Interaction of the SN1987A Neutrino with the Galaxy

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Abstract

In previous publications we have shown that long-ignored approximations in standard model calculations could have significant implications for very low mass particles such as neutrinos and photons. In particular we showed that, in a dilute plasma such as that in the solar corona, a significant decay probability of $\nu' \rightarrow \nu + \gamma$ will be possible as a consequence of the terms ignored in making the approximations. Here the $\nu'$ and $\nu$ are high and low mass eigenstates of the neutrino.

In this paper, we investigate the effect in the vicinity of an expanding supernovae remnant such as SN1987A. We show that, in the dilute plasma external to the remnant, such decays are possible and significant. We describe a calculation of effects of such decays on the anti-neutrinos from SN1987A. The calculated anti-neutrino energy against arrival time agrees reasonably well with that observed, assuming that the expansion velocity of the remnant is $\approx 0.2c$ and that the plasma density is high within the expanding remnant.
I. INTRODUCTION

Anti-neutrinos from the supernovae SN1987A in the Large Magallanic Cloud (LMC) have been detected and measured by the Baksan, IMB, and Kamiokande Collaborations [1–3]. The results indicate that their mean energy falls with time. In this paper an explanation of the phenomenon is given involving a new process.

Calculation of the transition rate, $\Gamma$, for the process

$$\nu' \rightarrow \nu + \gamma \quad (1)$$

using the Standard Model (SM) of particle physics shows that it is very small. However, the SM calculation assumes plane waves for the wave functions of the particles which quickly become independent of the source and the integrations are made over infinite space and momentum [4–7]. This is an approximation since the sources of the particles involve finite distances and sizes and the waves are not exactly plane. The difficulty of the calculation using plane waves was found sometime ago by Stueckelberg [8]. Making exact calculations with the appropriate boundary conditions and without these approximations shows that the transition probability, $P$, becomes

$$P = \Gamma T + P^{(d)} \quad (2)$$

where $T$ is a time interval for $P < 1$ [9–11]. The term $\Gamma T$ results from the SM calculation while the term $P^{(d)}$, which is time independent, is an extra term resulting from the ignored approximations in the SM calculation. In most SM applications involving heavy particles the probability $P^{(d)}$ is small and undetectable. However, this is not always the case for very low mass particles, such as neutrinos [12] or where $\Gamma T$ is small [13–17]. It has been shown in a previous publication that if neutrinos pass through a plasma of low enough density, such that the effective mass of the photon is smaller than the neutrino mass difference, the probability $P^{(d)}$ is finite and can have observable consequences. Here the photon effective mass is $m_{\gamma, eff} = \hbar \omega_p$ where $\omega_p$ is the plasma frequency. One such consequence is the stimulation of the decay(equation 1) for neutrinos in the low density plasma of the solar corona. The decay rate predicted by the theory is sufficient to account for the heating of the corona to very high temperatures, a so-far unexplained phenomenon [18]. The effect in the solar corona is analogous to the Hall effect for electrons in solids and it is termed the Electroweak Hall Effect (EHE).
The assumption of plane waves and asymptotic independence of the wave functions in the SM calculation results in a Dirac Delta function in the transition probability. This automatically conserves the measurable energy for finite times. However, this is not the case for neutrinos in a dilute plasma where $\Gamma$ is small and $P^{(d)}$ is finite. An interaction energy of waves then enters through the many-body interactions. Consequently the total energy is conserved but the visible energy/momentum, termed kinetic energy, are not conserved since the interaction potential energy term is not accounted for \[18\]. A consequence of this is that in the plasma which stimulates the decay in equation 1, the outgoing $\nu'$ and $\gamma$ waves are coherent only at small angles less than of order $\frac{m^2_{\nu}}{E^2_{\nu}}$ where $m_{\nu}$ and $E_{\nu}$ are the mass and energy of the decaying neutrino. The $P^{(d)}$ term should also exist for photons interacting via either Thompson or Compton scattering. The photon and scattered electron waves will be coherent and depend on the source size. Due to the large source sizes deviations from the standard formula for these processes occur in the far forward directions. It is shown that in the dilute plasma external to SN1987A the coherent electron-photon wave packets move at reduced velocity. In this way the photons from the decay of the anti-neutrinos will be delayed in their arrival time at the Earth. This accounts for the failure of the Solar Maximum Mission (SMM) \[21-23\] to detect prompt gamma rays from the decays of the anti-neutrinos from SN1987A. In this paper the details of the calculation of $P^{(d)}$ for the anti-neutrinos from SN1987A are given and the spectrum of energies and arrival times are computed and compared with the measurements.

In Section 2, the electroweak Hall interaction and the wave functions of the decaying neutrino are presented. The transition amplitude and probability are computed in Section 3, and the comparison with the observation is made in Section 4. The summary is given in Section 5.

II. ELECTROWEAK HALL EFFECT AND ANOMALOUS RADIATIVE TRANSITION

$P^{(d)}$ has different properties from $\Gamma T$. For its computation, we follow the von Neumann’s fundamental principle of quantum mechanics (FQM), that connects the probability $P$ with the state vectors, $P = |\langle \alpha | \beta \rangle|^2$, for normalized states. Since states in nature or experiments have finite sizes, the probability thus computed is compared with the observations. The
plane waves, which are idealistic for theoretical studies, are not appropriate owing to its non-normalized nature.

It was found before \[9, 10\] that \(P(d)'s\) characteristic length is \(\lambda(d) = \frac{2\nu c E}{m_c^2}\), where \(c, m\), and \(E\) are the light velocity, mass, and the energy which depend on the absolute mass \(m_{\nu}\) and \(m_{\gamma}\) the effective photon mass \(m_{\gamma}\) determined from the plasma frequency. This is much longer than the de Broglie length \(\lambda_{dB} = \frac{\hbar}{p}\), where \(\hbar = \frac{h}{2\pi}\) and \(h\) is the Plank constant, and \(p\) is a momentum for light particles, and can be extremely long. Hence, the effect appears as a macroscopic quantum phenomenon. \(P(d)\) is independent of \(T\), and important in the processes for which \(\Gamma\) is very small such as \(\nu + \gamma \rightarrow \nu + \gamma, \nu + B(E) \rightarrow \nu + \gamma\) \[15–17\]. \(P(d)\) derived from the vacuum fluctuation of the tri-angle electron loop which would be useful for relic neutrino observations \[11\], now is further enhanced by the electroweak Hall effect in the dilute weak magnetized plasma.

The interaction Hamiltonian of the electroweak Hall effect is obtained from the one-loop effect of the electrons in the magnetic field in the form, \[18\]

\[
H_{\text{int}} = H_{\text{Faraday}} + G_{\nu,\gamma} \int d\vec{x} \left( \bar{\nu}(x) (1 - \gamma_5)\gamma_\mu \nu'(x) \right) e^{\mu\alpha\beta} \partial_\alpha A_\beta, \tag{3}
\]

where \(H_{\text{Faraday}}\) is the Chern-Simons term of the electromagnetic potential, \(G_F\) is Fermi coupling constant, \(n_e\) is the electron density, and \(B\) is the magnetic field. Electrons in each Landau level give independent contributions in the loop integral and the coupling strength is proportional to the filling fraction and sizable in a dilute plasma in a weak magnetic field. \(H_{\text{int}}\) derives from the quantum fluctuation of the electrons in the magnetic field, and \(H_{\text{Faraday}}\) gives the Hall effect for the electromagnetic current and the Faraday rotation of the electromagnetic waves, which is useful for the measurement of the magnetic field in the Galaxy \[24\]. The rest gives the neutrino-photon interaction, and is applied to the neutrinos from SN1987A. For the magnetic field in the \(z = x_3\)-direction, \((\mu, \alpha, \beta)\) is the space perpendicular to \(x_3\)-axis, i.e., \((0, 1, 2)\).

