THE KINETIC SUNYAEV–ZEL'DOVICH EFFECT FROM REIONIZATION: SIMULATED FULL-SKY MAPS AT ARCMINUTE RESOLUTION

MARCELO A. ALVAREZ
Canadian Institute for Theoretical Astrophysics, 60 St. George Street, Toronto, ON M5S 3H8, Canada

Received 2015 November 24; accepted 2016 March 22; published 2016 June 21

ABSTRACT

The kinetic Sunyaev–Zel'dovich (kSZ) effect results from Thomson scattering by coherent flows in the reionized intergalactic medium. We present new results based on ray-tracing an 8 Gpc/h realization of reionization with resolution elements 2 Mpc/h (subtending ~1′ at z = 6) on a side to create a full-sky kSZ map. The realization includes, self-consistently, the effects of reionization on scales corresponding to multipolos 10 < ℓ < 5000. We separate the kSZ map into Doppler ⟨v⟩, Ostriker–Vishniac ⟨dv⟩, patchy ⟨vv⟩, and third-order ⟨avv⟩ components, and compute explicitly all the auto- and cross-correlations (e.g., ⟨vv⟩, ⟨dvvv⟩, etc.) that contribute to the total power. We find a complex and nonmonotonic dependence on the duration of reionization at ℓ ∼ 300 and evidence for a non-negligible (10%–30%) contribution from connected four-point correlations, ⟨vvvv⟩, usually neglected in analytical models. We also investigate the cross-correlation of linear matter and large-scale kSZ temperature fluctuations, focusing on (1) cross-power spectra with biased tracers of the matter density and (2) cold spots from infall onto large, rare H II regions centered on peaks in the matter distribution at redshifts z > 10 that are a generic non-Gaussian feature of patchy reionization. Finally, we show that the reionization history can be reconstructed at 5σ–10σ significance by correlating full-sky 21 cm maps stacked in bins with Δν = 10 MHz with existing cosmic microwave background (CMB) temperature maps at ℓ < 500, raising the prospects for probing reionization by correlating CMB and LSS measurements. The resulting kSZ maps have been made publicly available at www.cita.utoronto.ca/~malvarez/research/ksz-data/.

Key words: cosmic background radiation — cosmology: theory — dark ages, reionization, first stars — intergalactic medium — large-scale structure of universe — methods: numerical

1. INTRODUCTION

Secondary anisotropies of the cosmic microwave background (CMB) are an excellent probe of reionization, mainly because scattering of photons by free electrons during reionization affects the observed temperature and polarization in a predictable fashion that is sensitive to the spatial structure of reionization and its correlation with the underlying potential fluctuations. For reviews of the main effects on secondary anisotropies associated with reionization, in the larger context of the effects associated with large-scale matter fluctuations, dark matter and gas in halos and filaments, and the relation between cosmic expansion and growth of structure, see Hu & Dodelson (2002) and Aghanim et al. (2008).

Of particular focus in the present work is the kinetic Sunyaev–Zel'dovich (kSZ) effect, which refers to blackbody temperature fluctuations induced by the Doppler shift of CMB photons scattering off of electrons in coherent bulk flows. Although it has long been recognized as one of the most promising probes of the intergalactic medium during and after reionization (e.g., Sunyaev et al. 1978; Kaiser 1984; Ostriker & Vishniac 1986), it has begun to be used only recently to provide constraints on reionization through the analysis of the angular power spectrum of the CMB temperature at ℓ ≈ 3000 (Reichardt et al. 2012; Zahn et al. 2012; George et al. 2015). Significant gains in the accuracy of kSZ power spectrum measurements are expected from new CMB experiments coming online in the near term (e.g., Calabrese et al. 2014). A main goal of this work is to refine theoretical predictions for the kSZ effect from patchy reionization and extend existing results to larger scales. Before doing so, we give below a brief historical overview of the development of the field.

Purely blackbody temperature fluctuations arising from the Doppler shift of CMB photons scattered by coherent motions were first discussed by Chibisov & Ozernoy (1969), in the context of “whirl” turbulent velocity perturbations on cosmological scales. Sunyaev & Zeldovich (1970a) also discussed the Doppler effect, but only for velocity fluctuations at recombination; their discussion of anisotropies generated during or after reionization was limited to Comptonization as opposed to coherent Doppler shifting. The effect from coherent motions of galaxy clusters was first discussed by Sunyaev & Zeldovich (1972) and has come to be known as the kSZ effect. Sunyaev & Zel’dovich argued that the temperature fluctuation in the Rayleigh–Jeans part of the spectrum in the direction of galaxy clusters should be dominated by Comptonization, rather than bulk motion of the cluster itself—Thermal Sunyaev–Zel'dovich Effect (tSZ) should dominate over kSZ for individual galaxy clusters, resulting generically in a decrement at low frequencies.

The first appearance in the literature of the secondary temperature anisotropies arising from linear velocity perturbations due to the growth of structure in a diffuse reionized intergalactic medium was in Sunyaev (1977), where it was estimated that secondary temperature anisotropies from the Doppler effect could conceivably exceed those generated at recombination, provided that τ ∼ 1 and density perturbations on scales corresponding to galaxy clusters were of order unity at reionization. Obviously this is not the case, and the strong sensitivity to the amplitude of perturbations, density of the universe, and timing of reionization was pointed out by Sunyaev et al. (1978), where the now well-known line-of-sight Doppler cancellation at small scales was first sketched out. Later calculations, in particular by Kaiser (1984), Ostriker &
Vishniac (1986), and Vishniac (1987), improved greatly on the accuracy of Sunyaev’s initial estimates.

In a somewhat arbitrary adoption of nomenclature, the Doppler effect has come to describe large-scale anisotropies from purely linear velocity perturbations, while the kSZ effect has come to correspond to all secondary anisotropies that depend on electron density fluctuations, including those due to linear and nonlinear gas density fluctuations, individual clusters and groups of galaxies, and the patchiness of reionization. In this paper, the traditional usage of “Doppler” will be retained, but “kSZ” will be used to refer to any blackbody temperature fluctuation arising from bulk motion integrated along the line of sight, including the Doppler effect. It is in this sense that we refer to the Doppler effect as responsible for large-angle kSZ anisotropies in the rest of the paper.

The power spectrum of kSZ fluctuations is a sensitive probe of the patchiness and duration of reionization at small to intermediate angular scales, ℓ ∼ 1000, due to the transfer of large-scale velocity perturbations to small scales by fluctuations in the ionized fraction on scales l ∼ R/Mpc ≲ 100 (e.g., Gruzinov & Hu 1998; Jaffe & Kamionkowski 1998; Knox et al. 1998; Gnedin & Jaffe 2001). Development in the subject has been mostly theoretical, with numerical simulations playing an increasingly important role in refining the likely shape and amplitude of the angular power spectrum (Valageas et al. 2001; Zhang et al. 2004; McQuinn et al. 2005; Iliev et al. 2007; Jelić et al. 2010; Tashiro et al. 2011; Mesinger et al. 2012; Visbal & Loeb 2012; Battaglia et al. 2013; Park et al. 2013).

Preliminary analysis by the SPT collaboration (Reichardt et al. 2012) determined a 2σ upper limit on D_{3000}, where D_{7} ≡ ℓC_{ℓ}(2π), of 2.8 μK^2 in the case where the thermal SZ—Cosmic Infrared Background (CIB) correlation is assumed to be zero, and 6.7 μK^2 when a tSZ—CIB correlation is allowed. After additional observation and analysis, the SPT constraint was considerably tightened. When the bispectrum of the tSZ is used as an additional constraint, SPT data imply D_{3000} = 2.9 ± 1.3 μK^2 (George et al. 2015). Several authors have discussed the implications of the ℓ ∼ 3000 kSZ measurements for models of patchy reionization, generally interpreting upper limits on D_{3000} as upper limits on the duration of reionization (Mesinger et al. 2012; Zahn et al. 2012; Battaglia et al. 2013), although it has been pointed out that this is only generally true for reionization scenarios in which the bulk of ionization is from UV photons produced by a quickly growing population of galaxies in dark matter halos with masses ≥10^{9} M_☉ (Park et al. 2013).

The kSZ anisotropies on larger scales are much more challenging to detect, due to line-of-sight cancelation and the dominance of the primary temperature fluctuations (Sunyaev et al. 1978; Kaiser 1984; Vishniac 1987; Hu et al. 1994), and are thus usually considered entirely negligible (Dodelson & Jubas 1995; Knox et al. 1998; Valageas et al. 2001; Hu & Dodelson 2002; Ma & Fry 2002). Nevertheless, the kSZ component could be isolated by reconstructing the velocity field on fairly large scales, by using the fact that density and velocity are correlated. Tidal reconstruction in general is a method that is promising due to its quadratic dependence on the underlying field to be estimated, making it less susceptible to systematic effects such as foreground contamination than linear cross-correlations (e.g., Pen et al. 2012). The cross-correlation is still an important quantity to characterize, however, due to its straightforward interpretation. The most likely tracer of large-scale structure in the Epoch of Reionization (EOR) to be correlated with CMB is the 21 cm background, as first discussed by Cooray (2004), followed by initial attempts at correlating simulated maps by Salvaterra et al. (2005).

On large scales (ℓ ∼ 100), where the patchiness of reionization averages out, Alvarez et al. (2006) found a substantial CMB—21 cm cross-correlation due to the Doppler effect, sensitive to the H II region bias and reionization history, and first demonstrated that linear matter and kSZ temperature fluctuations induced by peculiar velocity effects are anticorrelated, such that density enhancements during reionization result in cold spots in CMB secondary anisotropies. They found that such a correlation would be detectable with a futuristic experiment like SKA. Giannantonio & Crittenden (2007) calculated the Doppler-matter cross-correlation in a more general context, confirming the earlier results of Alvarez et al. (2006) on the sign and shape of the correlation. Adshead & Furlanetto (2008) and Tashiro et al. (2010) made similar predictions but were pessimistic about its detectability, due to cosmic variance. However, these studies used simplified analytical expressions for the ionization—density correlation to estimate the CMB—21 cm power spectrum at a given frequency, without stacking frequency maps to increase the strength of the correlation.

