Research Article
Partial Relay Selection with Feedback Delay and Cochannel Interference: Performance Analysis and Power Optimization

Xuanxuan Tang, Weifeng Mou, Fang Fang, Yueming Cai, Weiwei Yang, and Tao Zhang

College of Communication Engineering, PLA University of Science and Technology, Nanjing 210007, China

Correspondence should be addressed to Yueming Cai; caiym@vip.sina.com

Received 12 July 2013; Accepted 11 December 2013; Published 9 January 2014

Academic Editor: Shukui Zhang

Copyright © 2014 Xuanxuan Tang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The performance of a dual-hop amplify-and-forward (AF) relay system is studied with the kth worst partial relay selection (PRS) protocol under the joint impact of feedback delay and multiple cochannel interference (CCI) at the relay nodes. In our analysis, the closed-form and asymptotic expressions of the outage probability are derived and the achievable coding gain and diversity gain of the system are accurately revealed. Further, based on the asymptotic analysis we propose a power allocation scheme, which can lead to SNR performance gains of more than 1 dB compared to the equal power allocation scheme. Finally, Monte-Carlo simulation results are presented to verify the validity of our theoretical analysis.

1. Introduction

Deployment of multiple relays in the wireless communication offers a series of significant performance improvements, including the systems’ throughput ability enhancement and the signal coverage advancement. In such systems, performance can be improved by selecting one of the relay nodes [1–7]. And two popular protocols of relay selection have been presented, that is, opportunistic relay selection (ORS) [1] and partial relay selection (PRS) [2]. In the opportunistic relaying, the source always selects the best relay according to the global (two-hop) channel state information, which results in the great increase of system’s complexity and cost in ad hoc and wireless sensor networks. Such problems have promoted the development of PRS protocol, which requires only single hop information and the network lifetime can be prolonged in resource-constrained wireless systems such as wireless sensor networks.

In practical partial relay system, the PRS is implemented in time-varying channels and the outdated channel state information (CSI) would be used for relay selection due to feedback delay. In other words, the selected relay may not be the best at the instant of actual transmission. In [3], asymptotic lower and upper system capacity of a fixed-gain PRS scheme was investigated with feedback delay. The performance of PRS system affected by feedback delay was quantified by obtaining outage probability and average bit error rate (BER) and further analyzed at high SNR regime in [4]. On the other hand, frequency reuse gives rise to the so-called cochannel interference (CCI) in the cellular network. The authors of [5] investigated the performance of PRS scheme affected by CCI with variable gain relaying and derived the probability density function (PDF) of the received SNR and an asymptotic exponential expression. Kim and Heo studied the performance of PRS system under the impact of CCI and derived the closed-form expression for the outage probability [6]. However, to our knowledge, the works that investigate the joint impact of feedback delay and multiple CCI at relay node have not come out yet.

In this paper, we analyze the performance of a dual-hop amplify-and-forward (AF) relay system under the joint impact of feedback delay and CCI at the relay node. The contributions of this paper are summarized as follows.

(i) With the kth worst PRS protocol [7], the closed-form expression for outage probability of partial relay selection system with feedback delay and CCI is derived, which provides an efficient means to investigate the
impact of the number and power levels of CCI as well as the correlation coefficient $\rho$ between the actual and the outdated CSI on the outage performance.

(ii) We derive the asymptotic expression for outage probability of the system with feedback delay and CCI based on the closed-form expression of outage probability. The results show that the achievable coding gain has much to do with the number and the power levels of CCI and the correlation coefficient $\rho$, and these factors do not affect the diversity gain.

(iii) Based on the asymptotic analysis of the outage probability, we propose a power allocation scheme to improve the system’s performance and the proposed scheme can lead to SNR performance gains of more than 1 dB compared to the equal power allocation scheme. Finally, Monte-Carlo simulation results are presented to verify the validity of our theoretical analysis.

