MODELLING ANISOTROPIC PHENOMENA OF FRICTION OF DEEP-DRAWING QUALITY STEEL SHEETS USING ARTIFICIAL NEURAL NETWORKS

ABSTRACT

This paper presents a method of determining the coefficient of friction in metal forming using multilayer perceptron based on experimental data obtained from the pin-on-disk tribometer. As test material, deep-drawing quality DC01, DC03 and DC05 steel sheets were used. The experimental results show that the coefficient of friction depends on the measured angle from the rolling direction and corresponds to the surface topography. The number of input variables of the artificial neural network was optimized using genetic algorithms. In this process, surface parameters of the sheet, sheet material parameters, friction conditions and pressure force were used as input parameters to train the artificial neural network. Some of the obtained results have pointed out that genetic algorithm can successfully be applied to optimize the training set. The trained multilayer perceptron predicted the value of the friction coefficient for the DC04 sheet. It was found that the tested steel sheet exhibits anisotropic tribological properties. The highest values of the coefficient of friction under dry friction conditions were registered for sheet DC05, which had the lowest value of the yield stress. Prediction results of coefficient of friction by multilayer perceptron were in qualitative and quantitative agreement with the experimental ones.

Keywords: artificial neural networks; coefficient of friction; genetic algorithm; friction; sheet metal forming

INTRODUCTION

The analysis and design of sheet metal forming (SMF) operations require knowledge of the tribological phenomena of the surface layer and the properties of the deformed material [1, 2]. The values of the coefficient of friction (COF) of sheets in plastic working processes are determined by many tribological tests, including: strip-drawing test, bending under tension...
test, draw-bead test, bending with tangential compression, strip-tension test, drawing with tangential compression, hemispherical stretching and strip-reduction testing [3]. However, anisotropy of COF can only be determined using pin-on-disk test [4]. Apart from the friction conditions, the second important parameter determining the sheet deformability is the anisotropy of the mechanical properties of the sheet material [5]. The forming conditions that determine the geometric and shape quality of drawpieces depend on four groups of factors related to (1) lubrication conditions, (2) forming conditions, (3) phenomena related to the type of material being deformed, and (4) tool related factors. The first group of factors includes the viscosity of the lubricant and the thickness of its layer [6]. The operating parameters include the forming temperature, duration of contact, forming speed and character of contact (static or dynamic) and the value of normal pressures [7]. The third group includes the mechanical properties of the sheet material (yield stress, ultimate tensile stress, strain hardening etc.), the type and properties of the protective coating (chemical composition, microhardness, thermal properties) [8]. The tool properties that significantly determine the tribological system in SMF include the properties of the tool material (hardness, surface quality, physical properties), the type of protective coating (microhardness, delamination strength, thermal properties) [9]. One of the main factors influencing the COF is the surface topography of the deformed sheet. Since the friction between the tool and the workpiece is one of the important factors influencing the geometric and shape quality of the drawpiece, it is very important to explain the friction conditions for the modeling and analysis of SMF processes. The synergistic effect of the surface topography of both the tool and workpiece is also an important factor that controls the lubrication mechanisms in the metal forming process. The size of the oil pockets is closely related to the occurrence of mixed friction [10].

Due to the complexity of interactions between many friction parameters and the COF value, analytical models were developed for the statistical description of tribological phenomena. Mathematical models based on stochastic methods should have such a level of complexity that they can reliably describe the phenomena occurring during the friction. Such methods include, among others, multiple regression, artificial neural networks (ANNs) and multivariate methods [11, 12]. The regression equation may not make any physical sense, but under certain assumptions it allows the prediction of quantities determined on the basis of knowledge of other variables. When selecting the factors influencing the COF of the sheet, the requirements related to the construction of a regression model describing their impact should be taken into account. These requirements come down to the selection of such factors which significantly affect friction, but at the same time are independent of each other. Among the wide application of multivariate analysis in tribology, particular attention should be paid to determining the value of the COF and to determining the influence of the load and friction path on the wear rate [13]. Many authors have successfully applied ANNs to nonlinear regression analysis. Investigations on the frictional wear of materials used for vehicle brake pads has shown a very good ability of the network to regression, taking into account more than 20 input variables [14]. The number of variables in the model is not a determinant factor of the quality of the regression model. The regression model proposed by Jurkovic et al. [15] took into account the influence of only two factors, the lubrication conditions and sheet strain, on the value of the COF occurring at the die edge in deep-drawing process.

