Modular Representation Underlies Systematic Generalization in Neural Natural Language Inference Models

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Abstract

In adversarial (challenge) testing, we pose hard generalization tasks in order to gain insights into the solutions found by our models. What properties must a system have in order to succeed at these hard tasks? In this paper, we argue that an essential factor is the ability to form modular representations. Our central contribution is a definition of what it means for a representation to be modular and an experimental method for assessing the extent to which a system’s solution is modular in this general sense. Our work is grounded empirically in a new challenge Natural Language Inference dataset designed to assess systems on their ability to reason about entailment and negation. We find that a BERT model with fine-tuning is strikingly successful at the hard generalization tasks we pose using this dataset, and our active manipulations help us to understand why: despite the densely interconnected nature of the BERT architecture, the learned model embeds modular, general theories of lexical entailment relations.

1 Introduction

Deep learning models now routinely achieve outstanding scores on benchmark tasks, sometimes even surpassing our estimates of how consistent humans are when annotating task data. At the same time, challenge and adversarial tests (Jia and Liang, 2017) have revealed how narrow these systems’ abilities actually are, as compared to the complex human abilities that we hoped to capture (Levesque, 2013). Furthermore, inoculation testing (Liu et al., 2019) has helped to distinguish shortcomings of the available data from deeper conceptual problems with the models. All these efforts are seeking to ensure that we pose generalization tasks that are actually good proxies for the capabilities that we want these models to acquire.

What properties must a system have in order to succeed at the very hard generalization tasks we are now posing? In this paper, we argue that an essential factor is the ability to combine previously learned capabilities in new ways to define more complex capabilities. This idea is intimately related to the view of Fodor (1975) and others that human cognition is modular, and to the related ideas from Fodor and Pylyshyn (1988) concerning the systematicity of human cognition (roughly, that certain abilities necessarily imply others). Our central contribution is a method for assessing the modularity of systems in this sense, even if those systems are highly interconnected deep learning models for which it would be a mistake to presuppose any particular kind of structure.

Our empirical focus for this work is the role of
monotonicity reasoning in Natural Language Inference (MacCarty, 2009; Icard and Moss, 2013). More specifically, we study the interactions of entailment reasoning with negation, as one instance of systematicity in Fodor and Pylyshyn’s sense. We would like to determine whether a system can actually reason about lexical entailment and, furthermore, whether it has learned that negation is downward monotone (roughly, that $A$ entails $B$ if, and only if, not-$B$ entails not-$A$, for all $A$ and $B$).

To facilitate this work, we present Monotonicity NLI (MoNLI), a new naturalistic NLI dataset for training and assessing systems on these semantic notions. MoNLI extends SNLI (Bowman et al., 2015) with examples that allow us to pose generalization tasks concerning entailment and negation that are hard but fair. The tasks are hard in that they require models to generalize to entirely new pairs of lexical items in negated linguistic contexts. The problems are fair in that the desired learning targets are fully specified (Geiger et al., 2019), which ensures that failures do not simply trace to deficiencies in the datasets.

Using MoNLI, we evaluate Enhanced Sequential Inference Models (Chen et al., 2016), Decomposable Attention Models (Parikh et al., 2016), and BERT-based models (Devlin et al., 2019). These experiments lead to the positive result that a BERT-based NLI model can solve MoNLI with only a modest amount of task-specific fine-tuning and without losing performance on its original evaluation data. The other NLI models successfully solve the generalization task, but at the cost of a performance drop on the original evaluation data.

This leads us back to our central question: what properties does a neural model have that allows for this striking level of generalization? For this, we must pry open the black box and examine the intermediate representations created by the model. For a model like BERT, it is not immediately evident that this can be done; a hallmark of all such Transformer-based models is their dense interconnections. In this setting, it would be a mistake to assume that, for example, there are intermediate vectors that encode a modular notion of lexical relations. To avoid this trap, we develop new methods for discovering modular representations, actively manipulating model internal representations to reveal the underlying causal dynamics of a model. This method is depicted schematically in Figure 1. These manipulations reveal a striking degree of modularity in BERT.

2 Related work

Systematicity and Modularity in Cognition Fodor and Pylyshyn (1988) offer systematicity as a hallmark of human cognition. Systematicity says that certain abilities necessarily imply others. For example, the ability to understand the puppy loves Sandy entails the ability to understand Sandy loves the puppy, and the additional ability to understand how puppy and kitten relate to each other entails understanding the kitten loves Sandy, Sandy loves the kitten, the kitten loves the puppy, and so forth. For Fodor and Pylyshyn, these observations trace to the ability to combine previously learned capabilities in novel ways to do novel things.

