Topological lumps and Dirac zero modes in SU(3) lattice gauge theory on the torus

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Abstract

We compute eigenmodes of the lattice Dirac operator for quenched SU(3) gauge configurations on the 4-torus with topological charge ±1. We find a strong dependence of the zero modes on the boundary conditions which we impose for the Dirac operator. The lumps seen by the eigenmodes often change their position when changing the boundary conditions, while the local chirality of the lumps remains the same. Our results show that the zero mode of a charge ±1 configuration can couple to more than one object. We address the question whether these objects could be fractionally charged lumps.

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Introduction

Understanding the nature of the QCD vacuum still is an open and challenging problem. In particular, a possible connection between confinement and chiral symmetry breaking seems plausible. A hint that such a connection might exist can be seen in the fact that at the QCD phase transition deconfinement and chiral symmetry restoration take place at the same temperature.

A step in the direction of getting a grip on such a mechanism was the finding of so-called Kraan - van Baal (KvB) solutions [1]. KvB solutions are self dual solutions of the classical Yang-Mills equations on a euclidean cylinder $(S^1 \times \mathbb{R}^3)$. Thus they correspond to SU(3) gauge theory\(^1\) with temperature. A remarkable property of KvB solutions is that they consist of 3 sub-lumps and these three constituents together build up an object of topological charge 1. Furthermore these lumps were found to be BPS monopoles, a property that might lead to the possible connection to the confinement mechanism. Recently also KvB solutions with higher topological charge were found [2], giving rise to $3|Q|$ monopoles for the solutions with topological charge $|Q|$.

Strong evidence that KvB solutions play a role also in the quantized theory was found by several studies on the lattice. For SU(2) analyses based on cooling with twisted boundary conditions [3] and with regular boundary conditions [4] provided configurations that show the features of KvB solutions. The existence of constituents was established and the characteristic behavior of the Polyakov loop was seen. The zero modes of the Dirac operator showed the expected change of location as a function of the boundary condition. For SU(3) evidence for KvB solutions comes from a study of eigenvectors of the Dirac operator [5, 6] in thermalized (not cooled) configurations. In [7] the zero mode of the Dirac operator in the background of a KvB solution was computed and it was found that it strongly depends on the fermionic boundary condition used. The zero mode is localized on only one of the lumps but the particular constituent it chooses is determined by the boundary condition. This behavior of the zero mode was found for thermalized configurations in [5, 6] and good agreement with the properties of KvB zero modes was established. It is furthermore known that the gauge lumps are predominantly (anti-) self dual [8] and a first step towards matching the results from cooling with fermionic methods is documented in [9].

The analysis of Dirac eigenmodes in thermalized configurations makes

\(^1\)Actually the KvB solutions are known for all SU(N), but here we only address the case of SU(3).
use of the fact that the low lying modes couple only to the long range structures of the gauge field and thus are an efficient filter for removing the UV fluctuations. Allowing for more general fermionic boundary conditions (in particular an arbitrary complex phase) makes this method even more powerful and led to establishing the KvB behavior of zero modes in finite temperature gauge configurations.

It is natural to apply the generalized boundary condition technique also to gauge ensembles at zero temperature. A first step in this direction was made in [6] where for a few configurations with topological charge $\pm 1$ it was shown that again the zero mode can be localized on different lumps when changing the boundary conditions. This hints at the existence of constituents also for charge $\pm 1$ configurations on the 4-torus $T^4$. For this manifold no analytic solution is known (see [10] for a discussions of the classical Yang-Mills fields on the 4-torus) and a lattice study has a more exploratory aspect in this case. However, for the case of two compact directions ($T^2 \times \mathbb{R}^2$) the recently found analytic solution [11] again shows constituents and is already one step closer to the 4-torus analyzed here.

In this article we present a large scale study of the Dirac eigenmodes for thermalized SU(3) configurations with topological charge $\pm 1$ on the 4-torus and analyze the dependence of the eigenmodes on the boundary conditions. We compare ensembles at two different volumes to analyze finite size effects. We show that in both volumes about 40% of the configurations have zero modes that change their position when switching from periodic to anti-periodic boundary conditions. This finding indicates that also on the torus a large portion of charge $\pm 1$ configurations gives rise to zero modes that couple to more than one object.

