Fast and slow two-fluid magnetic reconnection

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Abstract

We present a two-fluid magnetohydrodynamics model of quasi-stationary, two-dimensional magnetic reconnection in an incompressible plasma composed of electrons and ions. We found that there are two distinct regimes of slow and fast reconnection. The presence of these two regimes can provide a possible explanation for the initial slow build-up and the subsequent rapid release of magnetic energy frequently observed in cosmic and laboratory plasmas.

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1. Introduction

Magnetic reconnection is the physical process by means of which magnetic field lines join one another and rearrange their topology. Magnetic reconnection is believed to be the mechanism by which magnetic energy is converted into kinetic and thermal energy in the solar atmosphere, in the Earth’s magnetosphere and in laboratory plasmas [1–7]. Even though magnetic reconnection normally happens locally, in small dissipation regions, it frequently controls global dynamics and the overall energy balance of (astro)physical systems. As a result, theoretical understanding of magnetic reconnection is essential for understanding many other physical processes, such as the transport of charged particles and heat in laboratory and astrophysical plasmas, dynamo and turbulence in magnetized fluids and acceleration of charged particles.

Many reconnection-related physical phenomena observed in cosmic and laboratory plasmas exhibit a two-stage behavior. During the first stage, magnetic energy slowly builds up and gets stored in the system with relatively little reconnection occurring. The second stage is characterized by a sudden and rapid release of the accumulated magnetic energy due to a fast reconnection process. For example, a solar flare is powered by a sudden (on a timescale ranging from minutes to tens of minutes) release of magnetic energy stored in the upper solar atmosphere [4].

Because the value of the Spitzer electrical resistivity is very low in hot plasmas, magnetic energy release rates predicted by a simple single-fluid magnetohydrodynamics (MHD) description of magnetic reconnection are much lower than the rates observed during fast reconnection events in astrophysical and laboratory plasmas [1, 3–7]. One of the most promising solutions to this discrepancy is the two-fluid MHD theoretical approach to magnetic reconnection ([1, 4–7] and references therein). Recently, a model of two-fluid reconnection in an electron–proton plasma was presented in [8]. In this paper, we consider the more general case of two-fluid reconnection in electron–ion and electron–positron plasmas, and we present derivations in detail. In section 6 we also argue that the slow and fast reconnection regimes predicted by our model can provide a possible explanation for the observed two-stage reconnection behavior.

2. Two-fluid MHD equations

In this study, we use physical units in which the speed of light $c$ and four times $\pi$ are replaced by unity, $c = 1$ and $4\pi = 1$. To rewrite our equations in Gaussian centimeter–gram–second (CGS) units, we need to make the following substitutions: magnetic field $\mathbf{B} \rightarrow \mathbf{B}/\sqrt{4\pi}$, electric field $\mathbf{E} \rightarrow e\mathbf{E}/\sqrt{4\pi}$ and electric current $\mathbf{j} \rightarrow \sqrt{4\pi} \mathbf{j}/c$, electrical resistivity $\eta \rightarrow \eta c^2/4\pi$ and proton electric charge $e \rightarrow \sqrt{4\pi} e/c$.

