Entropic gravity from noncommutative black holes

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In this paper we will investigate the effects of a noncommutative (NC) space-time on the dynamics of the universe. We will generalize the black hole entropy formula for a NC black hole. Then, using the entropic gravity formalism, we will show that the noncommutativity changes the strength of the gravitational field. By applying this result to a homogeneous and isotropic universe containing nonrelativistic matter and a cosmological constant, we will show that the model modified by the noncommutativity of the space-time is a better fit to the obtained data than the standard one.

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1. INTRODUCTION

In the last 40 years, detailed investigation of several aspects concerning the black hole (BH) physics were carried out. The thermodynamics of BHs has been one of most active research lines in theoretical physics. The pioneering work of Bekenstein [1] has shown that the BH entropy is proportional to its surface area. Thereafter, analyzing the origin of BH’s entropy following the quantum mechanics, Hawking [2] showed that the BH has a thermal radiation with a temperature $T_H = \kappa/2\pi$ ($\kappa = $ surface gravity). Hawking also suggested that the majority of the information concerning the initial states is protected behind the event horizon. The information will not be back to the asymptotic region far from the evaporating BH [3]. From these initial works, several discussions on the thermodynamics of BHs [4] have arisen.

However, in spite of the progress that the theoretical physics has accomplished in the understanding of this issue, a complete explanation for the final state of a BH after the evaporation remains unknown. The final piece of the puzzle should be a quantum gravity theory (QGT). One of the main candidates for QGT is the string theory. It is known that when we have a magnetic background in string theory, the resulting algebra acquires noncommutative (NC) features. Due to string-BH correspondence principle [2] and some other results concerning noncommutativity [5], there is a whole literature approaching the so-called noncommutative black holes (NCBH) [6].

A powerful way to describe NCBHs is through the so-called generalized uncertainty principle (GUP) [6]. When quantum gravitational effects are taken into account the usual Heisenberg uncertainty principle is modified [6]. Thus GUP provides a minimal length scale and therefore change the thermodynamics of a singular BH at the Planck scale. The GUP imply yet in the nonvanishing of the position coordinates commutator. This noncommutative property is described by a NC parameter, which lies at the Planck scale. The NCBHs that emerges from the noncommutativity of the space-time at small scales [10] are analogous to a nonsingular BH with two horizons [11].

Alternatively, another way to deal with a BH is through the holographic principle, which merges the effects of quantum mechanics and gravity [12].

In this work we will use the entropic gravity formalism, in which gravitation is obtained from thermodynamic properties of a holographic surface [13] to obtain the modifications on the gravitational field from the thermodynamics of NCBHs. From the NC corrections we will obtain a modified Friedmann equation and the we will constrain the NC parameter to some of the most recent observational data.

This paper is organized as follows: in Section 2 we will discuss briefly the thermodynamics of NCBHs; in Section 3 we will present the NC corrections on the gravitational field following the entropic gravity formalism and derive the equations for Friedmann-Robertson-Walker universe; in Section 4 we will investigate the feasibility of the model connected to the latest observational data; in Section 5 we will present our conclusions.
2. NONCOMMUTATIVE BLACK HOLE THERMODYNAMICS IN A NUTSHELL

In a NC space-time, the commutator between the position coordinates is given by

\[ [x^\mu, x^\nu] = i \theta^{\mu\nu}, \quad (1) \]

where \( \theta^{\mu\nu} \) is the so-called NC parameter. In the string theory, this commutator shows that the coordinates of the target space-time become NC operators on a D-brane. The product of two fields on this NC space-time is replaced by the Moyal product of commutative fields \([14]\), given by

\[ f(x) \star g(x) = \left\{ \exp \left[ i \frac{1}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial y^{\beta'}} \right] f(x + \alpha)g(x + \beta) \right\}_{\alpha = \beta = 0}. \]

Thus, a NCBH solution can be obtained by redefining the metric tensor \( g_{\mu\nu} \) non-vanishing components of a NC Schwarzchild BH is given by \([10]\),

\[ g_{\mu\nu} = \eta_{\mu\nu} e^a e^b \eta_{ab}. \quad (4) \]