ANNs are tools enabling the construction of linear and nonlinear models that solve complex classification and regression tasks. Calculations performed by neural networks belong to the group of so-called soft computing. The application of ANNs made it possible to find the relationship between the surface roughness value and the real contact area for various friction conditions [16]. The analysis of the possibility of using ANNs in tribological studies was carried out by Grymek et al. [17], pointing to the possibilities of using this technique in
tribological problems. According to the authors, the main error affecting the reduction of the quality of neural network predictions is the insufficient amount of training data used in the network training process. Increasing the accuracy of the neural model was achieved by optimizing the vector of explanatory variables with the use of genetic algorithms [18]. A literature review on the use of ANNs in tribological applications was carried out by Frangu and Ripa [19]. Trzos [20] presented the use of computer techniques, including information systems, in modeling tribological objects.

The aim of the experimental research and neural modeling presented in this article is to determine the COF for various types of deep-drawing quality steel sheets used in SMF. Given the large number of factors influencing COF during sheet forming, analytical determination of the relationships used to calculate the friction coefficient is practically impossible. For this reason, to determine the relationship between the selected parameters of the friction process and the value of COF, a multi-layer perceptron (MLP) was used. The important factors influencing the correct operation of ANN were selected using genetic algorithm (GA).

**MATERIAL AND METHODS**

As test material, deep-drawing quality cold-rolled low carbon DC01, DC03, DC04 and DC05 steel sheets were used. Tensile test was carried out in universal testing machine to determine mechanical parameters of the samples including hardening properties (Table 1). The specimens for tensile tests were cut at angle of 0° and 90° with respect to the rolling direction of the sheet metal. The main mechanical parameters determined through tensile tests at room temperature are: the yield stress \( R_{p0.2} \), ultimate tensile strength \( R_m \), elongation \( A_50 \), material constant \( K \), hardening exponent \( n \) and anisotropy factor \( r \). Values of work hardening parameters were determined based on the approximation of the true stress-true strain curves by the power law equation, also known as Hollomon equation \( \sigma = K \cdot \varepsilon^n \) (where \( \sigma \) is true stress and \( \varepsilon \) is the true strain). Three specimens were tested for each direction and average values of specific parameters have been determined. Uniaxial tensile tests were conducted at 5 mm/min tensile speed. Chemical composition of tested sheets according to the EN 10130:2006 [21] standard is listed in Table 2.

**Table 1. Mechanical properties of the tested sheets**

| Material | Specimen orientation | \( R_{p0.2} \) (MPa) | \( R_m \) (MPa) | \( A_50 \) | \( K \) (MPa) | \( n \) |
|----------|----------------------|-----------------------|----------------|----------|--------------|------|
| DC01     | 0°                   | 193                   | 351            | 0.36     | 554          | 0.166|
|          | 45°                  | 197                   | 372            | 0.32     | 591          | 0.171|
|          | 90°                  | 193                   | 353            | 0.34     | 563          | 0.174|
| DC03     | 0°                   | 196                   | 336            | 0.42     | 557          | 0.192|
|          | 45°                  | 196                   | 336            | 0.38     | 547          | 0.183|
|          | 90°                  | 198                   | 311            | 0.41     | 526          | 0.177|
| DC04     | 0°                   | 162                   | 310            | 0.42     | 554          | 0.21 |
|          | 45°                  | 163                   | 322            | 0.41     | 542          | 0.20 |
|          | 90°                  | 168                   | 312            | 0.41     | 530          | 0.21 |
| DC05     | 0°                   | 151                   | 282            | 0.44     | 494          | 0.221|
|          | 45°                  | 153                   | 293            | 0.40     | 494          | 0.207|
|          | 90°                  | 153                   | 287            | 0.42     | 487          | 0.211|
Table 2. Chemical composition of tested sheets (wt.%)

| Grade | C   | Mn  | P   | S   | Fe    |
|-------|-----|-----|-----|-----|-------|
| DC01  | max. 0.12 | max. 0.6 | max. 0.045 | max. 0.045 | balance |
| DC03  | max. 0.1 | max. 0.45 | max. 0.035 | max. 0.035 | balance |
| DC04  | max. 0.08 | max. 0.40 | max. 0.03 | max. 0.03 | balance |
| DC05  | max. 0.06 | max. 0.35 | max. 0.025 | max. 0.025 | balance |

Surface roughness was measured using Taylor Hobson Subtronic 3+ instrument and the values of basic surface roughness parameters were registered. The standard 2D parameters determined by this measurement are the arithmetical ($Ra$) and root ($Rq$) mean deviation of the assessed profile, the total height of the profile $Rt$, the maximum profile peak height $Rp$, the maximum profile valley depth $Rv$, the mean height of profile elements $Rc$ and the root mean square slope $Rdq$. The surface parameter values were measured before the friction tests at 15° between both the rolling and transverse direction according to the schematic shown in the Fig. 1. The selected measured surface roughness parameters for DC01 steel sheet which are the most suitable for surface topography description [22] are presented in Table 3. It has been confirmed that steel sheets are characterized by the anisotropy of surface topography.

