Radiation of the Inner Horizon of the Reissner-Nordström Black Hole

Ari Peltola and Jarmo Mäkelä

Department of Physics, University of Jyväskylä, PB 35 (YFL), FIN-40351 Jyväskylä, Finland

Despite of over thirty years of research of the black hole thermodynamics our understanding of the possible role played by the inner horizons of Reissner-Nordström and Kerr-Newman black holes in black hole thermodynamics is still somewhat incomplete: There are derivations which imply that the temperature of the inner horizon is negative and it is not quite clear what this means. Motivated by this problem we perform a detailed analysis of the radiation emitted by the inner horizon of the Reissner-Nordström black hole. As a result we find that in a maximally extended Reissner-Nordström spacetime virtual particle-antiparticle pairs are created at the inner horizon of the Reissner-Nordström black hole such that real particles with positive energy and temperature are emitted towards the singularity from the inner horizon and, as a consequence, antiparticles with negative energy are radiated away from the singularity through the inner horizon. We show that these antiparticles will come out from the white hole horizon in the maximally extended Reissner-Nordström spacetime, at least when the hole is near extremality. The energy spectrum of the antiparticles leads to a positive temperature for the white hole horizon. In other words, our analysis predicts that in addition to the radiation effects of black hole horizons, also the white hole horizon radiates. The black hole radiation is caused by the quantum effects at the outer horizon, whereas the white hole radiation is caused by the quantum effects at the inner horizon of the Reissner-Nordström black hole.

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I. INTRODUCTION

Hawking’s celebrated paper on black hole radiation came as a great surprise to almost everyone working in the field of general relativity. Till then it was strongly believed that black holes are totally black, i.e., no matter nor radiation can be emitted by black holes. In fact, this is what one would expect on the purely classical grounds. However, when one takes into account the quantum mechanical effects near the event horizon of a hole, one finds that the black hole does radiate thermal radiation with a spectrum similar to that of a black body.

One way to understand the origin of the radiation is to consider spontaneous particle-antiparticle pair production near the event horizon of a black hole. Normally, such a pair annihilates itself very rapidly. It is possible, however, that one of them—particle or antiparticle—is swallowed by the hole before the annihilation such that the other one is free to escape away from the hole. This event is illustrated in Fig. 1(a) in the case of a Schwarzschild black hole. It can be shown that as a net effect more antiparticles than particles fall through the horizon towards the singularity. Therefore an observer outside the hole, that is, at the region I of the Fig. 1(a), observes a particle flux which seems to come out from the black hole.

It is well known that the outer horizon of a Reissner-Nordström black hole radiates in a similar way as the event horizon of a Schwarzschild black hole. However, it is interesting to see what kind of phenomena is predicted by the virtual pair production mechanism if one looks at the inner horizon of the Reissner-Nordström black hole. Consider

*Electronic address: ari.peltola@phys.jyu.fi
†Electronic address: jarmo.makela@phys.jyu.fi
FIG. 1: (a) Conformal diagram of the maximally extended Schwarzschild spacetime. In this diagram the regions I and III represent spacetime surrounding the regions II (black hole) and IV (white hole). However, the regions I and III are causally separated. If a particle-antiparticle pair is spontaneously created near the event horizon of the hole in region I, it is possible that either a particle or an antiparticle is swallowed by the hole such that the other one is free to escape to the infinity at $\mathbb{I}^+$. (b) Maximally extended Reissner-Nordström spacetime. Similarly as in the Schwarzschild spacetime, a virtual pair created near the inner horizon $r = r_-$ may avoid annihilation if the particle and the antiparticle are separated by the horizon.

A maximally extended Reissner-Nordström spacetime (see Fig. 1(b)). It is easy to see that the causal relationship between the regions $V'$ and $IV'$ is similar to that between the regions I and II, respectively. Therefore, as shown in the Fig. 1(b), a virtual particle-antiparticle pair which emerges very close to the inner horizon $r = r_-$ in the region $V'$ can avoid annihilation if either the particle or the antiparticle falls into the region $IV'$ and the other one remains in $V'$. Therefore the pair production mechanism implies that the inner horizon does radiate and, moreover, that the radiation is directed inwards, towards the singularity. This line of reasoning, however, provides no information about the radiation itself. Especially, it remains unclear whether the inner horizon radiates particles or antiparticles.

After Hawking’s original work there have been various derivations of the Hawking effect with different physical assumptions [2]. Curiously, very little is known about the radiation of the inner horizons of the Reissner-Nordström and the Kerr-Newman black holes. This feature can, at least to some extent, be regarded as a consequence of the fact that inside the inner horizon there are no spacetime regions analogous to the regions $\mathbb{I}^+$ or $\mathbb{I}^-$. After all, Hawking’s original work was based on the analysis of the properties of the Klein-Gordon field at $\mathbb{I}^+$ and $\mathbb{I}^-$. To the best of our knowledge, the only explicit calculation considering the radiation of the inner horizons has been performed by Wu and Cai by means of the analytic continuation of the Klein-Gordon field [3]. However, as a result of their analysis they found that the temperature of the inner horizon is negative and this seems to contradict the general attitude towards the black hole thermodynamics [4], as well as the very foundations of thermodynamics itself. Thus, the true nature of the radiation of the inner horizon is still somewhat unclear.

The aim of this paper is to perform a detailed analysis of the radiation of the inner horizon of the Reissner-Nordström black hole. We would like to point out, however, that our analysis predicts very little astrophysical consequences because no mechanism for the formation of Reissner-Nordström black holes is known. One of the main reasons why the full Reissner-Nordström spacetime is not considered astrophysically real is the phenomenon called
mass inflation near the inner horizon of the Reissner-Nordström black hole. It is known from the works of Poisson and Israel that when one considers the spherical collapse of a charged star then, at least in a somewhat idealized situation, the flux of particles emitted by the collapsing star and its backscattered counterpart near the inner horizon of the Reissner-Nordström spacetime provoke an enormous inflation of the internal mass parameter of the black hole. Eventually the mass parameter becomes large enough to form a singularity at the inner horizon and in effect to freeze the evolution of spacetime. The inflation of the mass parameter at the inner horizon does not have any implications in the region outside the black hole since these regions are causally separated.

In this paper we take the maximally extended Reissner-Nordström spacetime as the starting point of our analysis. We would like to emphasize that here the Reissner-Nordström spacetime is considered only as a mathematical solution to the combined Maxwell-Einstein field equations, and the whole problem concerning the formation of such a spacetime is completely ignored. One could ask, of course, why are we interested in the properties of the full Reissner-Nordström spacetime if it is not considered astrophysically relevant. The answer to this question lies in the fact that Reissner-Nordström spacetime provides an explicit example of a spacetime geometry which contains two horizons, of which one is hidden from the outside observer. The problem we are interested in is the following: Does only one of the horizons emit Hawking radiation, as it is generally believed, or do both of the horizons radiate? This is an intriguing question, and if one is able to show that both of the horizons radiate, then this result may be seen to support the idea that all horizons of spacetime emit radiation. It is possible that this idea, if true, may provide useful clues in the search for quantum gravity. The main object of interest in our paper is therefore not the Reissner-Nordström black hole itself, but the general semi-classical properties of gravity. One may also hope that our results are qualitatively the same for the more realistic Kerr black hole solution. Indeed, this might be the case because the causal structures of the Reissner-Nordström and the Kerr spacetimes are very similar. In that case the radiation of the inner horizon may cause even certain astrophysical effects since there is not necessarily mass inflation in the Kerr spacetime.

