Design, fabrication and modeling analysis of a spiral support structure with superelastic Ni-Ti shape memory alloy for continuum robot

Jiawen Tian\(^1,3\), Tianmiao Wang\(^1,3\), Xi Fang\(^2,3\) and Zhenyun Shi\(^3,4\)

\(^1\) Intelligent Technology and Robotics Lab, Beihang University, Beijing, People’s Republic of China
\(^2\) Biomechanics and Soft Robotics Lab, Beihang University, Beijing, People’s Republic of China
\(^3\) School of Mechanical Engineering and Automation, Beihang University, Beijing, People’s Republic of China

E-mail: tianjiawen@buaa.edu.cn, itm@buaa.edu.cn, fangxi@buaa.edu.cn and shichong1983623@hotmail.com

Received 26 May 2019, revised 12 December 2019
Accepted for publication 4 February 2020
Published 25 February 2020

Abstract

In order to improve the working ability of continuum robots in confined space, a novel central support structure with the spiral cutting pattern is proposed in this paper. The spiral hollow scheme was selected through analysing different cutting patterns of the thin-walled superelastic Ni-Ti shape memory alloy (SMA) tube. Then the laser cutting process and heat treatment for fabricating the structure were analysed. The analysis results of their impact on the material characteristics proves that the laser cutting and heat treatment ensure the performance of the material. The single-segment and multi-segment prototypes both showed prospective motion ability in the flexural rigidity and continuous deformation tests respectively. Then the flexural rigidity model of the spiral structure was proposed, which was proved effective by comparing the model simulation and finite element analysis results. Furthermore, the central support structures with different flexural rigidities can be customized, which has broad application prospects in continuum robots.

Supplementary material for this article is available online

Keywords: superelastic shape memory alloy, flexural rigidity model, continuum robot

(Some figures may appear in colour only in the online journal)

1. Introduction

Compared with traditional robots, continuum robots have many advantages, such as flexible bending with large curvature. It can work in confined space such as pipes, with strong adaptability in multi-barrier and non-structural environments. The whole body of continuum robots can be directly used as an actuator to complete the grasping work. These advantages enable continuum robots to have a wide range of applications, such as industrial inspection and repair [1, 2], minimally invasive abdominal surgery [3, 4], anti-terrorism rescue [5] and military. Because of the continuous deformation ability of continuum robots, it can mimic biological morphology in terms of bionic structures [6, 7] and bionic movements [8–10].

The existed continuum robots can be roughly classified into three types based on their mechanical structures: (1) Modular hinge-joint structure [11, 12], (2) Circumferential structure with flexible support [13–15], (3) The central support
structure. Modular hinge-joint structure is usually complicated and has a large mass, resulting in that it requires a large driving force to achieve effective continuous motion. So continuum robots with modular hinge-joint structure are not suitable in a narrow space. Circumferential structure usually has low motion control accuracy, low load capacity and limited self-restoring performance due to the nonlinear characteristics of silicon. The flexural rigidity of central support structure is difficult to control. Insufficient flexural rigidity results in difficulty to maintain the shape of the robot. And excess flexural rigidity will reduce the motion performance of the robot. In addition, the support structure also requires light weight, good fatigue resistance and self-restoring ability. To our knowledge, until now there is no perfect solution. Compared to the other two support structures, the central support structure is more appropriate for design of continuum robots woking in confined space. These robots usually have high motion control accuracy with small size and light weight, and have broad application prospects in medical fields.

The central support structure was adopted for continuum robots in a few typical examples. Choi et al of Hanyang University developed a flexible endoscopic robot supported by an elastic skeleton structure [16]. The endoscopic robot is supported by a spring bracket that functions to maintain the shape of the robot and provide flexural rigidity for the bending motion. Alambeigi et al employed a central support structure composed of silicone tube and spring, and the flexural rigidity can be changed by heating low melting-point alloy [17]. In the EU-funded project STIFF-FLOP [18, 19], the soft manipulator module has a central stiffening channel and three pneumatic channels inside for actuation. Junius Santoso et al designed a multiple degree-of-freedom soft pneumatic actuator with singel chamber which consists of silicon resin and shape memory polymer, and flexural rigidity of the central support structure is variable [20]. In addition, Huaming Wang et al developed a rolled dielectric elastomer (DE) actuator with the central support structure, which is fabricated with springs and DE material [21]. The continuum robots mentioned above usually adopt complex central support structures, which are made up of materials with different flexural rigidity. If single material is utilized for the central support structure, the flexural rigidity of the material should be low in order to achieve a better motion performance for the robot. And if superalistic Ni-Ti SMA is adopted for the central support structure, the flexural rigidity of the structure would reduce with the deformation increasing, which enables smaller driving force and enough flexural rigidity to maintain the shape of the robot after unloading the driving force [22, 23].

In this paper, a novel hollow spiral support structure using Ni-Ti superelastic SMA is proposed. In section 2, the structure design and fabrication process of the central support structure for continuum robots are illustrated in detail. In section 3, fabrication analysis and performance test of the support structure were carried out. The results represent that the hollow spiral support structure can provide sufficient flexural rigidity and the multi-segment prototype achieved continuous large deformation, which is suitable for design of continuum robots. In section 4, the flexural rigidity model of the support structure was proposed. By comparison of the flexural rigidity model simulation and finite element analysis of the spiral structure, the flexural rigidity model is reasonable. Furthermore, using the structure design method and flexural rigidity model proposed in this paper, the central support structures with different flexural rigidity can be customized.

