Reducing Influence of Gravity Model Error in Precise Orbit Determination of Low Earth Orbit Satellites

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Abstract  Based on the orbit integration and orbit fitting method, the influence of the characters of the gravity model, with different precisions, on the movement of low Earth orbit satellites was studied. The way and the effect of absorbing the influence of gravity model error on CHAMP and GRACE satellite orbits, using linear and periodical empirical acceleration models and the so-called “pseudo-stochastic pulses” model, were also analyzed.

Keywords  precise orbit determination; gravity model; orbit fitting

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Introduction

Earth-oriented observation systems were improved significantly during the last decades because of people’s thirst for knowledge of the planet we are living in. With a number of satellite objects that were carried into execution, the quality of satellite applications was brought into a higher level and satellite orbit precision has been necessary. For example, the radial direction accuracy of the orbits of the T/P satellite is 5 cm[1] and more rigorous for CHAMP and GRACE gravity observation satellite systems[2].

The error of satellite orbits caused by the gravity model are one of the main factors that need to be taken into account, and is not accurate enough for the precise orbit determination (POD) of low Earth orbit (LEO) satellites, though it has been improved significantly by several gravity recovery satellite projects[3]. In order to obtain a cm-level precision orbit of different kinds of LEO satellites, some empirical acceleration models need to be used in the POD procedure to absorb the un-modeled error[4].

Based on the analysis of the accuracy of the predicted orbits of satellites with different altitudes in different gravity models, the effect of different empirical accelerations, such as in the linear, periodical empirical acceleration model and the so-called “pseudo-stochastic pulses” model[4] in absorbing gravity error, were studied using orbit integration and dynamic smoothing method. All the computations were done on the basis of PANDA software, which was developed in Wuhan University, and the products used in choosing the empirical acceleration models and deciding the number of parameters to be involved in the POD procedure.

1  Empirical acceleration models

There are three kinds of empirical acceleration models, including piece wise linear acceleration, periodical empirical acceleration in radial, along and
across-track direction and the so-called “pseudo-stochastic pulses” model\textsuperscript{[5,6]}, which was first applied in CODE for GPS satellites orbit determination\textsuperscript{[7,8]} and for CHAMP satellite in TUM\textsuperscript{[9]}.

1.1 Piece wise linear empirical acceleration model

The movements of satellites are very complex and cannot be modeled exactly, especially in the along-track direction, so empirical acceleration parameters are used to absorb the un-modeled error. The un-modeled error in radial and cross-track directions also cannot be neglected when satellites became slow, so the piece wise linear acceleration model is given in three directions as Eq. (1).

\[
P_{\text{line}} = \begin{bmatrix}
 t - t_{i-1} & \frac{C_i}{C_n} & \frac{C_i}{C_n} \\
 t_{i+1} - t & \frac{C_{i+1}}{C_n} & \frac{C_{i+1}}{C_n} \\
 t_{i+1} - t_{i-1} & \frac{C_{i+1}}{C_n} & \frac{C_{i+1}}{C_n}
\end{bmatrix} \cdot \begin{bmatrix}
u_r \\
u_t \\
u_n
\end{bmatrix}
\]

\((t_i \leq t < t_{i+1})\)

where \(C_r, C_t, C_n\) are acceleration parameters in radial, along-track and cross-track direction; \(u_r, u_t, u_n\) are the unite direction vectors.

1.2 Periodical empirical acceleration model

Un-modeled error in radial, along-track and cross-track direction can also be observed by periodic empirical acceleration parameters as Eq. (2).

\[
P_{\sin} = \begin{bmatrix}
P_r \\
P_t \\
P_n
\end{bmatrix} = \begin{bmatrix}
C_r \cos u + S_r \sin u \\
C_t \cos u + S_t \sin u \\
C_n \cos u + S_n \sin u
\end{bmatrix}
\]

where \(P_r, P_t, \) and \(P_n\) are disturbing force in radial, along-track and cross-track direction; \(u\) is the latitude of satellites; \(C_r, S_r\) are coefficients in radial direction; \(C_t, S_t\) are coefficients along-track direction and \(C_n, S_n\) are coefficients in cross-track direction.

Above empirical model coefficients are always estimated each cycle, so they are called “one-cycle-per-resolution” parameters\textsuperscript{[5]}.

