Bose-Einstein condensation into non-equilibrium states studied by condensate focusing

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We report the formation of Bose-Einstein condensates into non-equilibrium states. Our condensates are much longer than equilibrium condensates with the same number of atoms, show strong phase fluctuations and have a dynamical evolution similar to that of quadrupole shape oscillations of regular condensates. The condensates emerge in elongated traps as the result of local thermalization when the nucleation time is short compared to the axial oscillation time. We introduce condensate focusing as a powerful method to extract the phase-coherence length of Bose-Einstein condensates.

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Among the quantum fluids, the quantum gases are especially suited to study the kinetics of Bose-Einstein condensation (BEC). The modest density of the quantum gases makes it possible to stretch the time of condensate formation to values allowing detailed experimental investigation. In external potentials condensates have a characteristic equilibrium shape, known as the Thomas-Fermi shape, that differs markedly from the shape of thermal clouds. Since the pioneering experiments on BEC in the alkali systems [4, 5, 6, 7], this feature has been observed by many groups and is used routinely to measure the condensate fraction [4]. Phase coherence is another key property of equilibrium condensates and was established in the first interference experiments [5, 6, 7, 8].

Equilibrium condensates [4, 5] are produced by quasi-static growth, where heat extraction limits the formation rate. The condensate nucleates as a small feature in the center of the trap and grows as long as heat is extracted from the sample. To observe the formation kinetics, the gas has to be brought out of equilibrium, in practice by shock cooling. Since the first experiment on condensate growth, by Miesner et al. [4], this is done by fast RF removal of the most energetic atoms from the trap. Starting from a thermal gas just above the phase transition temperature (Tc), the condensate appears as the result of thermalization. Miesner et al. [4] observed the growth under adiabatic conditions. Köhl et al. [8] continued the extraction of heat and atoms, also during growth. In both experiments, the condensate was observed to grow from the center of the trap, like in the quasi-static limit.

Kagan et al. [9] pointed out that qualitatively different stages have to be distinguished in the formation of equilibrium condensates with a large number of atoms. The early stage (kinetic stage) is governed by Boltzmann kinetic processes and leads to a preferential occupation of the lowest energy levels. Once a substantial fraction of the atoms gathers within an energy band of the order of the chemical potential of the emerging condensate during formation, their density fluctuations are suppressed in a fast interaction-dominated regime governed by a non-linear equation for the boson field. The appearing phase-fluctuating condensate then grows and the condensed fraction approaches its equilibrium value. However, the phase fluctuations still persist, giving rise to dynamically evolving flow patterns in search for the true equilibrium state. In elongated 3D trapped gases the phase fluctuations can be pronounced even under equilibrium conditions as was predicted by Petrov et al. [10] and observed experimentally by Dettmer et al. [11].

In this Letter we report the formation of condensates into non-equilibrium states and a new path towards equilibrium in elongated traps. In contrast to the previous experiments our results were obtained starting from thermal clouds deep in the cross-over regime to hydrodynamic behavior. The condensates are much longer than equilibrium condensates with the same number of atoms. Moreover, they display strong phase fluctuations and a dynamical evolution similar to that of a quadrupole shape oscillation decaying towards equilibrium. We identify 1/ωz as a characteristic time that should be addressed explicitly for elongated cylindrical harmonic traps, i.e. for traps with ωp ≫ ωz, where ωp and ωz are the radial and axial angular frequencies, respectively. We show that these exotic condensates emerge as the result of local thermalization when the nucleation time is short as compared to 1/ωz. The dynamical evolution of the condensate in the trap has to be dealt with explicitly to properly interpret time-of-flight absorption images. In this context we introduce condensate focusing as an alternative to Bragg scattering [12] for measuring the phase-coherence length of phase-fluctuating Bose-Einstein condensates.

In the previous experiments on condensate formation the phase fluctuations were not studied. The results of Miesner et al. [4] were compared to an analytical expression for adiabatic growth of a condensate from a thermal cloud, derived by Gardiner et al. [13]. Although a qualitative agreement between theory and experiment was readily obtained it turned out to be impossible to obtain detailed agreement at the quantitative level [13, 14]. In the
experiment of Köhl et al. [10] quantitative agreement with the quantum kinetic approach (see refs. [14,15,16]) was obtained for strong truncation, whereas for weak truncation the observed behavior differed distinctly from theory.

