Competing Persuaders in Zero-Sum Games*

Dilip Ravindran† and Zhihan Cui‡

August 20, 2020

Abstract

We study a Bayesian Persuasion game with multiple senders employing conditionally independent experiments. Senders have zero-sum preferences over what information is revealed. We characterize when a set of states cannot be pooled in any equilibrium, and in particular, when the state is (fully) revealed in every equilibrium. The state must be fully revealed in every equilibrium if and only if sender utility functions are sufficiently nonlinear. In the binary-state case, the state is fully revealed in every equilibrium if and only if some sender has nontrivial preferences. Our takeaway is that ‘most’ zero-sum sender preferences result in full revelation.

*We thank Navin Kartik, Elliot Lipnowski, Alessandro Pavan, Jacopo Perego, and seminar participants for very helpful comments.

†Department of Economics, Columbia University. Email: drr2130@columbia.edu

‡School of International and Public Affairs, Columbia University. Email: zc2322@columbia.edu
1 Introduction

A key question in the economics of persuasion is the effect of competition on information provision. While it has been shown that information disclosure increases with competition in some settings (Battaglini (2002); Milgrom and Roberts (1986); Shin (1998)), in others competition has the opposite effect (Emons and Fluet (2019); Kartik et al. (2017)). In this paper we address the question by modelling two or more senders persuading one (or multiple) receiver(s) about an unknown state. The senders influence the receiver’s beliefs by disclosing information in the manner of Bayesian Persuasion. Unlike existing work, our senders simultaneously choose conditionally independent experiments; the receiver observes these experiments and their realizations and updates her belief. To fix ideas, consider competing lobbyists commissioning reports to persuade a politician (or entire legislature) to vote yes/no on a climate change bill. Here the state may be whether climate change is a threat; the politician would only like the bill to pass if it is while lobbyists have differing interests in it passing. Our analysis applies equally well to prosecutor and defense attorneys persuading a judge or media outlets persuading voters.

We consider an environment where senders are maximally-competitive —the senders’ payoffs are zero-sum functions of the receiver’s posterior. This assumption is natural for the lobbyist example as lobbyists may only care about the probability the bill is successfully passed, which varies with the politician’s posterior. Our question is: how does competition affect how much information is revealed in equilibrium and how does this change with the number of senders?

There is always an equilibrium of this game in which all senders fully reveal the state. Our main result is that typically the state is fully revealed in every equilibrium. We find that —under mild technical assumptions —when sender utility functions are sufficiently nonlinear (in particular are nonlinear on every edge of the simplex) then regardless of the number of senders the state is fully revealed in every equilibrium. If utility functions are sufficiently linear, there are equilibria in which the receiver does not always learn the state. Two implications are worth mentioning. In the binary-state case, the state is fully revealed in every equilibrium if and only if some sender has nontrivial preferences. If the receiver chooses among a finite set of actions, then generically the receiver learns enough to take her first best action; furthermore, the state is fully revealed in every equilibrium if and only if the receiver prefers a different action in every state.

\[^1\text{The every quantifier implies, by standard arguments, that if the conditions for full revelation are met for zero-sum utilities, then for utilities close to those all equilibria are almost fully revealing.}\]
The intuition for our results can be seen from the two-sender binary-state case. The first observation is that, as a sender is always free to fully reveal the state and the game is zero-sum, each sender must do exactly as well in any equilibrium as she would from full revelation. When utility functions are nonlinear, we show that some sender $i$ can guarantee a payoff strictly larger than from full revelation whenever her opponent $j$ does not fully reveal the state. Sender $i$ can do this by choosing an experiment that ensures the posterior will fall in regions she has an ‘advantage’ and not in regions her opponent has an advantage.

This idea extends to arbitrary finite state spaces and more than two senders. Given choices of experiments for each sender, we say that a set of states is not pooled if the receiver never assigns positive probability to all of them. We show that a subset of states is not pooled in every equilibrium if and only if conditional on the receiver learning the state is in this subset, some sender has strict preferences over what further information to reveal. For instance, a pair of states is not pooled in all equilibria if any only if some sender’s utility is nonlinear on the edge of the simplex between those two states.

It follows that nonlinearity on every edge is necessary and sufficient for full revelation in every equilibrium.

Related Literature. This paper relates most closely to work in the multiple sender Bayesian Persuasion literature, most notably Gentzkow and Kamenica (2016) and Gentzkow and Kamenica (2017) (henceforth GK). These authors’ main finding is that more competition leads to no less equilibrium information. GK (2017), like us, find that in zero-sum games sufficient nonlinearity in sender preferences ensures full revelation in every equilibrium. However, both GK papers make a strong and arguably unrealistic assumption that senders can choose from a set of ‘Blackwell connected’ experiments that allow for arbitrary correlation in senders’ experiments. In contrast we study the case of conditionally independent experiments; conditional independence is a common assumption in information economics. We discuss the distinction in Section 5 and show that, but for a few additional assumptions we make, our results generalize that of GK (2017).

Boleslavsky and Cotton (2018) and Au and Kawai (2020) study two senders persuading a receiver. However their setups are substantially different from ours because each sender can only reveal information about part of the state (their own type); as a consequence, they find unique non-fully revealing equilibria. Li and Norman (2018a), Li and Norman (2018b), and Wu (2017) consider Bayesian Persuasion with multiple senders moving sequentially. Finally, in a concurrent paper, Dworczak and Pavan (2020) (hence-

\footnote{i.e. the line joining degenerate beliefs on the two states.}
forth DP) study a single persuader who is uncertain about what additional information nature may give the receiver and chooses an experiment to maximize her worst-case pay-off. This setting is related to competition between two senders in our model (our case of more than two senders is less related). While their baseline model allows nature to arbitrarily correlate her experiment with the persuader’s, they address the case of conditionally independent experiments in a supplementary appendix and obtain results close to ours. However, due to differences between the models, our results concerning the total information revealed in equilibrium are stronger. See Section 5 for discussion.

2 Model

There is a state \( \omega \in \Omega = \{1, \ldots, N\} \). All agents have a common prior belief on \( \omega \) with full support \( \pi \in \text{int}\Delta(\Omega) \). There are \( M > 1 \) senders, \( 1, \ldots, M \), who persuade a receiver. \(^3\)

Fix a set of signals \( S \) with \( |S| = |\Delta(\Omega)| \). The game starts with each sender \( i \) simultaneously choosing a set \( S_i \subset S, |S_i| < \infty \), and an experiment \( \Pi_i : \Omega \to \Delta(S_i) \). Each \( \Pi_i \) gives the probability of the receiver receiving each signal in \( S_i \) conditional on each state. As \( |S_i| < \infty \), senders may only choose \textit{finite signal} experiments; we relax this assumption in Supplementary Appendix B. \(^4\) Implicit in this definition of experiments is that senders’ experiments are independent conditional on the state.

The receiver observes the choices of \( \Pi_1, \ldots, \Pi_M \) (and implicitly \( S_1, \ldots, S_M \)). Then, the state is realized (but not observed by the receiver) and signals from each of the \( M \) experiments, \( s_1 \in S_1, \ldots, s_M \in S_M \), are realized and observed by the receiver. The receiver is Bayesian and updates his belief on \( \omega \) to some posterior \( \beta \in \Delta(\Omega) \). Senders receive their payoffs and the game ends.

Senders’ payoffs depend only on the receiver’s posterior belief \( \beta \). Each sender \( i \) has a piecewise analytic utility function \( u_i : \Delta(\Omega) \to \mathbb{R} \). \(^5\) Crucially, we assume senders’ payoffs are zero-sum: \( u_1(\beta) + \ldots + u_M(\beta) = 0 \) for all \( \beta \in \Delta(\Omega) \). \(^6\) For any state \( l = 1, \ldots, N \) let \( \delta_l \in \Delta(\Omega) \) represent the belief that puts probability 1 on state \( l \). Due to the structure of

\(^3\)As we do not explicitly model the receiver acting, the model allows for any number of receivers.

\(^4\)All equilibria with the finite signal restriction are equilibria without it.

\(^5\)That is, each \( u_i \) is defined by a finite partition of \( \Delta(\Omega) \) into convex sets and a real analytic function for each element of the partition. Note this restriction is not necessary; see Section 5 for discussion.

\(^6\)This could represent the reduced form of a game where the receiver chooses an action \( a \in A \) after observing experiment realizations. The receiver has preferences \( u_r(a, \omega) \) and the senders may also have state dependent preferences \( \{u_i(a, \omega)\}_i \), which are zero-sum: \( \sum_i u_i(a, \omega) = 0 \) for all \( a \in A, \omega \in \Omega \).
the game, we can make the following normalization: \( u_i(\delta_l) = 0 \) for all senders \( i = 1, \ldots, M \) and all states \( l = 1, \ldots, N \).

A strategy profile is a choice of experiment for each sender \( (\Pi_1, \ldots, \Pi_M) \). Let \( U_i(\Pi_1, \ldots, \Pi_M) = \mathbb{E}_{\Pi_1, \ldots, \Pi_M}[u_i(\beta)] \) be sender \( i \)'s ex-ante expected utility from \( (\Pi_1, \ldots, \Pi_M) \); the expectation is over experiment realizations, of which \( \beta \) is a function. Senders choose experiments to maximize their ex-ante expected utility.

**Interim Beliefs.** Instead of thinking of sender \( i \) picking \( \Pi_i \), it is easier to think of \( i \) choosing a distribution over the receiver’s interim beliefs. For any \( i \) and choice of \( \Pi_i \), let \( \Gamma_i \in \Delta(\Omega) \) be the random variable representing the receiver’s belief on \( \omega \) if she observes only the realization of \( \Pi_i \), \( s_i \in S \). \( \Gamma_i \) represents the interim belief of the receiver after she observes information from \( \Pi_i \) but before viewing the realizations from \( \{\Pi_j\}_{j \neq i} \) and updating to her posterior belief.

Following [Kamenica and Gentzkow (2011)](#), it is without loss for us to recast the choice of experiment of each sender \( i \) as a selection of a Bayes-plausible distribution of the interim beliefs, \( \Gamma_i \), the experiment induces. As we have restricted senders to picking finite signal experiments, a pure strategy for sender \( i \) is a selection of a Bayes-plausible \( \Gamma_i \) with finite support. Henceforth, when we use \( \Gamma_i \), it implicit that this random variable is Bayes-plausible and has finite support. A strategy profile is a vector \( (\Gamma_1, \ldots, \Gamma_M) \). Fixing any strategy profile and sender \( i \), let \( \Gamma_{-i} \) denote the experiment induced by observing realizations \( \{\Gamma_j\}_{j \neq i} \).

There are two benchmark experiments to consider. We say \( \Gamma_i \) is fully revealing, or \( \Gamma_i = \Gamma_{FR} \), if \( \Pr(\Gamma_i = \delta_l) = \pi_l \forall l \in \Omega \). If any sender chooses a fully revealing experiment the receiver learns the state with certainty. The second benchmark is the fully uninformative experiment which we will denote \( \Gamma_U \); \( \Gamma_i = \Gamma_U \) if \( \Pr(\Gamma_i = \pi) = 1 \).

**Equilibrium.** A Nash Equilibrium of this game is a vector of random variables \( (\Gamma_1, \ldots, \Gamma_M) \) such that no sender \( i \) can strictly improve her ex-ante expected utility, \( U_i(\Gamma_1, \ldots, \Gamma_M) \), by deviating.

There is a trivial NE of this game: \( (\Gamma_{FR}, \ldots, \Gamma_{FR}) \). All senders are left indifferent across all experiment choices as the state will be fully revealed by other senders’ ex-
periments regardless. Our results characterize when the state is fully revealed in every equilibrium.

3 Two Senders and a Binary State Space

First we derive the main results for the two-sender binary-state case. The intuition will extend to the general case.

Let \( \Omega = \{0, 1\} \). A belief here is a scalar representing the probability the state is \( \omega = 1 \). Figure 1 shows an example of sender preferences. A sender \( i \)'s strategy is a choice of interim belief random variable \( \Gamma_i \in [0, 1] \). Note that for any \( \Gamma_1, \Gamma_2 \) chosen, the receiver’s posterior belief can be written as a function of the interim beliefs realized from both experiments. If \( \Gamma_1 = x \) and \( \Gamma_2 = y \), then the posterior is:

\[
\beta(x, y) = \frac{(1 - \pi)xy}{xy - \pi x - \pi y + \pi} \quad (1)
\]

**Figure 1:** Example of \( u_1 \) (blue) and \( u_2 \) (red). \( u_1(\beta) = \beta \) for \( \beta < 0.6 \) and \( u_1(\beta) = 1 - \beta \) for \( \beta \geq 0.6 \). Sender 1’s preferences are those in Kamenica and Gentzkow (2011)’s leading example with discontinuity at 0.6 and normalization \( u_1(0) = u_1(1) = 0 \).
Note that $\beta(1, y) = \beta(x, 1) = 1$ and $\beta(0, y) = \beta(x, 0) = 0$; if either interim belief fully reveals the state, the other is irrelevant. Note $\beta(0, 1)$ and $\beta(1, 0)$ are not well defined but this is not an issue as it is impossible for one sender to fully reveal $\omega = 0$ while the other reveals $\omega = 1$.

For either sender $i$, given any strategy $\Gamma_i$, consider the distribution of $\Gamma_i$ conditional on $\Gamma_j = x$ ($j \neq i$): $Pr(\Gamma_i = y|\Gamma_j = x)$. $Pr(\Gamma_i = y|\Gamma_j = x)$ can be constructed by taking the signal structure $\Pi_i$ that corresponds to $\Gamma_i$ and deriving the distribution over interim beliefs it induces if $x$ and not $\pi$ was the receiver’s prior.

Given an opponent choice of $\Gamma_j$, define $W_i(x)$ as sender $i$’s expected payoff conditional on generating $\Gamma_i = x$. For a fixed $\Gamma_2$ $W_1(x)$ is written:

$$W_1(x) = \sum_{y \in \text{supp} [\Gamma_2]} u_1(\beta(x, y)) Pr(\Gamma_2 = y|\Gamma_1 = x) dy$$

(2)

Note that $W_1(1) = W_2(0) = W_1(0) = W_2(1) = 0$; if either players’ experiment generates a fully revealing interim belief then the other experiment is irrelevant. Two special cases are important. When $\Gamma_2 = \Gamma^{FR}$ then, regardless of $u_1$ or the prior, $W_1(x) = 0$ for all $x$. This is because $\Gamma_2$ will reveal the state to be 0 or 1; any interim belief sender 1 produces can only affect the relative probability of these events, both of which yield $u_1 = 0$. Meanwhile, when $\Gamma_2 = \Gamma^U$, then $W_1(x) = u_1(x)$ as $x$ will be the receiver’s posterior.

3.1 Analysis

The result below will be useful.

**Lemma 1.** In any equilibrium $(\Gamma_1, \Gamma_2)$:

1. $U_1(\Gamma_1, \Gamma_2) = U_2(\Gamma_1, \Gamma_2) = 0$.
2. $W_1(x) \leq 0$ and $W_2(x) \leq 0$ for all $x \in [0, 1]$.

We provide a formal proof in Supplementary Appendix A, but the logic is straightforward. Property (1) follows from the game being zero-sum and the observation that each sender $i$ can guarantee a payoff $U_i = 0$ by fully revealing the state. While property (1) says that sender equilibrium payoffs equal those from full revelation, it does not say...
that we must have full revelation in equilibrium: for instance if the \( u_1 \) and \( u_2 \) are linear, any \((\Gamma_1, \Gamma_2)\) constitute an equilibrium.

Property (2) holds because any violation leads to a contradiction of (1). Fix any \( \Gamma_j \) such that sender \( i \neq j \) has \( W_i(x) > 0 \) for some \( x \). We can find a \( \Gamma_i \) with support only on \( \{x, 0, 1\} \); as \( i \) gets strictly positive expected utility whenever \( x \) is realized and 0 otherwise, \( U_i(\Gamma_i, \Gamma_j) > 0 \). Hence such \( \Gamma_j \) cannot be played in equilibrium.\(^{11}\)

The main result for the two-sender binary-state case relies on Lemma 1. We show that if utility functions are nonlinear, then in equilibrium at least one sender \( i \) must choose \( \Gamma_i = \Gamma^{FR} \), or else \( W_j(x) \) will violate property (2).

**Proposition 1.** The state is fully revealed in every equilibrium if and only if \( u_1 \) is nonlinear.

The ‘only if’ direction is trivial —if \( u_1 \) is linear then senders are indifferent between all strategy profiles. Hence the result can be restated as:

There is full revelation in every equilibrium

\[
\iff \exists (\Gamma_1, \Gamma_2), (\Gamma'_1, \Gamma'_2) \text{ and a sender } i \text{ with } U_i(\Gamma_1, \Gamma_2) \neq U_i(\Gamma'_1, \Gamma'_2).
\]

We discuss the intuition and sketch the proof but leave details to Supplementary Appendix A. In the rest of this section we discuss the ‘if’ direction: \( u_1 \) nonlinear \( \implies \) full revelation in every equilibrium.

The idea can be seen using the example in Figure 1 with any prior. Note that for all \( r \in [0.6, 1) \), \( u_1(\beta) > 0 \) for all \( \beta \in [r, 1) \); fix any such \( r \). Suppose for contradiction that sender 2 plays a non-fully revealing strategy \( \Gamma_2 \) in some equilibrium. As \( \Gamma_2 \neq \Gamma^{FR} \), \( \Pr(0 < \Gamma_2 < 1) > 0 \); let \( \chi = \min \text{ supp}[\Gamma_2] \setminus \{0, 1\} \in (0, 1) \) be in the smallest interior belief in the support of \( \Gamma_2 \). Using the definition of \( \beta(x, y) \), define \( \chi \) by \( \beta(\chi, y) = r \). Conditional on \( \Gamma_1 = x \in [\chi, 1) \), \( \beta(x, y) \in [r, 1) \) for all interior \( y \) in \( \Gamma_2 \)'s support. But then for all \( x \in [\chi, 1) \) we have:

\(^{11}\)As an example consider \( u_1 \) in Figure 1 with \( \Gamma_2 = \Gamma^U \). Then \( W_1(0.6) = 0.4 \) so sender 1 can generate \( U_1(\Gamma_1, \Gamma_2) > 0 \) with \( \Gamma_1 \) putting support on \( \{0, 0.6, 1\} \).
\[
W_1(x) = u_1(\beta(x,0)) \Pr(\Gamma_2 = 0 | \Gamma_1 = x) + u_1(\beta(x,1)) \Pr(\Gamma_2 = 1 | \Gamma_1 = x) + \\
\sum_{y \in \text{supp}([\Gamma_2] \setminus \{0,1\}) > 0} u_1(\beta(x,y)) \Pr(\Gamma_2 = y | \Gamma_1 = x) > 0.
\]

This contradicts Lemma 1 property (2).

The broader intuition is as follows. We say a sender \(i\) has an advantage on any subset of \([0,1]\) on which \(u_i\) is strictly positive; for instance in the example, sender 1 has an advantage on \([0.6,1)\). While senders would like the receiver’s posterior to fall in their regions of advantage with high probability, neither sender’s experiment unilaterally controls the posterior. However, in the example sender 1 has an advantage at the end of the unit interval, \([r, 1]\). If sender 2 chooses \(\Gamma_2\) that is interior with positive probability then sender 1 can find extreme enough interim beliefs \(x \geq x\) guaranteeing that, conditional on \(x\) being realized and \(\Gamma_2\) being interior, the receiver’s posterior is in \([r, 1]\). Whenever \(\Gamma_2\) fully reveals the state both senders get utility 0, and so overall sender 1 gets a strictly positive expected payoff from generating an interim belief \(x \in [x, 1]\); Lemma 1 says this is not possible in equilibrium.

This argument does not depend on the specific \(u_1\) in the example. As \(\beta \to 1\), whenever \(u_1\) approaches \(u_1(1) = 0\) from above, as is the case in the example, we can find an \(r \in (0, 1)\) such that sender 1 has an advantage on \([r, 1]\). Hence for any such \(u_1\), we can replicate the same argument to show that any \(\Gamma_2 \neq \Gamma^{FR}\) cannot be played in equilibrium. If \(u_1\) approaches 0 from below as \(\beta \to 1\) then \(u_2\) must approach from above, and so we must have \(\Gamma_1 = \Gamma^{FR}\) in any equilibrium. The same argument applies whenever \(u_1\) or \(u_2\) approach 0 from above as \(\beta \to 0\).

