We propose a holographic dual of large $N_c$ quantum chromodynamics (QCD) with the gauge groups $O(N_c)$ and $USp(N_c)$ and $N_f$ flavors of massless quarks. This is constructed by adding O6-planes to an intersecting D4-D8 system in type IIA superstring theory. The holographic dual description is formulated in Witten’s D4-brane background with D8-branes and O6-planes embedded in it as probes. The D4-brane background gives rise to a smooth interpolation of D8-D8 pairs and an O6-O6 pair. We show that the resultant brane configuration explains geometrically the flavor symmetry breaking patterns in $O(N_c)$ and $USp(N_c)$ QCD, which are caused by quark bilinear condensates. We next discuss that baryons can be realized as D4-D4 pairs wrapped on $S^4$, which intersect with the O6-plane. By analyzing the tachyons on it, we reproduce the stability conditions of the baryons that are expected from the gauge theory viewpoint. The stable baryon configurations are classified systematically using K-theory. We also give a similar analysis of the flux tubes and again reproduce the results that are consistent with QCD.

Subject Index: 101, 121

§1. Introduction

Recently, there has been remarkable progress in the study of the strong coupling dynamics of gauge theories by employing duality between gauge theory and string theory. Following the discovery of the AdS/CFT correspondence\(^1\)–\(^3\) (for a review, see Ref. 4), intensive attempts to apply the idea of the gauge/string duality to QCD have been made. In Ref. 5), a holographic dual of QCD with massless quarks is proposed on the basis of a D4-D8 configuration in type IIA superstring theory. One of the most insightful results of this model is that it gives a simple geometric explanation of the chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$.

From the gauge theory viewpoint, many other interesting models that are considered to exhibit spontaneous symmetry breaking via strong gauge dynamics are known. Among them is massless QCD with the gauge group $G = O(N_c)$ and $G = USp(N_c)$.\(^1\) The fundamental degrees of freedom of this theory are the gluon and massless quarks $q^a_{\alpha \dot{\alpha}}$, where $\alpha = 1, 2$ denotes the undotted spinor index, $a = \ldots$
1, 2, \cdots, N_c is the color index of the gauge group, and $i = 1, 2, \cdots, N_f$ is the flavor index. Since the fundamental representation for these gauge groups is vector-like, there is no distinction between left-handed and right-handed components of the quark fields, and hence, the flavor symmetry is $U(N_f)$ for both $G = O(N_c)$ and $G = USp(N_c)$ cases. Note that $N_f$ must be even for the $G = USp(N_c)$ case to avoid the global anomaly.\footnote{Here, we include the anomalous $U(1)$ part of the flavor symmetry, since the effect of the anomaly vanishes in the large $N_c$ limit. The effect of the anomaly can be incorporated as studied in §5.8 of Ref. 5.)}

Like $SU(N_c)$ QCD, these models are considered to develop non-vanishing condensates:

\[
\langle \epsilon^{\alpha\beta} \delta_{ab} q_{\alpha i}^a q_{\beta j}^b \rangle = c \delta_{ij} , \quad (\text{for } G = O(N_c)) \\
\langle \epsilon^{\alpha\beta} J_{ab} q_{\alpha i}^a q_{\beta j}^b \rangle = c J_{ij} . \quad (\text{for } G = USp(N_c))
\] (1.1)

Here, $c$ is a non-vanishing constant and $J$ is the $USp$ invariant anti-symmetric tensor.

This implies that the flavor symmetry $U(N_f)$ is spontaneously broken as

\[
U(N_f) \rightarrow O(N_f) , \quad (\text{for } G = O(N_c)) \\
U(N_f) \rightarrow USp(N_f) . \quad (\text{for } G = USp(N_c))
\] (1.2)

This phenomenon should result from strong coupling gauge dynamics, and hence, it is rather difficult to prove that this really occurs by making a full analysis of the gauge dynamics. A natural question to ask then is whether we can demonstrate this phenomenon from string theory, following the same line in Ref. 5). The purpose of this paper is to show that it is indeed possible to analyze $O(N_c)$ and $USp(N_c)$ QCD using string theory. To this end, we begin by constructing a brane configuration that realizes $O(N_c)$ and $USp(N_c)$ QCD with massless flavors, by adding O6-planes in the D4-D8 configuration presented in Ref. 5). Then, the D4-branes are replaced with the corresponding supergravity (SUGRA) background obtained in Ref. 7), which is a holographic dual of large $N_c$ strongly coupled Yang-Mills (YM) theory. As in Ref. 5), the quarks are incorporated into the model by embedding probe D8-branes into the D4-brane background. Here, we use the probe approximation and ignore the backreaction of the O6-plane and D8-branes, which can be justified when $N_f \ll N_c$. It is found that a brane interpolation mechanism that is observed in Ref. 5) leads to a natural explanation of the symmetry breaking pattern (1.2) from the physics of intersecting D-branes and O-planes.

We also study the stability of baryons and flux tubes in $O(N_c)$ and $USp(N_c)$ QCD using our holographic description. The properties of these objects are quite different from the cases with $G = SU(N_c)$. As discussed in Ref. 8), the baryon number in $O(N_c)$ QCD is $\mathbb{Z}_2$-valued; that means a single baryon is stable, while two baryons can decay. The flux tubes also behave in a similar way. On the other hand, the baryons and flux tubes are totally unstable in $USp(N_c)$ QCD. In the holographic description of QCD, the baryons and flux tubes can be realized as D-branes wrapped on $S^4$ in the D4-brane background. We show that the stability conditions of these...
D-branes are in agreement with what we expect in QCD. Furthermore, since the stable D-brane configurations are classified using K-theory, the baryons and flux tubes correspond to the elements of K-groups. We find that the relevant K-groups that are used to classify the stable baryons and flux tubes again reproduce the above results. This refines the classification via homotopy groups given in Ref. 8), in which there are some discrepancies for \( N_f \leq 3 \).

This paper is organized as follows. In §2, we present a brane configuration that defines massless QCD with the gauge group \( G = O(N_c) \) and \( G = USp(N_c) \). In §3, we formulate the holographic dual description and show that it nicely explains the flavor symmetry breaking and the stability of baryons and flux tubes in QCD. We end this paper with a summary and discussion in §4. In Appendices A and B, we summarize the properties of intersecting D\(_p\)-O\(_p\)' systems and K-groups that are used in this paper, respectively.

§2. Brane configuration of \( O(N_c) \) and \( USp(N_c) \) QCD

In this section, we construct a brane configuration of \( O(N_c) \) and \( USp(N_c) \) QCD with massless flavors, by generalizing the model given in Ref. 5), which is proposed as a holographic dual of \( U(N_c) \) QCD with massless flavors. For this purpose, let us first review some key results in Ref. 5) with an emphasis on how the gluon and quarks emerge. This model is composed of \( N_c \) D4-branes and \( N_f \) pairs of D8- and \( \overline{D8} \)-branes:

\[
\begin{array}{cccccccccc}
 & x^0 & x^1 & x^2 & x^3 & (x^4) & x^5 & x^6 & x^7 & x^8 & x^9 \\
D4 & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
D8-\overline{D8} & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\end{array}
\]  

(2.1)

Here, the \( x^4 \) direction is compactified on \( S^1 \) of radius \( M_{KK}^{-1} \). In this paper, we work in the \( M_{KK} = 1 \) unit. The D4-branes are wrapped around this circle, while the D8-branes and \( \overline{D8} \)-branes are located at the antipodal points \( x^4 = \pi/2 \) and \( x^4 = -\pi/2 \), respectively. Following Ref. 7), we impose the anti-periodic boundary condition along the \( S^1 \) parametrized by \( x^4 \) on all the fermions in the system, while all the bosonic fields are kept periodic. Then, the gluinos as well as the scalar fields on the D4-brane, which belong to the adjoint representation of the \( SU(N_c) \) gauge symmetry, become massive. In addition, the left-handed and right-handed components of the quark fields (\( q_L \) and \( q_R \), respectively) are created as the massless modes in the 4-8 strings (open strings stretching from the D4-branes to the D8-branes) and 4-\( \overline{8} \) strings, respectively. Then, the D4-brane world-volume theory flows to four-dimensional \( U(N_c) \) QCD with \( N_f \) massless quarks at low energy.\(^*\) Note that the gauge symmetries on the D8-branes and \( \overline{D8} \)-branes correspond to the chiral symmetries \( U(N_f)_L \) and \( U(N_f)_R \), respectively.