Fodor and Pylyshyn contend that connectionist (deep learning) models are not systematic on the grounds that, for example, it is easy to train a model that seems to understand the puppy loves Sandy but transparently has no abilities with regard to Sandy loves the puppy. While this is certainly true, it leaves open whether we can train networks that do display such systematicity. One of our main objectives is to argue that, when networks do achieve systematicity, they do so using modular representations.

Compositionality in Semantics Systematicity is closely related to the compositionality principle in semantics, which says that the meanings of complex linguistic units are derived from (functions of) their parts (Partee, 1984; Janssen, 1997). Compositionality is a very restrictive notion of systematicity; while all compositional systems are systematic, it is possible to be systematic without adhering to the strict tenets of compositionality (Potts, 2019). We do not take a stand on whether the models we evaluate can be seen as compositional, opting instead for the more general claim that they display non-trivial degrees of systematicity.

Adversarial Testing Adversarial (or challenge) test datasets are supplementary evaluation resources that test the ability of a model to generalize to examples outside the distribution of the data the model was trained, developed, and (standardly) tested on. These tests can be seen as attempts to probe the generalization capabilities of state-of-the-art models with respect to the tasks they have been trained on, by focusing on difficult or un-
derrepresented examples in a model’s training set. Many NLI adversarial tests have drawn directly on intuitions like those of Fodor and Pylyshyn. For example, Glockner et al. (2018) create variants of SNLI examples by replacing individual words with others in ways that should systematically determine a new label for the example (e.g., substitution of synonyms should preserve labels). Naik et al. (2018) pursue similar strategies for MNLI, and Nie et al. (2019) perform syntactic manipulations that depend on how syntactic constructions relate to each other systematically. In general, these tests have seemed to reveal a disappointing lack of systematicity, although the rise of pretrained language models has led to at least some adversarial datasets being solved (Richardson et al., 2019) and the inoculation tests of Liu et al. (2019) suggest that some, but not all, of these failings trace to the data rather than the underlying architectures.

### Monotonicity

Our empirical focus is entailment and negation. This is one (highly prevalent) aspect of monotonicity reasoning, which governs many aspects of lexical and constructional meaning in natural language (Sánchez-Valencia, 1991; van Benthem, 2008). There is an extensive literature on monotonicity logics (Moss, 2009; Icard, 2012; Icard and Moss, 2013; Icard et al., 2017). Within NLP, MacCartney and Manning (2008, 2009) apply very rich monotonicity algebras to NLI problems, Hu et al. (2019a,b) create NLI models that use polarity-marked parse trees, and Yanaka et al. (2019b,a) investigate the ability of neural models to understand monotonicity reasoning. While we consider only a small fragment of these approaches, the methods we develop should apply to more complex systems as well.

### Probing

The techniques we develop for assessing modularity bear a superficial resemblance to recent efforts to probe the internal structure of Transformer-based models to try to determine how much linguistic structure they implicitly encode (Peters et al. 2018; Tenney et al. 2019; Clark et al. 2019; for a full review, see Belinkov and Glass 2019). In aggregate, this work has provided nuanced insights into the internal representations of these models, as well as their capacity to directly support learning diverse NLP tasks via fine-tuning (Hewitt and Liang, 2019). In Section 7, we go beyond probing, actively manipulating representations to understand the causal dynamics of the model.

### 3 Monotonicity NLI dataset

We created the MoNLI corpus to investigate the ability of models to perform systematic generalizations and create modular representations. The corpus contains 2,678 NLI examples in the usual format for NLI datasets like SNLI.

In each example, the hypothesis is the result of substituting a single word \(w_p\) in the premise for a hypernym or hyponym \(w_h\) in the hypothesis. We refer to \(w_h\) and \(w_p\) as the substituted words in an example. In 1,202 of these examples, the substitution is performed under the scope of the downward monotone operator \(\not=\). We refer to these examples collectively as NMoNLI. In the remaining 1,476 examples, this substitution is performed under the scope of no downward monotone operator. We refer to these examples collectively as PMoNLI.

MoNLI was generated according to the following procedure. First, randomly select a premise or hypothesis sentence \(s\) from the SNLI training dataset. Second, select a noun in \(s\), and, using WordNet (Fellbaum, 1998), select all hypernyms and hyponyms of the noun subject to two conditions: (1) the hypernym or hyponym appears in the SNLI training data, and (2) substituting the hypernym or hyponym results in a grammatical, coherent sentence \(s’\). Finally, for each substitution, generate two examples for the corpus – one where the original sentence is the premise and the edited sentence is the hypothesis, and one example with those roles reversed. Each of these example pairs has one example with the label \textit{entailment} and one example with the label \textit{neutral}, resulting in a dataset perfectly balanced between the two labels.