As \(u_1, u_2\) are piecewise analytic, there is only one other case to consider: \(u_1, u_2\) nonlinear and \(u_1(\beta) = u_2(\beta) = 0\) for all \(\beta\) in some neighborhoods of both 0 and 1. Here too there will be a sender with a region of advantage closest to the ends of \([0, 1]\) who can find a violation of Lemma 1 property (2) whenever her opponent does not fully reveal the state. Let \(r = \sup\{\beta \in [0, 1] : u_1(\beta) \neq 0\}\) be the supremum of posteriors at which \(u_1, u_2\) are nonlinear (note \(r < 1\)). WLOG, assume either \(u_1(r) > 0\) or \(u_1(r^-) > 0\) (piecewise

\[12\]We use the word advantage because Lemma 1 tells us that both senders will get ex-ante expected utility 0 in equilibrium. Any posteriors that yield strictly better utility than this for a sender are relatively advantageous to that sender.
analycity implies this must hold for either \( u_1 \) or \( u_2 \). Suppose \( \Gamma_2 \neq \Gamma^{FR} \). Defining \( y \) and \( x \) as before, if \( u_1(r) > 0 \), then \( W_1(x) = u_1(r)Pr(\Gamma_2 = y|\Gamma_1 = x) > 0 \). If \( u_1(r^-) > 0 \), then \( W_1(x - \epsilon) > 0 \) for small enough \( \epsilon > 0 \).

4 Main Result

Now we apply the logic from the previous section to \( N \geq 2 \) states and \( M \geq 2 \) senders. For any \( T \geq 1 \) and experiments \( \Gamma_1,...,\Gamma_T \), let \( \beta(\Gamma_1,...,\Gamma_T) \) be the receiver’s posterior belief after observing all \( T \) realizations. First note Lemma 1 extends to this setting (the logic is the same).

For any strategy profile \( (\Gamma_1,...,\Gamma_M) \) and subset of states \( \Omega' \subseteq \Omega \), we say \( \Omega' \) is not pooled if \( Pr(\beta_l(\Gamma_1,...,\Gamma_M) > 0 \forall l \in \Omega') = 0 \) (otherwise, \( \Omega' \) is pooled). When \( \Omega' \) is not pooled, the receiver will always be able to rule out at least one of the states in the set. For any \( \Omega' \subseteq \Omega \), let \( \Delta(\Omega') = \{ \gamma \in \Delta(\Omega) : \sum_{l \in \Omega'} \gamma_l = 1 \} \) be the subset of the simplex assigning probability 1 to \( \omega \in \Omega' \). Note that for two states \( l, k \), \( \Delta(\{l,k\}) \) is the edge of the simplex between \( \delta_l \) and \( \delta_k \). Hence Proposition 1 can be restated as: states 0, 1 are not pooled in every equilibrium if and only if some \( u_i \) is nonlinear on \( \Delta(\{0,1\}) \). Proposition 2 generalizes Proposition 1 and characterizes when any subset of states is not pooled in every equilibrium.

**Proposition 2.** Consider any \( \Omega' \subseteq \Omega \). No equilibrium pools \( \Omega' \) if and only if for some sender \( i \) \( u_i \) is nonlinear on \( \Delta(\Omega') \).

Suppose the receiver learns that \( \omega \in \Omega' \subseteq \Omega \). Conditional on this event, if \( u_i \) is linear on \( \Delta(\Omega') \) for all \( i \) then all senders are indifferent across all additional information that can be revealed. Meanwhile, if for some \( i \) \( u_i \) is nonlinear on \( \Delta(\Omega') \), then there is some additional experiment that \( i \) either strictly prefers or disprefers to not providing any additional information. Proposition 2 says that conditional on the receiver learning that \( \omega \in \Omega' \), some sender having strict preferences over revealing additional information characterizes \( \Omega' \) being not pooled in every equilibrium. One important implication is that revealing no information, the strategy profile \( (\Gamma^U,...,\Gamma^U) \), is an equilibrium if and only if all senders are indifferent across all strategy profiles.

We give the broad idea of the proof here and give a detailed proof sketch in the Appendix. The full proof is in Supplementary Appendix A. The ‘only if’ direction is straightforward; if all \( u_i \) are linear on \( \Delta(\Omega') \) then all senders fully revealing the state
whenever $\omega \in \Omega \setminus \Omega'$ and revealing no further information whenever $\omega \in \Omega'$ is a non-fully revealing equilibrium.

Now for the ‘if’ direction. For any $\Omega' \subseteq \Omega$ and sender $i$, fixing an opponent strategy $\Gamma_{-i}$ consider $W_i(x)$ on $\Delta(\Omega')$. First note that $\Gamma_i = x \in \Delta(\Omega') \implies \beta(x, \Gamma_{-i}) \in \Delta(\Omega')$ w.p. 1. Generating an interim belief in $\Delta(\Omega')$ tells the receiver that $\omega \in \Omega'$, ensuring the posterior is also on this set. Conditional on $x \in \Delta(\Omega')$, the only information $\Gamma_{-i}$ can convey is relative probabilities of states in $\Omega'$. When evaluating $W_i(x)$ on $\Delta(\Omega')$, sender $i$ can treat $\Gamma_{-i}$ as an experiment just about states in $\Omega'$. If some $u_i$ is nonaffine on $\Delta(\Omega')$ we can apply a similar argument to Proposition 1. Firstly, at least one sender has an advantage somewhere on $\Delta(\Omega')$. We can find some sender $j$ such that whenever $\Pr(\Gamma_{-j} = y \text{ s.t. } y_l > 0 \forall l \in \Omega') > 0$ (i.e. $\Gamma_{-j}$ pools $\Omega'$), $W_j(x') > 0$ for some $x' \in \Delta(\Omega')$. Like in Proposition 1, $x'$ will be extreme enough — close enough to the boundaries of $\Delta(\Omega')$ — to ensure that whenever $\Gamma_{-j}$ assigns positive probability to all states in $\Omega'$, $\beta(x', \Gamma_{-j})$ falls in a region of $j$’s advantage with positive probability. Otherwise, $\beta(x', \Gamma_{-j})$ will fall where $j$ gets 0 utility. Hence $W_j(x') > 0$, violating Lemma 1 and implying that $\Gamma_{-j}$ must not pool $\Omega'$ in equilibrium.

Whenever no pair of states can be pooled in any equilibrium, the state is fully revealed all equilibria. Applying Proposition 2 to every pair of states:

**Theorem 1.** The state is fully revealed in every equilibrium if and only if for every pair of states $l$ and $k$ there exists sender $i$ with $u_i$ nonlinear on $\Delta(l, k)$.

This is an immediate corollary of Proposition 2. Theorem 1 shows that preferences being sufficiently nonlinear characterizes all equilibria being fully revealing. The state may not be fully revealed only if all senders have linear preferences on an edge of the simplex. Linearity along any edge for any sender, let alone all senders, is knife-edge and so for typical sender preferences the state is fully revealed in all equilibria.

Thus far, senders’ preferences have been defined over posterior beliefs. Theorem 1 yields a clean result when preferences can be microfounded by modeling a single receiver choosing from a finite set of actions. Suppose after observing all experiment realizations the receiver takes an action $a$ and receives a payoff $u_r(a, \omega)$ while each sender $i$ gets payoff $u_i(a, \omega)$. We make the generic assumption that no agent is indifferent between any actions at any state and assume the receiver uses a fixed tie-breaking rule when indifferent between actions.

In the space of posteriors, senders have piecewise linear utility functions. These functions are linear on subset $\Delta(\Omega')$ if and only if the receiver has the same best action.
at every state in $\Omega'$. By Proposition 2, $\Omega' \subseteq \Omega$ is not pooled in every equilibrium if and only if the receiver has different best actions at some states in $\Omega'$. This implies a version of Theorem 1: the state is fully revealed in every equilibrium if and only if the receiver has a different best action at every state. Further, when states are pooled in equilibrium, additional information would not change the receiver’s action. Hence the receiver always learns enough take her first best action. See Supplementary Appendix B for formal details.

## 5 Discussion

### Robustness to zero-sumness.

Using standard upper hemicontinuity arguments, we can show Theorem 1 is robust.

**Result.** Suppose senders’ utility functions converge to zero-sum and utilities are sufficiently nonlinear in the limit. Whenever convergent, the information revealed along any sequence of equilibria converges to full revelation.\(^{13}\)

Robustness is one reason we focus on conditions for full revelation in all equilibria. Note that if the limiting preferences are linear on every edge of the simplex, it is still possible for the information revealed in all equilibria to converge to full revelation. Examples are provided and the result is formalized in Supplementary Appendix B.

### Private Information.

Suppose when the game begins each sender receives a private signal. We assume these signals are bounded\(^{14}\) and realized from finite signal conditionally independent experiments. In equilibrium, senders could potentially signal their private information through their choice of experiment. However, for ‘most’ sender preferences the takeaway from Theorem 1 remains the same.

**Result.** Suppose senders receive private signals before the game. In all but a knife-edge case of preferences, the state is fully revealed in every equilibrium.

The knife-edge case includes all senders having linear utility on every edge of the simplex. Details are in Supplementary Appendix B but the logic is similar to the baseline model.

### Experiments without finite signals.

We have focused on finite signal equilibria

\(^{13}\)Convergence for utilities is in the sup norm. For information, the notion is convergence in distribution of the receiver’s posterior.

\(^{14}\)They induce beliefs bounded away from the simplex’s boundaries.
because the results are clean. We show in Supplementary Appendix B that the same intuition applies when senders can choose any experiments. However in this case we have a sufficient condition for full revelation in every equilibrium —satisfied in all but a knife-edge case —but not a necessary one. This result is most similar to that of Dworczak and Pavan (2020).

**Piecewise analytic utility.** Our assumption that utility functions are piecewise analytic is not necessary for Theorem 1. We just need to rule out pathological utility functions that, under our normalization, take values oscillating infinitely about 0 on any edge of the simplex.

**Comparison to Gentzkow and Kamenica (2017).** GK (2017) obtains a very similar result to ours in a different setting. They consider a game in which senders are allowed to arbitrarily correlate their signal realizations and show that nonlinearity on every edge of the simplex guarantees full revelation in every equilibrium. Our conditional independence assumption makes this paper different for three reasons. For applications, conditional independence corresponds to senders conducting investigations independently, which is an important benchmark. Secondly, our results still go through if senders are allowed to correlate (to any extent) their experiments’ realizations. This is because our proofs primarily relied on senders being able to deviate to experiments which are conditionally independent of their opponents’. Even when senders are allowed to correlate signal realizations, this is still possible.

**Result:** Suppose each sender’s strategy space contains only finite signal experiments and includes every finite signal conditionally independent experiment. Then Propositions 1 and 2 and Theorem 1 hold (regardless of what correlated experiments are also available).

This result combined with Theorem 1 nests the zero-sum result of GK (2017) with the caveats that we assume piecewise analytic utility functions and restrict strategy spaces to finite signal experiments.

Finally it’s important to note the forces that deliver the full revelation results in both papers are different. In GK (2017), ability to correlate experiments gives each sender much more control over the posterior. Given any $\Gamma_{-i}$ played by her opponents, a sender $i$ can play a different experiment for each realization of $\Gamma_{-i}$. Senders’ ability to manipulate the posterior belief by belief makes Proposition 2, and hence Theorem 1, much easier to prove. In our setting senders have less control over posteriors; the strongest tool a sender has is using extreme interim beliefs to ensure posteriors are similarly extreme.

\[^{15}\text{The sufficient condition is the same as that in our private information result.}\]
Timing and Relationship to Dworczak and Pavan (2020). See Section 1 for a description of DP (2020). One difference between our model and DP’s is timing. In their model with conditionally independent experiments, nature can condition her choice of experiment on the persuader’s choice. This is similar to a version of our model with two senders moving sequentially. In a sequential version of our model, senders 1, ..., M move in order observing all previous experiment choices (but not realizations); we are interested in subgame perfect Nash Equilibria (SPNE) of this game. Note that for each simultaneous game there are multiple corresponding sequential games, one for each ordering of senders. The following result helps clarify the relationship between the our baseline model and a sequential version.

**Result.** If for \( u_1, ..., u_M \) there is full revelation in every SPNE of the sequential game with the senders moving in some order, then there is full revelation in every equilibrium of the simultaneous game.

We prove the result Supplementary Appendix B and also show that the converse does not hold. The result shows that we are guaranteed full revelation in equilibrium for a (weakly) larger set of sender preferences with simultaneity than with sequentiality. This is in line with Norman and Li (2018a) and Wu (2017), which show that simultaneous persuasion cannot generate less information than sequential.

Differences in timing are one reason our full revelation results are starker than those of DP. Our results are also stronger due to our focus on equilibria in finite signals. Finally, while our results concern the total information revealed by multiple senders, their results emphasize only the information revealed by a single persuader (less so their opponent — nature). Formally, our condition for a set \( \Omega' \subseteq \Omega \) to be not pooled in every equilibrium is equivalent to the persuader in DP’s model having a unique optimal strategy of not pooling \( \Omega' \) or nature minimizing her payoff by not pooling \( \Omega' \).

6 Appendix

Proof Sketch of Proposition 2.

The ‘only if’ direction is straightforward. The approach to proving the ‘if’ direction is similar to that of Proposition 1. Suppose some \( u_j \) is nonlinear on \( \Delta(\Omega') \). Fix a strategy

\[16\] However both papers allow senders to correlate experiments arbitrarily. Wu (2017) also considers zero-sum games, but only shows existence of a fully revealing equilibrium.
profile \((\Gamma_1, \ldots, \Gamma_M)\) that pools \(\Omega'\) and let \(\Gamma\) be the experiment induced by observing the realizations of all \(M\) experiments; we show, via violation of Lemma 1 property (1) (generalized to \(N \geq 2, M \geq 2\), that this is not an equilibrium. To do this it suffices to identify a sender \(i\) and an interim belief \(\bar{x}\) such that \(\mathbb{E}[u_i(\beta(\bar{x}, \Gamma))] > 0\). As in the proof of Lemma 1 property (2), \(i\) can then construct an experiment \(\Gamma'_i\) with support on \(\{\delta_1, \ldots, \delta_N, \bar{x}\}\) and obtain a strictly positive payoff by playing \(\Gamma'_i\) in addition to, conditionally independently, \(\Gamma_i\). In the binary-state case \(i\) was a sender who had an advantage at points closest to the extremes of the \([0, 1]\) interval. This idea is simply generalized when \(N\) and/or \(\Omega'\) are larger.

Let \(\Delta^{int}(\Omega') = \{\gamma \in \Delta(\Omega') : \gamma_l > 0 \ \forall l \in \Omega'\}\); this is the set of beliefs in \(\Delta(\Omega')\) whose support is \(\Omega'\). We first make some simple observations (proofs are trivial and omitted).

**Observation 1.** Conditional on an interim belief \(x \in \Delta^{int}(\Omega')\) being realized, the following hold. (1) \(\beta(x, \Gamma) \in \Delta(\Omega')\) w.p 1 as \(x\) has ruled out states \(\Omega \setminus \Omega'\). (2) \(\beta(x, \Gamma) \in \Delta^{int}(\Omega')\) if and only if \(\Gamma\) assigns positive probability to all states in \(\Omega'\). (3) Realizations of \(\Gamma\) that assign probability 0 to \(\Omega'\) occur w.p. 0.

The case of \(|\Omega'| = 2\) is simple\(^{17}\). Conditional on interim beliefs \(x \in \Delta^{int}(\{l, k\})\), \(\beta(x, \Gamma) \in \Delta(\{l, k\})\) (Observation 1) and so we are in a binary-state world. We can identify sender \(i\) and \(\bar{x}\) similarly Proposition 1 — the extension from two to \(M \geq 2\) senders is straightforward.

To see how to extend the result to \(|\Omega'| > 2\), first consider an example with \(N = 3\) and \(\Omega' = \Omega\). Let \(A = \{\gamma \in \Delta(\Omega) : u_j(\gamma) > 0 \text{ for some } j\}\) be the set of posteriors at which some sender has an advantage and let \(cl(A)\) be its closure. Note that by zero-sumness, at all posteriors outside \(A\) all senders get 0 utility. Suppose \(cl(A)\) is as shown in Figure 2.

\(^{17}\)Note we only need Proposition 2 with \(|\Omega'| = 2\) to prove Theorem 1.
It is convenient to represent each belief $\gamma \in \Delta(\Omega)$ by two ratios: $r_2(\gamma) = \frac{\gamma_2}{\gamma_3}$ and $r_1(\gamma) = \frac{\gamma_1}{\gamma_2 + \gamma_3}$. As in Figure 2 $cl(A)$ does not touch the boundaries of $\Delta(\Omega)$, we can find $\bar{r}_2 = \max_{\gamma \in cl(A)} r_2(\gamma)$ with $0 < \bar{r}_2 < \infty$; the dotted line shows points $\gamma \in \Delta(\Omega)$ with $r_2(\gamma) = \bar{r}_2$. Amongst the points in $\arg\max_{\gamma \in cl(A)} r_2(\gamma)$, we can then identify a unique point, $\bar{\beta} \in \Delta^{int}(\Omega)$, with the largest $r_1$ ratio (see Figure 2). We will find $x^* \in \Delta^{int}(\Omega)$ conditional on which the posterior will either fall outside of $cl(A)$ (yielding payoff 0) or will equal $\bar{\beta}$. A sender who has an advantage at or near $\bar{\beta}$ will hence be able to get strictly positive utility by generating an interim belief at, or near, $x^*$.

Let $Z = \{\gamma \in supp[\Gamma] : \gamma_1, \gamma_2, \gamma_3 > 0\}$ (note $Z$ is nonempty). Conditional on generating $x \in \Delta^{int}(\Omega)$, all beliefs in the support of $\Gamma$ that are not in $Z$ can be ignored as they will result in posteriors on the boundaries of $\Delta(\Omega)$ and hence, in the example, yield utility 0. Consider interim beliefs $x \in \Delta\{2, 3\}$; for all $y \in Z$, as $x \to \delta_2$, $\beta(x, y) \to \delta_2 \implies r_2(\beta(x, y)) \to \infty$ and as $x \to \delta_3$, $\beta(x, y) \to \delta_3 \implies r_2(\beta(x, y)) \to 0$. The finiteness of $Z$, continuity of Bayesian updating, and intermediate value theorem together imply that there exists $x' \in \Delta\{2, 3\}$ such that $\min_{y \in Z} r_2(\beta(x', y)) = \bar{r}_2$. Next consider taking a convex combination of $x'$ and $\delta_1$: $\lambda \delta_1 + (1 - \lambda)x'$; as $\lambda \to 1$, $\beta(\lambda \delta_1 + (1 - \lambda)x', y) \to \delta_1$ for all $y \in Z$. While $r_1(\lambda \delta_1 + (1 - \lambda)x')$ changes in $\lambda$, $r_2(\lambda \delta_1 + (1 - \lambda)x', y)$ does not as $\lambda$ does not affect the relative probabilities assigned to states 2 and 3. As a consequence, $\lambda$ also does not affect $r_2(\beta(\lambda \delta_1 + (1 - \lambda)x', y))$ for any $y \in Z$. Hence by the intermediate

---

18These are not well defined for all $\gamma$, but this won’t be a problem in our example.
value theorem we can find $\lambda^* \in (0, 1)$ and $x^* = \lambda^* \delta_1 + (1 - \lambda^*) x'$ such that:

\[
\begin{align*}
(1) \quad \min_{y \in Z} r_2(\beta(x^*, y)) &= \bar{r}_2 \\
(2) \quad \min_{y' \in \text{argmin}_{y \in Z} r_2(\beta(x^*, y))} r_1(\beta(x^*, y')) &= r_1(\bar{\beta}) \\
(3) \quad \text{Hence for all } y \in Z, \text{ we have either } \beta(x^*, y) = \bar{\beta} \text{ or } \beta(x^*, y) \notin cl(A). \text{ Posteriors for which the latter is true will fall either to the left of the dotted line in Figure } 2 \text{ or on the dotted line and (strictly) above } \bar{\beta}. \text{ If } \bar{\beta} \in A, \text{ then set } x = x^* \text{ and } E\left[u_i(\beta(x, \Gamma))\right] > 0 \text{ for some } i. \text{ If } \bar{\beta} \notin A, \text{ then there exists some } \beta' \in A \text{ close to } \bar{\beta} \text{ and } x \text{ close to } x^* \text{ such that either } \beta(x, y) = \beta' \text{ or } \beta(x, y) \notin cl(A) \text{ for all } y \in Z.
\end{align*}
\]

The steps in proving the existence of $x$ generalize beyond this example. Due to Observation 1, the same logic applies for $N > 3$ and $\Omega' = \{1, 2, 3\}$ whenever $cl(A) \cap \Delta(\Omega') \subset \Delta^{int}(\Omega')$ (i.e. $cl(A)$ does not touch the boundaries of $\Delta(\Omega')$). If $|\Omega'| = K > 3$, whenever $cl(A) \cap \Delta(\Omega') \subset \Delta^{int}(\Omega')$ we can extend the ratio representation of a belief $\gamma$ to ratios $r_1(\gamma), ..., r_{K-1}(\gamma)$ and iterate the same procedure to find $\bar{r}_1, ..., \bar{r}_{K-2}$ and $\bar{\beta}$. $x^*$ will now have to satisfy $K - 1$ equations analogous to those in equation 3. The case of $cl(A) \cap \Delta(\Omega') \notin \Delta^{int}(\Omega')$ requires additional details and we leave it to Supplementary Appendix A.

