A Bayesian Machine Learning Algorithm for Predicting ENSO Using Short Observational Time Series

Nan Chen, Faheem Gilani, and John Harlim

Abstract A simple and efficient Bayesian machine learning (BML) training algorithm, which exploits only a 20-year short observational time series and an approximate prior model, is developed to predict the Niño 3 sea surface temperature (SST) index. The BML forecast significantly outperforms model-based ensemble predictions and standard machine learning forecasts. Even with a simple feedforward neural network (NN), the BML forecast is skillful for 9.5 months. Remarkably, the BML forecast overcomes the spring predictability barrier to a large extent: the forecast starting from spring remains skillful for nearly 10 months. The BML algorithm can also effectively utilize multiscale features: the BML forecast of SST using SST, thermocline, and windburst improves on the BML forecast using just SST by at least 2 months. Finally, the BML algorithm also reduces the forecast uncertainty of NNs and is robust to input perturbations.

Plain Language Summary One major challenge in applying machine learning algorithms for predicting the El Niño-Southern Oscillation is the shortage of observational training data. In this article, a simple and efficient Bayesian machine learning (BML) training algorithm is developed, which exploits only a 20-year observational time series for training a neural network. In this new BML algorithm, a long simulation from an approximate parametric model is used as the prior information while the short observational data plays the role of the likelihood which corrects the intrinsic model error in the prior data during the training process. The BML algorithm is applied to predict the Niño 3 sea surface temperature (SST) index. Forecast from the BML algorithm outperforms standard machine learning forecasts and model-based ensemble predictions. The BML algorithm also allows a multiscale input consisting of both the SST and the wind bursts that greatly facilitate the forecast of the Niño 3 index. Remarkably, the BML forecast overcomes the spring predictability barrier to a large extent. Moreover, the BML algorithm reduces the forecast uncertainty and is robust to the input perturbations.

1. Introduction

As the most prominent interannual climate variability, the El Niño–Southern Oscillation (ENSO) manifests as a basin-scale air-sea interaction phenomenon characterized by sea surface temperature (SST) anomalies in the equatorial central to eastern Pacific (Clarke, 2008; Rasmusson & Carpenter, 1982; Zebiak & Cane, 1987). It has a strong impact on climate, ecosystems, and economies around the world through global circulation (Ashok & Yamagata, 2009; Ropelewski & Halpert, 1987). Classically, ENSO is regarded as a cyclic phenomenon (Jin, 1997; Wyrski, 1975), in which the positive and negative phases are known as El Niño and La Niña, respectively.

The traditional ensemble forecast using physics-based models has been widely used for predicting the ENSO (Moore & Kleeman, 1998; Tang et al., 2018; B. P. Kirtman & Min, 2009). A hierarchy of models ranging from the general circulation models (GCMs) to many intermediate and low-order models are employed for forecasting the refined and large-scale ENSO features, respectively. However, model error, which leads to large predictive uncertainty, is ubiquitous in these parametric models and often results in ineffective forecasts. The model error often comes from the incomplete understanding of nature and/or the inadequate spatiotemporal resolutions in these models (Kalnay, 2003; Majda & Chen, 2018; Palmer, 2001). More recently, machine learning (ML) techniques have become prevalent in forecasting ENSO and many other climate...
phenomena (Ding et al., 2018; Ham et al., 2019; LeCun et al., 2015; Wang et al., 2020). These ML approaches exploit sophisticated neural networks (NNs) or other nonparametric methods to recover the complex dynamics in nature. Given sufficient training data, these approaches can achieve state-of-the-art numerical performance, beating traditional physics-based models.

However, one major challenge in applying ML algorithms for predicting ENSO is the shortage of observational training data. In fact, only three extreme El Niño events and a few moderate ones were observed during the satellite era (i.e., from 1980 to the present). To augment the training data set, commonly used strategies include concatenating the satellite observations with either the proxy-based reconstructed data (Barrett et al., 2018; Emile-Geay et al., 2013; Gergis & Fowler, 2009; McGregor et al., 2010; Rayner et al., 2003) or with time series generated from certain parametric models (such as a GCM) (Ham et al., 2019). Yet, the augmented data from both sources often contain a range of uncertainties and inaccuracies when compared with the high-resolution satellite observations for characterizing the ENSO features. Therefore, developing a new ML training algorithm that systematically reduces the forecasting errors and uncertainties in the augmented training data set becomes essential for extending ENSO forecasting skills.

