How magnetic field can transform a superconductor into a Bose metal

Tianhao Ren and Alexei M Tsvelik
Condensed Matter and Materials Physics Division, Brookhaven National Laboratory, Upton, NY 11973-5000, United States of America

Abstract
We discuss whether a simple theory of superconducting stripes coupled by Josephson tunneling can describe a metallic transport, once the coherent tunneling of pairs is suppressed by the magnetic field. For a clean system, the conclusion we reached is negative: the excitation spectrum of preformed pairs consists of Landau levels, and once the magnetic field exceeds a critical value, the transport becomes insulating. As a speculation, we suggest that a Bose metal can exist in disordered systems provided that the disorder is strong enough to localize some pairs. Then the coupling between propagating and localized pairs broadens the Landau levels, resulting in a metallic conductivity. Our model respects the particle–hole symmetry, which leads to a zero Hall response. And intriguingly, the resulting anomalous metallic state has no Drude peak and the spectral weight of the cyclotron resonance vanishes at low temperatures.

1. Introduction
The discovery of the anomalous superconducting state in the stripe-ordered lanthanum barium copper oxide (LBCO) [1, 2] has gradually aroused interest in the possible pair density wave (PDW) state—a superconducting state where pairs carry finite momentum (see [3] for the most recent review of the field). At zero field this 3D layered material exhibits an unusual 2D superconductivity with an in-plane Berezinskii–Kosterlitz–Thouless (BKT) transition coexisting with a finite resistivity along the c-axis. This resistivity vanishes at a much smaller temperature, marking an onset of 3D superconductivity with the Meissner effect.

The recent experiments on several stripe-ordered superconductors have revealed that once the superconductivity is destroyed by the applied magnetic field, a peculiar resistive metallic state with zero Hall response emerges, which persists down to lowest temperatures [4–6]. The Hall response vanishes at the BKT temperature, and it remains zero throughout the entire low temperature region even when the superconductivity is destroyed. The critical field is relatively small. The electrical resistance gradually increases with the field, and at fields around 25–30 T, the sheet resistance reaches a plateau at $R_{\square} \approx 2\pi \hbar / 2e^2$.

Similar anomalous metallic states have been observed in disordered thin films (see, for instance, [7] and references therein) and proximity Josephson junction arrays [8]. The situation with proximity Josephson junction arrays bears the closest resemblance to the one which takes place in the stripe-ordered LBCO, with the difference that the space between the superconducting granules is filled with an ordinary metal [9–11]. Since the fermionic quasiparticles are necessary there, one cannot call such state a Bose metal.

The purpose of this paper is to find out whether one can describe the anomalous metal without invoking quasiparticles. Experimentally, the stripe-ordered LBCO appears to be a good candidate for this. Its simplest description [12, 13] does not invoke quasiparticles and this is the prism through which we will consider this system. We augment the model by the long-range Coulomb interaction and call it the wire theory (WT for short). WT may be considered as a minimalistic model of PDW since it does not introduce any other entity besides preformed pairs. It describes the stripe-ordered state as an assembly of...
one-dimensional superconducting wires (stripes) separated by insulating regions and coupled by Josephson tunneling. Each wire constitutes a Luther–Emery liquid, where the interactions generate a spin gap responsible for the superconducting pairing. The bulk superconductivity emerges when the pair tunneling establishes a global coherence. The distinguishing feature of WT is the suggestion that the Josephson coupling has the wrong sign so that the sign of the superconducting order parameter alternates between the neighboring stripes. Since the stripe orientation changes along the c-axis, the sign alternation frustrates the pair tunneling in this direction. It is also assumed that the low energy sector of the system is occupied by bosonic excitations—charged pairs and spin fluctuations from the undoped copper oxide chains between the stripes. If WT is correct and there are no fermionic quasiparticles, then the low temperature resistive state is some kind of Bose metal, where the transport is carried by incoherent pairs.

Below we will study WT both in zero and finite magnetic field. We demonstrate that in its simplest form the theory cannot explain the experiments and hence requires certain modifications. It is shown that the excitation spectrum of pairs experiences Landau quantization. Although the nonlinear effects lead to a certain broadening of the Landau levels, in the clean system this effect is not sufficiently strong to lead to metallic transport. As a consequence, once the magnetic field destroys the coherence of pairs, the system becomes an insulator. We suggest as a speculation that a Bose metal can exist in disordered systems provided that the disorder is strong enough to localize some pairs. Due to their small size, these states do not experience Landau quantization and serve as a reservoir of low energy states for the transport. This mechanism allows the existence of a resistive state in the purely bosonic theory of PDW, though it does not explain all the universal features of the transport. An alternative possibility was described in section III.A.3 of [14], where one of the authors suggested that the resistive state in the stripe-ordered LBCO owes its existence to the presence of fermionic quasiparticles. This brings us to theories described in [9–11], where superconducting islands embedded in a metallic environment, instead of superconducting stripes coupled by Josephson tunneling, are discussed.

This paper is organized as follows. In section 2, we introduce the model and discuss its properties in zero magnetic field. In section 3, we discuss the model in finite magnetic field when superconductivity is suppressed. The calculation within the random phase approximation (RPA) gives us an insulator in a clean system. In section 4, we discuss the above-mentioned speculation to obtain a Bose metal by introducing localized pairs due to strong disorder. The resulting anomalous metallic state has a zero Hall response due to the particle–hole symmetry, possesses no Drude peak, and has a vanishing cyclotron resonance at low temperatures. These features are consistent with the experiments [4, 6, 7, 15]. Finally in section 5, we make the conclusion. Extra technical details are relegated to the appendices.

2. The model

The model of the stripe-ordered state we consider is of a 3D array of coupled Luther–Emery liquids augmented with the long-range Coulomb interaction, where the c-axis coupling is frustrated and can be set to zero. One can consider two versions of it: one where the stripes consist of single doped chains and another where they consist of double chains (see figure 1). In the first (second) case, the superconductivity competes with \(2k_F(4k_F)\) charge density wave, or CDW for short, since at weak interactions both susceptibilities are singular. If the pairing susceptibility is singular in the second case, the charge density correlations at \(2k_F\) are short-ranged (see, for example, [16]). To simplify matters, we will consider the case when the CDW matrix elements are zero. In this case, the Lagrangian can be written as

\[
L = \sum_j \int dx \left\{ \frac{\nu}{2\pi} \left( \partial_i \theta_j - \frac{1}{c} A_i^j \right)^2 + i \left( \partial_i \phi_j - A_i^j \right) \partial_i \phi_j \right. \\
+ \left. \frac{\pi \nu}{2} \left( \partial_i \phi_j \right)^2 - J_{\langle i \rangle} \cos \left( \theta_i - \theta_j + \frac{1}{c} \int_{i}^{j} ds A'(s) \right) \right]\right\} \\
+ \frac{1}{32e^2 \pi} \int dx \ dy \ dz \left[ (\nabla \times A)^2 + \left( \nabla A^0 - \frac{1}{c} \partial_j A \right)^2 \right],
\]

(2.1)

where a single layer of arrays is considered, since the c-axis coupling is frustrated and can be set to zero. Also, we find this form suggested in [17] where dual bosonic fields \(\theta, \phi\) are both present more convenient.