**References**

Marco Battaglini. Multiple referrals and multidimensional cheap talk. *Econometrica*, 70(4):1379–1401, 2002.

Paul Milgrom and John Roberts. Relying on the information of interested parties. *The RAND Journal of Economics*, pages 18–32, 1986.

Hyun Song Shin. Adversarial and inquisitorial procedures in arbitration. *The RAND Journal of Economics*, pages 378–405, 1998.

Winand Emons and Claude Fluet. Strategic communication with reporting costs. *Theory and Decision*, 87(3):341–363, 2019.

Navin Kartik, Frances Xu Lee, and Wing Suen. Investment in concealable information by biased experts. *The RAND Journal of Economics*, 48(1):24–43, 2017.
A Supplementary Appendix A: Proofs

Definitions and Facts. The following definitions and facts are used in both Supplementary Appendix A and B.

Let $P$ be the set of Bayes-plausible finite support elements of $\Delta(\Delta(\Omega))$. It will be convenient to talk about a strategy for sender $i$ as a choice of interim belief $\Gamma_i$ with probability mass function $p_i \in P$ (in the text of the paper we did not introduce notation for the distribution of $\Gamma_i$).

For any strategy profile $(\Gamma_1, \ldots, \Gamma_M)$ and subset of senders $S \subseteq \{1, \ldots, M\}$, let the random variable $\Gamma_S$ be the receiver’s belief after observing realizations of $\{\Gamma_j\}_{j \in S}$ but not the realizations of $\{\Gamma_j\}_{j \in S^c}$; let $p_S$ be it’s probability mass function and $p_S(\cdot|\omega = k)$ be its
probability mass function conditional on the state being \( k \). Let \( \Gamma_{-S} \) and \( p_{-S} \) be the same objects for the complementary set of senders.

For any disjoint subsets of senders \( S, S' \subset \{1, \ldots, M\} \) and any fixed strategy profile \((\Gamma_1, \ldots, \Gamma_M)\), let \( p_{S'}(\cdot|x) \) be the probability mass function of \( \Gamma_{S'} \) conditional on \( \Gamma_S = x \).

\[
p_{S'}(y|x) = \sum_{k \in \Omega} p_{S'}(y|\omega = k, x) Pr(\omega = k|\Gamma_S = x) = \sum_{k \in \Omega} p_{S'}(y|\omega = k) x_k = \sum_{k \in \Omega} \frac{Pr(\omega = k|y)p_{S'}(y)}{Pr(\omega = k)} x_k
\]

(A.1)

Where the second equality comes from conditional independence of \( \Gamma_S \) and \( \Gamma_{S'} \). Claim [A.1] tells us that conditional on \( \Gamma_S \), with probability 1 \( \Gamma_{S'} \) assigns positive probability to at least one state that \( \Gamma_S \) assigns positive probability to (i.e. \( \Gamma_{S'} \) cannot contradict \( \Gamma_S \)). This is a simple implication of Bayesian updating.

**Claim A.1.** For any disjoint subset of senders \( S, S', \Gamma_S, \Gamma_{S'} \), and \( x \in \Delta(\Omega) \): \( p_{S'}(y|x) = 0 \) for all \( y \) s.t. \( y_l = 0 \) for all \( l \in \Omega \) for which \( x_l > 0 \). Further, there exists \( y \in \text{supp}[\Gamma_{S'}] \) such that \( p_{S'}(y|x) > 0 \).

**Proof.** The first statement, that \( p_{S'}(y|x) = 0 \) for all \( y \) such that \( y_l = 0 \) for all \( l \) for which \( x_l > 0 \), follows immediately from Equation [A.1]. The second statement follows from Bayes-plausibility of \( \Gamma_{S'} \). For every \( l \) such that \( x_l > 0 \), as \( \pi_l > 0 \), there exists \( y \in \text{supp}[\Gamma_{S'}] \) with \( y_l \geq \pi_l > 0 \); by Equation [A.1] \( p(y|x) > 0 \) for such a \( y \). \( \square \)

Let \( \beta_l(x^1, \ldots, x^M) = Pr(\omega = l|x^1, \ldots, x^M) \) be the receiver’s posterior belief that \( \omega = l \) after observing experiment realizations \( \Gamma_1 = x^1, \ldots, \Gamma_M = x^M \). By Bayes rule:

\[
\beta_l(x^1, \ldots, x^M) = \frac{Pr(\Gamma_1 = x^1, \ldots, \Gamma_M = x^M|\omega = l) Pr(\omega = l)}{Pr(\Gamma_1 = x^1, \ldots, \Gamma_M = x^M)} = \frac{\Pi_{i=1}^M p_i(x^i|\omega = l) \pi_l}{\sum_{i=1}^N \Pi_{i=1}^M p_i(x^i|\omega = k) Pr(\omega = k)}
\]

(A.2)

Where the second equality uses the conditional independence of \( \Gamma_1, \ldots, \Gamma_M \). Note that \( \beta_l \) is not well defined when for each state \( k \in \Omega \) there exists sender \( j \) with \( x^j_k = 0 \). However
it is straightforward to see by applying Claim A.1 that such a realization of \((\Gamma_1, \ldots, \Gamma_M)\) occurs with zero probability; after viewing the realizations of any number of experiments, the Bayesian receiver will have a well defined posterior w.p. 1.

For any strategy profile \((\Gamma_1, \ldots, \Gamma_M)\) and disjoint sets of senders \(S_1, \ldots, S_T\), we similarly define the receiver’s posterior as a function of interim belief realizations from each experiment: \(\{\Gamma_{S_s} = y^{S_s}\}_{s=1,\ldots,T}\).

\[
\beta_l(y^{S_1}, \ldots, y^{S_T}) = \frac{\prod_{s=1}^{T} y_{l}^{S_s}/\pi_l^{T-1}}{\sum_{k=1}^{N} \prod_{s=1}^{T} y_{k}^{S_s}/\pi_k^{T-1}} \tag{A.3}
\]

Note: \(\Gamma_{S_1 \cup \ldots \cup S_T} = \beta(\Gamma_{S_1}, \ldots, \Gamma_{S_T})\), as both define the receiver’s belief after observing realizations of \(\Gamma_{S_1}, \ldots, \Gamma_{S_T}\).

Claim A.2 shows that if any subset of experiments in a strategy profile generate an interim belief in \(\Delta(\Omega')\) then the posterior will fall in \(\Delta(\Omega')\) w.p. 1.

**Claim A.2.** For any strategy profile \((\Gamma_1, \ldots, \Gamma_M)\), disjoint subsets of senders \(S_1, \ldots, S_T\), and states \(\Omega' \subseteq \Omega\), if \(\Gamma_{S_1} \in \Delta(\Omega')\) then \(\beta(\Gamma_{S_1}, \ldots, \Gamma_{S_T}) \in \Delta(\Omega')\) w.p. 1.

**Proof.** This can be seen from the definition of \(\beta(y^{S_1}, \ldots, y^{S_T})\) which implies \(\beta_l(y^{S_1}, \ldots, y^{S_T}) = 0\) for all \(l \notin \Omega'\). After observing \(\Gamma_{S_1} \in \Delta(\Omega')\), the receiver updates to an interim belief assigning 0 probability to all states outside of \(\Omega'\). No additional information can change this.

**Claim A.3.** For any strategy profile \((\Gamma_1, \ldots, \Gamma_M)\), \(\Omega' \subseteq \Omega\), and any subsets of senders \(S\): If \(\Gamma_S\) does not pool \(\Omega'\) then \((\Gamma_1, \ldots, \Gamma_M)\) does not either.

**Proof.** Let \(S' = \{1, \ldots, M\} \setminus \{S\}\). As \(\Gamma_S\) does not pool \(\Omega'\), then \(Pr(\Gamma_S = y : \text{s.t. } y_l > 0 \ \forall l \in \Omega') = 0\). If \(\Gamma_S = y\), \(\Gamma_{S'} = y'\), then by Equation A.3 if \(y_l = 0\) then \(\beta_l(y, y') = 0\). Hence as w.p. 1 \(\Gamma_S\) assigns 0 probability to at least one state in \(\Omega'\), \(\beta(\Gamma_S, \Gamma_{S'})\) does as well and so \((\Gamma_1, \ldots, \Gamma_M)\) does not pool \(\Omega'\).

**A.1 Section 2**

**Normalization of utility functions.** Here we show that we can normalize \(u_i(\delta_l) = 0\) for all \(i = 1, \ldots, N, \ l = 1, \ldots, M\) without changing senders’ preferences over strategy profiles or the zero-sumness of the game.
Suppose senders have utility functions \( u'_1, ..., u'_M \) with \( u'_1 + ... + u'_M = 0 \) for all \( \beta \). For \( i = 1, ..., M \) let \( \alpha_i : \Delta(\Omega) \to \mathbb{R} \) be the affine function \( \alpha_i(\beta) = -\sum_l \beta_l u_i(\delta_l) \). For each \( i \), define the function \( u_i : \Delta(\Omega) \to \mathbb{R} \) as \( u_i = u'_i + \alpha_i \). Then \( u_i(\delta_l) = 0 \) for all \( i, l = 1, ..., N \). Note that utility function \( u_i \) preserves the same preferences over strategy profiles as \( u'_i \); as for any strategy profile \( (\Gamma_1, ..., \Gamma_M) \), \( \mathbb{E}_{\rho_1,...,\rho_M}[u_i(\beta(\Gamma_1, ..., \Gamma_M))] = \mathbb{E}_{\rho_1,...,\rho_M}[u'_i(\beta(\Gamma_1, ..., \Gamma_M))] - \sum_l \pi_l u_i(\delta_l) \) - the latter term is a constant. Finally note that \( \alpha_1(\beta) + ... + \alpha_M(\beta) = 0 \) for all \( \beta \in \Delta(\Omega) \), so \( u_1 + ... + u_M = 0 \).

### A.2 Section 3

**Lemma 1. General Case:** In any equilibrium \( (\Gamma_1, ..., \Gamma_M) \): (1) \( U_i(\Gamma_1, ..., \Gamma_M) = 0 \) for \( i = 1, ..., M \) and (2) \( W_i(x) \leq 0 \) for all \( x \in \Delta(\Omega) \) and \( i = 1, ..., M \).

**Proof.** We prove (1) first. First note that as the functions \( \{u_i\}_{i=1,...,M} \) are zero-sum, so are \( \{U_i\}_{i=1,...,M} \). To see this, fix any \( (\Gamma'_1, ..., \Gamma'_M) \) and let \( F' \) be the random variable representing the receiver’s posterior after viewing all \( M \) experiment realizations and \( p' \) be its pmf. Then \( \sum_{i=1}^M U_i(\Gamma'_1, ..., \Gamma'_M) = \sum_i \sum_{\beta \in \text{supp}[\Gamma'_i]} u_i(\beta)p'(\beta) = \sum_{\beta \in \text{supp}[\Gamma'_i]} \beta \sum_i u_i(\beta) = 0 \). Next note that any sender \( i \) choosing \( \Gamma_i = \Gamma_{FR} \) yields \( U_i(\Gamma_{FR}, \Gamma_{-i}) = 0 \) for all \( \Gamma_{-i} \). Hence in any equilibrium \( (\Gamma_1, ..., \Gamma_M) \), each sender gets \( U_i(\Gamma_1, ..., \Gamma_M) \geq 0 \). Finally no sender can have \( U_i(\Gamma_1, ..., \Gamma_M) > 0 \) as this would imply \( U_j(\Gamma_1, ..., \Gamma_M) < 0 \) for some \( j \neq i \).

For (2) we prove the contrapositive. Fix any sender \( i \) and opponents’ strategy profile \( \Gamma_{-i} \) such that \( W_i(x) > 0 \) for some \( x \in \Delta(\Omega) \); we will show \( \Gamma_{-i} \) cannot be played in equilibrium. Consider the strategy \( \Gamma'_i \) with distribution \( p'_i \) and support only on \( x \) and \( \{\delta_l\}_{l=1,...,N} \). Set \( p'_i(x) > 0 \) small enough such that \( \pi_l - x p'_i(x) > 0 \) for all \( l \) (such a value exists as \( \pi_l > 0 \) for all \( l \)). Bayes-plausibility implies we must have: \( p'_i(\delta_l) = \pi_l - x p'_i(x) > 0 \) for all states \( l \) (as the support of \( \Gamma'_i \) is \( \{x, \delta_1, ..., \delta_M\} \)). Then \( U_i(\Gamma'_i, \Gamma_{-i}) = W_i(x)p'_i(x) + \sum_l u_i(\delta_l)p'_i(\delta_l) > 0 \). Property (1) of the lemma implies \( \Gamma_{-i} \) cannot be played in equilibrium; hence \( W_i(x) \leq 0 \) for all \( i \) in any equilibrium.

**Proposition 1.** Proposition 1 is implied by Proposition 2, proven in the next section. However, as the proving Proposition 1 is much simpler than Proposition 2, we provide a proof here for exposition.

**Proof.** The ‘only if’ direction is trivial. If \( u_1 \) is linear so is \( u_2 \). Under our normalization of \( u_1(0) = u_1(1) = 0 \), this implies that \( u_1(\beta) = u_2(\beta) = 0 \) for all \( \beta \in [0, 1] \). Hence both senders are indifferent across all strategy profiles and any \((\Gamma_1, \Gamma_2)\) is an equilibrium.
Now for the ‘if’ direction. Suppose $u_1$ (and hence $u_2$) are nonlinear. Let $q = \sup\{\beta \in [0,1] : u_1(\beta) \neq 0\}$ be the supremum of posteriors at which $u_1, u_2$ are nonlinear. We prove the result in two cases.

**Case 1:** $q = 1$. If $q = 1$, then by the piecewise analyticity of $u_1, u_2$, there exists $r < 1$ such that either $u_1(\beta) > 0$ or $u_1(\beta) < 0$ for all $\beta \in [r,1)$. If $u_1(\beta) < 0$ then $u_2(\beta) > 0$, and so WLOG (we can always relabel senders) we assume $u_1(\beta) > 0$ for all $\beta \in [r,1)$. Suppose for contradiction that sender 2 plays a non-fully revealing strategy $\Gamma_2$ in some equilibrium. As $\Gamma_2 \neq \Gamma^{FR}$, $\Pr(0 < \Gamma_2 < 1) > 0$; let $\underline{\beta} = \min \text{supp}[\Gamma_2] \setminus \{0,1\} \in (0,1)$ be in the smallest interior belief in the support of $\Gamma_2$. Using the definition of $\beta(x,y)$, define $\underline{x}$ by $\beta(\underline{x},\underline{\beta}) = r$. Conditional on $\Gamma_1 = x \in [\underline{x},1)$, $\beta(x,y) \in [r,1)$ for all interior $y$ in $\Gamma_2$’s support. But then for all $x \in [\underline{x},1)$ we have:

$$W_1(x) = u_1(\beta(x,0)) \Pr(\Gamma_2 = 0|\Gamma_1 = x) + u_1(\beta(x,1)) \Pr(\Gamma_2 = 1|\Gamma_1 = x) + \\
\sum_{y \in \text{supp}[\Gamma_2] \setminus \{0,1\}} u_1(\beta(x,y)) \Pr(\Gamma_2 = y|\Gamma_1 = x) > 0.$$

This contradicts Lemma 1 property (2) and hence $\Gamma_2 = \Gamma^{FR}$ in all equilibria.

**Case 2:** $q < 1$. We break this case into two subcases.

First suppose $u_1(q) \neq 0$. WLOG assume $u_1(q) > 0$ (if not then $u_2(q) > 0$). Suppose for contradiction $\Gamma_2 \neq \Gamma^{FR}$ in some equilibrium. Then let $r = q$ and define $\underline{\beta}, \underline{x}$ as before. Again Lemma 1 property (2) is violated as:

$$W_1(\underline{x}) = u_1(\beta(x,0)) \Pr(\Gamma_2 = 0|\Gamma_1 = \underline{x}) + u_1(\beta(x,1)) \Pr(\Gamma_2 = 1|\Gamma_1 = \underline{x}) + \\
\sum_{y \in \text{supp}[\Gamma_2] \setminus \{0,1,\underline{\beta}\}} u_1(\beta(x,y)) \Pr(\Gamma_2 = y|\Gamma_1 = \underline{x}) > 0 + u_1(\beta(\underline{x},\underline{\beta})) \Pr(\Gamma_2 = \underline{\beta}|\Gamma_1 = \underline{x}) > 0.$$

Now suppose $u_1(q) = u_2(q) = 0$. Then by piecewise analyticity of utilities either $u_1(q^-) > 0$ or $u_2(q^-) > 0$. WLOG assume $u_1(q^-) > 0$ and suppose for contradiction $\Gamma_2 \neq \Gamma^{FR}$ in some equilibrium. Define $\underline{\beta}$ as before. There exists $r < q$ and $\underline{x}$ such that
$u_1 > 0$ on interval $[r, q)$ and $\beta(x, y) = r$. Then we have $W_1(x) > 0$, violating Lemma 1 property (2).

\[\square\]

A.3 Proof of Proposition 2

A.3.1 ‘Only if’ direction.

Suppose for some $\Omega' \subseteq \Omega$ all senders have linear utilities on $\Delta(\Omega')$. Let $x' \in \Delta(\Omega')$ with $x'_k = \frac{\pi_k}{\sum_{l \in \Omega'} \pi_l} \forall k \in \Omega'$. Consider the experiment $\Gamma'$ with $Pr(\Gamma' = \delta_l) = \pi_l$ for all $l \notin \Omega'$ and $Pr(\Gamma' = x') = \sum_{n \in \Omega'} \pi_n$. $\Gamma'$ is Bayes-plausible and has finite support. The strategy profile $(\Gamma', ..., \Gamma')$ is an non-fully revealing equilibrium. To see this consider a sender $i$’s incentive to deviate. If $\omega \in \Omega'$ then $\Gamma_i \in \Delta(\Omega') \implies \beta(\Gamma_1, ..., \Gamma_M) \in \Delta(\Omega')$ w.p. 1 (Claim 2); as $u_i$ is linear on $\Delta(\Omega')$, $i$ has no profitable deviation conditional on $\omega \in \Omega'$. Conditional on $\omega \notin \Omega'$, $\Gamma_i$ fully reveals the state and no deviation from $i$ can change this.

A.3.2 ‘If’ direction.

Let $\Delta^\text{int}(\Omega') = \{\gamma \in \Delta(\Omega') : \gamma_l > 0 \forall l \in \Omega'\}$; this is the set of beliefs in $\Delta(\Omega')$ whose support is $\Omega'$.

We first prove the result for the case of $|\Omega'| = 2$. This case is simpler than the case of $|\Omega'| > 2$ and is of particular interest because Theorem 1 only relies on Proposition 2 with $|\Omega'| = 2$.

Proof for $|\Omega'| = 2$.

Proof. WLOG let $\Omega' = \{1, 2\}$. Suppose some sender $i$ has $u_i$ nonlinear on $\Delta(\{1, 2\})$. For each sender $j'$ let $r'_{j'} = \sup\{t \in [0, 1] : u_{j'}(t\delta_2 + (1-t)\delta_1) > 0\}$. Let $r' = \max_{j' = 1, ..., M} r'_{j'}$, and $j \in \arg\max_{j' = 1, ..., M} r'_{j'}$. As $u_i$ is nonlinear there exists $\gamma \in \Delta^\text{int}(\{1, 2\})$ with $u_i(\gamma) \neq 0$. If $u_i(\gamma) < 0$ then $u_i'(\gamma) > 0$ for some sender $i'$ (zero-sumness); otherwise $u_i(\gamma) > 0$. Regardless, we have that $r'$ exists and is $> 0$.

WLOG let $j = 1$. We prove the ‘if’ direction in 2 cases.

Case 1: $r' = 1$. $u_1$ is piecewise real analytic and so $\Delta(\{1, 2\})$ can be partitioned into...
intervals each of which $u_1$ is real analytic on. Each $\gamma \in \Delta(\{1, 2\})$ can be represented by scalar $\gamma_2$—how close it is to $\delta_2$. For some $a \in [0, 1)$, $u_1$ is real analytic on an interval $\{\gamma \in \Delta(\{1, 2\}) : \gamma_2 \in (a, 1)\}$ (this $a \in [0, 1)$ is not unique; any selection will do). This implies that there are a finite number of points (possibly zero) on $\{\gamma \in \Delta(\{1, 2\}) : \gamma_2 \in (a, 1)\}$ at which $u_1 = 0$. As $r' = 1$, there exists $r \in (a, 1)$ such that $u_1(\gamma) > 0$ for all $\gamma \in \Delta(\{1, 2\})$ s.t. $\gamma_2 \in [r, 1)$ (again, this $r$ will not be unique; any selection will do).

