Analysis of a Turbulent Boundary Layer Subjected to a Strong Adverse Pressure Gradient

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Abstract. A strongly decelerated turbulent boundary layer is investigated by direct numerical simulation. Transition to turbulence is triggered by a trip wire which is modelled using the immersed boundary method. The Reynolds number close to the exit of the numerical domain is $Re_\theta = 2175$ and the shape-factor is $H = 2.5$. The analysis focuses on the latter portion of the flow with large velocity defect, at higher Reynolds numbers and further from the transition region. Mean velocity profiles do not reveal a logarithmic law. Departure from the law of the wall occurs throughout the inner region. The production and Reynolds stress peaks move to roughly the middle of the boundary layer. The profiles of the $uv$ correlation factor reveal that $u$ and $v$ become less correlated throughout the boundary layer as the mean velocity defect increases, especially near the wall. The structure parameter is low in the present flow, similar to equilibrium APG flows and mixing layers, and decreases as the mean velocity defect increases. The statistics of the upper half of the boundary layer resemble those of a mixing layer. Furthermore, various two-dimensional two-point correlation maps are obtained. The $C_{vv}$ and $C_{ww}$ correlations obtained far from the transition region at $Re_\theta = 2175$ and at $y/\delta = 0.4$ coincide with results obtained for a ZPG boundary layer, implying that the structure of the $v, w$ fluctuations is the same as in ZPG. However, $C_{uu}$ indicates that the structure of the $u$ fluctuation in this APG boundary layer is almost twice as short as the ZPG one. The APG structures are also less correlated with the flow at the wall. The near-wall structures are different from ZPG flow ones in that streaks are much shorter or absent.

1. Introduction

Wall bounded turbulent flows are important to understand in order to be able to reduce the energy losses associated with these turbulent flows, which have a wide range of practical applications. An important sub-group of wall-bounded turbulent flows are adverse-pressure-gradient (APG) boundary layers. These flows are found for instance around surfaces with curvature, as encountered in many aerodynamic applications such as airplane wings, cars and rotors in turbines. Although a significant amount of research has been devoted to understanding channel flows, pipe flows and zero-pressure-gradient (ZPG) boundary layers, the same cannot be said for APG boundary layer flows. A brief overview will be presented of the most significant results which are consequential to the study reported in this article.
The velocity components of the boundary layer flow will be denoted by \( u, v, w \), and the respective spatial coordinates by \( x, y, z \) for the streamwise, wall-normal and spanwise directions, respectively. The pressure is denoted by \( p \). Capital letters refer to mean quantities, and primed ones to root-mean-squared fluctuation intensities. The superscript, ‘\( + \)’, denotes viscous-friction scaled quantities, normalized with the \( x \)-dependent friction velocity \( u_\tau \) and the kinematic viscosity \( \nu \). The 99\% boundary-layer thickness is \( \delta \).

Although studied less, there is still a wealth of experimental and numerical data on first- and second-order statistics available for a range of APG boundary layer flows, for example [1–7]. Among other things, these studies clearly demonstrate that the important problem in APG boundary layer flow research is the lack of a canonical set-up and the appropriate design for the corresponding experiments. The lack of such a canonical set-up is to a large extent related to the lack of understanding of which parameters are paramount for the development of the APG boundary layer. This might also explain the reason why APG boundary layer flows have been studied less than other turbulent wall-bounded flows.

One of the more significant results from these previous studies is the observation that, under certain conditions, the near-wall \( u' \)-peak is absent and \( u' \) reaches a maximum in the middle of the boundary layer. Although the absence of the near-wall \( u' \)-peak seems related to a high or prolonged APG [8], one of the outstanding questions that needs to be addressed is: what exactly causes the disappearance of the near-wall \( u' \)-peak? Furthermore, it is unknown how these APG boundary layer flows relate to other types of canonical flows, like the ZPG boundary layer flow and free shear flows.

It has long been recognized that turbulent flows contain coherent 3-D structures [9]. Due to the deterministic temporal and spatial behaviour of these structures in a chaotic background environment it has also long been recognized that these structures offer the best opportunity to control the flow. Recently statistical information on these structures in a 3-D framework has been studied [10–12] for the outer part of turbulent channel flow and for APG boundary layer flows.