To obtain \( O(N_c) \) and \( USp(N_c) \) QCD, we consider an orientifold defined on the basis of the \( \mathbb{Z}_2 \) action \( (x^4, x^8, x^9) \rightarrow (-x^4, -x^8, -x^9) \) together with the world-sheet

\(^*\) We regard the diagonal \( U(1) \) part of the \( U(N_c) \) gauge symmetry on the D4-brane as a global symmetry, since it decouples in the IR limit.
parity transformation. There are two \((6 + 1)\)-dimensional fixed planes, called O6-plane, located at \(x^4 = x^8 = x^9 = 0\) and \(x^4 = \pi, x^8 = x^9 = 0\), which are invariant under the \(\mathbb{Z}_2\) action. To be consistent with the anti-periodic boundary condition for the fermions around the \(S^1\) parametrized by \(x^4\), the two O6-planes should break different halves of the supersymmetry. Whenever we need to distinguish the two, we call the fixed planes at \(x^4 = x^8 = x^9 = 0\) and \(x^4 = \pi, x^8 = x^9 = 0\) O6-planes, respectively. An O6-plane carries an RR charge opposite to that of an O6-plane with their world-volume orientations opposite to each other. There are two basic choices of the orientifold \(p\)-planes called \(O^\pm\) and \(O^\mp\)-planes. One way to characterize the \(O^\pm\)-planes is to consider the world-volume gauge theory of probe D-branes, as described in Appendix A. (See Refs. 12),13) for reviews.)

Now, we argue that massless QCD with \(\mathcal{G} = O(N_c)\) and \(\mathcal{G} = USp(N_c)\) can be obtained by placing \(O^+\,\overline{O}^+\) and \(O^-\,\overline{O}^-\) into (2.1), respectively. The brane configuration we consider is

\[
\begin{array}{c|cccccccc}
\text{D4} & x^0 & x^1 & x^2 & x^3 & (x^4) & x^5 & x^6 & x^7 & x^8 & x^9 \\
\text{D8-D8} & & & & & & & & & & \\
\text{O6} & \text{O6} & & & & & & & & & \\
\end{array}
\]

(2.2)

(See also Fig. 1). In general, a system with a \(Dp\)-brane and an \(Op\)-plane that are extended along \((q + 1)\) common directions is characterized by the number of relative transverse directions \(D_{rel} = p + p' - 2q\). For our D4-O6 and D8-O6 configurations in (2.2), we have \(D_{rel} = 4\) and \(D_{rel} = 2\), respectively. Then, as explained in Appendix A, the \(U(N_c)\) gauge field on the D4-branes obeys

\[
A_\mu(x^\mu, x^4) = -\gamma_\pm A_\mu^T(x^\mu, -x^4)\gamma_\pm^{-1}
\]

(2.3)

\((\mu = 0, \ldots, 3)\) for the cases with O6\(^\pm\)-planes, together with the periodic boundary condition \(A_\mu(x^\mu, x^4 + 2\pi) = A_\mu(x^\mu, x^4)\). Here, \(\gamma_\pm\) is defined by

\[
\gamma_+ = I_{N_c}, \quad \gamma_- = J_{N_c} \equiv \begin{pmatrix} 0 & I_{N_c/2} \\ -I_{N_c/2} & 0 \end{pmatrix}
\]

(2.4)
with $I_k$ being the identity matrix of rank $k$. Note that $N_c$ should be even for the case with $O6^-$-plane. Therefore, the zero mode of the gauge field along $x^4$ that survives the orientifold projection (2-3) gives four-dimensional $O(N_c)$ and $USp(N_c)$ gauge fields for the cases with $O6^+$- and $O6^-$-planes, respectively. On the other hand, the orientifold action ($Z_2$ action associated with the orientifold) maps D8-branes to $\overline{D8}$-branes and vice versa. This implies that the quark fields are given by $Z_2$-invariant linear combinations of $q_L$ and $q_R^*$, because the $Z_2$ maps a 4-8 string to an $\overline{8}$-4 string, and the flavor symmetry is reduced to $U(N_f)$. Therefore, we conclude that the brane configuration (2.2) with $O6^+$-plane and $O6^-$-plane yields $O(N_c)$ and $USp(N_c)$ QCD with $N_f$ massless quarks, respectively.

A few comments are in order. The number of Weyl fermions in the fundamental representation of $USp(N_c)$ gauge group must be even to avoid the global anomaly. This fact can be understood in the brane setup (2.2) as follows. Recall that a D8-brane is a source of the RR nine-form potential and its field strength is dual to RR zero-form field strength $F_0$, which takes an integer value if we normalize $F_0 = 1$ for a unit flux. In (2.2), the ten-dimensional space-time is divided by the D8- and $\overline{D8}$-branes into two regions, $-\pi/2 < x^4 < \pi/2$ and $\pi/2 < x^4 < 3\pi/2$. The difference in the $F_0$ flux between the two regions is $N_f$. Let us consider what happens if $N_f$ is odd. Without loss of generality, we assume that $F_0$ is even in the region $\pi/2 < x^4 < 3\pi/2$. Then, the $O6^-$-plane at $x^4 = 0$ is embedded in the region with odd $F_0$ flux. However, it is known that the $O6^-$-plane is allowed to exist only in the background with even $F_0$ flux. This is exactly what we expect to avoid the global anomaly. Instead of the $O6^-$-plane, one could consider an $\overline{O6}^-$-plane, which is an $O6^-\overline{O6}^-$-plane with a D6-brane stuck on it and allowed to exist in the background with odd $F_0$ flux. In this case, we have an additional flavor of quarks created by the open string stretched between the $\overline{O6}^-$-plane and the D4-branes, and the number of flavors will again become even. In this way, the number of flavors is always even and the global anomaly is automatically avoided.

§3. Analysis in holographic dual of QCD

In this section, we study the properties of $O(N_c)$ and $USp(N_c)$ QCD with massless flavors by working in the holographic dual of the brane configuration given in (2.2).

3.1. Flavor symmetry breaking

We start by discussing how the flavor symmetry breaking patterns of $O(N_c)$ and $USp(N_c)$ QCD in (1.2) can be seen in the holographic dual description obtained from the brane setup (2.2). For this purpose, we first provide a brief review of holographic chiral symmetry breaking of $U(N_c)$ QCD.

In general gauge/string duality, the holographic dual description is obtained by replacing the D-branes representing the gauge theory with the corresponding curved

\footnote{It is also possible to construct $O(N_c)$ and $USp(N_c)$ QCD by adding orientifolds in the D4-D6 system considered in Ref. 14). However, the full flavor symmetry $U(N_f)$ is not manifestly realized in such brane configurations.}
background that solves the SUGRA equations of motion. In our brane configuration (2.2), the SUGRA background corresponding to the \( N_f \) D4-branes is given by Witten.\(^7\) Although the explicit solution is known, we only need information on the topology of the background, which is \( \mathbb{R}^{1,3} \times \mathbb{R}^2 \times S^4 \). Here, \( \mathbb{R}^{1,3} \) is the four-dimensional Minkowski space-time \( \{x^{0-3}\} \) and \( \mathbb{R}^2 \) is the two-dimensional cigar-like geometry parametrized by \( x^4 \) and the radial coordinate \( r \) of the five-dimensional plane \( \{x^{5-9}\} \) transverse to the D4-branes.\(^\ast\) We also use the coordinates \((y, z)\) related to \((r, x^4)\) as
\[
y = r \cos x^4, \quad z = r \sin x^4.
\]
The \( S^4 \) corresponds to the angular directions in the five-dimensional plane \( \{x^{5-9}\} \), around which \( N_c \) units of an RR four-form field strength are turned on. This flux plays a substantial role in analyses of baryons to be made in §3.3.