2. Design and fabrication of the support structure

In order to realize the operation of the robot in a narrow space, such as endoscopic detection in the field of medical robots, a continuum robot with a central support structure is a feasible design, and the radial dimension of the robot should be less than 30 mm, which is applicable in the endoscopic detection and other application scenarios. However, the material selection and structural design of the central support structure can greatly affect the motion performance of the robot. For this reason, this study attempts to establish a general design method to provide a reference for the design of the support structure for continuum robots.

2.1. Material selection

The central support structure fabricated with flexible material has low flexural rigidity, thus requires external auxiliary structure to increase the flexural rigidity of the entire robot. At the same time, its self-restoring ability is limited, and it cannot recover to the original state after some repeated movements, so the control accuracy of the robot is usually low. If common metal materials are used for the design of the central support structure, the rigidity of the structure is usually large. But the fatigue resistance under large deformation is low, and fatigue fracture is likely to occur.

To meet the above requirements, we have considered using SMA materials for the central support structure. Ni-Ti SMA is one of the typical SMA material, which is currently widely studied. Because of its superelasticity in the austenite phase state, Ni-Ti SMA can generate large deflection deformation when the driving force is applied, and can restore to the initial state when the driving force is unloaded. In addition, its fatigue resistance is also very good. Ni-Ti SMA material can maintain the austenite phase at a temperature higher than 20 °C after heat treatment, which meets general application requirements. Because of the low density of the Ni-Ti SMA material, the mass of the robot can be effectively reduced, thereby reducing the impact of gravity on the structure. But neither the Ni-Ti SMA wire nor the Ni-Ti SMA stick can provide the flexural rigidity required for the movement of continuum robots, so the thin-walled Ni-Ti SMA tube becomes the focus of our study.

2.2. Design and comparison of the hollow schemes

To meet the application requirements of continuum robots in narrow space such as industrial endoscope detection, the overall radius of the robot is usually less than 30 mm, so in this situation the outer diameter of the central support structure is usually about 10 mm.
Through the finite element analysis of thin-walled Ni-Ti tube with an inner diameter of 10 mm, a length of 100 mm and different wall thicknesses, we found that when the wall thickness is 0.1 mm, the axial strain is about 0.158% when 10 N axial pressure is applied. And the thin-walled Ni-Ti tube cannot achieve continuous and effective large deformation because of the extremely large flexural rigidity. At the same time, for conventional Ni-Ti SMA thin-walled tube, the processing limitation of the wall thickness is about 0.15 mm. Considering the workability, a thin-walled Ni-Ti SMA tube with a wall thickness $t$ of 0.2 mm and an inner diameter $d$ of 10 mm are selected as the substrate. And the Ni-Ti SMA tube needs to be further processed for the application in continuum robots.

In order to reduce the flexural rigidity of the tube while preserving the superelasticity of the material, hollowing treatment is a feasible way. When the tube is hollowed out, the extrusion deformation of the material only exists at a few joints when the bending moment is applied, which enables the tube to perform continuous bending effectively. For this purpose, this study explores three common hollow schemes.

The Ni-Ti SMA support structures adopting the three schemes are composed of three parts. There are two non-hollowed parts with a height of $h$ ($h = 6$ mm) at both ends of the support structure for installation. The hollowed part is between the two non-hollowed parts, and the diameter of the hollowed part is $D$ ($D = d + 2t$). The length of the three structures can be customized by changing the number of elements in array of the cutting pattern.

The structure in figure 1 adopts a spindle-apparatus cutting pattern. The radial angle between two adjacent cuttings is $90^\circ$ and the axial distribution is staggered. The radius of the spindle apparatus $R_s$, the length of the spindle apparatus $L_s$, the array pitch $P_a$ and the number of elements in array $n_a$ are demonstrated in table 1. Note that the length of the entire support structure is $L_1 (L_1 = 2 \cdot h + (n_a - 1) \cdot P_a + L_s)$ and the material removal rate is 70.2%.

The structure in figure 2 adopts a peanut cutting pattern, the array distribution of which is the same as that of figure 1. The radius of the arc on middle dent of the peanut $R_{p2}$, the center distance of the arcs on both sides of the peanut $d_p$, the array pitch $P_a$ and the number of elements in array $n_a$ are demonstrated in table 2. Note that the length of the entire support structure is $L_2 (L_2 = 2 \cdot h + n_a \cdot P_a)$ and the material removal rate is 55.5%.

The structure in figure 3 adopts a spiral cutting pattern with four-start thread. The axial pitch $p$, the effective number of turns $n$, the thread cutting angle $\alpha$ and the number of thread starts $s$ are demonstrated in table 3. Note that the length of the entire support structure is $L_3 (L_3 = 2 \cdot h + n \cdot p)$ and the material removal rate is 50%.

When the support structures are applied to continuum robots, the flexural rigidity, stress concentration and fatigue resistance should be considered. A suitable flexural rigidity enables the continuum robots to achieve large deformation. The robot will suffer from stress fracture if the maximum stress exceeds allowable stress. And the fatigue resistance makes the central support structure of the continuum robot durable. To this end, we conducted finite element analysis of the three structures in the software ANSYS 18.2 (ANSYS, Inc., USA). The settings for the finite element analysis are as follows. The shape of the mesh is tetrahedron. The mesh size is self adaptive according to the shape and size of the model in the software. The relevance center is coarse. The max face size, mesh defeature size, growth rate and max tet size are default.