We consider an incompressible two-component plasma, composed of electrons and ions. We assume that the plasma is non-relativistic and therefore quasi-neutral. The ions are assumed to have mass $m_i$ and electric charge $Ze$, while the electrons have mass $m_e$ and charge $-e$. Because of incompressibility, the electron and ion number densities are...
constant:
\[ n_e \equiv n = \text{const}, \quad n_i = Z^{-1}n = \text{const}, \] (1)
where the last formula follows from the plasma quasi-neutrality condition \( \sum_{z} e_n z = 0 \). The plasma density \( \rho \), the electric current \( \mathbf{j} \) and the plasma (center-of-mass) velocity \( \mathbf{V} \) are
\[ \rho = m_e n_e + m_i n_i = n(Z^{-1}m_i + m_e) = \text{const}, \] (2)
\[ \mathbf{j} = Z e_n \mathbf{u}^e - e_n \mathbf{u}^i = ne(\mathbf{u}^e - \mathbf{u}^i), \] (3)
\[ \mathbf{V} = (m_e n_e \mathbf{u}^e + m_i n_i \mathbf{u}^i)/\rho = n(Z^{-1}m_i \mathbf{u}^i + m_e \mathbf{u}^e)/\rho. \] (4)
Here, \( \mathbf{u}^e \) and \( \mathbf{u}^i \) are the mean electron and ion velocities, which can be found from the above equations:
\[ \mathbf{u}^e = \mathbf{V} - (m_i/Z \mathbf{e} \mathbf{p}) \mathbf{j}, \quad \mathbf{u}^i = \mathbf{V} + (m_e/e \mathbf{p}) \mathbf{j}. \] (5)
The equations of motion for the electrons and ions are [9, 10]
\[ n_e m_e [\partial_t \mathbf{u}^e + (\mathbf{u}^e \nabla) \mathbf{u}^e] = -\nabla P_e - n_e e(\mathbf{E} + \mathbf{u}^e \times \mathbf{B}) - \mathbf{K}, \] (6)
\[ n_i m_i [\partial_t \mathbf{u}^i + (\mathbf{u}^i \nabla) \mathbf{u}^i] = -\nabla P_i + n_i Ze(\mathbf{E} + \mathbf{u}^i \times \mathbf{B}) + \mathbf{K}. \] (7)
where \( P_e \) and \( P_i \) are the electrons and ion pressure tensors, and \( \mathbf{K} \) is the resistive frictional force due to electron–ion collisions. The force \( \mathbf{K} \) can be approximated as [9, 10]
\[ \mathbf{K} = n^2 e^2 \eta (\mathbf{u}^e - \mathbf{u}^i) = -ne\eta \mathbf{j}, \] (8)
where \( \eta \) is the electrical resistivity, and we use equation (3). For simplicity, we assume isotropic resistivity, and we also neglect ion–ion and electron–electron collisions and the corresponding viscous forces. Substituting equations (1), (5) and (8) into equations (6) and (7), we obtain
\[ nm_e [\partial_t \mathbf{V} + (\mathbf{V} \nabla) \mathbf{V}] - (nm_e m_i / Z \mathbf{e} \mathbf{p}) [\partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j}] + (\mathbf{V} \nabla) [\nabla P_e - ne \mathbf{E} - ne \mathbf{V} \times \mathbf{B} + (m_i n_i / Z \mathbf{p}) \times \mathbf{B} + ne \eta \mathbf{j}], \] (9)
\[ Z^{-1} nm_i [\partial_t \mathbf{V} + (\mathbf{V} \nabla) \mathbf{V}] + (nm_e m_i / Z \mathbf{e} \mathbf{p}) [\partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j}] + (\mathbf{V} \nabla) [\nabla P_i + ne \mathbf{E} + ne \mathbf{V} \times \mathbf{B} + (m_i n_i / \rho) \times \mathbf{B} - ne \eta \mathbf{j}]. \] (10)
We sum equations (9) and (10) together and obtain the plasma momentum equation
\[ \rho [\partial_t \mathbf{V} + (\mathbf{V} \nabla) \mathbf{V}] + (m_e m_i / Z \mathbf{e} \mathbf{p}) [\partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j}] = -\nabla P + \mathbf{j} \times \mathbf{B}, \] (11)
where \( P = p_e + p_i \) is the total pressure. Next we subtract equation (10) multiplied by \( Z m_e / m_i \) from equation (9) and obtain the generalized Ohm’s law
\[ \mathbf{E} = \eta \mathbf{j} - \mathbf{V} \times \mathbf{B} + (m_i / Z \mathbf{e} \mathbf{p}) [(1 - Z m_e / m_i) \times \mathbf{B} - (m_e / Z \mathbf{e} \mathbf{p}) \nabla P_e - (Z m_e / m_i) \nabla P_i] + (m_e m_i / Z \mathbf{e} \mathbf{p}^2) \left[ \partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j} + (\mathbf{V} \nabla) \mathbf{V} - (m_e / Z \mathbf{e} \mathbf{p}) (1 - Z m_e / m_i) (\mathbf{V} \nabla) \mathbf{j} \right]. \] (12)
It is convenient to introduce the ion and electron inertial lengths
\[ d_i \equiv (m_i / n_e Z^2 e^2)^{1/2} = (m_i / Z n e^2)^{1/2}, \] (13)
\[ d_e \equiv (m_e / n_e e^2)^{1/2} = (m_e / n e^2)^{1/2} \leq d_i, \]
and constants
\[ \omega_e^2 \equiv (1 + Z m_e / m_i)^{-1} = (1 + d_i^2 / d_e^2)^{-1}, \] (14)
\[ \omega_i^2 \equiv 1 - Z m_e / m_i = 1 - d_i^2 / d_e^2 \geq 0. \]
Here we consider a physically relevant case of \( Z m_e \ll m_i \), so that \( d_e \ll d_i \), \( 0 < \omega_e^2 < 1 \) and \( 1/2 < \omega_i^2 < 1 \). Note that \( \omega_e^2 \approx \omega_i^2 \approx 1 \) in the case of electron–ion plasma \( (Z m_e \ll m_i) \), and \( \omega_e^2 = 1/2 \) and \( \omega_i^2 = 0 \) in the case of positron–electron plasma \( (Z = 1 \text{ and } m_i = m_e) \).
Using definitions (13) and (14), we obtain for the plasma density (2) expression
\[ \rho = m_e n_e + Z \omega_i^2 \equiv n^2 e^2 d_i^2 / \omega_i^2, \] (15)
and we rewrite the plasma momentum equation (11) and Ohm’s law (12) as
\[ \rho [\partial_t \mathbf{V} + (\mathbf{V} \nabla) \mathbf{V}] + \omega_i^2 \omega_e^2 \left[ \partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j} + (\mathbf{V} \nabla) \mathbf{V} - (\omega_i^2 / n e)(\mathbf{V} \nabla) \mathbf{j} \right] = -\nabla P + \mathbf{j} \times \mathbf{B}, \] (16)
\[ \mathbf{E} = \eta \mathbf{j} - \mathbf{V} \times \mathbf{B} + (\omega_i^2 / n e) \mathbf{j} \times \mathbf{B} - (\omega_i^2 / n e)(\mathbf{V} \nabla) \mathbf{j} \]
\[ + \omega_i^2 \omega_e^2 \left[ \partial_t \mathbf{j} + (\mathbf{V} \nabla) \mathbf{j} + (\mathbf{V} \nabla) \mathbf{V} - (\omega_i^2 / n e)(\mathbf{V} \nabla) \mathbf{j} \right]. \] (17)
It is noteworthy that the electron inertia terms, proportional to \( d_i^2 \), enter both Ohm’s law and the momentum equation. Although these terms are important for fast two-fluid reconnection (as we shall see below), they have been frequently neglected in the momentum equation in the past. In addition, we note that \( \nabla \cdot \mathbf{B} = 0 \) and also \( \nabla \cdot \mathbf{V} = 0 \) and \( \nabla \cdot \mathbf{j} = 0 \) for incompressible and non-relativistic plasmas.
For the convenience of the presentation, below we will refer to the plasma as being electron–ion, even though, unless otherwise stated, our derivations in the next two sections are valid for reconnection in an electron–positron plasma as well.

3. Reconnection layer

We consider two-fluid magnetic reconnection in the classical two-dimensional Sweet–Parker–Petschek geometry, which is shown in figure 1. The reconnection layer is in the \( x \)-\( y \) plane with the \( x \) - and \( y \) -axis perpendicular to and along the reconnection layer, respectively. The \( z \) derivatives of all physical quantities are zero.