Assuming \( N_c \gg N_f \), we use the probe approximation to include the D8-branes. Namely, the D8-branes are embedded in Witten’s D4-brane background as probes and their backreaction is neglected. This probe approximation is related to the quenched approximation, which is widely used in lattice QCD, since quark loops in QCD correspond to holes in the string world-sheet attached to the D8-branes. Then, we find that the D8-branes and \( \overline{\text{D}8} \)-branes in the previous brane configuration (2.1) must be smoothly connected when we replace the D4-brane with the corresponding SUGRA background described above and we end up with the configuration with only one connected component of \( N_f \) D8-branes extended along the \( x^{0-3} \), \( z \), and \( S^4 \) directions. It is shown in Ref. 5 that this smooth interpolation of the \( \text{D8-} \overline{\text{D}8} \) pairs gives the geometric realization of the chiral symmetry breaking. Since D8-branes and \( \overline{\text{D}8} \)-branes are located at \( x^4 = \pi/2 \) and \( x^4 = -\pi/2 \) in the previous brane configuration (2.1), the chiral symmetries \( U(N_f)_L \) and \( U(N_f)_R \) correspond to the gauge symmetry at the two boundaries \( z \to + \infty \) and \( z \to - \infty \), respectively.\(^\ast\ast\) Now, the \( z \)-dependent gauge transformation shifts the \( z \) component of the gauge field \( A_z \) on the D8-brane world-volume, and hence, it is spontaneously broken. (See §3.2 for more on this point.) The unbroken part is given by the constant gauge transformation that is \( U(N_f)_V \). Therefore, the spontaneous chiral symmetry breaking \( U(N_f)_L \times U(N_f)_R \to U(N_f)_V \) is caused by the fact that the geometry of the D4-brane background requires that the D8-branes and \( \overline{\text{D}8} \)-branes be smoothly connected.

Next, we argue that the flavor symmetry breaking in \( O(N_c) \) and \( USp(N_c) \) massless QCD can again be understood geometrically. As explained in the previous section, we consider the orientifold defined by the \( \mathbb{Z}_2 \) action \((x^4, x^8, x^9) \to (-x^4, -x^8, -x^9) \). Since the D4-brane background has this \( \mathbb{Z}_2 \) symmetry, we can consistently impose the orientifold projection in the holographic dual description.\(^\ast\ast\ast\)

\(^\ast\) Here, this \( r \) is related to \( \sqrt{(U/U_{\text{KK}})_{x^4} - 1} \) in the coordinates used in Ref. 5).
\(^\ast\ast\) The gauge symmetry at the boundaries \( z \to \pm \infty \) yields the global symmetry in QCD, since the corresponding gauge couplings vanish. (See Ref. 16.)
\(^\ast\ast\ast\) Here, the backreaction of the O6-planes on the geometry is also neglected by assuming large \( N_c \).
Then, we find that the fixed plane of the $\mathbb{Z}_2$ orientifold action is an O6$^\pm$-plane extended along the $x^0$-3, $y$, and $S^2$ directions located at $z = x^8 = x^9 = 0$. (See Fig. 2.) This means that the O6$^\pm$-O6$^\mp$ pair in the previous configuration (2.2) (see also Fig. 1) is now smoothly connected in the D4-brane background, which is possible because the world-volumes of the O6$^\pm$-plane and O6$^\mp$-plane in (2.2) are oriented opposite to each other, just as in the case of the D8-D8 pairs. Together with the probe D8-branes considered above, we obtain an intersecting D8-O6 system with $D_{\text{rel}} = 4$. Then, as explained in Appendix A, the $U(N_f)$ gauge field on the D8-brane world-volume satisfies

$$A_\mu(x^\mu, z) = -\gamma_\pm A_\mu^T(x^\mu, -z)\gamma_\pm^{-1}, \quad A_z(x^\mu, z) = \gamma_\pm A_z^T(x^\mu, -z)\gamma_\pm^{-1} \quad (3.2)$$

($\mu = 0, \ldots, 3$) for the cases with O6$^\pm$-planes, where $\gamma_\pm$ is defined by replacing $N_c$ with $N_f$ in (2.4). Here, we have omitted the components of the gauge field along the $S^4$ as well as the $S^4$ coordinate dependence for simplicity. The gauge transformation consistent with the constraint (3.2) is given by a gauge function $g(x^\mu, z) \in U(N_f)$ satisfying

$$g(x^\mu, z) = \gamma_\pm g^*(x^\mu, -z)\gamma_\pm^{-1}, \quad (3.3)$$

where $g^* = (g^{-1})^T$ is the complex conjugate of $g$. In particular, the gauge transformation at $z = 0$ is restricted to $O(N_f)$ or $USp(N_f)$ for O6$^+$-plane and O6$^-$-plane, respectively.

With these setups, we are now ready to show the flavor symmetry breaking patterns in $O(N_c)$ and $USp(N_c)$ QCD. First, the D8-branes have only a single boundary, because the apparent two boundaries at $z \to \pm \infty$ are identified with each other by the orientifold projection, which involves $z \to -z$. Thus, the flavor symmetry is $U(N_f)$, as discussed in the previous section. Next, by noting that the unbroken flavor symmetry results from constant gauge transformations on the D8-branes, we find that the unbroken flavor symmetry consistent with the constraint (3.3) is $O(N_f)$ for $O(N_c)$ QCD (with O6$^+$-plane) and $USp(N_f)$ for $USp(N_c)$ QCD (with O6$^-$-plane). This is exactly what we have observed in (1.2). In the holographic description, the
spontaneous flavor symmetry breaking (1.2) is caused by the fact that the D8-branes must intersect with the O6±-plane in the D4-brane background.

3.2. Meson effective action

As argued in Refs. 5),16), the open strings attached to the probe D8-branes are interpreted as mesons. The meson effective action is given by the effective action on the D8-brane world-volume. After reducing the $S^4$ directions, the effective theory of the mesons in $U(N_c)$ QCD turned out to be a five-dimensional $U(N_f)$ Yang-Mills (YM) - Chern-Simons (CS) theory. The action is given by Ref. 5)

\[ S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}, \]

\[ S_{\text{YM}} = -\kappa \int d^4x dz \text{Tr} \left( \frac{1}{2} h(z) F^{2}_{\mu\nu} + k(z) F^{2}_{\mu z} \right), \]

\[ S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_{5\text{dim}} \omega_5(A), \] (3.4)

where $\mu, \nu = 0, \cdots, 3$ are the Lorentz indices for the four-dimensional space-time and $z$ is the coordinate of the fifth dimension along the D8-brane world-volume. The warp factors in the YM action are given by $k(z) = 1 + z^2$ and $h(z) = (1 + z^2)^{-1/3}$, and $\omega_5(A)$ is the Chern-Simons five-form. The Kaluza-Klein decomposition of the five-dimensional gauge field with respect to the warped $z$-direction yields an infinite tower of massive vector/axial-vector mesons as well as a massless pion in four-dimensional space-time. It was shown in Refs. 5),16) that the resultant four-dimensional effective action for these mesons reproduces various properties of the mesons found in the experiments.