If we adopt the gauge where \(\nabla \cdot A = 0\), then the \(A^0\) component decouples and can be integrated out. The resulting contribution to the Lagrangian density is
two relevant issues: the plasmon mode in zero magnetic field and the pairing susceptibility of a single stripe.

2.1. The plasmon mode in zero magnetic field
In the superconducting phase where \( \theta \) fields are pinned either at 0 (\( J > 0 \)) or at 0 and \( \pi \) on alternating stripes (\( J < 0 \)). The later case can be reduced to the former by the substitution \( \theta_j = j\pi + \bar{\theta}_j \) where \( \bar{\theta}_j \) is a slow function of coordinates. Then we can expand the cosine term and obtain the spectrum

\[
\omega^2 = C(|q|) \left[ \pi^{-1} v_x q_y^2 + 4J/|\sin^2(q_x a_0/2)| \right],
\]

where \( a_0 \) is the distance between the stripes and \( C(|q|) \) is the Fourier component of the matrix \( C_{ij} \). In the arrangement relevant to stripe-ordered cuprates, the stripes comprise a three dimensional structure and the Fourier transform of \( C_{ij} \) depends on all three components of the wave vector. At small wave vectors \( C(|q|) \sim |q|^{-2} \), therefore the spectrum in 3D is gapped, as is customary for 3D plasmons. In literature related to the stripe-ordered states, the Goldstone excitations in Fulde–Ferrel–Larkin–Ovchinnikov superconductor have been considered [18], but the issue of the Coulomb interaction were not addressed.

As for experimental data, the value of the in-plane plasma frequency for \( x = 1/8 \) LBCO extracted from the optical measurements [19] is around 1600 cm\(^{-1} \) (200 meV). The same experiments give the spectral gap the estimate around 20 meV so that the plasma frequency is well above this cutoff, and we can ignore the effect of the plasmon mode in the following discussions.

2.2. Pairing susceptibility for a single stripe
As far as pairing susceptibility is concerned, we will show that the long-range nature of the Coulomb interaction does not change matters qualitatively. For this, we calculate the pairing susceptibility for a single stripe. We assume that the system is 3D and integrate over the transverse momentum Q over the 2D slice of the Brillouin zone. In what follows, we neglect the \( q_x \)-dependence of \( C(Q, q_z) \). Then the pairing susceptibility for a single stripe is obtained as

\[
\chi_p(\tau, x) \sim \exp \left\{ -N \int \frac{d^2 Q}{(2\pi)^2} \frac{d\omega}{(2\pi)^2} \frac{1 - e^{-i\omega \tau + ipx}}{\omega^2 / vC(Q) + vq_y^2 / \pi} \right\}
\]

\[
= \exp \left\{ -\frac{N}{4v} \int \frac{d^2 Q v(Q)}{(2\pi)^2} \ln \left( \frac{v^2(Q)\tau^2 + x^2}{a_0^2} \right) \right\},
\]

where \( a_0 \) is the distance between the stripes. Since the backscattering term is absent in the current case, we can also integrate over \( \phi \) fields and obtain a closed expression for the Lagrangian:

\[
L = \sum_i \int dx \left[ \frac{1}{2} \partial_i \theta_i C_{ij} \partial_j \theta_j + \frac{v}{\pi} \left( \partial_i \theta_i - \frac{1}{c} A_i^2 \right)^2 + J_{ij} \cos \left( \theta_i - \theta_j + \frac{1}{c} \int_j^i ds A^i(s) \right) \right]
\]

\[
+ \frac{1}{32e^2\pi} \int dx\, dy\, dz(\nabla \times A)^2,
\]

where the matrix \( C_{ij} \) is defined through

\[
\sum_{ij} \partial_i \phi_i C_{ij} \partial_j \phi_j = \sum_i \pi v(\partial_i \phi_i)^2 + \sum_{ij} \int d\xi \partial_x \phi_i(x) \frac{4e^2}{(\xi^2 + a_0^2 (i-j)^2)^{1/2}} \partial_x \phi_j(x + \xi).
\]
where \( \nu(Q) = \nu[Q/(\pi)^{1/2}] \) and \( N \) is a normalization factor such that we get back to the noninteracting result when we set \( \nu = 0 \). This function can be approximately replaced by the standard power law with the scaling dimension

\[
d = d_0 \left[ \sqrt{1 + \alpha + \alpha \ln(\alpha^{-1/2} + \sqrt{1 + \alpha^{-1/2}})} \right],
\]

where \( \alpha = 16e^2/\pi, d_0 = 1/4 \). As a result, we can imitate the effect of the long-range Coulomb interaction as equivalent to the renormalization of the Luttinger parameter. In what follows, we focus on the case \( d \ll 1 \) when the Josephson tunneling is relevant.