More accurate theoretical predictions for the kSZ fluctuations on intermediate to large angular scales require realistic realizations in large volumes, in order to capture large-scale fluctuations in both the ionization field and velocity, which are generally correlated. Jelić et al. (2010) carried out simplified radiative transfer simulations in boxes of size 10^3/h Mpc on a side, corresponding roughly to an angular scale of about a degree. They found that on intermediate scales of ℓ ∼ 10^3 the correlation will be swamped by the primary CMB fluctuations, while at smaller scales the signal is too small to be detected. They were unable to probe to larger scales, where the cross-correlation is expected to be significantly enhanced, because of the limited simulation volume. The need for more realistic calculations of the kSZ effect and its correlation with the redshifted 21 cm background during reionization served as one of the original motivations for the present work.

As an aside, it is worth mentioning spectral distortions arising from Comptonization of CMB photons in the context of reionization. Spectral distortions from hot ionized gas are referred to historically as the thermal Sunyaev—Zel’dovich effect (tSZ; Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1970b) and are most readily detected in the direction of galaxy clusters (Sunyaev & Zeldovich 1972). However, the energy release associated with feedback from early structure formation was in fact the initial motivation for suggesting that deviations from the Planck spectrum in the CMB could result from Compton scattering off of hot electrons at z < 1000 (Weymann 1966). Bulk motions along the line of sight, as discussed by Zel’dovich et al. (1972), produce a nearly identical y-type distortion to that produced by Comptonization, the crucial difference being that, in the latter case, the effect depends on the optical-depth-weighted line-of-sight velocity dispersion rather than electron temperature (Hu et al. 1994; Chluba & Sunyaev 2004). While such y-type spectral distortions could in principle be detected with advanced CMB experiments (e.g., Hill et al. 2015), the present work is limited to blackbody temperature fluctuations produced by coherent motions.
The outline of this paper is as follows. In Section 2 we give a brief review of the basics of the kSZ effect, followed by simplified expressions for the large-scale ($\ell \lesssim 200$) CMB temperature fluctuations expected in currently favored reionization scenarios, ending with some new estimates of the signature of individual HII regions in the CMB. In Section 3 we describe our method of creating kSZ and 21 cm maps from parameter sets, consistent with results from Hinshaw et al. 2013, "WMAP 2013" and Planck Collaboration et al. (2015, “Planck 2015”).

2. LARGE-SCALE kSZ FROM REIONIZATION

In this section, the expressions that determine the kSZ fluctuation along a given line of sight, as well as its power spectrum, are presented first for an arbitrary electron momentum field separated into curl and divergence-free components. Analytical expressions are obtained using linear theory, and it is shown how the fluctuations created while the ionized fraction is evolving rapidly, as expected during reionization, are qualitatively different from those generated at later times, when the universe is nearly fully ionized. Finally, an estimate of the size and amplitude of temperature fluctuations created by rare, isolated HII regions along the line of sight is given.

2.1. Basic Expressions

The optical depth to Thomson scattering along a direction $\hat{\gamma}$ in the sky is given by

$$\tau(\hat{\gamma}) = \int dz g(z)[1 + \delta_e(\chi)],$$

(1)

where $\delta_e$ is the electron density contrast, $\delta_e = \delta + \delta_i + \delta_e$, $\delta$ is the gas density contrast, and the ionization contrast is defined to be $\delta_i = x_i - 1$, where $x_i$ is the ionized fraction and $\chi(z)$ is the mean, volume-averaged, ionized fraction, and $n_{e,0} = [1 - (4 - N_{\text{He}})Y/4] \Omega_b h^2_{\text{ref}}/m_p$ is the mean electron number density, with a helium mass fraction of $Y = 0.24$. The number of helium ionizations per hydrogen ionization is set to $N_{\text{He}} = 1$ and 2 for $z > 3$ and $z < 3$, respectively, so that helium is singly ionized along with hydrogen and He II reionization occurs instantaneously at $z = 3$. The visibility function is defined to be

$$g(z) = \frac{\partial \langle \tau \rangle}{\partial \chi} = \sigma_T n_{e,0} x(z)(1 + z)^2 \equiv g_0(z)x(z),$$

(2)

where $\langle \tau \rangle$ is the mean optical depth over the sky.

The kSZ temperature fluctuation in a direction $\hat{\gamma}$ is

$$\frac{\Delta T(\hat{\gamma})}{T_{\text{cmb}}} = \int dz v(\chi) \cdot \hat{\gamma} = \int dz g(z) q(\chi) \cdot \hat{\gamma},$$

(3)

where the specific momentum $q = (1 + \delta_e) v = (1 + \delta)(1 + \delta_e) v$. This momentum can be separated into transverse and longitudinal components, according to whether its Fourier transform is perpendicular or parallel to wavenumber, respectively, $q_{\perp} = q - (q \cdot \hat{k}) \hat{k}$ and $q_{\parallel} = (q \cdot \hat{k}) \hat{k}$.

The expression for the angular power spectrum is

$$C_\ell = C_\perp^0 + C_\parallel^0 = \int dz \int d\chi \int d\chi'[F_\perp^0(\chi, \chi') + 2F_\perp^0(\chi, \chi')],$$

(4)

where

$$F_\perp^0(\chi, \chi') = \frac{1}{\pi} \int d\mathbf{k} P_f(k, \chi) j_{\perp}(k) j_{\perp}(k'),$$

(5)

$$(2\pi)^3 P_f(k, \chi, \chi') \delta(k - k') = \langle f(k, \chi)f^*(k', \chi') \rangle$$

$$f_{\perp}(k, \chi) \equiv g(\chi) q_{\perp}(k, \chi),$$

(6)

and

$$f_{\parallel}(k, \chi) \equiv \frac{\partial}{\partial \chi} g(\chi) q_{\parallel}(k, \chi).$$

(7)

Note that $C_\parallel^0$ is an integral over the power spectrum of the derivative of $g(z)q_{\parallel}$ with respect to comoving distance. It falls off strongly toward small scales (Dodelson & Jubas 1995; Knox et al. 1998; Valageas et al. 2001; Hu & Dodelson 2002; Mu & Fry 2002) because the visibility function changes slowly along the line of sight, in comparison to the wavelength of the velocity perturbation, leading to nearly complete cancellation (Sunyaev et al. 1978; Kaiser 1984; Vishniac 1987; Hu et al. 1994).
2.2. Angular Power Spectrum of Doppler Effect

Approximating the velocity using linear perturbation theory, \(v(k) = -i a H_\ell \delta(k)/k^2\), where \(f(z) \equiv d \ln D / d \ln a\), and only keeping the leading-order term in \(q_\parallel(k)\), so that \(f_\parallel(k) \rightarrow \partial g(z) v(k) / \partial \chi\), we obtain

\[
F_\parallel \approx F_\parallel^{(1)} = \frac{1}{\pi} \frac{\partial u}{\partial \chi} \int \frac{dk}{k^2} P(k) \delta_\parallel(k) \delta_\ell(k),
\]

where \((2\pi)^3 P(k) \delta^3(k - k') = \langle \delta(k) \delta^*(k') \rangle\),

\[
U(\chi, \chi') \equiv \frac{\partial u}{\partial \chi} \frac{\partial u'}{\partial \chi'}
\]

and

\[
u(z) \equiv g(z) D(z) / D(z) / (1 + z) = u_0(z) x(z).
\]

Shown in Figure 1 is the contribution per redshift interval to the total signal,

\[
\frac{\partial^2 \ell^2 C_\ell}{2\pi \partial \ell \partial \ell'} = \frac{1}{\pi^2} \frac{\partial u}{\partial \chi} \frac{\partial u'}{\partial \chi'} W_\ell(\chi, \chi').
\]

Two cases are shown: uniform ionization (i.e., no recombination, \(x = 1\); upper left in each panel), and an analytical reionization history \(^7\) given by

\[
x(z) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{y(z) - y}{\Delta y} \right) \right],
\]

where \(y(z) = (1 + z)^{1/2}\) and \(\Delta y = 3/2(1 + z_r)^{1/2}\Delta z\), and we have set \(z_r = 10\) and \(\Delta z = 0.5\) (lower right in each panel).

There is a stark difference between a reionizing universe, in which the ionization fraction goes from \(\sim 0\) to \(\sim 1\) over a relatively short interval of redshift, and one in which the ionization fraction is constant, \(x = 1\). The realistic scenario exhibits a strongly peaked contribution produced at the “surface of re-scattering” at \(z \sim z_r = 10\) (lower right half of each panel). This is due to incomplete cancelation of the line-of-sight velocity modes, as the ionization fraction evolves significantly even across the velocity perturbation itself (Alvarez et al. 2006). The “no-reionization” scenario, however, shows no such peak, and the contribution is much more smoothly distributed with redshift and confined to relatively larger scales.