The low energy effective theory of mesons for $O(N_c)$ and $USp(N_c)$ QCD can be obtained by simply imposing the constraint (3.2) on the five-dimensional gauge field in (3.4) and restrict the $z$ integral to $0 \leq z < +\infty$. In fact, the $Z_2$ action associated with the O6±-plane and O6∓-plane corresponds to the charge conjugation and G-parity transformation, respectively, and it is easy to check that the action (3.4) is invariant under them.

It is interesting to show how to extract the pion degrees of freedom that appear as the Nambu-Goldstone modes associated with the spontaneous flavor symmetry breaking (1.2). Following Ref. 5), let us define $U(N_f)$-valued fields

\[ \xi_{\pm}^{-1}(x^\mu) = \text{P exp} \left( - \int_0^{\pm\infty} dz A_z(x^\mu, z) \right). \] (3.5)

Here, $\xi_+(x^\mu)$ and $\xi_-(x^\mu)$ are related to each other by the constraint (3.2) as

\[ \xi_+(x^\mu) = \gamma_\pm \xi_-(x^\mu) \gamma_\pm^{-1}. \] (3.6)

These fields transform as

\[ \xi_{\pm}(x^\mu) \rightarrow h(x^\mu) \xi_{\pm}(x^\mu) g_{\pm}^{-1} \] (3.7)

*) Here, we only consider the five-dimensional components of the nine-dimensional gauge field on the D8-brane world-volume that is constant along the $S^4$. Then, the constraint (A.2) imposed on the gauge field is reduced to (3.2). The higher KK-modes associated with the $S^4$ are the artifacts of the model that cannot be interpreted as the bound states in QCD.
under the five-dimensional gauge transformation given by the gauge function satisfying constraint (3.3), where\(^1\)

\[ h(x^\mu) = g(x^\mu, 0), \quad g_\pm = \lim_{z \to \pm\infty} g(x^\mu, z). \quad (3.8) \]

As discussed in §3.1, the gauge transformation at the boundary \(g_\pm \in G\) corresponds to the flavor symmetry \(G = U(N_f)\) of the \(O(N_c)\) and \(USp(N_c)\) QCD. On the other hand, \(h(x^\mu)\) corresponds to the hidden local symmetry introduced in Ref. 18 to write down an effective action including pion and rho mesons. In our case, the constraint (3.3) implies \(h(x^\mu) \in H\) with \(H = O(N_f)\) and \(H = USp(N_f)\) for \(O(N_c)\) and \(USp(N_c)\) QCD, respectively. The transformation property (3.7) is exactly the same as that for the Nambu-Goldstone mode associated with the spontaneous symmetry breaking \(G \to H\) in the general formulation of the hidden local symmetry approach. (See Ref. 19 for a review.) Since \(H\) here is a local symmetry, the physical Nambu-Goldstone mode parametrizes the quotient space \(G/H\).

3.3. Baryons and flux tubes

As in the \(G = SU(N_c)\) case, a baryon in QCD with \(G = O(N_c)\) and \(G = USp(N_c)\) is composed of \(N_c\) quarks with the color indices totally anti-symmetrized by the \(\epsilon\)-tensor. It is argued, however, that the baryons exhibit novel features regarding the stability. For \(G = O(N_c)\), the conserved baryon number is \(\mathbb{Z}_2\)-valued. Namely, a single baryon is stable, while two baryons can decay. This is because the product of two \(\epsilon\)-tensors can be decomposed as

\[ \epsilon_{a_1 \cdots a_{N_c}} \epsilon_{b_1 \cdots b_{N_c}} = \delta_{a_1 b_1} \delta_{a_2 b_2} \cdots \delta_{a_{N_c} b_{N_c}} \pm \cdots, \quad (3.9) \]

implying that two baryons can decay into \(N_c\) mesons. In contrast, none of the \(G = USp(N_c)\) baryons is stable, because the \(\epsilon\)-tensor can be decomposed as

\[ \epsilon_{a_1 \cdots a_{N_c}} = J_{a_1 a_2} J_{a_3 a_4} \cdots J_{a_{N_c/2-1} a_{N_c/2}} \pm \cdots. \quad (3.10) \]

This implies that even a single baryon is unstable against decay to \(N_c/2\) mesons.

Now, we demonstrate that the stability of baryons in \(O(N_c)\) and \(USp(N_c)\) QCD can be understood geometrically from the holographic description formulated in §3.1. To this end, it is helpful to recall first that the baryons in QCD with \(G = U(N_c)\) can be realized as a D4-brane wrapped around \(S^4\) at the tip of the cigar \((y = z = 0)\) in the D4-brane background.\(^5,20\) It was shown in Ref. 20) that \(N_c\) units of electric charge are induced on the wrapped D4-brane through the RR four-form field strength around \(S^4\), and the Gauss law constraint requires that the induced charge be cancelled by \(N_c\) open strings ending on the D4-brane. The resultant configuration is interpreted as a bound state of \(N_c\) quarks, namely, a baryon. \(n\)-baryon systems can be obtained by wrapping \(n\) D4-branes around \(S^4\). Without the probe D8-branes, the \(N_c\) strings attached to a D4-brane are extended to infinity and the baryon is

\(^1\) Here, we assume \(g_\pm\) to be constant, since the gauge symmetry at \(z \to \pm\infty\) corresponds to the global symmetry in QCD. It is also useful to gauge it by allowing the \(x^\mu\)-dependence to \(g_\pm\) in order to extract the current associated with the flavor symmetry. (See Refs. 5, 16, 17.)
infinitely massive. In our brane configuration with the D8-branes, the fundamental strings can end on D8-branes and the baryon becomes dynamical.

It would be natural to guess that the baryons in $O(N_c)$ and $USp(N_c)$ QCD can also be obtained by wrapping D4-branes around the $S^4$. However, it turns out that the baryons are constructed using D4-D4 pairs wrapped around the $S^4$. A key ingredient here is that a D4-brane wrapped on $S^4$ intersects with the O6±-plane with $D_{\text{rel}} = 6$. Then, the orientifold action maps the D4-branes to D4-branes, and they should be paired in order to be invariant under the orientifold action. If we consider $n$ D4-D4 pairs wrapped on the $S^4$, the tachyon field $T$ created by the 4-D and D-4 strings becomes an $n \times n$ complex matrix satisfying

$$T(x^8, x^9) = \mp T^T(-x^8, -x^9) \quad (3.11)$$

for the case with O6±-plane, respectively. (See (A.4).) For the case with O6−-plane, the tachyon field is allowed to take a constant non-vanishing value for any $n$. Thus, we conclude that the baryons in $USp(N_c)$ QCD are unstable, as expected in the field theoretical consideration reviewed above. Let us next consider the case with O6+−-plane that corresponds to the $O(N_c)$ QCD. When $n = 1$, the constraint (3.11) implies that the tachyon field vanishes at $x^8 = x^9 = 0$ and the D4-D4 pair cannot be annihilated completely. A tachyon profile consistent with (3.11) for $n = 1$ is given by a $k$-vortex configuration $T(x^8, x^9) \propto (x^8 + ix^9)^k$ with odd $k$, which gives rise to the decay of the D4-D4 pair to $k$ D2-branes wrapped around $S^2$ in $S^4$ at $x^8 = x^9 = 0$.

