Research Article

Risk-Averse Pricing Decisions Related to Recyclables’ Quality in a Closed-Loop Supply Chain

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In a closed-loop supply chain, uncertainty of recyclables’ quality is a major factor of supply chain members’ decision-making. Because of this uncertainty, manufacturers must pay varying manufacturing costs for remanufacturing recyclables. Our study assumed that manufacturers are risk-averse towards uncertainty in manufacturing costs and constructed a retailer recycling model and a third-party recycling model to investigate pricing decisions in a decentralized closed-loop supply chain under uncertainty about recyclables’ quality. Our findings can be summarized as follows: (1) the higher the degree of consumer preference for remanufactured products, the higher the wholesale and retail prices of remanufactured products and the higher the recycling price of used products; (2) the two recycling models showed a U-shaped relationship between supply chain revenue and the degree of consumer preference for remanufactured products, and this supply chain revenue is related to the consumer preference coefficient; (3) there is a U-shaped relationship between the retailer’s expected revenue and the degree of consumer preference for remanufactured products in the R mode and an M-shaped relationship between them in the 3P mode; (4) in both recycling modes, the manufacturer’s risk aversion is inversely proportional to supply chain revenue, and supply chain revenue in the R mode is higher than that in the 3P mode; and (5) the higher the uncertainty of recyclables’ quality, the lower the recycling price of used products and the lower the manufacturer’s enthusiasm for recycling or for used products.

1. Introduction

Along with rapid economic and technological development, changes in consumer demand and product updates are increasingly rapidly. To gain better experiences from products, consumers choose to shorten the product life cycle and buy new products, resulting in a sharp increase in the number of waste products in the market. According to the White Paper on WEEE Recycling Industry in China 2017, the theoretical scrappage of electrical and electronic products in China amounted to 500.04 million items in 2017, an increase of approximately 33% from 2016. According to the Annual Report of 2017 on Prevention and Control of Environmental Pollution by Solid Waste in China’s Large and Medium-sized Cities, 79.35 million electrical and electronic waste items were recycled and disposed of by 103 pollution treatment enterprises countrywide in 2016, with a recycling ratio of only 21%. In addition, the number of recycled electrical and electronic waste products in 2017 was virtually the same as that of 2016. Recycling of used products saves material and energy costs and is conducive to sustainable development [1]. Due to the differences in history and habits of product use, the quality of used products is uncertain and varies widely; therefore, their remanufacturing costs also differ [2, 3]. The wide variation in the quality of used products brings about uncertainty in remanufacturing costs paid by manufacturers. Hence, it is a matter of importance to study which attitude manufacturers will take toward uncertainty in remanufacturing costs to protect their own interests.

There have been a number of studies on the pricing of recycled used products. For example, Savaskan et al. [4] and Chuang et al. [5] studied pricing and coordination in closed-
loop supply chains under three conditions: recycling by manufacturers, recycling by retailers, and recycling by third parties. On this basis, Gong and Jiang [6] studied two closed-loop supply chain models (including one model for manufacturer and retailer hybrid recycling and one model for manufacturer, retailer, and third-party hybrid recycling) and proposed respective optimal decisions. Zhu et al. [7] investigated the pricing decisions in supply chains when the costs of recycling by traditional channels differ from those of online recycling. However, most of these studies focused on recycling channels and assumed that the quality of used products is consistent while ignoring the impact of the varying quality of used products in supply chains on remanufacturing.

The varying quality of used products may affect remanufacturing costs in supply chains, as well as the pricing of new and remanufactured products. Cheng et al. [8] studied the role of government subsidies in the pricing decisions in closed-loop supply chains when the quality of used products is uncertain. They found that the recycling price of used products decreases when the threshold of remanufacturing quality increases but increases with a rise in the subsidy coefficient. Examining government subsidies, Deng et al. [9] also considered carbon taxes, finding the following: (1) the optimal recycling quality coefficient is affected by carbon taxes; (2) government subsidies combined with carbon taxes can encourage the recycling of used products, reduce total carbon emissions, and encourage manufacturers to remanufacture more effectively. Guo and Tan [10] investigated the role of governmental regulation, which is manifested as the used recycling ratio specified by governments. They found that governmental regulation can effectively improve the quality of recycled used products; however, the difference between the degree of governmental regulation and the recycling payment coefficient at different levels must stay within a certain range to ensure the effectiveness of trading markets. Gao and Li [11] and Huang et al. [12] studied the pricing of recycled used products from the perspective of recycling channels, and both studies classified used products into different grades. In Gao and Li’s work, used products were recycled from consumers at the same prices, and then the recycled used products were graded. According to Huang et al., when used products were recycled in a competitive manner between manufacturers, retailers, and third parties, used products were graded according to their quality and were then priced. Most of the aforementioned studies assume that supply chain members are risk-neutral. However, Jammernegg and Kischka [13] found that decision makers’ risk appetite affects the pricing of used products in supply chains. By combining the risk appetite of supply chain members with the selection of recycling channels, Chen [14] investigated whether manufacturers resort to the direct selling channel when retailers are risk-averse. Their findings showed that (1) the direct selling channel does not always damage the revenue of retailers and that (2) manufacturers resort to the direct selling channel only when the costs of direct selling meet certain conditions; otherwise, manufacturers will sustain losses. Li et al. [15] constructed a two-level supply chain model based on dual-source channels. Their study showed that the optimal retail prices of products under a direct selling channel and those under traditional channels are not related to whether retailers cooperate with manufacturers, but rather to the degree of risk aversion on both sides. Most of the studies cited above focus on the risk resulting from fluctuations in new product demand in the market rather than the risk of uncertainty about the quality of used products.

In summary, this study investigated the pricing differences between new products and remanufactured products under two recycling modes, retailer recycling and third-party recycling. Our study considered the uncertainty of used products’ quality and the risk appetite of manufacturers and assumed that manufacturers are risk-averse to the uncertainty of used products’ quality. Accordingly, we discuss the following three issues: (1) how are recycled products and new products priced under different recycling modes? (2) How does the risk appetite of manufacturers affect the pricing of recycled products and new products? (3) How does the quality of used products affect closed-loop supply chains? Retailer recycling and third-party recycling are common in the market. Therefore, this study considered two recycling modes, retailer recycling (referred to as R mode hereinafter) and third-party recycling (referred to as 3P mode hereinafter).

The remainder of the paper is organized as follows. Section 2 presents the problem description and basic hypotheses. Section 3 constructs the two models of retailer recycling (R mode) and third-party recycling (3P mode), respectively, and their optimal pricing decisions are analyzed. Section 4 analyzes the property of the two models. Numeric analysis is given in Section 5. Finally, Section 6 concludes the paper.

