Adaptive Nonlinear Control for the Stabilized Platform With Disturbance and Input Saturation

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\textbf{ABSTRACT} This article proposes a control strategy including super-twisting extended state observer (STESO) and sliding mode controller based on backstepping technique for the stabilized platform with disturbance and input saturation. Firstly, the system is modelled to obtain a non-matched nonlinear model. Secondly, two STESOs are designed to estimate the disturbances in the matched channel and the unmatched channel. And then, multi-sliding mode controller based on backstepping technique is designed for non-matched nonlinear system; and the first-order auxiliary system is introduced to alleviate the effect of the input saturation. Furthermore, Lyapunov function is used to prove the stability of the closed system with the proposed controller and finite-time convergence proof is given in this article. Finally, comparative simulations are carried out and the results show the effectiveness of the proposed controller. It is proved that the control accuracy is improved effectively and the proposed controller can guarantee the consistent tracking error of the system.

\textbf{INDEX TERMS} Stabilized platform, super-twisting extended state observer, input saturation, disturbance rejection, sliding mode controller, backstepping technique.

\textbf{I. INTRODUCTION}

The goal of the seeker is to search, capture and track the target, and then provide guidance information for the missile to strike accurately in real time. The stabilized platform is the most important part of the seeker. However, the stabilized platform is subject to modeling uncertainty, disturbance and input saturation due to the random base motion, coupling torque and nonlinear factor of the system. The controller design of the stabilized platform aims to provide anti-disturbance ability and stabilize the line of sight (LOS) of the system. In order to get an accurate tracking performance of the stabilized platform, a robust controller should be designed to isolate disturbances and other nonlinear factors.

As is well known, the research on disturbance rejection is the priority in the control of the stabilized platform. In order to reject the disturbances, various disturbance observers are introduced by scientists to estimate the disturbance of the system. Some disturbance observers depend on the system’s accurate model, such as disturbance observer (DOB) \cite{1}.

Due to the uncertainty caused by time-varying loads and disturbances, it is difficult to obtain the accurate model. Extended state observer (ESO) \cite{2} proposed by Jingqing Han does not depend on the model of the system. And it can estimate disturbance, unmeasured state simultaneously. ESO and its various forms are widely used in direct current (DC) motors. In \cite{3}, a fixed-time ESO is designed to estimate the total disturbance. In \cite{4}, a nonlinear ESO estimates the disturbances of the manipulator joint. In \cite{5}, a linear extended state observer (LESO) is used to estimate the lumped uncertainties. In \cite{6}, an ESO is employed to handle all modeling uncertainties in feedback linearization control. In \cite{7}, an ESO is used to predict the current in the next instant and the disturbance. In \cite{8}, a new high order extended state observer (HOESO) has been developed by means of input and output signals to estimate the unmeasurable states as well as the lumped disturbances synchronously. From the above analysis, we can see that ESO has two forms: linear and nonlinear. Non-linear ESO \cite{9} is effective in improving disturbance estimation capability and dynamic performance. But it is very difficult to adjust and analyze the parameters. LESO \cite{10} proposed by Zhiqiang Gao is superior to the nonlinear ESO.
in terms of parameter adjustment and theoretical analysis, but it also exhibits deficiencies under multiple time-varying disturbances. The linear gain is often set to a very large value to resolve rapidly changing disturbances, which introduces larger noise and reduces the estimation accuracy of the system. Sliding mode observer (SMO) [11], [12] is also proposed to estimate the disturbance. However, due to the existence of discontinuous terms, the disturbance observer may have a chattering problem. In order to attenuate the problem and simplify the control system, various second-order SMO are proposed, such as twisting and super-twisting algorithms. The super-twisting algorithm is a widely used in SMO which does not need to measure the derivative of the sliding surface. And the algorithm does not have the problem of chattering phenomena and has better ability to suppress noise. Super-twisting observer (STO) [13] is used in many applications [14], [15]. Considering the characteristics of STO and ESO, super-twisting extended state observer (STESO) [16] is designed to estimate states and disturbances. STESO has better performance for nonlinear systems than general ESO, mainly due to its limited time convergence.

As a kind of space moving component, the stabilized platform of the seeker not only has uncertainty and disturbance, but also has input saturation phenomenon due to the limitation of the system hardware during the movement. In order to quickly track the target, a larger angle of the stabilized platform needs to be generated, so that it is easier to reach the amplitude limit of the controller, so the control problem of the stabilized platform can be described as a special type of nonlinear control problem: non-linear disturbances and input saturation. The input saturation of the stabilized platform is a very important issue in the design of the controller. If the influence of input saturation is not considered when designing the controller, once the saturation limit value is reached during system operation, it will seriously affect the control performance of the system. However, to our best knowledge, the previous controller is designed to improve the disturbance rejection ability, and the input saturation problem is often ignored. Although a control method of comprehensive backstepping technique, sliding mode control and adaptive neural network for saturation is designed in [17], because neural network training needs a large number of samples, the proposed controller is not suitable for the stabilized platform of the seeker. In recent years, extensive research has been conducted on the problem of saturation control. Many approaches have been designed to deal with input saturation. If the control gain is known, the method of designing a low gain controller [18], treating saturation problem as the disturbance [19], and designing auxiliary design system [20] are proposed to avoid saturation effect. If the gain is unknown, hyperbolic tangent curves [21], [22], exponential function [23], [24], Gaussian function [25], [26] and explicit reference governor [27] are used to analyze the saturation phenomenon. And then, the method of fuzzy [19], [28] or neural network controller [29] is designed to avoid the saturation. Among all the method, designing an auxiliary system is a common method to solve input saturation. There are two types of auxiliary system. One is the same order as the controlled object [30] and the other one [31], [32] is the first-order auxiliary system model [33], [34]. The control gains of this article can be known, and considering the feasibility of system implementation, a first-order auxiliary system is used to compensate the input saturation in this article.

To complete the disturbance estimation and input saturation analysis, it is also necessary to design robust controllers to complete the control of the system. Many controllers are designed for the stabilized platform, such as proportional integral derivative(PID) controller [35], continuous finite-time sliding mode controller (FTSMC) [36], and improved cerebellar model articulation controller [37]. In recent years, because of its better performance, such as rapid response, insensitiveness of the disturbance, the sliding mode controller has been widely used in many applications. Considering the complexity of the stabilized platform, backstepping technique is a suitable control method. It is simple in the systematic design process and can also prove the stability of the system. However, although the backstepping control method is widely used in nonlinear control, it requires repeated differentiation of the virtual controller, which inevitably introduces the problem of computational explosion. Many methods have been adopted to deal with the computational explosion, such as tracking differentiator filtering [30], dynamic surface technique (DSC) [33], command filtering [34]. Considering the non-matched characteristics and multiple non-linear factors of the stabilized platform, the backstepping technique based on DSC technique is adopted to carry out the sliding mode controller.