Note that these D2-branes are embedded in the O6±-plane with $D_{\text{rel}} = 4$. It is found from (A-2) that the gauge symmetry on the D2-brane world-volume is $O(k)$, and the scalar fields on it corresponding to the position of the D2-branes transverse to the O6±-plane are in the anti-symmetric representation of the gauge group $O(k)$. For $k = 1$, because there is no anti-symmetric representation in this case, a single D2-brane cannot move away from the O6+−-plane and gives a stable configuration. However, an even number of D2-branes can be separated from the O6+−-plane and they will eventually decay, since the two-cycles in $S^4$ are contractible. After all, for odd $k$, we end up with a single D2-brane stuck on the O6+−-plane, which is a stable configuration interpreted as a baryon. When $n = 2$, a constant non-vanishing tachyon field is consistent with the constraint (3.11) and the D4-D4 pairs can be annihilated completely. Therefore, we see that a single baryon in $O(N_c)$ QCD is stable, while two baryons can decay. This is exactly what is expected from the field theoretical argument.

To examine the nature of the baryon vertex in more detail, we study the effects of open strings attached on a D4-D4 pair wrapped on the $S^4$ for the $G = O(N_c)$ case. As in the $G = U(N_c)$ case reviewed above, there exist $N_c$ open strings attached to the D4-brane wrapped on the $S^4$, and the other end points of these $N_c$ open strings can be attached to the probe D8-branes. These 4-8 strings are mapped to 8-4 strings under the orientifold action. Then, this configuration is interpreted as a color singlet bound state of $N_c$ quarks. Note that a 4-8 string and an 8-4 string that are not mirror

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*See, for example, Ref. 21) for a review of tachyon condensation in unstable D-brane systems.

** The number of strings here is counted in the covering space before the orientifold projection.
to each other under the orientifold action can be recombined to form a 4-\(T\) string and an 8-8 string.\(^*\) Here, the 8-8 string is interpreted as a meson and it can move away from the baryon, and the 4-\(T\) string is no longer carrying the flavor indices. This is interpreted as the process that two quarks in the baryon are transmuted to a gluon through the gauge interaction. Unlike the \(U(N_c)\) QCD, the number of quarks trapped in a baryon is not conserved, since there is no \(U(1)\) symmetry associated with the quark number charge conserved in a baryon.\(^**\) Repeating this process, we can construct a bound state of \(k\) gluons and \((N_c - 2k)\) quarks

\[
\epsilon_{a_1a_2\cdots a_{Nc-1}a_{Nc}}F_{\mu_1\mu_2}^{a_1a_2}\cdots F_{\mu_2k-1\mu_{2k}}^{a_{2k-1}a_{2k}}q^{a_{2k+1}i_{2k+1}}\cdots q^{a_{Nc}i_{Nc}},
\]

where \(F_{\mu\nu}^{ab}\) is the field strength of the \(O(N_c)\) gauge field and \(q^{ai}\) is the quark field. This state corresponds to a D4-D4 pair with \(2k\) 4-\(T\) strings and \((N_c - 2k)\) 4- and 8-\(T\) strings attached to it in the covering space of the orientifold.

For even \(N_c\), we can take \(k = N_c/2\) in (3.12) and construct a bound state given by the Pfaffian of the gluon field. This Pfaffian particle exists even for pure Yang-Mills theory without quarks, and can be constructed in an alternative manner parallel to that shown in Ref. 20), in which a holographic dual of \(N = 4\) super Yang-Mills theory with \(G = O(N_c)\) and \(G = USp(N_c)\) is constructed by considering D3-O3 systems. To see this, we note that pure Yang-Mills theory with gauge group \(G = O(N_c)\) and \(G = USp(N_c)\) can also be formulated by placing an O4-\(\bar{T}\)-plane parallel to the D4-branes instead of the O6\(\pm\)-O6\(\pm\) pair in (2.2) without adding D8-\(D\bar{4}\) pairs. Then, the holographic dual of this system is given by Witten’s D4 background with the \(S^4\) divided by the \(Z_2\) orientifold action, which maps a point in the \(S^4\) to the antipodal point. The Pfaffian particle in this setup is given by a D4-D\(\bar{4}\) pair wrapped around the \(S^4\), which will decay to a stable D2-brane wrapping a \(Z_2\) invariant \(S^2\) in the \(S^4\) for the case with O4-\(\bar{T}\)-plane.

When \(N_c\) is odd, \((N_c - 1)\) pairs of 4-8 and 8-\(T\) strings can be converted to \((N_c - 1)\) 4-\(T\) strings through the process considered above, but at least a pair of 4-8 and 8-\(T\) strings that are mirror to each other under the orientifold action remains connected to the D8-brane. This is plausible because the baryons are fermionic, forbidding all the quarks in the baryons to be replaced with gluons. This fact leads us to an interesting consequence. As discussed above, the tachyon condensation with a one-vortex profile gives rise to the decay of the D4-D\(\bar{4}\) pair to a D2-brane wrapped on the \(S^2\) along the O6\(^+\)-plane inside the \(S^4\). The above consideration implies that for odd \(N_c\), there must be an odd number of 2-8 strings (together with the 8-2 strings that are their images of the orientifold action) attached on the D2-brane. It would be interesting to see if this fact can be understood directly from the D2-brane world-volume theory.

\(^*\) Here, if the 4-8 and 8-\(T\) strings were mirror to each other, the midpoint of the resultant 4-\(T\) string would be confined at the O6\(^+\)-plane, which is not allowed unless there are D-branes stuck on the O6-plane.

\(^**\) In this paper, we use the terminology “baryon” for a color singlet bound state of quarks and/or gluons with \(N_c\) color indices contracted by an epsilon tensor. The \(Z_2\)-valued baryon number in the \(O(N_c)\) QCD can be defined by the \(Z_2\) generated by the element of \(O(N_c)/SO(N_c)\).
The stability of the baryons can also be understood from the topological viewpoint. A simple argument for it was given in Ref. 8) assuming that the baryons are described as a topological soliton in the effective theory of Nambu-Goldstone boson associated with the spontaneous flavor symmetry breaking as in the Skyrme model.\(^{22}\) The Nambu-Goldstone mode takes a value in the coset space \(G/H\), where \(G = U(N_f)\) is the flavor symmetry group and \(H\) is the unbroken subgroup, that is, \(H = O(N_f)\) and \(H = USp(N_f)\) for \(O(N_c)\) and \(USp(N_c)\) QCD, respectively. Then, the classification of the point-like topological solitons in \((3 + 1)\)-dimensions is given by the homotopy group

\[
\begin{align*}
\pi_3(U(N_f)/O(N_f)) &= \mathbb{Z}_2, \quad \text{(for } G = O(N_c) \text{ with } N_f \geq 4) \\
\pi_3(U(N_f)/USp(N_f)) &= 0. \quad \text{(for } G = USp(N_c))
\end{align*}
\]

(3.13)

This is in accord with the previous results on the stability of baryons. However, for the cases with less than four flavors, we have \(\pi_3(U(N_f)/O(N_f)) = \mathbb{Z}_4\) for \(N_f = 3\) and \(\pi_3(U(N_f)/O(N_f)) = \mathbb{Z}\) for \(N_f = 2\), which do not agree with what we expect in \(O(N_c)\) QCD. It was argued in Ref. 8) that this discrepancy might be due to the fact that the topological classification in (3.13) only takes into account the Nambu-Goldstone boson, which is only a part of the whole configuration space including an infinite tower of massive mesons.