The rest of the paper is organized as follows. Section 2 describes the system model. Section 3 derives the exact and asymptotic outage probability of partial relay selection system with feedback delay and CCI. Section 4 investigates the power allocation problem and presents an optimum solution for the system based on the analysis of outage probability. Section 5 gives the simulation results which verify our analysis. Section 6 concludes the whole paper.

2. System Model

We consider a dual-hop wireless communication network with one source $S$, one destination $D$, and $M$ relays $R_k$ ($k = 1, 2, \ldots, M$); each relay is, respectively, affected by $N$ interferers as shown in Figure 1. There is no direct link between $S$ and $D$. $S$ periodically monitors and arranges the quality of the system with feedback delay and CCI. Particularly, the best one is chosen when $m = M$, similar to [4, 7]. Each node is equipped with a single antenna and works in the half-duplex mode.

The whole communication of $S$ to $D$ takes place in two time slots. In the first time slot, $S$ broadcasts the signal $x(t)$ to the relay $R_k$. In the second time slot, $R_k$ amplifies the received signal by a forward factor $G$ and transmits it to $D$. The received signal at $R_k$ can be represented as

$$y_{R_k}(t) = \sqrt{P_S|h_{SR_k}(t)|^2} x(t) + \sum_{n=1}^{N} \sqrt{P_n|h_{kn}(t)|^2} x_{kn}(t) + n_{R_k}(t),$$

where $P_S$ stands for the transmit power of $S$ and $P_{kn}$ is the power of the interference source $I_{kn}$ ($n = 1, 2, \ldots, N$) at $R_k$ ($k = 1, 2, \ldots, M$). $h_{SR_k}(t)$ and $h_{kn}(t)$ are the channel coefficient between $S-R_k$ and $R_k-I_{kn}$, respectively, which satisfies $E[|h_{SR_k}(t)|^2] = \sigma_{SR_k}^2$, $E[|h_{kn}(t)|^2] = \sigma_{kn}^2$ where $E[\cdot]$ denotes the expectation operation. And $\sigma_{kn}^2 \propto d_{kn}^{-\mu}$, where $d_{ab}$ is the distance between node $a$ and node $b$ and $\mu$ is the path loss factor. $x_{kn}(t)$ is the signal that $I_{kn}$ transmits to $R_k$. $n_{R_k}(t)$ is the additive white gaussian noise (AWGN) which has zero mean and variance $N_0$.

The received signal at $D$ can be written as

$$y_D(t) = G h_{RD}(t) y_{R_k}(t) + n_D(t),$$

where $h_{RD}(t)$ is the channel coefficient between $R_k$ and $D$ satisfying $E[|h_{RD}(t)|^2] = \sigma_{RD}^2$ and $n_D(t)$ is the AWGN which has zero mean and variance $N_0$. The forward factor $G$ is given by

$$G = \frac{P_k}{\sqrt{P_S|h_{RD}(t)|^2 + \sum_{n=1}^{N} P_{kn}|h_{kn}(t)|^2 + N_0}},$$

where $P_k$ stands for the transmit power of $k$th relay $R_k$.

We assume that the instantaneous SNR of $S-R_k$ link is $\gamma_{1(k)} = P_S|h_{SR_k}(t)|^2/N_0$. Arrange $\gamma_{1(k)}$ in an increasing order of magnitude; that is, $\gamma_{1(1)} \leq \gamma_{1(2)} \leq \cdots \leq \gamma_{1(M)}$. The acquired actual CSI by $S$ is outdated in the next transmission time slot and the performance of the system is analyzed according to the outdated CSI with a time delay of $T_d$. Let $\gamma'_{1(k)} = P_S|h_{SR_k}(t)|^2/N_0$ be the delay version of $\gamma_{1(k)}$ of which the mean satisfies $\bar{\gamma}'_{1(k)} = P_S\sigma_{SR_k}^2/N_0$. We assume that the correlation coefficient between $\gamma'_{1(k)}$ and $\gamma_{1(k)}$ is $\rho$ which can be expressed as $\rho = J_0(2\pi f_{dSR_k} T_d)$ according to Jakes’ autocorrelation model, where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind and $f_{dSR_k}$ is the maximum Doppler frequency on the link $S-R_k$.