The purpose of this article is to review results and present new results on strong APG boundary layers that relate the statistical results of these flows with those from other canonical flows, like free shear flows and ZPG boundary layer flows, and to study the velocity correlations and coherent structures for the APG boundary layer flow simulated especially for this study.

The article begins with a discussion of the numerical methodology employed to undertake the DNS of the APG boundary layer flow. This is followed by a discussion of the results, beginning with a general overview of the flow field, followed by a more in-depth presentation of the first- and second-order statistics, as well as a comparison with other results. Then the structure of the boundary layer is examined, by considering the various two-point correlations and \( uv \)-structures. The article concludes with a summary of the main results.

2. Numerical methodology

The boundary layer subjected to a strong APG is simulated in a parallelepiped domain over a smooth no-slip wall, with spanwise periodicity and streamwise non-periodic inflow and outflow. The numerical scheme used for the simulation is similar to the one described in Simens et al [13]. The Navier-Stokes equations are integrated using a fractional step method on a staggered grid, with third-order Runge-Kutta time integration, fourth-order compact spatial discretisation for the convective and viscous terms, and second-order discretisation for the pressure in the directions perpendicular to the span, which is spectral. The simulation parameters are summarized in table 1.

The desired constant streamwise velocity gradient is controlled by imposing a constant uniform suction at the top simulation boundary. The streamwise and spanwise velocities at the top boundary satisfy free-slip conditions. The laminar Hiemenz profile is prescribed at the
Table 1. Parameters of the turbulent boundary layer simulations. $L_x$, $L_y$ and $L_z$ are the box dimensions along the three axes. $N_x$, $N_y$, and $N_z$ are the corresponding grid sizes, and the $\Delta$ values are the resolutions in wall units. The momentum thickness, $\theta_0$, is measured at the inflow. The Kolmogorov length $\eta$ is computed from the local energy dissipation. The coarsest resolution along $x$ and $z$ in terms of $\eta$ is found near the wall at reattachment, where $\Delta x/\eta \approx 10$.

| Re$\theta$ | $(L_x, L_y, L_z)/\theta_0$ | $\Delta x^+, \Delta y_{min}^+, \Delta z^+$ | $N_x, N_y, N_z$ | $\beta$ |
|------------|-----------------------------|---------------------------------|----------------|------|
| 150-2200   | $2380 \times 450 \times 1100$ | $2.2 \times 0.2 \times 2.0$ | $1537 \times 201 \times 768$ | 5-100 |

inflow, and the velocities at the outflow are estimated by a convective boundary condition, with small corrections to enforce global mass conservation [13]. The turbulent transition is triggered by a disturbance strip with a height of $1.16\theta_0$ located close to the inflow, and modelled using the immersed boundary method [14–18]. The implementation of the immersed boundary method is discussed in detail in Simens and Gungor [19], including a full discussion of the effect of roughness on the development of APG boundary layers.

3. Results and discussion
3.1. Flow description
The flow is initially laminar, separates, transitions within the separation bubble, reattaches and develops into an attached turbulent APG boundary layer, the latter being the flow zone of interest for this study. The presence of a small transitional separation bubble near the inflow helps achieve the fully turbulent regime within a shorter distance than what would be obtained with attached transition through Tollmien-Schlichting waves. The disturbance strip located upstream of the bubble does not cause the flow to transition, but is responsible for generating perturbations that hasten the reattachment of the separation bubble [19]. The flow after the trip element is still laminar, and perturbations due to the trip element hardly grow until the bubble starts to form. The turbulent kinetic energy distribution in the lower half of the domain is shown in figure 1. The time-averaged separated region and the boundary layer growth are also indicated in the same figure with thick solid lines. The flow fluctuations originate at the inflection point of the shear layer formed by the separation bubble. The perturbations generated by the disturbance strip grow in the separated shear layer, and trigger the inviscid Kelvin–Helmholtz instability that is responsible for the breakdown to turbulence. Kelvin–Helmholtz instability is characterized by the formation of large, two-dimensional vortices in the separated shear layer. The large amplitude fluctuations seen in figure 1 are due to the regular shedding of these vortical structures. The growth of these fluctuations in the initial part of the separated region is slow, whereas they grow much faster close to reattachment. This sudden growth of the fluctuations triggers a slowdown of the bubble growth due to the turbulent energy diffusion, and reattachment occurs. The reattachment of the flow is accompanied by an overall maximum in the turbulent intensity. This intensity decreases slowly downstream, while the peak location is shifted outward from the reattachment point to the end of the solution domain.

