Numerical Analysis of Wave Run-up Characteristics on Dual Non-submerged Vertical Cylinders

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Abstract. In this paper, a numerical model is set up to solve the mild slope equation proposed by Berkhoff using the finite element method. The computational result of wave run-up on a single non-submerged vertical cylinder matches well with the analytical solution, verifying the validity of the numerical model. The effect of the spacing between two cylinders on wave run-up is discussed. The results show that the position of maximum and minimum wave run-up on the upstream cylinder is almost the same with that of a single cylinder, but when the spacing is odd or even times of one quarter of incident wavelength, the secondary maximum or minimum wave run-up appears at the both sides of the cylinder. The curves of wave run-up on the downstream cylinder is similar but smaller than that of a single cylinder.

1. Introduction
Non-submerged vertical cylinders system is a common constitution in offshore structures and ocean platforms. When water waves meet these cylinders during propagation, wave reflection, diffraction and run-up occurs. The wave run-up will slam the ocean structures and have an effect on bearing capacity and stability. With respect to this problem, McCamy and Fuchs proposed a diffraction theory for wave run-up and forces on piles [1]. Galvin and Hallermeier researched the wave of large steepness run-up on vertical cylinders by physical model test [2]. Niedzwecki and Duggal conducted the experiment on regular and random waves run-up on cylinders [3]. Morris-Thomas also investigated into wave height distribution around vertical surface piercing cylinders in monochromatic waves [4]. Recently, with the rapid development of computer technology, the computational fluid dynamics method, based on the three-dimensional numerical wave flume, is applied to further discuss the problem. Morgan and Zang used the OpenFOAM to discuss the effect of grid size, discrete scheme, and time step on non-linear wave action on a cylinder [5]. Tang, et al [6], Liu and Wan [7] employed Navier-Stokes equations and VOF technique to trace free surface and simulate wave height distribution on a vertical cylinder.

Meanwhile, the mild slope equation is widely applied with its accurate solutions and small computation to study wave-structure interactions. The mild slope equation is a depth integrated combined refraction-diffraction equation, originally derived by Berkhoff with the aid of a small parameter development based on the linear wave theory and the mild slope assumption [8]. Booij examined the accuracy of the mild slope equation, demonstrating the equation is valid for a seabed slope of up to 1:3 [9]. Considering most of the offshore structures and platforms composed of more
than two vertical cylinders, the present study is concerned with the wave run-up characteristics for normally incident waves on dual non-submerged vertical cylinders by the finite element method to obtain the solution of the mild slope equation.

2. Numerical Model

2.1. Governing Equation

The original mild slope equation derived by Berkhoff is

\[ \nabla \cdot (c_s^2 \nabla \phi) + \frac{c_s \omega^2}{c} \phi = 0 \]  

(1)

where \( \nabla \) is the horizontal gradient operator; \( \phi \) is the spatial velocity potential; \( \omega = 2 \pi / T \) is the wave circular frequency, and \( T \) is the wave period; \( c \) and \( c_s \) is the phase velocity and group velocity, respectively; \( k = 2 \pi / L \) is the wave number, \( L \) is the wavelength, and the dispersion relation is

\[ \omega = gk \tanh kh \]  

(2)

in which \( g \) is the acceleration of gravity and \( h \) is the depth.

When the seabed is horizontal, the Equation (1) is reduced to Helmholtz equation

\[ \nabla^2 \phi + k^2 \phi = 0 \]  

(3)

2.2. Finite Element Solution

The finite element method (FEM) is applied to solve the mild slope equation as proposed in literatures [10-12]. The whole domain is divided into the inner region and the outer region, shown in Figure 1. The inner region is finite, with the variable depth and the body surface boundary \( B_1 \), satisfying the total reflection condition. The outer region is infinite, with the constant depth and the common boundary \( B_\infty \), satisfying the velocity potential and velocity continuous condition. \( B_\infty \) is the infinite boundary, satisfying the Sommerfeld radiation condition.

The inner region is cover by six node triangles, namely the finite elements, with simple shape functions. The outer region uses a super-element. Since the seabed in the outer region is horizontal, the governing equation is Equation (3), and its velocity potential can be written as follow

\[ \phi_{out} = A_0 H_0 (kr) + \sum_{n=1}^{N} H_n (kr) \left( A_n \cos n\theta + B_n \sin n\theta \right) \]  

(4)

where \( A_n \) (n=0, 1, 2, ..., N) and \( B_n \) (n=1, 2, ..., N) are unknowns, \( N \) is the number of terms after truncation; \( H_n (kr) \) (n=0, 1, 2, ..., N) are Hankel functions of the first kind, and \( r \) and \( \theta \) are radial and angular variables in polar coordinates.

Thus, the linear equations for \( M \) unknowns plus \( N \), where \( M \) is the amount of nodes in discrete elements, can be solved by Gaussian elimination.

