RING FORMATION IN MAGNETICALLY SUBCRITICAL CLOUDS AND MULTIPLE-STAR FORMATION

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ABSTRACT

We study numerically the ambipolar diffusion–driven evolution of nonrotating, magnetically subcritical, disklke molecular clouds, assuming axisymmetry. Previous similar studies have concentrated on the formation of single magnetically supercritical cores at the cloud center, which collapse to form isolated stars. We show that for a cloud with many Jeans masses and a relatively flat mass distribution near the center, a magnetically supercritical ring is produced instead. The supercritical ring contains a mass well above the Jeans limit. It is expected to break up, through both gravitational and possibly magnetic interchange instabilities, into a number of supercritical dense cores, whose dynamic collapse may give rise to a burst of star formation. Nonaxisymmetric calculations are needed to follow in detail the expected ring fragmentation into multiple cores and the subsequent core evolution. Implications of our results on multiple star formation in general and the northwestern cluster of protostars in the Serpens molecular cloud core in particular are discussed.

Subject headings: ISM: clouds — ISM: magnetic fields — MHD — stars: formation

1. INTRODUCTION

At the heart of multiple-star formation lies cloud fragmentation. The role of magnetic fields in cloud fragmentation is not well explored (Boss 2000), even though present-day molecular clouds are thought to be strongly magnetized. Reliable Zeeman measurements of the magnetic field strength of molecular clouds made to date, as compiled by Crutcher (1999), suggest that after geometric corrections (Shu et al. 1999) the clouds are remarkably close to being magnetically critical. These measurements reinforce the oft-expressed view that magnetic fields in molecular clouds are strong enough to be dynamically important.

Dynamically important magnetic fields can change the characteristics of cloud fragmentation fundamentally. The magnetic and gravitational forces are often comparable in magnitude but opposite in direction. The near cancellation of forces (Shu & Li 1997) can in principle keep the clouds in a magnetically levitated state over many dynamic times, allowing more time for overdense substrutures to develop and fragment. To isolate the effects of magnetic fields on cloud fragmentation from those of turbulence (which have been the subject of intensive numerical simulations, as reviewed by Vazquez-Semadeni et al. 2000), we will concentrate on the relatively quiescent regions of molecular clouds. Such regions are capable of producing binaries, multiple stellar systems, as well as groups or small clusters—all of which we broadly term “multiple stars”—but not rich clusters of hundreds to thousands of stars, which tend to form in more turbulent regions (Myers 1999). Our goal is to extend the reasonably successful scenario of single-, isolated-star formation, based on ambipolar diffusion (Shu, Adams & Lizano 1987; Mouschovias & Ciolek 1999; see, however, Nakano 1998 for a different opinion), to the formation of multiple stars. In the longer term, we hope to apply the insight gained on magnetically controlled fragmentation and multiple-star formation to the more difficult problem of rich cluster formation, where turbulence plays a more dominant role (e.g., Klessen, Burkert, & Bate 1998).

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(e.g., Black & Bodenheimer 1976). They are shown to be strongly susceptible to dynamical nonaxisymmetric fragmentation into small pieces (Norman & Wilson 1978). These rotating rings were widely discussed in connection with the production of multiple stellar systems in the late 1970s and early 1980s, although their formation depends sensitively on the numerical treatment of angular momentum transport (Norman, Wilson, & Barton 1980). The ring formation to be discussed in this paper does not depend on rotation. It represents a first step toward a theory of multiple-star formation in a strongly magnetized cloud.

The rest of the paper is organized as follows: The mathematical formulation of the problem of ring formation in a strongly magnetized cloud is given in § 2, and numerical examples are presented in § 3. We discuss in § 4 ring fragmentation and its implications for multiple-star formation in general and the northwestern cluster of the Serpens molecular cloud core in particular.

2. FORMULATION OF THE PROBLEM

2.1. Governing Equations

We adopt the standard thin-disk approximation (e.g., Nakamura, Hanawa, & Nakano 1995), and cast the MHD equations that govern the evolution of magnetized clouds in a vertically integrated form. Mass conservation of the disk material yields

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma V) = 0,$$

(1)

where $\Sigma$, $t$, $r$, and $V$ are, respectively, the (mass) column density, time, cylindrical radius, and radial component of disk velocity. Axisymmetry and a cylindrical coordinate system ($r$, $\phi$, $z$) are adopted throughout the paper.

We assume that the disk is isothermal with an effective sound speed of $a$ and is threaded by an ordered magnetic field with a (cylindrically) radial component $B_r$ and a vertical component $B_z$. The vertically integrated momentum equation in the radial direction then becomes

$$\frac{\partial (\Sigma V)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma V^2) = \Sigma g_r + \frac{B_r B_z}{2\pi} - \frac{\partial (\Sigma a^2)}{\partial r} - H \frac{\partial}{\partial r} \left( \frac{B_z^2}{4\pi} \right),$$

(2)

where $g_r$ is the radial component of gravity and $H$ is the disk half-thickness. The terms on the right-hand side of the equation represent the forces associated with, respectively, the gravity, magnetic tension, thermal (and possibly turbulent) pressure, and magnetic pressure. We have kept only the leading terms for the magnetic tension and pressure force and have ignored cloud rotation, which is dynamically unimportant in general before the formation of compact stellar objects (i.e., protostars and their disks; see Basu & Mouschovias 1994). Rotation can easily be included if necessary.