Motivated by the above analysis, the disturbance and input saturation of the stabilized platform are comprehensively considered in this article, and an effective controller is proposed. For the matched, unmatched disturbances, two STESOs are proposed to estimate the total disturbance. Lyapunov method is used to prove the convergence of the estimation error in finite time. As for the input saturation problem, first-order auxiliary system is adopted to compensate for saturation. The adaptive sliding mode controller based on DSC’s backstepping technique is designed and the proof of the system’s finite time convergence is given. Lyapunov stability theorem is used to prove the controller tracking error can converge to zero in finite time. Simulations are carried out to compare the proposed controller with the traditional algorithm. Isolation index, step response and coupling characteristics of the stabilized platform are implemented respectively.

The contribution of this article is as follows.

1. This work not only focuses on the matched disturbance, but also analyzes the mismatched disturbance. The two STESOs are firstly used to estimate the system disturbances.

2. Disturbance and saturation are considered simultaneously in the stabilized platform. And the convergence of the synthesized algorithm is proved by Lyapunov equation. This article provides a new idea for the control of stabilized platform.
In this article, there are two improvements in the proposed controller, gain adaptation and tanh function instead of sign function are used to effectively alleviate chattering. The proposed controller in this article is insensitive to noise and dynamic changes.

The organization of this article is as follows. Section 2 gives the model of the two-axis stabilized platform system. Based on the rigid body motion mechanics, the model of the full-state seeker system is derived, and the model with non-matched characteristics is given. Section 3 gives the controller design process and stability conclusions based on the characteristics of disturbance and input saturation for the two-axis stabilized platform system. Section 4 verifies the effectiveness of the control strategy by simulation.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

In this article, we focus on the study on two-axis stabilized platform system. Based on the rigid body motion mechanics, the model of the full-state seeker system is derived, and the model with non-matched characteristics is given. Section 3 gives the controller design process and stability conclusions based on the characteristics of disturbance and input saturation for the two-axis stabilized platform system. Section 4 verifies the effectiveness of the control strategy by simulation.

A. STABILIZED PLATFORM KINEMATICS

The system performs kinematics modeling by Euler angle and coordinates transformation matrix. As shown in Fig. 1, there are three coordinate frames, O-x0y0z0, O-x1y1z1 and O-x2y2z2 which represent base coordinate, outer gimbal coordinate and inner gimbal coordinate, respectively.

Right-handed rule is used to define all the coordinate frames. Here, to simplify the problem, all coordinate frames are set to have the same origin. \( \dot{\theta}_a \) and \( \dot{\theta}_e \) are the angular velocity of the azimuth and elevation gimbals, respectively. \( \omega_0 \), \( \omega_1 \) and \( \omega_2 \) are introduced to denote the angular velocity of the base coordinate, outer gimbal coordinate and inner gimbal coordinate relative to the inertia space, respectively.

The kinematics of the azimuth and elevation gimbals are as follows

\[
\begin{align*}
\omega_1 &= R_{0,1}(\theta_a)\omega_0 + w_1\dot{\theta}_a \\
\omega_2 &= R_{1,2}(\theta_e)\omega_1 + w_2\dot{\theta}_e
\end{align*}
\]

where \( R_{0,1}(\theta_a) = \begin{bmatrix} \cos \theta_a & \sin \theta_a & 0 \\ -\sin \theta_a & \cos \theta_a & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( R_{1,2}(\theta_e) = \begin{bmatrix} \cos \theta_e & 0 & \sin \theta_e \\ 0 & 1 & 0 \\ -\sin \theta_e & 0 & \cos \theta_e \end{bmatrix} \) are the transformation matrix from the base to the outer gimbal, and from the outer gimbal to the inner gimbal, respectively.

Here, \( \omega_0 = [\omega_{0x} \omega_{0y} \omega_{0z}]^T \in \mathfrak{R}^{3 \times 1} \), \( w_1 = [0 \ 1 \ 0]^T \), \( \omega_1 = [\omega_{1x} \omega_{1y} \omega_{1z}]^T \), \( \omega_2 = [\omega_{2x} \omega_{2y} \omega_{2z}]^T \), \( w_2 = [0 \ 0 \ 1]^T \).

B. STABILIZED PLATFORM DYNAMICS

In the actual stabilized platform, the system is subject to non-linear factors. In order to establish the dynamics of the stabilized platform, the non-linear factors of the system must be analyzed. The main non-linear factors of the system are friction torque, mass imbalance torque, coupling torque and wire winding torque.

Friction torque is a very important factor that affects the stabilized platform. Friction torque comes from bearing friction and motor brush friction. Both are related to the assembly preload of the system, and unavoidable. There are many friction models used for analysis, such as Coulomb friction, Strubeck friction and LuGre friction model \([38],[39]\). Generally, the complex friction model can express the accurate friction characteristics, but the model contains piecewise function, and the realization is complex. If the smooth method is used to analyze the friction, the model will be simplified \([39]\). Considering the characteristics of the system, in this article, viscous friction which is continuously differential is used for analysis. The other complex parts of the friction torque are not modeled and treated as part of the disturbance.

Each rotation axis of the stabilized platform is arranged according to the static balance standard. Due to the strict limitation of mass and volume, it is difficult to obtain the true static balance. As the two-axis frames have an eccentricity, when the system rotates, there is a mass imbalance torque. The amplitude of the imbalance torque is related to the imbalance mass distribution and the angular velocity of the rotating shaft. Since the overload coefficient of the missile can reach 5g to 40g, the imbalance torque cannot be ignored.

The coupling torque between the frames is derived from the kinematic coupling. The base motion, the frames motion
and the combined effect between both of them will produce a coupling torque on the rotating shaft system of the stabilized platform, which reducing system performance.

Because the gyroscope of the stabilized platform is located on the inner gimbal, the wire-winding torque will be generated on the rotating shaft during the relative motion. This torque is inevitable and can be reduced by wire selection and winding design but not eliminated.

