Supersymmetric gyratons in five dimensions

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Abstract

We obtain the gravitational and electromagnetic field of a spinning radiation beam-pulse (a gyraton) in minimal five-dimensional gauged supergravity and show under which conditions the solution preserves part of the supersymmetry. The configurations represent generalizations of Lobatchevski waves on AdS with nonzero angular momentum, and possess a Siklos–Virasoro reparametrization invariance. We compute the holographic stress–energy tensor of the solutions and show that it transforms without anomaly under these reparametrizations. Furthermore, we present supersymmetric gyratons both in gauged and ungauged five-dimensional supergravity coupled to an arbitrary number of vector supermultiplets, which include gyratons on domain walls.

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1. Introduction

Supersymmetry plays an important role in theoretical physics, because solutions preserving part of the supersymmetry of the theory under consideration benefit from good perturbative properties which permit the probing of non-perturbative effects thanks to non-renormalization theorems. These remarkable properties allow, for example, a microscopic explanation of black hole entropy in string theory [1]. For this reason, much effort has been dedicated to finding and classifying all supersymmetric solutions to various supergravity theories, first using the mathematical concept of G-structures [2] and more recently applying the more efficient spinorial geometry techniques [3]. This task has been successfully carried out for several supergravity theories in diverse dimensions, cf e.g. [4–7]. However, the results are rather implicit, since the construction of the full geometry requires the solution of systems of differential equations that can be fairly complicated. This makes it difficult to explore the physics behind these large classes of supersymmetric solutions, but trying to investigate particular solutions can be rewarding, as shown by the discovery of five-dimensional supersymmetric anti-de Sitter black holes [8, 9], supersymmetric black rings...
In this paper we shall focus on both ungauged and gauged matter-coupled supergravity in five dimensions. The latter theory plays a central role in the AdS/CFT correspondence [14], since its asymptotically AdS solutions are dual to four-dimensional field theories. The general supersymmetric solutions to these theories are known [9, 15], and we shall extract a particular subclass of geometries having the interpretation of the gravitational field generated by gyratons in flat space or in AdS.

Gyratons are ultrarelativistic pulsed beams of finite duration and finite cross-section, carrying a finite amount of energy and angular momentum [16, 17]. The gravitational field generated by such sources is important in the study of mini black hole production in colliders or cosmic ray experiments. Usually, the corresponding amplitudes are computed using the Aichelburg–Sexl metric, which is the gravitational field generated by spinless ultrarelativistic particles (see for example [18] for a general review on the subject). However, spin–spin and spin–orbit interactions can be important in certain regimes, and the full gravitational field generated by the gyratons is important to study the effect of the spin on the interaction of ultrarelativistic particles with spin [16].

An important property of these gyraton solutions in the gauged theories is that all curvature invariants that can be built from the metric are independent of the functions $\Phi_1$ and $\Psi_1$ that characterize the geometry, and assume the same value as in pure AdS space. Therefore, the spacetimes (2.1) are free of curvature singularities, and regular everywhere. This implies that these solutions do not get any $\alpha'$ correction [19] and are perturbatively exact in string theory.

In this paper, we shall first find the general form of the gyraton solutions in minimal supergravities in five dimensions, and the conditions they must satisfy in order to preserve some supersymmetry. We then generalize these solutions by including general vector multiplets. We obtain the supersymmetric gyratonic solutions of the ungauged supergravities coupled to vector multiplets, and work out the case of the STU model. Then, by taking the $U(1)$ truncation of the latter theory we recover the result of the minimal case. We study then supersymmetric gyratons in the context of gauged supergravities coupled to an arbitrary number of vector multiplets. In this case the equations are more complicated, but in the special case of the gauged STU model explicit supersymmetric gyraton solutions with nonconstant scalar fields are obtained. Finally, we compute the holographic stress tensor associated with the AdS gyratons and show that it transforms without anomaly under the Siklos–Virasoro transformations that leave the form of the metric invariant.

### 2. Supersymmetric gyratons in minimal gauged supergravity

Following [19], the geometry of five-dimensional AdS gyratons can be put in the form

$$
\text{d}s^2 = -\frac{r^2}{z^2}(\text{-}2\, \text{d}u \, \text{d}v + \Phi \, \text{d}u^2 + 2(\Psi_i \, \text{d}x^i + \Psi_j \, \text{d}y^j + \Psi_k \, \text{d}z) \, \text{d}u + \text{d}x^2 + \text{d}y^2 + \text{d}z^2),
$$

(2.1)

---

Our notation and conventions are as follows: the signature is mostly minus. The five-dimensional geometries are described by the coordinates $(u, v, x^1, x^2, x^3)$, where $x^1 = z, x^2 = x, x^3 = y$. Late greek letters $\mu, \nu, \ldots$ denote curved indices in five dimensions. We will often consider lower-dimensional sections of this geometry; latin letters $i, j, \ldots$ are indices on the three-dimensional flat space parametrized by $(x^1, x^2, x^3)$. $\Delta^{(2)} = \partial_x \partial_y$ is the flat Laplacian, $\hat{d}$ the external derivative and $\ast$ the corresponding Hodge operator. Early greek letters $\alpha, \beta, \ldots$ are indices of the two-dimensional space $(x^2, x^3)$, again with flat metric. The antisymmetric tensor $\varepsilon_{\alpha \beta}$ on this space is defined such that $\varepsilon_{23} = 1$, and $\Delta^{(2)}$ is the flat Laplacian in two dimensions.
where the functions $\Phi$ and $\Psi$ depend on $(u, x, y, z)$, but not on $v$. We use this ansatz for the metric in minimal gauged supergravity, whose action reads

$$S = -\frac{1}{16 \pi G_5} \int \left( R - 2\Lambda + F^2 + \frac{2}{3\sqrt{3}} \epsilon^{\mu
u\rho\tau} F_{\mu\nu} F_{\rho\tau} A_5 \right) \sqrt{|g|} d^5x,$$

where the cosmological constant $\Lambda$ is linked to the minimal gauge coupling $\chi$ and the curvature radius $l$ of AdS by the relations

$$\Lambda = \frac{\chi^2}{2} = \frac{6}{l^2}.$$

Then, with the additional ansatz $A = A_u(u, x, y, z) du$ for the gauge field, the Maxwell equations become

$$\Delta A_u - \frac{1}{z} A_{u,z} = 0.$$

Note that this is just the Laplace equation on the hyperbolic space $H^3$ with metric

$$ds^2 = \frac{dx^2 + dy^2 + dz^2}{z^2}.$$

If we define $\Omega$ as the curvature tensor associated with the Sagnac connection $\Psi$,

$$\Omega_{ij} = \Psi_{j,i} - \Psi_{i,j},$$

the $(ui)$ components of Einstein’s equations read

$$\Omega_{i,x} + \Omega_{j,y} + \Omega_{k,z} = \frac{3}{\ell} \Omega_{i,z},$$

while their $(mu)$ component, since the $R_{uu}$ component of the Ricci tensor is given by

$$R_{uu} = -\frac{1}{2} \Delta (\Phi + \frac{3}{2z} \Phi_{,z} - \frac{\Phi_{,z}}{\ell^2} - \frac{1}{4} \Omega_{ij} \Omega_{ij}),$$

imposes an equation for $\Phi$,

$$\Delta (\Phi + \frac{3}{2z} \Phi_{,z} - \frac{\Phi_{,z}}{\ell^2} - \frac{1}{4} \Omega_{ij} \Omega_{ij}) = -\frac{4\ell^2}{l^2} A_{u,i} A_{u,i}.$$

If equations (2.4), (2.6) and (2.8) are satisfied, then the fields represent a solution of minimal gauged supergravity in five dimensions describing a gyraton propagating in anti-de Sitter space.