An important question is whether these topological lumps have fractional charge and are truly constituents of a charge 1 object. The fact that the different lumps show up in the one and only zero mode we see supports this interpretation. Also the KvB solutions and a finite volume argument we discuss give hints in this direction.

**Description of observables**

For our calculations we use two ensembles of quenched gauge configurations generated with the Lüscher-Weisz gauge action [12] on lattices with sizes $12^4$ and $16^4$. For both lattices we use a coupling of $\beta = 8.45$ which gives rise to a lattice spacing of $a = 0.094(1)$ as determined in [13] from the Sommer parameter. For the gauge fields the boundary conditions are periodic.

We compute eigenvectors $\psi$ of the chirally improved lattice Dirac opera-
tor [14] which has good chiral properties and was shown [15] to reproduce the zero mode of a discretized instanton down to radii relatively small in lattice units. In our calculation we allow for a general phase at the temporal boundary of the lattice Dirac operator. The gauge fields have periodic boundary conditions and they do not single out a time direction so we choose the 4-direction to be the time-direction where we impose the general boundary condition for the Dirac operator and its eigenmodes

$$\psi(\vec{x}, L_4 + 1) = e^{i 2\pi \zeta} \psi(\vec{x}, 1).$$  \hspace{1cm} (1)

The boundary phase parameter $\zeta$ can assume values in the interval $[0,1]$ with the two endpoints 0 and 1 giving periodic boundary conditions. Anti-periodic temporal boundary conditions correspond to $\zeta = 0.5$. For all gauge configurations generated we compute 50 (30 for $16^4$) eigenvalues and eigenvectors with periodic boundary conditions ($\zeta = 0$) using the Arnoldi method [16]. From the complete ensemble we then select those configurations which have exactly one zero-mode, i.e. have topological charge $Q = \pm 1$ according to the index theorem. For this subset of configurations we then solve the eigenvalue problem a second time now using anti-periodic boundary conditions ($\zeta = 0.5$). The restriction to configurations with topological charge $Q = \pm 1$ avoids possible problems with mixing of the zero modes and allows for a clean interpretation of the findings. Our statistics is 291 configurations with charge $Q = \pm 1$ for $12^4$ and 199 configurations for $16^4$. For a small subsample we also analyzed the eigenvalue problems with increasing $\zeta$ in small steps $\zeta = 0, 0.1, 0.2 ... 0.9$.

From the eigenmodes $\psi$ we can construct the scalar density $\rho$ and the pseudoscalar density $\rho_5$ as

$$\rho(x) = \sum_{\alpha, c} \psi(x)^\ast_{\alpha, c} \psi(x)_{\alpha, c} , \hspace{1cm} (2)$$

$$\rho_5(x) = \sum_{\alpha, \alpha', c} \psi(x)^\ast_{\alpha, c} (\gamma_5)_{\alpha, \alpha'} \psi(x)_{\alpha', c} . \hspace{1cm} (3)$$

The star denotes complex conjugation. Both densities are gauge invariant observables. The scalar density $\rho$ is always non-negative, while the pseudoscalar density can have both signs, i.e. it is sensitive to the chirality of the lumps. Note that for the scalar density we have $\sum_x \rho(x) = 1$, since the eigenvectors are normalized to 1.

As we have discussed above, we find localized structures in the eigenmodes, i.e. localized lumps in $\rho$ and $\rho_5$. A first observable is the position $x^{\text{max}}$ of the peak of the lump seen in the densities. As announced the
eigenmode can be located at different positions when comparing different boundary conditions. We will be particularly interested in the distance between the maxima seen by the periodic and the anti-periodic zero mode which is given by

\[ d = \| x_{\text{max, periodic}} - x_{\text{max, antip.}} \|_{\text{torus}}. \]  

(4)

Here \( \| .. \| \) denotes the 4-dimensional euclidean distance. The subscript \text{torus} indicates that we always take the shortest distance possible on the 4-torus. We also find that the eigenmodes come in different sizes ranging from tall and narrow (localized) to wide spread (delocalized). A convenient measure for the localization of an eigenmode is its inverse participation ratio \( I \). It is defined as

\[ I = V \sum_x \rho(x)^2. \]  

(5)

Here \( V \) denotes the number of lattice points. It is easy to verify that for a maximally localized lump (\( \rho(x) = \delta(x, x_0) \)) one finds \( I = V \), while for a maximally spread out lump (\( \rho(x) = 1/V \)) one obtains \( I = 1 \). Thus a large value of \( I \) corresponds to a localized, i.e. tall and narrow peak, while small values of \( I \) come from widely spread, delocalized modes.