In this article, we develop a new Bayesian machine learning (BML) training algorithm that utilizes only short observational time series for an effective prediction of the ENSO. The focus here is on predicting the Niño 3 SST index, which is a commonly used ENSO index for characterizing the large-scale features of the eastern Pacific El Niño. A simple feedforward NN (Fine, 2006) is adopted as the prediction model. In this BML algorithm, a long simulation from a parametric model is used as the prior information for training the NN while the short observational data plays the role of the likelihood which corrects the intrinsic model error in the prior data (Bernardo & Smith, 2009; Box & Tiao, 2011) during the training process. The NN model trained using the BML framework outperforms the same NN model trained with the standard procedure and model-based ensemble predictions. Note that the new BML algorithm is adaptable to any NN architecture and any geophysical system with limited observations (Watson et al., 2017), provided that a reasonable approximate parametric model is in hand.

The rest of the article is organized as follows. The observational data sets are described in Section 2. A simple three-dimensional parametric model that is, utilized to generate the long time series as the prior information for training the NN is introduced in Section 3. The new BML training algorithm is developed in Section 4. The forecast results are shown in Section 5. The article is concluded in Section 6.

2. The Observational Data Sets

In this study, we use reanalysis data from satellite observations. The SST data is from the Optimum Interpolation Sea Surface Temperature (OISST) reanalysis (Reynolds et al., 2007) while the zonal winds are at 850 hPa from the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis (Kalnay et al., 1996). The thermocline depth is computed from the potential temperature as the depth of the 20°C isotherm using the NCEP Global Ocean Data Assimilation System (GODAS) reanalysis data (Behringer et al., 1998). The temporal resolution of all the data is daily and they cover the period from 1982/01/01 to 2020/02/29. The spatial resolutions are 0.25°, 1°, and 2.5°, respectively, for the SST, the thermocline depth, and the zonal winds. All data sets are averaged meridionally within 5°N-5°S in the tropical Pacific (120°E–80°W), which is followed by removing the climatology mean and the seasonal cycle.

The following three indices are derived from the above data sets: the averaged SST in the Niño 3 region (150–90°W; which is essentially the eastern Pacific) \( T_p \), the averaged thermocline depth in the western Pacific region (120°E–160°W) \( H_p \), and the averaged wind bursts over the western Pacific region \( \tau \).

The Niño 3 SST index \( T_p \) characterizes the eastern Pacific El Niños and rules out most of the central Pacific events (Di Lorenzo et al., 2010; Yeh et al., 2009). The main reason for choosing such a simple index as the prediction target is that the prior information of \( T_p \) is naturally obtained from the recharge-discharge paradigm (Jin, 1997), which facilitates the understanding of the BML algorithm. The BML algorithm can be easily applied to predict the Niño 3.4 index and the spatiotemporal evolutions of the ENSO.
3. A Simple Non-Gaussian Parametric Model for the Prior Information

The BML algorithm trains the NN model in a Bayesian framework, using a long time series generated from a (prior) parametric model. To this end, the parametric model and the associated time series are called the “prior parametric model” and the “prior data,” respectively. Note that this prior parametric model does not need to be perfect, as it is often the case in practice. Yet, it has to be reasonable in the following sense. While the prior model alone may not produce an effective predictive skill, it will provide a set of training data that reflects some qualitative features of the input variables and climatological statistics, which in turn, produces skillful forecasts when the neural-network model is trained using the proposed BML algorithm, as we shall see.