The original commutative metric for a Schwarzchild BH solution is

\[ ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (3) \]

where \( M \) is the BH mass. In the vierbeins representation, the metric tensor \( g_{\mu\nu} \) can be written as

\[ g_{\mu\nu} = \epsilon^a_{\mu} \epsilon^b_{\nu} \eta_{ab}. \quad (4) \]

Thus, a NCBH solution can be obtained by redefining Eq.\([1]\) as \([10]\):

\[ \tilde{g}_{\mu\nu} = \epsilon^a_{\mu} \epsilon^b_{\nu} \eta_{ab}. \quad (5) \]

In what follows we will consider the case in which the non-vanishing components of \( \theta^{\mu\nu} \) are \( \theta^{23} = \beta \) and \( \theta^{12} = -\beta \). In this case, the non-vanishing components of the metric tensor \([5]\) are given by

\[ \tilde{g}_{00} = g_{00}, \]
\[ \tilde{g}_{11} = g_{11} + \frac{1}{4} \beta^2 g_{11} \cos(2\theta), \]
\[ \tilde{g}_{22} = g_{22} - \frac{1}{4} \beta^2 g_{22} \cos(2\theta), \]
\[ \tilde{g}_{33} = g_{33} + \frac{1}{8} \beta^2 \frac{\partial^2 g_{33}}{\partial \theta^2}, \]
\[ \tilde{g}_{12} = -\frac{\beta^2}{4} \sqrt{g_{11}g_{22}} \sin(2\theta). \]

From Eqs. \([5]\), it is possible to show that the entropy of a NC Schwarzchild BH is given by \([10]\):

\[ S'(r_+) = \left( 1 - \frac{\beta^2}{4} \right) S(r_+). \quad (7) \]

where \( S(r_+) = A/4 \) is the entropy of the commutative Schwarzchild BH and \( S'(r_+) \) is the entropy of the NC Schwarzchild BH.

3. NONCOMMUTATIVE ENTROPIC GRAVITY

According to Verlinde’s hypothesis \([13]\), the tendency of any system to increase their entropy (the second law of thermodynamics) is the origin of gravity and leads to the emergence of space-time. In fact, this approach helps us to provide a thermodynamic description of the gravitational field equations in various theories \([13,32]\). Since the entropy formula plays a key role in this approach, any modification of the system entropy may affect the gravitational field equations and therefore the corresponding Friedmann equations \([16,19]\). Moreover, it is worthwhile mentioning here that Verlinde’s interpretation of the origin of gravity and space-time is in line with both the generalized entropy formula and its corresponding cosmology \([30]\).

In order to use Verlinde’s approach, we need to evaluate the entropy of the system. For this purpose, consider a system of energy \( E \) enclosed by the surface \( A \). By generalizing Eq. \([7]\) to the system boundary, a surface of radii \( r_h = 2M \) assumed to be the holographic screen \([13]\), the entropy of the system is given by

\[ S_A = \left( 1 - \frac{\beta^2}{4} \right) \frac{A}{4}, \quad (8) \]

where \( A = 4\pi r_h^2 \) is the surface area of the boundary and \( \beta = \text{const.} \) is the NC parameter. Since the boundary surface consists of \( N \) degrees of freedom, we can write \([37]\)

\[ A = Q N, \quad (9) \]

where \( Q \) is a constant proportional to the square of Planck length \( l_p \). According the energy equipartition theorem, the source energy content is distributed on the surface degrees of freedom as \([37]\)

\[ E = \frac{1}{2} N T, \quad (10) \]

where \( T \) is the surface temperature. Combining Eqs. \([9]\) and \([10]\) we have that

\[ T = \frac{Q M}{2\pi l_p^2}. \quad (11) \]

where \( M = E \) is the gravitational mass of the source. According to Verlinde’s approach \([13]\), this tendency of the source to increase its entropy implies that the force acting on a test particle of mass \( m \) is given by

\[ F \Delta x = -T \Delta S_A, \quad (12) \]