For the case of instantaneous reionization, in which the ionized fraction goes from zero to one instantaneously at some redshift \(z_\star\),

\[
U = U_0 + \delta(z_\star - \chi) \delta(\chi' - \chi_\star) u_0 u_0' - 2\delta(z - \chi_\star) u_0 \frac{\partial u_0'}{\partial \chi'},
\]

where \(U_0 \equiv (\partial u_0 / \partial \chi)(\partial u_0' / \partial \chi')\). Combining Equations (4), (8), and (13), the power spectrum from linear longitudinal velocity fluctuations induced by instantaneous reionization is

\(\text{Figure 2. Angular power spectrum of the Doppler effect for instantaneous reionization occurring at } z = 10. \text{ The dotted curve labeled } C_\ell^{R}\text{ corresponds to the contribution from velocity fluctuations projected onto the re-scattering surface. Note that those fluctuations are larger and peak at smaller scales than the integrated term, } C_\ell^{R}. \text{ The factor } 2C_\ell^{RD}\text{ accounts for the partial cancellation of the re-scattering surface fluctuations by flows on the near side.}\)

\(\text{Figure 3. Angular power spectrum of the Doppler effect for different reionization histories given by Equation (12), with } z_r = 10\text{ and } \Delta z_{\text{reion}} = 0.1, 1, \text{ and } 2, \text{ as labeled. The inset shows the reionization history for each case.} \text{ The shorter the duration of reionization at fixed redshift, the larger the amplitude of the power spectrum.}\)

\(\text{The term } C_\ell^{R}\text{ corresponds to velocity fluctuations projected onto the re-scattering surface at } z_\star, \text{ while } C_\ell^{RD}\text{ corresponds to fluctuations produced from growing velocity fluctuations, i.e., when } x = \text{ constant and density peaks correspond to hot spots. Shown in Figure 2 is the angular power spectrum for instantaneous reionization at } z_\star = 10, \text{ for which } \tau_{\text{cs}} \approx 0.09.\)
A broad peak is evident at $\ell \sim 20$–30, with the re-scattering surface term, $C_\ell^s$, dominating over the other terms for $\ell \gtrsim 10$.

Shown in Figure 3 are the angular power spectra obtained for $z_r = 10$ and $\Delta z = 0.1, 0.5$, and 0.967 in Planck 2015. The strength of the fluctuations is also linearly dependent on $\Omega_m h^2$ and $\sigma_8$, which reflect the strength of the velocity at fixed perturbation amplitude and the amplitude of perturbations, respectively.

The latest results from the Planck Collaboration indicate a value of $\tau = 0.066 \pm 0.013$ at 1σ for the “Planck TT+lowP+lensing+BAO” data combination, with a 2σ upper limit of $\tau = 0.092$. When the Planck temperature and polarization data are considered without lensing (“Planck+lowP”), the constraint changes to $\tau = 0.078 \pm 0.019$, implying a 2σ upper limit of $\tau = 0.116$. The strong dependence of the large-scale signal on optical depth is shown in Figure 4, where the peak amplitude of the angular power spectrum, $\ell^2 C_\ell^{\text{pk}}/(2\pi)$, is plotted versus $\tau$ for the two fiducial cosmologies calculated. For currently favored values of $\tau \sim 0.07$–0.08, the peak values are $\sim 15$–20 $\mu K^2$, while for $\tau = 0.15$ the peak amplitude is expected to lie in the range 60–70 $\mu K^2$. Its dependence on $\tau$ is approximated with a nearly quadratic function,

$$\frac{\ell^2 C_\ell^{\text{pk}}}{2\pi} \approx 30 \mu K^2 \left(\frac{\tau}{0.1}\right)^{1.9},$$

shown as the dashed line in Figure 4. This is quite close to the simple scaling relationship given by Kaiser (1984), $C_\ell \approx \langle v^2 \rangle \tau^2$, just slightly shallower. This is consistent with $\langle v^2 \rangle$ having a weak inverse dependence on $\tau$, since higher $\tau$ corresponds to somewhat higher redshifts, when the linear velocity perturbations are smaller.

Figure 4. Peak amplitude of the Doppler effect power spectrum vs. $\tau$ for the two fiducial cosmological parameter sets considered. The dotted line corresponds to the fit given in Equation (15).

2.3. Doppler-LSS Cross-correlation

The kSZ-LSS cross-correlation can be understood qualitatively by examining the behavior of the quantity

$$\frac{\partial u}{\partial z} = x \frac{\partial u_0}{\partial z} + u_0 \frac{\partial \tau}{\partial z},$$

since

$$\Delta T \propto \left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial z}{\partial \ln \chi}\right)\delta,$$

for a linear density perturbation with amplitude $\delta$ and size $\chi$. Since in general $\partial z/\partial \ln \chi > 0$, the sign of $\partial u/\partial z$ determines whether a linear density enhancement appears as a hot spot or cold spot in the CMB. When $\partial u/\partial z > 0$, the far side of a density perturbation contributes more, and peaks in the density field will appear as hot spots with $\Delta T > 0$. This corresponds to when $\partial \chi/\partial z > 0$, as is the case during recombination or after reionization. For sufficiently short ionization histories, however, it is possible for the condition

$$u_0 \frac{\partial \tau}{\partial z} < -x \frac{\partial u_0}{\partial z}$$

to be satisfied during reionization. In this case, $\partial u/\partial z < 0$, and density enhancements will appear as “cold spots.” This is illustrated in Figure 5 for a reionization history parameterized by Equation (12). The implications of this for temperature
features induced by H\textsc{ii} in the CMB will be further discussed in Section 2.4.

Any arbitrary linearly biased tracer of the matter density can be written as

\[ f(\gamma) = \int dz \frac{d\chi}{dz} w(z) \left[ b(z) \delta(\chi) - \frac{1 + z}{H(z)} \frac{\partial v}{\partial \chi} \cdot \gamma \right], \]  

where \( b(z) \) is the linear bias and \( w(z) \) is the redshift weighting. The second factor accounts for the redshift-space distortion in the linear regime (Kaiser 1987). The cross-correlation between \( f(\gamma) \) and \( \Delta T(\gamma)/T \) is (see derivation of Equation (70) in the Appendix)

\[ C_{TT}^f = 2 \int \frac{dk}{k} \frac{\delta_f(k)}{k} \sum_{l=0}^{\infty} C_{l}^\ell \cdot \langle \delta_0(k) \rangle. \]  

2.4. Doppler Effect from a Single Perturbation

Consider a uniform density perturbation with amplitude \( \delta_0 \) at some redshift \( z_0 \), located at a comoving distance \( \chi_0 \equiv \chi(z_0) \) from the observer along the direction \( \gamma_0 \), with comoving radius \( R \). On scales much smaller than the horizon, we can treat cosmological evolution to first order in conformal time across the perturbation. The observer will see a temperature fluctuation along the line of sight given by

\[ \frac{\Delta T_{\text{cmb}}}{T_{\text{cmb}}} \approx -\int_{\chi_{\nu}-R}^{\chi_{\nu}+R} d\chi g_\nu(z) x(z) v(\gamma) \cdot \gamma \approx -\int_{-R}^{R} dR (g_\nu + Rg_\nu') x(z) v(\gamma(R)), \]

where \( v(\gamma(R)) = v((\chi_0 + R)\gamma_{\nu}) \cdot \gamma_{\nu}, g_\nu \equiv g_0(z_\nu), \) and \( g_\nu' \equiv \frac{dg_\nu(z_\nu)}{d\chi} \).

To obtain the component of the peculiar velocity along the line of sight, one can assume that the perturbation has a small amplitude, \( \delta \ll 1 \). Using linear theory in a matter-dominated universe, \( \nabla \cdot v = -\delta/(1+z) = -\delta(D/D)/(1+z) \), one obtains

\[ v(\gamma) = -\frac{D(\gamma)}{3D(z)(1+z)} \delta_\nu R - \frac{\delta_\nu}{3} (D_\nu + R D_\nu'), \]

where \( D(z) = D(z)/(1+z) \) and \( D_\nu = dD/d\chi \). The assumption that the perturbation itself is at rest with respect to the CMB is likely to be a good approximation on large scales, and certainly true on average. Thus, photons scattered into the line of sight on the near side of the perturbation, with \( \chi < \chi_\nu \), experience a redshift, \( v \cdot \gamma > 0 \), and the temperature fluctuation is given by

\[ \frac{\Delta T_{\text{cmb}}}{T_{\text{cmb}}} = \frac{\delta_\nu}{3} \int_{-R}^{R} dR (g_\nu + Rg_\nu') (D_\nu + R D_\nu') x(R). \]  

The largest possible kSZ effect arising from the perturbation occurs if it is ionized at \( z_\nu \) on a timescale much shorter than its light-crossing time. Such a condition does not violate causality, because reionization is a locally driven process until percolation. In this case, the ionized fraction changes from zero to one very nearly instantaneously, and the ionized fraction along the line of sight is \( x \sim 1 \) for \( R < 0 \) and \( x = 0 \) otherwise, so that to leading order in \( R \nu \), Equation (24) simplifies to

\[ \frac{\Delta T_{\nu}}{T_{\text{cmb}}} \approx \frac{g_\nu D_{\nu} \delta_{\nu}}{3} \int_{0}^{R_\nu} RdR = -\frac{g_\nu D_{\nu} \delta_{\nu} R^2}{6}, \]

and the associated temperature fluctuation is

\[ \Delta T_{\nu} \approx -10 \mu K \left( \frac{\delta_\nu}{10^{-2}} \right) \left( \frac{1 + z_\nu}{16} \right)^{1/2} \left( \frac{R_\nu}{200 \text{ Mpc}} \right)^2, \]

where matter domination during reionization has been used. Because rapidly expanding H\textsc{ii} regions are expected to be in highly biased overdense regions, with \( \delta > 0 \), cold spots are a generic imprint from early, rare H\textsc{ii} regions.

3. NUMERICAL APPROACH

In this section, it is first explained how full-sky kSZ and 21 cm maps are obtained from a simulation of reionization. Maps of the kSZ effect are calculated with and without the linear density fluctuations and patchy ionization fields included, in order to separate first- from second-order contributions to the kSZ power spectrum and illustrate visually how large-scale velocity anisotropies are partially transferred to smaller scales by density and ionization fluctuations. Finally, auto- and frequency-dependent power spectra are presented for the kSZ and 21 cm maps.

The full mapmaking procedure, including random field generation, the patchy reionization calculation, and light-cone projection, takes about 1 hr on 128 nodes (8 cores and ~12 GB available RAM each) with a memory footprint of five float (density contrast, reionization redshift, and linear velocity) for each of the 40963 resolution elements, or 1.25 TB.

