Multifrequency Method for Mapping Active Galactic Nuclei with Allowance for the Frequency-Dependent Image Shift

A. T. Bajkova\textsuperscript{1} and A. B. Pushkarev\textsuperscript{1,2,3}

\textsuperscript{1}Pulkovo Astronomical Observatory of RAS, Pulkovskoe sh. 65, St. Petersburg, 196140 Russia
e-mail: bajkova@gao.spb.ru

\textsuperscript{2}Max Planck Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany

\textsuperscript{3}Crimean Astrophysical Observatory, pos. Nauchnyi, Crimea, 98409 Ukraine

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Abstract — We consider the problem of multifrequency VLBA image synthesis and spectral-index mapping for active galactic nuclei related to the necessity of taking into account the frequency-dependent image shift. We describe our generalized multifrequency synthesis algorithm with a spectral correction based on the maximum entropy method. The results of our processing of multifrequency VLBI data for the radio sources J2202+4216, J0336+3218, and J1419+5423 are presented.

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INTRODUCTION

Active galactic nuclei (AGN) (quasars, blazars, radio galaxies, and Seyfert galaxies) are among the most enigmatic objects in our Universe. The AGN phenomenon consists in colossal energy release (up to $10^{46-47}$ erg s\textsuperscript{-1}) from relatively small spatial scales ($R < 1$ pc). Several theoretical models have been proposed to explain the nature of AGN, with the model of a supermassive (up to $10^{8\pm2}$M\textsubscript{☉}) black hole at the galactic center onto which matter accretes having gained the widest acceptance. As a result of accretion, a colossal energy emitted by AGN is generated; the flows of magnetized ultra-relativistic plasma are ejected along the rotation axis of the accreting disk with a speed close to the speed of light to produce jets observed at parsec and kiloparsec distances from the ”central engine” (Marscher and Erstad 2007). The signatures of AGN activity include a non-thermal spectrum attributable mainly to synchrotron radiation generated by the motion of electrons with relativistic velocities in a magnetic field; strong variability; the presence of broad emission lines in optical spectra; the presence of relativistic jets; spectral and polarization peculiarities.

At present, very-long-baseline interferometry (VLBI) is the most powerful tool for studying the morphological structures and kinematic, polarization, and spectral characteristics of AGN; it allows objects to be imaged with a very high angular resolution reaching fractions of a milliarcsecond (mas). One of the topical problems of VLBI mapping for AGN is multifrequency image synthesis. Our interest in this method is mainly related to the peculiar geometry of the future high-orbit ground-space ”Radioastron” radio interferometer (Kardashev 1997), which will provide an ultrahigh resolution (microarcseconds), on the one hand, and poor aperture filling, on the other hand (Bajkova 2005).

Multifrequency synthesis in VLBI suggests mapping AGN at several frequencies simultaneously to improve the instrument aperture filling. This is possible, because the interferometer baselines are measured in wavelengths of the emission being received. The problem of multifrequency synthesis is complicated by the fact that the brightness of a radio source generally depends on the frequency and to avoid undesirable artifacts in the image, a spectral correction should be
made at the stage of its deconvolution.

Conway et al. (1990), Conway (1991), Sault and Wieringa (1994), and Sault and Conway (1999) investigated the influence of spectral effects on the image and developed the methods of their correction. These authors showed that if narrow frequency bands, up to ±12.5% of the reference frequency, are used, then the effects of the spectral dependence of the brightness of a radio source can be neglected in principle, because they are small and can be compensated at the calibration or self-calibration stage. However, in the case of broader bands, for example, ±30%, the spectral effects should be corrected.

The algorithm of a linear spectral correction based on the CLEAN method (Högbom 1974) and called "double deconvolution" (Conway et al. 1990) is the best-studied one. In this algorithm, the "dirty" image is first deconvolved with an ordinary "dirty" beam and the residual map is then deconvolved with the beam responsible for the first-order spectral term. The development of this method consisting in simultaneous reconstruction of the sought-for image and the map of the spectral term was proposed by Sault and Wieringa (1994). The vector relaxation algorithm developed by Likhachev et al. (2006) may be considered a generalized CLEAN-deconvolution method that includes the spectral terms of any order.