This subtlety can be improved by using our holographic description. We first recall that the stable baryons are described as stable D-brane configurations. It is now well established that the topological classification of the D-branes should be made using K-theory rather than the homotopy group.\(^9\) As shown in Appendix B, the topological classification of the baryons is given by the following K-groups:

\[
\begin{align*}
K_{\mathbb{R}}(\mathbb{R}^{1,3}) &= \mathbb{Z}_2, \quad \text{(for } O6^+ \text{)} \\
K_{\mathbb{R}}(\mathbb{R}^{1,3}) &= 0. \quad \text{(for } O6^-) 
\end{align*}
\]

(3.14)

Here, \(\mathbb{R}^{p,q}\) denotes \(\mathbb{R}^{p+q}\) with an involution acting on \(\mathbb{R}^p\). In (3.14), \(\mathbb{R}^{1,3}\) corresponds to the four-dimensional space \(\{(z, x^1, x^2, x^3)\}\) transverse to the baryon. Roughly speaking, the K-groups (3.14) classify the vector bundles associated with the D8-branes with an involution consistent with the constraint (3.2). (See Appendix B for more details.) Our result (3.14) is consistent with the classification of stable baryon configurations found above. It is known that the K-groups (3.14) are isomorphic to the homotopy groups (3.13) for large \(N_f\). The main difference between the homotopy group and K-theory is that the K-theory allows the creation and annihilation of D-brane - anti-D-brane pairs. A field configuration that looks stable in terms of the homotopy group may decay through the creation and annihilation of D-brane - anti-D-brane pairs, and our observation suggests that it actually happens in \(O(N_c)\) QCD with \(N_f < 4\).

There is another type of gauge-invariant configurations in QCD, i.e., flux tubes. Consider a pair of heavy quarks that belong to a representation \(R\) and its complex conjugate \(\overline{R}\) of the gauge group \(G\). Without the dynamical quarks, the electric flux sourced by the heavy quarks is squeezed to a straight line to form a flux tube. This is responsible for the linear potential between the quarks. The situation is changed
when the fundamental quarks are added. For $G = SU(N_c)$ and $G = USp(N_c)$, any heavy quark is screened by the pair-created fundamental quarks because any representation $R$ can be constructed from tensor products of an appropriate number of fundamental and anti-fundamental representations. Hence, any flux tube is unstable for $SU(N_c)$ and $USp(N_c)$ QCD. In contrast, flux tubes in $O(N_c)$ QCD exhibit a $Z_2$ stability, because the flux tube composed of a heavy quark pair in the spinor representation cannot be screened by the fundamental quarks, and therefore, is stable, while the tensor product of two spinors can be screened.\(^8\)

The stability of the flux tube in $O(N_c)$ and $USp(N_c)$ QCD can again be understood from our holographic description. The obvious string-like object in the holographic description is the fundamental string. However, this corresponds to the flux tube created by quarks in the fundamental representation of the gauge group, which is expected to be unstable. In fact, the fundamental strings can break up with the end points on the D8-brane, and are sent to infinity on the D8-brane world-volume. A possible candidate for the stable flux tube in the holographic description is a non-BPS D5-brane wrapped on the $S^4$. Since it is a six-dimensional object wrapped on a four-cycle, it also behaves as a string in the four-dimensional world. The number of relative transverse directions of the non-BPS D5-brane and O6\(^\pm\)-plane is $D_{rel} = 5$. The tachyon field $T$ on $n$ non-BPS D5-branes is an $n \times n$ Hermitian matrix satisfying

$$\tilde{T}(x^8, x^9) = \mp \tilde{T}(\mp x^8, \pm x^9),$$

(3.15)

where $\tilde{T} = T_{\gamma \pm}$, for the case with O6\(^\pm\)-plane, respectively. (See (A.7).) As in the discussion for baryons above, the tachyon field can take a constant non-vanishing value for the case with O6\(^-\)-plane, which results in the annihilation of the non-BPS D5-brane. In fact, by using the equation of motion for the gauge field on the non-BPS D5-brane

$$d \ast F \simeq c' F_4 \wedge e^{-T^2} dT,$$

(3.16)

with $F_4$ being the RR four-form field strength in the background and $c'$ a non-vanishing constant, it turns out that the electric flux along the flux tube is induced through the process of tachyon condensation. Here, we only consider the diagonal $U(1)$ part of the gauge field and tachyon field, and we have used the CS term for the non-BPS D-brane obtained in Refs. 23), 24). Then, because the electric flux is interpreted as fundamental strings, the non-BPS D5-brane is transmuted into fundamental strings, and hence, it is unstable in the presence of the D8-brane as discussed above. This is consistent with the fact that the flux tube is unstable for $USp(N_c)$ QCD. For the case with O6\(^+\)-plane, we see that an even number of non-BPS D5-branes are unstable for the same reason. When $n = 1$, the tachyon field is a real scalar field, and a profile consistent with the constraint (3.15) is given by a kink configuration $T(x^8, x^9) \propto x^8$. Then, the non-BPS D5-brane decays to a D4-brane wrapped on the $S^3 \subset S^4$ at $x^8 = 0$. Similar to the D2-brane representing a baryon discussed above, this D4-brane is hooked at the O6\(^+\)-plane with $D_{rel} = 4$ and gives a stable configuration. Therefore, we conclude that a single flux tube is stable, while two flux tubes can decay, which is again consistent with the $Z_2$ stability observed in the $O(N_c)$ QCD.
It is also interesting to note that the tension of the flux tube created by a heavy quark pair in the spinor representation in the $O(N_c)$ QCD is of order $N_c$. This fact is consistent with the above interpretation, because the tension of D-branes is proportional to $1/g_s \simeq O(N_c)^8,^{20}$

It was argued in Ref. 8) that the flux tubes can also be understood as topological solitons in the effective theory of the Nambu-Goldstone mode. Then, a topological classification is given by

$$\pi_2(U(N_f)/O(N_f)) = \mathbb{Z}_2, \quad \text{(for } G = O(N_c) \text{ with } N_f \geq 3)$$
$$\pi_2(U(N_f)/USp(N_f)) = 0. \quad \text{(for } G = USp(N_c)) \quad (3.17)$$

Again, this argument can be improved using K-theory. As discussed in Appendix B, the relevant K-groups for the flux tubes are

$$KR(\mathbb{R}^{1,2}) = \mathbb{Z}_2, \quad \text{(for } O6^+)$$
$$KH(\mathbb{R}^{1,2}) = 0. \quad \text{(for } O6^-) \quad (3.18)$$

This is in accord with the classification of stable flux tubes given above.

It is also possible to construct a domain wall by wrapping a D6-brane around the $S^4$. However, this domain wall is not stable in the presence of the probe D8-brane. The D6-brane can be represented as a magnetic flux in the D8-brane world-volume gauge theory. It can be shown that the magnetic flux is smeared in an infinite volume and the domain wall becomes infinitely thick. Then, the magnetic flux per unit volume will vanish and become invisible.

§4. Summary and discussion

In this paper, we have studied the strong coupling dynamics of large $N_c$ $O(N_c)$ and $USp(N_c)$ QCD with $N_f$ massless quarks using string theory. We formulated QCD with $G = O(N_c)$ and $G = USp(N_c)$ using the intersecting D4-D8-O6 system (2.2), and the holographic dual description was given by the intersecting D8-O6 system in Witten’s D4-brane background (Fig. 2), which is valid for $N_f/N_c \ll 1$ and large 't Hooft coupling.

With this machinery, we first showed that the flavor symmetry $U(N_f)$ is broken in a manner consistent with the gauge theory viewpoint (1.2). Although we have not given a direct proof of a non-vanishing quark bilinear condensate, our result can be regarded as a strong evidence of the spontaneous flavor symmetry breaking. Next, we discussed the stability of baryons and flux tubes. The baryons and flux tubes can be realized as D4-D4 pairs and non-BPS D5-branes wrapped on $S^4$ in the string dual picture, respectively. By examining the tachyon modes on them, we derived exactly the same stability conditions as those expected from the gauge theory viewpoint. The classification of the stable configurations is systematically given using K-theory, which improves the subtlety of the classification by homotopy groups.

One of the interesting features of the gauge/string duality is that information on the gauge group is translated to the geometry in the string theory description. As we have seen in (1.1), (3.9), and (3.10), the properties of the gauge-invariant quark
bilinear operators as well as the stability of the baryons and flux tubes in QCD follow from the structure of the gauge groups in addition to the strong dynamics of QCD. On the other hand, in the holographic dual description, the D-branes responsible for the color indices are replaced with the SUGRA background and all information on the gauge-invariant operators in the gauge theory is encoded in the geometry of the background. As we have seen, the flavor symmetry breaking and the stability of baryons and flux tubes are all understood from the topology of the background.

