A mathematical model for calculation of the influence of ferromagnetic components in Vertical Displacement Events and stability simulations of tokamak plasmas

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Abstract. Iron core transformer tokamaks have a distinct element, namely the iron core which can affect the plasma stability of the plasma. As a consequence of the high non-linear dependence of the magneto-hydrodynamic solutions on the iron permeability \( \mu_{Fe} \) in JET tokamak for example, Vertical Displacement Events (VDEs), equilibrium and stability calculations are more complicated and more time consuming than in air core transformer tokamaks. By considering the ferromagnetic components as a linear, isotropic and homogeneous media on subdomains, it is known that these media can be replaced by a homogeneous one (vacuum) and a surface-current density distribution \( i_{Fe}(l) \) on the separation surfaces between subdomains, with \( l \) the curve taken along the curve separating two different magnetic media. For the case of geometry with rotational symmetry, this surface-current density distribution along a curve, in a meridian plane, is given by a Fredholm integral equation of second kind. Practically, the advantage of this method is more obvious for the inverse formulation of the VDEs and stability problems by moving the non-linear term (due to the iron permeability \( \mu_{Fe} \)) from the differential operator to the r.h.s. of the equations. In this paper, we are presenting how the influence of ferromagnetic components in the equations of the surface currents developed in the vessel structures during Wall Touching Kink Modes (WTKMs) can be taken into account and are reviewing the equations to be solved in order to simulate the influence of the ferromagnetic components in VDEs and equilibrium stability calculations. The numerical results of these simulations for a real JET tokamak structure and plasma parameters will be reported in a future paper.

1. Introduction

Due to the necessarily large toroidal currents (15 MA in ITER) the tokamak concept suffers from a fundamental problem of stability. It is well known that the tokamak concept is based on Shafranov’s stability criterion for the free boundary kink modes \( q_a > 1 (q(\rho) \equiv \rho B_\phi/R B_\omega, \) with \( q \) the safety factor, \( a \), \( R \) the minor and major radius of the plasma, \( \rho, \omega, R_\phi \) the cylindrical coordinates and \( B_\phi, B_\omega \) are the toroidal and poloidal magnetic fields) [1, 2]. Later on, the edge value of \( q_a \) was increased to \( q_a > 2 \), but it is known that in tokamaks, the criterion \( q_a > 2 \)
cannot be approached: MHD instabilities in the form of disruptions terminate the discharge. The nonlinear evolution of MHD instabilities leads to a dramatic quench of the plasma current within milliseconds, i.e. a major disruption. If in currently operated tokamaks the damage is often large but rarely dramatic, in ITER tokamak, it is expected that the occurrence of a limited number of major disruptions will definitively damage the chamber with no possibility to restore the device. The magnetic effects of a disruption, associated with the sudden loss of the net plasma current $I_{pl}$, are known also as Vertical Displacement Events (VDE). The key basis for tokamak plasma disruption modeling is to understand how currents flow to the plasma facing surfaces during plasma disruption events.

In this paper we are presenting the equations to be solved in order to simulate the influence of the ferromagnetic components in VDEs and equilibrium stability calculations. As a consequence of the high non-linear dependence of the magneto-hydrodynamic solutions on the iron permeability $\mu_{Fe}$, in JET tokamak for example, Vertical Displacement Events (VDEs), equilibrium and stability calculations are more complicated and more time consuming than in air core transformer tokamaks.

In section 2, we are reminding the two kinds of surface current components developed during VDE (and presented previously by us in Ref. [3]) and we are presenting where in the equations describing these surface currents the influence of the ferromagnetic components has been considered. Section 3 describes how the influence of the ferromagnetic components has been taken into account by developing a boundary integral equation method instead of using the non-linear elliptical differential operator in the ferromagnetic domain. The summary and the next steps to be considered are given in section 4.

2. Electromagnetic thin wall model with two kinds of surface current components

Understanding that in disruptions the sharing of electric current between the plasma and the wall plays an important role in plasma dynamics, we have developed a wall model that covers both eddy currents, excited inductively, and source/sink currents due to current sharing between the plasma and the wall [3-12].

According to Helmholtz decomposition theorem, we have split the surface current density $d_w j$ in the conducting shell (wall) into the sum of an irrotational (curl-free) vector field component and a solenoidal (divergence-free) vector field component [3]:

$$\begin{align*}
d_w j &= i - d_w \sigma \nabla \Phi^S, \\
i &= \nabla I \times n, \quad (\nabla \cdot i = 0),
\end{align*}$$

where: $i$ is the divergence free surface current (eddy currents), $d_w \sigma \nabla \Phi^S$ is the source/sink current (S/SC) with potentially finite divergency to be able to describe the current sharing between plasma and wall, $d_w \sigma$ is the surface wall conductivity, $d_w$ is the thickness of the current distribution, $I$ is the stream function of the divergence free component (eddy currents), $n$ is the unit normal vector to the wall, and the surface function $\Phi^S$ is the source/sink potential.

