Polarization of Prompt $J/\psi$ at the Tevatron

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The polarization of prompt $J/\psi$ at the Fermilab Tevatron is calculated within the nonrelativistic QCD factorization framework. The contribution from radiative decays of P-wave charmonium states decreases, but does not eliminate, the transverse polarization at large transverse momentum. The angular distribution parameter $\alpha$ for leptonic decays of the $J/\psi$ is predicted to increase from near 0 at $p_T = 5$ GeV to about 0.5 at $p_T = 20$ GeV. The prediction is consistent with measurements by the CDF Collaboration at intermediate values of $p_T$, but disagrees by about 3 standard deviations at the largest values of $p_T$ measured.

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The production of charmonium and bottomonium states in high-energy collisions probes both the hard-scattering parton processes that create heavy quark-antiquark ($Q\bar{Q}$) pairs and the hadronization process that transforms them into color-singlet bound states. One particularly sensitive probe is the polarization of the $J^{PC} = 1^{-+}$ charmonium states. The nonrelativistic QCD (NRQCD) factorization approach to inclusive quarkonium production [1] makes the remarkable prediction that in hadron collisions these states should be transversely polarized at sufficiently large transverse momentum ($p_T$) [3]. Recent measurements at the Tevatron by the CDF Collaboration seem to be in dramatic contradiction with this prediction [3].

As first pointed out by Cho and Wise [2], the prediction of transverse polarization for $1^{-+}$ states at large $p_T$ follows from three simple features of the dynamics of massless partons and heavy quarks. First, the inclusive production of quarkonium (or any other hadron) at sufficiently large $p_T$ is dominated by fragmentation. In $p\bar{p}$ collisions at the Tevatron, the dominant contribution to the charmonium production rate at large $p_T$ comes from gluon fragmentation [2]. The gluon is almost on shell and thus predominantly transversely polarized. Second, a $Q\bar{Q}$ pair with small relative momentum created by the virtual gluon is, at leading order in $\alpha_s$, in a color-octet $3S_1$ state [2] with the same transverse polarization as the gluon. Third, the spin symmetry of nonrelativistic heavy quarks implies the suppression of spin-flip transitions in the binding of the $Q\bar{Q}$ pair into quarkonium. Thus, $1^{-+}$ states should have a large transverse polarization at sufficiently large $p_T$. A convenient measure of the polarization is the variable \[ \alpha = (\sigma_T - 2\sigma_L)/(\sigma_T + 2\sigma_L), \] where $\sigma_T$ and $\sigma_L$ are the transverse and longitudinal components of the cross section, respectively. Beneke and Rothstein studied the dominant fragmentation mechanisms for $\sigma_L$ [3], and concluded that, at sufficiently large $p_T$, $\alpha$ should be in the range $0.5 - 0.8$.

For charmonium production at the Tevatron, fragmentation does not yet dominate for most of the $p_T$ range that is experimentally accessible. In order to study the onset of the polarization effect, it is necessary to take into account the fusion contributions from parton processes $ij \to c\bar{c} + k$. Quantitative calculations of the polarization variable $\alpha$ for direct $\psi'$ mesons (i.e. those that do not come from decays) at the Tevatron have been carried out by Beneke and Krämer [4] and by Leibovich [5]. They predicted that $\alpha$ should be small for $p_T \lesssim 5$ GeV, but then should rise dramatically to $0.77 \pm 0.08$ at $p_T = 20$ GeV, according to Beneke and Krämer, and to $0.90 \pm 0.04$, according to Leibovich. The CDF Collaboration has measured the polarization of direct $\psi'$ [6], but the error bars are too large to draw any definitive conclusions.

The CDF Collaboration has also measured the polarization of prompt $J/\psi$ mesons [7] (i.e. those that do not come from the decay of $B$ hadrons). The number of $J/\psi$ events is larger than for $\psi'$ by a factor of about 100, allowing $\alpha$ to be measured more precisely and in more $p_T$ bins. They find that $\alpha$ has a positive value $0.32 \pm 0.10$ in the $p_T$ bin from 8 to 10 GeV. However, instead of increasing at larger $p_T$, $\alpha$ decreases to $\sim 0.29 \pm 0.23$ in the highest $p_T$ bin from 15 to 20 GeV. Theoretical predictions of the polarization of prompt $J/\psi$ are complicated by the fact that the prompt signal includes $J/\psi$ mesons that come from decays of the higher charmonium states $\chi_{c1}, \chi_{c2},$ and $\psi'$. They account for about 15%, 15%, and 10% of the prompt $J/\psi$ signal, respectively [7]. The polarization of $J/\psi$ from $\psi'$ not via $\chi_{c,J}$ is straightforward to calculate, since the spin is unchanged by the transition. The polarization of $J/\psi$ from $\chi_{c,J}$ and of $J/\psi$ from $\psi'$ via $\chi_{c,J}$ is more complicated, because the $\chi_{c,J}$ mesons are produced in various spin states and they decay into $J/\psi$ through radiative transitions.

