Non-Linear Pattern-Matching against Unfree Data Types with Lexical Scoping

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Abstract

This paper proposes a pattern-matching system which enables non-linear pattern-matching against unfree data types. Our system allows multiple occurrences of the same variables in a pattern, multiple results of pattern-matching and modularization of the way of pattern-matching for each data type at the same time. It enables us to represent pattern-matching against not only algebraic data types but also unfree data types such as sets, graphs and any other data types whose data have no canonical form and multiple ways of destruction. We realized that with a rule that pattern-matching is executed from the left side of a pattern and a rule that a binding to a variable in a pattern can be referred to in its right side of the pattern. Furthermore, we realized lexical scoping in these patterns. It is necessary for modularization of useful patterns. In our system, a pattern is not a first class object, but a pattern-function which obtains only patterns and returns a pattern is a first class object. This restriction simplifies the non-linear pattern-matching system with lexical scoping in patterns. We have already implemented the pattern-matching system in the Egison programming language.

1 Introduction

In this paper, we focus on the representation of non-linear pattern-matching against unfree data types. It enables to represent pattern-matching against data types whose data have no canonical form. A canonical form of an object is a standard way to represent that object. For instance, data of sets and graphs do not have a canonical form. For example, a collection \{a, b, c\} is equal to \{b, a, c\}, \{c, b, a\} and \{a, a, b, c\}, if it is regarded as a set.

Data types whose data have no canonical form often play important roles in expressing algorithms. Then, a natural way to handle these kinds of data is really important. Without it, we need to translate and regard them as a data type whose data have a canonical form when we treat them. For example, a set would be treated as a list. Many programmers think this is unavoidable, and in fact, it is a latent stress of programming.
We have designed and implemented a new pattern-matching system that allows multiple occurrences of the same variables in a pattern, multiple results of pattern-matching and modularization of the way of pattern-matching for each data type at the same time. With this system, we can treat unfree data types directly. We will show its expressive power and implementation in this paper.

Let us introduce our pattern-matching expression. We use Haskell-like pseudocode in this section. First, we introduce our `matchAll` expression to explain the concept of our pattern-matching system. We can explain the concept with only this expression.

```
matchAll [1, 2, 3] as (List Integer) of x:xs -> (x, xs) 
-- => [[(1, [2, 3])] ]
```

The characteristic of our pattern-matching expression is it takes a `matcher`. Our interpreter matches the target with the pattern in the way specified with the matcher. A matcher specifies the way of pattern-matching. We can modularize the way of pattern-matching for each data type in matchers.

In the following example, we do pattern-matching against a collection as a list, multiset, set respectively. The pattern constructor ‘:’ (cons) divides a collection into an element and the rest. The meaning of ‘:’ varies for each matcher and is expressed in the matcher definition. We explain how we define matchers in section 4.

```
matchAll [1, 2, 3] as (List Integer) of x:xs -> (x, xs) 
-- => [[(1, [2, 3])]]
macthAll [1, 2, 3] as (Multiset Integer) of x:xs -> (x, xs) 
-- => [[(1, [2, 3]), (2, [1, 3]), (3, [1, 2])] ]
macthAll [1, 2, 3] as (Set Integer) of x:xs -> (x, xs) 
-- => [[(1, [1, 2, 3]), (2, [1, 2, 3]), (3, [1, 2, 3])] ]
```

Note that we are handling pattern-matching with multiple results. This feature is necessary to pattern-matching against data types whose data have no standard form such as multisets and sets. This is because they have multiple ways of destruction. Pattern-matching with backtracking is really important to represent pattern-matching against unfree data types. We need backtracking in the pattern-matching process to try multiple forms.

The pattern constructor ‘++’ (join) divides a collection into two collections. If we introduce this pattern constructor, a list also has the multiple ways of destructions.

```
matchAll [1, 2, 3] as (List Integer) of xs ++ ys -> (xs, ys) 
-- => [[([], [1, 2, 3]), ([1], [2, 3]), ([1, 2], [3]), ([1, 2, 3], [])]]
```

Non-linear pattern-matching is also important to represent pattern-matching against unfree data types. Non-linear pattern-matching with backtracking eliminates deeply nested loops and conditional branches. For example, non-linear patterns enable us to represent a pattern that matches when the collection has multiple same elements.
matchAll [1, 2, 3, 3, 2, 4] as (Multiset Integer) of x:x:_ -> x
-- =>[2,3,3,2]

A part of the pattern that does not contain pattern constructors and free
variables is a value-pattern. In most of cases, the equality is checked for a value-
pattern. We can define the equality for each data type in the matcher. In the
above sample, the second appearance of ‘x’ is a value-pattern.

Our system can handle pattern-matching against infinite collections with in-
finitely results. The following sample enumerates all twin primes from the infinite
list of prime numbers. In the following sample, the second appearance of ‘p + 2’
is a value pattern. Note that, we can write an any expression in a value-pattern.
take 8 (matchAll primes as (List Integer) of (_ ++ (p:p + 2:_))) -> (p, p + 2)
-- =>[(3,5),(5,7),(11,13),(17,19),(29,31),(41,43),(59,61),(71,73)]

We can do pattern-matching also against nested data types such as lists of
sets and sets and sets. The following code enumerates common elem-
tections among
collections inside a collection that contains three collections.
matchAll [[1, 2, 3, 4, 5], [4, 5, 1], [6, 1, 7, 4]] as (List (Multiset Integer)) of
(n:_):(n:_):(n:_):[] -> n
-- =>(1 4)

Non-linear pattern-matching with backtracking is really challenging. We
have to invent new syntax and a new mechanism for that. We realized that
with a rule that pattern-matching is executed from the left side of a pattern
and a rule that a binding to a variable in a pattern can be referred to in its
right side of the pattern.

Furthermore, we have realized lexical scoping in patterns to modularize use-
ful patterns in many places of programs. Lexical scoping in patterns becomes a
challenging problem because we allow non-linear patterns.

We have solved all problems and created the new programming language
Egison. In this paper, we introduce our new pattern-matching system by show-
ing various programs in our new language and its implementation. We have
implemented it using Haskell.

