Spin-orbit scattering in $d$-wave superconductors

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Abstract. – When non-magnetic impurities are introduced in a $d$-wave superconductor, both thermodynamic and spectral properties are strongly affected if the impurity potential is close to the strong resonance limit. In addition to the scalar impurity potential, the charge carriers are also spin-orbit coupled to the impurities. Here it is shown that i) close to the unitarity limit for the impurity scattering, the spin-orbit contribution is of the same order of magnitude as the scalar scattering and cannot be neglected, ii) the spin-orbit scattering is pair-breaking and iii) induces a small $i d_{xy}$ component to the off-diagonal part of the self-energy.

In high-$T_c$ superconductors, disorder has important effects on both thermodynamic and spectral properties. The critical temperature $T_c$ and the superfluid density $\rho_s$ are lowered by non-magnetic impurity substitution [1-4] and disorder induced by irradiation [5, 6]. Recent ARPES data show clearly how disorder leads to a redistribution of spectral intensity by adding new states at the Fermi level [6]. The basic elements of the current theory have been inspired by previous studies on heavy-fermion superconductors and are given by the anisotropy of the order parameter and the strong resonance limit for the impurity potential [7, 8]. These elements, adjusted to describe condensates with a $d$-wave symmetry of the order parameter, are able to account for most of the features observed by experiments on high-$T_c$ $d$-wave superconductors [9-11]. However, discrepancies still exist, like the overestimation of the $T_c$ suppression [12]. In order to correct this situation, and to provide a more realistic picture, several improvements of the theory have been proposed [13-15], and, recently, the effect of spatial variation of the order parameter has been taken into account [12,16,17].

In addition to the scalar impurity potential, the charge carriers are also spin-orbit coupled to the impurities. So far, this additional scattering channel has not been considered because the spin-orbit interaction is believed to provide, if any, only negligible effects (at least in the absence of a Zeeman magnetic field). This argument is based on the observation that the spin-orbit potential is of order $v_{so} \sim \Delta g v$, where $v$ is the impurity potential and $\Delta g$ is the shift of the $g$-factor [18]. The value of $\Delta g$ depends on the specific impurity, however its order of magnitude is roughly $\Delta g \simeq 0.1$. From this estimate, it is expected therefore...
that the spin-orbit scattering rate $1/\tau_{so} \simeq N_0 v_{so}^2$, where $N_0$ is the charge carriers density of states, should be at most of order $1/\tau_{so} \sim 10^{-2}/\tau_{imp}$, therefore negligible with respect to the scalar impurity scattering rate $1/\tau_{imp}$ [19]. Although such an estimate is correct in the Born approximation (weak scattering) nevertheless it underestimates the effect by orders of magnitude in the strong resonance limit, believed to be valid for high-$T_c$ superconductors. To illustrate such a substantial discrepancy between the Born and the unitarity limit, let us first consider the self-consistent $t$-matrix solution of the impurity problem for $\Delta g = 0$. In this case the electron (hole) propagator is $G^{-1}(k, i\omega_n) = i\omega_n – \rho_{3\Sigma}(k) – \rho_1 \Delta(k)$, where $\Delta(k) = \Delta \cos(2\phi)$ is the $d$-wave order parameter and $\phi$ is the polar angle, $\epsilon(k)$ is the electron dispersion for a half-filled band and $\rho_1, \rho_1$ are Pauli matrices. The renormalized Matsubara frequency $i\tilde{\omega}_n$ satisfies the following equation [8,11]:

$$i\tilde{\omega}_n = i\omega_n + \Gamma g_0(i\omega_n)/(c^2 - g_0(i\omega_n)^2),$$

where $g_0(i\omega_n) = \langle i\tilde{\omega}_n^2 [\omega_n^2 + \Delta(k)^2]^{1/2} \rangle$ and $\langle \cdots \rangle$ is the average over the polar angle $\phi$. In eq. (1), $\Gamma = n_i/(\pi N_0)$ and $c = 1/(\pi N_0 v)$, where $n_i$ is the impurity concentration. For $c \gg 1$, eq. (1) reduces to the Born limit while for $c \ll 1$ it leads to the unitarity, or strong resonant, limit. Now, let us suppose that the spin-orbit impurity scattering leads to a renormalization contribution of the same form as eq. (1). Since $v_{so}/v \sim \Delta g$, the renormalization induced by both the impurity and the spin-orbit scatterings can be estimated by:

$$i\tilde{\omega}_n = i\omega_n + \Gamma g_0(i\omega_n)/(c^2 - g_0(i\omega_n)^2) + \Gamma g_0(i\omega_n)/((c/\Delta g)^2 - g_0(i\omega_n)^2).$$

