Dynamical origins of the community structure of multi-layer societies

Peter Klimek$^{1,†}$, Marina Diakonova$^{2,†}$, Víctor M. Eguíluz$^2$, Maxi San Miguel$^2$, Stefan Thurner$^{1,3}$

$^1$Section for Science of Complex Systems; CeMSIS; Medical University of Vienna; Spitalgasse 25; A-1090; Austria
$^2$Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB); E07122 Palma de Mallorca; Spain
$^3$IIASA, Schlossplatz 1; A 2361 Laxenburg; Austria

† These authors contributed equally to this work.

Social structures emerge as a result of individuals managing a variety of different social relationships. Societies can be represented as highly structured complex network systems. Here we study the dynamical origins of the specific community structures of a large-scale social multiplex network of a human society that interacts in a virtual world of a massive multiplayer online game. There we find substantial differences in the community structures of different social actions, represented by the various network layers in the multiplex. Community size distributions are either similar to a power-law or appear to be centered around a size of 50 individuals. To understand these observations we propose a voter model that is built around the principle of triadic closure. It explicitly models the co-evolution of node- and link-dynamics across different layers of the multiplex. Depending on link- and node fluctuation rates, the model exhibits an anomalous shattered fragmentation transition, where one layer fragments from one large component into many small components. The observed community size distributions are in good agreement with the predicted fragmentation in the model. We show that the empirical pairwise similarities of network layers, in terms of link overlap and degree correlations, practically coincide with the model. This suggests that several detailed features of the fragmentation in societies can be traced back to the triadic closure processes.

I. INTRODUCTION

Societies are organized dynamical patterns that emerge from the social actions of individuals. These arrange an array of different types of social relationships (e.g. friendship, marriage, co-workers, ...) to form stable groups, organizations, or institutions of various sizes. Each type of social relation defines a social network of its own. These networks are not independent of each other but co-evolve with the other networks in the society. Societies can be understood as the collection of these networks and can be represented as a dynamic, co-evolving multiplex network, i.e. a network where a set of nodes can be connected by links of more than one type [1–3]. The topological structures of these network layers can vary dramatically, depending on the type of the corresponding social interactions. For example, it has been shown that network layers corresponding to cooperative behavior can be characterized by high clustering, high reciprocity, and high link overlap, whereas layers encoding aggressive behavior exhibit pronounced power-law degree distributions [1 3]. It has also been shown that many characteristics of the individual multiplex layers, such as their degree distributions, clustering coefficients, or the probabilities for nodes to acquire new links, can be understood from the assumption that the link dynamics in networks is driven by the process of triadic closure (i.e. the tendency that nodes with common neighbors will establish links between themselves) together with a finite lifetime of links (i.e. the typical rate at they are added to and removed from the network) [3 4]. Another generic feature of social networks is that individuals tend to form communities, i.e. groups of nodes that share more links with each other than with nodes outside of the community [7 8]. There is evidence that the organization of community structure in human societies follows principles that are deeply rooted in human psychology, such as a hierarchically nested organization of communities of different sizes [9 10] or Dunbar’s number [11], a hypothesized upper cognitive limit to the number of people with whom humans can share stable social relationships.

In this work we want to understand if the empirical observations can be reminiscent of a so-called fragmentation transition. Fragmentation is the phenomenon in which a network might undergo a transition from a state where
almost all nodes belong to a single giant component to a \textit{fragmented} state in which the network breaks into many smaller components. To this end we propose a new type of voter model (VM). VMs introduced in \cite{15, 16} have repeatedly been shown to exhibit fragmentation transitions \cite{17, 18}. In the co-evolutionary VM (CVM) \cite{19} a node can either change its internal state to that of its neighbor or rewire one of its existing links towards a node that has the same internal state. Fragmentation transitions have been shown to be generic features of rewiring processes and largely independent of the underlying evolution rules for the internal states \cite{20, 21}. CVMs of this kind account for a wide class of phenomena ranging from opinion-formation \cite{22, 26} to speciation in ecosystems \cite{27, 28}. In an extension of the CVM to multiplex networks (MCVM) nodes have the same internal states in each layer and it is assumed that the driving force behind this transition is the asymmetry between rewiring rates in different network layers, i.e. the individual lifetime of links in each of the layers \cite{29}.