Defining $\tilde{\Omega} = z^{-1} \Omega$, (2.6) can be rewritten as

$$\partial_j \tilde{\Omega}_{ij} = 0,$$

or equivalently $\delta^j \tilde{\Omega} = 0$, which implies that locally $\star_j \tilde{\Omega} = \delta \psi$, where $\psi$ is some function of the coordinates $x^i$ and $u$. In components we have

$$\Omega_{u\alpha} = -2\sqrt{3} \frac{\epsilon_{\alpha\beta}}{l} \tilde{\psi}_{,\beta}, \quad \Omega_{23} = -2\sqrt{3} \frac{\psi_{,z}}{l} \left( \psi_{,z} - \frac{\psi}{z^2} \right),$$

where we defined $\tilde{\psi} = -z^2 l \psi/(2\sqrt{3})$ for later convenience. Note that (2.5) implies that the potential $\psi$ must satisfy equation (2.4), and thus is not completely arbitrary.

Actually, the ansatz on the gauge field is too restrictive. Starting with an arbitrary $A_u$ with no dependence on $v$ to respect the symmetry generated by $\partial_v$, and choosing the gauge in which $A_v = 0$, it follows immediately from Einstein’s equations that $F_{zx} = F_{zy} = 0$. With some additional work one finds then that the gauge field must be of the form

$$A = A_u(u, x, y, z) du + \epsilon_{\alpha\beta} \partial_\alpha p(u, x, y) dx^\alpha + A_z(u, z) dz,$$
where \( p(u, x, y) \) is a harmonic function on the two-space \((x, y)\),

\[
\Delta^{(2)} p = 0.
\]  

(2.12)

Then the general solution is given by solving the Maxwell equation

\[
\Delta^{(3)} A_{a} + \frac{1}{z}(\partial_{a} A_{z} - \partial_{z} A_{a}) - \partial_{b} \partial_{a} A_{b} = 0,
\]  

(2.13)

and the Einstein equations, whose \((u1)\) components, given in equation (2.6) remain unchanged, while its \((uu)\) component gets some new source from the additional components in the gauge field strength,

\[
\Delta^{(3)} \Phi = \frac{1}{2} \Omega_{ij} \Omega_{ij} - 2 \partial_{a} \Psi_{i,j} - \frac{3}{z}(\Phi, z - 2\Psi, u)
\]  

\[
= - \frac{4z^2}{l^2} [(\partial_{a} \partial_{z} p - \partial_{z} A_{a})^2 + (\partial_{a} \partial_{z} p + \partial_{z} A_{a})^2 + (\partial_{a} A_{z} - \partial_{z} A_{a})^2].
\]  

(2.14)

According to the analysis carried out in [19], such a solution describes a gyraton propagating in an asymptotically AdS5 spacetime.

Note that, if all functions \(\Psi_{i}\) vanish, then the metric reduces to the metric of travelling waves on AdS5 (Lobatchevski waves), and equation (2.8) reduces to Siklos’ equation [20]. It is interesting to note that all solutions of the supergravity equations of the travelling waves form are supersymmetric [21]. In the following, we shall determine which of the gyraton solutions preserve some supersymmetry.

We recall that supersymmetric solutions to supergravity theories are divided into timelike and null solutions, according to the nature of the Killing vector constructed as a bilinear from the Killing spinor. Solutions in the null class are all plane-fronted waves, and the gyratons are included among them. The general null solution has been obtained in [6] and reads

\[
ds^2 = H^{-1}(F_{a} du^2 + 2 da \ dv) - H^2[(dx^1 + a_1 du)^2 + e^{3\phi}(dx^a + e^{-3\phi} a_a du)^2],
\]  

(2.15)

\[A = A_a du + \frac{l}{\sqrt{3}} \frac{e^3}{4} \varepsilon_{a}^{\beta} \partial_{a} \phi \ dx^{\beta}.
\]  

(2.16)

The function \(\phi(u, x')\) is determined by the equation

\[e^{2\phi} \phi'^2 + \Delta^{(2)} \phi = 0.
\]  

(2.17)

Given a solution of (2.17), \(H(u, x')\) is obtained from

\[H = \frac{l}{2} \partial_{1} \phi,
\]  

(2.18)

and \(A_a(u, x')\) is found by solving the Maxwell equation

\[\partial_{a}[H^2 e^{2\phi} \partial_{a} (e^{\phi} A_a)] + \partial_{a}(H^2 \partial_{a} A_a) = \frac{l}{4} \frac{\sqrt{3}}{2} H \varepsilon_{a b} \partial_{a} \phi \partial_{b} H.
\]  

(2.19)

Then the functions \(a_a(u, x')\) are determined by the system

\[
\frac{1}{2}\sqrt{3} \varepsilon_{a b} \partial_{a} (H^2 a_b) = -H^2 e^{2\phi} \partial_{1} (e^{\phi} A_a),
\]  

(2.20)

\[
\frac{1}{2}\sqrt{3} [\partial_{a}(H^3 a_1) - \partial_{1}(H^3 a_a)] = H^2 \varepsilon_{a b} \partial_{b} A_a - \frac{l}{4} \frac{\sqrt{3}}{4} H^2 \partial_{a} \partial_{b} \phi.
\]  

4 The function \(S\) appearing in [6] is linked to our function \(\phi\) by the relation \(S = e^{3\phi/2}\).
whose integrability condition is (2.19). Finally, the function $\mathcal{F}(u, x')$ follows from the $uu$-component of the Einstein equations,

$$R_{uu} = -2 F_{u\sigma} F^{\sigma}_{u} + \frac{1}{2} g_{uu}(F^2 + 2\Lambda). \tag{2.21}$$

To find the subset of supersymmetric null solutions of the gyraton form, we introduce the new coordinate $z$ defined by

$$x^1 = l^2/(2z^2) \tag{2.22}$$

and choose

$$\phi(z) = \ln \left( \frac{l^2}{z^2} \right), \tag{2.23}$$

as the solution of (2.17). Then, the general metric (2.15) takes precisely the gyraton form\(^5\), where the functions $\Phi_1(u, x)$ and $\Psi_{1i}(u, x)$ are related to the functions $\mathcal{F}(u, x')$ and $a'(u, x')$ through

$$\Phi = \mathcal{F} + \Psi_1, \quad \Psi_1 = -\frac{z^3}{l^3} a_1, \quad \Psi_2 = \frac{z^6}{l^6} a_2, \quad \Psi_3 = \frac{z^6}{l^6} a_3. \tag{2.24}$$

With this ansatz, the Maxwell equation (2.19) for $A_u$ reduces to (2.4), equation (2.21) for $\Phi$ simplifies to (2.8) while the system (2.20) for $\Psi_i$ yields the relations

$$\Omega_{u\alpha} = -2\sqrt{3} \frac{z^2}{l} \epsilon_{\alpha\beta} A_{u,\beta}, \quad \Omega_{23} = -2\sqrt{3} \frac{z}{l} (A_{u,z} - \frac{2}{z} A_u). \tag{2.25}$$

Observe that equations (2.25) imply (2.6), and the supersymmetric configuration we constructed represents therefore a supersymmetric gyraton propagating in anti-de Sitter space. The converse is however not true, and not every gyratonic solution is supersymmetric. Comparing (2.25) with (2.10) we see that the gyraton is supersymmetric if the potential $\tilde{\psi}$ coincides with the gauge field $A_u$. This is to be opposed to the case of the Siklos solutions ($\Psi_i = 0$) which are all automatically supersymmetric [21].

