NUCLEAR TETRAHEDRAL SYMMETRY

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We recall the main features of the $T_d^P$ (tetrahedral) symmetry in atomic nuclei and present realistic mean-field calculations supporting the existence such a symmetry all over the nuclear chart. A few potential candidate-nuclei are investigated and the possible experimental signatures of the tetrahedral symmetry are also briefly discussed.

1. Introduction

It has been recently pointed out\(^1\) that atomic nuclei with tetrahedral symmetry could be encountered all over the nuclear chart. The predictions are based on a very general analysis of symmetries of the nuclear mean-field and are inspired by the group-theory considerations. The implied unique 4-fold degeneracies of the single-particle levels characteristic of tetrahedral (and/or octahedral) symmetry of the fermionic mean-field are thought to favour the appearance of large gaps in the shell structure thus leading to stable potential minima with the corresponding symmetry. [This mechanism is similar to that of the stabilizing mechanism related to the $(2j+1)$-fold degeneracies at spherical shapes]. Calculations performed for a few candidate-nuclei in various mass regions show indeed that the tetrahedral-shape isomers are relatively low in energy. They are separated from the ground-state minimum by a significant barrier (up to a couple of MeV). In this paper, we would like to focus on the nuclei whose hypothetical tetrahedral-symmetry states can be found in the dedicated experiments; we present examples of related mean-field calculations and briefly discuss the envisaged experimental challenges.
2. Theoretical Arguments Favouring the Tetrahedral Symmetry

In the following, the nuclear shape is parametrized using the standard expansion onto the basis of spherical harmonics $Y_{\lambda \mu}(\theta, \varphi)$:

$$R(\theta, \varphi) = R_0 c(\{\alpha\}) \left[ 1 + \sum_{\lambda=2}^{\lambda_{\text{max}}} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) \right] ; \quad R_0 = r_0 A^{1/3}. \quad (1)$$

In this expression, $R(\theta, \varphi)$ represents the nuclear surface in spherical coordinates, $r_0$ is the nuclear radius parameter, $c(\{\alpha\})$ accounts for the volume conservation when deforming the nucleus, and $\alpha_{\lambda\mu}$ are the deformation parameters. There exists a priori an infinite number of ways to generate a nuclear shape with the tetrahedral symmetry\(^a\). Various possible combinations of the deformation parameters as well as a detailed discussion of the geometrical properties of the implied shapes can be found elsewhere\(^2\). However, the lowest multipole-order to realize the symmetry in question is $\lambda = 3$ and one can demonstrate that any other allowed multipolarity must be greater or equal $\lambda = 7$. Thus it is believed that the high multipoles may contribute only negligibly. In this paper we consider tetrahedral symmetry realized by $\alpha_{32} \neq 0$ and every other $\alpha_{\lambda\mu}$ vanishing.

Single-particle levels show strong variation in function of $\alpha_{32}$ and it has been demonstrated using realistic mean-field calculations that huge gaps may appear in the corresponding spectra, comparable to, or larger than the known spherical gaps. The predicted\(^1\) tetrahedral-magic numbers for protons and neutrons are:

- $N = 16, 20, 32, 40, 56, 70, 90, 112, 136$
- $Z = 16, 20, 32, 40, 56, 70, 90, 112, 126$.

The corresponding nuclei should in principle be the best candidates for the tetrahedral symmetry. However, pairing effects and/or quasi-particle excitations can favour tetrahedral minima also in neighbouring nuclei. In addition, some nuclei are likely to present other shape isomers that compete with the tetrahedral one. It is therefore possible that the best candidates to look for may not be those with the tetrahedral magic numbers but in their vicinity. Case-by-case thorough calculations are therefore needed: here we limit ourselves to presenting a few illustrations only.