Considering all the disturbances above, rigid body dynamics and the Newton-Euler equation are adopted to establish the dynamics of the inner gimbal as follows

\[ J_I \dot{\omega}_2 + (\omega_2 \times J_I \omega_2) = L_I \]  

(2)

where

\[ J_I = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix} \]

is the inertial matrix of the inner gimbal. \( L_I \) is the total torque of the inner gimbal. As the inner gimbal’s y axis is the elevation control axis. Here, nonlinear factors such as friction \( T_f \), cable restraint torque \( T_{ICR} \) and mass imbalance torque \( T_{lu} \) are considered in this article. Subscribing Eq.(1) to (2), the dynamics equation of the inner gimbal is given as

\[ J_{Ixy} \dot{\omega}_2y = T_{Iy} + T_{I0} + T_{I1} + D_I \]

(3)

where \( T_{I0} \) is the disturbance torque caused by missile rotation, \( T_{Iy} \) is the output torque of motor, \( T_{I1} \) represents the reaction exerted by the outer gimbal on the inner gimbal,

\[ T_{I0} = -J_{Iyx} \sin \theta_e + J_{Ixy} \cos \theta_e (\dot{\omega}_{1x} + \omega_{1y} \omega_{1z}) + [(J_{Izz} - J_{Ixx}) \cos(2\theta_e) - 2J_{Ixz} \sin(2\theta_e)] \omega_{1y} \omega_{1z} \]

\[ + \frac{1}{2} [(J_{Izz} - J_{Ixx}) \sin(2\theta_e) + 2J_{Ixz} \cos(2\theta_e)] \omega_{1x}^2 \]

\[ + (J_{Iyy} \cos \theta_e - J_{Iyx} \sin \theta_e) \omega_{1x} \omega_{1z} = T_{I1} = (J_{Ixy} \sin \theta_e - J_{Iyx} \cos \theta_e) \dot{\omega}_{1x} \]

\[ - \frac{1}{2} [(J_{Izz} - J_{Ixx}) \sin(2\theta_e) + 2J_{Ixz} \cos(2\theta_e)] \omega_{1z}^2 \]

\[ D_I = T_{II} + T_{ICR} + T_{lu}. \]

The inertia of the outer gimbal is

\[ J_O = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix}. \]

According to the installation mode of gyroscope, the inner gimbal’s z axis is the azimuth control axis. The dynamic equation of the outer gimbal can be obtained by using the same analytical theory as the inner gimbal as follows

\[ J_o \dot{\omega}_2z = T_{oc} \cos \theta_e + (T_{001} + T_{002} + T_{003}) + T_{O2} + D_O \]

(4)

where \( J_o \) is equivalent inertia

\[ J_o = J_{Oxx} + J_{Ozz} \cos^2 \theta_e + J_{Oxx} \sin^2 \theta_e - J_{Ozz} \sin(2\theta_e), \]

\[ T_{001} = [J_{Oxx} + J_{Oxx} \cos^2 \theta_e] J_{Oxy} \sin^2 (\theta_e) + J_{Oxz} \sin(2\theta_e) - (J_{Oyy} + J_{Oyy} \cos \theta_e) \omega_{1x} \omega_{1y} \cos \theta_e, \]

\[ T_{002} = [(J_{Ixx} - J_{Izz}) \cos(2\theta_e) + 2J_{Ixz} \sin(2\theta_e) - J_{Iyy} + J_I]\]

\[ \hat{\theta}_e \omega_{1x} \cos \theta_e + J_x \sin \theta_e \omega_{1y} \]

\[ + [(J_{Ixx} - J_{Izz}) \sin(2\theta_e) - 2J_{Ixz} \cos(2\theta_e) + J_x \sin \theta_e / \cos \theta_e] \omega_{1y} \omega_{1z} \]

\[ + (J_{Iyy} \cos \theta_e - J_{Iyx} \sin \theta_e) \omega_{1x} \omega_{1z} - (J_{Iyz} \sin \theta_e + J_{Izy} \cos \theta_e) \omega_{1y} \omega_{1z} \]

\[ - (J_{Oyz} + J_{Oyz} \cos \theta_e - J_{Oyx} \sin \theta_e) \omega_{1x}^2 - \omega_{1z}^2 \cos \theta_e, \]

\[ T_{003} = -(J_{Oxx} + J_{Oyy} \cos \theta_e - J_{Oxx} \cos(2\theta_e)) \]

\[ \hat{\omega}_{1x} - \omega_{1y} \omega_{1z} \cos \theta_e \]

\[ - (J_{Oyz} + J_{Oyz} \cos \theta_e - J_{Oyx} \sin \theta_e) \omega_{1x}^2 - \omega_{1z}^2 \cos \theta_e, \]

\[ T_{02} = (J_{Ixy} \cos \theta_e - J_{Iyx} \cos \theta_e) \hat{\omega}_{1z} \cos \theta_e \]

\[ + (J_{Iy} \cos \theta_e + J_{Iyx} \sin \theta_e) \omega_{1x} \omega_{1z} + (J_{Ixx} - J_{Iyz} \sin \theta_e \omega_{1z}^2 \cos \theta_e. \]

\[ D_O = T_{O0} + T_{OCR} + T_{Ou}, \]

where \( T_{001}, T_{002} \) and \( T_{003} \) are the disturbances from base, and \( T_{O2} \) is the coupling torque from the inner gimbal.

C. DC DRIVE MODEL

The stabilized platform system uses two torque motors to drive the elevation and azimuth axes of the stabilized platform directly. The DC drive model is as follows

\[ \begin{cases} 
U = iR + L \frac{di}{dt} + K_m \omega_l \\
J_m \ddot{\omega}_l = T_m - T_l - T_{dm} \\
T_m = K_T i 
\end{cases} \]

(5)

where \( U \) is armature voltage, \( i \) is armature current, \( R \) is armature resistance and \( L \) is armature inductance. \( K_m, K_T \) represent the electro-mechanical constant and torque coefficient, respectively. \( J_m, T_{dm} \) represent the inertia of the motor and the nonlinear disturbance such as friction in the motor. In this article, the two-axis gimbals adopt the same torque motor, so the above parameters are the same. \( \omega_l \) and \( T_l \) represent the gimbal angular velocity and drive torque. Here, we define \( \omega_l = \omega_{2y}, T_l = T_{II} \) for the inner gimbal, \( \omega_l = \omega_{2z}, T_l = T_{O2} \) for the outer gimbal. The torque \( T_l \) produced by the motor is used to drive the gimbals.

D. SYSTEM MODEL

It is well known that the friction torque always consists of the linear part and the nonlinear part. For a more realistic representation of system friction torque, we define the substitute of the friction for the inner gimbal and outer gimbal as follows

\[ \begin{cases} 
B_{II} \dot{\omega}_{1y} + T_{II} (\dot{\theta}_e, r) = T_{I0} + T_{II} \\
B_{OII} \dot{\omega}_{1z} + T_{OII} (\dot{\theta}_e, r) = T_{O0} + T_{OII} 
\end{cases} \]

(6)

where \( B_{II} \) and \( B_{OII} \) are viscous friction coefficients and can be identified by rotating at constant angular velocity. The least-mean-square (LMS) algorithm is used, since it is relatively simple but effective for the identification of the friction torque.

