Interval-Valued Triangular Neutrosophic Linear Programming Problem

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Abstract

In this paper, we have proposed an Interval-valued triangular neutrosophic number (IV-TNN) as a key factor to solve the neutrosophic linear programming problem. In the present neutrosophic linear programming problem IV-TNN is expressed in lower, upper truth membership function, indeterminacy membership function, and falsity membership function. Here, we try the compare our proposed method with existing methods.

Keywords: Neutrosophic Set, Interval-valued triangular neutrosophic number, Neutrosophic Linear Programming Problem.

1. Introduction

A fuzzy set is defined by Zadeh[1] in the year 1965. Application of fuzzy set theory in various fields like decision making particularly in Linear programming problems (optimization problems) have been studied in a wide area after the introduction of a fuzzy set. As a result, many researchers have a contribution in this direction. Zadeh’s [1] definition on fuzzy set theory, it shows because of uncertainty for determining the degree of the membership function is not possible. In this regard, Zadeh[2] tried to short out the problem and in the year 1975, he proposed an interval-valued fuzzy set. After the discovery of the fuzzy set theory, Atanassov[3] introduced the concept of the Intuitionistic Fuzzy set which is an extension of the fuzzy set. The intuitionistic fuzzy set explains the degree of non-membership denoting the not belongs to the set. The important extension of the fuzzy set is the interval-valued fuzzy set, which is characterized by an interval-valued membership function. In [4], Atanassov discussed an idea on an interval-valued intuitionistic fuzzy set. D. Dubey [5] proposed an approach based on value and ambiguity in-dices to solve LPPs with data as Triangular Intuitionistic Fuzzy Numbers.

Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. By observing this in 1995, Smarandache [6-9] introduce neutrosophy, which is the study of neutralities as an extension of dialectics. Neutrosophic sets are characterized by three independent degrees namely truth- membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F), where T,I,F are standard or non-standard subsets of ]0;1[. The definition of single-valued neutrosophic sets were introduced by Wang [10], whose values belong to [0,1]. Single-valued neutrosophic sets were successfully applied to various decision-making problems [11–16].
Linear programming is most powerful technique, which occurs in decision-making. Bellman and Zadeh [17] introduced fuzzy optimization problems where they have stated that a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints. Many researchers such as Zimmermann [18], Tanaka, et al.[19], Campos and Verdegay [20], Rommelfanger et al.[21], Cadenas and Verdegay [22] who were dealing with the concept of solving fuzzy optimization problems, later studied this subject. Parvathi and Malathi [23] have done their work on intuitionist fuzzy linear optimization.

Then Abdel-Baset et al.[24] and pramanik [25] proposed neutrosophic linear programming methods based on the idea of neutrosophic sets. Also, Abdel-Baset et al.[26] introduced the neutrosophic linear programming models where their parameters are represented with the trapezoidal neutrosophic numbers and presented a technique for solving problems. Nafei et al. [27] presented a new method for solving interval neutrosophic linear programming problems. Abdel-Baset et al.[25] discussed a novel method for solving the fully neutrosophic linear programming problems. Khatter [39] proposed Neutrosophic linear programming using possibilistic mean according to Kirna[39] the proposed approach converts each triangular neutrosophic number in linear programming problem to weighted value using possibilistic mean to determine the crisp linear programming problem. Bera et al.[40] approach the application of neutrosophic linear programming problem to real life. They proposed an algorithm of the Big-M simplex method in this new climate and then it is applied to a real-life problem. Ye [41] studied on neutrosophic number linear programming method and its application under neutrosophic number environments.

In this paper, we proposed an interval-valued triangular neutrosophic number to solve the neutrosophic linear programming problem so that we could have a better result in comparison to other methods. The structure of the chapter is as follows: the next section is a preliminary discussion; the third section describes the interval-valued triangular neutrosophic number of the proposed model; the fourth section describes steps of the proposed model; the last section summaries the conclusion.