where \( \Delta x \) is the displacement of the test particle from the holographic screen \( A \) and the minus sign is due the
inward nature of the entropy flux \cite{34}. If the distance between the test particle and the holographic surface is of the order of magnitude of its Compton wavelength \( \lambda_m = 2\pi/m \), the particle is absorbed by the holographic screen leading to an increase of the system entropy \cite{13}. In this case, we can set \( \Delta x = \eta \lambda_m \), with \( \eta \sim 1 \). Finally, from \cite{33, 30, 11} and \cite{12} we obtain

\[
F = \frac{\Delta A \Delta S_A}{\Delta x} = -\frac{Q^2}{16\pi^2\eta}(1 - \frac{\beta^2}{4})(\frac{mM}{r_h^2}),
\]

where we have used the fact that \( \Delta A = A/N = Q \) \cite{17}. The Newtonian limit, \( F \to -mM/r^2 \), obtainable when \( \beta \to 0 \), yields \( Q = 4\pi\eta^{1/2} \). Thus, the gravitational force acting on a particle of mass \( m \) in the holographic screen is

\[
F = -(1 - \frac{\beta^2}{4})(\frac{mM}{r_h^2}). \tag{14}
\]

and, as the gravitational force is attractive, this fact constrains the NC parameter to have values in the range \(-2 \leq \beta \leq 2 \).

The gravitational potential energy and the kinetic energy of the test particle are, respectively,

\[
U = -(1 - \frac{\beta^2}{4})\frac{mM}{r_h}, \tag{15}
\]

and

\[
K = \frac{1}{2}m\dot{r}_h^2. \tag{16}
\]

Now, let us apply the above results for an homogeneous and isotropic expanding universe. In this case, the radius \( r_h \) can be written in terms of the comoving distance \( x \) as \( r_h = a(t)x \), where \( a(t) \) is the scale factor. Writing the source mass as \( M = 4\pi\rho a(t)^3 x^3/3 \), the energy conservation for the test particle can be written as

\[
E = \frac{1}{2}ma^2x^2 - \frac{4\pi}{3}(1 - \frac{\beta^2}{4})m\rho a^2x^2. \tag{17}
\]

Multiplying both sides by \( 2/ma^2x^2 \) and rearranging the terms we can write that

\[
H^2 = \frac{8\pi}{3}(1 - \frac{\beta^2}{4})\rho - \frac{\kappa}{a^2}, \tag{18}
\]

where \( H = \dot{a}/a \) is the Hubble parameter and \( \kappa = -2E/ma^2 \). As we can see, the above equation is a modified version of the Friedmann equation, which is obtained in the limit \( \beta \to 0 \). Thus, in the entropic gravity formalism, the NC of the space-time affects the way the universe evolves.

Since the energy stored in the source is \( U = M = 4\pi\rho a^3x^3 \), the first law of thermodynamics reads as:

\[
TdS = dU + pdV = 4\pi a^2x^3 \left[ \frac{1}{3}d\rho + (\rho + p)d\alpha \right]. \tag{19}
\]

Assuming a reversible expansion, \( dS = 0 \), is easy to show that

\[
\dot{\rho} + 3H(\rho + p) = 0. \tag{20}
\]

In the above equations, \( p \) is the pressure of the fluid. By combining \cite{18} and \cite{20} we obtain the acceleration equation as being

\[
\frac{\dot{a}}{a} = -\frac{4\pi}{3}(1 - \frac{\beta^2}{4})(\rho + 3p). \tag{21}
\]

Summarizing, the final effect of the NC entropic cosmology is the modification of the Newton constant, \( G_N \) to \( (1 - \beta^2/4)G_N \) in the usual field equations.

### 4. ENTROPIC COSMOLOGY WITH NONCOMMUTATIVE EFFECTS

In order to probe the viability of the cosmological scenario developed in the previous section we will consider a spatially flat Friedmann-Robertson-Walker universe dominated by a pressureless matter (baryonic plus dark matter) as well as the energy of the quantum vacuum \( (p = -\rho) \). Concerning these considerations the Eq. \cite{18} can be rewritten as

\[
H^2 = H_0^2\left[1 - \frac{\beta^2}{4}\right]\left[\Omega_m(1+z)^3 + \Omega_{\Lambda,0}\right], \tag{22}
\]

where \( \Omega_{t,0} \) denotes the current fractional densities of radiation, non-relativistic matter and quantum vacuum, respectively. In this case, the normalization condition reads

\[
\frac{4}{4 - \beta^2} = \Omega_{\Lambda,0} + \Omega_{m,0}; \quad \beta \neq \pm 2. \tag{23}
\]

Note that in the absence of the NC corrections, the standard \( \Lambda \)CDM model is recovered, as expected.

