GAUGE COUPLING UNIFICATION AND NEUTRINO MASSES IN 5D SUSY SO(10)

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In this talk I discuss the problems and virtues of SUSY GUTs in four dimensions. I then show how to solve some of these problems, without foregoing the virtues, by considering an SO(10) SUSY GUT in five dimensions. I discuss gauge coupling unification and neutrino masses. In particular, it is shown that the 5D compactification scale and cutoff scale are determined by fitting the low energy values of the three standard model gauge couplings. Neutrino masses can be accommodated.

1 Gauge coupling unification in 5D

We consider gauge coupling unification in $SO(10)$ in five dimensions. In particular we discuss hybrid gauge symmetry breaking with both orbifold and Higgs vevs on the brane. We calculate the GUT scale threshold corrections to gauge coupling unification. We then show that the compactification scale $M_c \approx 10^{14}$ GeV and the cutoff scale $M_* \approx 10^{17}$ GeV are fixed by the low energy data. Finally we consider neutrino masses and determine the Seesaw scale determining light neutrino masses. Let us first define some notation.

1.1 Charge quantization & Family structure

The Pati–Salam gauge symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$ unifies quarks and leptons of one family into two irreducible representations given by

$$\psi = (4, 2, 1) = \{Q = \begin{pmatrix} u \\ d \end{pmatrix}, \ L = \begin{pmatrix} \nu \\ e \end{pmatrix}\}$$

and

$$\psi^c = (\overline{4}, \overline{1}, \overline{2}) = \{Q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \ L^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}\}.$$ 

The two Higgs doublets of the minimal supersymmetric standard model are contained in one irreducible representation

$$\mathcal{H} = (1, \overline{2}, 2) = \{H_u, \ H_d\}.$$

Hence Pati-Salam naturally describes the family structure of the standard model. Moreover since there are no U(1) symmetries, charge quantization is
enforced. There are however three independent gauge couplings [two if one also demands parity] and thus no prediction for gauge coupling unification.

The gauge group SO(10) then unifies quarks and leptons into one irreducible spinor representation

\[ \psi + \psi^c \subset 16. \]

With the addition of Higgs triplets, the Higgs doublets are contained in the defining representation

\[ \mathcal{H} + (6,1,1) (= \{ T, \bar{T} \}) \subset 10_H. \]

Of course, SO(10) also predicts gauge coupling unification.

Finally both symmetry groups, Pati-Salam and SO(10), lead naturally to Yukawa unification for the third generation with

\[ \lambda \, 16_3 \, 10_H \, 16_3 \supset \lambda \, \psi_3 \, \mathcal{H} \, \psi^c_3 \]

and a single Yukawa coupling \( \lambda \). Given the above brief review, let us consider the virtues and problems of four dimensional SUSY GUTs.

### 1.2 Virtues and problems of 4D SUSY GUTs

Four dimensional SUSY GUTs have the following virtues.

- **Charge quantization** — No \( U(1) \) factors
- **Family structure** — Quarks and leptons are in the smallest chiral (i.e. non vector-like) representations
- **Neutrino Mass** \[ \nu m \nu^c + \frac{1}{2} \nu^c M \nu \] with \( m = \lambda \langle H_u \rangle \) and \( M = \) See–Saw scale \( \sim M_{\text{GUT}} \), we obtain \( m_\nu = m^2/M \) — A right-handed neutrino \( \nu_R = (\nu^c)^* \) is required in either PS or SO(10)
- **Gauge coupling unification** — Fits the low energy data
- **Yukawa coupling unification** — This is a prediction of minimal PS and SO(10)
- **Dark matter candidate** — With a conserved \( R \)-parity, the LSP (typically the lightest neutralino \( \chi^0_1 \)) is stable

4D SUSY GUTs have the following problems.

- **Gauge symmetry breaking requires a complicated symmetry breaking sector.**
- **Higgs doublet – triplet splitting can be accommodated but is not required by the theory.**
• A supersymmetric $\mu$ term, with a dimensionful parameter of order the electroweak scale, must be generated.

• Proton decay, due to dimension 5 operators, must be suppressed to satisfy the Super-Kamiokande bound - $1/\Gamma(p \rightarrow K^+ \bar{\nu}) > 1.9 \times 10^{32}$ yrs.

