The Parameters Ratio in the Strength of Bent Elements Calculations by the Deformation Model and the Ultimate Limit State Method

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Abstract. The relations between the deformation parameters and the forces in the normal section of the bent elements with the actual stress profile in the concrete compressed zone and the conventional rectangular stress profile shape of the limit stress method in the limiting state when the stress in the reinforcement is equal to the calculated resistance are determined in the article. The actual parameters in a normal section of a reinforced concrete element are calculated by a nonlinear deformation model using numerical methods of successive approximation using normalized diagrams of concrete and reinforcement according to domestic and foreign regulatory documents. The relations between the parameters in the limiting state are defined according to equality condition of the resultant efforts in the concrete compressed zone with the actual and rectangular stress plots depending on concrete diagrams used in calculations. Recommendations and suggestions are given on the use of the calculated dependencies of determining the compressed zone boundary height in strength calculations using the method of limiting efforts under the effect of the moment in the plane of symmetry of the section for concrete classes, including high-strength concrete.

1. Introduction

The deformation method for calculating reinforced concrete structures on strength using the deformation diagrams of concrete and reinforcement in recent years has acquired the priority status. In this case, regulatory documents [1,2,3,4] recommend different types of concrete deformation diagrams, among them two-linear and curvilinear.

There is an example of reinforced concrete bent elements with boundary reinforcement, after which comes the compressed zone destruction to reinforcement yield ($\sigma_s (R_s)$), and the authors establish the results of using traditional methods for calculating the strength of such elements [5] and the diagram method of their calculation using normalized diagrams. From the deformation scheme in the form of a normal section flat rotation, the dependence of the stresses in the reinforcement on the height of the compressed zone of concrete at the moment of destruction $\sigma_s = f (\xi)$, where $\xi = x/h_0$ is the compressed zone relative height is established. From the equation $\sigma_s = R_s$ it is obtained,
dependence is constructed for calculating the compressed zone boundary height \( \xi_R \), at which the limiting state occurs simultaneously with the achievement in the tension reinforcement, equal to the calculated resistance. It is defined the ratio between the compressed zone actual height \( \xi \) and the boundary height of the rectangular compressed zone in the form \( \xi_R = \omega \xi \), where \( \omega \) is the ratio of the stress distribution in the compressed zone. In [6,7] and in regulatory documents [3,5], in view of the adopted deformation scheme normal section, \( \omega \) and \( \xi_R \) is determined by the empirical dependencies obtained on the basis of processing experimental data and taking into account the type and strength of concrete. In the regulatory documents [1,4], these parameters with constant increase of the concrete class to B60 inclusively remain constant. Other values of ultimate deformations \( \varepsilon_{bu} \) are taken on the extreme fiber of the concrete compressed zone. There are questions in determining the boundary parameters for high-strength concrete.

2. Problem formulation
It is possible to obtain the actual parameters in the limiting condition provided that the tension in the reinforcement in the stretched zone is equal to the calculated resistance by calculating the deformation model using normalized diagrams of concrete, reinforcement and concrete classes, including high-strength concrete. Transition from the actual stress profile in the concrete compressed zone to the conventional rectangular shape of the stress diagram on the method of limiting state and establish the ratio between the parameters in the limiting state depending on the type of concrete diagrams used for the calculations. It should be clarified the parameters in the dependencies of determining the compressed zone boundary height for high-strength concrete.

3. The object of research
The research object is reinforced concrete bent elements with rectangular cross section, height is \( h = 60 \) cm, width is \( b = 30 \) cm. Reinforcement - longitudinal reinforcement class A400. The working conditions coefficient is \( \gamma_{bl} = 1.0 \) for a short (short-term) load action. Studies are performed in the range of concrete normalized classes B15-B100. The calculation by the deformation model is performed by numerical methods of successive approximation using a computer program. The concrete deformations on the extreme fiber of the compressed zone are assumed to be equal to their limiting values \( \varepsilon_{bu} \) according to the diagram description algorithm [1,2,3,4,8]. The stresses in the reinforcement reach values equal to the design strength \( R_s \). Reinforcement deformations are \( \varepsilon_s = \frac{R_s}{E_s} = 1.775\% \). The forces in the cross-section balance with a given accuracy when performing iterative processes is achieved by reaming the stretched zone for each concrete class.

