1 Introduction

Evolution of massive stars ($M > \sim 20M$) is accompanied by a mass loss, initiated by a high luminosity and high radiation pressure. In blue supergiants, situated near the main sequence, mass loss rate is moderate $M \sim 10^{-6} M_{\odot}/yr.$ and is connected with the outflow of layers having small optical depth. This mass loss rate is determined by radiation pressure in lines, where spectral absorption coefficient may be very high. Theory of the outflow is based here on Sobolev approximation of the lines treatment, (CAK theory), and velocity gradient in the flow increase essentially the pushing effect of the radiation pressure (Castor, Abbot, Klein, 1975). First attempt to construct a theory of such mass outflow have been done by [9], and for recent development of CAK theory see [10].

Evolved massive stars may lose mass with much higher rate then the blue supergiants. Formation of single Wolf-Rayet stars probably took place as a result of such intense mass loss (Bisnovatyi–Kogan, Nadyozhin, 1972). Outflowing atmospheres of luminous massive stars are described by equations of radiation hydrodynamics with radiative heat conductivity. The flux goes through a critical point where the velocity is equal to the local isothermal speed of the sound. The main difference between the outflowing atmospheres in blue and evolved supergiants is a value of the optical depth (average Rosseland) in the critical point - $\tau_{cr}$. In blue supergiants we have $\tau_{cr} < 1$, and acceleration is essential only in parts of the radiation spectrum around the spectral lines, but in evolved yellow supergiants there is $\tau_{cr} >> 1$ and the whole radiation flux takes part in the flow acceleration.

Theory of the mass outflow from the very luminous evolved stars was developed by Bisnovatyi-Kogan, Zeldovich, (1968), and self-consistent method for evolutionary calculations with a mass loss was proposed by (Bisnovatyi-Kogan, Nadyozhin, 1969, Bisnovatyi-Kogan, Nadyozhin, 1972). The method was based on finding the solution of the outflowing envelope which continuously passes the isothermal speed singular point and smoothly enters the second singular point at infinity where $v \rightarrow v_{\infty}$, $T \rightarrow 0$. The shortcoming of the consideration of [9], where evolution with self-consistent mass loss was first calculated, was ignorance of the fact, that the optical depth in the flow is decreasing and far from the star $\tau \rightarrow 0$ there is no influence of matter on the radiation. Formally equations used in this paper described only optically thick outflow and could not be extended to infinity.

The aim of this paper is to correct this shortcoming and to give a unified description of the flow which is started at large optical depth and goes to infinity at $\tau = 0$. The approximate system of equations based on Eddington approximation for the radiation is derived and solution is found which is continuous in both singular points.

The recipe based on the approximate treatment of the outer boundary condition was proposed for the problem of the mass loss by [13], [16]. Similarly to
the case of a static atmosphere it was suggested that at the stellar photosphere where $T = T_{\text{eff}} = (L/4\pi r^2\sigma)^{1/4}$ there is a fixed value of the optical depth $\tau_{ph} = 2/3$, or later $\tau_{ph} = 8/3$ in the papers of (Kato, 1985, Kato, Iben, 1992). The basic approximation made in these papers was based on the approximate choice of the value of $\tau_{ph}$, which instead of its expression $\tau = \int \kappa \rho dr$ was taken as $\tau_{ph} \simeq \tilde{\tau}_{ph} = (\kappa \rho r)_{ph}$. It looks out here, that we avoid all problems connected with the singular point at infinity, because the region of solution is restricted by $r < r_{ph}$. In fact this approximation creates another problems. In the case of incomplete ionization and rapid change of the opacity as a function of $T$ and $\rho$ this approximation could introduce rather big errors into the solution. The method considered is free from such shortcomings also. In the solution presented below we have $\tau_{ph} = 4$ and $\tilde{\tau}_{ph} = 3.75$. Recently another approximate approach for solution of the outflowing atmosphere was proposed by [13]. Expanding layers of the star are divided into subsonically extending photosphere where the stationary momentum equation is adopted, and the wind where the accelerating up to $v_\infty$ velocity of the outflow is approximated by the prescribed profile with 3 parameters which are determined from matching and additional conditions.