• In order to obtain Majorana neutrino masses consistent with atmospheric neutrino oscillations with $\Delta m^2_{\text{atm}} \sim 3 \times 10^{-3} \text{eV}^2$, one needs a See-Saw scale $M \sim 10^{-2} M_{\text{GUT}} \ll M_{\text{GUT}}$ for the tau neutrino (assuming the light neutrino spectrum is hierarchical).

We now consider SUSY SO(10) on an orbifold in 5D and show how some of these problems can be resolved, while at the same time retaining the virtues of 4D SUSY GUTs.

1.3 SUSY SO(10) on $\mathcal{M}_4 \times S_1/(Z_2 \times Z'_2)$

The 5D orbifold is a line segment $[0, \pi R/2]$ in the fifth dimension $y$ defined in terms of a $Z_2 \times Z'_2$ orbifolding of the circle with radius $R$. The first $Z_2$ breaks the effective 4D N=2 SUSY to N=1 SUSY, while the second breaks SO(10) to Pati-Salam [PS].\(^a\) Hence the brane at $y = 0$ has the full SO(10) symmetry, while the brane at $y = \pi R/2$ has only the PS symmetry. The 5D bulk fields (given in terms of 4D superfields) include the gauge sector $[V, \Phi]$ in the adjoint representation and the Higgs hypermultiplet $[10_H, \bar{10}_H]$ in the defining representation. In a standard notation we then have the $Z_2 \times Z'_2$ eigenstates $V_{++}, \Phi_{--} \subset \text{PS}; \ V_{+-}, \Phi_{-+} \subset \text{SO(10)/PS}; \ \mathcal{H}_{++}, \mathcal{H}_{--} \subset (1, 2, 2)$ and $T_{+-}, \bar{T}_{++}, T_{++}, \bar{T}_{--} \subset (6, 1, 1)$. Thus only $V_{++}$ (the PS gauge sector) and $\mathcal{H}_{++}$ (the Higgs doublets) contain zero modes. We then assume that the fields $\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle$, located on the PS brane, develop a vev of order $\sim M_* \ [\text{= cutoff scale}];$ spontaneously breaking PS to the standard model gauge group. The three families of quarks and leptons live either on the PS or SO(10) brane or in the bulk. They come in complete families either under SO(10) or PS. With this construction our 5D theory has the properties described in the following theoretical score card. [\(\sqrt{\ }\), means it is a property of the construction, while ?, will be discussed further in this talk.]

1.4 5D SO(10) — Theoretical Score Card

• Charge quantization & Family structure — $\sqrt{\ }$

• Gauge coupling unification — $?$

\(^a\)Orbifold breaking of SO(10) in 5D was discussed in Ref. 12 or in 6D in Refs. 11. We see no advantage in going to 6D.
• Yukawa coupling unification for the third generation — √
• R parity ⇒ dark matter candidate — √
• Neutrino mass (See–Saw mechanism) — ?
• Gauge symmetry breaking — √
• Higgs doublet–triplet splitting — √
• Proton decay ($p \rightarrow K^+ \bar{\nu}$) due to dim. 5 Operators — $R$ symmetry prevents dim. 5 ops. — √
• Proton decay ($p \rightarrow e^+ \pi^0$) due to dim. 6 operators — negligible in 4D, however in 5D one is now sensitive to physics at the cutoff and the effects are incalculable (and perhaps even observable ?)
• Right-handed neutrino mass scale — ?

1.5 Gauge coupling unification : Orbifold/Brane breaking

We use orbifold breaking to explicitly break SO(10) to PS and spontaneous breaking on the PS brane to break PS to the standard model. We now calculate the threshold corrections to gauge coupling unification, due to the tower of Kaluza-Klein modes. We show that the cutoff scale, $M_*$, the compactification scale, $M_c$, and the value of the unified gauge coupling at $M_*$ are fixed by the low energy data. This differs from 4D GUTs in only one respect. There too a precise fit to the low energy data requires three parameters, the GUT scale, $M_G$, the gauge coupling at the GUT scale, $\tilde{\alpha}_G$, and a perturbative threshold correction at $M_G$ due to the GUT and Higgs breaking sectors of the theory. If we define the GUT scale as the point where $\alpha_1 = \alpha_2 = \tilde{\alpha}_G$, then the necessary threshold correction is given by $\epsilon_3 = (\alpha_3(M_G) - \tilde{\alpha}_G)/\tilde{\alpha}_G \sim -4\%$. In the 5D case the “GUT scale” threshold corrections are solely due to the tower of KK states above $M_c$ and do not depend on arbitrary parameters in the superpotential responsible for GUT breaking.