A fixed constraint was applied to the bottom of the support structures, and a pushing force perpendicular to the central axis was applied to the point which is 2 mm upon the boundary of the hollowed part. The lengths of the three structures are separately 50 mm, 52 mm and 52 mm. In order to directly show the flexural rigidity of the three structures, after several trials in ANSYS, the horizontal pressure is chosen as 10 N for the spindle-apparatus structure and the peanut structure, and the horizontal pressure is chosen as 0.5 N for the spiral structure.
Density of the material is defined as 6.98 g cm$^{-3}$. In our ANSYS trials, the strain should be no more than 2.14%, and the material can maintain the austenite phase during the working process. When the strain is within the range of 10%, Young’s modulus of the superelastic Ni-Ti SMA will decrease by 1.5% with the strain increasing by 0.5% approximatively, the variation of Young’s modulus can be no more than 6% in the analysis. And for the spiral structure, the variation of Young’s modulus can be no more than 1.5% [24–26]. Young’s modulus of the superelastic Ni-Ti SMA is regarded as constant. According to test of the material, Young’s modulus of the material is 70 GPa and Poisson’s ratio is 0.33.

The analysis results are shown in figure 4. It can be seen from figure 4(a) that when 10 N pressure is applied to the structure, the maximum deformation is only 2.24 mm, thus the flexural rigidity is too large to meet the motion requirements of large deformation for continuum robots. Comparing figure 4(d) with figure 4(g), it can be found that when the pressure applied to the peanut structure is 20 times that of the spiral structure, the deformation of the peanut structure is still slightly smaller than that of the spiral structure. Comparing figure 4(f) with figure 4(i), when the maximum deformations of the peanut structure and the spiral structure are alike, it can be found that the peak stress of the peanut structure is 1484 MPa and the peak stress of the spiral structure is only 306.53 MPa. Furthermore, the stress of the peanut structure concentrated on the root of the hollowed part, while the stress of the spiral structure is more dispersed, which ensures that the structure won’t fracture due to stress concentration. The continuum robots with central support structures are usually cable-driven. To ensure that the outer diameter of the robot does not exceed 30 mm, the maximum arm of force of the cable is 15 mm. In order to reach the deformations in figure 4, for the peanut structure, the moment of driving force should be more than $F_n P_0$, so the pulling force of the cable should be no less than 28 N, which requires a higher output force of the driving device. While the required pulling force of the cable in the spiral structure is only 1.4 N.

The comparison of flexural rigidity, stress and other related mechanical properties for the three structures are described in table 4. Considering the finite element analysis results in figure 4 and the comparison results in table 4, we believe that the spiral structure is an ideal choice, for which we have conducted modeling analysis.

### 2.3. Multi-segment support structure

In order to explore the continuous deformation ability of the support structure, a multi-segment prototype was designed. As shown in figure 5, the model is composed of four series segments of spirally interleaved hollow structures. There are a 6 mm non-hollow part between two adjacent hollow structures and a 10 mm non-hollowed parts at each end of the

---

**Table 1. Structural parameters of the spindle-apparatus structure.**

| Parameter                                    | Value |
|----------------------------------------------|-------|
| Radius of the spindle apparatus (mm) $R_s$   | 12    |
| Array pitch (mm) $P_a$                       | 20    |
| Total length (mm) $L_t$                      | 50    |
| Length of the spindle apparatus (mm) $L_s$   | 18    |
| Number of elements in array $n_a$            | 2     |

**Table 2. Structural parameters of the peanut structure.**

| Parameter                                    | Value |
|----------------------------------------------|-------|
| Radius of the arc on both sides (mm) $R_{t1}$| 2.25  |
| Radius of the arc on middle dent (mm) $R_{t2}$| 8     |
| Center distance of the arcs on both sides (mm) $d_p$ | 5     |
| Array pitch (mm) $P_a$                       | 10    |
| Number of elements in array $n_a$            | 4     |
| Total length (mm) $L_3$                      | 52    |

**Table 3. Structural parameters of the spiral structure.**

| Parameter                                    | Value |
|----------------------------------------------|-------|
| Axial pitch (mm) $p$                         | 20    |
| Effective number of turns $n$                | 2     |
| Cutting angle $\alpha$                       | $\pi/4$|
| Number of thread starts $s$                  | 4     |
| Total length (mm) $L_3$                      | 52    |
whole tube for installation. The axial pitch is 20 mm and the effective number of turns is two. And the total length of the multi-segment prototype is 198 mm. The finite element analysis and modeling in the following parts are for the single segment structure.

2.4. Fabrication of the support structure

The Ni-Ti SMA tube is fabricated by drilling a solid Ni-Ti bar and then implementing the cold/hot drawing procedure with a mandrel. The Ni-Ti SMA material with 56.06% mass fraction of Ni was used. The wall thickness of the processed thin-walled tube is 0.20 mm, the error of which is ±0.05 mm. And the tube is supplied by the Beijing Gee Inc..

In order to increase the workability and meet the processing accuracy requirement, laser cutting was adopted during the hollow procedure of the Ni-Ti SMA tube with spiral structure.

The IPG fiber laser is employed for laser cutting and the laser is a continuous type. The laser is modulated to the pulse mode. And the parameters of the IPG fiber laser are demonstrated in table 5.
Finally, the motion performance of the prototype was tested. The sample was studied to explore the allowable working conditions. The topography and phase transition temperature of the heat treated support structure can be found in section 2.2.

3. Fabrication analysis and performance test of the heat affected zone

During the hollow process, the laser was fixed and remained perpendicular to the processed surface. The rotated Ni-Ti SMA tube was fed by the machine tool. The water cooling device was used to cool the processed surface during the hollow process to reduce the HAZ area. The cooling water temperature is 25 °C. After the laser cutting process, the Ni-Ti SMA hollowed tube was kept in the thermostat at 450 °C for 30 min to implement the annealing treatment [27].

To fabricate single segment and multi-segment prototypes with the spiral structure, the structural parameters can be found in section 2.2.