2 Preliminaries

Before explaining pattern-matching, we introduce basics of our language to un-
derstand the rest of this paper. Actually, samples in the previous section are just
pseudocode and our language has completely different syntax. Our language is
a purely functional programming language with a strong pattern-matching fa-
cility. In this section, we explain the ordinary purely functional aspect of the
language. We explain patterns and pattern-matching from the next section.

We demonstrate pattern-matching on the interpreter as below. ‘>’ is a
prompt. An expression after a prompt is input. Output is displayed from
the next line of the end of input. Our language has parenthesized syntax as
Lisp. We can add top-level bindings from the prompt with a `define` expression. Bindings added by a `define` expression can be referred from the next prompt.

```
> (define $x 10)
> x
10
> (+ x 100)
110
```

### 2.1 Built-in Data

In this paper, we use only booleans and integers for built-in data. Booleans are represented as `#t` and `#f`. We can represent numbers as other programming languages. For example, we represent negative numbers by adding ‘-’ ahead of a number literal as ‘-123’.

### 2.2 Objects

#### 2.2.1 Inductive Data

We can construct a complex object using *inductive data*. An inductive datum consists of a data constructor and its arguments enclosed with angled-brackets. It can have any inductive data as arguments. This is why it is called inductive. Note that the name of a constructor has to start with uppercase.

```
> <Nil>
<Nil>
> <Cons 1 <Cons 2 <Nil>>>  
<Cons 1 <Cons 2 <Nil>>>  
```

#### 2.2.2 Tuples (Multiple Values)

A tuple is expressed as a sequence of elements enclosed in square brackets. Note that a tuple consists of an element is treated as the same object with the element itself.

```
> [1 [[2]]]  
[1 2]
```

#### 2.2.3 Collections

A collection is a sequence of elements enclosed in braces. Note that an expression which has ‘∅’ is dealt not as an element but a *subcollection*, a segment of the collection. Using this notation, we can construct a collection from other collections, easily.

```
> {∅∅{1}} ∅{2 ∅{3}} 4}  
{1 2 3 4}
```
2.2.4 Functions

We define a function using \texttt{lambda} as other functional programming languages.

2.2.5 Pattern-Functions

A pattern-function is a function that gets only patterns and returns a pattern. We define it using a \texttt{pattern-function} expression. We demonstrate a lot of patterns and pattern-functions in detail in the next section.

2.2.6 Matchers

A matcher is defined to specify how to pattern-match for each data types. We define it using a \texttt{matcher} expression. We explain these expressions in detail from the next section.

2.3 Syntax

Our language has \texttt{if}, \texttt{let}, and \texttt{letrec} expressions as other ordinary functional programming languages. We omit explanation about these expressions.

\texttt{match-all} and \texttt{match} expressions are syntax for pattern-matching, the core of this paper. We explain these expressions in detail from the next section.

3 Pattern-Matching Expressions

This section explains how we express pattern-matching and demonstrates its expressive power.

3.1 Pattern-Matching with Backtracking

The following is syntax of the \texttt{match-all} expression. A \texttt{match-all} expression is composed of a \texttt{target}, a \texttt{matcher} and a match-clause which consists of a pattern and a body expression. A \texttt{match-all} expression evaluates the body of the match-clause for each pattern-matching result and returns the result as a collection. A matcher specifies the way to match the target with the pattern.

\[
\langle \text{match-all-expr} \rangle ::= \texttt{\{match-all\}(tgt-expr).matcher-expr.match-clause} ')'
\]

\[
\langle \text{match-clause} \rangle ::= \texttt{[\{pattern\}(expr) ',']}
\]

The following is the first samples of our \texttt{match-all} expression. The only difference among these three expressions is its matcher.

\begin{verbatim}
> (match-all {1 2 3} (list integer) [<cons $x $ts> [x ts]])
{{[1 {2 3}]}}
> (match-all {{1 2 3}} (multiset integer) [<cons $x $ts> [x ts]])
{{[1 {1 3}] [2 {1 3}] [3 {1 2}]}}
> (match-all {1 2 3} (set integer) [<cons $x $ts> [x ts]])
{{[1 {1 2 3}] [2 {1 2 3}] [3 {1 2 3}]}}
\end{verbatim}
<cons $x $ts> is a *inductive-pattern*. `cons` is a pattern-constructor. The name of a pattern-constructor starts with lowercase. The pattern-constructor `cons` takes patterns as arguments. It divides a collection into a head element and the rest. The meaning of a head differs for each matcher. For example, multisets ignore the order of the elements of the collection, so every element can be the head element. `$x` and `$ts` are called *pattern-variables*. We can access the result of pattern-matching by referring to them.

Sets ignore the order and the duplicates of the elements of the collection. Therefore, the target collection itself is bounded to `$ts`.

We can deal with pattern-matching that has infinite results. We explain this mechanism in section 5 in detail. `take` is a function that gets a number `n` and a collection `xs` and returns the first `n` elements of `xs`. `nat` is an infinite list which contains all natural numbers. `_' is a wildcard and matches with any object.

Note that we extract two elements from the collection with the nested `cons` inductive pattern.

```latex
> (take 10 (match-all nat (set integer) \[<cons $x <cons $n _>>[m n]\]))
\{[1 1] [1 2] [1 3] [2 2] [3 1] [1 4] [2 3] [3 2] [4 1]\}
```

We introduce other pattern-constructors `nil`, `join`. The `nil` pattern-constructor takes no arguments and matches when the target is an empty collection. The `join` pattern-constructor are defined only for the `list` matcher. The `join` pattern-constructor takes two arguments and divides a collection into two collections.