The Schrödinger equation for the processes Eqs. \[11\] is

\[
i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle = (H_0 + H_{\text{int}}) |\Psi(t)\rangle, \tag{4}
\]

\[
H_0 = \int d\vec{x} \sum_{l=e,\mu} \left( \bar{l}(x) \left( \vec{\alpha} \cdot \vec{\nabla} + \beta m_l \right) l(x) + \bar{\nu}_l(x) \left( \vec{\alpha} \cdot \vec{\nabla} + \beta m_{\nu_l} \right) \nu_l(x) \right).
\]
The case of no-mixing is considered first. Normalized $|\Psi(t)\rangle$ evolving from one neutrino state of momentum $\vec{p}_\nu$ coming from a source size of $\sigma_\nu$ at $t = 0$ is

$$|\Psi(t), \vec{p}_\nu, \vec{X}_\nu, \sigma_\nu\rangle = a_0(t)|\vec{p}_\nu, \vec{X}_\nu, \sigma_\nu\rangle + \int d\vec{p}_\nu d\vec{p}_\gamma a_1(t, \vec{p}_\nu, \vec{X}_\nu, \sigma_\nu, \vec{p}_\gamma)|\vec{p}_\nu, \vec{X}_\nu, \sigma_\nu, \vec{p}_\gamma, \sigma_\gamma\rangle + O(G_F^2),$$

$$a_0(t) = (1 + \zeta(t))^{-1/2} e^{-i\frac{E_\nu}{\hbar}t},$$

$$a_1(t, \vec{p}_\nu, \vec{X}_\nu, \sigma_\nu, \vec{p}_\gamma, \sigma_\gamma) = (1 + \zeta(t))^{-1/2} e^{-i\frac{E_\nu}{\hbar}t} e^{-i\frac{\omega t}{2\hbar}} \frac{1}{\omega} (\vec{p}_\nu, \vec{X}_\nu, \sigma_\nu, \vec{p}_\gamma, \sigma_\gamma | H_{\text{int}} | \vec{p}_\nu, \sigma_\nu\rangle, \ (6)$$

$$\zeta(0) = 1, \zeta(t) = \zeta < 1; \ (t > 0)$$

in the lowest order of $G_F$, where $\omega = E_\nu' + E_\gamma - E_\nu$, and

$$\zeta(t) = \int_{\omega \neq 0} d\vec{p}_\nu d\vec{p}_\gamma \frac{2 \sin \frac{\omega t}{2\hbar}}{\omega} |\vec{p}_\nu, \sigma_\nu, \vec{p}_\gamma, \sigma_\gamma | H_{\text{int}} | \vec{p}_\nu, \sigma_\nu\rangle|^2. \ (7)$$

$\sigma_\gamma$ is the range in space covered by the wave function that the photon interacts with in the microscopic process, which is called the photon wave packet hereafter. That is large and the waves deviate slightly from the plane waves. $\sigma_l$ stands for the wave packet size for particle $l$. $\sigma_\nu$ for the initial neutrino is determined by the size of the star, and $\sigma_\nu'$ and $\sigma_\gamma$ for the final neutrino or photon is determined by the range in space covered by the wave function of microscopic object which these interact. $\sigma_{a+b}$ stands for the cross section of the process $a + b$. In most of this paper $\sigma_\nu$, $\sigma_{\nu'}$, and $\sigma_{\gamma}$ are considered large. $\Gamma$ is proportional to $m_\nu^5$ and practically $\Gamma$ is negligibly small, $\tau = \infty$, hence $a_0(t)$ and $a_1(t)$ have no exponential damping factor in time of the Weisskopf-Wigner formula [5]. $\zeta(t)$ converges due to the wave packets and approaches a constant $\zeta$ rapidly. The square of norm of $\nu$ and $\nu' + \gamma$ at $t \to \infty$ is $(1 + \zeta)^{-1}$ and $(1 + \zeta)^{-1} \zeta$, where the former is the survival probability for the parent. $P^{(d)}$ is independent of $\Gamma$, and $\Gamma = 0$ but $P^{(d)} \neq 0$. [9–11].

The integral Eq. (7) from the region $\omega \neq 0$ diverges, if all the states are plane waves, and has been considered not to be relevant to a physical phenomenon. Accordingly any physical quantity has not been derived from the region $\omega \neq 0$. However the divergence is inherent to the plane waves and disappears in realistic situations. The final state is expressed by a wave packet of finite size in the physical process, where the decay product interacts with other microscopic states of the finite range in space. The integral is convergent then and the probability derived from the region $p_\nu \to \infty$ and that from the region $p_\gamma \to \infty$ become finite and possesses universal properties. They determine the probability of the events that the photon or the neutrino is measured or that they make reactions. In both cases, the
unmeasured state, i.e., the $\nu'$ in the former and the $\gamma$ in the latter includes the state $p \to \infty$ inherent in a relativistic invariance. Because the waves in two regions are different, they are independent each other, and are computed in the next section.

III. THE TRANSITION AMPLITUDE

The probability amplitude of $\nu \to \nu' + \gamma$ is determined by the initial and the final wave functions at finite time interval following FQM using $S[T]$, the matrix element which is determined from Eq. (6). $S[T]$ satisfies $[S[T], H_0] \neq 0$ due to the overlap of waves, whereas the standard S-matrix, $S[\infty]$, satisfies $[S[\infty], H_0] = 0$ from the asymptotic boundary condition at $T \to \infty$. $S[\infty]$ is useful for computing $\Gamma$ but useless for $P^{(d)}$. $S[T]$ is formulated with Møller operator, and the normalizable wave functions, wave packets that are localized in space and specified by their centers in the momentum and coordinate. The amplitude for an initial neutrino denoted as $|\nu\rangle$ to final neutrino $\nu'$ and a photon of the momentum $\vec{p}_\gamma$, $|\nu\rangle = |\vec{p}_\nu, \vec{X}_\nu, T_\nu; \sigma_\nu\rangle$, $|\nu'\gamma\rangle = |\vec{p}_\nu', \vec{X}_\nu', \sigma_\nu'; \vec{p}_\gamma, T_\gamma\rangle$; $\delta p = p_\nu - p_{\nu'} - p_\gamma$ is

$$M = G_{\nu, \gamma} \varrho_{\nu} \varrho_{\nu'} f I(\delta p), f = \bar{u}(\vec{p}_\nu)\gamma^\rho(1 - \gamma_5)u(\vec{p}_\nu')\epsilon^{\text{mag}}(p_\gamma), \quad (8)$$