The full-sky kSZ map generated according to the procedure outlined below is shown in Figure 6. Fluctuations of \( \mu K \) in amplitude are easily seen on scales of \( 5^\circ - 10^\circ \) for the kSZ map, corresponding to the “Doppler peak” at \( \ell \sim 20-30 \). Such features can be captured only with volumes larger than the line-of-sight distance over reionization (\( \sim 2 \text{ Gpc}/h \) for \( 6 < z < 20 \)) that subtend more than \( 10^5 \) (~4 Gpc at \( z = 20 \)). Thus, the 8/h Gpc size of the simulations presented here is more than sufficient, subtending 60° at \( z = 20 \) without repeating structure over the line of sight from \( z = 0 \) to \( z = 20 \) (\( \sim 7.5 \text{ Gpc}/h \)).
3.1. Simulations of Patchy Reionization

The simulation is carried out in a periodic box 8 Gpc/h on a side and with 40963 resolution elements. Once the background cosmology is fixed (WMAP13 is used; see the introduction for parameter values), there are three additional parameters that enter into the calculation: (1) $M_{\text{min}}$ — the minimum halo mass capable of hosting ionizing sources, (2) $\lambda_{\text{abs}}$ — the mean free path to Lyman-limit absorption systems, and (3) $\zeta_{\text{ion}}$ — the number of ionizing photons escaping halos per atom (Alvarez & Abel 2012). The first two are fixed at $M_{\text{min}} = 10^7 M_{\odot}$ and $\lambda_{\text{abs}} = 32 \text{ Mpc}/h$, and the efficiency parameter $\zeta_{\text{ion}}$ is varied so as to obtain a given value for the Thomson scattering optical depth $\tau = 0.1$, $\zeta_{\text{ion}} \approx 4000$. See Alvarez & Abel (2012) for details on the model and parameter dependence of the reionization history and morphology.

3.2. Light-cone Projection

The first step in obtaining the simulated maps is to generate a realization of the ionization field obtained from a random realization of the initial density field in a representative volume on the light cone. Such a realization is obtained with the excursion set reionization algorithm of Alvarez & Abel (2012), including a mean background level treatment of the opacity due to self-shielded absorption systems and a linear model for the density and, most crucially, velocity fluctuations.

The kSZ maps are obtained as follows. First, a random realization of the linear density contrast extrapolated to $z = 0$, $\delta_0(r)$ with the observer located at the origin, $r = 0$, is generated. The excursion set algorithm is then applied to obtain a reionization redshift field, $\hat{z}(r) \rightarrow z_t(r)$. The local density contrast at a distance $\chi$ along a line-of-sight direction $\hat{\gamma}$, $\delta(\chi \hat{\gamma}) = \delta_0(\chi \hat{\gamma})D(z)$, and the velocity, $v(\chi \hat{\gamma})$, are obtained using Eulerian linear perturbation theory, and the electron fraction is obtained directly from the reionization redshift field,

$$x_e(\chi \hat{\gamma}) = \Theta[z(\chi \hat{\gamma}) - z_t(\chi \hat{\gamma})],$$

where $\Theta$ is the Heavyside function. Finally, Equation (3) is integrated along the line of sight for each pixel center, using the gas density, velocity, and ionized fraction as determined above.

The mean differential brightness temperature at frequency $\nu$ in a direction $\hat{\gamma}$ corresponds to the deviation of the observed intensity, $I_\nu$, from that expected for the CMB,

$$\delta T_\nu(\hat{\gamma}) \equiv \frac{c^2 I_\nu(\hat{\gamma})}{2\nu^2 k_B} - T_{\text{cmb}} \equiv T_\nu(\hat{\gamma}) - T_{\text{cmb}}.$$

With this definition, differential brightness temperature can replace intensity in the equation of transfer along the line of sight,

$$T_\nu(\hat{\gamma}) = T_{\text{cmb}} e^{-\tau_\nu} + T_e(z, \hat{\gamma})(1 - e^{-\tau_\nu}),$$

with the spin temperature replacing the source function. We adopt the standard scenario for 21 cm in the epoch of reionization, i.e., $T_e \gg T_{\text{cmb}}$ and a large-scale optical depth that is small, $\tau_\nu \ll 1$. We ignore the effect of redshift-space distortions, which would only boost the strength of the correlation (Adshead & Furlanetto 2008). The final expression is (e.g., Alvarez et al. 2006)

$$\delta T_\nu(\hat{\gamma}) = T_0(z)[1 - x_e(\chi \hat{\gamma})][1 + \delta(\chi \hat{\gamma})],$$

where

$$T_0(z) = 23 \text{ mK} \left( \frac{\Omega_{\text{b}} h^2}{0.02} \right)^{1/2} \left[ \frac{0.15}{\Omega_m h^2} \right]^{1/2} \left( \frac{1 + z}{10} \right)^{1/2}.$$

See Furlanetto et al. (2006) and references therein for additional details on the redshifted 21 cm background from reionization.

3.3. Maps

Shown in Figure 7 is a $32 \times 32$ degree field of view from the full-sky kSZ maps obtained from the $8h^{-1} \text{ Gpc}$ box. Four maps are shown, the total signal and its separation into three components, each according to its contribution to the integrand in Equation (3), which can be written as

$$\frac{\Delta T(\hat{\gamma})}{T} = \int (1 + \delta + \delta_\nu \delta_\nu) \cdot \hat{\gamma} \, d\tau.$$
The Astrophysical Journal, 824:118 (14pp), 2016 June 20

Figure 8. Angular power spectra calculated from the maps shown in Figure 7, as labeled. The dotted blue line peaking at $\ell \sim 20 - 30$ is the Doppler component obtained with Equation (4) and the approximation of Equation (8), where $u(z)$ is determined from the mean ionization history in the simulation. Shaded regions show $1\sigma$ and $2\sigma$ cosmic variance limits, assuming a $\chi^2$ distribution with mean $C_\ell$ and $2\ell + 1$ degrees of freedom. The Ostriker–Vishniac component that has been subtracted out, consistent with Equation (34), is shown as the dotted red line.

The large-angle fluctuations on scales of $\sim 5^\circ - 10^\circ$, with fluctuation amplitudes reaching as large as $30 \mu K$, are due nearly entirely to velocity fluctuations. This is seen clearly by comparing the lower left panel (“Doppler”) with the lower right panel (“total”). Even in the uniform case, where density and ionization fluctuations are completely neglected, the large-scale fluctuation pattern stays essentially the same. Patchiness is certainly an important contribution to the fluctuations, but only on small scales, where it is the dominant source of kSZ fluctuations from reionization (e.g., compare upper right panel of Figure 7 with $\delta_s = 0$ to the lower left panel with $\delta = 0$).

3.4. Deconstructing the kSZ Angular Power Spectrum

In what follows it will be useful to refer to the angular power spectrum in terms of the sum of all possible correlations of the individual components of the map. Since the kSZ temperature is given by the optical-depth-weighted integral of four terms, $\mathbf{r}(1 + \delta + \delta_s + \delta_{ov})$ (Equation 32), the power spectrum will be the sum of 10 independent power spectra, represented schematically as

$$
\langle \Delta T \cdot \Delta T \rangle_{\ell} = \langle \mathbf{r} \cdot \mathbf{r} \rangle_{\ell} + \langle \mathbf{r} \delta \cdot \delta \rangle_{\ell} + \langle \mathbf{r} \delta_s \cdot \mathbf{r} \delta_s \rangle_{\ell}
+ 2 \langle \mathbf{r} \delta \cdot \mathbf{r} \delta_s \rangle_{\ell} + 2 \langle \mathbf{r} \delta \cdot \mathbf{r} \delta_{ov} \rangle_{\ell}
+ 2 \langle \mathbf{r} \delta_s \cdot \mathbf{r} \delta_{ov} \rangle_{\ell} + 2 \langle \mathbf{r} \delta \cdot \mathbf{r} \delta_{ov} \rangle_{\ell}
+ 2 \langle \mathbf{r} \cdot \mathbf{r} \delta_s \rangle_{\ell} + 2 \langle \mathbf{r} \cdot \mathbf{r} \delta_{ov} \rangle_{\ell}.
$$

The first two terms are easily identifiable as the Doppler and Ostriker–Vishniac effects, respectively. All of the terms were obtained explicitly by cross-correlating the four maps described in Section 3.3.

Shown in Figure 8 is the angular power spectrum of the simulated kSZ map over the full range of multipoles, $3 \leq \ell \leq 3000$. In order to focus on the properties of the patchy reionization signal, we have removed the components of the power spectrum that depend solely on gas (as opposed to electron) momentum fluctuations, $\delta\mathbf{v}$, as $\langle \delta\mathbf{v} \cdot \mathbf{r} \delta \rangle_{\ell}$ and $\langle \mathbf{r} \cdot \mathbf{r} \delta \rangle_{\ell}$. In practice, this amounts to subtracting out the linear

Figure 9. Deconstruction of patchy components of the kSZ map. “Total” in this figure corresponds to the angular power spectrum of the map, with “non-patchy” terms removed as in Equation (35). The approximation to the patchy–patchy fluctuations (shown by the solid green line) that neglects the connected part of the ionization-velocity correlations is shown by the dashed green line, indicating a non-negligible contribution from the connected part of the correlations to the total power.

Ostriker–Vishniac term,

$$
\ell^2 C_\ell^{ov} \equiv \frac{1}{2} \int \frac{dk}{k^3} P_{\ell k}(\ell/k)^2 (1 - \mu^2) / k^2 (k^2 + k^*^2 - 2kk^* \mu^2)
$$

and $\mu \equiv \mathbf{k} \cdot \mathbf{k}^*$. That is to say, $\langle \mathbf{r} \cdot \mathbf{r} \delta \rangle_{\ell}$ is entirely negligible since $\mathbf{v}$ and $\delta$ are only calculated to linear order in the simulation, and the nontrivial information content is contained in the non-Gaussian $\delta_s$ component.

