On asymmetry in inclusive pion production

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Abstract

On the basis of the mechanism proposed for one-spin asymmetries in inclusive hadron production we specify an $x$–dependence of asymmetries in inclusive processes of pion production. The main role in generation of this asymmetry belongs to the orbital angular momentum of quark-antiquark cloud in internal structure of constituent quarks. The $x$–dependence of asymmetries in the charged pion production at large $x$ reflects the corresponding dependence of constituent quark polarization in the polarized proton.

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1 Introduction

As it is widely known now, only part (less than one third in fact) of the proton spin is due to quark spins \([1, 2]\). These results can be interpreted in the effective QCD approach ascribing a substantial part of hadron spin to an orbital angular momentum of quark matter. It is natural to guess that this orbital angular momentum might be revealed in asymmetries in hadron production. In the recent paper \([3]\) we considered a possible origin of asymmetry in the pion production under collision of a polarized proton beam with unpolarized proton target and argued that the orbital angular momentum of partons inside constituent quarks leads to significant asymmetries in hadron production with polarized beam. This model has been successfully applied to an OZI-violating process of \(\varphi\)-meson production \([4]\).

Various mechanisms were proposed recently as a source of the significant asymmetries observed in pion production: higher twist effects \([5]\), correlation of \(k_{\perp}\) and spin in structure and fragmentation \([6, 8]\) functions, rotation of valence quarks inside a hadron \([9]\). Significant role in part of above references also belongs to orbital angular momentum. Recent reviews of theoretical and experimental aspects of single–spin asymmetry studies have been given in \([10, 11]\).

In the model \([3]\) the behavior of asymmetries in inclusive meson production was predicted to have a specific \(p_{\perp}\)–dependence, in particular, vanishing asymmetry at \(p_{\perp} < \Lambda_{\chi}\), its increase in the region of \(p_{\perp} \simeq \Lambda_{\chi}\), and \(p_{\perp}\)–independent asymmetry at \(p_{\perp} > \Lambda_{\chi}\). Parameter \(\Lambda_{\chi} \simeq 1\) GeV/c is determined by the scale of chiral symmetry spontaneous breaking. Such a behavior of asymmetry follows from the fact that the constituent quarks themselves have slow (if at all) orbital motion and are in the \(S\)–state, but interactions with \(p_{\perp} > \Lambda_{\chi}\) resolve the internal structure of constituent quark and feel the presence of internal orbital momenta inside this constituent quark.

We consider a nonperturbative hadron to be consisting of the constituent quarks located at the core of the hadron and quark condensate surrounding this core. Experimental and theoretical arguments in favor of such a picture were given, e.g. in \([12, 13]\). We refer to effective QCD and use the NJL model \([14]\) as a basis. The Lagrangian in addition to the 4–fermion interaction of the original NJL model includes 6–fermion \(U(1)_{A}\)–breaking term.

Strong interaction radius of this quark is determined by its Compton wavelength: \(r_Q = \xi/m_Q\), where the constant \(\xi\) is universal for different flavors. Spin of constituent quark \(J_U\) in this approach is determined by the sum \(J_U = 1/2 = J_u + J_{\bar{q}q} + \langle L_{\bar{q}q}\rangle\). The value of the orbital momentum contribution into the spin of constituent quark can be estimated with account for the experimental results from deep–inelastic scattering. The important point what the origin of this orbital angular momentum is. It was proposed \([3]\) to consider an analogy with an anisotropic generalization of the theory of superconductivity which seems to match well with the above picture for a constituent quark.

In Ref. \([3]\) we described how the orbital angular momentum, i.e. orbital motion of quark matter inside constituent quark, can be connected with the asymmetries in inclusive production at moderate and high transverse momenta and have given the predictions for \(p_{\perp}\)–dependence of asymmetry. In this note we specify \(x\)–dependence of asymmetries. In section 2 we give a brief outline of the model, section 3 is devoted to the description of an \(x\)–dependence of asymmetry in inclusive pion production and in section 4 we give short conclusion.
2 Outline of the model

We consider the hadron processes of the type
\[ h_1^\uparrow + h_2 \rightarrow h_3 + X \]
with polarized beam or target and \( h_3 \) being a charged pion.

In the model constituent quarks are supposed to scatter in a quasi-independent way by some effective field which is being generated at the first stage of interaction under overlapping of peripheral condensate clouds \[4, 13\]. Inclusive production of hadron \( h_3 \) results from recombination of the constituent quark (low \( p_\perp \)'s, soft interactions) or from the excitation of this constituent quark, its decay and subsequent fragmentation in the hadron \( h_3 \). The latter process is determined by the interactions at distances smaller than constituent quark radius and is associated therefore with hard interactions (high \( p_\perp \)'s). Thus, we adopt a two–component picture of hadron production which incorporates interactions at large and small distances. We supposed that \( \pi^+ \)–mesons are produced mostly by up flavors and \( \pi^- \)–mesons — by down flavors.