For example, suppose we select the hypothesis sentence (A) and we identify the noun \textit{plants} for substitution. Then we enter \textit{plants} into WordNet and find that \textit{flowers} is a hyponym of \textit{plants}, so we substitute \textit{flowers} for \textit{plants} to create the edited sentence (B):

\[(A)\] The three children are not holding \textit{plants}. \(\downarrow\)

\[(B)\] The three children are not holding \textit{flowers}.

This leads to two new MoNLI examples:

\[\text{In set-theoretic terms, a function } f \text{ is downward monotone iff, whenever } A \subseteq B, f(B) \subseteq f(A).\]
These two examples would belong to NMoNLI, due to the fact \textit{not} scopes over the substitution site. If \textit{not} were removed from both of these sentences, then their labels would be swapped and both examples would belong to PMoNLI.

MoNLI was generated by the authors by hand; examples judged to be unnatural were removed, and any grammatical or spelling errors in the original SNLI sentence were corrected.

This data generation process is similar to that of Glockner et al. (2018), except they focus on the lexical relations of exclusion and synonymy, while we focus on entailment relations. This difference prevents their dataset from capturing monotonicity reasoning, which involves entailment relations, but not exclusion or synonymy.

4 Models

We evaluated three models on MoNLI:

ESIM The Enhanced Sequential Inference Model (Chen et al., 2016) is a hybrid TreeLSTM-based and biLSTM-based model that uses an inter-sentence attention mechanism to align words across sentences.

DECOMP The Decomposable Attention model (Parikh et al., 2016) depends primarily on softly aligning words in the premise and hypothesis using attention mechanisms. This model can be seen as a precursor to Transformer-based models (Vaswani et al., 2017).

BERT A Transformer model trained to do masked language modeling and next-sentence prediction (Devlin et al., 2019). We rely on uncased BERT-base parameters as provided by the Hugging Face transformers library (Wolf et al., 2019).

We chose these models for two reasons. First, all of them achieve high and comparable scores on SNLI. Second, while they differ considerably in their internal structure, they all seek to achieve many dense interconnections between units, which could lead to two outcomes with respect to modularity: it could hinder modularity by encouraging distributed solutions, or it could facilitate modularity by allowing it to emerge naturally via data-driven learning.

|          | SNLI | PMoNLI | NMoNLI |
|----------|------|--------|--------|
| ESIM     | 87.9 | 86.6   | 39.4   |
| DECOMP   | 85.1 | 84.8   | 16.1   |
| BERT     | 90.8 | 94.4   | 2.2    |

Table 1: Adversarial testing results of our three models trained on SNLI. The numbers are accuracy values; all the datasets have balanced label distributions.

5 Adversarial Testing

5.1 Methods

We first use MoNLI as an adversarial test dataset where models trained on SNLI are expected to generalize to MoNLI. MoNLI can be considered an adversarial testing dataset that evaluates an NLI model’s ability to perform simple inferences founded in lexical entailments and monotonicity. However, as discussed in Section 3, it is not especially adversarial, in that we sampled sentences from the SNLI training set and only substituted in hypernyms and hyponyms that occur in SNLI training set. This keeps MoNLI as close as possible to the distribution of SNLI data. Thus, if a model fails on MoNLI, we can be confident that this failure stems from a lack of knowledge about monotonicity and lexical entailment relations, rather than some other confounding factor like syntactic structures or vocabulary items that were unseen in training.

5.2 Results

The results in Table 1 are stark. The three models achieve high accuracy on SNLI and PMoNLI, the examples where no downward monotone operators scope over the substitution site. However, they are well below chance accuracy on NMoNLI, the examples where \textit{not} scopes over the substitution site. BERT is more extreme than the other models, achieving a higher accuracy on PMoNLI than SNLI and almost zero accuracy on NMoNLI. High performance on PMoNLI shows that models have knowledge of the lexical relations between the substituted words, but low performance on NMoNLI shows the models have no knowledge of the downward monotone nature of \textit{not}. In fact, the below chance accuracy on NMoNLI indicates that these models are somewhat reliably (incredibly reliably in BERT’s case) predicting the wrong label on these examples, suggesting that they might be treating these examples the same as they do exam-
5.3 Discussion

Crucially, while these models trained on SNLI do not know that not is downward monotone in these examples, this is not conclusive evidence that these models are unable to learn this semantic property. It could well be that this ability is not necessary for success on SNLI.

For instance, a simple analysis of SNLI finds that negation (represented by not and n’t) appears in the premise and hypothesis in only 38 examples. Since NMoNLI has negation in the premise and hypothesis of every example, we believe the deficiency lies in the data, not the model.

A natural next step would be to train on MNLI, where the coverage with regard to negation is better: about 18K examples (∼4%) have negation in the premise and hypothesis. However, we tried this, by combining MNLI with SNLI, and the results were almost exactly the same; while negation is more present in MNLI, it seems that MNLI examples still do not manifest the kind of monotonicity reasoning that we are targeting.