2 Thermodynamic relations in outflowing envelope at arbitrary optical depth.

In deep layers, at large optical depth $\tau \gg 1$ we have a usual equation of state of a mixture of ideal gas with a black-body radiation.

$$P(\rho, T) = \frac{aT^4}{3} + \rho RT. \quad (1)$$

When $\tau$ is small it is not correct to speak about equilibrium between radiation and matter. Since local thermodynamical equilibrium is still presumed we take isotropic radiation component together with a gas pressure in the form

$$P_{\text{isotr.}}^{\tau \to 0} = \frac{aT^4}{3} \cdot \tau + \rho RT. \quad (2)$$

Multiplier $\tau$ in (2) ensures zero decreasing contribution of a $\frac{aT^4}{3}$ component at the infinity.

An increasing anisotropy of a pressure of radiation coming from the star at decreasing $\tau$ is taken into account in Eddington approximation. Basing on the solution of the radiative transfer equation in Eddington approximation for the spherically symmetric case with variable Eddington factor (Sobolev, 1967), we introduce the following approximate representation of thermodynamic relations for radiation.
\[ P_r = \frac{aT^4}{3} (1 - e^{-\tau}) + \frac{L(r)_{th}}{4\pi r^2 c}, \]  

(3)

\[ \rho E_r = aT^4 (1 - e^{-\tau}) + \frac{L(r)_{th}}{4\pi r^2 c}. \]  

(4)

Term in (3), (4) containing \( L_{th} \) is to be determined from a solution of self-consistent set of equations. This term essentially contributes only when \( \tau \) is small. This fact allows us to simplify further calculations by writing out \( L_{th}^\infty \) instead of \( L_{th}(r) \).

3 Basic equations.

A system of equations of radiation hydrodynamics describing continuous transition between optically thick and optically thin regions for the stationary outflow is written as:

\[ u \frac{du}{dr} = \frac{1}{\rho} \frac{dP_g}{dr} - \frac{GM(1 - \tilde{L}_{th})}{r^2}, \]

(5)

where \( \tilde{L}_{th} = \frac{L_{th}(r)}{L_{ed}} \), \( L_{ed} = \frac{4\pi cGM}{\kappa} \)

\[ L = 4\pi \mu \left( E + \frac{P}{\rho} - \frac{GM}{r} + \frac{u^2}{2} \right) + L_{th}(r) \]

(6)

\[ L_{th} = -\frac{4\pi r^2 c}{\kappa \rho} \left( \frac{dP_r}{dr} - \frac{E_r}{\rho} - 3P_r \right) \]

(7)

\[ \dot{M} \equiv \mu = \rho ur^2 \]

(8)

\[ P = \frac{aT^4}{3} (1 - e^{-\tau}) + \frac{L_{th}^\infty}{4\pi r^2 c} + P_g \]

(9)

\[ E_\rho = aT^4 (1 - e^{-\tau}) + \frac{L_{th}^\infty}{4\pi r^2 c} + E_g \]

(10)

\[ P_g = \rho RT \]

(11)

\[ E_g = \frac{3}{2} RT \]

(12)

\[ \tau = \int_r^\infty \kappa \rho dr \]

(13)

Where \( L \) - is a constant total energy flux consisting of the radiative energy transfer together with the energy of the matter current, \( u \) is a rate of the outflow,
\( \kappa \) is an opacity, assumed to be constant, \( a \) is the constant of a radiative energy density, \( \mathcal{R} \) is a gas constant.

We consider here the flow in the gravitational field of constant mass \( M \), neglecting self-gravity of the outflowing envelope outside the critical point \( m_{en} \). This approximation is very good in a realistic case of \( m_{en} \ll M \) (Zytkow, 1973).

This system of equations provides a description of a stationary outflowing envelope accelerated by a radiative force at arbitrary optical depth where continuum opacity is prevailed. In optically thick limit \( \tau \to \infty \), when terms in (9), (10) with \( L^\infty_{th} \) are negligible, and \( E_r \rho = 3P_r \) a solution of this system was obtained by [1]. In case of small \( \tau \) from (9), (10) for the anisotropic radiation flux we get: \( E_r \rho \simeq P_r \), what follows from transfer equation in Eddington approximation. (Sobolev, 1967).