Suppose, for contradiction, that some equilibrium $(\Gamma_1, ..., \Gamma_M)$ pools $\{1, 2\}$. Then we must have $Pr(\Gamma_{-1} = y \text{ s.t. } y_1, y_2 > 0) > 0$, i.e. $\Gamma_{-1}$ pools $\{1, 2\}$, by Claim A.3.

Let $Z = \{y \in supp[\Gamma_{-1}] : y_1, y_2 > 0\}$; $Z$ is nonempty. For any $x \in \Delta^{int}(\{1, 2\})$ and $y \in Z$, we have $p_{-1}(y|x) > 0$ and $\beta(x, y) \in \Delta^{int}(\{1, 2\})$ (by Claim A.2 and equation A.2). Note that as $x \to \delta_1$ we have $\beta(x, y) \to \delta_1 \Rightarrow \beta_2(x, y) \to 0$ and as $x \to \delta_2$ we have $\beta(x, y) \to \delta_2 \Rightarrow \beta_2(x, y) \to 1$. As $Z$ is finite, this implies $\min_{y \in Z} \beta_2(x, y)$ goes to 0 as $x \to \delta_1$ and $\min_{y \in Z} \beta_2(x, y)$ goes to 1 as $x \to \delta_2$. As for all $y \in Z \beta_2(x, y)$ is continuous in $x$ for $x \in \Delta^{int}(\{1, 2\})$, $\min_{y \in Z} \beta_2(x, y)$ is also continuous in $x$ for $x \in \Delta^{int}(\{1, 2\})$. By the intermediate value theorem there exists $x \in \Delta^{int}(\{1, 2\})$ such that $\min_{y \in Z} \beta_2(x, y) = r$.

Note that by equation A.2, $\beta(x, y) = \delta_1$ for all $y \in supp[\Gamma_{-1}]$ with $y_1 > 0$ and $y_2 = 0$; similarly $\beta(x, y) = \delta_2$ for all $y \in supp[\Gamma_{-1}]$ with $y_2 > 0$ and $y_1 = 0$. Finally by Claim A.1 $p_{-1}(y|x) = 0$ for all $y \in supp[\Gamma_{-1}]$ with $y_1 = y_2 = 0$.

Putting this together:

$$W_1(\bar{x}) = \sum_{y \in supp[\Gamma_{-1}], y_1=0, y_2>0} u_1(\delta_2) p_{-1}(y|\bar{x}) + \sum_{y \in supp[\Gamma_{-1}], y_2=0, y_1>0} u_1(\delta_1) p_{-1}(y|\bar{x})$$

$$+ \sum_{y \in Z, y_1=0, y_2>0} u_1(\beta(x, y)) p_{-1}(y|\bar{x}) > 0$$

This contradicts Lemma 1 property (2). Hence no equilibrium can pool $\{1, 2\}$.

**Case 2**: $r' < 1$. First, if $u_1(r') > 0$, then set $r = r'$ and derive $\bar{x}$ just as in Case 1. As in Case 1, we have $W_1(\bar{x}) > 0$, violating Lemma 1 property (2). Next if $u_1(r') < 0$, then some sender $i \neq 1$ must have $u_i(r') > 0$ (zero-sumness); we can relabel sender $i$ to 1 and repeat the same argument.

Next assume $u_1(r') = 0$. Now for some $a \in [0, r')$, $u_1$ is real analytic on an interval
\( \{ \gamma \in \Delta(\{1, 2\}) : \gamma_2 \in (a, r') \} \) (this \( a \in [0, r') \) is not unique; any selection will do). This implies that there are a finite number of points (possibly zero) on \( \{ \gamma \in \Delta(\{1, 2\}) : \gamma_2 \in (a, r') \} \) at which \( u_1 = 0 \). This implies that there exists \( r \in (a, r') \) such that \( u_1(\gamma) > 0 \) for all \( \gamma \in \Delta(\{1, 2\}) \) with \( \gamma_2 \in [r, r') \) (again, this \( r \) will not be unique; any selection will do). Note that \( u_1(\gamma) \geq 0 \) for all \( \gamma \in \Delta(\{1, 2\}) \) with \( \gamma_2 \geq r \).

Suppose, for contradiction, that in some equilibrium \( (\Gamma_1, ..., \Gamma_M) \) pools \( \{1, 2\} \). We follow identical steps in defining \( Z \) and \( x \). Note that for all \( y \in Z \), \( \beta_k(x, y) \in [r, 1) \implies u_1(\beta(x, y)) \geq 0 \). By the definition of \( x \), there exists \( y \in Z \) such that \( \beta(x, y) = r \implies u_1(\beta(x, y)) > 0 \). For all \( y \in supp[\Gamma |-1] \setminus Z \) either \( \beta(x, y) \in \{\delta_1, \delta_2\} \) or \( p_-1(y|x) > 0 \). Hence \( W_1(x) > 0 \), violating Lemma 1 property (2). \( \square \)

Now we proceed with the analysis for \( |\Omega'| \geq 2 \).

**General Analysis.**

Suppose some \( u_j \) is nonlinear on \( \Delta(\Omega') \). Fix a strategy profile \( (\Gamma_1, ..., \Gamma_M) \) that pools \( \Omega' \); let \( \Gamma \) be the experiment induced by observing the realizations of all \( M \) experiments and \( p \) be its probability mass function. We show, via violation of Lemma 1 property (1), that \((\Gamma_1, ..., \Gamma_M)\) is not an equilibrium. To do this it suffices to identify a sender \( j \) and an interim belief \( x \) such that when \( \Gamma_1, ..., \Gamma_M \) are played, conditional on generating interim belief \( x \) (from an experiment played additionally and conditionally independently to \( \Gamma_j \) \( j \) gets a strictly positive expected payoff: \( : \mathbb{E}[u_i(\beta(x, \Gamma))] > 0 \). As in the proof of Lemma 1 property (2), \( j \) can then construct an experiment \( \Gamma'_j \) with support on \( \{\delta_1, ..., \delta_N, x\} \) and obtain a strictly positive payoff by playing \( \Gamma'_j \) in addition to, conditionally independently, \( \Gamma_j \). The remainder of the proof shows that such a sender \( j \) and \( x \) exist.

For \( i = 1, ..., M \) let \( A_i = \{ \gamma \in \Delta(\Omega) : u_i(\gamma) > 0 \} \) and \( D_i = \{ \gamma \in \Delta(\Omega) : u_i(\gamma) < 0 \} \) be the sets of posteriors at which \( i \) has an advantage and disadvantage respectively. Let \( A = \cup_i A_i \) be the union of these advantage sets (also equal to the union of disadvantage sets as utilities are zero-sum) and \( cl(A) \) be its closure.

We say a subset of states \( \Theta \subseteq \Omega \ (|\Theta| > 1 \) is minimal if \( \Theta \cap A \neq \emptyset \) and \( \Theta' \cap A = \emptyset \) for all \( \Theta' \subset \Theta \). Note that if \( A \) is empty, then there are no minimal subsets. Meanwhile if \( A \) is nonempty, any subset of states that intersects \( A \) (i.e. any set \( \Theta \) for which some \( u_i \) is nonlinear on \( \Delta(\Theta) \)) is either minimal or has a minimal subset:

**Claim A.4.** Every subset \( \Theta \subseteq \Omega \) for which \( u_i \) (for some \( i \)) is nonlinear on \( \Delta(\Theta) \) is either minimal or has a subset \( \Theta' \) that is minimal.
Proof. If for some \( i \) \( u_i \) is nonlinear on \( \Delta(\Theta) \), then \( A \cap \Delta(\Theta) \neq \emptyset \). Either \( \Theta \) is minimal, or there exists a subset \( \Theta' \subset \Theta \) that intersects \( A \). Now set \( \Theta = \Theta' \) and repeat this process until \( \Theta \) is minimal; it must be minimal at some point because \( \Omega \) is finite and states are removed from \( \Theta \) each iteration. \( \square \)

By Claim A.4 it is sufficient to prove Proposition 2 for minimal \( \Omega' \) alone. If all minimal sets cannot be pooled in equilibrium, then any set on which there are nonlinear sender preferences cannot be pooled, as all such sets have a minimal subset. Henceforth we assume \( \Omega' \) is minimal.

Let \( |\Omega'| = K \leq N \) and WLOG let \( \Omega' = \{1, \ldots, K\} \).

It is convenient for us to represent any belief \( \gamma \in \Delta(\Omega') \) by the ratios \( (r_1(\gamma), \ldots, r_{K-1}(\gamma)) \in (\mathbb{R}_+ \cup \{\infty\})^{K-1} \), where for \( k = 1, \ldots, K-1 \): (1) \( r_k(\gamma) = \frac{\gamma_k}{1-\sum_{l=1}^{k} \gamma_l} \) when \( 1-\sum_{l=1}^{k} \gamma_l \) is nonzero, (2) when this doesn’t hold \( r_k(\gamma) = \infty \) if \( \gamma_k > 0 \) and \( r_k(\gamma) = 0 \) if \( \gamma_k = 0 \). We call this the ratio representation of \( \gamma \). The ratio \( r_k(\gamma) \) tells us the ratio of probability mass assigned to state \( k \) by \( \gamma \) to the mass assigned to states \( k+1, \ldots, K \).

Lemma A.1. Note for any \( \gamma, \gamma' \in \Delta(\Omega') \) we have \( r_k(\gamma) = r_k(\gamma') \) for all \( k = 1, \ldots, K-1 \) if and only if \( \gamma = \gamma' \); that is, ratio representations for beliefs in \( \Delta(\Omega') \) are unique.

Proof. The ‘if’ direction is trivial; we prove the ‘only if’ direction as follows. First suppose \( r_k(\gamma) < \infty \) for all \( k = 1, \ldots, K-1 \). This implies that \( 1-\sum_{l=1}^{k} \gamma_l \) is nonzero for all \( k \) (or else, let \( k' \) be the minimum \( k \) for which \( 1-\sum_{l=1}^{k} \gamma_l = 0 \); but then we must have \( \gamma_k > 0 \) \( \Rightarrow r_k(\gamma) = \infty \) — contradiction). But then from its definition, \( r_1 \) uniquely pins down \( \gamma_1 \) \((\gamma_1 = \frac{r_1(\gamma)}{1-r_1(\gamma)})\), after which \( r_2 \) pins down \( \gamma_2 \), \( r_{K-1} \) pins down \( \gamma_{K-1} \), and \( \gamma_K \) is pinned down by \( 1 = \sum_{l=1}^{K} \gamma_l \). Now suppose \( r_{k''}(\gamma) = \infty \) for some \( k'' \). Note that this implies \( \gamma_k = 0 \) for all \( k < k'' \); further, \( 1-\sum_{l=1}^{k} \gamma_l > 0 \) for all \( k < k'' \) and hence \( r_k(\gamma) < \infty \) for all \( k < k'' \). Then \( \gamma_1, \ldots, \gamma_{k''-1} \) are uniquely pinned down by using the definitions of \( r_1(\gamma), \ldots, r_{k''-1}(\gamma) \) (just as in the previous case). \( \gamma_{k''} \) is pinned down by \( 1 = \sum_{l=1}^{K} \gamma_l \). \( \square \)

The the continuity of \( r_k(\gamma) \) on part of \( \Delta(\Omega') \) will be useful later:

Claim A.5. For \( k = 1, \ldots, K-1 \), \( r_k(\gamma) \) is continuous in \( \gamma \) for \( \gamma \in \Delta(\{k, \ldots, K\}) \setminus \{\delta_k\} \).

Proof. \( r_k(\gamma) = \frac{\gamma_k}{1-\sum_{l=1}^{k} \gamma_l} \). As \( \gamma \in \Delta(\{k, \ldots, K\}) \), the denominator is strictly positive when \( \gamma_k < 1 \) and so \( r_k(\gamma) \) is continuous in \( \gamma \) on this domain. \( \square \)

The following simple results will be useful.
Lemma A.2. Suppose $K > 2$. For any $1 < L < K$, let $x \in \Delta(\{L, \ldots, K\})$, $x' \in \Delta(\{1, \ldots, L-1\})$ and $y \in \Delta(\Omega)$. If $\beta(x, y)$ is well defined then $r_k(\beta(x, y)) = r_k(\beta(x, y))$ for all $k = L, \ldots, K-1$ and $\lambda \in [0, 1)$.

Proof. For any $n \in \Omega$,

$$
\beta_n(\lambda x' + (1 - \lambda)x, y) = \frac{(\lambda x'_n + (1 - \lambda)x_n)y_n}{\sum_{n'=1}^N (\lambda x'_{n'} + (1 - \lambda)x_{n'})y_{n'}}.
$$

For $k \geq L$:

$$
r_k(\beta(\lambda x' + (1 - \lambda)x), y)) = \frac{\beta_k((\lambda x' + (1 - \lambda)x), y)}{\sum_{n=k+1}^N \beta_n((\lambda x' + (1 - \lambda)x), y)} = \frac{(\lambda x'_k + (1 - \lambda)x_k)y_k/\pi_k}{\sum_{n=k+1}^N (\lambda x'_{n} + (1 - \lambda)x_{n})y_{n}/\pi_{n}}
$$

whenever the denominator is nonzero; when the denominator is nonzero, this expression is equal to $r_k(\beta(x, y))$. When the denominator and numerator are zero, $r_k(\beta(\lambda x' + (1 - \lambda)x, y)) = r_k(\beta(x, y)) = 0$ and when the denominator is zero and the numerator is nonzero, $r_k(\beta(\lambda x' + (1 - \lambda)x, y)) = r_k(\beta(x, y)) = \infty$.

Claim A.6. Suppose $K > 2$. For any $1 < L < K$, let $x \in \Delta(\{L, \ldots, K\})$, $x' \in \Delta(\{1, \ldots, L-1\})$ and $y \in \Delta(\Omega)$. If $\beta(x, y)$ and $\beta(x', y)$ are well defined then $\beta_k(\lambda x' + (1 - \lambda)x, y) = \lambda \beta_k(x', y) + (1 - \lambda) \beta_k(x, y)$ for all $\lambda \in [0, 1]$, $k = 1, \ldots, N$.

Proof. Simple algebra.

Let $Z = \{ z \in \text{supp}[\Gamma] : z_n > 0 \text{ for all } n \in \Omega' \}$. Note $Z$ is nonempty as $(\Gamma_1, \ldots, \Gamma_M)$ pools $\Omega'$. For $k = 1, \ldots, K - 1$ and $x \in \Delta^{int}(\Omega')$ define $M_x(k)$ recursively starting with $k = K - 1$:

$$
M_x(K - 1) = \arg\min_{z \in \supp[\Gamma]}(\beta(x, z))
$$

\textit{\footnote{From its definition, one can see $\beta(x, y)$ is only not well defined when $y$ assigns probability 0 to every state that $x$ assigns strictly positive probability to.}}

27
$M_x(K - 1)$ is nonempty as $Z$ is. For $k < K - 1$, let $M_x(k) = \text{argmin}_{z \in M_x(k+1)} r_k(\beta(x, z))$; these sets are nonempty for all $k$. For $k = 1, \ldots, K - 1$ define $m_x(k)$ by: pick $z \in M_x(k)$ and let $m_x(k) = r_k(\beta(x, z))$. $m_x(k)$ is well defined for all $k$.

$M_x(K - 1)$ gives the set of realizations of $\Gamma$ that, conditional on interim belief $x$ being realized from a different experiment, would induce the lowest $r_{K - 1}$ ratio of posteriors among those in $Z$. $m_x(K - 1)$ gives the value of this lowest $r_{K - 1}$ ratio. $M_x(K - 2)$ gives the subset of $M_x(K - 1)$ that would result in lowest $r_{K - 2}$ ratio of posteriors conditional on $x$ being realized and $m_x(K - 2)$ gives this value, etc.

Note any $z \in M_x(1)$ must satisfy:

$$r_k(\beta(x, z)) = m_x(k) \text{ for all } k = 1, \ldots, K - 1 \quad (A.5)$$

As $\beta(x, y) \in \Delta(\Omega')$ ($x \in \Delta^{\text{int}}(\Omega')$ and Claim $[A.2]$, by Lemma $[A.1]$ ratios $m_x(1), \ldots, m_x(K - 1)$ uniquely pin down the value of $\beta(x, z)$ for all $z \in M_x(1)$. If we have $|M_x(1)| > 1$, this means that multiple realizations of $\Gamma$, $z \neq z'$, produce the same posterior conditional on $x$. This is possible when $x$ assigns probability 0 to states $z, z'$ do not $\rightarrow -z$ and $z'$ differing on these states may not affect the posterior.

Using the objects introduced above, we finish proving Proposition 2 in two cases. In the first case, $cl(A) \cap \Omega'' = \emptyset$ for all $\Omega'' \subsetneq \Omega'$. This case include the example in the main Appendix in the text of paper; the same logic generalizes. The second case to consider is $cl(A) \cap \Omega'' \neq \emptyset$ for some $\Omega'' \subsetneq \Omega'$.

**Case 1:** $cl(A) \cap \Omega'' = \emptyset$ for all $\Omega'' \subsetneq \Omega'$.

**Lemma A.3.** Suppose $cl(A) \cap \Delta(\Omega'') = \emptyset$ for all $\Omega'' \subsetneq \Omega'$. Then there exists $x^* \in \Delta(\Omega')$ and $\bar{\beta} \in cl(A)$ such that for all $y \in Z$ either: (1) $\beta(x^*, y) \notin cl(A)$ or (2) $\beta(x^*, y) = \bar{\beta}$.

**Proof.** Define the point $\bar{\beta} \in cl(A) \cap \Delta(\Omega')$ as follows. Let $E(K - 1) = \text{argmax}_{\gamma' \in cl(A) \cap \Delta(\Omega')} r_{K - 1}(\gamma)$ and $e(K - 1) = \text{max}_{\gamma' \in cl(A) \cap \Delta(\Omega')} r_{K - 1}(\gamma)$. For $k = 1, \ldots, K - 2$, let $E(k) = \text{argmax}_{\gamma' \in E(k+1)} r_k(\gamma)$ and $e(k) = \text{max}_{\gamma' \in E(k+1)} r_k(\gamma)$. Note that as $cl(A) \cap \Delta(\Omega'') = \emptyset$ for all $\Omega'' \subsetneq \Omega'$, we have $0 < e(k) < \infty$ for all $k = 1, \ldots, K - 1$. Further, by Lemma $[A.1]$, $|E(1)| = 1$ as $\gamma \in E(1)$ must satisfy $r_k(\gamma) = e(k)$ for all $k = 1, \ldots, K - 1$. Let $\bar{\beta}$ be the unique element in $E(1)$.

Consider $x \in \Delta^{\text{int}}(\{K - 1, K\})$. Note that as $x \rightarrow \delta_K$, $\beta(x, y) \rightarrow \delta_K \implies r_{K - 1}(\beta(x, y)) \rightarrow 0$ for all $y \in Z$. Similarly as $x \rightarrow \delta_{K - 1}$, $\beta(x, y) \rightarrow \delta_{K - 1} \implies r_{K - 1}(\beta(x, y)) \rightarrow \infty$ for all $y \in Z$. By the finiteness of $Z$, $x \rightarrow \delta_K \implies \text{min}_{y \in Z} r_{K - 1}(\beta(x, y)) \rightarrow \infty$. Note that this follows from the finiteness of $Z$ and the fact that the ratio of posteriors decreases to 0 as $x \rightarrow \delta_K$. Therefore, $\beta(x, y) \in M_x(1)$ for all $y \in Z$, which implies $\beta(x, y) \notin cl(A)$ for all $y \in Z$. 

28
0 and \( x \to \delta_{K-1} \implies \min_{y \in Z} r_{K-1}(\beta(x,y)) \to \infty \). The continuity of \( \beta(x,y) \) in \( x \), continuity of \( r_{K-1} \) in \( \beta(x,y) \) (Claim A.5), and finiteness of \( Z \) together imply the continuity of \( \min_{y \in Z} r_{K-1}(\beta(x,y)) \) in \( x \). By the intermediate value theorem, there exists \( x' \in \Delta^{\text{int}}(\{K-1, K\}) \) with \( \min_{y \in Z} r_{K-1}(\beta(x',y)) = e(K-1) \), or \( m_{x'}(K-1) = e(K-1) \).

We prove the result inductively, with the previous paragraph being the base case. Suppose we have found \( x' \in \Delta(\{k'+1, \ldots, K\}) \) such that for all \( k = k'+1, \ldots, K-1 \), \( m_{x'}(k) = e(k) \). We find \( x'' \in \Delta(\{k', \ldots, K\}) \) with \( m_{x''}(k) = e(k) \) for all \( k = k', \ldots, K-1 \).

