Error Propagation and Overhead Reduced Channel Estimation for RIS-Aided Multi-User mmWave Systems

(Invited Paper)

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Abstract—In this paper, we propose a novel two-stage based uplink channel estimation strategy with reduced pilot overhead and error propagation for a reconfigurable intelligent surface (RIS)-aided multi-user (MU) mmWave system. Specifically, in Stage I, with the carefully designed RIS phase shift matrix and introduced matching matrices, all users jointly transmit the pilot signals for the estimation of the correlation factors between different paths of the common RIS-base station (BS) channel, which achieves significant multi-user diversity gain. Then, the inherent scaling ambiguity and angle ambiguity of the mmWave cascaded channel are utilized to construct an ambiguous common RIS-BS channel composed of the estimated correlation factors. In Stage II, with the constructed ambiguous common RIS-BS channel, each user independently sends reduced pilots for estimating their specific user-RIS channel so as to obtain the entire cascaded channel. The theoretical number of pilots required for the proposed method is analyzed and the simulation results are presented to validate the effectiveness of this strategy.

Index Terms—Reconfigurable intelligent surface, channel estimation, multi-user systems, millimeter wave

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) technology is envisioned to be a promising technique for enhancing the spectrum and energy efficiency of 6G-and-beyond communications with relative low hardware cost and energy consumption [1]–[3]. To reap the benefits promised by RIS, accurate channel state information (CSI) is required, which is challenging to achieve due to the lack of complex signal processing ability of the RIS.

Recently, there have been many contributions on channel estimation for RIS-aided millimeter wave (mmWave) systems. Most work focused mainly on single-user system [4], [5], but it is not appropriate to apply these methods to the multi-user systems since the characteristic of the RIS-aided multi-user system that all users share the common RIS-BS channel is not utilized. Motivated by this observation, a significant breakthrough was made in reducing the channel estimation overhead for mmWave multi-user systems by revealing the correlation relationship among the cascaded channel matrices for multiple users [6]. Specifically, the authors of [6] fully exploited the angle and gain information of the common BS-RIS channel and proposed a two-phase based channel estimation protocol. In this protocol, other users’ cascaded channel can be estimated effectively in Stage II with low pilot overhead based on a re-parameterized common BS-RIS channel, which is constructed from the estimated typical user’s cascaded channel in Stage I. However, one issue of the two-phase based method is that the channel estimation error of the typical user in the previous stage will deteriorate the estimation accuracy of other users in the next stage, which is known as error propagation. An optional method to suppress this error propagation effect is the careful selection of the typical user. Furthermore, to ensure the good estimation performance of the typical user, allocating more pilots to the typical user is also a feasible solution. However, these schemes introduce excessive pilot overhead for the estimation of the typical user. More importantly, these schemes of selecting the typical user still suffer from the error propagation since the estimation performance of other users is limited by the estimation performance of the typical user. Once the estimated CSI of the typical user has sizable error, the estimation performance of the multi-user system will be severely degraded.

Against the above background, to reduce the error propagation, we propose a novel two-stage based channel estimation method without choosing a typical user for an RIS-aided mmWave multi-user system, where the proposed strategy benefits from the multi-user diversity gain. The theoretical number of pilots required by the strategy is analyzed and the corresponding simulation results for the proposed method are presented. Compared with the existing algorithms, the pilot overhead of the proposed strategy is extremely low, and the estimation performance of the multi-user system will be severely degraded.

Notations: For a matrix A of arbitrary size, A∗, AT , AH, A†, and vec(A) stand for the conjugate, transpose, conjugate transpose, pseudo-inverse, and vectorization of A. The m-th row of A is denoted by Am(·). Additionally, the Khatri-Rao product and Hadamard product between two matrices A and B are denoted by A ◦ B and A ⊙ B, respectively. I denote an identity matrix with appropriate dimensions. For a vector a,
\([a]_{m,n}\) denotes the subvector containing from the \(m\)-th element to the \(n\)-th element of \(a\). The symbol \(|a|\) represents the Euclidean norm of \(a\). \(\text{Diag}\{a\}\) is a diagonal matrix with the entries of \(a\) on its diagonal. The inner product between two vectors \(a\) and \(b\) is denoted by \((a,b) \triangleq a^Hb\), \(i \triangleq \sqrt{-1}\) is the imaginary unit. \(\mathbb{C}\) represents the set of complex numbers.

