Two-pion transitions from the $P$-wave to $S$-wave charmonium states

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we study the two-pion decays in the $P$-wave to $S$-wave charmonium transitions with the heavy meson effective theory, and make qualitative predictions for the ratios among the two-pion decay widths.

PACS numbers: 12.39.Hg; 13.25.Gv

Key Words: Charmonium states, Two-pion decays

1 Introduction

Recently, the BESIII collaboration searched for the two-pion transitions $\chi_{cj} \to \eta_c \pi^+ \pi^-$ ($j = 0, 1, 2$) using a sample of $1.06 \times 10^8 \psi(3686)$ events collected by the BESIII detector, and observed no signals for the three $\chi_{cj}$ states in the $\eta_c$ decay modes, and set the upper limits for the branching ratios $\text{Br}(\chi_{c0} \to \eta_c \pi^+ \pi^-) < 0.07\%$, $\text{Br}(\chi_{c1} \to \eta_c \pi^+ \pi^-) < 0.32\%$, $\text{Br}(\chi_{c2} \to \eta_c \pi^+ \pi^-) < 0.54\%$ at the 90\% confidence level \cite{1}. Taking into account the widths of the $P$-wave charmonium states $\Gamma(\chi_{c2}) = (1.95 \pm 0.13)\text{MeV}$, $\Gamma(\chi_{c1}) = (0.88 \pm 0.05)\text{MeV}$, $\Gamma(\chi_{c0}) = (10.5 \pm 0.8)\text{MeV}$ from the Particle Data Group \cite{2}, we can obtain the two-pion decay widths $\Gamma(\chi_{c2} \to \eta_c \pi^+ \pi^-) < 10.53\text{KeV}$, $\Gamma(\chi_{c1} \to \eta_c \pi^+ \pi^-) < 2.186\text{KeV}$ and $\Gamma(\chi_{c0} \to \eta_c \pi^+ \pi^-) < 7.35\text{KeV}$. While in previous studies, the BABAR collaboration searched for the processes $\gamma\gamma \to X \to \eta_c \pi^+ \pi^-$, where the $X$ stands for the resonances $\chi_{c2}$, $\eta_c'$, $\chi_{c2}'$, $X(3872)$, $X(3915)$, and set the upper limit $\text{Br}(\chi_{c2} \to \eta_c \pi^+ \pi^-) < 2.2\%$ at the 90\% confidence level \cite{3}. For the most promising process $\chi_{c1} \to \eta_c \pi^+ \pi^-$ dominated by the $E_1 - M_1$ transition, the upper limit $\text{Br}(\chi_{c1} \to \eta_c \pi^+ \pi^-) < 0.32\%$ is lower than the existing theoretical prediction $\text{Br}(\chi_{c1} \to \eta_c \pi^+ \pi^-) = (2.72 \pm 0.39)\%$ based on the QCD multipole expansion by almost an order of magnitude \cite{4, 5}. In the QCD multipole expansion, the $\chi_{c1} \to \eta_c \pi^+ \pi^-$ branching ratio (or width) dominated by the $E_1 - M_1$ transition is significantly smaller than that of the $E_1 - E_1$ transitions, for example, the transition $\psi' \to J/\psi \pi^+ \pi^-$ (or $\eta_c' \to \eta_c \pi^+ \pi^-$) \cite{2} \cite{5}. In Ref.\cite{6}, Voloshin studies the charmonium transitions $\chi_{cj} \to \eta_c$ ($j = 0, 1, 2$) with emission of one or two pions, and shows that only the decays $\chi_{c1} \to \eta_c \pi^+ \pi^-$ and $\chi_{c0} \to \eta_c \pi^0$ take place in the leading order in the QCD multipole expansion, and obtains the predictions $\frac{\Gamma(\chi_{c0} \to \eta_c \pi^0)}{\Gamma(\chi_{c1} \to \eta_c \pi^+ \pi^-)} \approx 13.7$ and $\frac{\Gamma(\psi' \to J/\psi \pi^+ \pi^-)}{\Gamma(\chi_{c1} \to \eta_c \pi^+ \pi^-)} \approx 0.1$. In this article, we study the two-pion decays in the $P$-wave to $S$-wave charmonium transitions with the heavy meson effective theory in the leading order approximation. The heavy meson effective theory has been successfully applied to study the pseudoscalar meson decays of the charmed mesons \cite{7}, and the radiative and vector-meson decays of the heavy quarkonium states \cite{8}.