### 3. Finite magnetic field. RPA approach

In this section we study the low temperature regime in strong magnetic field when superconductivity is suppressed. We treat the magnetic field inside the sample as uniform. We expect this approximation to be valid in strong magnetic fields. Below we consider the case with \( J < 0 \). In our analysis, we use the RPA, which replaces the original action for the order parameter field \( \Phi = \exp(i\tilde{d}) \) by the Gaussian one:

\[
S = \sum_{\omega,q,p} \Phi^+(\omega,q,p) \chi_p^{-1}(\omega,q) \Phi(\omega,q,p) - \int d\omega \int d\omega \int dx \left[ \Phi^+_j(\omega,x) e^{2\mu\Phi_j(x)/c} \Phi^+_{j+1}(\omega,x) + \text{H.c.} \right],
\]

where \( \chi_p \) is the pairing susceptibility for a single stripe calculated in section 2.2. The formal expansion parameter of RPA is the inverse number of nearest neighbors, so we expect our results to be valid only qualitatively. In the momentum representation things become even more convenient:

\[
S = \sum_{\omega,q,p} \Phi^+(\omega,q,p) \left[ \chi_p^{-1}(\omega,q) \Phi(\omega,q,p) - J e^{ip\Phi(\omega,q-p)} - J e^{-ip\Phi(\omega,q+p)} \right],
\]

where \( h = 2eHd_0/c \). To be compared with the experimental data, it is convenient to express it as

\[
\hbar d_0 = \frac{2\mu_B H}{\hbar^2/2m_e d_0},
\]

where \( \mu_B \) is the Bohr magneton and \( m_e \) is the electron mass. To calculate the Green’s function \( G \equiv -i\langle \Phi\Phi^\dagger \rangle \), we need to solve the equation

\[
\left[ \chi_p^{-1}(\omega,q) \delta_{q,q'} - J e^{ip\Phi(\omega,q-h)} - J e^{-ip\Phi(\omega,q+h)} \right] G(\omega,q',q'^\prime; p) = \delta_{q,q'^\prime}.
\]

The formal solution is expressed via normalizable eigenfunctions satisfying

\[
\left[ \chi_p^{-1}(\omega,q) \delta_{q,q'} - J \delta_{q,q'-h} - J \delta_{q,q'+h} \right] \Psi(q') = \lambda \Psi(q),
\]

such that the Green’s function can be expressed as

\[
G(\omega,q,q'; p) = \sum_\lambda e^{ip\Phi(q'-q)/\hbar} \Psi_\lambda(q) \Psi^\dagger_\lambda(q'),
\]

Taking into account that \( a_0 \approx 1.5 \text{ nm} \) we have \( \hbar^2/(4m_e a_0^2) \approx 10^2 \text{ K} \). For \( H = 30 \text{ T} \), we obtain \( \hbar d_0 \approx 1/3 \), which means that for the entire experimental range of \([4–6]\) it is reasonable to adopt \( \hbar d_0 \ll 1 \). Then we can reformulate the eigenvalue problem (3.5) as a differential equation:

\[
\left[ \chi_p^{-1}(\omega,q) - 2J \right] \Psi(q) - \hbar^2 \frac{d^2}{dq^2} \Psi(q) = \lambda \Psi(q).
\]

It is more convenient to perform the calculations in imaginary time, since then the Green’s function is real, and to do analytic continuation afterward. For \( T = 0 \) we have

\[
\chi_p^{-1}(i\omega_n,q) = -\left[ (\omega_n^2 + v^2 q^2)^{1/2} - \Lambda \right]^{-d/2},
\]

where \( \Lambda \) is the high energy cutoff, and we will set \( \Lambda = 1 \) and restore it when necessary. Let us perform the rescaling
\[
\omega_n = \epsilon_0 \Omega_n, \quad vq = \epsilon_0 x, \quad \epsilon_0 = (|J| h^2 v^2)^{\frac{1}{2m}},
\]

such that equation (3.7) assumes the dimensionless form:

\[
\left(\chi^2 + \Omega_n^2\right)^{1-d} - \frac{d^2}{dx^2} \Psi_k(x) = f_k(i\Omega_n)\Psi_k(x),
\]

\[
\lambda = 2|J| - (|J| h^2 v^2)^{\frac{1-d}{d}} f_k, \quad k = 0, 1, \ldots,
\]

where the eigenfunctions \(\Psi_k(x)\) can be chosen to be real. Then the Green’s function after analytic continuation \(i\Omega_n \to \omega + i0\) becomes

\[
G^0(\omega, q', q; p) = \sum_{k=0}^{\infty} \frac{\Psi_k(q' / \epsilon_0) \Psi_k(q / \epsilon_0)}{2|J| - (|J| h^2 v^2)^{\frac{1-d}{d}} f_k(\omega / \epsilon_0)}. \tag{3.11}
\]

The pole of this Green’s function at \(\omega = 0\) marks the critical magnetic field above which superconductivity will be lost. Since the eigenvalue in equation (3.10) for \(\Omega_n = 0\) is on the order of one, the critical magnetic field \(h_c\) can be determined as

\[
v h_c \sim \Lambda \left(\frac{|J|}{\Lambda}\right)^{\frac{1}{2d}}, \quad \epsilon_0 = \Lambda \left(\frac{2|J|}{f_0(0)}\right)^{\frac{1}{2d}}. \tag{3.12}
\]

It should be noted that the present calculation is valid only for \(h > h_c\), when the cyclotron radius for pairs \(v / \epsilon_0\) is finite.

To simplify the expressions, below we will set \(\epsilon_0 = v = 1\), and restore them when necessary. For real \(\Omega_n\), Schrödinger equation (3.10) has a discrete spectrum with eigenvalues depending on \(\Omega_n\). The wave functions for even \(k\) are even, for odd \(k\) are odd and vanish at \(x = 0\). It is interesting to express the Green’s function in real space:

\[
G^0(\omega, x_2, x_1, y) = \frac{e^{i\chi_{0}(x_{2} - x_{1})}}{2|J|} \int dq (2\pi)^d \Psi_k(q / \epsilon_0) \Psi_k(q + \frac{\Omega_n}{2}) - 1. \tag{3.13}
\]

It is obvious that for \(d \neq 0\), functions \(f_k(i\Omega_n)\) have branch cuts so that under analytic continuation \(i\Omega_n \to \omega + i0\) they become complex. This means that the discrete levels of the Schrödinger equation (3.10) acquire finite width. Moreover, since we are considering stripes made of double chains, CDW is suppressed and the CDW operator will not couple strongly to disorder.