When optical depth becomes small separation of radiation and matter should be taken into consideration. It means that only a part of radiation is interacting with outflowing gas envelope. For this part of quanta we still assume LTE to be valid, what means that we take the temperature in (3) - (4) to be the same as in (11) - (12). For the rest part of radiation that is not interacting with outflowing matter, another 'temperature'- mean energy of quanta should be introduced. This part of radiation transfers momentum to the outflowing matter and thus produces only the anisotropic part of the pressure, that is of the order of \( L^\text{th}/r^2 \). We avoid the problem of treating this another 'temperature' by solving the radiative transfer equation in momentum form (7). It is solved together with the equation of motion (5), energy conservation (6) and thermodynamical relations for the outflowing gaseous matter and radiation (9) - (10).

Equation of motion (5) is written in the form where the effect of the pressure gradient is explicitly written only for the gas component and all effects of radiation is embedded into the last term. This term is obtained from the transfer equation (7) when taking into account relations (9), (10). In case of large optical depths this approach insures that acceleration is providing by the total (isotropic = radiation + gas) pressure gradient, and in case of small \( \tau \) by gas pressure gradient and pressure of radiation flux. In intermediate region this terms will compete in accordance with (3), (4) and (7). About the difference of this system of equations from previously used see the Discussion.

Substituting (7) - (10) into (6) we get

\[
\frac{L}{4\pi} = \rho u r^2 \left( E + \frac{P}{\rho} - \frac{GM}{r} + \frac{\mu^2}{2\rho^2 r^4} \right) - \frac{r^2 c}{\kappa \rho} \left\{ \frac{d}{dr} \left[ \frac{(1 - e^{-\tau})aT^4}{3} + \frac{L^\infty_{th}}{4\pi r^2 c} \right] + 2 \frac{L^\infty_{th}}{4\pi r^2 c} \right\}. \tag{14}
\]

Differentiating in (14) with taking into account that \( d\tau = -\kappa \rho dr \) will result in a first differential equation.
\[ \lambda r^2 (1-e^{-\tau}) \frac{dT}{dr} = \mu \left\{ \frac{5}{2} R T - \frac{GM}{r} + \frac{\mu^2}{2 \rho^2 r^4} \right\} \]

\[ - \frac{L}{4\pi} + \mu \left[ 2 \frac{L_{th}^2}{4\pi r^2 c \rho} + \frac{4}{3\rho} (1-e^{-\tau}) aT^4 \right] \]

\[ + \frac{1}{3} a r^2 c T^4 e^{-\tau}. \quad (15) \]

Here we introduce a coefficient of a heat conductivity

\[ \lambda = \frac{4acT^3}{3\kappa \rho}. \quad (16) \]

In a limiting case of \( \tau \to \infty \) this equation coincides with a corresponding equation from [1], when taking into account that \[ \frac{L_{th}}{4\pi r^2} \ll caT^4. \]

Using (11), (8) in (5) will get

\[ \left( \frac{R T}{\rho} - \frac{\mu^2}{\rho^2 r^5} \right) \frac{d\rho}{dr} = -R \frac{dT}{dr} + \mu^2 \frac{2}{\rho^2 r^5} - \frac{GM}{r^2} (1 - \tilde{L}_{th}). \quad (17) \]

The system (15), (17) need to be completed from (13) by relation

\[ \frac{d\tau}{dr} = -\kappa \rho. \quad (18) \]

The equation (17) has a singular point, where the left hand side of it vanishes

\[ \frac{R T_{cr}}{\rho_{cr}} = \frac{\mu^2}{\rho_{cr}^2 r_{cr}^4}. \quad (19) \]

This point corresponds to the "isothermal sonic" point where

\[ u^2 = u_{cr}^2 = \left( \frac{\partial P}{\partial \rho} \right)_T. \]

Using (16), (7) gives

\[ L_{th} = -4\pi r^2 \left[ \lambda \frac{dT}{dr} (1-e^{-\tau}) - \frac{1}{3} a c T^4 e^{-\tau} \right]. \quad (20) \]

5
The second singular point of \((15) - (18)\) is situated at infinity, where
\[
T = 0, \quad \rho \sim \frac{1}{r^2} \to 0, \quad u \to \text{const} = u_\infty, \quad \text{when} \quad r \to \infty. \tag{21}
\]
This condition is related to the fact that far from the star the density in stellar wind is very small. In reality the wind may be treated as stationary only up to the limiting radius \(r_{lim} \sim v_\infty \tau > r_{cr}\), where \(t\) is the characteristic mass-loss time of the star. So the formal solution with outer boundary condition (21) is very close to the real solution with conditions at \(r_{lim}\). This approximation is a common way to consider a well-developed solar wind problem (Parker, 1963).

