2D Yang–Mills Theory as a Matrix String Theory

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Abstract

Quantization of two–dimensional Yang–Mills theory on a torus in the gauge where the field strength is diagonal leads to twisted sectors that are completely analogous to the ones that originate long string states in Matrix String Theory. If these sectors are taken into account the partition function is different from the standard one found in the literature and the invariance of the theory under modular transformations of the torus appears to hold in a stronger sense. The twisted sectors are in one-to-one correspondence with the coverings of the torus without branch points, so they define by themselves a string theory. A possible duality between this string theory and the Gross-Taylor string is discussed, and the problems that one encounters in generalizing this approach to interacting strings are pointed out. This talk is based on a previous paper by the same authors, but it contains some new results and a better interpretation of the results already obtained.

1 Matrix String Theory

This talk is based on Ref. [1], but it also includes some new results and, we hope, a better understanding of the results already contained in the original paper. This work has its origin in an old puzzle that dates back to 1993 and in some recent development. The puzzle consists in the following: while studying the functional integral approach to quantization of YM2 on a torus (see Ref. [2]) some of us noticed that in order to get the correct result for the partition function on the torus in the gauge where the field strength $F$ is diagonal (unitary gauge) we would have

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been obliged to neglect some contributions from twisted sectors that seem to arise naturally in that gauge. The problem was put aside and ascribed to some lack of understanding from us. Recently however the same type of contributions were considered in the Matrix String theory model of Ref. [3, 4] where they give origin to string configurations of different lengths. To be more specific let us consider the Matrix String theory action:

$$S = \frac{1}{2\pi} \int d\sigma d\tau \, \text{tr} \left( (D_\mu X^M)^2 + \theta^T \partial \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2} [X^M, X^S]^2 + \frac{1}{g_s} \theta^T \gamma_M[X^M, \theta] \right)$$

where the fields $X^M$ ($M = 1, 2, ... 8$) are $N \times N$ hermitian matrices, as are the 8 fermionic fields $\theta^\alpha_L$ and $\theta^\dot{\alpha}_R$. The two dimensional world sheet is an infinite cylinder parametrized by coordinates $(\sigma, \tau)$, with $\sigma$ between 0 and $2\pi$. This action can be obtained from the ten dimensional super Yang-Mills theory by dimensional reduction and it features the same set of fields as Green-Schwarz action for type II superstring, except that here the fields are matrices. In the limit where the string coupling constant $g_s$ goes to zero the fields $X$ and $\theta$ will commute and can be simultaneously diagonalized. The eigenvalues $x_i^M$ $(i = 1, 2, ..., N)$ of $X^M$ can be identified with string coordinates which describe the world sheets of a gas of $N$ Green-Schwarz light-cone strings. The key point is that the eigenvalues $x_i^M$ can be interchanged as one goes around the compact dimension $\sigma$:

$$x_i^M(\sigma + 2\pi) = P x_i^M(\sigma) P^{-1},$$

where $P$ is an element of $S_N$. In conclusion the fields $x_i^M(\sigma)$ take value on the orbifold space $S^N \mathbb{R}^8$, with strings of different lengths associated to the cycles of $P$, and the corresponding Hilbert space consists of twisted sectors in correspondence with the conjugacy classes of $S_N$.

## 2 YM2 in the unitary gauge

The same twisted sectors appear naturally in YM2 in the unitary gauge, where the field strength $F$ is diagonal. Consider the partition function of YM2 in the first order formalism on a general Riemann surface $\Sigma_g$ of genus $g$:

$$Z(\Sigma_g, t) = \int [dA][dF] \exp \left\{ -\frac{t}{2} \text{tr} \int_{\Sigma_g} d\mu F^2 + i \text{tr} \int_{\Sigma_g} f(A) F \right\},$$

where $d\mu$ is the volume form on $\Sigma_g$ and $f(A) = dA - iA \wedge A$. It is always possible, at least locally, to find a gauge transformation $g$ that diagonalizes $F$:

$$g^{-1} F g = \text{diag}(\lambda).$$

However $g$ is not unique: if $g$ diagonalizes $F$, so does any gauge transformation of the form $gP$, with $P \in S_N$: in general, there are $N!$ Gribov copies of the gauge-fixed
field strength $F$. As in the case of Matrix string theory, the twisted sectors appear because as we go around a homotopically non-trivial loop on $\Sigma_g$, the eigenvalues can be interchanged, namely we can go from one Gribov copy to another. Consider now the case where $\Sigma_g$ is a torus parametrized by coordinates $(\sigma, \tau)$ both ranging from 0 and $2\pi$. The twisted sectors are labelled by the pair of permutations $P$ and $Q$ associated to the two homotopically non-trivial loops, more precisely by the boundary conditions

$$
\lambda_i(\tau + 2\pi, x) = \lambda_{P(i)}(\tau, x), \\
\lambda_i(\tau, x + 2\pi) = \lambda_{Q(i)}(\tau, x),
$$

(5)

where consistency requires $P$ and $Q$ to commute: $PQ = QP$.

Pairs of commuting permutations also define $N$-coverings of the torus without branch points, the $N$ sheets of the covering at each point of the target space being labelled by one eigenvalue of $F$. This argument is easily generalized to higher genus Riemann surfaces, and one can say in conclusion that twisted sectors are in correspondence with the inequivalent $N$-coverings of $\Sigma_g$ in absence of branch points. However if the genus is greater than one the quantization in the unitary gauge leads to divergences whose regularization, according to Ref. [5], would amount to set to zero all twisted sectors. The argument is as follows: the BRST invariant action in the unitary gauge consists of two terms, the first, denoted by $S_{\text{Cartan}}$, depends only on the diagonal components of the gauge fields and is just the gauge action in the first order formalism for the residual $U(1)^N$ gauge invariance:

$$
S_{\text{Cartan}} = \int_{\Sigma_g} \sum_{i=1}^N \left[ \frac{t}{2} \lambda_i^2 d\mu - i \lambda_i dA^{(i)} \right],
$$

(6)

where $A^{(i)}$ is the $i$-th diagonal term of the matrix form $A$. The second term, named $S_{\text{off-diag}}$, contains the ghost and anti-ghost fields and the non–diagonal components of $A$:

$$
S_{\text{off-diag}} = \int_{\Sigma_g} d\mu \sum_{i \neq j} (\lambda_i - \lambda_j) \left[ \hat{A}_{ij}^0 \hat{A}_{ij}^0 - \hat{A}_{ij}^0 \hat{A}_{ij}^0 + i (c^{ij} \bar{c}^{ij} + \bar{c}^{ij} c^{ij}) \right],
$$

(7)

where $\hat{A}_{ij}^a = E_{\alpha}^a A_{ij}^\alpha$ and $E_{\alpha}^\mu$ denotes the inverse of the two dimensional vierbein. $S_{\text{off-diag}}$ has a fermionic symmetry, which exchanges gauge and ghost fields; hence one would expect the contributions to the partition functions from the ghost fields and the non–diagonal part of the gauge fields to cancel exactly.

However, this “supersymmetry” is in general broken by an anomaly in the functional measure, due to the fact that on a curved surface the number of degrees of freedom of a one-form (like $A_\mu$) and of two zero forms (like $c$ and $\bar{c}$) do not match exactly. In fact the corresponding functional integral has been calculated exactly in [5] and it is given by:

$$
\int \prod_{i \neq j} [dc^{ij}][d\bar{c}^{ij}][dA^{ij}_\mu] e^{-S_{\text{off-diag}}} = \exp \left[ \frac{1}{8\pi} \int_{\Sigma_g} R \sum_{i \neq j} \log(\lambda_i - \lambda_j) \right],
$$

(8)
where $R$ is the curvature scalar: only on flat Riemann surfaces like the torus or the cylinder the symmetry is preserved at the quantum level.