The following is a demonstration of `join`.

```latex
> (match-all \{1 2 3\} (list integer) \[<join $xs $ys> $xs $ys]\))
\{[\{} \{1 2 3\}] \{[1} \{2 3\}] \{[1 2} \{3\}] \{[1 2 3} \{\}\}
```

We can deal with pattern-matching that has infinite results. We explain this mechanism in section 5 in detail. `take` is a function that gets a number `n` and a collection `xs` and returns the first `n` elements of `xs`. `nat` is an infinite list which contains all natural numbers. `_' is a wildcard and matches with any object. Note that, we extract two elements from the collection with the nested `cons` inductive pattern.

```latex
> (take 10 (match-all nat (set integer) \[<cons $x <cons $n _>> [m n]\]))
\{[1 1] [1 2] [1 3] [2 2] [3 1] [1 4] [2 3] [3 2] [4 1]\}
```

We introduce other pattern-constructors `nil`, `join`. The `nil` pattern-constructor takes no arguments and matches when the target is an empty collection. The `join` pattern-constructor are defined only for the `list` matcher. The `join` pattern-constructor takes two arguments and divides a collection into two collections.

The following is a demonstration of `join`.

```latex
> (match-all \{1 2 3\} (list integer) \[<join $xs $ys> $xs $ys]\))
\{[\{} \{1 2 3\}] \{[1} \{2 3\}] \{[1 2} \{3\}] \{[1 2 3} \{\}\}
```

The order of the elements in the result of pattern-matching is very important. Specifically, why is not the result of the first expression as follow?

```latex
\{[\{} \{1 2 3\}] \{[1} \{2 3\}] \{[1 2} \{3\}] \{[1 2 3} \{\}\}
```

The reason is there is a case that the target collection is an infinite list. If we adopted the latter order, our interpreter tries to find the last element from the infinite list of natural numbers and cannot return any answers for the following pattern-matching.

```latex
> (take 10 (match-all nat (list integer) \[<join _ <cons $x _>> x]\))
\{1 2 3 4 5 6 7 8 9 10\}
```

### 3.2 Non-Linear Pattern-Matching

Non-linear pattern-matching is one of the most important features of our pattern-matching system. Non-linear pattern-matching is necessary to represent meaningful patterns against unfree data types. The following is an example of a non-linear pattern. The output of this example is the collection of numbers from which three number sequence starts.
Pattern-matching is executed from left to right, and the binding to a pattern-variable can be referred to in its right side of the pattern. In this example, at first, the pattern-variable $n$ is bound to any element of the collection. After that, the value-pattern $+(n\ 1)$ and $+(n\ 2)$ are examined. A value-pattern has ‘,’ ahead of it. The expression following ‘,’ can be any kind of expressions. A value-pattern is a pattern that matches if the object is equal with the content of the pattern. The meaning of “equal” is defined in matchers, and then varies by matchers. $+(n\ 1)$ and $+(n\ 2)$ place the right side of $n$. Therefore, after successful pattern-matching, $n$ is bound to an element from which three number sequence starts.

A value-pattern is one of the most important inventions of our proposal. Guard notation is not good with our system. This is because we would like to cut unnecessary backtracking in the middle of the pattern-matching process.

The following code is the second example of non-linear pattern-matching. It enumerates all twin primes from the infinite list of prime numbers with pattern-matching.

```
> (define $twin-primes
  (match-all primes (list integer)
    [<join _ <cons $p <cons ,(+ p 2) _>>>> [p (+ p 2)]]))
)> (take 10 twin-primes)
[[3 5] [5 7] [11 13] [17 19] [29 31] [41 43] [59 61] [71 73] [101 103] [107 109]]
```

We can write pattern-matching against nested unfree data types such as a list of multisets or a set of sets as the following sample code.

```
> (match-all {{1 2 3 4} {4 5 1} {6 1 7 4}} (list (multiset integer))
  [<cons <cons $n _> <cons <cons ,n _> <cons <cons ,n _> <nil>>>>> n])
)> (take 10 twin-primes)
[[[[1 2] [3 5]] [5 7] [11 13] [17 19] [29 31] [41 43] [59 61] [71 73] [101 103] [107 109]]]
```

Our language has also the match expression as ordinary functional languages. A match expression takes multiple match-clauses and tries pattern-matching for each pattern from the head of match-clauses. A match expression is useful to express conditional branches.

```
<match-expr> ::= ‘(match’ <tgt-expr> <matcher-expr> ‘{’ <match-clause>* ‘}’)
```

Figure 11 is a demonstration program that determines poker-hands. Note that, all poker-hands are represented in a single pattern. We explain the definition of the mod, suit and card matcher in section 4.

### 3.3 Or-Patterns, And-Patterns and Not-Patterns

We can enumerate prime triplets with pattern-matching using an or-pattern and and-pattern. An or-pattern matches with the object, if the object matches one
Matcher definitions

(define $mod
(lambda [$m]
matcher
{[$n []
  ([$tgt (if (eq? (modulo tgt m) (modulo n m))
     ([])
     ([]))
  )
  ($ [something] [($tgt {tgt}[])])
])}

(define $suit
(algebraic-data-matcher
({'<spade> <heart> <club> <diamond>})))

(define $card
(algebraic-data-matcher
({'<card suit (mod 13)>})))

A function that determines poker-hands

(define $poker-hands
(lambda [$cs]
(match cs (multiset card)
  {[<cons <card $s $n>]
    <cons <card ,s ,(n - 1)>
    <cons <card ,s,(n - 2)>
    <nil>>>>>>
    <Straight-Flush>
  }
  {[<cons <card _ $n>]
    <cons <card _ ,n>}
    <cons <card _ ,n>}
    <cons _}
    <cons _}
    <nil>>>>>>
    <Three-of-Kind>
  }
  {[<cons <card _ $n>]
    <cons <card _ ,n>}
    <cons _}
    <cons _}
    <nil>>>>>>
    <Two-Pair>
  }
  {[<cons _]
    <cons _}
    <cons _}
    <cons _}
    <nil>>>>>>
    <Nothing>
}))

Figure 1: Pattern-matching to determine poker hands
of given patterns. An and-pattern is a pattern that matches the object, if and only if all of the patterns are matched. An and-pattern is used like an as-pattern in the following code.

```scheme
> (define $prime-triplets
   (match-all primes (list integer)
     [x] join _ <cons $p
       <cons (& $m (| ,(+ p 2) ,(+ p 4)))
       <cons ,(+ p 6) _>>>
     [p m (+ p 6)])
>
> (take 7 prime-triplets)
[[5 7 11] [7 11 13] [11 13 17] [13 17 19] [17 19 23] [37 41 43] [41 43 47]]
```

A not-pattern matches with an object if the object does not match the pattern following after `^`. The following sample enumerates the elements of the collection that appears only once.

```scheme
> (match-all {1 2 3 3 2 4} (multiset integer) [<cons $x ^<cons ,x _>> x])
{1 4}
```

### 3.4 Modularization of Patterns with Lexical Scoping

Modularization of patterns is a necessary feature to reuse useful patterns. Non-linear patterns make modularization of patterns difficult.

