The cross-correlation of redshited 21-cm signal and Lyman-α forest: A cosmological probe

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Abstract. We have investigated the cross-correlation of the Lyman-α forest and redshifted 21-cm emission as a new observational probe of the large scale structures in the post-reionization era, with a significant advantage that the problems of continuum subtraction and foreground removal, and other systematics are expected to be considerably less severe in comparison to the respective auto-correlation signals. In this paper, we have explored the possibility of detecting the baryon acoustic oscillation in the cross-correlation signal. We have developed a formalism to calculate the expected cross-correlation signal and its covariance. We have used this to predict the expected signal and noise for a range of observational parameters.

1. Introduction
The cross-correlation of the 21-cm signal with the Lyman-α forest has recently been proposed as a new probe of the post-reionization era \[1\]. The 21-cm emission and the Lyman-α absorption signals, both originate from neutral hydrogen (HI) at the same redshift. However, these signals emanate from distinctly different sources. The 21-cm emission comes from the damped Lyman-α systems, which harbours bulk of HI at low redshifts. On the contrary, Lyman-α forest arises from small density fluctuations in the pre-dominantly ionized IGM. On large scales, however, the fluctuations in the 21-cm signal and the Lyman-α forest transmitted flux are both excellent tracers of the underlying dark matter density field and hence, the cross-correlation between the two can be used to probe the matter power spectrum. The foregrounds and systematics in the Lyman-α forest and the redshifted 21-cm emission are expected to be uncorrelated and therefore, pose much less severe problem for the cross-correlation. The signal, if detected, shall hence, ascertain its cosmological origin conclusively. Apart from being an independent probe of the large scale matter distribution, the same cosmological and astrophysical information as the individual signals is contained in the cross-correlation signal.

The sound horizon at recombination may be used as a standard ruler to calibrate cosmological distances. The baryon acoustic oscillation (BAO) imprinted in the late time clustering of matter allows a measurement of the angular diameter distance and the Hubble parameter as functions of redshift using the transverse and the longitudinal oscillations respectively. These provide means for constraining dark energy models. We have considered the possibility of detecting BAO using the cross-correlation signal. We have quantified the cross-correlation between the Lyman-α forest and the 21-cm emission using the multi-frequency angular power spectrum.
2. The cross-correlation signal

The fluctuations in the Lyman-α forest transmitted flux $\mathcal{F}(\hat{n}, z)$ along a line of sight $\hat{n}$ to a quasar is quantified using:

$$\delta \mathcal{F}(\hat{n}, z) = \mathcal{F}(\hat{n}, z)/\bar{\mathcal{F}} - 1.$$ 

On large scales, it is reasonable to adopt the fluctuating Gunn-Peterson approximation [3, 4] relating the flux to the matter density contrast $\delta$. If fluctuations $\delta \mathcal{F}$ are smoothened over sufficiently large length scales, then $\delta \mathcal{F} \propto \delta$ [3].

We have used $\delta T(\hat{n}, z)$ to quantify the fluctuations in the brightness temperature of redshifted 21-cm emission. In the redshift range of our interest ($z < 3.5$), it is reasonable to assume that $\delta T(\hat{n}, z)$ is a biased tracer of $\delta$. Simulations [5] indicate that the bias is scale independent on large scales. With these assumptions, we may express both $\delta \mathcal{F}$ and $\delta T$ in Fourier space as:

$$\Delta_\alpha(k) = C_\alpha [1 + \beta_\alpha \mu^2] \Delta(k),$$

where $\alpha = \mathcal{F}$ and $T$ refer to the Lyman-α forest and 21-cm signal respectively. $\Delta(k)$ is the matter density contrast in Fourier space and $\mu = \vec{k} \cdot \hat{n}$. We have used $C_\alpha, \beta_\alpha$ from [6] and [1]. These are the values at our fiducial redshift $z = 2.5$.