For $c \gg 1$, $i\tilde{\omega}_m \simeq i\omega_n + \Gamma g_0(i\omega_n)c^{-2}(1 + \Delta g^2)$ and, as expected in the Born limit, the contribution of spin-orbit scattering is $\Delta g^2$ times smaller than that of scalar impurity scattering. On the other hand, for strong scattering, $c/\Delta g$ can be very small and eventually it vanishes in the unitarity limit $c \to 0$. As a result, in this limit the spin-orbit scattering leads to a renormalization of the same order as the impurity scattering, namely of order $\Gamma/\Delta$.

To provide more solid grounds to the above simple picture, it is necessary to treat the impurity and spin-orbit interactions on the same level by generalizing the usual $t$-matrix approach for the impurity scattering also to the spin-orbit contribution. To this end, let us start by considering the spin-orbit impurity potential. Two-dimensionality is often assumed in describing the main electronic excitations at least for some of the high-$T_c$ superconductors. In the present context, the reduced dimensionality has the following implication. If the charge carriers are confined to move in the $x-y$ plane and the impurity potential is $V(r) = v \sum_{i,k} \exp[i\mathbf{k} \cdot (\mathbf{r} - \mathbf{R}_i)]$, where $\mathbf{R}_i$ denotes the impurity positions, the spin-orbit interaction assumes the following form:

$$V_{so}(r) = i\eta_{so} v \sum_{i,k} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{R}_i)} [\mathbf{k} \times \mathbf{p}] \cdot \sigma = i\Delta g v \sum_{i,k} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{R}_i)} [\mathbf{k} \times \mathbf{p}]_z / k_F^2 \sigma_z,$$

where $\mathbf{p} = -i\nabla_r$ is the momentum operator and the spin-orbit coupling has been parametrized by $\eta_{so} v = \Delta g v / k_F^2$, where $k_F$ is the Fermi momentum. The effect of two-dimensionality is therefore to couple the electron spin only along the $z$-direction. Hence, by choosing the $z$-axis as the direction of spin quantization, the spin-orbit interaction (3) does not mix the spin components. The generalized Green’s function in the particle-hole spin space is

$$G(k, i\omega_n)^{-1} = G_0(k, i\omega_n)^{-1} - \Sigma(k, i\omega_n),$$

where $G_0(k, i\omega_n)^{-1} = i\omega_n - \rho_{3\Sigma}(k) - \rho_2 \tau_2 \Delta(k)$ and the Pauli matrices $\rho_i$ and $\tau_j$ act on the particle-hole and spin subspaces, respectively [20]. In the self-consistent $t$-matrix approxima-
tion the self-energy is $\Sigma(k, \omega_n) = n_i T_{tot}(k, k, i\omega_n)$ where the $t$-matrix satisfies the following equation:

$$T_{tot}(k, k', i\omega_n) = u(k, k') + \sum_{k''} u(k, k'')G(k'', i\omega_n)T_{tot}(k'', k', i\omega_n),$$  \hspace{1cm} (5)$$

where $u(k, k') = \rho_3 v + i\tau_3 \Delta g v[k \times \hat{k}]_z$ is the impurity potential including the spin-orbit contribution. Because of the angular dependence of the spin-orbit interaction, it can be shown that the $t$-matrix (5) reduces to $T_{tot}(k, k', i\omega_n) = T(i\omega_n) + T_{so}(k, k', i\omega_n)$ where

$$T(i\omega_n) = \rho_3 v + \rho_3 v \sum_k G(k, i\omega_n)T(i\omega_n)$$  \hspace{1cm} (6)$$
is the usual momentum-independent contribution from non-magnetic impurities [8] and

