An Improved Method of Evidence Theory Based on Membership Function

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Abstract. Aimed at the two major problems of the basic probability assignment and conflict management in classical evidence theory, a basic probability assignment (BPA) based on membership function is proposed to alleviate the dilemma caused by the above two problems, and an example is given to show the effectiveness of this method.

1. Introduction

Data fusion method is a processing technology based on the data from different sources of the same thing or target, and fuses the data information, hoping to get a more objective and fundamental understanding of the thing or target. Dempster-Shafer evidence theory (hereinafter referred to as evidence theory) is one of the classical methods, which was proposed by Arthur Pentland Dempster in 1967 and promoted by his student Glenn Shafer in 1976 [1,2].

This theory broadens the basic event space in Probability theory to the power set of basic events (frame of discernment, FOD), and establishes the Basic Probability Assignment (BPA) on FOD. In addition, evidence theory provides a Dempster combination rule, which can realize evidence fusion without prior information. These advantages have made evidence theory a certain success in information processing. It should be noted that, compared with probability theory, Dempster combination rule has completely different calculation rules and results, which brings some advantages such as relative simplicity and reliability in arithmetic, but also causes certain defects. For example, Zadeh found that when there are huge differences in the tendency of evidences and serious conflicts of evidences, the theory may get counter-intuitive results [3]. This phenomenon is called Zadeh’s paradox, and it led to questions about the rationality of D - S evidence theory, so that the later study of the theory basically revolves around how to theoretically explain the combination results and technically improve the combination methods, such as Liu, et.al. proposed a new combination rule for the order of evidence to avoid the shortcomings of the original synthesis method They tried to assign weights to different evidences according to their credibility, and required all evidences to be combined after being modified based on the most credible evidence to alleviate conflicts. [5]

The main work of this paper is to introduce the concept of membership function in fuzzy set theory into evidence theory. With the help of the concept, the multi-source data of the same object is processed to obtain different levels of basic probability distribution function, and the value of spare basic probability distribution function is given to the complete sets of events to indicate the degree of awareness and uncertainty about the event itself, relieving the evidence conflict with simplifying operation process and improving application feasibility.
2. Theoretical Description

The application space of evidence theory is called frame of discernment, which is defined as follow:

Definition 1. Frame of Discernment. \( \Omega = \{ H_1, H_2, \ldots \} \) is the complete set of all possible values of the target proposition \( H \) and \( \Omega \) is called as a frame of discernment of proposition \( H \) in the evidence theory.

The calculation basis of evidence theory is the basic probability assignment function, which is defined as follows:

Definition 2. Basic Probability Assignment Function (BPA): Assumed that \( \Omega \) is a frame of discernment of proposition \( H \), if \( m:2^{\Omega} \rightarrow [0,1] \) and \( \forall A \subset \Omega \),

\[
\sum_{A \in \Omega} m(A) = 1, m(\emptyset) = 0
\] (1)

In Eq. (2), \( m \) is a BPA in FOD \( \Omega \), which can be used to represent a piece of evidence supporting the propositions under FOD.

Unlike probability theory, which uses a single probability value to indicate the probability of an event or proposition occurring, evidence theory uses an interval to represent this possibility. The upper and lower bounds of the interval are respectively called plausibility degree and belief degree, and are defined as follows:

Definition 3. Plausibility Degree and Belief Degree: Let \( A \) is denoted as a proposition, and it’s plausibility degree is defined as

\[
Pl(A) = \sum_{B \cap A \neq \emptyset} m(B)
\] (2)

The belief degree of Proposition \( A \) is defined as

\[
Bel(A) = \sum_{B \subset A} m(B)
\] (3)

The probability interval of Proposition \( A \) is \([Bel(A), Pl(A)]\).

For the evidence from two different sources (represented by BPA \( m_1 \) and \( m_2 \), respectively), Dempster combination rule can be used for data fusion, which is defined as follows:

Definition 4. Dempster combination rule: For the two BPA \( m_1 \) and \( m_2 \), the function \( m \) obtained by their fusion is defined as

\[
m(A) = \frac{1}{1-k} \sum_{B \cap C = A} [m_1(B) m_2(C)]
\] (4)

\[
k = \sum_{B \cap C = \emptyset} [m_1(B) m_2(C)]
\] (5)

The function \( m \) can be also written as

\[
m = m_1 \oplus m_2
\] (6)

Dempster combination rule satisfies the commutative law and associative law, providing the basic criteria for data fusion operation with multiple evidences.