Combining Eq.(3)-(6), and let \( x_1 = \omega_{2y}, x_2 = \dot{\theta}_e, x_3 = \omega_{2z}, x_4 = i_O \), the system can be described by the state variables
where \( i = 1 \) and \( i = 3 \) represent the inner gimbal and outer gimbal respectively. The system parameters are defined as

\[
[b_1, b_2, b_3, b_4]^T = \begin{bmatrix} 1 & 1 & K_T \end{bmatrix} J_{ryy} + J_m, \frac{1}{L}, J_s, J_m, \frac{1}{L},
\]

\[
f_1(\tilde{x}_1) = -\frac{B_f}{J_{ryy} + J_m} x_1,
\]

\[
f_2(\tilde{x}_2) = -\frac{K_m}{L} x_1 - R \frac{x_2}{L}, f_3(\tilde{x}_3) = -\frac{B_f}{J_{ozz} + J_m} x_3,
\]

\[
f_4(\tilde{x}_4) = -\frac{K_m}{L} x_3 - R \frac{x_4}{L},
\]

\[
d_1(t, x) = -\frac{T_{fin}(\tilde{\theta}_e, t) + T_{D1} + T_{D1} + T_{ICR} + T_{Lu} + D_{a1}}{J_{ryy} + J_m},
\]

\[
d_2(t, x) = D_{a2},
\]

\[
d_3(t, x) = -\frac{T_{O01} + T_{O01} + T_{O02} + T_{O02} + T_{Oy}}{J_{ozz} + J_m} - \frac{T_{OCR} + T_{Ou} + D_{a3}}{J_{ozz} + J_m},
\]

\[
d_4(t, x) = D_{a4},
\]

where \( D_{a1} \) and \( D_{a3} \) represent the uncertain inertia of the inner gimbal and the outer gimbal. \( D_{a2} \) and \( D_{a4} \) represent the uncertainty of the motor such as \( R, L, K_m, K_T \) of the inner gimbal and outer gimbal. As a result, \( R, L, K_m, K_T \) in the equation are nominal value that are marked on the motor parameters sheet. As can be seen from the above modeling method, the external disturbances, the inner coupling torque, the unmodeled dynamics, and unknown dynamics can be considered as an uncertain nonlinear \( d_i(x, t) \). The disturbance is not in the same channel as the control input, that is, the so-called unmatched disturbance. It indicates that the system has matched disturbances and unmatched disturbances. \( f_i(\cdot) \) is smooth scalar functions. \( u \) is the actual input of the system. Since the cross-couplings torque is very small, here we do not expand research for it, and regard it as the part of the disturbance.

Due to the limitation of the motor drive hardware, the maximum value of controller is limited. The form of the saturation function is as follows

\[
u = \begin{cases} u_{\text{max}}, & \text{if } u_c > u_{\text{max}} \\ u_c, & \text{if } u_{\text{min}} \leq u_c \leq u_{\text{max}} \\ u_{\text{min}}, & \text{if } u_c < u_{\text{min}} \end{cases}
\]

where \( u_c \) is the input control signal, we assume that \( u_{\text{max}} > 0 \) and \( u_{\text{min}} < 0 \) are known upper and lower bound parameters. The design of the speed loop is very important in the operation of the stabilized platform. The aim of this article is to design controller for the speed loop.

**Remark 1:** The detail information of \( d_i(x, t) \) shown in Eq.(7) is not discussed in the paper, all the system uncertainties, inner disturbances and external disturbances, and the effects due to parameter variations and nonlinearities are treated as the total disturbances.

**Remark 2:** As we know the two-axis gimbal can be controlled separately. Considering that the gyroscope is installed on the inner gimbal, the outer gimbal needs secant compensation. It can implies that from Eq. (7), two gimbals of the stabilized platform can use the similar control structures.

The control goal is to design a controller for a nonlinear system with disturbances and input saturation. The system output \( y \) can track a bounded reference signal by the controller. In order to analyze the system and design controller, the following assumption and lemmas are used in the paper.

**Assumption 1:** The unmatched disturbance and matched disturbance are continuously differential and are bounded. Thus is \( |d_i(x, t)| \leq L_i, (i = 1, 2, 3, 4), |L_i| > 0 \).

**Remark 3:** The characteristics of the seeker's structure and flight environment show that the disturbances on the stabilized platform are time-variable but restricted in finite energy, so they can be regarded as an unknown bounded signal, and the variation rate of the disturbance is limited. It can be concluded that Assumption 1 is reasonable.

**Assumption 2:** Input instruction \( \omega_d \) and its derivative \( \dot{\omega}_d \) exist and are bounded.

**Assumption 3:** Here, we define the disturbance error estimated by STESO as \( \tilde{d}_i(x, t) \). As verified by the convergences of the STESO, if the parameters of the STESOs are chosen appropriately, \( \| \tilde{d}_i(x, t) \| \leq \rho_i, \rho_i \geq 0, i = 1, 2, 3, 4 \) in finite time.

**Lemma 1 [40]:** Motivated by the performance of the function \( \tanh(\cdot) \), the following inequality holds

\[
0 \leq |z| - z \tanh(z/\varepsilon) \leq 0.2785 \varepsilon, \quad \forall \varepsilon > 0, z \in \mathbb{R} \tag{9}
\]

**Lemma 2 [41]:** Young’s inequality: set \( p > 1, q < \infty, \) and the two parameters can satisfy \( \frac{1}{p} + \frac{1}{q} = 1, \) if set \( a > 0, b > 0 \), and then

\[
ab \leq \frac{a^p}{p} + \frac{b^q}{q} \tag{10}
\]

**Lemma 3 [42]:** If a function inequality shown in Eq.(11) exists, and then it can converge to zero in finite time \( t_0 \).

\[
\dot{V}(t) < -aV(t) - bV^q(t) + bV^{q_1} < 0, t \geq t_0 \tag{11}
\]

where

\[
t_0 = t_0 + \frac{1}{a} \left( \frac{1 - \frac{p_1}{q_1}}{b} \right) \ln \left( \frac{1}{1 - \frac{p_1}{q_1}} \right),
\]

\[
(a, b, p_1, q_1) > 0, \quad \frac{1}{2} < \frac{p_1}{q_1} < 1.
\]

**III. CONTROLLER DESIGN**

The diagram of the proposed controller scheme for the system is shown in Fig.2.
A. SUPER-TWISTING EXTENDED OBSERVER

In this article, the angular velocity is available by the gyroscope. Two STESOs are applied to estimate the unmatched disturbance and matched disturbance. Considering the matched disturbance and unmatched disturbance of the system, the state equation of the system is reconstructed by taking the inner gimbal as an example as follows

\[ \begin{align*}
    \dot{z}_1 &= b_1x_2 + f_1(\xi_1) + z_2 + \lambda_1 |e_1|^\frac{1}{2} \text{sign}(e_1) \\
    \dot{z}_2 &= \beta_1 \text{sign}(e_1) \\
    \dot{z}_3 &= b_2u + f_2(x_2) + z_4 + \lambda_2 |e_3|^\frac{1}{2} \text{sign}(e_3) \\
    \dot{z}_4 &= \beta_2 \text{sign}(e_3)
\end{align*} \tag{12} \]

where \( z_2 = \hat{d}_1(x, t), z_4 = \hat{d}_2(x, t), e_1 = x_1 - z_1, e_2 = d_1(x, t) - z_2, e_3 = x_3 - z_3, e_4 = d_2(x, t) - z_4 \). Among them, the estimated state \( z_2, z_4 \) are used to estimate the unknown system disturbance \( d_1(x, t), d_2(x, t) \) respectively. \( \lambda_1, \lambda_2, \beta_1, \beta_2 \) are the positive observer gains.