2. Preliminaries

Definition 2.1 [6] Let X be a universe of discourse. A single-valued neutrosophic set N through X taking the form $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$, where $T_N(x): X \rightarrow [0, 1]$, $I_N(x): X \rightarrow [0, 1]$ and $F_N(x): X \rightarrow [0, 1]$ with $0 \leq T_N(x) \leq I_N(x) \leq F_N(x) \leq 3$ for all $x \in X$ and $T_N(x), I_N(x)$ and $F_N(x)$ represents truth membership function, indeterminacy membership function, and falsity membership function of x to N.

Definition:2.2[28] A triangular neutrosophic number (TNN) is denoted by $A^N = \{(a^t, a^m, a^u), (\mu, i, y)\}$ whose the three membership functions for the truth membership function, indeterminacy membership function and falsity membership function of x can be defined as

$$T_N^N(x) = \begin{cases} 
\frac{(x - a^i)}{(a^m - a^i)}, & a^i \leq x \leq a^m \\
\mu, & x = a^m \\
\frac{(a^u - x)}{(a^u - a^m)}, & a^m \leq x \leq a^u \\
0, & \text{otherwise}
\end{cases}$$

$$I_N^N(x) = \begin{cases} 
\frac{(x - a^i)}{(a^m - a^i)}, & a^i \leq x \leq a^m \\
\mu, & x = a^m \\
\frac{(a^u - x)}{(a^u - a^m)}, & a^m \leq x \leq a^u \\
0, & \text{otherwise}
\end{cases}$$

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Lower and upper truth membership functions are

\[
F_{A^N}(x) = \begin{cases} 
\dfrac{(x - a^t)}{(a^m - a^t)} \gamma, & a^t \leq x \leq a^m \\
\gamma, & x = a^m \\
\dfrac{(a^u - x)}{(a^u - a^m)} \gamma, & a^m \leq x \leq a^u \\
0, & \text{otherwise}
\end{cases}
\]

Where \(0 \leq T_{A^N}(x) + I_{A^N}(x) + F_{A^N}(x) \leq 3\), for \(x \in A^N\) and if \(a^t \geq 0\), \(A^N\) is called a nonnegative TNN and for \(a^t \leq 0\), \(A^N\) is called a negative TNN.

**Definition 2.3** [28] Let \(\tilde{A} = \{(a_1, a_2, a_3, (\mu_1, i_1, \gamma_1))\} \) and \(\tilde{B} = \{(b_1, b_2, b_3, (\mu_2, i_2, \gamma_2))\} \) be two TNN then the mathematical operation on two triangular neutrosophic numbers \(\tilde{A}\) and \(\tilde{B}\) are as follows

\[
\tilde{A} + \tilde{B} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3, (\mu_1 \wedge \mu_2, i_1 \lor i_2, \gamma_1 \lor \gamma_2))\}
\]

\[
\tilde{A} - \tilde{B} = \{(a_1 - b_1, a_2 - b_2, a_3 - b_3, (\mu_1 \lor \mu_2, i_1 \land i_2, \gamma_1 \land \gamma_2))\}
\]

\[
\lambda \tilde{A} = \{(\lambda a_1, \lambda a_2, \lambda a_3, (\mu_1, i_1, \gamma_1))\} \text{ if } \lambda \geq 0
\]

\[
\{(\lambda a_1, \lambda a_2, \lambda a_3, (\mu_1, i_1, \gamma_1))\} \text{ if } \lambda \leq 0
\]

3. **Interval Valued Triangular Neutrosophic Number (IV-TNN)**

**Definition 3.1** Interval-valued triangular neutrosophic number (IV-TNN) shows a more abundant and flexible result than the neutrosophic number. Many authors defined with a different type of triangular neutrosophic numbers in this work we will define interval-valued triangular neutrosophic number (IV-TNN).