Based on (1), we can obtain the signal-to-interference-plus-noise ratio (SINR) at $R_k$ as

$$\gamma_0 = \frac{\sum_{k=1}^{M} \gamma_{1(k)}}{\sum_{k=1}^{M} \gamma''_{1(k)}} = \frac{\gamma'_{1(k)}}{\sum_{k=1}^{M} \gamma''_{1(k)}},$$

where $\gamma_k = P_k|h_{kn}(t)|^2/N_0$ is the interference-to-noise ratio (INR) at $R_k$ of which the mean is $\bar{\gamma}_k = P_k\sigma_{kn}^2/N_0$. And $\gamma''_{1(k)} = \gamma_{1(k)} + 1/N$ means the approximate INR at the relay node.
according to [6, Equation (3)], and it can be seen that the results between analysis and simulation are perfectly matched in Section 5. And the mean of $\gamma^{(i)}_k$ is $\mu^{(i)}_k = \bar{\gamma}_k + 1/N$.

The SINR from S to D can be represented as

$$
\gamma_d = \frac{\gamma_0 y_2}{\gamma_0 + y_2 + 1},
$$

where $y_2 = R_k | h_{R_k,D}(t) |^2 / N_0$ is the instantaneous SNR of $R_k-D$ link of which the mean is $\gamma_2 = P_k \sigma^2_{R_k,D} / N_0$.

### 3. Performance Analysis

#### 3.1. Outage Probability Analysis

The outage probability $P_{\text{out}}$ is defined as the probability that the information of the selected channel falls below the transmission rate $R$ (bits/s/Hz). The outage probability can be expressed as

$$
P_{\text{out}} = \Pr (\log_2 (1 + \gamma_d) < R) = \Pr (\gamma_d < \gamma_{\text{th}}),
$$

where $\gamma_{\text{th}} = 2^R - 1$.

By (5), the outage probability can be expressed as

$$
P_{\text{out}} = \int_0^\infty \Pr (\frac{\phi y_2}{\phi + y_2 + 1} \leq \gamma_{\text{th}}) f_{\gamma_2}(\phi) \, d\phi. \tag{7}
$$

Denoting $\gamma = \phi - \gamma_{\text{th}}$ and after some manipulations, the expression above can be written as

$$
P_{\text{out}} = 1 - \int_0^\infty \Pr (\frac{\gamma_{\text{th}}^2 + \gamma_{\text{th}} y + \gamma y}{y}) f_{\gamma_2}(\gamma_{\text{th}} + y) \, dy. \tag{8}
$$

The cumulative distribution function (CDF) of $\gamma_2$ is

$$
F_{\gamma_2}(x) = 1 - e^{-x/\Gamma_2}. \tag{9}
$$

According to the CDF of $\gamma_2$ we can derive

$$
\Pr \left( \frac{\gamma_{\text{th}}^2 + \gamma_{\text{th}} y + \gamma y}{y} \right) = e^{-\gamma_{\text{th}} + \gamma_{\text{th}} y + \gamma y/\gamma_{\text{th}}}. \tag{10}
$$

To calculate (8), we need to further calculate $F_{\gamma_2}(x)$. Firstly, we get the probability density function (PDF) of $\gamma_{\text{th}}^{(k)}$ by [7, Equation (9)]

$$
f_{\gamma_{\text{th}}^{(k)}}(x) = \sum_{m=0}^{k-1} A e^{-C x}, \tag{11}
$$

where

$$
A = \frac{(-1)^m k \binom{M}{M-k} \binom{k-1}{m-1}}{\left[(M-k+m) (1-\rho) + 1 \right] \gamma_{\text{th}}^{(k)}},
$$

$$
C = \frac{M-k+m+1}{\left[(M-k+m) (1-\rho) + 1 \right] \gamma_{\text{th}}^{(k)}}.
$$