If the inner horizon of the Reissner-Nordström black hole really radiates, it is expected that this effect takes place during a very short time only. To see this, suppose that we begin with the purely classical Reissner-Nordström solution with two horizons, and apply the results of quantum field theory in the Reissner-Nordström spacetime. If, in the semiclassical limit, we find that the inner horizon radiates, then the backscattered part of that radiation is enough to trigger the mass inflation, and eventually the inner horizon disappears. In other words, if one is able to show that the inner horizon of the Reissner-Nordström black hole radiates, then it turns out that semiclassically the full Reissner-Nordström spacetime, even as a mathematical solution, is unstable. The radiation process of the inner horizon, however, should last as long as the inner horizon exists. We shall not discuss the backscattering effect and its consequences in more detail but we shall confine ourselves merely to the qualitative aspects discussed above. This approach is justified because we are interested in the radiation effects of the inner horizon of the Reissner-Nordström spacetime when the effects of mass inflation are still negligible.

In brief, the key points of our discussion can be expressed as follows: When the ideas of Hawking’s original work are utilized in Rindler spacetime, one recalls that the so-called Unruh effect, which is closely related to the Hawking effect, can be obtained by simply comparing the solutions to the Klein-Gordon equation for massless particles from the points of views of inertial and uniformly accelerated observers. As a preliminary we show this in Sec. III. In curved spacetime, however, the concept of inertial observer is replaced by the concept of freely falling observer. Inspired by this analogy we proceed to calculate the effective temperature of black hole horizons by comparing the solutions to the massless Klein-Gordon equation from the points of views of an observer in a radial free fall and an observer at rest with respect to the horizon. First, in Sec. III we perform, as an example, an analysis of the radiation of the outer horizon of a Reissner-Nordström black hole. After reproducing the well-known results, we proceed, in Sec. IV, to calculate the temperature of the radiation emitted by the inner horizon. In contrast to Wu and Cai, we find that the effective temperature for particles radiating from the inner horizon towards the singularity is not negative but positive: The inner horizon emits real particles with positive energy and temperature. To maintain the local energy
balance it is therefore necessary that the inner horizon emits antiparticles with negative energy in the direction away from the singularity. If one looks at the conformal diagram of a maximally extended Reissner-Nordström spacetime, one may speculate on the possibility that if the backscattering effects are neglected, the antiparticles emitted away from the singularity by the inner horizon will go through the intermediate region between the horizons, and finally they will come out through the white hole horizon. Motivated by this conjecture, we perform a detailed analysis of the antiparticle modes, and find that the antiparticles indeed come out of the white hole—at least when the black hole is almost extreme. In other words, our analysis predicts a new effect for maximally extended Reissner-Nordström spacetimes which we shall call “white hole radiation”. In the same way as is black hole radiation a consequence from the quantum effects at the outer horizon of the Reissner-Nordström black hole, the white hole radiation is a consequence from the quantum effects at the inner horizon of the hole. The black and the white hole radiations are separate and simultaneously ongoing processes in spacetimes containing a Reissner-Nordström black hole, and an observer situated at the exterior region of a Reissner-Nordström black hole observes the both types of radiation. An existence of the white hole radiation is the main result of this paper, and it seems that the same result holds also for Kerr-Newman black holes. We end our discussion in Sec. V with some concluding remarks.

II. PRELIMINARIES: THE UNRUH EFFECT

As a starting point of our analysis let us consider the thermal radiation of the Rindler horizon found by Unruh in 1976 [6]. Rindler horizons are such horizons of spacetime which appear in the rest frame of a uniformly accelerated observer. In general, the equation of the world line of a uniformly accelerated observer in flat two-dimensional Minkowski spacetime is (unless otherwise stated we shall always have $c = G = \hbar = k_B = 1$) [7]:

$$X^2 - T^2 = \frac{1}{a^2} \tag{2.1}$$

where $a$ is the proper acceleration of the observer, and $X$ and $T$, respectively, are the minkowskian space and time coordinates. The world line of the observer may be written in the parametrized form:

$$T(\eta) = \frac{1}{a} \sinh(a\eta), \quad (2.2a)$$

$$X(\eta) = \frac{1}{a} \cosh(a\eta). \quad (2.2b)$$

In this expression, $\eta$ is the proper time of the observer. If we define the Rindler coordinates $t$ and $x$ such that

$$x := \frac{1}{a}, \quad (2.3a)$$

$$t := a\eta, \quad (2.3b)$$

we can write the metric of two-dimensional Minkowski spacetime as

$$ds^2 = -x^2 dt^2 + dx^2. \quad (2.4)$$

The world line of a uniformly accelerated observer has been drawn in Fig. 2. In that figure we may also see the Rindler horizon of the accelerated observer. From the figure we can also see the four regions of Rindler spacetime, labelled as I, II, III and IV. Since this diagram is very similar to the Kruskal diagram of Schwarzschild spacetime, one would expect Rindler spacetime to have physical properties similar to those of Schwarzschild spacetime. In fact, it is
FIG. 2: Rindler spacetime. The curve C represents the worldline of a uniformly accelerated observer.

easy to see that the causal features of the regions II and IV are, respectively, similar to those of a black and a white hole.

The simplest way to obtain the Unruh effect is probably the following: Consider the Klein-Gordon equation of massless particles in the rest frame of an accelerated observer. In general that equation may be written as

$$\gamma^{\mu\nu} D_{\mu} D_{\nu} \phi = 0,$$

where $D_{\mu}$ denotes covariant derivative, and when spacetime metric is that of Eq. (2.4), Eq. (2.5) takes the form

$$\left( -\frac{1}{x^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} \right) \phi = 0.$$

If one defines

$$x^* := \ln x,$$

Eq. (2.6) becomes to:

$$\left( -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^*^2} \right) \phi = 0.$$

Orthonormal solutions to this equation are of the form

$$u_\omega = N_\omega e^{-i\omega U},$$

where $N_\omega$ is an appropriate normalization constant, and we denote

$$U := t - x^*.$$

These solutions represent, from the point of view of an accelerated observer in the region I, particles with energy $\omega$ propagating to the positive $X$-direction. In contrast, the corresponding solutions to the massless Klein-Gordon equation

$$\left( -\frac{\partial^2}{\partial T^2} + \frac{\partial^2}{\partial X^2} \right) \phi = 0,$$

written from the point of view of an inertial observer at rest with respect to the Minkoski coordinates $T$ and $X$, are of the form:

$$u'_\omega = N_\omega e^{-i\omega \tilde{a}},$$
where

\[ \tilde{u} := T - X. \]  

(2.13)

Again, these solutions represent particles with energy \( \omega \) propagating to the positive \( X \)-direction.

It is easy to see from Eqs. (2.2) and (2.10) that

\[ U = -\ln(-\tilde{u}), \]  

(2.14)

and therefore the Bogolubov transformation

\[ u_\omega = \sum_{\omega'} (A'_{\omega\omega'} u'_{\omega'} + B'_{\omega\omega'} u'_{\omega'}) \]  

(2.15)

between the orthonormal solutions \( u_\omega \) and \( u'_{\omega'} \) may be written as

\[ e^{i\omega \ln(-\tilde{u})} = \sum_{\omega'} (A'_{\omega\omega'} e^{-i\omega' \tilde{u}} + B'_{\omega\omega'} e^{i\omega' \tilde{u}}), \]  

(2.16)

where the Bogolubov coefficients \( A'_{\omega\omega'} \) and \( B'_{\omega\omega'} \) are expressible as Fourier integrals:

\[ A'_{\omega\omega'} = \frac{1}{2\pi} \int_{-\infty}^{0} d\tilde{u} e^{i\omega \ln(-\tilde{u})} e^{i\omega' \tilde{u}}, \]  

(2.17a)

\[ B'_{\omega\omega'} = \frac{1}{2\pi} \int_{-\infty}^{0} d\tilde{u} e^{i\omega \ln(-\tilde{u})} e^{-i\omega' \tilde{u}}. \]  

(2.17b)

The integration is performed from the negative infinity to zero because we are considering particles in the region I, and in that region \( \tilde{u} < 0 \). It is straightforward to show, by performing the integration in the complex plane, that

\[ |A'_{\omega\omega'}| = e^{\pi\omega} |B'_{\omega\omega'}| \]  

