On Surface Tension for Compact Stars

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Abstract. In an earlier analysis it was demonstrated that general relativity gives higher values of surface tension in strange stars with quark matter than neutron stars. We generate the modified Tolman–Oppenheimer–Volkoff equation to incorporate anisotropic matter and use this to show that pressure anisotropy provides for a wide range of behaviour in the surface tension than is the case with isotropic pressures. In particular, it is possible that anisotropy drastically decreases the value of the surface tension.

Key words. Relativity—pulsars—equation of state.

1. Introduction

Stars that are more compact than neutron stars, at present, have become a subject of considerable interest as they provide us natural laboratories for testing QCD. Over the last couple of decades, various models have been proposed to explain the compactness and properties of some of the observed compact objects. Pioneering works in this field have put forward new concepts of compact matter, namely strange stars (Witten 1984; Farhi & Jaffe 1984) and boson stars (Kaup 1968; Ruffini & Bonazzola 1969; Colpi et al. 1986). Due to the high matter densities within such stars one expects pressure to be anisotropic in general, i.e., in the interior of such stars the radial pressure and tangential pressure are different. An anisotropic energy momentum is a topic which is often ignored in the calculations of compact stars. However, since the pioneering work of Bowers & Liang (1974) there has been extensive research in the study of anisotropic relativistic matter in general relativity. The analysis of static spherically symmetric anisotropic fluid spheres is important in relativistic astrophysics. Ruderman (1972) showed that nuclear matter may be anisotropic in the high density ranges of order $10^{15}$ gm cm$^{-3}$ where nuclear interactions have to be treated relativistically. Anisotropy in compact objects may occur due to the existence of a solid core or the presence of type 3A superfluid (Kippenhahm & Weigert 1990), phase transition (Sokolov 1980), pion condensation (Sawyer 1972), slow rotation (Herrera & Santos 1997), mixture of two gases (Letelier 1980) or strong magnetic fields (Weber 1999). Also objects made up of self-interacting scalar particles known as boson stars are naturally anisotropic in their configurations. Anisotropic models for compact self gravitating objects have been studied by Herrera & Santos (1997); Rao et al. (2000); Corchero (2001); Mak & Harko
(2003); Ivanov (2002); Dev & Gleiser (2003); Hernández & Núñez (2004); Chaisi & Maharaj (2005), and many others. Anisotropic models for compact objects have been shown to achieve high red-shift values (Bowers & Liang 1974; Herrera & Santos 1997; Ivanov 2002; Mak & Harko 2003), and they are stable (Herrera & Santos 1997; Dev & Gleiser 2003). In this article, we show that pressure anisotropy may also affect the surface tension of compact stars. We believe that this aspect has not been considered yet in the context of anisotropic stellar models.

2. Surface tension of strange stars

In a recent paper by Bagchi et al. (2005), it has been shown that objects composed of u, d and s quarks popularly known as ‘strange stars’ give higher values of surface tension than neutron stars, a necessary criterion for the existence of stable strange stars in the Universe. This calculation is based on equations of state (EOS) for strange matter formulated by Dey et al. (1998). In an approximated linearized form, the EOS may be written as (Zdunik 2000; Gondek-Rosińska et al. 2000)

\[ p = a(\rho - \rho_b), \quad (1) \]

where \( \rho \) is the energy density, \( \rho_b \) is the density at the surface, \( p \) is the isotropic pressure, and \( a \) is a parameter related to the velocity of sound \( (a = dp/d\rho) \).

To calculate the surface tension, one assumes that the star is a huge spherical ball composed of strange matter which is self-bound and non-rotating. The excess pressure on the surface of the star can be expressed as

\[ |\Delta p|_{r=R} = \frac{2S}{R} , \quad (2) \]

where \( S \) is the surface tension of the star and \( R \) is the radius of curvature. At the surface

\[ |\Delta p|_{r=R} = r_n \frac{dp}{dr} |_{r=R}, \quad (3) \]

where \( r_n \) is the radius of the quark particle given by \( r_n = (1/\pi n)^{1/3} \) where \( n \) is the baryon number density. As strange stars are very compact, a relativistic treatment is necessary to find their configurations and other physical parameters. Thus for a given EOS, one uses the Tolman–Oppenheimer–Volkoff (TOV) equation (Oppenheimer & Volkoff 1939)

\[ \frac{dp}{dr} = - \frac{G(\rho + p)}{c^2 r} \left[ \frac{m(r)}{c^2 r} + \frac{4\pi r^2 p}{c^4} \right] \left( 1 - \frac{2Gm(r)}{r} \right) \quad (4) \]

to find the surface tension of the star, making use of equations (2) and (3). This method helps to yield higher values of surface tension as compared to neutron stars including the possible explanation for the existence of strange stars in the Universe and other related phenomena like delayed \( \gamma \)-ray bursts (Bagchi et al. 2005).

However, at very high densities, anisotropy may be significant in such stars which may contribute to the surface tension. If we assume that pressure within such a star is anisotropic in general then the TOV equation (4) gets modified yielding different results as obtained by Bagchi et al. (2005). In the following sections, we derive the modified TOV equation with anisotropic pressure and perform some numerical calculations to show the effects of pressure anisotropy on the surface tension of compact stars.