Effect of the stress triaxiality and Lode angle on the ductile damage evolution

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Damage in a structure is caused by material degradation due to initiation, growth and coalescence of micro-cracks/voids. In the recent years this topic acquired great importance in order to obtain a better design and to prevent the failure of components and structures. The present work aims to consider the influence of the stress triaxiality and the Lode angle effect on the ductile damage evolution, since it has been experimentally proved that the loading conditions highly affect the effective strain (i.e. cumulative plastic strain at failure). An ad hoc Lode angle function is adopted, together with the ductile damage evolution law proposed by Lemaitre, associating the decrease of the Lode angle influence with the generation of large plastic strain. On the other hands the choice of coupling the continuum damage mechanics framework together with an unconventional plasticity model is functional to investigate cyclic loading problems, since conventional plastic algorithms tend to overestimate the material ratcheting, leading to a wrong accumulation of the damage through cycles.

Key Words: Ductile damage, Lode angle effect, subloading surface, return mapping.

1. Introduction

Damage in a structure is caused by material degradation due to initiation, growth and coalescence of micro-cracks/voids. In the recent years this topic acquired great importance in order to obtain a better design and to prevent the failure of components and structures. The present work aims to consider the influence of the stress triaxiality and the Lode angle effect on the ductile damage evolution, since it has been experimentally proved that the loading conditions highly affect the effective strain (i.e. cumulative plastic strain at failure). On the other hands the choice of coupling the continuum damage mechanics framework together with an unconventional plasticity model is functional to investigate cyclic loading problems, since conventional plastic algorithms tend to overestimate the material ratcheting, leading to a wrong accumulation of the damage through cycles.

2. Constitutive model

2.1 Elasto-plastic and damage algorithm

Within the framework of elastoplasticity we adopted the Lemaitre’s approach, coupling the scalar internal damage variable together with the plastic ones by means of the plastic potential equation. The normal-yield and subloading surface of the Subloading Surface model were therefore modified as follows:

\[ f(\tilde{\sigma}) = (1 - D)F(H); \quad \tilde{\sigma} = \sigma - \alpha \]

\[ f(\bar{\sigma}) = (1 - D)F(H) \]

Here, \( \sigma \) is the Cauchy stress, \( \alpha \) is the back-stress, \( F \) is the isotropic hardening function, \( H \) is the isotropic hardening variable, \( D \) is the damage variable, \( s \) is the similarity center, \( R \) is the similarity transformation ratio, \( \bar{\sigma} \) and \( \tilde{\sigma} \) are the conjugate Cauchy stress and conjugate back-stress for the subloading surface, respectively, which are expressed as:

\[ \bar{\sigma} = \sigma - \bar{\alpha}, \quad \bar{\alpha} = s - R\tilde{s}, \quad \tilde{s} = s - \alpha \]

The constitutive equations were implemented with an incomplete implicit Euler scheme (i.e. cutting-plane return mapping) in order to speed up the computation without loss of accuracy. The material hardening is described by means of the Swift law:

\[ \sigma = F_0 + KH^n \]

Where \( K \) and \( n \) are two material constants, \( F_0 \) is the initial size of the plastic potential (i.e. yield stress). Their definition is reported in Table 1. No kinematic hardening was considered in this paper.

2.2 Lode angle effect

Recent works carried out by Bao and Wierzbicki, Xue, Li et al. and Korkolis and Kyriakides stressed out the importance of considering the Lode angle effect on the damage evolution since experimental tests evidenced that an evolution law based uniquely on plastic contribution and stress triaxiality is not enough to describe the crack formation under different loading configurations.

In the present work we assumed that a uniform distribution of defects is realized during the loading, allowing us to consider a scalar variable \( D \) for the degradation of the mechanical properties.
Following Lemaitre’s formulation, the damage evolution law can be written as:

$$D = \frac{\hat{\lambda}^+}{(1-D)} \left(-Y\right)^{s_2} \left[H - s_3\right] \frac{1}{\mu_\beta}$$

(4)

where $\lambda$ is the plastic multiplier and superscript $+$ indicates that just the tensile contributions are considered. $s_1$, $s_2$, and $s_3$ are material parameters; $s_1$ and $s_2$ affect the energy release rate $Y$:

$$Y = \left[ \frac{f(\sigma)^2}{6G(1-D)^2} + \frac{\sigma_m^2}{2K(1-D)^2} \right]$$

