THE POOLTABLE ANALOGY TO AXION PHYSICS *

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Abstract

An imaginary character named TSP finds himself in a playroom whose floor is tilted to one side. However, the pooltable in the playroom is horizontal. TSP wonders how this can be. In doing so, he embarks upon an intellectual journey which parallels that which has been travelled during the past two decades by physicists interested in the Strong CP Problem and axion physics.

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Consider the physics involved in playing snooker. The rules of the game require that the pooltable be horizontal. If the pooltable is not horizontal, a certain symmetry is broken. Let us call that symmetry $S$. If $S$ is broken, the balls tend to roll to one side, which is no good. The rules of pooltable physics require that $S$ be a good symmetry.

Similarly, the rules followed by the strong interactions obey discrete symmetries $P$ and $CP$. $P$ is parity and $CP$ is the product of parity with charge conjugation. These symmetries of the strong interactions have been known for a long time. In fact, the discovery that the weak interactions violate $P$ and $CP$ was a big surprise because physicists, used to seeing the strong interactions and also the electromagnetic interactions obey $P$ and $CP$, had a hard time conceiving that these symmetries could be violated at all.

We may imagine that the people playing snooker have done so for a very long time. Let us even imagine that they have always lived on the pooltable. They have a hard time conceiving that the $S$ symmetry could be broken. However, some day they discover the great wide world. They jump off the table and find themselves on the floor of the playroom. Now, to continue our analogy with the Standard Model of particle physics, we will assume that the playroom floor is not horizontal. The snookerplayers are astonished to discover that the wider world does not respect the $S$ symmetry they had become so used to. The playroom is skew somehow, which is very disconcerting. But after a while the snookerplayers become accustomed to this. They abandon the prejudice that $S$ should be a good symmetry.

The snookerplayers have become comfortable with the wider world and the fact that the $S$ symmetry is broken. However, one of them whom they call TSP (which could be short for Thinking Snooker Player) is deeply troubled. TSP realizes there is something wrong with the world he is living in. The playroom floor is not horizontal because the $S$ symmetry is broken. That’s fine. But why is the pooltable horizontal?

There is similarly something wrong with the Standard Model. This is called the “Strong $CP$ Problem”. The Standard Model violates $P$ and $CP$. How can the strong interactions, which are part of the Standard Model, conserve those symmetries? Within the Standard Model, it is as surprising to have the strong interactions conserve $P$ and $CP$ as it is surprising to find a horizontal pooltable in a playroom which is itself not horizontal. The Standard Model takes just pride in being able to explain the violation of $CP$ in an economical and natural way, by allowing the Yukawa couplings in the model to have arbitrary complex phases. This virtue is often emphasized. However, if the Yukawa couplings have arbitrary complex phases, then the $\theta$ angle of QCD has an arbitrary value as well, let us say any number between zero and $2\pi$, in which case the strong interactions violate $P$ and $CP$ in blatant fashion. This is contrary to observation. To be explicit, the upper limit on the neutron electric dipole moment, which provides the most sensitive test of $P$ and $CP$ violation by the strong interactions, requires that $\theta$ of QCD be less than $10^{-9}$.

His curiosity piqued, TSP sets out to check whether the pooltable is as horizontal as it appears. (Yeah, TSP is no casual observer. He’s got the soul of a physicist.) He finds that the pooltable is as horizontal as he can make out and, after much work, having pushed to its limits the measurement technology available to him, concludes that any deviation of the pooltable from perfect horizontality must be characterized by an angle less than $10^{-9}$. 

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Having been around and stuff, TSP knows that one part in a billion is easier said than done. He is astounded. His stomach cringes with the fear induced by the discovery of a fact at once bizarre and unexplained. “Someone is playing a trick on us, that’s for sure,” he thinks to himself.