As the study by Bajkova (2008) showed, applying Shannon’s maximum entropy method (MEM) (Frieden 1972; Skilling and Bryan 1984; Narayan and Nityananda 1986; Frieden and Bajkova 1994) allows tangible progress to be achieved in solving the problem of multifrequency synthesis owing to the possibility of a simple allowance for the spectral terms of any order. This, in turn, allows the range of synthesized frequencies to be extended significantly (up to 90% of the reference frequency).

However, the multifrequency synthesis algorithms based on both CLEAN and MEM deconvolution that we discuss here cannot be directly applied only to those radio sources for which no frequency-dependent image shift is observed. Otherwise, as was shown by Croke and Gabuzda (2008), an additional operation to align images at different frequencies should be performed to obtain the proper results of a multifrequency data analysis.

In this paper, our goal is to deduce our multifrequency synthesis algorithm based on MEM and to show how important the procedure for precorrecting the frequency-dependent image shift is in implementing multifrequency synthesis.

The paper is structured as follows. The frequency dependence of the image of a radio source is described in the next section. Subsequently, frequency dependent constraints on the visibility function are derived. The developed multifrequency synthesis algorithm with a frequency correction based on MEM is briefly deduced. We discuss the problem of aligning frequency-dependent images to properly construct the spectral-index distribution and, as an example, present the results of applying the developed method to multifrequency VLBI data for three radio sources: J2202+4216, J0336+3218, and J1419+5423.

\section{The Frequency Dependence of the AGN Radio Brightness}

The dependence of the intensity of a radio source on frequency \( \nu \) in the model of synchrotron radiation is given by (Conway et al. 1990)

\[ I(\nu) = I(\nu_0) \left( \frac{\nu}{\nu_0} \right)^\alpha, \]

where \( I(\nu_0) \) is the intensity of the radiation at the reference frequency \( \nu_0 \); and \( \alpha \) is the spectral index. Below, to simplify the writing, we will everywhere set \( I_0 = I(\nu_0) \).

Retaining the first \( Q \) terms in the Taylor expansion of (1) at point \( \nu_0 \), we can write the following approximate equality:

\[ I(\nu) \approx I_0 + \sum_{q=1}^{Q-1} I_q (\nu - \nu_0)^q, \]

where

\[ I_q = I_0 \frac{\alpha(\alpha - 1) \cdots [\alpha - (q - 1)]}{q!}. \]

In accordance with Eq. (2), for each point \((l,m)\) of the source's two-dimensional \((N \times N)\) brightness distribution (image, map), we have

\[ I(l, m) \approx I_0(l, m) + \sum_{q=1}^{Q-1} I_q(l, m) \left( \frac{\nu - \nu_0}{\nu_0} \right)^q, \]

where \( l, m = 1, ..., N \).

Thus, the derived brightness distribution over the source \( (3) \) is the sum of the brightness distribution at the reference frequency \( \nu_0 \) and the spectral terms, with the \( q \)-th-order spectral map depending on the spectral-index distribution over the source as follows:

\[ I_q(l, m) = I_0(l, m) \times \frac{\alpha(l, m) \alpha(l, m) - 1 \cdots [\alpha(l, m) - (q - 1)]}{q!}. \]

Of greatest interest is the first-order spectral map

\[ I_1(l, m) = I_0(l, m) \alpha(l, m). \]

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The spectral-index distribution over the source can be estimated from Eq. (5):

\[ \alpha(l, m) = \frac{I_q(l, m)}{I_0(l, m)} \]  

(6)

CONSTRAINTS ON THE VISIBILITY FUNCTION

The complex visibility function is the Fourier transform of the intensity distribution over the source that satisfies the spectral dependence (1) at each point of the map \((l,m)\). Given the finite number of terms in the Taylor expansion (3), the constraints on the visibility function can be written as

\[ V_{u_v, v_v} = F \{ I(l, m) \} \times D_{u_v, v_v} \]  

(7)

where \(F\) denotes the Fourier transform and \(D\) denotes the transfer function, which is the \(\delta\)-function of \(u\) and \(v\) for each measurement of the visibility function; different sets of \(\delta\)-functions correspond to different frequencies \(\nu\), as suggested by the indices of \(u\) and \(v\).