(5)

$s_3$ is a threshold for the cumulative plastic strain after which damage begins (the term in the Macaulay brackets is null until $H = s_3$). $\sigma_m$ is the mean stress and $G$ and $K$ are the shear and bulk moduli, respectively. The function $\mu_\beta$ is responsible for taking into account the Lode angle effect by accelerating the damage accumulation in dependence of the principal deviatoric stress ratio:

$$\chi = \frac{\sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (0 \leq \chi \leq 1)$$

(6)

It is in fact possible to assume symmetry in the deviatoric plane respect to the principal stress (i.e. $\chi=1$ or $\chi=0$), subdividing the octahedral plane in sextants. Moreover, it is also postulated that the symmetry holds in the plane strain axis ($\chi=0.5$). Experimental evidences carried out by Clausing\(^\text{13}\) and more recently by Li et al.\(^\text{11}\) showed a faster deterioration of the material performances for a plane strain case compare to the uniaxial extension, therefore the form of the function is chosen as:

$$\mu_\beta = 4\beta(1-\chi) + 1$$

(7)

where $\beta$ is a material parameter which modify the failure strain envelope (i.e. cumulative plastic strain at failure) depending on the loading path as indicated in Fig. 1. The effect of the material parameter $\beta$ can be observed in Fig. 2: for higher values the Lode angle influence is more pronounced, whereas for lower values it becomes smaller. Moreover, it respects the condition of the maximum Lode angle influence for the plane strain case, as experimentally proved by Xue\(^\text{10}\).

### 3. Numerical simulations

The work presented in this paper aims to test the Lode angle dependency of a Q460 steel reproducing the experimental and numerical results obtained in\(^\text{11}\). The material parameters for subloading surface are reported in Table 1.

| Table 1 material parameters for the subloading surface model. |
|---------------|-------------|
| $E$ (GPa) | 222.8 |
| $\nu$ | 0.3 |
| $K$ (MPa) | 969.14 |
| $n$ | 0.2 |
| $F_0$ (MPa) | 381.9 |
| $s_1, s_2, s_3$ | 9.5[MPa], 1.5, 0.01 |
| $\beta$ | 0.275 |

The set of samples used for testing the Lode angle dependency is the same as the one adopted by Li et al.\(^\text{11}\) and it is composed by: three notched bars, respectively with 6.25, 3.125 and 1.5 mm notch radius (named here UX625, UX3125 and UX15); three flat grooved plates with 10, 3 and 1 mm notch radius (i.e. PS10, PS3, PS1).

The geometry and the dimension of the samples is reported in the following Fig.3, whereas the mesh information are written in Table 2. For sake of simplicity a 2D axial symmetric model of half of the bars is adopted for the UX625, UX3125 and UX15 specimens whereas a 3D model of one-eighth of the whole geometry for the plane strain samples, applying the appropriate boundary conditions.

The idea is to calibrate the $s_1$, $s_2$, and $s_3$ parameters using the UX625 sample and the $\beta$ parameters of the Lode angle function with the flat grooved plate with 10 mm notch radius, subsequently we validated the numerical model on the PSI1.
problems. As it can be seen the simple damage evolution law cannot quite catch the experimental results, overestimating both the load after necking and the displacement at failure. On the other hand, the introduction of the Lode angle function improved the solution, displaying a better overlap with the experiments and a more correct cracking formation displacement. The solution numerically obtained by Li et al.\textsuperscript{11} cannot catch a correct post-necking behavior due to the fact that the constitutive model implements a Lode-dependent plasticity with no damage.

However, once we tried to validate the damage evolution law of Eq. (4) for the PS1 flat grooved plate the model gave a good but not perfect approximation of the experimental curve. Both the red and blue lines in fact, seems to overestimate the post-necking behavior and the elongation at crack. In details, the solution without the Lode angle effect overestimate of about 61% the experimental axial displacement at failure and the error is corrected to a 21% using the $\mu_l$ function, leaving open margins for improvement.

The experimental results carried out by Clausing\textsuperscript{13} on a set of notched bars and flat grooved specimens for seven types of steel revealed that the Lode angle effect seemed to become less relevant for high strain hardening materials suggesting a sort of mitigation of the function with the development of irreversible contributions. Other authors instead (i.e. Xue\textsuperscript{10},\textsuperscript{14}, Bao and Wierzbicki\textsuperscript{9}, Wierzbicki et al.\textsuperscript{15}) suggested a Lode angle dependency which is a function of the stress triaxiality, promoting a reduction of the effect for high stress triaxiality states, which has been validated for aluminium. A precise characterisation of the Lode angle function is still missing due to the difficulties of designing experiments where the stress varies on the same deviatoric plane, without variation of the pressure.