$$\varrho_\alpha = \left(\frac{m_\alpha}{(2\pi)^3 E_\alpha}\right)^{1/2} \left(\alpha = \nu, \nu'\right), \quad \varrho_\gamma = \left(\frac{1}{(2\pi)^3 2E_\gamma}\right)^{1/2},$$

$$\epsilon^{\text{mag}}(p_\gamma) = \langle \text{matter}'|\epsilon^{\alpha\beta} \partial_\alpha A_\beta|\text{matter}\rangle, \quad (9)$$

where $\bar{u}(p_\nu), u(p_{\nu'})$, and $\epsilon^{\text{mag}}(p_\gamma)$ are the spinors of the neutrinos and the photon coupling vector with matter in the magnetic field, and $(\rho, \alpha, \beta)$ is the three dimensional space of Eq.(4). In Eq.(8) the last term is,

$$I(\delta p) = \int_{T_\nu}^{T_\nu'} dt \int d\vec{x} e^{i\phi_\gamma(x)} w(x, X_\nu; \sigma_\nu) w^*(x, X_{\nu'}; \sigma_{\nu'})$$

$$\phi_\gamma(x, \vec{p}_\gamma) = E(\vec{p}_\gamma)t - \vec{p}_\gamma \cdot \vec{x}, \quad (10)$$

where the wave function is

$$\omega(x, X_\alpha, \sigma_\alpha) = \left(\frac{4\pi}{\sigma_\alpha}\right)^{1/2} e^{-\frac{1}{2\sigma_\alpha}(\vec{x} - \vec{X}_\alpha - \vec{v}_\alpha(t - T_\alpha))^2 - i\phi_\alpha(x, \vec{p}_\alpha)} \quad (11)$$

$$\phi_\alpha(x, \vec{k}_\alpha) = E(\vec{k}_\alpha)(t - T_\alpha) - \vec{k}_\alpha \cdot (\vec{x} - \vec{X}_\alpha), \quad (\alpha = \nu, \nu').$$
The photon’s coupling with matter through the magnetic coupling expressed by a normalized polarization vector $\epsilon_{\rho}^{\text{mag}}(p_{\gamma})$ and a coupling strength $f_{\gamma}$

$$\epsilon_{\rho}^{\alpha\beta}\partial_{\alpha}A_{\beta}(p_{\gamma}) = h_{\gamma}\epsilon_{\text{mag}}^{\rho}(p_{\gamma}), \quad h_{\gamma} = \sqrt{\frac{2p_{\gamma}^{2}}{3}},$$

of satisfying

$$\sum_{\rho} |\epsilon_{\rho}^{\alpha\beta}(p_{\gamma})_{\alpha}\epsilon_{\beta}(p_{\gamma})|^{2} = |\sum_{\rho} h_{\gamma}\epsilon_{\text{mag}}^{\rho}(p_{\gamma})|^{2},$$

where $\epsilon_{\beta}(p_{\gamma})$ is the photon’s polarization vector. The spreading of wave packet at large $|t - T_{\nu_e}|$ now is negligible [10].

The amplitude Eq.(8) is almost identical to that of the plane waves, but now the wave function is normalizable and the time interval is finite $T$. The overlapping waves interact each others, and the S-matrix $S[T]$ satisfies $[S[T], H_0] \neq 0$. Thus the transition to the kinetic energy non-conserving states is included and its probability $P^{(d)}$ is computed following the FQM.

A. Transition probability

The vector index in Eq.(8) is in (0, 1, 2), and the the spin average $\sum_{\text{spin}} |f|^{2} = \frac{2}{3} 2^{4} m_{\nu_{e}} m_{\nu_{\mu}} (\bar{p}_{\gamma}^{\cdot})^{2} (\bar{p}_{\nu_{\mu}}^{\cdot} - \frac{2}{7} \bar{p}_{\nu_{e}} \cdot p_{\nu_{e}})$, where $\bar{p}$ is a vector in this three dimension and a scalar products is that of the same three dimensional subspace. The probability for the event that the neutrino is measured or interacts, $P_{\nu}^{(d)}$, and that for the photon, $P_{\gamma}^{(d)}$, are computed following the method [9–11],

$$P = \int d\bar{p}_{\nu_{\mu}} \frac{d\bar{X}_{\nu_{\mu}}}{(2\pi)^{3}} d\bar{p}_{\gamma} |M|^{2}.$$  

1. Interaction of neutrino and photon waves with a large wave of matter

Neutrino probability $P_{\nu}^{(d)}$

A nucleus or atom in galaxy has a large mean free path due to the low density, and is expressed by the range in space covered by $\sigma_{\nu_{\mu}}$, which is different from that of the initial neutrino $\sigma_{\nu}$ determined by the size of the star and satisfies $\sigma_{\nu} \ll \sigma_{\nu_{\mu}}$. The probability is written with the smallest wave packet, $\sigma_{\nu}$ now. The photon is not measured and integrated

$$P_{\nu}^{(d)} = \int d\bar{p}_{\nu_{\mu}} \frac{d\bar{X}_{\nu_{\mu}}}{(2\pi)^{3}} d\bar{p}_{\gamma} |M|^{2}.$$
over the positive energy region. Hence $P^{(d)}_\nu$ is computed with the correlation function,

$$\Delta_{\nu,\gamma}(\delta x) = \frac{2}{3} \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_\nu}{2E_\gamma} 4^4(\vec{p}_\nu)^2(\vec{\bar{p}}_\nu \cdot \vec{\bar{p}}_\nu - 3/2p_{\nu, \cdot} \cdot p_{\nu'}) e^{-i(p_{\nu} - p_{\nu'})\delta x}. \quad (15)$$

The light-cone singular term inherent to the relativistic system from states $\omega = \infty$ in Eq.(7) couples with $\Delta_{\nu,\gamma}(\delta x)$ and gives the leading contribution,

$$P^{(d)}_\nu = \frac{2}{3} \frac{1}{(2\pi)^3} G_{\nu,\gamma}^2 \frac{1}{E_\nu} 2^4 \sigma_\nu \int \frac{d\vec{p}_\nu'}{E_{\nu'}} (\vec{p}_\nu - \vec{\bar{p}}_{\nu'})^2(\vec{p}_\nu \cdot \vec{\bar{p}}_{\nu'} - p_{\nu'} \cdot p_{\nu'}) T\tilde{g}(\omega_\nu T), \quad (16)$$

where $\omega_\nu = \frac{m^2}{2E_\nu}$, and $v_\nu = c = 1$ and the electron mass was neglected, and the fact that the wave packet vanishes at $(t - T_\nu)^2 - (\vec{x} - \vec{X}_\nu)^2 \leq 0$ is not important now, and is ignored. The asymptotic behavior of $\tilde{g}(\omega_\nu T)$ given in Appendix is substituted.