The effect of the separated region can be seen in figure 2, which displays the two-point cross-correlation coefficient

$$C_{vu}(x, y; x', y') = \frac{\langle u(x, y)u(x', y') \rangle}{\left(\langle u(x, y)^2 \rangle\langle u(x', y')^2 \rangle\right)^{1/2}},$$

where the averaging $\langle \rangle$ is taken over time and spanwise direction. The correlation is centred at the reattachment point of the separated region in the streamwise direction at $x/\theta_0 = 680$ and at $y/\delta = 0.6$, where $\delta$ is the local boundary layer thickness. The influence of the separated
Figure 1. Turbulent kinetic energy normalized with the inflow freestream velocity in the lower half of the domain. The axes are not to scale. The solid white line is the zero contour of the streamwise velocity $U$, and marks locations of the trip element ($80 \leq x/\theta_0 \leq 130$) and the separation bubble. The solid magenta line shows the boundary layer thickness. The vertical black lines are the selected two stations where ($Re_\theta, H$) = (1755, 2.0) and (2175, 2.5), respectively.

Figure 2. Two-point spatial cross-correlation coefficient of wall-normal and streamwise velocity fluctuations, $C_{vu}(x, y; x', y')$ in $xy$ plane. The correlation is centred at the reattachment location of the time-averaged separation bubble and at $y/\delta = 0.6$. The solid magenta line is the location of the boundary layer thickness. The dashed vertical lines mark the separation and reattachment locations of the averaged separated region. The dotted-dashed blue vertical lines are the selected two stations where ($Re_\theta, H$) = (1755, 2.0) and (2175, 2.5), respectively.

region survives up to $x/\theta_0 \approx 1800$, and only becomes small beyond that limit. The attached turbulent boundary layer is therefore probably still recovering from the effects of the shear layer instability all the way up to the end of the computational domain. The present study focuses on the region beyond $x/\theta_0 = 1800$, corresponding to $Re_\theta = 1700 – 2200$, where these effects should be relatively small.

In the case of boundary layers with an external flow with streamline curvature, like the present flow, the definition of the external velocity $U_e$ is not straightforward. In order to be as consistent as possible with boundary layer theory, $U_e$ is chosen as the maximum streamwise component of velocity in the wall-normal direction. Figure 3 shows the streamwise evolution of $U_e$ and of the pressure gradient parameter $\beta_{ZS} = -\frac{\Delta U_{zs}}{U_e}\frac{dU_e}{dx}$ in the region downstream of the separation bubble, where $U_{zs} = U_e\delta^*/\delta$ is the Zagarola-Smits velocity. Rotta–Clauser’s pressure gradient parameter $\beta = -\frac{\Delta u_z}{u_z}\frac{dU_e}{dx}$ is also shown in figure 3 for reference, where
Figure 3. Streamwise evolution of external velocity (——) and pressure gradient parameters downstream of the separation bubble. $\beta_{ZS}$, $\beta$, · · ·.

Figure 4. Streamwise evolution of the shape factor.

$\Delta = \delta^* U_e / u_\tau$ is the Rotta–Clauser length scale. $\beta$ is the traditional pressure gradient parameter that assumes the outer region velocity scale to be $u_\tau$. Because of flow curvature introduced by the outlet and top boundary conditions, $U_e$ does not decrease linearly, but, interestingly, the flow conditions lead to a pressure gradient parameter $\beta_{ZS}$ that increases in an almost linear fashion. A gradient of $\beta_{ZS}$ implies that the boundary layer is not in dynamic equilibrium [7]. Moreover, if a positive gradient of $\beta_{ZS}$ is maintained, the streamwise mean momentum defect of the boundary layer increases. These statements are reflected in the increase of the shape factor $H$ shown in figure 4 for $Re_\theta > 1000$. The increase of $H$ with respect to $Re_\theta$ is rapid, indicating the strong non-equilibrium nature of the flow.