Following [5] one finds, by gauge fixing the residual $U(1)^N$ invariance, that in the end only configurations where the eigenvalues $\lambda_i$ are constant and equal to integers $n_i$ contribute. So the r.h.s. of (8) gives the standard dependence of YM2 partition function from the genus, while the dependence from $t$ is given by the $U(1)^N$ action $S_{\text{Cartan}}$ thus reproducing the well known partition function of YM2 on a Riemann surface [6, 7, 8]:

$$Z(\Sigma_g, t) = \sum_{\{n_i\}} \frac{1}{\prod_{i>j} (n_i - n_j)^{2g-2}} e^{-2\pi^2 t \sum_i n_i^2}. \quad (9)$$

However in the present derivation nothing forbids $n_i = n_j$ for $i \neq j$, and such terms, that we shall name non-regular terms following [5], are divergent for $g > 1$. Notice that non-regular terms always appear in the twisted sectors, where at least two sheets of the $N$-covering are connected by going round some non contractible loop. Therefore the corresponding eigenvalues are given by the same integer. The regularization proposed in [5] is done by adding mass terms to the non diagonal part of the gauge fields; in this way the non-regular terms vanish due to the ghost contribution which is proportional to $(n_i - n_j)^2$, while the contribution from the gauge fields is now finite and proportional to $(n_i - n_j - m_{ij})^{-2g}$. The limit $m_{ij} \to 0$ is performed at the end. This regularization scheme, while preserving the $U(1)^N$ gauge symmetry, violates the original BRST invariance as well as the fermionic symmetry between gauge fields and ghosts discussed above. A fully consistent treatment of the non-regular terms in the unitary gauge for $g > 1$ is indeed lacking, and in our opinion this is a problem worth looking into in the future. The case $g = 1$ is special: non regular terms are finite, the fermionic symmetry is anomaly free and hence there is no need of regularization. Therefore we shall write the partition function as a sum over all sectors labelled by commuting pairs of permutations $(P, Q)$, thus including non regular terms. We will find that the result does not coincide with the standard partition function [6, 7], which can be reproduced only by limiting the sum to the subset of sectors of the type $(P, 1)$. This choice however is not invariant under modular transformations on the torus.

3 Free energy and partition function

The twisted sectors can also be labelled, as discussed earlier on, by the $N$-fold covers without branch points of the torus. In order to sum over all sectors it is convenient on one hand to work in the grand canonical formalism, in which $N$ is not fixed, and introduce the grand-canonical partition function $Z(t, q)$ and the corresponding free energy $F(t, q)$:

$$Z(t, q) = e^{F(t, q)} = \sum_N Z_N(t) q^N. \quad (10)$$
On the other hand it is convenient to work directly on the free energy $F(t,q)$, that receives contributions only from the connected coverings. The computation of the free energy entails two aspects: an “entropic” one, i.e. the counting of the inequivalent connected coverings, and the determination of the Boltzmann factor that the functional integral (3) implies for each covering. The counting of $N$-coverings of the torus without branch points, namely of the $N$-fold maps of a world sheet torus into the target torus, has already been discussed in the literature (see for instance [9, 10]) and its free energy is given by

$$F_{\text{cov}} = \sum_{r|N} \sum r q^r = -\sum_{k=1}^{\infty} \log(1 - q^{1/r})$$

where $r|N$ means that the sum is extended over the divisors $r$ of $N$. The coefficient of $q^N$ in Eq. (11), namely $\sum_{r|N} r$, enumerates the connected $N$-coverings of the torus. This result can be derived as follows: let the periods of the target space torus be $\pi_{1}^{\text{tar}} = 2\pi$ and $\pi_{2}^{\text{tar}} = 2\pi i$, the most general connected $N$-covering is then a torus of area $4\pi^2 N$ whose periods are given by