1 For particle species \( r \in \{e, i \} \), we use the standard definition of the pressure tensor as the density times the second moment of the particles velocity fluctuations relative to the mean velocity, \( P_r = n_r m_r (v^2 - \langle v^2 \rangle)(v^2 - \langle v^2 \rangle) \), where \( \mathbf{u} = (v^2) \) [9]. Instead, one could use velocity fluctuations relative to the plasma center-of-mass velocity (4) and define pressure as \( P_r = n_r m_r (v^2 - \langle v^2 \rangle)(v^2 - \langle v^2 \rangle) \) [10]. In this case, the total pressure tensor would be \( P = P_e + P_i = P + \omega_i^2 \omega_e^2 \mathbf{j} \), and therefore the electron inertia term \( \omega_i^2 \omega_e^2 (\mathbf{V} \nabla) \mathbf{j} \) in the momentum equation (16) would be absorbed into the pressure term \( \nabla P \). However, note that pressure \( P \) is strongly anisotropic.
The approximate thickness of the reconnection current layer is \(2\delta\), which is defined in terms of the out-of-plane current \((j_z)\) profile across the layer. The approximate length of the out-of-plane current \((j_z)\) profile along the layer is defined as \(2L\). Outside the reconnection current layer the electric currents are weak, the electron inertia is negligible, Ohm’s law (17) reduces to \(\mathbf{E} = -\nabla \times \mathbf{B} / ne - \mathbf{u}^2 / \mathbf{B}\) (in the case of electron–ion plasma, \(\omega_e^2 \approx \omega_i^2 \approx 1\)) and therefore the magnetic field lines are frozen into the electron fluid. Thus, \(2\delta\) and \(2L\) are also approximately the thickness and the length of the electron layer, where electron inertia is important and the electrons are decoupled from the field lines. The ion layer, where the ions are decoupled from the field lines, is assumed to have thickness \(2\Delta\) and length \(2L_{\text{ext}}\), which can be much larger than \(2\delta\) and \(2L\), respectively. The values of the reconnecting field in the upstream regions outside the electron layer (at \(x \approx \delta\)) and outside the ion layer (at \(x \approx \Delta\)) are almost the same: \(B_j \approx B_{\text{ext}}\) up to a factor of order unity. This result follows directly from the definition of \(2\delta\), and from the z-component of Ampere’s law, \(B_j(x, y = 0) = \int_0^\infty j_z(x', y = 0)\, dx'\). The out-of-plane field \(B_j\) is assumed to have a quadrupole structure (see figure 1) [5–7].

The reconnection layer is assumed to have a point symmetry with respect to its geometric center \(O\) (see figure 1) and reflection symmetries with respect to the \(x-\) and \(y\)-axis. Thus, the \(x, y\)- and \(z\)-components of \(\mathbf{V}, \mathbf{B}\) and \(\mathbf{j}\) have the following symmetries: \(V_x(\pm x, \mp y) = \pm V_x(x, y), V_y(\pm x, \mp y) = \mp V_x(x, y), V_z(\pm x, \mp y) = \mp V_x(x, y), B_x(\pm x, \mp y) = \pm B_x(x, y), B_y(\pm x, \mp y) = \mp B_x(x, y), B_z(\pm x, \mp y) = \mp B_x(x, y)\) and \(j_x(\pm x, \mp y) = j_z(x, y), j_y(\pm x, \mp y) = j_z(x, y)\). The derivations below extensively exploit these symmetries and are similar to the derivations in [8, 11, 12].

We make the following assumptions for the reconnection process. Firstly, resistivity \(\eta\) is assumed to be constant and very small, so that the characteristic Lundquist number \(S\) is very large:

\[
S = V_A L_{\text{ext}}/\eta \gg 1, \quad V_A \equiv B_{\text{ext}}/\sqrt{\eta}.
\] (18)

Here \(V_A\) is the Alfvén velocity. Secondly, the reconnection process is assumed to be quasi-stationary (or stationary), so that we can neglect time derivatives in the equations above and in the derivations below. This assumption is satisfied if there are no plasma instabilities in the reconnection layer, and the reconnection rate is low sub-Alfvénic, \(E_c \ll V_A B_{\text{ext}}\). Thirdly, we assume that the reconnection layer is thin, \(\delta \ll L\).

We use Ampere’s law and neglect the displacement current in a non-relativistic plasma to find the components of the electric current

\[
j_z = \partial_t B_z, \quad j_y = -\partial_z B_x, \quad j_z = \partial_t B_x - \partial_x B_z. \tag{19}
\]

The z-component of the current at the central point \(O\) (see figure 1) is

\[
j_o = (j_z)_o = (\partial_z B_x - \partial_x B_z)_o \approx (\partial_z B_x)_o \approx B_{\text{ext}}/\delta, \tag{20}
\]

where we use the estimates \((\partial_x B_x)_o \ll (\partial_z B_x)_o\) and \((\partial_z B_x)_o \approx B_{\text{ext}}/\delta\) at the point \(O\). The last estimate follows directly from the definition of \(\delta\) as being the half-thickness of the out-of-plane current profile across the reconnection layer.

In the case of a quasi-stationary two-dimensional reconnection, we neglect time derivatives, and Faraday’s law \(\nabla \times \mathbf{E} = -\partial_t \mathbf{B}\) for the \(x-\) and \(y\)-component of the magnetic field results in equations \(\partial_t E_x = -\partial_y B_x = 0\) and \(\partial_t E_y = -\partial_x B_y = 0\). Therefore, \(E_z\) is constant in space, and from the \(z\)-component of the generalized Ohm’s law (17) we obtain

\[
E_z = \eta j_z - V_x B_y + V_y B_x + (\omega_e^2/\rho_e)(j_y B_x - j_x B_y) + \omega_e^2 \rho_e^{-1/2} \left( V_{x} \partial_{y} j_{x} + V_{y} \partial_{x} j_{y} + j_{y} \partial_{x} V_{x} + j_{x} \partial_{y} V_{y} \right) + \left( \omega_e^2/\rho_e \right)(j_z \partial_{x} j_{y} + j_{y} \partial_{x} j_{z}) = \text{const}. \tag{21}
\]

The reconnection rate is determined by the value of \(E_z\) at the central point \(O\), that is,

\[
E_z = \eta j_o. \tag{22}
\]

We see that the electric field is balanced only by the resistive term \(\eta j_o\) at the central point \(O\); this is because we assume isotropic pressure tensors in this study. To estimate \(j_o\), in what follows we neglect time derivatives for a quasi-stationary reconnection and we use the symmetries of the reconnection layer.