The phase space is in the region $10$, $2p_\nu \cdot p_{\nu'} \leq m^2_\nu + m^2_{\nu'} - m^2_\gamma$ and the integral over the angle $\theta$ between the momenta of $\nu$ and $\nu'$ is made in the region,

$$1 - \cos \theta \leq \frac{1}{2E_\nu E_{\nu'}}[(1 - \frac{E_{\nu'}}{E_\nu})m^2_\nu + (1 - \frac{E_\nu}{E_{\nu'}})m^2_{\nu'} - \frac{m^2_\nu m^2_{\nu'}}{2E_\nu E_{\nu'}} - m^2_\gamma]. \quad (17)$$

We have the total probability expressed by the integral over the momentum fraction $x = \frac{\lvert\vec{\bar{p}}_{\nu'}\rvert}{\lvert\vec{p}_\nu\rvert}$

$$P^{(d)}_\nu = \frac{1}{(2\pi)^3} G_{\nu,\gamma}^2 \sigma_\nu E^4_\nu F_\nu(\xi), \xi = (m_\nu/m_{\nu'})^2, \quad (18)$$

$$F_\nu(\xi) = 2^6 \int_{1/\xi}^{1} dx x^3(x\xi - 1) \rightarrow \frac{8}{15} \xi \rightarrow \infty.$$

$P^{(d)}_\nu$ has unique properties; that is proportional to the range in space covered by the initial neutrino $\sigma_\nu$, the fourth power of the neutrino energy $E^4_\nu$, and the neutrino mass-squared ratio $\xi$. Here $\sigma_\nu$ is $\pi \times R^2$, where $R$ is the radius of the exploding star, and is a large macroscopic value. The average fraction, $\langle x \rangle = 3/7$ at $\xi \rightarrow \infty$ of about 0.5 is due to the modified phase space Eq.(17) that includes the region satisfying the inequality. The absolute value of momentum can deviate from the initial value despite $\theta \approx 0$. It is noted that a naive value of $\Gamma$ for a weak process $G_{\mu,\gamma}^2 m^5_\mu$ is negligible, but $P^{(d)}_\nu$ is different and can give significant effects. The $E^4_\nu$ is larger than $m^4_\nu$ by the factor $(E_\nu/m_\nu)^4$, which becomes now $(\frac{10^7}{10^{-3}})^4 = 10^{32}$. The enhancement due to the electroweak Hall effect is further amplified by the Lorentz non-invariance.

**Photon probability** $P^{(d)}_{\gamma}$

The probability that the photon interacts with matter is expressed by their wave functions, and the range in space covered by them is determined by that mean free path. They
satisfy $\sigma_\nu \approx \sigma_\nu' \gg \sigma_\nu$. The neutrino momentum in the final state is integrated in the phase space is replaced with, $2p_\nu \cdot p_\gamma \leq m_\nu^2 - m_{\nu'}^2 + m_\gamma^2$. Now $\tilde{\omega}_\gamma = E_\gamma (1 - \cos \theta) + \frac{m_\gamma^2}{2E_\gamma}$, where $\theta$ is the angle between $\vec{p}_\nu$ and $\vec{p}_\gamma$, which is almost zero from Eq. (20) discussed later. It follows that for a large $T$,

$$P^{(d)}_\gamma = \frac{1}{(2\pi)^3} G^{\gamma}_\nu G^{\nu}_\nu \frac{1}{E_\nu} 2^4 \sigma_\nu \int \frac{d\vec{p}_\nu}{E_\gamma} (\vec{p}_\nu - \vec{p}_\gamma) \cdot \vec{p}_\gamma \vec{p}_\gamma' \cdot \vec{p}_\nu (T \tilde{g}(\tilde{\omega}_\gamma T)).$$ (19)

Substituting the asymptotic form of $\tilde{g}(\tilde{\omega}_\gamma T)$, and integrating over the the region $1 - \cos \theta \leq \frac{1}{2p_\nu p_\gamma} [(1 - \frac{p_\gamma}{p_\nu})m_{\nu'}^2 - m_{\nu'}^2 + m_{\gamma}^2]$, (20)

we have the total probability expressed by the integral over the momentum fraction $x = \frac{p_\gamma}{p_\nu}$

$$P^{(d)}_\gamma = \frac{1}{(2\pi)^3} G^{\gamma}_\nu \sigma_\nu E_\nu^4 F_\gamma(\xi) \log \frac{2E_\nu^2}{m_\gamma^2},$$ (21)

$$F_\gamma(\xi) = 2^4 \int_{1/\xi}^{1-1/\xi} dx (1-x)(x-1/\xi) \rightarrow \frac{8}{3} \xi \rightarrow \infty,$$

which is proportional to the range in space covered by the initial neutrino $\sigma_\nu$ and the log factor of the initial momentum over the mass, the fourth power of the neutrino energy $E_\nu^4$. The probability is enhanced over the normal case by a factor $(\frac{E_\nu}{m_\nu})^4$ and by the large log factor of the momentum $\log \frac{2E_\nu^2}{m_\gamma^2} \approx 10^2$. For the small $T$, we have

$$P^{(d)}_\gamma = \frac{1}{(2\pi)^3} G^{\gamma}_\nu \sigma_\nu E_\nu^4 F_\gamma(\xi),$$ (23)

$$F_\gamma(\xi) = 2^4 \int_{1/\xi}^{1} dx (1-x)(x-1/\xi) \rightarrow \frac{8}{3} \xi \rightarrow \infty,$$

which is independent of $\sigma_\nu$. $m_\gamma$ is extremely small and $\frac{1}{m_\gamma} \approx (2 \times 10^8)^2 m_\gamma$ for $m_\gamma = 10^{-15} eV$. Thus $P^{(d)}$ of Eq. (23) is not very different from that of Eq. (21). The average energy fraction of the final neutrino, $\langle x \rangle = 1/2$ at $\xi \rightarrow \infty$ from the same reason as the previous case.

**Summary of $P^{(d)}$**

The overlapping waves of the initial and final neutrinos result to $P^{(d)}_\nu$ from the kinematic region $p_{\nu'} \leq p_\nu; p_\gamma \rightarrow \infty$, and those of the initial neutrino and the final photon result to
\( P^{(d)} \) from the region \( p_\gamma \leq p_\nu, \ p_\nu' \to \infty \). They are from different kinematic regions, and are added.

The satellite galaxy LMC is likely to have magnetic fields and electron densities similar to our own galaxy, the Milky Way. These will also affect the neutrino by the electroweak Hall effect so that the total value of \( P^{(d)} \) for the neutrinos will be the sum of the conventional probabilities in the LMC and the Milky Way Galaxy. We denote \( c_1 \) for the Galaxy and \( c_2 \) for LMC, and write the probability respectively as

\[
P^{(d)}(c_i) = P^{(d)}(c_i) + P^{(d)}(c_\gamma) = \frac{1}{(2\pi)^3} \frac{e^2 G_F^2}{2} \left( \frac{\nu^{(4)}(c_i)}{2\pi} \right)^2 \sigma_\nu E_\nu^4 F
\]

\[
F = F_\gamma(\xi) \log \frac{2 E_\nu^2}{m_\gamma^2} + F_\nu(\xi).
\]

\( F \) is around \( F \approx 10^3 \) in the Galaxy, where \( m_\gamma c^2 = 10^{-16} \text{eV} \), \( p_\nu = 20 \text{MeV} \), and \( \xi = 10^3 \) are used. In LMC, the density and the magnetic field are not known well and are left as parameters. The \( P^{(d)} \) is the sum of those of the Galaxy and LMC,

\[
P^{(d)} = P^{(d)}(c_1) + P^{(d)}(c_2) = \frac{1}{(2\pi)^3} \frac{e^2 G_F^2}{2} \sigma_\nu E_\nu^4 F \sum_i \left( \frac{\nu^{(4)}(c_i)}{2\pi} \right)^2
\]

Here \( \sigma_\nu \) depends on the radius. That varies slowly with the radius and the average value appears in the final expression. There is no contribution to \( P^{(d)} \) from the region \( E_\nu' \to \infty, E_\gamma \to \infty \).