![Fig. 1. Schematic of measurement of surface roughness parameters](image)

Table 3. Selected surface roughness parameters for DC01 steel sheet

| Angle of measurement | $Ra$ ($\mu$m) | $Rku$ | $Rsk$ |
|----------------------|--------------|-------|-------|
|                      | radial | tangent | radial | tangent | radial | tangent |
| 0°                   | 1.23   | 1.61    | 2.56   | 2.37    | 0.32   | 0.11    |
| 15°                  | 1.23   | 1.56    | 2.43   | 2.36    | 0.28   | 0.08    |
| 30°                  | 1.27   | 1.49    | 2.43   | 2.32    | 0.19   | 0.19    |
| 45°                  | 1.03   | 1.6     | 2.36   | 2.41    | 0.21   | 0.18    |
| 60°                  | 1.33   | 1.37    | 2.36   | 2.39    | 0.13   | 0.22    |
| 75°                  | 1.36   | 1.22    | 2.38   | 2.5     | 0.15   | 0.20    |
| 90°                  | 1.61   | 1.23    | 2.37   | 2.56    | 0.11   | 0.32    |

Friction properties of the DC01 steel sheet were determined by using the CSM Instruments pin-on-disk tribometer (Fig. 2). The tribometer conforms to both ASTM G99 and DIN 50324 designation according to which the friction coefficient may be also determined. In tribometer,
a pin is loaded onto the test sample with a precisely known force. The pin is mounted on a stiff lever, designed as a frictionless force transducer.

![View of measuring station](image)

**Fig. 2. View of measuring station**

To realize dry friction conditions, both rolls and sheet specimens were degreased using acetone, and for lubricant conditions conventional machine oil LAN-46 (Orlen Oil, Kraków, Poland) was used. The lubricant was distributed uniformly on the surface of the samples at 2 g/m². A 6 mm diameter pin made of bearing steel was held by a collet in an arm supported on two axes. Disk specimens were 50 mm in diameter and the initial thickness was 1 mm. They were fastened to the flat end of a 25.4 mm diameter horizontal shaft of a precision spindle, and thus the disk rotated in a vertical plane. Three normal forces with values of 6, 9 and 12 N were applied by using steel weights attached to the pin holder.

The value of the friction coefficient $\mu$ is determined according to Eq. (1):

$$\mu = \frac{F_T}{F_N}$$

where $F_T$ - friction force, $F_N$ - normal force.

**EXPERIMENTAL RESULTS**

The sheets tested under the dry friction conditions were characterized by a greater anisotropy of the coefficient of friction than the sheets tested under the lubrication conditions (Fig. 3 (a) –(c)). The differences in the minimum and maximum values of COF during one revolution of the samples made of DC01 sheet were 0.02 for the normal load 6 N and 0.023 for the normal load 12 N.

Under the conditions of lubricating the surface of the sheets with oil, the range of COF changes during one revolution was reduced by about a half. The greater reduction of COF value resulting from the application of lubrication was observed for the highest load used in the analysis, i.e., 12 N. In the case of DC03 (Fig. 3(b)) and DC05 (Fig. 3(c)) sheets tested under lubrication conditions, the values of the COF for the various loads were the closest. The highest values of the friction coefficient under dry friction conditions were observed for DC05 sheet (Fig. 3(c)), which is characterised by lowest value of the yield stress. In this way, the plasticized peaks increased the real contact area, thus intensifying the share of the adhesion mechanism in the total friction force.
To confirm that DC01 steel sheets are characterized by the anisotropy of tribological properties, friction anisotropy on a given surface has to be clearly distinguished from friction anisotropy for different perpendicular orientations between the pin and the surface. In this study, the friction coefficient as a function of angular position with respect to the rolling direction of the sheet metal was measured. Fig. 4 presents typical variation of friction coefficient value for the first rotation of the specimen at specified process conditions. There
are two maxima within a 360° rotation. These values correspond nearly to the transverse directions to the rolling direction.

