The Hierarchy Problem and an Exotic Bound State

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Abstract

The Multiple Point Principle, according to which there exist many vacuum states with the same energy density, is put forward as a fine-tuning mechanism. By assuming the existence of three degenerate vacua, we derive the hierarchical ratio between the fundamental (Planck) and electroweak scales in the Standard Model. In one of these phases, 6 top quarks and 6 anti-top quarks bind so strongly by Higgs exchange as to become tachyonic and form a condensate. The third degenerate vacuum is taken to have a Higgs field expectation value of the order of the fundamental scale.

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1 Introduction

The hierarchy problem refers to the long-standing puzzle of why the electroweak scale is so very small compared to the fundamental scale $\mu_{\text{fundamental}}$, which we shall identify with the Planck scale $\mu_{\text{Planck}}$. In particular, radiative corrections to the Standard Model (SM) Higgs mass diverge quadratically with the SM cut-off scale $\Lambda$; this is the so-called technical hierarchy problem. For example the top quark loop contribution to the SM Higgs mass is given by:

$$\delta M_H^2 = -\frac{3}{4\pi^2} g_t^2 \Lambda^2 \sim (300 \text{ GeV})^2 \left( \frac{\Lambda}{1 \text{ TeV}} \right)^2$$

This leads to a fine-tuning problem for $\Lambda > 1$ TeV, when the top quark loop contribution exceeds the physical SM Higgs mass. For a SM cut-off at the Planck scale $\Lambda = \mu_{\text{Planck}} \sim 10^{19} \text{ GeV}$, the quadratic divergencies have to be cancelled to more than 30 decimals at each order in perturbation theory.

The most popular resolution of this technical hierarchy problem is to introduce Supersymmetry or some other new physics (e.g. technicolor or a little Higgs model) at the TeV scale. Although SUSY stabilizes the hierarchy between the electroweak and Planck (or GUT) scales, it does not explain why the hierarchy exists in the first place. An alternative way to resolve the hierarchy problem is to accept the necessity for fine-tuning and to explicitly introduce a fine-tuning mechanism. The most well-known example is the anthropic principle\cite{1,2}. We shall discuss here another fine-tuning mechanism: the Multiple Point Principle. We shall apply it to the pure SM, with no new physics below the Planck scale except presumably for a minor modification at the see-saw scale to generate neutrino masses.

2 Multiple Point Principle and the Large Scale Ratio

According to the Multiple Point Principle\cite{3}, Nature chooses the values of coupling constants in such a way as to ensure the existence of several degenerate vacuum states, each having approximately zero value for the cosmological constant. This fine-tuning of the coupling constants is similar to the fine-tuning of the intensive variables temperature and pressure at the triple point of water, due to the co-existence of the three degenerate phases: ice, water and vapour. The triple point of water is an easily reproducible situation and
occurs for a wide range of the fixed extensive quantities: the volume, energy and number of moles in the system.

We do not really know what is the dynamics underlying the Multiple Point Principle, but it is natural to speculate that by analogy it arises from the existence of fixed, but not fine-tuned extensive quantities in the Universe, such as

\[ I_1 = \int d^4x \sqrt{g(x)} \quad \text{and} \quad I_2 = \int \sqrt{g(x)} |\phi(x)|^2 \]

where \( \phi(x) \) is the SM Higgs field. Such fixed extensive quantities, having the form of reparameterisation invariant integrals over space-time\[4\] \( I_i = \int d^4x \sqrt{g(x)} \mathcal{L}_i(x) \), can be imposed by inserting \( \delta \)-functions in the Feynman path integral, similar to the energy fixing \( \delta \)-function in the partition function for a microcanonical ensemble in statistical mechanics\[5, 6\]. Then the coefficient or coupling constant multiplying \( I_i \) in the effective action is constrained to lie in a very narrow range, analogous to the inverse temperature in the canonical ensemble for a macroscopic system with a fixed energy. The coupling constant acts as a Lagrange multiplier, which has to adjust itself to ensure that the extensive quantity \( I_i \) takes on its correct fixed value. There is then a generic possibility that, for a large range of values for \( I_i \), the Universe has to contain two or more degenerate phases in different space-time regions. The imposition of a fixed value for an extensive quantity is a non-local condition, which seems to imply some mildly non-local physics, such as wormholes or baby universes\[7\], must underlie the multiple point principle\[3, 5, 6\]. However we emphasize that the multiple point principle really has the status of a postulated new principle.