$$T_{so}(k, k', i\omega_n) = i\tau_3 \Delta g v[k \times \hat{k}]_z + i\Delta g v \sum_{k''}[k \times \hat{k}]_z \tau_3 G(k'', i\omega_n)T_{so}(k'', k', i\omega_n),$$  \hspace{1cm} (7)$$
is the $t$-matrix for the spin-orbit interaction. Before proceeding with the complete solution of eq. (7), it is useful to analyze the lowest-order contributions in $\Delta g$. By replacing $G(k, i\omega_n)$ with $G_0(k, i\omega_n)$, the expansion of eq. (7) up to the third order in $\Delta g$ leads to a spin-orbit part of the self-energy of the form $\Sigma_{so}(k, i\omega_n) = \Sigma_{so}^{(2)}(k, i\omega_n) + \Sigma_{so}^{(3)}(k, i\omega_n)$, where the first term is the usual Born contribution $\Sigma_{so}^{(2)}(k, i\omega_n) = n_i \Delta g^3 v^2 \sum_{k'}[k \times \hat{k}']^2 \tau_3 G_0(k', i\omega_n)\tau_3$ which renormalizes both the frequency and, contrary to the normal impurity scattering, the gap function. The term proportional to $\Delta g^3$ is instead

$$\Sigma_{so}^{(3)}(k, i\omega_n) = -in_i(\Delta g v)^3 \sum_{k', k''} [k \times \hat{k}]_z [k' \times \hat{k}']_z [k'' \times \hat{k}]_z \tau_3 G_0(k', i\omega_n)\tau_3 G_0(k'', i\omega_n)\tau_3$$

which contributes to the off-diagonal part of the self-energy. This term is proportional to $i\sin(2\phi)$ and has therefore a $id_{xy}$ symmetry which is a consequence of the angular dependence of the spin-orbit potential. Note that eq. (8) closely resembles the off-diagonal contribution found by Balatsky in the context of spin-orbit coupling of the condensate to magnetic scattering centres [21]. Although as a function of $\phi$, $\Sigma_{so}^{(3)}(k, i\omega_n)$ has maximum contribution where the gap function $\Delta\cos(2\phi)$ vanishes, it does not open any gap in the excitation spectrum. In fact, after analytical continuation $i\omega_n \rightarrow \omega$, it can be shown that at the nodes of $\Delta\cos(2\phi)$, $\phi = \pm \pi/4, \pm 3\pi/4$, the real part of the pole of the Green’s function for $k = k_F$ satisfies $\omega = \Sigma_{so}^{(3)}(k_F, \omega) \propto \Gamma(\Delta g v)^3(\omega/2\Delta)\log(2\Delta/|\omega|)$, so that $\omega = 0$ is the solution and no additional gap is opened. It is worth stressing that this result is based on the assumption that the system under investigation has particle-hole symmetry. If this condition is relaxed, the anomalous self-energy (8) could provide a finite gap of $d_{xy}$ symmetry.

Equation (8) suggests that the Green’s function solution of the $t$-matrix problem is of the form

$$G(k, i\omega_n)^{-1} = i\tilde{\omega}_n - \rho_3 \tilde{\epsilon}_n - \rho_2 \tau_2 [\tilde{\Delta}_n(\phi) + i\tau_3 \tilde{\Omega}_n(\phi)],$$  \hspace{1cm} (9)$$

where $\tilde{\omega}_n, \tilde{\epsilon}_n, \tilde{\Delta}_n(\phi)$ and $\tilde{\Omega}_n(\phi)$ are frequency-dependent quantities which must be calculated self-consistently. Note that in eq. (9), $\tilde{\Omega}_n(\phi)$ is the off-diagonal contribution which reduces to
eq. (8) at the lowest order in $\Delta g$. The $t$-matrix for the spin-orbit interaction (7) is solved by setting $T_{so}(k, k', i\omega_n) = i\Delta g v[\hat{k} \times t(k', i\omega_n)]_z \tau_3$, where
\[
t(k', i\omega_n) = k' + i\Delta g v \sum_{k''} \tilde{G}(k'' , i\omega_n)[\hat{k}'' \times t(k', i\omega_n)]_z.
\tag{10}
\]