Consider \( x'(\lambda) = \lambda \delta_{k'} + (1 - \lambda)x' \) for \( \lambda \in (0, 1) \). As \( \lambda \to 1 \), \( \beta(x'(\lambda), y) \to \delta_{k'} \implies r_{k'}(\beta(x'(\lambda), y)) \to \infty \) for all \( y \in Z \) and as \( \lambda \to 0 \), \( \beta(x'(\lambda), y) \to x' \implies r_{K-1}(\beta(x'(\lambda), y)) \to 0 \) for all \( y \in Z \). For all \( \lambda \in [0, 1] \), \( y \in Z \), \( k = k'+1, \ldots, K-1 \), \( r_{k}(\beta(x'(\lambda), y)) = r_{k}(\beta(x', y)) \) by Lemma A.2 hence changing \( \lambda \) will leave \( m_{x'}(k) = e(k) \) for \( k = k'+1, \ldots, K-1 \). By finiteness of \( M_{x'}(k'+1) \), continuity of \( \beta(x'(\lambda), y) \) for all \( y \in M_{x'}(k'+1) \), continuity of \( r_{k} \) in \( \beta(x'(\lambda), y) \) for all \( y \in M_{x'}(k'+1) \), and in the intermediate value theorem, there exists \( \lambda^* \in (0, 1) \) and \( x'' = \lambda^* \delta_{k'} + (1 - \lambda^*)x' \in \Delta(\{k', \ldots, K\}) \) such that \( m_{x''}(k') = e(k') \). Then \( m_{x''}(k) = e(k) \) for all \( k = k', \ldots, K-1 \).

Carrying this inductive process through until \( k' = 1 \), by equation A.5 we find \( x^* \in \Delta(\Omega') \) with, for all \( y \in M_{x^*}(1) \) and \( k = 1, \ldots, K-1 \): \( r_{k}(\beta(x^*, y)) = e(k) \). Hence for all \( y \in M_{x^*}(1), \beta(x^*, y) = \bar{\beta} \). Meanwhile for all \( y \notin M_{x^*}(1) \), there exists \( 1 \leq k' \leq K-1 \) such that \( r_{k}(\beta(x^*, y)) = e(k) \) for all \( k > k' \) and \( r_{k'}(\beta(x^*, y)) > e(k') \); this implies (by definition of \( e(k') \)) that \( \beta(x^*, y) \notin cl(A) \).

\[\square\]

The following result follows from Lemma A.3 almost immediately.

**Lemma A.4.** Suppose \( cl(A) \cap \Delta(\Omega'') = \emptyset \) for all \( \Omega'' \subsetneq \Omega' \). Then there exists \( \bar{x} \in \Delta(\Omega') \) and \( \beta' \in A \) such that for all \( y \in Z \) either: (1) \( \beta(\bar{x}, y) \notin cl(A) \) or (2) \( \beta(\bar{x}, y) = \beta' \).

**Proof.** Find \( x^* \) and \( \bar{\beta} \) as per Lemma A.3 If \( \bar{\beta} \in A \) then set \( \bar{x} = x^* \) and \( \beta' = \bar{\beta} \) and we’re done.

Assume now \( \bar{\beta} \notin A \). As \( Z \) finite, \( cl(A) \) is closed, and \( \beta(x,y) \) is continuous in \( x \) for all \( y \in Z \), there exists \( \epsilon > 0 \) and a set \( N_{\epsilon} = \{x \in \Delta(\Omega') : |x - x^*| < \epsilon\} \) such that for all \( x \in N_{\epsilon} \), and \( y \in Z \), \( \beta(x^*, y) \notin cl(A) \implies \beta(x,y) \notin cl(A) \).

By equation A.2 for any \( \gamma \in \Delta(\Omega') \) and \( y \in Z \) there exists \( x \in \Delta(\Omega') \) such that \( \beta(x, y) = \gamma \). Also by equation A.2 if \( \beta(x, y) = \beta(x, y') \) for some \( x \in \Delta^{\text{int}}(\Omega') \) and \( y, y' \in Z \), then \( \beta(x', y) = \beta(x', y') \) for all \( x' \in \Delta^{\text{int}}(\Omega') \).
As $\bar{\beta} \in cl(A)$, there is a sequence of beliefs in $A$ converging to $\bar{\beta}$. By continuity of $\beta(x,y)$ in $x$ for all $y \in Z$ and the facts in the previous paragraph, there exists $\beta' \in A$ close to $\bar{\beta}$ and $\bar{x} \in N_\epsilon$ such that for all $y \in Z$, $\beta(x^*, y) = \bar{\beta} \implies \beta(\bar{x}, y) = \beta'$.

Hence we have either $\beta(\bar{x}, y) = \beta' \in A$ or $\beta(\bar{x}, y) \not\in cl(A)$ for all $y \in Z$.

This next Lemma wraps up the proof of the ‘if’ direction of Proposition 2 for the case that $cl(A) \cap \Delta(\Omega'') = \emptyset$ for all $\Omega'' \subset \Omega'$:

**Lemma A.5.** Suppose $cl(A) \cap \Delta(\Omega'') = \emptyset$ for all $\Omega'' \subset \Omega'$. Then there exists a sender $j$ and $\bar{x} \in \Delta^{int}(\Omega')$ with $\mathbb{E}[u_j(\beta(\bar{x}, \Gamma))] > 0$.

**Proof.** Find $\beta'$ and $\bar{x}$ as per Lemma [A.4].

Note that for all $y \in supp[\Gamma] \setminus Z$, one of the following holds. (1) $y_l = 0$ for all $l \in \Omega'$. (2) $y_l = 0$ for some $l \in \Omega'$ but $y_k > 0$ for some $k \in \Omega'$. For $y$ in case (1), by Claim [A.1], $p(y|\bar{x}) = 0$. For $y$ in case (2), equation [A.2] implies $\beta(\bar{x}, y) \in \Omega''$ for some $\Omega'' \subset \Omega'$; as $\Omega'$ is minimal we have $u_i(\beta(\bar{x}, y)) = 0$ for all $i$.

For all $y \in Z$ such that $\beta(\bar{x}, y) \neq \beta'$, $u_i(\beta(\bar{x}, y)) = 0$ for all $i$.

Let $j$ be some sender with $u_j(\beta') > 0$. Then $\mathbb{E}[u_j(\beta(\bar{x}, \Gamma))] = u_j(\beta') Pr(\Gamma \in \{y \in Z : \beta(\bar{x}, y) = \beta'\}|\bar{x}) > 0$.

**Case 2: $cl(A) \cap \Omega'' \neq \emptyset$ for some $\Omega'' \subset \Omega'$.**

Note that it is still the case that $\Omega'$ is minimal, so $A \cap \Omega'' = \emptyset$ for all $\Omega'' \subset \Omega'$.

Let $\Omega'' \in \text{argmin}_{\Omega'' \subset \Omega': \text{cl}(A) \cap \Omega'' \neq \emptyset} |\Omega''|$ be one of the smallest subsets of $\Omega'$ that intersects $\text{cl}(A)$. Note that $1 \leq |\Omega''| < |\Omega'|$. Define $Z$, and all other necessary objects, as in the previous case. Then note that $\text{cl}(A) \cap \emptyset = \emptyset$ for all $\emptyset \subset \Omega''$ and so by an identical argument to Lemma [A.3] we can find $\bar{\beta} \in \text{cl}(A) \cap \Delta^{\text{int}}(\Omega'')$ and $x^* \in \Delta^{\text{int}}(\Omega'')$ such that for all $y \in Z$ either $\beta(x^*, y) = \bar{\beta}$ or $\beta(x^*, y) \not\in \text{cl}(A)$. Then by a similar argument to Lemma [A.4] we can find $\bar{x} \in \Delta^{\text{int}}(\Omega')$ close to $x^*$ and $\beta' \in A$ close to $\bar{\beta}$ such that either $\beta(\bar{x}, y) = \beta'$ or $\beta(\bar{x}, y) \not\in \text{cl}(A)$ for all $y \in Z$.

Following the same argument as in Lemma [A.5], all $y \in supp[\Gamma] \setminus Z$ either occur with
probability 0 conditional on \( x \) or result in a posterior outside of \( \Delta^{\text{int}}(\Omega') \) conditional on \( x \) (yielding 0 utility by minimality of \( \Omega' \)). Letting \( j \) be some sender with \( u_j(\beta') > 0 \), we have \( \mathbb{E}[u_j(\beta(x, \Gamma))] > 0 \) and we’re done.

A.3.3 Additional Claim

In the proof of Proposition 2 given, when utilities are nonlinear on some \( \Delta(\Omega') \) and \( (\Gamma_1, \ldots, \Gamma_M) \) pooled \( \Omega' \), we were able to find a sender \( j \) who could take advantage of \( \Omega' \) being pooled and find \( x \) conditional on which she gets strictly positive expected utility. Identifying this sender \( j \) did not depend on the strategy profile \( (\Gamma_1, \ldots, \Gamma_M) \). Hence, the claim below (which is useful for further results in Supplementary Appendix B) holds.

**Claim A.7.** Suppose for some sender \( i \) and \( \Omega' \subseteq \Omega \) that \( u_i \) is nonlinear on \( \Delta(\Omega') \). Then there exists a sender \( j \) such that for any \( (\Gamma_1, \ldots, \Gamma_M) \) that pools \( \Omega' \) \( j \) can find some \( x \) such that \( \mathbb{E}[u_j(\beta(x, \Gamma_1, \ldots, \Gamma_M))] > 0 \).

A.4 Proof of Theorem 1

**Theorem 1.**

**Proof.** ‘If’ direction. Proposition 2 implies that if for every \( l, k \) some \( u_i \) is nonlinear on \( \Delta(\{l, k\}) \), then in any equilibrium \( (\Gamma_1, \ldots, \Gamma_M) \), w.p. 1 \( \beta_n(\Gamma_1, \ldots, \Gamma_M) > 0 \) for only one \( n \in \Omega \). Hence w.p. 1 we must have \( \beta_n(\Gamma_1, \ldots, \Gamma_M) = 1 \) for some \( n \in \Omega \); the state is fully revealed.

‘Only if’ direction. Suppose for some \( l, k \in \Omega, u_i \) is linear along \( \Delta(\{l, k\}) \) for all \( i \). Then the equilibrium construction in the Proposition 2 ‘only if’ direction is a non-fully revealing equilibrium.

\( \Box \)
B Supplementary Appendix B: Extensions

B.1 Single Receiver with Finite Actions

Setup. There is a single receiver. Let $\mathcal{A} = \{a_1, ..., a_A\}$ be a finite set of actions; the game is as in the baseline model except after observing realizations of $\Gamma_1, ..., \Gamma_M$ the receiver picks an action $a \in \mathcal{A}$ after which the players get payoffs. The reciever’s utility function is $u_r : \mathcal{A} \times \Omega \to \mathbb{R}$ and senders’ utility functions are $u_i : \mathcal{A} \times \Omega \to \mathbb{R}$ for $i = 1, ..., M$. Sender’s preferences are zero-sum: for all $a \in \mathcal{A}$, $\omega \in \Omega$, $\sum_{i=1}^{M} u_i(a, \omega) = 0$. We make the assumption that for any $a \neq a' \in \mathcal{A}$, $\omega \in \Omega$, $u_r(a, \omega) \neq u_r(a', \omega)$ and $u_i(a, \omega) \neq u_i(a', \omega)$ for all $i \in \{1, ..., M\}$; this is generically true. The equilibrium concept is Perfect Bayesian Equilibrium (PBE).

For any $\beta \in \Delta(\Omega)$ let $A^*(\beta) = \arg\max_{a \in \mathcal{A}} \sum_{l=1}^{N} u_r(a, l) \beta_l$. In any PBE, after signals are realized and the receiver updates to posterior belief $\beta \in \Delta(\Omega)$, the receiver takes an action $a \in A^*(\beta)$ that maximizes expected utility (sequential rationality). Let $R_{indiff} = \{\beta \in \Delta(\Omega) : |A^*(\beta)| > 1\}$ be the set of posteriors at which the receiver is indifferent between multiple best actions. We assume that at posteriors in $\beta \in R_{indiff}$ the receiver breaks ties by choosing the lowest indexed action in $A^*(\beta)$. Hence in equilibrium the receiver takes action $a^*(\beta) = \arg\min_{a \in A^*(\beta)} b$; this is well defined and single valued for each $\beta \in \Delta(\Omega)$.

For $i = 1, ..., M$ define $v_i : \Delta(\Omega) \to \mathbb{R}$ as sender $i$’s expected utility from any posterior $\beta \in \Delta(\Omega)$ in a PBE following the specified tie breaking rule: $v_i(\beta) = \sum_{l=1}^{N} u_i(a^*(\beta), l) \beta_l$. First we show that in any PBE satisfying this tie-breaking rule, $v_i$’s are zero-sum and piecewise analytic. This will allow us to directly apply our previous results to them.

Claim B.1. In any PBE in which the receiver chooses action $a^*(\beta) = \arg\min_{a \in A^*(\beta)} b$ after signals are realized, $v_1, ..., v_M$ are each piecewise analytic and are zero-sum.

Proof. Zero-sumness is trivial. For all $\beta$, $\sum_{i=1}^{M} v_i(\beta) = \sum_{i=1}^{M} \sum_{l=1}^{N} u_i(a^*(\beta), l) \beta_l$

$$= \sum_{l=1}^{N} \beta_l \sum_{i=1}^{M} u_i(a^*(\beta), l) = 0.$$  

Now we show piecewise analyticity. For each $a_b \in \mathcal{A}$, let $\Delta_{a_b} = \{\beta \in \Delta(\Omega) : a_b = a^*(\beta)\}$. Note that for every $a_b \in \mathcal{A}$ the set $\Delta_{a_b}$ is convex; for any $\beta, \beta' \in \Delta_{a_b}$ we can see $a^*(\lambda \beta + (1 - \lambda) \beta') = a_b$ for all $\lambda \in (0, 1)$ as follows. For any $b' < b$ we have:


\[ \sum_{l=1}^{N} u_r(a_l, l)\beta_l > \sum_{l=1}^{N} u_r(a_{l'}, l)\beta_l \]
\[ \sum_{l=1}^{N} u_r(a_l, l)\beta'_l > \sum_{l=1}^{N} u_r(a_{l'}, l)\beta'_l \]
\[ \implies \sum_{l=1}^{N} u_r(a_l, l)(\lambda\beta_l + (1 - \lambda)\beta'_l) > \sum_{l=1}^{N} u_r(a_{l'}, l)(\lambda\beta_l + (1 - \lambda)\beta'_l) \]

For any \( b' > b \) the first two inequalities must hold weakly which implies the third holds weakly. So at \( (\lambda\beta_l + (1 - \lambda)\beta') \) \( a_b \) yields weakly higher utility for the receiver than all higher indexed actions. Together this implies \( \lambda^*(\lambda\beta + (1 - \lambda)\beta') = a_b \) and hence \( \Delta_{a_b} \) is convex. Note that \( \{\Delta_{a_i}\}_{b=1,...,A} \) partition \( \Delta(\Omega) \); hence they form a partition of \( \Delta(\Omega) \) into convex sets. On each \( \Delta_{a_b} \), each \( v_i = \sum_{l=1}^{N} u_i(a_l, l)\beta_l \) \((i = 1, ..., M)\) is linear in \( \beta \). Hence each \( v_i \) is piecewise linear on \( \Delta(\Omega) \). So \( \{v_i\}_{i=1,\ldots,M} \) are piecewise analytic.

Lemma [B.1] below shows that all \( v_i \)'s are linear on \( \Delta(\Omega') \) if and only if the receiver prefers the same action at all states in \( \Omega' \).

**Lemma B.1.** Fix \( \Omega' \subseteq \Omega \), \( |\Omega'| > 1 \). \( v_1, ..., v_M \) are all linear on \( \Delta(\Omega') \) if and only if \( a^*(\delta_l) = a^*(\delta_k) \) for all \( l, k \in \Omega' \).

**Proof.** We prove the ‘only if’ direction by proving the contrapositive. Consider any states \( l \neq k \in \Omega' \). Suppose \( a^*(\delta_l) \neq a^*(\delta_k) \). For \( \beta \in \Delta(\{l, k\}) \subseteq \Delta(\Omega) \), the receiver has expected utility \( u_r(a, l)\beta_l + u_r(a, k)\beta_k \). Note by our assumption that no agent is indifferent between any two actions at any state, we must have that \( u_r(a^*(\delta_l), l) > u_r(a', l) \) for all \( a' \neq a^*(\delta_l) \) and \( u_r(a', k) < u_r(a^*(\delta_k), k) \) for all \( a' \neq a^*(\delta_k) \). By continuity, this implies that there exists \( 1 > \beta^u_k > \beta^l_k > 0 \) such that for \( \beta \in \Delta(\{l, k\}) \): (1) \( \beta_k < \beta^l_k \) then \( a^*(\beta) = a^*(\delta_l) \) and (2) \( \beta_k > \beta^u_k \) then \( a^*(\beta) = a^*(\delta_k) \). For \( \beta \in \Delta(\{l, k\}) \) and \( i \in \{1, ..., M\} \), \( v_i(\beta) = \beta_k u_i(a^*(\delta_k), k) + (1 - \beta_k) u_i(a^*(\beta), l) = u_i(a^*(\beta), l) + \beta_k (u_i(a^*(\delta_k), k) - u_i(a^*(\beta), l)) \). When \( \beta_k > \beta^u_k \), \( v_i \) is linear in \( \beta_k \) with slope \( (u_i(a^*(\delta_k), k) - u_i(a^*(\delta_k), l)) \). When \( \beta_k < \beta^l_k \), \( v_i \) is linear in \( \beta_k \) with slope \( (u_i(a^*(\delta_l), k) - u_i(a^*(\delta_l), l)) \). These slopes are different because:

\[ u_i(a^*(\delta_k), k) - u_i(a^*(\delta_l), k) > 0 > u_i(a^*(\delta_k), l) - u_i(a^*(\delta_l), l) \]
\[ \implies u_i(a^*(\delta_k), k) - u_i(a^*(\delta_l), k) \neq u_i(a^*(\delta_l), k) - u_i(a^*(\delta_l), l) \]
Hence $v_i$ is nonlinear along $\Delta(\{l, k\})$ and hence nonlinear on $\Delta(\Omega') \supseteq \Delta(\{l, k\})$.

‘If’ direction. Suppose for all $l \in \Omega'$, $a^*(\delta_l) = a$. Note that $\Delta(\Omega')$ is the convex hull of $\{\delta_l : l \in \Omega'\}$. Then, by convexity of the set $\Delta_a$ (see proof of Claim B.1), we must have $\Delta(\Omega') \subseteq \Delta_a$. For all $i \in \{1, ..., M\}$ $v_i$ is linear on $\Delta_a$ (proof of Claim B.1) and hence $\Delta(\Omega')$.

Finally we prove our main results for the finite action model. Proposition B.1 is important as it says that even when the receiver does not fully learn the state, they learn adequately —that is, enough that further learning would not influence their action.

**Proposition B.1.** In any PBE in which the receiver chooses action $a^*(\beta) = \arg\min_{a,b \in A^*} b$, the receiver takes their first best action w.p. 1.

**Proof.** Fix a PBE and consider all posteriors induced by the equilibrium experiments with positive probability. At any posterior $\beta \in \{\delta_1, ..., \delta_N\}$, the receiver clearly takes their first best action. Now consider any posterior that occurs with positive probability that does not fully reveal the state. At such a $\beta$, there exists $\Omega' \subseteq \Omega$ with $|\Omega'| > 1$ and $\beta_l > 0$ for all $l \in \Omega'$. But then $\Omega'$ is being pooled in equilibrium which implies (Proposition 2) that $v_1, ..., v_M$ are all linear on $\Delta(\Omega')$ which implies (Lemma B.1) that there exists $a \in A$ such that $a^*(\delta_l) = a$ for all $l \in \Omega'$. Note that $\omega \in \Omega'$ (all other states are ruled out by $\beta$). By convexity of $\Delta_a$ (proof of Claim B.1), $a^*(\beta) = a = a^*(\delta_\omega)$. Hence, the receiver’s ex-post payoff is always their first best payoff: $u_r(a^*(\omega), \omega)$.

The finite action characterization of all equilibria being fully revealing follows immediately from Theorem 1 and Lemma B.1.

**Corollary B.1.** The state is fully revealed in every PBE in which the receiver chooses action $a^*(\beta) = \arg\min_{a,b \in A^*} b$ if and only if for every pair of states $l$ and $k$ $a^*(\delta_l) \neq a^*(\delta_k)$.

**Proof.** ‘If’ direction. If for a pair of states $l$ and $k$, $a^*(\delta_l) \neq a^*(\delta_k)$, then by Lemma B.1 some $v_i$ is nonlinear (and hence nonlinear) on $\Delta(\{l, k\})$. If this is true for every pair of states, then along every edge of the simplex we have nonlinearity of some sender’s utility function which implies (Theorem 1) full revelation in every equilibrium.

