Direct measurement and reconstruction of nonclassical features of twin beams generated in spontaneous parametric down-conversion

Ondřej Haderka, Jan Peřina Jr., Martin Hamar, Jan Peřina,
Joint Laboratory of Optics of Palacký University and
Institute of Physics of Academy of Sciences of the Czech Republic,
17. listopadu 50A, 772 00 Olomouc, Czech Republic

Correlations in twin beams composed of many photon pairs are studied using an intensified CCD camera. Joint signal-idler photon-number distribution and quantum phase-space quasi-distributions determined from experimental data have nonclassical features.

Quantum mechanics interprets nonlinear optical processes as physical effects composed of many elementary ‘quantum’ events in which one (several) photon is annihilated and several (one) photon emerge [1]. This behavior of nonlinearly interacting physical systems is completely unusual in classical physics and lies in heart of nonclassical-light generation. Generation of correlated photon pairs in the process of spontaneous parametric frequency down-conversion is probably the most common example [2]. Fundamental experiments performed for the first time with photon pairs have shown that these nonlinear elementary ‘quantum’ events have even an ‘internal structure’. Measurement using photon pairs in Hong-Ou-Mandel interferometer has revealed this structure demonstrating that photons in one photon pair (photon twins) are generated within a sharp time window typically of several tens of fs [3]. Later experiments have even shown that entangled photon pairs are the same fundamental entities as single photons. Similarly as a single photon can interfere only with itself, an entangled photon pair interferes only with itself [4]. These completely unusual properties of photon pairs have been experimentally tested in numerous experiments with the same qualitative conclusion - fundamental laws of quantum mechanics are a solid basis for the explanation of obtained experimental results. Among others, experimental confirmation of violation of Bell inequalities using photon pairs has excluded neoclassical theories with local hidden variables as a right tool for the description of Nature. Photon pairs are also an indispensable tool in quantum teleportation [5], quantum cryptography or dense coding.

The use of intense femtosecond pump fields together with availability of new materials with larger nonlinearities have opened a new area in investigation of twin beams. Nowadays even beams containing many photon pairs generated in a sharp time window can be obtained. The first experiments confirm in agreement with quantum theory that photon pairs inside such beams behave as independent entities. Coherence properties of twin beams reflect those of the pump beam. Moreover, it has been shown that photon pairs can have their origin also in stimulated emission [6] (laser-like generation of twin beams is sometimes mentioned).

These fundamental properties of photon pairs then determine statistical properties of ‘more intense’ twin beams. In this letter, we report on experimental determination of photocount statistics of twin beams using an intensified CCD camera (iCCD). An alternative approach to determine photocount statistics is based on homodyne detection and has revealed correlations in photon numbers of two fields comprising a two-mode squeezed state [7]. In our case, even raw experimental data show nonclassical properties of twin beams. The reconstructed photon-number distribution then enables to determine (radially symmetric) joint phase-space quasi-distributions of the twin beams.

In the reported experiment sketched in Fig. we use the second-harmonic of an amplified femtosecond Ti:sapphire laser system as pump source for the generation of down-converted light in a 5-mm long Li:IO₃ nonlinear crystal. Laser system operates at 800 nm and yields a train of 200 fs pulses with a repetition rate controlled by the amplifier and set to 11 kHz in our case. Their second-harmonic is produced in a 2-mm long BBO crystal and pulses of energy up to 1.3 µJ are obtained. Down-converted photons emerge from a crystal at a cone layer with vertex half-angle of 31 deg (see Fig. 1). One part (signal) of the cone layer is captured by the photocathode of the camera placed 10 cm behind the crystal. The corresponding opposite part of the cone layer (idler) is led to the camera after reflection on a high-reflectivity mirror placed as close to the crystal as possible to minimize path difference of the two beams. Prior to entering the camera, both beams are filtered by two edge filters (high-pass above 750 nm) and a 20 nm wide (FWHM) bandpass filter centered at 800 nm. The filters reduce the