2. **Survival probability**

The initial neutrino lowers the flux due to the transition \( \nu \to \nu' + \gamma \). From the unitarity

\[
\langle \nu | S[T]|\nu \rangle |\nu |S[T]|\nu \rangle + \langle \nu | S[T]|\nu' , \gamma \rangle |\nu', \gamma |S[T]|\nu \rangle = 1,
\]

where the second term in the left-hand side is computed from \( P^{(d)} \), the probability that the initial neutrino remains is given by

\[
|\langle \nu | S[T]|\nu \rangle^2| = 1 - |\langle \nu' , \gamma |S[T]|\nu \rangle|^2.
\]

For \( P^{(d)} \ll 1 \), the correction to the norm of the initial state is negligible and

\[
|\langle \nu', \gamma |S[T]|\nu \rangle| = P^{(d)}.
\]
For a larger $P^{(d)}$, including the norm’s correction, we have

$$|\langle \nu', \gamma | S[T] | \nu \rangle|^2 = \frac{P^{(d)}}{1 + P^{(d)}}.$$  \hfill (29)

The survival probability of the initial neutrino and the probability of the produced photon

$$P(\nu \rightarrow \nu) = \frac{1}{1 + P^{(d)}},$$  \hfill (30)

$$P(\nu \rightarrow \gamma) = \frac{P^{(d)} \gamma}{1 + P^{(d)}}$$

will be compared with the observations.

3. Mixing effect

There are three neutrinos and they mix each others. For mass eigenstates $\nu_i(x); i = 1, 3$ of the masses $m_{\nu_i}$, and the mixing matrix $U_{\alpha, i}$, the flavor neutrino fields $\nu_l(x)$ in Eq. (4) are the linear combination

$$\nu_l(x) = \sum_i U_{l, i} \nu_i(x), \ l = e, \mu, \tau,$$  \hfill (31)

where the best-fit values of mixing angles given in Ref. [12]

$$\sin^2 2\theta_{12} = 0.846 \pm 0.021,$$

$$\sin^2 2\theta_{23} = 0.999^{+0.001}_{-0.0018} \text{ (normal hierarchy)}, \ \sin^2 2\theta_{23} = 1.0000^{+0.000}_{-0.017} \text{ (inverted)},$$

$$\sin^2 2\theta_{13} = (9.3 \pm 0.8) \times 10^{-2},$$

are used and CP violation phase $\delta_{CP} = 0$ is assumed. The amplitude that the mass eigenstate $i$ makes the radiative transition is

$$\mathcal{M}_{i, \nu_e} = \mathcal{M}(\nu, i) U_{\nu_e, i}^*, $$  \hfill (32)

where the neutrino species is not specified in the final state. Thus the probability that the electron neutrino decays to a neutrino and a photon through $P^{(d)}_\gamma$ is given by the factorized form,

$$P^{(d)}_{e} = P^{(d)}_\gamma |U_{e, i}|^2.$$  \hfill (33)

The mixing modifies the probabilities slightly. We use a correction factor $1/2$. 

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4. Reactions of the decay products with a nucleus or an atom in the detectors

The probability that the neutrino or the photon directly detected with the detector on the earth is proportional to the range of space covered by the bound states. The wave functions of the nucleus or atoms are of microscopic sizes, and \( P^{(d)} \) proportional to these small sizes are much smaller than the previous cases.

For the event that the neutrino is detected, \( \sigma_{\nu'} \) is that of nucleus of the order \( \pi \times 10^{-30} \) meters\(^2\), and \( \sigma_{\nu'} \ll \sigma_{\nu} \). Accordingly the probability is smaller than Eq.(18) by the ratio \( \sigma_{\nu'}^{\text{nucl}} / \sigma_{\nu} \leq 10^{-50} \). Similarly for the event that the decay photon directly reacts with an atom in the gamma ray detector, which is a bound atom, \( \sigma_{\gamma}^{\text{atom}} \) is the atomic size of the order \( \pi \times 10^{-20} \) meters\(^2\), and \( \sigma_{\gamma} \ll \sigma_{\nu} \). The probability is smaller than Eq.(22) by the ratio \( \sigma_{\gamma}^{\text{atom}} / \sigma_{\gamma} \leq 10^{-40} \). This is also negligibly small.

During the long travel, actually, the photon interacts with the electrons with the strength of Quantum Electrodynamics(QED), and is affected by these reactions in the Galaxy. This effect is studied later.

IV. COMPARISON WITH THE SN1987A NEUTRINO

We study the events of antineutrino from SN1987A observed in the ground detectors. The number of the events is proportional to the survival probability \( P(\nu \to \nu) \), which depends upon the size \( \sigma_{\nu} \). \( \sigma_{\nu} \) shows the size of the area that the neutrino bursts takes places, which may be the size of the core of Supernovae, or the size of shock wave front. A current understanding based on the numerical simulations shows the latter of the velocity of about one tenth of the light velocity is favored.

A. Detection of the neutrino and the prompt gamma

We introduce the radius \( R \) of the relation

\[
\sigma_{\nu} = \pi R^2,
\]

and express hereafter the probability with it. This \( R \) may be around \( 10^4 \) meters for the supernove core or \( 10^7 - 10^8 \) meters for the expanding shock wave.
The probability in Eq. (23) is written as,
\[
P^{(d)}(R) = \left( \frac{R}{R_0} \right)^2, \quad (35)
\]
\[
R_0 = 1.37 \times 10^{10} (\frac{F}{10^3})^{-1/2} (\frac{20\,MeV}{E_\nu})^{3/2}, \quad (36)
\]
\[
r = \frac{0.4}{\sqrt{\nu^{(4)}(G)^2 + \nu^{(4)}(LMC)^2}}, \quad (37)
\]
where the units $\text{meter}^{-3}$ and Tesla are used for $n_e$ and $B$. For $\nu^{(4)}(LMC) = 0$, $r = 0.4$ corresponds to $B(G) = 10^{-10}$ Tesla and $n_e(G) = 10^4$ meter$^{-3}$. Thus the survival probability at the earth is
\[
P(\nu \to \nu) = \frac{1}{1 + \left( \frac{R}{R_0} \right)^2}, R \geq R_0. \quad (38)
\]

The probability of the neutrino reaction with the nucleus in the detector is determined by the standard cross section. Hence using the flux of the neutrino at the SN1987A, $\phi(E_\nu; SN1987A)$, the probability of the event that the SN1987A neutrino is detected at $t$, $N_\nu(t)$, is written as
\[
N_\nu(E_\nu, t) = N_\nu^{(0)} \sigma_{\nu+nucleus} n_{nucleus} L(\nu), \quad (39)
\]
\[
N_\nu^{(0)} = \phi(E_\nu; SN1987A) P(\nu \to \nu),
\]
where $\sigma_{\nu+nucleus}$ is the neutrino nucleus cross section, $n_{nucleus}$ is the nucleus density, and $L(\nu)$ is the total volume of the detector. The flux is modified from the naive value $\phi(E_\nu; 1987A)$ to $N_\nu^{(0)}$ by $P(\nu \to \nu)$ in the Galaxy, and will be compared with the observations.