TSP figures the person who made the pooltable compensated for the slant of the playroom floor by adjusting the lengths of the pooltable legs. This is illustrated in Fig. 1. To do this, the pooltable maker measured the angle between the vertical and the playroom floor. The vertical direction is determined by gravity and is manifested by the plumb, a well-known and wonderful tool. After having taken his measurements, he designed the pooltable legs accordingly, with a precision of one part per billion. TSP muses that if the pooltable maker has many customers, he must spend a lot of effort adjusting his pooltables to the various angles between the vertical and the floors of his customers’ playrooms. Each pooltable has to be individually build to insure the $S$ symmetry that the customers demand for their snooker playing, to the tune of one part per billion.

Some time passes by. One day, as TSP sat around thinking about the life of the pooltable maker, an idea occurred to him. If he himself TSP were in the pooltable making business, what he TSP would do is build each pooltable on a post that can pivot on an axle. At the end of the post opposite the pooltable is a big weight. The axle is mounted on a tripod. TSP’s contraption is illustrated in Fig. 2. The point is that gravity will automatically pull the weight down, the post vertical and the pooltable horizontal. Et voila! You see, all pooltables can be made identical now, with tremendous savings in effort and production costs. TSP gets excited at the idea of the fortune he could make in the pooltable manufacturing business. His pooltables would adjust themselves automatically in any playroom, just under the influence of gravity. The beauty of the scheme is that it is gravity which decides what’s vertical and what is not. So it can do the job of making the pooltable horizontal, just by itself!

What TSP just discovered is the analog of the Peccei-Quinn (PQ) solution to the strong $CP$ problem of the Standard Model of particle physics. Peccei and Quinn slightly modified the Standard Model in such a way as to make the $\theta$ angle of QCD a dynamical variable. There are non-perturbative effects which produce $P$ and $CP$ violation in QCD if the $\theta$ angle differs from zero or $\pi$. The analog of QCD is the physics on the pooltable; the analog of the $\theta$ angle is the misalignment of the pooltable from the horizontal; the analog of the non-perturbative effects that make QCD physics depend upon the $\theta$ angle is gravity which makes pooltable physics sensitive to lack of horizontalness of the table; the analog of $P$ and $CP$ symmetry in QCD is $S$ symmetry in the pooltable physics; and so on. In the PQ mechanism, the non-perturbative effects which make QCD physics depend upon $\theta$, pull $\theta$ to zero once the model has been arranged so that $\theta$ becomes a dynamical variable. In TSP’s contraption, gravity, which makes pooltable physics sensitive to a slant of the pooltable, removes any such slant once an axle is introduced to allow the pooltable to pivot.

TSP is pleased with himself, although it turns out he cannot make a fortune based on his insight. For some reason, he is confined to the playroom and this keeps him from going in the pooltable manufacturing business. More time passes by. One day, in a more humble mood than the one he got into following his theoretical discovery of the mechanism that
can straighten out pooltables (he had become very excited then), a fresh idea occurs to him. It might be that the pooltable maker who made the pooltable where TSP lives also discovered the mechanism for straightening pooltables and that he incorporated it into the pooltable in TSP’s room. TSP becomes very curious about this possibility. Unfortunately, all around the pooltable hangs a dark cloth which hides from view whatever supports it. But, after a while, TSP realizes that it is not necessary to see the support structure to deduce whether or not the pooltable has been build with the pooltable straightening mechanism. The point is that the physics of playing snooker on a pooltable with the mechanism differs from the physics of playing snooker on a regular pooltable, without the mechanism. On a regular table, as in Fig. 1, when the ball hits the rim, it bounces back with the same energy as it had before hitting the rim. (For the sake of argument, we are neglecting the absorption of energy by the rubber on the rim.) But on a pooltable which has the straightening mechanism of Fig. 2, a ball does not bounce off the rim with the same energy because some of the energy gets transferred to an overall oscillation of the pooltable about its horizontal equilibrium position. In the past, snookerplayers always perceived that the ball bounces back with the same energy but, of course, they had no reason to question whether this is true with infinite precision.