Patterns are not first class objects in our pattern-matching system. Therefore, for example, `(define $x _)` is illegal, because `_` is a pattern and not a first class object. However, a pattern-function, a function that takes patterns and returns a pattern, is a first class object. We can define pattern-functions in anywhere of programs, and use them to generate patterns or define other pattern-functions.

```
(pat-func-expr) ::= `(pattern-function' 'I' (pat-var)* 'I' (pattern') ')'
```

Since a pattern-function has lexical scoping as a normal function by `lambda`, the bindings for the pattern-variables in the argument patterns and the body of pattern-functions don’t conflict. Then, we don’t have to care about which pattern-variable occurs in a pattern-function. In the following sample, what is bound to $m$ and $n$ don’t matter in the body of pattern-function twin. What is bound to $pat$ does not also matter in the pattern of match-all, too. We can use a variable-pattern in the body of a pattern-function. We cannot use it in the pattern of a match-clause. In the following sample, pat1 and pat2 in the body of twin are variable-patterns. They must be the arguments of the pattern-function.

```scheme
> (define $twin
   (pattern-function [$pat1 $pat2]
     <cons (& $pat pat1) <cons ,pat pat2>>))
> (match-all {1 2 1 3} (multiset integer) [<cons $m (twin $n _)> [m n]]))
{{[2 1] [2 1] [3 1] [3 1]}}
```
Our pattern-matching system restricts use of patterns in match-clauses and bodies of pattern-functions. This restriction enables us to reuse our own patterns in a simple way. If we treat a pattern as a first class object as first class patterns [Tullsen(2000)], it is difficult to modularize patterns that contain pattern variables.

A pattern-function can take only patterns. If we would like to write a pattern that takes parameters, we write a function that obtains the objects as parameters and returns a pattern-function.

Figure 2 is a demonstration program that determines whether mahjong hands are finished or not. It is a very hard task if we write it without our pattern-matching system. We explain the definitions for each matcher in section 4.

### 3.5 Formal Definition of Patterns

This is the formal definition of the syntax of patterns of our pattern-matching system.

\[
\begin{align*}
\langle \text{pattern} \rangle &::= \_ \quad \text{(wildcard)} \\
&\mid \$ \langle \text{id} \rangle \quad \text{(pattern-variable)} \\
&\mid , \langle \text{expr} \rangle \quad \text{(value-pattern)} \\
&\mid < \langle \text{id} \rangle \langle \text{pattern} \rangle^* \_ \rangle \quad \text{(inductive-pattern)} \\
&\mid \langle \text{pat-func-expr} \langle \text{pattern} \rangle^* \_ \rangle \quad \text{(pattern-application)} \\
&\mid \langle \_ \rangle \langle \text{pattern} \rangle \quad \text{(or-pattern)} \\
&\mid \langle \& \rangle \langle \text{pattern} \rangle \quad \text{(and-pattern)} \\
&\mid ^\langle \text{pattern} \rangle \quad \text{(not-pattern)}
\end{align*}
\]

\[
\begin{align*}
\langle \text{pat-func-expr} \rangle &::= \langle \text{pattern-function} \_ \langle \text{pat-var} \rangle^* \_ \rangle \langle \text{pattern} \rangle \_ \langle \text{pattern} \rangle \_ \langle \text{pattern} \rangle \\
\langle \text{pattern} \rangle &::= \langle \text{id} \rangle \quad \text{(variable-pattern)} \\
&\mid \_ \quad \text{(wildcard)} \\
&\mid \$ \langle \text{id} \rangle \quad \text{(pattern-variable)} \\
&\mid , \langle \text{expr} \rangle \quad \text{(value-pattern)} \\
&\mid < \langle \text{id} \rangle \langle \text{pattern} \rangle^* \_ \rangle \quad \text{(inductive-pattern)} \\
&\mid \langle \text{pat-func-expr} \langle \text{pattern} \rangle^* \_ \rangle \quad \text{(pattern-application)} \\
&\mid \langle \_ \rangle \langle \text{pattern} \rangle \quad \text{(or-pattern)} \\
&\mid \langle \& \rangle \langle \text{pattern} \rangle \quad \text{(and-pattern)} \\
&\mid ^\langle \text{pattern} \rangle \quad \text{(not-pattern)}
\end{align*}
\]

### 4 Matcher Definitions

We define matchers to specify how to do pattern-matching for data of each type. In this section, we explain how to define matchers.
Matcher definitions

(define $suit
(algebraic-data-matcher
 {<wan> <pin> <sou>}))

(define $honor
(algebraic-data-matcher
 {<ton> <nan> <sha> <pe> <haku> <hatsu> <chun>}))

(define $tile
(algebraic-data-matcher
 {<num suit integer> <hnr honor>})))

Pattern modularization

(define $twin
(pattern-function [$pat1 $pat2]
 <cons (& $pat pat1)
 <cons ,pat pat2>)))

(define $shuntsu
(pattern-function [$pat1 $pat2]
 <cons (& <num $s $n> pat1)
 <cons <num ,s ,(+ n 1)>
 <cons <num ,s ,(+ n 2)>
 pat2>)))

(define $kohtsu
(pattern-function [$pat1 $pat2]
 <cons (& $pat pat1)
 <cons ,pat
 <cons ,pat
 pat2>)))

A function that determines whether the hand is completed or not.