The probability for the prompt gamma to be detected simultaneously with the neutrino is
\[
N_\gamma(E_\gamma, t) = N_\gamma^{(0)} \sigma_{\gamma+nucleus} n_{nucleus} L(\gamma), \quad (40)
\]
\[
N_\gamma^{(0)} = \phi(E_\gamma; SN1987A) P(\nu \to \gamma),
\]
where $\sigma_{\gamma+nucleus}$ is the gamma nucleus cross section, $L(\gamma)$ is the total volume of the detector, and others are the same as Eq. (39). The ratio $\sigma_{\gamma+nucleus}/\sigma_{\nu+nucleus}$ is about $10^{16}$, $(N_\gamma^{(0)}/N_\nu^{(0)})$ is much smaller than $10^{-20}$ from Eq. (21). The density is assumed same for both detectors, and the gamma detector is about 1 Kg, whereas the neutrino detector is more than $10^6$ Kg, and $L(\gamma)/L(\nu)$ is smaller than $10^{-6}$. Accordingly, $N_\gamma(E_\gamma, t)/N_\nu(E_\nu, t) \ll 10^{16-20-6} = 10^{-10}$. Thus the prompt gamma is not detectable.
FIG. 1. The photon from the neutrino decay has a large size and interacts with electrons by $P^{(d)}$ in the forward direction.

These gamma rays actually interact strongly with the electrons moving parallel, which were produced in the Supernovae, through the Compton or the Thomson processes, as in Fig.1. These have the enhanced probability and the photon loses the substantial energy. These overlapping photon and electron move with a central velocity $\vec{v}_0 = \frac{\sigma_e v_e + \sigma_\gamma v_\gamma}{\sigma_e + \sigma_\gamma}$, following the classical trajectory condition $[25, 28]$. The wave packet size of the electron, which is mainly the thermal one, is either macroscopic of the the size of the Supernovae or microscopic. In both cases, $\vec{v}_0$ is much less than the velocity of the light, because the electron’s velocity is substantially lower than the speed of the light, as $v/c = 10^{-3}$ or $v/c = 10^{-1}$ for the energy KeV or 100 KeV. Thus the velocity $\vec{v}_0$ is much lower than the speed of the light. Consequently the signal delays by a huge amount of time compared with the free photon of lower energy different from Eq.(21). The emergence of the low energy delayed photons instead of the prompt gamma rays is in accord with the observations. The detailed study of this process is outside of the scope of the present paper and will be studied elsewhere.

B. Expanding supernovae

From Eq.(38), $P^{(d)}$ depends on the radius $R$, and is negligibly small in the region $R \ll R_0$. The SN1987A neutrinos reaches the Earth unaffected by the Galaxy. The neutrino flux at the ground detector agrees with that of the initial neutrino. At a larger $R$, the effect becomes prominent, and in $R \approx R_0$ or $R \geq R_0$ $P^{(d)}$ becomes sizable, and the neutrino flux in the Earth decreases. Because $R_0$ is proportional to $E_\nu^{-3/2}$, the reduction rate increases in the high energy. The energy spectrum at $R > R_0$ becomes soft.
If the radius expands in time with a speed $v_{sv}$ and an initial radius $a_0$,

$$R = a_0 + v_{sv} t,$$  \hspace{1cm} (41)
the survival probability varies with time. Their magnitudes are considered as

$$a_0 = 10\text{Km},$$  \hspace{1cm} (42)
$$v_{sv} = 4.5 \times 10^6 - 3 \times 10^7 \text{meters/second},$$  \hspace{1cm} (43)
for the shock wave model. The velocity is around $1/10$ of the light velocity for the shock wave, and the maximum value allowed from the causality is the light velocity. $a_0$ is considered small generally.

Now we compare the theory with the observations. Parameters in the theoretical expressions Eq.(38) are the filling fraction and the size $\sigma_{\nu}$. Those in the Galaxy are known but those in LMC are unknown. So we compare the theoretical value from the Galaxy with the data [1–3].

Numerical simulations of supernovae explosion show that the total neutrino flux decreases rapidly with time but the energy spectrum remains or becomes wider at $t \leq 15$ seconds [29–31]. The average neutrino energy is either constant in or slightly increasing with time. Here focus to the average neutrino energy, and compare the theory with the observations. We study the simplest case that the SN1987A $\phi(E_{\nu}; AN1987A)$ does not vary with time [32]. The SN neutrino flux receives the absorption in the Galaxy and the flux detected by the ground detector is

$$\Phi_{\text{ground}}^{(d)}(E_{\nu}, t) = \phi(E_{\nu}; AN1987A) \times \frac{1}{1 + \left(\frac{a + v_{sv} t}{R_0}\right)^2}. \hspace{1cm} (44)$$

Due to low statistics, we compare a variation of the average energy in the period $2 \leq t \leq 12$ second. The average neutrino energy at $R \gg R_0$ from Eq.(44) or from Eq.(48) is

$$\langle E_{\nu} \rangle = 16.7\text{MeV},$$  \hspace{1cm} (45)
$$\langle E_{\nu} \rangle = 30\text{MeV}. \hspace{1cm} (46)$$

They are compared with the observations in Fig.(2). Our theory is in agreement with the observations, if the velocity is about one tenth of the light velocity. Data seems to show a reduction of the higher energy neutrino. From the fit, we have

$$v_s = 6 \times 10^7 \text{Meters/Seconds}(= 0.2c) \pm \delta v,$$  \hspace{1cm} (47)
FIG. 2. Time dependence of neutrino energy from SN1987A is compared with those computed from Eq. (44).

which is slightly larger than the theoretical shock front velocity Eq. (43). \( \phi(E_\nu; AN1987A) \) may change differently from Eq. (48) within ten seconds, then the velocity Eq. (47) should be considered the upper bound.

The neutrino spectrum

\[
\Phi^{nor}_{ground}(E_\nu, t) = \phi(E_\nu; SN1987A) \times e^{-\frac{t}{\tau}},
\]

(48)
is normally considered, where \( \tau \) is the relaxation time, and does not show the time-dependent average energy, which does not agree with the observation.

The energy transferred from the neutrino to the gamma ray is stored in extremely large waves of matters of the size \( \pi R^2 \). This photon interacts with another matter wave and is not directly detected. Assuming that is detected directly by the detector composed of bound atoms, we estimate the probability of the event. That is proportional to these sizes, and is too small to detect from Eqs. (21) and (40). Accordingly the present theory is consistent with the non-observation of the gamma rays from SN1987A [21].

