Probing modified dispersion relations in vacuum with high-energy $\gamma$-ray sources: review and prospects

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Abstract. A possible violation of Lorentz Invariance (LIV) in the form of Modified Dispersion Relations (MDR) for photons in vacuum appeared in the late 90s as a possible outcome of some models developed with the goal to provide a full theory of Quantum Gravity (QG). Since then, several ways to probe quantum spacetime from high-energy gamma-ray observations of distant sources have been followed and provide stringent limits on the characteristic energy scale of these QG-related phenomena.

In this paper, the use of astrophysical sources for constraining MDR will be described. In particular, strengths and weaknesses of each category of sources as well as temporal effects due to emission mechanisms will be discussed. The latest results obtained from observations of Gamma-Ray Bursts, flaring Active Galactic Nuclei and pulsars will be briefly reviewed. Then, efforts on-going to get more robust constraints on MDR and LIV will be discussed and put in context with the beginning of the Cherenkov Telescope Array operations in the next few years.

1. Introduction

Modified dispersion relations (MDR) for photons in vacuum have appeared in several theoretical approaches designed on the road to a full theory of Quantum Gravity (QG, see e.g. [1,2,3,4,5,6]). In these cases, Lorentz Invariance is violated but other models exist where Lorentz symmetry is deformed rather than violated such as Deformed Special Relativity (e.g. [7,8]).

A natural characteristic energy for these effects would be the Planck scale $E_P \sim 10^{19}$ GeV. Considering the extreme value of $E_P$, and disregarding any particular model, the MDR is generally expressed as a series expansion:

$$E^2 \simeq p^2 c^2 \times \left[1 \pm \sum_{n=1}^{\infty} k_n \left(\frac{E}{E_P}\right)^n\right], \quad (1)$$

where $c$ is the low energy limit of the speed of light, and $k_n$ coefficients to be measured, or constrained. The sign $\pm$ in this equation takes into account the possibility to have subluminal (the velocity decreases with increasing energy) or superluminal effects (the velocity increases with increasing energy).
From equation 1, the delay $\Delta t_n$ between two photons of energies $E_h$ and $E_l$ ($E_h > E_l$) can be expressed, assuming the two photons are emitted at the same time from a source at redshift $z$:

$$\Delta t_n \simeq s_\pm \frac{n + 1}{2} \frac{E_h^n - E_l^n}{E_{QG}^n} \kappa_n(z).$$

(2)

where $s_\pm$ allows subluminal ($s_\pm = +1$) or superluminal ($s_\pm = -1$) propagation and where $E_{QG}$ is the energy scale to be measured or constrained. Present-day experiments are sensitive only to linear ($n = 1$) and with a lower sensitivity to quadratic ($n = 2$) effects. Since the sensitivities to linear and quadratic effects are different, the notation $E_{QG,n}$ will be used in the following to give the results. For a source at redshift $z$,

$$\kappa_n(z) \equiv \int_0^z \frac{(1 + z')^n}{H(z')} dz',$$

(3)

where $H$ is the Hubble parameter takes into account Universe expansion while the photons propagate. This formulation was obtained by Jacob & Piran in [9], implicitly assuming translations are not affected by Planck scale effects. Getting a more general formulation for this parameter is an on-going effort (e.g. [10, 11]). For nearby sources such as galactic pulsars, $\kappa_n(z) \approx z \approx dH_0/c$, where $H_0$ is the Hubble constant and $d$ the euclidian distance to the source.

In this article, the emphasis will be put on one particular consequence of the MDR: energy-dependent time delays. The same kind of dispersion relation can lead to a modification of the kinematics of the $\gamma_{HE} + \gamma_{LE}$ interaction [12]. As a result, TeV photons would be less absorbed by Extragalactic Background Light (EBL): Universe would be more transparent. Several very stringent limits have been obtained following this approach (see e.g. [13, 14, 15]).