‘Only if’ direction. If $a^*(\delta_l) = a^*(\delta_k)$ for any pair of states $l$ and $k$, then Lemma B.1 implies that all $v_i$’s are linear on $\Delta(\{l, k\})$. This implies (Theorem 1) that there are non-fully revealing equilibria.
B.2 Robustness

Here we consider the robustness of our results to the assumption that preferences are zero-sum. Consider a game identical to the baseline model but with utility functions $u_1, ..., u_M : \Delta(\Omega) \to \mathbb{R}$ that need not be zero-sum. We assume that all utilities are piecewise analytic and make the normalization $u_i(\delta_l) = 0$ for all senders $i$ and states $l$. We adopt notation from the baseline model whenever it obviously carries over.

Before presenting the robustness results, which concern the information revealed in equilibrium as preferences approach zero-sum, we discuss what we can say in this more general setting. As in the baseline model, we have no issues with equilibrium existence; for any $u_1, ..., u_M$ there is a fully revealing equilibrium $(\Gamma^{FR}_1, ..., \Gamma^{FR}_M)$.

Of course our results in the paper, starting with Lemma 1, rely on the zero-sumness of preferences and do not generalize to this setting. While Lemma 1 says senders must get their full revelation payoff in every equilibrium of a zero-sum game, this is no longer true when preferences may be nonzero-sum. As an example, suppose all senders have the same preferences; then there will be an equilibrium in which one sender plays a single sender optimal experiment (a Bayesian Persuasion, or BP, solution) and all others play $\Gamma^U$. This BP solution may not fully reveal the state and may yield the senders strictly larger payoffs than the 0 payoff from full revelation. More generally, when preferences are not zero-sum, agreement between senders (even if it isn’t complete agreement) may allow them to play experiments that yield them strictly positive payoffs in equilibrium.

For any $\gamma \in \Delta(\Omega)$ and $u_1, ..., u_M$, let $\sum_i u_i(\gamma)$ be the total surplus shared among senders when the receiver’s posterior is $\gamma$. Note this surplus is 0 when $\gamma \in \{\delta_1, ..., \delta_N\}$. Let $MS(u_1, ..., u_M) = \sup_{\gamma \in \Delta(\Omega)} \sum_{i=1}^M u_i(\gamma)$ be the supremum of the total surplus senders get at any posterior. Note $MS(u_1, ..., u_M) = 0$ when the game is zero-sum. While we cannot pin down equilibrium payoffs as we did in Lemma 1, we can upperbound them for each sender. While senders may get above utility 0 in equilibrium, none can attain utility higher than the maximum surplus:

**Lemma B.2.** In any equilibrium $(\Gamma_1, ..., \Gamma_M)$, for all senders $i = 1, ..., M$: $0 \leq U_i(\Gamma_1, ..., \Gamma_M) \leq MS(u_1, ..., u_M)$.

---

20The reason this is an equilibrium is the same — no sender’s experiment is pivotal when others are full revealing the state.

21In fact whenever there exists a posterior $\gamma'$ that yields the senders utility larger than 0, then all BP solutions yield utility larger than 0. To see this, note that there exists an experiment that puts support only on $\gamma'$ and $\delta_1, ..., \delta_N$ (such a construction is shown in the proof of Lemma 1). This experiment yields all senders utility strictly greater than 0.
Proof. The first inequality, \(0 \leq U_i(\Gamma_1, \ldots, \Gamma_M)\), follows from the fact that each sender can always fully reveal the state and obtain payoff 0.

Note that for all strategy profiles \((\Gamma'_1, \ldots, \Gamma'_M)\) (with distributions \(p'_1, \ldots, p'_M\)), we have:

\[
\sum_i U_i(\Gamma_1, \ldots, \Gamma_M) = \sum_i E_{p'_1, \ldots, p'_M}[u_i(\beta(\Gamma_1, \ldots, \Gamma_M))] \leq E_{p'_1, \ldots, p'_M}[MS(u_1, \ldots, u_M)] = MS(u_1, \ldots, u_M).
\]

Suppose for contradiction that for equilibrium \((\Gamma_1, \ldots, \Gamma_M)\) and sender \(i\), \(U_i(\Gamma_1, \ldots, \Gamma_M) > MS(u_1, \ldots, u_M)\). Then the previous paragraph implies that there exists sender \(j\) with \(U_j(\Gamma_1, \ldots, \Gamma_M) < 0\). But then \(j\) can profitably deviate to \(\Gamma_j = \Gamma^{FR}\). Contradiction. \(\square\)

Lemma B.2 implies that when \(MS(u_1, \ldots, u_M) = 0\), all senders get payoff 0 in every equilibrium. In one special case, when only fully revealing posteriors generate this maximum surplus, all equilibria must be fully revealing:

**Corollary B.2.** If for all \(\gamma' \notin \{\delta_1, \ldots, \delta_M\}\) \(\sum_i u_i(\gamma') < 0\), then the state is fully revealed in every equilibrium.

Proof. Note that \(MS(u_1, \ldots, u_M) = 0\). For any strategy profile \((\Gamma_1, \ldots, \Gamma_M)\) that is fully revealing, \(U_i(\Gamma_1, \ldots, \Gamma_M) = 0\) for all \(i\). For \((\Gamma_1, \ldots, \Gamma_M)\) that is not fully revealing, \(\sum_i U_i(\Gamma_1, \ldots, \Gamma_M) < 0\), as with strictly positive probability the surplus at the posterior is strictly less than 0 (when the state is not fully revealed). This implies that for each non-fully revealing \((\Gamma_1, \ldots, \Gamma_M)\), there is a sender \(j\) such that \(U_j(\Gamma_1, \ldots, \Gamma_M) < 0\). Hence, by Lemma B.2, the state is fully revealed in equilibrium. \(\square\)

The logic of Corollary B.2 is similar to that of Proposition 1 of Gentzkow and Kamenica (2016). If sender surplus is uniquely maximized at fully revealing posteriors, any non-fully revealing strategy profile leaves at least one sender strictly worse off than full revelation, which is always an available strategy.

We now turn our attention to what we can say as preferences approach zero-sum. We consider convergence of utilities under the sup norm. A sequence of utility functions \(\{u^k\}_{k=1}^\infty\) converges to a function \(u\), or \(u_k \to u\), if \(\lim_{k \to \infty} \sup_{\gamma \in \Delta(\Omega)} |u^k(\gamma) - u(\gamma)| = 0\). For a sequence of profiles of utility functions \(\{(u^k_1, \ldots, u^k_M)\}_{k=1}^\infty\) (for notational convenience we will drop the limits), \((u^k_1, \ldots, u^k_M) \to (u_1, \ldots, u_M)\) if \(u^k_i \to u_i\) for \(i = 1, \ldots, M\).

For a sequence of strategies/experiments \(\{\Gamma^k\}_{k=1}^\infty\), with each \(\Gamma^k\) distributed according to pmf \(p^k\), we say \(\Gamma^k \to \Gamma\) (where \(\Gamma\) has distribution \(p\)), or \(\Gamma^k\) converges in distribution to
Γ, if for all γ ∈ Δ(Ω), lim_{κ→∞} p_k(γ) = p(γ). For any strategy profile (Γ_1, ..., Γ_M), let the random variable Γ(Γ_1, ..., Γ_M) denote the receiver’s posterior after observing realizations of all Γ_1, ..., Γ_M (i.e. the experiment induced by combining all M senders’ experiments).

The following result says that as utilities converge to zero-sum sufficiently nonlinear functions, the information revealed along any sequence of equilibria, whenever convergent, converges to full revelation.

**Proposition B.2.** Fix a sequence of games with utilities \{(u_1^k, ..., u_M^k)\} with (u_1^k, ..., u_M^k) → (u_1, ..., u_M) and \[\sum_i u_i(\gamma) = 0\] for all γ ∈ Δ(Ω). For each k let (Γ_1^k, ..., Γ_M^k) be an equilibrium of game (u_1^k, ..., u_M^k). Suppose for every pair of states \(l, k \in \Omega\) there exists an i with \(u_i\) nonlinear on \(Δ(\{l, k\})\). Then if Γ(Γ_1^k, ..., Γ_M^k) → Γ, Γ = Γ^{FR}.

Proposition B.2 is important, as indicates that Theorem 1 does not qualitatively rely on the knife-edge assumption of zero-sum preferences. As utilities get close to zero-sum and sufficiently nonlinear, the information revealed in every equilibrium (if it converges) gets close to full revelation. Before proving the result, we prove a lemma which shows Proposition 2 is similarly robust.

**Lemma B.3.** Fix a sequence of games with utilities \{(u_1^k, ..., u_M^k)\} with (u_1^k, ..., u_M^k) → (u_1, ..., u_M) and \[\sum_i u_i(\gamma) = 0\] for all γ ∈ Δ(Ω). For each k let (Γ_1^k, ..., Γ_M^k) be an equilibrium for (u_1^k, ..., u_M^k). Suppose for some \(Ω \subseteq \Omega\) and i, \(u_i\) is nonlinear on \(Δ(Ω)\). Then if Γ(Γ_1^k, ..., Γ_M^k) → Γ, Γ does not pool \(Ω\).

**Proof.** For strategy profile (Γ_1', ..., Γ_M') (with distributions \(p_1', ..., p_M'\)), let \(U_k^k(Γ_1', ..., Γ_M') = \mathbb{E}_{p_1', ..., p_M'}[u_i^k(β)]\) be sender i’s expected utility when she has preferences \(u_i^k\).

Let the distribution of Γ be p and for any k let the distribution of Γ(Γ_1^k, ..., Γ_M^k) be \(p^k\). Note that as Γ(Γ_1^k, ..., Γ_M^k) has finite support for every k, Γ must also finite support (by the definition of convergence in distribution). Define \(T = supp[Γ] \cup (\cup_{k=1}^∞ supp[Γ(Γ_1^k, ..., Γ_M^k)])\); note by the finiteness of all terms in the union, T is countable.

Suppose for contradiction that some \(u_i\) is nonlinear on \(Ω\) and Γ pools \(Ω\). By Claim A.7 there exists a sender j such who can find experiment Γ_j (with distribution \(p_j\)) such that in the limiting (zero-sum) game (u_1, ..., u_M), \(U_j(Γ_j, Γ) = c > 0\).

For any k, \(MS(u_1^k, ..., u_M^k) = \sup_γ \sum_i u_i^k(γ) = \sup_γ \sum_i u_i(γ) + (u_i^k(γ) - u_i(γ)) \leq \sup_γ \sum_i u_i(γ) + |u_i^k(γ) - u_i(γ)|\). As (u_1^k, ..., u_M^k) → (u_1, ..., u_M), the second term goes to 0 for all i as \(k \to ∞\) and hence \(MS(u_1^k, ..., u_M^k) → MS(u_1, ..., u_M) = 0\). This implies
there exists $K$ s.t. $\forall k > K$, $MS(u^k_1, \ldots, u^k_M) < c/2$. By Lemma B.2 for all $k > K$, $U_j^k(\Gamma^k_j, \Gamma^k_{-j}) < c/2$.

Consider $j$ playing experiment $\Gamma^k_j$ as well as (conditionally independently) playing $\Gamma_j$, while her opponents’ play $\Gamma^k_{-j}$. The expected payoff that $j$ gets from this can be written $U_j^k(\Gamma(\Gamma_j, \Gamma^k_j, \Gamma^k_{-j})) = U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M))$, as it does not affect $j$’s payoff if she plays $\Gamma^k_j$ or her opponents’ do. As $T \supset \text{supp}[\Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)]$ for each $k$, we can write $U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) = \sum_{x \in \text{supp}[\Gamma_j]} \sum_{y \in \text{supp}[T]} u_j^k(\beta(x, y)) p^k(y|x)p_j(x)$. Then:

$$\lim_{k \to \infty} U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) = \sum_{x \in \text{supp}[\Gamma_j]} \sum_{y \in \text{supp}[T]} \lim_{k \to \infty} u_j^k(\beta(x, y)) p^k(y|x)p_j(x)$$

For any $y \in T$, $\lim_{k \to \infty} u_j^k(\beta(x, y)) = u_j(\beta(x, y))$ and $\lim_{k \to \infty} p^k(y|x) = p(y|x)$ (by definition of $p^k(y|x)$ and convergence in distribution). Hence:

$$\lim_{k \to \infty} U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) = \sum_{x \in \text{supp}[\Gamma_j]} \sum_{y \in \text{supp}[T]} u_j(\beta(x, y)) p(y|x)p_j(x) = U_j(\Gamma_j, \Gamma) = c$$

This implies that there exists $K'$ such that $\forall k > K'$, $U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) > c/2$. Take $K'' = \max\{K, K'\}$. For all $k > K''$ we have: $U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) < c/2$ and $U_j^k(\Gamma_j, \Gamma(\Gamma^k_1, \ldots, \Gamma^k_M)) > c/2$. But for all $k > K''$, this contradicts that $(\Gamma^k_1, \ldots, \Gamma^k_M)$ is an equilibrium as $j$ has a profitable deviation of $\Gamma(\Gamma_j, \Gamma^k_j)$.

We now prove Proposition B.2.

**Proof.** Suppose not. Then $\Gamma(\Gamma^k_1, \ldots, \Gamma^k_M) \rightarrow \Gamma \neq \Gamma^{FR}$. Then $\Gamma$ pools some set of states $\Omega'$ with $|\Omega'| \geq 2$; let $l, k \in \Omega'$. There is a sender with $u_i$ nonlinear on $\Delta(\{l, k\})$; hence $u_i$ is nonlinear on $\Delta(\Omega')$. But then by Lemma B.3 $\Gamma$ must not pool $\Omega'$. Contradiction.

Proposition 4 relates to the standard results on the upper hemicontinuity of the set of equilibria (although here we are not concerned with the set of equilibrium actions themselves but instead the set of information that could be revealed in equilibrium). As is also standard, we do not have the corresponding lower hemicontinuity properties. In particular, it is possible for there to be non-fully revealing equilibria in the limit, but only fully revealing equilibria along the sequence. It is not hard to come up with examples of this; we provide a one here.
Example 1. Suppose \( \Omega = \{0, 1\} \) and there are two senders 1 and 2. Consider the sequence of utility functions \( \{ (u^k_1, u^k_2) \} \) with \( u^k_1(\beta) = \frac{\beta}{k} \) for all \( \beta \in [0, 1] \setminus \{0, 1\} \), \( u^k_1(0) = u^k_1(1) = 0 \), and \( u^k_2(\beta) = 0 \) for all \( \beta \in [0, 1] \). Define utility function \( u \) as \( u(\beta) = 0 \) for all \( \beta \in [0, 1] \). Then note \( (u^k_1, u^k_2) \to (u, u) \). Proposition 1 states that in the game \((u, u)\), there are non-fully revealing equilibria (any strategy profile is an equilibrium). Note for any \( k \), the game \((u^k_1, u^k_2)\) is fully revealing for all \( k \).

B.3 Infinite Signal Experiments

So far we have restricted senders to choosing experiments with a finite number of signals, or equivalently interim belief distributions \( p_i \in P \) that have finite support. In this section we demonstrate that our takeaway from the finite signal results—that for typical sender preferences we have full revelation in every equilibrium—extends when senders can choose from a more general set of experiments. Senders now choose any experiments \( \Pi : \Omega \to \Delta(S) \) (with no restrictions on the signal space). Again, we recast choices of experiments as choices of interim belief distributions. For technical convenience we restrict our attention to senders choosing interim belief distributions that can be written as the sum of absolutely continuous and discrete distributions.\(^{22}\)

We will call the space of pure strategies, or the set of Bayes-plausible distributions that satisfies this requirement \( G \). Formally \( G = \{ g \in \Delta(\Delta(\Omega)) : \mathbb{E}_g[\Gamma_i] = \pi, g = g_c + g_d \text{ for some abs. cont. and discrete (respectively) measures } g_c, g_d \in \Delta(\Delta(\Omega)) \} \). Each \( g \in G \) is a generalized density.\(^{23}\) A strategy for sender \( i \) is a choice of random variable \( \Gamma_i \) with generalized density \( g_i \in G \). Preferences over strategy profiles for a sender \( i \) are given by \( U_i(\Gamma_1,...,\Gamma_M) = \mathbb{E}_{g_1,...,g_M}[u_i(\beta)] \). The game and equilibrium concept are otherwise identical to the finite signal model (including our normalization of the \( u_i \)'s).

Remark: Note that our equilibrium analysis under both the finite signal restriction and under the technical restriction above can be seen as equilibrium selections. Any finite signal equilibrium will also be an equilibrium when senders are allowed to pick strategies from \( G \) and any equilibria with strategies selected from \( G \) will be equilibria in a game where senders can pick any distribution in \( \Delta(\Delta(\Omega)) \).

We make one additional technical assumption on utility functions:

\(^{22}\)As \( \Delta(\Omega) \subset \mathbb{R}^N \), by the Lebesgue Decomposition Theorem this only rules senders choosing distributions with singular continuous components.

\(^{23}\)\( g \) is the density function \( g_c \) on intervals where the distribution is absolutely continuous and the probability mass function \( g_d \) everywhere else.
Assumption B.1. For each \( l \in \Omega \) and \( i \in \{1, \ldots, M\} \), \( u_i \) is real analytic in some neighborhood of \( \delta_l \).

Note that Assumption B.1 only rules out piecewise analytic utility functions for which some \( \delta_l \) lies on the boundary between different pieces. For any states \( l, k \) let \( v^{l,k} \in \mathbb{R}^N \) be the vector from \( \delta_l \) to \( \delta_k \). For any sender \( i \) let \( \nabla_{v^{l,k}} u_i(\cdot) \) be the directional derivative of \( u_i \) moving along \( v^{l,k} \). Note for some \( i \) \( \nabla_{v^{l,k}} u_i(\cdot) \) may not be well defined at some points on \( \Delta(\Omega) \); but under Assumption 7.1 for all \( i \) and all \( l \) it is a well defined continuous function in some neighborhood of \( \delta_l \). With this assumption we can define Condition B.1.

Condition B.1 concerns the shape of utility functions on an edge of the simplex and will be sufficient for a pair of states \( l \) and \( k \) to not be pooled in every equilibrium.

Definition B.1. For any states \( l, k \) and sender \( i \) we say that \( u_i \) satisfies Condition B.1 on \( \Delta(\{l,k\}) \) if either \( \nabla_{v^{l,k}} u_i(\delta_l) \neq u_i(\delta_k) - u_i(\delta_l) \), \( \nabla_{v^{l,k}} u_i(\delta_k) \neq u_i(\delta_k) - u_i(\delta_l) \), or both.

For example, utility functions that look like Figure 7.2 along edge \( \Delta(\{l,k\}) \) do not satisfy Condition B.1 nor does a utility function that is linear along that edge. Functions that look this those in Figure 1 do satisfy Condition B.1.

\[ v_{l,k}^{l,k} = 1, \quad v_{l}^{l,k} = -1, \quad v_{n}^{l,k} = 0 \text{ for all } n \neq l, k. \]

\[ \text{Under the normalization we made, } u_i(\delta_k) - u_i(\delta_l) = 0. \]

Figure B.3

Proposition B.3.1. For any pair of states \( l \) and \( k \), if there exists a sender \( i \) with \( u_i \) satisfying Condition B.1 on \( \Delta(\{l,k\}) \), then \( l \) and \( k \) are not pooled in every equilibrium.
We prove Proposition [B.3.1] after redefining some objects for this setting. For any sender \( i \) and strategy profile for all opponents \( \{\Gamma_j\}_{j \neq i} \), we define \( W_i(x) \) for \( x \in \Delta(\Omega) \) analogously to the finite signal case.

\[
W_i(x) = \int_{\Delta(\Omega)} u_i(\beta(x,y))p_{-i}(y|x)dy \tag{B.1}
\]

For any vector \( v \in \mathbb{R}^N \) and \( y \in \Delta(\Omega) \), let \( \nabla_v p_{-i}(y|x) \) and \( \nabla_v \beta_k(x,y) \) (for any state \( k \)) be the directional derivates of these two functions with respect to \( x \) along vector \( v \). Note the following.

\[
\nabla_v p_{-i}(y|x) = \nabla_v \left( \frac{\sum_{l=1}^{N} x_l y_l p_{-i}(y|x)}{\pi_l} \right) = \sum_{l=1}^{N} \frac{v_l y_l p_{-i}(y|x)}{\pi_l} \tag{B.2}
\]

\[
\nabla_v \beta_k(x,y) = \nabla_v \left( \frac{\sum_{l=1}^{N} x_l y_l}{\pi_l} \frac{x_l y_l}{\pi_k} \right) = -\sum_{l=1}^{N} \frac{v_l y_l x_l y_l}{\pi_l (\sum_{n=1}^{N} x_n y_n)^2} + \frac{v_k y_k}{\pi_k} \tag{B.3}
\]

For any states \( l, k \in \{1, \ldots, N\} \) let \( \Delta^{int}({\{l, k\}}) = \Delta(\{l, k\}) \setminus \{\delta_l, \delta_k\} \) be the nondegenerate beliefs in \( \Delta(\{l, k\}) \). Let \( \Delta^0({\{l, k\}}) = \{\beta \in \Delta(\Omega) : \beta_l, \beta_k = 0\} \) and \( \Delta^1({\{l, k\}}) = \{\beta \in \Delta(\Omega) : \beta_l + \beta_k < 1; \beta_l \text{ and/or } \beta_k \neq 0\} \). Note \( \{\delta_l, \delta_k\} \cup \Delta^{int}(\{l, k\}) \cup \Delta^0(\{l, k\}) \cup \Delta^1(\{l, k\}) = \Delta(\Omega) \) and all four sets are disjoint.