(define $complete?
(match-lambda (multiset tile)
 {[(twin $th_1
 (| (shuntsu $sh_1 (| (shuntsu $sh_2 (| (shuntsu $sh_3 (| (shuntsu $sh_4 <nil>))
 (kohtsu $kh_1 <nil>)))))
 (kohtsu $kh_1 (kohtsu $kh_2 <nil>))))
 (kohtsu $kh_1 (kohtsu $kh_2 (kohtsu $kh_3 <nil>)))))
 (kohtsu $kh_1 (kohtsu $kh_2 (kohtsu $kh_3 (kohtsu $kh_4 <nil>)))))
 (kohtsu $kh_2 (kohtsu $kh_3 (kohtsu $kh_4 <nil>)))))
 (twin $th_2 (twin $th_3 (twin $th_4 (twin $th_5 (twin $th_6 (twin $th_7 <nil>)))))))
 #t]
 [_ #f]}))

Figure 2: Pattern-matching against mahjong tiles
4.1 Formal Definition and Simple Examples

At first, we briefly explain the formal definition of matchers.

\[
\langle \text{matcher-expr} \rangle ::= \left( \langle \text{matcher} \rangle \ \{ \langle \text{primitive-pmc} \rangle^{*} \} \right)
\]

\[
\langle \text{primitive-pmc} \rangle ::= \left( \langle \text{primitive-pp} \rangle \ \langle \text{next-matcher-expr} \rangle \ \{ \langle \text{primitive-dmc} \rangle^{*} \} \right)
\]

\[
\langle \text{primitive-dmc} \rangle ::= \left( \langle \text{primitive-dp} \rangle \ \langle \text{expr} \rangle \right)
\]

\[
\langle \text{primitive-pp} \rangle ::= \$ \ \langle \text{ident} \rangle \ (\text{value-pattern-pattern})
\]

\[
\langle \text{primitive-dp} \rangle ::= \langle \text{wildcard} \rangle \ |
\langle \text{primitive-data-variable} \rangle
\]

\[
\text{primitive-pmc and primitive-dmc are abbreviations of primitive-pattern-match-clause and primitive-data-match-clause, respectively. primitive-pp and primitive-dp are abbreviations of primitive-pattern-pattern and primitive-data-pattern, respectively. A primitive-pattern-pattern is a pattern that pattern-matches against a pattern. A primitive-data-pattern is a pattern that pattern-matches against a target datum.}
\]