The most noticeable feature is the broad peak in the total angular power spectrum at $\ell \sim 20 - 30$, corresponding to the Doppler effect. The solid blue line, obtained by calculating the power spectrum from maps in which $\delta = \delta_s = 0$, indicates that pure velocity correlations completely dominate the power spectrum at $\ell \lesssim 200$. The dotted blue line lying nearly on top of the solid one is the solution for the longitudinal term in the power spectrum from Equation (4), using the linearized mode-coupling term given in Equation (8). The only information used from the simulation in calculating the analytical power spectrum is the reionization history, $x(z)$, which enters into Equation (8) through $u(z) = \sigma_T n_{e,0} (1 + z)^2 \tilde{D}(z)/D(z)/(1 + z)x(z)$. This consistency between the analytical formula and ray-tracing result validates the simulation pipeline and shows that density and ionization fluctuations can be neglected for the kSZ power spectrum at $\ell \lesssim 200$, as expected. The agreement between the simulated and analytical power spectrum at $\ell \lesssim 200$ does not depend on whether the OV component (Equation (34)) is subtracted or not.

The situation becomes more interesting at $\ell \gtrsim 300$, as can be seen in Figure 9. For clarity, we have also subtracted out the Doppler term, so that the solid black line labeled “Total”
ionization fraction with density that persists to small scales, although more work will be necessary to fully understand the patchy-OV term. Finally, \( \langle \psi_{\delta x} \cdot \psi_{\delta y} \rangle_{\ell} \), while comparable in amplitude to the patchy-OV term at \( \ell \sim 3000 \), becomes negative at \( \ell \lesssim 900 \), corresponding to a projected comoving radius of \( \sim 15-20 \) Mpc, quite close to the typical H II region scale at the midpoint of reionization (e.g., Alvarez & Abel 2012).

Also shown in Figure (9) is an approximation for \( \langle \psi_{\delta x} \cdot \psi_{\delta y} \rangle_{\ell} \) that neglects the connected, or irreducible, fourth moment of the correlation (see Equation (65) in the Appendix):

\[
\ell^2 C_{\ell}^{(xy)}(x,y) \approx \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} (1 - \mu^2) \left[ (P(k')P_{\delta x}(|k - k'|) - k^2 P_{\delta x}(|k - k'|) P_{\delta y}(k') \right] - \frac{k^2 P_{\delta x}(|k - k'|) P_{\delta y}(k')}{k^2 + k'^2 - 2kk'/\mu} \]

\[
\ell^2 C_{\ell}^{(xy)}(x,y) \approx \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} (1 - \mu^2) \left[ (P(k')P_{\delta x}(|k - k'|) - k^2 P_{\delta x}(|k - k'|) P_{\delta y}(k') \right] - \frac{k^2 P_{\delta x}(|k - k'|) P_{\delta y}(k')}{k^2 + k'^2 - 2kk'/\mu} \]

This term was obtained by tabulating \( P_{\delta x} \) and \( P_{\delta y} \) from the simulation for all redshifts such that \( 0.01 < z(x) < 0.99 \) and integrating over the same range of redshift, since there should be no contribution to this term after reionization is complete. The contribution of the connected fourth moment is given by (Ma & Fry 2002; Park et al. 2016)

\[
P_{\delta x}^{(xy)} = \int \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k''}{(2\pi)^3} \sqrt{1 - \mu^2} \sqrt{1 - \mu'^2} \cos (\phi' - \phi'') P_{\delta x}(k - k', -k - k'', k', k'').
\]

In order to contribute significantly, \( P_{\delta x} \) would need to have some dependence on \( \phi' - \phi'' \) at fixed \( \mu' \) and \( \mu'' \). Ma & Fry (2002) pointed out that nonlinearities arising from non-overlapping, static spherical density enhancements—i.e., the halo model—exhibit no such dependence on \( \phi' - \phi'' \), since their internal structure has no explicit correlation with the velocity field. Recently, Park et al. (2015) pointed out that this is not generally the case, finding a contribution on the order of tens of percent lower redshifts, for the late-time kSZ effect, by using perturbation theory and N-body simulations. It seems plausible that even larger departures, such as those seen in Figure 9, could occur for such fluctuations induced by patchy reionization, but more detailed analysis will be required before any definitive statements can be made.

4. CMB–21 CM CROSS-CORRELATION

In this section we describe the large-scale CMB–21 cm cross-correlation, based on full-sky kSZ and 21 cm maps. We then give some preliminary estimates for its detectability under ideal observing conditions in which instrumental noise and foregrounds can be neglected and the observations are carried out over the full sky. Such observations will be considered to be cosmic variance limited.
4.1. Cross-correlation Power Spectrum

An ideal observation of CMB temperature is the sum of uncorrelated primary and kSZ fluctuations, \( \Delta T = \Delta T_p + \Delta T_{\text{kSZ}} \), with spherical harmonic coefficients \( a^T_{lm} = a^p_{lm} + a^k_{lm} \), for which the temperature power spectrum will have a mean value of \( C^T_{\ell} = C_p^{\ell} + C_k^{\ell} \) and variance \( V^T_{\ell} = 2(C^T_{\ell})^2 / (2\ell + 1) \).

An ideal observation of 21 cm differential brightness temperature at frequency \( \nu \) is \( \delta T^m_{\nu} \), with coefficients \( a^m_{\nu,lm} \) and power spectrum \( C^m_{\nu} \nu \) — i.e., the 21 cm observation is assumed to contain only the signal from H ii, without any noise. The observed cross-correlation will have a mean of \( C^T_{\nu,\ell} = C^T_{\ell} \) and a variance of

\[
V^T_{\ell} = C^T_{\ell} C^\nu_{\ell} + (C^T_{\ell})^2 / (2\ell + 1) \approx C^T_{\ell} C^\nu_{\ell} (2\ell + 1) .
\]

(41)

Defining the auto- and cross-correlation coefficients as

\[
(r^k_{\ell})^2 \equiv \langle C^k_{\ell} \rangle^2 / \langle C^T_{\ell} C^T_{\ell} \rangle = C^k_{\ell} / C^T_{\ell}
\]

(42)

and

\[
(r^{k,\nu}_{\ell})^2 \equiv \langle C^{k,\nu}_{\ell} \rangle^2 / \langle C^T_{\ell} C^\nu_{\ell} \rangle,
\]

(43)

one obtains a signal-to-noise ratio of

\[
(S/N)^k_{\ell,\nu} = \sum_{m} \langle C^k_{\ell,\nu} \rangle^2 / \langle V^T_{\ell} \rangle = \frac{1}{2} \sum_{\ell} (2\ell + 1)(r^k_{\ell})^2
\]

(44)

for the kSZ auto-correlation and

\[
(S/N)^{\nu}_{\ell,m} = \sum_{m} \langle C^{\nu}_{\ell,m} \rangle^2 \langle V^T_{\ell} \rangle
\]

\[
= \sum_{m} (2\ell + 1)(r^{\nu}_{\ell,m})^2 / [(r^{\nu}_{\ell,m})^2 + 1]
\]

(45)

for the kSZ–21 cm cross-correlation.

Shown in Figure 10 are the power spectrum, correlation coefficient (Equation (43)), and cumulative signal-to-noise ratios of the kSZ–21 cm cross-correlation (Equation (45)). The two maps show a significant correlation with the same shape as the matter power spectrum, peaking at \( \ell \sim 100 \), as predicted by Alvarez et al. (2006). The sign of the correlation can be understood in terms of correlated linear velocity and density fluctuations, with an ionized fraction that is strongly biased with respect to the density and smoothly distributed on large scales, hundreds of Mpc across. Although matter and Doppler fluctuations are negatively correlated during reionization (see, e.g., Figure 5), overdense regions contain less neutral hydrogen than underdense regions during reionization because of the bias of ionizing sources, and the overall effect leads to a positive correlation (Alvarez et al. 2006). We have verified that the cross-correlation in the case where ionization fraction fluctuations are ignored, \( \delta_i = 0 \), is negative, since in that case the 21 cm map traces the matter density (see, e.g., discussion in Section 2.3).

4.2. Dependence on Redshift

In order to explore the signal-to-noise ratio for a given spectral resolution, a Monte Carlo procedure was adopted. For each trial \( i \) of \( N \), a random realization of the primary CMB temperature fluctuations, \( \Delta T^p_{\ell,i}(\hat{\gamma}) \), was created. The simulated 21 cm and kSZ maps, \( \delta T^m_{\nu,i}(\hat{\gamma}) \) and \( \Delta T^{\text{kSZ}}_{\nu,i}(\hat{\gamma}) \), were held fixed since the sample variance of the primary CMB fluctuations is independent of the 21 cm signal and dominates the noisiness of the measured cross-correlation. The observed correlation for each trial is given by

\[
(2\ell + 1)C^{T,T}_{\ell,i} = \sum_{m=-\ell}^{\ell} a^T_{\ell,m} a^{\nu}_{\ell,m,i},
\]

(46)

where

\[
a^{\nu}_{\ell,m} = \int d^3 \gamma Y_{\ell,m}(\hat{\gamma}) \Delta T^p_{\ell,i}(\hat{\gamma}) \Delta T^{\text{kSZ}}_{\ell,i}(\hat{\gamma}).
\]

(47)

Because the dependence on frequency, \( \nu \) (or, equivalently, redshift, \( z \)), of the correlation, as opposed to its dependence on multipole, \( \ell \), contains the information that is most difficult to ascertain otherwise, namely, the shape of the reionization history, it makes sense to integrate over the multipoles of the correlation at fixed \( \nu \). For the frequency binning, one should use as large bins as possible, since that will decrease the instrumental noise (which is neglected in this analysis) and increase the signal in the correlation. Too large a bin, however, will lead to information about its redshift dependence being lost in the averaging process.

Figure 11 shows the redshift dependence of the integrated power of the cross-correlation in frequency bin \( \ell \),

\[
\mathcal{P}^{T,T}_{\ell}(\nu) = \sum_{\ell} \int_{\nu - \Delta \nu / 2}^{\nu + \Delta \nu / 2} w_{\ell}(\nu) C^{T,T}_{\ell}(\nu) d\nu,
\]

(49)

where the weight function, \( w_{\ell}(\nu) \), is normalized to unity. It can be chosen so as to optimize the overall signal-to-noise ratio of the cross-correlation (e.g., Peiris & Spergel 2000). Because the signal depends on a relatively undetermined reionization history, however, it is in practice not possible to know beforehand which
weighting function to use, and so \( w(\nu) = 1/\Delta \nu \) is used, with \( \Delta \nu \) the same for all frequency bins.