Let us introduce nondimensional variables
\[
\tilde{T}(r) = \frac{T(r)}{T_{cr}}, \quad \tilde{\rho}(r) = \frac{\rho(r)}{\rho_{cr}}, \quad \tilde{\mathcal{L}}_{th} = \frac{\mathcal{L}_{th}}{\mathcal{L}_{ed}}, \tag{22}
\]
\[
\tilde{x} = \frac{r_{cr}}{r}.
\]
After transformations we obtain dimensionless system of equations
\[
\frac{d\tilde{\rho}}{dx} = \left(\frac{x^4}{\rho^3} - \frac{T}{\rho}\right)^{-1} \left\{ \frac{d\tilde{T}}{dx} \left(1 + A_1 (1 - e^{-\tau}) \frac{T^3}{\rho}\right) - A_3 + \frac{1}{4} \frac{A_1 e^{-\tau} T^4}{A_5 x^2} + 2 \frac{x^3}{\rho^2} \right\}, \tag{23}
\]
\[
\frac{d\tilde{T}}{dx} = -\left(\frac{5}{2} T - A_3 x + \frac{1}{2} \frac{x^4}{\rho^2} + (1 - e^{-\tau}) A_1 \frac{T^4}{\rho} + \frac{e^{-\tau} T^4}{4 A_2 A_5 x^2} + 2 \frac{L^\infty}{A_3 A_5} \frac{x^2}{\rho} - \frac{A_4}{A_2} \right) \frac{A_2 \rho}{T^3 (1 - e^{-\tau})}, \tag{24}
\]
\[
\frac{d\tau}{dx} = \frac{\rho}{A_5 x^2} \tag{25}
\]
Where \(L^\infty = \tilde{\mathcal{L}}^\infty_{th}\). To simplify writing here and further we omit tilde. Dimensionless coefficients \(A_i\) are:
\[
A_1 = \frac{4a T^3_{cr}}{3 \rho_{cr} \mathcal{R}}, \quad A_2 = \frac{3 \kappa \mu}{4ac} \frac{\rho_{cr} \mathcal{R}}{r_{cr} T^4_{cr}}, \tag{26}
\]
\[
A_3 = \frac{GM}{r_{cr} \mathcal{R} T_{cr}}, \quad A_4 = \frac{3 \kappa L}{16ac \pi r_{cr} T^4_{cr}}. \tag{27}
\]
Physical sense of $A_i$ parameters have been revealed in [1]. Additional parameter is

$$A_5 = \frac{1}{r_{cr} \kappa \rho_{cr}},$$

the reciprocal $1/A_5$ is of the order of optical depth in the critical point.

Condition of transition of the solution through the critical point confines the number of dimensionless parameters. Equating to zero expression in the figure brackets of (23) in the critical point with account of (24) will get

$$A_4 = \left(4A_3A_5(1 - e^{-\tau_{cr}}) + 8A_5(-1 + e^{-\tau_{cr}}) + 4A_1^2A_2A_5(1 - 2e^{-\tau_{cr}} + (e^{-\tau_{cr}})^2) + A_2(12A_5 + A_3(-4A_5 + 8A_5^2L^\infty)) + A_1A_2(16A_5(1 - e^{-\tau_{cr}})) + A_3(4A_5(-1 + e^{-\tau_{cr}}) + A_5^2(8L^\infty - 8e^{-\tau_{cr}}L^\infty))\right)/A_5 + A_1A_5(1 - e^{-\tau_{cr}})).$$

All dimensional parameters of the flux: $T$, $\rho$, $r$ could be expressed as a function of dimensionless parameters $A_i$ and a dimensional combination of physical constants

$$r = \left(\frac{4\alpha \kappa}{3c}\right)^{2/5}\left(\frac{GM}{R^8/5}\right)^{1/5}\frac{1}{(A_1^2A_2A_3)^{2/5}A_3},$$

$$\rho = \left(\frac{3R}{4a}\right)^{3/5}\left(\frac{cR^{1/2}}{\kappa GM}\right)^{6/5}\left(A_1^2A_2A_3\right)^{6/5}\frac{\tilde{\rho}}{A_1}.$$