The evolution of the ordered magnetic field is governed by the magnetic flux conservation equation

$$\frac{\partial B_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r B_z V_y) = 0,$$

(3)

where $V_y$ is the velocity of magnetic field lines in the cross-field direction (Nakano 1984). In a lightly ionized medium such as a molecular cloud, the field lines slip relative to the neutral matter at a velocity

$$V_y - V = t \left[ \frac{B_y B_z}{2\pi} - H \frac{\partial}{\partial r} \left( \frac{B_z^2}{4\pi} \right) \right] / \Sigma,$$

(4)

where $t$ is the coupling time between the magnetic field and neutral matter. In the simplest case where the coupling is provided by ions that are well tied to the field lines and the ion density $\rho_i$ is related to the cloud density $\rho$ by the simple expression $\rho_i = C \rho^{1/2}$, one has

$$t = \frac{1.4}{\gamma C \rho^{1/2}},$$

(5)

where typically $\gamma C = 1.05 \times 10^{-2} \text{cm}^{3/2} \text{g}^{-1/2} \text{s}^{-1}$ (e.g., Shu 1991) and the factor 1.4 comes from the fact that the cross section for ion-helium collision is small compared to that of ion-hydrogen collision (Mouschovias & Morton 1991).

The disk half-thickness $H$ in equation (2) and the mass density $\rho$ in equation (5) are related through the definition

$$\Sigma = 2 \rho H.$$

(6)

To determine these two quantities separately, we assume that the disk is always in a static equilibrium in the disk-normal direction (Fiedler & Mouschovias 1993). Integration of the force balance equation vertically yields

$$\rho = \frac{\pi G \Sigma^2}{2a^2} \left( 1 + \frac{B_z^2}{4\pi^2 G \Sigma^2} \right) + \frac{P_e}{a^2},$$

(7)

to the lowest order in $(H/r)$. The two terms in the brackets represent, respectively, the gravitational compression and magnetic squeezing of the disk material. The magnetic squeezing term becomes important late in the cloud evolution, when the magnetic field configuration becomes highly pinched. The quantity $P_e$ denotes the ambient pressure that helps confine the disk, especially in low column density regions where gravitational compression is relatively weak.

2.2. Initial Conditions

The “initial” distributions of mass and magnetic flux of a star-forming cloud are not well determined either observationally or theoretically. For illustrative purposes, we prescribe them in a “reference” state, following Basu & Mouschovias (1994). We adopt a uniform distribution

$$B^f_{z}(r) = B_{\infty}$$

(8)

everywhere for the magnetic field, with $B_{\infty}$ denoting the background field strength, and a simple prescription

$$\Sigma^f(r) = \frac{\Sigma_0}{\left[ 1 + (r/r_0)^n \right]^{4/n}}$$

(9)

for the column density, with $\Sigma_0$ denoting the central value and $r_0$ a characteristic radius beyond which the column density drops off rapidly. We will sometimes refer to $\Sigma_0$ simply as “the reference column density” later. The prescriptions (8) and (9) are similar to those used by Basu & Mouschovias (1994), except that we leave the exponent $n$ as a free parameter to be specified. The exponent $n$ controls the amount of mass in the central “plateau” region where the mass distribution is more or less uniform. It will play a crucial role in ring formation (§ 3). The low column density “envelope” outside the radius $r_0$ is designed mainly to minimize the effects of the cloud boundary on the evolution of the central region. For the model clouds to be considered
in the next section, we will adopt a single cloud radius that is twice the characteristic radius \( r_0 \). The column density at the cloud edge will then be less than 1/16 of the central value in all cases. A region with such a low column density will be firmly controlled by magnetic fields, and its evolution will be effectively decoupled from that of the central region, where dynamic collapse and star formation occur.

The reference model clouds prescribed by equations (8) and (9) are not necessarily in mechanical equilibrium, since the thermal and magnetic forces are not designed to balance out the self-gravity exactly. We let these clouds adjust toward an equilibrium configuration under the constraints (1) that the magnetic field be frozen in matter and (2) that the field strength remain fixed at the cloud edge (so that continuity with the background field is preserved; see Basu & Mouschovias 1994). The first constraint guarantees that the mass-to-flux ratio is conserved for each individual mass element during the adjustment. The second constraint, together with the first, implies that the column density at the cloud edge remains fixed as well. Numerically, the equilibrium configurations are obtained by evolving the reference clouds in time according to the governing equations (1)–(3), setting the velocity of magnetic field lines \( v_B \) equal to the velocity \( V \) of neutral matter. An extra damping force proportional to \(- V\) is applied to the right-hand side of the momentum equation (2) during this adjustment phase to gradually bring the reference clouds into a static equilibrium. The final equilibrium configurations, in which \( V = 0 \) and the self-gravity is balanced exactly by a combination of thermal and magnetic forces, serve as the initial configurations for the subsequent cloud evolution driven by ambipolar diffusion. For the magnetically subcritical clouds that we are interested in, the adjustment is usually rather small, as noted previously by Basu & Mouschovias (1994).

