The accuracy comparison of the RK-4 and RK-5 method of SEIR model for tuberculosis cases in South Sulawesi

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Abstract. This research discusses about the accuracy of the 4th and 5th order Runge-Kutta methods (RK-4 and RK-5) as a numerical solution in the Susceptible-Exposed-Infected-Recovered (SEIR) mathematical model of Tuberculosis (TB) transmission in South Sulawesi. The data used are secondary data on the number of TB patients in South Sulawesi sourced from the South Sulawesi Health Office. The research begins by further examining the SEIR model on TB transmission, then looking for general solutions of the SEIR model with the 4th-order and 5th-order Runge-Kutta methods, parameter determination, simulation and results analysis. In this research, the movement graph is obtained from the SEIR model with real data. After analysing the numerical simulation, the accuracy of the 4th-order and 5th-order Runge-Kutta methods is compared, then the two methods are compared to observe a more accurate method. The result of this research shown that the simulation of SEIR model is accurate to predict TB cases in South Sulawesi. The Runge-Kutta method can used to observe trends in the spread of TB in South Sulawesi. The numerical simulation shown that the 5th-order more accurate than the 4th-order Runge-Kutta methods. The results of the modelling are simulated using Maple can predict the number of TB cases which can be used by the government as a consideration to prevent the spread of TB in South Sulawesi.

1. Introduction
The World Health Organization (WHO) 2009 states 9 million new Tuberculosis (TB) patients and 3 million deaths from TB worldwide, 95% of TB cases and 98% of deaths from TB occur in developing countries. Without handling and control over a period of 20 years, TB will kill 35 million people [1]. Seeing the condition, the WHO stated that TB became a global emergency since 1993 [2].

Based on data from South Sulawesi Health Minstry in 2011, TB disease number has reached 8,939 significant cases than the previous year, 7,783 cases. Takalar Regency ranks first number of cases with growth above 109%, following Pare-Pare 79%, Pinrang 75%, Makassar 70% and lowest Luwu 33% and Jeneponto 36%. The number of sufferers is due to various factors such as residential environment. In addition, the lack of home lighting makes the disease easy to spread. A TB sufferer is able to transmit to 10 people. Another factor is behavior. People with HIV/AIDS are very risky to Tuberculosis. Contribution of unhealthy behavior reaches 5-10% annually and the occurrence of nutritional malls [3]

Some researchers have made the model of infectious diseases [4-15], a TB-transmission model was conducted by [10,15], but had not yet searched for the numerical solution of the model, then the numerical solution using the perturbated method of the ascetic was carried out by [11, 13, 14], but did not find a model solution using Runge Kutta. Thus, this study will determine the numerical solution of
the SEIR model on transmission of TB using Runge Kutta method of order 4 and order 5 (RK-4 and RK-5). Parameter determination, analysis and simulation using secondary data number of TB cases in South Sulawesi. The result of the second numerical solution Runge Kutta method is then compared to see a more accurate method of controlling and handling TB transmission in South Sulawesi.

2. Method

This research is an applied study. SEIR mathematical model [15] was developed by conducting analysis and model simulation using Maple. The data used is the number of TB cases in South Sulawesi. The numerical solution of the SEIR model was obtained using Runge Kutta's Order 4 and order 5 methods. Furthermore, the numerical solutions of the SEIR model with Runge Kutta are compared with real data on the number of TB cases in South Sulawesi using Excel. Then both methods are compared to find out the RK-4 or RK-5 method, which is more accurate in predicting the number of TB cases in South Sulawesi.

3. Result and Discussion

3.1. SEIR Model for TB Transmission

The SEIR model that we use in this study is the SEIR model in tuberculosis transmission that has been previously studied by [15] in Figure 1.

![Figure 1](image)

Figure 1. Human population diagram of the SEIR model of TB transmission

Based on the Figure 1, the human population divided by four compartment are Suspend, Exposed, Infected, and Recovered and the SEIR mathematical model of the TB transmission can be interpreted as follow:

\[
\begin{aligned}
\frac{dS_h}{dt} & = \mu_h N_h - (\sigma_h + \beta_h + \mu_h)S_h \\
\frac{dE_h}{dt} & = \sigma_h S_h - \gamma \phi_h I_h E_h - \phi_h E_h - \mu_h E_h \\
\frac{dI_h}{dt} & = \beta_h S_h + \phi_h E_h - (\mu_h + \delta_h)I_h \\
\frac{dR_h}{dt} & = \gamma \phi_h I_h E_h - (\mu_h + \varphi_h)I_i \\
\end{aligned}
\]