Spontaneous symmetry breaking on the brane, i.e. brane breaking, is accomplished with the vevs of two fields, $\chi^c$, and its conjugate, $\bar{\chi}^c$, with the following transformation under PS $- \chi^c = (4, 1, 2)$.\footnote{For a nice discussion of brane breaking, see Ref.} Note, without loss of generality, $\chi^c$ is assumed to get a vev in the right-handed neutrino direction $\chi^c \supset \nu^c$ which breaks PS to the standard model. This spontaneous breaking generates a mass term for the PS gauge fields in PS/SM given by

$$\delta(y - \frac{\pi R}{2}) g_5^2 \left( \langle \chi^c \rangle^2 + \langle \bar{\chi}^c \rangle^2 \right) A_\mu^2.$$
The brane mass terms affect the KK spectrum. For $\langle \chi^c \rangle \sim M_*$ the effect is to repel the wave functions of fields in PS/SM away from the PS brane. In particular, we find $V_{++}, \Phi_{-+} \subset$ PS/SM goes to $\approx V_{+-}, \Phi_{+-}$, while $V_{++}, \Phi_{-+} \subset$ SM is unaffected. The effect on $\Phi$ is a consequence of the fact that N=1 supersymmetry is unbroken by the vevs. Finally, the Higgs sector is unaffected by the brane breaking. The resulting KK spectrum is illustrated in Fig. 1.

1.6 $\Delta_i, i = 1, 2, 3$ — Threshold corrections at $M_c$ due to the KK tower between $M_c$ and $M_*$

We are only interested in the running of the differences of gauge couplings, hence we subtract an overall constant from $\Delta_i$ such that $\Delta_1 \equiv 0$ (for more details see Ref. [1]). The differences run logarithmically above each KK mode. The quarks and leptons come in complete families and thus do not contribute at all to the running. We consider the gauge and Higgs contribution below.

The threshold corrections to gauge coupling unification defined at the compactification scale are given by $\Delta_i = \Delta_{i,\text{gauge}} + \Delta_{i,\text{Higgs}}$. In the paper we have performed a detailed calculation (see also Ref. [6] for similar analyses). The exact result is very well approximated by

$$\Delta_{i,\text{gauge}} \approx \frac{2}{3} b_i^\text{SM}(V) \log\left(\frac{M_*}{M_c}\right) \quad \text{and},$$

$$\Delta_{i,\text{Higgs}} \approx 0.$$  \hspace{1cm} (1) \hspace{1cm} (2)

This result is easy to understand. Consider first the Higgs sector. When an equal number of Higgs doublets and triplets contribute to the running, they form a complete SO(10) multiplet and thus give zero contribution to $\Delta_i$. From Fig. 1 we see that from $M_c$ to $2M_c$ there are more triplets than doublets. However from $2M_c$ to $3M_c$ the situation is reversed with more doublets than triplets. These two cases alternate as one goes up in energy and the net effect is to cancel each other. In the gauge sector the situation is slightly different. Complete gauge or $\Phi$ multiplets give zero contribution to the running. Again from Fig. 1 we see that from $M_c$ to $2M_c$ the $\Phi \subset$ SM contribution is missing, while from $2M_c$ to $3M_c$, there is an excess of $V \subset$ SM. The net result gives $2/3$ of the gauge sector contribution in the MSSM. With these results we can now compare 5D and 4D SO(10).