2.3. Dimensional Units and Dimensionless Quantities

The governing equations (1)–(3) are to be solved numerically. To facilitate the numerical attack, we first cast all dimensional quantities in these equations into a dimensionless form. We begin by normalizing the column density and radius with the central value \( \Sigma_0 \) and characteristic radius \( r_0 \) that appear in the reference column density distribution, equation (9), and denote the resultant dimensionless quantities by

\[
\sigma \equiv \frac{\Sigma}{\Sigma_0}, \quad \xi \equiv \frac{r}{r_0}.
\]

For typical values of \( \Sigma_0 \) and \( r_0 \), we adopt \( 10^{-2} \) g cm\(^{-2}\) (corresponding to a molecular hydrogen number column density of \( 2.1 \times 10^{21} \) cm\(^{-2}\), or roughly 2 mag of mean visual extinction; e.g., McKee 1989) and 1 pc. These typical values introduce two scaling factors \( \mu_2 \equiv \Sigma_0/(10^{-2} \text{ g cm}^{-2}) \) and \( \mu_r \equiv r_0/(1 \text{ pc}) \). The dimensional units for other quantities are then obtained from various combinations of \( \Sigma_0 \) and \( r_0 \). In terms of the scaling factors \( \mu_2 \) and \( \mu_r \), we have the following: \( M_\odot = 2 \pi \Sigma_0 r_0^2 = 3.0 \times 10^{12} (\mu_2) \) \( M_\odot \) for mass, \( V_B = (GM_\odot/r_0)^{1/2} = 1.1 (\mu_r)^{1/2} \) km s\(^{-1}\) for velocity, \( t_0 = r_0/V_B = 8.6 \times 10^4 (\mu_2/\mu_r)^{1/2} \) yr for time, \( B_0 = \Gamma (2\pi G^{1/2}/\Sigma_0) = 16 (\mu_r) \mu_2 \mu G \) for field strength, \( \rho_0 = \Sigma_0/r_0 = 3.2 \times 10^{-21} (\mu_2/\mu_r) \) g cm\(^{-3}\) for mass density, and \( P_0 = \pi G (\Sigma_0/2)^2 = 1.0 \times 10^{-11} \mu_2^2 \) dyn cm\(^{-2}\) for pressure. Note that the dimensionless parameter \( \Gamma \) is the ratio of the background field strength to the critical field strength, \( 2\pi G^{1/2}/\Sigma_0 \), associated with the reference column density \( \Sigma_0 \). It is a free parameter that characterizes the degree of cloud magnetization. Note also that the units \( t_0 \) and \( V_B \) are the characteristic free-fall time and free-fall speed of the reference cloud.

With the above-defined units, we obtain the following dimensionless quantities:

\[
\begin{align*}
\sigma & = \frac{\Sigma}{\Sigma_0}, & h & = \frac{H}{r_0}, & b_x & = \frac{B_x}{B_\odot}, & b_z & = \frac{B_z}{B_\odot}, & \tau & = \frac{\tau}{t_0}, \\
v & = \frac{V}{V_B}, & v_B & = \frac{V_B}{v}, & \dot{\alpha} & = \frac{\dot{\alpha}}{V_B}, & \dot{\rho} & = \frac{\dot{\rho}}{\rho_0}, & \dot{p} & = \frac{\dot{p}}{p_0}.
\end{align*}
\]

The dimensionless effective sound speed \( \dot{\alpha} \), which will play an important role in ring formation (§ 3), can be written in terms of the effective cloud temperature \( T_{\text{eff}} \) as

\[
\dot{\alpha} = 0.29 \left( \frac{1}{\mu_r \mu_2} \right)^{1/2} \left( \frac{T_{\text{eff}}}{30 \text{ K}} \right)^{1/2},
\]

where a helium abundance of 10% by number has been assumed. Combining equations (7) and (12), we find a molecular hydrogen number density of

\[
n_{H_2} = 2.2 \times 10^3 (1 + p_e) \mu_2^{1/2} \left( \frac{30 \text{ K}}{T_{\text{eff}}} \right) \text{ cm}^{-3}
\]

at the center of the reference clouds, where \( \Sigma = \Sigma_0 \) and \( B_0 = 0 \) (by symmetry). In the fiducial case of \( \mu_2 = 1 \), \( T_{\text{eff}} = 30 \text{ K} \), and \( p_e < 1 \) (i.e., gravity dominating external pressure in compressing the cloud center), the above number density is typical of the \(^{13}\text{CO} \) clumps found in many molecular clouds. The clumps are often taken as the starting point of isolated-star–formation calculations. For cluster-forming regions, somewhat higher reference density and effective temperature may be more appropriate.

2.4. Dimensionless Governing Equations and Boundary Conditions

We rewrite the governing equations of cloud evolution into a dimensionless, Lagrangian form using the dimensionless time \( \tau \) and mass \( m \) as independent variables:

\[
\begin{align*}
\frac{\partial \xi}{\partial \tau} &= \frac{1}{\sigma} \dot{\xi}, & \frac{\partial \xi}{\partial m} &= v, \\
\frac{\partial \sigma}{\partial \tau} &= \frac{\partial \sigma}{\partial m} - \Gamma^2 h_2 b_z \sigma - 2 \xi \frac{\partial \sigma}{\partial m} - \Gamma^2 h_2 \xi \frac{\partial (b_z^2)}{\partial m}, \\
\frac{\partial \sigma}{\partial \tau} &= - \frac{\partial \sigma}{\partial m} [\xi \dot{b}_z (v_B - v)].
\end{align*}
\]

The dimensionless drift velocity between magnetic field lines and neutral cloud matter that appears in the field diffusion equation (17) is given by

\[
v_B - v = \frac{\Gamma^2}{v_c \mu^{1/2}} \left[ \frac{b_z^2}{\sigma} - h_2 \xi \frac{\partial (b_z^2)}{\partial m} \right],
\]

associated with the reference column density \( \Sigma_0 \). It is a free parameter that characterizes the degree of cloud magnetization. Note also that the units \( t_0 \) and \( V_B \) are the characteristic free-fall time and free-fall speed of the reference cloud.
where the magnetic coupling coefficient \(v_c\) has a value of 11.6 for the simplest case of coupling given by equation (5). The dimensionless half-thickness and mass density of the disk are determined from

\[
h = \frac{2\hat{a}^2}{\sigma} \left(1 + \frac{\Gamma^2b^2 + p_e}{\sigma^2}\right)^{-1},
\]

