We compute the thermal axino production rate in supersymmetric QCD to leading order in the gauge coupling. Using hard thermal loop resummation and the Braaten–Yuan prescription, we obtain a finite result in a gauge-invariant way, which takes into account Debye screening in the hot quark–gluon–squark–gluino plasma. The relic axino density from thermal reactions in the early Universe is evaluated assuming the axino is the lightest supersymmetric particle and stable due to R-parity conservation. From the comparison with the WMAP results, we find that axinos could provide the dominant part of cold dark matter, for example, for an axino mass of 100 keV and a reheating temperature of $10^6$ GeV.

1. Introduction

In supersymmetric extensions of the standard model in which the strong CP problem is solved by the Peccei–Quinn (PQ) mechanism, the axino $\tilde{a}$ arises naturally as the fermionic superpartner of the axion. Thus, the axino is electrically and color neutral and its interactions with the MSSM particles are suppressed by the PQ scale $f_a/N \sim 5 \times 10^9$ GeV. If the axino is the lightest supersymmetric particle (LSP) and if R-parity is conserved, axinos are stable and could provide the dominant part of cold dark matter.

In this talk we present the computation of the thermal axino production rate at high temperatures and discuss its cosmological implications. With inflation governing the earliest moments of the Universe, any initial population of axinos was diluted away and the thermal production of axinos set in at the reheating temperature $T_R$. We restrict our investigation to $f_a/N > T_R \gtrsim 10^4$ GeV, where the $U(1)_{PQ}$ symmetry is broken and axino production from decays of particles out of equilibrium is negligible. The results presented are extracted from Ref. 3 where more details can be found.

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2. Thermal Axino Production in Supersymmetric QCD

We concentrate on the axino–gluino–gluon interactions given by the dimension-5 interaction term

\[ \mathcal{L}_{\tilde{a}gg} = i \frac{g^2}{64\pi^2 f_a/N} \tilde{a} \gamma_5 [\gamma^\mu, \gamma^\nu] \tilde{g} G^a_{\mu\nu} \]  

with the strong gauge coupling \( g \), the gluon field strength tensor \( G^a_{\mu\nu} \), the gluino \( \tilde{g} \), and \( N \) being the number of quarks with PQ charge. The resulting \( 2 \rightarrow 2 \) scattering processes in supersymmetric QCD and the corresponding squared matrix elements are listed in Table 1, where \( s = (P_1 + P_2)^2 \) and \( t = (P_1 - P_3)^2 \) with \( P_1, P_2, P_3 \), and \( P \) referring to the particles in the given order. Working in the limit, \( T \gg m_i \), the masses of all particles involved have been neglected. Sums over initial and final spins have been performed.

For quarks and squarks, the contribution of a single chirality is given.

The processes B, F, G, and H lead to a logarithmic collinear singularity due to \( t \)-channel and \( u \)-channel exchange of soft (massless) gluons. Here screening effects in the hot quark–gauge–squark–gluino plasma (QGSGP) have to be taken into account. With hard thermal loop (HTL) resummation,\(^4\) this can be done systematically in a gauge-invariant way. Following the Braaten–Yuan prescription,\(^5\) we introduce a momentum scale \( k_{\text{cut}} \) such that \( gT \ll k_{\text{cut}} \ll T \) in the weak coupling limit, \( g \ll 1 \). This separates soft gluons with momentum transfer of order \( gT \) from hard gluons with momentum transfer of order \( T \). In the region of soft momentum tran-

