Two-photon decay of a light scalar quark-antiquark state

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Abstract. The two-photon decay of a light scalar quarkonium is evaluated in a local and a nonlocal approaches. It is shown that the two-photon decay, driven by a triangle quark-loop diagram, is smaller than 1 keV for a mass below 0.7 GeV.

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INTRODUCTION

It is still debated if scalar resonances below 1 GeV are quarkonia [1], tetraquark [2], molecular states [3] or a mixing of these configurations [4]. If, as suggested in Ref. [5], the light scalar resonances are not quarkonia one is lead to identify the $qq$-states with resonances above 1 GeV where mixing with the scalar glueball takes place [6]. The two-photon decays of scalar resonances, both below and above 1 GeV, are regarded as an important source of informations toward the understanding of this puzzle [7]. However, as pointed out in Ref. [8], care is needed when dealing with this process.

In this work, following Refs. [8, 9, 10], I concentrate on the theoretical description of the two-photon decay of a light scalar-isoscalar quark-antiquark bound state (quarkonium). The amplitude for this process, occurring via a triangle-loop of constituent quarks as in Fig. 1, is studied in a local and a nonlocal field theoretical approaches. In the former case (Fig. 1.a) the scalar field, describing the quark-antiquark bound state, interacts locally with its constituents -the quarks. In the latter case (Fig. 1.b) a nonlocal interaction is introduced, which allows for a realistic treatment of the finite dimension (mean radius of about 0.5 fm) of the quark-antiquark bound state. For this reason, while interesting analytic formulas can be derived in the local treatment, the numerical results of the nonlocal approach represent the here outlined theoretical predictions.

In this work it is shown that the decay width of a scalar-isoscalar quark-antiquark state, with flavor wave-function $\bar{u}u = \frac{1}{2} (\bar{u}u + \bar{d}d)$ and a mass below 0.7 GeV, is smaller than 1 keV. This is against the common belief that a decay width of about 3 - 5 keV would favor a quarkonium interpretation of the resonance $\sigma f_0(600)$. A similar study is here performed for a $\bar{s}s$ scalar bound state: the decay into two photons via the quark-loop diagram of Fig. 1.b is rather small, thus a dominant $\bar{s}s$ wave function of the resonance $f_0(980)$ cannot explain the measured $\gamma\gamma$ decay as reported in Ref. [11].
SCALAR QUARKONIUM INTO $\gamma \gamma$

Amplitude in the local approach

Following Ref. [8] we consider a local interaction Lagrangian (L) describing the (point-like, zero-radius) scalar quarkonium field $\sigma$ with the constituent quarks $u$ and $d$:

$$\mathcal{L}_{\text{int}}^{\text{A}} = \frac{g_L}{2} \sigma (x) \bar{q}(x) q(x)$$  \hspace{1cm} (1)

where $q^T = (u; d)$ is the quark doublet with mass $m_q = m_u = m_d$ (isospin limit), which can be chosen between 0.25 and 0.45 GeV as a variety of low-energy effective approaches confirm. In this work I use $m_q = 0.35$ GeV (similar to the value of Ref. [12]). As shown in Ref. [8] varying the mass in the above range changes only slightly the results.

The decay of $\sigma$ into $\gamma \gamma$ is obtained by evaluating the diagram of Fig. 1.a:

$$\Gamma_{\sigma \gamma \gamma}^{\text{L}} = \frac{\pi}{4} \alpha^2 M_{\sigma}^3 A_{\text{L}}^2 \quad ; \quad A_{\text{L}} = \frac{8 g_{L}^{1} Q_{\sigma} I_{\sigma}^{L}}{2 \pi^2 N_c M_{\sigma}}$$  \hspace{1cm} (2)

where $\alpha$ is the fine structure constant, $N_c = 3$ the number of colors, the charge factor $Q_{\sigma} = \frac{3}{2} (\frac{1}{3} + \frac{1}{3}) = \frac{3}{2} \frac{5}{2}$ corresponds to the flavor wave functions $\sigma = \frac{1}{2} (u \bar{u} + d \bar{d})$ and $M_{\sigma}$ is the mass of the scalar state. The coupling constant $g_{L}^{1} = \sqrt{2} m_{q} F_{\pi}$ with $F_{\pi} = 92$ MeV is obtained by using the Goldberger-Treiman relation and linear realization of chiral symmetry. The amplitude $I_{\sigma}^{L}$ reads [8, 9]:

$$I_{\sigma}^{L} = \frac{2 m_{q}}{M_{\sigma}^2} \left( 1 + \frac{4 m_{q}^2}{M_{\sigma}^2} \right) \arcsin^2 \frac{M_{\sigma}}{2 m_{q}} :$$  \hspace{1cm} (3)

Note that $I_{\sigma}^{L}$ is a real number only if $M_{\sigma} < 2 m_{q}$. For $M_{\sigma} > 2 m_{q}$ an imaginary part, due to the absence of confinement and to the unphysical decay of the sigma meson into a quark-antiquark pair, arises. Thus, extending the calculation to $M_{\sigma} > 2 m_{q}$ is a dangerous step.
Moreover, as shown later the local approach can be trusted only if $M_\sigma$ is well below $2m_q$.