An IV-TNN is defined as \(\tilde{A} = \{(a^t, a, a^u, (\mu^t, \mu^u), (\mu^v, \mu^w))| (\mu^t, \mu^u, \mu^v, \mu^w)\} \) where \(a^t, a, a^u, \mu^t, \mu^u, \mu^v, \mu^w \) are belong to \(R\) and its lower, upper truth membership function, indeterminacy membership function, and falsity membership function is defined as follows

(a). Lower and upper truth membership functions are

\[
\tau_{A^L}(x) = \begin{cases} 
\dfrac{(x - a^t)}{(a^t - a^L)} \mu^t, & a^t \leq x \leq a^L \\
\mu^t, & x = a \\
\dfrac{a^L - x}{(a^L - a^L)} \mu^t, & a^L \leq x \leq a^t \\
0, & \text{otherwise}
\end{cases}
\]

\[
\tau_{A^U}(x) = \begin{cases} 
\dfrac{(x - a^u)}{(a^u - a^U)} \mu^u, & a^u \leq x \leq a^U \\
\mu^u, & x = a \\
\dfrac{a^U - x}{(a^U - a^U)} \mu^u, & a^U \leq x \leq a^u \\
0, & \text{otherwise}
\end{cases}
\]

(b). Lower and upper indeterminacy membership functions are

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\[ I^L_{A}(x) = \begin{cases} \frac{(x-e^L)}{(a-e^L)} l^L, & e^L \leq x \leq a \\ l^L, & x = a \\ \frac{(g^L-x)}{(g^L-a)} l^L, & a \leq x \leq g^L \\ 0, & \text{otherwise} \end{cases}, \quad \begin{cases} \frac{(x-e^U)}{(a-e^U)} l^U, & a^U \leq x \leq a \\ l^U, & x = a \\ \frac{(g^U-x)}{(g^U-a)} l^U, & a \leq x \leq g^U \end{cases} \]

(c) Lower and upper falsity membership functions are

\[ F^L_{A}(x) = \begin{cases} \frac{(x-i^L)}{(a-i^L)} y^L, & i^L \leq x \leq a \\ y^L, & x = a \\ \frac{(n^L-x)}{(n^L-a)} y^L, & a \leq x \leq n^L \\ 0, & \text{otherwise} \end{cases}, \quad \begin{cases} \frac{(x-i^U)}{(a-i^U)} y^U, & i^U \leq x \leq a \\ y^U, & x = a \\ \frac{(n^U-x)}{(n^U-a)} y^U, & a \leq x \leq n^U \end{cases} \]

**Remark 1:** If \( a^L = a^U = e^L = e^U = l^L = l^U = c^L = c^U = g^L = g^U = n^L = n^U \) then IV-TNN becomes a triangular neutrosophic number.

**Remark 2:** If \( a^L = a^U = e^L = e^U = l^L = l^U = c^L = c^U = g^L = g^U = n^L = n^U \) and \( \mu^L = \mu^U, l^L = l^U \) and \( y^L = y^U \) then IV-TNN becomes neutrosophic set.

**Definition 3.2**

Let

\[ \tilde{M} = \{(a_M^{l^L}, a, c_M^{l^L}, (a_M^{u^L}, a, c_M^{u^L}) | \mu_M^{l^L}, \mu_M^{u^L}, (e_M^{l^L}, a, g_M^{l^L}) | l_M^{l^L}, l_M^{u^L}, (l_M^{l^L}, a, n_M^{l^L}) | l_M^{l^L}, l_M^{u^L}) \} \text{ and} \]

\[ \tilde{N} = \{(a_N^{l^L}, a, c_N^{l^L}, (a_N^{u^L}, a, c_N^{u^L}) | \mu_N^{l^L}, \mu_N^{u^L}, (e_N^{l^L}, a, g_N^{l^L}) | l_N^{l^L}, l_N^{u^L}, (l_N^{l^L}, a, n_N^{l^L}) | l_N^{l^L}, l_N^{u^L}) \} \]

then

\[ \tilde{M} \oplus \tilde{N} = \left\{ (a_M^{l^L} + a_N^{l^L}, 2a, c_M^{l^L} + c_N^{l^L}) | \mu_M^{l^L} + \mu_N^{l^L}, \mu_M^{u^L} + \mu_N^{u^L} \right\} \left( e_M^{l^L} + e_N^{l^L}, 2a, g_M^{l^L} + g_N^{l^L} \right) \left( l_M^{l^L} + l_N^{l^L}, 2a, n_M^{l^L} + n_N^{l^L} \right) \]

\[ \text{if } k > 0 \]

\[ = \left\{ (ka_M^{l^L}, ka, kc_M^{l^L}) | (ka^U, k, ka^L) | (ka_N^{l^L}, ka, kc_N^{l^L}) | (ka^U, k, ka^L) \right\} \text{ if } k < 0 \]

\[ = \left\{ (ka_M^{l^L}, ka, kc_M^{l^L}) | (ka^U, k, ka^L) | (ka_N^{l^L}, ka, kc_N^{l^L}) | (ka^U, k, ka^L) \right\} \text{ if } k = 0 \]
3.3. Neutrosophic Linear Programming Problem (NLPP)