The initial values of the variable and parameter used in the SEIR model for TB disease in South Sulawesi presented in Table 1.
Table 1. Initial Values of SEIR Model for TBC transmission in South Sulawesi

| Variable | Estimated Value | Source | Parameter | Estimated Value | Source |
|----------|-----------------|--------|-----------|-----------------|--------|
| $N_h$    | 8034776         | [13]   | $\mu_h$   | 3.5x10^{-4}     | [13]   |
| $S_h$    | 15073           | [13]   | $\sigma_h$| 3.1 x10^{-3}    | [13]   |
| $E_h$    | 8060796         | [13]   | $\beta_h$ | 3.267 x10^{-3}  | [13]   |
| $I_h$    | 7087            | [13]   | $\phi_h$  | 8.8 x10^{-2}    | [13]   |
| $I_l$    | 1417            | [13]   | $\gamma\phi_h I_h$ | 5.08 x10^{-5} | [13]   |
| $R_h$    | 3771            | [13]   | $\delta_h$ | 6.265 x10^{-4}  | [13]   |

Based on the data and parameters in Table 1, the simulation model using Maple found the prediction of the SEIR model for TB transmission. Result of the simulation model are given by Table 2.

Table 2. Rate of case of transmission TBC based on SEIR model

| t  | $S_h$ | $E_h$ | $I_h$ | $I_l$ | $R_h$ |
|----|-------|-------|-------|-------|-------|
| 0  | 0.00187597 | 0.006772654 | 0.000705682 | 0.000176358 | 0.00046934 |
| 1  | 0.002212239 | 0.006205958 | 0.001276469 | 0.000175686 | 0.00047547 |
| 2  | 0.002546258 | 0.005688153 | 0.001797053 | 0.000175045 | 0.00048501 |
| 3  | 0.002878084 | 0.005215108 | 0.002271856 | 0.000174429 | 0.00049760 |
| 4  | 0.003207602 | 0.004783027 | 0.002704945 | 0.000173830 | 0.00051317 |
| 5  | 0.003534957 | 0.004388443 | 0.003100026 | 0.000173242 | 0.00053125 |
| 6  | 0.003860121 | 0.004028193 | 0.003460479 | 0.000172662 | 0.00055169 |
| 7  | 0.004183109 | 0.003699368 | 0.003869411 | 0.000172087 | 0.00057428 |
| 8  | 0.004503934 | 0.003399296 | 0.004086669 | 0.000171515 | 0.00059883 |
| 9  | 0.004822612 | 0.003125549 | 0.004363842 | 0.000170942 | 0.00062516 |
| 10 | 0.005139156 | 0.002875902 | 0.004614298 | 0.000170369 | 0.00065314 |

3.2. Numerical Solutions of SEIR Models for TB transmission in South Sulawesi

3.2.1. Numerical Solution of SEIR Models for TB in South Sulawesi by RK-4
The standard formula of the RK-4 method as in Equation (6).

\[ y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \]  \hspace{1cm} (6)

with
\[ k_1 = hf(x, y) \]
\[ k_2 = hf(x + \frac{1}{2}h, y + \frac{1}{2}k_1) \]
\[ k_3 = hf(x + \frac{1}{2}h, y + \frac{1}{2}k_2) \]
\[ k_4 = hf(x + h, y + k_3) \]

(7)

Then we have numerical solution of SEIR model by RK-4 shown by Table 3

| T  | S_h | E_h | I_h | I_i | R_h |
|----|-----|-----|-----|-----|-----|
| 0  | 15073 | 54417 | 5670 | 1417 | 3771 |
| 1  | 19178 | 50596 | 10258 | 1412 | 3820 |
| 2  | 23283 | 46776 | 14845 | 1406 | 3870 |
| 3  | 27387 | 42955 | 19433 | 1401 | 3919 |
| 4  | 31492 | 39135 | 24021 | 1395 | 3968 |
| 5  | 35597 | 35314 | 28608 | 1390 | 4017 |
| 6  | 39702 | 31494 | 33196 | 1385 | 4067 |
| 7  | 43807 | 27673 | 37783 | 1379 | 4116 |
| 8  | 47911 | 23853 | 42371 | 1374 | 4165 |
| 9  | 52016 | 20033 | 46959 | 1368 | 4215 |
| 10 | 56121 | 16212 | 51546 | 1363 | 4264 |

3.2.2. Numerical Solution of SEIR model for TB in South Sulawesi by RK-5

The standard formula of the RK-5 method as in Equation (8).
\[ k_1 = hf(t_i, x_i) \]
\[ k_2 = hf(t_i + \frac{h}{2}, x_i + \frac{k_1}{2}) \]
\[ k_3 = hf(t_i + \frac{3h}{4}, x_i + \frac{3k_1 + k_2}{16}) \]
\[ k_4 = hf(t_i + \frac{h}{2}, x_i + \frac{k_3}{2}) \]
\[ k_5 = hf(t_i + \frac{3h}{4}, x_i + \frac{-3k_2 + 6k_3 + 9k_4}{16}) \]
\[ k_6 = hf(t_i + h, x_i + \frac{k_1 + 4k_2 + 6k_3 - 12k_4 + 8k_5}{7}) \]

\[ x_{i+1} = x_i + \frac{1}{90} (7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \]