3. Difficulties and Solutions

In view of the two application difficulties inherent in evidence theory, this paper gives corresponding solutions.

3.1. Basic Probability Assignment (BPA)

In evidence theory, there is no specific implementation method of BPA. This paper introduces the concept of membership function in fuzzy set theory to solve this problem.
We consider that FOD is a closed interval and $\Omega = [0,100]$, and the value of evidence $X$ considering that the target proposition holds is $x$. Then for any proposition $A = [a_1, a_2]$, the support degree of evidence $X$ for proposition $A$ can be expressed in the membership function, as shown in the following formula.

$$ y = m_X(x|A) = \begin{cases} \frac{x-a_1}{a-a_1} & a_1 \leq x < a \\ \frac{a_2-x}{a_2-a} & a \leq x \leq a_2 \\ 0 & \text{otherwise} \end{cases} $$ \hfill (7)

Note that $a = (a_1 + a_2) / 2$.

This formula can be represented by the following figure1.

![Figure 1. Membership function for proposition A](image)

For any adjacent proposition $A = [a_1, a_2]$ and $B = [b_1, b_2]$, Similarly, the membership function can be used to represent the support degree of evidence $X$ for proposition $(A \cup B)$, as shown in the following formula.

$$ y = m_X(x|A \cup B) = \begin{cases} \frac{x-a}{c-a} & a \leq x < c \\ \frac{b-x}{b-c} & c \leq x \leq b \\ 0 & \text{otherwise} \end{cases} $$ \hfill (8)

Note that $a = (a_1 + a_2) / 2$, $b = (b_1 + b_2) / 2$, $c = (c_1 + c_2) / 2$.

This formula can be represented by the following figure2.
Using the above two formula, FOD $\Omega$ can be divided into $n$ disjoint propositions $A$, $B$, $C$, ..., $N$, to receive obtain the support degree of evidence $X$ for proposition $A$, $B$, $C$, ..., $N$ and $(A \cup B)$, $(B \cup C)$, ..., $(M \cup N)$. The support degree is shown in Figure 3.

3.2. Evidence Conflict
To avoid the paradox caused by the evidence conflicts in the evidence theory, we assigns a part of the basic probability to FOD $\Omega$, which shows the degree of uncertainty about all propositions. The degree of support of evidence $X$ for FOD $\Omega$ is

$$m_X(\Omega) = 1 - \sum_{A \subseteq \Omega} m(A) \quad (9)$$

4. Experiments
In this paper, we validate whether the above method can alleviate the application difficulties of evidence theory by an experiment. We assumed that FOD $\Omega$ is divided into five propositions, i.e. $A$, $B$, $C$, ..., $N$. The partition intervals of the BPA of each proposition is $[80,100]$, $[60,80]$, $[40,60]$, $[20,40]$ and $[0,20]$, showed by the following figure 4.
And we assumed that there are three pieces of evidence X, Y, Z. The evidence thinks that the value of the target proposition should be 93, 84 and 72, respectively. After calculation, the BPA of each evidence is described by the following table.

**Table 1. BPA of Evidences**

| Evidence | Propositions | Ω  | A  | A UB | B  |
|----------|--------------|----|----|------|----|
| X        |              | 0.3| 0.7| 0    | 0  |
| Y        |              | 0  | 0.4| 0.6  | 0  |
| Z        |              | 0  | 0  | 0.2  | 0.8|

The BPA after fusing three pieces of evidence by Dempster combination rule, the plausibility degree and the belief degree of the propositions are shown in the following table.

**Table 2. Result of Evidence Fusion**

| Result of Evidence Fusion | Propositions | Ω  | A   | A UB | B  |
|---------------------------|--------------|----|-----|------|----|
| BPA                       |              | 0  | 0.477| 0.105| 0.418|
| Plausibility Degree       |              | 0  | 0.582| 0.105| 0.523|
| Belief Degree             |              | 0  | 0.477| 0.105| 0.418|

The above process shows that this method provides a method to obtain the BPA, and avoids the paradox caused by evidence conflict.

5. Conclusion
In this paper we proposed a method of obtaining the BPA of evidence theory. The proposed method is based on the membership function, and avoids the Zadeh paradox by giving the FOD a BPA, which improves the classical evidence theory.

References
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