We can calculate \(\delta f_k(\omega) = f_k(\omega) - f_k(0)\) for small \(\omega\) using the first order perturbation theory:

\[
\delta f_k(\Omega_n) = \int dx \Psi_k^2(x) \left[\chi^2 + \Omega_n^2\right]^{1-d} - |x|^{2(1-d)}]. \tag{3.14}
\]

followed by the analytic continuation \(i\Omega_n \to \omega + i0\). Then the imaginary and real parts of \(\delta f_k(\omega)\) are

\[
\Im \delta f_k \approx -2 \sin(\pi d) \text{sgn}(\omega) \int_0^{\omega} \left[\omega^2 - x^2\right]^{1-d} \Psi_k^2(x) dx,
\]

\[
\Re \delta f_k \approx -(1 - d) \omega^2 \int dx \Psi_k^2(x) x^{-2d} dx. \tag{3.15}
\]

The calculation of the imaginary part depends on \(k\). For even \(k\), we have

\[
\Im \delta f_k \approx -2 \sin(\pi d) \text{sgn}(\omega) \Psi_k^2(0) \int_0^1 (1 - x^2)^{1-d} dx |\omega|^{3-2d}, \tag{3.16}
\]

while for odd \(k\), we have

\[
\Im \delta f_k \approx -\sin(\pi d) \text{sgn}(\omega) \int_0^1 (1 - x^2) x^2 dx |\omega|^{3-2d}. \tag{3.17}
\]

To summarize, we use the following short notations:
The complete Kubo formula for the conductivity is

\[ \Im m \delta k \approx \begin{cases} -\alpha_k |d\omega|^{3/2d} \text{sgn}(\omega) & \text{if } k \text{ is even} \\ -\alpha_k |d\omega|^{3/2d} \text{sgn}(\omega) & \text{if } k \text{ is odd} \end{cases} \]

(3.18)

where \( \alpha_k, \beta_k \) are dimensionless constants depending on the scaling dimension \( d \). Close to the critical field at frequencies \( \omega \ll \epsilon_0 \) we can write the Green’s function as

\[ G^R(\omega, q, q'; p) = \frac{e^{ipn_0(q-d)/h}}{2J} \sum_{k=0}^{\infty} \Phi_k(q) \Psi_k(q') \chi_k(\omega), \]

(3.19)

where the first term in the summation is

\[ \chi_0(\omega) = \frac{1}{\left(h/h_c \frac{1+2d}{1-2d} - 1\right)} - \beta_0 \omega^2 - i\epsilon_0 |d\omega|^{3/2d} \text{sgn}(\omega), \]

(3.20)

and the other terms with nonzero \( k \) are similar. It can be seen from the poles of \( G^R \) that the excitation spectrum of the pairs corresponds to damped Landau levels. Another important feature of the Green’s function is that it possesses the particle–hole symmetry such that \( G^R(\omega) = [G^R(-\omega)]^* = G^A(-\omega) \). This will lead to a zero Hall response, which we will discuss in section 3.1.

### 3.1. Conductivities

The current operators for currents along and transverse to the stripe direction are given by the following expressions:

\[ J_x = i\Phi^+_x \partial_t \Phi_x, \quad J_y = i \left( \Phi^+_x \Phi_y + e^{i\hbar x} - \text{H.c.} \right). \]

(3.21)

The complete Kubo formula for the conductivity is

\[ \sigma_{\alpha\beta}(q, \omega) = \frac{1}{\omega} \int_0^\infty dt e^{i\omega t} \langle [J^\alpha(t, q), J^\beta(0, 0)] \rangle + \frac{eK}{\pi \omega} \delta_{\alpha\beta}, \]

(3.22)

where \( K = d\hbar_0 / d \) is the Luttinger parameter. The second term in \( \sigma_{\alpha\beta}(q, \omega) \) is the diamagnetic term, and it will be canceled by a corresponding contribution from the first term, since we are considering the case \( h > h_c \) where superconductivity is lost. Consequently, only the real part of the conductivity remains:

\[ \Re \sigma_{\alpha\beta}(q, \omega) = \frac{1 - e^{-\hbar_0 \omega}}{2\omega} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle J^\alpha(t, q) J^\beta(0, 0) \rangle. \]

(3.23)

This form of Kubo formula can be understood as a kind of fluctuation–dissipation theorem. We now focus on the DC conductivity at \( h > h_c \), which can be calculated by setting \( q = 0 \) in equation (3.23) and taking the limit \( \omega \to 0 \). Firstly, the conductivity along the stripe direction is obtained as

\[ \sigma_{xx}(\omega) = \sum_{p, k_1, k_2} \left( \int dq \Psi^{\dagger}_{k_1}(q) \Psi_{k_2}(q) \right)^2 F_{k_1, k_2}(\omega), \]

(3.24)

\[ F_{k_1, k_2}(\omega) = \frac{i}{\omega} \Pi_{k_1, k_2}(\omega), \]

\[ \Pi_{k_1, k_2}(i\Omega_n) = \frac{T}{4\Omega_n} \sum_{\omega_m} \chi_{k_1}(\omega) \chi_{k_2}(i\Omega_n + \omega_m). \]

As usual, the summation over \( p \) gives \( \Phi / \Phi_0 \), where \( \Phi = HS, \Phi_0 = 2\pi \hbar c / (2e) \). The matrix elements in equation (3.24) are nonzero only when the eigenfunctions \( \Psi_k \) have different parity. The frequency sum gives

\[ \Pi_{k_1, k_2}(i\Omega_n) = \frac{1}{4f^2\pi} \int d\xi d\xi' \frac{\coth(\xi/2T) - \coth(\xi'/2T)}{i\Omega_n + \xi - \xi'} \Im m \chi_{k_1}(\xi) \Im m \chi_{k_2}(\xi'). \]

(3.25)

Performing the analytic continuation \( i\Omega_n \to \omega + i0 \) and we obtain

\[ F_{k_1, k_2}(\omega) = \frac{1}{4f^2\pi} \int dy \frac{\coth(y/2T) - \coth[(y + \omega)/2T]}{\omega} \Im m \chi_{k_1}(y) \Im m \chi_{k_2}(y + \omega). \]

(3.26)

If we perform a shift of the integration variable \( y \to y - \omega \) followed by a reflection \( y \to -y \), using the particle–hole symmetry that \( \Im m \chi_{k_1}(\omega) = -\Im m \chi_{k_1}(-\omega) \), we then obtain \( F_{k_1, k_2}(\omega) = F_{k_2, k_1}(\omega) \). This ensures that the summation over \( k_1 \) and \( k_2 \) will not make the longitudinal conductivity \( \sigma_{xx}(\omega) \) vanish.
For magnetic fields above the transition point in the limit of zero frequency we obtain

\[
\sigma_{xx}(\omega \to 0) \sim \int \frac{dy |y|^{8-4d}}{2T \sinh^2(y/2T)} \sim \sigma_0(T/\epsilon_0)^{8-4d},
\]  

(3.27)

where it is assumed that the scaling dimension \( d \ll 1 \). We can see that the DC conductivity \( \sigma_{xx}(\omega \to 0) \) decreases with temperature, resulting in an insulator, though without a sharp gap.