2. Problem Description and Basic Hypotheses

With the recent changes in the market structure and the relative strength of supply chain members, manufacturers attach more importance to business innovation and R&D of new products. Moreover, the role of the recycler in a closed-loop supply chain has gradually shifted from manufacturers to third-party recyclers and retailers. Compared with manufacturers, third-party recyclers possess a more stable recycling network, and retailers are closer to markets; therefore, commissioned third-party recycling and retailer recycling have gradually gained popularity. Considering the two recycling modes, R and 3P, this study presents a closed-loop supply chain model comprising a single manufacturer, a single retailer, a single third-party recycler, and a single product (Figure 1). In a traditional logistics (forward logistics) process, the manufacturer sells the new product and the remanufactured product to the retailer at the wholesale prices of $w_n$ and $w_r$, respectively; then, the retailer sells them to consumers at the retail prices of $p_n$ and $p_r$. In a reverse logistics process, the retailer or third-party recycler recycles the used product from consumers at the price of $b_r$, and the manufacturer purchases the used product from the retailer or third-party recycler at the transfer price of $b_m$. The manufacturer turns the used product into a remanufactured product. This study will investigate the problem of pricing new and remanufactured products under the two recycling modes.
product and sells the remanufactured product in a forward logistics process.

Under the condition of complete market information disclosure, the manufacturer is the market dominator, and optimal revenue models under respective recycling modes are built according to game theory. The symbolic variables and model hypotheses are described as follows.

The symbolic variables are introduced and described as follows:

- \(d_n\) and \(d_r\), respectively, denote the market demand for the new product and remanufactured product, where \(d_n > d_r\).
- \(c_n\) and \(c_r\), respectively, denote the unit manufacturing cost of the new product made from new materials and the unit manufacturing cost of the remanufactured product made from a used product, where \(c_n > c_r\).
- \(ω_n\) and \(ω_r\), respectively, denote the unit wholesale prices of the new product and remanufactured product.
- \(p_n\) and \(p_r\) respectively, denote the unit retail prices of the new product and remanufactured product.
- \(b_m^r(> 0)\) denotes the unit price at which the manufacturer transfers a used product from the retailer, namely, the transfer price.
- \(b_r(>0)\) denotes the unit price at which the retailer recycles a used product from consumers, namely, the recycling price.
- \(q_r\) denotes the quantity of used products recycled from consumers, and \(dq_r = h + yb_r\), where \(h > 0\) indicates the quantity of used products that consumers return for recycling voluntarily and \(y\) denotes the price elasticity of consumers to recycled used products.
- \(u_n\) and \(u_r\), respectively, denote the net utility of the new product and the remanufactured product to consumers; consumers decide to purchase the new product or remanufactured product based on a comparison between \(u_n\) and \(u_r\).

\[\prod_j\] denotes the profit function of supply chain member \(j\) under recycling mode \(i\) in the closed-loop supply chain \((i = R \text{ or } 3P)\); the subscript \(j = R, 3P, T\), respectively, denote the retailer, the third-party retailer, and the supply chain; the superscript * denotes the optimal decision.

2.1. Model Hypotheses

**Hypothesis 1.** The manufacturer is the market dominator and is risk-averse.

**Hypothesis 2.** The quality of recycled used products is uncertain, and the quality of used products \((x)\) follows normal distribution, in which the expectation is \(μ\) and the variance is \(σ^2\), namely, \(x \in N(μ, σ^2)\). Here, \(μ\) denotes the average quality of \(μ\), and \(σ\) denotes the fluctuation in the quality of used products around the average quality. It is uncertain whether the \(σ\) value is positive or negative, and \(|σ| > |μ| > 0\). The unit manufacturing cost of the remanufactured product \((c_r)\) is related to the quality of used products. In other words, \(c_r = c_n − vx\), where \(v(v > 0)\) denotes the cost-of-quality coefficient and \(vx\) denotes the cost saved by recycling and remanufacturing the used product with quality \(x\).

**Hypothesis 3.** At the transfer price of \(b_m^r\), the manufacturer transfers the used products recycled by the retailer; at the price of \(b_r\), the retailer recycles the used products from consumers. In other words, \(b_r = (1 − r)b_m^r\), where \(r\) denotes the profit per unit of the used product recycled by the retailer.

**Hypothesis 4.** To encourage the recycler to recycle a product with higher quality, the manufacturer “rewards” the retailer or third-party retailer with the amount of the cost saved from the remanufactured product in the proportion of \(θ(0 < θ < 1)\), namely, \(θxd_r\); \(kx^2/2\) denotes the effort cost that the retailer or third-party recycler pays to recycle a product with higher quality \((k\) is the effort cost coefficient).

**Hypothesis 5.** The recycled used products can all be remanufactured at varying costs, the market demand for remanufactured products is smaller than or equal to the quantity of recycled used products, and the manufacturer treats the value of non-remanufactured used products as the scrap value \(s\). The scrap value is small enough so that it is not considered in any model used in this study.

**Hypothesis 6.** Consumers’ valuation of the new product \((φ)\) obeys a uniform distribution in the range of \([A, B]\), and the probability density function is as follows:

\[
f(x) = \begin{cases} 
\frac{1}{(B - A)} & x \in [A, B], \\
0, & x \notin [A, B]. 
\end{cases}
\]

\[1\]
Consumers’ utility valuation of a remanufactured product is \( a \) times as much as that of a new product; \( a \) denotes the coefficient of consumer preference for a remanufactured product \((0 < a < 1)\).

**Hypothesis 7.** The manufacturer is risk-averse about the quality of used products, and the expected utility of the closed-loop supply chain and its members is measured using the mean-variance method. The utility function is \( U(\prod^*) = E(\prod^*) - \lambda \text{Var}(\prod^*) \), in which \( \lambda \) denotes the degree of risk aversion \((0 \leq \lambda \leq 1)\) [16, 17].

