Short communication

Aggregate download throughput for TCP-controlled long file transfers in a WLAN with multiple STA-AP association rates

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1. Introduction

IEEE 802.11a/b/g/n based Wireless Local Area Networks (WLANs) in “infrastructure mode” are very common in many places. In this paper, we are concerned with an analytical model for evaluating the performance of TCP-controlled downloads in a WLAN. “TCP” is the Transmission Control Protocol, which is regarded as the workhorse of the Internet; numerous applications, including web browsing, file transfer, and secure e-commerce, rely on TCP as the transport protocol. The system we consider is shown in Fig. 2. A detailed analysis of the aggregate throughput of TCP-controlled file downloads for a “single rate” Access Point (AP) is given in [1]; in this, all STAs are assumed to be associated with the AP at the same rate. In practice, many data rates are possible and hence considering multiple rates is important. The aggregate download throughput is evaluated for the two rates case in [2]. In this paper, we consider an arbitrary but finite number of possible rates of association between stations (STA) and a single AP.

We are motivated to study an analytical model because of the improved understanding that it leads to, and the useful insights that it can provide. Closed-form expressions or numerical calculation procedures are helpful because other features and capabilities can be built upon them. One possible application, which we are studying now, is to utilize the results reported here in devising a better AP-STA association policy.

Our approach is to model the number of STAs with TCP Acknowledgements (ACKs) in their Medium Access Control (MAC) queues as an embedded discrete time Markov chain (DTMC), embedded at the instants of successful transmission events. We consider a successful transmission from the AP as a reward. This leads to viewing the aggregate TCP throughput in the framework of Renewal Reward theory given in [3].

Almost the entire existing literature considers a single rate of association only. This is rather limiting, because in practice, it is extremely likely that a WLAN will have STAs associated at a number of rates allowed by the technology (for example, one of 6 Mbps, 12 Mbps, 18 Mbps, 24 Mbps, 30 Mbps, 36 Mbps, 48 Mbps or 54 Mbps in 802.11g, and one of 1 Mbps, 2 Mbps, 5.5 Mbps or 11 Mbps in 802.11b). A first step towards considering multiple association rates was taken in [2], but there, only
2 possible association rates were considered. In this paper, any number of association rates is allowed. Because of this, our model is applicable to any variant of WLAN technology, for example: 802.11a/b/g/n.

The contributions of this paper are as follows. We present a model for analyzing the performance of TCP-controlled file transfers with $0 < k < \infty$ rates of association. This generalizes earlier work. Secondly, our model incorporates TCP-specific aspects like "delayed ACKs"; this is a technique to reduce the frequency of TCP ACK generation by a TCP receiver. In most implementations, a TCP receiver generates a TCP ACK for every 2nd TCP packet received; our model is general, and considers that one TCP ACK is generated for every $d$ TCP packets. Our analytical results are in excellent agreement with simulations, with the discrepancy being less than 1% in all cases.

The paper is organized as follows: In Section 2, related works are discussed. In Section 3, we state the assumptions in first part and then present our analysis. In Section 4, we present performance evaluation results. In Section 5 we discuss the results. Finally, the paper is concluded in Section 6.

2. Related work

The literature on throughput modeling in a WLAN can be classified into several groups depending on the approach.

In the first group, all WLAN entities (STAs and the AP) are assumed to be saturated, i.e., each entity is backlogged permanently. Bianchi [4], Kumar et al. [5] and Cali et al. [6] consider this saturated traffic model. However, our interest is in modeling aggregate TCP throughput, and the saturated traffic model does not capture the situation well.

To see why unsaturated traffic makes a difference, we consider Fig. 1. The left part shows a saturated traffic scenario, where all WLAN entities have packets to transmit; therefore, $(N + 1)$ entities contend for the channel. The right part shows the situation with TCP in the picture. Essentially, for many TCP connections, the entire window of packets sits at the AP, leaving the corresponding STAs with nothing to send. This means that the number of contending WLAN entities is much smaller as mentioned in [1]. This indicates why approaches relying on a model with saturated nodes are inadequate.

The second group considers TCP traffic. Kuriakose et al. [1] propose a model for TCP-controlled file downloads in a single rate WLAN; i.e., one in which all STAs are associated with a single AP at the same rate. Bruno et al. [7–11], and Vendictis et al. [12] generalize this and analyze TCP-controlled file uploads as well as downloads; however, it is assumed again that all STAs are associated at the same rate. Similarly, Yu et al. [13] provide an analysis for a given number of STAs and a maximum TCP receive window size by using the well-known $p$-persistent model proposed in [6]. As noted above, these papers analyze TCP-controlled file transfers (in some cases UDP traffic is allowed as well) but limit themselves to a single rate of association.