5. SUMMARY

The angular power spectrum for the kSZ effect due to reionization was calculated on all scales \( \ell \lesssim 3000 \), using analytical models and numerical simulations of patchy reionization. The angular power spectrum generally exhibits two main features, a broad peak at \( \ell \sim 20-30 \) with an amplitude \( \sim 10-30 \mu K^2 \), and a small-scale plateau at \( \ell \gtrsim 300 \), with an amplitude of \( \sim 1-5 \mu K^2 \). The low-\( \ell \) peak is caused by the so-called “Doppler effect,” arising from longitudinal modes in the velocity field \( v_\parallel = (v \cdot \hat{k}) \), while the broad plateau at higher multipoles is a superposition of transverse momentum correlations seeded by patchy ionization and density fluctuations, \( \nu v \) and \( \delta v \), respectively.

These results are in good agreement with previous analyses from small-scale simulations of patchy reionization, \( \ell \gtrsim 1000 \), with a roughly constant amplitude of order a few \( \mu K^2 \) (Santos et al. 2003; McQuinn et al. 2005; Zahn et al. 2005; Iliev 2007; Trac et al. 2011; Mesinger et al. 2012; Battaglia et al. 2013; Park et al. 2013). For reionization driven by UV sources located in relatively rare dark matter halos—the scenario favored by existing data—the patchiness of reionization is “seeded” by large-scale velocity modes, and the amount of small-scale power is dependent mainly on the duration and redshift of reionization, such that more extended reionization histories lead to larger fluctuations at \( \ell \sim 3000 \) at fixed \( \tau \).

On large scales, \( \ell \lesssim 300 \), where the Doppler effect is the dominant source of kSZ fluctuations, the situation is reversed—shorter reionization histories lead to larger fluctuations at fixed \( \tau \). Unlike the small-scale fluctuations that are created by inhomogeneities in the electron density that quickly lose coherence and accumulate along the line of sight, the large-scale velocities are coherent on large scales, such that longer reionization histories lead to more cancelation, instead.

At intermediate scales, \( 300 \lesssim \ell \lesssim 1000 \), the signal exhibits features that could in principle provide additional constraints on reionization from CMB data alone. For example, it has been recently suggested that one can distinguish between primary and secondary temperature fluctuations by subtracting the theoretical power spectrum for primary fluctuations from the actual observed temperature fluctuations, using independent cosmological parameter constraints obtained from polarization measurements—what’s left would be the kSZ power spectrum (Calabrese et al. 2014). The analysis presented here indicates that such an approach would be quite sensitive to the duration of reionization, since the low- and high-\( \ell \) peak and plateau, respectively, have opposite dependences on \( \Delta \tau \).

We performed a novel, map-based separation of the patchy kSZ isoclinics on scales \( 300 \lesssim \ell \lesssim 3000 \), finding several new results. First, the fluctuations from the patchy component are dominated by correlations of the form \( \langle \nu \delta_\parallel \cdot \nu \delta_\parallel \rangle \), which compose 60%-70% of the total power over the full range. Surprisingly, the six-point correlation, \( \langle \nu \delta_\parallel \cdot \nu \delta_\parallel \rangle \), contributes as much as 5% at small scales, even for the linear density fluctuations we assumed. The term \( \langle \nu \delta_\parallel \cdot \nu \delta_\parallel \rangle \) exhibits an interesting feature at \( \ell \sim 900 \), where it changes sign. This scale is quite close to the typical radius of \( \text{H} \text{II} \) regions at the half-ionized epoch, \( 15-30 \) comoving Mpc. Finally, by comparing the leading-order patchy term, \( \langle \nu \delta_\parallel \cdot \nu \delta_\parallel \rangle \), to the prediction based on the unconnected part of the four-point function, we find tentative evidence for a negative contribution from the connected part at \( \ell \lesssim 2000 \) and a positive contribution at \( \ell \gtrsim 2000 \). More detailed analysis is required, however, to test this hypothesis and the dependence on the sources and sinks of reionization.

The results presented here are for a specific reionization model, with reionization occurring relatively rapidly and with relatively rare sources \( (M_{\text{halo}} > 10^9 M_\odot) \). As shown in Alvarez & Abel (2012), even a simplified three-parameter model, in which the mean free path and halo ionizing efficiency do not depend on time and position, displays a strong variation in the timing and morphology of reionization. In more extended reionization scenarios with less efficient sources than the ones considered here, in which recombinations can slow ionization fronts considerably, the finite travel time of ionization fronts can lead to partially ionized cells near the boundaries of \( \text{H} \text{II} \) regions. This was accounted for by Mesinger et al. (2012), who used a parameterization similar to that of Alvarez & Abel (2012) to explore the dependence of the small-scale kSZ fluctuations on these parameters, the major differences being their use of a minimum virial temperature rather than halo mass, using the nonlinear density field for the excursion set data, and allowing for partial ionization of individual cells. New simulations for scenarios that go beyond the simplified three-parameter models (e.g., Alvarez et al. 2012), with a resolution and dynamic range similar to those presented here, will be necessary to understand the implications of CMB observations for reionization and build realistic extragalactic sky models for component separation and de-lensing (e.g., Amblard et al. 2006; Lewis & Challinor 1998).

Going beyond CMB observations alone, the kSZ effect can provide constraints on reionization via cross-correlation with tracers of the large-scale density field, particularly the redshifted 21 cm background. At very high redshifts, it is possible that rare, quicky growing \( \text{H} \text{II} \) regions centered on density peaks could have created large features in the CMB. More work will be required to see just how likely the types of \( \text{H} \text{II} \) region abundances, sizes, growth rates, and biases that are required are to actually occur. At lower redshifts, we confirm with three-dimensional simulations the prediction of Alvarez et al. (2006) that the cross-correlation between the CMB blackbody temperature and redshifted 21 cm intensity from redshifts \( z \sim 10 \) exhibits a broad positive peak at \( \ell \sim 100 \), and that the 21 cm frequency dependence of the correlation traces the reionization history with a resolution of \( \Delta \ell \sim 1 \). Such a correlation is attractive as well because the 21 cm signal naturally contains redshift information, and the systematics of the GHz and MHz band observations are likely to be of quite different origins and therefore uncorrelated. The difficulty lies, however, in the “noise” contributed by the much larger primary CMB fluctuations that are peaking at the same multipoles. Indeed, assuming that correlated foregrounds can be cleaned and a sufficiently sensitive instrument can observe over half of the sky, the cross-correlation can be detected at 5σ–10σ significance, for reionization occurring at \( z \sim 10 \) in our fiducial scenario. This raises the prospect of using more precise large-scale structure tracers of the velocity, such as large \( \text{H} \text{II} \) regions and tidal reconstructions, as a new window into the reionization epoch with CMB temperature measurements.
I would like to thank J. R. Bond, N. Battaglia, E. Komatsu, P. D. Meerburg, H. Park, U.-L. Pen, and A. van Engelen for helpful discussions and encouragement. Research in Canada is supported by NSERC and CIFAR. The reionization simulations were performed on the GPC supercomputer at the SciNet HPC Consortium. SciNet is funded by the Canada Foundation for Innovation under the auspices of Compute Canada; the Government of Ontario; Ontario Research Fund—Research Excellence; and the University of Toronto.

APPENDIX

The kSZ angular auto- and cross-power spectra are derived first for the case where the electron density is spatially homogeneous, where only the longitudinal component of the specific electron momentum contributes, in Appendix A. Appendix B contains a derivation of the angular power spectrum for the transverse component, relevant to the patchy signal, up to leading order in density and ionization fluctuations.

The specific momentum, \( q = (1 + \delta_c) v \), can be written, to leading order in \( \delta \) and \( \delta_c \), as

\[
q = v(1 + \delta + \delta_c),
\]

which, when Fourier transformed, is

\[
q(k) = v(k) + \int \frac{d^3k'}{(2\pi)^3} \{ \delta(k - k') + \delta_c(k - k') \} v(k')
\]

The momentum is further decomposed into longitudinal and transverse components using

\[
q(k) = \vec{q} = q - (q \cdot \hat{k}) \hat{k},
\]

\[
q_{\perp}(k) = \int \frac{d^3k'}{(2\pi)^3} \{ \delta(k - k') + \delta_c(k - k') \} v(k') \hat{k}' - \mu \hat{k}
\]

and

\[
q_l(k) = v(k) + \int \frac{d^3k'}{(2\pi)^3} \{ \delta(k - k') + \delta_c(k - k') \} v(k') \mu' \hat{k},
\]

where \( \mu' = \hat{k}' \cdot \hat{k} \) and \( v(k) = v(k) \hat{k} \) holds on all scales of interest.

APPENDIX A

LONGITUDINAL MOMENTUM FLUCTUATIONS

The leading-order contribution to the fluctuations arising from \( q_l \) depends only on the linear velocity, \( v = |v| \hat{k} \). This greatly simplifies calculation of the momentum fluctuations that contribute to the Doppler component. On large scales, however, the Limber approximation breaks down and the full integral over pairs of lines of sight must be retained.

