Self-similar turbulent boundary layer in pressure gradient. Four flow regimes

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Abstract. Self-similar flows in a turbulent boundary layer when the free-stream velocity is specified as a power, with the exponent $m$, function of longitudinal coordinate are investigated. The self-similar formulation not only simplifies solving of the problem by reducing the equations of motion to ordinary differential equations but also provides a mean for formulating closure conditions. It is shown that for the class of flows under consideration that depend on three governing parameters the dimensionless mixing length is a function of the normalized distance from the wall and the exponent $m$ in the outer region and a universal function of local Reynolds number in the wall region, the latter corollary being true even when the skin friction vanishes. In calculations this function is set to be independent of pressure gradient, which gives the results very close to experimental data.

There exist four different self-similar flow regimes. Each regime is related to its similarity parameter, one of which is the well-known Clauser equilibrium parameter and the other three are established for the first time. In case of adverse pressure gradient when the exponent $m$ lies within certain limits, which depend on Reynolds number, the problem has two solutions with different values of the boundary layer thickness and the skin friction, which points out the possibility of hysteresis in near-separating flow. Separation occurs not at the minimal value of $m$ that corresponds to the strongest adverse pressure gradient but at $m = -0.216 - 0.4 \Re_{\eta}^{-1/3} + O(\Re_{\eta}^{-2/3})$, where $\Re_{\eta}$ is the Reynolds number based on the longitudinal pressure gradient. The results of the theory are in good agreement with experimental data.

1. Main results

There are four self-similar flow regimes for turbulent boundary layers with a power-law free-stream velocity distribution $U(x) = B x^m$, where $B$ is some dimensional constant. In favorable and moderate adverse pressure gradients (the first regime), the boundary layer has a normal two-layer structure, the same as for zero-pressure-gradient flows. A similarity parameter is the well-known Clauser equilibrium parameter $\beta = -2\delta^* U' / c_f U$, where $c_f$ is the skin friction coefficient and $\delta^*$ is the displacement thickness.

Under strong adverse pressure gradient (the second regime), the boundary layer becomes three-layered. Near the wall over the logarithmic sublayer, there arises an intermediate region — a gradient sublayer, in whose outer part the velocity profile satisfies a square-root law. In the outer boundary-layer region, the velocity profile is described, in similarity variables, by a universal (depending on the normalized distance from the wall alone) function. The quantity $\omega = (c_f/2)^{1/6} \sqrt{1 + \beta}$ serves as a similarity parameter. Under this regime, there exist exactly
two solutions having different values of boundary-layer thickness ratio and skin friction for each value of the exponent $m$. The transition from one branch of the solution to another occurs at the threshold similarity-parameter value $\omega \approx 0.67$. The boundary-layer thickness ratio depends on longitudinal coordinate and decreases downstream proportional to $(\ln Re_x)^{-1}$ and $(\ln Re_x)^{-2/3}$ for the first and second regimes, respectively.

The regime of near-separating flow is the third one. The boundary layer still exhibits a triple-layer structure. The structure is such that the velocity profile obtained from the solution for the outer region satisfies a slip condition at the wall and furthermore follows the square-root law. As a similarity parameter $\Omega = \sqrt{\delta_\ast U'/U}$ increases, the slip magnitude decreases and vanishes at the value $\Omega_\ast = 0.0911$ that corresponds to separation. The near-separating regime is the last one when the velocity profile has a logarithmic portion.

The logarithmic sublayer completely disappears under the fourth flow regime when the skin friction can vanish. Its place is taken by an intermediate region where the velocity profile obeys the square-root law, the boundary layer acquiring a two-layer structure again. In the outer region, the velocity profile is described, in similarity variables, by one universal function obtained at $\Omega = \Omega_\ast$. A similarity parameter is the quantity $\tau = Re_p^{2/3} \sigma_f / 2$. In the case of third and fourth regimes, the boundary-layer thickness and integral parameters grow in the first approximation linearly along the length, however the skin friction is not constant and depends on the longitudinal coordinate for all four regimes.

A characteristic scale for the velocity defect (a velocity-defect law) valid for four flow regimes is established as a result of an exact asymptotic solution of the problem. Two families of dimensionless velocity profiles are calculated depending on the parameters $\beta$ and $\Omega$. Similarity laws for the Reynolds-stress components are formulated. A skin-friction law valid in the entire range of pressure-gradient variation, from favorable values till the ones giving rise separation is established. The law formulation contains three universal functions of the variables $\beta$, $\Omega$ and $\tau$, respectively. The exponent $m$ is a function of two similarity variables $\beta$ and $\Omega$.

2. Comparison with experimental data

Figure 1 shows the mean-velocity and shear-stress profiles measured by Andersen et al (1972) in different cross-sections of self-similar turbulent boundary layers and the calculated curves corresponding to the first flow regime. The comparison, carried out in similarity variables, exhibits excellent agreement between the theory and experiment. In Fig. 1a,b, the experimental points diverge from the theoretical curves only near the wall in the area of a viscous sublayer where the established velocity defect law must not hold.

The theoretical conclusions are also confirmed by the experimental data of Skåre & Krogstad (1994) displayed in Fig. 2. These data and the calculated curves correspond to the near-separating flow regime and are represented in appropriate similarity variables. Figures 1a,b and 2a depict the square root of the velocity defect plotted versus the square root of the normalized distance from the wall. It is to exhibit the presence of a gradient sublayer near the wall where the square-root law is valid, this law corresponds to a straight portion on the velocity profile.
Figure 1. Experimental mean-velocity ($a, b$) and shear-stress ($c, d$) distributions obtained by Andersen et al (1972) and the calculated curves for $\beta \approx 0.7$ ($a, c$) and $\beta \approx 1.6$ ($b, d$) represented in the similarity variables for the case of moderate adverse pressure gradient.

Acknowledgments

This work was partially supported by the Russian Foundation for Basic Research, Project No. 09-08-00307.
Figure 2. Mean velocity (a) and shear-stress (b) profiles obtained by Skåre & Krogstad (1994) and the calculated curves for $\Omega = 0.076$ represented in the similarity variables for the case of near-separating flow.

References

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