We utilize the following three-dimensional (3D) stochastic differential equations (SDEs) as the prior model:

\[
\frac{dT_E}{dt} = (-d_{\tau}T_E + \omega H_W + \alpha_T \tau) + \sigma_T \dot{W}_T, \tag{1a}
\]

\[
\frac{dH_W}{dt} = (-d_H H_W - \omega T_E + \alpha_H \tau) + \sigma_H \dot{W}_H, \tag{1b}
\]

\[
\frac{d\tau}{dt} = (-d_{\tau} \tau + \sigma_\tau(T_E) \dot{W}_\tau. \tag{1c}
\]

where, as was defined in Section 2, \( T_E, H_W \), and \( \tau \) represent the averaged SST in the eastern Pacific, the averaged thermocline depth in the western Pacific, and the averaged wind bursts in the western Pacific, respectively. The constants \( d_{\tau}, d_H \) and \( d_T \) are the damping coefficients while the constant \( \omega \) characterizes the oscillation frequency. The constants \( \alpha_T \) and \( \alpha_H \) are the coefficients that couple the wind bursts to the interannual variables. The two noise coefficients \( \sigma_T \) and \( \sigma_H \) are constants while the remaining noise coefficient \( \tau(T_E) \) is a function of \( T_E \). The terms \( \dot{W}_T, \dot{W}_H \) and \( \dot{W}_\tau \) are independent white noise. The parametric model in Equation 1 can be regarded as the recharge-discharge model (Jin, 1997) augmented by a random wind burst model. Note that the increased SST anomaly enhances the convective activity, which results in more active wind bursts (Hendon et al., 2007; Puy et al., 2016; Tziperman & Yu, 2007). Therefore, it is essential to adopt a state-dependent (i.e., multiplicative) noise \( \sigma_\tau \) in Equation 1c, which assumes that the wind burst is positively correlated with the SST anomalies. Since \( T_E \) is the only SST variable in Equation 1, it is used as an approximation to the basin-averaged SST that influences the wind burst amplitudes. The SDE (Equation 1c) can generate both the westerly wind bursts (WWBs) and the easterly wind bursts (EWBs), corresponding to \( \tau \) with positive and negative values, respectively. It is also this state-dependent noise that allows the coupled model (Equation 1) to generate non-Gaussian features of the observed ENSO. One time unit in the model stands for one month. The units of \( T_E \) and \( \tau \) are °C and m/s, respectively. On the other hand, one unit of \( H_W \) in the model corresponds to 15 m, which allows \( H_W \) and \( T_E \) to have similar amplitudes in the model simulation. The parameter values are determined by minimizing the errors between the prior model’s and the observed time series’ probability density functions and the autocorrelation functions. The estimated parameters, which we will refer to as the reference parameters in this paper, are given as follows,

\[
d_{\tau} = 1.5, \quad d_H = 4, \quad \omega = -1.5, \quad \alpha_T = 1, \quad \alpha_H = -0.4, \quad \sigma_T = \sigma_H = 0.8, \quad \text{and} \quad \sigma_\tau(T_E) = 4.5 \tanh(T_E + 1) + 4. \tag{2}
\]

As will be shown in Section 5.5, the BML forecasts are quite robust to perturbations of these parameters, which allow for a wide range of application of the BML algorithm in practice.

Figure S1 shows that the prior model (Equation 1) can reproduce the qualitative features of the Niño 3 SST index and the associated non-Gaussian statistics. Yet, as we shall see, the model alone cannot produce effective forecasts or more complex features. Consequently, NNs trained using the prior data alone will not exhibit the most effective prediction skill (See Section 5.4). This further motivates the development of a BML framework that combines the observational time series with the model simulation to reduce the model error and improve the forecast skill of the Niño 3 index.
4. A Bayesian Machine Learning (BML) Algorithm

We now discuss a BML algorithm which takes into account both the prior data described in Section 3 with the observation data described in Section 2. Let \( \mathbf{v}(t) \) be the prior data, which is a long time series from the prior parametric model (Equation 1), and \( \mathbf{u}(t) \) be the short observational time series. Here \( \mathbf{v}(t) \) and \( \mathbf{u}(t) \) can both be one-dimensional, representing the time series of \( T_e \), or they can be multi-dimensional time series containing any subset of the three variables \( T_e, H_k, \) and \( \tau \).