The article is arranged as follows: we study the two-pion decays in the $P$-wave to $S$-wave charmonium transitions with the heavy meson effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

\footnote{1E-mail: wangzgyiti@yahoo.com.cn.}
2 The two-pion transitions with the heavy meson effective theory

The charmonium states have the same radial quantum number \( n \) and orbital angular momentum \( L = 0, 1 \) can be expressed by the superfields \( J, J^\mu \) [9, 10],

\[
J = \frac{1 + \gamma}{2} \{ \psi \gamma - \eta \gamma 5 \} \frac{1 - \gamma}{2},
\]

\[
J^\mu = \frac{1 + \gamma}{2} \left\{ \chi_{\mu 0} \gamma^\mu + \frac{1}{\sqrt{2}} \epsilon_{\alpha \beta \lambda} \bar{\psi} \gamma^\alpha \chi_{\lambda} + \frac{1}{\sqrt{3}} (\gamma^\mu - \gamma^\nu) \chi_{\nu 0} + h_c^\mu \gamma_5 \right\} \frac{1 - \gamma}{2},
\]

where the \( \psi^\mu \) denotes the four-velocity associated to the superfields, the charmonium states \( \chi_{\mu 0}, \chi_{\nu 0}, h_c^\mu, \psi^\mu, \eta_c \) have the total angular momentum \( j = 2, 1, 0, 1, 0, \) respectively, and belong to different multiplets. We multiply those charmonium fields with the factors \( \sqrt{M_{\chi_{0 2}}, \sqrt{M_{\chi_{0 1}}, \sqrt{M_{h_c}}, \sqrt{M_{\psi}}, \sqrt{M_{\eta_c}}, \sqrt{M_{h_b}}, \sqrt{M_{\bar{b} b}}}, \) respectively, and they have dimension of mass \( \frac{2}{3} \). The superfields \( J^{(\mu)} \) have the following properties under the parity \( (P) \), charge conjugation \( (C) \), heavy quark spin transformations \( (S) \),

\[
J^{(\mu)} \rightarrow P \gamma^0 J^{(\mu)} \gamma^0,
J^{(\mu)} \rightarrow C (-1)^{L+1} C([J^{(\mu)}]^T) C,
J^{(\mu)} \rightarrow S J^{(\mu)} S^T,
v^\mu \rightarrow PV,\]

where \( S, S' \in SU(2) \) heavy quark spin symmetry groups, and \( [S, \gamma] = [S', \gamma] = 0 \).

The \( \pi^+ \pi^- \) transitions between the \( P \)-wave and \( S \)-wave charmonium states can be described by the following phenomenological Lagrangians \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \),

\[
\mathcal{L}_1 = \frac{g_c}{\Lambda^2} \text{Tr} \left[ J_\mu \sigma^{\rho \sigma} J^\rho \right] \epsilon^{\beta \gamma} \text{Tr} \left[ A^\rho \tilde{A}_\beta \right] - \frac{ig_d}{\Lambda} \text{Tr} \left[ J_\mu \sigma^{\rho \tau} \tilde{J} \sigma_{\tau \rho} \right] \epsilon^{\beta \gamma} \text{Tr} \left[ A^\rho \tilde{A}_\beta \right] + h.c.,
\]

\[
\mathcal{L}_2 = -\frac{ig_f}{\Lambda^2} \text{Tr} \left[ J_\mu \sigma^{\lambda \beta} \partial_\lambda J_\beta \right] \epsilon^{\beta \gamma} \text{Tr} \left[ A^\rho \tilde{A}_\beta \right] - \frac{g_h}{\Lambda^2} \text{Tr} \left[ J_\mu \sigma^{\rho \tau} \partial_\tau \tilde{J} \sigma_{\tau \rho} \right] \epsilon^{\beta \gamma} \text{Tr} \left[ A^\rho \tilde{A}_\beta \right] + h.c.,
\]