Let us rewrite Eq. (7) for the real and imaginary parts of the visibility function \(V_{u_v, v_v} = A_{u_v, v_v} + jB_{u_v, v_v}\) by taking into account the measurement errors as

\[ \sum_{q=0}^{Q-1} a_{u_v, v_v}^l \nu^q + \eta_{u_v, v_v}^r = A_{u_v, v_v}, \]

\[ \sum_{q=0}^{Q-1} b_{u_v, v_v}^l \nu^q + \eta_{u_v, v_v}^i = B_{u_v, v_v}, \]

(8)

where \(a_{u_v, v_v}^l\) and \(b_{u_v, v_v}^l\) are the constant coefficients (cosines and sines) that correspond to the Fourier transform; \(\eta_{u_v, v_v}^r\) and \(\eta_{u_v, v_v}^i\) are the real and imaginary parts of the instrumental additive noise distributed normally with a zero mean and a known dispersion \(\sigma_{u_v, v_v}\).

THE MULTIFREQUENCY SYNTHESIS ALGORITHM

In this case, the distributions \(I_q(l, m), q = 0, \ldots, Q -1; \ l, m = 1, \ldots, N\), and the measurement errors of the visibility function \(\eta_{u_v, v_v}^r, \eta_{u_v, v_v}^i\) are unknown. Note that although the brightness distribution over the source is described by a nonnegative function, the spectral maps of arbitrary order (4) can generally take on both positive and negative values, because, in particular, the spectral-index distribution over the source is an alternating one. Since the logarithm of a negative value is not defined on the set of real numbers, we will form a functional in which the spectral maps appear in absolute value to find the solutions for \(I_q(l, m), q = 0, \ldots, Q - 1:\)

\[ E = \{ \sum_{l,m} I_0(l, m) \ln[I_0(l, m)] + \sum_{q=1}^{Q-1} I_q(l, m) \ln[|I_q(l, m)|] \}

+ \rho \sum_{u_v, v_v} \frac{(\eta_{u_v, v_v}^r)^2 + (\eta_{u_v, v_v}^i)^2}{\sigma_{u_v, v_v}^2}, \]

(10)

\[ I_0(l, m) \geq 0, \quad I_q^+(l, m) \geq 0, \quad I_q^-(l, m) \geq 0, \]

(14)

where \(\rho\) is a positive weighting factor.

Minimizing functional (10) with constraints (8), (9), and (11) constitutes the essence of the MEM-based multifrequency synthesis algorithm.

The last term in (10) is an estimate of the disagreement of the reconstructed spectrum with the measured data according to the \(\chi^2\) test and may be considered as an additional stabilizing term. In this case, the influence of this term on the resolution of the reconstruction algorithm should be kept in mind.

For a practical solution of the reconstruction problem, let us pass to the generalized maximum entropy method described in detail by Bajkova (1992, 1993, 2005, 2008) and Frieden and Bajkova (1994). It consists in the substitution

\[ I_q(l, m) = I_q^+(l, m) - I_q^-(l, m), \]

(12)

where the superscripts \(+\) and \(-\) denote the positive and negative parts of the function, respectively, and the following modification of functional (10):

\[ E = \sum_{l,m} I_0(l, m) \ln[aI_0(l, m)] + \sum_{q=1}^{Q-1} I_q^+(l, m) \ln[aI_q^+(l, m)]

+ I_q^-(l, m) \ln[aI_q^-(l, m)] \}

+ \rho \sum_{u_v, v_v} \frac{(\eta_{u_v, v_v}^r)^2 + (\eta_{u_v, v_v}^i)^2}{\sigma_{u_v, v_v}^2}, \]

(13)

\[ I_0(l, m) \geq 0, \quad I_q^+(l, m) \geq 0, \quad I_q^-(l, m) \geq 0, \]

(14)

where \(a \gg 1\) s the parameter responsible for the accuracy of separating the positive, \(I_q^+(l, m)\), and negative, \(I_q^-(l, m)\), parts of the solution for \(I_q(l, m)\) (Bajkova 1992).