4. Calculations using the two-line diagram of concrete according to [1,4]
The parameters for calculating an element according to the deformation model using two linear diagrams of concrete and normalized ultimate deformations \( \varepsilon_{b2} \), are presented in Table 1 in the following sequence: efforts in the concrete compressed zone \( N_{bf} \), equal to the forces in the reinforcement \( N_s \) (\( N_{bf} = N_s \)); compressed zone relative height \( \xi \); the coefficient of diagram completeness \( \omega \), as the ratio of the actual stress diagram area to the rectangular stress diagram area; the distance from the reinforcement axis to gravity center in the concrete compressed zone \( z_{sf} \).
ultimate moment \( M_{ult,\phi} \). Inclusively to the class of concrete B60 element curvature \( \eta \), parameters \( \xi \), \( \omega \), \( z_{sph} \) remain as constant values. The inelastic deformations proportion in total deformations decreases and the condition for the constancy of these parameters is violated in high-strength concretes.

**Table 1.** Deformation parameters and forces in the limiting state, obtained by calculating the subformation model using two linear diagrams of concrete [1] and stress plots in concrete compressed zone.

| Concrete class | \( N_{psh} \), kN | \( \xi \) | \( \Omega \) | \( z_{sph} \), cm | \( M_{ult,\phi} \), kNm | \( \xi \) | \( z_s \), cm | \( M_{ult} \), kNm |
|----------------|------------------|--------|--------|-----------------|-----------------|--------|---------|----------------|
| B15            | 744              | 0.66   | 0.786  | 41.0            | 305             | 0.521  | 41.4    | 308            |
| B30            | 1489             | 0.66   | 0.786  | 41.0            | 611             | 0.521  | 41.4    | 617            |
| B45            | 2190             | 0.66   | 0.786  | 41.0            | 898             | 0.521  | 41.4    | 907            |
| B60            | 2890             | 0.66   | 0.786  | 41.0            | 1186            | 0.521  | 41.4    | 1198           |
| B70            | 3123             | 0.65   | 0.773  | 41.5            | 1297            | 0.502  | 41.9    | 1311           |
| B80            | 3342             | 0.64   | 0.760  | 41.9            | 1402            | 0.485  | 42.4    | 1417           |
| B90            | 3450             | 0.63   | 0.747  | 42.4            | 1464            | 0.467  | 42.9    | 1482           |
| B100           | 3576             | 0.61   | 0.732  | 42.9            | 1533            | 0.448  | 43.5    | 1554           |

The boundary height \( z_R \) is calculated from the equality condition of the resultant forces \( N_b \) - the conditional rectangular shape of the stress plot in the compressed zone and the non-linear form with the actual height of the compressed zone. The points of resultants application do not coincide (\( z_s \)) with these efforts’ equality. However, this does not lead to a significant increase in the limiting moments \( M_{ult} \approx M_{ult,\phi} \). The ratio \( M_{ult} \approx M_{ult,\phi} \) remains.

5. **Calculations using a concrete curvilinear diagram**

The curvilinear diagram of concrete in compression according to [1] has ascending and falling branches and the analytical dependence of the compression diagram is recommended to be:

\[
\varepsilon_b = \frac{\sigma_{b}}{E_{b} V_{b}},
\]

where \( E_{b} \) - the initial modulus concrete of elasticity;

\( V_{b} \) - the coefficient of the secant modulus change, which is calculated by the formula presented in [8, 9, 10].