3.1. Analysis of laser cutting impact on the material characteristics

HAZ on the lancing side of the material and the recast layer formed during the hot working process will directly affect the strength and fatigue resistance of the material, and also destroy the superelasticity [28]. In order to maximize the performance of the Ni-Ti SMA tube with spiral structure, the HAZ must be controlled within a tolerable range, that is, the width of the HAZ is within 15 μm [29, 30].

The x-ray energy spectrum analysis of the non-heat treated sample is shown in figure 6. In the cutting section, a sampling line with a length of 30 μm perpendicular to the cutting surface was taken for analysis, as shown in figure 6(a). As can be seen from figure 6(c), the HAZ depth of the non-heat treated sample is 15 μm. For the Ni-Ti SMA tube with spiral structure, the volume of the HAZ accounts for less than 0.96% of the whole volume of the tube, which does not have a great influence on the superelasticity and other properties of the material. The x-ray energy spectrum analysis of the heat treated sample is shown in figure 7. In the cutting section, a sampling line with a length of 22 μm perpendicular to the cutting surface was taken for analysis, as shown in figure 7(a). As can be seen in figure 7(c), the HAZ depth of heat treated sample is 10 μm. For the Ni-Ti SMA tube with spiral structure, the volume of the HAZ accounts for less than 0.65% of the whole volume of the tube. This reveals that heat treatment effectively reduces the HAZ depth and further ensures that the HAZ of laser cutting won’t have much influence on the superelasticity and other properties of the prototype, which guarantees the reliability of the material.

3.2. Surface topography analysis

Scanning electron microscopy (SEM) observations of the cutting sections of the sample after heat treatment and the unheated sample are shown in figure 8. As can be seen from figures 8(a), (b), the surface roughness of the cutting section of the sample after heat treatment is low, which is within allowable range and won’t have much influence on the performance of the prototype. After heat treatment, the surface appearance of the cutting section is smoother compared to the non-heat treated sample, which further ensures that the lancing quality of the prototype.

The surface appearance of the non-heat treated laser cutting sample after acid etching is shown in figure 9(a), and the corrosion resistance of the HAZ is not that good. Compared to figures 8(a), (b), the non-HAZ of the Ni-Ti SMA material almost do not react to acid etching and has strong corrosion resistance. This indicates that for the non-heat treated sample, the corrosion resistance of the non-HAZ is far greater than that of the HAZ. Therefore the non-HAZ of Ni-Ti SMA material has a relatively good corrosion resistance. As mentioned in 3.1.2, the HAZ volume accounts for a very small proportion of the whole volume of the prototype, indicating that the prototype is not susceptible to corrosion damage when working in the corrosive environment.

The surface appearance of the heat treated laser cutting sample after acid etching is shown in figure 9(b). Similar to figure 9(a), the HAZ has a low acid-etched resistance. Compared to figures 8(c), (d), the non-HAZ of the Ni-Ti SMA material is almost unaffected by acid etching. This indicates that for the heat treated sample, the corrosion resistance of the non-HAZ is also far greater than that of the HAZ.

Comparing figure 9(a) with figure 9(b), it can be concluded that the HAZ corrosion resistance of the heat treated...
sample is better than that of the non-heat treated sample, indicating that the heat treatment has improved the HAZ corrosion resistance of the Ni-Ti SMA material, which further ensures that the prototype can work in environments such as medical treatment and chemical industry.

3.3. Analysis of phase transition temperature

In order to ensure that the continuum robot with the prototype in this paper can work well under the room temperature condition (18 °C–26 °C), the supporting structure should be in...
3.4. The flexural rigidity and continuous deformation ability tests

3.4.1. Test setup. To verify the reasonability of the flexural rigidity model, a flexural rigidity experiment for the single segment prototype was carried out to obtain the relationship between the deflection and the pushing force. And to test the continuous deformation ability of the multi-segment prototype, a experiment was conducted to measure the effective bending angle of the four-segment structure.

During the tests, the Ni-Ti SMA prototype was fixed on the platform. The robot arm (MOTOMAN MH3F, YASKAWA Inc., Japan) provided the pushing force input and deflection feedback. A 6-axis force sensor (Mini 40 FT−1 sensor, ATI, USA) was installed at the end of the robot arm to collect the pushing force data. A M5 screw was fixed on the tip of the force sensor, serving as the pushing force output point of the robot arm. During the movement, the robot arm remained at the same horizontal plane. Because the screw has a cylindrical surface which contacts with the prototype during experiments, the direction of the pushing force is always horizontal and perpendicular to the axis of the prototype. After several trials, to keep the contact point fixed, the tip coordinate of the robot arm was adjusted and then recorded. Note that the contact point is 2 mm upon the boundary of the hollowed part.

In the flexural rigidity experiment for the single segment prototype, four points with related deflection ranging from 5 mm to 20 mm approximately with the interval of 5 mm were collected. In the continuous deformation ability test of the multi-segment prototype, the bending angle of the terminal plane of the prototype ranged from 0° to 180°. 20 and 10 data points distributed uniformly within the range from 0° to 90° and from 90° to 180°, separately. The experimental platform is shown in figure 12.

3.4.2. Flexural rigidity test of the single segment support structure. The relationship between the deflection of the force bearing point and the pushing force is shown in figure 13. As the deflection changing from 0 mm to 20.724 mm with an interval of 5 mm approximately, five pushing force data were recorded by the six-axis force sensor. In the experiment, when the maximum deflection was 20.724 mm, the pushing force reached 0.7641 N. Three repeated experiments were conducted and average values of the recorded data were taken as final results. And the deflection data were fitted with a zeroaxial straight line, the linearly dependent coefficient of which is more than 0.9993, and the function of the deflection with the pushing force as independent variable is shown in equation (1).