The \(z\)-component of the momentum equation (16) is

\[
\rho V_x \partial_x V_x + V_y \partial_y V_x + \omega_e^2 \rho_e^{-1/2} \left( j_y \partial_x j_x + j_x \partial_y j_x \right) = j_x B_y - j_y B_x. \tag{23}
\]

Taking the second derivatives of this equation with respect to \(x\) and \(y\) at the point \(O\), we obtain

\[
\rho \left( \partial_x V_x \right)_o (\partial_x^2 V_x)_o + \omega_e^2 \rho_e^{-1/2} \left( \partial_y j_x \right)_o (\partial_{xx} j_x)_o = (\partial_x j_x)_o (\partial_x B_y)_o, \quad \rho \left( \partial_y V_x \right)_o (\partial_x^2 V_x)_o + \omega_e^2 \rho_e^{-1/2} \left( \partial_x j_x \right)_o (\partial_{xx} j_x)_o = -(\partial_x j_x)_o (\partial_x B_y)_o.
\]

Therefore,

\[
(\partial_x V_x)_o = - (\partial_x B_y)_o (\partial_x^2 V_x)_o + \omega_e^2 \rho_e^{-1/2} (\partial_{xx} j_x)_o/\rho (\partial_y V_x)_o, \quad (\partial_y V_x)_o = (\partial_x B_y)_o (\partial_x^2 V_x)_o + \omega_e^2 \rho_e^{-1/2} (\partial_{xx} j_x)_o/\rho (\partial_y V_x)_o.
\]

where we use equations (19) and the plasma incompressibility relation \(\partial_y V_x = -\partial_x V_y\).
Next, we substitute the second derivatives of equation (21) with respect to $x$ and $y$ at the central point $O$ and obtain

$$0 = \eta (\partial_{xx} j_y) - 2 (\partial_y V_y) - (\omega^2 B_x^2) \frac{z}{ne}(\partial_{xx} j_x) + (\partial_{yy} j_y) \frac{z}{ne}(\partial_{xx} V_x).$$

Substituting expressions (23) into these equations and using equations (15) and (19) and $\partial_y V_y = -\partial_x V_x$, we obtain

$$-\eta (\partial_{xx} j_y) = 2 (\partial_y V_y) - \omega^2 d^2 e^2 (\partial_{xx} j_x) \times \left[1 + \frac{x}{\tilde{\gamma} (\omega^2 - d^2 \tilde{\gamma} / d^2 y)}\right],$$

$$-\eta (\partial_{yy} j_y) = 2 (\partial_y V_y) + \omega^2 d^2 e^2 (\partial_{yy} j_y) \times \left[1 + \frac{z}{\tilde{\gamma} (\omega^2 - d^2 \tilde{\gamma} / d^2 y)}\right].$$

where we introduce a useful dimensional parameter

$$\tilde{\gamma} = \frac{\omega^2 e^2 (\partial_y V_y)}{ne\eta (\partial_y V_y)}.$$

In the case of electron-ion plasma ($Zm_e \ll m_i$ and $\omega_i \approx \omega_e \approx 1$), the parameter $\tilde{\gamma}$ represents the relative strength of the Hall term ($J \times B_x$) per unit electron density and the ideal MHD term ($V_x \times B_x$) inside the electron layer.

Taking the ratio of equations (24) and (25), we obtain

$$\frac{\partial_y B_x}{\partial_y V_y} \approx \frac{B_{ext} e^{-2}}{L^2} (1 + 2 \omega^2 d^2 e^2 / \delta^2),$$

where we use the estimates $(\partial_{xx} j_x) \approx -j_0 / \delta^2$ and $(\partial_{yy} j_y) \approx -j_0 / L^2$, and equation (20).

In equation (21), the electric field $E_z$ is balanced by the ideal MHD and Hall terms outside the electron layer, where the resistivity and electron inertia terms are insignificant. Therefore,

$$E_z \approx -V_x B_z \left[1 - \omega^2 (\partial_{yy} j_y) / ne j_x / V_y\right]$$

$$\approx (\partial_y V_y) (\partial_y V_y) \frac{d}{\delta^2} (1 + \omega^2 \tilde{\gamma}),$$

$$E_z \approx V_x B_z \left[1 - \omega^2 (\partial_{yy} j_y) / ne j_x / V_y\right]$$

$$\approx (\partial_y V_y) (\partial_y V_y) \frac{d}{\delta^2} (1 + \omega^2 \tilde{\gamma}).$$

where at the points $(x \approx \delta, y = 0)$ and $(x = \delta, y \approx L)$, respectively. Here we use the estimates $j_x \approx (\partial_y V_y) / \delta, j_y \approx - (\partial_y V_y) L, V_x \approx - (\partial_y V_y) \delta, V_y \approx (\partial_y V_y) L, B_z \approx B_{ext}$, and equation (26). The ratio of equations (28) and (29) gives

$$\frac{(\partial_y B_z)_o}{\partial_y V_y} \approx \frac{B_{ext} e^{-2}}{L^2} \approx \frac{B_{ext} e^{-2}}{j_x L^2},$$

where we use equation (20). Comparing this estimate with equation (27), we find $\delta \approx \omega_e d_e \approx d_e$. Therefore, using equation (20), we obtain

$$j_e \approx B_{ext} / d_e.$$ 

and $E_z \approx \eta B_{ext} / d_e$ \[13\]. The estimate $B_z \approx \frac{\partial_y B_z}{\partial_y V_y} L \approx B_{ext} e^{-2} / L$ for the value of the perpendicular magnetic field is in agreement with the geometrical configuration of the magnetic field lines inside the electron layer of thickness $\delta$ and length $L$.

Combining equations (20), (22) and (28), we obtain

$$\eta j_e^2 \approx \frac{(\partial_y V_y) B_{ext}^2 (1 + \omega^2 \tilde{\gamma})}{\delta}.$$

This equation describes the conversion of the magnetic energy into Ohmic heat inside the electron layer with rate $\approx |(\partial_y V_y)| \approx |(\partial_y V_y) - \omega^2 j_x / ne | \approx \frac{\partial_y V_y}{(1 + \tilde{\gamma})}$ in the case of electron-ion plasma ($\omega_i \approx 1$) \footnote{In the case of electron-ion plasma, in the upstream region outside the electron layer the magnetic field lines are frozen into the electron fluid and inflow with the electron velocity $v_e$.}, and with rate $\approx |(\partial_y V_y)| \approx \frac{\partial_y V_y}{(1 + \tilde{\gamma})}$ in the case of electron–positron plasma ($\omega_p \approx 0$).