Consumers decide whether to purchase new products or remanufactured products based on a comparison of utility between the products. This section focuses on the pricing decisions for new products and remanufactured products; therefore, we ignored the time and distance costs from the purchase of products. These include the differences between product utility (i.e., product valuation) and retail prices. Therefore, the utility of purchased new products is expressed as follows:

\[
\begin{align*}
  u_n &= \phi - p_n, \\
  u_r &= a\phi - p_r,
\end{align*}
\]

and \((p_n - p_r)/(1 - a) \leq B\), namely, when \( a \leq 1 - (p_n - p_r)/B \), the quantity of purchased new products is expressed as follows:

\[
d_n = (B - A) \int_{(p_n - p_r)/(1 - a)}^{B} f(x)dx = B - \frac{p_n - p_r}{1 - a} \quad (3)
\]

\[
\bigcirc \quad (p_n - p_r)/(1 - a) \leq B, \quad \text{nearly, when} \quad a \leq 1 - (p_n - p_r)/B, \quad \text{the quantity of purchased remanufactured products is expressed as follows:}
\]

\[
d_r = (B - A) \int_{p_n/a}^{p_n/a} f(x)dx = \frac{ap_n - p_r}{a(1 - a)}. \quad (4)
\]

3. Optimal Pricing Decisions in the Closed-Loop Supply Chain under Decentralized Decision-Making

3.1. Optimal Pricing Decisions under Retailer Recycling (R Mode). Under the R mode, the manufacturer recycles used products through the retailer. The manufacturer’s revenue includes the revenue from the wholesale of new products and remanufactured products, as well as the costs saved by the remanufacturing of used products. The manufacturer’s costs include the manufacturing cost of new products and remanufactured products, the transfer cost of the used products recycled by the retailer, and the reward given to the retailer. The retailer’s revenue includes the revenue from the sale of new products and remanufactured products to consumers, the manufacturer’s transfer cost, and the reward for recycling of used products. The retailer’s costs include the wholesale cost of new products and remanufactured products paid to the manufacturer, the recycling cost paid to consumers, and the cost of the effort to recycle used costs with higher quality. Accordingly, the revenue function models for the manufacturer and retailer under the R mode can be expressed as follows:

\[
\begin{align*}
  \prod^R &= \left(\omega_n - c_n\right)d_n^R + \left(\omega_r - c_r\right)d_r^R - b_mq_r - \theta vxd_r^R, \\
  \prod^R &= \left(\omega_n - c_n\right)d_n^R + \left(\omega_r - c_r\right)d_r^R + b_mq_r + \theta vxd_r^R - b_rq_r - \frac{k\mu^2 + \sigma^2}{2} \quad (5)
\end{align*}
\]

In this model, the manufacturer is the leader of the supply chain, so the decision-making sequence is as follows: (1) the manufacturer decides the wholesale prices of new products and remanufactured products and the transfer prices of recycled used products; (2) the retailer decides the retail prices of new products and remanufactured products and the recycling prices of recycled used products. Using the backward induction method, the retailer’s expected profit \( E(\prod^R) \) is expressed as follows:

\[
E(\prod^R) = \left(\mu^R - \omega_n\right)d_n^R + \left(\mu^R - \omega_r\right)d_r^R + b_mq_r + \theta vxd_r^R - b_rq_r - \frac{k\mu^2 + \sigma^2}{2}. \quad (7)
\]

We calculate the first-order partial derivative of the retailer’s utility with respect to the retail price of new products \(p_n^R\), retail price of remanufactured products \(p_r^R\), and per-unit profit \((\mu^R)\), construct their simultaneous equations, and calculate the solutions as follows:

\[
\begin{align*}
  p_n^R &= \frac{B + \omega_n}{2}, \\
  p_r^R &= \frac{aB - v\mu + \omega_r}{2}, \quad (8)
\end{align*}
\]
Next, we substitute equations (8) to (10) into the manufacturer's expected utility function and calculate the first-order partial derivative of the wholesale price of new products ($\omega^*_n$), wholesale price of remanufactured products ($\omega^*_R$), and transfer price ($b^*_m$). Then, we determine the optimal solutions of simultaneous equations. The manufacturer is risk-averse, so the manufacturer’s utility can be calculated according to the criterion $U(\prod^R_M) = E(\prod^R_M) - \lambda \sqrt{\text{Var}(\prod^R_M)}$. Using the manufacturer’s expected utility function (equation (5)), we calculate the manufacturer’s expected utility as follows:

$$U\left(\prod^R_M\right) = (\omega^*_n - c_n) d_n^R + (\omega^*_R - c_n + \nu \mu) d^R - b^*_m d^R - \theta \nu \mu d^R - \lambda \nu \sigma \frac{(a p^*_R - p^*_R)(1 - \theta)}{(1 - a)a}. \quad (11)$$

We then substitute equations (8) to (10) into equation (11); thus, the manufacturer’s expected utility is as follows:

$$U\left(\prod^R_M\right) = (\omega^*_n - c_n) d_n^R + (\omega^*_R - c_n + \nu \mu) d^R - b^*_m d^R - \theta \nu \mu d^R - \lambda \nu \sigma \frac{(a \omega^*_n - \omega^*_R - \nu \mu)(1 - \theta)}{2a - 2a^2} \geq 0. \quad (12)$$

Before solving the above equation, we solve the Hesse matrix of $U(\prod^R_M)$ and determine whether it has an optimal solution. The Hessian matrix is expressed as follows:

$$H^R_M = \begin{cases} 1 & 1 & 0 \\ a - 1 & 1 - a & 0 \\ 1 & 1 - a & (a - 1)a \\ 0 & 0 & -\gamma \end{cases}. \quad (13)$$

As $a < 1$, $|H^R_M| = -\gamma/(1 - a)a < 0$, the second-order principal minor is greater than 0, and the first-order principal minor is smaller than 0. Therefore, $U(\prod^R_M)$ is a strictly concave function, which has a unique optimal solution.

The demand for remanufactured products should be smaller than or equal to the quantity of recycled used products. Therefore, the Lagrange multiplier $\xi_1 \geq 0$ is introduced to the constraint condition in $U(\prod^R_M)$ to construct the following Lagrange function:

$$L\left(\omega^*_n, \omega^*_R, b^*_m, \xi_1\right) = (\omega^*_n - c_n) d_n^R + (\omega^*_R - c_n + \nu \mu) d^R - b^*_m d^R - \theta \nu \mu d^R - \lambda \nu \sigma \frac{(a \omega^*_n - \omega^*_R - \nu \mu)(1 - \theta)}{2a - 2a^2}$$

$$+ \left(\omega^*_R - c_n + \nu \mu\right) \frac{(\nu \theta \mu + a \omega^*_n - \omega^*_R)}{2a - 2a^2} - \theta \nu \mu \frac{(a \omega^*_n - \omega^*_R + \nu \mu)(1 - \theta)}{2a - 2a^2} + \xi_1 \left(h + b^*_m \nu \mu - \nu \theta \mu + a \omega^*_n - \omega^*_R\right). \quad (14)$$

According to the KT condition, two circumstances are considered (i.e., $\xi_1 = 0$ and $\xi_1 \neq 0$). When $\xi_1 = 0$, $L\left(\omega^*_n, \omega^*_R, b^*_m, \xi_1\right) = U(\prod^R_M)$; then, the solution is as follows:
\[ \omega^R_n = \frac{B + c_n}{2}, \]
\[ \omega^R_r = \frac{1}{2} (aB + c_n + s_v (1 - 2\theta)\mu + v (1 - \theta)\lambda\sigma) \tag{15} \]
\[ b^R_m = -\frac{h}{2\gamma} < 0. \]

The transfer price \( b^R_m \) cannot be negative; therefore, it is omitted when \( \epsilon_1 = 0 \).