FIG. 1: Scheme of the experiment. The inset shows accumulated picture after 240,000 frames.
noise coming from fluorescence in the nonlinear crystal and stray light in the laboratory to an acceptable level and bandpass filter defines the thickness of the cone layer. In the software of the camera, three regions of interest are defined, two for the signal and idler strips and a third one that serves for monitoring the noise level (see inset in Fig. 1a). The overall quantum efficiency of the iCCD camera has been found to be 9.5%. Quantum state of twin beams at the output plane of the crystal can be described by the following statistical operator $\hat{\rho}_{SI}$:

$$\hat{\rho}_{SI} = \sum_{n_S=0}^{\infty} \sum_{n_I=0}^{\infty} p(n_S, n_I) |n_S\rangle_S \langle n_I|_I \otimes |n_I\rangle_I |n_S\rangle_S;$$  (1)

$|n_S\rangle_S (|n_I\rangle_I)$ denotes Fock state with $n_S$ ($n_I$) photons and then $p(n_S, n_I)$ means the joint signal-idler photon-number distribution. The down-converted photon pairs then undergo several loss mechanisms before they are registered. Taking into account losses in the signal and idler paths (expressed using effective transmittances $T_S, T_I$), quantum efficiency of the camera ($\eta$), and noise due to other light sources and internal noises of the camera ($D$), the measured probabilities $f(c_S, c_I)$ of having $c_S$ detections in the signal strip and $c_I$ detections in the idler strip at the camera are determined as:

$$f(c_S, c_I) = \sum_{n_S=0}^{\infty} \sum_{n_I=0}^{\infty} p(n_S, n_I) \times K^S(c_S, n_S) K^I(c_I, n_I);$$  (2)

$$K^i(c_i, n_i) = \frac{\min(c_i, n_i)}{c_i!} \left( \frac{n_i}{c_i} \right) (T_i \eta)^{c_i} (1 - T_i \eta)^{n_i - c_i} \times \frac{D^{n_i - c_i}}{(n_i - c_i)!} \exp(-D), \quad i = S, I. \quad (3)$$

This formula holds provided that the number of photons detected by the camera is much lower than the number of active pixels $K$.

An example of the measured photon-number distribution $f(c_S, c_I)$ is given in Fig. 2a. A correlated character of twin beams is clearly visible in the graph in Fig. 2b, showing the difference between the measured photon-number distribution $f(c_S, c_I)$ and that one composed of two independent Poissonian distributions with mean values equal to those of the measured signal and idler fields. We can see that elements lying on diagonal or near diagonal are enhanced whereas those lying far from diagonal are significantly suppressed. This is a direct experimental manifestation of the fact that signal and idler photons are generated in pairs.

Covariance $C_p$ of the signal and idler photon numbers determined along the formula $10$

$$C_p = \frac{\langle \Delta n_S \Delta n_I \rangle}{\sqrt{\langle \Delta n_S^2 \rangle \langle \Delta n_I^2 \rangle}}, \quad \langle \Delta n_i \rangle = n_i - \langle n_i \rangle, i = S, I \quad (4)$$

equals $0.0435 \pm 0.008$ for the data shown in Fig. 2. Type of photon-number statistics can be judged according to the value of coefficient $K$,

$$K = \frac{\langle n^2 \rangle}{\langle n \rangle^2} - 1; \quad (5)$$

$K = 1$ for Poissonian statistics, $K = 2$ characterizes Gaussian statistics, and $K < 1$ for a Fock state. The experimentally determined marginal signal-field (idler-field) photon-number distribution has $K_S (K_I)$ equal to $0.997 \pm 0.030$ ($0.994 \pm 0.030$), i.e. the statistics can be considered to be Poissonian within the experimental error. Statistics of photon pairs deviate from the Poissonian ones if the number of independent modes constituting the field is small $11$. Taking into account the fact that the measured marginal signal and idler photon-number distributions are Poissonian, the measured nonzero value of covariance $C_p$ also indicates nonclassical properties of detected twin beams. We note that a classical coherent field (with Poissonian statistics) cannot have any correlations in photon numbers at two distinct spatial points.

Pair character of twin beams is clearly revealed in the original joint signal-idler photon-number distribution $p(n_S, n_I)$ characterizing the light field as it occurs at the output plane of the crystal. The method of maximum likelihood estimation has proven to be extraordi-

![Fig. 2: (a) Measured joint signal-idler photon-number distribution $f$ as a function of the signal $(n_S)$ and idler $(n_I)$ photon numbers, (b) difference between the measured photon-number distribution $f$ and the distribution given by direct product of two independent Poissonian distributions with means equal to those of experimental data. Solid line denotes zero contour.](image-url)
reconstruction is not perfect due to the impossibility to describe precisely all noises occurring in the experiment. Mixing the signal and idler fields together, the resultant photon-number distribution preferring even photon-numbers also reflects these correlations, as has been observed in [14] using a special photon-number resolving detector.

