The determination of friction coefficient in Nakazima test and its application in predicting FLCs with modified M-K model

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Abstract. In this work, the friction coefficient between the punch and the specimen during the Nakazima tests is determined by comparing the experimental and the numerical strain paths. Afterwards, the FLCs of the Al-Mg-Li alloy sheet are predicted with the modified M-K model, which considers the through-thickness normal stress and the friction through introducing the expressions of the normal stress and the friction stress into M-K model. Finally, the predicted FLCs of the sheet are compared with the forming limits determined by Nakazima tests.

1. Introduction
The Al-Mg-Li alloy sheet has obtained wide application in many fields such as aircraft for its excellent properties of low density, high specific stiffness and high specific strength [1-3]. Forming limit curves (FLCs) are important methods to judge the formability of sheet metals and it is specified in ISO 12004-2 standard that the FLCs of sheet metal can be determined by the Nakazima test. However, the experimental procedure to obtain FLCs is complicated and time-consuming, because large quantities of specimens with a wide range of sizes are needed to take various strain paths into account [4]. Therefore, several theoretical models such as the MMFC model [5] and the M-K model [6] have been proposed based on the deformation theory to calculate the forming limits of sheet metal. Therein, the M-K model which is established based on the groove theory has been widely applied to predict the forming limits of different sheet metals such as AA5086 [7], Ti6Al4V alloy [8] and Qste600TM steel [9] for its simplicity in calculation process and high prediction accuracy.

Although the M-K model has been used in many literatures to predict the forming limits of different sheet metals, there are still differences between the predicted FLCs and the experimental forming limits. As a result, scholars have made many resourceful improvements on the original M-K model to improve the prediction accuracy of the FLCs. In the original M-K model, the stress state of the specimen is supposed to be planar, but the sheet during the Nakazima test is under through-thickness normal stress, which has been proved to affect the FLCs of sheet metal [10]. Given the ratio of the normal stress to the major stress, or simplifying the normal stress in the deformation process to be constant is one method to consider the through-thickness normal stress in the M-K model [11, 12]. Besides, Wang et al. [13] established the expression of normal stress through analyzing the stress state in the Nakazima test and introduced it into the M-K model. During the Nakazima test, the friction between the punch and the specimen is inevitable and it has been identified that the friction has influence on the strain paths and the FLCs of sheet metal [14]. Thus, it should be considered in the M-K model to improve the accuracy of FLCs predicted by M-K model. Based on the experimental and...
numerical investigation for the friction influence on the FLCs [15, 16], Wang et al. [13] established the friction stress expression and introduced it into the M-K model. Results showed that this modified M-K model where the expressions of the normal stress and the friction stress were introduced could improve the prediction accuracy of FLCs for the IF and DP600 steels. However, the friction coefficient in the friction stress expression which denotes the lubrication condition between the specimen and the tooling during Nakazima tests is difficult to directly determine through experiments.

In this work, the friction coefficient is determined by comparing the strain paths of specimens which are respectively obtained by the finite element simulation method and the Nakazima tests. Furthermore, the FLCs of the Al-Mg-Li alloy sheet are predicted by the modified M-K model and compared with the experimental forming limits.

2. The through-thickness normal stress and friction stress expressions
During Nakazima test process, the fracture of specimen does not always occur at the apex of the dome because of the friction between the specimen and the punch. Thus, as shown in figure 1, one small force body near the apex of the dome is chosen, where the fracture is assumed to occur.

![Figure 1. The location and the stress state of the force body in the Nakazima test [13].](image)

The force body is assumed to be small enough that the increments of radians $d\theta$ and $d\phi$ are infinitesimal. Based on this assumption and considering the stress gradient along the thickness direction, the force equilibrium equation is established as shown in equation (1) and the through-thickness normal stress expression is established in the form of equation (2).

$$\frac{d\tau_{\phi}}{d\phi} \sin \theta_R = 2\sigma_0 \sin \theta_R \frac{d\theta}{2} = 2\sigma_\phi S_\phi \sin \frac{d\theta}{2},$$  \hspace{1cm} (1)

$$\sigma_3 = -\frac{2\tau}{R} \left(\sigma_1 + P\sigma_2\right),$$ \hspace{1cm} (2)

where the parameter $P$ represents the position of sheet failure in the deformation process and it is associated with the geometry around the apex. When the fracture occurs within a distance less than 15% of the punch diameter away from the apex of the dome, the $\phi$ value is close to $\pi$, so the value of $P$ could be set to 1 in the computational process [13].

Based on the analysis of the contact condition between the punch and the specimen during Nakazima tests, the friction between the sheet and the punch is usually considered as the Coulomb friction and the friction force can be defined by

$$f = \mu F,$$ \hspace{1cm} (3)

where $f$ is the friction force, $\mu$ is the friction coefficient and $F$ is the compression force on the contact surface, respectively. The friction force on the contact surface of the force body is shear force, but it is difficult to be directly considered in the M-K model. In order to take into account the effect of friction on the forming limits in the M-K model, the symbol $\sigma_f$ is defined as the friction stress along the principal stress $\sigma_1$ direction by means of equaling the friction force $f$ on the internal surface to the increment along the principal stress [13]. The defined friction stress $\sigma_f$ is determined by

$$\sigma_f = 2\mu d\theta (\sigma_1 + P\sigma_2).$$ \hspace{1cm} (4)
3. The determination of friction coefficient between the punch and the specimen

In this section, the friction coefficient between the punch and the tooling in the Nakazima test is determined by comparing the strain paths of samples, which are obtained by the experiment and the finite element simulation method, respectively.