An important property of the full family of gyraton solutions is that it enjoys a large reparametrization invariance, called Siklos–Virasoro invariance. Indeed, if we perform the diffeomorphism

$$\bar{u} = \chi(u), \quad \bar{v} = v + \frac{\chi''}{4\chi} x_j x_i + \lambda(u, x'), \quad \bar{x}^i = \sqrt{\chi} x^i, \tag{2.26}$$

defined by two arbitrary functions $\chi(u)$ and $\lambda(u, x')$, the metric and the field equations remain invariant in form if the functions $\Phi, \Psi_i$ and $A_u$ transform according to

$$\Phi = \frac{1}{\chi} \left[ \Phi + \frac{1}{2} \{ \chi; u \} x_i x_i - \frac{\chi''}{\chi} (\Psi_i + \lambda, x_j) x_j + 2\lambda, u \right], \tag{2.27}$$

$$\Psi_i = \frac{1}{\sqrt{\chi}} (\Psi_i + \lambda, j), \tag{2.28}$$

$$A_{\pi} = \frac{1}{\chi} A_u, \tag{2.29}$$

where

$$\{ \chi(u); u \} = \frac{\chi''(u)}{\chi'(u)} - \frac{3}{2} \left( \frac{\chi''(u)}{\chi'(u)} \right)^2 \tag{2.30}$$

\(^5\) We also have to take $v \mapsto -v$. 

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\(R_{uu} = -2 F_{u\sigma} F^{\sigma}_{u} + \frac{1}{2} g_{uu}(F^2 + 2\Lambda)\)
denotes the Schwarzian derivative. For Lobatchevski waves, when $\Psi_1 = 0$, this invariance was first obtained by Siklos\cite{20}. Note from (2.28) that the $\lambda$-transformation (with $\chi (u) = u$) acts as a gauge transformation on the connection $\Psi_1$, leaving its field strength $\Omega_{ij}$ unchanged. Therefore, if $\Omega_{ij} = 0$ we can always perform (at least locally) a diffeomorphism to eliminate the connection $\Psi_1$ from the metric, and the solution is isometric to a Lobatchevski wave. The solution represents a genuine gyraton—and carries angular momentum—if and only if $\Omega \neq 0$.

A particular solution of (2.4) is given by $A_u = b(u, x^\mu) z^2$, with $b(u, x^\mu)$ harmonic in $x^\mu$, $\Delta^{2)} b(u, x^\mu) = 0$. In the gauge $\Psi_1 = 0$, which can always be achieved by a suitable $\lambda$-transformation (2.28), equations (2.25) yield for the Sagnac connection

$$\Psi_1 = -\frac{\sqrt{3} z^4}{2 l} \epsilon_{\alpha \beta} \partial_\beta b.$$  

(2.31)

Finally, the Siklos equation (2.8) is solved by

$$\Phi = \alpha (u, x, y) z^4 + \beta (u, x, y) z^6,$$  

(2.32)

where the functions $\alpha$ and $\beta$ obey

$$\Delta^{(2)} \beta - \frac{8}{l^2} \partial_\alpha b \partial_\alpha b = 0, \quad \Delta^{(2)} \alpha + 12 \beta + \frac{16}{l^2} b^2 = 0.$$  

(2.33)

In the special case $b = \beta = 0, \alpha = \text{const}$, this solution reduces to the five-dimensional generalization of the Kaigorodov spacetime [23], constructed in [24].

Finally, let us consider the ungauged limit. To obtain a finite result we first have to introduce the new coordinate $\eta$ defined by

$$\frac{z}{l} = \exp \left( \frac{\eta}{l} \right),$$  

(2.34)

then the $l \to \infty$ limit can be performed safely. We obtain the geometry of plane-fronted waves with parallel rays (pp-waves), whose metric is

$$ds^2 = F du^2 + 2 du dv - (dx^i + \Psi_i du)^2,$$  

(2.35)

and the gauge potential reads

$$A = -\frac{1}{2 \sqrt{3}} \psi (u, x^i) du,$$  

(2.36)

where we have defined $F = \Psi_1 \Psi_5 - \Phi$ and $\psi = -2 \sqrt{3} A_u$. Then, Maxwell’s equations (2.13) imply that $\psi$ has to be a harmonic function on the flat three-dimensional base space, $\Delta^{(3)} \psi = 0$, while the $(ii)$ components of the Einstein equations are solved iff the curl of $\Psi$ is the gradient of some function $\bar{\psi} (u, x^i)$ on $\mathbb{R}^3$,

$$\tilde{\nabla} \times \bar{\Psi} = \tilde{\nabla} \bar{\psi}.$$  

(2.37)

Finally, the $(uu)$ component of Einstein’s equations reads

$$\Delta^{(3)} F + 2 D \tilde{\nabla} \cdot \Psi = 2 \partial_\mu \Psi_\mu \partial_\nu \Psi_\nu + \frac{4}{3} (\nabla \psi)^2,$$  

(2.38)

where we have defined the operator $D$ acting on scalar functions $f$ as $D f = \partial_\mu f - \Psi^\mu \partial_\mu f$. Therefore, one obtains a solution of minimal ungauged supergravity with a metric of the form (2.35) by choosing a function $\psi (u, x^i)$ harmonic on $\mathbb{R}^3$ and an arbitrary function $\bar{\psi}$, and then solving (2.38) for $F (u, x^i)$. It is easy to show that this is the most general solution of minimal supergravity with the metric of the form (2.35) up to gauge transformations. The supersymmetry condition then simply requires the equality of the functions $\psi$ and $\bar{\psi}$, $\psi = \bar{\psi}$. This condition can be equivalently obtained by taking the ungauged limit of equations (2.25).

\footnote{In the original paper by Siklos there is a typeset error, corrected in [22].}

\footnote{The functions $p(u, x, y)$ and $A_z (u, z)$ appearing in (2.11) can indeed be gauged away in the $l \to \infty$ limit.}
Observe that we are free to impose the gauge condition $\vec{\nabla} \cdot \Psi = 0$ by performing a diffeomorphism; then the equation for $F$ simplifies and reads
\[
\Delta^{(3)F} = 2\partial_j\Psi_1\partial_j\Psi_1 + \frac{1}{4}(\nabla\Psi)^2.
\] (2.39)

This family of supersymmetric solutions to minimal supergravity in five dimensions was already obtained in [4], as part of the general supersymmetric configuration in the null class, and generalized in the non-supersymmetric case to arbitrary dimension in [25].