**Table of physical quantities in the SN1987A and the galaxy**

1. the density of neutral atoms; \( n_{\text{neutral}} \approx 1 \text{[m}^{-3}\text{]} \)
2. the density of electrons; \( n_e = 10^4 \text{[m}^{-3}\text{]} \)
3. the magnetic field; \( B \text{[Tesla]} = 10^{-10} - 10^{-9} \text{[Tesla]} \)
4. filling fraction; \( 0.4 - 0.04/\text{m} \)
5. typical radius; \( R_0 \), \( 10^7 \text{ m} \)
C. Other processes

The enhanced $P^{(d)}$ is studied further in systems other than the SN1987A neutrino.

1. Dilute gas

High energy neutrinos produced inside the galaxy emit the photon by the electroweak Hall interaction with the probability $P^{(d)}$ and lose their energies. The energy carried originally by the neutrino is partly transmitted to the photon first and to electrons, molecules, or larger objects later in the galaxy. These photons are out of equilibrium, and do not follow the Planck distribution. At a higher energy, $P^{(d)}$ becomes larger. The high energy neutrino has a large component of $\nu + \gamma$ in the galaxy or in the earth’s ionosphere. These would be observed by large ground detectors such as Icecube, Telescope Array, and . Owing to the geometry dependence and other unique features of $P^{(d)}$, careful analysis is required.

2. Dense gas

In a star of high density, the photon’s effective mass is larger and satisfies

$$m_\gamma > m_\nu + m_\nu',$$  \hfill (49)

hence the transitions

$$\nu + \bar{\nu}' \rightarrow \gamma, \ \gamma \rightarrow \nu + \bar{\nu}'$$  \hfill (50)

occur. $\Gamma$ of these processes from the anomalous moments have been studied, and a weaker constraint than those of [21–23] was obtained [33]. If this star has the magnetic field, the electroweak Hall effect takes place. However, $\sigma_\gamma$ in the high density is much smaller compared with that in the galaxy, $P^{(d)}$ is not much enhanced.

V. SUMMARY

The anomalous radiative transition of the anti-neutrino from the supernovae 1987A in the Galaxy is studied. The survivable probability observed in the earth reflects the transition and causes the distortion of the energy spectrum. The theoretical mean energy agrees with
the observations if the expanding velocity of the region that the neutrino burst takes place is $1/5c$. This velocity is slightly larger than the standard shock front velocity, $v = 0.01 - 0.1c$. There are two possibilities to reconcile the disagreement. One is to include the absorption in the LMC, and the other is to modify the shock front propagation. In the former one, the filling fraction in LMC, which is unknown now, can be estimated. Theoretical results become to agree with the observations, if they are larger than the values in the Galaxy. The best fit is obtained with

$$\nu^{(4)}(\text{LMC}) = 10 \times \nu^{(4)}(\text{Galaxy}).$$

This value is understandable from the size of LMC and and the period of its rotation. LMC has the size of one third and the period of three times of those of the Galaxy. Then the filling fraction is expected to be about ten times of the Galaxy, which seems consistent. The second possibility of the higher speed of the shock front suggests that the dynamics of the shock front is modified. This may in fact happen if $P^{(d)}$ is included in the shock front dynamics.

The strength of the electroweak Hall effect is determined by the filling fraction $\nu^{(4)}$ and sizable in the system of the low electron density and weak magnetic field, if their ratio is sizable. The anomalous transition $\nu \rightarrow \nu' + \gamma$ of enormously enhanced probability is induced, and gives the sizable effect to the neutrino from SN1987A. The density and the magnetic field are extremely low but their ratio is not so small in fact in the Galaxy. The transition probability from this interaction is, $P = P^{(d)}$, instead of the standard $P = \Gamma T$, and does not increase with time interval, hence the present analysis differs drastically from the previous one. $P^{(d)}$ is proportional to the overlap of wave functions, which extends to the gigantic area, and enhanced anomalously. The theoretical energy spectrum in the time interval $T \leq 12$ Seconds varying with time rapidly is consistent with the previous experiments and observations, and gives the unique information through the survivable probability $P(\nu \rightarrow \nu')$ on the SN1987A radius. The expanding speed of the exploding star, obtained from the comparison of our theory with the observations, is in agreement with the speed of the shock front.

The small detection probability of the prompt gamma from the process $\nu \rightarrow \nu' + \gamma$ is in accord with the non-observation of the prompt gamma rays during the neutrino burst by the Solar Maximum Mission (SMN) Gamma Ray Spectrometer (GRS) \cite{21,23}. These
photons interact with matters in the Galaxy and produce the delayed gamma-rays, x-rays, and others. Those that are produced by the interaction of the high energy gamma with matters in the Galaxy through $P^{(d)}$ will be studied in a subsequent publication.

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**Appendix A: Integrals**

The light-cone singularities used are partly given in many textbooks and in Ref. [9, 10]. The integral over the coordinates $x_1$, $x_2$ and $\tilde{X}_{\nu_e}$ is written as

$$
\int d\tilde{X}_{\nu_e} \int d^4x_1 d^4x_2 e^{ip_{\nu_e} \cdot \delta x} f(\delta x) \prod_{i=1,2} w(x_i, X_{\nu}; \sigma_{\nu}) w(x_i, X_{\nu'}; \sigma_{\nu'}) 
\begin{align*}
= & \left( \frac{\pi \sigma_{\nu} \sigma_{\nu'}}{\sigma_{\nu} + \sigma_{\nu'}} \right)^2 \int d\tilde{X}_{\nu} e^{\frac{1}{2} \frac{(\tilde{X}_{\nu} - \tilde{X}_{\nu'})^2}{\sigma_{\nu} + \sigma_{\nu'}}} \int dt_1 dt_2 d\delta \tilde{x} e^{ip_{\nu} \cdot \delta x} e^{-\frac{1}{4\sigma_{\nu}} (\delta \tilde{x} - \tilde{v}_{\nu} \delta t)^2 - \frac{1}{4\sigma_{\nu'}} (\delta \tilde{x} - \tilde{v}_{\nu'} \delta t)^2} \\
\times & \exp \left[ -\frac{(\tilde{v}_{\nu} - \tilde{v}_{\nu'})^2}{\sigma_{\nu} + \sigma_{\nu'}} \left( \frac{t_1 + t_2}{2} - \tilde{T}_L \right)^2 \right] f(\delta x), 
\end{align*}
$$
(A1)

and using Gaussian approximation for integration in $\tilde{X}_{\nu'}$, we have

$$
\left( \frac{\pi^2 \sigma_{\nu} \sigma_{\nu'}}{2} \right)^2 \int dt_1 dt_2 d\delta \tilde{x} e^{ip_{\nu} \cdot \delta x} e^{-\frac{1}{4\sigma_{\nu}} (\delta \tilde{x} - \tilde{v}_{\nu} \delta t)^2 - \frac{1}{4\sigma_{\nu'}} (\delta \tilde{x} - \tilde{v}_{\nu'} \delta t)^2} f(\delta x). 
(A2)
$$