In numerical calculations coefficient $A_5$ should be expressed as a function of $A_1 - A_3$ and a nondimensional combination of physical constants and thus it is not an independent parameter

$$A_5 = \left(\frac{3}{4}\right)^{1/5}\frac{A_3^{1/5}R^{4/5}}{A_1^{3/5}A_2^{4/5}k^{1/5}a^{1/5}c^{4/5}(GM)^{1/5}}.$$  

4 Numerical solution

In order to satisfy boundary conditions far from the star we need to integrate (23) - (25) from the critical point outward to the infinity. We exit the critical
point by means of expansion formula. Expanding the solution in critical point
\[ x = T = \rho = 1 \] in powers of \((1 - x)\) we have
\[ T = 1 + a(1 - x), \quad (31) \]
\[ \rho = 1 + b(1 - x). \quad (32) \]
Similarly
\[ e^{-\tau} \approx e^{-\tau_{cr}} (1 + \frac{y}{A_5}), \]
where \(y = 1 - x\).

For the \(a\) and \(b\) coefficients we get
\[ b = (-12A_2A_5 - 4A_1A_2A_5 + 4A_2A_3A_5 + 4A_4A_5
+ 4A_1A_2A_5e^{-\tau_{cr}} - 8A_2A_3A_5^2L_\infty
- e^{-\tau_{cr}})/(4A_5((-1 + e^{-\tau_{cr}}))), \quad (33) \]
\[ a = (-c_1 - (c_1^2 - 4c_2c_0)^{1/2})/(2c_2). \quad (34) \]

The coefficients \(c_i\) due to their complicated form are adduced in Appendix.

Numerically integrating we escape the critical point by means of the expansion formulas (31), (32) . Then integrating outward to the infinity we satisfy the boundary conditions (21). Results of the numerical calculations are shown in Fig.1-2. Curves at this figures correspond to the following dimensional values of the dimensionless parameters:
\[ A_1 = 50, \quad A_2 = 10^{-4}, \quad A_3 = 43.88, \quad \tau_{cr} = 125, \quad L_\infty = 0.6. \] This set of parameters corresponds to the following values at critical point:
\[ T_{cr} = 1.4 \cdot 10^4 K, \quad \rho_{cr} = 2.6 \cdot 10^{13} cm, \quad \rho_{cr} = 6.6 \cdot 10^{-12} g/cm^3. \]

The behavior of the solution with rapidly decreasing Mach number at \(r < r_{cr}\) shows, that it may be matched to the static solution for the core. In reality the opacity peak is situated near critical point, opacity inside is decreasing and the inside velocity drop is more rapid, then in the case of \(\kappa = \text{const}\) (Bisnovatyi - Kogan and Nadyozhin, 1972). The effective temperature of the photosphere may be obtained from:
\[ L_{ph}/4\pi r^2 = \sigma T^4. \] For the given set of parameters we get:
\[ x_{ph} = 0.03, \quad \tau_{ph} = 4, \quad T_{eff} = 0.06, \quad \tilde{\tau}_{ph} = 3.75. \] Conditions in critical points impose two relations upon the set of nondimensional parameters. As it was shown the solution depends upon the following nodimensional parameters:
\[ A_1, \quad A_2, \quad A_3, \quad A_4, \quad \tau_{cr}, \quad L_\infty. \] When numerically treated \(A_4\) was expressed as a function of the remaining dimensionless parameters when taking into account that in sonic point \(x = \rho = T = 1 \) (22). Parameter \(A_3\) is to be numerically determined from the condition at the infinity where \(T = 0 \) (22).

When \(r \to \infty\) velocity tends to constant and thus \(\rho \sim 1/r^2\). Only the unique value of \(\tau_{cr}\) allows to obtain the proper behavior of \(u\) (or \(\rho\)) at the infinity. Varying \(\tau_{cr}\), we adjust the behavior of \(u\) to get \(u \to u_\infty\) at \(r \to \infty\).
At \( r = \infty \) the solution could be represented in an expansion form. In this case \( L_{\infty}^* \) could be directly determined from (20), and the appropriate expansion.