In 5D we have (for $\mu \leq M_c$)

$$\frac{2\pi}{\alpha_i(\mu)} \approx \frac{2\pi}{\alpha(M_*)} + b_i^\text{MSSM} \log\left(\frac{M_c}{\mu}\right) + \Delta_i,$$

$$= \frac{2\pi}{\alpha(M_*)} + b_i^\text{MSSM} \log\left(\frac{M_\mu}{\mu}\right) + \frac{2}{3} b_i^\text{SM}(V) \log\left(\frac{M_*}{M_c}\right).$$  \hspace{1cm} (3)

Whereas in the 4D MSSM we have

$$\frac{2\pi}{\alpha_i(\mu)} \approx \frac{2\pi}{\alpha(M_{\text{GUT}})} + b_i^\text{MSSM} \log\left(\frac{m_T}{\mu}\right) + b_i^\text{SM}(V) \log\left(\frac{M_{\text{GUT}}}{m_T}\right).$$  \hspace{1cm} (4)
Using two loop RG running above the weak scale and one loop threshold corrections at $M_Z$ we find $M_{GUT} \simeq 3 \times 10^{16}$ GeV. In addition the necessary GUT threshold correction, $\epsilon_3$, can be obtained with the color triplet Higgs $T, \bar{T}$ mass $m_T \simeq 2 \times 10^{14}$ GeV (see for example, Ref. 7). From Eqns. 8.
it is easy to see that the 5D RG equation is equivalent to the 4D MSSM result if we take
\[ M_c = m_T, \quad \frac{M_s}{M_c} = \left( \frac{M_{GUT}}{m_T} \right)^2, \quad \alpha(M_{GUT}) = \alpha(M_s). \] (5)
We thus obtain gauge coupling unification with \( M_c \simeq 10^{14} \) GeV and \( M_s \simeq 10^{17} \) GeV. The differential running of the gauge couplings above \( M_c \) is illustrated in Fig. 2 for the 5D and 4D cases.

2 Neutrino mass in 5D SO(10)

For the See-Saw mechanism to work in a 4D SO(10) model, we need to give the right-handed neutrino a Majorana mass of order the GUT scale. There are two methods for obtaining this:

- using a higher dimension operator
  \[ \overline{16} 16 \overline{16} \overline{16} \overline{16} \]
  where \( M_s \) is the cutoff scale of the theory, or

- adding an SO(10) singlet \( N \) and the renormalizable interactions
  \[ \lambda_N \overline{16} 16 N + \frac{1}{2} M_2 N N. \] (7)
In both cases we assume a non-zero vacuum expectation value [vev]
\[ \lambda_N \langle \overline{16} \rangle = M_1 \neq 0 \]
where, in the right-handed neutrino direction. In the first case, the product of fields has, in general, several inequivalent SO(10) invariant combinations. One particularly simple combination in the first case, with the first two fields combined to make an SO(10) singlet, can be obtained as an effective interaction after integrating out \( N \) in the second case.

We now include the usual, electroweak scale, Dirac mass coming from the Yukawa term \( \lambda \overline{16}_3 10_H \overline{16}_3 \) (with 3 denoting the third generation). After electroweak symmetry breaking we have
\[ W = m_D \nu \nu^c \] (8)
(with \( m_D = \lambda \sqrt{2} \sin \beta \)) and we obtain a \( 3 \times 3 \) neutrino mass matrix, rather than a \( 2 \times 2 \) matrix, given by
\[ \mathcal{M} \begin{pmatrix} \nu \\ \nu^c \\ N \end{pmatrix} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_1 \\ 0 & M_1 & M_2 \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ N \end{pmatrix}. \] (9)

\[ ^c \text{This is similar to the double see-saw mechanism suggested by Mohapatra and Valle} \]
Figure 2. Differential running of $\delta_i = 2\pi(\frac{1}{\alpha_i} - \frac{1}{\alpha_1})$ for $i = 2, 3$.

Note, $|\text{Det}\mathcal{M}| = m_D^2 M_2$, $\text{Tr}\mathcal{M} = M_2$ and $m_D \ll M_1, M_2$. Hence the effective See-Saw scale $M_{\text{eff}}$ in this case may be obtained by evaluating the inverse of
the heavy $2 \times 2$ mass matrix

$$
\begin{pmatrix}
0 & M_1 \\
M_1 & M_2
\end{pmatrix}
$$

(10)
in the $\nu^c$ direction. We find $M_{\text{eff}} = M_1^2 / M_2$. Note, the result is independent of the mass ordering, i.e. $M_1 \ll M_2$, $M_1 \gg M_2$ or $M_1 \approx M_2$. Finally, we obtain the light neutrino mass given by

$$
m_{\nu} \simeq \frac{m_D^2 M_2}{M_1^2},
$$

(11)
irrespective of the ratio $M_1/M_2$, as long as $m_D \ll M_1, M_2$.