Next, we use the z-component of Faraday’s law, $\partial_y E_y - \partial_x E_z = -\partial_t B_z = 0$, where the time derivative is set to zero because we assume that the reconnection is quasi-stationary. We substitute $E_x$ and $E_z$ into this equation from Ohm’s law (17) and, after tedious but straightforward derivations, we obtain

$$\eta (\partial_y j_y - \partial_x j_x) + \omega_e (\omega_e e^2 / ne) (B_z j_x + B_x j_y)$$

$$+ V_x B_z + V_y (\partial_x B_z) - B_z \partial_x V_z - B_y \partial_y V_z$$

$$+ \omega^2 d^2 e^2 (V_x (\partial_{xx} j_x - \partial_{yy} j_y) + V_y (\partial_{yy} j_y - \partial_{yy} j_x))$$

$$+ j_x \partial_y V_y - \partial_y V_y (\partial_y V_y - \partial_y V_x)$$

$$+ j_y \partial_x V_x - \partial_x V_x (\partial_x V_x - \partial_x V_y)$$

$$\approx \frac{\eta e}{(\partial_y V_y) \tilde{\gamma} / \omega^2 \delta^2}$$

$$- \left(\frac{\omega^2 - d^2 \tilde{\gamma} / d^2 y}{d^2 e^2 / ne}\right) \frac{j_e}{L^2} (\partial_y V_y + \partial_x V_x) \lt / \delta^2.$$

Taking the $\partial_y$ derivative of this equation at the central point O and using equations (19) and (23), we obtain

$$0 = -\eta \left[\partial_{xx} B_z, \partial_{yy} B_z\right] + \omega^2 e^2 / ne \left[(\partial_y B_x) \partial_y j_y + (\partial_y B_y) \partial_y j_y \right]$$

$$\approx \frac{\eta e}{(\partial_y V_y) \tilde{\gamma} / \omega^2 \delta^2}$$

\[33\]

To derive the final expression, we use equation (26) and the estimates $(\partial_{xx} B_z, \partial_{yy} B_z) \approx (\partial_{xx} B_z, \partial_{yy} B_z) / \delta^2 \approx (\partial_{xx} B_z, \partial_{yy} B_z) \approx - j_0 / \delta^2$, $(\partial_{yy} j_y) \approx - j_0 / L^2$, and $(\partial_y B_y) \approx j_0$. Using equations (15), (18), (20) and (30), we rewrite equation (33) as

$$\omega^2 - d^2 \tilde{\gamma} / d^2 y \approx \eta L^2 (\partial_y V_y) \tilde{\gamma} / \omega^2 d^2 e^2 V_x^2.$$

Note that equations (33) and (34) result in

$$0 \leq \tilde{\gamma} \leq \omega^2 d^2 e^2 / d^2 y.$$

Equation (16) for the plasma (ion) acceleration along the reconnection layer in the $y$-direction gives

$$\rho (V V_y) = \omega^2 d^2 e^2 / \omega (\partial_y V_y) j_y = -\partial_x P + j_x B_x - j_x B_z.$$

Taking the $y$ derivative of this equation at the central point O and using equations (15), (19) and (26), we obtain

$$\rho (\partial_y V_y) \approx (1 + d^2 \tilde{\gamma} / d^2 y) \approx B_{ext}^2 / L^2 + j_0 (\partial_y B_x) / L^2.$$
In the derivation of this equation, we use the estimate \((\partial_y P) \approx (\partial_y B^2 / 2) \simeq -B^2 \partial_y L^2 / 2\), which reflects the fact that the pressure drop is approximately equal to the drop in the external magnetic field pressure. This estimate follows from the force balance condition for the slowly inflowing plasma across the layer, in analogy with the Sweet–Parker derivations\(^5\). Using equations (18) and (30) and neglecting factors of order unity, we rewrite equation (37) as
\[
(\partial_y V_y)_0 \approx (V_A/L)(1 + d^2 \gamma / d^2 y)^{-1 / 2}.
\]

Now we note that on the y-axis \((x = 0)\), equation (36) reduces to \(\rho V_y \partial_y V_y = -\omega \delta^2 / \delta_j \partial_y \delta_j - \partial_y P + j_y B_y\). We integrate this equation from the central point \(O\) to the downstream region outside of the ion layer, \(x = 0\) and \(y \approx L_{ext}\), where ideal MHD applies and \(j_y \approx 0\). The plasma inertia term \(\rho V_y \partial_y V_y\) integrates to \(\rho V^2 / 2 = (1 / 2) (B_{ext} V_y / V_y)^2\), the electron inertia term \(\omega \delta^2 / \delta_j \partial_y \delta_j\) integrates to zero, the pressure term \(-\partial_y P\) integrates to \(\approx B^2_{ext}/2\), and the magnetic tension force term \(j_y B_y\) integrates to \(\approx B^2_{ext}/2\). As a result, we find that the plasma electron outflow velocity is approximately equal to the Alfvén velocity, \(V_e \approx V_A\), in the downstream region outside of the ion layer (at \(y \approx L_{ext}\)).