When \( \epsilon_1 \neq 0 \), the solution is as follows:

\[ \omega^R_n = \frac{B + c_n}{2}, \tag{16} \]
\[ \omega^R_r = \frac{\nu \theta \mu + ac_n + \nu \lambda \sigma (1 - \theta)(1 - a)\gamma + a(a - 1) (h - \gamma (c_n - \nu \mu (1 - \theta))) + aB + \nu \theta \mu}{2(1 + (1 - a)\gamma)}, \tag{17} \]
\[ b^R_m = \frac{\gamma (\nu (\mu - (1 - \theta)\lambda\sigma) - c_n(1 - a)) + h - \frac{h}{\gamma}}{2\gamma(1 + (1 - a)\gamma)}, \tag{18} \]
\[ \epsilon_1^* = \frac{\nu (\mu - (1 - \theta)\lambda\sigma) - (1 - a)(c_n + ah)}{1 + (1 - a)\gamma}. \tag{19} \]

Therefore, the optimal pricing decision under the R mode is as follows:

\[ \omega^R_n = \frac{B + c_n}{2}, \]
\[ \omega^R_r = \frac{\nu \theta \mu + ac_n + \nu \lambda \sigma (1 - \theta)(1 - a)\gamma + a(a - 1) (h - \gamma (c_n - \nu \mu (1 - \theta))) + aB + \nu \theta \mu}{2(1 + (1 - a)\gamma)}, \]
\[ b^R_m = \frac{\gamma (\nu (\mu - (1 - \theta)\lambda\sigma) - c_n(1 - a)) + h - \frac{h}{\gamma}}{2\gamma(1 + (1 - a)\gamma)}, \]
\[ \epsilon_1^* = \frac{\nu (\mu - (1 - \theta)\lambda\sigma) - (1 - a)(c_n + ah)}{1 + (1 - a)\gamma}. \]

We substitute equations (15) to (18) into equations (8) to (10); then, the retail prices of new products and remanufactured products and per-unit profit of recycled used products are as follows:

\[ p^R_n = \frac{1}{4} (3B + c_n), \tag{21} \]
\[ p^R_r = \frac{3aB}{4} + \frac{a(c_n(1 - a)(h - \gamma(c_n - \nu (\mu - (1 - \theta)\lambda\sigma)) + h)}{4(1 + (1 - a)\gamma)}, \tag{22} \]
\[ r^R_R = \frac{h - (1 - a)c_n\gamma + \nu \gamma(\mu - (1 - \theta)\lambda\sigma)}{2(\nu \gamma(\mu - (1 - \theta)\lambda\sigma) - (1 - a)c_n\gamma - h(1 + 2\gamma - 2a^2\gamma))}. \tag{23} \]

We substitute equations (21) to (23) into equations (3) and (4); then, the demand for new products and demand for remanufactured products, respectively, are as follows:

\[ d^R_n = \frac{h + \gamma b^R_n}{4(1 + (1 - a)\gamma)}, \tag{24} \]
\[ d^R_r = \frac{h + \gamma b^R_r}{4(1 + (1 - a)\gamma)}, \tag{25} \]
\[ b^R_n = \frac{\gamma (\nu (\mu - (1 - \theta)\lambda\sigma) - c_n(1 - a)) + h(3 + 4(1 - a)\gamma)}{4\gamma(1 + (1 - a)\gamma)}. \tag{26} \]

Therefore, the quantity of recycled used products is as follows:

\[ q^R = \frac{h + \gamma b^R}{4(1 + (1 - a)\gamma)}. \tag{27} \]
We substitute equations (15) to (27) into equations (11) and (7); then, the manufacturer’s expected utility and the retailer’s expected profit, respectively, are as follows:

\[
U\left(\frac{R^*}{M}\right) = \frac{(h + yac_n)^2 + (h - y(c_n - v(\mu - (1 - \theta)\lambda\sigma)))^2 - h^2}{8\gamma (1 + (1 - a)\gamma)} + \frac{(B - c_n)^2}{8} - \frac{c_n a y (c_n - v(\mu - (1 - \theta)\lambda\sigma))}{1 + (1 - a)\gamma},
\]

(28)

\[
E\left(\frac{R^*}{R}\right) = \frac{(h - y(1 - a)c_n)^2 - y^2(1 - a)c_n - v(\mu - (1 - \theta)\lambda\sigma))^2}{16\gamma (1 + (1 - a)\gamma)} + \frac{(c_n + B)^2}{16} - \frac{k(\mu^2 + \sigma^2)}{2}.
\]

(29)

### 3.2. Optimal Pricing Decisions under Third-Party Recycling (3P Mode)

Under the 3P mode, the manufacturer commissions a third-party recycler to recycle used products. The financial gains realized by the manufacturer include revenue from the wholesale of new products and remanufactured products and manufacturing costs saved by the remanufacturing of used products. The manufacturer’s costs include the manufacturing cost of new products and remanufactured products, the transfer cost of used products recycled by the third-party recycler, and the reward given to the third-party recycler. The retailer’s revenue is the revenue from the sale of new products and remanufactured products to consumers. The retailer’s cost is the wholesale cost of new products and remanufactured products paid to the manufacturer. The third-party recycler’s revenue includes the transfer cost paid by the manufacturer and the reward given by the manufacturer for recycling used products. The third-party recycler’s costs include the wholesale cost paid to consumers and effort cost paid to recycle used costs with higher quality. Accordingly, the revenue function models for the manufacturer, retailer, and third-party recycler are, respectively, expressed as follows:

\[
\prod_m = (\omega_n^3 - c_n^{3P})d_n^{3P} + (\omega_r^3 - c_r^{3P})d_r^{3P} - b_m^{3P}q_r^{3P} - \theta v_d q_r^{3P}, q_r^{3P} \geq d_r^{3P},
\]

\[
\prod_r = (\prod_m^3 - \omega_n^3) d_n^{3P} + (\prod_r^3 - \omega_r^3) d_r^{3P},
\]

\[
\prod_r^3 = b_m^{3P} q_r^{3P} - b_r q_r^{3P} + \theta v_d q_r^{3P} - \frac{k(\mu^2 + \sigma^2)}{2}.
\]