where

\[
A_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) = \frac{im}{f_\pi \sigma^\mu} + \cdots,
\]

and \( \tilde{J} = \gamma^0 J^\dagger \gamma^0 \). We introduce an energy scale \( \Lambda = 1 \) GeV to warrant that the strong coupling constants \( g_c, g_d, g_f \) and \( g_h \) are dimensionless quantities. The light pseudoscalar mesons are described by the fields \( \xi = \exp \left( \frac{iM}{f_\pi} \right) \), where

\[
M = \begin{pmatrix} \sqrt{\frac{3}{2}} \pi^0 + \sqrt{\frac{1}{2}} \eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{3}{2}} \pi^0 + \sqrt{\frac{1}{2}} \eta & K^0 \\ K^- & K^0 & -\sqrt{\frac{3}{2}} \eta \end{pmatrix},
\]

and the decay constant \( f_\pi = 130 \) MeV. The Lagrangians \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) violate the heavy quark spin symmetry, and describe the \( E_1 - M_1 \) and \( E_1 - M_2 \) transitions between the \( P \)-wave and \( S \)-wave charmonium states, respectively. For review of the QCD multipole expansion, one can consult Ref.[12]. The Lagrangian \( \mathcal{L}_1 \) is taken from Ref.[9] and the Lagrangian \( \mathcal{L}_2 \) is constructed in this
article. We carry out the trace in the heavy meson effective Lagrangians $L_1$ and $L_2$, and observe that the decays $\chi_{c2} \rightarrow J/\psi \pi^+\pi^-$, $\chi_{c1} \rightarrow J/\psi \pi^+\pi^-$, $\chi_{c0} \rightarrow J/\psi \pi^+\pi^-$, $h_c \rightarrow J/\psi \pi^+\pi^-$, $\chi_{c1} \rightarrow \eta_c \pi^+\pi^-$, $h_c \rightarrow \eta_c \pi^+\pi^-$ receive contributions from both the $E_1 - M_1$ and $E_1 - M_2$ transitions, while the decay $\chi_{c2} \rightarrow \eta_c \pi^+\pi^-$ only receives contribution from the $E_1 - M_2$ transition. The charmonium parts $\text{Tr} \left[ J^a J^a \right]$, $\text{Tr} \left[ J^a \sigma^{\mu\nu} \sigma_{\mu\nu} J^a \right]$, $\text{Tr} \left[ J^a \sigma^{\mu\nu} \partial_{\mu} J^a \right]$ and $\text{Tr} \left[ J^a \sigma^{\mu\nu} \partial_{\mu} J_{\sigma\rho} \right]$ in the effective Lagrangians lead to the spin-conserved (electro-like) transitions $\chi_{c2} \rightarrow J/\psi$, $h_c \rightarrow \eta_c$ and spin-violated (magnetic-like) transitions $\chi_{c1} \rightarrow \eta_c$, $h_c \rightarrow J/\psi$. The electro-like ($E_1$) and magnetic-like ($M_1$, $M_2$) processes manifest themselves through $\chi_{c2} \rightarrow J/\psi$, $h_c \rightarrow \eta_c$ and $\chi_{c1} \rightarrow \eta_c$, $h_c \rightarrow J/\psi$ respectively in the $E_1 - M_1$ (or $E_1 - M_2$) transitions, just like the children always manifest the feature of one parent in biology; the spin-conserved transitions violate isospin symmetry while the spin-violated transitions conserve isospin symmetry. The decays $\chi_{cj} \rightarrow J/\psi \pi^+\pi^-$ and $h_c \rightarrow \eta_c \pi^+\pi^-$ violate isospin symmetry or $G$-parity, and can also take place through the $E_1$ electromagnetic interactions with the intermediate $\rho$ meson, this mechanism corresponds to the effective Lagrangian $g_c \text{Tr} \left[ J^a J^a \right] \tilde{v}^b \text{Tr} \left[ A_\alpha A_b \right]$, the charmonium part $\text{Tr} \left[ J^a J^a \right]$ only leads to the spin-conserved transitions $\chi_{cj} \rightarrow J/\psi$ and $h_c \rightarrow \eta_c$. We can draw the conclusion tentatively that the electro and electro-like interactions lead to the isospin-violated decays, and the $g_c \text{Tr} \left[ J^a J^a \right] \tilde{v}^b \text{Tr} \left[ A_\alpha A_b \right]$ can be absorbed in the effective Lagrangian $L_1$. The isospin-violated processes can take place, for example, the BESIII collaboration had precisely measured the branching ratio $\text{Br}(\psi' \rightarrow J/\psi \pi^0) = (1.26 \pm 0.02 \pm 0.03) \times 10^{-3}$.