(2.18)

(see Fig. 3). Because of the well-known relation between the Bogolubov coefficients,

\[ \sum_{\omega'} \left( |A'_{\omega\omega'}|^2 - |B'_{\omega\omega'}|^2 \right) = 1, \]  

(2.19)

one finds that when the field is in vacuum from the point of view of an inertial observer, the number of particles with energy \( \omega \) is, from the point of view of an accelerated observer,

\[ n_\omega = \sum_{\omega'} |B'_{\omega\omega'}|^2 = \frac{1}{e^{\pi\omega} - 1}. \]  

(2.20)

This is the Planck spectrum at the temperature \( T_0 = \frac{1}{2\pi} \), which is related to the temperature experienced by an observer situated at a given point in space by the Tolman relation [8]:

\[ T = (g_{00})^{-\frac{1}{2}} T_0. \]  

(2.21)

Hence it follows that a uniformly accelerated observer observes particles coming out from the Rindler horizon with the black-body spectrum corresponding to the characteristic temperature

\[ T_U := \frac{1}{2\pi x} = \frac{a}{2\pi}, \]  

(2.22)

even when, from the point of view of an inertial observer, the field is in vacuum. This result is known as the Unruh effect, and it is one of the most remarkable outcomes of quantum field theory.
FIG. 3: Integration contours in the complex plane. In this figure \( \gamma_+ \) and \( \gamma_- \) are closed contours circulating the shaded regions in the upper and the lower half of the complex plane, respectively. When the integral in Eq. is calculated along the contour \( \gamma_+ \), one easily sees that, in the limit where \( R \to \infty \) and \( r \to 0 \), the integrals along the arcs of the circles vanish. The analyticity of the functions under consideration in the shaded regions implies that the contour integral around \( \gamma_+ \) vanishes, and therefore the integral from negative infinity to zero along the real axis may be transformed into an integral from positive infinity to zero along the imaginary axis. Similar result holds for the integral in Eq. along the path \( \gamma_- \), except that now the integral from negative infinity to zero along the real axis may be transformed to an integral from negative infinity to zero along the imaginary axis. The integrals along the imaginary axis lead directly to Eq. 

**III. RECONSIDERATION OF THE HAWKING EFFECT FOR THE OUTER HORIZON OF A REISSNER-NORDSTRÖM BLACK HOLE**

The ideas inspired by the properties of Rindler spacetime can be easily utilized when one investigates the thermal properties of Reissner-Nordström horizons in the following manner: At first one constructs a certain *geodesic system of coordinates* for the neighborhood of the horizon under scrutiny. The geodesic coordinates are constructed such that an observer in a *radial* free fall remains at rest with respect to those coordinates. Such an observer does not observe the horizon, and therefore one expects that no radiation effects should be experienced by him. Because of that one may view the particle vacuum of the freely falling observer as the vacuum that would exist in spacetime if there were no horizon at all. In this sense the observer in a radial free fall is analogous to the inertial observer in flat spacetime, and we take this similarity as a starting point of our analysis.

To calculate the particle flux emitted by the horizon one compares the solutions to the massless Klein-Gordon equation very close to the horizon in two different coordinate systems. One of these systems is the geodesic system of coordinates and the other one is a coordinate system at rest with respect to the horizon. The analysis of the Klein-Gordon modes must be performed infinitesimally close to the horizon for the very reason that only in that case one is able to solve the Klein-Gordon equation analytically (excluding, of course, the solutions at the asymptotic infinities). Then one can obtain the Bogolubov transformations between these solutions and infer the effective temperature of the radiation flux from the point of view of an observer at rest with respect to the horizon.

To see what all this really means consider, as an example, the outer horizon of a Reissner-Nordström black hole.
The Reissner-Nordström metric can be written as

\[
ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2),
\]

(3.1)

where \(M\) is the mass and \(Q\) is the electric charge of the hole. In addition to the physical singularity at \(r = 0\), this metric has two coordinate singularities when

\[
r = r_{\pm} := M \pm \sqrt{M^2 - Q^2}.
\]

(3.2)

The two-surfaces where \(r = r_+\) and \(r = r_-\) are called, respectively, the outer and the inner horizons of the Reissner-Nordström black hole. It is well known that when \(r > r_+\), and the backscattering effects are neglected, an observer at rest with respect to the coordinates \(r, \theta, \text{and } \varphi\) observes thermal radiation emitted by the hole with a characteristic temperature

\[
T_+ := \frac{\kappa_+}{2\pi \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}} = \frac{\sqrt{M^2 - Q^2}}{2\pi (M^2 + \sqrt{M^2 - Q^2})^2 \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}},
\]

(3.3)

where

\[
\kappa_+ := \frac{r_+ - r_-}{2r_+^2}
\]

(3.4)

is the surface gravity of the outer horizon. The factor \((1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1/2}\) is due to the “redshift” of the radiation. At asymptotic infinity the redshift factor is equal to one, but at the horizon \(r = r_+\) it becomes infinitely large. In other words, an observer at rest very close to the horizon may measure an infinite temperature for the black hole radiation.

We shall now show how Eq. (3.3) may be obtained by means of the method we explained at the beginning of this section. As the first step we separate the Klein-Gordon field \(\phi\) of massless particles such that

\[
\phi(t, r, \theta, \varphi) = \frac{1}{r} f(t, r) Y_{lm}(\theta, \varphi),
\]

(3.5)

where \(Y_{lm}\) is the spherical harmonic function satisfying the differential equation

\[
\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta}\right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2}\right] Y_{lm} = -l(l+1) Y_{lm},
\]

(3.6)

where, as usual, the allowed values of \(l\) are 0, 1, 2, \(\cdots\), and those of \(m\) are 0, \(\pm 1, \pm 2, \cdots, \pm l\). In that case the massless Klein-Gordon equation, when written in terms of the coordinates \(t, r, \theta, \text{and } \varphi\), implies that

\[
\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} - V(r)\right] f = 0,
\]

(3.7)

where we have defined the “tortoise coordinate” \(r_+\) such that

\[
r_+ := \int \frac{dr}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} = r - \frac{r_+^2}{r_+ - r_-} \ln |r - r_-| + \frac{r_-^2}{r_+ - r_-} \ln |r - r_+|,
\]

(3.8)

and the “potential term”

\[
V(r) := \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) \left[\frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4}\right].
\]

(3.9)

Very close to the horizon, where

\[
\Delta := \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}
\]

(3.10)
is infinitesimally small, the potential \( V(r) \) vanishes, and the solutions to Eq. (3.7) corresponding to the particles with energy \( \omega \) moving towards the horizon are of the form

\[
f(t, r) \sim e^{-i\omega V},
\]

and for the solutions coming outwards from the horizon we have

\[
f(t, r) \sim e^{-i\omega U},
\]

where the coordinates \( V \) and \( U \) are the advanced and the retarded coordinates defined as:

\[
V = t + r_*, \tag{3.13a}
\]

\[
U = t - r*. \tag{3.13b}
\]

Therefore, from the point of view of the observer at rest very close to the horizon, the ingoing and the outcoming solutions to the massless Klein-Gordon equation are, respectively,

\[
\phi_{\text{in}} \approx N_{\omega l m} Y_{lm} \frac{1}{r} e^{-i\omega V}, \tag{3.14a}
\]

\[
\phi_{\text{out}} \approx N_{\omega l m} Y_{lm} \frac{1}{r} e^{-i\omega U}, \tag{3.14b}
\]

where \( N_{\omega l m} \) is an appropriate normalization constant.