Denote by \( \theta \) the parameters in the NN and \( \theta_k \) the estimated parameters after the \( k \)-th iteration in the stochastic gradient descent (SGD) method (Bottou, 2010). Next, denote by \( \mathbf{x}_i^D \) and \( \mathbf{y}_i^D \) for \( i = 1, \ldots, N_D \), the input and output data, constructed from the prior time series \( \mathbf{v}(t) \), where each \( \mathbf{x}_i^D \) is a delay embedded time series of \( \mathbf{v}(t) \) with an appropriate delay embedding time and each \( \mathbf{y}_i^D \) is a corresponding forecast value. Denote by \( \mathbf{x}_i^O \) and \( \mathbf{y}_i^O \) for \( i = 1, \ldots, N_O \), the input and output data constructed from \( \mathbf{u}(t) \) in the same way that \( \mathbf{x}_i^D \) and \( \mathbf{y}_i^D \) are constructed from \( \mathbf{v}(t) \). Since the observational time series is much shorter than the prior time series, we have \( N_O \ll N_D \). Note that the input and output data will be further specified in Section 5.1.

Define two loss functions \( L^D \) and \( L^O \) in training the NN. The first loss function is exploited to provide a potential update to the parameters \( \theta \),

\[
L^D = \sum_i \| \mathbf{y}_i^D - \text{NN}(\mathbf{x}_i^D; \theta) \|^2 \tag{3a}
\]

\[
\theta_{k+1}^D = \theta_k^D - \eta_k^D \frac{\partial L^D}{\partial \theta} \bigg|_{\theta = \theta_k^D}, \tag{3b}
\]

where \( \| \cdot \| \) in Equation 3a is a given metric for computing the loss function, such as the mean-squared error. We should point out one can always define a computationally reasonable loss function with a metric that is, adequate for quantifying the error of the point estimator. The loss function in Equation 3a corresponds to the standard empirical least-squares regression in supervised learning.

The Equation 3b is the SGD for updating the parameters in the NN, where \( \eta_k^D \) is the standard learning rate. Note that the observational information is not involved in Equation 3; it only appears in the second loss function,

\[
L^O = \sum_i \| \mathbf{y}_i^O - \text{NN}(\mathbf{x}_i^O; \theta) \|^2, \tag{4}
\]

which is used to validate the proposed parameters formulated in Equation 3b. The BML training algorithm contains two steps in each iteration cycle for updating \( \theta \). Denote by \( \theta_k^* \) the current parameter estimate.

**Step 1: Proposal.** Generate a proposal \( \theta_{k+1}^D \) as a potential update by utilizing (3b) with \( \theta_k^D = \theta_k^* \). Note that only the prior information is used in this step. **Step 2: Validation.** Evaluate the likelihood function (Equation 4) on the proposal \( \theta_{k+1}^D \) obtained from Step 1. Then the resulting value of the loss function \( L^O(\theta_{k+1}^D) \) is compared with \( L^O(\theta_k^*) \). If the relationship \( L^O(\theta_{k+1}^D) \geq L^O(\theta_k^*) \), we reject the proposal and repeat Step 1 with \( \theta_{k+1}^* = \theta_k^* \). Otherwise, accept the proposal and let \( \theta_{k+1}^* = \theta_{k+1}^D \) and repeat Step 1. If the proposed parameters are rejected in the last 5 consecutive cycles, which means the proposed parameters fail to improve the loss function (Equation 4) (i.e., the likelihood based on the observational data), then the training process will be terminated.