Interestingly, the $\gamma\gamma$ decay of the Higgs particle in the Standard Model \cite{13} proceeds similarly via leptonic loops as in Fig. 1.a. In most cases $M_{\text{Higgs}} > 2m_{\text{lepton}}$ and an imaginary part is present. Being the Higgs an elementary particle interacting locally with the leptons the use (with due changes) of Eq. (2) is allowed below and above threshold. This, however, is not the case for the ‘Higgs’ of QCD, i.e. the scalar quarkonium state, for essentially two reasons: (i) Confinement, as mentioned above. No imaginary part shall arise. (Moreover, the quark propagator and the coupling constants run already at low energy.) (ii) The scalar quark-antiquark bound state is not elementary: a nonlocal interaction with its constituents should be considered as I do in the following.

**Amplitude in the nonlocal approach**

I turn to the scalar quark-antiquark field $\sigma$ by using the following nonlocal (NL) interaction Lagrangian \cite{8, 9, 10}

$$L_{\text{NL}}^{\text{int}}(x) = \frac{g_{\sigma}^{\text{NL}}}{2} \sigma(x) \int d^4y \Phi(y^2) \bar{q}(x+y=2)q(x+y=2);$$

where the delocalization takes account of the extended nature of the quarkonium state by the covariant vertex function $\Phi(y^2)$. The (Euclidean) Fourier transform of this vertex function is taken as $e^{\Phi(k^2)} = \exp(-k^2E^2)$; also assuring UV-convergence of the model. The cutoff parameter $\Lambda$ can be varied between 1 and 2 GeV, corresponding to a spatial extension of the $\sigma$ of about $l = \Lambda / 1.5$ fm. Previous studies \cite{14} have shown that the precise choice of $\Phi(k^2)$ affects only slightly the result, as long as the function falls off sufficiently fast at the energy scale set by $\Lambda$. Moreover, the dependence of the results on $\Lambda$ is very soft \cite{8}. In this work the intermediate value $\Lambda = 1.5$ GeV is used for numerical evaluations.

Within the nonlocal treatment the coupling $g_{\sigma}^{\text{NL}}$ is determined by the so-called compositeness condition $Z_\sigma = 1$ \cite{9, 15}, where $\Sigma_0$ is the derivative of the $\sigma$-meson mass operator given by

$$\Sigma_\sigma(p^2) = N_c \frac{2d^4k}{(2\pi)^4}i \Phi^2(k^2) \text{tr} S_q(k + p = 2)S_q(k + p = 2);$$

and $S_q(k) = (m_q \gamma^\mu k^\mu)^{-1}$ is the quark propagator. Note, the compositeness condition is equivalent to the hadron wave function normalization in quantum field approaches based on the solution of the Bethe-Salpeter/Faddeev equation \cite{16}. At this level $g_\sigma^{\text{NL}}$ is a slowly decreasing function of $M_\sigma$ (details in Refs. \cite{8, 17}).
FIGURE 2. Left panel: $\gamma\gamma$-amplitude in the local (thin solid line below $2m_q$ and dashed line above $2m_q$) and in the nonlocal (thick line) cases. Right panel: decay width within the local (thin line) and nonlocal (thick line) approaches. The values $m_q = 0.35$ GeV and $\Lambda = 1.5$ GeV have been used.

Following Refs. [9, 10] the contribution of the gauge-invariant part\(^1\) of the triangle diagram of Fig. 1.b to the two-photon decay width is given by:

\[ \Gamma_{\sigma\gamma\gamma}^{NL} = \frac{\pi}{4} \alpha^2 M^3_\sigma A_{\sigma}^{NL} \; \Gamma_{\sigma\gamma\gamma}^{NL} = \frac{8\sigma_0}{2\pi^2} N_c Q_\sigma I_{\sigma}^{NL} \]

with \(I_{\sigma}^{NL} = I_{\sigma}^{(1)} + I_{\sigma}^{(2)}\) and

\[ I_{\sigma}^{(1)} = m_q \int \frac{d^4k}{\pi^2 i} \Phi(\frac{q^2}{M_\sigma^2}) \frac{1}{(m^2_q - p_1^2)(m^2_q - p_2^2)(m^2_q - p_3^2)} \]

and

\[ I_{\sigma}^{(2)} = m_q \int \frac{d^4k}{\pi^2 i} \Phi(\frac{q^2}{M_\sigma^2}) \frac{4}{M_\sigma^2} \frac{32}{M_\sigma^2} (kq_1)(kq_2) \frac{1}{(m^2_q - p_1^2)(m^2_q - p_2^2)(m^2_q - p_3^2)} \]

where \(q_1\) and \(q_2\) are the photon momenta and \(p_1 = k + q_1\); \(p_2 = k\); \(p_3 = k\); \(q_2 = (p_1 + p_3) = 2\); Clearly the limit \(\lim_{\Lambda! \to \infty} I_{\sigma}^{NL} = I_{\sigma}^L\) holds.