3. Inclusion of vector multiplets

The general supersymmetric solution for $N = 2, D = 5$ supergravity, coupled to $n - 1$ abelian vector multiplets was obtained in [15], both for the ungauged and the gauged theories. The action for these supergravity theories is given by [26, 27]
\[
S = -\frac{1}{16\pi G} \int \left[ (R - 2\kappa^2 V) \star 1 + Q_{IJ} F^I \wedge \star F^J 
- Q_{IJ} dX^I \wedge \star dX^J + \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \right].
\] (3.1)

Here the indices $I, J, K, \ldots$ label the vector multiplets, $A^I$ are the $n U(1)$ gauge fields and $F^I$ denote the corresponding field strengths. The graviphoton of the theory is given by the linear combination of the gauge fields $A = V_I A^I$, determined by the constant coefficients $V_I$. The scalars $X^I$ are constrained by the relation
\[
V = \frac{1}{6} C_{IJK} X^I X^J X^K = 1,
\] (3.2)
so that one has $n - 1$ independent scalars. The homogeneous cubic polynomial $V$ defines a 'very special geometry' [28]. The coefficients $C_{IJK}$ are constants that are symmetric in $I, J, K$, and can be used to define the fields $X_I$ with lower indices,
\[
X_I = \frac{1}{3} C_{IJK} X^J X^K,
\] (3.3)
in terms of which the scalar constraint becomes $X_I X^I = 1$. The metric $Q_{IJ}$ depends on the $X^I$ via
\[
Q_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{3} C_{IJK} X^K.
\] (3.4)

We will assume it is invertible, and denote its inverse by $Q^{IJ}$. The following relations are then shown to hold,
\[
Q_{IJ} X^I = \frac{3}{2} X_J, \quad Q_{IJ} \partial_\mu X^I = -\frac{3}{2} \partial_\mu X_J.
\] (3.5)

Finally, the potential for the scalar fields is given by
\[
V = 9V_I V_J (X^I X^J - \frac{1}{2} Q^{IJ}).
\] (3.6)

For Calabi–Yau compactifications of M-theory, $V$ denotes the intersection form, $X^I$ and $X_I$ correspond to the size of the two- and four-cycles of the Calabi–Yau threefold respectively, and $C_{IJK}$ are the intersection numbers of the threefold. In this case, $n - 1$ is given by the Hodge number $h(1,1)$.

The solutions of these models in which only the photon and graviton are excited solve as well the minimal model (2.2). These are the subclass of solutions with constant scalar fields and all vector fields proportional to the graviphoton. Indeed, setting
\[
X_I = \Sigma V_I, \quad A^I = \frac{2}{3} X^I A,
\] (3.7)
we recover the minimal gauged supergravity action \((2.2)\), with the normalization \(A = (\sqrt{3} \Xi/2)A\) for the graviphoton and an effective gauge coupling constant \(\chi_{\text{min}}\) given by

\[
\chi_{\text{min}}^2 = \frac{12 \chi^2}{\Xi^2}.
\]

(3.8)

On the other hand, ungauged \(N = 2\) supergravities coupled to vector multiplets are obtained by taking the limit \(\chi \to 0\).

Of particular interest are the STU models, which represent truncations of the dimensional reduction of higher-dimensional supergravities coming from string theory. These models, in both the ungauged and gauged case, are obtained for \(n = 3\) by taking \(C_{123}\) and its permutations equal to one and otherwise zero. Furthermore one has \(V_1 = 1/3\) in these models. Then

\[
Q_{IJ} = \frac{1}{2(n+1)^2} \delta_{IJ},
\]

(3.9)

\(X_I = 1/(3X^I)\), and the scalar potential reads

\[
V = 2 \left( \frac{1}{X^1} + \frac{1}{X^2} + \frac{1}{X^3} \right).
\]

(3.10)

The minimal supergravities of the previous section can then be obtained as truncations of the STU models using the prescription \((3.7)\), for instance by taking \(\Xi = 1\), all gauge fields equal and normalized as in \((3.7)\), and constant scalar fields \(X^I = 1\). Moreover, one also needs to rescale the gauge coupling, \(\chi_{\text{min}}^2 = 12 \chi^2\).

In the following, we shall find supersymmetric gyraton solutions to these supergravity theories.

### 3.1. Ungauged supergravity coupled to vector multiplets

We first consider the ungauged model, and subsequently generalize the results to the gauged case. The general null solution in the ungauged supergravity has a geometry described by the metric \([15]\)

\[
ds^2 = H^{-1}(F \, du^2 + 2 \, du \, dv) - H^2(dx^i + a_i \, du)^2,
\]

(3.11)

and the gauge fields are of the form

\[
F^I = du \wedge Y^I + \ast_3 \hat{d}(H X^I),
\]

(3.12)

where \(Y^I(u, x^i)\) are \(n\) one-forms on the base space \(\mathbb{R}^3\). The scalars \(X^I\) are given in terms of \(n\) arbitrary \(u\)-dependent harmonic functions \(K^I(u, x^i)\) on \(\mathbb{R}^3\),

\[
X^I = H^{-1} K^I.
\]

(3.13)

The scalar constraint \((3.2)\) translates therefore into the relation

\[
H^3 = \frac{1}{6} C_{IMN} K^I K^M K^N,
\]

(3.14)

which determines the function \(H(u, x^i)\). The one-forms \(Y^I\) on \(\mathbb{R}^3\) have to satisfy the constraint

\[
X_I Y^I = \frac{1}{2} H^{-2} \ast 3 \hat{d}(H^3 u),
\]

(3.15)

the Bianchi identity

\[
\hat{d} Y^I = \ast_3 \hat{d}(\partial_u K^I),
\]

(3.16)

and the Maxwell equations

\[
C_{IJK} \left( Y^J, \nabla^K K^K + \frac{1}{2} \nabla^J Y^I K^K \right) = 0.
\]

(3.17)
Finally, the function $F(u, x^l)$ is determined by the $(uu)$ component of Einstein’s equations,
\[ \Delta^{(3)} F = -2H(\mathcal{D}\mathcal{W} - W_{ij}W^{ij}) + 2HQ_{ij}(U^l_iU^{l*}_j + H^2\mathcal{D}X^l\mathcal{D}X^l), \] (3.18)
where we have defined
\[ W_{ij} = H\partial_i(a_j) - D_i\delta_{ij}, \quad \mathcal{W} = W_{ij}, \] (3.19)
\[ U^l = Y^l + \ast[a \wedge \hat{d}(H^lX^l)], \] (3.20)
and the operator $\mathcal{D}$ acts on scalar functions $f$ as $\mathcal{D}f = \partial_a f - a^i\partial_i f$.

A plane-fronted wave is said to be a plane-fronted wave with parallel rays (pp-wave) if $du$ is covariantly constant. This condition imposes without loss of generality $H = 1$. We also make the ansatz $A^l = A^l_u du$ for the gauge fields, by analogy with the minimal case, where the supersymmetric gyratons have only this component of the gauge field.\(^8\) Then, comparing with (3.12), we deduce that $Y^l = -\hat{a}_iA^l_i du$ and that $X^l$, and therefore $K^l$, are functions of $u$ only. The Bianchi identity (3.16) is then automatically satisfied, the Maxwell equations reduce to
\[ C_{IMN}K^M\Delta^{(3)}A^N_u = 0, \] (3.21)
and the connection $a_i$ is determined by equation (3.15).

