Stress Analysis of the Solar Cells in PV Modules

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Abstract: In this paper, an analytical solution for evaluation of the stress in the solar cells was developed. The stresses of the solar cells in PV module of 1580mm × 808mm were calculated by the present solution and the wind pressures and the effects of the storage shear modulus of the EVA were considered. The results by the present solution were in good agreement with those from FE.

1. Introduction
Today, the application of the PV (Photovoltaic) is very popular in China. The PV module is a laminated structure and the solar cells are embedded in the EVA (Ethylene-Vinyl Acetate) layer separated by small gaps. The configuration of the structure with non-continue cell layer results in the difficulty for the stress analysis and measurement of the solar cells. On the other hand, the EVA is a visco-elastic material and its storage shear modulus will increase dramatically as the temperature decreases [1], which results in the significant rising of the stress in solar cells under the external loads such as the wind loads.

As so far, the investigations related to the mechanical behavior of the PV modules are limited in literature and those studies mainly in three categories, 1) The stress analysis of the module structure [2~3]; 2) The stress analysis of the solar cells under mechanical loads [4~8] in which the cell layer was treated as a continue layer, and 3) The stress analysis of the solar cells under the temperature cycles [9~12].

Li and Yang [13] investigated the stress of the solar cell using ANSYS where the cell layer was treated as a layer with separated cells and the effects of the storage shear modulus variation of the EVA layer and the different wind pressures were considered. In this paper, the solutions for the evaluation of the stresses in solar cell were established and the results from the present solution were compared to those by ANSYS.

2. Formulation
Figure 1 shows the laminated configuration of the PV module in which the layers from the top to bottom are 3.2mm glass panel, 0.5mm EVA, solar cells, 0.5mm EVA and 0.35mm TPT (Tedlar/Pet/Tedlar), respectively. The cells are separated by small gaps and the following assumptions are accepted in analysis

1. The stress \( \sigma_z \) which is normal to the surface of the cell is a small term compared to the other stresses and its effect is neglected in analysis, i.e., \( \sigma_z = 0 \);

2. The cell is loaded by the shear force acting on the upper and lower surfaces when the module is bended. The elastic modulus of the TPT is, however, two orders of magnitude smaller compared to that of the cell material and thus the effect of the TPT layer on the cell is neglected.

3. When the module is bended, the solar cells are under the combination of the bi-tension and bending, however, the stresses due to the bending of the cell are small terms and are neglected in our
analysis, and further assumed that the stresses through the thickness of the cell are constants.

4. In general, the size of the cell is one order smaller compared to that of the module, the strains on the low surface of the glass panel are thus regarded as constants in the area corresponding to a cell.

The boundary conditions and the stress symmetry of the cell can be expressed as

\[ x = \pm a, \sigma_x = 0, \tau_{xy} = 0; y = \pm a, \sigma_y = 0, \tau_{yx} = 0 \]  
(1)

\[ \sigma_x(x, y) = \sigma_x(-x, y), \sigma_y(x, y) = \sigma_y(-x, -y) \]  
(2)

\[ \sigma_y(x, y) = \sigma_y(-x, y), \sigma_x(x, y) = \sigma_x(-x, -y) \]  
(3)

\[ \gamma_{xy} = \frac{1 + b/\delta}b \epsilon_x^e x - u_x \]  
(4)

\[ \gamma_{yx} = \frac{1 + b/\delta}b \epsilon_y^e y - u_y \]  
(5)

where \( u_x \) and \( u_y \) represent the displacements of the cell, \( b \) is the thickness of the EVA layer, \( \delta \) denotes the distance from the curved middle surface of the module to the lower surface of the glass panel. The shear forces acting on the top surface of the cell in \( x \) and \( y \) direction are written as

\[ p_x = G_{EVA} \frac{(1 + b/\delta)\epsilon_x^e x - u_x}b \]  
(6)

\[ p_y = G_{EVA} \frac{(1 + b/\delta)\epsilon_y^e y - u_y}b \]  
(7)

in which \( G_{EVA} \) is the storage shear modulus of EVA layer. Consider a cell unit with \( dx \), \( dy \) and \( t \) where \( t \) is the thickness of the cell, the equilibrium of the unit in \( x \) and \( y \) direction can be expressed as

\[ \frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{p_x}{t} = 0 \]  
(8)

\[ \frac{\partial\tau_{yx}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{p_y}{t} = 0 \]  
(9)

and the compatibility relation

\[ \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = -(1 + \mu) \left(\frac{\partial p_x}{\partial x} + \frac{\partial p_y}{\partial y}\right) \]  
(10)
where \( \mu \) is the Poison ratio of the cell material. The equations (8) and (9) are performed the derivative with respect to \( x \) and \( y \), respectively, and then subtracted from each other, we have

\[
\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial \sigma_y}{\partial x} - \frac{\partial \sigma_x}{\partial y} = 0
\]

(11)

With inserting of (6) and (7) into (10) and (11), then the Hooke’s law is applied, we arrive at the following equations

\[
\nabla^2 (\sigma_x + \sigma_y) = \frac{G_{EVA} (1 - \mu^2)}{E b t} (\sigma_x + \sigma_y) = -G_{EVA} \frac{(1 + \mu)( 1 + b/\delta)(\varepsilon_x + \varepsilon_y) }{b t} 
\]

(12)

\[
\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2} - \frac{(1 + \mu)G_{EVA} (\sigma_x - \sigma_y) }{E b t} = \frac{G_{EVA} (1 + b/\delta)(\varepsilon_x - \varepsilon_y) }{b t} 
\]

(13)