(30)

(31)

(32)

At this time, the manufacturer is the leader of the supply chain, so the decision-making sequence is as follows: (1) the manufacturer decides the wholesale prices of new products and remanufactured products and transfer prices of used products recycled from the third-party recycler; (2) after being informed of the manufacturer’s pricing decisions, the retailer and third-party recycler, respectively, decide the retail prices of new products and remanufactured products and the recycling prices of used products based on the principle of self-interest maximization. Using the backward induction method, the retailer’s expected profit \(E(\prod_r^3)\) and the third-party recycler’s expected profit \(E(\prod_r^3)\) are calculated according to equations (31) and (32). \(E(\prod_r^3)\) and \(E(\prod_r^3)\) are expressed as follows:

\[
E(\prod_r) = (p_n^{3P} - \omega_n^{3P}) d_n^{3P} + (p_r^{3P} - \omega_r^{3P}) d_r^{3P},
\]

(33)

\[
E(\prod_r^3) = b_m^{3P} q_r^{3P} - b_r q_r^{3P} + \theta v_d q_r^{3P} - \frac{k(\mu^2 + \sigma^2)}{2}.
\]

(34)

We calculate the first-order partial derivative of the retailer’s utility with respect to the retail price of new products \(p_n^{3P}\) and retail price of remanufactured products \(p_r^{3P}\) and calculate the first-order partial derivative of the third-party recycler’s utility with respect to the per-unit profit of recycled products \(r_r^{3P}\). We construct their simultaneous equations and calculate the solutions as follows:

\[
p_n^{3P*} = \frac{B + \omega_n^{3P}}{2},
\]

(35)

\[
p_r^{3P*} = \frac{aB + \omega_r^{3P}}{2},
\]

(36)

\[
r_r^{3P*} = \frac{h + b_m^{3P} v_d}{2b_m^{3P}},
\]

(37)

We substitute equations (35) to (37) into the manufacturer’s expected utility function and calculate the first-order partial derivatives of the wholesale price of new products \(\omega_n^{3P*}\), the wholesale price of remanufactured products \(\omega_r^{3P*}\), and the transfer price \(b_r^{3P*}\). We can then determine the optimal solutions of their simultaneous equations. The manufacturer is risk-averse, so the manufacturer’s utility can be calculated according to the criterion \(U(\prod_m^3) = E(\prod_m^3) - \lambda Var(\prod_m^3)\). Using the manufacturer’s expected utility function (equation (25)), we determine the manufacturer’s expected utility as follows:

\[
U(\prod_m^3) = \frac{(h + yac_n)^2 + (h - y(c_n - v(\mu - (1 - \theta)\lambda\sigma)))^2 - h^2}{8\gamma (1 + (1 - a)\gamma)} + \frac{(B - c_n)^2}{8} - \frac{c_n a y (c_n - v(\mu - (1 - \theta)\lambda\sigma))}{1 + (1 - a)\gamma},
\]

(28)
We substitute the determined retail price of new products ($p_{nM}^{3P}$) and retail price of remanufactured products ($p_{rM}^{3P}$)—namely, equations (35) and (36)—into equation (38); thus, the manufacturer’s expected utility is as follows:

\[
U \left( \prod_{M}^{R} \right) = \left( \omega_{nM}^{3P} - c_{nM} \right) \left( B - \omega_{nM}^{3P} - \omega_{rM}^{3P} \right) + \left( \omega_{rM}^{3P} - c_{nM} + \nu \mu \right) \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) - b_{m}^{3P} \left( h - \frac{y b_{mM}^{3P} - h}{2} \right) - \theta \nu \mu \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) - \nu^{2} \sigma^{2} \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) (1 - \theta)^{2} - \nu^{2} \sigma^{2} \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) (1 - \theta)^{2}
\]

Before solving the above equation, we solve the Hessian matrix of $U \left( \prod_{M}^{R} \right)$ and judge whether it has an optimal solution. The Hessian matrix is expressed as follows:

\[
H_{MM}^{3P} = \begin{pmatrix}
1 & 1 & 0 \\
1 & 1 - a & 0 \\
1 - a & (a-1)a & 0 \\
0 & 0 & -\gamma
\end{pmatrix}
\]

As $a < 1$, $|H_{MM}^{3P}| = -\gamma^{2}(1-a)<0$; the second-order principal minor is greater than 0, and the first-order principal minor is smaller than 0. Therefore, $U \left( \prod_{M}^{3P} \right)$ is a strictly concave function, which has a unique optimal solution.

The demand for remanufactured products should be smaller than or equal to the quantity of recycled used products. According to this constraint condition, the Lagrange multiplier $\varepsilon_2$ is introduced to the constraint condition in $U \left( \prod_{M}^{3P} \right)$ to construct the following Lagrange function:

\[
L \left( \omega_{nM}^{3P}, \omega_{rM}^{3P}, b_{mM}^{3P}, \varepsilon_2 \right) = \left( \omega_{nM}^{3P} - c_{nM} \right) \left( B - \omega_{nM}^{3P} - \omega_{rM}^{3P} \right) + \left( \omega_{rM}^{3P} - c_{nM} + \nu \mu \right) \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) - b_{m}^{3P} \left( h - \frac{y b_{mM}^{3P} - h}{2} \right) - \theta \nu \mu \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) - \nu^{2} \sigma^{2} \left( \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right) (1 - \theta)^{2} + \varepsilon_2 \left( \frac{h + b_{m}^{3P} \nu}{2} - \frac{a \omega_{nM}^{3P} - \omega_{rM}^{3P}}{2a - 2a^{2}} \right)
\]

According to the Kuhn–Tucker condition, we consider two circumstances (i.e., $\varepsilon_2 = 0$ and $\varepsilon_2 \neq 0$).
We substitute equations (43)–(45) into equations (35)–(37); thus, the retail prices of new products and remanufactured products and per-unit profit of recycled used products are as follows:

\[ p_{n}^{3P^*} = \frac{1}{4} (3B + c_{n}), \]  

\[ p_{r}^{3P^*} = \frac{3aB}{4} + \frac{a(c_{n} - (1 - a)(h - \gamma(c_{n} - \nu(1 - \theta)(\mu - \lambda\sigma))))}{4 + 4(1 - a)\gamma} \]  

\[ r_{r}^{3P^*} = \frac{\nu\gamma(1 - \theta)(\mu - \lambda\sigma) - (1 - a)c_{n}\gamma + h}{2(\nu\gamma(1 - \theta)(\mu - \lambda\sigma) - (1 - a)c_{n}\gamma - h(1 + 2a\gamma - 2a^{2}\gamma))} \]