Linear programming is an optimization technique widely used in practical problem In this section we generalize the LPP term as the interval-valued triangular neutrosophic programming problem, denoted as IV-TNLP problem and defined as

Maximize/minimize $\tilde{Z} = \tilde{c} x$

Such that $\tilde{A} x \leq \tilde{b}, x \geq 0$

Where $\tilde{c}, \tilde{A}, \tilde{b}$ are interval-valued triangular neutrosophic numbers.

4. Proposed IV-TNLP Method

Step 1 Let the decision-makers insert their IV-TNLP problem. Because we always try to maximize truth membership function and minimize indeterminacy membership function and falsity membership, then inform decision-makers to apply the concept when entering triangular neutrosophic numbers of the IV-TNLP problem.

Step 2 Convert IV-TNLP problem to its crisp model by using the following method

Let $\tilde{A} = \{(a^l, a^u, a^i)\}$ be an interval-valued triangular neutrosophic number where $\mu, \nu$ and $\gamma$ are truth membership function, indeterminacy membership function and falsity membership function of $A$. The ranking function for an interval-valued neutrosophic number $\tilde{A}$ will be defined as

$$ R(\tilde{A}) = \frac{a^l + a^u + a^i + a^l + a^u + a^i + 2a + g^l + g^u + l^i + l^u + n^i + n^u}{24} + \frac{\mu^l + \mu^u}{2} + \frac{\nu^l + \nu^u}{2} + \frac{\gamma^l + \gamma^u}{2}. $$

Step 3 By applying the proposed ranking function converts each interval-valued triangular neutrosophic number to its equivalent crisp value. This lead to convert the IV-TNLP problem to its crisp model.

Step 4 Solve the crisp model using the standard method and obtained the optimal solution to the problem.

5 Comparison of the proposed method with existing methods

Comparison of the proposed method with [23]

Maximize $\tilde{Z} = \tilde{3} x_1 + \tilde{4} x_2$

Such that,

$\tilde{1} x_1 + \tilde{2} x_2 \leq \tilde{2} 0,$

$\tilde{3} x_1 + \tilde{5} x_2 \leq \tilde{6} 0$

$x_1, x_2 \geq 0$

According to [23] the optimal value of the objective function is $Z=21.45$. Now we assume the parameters of LPP according to the proposed method which are represented as follows –

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\[ \bar{3} = ((2.55, 3, 6.55)(2.65, 3.6) [0.40, 0.60]; \ (2.25, 3, 4.80)(2.65, 3, 6.80) \\
\quad [0.1, 0.2]; \ (2.75, 3, 6.50)(2.80, 3, 7.85) [0.3, 0.5]) \]

\[ \bar{4} = \{(2.5, 4, 5.9)(2.9, 4, 6)[0.3, 0.5]; \ (2.1, 4, 5.6)(2.4, 4, 5.8)[0.2, 0.3]; \]
\[ (3, 4, 4.5), (3.5, 4.5)[0.7, 0.9] \]

\[ \bar{1} = ((0.2, 1, 1.9)(0.3, 1, 2)[0.8, 0.9]; \ (0.1, 1, 1.2)(0.2, 1, 1.3)[0.1, 0.3]; \]
\[ (0.4, 1, 1.2)(0.8, 1, 1.8)[0.3, 0.4] \]

\[ \bar{2} = ((1.4, 2, 2.4)(1.7, 2, 3)[0.3, 0.4]; \ (1.1, 2, 2.5)(1.5, 2, 2.6)[0.1, 0.3]; \]
\[ (1.8, 2, 2.1)(1.9, 2, 2.9)[0.1, 0.3] \]