**Artificial neural network modelling**

Artificial neural networks are used to analyse the relations between parameters particularly when dependence between inputs and outputs are very complicated. The network consists of elements named neurons that are connected together and the processing data is supplied as input. A multilayer perceptron with a suitable number of hidden layers and neurons is theoretically sufficient to approximate any nonlinear function [23]. In order to calculate the output value of neuron of MLP network, the hyperbolic tangent function at the points $a$ and $-a$ (Eq. 2) is applied:

$$f(a) = \tanh(x) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$$

In this study, the following input sets of variables were assigned as input signals:
- measurement orientation of surface roughness parameters in relation to the rolling direction of the specimens $\alpha$,
- pin load $p_N$,
- roughness parameters of the as-received specimens: $Ra$, $Rq$, $Rt$, $Rp$, $Rv$, $Rc$, $Rsk$, $Rku$ and $Rdq$.
- mechanical parameters of the specimen material: $R_{p0.2}$, $R_m$, $A_{50}$, $K$, $n$,
- friction conditions: lubricated conditions and dry friction.

The selection of variables that significantly influence the value of the COF is difficult due to the complex interactions of input and output variables. There are no universal guidelines for the selection of input variables which, due to their correlation, can produce a complex synergistic effect. On the one hand, the lack of important variables can lead to misinterpretations, while, on the other hand, taking into account too many variables introduces information noise.

**Selection of input parameters**

Genetic algorithms imitating natural evolution and they are methods of solving problems, mainly in optimization tasks. They are characterized by high versatility and simplicity of search procedures for the best solutions using the stochastic method [24]. GAs are search procedures based on the mechanisms of natural selection and inheritance. Their operation is based on the evolutionary principle of survival of the fittest individuals. At the beginning of GA operation, a random selection of individuals (chromosomes) for the initial population is performed [25]. Then, the fitness of the individual in the population is assessed on the basis of the calculated fitness function. If the stop condition is met, the best chromosome is remembered. If not, the next step is to select individuals [26].

For optimization of a number of input variables, classical Holland’s genetic algorithm was used. The initial population of the genetic algorithm in this study has 150 individuals with mutation coefficient $p_m = 0.1$ and crossover coefficient $p_k = 0.5$ [18]. Different values of unit penalty (Table 4) were analysed. When solving the optimization problem with the use of a genetic algorithm, the value of the fitness function must be positive and it is subject to maximization. An additional optimization criterion is the relative error between the given total speed increment and the increment returned in a given iteration by the genetic algorithm. If the criterion is not met, then a penalty function is superimposed on the local solution (e.g. a
fixed-value function). The unit penalty coefficient is multiplied by a number chosen as a mask for each of the input variables, and then added to the value of validation error.

With a large number of input variables determined by a small value of unit penalty appears a high value of a genetic algorithm error, which next decreases until it reaches local minimum for unit penalty value of 0.001 (Table 4).

| Table 4. Influence of unit penalty value on the choice of input parameters by GA |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Parameter         | 0.0001  | 0.0003  | 0.0005  | 0.001  | 0.003  | 0.005  |
| $R_{\phi 2}$      | +       | +       | +       | +     | +     | –     |
| $R_m$             | –       | –       | –       | –     | –     | –     |
| $A_{50}$          | +       | +       | –       | –     | –     | –     |
| $K$               | +       | +       | +       | +     | –     | –     |
| $n$               | +       | +       | –       | –     | –     | –     |
| $\alpha$         | +       | +       | +       | +     | +     | +     |
| $p_N$            | +       | +       | +       | +     | +     | +     |
| $R_a$            | +       | +       | +       | +     | +     | +     |
| $R_q$           | –       | –       | –       | –     | –     | –     |
| $R_t$           | +       | –       | –       | –     | –     | –     |
| $R_p$           | +       | +       | +       | +     | +     | –     |
| $R_v$           | +       | +       | –       | –     | –     | –     |
| $R_c$           | +       | +       | +       | +     | –     | –     |
| $R_{sk}$         | +       | +       | +       | –     | –     | –     |
| $R_{ku}$         | +       | +       | +       | +     | –     | –     |
| $R_{dq}$        | +       | +       | –       | –     | –     | –     |
| friction conditions | +     | +       | +       | +     | +     | +     |
| Error            | 1.256   | 1.173   | 1.218   | 1.153 | 1.334 | 1.279 |

The error values increase with the value of the unit penalty. The high error value with a great number of variables can be explained by noise of variables which can be in certain range of values correlated with each other. Then a high value of unit penalty causes, from the viewpoint of the quality of algorithms, the number of variables more important [27]. Local increase of the error value with the value of unit penalty can be explained by the fact that removing two variables makes the correlation with other variables dominating. For further analysis, a set of 9 input variables characterized by the smallest error value was chosen. The correctness of the analysis of the selection of variables is confirmed by the rejection of the roughness parameter $R_q$ by all analyzed genetic algorithms. It is well known that the parameters $R_a$ and $R_q$ are strongly correlated. Moreover the value of ultimate tensile stress $R_m$ is proportional to the value of yield stress $R_{p02}$. So this parameter was not important in the analyses. The parameters that carry the relevant information confirmed by all GA analysis are pin load, measurement orientation of surface roughness parameters, the roughness average $R_a$ and friction conditions.