By separation of variables, the solutions for the problem can be found in the forms as

\[
\sigma_x = \sum_{n=1}^{\infty} \frac{r_n e_n \mu A}{b_n e_n - c_n d_n} (\mu \xi_n^2 - \xi_n^2) \cos \xi_n x \cosh \xi_n y + \sum_{n=1}^{\infty} \frac{r_n d_n \mu B}{b_n e_n - c_n d_n} \xi_n \eta_n \cos \xi_n y \cosh \xi_n x 
\]

(14)

\[
\sigma_y = \sum_{n=1}^{\infty} \frac{r_n e_n \mu A}{b_n e_n - c_n d_n} (\xi_n^2 + \lambda^2) \cos \xi_n x \cosh \xi_n y + \sum_{n=1}^{\infty} \frac{r_n d_n \mu B}{b_n e_n - c_n d_n} \xi_n \eta_n \cos \xi_n y \cosh \xi_n x 
\]

in which

\[
A = \frac{(1 + b/\delta) E e^f}{1 - \mu^2}
\]

(17)

\[
B = \frac{(1 + b/\delta) E e^f}{1 - \mu^2}
\]

(18)

\[
k^2 = \frac{(1 - \mu^2) G_{EVA}}{E b t}
\]

(19)

\[
\lambda^2 = \frac{(1 + \mu) G_{EVA}}{E b t}
\]

(20)
\[ a_n = \frac{(2n-1)\pi}{2a} \]  
(21)

\[ \zeta_n = \sqrt{\xi_n^2 + k^2} \]  
(22)

\[ \eta_n = \sqrt{\xi_n^2 + 2\lambda^2} \]  
(23)

\[ b_n = (\lambda^2 + \xi_n^2) \cosh \zeta_n a \]  
(24)

\[ c_n = \xi_n \eta_n \cosh \eta_n a \]  
(25)

\[ d_n = \xi_n \zeta_n \sinh \zeta_n a \]  
(26)

\[ e_n = (\lambda^2 + \xi_n^2) \sinh \eta_n a \]  
(27)

\[ r_n = (-1)^n \left[ \frac{4}{(2n-1)\pi} - \frac{4(2n-1)\pi}{(2n-1)^2 \pi^2 + 4a^2 k^2} \right] \]  
(28)

3. Results and discussion

Li and Yang [13] investigated the stress distribution of the Monocrystalline silicon solar cell in the standard PV module of 1580mm \( \times \) 808mm with 72 pieces of 125mm \( \times \) 125mm solar cells under wind loads. The storage shear modulus variation of the EVA layer and the different wind pressures were considered in FE calculation. The results shown that the maximum von Mise stress appears at the middle of the cell. In this section, the stresses of the solar cell in the same PV module under different wind loads are evaluated by present solution and the results are compared with those from the ANSYS in Li and Yang [13].

In our calculation, the \( x \) axis is along the width direction of the module while the \( y \) axis in the length direction. Five wind pressures, 0.5 \( \text{kN/m}^2 \), 0.65 \( \text{kN/m}^2 \), 0.8 \( \text{kN/m}^2 \), 0.95 \( \text{kN/m}^2 \) and 1.1 \( \text{kN/m}^2 \), are considered. The normal strains of the lower surface at the middle of the glass panel corresponding to the wind pressures are shown in Table 1 [13]

Table 1 The normal strains of the lower surface at the middle of the glass panel

| Wind pressures | \( \epsilon_{fg} \) | \( \epsilon_{g} \) |
|----------------|-----------------|----------------|
| 0.5kN/m²       | 3.029×10⁻⁴     | 1.645×10⁻⁴    |
| 0.65kN/m²      | 3.938×10⁻⁴     | 2.138×10⁻⁴    |
| 0.8kN/m²       | 4.847×10⁻⁴     | 2.631×10⁻⁴    |
| 0.95kN/m²      | 5.756×10⁻⁴     | 3.125×10⁻⁴    |
| 1.1kN/m²       | 6.664×10⁻⁴     | 3.618×10⁻⁴    |

For the solar cell under bi-tension, the von Mise equivalent stress is used to describe the stress state of the cell

\[ \sigma_{eq} = \left( \frac{3}{2} s_{ij} s_{ij} \right)^{1/2} \]  
(29)
$t = 0.19\text{mm}$, $b = 0.5\text{mm}$, respectively.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{stress_graph.png}
\caption{The stress of the solar cell versus $G_{\text{EVA}}$ for 125mm$\times$125mm solar cell under different wind pressures}
\end{figure}

The stresses evaluated from the present solution versus the EVA storage shear modulus, $G_{\text{EVA}}$, for the wind pressures of 0.5 kN/m$^2$, 0.65 kN/m$^2$, 0.8 kN/m$^2$, 0.95 kN/m$^2$ and 1.1 kN/m$^2$ are plotted in Fig. 2 where the results by ANSYS [13] are also plotted for comparison.

It is seen that the results from the present solution are slightly lower (about 2.5%) than those from the ANSYS in Li and Yang [13], the reason is that the effect due to the TPT layer is neglected in present model.

4. Conclusion
The analytical solution for the stress evaluation of the solar cell in PV module was developed. The stress of the solar cell in PV module with the size of 1580mm$\times$808mm was evaluated by present solution and the results were compared to those from ANSYS. It shown that the present solution can calculate correctly the stress of the solar cell in PV module under wind pressures.

The results from the present solution show that the stress of the cell increases significantly as the storage shear modulus of the EVA increases, which implies that the environment temperature will affect the stress of the cell dramatically. The factors such as the wind pressure and the environment temperature in the PV application region should be considered in design of the PV system.

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6. Reference
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