To summarize, the generic supersymmetric gyraton is obtained by choosing $n$ arbitrary functions $X^l = X^l(u)$ subject to the constraint $C_{IJK}X^lX^jX^K = 6$. Then the gauge potentials are determined by
\[ C_{IJK}X^j\Delta^{(3)}A^K_u = 0, \] (3.22)
and the connection $a_i$ by
\[ \epsilon_{ijk}\partial_ja_k = -\frac{1}{2}C_{IJK}X^jX^K\partial_iA^l_u. \] (3.23)

The equation for $F$ follows then from the $(uu)$ component of Einstein’s equations,
\[ \Delta^{(3)} F = -2(\mathcal{D}\mathcal{D}a_i - \partial_i(a_j\partial_ja_i)) + 2Q_{ij}(\partial_iA^l_u\partial_jA^l_u + \partial_iX^l\partial_jX^l). \] (3.24)

Note that, by an appropriate diffeomorphism, we can always choose coordinates such that $\nabla \cdot \hat{a} = 0$, so that (3.24) simplifies to
\[ \Delta^{(3)} F = 2\partial_i(a_j\partial_ja_i) + 2Q_{ij}(\partial_iA^l_u\partial_jA^l_u + \partial_iX^l\partial_jX^l). \] (3.25)

More progress can be done in the STU model. Recalling $n = 3$ and $C_{123} = 1$ is the only non-vanishing component of $C_{IJK}$, up to permutations, the solution is given by three arbitrary functions $X^l(u)$ subject to the constraint $X^lX^2X^3 = 1$ and three $u$-dependent harmonic functions $A^l_u(u, x^l)$ on the flat base space $\mathbb{R}^3$, $\Delta^{(3)}A^l_u = 0$. Finally, the connection $a_i$ is determined by the equation
\[ \nabla \wedge a = -\nabla \sum_{l=1}^3 (X^l)^{-1}A^l_u. \] (3.26)

Using (3.4) the equation for $F$ becomes (in the $\nabla \cdot a = 0$ gauge)
\[ \Delta^{(3)} F = 2\partial_i(a_j\partial_ja_i) + \sum_{l=1}^3 \frac{1}{(X^l)^2}[(\nabla A^l_u)^2 + (\partial_uX^l)^2]. \] (3.27)

The minimal supergravity gyraton of the previous section is then easily recovered taking $X^l = 1$ and all gauge fields $A^l_i$ equal and proportional to the graviphoton field $A$, as in equation (3.7). Then, the potential $\psi$ defined in (2.36) is $\psi = -3A^l_u$ for any $I$. With these definitions, equations (3.26) and (3.27) reduce respectively to (2.37) and (2.39), and the complete solution corresponds to a gyraton of minimal ungauged supergravity.

\(^8\) In presence of vector multiplets, this is not the most general supersymmetric solution of the gyratonic form.
3.2. Gauged supergravity coupled to vector multiplets

Let us turn now to the gauged case. The general null solution is given by the metric [15]
\[ ds^2 = H^{-1}(\mathcal{F} du^2 + 2 du dv) - H^2[(dx^1 + a_1 du)^2 + (S dx^2 + S^{-1}a_2 du)^2 + (S dx^3 + S^{-1}a_3 du)^2], \] (3.28)
and the gauge fields are of the form
\[ F^I = du \wedge Y^I + *3[\hat{\partial}(HX^I) - 3\chi H^2Q^{IJ}V_I dx^J], \] (3.29)
where \( Y^I(u, x^j) \) are \( n \) one-forms on the three-dimensional base space
\[ ds_3^2 = (dx^1)^2 + S^2(1(dx^2)^2 + (dx^3)^2), \] (3.30)
whose metric tensor will be denoted by \( h_{ij} \). Covariant derivatives with respect to this metric are indicated by \( \hat{\partial} \), and external derivatives by \( \hat{d} \) and from now on \( *3 \) denotes the Hodge dual on the curved base space. The corresponding gauge potentials \( A^I \) have to satisfy the constraint
\[ V_I A^I = V_I A^I_{\ u} du + \frac{1}{3\chi} S^{-1}(\partial_2 S \, dx^3 - \partial_3 S \, dx^2). \] (3.31)
The functions \( S \) and \( H \) are linked by
\[ \partial_1 S = -3\chi V_I X^I SH, \] (3.32)
\[ \Box \ln S = -9\chi^2 H^2(Q^{IJ} - 2X^I X^J)V_I V_J = 2\chi^2 H^2 V. \] (3.33)
Here, \( \Box \) is the Laplace operator on the three-dimensional base space \( ds_3^2 \). The one-forms \( Y^I \) have to satisfy the constraint
\[ X_I Y^I = \frac{1}{2} H^{-2} *3 \hat{\partial}(H^3 a) - 2\chi HV_I A^I_{\ u} \, dx^1, \] (3.34)
which yield an equation for \( a^I \), while the Bianchi identity imposes the conditions
\[ \hat{d} Y^I = \partial_a[*3(\hat{\partial}(HX^I) - 3\chi H^2Q^{IJ}V_J dx^J)], \] (3.35)
\[ \hat{d} *3 \hat{\partial}(HX^I) = 3\chi S^{-2}\partial_1(S^2 H^2Q^{IJ} V_J d\text{Vol}_3), \] (3.36)
and the Maxwell equations read
\[ \hat{d}(HQ_{IJ} *3 Y^J) = 3\hat{\partial}(H^3 a) \wedge [\frac{1}{2} \hat{\partial}(H^{-1}X_I) + \chi V_I \, dx^1] \]
\[ + \frac{1}{2} C_{JK} *3 Y^J \wedge [\hat{\partial}(HX^K) - 3\chi H^2 Q^{KP} V_P \, dx^J]. \] (3.37)
Finally, the function \( \mathcal{F}(u, x^j) \) is determined by the \((uu)\) component of Einstein’s equations
\[ \Box \mathcal{F} = -2H(\hat{\mathcal{D}} \hat{\mathcal{W}} - W_{ij}W^{ij}) + 2HQ_{IJ}(U^I_{\ u}U^{J}_{\ u} + H^2DX^I DX^J), \] (3.38)
where we have defined
\[ W_{ij} = H\hat{\mathcal{V}}_{(a_j)} - \langle \mathcal{D}H \rangle h_{ij} - \frac{1}{2} H \partial_a h_{ij}, \hspace{1cm} \hat{\mathcal{W}} = W_{ij}, \] (3.39)
\[ U^I = Y^I + *3[a \wedge \hat{\partial}(HX^I) - 3\chi H^2 Q^{IJ} V_J \, dx^J)], \] (3.40)
and the operator \( \mathcal{D} \) acts on scalar functions \( f \) as \( \mathcal{D} f = \partial_a f - a^I \partial_I f \).
To obtain a supersymmetric solution with metric of the gyratonic form in presence of vector multiplets, we make the following ansatz for the functions \( H \) and \( S \),
\[ H = -\frac{l}{2\chi^2}, \hspace{1cm} S = \left( \frac{2\chi}{l} \right)^{3/2}, \] (3.41)
in analogy with what we found in the minimal case. Then, equations (3.32) and (3.33) are satisfied if we impose the constraints
\[ V_t X^I = \frac{1}{\chi^2}, \quad Q^{IJ} V_I V_J = \frac{2}{3\chi^2} \]
onumber (3.42)
onumber
on the scalar fields. It immediately follows that the scalar potential is constant with value
\[ V = \frac{6}{\chi^2}. \]
(3.43)
onumber
so it acts as an effective cosmological constant. Moreover, the scalar equation (3.36) can be rewritten as the vanishing of the external derivative of a two-form, and is solved if there exist \( n \) one-forms \( \psi^I(u, x^t) \) on the three-dimensional base space such that
\[ \ast_3 [\hat{d}(H X^I) - 3\chi H^2 Q^{IJ} V_J \, dx^1] = \hat{d}\psi^I(u, x^t). \]
(3.44)
onumber
Comparing with equation (3.29), we see that
\[ F^I = du \wedge Y^I + \hat{d}\psi^I, \]
onumber
while the Bianchi identity (3.35) implies the existence of functions \( \lambda^I(u, x^t) \) such that \( Y^I = \partial_u \psi^I - \hat{d}\lambda^I \). Plugging back in (3.29) one discovers then that the newly introduced quantities are the components of the gauge fields,
\[ A^I u = \lambda^I, \quad A^I_i \, dx^i = \psi^I. \]
(3.45)
onumber
Finally, since there is no dependence on \( x^\alpha \) in \( S \), equation (3.31) implies the vanishing of the spatial components of the graviphoton,
\[ V_I \psi^I = 0. \]
(3.46)
onumber
The general solution can then be found by solving Maxwell’s equation (3.37) for \( \lambda^I \) and \( \psi^I \), equation (3.34) for \( a_i \) and finally (3.38) for \( F \). Observing that in the minimal supergravities the supersymmetric solutions have \( A_i = 0 \), we make the simplifying ansatz \( \psi^I = 0 \). Then, equation (3.44) states the independence of the scalar fields on the variables \( x^\alpha, X^I(u, x^t) \), and its \( x^1 \) component imposes
\[ \partial_1 X_I = -2\chi H \left( V_I - \frac{H'}{2\chi H^2} X_I \right), \]
(3.47)
onumber
with \( H' = \partial_1 H \). This can be integrated to give
\[ X_I = \chi V_I + \frac{\kappa_I(u)}{x^1}, \]
(3.48)
onumber
where \( \kappa_I(u) \) are \( u \)-dependent integration constants. Contracting with \( X^I \) and using the constraints (3.42) leads to \( X^I X^I = 0 \), whereas contraction with \( Q^{IJ} V_J \) implies \( Q^{IJ} \kappa_j V_J = 0 \). Using these identities and contracting finally with \( Q^{IJ} \kappa_J \) one gets \( Q^{IJ} \kappa_I \kappa_J = 0 \) and thus \( \kappa_I = 0 \), because \( Q_{IJ} \) is positive definite. The scalars \( X_I \) are thus constant. The Maxwell equations (3.37) imply that the gauge potentials \( \lambda^I = A^I u \) satisfy (2.4), where the coordinate \( z \) is defined by (2.22). Note that this does not mean that the vector fields \( A^I \) can be written as in (3.7), so we are not necessarily in the minimal supergravity theory.