(19)

and

\[
\hat{\rho} = \frac{\sigma^2}{4\hat{a}^2} \left(1 + \frac{\Gamma^2b^2 + p_e}{\sigma^2}\right).
\]

(20)

Together with the auxiliary equations (18)–(20), the four governing equations (14)–(17) completely determine the time evolution of four cloud quantities: column density \(\sigma\), radius \(r\), radial velocity \(v\), and the vertical field strength \(b_z\), provided that the radial component of gravity \(\hat{g}_r\) and the radial field strength \(b_r\) are determined. These two quantities are determined approximately as follows.

It is well known that the radial component of gravity on an infinitely thin disk with column density \(\sigma(\zeta)\) is given by

\[
\hat{g}_r = \int_0^\infty \zeta \sigma(\zeta) M(\zeta, \zeta) \, d\zeta,
\]

(21)

where the integral kernel is obtained from

\[
M(\zeta, \zeta') = \left(\frac{2}{\pi}\right) \frac{d}{d\zeta} \left[\frac{1}{\zeta_{\text{max}}} - K\left(\frac{\zeta_{\text{min}}}{\zeta_{\text{max}}}\right)\right],
\]

(22)

with \(\zeta_{\text{min}} = \min(\zeta, \zeta')\), \(\zeta_{\text{max}} = \max(\zeta, \zeta')\), and \(K\) being the complete elliptic integral of the first kind. Following Ciolek & Mouschovias (1993), we shall use equation (21) to approximate the radial component of gravity for our thin disk of finite thickness.

On an infinitely thin disk, the radial field strength \(b_r\) can be determined from the vertical field strength \(b_z\). We adopt the formalism of Lubow, Papaloizou, & Pringle (1994), who assumed that the magnetic field consists of two parts: a uniform background and a potential field due to a toroidal current confined entirely to the disk. The first part is prescribed, and the second part is determined solely by the (vertically integrated) toroidal current density \(J_\phi\), which is linearly proportional to \(b_z\) on the disk. In particular, \(b_z\) on the disk can be expressed as an integral over disk radius with an integrand that is linearly proportional to \(J_\phi\) (Jackson 1975) and thus \(b_r\). A simple matrix inversion then allows one to calculate \(b_r\) in terms of \(b_z\) (see Lubow et al. 1994 for details). As an approximation, we shall use the above formalism to compute the radial field strength for our thin disk of finite thickness.

For boundary conditions, we impose at the center of the cloud (where \(m = 0\)) the conditions of symmetry: \(\zeta = 0, v = 0, \partial b_z/\partial m = 0\), and \(\partial \sigma/\partial m = 0\). At the outer edge, we let the cloud boundary move freely while holding the vertical component of magnetic field \(b_z\) and the column density of the disk \(\sigma\) fixed at their initial values at all times, consistent with the boundary conditions we imposed in § 2.2 during the adjustment from the reference state toward the equilibrium configuration. These specifications complete our discussion of governing equations and initial and boundary conditions.

### 2.5. Numerical Method

The numerical methods for Lagrangian hydrodynamics and magnetohydrodynamics are well documented in chapters 4 and 8 of Bowers & Wilson (1991), respectively. We follow the procedures outlined in those chapters closely, except for the treatment of magnetic field diffusion equation (17). The field diffusion is driven by a combination of magnetic pressure, gravitational, and net kinetic energy terms. For the two driving terms separately. While the usual finite differencing is adequate for the magnetic pressure term to ensure stability; for the pressure term, we employ the so-called Gaussian elimination technique, as described by Bowers & Wilson (1991, p. 408).

### 3. Ring Formation in Magnetically Subcritical Clouds

In this section, we numerically integrate the non-dimensional governing equations (14)–(17), subject to the initial and boundary conditions described in §§ 2.2 and 2.4. Two dimensionless constants appear in the governing equations: \(\Gamma\), the ratio of the background field strength \(B_\phi\) to the critical field strength \(2\pi G^{1/2} \Sigma_0\), and \(\hat{a}\), the dimensionless effective sound speed. The quantity \(\Gamma\) controls the degree of cloud magnetization and must be greater than unity in order for the clouds to be mainly supported by (ordered) magnetic fields. Although Nakano (1998) presented theoretical arguments against star-forming clumps being magnetically subcritical, the situation is less clear observationally: Zeeman measurements of the field strength in molecular clouds, as compiled by Crutcher (1999), is roughly consistent with the clouds being magnetically critical, after likely geometric corrections (Shu et al. 1999). Uncertainties involved in estimating the mass-to-flux ratio preclude a firmer conclusion. We believe that at least some star-forming clumps are magnetically subcritical to begin with and adopt for definitiveness a round value of \(\Gamma = 2\). Clouds with values of \(\Gamma\) substantially less than 2 will not be disklike, and the thin-disk approximation adopted here may not be applicable. Values of \(\Gamma\) much larger than 2, on the other hand, are difficult to reconcile with the Zeeman measurements. For the dimensionless effective sound speed, we will first adopt a round value of \(\hat{a} = 0.3\) (corresponding to an effective temperature of 32 K for the canonical choice of the scaling factors \(\mu_0 = \mu_e = 1\)) and then consider a smaller value of \(\hat{a} = 0.2\) for comparison. A third constant, the dimensionless external pressure \(p_e\), appears in auxiliary equations (19) and (20). We adopt, for simplicity, a value of \(p_e = 0.1\), which corresponds to a reasonable dimensional value of \(10^{-12} \mu_e^2\) dyn cm\(^{-2}\).