Consider now an observer in a radial free fall in the region I of the Reissner-Nordstr"om spacetime, infinitesimally close to the point \( P_1 \) in Fig. 4. In other words, the observer is in a radial free fall just outside the outer horizon. At first let us introduce in the Reissner-Nordstr"om spacetime the coordinates \((u, v)\) which are similar to the Kruskal coordinates in Schwarzschild spacetime. In general, these “Kruskal-type coordinates” may be defined in the different
regions of the Reissner-Nordström spacetime such that
\[
\begin{align*}
\{ u = \frac{1}{2} (e^{\alpha V} + e^{-\alpha U}), \\
v = \frac{1}{2} (e^{\alpha V} - e^{-\alpha U}), \\
\} & \quad \text{(Region I, I', \cdots)} \\
\{ u = \frac{1}{2} (e^{\alpha V} - e^{-\alpha U}), \\
v = \frac{1}{2} (e^{\alpha V} + e^{-\alpha U}), \\
\} & \quad \text{(Region II, II', \cdots)} \\
\{ u = -\frac{1}{2} (e^{\alpha V} + e^{-\alpha U}), \\
v = -\frac{1}{2} (e^{\alpha V} - e^{-\alpha U}), \\
\} & \quad \text{(Region III, III', \cdots)} \\
\{ u = -\frac{1}{2} (e^{\alpha V} - e^{-\alpha U}), \\
v = -\frac{1}{2} (e^{\alpha V} + e^{-\alpha U}), \\
\} & \quad \text{(Region IV, IV', \cdots)}
\end{align*}
\]
where \( \alpha \) is an appropriate constant. When we study the physical properties of the outer horizon, the constant \( \alpha \) must be chosen such that the metric on the two-surface \( r = r_+ \) is regular. The most natural choice is
\[
\alpha = \kappa_+,
\]
and this choice leads to the non-singular metric
\[
ds^2 = \frac{1}{\kappa_+^2 r^2} e^{-2\kappa_+ r (r - r_-)} \left[ \frac{r^2}{r_+^2} + 1 \right] (-dv^2 + du^2) + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).
\] (3.17)

It is now easy to construct a geodesic coordinate system for an infinitesimal neighborhood \( \mathcal{U}(P_1) \) of the point \( P_1 \). By infinitesimal geodesic coordinate system we mean coordinates \( X^I \) \((I = 0, 1, 2, 3)\) in \( \mathcal{U}(P_1) \) such that, at the point \( P_1 \), the metric takes the form of that of flat spacetime, i.e.,
\[
ds^2 = \eta_{IJ} dX^I dX^J,
\] (3.18)
where \( \eta_{IJ} = \text{diag}(-1, 1, 1, 1) \) is the flat Minkowski metric and the derivatives of the metric vanish. Let us define the coordinates
\[
X^0 := l_+ v, 
\] (3.19a)
\[
X^1 := l_+ u, 
\] (3.19b)
where
\[
l_+ := \frac{1}{\kappa_+ r_+} e^{-\kappa_+ r_+ (r_+ - r_-)} \left[ \frac{r_+^2}{r_+^2} + 1 \right].
\] (3.20)
By using these definitions one finds that at the point \( P_1 \) the metric can be written as
\[
ds^2 = -(dX^0)^2 + (dX^1)^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2),
\] (3.21)
and the derivatives of the metric with respect to \( X^0 \) and \( X^1 \) vanish (for details, see Appendix). Therefore, the geodesic coordinates of the freely falling observer can be chosen to be \( X^0 \) and \( X^1 \). (Note that even though the above metric is not strictly of the form of Eq. (3.18), for the observer in a radial free fall these coordinates provide a geodesic coordinate system since in that case \( \theta \) and \( \varphi \) are constants.)

If the massless Klein-Gordon field is now separated such that
\[
\phi(X^0, X^1, \theta, \varphi) = \frac{1}{r} \tilde{f}(X^0, X^1) Y_{lm}(\theta, \varphi),
\] (3.22)
the Klein-Gordon equation, when written in terms of the coordinates $X^0, X^1, \theta, \varphi$, implies:

$$
-\frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} + \frac{1}{r} \left( \frac{\partial^2 r}{\partial (X^0)^2} - \frac{\partial^2 r}{\partial (X^1)^2} \right) - \frac{l(l+1)}{r^2} F_+(r) \tilde{f}(X^0, X^1) = 0,
$$

(3.23)

where we have denoted:

$$
F_+(r) := \frac{1}{\kappa_+^2 r^2} e^{-2\kappa_+ (r - r_-)} r^{l+1}.
$$

(3.24)

It follows from Eqs. (3.19), (A.5) and (A.6) that

$$
\frac{\partial r}{\partial X^0} = -\kappa_+ F_+(r) X^0,
$$

(3.25a)

$$
\frac{\partial r}{\partial X^1} = \kappa_+ F_+(r) X^0,
$$

(3.25b)

and therefore:

$$
\frac{\partial^2 r}{\partial (X^0)^2} - \frac{\partial^2 r}{\partial (X^1)^2} = \kappa_+ F_+(r) \left\{ \kappa_+ F_+(r) \left[ (X^0)^2 - (X^1)^2 \right] - 2 \right\},
$$

(3.26)

where the prime means derivative with respect to $r$. Using Eqs. (A.4)–(A.6) and (3.8) one finds:

$$
F_+(r) \left[ (X^0)^2 - (X^1)^2 \right] = \frac{2}{\kappa_+^2} + \frac{1}{\kappa_+^2} \left( \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right),
$$

(3.27)

and therefore Eq. (3.23) takes the form:

$$
\left[ -\frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} - \tilde{V}(r) \right] \tilde{f}(X^0, X^1) = 0,
$$

(3.28)

where the “potential term” is

$$
\tilde{V}(r) := \left[ -\frac{2M}{r^3} + \frac{2Q^2}{r^4} + \frac{l(l+1)}{r^2} \right] F_+(r).
$$

(3.29)

The function $F_+(r)$ has the property

$$
F_+(r_+) = 1,
$$

(3.30)

and therefore Eq. (3.28) takes, at the outer horizon of the Reissner-Nordström black hole, the form:

$$
\left[ -\frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} + \frac{2M}{r_+^3} - \frac{2Q^2}{r_+^4} - \frac{l(l+1)}{r_+^2} \right] \tilde{f}(X^0, X^1) = 0.
$$

(3.31)

So we see that, in contrast to Eq. (3.7), the “potential term” does not vanish at the horizon. For a macroscopic hole, however, the “potential term” may be neglected: For Reissner-Nordström black holes $r_+ \geq M$ and $0 \leq |Q| \leq M$, and so it follows that

$$
\left| \frac{2M}{r_+} - \frac{2Q^2}{r_+^2} \right| \leq \frac{2}{M^2},
$$

(3.32)
which means that when, in Planck units, $M \gg 1$, the terms involving $M$ and $Q$ will vanish. Moreover, if the orbital angular momentum $l$ of the Klein-Gordon particle is sufficiently small, we may neglect the term $l(l+1)/r^2$. In other words, we may write Eq. (3.31), in effect, as:

$$\left[-\frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2}\right] \tilde{f}(X^0, X^1) = 0. \quad (3.33)$$

For very small $l$ the ingoing and the outcoming solutions to the massless Klein-Gordon equation very close to the horizon $r = r_+$ are:

$$\phi'_{\text{in}} \approx N_{\omega lm} Y_{lm} \frac{1}{r} e^{-i\omega \tilde{v}}, \quad (3.34a)$$

$$\phi'_{\text{out}} \approx N_{\omega lm} Y_{lm} \frac{1}{r} e^{-i\omega \tilde{u}}, \quad (3.34b)$$

where

$$\tilde{v} = X^0 + X^1, \quad (3.35a)$$

$$\tilde{u} = X^0 - X^1, \quad (3.35b)$$

and $N_{\omega lm}$ is the normalization constant corresponding to the fixed values of $l$, $m$, and $\omega$. These solutions represent, from the point of view of the freely falling observer, particles with energy $\omega$ moving towards and out of the horizon, respectively. From Eqs. (3.13b), (3.19), and (3.35b) one easily finds that in the region $I$

$$U = -\kappa_+^{-1} \ln(-\tilde{u}) + \kappa_+^{-1} \ln \kappa_+ l_+. \quad (3.36)$$