In the third category, Bharadwaj et al. [14] consider finite AP buffers, in contrast to the previous two, where AP buffers were assumed to be infinite; however, the single rate assumption is retained.

The three groups mentioned above focus on long file transfers, where the TCP sender is assumed to have a file that is infinite in size. Miorandi et al. [15] model a different situation motivated by web browsing over a WLAN. A queuing model is proposed to compute the mean session delay for short-lived TCP flows. The impact of maximum TCP congestion window size on this delay is studied as well.

Even though a fair amount of work modeling TCP-controlled transfers has been done, we are unaware of any work that allows multiple AP-STA association rates. Clearly, this is the situation observed most often in practice, where the distance between the AP and a STA governs the rate of association. In this paper, we consider an arbitrary (but finite) number of rates of association between STAs and the AP; to the best of our knowledge, this is the first paper to consider this general model.

3. System model

3.1. Assumptions

We consider $M$ stations associated with an AP as shown in Fig. 2. All the nodes contend for the channel using the Distributed Coordination Function (DCF) mechanism as given in IEEE 802.11a/b/g/n. The stations are associated with the AP at $k$ different physical rates ($m_1$ STAs at rate $r_1$, $m_2$ STAs at rate $r_2$, ..., $m_k$ STAs at rate $r_k$). We assume that there are no
link errors. This is not merely a simplifying assumption; the “auto rate fallback” mechanism, implemented widely in STAs and APs, is intended to ensure that we have an error-free but lower rate channel rather than a higher rate but error-prone channel. Thus, our assumption of no link errors is consistent with the usual mode of WLAN operation.

Packets in the medium are lost only due to collisions. Each station has a single TCP connection to download long files from the server and all TCP connections have equal window sizes. The AP transmits TCP packets for the stations and the stations return TCP-ACK packets. Further, we assume that the AP uses the RTS-CTS mechanism while sending packets to stations and stations use basic access to send ACK packets (RTS: Request to Send, CTS: Clear to Send; these are control packets that reserve the wireless medium for the subsequent long data packet). In IEEE 802.11 WLANs, the RTS Threshold parameter determines whether the RTS-CTS exchange will precede a packet transmission. In most operational WLANs, the RTS Threshold is set such that TCP data packets are larger than the RTS Threshold, and hence sent after RTS-CTS exchange, while TCP ACK packets are smaller than the RTS Threshold, and hence sent without RTS-CTS exchange (the latter is referred to as “basic access”).

Upon reception of $d$ data packets, a STA generates an ACK packet and it is enqueued at the MAC layer for transmission. We assume that all nodes have sufficiently large buffers, so that packets are not lost due to buffer overflow. Also, TCP timeouts do not occur. TCP start-up transients are ignored by considering all connections to be in Congestion Avoidance. For long file transfers (which we are considering in this paper), this is a reasonable assumption because the initial start-up phase of a TCP connection lasts for a time that is completely negligible compared to the connection lifetime; the TCP connection moves quickly to the Congestion Avoidance phase and remains there. The value of RTT is very small, since files are downloaded from a server located on the LAN as shown in Fig. 2.

Thus, several TCP connections exist simultaneously and all STAs with TCP ACK packets, and the AP (which is full of TCP data packets for the STAs), contend for the channel. Since no preference is given to the AP, and it has to serve all STAs, the AP becomes a bottleneck, and it is modeled as being backlogged permanently. The aggregate throughput of the AP is shared equally among all $M$ stations. This is true by virtue of the assumption that all TCP connections have equal window sizes. When the AP wins the channel, it picks a packet from the ones stored in its buffer and transmits it. For the case in which the server is located on the LAN, the full windows of all the connections are stored at the AP buffer. Because of equal window sizes, the probability that the transmitted packet is for a station $i$ is simply $\frac{1}{M}$, for any $i \in \{1, 2, \ldots, M\}$. Hence, the aggregate throughput of the channels is shared equally among all the $M$ STAs. The same argument indicates that if the connection windows are unequal, then the individual throughputs will differ in the same ratio.