At the top of the stress diagram, \( \sigma_{b} \) is equal to calculated characteristics of compressed concrete in the first group (\( R_b \)) and the second group (\( R_{b,ser} \)) in the limit states. Deformations at the top of the diagram \( \varepsilon_{b} \) are calculated with the empirical dependence, built on the basis of experimental data, taking into account the type of concrete, strength and deformation properties of concrete. The falling branch is limited to maximum deformations \( \varepsilon_{bu} \) at stress values \( \sigma_{bu} = 0.85 R_b \) and is taken into account in the calculation by the deformation model for concrete with concrete, not exceeding the
class B80. It is obtained reasonable values corresponding to the experimental data of the parameters in the automated calculations of structures for deformations with the use of a curvilinear diagram. The parameters of the curvilinear diagram in the limiting state are studied and the relationship with their values in the calculation of elements is determined using the limiting efforts method for the elements of the first group calculations limit states. The parameters for calculating the element of the deformation model using a curved concrete diagram are presented in Table 2 in the same sequence as in the calculations using two linear concrete diagrams.

Table 2. Deformation parameters and forces in the limiting state, obtained by calculating the subformation model using a curved diagram of concrete [1] and stress plots in concrete compressed zone.

| Concrete class | Actual | Rectangular |
|----------------|--------|-------------|
| N_{bφ} (kN)   | ξ_{bφ} | z_{bφ} (cm) | M_{ult,bφ} (kN·m) | ξ_{R} | z_{R} (cm) | M_{ult,R} (kN·m) |
| B15            | 740    | 0.67        | 36.9 | 273.0 | 0.52 | 41.5 | 307 |
| B30            | 1480   | 0.67        | 38.3 | 566.8 | 0.52 | 41.5 | 614 |
| B45            | 2147   | 0.67        | 38.4 | 823.4 | 0.51 | 41.7 | 895 |
| B60            | 2899   | 0.68        | 38.8 | 1125  | 0.52 | 41.4 | 1199|
| B70            | 3287   | 0.68        | 38.9 | 1279  | 0.53 | 41.2 | 1354|
| B80            | 3557   | 0.69        | 38.4 | 1366  | 0.52 | 41.5 | 1478|
| B90            | 3111   | 0.59        | 43.0 | 1340  | 0.42 | 44.2 | 1376|
| B100           | 3368   | 0.6         | 43.0 | 1448  | 0.42 | 44.2 | 1488|

The deformations at the gravity center $\varepsilon_{bc} \wedge$ and at the top of the diagram $\varepsilon_{b} \wedge$ increase with increasing strength of concrete, while up to the class of concrete B70 inclusively $\varepsilon_{bc} \approx \varepsilon_{b} = 0.89$. Due to the diagram flatness, the condition $\sigma_{bc} \approx R_{b}$ is observed at its top. The height of the compressed zone $\xi$ and the coefficient of diagram completeness $\varnothing$ change slightly. These parameters combination determines the resultant force in the concrete $N_{bφ}$. Their ratio ensures effort equality in comparison with the two-line diagram.

However, due to the shift of the gravity center of the stress diagram to the neutral axis, ultimate moments are less than the two-line diagram. Above the concrete class B70 and, with the restriction on the use of the diagram falling branch for concrete classes B80 and higher, the gravity center in the diagram shifts from the top to the stress axis and the condition $\sigma_{bc} = R_{b}$ is violated. The coefficient of diagram completeness and the compressed zone height are reduced. The boundary height of the compressed zone, calculated from the condition of the resultant efforts’ equality in the stress and rectangular and nonlinear forms, to a concrete class B60 take a constant value and decrease as an increase in the concrete class in the two linear diagrams, which should be taken into account in the calculated dependencies of determining the compressed zone boundary height for high strength concrete.

6. Calculations using a curvilinear diagram of concrete according to the National Security Council of Ukraine

The purpose for comparing the parameters in the limiting state in the calculations using the deformation model is the analytical dependence $\sigma_{b} - \varepsilon_{b}$, presented in the foreign normative literature [2,3]. The calculation parameters are presented in Table. 3 in the same sequence as in...
calculations using previously considered concrete diagrams. Non-linear properties of concrete are used to a greater extent for low strength concrete. It ensures the growth of the forces perceived by concrete in the ultimate state.