At the end of this section, we derive an estimate for the ion layer half-thickness \(\Delta\). In these derivations, we proceed as follows. Outside the electron layer, the electron inertia and magnetic tension terms can be neglected in equation (36), and we have \(\rho (\nabla V V_y) \approx -\partial_y P\). Taking the y derivative of this equation at \(y = 0\), we obtain \(\rho (\partial_y (V_y V_y) + (\partial_y V_y)^2) \approx -\partial_y P \partial_y\). Here the term \(V (\partial_y V_y)\) is about the same size as the term \((\partial_y V_y)^2\). Therefore, we find that \((\partial_y V_y) \approx V_A / L\) outside the electron layer (but inside the ion layer). Next, in the upstream region outside the ion layer, ideal single-fluid MHD applies. Therefore, at \(x \approx \Delta\) and \(y = 0\) equation (21) reduces to \(E_z \approx -V_y B_y \approx -\partial_y (V_y \partial_y \Delta B_{ext} = \partial_y V_y) \Delta B_{ext} \approx V_A \Delta B_{ext} / L\), where \(E_z\) is given by equation (22). As a result, we obtain
\[
(\partial_y V_y) \approx V_A / L, \quad \Delta \approx \eta j_e L / V_A B_{ext}.
\]

5. Solution for two-fluid reconnection

To be specific, hereafter, unless otherwise stated, we will focus on two-fluid reconnection in electron–ion plasma and will assume \(z = e < m_i, d_i < d_e\) and \(\omega_e^2 = \omega_i^2 = 1\). In this case, equations (32) and (34) reduce to
\[
\eta_j^2 \approx (\partial_y V_y)_0 B_{ext}^2 (1 + \gamma), \quad 1 - d^2 \gamma / d^2 y \approx \gamma L^2 (\partial_y V_y) \gamma / d^2 y L^2_{A}.
\]

We solve these equations and equations (20), (26), (30), (38) and (39) for unknown physical quantities \(j_e, \delta, \Delta, L, \gamma\), \((\partial_y V_y)_0, (\partial_y B_y)_0\), and \((\partial_y j)_0\). We calculate the reconnection rate \(E_r\) by using equation (22). We neglect factors of order unity, and we treat the external field \(B_{ext}\) and scale \(L_{ext}\) as known parameters. Recall that parameter \(\gamma\), given by equation (26), measures the relative strength of the Hall term and the ideal MHD term in the z-component of Ohm’s law (in the case of electron–ion plasma). Depending on the value of parameter \(\gamma\), we find the following reconnection regimes and the corresponding solutions for the reconnection rate.

5.1. Slow Sweet–Parker reconnection

When \(\gamma \lesssim 1\), both the Hall current and the electron inertia are negligible, the electrons and ions flow together, and the electron and ion layers have the same thickness and length. In this case, equations (38) and (40) become \((\partial_y V_y)_0 \approx V_A / L\), \(\eta_j^2 \approx (\partial_y V_y)_0 B_{ext}^2\), and \(1 \approx \eta L^2 (\partial_y V_y) \gamma / d^2 y L^2_{A}\), respectively. As a result, we obtain the Sweet–Parker solution [14, 15]:
\[
1 \ll S = V_A L_{ext} / \eta \lesssim L_{ext} / d_i^2, \quad \gamma \approx V_A d_i^2 / \eta L_{ext} = S d_i^2 / L_{ext}, \quad E_z \approx \eta L^2 / \eta L_{ext} / \gamma L_{ext} / d_i^2, \quad j_e \approx \eta L^2 / \eta L_{ext} / d_i^2 = \gamma L_{ext} / d_i L_{ext},
\]
where the Lundquist number \(S \gg 1\) is defined by equation (18). The condition \(S \lesssim L_{ext} / d_i^2\) is obtained from \(\gamma \lesssim 1\). From this condition for \(S\) we find that Sweet–Parker reconnection takes place when \(d_i\) is less than the Sweet–Parker layer thickness, \(d_i \lesssim L_{ext} / S^{1 / 2}\), which is a result observed in numerical simulations [5–7]. Note that the quadrupole field is small in the Sweet–Parker reconnection case, \(B_{quad} \approx \gamma L_{ext} L_{ext} L_{ext} / \gamma S^{1 / 2} d_i L_{ext} \lesssim B_{ext}\), and the ion and electron outflow velocities are approximately equal to the Alfvén velocity, \(V_e \approx (\partial_y V_y)_0 L \approx V_A [6, 7]\).

Now, let us for a moment consider the case of reconnection in electron–positron plasma. In this case \(d_e = d_i\), \(\omega_e^2 = 1 / 2\), \(\omega_i^2 = 0\) and equation (35) gives \(\gamma = 0\). This result represents an absence of the quadrupole field \(B_{quad}\) (refer to equation (26)), which is known from numerical simulations [16–18]. Therefore, our model predicts the slow Sweet–Parker reconnection solution for reconnection in electron–positron plasmas, which is in disagreement with the results of kinetic numerical simulations [16–18]. A likely reason for this discrepancy is that our model neglects pressure tensor anisotropy, which plays an important role in reconnection in electron–positron plasma.

5.2. Transitional Hall reconnection

When \(1 \approx \gamma \lesssim d_i / d_e\), the Hall current is important but the electron inertia is negligible. In this case, equations (38) and (40) become \((\partial_y V_y)_0 \approx V_A / L\), \(\eta_j^2 \approx (\partial_y V_y)_0 B_{ext}^2\), and \(1 \approx \eta L^2 (\partial_y V_y) \gamma / d^2 y L^2_{A}\). As a result, we obtain the following solution: \(1 \lesssim \gamma \approx d_i / d_e\)
\[
E_z \approx (d_i / L) V_A B_{ext}, \quad j_e \approx d_i V_A B_{ext} / L \eta L = S d_i B_{ext} / L L_{ext},
\]
Table 1. Solution for two-fluid reconnection.

|                | Slow Sweet–Parker | Hall | Fast |
|----------------|-------------------|------|------|
| $S$            | $1 \ll S \lesssim L_{ext}^2/d_i^2$ | $L_{ext}/d_i^2$ | $L_{ext}/d_i$ |
| $\gamma$       | $Sd_i^2/L_{ext}$  | $L_{ext}/L$     | $d_i/d_e \lesssim \gamma < d_i^2/d_e^2$ |
| $E_z$          | $V_A B_0/L_{ext}^{1/2}$ | $(d_i/L)V_A B_{ext}$ | $(L_{ext}/Sd_i)V_A B_{ext}$ |
| $j_0$          | $S^{1/2} B_{ext}/L_{ext}$ | $B_{ext} L_{ext}/d_i L$ | $B_{ext}/d_i$ |
| $\delta$       | $L_{ext}/S^{1/2} \approx \delta$ | $d_i L_{ext}/L \gtrsim d_i$ | $d_i \gg \delta$ |
| $L$            | $L_{ext}$         | $L \gtrsim d_i L_{ext}/d_i$ | $Sd_i/d_{ext}$ |
| $(\partial_y V_y)_o$ | $V_A/L$         | $V_A/L$         | $V_A L_{ext}/Sd_i \lesssim V_A/L$ |
| $(\partial_y B_y)_o$ | $V_A/L$         | $V_A/L$         | $V_A L_{ext}/Sd_i \approx V_A/L$ |
| $(\partial_y B_z)_o$ | $B_{ext}/L_{ext} S^{1/2}$ | $B_{ext} d_i/L_{ext}$ | $B_{ext} L_{ext}^2/S^2 d_i^2$ |
| $(\partial_y B_z)_o$ | $S B_{ext} d_i/L_{ext}^3$ | $B_{ext} L_{ext}/L_{ext}$ | $B_{ext} L_{ext}/Sd_i d_i$ |