3.2. Analysis

Let $m_i$ be the number of stations associated with the AP at the physical transmission rate $r_i$, where $i \in \{1, 2, \ldots, k\}$ with $r_1 > r_2 > \cdots > r_k$. We assume that $m_i$ is large. Our results will show that $m_i \geq 3$ or $4, 1 \leq i \leq k$ (with $k \geq 4$) suffices for the analysis to be applicable. Given that the AP wins the channel, the conditional probability that it sends a TCP data packet to a station at rate $r_i$ is $p_i$.

To obtain $p_i$, we proceed as follows. The AP is backlogged permanently. As we have noted above, when it wins the channel, it picks a packet from the ones stored in its buffer and transmits it. For the LAN case, the full windows of all the connections are stored at the AP buffer. By virtue of equal window sizes, the probability $p_i$ that the AP sends a packet to a STA associated at rate $r_i$ is given by the fraction of STAs at rate $r_i$. This is what we have used in our numerical calculations later.

Fig. 3 shows a possible sample path of the events on the WLAN channel. The random epochs $G_j$ indicate the end of the $j$th successful transmission from either the AP or one of the stations. We observe that most STAs have empty MAC queues, because, in order for many STAs to have TCP-ACK packets, the AP must have had a long run of successes—and this is unlikely because no special preference is given to the AP. So when the AP succeeds in transmitting, the packet is likely to be for a STA with an empty MAC queue.

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1 This can be generalized, as in [16].
The state 

generates a TCP ACK after receiving TCP packets; this is incorporated in our model in the following way. When the DTMC evolves as a Discrete Time Markov Chain (DTMC) over the epochs 

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Fig. 3. A possible sample path of events in WLAN shows the backoffs and the channel activity.

\[ \text{STAs. If there are } \sum_{n_i=1}^{N} S_{i,j} = N_i \text{ be the number of nonempty STAs. If there are } N \text{ nonempty STAs and a nonempty AP, each nonempty WLAN entity attempts to transmit with probability } \beta_{N+1} \text{ as in [5], where } \beta_{N+1} \text{ is the attempt probability with } (N+1) \text{ saturated entities.} \]

To obtain \( \beta_N \), we have followed the method given in [5]; Eqs. (1), (2) and (4) of the paper are used. The approach is as follows. There are \( N \) contending nodes in the WLAN system. All \( N \) nodes have packets to send, i.e., they are saturated. Assuming that all nodes see a collision probability \( \gamma \), Eq. (1) gives the attempt probability of a node \( G(\gamma) \). Next, the “decoupling approximation” referred to in [5] is used, and Eq. (2) provides an expression for the collision probability experienced by a node, given that its attempt probability is \( \beta \); this probability is \( \Gamma (\beta) \). These two equations lead to the fixed point equation (Eq. (4)):

\[ \gamma = \Gamma (G(\gamma)). \]

Brouwer’s Fixed Point Theorem assures us that a solution exists, and Theorem 5.1 of [5] allows us to conclude that a unique fixed point exists. Fig. 5 in the paper shows how numerical solutions to the equation can be obtained. The intersection of the straight line and the decreasing curves (there is one curve for each value of \( N \), the number of contending nodes) determines the collision probability for \( N \) contending nodes. Plugging this value into Eq. (1), \( G(\gamma) \) yields the attempt probability \( \beta_N \) for \( N \) nodes.

It can be seen that \( (S_{1,j}, S_{2,j}, \ldots, S_{k,j}) \) evolves as a Discrete Time Markov Chain (DTMC) over the epochs \( G_j \). This allows us to consider \( (S_{1,t}, S_{2,t}, \ldots, S_{k,t}) \) as a Markov Renewal Sequence, and \( (S_{1}(t), S_{2}(t), \ldots, S_{k}(t)) \) as a semi-Markov process.

We have a multidimensional DTMC which is shown in Fig. 4; transition probabilities are indicated as well. An STA \( \gamma = \Gamma (G(\gamma)). \)

By inspection, we can say that the DTMC is irreducible; further, the Detailed Balanced Equation holds for a properly chosen set of equilibrium probabilities. The Detailed Balance Equation (DBE) is

\[ \forall n = (n_1, n_2, \ldots, n_k) : n_i \geq 0, \quad 1 \leq i \leq k, \]

\[ \pi (n_1, n_2, \ldots, n_i, \ldots, n_k) \frac{p_i}{(N+1)} = \frac{1}{d} \pi (n_1, n_2, \ldots, (n_i+1), \ldots, n_k, \ldots) \frac{(n_i+1)}{(N+2)} \]