For $f(x) = \frac{i}{4\pi} \delta(\lambda)$, integral in Eq. (A2) is written as

$$
\int dt_1 dt_2 e^{-\frac{1}{4\sigma_{\nu}} (\delta \tilde{x} - \tilde{v}_{\nu} \delta t)^2 - \frac{1}{4\sigma_{\nu'}} (\delta \tilde{x} - \tilde{v}_{\nu'} \delta t)^2} \frac{i}{4\pi} \delta(\lambda) \delta(\delta t) 
\begin{align*}
= & \int_0^T dt_1 dt_2 e^{-\frac{(\tilde{v}_{\nu} - \tilde{v}_{\nu'})^2}{4\sigma_{\nu}} \delta t^2} e^{-\frac{(\tilde{v}_{\nu} - \tilde{v}_{\nu'})^2}{4\sigma_{\nu'}} \delta t^2} \frac{i}{4\pi} \delta(\lambda) \delta(\delta t) \\
\simeq & \frac{i}{2} \sigma_{\nu'} \int_0^T dt_1 dt_2 e^{-\frac{(\tilde{v}_{\nu} - \tilde{v}_{\nu'})^2}{4\sigma_{\nu}} \delta t^2} e^{-\frac{(1 - |\tilde{v}_{\nu} \delta t|)^2}{4\sigma_{\nu'}} \delta t^2} \frac{i}{\delta t}., 
\end{align*}
(A3)
where \( \omega_{\nu} = \frac{m^2}{2E_{\nu}}, \) and \( \sigma_{\nu}/|\vec{p}_{\nu}| \ll T \) is used. Due to the small mass of neutrino, \( e^{-\frac{(\varepsilon_{\nu} - \varepsilon_{\nu}^*)^2}{4\sigma_{\nu}} \delta t^2} = e^{-\frac{(1-|\vec{p}_{\nu}|)^2}{4\sigma_{\nu}}} = 1 \), but this suppression factor cannot be ignored for massive particles.

**Appendix B: Universal function \( \tilde{g}(\omega, T) \)**

Due to the small mass of neutrino, the approximation \( e^{-\frac{(\varepsilon_{\nu} - \varepsilon_{\nu}^*)^2}{4\sigma_{\nu}} \delta t^2} = e^{-\frac{(1-|\vec{p}_{\nu}|)^2}{4\sigma_{\nu}}} \) is good, which cannot be used for massive particles, and we have

\[
g(\omega_{\nu}, T) = i \int_0^T dt_1 dt_2 e^{-i\omega_{\nu}(t_1 - t_2)} t_1 - t_2 = -2 \left( \int_0^T dt \frac{\sin(\omega_{\nu} t)}{t} - \frac{1 - \cos(\omega_{\nu} T)}{\omega_{\nu} T} \right),
\]

where \( t_+ = \frac{t_1 + t_2}{2}, \ t_- = t_1 - t_2 \). Since \( g(\omega_{\nu}, \infty) = -\pi \) is cancelled with the short-range term from \( J_{\text{regular}} \), we write

\[
\tilde{g}(\omega_{\nu}, T) = \pi - 2 \left( \int_0^T dt \frac{\sin(\omega_{\nu} t)}{t} - \frac{1 - \cos(\omega_{\nu} T)}{\omega_{\nu} T} \right).
\]

**Appendix C: integral**

The integral over the relative coordinates is given by

\[
\int d\vec{r} e^{i\vec{p}_1 \cdot \vec{r}} e^{-\frac{1}{2\sigma}(\vec{r} - \vec{v}_1 t)^2} \delta(s^2 - c^2 t^2) = \\
\int d\vec{s} e^{i\vec{p}_2 \cdot \vec{s}} e^{-\frac{1}{2\sigma}(\vec{s}^2)} \delta(s^2 + \vec{v}_1 t^2 - c^2 t^2 + 2\vec{v}_1 t) = \\
e^{ip\vec{n}_2(t_1)} \int d\vec{s} e^{ip(-\vec{v}_1 + \vec{n}_2) \cdot \vec{s}} e^{ip\vec{n}_2 \cdot \vec{s}} e^{-\frac{1}{2\sigma}(\vec{s}^2)} \delta(s^2 + 2\vec{v}_1 t) = \\
e^{ip\vec{n}_2(t_1)} \int d\vec{s} \sum_{l} \frac{1}{l!} (ip(-\vec{v}_1 + \vec{n}_2) \cdot \vec{s})^l e^{ip\vec{n}_2 \cdot \vec{s}} e^{-\frac{1}{2\sigma}(\vec{s}^2)} \delta(s^2 + 2\vec{v}_1 t) = \\
e^{ip\vec{n}_2(t_1)} 2\pi d\cos \theta s^2 ds e^{ipv_{1s} \cos \theta} e^{-\frac{1}{2\sigma}s^2} (\delta(s^2 + 2sv_1 t \cos \theta)(1 + \epsilon) = \\
e^{ip\vec{n}_2(t_1)} 2\pi \frac{1}{2s|t|v_1} s^2 ds e^{-ipv_{1s} \frac{s^2}{2s|t|v_1}} e^{-\frac{1}{2\sigma}s^2} (1 + \epsilon) = \\
e^{ip\vec{n}_2(t_1)} 2\pi \frac{1}{4|t|v_1} 1 \int ds e^{-\frac{1}{2\sigma}(1 + \frac{|\vec{n}_2|}{|t|v_1})s^2} (1 + \epsilon) = \\
e^{ip\vec{n}_2(t_1)} 2\pi \frac{1}{4|t|v_1} \frac{1}{\frac{1}{2\sigma} + \frac{|\vec{n}_2|}{|t|v_1}} (1 + \epsilon),
\]
where the variable \( \vec{s} = \vec{r} - \vec{v}_1 t \) is used and a small quantity \( \epsilon \) is ignored.

The integral over the times

\[
\int_0^T dt_1 dt_2 e^{-i(E-p(\vec{n}_2 \vec{v}_1))t} \frac{1}{4|t|v_1} \frac{1}{\frac{1}{2\sigma} + \frac{\mu^2}{4t}} (1 + \epsilon) \text{sign } t,
\]

in the region \( \frac{p}{t} \ll \frac{2}{\sigma} \) is,

\[
\int_0^T dt_1 dt_2 e^{-i(E-p(\vec{n}_2 \vec{v}_1))t} \frac{1}{4tv_1} \frac{1}{\frac{1}{2\sigma}} = -i \frac{\sigma}{2v_1} T(\pi/2 - \tilde{g}(\tilde{\omega} T))
\]

and in the region \( \frac{1}{2\sigma} \ll \frac{p}{t} \) is,

\[
\int_0^T dt_1 dt_2 e^{-i(E-p(\vec{n}_2 \vec{v}_1))t} \frac{1}{4tv_1} \frac{1}{\frac{1}{2\sigma}} = -i \frac{p v_1}{2p v_2} \frac{4(\sin \tilde{\omega} T)^2}{\tilde{\omega}^2},
\]

where \( \theta \) is the angle between \( \vec{n}_2 \) and \( \vec{v}_1 \) and

\[
\tilde{\omega} = E(p) - p \vec{n}_2 \vec{v}_1 = p(1 - \cos \theta) + \frac{m^2}{2E}.
\]

\[\text{C2}\]

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