To reconcile the dynamics of the CVM with the empirical dominance of triadic closure, where a substantial amount of newly created links in social networks connect nodes that already share common neighbors \cite{1, 4, 30, 31}, we propose a novel type of VM in which the rewiring step is carried out by a triadic closure process, and call it the triadic closure voter model (TCVM). We study the dynamics of the TCVM on a single-layer and on a multiplex network (MTCVM). We show that the MTCVM displays a novel fragmentation transition that is again different from shattered fragmentation in the MCVM. We investigate whether the fragmentation behavior of the MTCVM is compatible with the community structure observed in \textit{Pardus} multiplex network data. To understand the extent to which the microscopic dynamics of the model multiplex is compatible with the data, we compare results for pairwise similarity measures of layers in terms of the properties of nodes (states) and links (overlap and degree).

\section{DATA AND METHODS}

\subsection{Data}

The Pardus dataset contains all actions of more than 380,000 players in a massive multiplayer online game. The players interact in an virtual, open-ended game universe to connect with other players to achieve self-posed goals, such as accumulating wealth and influence. Players can engage in three different types of cooperative interactions. They can establish mutual friendship links, exchange private, one-to-one messages and trade with each other in the game. The data can be represented as a dynamic multiplex network $M^\alpha(t)$ where the index $\alpha$ labels the adjacency matrix of the network given by interactions of type $\alpha$ at time $t$. The multiplex $M^\alpha(t)$ is constructed for each month (30 days) over one year of data, from Sep 2007 to Sep 2008. Two players are linked in a corresponding multiplex layer if they had a friendship link ($\alpha = \text{friend}$), traded with each other ($\alpha = \text{trade}$), or exchanged a private message ($\alpha = \text{communication}$) within a given month. For a particular $t$ we include all players that have at least one link in each of the multiplex layers. For more information on the topology and structure of the Pardus multiplex network see \cite{11, 14}. Per construction, each layer has the same average number of nodes $N = 3.1(0.2) \cdot 10^3$. Numbers in brackets denote standard deviations. Table \ref{tab:rewiring_rates} shows results for the average number of links in each layer, $L^\alpha$. The trade network shows the highest link density, the friendship and communication network have similar numbers of links.

It has been shown that for the given time-span the three network layers $\alpha$ are in a stationary state in the sense that links are added and removed with comparable rates. These rates are orders of magnitude larger than the rates at which nodes are added or removed, see \cite{6}. The dynamics within the network layers is therefore dominated by rewiring processes. The rewiring rate $p_\alpha$ is defined as the average value of the probabilities that a link will be added or removed, that is rewired, in layer $\alpha$. Table \ref{tab:rewiring_rates} shows results for the rewiring rates $p_\alpha$ in the three layers. $p_\alpha$ in the friendship network is orders of magnitude smaller than in the communication and trade networks. This can be understood by the difference in processes that govern the interactions in these layers. In the friendship network a link persists after it has been formed until the link is removed or one of the players leaves the game. In the message and communication network, on the other hand, a link is only formed between two players if at least one interaction took place within the considered time interval. This results in a substantially lower turnover of links in the friendship network than in the other layers. We will therefore refer to the friendship layer as being \textit{slow} and to the trade and message layers as being \textit{fast} in terms of the average time between two consecutive rewiring events in the given layer. Note that although friendship links have the longest survival time (i.e. lowest turnover $p_\alpha$), the friendship network has also the smallest number of links $L^\alpha$.