These successes lead us to expect that the gauge/string duality is sufficiently powerful to explore strong coupling dynamics of a wide class of gauge theories including non-supersymmetric and non-conformal field theories. A deeper investigation along this line would be worthwhile.

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Appendix A

—— Dp-Op′ System ——

In this appendix, we summarize some important properties of intersecting Dp-Op′ systems that are used in this paper. Here, p′ is assumed to be even or odd for type IIA or IIB superstring theory, respectively.

Before considering the general Dp-Op′ system, let us first recall the results in the cases of p′ = 9. The string theory with O9−-plane is known as the type I string theory and the case with O9+-plane is the string theory with USp(32) gauge symmetry considered in Refs. 26,27. The gauge groups, tachyon, and scalar fields on the Dp-brane world-volume in the presence of the O9±-plane are summarized in Table I. See Refs. 28–30 for the derivation.

Next, consider an intersecting Dp-Op′ system such that the Dp-brane and Op′±-plane are extended along (q + 1) common directions. To be more specific, we consider N Dp-branes extended along x0, x1, ..., xp directions and an Op′±-plane extended along x0, x1, ..., xq, xp+1, ..., xp+p′−q directions. In this appendix, we use the indices μ, i, I, and M for the coordinates xμ = (x0, ..., xq), xi = (xp+1, ..., xp), xI = (xp+1, ..., xp+p′−q), and xM = (xp+p′−q+1, ..., x9), respectively. The Z2 orientifold
invariant under T-duality. The D- and \( \Phi \) in the T-dualized picture as
\[
\gamma (x) = \gamma^{-1} (x') \quad \text{and} \quad \Phi (x) = (\Phi^T (x^\mu, -x^i))_{\gamma^+}^{-1},
\]
where \( \gamma_+ = I_N \) and \( \gamma_- = J_N \) are defined in (2.4).

For \( D_{\text{rel}} \equiv 2 \) (mod 4), the orientifold action maps Dp-brane to Dp'brane, and hence, we ought to consider \( N \) Dp-Dp' pairs to obtain a \( \mathbb{Z}_2 \) invariant brane configuration. Then, the gauge group of the system becomes \( U(N) \times U(N) \), and the gauge
fields and scalar fields are paired as \((\mathcal{A}, \bar{\mathcal{A}})\) and \((\varphi, \bar{\varphi})\). In addition, it is important to consider the tachyon field \(T\) created by the open strings stretched between the Dp-branes and \(\overline{\text{D}p}\)-branes, which belongs to the bifundamental representation of the \(U(N) \times U(N)\) gauge group. The constraints for these fields are
\[
\mathcal{A}(x^\mu, x^i) = -\bar{\mathcal{A}}^T(x^\mu, -x^i), \quad \varphi(x^\mu, x^i) = +\bar{\varphi}^T(x^\mu, -x^i) \quad \text{(A.3)}
\]
and
\[
T(x^\mu, x^i) = \pm T^T(x^\mu, -x^i), \quad \text{(for } \text{ Op}^{\pm}, D_{\text{rel}} = 2, 10) \quad \text{(A.4)}
\]
Again, these relations are obtained from the consistency with Table I in the T-dualized picture.

For odd \(D_{\text{rel}}\), the Dp-brane is a non-BPS D-brane whose world-volume theory is a \(U(N)\) gauge theory with a tachyon field in the adjoint representation. The constraints are obtained as
\[
\mathcal{A}(x^\mu, x^i) = -\gamma_+^\mp \mathcal{A}^T(x^\mu, -x^i)\gamma_-^{-1}, \quad \varphi(x^\mu, x^i) = +\gamma_+^\mp \varphi^T(x^\mu, -x^i)\gamma_-^{-1}, \quad \text{(for Op}^{\pm}, D_{\text{rel}} = 1, 7, 9) \quad \text{(A.5)}
\]
\[
T(x^\mu, x^i) = -\gamma_+ T^T(x^\mu, -x^i)\gamma_-^{-1}, \quad \text{(for Op}^{\pm}, D_{\text{rel}} = 1, 9) \quad \text{(A.6)}
\]
Or, if we define \(\widetilde{T} \equiv T\gamma_\pm\) (for \(D_{\text{rel}} = 3, 5\)) or \(\widetilde{T} \equiv T\gamma_\mp\) (for \(D_{\text{rel}} = 1, 7, 9\), the condition (A.6) can be written as
\[
\widetilde{T}(x^\mu, x^i) = \pm \widetilde{T}^T(x^\mu, -x^i), \quad \text{(for Op}^{\pm}, D_{\text{rel}} = 1, 3, 9) \quad \text{(A.7)}
\]
**Appendix B**

**Real K-Theory**

Here, we summarize some properties of the K-theory, which are used in the paper.*)

Let \(X\) be a manifold with an involution. The Real K-theory group \(KR(X)\) is defined as the Grothendieck group of the category of complex vector bundles \(E\)

*) S. S. thanks A. Tsuchiya and K. Hori for the discussion on K-theory, which was crucial for this appendix.
over $X$ provided with an anti-linear involution that commutes with the involution of $X$. (See Refs. 31,32 for more details.) The physical interpretation is as follows. Consider a topological classification of the gauge configurations on the D-brane - anti-D-brane system filling $X$. The vector bundle $E$ corresponds to the Chan-Paton bundle of the D-brane on which the gauge field is defined as a connection. The involution of $X$ corresponds to the orientifold action $x^i \rightarrow -x^i$ considered in Appendix A and the gauge field is required to obey the constraint (A.2) with $\gamma_+$. $KR(X)$ is the group that classifies the configuration of this D-brane - anti-D-brane system allowing the creation and annihilation of D-brane - anti-D-brane pairs. As proposed in Ref. 9), this is considered to be equivalent to the classification of D-brane charge in this system. If $X$ is non-compact, this K-group only takes into account the charge of the D-branes whose world-volumes are not extended to infinity. It is also possible to define a similar group denoted as $KH(X)$ by using $\gamma_-$ in the constraint (A.2). (See Refs. 33–36 for more details.)

We use the notation $\mathbb{R}^{p,q}$ that denotes $\mathbb{R}^{p+q} = \mathbb{R}^p \times \mathbb{R}^q$ with an involution acting on the first $\mathbb{R}^p$ factor, and $S^{p,q}$ is the $(p + q - 1)$-dimensional unit sphere in $\mathbb{R}^{p,q}$. It can be shown that $KR(X \times \mathbb{R}^{p,q})$ only depends on $p - q$, and we define

$$KR^{p-q}(X) = KR(X \times \mathbb{R}^{p,q}),$$

and similarly for $KH^{-n}(X)$. Some useful properties are

$$KR^{-n}(X) = KR^{-n+8}(X), \quad KH^{-n}(X) = KR^{-n+4}(X).$$

These groups $KR^{-n}(X)$ and $KH^{-n}(X)$ are used to classify the D-brane charge in type II string theory with an orientifold in the following way. Let us consider a ten-dimensional space-time of the form $\mathbb{R}^{r,s} \times \mathbb{R}^{r',s'} \times X$, where $r + s + r' + s' + \dim X = 10$. We assume that the set of fixed points of the involution on $X$ is a connected $k$-dimensional submanifold of $X$ and consider an $Op^{r'}$-plane with $p' + 1 = s + s' + k$ placed at the fix point of the involution in the ten-dimensional space-time. Then, $KR^{-n}(X)$ with $n = 9 - p' - r' + s'$ is used to classify the D-brane charge for stable D-branes whose world-volumes are uniformly extended along $\mathbb{R}^{r,s}$ and localized in $\mathbb{R}^{r',s'}$. Similarly, $KH^{-n}(X)$ is used for the case with an $Op^{p'}$-plane. It is easy to see that this statement agrees with the proposal given in Ref. 34) by noting

$$KR^{-n}(X) = KR^{-(9-p')}(\mathbb{R}^{r',s'} \times X), \quad \text{(for $Op^{r'}$)}$$

$$KH^{-n}(X) = KH^{-(9-p')}(\mathbb{R}^{r',s'} \times X), \quad \text{(for $Op^{p'}$)}$$