| process \( i \) | \( |\mathcal{M}_i|^2/8\pi^2 T^2_{ji}(f_a/N)^2 \) |
|-----------------|------------------------------------------|
| A \( g^a + g^b \rightarrow g^c + \tilde{a} \) | \( 4(s + 2t + 2\frac{s^2}{t})|f^{abc}|^2 \) |
| B \( g^a + g^b \rightarrow g^c + \tilde{a} \) | \(-4(t + 2s + 2\frac{s^2}{t})|f^{abc}|^2 \) |
| C \( \tilde{q}_i + q_j \rightarrow q_j + \tilde{a} \) | \( 2s|T^a_{ji}|^2 \) |
| D \( g^a + q_i \rightarrow q_j + \tilde{a} \) | \(-2t|T^a_{ji}|^2 \) |
| E \( \tilde{q}_i + q_j \rightarrow g^a + \tilde{a} \) | \(-2t|T^a_{ji}|^2 \) |
| F \( g^a + g^b \rightarrow g^c + \tilde{a} \) | \(-s(s + 2t + 2\frac{s^2}{t})|f^{abc}|^2 \) |
| G \( q_i + g^a \rightarrow q_j + \tilde{a} \) | \(-4(s + \frac{s^2}{t})|T^a_{ji}|^2 \) |
| H \( \tilde{q}_i + q_j \rightarrow \tilde{a} + \tilde{a} \) | \(-2(\frac{s}{t} + 2s + 2\frac{s^2}{t})|T^a_{ji}|^2 \) |
| I \( q_i + \tilde{q}_j \rightarrow \tilde{g}^a + \tilde{a} \) | \(-4(t + \frac{s^2}{t})|T^a_{ji}|^2 \) |
| J \( \tilde{q}_i + \tilde{q}_j \rightarrow \tilde{g}^a + \tilde{a} \) | \(2(s + 2t + 2\frac{s^2}{t})|T^a_{ji}|^2 \) |
fer, \( k < k_{\text{cut}} \), we use the HTL-resummed gluon propagator, which takes into account Debye screening in QGSGP in terms of the supersymmetric thermal gluon mass \( m_g = gT \sqrt{(N_c + n_f)/6} \), where \( N_c = 3 \) and \( n_f = 6 \) are respectively the number of colors and color triplet and anti-triplet chiral multiplets. Computing the imaginary part of the thermal axino self-energy with the ultraviolet (UV) momentum cutoff \( k_{\text{cut}} \), the soft contribution to the production rate of axinos with energies \( E > \sim T \) is obtained

\[
\frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{soft}} = f_F(E) \frac{3g^4(1-N_c^2)m_g^2T}{4096\pi^8(f_a/N)^2} \left[ \ln \left( \frac{k_{\text{cut}}^2}{m_g^2} \right) - 1.379 \right],
\]

where \( f_F(E) = \left[ \exp(E/T + 1) \right]^{-1} \). In the region of hard momentum transfer, \( k > k_{\text{cut}} \), the bare gluon propagator can be used. From the squared matrix elements of the \( 2 \rightarrow 2 \) processes (cf. Table 1) weighted with appropriate multiplicities, statistical factors, and phase space densities, the corresponding hard contribution can then be obtained conveniently

\[
\frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{hard}} = \frac{g^6}{32\pi^4(f_a/N)^2} \left[ (N_c + n_f) \frac{T^3f_F(E)}{128\pi^4} \ln \left( \frac{2^{1/3}T}{k_{\text{cut}}} \right) + \ldots \right]
\]

with \( k_{\text{cut}} \) as the infrared (IR) cutoff for gluon momentum transfers. The expression represented by the ellipses can be found in Ref. 3. It is independent of \( k_{\text{cut}} \). Thus, by summing the soft and hard contributions, the artificial \( k_{\text{cut}} \) dependence cancels and the finite leading order rate is obtained

\[
\frac{d\Gamma_{\tilde{a}}}{d^3p} = \frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{soft}} + \frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{hard}}.
\]

In Fig. 1a, we show the normalized thermal axino production rate \( 1/\Gamma_{\tilde{a}}d\Gamma_{\tilde{a}}/d(E/T) \) as a function of \( E/T \) for temperatures of \( T = 10^6 \) GeV, \( 10^7 \) GeV, \( 10^8 \) GeV, and \( 10^9 \) GeV. For \( E \lesssim T \), we expect modifications as our computation is restricted to the thermal production of hard \( (E \gtrsim T) \) axinos. Here and below the 1-loop running of the strong coupling in the MSSM is taken into account by replacing \( g \) with \( g(T) = [g^{-2}(M_Z) + 3\ln(T/M_Z)/(8\pi^2)]^{-1/2} \), where the value of the strong coupling at the Z-boson mass \( M_Z \), \( g^2(M_Z)/(4\pi) = 0.118 \), is used as input. For \( E/T \lesssim 2 \), the rate turns negative except for very high temperatures. With \( g(T = 10^6 \) GeV) = 0.986 and \( g(T = 10^9 \) GeV) = 0.880, this unphysical behavior follows from the extrapolation of our result from \( g \approx 1 \) to \( g \approx 1 \). As higher-order corrections might become sizable for \( T \lesssim 10^6 \) GeV, new techniques are needed to compute the thermal production rate reliably also for \( g \gtrsim 1 \). At present, however, our result derived in a gauge-invariant way supersedes previous gauge-dependent and cutoff-dependent estimates.
3. Stable Axinos as Dark Matter in the Universe