3.2. Comparison with other flows

In this section, comparisons with other flows of interest are made at the same time as the velocity statistics of the present flow are discussed. The data for the present flow is taken at two streamwise positions near the end of the domain, where the mean velocity defect is large, $H = 2$ and 2.5 respectively, and the influence of the shear-layer instability of the upstream bubble is expected to be small. The other turbulent boundary layers considered are: the ZPG TBL of Simens et al. [13] at a comparable Reynolds number ($Re_\theta = 1975$); the equilibrium APG TBL of Skare and Krogstad [2] (SK), which has a relatively large velocity defect ($H \approx 2$) and is at high Reynolds numbers ($Re_\theta = 39000–51000$), and the non-equilibrium APG TBL of Maciel et al. [6] (MRL) with increasing velocity defect that includes data at detachment ($H = 1.72–3.85$, $Re_\theta = 3350–12691$). Finally, the single-stream mixing layer of Wygnanski and Fiedler [20] (WF) is also considered because the mean velocity profile of the outer region of a large defect boundary layer resembles that of the high-speed side of a mixing layer.

Outer (or high-speed region) scales common to both types of flows have to be defined in order to be able to compare the mixing layer with boundary layers. For simplicity, we choose scales that are similar to those frequently used for free shear layers. For the boundary layers, the outer length scale remains the boundary layer thickness $\delta$ as defined previously, that is the wall-normal distance from the wall to the point where $U = 0.99U_e$. For the single-stream mixing layer, since the data is not accurate in the low-speed region, the outer length scale is defined
Figure 5. Mean velocity profiles of the present flow for: Blue line, $H = 2$; red line, $H = 2.5$; black line, ZPG TBL; green □, equilibrium APG TBL of SK; red □, profile at detachment of the non-equilibrium APG TBL of MRL; blue ■ mixing layer of WF.

Figure 6. Velocity defect profiles normalized with $U_o$. Lines and symbols as in figure 5.

only with the high-speed part of the flow:

$$\delta = 2(y_{0.99} - y_{0.5}), \quad \text{(only for the mixing layer)}$$

where $y_{0.5}$ and $y_{0.99}$ correspond to the positions where $U = 0.5U_e$ and $0.99U_e$, respectively. The outer velocity scale, $U_o$, is used for both TBLs and mixing layer and is defined as twice the velocity defect at the middle of the shear layer:

$$U_o = 2[U_e - \langle y = 0.5\delta \rangle].$$

It is (only approximately) equivalent to $U_{zs}$, in the sense that both velocity scales are proportional to the mean velocity deficit of the outer flow.

Selected mean velocity profiles of these various flows are shown in figure 5. The profile shown for the APG TBL of MRL is the one at detachment ($C_f = 0$). Excluding the wall region, the velocity profiles of the large-defect boundary layers clearly resemble the single-stream mixing layer one, with large velocity gradients and the presence of an inflection point. Two of these velocity profiles are at the same shape factor $H = 2$, namely one for the present flow and the equilibrium one of SK, but the shapes of these profiles are very different. Such a shape difference is probably more related to the different streamwise evolution of the flows than to the dissimilarity in Reynolds number. The corresponding velocity defect profiles normalized with $U_o$ are presented in figure 6. With the exception of the ZPG TBL, the velocity defect profiles of all other flows are similar on the high-speed side. It can be seen from figure 6 that the choice of $U_o$ and $\delta$ as common outer scales is adequate to compare velocity statistics between these flows.