In what follows, we will perform an observational analysis of the above model. In order to constrain the model parameters we use some cosmological probes that map the late-time universe expansion history. The data used in our analysis are: the “joint light curves” (JLA) sample \cite{38} which comprises 740 type Ia supernovae in the redshift range \( \langle z \rangle < 2 \); and the recent measurement of the Hubble constant, \( H_0 = 73.24 \pm 1.74 \) km·s\(^{-1}\)Mpc\(^{-1}\) \cite{44}.

We will use CLASS \cite{45} and Monte Python \cite{46} codes to perform the statistical analysis of the model for the...
FIG. 1: 1σ and 2σ confidence contours for the model parameters obtained from a joint analysis with JLA, BAO, CC and $H_0$ data sets.

combined data set: JLA + BAO + CC + $H_0$. In order to obtain correlated Markov Chain Monte Carlo samples from CLASS/Monte Python code, we use the Metropolis Hastings algorithm with uniform priors on the model parameters.

Parameter best fit $\pm 1\sigma$

| Parameter | Best Fit | $1\sigma$ Interval |
|-----------|----------|--------------------|
| $\beta^2$ | 0.3922$^{+0.07}_{-0.67}$ |  |
| $h$       | 0.728$^{+0.042}_{-0.040}$ |  |
| $\Omega_m$| 0.3034$^{+0.045}_{-0.043}$ |  |
| $\Omega_\Lambda$ | 0.6966$^{+0.043}_{-0.046}$ |  |

TABLE I: Constraints on the free parameters of the model from the combined JLA + BAO + CC + $H_0$ data set.

Table I summarizes the main results of our statistical analysis. Figure I shows the parametric spaces $h - \beta^2$ ($h = H_0/100\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$), $\Omega_{m,0} - \beta^2$ and $\Omega_{m,0} - h$. The results of our analysis shows that, at $1\sigma$ confidence level, the modified $\Lambda$CDM model fits the data better than the standard $\Lambda$CDM model, i.e., without the NC effects. However, the standard $\Lambda$CDM model still remains in good agreement with the data since $-0.08 \leq \beta^2 \leq 0.92$ at $2\sigma$ confidence level. Therefore, it’s characterized, at least for the data combination used in this paper, that NC effects, encoded in the parameter $\beta$, can not be ruled out. Figure 2 shows the effects of the parameter $\beta^2$ on the expansion rate of the universe front to the CC data.

5. FINAL REMARKS

To explain the current accelerated phase of the universe is one of the main challenges of the theoretical physics nowadays. The knowledge acquired in many areas of physics has been put together and it was applied to cosmology in the hope that a better understanding of the physics behind the cosmic acceleration can be achieved.

Considering this line of research, we have investigated the implications of a NC geometry to the late time expansion of the universe from the entropic gravity per-
FIG. 2: Evolution of the function $H(z)$ in the units of km s$^{-1}$ Mpc$^{-1}$ for some values of $\beta^2$.

In the entropic gravity formalism developed by Verlinde [13], the origin of the gravitational field is associated with the perturbations in the information manifold due to particles’ motion connected to the holographic screen. Since the noncommutativity of the space-time changes the value of the black hole entropy, the gravitational field emerging from the entropic principle will be different from the usual Newtonian gravitational field.

In this paper, we have investigated the consequences of this modification to the late time expansion of the universe. The correspondent NC correction of the Friedmann equation is measured by the NC parameter $\beta$. Assuming flatness and that the universe contains only non-relativistic matter and a cosmological constant, we constrain the NC parameter with the latest observational data of type Ia supernovae distance, baryon acoustic oscillations and cosmic chronometers. Our results show that the NC corrected ΛCDM model is favored compared with the standard one. The results obtained here are in agreement with the results obtained from the entropic gravity formalism in the framework of nongaussian statistics which also favor a weaker gravitational field [17].

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