A more interesting solution with nonconstant scalar fields can be found for the STU model. To this end we have to relax the ansatz (3.41) and allow for an arbitrary dependence of the functions \( H, S \) and \( X^I \) on the coordinates \( u \) and \( x^t \) while keeping them independent of \( x^\alpha \). Then equation (3.36) can be integrated once,
\[ \partial_1 \xi^I - 2\chi \left( \frac{\xi^I}{\xi^t} \right)^2 = S^{-2} s^I, \]
(3.49)
where we have defined for convenience \( \xi^I(u, x^t) = H X^I \) and the functions \( s^I(u) \) are arbitrary integration constants. It follows from (3.44) that
\[ \hat{d}\psi^I = s^I(u) \, dx^2 \wedge dx^3, \]
(3.50)
and therefore $s' (u)$ represent the magnetic fields in the transverse space. It is however difficult to solve the equations in this case, and we shall take $s' (u) = 0$ for simplicity. Then, equation (3.49) can be integrated,

$$\xi^I (u, x^1) = \frac{1}{c^I (u) - 2 \chi x^1},$$

(3.51)

where the arbitrary functions $c^I (u)$ are integration constants. Integration of (3.32) yields $S(u, x^1),$

$$S(u, x^1) = \alpha (u) \prod_I \sqrt{c^I (u) - 2 \chi x^1},$$

(3.52)

and the condition $\xi^I \xi^2 \xi^3 = H^3$ gives for $H(u, x^1)$

$$H(u, x^1) = \prod_I \frac{1}{(c^I (u) - 2 \chi x^1)^{1/3}};$$

(3.53)

so that the scalar fields take the form

$$X^I (u, x^1) = \prod_J \frac{(c^J (u) - 2 \chi x^1)^{1/3}}{c^I (u) - 2 \chi x^1}.$$  

(3.54)

Equation (3.33) is then satisfied without further restrictions. Then, Maxwell’s equations (3.37) read

$$\partial_2 A^I_\alpha - \frac{4 \chi}{c^I (u) - 2 \chi x^1} \partial_1 A^I_\alpha + S^2 \Delta^{(2)} A^I_\alpha = 0,$$

(3.55)

and the functions $\Psi_i = H^2 a_i$ are determined by solving the system

$$\partial_2 \Psi_3 - \partial_3 \Psi_2 = -\alpha^2 (u) \sum_I [(c^I - 2 \chi x^1) \partial_1 A^I_\alpha - 2 \chi A^I_\alpha],$$

(3.56)

$$\partial_1 \Psi_3 - \partial_3 \Psi_1 = -\frac{\alpha^2}{S^2} \sum_I (c^I - 2 \chi x^1) \epsilon_{\alpha \beta} \partial_\beta \sum_J [(b^J + a^J (c^J - 2 \chi x^1))].$$

(3.57)

Finally, the function $\mathcal{F}$ is obtained by solving the Siklos equation (3.38).

A nontrivial solution of (3.55) is given by

$$A^I_\alpha = \frac{b^I (u, x^\alpha)}{c^I (u) - 2 \chi x^1} + a^I (u, x^\alpha),$$

(3.58)

where the integration constants $a^I (u, x^\alpha)$ and $b^I (u, x^\alpha)$ are arbitrary functions of $u$ and harmonic functions in the two-dimensional flat space $(x^2, x^3)$,

$$\Delta^{(2)} a^I = 0, \quad \Delta^{(2)} b^I = 0.$$  

(3.59)

Then the system of equations for $\Psi_i$ reduces to

$$\partial_2 \Psi_3 - \partial_3 \Psi_2 = 2 \chi \alpha^2 (u) \sum_I a^I (u, x^\alpha),$$

(3.60)

$$\partial_1 \Psi_3 - \partial_3 \Psi_1 = -\frac{1}{\prod_I (c^I - 2 \chi x^1)} \epsilon_{\alpha \beta} \partial_\beta \sum_I [b^I + a^I (c^I - 2 \chi x^1)],$$

(3.61)

that can be readily integrated in the $\Psi_1 = 0$ gauge (which can always be chosen by a suitable diffeomorphism [15]) to give

$$\Psi_\alpha = -\epsilon_{\alpha \beta} \partial_\beta \int \frac{b^I + a^I (c^I - 2 \chi x^1)}{\prod_I (c^I - 2 \chi x^1)} \ dx^1 + f_\alpha (u, x^\beta),$$

(3.62)
where the functions $f_\alpha(u, x^\beta)$ are solutions of the equation
\[ \partial_2 f_3 - \partial_3 f_2 = 2\chi \alpha^2(u) \sum_i a^i(u, x^a). \]  
(3.63)

The explicit evaluation of the integral (3.62) depends on the number of functions $c^I(u, x^a)$ that coincide, and we have therefore three possible cases. If all functions $c^I$ are distinct, we have
\[ \Psi_a = \frac{1}{2\chi} \sum_I b^I \ln(c^I - 2\chi x^I) - \frac{1}{2} \sum_I c^I \ln(c^I - c^I_0). \]