Take STESO1 as an example. The observer error dynamics are as follows

\[ \begin{align*}
    \dot{e}_1 &= e_2 - \lambda_1 |e_1|^\frac{1}{2} \text{sign}(e_1) \\
    \dot{e}_2 &= d_1(x, t) - \beta_1 \text{sign}(e_1)
\end{align*} \tag{13} \]

In order to prove the convergence of the observer in finite time, define the following matrix

\[ \zeta_1 = \begin{bmatrix} |e_1|^\frac{1}{2} \text{sign}(e_1) \quad e_2 \end{bmatrix}^T = [\zeta_{11}, \zeta_{12}]^T \tag{14} \]

According to the super-twisting algorithm [43], [44], Lyapunov method is introduced to prove the stability of the observer. Lyapunov function is shown in Eq. (15).

\[ \begin{align*}
    V_{E1} &= \zeta_1^T \mathbf{P}_1 \zeta_1 \tag{15} \\
    \dot{V}_{E1} &= 2\zeta_1^T \mathbf{P}_1 \zeta_1 = 2|e_1|^\frac{1}{2} \zeta_1^T \mathbf{P}_1 \mathbf{A}_1 \zeta_1 \\
    &= -|e_1|^{(-1/2)} \zeta_1^T \left( \mathbf{A}_1^T \mathbf{P}_1 + \mathbf{P}_1 \mathbf{A}_1 \right) \zeta_1 \\
    &= -|e_1|^{(-1/2)} \zeta_1^T \mathbf{Q}_1 \zeta_1 \tag{16}
\end{align*} \]

where \( \mathbf{P}_1 \) is a symmetric positive definite matrix,

\[ \mathbf{P}_1 = \begin{bmatrix} 2\beta_1 + \frac{\lambda_1^2}{2} & \frac{-\lambda_1}{2} \\ \frac{-\lambda_1}{2} & 1 \end{bmatrix}, \quad \mathbf{A}_1^T \mathbf{P}_1 + \mathbf{P}_1 \mathbf{A}_1 = -\mathbf{Q}_1. \]

By choosing values of \( \lambda_1, \beta_1 \) reasonably, \( \dot{V}_{E1} \leq -\frac{1}{|e_1|^2} \lambda_{\text{min}} (\mathbf{Q}_1) \| \zeta_1 \|^2 \) will be satisfied.

Considering the following facts

\[ \lambda_{\text{min}} (\mathbf{P}_1) \| \zeta_1 \|^2 \leq \zeta_1^T \mathbf{P}_1 \zeta_1 \leq \lambda_{\text{max}} (\mathbf{P}_1) \| \zeta_1 \|^2 \tag{17} \]

\[ \| \zeta_{11} \| \leq \| \zeta_1 \| \leq \frac{V_{E1}^{1/2}}{\lambda_{\text{min}} (\mathbf{P}_1)} \tag{18} \]

The time derivative will become [44]

\[ \dot{V}_{E1} \leq -\frac{1}{|e_1|^2} \lambda_{\text{min}} (\mathbf{P}_1) \lambda_{\text{min}} (\mathbf{Q}_1) V_{E1}^{1/2} \frac{V_{E1}^{1/2}}{\lambda_{\text{max}} (\mathbf{P}_1)} \tag{19} \]

where \( \lambda_{\text{min}} \{ \cdot \} \) and \( \lambda_{\text{max}} \{ \cdot \} \) are the minimum and maximum eigenvalues for matrices. The observer estimation error can converge to zero in finite time [44]. By using the same method to analyze STESO and constructing Lyapunov function for the matched channel, it can be concluded that by choosing the values of \( \lambda_2 \) and \( \beta_2 \) reasonably, the observer estimation error can converge to zero in finite time.

Remark 4: STESO is the dynamic process, which only uses the input-output information of the original object, and does not use any information of the model function describing the transfer relationship of the object. There is no need to assume that the model function is continuous or discontinuous, whether it is known or unknown, as long as the real-time effect of the disturbance in the process is bounded, and the parameter is known, it can always be selected appropriately.
Remark 5: Note that two STESOs are designed for one gimbal of the stabilized platform. The multiple STESO method is very suitable to solve the problem of system with unmatched and matched disturbance.

B. ADAPTIVE SLIDING CONTROLLER BASED ON BACKSTEPPING

Considering the unmatched and matched disturbances in the model. Multi-sliding controller based backstepping method is proposed in this article. Here, inner gimbal model is still chosen as the controlled object. The sliding manifold is defined for every state variable.

\[ s_i = x_i - \alpha_{i−1} \quad (i = 1, 2), \quad \alpha_0 = y_d \]  

where \( \alpha_{i−1} \) is the desired value of \( x_i \), and \( y_d \) is the input instruction.

Step 1. The time derivative of the \( s_1 \) is as follows:

\[
\dot{s}_1 = \dot{x}_1 - \dot{\tilde{a}}_1 = b_1(x_2) + f_1(\tilde{x}_1) + \tilde{d}_1(x, t) - \dot{y}_d = \dot{x}_2 + \tilde{a}_1 + \tilde{d}_1(x, t) - \dot{y}_d
\]  

Here, in order to solve the problem of computational explosion of the backstepping method, the DSC technique is used to get the differentiation of \( \alpha_1 \), so \( s_2 = x_2 - \tilde{a}_1 \). One order filter is defined as

\[
\tau_1 \dot{\alpha}_1 + \tilde{a}_1 = \alpha_1, \quad \tilde{a}_1(0) = \alpha_1(0)
\]  

where \( \tau_1 \) is a positive constant, \( \alpha_1 \) is the input of the filter and \( \tilde{a}_1 \) is the state of the filter and the output of the filter. The error of the filter is set as \( \phi_1 = \alpha_1 - \tilde{a}_1 \) and \( \tilde{\phi}_1 = \dot{\tilde{a}}_1 - \dot{\alpha}_1 = -\phi_1 + C_1 \), where \( C_1 \) describe \( \tilde{\dot{a}}_1 \). As the assumption 1 shown, the virtual input is continuous, as a result, the function of \( C_1 \) satisfies the expression \( |C_1| \leq C_{1\text{max}}, C_{1\text{max}} > 0 \). This conclusion can refer to the theorem: in the metric space, it is proved that a continuous function on a compact set must be bounded and can reach its supremum and infimum. The details of the theorem can be referred to [28], [45].

Remark 6: \( \tilde{a}_1 \) is obtained easily by the Eq. (22) to replace the differential term \( \alpha_1 \), and this algebraic operation is easy to implement. Considering the system order of the system and the method easy to implement, the problem of “explosion of complexity” is avoided by DSC technique.