The above equation is actually a system of two coupled equations for the components $t_{xz}(k', i\omega_n)$ and $t_{yz}(k', i\omega_n)$ which can be explicitly solved and, after some algebra, the spin-orbit contribution $T_{so}(k, k, i\omega_n)$ to the total $t$-matrix becomes
\[
T_{so}(k, k, i\omega_n) = i\Delta g v [\hat{k}_x \hat{k}_y][A^{-1}_{yx}(i\omega_n) - A^{-1}_{xy}(i\omega_n)]\tau_3 + 
+ (\Delta g v)^2 \sum_{k'} \left[ A^{-1}_{xx}(i\omega_n)(\hat{k}_x \hat{k}_y)^2 + A^{-1}_{yy}(i\omega_n)(\hat{k}_y \hat{k}_x)^2 \right] \tau_3 \tilde{G}(k', i\omega_n) \tau_3,
\tag{11}
\]
where $A^{-1}_{xy}(i\omega_n)$ and $A^{-1}_{yx}(i\omega_n)$ are the inverse of the $4 \times 4$ matrices
\[
A_{xy}(i\omega_n) = 1 - (\Delta g v)^2 \left[ \sum_k (\hat{k}_x)^2 \tau_3 \tilde{G}(k, i\omega_n) \right] \left[ \sum_k (\hat{k}_y)^2 \tau_3 \tilde{G}(k, i\omega_n) \right],
\tag{12}
\]
\[
A_{yx}(i\omega_n) = 1 - (\Delta g v)^2 \left[ \sum_k (\hat{k}_y)^2 \tau_3 \tilde{G}(k, i\omega_n) \right] \left[ \sum_k (\hat{k}_x)^2 \tau_3 \tilde{G}(k, i\omega_n) \right].
\tag{13}
\]

Note that the first term in the right-hand side of eq. (11) is proportional to $\hat{k}_x \hat{k}_y = (1/2) \sin(2\phi)$ and, in fact, it reduces to eq. (8) at the lowest order in $\Delta g$. Moreover, this term would be absent for $s$-wave superconductors since in this case $A_{xy}(i\omega_n) = A_{yx}(i\omega_n)$.

Plugging eq. (9) into (12) and (13) permits to invert $A_{xy}(i\omega_n)$ and $A_{yx}(i\omega_n)$, and the final equations for $\tilde{\omega}_n$, $\tilde{\epsilon}_n$, $\tilde{\Delta}_n(\phi)$ and $\tilde{\Omega}_n(\phi)$ are obtained by demanding self-consistency for eqs. (4), (6), (9), (11)-(13). A consistent solution requires that $\tilde{\Delta}_n(\phi) = \tilde{\Delta}_n \cos(2\phi)$ and $\tilde{\Omega}_n(\phi) = \tilde{\Omega}_n \sin(2\phi)$, where
\[
\tilde{\Delta}_n = \Delta + \Gamma \frac{(c/\Delta g)^2 - (f_n^2 - g_n^2)}{[(c/\Delta g)^2 - (f_n^2 + g_n^2)]^2 - 4f_n^2g_n^2} f_n,
\tag{14}
\]
\[
\tilde{\Omega}_n = -2\Gamma \frac{f_n g_n (c/\Delta g)}{[(c/\Delta g)^2 - (f_n^2 + g_n^2)]^2 - 4f_n^2g_n^2},
\tag{15}
\]
where
\[
g_n = \left< \frac{i\tilde{\omega}_n \sin(\phi)^2}{\sqrt{\tilde{\Delta}_n(\phi)^2 + \tilde{\Omega}_n(\phi)^2 + \tilde{\omega}_n^2}} \right>, \quad f_n = \left< \frac{\tilde{\Delta}_n(\phi) \sin(\phi)^2}{\sqrt{\tilde{\Delta}_n(\phi)^2 + \tilde{\Omega}_n(\phi)^2 + \tilde{\omega}_n^2}} \right>,
\tag{16}
\]
while the equation for the renormalized frequency $\tilde{\omega}_n$ is
\[
i\tilde{\omega}_n = i\omega_n + \frac{g_{0n}}{c^2 - g_{0n}} + \Gamma \frac{(c/\Delta g)^2 + (f_n^2 - g_n^2)}{[(c/\Delta g)^2 - (f_n^2 + g_n^2)]^2 - 4f_n^2g_n^2} g_n,
\tag{17}
\]