Here we suggested an approach based on the mitigation of the Lode angle dependency with the evolution of the plastic strain, since an approach based on the stress triaxiality didn’t lead to an improvements of the results. In particular, we modified the expression of the damage evolution law in Eq. (3) as:

$$
\dot{D} = \frac{\mu^+}{(1-D)} \left( \frac{\gamma^+}{\gamma^+} \right)^2 \left( H - s_3 \right) \frac{1}{\mu_{\text{ef}}} 
$$

where the $\mu_{\text{ef}}$ was introduced to indicate that now the Lode angle effect is also function of the plastic strain and it can be expressed as follows:

$$
\mu_{\text{ef}} = \mu_{\text{ef}} + \mu_l 
$$

where $\mu_{\text{ef}}$ is exactly the same function as in Eq. (7). The qualitative form of the failure strain envelope for one sextant can be seen in Fig. 6. $\mu_{\text{ef}}$ can be written as:
where $\alpha_1$ and $\alpha_2$ are two material constants that respectively indicates a cumulative plastic strain threshold after which the Lode angle effect can be assumed as negligible and the rate at which that condition is reached. The definition of these two additional material parameters was realized by calibrating once again the load displacement curve for the PS10 flat grooved specimen and subsequently validated on the PS1. The results of the calibration are reported in Fig. 7 where the blue and green lines highlight the difference between the damage evolution law in Eq. (4) and the one in Eq. (8). The damage itself is reported with dashed lines showing that the accumulation seems to be faster at the beginning with the $\mu_{slr}$ function, due to the higher value of the $\beta$ parameter, but then it tends to slow down due to the high amount of plastic deformation induced. This allows to have a better overlap with the experimental results maintaining the same value for the axial displacement at failure. The values for the Lode angle function are reported in Table 3.

**Table 3** Lode angle function parameters.

| $\alpha_1$, $\alpha_2$ | 1.0, 0.85 |
|------------------------|-----------|
| $\beta$                | 0.55      |

Again, the model was validated for the PS1 flat grooved plate using the parameters obtained in the calibration. The results are reported in Fig. 8. As it is possible to see the numerical simulation with the new definition of the Lode function (i.e. green line) approximates quite well the experimental results; the post-necking behavior of the elasto-plastic and damage algorithm catches quite well the decrease of stiffness and moreover the overestimation of the displacement at crack is reduced to just $+5.7\%$ of the experimental one. An additional check is performed testing the damage evolution law for the PS3 flat grooved specimen as shown in Fig. 9, proving the reliability of the new function. All the results for the notched bars and plates are summarized in Fig 10.
Fig. 10 Load-displacements curves for the samples of Fig. 3.

Fig. 11 Equivalent plastic strain vs. stress triaxiality.

Fig. 11 reports the average stress triaxiality and cumulative plastic strain path until the material failure together with the results obtained by Li et al.\textsuperscript{11).} It has to be said that Li et al. didn’t adopt an elasto-plastic and damaged constitutive model and the points in Fig. 11 are obtained by reporting the equivalent plastic strain against the stress triaxiality for the same total displacement at failure registered in the experiments. The little mismatch with our numerical results is therefore justifiable by the different modeling strategy adopted, however they show the same decreasing tendency with the increase of stress triaxiality.

4. Conclusions

The present paper proved the necessity of considering the Lode angle effect for the damage accumulation showing how a criteria based uniquely in the stress triaxiality is not sufficient for the description of the material failure. In addition, a modification of the Lode angle function has been introduced to get a better agreement with the experimental results obtained by Li et al.\textsuperscript{11).} In particular the ductile damage evolution law is characterized with a function that, instead of reducing the Lode angle effect due to an increase of stress triaxiality (i.e. conventional approach: Xue\textsuperscript{10,14), Bao and Wierzbicki\textsuperscript{9), Wierzbicki et al.\textsuperscript{11), Papasidero et al.\textsuperscript{16), Cortese et al.\textsuperscript{17),} it reduces its influence following the generation of plastic deformations. A future experimental campaign is however necessary to better characterize the Lode angle dependence of steels.

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