At the stellar core optical depth may be taken arbitrary large, and it is necessary to match only \( T \) and \( \rho \). Above mentioned method of obtaining the unique value of \( \tau_{cr} \) allows to avoid the problem of fitting \( \tau \) when integrating to the stellar core.

All other dimensionless parameters of the envelope and parameters of the static core could be obtained only when matching the solution for the stellar core with the solution for the expanding layers. In this approach all the treatment will be fully self-consistent.

In our treatment we have not used the second expansion and thus could not specify uniquely the \( L_{\infty}^* \).

Our aim was to develop a method of calculating the parameters of the flux in arbitrary \( \tau \). Fully self-consistent treatment which may be applied to the real star is under the consideration.

## 5 Discussion

Solution for the spherically-symmetrical stationary outflowing envelope accelerated by the radiative force in arbitrary optical depth case was obtained in this work.

We have introduced thermodynamical relations for the matter flux with partially separated radiation. This approach provides satisfactory description of the problem for the arbitrary \( \tau \). In case of \( \tau \to \infty \) equation of state yields \( E_r \rho = 3P_r \) and our method reproduces results of [1]. Also when \( \tau \to 0 \): \( E_r \rho \simeq P_r \) accords the result obtained from the radiative transfer equation in Eddington approximation.

As a result of this treatment we have introduced a system of differential equations. This system provides a continuous transition of the solution between optically thin and optically thick regions. To satisfy boundary conditions the solution should proceed through the critical point where the speed of the flux equals the local isothermal sound speed. We have derived analytically approximate representation of the solution at the vicinity of the sonic point. Using this representation we numerically integrate the system of equation from the critical point to satisfy conditions at the infinity.

Beginning from (Zhytkow, 1973), the obtaining of outer boundary conditions was oversimplified, making a wrong impression, that regions with \( \tau < 1 \), have no influence on the mass outflow. Zhytkow imposed boundary conditions at the photosphere. That was also the result of that there was no method for the self-consistent description of radiation and matter at small \( \tau \). It should be mentioned that most of the papers only inherited the method developed by Zhytkow. As soon as it is possible to describe correctly the whole region of the flow one should forthwith impose correct boundary conditions. Problem of the
infinite boundary conditions was elaborated for the theory of Solar wind.

In papers of Zytkow, it was made an attempt to describe the whole flow. The description was based on Paczynsky approximation (Paczynski, 1969), that roughly takes into account the dilution of the radiation flux. For $\tau < 2/3$ for the part of the equation of motion, that describes pressure, it was taken:

$$\frac{d\rho}{dx} \sim - \frac{\partial P_g}{\partial T} \frac{dT}{dr} + \frac{\kappa \rho L}{4\pi c r^2},$$

where

$$\frac{dT}{dr} = - \frac{L}{4\pi r^2 \lambda} - \frac{1}{2} f(\tau) T_0^{1/2} r^{-3/2},$$

where $T_0^4 = L/(4\pi ac R_0^2)$ - the effective temperature of the photosphere, and $L$ is constant for $\tau < 2/3$, $f = 1 - (3/2)\tau$. The second term in the relation for the temperature gradient originated from the approximation of regions of small $\tau$ (with Eddington factor 1), with regions of $\tau >> 1$ (with factor 1/3) and is to describe roughly the process of the radiation dilution. The shortcoming descended from the description of Zhytkow concerns this last effective surface ($T = T_{eff}$, $\tau = 2/3$). The 'temperature' of free, not interacting quanta corresponds to the prescribed $T_{eff}$ of the photosphere. It can be not a bad approximation, if mostly neglecting the influence of the regions with small $\tau$, but to describe the whole flow, this approach seems to be too rude. It seems obvious that the solution is artificially restricted by taking this prescribed separation with the certain value of the effective temperature of the photosphere.

When $\tau$ becomes small, separation of the radiation and matter progresses in two parts (two 'temperatures', see appropriate discussion), with accordance to the transfer equation in momentum form (7). If not treating the transfer equation, together with equations of hydrodynamics, the energy conservation will be broken for the radiation at the regions of small $\tau$. That will only enlarge the uncertainty within the extended atmosphere.