$$\pi_{1}^{\text{ws}} = 2\pi k; \quad \pi_{2}^{\text{ws}} = 2\pi s + i 2\pi r,$$

with $kr = N$ and $s = 0, 1, \ldots, k-1$. An example with $N = 12$, $k = 4$ and $s = 3$ is given in Figure 1. There are $\sum_{r|N} k = N \sum_{r|N} 1/r$ choices of the integers $k,r$ and $s$ that satisfy these conditions, but one has to divide by the symmetry factor $N$ to account for the fact that coverings corresponding to different labelings of the sheets (i.e. of the eigenvalues) have to be identified. Notice that the world sheet torus has a modular parameter $\tau = s/k + i r/k = s/k + iN/k^2$ and that summing over these tori with the weight $1/r = k/N$ is the discrete version of integrating over the modular parameter $\tau$ with the usual modular-invariant measure. In fact from $\text{Im}\tau = N/k^2$ and $N$ fixed we have $\int d\text{Im}\tau/(\text{Im}\tau)^2 \to \sum_{r|N} \frac{k}{N}$.
It is easy to see that in terms of the field $\lambda$, the partition function (3) becomes the partition function of a $U(1)$ theory on the world sheet to rus. This partition function depends only on the area; it is well-known to be $\theta_3(0|2\pi Nt) = \sum_{n=-\infty}^{\infty} \exp(-2\pi^2 Ntn^2)$. This is the Boltzmann weight to be associated to the connected coverings of degree $N$.

We are now in the position of writing down the free energy, and thus automatically the grand-canonical partition function:

$$F^\pm(t, q) = \pm \sum_N \sum_{r[N]} \frac{1}{r} \sum_{n=-\infty}^{\infty} e^{-2\pi^2 rNtn^2} q^N = \pm \sum_{n=-\infty}^{\infty} \sum_{k=1}^\infty \log(1 - e^{-2\pi^2 ktn^2} q^k) ,$$

$$Z^\pm(t, q) = \prod_{n=-\infty}^{\infty} \prod_{k=1}^{\infty} (1 - e^{-2\pi^2 ktn^2} q^k)^{\mp 1} .$$

We have allowed in front of the free energy $F(t, q)$ a sign ambiguity, which corresponds to different choices of the a priori undetermined weights with which the contributions from the different twisted sectors are added. It is clear from (13) that the plus sign leads to partition function $Z^+$ of “bosonic” type (a “state” with fixed $k$ and $n$ may appear any number of times), while $Z^-$ is “fermionic” (the exclusion principle holds).

If we consider only the contribution of a subset of connected coverings, namely those with $k = 1$, $r = N$, that are associated to permutations of the type $(P, 1)$, we obtain the following partition function:

$$Z^\pm(t, q) = \prod_{n=-\infty}^{\infty} (1 - e^{-2\pi^2 tn^2} q^{1}) .$$

These expressions are known in the literature. $Z^-(t, q)$ is the grand-canonical expression that encodes, as shown in [2], the standard partition function for the $U(N)$ theory on the torus [6, 7]. $Z^+$ reproduces the partition function obtained in [11] by quantizing on the algebra rather than on the group. By comparing Eqs. (13) and (14) we find

$$Z^\pm(t, q) = \prod_{k=1}^{\infty} Z(kt; q^k) .$$

An expansion in powers of $q$ of both sides of Eq. (13) leads for the fermionic case to the following relation:

$$(-1)^N Z^-_{\{\cdot\}}(t) = \sum_{\{r_k\}} \delta(\sum_{k=1}^{N} ktr_k - N) \prod_{k=1}^{N} (-1)^{r_k} Z^{-}_{r_k}(kt) ,$$

$^2$The coefficient of $q^N$ in the power series expansion of $Z^-(t, q)$ coincides, up to a sign and an overall normalization factor, with the standard partition function of $U(N)$ only for odd values of $N$. For even values of $N$ the integers $n_i$ are replaced by half-integers in the standard YM partition function. This half-integer shift however can be re-absorbed by adding to the action a term proportional to $\text{tr} F$ (which is entirely in the $U(1)$ factor of $U(N)$). The expansion of $Z^-(t, q)$ for even $N$ gives such modified theory. Notice that in the case of $SU(N)$ the problem does not arise and the sum over the $(P, 1)$ sectors correctly reproduces the standard result for all values of $N$. 6
where $Z_{N}(t)$ is the $U(N)$ partition function including all sectors and $Z_{r_{k}}^{-}(kt)$ is the standard $U(r_{k})$ partition function (see however the discussion in the footnote). As an example we give the partition function for the case $N = 3$: 

$$Z_{3}^{-}(t) = Z_{3}^{-}(t) - Z_{1}^{-}(t)Z_{1}^{-}(2t) + Z_{1}^{-}(3t).$$ \hspace{1cm} (17)

The extra terms at the r.h.s. of (17) are related to the states with $k > 1$ and are not present in the conventional approach.