After some manipulations, we derive the CDF of $\gamma_{\text{th}}^{(k)}$ as

$$
F_{\gamma_{\text{th}}^{(k)}}(x) = \sum_{m=0}^{k-1} A \left(1 - e^{-C x} \right). \tag{13}
$$

Secondly, with the help of [8], the PDF of $\sum_{n=1}^{N} \gamma_{\text{th}}^{(n)}$ is given as

$$
f_{\sum_{n=1}^{N} \gamma_{\text{th}}^{(n)}}(y) = \frac{\rho(E) \tau(E)}{\Gamma(j)} \prod_{i=1}^{N} \gamma_{\text{th}}^{(1)} y_{i}^{j-1} e^{-y/\Gamma_2}, \tag{14}
$$

where $E = \text{diag} (\gamma_{1}, \gamma_{2}, \ldots, \gamma_{N})$, $\rho(E)$ is the number of distinct diagonal elements of $E$, $\gamma_{i}$ is the multiplicity of $\gamma_{i}$, and $\chi_{i,j}(E)$ is the $(i,j)$th characteristic coefficient of $E$.

According to (4), the CDF of $\gamma_0$ is

$$
F_{\gamma_0}(x) = \int_0^\infty \Pr \left( \gamma_{\text{th}}^{(k)} < xy \right) f_{\sum_{n=1}^{N} \gamma_{\text{th}}^{(n)}}(y) \, dy. \tag{15}
$$

By [9, Equation (3.351.3)] and after some manipulations we get

$$
F_{\gamma_0}(x) = \sum_{m=0}^{k-1} A \sum_{j=1}^{\rho(E)} \sum_{i=1}^{\tau(E)} \chi_{i,j}(E) \left( \frac{C x + 1}{\gamma_{\text{th}}^{(1)}} \right)^{-j}. \tag{16}
$$

Then, the PDF of $\gamma_0$ can be found by taking the derivative of $F_{\gamma_0}(x)$ with respect to $x$. After some manipulations, the PDF of $\gamma_0$ can be written as

$$
f_{\gamma_0}(x) = \sum_{m=0}^{k-1} A \sum_{j=1}^{\rho(E)} \sum_{i=1}^{\tau(E)} \chi_{i,j}(E) \left( \frac{C x + 1}{\gamma_{\text{th}}^{(1)}} \right)^{-j-1}. \tag{17}
$$

Substituting (10) and (17) into (8) and by [9, Equation (3.471.7)] we get

$$
P_{\text{out}} = 1 - \sum_{m=0}^{k-1} \left( \frac{(-1)^m k \binom{M}{M-k} \binom{k-1}{m-1}}{\left[(M-k+m) (1-\rho) + 1 \right] \gamma_{\text{th}}^{(k)}} \right) \times \frac{\rho(E) \tau(E)}{\Gamma(j)} \prod_{i=1}^{N} \gamma_{\text{th}}^{(1)} y_{i}^{j-1} e^{-y/\Gamma_2},
$$

$$
x = \frac{\gamma_{\text{th}} + \left( \frac{(M-k+m) (1-\rho) + 1}{\gamma_{\text{th}}^{(1)}} \right)}{\Gamma(j)}.
$$
where $\Phi = 1$ is the diversity gain, which determines the slope of the asymptotic outage probability curve, and

$$
\Psi = \frac{y_{th}}{\sigma_{RD}^2} + \frac{y_{th}^{k-1}}{\sigma_{SR}^2} \sum_{m=0}^{k-1} \frac{(-1)^m k \left(\begin{array}{c} M \\k \end{array}\right) \left(\begin{array}{c} k-1 \\m \end{array}\right)}{(M-k+m)(1-\rho)+1} \\
\times \sum_{i=1}^{\rho(E)} \sum_{j=1}^{\tau_i(E)} X_{i,j} \left(Y_{th} f_{R_{th}}\right),
$$

(24)

is the coding gain, which characterizes the SNR advantage of the asymptotic outage probability relative to the reference curve $f_{th}$, where $f_{th} = P_{th}/N_0$. From (23) we can see that the coding gain has much to do with the number and the power levels of CCI and the correlation coefficient $\rho$, and these factors do not affect the diversity gain. This is because our analysis is based on the kth worst PRS protocol, which is low-complexity and unbenevolent to the diversity gain.