Invoking Lemma 2, we have

\[
\phi_1^T C_{1\text{max}} \leq \frac{\|\phi_1\|^2 \|C_{1\text{max}}\|^2}{2\tau} + \frac{\tau}{2}, \tau > 0
\]

\[
\phi_1^T \dot{\phi}_1 = -\frac{\phi_1^T \phi_1}{\tau_1} + \frac{1}{\tau_1} C_1 \leq -\frac{\phi_1^T \phi_1}{\tau_1} + \frac{1}{\tau_1} C_{1\text{max}}^2 \phi_1^2 + \frac{\tau}{2}
\]

Considering the system disturbances are unknown, an augmented Lyapunov function is designed as following

\[
\dot{V}_1 = \frac{1}{2} s_1^T s_1 + \frac{\tau}{\gamma_1} \dot{\tilde{a}}_1 + \frac{1}{\gamma_1} \dot{\tilde{a}}_1 \dot{\tilde{a}}_1 + \frac{1}{\gamma_1} \dot{\tilde{a}}_1 \dot{\tilde{a}}_1
\]

where \( \phi_1 \) is the error of the one order filter. Considering the Eq. (23)-(24), the time derivative of Eq. (25) is given by

\[
\dot{V}_1 = \frac{1}{2} s_1^T s_1 + \phi_1^T \dot{\phi}_1 + \frac{1}{\gamma_1} \dot{\tilde{a}}_1 \dot{\tilde{a}}_1 + \frac{1}{\gamma_1} \dot{\tilde{a}}_1 \dot{\tilde{a}}_1
\]

In order to make sure the first subsystem converge to the sliding manifold, the virtual control law is set as follows

\[
\alpha_1 = \frac{1}{\gamma_1} \left[-K_1 s_1 - f_1(\tilde{x}_1) + \dot{y}_d - \tilde{\rho}_1 \tanh \left( \frac{s_1}{\epsilon_1} \right) \right]
\]

where \( \tilde{\rho}_1 \) is the estimation of the upper value of the total disturbance \( \dot{d}_1(x, t) \). As we know, the upper value of \( \rho_1 \) (1, 2, 3, 4) are not known in advance. In order to overcome this problem, adaptive law is proposed in this article. As is known, the chattering problem caused by the switching control can be attenuated by using \( \tanh(\cdot) \). Define the error \( \hat{\rho}_1 = \rho_1 - \tilde{\rho}_1 \), and the adaptive law is

\[
\dot{\hat{\rho}}_1 = \gamma_1 \left( s_1^T \tanh \left( \frac{s_1}{\epsilon_1} \right) - \alpha_1 \hat{\rho}_1 \right), \gamma_1, \epsilon_1, \sigma_1 > 0
\]

Invoking Lemma 1 and Lemma 2, the following equation is obtained

\[
|s_1| \rho_1 \leq \left( k_p \epsilon_1 + s_1 \tanh \left( \frac{s_1}{\epsilon_1} \right) \right) \rho_1
\]

\[
\leq s_1 \tanh \left( \frac{s_1}{\epsilon_1} \right) \rho_1 + \frac{1}{2} \|k_p \epsilon_1 \|^2 + \frac{1}{2} \|\rho_1\|^2
\]

\[
- \frac{1}{\gamma_1} \rho_1 \dot{\rho}_1 = - \frac{1}{\gamma_1} \rho_1 \dot{\gamma}_1 \left( s_1 \tanh \left( \frac{s_1}{\epsilon_1} \right) - \sigma_1 \rho_1 \right)
\]

\[
- \rho_1^T \dot{s}_1^T \tanh \left( \frac{s_1}{\epsilon_1} \right) + \dot{\tilde{\rho}_1} \sigma_1 \rho_1
\]

\[
- \dot{\rho}_1 \dot{s}_1^T \tanh \left( \frac{s_1}{\epsilon_1} \right) - \frac{\sigma_1}{\epsilon_1} \|\dot{\tilde{\rho}_1}\|^2 + \frac{\sigma_1}{\epsilon_1} \|\rho_1\|^2
\]

Substituting Eq. (29) and Eq. (30) into Eq. (26) yields:

\[
\dot{V}_1 \leq - s_1^T K_1 s_1 + \frac{1}{2} \|k_p \epsilon_1 \|^2 + \frac{1}{2} \|\rho_1\|^2 - \frac{\sigma_1}{\epsilon_1} \|\dot{\tilde{\rho}_1}\|^2 + \frac{\sigma_1}{\epsilon_1} \|\rho_1\|^2
\]

\[
+ \frac{\sigma_1}{\epsilon_1} \|\rho_1\|^2 + \frac{1}{2} \frac{C_{1\text{max}}^2 \phi_1^2}{\tau} + \frac{\tau}{2}
\]

Step 2. Differentiating the error surface \( s_2 = x_2 - \tilde{a}_1 \), the following equation is obtained

\[
\dot{s}_2 = \dot{x}_2 - \dot{\tilde{a}}_1 = \dot{x}_2 - (\phi_1 + \tilde{\phi}_1)
\]

\[
= b_2 u + f_2(\tilde{x}_2) + \tilde{d}_2(x, t) - (\phi_1 + \tilde{\phi}_1)
\]

where \( \tilde{a}_1 \) is the output of the one order filter and the input is \( \alpha_1 \). In this step, the final controller output \( u_c \) will be designed. Considering the input saturation of the system, the auxiliary system against input saturation is designed in this section.

Assumption 4: Here, \( \Delta u = u - u_c \), and this variable satisfies the following expression: \( \|\Delta u\|_2 \leq \beta \), where \( \beta \) is non-negative real value.
Considering the effect of the input saturation of the system, the auxiliary system is developed as following
\[
\begin{align*}
\dot{\xi} &= -B\xi + b_2\Delta u, \quad u_c \geq u_{\max} \quad \text{or} \quad u_c \leq u_{\min} \quad (\text{33}) \\
\xi &= 0, \quad u_{\min} < u_c < u_{\max}
\end{align*}
\]
where \(\xi\) is the state variable of the auxiliary system, \(B\) is the state parameter of the auxiliary system, and the selection of \(B\) usually needs to be large enough to prevent the system instability caused by too large \(\Delta u\). The advantage of this anti-saturation auxiliary system is that when the input quantity does not exceed the system limitation, the auxiliary system has no effect on the whole system. Once when the limit conditions are exceeded, the anti-saturation effect will be exerted on the system. Considering the effect of the input saturation, the control law is designed as follows
\[
u_c = -\frac{1}{b_2} \left( K_2 s_2 + b_1 s_1 + f_2(\hat{s}_2) + \hat{\rho}_2 \tan \left( \frac{s_2}{\epsilon_2} \right) \right) - \hat{\alpha}_1 - k_\xi \xi
\]
(34)