We now prove Proposition [B.3.1]

**Proof.** First note Lemma 1 still holds in this context by an identical proof. In any equilibrium \( (\Gamma_1, \ldots, \Gamma_M) \) all senders \( i \) have: \( U_i(\Gamma_1, \ldots, \Gamma_M) = 0 \) and \( W_i(x) \leq 0 \) for all \( x \in \Delta(\Omega) \).

WLOG let \( l = 1, k = 2 \). For notational convenience let \( v = v^{1.2} \). Suppose \( u_i \) satisfies Condition [B.1] on \( \Delta(\{1, 2\}) \) for some \( i \). WLOG we consider the case \( \nabla_v u_i(\delta_2) \neq u_i(\delta_2) - u_i(\delta_1) = 0 \). Then by zero-sumness (derivatives must also be zero-sum where they exist for all senders) there exists a sender \( j \) with \( \nabla_v u_j(\delta_2) = c < 0 \). We will prove that \( \Gamma_{-j} \) must not pool \( \{l, k\} \); this clearly implies the Proposition [B.3.1] as by Claim [A.3] \( (\Gamma_1, \ldots, \Gamma_M) \) also will not pool \( \{l, k\} \).

Suppose for contradiction there is an equilibrium \( (\Gamma_1, \ldots, \Gamma_M) \) such that \( \Gamma_{-j} \) pools \( \{l, k\} \). For \( x \in \Delta(\Omega) \), by the product rule and the partition of \( \Delta(\Omega) \) into \( \{\delta_1, \delta_2\}, \Delta^{int}(\{1, 2\}), \Delta^0(\{1, 2\}), \Delta^1(\{1, 2\}) \):
\[ \nabla_v W_j(x) = \nabla_v (u_j(\beta(x, \delta_2))p_{-j}(\delta_2|x) + u_j(\beta(x, \delta_2))\nabla_v p_{-j}(\delta_2|x) + \nabla_v u_j(\beta(x, \delta_1))p_{-j}(\delta_1|x) + u_j(\beta(x, \delta_1))\nabla_v p_{-j}(\delta_1|x) \]
\[ + \int_{\Delta^*(\{1,2\})} \nabla_v u_j(\beta(x, y))p_{-j}(y|x)dy + \int_{\Delta^*(\{1,2\})} u_j(\beta(x, y))\nabla_v p_{-j}(y|x)dy \]
\[ + \int_{\Delta^0(\{1,2\})} \nabla_v u_j(\beta(x, y))p_{-j}(y|x)dy + \int_{\Delta^0(\{1,2\})} u_j(\beta(x, y))\nabla_v p_{-j}(y|x)dy \]
\[ + \int_{\Delta^1(\{1,2\})} \nabla_v u_j(\beta(x, y))p_{-j}(y|x)dy + \int_{\Delta^1(\{1,2\})} u_j(\beta(x, y))\nabla_v p_{-j}(y|x)dy \]

though this may not be well defined for some \( x \). Note:
\[ \nabla_v (u_j(\beta(x, y))p_{-j}(\delta_2|x)) = \sum_{k=1}^N \frac{\partial u_j(\beta(x, y))}{\partial \beta_k} \nabla_v \beta_k(x, y)p_{-j}(y|x) \]

(again this may not be well defined for some \( x \)). Now consider \( x \in \Delta(\{1,2\}) \). For such \( x \), with some algebra:
\[ \nabla_v \beta_k(x, y)p_{-j}(y|x) = \frac{(y_1/\pi_1 - y_2/\pi_2)x_{jk}}{\pi_1} + \frac{v_ky_k}{\pi_k} \] (B.4)

One can check that for \( y \in \Delta^0(\{1,2\}) \) and for \( y \in \{\delta_1, \delta_2\} \) this expression is 0 for all \( k = 1, ..., N \). This tells us that the 1st, 3rd, and 7th terms of \( \nabla_v W_j(x) \) are 0 for \( x \in \Delta(\{1,2\}) \). Note that for \( y \in \Delta(\{1,2\})^0 \), \( \nabla_v p_{-i}(y|x) = 0 \), and so the 8th term is also 0.

We consider the limit of \( \nabla_v W_j(x) \) for \( x \in \Delta(\{1,2\}) \) as \( x \to \delta_2 \). We show this limit exists and is negative.

\[ \lim_{x \to \delta_2} \nabla_v W_j(x) = \lim_{x \to \delta_2} u_j(\beta(x, \delta_2))\nabla_v p_{-j}(\delta_2|x) + \lim_{x \to \delta_2} u_j(\beta(x, \delta_1))\nabla_v p_{-j}(\delta_1|x) \]
\[ + \int_{\Delta^*(\{1,2\})} \sum_{k=1}^N \lim_{x \to \delta_2} \frac{\partial u_j(\beta(x, y))}{\partial \beta_k} \nabla_v \beta_k(x, y)p_{-j}(y|x)dy + \int_{\Delta^*(\{1,2\})} \lim_{x \to \delta_2} u_j(\beta(x, y))\nabla_v p_{-j}(y|x)dy \]
\[ + \int_{\Delta^1(\{1,2\})} \sum_{k=1}^N \lim_{x \to \delta_2} \frac{\partial u_j(\beta(x, y))}{\partial \beta_k} \nabla_v \beta_k(x, y)p_{-j}(y|x)dy + \int_{\Delta^1(\{1,2\})} \lim_{x \to \delta_2} u_j(\beta(x, y))\nabla_v p_{-j}(y|x)dy \]
We evaluate this term by term. The first term is 0 as \( \nabla vp_j(\delta_2|x) \) is finite and does not depend on \( x \) and \( \lim_{x \to \delta_2} \beta(x, \delta_2) = \delta_2 \) which implies (by continuity of \( u_j \) in a neighborhood of \( \delta_2 \)) \( \lim_{x \to \delta_2} u_j(\beta(x, \delta_2)) = 0 \). The second term is also 0 as \( \nabla vp_j(\delta_1|x) \) is finite and does not depend on \( x \) and \( \lim_{x \to \delta_2} \beta(x, \delta_1) = \delta_1 \) (by L’Hopital’s rule) which implies (by continuity of \( u_j \) in a neighborhood of \( \delta_1 \)) \( \lim_{x \to \delta_2} u_j(\beta(x, \delta_1)) = 0 \).

The fourth term is also 0 as for all \( y \in \Delta^{int}(\{1, 2\}) \), \( \lim_{x \to \delta_2} \beta(x, y) = \delta_2 \) and \( u_j(\delta_2) = 0 \) while \( \nabla vp_j(y|x) \) is finite and does not depend on \( x \). The sixth term is also 0 for the following reason. For \( y \in \Delta^i(\{1, 2\}) \) with \( y_2 > 0 \), \( \lim_{x \to \delta_2} \beta(x, y) = \delta_2 \); as \( u_j(\delta_2) = 0 \), the terms inside the integral are 0 when \( y_2 > 0 \). For \( y \in \Delta^i(\{1, 2\}) \) with \( y_2 = 0 \), we must have \( y_1 > 0 \); then by L’Hopital’s rule \( \lim_{x \to \delta_2} \beta(x, y) = \delta_1 \) and \( u_j(\delta_1) = 0 \) meaning terms inside the integral are 0 when \( y_2 = 0 \).

Using (B.4) note that \( \nabla v\beta_k(x, y)p_j(y|x) = 0 \) for all \( k \neq 1, 2 \) as for such \( k x_k = v_k = 0 \). For \( y \in \Delta(\{1, 2\})^{int} \) we have: \( \lim_{x \to \delta_2} \nabla v\beta_1(x, y)p_j(y|x) = \frac{-\partial u_j(\delta_2)}{\partial \beta_2} \) and \( \lim_{x \to \delta_2} \nabla v\beta_2(x, y)p_j(y|x) = \frac{\partial u_j(\delta_2)}{\partial \beta_1} \). The same holds for \( y \in \Delta^i(\{1, 2\}) \) with \( y_2 > 0 \).

For \( y \in \Delta^i(\{1, 2\}) \) with \( y_2 = 0 \), we have: \( \lim_{x \to \delta_2} \nabla v\beta_1(x, y)p_j(y|x) = \lim_{x \to \delta_2} \nabla v\beta_2(x, y)p_j(y|x) = 0 \) (applying L’Hopital’s rule).

Putting this together:

\[
\lim_{x \to \delta_2} \nabla vW_j(x) = \int_{\Delta^{int}(\{1, 2\}) \cup \{y \in \Delta^i(\{1, 2\}) : y_2 > 0\}} \frac{\partial u_j(\delta_2)}{\partial \beta_2} \nabla v\beta_1(x, y)p_j(y|x) dy + \frac{\partial u_j(\delta_2)}{\partial \beta_1} \nabla v\beta_2(x, y)p_j(y|x) dy
\]

\[
= \nabla u_j(\delta_2) \int_{\Delta^{int}(\{1, 2\}) \cup \{y \in \Delta^i(\{1, 2\}) : y_2 > 0\}} \frac{\partial u_j(\delta_2)}{\partial \beta_1} \nabla v\beta_2(x, y)p_j(y|x) dy
\]

\[
= \frac{\partial u_j(\delta_2)}{\partial \beta_1} \int_{\Delta^{int}(\{1, 2\}) \cup \{y \in \Delta^i(\{1, 2\}) : y_2 > 0\}} \frac{y_1}{\pi_1} p_j(y|x) dy
\]

As \( \Gamma_{-j} \) pools \( \Omega' \), there exists \( y \in \text{supp}[\Gamma] \) for which \( y_1, y_2 > 0 \). Such a \( y \) must fall inside the set \( (\Delta^{int}(\{1, 2\}) \cup \{y \in \Delta^i(\{1, 2\}) : y_2 > 0\}) \) (any point in the complement of this set assigns probability 0 to at least one of states 1, 2.). This implies that there are \( y \in (\Delta^{int}(\{1, 2\}) \cup \{y \in \Delta^i(\{1, 2\}) : y_2 > 0\}) \) for which \( p_j(y) > 0 \) and \( y_1 > 0 \). As \( y'_1 \geq 0 \) for all \( y' \in \Delta(\Omega) \), the integral on the righthand side of equation (B.5) is strictly positive. As \( c < 0 \), we have \( \lim_{x \to 0} \nabla vW_j(x) < 0 \).

But as \( W_j(\delta_2) = 0 \), this implies that for some \( x^* \in \Delta(\{1, 2\}) \) close enough to \( \delta_2 \), we must have \( W_j(x^*) > 0 \), contradicting Lemma 1.

\[ \square \]

Condition (B.1) holding on each edge for some sender is a sufficient condition for full
revelation in any equilibrium.

**Theorem B.3.1.** If for every pair of states \( l \) and \( k \) there exists a sender \( i \) such that \( u_i \) satisfies Condition [B.1] on \( \Delta(\{l, k\}) \) then the state is fully revealed in every equilibrium in which senders choose experiments from \( G \).

**Proof.** If for every pair \( l, k \) there is a sender with \( u_i \) satisfying Condition [B.1] on \( \Delta(\{l, k\}) \), then by Proposition [B.3.1] no pair of states is pooled in any equilibrium. Hence w.p. 1 the posterior assigns positive probability to only 1 state and hence must fully reveal that state.

Note that a function not satisfying Condition [B.1] along a given edge is knife-edge—it requires a particular directional derivative to take a certain value at two points. If no sender has a utility function satisfying Condition [B.1] along an edge, this is even more particular. Hence we ‘typically’ expect Condition [B.1] to be satisfied on each edge for some \( u_i \) and so Theorem [B.3.1] says we should typically expect fully revelation in every equilibrium.

### B.4 Privately Informed Senders

Consider our baseline model with one modification: each sender receives a private signal before the game. For simplicity, senders’ private signals are realizations of finite signal experiments that are conditionally (on \( \omega \)) independent across senders.\(^{26}\) We think of the experiments in terms of the beliefs they induce. Formally each sender \( i \) draws a private belief \( b_i \in \Delta(\Omega) \) with \( b_i \sim B_i \in \mathcal{P} \), \( |\text{supp}[b_i]| < \infty \), and \( \mathbb{E}[b_i] = \pi \) (Bayes-plausibility). The distributions \( \{B_i\}_{i=1}^N \) are conditionally independent. We make one more assumption: that for each \( \Omega' \subsetneq \Omega \), \( \text{supp}[b_i] \cap \Delta(\Omega') = \emptyset \) for all \( i \); no sender’s private information rules out any states.\(^{27}\)

A pure strategy for sender \( i \) is a mapping from private beliefs (or types) to choices of finite signal experiments: \( \sigma_i : \text{supp}[b_i] \to \mathcal{P} \). As before, we use \( \Gamma_i \) to denote the interim belief produced by sender \( i \)'s experiment. \( i \) chooses the distribution of \( \Gamma_i \), \( p_i \in \mathcal{P} \) after observing her own type. Importantly, we define \( \Gamma_i \) to be the interim belief the receiver holds after viewing realization of \( i \)'s experiment but without updating her belief on \( \Omega \) from observing the choice of \( p_i \) (we formalize this updating in the next paragraph). Hence \( \Gamma_i \) is

\(^{26}\)These assumptions are not necessary.

\(^{27}\)Having already made the assumption of finite signals, this assumption is equivalent saying signals are bounded.
the receiver’s learning from $i$’s experiment ignoring information from signalling. A pure strategy profile is a vector $(\sigma_1, ..., \sigma_M)$.

The receiver’s posterior belief $\beta$ is a function of the signal realizations she observes as well as the experiment choices she observes. The receiver will form beliefs about each $\{b_i\}_{i=1,...,M}$ independently via a belief function $\mu^i : \Delta(\Delta(\Omega)) \rightarrow \Delta(\text{supp}[b_i])$ ($i = 1, ..., M$) which maps choices of experiment to a belief on the sender’s type. For an experiment choice of $p_i$ by sender $i$, let $\mu_i(p_i)[b]$ denote the probability the receiver assigns to $b_i = b$ under belief function $\mu^i$. For $i = 1, ..., M$ let $\tau^i(\mu^i, p_i) \in \Delta(\Omega)$ be the receiver’s belief on $\omega$ given belief function $\mu^i$ after observing experiment choice $p_i$ from sender $i$ but not its signal realization or any other senders’ experiment choices are realizations. Then for each state $l \in \Omega$:

$$\tau^i(\mu^i, p_i) = \sum_{b \in \text{supp}[b_i]} Pr(\omega = l | b_i = b) Pr(b_i = b | p_i) = \sum_{b \in \text{supp}[b_i]} b_i \mu_i(p_i)[b] \quad (B.6)$$

For any sender $i$ let $\alpha^i(\Gamma_i, p_i, \mu^i) \in \Delta(\Omega)$ be the random variable representing the receiver’s interim belief given $\mu^i$ after observing just the choice $p_i$ and the realization of $\Gamma_i$. $\alpha^i$ hence captures the receiver’s belief after taking into account all information—signalling and otherwise—from sender $i$. For any $l \in \Omega$:

$$\alpha^i_l(\Gamma_i, p_i, \mu^i) = \frac{\tau^i_l(\mu^i, p_i) \Gamma_{i,l}/\pi_l}{\sum_{k=1}^N \tau^i_k(\mu^i, p_i) \Gamma_{i,k}/\pi_k} \quad (B.7)$$

For fixed $\{\mu^i\}_{i=1,...,M}$, after observing $\{p_i\}$ and realizations $\{\Gamma_i\}$, the receiver updates by Bayes rule to a posterior belief for each $l \in \Omega$:

$$\beta^i(\{\Gamma_i\}, \{p_i\}, \{\mu^i\}) = \frac{\prod_{i=1}^M \alpha^i_l(\Gamma_i, p_i, \mu^i)/\pi_i^{M-1}}{\sum_{k=1}^N \prod_{i=1}^M \alpha^i_k(\Gamma_i, p_i, \mu^i)/\pi_k^{M-1}} \quad (B.8)$$

A PBE (in pure strategies) is a strategy profile $(\sigma_1, ..., \sigma_M)$ and a set of belief functions $(\mu^1, ..., \mu^M)$ satisfying two conditions. First, no sender $i$ can strictly gain from deviating from $\sigma_i(b_i)$ for any $b_i \in \text{supp}[b_i]$:

\[\text{28While other senders’ experiment choices and the realizations of } (\Gamma_1, ..., \Gamma_M) \text{ will affect the receiver’s belief about each } b_i, \text{ as players’ types and experiment realizations are conditionally independent they will only affect the receiver’s beliefs through learning about } \omega. \text{ This updating will hence not affect the receiver’s belief on } \omega, \text{ which is all that players care about. The functions } \{\mu^i\} \text{ are what is important for evaluating senders’ payoffs.}\]
∀i ∈ {1, ..., M}, b_i ∈ supp[b_i] : \( \mathbb{E}_{\{B_j\}_{j \neq i}}[\mathbb{E}_{\sigma_i(b_i), \{p_j\}_{j \neq i}}[u_i(\beta)]]b_i \geq \mathbb{E}_{\{B_j\}_{j \neq i}}[\mathbb{E}_{p'_i, \{p_j\}_{j \neq i}}[u_i(\beta)]]b_i \) for all \( p'_i \in P \) (B.9)

where the receiver’s posterior \( \beta \) is formed using [B.8]. It is important to note that sender \( i \) may not know \( \{p_j\}_{j \neq i} \) but forms beliefs about these given her own private information to evaluate expected utility.

Second, beliefs must follow Bayes rule on path:

\[ \forall i \in \{1, ..., M\} \text{ and } p \in P \text{ s.t. } \exists b_i \in supp[b_i] \text{ with } \sigma_i(b_i) = p : \]
\[ \forall b \in supp[b_i], \mu^1(p)[b] = \frac{1_{\sigma(b) = p} B_i(b)}{\sum_{b' \in supp[b_i]} 1_{\sigma(b') = p} B_i(b')} \] (B.10)

The result below is Lemma 1 adapted to this setting with private information.

**Lemma B.4.** Take any equilibrium \( (\sigma_1, ..., \sigma_M), (\mu^1, ..., \mu^M) \). For any sender \( i \) and \( b \in supp[b_i] \), conditional on \( b_i = b \) sender \( i \) gets expected utility 0.

**Proof.** Fix any equilibrium. No sender \( i \) can get expected utility strictly less than 0 conditional on any \( b_i = b \in supp[b_i] \), as playing the fully revealing experiment guarantees expected utility 0. This means each sender \( i \)'s expected utility unconditional on type,

\[ \sum_{b' \in supp[b_i]} \mathbb{E}_{\{B_j\}_{j \neq i}}[\mathbb{E}_{\sigma_i(b'), \{p_j\}_{j \neq i}}[u_i(\beta)]]B_i(b') \] (B.11)

is weakly positive. As the game is zero-sum, the sum of all senders’ unconditional expected utilities must be 0; as each of these payoffs is weakly positive, it must be Equation B.11 is equal to 0 for each \( i \). But then as each term in the summation of Equation B.11 is weakly positive, sender \( i \)'s expected utility conditional on \( b_i = b \) cannot be strictly positive, and hence it must be 0. \( \square \)

We redefine state pooling in the game with private information as follows. An equilibrium \( (\sigma_1, ..., \sigma_M), (\mu^1, ..., \mu^M) \) does not pool a set of states \( \Omega' \subseteq \Omega \) if \( Pr(\beta_l > 0 \forall l \in \Omega') = 0 \). Otherwise, the equilibrium pools \( \Omega' \). The following Lemma is useful for the main results; it says an equilibrium pools \( \Omega' \) if and only if \( \alpha^i \) does for every sender \( i \).
Lemma B.5. An equilibrium $(\sigma_1, \ldots, \sigma_M), (\mu^1, \ldots, \mu^M)$ pools $\Omega' \subseteq \Omega$ if and only if for all $i$ $\Pr(\alpha_i^l(\Gamma_i, p_i, \mu^i) > 0 \forall l \in \Omega') > 0$.

Proof. ‘If’ direction. If for all $i$ $\Pr(\alpha_i^l(\Gamma_i, p_i, \mu^i) > 0 \forall l \in \Omega')$, then as $\alpha^i$ is conditionally (on state) across senders (as $b_i$ and $\Gamma_i$ are), $\Pr(\alpha_i^l(\Gamma_i, p_i, \mu^i) > 0 \forall l \in \Omega' \forall i = 1, \ldots, M) > 0$. By equation B.8 $\Pr(\beta_i > 0 \forall l \in \Omega') > 0$.