By comparing the axino interaction rate for \( f_a/N = 10^{11} \) GeV with the Hubble parameter \( H \) for the early radiation-dominated epoch, an axino decoupling temperature of \( T_D \approx 10^9 \) GeV is estimated. For \( T_R < T_D \), axinos have not been in thermal equilibrium after inflation. Here the evolution of the axino number density \( n_\tilde{a} \) with cosmic time \( t \) is described by the Boltzmann equation with a collision term \( C_\tilde{a} \) accounting for both the axino production and disappearance processes in the primordial plasma,

\[
\frac{dn_\tilde{a}}{dt} + 3Hn_\tilde{a} = C_\tilde{a} .
\]

The disappearance processes can be neglected for \( T_R \) sufficiently below \( T_D \). Then, by integrating the production rate (4), we obtain the collision term

\[
C_\tilde{a} \approx \frac{(N^2 - 1) 3\zeta(3) g^6 T^6}{(f_a/N)^2} \left[ \ln \left( \frac{1.380 T^2}{m_\tilde{g}^2} \right) \right] \left( N_c + n_f \right) + 0.4336 n_f ,
\]

where \( \zeta(3) \approx 1.2021 \). Assuming conservation of entropy per comoving volume, the Boltzmann equation can be solved analytically. The resulting present \( (t_0) \) axino density parameter \( \Omega_\tilde{a} h^2 = m_\tilde{a} n_\tilde{a}(t_0) h^2/\rho_c \) with \( \rho_c/h^2 = 3.6 \times 10^{-9} \) GeV depends on the axino mass \( m_\tilde{a} \), the PQ scale \( f_a/N \), and
the reheating temperature \( T_R \) in the following way
\[
\Omega_\tilde{a} h^2 = 5.5 g^6 \ln \left( \frac{1.108}{g} \right) \left( \frac{m_\tilde{a}}{0.1 \text{ GeV}} \right) \left( \frac{10^{11} \text{ GeV}}{f_a/N} \right)^2 \left( \frac{T_R}{10^4 \text{ GeV}} \right),
\]
where \( g = g(T_R) \). In Fig. 1b, this result is illustrated as a function of \( T_R \) for \( f_a/N = 10^{11} \text{ GeV} \) and \( m_\tilde{a} = 1 \text{ keV}, 100 \text{ keV}, \) and 10 MeV. For \( T_R \) above \( T_D \), axinos are in thermal equilibrium so that \( \Omega_\tilde{a} h^2 \) is independent of \( T_R \) as shown for \( m_\tilde{a} = 1 \text{ keV} \) by the horizontal line. There will be a smooth transition instead of a kink once the axino disappearance processes are taken into account. The grey band indicates the WMAP result on the cold dark matter density (2\( \sigma \) error) \( \Omega_{CDM} h^2 = 0.113^{+0.016}_{-0.018} \).

4. Conclusion
For \( m_\tilde{a} = 100 \text{ keV} \) and \( T_R \approx 10^6 \text{ GeV} \), the relic axino density (obtained with \( f_a/N = 10^{11} \text{ GeV} \)) agrees with the WMAP result on the cold dark matter density. Although relatively light for being cold dark matter, axinos with \( m_\tilde{a} = 100 \text{ keV} \) could still explain large-scale-structure formation, the corresponding power spectrum, and the early reionization observed by WMAP. Thus, as far as abundance and structure formation are concerned, axinos from thermal production in the early Universe are indeed a viable solution of the cold dark matter problem. This conclusion has already been drawn in Ref. 2 based on a gauge-dependent and cutoff-dependent estimate of the relic axino abundance from thermal production. Since our result for \( \Omega_\tilde{a} h^2 \) is smaller by a factor of ten, the upper limit on \( T_R \) for a given value of \( m_\tilde{a} \) is relaxed by one order of magnitude. This can be important for models of inflation and for the understanding of the baryon asymmetry in the Universe. Already \( T_R \approx 10^6 \text{ GeV} \) is relatively small and excludes, for example, thermal leptogenesis.

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