If two of them coincide, say $c^1 = c^2 = c$ and $c^3 = \tilde{c}$ with $\tilde{c} \neq c$, then
\[ \Psi_a = -\frac{1}{2\chi} \sum_I b^I \left[ c^I - c^I_0 \right] \ln\left(\frac{c^I - 2\chi x^I}{\tilde{c} - 2\chi x^I} + \left(a^3 - \sum_I b^I\right) \ln\left(\frac{c^I - 2\chi x^I}{\tilde{c} - 2\chi x^I}\right) \right] + f_a. \]

Finally, if the three functions coincide, $c^1 = c^2 = c^3 = c$, we obtain
\[ \Psi_a = -\frac{1}{4\chi} \sum_I b^I \left[ c^I - c^I_0 - 2a^I \right] + f_a. \]

We finally note that in the special case $A^I = 0$, $\Psi_t = 0$, $c^I(u) = c^I = \text{const}$, $\alpha(u) = 1$, the metric reduces to
\[ ds^2 = H^{-1}(F du^2 + 2 du^2 dv + (dx^2)^2 + (dx^3)^2) - H^2(dx^1)^2, \]  
(3.64)

where $H$ is given by (3.53) with constant $c^I$, and $F$ obeys
\[ \partial_t (H^{-1}\partial_t F) + \Delta^{(2)} F = 0. \]
(3.65)

The geometry (3.64) represents a wave propagating on a domain wall\(^9\), whereas the more general solutions with $\Psi_t \neq 0$ presented above describe gyratons on a domain wall background.

### 4. Holographic stress tensor

In this section we compute the holographic stress tensor [30] associated with the solutions (2.1) of minimal gauged supergravity. Let us consider the hypersurface $z = \text{const}$, with unit normal vector $u^a = (0, 0, 0, z/l)$, and induced metric
\[ ds^2 = -\frac{l^2}{z^2}(-2 du^2 + (\Phi - \Psi_z^2) du^2 + 2(\Psi_t dx + \Psi_z dy) du + dx^2 + dy^2). \]  
(4.1)

It is convenient to introduce a new function $\xi = \Phi - \Psi_z^2$, since $\Phi$ enters the induced metric (and the extrinsic curvature, as we will shortly see) only through this combination. Then, the extrinsic curvature $K_{ab} = \sigma_a \sigma_b \nabla_c u_d$ of the $z = \text{const}$ hypersurface is\(^10\)
\[ K_{ab} = \begin{pmatrix} \frac{l}{z^2}(z\xi_z - 2\xi) & \frac{l}{z^2}(z\Psi_{t,z} - 2\Psi_t) & \frac{l}{z^2}(z\Psi_{y,z} - 2\Psi_y) \\ \frac{l}{z^2}(z\Psi_{x,z} - 2\Psi_x) & 0 & 0 \\ \frac{l}{z^2}(z\Psi_{y,z} - 2\Psi_y) & 0 & -\frac{l}{z^2} & 0 \\ \frac{l}{z^2}(z\Psi_{x,z} - 2\Psi_x) & 0 & 0 & -\frac{l}{z^2} \end{pmatrix}. \]  
(4.2)

\(^9\) For $F = 0$, (3.64) reduces to a subclass of the domain wall solutions found in [29].

\(^10\) Early Latin indices $a, b, \ldots$ are used only in this section; $x^a$ are the coordinates $u, v, x, y$. 

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Supersymmetric gyratons in five dimensions
Its trace is $K = K^a_a = 4$, and the only non-vanishing components of the Einstein tensor of the $z = \text{const}$ hypersurface are

$$G_{uu} = -\frac{1}{2} \Delta(2) \zeta + \Psi_{z,uu} + \Psi_{y,uu} + \frac{1}{2} (\Omega_{xy})^2, \quad G_{ux} = \frac{1}{2} \Omega_{xy,y}, \quad G_{uy} = \frac{1}{2} \Omega_{yx,x}.$$  \hfill (4.3)

The holographic energy–momentum tensor is given by \cite{30}

$$T_{ab} = \frac{1}{8 \pi G_5} \left[ (K_{ab} - K \sigma_{ab}) + \frac{3}{l} \sigma_{ab} + \frac{1}{2} G_{ab} \right].$$  \hfill (4.4)

Using (4.2) and (4.3), we find that the only non-vanishing components read

$$T_{uu} = \frac{l}{16 \pi G_5} \left[ \frac{1}{z^2} \left( \zeta, \zeta \right) + \frac{1}{2} \Delta(2) \zeta + \Psi_{x,uu} + \Psi_{y,uu} + \frac{1}{2} (\Omega_{xy})^2 \right],$$

$$T_{ux} = \frac{l}{16 \pi G_5} \left[ \frac{1}{z} \left( \Psi_{x, \zeta} + \frac{1}{2} \Omega_{xy,y} \right) \right],$$

$$T_{uy} = \frac{l}{16 \pi G_5} \left[ \frac{1}{z} \left( \Psi_{y, \zeta} + \frac{1}{2} \Omega_{yx,x} \right) \right].$$  \hfill (4.5)

The stress tensor $\tilde{T}_{ab}$ of the associated conformal field theory is then obtained by

$$\tilde{T}_{ab} = \lim_{z \to 0} l^2 T_{ab},$$  \hfill (4.6)

and its components are

$$\tilde{T}_{uu} = \frac{N^2}{8 \pi^2} \lim_{z \to 0} \frac{1}{z^2} \left[ \frac{1}{z^2} \left( \zeta, \zeta \right) + \frac{1}{2} \Delta(2) \zeta + \Psi_{x,uu} + \Psi_{y,uu} + \frac{1}{2} (\Omega_{xy})^2 \right],$$

$$\tilde{T}_{ux} = \frac{N^2}{8 \pi^2} \lim_{z \to 0} \frac{1}{z} \left( \Psi_{x, \zeta} + \frac{1}{2} \Omega_{xy,y} \right),$$

$$\tilde{T}_{uy} = \frac{N^2}{8 \pi^2} \lim_{z \to 0} \frac{1}{z} \left( \Psi_{y, \zeta} + \frac{1}{2} \Omega_{yx,x} \right),$$  \hfill (4.7)

where we used the AdS/CFT dictionary $l^3 / G_5 = 2N^2 / \pi$. Note that the energy–momentum tensor is traceless, $\tilde{T} = \sigma_{ab} \tilde{T}_{ab} = 0$, so there is no conformal anomaly in the dual CFT.