II. System Model

We consider a narrow-band time-division duplex (TDD) mmWave system, in which \(K\) single-antenna users communicate with a BS equipped with an \(N\)-antenna uniform linear array (ULA). To improve the communication performance, an RIS equipped with a passive reflecting ULA is deployed. In addition, the direct channel between the BS and users are assumed to be blocked.

Denote \(H \in \mathbb{C}^{N \times M}\) as the channel matrix between the RIS and the BS, and \(h_k \in \mathbb{C}^{M \times 1}\) as the channel matrix between user \(k\) and the RIS, respectively. The set of users is defined as \(\mathcal{K} = \{1, \ldots, K\}\). Due to the limited scattering characteristics in the mmWave environment, we use the geometric channel model to express the channel matrices \(H\) and \(h_k\) as

\[
H = \sum_{l=1}^{L} \alpha_l a_N(\psi_l)a_M^H(\omega_l) = A_N A_M^H, \quad (1)
\]

\[
h_k = \sum_{j=1}^{J_k} \beta_{k,j} a_M(\varphi_{k,j}) = A_{M,k} \beta_k, \quad \forall k \in \mathcal{K}, \quad (2)
\]

where \(L\) and \(J_k\) denote the number of propagation paths (scatterers) between the BS and the RIS, and the number of propagation paths between the RIS and user \(k\), respectively. In addition, \(\alpha_l, \psi_l\) and \(\omega_l\) are the complex path gain, angle of arrival (AoA), and angle of departure (AoD) of the \(l\)-th path in the RIS-BS channel, respectively. Similarly, \(\beta_{k,j}\) and \(\varphi_{k,j}\) represent the complex path gain and AoA of the \(j\)-th path in user \(k\)-RIS channel, respectively. Moreover, \(A_N = [a_N(\psi_1), \ldots, a_N(\psi_L)] \in \mathbb{C}^{N \times L}\), \(A_M = [a_M(\omega_1), \ldots, a_M(\omega_L)] \in \mathbb{C}^{M \times L}\), and \(A = \text{Diag}\{a_1, \ldots, a_L\} \in \mathbb{C}^{L \times L}\) are the AoA steering (array response) matrix, AoD steering matrix and complex gain matrix of the common RIS-BS channel, respectively, and \(A_{M,k} = [a_M(\varphi_{k,1}), \ldots, a_M(\varphi_{k,J_k})] \in \mathbb{C}^{M \times J_k}\) and \(\beta_k = [\beta_{k,1}, \ldots, \beta_{k,J_k}]^T \in \mathbb{C}^{J_k \times 1}\) are the AoA steering matrix and complex gain vector of user \(k\)-RIS channel, respectively.

Here, \(a_N(\psi)\) represents the array steering vector connected to the BS with \(N\) elements, while \(a_M(\omega)\) and \(a_M(\varphi)\) represent the array steering vectors connected to the RIS with \(M\) elements. For a typical ULA with \(Q\) elements, whose steering vector \(a_Q(x) \in \mathbb{C}^{Q \times 1}\) can be represented by

\[
a_Q(x) = [1, e^{-i2\pi x}, \ldots, e^{-i2\pi(Q-1)x}]^T.
\]

The variable \(x\) is regarded as the spatial frequency and there exists a relationship between the spatial frequency \(x\) and the physical angle \(\vartheta\) as \(x = \frac{\vartheta}{\lambda_c} \cos(\vartheta)\) when assuming \(d \leq \lambda_c/2\). Here, \(\lambda_c\) is the carrier wavelength and \(d\) is the element spacing. In this paper, we refer to the angle information as spatial frequency for simplicity.