and
\[
g_{0n} = \left< \frac{i\tilde{\omega}_n}{\sqrt{\tilde{\Delta}_n(\phi)^2 + \tilde{\Omega}_n(\phi)^2 + \tilde{\omega}_n^2}} \right>.
\tag{18}
The set of equations (14)-(18) represents the main result of this paper and several features can be already outlined. First, although the equation for the renormalized frequency (17) is more complex than the simple minded eq. (2), the conjecture discussed in the introduction is confirmed, i.e., as long as $c/\Delta g \ll 1$ the renormalization due to spin-orbit interaction is of the same order of magnitude as the one induced by the impurity scattering. In addition, the spin-orbit scattering renormalizes also the gap function and induces the additional off-diagonal self-energy $\tilde{\Omega}_n$. From eq. (15) it can be seen that $\tilde{\Omega}_n \propto (\Gamma/\Delta)(\Delta g/c)^3$ for $\Delta g/c \ll 1$ and $\tilde{\Omega}_n \propto (\Gamma/\Delta)c/\Delta g$ for $c/\Delta g \ll 1$ and is therefore much smaller than the spin-orbit parts of $\Delta_n$ and $\tilde{\omega}_n$ which are of order $(\Gamma/\Delta)(\Delta g/c)^2$ for $\Delta g/c \ll 1$ and $\Gamma/\Delta$ for $c/\Delta g \ll 1$, respectively.

Equations (14)-(18) must be completed with the equation for the order parameter $\Delta$ which, if the pairing interaction is $V_{\text{pair}}(k, k') = -V_{\text{pair}} \cos(2\phi) \cos(2\phi')$, reduces to

$$\Delta = \frac{V_{\text{pair}}}{4} T \sum_n \sum_k \cos(2\phi) \text{Tr} [\rho_2 \tau_2 G(k, i\omega_n)] = \lambda \pi T \sum_n \left( \frac{\tilde{\Delta}_n(\phi) \cos(2\phi)}{\tilde{\Delta}_n(\phi)^2 + \tilde{\Omega}_n(\phi)^2 + \tilde{\omega}_n^2} \right), \quad (19)$$

where $\lambda = V_{\text{pair}} N_0$ is the coupling constant. The critical temperature $T_c$ is obtained from eq. (19) by setting $\Delta_n \to 0$ and $\tilde{\Omega}_n \to 0$ in eqs. (14)-(19). The resulting critical temperature satisfies an Abrikosov-Gorkov type of relation [22]:

$$\log \left( \frac{T_c}{T_{c0}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{\Gamma_n + \Gamma_{so}}{2\pi T_c} \right), \quad (20)$$

where $\psi$ is the di-gamma function and $T_{c0}$ is the critical temperature for the pure system ($\Gamma = 0$). The impurity and spin-orbit scattering parameters are

$$\Gamma_n = \frac{\Gamma}{c^2 + 1}, \quad \Gamma_{so} = \frac{3(2c/\Delta g)^2 + 1}{[(2c/\Delta g)^2 + 1]^2}. \quad (21)$$

Superconductivity is destroyed for the critical value $\Gamma^*_n + \Gamma^*_{so} = \pi T_{c0}/2\gamma$ ($\gamma \simeq 1.781$). For $\Delta g = 0$, the $c \to 0$ limit leads to the usual result for scalar impurities $\Gamma^* = \pi T_{c0}/2\gamma$ [11] while, for $\Delta g \neq 0$, $\lim_{c \to 0} \Gamma^* = \pi T_{c0}/4\gamma$, i.e., half the value of the impurity concentration in the unitarity limit without spin-orbit scattering. This result clearly shows that, in the unitarity limit, spin-orbit scattering has a strong pair-breaking effect of the same size as scalar impurity scattering. This feature is a consequence of the infinite resummation of the $t$-matrix which gives rise to a non-analytic point at $c = 0$, $\Delta g = 0$, and the limit $\Delta g \to 0$, $c = 0$ differs from $c \to 0$, $\Delta g = 0$. The reduced critical-temperature order parameter $\Delta/\Delta_0$ (solid lines) and the reduced zero-temperature order parameter $\Delta/\Delta_0$ (dashed lines) are shown in fig. 1 as a function of $\Gamma/\Delta_0$ for $c = 0.1$ and $\Delta g = 0.0, 0.05, 0.1, 0.15$ and 0.2 (from right to left). $\Delta$ is calculated numerically from the $T \to 0$ limit of eq. (19) and $\Delta_0$ is the gap for the pure case (in the weak coupling limit). As a function of impurity concentration, the suppression of both $T_c/T_{c0}$ and $\Delta/\Delta_0$ is stronger for smaller values of $c/\Delta g$. In the inset of fig. 1, $T_c/T_{c0}$ and $\Delta/\Delta_0$ are shown for the limiting cases $c \to 0$, $\Delta g = 0$ (a) and $\Delta g \to 0$, $c = 0$ (b).