Based on the EC600 sheet test platform and the ARAMIS non-contact strain measurement system, the Nakazima tests are performed with the samples of two sizes which are named R60 and R0 as shown in figure 2. According to our previous study, the R60 and R0 specimens are under tensile-compressed and tensile-tensile stress states during the Nakazima test, which indicates that the strain paths on the left and the right sides of forming limit diagram (FLD) are described, respectively. In the Nakazima test, a composite lubrication of grease and 0.05mm thick polytetrafluoroethylene (PTFE) film is used. The ARAMIS non-contact strain measurement system is used to continuously record the deformation state of the samples during experiment process.

![Figure 2. Dimensions of the specimens for Nakazima tests.](image)

Afterwards, with the ABAQUS finite element software, the numerical simulation of the Nakazima test process is performed, which includes a spherical punch, a die and a specimen as shown in figure 3. In order to shorten the calculation time, a 1/4 model is used for the simulation considering the axial symmetry of the sample in the Nakazima test, and the blank holding force on the specimen is applied by dividing the specimen surface instead of defining the blank holder parts. In the finite element simulation, the boundary conditions and the load settings are consistent with the experiment to obtain more accurate simulation results. In the Nakazima test, there are two kinds of friction contacts between the sheet and the tooling, one is between the punch and the sheet, the other is between the sheet and the die. Here, for simplification, the same value is defined for these two kinds of friction coefficients. In the simulation, three different friction coefficients of 0, 0.1 and 0.2 are used, respectively.

![Figure 3. The finite element simulation model of Nakazima test.](image)

The strain paths at the fracture locations of samples are shown in figure 4, which are respectively determined by the experiment and the finite element simulation with different friction coefficients of 0, 0.1 and 0.2. It can be found from the comparison between simulated and experimental strain paths that the experimental strain paths of the R0 and R60 samples are between the simulated strain paths obtained at the friction coefficient of 0 and 0.1. As a result, it can be concluded that the friction coefficient on the contact surface of the specimen and the tooling is between 0 and 0.1 when the composite lubrication of PTFE film and grease is applied in the Nakazima tests. Furthermore, the friction coefficient in the friction stress expression should be set according to the comparison results when the modified M-K model is applied to improve the prediction accuracy of FLCs.
4. The predicted FLCs of Al-Mg-Li alloy sheet

With the modified M-K model and the determined friction coefficient, the FLCs of the Al-Mg-Li alloy sheet are predicted and compared with the experimental results of Nakazima tests in this section.

The modified M-K model is established by introducing the expressions of normal stress and friction stress into the original M-K model as shown in figure 5. The Al-Mg-Li alloy sheet has the thickness of 1.8 mm and table 1 shows its composition. Moreover, the Swift constitutive model as shown in equation (5) and the Hill’s 48 yield criterion as shown in equation (6) are applied to respectively describe the stress-strain relationship and the anisotropy of the sheet, where the material parameters have been obtained in our previous work as listed in table 2.

\[
\bar{\sigma} = K(\bar{\varepsilon}_0 + \varepsilon)^n, \\
\bar{\sigma} = \sqrt{F\sigma_2^2 + G\sigma_1^2 + H(\sigma_1 - \sigma_2)^2 + 2N\sigma_1^2},
\]

where K is hardening coefficient, n is hardening index, F, G, H and N are anisotropy coefficients.

| Element | Mg | Li | Zr | Fe | Si | Cu | Ti | Al |
|---------|----|----|----|----|----|----|----|----|
| wt %    | 5.22 | 2.19 | 0.10 | 0.04 | 0.02 | 0.01 | 0.01 | Bal |

Finally, the comparison between the predicted FLCs of modified M-K model and the experimental results are shown in figure 6. It can be found that the modified M-K model can accurately predicted the FLCs of the Al-Mg-Li alloy sheet at the left side and under small strain ratios. When the strain
path increases, the difference between the predicted FLCs and the experimental forming limits becomes significant. The reason that causes this problem may be the material parameters in the constitutive model, which will be analyzed in the next work.

![FLC comparison](image)

**Figure 6.** The predicted FLCs and the experimental forming limits of the Al-Mg-Li alloy sheet.

5. Conclusions

In this work, the friction coefficient in the friction stress expression is determined between 0 and 0.1 when the composite lubrication of PTFE film and grease is applied in the Nakazima tests. Furthermore, the forming limits of the Al-Mg-Li alloy sheet are predicted by the modified M-K model and compared with the experimental results of the Nakazima tests. The comparison shows that the FLCs of the Al-Mg-Li alloy sheet are basically accurately predicted.

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