Next we consider the conductivity transverse to the stripe direction. It is obtained as

\[
\sigma_{yx}(\omega) = \sum_{p,k_1,k_2} \left\{ \int dq \Psi_{k_1}(q) \left( \Psi_{k_2}(q-h) - \Psi_{k_2}(q+h) \right) \right\}^2 F_{k_1,k_2}(\omega).
\]  

(3.28)

This expression is even in \( h \) as it should be, and it has a similar structure as \( \sigma_{xx}(\omega) \), where the matrix elements are nonzero only when the eigenfunctions \( \Psi_k \) have different parity. Consequently, \( \sigma_{yx}(\omega \to 0) \) has the same behavior as \( \sigma_{xx}(\omega \to 0) \) at low temperatures, vanishing with the same power as shown in equation (3.27).

Finally, we consider the Hall conductivity, which is obtained as

\[
\sigma_{xy}(\omega) = -i \sum_{p,k_1,k_2} \left\{ \int dq \Psi_{k_1}(q) \left( \Psi_{k_2}(q-h) - \Psi_{k_2}(q+h) \right) \right\} \cdot \left( \int dq \Psi_{k_2}(q) \Psi_{k_1}(q) \right) F_{k_1,k_2}(\omega).
\]  

(3.29)

This expression is odd in \( h \) as it should be. Since \( F_{k_1,k_2}(\omega) = F_{k_2,k_1}(\omega) \) as shown above, the summand in this expression is odd under the exchange \( k_1 \leftrightarrow k_2 \). As a result, the summation over \( k_1 \) and \( k_2 \) makes \( \sigma_{xy}(\omega) \) vanish identically. This shows that the particle–hole symmetry ensures a zero Hall conductivity, which is consistent with the experiments \([4, 6, 15]\).

To summarize, the DC transport calculation in this section shows that both longitudinal conductivities, along and transverse to the stripe direction, vanish as powers of \( T \) at low temperatures, while the Hall conductivity vanishes identically. As a result, when superconductivity is suppressed, WT gives out an insulator with a soft gap on the level of RPA, excluding the possibility of a Bose metal in a clean system.

4. A possible escape route

So it appears that the simple WT in a clean system is not capable to describe the metallic state. A possible mechanism to produce a metallic state on top of WT is to imagine that the system supports localized pairs due to strong disorder. Here we just suggest this mechanism as a possibility without providing any microscopic justification for it.

Our speculation comes from the suggestion that above some critical magnetic field \( \sim h_c \), there is an intermediate region between the superconducting stripes, termed as charged insulator \([20]\). This region can accommodate charges in the form of pairs localized by quenched disorder (see figure 2). Due to the small size of these localized pairs, they are not subject to Landau quantization and can act as a reservoir of low energy state necessary for the broadening of the Landau levels. Localized pairs are coupled to the order parameter field \( \Phi_j(x) \) through the Andreev reflection mechanism:

\[
H_{\text{res}} = \frac{g}{2} \sum_j \int dx \left[ \Phi_j^+(x) \sigma_j^-(x) + \sigma_j^+(x) \Phi_j(x) \right] + \frac{1}{2} \sum_j \int dx \tilde{h}_j(x) \sigma_j^+(x),
\]  

(4.1)

where the Pauli matrix operator \( \sigma_j^+(x) \) creates a localized pair at point \( x \) within the charged insulator adjacent to the \( j \)th superconducting stripe, and the localized level \( \tilde{h}_j(x) \) suffers from quenched disorder. To the leading order in the small parameter \( g \), the effect of the localized pairs can be represented by a local potential:

\[
\tilde{V}_{\text{res}} = - \sum_{\omega} \int dx \Phi_j^+(\omega,x) V(\omega,x) \Phi_j(\omega,x),
\]  

(4.2)

\[
V(i\omega_n,x) = - \frac{g^2}{4} \langle \sigma^- \sigma^+ \rangle = g^2 \tanh(\tilde{h}_j(x)/2T) \frac{\omega_n}{\omega_n - \tilde{h}_j(x)},
\]

where the random variable \( \tilde{h}_j(x) \) is locally correlated in real space:

\[
\tilde{h}_j(x) \tilde{h}_j(x') = \Delta \delta(x-x') \delta_{jj'}.
\]  

(4.3)
As is shown in appendix B, the disorder-averaged conductivity receives higher order in $g^2$ contributions from the vertex corrections. As a result, to the leading order in $g^2$, it suffices to use the Drude approximation, where we can average the Green’s function individually.

Now we calculate the disorder-averaged Green’s function. The leading order contribution from the local potential to the RPA self energy is

$$\Sigma(i\omega_n) = \frac{V(i\omega_n, x)}{\omega}.$$ (4.4)

After analytic continuation, its imaginary part is

$$\Im m \Sigma^R(\omega) = -ic\pi g^2 K(\omega), \quad K(\omega) \propto \tanh(\omega/2T),$$ (4.5)

where $c$ is the concentration of localized pairs, provided the distribution of disorder is wide enough to encompass the frequency $\omega$. As a result, to the leading order in $g^2$, equation (3.4) is modified as

$$\left\{ \chi^{-1}_b(\omega, q) - f e^{i\delta_0q} \delta_{q, -h} - f e^{-i\delta_0q} \delta_{q, h} + ic\pi g^2 K(\omega) \right\} \Sigma(\omega, q', p) = \delta_{q, q'},$$ (4.6)

where we have ignored the effect of $\Re \Sigma^R$, which can be incorporated into renormalization of the Josephson coupling constant. The resulting disorder-averaged Green’s function for $h > h_c$ still has the form in equation (3.19), but the function $\chi_b(\omega)$ is modified by the extra imaginary term in equation (4.6). For example, the function $\chi_b(\omega)$ in equation (3.20) is changed to

$$\chi_0 = \frac{1}{(h/h_c)^{\beta d}} - \beta c(\omega^2 - i\alpha_0) |\omega|^{3-2d} \text{sgn} (\omega) - i\gamma K(\omega),$$ (4.7)

where $\gamma = c\pi g^2/(2|J|)$. Notably, the disorder-averaged Green’s function still respects particle–hole symmetry.