We write down the two-pion transition amplitudes $T$, and obtain the decay widths $\Gamma$,

$$\Gamma = \frac{1}{(2j+1)2M_i} \int \sum |T|^2 \frac{d^2l}{2\pi} d\Phi(P \rightarrow q, l) d\Phi(l \rightarrow r, t), \quad (7)$$

where the $d\Phi(P \rightarrow q, l)$ and $d\Phi(l \rightarrow r, t)$ are the two-body phase factors,

$$d\Phi(P \rightarrow q, l) = (2\pi)^4 \delta^4(P - q - l) \frac{d\vec{q}}{(2\pi)^3 2q_0} \frac{d\vec{l}}{(2\pi)^3 2l_0}, \quad \text{and}$$

$$d\Phi(l \rightarrow r, t) = (2\pi)^4 \delta^4(l - r - t) \frac{d\vec{r}}{(2\pi)^3 2r_0} \frac{d\vec{t}}{(2\pi)^3 2t_0}, \quad (8)$$

the $P_\mu$ and $q_\mu$ are the momenta of the initial and final charmonium states respectively, the $M_i$ is the mass of the initial charmonium state, the $j$ is the total spin of the initial charmonium state, the $r_\mu$ and $t_\mu$ are the momenta of the $\pi^+$ and $\pi^-$ respectively, and the $l^2$ is the invariant mass of the $\pi^+\pi^-$ system. We carry out the integrals $d\Phi(l \rightarrow r, t)$, $d\Phi(P \rightarrow q, l)$ and $dl^2$ sequentially. In calculations, we have used the following formula,

$$\int t_{\alpha\mu\beta\nu} r_{\beta\lambda} r_{\nu\delta} d\Phi(l \rightarrow r, t) = \frac{i}{4\pi \sqrt{l^2}} \left\{ A(g_{\alpha\mu} g_{\beta\nu} + g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\beta\mu}) + B g_{\alpha\mu} l_{\beta\nu} + g_{\alpha\beta} l_{\mu\nu} + g_{\alpha\nu} l_{\beta\mu} + g_{\beta\mu} l_{\alpha\nu} + D l_{\alpha\beta} l_{\mu\nu} \right\} \frac{l^2}{l^4}, \quad (9)$$

where

$$A = \frac{l^4 - 8l^2 m_\pi^2 + 16m_\pi^4}{240},$$

$$B = -\frac{3l^4 - 14l^2 m_\pi^2 + 8m_\pi^4}{120},$$

$$C = \frac{l^4 - 3l^2 m_\pi^2 - 4m_\pi^4}{60},$$

$$D = \frac{l^4 + 2l^2 m_\pi^2 + 6m_\pi^4}{30}, \quad (10)$$

and we carry out the sum of all the polarization vectors of the charmonium states using the FeynCalc.
3 Numerical Results