In our experiments we typically load 4 × 10^9 atoms of ^87^Rb in the |S_z/2, F = 2, m_F = 2⟩ state into a horizontal Ioffe-Pritchard quadrupole trap with ω_p = 2π × 477(2) Hz and ω_z = 2π × 20.8(1) Hz (no RF dressing) [17]. The trap minimum B_0 = 88.6(6) μT corresponds to a radio frequency of ν_0 = 620 kHz as calibrated against atom laser output coupling [18]. The trap minimum shows a long term drift of 5 kHz/hr. At full current thermal drift effects are less than 1 kHz/s.

The gas is prepared by forced RF evaporation at a final rate of ν_tr ≈ −433 kHz/s to a value ν_tr,a ≈ 740 kHz, followed by 20 ms of plain evaporation at ν_tr,a. As

$$|\dot{\nu}_{tr}/(\nu_{tr,a} - \nu_0)| \ll \omega_z$$

this yields a static, purely thermal cloud of N_i ≈ 5 × 10^6 atoms at a temperature T_0 = 1.3(1) μK as determined from the axial size l_z = [2kT_0/mω_z^2]^{1/2} of the cloud in the trap, measured by time of flight absorption imaging shortly (t < 1/ω_z) after release from the trap. We calculated a central density n_0 = 4 × 10^{14} cm^{-3}, corresponding to a mean free path λ_0 = (2^{1/2}n_0σ)^{-1} ≈ 3 μm and a collision rate γ_c = n_0ν_0ν_0σ ≈ 5000 s^{-1} [10]. Radially we find λ_0/λ_r ≈ 0.5. Axially we have λ_0/l_z ≈ 0.2 to be compared with the values λ_0/l_z ≈ 0.5 and 0.3 used in previous experiments [9] and [10] on condensate formation. Hence, our thermal samples are prepared far deeper in the cross-over to the hydrodynamic regime.

The hydrodynamic behavior manifests itself in the damping time τ_Q and a shift of the frequency ω_Q of the quadrupole shape oscillation as well as in an anisotropic expansion of the cloud after release from the trap. We measured ν_ω/ω_Q = 1.56(5). For ω_z/ω_Q = 10(3) theory predicts ω_ω/ω_z = 1.57(2) [20] and ω_Q/ω_z = \sqrt{12/5} ≈ 1.55 for the hydrodynamic limit in very elongated traps [21]. Further, writing β ≡ ω_z/ω_p, from the scaling theory [21], we find in this limit v_z/v_p ≈ 2.6β ≈ 0.11 for the ratio of axial to radial expansion velocity after release from the trap. We measured v_z/v_p = 0.78(2) and a final temperature T_∞ = 0.94(4) μK as determined from the axial expansion, which implies that the gas expands hydrodynamically and cools only briefly before the expansion becomes ballistic. For isentropic hydrodynamic expansions the degeneracy parameter is conserved: n(τ)/n_0 = (T_∞/T_0)^{3/2}. Then, assuming that the expansion is hydrodynamic initially and becomes ballistic after a time τ_{freeze}, the scaling theory [21] gives a relation between T_0/T_∞ and v_z/v_p. In elongated traps, for v_z/v_p > β, we find

$$T_0/T_∞ ≈ [1 + 2(v_p/v_z)^2]/3.$$  \(2\)

The observed ratio v_z/v_p and temperature T_0 lead to T_∞ = 0.91(7), which coincides with T_∞ measured from the axial expansion. From the obtained value of n(τ_{freeze})/n_0 we calculate τ_{freeze} ≈ 0.3 ms, i.e. a couple of collision times. This is consistent with the ratio λ_0/λ_p ≈ 0.5 mentioned above and confirms the picture that we are operating near the onset of hydrodynamic behavior in the radial direction. For completeness we verified that the expansion becomes isotropic, v_z/v_p = 1.02(4), when the atom number is reduced by a factor of 30.