Three model clouds are considered. They have, respectively, \(n = 2, 4, \) and 8, where \(n\) is the exponent that specifies the column density distribution in the reference state. A quick inspection of the prescription of the reference column density distribution, equation (9), reveals that the \(n = 8\) cloud has the largest cloud mass for the same cloud radius (taken to be twice the characteristic radius \(r_0\) for all clouds, as mentioned earlier). Its dimensionless mass is 0.80, compared with 0.40 and 0.66 for the \(n = 2\) and 4 clouds, respectively. The Jeans mass is, on the other hand, much smaller,
at least near the cloud center, where

\[ M_J = \frac{1.17a^4}{G^2 \Sigma_0}, \]

(23)

according to Larson (1985; his eq. [9]). The dimensionless central Jeans mass takes the following simple form:

\[ m_J = 7.35\dot{\alpha}^4, \]

(24)

which has a value of 0.06 for our standard choice of dimensionless sound speed \( \dot{\alpha} = 0.3 \).

The reference clouds are allowed to settle toward an equilibrium configuration under the influence of an artificial damping force, with magnetic flux frozen in, as described in § 2.2. After the equilibrium state is reached, we reset the time \( t \) to zero and turn on ambipolar diffusion. The clouds then evolve on the magnetic flux redistribution timescale, which is typically an order of magnitude longer than the dynamic timescale. In what follows, we will illustrate the main features of cloud evolution by displaying various cloud quantities at the initial equilibrium time \( t = 0 \) and two representative times \( t_1 \) and \( t_2 \), when the maximum column density reaches, respectively, 10 and 10\(^2\) times the reference value \( \Sigma_0 \). Typically, by the time \( t_1 \), the clouds start to become magnetically supercritical and are about to collapse dynamically. By the time \( t_2 \), the collapse is well into the dynamic phase, with a maximum infall speed comparable to or greater than the effective sound speed \( a \).

First, we plot the distributions of column density and infall velocity at the time \( t = 0 \) and \( t_1 \) in Figure 1. The latter time corresponds to 6.75, 7.87, and 8.63 million yr for the \( n = 2, 4, \) and \( 8 \) clouds, respectively. The column density distributions at time \( t_1 \) illustrate clearly two distinctive modes of cloud evolution: in the \( n = 2 \) cloud with a relatively peaky initial mass distribution (see insert), a dense core has formed at the center, while the more massive \( n = 8 \) cloud with a flatter central mass distribution has produced a dense ring. The intermediate \( n = 4 \) cloud is close to the borderline between core forming and ring forming; a slight increase of the exponent \( n \), to a value of 4.2 for example, would induce the cloud to form a ring instead of a core.

Fig. 1.—Evolution of three magnetized clouds with different initial mass distributions as specified by the exponent \( n \) in eq. (9), (a) Column density distributions at the initial equilibrium \( t = 0 \) and the time \( t_1 \), when the maximum column density reaches 10 times the reference value \( \Sigma_0 \). Note that a dense core is formed at the center of the \( n = 2 \) and \( 4 \) clouds whereas a dense ring is produced in the \( n = 8 \) cloud. For clarity, the distributions at \( t = 0 \) are lowered by one unit. Their differences show up more clearly in the insert. (b) Distributions of infall speed at the same two times as in (a), showing the quasi-static nature of ring and core formation, although substantial infall motion is clearly present on the (sub)parsec scale.

We set the scaling factors \( \mu_\epsilon \) and \( \mu_\mu \) to unity hereafter (including in the figures) to obtain concrete dimensional quantities from the dimensionless numerical solutions. The dependences of dimensional units on \( \mu_\epsilon \) and \( \mu_\mu \) are given in § 2.3.
Both the central cores and dense ring are formed quasi-statically, as indicated by the velocity distributions shown in Figure 1b. Although the maximum infall speeds are substantial, ranging from $\sim 0.05$ to $0.22$ km s$^{-1}$ in our particular examples, they remain below the effective sound speed (and should be even smaller compared with the magneto-sonic speed, the relevant signal speed in a magnetized cloud). Note that the (subsonic) infall regions are clearly "large scale", spanning a good fraction of a parsec, even during this relatively early "starless" phase of evolution (see also Li 1999; Ciolek & Basu 2000). Such an extended infall motion may have been detected in the starless core L1544 (Tafalla et al. 1998; Williams et al. 1999). For starless cores formed in magnetically subcritical clouds, Ciolek & Basu (2000) showed that the infall speed is sensitive to the background field strength (i.e., the value of $\Gamma$ in our notation). Figure 1b demonstrates that the speed depends rather strongly on the initial mass distribution as well.

Second, we plot in Figure 2 the distributions of the mass-to-flux ratio and cloud shapes at the times $t = 0$ and $t_2$. The latter time corresponds to 6.95, 8.10, and 8.94 million yr for the $n = 2, 4, \text{and} 8$ clouds, respectively. By the time $t_2$, a substantial amount of mass ($14$, $55$, and $90\ M_\odot$, respectively) has become magnetically supercritical, inside either the central core or dense ring, according to Figure 2a. Note that the mass-to-flux ratio of the $n = 8$ cloud peaks inside the ring and that the central region interior to the ring remains magnetically subcritical. The relatively strong central field should cushion the contraction of the ring toward the origin as a whole, with potentially observable signatures.