\subsection{Community detection}

From the network layers $M^\alpha(t)$, for data and model, we identify the communities $C^\alpha(t) = \{C^\alpha(1,t), C^\alpha(2,t), \ldots, C^\alpha(N^\alpha_c, t)\}$, where the $i$-th community $C^\alpha(i, t)$ is the set with a number of $n^\alpha_i(t)$ nodes within community $i$ at time $t$. We use the OSLOM community detection algorithm \cite{32}, which
TABLE I: Overview of characteristics of the Pardus multiplex network layers. For each layer \(\alpha\) we show the values of the rewiring probability \(p_\alpha\) and the average number of links \(L^\alpha\). The rewiring probability \(p_\alpha\) is orders of magnitude smaller for the friendship network, compared to communication and trade. Each layer has the same average number of nodes, \(N = 3.1(0.2) \cdot 10^3\).

| \(\alpha\) | friendship | trade | communication |
|---|---|---|---|
| \(p_\alpha\) | 0.004(1) | 0.27(1) | 0.35(2) |
| \(L^\alpha\) | \(1.5(0.2) \cdot 10^4\) | \(5.6(0.2) \cdot 10^4\) | \(1.9(0.3) \cdot 10^4\) |

retrieves only statistically significant communities and which does not suffer from the so-called resolution limit problem (failure of the detection of small communities) of other approaches \([33]\). It also allows to detect the absence of community structure as well as homeless nodes, that do not belong to any community. We use the OSLOM implementation as provided by the authors for unweighted networks with a coverage parameter of 1 and a standard significance threshold of \(p < 0.1\). \([32]\).

C. Components in the model

To understand the fragmentation behavior of the TCVM we describe the organization of the model-networks into components by the following observables. \(N^\alpha_c(t)\) is the number of components in network layer \(\alpha\) at time \(t\). We refer to the time average of \(N^\alpha_c(t)\) over \(T\) consecutive time-steps by dropping the dependence on \(t\), i.e. \(\bar{N}^\alpha_c \equiv \frac{1}{T} \sum_t N^\alpha_c(t)\). The size of the \(k\)-largest component at \(t\) is denoted \(S_k^\alpha(t)\). The time average of the \(k\)-largest component size, \(\bar{S}_k^\alpha\), is again denoted by dropping the dependence on \(t\).

D. Similarity measures

The similarity of the sets of links in two layers, \(M^\alpha(t)\) and \(M^\beta(t)\), is measured by the Jaccard coefficient \(J(\alpha, \beta, t)\). Let \(E^{\alpha(\beta)}(t)\) be the set of links in \(M^{\alpha(\beta)}(t)\). The Jaccard coefficient is given by \(J(\alpha, \beta, t) = \frac{|E^{\alpha(\beta)}(t)| \cap |E^{\alpha(\beta)}(t)|}{|E^{\alpha(\beta)}(t)| \cup |E^{\alpha(\beta)}(t)|}\). Let us further denote the degree sequence of \(M^{\alpha(\beta)}(t)\) by \(k^{\alpha(\beta)}(t)\) and the sequence of the ranks of the degrees by \(Rk(k^{\alpha(\beta)}(t))\). The degree correlation, \(\rho(k^{\alpha(\beta)}(t), k^{\alpha(\beta)}(t))\), is Pearson’s correlation coefficient between the degree sequences \(k^{\alpha}(t)\) and \(k^{\beta}(t)\). Similarly, the degree rank correlation, \(\rho(Rk(k^{\alpha}(t)), Rk(k^{\beta}(t)))\), is Pearson’s correlation coefficient between the degree rank sequences \(Rk(k^{\alpha}(t))\) and \(Rk(k^{\beta}(t))\). As before, we refer to the time averages of \(J(\alpha, \beta, t)\), \(\rho(k^{\alpha}(t), k^{\beta}(t))\), and \(\rho(Rk(k^{\alpha}(t)), Rk(k^{\beta}(t)))\), by dropping the time dependence in the variables, respectively.