We take the input parameters from the Particle Data Group [2], $M_{\chi^0} = 3556.20$ MeV, $M_{\chi^+} = 3510.66$ MeV, $M_{\chi^0} = 3414.75$ MeV, $M_{h^+} = 3525.41$ MeV, $M_{J/\psi} = 3096.916$ MeV, $M_{h^+} = 2981.0$ MeV, and $M_{\pi} = 139.57$ MeV, and obtain the numerical values of the two-pion widths,

\[
\Gamma(\chi c_2 \rightarrow J/\psi \pi^+ \pi^-) = \left\{ 43.8353(g_c + g_d)^2 + 3.7406(g_c + g_d)g_f + 9.6279(g_c + g_d)g_h \\
+ 0.3957g_f^2 + 0.5224g_f g_h + 2.9811g_h^2 \right\} \text{KeV},
\]

\[
\Gamma(\chi c_1 \rightarrow J/\psi \pi^+ \pi^-) = \left\{ 14.2933(g_c + g_d)^2 + 0.9305(g_c + g_d)(g_f - g_h) + 0.0546(g_f - g_h)^2 \right\} \text{KeV},
\]

\[
\Gamma(\chi c_0 \rightarrow J/\psi \pi^+ \pi^-) = \left\{ 0.7121g_c^2 - 2.8483g_c g_d - 0.0136g_c g_f - 0.0347g_c g_h + 2.8483g_d^2 - 0.0273g_d g_f \\
+ 0.0693g_d g_h + 0.0001g_f^2 - 0.0002g_f g_h + 0.0009g_h^2 \right\} \text{KeV},
\]

\[
\Gamma(h_c \rightarrow J/\psi \pi^+ \pi^-) = \left\{ 42.6521(g_c - g_d)^2 + 3.0694(g_c - g_d)(g_f - g_h) + 0.1978(g_f - g_h)^2 \right\} \text{KeV},
\]

\[
\Gamma(h c_2 \rightarrow \eta_c \pi^+ \pi^-) = \left\{ 3.3352(g_f + g_h)^2 \right\} \text{KeV},
\]

\[
\Gamma(h c_1 \rightarrow \eta_c \pi^+ \pi^-) = \left\{ 340.7208(g_c + g_d)^2 + 40.3081(g_c + g_d)(g_f + g_h) + 2.0529(g_f + g_h)^2 \right\} \text{KeV},
\]

\[
\Gamma(h c_0 \rightarrow \eta_c \pi^+ \pi^-) = \left\{ 868.4603g_d^2 + 356.3769g_d g_h + 52.2813g_h^2 \right\} \text{KeV}.
\]  

(11)

In general, we expect to fit the coupling constants $g_c$, $g_d$, $g_f$ and $g_h$ to the precise experimental data, however, in the present time the experimental data are rare.

In the QCD multipole expansion, the Hamiltonians for the chromo-electric dipole $E_1$, the chromo-magnetic dipole $M_1$ and the chromo-magnetic quadrupole $M_2$ transitions are

\[
H_{E_1} = -\frac{1}{2}\xi^a \vec{r} \cdot \vec{E}^a,
\]

\[
H_{M_1} = -\frac{1}{2m_Q}\xi^a \vec{\Delta} \cdot \vec{B}^a,
\]

\[
H_{M_2} = -\frac{1}{4m_Q}\xi^a S_{ij}r_i \{D_iB_j(0)\}^a,
\]  

(12)

where the $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark, the $\vec{\Delta} = \vec{\sigma}_1 - \vec{\sigma}_2$ is the spin operator with $\vec{\sigma}_1$ and $\vec{\sigma}_2$ acting on the quark and antiquark respectively, the $\vec{r}$ is the relative vector of the quark and antiquark, the $\vec{S} = \frac{1}{2}t^a \vec{r}$ is the operator of the total spin of the quark-antiquark pair, and the $\vec{D}$ is the QCD covariant derivative, the $\vec{E}^a$ and $\vec{B}^a$ are the chromo-electric and chromo-magnetic components of the gluon field strength tensor respectively [12]. The $E_1 - M_1$ and $E_1 - M_2$ transitions both take place at the next-to-leading order $O(\frac{1}{m_Q})$, they are suppressed compared to the $E_1 - E_1$ transitions, which take place in the leading order $O(1)$. There is an additional covariant derivative $\vec{D}$ in the $H_{M_2}$, the $E_1 - M_2$ transitions are suppressed in the phase-space compared to the $E_1 - M_1$ transitions, if the momentum transfer in the $P$-wave to $S$-wave charmonium transitions is small. From Eq.(11), we can see that the coefficients (which can be denoted as $X$) of the coupling constants $g_c$, $g_d$, $g_f$ and $g_h$ have the hierarchy $X(g_d^2), |X(g_c g_d)|, X(g_d^2) \gg |X(g_c g_f)|, |X(g_d g_f)|, |X(g_c g_h)|, |X(g_d g_h)| \gg X(g_f^2), |X(g_f g_h)|, X(g_h^2)$, the $E_1 - M_2$ transitions are suppressed indeed in the phase-space.