Let me digress briefly to explain what is going through TSP’s mind at this moment. TSP’s fellow snookerplayers have always thought him a bit odd because, although TSP was recognized from early on to be quite smart, he didn’t achieve much in real life. TSP just sits around thinking about this and that but he never does much. His fellow snookerplayers had thought TSP was acting very strangely when he had insisted that there is something “terribly wrong” about a horizontal pooltable in a room which is itself not horizontal. “What’s so wrong about that?” they said to each other, “It’s actually good to have a horizontal pooltable to play snooker”. Their viewpoint is just so, oh, totally different from TSP’s. Of course, TSP enjoys thinking and that’s why he does that rather than anything else. So, contrary to what his fellow snookerplayers believe, TSP has a happy life. Still, he would like it better if he were more appreciated. Now, with his theoretical discovery of the pooltable straightening mechanism, TSP sees an opportunity to impress his fellows. If he can show that the rules of snooker are not quite what they appear to be and hence that there are new possibilities to the game, that is something his fellow snookerplayers would appreciate. They did not care to wonder why the pooltable is horizontal even though the playroom floor is tilted, but if balls can give up some of their energy to an oscillation of the pooltable and hence an oscillation of the pooltable can give extra energy to the balls, well, of course, that is very important and they will want to know about that.

So TSP sets hard to work. His goal is simple. He wants to produce an oscillation of the pooltable and then put into evidence that such an oscillation is occurring. For example, he puts one ball some place on the pooltable next to the rim. Then he shoots another ball very hard against the rim on the opposite side of the table. Some of that energy gets absorbed into an oscillation of the table. Then some of the energy in the table oscillation gets transferred to the first ball which was sitting next to the table rim. This would be the experimental signature of the fact that the pooltable is built with the pooltable straightening mechanism. When TSP’s fellow snookerplayers see that energy can be transferred from one ball to another without the balls actually touching each other,
they will be astounded. They will want to know how this happens. TSP will give them lectures. TSP will become famous. So he hopes.

The analog of the pooltable oscillation in case the pooltable is built with the pooltable straightening mechanism of Fig. 2 is, of course, an oscillation of the $\theta$ parameter of QCD if the Standard Model has incorporated into it the Peccei-Quinn mechanism described earlier. The axion is the quantum of oscillation of the $\theta$ parameter of QCD. It is a particle in the same way that the quantum of oscillation of the electromagnetic field, the photon, is a particle. To discover whether the Peccei-Quinn mechanism has been incorporated into the Standard Model, one searches for the axion. To search for the axion, one tries to produce a few and then detect them. It is necessary to produce them first because they are unstable and hence cannot be around for a long time. (This last statement is not always true but let’s accept it for the moment. We will return to this point later.) To produce axions, one may take a beam of protons and dump it into a block of material. The axions produced may then be put into evidence by a detector that converts their energy back into more immediately visible forms of energy such as photons. This experiment and many others which try to produce and detect axions were carried out in the late 70’s and early 80’s but no axions were found. They are the analog of the experiment TSP proposes to carry out to put into evidence the pooltable straightening mechanism.

As it turns out, TSP’s hopes are dashed, totally, mercilessly ... No matter how hard he tries, he does not manage to produce an oscillation of the pooltable that is sufficiently large for him to detect. What is he to make of that? What makes the pooltable horizontal to one part in a billion if not the mechanism of Fig.2? Must he return to the idea that the pooltable maker adjusted the lengths of the legs with the required precision? At this point, TSP realizes that his ability to put into evidence oscillations of the pooltable depends upon the length $l$ of the lever arm between the axle and the big weight. If the length $l$ is very large, it becomes very difficult to produce pooltable oscillations by hitting balls against the pooltable rim. TSP also notices that when $l$ is very large, the oscillation frequency of the pooltable is very low. TSP now carries out detailed calculations. He finds that if the length $l$ is more than about three meters, his attempts to produce and detect pooltable oscillations must fail even if the pooltable is constructed with the pooltable straightening mechanism. Thus, his experiments only rule out the mechanism if $l$ is less than three meters or, equivalently, if the oscillation frequency of the pooltable is more than 0.18 cycles per second, which is the oscillation frequency of a pendulum of length three meters, on Mars where TSP and his fellow snookerplayers happen to be living.