Thus, while these adversarial testing datasets do reveal that state-of-the-art NLI models trained on SNLI and MNLI are unable to perform inferences requiring the knowledge that negation is downward monotone, it is plausible that this is a failing of the data rather than the models. Since we are interested in the underlying capacity of these models to learn generalized solutions to MoNLI, we need to push past this analytic limitation. This is the goal of our generalization tests.

6 Generalization Tests

We now know our three models trained on SNLI have knowledge of the lexical relations between substituted words, but do not know that the presence of not reverses the relationship between the word-level relation and the sentence-level relation. We believe this inability is due to a lack of exposure to examples that require such knowledge, and so we now investigate whether models can learn generalized solutions when provided with minimal, but sufficient, data.

| NMoNLI Train | NMoNLI Test |
|--------------|-------------|
| person       | 198         | dog          | 88          |
| instrument   | 100         | building     | 64          |
| food         | 94          | ball         | 28          |
| machine      | 60          | car          | 12          |
| woman        | 58          | mammal       | 4           |
| music        | 52          | animal       | 4           |
| tree         | 52          |              |             |
| boat         | 46          |              |             |
| fruit        | 42          |              |             |
| produce      | 40          |              |             |
| fish         | 40          |              |             |
| plant        | 38          |              |             |
| jewelry      | 36          |              |             |
| anything     | 34          |              |             |
| hat          | 20          |              |             |
| man          | 20          |              |             |

Table 2: The hyponyms that occur in the train-test split of NMoNLI described in Section 6. The number next to each hyponym corresponds to the number of examples that hyponym occurs in. For NMoNLI train, we left out the 11 hyponyms that occurred in less than 20 examples.

6.1 Challenging Test Splits

Standard practice would be to randomly partition NMoNLI into training, development, and testing sets. However, we do not believe this generalization task inherently requires the model to leverage its knowledge of lexical entailment to learn a proper semantics for negation. Such a random partition could result in many of the same substituted words being present in training and testing. This would mean that a model that relearns the relation between every pair of substituted words in reverse and accesses this knowledge when not scopes over the substitution site would be successful at this task. Such a model would not have learned to reverse the lexical entailment relation when not scopes over the substitution site, but instead would have learned how to handle new special cases for the examples in NMoNLI and would not be able to generalize to lexical entailment relations it was not exposed to during training.

For this reason, we propose a generalization task where NMoNLI is partitioned into training, development, and testing sets such that the substituted words in the training set and the substituted words in the development and test sets are entirely disjoint. The specific train/test split we used is de-
Table 3: Ablation study of BERT training regimes.

| Model                              | NMoNLI Test |
|------------------------------------|-------------|
| BERT w/o pretraining or SNLI training | 62.3        |
| BERT with pretraining but no SNLI training | 98.5        |

Is this an unfair task that is too difficult to expect any model to solve? We do not believe so. The models know the relations between all substituted lexical items from their training on SNLI, as demonstrated by their near-perfect performance on PMoNLI. It isn’t crucial that models be shown every pair of substituted words in the presence of not. The models only need to be shown every relation that can hold between two substituted words in the presence of not, which fully determines the downward monotone property of not. For our purposes, there are two relations that hold between substituted lexical items, entailment and reverse entailment, and the NMoNLI training dataset is perfectly balanced between the two. This allows us to conclude that our task is fair in the sense of Geiger et al. 2019.

6.2 Inoculation by Fine-Tuning

Ideally, a model trained on SNLI that is further trained on NMoNLI will still maintain strong performance on SNLI. We use inoculation by fine-tuning (Liu et al., 2019) to evaluate models on this ability. In this method, a pretrained model is further fine-tuned on different small amounts of adversarial data while performance on the original dataset and the adversarial dataset is tracked. For each amount of adversarial data, a hyperparameter search is run and the model with the highest average performance on the original dataset and adversarial dataset is selected. Optimizing for the average accuracy is what Richardson et al. (2019) refers to as lossless inoculation and we perform the same hyperparameter searches that they do.

6.3 Results

We present our results in Figure 2. All three of our models are able to solve our challenging generalization task after inoculation by fine-tuning. However, the results are mixed when it comes to the ability to maintain performance on the SNLI test set. ESIM and DECOMP both show clear trade-offs; for them, succeeding on NMoNLI leads to reduced performance on SNLI, and vice versa. In contrast, the BERT model is able to succeed at both task simultaneously.