Fluctuations on the sky of the kSZ temperature can be expressed in terms of spherical harmonics \( C_{\ell} = \langle a_{\ell m} a_{\ell m}^{*} \rangle \), where

\[
a_{\ell m} = \int dY Y_{\ell m}(\hat{\gamma}) \Delta T(\hat{\gamma}) / T.
\]

The kSZ temperature fluctuation can be written in terms of the Fourier transform of the momentum,

\[
\Delta T(\hat{\gamma}) = \int d\chi g(z) q(z) \cdot \hat{\gamma} = \int d\chi g(z) \int \frac{d^3k}{(2\pi)^3} e^{-ik \cdot \hat{\gamma}} \times (q_{\perp} \cdot \hat{\gamma} + q_l \hat{k} \cdot \hat{\gamma} \cdot \hat{k} ) \\
\]

\[
\equiv \Delta T_{\perp}(\hat{\gamma}) + \Delta T_{l}(\hat{\gamma}),
\]

from which it follows that

\[
C_{\ell} = \langle a_{\ell m} a_{\ell m}^{*} \rangle \equiv C_{\ell}^{\perp} + C_{\ell}^{l}.
\]

The parallel component is given by

\[
\frac{\Delta T_{l}}{T} = \int d\chi g(z) \int \frac{d^3k}{(2\pi)^3} q_l(k) \hat{k} \cdot \hat{\gamma} e^{-ik \cdot \hat{\gamma}}
\]

\[
= \int \frac{d^3k}{(2\pi)^3} \frac{1}{k} \int q_l(k) g(z) \frac{\partial e^{-ik \cdot \hat{\gamma}}}{\partial \chi} d\chi.
\]

Integrating by parts, one obtains

\[
\frac{\Delta T_{l}}{T} = -\int \frac{d^3k}{(2\pi)^3} \frac{1}{k} \int d\chi e^{-ik \cdot \hat{\gamma}} f_l(k),
\]

where \( f_l(k) \equiv \partial[q_l(k)(\hat{\gamma} \cdot \hat{\gamma})] / \partial \chi \). Using one of Rayleigh’s identities,

\[
\frac{\Delta T_{l}}{T} = -4\pi \sum_{\ell m} i\ell Y_{\ell m}(\hat{\gamma}) \int \frac{d^3k}{(2\pi)^3} Y_{\ell m}^{*}(\hat{k}) \int d\chi f_{l}(k) j_{\ell}(k \chi),
\]

and the spherical harmonic coefficients are therefore

\[
a_{\ell m} = 4\pi (-i)^{\ell} \int d\chi \int \frac{d^3k}{(2\pi)^3} f_{l}(k) j_{\ell}(k \chi) Y_{\ell m}^{*}(\hat{k}).
\]

The angular power spectrum of fluctuations due to the longitudinal component is

\[
C_{\ell}^{\perp} = \langle a_{\ell m} a_{\ell m}^{*} \rangle = (4\pi)^2 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\langle f_{l}(k)f_{l}^{*}(k) \rangle}{k^2} \right]
\]

\[
= \frac{2}{\pi} \int d\chi \int dk P_{f_{l}}(k, \chi) j_{\ell}(k \chi) j_{\ell}(k \chi)
\]

\[
\int \left[ \frac{\langle f_{l}(k, \chi) f_{l}(k', \chi') \rangle \delta(k - k')}{k^2} \right]
\]

\[
= (2\pi)^3 P_{f_{l}}(k, \chi, \chi') \delta(k - k') = \langle f_{l}(k, \chi) f_{l}(k', \chi') \rangle.
\]

This is the \( C_{\ell}^{\perp} \) term in Equation (4) and the main result of this section. The Limber approximation can be obtained by using the relationship \( \ell^2 C_{\ell}^{\perp}(k, \ell / \chi, \chi) \approx \pi \delta(\chi - \chi') P(\ell / \chi, \chi) / 2 \), which implies

\[
\ell^2 C_{\ell}^{\perp} = \int d\chi P_{f_{l}}(k, \ell / \chi, z).
\]

This is only a good approximation when \( \ell \gg H(z) \chi(z) / \delta \chi \), where \( \delta \chi = f_{l} / (\partial f_{i} / \partial z) \).

APPENDIX B

TRANSVERSE MOMENTUM FLUCTUATIONS

The Limber approximation is generally valid at multipoles where the transverse momentum fluctuations contribute to kSZ fluctuations. Because the velocity field is longitudinal, however, the momentum fluctuations are second order and depend

\[\text{on the lower and upper bounds of the line-of-sight integral, are omitted, since they generally negligible along the full line of sight. This is not necessarily the case when dealing with simulated maps that only include a range of redshifts. In that case, it can be important to take into account “surface fluctuations” at the boundaries of the projected region along the line of sight.}\]
on the density and ionization fluctuations and their cross-correlations with each other and the velocity.

The angular power spectrum of the transverse component is given accurately by Limber’s approximation (e.g., Park et al. 2013),

\[ C_\ell^\perp = \frac{1}{2} \int \frac{d\chi}{\chi^2} g^2(z) P_{\kappa\perp}(\kappa = \ell / \chi, z), \]

where \((2\pi)^3 P_{\kappa\perp}(k, z) \delta_D(k - k') = \langle \mathbf{q}(\mathbf{k}) \mathbf{q}(\mathbf{k'})^* \rangle\). Analytical expressions have been derived previously for the \(q_\perp\) terms in the angular power spectrum (e.g., Ma & Fry 2002; Santos et al. 2003; Park et al. 2013). Ma & Fry (2002) derived an expression for the power spectrum of \(q_\perp(k)\) in linear theory (their Equation (7)), but neglecting patchiness (i.e., \(\delta_p = 0\) in Equation (53)). In what follows we derive the expression for the power spectrum with patchy terms included.

The power spectrum can be obtained from Equation (51) (see also Hu 2000; McQuinn et al. 2005):

\[
\langle q_\perp(k) q_\perp(k') \rangle = \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3k''}{(2\pi)^3} \langle \delta(k') \delta(k'') \rangle \\
+ \delta_p(k'_0) \langle \mathbf{v}(k') \delta(k''_0) + \delta_p(k''_0) \rangle \\
x \mathbf{v}(k'') \langle \mathbf{k}' - \mu' \mathbf{k}' \cdot \mathbf{k}' - \mu' \mathbf{k}' \rangle, \\
\]

where \(k'_0 \equiv k - k'\) and \(k''_0 \equiv k - k''\). In general, a four-point function can be written as

\[
\langle \mathbf{A} \mathbf{B} \mathbf{C} \mathbf{D} \rangle = \langle \mathbf{A} \mathbf{B} \rangle \langle \mathbf{C} \mathbf{D} \rangle + \langle \mathbf{A} \mathbf{C} \rangle \langle \mathbf{B} \mathbf{D} \rangle + \langle \mathbf{A} \mathbf{D} \rangle \langle \mathbf{B} \mathbf{C} \rangle + \langle \mathbf{A} \mathbf{B} \mathbf{C} \mathbf{D} \rangle, 
\]

where \(\langle \mathbf{A} \mathbf{B} \mathbf{C} \mathbf{D} \rangle\) is the irreducible or connected fourth moment. It has been pointed out that the connected term is generally subdominant with respect to the others at late times, due to sphericallyization of density perturbations in the nonlinear regime (Ma & Fry 2002), with a contribution on the order of 10% (Park et al. 2016). Isotropy implies that terms containing factors with the form \(f(k - k')g(k - k'')\) are only nonzero for \(k = 0\), so only terms containing combinations of factors of the form \(f(k - k')g(k - k'')\), \(\langle f(k')g(k'') \rangle\), and \(\langle f(k'')g(k - k'') \rangle\) remain. The resulting expression is

\[
P_{q_\perp}(k) = \frac{1}{(2\pi)^2} \int k'^2 dk' \int d\mu(1 - \mu^2) P_{q_\perp}(k') \times \left\{ P_{\delta\delta}(k - k') + P_{\kappa\kappa}(k - k') + 2P_{\delta\kappa}(k - k') \right\} \\
- \frac{k'}{|k - k'|} \left[ P_{\delta\delta}(k - k')P_{\kappa\kappa}(k') + P_{\kappa\kappa}(k - k')P_{\delta\delta}(k') + P_{\delta\kappa}(k - k')P_{\kappa\delta}(k') + P_{\kappa\delta}(k - k')P_{\delta\kappa}(k') \right],
\]

where \(|k - k'| = (k^2 + k'^2 - 2kk'\mu)^{1/2}\). The velocity obtained from linear perturbation theory is an excellent approximation and gives \(P_{v_\perp}(k) = \alpha(z) P_{\delta\delta}(k)/k\), \(P_{v_\parallel}(k) = \alpha^2(z) P_{\delta\delta}(k)/k^2\), and \(P_{\delta\kappa}(k) = \alpha(z) P_{\kappa\kappa}(k)\), where \(\alpha(z) \equiv D(z)/D(z)/(1 + z)\). The expression is further simplified to

\[
P_{q_\perp}(k) = \frac{\alpha^2}{(2\pi)^2} \int k'^2 dk' \int d\mu(1 - \mu^2) \left\{ P_{\delta\delta}(k - k') \right\} \\
+ P_{\kappa\kappa}(k - k') + 2P_{\delta\kappa}(k - k') \\
- \frac{k'^2}{|k - k'|^2} \left[ P_{\delta\delta}(k - k')P_{\kappa\kappa}(k') + P_{\kappa\kappa}(k - k')P_{\delta\delta}(k') + P_{\delta\kappa}(k - k')P_{\kappa\delta}(k') + P_{\kappa\delta}(k - k')P_{\delta\kappa}(k') \right].
\]

Finally, we group the terms by their dependence on density and ionization fluctuations,

\[
P_{q_\perp}(k) = \frac{\alpha^2}{(2\pi)^2} \int k'^2 dk' \int d\mu(1 - \mu^2) \\
\times \left\{ P_{\delta\delta}(k - k') + P_{\kappa\kappa}(k - k') + P_{\delta\kappa}(k - k') \right\},
\]

where

\[
P_{\delta\delta}(k - k', \mu) \equiv P_{\delta\delta}(k') P_{\delta\delta}(k - k') \frac{k(k - 2k\mu)}{k^2 + k'^2 - 2kk'\mu} 
\]

depends only on matter fluctuations \((\mathbf{v}\mathbf{v}\delta\delta)\),

\[
P_{\delta\kappa}(k - k', \mu) \equiv P_{\delta\kappa}(k') P_{\kappa\delta}(k - k') \frac{k^2 P_{\delta\delta}(k - k')}{k^2 + k'^2 - 2kk'\mu}
\]

depends only on patchiness \((\mathbf{v}\mathbf{v}\delta\kappa)\), and

\[
P_{\kappa\kappa}(k - k', \mu) \equiv 2P_{\kappa\kappa}(k') P_{\kappa\kappa}(k - k') \frac{k^2 P_{\delta\delta}(k - k')}{k^2 + k'^2 - 2kk'\mu}
\]

depends on the cross terms \((\mathbf{v}\mathbf{v}\delta\kappa)\). When the ionization field is assumed to be uniform, i.e., \(x = \langle x \rangle \implies P_{\delta\delta} = P_{\kappa\kappa} = 0\), the only term remaining is \(P_{\delta\kappa}\), corresponding to the “linear Ostriker–Vishniac effect,” i.e., \(\langle \delta v \rangle \langle \delta v \rangle\).