III. RESULTS

A. Community structure in the Pardus multiplex

The time-averaged distribution functions for the community sizes in the three network layers are shown in figs. 1(A)-(D). There is a clear discrepancy between the distribution observed in the slow friendship layer when compared to the trade and message networks, that are characterized by substantially larger rewiring rates. For the friendship layer the frequencies of community sizes are clearly a decreasing function in size (roughly following a power-law), whereas there exist distinct peaks in the distributions of community sizes in the trade and message layers. In both fast layers these peaks are centered around a community size of 50.

B. The Triadic Closure Voter Model (TCVM)

The standard binary-state CVM on a single network as introduced in \([10, 19]\) is given as follows. Each node,
\(\sigma_i(t)\) is described as a time-dependent, binary, internal state \(\sigma_i(t) \in \{0, 1\}\) subject to the following update rule. (i) Pick node \(i\) and one of its neighbors, \(j\), at random. If the internal states of these nodes differ, \(\sigma_i(t) \neq \sigma_j(t)\), then (iia) with probability \(p\) the link between \(i\) and \(j\) is removed and a link between \(i\) and a different node \(k\) is formed. \(k\) is randomly chosen from the set of all nodes disconnected to, but in the same internal state as node \(i\), \(\sigma_i(t) = \sigma_k(t)\). If no such node \(k\) exists, the link is not re-wired. Otherwise, (iib) with probability \(1 - p\), the state of node \(i\) is changed to \(\sigma_j(t)\). This model has one parameter, the rewiring rate \(p\), which defines the preference of a node to re-wire the link over changing its state.

We now introduce the triadic closure voter model (TCVM) on a single network. The TCVM is motivated by the empirical fact that links that connect nodes that share common neighbors are more likely to form than links that do not connect such nodes, i.e. the process of triadic closure \([6, 29, 31]\). In the TCVM the rewiring step (iib) of the CVM is replaced by the following update rule. With the triadic closure probability, \(p_c\), the new link is made between node \(i\) and a different node \(l\) that is randomly chosen from the set of all nodes that share at least one common neighbor with \(i\) (but there is no connection between \(i\) and \(l\), yet). If no such node \(l\) exists, no rewiring takes place. Otherwise, with probability \(1 - p_c\), we follow the rewiring rule from step (iib) of the CVM.

The network is initialized as a random graph with size \(N\) and average degree \(\mu = 4\), which results in a network that is initially connected. The initial distribution of internal states is random, so that each node has the same probability \(1/2\) to be in one of the two possible states. Results for the dynamics and phase diagrams of the TCVM on single networks are discussed in the supplementary information, where it is shown that the system undergoes a fragmentation transition at a critical rewiring rate \(p_c\). For \(p < p_c\), the network freezes into one giant component with all nodes in the same state, i.e. the system reaches consensus. For \(p > p_c\), the network splits into two components of roughly equal size, but differing internal states (one component is composed of nodes with \(\sigma_i(t) = 0\), the other with \(\sigma_i(t) = 1\)). In this regime the internal states initially present in the system are preserved, but those who hold them become segregated from each other. Such fragmentation at high rewiring rates is typical for a range of dynamical and evolution rules \([21]\). For the single layer case, this fragmentation transition is largely independent of the triadic closure probability, \(p_c\). In the following we set \(p_c = 1\).

### C. The TCVM on multiplex networks (MTCVM)

We now study the TCVM on a multiplex network, the MTCVM, by joining two networks that both follow the update rules of the TCVM. The multiplex evolves by, first, picking a layer randomly and, secondly, by evolving that layer by using the update rules from the TCVM, see fig. 2. This introduces a co-evolution of the two layers because the formation of new links depends on the internal state of nodes which is the same (!) in both layers for each of the nodes. Each of the layers thus has its own re-wiring probability, \(p_1\) and \(p_2\), respectively.