(1)

\[ 1 \leq i \leq k. \]

Here \( \pi (n_1, n_2, \ldots, n_i, \ldots, n_k) \), \( n_1, n_2, \ldots, n_k \in \{0, 1, 2, \ldots\} \) is the stationary distribution of the DTMC. From the set of equations given in (1) and

\[ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots \sum_{n_k=0}^{\infty} \pi (n_1, n_2, \ldots, n_i, \ldots, n_k) = 1, \]

the stationary distribution is

\[ \pi (n_1, n_2, \ldots, n_i, \ldots, n_k) = (N + 1)^{-k} \int_{\Gamma_{i=1}^{k}} \frac{d^n (p_i)^n_i}{(n_i!)} \ast \frac{1}{(2e)}. \]

(2)

Fig. 4. Embedded Markov chain formed by the AP and \( n_1 + n_2 + \cdots + n_k = N \) stations associated with the AP at \( k \) different data rates.
To obtain the throughput, we use Markov regenerative analysis, culminating in the Renewal Reward Theorem given in [17,3]. For a given state \((n_1, \ldots, n_k)\), successive entries into state \((n_1, \ldots, n_k)\) form renewal epochs. To obtain the mean time between successive entries (the mean renewal cycle lengths), we obtain the mean sojourn time in the state \((n_1, \ldots, n_k)\).

Let \(X\) be the sojourn time in a state \((S_{i_1}, \ldots, S_{i_j})\). Conditioning on various events (idle slot, collision or successful transmission) that can happen in the next time slot, the following expression for the mean cycle length can be written down:

\[
E_{n_1, n_2, \ldots, n_k} = P_{\text{idle}}(\delta + E_{n_1, n_2, \ldots, n_k}) + \sum P_{\text{STA}}^i (T^i_{c} + E_{n_1, n_2, \ldots, n_k}) + \sum P_{\text{STA}}^i (T^i_{c} + E_{n_1, n_2, \ldots, n_k}) + \sum (P_{\text{STA}}^i (T^i_{c} + E_{n_1, n_2, \ldots, n_k})).
\]

In the above expression (3), \(P_{\text{idle}}\) is the probability of the slot being idle, \(P_{\text{STA}}^i\) is the probability that the AP wins the contention and transmits the data packet at rate \(r_i\) and \(P_{\text{STA}}^i\) is the probability that a STA associated at rate \(r_i\) wins the channel ("\(s\)" in the suffix stands for "success"). Correspondingly, the conditional expected sojourn times in state \((n_1, n_k)\), given the events are, respectively, \((\delta + E_{n_1, n_2, \ldots, n_k})\), \(T_{\text{AP}}^i\) and \(T_{\text{STA}}^i\). Detailed expressions for these quantities are provided in the Appendix.

The 3rd and 5th terms on the right side of (3) correspond to collision events. The third term in (3) arises when the AP transmits a TCP data packet to a station at rate \(r_i\) and some other stations are involved in a collision; in other words, the third term captures the situations in which the AP is involved in a collision. The fifth term in (3) captures collision events in which the AP is not involved; we have a STA transmitting a TCP ACK packet to the AP at rate \(r_i\) and one or more other STAs transmitting simultaneously. The various probabilities have been obtained by using the attempt probability \(\beta_{n+1}\), when there are \((N+1)\) contending nodes. From Eq. (3) we have \(E_{n_1, n_2, \ldots, n_k} = \frac{P_{\text{idle}} + \sum P_{\text{STA}}^i (T_{c} + E_{n_1, n_2, \ldots, n_k})}{1 - P_{\text{idle}} - \sum P_{\text{STA}}^i - \sum P_{\text{STA}}^i - \sum P_{\text{STA}}^i} + \sum P_{\text{STA}}^i (T_{c} + E_{n_1, n_2, \ldots, n_k})} + \sum P_{\text{STA}}^i (T_{c} + E_{n_1, n_2, \ldots, n_k}).
\]