$\delta \approx \eta L/d_i V_A = L_{ext}/Sd_i$, \(\Delta \approx d_i\), \(\gamma \approx \gamma_{ext}\) \(\gamma \approx \gamma_{ext}\) \(\gamma \approx \gamma_{ext}\).

As the electron layer length $L$ decreases from its maximal value $L \approx L_{ext}$ to its minimal value $L \approx d_i L_{ext}/d_i$, the transitional Hall reconnection solution (42) changes from the slow Sweet–Parker solution (41) to the fast collisionless reconnection solution presented below (see equations (43)–(53) and table 1).

5.3. Fast collisionless reconnection

When $d_i/d_e \lesssim \gamma < d_i^2/d_e^2$ (compare to equation (35)), the electron inertia and the Hall current are important inside the electron layer and the ion layer, respectively. In this case, equations (38) and (40) become $(\partial_y V_y)_o \approx d_i V_A/d_i L$, $\eta_{ext}^2 \approx (\partial_y V_y)_o B_{ext} \gamma$ and $1 - d_i^2 \gamma/d_i^2 \approx \eta L^2 (\partial_y V_y)_o \gamma/d_i^2 V_A^2$. As a result, taking into consideration equation (31), we obtain the following solution:

$$L_{ext}/d_i \ll S = V_A L_{ext}/\eta \lesssim L_{ext}^2/d_i d_i$$

$$d_i/d_e \lesssim \gamma < d_i^2/d_e^2$$

$$E_z \approx \eta B_{ext}/d_i = (L_{ext}/Sd_i)V_A B_{ext}$$

$$\approx (\Delta/L)V_A B_{ext} \approx (d_i/L)V_A B_{ext}$$

$$j_0 \approx B_{ext}/d_i$$

$$\delta \approx d_i$$

$$L \approx V_A d_i L_{ext}/d_i$$

$$d_i V_A L_{ext}/Sd_i \lesssim V_A/L$$

$$(\partial_y V_y)_o \approx \eta d_i^2 \gamma_{ext} = V_A L_{ext}/Sd_i \gamma_{ext} \lesssim V_A/L$$

$$(\partial_y B_y)_o \approx \eta d_i^2 d_i L_{ext}/d_i = V_A L_{ext}/Sd_i$$

$$(\partial_y B_z)_o \approx B_{ext} L_{ext}/Sd_i d_i$$

Here the limits on the Lundquist number given in equation (43), $L_{ext}/d_i \ll S \lesssim L_{ext}^2/d_i d_i$, are obtained from the conditions $E_z \ll V_A B_{ext}$ (slow quasi-stationary reconnection).
and $L \lesssim L_{\text{ext}}$ (the electron layer length cannot exceed the ion layer length). Except for the definition of the reconnecting field $B_{\text{ext}}$, equations (45)–(47) and (49) essentially coincide with the results obtained in [13] for a model of electron MHD (EMHD) reconnection. The collisionless reconnection rate, given by equation (45), is much faster than the Sweet–Parker rate $E_c \approx V_{\Lambda} B_{\text{ext}}/\sqrt{3}$ (see equations (41)).

Note that the value of $\gamma$ or, alternatively, the value of the ion acceleration rate $(\partial_t V_i)_{\text{obs}} \approx \eta d_i^2 \gamma$ at the point O cannot be determined exactly. This is because in the plasma momentum equation (36), the magnetic tension and pressure forces are balanced by the electron inertia term $d_i^2 (\mathbf{j} \cdot \nabla) j_i$ inside the electron layer. The ion inertia term $\rho_i (\mathbf{V} \cdot \nabla) V_i$ can be of the same order or smaller, resulting in the upper limit $(\partial_t V_i)_{\text{obs}} \lesssim V_{\Lambda}/L$. In other words, inside the electron layer the magnetic energy is converted into the kinetic energy of the electrons (and Ohmic heat), while the ion kinetic energy can be considerably smaller. Therefore, the ion outflow velocity can be significantly less than $V_{\Lambda}$ in the downstream region outside the electron layer (at $y \approx L$). At the same time, the electron outflow velocity is much larger than $V_{\Lambda}$ and is approximately equal to the electron Alfvén velocity.

$$u_e^c \approx (m_e/Ze\rho) j_e = (d_i V_{\Lambda}/B_{\text{ext}}) (\partial_t V_i)_{\text{obs}} \approx d_i V_{\Lambda}/d_i \approx V_{\Lambda} \approx B_{\text{ext}}/\sqrt{\eta m_e} \gg V_{\Lambda}.$$ However, further in the downstream region, at $y \gtrsim L$, as the electrons gradually decelerate, their kinetic energy is converted into ion kinetic energy. As a result, the eventual ion outflow velocity becomes $\approx V_{\Lambda}$, as was estimated in the end of section 4. These results emphasize the critical role that electron inertia plays in the plasma momentum equation (16). These results also agree with simulations [27], which found the ion outflow velocity to be significantly less than $V_{\Lambda}$ in the downstream region outside of the electron layer and found acceleration of ions further downstream (in the decelerating electron outflow jets).