We call this training procedure the BML algorithm since the two steps above are very similar to those of the Bayesian Markov chain Monte Carlo algorithm. We should point out that the resulting predictor is a point estimator (one neural-network model). Once the model is trained via BML, the prediction of the resulting NN model is no different than the standard ML testing procedure. It is worthwhile to point out that the proposed training algorithm is not similar to the well-known Bayesian neural networks (BNN) (Lampinen & Vehtari, 2001; MacKay, 1995) which has widely been used, such as in language modeling and sentiment analysis (Gal & Ghahramani, 2016; Mukhoti & Gal, 2018). The BNN aims to construct a posterior distribution of hyperparameters \( \theta \), from which one can employ an ensemble of NN models with \( \theta \) sampled from the estimated posterior distribution and subsequently uses the ensemble average as a point estimator or
ensemble variance to quantify uncertainties of the point estimator. Technically, BNN requires a specification of the prior distribution of the hyperparameter $\theta$ and employs the optimization on the observed data (so it is not designed to overcome a shortage of observation data).

The BML algorithm is especially useful when only short observations and an approximate parametric model that contains model error are available. Note that the loss function in Equation 4 is not directly used with the SGD to update $\theta$ as would be the case when using the observation datasets to train a NN. This is because the limited observational data results in large variance and, due to the bias-variance tradeoff (Geman et al., 1992), larger expected error, thereby degrading forecast skill. On the other hand, although training with a sufficient amount of prior data results in lower variance and a robust training algorithm, the model error in the prior data may result in biased estimation of $\theta$. The BLM algorithm attempts to de-bias this estimate by accepting/rejecting the proposed network parameters based on the value of the loss function on the observation data.

As a remark, the second step of the above algorithm, namely (Equation 4), can also be understood as a special validation procedure (El Naqa et al., 2018; Mello & Ponti, 2018) in the NN training process. Since it utilizes only observational data, we call it the “data-driven validation.” As will be seen in Section 5.4, the proposed BLM algorithm outperforms standard NN training and validation procedures in the case of ENSO forecasting.

## 5. Results

### 5.1. Setup

We employ a feedforward net with three hidden layers. The first two hidden layers have 32 tanh units each and the final hidden layer has 128 linear units. The output layer consists of 50 linear units. The network uses SGD to minimize the root-mean-square error (RMSE) with batch sizes of 128. In the following experiments, different input data are used, which may contain only a single variable or three variables $(T_E, H_W, \tau)$. Each input, corresponding to an $x^D_i$ in Section 4, takes into account a delay embedded time series. Here, the delay embedding time for $T_E$ and $H_W$ is the past 8 weeks, and that for $\tau$ is the past 8 days. Weekly data is used for $T_E$ and $H_W$ while daily data is used for $\tau$. The output contains only $T_E$. The $i$-th output $y_i^D$ represents the forecast value of $T_E$ at the lead time of the $i$-th week, where $i = 1, \ldots, 50$. As we stated before, $\{x_i^D, y_i^D\}$ will be constructed in the same way as $\{x_i^O, y_i^O\}$, except that the former consists of only the observed data, $u(t)$. The parameter $S$ in the BML is set to be $S = 20$. This parameter was determined by a quick test on validation data but the authors found that the $S = 15$ and $S = 25$ yield similar results so the performance of the BML algorithm using the suggested network is not too sensitive to the choice of $S = 20$.

In each experiment, the training data is generated by integrating the prior model (Equation 1) forward in time until there are approximately 30,000 weekly samples of $T_E$ and $H_W$ for training (with daily windburst values chosen appropriately to match current time values of $T_E$ and $H_W$ during training). This is done by integrating forward on a daily scale and keeping every seventh observation. The efficacy of the learning algorithm is checked using a two-fold test by splitting the observation data into two time periods: first using the 1983–1999 data as training and the 2001–2020 data as testing and then vice-versa. The data in the year 2000 is excluded to prevent the overlap between the training and testing periods. The overall prediction skill is measured by evaluating the normalized RMSE (NRMSE) and pattern correlation (Corr) between the truth and the predicted time series in the entire period (1983–2020 except the year 2000). The Corr and NRMSE are defined as:

$$\text{NRMSE} = 1 - \frac{\text{RMSE}}{\text{std}(u^O)}, \quad \text{where RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(u_i^f - u_i^O)^2}{n}}.$$  

$$\text{Corr} = \frac{\sum_{i=1}^{n}(u_i^f - \bar{u}^f)(u_i^O - \bar{u}^O)}{\sqrt{\sum_{i=1}^{n}(u_i^f - \bar{u}^f)^2\sum_{i=1}^{n}(u_i^O - \bar{u}^O)^2}}, \quad \text{for } n \geq 2.$$  