We now wish to apply this fine-tuning principle to the problem of the huge scale ratio between the Planck scale and the electroweak scale: \( \mu_{\text{Planck}} / \mu_{\text{weak}} \sim 10^{17} \). It is helpful at this point to recall that another large scale ratio, \( \mu_{\text{Planck}} / \Lambda_{\text{QCD}} \sim 10^{20} \), is generally considered to be a natural consequence of the SM renormalisation group equation (RGE) for the QCD fine structure constant:

\[ \frac{d}{d\mu} \left( \frac{1}{\alpha_3(\mu)} \right) = \frac{7}{2\pi} \]

Then taking \( \alpha_3(\mu_{\text{Planck}}) \approx 1/50 \), which corresponds to an order of unity value for the coupling constant at the Planck scale \( g_3(\mu_{\text{Planck}}) \approx 1/2 \), the RGE gives \( \mu_{\text{Planck}} / \Lambda_{\text{QCD}} = \exp(2\pi/7\alpha_3(\mu_{\text{Planck}})) \approx \exp(45) \). A full understanding would of course require a derivation of the value \( g_3(\mu_{\text{Planck}}) \approx 1/2 \) from
physics beyond the SM, such as is done in the family replicated gauge group model\cite{8}.

Our proposed explanation for the large scale ratio $\mu_{\text{fundamental}}/\mu_{\text{weak}}$ is similarly based on the use of the RGE for the running top quark Yukawa coupling $g_t(\mu)$ in the SM:

$$\frac{dg_t}{d\ln \mu} = \frac{g_t}{16\pi^2} \left( \frac{9}{2} g_3^2 - 8 g_2^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right)$$  \(4\)

Here $g_3$, $g_2$ and $g_1$ are the $SU(3) \times SU(2) \times U(1)$ running gauge coupling constants, which we shall consider as given. The multiple point principle is used to fine-tune the boundary values of $g_t(\mu)$ at \textit{both} the fundamental and weak scales, due to the existence of 3 degenerate SM vacua. Note that we \textbf{do not} use the physical top quark mass as an input. These boundary values, $g_t(\mu_{\text{fundamental}})$ and $g_t(\mu_{\text{weak}})$, then fix the amount of running needed from the RGE \(4\) and hence the required scale ratio $\mu_{\text{fundamental}}/\mu_{\text{weak}}$.

3 Two degenerate minima in the SM effective potential

In order to fine-tune the value of $g_t(\mu_{\text{fundamental}})$ using the multiple point principle, we postulate the existence of a second degenerate vacuum\cite{3,9}, in which the SM Higgs field $\phi$ has a vacuum expectation value of order $\mu_{\text{fundamental}}$. This requires that the renormalisation group improved effective potential $V_{\text{eff}}(\phi)$ should have a second minimum near the fundamental scale, where the potential should essentially vanish in order to be degenerate with the usual electroweak scale minimum.

For large values of the SM Higgs field $\phi \sim \mu_{\text{fundamental}} \gg \mu_{\text{weak}}$, the renormalisation group improved effective potential is well approximated by

$$V_{\text{eff}}(\phi) \simeq \frac{1}{8} \lambda(\mu = |\phi|)|\phi|^4$$  \(5\)

and the degeneracy condition means that $\lambda(\mu_{\text{fundamental}})$ should vanish to high accuracy. The effective potential $V_{\text{eff}}$ must also have a minimum and so its derivative should vanish. Therefore the vacuum degeneracy requirement means that the Higgs self-coupling constant and its beta function should vanish near the fundamental scale:

$$\lambda(\mu_{\text{fundamental}}) = \beta\lambda(\mu_{\text{fundamental}}) = 0$$  \(6\)
This leads to the fine-tuning condition

\[ g_t^4(\mu_{\text{fundamental}}) = \frac{1}{48} \left( 9g_2^4 + 6g_2^2g_1^2 + 3g_1^4 \right) \]  

(7)

relating the top quark Yukawa coupling \( g_t(\mu) \) and the electroweak gauge coupling constants \( g_1(\mu) \) and \( g_2(\mu) \) at \( \mu = \mu_{\text{fundamental}} \). We must now input the experimental values of the electroweak gauge coupling constants, which we evaluate at the Planck scale using the SM renormalisation group equations, and obtain our prediction:

\[ g_t(\mu_{\text{fundamental}}) \approx 0.39. \]  

(8)

However we note that this value of \( g_t(\mu_{\text{fundamental}}) \), determined from the right hand side of Eq. (7), is rather insensitive to the scale, varying by approximately 10% between \( \mu = 246 \text{ GeV} \) and \( \mu = 10^{19} \text{ GeV} \).