### 4. Optimum Power Allocation

In this section, the problem of power allocation between the source and the relay is investigated for the improvement of system performance. The total power of system is set to be $P_T$ and the power of source and relay is $P_S = \beta P_T$, $P_k = (1-\beta)P_T$ (0 < $\beta$ < 1), respectively; thus $f_{th}$ and $f_{2}$ can be written as

$$
\bar{f}_{th}(k) = \frac{P_S\sigma_{SR}^2}{N_0} = \frac{\beta P_T\sigma_{SR}^2}{N_0},
$$

(25)

$$
\bar{f}_{2} = \frac{P_k\sigma_{RD}^2}{N_0} = \frac{(1-\beta)P_T\sigma_{RD}^2}{N_0}.
$$

(26)

Substituting (25) into (23) we get

$$
P^{\infty}_{out} \approx \frac{M_1}{(1-\beta) + M_1},
$$

(27)

where

$$
M_1 = \frac{y_{th} N_0}{P_T \sigma_{SR}^2} \sum_{i=1}^{\rho(E)} \sum_{j=1}^{\tau_i(E)} X_{i,j} \left(Y_{th} f_{R_{th}}\right) \left(Y_{th} f_{R_{th}}\right),
$$

(28)

The derivation of $P^{\infty}_{out}(y_{th})$ with respect to $\beta$ can be calculated as

$$
\frac{\partial P^{\infty}_{out}}{\partial \beta} = \frac{\partial \left(\left(M_2/(1-\beta)\right) + (M_1/\beta)\right)}{\partial \beta} = \frac{M_2}{(1-\beta)^2} - \frac{M_1}{\beta^2}.
$$

(29)

Let (28) be zero; then the optimum $\beta^*$ can be given by

$$
\beta^* = \frac{M_1 - \sqrt{M_1 M_2}}{M_1 - M_2}.
$$

(30)
It is easy to note that the optimum power allocation solution is related to feedback delay and CCI as well as the channel information. When the power allocation of the system is set to be $P_S = \beta^2 P_T$, $P_k = (1 - \beta^2) P_T$, the performance can be improved, which can be verified in Section 5.

5. Simulation Results and Discussions

In this section, numerical results are carried out to demonstrate the validity of our theoretical analysis. In all cases, we assume that $M = 3$ and the relay node is, respectively, affected by $N = 3$ interferers and we select the $k$th relay $R_k$ to forward the information. In our simulation, we set $k = M$ and all the interferers have the same transmit power, which is $\alpha$ times the average power of the noise. We assume that the distance of $S-R_k$, $d_{SR_k} = 1$ and the distance of $R_k-D$, $d_{R_kD} = 0.9 d_{SR_k}$. The distance between interferers and $k$th relay node forms vector $d = [3.5, 3.5, 2.5]$. The path loss factor $\mu$ is assumed to be $\mu = 3$. The transmit rate is set to be $R = 2$ (bits/s/Hz).

Figures 2–4 present the joint impact of feedback delay and CCI on the outage probability versus the total SNR ($P_T/N_0$) of the transmission path. As we can see, the simulation is in exact agreement with theoretical analysis and the asymptotic curve is very tight at high SNR regime, which verifies the correctness of our analysis.

In Figure 2, the outage probability versus the average SNR for different power level of the interference $\alpha$ is shown. We assume that $P_S = P_k = 0.5P_T$, $\rho = 0.8$, and $\alpha = 0, 10, 100$, respectively. It can be seen that the performance of the system decreases rapidly with the increase of the power of interference at almost the whole SNR regime. From the asymptotic results, we can see that the diversity is 1 for different power level of the interference and the coding gain decreases with the increase of the power of interference, which verify the validity of the theoretical analysis in (23).