Further, denote \(e_t \in \mathbb{C}^{M \times 1}\) as the phase shift vector of the RIS in time slot \(t\), due to the property that \(h_k^T \text{Diag}\{e_t\} = e_t^H \text{Diag}\{h_k^T\}\), it has been shown that the joint BS precoding and RIS phase shift coefficient design only depends on the cascaded user-RIS-BS channel as [4]:

\[
G_k = H \text{Diag}\{h_k\} \in \mathbb{C}^{N \times M}, \forall k \in \mathcal{K}. \quad (4)
\]

Combining (1) with (2), the cascaded channel \(G_k\) in (4) can be expressed as

\[
G_k = A_N A_M^H \text{Diag}\{A_{M,k} \beta_k\}, \forall k \in \mathcal{K}. \quad (5)
\]

We will develop a novel cascaded channel estimation strategy with reduced error propagation and low pilot overhead to estimate the cascaded channel \(G_k\).

III. Two-Stage Based Cascaded Channel Estimation for a Multi-user System without Choosing a Typical User

In this section, a two-stage based uplink cascaded channel estimation strategy without choosing a typical user is proposed. Specifically, in Stage I, an ambiguous common RIS-BS channel is constructed based on the pilot signals transmitted by all users jointly to achieve multi-user diversity gain and suppress the impact of error propagation. In Stage II, each user only needs to transmit the reduced pilots for estimating the specific user-RIS channel to obtain full CSI of the cascaded channel. Finally, the required pilot overhead is analyzed.

A. Stage I: Estimation of the Ambiguous Common RIS-BS Channel

In Stage I, with the carefully designed RIS phase shift coefficients, all users transmit the training pilots simultaneously so as to achieve the multi-user diversity gain. Assume that the pilot symbols satisfy \(s_k(t) = 1\) for \(\forall k \in \mathcal{K}\), and the transmitted power of each user \(P\) equals to 1. Then, the received signal at the BS can be expressed as

\[
y(t) = y_1(t) + \ldots + y_K(t) + \ldots + y_K(t) + n(t) = H \text{Diag}\{e_t\} h_1 + \ldots + H \text{Diag}\{e_t\} h_K + n(t)
\]

\[
= H \text{Diag}\{h_1\} e_t + \ldots + H \text{Diag}\{h_K\} e_t + n(t)
\]

\[
= H \text{Diag}\{h\} e_t + n(t), \quad (6)
\]

where \(h \triangleq \sum_{k=1}^{K} h_k\) is treated as the equivalent user-RIS channel, \(n(t) \in \mathbb{C}^{N \times 1} \sim \mathcal{C}\mathcal{N}(0, \Omega^2)\) is the additive white Gaussian noise (AWGN) at the BS. Stacking \(V\) time slots of (6), the received matrix \(Y = [y(1), y(2), \ldots, y(V)] \in \mathbb{C}^{N \times V}\) is expressed as

\[
Y = H \text{Diag}\{h\} [e_1, e_2, \ldots, e_V] + [n(1), n(2), \ldots, n(V)]
\]

\[
= A_N A_M^H \text{Diag}\{h\} E + N, \quad (7)
\]

where \(E = [e_1, e_2, \ldots, e_V] \in \mathbb{C}^{M \times V}\) is the RIS phase shift matrix during this stage and \(N = [n(1), \ldots, n(V)] \in \mathbb{C}^{N \times V}\).
1) Common AoAs estimation: Estimating the AoA steering matrix of the common RIS-BS channel, i.e., $\hat{A}_N$, from (7) is a classical direction of arrival (DOA) estimation problem in array processing, and can be solved by many mature signal processing techniques [5]. Due to the large scale antenna arrays employed at the BS with typically $L \ll N$, the DFT-based method in [6] can be adopted to obtain the estimate of the common AoA steering matrix.

Denote the estimate of $A_N$ as $\hat{A}_N = \left[ a_N(\tilde{\psi}_1), \ldots, a_N(\tilde{\psi}_L) \right] \in \mathbb{C}^{N \times L}$ and introduce $\Delta A_N = A_N - \hat{A}_N$ to represent the estimation error between the common AoA $A_N$ and its estimate $\hat{A}_N$. Then, due to the property that $\text{rank}(A_N) = L$, by using $A_N = \hat{A}_N + \Delta A_N$, we can obtain the equivalent received matrix $\hat{A}_N^H Y \in \mathbb{C}^{L \times V}$ as

$$\hat{A}_N^H Y = \Lambda A_N \text{Diag} \{ h \} E + \hat{A}_N^H \tilde{N} \in \mathbb{C}^{L \times V},$$

where $\tilde{N} \triangleq N + \Delta A_N \Lambda A_N \text{Diag} \{ h \} E$ represents the equivalent noise matrix.