In this letter, we present a quantitative analysis of the polarization of prompt $J/\psi$ using the NRQCD factorization formalism. We reanalyze the CDF data on the $p_T$ distributions for $J/\psi, \chi_{c_J}$, and $\psi'$ to determine the relevant color-octet NRQCD matrix elements (ME’s). The cross sections for the spin states of $J/\psi, \chi_{c_J}$, and $\psi'$ are calculated using these ME’s and the appropriate parton cross sections. The cross sections for the spin states of
the $\chi_cJ$ required the calculation of new parton cross sections, which will be published elsewhere \[10\]. The variable $a$ is then obtained by combining these cross sections with the appropriate branching ratios into longitudinally polarized $J/\psi$ ($\psi_L$).

The NRQCD factorization formula for the differential cross section for the inclusive production of a charmonium state $H$ of momentum $P$ and spin quantum number $\lambda$ has the schematic form

$$d\sigma^{H_{\lambda}}(P) = d\sigma^{c\bar{c}_{\lambda}}(P) \langle O_n^{H_{\lambda}}(P) \rangle,$$

where the summation index $n$ runs over all the color and angular momentum states of the $c\bar{c}$ pair. The $c\bar{c}$ cross sections $d\sigma^{c\bar{c}_{\lambda}}$ can be calculated using perturbative QCD. All dependence on the state $H$ is contained within the nonperturbative ME’s $\langle O_n^{H_{\lambda}}(P) \rangle$. In general, they are Lorentz tensors that depend on the momentum $P$ and the polarization tensor of $H_{\lambda}$. The Lorentz indices are contracted with those of $d\sigma^{c\bar{c}_{\lambda}}$ to give a scalar cross section. The symmetries of NRQCD can be used to reduce the tensor ME’s $\langle O_n^{H_{\lambda}}(P) \rangle$ to scalar ME’s $\langle O_n^{H} \rangle$ that are independent of $P$ and $\lambda$. Thus one may calculate the cross section for polarized quarkonium once the relevant scalar ME’s are known. A nonperturbative analysis of NRQCD reveals how the various ME’s scale with the typical relative velocity of the heavy quarks. It also gives exact and approximate symmetry relations that can be used to simplify the ME’s. The most important ME’s for the production of $J/\psi = \psi(1S)$ or $\psi' = \psi(2S)$ can be reduced to one color-singlet parameter $\langle O_{1}^{\psi(nS)}(3S_{1}) \rangle$ and three color-octet parameters $\langle O_{8}^{\psi(nS)}(3S_{1}) \rangle$, $\langle O_{8}^{\psi(nS)}(1S_{0}) \rangle$, and $\langle O_{8}^{\psi(nS)}(3P_{0}) \rangle$. The most important ME’s for $\chi_cJ$ production can be reduced to a color-singlet parameter $\langle O_{1}^{\chi_{cJ}}(3P_{0}) \rangle$ and a single color-octet parameter $\langle O_{8}^{\chi_{cJ}}(3S_{1}) \rangle$. The ME’s enumerated above should be sufficient for a calculation of the polarization of prompt $J/\psi$.

In $p\bar{p}$ collisions, the parton processes that dominate the $c\bar{c}$ cross section depend on $p_T$. If $p_T$ is of order $m_c$, those which dominate are fusion processes, whose contributions can be expressed as

$$d\sigma^{H_{\lambda}}_{fr}(P) = f_{i/p} \otimes f_{j/\bar{p}} \otimes d\sigma^{c\bar{c}_{\lambda}}(P) \langle O_n^{H_{\lambda}}(P) \rangle,$$

where $f_{i/p}(x, \mu)$ and $f_{j/\bar{p}}(x, \mu)$ are parton distribution functions (PDF’s) and a sum over the partons $i, j$ is implied. The leading-order parton cross sections $d\hat{\sigma}$ are proportional to $\alpha_s^2(\mu)$. These cross sections are given in Refs. \[3\] and \[4\] for all the relevant $c\bar{c}$ spin states with the exception of color-singlet $3P_{0}$ states, which required a new calculation. For $p_T \gg m_c$ the parton cross sections are dominated by fragmentation processes with the scaling behavior $d\hat{\sigma}/dp_T^2 \sim 1/p_T^4$. These contributions can be expressed as