In Figure 3, the outage probability versus the average SNR for different correlation coefficient $\rho$ is shown. We assume that $P_S = P_k = 0.5P_T$, $\alpha = 100$, and $\rho = 0, 0.8, 1$, respectively. We can see that small $\rho$ leads to high outage probability, which means that high feedback delay will significantly decrease the performance of system. In addition, with the increase of $\rho$, the performance gain is not so noticeable at low SNR regime, which means that at low SNR regime the system performance becomes less sensitive to the feedback delay. Especially, when $\rho = 1$ (which means there is no feedback delay) the system can gain the best outage performance. What is more, from the asymptotic results we can see that the diversity is 1 for different $\rho$ and the coding gain increases with the increase of the correlation coefficient $\rho$, which demonstrate the validity of the theoretical analysis in (23).

In Figure 4, we compare the system with optimum power allocation to that with equal power allocation in terms of outage probability. From (29), we know that the optimum power allocation has much to do with the number and the power levels of CCI and the correlation coefficient $\rho$. In our analysis, we assume that $\alpha = 100$ and $\rho = 0.5$. And at an outage probability level of $10^{-2}$, the performance of the system with optimum power allocation leads to SNR gains of more than 1 dB compared to the equal power allocation. As we can see, there is noticeable performance gain after power optimization.
6. Conclusion

In this paper, a dual-hop AF wireless network is studied with the joint impact of interference and feedback delay. We derived the closed-form and asymptotic expressions of outage probability and further discussed the diversity order of the system at high SNR regime. Monte-Carlo simulation results were presented for the validity of theoretical analysis. The findings suggest that the diversity order is one under the joint impact of interference and feedback delay regardless of the number of relays. In addition, the performance of system is affected by both the interference and feedback delay, but at low SNR regime, the impact of feedback delay is not very noticeable while the interference functions almost at the whole SNR regime. Finally, we investigated the power optimization problem for the system and found that the optimum solution is related to feedback delay and CCI. The system performance of our specific optimum solution leads to SNR gains of more than 1 dB compared to the equal power allocation, which is a noticeable performance gain, indicating that our solution is valuable to practical system designing.

Conflict of Interests

The authors declare that they have no competing interests.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (nos. 61001107, 61301162, and 61301163) and the Jiangsu Provincial Natural Science Foundation of China (no. BK20130067).

References

[1] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 659–672, 2006.
[2] I. Krikidis, J. S. Thompson, S. Michalopoulos, and G. K. Karagiannidis, "Semi-blind amplify and forward with partial relay selection," *IEEE Communications Letters*, vol. 45, pp. 235–237, 2008.
[3] E.-H. Shin and D. Kim, "On the capacity growth of opportunistic single-relay selection using quality-based feedback in fixed-gain amplify-and-forward relaying," *IEEE Communications Letters*, vol. 13, no. 11, pp. 835–837, 2009.
[4] H. A. Suraweera, M. Soysa, C. Tellambura, and H. K. Garg, "Performance analysis of partial relay selection with feedback delay," *IEEE Signal Processing Letters*, vol. 17, no. 6, pp. 531–534, 2010.
[5] I. Krikidis, J. S. Thompson, S. McLaughlin, and N. Goertz, "Amplify and forward with partial relay selection," *IEEE Communications Letters*, vol. 12, no. 2, pp. 235–237, 2008.
[6] S.-I. Kim and J. Heo, "Outage probability of interference-limited amplify-and-forward relaying with partial relay selection," in *Proceedings of the IEEE 73rd Vehicular Technology Conference (VTC '11)*, pp. 1–5, Yokohama, Japan, May 2011.
[7] M. Soysa, H. A. Suraweera, C. Tellambura, and H. K. Garg, "Amplify-and-forward partial relay selection with feedback delay," in *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC '11)*, pp. 1304–1309, Cancun, Mexico, March 2011.
[8] D. Lee and J. H. Lee, "Outage probability of decode-and-forward opportunistic relaying in a multicell environment," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 4, pp. 1925–1930, 2011.
[9] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, San Diego, Calif, USA, 7th edition, 2007.