\[ \bar{3} = ((2.6, 3, 3.9)(2.7, 3, 4)[0.3, 0.4]; \ (2.4, 3, 3.2)(2.5, 3, 3.5)[0.2, 0.3]; \]
\[ (2.8, 3, 3.1)(2.9, 3, 3.9)[0.5, 0.6] \]

\[ \bar{5} = ((4.3, 5, 6.1)(4.4, 5, 6.3)[0.2, 0.6]; \ (4.5, 6)(4.2, 5, 4.8)[0.4, 0.6]; \]
\[ (4.5, 5, 5.6)(4.8, 5, 6)[0.3, 0.4] \}

\[ \overline{20} = ((19.1, 20, 20.4)(19.2, 20, 20.6)[0.4, 0.6]; \ (19, 20, 20.2)(19.5, 20, 20.5) \\
\quad [0.1, 0.2]; \ (19.5, 20, 20.1)(19.9, 20, 20.3)[0.2, 0.6]) \]

\[ \overline{60} = ((59.50, 60, 71.80)(58.50, 60, 72.50)[0.5, 0.6]; \ (59, 60, 61.50) \\
\quad (59.85, 60, 62.50)[0.2, 0.3]; \ (59.20, 60, 62.85)(59.70, 60, 70.50)[0.3, 0.4]) \]

By using proposed ranking function the problem will be converted to the crisp model as follows

Maximize \( \bar{Z} = 3.77x_1 + 3.95x_2 \)

Such that,

\[ 0.78x_1 + 1.96x_2 \leq 19.88, \]
\[ 2.51x_1 + 4.59x_2 \leq 61.51 \]
\[ x_1, x_2 \geq 0 \]

Solving the problem the optimal solution is \( x_1 = 24.51; x_2 = 0 \) with optimal objective value 92.39.

By comparing the proposed model results with the results of [23] of the same problem, we observed that our proposed model results are better than the results of [23]. Also, if we see the optimal solution of the existing solution under the intuitionistic fuzzy system are \( x_1 = 7.15, x_2 = 0 \) and \( Z_{[23]} = 21.45 \) and from the optimal solution of the proposed method linear programming problem the objective function value equals 92.39 which is a problem of maximization. The proposed approach is smoother than the existing method in [23]. The existing method in [23] is only able to solve the problem but we can handle the situation in interval-valued neutrosophic number and due to this we can convert each interval-valued triangular neutrosophic number to its equivalent crisp value in a better way in comparison to

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existing method. In addition, due to the explanation of determining truth membership function, falsity membership function, and indeterminacy-membership function, the proposed model is more effective than their method in [23].

**Comparison of the proposed method with [27]**

Maximize $\tilde{Z} = \tilde{7} x_1 + \tilde{6} x_2 + \tilde{4} x_3$

Such that,

\begin{align*}
\tilde{15} x_1 + \tilde{1} x_2 & \leq \tilde{10} , \\
\tilde{9} x_1 + \tilde{4} x_2 + \tilde{8} x_3 & \leq \tilde{2} \\
\tilde{9} x_1 + \tilde{11} x_3 & \leq \tilde{4} \\
x_1, x_2, x_3 & \geq 0
\end{align*}

Following the method [27], we can observe when $a^l = a^u = e^l = e^u = l^l = l^u, c^l = c^u = g^l = g^u = n^l = n^u$ the proposed method and existing method in [27] shows the same result.

Now to get a more convenient optimal solution let us apply the proposed approach in comparison with the example of [27]. Let us assume IV-TNN as follows