If we take the approximation $g_f \approx g_h \approx 0$, i.e. neglect the $E_1 - M_2$ transitions, the two-pion
decay widths can be simplified as

\[ \Gamma(\chi_{c2} \rightarrow J/\psi\pi^+\pi^-) = 43.8353(g_c + g_d)^2 \text{KeV}, \]
\[ \Gamma(\chi_{c1} \rightarrow J/\psi\pi^+\pi^-) = 14.2933(g_c + g_d)^2 \text{KeV}, \]
\[ \Gamma(\chi_{c0} \rightarrow J/\psi\pi^+\pi^-) = 0.7121g_c^2 - 2.8483g_d(g_c - g_d) \text{KeV}, \]
\[ \Gamma(h_c \rightarrow J/\psi\pi^+\pi^-) = 42.6521(g_c - g_d)^2 \text{KeV}, \]
\[ \Gamma(\chi_{c2} \rightarrow \eta_c\pi^+\pi^-) = 0 \text{KeV}, \]
\[ \Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-) = 340.7208(g_c + g_d)^2 \text{KeV}, \]
\[ \Gamma(h_c \rightarrow \eta_c\pi^+\pi^-) = 868.4603g_c^2 \text{KeV}, \]

then we obtain the ratios

\[ \frac{\Gamma(\chi_{c2} \rightarrow J/\psi\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} = 0.129, \ 0.129, \ 0.129, \]
\[ \frac{\Gamma(\chi_{c1} \rightarrow J/\psi\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} = 0.042, \ 0.042, \ 0.042, \]
\[ \frac{\Gamma(\chi_{c0} \rightarrow J/\psi\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} = 0.001, \ 0.002, \ 0.008, \]
\[ \frac{\Gamma(h_c \rightarrow J/\psi\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} = 0.0, \ 0.125, \ 0.125, \]
\[ \frac{\Gamma(h_c \rightarrow \eta_c\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} = 0.637, \ 0.0, \ 2.549, \]

with the additional approximation \( g_c = g_d, \ g_d = 0, \ g_c = 0 \), respectively. The present prediction \( \Gamma(h_c \rightarrow J/\psi\pi^+\pi^-) = 0.125 \) with the coupling \( g_c = 0 \) (or \( g_d = 0 \)) is consistent with the value 0.1 from the QCD multipole expansion [6]. By measuring the ratio \( \frac{\Gamma(h_c \rightarrow \eta_c\pi^+\pi^-)}{\Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-)} \), we can obtain powerful constraint on the couplings \( g_c \) and \( g_d \). If the value \( g_c = 0 \) is excluded, the transition \( \chi_{c1} \rightarrow \eta_c\pi^+\pi^- \) is the most promising process, and the transitions \( \chi_{c0} \rightarrow J/\psi\pi^+\pi^- \), \( h_c \rightarrow J/\psi\pi^+\pi^- \), \( \chi_{c2} \rightarrow \eta_c\pi^+\pi^- \) are greatly suppressed. The upper limits of the two-pion decay widths \( \Gamma(\chi_{c2} \rightarrow \eta_c\pi^+\pi^-) < 10.53 \text{KeV} \) and \( \Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-) < 2.186 \text{KeV} \) lead to the prediction \( |g_f + g_h| < 1.7769 \) and \( |g_c + g_d| < 0.0801 \) from the formulae \( \Gamma(h_c \rightarrow \eta_c\pi^+\pi^-) = 3.3352(g_f + g_h)^2 \text{KeV} \) and \( \Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-) = 340.7208(g_c + g_d)^2 \text{KeV} \), respectively, which are inconsistent with our naive expectation based on the QCD multipole expansion. Precise measurements are needed. The transitions \( h_c \rightarrow J/\psi\pi^+\pi^- \) and \( \chi_{c1} \rightarrow \eta_c\pi^+\pi^- \) are of particular interesting, the decay widths