In the Drude approximation, the disorder-averaged conductivity receives main contributions from products of disorder-averaged Green’s functions:

$$\Im m \chi_b(\xi) \Im m \chi_b(\xi') \approx \Im m \chi_b(\xi) \cdot \Im m \chi_b(\xi').$$ (4.8)

Then the extra imaginary term in equation (4.6) introduces another contribution to the longitudinal conductivity $\sigma_{xx}$ along the stripe direction in equation (3.27):

$$\sigma_{xx}(\omega) \sim \gamma^2 \frac{\sinh(\omega/2T)}{\omega} \int dy \frac{K(y)K(y + \omega)}{\sinh(y/2T) \sinh((y + \omega)/2T)} + O(T^{8-4d})$$

$$\sim \sigma_0 + O(T^{8-4d}),$$ (4.9)

where $\omega_0$ corresponds to the lowest Landau level and $\sigma_0 \sim \gamma^2 \propto g^4$ is a constant. Consequently, the leading term for $\sigma_{xx}(\omega \to 0)$ is finite even at $T \to 0$, resulting in a Bose metal. As we can see, this anomalous metal does not have a Drude peak. The same analysis also applies to the calculation of the longitudinal conductivity $\sigma_{yy}(\omega \to 0)$ transverse to the stripe direction, while the Hall conductivity $\sigma_{yx}(\omega \to 0)$ still vanishes due to the symmetry argument stated below equation (3.29). It should be noted, however, that in contrast to the experiment which shows that at high fields the sheet conductance approaches the universal value $\sim 2e^2/2\pi h$, the mechanism we discuss does not lead to such a saturation.
As far as AC conductivity is concerned, we will be interested in the cyclotron resonance peak. Assuming that the transition frequency $\omega_k - \omega_0 \gg T$, from (3.26) we obtain for $\omega$ near the cyclotron resonance that

$$F_{\text{sd}}(\omega) \sim \frac{\sinh(\omega/2T)}{\omega} \int_{-\infty}^{\infty} dy \frac{dy}{\cosh(y/2T)\cosh((y+\omega)/2T)} \left[ (y^2 - \omega_0^2)^2 + \gamma^2 \tanh^2(y/2T) \right]$$

$$\times \left[ (y + \omega)^2 - \omega_0^2 \right] + \gamma^2 \left[ (y + \omega)^2 - \omega_0^2 \right] + \gamma^2.$$  

(4.10)

Since the integrand contains no divergence in the limits of integration, this integral is nonsingular.

Therefore, there is no cyclotron resonance at $T \to 0$. It is easy to see that the absence of the cyclotron resonance at $T \to 0$ is related to the absence of the Fermi sea for particles with Bose statistics. Hence for temperatures much smaller than the energy of the first peak, the spectral weight of the cyclotron resonance is small $\sim T$. This is consistent with the experiment [7]. It would be interesting to measure the terahertz conductivity for the stripe-ordered LBCO of high disorder level, where the sheet resistance $R_{\|} \gtrsim 6000 \Omega$, with a magnetic field well above 15 T. At low temperatures around several to 10 K, our theory suggests that the spectral weight of the cyclotron resonance is negligible. While for higher temperatures, a linear in $T$ dependence starts to emerge. This is in contrast to the more slow development of the Drude peak at higher temperatures as predicted in equation (4.9).

5. Discussion and conclusions

In this paper, we discussed a simple theory of superconducting stripes separated by insulating regions and coupled by Josephson tunneling. Our results suggest that in a sufficiently strong magnetic field the excitation spectrum of the preformed bosonic pairs consists of Landau levels. The corresponding excitations have a finite lifetime due to the interactions, disorder, and temperature effects. In a clean system, the broadening is modest, leading to a power-law temperature dependence of the longitudinal conductivity, which is insufficient to keep the sample metallic above the critical magnetic field when superconductivity is lost. Thus, in a magnetic field, the clean system undergoes a transition from a 2D superconductor to a weak insulator.

One way to compromise on the observations of the anomalous metal in recent experiments of the stripe-ordered LBCO [4] is to invoke quasiparticles as was suggested in Tsvelik’s earlier paper [21]. The role of quasiparticles has been discussed in the context of thin films and proximity Josephson junction arrays [9–11], and there are theories where PDW coexists with fermionic quasiparticles [21–23]. Some of them [21, 22] respect the particle–hole symmetry, while others [23] do not.

It is then legitimate to ask whether there is any possibility to have a situation where all low energy excitations remain bosonic, yet we still have a metallic state. We have proposed such a possibility in this paper. The essence of our proposal is that strong disorder leads to the creation of localized pairs or, perhaps, small superconducting grains. Due to their small size, their spectrum does not experience Landau quantization and hence provides a reservoir of low energy states for metallic transport. Our proposal provides a mechanism for the Bose metal other than the exotic one of quantum phase glass [24, 25], wherein the latter the Josephson coupling constant is random in sign. The anomalous metallic state resulting from our proposal possesses some of the features observed in the stripe-ordered LBCO [4] and the indium oxide films [7]. It has zero Hall response, no Drude peak, and no cyclotron resonance at low temperatures. It would be interesting to measure terahertz conductivity for LBCO to test the relevance of the theory described in this paper.

Acknowledgments

We are grateful to Andrey Chubukov, Theirry Giamarchi, and John Tranquada for very valuable discussions. We thank Peter Armitage for attracting our attention to reference [7]. This work was supported by the Office of Basic Energy Sciences, Material Sciences and Engineering Division, U.S. Department of Energy (DOE) under Contract No. DE-SC0012704.
Appendix A. Effect of disorder in Josephson coupling

In this appendix, we consider the effect of disorder in Josephson coupling on the simple WT. The quenched disorder in the Josephson coupling between superconducting stripes is assumed to be $\delta$-correlated in space:

$$I \to I + \delta I(x), \quad \delta_{I}(x)\delta_{I}(x') = \Delta \delta(x - x')\delta_{I},$$

(A.1)

where we also assume that the disorder is weak $\Delta I \ll \bar{f}$ such that it will not change the sign of $I$. We use the replica trick to calculate the disorder-averaged Green’s function:

$$\overline{G}(t - t', x, x'; j - f) = -i \lim_{n \to 0} \prod_{a=1}^{n} \left[ \right],$$

where the replicated action is

$$S_{R} = \sum_{\omega,q_{0},p_{0}} \Phi_{(a)}^{(a)}(\omega, q_{0}, p_{0} = 0) \left\{ \chi_{R}^{-1}(\omega, q_{0}, p_{0}) \Phi_{(a)}^{(a)}(\omega, q_{0}, p_{0}) - Ie^{i\theta_{0}} \Phi(\omega, q_{0}, p_{0}) \right\}$$