Note that the effective See-Saw mass $M_{\text{eff}}$, in either case, can be lower than the cutoff scale of the theory. For example, in 4D, the natural size of the right-handed tau neutrino Majorana mass is determined by taking the cutoff scale $M_* = M_{\text{Pl}}$ in the effective higher dimension operator. By replacing $\langle \overline{16} \rangle \sim M_{\text{GUT}}$ one obtains

$$
M_{\nu^c} \equiv M_{\text{eff}} \sim \frac{M_{\text{GUT}}^2}{M_{\text{Pl}}} \sim 10^{14} \text{ GeV}.
$$

In 5D, we have several possible choices for locating the matter multiplets on the SO(10) or PS brane or in the bulk. We also can imagine SO(10) singlet fields, living in the bulk and giving light neutrino masses via a double See-Saw mechanism. We have considered a detailed analysis of all possible ways of obtaining neutrino masses in 5D SO(10) models.\(^{24}\) The bottom line can be obtained with simple dimensional analysis.

Consider an effective higher dimensional right-handed neutrino mass operator on the PS brane given by

$$
W = \frac{C}{2M_*^n} \overline{\chi}^c \psi^c \overline{\chi}^c \psi^c \delta(y - \frac{\pi R}{2}).
$$

(12)
The constant $C$ is fixed by Naive Dimensional Analysis with $C = 16 \pi^2 c$ and $n = 1$ (for $\psi^c$ on the PS brane) or $C = 24 \pi^3 c$ and $n = 2$ (for $\psi^c$ in the bulk). Thus when $4 \pi \langle \overline{\chi}^c \rangle \sim M_*$ in the right-handed neutrino direction and $c \sim 1$ (i.e. the “natural” values) we find a right-handed neutrino mass

$$
W = \frac{1}{2} M_{\text{eff}} \nu^c \nu^c
$$

(13)
with

$$
M_{\text{eff}} \sim \left( \frac{M_*}{\frac{4}{7} M_c} \right) \text{ for } \psi^c \left( \text{on the PS brane in the bulk} \right).
$$

We find that the value of $M_{\text{eff}}$ for $\psi^c$ in the bulk is of order $M_c$, but for $\psi^c$ on the PS brane we can only obtain this desired value with a small value of $c \sim 10^{-3}$. Thus for $\psi^c$ on the PS brane we need to suppress $M_{\text{eff}}$. This can be accomplished in two possible ways. In the first way, by assuming a spontaneously broken U(1) symmetry which suppresses the effective operator coefficient $C$. This suppression can be the consequence of a U(1) symmetry requiring the insertion of a singlet field $S$ in the effective operator. Then
a suppression factor $\langle S \rangle / M_\nu$ is obtained. In the second case, a Majorana mass $\ll M_\nu$ is assumed for the bulk singlet field. This seems to be the only situation where the Majorana mass of an SO(10) singlet, located in the bulk, is important.

2.1 Conclusion — Theoretical Score Card

Let me conclude by listing the virtues and problems of 5D SUSY SO(10).

Virtues of the 5D theory

- Charge quantization & Family structure — √
- Gauge coupling unification — √
- Yukawa coupling unification for the third generation — √
- R parity $\implies$ dark matter candidate — √
- Neutrino mass (See–Saw mechanism) — √
- Gauge symmetry breaking — √
- Higgs doublet–triplet splitting — √
- Proton decay ($p \to K^+ \bar{\nu}$) due to dim. 5 Operators — R symmetry prevents dim. 5 ops. — √

Problems of the 5D theory

- Right-handed neutrino mass scale of order $M_\nu$ can be obtained. It is only natural if the right-handed components of quarks and leptons are located in the bulk.
- Proton decay ($p \to e^+ \pi^0$) due to dim. 6 operators — negligible in 4D, however in 5D one is now sensitive to physics at the cutoff and the effects are incalculable (and perhaps even observable ?)
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