By applying the continuity equation for the surface currents, the Faraday law and applying the operators $n \cdot \nabla \times$ on the equation resulted by applying Faraday law, we are obtaining the following equations to be solved [3]:

$$\begin{align*}
\nabla \cdot (d_w j) &= -\nabla \cdot (d_w \sigma \nabla \Phi^S) = j_\perp, \\
-\frac{\partial A}{\partial t} - \nabla \Phi^E &= \frac{1}{d_w \sigma} (\nabla I \times n) - \nabla \Phi^S, \\
\nabla \cdot \left( \frac{1}{d_w \sigma} \nabla I \right) &= \frac{\partial B_\perp}{\partial t} = \frac{\partial (B^pl_\perp + B^{coil}_\perp + B^I_\perp + B^S_\perp + B^{Fe}_\perp)}{\partial t},
\end{align*}$$
where: \( \mathbf{j}_\perp = -(\mathbf{j} \cdot \mathbf{n}) \) is the current density coming from/to the plasma, \( \mathbf{A} \) is the magnetic vector potential and \( \Phi^E \) is the electric potential. On the right-hand side of Eq. (5), the normal component of the magnetic field is given by: plasma \( B^\perp_{\text{pl}} \), external coils \( B^\perp_{\text{coil}} \), the two components of the surface currents \( B^\perp_I \) and \( B^\perp_S \) and the newly introduced ferromagnetic components (of JET for example) \( B^\perp_{\text{Fe}} \). In the next section, a method to calculate \( B^\perp_{\text{Fe}} \) as a function of space and time will be presented.

3. Calculation of the ferromagnetic components influence

There are many numerical simulations to investigate the influence of the iron circuit on plasma equilibrium [13-19]. Most of them are based on solving the non-linear elliptical differential operator (10) in the iron region.

In an equivalent two-dimensional geometry with rotational symmetry (Fig. 1), the equations for the flux function \( \Psi \) of the poloidal magnetic field are:

\[
\Delta^* \Psi = -\mu_0 r J_\phi(r, \Psi), \quad r, z \in \Omega_{\text{pl}},
\]

\[
\Delta^* \Psi = -\mu_0 r \sum_k I_k \delta(r - r_k; z - z_k), \quad r, z \in \Omega_i - \Omega_{\text{pl}},
\]

\[
\Delta^*_\mu \Psi = 0, \quad r, z \in \Omega_f
\]

where \( J_\phi \) is the toroidal plasma current density, \( I_k \) is the current flowing through the \( k\text{th} \) coil, \( \Delta^* \) is the well known linear elliptical operator

\[
\Delta^* = r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2},
\]

while \( \Delta^*_\mu \) is the non-linear elliptical operator

\[
\Delta^*_\mu = r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{1}{\mu(B)} \frac{\partial}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu(B)} \frac{\partial}{\partial z} \right).
\]

Eq. (8) has been obtained by making several stages of simplification. The main assumption is a piece-wise homogeneous \( \mu(B) \) in volumetric layers in the iron core. For explanation only, the central iron core can be represented by layers \( z_i \leq z \leq z_{i+1} \) with \( \mu = \mu_i = \text{const} \). In this case, the nonlinear equation in the iron core can be simplified to

\[
\Delta^*_\mu \Psi = \frac{1}{\mu} \Delta^* \Psi + \sum_i \left( \frac{1}{\mu_{i+1}} - \frac{1}{\mu_i} \right) \delta(z - z_i) \cdot \frac{1}{r} \frac{\partial \Psi}{\partial z} = 0.
\]

The \( \frac{1}{r} \frac{\partial}{\partial z} \) derivative in the \( \delta\)-functional term represents the tangential to the layer component of the magnetic field (\( B_r \)), which can be neglected in comparison to the normal component. As a result, the equation for magnetic field inside the core is reduced to the vacuum field equation \( \Delta^*_\mu \Psi = 0 \) and can be represented by a surface current \( i(l) \) on the contour of the iron core.

In a previous step, we have developed a boundary integral equations method to calculate the magnetostatic part of the MHD equations and have used it first in a simplified form for T-15 tokamak equilibrium calculation [20, 21, 22]. For exemplification of our present approach only, we have considered the JET iron core geometry (Fig. 1).