2) Correlation relationship between different paths: With the estimated common AoA, i.e., $\hat{A}_N$, a correlation relationship between different paths in the common RIS-BS channel will be revealed, which helps us to construct the ambiguous common RIS-BS channel. Specifically, by calculating the $l$-th row and the $r$-th row of $\hat{A}_N^H Y$ in (8), and then taking their conjugate transpose, we have

$$[(\hat{A}_N^H Y)_{l,:}]^H = [(\Lambda A_N \text{Diag} \{ h \} E + \hat{A}_N^H \tilde{N})_{l,:}]^H = \text{E}^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* + \tilde{n}_l,$$

$$[(\hat{A}_N^H Y)_{r,:}]^H = [(\Lambda A_N \text{Diag} \{ h \} E + \hat{A}_N^H \tilde{N})_{r,:}]^H = \text{E}^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* + \tilde{n}_r,$$

where $\tilde{n}_l \triangleq [(\hat{A}_N^H \tilde{N})_{l,:}]^H$ and $\tilde{n}_r \triangleq [(\hat{A}_N^H \tilde{N})_{r,:}]^H$. Apparently, the dominant terms of $[(\hat{A}_N^H Y)_{l,:}]^H$ and $[(\hat{A}_N^H Y)_{r,:}]^H$, i.e., $\text{E}^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^*$ and $\text{E}^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^*$, contain the whole information regarding the $l$-th path and the $r$-th path, respectively, and there exists a relationship between these two terms, reflecting the correlation between the corresponding two paths.

To illustrate this correlation relationship, define a matching matrix $A_l$ for the $l$-th path with respect to the $r$-th path as

$$A_l \triangleq A(\varpi_l) x_l = [U_V \odot (a_V(\varpi_l) \mathbf{1}_V^T)]^H \frac{1}{V} U_V x_l,$$

where $A(\varpi_l) \triangleq [U_V \odot (a_V(\varpi_l) \mathbf{1}_V^T)]^H \frac{1}{V} U_V \in \mathbb{C}^{V \times V}$ is a complex nonlinear function of $\omega_l$. $U_V$ is a $V \times V$ DFT matrix with the $(n,m)$-th entry given by $U_{V,n,m} = e^{-j2\pi (n-1)(m-1)}$. $a_V(\varpi_l)$ can be regarded as the array manifold with dimension $V$ and $\mathbf{1}_V$ is an all-one vector of size $V$. Here, the $r$-th path is treated as the reference path. $\varpi_l$ and $x_l$ are the rotation factor and scaling factor for the $l$-th path with respect to the $r$-th path, respectively, which are given by

$$\varpi_l = \omega_r - \omega_l, \quad x_l = \frac{\alpha_r^*}{\alpha_l^*}. \quad (12)$$

Clearly, $\varpi_l \in [-\frac{2\pi}{N}, \frac{2\pi}{N}]$. In this stage, the RIS phase shift matrix $E$ in (7) needs to be designed carefully, which should satisfy the following structure

$$E = \left[ U_V^H \odot 0_{V \times (M-V)} \right]^H,$$

where $E \in \mathbb{C}^{M-V \times V}$ represents the remaining part of $E$. For simplicity, we set $E$ as a zero matrix, i.e., $0_{(M-V) \times V}$. Then, we will show how the matching matrix $A_l$ works. For notation simplicity, the noise terms of (9) and (10) are momentarily omitted. Then, we have

$$A_l E \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* = [U_V \odot (a_V(\varpi_l) \mathbf{1}_V^T)]^H \frac{1}{V} U_V x_l E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* = \left[ (I_V a_V(\varpi_l) \odot U_V^H) [I_V \odot E_{V \times (M-V)}] \right] \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* = \left[ I_V U_V^H \text{Diag} \{ a_V(\varpi_l) \} \odot 0_{V \times (M-V)} \right] \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* = E^H \text{Diag} \{ a_M(\varpi_l) \} \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* = E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^*.$$

where the third equality is obtained using $(yx^H) \odot A = \text{Diag} \{ y \} A \text{Diag} \{ x \}$. From Eq. (14), the term $E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^*$ can be expressed as the result of the term $E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^*$ via the linear transformation $A_l$. Thus the correlation between the $l$-th path and the $r$-th path is depicted by the two variables of matching matrix $A_l$, i.e., $\varpi_l$ and $x_l$. By analogy, we conclude that the correlation relationship between any two paths in the common RIS-BS channel can be described by their rotation factors and scaling factors. Therefore, we name these two kinds of factors as correlation factors and we will show how to estimate the correlation factors from the equivalent received signal $\hat{A}_N^H Y$.