Table 3. Deformation parameters and forces in the limiting state, obtained by calculation from the deformation model using a concrete curvilinear diagram according to the National Security Service [3] and stress plots in the concrete compressed zone.

| Concrete class | actual | rectangular |
|----------------|--------|-------------|
|                | $N_{bc}$, kN | $\zeta$ | $\Omega$ | $z_{sqb}$, cm | $M_{ult,\phi}$, kN\(m\) | $\xi_R$ | $z_s$, cm | $M_{ult}$, kN\(m\) |
| B15            | 792    | 0.66        | 0.84    | 39.0         | 310 | 0.55 | 40.5 | 321 |
| B30            | 1611   | 0.66        | 0.83    | 39.5         | 636 | 0.55 | 40.6 | 655 |
| B50            | 2518   | 0.66        | 0.82    | 40.0         | 1008 | 0.54 | 40.7 | 1026 |
| B60            | 2825   | 0.64        | 0.79    | 41.0         | 1158 | 0.51 | 41.7 | 1179 |
| B70            | 2997   | 0.63        | 0.77    | 41.7         | 1248 | 0.482 | 42.5 | 1274 |
| B80            | 3105   | 0.61        | 0.74    | 42.3         | 1314 | 0.45 | 43.4 | 1347 |
| B90            | 3275   | 0.61        | 0.72    | 42.4         | 1389 | 0.443 | 43.6 | 1428 |
| B100           | 3492   | 0.61        | 0.72    | 42.5         | 1485 | 0.438 | 43.8 | 1528 |

The completeness coefficient of the plot $\Omega$ in the considered range of concrete classes decreases with increasing concrete strength, the gravity center of the concrete diagram $\varepsilon_{bc}$ shifts to the stress axis. However, up to the class of concrete B55 inclusively complied with the condition $\sigma_{bc} \approx R_b$.

The compressed zone boundary height is calculated under the condition of resultant efforts equality in the actual and conditional rectangular stress plots in the concrete compressed zone. By reducing the distance from the axis of the reinforcement to the center of gravity of the stress profile in concrete, the difference in the values of the limiting moments is compensated. The difference in the limiting moments $M_{ult}$ is compensated by reducing the distance from the reinforcement axis to the gravity center of the stress profile in concrete $z_s$. It is complied with the condition $\xi_R = \alpha \xi$. The dependence on the concrete strength $\Omega$ should be considered in the formulas for calculating the compressed zone boundary height.

Calculation formulas of $\xi_R$. Regulatory documents offer different approaches to the calculation of the compressed zone boundary height. The calculation of $\xi_R$ is made according to the formula [1,4]

$$\xi_R = \frac{0.8}{1 + \varepsilon_{s,el}/\varepsilon_{b2}}$$

(2)

where inclusive up to the class of concrete B60, the formula parameters are: $\varepsilon_{s,el} = 1.775\% \varepsilon_{b2} = 3.5\%$ and $\xi_R$ is a constant value. For example, for reinforcement class A400, it is assumed $\xi_R = 0.531$. If for concrete classes inclusive up to B60 the use of a constant value $\xi_R$ is justified, then not taking into account the physical properties of high-strength concrete leads to unreasonably overestimated ultimate forces. Therefore, for heavy concrete classes B70-B100, the parameters in the formula (2) are recommended to be adjusted: some non-linear deformations are reduced by
multiplying the relative deformations $\varepsilon_{b2}$ by the ratio $(270-B)/210$; instead of 0.8 is taken 0.7 in the numerator.

The calculation results according to the formula (2) taking into account the recommendations [1] are presented in Table 4. The parameters $\xi_R$, $N_b$, $M_{ult}$ are consistent with the calculated values obtained from the deformation model using a double-line diagram of concrete, although for concrete classes up to B60 inclusively, their values should be reduced, and for high-strength concrete, opposite should be increased. It is recommended in the numerator in the formula (2) for heavy concrete classes up to B60 instead of 0.8, accept 0.79. It is consistent with studies [11], and for high-strength concrete instead of 0.7, respectively, 0.74.