Thus, in the spherical symmetric case, the Bogolubov transformation between the outcoming modes in Eqs. (3.14b) and (3.34b) can be written in the form

$$e^{i\omega \kappa_+^{-1} \ln(-\tilde{u})} e^{-i\omega \kappa_+^{-1} \ln \kappa_+ l_+} = \sum_{\omega'} \left(A'_{\omega \omega'} e^{-i\omega' \tilde{u}} + B'_{\omega \omega'} e^{i\omega' \tilde{u}}\right), \quad (3.37)$$

and, moreover, we can express the Bogolubov coefficients $A'_{\omega \omega'}$ and $B'_{\omega \omega'}$ as Fourier integrals such that

$$A'_{\omega \omega'} = \frac{1}{2\pi} e^{-i\omega \kappa_+^{-1} \ln \kappa_+ l_+} \int_{-\infty}^0 d\tilde{u} e^{i\omega \kappa_+^{-1} \ln(-\tilde{u})} e^{i\omega' \tilde{u}}, \quad (3.38a)$$

$$B'_{\omega \omega'} = \frac{1}{2\pi} e^{-i\omega \kappa_+^{-1} \ln \kappa_+ l_+} \int_{-\infty}^0 d\tilde{u} e^{i\omega \kappa_+^{-1} \ln(-\tilde{u})} e^{-i\omega' \tilde{u}}. \quad (3.38b)$$

As in the previous section, the integration is performed from the negative infinity to zero because we are considering particles in the region $I$, and in that region $\tilde{u} < 0$.

The integrals in Eqs. (3.38) are similar to those found in Eqs. (2.17), and the integration in the complex plane gives

$$|A'_{\omega \omega'}| = e^{\pi \kappa_+^{-1} \omega} |B'_{\omega \omega'}|. \quad (3.39)$$
Therefore, by using Eq. (2.19), we find that when the field is in vacuum from the point of view of a freely falling observer, the number of the particles with energy $\omega$ observed by an observer at rest very close to the horizon is

$$n_\omega = \sum_{\omega'} |B'_{\omega\omega'}|^2 = \frac{1}{e^{2\pi \kappa_+ \omega} - 1}. \quad (3.40)$$

This is the Planck spectrum at the temperature

$$T = \frac{\kappa_+}{2\pi}, \quad (3.41)$$

which represents the temperature of the outer horizon experienced by an observer at rest with respect to the horizon when the redshift effects of the radiation are ignored. The redshift factor can be recovered by the Tolman relation (2.21), and as a result we find that, from the point of view of an observer at rest very close to the outer horizon, the Reissner-Nordström black hole emits radiation with a characteristic temperature

$$T_+ = (g_{00})^{-\frac{1}{2}} \frac{\kappa_+}{2\pi} = \frac{\kappa_+}{2\pi \Delta} = \frac{\sqrt{M^2 - Q^2}}{2\pi r^2 + \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}}, \quad (3.42)$$

which is Eq. (3.3). In other words, we have shown that our method, which relies on the comparison of the solutions to the Klein-Gordon equation from the points of views of two observers close to the horizons, reproduces the familiar result which is usually obtained by means of the comparison of the solutions to the Klein-Gordon equation at $\Im^+$ and $\Im^-$. Encouraged by this welcome outcome of our analysis we now proceed to apply our method for an analysis of the properties of the inner horizon of the Reissner-Nordström black hole.

IV. HAWKING EFFECT FOR THE INNER HORIZON OF THE REISSNER-NORDSTRÖM BLACK HOLE

An analysis of the radiation emitted by the inner horizon of a Reissner-Nordström black hole can now be performed in a very similar way as that of the outer horizon. In essence, the ingoing and the outcoming solutions to the Klein-Gordon equation, when written in terms of the coordinates $t$, $r$, $\theta$, and $\varphi$, can be obtained directly from Eqs. (3.14). However, since the radiation is now directed towards the singularity $r = 0$, the observer at rest with respect to the inner horizon must be situated inside the two-sphere $r = r_-$. Therefore the roles of the ingoing and the outcoming modes interchange. More precisely, the solutions representing particles with energy $\omega$ are, from the point of view of the observer at rest very close to the inner horizon,

$$\phi_{\text{in}} \approx N_{\omega l m} Y_{l m} \frac{1}{r} e^{-i\omega U}, \quad (4.1a)$$

$$\phi_{\text{out}} \approx N_{\omega l m} Y_{l m} \frac{1}{r} e^{-i\omega V}. \quad (4.1b)$$

The solution $\phi_{\text{in}}$ represents a particle which moves towards the horizon, and therefore away from the singularity. The solution $\phi_{\text{out}}$, in turn, represents a particle which moves out of the horizon, and therefore towards the singularity.

As it comes to the freely falling observer near the inner horizon, we cannot use the same geodesic coordinate system as we did in the previous section. This is a consequence of the fact that the Kruskal-type coordinates $u$ and $v$ of Eqs. (3.15) with the choice $\alpha = \kappa_+$ lead to the metric which is not regular at $r = r_-$. A remedy to this problem can be obtained by choosing

$$\alpha = \kappa_- := \frac{r_+ - r_-}{2r_-} \quad (4.2)$$
and defining a new geodesic system of coordinates based on this choice. Consider now an observer in a radial free fall in the region VI’ of Reissner-Nordström spacetime infinitesimally close to the point $P_2$ (see Fig. 5). When written in terms of the coordinates $u$ and $v$, the spacetime metric takes the form:

$$ds^2 = \frac{1}{\kappa^2 r^2} e^{-2\kappa - r} \left( \frac{r^2}{2} + 1 \right) \left( -du^2 + dv^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \right). \tag{4.3}$$

In this expression the coordinates $u$ and $v$ have been defined in such a way that in the regions V’, IV’, VI’ and II of Fig. 1(b), respectively, the coordinates $u$ and $v$ are given in terms of $U$ and $V$ by Eqs. (3.15a), (3.15b), (3.15c), and (3.15d). One may also easily check that in the region VI’ the coordinates $u$ and $v$ are increasing functions of $r$ and $t$, respectively. More precisely, when $u$ is taken to be a constant, the coordinate $v$ increases as a function of $t$, whereas the coordinate $u$ increases as a function of $r$, when $v$ is constant.

Similarly as in the case of the outer horizon, we define a geodesic system of coordinates for an infinitesimal neighborhood of the point $P_2$ such that

$$X^0 := l_- v, \tag{4.4a}$$

$$X^1 := l_- u, \tag{4.4b}$$

where

$$l_- := \frac{1}{|\kappa - r^-|} e^{-\kappa - r^-} \left( r^++ r^- \right) \frac{1}{2} \left( \frac{r^2}{2} + 1 \right). \tag{4.5}$$

When the remaining coordinates are chosen to be the spherical coordinates $\theta$ and $\varphi$, the metric is given by Eq. (3.21), and the derivatives of the metric vanish when $r = r_-$ (see Appendix). Therefore the coordinates $X^0$ and $X^1$ provide a geodesic system of coordinates. Furthermore, when the massless Klein-Gordon field is separated as in Eq. 3.22, one finds that the massless Klein-Gordon equation implies:

$$\left[ - \frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} + \left( \frac{2M}{r^2} - \frac{2Q^2}{r^4} - \frac{l(l + 1)}{r^2} \right) F_- (r) \right] \tilde{f} (X^0, X^1) = 0, \tag{4.6}$$

where we have defined:

$$F_- (r) := \frac{1}{\kappa^2 r^2} e^{-2\kappa - r} \left( r^2 + 1 \right). \tag{4.7}$$
Again, one finds that
\[ F_-(r_-) = 1, \]  
(4.8)
and therefore Eq. (4.6) takes, at the point \( P_2 \), the form
\[
\left[ - \frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} + \frac{2M}{r_-^2} - \frac{2Q^2}{r_-^4} - \frac{l(l + 1)}{r_-^2} \right] \tilde{f}(X^0, X^1) = 0. \]  
(4.9)