For the beginning, we have considered the problem of surface currents distribution \( i_0(l_1,2) \) on contours \( L_1 \) and \( L_2 \) which are not producing a magnetic field in the \( \Omega_i \) and \( \Omega_e \) domains (Fig. 1) and all the magnetic field due to the currents \( i_0(l_1) \) and \( i_0(l_2) \) is limited to the \( \Omega_f \) domain only. By considering in the \( \Omega_i \) and \( \Omega_e \) domains a zero value for the components of the magnetic
field tangential to the contours \( l_1 \) and \( l_2 \), we have obtained the equations for the determination of currents \( i_0(l) \) and \( i_0(l) \):

\[
B_{\tau 1}(l_1) = -\frac{\mu_0}{2}i_0(l_1) + \int_{L_1} b_\tau(l_1, l'_1)i_0(l'_1)dl'_1 + \int_{L_2} b_\tau(l_1, l_2)i_0(l_2)dl_2 = 0, \tag{12}
\]

\[
B_{\tau e}(l_2) = \int_{L_1} b_\tau(l_2, l_1)i_0(l_1)dl_1 + \frac{\mu_0}{2}i_0(l_2) + \int_{L_2} b_\tau(l_2, l'_2)i_0(l'_2)dl'_2 = 0, \tag{13}
\]

with \( b_\tau(l, l') \) the tangential magnetic field component in point \( l \) produced by a unit circular current passing via point \( l' \).

It is known that the boundary integral equations method can be applied only in linear, isotropic and homogeneous media. But if a magnetic medium is homogeneous on subdomains, it can be replaced by a homogeneous one (vacuum) and a surface-current density distribution on the separation surfaces between subdomains. If we are considering a geometry with rotational symmetry, then the surface-current density distribution along a \( L \) curve (any one from the \( L_1 \) or \( L_2 \) curves given in Fig.1) in a meridian plane, separating two homogeneous media, can be calculated with a Fredholm integral equation of the second kind [20]:

\[
\frac{1}{2}\mu_0i(l) = \frac{\mu_{\text{out}}(l) - \mu_{\text{inn}}(l)}{\mu_{\text{out}}(l) + \mu_{\text{inn}}(l)} \left( B_{\tau}^{\text{ext}}(l) + \int_L b_\tau(l, l')i(l')dl' \right), \tag{14}
\]

where \( b_\tau \) is the tangential component of the magnetic field at the \( L \) curve, produced in \( l \) (a contour coordinate) by a unit current located in \( l' \). \( B_{\tau}^{\text{ext}}(l) \) represents the tangential component of the magnetic field at the \( L \) curve, given by an external known current distribution such as the plasma and the coils. The superscripts \( \text{out} \) and \( \text{inn} \) denote the outer and the inner domain of the \( L \) curve.

For the solution of the integral equation (14) two approaches have been considered. In the first one, presented in detail in Refs. [20, 21] to underline the iron influence, the \( L \) curve represents only the sum of the external vacuum-iron interfaces (\( L_1 \) and \( L_2 \) in Fig. 1), where in the second approach, the \( L \) curve considered in Eq. (14) includes also the interfaces between the different iron subdomains over which the magnetic permeability is considered homogeneous.

If on each segment of a curve, which must be a Liapunov curve [23] (i.e. at every point of \( L \) there is a well-defined tangent - consequently a well-defined normal - and the angle between the tangents or normals are satisfying a simple Liapunov relation; all curves \( L_1, L_2, l_k \) from Fig. 1 are satisfying these conditions) the surface-current density \( i(l) \) is considered continuous (Hölder continuity [23]) and of bounded variation, then it admits a uniformly convergent expansion.

By using Legendre polynomials \( P_k(x) \), \( k \) being the polynomial degree (the weight function for these polynomials are equal to one) in the interval \(-1 \leq x \leq 1\), with the standardization \( P_{k1}(x) = 1 \), we have the following orthogonality relation [24]

\[
\int_{-1}^{1} P_{k1}(x)P_{k2}(x)dx = \frac{2}{2k1 + 1} \delta_{k1,k2}, \quad \delta_{k1,k2} = 1 \text{ if } k1 = k2 \text{ and } \delta_{k1,k2} = 0 \text{ if } k1 \neq k2, \tag{15}
\]

with \( \delta \) the Kronecker delta function. Then the sequence of sums for the unknown function \( i(x) \) is

\[
i(x) = \sum_{k1=0}^{K1} \alpha_{k1}P_{k1}(x) \tag{16}
\]

converges to \( i(x) \) in the interval \([-1, 1] \) as \( K1 \to \infty \) if \( i(x) \) and \( i'(x) \) are at least sectionally continuous in this interval. By multiplying Eq. (16) by \( P_{k2}(x) \), integrating term by term and
using the orthogonal property given by Eq. (15), we obtain

\[
\int_{-1}^{1} \sum_{k_1=0}^{K_1} \alpha_{k_1} \bar{P}_{k_1}(x) P_{k_2}(x) \, dx = \int_{-1}^{1} \bar{i}(x) P_{k_2}(x) \, dx \quad \Rightarrow
\]

\[
\alpha_{k_2} = \frac{2k_2 + 1}{2} \int_{-1}^{1} \bar{i}(x) P_{k_2}(x) \, dx.
\]  \hspace{1cm} (17)