Here is the first sample of a matcher definition. With unordered-pair, we can pattern-match a pair of data ignoring the order of the elements of the pair. For example, the datum \langle Pair 2 5 \rangle is pattern-matched with the pattern \langle Pair ,5 $x$ \rangle.

```lisp
> (define $unordered-pair
  (lambda [a]
    (matcher {
       [<pair $ $> [a a] {
         [<Pair $x $y> {
           [x y] [y x]]}
       }]
       [$ [something] {[$tgt (tgt)]}]}
    )))

> (match-all <Pair 2 5> (unordered-pair integer) \[<pair ,5 $x$> x]\)
(2)
> (match-all <Pair 2 5> (unordered-pair integer) \[$p p$\]
\[<Pair 2 5>\]

unordered-pair is defined as a function that gets a matcher and returns a matcher. It is to specify how to pattern-match against the elements of the pair.

A matcher expression defines the way of pattern-matching. First, the pattern is pattern-matched with each primitive-pattern-pattern. \langle Pair $y$ \rangle is a primitive-pattern-pattern, and this primitive-pattern-pattern matches with the pattern \langle Pair ,5 $x$ \rangle in the first match-all expression. \$ is called a
primitive-pattern-variable and extracts a pattern. The patterns bound to ‘$’
are called next-patterns. We can create as many next-patterns as we want.
The first ‘$’ pattern-matches with ‘,5’ and the second ‘$’ matches with ‘$x’.
[a a] is a next-matcher-expression. A next-matcher-expression returns a tu-
ple of matchers. These are called next-matchers. In this case, ‘a’ is bound
to integer, and it means both ‘,5’ and ‘$x’ is matched as integer. <Pair $x $y> {{x y} {y x}} is a primitive-data-match-clause. <Pair $x $y> is
matched with the target datum <Pair 2 5>, and ‘$x’ and ‘$y’ is matched with
‘2’ and ‘5’, respectively. The pattern-matching of primitive-data-patterns is sim-
ilar with the pattern-matching of ordinary functional programming languages.
The primitive-data-match-clause returns {{2 5} {5 2}}. The primitive-data-
match-clause returns a collection of next-targets. This means ‘,5’ and ‘$x’ are
matched with ‘2’ and ‘5’ or ‘5’ and ‘2’ using integer, respectively.

The pattern of the second match-all expression is a single pattern-variable.
When the pattern is a pattern-variable, the second primitive-pattern-pattern
matches with the pattern and the second primitive-pattern-match-clause is used.
The next-matcher is something. The next-pattern and the next-target do not
change.

Next, we introduce the matcher of integers. eq? is a built-in function that
determines equality of built-in data. It returns #t if two arguments are equal,
otherwise it returns #f.

(define $integer
(matcher {{[$n []] [{[$tgt (if (eq? tgt n) {}])}]}
[[$something] [{[$tgt {tgt}]}]])

In the definition of integer, there is an example of a value-pattern-pattern.
The primitive-pattern-pattern, $n is a value-pattern-pattern. The value bounded
to the variable n can be referred in the body of the primitive-data-match-clause.
There are no next-patterns. The next-matchers is an empty tuple. If the
pattern-matching succeeds, the next-targets is a collection consists of an empty
tuple. Otherwise, the next-targets is an empty collection.

We can define the mod matcher as follow. mod is a function that takes a
number and return a matcher. (mod m) is a matcher for the quotient ring
modulo m.

(define $mod
(lambda [$m]
(matcher
{{[$n []] [{[$tgt (if (eq? (modulo tgt m) (modulo n m)) {}])}]}
[[$something] [{[$tgt {tgt}]}])

We can define the card matcher that has appeared in the poker hands ana-
lyzer in figure 1 using mod.

(define $suit
(matcher {{<spade> []} {{<Diamond> {}}}} [_ {}] [[<Heart> {}] {}])

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Matcher definitions for algebraic data types are verbose. We have a syntax sugar for that.

\[
\text{\langle algebraic-data-matcher-expr \rangle} ::= \text{`} (\text{\langle constructor-definition \rangle}^* \text{'})
\]

\[
\text{\langle constructor-definition \rangle} ::= \text{`} (\text{\langle ident \rangle} \text{\langle matcher-expr \rangle}^* \text{'})
\]

We define the suit and card matcher using the algebraic-data-matcher expression in figure 1. We also define the tile matcher that has appeared in figure 2 using the algebraic-data-matcher expression.

### 4.2 Matchers for Collection Data Types

In this section, we explain matchers that handle collections.

Our pattern-matching system handles collections as primitive and prepares primitive-pattern-patterns for them. The list matcher is defined using them. In the following code, we omit the piece of code that handles the pattern constructor join.

\[
\text{(define \$list \{lambda [\$a] \text{matcher}}
\]

\[
\text{\{[\$tgt \langle \text{match \[\text{val \ tgt} \ (\text{list} \ a) \ (\text{list} \ a)\]}\}
\]
\]

\[
\text{\{\text{[nil]} \ \text{[{} \ (\text{[]} \ (\text{[])})]}\}}
\]

\[
\text{\{\text{[nil]} \ \text{[{} \ (\text{[]} \ (\text{[])})]}\}}
\]

\[
\text{\{\text{[nil]} \ \text{[{} \ (\text{[]} \ (\text{[])})]}\}}
\]

\[
\text{\{\text{[\text{\$tgt} \ \text{letrec \[\text{[unjoin'} \ (\text{lambda \ [\text{\$xs} \ \text{\$ys}] \ (\text{match \ ys} \ (\text{list} \ a) \ ([\text{\text{[\text{\$tgt}} \ (\text{[]} \ (\text{[]})))\]
\]

\[
\text{\{\text{[unjoin'} \ (\text{\$tgt})\]})\}]}\}
\]

\[
\text{\{\text{\$ [something]} \ (\text{[\text{\$tgt} \ \text{[\text{tgt}]})])\]}}
\]

A Definition of the matcher multiset and set are given as follows. We can define them simply using the list matcher.

\[
\text{(define \$multiset \{lambda [\$a] \text{matcher}}
\]

\[
\text{\{\text{[\text{\$tgt} \ \text{letrec \[\text{[unjoin'} \ (\text{lambda \ [\text{\$xs} \ \text{\$ys}] \ (\text{match \ ys} \ (\text{list} \ a) \ ([\text{\text{[\text{\$tgt}} \ (\text{[]} \ (\text{[]})))\]
\]

\[
\text{\{\text{[unjoin'} \ (\text{\$tgt})\]})\}]}\}
\]

\[
\text{\{\text{\$ [something]} \ (\text{[\text{\$tgt} \ \text{[\text{tgt}]})])\]}}
\]

4
Note the importance of value-pattern-patterns, we cannot realize non-linear pattern-matching without them. This is because we need to handle values in a pattern to realize non-linear pattern-matching. Associating the definition for the way to handle values to the matcher using value-pattern-pattern is one of the possible solution to realize that.

5 Mechanism of Pattern-Matching

In this section, we explain the implementation of our pattern-matching system.

5.1 Notions

We introduce several notions to explain our pattern-matching mechanism. Here is a really brief explanation of each notion. We will deepen the understanding of these notions, examining the examples in the following sections.

Matching-State Our pattern-matching process is reduction of a collection of matching-states. Each matching-state has a stack of matching-trees and data to proceed pattern-matching.

Matching-Tree A matching-tree has two kinds of forms, a matching-atom and a matching-node.

Matching-Atom A matching-atom is a tuple of a pattern, a target, and a matcher.

Matching-Node A matching-node has a stack of matching-trees as a matching-state. The structure of matching-node is similar with a matching state.
5.2 Simple Non-Linear Patterns

In this section, we explain how pattern-matching is executed for a simple non-linear pattern. Let us examine what will happen when our system evaluates the following pattern-matching expression.