It is more convenient to express it in momentum space:

$$S_{R} = \sum_{\omega,q_{0},p_{0}} \Phi_{(a)}^{(a)}(\omega, q_{0}, p_{0} = 0) \left\{ \chi_{R}^{-1}(\omega, q_{0}, p_{0}) \Phi_{(a)}^{(a)}(\omega, q_{0}, p_{0}) - Ie^{i\theta_{0}} \Phi(\omega, q_{0}, p_{0}) \right\}$$

We calculate the one-loop correction to the diagonal Green’s function, assuming that the replica symmetry is unbroken. By taking the limit $n \to 0$, we obtain the equation for the disorder-averaged Green’s function:

$$G(\omega_{2}, q_{2}, p_{2}) = \delta_{q_{2}, q_{1}''},$$

(A.5)

where $G(\omega_{2}, q_{2}, p_{2})$ is the disorder-free Green’s function in equation (3.6). By performing the summation, we arrive at essentially the same equation as in equation (3.4), only the Josephson coupling constant $I$ is replaced by an effective one:

$$J_{eff} = J \left\{ 1 + \frac{\Delta I}{f} \sum_{k=0}^{\infty} \left[ \int \frac{d\omega}{2\pi} \chi_{k}(\omega) \left[ \int \frac{dq}{2\pi} \Psi_{k}(q) (\Psi_{k}(q - h) + \Psi_{k}(q + h)) \right] \right] \right\}.$$ (A.6)
The effective critical magnetic field is determined by the Josephson coupling constant via equation (3.12). For two stripes where the Josephson coupling between them is strong such that the effective critical magnetic field exceeds the applied magnetic field $h_i > h_i$, we have superconducting correlation across them and we can fuse them into a single stripe. After one step of such coarse-graining, we end up with exactly the same WT defined in equation (2.1), only with a smaller Josephson coupling constant between the effective stripes. If this coarse-graining procedure can be carried on until we have a single stripe, we are in the superconducting phase. If otherwise, we obtain WT with an effective critical magnetic field $h_i < h_i$ then the calculation of the conductivities in section 3.1 tells us that we are in the insulating phase. As a result, a Bose formula in equation (3.23) we obtain

$$
\sigma_{xx}(\omega) = \frac{1}{4J^2\pi} \int \frac{d^2 q d^2 q'}{\omega} \frac{\coth(y/2T) - \coth[(y + \omega)/2T]}{y + \omega, q', q; 0),}
\times \int d^2 q d^2 q' \Im m G^R(y, q, q'; 0) \Im m G^R(y + \omega, q', q; 0),
$$

where the products of the imaginary part of the Green’s functions can be rewritten using both the retarded and advanced Green’s functions:

$$
\Im m G^R \Im m G^R = \Im m G^R \Im m G^A = \frac{1}{4} \left( G^R - G^A \right) \left( G^R - G^A \right) = -\frac{1}{2} \Re m \left( G^R G^R - G^R G^A \right).
$$

The Drude approximation for the disorder-averaged conductivity replaces the average over products of Green’s functions by products of disorder-averaged individual Green’s functions:

$$
G^R G^R \approx G^R \cdot G^R, \quad G^R G^A \approx G^R \cdot G^A.
$$

We have seen in section 4 that at low temperatures this results in the conductivity $\sigma(\omega \to 0) \propto \gamma^2 \propto g^4$. The corrections beyond the Drude approximation come from multiple scatterings on the same localized pair across different propagators, known as the vertex corrections. Two famous examples for $G^R G^R$ are the diffuson and the cooperon shown in figure 3. Besides, in our situation there are also similar contributions for $G^R G^A$. These vertex corrections contain the basic element:

$$
V(\xi, x) V(\xi', x') = V(\xi, x) V(\xi', x) \delta(x - x'),
$$

which is represented by a dotted line with a cross in figure 3. When substituting it into the integrals of equation (B.1), we just take $\xi = y, \xi' = y + \omega$. The local potential $V(\xi, x)$ is defined in equation (4.2), and the analytic continuation from $i\xi_n$ to $\xi \pm i0$ obeys the following rule: if we are dealing with $G^R G^R$, then both frequencies are analytically continued as $i\xi_n \to \xi + i0, i\xi'_n \to \xi' + i0$; if we are dealing with $G^R G^A$, then the two frequencies are analytically continued differently $i\xi_n \to \xi + i0, i\xi'_n \to \xi' - i0$. 

\[ J_{\text{eff}} = J \left\{ 1 - \frac{\Delta J}{4\pi^2 J^2} \sum_{k=0}^{\infty} \text{arctan} \left( \frac{\sqrt{\beta_k}}{2} \right) \left[ \int \text{d}x \Psi_k^2(x) \right] \right\}, \]  
(A.7)
Figure 3. Vertex corrections to $G^R G^A$ beyond the Drude approximation in real space representation: (a) the diffuson (b) the cooperon. In the figure, $r = (x, j)$ and the cross represents the local potential $V$. The solid and dashed lines represent the retarded Green’s function $G^R$ and advanced Green’s function $G^A$, respectively.

Figure 4. Disorder-averaged individual Green’s function. In section 4, we only considered the leading order correction shown in (a). The next order correction includes two terms, shown in (b) and (c).

Given the random distribution characterized by equation (4.3), we obtain the following expressions for $f(\xi, \xi') \equiv V(\xi, x) V(\xi', x)$:

$$f^{RR}(\xi, \xi') = \Re e f - i c R e g^4 \frac{\tanh^2(\xi/2T) - \tanh^2(\xi'/2T)}{\xi' - \xi},$$  
(B.5)

$$f^{RA}(\xi, \xi') = \Re e f - i c R e g^4 \frac{\tanh^2(\xi/2T) + \tanh^2(\xi'/2T)}{\xi' - \xi},$$

and $\Re e f$ is the principal integral

$$\Re e f = g^4 \int_{\mathbb{R}L} d\tilde{h} P(\tilde{h}) \frac{\tanh^2(\tilde{h}/2T)}{(\xi - \tilde{h})(\xi' - \tilde{h})},$$  
(B.6)

where $P(\tilde{h})$ is the distribution of the random variable $\tilde{h}(x)$. The corresponding vertex corrections have the important feature that the momentum integration can still be factored out, similar to that in equation (3.24), such that we can only focus on the frequency integration, which we denote as $\Delta F_{k_1, k_2; k'_1, k'_2}(\omega)$. The leading order term of the diffuson-type correction in it is

$$\Delta F^{RR}_{k_1, k_2; k'_1, k'_2}(\omega) = \frac{1}{8 g^4 T^2} \int dy \frac{\coth(y/2T) - \coth((y + \omega)/2T)}{\omega} D(y, y + \omega),$$