**Properties of zero-modes with general boundary conditions**

Let us begin with describing some general features which we observed for the zero modes on the torus. We find that the zero modes show a single lump in both densities \( \rho \) and \( \rho_5 \). For all values of \( \zeta \) we find that the lumps are localized in space and time and on the average are rotationally invariant, i.e. no space-time direction is singled out by the shape of the lumps.

When we change the boundary condition the number of zero modes remains invariant, i.e. we always find exactly 1 zero mode\(^2\). When we now compare the zero modes with different boundary conditions but on the same configurations we often find drastic differences: The position of the lump in space time can be different and also the inverse participation ratio. In Fig. 1 we show a schematic plot of the typical behavior often observed when comparing the zero mode for periodic and anti-periodic boundary conditions.

In the two plots of Fig. 1 on the horizontal axes we show all of space time - the corresponding variable is \( x \). On the vertical axes we plot some general density \( \rho \). The zero mode is represented by the full curve and in this case the density \( \rho \) on the vertical axes is the scalar density defined in Eq. (2). The left

\(^2\)We remark that for both lattice sizes we found for about 1\% of the configurations an exception, i.e. a single value of \( \zeta \) where there was no or two zero modes.
Figure 1: Schematic sketch of the motion of the zero mode when switching the fermionic boundary condition from $\zeta = 0$ (l.h.s.) to $\zeta = 0.5$ (r.h.s.).

Hand side (l.h.s.) plot shows the case of periodic boundary conditions ($\zeta = 0.0$) while the r.h.s. plot is for anti-periodic boundary conditions ($\zeta = 0.5$). For the different boundary conditions the zero mode is located at different positions in space time. When scanning through the boundary parameter $\zeta$ in small steps of 0.1 we found that the zero mode can be located at up to three different positions. To be more specific, for a subensemble of 10 configurations on the $16^4$ lattice where we computed eigenvectors with small steps of $\zeta$ we found that 4 zero modes visited 3 different locations, 2 zero modes visited 2 locations and 4 zero modes remained at the same position for all values of $\zeta$.

For cooled configurations it has been established that lumps seen by the zero modes are associated with localized structures in the gauge fields. For thermalized configurations quantum fluctuations are present, but the zero modes and also the low lying modes couple only to the infrared structures in the gauge field [17]. In Fig. 1 we indicate such localized lumps in the infrared part of the gauge field using dashed curves. Here the density on the vertical axis is e.g. the gluonic action density a quantity that is often used in the study of gauge lumps in cooled configurations. In the r.h.s. plot we also show the distance $d$ (compare Eq. (4)) between the lump seen by the zero mode with periodic boundary conditions and the anti-periodic lump.

We stress, that when the zero mode visits different lumps when changing $\zeta$, the lumps always have the same chirality. When inspecting the pseudoscalar density $\rho_5$ we find that all lumps are seen with the same sign of $\rho_5$. This is in agreement with the index theorem, which implies that if there is only one zero mode, its chirality is given by minus the topological charge. Since we do not touch the gauge configuration and only change $\zeta$, the in-
dex theorem requires the chirality of the zero mode to be the same for all fermionic boundary conditions.

Let us discuss an important consistency check we performed. The boundary conditions for the gauge fields are periodic in all directions. In order to use the zero modes as an analyzing tool we single out one of the directions by applying the boundary condition. However, the infrared structures of the gauge field should not depend on this choice. In order to check this, we applied the non-trivial fermionic boundary conditions not only in the 4-direction, but compared also with the zero-modes computed with the boundary condition applied to one of the other directions. We find that the same positions for the constituents are identified when applying the boundary conditions in different directions.