The experimental result indicates that the deflection and the pushing force has a linear relationship.

\[ w = 26.7644F(R^2 = 0.9993) \]  

(1)

3.4.3. Continuous deformation ability test of the multi-segment support structure. The motion trajectory of the force bearing point of the multi-segment prototype is shown in figure 14. During the three repeated back-and-forth experiments, the bending angle of the terminal plane of the prototype varies continuously within the range from 0° to 180°. In the range of 0°~90°, the curvature of the motion trajectory increases slowly. While in the range of 90°~180°, the curvature increases more rapidly. Thus in the application of continuum robots, the control accuracy of the continuum robot with multi-segment prototype is higher, if the bending angle of the prototype is within the range of 0°~90°. In figure 13, when the bending angle reaches 90°, the terminal plane of the prototype goes farthest away from the x axis and the distance is 148.6 mm.

In the experiment, the support structure can complete large-scale continuous deformation under the total length of 198 mm, the bending angle of the prototype can achieve 180°. The continuous deformation ability of the multi-segment prototype can meet the requirements for the application in continuum robots with central support structure. What’s more, after unloading the pushing force, the support structure can almost restore to its initial shape, which can be applied to high-precision control in continuum robots.

4. Modeling and analysis of flexural rigidity of the support structure

4.1. Flexural rigidity model of the support structure

For the calculation of the flexural rigidity of the structure, the Ni-Ti SMA tube can be regarded as a slender beam model. For slender beam model, the effect of shear on the deformation can be neglected. When bending, the cross section of the slender beam model remains flat, and is orthogonal to the bending beam axis. For the spiral structure in this study, the
hollowed part can be regarded as a spring with a rectangular cross section. The force analysis of the hollowed part is shown in figure 15.

When the spiral structure is subjected to radial load $F$, at the section which is $\zeta (\zeta = H - \frac{\varphi}{2}p)$ away from $F$, there are bending moment $M (M = F \cdot \zeta)$ and horizontal force $F$. $M$ and $F$ can be decomposed into the following components.

Torque around the $t$-axis $T_t$,

$$T_t = M \cos \varphi \cos \beta - \frac{FD}{2} \sin \varphi \sin \beta$$
$$= F\zeta \cos \varphi \cos \beta - \frac{FD}{2} \sin \varphi \sin \beta$$

Moment of rotation around the $b$-axis $M_b$,

$$M_b = -M \cos \varphi \sin \beta - \frac{FD}{2} \sin \varphi \cos \beta$$
$$= -F\zeta \cos \varphi \sin \beta - \frac{FD}{2} \sin \varphi \cos \beta$$

Moment of rotation around the $n$-axis $M_n$,

$$M_n = M \sin \varphi = F\zeta \sin \varphi$$

Normal force acting along the $t$-axis $F_t$,

$$F_t = F \sin \varphi \cos \beta$$

Radial force acting along the $b$-axis $F_b$,

$$F_b = -F \sin \varphi \sin \beta$$

Radial force acting along the $n$-axis $F_n$,

$$F_n = -F \cos \varphi$$

When unit force $(F = F_t = 1)$ acts on the spring,

$$M_t = F_t \zeta = \zeta$$

$$T_{tt} = M_t \cos \varphi \cos \beta - \frac{D}{2} \sin \varphi \sin \beta$$
$$= \zeta \cos \varphi \cos \beta - \frac{D}{2} \sin \varphi \sin \beta$$

$$M_{tb} = -M_t \cos \varphi \sin \beta - \frac{D}{2} \sin \varphi \cos \beta$$
$$= -\zeta \cos \varphi \sin \beta - \frac{D}{2} \sin \varphi \cos \beta$$

$$M_{tn} = M_t \sin \varphi = \zeta \sin \varphi$$

Figure 8. Laser cutting surface topography of the two samples. (a) Non-heat treated sample. (b) Non-heat treated sample. (c) Heat treated sample. (d) Heat treated sample.
According to the energy law, the deformation formula of the spiral structure under the bending moment can be obtained.

\[
\begin{align*}
W &= \frac{1}{8} \int_0^{2\pi n} \frac{T_0 T_1}{G I_n} \frac{D d \varphi}{2 \cos \beta} + \int_0^{2\pi n} \frac{M_0 M_{1b}}{E I_b} \frac{D d \varphi}{2 \cos \beta} \\
&+ \int_0^{2\pi n} \frac{M_0 M_{1n}}{E I_n} \frac{D d \varphi}{2 \cos \beta}
\end{align*}
\]  

(12)

In equation (12), \( E \) is the modulus of elasticity of the material, \( G \) is the shear modulus of the material, \( D \) is the diameter of the spring, \( n \) is effective number of turns, \( I_n \) is the moment of inertia of the spring material section rotating around the n-axis, \( I_b \) is the moment of inertia of the spring material section rotating around the b-axis, \( I_p \) is the polar moment of inertia of the spring material section. When \( u \) is Poisson’s ratio of the material, \( G \) can be calculated by equation (13).

\[
G = \frac{E}{2(1 + u)}
\]  

(13)

In figure 15, \( b(b = t) \) is the width of the rectangular cross section, and \( h_t \) is the length of the rectangular cross section.