(5)
where $f_i$ and $o_i$ are the forecast and the truth, respectively, at time $t = t_i$. The time averages of the forecast and the true time series are denoted by $\bar{f}$ and $\bar{o}$ while $\text{std}(o)$ is the standard deviation of the truth. The NRMSE starts from $\text{NRMSE} = 1$ and loses its skill once it reaches the value $\text{NRMSE} = 0$, at which point the RMSE in the forecasted time series is equal to the standard deviation of the truth. The pattern correlation loses its skill when $\text{Corr} < 0.5$.

### 5.2. Forecasts Using Different Inputs

Figure 1 shows the forecast skill of different inputs, using the BML algorithm, and compares them to the classical approach of training the network for a fixed number of epochs.

The skill scores in Panels (a and b) indicate that the persistence forecast (purple curves) is skillful only up to 5 months. The traditional ensemble forecast (green curves) based on the 3D model (Equation 1) outperforms the persistence forecast and it remains effective for up to about 6 months. Yet, this purely
model-based ensemble forecast is far less skillful than the BML forecast. Even in the situation with only $T_E$ being the input (blue curves), the BML forecast remains skillful for up to nearly 7.5 months. This indicates the advantage of the ML forecast over the traditional ensemble forecast based on parametric models.

Next, if all the three variables $(T_E, H_W, \tau)$ are used as the input, then the BML forecast skill can be further extended from 7.5 to 9.5 months (red curves). The WWBs are known to be important triggering effects of the El Niño (Hu et al., 2014; Lian et al., 2014; Seiki & Takayabu, 2007; Thual et al., 2016) and this is reflected in the BML forecast shown here. Specifically, Panels (c and d) of Figure 1 indicate that the forecast with the additional input of the wind bursts can capture the timing of the strong El Niño (1987–1988 and 1997–1998) more accurately. It is important to note that since the wind bursts lie in the intraseasonal time scale, only the wind burst data during the past 8 days is used as the BML input, which is much shorter than the 8-week input data of $T_E$ and $H_W$. Such a multiscale feature for the input is crucial in facilitating the effective BML forecast.

We also found that the BML forecast with $(T_E, H_W)$ as input has about the same skill as the forecast where $T_E$ is the only input (see Figure S2). A plausible explanation for this result is as follows. According to the celebrated recharge-discharge theory (Jin, 1997), the variables $T_E$ and $H_W$ are the two components of a coupled system that forms an oscillator. Since the input of the BML contains the historic data up to the past 8 weeks, the information in $H_W$ has been characterized by such historic data of $T_E$ according to the Takens’ delayed embedding theorem (Takens, 1981). Thus, including the additional variable $H_W$ as the input provides little improvement for the BML forecast. Note that the use of both the current and the past data as the BML input shares a similar mechanism as the delayed oscillator theory (Battisti & Hirst, 1989; Suarez & Schopf, 1988), which involves only a single variable but contains the historical values in the differential equations.

5.3. Reducing the Spring Predictability Barrier

Figure 2 shows the forecasts starting from different months. Both the persistence and the ensemble forecast using the 3D model suffer from the so-called spring predictability barrier (Barnston et al., 2012; Duan & Wei, 2013). In particular, the forecast is quite challenging when starting from February, March, or April, where the skillful forecast only lasts for up to 4 months. The BML forecast with a single variable $T_E$ as the input slightly improves the forecast skill starting from February and March while the useful forecast starting from April is significantly extended to 9 months. The more improved forecast results are provided by the BML with $(T_E, H_W, \tau)$ being the input. In this case, the skillful forecast lasts for nearly 10 months starting from any time between February to August. In particular, the Corr remains above 0.75 at a lead time of 4 months when the starting time is boreal spring and the threshold of $\text{Corr} = 0.5$ is nearly uniform for different starting months. Since the wind accounts for a large portion of the uncertainty in the ENSO forecast, the improved results here are consistent with the viewpoints in (Mu et al., 2007; Yu et al., 2009) that exploited an advanced approach to study different initial uncertainties in affecting the ENSO forecast. Notably, the forecast skill starting from April is significantly improved using the BML. This finding supports the argument in (B. Kirtman et al., 2001; Duan & Wei, 2013) that indicates the forecast starting in this season is relatively easy and there is no notable spring predictability barrier phenomenon.