### 3.1 Modular invariance

One of the new features of our approach is that the ensemble of twisted sectors that contribute to the partition function is invariant under modular transformation on the cylinder, while the subset that reproduces the conventional result is not. In fact for instance a Dehn twist maps the sector labelled by $(1, P)$ into one labelled by $(P, P)$. As a result the conventional formulation is not modular invariant. This does not show in the partition function, which depends only on the area, but it should appear at the level of correlation function. For instance correlation functions of Wilson loops that are mapped into each other by modular transformations on the torus (like the two sets depicted in Figure 2) should coincide in our approach but not in the conventional formulation. Work is in progress to verify this point.

![Figure 2: Two sets of Wilson loops that are related by a Dehn twist.](image)

### 3.2 Quantization on the cylinder

The partition function on a torus is often calculated by first considering a cylinder with fixed holonomies at the edges and then by sewing the two ends together. This involves identifying the two end holonomies up to a gauge transformation, namely identifying their eigenvalues up to a permutation $P$. Hence a sum over $P$ appears in the final result. It is clear that in this way only the $(P, 1)$ sectors are taken into account, and in order to consider also sectors corresponding to non trivial commuting pairs $(P, Q)$, with $Q$ associated to the compact dimension of the cylinder, one has somehow to generalize the possible boundary conditions at
the edges of the cylinder which in the conventional approach are just the U(N)
holonomies. In order to do that let us choose the unitary gauge on the cylinder as in
Eq. (4), and consider a non trivial sector labelled by a permutation $Q$ that defines,
according to the second of Eq.s (5), the boundary conditions for the eigenvalues
$\lambda_i(\tau, x)$ as we go round the compact dimension parametrized by the coordinate $x$.
We refer to the original paper [1] for the details of the calculation and give here
only the main result. This can be summarized by saying that for non trivial $Q$
there are as many independent invariant angles in the holonomies as the number of
cycles in $Q$. More precisely, supposing that $Q$ has $r_k$ cycles of order $k$ the invariant
angles $\theta_i$ of a Wilson loop winding round the compact dimension have the following
structure:

$$\theta_{k,\alpha,n} = \theta_{k,\alpha} + \frac{2\pi in}{k},$$ (18)

where we have made the replacement $i \rightarrow (k, \alpha, n)$ with $\alpha = 1, \ldots, r_k$ and $n =
0, 1, \ldots, k - 1$ to denote that $i$ is the $n$-th element of the $\alpha$-th cycle of order $k$. The
independent invariant angles are just the $\theta_{k,\alpha}$, and are associated to the cycles, the
other eigenvalues within each cycle being spaced like the $k$-th roots of 1. When
sewing the cylinder the invariant angles are identified up to a permutation $P$ that
preserves the cycle structure, namely

$$(k, \alpha, n) \xrightarrow{P} (k, \pi_k(\alpha), n + s(k, \alpha)),$$ (19)

where $\pi_k \in S_{r_k}$ is a permutations of the $r_k$ cycles of order $k$ and $s(k, \alpha)$ is an
integer shift mod $k$. Eq. (19) is equivalent to the statement that $P$ commutes
with $Q$, and hence it reproduces the by now familiar pattern of the twisted sectors.
From the previous discussion it is clear that the end states on the cylinder are not
parametrized by the U(N) holonomies but rather by the holonomies of $U(r_1) \otimes
U(r_2) \otimes \ldots$. It is also clear that by considering just the U(N) holonomies one is
automatically projecting on the trivial sector $Q = 1$. Although this projection
appears the most natural thing to do in the framework of gauge theories, and it
is so far also the only approach that we know how to implement on a lattice, it
introduces an asymmetry between the two generators of the torus and ultimately
breaks the modular invariance in the sense mentioned above.