To gain further insights into which aspects of BERT enable this exceptional performance, we explored two ablations to our BERT pretraining and fine-tuning regimes. First, we trained BERT from scratch on our generalization task, with no general pretraining (i.e., no masked language modeling with next-sentence prediction) or training on SNLI. This model achieves 62.3% accuracy, which is very poor, but still indicates there are some artifacts in our data that allow a model with no prior knowledge of lexical relations to get some
We believe that the ability of BERT models to leverage. Second, we tried using pretrained BERT but without any SNLI training. Surprisingly, this model achieved near-perfect accuracy on our generalization task. This finding indicates that BERT might acquire knowledge of lexical relations during its general pretraining phase.

6.4 Discussion

We believe that the ability of BERT models to solve this more difficult generalization task is evidence that they create modular, effective representations of lexical relations. However, we also believe this evidence is weak, as there is no formal relationship between a model solving a generalization task and that model implementing modular representations. In order to further support our claim, we must pry open the black box and investigate the underlying causal structure of BERT.

7 Prying Open the Black Box

We now know that BERT is able to leverage its knowledge of lexical relations to achieve a universal solution to NMoNLI after only being trained on a strict subset of the lexical relations it contains. We hypothesize that modular representations underlie this ability to systematically generalize. To investigate this hypothesis, we develop an intuitive definition of modular representations that is grounded in the effect that interchanging representations has on the model output. We then present experimental evidence that reveals some of the underlying modular structure in BERT.

We focus our analysis on BERT for two reasons. First, there is no obvious method to determine whether a model like BERT has aligned and merged two words to form a modular representation of the pair, as is necessary for the monotonicity computations in NMoNLI. Second, BERT is in a class of wildly successful models in general, and it was the model with the strongest results in the inoculation experiments. We believe an ability to create modular representations underlies some of this success.

The BERT model we analyze is the one from the general pretraining phase.

7.1 Defining Modularity for BERT

Notation Recall that the structure of MoNLI examples is such that the premise and hypothesis token sequences in each example differ by exactly one word. That is, for a given premise sequence \( w_1, \ldots, w_p, \ldots, w_n \), we have a hypothesis \( w_1, \ldots, w_h, \ldots, w_n \) differing only in the tokens \( w_p \) and \( w_h \) corresponding to the core MoNLI manipulation described in Section 3.

To simplify notation, then, let us use \( w_{w_p}, w_{w_h} \) to abbreviate \( w_1, \ldots, w_p, \ldots, w_n \) and \( w_{w_h}, w_{w_p} \) to abbreviate \( w_1, \ldots, w_h, \ldots, w_n \). This allows us to specify MoNLI BERT input sequences as

\[
e = \{[CLS], w_{w_p}, [SEP], w_{w_h}, [SEP]\}
\]

Let \( BERT[e] \) be the grid of Transformer-layer output representations that BERT determines based on the input sequence \( e \), which we can represent schematically for \( R \) transformer layers:

\[
h_1^R \ h_2^R \ldots \ h_n^R \\
\vdots \ \vdots \ \vdots \\
h_1^1 \ h_2^1 \ldots \ h_n^1 \\
inputs: \ e_1 \ e_2 \ldots \ e_n
\]

Further, let \( BERT_{e_j}^r[e] \) be the vector in row \( r \) for input token \( e_j \) that is created when BERT processes example \( e \). For example, in the above, \( BERT_{e_2}^1[e] = h_2^1 \).

For two examples with different numbers of tokens, \( e \) and \( e' \), the grids \( BERT[e] \) and \( BERT[e'] \) will be of different widths, but we still want to be able to refer to parallel locations in each example. So we define \( BERT_{w_p}^r \) to be the locations where the vectors in row \( r \) are created for the input token \( w_p \) across all examples. We similarly define \( BERT_{w_h}^r \) and \( BERT_{[CLS]}^r \).

Our goal is to define what it would mean for the vectors created at the locations \( BERT_{w_p}^r, BERT_{w_h}^r, \) or \( BERT_{[CLS]}^r \) to define modular representations.

Input Swapping Swapping is a systematic manipulation of the input sequence of one example based on the input sequence of another. For two examples \( e \) and \( e' \),

\[
e = \{[CLS], w_{w_p}, [SEP], w_{w_h}, [SEP]\} \\
e' = \{[CLS], x_{x_p}, [SEP], x_{x_h}, [SEP]\}
\]

let \( Swap(e, e') \) be the version of \( e \) in which the \( w_p \) and \( w_h \) tokens in \( e \) are replaced by the corresponding tokens \( x_p \) and \( x_h \) in \( e' \), with no other changes to \( e \):

\[
Swap(e, e') = \{[CLS], w_{x_p}, [SEP], w_{x_h} [SEP]\}
\]
**Representation Replacement** Replacement is an operation on BERT parameter grids. We define \( \text{Replace}(\text{BERT}[e], \text{BERT}_w^r[e']) \) to be the grid of Transformer-layer representations that is just like \( \text{BERT}[e] \) except while BERT processes \( e \), \( \text{BERT}_w^r[e] \) is replaced with \( \text{BERT}_w^k[e] \), which will change the identity of \( \text{BERT}_w^r[e] \) for \( k > r \).