The linear constraints (8) and (9) on the measured visibility function will be rewritten accordingly:

\[ R_A = \sum_{l,m} I_0(l, m)a_{u_v, v_v}^l \]

(15)
The spectral-index distribution over the source can be constructed by various methods. The traditional method suggests: (1) the formation of images at two separate frequencies ($\nu_1$ and $\nu_2$), with the solutions of the deconvolution problem (CLEAN or MEM) being convolved with the same clean beam corresponding to the lower observing frequency; (2) determining the two-dimensional spectral-index distribution over the source from Eq. (1). Obviously, this sequence of operations is legitimate only when the positions of the VLBI cores of sources (not to be confused with the physical core of the source that is undetectable due to absorption effects) are frequency-independent.

The image reconstruction using the iterative self-calibration procedure is known (Thompson et al. 2003) to lead to the loss of information about the absolute position of the source in the sky: during the phase self-calibration, the centroid of the object is placed at the phase center of the map with coordinates $(0,0)$. However, since most of the radio-loud AGN are characterized by a dominant compact core (Kovalev et al. 2005; Lee et al. 2008; Pushkarev and Kovalev 2009), the VLBI core of the source coincides with the peak radio brightness of the source in an overwhelming majority of cases.

Nevertheless, the standard theory of extragalactic radio sources (Blandford and Konigl 1979) predicts a frequency-dependent VLBI core shift due to opacity effects in the source’s core region. Synchrotron self-absorption takes place in an ultra-compact region near the “central engine” of AGN, whose mechanism is most efficient at low frequencies. As a result, the maximum brightness manifests itself further from the core along the jet axis at lower frequencies. This theoretical prediction was confirmed by observations: the frequency-dependent shift in the core position was measured for several quasars by Lobanov (1998). In the literature, this phenomenon is actively debated from the viewpoint of the accuracy of astrometric measurements (Charlot 2002; Boboltz 2006; Kovalev et al. 2008).

It thus follows that the multifrequency data analysis must be preceded by the alignment of images at different frequencies. This can be achieved in three ways: (1) performing VLBI observations of the objects under study together with reference sources; (2) finding the parameters of the shift of one image relative to the other by aligning compact features of the optically thin jet, which are not subjected to absorption effects to the same extent as in the source’s core (Paragi et al. 2000; Kovalev et al. 2008a); (3) finding the shift parameters using a cross-correlation analysis (Croke and Gabuzda 2008). Being laborious from the viewpoint of performing observations and their subsequent reduction, the first method gives no significant advantage.

\[
+ \sum_{q=1}^{Q-1} \sum_{l,m} [(I^+_q(l,m) - I^-_q(l,m))]a_{u_v,v_\nu}
\times \left(\frac{\nu - \nu_0}{\nu_0}\right)^q + \eta_{u_v,v_\nu} = A_{u_v,v_\nu},
\]

where $R_A$ and $R_B$ denote the left-hand sides of the equations.

Thus, reconstructing the image $I_0(l,m)$ requires optimizing functional (13), i.e., finding

\[
\min E \quad (17)
\]

under constraints (14)-(16) for all unknown $I_0(l,m)$, $I_{q}^{(+)}(l,m)$, $q = 1, ..., Q - 1$, $l$, $m = 1, ..., N$ and $\eta_{u_v,v_\nu}$, $\eta_{u_v,v_\nu}^{lm}$. Note that requirements (14) can be omitted due to the peculiarity of the entropy solution, which is purely positive, and then only the linear constraints (15) and (16) on the measured complex visibility function remain. A detailed algorithm for numerical implementation of the proposed multifrequency synthesis method is given in Bajkova (2008).

The advantage of the proposed multifrequency synthesis algorithm is that the spectral terms of any order can be easily taken into account in functional (13) being minimized. This allows the spectral correction of images to be made both in a wide frequency range and for large spectral indices. The results of careful modeling of the method and estimates for the quality of the spectral correction of images depending on the size of the frequency band being synthesized, the spectral index, and the quality of the measurements of the visibility function are presented in Bajkova (2008).