Any photon-number distribution \( p(n_S, n_I) \) originating in a classical field has to fulfill the following inequality [14]:

\[
p(n_S, n_I) \leq \frac{n_S^{n_S}}{n_S!} \exp(-n_S) \frac{n_I^{n_I}}{n_I!} \exp(-n_I);
\]

i.e. this inequality represents a criterion of nonclassicality. The reconstructed probabilities lying inside the bold curve in Fig. 3b exhibit violation of this inequality. The violation is a consequence of the generation of photons in pairs (probabilities tend to be concentrated towards the diagonal where they reach greater values).

Experimental marginal signal and idler photon-number distributions derived from the reconstructed joint signal-idler photon-number distribution are Poissonian (\( K_S = 0.997, K_I = 1.000 \)). This means that also the probability distribution of generated photon pairs is Poissonian. Poissonian statistics reflects the fact that photon pairs can be generated into many independent modes distinguishable in space and time. In case of our experimentally obtained twin beams having typically tens or hundreds of photon pairs, each pair occupies its own mode with a high probability.

The reconstructed joint signal-idler photon-number distribution \( \rho^{(\infty)} \) enables to determine the joint distribution of integrated intensities \( W_S \) and \( W_I \) of the signal and idler fields at the output plane of the crystal:

\[
W_l = \int_{-\infty}^{\infty} d\tau \hat{E}_l^{-}(\tau) \hat{E}_l^{+}(\tau), \quad l = S, I.
\]

Inverting photodetection equation we have for the joint signal-idler integrated-intensity distribution \( P(W_S, W_I, s, M) \) related to \( s \)-ordering of field operators and assuming \( M \) independent modes [14, chap. 4]:

\[
P(W_S, W_I, s, M) = \frac{1}{W_S W_I} \exp \left[ -\frac{2(W_S + W_I)}{1 - s} \right]
\]

\[
\times \left[ \frac{4W_S W_I}{(1 - s)^2} \right]^M \sum_{n_S=0}^{\infty} \sum_{n_I=0}^{\infty} \frac{\rho^{(\infty)}(n_S, n_I) n_S! n_I!}{\Gamma(n_S + M) \Gamma(n_I + M)}
\]

\[
\times \left( \frac{s + 1}{s - 1} \right)^{n_S + n_I} L_{n_S}^{M-1} \left( \frac{4W_S}{1 - s^2} \right) L_{n_I}^{M-1} \left( \frac{4W_I}{1 - s^2} \right); \quad (9)
\]

\( L_n^{M-1} \) are Laguerre polynomials and \( \Gamma \) means the gamma function. The parameter \( s \) equals -1, 0, and 1 for antinormal, symmetric, and normal ordering of field operators, respectively.

Since both signal and idler fields are phase independent (they are generated in a multimode spontaneous process), the joint signal-idler phase-space quasi-distribution \( \Phi(\alpha_S, \alpha_I, s, M) \equiv \Phi(|\alpha_S|, |\alpha_I|, s, M) \) related
to $s-$ordering can be simply determined:

$$
\Phi(|\alpha_S|, |\alpha_I|, s, M) = \frac{1}{\pi^2} P(|\alpha_S|^2, |\alpha_I|^2, s, M)
$$

(10)

FIG. 4: Joint signal-idler quasidistribution $\Phi(|\alpha_S|, |\alpha_I|, s = 0.3, M = 50)$ determined from (a) experimental data and (b) assuming an ideal signal-idler photon-number distribution $p(n_S, n_I) = \delta_{n_S,n_I} \exp(-\mu)\mu^{n_S}/n_S!$, $\mu = 14.5$.

The iCCD camera is also able to reveal spatial correlations of photon pairs that can be characterized by entanglement area of two photons in a photon pair. Spatial correlations can then be used for further studies of photon-number statistics. These investigations are in progress.

In conclusion, we have experimentally demonstrated that twin beams composed of many photon pairs are of nonclassical origin that is manifested both in joint photon-number distributions and joint quantum phase-space quasi-distributions.

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