Figure 7 shows the mean velocity profiles normalized with the viscous-friction scales ($u_\tau$ and $\nu/u_\tau$) of the present flow and of the ZPG TBL of Simens et al. at a comparable Reynolds number. Even if at a low Reynolds number, the ZPG TBL profile follows fairly closely the law of the wall throughout the inner region. In contrast, the present non-equilibrium TBL deviates from it and the departure increases as the velocity defect increases. It cannot be excluded that a departure from the law of the wall is due or partly due to the presence of the separation bubble upstream, even if the profiles considered are relatively far from it. But because this departure
exists even very close to the wall, where the impact of the bubble shear layer instability should be small, it might rather be linked to the non-equilibrium state of the boundary layer, while it could also be amplified by the fact that the Reynolds number is low. Such a departure has been observed in many other APG TBLs with strong non-equilibrium conditions [3, 5, 7, 21–26]. Moreover, it is expected near detachment for all TBLs at finite Reynolds number, since in these conditions $u_\tau$ and $\nu/u_\tau$ no longer scale the mean velocity profile near the wall. In contrast, it is not clear if the law of the wall could still hold for equilibrium APG TBLs with large velocity defects. For instance, the equilibrium APG TBL of SK follows the logarithmic law, at least approximately, even if it has a relatively large velocity defect. It is therefore important to stress that it is not so much the strength of the pressure gradient that matters here, but the fact that the pressure gradient parameter is changing. In other words, the fact that the flow is not in dynamic equilibrium.

Selected profiles of the streamwise Reynolds normal stress of the various flows are shown in figure 8. In order to be able to compare the mixing layer and the boundary layers, $\langle u^2 \rangle$ is normalized with the common outer scale $U_o$ and the plot is linear-log in order to clearly reveal the near-wall behaviour. Note that in the case of the TBLs the trends shown in this figure remain the same when $\langle u^2 \rangle$ is normalized with $U_{zs}$. The five profiles of MRL are included in this figure in order to show the behaviour observed in a non-equilibrium boundary layer with

**Figure 7.** Mean velocity profiles normalized with viscous-friction scales. Near-wall region is shown in the inset. Lines as in figure 5.

**Figure 8.** Streamwise Reynolds normal stress normalized with $U_o$. Lines and symbols as in figure 5.

**Figure 9.** Reynolds shear stress normalized with $U_o$. Lines and symbols as in figure 5.
increasing velocity defect up to detachment. The trend observed by MRL is one of a gradual streamwise decrease of all Reynolds stresses throughout the boundary layer, but especially near the wall. The two most downstream profiles are actually superposed, indicating a TBL that reaches a state of near equilibrium before detachment (the last profile is at detachment). The near-wall peak of $\langle u^2 \rangle$, which is a common feature of canonical wall flows, is not present in the case of all the TBLs subjected to a strong APG, with the possible exception of the first profile of MRL, which is at the beginning of the APG region of the flow ($H = 1.72$). The maximum of $\langle u^2 \rangle$ is in the middle of the boundary layer for all these flows. Another particularity of these strong APG TBLs is that $\langle u^2 \rangle/U_0^2$ is lower than in the ZPG case for $y < 0.5\delta$. In the case of the present flow, the stronger levels of fluctuation in the outer region could be due to the presence of the shear layer instability upstream. This seems to be corroborated by the fairly strong decrease of $\langle u^2 \rangle/U_0^2$ from the first profile to the second one. Interestingly, the profile of $\langle u^2 \rangle/U_0^2$ of the mixing layer is similar to the detachment profile of the MRL flow. In the following, it will be seen that other turbulence statistics are similar between these two flows.

All the observations that have been made above for $\langle u^2 \rangle/U_0^2$ can be repeated for the Reynolds shear stress normalized by $U_o$, as shown in figure 9. However, the similarity between the mixing layer and the TBL detachment profile is even better, in particular closer to the wall. Another difference is that, in the case of the large-velocity defect boundary layers, the dominance of $-\langle uv \rangle$ in the outer region in comparison to the wall region is even more pronounced than for $\langle u^2 \rangle$. Since $U_o$ is proportional to the mean strain rates present in the outer region, figures 8 and 9 indicate that the strong APG flows are globally less efficient in extracting turbulent energy from the mean flow than the ZPG one.

