + \lambda^{a'b'} \cosh x - 1 \quad \lambda^{a'b'} + \lambda^{a'b'} \cosh x - 1$$

(A.7)

The Lorentz variation rules are obtained as

$$\left\{ \begin{array}{ll}
\delta_\lambda x^m = \lambda^m n_x^n \\
\delta_\lambda \theta^\mu = \frac{1}{2}(\gamma_{ab} \lambda^{ab} + \gamma_{a'b'} \lambda^{a'b'}) \theta \end{array} \right.. \quad (A.8)$$

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13