Then we have numerical solution of SEIR model by RK-4 shown by Table 4.

| T  | S_h | E_h | I_h | I_i | R_h |
|----|-----|-----|-----|-----|-----|
| 0  | 15073 | 54417 | 5670 | 1417 | 3771 |
| 1  | 17775 | 49864 | 10256 | 1422 | 3820 |
| 2  | 20477 | 45310 | 14842 | 1426 | 3870 |
| 3  | 23179 | 40757 | 19428 | 1431 | 3919 |
| 4  | 25880 | 36204 | 24014 | 1436 | 3968 |
3.3. Accuracy comparison of the RK-4 and RK-5 Method of the SEIR for TB in South Sulawesi

The comparison of the mean errors in Table 3 and Table 4 is presented in Table 5.

Table 5. The number comparison error of the infected of infected population of TB in South Sulawesi

| t  | Sb  | Eb  | Ib  | Ii  | Rb  |
|----|-----|-----|-----|-----|-----|
| 5  | 28582 | 31651 | 28600 | 1440 | 4017 |
| 6  | 31284 | 27098 | 33186 | 1445 | 4067 |
| 7  | 33986 | 22544 | 37772 | 1450 | 4116 |
| 8  | 36688 | 17991 | 42359 | 1455 | 4165 |
| 9  | 39390 | 13438 | 46945 | 1459 | 4215 |
| 10 | 42091 | 8885  | 51531 | 1464 | 4264 |

Based on the Table 5, the RK-5 method is more accurate than RK-4 method in calculating the number of suspect, infected of human populations, and the number of recovered populations. While the RK-4 method is more accurate than RK-5 method for calculating the number of populations that are exposed (have symptoms), and populations that are infected of infected (directly infected by bacteria).

3.4. Discussion

Research on the model of SIR and SEIR on transmission of TB by [10.15], model [10] divide the compartments into three namely suspected, infected and recovered while model [15] divide the compartments into four parts namely suspected, exposed, infected and recovered. The analysis result of SEIR Model using the Lyapunov function method. The result of this research is a model of SEIR, which simulated to get prediction of the number of TB cases in South Sulawesi that shown in Table 3 and Table 4, then obtained the numerical solution of SEIR model using method RK-4 and RK-5. The obtained result indicates that the RK-5 method is more accurate than the RK-4 method.

4. Conclusion

Based on the results and discussion can be concluded that the spread of TB can be formed in the math model SEIR, analysis and model simulation can predict the number of TB cases in South Sulawesi in the future. The model numeric solution SEIR transmission TB in South Sulawesi can be done using the RK-4 and RK-5 methods, the solution results show that the RK-5 method is more accurate than the RK-4 method, this is demonstrated in the trend of the smaller value Δξ compared to the RK-4 method.

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References

[1] Lisa P 2009 Analysis of the stability of the model of tuberculosis spread (Semarang: Universitas Diponegoro)
[2] Arfandi S 2012 Synergy Partnership in Tuberculosis Control (Makassar: Universitas Hasanudin)

[3] Hermi A 2012 TB mortality rate in South Sulawesi is high Online. http://www.aisyiyahsulselpeduli.com/2012/12/angka-penderita-tbc-di-sulsel-tinggi.html. [accessed 9 March 2014].

[4] Van Den P D, Lin W and Xingfu Z 2007 Math. Biosci. Eng. 4(2) 205

[5] Tracy A 2008 Spring Term

[6] Ashley T, Jacqueline S and John S 2010 Modeling the spread of Tuberculosis in a Closed Population Online. http://educ.jmu.edu/~strawbem/math_201/final_reports/Scotti_Takahashi_Spreadbury_Final.pdf. [accessed, 20 April 2014].

[7] Dontwi I. K, W Obeng-Denteh, E A Andam and L Obiri-Apraku 2014 Br. J. Math. Comput. Sci. 4(3) 402

[8] Idianto, Bayu and Nilamsari K 2013 Bul. ILM. Mat. Stat. Ter. 2(3) 173

[9] Queena K, Tjokorda B.O and I Made E.D 2012 e-J. Mat. 1(1) 52

[10] Syafruddin S 2015 Adv. Sci. Lett. 21(2) 137

[11] Rangkuti Y.M, Side S, and Noorani M.S.M 2014 J. Math. Fundam. Sci. 46A(1) 91

[12] Syafruddin S and M.S.M Noorani 2013 Int. J. Simul. Process Model. (IJSPM) 8(2/3) 177

[13] Side S, Pratama M.I, Ramadhan N.R, and Sanusi W 2020 Int. J. Sci. Technol. Res. 9(2) 816

[14] Ramadhan N.R., Side S, Sidjara S, Irwan, Sanusi W 2019. AIP Conf. Proc. 2192(1) 060015

[15] Side S, Mulbar U, Sidjara S, and Sanusi W 2017 AIP Conf. Proc. 1830(1)