If the spiral of the structure unfolds along the cylindrical surface within the range of a thread pitch, it becomes a
straight line. And the straight line, the thread pitch \( p \) and \( \pi D \) form a right triangle with \( p \) and \( \pi D \) serving as the right-angle sides. So the length of the rectangular cross section \( h_r \) can be calculated as follows in equation (14).

\[
h_r = D \cdot \left( \frac{\pi}{s} - \frac{\alpha}{2} \right) \cdot \sin \beta
\]  
(14)

\[
\beta = \arctan \frac{p}{\pi D}
\]  
(15)

The moment of inertia \( I_n \) is as follows,

\[
I_n = \frac{bh_r^3}{12}
\]  
(16)

The moment of inertia \( I_b \) is as follows,

\[
I_b = \frac{b^3h_r}{12}
\]  
(17)

The moment of inertia \( I_p \) is as follows,

\[
I_p = I_n + I_b
\]  
(18)

Equation (12) can be calculated as follows,

\[
w = \frac{1}{s} \left( \int_0^{2\pi} \frac{T_i T_j}{GI_p} \frac{Dd\varphi}{2 \cos \beta} + \int_0^{2\pi} \frac{M_i M_j}{E_b} \frac{Dd\varphi}{2 \cos \beta} + \int_0^{2\pi} \frac{M_i M_j}{E_b} \frac{Dd\varphi}{2 \cos \beta} \right)
\]

\[
= \frac{1}{s} \left( \int_0^{2\pi} \frac{F_c^2 \cos^2 \varphi \cos^2 \beta - F D \sin \varphi \cos \varphi \sin \beta \cos \beta + \frac{F D^2}{4} \sin^2 \varphi \sin^2 \beta}{G I_p} \frac{Dd\varphi}{2 \cos \beta} \right)
\]

\[
+ \int_0^{2\pi} \frac{F_c^2 \cos^2 \varphi \sin^2 \beta + F D \sin \varphi \cos \varphi \sin \beta \cos \beta + \frac{F D^2}{4} \sin^2 \varphi \cos^2 \beta}{E b} \frac{Dd\varphi}{2 \cos \beta} \right)
\]

\[
= \frac{1}{s} \left( \frac{DF}{2GI_p \cos \beta} \left( \frac{np^2 \cos^2 \beta}{8\pi} + \frac{n^3p^2 \pi^2 \sin^2 \beta}{3} + \frac{D P \sin \beta \cos \beta}{4} \right)
\]

\[
+ \frac{DF}{2E_b \cos \beta} \left( \frac{np^2 \sin^2 \beta}{8\pi} + \frac{n^3p^2 \pi^2 \sin^2 \beta}{3} + \frac{D P \sin \beta \cos \beta}{4} \right)
\]

\[
+ \frac{nD^3 \pi^2 \cos^2 \beta}{4} \right) \right) \right) \right)
\]

\[
= \frac{DF}{2E_b \cos \beta} \left( \frac{np^2 \sin^2 \beta}{8\pi} + \frac{n^3p^2 \pi^2 \sin^2 \beta}{3} + \frac{D P \sin \beta \cos \beta}{4} \right)
\]

\[
+ \frac{nD^3 \pi^2 \cos^2 \beta}{4} \right) \right) \right)
\]  
(19)
The non-hollowed part of the spiral structure used for installation is a thin-walled circular ring whose moment of inertia is
\[ I = \pi D^3 t E \] and its flexural rigidity is much larger than that of the hollowed part. So it can be assumed that the deflection of the non-hollowed part can be ignored.

4.2. Simulation of the flexural rigidity model and comparison with the finite element analysis and test results

In order to verify the validity of the flexural rigidity model described in section 4.1, the comparison between the model simulation, the finite element analysis of the spiral structure and experimental results is vital, so the simulation of the flexural rigidity model should be calculated first.

According to the flexural rigidity model mentioned in section 4.1, equation (19) is for calculating the deflection of the spiral structure.

The simulation conditions are as follows. The spiral structure was placed horizontally and one end was fixed. A horizontal pushing force of 0.5 N, which is perpendicular to the axis of the structure, was applied to the point on the other end of the structure. The force bearing point is 2 mm upon the boundary of the hollowed part. Substituting the value of \( I_h \) into equation (3), the deflection of each point \( x \) on the spiral structure can be integrated. Under this circumstance, with the gravity of the structure neglected, the deflection of the force bearing point is 11.102 mm. And the deflection curve of the whole spiral structure is illustrated in figure 16.

In order to verify the validity of the flexural rigidity model, as mentioned in section 4.1, finite element analysis of the support structure in ANSYS needs to be carried out. The simulation result of the support structure is shown in figure 17. When the pushing force is 0.5 N, the maximum deformation of the support structure is 13.751 mm, and the deflection at the force bearing point is 12.123 mm. As the pushing force changing from 0 N to 0.5 N with an interval of 0.05 N, the deflections of the force bearing point are recorded, as shown in figure 17. And the deflection data were fitted with a zeroaxial straight line, the linearly dependent coefficient of which is more than 0.9999, and the function of the deflection with the pushing force as independent variable is shown in equation (21). The result indicates that the deflection and the pushing force has a linear relationship.

\[ w = 24.2938 F \quad \text{(} R^2 = 0.9999 \text{)} \] (21)

In the flexural rigidity model simulation, under the same conditions and data processing method, the relationship between the deflection of the force bearing point and the pushing force can be obtained. And the linearly dependent coefficient of the fitted line is also more than 0.9998. The function of the deflection with the pushing force as independent variable is shown in equation (22). The result is demonstrated in figure 17.

\[ w = 21.9514 F \quad \text{(} R^2 = 0.9998 \text{)} \] (22)
In Figure 17, the gap between the finite element analysis result and model simulation is small, and the gradient error of the model simulation is 9.64% compared to the finite element analysis result, which indicates that the flexural rigidity model agrees with the finite element analysis result. The gradient error of the model simulation is 17.98% compared to the experimental result. And the gradient error of the finite element analysis is 9.23% compared to the experiment result. Therefore the validity of the flexural rigidity model is proved. In the finite element analysis and the flexural rigidity model of the support structure, we neglected the decrease of Young’s modulus caused by strain. And in the flexural rigidity model, we regarded that the flexural rigidity and the number of thread starts have approximate linear relationship. This is the main error source in the flexural rigidity model. In the future use, a correction factor should be considered to compensate for the flexural rigidity model error.