RESULTS AND DISCUSSIONS

We report in Fig. 2 the amplitude and the two-photon decay width in both the local and nonlocal theories as a function of the mass \(M_\sigma\). A number of comments is in order:

(a) The nonlocal amplitude \(A_{\sigma}^{NL}\) of Eq. (6) (thick line in Fig. 2.a) shows an almost constant behavior from 0 up to threshold. This is a remarkable and stable result. Notice that \(A_{\sigma}^{NL} = A_{\sigma}^L = 0.73\) for \(M_\sigma = 0\) because of the cutoff \(\Lambda\) (i.e., finite dimension) in

\(^1\) Gauge invariance in a nonlocal approach implies that other diagrams, in which the photon couples directly to the nonlocal interaction vertex, are present [9]. However, their contribution is numerically suppressed by a factor $10^{-5}$ and thus is omitted here.
Eqs. (7)-(8). Contrary to the nonlocal case, the local amplitude $A^L$ of Eq. (2) (solid thin line in Fig. 2.a) is enhanced at threshold. This fact is however connected to the non-realistic point-like nature of the sigma field and to the, also non-realistic, constant (not running) behavior of the coupling strength $g^L_{\sigma}$. In fact, while $g^NL_{\sigma}(M_{\sigma} = 0) = 5.56$ 
$\quad g^L_{\sigma} = \frac{1}{\sqrt{2}}m_q = 5.36$, one has just below threshold $g^NL_{\sigma}(M_{\sigma} = 0.69) = 3.48$; thus sizably reduced.

(b) The dashed line in Fig. 2.a describes $A^L$ above threshold, see Eq. (3). As remarked previously the local amplitude $A^L$ is a complex number above $2m_q$; this is a consequence of lack of confinement. The nonlocal amplitude $A^NL$ has been plotted only up to threshold. Although it would be appealing to continue the constant behavior even above $2m_q$, this cannot be done at the present level. A consistent modification of the quark propagator should be applied to study the reaction above $2m_q$.

(c) In Fig. 2.b the decay width as function of $M_{\sigma}$ is shown. While the local and nonlocal approaches deliver similar results for small $M_{\sigma}$ large differences arise for $M_{\sigma} > 2m_q$; where $\Gamma^L_{\sigma\gamma\gamma}$ overshoots $\Gamma^NL_{\sigma\gamma\gamma}$ of a factor 4.

Thus, the final result of the present study is summarized in the following equation:

$$\Gamma^NL_{\sigma\gamma\gamma} < 1 \text{ keV for } M_{\sigma} < 0.7 \text{ GeV.} \quad (9)$$

In Ref. [8] a more detailed study on parameter variations confirms this result. In Ref. [1] the amplitude is calculated within the local treatment in the limit $M_{\sigma} > 2m_q$ and thus a decay widths larger than 1 keV is obtained. However, for all the reasons discussed above I would like to point out that the local approach should not be used for $M_{\sigma} > 2m_q$ where the discrepancy with the nonlocal treatment is most evident. Above threshold the application of the local approach is even more problematic. I conclude this discussion by stressing that the region of applicability of the local approach is restricted to small $M_{\sigma}$ (safely below $2m_q$).

As a last step I evaluate the $\gamma\gamma$ decay width of a scalar state $S$ - $\bar{s}s$ within the nonlocal approach. The corresponding $\Gamma^NL_{S\gamma\gamma}$ has the same form of Eq. (6) up to the charge factor which is now $Q_S = 1.9$: By using a constituent strange mass $m_s = 550$ MeV (close to the value of Ref. [12]) and $\Lambda = 1.5$ GeV a decay width $\Gamma^NL_{S\gamma\gamma} = 0.062$ KeV is obtained. Thus, a dominant $\bar{s}s$ component of the resonance $f_0(980)$ cannot explain -via the quark-loop of Fig. 1.b - the experimental value $\Gamma_{f_0(980)} = 0.39^{+0.10}_{-0.13}$ [11]. The reason why in Ref. [18] a larger value of 0.3 keV is obtained has essentially the same origin (local treatment at threshold) explained in the non-strange case.

CONCLUSIONS

The $\gamma\gamma$ transition of a scalar quarkonium has been studied in a local and a nonlocal field theoretical approaches. The limit of validity of the local approach has been carefully addressed. Within the nonlocal treatment, which takes into account the finite dimension

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2 Note also that a small change in $M_{\sigma}$ causes a large change of $\Gamma^L_{\sigma\gamma\gamma}$ (Fig. 2.b) thus making a clear prediction close to threshold difficult.
of the quarkonium state, it is shown that the quark-loop contribution to the decay of a scalar quarkonium with wave function $\frac{1}{2}(uu + dd)$ into two-photons is smaller than 1 keV for a mass below 0.7 GeV. Thus, an eventual confirmation of a large two-photon width of the resonance $f_0(600)$ does not favor a quarkonium interpretation of the latter. A similar consideration holds for an $ss$ interpretation of the resonance $f_0(980)$. Loops of kaons and pions should be included as a further contribution to $\gamma\gamma$ decay width which, although suppressed according to large-$N_c$ counting rules, may play an important role. The corresponding evaluation within the nonlocal treatment represent an interesting outlook.

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