The main difference of our approach with previously published is that we use the equation of motion, in which the effects of the gas pressure are separated from the effects of the radiation, together with the transfer equation (7). The effect of the gas pressure gradient that is valid at small $\tau$ as well as in deep interior is explicitly written. At $\tau >> 1$ the main driving force originates from the gradient of the pressure of the gas together with the radiation. Far from the star acceleration occurs mainly due to the momentum transfer from the radiation to matter. In the way the equation (5) is written it is correct for arbitrary optical depth, if, of course, to calculate $L_{th}$ from (7). This description provides the proper competition of this essentially different effects of the radiation. Contrary to other authors which used approximations obtained from (7), we are resulting from the 'equations of state' (9) - (10). We believe, that this approach is almost free from the shortcomings mentioned above.

We have used equation of state which corresponds to constant $R_g$. Account of variable $R_g$, $\dot{M}$ and $\kappa$ are of principal importance for the real stars as it was described by (Bisnovatyi-Kogan, Nadyozin, 1972). This complication does not change our method and qualitative results. When one will take into account...
lines effects, the importance to describe correctly the region of small $\tau$ will forthwith be greater.

As it was mentioned at the beginning, the problem of taking $\tilde{\tau}$ instead of $\tau$ needs to be treated more carefully. In this paper we simply assumed $\kappa$ to be constant. When considered far from the star velocity of the flux may be approximately taken constant, and thus from $M = 4\pi \rho u r^2$, get $\rho \sim 1/r^2$. In this case $\tau \simeq \tilde{\tau}$. Otherwise if we consider a power law of $\kappa = \rho^\alpha$ it is easy to obtain $\tau_{ph} = (\kappa \rho r)_{ph}/(1 + 2\alpha)$, what is $(1 + 2\alpha)$ times smaller than $\tilde{\tau}_{ph}$. For steeper (exponential) decrease of $\kappa$ with $r$ the difference between $\tau_{ph}$ and $\tilde{\tau}_{ph}$ may be much larger.

In real stars opacity may increase more steep. Even if temperature is decreasing smoothly, partial recombination of ions will cause an increase of $\kappa$ (Cox, Tabor 1975). Hence it will create regions where flow is accelerated and $u$ can not be taken constant and thus law for $\rho$ will be far from $1/r^2$. In this case rather big errors could be introduced when taken $\tilde{\tau}$ instead of $\tau$.

On a supergiant phase massive stars may have regions of not fully ionized H and He what cause an increase of the opacity which leads to the acceleration in continuum (Bisnovatyi - Kogan, Nadyozhin 1969, 1972).

To take into account real physical effects our method should be improved by taking non-constant $\kappa$ and variable ionization degree. If improved this way theoretical approach presented in this work should be useful for self-consistent simulations of the massive stars evolution with mass loss.

This work was partly supported by RFBR grant 96-02-17231, grant 96-02-16553, and CRDF grant RP1-173, Astronomical Program 1.2.6.5.

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Further sophisticated calculations were made using Mathematica 2.2.