The cloud shapes, as outlined by the half-thickness $H$ defined in equation (6) and shown in Figures 2b–2d, change dramatically as the clouds evolve. The minimum thickness occurs at the column density maximum inside either the central cores or dense ring, where self-gravity squeezes on the disk material the hardest. The lower column density "envelope" is compressed, in contrast, mainly by external pressure. Together, the self-gravity and external pressure keep the cloud material flattened at all times, justifying the thin-disk approximation adopted. Also shown in Figures 2b–2d are magnetic field lines at time $t_2$. It is clear that mass accumulation in the core ($n = 2$ and 4) and the ring ($n = 8$) has also led to an accumulation of magnetic flux in these overdense regions, creating a pinched magnetic configuration. Indeed, the mass-to-flux ratio at time $t_2$ remains less than twice the critical value everywhere (see Fig. 2a), even though the column density has increased by a factor of

![Graph showing distributions of mass-to-flux ratio](image)
nearly $10^2$ from its initial value and the volume density by an even larger factor, close to $10^4$. The near-critical mass-to-flux ratio and associated strong magnetic field are characteristic of dense cores (see, e.g., Lizano & Shu 1989; Basu & Mouschovias 1994) and rings formed out of magnetically subcritical clouds, in contrast with those formed in other (weakly magnetic or nonmagnetic) scenarios.

The dichotomy of core and ring formation is illustrated most vividly by Figure 3, where the column density distributions of the core-forming $n = 2$ and ring-forming $n = 8$ clouds are represented graphically at the times $t = 0, t_1,$ and $t_2$. The dense, opaque central core of the $n = 2$ cloud is expected to collapse quickly, on a (very short, local) dynamic timescale, to produce a single, isolated star. What happens to the ring? In § 4.1 below, we will argue that the dense, self-gravitating ring is likely to fragment into smaller pieces, which collapse to form more than one star. Therefore, the dichotomy of core and ring formation should be indicative of two modes of star formation in a strongly magnetized cloud: a single, isolated mode and a multiple, clustered mode, depending on the mass of the cloud and its distribution.

So far, we have concentrated on how mass distribution, as specified by the exponent $n$ in equation (9), affects core and ring formation. We now wish to demonstrate that the effective sound speed, $a$, has a profound effect as well. For...
this purpose, we repeat the evolution of the same three clouds shown in Figures 1–3 with a smaller value of dimensionless sound speed \( \tilde{a} = 0.2 \) (instead of 0.3). We find that whereas the \( n = 2 \) \((n = 8)\) cloud forms a central core (dense ring) as before, the borderline \( n = 4 \) cloud is induced by the lower sound speed to collapse off-center into a ring instead of a central core. This difference is consistent with the trend found by Bastien (1983) for flattened, nonmagnetic clouds: decreasing the sound speed (and thus the Jeans mass) makes ring formation easier. Indeed, if we were to remove all of the magnetic fields from our magnetically supported clouds suddenly (assuming \( \tilde{a} = 0.2 \)), they would collapse promptly, with the \( n = 2 \) cloud forming a dense core and the \( n = 4 \) and 8 clouds each forming a dense ring, just as their magnetized counterparts. The strong magnetic fields lengthen the ring formation time (by roughly an order of magnitude) but do not appear to suppress the ring formation tendency of a multi-Jeans mass cloud with a relatively flat mass distribution. Indeed, it is the magnetic fields that are responsible for the cloud flattening and thus the ring formation in the first place. We have explored other forms of mass and magnetic flux distribution as well as other values of the dimensionless sound speed \( \tilde{a} \) and come to the same general conclusion.

Finally, we note that Mouschovias and collaborators have studied the ambipolar diffusion-driven evolution of disklke magnetic clouds extensively (e.g., Ciolek & Mouschovias 1994; Basu & Mouschovias 1994, 1995), although none of their models produced a dense ring. The lack of ring formation in their models can be traced to the particular form of reference mass distribution adopted, which is similar to the relatively peaked distribution of the core-forming \( n = 2 \) cloud shown in Figures 1–3 (see eq. [31] of Basu & Mouschovias 1994). Our more flexible prescription of mass distribution enables us to uncover a new outcome to the ambipolar diffusion-driven evolution of magnetically subcritical clouds: ring formation, which has implications for cloud fragmentation and multiple-star formation.

4. DISCUSSION AND IMPLICATIONS

4.1. Ring Fragmentation

Dense, self-gravitating rings like the one shown in the right column of Figure 3 contain many Jeans masses and are thus susceptible to breaking up gravitationally into a number of smaller pieces. The fact that they are gradually condensed out of strongly magnetized clouds modifies their fragmentation properties in two important ways:

First, the strong magnetic fields present during the initial magnetically subcritical, quasi-static phase of cloud evolution prevents the Jeans instability from developing on a dynamic timescale everywhere, including inside the forming rings (Langer 1978; Nakano 1988). On the other hand, after becoming substantially supercritical, the dense rings collapse dynamically, leaving little time for small fragments to grow. It therefore appears that the best time for the rings to break up gravitationally is during the transitional period when they are marginally magnetically critical, after being freed from the firm grips of magnetic fields through ambipolar diffusion, but before embarking on a runaway collapse. If this is indeed the case, then we can estimate the number of fragments expected from the breakup. For typical dark-cloud conditions, the transition occurs roughly around the time \( t_1 \), when the maximum column density reaches 10 times the reference value \( \Sigma_0 \). Since the ring is narrow and self-gravitating at this time, it should fragment in a way similar to that of an infinitely long self-gravitating cylinder. It is well known (Larson 1985) that (nonmagnetic) cylinders with a sound speed \( a \) and central mass density \( \rho_c \) are unstable to perturbations with wavelength exceeding about

\[
\lambda_{ct} = 4\left(\frac{2a^2}{\pi G\rho_c}\right)^{1/2}.
\]