Figure 10 presents selected terms of the turbulent kinetic energy budgets normalized with $U_o$ and $\delta$ for four flows: the present flow at $H = 2.5$, the ZPG TBL of Simens et al., the equilibrium APG TBL of SK and the mixing layer of WF. It is important to note that these budgets are very different from those of canonical wall flows. In the latter, the maximum of the various terms occur in the near-wall region and with values an order of magnitude higher than in the outer region. These facts can be partly appreciated with the ZPG TBL profiles of production and dissipation shown in the figure. In large-velocity-defect TBLs, the maximum of production and dissipation is in the middle of the boundary layer. But this does not necessarily mean that the levels of production and dissipation in the outer region of large-velocity-defect TBLs are higher than those of ZPG TBLs. Indeed, figure 10 shows that these levels are comparable in the upper half of all boundary layers, with the exception of the experimentally deduced dissipation of the equilibrium APG TBL of SK. Note also that a small peak of production is present near
the wall in the case of the equilibrium APG TBL. Such a peak also exists further upstream in the present flow, but it gradually decreases and disappears as the outer peak develops.

The budget terms of the mixing layer are seen to be qualitatively similar to those of the large-velocity-defect TBLs. The levels are however higher and the maxima are closer to the edge of the shear layer. Another noticeable difference is the higher level (in relative terms) of turbulent transport of turbulent kinetic energy from the middle of the mixing layer to both edges.

![Figure 11.](image)

**Figure 11.** $uv$-correlation factor. Lines and symbols as in figure 5.

![Figure 12.](image)

**Figure 12.** Structure parameter. Lines and symbols as in figure 5.

Figure 11 shows the $uv$ correlation factor for the various flows. The level of correlation between $u$ and $v$ is higher in the ZPG TBL than in all other flows. In the case of the APG TBLs, the correlation of $u$ and $v$ is seen to deteriorate as the mean velocity defect increases, especially near the wall. The correlation profiles of the mixing layer and of the APG TBL of MRL at detachment are strikingly similar. They suggest that turbulence significantly loses its coherence in the low-speed side of these flows.

The structure parameter $-\langle uv \rangle / 2k$, where $k$ is the turbulent kinetic energy, is shown in figure 12. With the exception of the detachment profile of MRL in the outer region, all other flows have a lower structure parameter than the ZPG TBL. The structure parameter is particularly low in the present flow. Unlike the Reynolds stresses and the $uv$ correlation factor, the structure parameter of the mixing layer is closer to that of the equilibrium APG TBL of SK than to that of the detachment profile of MRL, except near the wall.

3.3. Flow structure

Figure 3.3 shows the fractional contributions from the four quadrants to the Reynolds shear stress for both ZPG and APG flows: $(-\langle uv \rangle_{Qi})/(-\langle uv \rangle)$ where $Qi$ with $i = 1, 2, 3, 4$ denotes a quadrant in the parameter space of $u$ and $v$. The pressure gradient is seen to increase the fractional contribution of all quadrants. In the ZPG case, the sweeps ($Q4$ events) contribute the most to the Reynolds shear stress very near the wall while ejections ($Q2$ events) dominate slightly elsewhere. Whereas, for the APG case sweeps contribute more than ejections in the near-wall region and in the lower part of the outer region as well. Correspondingly, the negative contribution of $Q1$ and $Q3$ motions increase near the wall due to the increased energy transfer through the turbulent transport toward the wall [23]. Also as suggested in Krogstad and Skare [27], the strength of $Q1$ motions might be due to reflections of $Q4$ motions at the wall.
The crossing between the sweep-dominated region and the ejection-dominated region takes place much farther from the wall in the APG case. This point is very close to the maximum of the Reynolds stress.