4 YM2 as a Matrix String Theory.

The grand canonical partition function $Z^\pm(t, q)$ given in Eq. (13) describes in the
large $t$ limit the coverings of the torus. In fact in that limit the Boltzmann factor
given by the partition function of the U(1) gauge theory tends to 1 and we are left
with the partition function that simply counts the homotopically distinct coverings
of the $g = 1$ target space by a $G = 1$ world-sheet: $Z^\pm(t, q) \xrightarrow{t \rightarrow \infty} Z_{\text{cov}}(q)$. So in this
limit the twisted sectors define a string theory, exactly in the same way as they
do in Matrix string theory. It can also be argued, and in the fermionic case it is
obvious from the exclusion principle, that in this limit large values of \( k \), namely long strings, are relevant. Quite the opposite happens in the limit \( t \to 0 \), in fact one can see from the Poisson re-summation formula that in this limit the Boltzmann weight behaves like \( 1/\sqrt{t} \), and so the leading contributions to the partition function come from coverings that maximize the number of disconnected world sheets. So only states with \( k = 1 \) survive in the \( t \to 0 \) limit, and the theory reduces to the conventional one without twisted sectors. It is possible, but yet not proved, that a phase transition separates the two phases dominated respectively by long and short strings. The small \( t \) region is also the relevant one for the large \( N \) limit of YM2 studied by Gross and Taylor in a series of papers \([1]\). In fact the limit is done, following the original idea of ’t Hooft, by taking \( t = t/N \) and keeping \( \tilde{t} \) fixed. Gross and Taylor proved that the partition function of \( U(N) \) Yang-Mills theory on a two dimensional Riemann surface \( M_g \) of genus \( g \) counts the homotopically distinct maps from a world-sheet \( W_G \) of genus \( G \) to \( M_g \). In particular if the target space is a torus and one considers only the leading term in the large \( N \) expansion, which means considering only world sheets of genus 1, one finds the partition function

\[
Z_{GT} = \prod_{k=1}^{\infty} (1 - e^{-kt})
\]

This is the same string partition function that is found from the twisted sectors in the large \( t \) limit, with \( q \) replaced by \( e^{-\tilde{t}} \). This coincidence is suggestive of some kind of underlying duality in the theory. However for this duality to exists beyond the rather trivial case of \( W_{G=1} \to M_{g=1} \) maps we should be able to extend the twisted sectors introduced in \([1]\) to include coverings with branch points, which correspond to higher genus world sheets, namely to the possibility for strings to split and join. These would be dual to the sub-leading terms in the \( 1/N \) expansion of \([1]\). However branch points correspond to points where the curvature of the world sheet becomes a delta function, leading through Eq. (8) to terms coupling different sheets of the coverings. The problem here is of the same nature as the one that is encountered if one quantizes YM2 in the unitary gauge on a surface with \( g > 1 \), namely the appearance of logarithmic interactions between different eigenvalues, that eventually lead to divergences. A better understanding of this point is then crucial also for a consistent formulation of YM2 as a matrix string theory on a torus, so it is not yet clear to us if such formulation exists also at the level of string interactions, or weather this can be done only by embedding YM2 in the more general framework of the Matrix String Theory given by \([1]\), where it is known that string interactions can be consistently introduced \([12]\). In both cases the analysis developed in \([1]\) is relevant. In fact gauge degrees of freedom are crucial in Matrix String theory for the description of string interactions as well as for the relation of non-trivial fluxes with D-brane charges \([4]\). Shortly after our paper appeared on the hep-th archive, Kostov and Vanhove \([13]\) calculated the partition function of the Matrix String Theory of Eq. (11). They took advantage of the fact that the contributions to the partition function of the ”matter fields” \( X^M \) and \( \theta \) cancel due
to supersymmetry, so that in the end the only contributions come from the different topological sectors of YM2. In fact their result coincides with ours, except that they obtain our free energy rather than the partition function because the structure of the fermionic zero-modes in the matter sector effectively kills the contributions from disconnected world sheets. In [13] the dimensional reduction of this free-energy to zero dimensions is shown to correctly reproduce the partition function for the IKKT model [14], namely
\[ Z_{\text{IKKT}} \propto \sum_{r|N} \frac{1}{r} \]
The latter [15] is related to the moduli space of D-instantons or, by T-duality, to the counting of bound states of D0-branes.

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