The instability grows fastest at a wavelength near \( 2\lambda_{ct} \) (Inutsuka & Miyama 1992). For a ring centered at \( r_r \), one expects the number of fragments to be roughly

\[
N \approx \frac{2\pi r_r}{2\lambda_{ct}} \approx \frac{\pi \sigma_r \xi_r}{16a^2},
\]

where \( \sigma_r \) and \( \xi_r \) are the dimensionless column density and radius at the ring location. We have used equations (7) and (25) in deriving the second expression, also taking into account the fact that the ring is mainly compressed by self-gravity. Applying the above formula to the marginally critical ring shown in the middle right panel of Figure 3, we find the expected number of fragments to be \( \sim 5 \) (with \( \sigma_r = 10 \), \( \xi_r = 0.23 \), and \( \tilde{a} = 0.3 \)). If the dimensionless effective sound speed \( \tilde{a} \) is lowered to 0.2, then the same cloud would produce \( \sim 19 \) fragments instead (with \( \sigma_r = 10 \) and \( \xi_r = 0.38 \)). Clearly, the number of fragments increases quickly with decreasing sound speed, as one might expect intuitively.

The second modification introduced by a strong magnetic field is the possibility of interchange instability. In the simplest case of an infinitely thin disk supported entirely (and statically) by a magnetic field, the square of the growth rate \( \gamma \) is given by (see Spruit & Taam 1990)

\[
\gamma^2 = -\frac{d[\ln(\Sigma/B_r)]}{dr} g_r,
\]

where \( g_r \) denotes the radial component of gravity as before. The above equation implies that this (local) instability will grow (i.e., \( \gamma^2 > 0 \)) as long as the mass-to-flux ratio, \( \Sigma/B_r \), decreases in the direction of gravity, \( g_r \). This criterion will be satisfied in part of the ring during its (long) quasi-static phase of formation. The reason is simply that the mass-to-flux ratio peaks inside the ring (see Fig. 2a) and thus decreases toward the origin, which is also the direction of gravity during most of the quasi-static phase of cloud evolution. The instability is demonstrated explicitly in Figure 4, where \( \gamma^2 \) is plotted against the radius and mass for the ring-forming \( n = 8 \) cloud shown in Figures 1–3 at four relatively early times: the initial equilibrium time \( t = 0 \) and the three times when the maximum column density reaches 2, 4, and 8 times the reference value \( \Sigma_0 \) (corresponding to 5.39, 7.76, and 8.52 million \( yr \), respectively). Note that the inner, plateau part of the cloud (where star formation occurs) starts out close to being marginally stable. It becomes increasingly more unstable as the cloud condenses out, until enough mass has accumulated in the ring to reverse the direction of the gravity interior to the ring (to outward pointing). The reversal explains the suppression of instability near the center at the last time shown. It is likely that magnetic interchange instability, which is intrinsic to the ring formation process, contributes significantly to ring fragmentation, once the axisymmetry is removed. The situ-
cally subcritical, multi-
following scenario for multiple-star formation: As magneti-
mentation to nonaxisymmetric clouds, we propose the
self-gravitating rings that are prone to breaking up into
of this magnetically retarded Jeans instability lead, in a
Langer (1978). We have shown that nonlinear developments
mass) for instability, although the growth time could be
cloud does not change the minimum wavelength (and thus
netic Ðeld in a lightly ionized medium such as a molecular
ple fragments of Jeans mass. The presence of a strong mag-

Fig. 4.—Square of the growth rate of magnetic interchange instability, $\gamma^2$, in units of the inverse of the square of a million yr, plotted against (a) radius and (b) mass of the ring-forming $n = 8$ cloud shown in Fig. 3, at the initial equilibrium time $t = 0$ (dotted line) and the three times when the maximum column density reaches 2 (dashed line), 4 (dash-dotted line), and 8 (solid line) times the reference value $\Sigma_0$, corresponding to 5.39, 7.76, and 8.52 million yr. Regions with positive $\gamma^2$ are unstable, according to the simple criterion given in the text.

ation may be complicated, however, by ambipolar diffusion
and the associated drift between magnetic field lines and
cloud matter. Nonaxisymmetric models are required to
examine the interplay between magnetic interchange insta-
and cloud fragmentation in detail.

4.2. A Scenario of Multiple-Star Formation

Multi–Jeans mass clouds are unstable to forming multi-
ple fragments of Jeans mass. The presence of a strong mag-
netic field in a lightly ionized medium such as a molecular
cloud does not change the minimum wavelength (and thus
mass) for instability, although the growth time could be
greatly lengthened, according to the linear analysis of
Langer (1978). We have shown that nonlinear developments
of this magnetically retarded Jeans instability lead, in a
strict axisymmetric geometry, to the formation of dense,
self-gravitating rings that are prone to breaking up into
smaller pieces. Generalizing the ring formation and frag-
mentation to nonaxisymmetric clouds, we propose the
following scenario for multiple-star formation: As magneti-
cally subcritical, multi–Jeans mass clouds evolve due to
ambipolar diffusion, dense elongated substructures develop
quasi-statically. These substructures break up, through
Jeans and possibly magnetic interchange instabilities, into
a number of smaller pieces, creating a cluster of dense, mag-
ettically supercritical cores. The supercritical cores collapse
quickly in a (local) dynamic time, leading to a burst of star
formation.