We are interested in finding the long run time average of successful transmissions from the AP. We obtain this by applying the Renewal Reward Theorem of Wolff [17]. To get the mean renewal cycle length, we can use the mean sojourn time given in Eq. (4) and use Theorem 5.3 in [3]. The mean reward in a cycle can be obtained as follows. A reward of 1 is earned when the AP transmits a TCP packet successfully by winning the channel. The probability of the AP winning the channel is \(\frac{1}{1+n_1+n_2+\ldots+n_k+1}\). Similarly, a reward of 0 is earned with probability \(\frac{1}{n_1+n_2+\ldots+n_k+1}\). Therefore, the expected reward is

\[
\Phi_{\text{AP-TCP}} = \frac{\sum \sum \sum \sum \pi(n_1, \ldots, n_k) \frac{1}{n_1+n_2+\ldots+n_k+1}}{\sum \sum \sum \sum \pi(n_1, \ldots, n_k) E_{n_1, n_2, \ldots, n_k}}.
\]

4. Evaluation

To verify the accuracy of the model, we performed experiments using the Qualnet 4.5 network simulator [18]. We considered 802.11b physical data rates: 1 Mbps, 2 Mbps, 5.5 Mbps and 11 Mbps; higher rates correspond to smaller distances between the STAs and the AP. In Table 1, results are given for a few cases of this multirate scenario. For example, the first row considers a total of 10 STAs associated with the AP, out of which 2 STAs are associated at 11 Mbps and 2 Mbps respectively, while 3 STAs are associated at 5.5 Mbps and 1 Mbps, respectively. The values of \(p_i\) in Eq. (2) are calculated by using the number of STAs associated with the AP. For example, in the first row in Table 1, \(p_1\) (for 11 Mbps) is \(\frac{2}{10}\).

In 802.11g, the different possible data rates are 54, 48, 36, 24, 18, 12 and 6 Mbits/s. Qualnet 4.5 is configured to this mode by setting the channel frequency for the 802.11a radio as 2.4 GHz. In Table 2, comparisons between analytical and simulation values are given.

In Tables 3 and 4 comparisons between analytical and simulation values are given for TCP with delayed ACKs. We observe that in all cases, the analytical results are in excellent agreement with simulations.
Table 2
Throughput (Mbps) of multirate AP by analysis and simulation for IEEE 802.11g.

| M | No. of STAs at rate (Mbps) | Aggregate throughput (Mbps) |
|---|--------------------------|-----------------------------|
|   | 54 48 36 24 18 6         | Analysis | Simulation | Error% |
| 15| 1 2 3 4 2 3             | 8.14    | 8.18         | 0.49   |
|   | 2 1 3 4 2 3             | 8.16    | 8.20         | 0.48   |
|   | 3 2 1 4 2 3             | 8.31    | 8.32         | 0.12   |
|   | 4 3 2 1 3 2             | 10.22   | 10.25        | 0.29   |
|   | 3 2 4 3 1 2             | 10.38   | 10.41        | 0.29   |
|   | 3 2 4 3 2 1             | 12.27   | 12.31        | 0.24   |

Table 3
Analysis and simulation results (Mbps) for multirate AP in IEEE 802.11b. Delayed ACKs are implemented at the receiver with alternate segment being acknowledged ($d = 2$).

| M | No. of STAs at rate (Mbps) | Aggregate throughput (Mbps) |
|---|--------------------------|-----------------------------|
|   | 11 5.5 2 1              | Analysis | Simulation | Error% |
| 10| 2 3 2 3                | 1.1221  | 1.1131      | 0.80   |
|   | 1 2 3 4                | 0.8889  | 0.8814      | 0.81   |
| 12| 2 2 4 4                | 0.9715  | 0.9617      | 0.81   |
|   | 4 4 2 2                | 1.5647  | 1.5523      | 0.80   |

Table 4
Throughput (Mbps) of multirate AP by analysis and simulation for IEEE 802.11g. Delayed ACKs are implemented at the receiver with alternate segment being acknowledged ($d = 2$).

| M | No. of STAs with at (Mbps) | Aggregate throughput (Mbps) |
|---|--------------------------|-----------------------------|
|   | 54 48 36 24 18 6         | Analysis | Simulation | Error% |
| 15| 1 2 3 4 2 3             | 8.14    | 8.18         | 0.49   |
|   | 2 1 3 4 2 3             | 8.16    | 8.20         | 0.48   |
|   | 3 2 1 4 2 3             | 8.31    | 8.32         | 0.12   |
|   | 4 3 2 1 3 2             | 10.22   | 10.25        | 0.29   |
|   | 3 2 4 3 1 2             | 10.38   | 10.41        | 0.29   |
|   | 3 2 4 3 2 1             | 12.27   | 12.31        | 0.24   |

5. Discussion

In this work, we presented an analytical model to obtain the aggregate throughput when several TCP-controlled long file downloads are going on. Now let us consider simultaneous TCP uploads and downloads. The attempt behavior of nodes is independent of the packet length. If we interchange downlink data packets sent by the AP with ACK packets and uplink ACK packets sent by stations with the TCP data packets, the same analysis holds good for the TCP-controlled file uploads.