**Table 4.** Calculated deformation parameters and efforts in the limit state according to the formulas (2), (3) regulatory documents and (4) - the authors proposals.

| Concrete class | Formula (2) | Formula (3) | Formula (4) |
|----------------|-------------|-------------|-------------|
|                | $\xi_R$    | $N_b$, K_H | $M_{ult}$, K_H | $\xi_R$    | $M_{ult}$, K_H | $\xi_R$    | $M_{ult}$, K_H |
| B15            | 0.53        | 758        | 312         | 0.78       | 0.65         | 351        | 0.80       | 0.53        | 312         |
| B30            | 0.53        | 1514       | 623         | 0.71       | 0.57         | 653        | 0.80       | 0.53        | 623         |
| B45            | 0.53        | 2230       | 917         | 0.65       | 0.50         | 887        | 0.80       | 0.53        | 917         |
| B60            | 0.53        | 2944       | 1211        | 0.59       | 0.44         | 1066       | 0.80       | 0.53        | 1211        |
| B70            | 0.46        | 2829       | 1224        | 0.55       | 0.41         | 1133       | 0.769      | 0.49        | 1294        |
| B80            | 0.45        | 3077       | 1338        | 0.52       | 0.38         | 1189       | 0.755      | 0.47        | 1383        |
| B90            | 0.44        | 3235       | 1415        | 0.50       | 0.36         | 1219       | 0.744      | 0.45        | 1447        |
| B100           | 0.43        | 3419       | 1504        | 0.47       | 0.33         | 1242       | 0.729      | 0.43        | 1507        |

The calculation of $\xi_R$ is made according to the empirical dependence and the recommendations [1,4,6,7]:

$$\xi_R = \frac{\omega}{1 + R_b / \sigma_{sc,u} (1 - \omega / 1,1)} ,$$

where: $\omega = \alpha - 0,008 R_b ; \sigma_{sc,u} = 400$MPa.

The value $\omega$ depends on the type and strength of concrete. The type of concrete is taken into account by the coefficient $\alpha$. For example, for heavy concrete $\alpha = 0.85$, and for lightweight concrete $\alpha \leq 0.8$ since the curvature of the diagram is slightly less than for heavy concrete. The calculation results of $\omega, \xi_R$, according to the formula (3) and the moment values in the limit state are presented in Table 4. For concrete class of B100 $\omega=0.47$, it is less than the theoretically possible minimum value $\omega = 0.5$. The dependence structure (3) does not allow to ultimately obtain the limiting forces in accordance with the diagram method and, above all, for high-strength concrete.

7. **Recommendations**

Calculation result analysis of the deformation parameters and the efforts in the limiting state shows that their values, obtained by calculation using the deformation model, correspond to the type of normalized diagrams. However, the mutual parameters deviations in the range of concrete classes B15-B100 do not exceed 7% and the general regularity of their changes is maintained. It is proposed to take the normalized boundary height of the compressed zone up to the concrete class B60 in accordance with Table 3.2 in [4].
In the calculations $R\xi$ for high-strength concrete it should be used empirical dependence (3) with some individual parameters’ adjustment. Earlier studies by research scientists and educational institutions prove the high reliability and relevance of this dependence (3). The calculation results of the numerous tested flexible and eccentrically compressed elements with a wide range of concrete strength and percent reinforcement showed good consistency with experienced bearing capacity.

It is necessary to match the strain values at the extreme fiber of the compressed zone, adopted for deriving the dependence $\sigma_x = f(\xi)$, with normalized values for concrete diagrams in accordance with modern regulatory documents if we agree on the calculation results using formula (3) and the deformation model. In addition, it is possible to clarify the calculation formula of $\omega$. Based on the above, the dependence (3) takes the form:

$$\xi_R = \frac{k_\omega \omega}{1 + (R_k / \sigma_{sc,0})(1 - \omega/1,1)} \quad (4)$$

where $\omega = 0,85 - 10^4 \left(\frac{R_k}{E_b}\right)^2; k_\omega = 1,05 \omega$.

The results of calculations $\xi_R$ by the formula (4) for high-strength concrete and for classes of concrete up to B60, taking $\xi_R = 0,53$, are presented in Table 3.

8. Conclusions

The deformation parameters and forces values in concrete and reinforcement in strength calculations based on the deformation model depend on the type of material deformation diagrams and may differ for the same concrete classes.

The calculated dependences for determination of the compressed zone boundary height for high-strength concrete should be adjusted taking into account the reduction in the share of non-linear deformations in the concrete deformation diagrams.

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