Another thermodynamic quantity of interest is the superfluid density $\rho_s$ which is calculated from

$$\rho_s = 2\pi m N_0 v_F^2 T \sum_m \left( \frac{\tilde{\Delta}_n(\phi) \cos(\phi)^2}{[\tilde{\Delta}_n(\phi)^2 + \tilde{\Omega}_n(\phi)^2 + \tilde{\omega}_n^2]^{3/2}} \right), \quad (22)$$

where $v_F$ and $m$ are the electron Fermi velocity and mass, respectively [23]. In fig. 2 it is shown $T_c/T_{c0}$ as a function of the zero-temperature limit of $\rho_s/\rho_{s0}$, where $\rho_{s0}$ is the superfluid density for the pure system, for $\Delta g = 0.1$ and different values of $c$. Note that, by lowering
c. grimaldi: spin-orbit scattering in d-wave superconductors

Fig. 1

Fig. 1. – Reduced critical temperature (solid lines) and order parameter (dashed lines) as a function of $\Gamma/\Delta_0$ for $c = 0.1$ and $\Delta g = 0.0, 0.05, 0.1, 0.15$, and $0.2$ (from right to left). Inset: the same quantities for the $c \to 0, \Delta g = 0$ limit (a) and for the $\Delta g \to 0, c = 0$ limit (b).

Fig. 2

Fig. 2. – Reduced critical temperature as a function of the reduced superfluid density for $\Delta g = 0.1$ and different values of $c$. Inset: superfluid density in the $c \to 0, \Delta g = 0$ limit (a) and in the $\Delta g \to 0, c = 0$ limit (b).

c, the curves move upwards and that this effect is visible also for the limiting cases $c \to 0, \Delta g = 0$ (a) and $\Delta g \to 0, c = 0$ (b) reported in the inset. This feature is given by the spin-orbit renormalization of $\tilde{\Delta}_n$, otherwise absent when $\Delta g = 0$. In ref. [12], it has been stressed that when $T_c/T_{c0}$ is plotted against the zero-temperature limit of $\rho_s/\rho_{s0}$ the experimental data lie above the theoretical curve corresponding to the unitarity limit. As inferred from both fig. 2 and the inset, the inclusion of spin-orbit scattering tends to cure this discrepancy.

In addition to thermodynamic quantities, the spin-orbit interaction affects also the spectral properties of d-wave superconductors. In fact, nonzero values of $\Delta g$ contribute to the amount of gapless excitations already provided by the resonant scattering with impurities. This feature can be investigated by performing the analytical continuation of eqs. (14)-(18) by setting $i\omega_n \to \omega$ and $i\tilde{\omega}_n \to \tilde{\omega}$ and then plugging the results into the quasiparticle density of states

![Fig. 3](image)

Fig. 3. – Quasiparticle density of states for $\Gamma = 0.05$, $c = 0.05$ (a) and $\Gamma = 0.1$, $c = 0.1$ (b) and different values of $\Delta g$. Insets: corresponding zero-energy density of states as a function of $\Delta g$. 
\[ N(\omega) \text{ given below:} \]
\[ \frac{N(\omega)}{N_0} = \text{sgn}(\tilde{\omega}) \text{Re} \left\langle \frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2 - \tilde{\Delta}(\phi)^2 - \Omega(\phi)^2}} \right\rangle. \]  

The resulting density of states is plotted in fig. 3 for \( \Gamma/\Delta_0 = 0.05, c = 0.05 \) (a) and for \( \Gamma/\Delta_0 = 0.1, c = 0.1 \) (b). In both cases, nonzero values of \( \Delta g \) provide an additional contribution to the gapless states lowering at the same time the intensity of the coherence peaks at \( \omega \approx \Delta \). In the insets of fig. 3 it is also shown how the zero-energy density of states \( N(0)/N_0 \) is affected by the spin-orbit coupling.

In summary, it has been shown that, if the impurity potential is close to the unitarity limit, the spin-orbit coupling to the impurities is as important as the scalar impurity potential and its effects cannot be neglected. This result points toward a critical re-examination of the existing experimental data in addition to the recent theoretical developments based on the spatial variation of the order parameter. As a final remark, it should be noted that a promising experimental route for the estimation of the spin-orbit interaction in \( d \)-wave superconductors is provided by the Zeeman response under an external magnetic field \([19, 24]\). Preliminary results suggest that nonzero values of \( \Delta g \) drastically affect the Zeeman response of \( d \)-wave superconductors \([25]\).

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