Another case arises when some stations are uploading and some are downloading long files. Here also our basic Markov model for the number of stations with packets to send remains the same, if all the TCP windows are equal. Even different window sizes can be taken care of by this approach. Some of these extensions have been analyzed in [16].

In our simulation and numerical evaluation, we used the 802.11b and 802.11g standards. However, our mathematical expressions are independent of these standards, and hence the model can be applied to any other standard that has different number of physical data rates.

6. Conclusion

In this work, we have presented a simple analytical model for the aggregate throughput for TCP-controlled long file transfers in a “multirate” AP. We verified the correctness of the analytical model with the simulation results. As future work, we plan to consider short file transfers. This can be used to estimate the delay seen by stations. Further, association schemes can be built upon this.

Appendix

Expressions for probabilities and times discussed in Section 3.2

\[ P_{\text{idle}} = (1 - \beta_{N+1}) W + 1 \]

\[ P_{\text{AP}}^{(i)} = p_i \beta_{N+1} (1 - \beta_{N+1})^W. \]
To verify that the sum of all the probabilities is 1:

$$P_{\text{idle}} + \sum_{i=1}^{k} P_{\text{AP}}^{i} + \sum_{i=1}^{k} P_{c_{\text{STA}}}^{i} + \sum_{i=1}^{k} P_{\text{cSTA}}^{i}$$

$$= (1 - \beta_{N+1})^{N+1} + \beta_{N+1}(1 - \beta_{N+1})^{N} + N\beta_{N+1}(1 - \beta_{N+1})^{N}$$

$$+ \beta_{N+1}(1 - (1 - \beta_{N+1})^{N}) + \sum_{i=1}^{k} (1 - \beta_{N+1})^{n_{i}+\cdots+n_{k}+1} - \sum_{i=1}^{k} (1 - \beta_{N+1})^{n_{i}+\cdots+n_{k}+1} - \sum_{i=1}^{k} n_{i}\beta_{N+1}(1 - \beta_{N+1})^{N}$$

$$= (1 - \beta_{N+1})^{N+1} + \beta_{N+1}(1 + N(1 - \beta_{N+1})^{N}) + (1 - \beta_{N+1})^{N+1} - (1 - \beta_{N+1})^{N+1} - N\beta_{N+1}(1 - \beta_{N+1})^{N+1}$$

$$= 1.$$  

From the above verification, it is clear that all possibilities events have been considered in Eq. (3).

$$T_{i}^{c}$$ is the collision duration when the AP and STAs at rate $r_{i}$ are involved.

$$= T_{p} + T_{\text{PHY}} + \frac{L_{\text{MAC}+\text{PHY}+\text{TCP-ACK}}}{c_{r_{i}}} + T_{\text{EIFS}}$$

$$T_{i}^{\text{AP}}$$ is the time taken by AP to send a packet to an STA at rate $r_{i}$

$$= T_{p} + T_{\text{PHY}} + \frac{L_{\text{TCP-ACK}}}{c_{r_{i}}} + T_{\text{SIFS}} + T_{p} + T_{\text{PHY}} + \frac{L_{\text{MAC}+\text{PHY}+\text{TCP-ACK}}}{c_{r_{i}}} + T_{\text{SIFS}} + T_{p} + T_{\text{PHY}} + \frac{L_{\text{ACK}}}{c_{r_{i}}} + T_{\text{DIFS}}$$

$$T_{i}^{\text{STA}}$$ is the time required to transmit one TCP-ACK packet from an STA at rate $r_{i}$, including overhead

$$= T_{p} + T_{\text{PHY}} + L_{\text{MAC}} + \frac{L_{\text{TCP-ACK}}}{c_{r_{i}}} + T_{\text{SIFS}} + T_{p} + T_{\text{PHY}} + \frac{L_{\text{ACK}}}{c_{r_{i}}} + T_{\text{DIFS}}$$

$$T_{i}^{c_{\text{STA}}}$$ is the collision duration of STAs at rate $r_{i}$

$$= T_{p} + T_{\text{PHY}} + \frac{L_{\text{MAC}+\text{PHY}+\text{TCP-ACK}}}{c_{r_{i}}} + T_{\text{EIFS}}.$$
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