**Systematic comparison of periodic and anti-periodic zero modes**

Let us now come to presenting the results of a large scale comparison of anti-periodic and periodic zero modes. We begin with a discussion of the distribution of the distance $d$ (compare Eq. (4)) between the peaks in the periodic and anti-periodic modes. The corresponding histograms for our $12^4$ ensemble are shown on the l.h.s. and for the $16^4$ ensemble on the r.h.s. of Fig. 2. The distance is measured in lattice units $a = 0.094(1)$ fm and the histograms are normalized to 1.

For both lattice sizes we find that the histogram has large contributions for very small distances between $d = 0$ and $d = 2a$. These contributions come from configurations where the periodic and the anti-periodic zero mode sit at essentially the same position, with the small discrepancies in the position partly due to quantum fluctuations. Clearly separated from these modes we find a well pronounced peak in the histogram containing configurations which show distances of at least $d = 3a$. For both volumes the number of configurations contributing to this peak is about half of the total statistics.

An interesting question is whether one can understand the distribution of distances and the peak of the histograms. We experimented with a simple model where the positions of the two lumps are distributed independently on the lattice. In $\mathbb{R}^4$ such a distribution would be proportional to $d^3$ but on the toroidal lattice we obtain a distribution which has a maximum at $L/2 + 1$ where $L$ is the linear extension of the lattice. The corresponding curves are shown in the plots as connected dots. Note that these model curves are not normalized to 1. Instead their normalization was chosen such that they optimally describe (using a $\chi^2$ optimization) the data in
the range \(d \geq 2a\). Using this procedure we find that the area under the model curve is 0.425 for \(12^4\) and 0.439 for \(16^4\). For both volumes the model describes reasonably well the part of the histogram coming from separated periodic and anti-periodic zero modes. This implies that at least 40 percent of the charge \(\pm 1\) configurations show two or more locations for their zero mode and the corresponding lumps seem to be distributed independently. Note that we only compared periodic and anti-periodic boundary conditions, and we thus do not count cases where the periodic and anti-periodic zero modes are located at the same position, but a different position is visited for some \(\zeta \neq 0, 0.5\). We thus expect that the fraction of configurations with “jumping” zero modes is actually larger than 40 percent.

In Fig. 3 we compare the distribution of the inverse participation ratio of the periodic zero modes to the distribution for the anti-periodic zero modes. The l.h.s. plot is for lattice size \(12^4\), the r.h.s. plot for \(16^4\). For both volumes we find that the distribution peaks at a relatively low value, with the distribution shifted to larger values of \(I\) for \(16^4\). The remarkable feature of the plots is the good agreement of the distribution for the periodic zero modes with the distribution for the anti-periodic modes. This agreement demonstrates again that the size distribution of the lumps is governed by the infrared structures of the underlying gauge field and is not distorted by
the zero modes with different boundary conditions which we use as analyzing tool. Such a distortion could manifest itself by a different distribution of $I$ for periodic and anti-periodic zero modes, which is clearly ruled out by our data.

The nature of the topological lumps

In the previous sections we have demonstrated that the zero mode can be located at different positions if one changes the fermionic boundary conditions. We have shown that for at least 40% of the configurations the maximum of the scalar density visits two or more different space-time points. The zero modes are localized in all 4 directions and the chirality of the zero mode remains unchanged in accordance with the index theorem. The crucial question is now whether the zero modes hop between gauge lumps of fractional charge or simply move from one charge $\pm 1$ object to another.

Let us first try to find a possible mechanism for the latter scenario, where the hopping comes about through some intricate interplay of independent topological objects with charge $\pm 1$. For example the appearance of three lumps with positive chirality seen by the zero mode could come from a configuration with three anti instantons and two instantons. Such a configuration would have net charge $-1$ and naively one would expect 5 zero modes. However, quantum fluctuations could distort 4 of the zero modes, shift their eigenvalues up or down the imaginary axis and turn them into
near zero modes. The remaining single zero mode could then couple only to
the three anti instantons and in this way show only positive chirality for all
three lumps.