The MTCVM displays a novel fragmentation transition that is visible in the phase diagrams of the observables \(S_1^c\) (size of largest component) and \(N_c^a\) (number of components). Fig. 3 shows the phase diagrams of (a) \(S_1^c\) and (b) \(N_c^a\) in layer 1 of the TCVM as a function of the rewiring probabilities \(p_1\) and \(p_2\). The results have been averaged over \(10^5\) realizations of the final (absorbing) configurations of networks of size \(N = 250\). Consider the case where layer 1 is connected to a static layer with \(p_2 = 0\). With increasing \(p_1\) the largest component of layer 1 shrinks to a value around 0.5, similar to the single-layer case. \(N_c\), on the other hand, increases by increasing \(p_1\) in the same way as \(S_1\) decreases. This means that with increasing \(p_1\) nodes leave the largest component and form small or isolated components of their own (and not a second, large component), i.e. \(N_c\) increases. This process has been called shattered fragmentation \([29]\). From the phase diagrams in fig. 3 follows by symmetry that in the static layer 2 the nodes remain in one giant component as layer 1 undergoes this shattered fragmentation.

Note that this fragmentation transition in the MTCVM is different from the fragmentation transition of the MCV. The MCV can be recovered by setting \(p_c = 0\) in both layers and by introducing an additional parameter, the multiplexity \(q\). In the MCV both layers contain \(N\) nodes, but only a fraction \(qN\) nodes exist in both layers, i.e. the internal state of the remaining \((1 - q)N\) nodes depends only on the dynamics in their own layer. The MCV also displays a shattered fragmentation transition that is, however, only encountered below a critical value of \(q\), \(q < q_c < 1\). Under triadic closure, partial multiplexing (i.e. \(q < 1\)) is no longer required to see shattered fragmentation.

![FIG. 2: Schematic representation of the coevolving voter model with triadic closure on a multiplex (MTCVM). We pick a layer at random (step 1) and evolve it according to the dynamics of the CVM with triadic closure (step 2). As a result the state of a node may change in both layers (step 3).](image-url)
The community size distributions obtained from the model agrees well with the data. In fig. 1 we show the frequency of community sizes observed from 1000 realizations using $N = 3100$ and the rewiring rates $p_\alpha$ of the two layers as measured for the respective layers in the data and compare these results to the data. Results are shown for the friendship-trade multiplex (top row), namely (A) friendship and (B) trade, and for the friendship-communication multiplex (bottom row), (C) friendship and (D) communication. In both model multiplexes the frequencies of community sizes decreases as a function of their size in strong resemblance to results observed in the data. For the trade and communication layers, respectively, we also observe a clear peak in the frequencies of community sizes that coincides with the peaks observed in the data around a community size of 50. Note that here we compare communities in the data to communities in the model (and not to components). Clearly, the fragmentation behavior into components in the model is closely related to its community structure. Given the rewiring probabilities $p_\alpha$ in the data, we would expect from the MTCVM that the friendship layer ($p_{\text{friend}} = 0.004(1)$) shows one large component whereas the trade ($p_{\text{trade}} = 0.27(1)$) and communication ($p_{\text{comm}} = 0.35(2)$) layers display fragmentation. Indeed, we observe for the communities in, both, data and model for the friendship layer a roughly power-law-like size distribution with a small number of very large communities, whereas the trade and message layers fragment into a large number of smaller communities with a peak centered at around 50. Data and model show the same behavior in terms of fragmentation into communities of various sizes.