Our theoretical results for collisionless reconnection are in good agreement with numerical simulations and/or laboratory experiments. Indeed, the estimates $\Delta \approx d_i$ for the ion layer thickness, $\delta \approx d_e$ for the electron layer thickness, $B_e \approx (\partial_t V_i)_{\text{obs}}$, $L \approx B_{\text{ext}}$ for the quadrupole field, and $u_e^c \approx V_{\Lambda} \approx B_{\text{ext}}/\sqrt{\eta m_e}$ for the electron outflow velocity agree with simulations [5–7, 25–28]. The estimates $\Delta \approx d_i$ and $B_e \approx B_{\text{ext}}$ also agree with experiment [6]. However, the experimentally measured thickness of the electron layer is about eight times larger than what our theoretical model and numerical simulations predict [29, 30]. This discrepancy can be due to three-dimensional geometry effects and plasma instabilities that may play an important role in the experiment [6, 30].

Our results are also in qualitative agreement with recent numerical findings of an inner electron dissipation layer and of electron outflow jets that extend into the ion layer [25–28]. We note that the estimated electron layer length $L \approx V_{\Lambda} d_e/d_i/\eta$ is generally much larger than both the electron layer thickness $\delta \approx d_e$ and the ion layer thickness $\Delta \approx d_i$, which is consistent with numerical simulations [25–27]. However, if resistivity $\eta$ becomes anomalously enhanced and considerably over the Spitzer value, then $L$ can theoretically become of the order of $d_i$ and the reconnection rate can become comparable to the Alfvén rate $V_{\Lambda} B_{\text{ext}}$, which is also observed in numerical simulations [22, 28].

Unfortunately, a detailed quantitative comparison of our theoretical results to the results of kinetic numerical simulations is not possible because these simulations do not explicitly specify constant resistivity $\eta$. In addition, in the simulations the anisotropy of the electron pressure tensor anisotropy was found to play an important role inside the electron layer and in the electron outflow jets [27, 28]. In contrast, in this study, we assume an isotropic pressure and the electrons are coupled to the field lines everywhere outside the electron layer (including the jets).

In our model, the electric field $E_c$ is supported by the Hall term $(\mathbf{j} \times \mathbf{B})/ne$ in the downstream region $L \lesssim y \lesssim L_{\text{ext}}$. Therefore, in the collisionless reconnection regime, our model predicts the existence of Hall–MHD Petschek shocks that are attached to the two ends of the electron layer and separate the two electron outflow jets and the surrounding plasma. Note that, for electron–ion plasma ($Z m_e \ll m_i$), the ideal MHD and Hall terms in Ohm’s law (12) can be combined together as $-\mathbf{V} \times \mathbf{B} + (m_i/Ze\rho) \mathbf{j} \approx -\mathbf{u}^e \times \mathbf{B}$, where $\mathbf{u}^e$ is the electron velocity given by equation (5). Therefore, all results for the Hall–MHD Petschek shocks can be obtained from the corresponding results derived for the standard MHD Petschek shocks by replacing the plasma velocity $\mathbf{V}$ with the electron velocity $\mathbf{u}^e$. In particular, the parallel components of the magnetic field and electron velocity jump across the Hall–MHD Petschek shocks, the velocity of the shocks is $|u_e^c| \approx (m_i/Ze\rho) j_e \approx (d_i V_{\Lambda}/B_{\text{ext}})(\partial_t V_i)_{\text{obs}} \approx V_{\Lambda} \approx B_{\text{ext}}/\sqrt{\eta m_e}$, and the opening angle between the shocks is $\approx \theta_e B_e \approx B_{\text{ext}}/L_{\text{ext}} \approx L_{\text{ext}}/S d_i \ll 1$. Shocks were indeed observed in numerical simulations [31]. However, in these simulations, a spatially localized anomalous resistivity was prescribed, resulting in a short layer length, while in our study resistivity $\eta$ is assumed to be constant.

6. Discussion

The solution for two-fluid reconnection is summarized in table 1. This table includes solution formulae for three reconnection regimes: the slow Sweet–Parker reconnection regime, the transitional Hall reconnection regime and the fast collisionless reconnection regime. The reconnection rates for these three regimes are, respectively, shown by the solid, dotted and dashed lines in figure 2.

It is well known that resistivity $\eta$ can be considerably enhanced by current-driven plasma instabilities [6, 7, 24]. Because the collisionless reconnection rate $E_c \approx \eta B_{\text{ext}}/d_i$ is proportional to the resistivity (see equation (45)), this rate can increase significantly as well. As a result, we propose below a possible theoretical explanation for the two-stage reconnection behavior (fast and slow) frequently observed in cosmic and laboratory plasma systems undergoing reconnection processes.

During the first stage, such a system is in the very slow Sweet–Parker reconnection regime, during which magnetic energy slowly builds up and gets stored in the system. The magnetic energy and electric currents build up, the field...
strength increases and the resistivity decreases [32]. As a result, the Lundquist number $S$ increases and the system moves to the right along the solid line in figure 2.

When the Lundquist number $S$ becomes comparable to $L_{\text{ext}}^2/d_e^2$ and the thickness of the current layer $L_{\text{ext}}/S^{1/2}$ becomes comparable to $d_i$, the system reaches point A in figure 2. Next, the system goes into the transitional Hall reconnection regime and quickly moves up along the vertical dotted line in figure 2. During this transition, the length of the electron layer shrinks from $\approx L_{\text{ext}}$ to $\approx (d_i/d_e)L_{\text{ext}}$, the electron layer thickness decreases from $\approx d_i$ to $\approx d_e$, and both the electric current and the reconnection rate increase by a factor of $\approx d_i/d_e \gg 1$. The system ends up in the fast collisionless reconnection regime at point B in figure 2.

Because of the considerable increase in electric current during the Hall reconnection transition from point A to point B, plasma instabilities develop and, consequently, resistivity $\eta$ becomes anomalous and rises in value. As a result, the reconnection rate $E_z \approx \eta B_{\text{ext}}/d_e$ increases, the Lundquist number $S \approx V_A L_{\text{ext}}/\eta$ and the electron layer length $L \approx V_A d_i/d_e/\eta$ decreases, and the system moves from point B to the left along the dashed line in figure 2. The system enters the second stage characterized by a rapid release of the accumulated magnetic energy. Even though our theoretical model is stationary, assumes constant resistivity and cannot describe this stage in detail, the physical mechanism of slow and fast reconnection outlined above is self-consistent and may take place in nature.

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