4 Three degenerate vacua and the exotic bound state

We now want to fine-tune the value of \( g_t(\mu_{\text{weak}}) \) using the multiple point principle. In order to achieve this, it is necessary to have 2 degenerate vacua which only deviate by their physics at the electroweak scale. So what could the third degenerate SM vacuum be? Different phases are most easily obtained by having different amounts of some Bose-Einstein condensate. We are therefore led to consider a condensate of a bound state made out of some SM particles. We actually propose\(^8,10\) a new exotic strongly bound state made out of 6 top quarks and 6 anti-top quarks – a dodecaquark! The reason that such a bound state was not considered previously is that its binding is based on the collective effect of attraction between several quarks due to Higgs exchange.

The virtual exchange of the Higgs particle between two quarks, two anti-quarks or a quark anti-quark pair yields an attractive force in each case. For top quarks Higgs exchange provides a strong force, since we know phenomenologically that \( g_t(\mu) \sim 1 \). So let us consider putting more and more \( t \) and \( \bar{t} \) quarks together in the lowest energy relative S-wave states. The Higgs exchange binding energy for the whole system becomes proportional to the number of pairs of constituents, rather than to the number of constituents.
So a priori, by combining sufficiently many constituents, the total binding energy could exceed the constituent mass of the system! However we can put a maximum of $6t + 6\bar{t}$ quarks into the ground state S-wave. So we shall now estimate the binding energy of such a 12 particle bound state.

As a first step we consider the binding energy $E_1$ of one of them to the remaining 11 constituents treated as just one particle analogous to the nucleus in the hydrogen atom. We assume that the radius of the system turns out to be reasonably small, compared to the Compton wavelength of the Higgs particle, and use the well-known Bohr formula for the binding energy of a one-electron atom with atomic number $Z = 11$ to obtain the crude estimate:

$$E_1 = -\left(\frac{11g_t^2}{4\pi}\right)^2 \frac{11m_t}{24}. \tag{9}$$

Here $g_t$ is the top quark Yukawa coupling constant, in a normalisation in which the top quark mass is given by $m_t = g_t 174$ GeV.

The non-relativistic binding energy $E_{\text{binding}}$ of the 12 particle system is then obtained by multiplying by 12 and dividing by 2 to avoid double-counting the pairwise binding contributions. This estimate only takes account of the $t$-channel exchange of a Higgs particle between the constituents. A simple estimate of the $u$-channel Higgs exchange contribution increases the binding energy by a further factor of $(16/11)^2$, giving:

$$E_{\text{binding}} = \left(\frac{11g_t^4}{\pi^2}\right) m_t \tag{10}$$

We have so far neglected the attraction due to the exchange of gauge particles. So let us estimate the main effect coming from gluon exchange with a QCD fine structure constant $\alpha_s(M_Z) = g_s^2(M_Z)/4\pi = 0.118$, corresponding to an effective gluon $t - \bar{t}$ coupling constant squared of:

$$e_{tt}^2 = 4\frac{g_s^2}{3} \simeq \frac{4}{3} 1.5 \simeq 2.0 \tag{11}$$

For definiteness, consider a $t$ quark in the bound state; it interacts with 6 $\bar{t}$ quarks and 5 $t$ quarks. The 6 $\bar{t}$ quarks form a colour singlet and so their combined interaction with the considered $t$ quark vanishes. On the other hand the 5 $t$ quarks combine to form a colour anti-triplet, which together interact like a $\bar{t}$ quark with the considered $t$ quark. So the total gluon interaction of the considered $t$ quark is the same as it would have with a single
quark. In this case the $u$-channel gluon contribution should equal that of the $t$-channel. Thus we should compare the effective gluon coupling strength $2 \times e_t^2 \simeq 2 \times 2 = 4$ with $(16/11) \times Z g_t^2/2 \simeq 16 \times 1.0/2 = 8$ from the Higgs particle. This leads to an increase of $E_{\text{binding}}$ by a factor of $(4+8)/8 = (3/2)^2$, giving our final result:

$$E_{\text{binding}} = \left( \frac{9g_t^4}{4\pi^2} \right) m_t \quad (12)$$

We are now interested in the condition that this bound state should become tachyonic, $m_{\text{bound}}^2 < 0$, in order that a new vacuum phase could appear due to Bose-Einstein condensation. For this purpose we consider a Taylor expansion in $g_t^2$ for the mass squared of the bound state, crudely estimated from our non-relativistic binding energy formula:

$$m_{\text{bound}}^2 = (12m_t)^2 - 2(12m_t) \times E_{\text{binding}} + ...$$

$$= (12m_t)^2 \left( 1 - \frac{33}{8\pi^2} g_t^4 + ... \right) \quad (14)$$

Assuming that this expansion can, to first approximation, be trusted even for large $g_t$, the condition $m_{\text{bound}}^2 = 0$ for the appearance of the above phase transition with degenerate vacua becomes to leading order:

$$g_t \bigg|_{\text{phase transition}} = \left( \frac{8\pi^2}{33} \right)^{1/4} \simeq 1.24 \quad (15)$$

We have of course neglected several effects, such as weak gauge boson exchange, $s$-channel Higgs exchange and relativistic corrections. In particular quantum fluctuations in the Higgs field could have an important effect in reducing $g_t \big|_{\text{phase transition}}$ by up to a factor of $\sqrt{2}$. It is therefore quite possible that the value of the top quark running Yukawa coupling constant, predicted from our vacuum degeneracy fine-tuning principle, could be in agreement with the experimental value $g_t(\mu_{\text{weak}})_{\exp} \approx 0.98 \pm 0.03$. Assuming this to be the case, we can now estimate the fundamental to weak scale ratio by using the leading order RGE for the top quark SM Yukawa coupling $g_t(\mu)$. It should be noticed that, due to the relative smallness of the fine structure constants $\alpha_i = g_i^2/4\pi$ and particularly of $\alpha_3(\mu_{\text{fundamental}})$, the beta function $\beta_{g_t}$ for the top quark Yukawa coupling constant, Eq. (4), is numerically rather small at the fundamental scale. Hence we need many $e$-foldings between the two scales, where $g_t(\mu_{\text{fundamental}}) \simeq 0.39$ and $g_t(\mu_{\text{weak}}) \simeq 1.24$. The predicted
Figure 1: Plots of $g_t$ and $\lambda$ as functions of the scale of the Higgs field $\phi$ for degenerate vacua with the second Higgs VEV at the Planck scale $\phi_{\text{vacuum}_2} = 10^{19}$ GeV. The second order SM renormalisation group equations are formally applied up to a scale of $10^{25}$ GeV.

The scale ratio is quite sensitive to the input value of $\alpha_3(\mu_{\text{fundamental}})$. When we input the value of $\alpha_3 \simeq 1/54$ evaluated at the Planck scale, from the phenomenological value of $\Lambda_{QCD}$ using the RGE for the SM fine structure constants, we predict the scale ratio to be:

$$\mu_{\text{fundamental}}/\mu_{\text{weak}} \sim 10^{16} - 10^{20}$$

The running of the top quark Yukawa coupling is shown in Figure 1 as a function of $\log_{10} \phi$. We note that, as can be seen from Eq. (11), the rate of logarithmic running of $g_t(\mu)$ increases as the QCD gauge coupling constant $g_3(\mu)$ increases. Hence the value of the weak scale is naturally fine-tuned to be a few orders of magnitude above the QCD scale. Using the RGE for the SM Higgs self-coupling

$$\frac{d\lambda}{d\ln \mu} = \frac{1}{16\pi^2} \left[ 12\lambda^2 + 3(4g_t^2 - 3g_2^2)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 \right]$$

and the boundary value at the fundamental scale, Eq. (13), we can calculate the running of $\lambda(\mu)$. The results are also shown in Figure 1. The value of
\( \lambda(\mu_{\text{weak}}) \) obtained can be used to predict the SM Higgs mass

\[
M_H = 135 \pm 9 \text{ GeV}
\] (18)

5 Properties of the exotic bound state

Strictly speaking, it is \textit{a priori} not obvious within our scenario in which of the two degenerate electroweak scale vacua discussed in Section 4 we live. There is however good reason to believe that we live in the usual Higgs phase without a condensate of new bound states rather than in the one with such a condensate. The point is that such a condensate is not invariant under the \( SU(2) \times U(1) \) electroweak gauge group and would contribute to the squared masses of the \( W^\pm \) and \( Z^0 \) gauge bosons. Although these contributions are somewhat difficult to calculate, preliminary calculations indicate that these contributions would make the \( \rho \)-parameter deviate significantly from unity, in contradiction with the precision electroweak data.