(B.3)

which follow from the relation (B.1). Note that if we consider $Dp$-branes whose world-volume is given by $\mathbb{R}^{r,s} \times X$, where $r + s + \dim X = p + 1$, the $Dp$-$Op'$ system has $D_{\text{rel}} = r + s' + \dim X - k = n$. The elements of $KR^{-n}(X)$ and $KH^{-n}(X)$ are interpreted as the charge of stable D-branes made by the $Dp$-branes (or $Dp$-$Dp'$ system) whose world-volume field configuration is uniform along $\mathbb{R}^{r,s}$.

To calculate the K-groups relevant to our system, the following relation will be useful.

$$KR^{-n}(\mathbb{R}^{p,q} \times S^{r,s}) = KR^{-n+p-q}(pt) \oplus KR^{-n+p-q+r-s+1}(pt). \quad \text{(for } s \geq 1)$$

(B.4)
Here $pt$ denotes a point. (See below for a derivation.) Note that (B.4) also holds for $s = 0$ with $r \geq 3$.\(^\dagger\) The $KR^{-n}(pt)$ is known as

| $n \pmod{8}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|------------|---|---|---|---|---|---|---|---|
| $KR^{-n}(pt)$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 |

Note that (B.4) can be written as

$$KR^{-n}(\mathbb{R}^{p,q} \times S^n) = KR^{-n}(\mathbb{R}^{p,q}) \oplus KR^{-n}(\mathbb{R}^{p+r,q+s-1}),$$

$$KH^{-n}(\mathbb{R}^{p,q} \times S^n) = KH^{-n}(\mathbb{R}^{p,q}) \oplus KH^{-n}(\mathbb{R}^{p+r,q+s-1}),$$

from which we interpret that $KR^{-n}(\mathbb{R}^{p,q})$ and $KH^{-n}(\mathbb{R}^{p,q})$ correspond to the D-branes localized along $\mathbb{R}^{p,q}$ and wrapped on the $S^{r,q}$, while $KR(\mathbb{R}^{p+r,q+s-1})$ and $KH(\mathbb{R}^{p+r,q+s-1})$ correspond to the D-branes localized along $\mathbb{R}^{p,q} \times S^{r,s}$.

Let us apply these results to the system considered in §3.1. We are interested in the space-time $\mathbb{R}^{1,5} \times S^{2,3}$ where $\mathbb{R}^{1,5} = \{(z, y, x^0, \ldots, x^3)\}$ and $S^{2,3}$ is the unit four-sphere in $\{(x^5, \ldots, x^9)\}$ with the involution $(z, x^8, x^9) \rightarrow (-z, -x^8, -x^9)$.

The point-like particles are classified by choosing $X$ to be $\mathbb{R}^{1,3} \times S^{2,3}$, where $\mathbb{R}^{1,3} = \{(z, x^1, x^2, x^3)\}$ and considering stable D-branes whose world-volumes are of the form $\mathbb{R}^{0,1} \times X'$, where $\mathbb{R}^{0,1}$ is the time direction and $X'$ is a compact submanifold of $X$. For this configuration, we have $n = D_{\text{rel}} = 4$ and the classification of the particles is given by

$$KH(\mathbb{R}^{1,3} \times S^{2,3}) = KH(\mathbb{R}^{1,3}) \oplus KH(\mathbb{R}^{3,5}) = 0, \quad \text{(for O6$^-$)}$$

$$KR(\mathbb{R}^{1,3} \times S^{2,3}) = KR(\mathbb{R}^{1,3}) \oplus KR(\mathbb{R}^{3,5}) = \mathbb{Z}_2 \oplus \mathbb{Z}_2. \quad \text{(for O6$^+$)}$$

Here, $KH(\mathbb{R}^{1,3})$ and $KR(\mathbb{R}^{1,3})$ correspond to the D4$-$D4 pairs wrapped on the $S^4$, which are interpreted as the baryons. The elements of $KH(\mathbb{R}^{3,5})$ and $KR(\mathbb{R}^{3,5})$ correspond to the D0$-$D0 pairs, which do not correspond to bound states of quarks and/or gluons in QCD.$^*$

The string-like objects extended along $(x^0, x^1)$ are classified by choosing $X$ to be $\mathbb{R}^{1,2} \times S^{2,3}$, where $\mathbb{R}^{1,2} = \{(z, x^2, x^3)\}$. They are classified by

$$KH(\mathbb{R}^{1,2} \times S^{2,3}) = KH(\mathbb{R}^{1,2}) \oplus KH(\mathbb{R}^{3,4}) = 0, \quad \text{(for O6$^-$)}$$

$$KR(\mathbb{R}^{1,2} \times S^{2,3}) = KR(\mathbb{R}^{1,2}) \oplus KR(\mathbb{R}^{3,4}) = \mathbb{Z}_2 \oplus \mathbb{Z}_2. \quad \text{(for O6$^+$)}$$

Here, $KH(\mathbb{R}^{1,2})$ and $KR(\mathbb{R}^{1,2})$ correspond to the non-BPS D5-branes wrapped on the $S^4$, that are interpreted as the flux tube in QCD. The elements of $KH(\mathbb{R}^{3,4})$ and $KR(\mathbb{R}^{3,4})$ correspond to the non-BPS D1-branes, which do not have any counterparts in QCD.

For completeness, let us sketch the derivation of (B.4). From (B.1), it is sufficient to show the $p = q = 0$ case in (B.4). We use the fact that there is an exact sequence

$$KR^{-n-1}(X) \rightarrow KR^{-n-1}(Y) \rightarrow KR^{-n}(X, Y) \rightarrow KR^{-n}(X) \rightarrow KR^{-n}(Y),$$

$^*$ The D0-brane also exists in the case without orientifold plane. Since it has a net charge for RR 1-form potential, it cannot be considered as a bound state made by quarks and/or gluons.
where $Y \subset X$, and $(X, Y)$ is a pair of compact spaces with involution. It follows from this exact sequence that
\[ KR^{-n}(X) = KR^{-n}(X, Y) \oplus KR^{-n}(Y), \tag{B.10} \]
when $Y$ is a retract of $X$, i.e., there is a continuous map $r : X \to Y$ such that $r|_Y = 1_Y$. Applying (B.10) for $X = S^{r,s}$ and $Y = pt$, we obtain
\[ KR^{-n}(S^{r,s}) = KR^{-n}(S^{r,s}, pt) \oplus KR^{-n}(pt). \tag{B.11} \]
Here, $Y = pt$ is chosen to be a fixed point of the involution in $X = S^{r,s}$, which exists for $s \geq 1$.

Again, applying (B.10) for $X = B^{r,s}$ (a unit ball in $\mathbb{R}^{r,s}$) and $Y = S^{r,s}$, and using the facts $KR^{-n}(B^{r,s}) = KR^{-n}(pt)$ and $KR^{-n}(B^{r,s}, S^{r,s}) = KR^{-n}(\mathbb{R}^{r,s}) = KR^{-n+r-s}(pt)$, we can show
\[ KR^{-n}(S^{r,s}, pt) = KR^{-n+r-s+1}(pt). \tag{B.12} \]
From (B.11) and (B.12), we obtain (B.4).

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