We have used the smaller field of view $L \times L$ (in radians) of the radio telescope to estimate the cross-correlation signal. In the flat sky approximation ($L \ll 1$), it is convenient to decompose $\delta_\alpha(\bar{\theta}, z)$ into Fourier modes $\Delta_\alpha(U, z)$, where $U$ is conjugate to $\theta$. The multi-frequency angular power spectrum (MAPS) is the correlation of angular modes on two planes at comoving distances $r$ and $r + \Delta r$, defined as:

$$\langle \Delta_\alpha(U, z) \Delta_\gamma^*(U', z + \Delta z) \rangle = L^2 \delta_{UU'} P_{\alpha\gamma}(U, \Delta z),$$

where the indices $\alpha$ and $\gamma$ both assume values $T$ and $\mathcal{F}$. The entire three dimensional information is contained in $P_{\alpha\gamma}(U, \Delta z)$ through the $U$ and $\Delta z$ dependence. It is related to the 3D matter power spectrum $P(k)$ as [2]:

$$P_{\alpha\gamma}(U, \Delta z) = \frac{1}{\pi r^2} \int_0^\infty dk_\parallel \cos(k_\parallel \Delta r) F_{\alpha\gamma}(\mu) P(k),$$

where $\Delta r = c\Delta z/H(z)$, $k = \sqrt{k_\parallel^2 + (2\pi U/r)^2}$, $\mu = k_\parallel/k$, and

$$F_{\alpha\gamma}(\mu) = C_\alpha C_\gamma [1 + \beta_\alpha \mu^2][1 + \beta_\gamma \mu^2].$$

The angular mode $U$ may be identical with the angular multipole $\ell$ as $2\pi U = \ell$. We have referred the 2D transverse angular power spectrum as:

$$P_{\mathcal{F}T}(\ell) \equiv P_{\mathcal{F}T}(U, \Delta z = 0).$$

This has contribution from all the 3D modes $k$, whose projection on the transverse plane matches $\ell/r \equiv 2\pi U/r$, such that only modes $k > \ell/r$ contribute to $P_{\mathcal{F}T}(\ell)$. The amplitude of $P_{\mathcal{F}T}(\ell)$ depends on $C_T$ and $C_\mathcal{F}$, whose values are highly uncertain. On the contrary, the shape of $P_{\mathcal{F}T}(\ell)$ is related to the matter power spectrum and the comoving distance $r$. At large $\ell$ ($> 1000$), we have $P_{\mathcal{F}T}(\ell) \propto \ell^{-1.76}$. The angular power spectrum flattens out at $\ell \sim k_{\text{eq}} r$, which corresponds to the peak in $P(k)$.
Figure 1. This shows the imprint of the BAO on the transverse angular power spectrum $P_{FT}(\ell)$ and the radial de-correlation $\kappa(\Delta z)$ for the cross-correlation signal. To highlight the BAO, we have divided $P_{FT}(\ell)$ by $P^{nw}_{FT}(\ell)$, which corresponds to $P(k)^{nw}$ [7]. This is shown for three redshifts $z = 1.5, 2.5$, and $3.5$. The $\kappa(\Delta z)$ is shown for the fiducial redshift $z = 2.5$ for several $\ell$ values. The BAO in $\kappa$ shows ringing around its ‘no wiggles’ counterpart.

At fixed $\ell$, $P_{FT}(U, \Delta z)$ is the correlation between $\Delta F(U, z)$ and $\Delta T(U, z + \Delta z)$ at fixed $U$, but located on two planes separated by $\Delta r$. The cosine term arises due to a single 3D mode projecting onto planes with a phase difference of $k_\parallel \Delta r$. We have introduced the de-correlation function as:

$$\kappa_{\ell}(\Delta z) = P_{FT}(\ell, \Delta z)/P_{FT}(\ell),$$

which varies in the range $0 \leq |\kappa_{\ell}(\Delta z)| \leq 1$. With increasing $\Delta z$, $\kappa_{\ell}(\Delta z)$ falls, crosses zero and then gets anti-correlated. The de-correlation is faster for large $\ell$ values in the entire $z$ range.