$$d\sigma^{H_{\lambda}}_{fr}(P) = f_{i/p} \otimes f_{j/\bar{p}} \otimes d\hat{\sigma}^{k(p/z)}(P) \otimes D_k^{c\bar{c}_{\lambda}} \langle O_n^{H_{\lambda}}(P) \rangle,$$

where $D_k^{c\bar{c}_{\lambda}}(z, \mu_t)$ is a fragmentation function (FF). We use a common renormalization and factorization scale $\mu$ for $f_{i/p}$, $f_{j/\bar{p}}$, and $d\hat{\sigma}$, but we allow $\mu_t$ to be different. The momentum $k$ of the fragmenting parton is denoted by $P/z$ in \[3\]. However, it is inconsistent to set $k^\mu = P^\mu / z$, since the parton is massless while $P^\mu = 4m_c^2$. We choose $k^\mu$ so that $z$ is the fraction of the light-cone momentum of the parton $k$ that is carried by the $c\bar{c}$ pair in the parton CM frame. The covariant expression is $k^\mu = [(\Delta + K \cdot P)P^\mu - P^2 K^\mu] / (2z\Delta)$, where $K^\mu$ is the total momentum of the colliding partons $i$ and $j$, and $\Delta = [(K \cdot P)^2 - K^2 P^2]^{1/2}$.

In order to predict the polarization of prompt $J/\psi$ at the Tevatron, we need values for the scalar ME’s. The color-singlet ME’s $\langle O_{1}^{\chi_{cJ}}(3S_{1}) \rangle$ and $\langle O_{1}^{\psi(nS)}(3S_{1}) \rangle$ can be determined phenomenologically from the decay rates for $\psi(nS) \rightarrow \ell^+\ell^-$ and $\chi_{c2} \rightarrow \gamma\gamma$ \[12\]. Using the vacuum saturation approximation and spin symmetry in the NRQCD factorization formulae and including NLO QCD radiative corrections \[13\], we obtain the values in Table \[1\]. The errors come from the experimental errors in the decay rates only.

The color-octet ME’s are phenomenological parameters that must be determined from production data. To predict the polarization at the Tevatron, it is preferable to use ME’s extracted directly from Tevatron data in order to cancel theoretical errors associated with soft gluon radiation. There have been several previous extractions of the color-octet ME’s \[14\] from the CDF data on the $p_T$ distributions of $J/\psi$, $\chi_c$, and $\psi'$. We carry out an updated analysis largely following the strategy used in Ref. \[10\]. In the fusion cross section \[3\], we include the parton processes $ij \rightarrow c\bar{c} + k$, with $i, j = q, g, q, q$ and $q = u, d, s$. In the fragmentation cross section \[3\], we include only the $y \rightarrow c\bar{c}_{\lambda} + k$ term, since this is the only fragmentation process for which $D_k^{c\bar{c}_{\lambda}}$ is of order $\alpha_s$. The FF $D_k^{c\bar{c}_{\lambda}}(3S_{1})$ is evolved in $\mu_t$ using the standard homogeneous timelike evolution equation. The effects of the violation of the phase-space constraint $\mu_t > 4m_c^2 / z$ are negligible at the Tevatron due to the rapid fall-off of the $p_T$ distribution \[7\].

We consider two choices for the PDF’s: MRST98LO as our default and CTEQ5L for comparison \[18\]. We evaluate $\alpha_s$ from the one-loop formula using the value of $\Lambda_{QCD}$ appropriate for the PDF set \[18\]. We set $\mu = (4m_c^2 + p_T^2)^{1/2}$ and $m_c = 1.5$ GeV. The cross section for $\psi(nS)$ depends on the linear combination $M_r = \langle O_{8}^{\psi(nS)}(3S_{1}) \rangle + r\langle O_{8}^{\psi(nS)}(3P_{0}) \rangle / m_c^2$, where $r$ varies from about 3.6 at $p_T = 5.5$ GeV to about 3.0 at $p_T = 18$ GeV, so we can only determine $M_r$ at some optimal value of $r$. We determined $\langle O_{8}^{\psi(nS)}(3S_{1}) \rangle$ and $M_r$ for $\psi(nS)$ by fitting the $p_T$ distributions from CDF following the strategy in \[13\]. We determined $\langle O_{8}^{\psi(nS)}(3P_{0}) \rangle$ by fitting the $p_T$ distribution for $\chi_c$ together with the constraint from the preliminary CDF measurement of $\sigma_{\chi_c1}/\sigma_{\chi_c2}$ \[1\]. Our values
for the color-octet ME's are summarized in Table I. The error bars take into account the statistical errors only. Our default \( \psi' \) color-octet ME's agree within errors with those of Ref. [8] used by Leibovich [8] and with those for 2 of the 3 PDF sets used by Beneke and Krämer [7]. Our default \( J/\psi \) color-octet ME's agree within errors with those of Ref. [8]. Our value for \( \langle O_S^{J/\psi}(3S_1) \rangle \) is about a factor of 3 smaller than in Ref. [8], while \( M^*_{J/\psi} \) is about a factor of 2 larger.