We then substitute equations (48) and (49) into equations (3) and (4); then, the demand for new products and demand for remanufactured products are, respectively, as follows:

\[ d_{n}^{3P^*} = \frac{B}{4} - \frac{c_{n} + a(h + \nu\gamma(1 - \theta)(\mu - \lambda\sigma))}{4(1 - a)\gamma} \]  

\[ d_{r}^{3P^*} = \frac{h + \gamma(1 - \theta)(\mu - \lambda\sigma) - c_{n}(1 - a)}{4(1 + (1 - a)\gamma)} \]

Then, the quantity of recycled used products is as follows:

\[ q_{r}^{3P^*} = \frac{h + \gamma(1 - \theta)(\mu - \lambda\sigma) - c_{n}(1 - a)}{4(1 + (1 - a)\gamma)} \]

We substitute equations (43)–(53) into equations (33), (34), and (38); then, the manufacturer’s expected utility, the retailer’s expected profit, and the third-party recycler’s expected profit, respectively, are as follows:

\[ U\left(3P^*\right) = \frac{(B - c_{n})^{2} + (h + \gamma c_{n}a)^{2} - (h - \gamma(c_{n} - \nu(1 - \theta)(\mu - \lambda\sigma)))^{2}}{8(1 + (1 - a)\gamma)} \]  

\[ - \frac{\nu ac_{n}(c_{n} - \nu(1 - \theta)(\mu - \lambda\sigma))}{4(1 + (1 - a)\gamma)} \]

\[ E\left(3P^*\right) = \frac{(B - c_{n})^{2}}{16} - \frac{(1 - a)c_{n}\gamma(h + \nu\gamma(1 - \theta)(\mu - \lambda\sigma))}{8(1 + (1 - a)\gamma)^{2}} \]  

\[ - \frac{(h + \nu\gamma(1 - \theta)(\mu - \lambda\sigma))^{2}(1 - a)a}{16(1 + (1 - a)\gamma)^{2}} \]
4. Property Analysis

Property 1. Under R and 3P modes, both optimal wholesale price and optimal retail price of remanufactured products are directly proportional to the coefficient of consumer preference for remanufactured products.

Proof. Under the R mode, the first-order partial derivative of the optimal wholesale price of remanufactured products concerning the consumer preference coefficient is as follows:

\[
\frac{\partial \omega^*_R}{\partial \alpha} = \frac{B}{2} + c_n + c_n \gamma -(h + \nu \gamma ((\mu - (1 - \theta) \lambda \sigma)) \frac{B}{2} + c_n + \gamma (c_n - \nu \mu) - h
\]

According to equation (3), the demand for new products is \(B - (p_n - p_r)/(1 - a) < B\), and the demand for remanufactured products is smaller than the demand for new products: \(d_R^* < d_{Rn}^*\). Under the R mode, \(d_R^* = q_{Rn}^* = h + b_R^*\). Therefore, \(q_{Rn}^* = h + b_R^* < d_{Rn}^* = B - (p_n - p_r)/(1 - a) < B\), proving that the quantity of used products voluntarily recycled by consumers \((h)\) is far smaller than consumers’ maximum valuation of new products \((B)\); \(c_n - \nu \mu\) denotes the average manufacturing cost of remanufactured products. Therefore, \(\partial \omega^*_R / \partial \alpha \geq B/2 + (c_n + \gamma (c_n - \nu \mu) - h)/2 > 0\). Under the R mode, the optimal wholesale price of remanufactured products is directly proportional to the degree of consumer preference for remanufactured products. Likewise, it can be said that, under the 3P mode, the optimal wholesale price of remanufactured products is directly proportional to the consumer preference coefficient.

Property 1 shows that, under R and 3P modes, the optimal wholesale and retail prices of remanufactured products will increase if the degree of consumer preference for remanufactured products rises. A rise in the degree of consumer preference for remanufactured products implies an increase both in consumers’ utility valuation and their degree of acceptance of remanufactured products; in this case, the manufacturer wants to earn more revenue by raising the optimal wholesale and retail prices of remanufactured products.

Property 2. Under R and 3P modes, the optimal recycling price of used products depends on the coefficient of customer preference for remanufactured products \((a)\): (1) when \(a > 1/2\), the optimal recycling price of used products is directly proportional to the coefficient of customer preference for remanufactured products under R and 3P modes; (2) when \(a < 1/2\), the optimal recycling price of used products in

The four models is directly proportional to the coefficient of customer preference for remanufactured products only if \(h < (c_n + 2(1 - a)c_n \gamma)/(2 - 4a) - \nu \gamma (\mu - \lambda \sigma)\).

Proof. In the four models, the first-order partial derivative of the optimal recycling price of used products concerning the consumer preference coefficient is as follows:

\[
\frac{\partial h^*_{3P}}{\partial \alpha} = c_n + (1 - a)^2 c_n \gamma - (1 - 2a)(h + \nu \gamma (\mu - (1 - \theta) \lambda \sigma)) \frac{4}{2(1 + (1 - a)\gamma)^2}.
\]

Then, the positive or negative characteristic of \(\partial h^*_{3P} / \partial \alpha\) and \(\partial h^*_{3P} / \partial \alpha\) is related to the positive or negative characteristic of \(1 - 2a\). When \(1 - 2a > 0\) (namely, \(a < 1/2\)), \(\partial h^*_{3P} / \partial \alpha\) and \(\partial h^*_{60} / \partial \alpha\) are both greater than 0. When \(1 - 2a < 0\) (namely, \(a > 1/2\)), they cannot be judged directly. If \(\partial h^*_{3P} / \partial \alpha\) and \(\partial h^*_{60} / \partial \alpha\) are still greater than 0, it is necessary to specify the value range of \(h\). Make \(\partial h^*_{3P} / \partial \alpha\) and \(\partial h^*_{60} / \partial \alpha\) greater than 0; then, the \(h^*\) value is as follows:

\[
h^*_{3P} = \frac{c_n(1 + (1 - a)^2) \gamma}{1 - 2a} - \nu \gamma (\mu - (1 - \theta) \lambda \sigma),
\]

The value range of \(h^*_{3P}\) is greater than that of \(h^*_{3P}\), namely, \(h^*_{3P} > h^*_{60}\). To ensure that the optimal recycling price of used products is directly proportional to the coefficient of customer preference for remanufactured products under the
R and 3P modes when \( a < 1/2 \), \( h^* \) must be equal to 
\[
c_n(1 + (1 - a)^2\gamma)/2 - 2a - (1 - \theta)(\mu - \lambda \sigma).
\]