Figure 14 presents the joint p.d.fs of the logarithm of the size in $xy$ and $zy$ of the circumscribing boxes for $Q2$ and $Q4$ three-dimensional structures. The clusters are obtained following the method described in [11]. In particular we use the same threshold $|u(x, y)v(x, y)| > Hu'(x, y)v'(x, y) + 10^{-6}$, with $H = 1.75$ and adding $10^{-6}$ to prevent anomalies in the free-stream part of the flow. The value for $H$ was chosen in order to directly compare with [11] and was therefore not based on an analysis of the percolation threshold. The clusters are obtained in the region of the flow where $Re_\theta = 1700 - 2200$, which corresponds to the region that is decorrelated from the separation bubble. The size of the boxes was scaled with a $\delta$ averaged over this region.

Both the $Q2$ and the $Q4$ structures have very similar sizes, which is in agreement with what was obtained in [11] for turbulent channel flows. However, contrary to channel flow the structures are as long as they are tall and wide, $\Delta_x \approx \Delta_y \approx \Delta_z$. In general the $Q2$ and $Q4$ structures are less elongated than in the channel flow. On the other hand the behaviour for the largest structures is similar to what is found in channel flow, in the sense that the $Q2$ structures are larger than the $Q4$ structures, especially in height. The largest structures are essentially twice as large as the boundary layer thickness.

Figure 14. Joint p.d.f. of the logarithms of the sizes of the boxes circumscribing both detached and attached $Q2$ and $Q4$ structures, $\cdot\cdot\cdot : Q2$, $\cdot\cdot\cdot : Q4$. (a) $p(\Delta_x, \Delta_y)$. (b) $p(\Delta_z, \Delta_y)$. Contours are plotted between 0.03 and the maximum level in five equidistant steps. The straight dashed lines are $\Delta_y = \Delta_x$ and $\Delta_y = \Delta_z$, respectively.
Figure 15. Two-point spatial correlation coefficient centred at $Re_\theta = 2175$ and at $y/\delta = 0.4$. The axes are not to scale. (a) $C_{uu}$, (b) $C_{vv}$, (c) $C_{ww}$. Black: 0.02(0.05)1, Red: -0.02(0.03)1.01. (d) $C_{vu}$. Black: 0.01(0.01)0.4, Red: −0.01(0.02)0.41.

Figure 16. Two-point spatial correlation coefficient of streamwise velocity fluctuations $C_{uu}$ in $xz$ planes for the ZPG TBL, the present APG TBL and the APG TBL of MRL. The axes are not to scale. Black: 0.02(0.05)0.97, Red: −0.02(0.03)0.98.

Figure 15 shows contours of the two-point spatial correlation coefficients of the streamwise, wall-normal and spanwise fluctuating velocities obtained at $Re_\theta = 2175$ and at $y/\delta = 0.4$ in $xy$ planes. Like for ZPG TBLs [28], the contours of $C_{uu}$ are roughly elliptical in shape with the major axis tilted at an angle to the streamwise direction, but they are more inclined than the ZPG TBL ones. The correlation length in the streamwise direction is large. The vertical velocity correlation contours, $C_{vv}$, are quite different from the streamwise velocity correlations in that the length scales are more limited, and the correlations are slightly elongated in the vertical direction. For ZPG flows, the eddy structures have large correlation lengths in the streamwise direction, indicating that the eddies are more elongated than those in the APG flows. This follows the observation of Krogstad and Skare [27] who show that large-scale structures were shortened and structure inclination angle increased in APG flows. A recent experiment on a
strongly decelerated boundary layer by Rahgozar and Maciel [29] has shown that $u$-structures in the ZPG cases are more elongated in the streamwise direction than those in the APG cases and that, as the turbulent boundary layer continues to decelerate, $u$-structures become less streamwise elongated.

Figure 16 shows the correlation coefficient of the streamwise velocity fluctuations, $C_{uu}$ in $xz$ planes in the outer layer at $y/\delta = 0.5$, to match the available experimental data [6], and at $y/\delta = 0.2$ at Reynolds numbers of $Re_\theta = 1755$ and 2175 corresponding to $H = 2$ and 2.5, respectively. It can be seen that the correlation contours in the outer part of the boundary layer are shorter and wider compared to the ones closer to the wall. This is consistent with what was found in [27]. It is generally thought that the structures appear less frequently or meander more in an APG flow as compared to the ZPG ones [5, 29, 30]. This is illustrated in the figure, where a ZPG [28] and several strongly decelerated boundary layers with very large velocity defect [6] are shown. The $u$-fluctuations correlate over much shorter lengths as the velocity defect increases, showing the effect of the APG on the structures.