We expect the new bound state to be strongly bound and relatively long lived in our vacuum; it could only decay into a channel in which all 12 constituents disappeared together. The production cross-section of such a particle would also be expected to be very low, if it were just crudely related to the cross section for producing 6 \( t \) and 6 \( \bar{t} \) quarks. It would typically decay into 6 or more jets, but it would probably not be possible to reconstruct the multi-jet decay vertex precisely enough to detect its displacement from the bound state production vertex. There would be a better chance of observing an effect, if we optimistically assume that the mass of the bound state is close to zero (i.e. very light compared to \( 12m_t \approx 2 \text{ TeV} \)) even in the phase in which we live. In this case the bound state obtained by removing one of the 12 quarks would also be expected to be light. These bound states with radii of order \( 1/m_t \) might then be smaller than or similar in size to their Compton wavelengths and so be well described by effective scalar and Dirac fields respectively. The 6 \( t \) + 6 \( \bar{t} \) bound state would couple only weakly to gluons whereas the 6\( t \) + 5 \( \bar{t} \) bound state would be a colour triplet. So the 6\( t \) + 5 \( \bar{t} \) bound state would be produced like a fourth generation top quark\(^1\) at the LHC. If these 11 constituent bound states were pair produced, they would presumably decay into the lighter 12 constituent bound states with

\(^1\)However there would be very little mixing with the top quark, due to the small overlap of their wave functions.
the emission of a $t$ and a $\bar{t}$ quark. The $6t + 6\bar{t}$ bound states would in turn decay into multi-jets, producing a spectacular event.

6 Summary and Conclusion

In this talk, we have put forward a scenario for how the huge scale ratio between the fundamental scale $\mu_{\text{fundamental}}$ and the electroweak scale $\mu_{\text{weak}}$ may come about in the pure SM. We appeal to a fine-tuning postulate – the Multiple Point Principle – according to which there are several different vacua, in each of which the energy density (cosmological constant) is very small. In fact our scenario requires a landscape of 3 degenerate SM vacua, in contrast to the $10^{1000}$ or so string vacua\cite{1, 2}.

The existence of an exotic bound state of six top quarks and six anti-top quarks is crucial to our scenario. Furthermore the binding of this dodecaquark state, due mainly to Higgs particle exchange, must be so strong that a condensate of such bound states can form and make up a phase in which essentially tachyonic bound states of this type fill the vacuum. The calculation of the critical top quark Yukawa coupling $g_t|_{\text{phase transition}}$ for which such a vacuum should appear involves no fundamentally new physics. It is a very difficult SM calculation, but would provide a clean test of the Multiple Point Principle as $g_t|_{\text{phase transition}}$ is predicted to equal the experimentally measured value $g_t(\mu_{\text{weak}})_{\exp}$. Within the accuracy of our crude extrapolation (15) of the non-relativistic Bohr formula, our Multiple Point Principle estimate of $g_t(\mu_{\text{weak}})$ is in agreement with experiment.

In addition to the 2 degenerate electroweak scale vacua, we postulate the existence of another degenerate vacuum in which the SM Higgs field has a vacuum expectation value of order the fundamental scale. We thereby obtain a prediction (8) for the value of $g_t(\mu_{\text{fundamental}})$ in terms of the electroweak gauge coupling constants. The crucial point now is that we need an appreciable running of $g_t(\mu)$, in order to make its fine-tuned values at $\mu_{\text{weak}}$ and at $\mu_{\text{fundamental}}$ compatible. That is to say we need a huge scale ratio (16), since the running is rather slow due to the smallness of the SM fine structure constants $\alpha_i$ from the renormalisation group point of view. It also naturally follows that the electroweak scale lies within a few orders of magnitude above $\Lambda_{\text{QCD}}$.

Finally we remark that, in our scenario, there are still quadratic divergences in the radiative corrections to the Higgs mass squared at each order.
of perturbation theory. However the Multiple Point Principle fine-tunes the bare parameters at each order of perturbation theory, so as to ensure the equality of the energy densities in the three different SM vacua. Indeed we obtain a prediction for the Higgs mass: \( M_H \sim 135 \) GeV.

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