Once the thermal cloud is prepared we distinguish three distinct stages. First, in the truncation stage, the radio frequency is set to the value ν_tr,b = 660 kHz. This stage has a duration t_tr = 1 ms, which is chosen to be long enough (t_tr > 1/ω_p) to allow atoms with radial energy ε_r larger than the RF truncation energy ε_tr to escape from the trap, yet is short enough to disallow evaporative cooling. We found that in this stage 50% of the atoms are removed. Notice that due to the finite radial escape horizon (λ_0/λ_r ≈ 0.4) the ejection is not expected to be complete. Furthermore, the escape efficiency is anisotropic as a result of gravitational sag. The truncation energy ε_tr covers the range 3 μK - 5 μK depending on the position of the truncation edge in the gravity field and is lowered by an additional 1 μK due to RF-dressing (Rabi frequency Ω_Ω ≈ 2π × 14 kHz). At the start of the second stage, the thermalization stage, the radio frequency is stepped back up for a time t_{th} to the frequency ν_tr,a to allow the gas to thermalize under formation of a condensate. The value ν_tr,a is chosen to eliminate any appreciable evaporative cooling. The third stage, the expansion stage, starts by switching off the trap and covers the time of flight τ after which the sample is absorption imaged on the |S_z/2, F = 2⟩ → |S_z/2, F = 1, 2 or 3⟩ transition.

To follow the evolution of the trapped gas after the truncation we took time-of-flight absorption images for a range of evolution times t ≡ t_{tr,a} + t_{th} and a fixed expansion time τ. The images show a bimodal distribution, indicating that the truncation procedure results in BEC. The condensate fraction grows to a final value of 6% with a characteristic time of 6 ms. This corresponds to 30τ_{col}, in accordance with previous experiments.

Rather than discussing the details of the growth kinetices we emphasize that our condensates nucleate into non-equilibrium states. In Fig.1 we plot the Thomas-Fermi half-length L_0 obtained with the standard fitting procedure of a bimodal distribution to our data [3]. For the shortest expansion time, τ = 2.8 ms, the axial size of the condensate image equals to good approximation (τ ≪ 1/ω_z) the axial size of the condensate in the trap. We see that L_z(t) is initially oversized by a factor L_z(0)/L_z(∞) = 2.2(3) and rapidly decreases to reach its equilibrium size after roughly one strongly damped shape oscillation (see open triangles Fig.1). Hence, the condensate is clearly not in equilibrium [22].

The formation of oversized condensates follows from the local formation concept underlying ref. [11]. Starting
with a thermal cloud of \( N_1 = 5 \times 10^6 \) atoms in an 11.5 mK deep harmonic trap at a temperature \( T_0 = 1.3 \) \( \mu \)K, we calculate that 55\% of the atoms remain trapped after all atoms with energy \( \varepsilon > \varepsilon_t = 3.4 \) \( \mu \)K are removed. The gas will rethermalize well within a time short compared to \( 1/\omega_z \). Hence, the resulting temperature varies along the trap axis. In this respect the thermalization is a local phenomenon. The local \( T_C \) is given by \[ kT_C(z) \approx 1.28 \hbar \omega_p \left[ n_{1D}(z) r_p \right]^{2/5}, \tag{3} \] where \( r_p = [\hbar/m\omega_p]^{1/2} \) is the radial oscillator length and \( n_{1D}(z) \) the atom number per unit length at position \( z \). We find that the local temperature \( T(z) \) is lower than the local \( T_C(z) \) over a length of order \( l_z \). In view of the simplicity of this model we consider this as good qualitative agreement with experiment.

To further investigate the formation process we introduce condensate focusing. In our case one-dimensional focusing results from axial contraction of the expanding cloud when the gas is released from the trap during the inward phase of a shape oscillation. The focus is best demonstrated by plotting the axial size \( L_z(t, \tau) \) of the condensate as a function of expansion time \( \tau \) after a fixed evolution of \( t = 11 \) ms in the trap (see Fig.2). The axial size is seen first to decrease and to increase again later as expected for a focus \[ b_2(t, \tau) \approx 1 + (\pi \beta \omega_z/2 - a_z \omega_Q \sin \omega_Q t) \tau. \tag{4} \]

The first term under the brackets corresponds to the axial expansion kick caused by the declining chemical potential at trap release. The second term represents the scaled axial dilatation velocity in the trap at the moment of release. From Eq.(4) we see that the axial dilatation field reaches a real (virtual) focus for positive (negative) values of the expansion time \( \tau_{\text{focus}} = (a_z \omega_Q \sin \omega_Q t - \frac{\pi}{2} \omega_z)^{-1} \). A real focus can be obtained already for small amplitudes, \( a_z > \beta \).