3) Estimation of the rotation factors and scaling factors: We still take the $l$-th path and the $r$-th path, i.e., the reference path, as an example to illustrate the method for estimating the rotation factors and scaling factors. Similar to Eq. (14), we process the received signal $[(\hat{A}_N^H Y)_{r,:}]^H$ in (10) via the linear transformation $A_l$ as

$$A_l [(\hat{A}_N^H Y)_{r,:}]^H = A_l E \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* + A_l \tilde{n}_r,$$

$$= E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* - \Delta \tilde{n}_r,$$

where $\Delta \tilde{n}_r \triangleq -A_l \tilde{n}_r$. By using (15) and $A_l = A(\varpi_l) x_l$, (9) is re-expressed as

$$[(\hat{A}_N^H Y)_{l,:}]^H = E^H \text{Diag} \{ h^* \} a_M(\omega_r) \alpha_r^* + \tilde{n}_l = A(\varpi_l) x_l (\hat{A}_N^H Y)_{r,:}]^H + n_{\text{noise}},$$

where $n_{\text{noise}} \triangleq \Delta \tilde{n}_r + \tilde{n}_l$ represents the corresponding noise vector. Moreover, denote $[(\hat{A}_N^H Y)_{l,:}]^H$ as $\tilde{y}_l$ and $A(\varpi_l)(\hat{A}_N^H Y)_{r,:}]^H$ as $b(\varpi_l) \in \mathbb{C}^V$, (16) can be written as

$$\tilde{y}_l = b(\varpi_l) x_l + n_{\text{noise}}.$$

So our aim is to estimate the rotation factor $\varpi_l$ and scaling factor $x_l$ with the received $\tilde{y}_l$, which can be achieved by solving the problem shown below as

\[ \text{arg max}_{x_l} \left\{ \| \tilde{y}_l - b(\varpi_l) x_l \| \right\}^2. \]
It is observed that the relationship between $A_s$ and $A$, and the relationship between $A_s$ and $A_M$ can be described as
\[
A_s = \text{Diag}\{\alpha_1, \alpha_2, \ldots, \alpha_L\}/\alpha_r = A_M (\omega_1, \ldots, \omega_L),
\] (25)
\[
A_s = [a_M(\omega_1 - \omega_r), a_M(\omega_2 - \omega_r), \ldots, a_M(\omega_L - \omega_r)].
\] (26)

With the obtained $A_N$, $A_s$ and $A$, the corresponding ambiguous common RIS-BS channel, denoted by $H_s$, is naturally constructed as
\[
H_s = A_N A_s A_s^H.
\] (27)
Then, substituting $A = \alpha_r A_s$ and $A_M = \text{Diag}\{a_M(\omega_r)\} A_s$ into $G_k$ in (5), we have
\[
G_k = A_N A_s H_s \text{Diag}\{A_{M,k} \beta_k\}
= A_N (\alpha_r A_s) A_s^H \text{Diag}\{A_{M,k} \beta_k\} \text{Diag}\{A_M(\omega_r)\}
= A_N A_s A_s^H \text{Diag}\{A_{s,k} \beta_k\}
= A_N A_s A_s^H \text{Diag}\{A_{s,k} \beta_k\}
= H_s \text{Diag}\{h_{s,k}\}, \quad \forall k \in \mathcal{K},
\] (28)
where $A_{s,k} \triangleq \text{Diag}\{a_M(\omega_r)\} A_{M,k} = [a_M(\varphi_{k,1} - \omega_r), \ldots, a_M(\varphi_{k,J_k} - \omega_r)] \in \mathbb{C}^{M \times J_k}$ and $\beta_k \triangleq \alpha_r \beta_k = [\alpha_r \beta_{k,1}, \ldots, \alpha_r \beta_{k,J_k}]^T \in \mathbb{C}^{J_k \times 1}$ are the corresponding ambiguous AoA steering matrix and ambiguous complex gain vector of the specific user-RIS channel for user $k$, respectively. Accordingly, $h_{s,k} \triangleq A_{s,k} \beta_k = \alpha_r \text{Diag}\{a_M(\omega_r)\} h_k$ is the corresponding ambiguous specific user-RIS channel for user $k$, that needs to be estimated. In next subsection we will show how to estimate $h_{s,k}$ for $\forall k \in \mathcal{K}$ with the constructed $H_s$, leading to a significant reduction in the pilot overhead.