THE PROBLEM OF SPECTRAL-INDEX MAPPING

One of the most important sources of information about the physical conditions in the radio-emission regions of AGN is the spectral-index distribution over the source. The core region is usually characterized by a large optical depth and an almost flat or inverted spectrum, while the jets are optically thin with respect to synchrotron radiation and have steeper spectra (Pushkarev and Kovalev 2009; Croke and Gabuzda2008).
in determining the shift; therefore, the second and/or third methods are used more often.

Recall that the alignment procedure implemented by shifting one image relative to the other is equivalent to the phase correction of the spectrum (or visibility function) of the image being shifted relative to the fixed one. The necessity of precorrecting the data for the source’s visibility function at different frequencies makes the direct use of the multifrequency synthesis algorithm described above problematic, because the frequency dependence of the core shift is not known in advance. It can be determined by forming the images at each frequency and determining the corresponding shifts. As was shown by Kovalev et al. (2008b) and O’Sullivan and Gabuzda (2009), the frequency dependence of the VLBI core position is well fitted by a hyperbolic dependence of the form \( r \propto \nu^{-1} \). Thus, our multifrequency synthesis procedure can be used after allowance for the shifts in the positions of the VLBI cores at different frequencies and their coordinates relative to the phase center and applying the corresponding frequency-dependent phase corrections to the visibility function.

**RESULTS OF OUR PROCESSING OF REAL DATA**

We will present the results of applying the developed multifrequency image synthesis algorithm to three representatives of AGN: J2202+4216, J0336+3218, and J1419+5423. All these sources have a fairly complex structure that includes an optically thick VLBI core and an optically thin extended jet, which must manifest itself in the spectral-index distribution.

The observations performed with the VLBA antennas and several antennas of the global VLBI network were taken from the NRAO archive.[1] The data on the visibility function at various frequencies were obtained simultaneously in “snapshot” mode. The data were calibrated using standard procedures from the AIPS package. The images were formed using procedures based on MEM and its generalizations within the framework of the Pulkovo “VLBIImager” software package.

The object names, dates and frequencies of observations, and parameters of the synthesized maps and the smoothing “clean” beam that determines the system’s resolution are given in Table 1. The parameters of the maps synthesized from the observational data at different frequencies and the parameters of the frequency-dependent image shift found by aligning compact features of the optically thin jet are given in Table 2. As expected, the direction of the shift coincides with the jet direction in all cases. The images of the sources obtained from both single-frequency and multi-frequency data and the two-dimensional spectral-index distributions over the source are shown in Figs. 1–6.

Let us analyze the results obtained.

1) **J2202+4216.** Figure 1 presents the images of the radio source obtained separately at 2.3 and 8.6 GHz. The source consists of an optically thick core and an optically thin jet directed southward of the core. The apparent extent of the jet reaches 36 and 8 mas at the low and high frequencies, respectively.

Figure 2 shows the intensity map and the map of the spectral-index distribution over the source synthesized at a reference frequency of 5.5 GHz from the data at 2.3 and 8.6 GHz without any allowance for the image shift found. As can be seen, the spectral-index map does not correspond to the physical meaning of an optically thick core and an optically thin jet: there are segments with negative spectral indices in the core region and with positive spectral indices in the jet region. The source’s image is not compact either: the core shape is severely distorted.

Figure 3 shows the result of two-frequency synthesis with the correction of the shift found by aligning the peak values of the single-frequency images. As we see, we managed to improve noticeably the previous result, but it is not yet quite proper, because the frequency-dependent shift of the peak values does not reflect the real shift of the images relative to each other.

Only allowance for the real shift found by aligning features of the optically thin jet, which is virtually unaffected by absorption effects, yielded the proper result shown in Fig. 4. As we see, as a result of the synthesis, we managed to reconstruct a more extended jet structure (up to 14 mas) than in the case of using only the high-frequency data but with the same high angular resolution. The spectral-index map adequately reflects the physical characteristics of the regions of the optically thick compact VLBI core and the optically thin extended jet. We see a fairly regular structure with smooth transitions between segments of different intensities along the entire source. The presented result agrees well with that obtained by Croke and Gabuzda (2008).