4. Conclusions

A direct numerical simulation of a non-equilibrium boundary layer subjected to strong adverse pressure gradient is investigated. The DNS is at fairly low Reynolds number ($Re_\theta = 150 − 2200$) and distorted due to the upstream boundary layer which is initially laminar and separates due to the strong pressure gradient. Transition to turbulence is triggered using a trip wire, positioned upstream of the unforced separation point. The two-point cross-correlation coefficient, $C_{vu}$, with a reference point close to the transition region shows flow periodicity persisting to $Re_\theta \approx 1600$. This periodicity is related to the shear layer instability of the separation bubble created as a result of the APG, and limits the useful range of the attached turbulent boundary layer to the approximate range of $Re_\theta = 1700 − 2200$.

The first- and second-order velocity statistics of the present APG TBL follow the trends generally encountered when the following two conditions are present in an APG TBL: strong non-equilibrium state of the TBL in dynamic terms, and a large mean velocity defect. Note that these two conditions are not necessarily always present at the same time in a given TBL. Like other non-equilibrium TBLs with large velocity defect, the present flow does not follow the law of the wall, even very close to the wall. Similarly to large-velocity-defect TBLs, in this case irrespective of their equilibrium state, the maximum turbulence activity is found to be in the middle of the boundary layer and not near the wall. A near-wall peak of streamwise Reynolds normal stress and of turbulence production does not exist in the present non-equilibrium flow, while the results of Skare and Krogstad [2] suggest that a near-wall production peak might be present in an equilibrium APG TBL with a fairly large velocity defect. The present analysis also shows that $u$ and $v$ are more decorrelated throughout the boundary layer than in ZPG TBLs. Near the wall, $u$ and $v$ become more and more decorrelated as the velocity defect increases.

In connection to the previous observations, an important conclusion that can be drawn from the present analysis is that, in part due to the absence of strong velocity gradients near the wall, large-velocity-defect boundary layers are globally less efficient in extracting turbulent energy from the mean flow than the ZPG TBL. When normalized with an adequate outer region velocity scale ($U_o$ or $U_{zs}$), the Reynolds stresses and the production of turbulent kinetic energy are weaker in the lower half of the large-velocity-defect boundary layers than in the ZPG TBL.

Due to the resemblance of mean velocity profiles in the outer region of large-velocity-defect TBLs to the high speed part of mixing layers, the present flow and three other TBLs are also compared with a single-stream mixing layer. To be able to do so, an outer region velocity scale analogous to the commonly defined free shear layer velocity scales is introduced. The resemblance between these two types of flows is striking in terms of the distributions of Reynolds stresses, production, dissipation and turbulent transport of turbulent kinetic energy.
uv correlation, and structure parameter $-\langle uv \rangle/2k$. Although more data is needed to draw a definitive conclusion, the outer-region turbulent statistics of TBLs close to detachment appear to have a similar character to those of the single-stream mixing layer.

The fractional quadrant decomposition of the Reynolds shear stress indicates that sweeps seem to dominate ejections in the near-wall region for the strong APG boundary layers, whilst ejections dominate the outer region. The switch between these two regions is observed to take place close to the maximum of the Reynolds shear stress in the outer layer. The $Q2$ and $Q4$ structures are less elongated than the same structures in channel flow.

The two-point spatial correlation $C_{uu}$ obtained far from the transition region at $Re_\theta = 2175$ and at $y/\delta = 0.4$ indicates that the correlation length of the $u$ fluctuations in an APG boundary layer is almost half that observed in the ZPG TBL. The wall-normal and spanwise correlations, $C_{vv}$ and $C_{ww}$, are quite different from the $C_{uu}$ correlations in that the length scales are more limited and the correlations are slightly elongated in the vertical direction, as found in ZPG boundary layers, implying that the structure of the $v, w$ fluctuations is the same as in ZPG.

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