1.3 Pseudo-stochastic-pulses model

The pseudo-stochastic-pulses model, or the use of so-called pseudo-stochastic pulses or small changes in the velocity at user-determined epochs, was developed for GPS orbits by the CODE Analysis Center Team\textsuperscript{[7,8]}. The estimation of such pulses can easily be implemented into the orbit determination procedure due to the nice feature that the partials for such pulses can be computed as a linear combination of the partials with respect to the initial conditions. This approach was applied in the Bernese software, and many high precision results were obtained\textsuperscript{[9,10]}.

2 Orbits prediction with different gravity models

With the actualization of the CHAMP and GRACE satellites plan, the quality of gravity fields models were improved significantly. The gravity field model EIGEN-CG03C with full 360 degrees, which includes the information collected by GRACE and CHAMP and altimeter satellites, and also the terrestrial gravity field data, was published by GFZ in May of 2005. So the gravity field model EIGEN-CG03C was chosen as the “ground field” and full 120 degrees were selected for grace satellite with altitude 400 km. In order to analyze the validity of different approaches, three different quality gravity models, including JEM, EGM96 and EIGEN-CHAMP02 are used in this research.

The orbits in different gravity models were obtained by integrating from the same point wise orbit elements which were derived from the real CHAMP satellite orbit. The orbit in gravity model EIGEN-CG03C was used as the ground one, while other orbits were compared with this one. The maximal differences of the orbit comparisons are given in Table 1.

| Gravity model | Degree | Maximal difference in 6 h/m | Maximal difference in 24 h/m |
|---------------|--------|-----------------------------|-----------------------------|
|               |        | Radial | Along | Cross | Radial | Along | Cross |
| EIGEN2        | 70×70  | 0.56   | 3.10  | 0.36  | 1.27   | 3.75  | 0.88  |
| EGM96         | 70×70  | 3.60   | 10.9  | 2.17  | 5.79   | 30.4  | 2.93  |
| JGM2          | 70×70  | 3.51   | 23.3  | 2.3   | 5.33   | 41.4  | 4.89  |

From Table 1, we can find that the quality of the EIGEN2-CHAMP gravity model is much better than EGM96 and JGM2, but it’s still not good enough for orbit determination if no approaches are used to ab-
sorb the model error. In order to study the trend of the orbit difference, the residuals were given in Figs.1-3. Because of the accumulation of the gravity field model, the difference keeps increasing during the orbit integration, so the acceleration between the different models cannot be derived from the different orbits directly. In this research, the acceleration was calculated from different models along the reference orbits, and were given in the right of Figs.1-3, and only one cycle (1.5 h) was selected.

The acceleration difference in Figs.1-3 show that they are in the level of μm/s² for satellite with 400 km altitude. We can find that the EIGEN2-CHAMP model has better low degree parameters because the acceleration difference between it and the ground model was presented with high frequency. No distinct trend appeared in the residuals serials, so more simulation is needed to determine which approach is better.

3 Compensating for the error of gravity field

During the procedure of precise orbit determination, if the gravity parameters are fixed without turning,
some approaches need be applied to absorb the error of the gravity model. In order to make good use of kinematic and dynamic information, the approaches used to absorb the error of the gravity model has the following rules.

1) The less parameters that need to be estimated in the POD procedure, namely that the approach that describes the error of the gravity model more precisely, the better.

2) The one easier to be applied is better.

3) If the orbit obtained with the approach is smoother, then that approach is the better one.

In this research both CHAMP and GRACE satellites were studied. The orbits of the CHAMP satellite are obtained by simulation while the orbits of the GRACE satellite used are those obtained through real observed data.

### 3.1 Orbit fitting of CHAMP satellite

The orbits obtained in EIGEN2-CHAMP and EGM96 gravity models were fitted to the reference orbit with different compensating methods in seven cases. $A$ is pseudo-stochastic-pulses (3 parameters per 9 min); $B$ is piece wise linear empirical acceleration (3 parameters per 30 min); $C$ is periodical empirical piece-wise linear acceleration (9 parameters per 90 min); $D$ is pseudo-stochastic-pulses (3 parameters per 4 min); $E$ is piece wise linear empirical acceleration (3 parameters per 18 min); $F$ is periodical empirical piece-wise linear acceleration (9 parameters per 90 min); $G$ is periodical empirical piece-wise linear acceleration (6 parameters per 90 min+ 3 parameters per 30 min).