TSP believes he understands everything now. The reason the pooltable is horizontal is the mechanism of Fig. 2. The reason he cannot put into evidence pooltable oscillations is that the length $l$ of the lever arm is more than three meters. TSP thinks he ought to be pleased with his insight, but actually he feels pretty frustrated. He understands why the pooltable is horizontal but he cannot produce the pooltable oscillations which would confirm his understanding and surprise his fellow snookerplayers. If the mechanism is implemented with a very long lever arm $l$, there’s just no way anyone will ever be able to put into evidence pooltable oscillations. Yet, the mechanism works! With some bitterness, he mutters to himself a name for his invention. He calls it the “invisible pooltable straightening mechanism”, because it works yet it can not be visibly demonstrated.
The analog of the “invisible pooltable straightening mechanism” in the world of particle physics is the PQ mechanism with an “invisible” axion. The properties of the axion depend upon a parameter $f$, called the axion decay constant, which is analogous to the length $l$ in the pooltable straightening mechanism. If $f$ is very large, then the axion becomes very light and very weakly coupled. The axion mass $m$ is related to the minimum oscillation frequency $\nu$ of the $\theta$ parameter of QCD by the famous relation: $mc^2 = h\nu$ where $h$ is Planck’s constant. So, the small mass of the axion if $f$ is large is analogous to the low pooltable frequency if $l$ is large. Also, the fact that the axion is weakly coupled is analogous to the fact that it is difficult to produce pooltable oscillations. If $f$ is large, the axion production and detection rates in the axion search experiments described earlier are so low that these experiments cannot find axions even if axions exist. But the PQ mechanism still works!

TSP ponders his fate. What is the worth of theoretical insight without experimental confirmation? Einstein discovered General Relativity and very soon afterwards his theory was confirmed by the measurement of the deflection of starlight by the sun. Democritus discovered (correctly guessed? What is the difference between a theoretical discovery and a good guess?) that matter is made of atoms. At the time there were no experiments that could put atoms directly into evidence. Those experiments came twenty-three centuries later ... As he walks around the playroom, pondering this and other questions, TSP glances for the umptieth time at a copper plate that is affixed to the side of the pooltable. It reads: ‘Made in Minneapolis, Minnesota, USA, home of “Minnesota Fats”. TSP always wondered what is ‘Minnesota Fats’ ... But he does know about the USA. The USA is a country on Earth.

TSP imagines how the pooltable was brought to Mars from Earth on a spaceship. That’s actually pretty interesting because during the trip between Earth and Mars, when the spaceship is just coasting along, there is ‘no gravity’. In that situation, the pooltable is not oriented in any particular direction. It would seem impossible to play snooker then ..., but TSP is thinking about something else altogether. What strikes him is that when the spaceship approaches Mars and prepares for landing by firing its retrorockets, the pooltable is not initially horizontal with respect to the direction of gravity at the place on Mars where the spaceship is going to land. The landing on Mars is illustrated in Fig. 3. Only when the rockets are fired, does the big weight of the pooltable straightening mechanism begin to feel Mars’ gravity. It then begins to pull the pooltable horizontal with respect to the direction of gravity on Mars, but it overshoots! The pooltable does not get pulled nicely to a horizontal position. Instead, because there is no damping mechanism, it oscillates about the horizontal. Once it has landed, it will oscillate about the horizontal with constant amplitude indefinitely because it turns out that, if the length $l$ is longer than three meters, the pooltable oscillations are so weakly coupled that they continue for very long times, much longer than the present age of the solar system. (We are assuming for the analogy’s sake that there is no friction on the axle about which the pooltable pivots and that the only way pooltable oscillations get damped is by giving off energy to the large collection of billiard balls on the table.)