B. Stage II: Estimation of the Ambiguous Specific User-RIS Channel

In Stage II, the users are required to transmit the pilot sequences one by one for the estimation of the ambiguous specific user-RIS channel.

Without loss of generality, we consider an arbitrary $k$ from $\mathcal{K}$ and show how to estimate user $k$'s ambiguous specific user-RIS channel, i.e., $h_{s,k}$. Assume $\tau_k$ pilots are allocated for user $k$ in this stage. In addition, assume the pilot symbols satisfy $s_k(t) = 1$ and the transmission power $P$ is equal to 1. Then, the received signal from user $k$ at the BS in time slot $t$ can be expressed as
\[
y_k(t) = H_k \text{Diag}\{e_k\} h_k + n_k(t)
= G_k e_t + n_k(t)
= H_k \text{Diag}\{h_{s,k}\} e_t + n_k(t).
\] (29)

For clear illustration, we still assume that the BS receives the pilot sequence from time slot 1 to time slot $\tau_k$, and thus the received matrix $Y_k = [y_k(1), \ldots, y_k(\tau_k)] \in \mathbb{C}^{N \times \tau_k}$ during user $k$'s pilot transmission is expressed as
\[
Y_k = H_k \text{Diag}\{h_{s,k}\} E_k + N_k
= A_N A_s A_s^H \text{Diag}\{h_{s,k}\} E_k + N_k,
\] (30)
where $E_k = [e_1, \ldots, e_{\tau_k}] \in \mathbb{C}^{M \times \tau_k}$ and $N_k = [n_k(1), \ldots, n_k(\tau_k)] \in \mathbb{C}^{N \times \tau_k}$.

With the obtained $A_N$ in Stage I, user $k$'s received matrix is processed as follows
\[
\hat{A}_N^+ Y_k = A_s A_s^H \text{Diag}\{h_{s,k}\} E_k + \hat{N}_k \in \mathbb{C}^{L \times \tau_k},
\] (31)
where \( N_k = N_k + \Delta A_N A_s A_s^H \text{Diag}\{h_{s,k}\} \). Then, vectorizing (31) and defining \( w_k = \text{vec}(\hat{A}_N Y_k) \in \mathbb{C}^{L \tau_k \times 1} \), we have
\[
\begin{align*}
  w_k & = \text{vec}(\hat{A}_N A_s^H \text{Diag}\{h_{s,k}\} E_k) + \text{vec}(\hat{A}_N N_k) \\
  & = (E_k^T \circ \Lambda_s A_s^H) h_{s,k} + n_k \\
  & = W_k h_{s,k} + n_k,
\end{align*}
\]
where \( W_k = (E_k^T \circ \Lambda_s A_s^H) \in \mathbb{C}^{L \tau_k \times M} \) and \( n_k \) is the corresponding equivalent noise vector given by \( \text{vec}(\hat{A}_N N_k) \in \mathbb{C}^{L \tau_k \times 1} \). The second equality is obtained via \( \text{vec}(\Lambda A \text{Diag}(b) C) = (C^T \circ A) b \).