‘Only if’ direction. If for some sender $i$, $\Pr(\alpha_i^l(\Gamma_i, p_i, \mu^i) > 0 \forall l \in \Omega') = 0$, then by equation B.8 $\beta_i = 0$ for some $l \in \Omega'$ w.p. 1. □

We now provide a sufficient condition for a pair of states to be not pooled in every equilibrium. The sufficient condition is the same as that in Section B.3 and will lead to the same sufficient condition for full revelation in all equilibria. We note as before that this condition is satisfied for all but a knife-edge case of sender preferences. As in Section B.3 we make the mild technical assumption that all sender utilities are real analytic in some neighborhood of $\delta_i$ for all states $l$ (Assumption B.1).

Lemma B.6. Suppose Assumption B.1 holds. Consider any pair of states $l$ and $k$. $\{l, k\}$ is not pooled in every equilibrium if for some $i$ $u_i$ satisfies Condition B.1 on $\Delta(\{l, k\})$.

Proof. WLOG let $l = 1, k = 2$. For notational convenience let $v = v^{1,2}$. Suppose $u_i$ satisfies Condition B.1 on $\Delta(\{1, 2\})$ for some $i$. WLOG we consider the case $\nabla_v u_i(\delta_2) \neq u_i(\delta_2) - u_i(\delta_1) = 0$. Then by zero-sumness (derivatives must also be zero-sum where they exist for all senders) there exists a sender $j$ with $\nabla_v u_j(\delta_2) = c < 0$. This implies there exists $r \in (0, 1)$ such that for all $\gamma \in \Delta(\{1, 2\})$ with $\gamma_2 > r$, $u_j(\gamma) > 0$. In other words, $j$ has a region of advantage along $\Delta(\{1, 2\})$ close to $\delta_2$.

Suppose for contradiction there is an equilibrium $(\sigma_1, \ldots, \sigma_M), (\mu^1, \ldots, \mu^M)$ that pools $\{l, k\}$. For each $j' \neq j$, let $\Lambda_{j'} = \alpha^{j'}(\Gamma_{j'}, p_{j'}, \mu^{j'})$. By Lemma B.5 for all $j' \neq j$, $\Lambda_{j'}$ pools $\{1, 2\}$. Note $\Lambda_{j'}$ is also a random variable (where randomness is over $p_{j'}$ and the realization of $\Gamma_{j'}$) with finite support (due to finite support of $b_{j'}$ and $\Gamma_{j'}$). If all senders $j' \neq j$ follow the equilibrium play, $\Lambda_{j'}$ is also Bayes-plausible (with mean $\pi$); this is because $\Gamma_{j'}$ Bayes-plausible, any learning the receiver does about $b_{j'}$ must follow Bayes rule on path, and the distribution of $b_{j'}$ has mean $\pi$. Let $\Lambda_{-j}$ denote the interim belief induced by viewing realizations of all $\{\Lambda_{j'}\}_{j' \neq j}$; note this experiment also pools $\{1, 2\}$; this is also a finite signal Bayes-plausible experiment.

We will now find a profitable deviation for sender $j$. This deviation will take the form of an experiment $p_{j'}$, which we will construct, that generates strictly positive expected
utility no matter what \(j\)’s type is.

We can rewrite \(\beta_l\) for \(l \in \Omega\) (from equation [B.8]) conditional on this deviation \(p'_j\) as:

\[
\beta_l(\Lambda_{-j}, \alpha^j(\Gamma_j, p'_j, \mu^j)) = \frac{\Lambda_{-j, \mu}(\Gamma_j, p'_j, \mu^j)/\pi_l}{\sum_{k=1}^{N} \Lambda_{-j, \mu}(\Gamma_j, p'_j, \mu^j)/\pi_k}
\]  
(B.12)

It is also useful to write down the probability distribution of \(\Lambda_{-j}\) conditional on \(\alpha^j(\Gamma_j, p'_j, \mu^j)\):

\[
Pr(\Lambda_{-j} = y|\alpha^j(\Gamma_j, p'_j, \mu^j)) = \frac{\sum_{l=1}^{N} \alpha^j_l(\Gamma_j, p'_j, \mu^j)y_l}{\pi_l}
\]  
(B.13)

Let \(Y = \{y \in supp[\Lambda_{-j}] : y_1, y_2 > 0\}\); note this set is nonempty as \(\Lambda_{-j}\) pools \(\{1, 2\}\). Let \(Y_0 = \{y \in supp[\Lambda_{-j}] : y_1, y_2 = 0\}\), \(Y_1 = \{y \in supp[\Lambda_{-j}] : y_1 > 0, y_2 = 0\}\), and \(Y_2 = \{y \in supp[\Lambda_{-j}] : y_1, y_2 > 0\}\). These sets partition the support of \(\Lambda_{-j}\).

Consider \(j\) generating interim belief \(\Gamma_j = x \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\). As in Proposition 2’s proof, we will try and find such an \(x\) conditional on which \(j\) gets a strictly positive expected payoff. We will then construct \(p'_j\) which assigns positive probability to \(x\). Note that \(Pr(\alpha^j(\Gamma_j, p'_j, \mu^j) \in \Delta(\{l, k\}) \setminus \{\delta_1, \delta_2\}|\Gamma_j = x) = 1\). This can be seen from the definition of \(\alpha^j\) (equation [B.7]) and is a consequence of every private belief \(b \in supp[b]\) having \(b_n > 0\) for all \(n \in \Omega\) which implies \(\tau'_{l}(\mu^j, p'_j) > 0\) for all \(l \in \Omega\). This implies that \(Pr(\Lambda_{-j} \in Y_0|\Gamma_j = x \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}) = 0\) (equation [B.13]). Also note that conditional on \(\Gamma_j = x \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\), \(\beta = \delta_1\) when \(\Lambda_{-j} \in Y_1\) and \(\beta = \delta_2\) when \(\Lambda_{-j} \in Y_2\) (see equation [B.12]); these posteriors both yield utility 0.

Finally, note that conditional \(\Gamma_j = x \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\), we have \(Pr(\Lambda_{-j} \in Y) > 0\) (by equation [B.13] and \(\alpha^j \in \Delta(\{l, k\}) \setminus \{\delta_1, \delta_2\}\)). Note that for each \(y \in Y\) and \(\alpha^j(\Gamma_j, p'_j, \mu^j) \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\), \(\beta_k(y, \alpha^j(\Gamma_j, p'_j, \mu^j))\) is continuous in \(\alpha^j(\Gamma_j, p'_j, \mu^j)_k\), \(\beta_k(y, \alpha^j(\Gamma_j, p'_j, \mu^j)) \rightarrow 1\) as \(\alpha^j(\Gamma_j, p'_j, \mu^j) \rightarrow 1\), and \(\beta_k(y, \alpha^j(\Gamma_j, p'_j, \mu^j)) \rightarrow 0\) as \(\alpha^j(\Gamma_j, p'_j, \mu^j) \rightarrow 0\). As \(Y\) is finite, the function min\(_{y \in Y}\) \(\beta_k(y, \alpha^j(\Gamma_j, p'_j, \mu^j))\) is also continuous in its second argument for \(\alpha^j(\Gamma_j, p'_j, \mu^j) \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\) and goes to 0 or 1 as \(\alpha^j(\Gamma_j, p'_j, \mu^j)\) goes to 0 or 1 respectively. By the intermediate value theorem, there exists a \(\alpha_r \in (0, 1)\) such that min\(_{y \in Y}\) \(\beta_k(y, \alpha_r) = r\). When \(1 > \alpha^j(\Gamma_j, p'_j, \mu^j) > \alpha_r\), we have \(\beta_k(y, \alpha^j(\Gamma_j, p'_j, \mu^j)) \in (r, 1)\) for all \(y \in Y\).

Note that for \(\Gamma_j = x \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\}\), we have \(\alpha^j_2(x, p'_j, \mu^j) = \frac{\tau'_{2}(\mu^j, p'_j)x_2/\pi_2}{\tau'_{1}(\mu^j, p'_j)x_1/\pi_1 + \tau'_{2}(\mu^j, p'_j)x_2/\pi_2}\),
We can rewrite this as: \( \alpha_2^j(x, p'_j, \mu^j) = \frac{\tau_1^j(\mu^j, p'_j)}{\tau_i^j(\mu^j, p'_j) x_2/\pi_2} \). Note that as private beliefs have finite support and cannot rule out any state, for all beliefs the receiver may hold about \( j \)'s type, \( \mu \in \Delta(\text{supp}[b_j]) \), the corresponding belief \( \tau \) this induces on \( \Omega (\tau_1 = \sum_{b \in \text{supp}[b_j]} b \mu[b]) \) must have \( \frac{\tau_2^j}{\tau_1^j} \geq d > 0 \) for some \( d \in (0, 1) \). Hence:

\[
\alpha_2^j(x, p'_j, \mu^j) = \frac{\tau_1^j(\mu^j, p'_j) x_2/\pi_2}{x_1/\pi_1 + \frac{\tau_1^j(\mu^j, p'_j) x_2/\pi_2}{\tau_i^j(\mu^j, p'_j) x_2/\pi_2}} \geq \frac{dx_2/\pi_2}{x_1/\pi_1 + dx_2/\pi_2} \tag{B.14}
\]

\( \alpha_2^j(x, p'_j, \mu^j) \) is continuous in \( x \) on \( \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\} \) and will also fall in \( \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\} \). By equation \( \text{B.14} \) as \( x_2 \to 1 \), \( \alpha_2^j(x, p'_j, \mu^j) \to 1 \) regardless of what beliefs the receiver holds. Hence there exist \( x^* \in \Delta(\{1, 2\}) \setminus \{\delta_1, \delta_2\} \) such that \( 1 > \alpha_2^j(x^*, p'_j, \mu^j) > \alpha_r \).

Hence we have \( \beta_k(y, \alpha^j(x^*, p'_j, \mu^j)) \in (r, 1) \) for all \( y \in Y \). Conditional on \( x^* \), \( j \) gets expected utility:

\[
\sum_{y \in Y_1} u_j(\delta_1) \Pr(\Lambda_{-j} = y | \alpha^j(x^*, p'_j, \mu^j)) + \sum_{y \in Y_2} u_j(\delta_2) \Pr(\Lambda_{-j} = y | \alpha^j(x^*, p'_j, \mu^j))
\]

\[
+ \sum_{y \in Y} u_j(\beta(y, \alpha^j(x^*, p'_j, \mu^j))) \Pr(\Lambda_{-j} = y | \alpha^j(x^*, p'_j, \mu^j)) > 0
\]

The proof of Lemma 1 demonstrates how any type of sender \( j \) can construct a strategy \( p'_j \) which assigns positive probability only to \( x^* \) and \( \{\delta_1, ..., \delta_N\} \). \( p'_j \) yields \( j \) strictly positive expected utility conditional on \( x^* \) being realized and 0 utility otherwise. Hence Lemma \( \text{B.4} \) is violated. Contradiction. Hence no equilibrium can pool \( \{1, 2\} \).

Lemma \( \text{B.6} \) implies that Condition \( \text{B.1} \) being satisfied by some \( u_i \) on each edge of the simplex is sufficient for full revelation in all equilibria. It is worth noting again that is sufficient condition is satisfied for all but a knife-edge case of sender preferences.

**Proposition B.3.** Suppose Assumption \( \text{B.1} \) holds. The state is fully revealed in every equilibrium if for all pairs of states \( l \) and \( k \), there is some \( u_i \) that satisfies Condition \( \text{B.1} \) on \( \Delta(\{l, k\}) \).
Proof. Argument is identical to Theorem B.3.1’s proof. □

B.5 Sequential Moving Senders

Consider a sequential version of our baseline model. Senders 1,...,M move in order, observing all previous experiment choices (but not realizations); we are interested in pure strategy subgame perfect Nash Equilibria (henceforth just SPNE) of this game. Note that for a simultaneous game, there are multiple corresponding sequential games, one for each ordering of senders.

A few facts easily carry over from the simultaneous case. First, all senders must get utility 0 in equilibrium (as the game is zero-sum and anyone can fully reveal the state). Second, full revelation can be supported as an SPNE outcome in a game with senders moving in any order. We can construct such an equilibrium with all senders playing \( \Gamma^{FR} \) on path and playing any sequentially rational strategies off path. No sender after the first has an incentive to deviate if those upstream from them have not (as the receiver will the learn the state from upstream senders). The first sender cannot strictly gain from deviating, as a strict gain would imply a strict loss for a downstream sender; sequential rationality rules this out as the downstream sender can fully reveal the state to avoid a loss. As for the simultaneous game, the interesting question is when all equilibria (here SPNE) are fully revealing. The following results and discussion clarify the relationship between the our simultaneous model and a sequential version.

Proposition B.5.1. If for \( u_1,\ldots,u_M \) there is full revelation in every SPNE of the sequential game with the senders moving in some order, then there is full revelation in every equilibrium of the simultaneous game.

Proof. We prove the following statement, from which the result follows: if there exists a non-fully revealing equilibrium in the simultaneous game, then, for any order of senders, there exists a non-fully revealing SPNE in the sequential game.

Choose any ordering of senders 1,...,M. Consider any non-fully revealing equilibrium of the simultaneous game, \( (\Gamma_1,\ldots,\Gamma_M) \) and let \( \Gamma \) be the experiment induced by observing the realizations of \( \Gamma_1,\ldots,\Gamma_M \). In the sequential game, consider the following strategy profile: (1) sender 1 plays \( \Gamma \). (2) each sender \( i = 2,\ldots,M \) plays \( \Gamma^U \) if all previous senders haven’t deviated and play some sequentially rational strategies otherwise. We will show this is an SPNE. By Lemma 1 all senders get utility 0 from following perscribed play as \( \Gamma \) is the information revealed in an equilibrium of the simultaneous game and no additional
information is revealed. First note sender 1 has no strict incentive to deviate as any profitable deviation would give some downstream sender strictly negative utility. This is not possible along any path of play in an SPNE as this downstream sender can always fully reveal the state.

If sender 1 plays $\Gamma$, senders 2, ..., $M$ have no incentive to deviate for the following reason. Consider any deviation $\Gamma'_j \neq \Gamma^U$ for sender $j$, $2 \leq j \leq M$. This deviation leads to a path of play producing information from $\Gamma$ as well as additional conditionally independent experiments. Suppose, for contradiction, this deviation yields $j$ strictly positive expected utility. But then $j$ has a profitable deviation from the simultaneous game equilibrium $(\Gamma_1, ..., \Gamma_M)$; if $j$ unilaterally plays these additional conditionally independent experiments in addition to $\Gamma_j$, she gets strictly positive utility.

Proposition B.5.1 implies that the set of (zero-sum) utility functions under which there is full revelation in all equilibria in the simultaneous game contains the set under which for some order of senders there is full revelation in all SPNE of the sequential game. The converse is not true. It is possible for there to be full revelation in every equilibrium of the simultaneous game but, for every order of senders, non-fully revealing SPNE in sequential game. The following example demonstrates this.

**Example 2.** There are two senders 1, 2 and three possible states 1, 2, 3. Assume that $\pi_1 < \pi_2 < \pi_3$ (this is not necessary, but eases exposition; the assumption rules out any two states having equal prior probabilities but is otherwise without loss). Suppose $u_1((1/2, 1/2, 0)) = u_1((1/2, 0, 1/2)) = 1$ and $u_2((1/2, 1/2, 0)) = u_2((1/2, 0, 1/2)) = -1$. Also, $u_2((0, 1/2, 1/2)) = 1$ and $u_1((0, 1/2, 1/2)) = -1$. At all other $\gamma \in \Delta(\Omega)$, $u_1(\gamma) = u_2(\gamma) = 0$. Sender 1 has an advantage at single points along edges $\Delta(\{1, 2\})$ and $\Delta(\{1, 3\})$ and sender 2 has a single advantage on edge $\Delta(\{2, 3\})$; on the rest of the simplex, neither sender has an advantage. Figure B.4 summarizes this. By Theorem 1, the state is fully revealed in all equilibria of the simultaneous game. However we will show that regardless of the order senders move in, there is always a non-fully revealing SPNE.
First suppose sender 1 plays first, then sender 2. Consider the sender 1 playing $\Gamma_1$ s.t. $Pr(\Gamma_1 = \delta_3) = \pi_3$ and $Pr(\Gamma_1 = (\frac{\pi_1}{\pi_1+\pi_2}, \frac{\pi_2}{\pi_1+\pi_2}, 0)) = 1 - \pi_3$ (this distribution satisfies Bayes-plausibility). Suppose sender 2 plays $\Gamma_2 = \Gamma_U$ whenever sender 1 plays this and following a deviation plays any sequentially rational $\Gamma_2$. Note the posterior when following prescribed play will either be $\delta_3$ or $((\frac{\pi_1}{\pi_1+\pi_2}, \frac{\pi_2}{\pi_1+\pi_2}, 0)) = 1 - \pi_3$. The latter is not equal to $(1/2, 1/2, 0)$ by our assumption on the prior, and hence both senders get utility 0 at all posteriors. Sender 1 hence has no incentive to deviate as no experiment can yield strictly positive utility (sender 2 can always fully reveal the state and is playing sequentially rationally). Sender 2 has no incentive to deviate as conditional on $\omega = 3$ the receiver learns the state and conditional on $\omega \in \{1, 2\}$ the posterior will lie on $\Delta(\{1, 2\})$, on which sender 2 has no points of advantage, w.p. 1 (by Claim A.2). This is a non-fully revealing equilibrium.

If sender 2 plays first, we can construct an analogous non-fully revealing equilibrium with $\Gamma_2$ s.t. $Pr(\Gamma_2 = \delta_1) = \pi_1$ and $Pr(\Gamma_2 = (0, \frac{\pi_2}{\pi_2+\pi_3}, \frac{\pi_3}{\pi_2+\pi_3})) = 1 - \pi_1$. Sender 1 plays $\Gamma_1 = \Gamma_U$ on path and any sequentially rational $\Gamma_1$ otherwise.

In the simultaneous version of Example 2, the state is fully revealed in every equilibrium for the following reasons. States 2 and 3 must not be pooled in equilibrium because if not sender 2, who has an advantage along $\Delta(\{2, 3\})$, could find an experiment yielding a strictly positive payoff (violating Lemma 1). More precisely, $\Gamma_1$ cannot pool states $\{2, 3\}$, because if not sender 2 can gain a strictly positive payoff. Similarly, $\Gamma_2$ cannot pool $\{1, 2\}$ or $\{1, 3\}$ or sender 1 take advantage. Each sender is ‘responsible’ for not pooling some states in equilibrium because their opponent has an advantage.

More generally, when the state must be fully revealed in every equilibrium of a simultaneous game, for every subset of states $\Omega'$ there is some sender $j$ who could take advantage of $\Gamma_{-j}$ pooling $\Omega'$. This is shown by Claim A.7 (in the proof of Proposition 2); sender $j$ must have an advantage somewhere on $\Delta(\Omega)$. When sender $j$ moves last in the sequential game, then by the same argument, all upstream senders must (collectively) not pool $\Omega'$ in any SPNE. For example, when sender 2 moved second in the example, sender 1 did not pool $\{2, 3\}$. However, as the last moving sender may only be able to take advantage of some subsets of states being pooled, we need not get full revelation. If there is a sender $j$ who can take advantage of $\Omega'$ being pooled for any $\Omega' \in 2^\Omega$ s.t. $|\Omega'| > 1$, then when this sender moves last, every SPNE fully reveals the state. As in Example 2 it is when there is no such sender exists that there are non-fully revealing equilibria for all orders of senders.
This logic implies the following result: if any set of states is not pooled in every equilibrium of the simultaneous game, there is an ordering of senders such that those states are not pooled in every SPNE of the sequential game.

**Proposition B.5.2.** If for utility functions $u_1, \ldots, u_M$ a set of states $\Omega' \subseteq \Omega$ is not pooled in every equilibrium of the simultaneous game, then, for some ordering of senders, $\Omega'$ is not pooled in every SPNE of the sequential game.

**Proof.** Suppose $\Omega' \subseteq \Omega'$ is not pooled in every equilibrium of the simultaneous game. Then by Proposition 2, $u_i(\gamma) > 0$ for some sender $i$ and some $\gamma \in \Delta(\Omega')$. By Claim A.7 there exists a sender $j$ such that for every $\Gamma_{-j}$ that pools $\Omega'$, there exists a $\Gamma_j$ such that $U_j(\Gamma_j, \Gamma_{-j}) > 0$. Consider any ordering of senders with $j$ moving last. Then as all senders must get utility 0, in any SPNE senders upstream from $j$ (collectively) do not pool $\Omega'$ (or else $j$’s best response yields strictly positive utility). Hence, by Lemma A.3 all SPNE do not pool $\Omega'$.

□