Therefore, if the pooltable in the playroom where TSP lives is horizontal because of the so-called ‘invisible’ pooltable straightening mechanism, then it should be still oscillating
now. The oscillation is a relic of the epoch when the pooltable was brought to Mars. What is the amplitude of this relic oscillation? TSP realizes that the crucial parameter is the ratio of the pooltable oscillation period to the time scale over which Mars’ gravity gets effectively turned on when the spaceship bringing the pooltable landed on Mars. If the landing is very sudden as compared to the oscillation period of the pooltable, for example if the spaceship is in free fall towards the Martian surface till the very last millisecond when the retrorockets are turned on full blast to decelerate the spaceship and bring it to zero velocity just before it lands, then the final amplitude of the oscillation is the initial misalignment angle which is a random angle between zero and 180 degrees and which we have no reason to assume to be particularly small. This possibility is incompatible with the present state of the pooltable because the pooltable does not appear to oscillate at all now. If, on the other hand, the landing occurs very slowly, i.e. if the retrorockets are fired long before the spaceship lands and therefore Mars’ gravity is turned on very progressively, then the amplitude of oscillation decreases while the landing occurs. The switch-on of gravity is adiabatic in this case, and the oscillation amplitude decreases as the inverse of the square root of the oscillation frequency, and the latter increases as the square-root of Mars’ apparent gravity.

TSP carries out careful observations on the pooltable to determine whether it is oscillating at present since he realizes now that a relic oscillation is the telltale sign of the pooltable straightening mechanism. He does not detect any and places an upper limit of $10^{-12}$ on the present oscillation amplitude of the pooltable. This rules out the possibility of making the pooltable straightening mechanism invisible at will by lengthening the lever arm $l$ because the longer $l$, the lower the oscillation frequency of the pooltable and, comparatively, the more sudden the switch-on of gravity when the spaceship bringing the pooltable landed on Mars, and hence the larger the amplitude of relic pooltable oscillations. From a NASA publication that happens to be laying around in the playroom, TSP can deduce the time scale over which the retrorockets are fired when landing on Mars which is also the time scale over which Mars’ gravity gets effectively turned on. From this he can figure out the amplitude of relic pooltable oscillations as a function of $l$. He finds that the upper limit of $10^{-12}$ on relic pooltable oscillations requires that $l$ be smaller than 10 meters. TSP is very excited about this result. On the one hand, $l$ must be larger than 3 meters because he was unable to produce and detect pooltable oscillations. On the other hand, $l$ must be smaller than 10 meters because he was unable to detect relic pooltable oscillations. It seems TSP is closing in on a resolution of the horizontal pooltable mystery.

The switch-on of gravity when the spaceship approaches Mars is analogous to the switch-on of non-perturbative QCD effects when the universe is about $10^{-7}$ seconds old and the temperature is about one GeV. The relic pooltable oscillations are analogous to the coherent axion field oscillations that constitute the present axion cosmological axion energy density if $f$ is large. The requirement that the axion cosmological energy density not overclose the universe puts an upper limit on $f$ and hence a lower limit on the axion mass. Just as TSP found lower and upper limits on the length $l$ of the lever arm in the pooltable straightening mechanism, there are lower and upper limits on the axion decay constant $f$. If the axion mass is near its lower limit, axions may be the dark matter of the universe. Experiments presently under way attempt to detect the axion field oscillations...
that constitute the dark matter in our galaxy.

TSP figures out a means of detecting relic pooltable oscillations if \( l \) is in the range 3 to 10 meters, or equivalently, if the frequency of relic pooltable oscillations is in the range 0.18 to 0.097 cycles per second. His device is just a simple high quality oscillator placed on the pooltable. See Fig. 4. The oscillator frequency is tunable by changing the mass at the end of the spring. TSP plans to slowly change the frequency. When it equals that of relic pooltable oscillations, his oscillator will get excited. TSP should be able to see this effect.

Will TSP succeed in his latest venture and solve at last the mystery of the horizontal pooltable? We don’t know yet but if he succeeds, there may be a sequel to this story.

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Figure Captions:

Fig. 1. A horizontal pooltable on a slanted floor.
Fig. 2. The pooltable straightening mechanism.
Fig. 3. The pooltable in the spaceship about to land on Mars.
Fig. 4. A detector of relic pooltable oscillations.

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