\begin{align*}
\tilde{7} &= ((5, 7, 9)(5.2, 7, 9.50)[0.70, 0.90]; (5.10, 7, 9.10)(5.50, 7, 9.50)[0.10, 0.40]; \\
& \quad (5, 7, 8.50)(6.50, 7, 9)[0.20, 0.40]) \\
\tilde{6} &= ((5.20, 6, 7.2)(5.8, 6, 7.8)[0.20, 0.60]; (5.30, 6, 6.85)(5.90, 6, 7)[0.20, 0.50]; \\
& \quad (4.95, 6, 7.10), (5.50, 6, 7.25)[0.10, 0.80]) \\
\tilde{4} &= ((10.55, 14, 17.50)(11.35, 14, 18.25)[0.50, 0.70]; (10.75, 14, 17.85) \\
& \quad (11.80, 14, 18.55)[0.40, 0.60]; (11.95, 14, 17.55)(12.15, 14, 18.25)[0.30, 0.40]) \\
\tilde{5} &= ((13.5, 15, 16.50)(14.10, 15, 16.85)[0.60, 0.80]; (14.20, 15, 17) \\
& \quad (14.80, 15, 18.20)[0.10, 0.40]; (14.30, 15, 16.30)(14.55, 15, 17.4)[0.40, 0.90]) \\
\tilde{1} &= ((0.85, 1, 1.95)(0.90, 1, 2.90)[0.20, 0.70]; (0.55, 1, 2.15)(0.80, 1, 3.50) \\
& \quad [0.10, 0.40]; (0.55, 1, 1.85)(0.80, 1, 2.85)[0.40, 0.95]) \\
\tilde{0} &= ((5.85, 10, 16.25)(7.65, 10, 17.55)[0.1, 0.5]; (5.95, 10, 16.55) \\
& \quad (7.55, 10, 17.85)[0.10, 0.40]; (6.80, 10, 16.85)(7.85, 10, 17.85)[0.60, 0.90]) \\
\tilde{9} &= ((7.95, 9, 11.85)(8.55, 9, 13.55)[0.40, 0.50]; (8.55, 9, 11.85)(8.25, 9, 12.55) \\
& \quad [0.50, 0.80]; (8.50, 9, 12.65)(8.35, 9, 14.85)[0.40, 0.80]) \\
\tilde{3} &= ((3.55, 4, 6.95)(3.65, 4, 7.95)[0.10, 0.90]; (3.35, 4, 7.65)(3.65, 4, 8.85) \\
& \quad [0.40, 0.5]; (3.25, 4, 6.85)(3.85, 4, 8.55)[0.30, 0.40])
\end{align*}
\[ \delta = [(6.85, 8.95)(7.55, 8.1125)|0.70, 0.80]; (6.25, 8, 10.85)(7.25, 8, 11.55) \]
\[ \delta = [(1.55, 2, 4.55)(1.75, 2, 5.55)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]
\[ \delta = [(1.5, 2, 4.5)(1.7, 2, 5.5)|0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \]

Now by using proposed ranking function the previous problem will be converted to the crisp model as follows

Maximize \( Z = 7.30x_1 + 6x_2 + 14.13x_3 \)

Such that,
\[ 15.01x_1 + 0.30x_2 \leq 10.26 \]
\[ 9.01x_1 + 4.40x_2 + 8.36x_3 \leq 2.20 \]
\[ 19.05x_1 + 11.15x_3 \leq 4.66 \]
\[ x_1, x_2, x_3 \geq 0 \]

Solving the problem the optimal objective value is 3.72.

This clears that the proposed method shows a more convenient optimal value than the existing method in [27]. Though both the methods (existing and proposed) are the same when \( d^l = d^u = e^l = e^u = l^l = l^u, c^l = c^u = g^l = g^u \) \( n^l = n^u \) but due to having more chances for taking lower and upper triangular neutrosophic numbers for determining truth membership function, falsity membership function, and indeterminacy-membership function the proposed method shows the more suitable result. Hence our method is more applicable for solving real-life problems.

**Conclusion:**

In this paper, we have tried to discuss the neutrosophic linear programming problem concerning the interval-valued triangular neutrosophic number and compare the proposed method with some papers. In the proposed method interval-valued triangular neutrosophic number is expressed in lower, upper truth membership function, indeterminacy membership function, and falsity membership function. It is seen that the proposed method shows better results in comparison to [23]. The second comparison with [27] shows almost the same result when \( d^l = d^u = e^l = e^u = l^l = l^u, c^l = c^u = g^l = g^u = n^l = n^u \) for the interval-valued triangular number that is when lower and upper truth membership function, indeterminacy membership function and falsity membership function, and interval-valued triangular number become the same. However, when we go through the proposed method it shows different in solution. In the future study, we extend the IV-TNLP algorithm in an interval-valued Neutrosophic Linear Fractional

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Programming Problems. Hope our method would help in the future for getting better results in the decision-making system.

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