In the next section, the different astrophysical sources to be used for MDR searches will be discussed. In particular, their strengths and weaknesses will be given. Then, in section 3 published constraints on $E_{QG}$ will be quickly reviewed. Source intrinsic effects, delays originating from emission mechanisms will be considered in section 4 as well as how they can be dealt with. A possibility to do so is to get a full modeling of emission mechanisms and the delays they can produce. A first attempt in that direction will be described in section 4.2. To conclude, and considering the future of MDR searches, some issues to be solved and some prospects will be discussed.

2. Astrophysical sources for MDR searches

2.1. General characteristics

Obtaining the best constraints on energy scales $E_{QG,n}$ relies on a combination of different characteristics for the sources. The best limits will be obtained with very bright, very distant, very energetic sources showing extreme variability.

It is commonly assumed that the tiny effects due to the quantum nature of spacetime add up during the propagation of photons. This translates in the fact the integral in equation 3 is increasing with increasing redshifts.

The energy content of the data (and the so-called energy “lever-arm” $\Delta E_n \equiv E_h^n - E_l^n$) depends on different factors such as the intrinsic spectrum of the source, the detector performance and observation conditions. $\Delta E_n$ will be maximized for hard spectra, and for detectors with large effective area over a wide energy range. However, absorption by the EBL reduces the flux of TeV photons and limits the energy lever-arm. High variability translates into the presence of fine peaks in the light curve and thus into a better accuracy when measuring the lag. Access to extreme variability is not only related to sources, but also to statistics of detected photons and to signal-to-background ratio. In
particular, since most methods for lag measurement start from binned light curves, high statistics allow to use a fine time binning.

Pulsars (PSR), Gamma-Ray Bursts (GRB) and flaring Active Galactic Nuclei (AGN), observed by satellites and ground-based experiments have all the characteristics mentioned above and have been used extensively for MDR searches.

2.2. GRB, AGN and pulsars: complementarity

Due to their fast, transient and random nature, GRB are mostly detected by satellites because of their large field of view, but only below a few tens of GeV. At these energies, the EBL absorption is negligible and brightest GRB have been observed up to redshifts of $\sim 8$. The detection of GRB190114C ($z = 0.4245$) by MAGIC in January 2019 [39] opened a new era for gamma-ray astronomy and probably also for MDR searches. This GRB, the first one detected by a ground-based experiment, will certainly provide strong constraints on $E_{QG}$.

Contrary to GRB, even during flares, AGN have generally a flux which is too low to ensure high enough statistics when observed with satellites, due to their limited effective area. On the contrary, arrays of Imaging Atmospheric Cherenkov Telescopes (IACT) such as H.E.S.S., MAGIC or VERITAS have a large collection area which make them very efficient to detect AGN flares. But, due to their limited field-of-view (a few degrees), they rely on alerts issued by satellites to catch random and transient events.

Since almost all of the observed pulsars are within our own galaxy, and so at small distances, they are disadvantaged for MDR searches as compared to GRB and flaring AGN. However, their emission is permanent and their short periods translate into an extreme variability of the order of a few milliseconds. This in turn allows a very precise lag measurement, which make them very useful to constrain $E_{QG,n}$. In addition, they are observed on a wide energy range with both satellites and Cherenkov telescopes.

GRB, AGN and pulsar complementarity can be summarized by the following points:

- GRB and AGN flares happen randomly while PSR emit gamma-rays all the time;
- GRB and AGN are located at cosmological distances while PSR are galactic objects;
- GRB and PSR are detected with high statistics by satellites in the MeV-GeV range while flaring AGN are detected with high statistics by IACT in the GeV-TeV range.

3. Overview of the most important results

Table 1 (next page) summarizes most of the results obtained in the last 20 years. See also figure [1].

The limits listed in table [1] are given for four distinct categories: “Individual GRB”, “Several GRB”, “Individual flaring AGN” and “Individual PSR”. The adjective “individual” is used when only one source was analyzed to derive the limit. On the contrary, for the category called “Several GRB”, different sources at various distances are used to perform a global fit to set limits on $E_{QG,n}$, taking into account a dependance of the time-lag with redshift. These proto-population studies result in limits which are less constraining. In addition, the limits have been derived considering the intrinsic effects have the same magnitude for all sources in the studied sample.