> (match-all {2 8 2} (multiset integer)
  [<cons $m <cons ,m _>> m])
{2 2}

At first, the initial matching-state is generated. It is as follow. The data constructor MState takes three arguments, a stack of matching-trees, an environment, and a result in the middle of the pattern-matching. The ‘env’ is the environment when the evaluation process enters the match-all expression. The stack of matching-tree contains a single matching-atom whose pattern, target and matcher are same with the arguments of match-all.

MState {{<cons $m <cons ,m _>> {2 8 2} (multiset integer)}} env {}  

The stack of the matching-tree contains only one matching-atom. This matching-atom is reduced with the matcher (multiset integer) as specified in the matching-atom. The matching-states increases to 3 with this reduction as follow.

MState {{<cons $m 2 integer} [ <cons ,m _> (8 2) (multiset integer)]} env {}  
MState {{<cons $m 8 integer} [ <cons ,m _> (2 2) (multiset integer)]} env {}  
MState {{<cons $m 2 integer} [ <cons ,m _> (2 8) (multiset integer)]} env {}

We focus on the first matching-state, for now. This matching-state is reduced as follow in the next reduction step. The matcher of the matching-atom of the top of the stack is changed to ‘something’ from ‘integer’. something is the built-in matcher of our pattern-matching system. It can match only with a wildcard or a pattern variable.

MState {{<cons $m 2 something} [ <cons ,m _> (8 2) (multiset integer)]} env {}  

This matching-state is reduced as follow in the next reduction step. The matching atom of the top of the stack is popped out, and a new binding [m 2] is appended to the result of the middle of pattern-matching. something can only append a new binding to the result of pattern-matching.

MState {{<cons ,m _> (8 2) (multiset integer)}} env {[m 2]}  

This matching-state is reduced as follow in the next reduction step. The matching-states increases to 2 with this reduction.

MState {{,m 8 integer} [ _ (2) (multiset integer)]} env {[m 2]}  
MState {{,m 2 integer} [ _ (8) (multiset integer)]} env {[m 2]}
In the above matching-states, \(m\) is pattern-matched with 8 and 2 respectively as \texttt{integer}. When we do pattern-matching with the value pattern, the result of the middle of pattern-matching is used to evaluate it. Therefore, in this case, \(m\) is evaluated to 2. The first matching-state fails to pattern-match. The second matching-state succeeds in pattern-matching and be reduced as follow in the next reduction step.

\[
\text{MState} \{\_ \{8\} \text{(multiset integer)}\} \text{ env } \{\text{m} \ 2\}
\]

This matching-state is reduced as follow in the next reduction step. The pattern is a wildcard and matches with any object. No new binding is appended to the result of pattern-matching.

\[
\text{MState} \{\} \text{ env } \{\text{m} \ 2\}
\]

When the matching-tree stack is empty, the reduction finish. This result of pattern-matching \([\text{m} \ 2]\) is added to the final result.

5.3 Or-Patterns, And-Patterns and Not-Patterns

Or-patterns, and-patterns and not-patterns are specially handled. In this section, we explain them.

5.3.1 Or-Patterns

Let us examine what will happen when our system evaluates the following pattern-matching expression.

\[
> \text{(match-all \{1 1 2\} \text{(list integer)}
    \[<\text{cons} \$m \ (| <\text{nil}> <\text{cons} \ ,m\ _>)> m\]\)
\{1\}
\]

Our system reaches the following matching-state.

\[
\text{MState} \{(| \text{nil}\> <\text{cons} \ ,m\ _>) \{1 \ 2\} \text{(list integer)}\} \text{ env } \{\text{m} \ 1\}
\]

This matching-state is reduced as follow in the next reduction step.

\[
\text{MState} \{\text{nil}\> \{1 \ 2\} \text{(list integer)}\} \text{ env } \{\text{m} \ 1\}
\]

\[
\text{MState} \{<\text{cons} \ ,m\ _> \{1 \ 2\} \text{(list integer)}\} \text{ env } \{\text{m} \ 1\}
\]

5.3.2 And-Patterns

Let us examine what will happen when our system evaluates the following pattern-matching expression.

\[
> \text{(match-all \{1 2 3\} \text{(list integer)}
    \[<\text{cons} \$n \ (& <\text{cons} \ _ _> $rs)> [n \ rs]\]\)
\{[1 \ {2 \ 3}]\}
\]

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Our system reaches the following matching-state.

\[
\text{MState} \left\{ \left[ (\& \ <\text{cons} \ _ \ _ > \ $rs) \ \{2 \ 3\} \ \text{(list integer)} \right] \ \\text{env} \ \{ [n \ 1] \} \right\}
\]

This matching-state is reduced as follow in the next reduction step.

\[
\text{MState} \left\{ \left[ (\<\text{cons} \ _ \ _ > \ {2 \ 3} \ \text{(list integer)} \right] \ \left[ \$rs \ \{2 \ 3\} \ \text{(list integer)} \right] \ \\text{env} \ \{ [m \ 1] \} \right\}
\]

5.3.3 Not-Patterns

Let us examine what will happen when our system evaluates the following pattern-matching expression.

\[
> \text{(match-all} \ \{2 \ 8 \ 2\} \ \text{(multiset integer)}
\quad \left[ (\<\text{cons} \ \_ \ _ > \ \$m \ \& \ ^\text{<\text{cons} \ ,m \ _ >} \ $rs) > \ [m \ rs]] \ \right] \ \{[8 \ {2 \ 2}]\}
\]

Our system reaches the following matching-state.

\[
\text{MState} \left\{ \left[ ^\text{<\text{cons} \ ,m \ _ >} \ {2 \ 2} \ \text{(multiset integer)} \right] \ \left[ \$rs \ \{2 \ 2\} \ \text{(multiset integer)} \right] \ \\text{env} \ \{ [m \ 8] \} \right\}
\]

When our system reaches the matching-state whose top matching-atom is a not-pattern, our system generates a new matching-state which contains only the matching-atom with the not-pattern as follow. All information of the matching-state and the matching-nodes except about the rest of matching-tree stack are retained.

\[
\text{MState} \left\{ \left[ ^\text{<\text{cons} \ ,m \ _ >} \ {2 \ 2} \ \text{(multiset integer)} \right] \ \left[ \$rs \ \{2 \ 2\} \ \text{(multiset integer)} \right] \ \\text{env} \ \{ [m \ 8] \} \right\}
\]

Our system proceeds the pattern-matching on the new generated matching-state, and if it fails pattern-matching our system pops out the matching-atom of the not-pattern from the original matching-state as follow and proceeds the pattern-matching. Otherwise our system fails the pattern-matching.

In this case, the above matching-state succeeds pattern-matching. Therefore, the matching-state with the not-pattern is reduced as follow.

\[
\text{MState} \left\{ \left[ \$rs \ \{2 \ 2\} \ \text{(multiset integer)} \right] \ \\text{env} \ \{ [m \ 8] \} \right\}
\]

5.4 Application of Pattern-Functions

In this section, we explain how our system deals with modularization of patterns. Let us examine what will happen when our system evaluates the following pattern-matching expression.
> (define $twin
(pattern-function [[$pat1 $pat2]
<cons ($pat pat1)
<cons ,pat
pat2>])
> (match-all {1 2 1 3} (multiset integer)
[<cons $m (twin $n _)> [m n]])
{[2 1] [3 1] [2 1] [3 1]}

Our system reaches the following matching-state.

MState {[(twin $n _) {1 1 3} (multiset integer)]} env {[m 2]}

This matching-state is reduced as follow in the next reduction step. A
matching-node has extra information, a pattern-environment. In this case, the
pattern-environment is {[pat1 $n] [pat2 _]}.

MState {(MNode {[<cons (& $pat pat1) <cons ,pat pat2>>
{1 1 3} (multiset integer)])