Recall \( A_{s,k} = a_M(\varphi_{k,j} - \omega_r), \ldots, a_M(\varphi_{k,j} - \omega_f) \) where \( \varphi_{k,j} - \omega_r \) for \( \forall j \in \{1, \ldots, J_k\} \) lies within \([-2 \frac{d_{\text{BS}}}{\lambda}, 2 \frac{d_{\text{BS}}}{\lambda}]\), thus we can formulate (32) as a \( J_k \)-sparse signal recovery problem by using \( h_{s,k} = A_{s,k} \beta_{s,k} \):
\[
\begin{align*}
  w_k & = W_k A_{s,k} \beta_{s,k} + n_k \\
  & = W_k A d_k + n_k,
\end{align*}
\]
where \( A d_k \) in the third equality is the virtual angular domain (VAD) representation of \( h_{s,k} \). \( A \in \mathbb{C}^{M \times D} \) is an overcomplete dictionary matrix (\( D \geq M \)), and the columns of \( A \) contain values for \( a_M(\varphi_{k,j} - \omega_r) \) on the angle grid. \( d_k \in \mathbb{C}^{D \times 1} \) is a sparse vector with \( J_k \) nonzero entries corresponding to the ambiguous gains \( \{\alpha_j, \beta_{s,j}\}_{j=1}^{J_k} \). Accordingly, \( w_k \) is regarded as the equivalent measurement vector for the estimation of \( h_{s,k} \). Hence, the estimation problem of \( h_{s,k} \) corresponding to (33) can be solved using CS-based techniques. To obtain the best CS-based estimation performance, the RIS phase shift matrix \( E_k \) should be designed to ensure that the columns of the equivalent dictionary \( W_k \) are orthogonal. A detailed design for \( E_k \) that achieves this goal can be found in [6]. A simpler method is to choose the random Bernoulli matrix as \( E_k \), i.e., randomly generate the elements of \( E_k \) from \([-1, +1]\) with equal probability [7].

By using the above CS-based method, we obtain the estimate of \( A_{s,k} \) and \( \beta_{s,k} \), and thus the estimate of the ambiguous specific user-RIS channel, denoted by \( h_{s,k} \), can be obtained directly. Denote the estimated common RIS-BS channel in (27) as \( H_k \), the estimate of the cascaded channel for user \( k \) is given by \( \hat{G}_k = H_k \text{Diag}\{h_{s,k}\} \). Finally, the completed estimation of \( G_k \) for \( 1 \leq k \leq K \) is summarized in Algorithm 1.

### Algorithm 1: Estimation of \( G_k \), \( 1 \leq k \leq K \)

**Input:** \( Y \) in (7), \( Y_k \) in (30) for \( 1 \leq k \leq K \).

**Stage I: Estimation of \( H_k \)**

1. Obtain the estimate \( \hat{A}_N \) via the DFT-based method in [6];
2. Choose the reference path, denote its index as \( r \);
3. for \( 1 \leq l \leq L, l \neq r \) do
4. Obtain \( \hat{\alpha}_l \) and \( \hat{\beta}_l \) according to (21) and (22);
5. end for
6. Construct the estimate \( \hat{A}_s \) and \( \hat{A}_s \) based on (23) and (24);
7. Obtain the estimate of the ambiguous common RIS-BS channel, i.e., \( \hat{H}_k = \hat{A}_N \hat{A}_s^H \);

**Stage II: Estimation of \( h_{s,k} \)**

8. for \( 1 \leq k \leq K \) do
9. Estimate the ambiguous specific user-RIS channel \( h_{s,k} \) from the sparse recovery problem associated with (33);
10. Obtain the estimate of the cascaded channel, i.e., \( \hat{G}_k = \hat{H}_k \text{Diag}\{h_{s,k}\} \);
11. end for

**Output:** \( G_k, 1 \leq k \leq K \).

### IV. Simulation Results

In this section, we present simulation results to validate the effectiveness of the proposed method. The channel gains \( \alpha_l \) and \( \beta_{s,j} \) follow a complex Gaussian distribution with zero mean and variance of \( 10^{-3}d_{\text{BS}}^{-2.2} \) and \( 10^{-3}d_{\text{RU}}^{-2.8} \), respectively. Here, \( d_{\text{BS}} \), defined as the distance between the BS and the RIS, is set to 10 m, while, \( d_{\text{RU}} \), defined as the distance between the RIS and the users, is assumed to be 100 m. The antenna spacing at the BS and the element spacing at the RIS are set to \( d_{\text{BS}} = d_{\text{RIS}} = \frac{\lambda}{2} \). The random Bernoulli matrix is chosen as the initial RIS phase shift training matrix \( E_k \) in Stage II of the proposed strategy [7]. The number of paths between the BS and the RIS, and the number of paths between the RIS and users are set to \( L = 5 \) and \( J_1 = \cdots = J_K = 4 \), respectively. The normalized mean square error (NMSE) is chosen as the performance metric, which is defined by
\[
\text{NMSE} = \mathbb{E}\{(\sum_{k=1}^{K} |G_k - \hat{G}_k|^2)/(\sum_{k=1}^{K} |G_k|^2)\}.
\]
We compare the proposed method with the following three channel estimation methods: direct-OMP method [8], DS-OMP method [7], and typical user required method [6].