2) **J0336+3218.** Figure 5 presents both the single-frequency images of the radio source (upper panel) obtained separately at 2.3 and 8.6 GHz and the maps of the intensity and spectral-index distributions over the source obtained through two-frequency synthesis at a reference frequency of 5.5 GHz (lower panel). The source consists of an optically thick core and an optically thin jet extending roughly southwestward of the core. The jet extent reaches about 27 and 10 mas at the low and high frequencies, respectively.
As a result of the two-frequency synthesis with allowance made for the shift found, we managed to obtain the proper maps of the intensity and spectral-index distributions over the source. As we see from Fig. 5c, we managed to reconstruct a more extended jet structure (up to 15 mas) than in the case of using only the high-frequency data, with the synthesized map having the angular resolution corresponding to the high-frequency data. The spectral-index map adequately reflects the physical characteristics of the regions of the optically thick compact VLBI core, where the spectral index is positive, and the optically thin extended jet, where the spectral index is negative. We see that the spectral-index distribution reflects the spatial structure of the jet that consists of several compact features, showing smooth transitions between segments of different intensities.

(3) J1419+5423. This source is represented by observational data simultaneously at three frequencies: 5, 8.4, and 15.3 GHz (see Table 1). As a result, we have the possibility of three-frequency synthesis. The intensity maps obtained at individual frequencies are shown in the upper panel of Fig. 6. As can be seen, the source consists of a compact core and an extended jet extending southwestward of the core to a distance of about 25 mas. The compact features of the jet at distances of about 5 and 18 mas from the core are clearly seen on the maps constructed from the data at 5 and 8.4 GHz. The map constructed from the data at 15.3 GHz has a higher angular resolution and clearly shows the inner component of the jet at a distance of about 1 mas from the core, but, at the same time, the component at 5 mas is barely seen and the component at 18 mas is completely invisible.

The lower panel of Fig. 6 shows the results of our three-frequency synthesis with a reference frequency of 8.4 GHz obtained by taking into account the relative shift of the single-frequency images found. As can be seen from our intensity map, as a result of the three-frequency synthesis, we managed to reconstruct all jet components, both the component closest to the core and the farther ones. Thus, just as for the two previous sources, both a high resolution of the source’s components and a high sensitivity to the extended jet component can be achieved through the use of both low-frequency and high-sensitivity measurements.

Figure 6e shows the spectral-index map corresponding to the resolution of the synthesized image (Fig. 6d), from which the spectral-index distribution in the region of the resolved inner jet component can be judged. Figure 6f shows the map obtained with a low resolution corresponding to the measurements at the lowest observing frequency (in the C band), from which the larger-scale spectral-index distribution over the source can be judged. Thus, it is pertinent to note that, in contrast to the traditional method dealing with single-frequency images convolved with a wide beam corresponding to the lowest observing frequency, multifrequency synthesis allows the spectral index to be mapped with a high spatial resolution corresponding to that of the synthesized image.

CONCLUSIONS

We developed and tested an efficient multifrequency image synthesis algorithm with the correction of the frequency dependence of the radio brightness of a source. The algorithm is based on the maximum entropy method; it allow one to take into account the spectral terms of any order and to map the spectral index, which is of great importance in investigating the physical characteristics of AGN.

We showed how important the allowance for the frequency-dependent image shift is in applying the multifrequency synthesis algorithm. Our conclusions are based on the results of processing multifrequency VLBA data for the radio sources J2202+4216, J0336+3218, and J1419+5423 with a fairly complex extended jet structure, which also manifests itself in the spectral-index distribution over the source.

Analysis of the results obtained shows that multifrequency synthesis is an efficient method for improving the mapping quality; low-frequency data allow the extended structure of a source to be reconstructed more completely, while high-frequency data allow a high spatial resolution to be achieved.

As an additional advantage of the multifrequency synthesis algorithm, it should also be emphasized that the spectral-index distribution can be mapped with a high resolution corresponding to that of the synthesized image of the source, while the traditional spectral-index mapping method deals with images whose resolution is determined by the lowest observing frequency.