To study whether the dynamics of the MTCVM is able to describe the observed similarities between pairs of network layers in the data, we introduce a calibrated version of the MTCVM. We model the friendship-trade and the friend-communication multiplexes with corresponding $p_\alpha$ and initial conditions that are given by snapshots of the respective data layers at $t_1 = 1000$. For the friendship-communication multiplex the absorbing state was typically reached after $t \sim 10^5$, the friendship-trade multiplex did not do so within any reasonable time, i.e. for at least several orders of magnitude of simulation time. Therefore the friendship-trade multiplex was compared to snapshots taken at times ranging over several orders of magnitude, namely at simulation times $t = 10^3, 10^4, 10^5$. In the data we found substantially higher levels of similarity in terms of the Jaccard coefficients, degree correlations, and rank degree correlations than in the model. This can be understood by the fact that the MTCVM does not contain any mechanism that explicitly increases the similarity of layers, such as by copying one link from one layer to the other. Such an inter-layer link correlation mechanism can be easily added to the MTCVM: In the rewiring step a probability $p_{\text{sim}}$ is introduced to re-wire the link to another node in the same state to which the node is already connected in the other layer. We find that by introducing even a small modification of this type ($p_{\text{sim}} = 0.1$) we are able to account for the observed values of the Jaccard similarity, degree correlations, and degree rank correlations, which are shown in fig. 1 for data and model. Although the friendship-trade multiplex has not reached its absorbing state in these simulations, the variance of the similarity measures shows that they do not considerably change over time. The model exhibits the same trends as seen in the data, with the overlap between data and model being higher in the friend-trade multiplex than the friend-message multiplex, in particular for the degree correlation and the degree rank correlation. This shows that the modified co-evolutionary dynamics of the MTCVM preserves the observed similarities between pairs of layers already for a very small probability $p_{\text{sim}}$ to “copy” a link from one layer to the other one. We have confirmed that this value of $p_{\text{sim}}$ is indeed small enough that the community size distributions are unaltered by this inter-layer link correlation modification. This link-copying mechanism has also no substantial impact on the phase diagrams of the
FIG. 4: Similarity between pairs of multiplex layers for data (blue) and model (red). We show the Jaccard coefficient of the edge sets of the friendship-communication multiplex (left) and the friendship-trade multiplex (right), together with the degree correlation $\rho(k_\alpha, k_\beta)$ and the degree rank correlation $\rho(Rk(k_\alpha), Rk(k_\beta))$. The model follows the similarities found in the data, with the overlap between data and model being higher in the friendship-trade multiplex than the friendship-communication multiplex.

model, since the mechanism is similar to a random linking process (i.e. a triadic closure probability, $p_{tc}$, smaller than one). As already discussed, the phase diagrams are largely independent of such small variations of $p_{tc}$.

IV. DISCUSSION

We investigated the dynamical origins of the community structure of societies represented as dynamical multiplex networks. In empirical data from the large-scale online game society Pardus we observed substantial differences in the community structures of individual network layers of this multiplex. While one layer is characterized by a small number of large communities and a power-law-like distribution of community sizes, in the other layers we find a peak of communities of intermediate size around 50. We found that the time-scales on which the link dynamics takes place in the various network layers differ by several orders of magnitude. Remarkably, we find that the power-law like distribution of community sizes is found in a layer with a very small re-wiring rate, whereas layers with the centered distribution are characterized by rates that are orders of magnitude higher.

To understand these empirical findings we proposed a generalization of the co-evolutionary voter model on multiplex networks which incorporates the process of triadic closure, the MTCVM. This process has been shown to be crucial in modeling the structure formation of individual layers in the Pardus society. We studied the phase diagram of the new model and found that it exhibits an anomalous fragmentation transition for multiplex networks that makes the model interesting in its own right. This transition is characterized by a break-up of the largest component of a network layer into a large number of small components. Intriguingly, the crucial parameters of the model turn out to be the differences of time-scales on which the link re-wiring dynamics takes place in the individual layers. When the model is calibrated to mimic the Pardus data on two different two-layer multiplex networks, community size distributions are perfectly compatible with those found in the data.