APPENDIX C

LARGE-SCALE CROSS-CORRELATION WITH LINEARLY BIASED TRACERS

Consider a projected map of some biased tracer of large-scale structure,

\[
f(\hat{\gamma}) = \int d\chi w(z) \left\{ b(z) \delta(\chi \hat{\gamma}) - \frac{1}{aH} \frac{\partial}{\partial \chi} \nu \cdot \hat{\gamma} \right\},
\]

where \(b(z)\) is the linear bias and \(w(z)\) is a redshift weighting that encodes the properties of the particular survey. The second factor in the integral accounts for the redshift-space distortion from linear velocity fluctuations at first order in the density fluctuation (Kaiser 1987). We derive in what follows an expression for the angular power spectrum of the cross-correlation with the longitudinal (i.e., Doppler) component of the kSZ signal, \(\Delta T(\hat{\gamma})/T\), as defined in Equation (55). The cross-correlation in this limit is given by \(C^{T\ell} = \langle a_{lm} a_{lm}^* \rangle\). Fourier transforming \(\delta\) and \(\nu\) in Equation (67), and using...
$$v(k) = -ik \delta(k) \dot{D}(z)/[D(z)k^2(1 + z)] = -ifaH\delta(k)k/k^2,$$
where \( f = d \ln D/d \ln a \), we obtain

\[
f(\hat{\gamma}) = \int d\chi w(z) \int \frac{d^3k}{(2\pi)^3} \delta(k) \times \left[ b(z)e^{-i k \hat{\gamma}} + \frac{ik \cdot \hat{\gamma}}{k^2} \frac{\partial}{\partial \chi} e^{-i k \hat{\gamma}} \right] = \int d\chi w(z) \int \frac{d^3k}{(2\pi)^3} \delta(k) \times \left[ b(z)e^{-i k \hat{\gamma}} - \frac{f}{k^2} \frac{\partial^2}{\partial \chi^2} e^{-i k \hat{\gamma}} \right] = -4\pi \sum_\ell \ell Y_{\ell m}(\hat{\gamma}) \int d\chi w(z) \int \frac{d^3k}{(2\pi)^3} Y_{\ell m}^*(\hat{k}) \delta(k) \times \left[ b(z)j_\ell(k\chi) - \frac{f}{k^2} \frac{\partial^2 j_\ell(k\chi)}{\partial \chi^2} \right]. \tag{68}
\]

The spherical harmonic coefficients are obtained in the same way as in Equation (56),

\[
a_{\ell m}^f = 4\pi (-i)^\ell \int d\chi w(z) \int \frac{d^3k}{(2\pi)^3} \delta(k) \times \left[ b(z)j_\ell(k\chi) - \frac{f}{k^2} \frac{\partial^2 j_\ell(k\chi)}{\partial \chi^2} \right] \bar{Y}_{\ell m}(\hat{k}). \tag{69}
\]

The cross-correlation is obtained by using Equation (69) and the first-order expression \( f||(k) \rightarrow \partial[g(z)v(k)]/\partial \chi \) in Equation (56):

\[
C^{TT}_\ell = \langle a_{\ell m}^f a_{\ell m}^{*g} \rangle = \frac{2}{\pi} \int d\chi w(z) \int d\chi' \frac{\partial u}{\partial \chi} \int d\chi P(k) \times \left[ b(z)j_\ell(k\chi) - \frac{f}{k^2} \frac{\partial^2 j_\ell(k\chi)}{\partial \chi^2} \right] j_\ell(k\chi'). \tag{70}
\]

REFERENCES

Adshead, P. J., & Furlanetto, S. R. 2008, MNRAS, 384, 291
Aghanim, N., Majumdar, S., & Silk, J. 2008, RPPh, 71, 066902
Alvarez, M. A., & Abel, T. 2012, ApJ, 747, 126
Alvarez, M. A., Finlator, K., & Trenti, M. 2012, ApJL, 759, L38
Alvarez, M. A., Komatsu, E., Doré, O., & Shapiro, P. R. 2006, ApJ, 647, 840
Amblard, A., Vale, C., & White, M. 2006, NewA, 9, 687
Battaglia, N., Natarajan, A., Trac, H., Cen, R., & Loeb, A. 2013, ApJ, 776, 83
Calabrese, E., Hložek, R., Battaglia, N., Bond, J. R., et al. 2014, JCAP, 8, 10
Chibisov, G. V., & Özerney, L. M. 1969, ApL, 3, 189
Chluba, J., & Sunyaev, R. A. 2004, A&A, 424, 389
Cooray, A. 2004, PhRvD, 70, 063509
Dodelson, S., & Jabas, J. M. 1995, ApJ, 439, 503
Furlanetto, S. R., Oh, S. P., & Briggs, F. H. 2006, PhR, 433, 181
George, E. M., Reichardt, C. L., et al. 2015, ApJ, 799, 177
Giannantonio, T., & Crittenden, R. 2007, MNRAS, 381, 819
Gnedin, N. Y., & Jaffe, A. H. 2001, ApJ, 551, 3
Gruzinov, A., & Hu, W. 1998, ApJ, 508, 435
Hill, J. C., Battaglia, N., Chluba, J., et al. 2015, PhRvL, 115, 261301
Hinshaw, G., Larson, D., Komatsu, E., et al. 2013, ApJS, 208, 19
Hu, W. 2000, ApJ, 529, 12
Hu, W., & Dodelson, S. 2002, ARA&A, 40, 171
Hu, W., Scott, D., & Silk, J. 1994, PhRvD, 50, 468
Iliev, I. T., Pen, U.-L., Bond, J. R., Melnem, G., & Shapiro, P. R. 2007, ApJ, 660, 933
Jaffe, A. H., & Kamionkowski, M. 1998, PhRvD, 58, 043001
Jelic, V., Zaroubi, S., Aghanim, N., et al. 2010, MNRAS, 402, 2279
Kaiser, N. 1984, ApJ, 282, 374
Kaiser, N. 1987, MNRAS, 227, 1
Knox, L., Scocciommaro, R., & Dodelson, S. 1998, PhRvL, 81, 2004
Lewis, A., & Challinor, A. 1998, PhR, 429, 1
Ma, C.-P., & Fry, J. N. 2002, PhRvL, 88, 211301
McQuinn, M., Furlanetto, S. R., Hernquist, L., Zahn, O., & Zaldarriaga, M. 2005, ApJ, 630, 643
Mesinger, A., McQuinn, M., & Spergel, D. N. 2012, MNRAS, 422, 1403
Ostriker, J. P., & Vishniac, E. T. 1986, ApJL, 306, L51
Park, H., Komatsu, E., Shapiro, P. R., Koda, J., & Mao, Y. 2016, ApJ, 818, 37
Park, H., Shapiro, P. R., Komatsu, E., et al. 2013, ApJL, 769, 93
Peiris, H. V., & Spergel, D. N. 2000, ApJ, 541, 605
Pen, U.-L., Sheth, R., Harisson-Derspas, J., Chen, X., & Li, Z. 2012, arXiv:1202.5804
Planck Collaboration et al. 2015, arXiv:1502.01589
Reichardt, C. L., Shaw, L., Zahn, Z. O., et al. 2012, ApJ, 755, 70
Salvaterra, R., Ciardi, B., Ferrara, A., & Baccigalupi, C. 2005, MNRAS, 360, 1063
Santos, M. G., Cooray, A., Haiman, Z., Knox, L., & Ma, C.-P. 2003, ApJ, 598, 756
Sunyaev, R. A. 1977, SvAL, 3, 268
Sunyaev, R. A. 1978, in IAU Symp. 79, Large Scale Structures in the Universe, ed. M. S. Longair, & J. Einasto (Dordrecht: Reidel), 393
Sunyaev, R. A., & Zeldovich, Y. B. 1970a, ApSS, 7, 3
Sunyaev, R. A., & Zeldovich, Y. B. 1970b, CoASP, 2, 66
Sunyaev, R. A., & Zeldovich, Y. B. 1972, CoASP, 4, 173
Tashiro, H., Aghanim, N., Langer, M., et al. 2010, MNRAS, 402, 2617
Tashiro, H., Aghanim, N., Langer, M., et al. 2011, MNRAS, 414, 3424
Trac, H., Bode, P., & Ostriker, J. P. 2011, ApJ, 727, 94
Valageas, P., Balbi, A., & Silk, J. 2001, A&A, 367, 1
Vishal, E., & Loeb, A. 2012, JCAP, 5, 7
Vishniac, E. T. 1987, ApJ, 322, 597
Weymann, R. M. 1966, ApJ, 145, 560
Zahn, O., Reichardt, C. L., Shaw, L., Lidz, A., et al. 2012, ApJ, 756, 65
Zahn, O., Zaldarriaga, M., Hernquist, L., & McQuinn, M. 2005, ApJ, 630, 657
Zel’dovich, Y. B., Illarionov, A. F., & Sunyaev, R. A. 1972, JETP, 35, 643
Zeldovich, Y. B., & Sunyaev, R. A. 1969, ApSS, 4, 301
Zhang, P., Pen, U.-L., & Trac, H. 2004, MNRAS, 347, 1224