env1 {[]} {[pat1 $n] [pat2 _]})
env {[m 2]}

Our system reaches the following matching-state. When the top of the
matching-tree stack of the matching-state is a matching-node, our system pops
the matching-atom of the top of the matching-tree stack of the matching-node. If the top of the matching-tree stack of the matching-node is a matching-node
again, our system pops out the matching-atom from the top of the matching-tree
stack of that matching-node.

MState {(MNode {[pat1 1 integer]
[<cons ,pat pat2> {1 3} (multiset integer)]})
env1 {[pat 1] {[pat1 $n] [pat2 _]})
env {[m 2]}

This matching-state is reduced as follow in the next reduction step. pat1
is called a variable-pattern. It can appear only in the body of pattern-functions. When the matching-atom whose pattern is a variable-pattern is popped out,
our system gets what pattern is bound to the variable-pattern from the pattern-
environment, and push a new matching-atom to the matching-tree stack of the
one level upper matching-node or matching-state.

MState {[$n 1 integer]
(MNode {[<cons ,pat pat2> {1 3} (multiset integer)]})
env1 {[pat 1] {[pat1 $n] [pat2 _]})
env {[m 2]}

The arguments of a pattern-function are handled in special way as above.
This is the reason why the pattern-function can take only patterns. A pattern
must be bound to a variable-pattern.
5.5 Pattern-Matching with Infinite Results

In this section, we explain how our system executes pattern-matching which has infinite results. Let us examine what will happen when our system evaluate the following pattern-matching expression.

> (take 10 (match-all nat (set integer) [<cons $m <cons $n _>>[m n]])

Figure 3 is the reduction tree of matching-states when we execute the pattern-matching above. Rectangles stand for matching-states. The rectangle at the upper left is the initial matching-state. Circles stand for final matching-states that succeed pattern-matching.

The width of a reduction tree of matching-states can be infinite because there are cases that a matching-state is reduced to infinite matching-states. The depth of a reduction tree also can be infinite if we use a recursive pattern-function in a pattern. We need to think on the order of reduction to examine all nodes of a reduction tree. The numbers on rectangles and circles denote the order of reduction. If we see a reduction tree obliquely, it can be regarded as a binary tree. Therefore, we can trace all nodes of reduction trees if we do breadth-first search on the tree, though it will use a lot of memory.

6 Related Work

In this section, we introduce existing studies in the field of pattern-matching. Miranda laws [Thompson(1990), Turner(1985)] and Wadler’s views [Wadler(1987)] are famous work. These proposals provide the way to destruct data which have
multiple representations, by declaring transformation between each representations. Data are automatically transformed in the matching process. However, the pattern-matching systems of these proposals treat neither multiple results of pattern-matching nor non-linear patterns. Therefore, these studies are not useful for pattern-matching with unfree data types.

Active patterns [Erwig(1996)] provide a way to destruct unfree data. We define a *match function* for each pattern constructor to destruct unfree data. In the following sample code, Add' is a match function. With the match function Add', we can extract an element ignoring the order of elements from the target which is constructed with the Add constructor.

\[
\text{pat Add’ (x,_) =} \\
\quad \text{Add (y,s) => if x == y then (y,s)} \\
\quad \quad \text{else let Add’ (x,t) = s} \\
\quad \quad \quad \text{in Add (x, Add (y, t)) end}
\]

\[
\text{fun member x (Add’ (x,s)) = true} \\
\quad \mid \text{member x s = false}
\]

The weakness of active patterns is that it does not support backtracking in the pattern-matching process. The value bound to pattern variables must be fixed from the left side of a pattern, though many forms should be tried for pattern-matching with unfree data types. Active patterns also do not support pattern-matching with nested unfree data types, such as sets of sets. This is because a pattern-matching method is defined for each constructor, and a head argument of a match function must be constant.

First class patterns [Tulsen(2000)] propose a sophisticated system which treats patterns as first class objects. The essence of this study is a *pattern-function* that defines how to destruct data with each data constructor. In the following sample code, cons# is a pattern-function. The pattern function cons# helps to destruct a list in the join representation.

\[
\text{data List a = Nil | Unit a | Join (List a) (List a)}
\]
\[
\text{cons x xs = Join (Unit x) xs}
\]
\[
\text{cons# Nil = Nothing} \\
\text{cons# (Unit a) = Just (a,Nil)} \\
\text{cons# (Join xs ys) = case cons# xs of} \\
\quad \text{Just (x,xs’)} \rightarrow \text{Just (x, Join xs’ ys)} \\
\quad \text{Nothing} \rightarrow \text{cons# ys}
\]

First class patterns can deal with pattern-matching which generates multiple results. To generate multiple results, a pattern-function returns a list, not a datum of the type Maybe. However pattern-matching with this proposal also has a weak point. First class patterns do not support non-linear pattern-matching, though non-linear patterns are necessary to express meaningful patterns for unfree data types.

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7 Conclusion

The contribution of the proposal of this paper can be divided into two.

The first contribution is the realization of non-linear pattern-matching against unfree data types. It is done by realizing non-linear pattern-matching, pattern-matching with multiple results and modularization of the way of pattern-matching for each data type at the same time. The existing pattern-matching systems does not support all of them at the same time. Non-linear pattern-matching is realized with a rule that pattern-matching is executed from the left side of the pattern. Non-linear patterns are represented with value-patterns that match if the target is equal with the content of the pattern. One of the characteristics of our method is it specifies pattern-matching methods with matchers for each data type not for each pattern-constructor. It enables us to reuse pattern-constructors and pattern-functions for similar data types. For example, we can use the same pattern-constructors and pattern-functions, such as \texttt{nil}, \texttt{cons} and \texttt{twin} for lists and multisets. This is very useful because unfree data can be pattern-matched using different matchers in different places of the program.

The second contribution is the realization of non-linear pattern-matching with lexical scoping. It enables us to modularize and reuse patterns. Lexical scoping in patterns became difficult and necessary because of non-linear patterns. It is realized with a restriction that a pattern is not a first class object but a pattern-function that obtains only patterns and returns a pattern is a first class object. The tree-shaped matching-tree stack mechanism realizes lexical scoping in patterns.

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