\[ \begin{align*}
\mathbf{c}_0 &= (A_1A_2^2A_3^3A_5^3(1 - e^{-\tau \epsilon})(L^\infty)^2 + A_2^3(-40A_2^2A_3L^\infty + A_2^3A_3(-160 + 320e^{-\tau \epsilon} - \\
&160(e^{-\tau \epsilon})^2L^\infty + A_2A_3176(1 - e^{-\tau \epsilon})L^\infty + A_2^3(-48 + 48e^{-\tau \epsilon}L^\infty) + \\
&A_1(2A_2A_3(80 + 96A_4 + (-160 - 96A_4)e^{-\tau \epsilon} + 80(e^{-\tau \epsilon})^2)L^\infty + A_2^3A_3^296(1 - e^{-\tau \epsilon})L^\infty + A_3392(-1 + \\
&2(e^{-\tau \epsilon})^2 + A_5(2(e^{-\tau \epsilon})^2 + e^{-\tau \epsilon}(-2 + 8A_4) + A_2e^{-\tau \epsilon}(-29 + 8A_3) + 2A_3(1e^{-\tau \epsilon} - (e^{-\tau \epsilon})^2) + \\
&2A_2^2A_2e^{-\tau \epsilon}(1 - 2(e^{-\tau \epsilon}) + (e^{-\tau \epsilon})^2) + A_1(-2A_4e^{-\tau \epsilon} + 2A_4(e^{-\tau \epsilon})^2 + A_2(-7e^{-\tau \epsilon} + 7(e^{-\tau \epsilon})^2 + A_3 \\
&(e^{-\tau \epsilon})^2)))) + A_3^2(-96 + A_2^3(-60 + 20A_3) - 72A_4 + (192 + 72A_4)e^{-\tau \epsilon} - 96(e^{-\tau \epsilon})^2 + A_3(24 + 24A_4 \\
&(-48 - 24A_4)e^{-\tau \epsilon} + 24(e^{-\tau \epsilon})^2) + A_4^3A_3^2(-56 + 168e^{-\tau \epsilon} - 168(e^{-\tau \epsilon})^2 + 56(e^{-\tau \epsilon})^3) + \\
&A_3^3(2A_2(24 + 80A_4 + (-72 - 160A_4)e^{-\tau \epsilon} + (72 + 80A_4)(e^{-\tau \epsilon})^2 - 24(e^{-\tau \epsilon})^3) + A_2^3(-292 + 584e^{-\tau \epsilon} \\
&A_2A_3(80 - 160e^{-\tau \epsilon} + 80(e^{-\tau \epsilon})^2)))+ A_2(232 + 20A_4 + A_3^3(24 - 24e^{-\tau \epsilon}) - 232e^{-\tau \epsilon} + \\
A_3(-152 + 152e^{-\tau \epsilon} - 16e^{-\tau \epsilon}L^\infty)) + A_4(-24A_4 - 24A_4^2 + (48A_4 + 24A_4^2)e^{-\tau \epsilon} - 24A_4(e^{-\tau \epsilon})^2 + \\
A_3^2(-392 + A_3(196 + 196e^{-\tau \epsilon}) + 392e^{-\tau \epsilon} + A_3^3(-24 + 24e^{-\tau \epsilon}) + A_2(160 + 196A_4 + (-320 - 196A_4 \\
&160(e^{-\tau \epsilon})^2 + A_3(-56 - 48A_4 + (112 + 48A_4)e^{-\tau \epsilon} - 56(e^{-\tau \epsilon})^2 + L^\infty(4e^{-\tau \epsilon} - 4(e^{-\tau \epsilon})^2))))))/(2(- \\
\mathbf{c}_1 &= A_2^3(8 + A_2(56 - 20A_3) - 20A_4 + A_3(12 - 12e^{-\tau \epsilon}) - 8e^{-\tau \epsilon} + \\
&A_2^2A_2(8 - 16e^{-\tau \epsilon}e^{-\tau \epsilon})^2) + \\
&A_1(-12A_4 + 12A_4e^{-\tau \epsilon} + A_2(48 - 48e^{-\tau \epsilon} + A_3(-12 + 12e^{-\tau \epsilon}))) + \\
&2A_2^3(32A_2A_3L^\infty + A_1A_2A_3(16 - 16e^{-\tau \epsilon})L^\infty) + 5A_2e^{-\tau \epsilon} ,
\end{align*} \]
7 Figure captions

**Fig. 1** Results of the integration of equations (23) - (25) from the critical point \( x = 1 \) outward to the infinity \( x = 0 \). Points \( x = 1 \) and \( x = 0 \) are the singular points of our system. A solution shown for \( A_1 = 50, A_2 = 10^{-4}, A_3 = 43.89, \tau_{cr} = 125, L_\infty^{th} = 0.6, \) passes through the critical point \( (T_{cr} = 1.4 \cdot 10^4 K, r_{cr} = 2.6 \cdot 10^{13} cm, \rho_{cr} = 6.6 \cdot 10^{-12} g/cm^3) \) and satisfies boundary conditions \([21]\).

**Fig. 2** Dimensionless velocity (solid line) and Mach number \( u \cdot \left( \frac{\partial \rho}{\partial \rho} \right)_s^{-\frac{1}{2}} \) (dashed line). Inwardly decreasing Mach number ensures matching the static core even in the case of \( \kappa = const \). When \( r \to \infty \) velocity tends to constant \( v_\infty \simeq 16 km/s \) and \( \rho \sim 1/r^2 \). Solution is shown for the same values of nondimensional parameters as on Fig. 1.