The BAO scale $s$ defines an angular scale $\theta_s = s[(1 + z)D_A(z)]^{-1}$ and a redshift interval $\Delta z_s = sH(z)\theta_s$, where $D_A(z)$ and $H(z)$ are the angular diameter distance and Hubble parameter respectively. The MAPS allows us to measure $\theta_s$ and $\Delta z_s$ separately, and thereby, $D_A(z)$ and $H(z)$. Figure 1 shows the BAO feature in $P_{FT}(\ell)$. The BAO appears as a series of oscillations in $P_{FT}(\ell)$, the positions of the peaks being consistent with $\ell \sim kr$. The amplitude of the first oscillation in $P_{FT}(\ell)$ is around 1%, in contrast to the $\sim 10\%$ feature in $P(k)$. The reduction is caused by the projection onto plane. At $z = 2.5$, the first peak occurs at $\ell \sim 270$, and has a full width of $\Delta \ell \sim 200$. These values scale as $r$ if the redshift is changed.

The first BAO peak imprints on $\kappa_{\ell}(\Delta z)$ only if $\ell < k_1 r$, where $k_1$ is the position of the first peak in the 3D matter power spectrum. Thus, for $z = 2.5$, the first peak has no impact on $\kappa_{\ell}(\Delta z)$ at $\ell > 500$, and has an impact at all angular modes $\ell \leq 500$. Figure 1 shows $\kappa_{\ell}(\Delta z)$ along with $\kappa_{\ell}(\Delta z)$ calculated using $P^{nw}(k)$, which does not have the BAO features. The BAO has little impact near $\Delta z \approx 0$, where $\kappa_{\ell}(\Delta z)$ is positive. The significant effect (as large as 40% to $\sim 100\%$ relative to the no-wiggles model) is seen only at large $\Delta z$, typically in the range $\Delta z = 0.04$ to $0.16$, where $\kappa_{\ell}(\Delta z)$ is negative. The BAO ringing feature around the smooth ‘no wiggles’ $\kappa_{\ell}(\Delta z)$, is distinct from the slow oscillation seen in the no-wiggles model.
2.1. Detecting the cross-correlation signal

The observed Lyman-α flux $\delta_{F0}(\vec{\theta}, n)$ can be written as:

$$\delta_{F0}(\vec{\theta}, n) = \rho(\vec{\theta}) \left[ \delta_F(\vec{\theta}, z_n) + \delta_{FN}(\vec{\theta}, n) \right],$$

with $z_n = z_0 + n\Delta z_c$ and $n = 1, 2, \cdots, N_c$. Here, $z_0$ is a reference redshift $N_c$, which is the total number of frequency channels, $\delta_{FN}(\vec{\theta}, z)$ is the pixel noise, and $\rho(\vec{\theta}) = \sum_a \omega_a \delta_D^2(\vec{\theta} - \vec{\theta}_a)/\sum \omega_a$ is the sampling function. Here, $\vec{\theta}_a$ and $\omega_a$ respectively denote the positions and weights of the quasars with $a = 1, 2, \cdots, N$ and

$$\langle \delta_{FN}(\vec{\theta}_a, n) \delta_{FN}(\vec{\theta}_b, m) \rangle = \delta_{a,b} \delta_{n,m} \sigma_{FN}^2,$$

where $\sigma_{FN}^2$ is the noise variance. We have used weights $\omega_a = 1$ assuming high SNR of flux for all the quasars. In Fourier space, we have:

$$\Delta_{F0}(U, z) = \rho(U) \otimes \left[ \Delta_F(U, z) + \Delta_{NF}(U, z) \right].$$

Radio observations directly measure $\Delta_T(U, z_n)$. Each pair of antennas measure:

$$\Delta_{T0}(U, n) = \Delta_T(U, z_n) + \Delta_{TN}(U, n),$$

where $\Delta_{TN}$ is a noise term. We have defined an estimator $\hat{E}$ as:

$$\hat{E}(U, p) = \sum_{n=1}^{N_c} \sum_{m=1}^{N_c} \frac{1}{2} \left[ \Delta_{F0}(n) \Delta_{T0}^*(m) + \Delta_{F0}^*(n) \Delta_{T0}(m) \right] \delta_{|n-m|, p} / \sum_{n=1}^{N_c} \sum_{m=1}^{N_c} \delta_{|n-m|, p},$$