Let us now consider the effect of a Siklos–Virasoro transformation (2.26) on the components of the holographic stress tensor. To this end, we choose the gauge $\Psi_z = 0$, which can always be achieved by a gauge transformation (2.28). The transformations that preserve this gauge obey $\partial_z \lambda = 0$. It is then straightforward to show that under (2.26)

$$T_{uu} \to \tilde{T}_{uu} = \frac{1}{\lambda} T_{uu} + \frac{l}{32 \pi G_5} \frac{x^\gamma}{x^\alpha} \left( \partial_z - \frac{1}{z} \right) x^\alpha \Omega_{uu},$$

$$T_{ux} \to \tilde{T}_{ux} = \frac{1}{\lambda^2} T_{ux}, \quad T_{uy} \to \tilde{T}_{uy} = \frac{1}{\lambda^{2/3}} T_{uy}.$$  \hfill (4.10)

Contrary to the wave profile $\Phi$ (cf (2.27)), the component $T_{uu}$ transforms without anomaly term proportional to the Schwarzian derivative. The reason for this is that the Schwarzian derivative coming from the term $z^{-1} \partial_z \Phi$ cancels that appearing in $-\Delta(2) \tilde{\Phi} / 2$. The stress tensor proposed in \cite{31} has only the former term and transforms therefore with anomaly. As the second piece (which stems from the Einstein tensor, i.e., from the counterterms that are necessary to cancel divergences) is absent in \cite{31}, their stress tensor is divergent in the $z \to 0$ limit.
In order to obtain a more explicit expression for the holographic energy–momentum tensor, we use the Fefferman–Graham expansion [32],
\[
\mathcal{g}_{\mu\nu} = \frac{l^2}{z^2} \mathcal{g}_{\mu\nu}^{(0)} + \frac{z^2}{l^2} \mathcal{g}_{\mu\nu}^{(4)} + \frac{z^2}{l^2} \ln \frac{z}{l} \mathcal{g}_{\mu\nu} + \cdots, \tag{4.11}
\]
which implies\(^{11}\)
\[
\Phi = \Phi_0 + \frac{z^2}{l^2} \Phi_2 + \frac{z^4}{l^4} \Phi_4 + \frac{z^4}{l^4} \ln \frac{z}{l} \Phi + \cdots,
\]
\[
\Psi_a = \Psi_a^{(0)} + \frac{z^2}{l^2} \Psi_a^{(2)} + \frac{z^4}{l^4} \Psi_a^{(4)} + \frac{z^4}{l^4} \ln \frac{z}{l} \Psi_a + \cdots,
\]
\[
\Omega_{xy} = \Omega_{xy}^{(0)} + \frac{z^2}{l^2} \Omega_{xy}^{(2)} + \frac{z^4}{l^4} \Omega_{xy}^{(4)} + \frac{z^4}{l^4} \ln \frac{z}{l} \Omega_{xy} + \cdots,
\]
where all coefficients are functions of \(u, x, y\), and we defined
\[
\Omega_{xy} = \partial_x \Psi_y - \partial_y \Psi_x, \quad \Omega_{xy}^{(2n)} = \partial_x \Psi_y^{(2n)} - \partial_y \Psi_x^{(2n)}, \quad n = 0, 1, \ldots.
\]
Equation (2.6) yields then
\[
\Psi_{a,a} = \Psi_{a,a}^{(2)} = \Psi_{a,a}^{(4)} = 0,
\]
and
\[
\Omega_{xy,\alpha}^{(0)} = \frac{4}{l^2} \epsilon_{\alpha\beta} \Omega_{xy}^{(2)} \Psi_{\beta}, \quad \Omega_{xy,\alpha}^{(2)} = -\frac{4}{l^2} \epsilon_{\alpha\beta} \Omega_{xy}^{(2)} \Psi_{\beta}.
\]

The Fefferman–Graham expansion of the gauge field \(A_u\) is obtained from (2.25),\(^{12}\) and reads
\[
A_u = \frac{l}{4\sqrt{3}} \Omega_{xy}^{(0)} + \frac{z^2}{l^2} A_u^{(2)} - \frac{l}{2\sqrt{3}} \frac{z^2}{l^2} \ln \frac{z}{l} \Omega_{xy}^{(2)}
- \frac{l}{4\sqrt{3}} \frac{z^4}{l^2} \left( \Omega_{xy}^{(4)} - \frac{1}{2} \Omega_{xy}^{(2)} \right) - \frac{l}{4\sqrt{3}} \frac{z^4}{l^2} \ln \frac{z}{l} \Omega_{xy} + \cdots,
\]
where
\[
A_u^{(2)} = \frac{1}{2\sqrt{3} l} \epsilon_{\alpha\beta} (4 \Psi_{\beta}^{(4)} + \Psi_{\beta}).
\]

Finally, the generalized Siklos equation (2.8) yields
\[
\Delta^{(2)} \Phi_0 - \left( \Omega_{xy}^{(0)} \right)^2 - 2 \partial_u \Psi_{a,a}^{(0)} - \frac{4}{l^2} \Phi_2 = 0.
\]
Plugging these results into equations (4.7), (4.8) and (4.9), and subtracting the logarithmically divergent terms that cannot be removed by adding local counterterms [33], one finally obtains the holographic stress tensor
\[
\tilde{T}_{uu} = \frac{N^2}{8\pi l^2} \left[ \frac{4}{l^2} \Phi_4 + \frac{1}{l^2} \Phi - \frac{1}{2} \Delta^{(2)} \Phi_2 + \Omega_{xy}^{(0)} \Omega_{xy}^{(2)} \right], \tag{4.12}
\]
\[
\tilde{T}_{u\alpha} = \frac{N^2}{16\pi l^2} \epsilon_{\alpha\beta} \partial_\beta \left[ \Omega_{xy}^{(2)} - \frac{4}{l} A_u^{(2)} \right]. \tag{4.13}
\]

Using the CFT metric
\[
\text{d}s^2 = -2 \text{d}u \text{d}v + \Phi_0 \text{d}u^2 + 2(\Psi_x^{(0)} \text{d}x + \Psi_y^{(0)} \text{d}y) \text{d}u + \text{d}x^2 + \text{d}y^2, \tag{4.14}
\]
\(^{11}\)We use the gauge \(\Psi_t = 0\).
\(^{12}\)Here we are interested in the supersymmetric case only.
one easily checks that this energy–momentum tensor is covariantly conserved, \( \nabla_a T^a_b = 0 \). Note that (4.14) describes itself a gyraton (in flat space), unless the functions \( \Phi_0 \) and \( \Psi_0(0) \) vanish. (For \( \Psi_0(0) = 0 \) and \( \Phi_0 \neq 0 \) the CFT would live on a pp-wave background.)

For the Kaigorodov spacetime, which is given by \( \Psi_1(0) = 0 \) and \( \Phi_1 = z^4 \Phi_4 / l^4 \) with constant \( \Phi_4 \), (4.12) reduces to the result found in [21], namely that the dual CFT has constant null momentum density. We see that in our case one has in addition a nonvanishing angular momentum density. It would be interesting to explore the AdS/CFT interpretation of our gyratons in more detail.

5. Final remarks

In this paper we obtained gyraton solutions of minimal gauged and ungauged supergravity in five dimensions and analysed under which conditions they preserve some supersymmetry. It was shown that the gyratons on AdS enjoy a Siklos–Virasoro reparametrization invariance, and that the holographic stress tensor associated with these solutions transforms without anomaly under these transformations, contrary to claims that appeared previously in the literature [31].

We furthermore obtained supersymmetric gyratons in both gauged and ungauged \( \mathcal{N} = 2 \) five-dimensional supergravity coupled to an arbitrary number of vector multiplets. It would be interesting to study the asymptotically AdS gyratons from the point of view of the AdS/CFT correspondence. In this context it has been conjectured that gravity on the Kaigorodov spacetime, which represents a special case of a pp-wave in AdS, is dual to a conformal field theory in the infinite momentum frame with constant momentum density [24].

Another interesting point would be to look for gyraton solutions in ten- and eleven-dimensional supergravity and to see how much supersymmetry they preserve. Since string theory on the maximally supersymmetric IIB plane wave background is exactly quantizable [34], one could ask the question if this is still the case for a gyraton background, which represents a generalization of the pp-wave. Work in this direction is in progress [35].

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