5.4. Comparison to Standard Machine Learning Training Procedures

Now we report more numerical experiments to get a better sense of the BML algorithm. All the experiments in this subsection employ the variables $(T_E, H_W, \tau)$ as the input. Note that the SGD can approach a local minimum of the loss function, which is generally nonconvex, so the initial value of $\theta$ is a source of uncertainty in the NN training. Another source of the uncertainty comes from the random realizations of the prior time series. To account for these uncertainties, we run the training and forecasting algorithm 10 times for each experiment with different initial guesses of $\theta$ and different realizations of the prior time series. The shading area in Figure 3 shows the 95% confidence interval of the forecast skill for each experiment.

Figure 3 compares the BML forecast (panel (b)) with the forecast of the same NN architecture trained in a standard fashion (panel (a)), using Equation 3 for a fixed number of epochs instead of the validation Step 2. The red curve in Panel (a) shows that the NN trained for a fixed number of epochs on only the prior time series results in a forecast that remains skillful for only 7.5 month, which is much shorter than the BML.
forecast (9.5 months) shown in Panel (b). Importantly, as is shown by the green curve in Panel (a), in the absence of the data-driven stopping criterion (Equation 4), even if the observational and the prior data are both included in Equation 3 for training the network, the resulting forecast skill has little improvement. These findings indicate the importance of using the limited observational data to validate the proposals using Equation 4. On the other hand, as anticipated, when the prior time series is replaced by the short observations in Equation 3 for updating $\theta$, the small amount of training data leads to an extremely unskillful forecast (the yellow curves).

One important issue is to understand the robustness of the proposed scheme to the choice of the validation set in applying the second step of BML. In Panel (c) of Figure 3, it is shown that if the short observations are replaced by a prior time series from a time interval independent from the one used in Equation 3, then the skillful forecast becomes less than 8.5 months (blue curve). Notably, the confidence interval associated with such an experiment has no overlap with that using the short observations as the validation set (red curve) at the Corr $= 0.5$ threshold, which indicates the statistical significance of the difference between these two methods. Another test is to first mix the long prior data and the short observational data, which is then followed by splitting the mixed data into the training and validation sets for Equations 3 and 4, respectively. However, since the prior data is much longer than the observations, such an approach (green curve) leads to essentially the same forecast result as the one without the observational data (blue curve). These facts
highlight the importance of the data-driven validation (Equation 4) that serves as the likelihood function in the BML framework.

### 5.5. Sensitivity of BML Algorithm Under Perturbations of the Prior Model

The prior time series in all the tests that we have shown so far were generated from the 3D parametric model (Equation 1) with the reference parameters (Equation 2). While the modeling error prohibits this model to reflect more complex features, such as the spatiotemporal patterns of the ENSO diversity, it qualitatively reproduces the characteristics of the Niño 3 SST time index (as shown in Figure S1). It is therefore critical to understand the sensitivity of the BML algorithm to the choice of prior models. With this goal in mind, we randomly perturb the reference parameters and check the forecast skills of the standard ML and the BML algorithms, when both models are trained using the training data from the perturbed prior models. We repeat the experiments with the perturbed parameters 10 times. In each experiment, we add random Gaussian noise to each of the reference parameters in Equation 2, where the random Gaussian noise has zero mean and variance 0.8. The model simulation with the perturbed parameters is illustrated in the supporting information.