**Interchange and Causal Interchange** Interchange is the central notion for modularity. It relates \( \text{Swap} \) and \( \text{Replace} \).

**Interchange** Examples \( e \) and \( e' \) interchange at \( \text{BERT}_w^r \) if, and only if, \( \text{BERT}[\text{Swap}(e, e')] \) and \( \text{Replace}(\text{BERT}[e], \text{BERT}_w^r[e']) \) are models that lead to the same output label predictions.

The intuition behind this definition of interchange is that it connects a systematic manipulation of the inputs with a systematic manipulation of the internal representations of the model. To the extent that these manipulations lead to the same output predictions, we can say that we have identified a systematic aspect of the representation of a certain pair of input tokens in the context of the full example. Conversely, if interchange fails for some examples, then the effects of the input manipulation represented by \( \text{Swap}(e, e') \) are certainly not localized at representation \( \text{BERT}_w^r \). They may be elsewhere in the network, or there may be no such representation.

Crucially, we only get this kind of insight if the process of interchanging has an effect on the model’s final output. For this reason, we are concerned with sets of examples that interchange and have the following property: for some \( e, e' \in E \), interchange holds and, furthermore, \( \text{BERT}[\text{Swap}(e, e')] \) leads to a different output prediction\(^3\) from \( \text{BERT}[e] \). We call this causal interchange because of the detectable causal effect it has on the outputs.

**Measuring Interchange** We have no guarantee that BERT parameters will satisfy (causal) interchange for any examples. Indeed, the nature of BERT might seem to resist such localization of specific semantic effects, leading to strong performance but no predictable localization in any specific internal representations. However, we observe is in fact a strikingly high level modularity in BERT models trained on MoNLI.

To quantify this, we create a graph in which the examples of MoNLI are the nodes and there is an edge between two nodes if and only if those two examples interchange at \( \text{BERT}_w^r \). Clique this graph will, in turn, correspond to sets of examples that interchange at \( \text{BERT}_w^r \).

To see the logic behind this graph, it is helpful to consider some logically possible scenarios. First, if no examples interchange at our chosen position \( \text{BERT}_w^r \), then our graph for that position will have no edges at all. Second, if all examples interchange at \( \text{BERT}_w^r \), then our graph will be one enormous clique. This would represent perfect modularity.

### 7.2 Experiments
MoNLI contains 2,678 examples. Thus, for a given vector location, 7 million interchange experiments must be run to construct the full graph. Under the constraint of resources, we conducted interchange experiments at random locations among the 36 locations defined by \( \text{BERT}_{wp}^r \), \( \text{BERT}_{w_1}^r \), and \( \text{BERT}_{[CLS]}^r \) with random pairs of inputs and selected the location with the most clustering, which was \( \text{BERT}_{wp}^3 \).

The problem of finding the largest clique in a graph is NP-complete, so only heuristics are available, but heuristics are fine for the purpose of finding a clique that is large enough. Some edges correspond to interchanges that are causal, and some correspond to interchanges that are not causal. To ensure we identify sets of examples that interchange causally, we use the following greedy algorithm: begin with the full graph, and then remove the node with the least number of causal edges until the node with the least number of causal edges is less than \( \alpha \), then remove the node with the least number of edges until only a clique remains. We tested \( \alpha \) values between 1 and 10 and chose the best results.

### 7.3 Results
We ran our interchange method at the location \( \text{BERT}_{wp}^3 \) to construct a graph which we partitioned into cliques using our simple, greedy algorithm. We discovered several large disjoint cliques corresponding to sets of examples that interchange causally. These cliques had size 98, 63, 47, and 37.
To put these results in context, we created a graph with the same number of nodes as the original and edges that were assigned randomly with a 50% probability. This baseline tells us the level of modularity that would be expected if interchanging a representation randomized the output of the model for its binary classification task.

The expected number of cliques of size k for this graph (2,678 nodes; edge probability of 0.5) is \(\binom{n}{k} \times 2^{\binom{k}{2}}\). Thus, for \(k > 20\), the expected number of cliques with \(k\) nodes is less than \(10^{-8}\).

**7.4 Discussion**

Our random graph baseline has almost no chance of forming cliques with more than 20 nodes, so our cliques of size 98, 63, 47, and 37 are not the result of random chance. This is conclusive evidence that BERT creates modular representations of substituted words for several large sets of examples.