Remark 7: The characteristics of the controller in this article are that, for the unmatched disturbances of the system, the proposed method eliminates them in the virtual article are that, for the unmatched disturbances of the stabilized platform. The time derivative of the augmented Lyapunov function is as following
\[
\dot{V}_2 = s_2^T [b_2 u_c + b_2 \Delta u + f_2(\hat{s}_2) + \hat{d}_2(x, t) - (\hat{\phi}_1 + \hat{\alpha}_1)] - \frac{1}{2} \hat{\rho}_2^T \hat{\rho}_2 + \frac{1}{2} \xi^T \xi
\]
(36)

Referring to the method of Eq. (29)-(30), substituting Eq. (35) into Eq. (36), and invoking assumption 4 and lemma 1, the time derivative of the \(V_2\) is as following
\[
\dot{V}_2 = s_2^T [b_2 u_c + b_2 \Delta u + f_2(\hat{s}_2) + \hat{d}_2(x, t) - (\hat{\phi}_1 + \hat{\alpha}_1)] - \frac{1}{2} \hat{\rho}_2^T \hat{\rho}_2 + \frac{1}{2} \xi^T \xi
\]
(37)

Invoking Eq.(30) and noting the following equations
\[
\begin{align*}
\hat{\rho}_2 &= \gamma_2 \left( s_2^T \tan \left( \frac{s_2}{\epsilon_2} \right) - \sigma_2 \hat{\rho}_2 \right), \quad \gamma_2, \epsilon_2, \sigma_2 > 0 \\
\xi^T \xi &= -B\xi^T \xi + b_2\Delta u \leq -B\xi^T \xi + \frac{1}{2} \xi^T \xi
\end{align*}
\]
(38)

\[
\begin{align*}
\eta &= \min \left( 2K_1, 2 \left( \frac{1}{\tau_1} - \frac{1}{2} \epsilon_2 C_1 \max \right), 2 \left( K_2 - 1 \right), \right) \\
M &= \frac{\tau}{2} + b_2^2 \beta^2 + \frac{\|k_\phi \xi\|^2}{2} + \frac{\|k_\phi \rho\|^2}{2} + \frac{\epsilon_2}{\gamma_2} \frac{\|\rho\|^2}{2} + \frac{\|\rho\|^2}{2}
\end{align*}
\]
(39)

The following theorem is designed to prove the stability of the closed-loop system.

**Theorem 1:** As shown in Eq. (7), considering the internal and external disturbances, system uncertainty, unmatched disturbances, and input saturation of the stabilized platform. If assumptions 1-6, lemma1,lemma2 are all satisfied, the state observer is designed as Eq.(10), the designed control scheme Eq.(28), Eq.(34) and Eq.(35), the virtual control law Eq.(27), the auxiliary system shown in Eq.(33) and adaptation laws shown in Eq.(28) and Eq.(35) guarantee the stability of the closed-loop system. By designing appropriate parameters, all variables in the closed-loop system can be bounded, and the control performance can meet the requirements.

**Proof:** The total Lyapunov function is defined as
\[
V = V_1 + V_2
\]
(44)

The time derivative of the augmented Lyapunov function candidate is shown as follows
\[
\dot{V} = -s_2^T s_1 - \phi_1^T \left( \frac{1}{\tau_1} - \frac{1}{2 \epsilon_2} C_1 \max \right) \phi_1 - s_2^T (K_2 - 1)s_2 - \frac{1}{2} \xi^T \xi
\]
(45)

It can be concluded that the parameters as shown in Eq. (45) should be chosen to make \(\eta > 0\), and invoking Lemma 3, the system can converge to zero after the finite time. All the parameters will satisfy the conditions shown in Eq. (45), and they can be finally determined by trial and error. where
\[
\begin{align*}
\eta &= \min \left( 2K_1, 2 \left( \frac{1}{\tau_1} - \frac{1}{2} \epsilon_2 C_1 \max \right), 2 \left( K_2 - 1 \right), \right) \\
M &= \frac{\tau}{2} + b_2^2 \beta^2 + \frac{\|k_\phi \xi\|^2}{2} + \frac{\|k_\phi \rho\|^2}{2} + \frac{\epsilon_2}{\gamma_2} \frac{\|\rho\|^2}{2} + \frac{\|\rho\|^2}{2}
\end{align*}
\]
Then, integrating the expression $\dot{V}$ from 0 to $t$ yields

$$0 \leq V \leq e^{-\eta(t-t_0)} V(t_0) + \frac{M}{\eta} \left(1 - e^{-\eta(t-t_0)}\right)$$

Considering Eq. (46), we have

$$\lim_{t \to \infty} V \leq \frac{M}{\eta}$$  (47)

This Eq. (47) implies that $V$ is bounded. Therefore, the stability of the controller is proved.

**Remark 8:** Compared with the existing controller for the stabilized platform, the proposed control scheme is carried out based on the problem of disturbance and input saturation. The synthesis of the two STESOs, sliding mode controller based backstepping technique and the auxiliary system for saturation are designed to provide the robust controller.

**Remark 9:** The tracking error can be converged to zero by choosing the suitable parameters of the controller. The conclusions is the same as in [46] that a solution of disturbance and input saturation can be found in output feedback control of integrator systems with disturbance and input saturation.

**IV. SIMULATION AND ANALYSIS**

In this section, the effectiveness of the proposed controller scheme is verified by simulation. The system uses an infrared seeker as an example. The system first drives the tracking loop by the offset angle of the infrared tracker, and then uses it as the stabilizing loop command after the tracking controller. The inner gimbal is used as an example to illustrate the stabilized loop. It is identical to the outer gimbal except for speed coordinate transformation and secant compensation. Stabilized platform adopts two loop control schemes, which are speed loop and position loop. Parameters of the system are shown in Table 1.

As shown above, the same type of DC torque motor is selected for the two gimbals. All the initial states of the stabilized platform and STESO are set as zero. The upper and lower limits of the saturation are defined as 5 and -5, respectively. The sampling time is set to 0.1ms. The comparative simulations are carried out by five different controllers with same initial states. Trial and error method is used to tune the parameters of the controllers.

**A. ELEVATION AXIS CONTROL ANALYSIS**

In order to simulate the stabilized platform more realistically, the whole system simulation with two gimbals is carried out. Firstly, inner gimbal is used as the example to verify the proposed controller. Step input signal is taken as the reference signal to verify the control performance and anti-disturbance ability. Here we set $d_1 = d_2 = 15 \sin(2\pi t)$. The step response curves are depicted in Fig.3. The performance comparison is shown in Table 3. Standard deviation of the output is from 2s to 4s. Fig.3(c) demonstrates that the tracking error of the stabilized platform converges to a small region with less time by proposed controller. It can be seen from the
Fig.3 and Table 3 that the proposed controller is better than the other controller.

In addition, the performance of the proposed STESO is verified under different disturbances. Fig.4 describes the estimation value of the sinusoidal disturbance by STESOs and the estimation errors are also depicted in Fig.4. It can be concluded that STEO1 and STESO2 can better estimate the sinusoidal input by selecting appropriate parameters.