Let us focus on the gaps 40 and 70. Two types of mean-field calculations have been performed. One of them is based on the macroscopic-microscopic approach where the energy is given by the sum of a liquid drop term and the shell-supplemented by a pairing-energy terms. In such an approach, the nuclear shape is an input, and to find physical solutions one has to minimize the energy as a function of the deformation parameters. Another type, the self-consistent Hartree-Fock technique based on an effective (Skyrme) interaction do not explicitly define any dependence on the deformation, and the deformation parameters are extracted from the selfconsistent solutions: it is thus an output of the theory. However, the

\(^a\)In other words: a shape which is invariant under every symmetry operation of the group $T_d^D$. 
single-particle degeneracies in a nucleus do not depend on which mean-field is used, but exclusively on the particular symmetry of the nucleus. Indeed calculations show that the four-fold degeneracies in the spectra are independent of the type of the mean-field used, while the exact values of the tetrahedral gaps of course do depend somewhat on the model and its parametrisation\textsuperscript{b}.

In Figure 1, we show the total energy of two isotopes of Zirconium as a function of the elongation $\beta_2$. At each point, the energy is minimized over the $\gamma$ angle and all octupole and hexadecapole degrees of freedom using the gradient method. The energy is calculated in the macroscopic-microscopic approach, with a liquid drop term including curvatures effects\textsuperscript{3}, a shell correction\textsuperscript{4} based on the non-relativistic Woods-Saxon potential with the universal parametrization\textsuperscript{5} and a pairing correction\textsuperscript{3} with particle-number projection (before variation). Two minima clearly appear in both nuclei: a prolate ground-state and a minimum at $\beta_2 = 0$ which turns out to be tetrahedral (with $\alpha_{32} \sim 0.15$). These calculations are very much consistent with those published earlier\textsuperscript{6}, the latter obtained independently using self-consistent Skyrme-Hartree-Fock technique.

We performed also Skyrme-Hartree-Fock-Bogoljubov calculations in the region of $^{110}$Zr, the results of which are presented in Table 2. The most striking result shown in the Table is that with the parameterization SLy4, Ref.\textsuperscript{7}, the tetrahedral minima appear to be the ground-state in both $^{110}$Zr and $^{112}$Zr. We have verified using other Skyrme parametrizations that the tetrahedral minima persist although they lie above the corresponding ground-states.

\textsuperscript{b}The situation is very similar to that of spherical nuclei, where the magic numbers are the same for all kinds of mean-fields except for the very large particle numbers (e.g. superheavy nuclei).
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Table 1. HFB solutions, SLy4 parametrization, in heavy Zr isotopes for various energy minima.

| Nucleus | Tetrahedral | Spherical | Prolate | Oblate |
|---------|-------------|-----------|---------|--------|
| $^{108}\text{Zr}$ | 0.0 | +0.391 | -1.099 | -0.679 |
| $^{110}\text{Zr}$ | 0.0 | +0.431 | +0.072 | +0.266 |
| $^{112}\text{Zr}$ | 0.0 | +0.027 | +0.299 | +1.006 |

3. Experimental Signatures of Tetrahedral States

Tetrahedral minima may lead to shape isomerism; they correspond to a non-axial octupole shape. Consequently, if rotational bands are built on top of these configurations they should lead to parity doublets. However, contrary to the better-known case of axial-octupole deformations, no E1 inter-band transitions should be observed due to the vanishing dipole moment. The bands should be sought at relatively low spins since the angular momentum alignment will tend to break the tetrahedral symmetry, and at relatively high excitation energy. Indeed, tetrahedral minima are predicted at about 1-2 MeV above the ground-states in most of the cases; the corresponding moments of inertia are expected to be smaller than those in the prolate minima, the intraband decay energies should be correspondingly larger. Also: the single-particle energies in a tetrahedral nucleus being up to 4-fold degenerate, one may expect to observe a unique 16-fold, approximately degenerate, quasi-particle pattern.

We have presented results of microscopic calculations supporting the existence of very low-lying tetrahedral shape minima in Zirconium isotopes. The eventuality of tetrahedral ground-states in the heaviest Zr must also be considered. These nuclei are not beyond the range of the current experimental facilities.

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