In particular the model confirms that slow layers in terms of the time-span between two re-wiring events, show a power-law-like distribution of community sizes, whereas the fast layers display an additional peak around community sizes of 50. This means that the empirical community structure of the Pardus virtual society indeed resembles the fragmentation behavior predicted by the MTCVM. Note that for these results the model layers only differ in their re-wiring rates (but not, for instance, in their degree), so that these results can only be attributed to the multiplex interaction of dynamics on different time-scales. We further confirmed for an extended and calibrated version of the MTCVM that node- and link-based similarities between two pairs of layers in the data are indeed in good agreement with results from the model. These results suggest that the dynamical origin of the community structure of societies can be understood through the interplay of triadic closure processes taking place on different time-scales, which manifests itself in the phenomenon of shattered fragmentation. Whether these results hold for empirical data in real-world societies remains to be seen.

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average of such times is identified with the characteristic time, i.e. the average time until absorption, for the same system (center panel), normalized by system size. The maximum (traced in dark circles) is associated with the fragmentation transition. Right panel: The variation of the maximum of characteristic time for various system sizes, computed from ensembles with $10^4$ elements.

V. SUPPLEMENTARY INFORMATION

1. Further Results on the Co-evolving Voter Model with Triadic Closure (TCVM)

Triadic closure in the monoplex. A finite monoplex with standard, unlimited rewiring range ($p_{tc} = 0$) always reaches an absorbing state. This holds for the TCVM model except for the extreme $p = 1, p_{tc} = 1$ case. Here, limiting the rewiring range introduces dynamical traps, where ‘active’ links between nodes in differing states continue to exist as they cannot be changed. This happens when the active link between $i$ and $j$ is also both $i^{th}$ and $j^{th}$ only active link. An isolated node pair in differing states is one such example. Clearly as long as $p_{tc} < 1$ or $p = 1$ this trap is avoided as either one of the nodes rewire to someone further away, or eventually one of the nodes changes states. Existence of dynamical traps is just one of the consequences of a limited rewiring range. Another implication is that once a node is isolated, it cannot be brought back into a component, and hence the number of isolated components is non-decreasing.

Activity in the model is measured by the interface density, i.e. the fraction of active links which are the ones connecting nodes in different states. Figure 5 shows the activity of the system in terms of asymptotic of the interface density averaged over surviving runs. When the interface density is zero the system is in an absorbing state. The $p_{tc} = 0$ limit corresponds to the CVM ([19]), where activity decreases with rewiring and falls to zero around $p_c$, defined in the limit of infinite systems. This critical rewiring denotes the absorbing transition and is accompanied by the critical slowing down of the time it takes for the system to reach an absorbing state. The average of such times is identified with the characteristic time, plotted in the middle panel of fig. 5. We associate its peak, $T_{max}$, with the finite-size approximation of the absorbing transition. Fig. 5 shows that increasing the fraction of triadic closure decreases activity, and brings down the critical rewiring $p_c$. In other words, limiting the range of rewiring causes systems to freeze for an even lower rewiring range. The right panel of fig. 5 which traces the effect of system size of $p_{tc}$, suggests that the absorbing transition does not exist when $p_{tc} = 1$; large systems rewiring with only triadic closure do not sustain a constant level of activity, and will always reach an absorbing state in finite time.

The corresponding fragmentation diagram, overlayed by the trend of $T_{max}$ computed earlier, (fig. 6) shows that for the vast majority of parameters in the TCVM the absorbing and the fragmentation transitions once more coincide: active systems ($p < p_c$) will reach an absorbing state with one giant component with all nodes in the same state, and frozen systems ($p > p_c$) will split into two components corresponding to the two initially present states (this is corroborated by examining the size of the second largest component, figure not shown). The novel aspect is the different nature of fragmentation observed for large values of $p_{tc}$, and maximized at $p_c$. There the decrease in the size of the largest component is no longer compensated only by the growth of the second largest component, but is accompanied by an increase in the number of small components that tend to be isolated nodes or pairs of isolated nodes. This is an example of shattered fragmentation: topologies with one or two giant components and a multiplicity of components of negligible size. This was first observed in the CVM on a multiplex ([20] for incomplete interlayer connectivity $q < 1$, and in ([21]) for the CVM on a monoplex with noise. Here we see that a similar effect can be caused by limiting the scope of rewiring.