In Panels (d and e) of Figure 3, we show the averaged skill score over the 10 outcomes for each experiment as well as the associated uncertainty (shading area), which mimic the scenario of the multi-model forecast.
except that each “model” here is a ML forecast, obtained based on training data generated by the prior model with different parameters. Panel (d) shows the results using the standard ML forecast algorithm (red curve). Due to the relatively large model error in the prior model, the averaged skill score using the prior time series as the training data is 5 months, which is only slightly better than the forecast using the short observational data for training (yellow curve). The associated uncertainty (red shading area) using the standard ML forecast is also quite large. The forecast results using the standard ML algorithm can be improved if the prior time series are concatenated with the so-called posterior time series. These posterior time series are obtained by using a recently developed sampling algorithm (N. Chen, 2020), which exploits data assimilation to sample a collection of time series based on the prior model and the short observations. See the supporting information for technique details. Using these posterior time series as the training data, the skillful forecast of the standard ML algorithm can be extended to about 6 months together with a reduction of the forecast uncertainty.

In contrast, despite the large model error in the prior model with the perturbed parameters, the forecast skill score using the BML, as is shown in the red curve in Panel (e) of Figure 3, is still skillful up to 7 months. Also, the uncertainty (red shading area) is significantly smaller than the uncertainty of the standard ML forecast, which indicates that the BML algorithm is robust. These merits are due to the data-driven validation (Equation 4), which significantly reduces the model error in the prior time series. One interesting finding is that even if one includes posterior time series obtained using a class of data assimilation schemes (see the supporting information for details) into the training data for updating $\theta$ of Equation 3 (blue curve in Panel (e)), the skill score remains almost the same as that using only the prior data in Equation 3. This is because the BML algorithm has already taken into account the combined information from the prior model and the observations through the two steps in the training procedure, which captures the information contained in the posterior time series with less uncertainty. This suggests another practical advantage of BML: it does not depend on posterior time series produced by a separate “data assimilation” scheme that may involve non-negligible computational efforts, depending on the complexity of the prior model.

6. Conclusion

In this article, a simple and efficient BML training algorithm, which exploits only short observational time series and an approximate parametric model, is developed to provide effective predictions of the Niño 3 SST index. The BML forecast significantly outperforms the model-based ensemble predictions and standard ML forecasts. It also overcomes the spring predictability barrier to a large extent. The BML algorithm allows a multiscale input consisting of both the SST and the wind bursts, which improved prediction skills over the forecasts where SST is the sole variable. Finally, the BML algorithm also reduces the forecast uncertainty and produces robust results even when the training data is generated by prior models that are significantly perturbed from its reference parameter.

The primary goal here is to provide a new and efficient training framework that facilitates a predictive ML model even when there is a small amount of available training data. The main idea is to supplement the training strategy with a set of computer-generated data from a model to reduce the large variance error usually present in an ML model trained using a standard training procedure. While the BML may not be better than the standard training procedure when a large data set becomes available, it is worthwhile to try and determine the length of observational data required so that the two methods are comparable. This is a complicated study to undertake since it may depend on the class of NN models and the choice of prior models. Unfortunately, in the context of SST prediction, such a study may not be conclusive due to the shortage of available data and unknown underlying dynamics. We plan to address this issue in a more controlled environment, where the underlying dynamics are known. Finally, while the framework is rather simple and can be used in any application where there are a small number of observations, the effectiveness of the method depends on the choice of prior model. In real applications, since the underlying dynamical model is not available, the main task is to identify an effective prior model. In the context of SST prediction with time series training data, we have shown the effectiveness of BML over standard training procedure using the SDE in Equation 1 as prior. For future work, we plan to inspect whether other variables, such as the tropical and extratropical precursors (H.-C. Chen et al., 2020; Boschat et al., 2013) can improve the SST forecast. It
is also interesting to use operational models as the prior to test the BML algorithm. Another natural future work using the new BML framework is to skillfully forecast spatiotemporal patterns of the ENSO diversity.

Data Availability Statement

The source codes and data are included in the supporting information for the purposes of review, but will be made available at (Gillani, 2021).

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