**Formulating neural friction model**

During the operation of the genetic algorithm optimization, the number of evaluated neuron networks is the product of chromosome numbers in the population and the number of considered generations. In the early phase of the genetic algorithm, set of input variables were
investigated using a series of analyses of different ANN architecture in *Statistica Neural Networks*. The objective of these analyses was to find the network architecture that ensures the smallest value of standard deviation ratio in connection with high value of Pearson’s correlation coefficient $R$.

The training set consisted of experimental data on DC01, DC03 and DC05 sheets. From the experimental data belonging to training set (TR), $10\%$ was separated and assigned as validation set (TV). Among all experimental sets of input data that correspond with output signal, $24$ element sets were separated and assigned as test set (TS). This set corresponds to the experimental data of DC04 steel sheet. Data vectors from a test set did not participate in the training process and served for ANN prognostic evaluation purpose only. Data from this group were used for independent check of back propagation training algorithm. The learning rate was equal to $0.1$. The training process was stopped when the value of $RMS$ error (Eq. 3) for validation set was stopped dropping $18$.

$$RMS = \sqrt{\frac{\sum_{i=1}^{N} (z_i - y_i)^2}{N}}$$

(3)

where: $N$ is number of vectors of training set, $y_i$ is signal of output signal for $i^{th}$ standard, $z_i$ is expected signal of output neuron for $i^{th}$ standard.

The network characterised by the lowest value of Standard Deviation (SD) ratio and the highest amount of Pearson-R correlation for training set is multilayer perceptron with architecture MLP 9:9-12-1:1. The correlation coefficient for all sets values are greater than $0.98$ (Table 5), so the constructed neural model can be considered valid. The lowest value of SD ratio was observed for the test set not participating in the learning process. It is recommended [18] that if the value of this parameter is below $0.1$, it confirms a very good prognostic quality of the ANN. Results COF forecasting by ANN are in qualitative and quantitative agreement with the experimental ones (Fig. 5).

| Regression statistics | Set   | TS      | TR       | TV       |
|-----------------------|-------|---------|----------|----------|
| Error SD              |       | 0.005344| 0.00557 | 0.005993|
| Absolute error mean   |       | 0.004218| 0.005508| 0.006069|
| RMS error             |       | 0.006753| 0.00599 | 0.00736 |
| SD ratio              |       | 0.119563| 0.17262 | 0.199907|
| Correlation R         |       | 0.992831| 0.984996| 0.980221|

The changes of the COF as a function of measurement orientation showed two minima and two maxima during one sample rotation. Curve generated by MLP is more symmetrical with respect to both directions $0^\circ$-$180^\circ$ and $90^\circ$-$270^\circ$. 
Fig. 5. Variation of COF value as a function of measurement orientation according to the rolling direction of the DC04 steel specimen, load 9 N, dry friction conditions

CONCLUSIONS

The application of ANN for modelling the COF in sheet forming method allows avoiding the time-consuming testing of neural networks with different architecture in order to find the optimum network for description variation of friction coefficient value determined using pin-on-disk tribometer. The following conclusions can be drawn from the research:

- tested deep-drawing steel sheets showed anisotropy of COF,
- the greatest differences in the COF value during the sample rotation were observed under dry friction conditions,
- in the lubricated conditions, the range of COF changes during one sample rotation was reduced by about a half compared to the dry friction conditions,
- the highest COF values under dry friction conditions were observed for the DC05 sheet, which materials is characterised by the lowest value of yield stress,
- input parameters $R_q$ and $R_m$ are not considered significant by the genetic algorithm because they are strongly correlated with values of $R_a$ and $R_{p,0.2}$, respectively,
- parameters that are carriers of relevant information confirmed by all analyzes are the roughness average $R_a$, pin load, friction conditions and measurement orientation of surface roughness parameters,
- the ANN model was characterized by ability to smooth experimental data noised by system that was recording friction force value.

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