In Figure 3, we show a visualization of the substituted words that the largest modular representation abstracts over. We clustered the pairs of substituted words based on their hyponyms and found that many substituted words shared the same hyponym. When we randomly sample 98 examples from MoNLI, we consistently found over 30 of the 69 distinct hyponyms in MoNLI appear in the sample. The 98 pairs of substituted words that this modular representation abstracts over contains only 13 of the 69 distinct hyponyms in MoNLI, which makes it clear this modular representation does not abstract over a random sample. It is hard to say exactly what this tells us about how BERT makes representations.

Importantly, these results are not evidence that BERT fails to create modular representations of substituted words from a larger subset of examples. First, we only investigated the representations at BERT's \(w_p, w_h\). BERT could be making modular representations at other vector locations. Second, BERT could be making modular representations using parts of vectors at various locations. Third, even if we chose the correct location, we used a greedy algorithm to discover cliques, which doesn’t guarantee that we found the largest clique.

We did not exhaustively analyze BERT to find the largest possibly modular representation; such an analysis is likely computationally impossible as it would require testing every possible combination of vector elements to see if they form a modular representation. What we did do is perform an efficient analysis that was able to find a large modular representation.
8 Conclusion

We presented a new method for actively manipulating BERT internal representations to determine whether they encode lexical realizations in a modular way. The goal of these methods is to go beyond performance metrics on standard evaluation tasks to understand the abstract capacity of our models to reason about language. We reported on experiments involving MoNLI, a new challenge dataset for evaluating a model’s ability to reason about lexical entailment and negation. Our findings suggest that BERT-based internal representations show a high degree of modularity when fine-tuned on MoNLI. We expect the general logic of these methods and experiments to extend to a wide range of neural architectures and semantic tasks.

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References

Yonatan Belinkov and James Glass. 2019. Analysis methods in neural language processing: A survey. Transactions of the Association for Computational Linguistics, 7:49–72.

Johan van Benthem. 2008. A brief history of natural logic. In Logic, Navya-Nyaya and Applications: Homage to Bimal Matilal.

Samuel R. Bowman, Gabor Angeli, Christopher Potts, and Christopher D. Manning. 2015. A large annotated corpus for learning natural language inference. In Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing, pages 632–642, Lisbon, Portugal. Association for Computational Linguistics.

Qian Chen, Xiaodan Zhu, Zhen-Hua Ling, Si Wei, and Hui Jiang. 2016. Enhancing and combining sequential and tree LSTM for natural language inference. CoRR, abs/1609.06038.

Kevin Clark, Urvashi Khandelwal, Omer Levy, and Christopher D. Manning. 2019. What does BERT look at? an analysis of BERT’s attention. In Proceedings of the 2019 ACL Workshop BlackboxNLP: Analyzing and Interpreting Neural Networks for NLP, pages 276–286, Florence, Italy. Association for Computational Linguistics.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. BERT: Pre-training of deep bidirectional transformers for language understanding. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4171–4186, Minneapolis, Minnesota. Association for Computational Linguistics.

Christiane Fellbaum, editor. 1998. WordNet: An Electronic Database. MIT Press, Cambridge, MA.

Jerry A. Fodor. 1975. The Language of Thought. Thomas A. Crowell Co., New York.

Jerry A. Fodor and Zenon W. Pylyshyn. 1988. Connectionism and cognitive architecture: A critical analysis. Cognition, 28(1):3–71.

Atticus Geiger, Ignacio Cases, Lauri Karttunen, and Christopher Potts. 2019. Posing fair generalization tasks for natural language inference. In Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP), pages 4485–4495, Hong Kong, China. Association for Computational Linguistics.

Max Glockner, Vered Shwartz, and Yoav Goldberg. 2018. Breaking NLI systems with sentences that require simple lexical inferences. In Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 2: Short Papers), pages 650–655, Melbourne, Australia. Association for Computational Linguistics.

John Hewitt and Percy Liang. 2019. Designing and interpreting probes with control tasks. In Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP), pages 2733–2743, Hong Kong, China. Association for Computational Linguistics.

Hai Hu, Qi Chen, and Larry Moss. 2019a. Natural language inference with monotonicity. In Proceedings of the 13th International Conference on Computational Semantics - Short Papers, pages 8–15, Gothenburg, Sweden. Association for Computational Linguistics.

Hai Hu, Qi Chen, Kyle Richardson, Atreyee Mukherjee, Lawrence S. Moss, and Sandra Kübler. 2019b. MonaLog: A lightweight system for natural language inference based on monotonicity. ArXiv, abs/1910.08772.

Thomas Icard, Lawrence Moss, and William Tune. 2017. A monotonicity calculus and its completeness. In Proceedings of the 15th Meeting on the Mathematics of Language, pages 75–87, London, UK. Association for Computational Linguistics.