The above scenario is a direct extension of the standard
picture of isolated-star formation (Shu et al. 1987; Mous-
chovias & Ciolek 1999) to the formation of multiple stars. A
key ingredient is the gradual condensation of elongated
multi–Jeans mass substructures capable of breaking up into
multiple dense cores. If the breakup occurs mostly during
the transitional period when the substructure is approx-
imately magnetically critical, as we speculated in the last
subsection, then the initial size of the dense cores and the
separation between neighboring cores would be compara-
tible to the Jeans length scale evaluated at the magnetically
critical density (see eq. [25]). The same size scale applies to
supercritical dense cores formed in isolation, as pointed out
by Basu & Mouschovias (1995).

Although details remain to be worked out, we anticipate
two potentially observable features of the multiple cores
formed in the above scenario. First, as with single cores
formed through ambipolar diffusion in isolated-star forma-
tion (e.g., Lizano & Shu 1989; Basu & Mouschovias 1994),
the multiple cores should have magnetic field strength close
to (say, within a factor of 2 of) the critical value, even after
they become supercritical and after star formation. Second,
the cores should have small motions relative to one another,
since the motions are cushioned by both the magnetic flux
trapped inside the cores and the flux held in between them
(see Fig. 2d). These features are not expected from cores
formed, e.g., in prompt collapse of nonmagnetic Jeans-
unstable clouds (Klessen et al. 1998). Observations of mass-
to-flux ratio and relative motions of dense cores can provide
key tests of various scenarios of multiple-star formation.

To make the scenario more quantitative, one needs to
construct nonaxisymmetric models. Such models will allow
one to follow the ring fragmentation process and, more
importantly, to study the formation and fragmentation of
 overdense substructures in more realistic clouds that are
irregular in shape and/or contain appreciable substructures
in mass distribution to begin with. Moreover, once formed,
the fragments (i.e., multiple cores) are expected to interact
with one another and with their common envelope, both
gravitationally and magnetically. The interaction should
play a role in determining the mass spectrum and motions
of the cores and thus stars (Motte, Andre, & Neri 1998;
Testi & Sargent 1998). In particular, a gravitational drag
effect analogous to the “dynamic friction” in stellar
dynamics could in principle bring the cores closer together
to form binaries and multiple stellar systems (Larson 2001).
Nonaxisymmetric models will also be required to follow the
nonlinear developments of the (dynamic) Jeans and mag-
ettice interchange instabilities, which could provide an
important source of turbulence (see also Zweibel 1998).
They may explain, at least in part, why cluster-forming
regions are more turbulent than isolated-star-forming
regions, even before stars are formed.

We should stress that the above scenario is intended
mainly for the formation of stellar groups and small clusters
in relatively quiescent regions of molecular clouds, such as
the fragmented starless cores observed in millimeter contin-
um by Ward-Thompson, Motte, & Andre (1999), and
perhaps the $\rho$ Oph B2 core studied in detail by Motte et al.
(1998). The $\rho$ Oph B2 core has apparently broken up into a
dozen or so starless clumps. It may represent a short-lived
phase during the evolution of an initially magnetically sub-
critical, multi–Jeans mass cloud in which supercritical cores
have already formed through ambipolar diffusion and frag-
mentation but yet to collapse into a group of stars. The
well-studied starless core L1544 may represent a similar
short-lived phase during the formation of an isolated mag-
etized core leading to single-star formation (e.g., Williams
et al. 1999; Li 1999; Ciolek & Basu 2000). Our scenario for
multiple-star formation is not directly applicable to the for-
mation of rich clusters, such as the Trapezium cluster in Orion, where turbulence-induced fragmentation probably plays a dominant role (Klessen et al. 1998; Padoan et al. 2001).

4.3. Northwestern Cluster of the Serpens Molecular Cloud Core

Although one does not expect to find many ringlike structures in real star-forming regions because of the high degree of geometric symmetry required, they are sometimes observed. The most famous example is perhaps the ring of massive stars in the massive cluster-forming region W49N (Welsh et al. 1987). A more recent example is the northwestern cluster of protostars and dense cores in the Serpens molecular cloud core—another active (although less massive) cluster-forming region (e.g., Testi & Sargent 1998; Davis et al. 1999). The cluster has the appearance of a tilted, fragmented ring and could be the result of the fragmentation of a dense ringlike structure formed quasi-statically in a strongly magnetized cloud. This interpretation is strengthened by polarization measurements of thermal dust emission, which indicate a large-scale magnetic field threading the ring plane more or less perpendicularly (Davis et al. 2000), as expected in our scenario (see Fig. 2d). Moreover, the velocity dispersion among the cores is rather small, on the order of 0.25 km s⁻¹ (J. Williams 2000, private communication), which could result from the magnetic cushion effect mentioned earlier. Infall motions are observed on both the large, cluster scale (~0.2 pc) and the small, individual core/proto star scale (~0.02 pc; Williams & Myers 2000). In our picture, the large- and small-scale infall motions would be associated, respectively, with the process of ring formation as a whole (similar to that shown in Fig. 1b) and with gravitational contraction onto individual cores/proto stars, although substantial contributions from localized turbulence dissipation are also possible (Myers & Lazarian 1998). Our interpretation is complicated, however, by the strong turbulent and outflow motions present in the region.

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