It’s shown in Table 2 that more parameters need to be estimated when pseudo-stochastic-pulses are used if similar RMS of the residuals is required. This is demonstrated by the results obtained in Reference [9], in which the orbits with 3 pseudo-stochastic-pulses parameters per 9 min are better than that with 3 pseudo-stochastic-pulses parameters per 15 min. Residuals of orbit fitting are given in Fig.4 and Fig.5.

| Gravity model | Case | Parameters per 24 h | RMS of fitting residuals/cm |
|---------------|------|---------------------|----------------------------|
|               |      | Radial | Along | Cross |
| EIGEN2        | $A$  | 246    | 1.41  | 1.64  | 0.47  |
| 70x70         | $B$  | 78     | 1.49  | 0.93  | 1.48  |
| EGM96         | $C$  | 78     | 1.56  | 1.19  | 1.83  |
| 70x70         | $D$  | 546    | 1.71  | 1.80  | 0.62  |
|                | $E$  | 150    | 2.17  | 0.87  | 3.39  |
|                | $F$  | 78     | 7.33  | 5.95  | 11.6  |
|                | $G$  | 132    | 3.09  | 1.44  | 3.70  |

### 3.2 Orbit fitting of GRACE satellite

Because the accelerometers on GRACE satellites are good enough, the orbit used in this research are JPL reduced dynamic orbits obtained from real data with 2-3 cm accuracy. The orbit of GRACE-B satellite on day 122 of 2004 was fitted. The no-potential force is observed instead by accelerometer and potential models are used, including third body force (sun and moon), solid, sea tides and relative effect. Only 6 orbit elements and 3 bias and 3 rate parameters were estimated for 24 h orbit. The residuals are given in Fig.6.
Fig. 6 Orbit fitting residuals of GRACE satellite without empirical force model in EIGEN2-CG03C gravity model

From the orbit fitting residuals, we can find the RMS is 1.37 cm, 3.48 cm and 4.74 cm in radial, along and across the track direction, through only 12 parameters estimated per day. This demonstrates the high quality of gravity field model EIGEN-CG03c. Through some periodical error can be found with low frequency, it may be caused by the gravity variation.

If the EIGEN2-CHAMP gravity model is used instead of the EIGEN-CG03C gravity field model, other models are kept and a compensating method used with Cases A, B and C. The residuals are given in Figs. 7-8 and the statistics are listed in Table 3.

Table 3 Orbit fitting result of GRACE satellite in different cases

| Gravity model | Case | Parameters per 24 h | Radial | Along | Cross |
|---------------|------|---------------------|--------|-------|-------|
| EIGEN2 70×70  | A    | 492                 | 0.84   | 1.43  | 0.38  |
|               | B    | 156                 | 0.81   | 0.69  | 0.99  |
|               | C    | 156                 | 0.99   | 1.01  | 0.91  |

From the orbit fitting results, it can be concluded that if EIGEN2-CHAMP gravity model is used, the orbit of satellite with 400 km altitude can be well fitted in Cases A, B and C. However, we found that if the orbit is fitted with the pseudo-stochastic-pulses, more parameters need to be estimated, and the orbits are not as smooth as that fitted with periodical empirical piece wise linear acceleration. So it’s better to use the empirical acceleration model to absorb the error of the gravity model when POD of LEO satellites are applied with real data. High quality orbits obtained in Reference [4] with a compensating
method in Case C can support the results obtained in this research.

4 Conclusions

The error of gravity model becomes one of the main sources that influence the orbit of LEO when the altitude of orbit is as low as 400 km or even lower. If the accuracy required is several centimeters, the approaches used to absorb the error of the gravity model are necessary. On the basis of PANDA software, orbits of both the CHAMP and GRACE satellites are studied in this paper. The results show that the orbit difference can reach several meters or even more if the satellite with altitude of 400 km is predicted for 24 h. The gravity error of different gravity models e.g. EGM96 or EIGEN2-CHAMP with 70 full degrees in 400 km altitude is several million meters and without any obvious characteristics. The pseudo-stochastic- pulses approach, periodical empirical and piece wise linear acceleration model can be used to absorb the error of gravity field model. The pseudo-stochastic-pulses approach needs more parameters if similar results are required, and the orbits obtained with this approach is not as smooth as other methods, so it’s better to use periodical empirical or piece wise linear acceleration model to absorb the error of the gravity model when the altitude of satellites are as low as 400 km.

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