The table also gives the different methods which were used to measure the lags. Each method has its advantages and drawbacks. As an example, Cross Correlation Function (CCF, MCCF) and PairView are very simple to implement, while Wavelet Transform requires multiple steps and complex algorithms. The maximum-likelihood technique [29], widely used in the most recent papers, relies on a parameterization of a binned light curve at low energies which in turn relies on some assumptions for the peak shapes, e.g. the fact that the shape do not change much in different energy bands.
Table 1. A selection of limits (95% CL) obtained with various instruments and methods, for linear \( E_{QG,1} \) and quadratic \( E_{QG,2} \) terms of the MDR, for a subluminal effect.

| Source(s) | Experiment | Method | Results | Reference | Note |
|-----------|------------|--------|---------|-----------|------|
| GRB 021206 | RHESSI     | Fit + mean arrival time in a spike associating a 13 GeV photon with the trigger time | \( E_{QG,1} > 1.8 \times 10^{17} \) GeV \( E_{QG,2} > 0.8 \times 10^{10} \) GeV | 10       |      |
| GRB 080916C | Fermi GBM + LAT | associating a 31 GeV photon with the start of any observed emission, DisCan | \( E_{QG,1} > 1.5 \times 10^{20} \) GeV \( E_{QG,2} > 3.0 \times 10^{10} \) GeV | 18       | c    |
| GRB 090510 | Fermi LAT   | PairView, SMM, likelihood | \( E_{QG,1} > 9.3 \times 10^{19} \) GeV \( E_{QG,2} > 1.3 \times 10^{11} \) GeV | 19       | i    |
| 9 GRBs    | BATSE + OSSE | Fit | \( E_{QG,1} > 10^{15} \) GeV | 20       | a    |
| 15 GRBs   | HETE-2      | wavelets | \( E_{QG,1} > 0.7 \times 10^{16} \) GeV \( E_{QG,2} > 2.9 \times 10^{6} \) GeV | 21       | a    |
| 17 GRBs   | INTEGRAL    | wavelets | \( E_{QG,1} > 0.4 \times 10^{16} \) GeV \( E_{QG,2} > 3.2 \times 10^{11} \) GeV | 22       | a    |
| 35 GRBs   | BATSE + HETE-2 + Swift | wavelets | \( E_{QG,1} > 1.4 \times 10^{16} \) GeV \( E_{QG,2} > 2.3 \times 10^{11} \) GeV | 23, 25   | f, g |
| 15 GRBs   | SWIFT       | CCF   | \( E_{QG,1} > 1.4 \times 10^{16} \) GeV | 26       |      |
| Mrk 421   | Whipple     | average time of the main pulse in different energy bands | \( E_{QG,1} > 0.4 \times 10^{17} \) GeV | 22       | a, h |
| Mrk 501   | MAGIC       | ECP, likelihood | \( E_{QG,1} > 0.2 \times 10^{18} \) GeV \( E_{QG,2} > 2.6 \times 10^{10} \) GeV | 28       |      |
| PKS 2155-304 | H.E.S.S.  | likelihood | \( E_{QG,1} > 0.3 \times 10^{18} \) GeV \( E_{QG,2} > 5.7 \times 10^{10} \) GeV | 29       |      |
| PG 1553+113 | H.E.S.S.  | likelihood | \( E_{QG,1} > 3.6 \times 10^{17} \) GeV \( E_{QG,2} > 8.5 \times 10^{10} \) GeV | 13       |      |
| 3C 279    | H.E.S.S.    | likelihood | \( E_{QG,1} > 7.2 \times 10^{17} \) GeV \( E_{QG,2} > 0.1 \times 10^{10} \) GeV | 23       |      |
| Crab pulsar | EGRET      | average time of the main pulse in different energy bands, fit of main pulse | \( E_{QG,1} > 0.2 \times 10^{16} \) GeV | 34       |      |
| Vela pulsar | H.E.S.S.   | likelihood | \( E_{QG,1} > 1.9 \times 10^{17} \) GeV \( E_{QG,2} > 5.9 \times 10^{10} \) GeV | 34       |      |
| Vela pulsar | MAGIC      | DisCan | \( E_{QG,1} > 5.5 \times 10^{17} \) GeV \( E_{QG,2} > 6.4 \times 10^{9} \) GeV | 34       |      |