$$D(\xi, \xi') = \Re e \left[-P^{RR}_{k_1, k_2}(\xi, \xi') f^{RR}_{k'_1, k'_2}(\xi, \xi') + P^{RA}_{k_1, k_2}(\xi, \xi') f^{RA}_{k'_1, k'_2}(\xi, \xi') P^{RA}_{k'_1, k'_2}(\xi, \xi') \right],$$  
(B.7)
where \( R^R_{k_1,k_2}(\xi,\xi') \), \( R^A_{k_1,k_2}(\xi,\xi') \) are products of disorder-averaged Green’s functions:

\[
P^R_{k_1,k_2}(\xi,\xi') \equiv \bar{\chi}^R_{k_1}(\xi) \cdot \bar{\chi}^R_{k_2}(\xi'),
\]

\[
P^A_{k_1,k_2}(\xi,\xi') \equiv \bar{\chi}^A_{k_1}(\xi) \cdot \bar{\chi}^A_{k_2}(\xi').
\]  

(B.8)

Using equations (4.6) and (4.7) for the disorder-averaged Green’s function, we can evaluate equation (B.7), the leading-order diffusion-type correction to the disorder-averaged conductivity as:

\[
\sigma_D(\omega \to 0) \sim C_1 g^4 \int dy \frac{y^{8-4d}}{2T \sinh^2(y/2T)} \frac{1}{T} + C_2 g^4 \int dy \frac{|y|^{3-2d} \text{sgn} y \tanh(y/2T)}{2T \sinh^2(y/2T) 2T \cosh(y/2T)}
\]

\[
+ C_3 g^4 \int dy \frac{(3-2d)|y|^{2-2d} \tanh^2(y/2T)}{2T \sinh^2(y/2T)} + O(g^6),
\]  

(B.9)

where \( C_1, C_2, C_3 \) are some constants. At low temperatures, we then obtain

\[
\sigma_D(\omega \to 0) \sim g^4(T/\epsilon_0)^{2-2d} + O(g^6).
\]  

(B.10)

In the zero temperature limit \( T \to 0 \), this result is of higher order in \( g^2 \), compared with the result in the Drude approximation. The same line of calculations can be directly applied to the Cooperon-type correction. The leading order term then contains two factors of \( f(\xi,\xi') \), as can be seen from figure 3. Thus, the leading order term of the Cooperon-type correction is already of order \( g^2 \).

There are two divergences that one must be careful with, though. One is due to the \( 1/\omega \) dependence in the imaginary part of \( f^A(y,y+\omega) \), and the other is due to the \( 1/T \) dependence of all the \( f(\xi,\xi') \). In the limit \( \omega \to 0, T \to 0 \), both of them will lead to divergence of the conductivity in higher order corrections beyond equation (B.10), which must be canceled order by order. The former divergence only happens in diagrams containing no less than two factors of \( f^A(y,y+\omega) \), and they will be canceled off between the diffusion and the Cooperon. The latter divergence is canceled off by the same order correction in the Drude approximation, of which one example is shown in figure 4. There, the next order correction to the disorder-averaged Green’s function contains a term \( (c) \), which contains the same \( 1/T \) divergence as that in the next order vertex correction.

In summary, all the vertex corrections are regular and smaller than the leading Drude approximation by factors of \( g^2 \), so for the disorder-averaged conductivity, the vertex corrections are negligible and it suffices to use the Drude approximation for our model.

**ORCID iDs**

Tianhao Ren https://orcid.org/0000-0003-1265-4442

Alexei M Tselvik https://orcid.org/0000-0002-7478-670X

**References**

[1] Li Q, Hücke M, Gu G D, Tselvik A M and Tranquada J M 2007 Phys. Rev. Lett. 99 067001

[2] Tranquada J M et al 2008 Phys. Rev. B 78 174529

[3] Agterberg D F et al 2020 Annu. Rev. Condens. Matter Phys. 11 231

[4] Li Y, Terzić J, Baity P G, Popović D, Gu G D, Li Q, Tselvik A M and Tranquada J M 2019 Sci. Adv. 5 eaav7686

[5] Shi Z, Baity P G, Terzić J, Sasagawa T and Popović D 2019 Signatures of a pair density wave at high magnetic fields in cuprates with charge and spin orders (arXiv:1907.11708)

[6] Shi Z, Baity P G, Sasagawa T and Popović D 2020 Sci. Adv. 6 eaav8946

[7] Wang Y, Tamir I, Shahr D and Armitage N P 2018 Phys. Rev. Lett. 120 167002

[8] Han Z, Allain A, Arijmandi-Tash H, Tikhonov K, Feigel’man M, Sacépé B and Bouchiat V 2014 Nat. Phys. 10 386

[9] Feigel’man M V and Larkin A I 1998 Chem. Phys. 235 107

[10] Feigel’man M V, Larkin A I and Skvortsov M A 2001 Phys. Rev. Lett. 86 1869

[11] Feigel’man M V, Skvortsov M A and Tikhonov K S 2008 JETP Lett. 88 747

[12] Himeda A, Kato T and Ogata M 2002 Phys. Rev. Lett. 88 117001

[13] Berg E, Fraïdine E, Kim E-A, Kivelson S A, Oganesyan V, Tranquada J M and Zhang S C 2007 Phys. Rev. Lett. 99 127003

[14] Kapitulnik A, Kivelson S A and Spivak B 2019 Rev. Mod. Phys. 91 011002

[15] Breznay N P and Kapitulnik A 2017 Sci. Adv. 3 e1700612

[16] Controzzi D and Tselvik A M 2005 Phys. Rev. B 72 035110

[17] Aleiner I L, Kharzeev D E and Tselvik A M 2007 Phys. Rev. B 76 195145

[18] Samokhin K V 2010 Phys. Rev. B 81 224507

[19] Homes C C, Hücke M, Li Q, Xu Z J, Wen J S, Gu G D and Tranquada J M 2012 Phys. Rev. B 85 134510

[20] Ren T and Aleiner I 2019 unpublished

[21] Tselvik A M 2019 Proc. Natl Acad. Sci. USA 116 12729
[22] Tselik A M 2016 Phys. Rev. B 94 165114
[23] Lee P A 2014 Phys. Rev. X 4 031017
[24] Dalidovich D and Phillips P 2002 Phys. Rev. Lett. 89 027001
[25] Wu J and Phillips P 2006 Phys. Rev. B 73 214507