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Table 1. Parameters of single-frequency maps

| Object     | Date of observations | Observing frequency, GHz | Frequency range | Beam, mas × mas,[°] | Peak flux Jy/beam |
|------------|----------------------|--------------------------|-----------------|----------------------|-------------------|
| J2202+4216 | 10.05.1999           | 2.292                    | S              | 3.40 × 2.50, −14°   | 1.58              |
|            |                      | 8.646                    | X              | 1.00 × 0.70, −14°   | 1.05              |
| J0336+3218 | 05.07.2001           | 2.302                    | S              | 4.00 × 2.43, −12°   | 0.83              |
|            |                      | 8.646                    | X              | 1.04 × 0.67, −13°   | 0.76              |
| J1419+5423 | 09.02.1997           | 4.971                    | C              | 2.18 × 1.93, 84°    | 0.58              |
|            |                      | 8.405                    | X              | 1.31 × 1.14, 85°    | 0.57              |
|            |                      | 15.349                   | U              | 0.70 × 0.62, 75°    | 0.56              |

Table 2. Parameters of multifrequency maps

| Object     | Synthesized frequencies | Reference frequency, GHz | Shift Δx, mas | Shift Δy, mas | Beam, mas × mas,[°] | Peak flux Jy/beam |
|------------|-------------------------|--------------------------|---------------|---------------|----------------------|-------------------|
| J2202+4216 | S+X                     | 5.469                    | −0.185 (S–X)  | −2.300 (S–X)  | 1.58 × 1.00, −14°   | 0.77              |
|            |                         |                          |               |               | 1.04 × 0.67, −13°   |                   |
| J0336+3218 | S+X                     | 5.474                    | 1.230 (S–X)   | −0.680 (S–X)  | 1.04 × 0.67, −13°   | 0.61              |
|            |                         |                          |               |               |                      |                   |
| J1419+5423 | C+X+U                   | 8.405                    | 0.190 (C–U)   | −0.170 (C–U)   | 0.70 × 0.62, 75°    | 0.50              |
|            |                         |                          | 0.060 (X–U)   | −0.050 (X–U)   |                      |                   |
Figure 1: Intensity maps for the radio source J2202+4216 at frequencies of (a) 2.3 and (b) 8.6 GHz. The lower level of the contour lines is (a) 0.125\% and (b) 0.25\% of the peak flux density (the peak flux densities are listed in Table 1); the values of the succeeding levels are doubled.

Figure 2: Results of two-frequency image synthesis for the radio source J2202+4216 without any allowance for the mutual shift of its single-frequency images: (a) intensity map (the lower level of the contour lines is 0.25\% of the peak flux density, the values of the succeeding levels are doubled), the reference frequency is 5.47 GHz; (b) spectral-index map.
Figure 3: Results of two-frequency image synthesis for the radio source J2202+4216 with the pre-alignment of the peak values of its single-frequency images: (a) intensity map (the lower level of the contour lines is 0.25% of the peak flux density, the values of the succeeding levels are doubled); (b) spectral-index map.

Figure 4: Results of two-frequency image synthesis for the radio source J2202+4216 with the pre-alignment of optically thin jet features of its single-frequency images: (a) intensity map (the lower level of the contour lines is 0.25% of the peak flux density (see Table 2), the values of the succeeding levels are doubled); (b) spectral-index map.
Figure 5: Mapping results for the radio source J0336+3218: (a) intensity map at 2.3 GHz (the lower level of the contour lines is 0.5% of the peak flux density on the map, the values of the succeeding levels are doubled); (b) intensity map at 8.6 GHz (the lower level of the contour lines is 0.25%) (the peak fluxes are listed in Table 1); (c) two-frequency intensity map (the reference frequency is 5.47 GHz) synthesized by taking into account the mutual shift of the single-frequency images found by aligning optically thin jet features (the lower level of the contour lines is 0.25%) (the peak flux is given in Table 2); (d) spectral-index map.
Figure 6: Mapping results for the radio source J1419+5423: intensity map at (a) 5, (b) 8.4, and (c) 15.3 GHz (the lower level of the contour lines is 0.25% of the peak flux density on the map, the values of the succeeding levels are doubled) (the peak fluxes are given in Table 1); (d) three-frequency intensity map (the reference frequency is 8.4 GHz) synthesized by taking into account the mutual shift of the single-frequency images found by aligning optically thin jet features (the lower level of the contour lines is 0.125%)(the peak flux is given in Table 2); (e) and (f) spectral-index maps with the resolutions corresponding to the data in the U and C bands, respectively.