Property 2 shows that, under \( R \) and 3P modes, the optimal recycling price of used products is related to the customer's preference for remanufactured products. When \( a > 1/2 \), a high degree of customer preference for remanufactured products creates a large demand, and under \( R \) and 3P modes, the optimal recycling price of used products will increase with the rise in the degree of customer preference for remanufactured products. In this case, there is an increasing demand for remanufactured products among consumers, so the manufacturer and recycler will raise the recycling price of used products to acquire more used products for remanufacturing. When \( a < 1/2 \), the degree of customer preference for remanufactured products is low. The optimal recycling price of used products will increase with the rise in the degree of customer preference for remanufactured products only if 
\[
h < (c_n + 2(1 - a)^2c_n\gamma)/(2 - 4a) - (1 - \theta)(\mu - \lambda \sigma),
\]
\( h \) denotes the quantity of used products voluntarily handed in by consumers for recycling. When 
\[
h > (c_n + 2(1 - a)^2c_n\gamma)/(2 - 4a) - (1 - \theta)(\mu - \lambda \sigma),
\]
there are many highly environmentally conscious consumers in the market, and many used products can be recycled without a reward or price incentive. In this case, a low recycling price is enough to ensure recycling of the required quantity of used products for remanufacturing. When \( h \) is smaller than the above value, consumers are not highly environmentally conscious. To recycle a larger quantity of used products, a certain price incentive is required. Given a rise in the degree of customer preference for remanufactured products, the manufacturer and the recycler can simply raise the recycling price of used products to recycle more used products, thus satisfying the increasing demand for remanufactured products.

**Property 3.** Under \( R \) and 3P modes, the optimal retail price of remanufactured products \( (p^*_r) \) is inversely proportional to the average quality of used products \( (\mu) \) but directly proportional to the quality fluctuation margin of used products \( (\sigma) \); in contrast, the recycling price of used products \( (b^*_r) \) is directly proportional to the average quality of used products \( (\mu) \) but inversely proportional to the quality fluctuation margin of used products \( (\sigma) \).

**Proof.** In the four models, the first-order partial derivatives of the optimal retail price of remanufactured products \( (p^*_r) \) concerning the average quality \( (\mu) \) and quality fluctuation margin \( (\sigma) \) of used products are as follows:
\[
\frac{\partial p^*_r}{\partial \mu} = \frac{(1 - a)\alpha \gamma}{4 + 4(1 - a)\alpha \gamma} < 0, \\
\frac{\partial p^*_r}{\partial \sigma} = \frac{(1 - a)\alpha \gamma(1 - \theta)\lambda}{4 + 4(1 - a)\alpha \gamma} > 0,
\]
Under \( R \) and 3P modes, the first-order partial derivatives of the optimal recycling price of used products \( (b^*_r) \) concerning the average quality \( (\mu) \) and quality fluctuation margin \( (\sigma) \) of used products are as follows:
\[
\frac{\partial b^*_r}{\partial \mu} = \frac{\nu}{4 + 4(1 - a)\alpha \gamma} > 0, \\
\frac{\partial b^*_r}{\partial \sigma} = \frac{\nu(1 - \theta)\lambda}{4 + (1 - a)\alpha \gamma} < 0.
\]

**Property 4.** Under \( R \) and 3P modes, the optimal total revenue of the supply chain and the optimal recycling price of used products are inversely proportional to the manufacturer’s risk aversion coefficient, whereas the optimal retail price of remanufactured products is directly proportional to the manufacturer’s risk aversion coefficient.

**Proof.** Under \( R \) and 3P modes, the first-order partial derivatives of the optimal recycling price of used products considering the manufacturer’s risk aversion coefficient are as follows:
\[
\frac{\partial b^*_r}{\partial \lambda} = \frac{\nu(1 - \theta)\sigma}{4 + 4(1 - a)\alpha \gamma} < 0, \\
\frac{\partial b^*_r}{\partial \lambda} = \frac{\nu(1 - \theta)\sigma}{4 + 4(1 - a)\alpha \gamma} < 0.
\]
Under $R$ and $3P$ modes, the first-order partial derivatives of the optimal wholesale price of remanufactured products concerning the manufacturer’s risk aversion coefficient are as follows:

\[
\frac{\partial P^*_R}{\partial \lambda} = \frac{(1 - a)av(1 - \theta)\sigma}{4 + 4(1 - a)ay} > 0, \\
\frac{\partial P^*_R}{\partial \lambda} = \frac{(1 - a)av(1 - \theta)\sigma}{4 + 4(1 - a)ay} > 0.
\]

\[
\frac{\partial E(\Pi^*_R)}{\partial \lambda} = 3\nu(1 - \theta)\sigma(h + \nu\nu(\mu - (1 - \theta)\lambda) - (1 - a)c_n) \times \frac{8 + 8(1 - a)ay}{8 + 8(1 - a)ay} < 0,
\]

\[
\frac{\partial E(\Pi^*_R)}{\partial \lambda} = -\nu(1 - \theta)\sigma(3h + \nu(3 - \theta)\mu - 3(1 - \theta)\lambda) \times \frac{8 + 8(1 - a)ay}{8 + 8(1 - a)ay} < 0.
\]

4.1. QED. Property 4 shows that the degree of the manufacturer’s risk aversion affects optimal decisions in the supply chain. When the degree of risk aversion increases, the manufacturer is less willing to assume risk related to the quality uncertainty of used products; therefore, the manufacturer will reduce the quantity of recycled used products by reducing their optimal recycling price. In this case, the cost saved in the remanufacturing process and the quantity of remanufactured products will be reduced, and the manufacturer will raise the optimal retail price of remanufactured products to maintain their revenue and to reduce consumers’ demand for remanufactured products. As a result, the revenue accruing to the supply chain will decrease during the process of the remanufacturing and sale of remanufactured products, and the total revenue of the supply chain will decrease with the rise in the degree of the manufacturer’s risk aversion.

5. Numeric Analysis

5.1. Impact of Consumer Preference for Remanufactured Products. To investigate the impact of consumer preference for remanufactured products on the revenue of the supply chain and on pricing decisions, this study assumes that the values of other variables remain constant. Specifically, $B = 250$, $h = 5$, $y = 4$, $c_n = 50$, $v = 40$, $\theta = 0.2$, $\mu = 0.8$, $\sigma = 0.1$, $\lambda = 0.2$, $k = 2$, and $a \in [0, 1]$.