a Limit obtained not taking into account the factor \( (1 + z) \) in the integral of Eq. 3.
b The pseudo-redshift estimator \([38]\) was used. This estimator can be wrong by a factor of 2.
c Only the most conservative limit is given here.
d Photon tagged data was used.
e The pseudo-redshift estimator \([38]\) was used for 6 GRB out of 11.
f The limits of \([24]\) were corrected in \([25]\) taking into account the factor \( (1 + z) \) in the integral of Eq. 3. Only the limit obtained for a linear correction is given.
g \( E \) A likelihood procedure was used, but not on an event-by-event basis.
h In this study, four bursts were analyzed. Only the best limits are given here, obtained for GRB 090510 with the PairView \((n = 1)\) and SMM \((n = 2)\) methods.
i Preliminary.
Figure 1. A selection of limits obtained with the three categories of sources: PSR (green ■), AGN (black ●) and GRB (red ×). The blue line corresponds to the best limit obtained analyzing several GRB [26]. Its width gives the redshift range of the 15 GRB considered in the study. Other sources and references: 1. Vela pulsar [37]; 2. Crab pulsar [36]; 3. Mrk 421 [27]; 4. Mrk 501 [29]; 5. PKS 2155-304 [31]; 6. PG 1553+113 [32]; 7. 3C 279 [33]; 8. GRB 090510 [19]; 9. GRB 090902B [19]; 10. GRB 090926A [19]; 11. GRB 080916C [19].

The most stringent limits available so far were obtained from observations of the very short and very energetic burst GRB 090510 by the Fermi satellite [18, 19]. If the Planck scale is really the characteristic energy for QG effects, limits above \( E_p \) tend to disfavor classes of models where only linear dispersion is obtained. Though, they still need to be confirmed with population studies. Despite their proximity, which penalizes them for constraints on the linear effect, pulsars allow to obtain limits on \( E_{QG,2} \) which are as constraining as some limits derived with GRB. As already discussed above (section 2.2), this is due to the extreme variability of PSR.

4. Source intrinsic effects and how to deal with them

4.1. Source intrinsic effects

To obtain equation 2, the hypothesis that low and high-energy photons are emitted at the same time is made. This results in neglecting any time delay which can originate from emission mechanisms, also called “source intrinsic effects”. In general, the measured delay is rather the sum of source intrinsic effects (hereafter \( \Delta t_{\text{source}} \)) and propagation effects such as the one due to LIV:

\[
\Delta t_{n,\text{total}} = \Delta t_{n,\text{LIV}} + (1 + z) \Delta t_{\text{source}},
\]

where \( n \) is the order of the correction in the MDR and \( z \) the redshift.

Time delays between low and high energy photons have been observed in all three kinds of sources in use for MDR searches (see e.g. [40, 41, 42]), but each time considering a wide energy range so that several spectral components and emission mechanisms are at play, e.g. between radio and GeV domains for PSR.

In most MDR searches performed so far, energies are usually limited to a range small enough so that, in principle, only one emission process is responsible for the emission. In that case, it is safe to neglect intrinsic effects. Yet, a significant lag was measured at one occasion in the TeV
range for a flare of blazar Mkn 501 observed by MAGIC [43, 29]. No delay was measured for other AGN or even for another flare of the same source [14]. This suggest that intrinsic effects could be different for each flare of the same AGN. A fortiori, intrinsic effects are most probably different for each type of source.