4.3. Discussion of the structural parameters

In equation (19), the deflection of the support structure \( w \) is related to the axial pitch \( p \), effective number of turns \( n \), the cutting angle \( \alpha \), the number of thread starts \( s \), the inner diameter \( d \) and the wall thickness \( t \). Model simulation and finite element analysis results of the relationship between structural parameters and deflection of the spiral structure are shown in Figure 18. Among the above parameters, \( d, t, p \) and \( n \) are determined by the application scenarios, and the parameters \( d \) and \( t \) decide the radial size of the Ni-Ti SMA tube, the parameters \( p \) and \( n \) decide the axial dimension of the Ni-Ti SMA tube.

The relationship of deflection and the inner diameter \( d \) is shown in Figure 18(a). When other parameters are constant, as the inner diameter \( d \) changing from 5 mm to 15 mm with an interval of 1 mm, the deflection first decreases when \( d \) is no more than 13 mm, and the minimum deflection is 10.5 mm. Then the deflection slightly increases with the increase of \( d \).

The relationship of deflection and the wall thickness \( t \) is

![Figure 15. Force analysis of the single-segment spiral structure.](image)

![Figure 16. Flexural rigidity model simulation of the single-segment spiral structure in Matlab.](image)
shown in figure 18(b). When other parameters are constant, as the wall thickness $t$ changing from 0.15 mm to 0.3 mm with an interval of 0.05 mm, the deflection decreases sharply from 32 mm to 4 mm approximately with the increase of $t$.

The relationship of deflection and the axial pitch $p$ is shown in figure 18(c). When other parameters are constant, as the axial pitch $p$ changing from 10 mm to 30 mm with an interval of 2.5 mm, the deflection increases from 5 mm to 30 mm approximately with the growth rate of the deflection increasing gradually. When $p$ is determinate, the total length of this structure is determined by the effective number of turns $n$. The relationship of deflection and the effective number of turns $n$ is shown in figure 18(d). As the effective number of turns $n$ changing from 1 to 3 with an interval of 0.25, the relationship between deflection and $n$ is nearly linear, which indicates that the flexural rigidity is irrelevant to $n$.

Figure 17. Results of finite element analysis and model simulation of the deflection on force bearing point under different pushing forces.

Figure 18. Model simulation and finite element analysis results of the relationship between structural parameters and deflection of the spiral structure.
When \( d, t, \alpha \) and \( s \) are constant, the flexural rigidity of the hollowed part is determined by \( \alpha \) and \( s \). To ensure a proper ratio of the spiral part, the product of \( s \) and \( \alpha \) must be no more than 2\( \pi \). The less the product of \( s \) and \( \alpha \), the less the removal rate of the material. And the flexural rigidity of the hollowed part increases with the decrease of the product of \( s \) and \( \alpha \). As figure 18(c) shows, when other parameters keep constant and cutting angle \( \alpha \) changes from 25° to 65° with an interval of 5°, the material removal rate increases linearly with the increase of \( \alpha \), and the deflection increases more and more rapidly with the increase of \( \alpha \).

Once \( d, t, \alpha \) and \( s \) are decided, when the total length of the structure remains unchanged, the relationship between the axial pitch \( p \) and the deflection \( w \) can be obtained. When the length of the hollowed part is 40 mm, the relationship between deflection and the axial pitch \( p \) is shown in figure 18(f). It can be seen that, with the increase of the axial pitch \( p \), the deflection of the spiral structure first decreases when \( p \) is no more than 40 mm, and the reducing rate of the deflection decreases gradually. Then the deflection gradually increases with the increase of \( p \).

In figure 18, we can know that when the structural parameters change within the ranges mentioned above, all the flexural rigidity model simulation results agree with the finite element analysis results basically, which reveals that the flexural rigidity model of the support structure is effective within the discussed ranges of structural parameters.

5. Conclusion

In this paper, a central support structure with the spiral cutting pattern for continuum robots is proposed. The flexural rigidity and stress concentration of three hollow schemes of the Ni-Ti SMA tube were analysed through finite element analysis, through which we find that, flexural rigidity of the spiral structure can meet the large deformation demand of continuum robots, and there is no obvious stress concentration in the spiral structure. Then the fabrication process was described, and the influences of laser cutting and heat treatment on material properties were analysed. By employing the laser cutting and heat treatment, the ratio of HAZ is less than 0.65%, which maintains the superelastic properties of the material.

Then the flexural rigidity and continuous deformation ability tests were carried out to verify the reasonability of the spiral structure. Experimental results show that the effective bending angle of the multi-segment prototype is more than 180°, and its continuous deformation ability is encouraging.

And flexural rigidity model of the support structure is proposed. The flexural rigidity model is related to the axial pitch \( p \), the cutting angle \( \alpha \), the number of thread starts \( s \), the inner diameter \( d \) and the wall thickness \( t \). The difference between the flexural rigidity model and experimental results is less than 17.98%. The difference between the results of finite element analysis and the experimental results is less than 9.23%. The flexural rigidity model error can be reduced by add a correction factor. The relationship between those structural parameters above and deflection was explored, which indicated that the flexural rigidity model of the support structure is effective within the discussed ranges of structural parameters.

This paper systematically introduces the design method, fabrication process as well as the flexural rigidity model of the central support structure using superelastic SMA material, which provides a reference for the design and modelling of continuum robots.

To explore the potential of superelastic SMA material, future work includes applying the support structure to continuum robots and searching for the actuation and control methods to accomplish working in confined space, such as endoscope inspection.