By replacing the integration variables \(x\) by \(y\) and \(k_2\) by \(k_1\) and substituting Eq. (16) in Eq. (17) we have

\[
i(x) = \sum_{k_1=0}^{K_1} \alpha_{k_1} \bar{P}_{k_1}(x) = \sum_{k_1=0}^{K_1} \frac{2k_1 + 1}{2} \left( \int_{-1}^{1} \bar{i}(y) P_{k_1}(y) \, dy \right) P_{k_1}(x).
\]  \hspace{1cm} (18)

Considering now the general case of \(L_k, k = 1, M\) contours, each one with \(n = 1, N_k\) discrete segments of length \(2\Delta l^n_k\), we have

\[
i(l^n_k) = \sum_{i=0}^{p^n_k} \alpha^n_{k,i} P_i(x), \quad x \in [-1, 1],
\]  \hspace{1cm} (19)

where \(\alpha^n_{k,i}\) are the unknown coefficients to be determined and \(P_i(x)\) are orthogonal Legendre polynomials of order \(p^n_k\).

Introducing eq. (19) in eq. (14), written for the general case of \(M\) contours \(L_k\) one obtains:

\[
\alpha^n_{k,i} = \frac{2i + 1}{\mu_0} \left\{ \frac{\mu^{out}(l^n_k)}{\mu^{out}(l^n_k)} - \frac{\mu^{inn}(l^n_k)}{\mu^{inn}(l^n_k)} \left[ \int_{-1}^{1} B^{ext}_r(l^n_k) P_i(x) \, dx \right] \right. \\
+ \left. \sum_{j=1}^{M} \sum_{n=1}^{N_j} \sum_{l=0}^{p_j} \alpha^j_{n,l} \Delta l^n_j \int_{-1}^{1} b_r(l^n_k, l^n_j) P_i(x) P_l(y) \, dx \, dy \right\}, \quad x, y \in [-1, 1].
\]  \hspace{1cm} (20)

The Legendre polynomials \(P_i(x)\) of degree \(i\) are given by the known Rodrigues’ formula

\[
P_i(x) = \frac{1}{2^i i!} \frac{d^i}{dx^i} (x^2 - 1)^i.
\]  \hspace{1cm} (21)

The double integral in Eq. (20) can be computed as a limit of a proper integral [25] and the result of the integration being a continuous function, for the second integral a Gauss quadrature, of relatively low order, can be used.

4. Conclusions and next steps
In the present paper, a boundary integral equations method used to calculate the influence of ferromagnetic components in Vertical Displacement Events (VDEs), magnetostatic part of the equilibrium configurations and stability condition is presented. Using this method, the computations for an iron-core transformer tokamak can be performed in the same way as for air-core transformer tokamaks. The advantage of this approach is more obvious for the inverse formulation of the equilibrium problem, where the currents in the external coils, required to ensure a particular shape of the plasma, have to be determined directly. In addition, for a given geometry of the magnetic circuit, the calculations which take most of the running time must be performed only once, while the resulting influence matrix, stored on files, can serve to investigate all cases of interest.
With the method presented above, the equilibrium configurations and the stability of a plasma column with arbitrary cross section can be determined in the same manner as for air-core transformer tokamaks. (Refs. [26-28] for equilibrium calculation and Ref. [29] for stability calculation). For a plasma with a circular cross section and large aspect ratio (like the T15 tokamak plasma) the stability of the plasma equilibrium with respect to axisymmetric perturbations can be investigated via the stability criteria for radial displacement and vertical displacement given in Ref. [30].

As a potential improvement, the contribution of the δ-functional term in Eq. (11), which is equivalent to a surface current density, can be taken explicitly as the next step in development of the algorithm. This consideration can extend the boundary integral equation method used for homogeneous isotropic media for the case of piece-wise uniform iron core.

For the real JET tokamak iron core geometry and JET plasma parameters, numerical results of calculation the iron core influence on VDE, equilibrium and stability are in the process of being completed and will be reported in an future paper.

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