As shown in Figure 2, with the rise in the degree of consumer preference for remanufactured products, the wholesale price of remanufactured products increases under both recycling modes, and the wholesale price of remanufactured products under the $R$ mode is always higher than that under the $3P$ mode.

As shown in Figure 3, the recycling price of used products increases with the rise in the degree of consumer preference for remanufactured products under both recycling modes, and the recycling price of used products under the $R$ mode is always higher than that under the $3P$ mode. However, the recycling price of used products is positive (i.e., the recycler is willing to recycle used products at their own cost) only when the degree of consumer preference for remanufactured products reaches a certain level; otherwise, the recycler’s benefit will be reduced.

As shown in Figure 4, the manufacturer’s expected utility shows a U-shaped variation under both recycling modes with the rise in the degree of consumer preference for remanufactured products. Specifically, a low degree of consumer preference for remanufactured products inhibits the manufacturer’s expected utility, whereas a high degree of consumer preference for remanufactured products increases the manufacturer’s expected utility.

As shown in Figure 5, under the $R$ mode, there is a U-shaped relationship between the retailer’s expected revenue and the degree of consumer preference for remanufactured products. The optimal decisions of the retailer responsible for recycling of used products are primarily affected by the remanufacturer, so the retailer’s profit shows a U-shaped variation. However, under the $3P$ mode, there is an M-shaped relationship between the retailer’s expected revenue and the degree of consumer preference for remanufactured products. In the two recycling models, the retailer is only responsible for the sale of products and does not participate in any reverse logistics process. Furthermore, the wholesale and retail prices of new products are only related to their manufacturing costs.

As shown in Figure 6, there is a U-shaped relationship between the total utility of the supply chain and the degree of consumer preference under two recycling modes for remanufactured products; moreover, the total utility of the supply chain is related to the consumer preference coefficient.

5.2. Impact of the Degree of Manufacturer’s Risk Aversion.
Figure 2: Impact of the coefficient of consumer preference for remanufactured products on the wholesale price of remanufactured products under two recycling modes.

Figure 3: Impact of the coefficient of consumer preference for remanufactured products on the recycling price of used products under two recycling modes.

Figure 4: Impact of the coefficient of consumer preference for remanufactured products on the manufacturer’s utility under two recycling modes.

Figure 5: Impact of the coefficient of consumer preference for remanufactured products on the retailer’s utility under two recycling modes.

Figure 6: Impact of the coefficient of consumer preference for remanufactured products on the total utility of the supply chain under two recycling modes.

Figure 7: Impact of the risk aversion coefficient on the recycling price of used products under two recycling modes.
To investigate the impact of the manufacturer’s risk aversion coefficient on the revenue of the supply chain and on pricing decisions, this study assumes that the values of other variables remain constant. Specifically, $B = 250$, $h = 5$, $y = 4$, $c_n = 50$, $v = 40$, $\theta = 0.2$, $\mu = 0.8$, $\sigma = 0.1$, $a = 0.8$, $k = 2$, and $\lambda \in [0, 1]$.

As shown in Figure 7, the recycling price of used products is inversely proportional to the manufacturer’s risk aversion coefficient. Furthermore, the recycling price of used products under the $3P$ mode is lower than that under the $R$ mode.

As shown in Figure 8, the retail price of used products is directly proportional to the manufacturer’s risk aversion coefficient, and the retail price of used products under the $3P$ mode is higher than that under the $R$ mode.

Figures 9 and 10 show that both the manufacturer’s expected utility and the retailer’s expected revenue are inversely proportional to the manufacturer’s risk aversion coefficient, and the profit earned by the manufacturer is higher than the profit earned by the retailer. In addition, both the manufacturer’s expected utility and the retailer’s expected revenue under the $R$ mode are higher than those under the $3P$ mode.

5.3. Impact of Quality Uncertainty of Used Products. To investigate the impact of the manufacturer’s reward ratio on supply chain revenue and pricing decisions, we assumed that the values of other variables remain constant. Specifically, $B = 250$, $h = 5$, $y = 4$, $c_n = 50$, $v = 40$, $\theta = 0.2$, $\mu = 0.8$, $a = 0.8$, $\lambda = 0.2$, and $k = 2$. Considering that the quality of used products fluctuates around the average quality, the $\sigma$ value can be either positive or negative. We further assumed that $\sigma \in [-0.5, 0.5]$.

As shown in Figure 11, the recycling price of used products is inversely proportional to the quality fluctuation margin of used products ($|\sigma|$); moreover, it is highest in the
As shown in Figure 12, both wholesale and retail prices of remanufactured products are directly proportional to the quality fluctuation margin of used products (|σ|); in the decentralized decision-making model, the wholesale price of remanufactured products is highest under the R mode, whereas the retail price of remanufactured products is highest under the 3P mode.

As shown in Figures 13 and 14, both the manufacturer’s expected utility and the retailer’s expected profit are inversely proportional to the quality fluctuation margin of used products (|σ|).

6. Conclusions

This study investigated risk-averse pricing decisions associated with product recycling in a closed-loop supply chain. Through theoretical analysis combined with numerical analysis, we reach the following conclusions: (1) the degree of consumer preference for remanufactured products is directly proportional to the wholesale and retail prices of remanufactured products, as well as the recycling price of used products; under the same degree of consumer preference for remanufactured products, the optimal wholesale and retail prices of remanufactured products under the R mode are higher than those under the 3P mode. (2) Under these two recycling modes, there is a U-shaped relationship between the revenue of the supply chain and the degree of consumer preference for remanufactured products; when the coefficient of consumer preference for remanufactured products is high, the total revenue of the supply chain under the R mode is higher than that under the 3P mode; thus, R mode should be selected. When the coefficient of consumer preference for remanufactured products is low, 3P mode should be selected. (3) There is a U-shaped relationship between the retailer’s expected revenue and the degree of consumer preference for remanufactured products under the R mode, whereas there is an M-shaped relationship between them under the 3P mode. (4) Under both recycling modes, the degree of the manufacturer’s risk aversion is inversely proportional to the revenue of the supply chain; however, the revenue of the supply chain under the R mode is higher than that under the 3P mode. (5) The higher the uncertainty of used products’ quality, the lower the recycling price of used products, the lower the manufacturer’s enthusiasm for recycling or for used products, and the lower the manufacturer’s and retailer’s revenue. However, the R mode is superior to the 3P mode.

Despite its significant contributions, this study has a few limitations. For example, this study only considers a closed-
loop supply chain comprising a single manufacturer and a single retailer, not a closed-loop supply chain comprising multiple manufacturers and retailers. This also indicates a direction for subsequent studies.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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