The question about whether the terms involving \( r_- \) are negligibly small or not, is a very delicate one, and when the absolute value of the electric charge \( Q \) is very small, those terms will certainly not vanish. We may, however, consider a special case where \( |Q| \) is “reasonably big”. More precisely, we shall assume that there is a fixed positive number \( \gamma \leq 1 \) such that between \( |Q| \) and \( M \) there is, in Planck units, the relationship:
\[ |Q| = \gamma M. \]  
(4.10)

In that case
\[ r_- = (1 - \sqrt{1 - \gamma^2})M, \]  
(4.11)
and because
\[ \sqrt{1 - \gamma^2} = 1 - \frac{1}{2} \gamma^2 - \frac{1}{8} \gamma^4 - \frac{1}{16} \gamma^6 - \ldots < 1 - \frac{1}{2} \gamma^2, \]  
(4.12)
we find that
\[
\left| \frac{2M}{r_-^2} - \frac{2Q^2}{r_-^4} \right| < \frac{2M}{r_-^2} + \frac{2Q^2}{r_-^4} < \frac{48}{\gamma^6 M^2}. \]  
(4.13)

Hence it follows that if \( \gamma \) is “reasonably big”, and \( M \gg 1 \) in Planck units, the terms involving \( M \) and \( Q \) are negligible. The same line of reasoning implies that for “sufficiently small” \( l \) the term \( l(l + 1)/r_-^2 \) may be neglected, and Eq. (4.9) may be written, in effect, in the form:
\[
\left[ - \frac{\partial^2}{\partial (X^0)^2} + \frac{\partial^2}{\partial (X^1)^2} \right] \tilde{f}(X^0, X^1) = 0. \]  
(4.14)

One easily sees that in the region \( VI' \) the solutions corresponding to the particles with energy \( \omega \) going in and out of the horizon are
\[
\phi'_\text{in} \approx N_{\omega lm} Y_{lm} \frac{1}{r} e^{-i\omega \tilde{v}}, \]  
(4.15a)
\[
\phi'_\text{out} \approx N_{\omega lm} Y_{lm} \frac{1}{r} e^{-i\omega \tilde{u}}, \]  
(4.15b)
where \( \tilde{v} \) and \( \tilde{u} \) are defined as in Eqs. (3.35).

It is now possible to write the Bogolubov transformation between the outcoming solutions (4.15) and (4.15a) in the spherical symmetric case. One easily finds that in the region \( VI' \),
\[
V = \kappa_-^{-1} \ln(-\tilde{v}) - \kappa_-^{-1} \ln l_- \]  
(4.16)
Moreover, the Bogolubov transformation takes the form
\[
e^{-i\omega \kappa_-^{-1} \ln(-\tilde{v})} e^{i\omega \kappa_-^{-1} \ln l_-} = \sum_{\omega'} \left( A'_{\omega \omega'} e^{-i\omega' \tilde{v}} + B'_{\omega \omega'} e^{i\omega' \tilde{v}} \right), \]  
(4.17)
and the Bogolubov coefficients can be expressed as:

\[ A_{\omega\omega'} = \frac{1}{2\pi} e^{i\omega\kappa^{-1} \ln l_-} \int_{-\infty}^{0} d\tilde{v} e^{-i\omega\kappa^{-1} \ln(-\tilde{v})} e^{i\omega'\tilde{v}}, \]  

(4.18a)

\[ B_{\omega\omega'} = \frac{1}{2\pi} e^{i\omega\kappa^{-1} \ln l_-} \int_{-\infty}^{0} d\tilde{v} e^{-i\omega\kappa^{-1} \ln(-\tilde{v})} e^{-i\omega'\tilde{v}}. \]  

(4.18b)

As before, these integrals yield the result

\[ |A_{\omega\omega'}| = e^{-\pi\kappa^{-1} \omega} |B_{\omega\omega'}|. \]  

(4.19)

Therefore, by using Eq. (2.19), we see that the number of particles with energy \( \omega \) is, from the point of view of the observer at rest very close to the horizon \( r = r_- \), when, from the point of view of the freely falling observer, the field is in vacuum,

\[ n_\omega = \sum_{\omega'} |B_{\omega\omega'}|^2 = \frac{1}{e^{-2\pi\kappa^{-1} \omega} - 1}. \]  

(4.20)

From this distribution one may infer that the temperature concerning the particle radiation emitted by the inner horizon is, when the redshift effects are ignored,

\[ T = \frac{\kappa_-}{2\pi}, \]  

(4.21)

which is positive.

The result in Eq. (4.21) is in agreement with the findings of Ref. [4]. Since the temperature is positive, there are no interpretative problems concerning the thermodynamical properties of the radiation of the inner horizon. Again, the redshift factor can be recovered by using Eq. (2.21), and as the result one finds that the temperature, from the point of view of an observer at rest very close to the inner horizon, is

\[ T_- := \frac{-\kappa_-}{2\pi\Delta} = \frac{\sqrt{M^2 - Q^2}}{2\pi r_-^2 \sqrt{1 - \frac{4M}{r_-} + \frac{Q^2}{r_-^2}}}. \]  

(4.22)

As we can see, our expression for the temperature of the particles emitted by the inner horizon inside the inner horizon is very similar to Eq. (3.42), which gives the temperature of the particles emitted by the outer horizon outside the outer horizon. The only difference is that \( r_+ \) has been replaced by \( r_- \). Our result that the inner horizon emits particles inside the inner horizon with a positive temperature given by Eq. (4.22) has a most important consequence: To maintain a local energy balance it is necessary that when real particles with energy \( \omega \) are emitted towards the singularity from the inner horizon, antiparticles with energy \( -\omega \) are emitted away from the singularity through the inner horizon. The process is similar to the one which, according to the Hawking effect, takes place at the outer horizon of the Reissner-Nordström black hole: At the outer horizon antiparticles go in and particles come out, and now we found that this is true at the inner horizon as well. According to our best knowledge this phenomenon, despite of its apparent triviality, has not been noticed before.

An intriguing question now arises: What happens to the antiparticles which are emitted away from the singularity through the inner horizon? A hint for the answer to this question may be found if we consider the conformal diagram of the maximally extended Reissner-Nordström spacetime of Fig. 6. In that figure we have drawn the world lines of the particle and the antiparticle which are created at the inner horizon. The real particle, as we found in our analysis, remains inside the inner horizon, and finally meets with the singularity of the Reissner-Nordström hole. The
antiparticle, in turn, enters the intermediate region between the horizons and one may—if the backscattering effects are neglected—speculate on the possibility that it travels across the intermediate region and finally comes out from the white hole horizon. The situation is, however, quite complicated since the vacuum states corresponding to a freely falling observer near the inner horizon of Reissner-Nordström black hole and the white hole horizon are completely different. Therefore, a detailed analysis of the antiparticle modes travelling through the intermediate region is needed.