2.3. Validation

To validate the numerical model, comparison is made between the current result and an analytical result for wave run-up on a single non-submerged vertical cylinder. As depicted in Figure 2, the cylinder with \( d=0.522 \) m in diameter is located in a horizontal bottom of \( h=0.165 \) m, and the size of the inner region is \( d/2 \leq \sqrt{x^2 + y^2} \leq 2L \). The incident wave travels in the positive direction of the \( x \)-axis, with its height \( H_0=0.079 \) m, wave period \( T=0.94 \) s, wave length \( L=1.046 \) m. Gambit software is introduced to generate meshes, and to balance between calculation accuracy and computational efficiency, 15 grids is set within a wavelength range. There are total 9098 element grids with 18444 nodes.

Figure 3 shows a comparison between the current numerical solution and the analytical result proposed by McCamy and Fuchs [1], and the agreement is excellent. The maximum wave run-up is at
the front point (θ=0°), which is 1.8 times the incident wave height. The minimum wave run-up, about 50% the incident wave height, is at the points (θ=145°), that is 35 degrees on both side of the rear of the cylinder. This demonstrates the correctness of the present model.

Figure 1. A sketch of computational region

Figure 2. Wave run-up on a single non-submerged vertical cylinder

Figure 3. Wave height distribution around a single non-submerged vertical cylinder

3. Numerical Simulation
For further study the effect of spacing between the dual non-submerged vertical cylinders on wave run-up, two cylinders, labelled as upstream cylinder and downstream cylinder, are arranged in wave-propagation direction (see Figure 4). The size of the two cylinders and wave elements are the same with the validation case. And the spacing $S$ between the two cylinders is ranging from $1.25L$ to $3L$ with the step $0.25L$ (see Table 1).

Figure 4. Wave run-up on dual non-submerged vertical cylinders
Table 1. Spacing for calculation cases.

| Case | Spacing (m) |
|------|-------------|
| 1    | 1.25L       |
| 2    | 1.50L       |
| 3    | 1.75L       |
| 4    | 2.00L       |
| 5    | 2.25L       |
| 6    | 2.50L       |
| 7    | 2.75L       |
| 8    | 3.00L       |

3.1. Wave Run-up on Upstream Cylinder

Figure 5 gives the comparison of wave run-up on upstream cylinder for different spacing. Compared with the single cylinder:

- When the spacing is an integral multiple of one quarter of incident wavelength, i.e. $S=nL/4$ ($n=5, 6, \ldots, 12$), the position and the value of the maximum wave run-up is the identical.
- When the spacing is odd times of one quarter of incident wavelength, i.e. $S=(2n+1)L/4$ ($n=2, 3, 4, 5$), the position of the minimum wave run-up is basically the same ($\theta=145^\circ$), while the value is smaller, and the difference decreases with the increase in spacing. Moreover, the secondary maximum wave run-up appears at the location of $\theta=90^\circ$. The results are caused by superposition of the diffraction from upstream and the reflection from downstream.
- When the spacing is even times of one quarter of incident wavelength, i.e. $S=2nL/4$ ($n=3, 4, 5, 6$), the location of the minimum wave run-up is also basically the same ($\theta=145^\circ$), while the value is larger, and the difference decreases with the increase in spacing. Moreover, the secondary minimum wave run-up appears at the point of $\theta=100^\circ$. The results are also related to wave diffraction and reflection.

![Wave run-up on upstream cylinder](image)

(a) $S=(2n+1)L/4$ ($n=2, 3, 4, 5$)  
(b) $S=2nL/4$ ($n=3, 4, 5, 6$)

Figure 5. Wave run-up on upstream cylinder

3.2. Wave Run-up on Downstream Cylinder

Figure 6 gives the comparison of wave run-up on downstream cylinder for different spacing. And the values of maximum and minimum wave run-up are listed in Table 2. It is observed that the curves of wave run-up are similar with that of a single cylinder, but

- Under the cover of the upstream cylinder, both the maximum and minimum wave run-up are smaller than that of a single cylinder.
• When the spacing is even times of one quarter of incident wavelength, both the maximum and minimum wave run-up are larger than that of the spacing odd times of one quarter of incident wavelength.

**Table 2.** Spacing for calculation cases.

| Case | S/L  | Maximum run-up | Minimum run-up |
|------|------|----------------|----------------|
| single | /    | 1.801          | 0.517          |
| 1     | 1.25 | 1.609          | 0.444          |
| 2     | 1.50 | 1.749          | 0.485          |
| 3     | 1.75 | 1.649          | 0.456          |
| 4     | 2.00 | 1.754          | 0.491          |
| 5     | 2.25 | 1.672          | 0.467          |
| 6     | 2.50 | 1.754          | 0.491          |
| 7     | 2.75 | 1.684          | 0.473          |
| 8     | 3.00 | 1.754          | 0.490          |

**Figure 6.** Wave run-up on downstream cylinder

4. **Conclusions**
A numerical model of the mild-slope equation is set up applying the finite element method. Based on the model, wave run-up on a single non-submerged vertical cylinder is calculated, and the result agrees well with the analytical solution, demonstrating the correctness of the model. For the upstream cylinder, the position of maximum and minimum wave run-up almost the same with that of a single cylinder, but when the spacing is odd or even times of one quarter of incident wavelength, the secondary maximum or minimum wave run-up appears at the point of \( \theta \approx 90^\circ \). For the downstream cylinder, the curves of wave run-up are similar with that of a single cylinder.

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