Acknowledgments

This research was supported by the National Science Foundation project, China under contract number 61603015.

ORCID IDs

Jiawen Tian @ https://orcid.org/0000-0003-3595-6099
Xi Fang @ https://orcid.org/0000-0002-3202-8708

References

[1] Buckingham R and Graham A 2005 Snaking around in a nuclear jungle Industrial Robot: An International Journal 32 120–7
[2] Buckingham R 2002 Snake arm robots Industrial Robot: An International Journal 29 242–5
[3] Nakamura Y et al 1995 Shape-memory-alloy active forceps for laparoscopic surgery Robotics and Automation, 1995. Proceedings., 1995 IEEE International Conference on. IEEE 3 2320–7
[4] Ranzani T et al 2015 A bioinspired soft manipulator for minimally invasive surgery Bioinsp. Biomim. 10 035008
[5] Luo M et al 2014 Design improvements and dynamic characterization on fluidic elastomer actuators for a soft robotic snake IEEE Int. Conf. on Technologies for Practical Robot Applications (TePRA) (Woburn, MA) 1–6
[6] Andrikopoulos G, Nikolakopoulos G and Manesis S 2011 A survey on applications of pneumatic artificial muscles 2011 19th Mediterranean Conference on (MED) (Corfu) 1439–46
[7] Mosadegh B et al 2014 Soft robotics: pneumatic networks for soft robotics that actuate rapidly (Adv. Funct. Mater. 15/2014) Adv. Funct. Mater. 24 2163–70
[8] Marchese A D, Onal C D and Rus D 2014 Autonomous soft robotic fish capable of escape maneuvers using fluidic elastomer actuators Soft Robotics 1 75–87
[9] Sfakiotakis M et al 2013 Octopus-inspired eight-arm robotic swimming by sculling movements IEEE Int. Conf. on Robotics & Automation (https://doi.org/10.1109/ICRA.2013.6631314)
[10] Shepherd R F et al 2011 Multigait soft robot Proc. Natl Acad. Sci. USA 108 20400–3
[11] Gravagne I A and Walker I D 2000 Kinematic transformations for remotely-actuated planar continuum robots IEEE Int. Conf. on Robotics and Automation, 2000. Proc.. ICRA vol 1, 19–26
[12] Evangelou N and Tzes A 2016 Development of an SMA-actuated redundant robotic platform for minimally invasive
surgery IEEE Int. Conf. on Biomedical Robotics & Biomechatronics (https://doi.org/10.1109/BIOROB.2016.7523651)

[13] Margheri L, Laschi C and Mazzolai B 2012 Soft robotic arm inspired by the octopus: I. From biological functions to artificial requirements Bioinsp. Biomim. 7 025004

[14] Mazzolai B et al 2012 Soft-robotic arm inspired by the octopus: II. From artificial requirements to innovative technological solutions Bioinspir Biomim 7 025005

[15] Laschi C et al 2012 Soft robot arm inspired by the octopus Adv. Robot. 26 709–27

[16] Choi D G, Yi B J and Kim W K 2007 Design of a spring backbone micro endoscope IEEE/rsj Int. Conf. on Intelligent Robots and Systems 1815–21

[17] Alambeigi F, Seifabadi R and Armand M 2016 A continuum manipulator with phase changing alloy 2016 IEEE Int. Conf. on Robotics and Automation (ICRA). IEEE 758–64

[18] Cianchetti M et al 2013 STIFF-FLOP surgical manipulator; mechanical design and experimental characterization of the single module 2013 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems. IEEE 3576–81

[19] Noh Y et al 2014 A continuum body force sensor designed for flexible surgical robotics devices 36th Inte. Conf. IEEE Engineering in Medicine and Biology Society (Chicago, IL) 3711–14

[20] Santoso J et al 2019 Single chamber multiple degree-of-freedom soft pneumatic actuator enabled by adjustable stiffness layers Smart Mater. Struct. 28 125008

[21] Wang H et al 2016 Analysis and application of a rolled dielectric elastomer actuator with two degrees of freedom Smart Mater. Struct. 25 125008

[22] Liu Y and Xiang H 1998 Apparent modulus of elasticity of near-equiatomic NiTi Journal of Alloys & Compounds 270 154–9

[23] Suresh K S et al 2015 Microstructure dependent elastic modulus variation in NiTi shape memory alloy J. Alloys Compd. 633 71–4

[24] Huang W M et al 2005 V-shape in Young’s modulus versus strain relationship in shape memory alloys upon mechanical loading Journal of Alloys & Compounds 390 175–81

[25] Sittner P et al 2014 Young’s modulus of austenite and martensite phases in superelastic NiTi wires J. Mater. Eng. Perform. 23 2303–14

[26] Wang Y F, Yue Z F and Wang J 2007 Experimental and numerical study of the superelastic behaviour on NiTi thin-walled tube under biaxial loading Comput. Mater. Sci. 40 246–54

[27] Huang X and Liu Y 2001 Effect of annealing on the transformation behavior and superelasticity of NiTi shape memory alloy Scr. Mater. 45 153–60

[28] Vieira L A et al 2011 Mechanical behaviour of Nd:YAG laser welded superelastic NiTi Materials Science and Engineering: A 528 5560–5

[29] Mehropuaya M et al 2018 Laser welding of NiTi shape memory sheets using a diode laser Optics & Laser Technology 2018 108 142–9

[30] Liu L et al 2016 Fiber laser micromachining of thin NiTi tubes for shape memory vascular stents Applied Physics a-Materials Science & Processing 122 9

[31] Uchil J, Fernandes F M B and Mahesh K K 2007 X-ray diffraction study of the phase transformations in NiTi shape memory alloy Mater. Charact. 58 243–8