Let us consider antiparticle solutions with energy $-\omega$ moving towards the inner horizon in the region VI’ of the Fig. 6. We shall adhere a convention where the Kruskal type coordinates defined at the neighborhoods of the points $P_2$ and $P_1'$ are labelled by minus and plus sign, respectively. In that case the antiparticle solutions may be written as

$$\phi' \approx N_{\omega lm} Y_{lm} \frac{1}{r} e^{i\omega \tilde{u}}.$$  

(4.23)

where we have defined

$$\tilde{u} := l_- (v_- - u_-).$$  

(4.24)

The Reissner-Nordström metric is an analytic function of the coordinates $u_-$ and $v_-$ at the inner horizon, and therefore the antiparticle solutions have exactly the same form as in Eq. (4.23), when they are transferred across the horizon to the region IV’. The definitions of $u_-$ and $v_-$, however, change according to the Eqs. (3.15), and in the region IV’ we have:

$$\begin{cases}
u_- = \frac{1}{2} (e^{\kappa_- V} - e^{-\kappa_- U}), \\
u_+ = \frac{1}{2} (e^{\kappa_+ V} + e^{-\kappa_+ U}), \\
u_+ = \frac{1}{2} (e^{\kappa_+ V} - e^{-\kappa_+ U}), \\
u_+ = \frac{1}{2} (e^{\kappa_- V} + e^{-\kappa_- U}),
\end{cases}$$

(4.25a)

$$\begin{cases}
u_- = \frac{1}{2} (e^{\kappa_- V} + e^{-\kappa_- U}), \\
u_+ = \frac{1}{2} (e^{\kappa_+ V} + e^{-\kappa_+ U}),
\end{cases}$$

(4.25b)

In general, the relationships between the coordinates (4.25a) and (4.25b) are quite complicated. Therefore, for the sake of simplicity, we confine ourselves to a special case where the Reissner-Nordström black hole is almost extreme.
More precisely, we shall assume that

\[ \epsilon := \frac{r_+ - r_-}{r_+} << 1. \]  

(4.26)

In that case we have

\[ \kappa_-(\epsilon) = \frac{-\epsilon}{2r_+(1 - \epsilon)^2}, \]  

(4.27)

and \( \kappa_- \) can be expanded as series with respect to \( \epsilon \) resulting

\[ \kappa_-(\epsilon) = -\kappa_+ + \mathcal{O}(\epsilon^2). \]  

(4.28)

By using the above result one easily finds that

\[ u_- = \frac{-u_+}{u_+^2 - v_+^2} + \mathcal{O}(\epsilon^2), \]  

(4.29)

\[ v_- = \frac{v_+}{u_+^2 - v_+^2} + \mathcal{O}(\epsilon^2), \]  

(4.30)

and, moreover,

\[ \tilde{u}_- = \frac{l_-}{u_+ - v_+} + \mathcal{O}(\epsilon^2). \]  

(4.31)

When we define

\[ \tilde{u}_+ := l_+(v_+ - u_+) \]  

(4.32)

we finally get

\[ \tilde{u}_- = -\frac{l_- l_+}{\tilde{u}_+} + \mathcal{O}(\epsilon^2). \]  

(4.33)

Therefore, at the region IV', one may write Eq. (4.23) as:

\[ \phi' \approx N_{\omega lm}Y_{lm} \frac{1}{r} e^{-i\frac{l_+ - l_+}{l_+ + \epsilon}}, \]  

(4.34)

when \( \epsilon \) is sufficiently small.

When the solutions in Eq. (4.34) are transferred from the region IV' into the region I', they have again exactly the same form as above but the coordinates \( u_+ \) and \( v_+ \) are defined differently according to Eqs. (4.30). The antiparticle solutions coming out from the white hole near the horizon in the region I' are, according to the observer at rest with respect to the horizon,

\[ \phi \approx N_{\omega lm}Y_{lm} \frac{1}{r} e^{i\omega U}. \]  

(4.35)

From Eq. (4.36) one finds that

\[ U = -\kappa_+^{-1} \ln(-\tilde{u}_+) + \kappa_+^{-1} \ln l_+. \]  

(4.36)

Thus we arrive at the Bogolubov transformation

\[ e^{-i\omega \kappa_+^{-1} \ln(-\tilde{u}_+)} e^{i\omega \kappa_+^{-1} \ln l_+} = \sum_{\omega'} \left( A'_{-\omega \omega'} e^{-i\omega \kappa_+^{-1} \ln l_+} + B'_{-\omega \omega'} e^{i\omega \kappa_+^{-1} \ln l_+} \right). \]  

(4.37)
When one denotes
\[ z := \frac{l - l_+}{u_+} \]  
(4.38)
the above transformation becomes to:
\[ e^{i \omega \kappa_+^{-1} \ln(-z)} e^{-i \omega \kappa_+^{-1} \ln l_-} = \sum_{\omega'} \left( A'_{-\omega'} e^{-i \omega' z} + B'_{-\omega'} e^{i \omega' z} \right). \]  
(4.39)
The Bogolubov coefficients can be obtained from the integrals
\[ A'_{-\omega'} = \frac{1}{2\pi} e^{-i \omega \kappa_+^{-1} \ln l_-} \int_{-\infty}^{0} dz e^{i \omega \kappa_+^{-1} \ln(-z)} e^{i \omega' z}, \]  
(4.40a)
\[ B'_{-\omega'} = \frac{1}{2\pi} e^{-i \omega \kappa_+^{-1} \ln l_-} \int_{-\infty}^{0} dz e^{i \omega \kappa_+^{-1} \ln(-z)} e^{-i \omega' z}. \]  
(4.40b)
These integrals lead to the result
\[ |A'_{-\omega'}| = e^{\pi \kappa_+ \omega} |B'_{-\omega'}|. \]  
(4.41)
Therefore, by using Eq. (2.19), we see that the number of antiparticles with energy \(-\omega\) is, from the point of view of the observer at rest very close to the white hole horizon, when, from the point of view of the freely falling observer, the field is in vacuum, 
\[ n_{-\omega} = \sum_{\omega'} |B'_{-\omega'}|^2 = \frac{1}{e^{2\pi \kappa_+ \omega} - 1}. \]  
(4.42)
This is the Planck spectrum at the temperature
\[ T = \frac{\kappa_+}{2\pi} \]  
(4.43)
corresponding to the antiparticle flow coming out from the white hole.

When redshift effects are taken into account, one finds that the temperature of the white hole horizon for antiparticles is
\[ T_{WH} := \frac{\sqrt{M^2 - Q^2}}{2\pi r_+ \sqrt{1 - \frac{2M}{r_+} + \frac{Q^2}{r_+^2}}}. \]  
(4.44)
In other words, our analysis predicts that not only do the black hole horizons of the Reissner-Nordström spacetime emit thermal radiation with a black body spectrum but thermal radiation is emitted by the white hole horizons as well. This means that outside the Reissner-Nordström black hole there exists two simultaneous radiation processes. They are the normal black hole radiation, and the “white hole radiation” which is caused by the pair creation effects at the inner horizon. The “white hole radiation” predicted by our analysis is a new effect which according to our best knowledge has not been mentioned in the literature before. However, the white hole radiation does not consist of particles with positive energy, but of antiparticles with negative energy. The emission of antiparticles out of the white hole, in turn, may be understood as an absorption of energy by the white hole horizon. This feature contradicts with the classical results (classically, no energy may be absorbed by the white hole horizon) in a similar way as does the evaporation process at black hole horizons.

In our analysis we considered the special case where the Reissner-Nordström black hole was almost extreme. This was indeed a very strong assumption but it was needed for analytical calculations. It is, however, natural to expect that there exists a certain antiparticle flow from a white hole horizon even when the hole is quite far from extremality. In that case one may also expect that the energy spectrum of the antiparticles differs from the spectrum (4.42), thus leading to a different temperature as well. These kinds of effects may take place especially when the absolute value of the electric charge \(Q\) is, in natural units, much smaller than \(M\).
V. CONCLUDING REMARKS

In this paper we have found that in maximally extended Reissner-Nordström spacetime both the black and the white hole horizons emit thermal radiation which, when the possible backscattering effects are neglected, obeys the normal black body spectrum. The analysis implying an existence of the white hole radiation was based, for the sake of simplicity, on the assumption that the Reissner-Nordström black hole was almost extreme. However, it is natural to expect that this phenomena also exists for black holes far from extremality. In that case, the situation becomes much more complicated and one may anticipate corrections to the temperature of the white hole horizon. The radiation coming out from the white hole horizons is caused by the pair creation effects at the inner horizon of the Reissner-Nordström black hole. When a particle-antiparticle pair is created just inside the inner horizon of the Reissner-Nordström hole, the real particle is emitted towards the singularity from the inner horizon, whereas the antiparticle is emitted away from the singularity through the inner horizon. The particle finally meets with the black hole singularity, whereas the antiparticle travels across the intermediate region between the horizons of the Reissner-Nordström black hole, and finally comes out from the white hole horizon. In other words, we have found that outside the Reissner-Nordström black hole there occurs two simultaneous radiation processes which are caused by the pair creation effects at the both horizons of the hole. Since the white hole radiation consists of antiparticles, the white hole radiation process may be understood as an absorption of energy by the white hole horizon.