Thomas F. Icard. 2012. Inclusion and exclusion in natural language. Studia Logica, 100(4):705–725.
Thomas F. Icard and Lawrence S. Moss. 2013. Recent progress on monotonicity. *Linguistic Issues in Language Technology*, 9(7):1–31.

Theo M. V. Janssen. 1997. Compositionality. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, pages 417–473. MIT Press and North-Holland, Cambridge, MA and Amsterdam.

Robin Jia and Percy Liang. 2017. Adversarial examples for evaluating reading comprehension systems. In *Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing*, pages 2021–2031, Copenhagen, Denmark. Association for Computational Linguistics.

Hector J. Levesque. 2013. On our best behaviour. In *Proceedings of the Twenty-third International Conference on Artificial Intelligence*, Beijing.

Bill MacCartney. 2009. *Natural Language Inference*. Ph.D. thesis, Stanford University.

Bill MacCartney and Christopher D. Manning. 2008. Modeling semantic containment and exclusion in natural language inference. In *Proceedings of the 22nd International Conference on Computational Linguistics (Coling 2008)*, pages 521–528, Manchester, UK. Coling 2008 Organizing Committee.

Bill MacCartney and Christopher D. Manning. 2009. An extended model of natural logic. In *Proceedings of the Eight International Conference on Computational Semantics*, pages 140–156, Tilburg, The Netherlands. Association for Computational Linguistics.

Lawrence S Moss. 2009. Natural logic and semantics. In *Proceedings of the 18th Amsterdam Colloquium: Revised Selected Papers*, pages 71–80, Berlin. University of Amsterdam, Springer.

Aakanksha Naik, Abhilasha Ravichander, Norman Sadeh, Carolyn Rose, and Graham Neubig. 2018. Stress test evaluation for natural language inference. In *Proceedings of the 27th International Conference on Computational Linguistics*, pages 2340–2353, Santa Fe, New Mexico, USA. Association for Computational Linguistics.

Yixin Nie, Yicheng Wang, and Mohit Bansal. 2019. Analyzing compositionality-sensitivity of NLI models. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pages 6867–6874.

Ankur P. Parikh, Oscar Täckström, Dipanjan Das, and Jakob Uszkoreit. 2016. A decomposable attention model for natural language inference. *CoRR*, abs/1606.01933.

Barbara H. Partee. 1984. Compositionality. In Fred Landman and Frank Veltman, editors, *Varieties of Formal Semantics*, pages 281–311. Foris, Dordrecht.

Matthew Peters, Mark Neumann, Luke Zettlemoyer, and Wen-tau Yih. 2018. Dissecting contextual word embeddings: Architecture and representation. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 1499–1509, Brussels, Belgium. Association for Computational Linguistics.

Christopher Potts. 2019. A case for deep learning in semantics: Response to Pater. *Language*, 95(1):e115–e125.

Kyle Richardson, Hai Hu, Lawrence S. Moss, and Ashish Sabharwal. 2019. Probing natural language inference models through semantic fragments.

Victor Sánchez-Valencia. 1991. *Studies in Natural Logic and Categorial Grammar*. Ph.D. thesis, University of Amsterdam.

Ian Tenney, Dipanjan Das, and Ellie Pavlick. 2019. BERT rediscovers the classical NLP pipeline. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 4593–4601, Florence, Italy. Association for Computational Linguistics.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Ł ukasz Kaiser, and Illia Polosukhin. 2017. Attention is all you need. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, *Advances in Neural Information Processing Systems 30*, pages 5998–6008. Curran Associates, Inc.

Thomas Wolf, Lysandre Debut, Victor Sanh, Julien Chaumond, Clement Delangue, Anthony Moi, Pierric Cistac, Tim Rault, R’emi Louf, Morgan Funtowicz, and Jamie Brew. 2019. Huggingface’s transformers: State-of-the-art natural language processing. *ArXiv*, abs/1910.03771.

Hitomi Yanaka, Koji Mineshima, Daisuke Bekki, Kentaro Inui, Satoshi Sekine, Lasha Abzianidze, and Johan Bos. 2019a. Can neural networks understand monotonicity reasoning? In *Proceedings of the 2019 ACL Workshop BlackboxNLP: Analyzing and Interpreting Neural Networks for NLP*, pages 31–40, Florence, Italy. Association for Computational Linguistics.

Hitomi Yanaka, Koji Mineshima, Daisuke Bekki, Kentaro Inui, Satoshi Sekine, Lasha Abzianidze, and Johan Bos. 2019b. HELP: A dataset for identifying
shortcomings of neural models in monotonicity reasoning. In *Proceedings of the Eighth Joint Conference on Lexical and Computational Semantics (*SEM 2019)*, pages 250–255, Minneapolis, Minnesota. Association for Computational Linguistics.