with the property that:

$$\langle E(U, p) \rangle = P_{FT}(U, p \Delta z_c),$$

and a covariance matrix is given by:

$$\text{Cov}(p, q) = \sum_{a, \beta} \sum_{n=1}^{N_c} \sum_{m=1}^{N_c} P_{a \beta 0}(p + m - n)P_{a' \beta' 0}(m - n - q)/(N_c - p)(N_c - q).$$
where the variable $\alpha'$ has the value of $F$ when $\alpha = T$ and vice-versa, and $\beta$, $\beta'$ are defined in a similar way. Here,

$$P_{\mathcal{F}F_0}(U,p) = P_{\mathcal{F}F}(U,p\Delta z_c) + \frac{1}{\bar{n}_Q} \left[ \xi_F(p\Delta z_c) + \delta_{p,0}\sigma_{F,N}^2 \right],$$

where $\xi_F(\Delta z)$ is the 1D flux correlation function and $\bar{n}_Q = N/L^2$ is the quasar density on the sky. Further,

$$P_{TT_0}(U,p) = P_{TT}(U,p\Delta z_c) + \delta_{p,0}N_T(U),$$

where $N_T(U)$ is the 21-cm noise power spectrum.

We have considered the quasars in the redshift range $2 < z < 3$. The region 10000 km s$^{-1}$ blue-wards of the quasar’s Lyman-α emission is excluded due to the quasar’s proximity effect and only the pixels at least 1000 km s$^{-1}$ red-ward of the quasar’s Lyman-β and O-VI lines are considered. Also, we have considered 21-cm observations of bandwidth $B = 128$ MHz centred around 406 MHz. Only a fraction (approximately 40%) of the total number of quasars will contribute the cross-correlation in the entire $z$ range 2 to 3. We have considered a radio array with antennas at the right corner. We have found that a 5σ detection of $P_{TT}(\ell)$ is possible for $\bar{n}_Q \sim 0.1$ deg$^{-2}$ and $N_T \sim 10^{-4}$ mK$^2$.

The first BAO peak is a 1% feature in $P_{TT}(\ell)$, and a 5σ detection of the BAO peak requires an SNR of 500 for $P_{TT}(\ell)$, and it is necessary to consider multiple pointings. The BOSS$^1$ is expected to have a QSO density of 16 deg$^{-2}$, which corresponds to $\bar{n}_Q = 6.4$ deg$^{-2}$. This survey is expected to cover $\sim 10000$ deg$^2$ of the sky, and we could ideally have $N_{\text{point}} = 25$ independent pointings of the $20^\circ \times 20^\circ$ field of view. We have SNR = 100 for $N_T = 1.1 \times 10^{-5}$ mK$^2$, and a 5σ detection of the first BAO peak is possible with $N_{\text{point}} = 25$ fields of view. BigBOSS [8] may achieve a QSO density of $\sim 64$ deg$^{-2}$, which corresponds to $\bar{n}_Q = 25.6$ deg$^{-2}$. At $N_T = 3.3 \times 10^{-5}$ mK$^2$, 6σ detection is possible with $N_{\text{point}} = 25$. The radial oscillations is a $\sim 1\%$ deviation, which is comparable to the deviation introduced by the angular oscillations. The BAO signal is maximum in the vicinity of $\ell \approx 250$. We have collapsed the three central $U$ bins in order to enhance the SNR for the radial oscillations. We have found that a 5σ detection is possible with BOSS if we observe 25 fields of view with $N_T = 6.25 \times 10^{-6}$ mK$^2$. We require $N_T = 1.7 \times 10^{-5}$ mK$^2$ for a similar detection with BigBOSS.

BAO observations can be used to constrain cosmological parameters and the equation of state of the dark energy. In this paper, we have estimated the range of observational parameters, for which it will be possible to detect the BAO using the cross-correlation signal. Despite being a promising tool, several observational issues including the 21-cm foreground subtraction shall, however, require serious consideration towards detecting the signal.

References

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