B. SIMULATION OF TWO-AXIS STABILIZED PLATFORM SYSTEM

Two-axis gimbals of the stabilized platform system are considered to control at the same time. Two control loops which are tracking loop(position loop) and stabilized loop(speed loop) for each axis are proposed. The controller of the stabilized loop chooses (2) PI-ESO controller with input saturation compensation and (5) proposed controller as the controller to compare.

The controller of the tracking loop are $P$ controllers. For the stabilized loop with controller 2, the controller (2) parameters for stabilized loop are $K_{pp1} = 13$ and $K_{pp0} = 13$ for two gimbals. For the stabilized loop with controller 5, the controller (5) parameters for stabilized loop are $K_{pp1} = 20$ and $K_{pp0} = 20$ for two gimbals. The first subscript $p$ denotes proportional item. The second subscript $p$ denotes position loop. The control scheme is shown in Fig.5.

1) BANDWIDTH COMPARISON

Frequency response method is used to obtain the frequency characteristic of the stabilized platform. The input signal is $\sin(2\pi ft)$, and $f = 0.1 - 50$ Hz. Frequency response can be described by bode plot as shown in Fig.6.

From Fig.6, the bandwidth of the proposed controller exceeds the PI-ESO-SAT controller for the two gimbals.

2) ISOLATION COMPARISON

In actual engineering, the anti-disturbance ability of the control system is usually evaluated by the isolation index. The isolation $J$ is defined as

$$J = \frac{|\omega_{out}|}{|\omega_{b}|} \times 100\%$$ (48)
where $\omega_{out} = \omega_{2y}$ or $\omega_{2z}$ is the angular velocity of two frames, and $\omega_b = \omega_{0y}$ or $\omega_{0z}$ is the angular velocity of the three-axis turntable. According to different seeker state of the missile, a typical disturbance signals is imposed on the system. The base disturbance signals is set as $\theta_0x = \theta_0y = \theta_0z = 7 \sin(4\pi t)^\circ$. And the position inputs of the system for the two-axis frames are set as zero. The velocity responses of the two-axis are shown in Fig.7. The control performance for the two-axis are shown in Table 4 and Table 5. Besides $J$, standard deviation (std), integral absolute error (IAE), and integral squared error (ISE), are used as the performance indexes to verify the controller performance. The unit of std, IAE, ISE are $\circ$/s, $\circ$/s, and ($\circ$/s)$^2$. It illustrates that the proposed controller has the better isolation index and steady performance.

3) COUPLING PERFORMANCE ANALYSIS

In order to verify the coupling performance by the proposed controller, two-axis gimbals and two control loops are considered in this subsection. There are two scenarios to check the proposed controller. First, the square input with amplitude 1$^\circ$ is given to the elevation axis, and the azimuth axis is set as 0$^\circ$. Second, the square input with amplitude 1$^\circ$ is given to the azimuth axis, and the elevation axis is set as 0$^\circ$. The base disturbance signal is set as $\theta_{0x} = \theta_{0y} = \theta_{0z} = 7 \sin(4\pi t)^\circ$. The response curves are depicted in Fig.8 and Fig.9. $R$ represents the reference signal. Angular curve of the two gimbals illustrates the proposed controller can has fast response and anti-disturbance ability. The coupling angular velocity controller by PI-ESO-SAT is bigger than the output controlled by the proposed controller. It can be seen that the proposed controller has good performance for disturbance and saturation.

4) SATURATION COMPENSATION ANALYSIS

In order to verify the performance of the saturation compensation, the following simulations are carried out. Take the elevation axis as an example, the square input with amplitude 10$^\circ$ is given as the reference signal. The base disturbance signal is set as $\theta_{0x} = \theta_{0y} = \theta_{0z} = 0^\circ$. It should be noted that, as shown in Fig. 3, the performance of controller (4) without observer is very poor. Here we only discuss controller 1,2,3,5. The response curve is shown in Fig.10. The control input by different controllers are depicted in Fig.11. It can be seen from Fig.10 that the tracking performance of the proposed controller has good performance for disturbance and saturation.
controller is satisfied the demand. From Fig.11, it illustrates the control input by the proposed controller are below the control input by PI-ESO-SAT controller. The proposed controller can alleviate input saturation problem. It can be known that the proposed controller for the saturation can guarantee the system converge to a small neighborhood of zero in a shorter time.

5) STEADY STATE ANALYSIS
As we know, due to the existence of the sign function, traditional sliding mode controller has the chattering phenomenon. As a result, the controlled system is sensitive to noise and high-order dynamic. In the previous section, the response of the controlled system to disturbances (which can be regarded as including unmodeled high-order dynamics) has been analyzed, and the effectiveness of the controller has been proved. Here, the response of the controlled system to noise is verified by simulation. According to the real output of gyroscope used in practice, the limited bandwidth white noise is selected to simulate the system noise. Noise power is set as 0.000001. The step response of the system with noise by different controllers are depicted in Fig.12.

The std of the output data is from 0.3s to 1s. The std of system controlled by the four controllers are 0.0994°/s, 0.0994°/s, 0.0987°/s, 0.0990°/s, respectively. It can be seen that the output of the system with noise is consistent with
that of the traditional control method. There is little difference between them. The proposed controller does not reduce the stability of the system.

The above results are reasonable. Although STESO is used in this article, the adaptive algorithm is also applied to further control the disturbance error and effectively alleviate the chattering phenomenon. This idea is also well described in [48]. The main objective of this reference is to bring together two of the previous chattering reduction approaches, gain adaptation and high order sliding mode control. As for sliding mode controller, tanh function instead of sign function is used to effectively alleviate chattering. It can be seen...
that there are two improvements in the proposed controller. The proposed controller in this article is insensitive to noise and dynamic changes, and can achieve the same effect as traditional controller.

All the simulation results demonstrate that the proposed controller has better tracking performance for the stabilized platform with lumped disturbance and input saturation. The converge time is short and the steady error is smaller.

V. CONCLUSION
In this article, sliding mode controller based on backstepping technique and two STESOs has been proposed for the stabilized platform subject to disturbance (friction, imbalance torque, external and internal disturbance) and input saturation. By introducing two STESOs, the unmatched disturbance and matched disturbances has been estimated. The estimation error can converge to zero in finite time. By introducing the one order auxiliary system, the effect of saturation is alleviated. Furthermore, a multi-sliding mode controller based on backstepping method has been designed to control the stabilized platform. Lyapunov function has employed to analyze the stability of the closed-loop control system of the stabilized platform. Simulations have been carried out to verify the performance of the proposed controller. The simulation results demonstrate that the proposed controller has superior performance. It shows that the proposed algorithm effectively improves the system performance. The stabilized platform is widely used in many optical-electrical devices to stabilize the optical device. The control scheme proposed in this article can also be used for reference in other applications.

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