This scenario cannot be entirely ruled out, but arguments were given in
\cite{15} that such behavior is unlikely to occur, at least so at finite temperature.
Also our comparison of two different volumes makes this scenario unlikely.
If one increases the volume, more topological objects with charge \( \pm 1 \) can fit
into our box. This would imply that the scenario based on the interplay of
charge \( \pm 1 \) objects could happen more often for the larger volume. Our data
show however, that the rate remains close to 40\% for both volumes although
the larger lattice has a space-time volume more than three times as big as
the smaller lattice. If on the other hand the hopping of the zero-modes
is a genuine property of charge \( \pm 1 \) configurations, caused e.g. by fraction-
ally charged constituents, then the percentage of charge \( \pm 1 \) configurations
showing hops should be independent of the volume. This is what we observe.

At this point we would like to comment on related work by Ilgenfritz,
Martemyanov, Müller-Preussker and Veselov \cite{19}. In their article SU(2)
gauge fields at varying temperature are subjected to a standard cooling
technique. The authors concentrate on (approximate) solutions of the lat-
tice field equations with topological charge \( \pm 1 \). Whereas in the confinement
phase near the critical temperature a considerable fraction of configura-
tions is seen to consist of separated constituents as static BPS monopoles,
at lower temperatures topological lumps of integer topological charge, i.e.
non-dissociated calorons show up. In terms of the Polyakov loop variable
it turns out that these configurations have an intrinsic “\( \pm 1 \) dipole” struc-
ture connected to the nontrivial asymptotics of the holonomy. The form
of the solutions is in one-to-one correspondence to the analytically known
KvB solutions. In an analogous manner they have also studied the case
of the 4-torus. They have found always non-dissociated “instantons” but
with a similar dipole structure in all space-time directions for which the
asymptotic holonomy is non-trivial. In this sense an interpretation of the
topological lumps in terms of constituents is proven also in their paper. The
fact that these lumps are not seen dissolved into constituents at \( T = 0 \) is not
straightforward to reconcile with our constituent interpretation of the zero-
mode hopping for equilibrium fields. The lack of separate constituents in
the cooled configurations may be an artifact of the standard cooling method
which could drive the constituents into such a “bound state”. Together
with the authors of \cite{19} we are currently beginning a detailed analysis of
this issue using APE blocking and improved cooling techniques combined
with fermionic methods.
Summary and outlook

In this article we show that for SU(3) gauge theory on the torus the zero mode of charge ±1 configurations can couple to different topological objects. In particular we find that the zero mode often is localized at different positions in space-time when changing the boundary conditions, while the chirality of the lumps seen by the zero modes remains the same. For two different volumes we compared the periodic and the anti-periodic zero modes for a large number of charge ±1 configurations. For both volumes we find that for about 40% of the configurations the periodic and the anti-periodic zero modes are located at different positions. We compare the distribution of the distance between the different lumps to a simple model and find that the behavior of the constituents is compatible with independent placement on the torus.

Although we cannot entirely rule out the possibility that the hopping is between integer charged objects, we lean towards the interpretation that it is true fractionally charged constituents we observe. This interpretation is supported by analytic arguments at finite $T$ showing that hopping between integer lumps is unlikely to occur. Also the absence of a volume dependence favors an explanation by fractionally charged constituents.

Our results support a possible extension of the monopole constituent picture known for manifolds with one or two compact dimensions to the 4-torus. Such a generalization of the known classical solutions to the torus (with twisted boundary conditions) is certainly a challenging and interesting problem. On the numerical side also several interesting questions remain to be studied. In order to avoid problems with mixing of zero modes we have so far restricted ourselves to configurations with exactly one zero mode. It would be interesting to analyze also the zero modes for higher topological sectors and check whether a similar behavior can be identified. Another direction for research would be an analysis of the near zero modes. They are expected to emerge when topological objects start to overlap. For the lowest modes one expects that the behavior of the lumps seen by the eigenvectors still resembles the properties of the exact zero modes. Showing constituent-like behavior for these near zero modes, which through the Banks-Casher relation are responsible for chiral symmetry breaking, would lead one step closer to a possible unification of the mechanisms for confinement and chiral symmetry breaking.
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