\[ \Gamma(h_c \rightarrow J/\psi\pi^+\pi^-) = \{ 42.6521(g_c - g_d)^2 + 3.0694(g_c - g_d)(g_f - g_h) + 0.1978(g_f - g_h)^2 \} \text{KeV}, \]
\[ \Gamma(\chi_{c1} \rightarrow \eta_c\pi^+\pi^-) = \{ 340.7208(g_c + g_d)^2 + 40.3081(g_c + g_d)(g_f + g_h) + 2.0529(g_f + g_h)^2 \} \text{KeV}, \]

depend heavily on the relative sign of the coupling constants. If \( |g_c| \sim |g_d| \) and \( |g_f| \sim |g_h| \), the special combinations \( g_c \pm g_d \) and \( g_f \pm g_h \) can lead to large augment or depression in those two-pion transitions. In the case that the coupling constants \( g_c \) and \( g_d \) (\( g_f \) and \( g_h \)) have the same sign, the prediction is consistent with the value from the QCD multipole expansion [6]. We can fit the coupling constants \( g_c, g_d, g_f \) and \( g_h \) to the experimental data at the BESIII, KEK-B, RHIC, PANDA and LHCb in the future, and obtain quantitative predictions.

## 4 Conclusion

In this article, we study the two-pion decays of the \( P \)-wave to \( S \)-wave charmonium transitions with the heavy meson effective theory, and make qualitative predictions for ratios among the two-pion
decay widths. We can confront the decay widths with precise experimental data in the future to fit the coupling constants and obtain quantitative predictions.

Acknowledgement

This work is supported by National Natural Science Foundation of China, Grant Number 11075053, and the Fundamental Research Funds for the Central Universities.

References

[1] M. Ablikim et al, arXiv:1208.4805.
[2] J. Beringer et al, Phys. Rev. D86 (2012) 010001.
[3] J. P. Lees et al, Phys. Rev. D86 (2012) 092005.
[4] Q. Lu and Y. P. Kuang, Phys. Rev. D75 (2007) 054019.
[5] Z. G. Wang, Eur. Phys. J. A47 (2011) 94.
[6] M. B. Voloshin, Phys. Rev. D86 (2012) 074033.
[7] P. Colangelo, F. De Fazio and R. Ferrandes, Phys. Lett. B634 (2006) 235; P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B642 (2006) 48; P. Colangelo, F. De Fazio, S. Nicotri and M. Rizzi, Phys. Rev. D77 (2008) 014012; P. Colangelo and F. De Fazio, Phys. Rev. D81 (2010) 094001; Z. G. Wang, Phys. Rev. D83 (2011) 014009; P. Colangelo, F. De Fazio, F. Giannuzzi and S. Nicotri, Phys. Rev. D86 (2012) 054024.
[8] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B302 (1993) 95; F. De Fazio, Phys. Rev. D79 (2009) 054015; Z. G. He, X. R. Lu, J. Soto and Y. Zheng, Phys. Rev. D83 (2011) 054028; Z. G. Wang, Int. J. Theor. Phys. 51 (2012) 1518; Z. G. Wang, Mod. Phys. Lett. A27 (2012) 1250197; Z. G. Wang, Commun. Theor. Phys. 57 (2012) 93.
[9] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B309 (1993) 163.
[10] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto and G. Nardulli, Phys. Rept. 281 (1997) 145.
[11] M. Ablikim et al, Phys. Rev. D86 (2012) 092008.
[12] M. B. Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455; Y. P. Kuang, Front. Phys. China. 1 (2006) 19.