Since the number of pilots allocated in different stages for the proposed two-stage based method and the typical user required method are different, we consider the users’ average pilot overhead, denoted as \( T \). Moreover, the number of pilots allocated in Stage I and the number of pilots allocated to each user in Stage II for the above-mentioned two methods are denoted as \( T_1 \) and \( T_2 \), respectively. To reduce the error propagation, we allocate more pilots in Stage I and fewer pilots in Stage II. In addition, for fairness, the number of pilots allocated to the users in Stage II of the proposed two-stage based method is the same as that allocated to other users of the typical user required method. For example, 20 pilots and 28 pilots are allocated in Stage I of the proposed two-stage based method and the typical user required method, respectively. The
same 8 pilots are allocated to each user in Stage II of these two methods. Therefore, when the number of users is set to $K = 8$, the average number of pilots for these two methods is given by $T = 10.5$.

Fig. 1 shows the NMSE performance versus signal-to-noise ratio (SNR) for various methods. As expected, the NMSEs of all algorithms decrease with the SNR. In particular, when the SNR is larger than $-5$ dB, the NMSE of the proposed method with $T = 10.5$ decreases linearly with the SNR, which implies that the multi-user diversity gain of the proposed method enables us to estimate the angle information accurately. On the other hand, in the low SNR case, the gap between the proposed method with $T = 10.5$ and the three benchmark methods, i.e., the typical user required method with $T = 10.5$, the direct-OMP and the DS-OMP methods with $T = 12$, is large. Furthermore, at the high SNR region, it is observed that the NMSEs of the direct-OMP method and the DS-OMP method decrease slowly and become saturated. Similar to the trends of these two methods, the NMSE of the typical user required method with $T = 7.5$ ($T_1 = 16$, $T_2 = 6$) is also reduced slightly. This behaviour means that 20 pilots per user for the direct-OMP and the DS-OMP methods, and 7.5 pilots per user for the typical user required method are not enough. On the contrary, the estimation performance of the proposed two-stage method with the same number of pilots, i.e., $T = 7.5$ ($T_1 = 12$, $T_2 = 6$), is improved significantly with the increase of SNR because of the multi-user diversity gain achieved in Stage I.

Fig. 2 shows the NMSE performance of various methods versus the number of users for the ULA-type RIS case. It is observed that the proposed two-stage based method presents a totally different trend compared with the other three benchmark methods. Specifically, the NMSE of the proposed method first reduces rapidly with the number of users, and then decreases marginally when the number of users exceeds 8. By contrast, the NMSEs of the other methods keep generally constant. The reasons for this behavior can be explained as follows. First, the increasing number of users brings larger multi-user diversity gain, which indirectly increases the SNR at the BS in Stage I of the proposed method, and thus its estimation performance is improved significantly. Second, when the number of users increases to a certain value, see $K = 8$ in Fig. 2; the estimation accuracy in Stage I of the proposed method is good enough. In this case, the estimation performance of the proposed method is limited by the estimation error in Stage II for the estimation of the specific user-RIS channel.

V. CONCLUSIONS

In this paper, we proposed a novel two-stage based uplink channel estimation strategy for an RIS-aided multi-user mmWave system, which was shown to fully exploit the correlation among different user’s cascaded channel without choosing a typical user. Due to the inherent ambiguity of the mmWave cascaded channel and the multi-user diversity gain, the proposed strategy achieved pilot overhead reduction and alleviated the error propagation effect significantly. Simulation results validated that the proposed strategy outperforms other existing algorithms in terms of pilot overhead, and its estimation performance can be improved further with the increase of the number of users.

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