For \( \tau \gg 1/\omega_p \) the radial expansion is described by \[ b_p(t, \tau) \approx \omega_p \tau (1 - \frac{1}{4} a_z \cos \omega_Q t), \] i.e. shows no focus. As the radial size remains finite, the chemical potential will build up near the focus until the compression is balanced and the axial size starts to expand again. As follows from the scaling equations (see ref. \[ 22 \] or \[ 21 \]), at maximum compression the axial size is reduced by a factor of \( \beta^2 \).

Our data match a scaled focal size \( b_2(t, \tau_{\text{focus}}) = 0.49(6) \), i.e. the focus is strongly broadened as compared to the minimum scaled size \( \beta^2 \). The broadening is attributed to local variations in expansion velocity caused by the presence of phase fluctuations in our condensates. After some expansion these variations give rise to irregular stripes (see the inset of Fig.2) as previously observed in Hannover \[ 13 \]. At the focus, the axial distribution maps linearly onto the momentum distribution due to the phase fluctuations in our original condensate. The scaled focal size is given by \( b_2(t, \tau_{\text{focus}}) \sim (\hbar/mL_\phi) \tau_{\text{focus}}/L_z \), where \( L_\phi \) is the phase coherence length and \( (\hbar/mL_\phi) \) characterizes the expansion velocity due to the phase fluctuations. For equilibrium phase fluctuations close to \( T_C \) we estimate \( L_z/L_\phi = 7(4) \) (see ref. \[ 22 \]) and \( b_2 \sim 0.05 \). As the observed focal size is larger by an order of magnitude, the phase fluctuations have a non-equilibrium origin with a phase coherence length of only \( L_\phi \sim 1 \) \( \mu \)m. The decay of the phase fluctuations as a function of evolution time is subject of further investigation in our group.
We point out that for a thermal cloud driven on the low-frequency quadrupole mode the phenomenon of focusing is absent except deeply in the hydrodynamic regime. For a small ratio $T_{\infty}/T_0$ the scaled focal size will be given by

$$b_z(t, \tau_{\text{focus}}) \approx v_{th}(T_{\infty}) \tau_{\text{focus}}/I_z(T_0) \approx (T_{\infty}/T_0)^{1/2} \omega_z \tau_{\text{focus}} < 1.$$ 

In Fig.1 we also show the oscillation in the axial size of the condensate as observed for 15 ms (open squares) and 25.3 ms (crosses) of expansion [2]. Due to enhancement by focusing the amplitude of the oscillation has increased as compared to the 2.8 ms results. For $\tau = 25.3$ ms the shape oscillation is seen to exceed the noise for at least 100 ms. This oscillation can be described by a linear response expression for evolution times $t \geq 20$ ms where $a_z \leq 0.2$. We measure a damping time of $\tau_Q = 50(9)$ ms and a frequency ratio $\omega_Q/\omega_z = 1.54(4)$. The latter is slightly lower than the frequency expected for a quadrupole shape oscillation of a pure condensate in very elongated traps, $\omega_Q/\omega_z \approx 5/2 \approx 1.58$ [21, 22]. A 5% negative frequency shift was observed for the quadrupole mode in Na condensates just below $T_C$ [29] and is consistent with theory [30]. The condensate focusing is a particularly useful tool. It separates the condensate from the thermal cloud in time-of-flight absorption imaging. This enables the observation of small condensate fractions and measurements of time-of-flight thermometry. The focal size and shape allow to quantify the phase fluctuations inside condensates. Extended to two dimensions the focusing can serve imaging applications, with the mean field of the quantum fluid acting as a tunable component in atom optics.

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