We can calculate the cross sections for the polarized states \( H_\Lambda \) using the scalar ME's in Table I. The cross section [8] can be reduced to an expression linear in the scalar ME's, with coefficients that involve the polarization tensor of \( H_\Lambda \). In the channel \( c\bar{c}_S(3S_1) \to \psi_\lambda(nS) \), we interplay between the fusion cross section at low \( p_T \) and the fragmentation cross section at high \( p_T \) using the prescription

\[ d\sigma^{H_\Lambda} = d\sigma^{H_\Lambda}_{tr} \times \left( d\sigma^{H_\Lambda}_{tr}[\mu_{tr} = \mu] / d\sigma^{H_\Lambda}_{tr}[\mu_{tr} = 2m_c] \right). \tag{4} \]

We proceed to summarize our calculation of the errors in \( \sigma_L \) and \( \sigma_T \). The errors in the ME's in Table I are taken into account. We take the central values of \( \mu \) and \( m_c \) to be \( \mu_T = (4m_c^2 + p_T^2)^{1/2} \) and 1.5 GeV and allow them to vary within the ranges \( \frac{1}{2} \mu_T - 2\mu_T \) and 1.45 - 1.55 GeV, respectively. We take MRST98LO as our default PDF, and we treat the difference between it and CTEQ5L as an error. The cross section \( \sigma_L \) for \( \psi(nS) \) is sensitive to a different linear combination of \( \langle O_S(1S_0) \rangle \) and \( \langle O_S(3P_0) \rangle \) than appears in \( M_\psi \). We take this into account by expressing the cross section as a function of \( M_\psi \) and \( x = \langle O_S(1S_0) \rangle / M_\psi \), taking the central value of \( x \) to be \( \frac{1}{2} \), and allowing \( x \) to vary between 0 and 1.

We first consider the polarization of direct \( \psi' \), since it is not complicated by feeddown from higher charmonium states. The polarization variable \( \alpha \) measured by the CDF Collaboration [8] describes the angular distribution of leptons from the decay of the \( \psi' \) with respect to the \( \psi' \) momentum in the hadron CM frame. The covariant expression for the polarization vector of \( \psi'_L \) is

\[ (P^2 Q^\mu - P \cdot Q P^\mu) / (\sqrt{P^2} \Delta), \]

where \( Q = p + \bar{p} \) is the total hadron momentum and \( \Delta = [(P \cdot Q)^2 - P^2 Q^2]^{1/2} \). In Fig. 1(a), we compare our result for \( \alpha \) as a function of \( p_T \) with the CDF data [8] and with previous predictions from Refs. [7] and [8]. We present our result in the form of an error band obtained by combining in quadrature all the errors described above. The most important errors are those from the \( \psi' \) ME's, the PDF's, and \( x \). The error bars in the CDF data are too large to draw any definitive conclusions. Our result for \( \alpha \) is close to the prediction of Leibovich [8], and significantly larger than that of Beneke and Krämer [7]. Their calculations differ in the treatment of terms of order \( \alpha_\psi^2 \) in the gluon FF [8]. Beneke and Krämer included these terms in \( \sigma_L \) but neglected them in \( \sigma_T \), while Leibovich neglected them in both \( \sigma_L \) and \( \sigma_T \). We have adopted the strategy of Beneke and Krämer, since these terms give a significant increase in \( \sigma_L \) at large \( p_T \) but have only a small effect on \( \sigma_T \). Although this tends to decrease \( \alpha \), our smaller value of \( M_\psi \) tends to decrease \( \alpha \), and the net result is close to the prediction of Leibovich.