A better understanding of intrinsic effects is essential and requires a detailed modeling of emission mechanisms.

4.2. Modeling source intrinsic effects in flaring AGN
Extending the energy range is a way to improve sensitivity to MDR effects, both through higher statistics, and a larger $\Delta E_n$. However, a full interpretation of delays requires a good understanding of intrinsic effects. Even though progress is made on a regular basis, a detailed understanding of emission mechanisms for GRB, flaring AGN and PSR is still far from being complete.

In particular for AGN, no extensive model exists yet allowing to fully reproduce both the temporal and spectral evolution of observed flares. However, an effort is on-going with the goal to characterize intrinsic effects [44, 45, 46]. This attempt is based on the use of a leptonic model where temporal evolution is due to (i) electron acceleration for the flux increase phase, and (ii) electron energy losses and decrease of magnetic field for the flux decrease phase. The spectral energy distribution is obtained from a Synchrotron Self-Compton model [47].

This model produces time delays between flux maxima in different energy bands, with the possibility to test different initial values for the acceleration of the electrons, for the magnetic
Time delays are found to be mainly driven by acceleration or by radiative cooling. In the first case, electrons are still accelerating when light curves start to decay. Thus, they need more time to be accelerated enough so they emit the highest energy photons. As a consequence, low energy light curves reach their maximum first. In the second case, electrons have started to cool down when light curves start to decay. Electrons emitting the highest energy photons lose their energy faster than electrons emitting low energy photons. As a result, high energy light curves reach their maximum first.

Focusing on high energies, delays are found to vary with energy according to a power law

\[ \Delta t = \xi \times (E^\alpha - E_0^\alpha) \].

In all cases studied so far, \( \alpha \) is found to be in the range 0.4 – 0.9, while \( \xi \) can be positive (mimicking a subluminal LIV) or negative (mimicking a superluminal LIV).

Since no lag have been measured at GeV–TeV energies except for Mrk 501 so far, it is foreseen that it will be possible to put severe constraints on emission mechanisms in a near future.

5. Prospects

Three closely connected fronts related to modified dispersion relations searches with astrophysical sources are opened.

One of them is on the theoretical and phenomenology side. In particular, and from the experimentalist’s point of view, a number of questions directly connected with experimental searches remain open, such as: what is the true dependence of the lag with the redshift and with the energy? What is the real significance of the Planck scale, or what does it really mean when limits on \( E_{QG} \) reach the Planck scale?

The MDR of equation 1 is only a series expansion which emphasizes, by construction, integer powers of the energy. Integer powers of the energy were indeed obtained in some models, but maybe because of some simplifying assumptions. A full theory of Quantum Gravity may lead to more complex expressions, e.g., to non-integer powers of the energy. As already mentioned, the variation of the lag with respect to the redshift may be different than the one used in all MDR searches. Changing the expression of \( \kappa_n(z) \) would automatically lead to a modification of all the limits published so far. For this reason, and also because the Planck scale should probably be considered only as a rough order of magnitude for QG effects, limits that have reached \( E_P \) may not really exclude any model.

The two other fronts lie on the experimental side. The first one aims at providing more and more constraints on MDR with as much sources as possible. The second one deals with the modeling of emission mechanisms and source intrinsic effects. These two topics are of course closely connected and population studies will help for both. New instruments such as the Cherenkov Telescope Array (CTA) \[48]\, with increased sensitivity and energy coverage, and dedicated strategies to observe more and more transient events, will hopefully allow breakthroughs concerning both propagation effects searches and intrinsic effects understanding.

A European COoperation in Science and Technology (COST) action\[1\] has started very recently on the topic of “Quantum gravity phenomenology in the multi-messenger approach”. It gathers theorists, phenomenologists and observers who will work for the first time in a close collaboration to address the problems ahead on the road to Quantum Gravity.

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\[1\] https://www.cost.eu/actions/CA18108