We obtained our results by means of an analysis which was similar to the normal derivation of the Unruh effect. More precisely, we considered the quantum-mechanical properties of the massless Klein-Gordon field in the vicinity of the horizons from the points of views of two different observers. One of these observers was at rest with respect to the Reissner-Nordström coordinates either just outside the outer horizon or inside the inner horizon, whereas another observer was in a free fall through the horizon. We found that an observer at rest observes particles even though, from the point of view of an observer in a free fall, the field is in vacuum. The observer at rest just outside the outer horizon observes outcoming real particles. Similarly an observer at rest just inside the inner horizon observes particles propagating towards the singularity.

The most remarkable result of our analysis is that, in contrast to common beliefs, the inner horizon is not a passive spectator but an active participant in the radiation processes of the Reissner-Nordström black hole. Although this result is based on an almost trivial observation that both of the horizons of the Reissner-Nordström black hole emit particles, there may also be some element of surprise in it, and therefore the first question concerns the physical and the mathematical validity of our analysis. After all, we did not follow the usual route with an analysis based on a comparison of the solutions to the Klein-Gordon equation in the past and in the future light-like infinities. Since this kind of an analysis would have been impossible when considering the radiation emitted by the inner horizon, we instead compared the solutions in the rest frames of two observers. Is this kind of approach valid?

The physical validity of this kind of approach has been considered, in the case of the Schwarzschild black hole, by Unruh, and similar arguments also apply here. The best argument in favor of the validity of our analysis is probably given by the fact that exactly the same methods which were used in an analysis of the radiation of the inner horizon produced the well-known results for the radiation emitted by the outer horizon. Another problem is that we have simply ignored all possible backscattering effects. To consider such effects one should perform a numerical analysis of the solutions to the Klein-Gordon equation. However, as mentioned in Introduction, it is expected that the backscattered particles will trigger the mass inflation, and therefore the radiation effects of the inner horizon of the Reissner-Nordström black hole are of very brief duration. As a consequence, the full Reissner-Nordström spacetime, even as a mathematical solution, is unstable at the semiclassical limit. Since the Reissner-Nordström spacetime is not considered astrophysically relevant, we do not expect that our analysis has direct astrophysical consequences. If, however, our results are qualitatively the same for more realistic Kerr black holes, the phenomena discussed in this paper might possess some astrophysical implications. Nevertheless, the radiation effects of the inner horizon have
much importance in their own right since they support the idea that all horizons of spacetime emit radiation.

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APPENDIX: INFINITESIMAL GEODESIC COORDINATES NEAR THE HORIZONS OF THE REISSNER-NORDSTRÖM SPACETIME

Consider geodesic coordinates near the outer horizon of a Reissner-Nordström black hole. More precisely, consider a geodesic coordinate system in the region I infinitesimally close to the point $P_1$ of Fig. 4. By using the definitions such that $\alpha = \kappa_+$ in Eqs. (3.19), the metric in the region I of Reissner-Nordström spacetime can be written as
\[
ds^2 = \frac{1}{\ell^2} \frac{1}{\kappa_+^2 r^2} e^{-2\kappa_+ r (r - r_-)} \left( \frac{r^2}{r_+^2} + 1 \right) \left( - (dX^0)^2 + (dX^1)^2 \right) + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2). \tag{A.1}\]

Especially, on the two-surface $r = r_+$ the metric takes a very simple form:
\[
ds^2 = -(dX^0)^2 + (dX^1)^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2), \tag{A.2}\]

Thus, the coordinates $X^0$ and $X^1$ provide an infinitesimal geodesic coordinate system for the freely falling observer if only the derivatives of the metric with respect to $X^0$ and $X^1$ vanish at the point $P_1$. According to Eq. (A.1) the components of the metric depend merely on $r$. Moreover, in order to show that the derivatives of the metric vanish, it is sufficient to show that
\[
\frac{\partial r}{\partial u} = \frac{\partial r}{\partial v} = 0 \tag{A.3}\]
at the point $P_1$.

Let us first note that the relationship between the coordinates $u$, $v$, and $r$ can be expressed in an implicit form such that
\[
u^2 - u^2 = e^{2\kappa_+ r_+}. \tag{A.4}\]

By differentiating both sides with respect to $u$, one gets
\[
2u = 2\kappa_+ e^{2\kappa_+ r_+} \frac{dr}{dr} \frac{\partial r}{\partial u}, \tag{A.5}\]

and similarly differentiation with respect to $v$ gives
\[
-2v = 2\kappa_+ e^{2\kappa_+ r_+} \frac{dr}{dr} \frac{\partial r}{\partial v}. \tag{A.6}\]

Since all equations (A.1-6) must be satisfied at $P_1$, one finds that $u = v = 0$ at $P_1$. However, it is easy to see that when $r = r_+$,
\[
2\kappa_+ e^{2\kappa_+ r_+} \frac{dr}{dr} = 2\kappa_+ r_+^2 e^{2\kappa_+ r_+} (r_+ - r_-) \left( \frac{r^2}{r_+^2} + 1 \right) \neq 0. \tag{A.7}\]

Hence Eq. (A.3) is satisfied at the point $P_1$. 

Next, let us concentrate on the geodesic coordinates near the inner horizon. Consider a geodesic coordinate system in the region VI’, infinitesimally close to the point $P_2$ of Fig. 5. In this case, we choose $\alpha = \kappa_-$ for the Kruskal-type coordinates. By using the definitions (4.4) and (4.5) the metric in the region VI’ can be written as

$$ds^2 = \frac{1}{l^2_\Sigma} \frac{1}{\kappa_- r^2} e^{-2\kappa_- r} (r_+ - r)^{\frac{r^2}{2} + 1} \left( - (dX^0)^2 + (dX^1)^2 \right) + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2).$$  \hspace{1cm} (A.8)

When $r = r_-$, the metric has the form of that of Eq. (A.2), and, therefore, the coordinates $X^0$ and $X^1$ provide an infinitesimal geodesic coordinate system if the derivatives of the metric vanish at the point $P_2$. Again, one easily sees that it is sufficient to show that Eq. (A.3) holds also at $P_2$.

To show this, we proceed as before. The relationship between the coordinates $u$, $v$, and $r$ can now be expressed as

$$u^2 - v^2 = e^{2\kappa_- r_*}.$$  \hspace{1cm} (A.9)

Differentiating both sides with respect to $u$ and $v$ gives, respectively,

$$2u = 2\kappa_- e^{2\kappa_- r_*} \frac{dr}{dr} \frac{\partial r}{\partial u},$$  \hspace{1cm} (A.10)

$$-2v = 2\kappa_- e^{2\kappa_- r_*} \frac{dr}{dr} \frac{\partial r}{\partial v}.$$  \hspace{1cm} (A.11)

Similarly as before, we have $u = v = 0$ at $P_2$. Furthermore, one easily finds that, when $r = r_-,$

$$2\kappa_- e^{2\kappa_- r_*} \frac{dr}{dr} = 2\kappa_- r_-^2 e^{2\kappa_- r_-} (r_+ - r_-)^{-\left(\frac{r^2}{2} + 1\right)} \neq 0.$$  \hspace{1cm} (A.12)

Hence Eq. (A.3) is satisfied also at the point $P_2$.

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