We next consider the polarization variable \( \alpha \) for prompt \( J/\psi \). The prompt cross section \( \sigma_T + \sigma_L \) is the sum of the direct cross section for \( J/\psi \) and the cross sections for \( \chi_{cJ}(\lambda) \) and \( \psi' \) weighted by the branching fractions \( B_{J/\psi} \to J/\psi \). The prompt longitudinal cross section \( \sigma_L \) is the sum of the direct cross section for \( \psi_L \) and the cross sections for each of the spin states \( \chi_{cJ}(\lambda) \) and \( \psi' \) weighted by \( B_{H_\Lambda} \to J/\psi \) and by the probability \( P_{H_\Lambda} \to \psi_L \) for the polarized state to decay into \( \psi_L \). The observed transitions of \( \psi' \) to \( J/\psi \) involve no spin flips, so that \( P_{\psi'_L \to \psi_L} = 1 \) for \( \psi'_0 \) and 0 for \( \psi'_\perp 1 \). For the radiative decay of \( \chi_{cJ}(\lambda) \) into \( J/\psi \), the probability \( P_{\chi_{cJ}(\lambda) \to \psi_L} = 1/3 \) for \( \chi_{cJ}(1^{11}) \), \( 2/3 \) for \( \chi_{cJ}(00) \), \( 1/2 \) for \( \chi_{cJ}(2^{11}) \), and 0 for the other spin states [19]. In Fig. 1(b), we compare our result for \( \alpha \) as a function of \( p_T \) with the CDF data [8]. The shaded area indicates the error band obtained by adding the errors in quadrature. The most important errors are those from the PDF's, the \( J/\psi \) ME's, and \( x \). Our result for \( \alpha \) is small around \( p_T = 5 \) GeV, but it increases with \( p_T \) to a value around 0.5 at \( p_T = 20 \) GeV. Our result is in good agreement with the CDF measurement at intermediate values of \( p_T \), but it disagrees by about 3 standard deviations in the highest \( p_T \) bin. The three solid lines in Fig. 1(b) are the central curves of \( \alpha \) for direct \( J/\psi \), \( J/\psi \) from \( \chi_{cJ} \), and \( J/\psi \) from \( \psi' \). The \( \alpha \) for direct \( J/\psi \) is smaller than that for direct \( \psi' \), because \( \langle O_S(3S_1) \rangle \) is comparable for \( J/\psi \) and \( \psi' \), while \( M_\psi \) is significantly larger for \( J/\psi \). In the moderate-\( p_T \) region, the contributions from \( \psi' \) and \( \chi_{c} \) add to give an increase in the transverse polarization of prompt \( J/\psi \) compared to direct \( J/\psi \). In the high-\( p_T \) region, the contributions from \( \psi' \) and \( \chi_{c} \) tend to cancel. The prediction of Beneke and Krämer for \( \alpha \) for direct \( J/\psi \) is identical to their prediction for direct \( \psi' \) in Fig. 1(a). At \( p_T = 20 \) GeV, it is significantly larger than our prediction for direct \( J/\psi \). The difference comes from our smaller value for \( \langle O_S(3S_1) \rangle \) and our larger value for \( M^*_{J/\psi} \). Beneke and Krämer's prediction for \( \alpha \) for \( J/\psi \) from \( \psi' \) would be significantly lower than our result in Fig. 1(b), but it would have a small effect on the value of \( \alpha \) for prompt \( J/\psi \). The discrepancies between their predictions and ours could be eliminated by more accurate data on the \( J/\psi \) and \( \psi' \) cross sections, which would decrease some of the ambiguities in the analysis.

The CDF measurement of the polarization of prompt \( J/\psi \) presents a serious challenge to the NRQCD factorization formalism for inclusive quarkonium production. There are many effects that could change our quantitative prediction for \( \alpha \), such as next-to-leading order radiative corrections, but the qualitative prediction that \( \alpha \) should increase at large \( p_T \) seems inescapable. In Run II
of the Tevatron, the data sample for J/ψ should be more than one order of magnitude larger than in Run I, allowing the polarization to be measured with higher precision and out to larger values of pT. If the result continues to disagree with the predictions of the NRQCD factorization approach, it would indicate a serious flaw in our understanding of inclusive charmonium production. The predictions of low-order perturbative QCD for the spin-dependence of c ¯ c cross sections could be wrong, or the use of NRQCD to understand the systemsatics of the formation of charmonium from the c ¯ c pair could be flawed, or m c could simply be too small to apply the factorization approach to the charmonium system.

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