In the expression for asymmetry \( A_N \)[3]
\[
A_N(s, \xi) = \frac{\int_0^\infty db \left[I_-(s, b, \xi)/|1 - iU(s, b)|^2 - I_+(s, b, \xi)/|1 - iU(s, b)|^2\right]}{\int_0^\infty db \left[I_+(s, b, \xi)/|1 - iU(s, b)|^2 + I_-(s, b, \xi)/|1 - iU(s, b)|^2\right]}
\] (1)

the function \( U(s, b) \) is the generalized reaction matrix \[13\] and the functions \( I_\pm \) can be expressed through its multiparticle analogs \[3\], \( \xi \) denotes the set of kinematical variables for the detected meson. In the model the spin–independent part \( I_+(s, b, \xi) \) gets contribution from the processes at small (hard processes) as well as at large (soft processes) distances, i.e. \( I_+(s, b, \xi) = I^h_+(s, b, \xi) + I^s_+(s, b, \xi) \), while the spin–dependent part \( I_-(s, b, \xi) \) gets contribution from the interactions at short distances only \( I^h_-(s, b, \xi) = I^h_-(s, b, \xi) \). The function \( I^h_+(s, b, \xi) \) gets a nonzero value due to interference between the two helicity amplitudes, which gain different phases due to internal motion of partons inside the constituent quark. The following relation between the functions \( I^h_-(s, b, \xi) \) and \( I^h_+(s, b, \xi) \) has been proposed assuming the effect of internal motion of partons inside constituent quark leads to a shift in the produced meson transverse momentum:
\[
I^h_-(s, b, \xi) = \sin[P_{\tilde{Q}}(x)\langle L_{\bar{q}q}\rangle]I^h_+(s, b, \xi),
\] (2)

where \( P_{\tilde{Q}}(x) \) is the polarization of the leading constituent quark \( \tilde{Q} \) (in the process of the meson \( h_3 \) production) and \( \langle L_{\bar{q}q}\rangle \) is the mean value of internal angular momentum inside the constituent quark. This relation allowed to get a parameter–free prediction for the \( p_\perp \)–dependence of asymmetries in inclusive pion production \[3\]. We have considered there \( P_{\tilde{Q}} \) being a constant. Here we assume \( P_{\tilde{Q}} \) being a function of \( x \) and consider the behavior of asymmetry in the beam fragmentation region (where \( x \approx x_F \)).

The \( x \)–dependencies of the functions \( I^s_+(s, b, \xi) \) and \( I^h_+(s, b, \xi) \) are determined by the distribution of constituent quarks in a hadron and by the structure function of constituent quark respectively:
\[
I^s_+(s, b, \xi) \propto \omega_{\bar{q}/h_1}(x)\Phi^s(s, b, p_\perp) \quad \text{and} \quad I^h_+(s, b, \xi) \propto \omega_{\bar{q}/\tilde{Q}}(x)\Phi^h(s, b, p_\perp).
\] (3)
Taking into account the above relations, we can represent inclusive cross-section in the unpolarized case \(d\sigma/d\xi\) and asymmetry \(A_N\) in the following forms:

\[
d\sigma/d\xi = 8\pi[W_s^+(s, \xi) + W_h^+(s, \xi)],
\]

\[
A_N(s, x, p_\perp) = \frac{\sin[\mathcal{P}_Q(x)\langle L_{(\bar{q}q)}\rangle]W_h^+(s, \xi)}{[W_s^+(s, \xi) + W_h^+(s, \xi)]},
\]

where the functions \(W_s^+, W_h^+\) are determined by the interactions at large and small distances:

\[
W_s^+, h^+(s, \xi) = \int_0^\infty bdbI_{s, h}^+(s, b, \xi)/|1 - iU(s, b)|^2.
\]

3 The x-dependence of asymmetry

The asymmetry in the model has a significantly different \(x\)-dependence in the regions of transverse momenta \(p_\perp \leq \Lambda_\chi\) and \(p_\perp \geq \Lambda_\chi\). Therefore we consider these two kinematical regions separately. For that purpose it is useful to introduce the ratio

\[
R(s, \xi) = \frac{W_h^+(s, \xi)/W_s^+(s, \xi)}{r(s, p_\perp)},
\]

where the function \(r(s, p_\perp)\) in its turn is the \(x\)-independent ratio

\[
r(s, p_\perp) = \frac{\int_0^\infty bdb\Phi^+(s, b, p_\perp)/|1 - iU(s, b)|^2}{\int_0^\infty bdb\Phi^+(s, b, p_\perp)/|1 - iU(s, b)|^2}.
\]

The expression for the asymmetry \(A_N(s, \xi)\) can be rewritten in the form

\[
A_N(s, x, p_\perp) = \sin[\mathcal{P}_Q(x)\langle L_{(\bar{q}q)}\rangle]R(s, x, p_\perp)/[1 + R(s, x, p_\perp)],
\]

The function \(R(s, x, p_\perp) \gg 1\) at \(p_\perp > \Lambda_\chi\) since in this region dominate short distance processes and due to the similar reason \(R(s, x, p_\perp) \ll 1\) at \(p_\perp \leq \Lambda_\chi\). Thus we have simple \(p_\perp\)-independent expression for asymmetry at \(p_\perp > \Lambda_\chi\)

\[
A_N(s, x, p_\perp) \simeq \sin[\mathcal{P}_Q(x)\langle L_{(\bar{q}q)}\rangle]
\]

and a more complicated one for \(p_\perp \leq \Lambda_\chi\)

\[
A_N(s, x, p_\perp) \simeq \sin[\mathcal{P}_Q(x)\langle L_{(\bar{q}q)}\rangle]\frac{\omega_{\bar{q}/Q}(x)}{\omega_{{\bar{q}}/h_1}(x)}r(s, p_\perp).
\]

As it is clearly seen from Eq. (6) the asymmetry at \(p_\perp \leq \Lambda_\chi\) has a nontrivial \(p_\perp\)-dependence. In this region asymmetry vanishes at small \(p_\perp\) and is suppressed also by the factor \(\omega_{\bar{q}/Q}(x)/\omega_{{\bar{q}}/h_1}(x)\) which can be considered as the ratio of sea and valence quark distributions in hadron. The \(x\)-dependence of asymmetry in this kinematical region strongly depends on particular parameterization of these distributions. We therefore will consider the region.
of transverse momenta \( p_\perp > \Lambda_\chi \) where the \( x \)-dependence of asymmetry has a simple form reflecting corresponding dependence of leading constituent quark polarization. In [3] the two different cases were considered \( \mathcal{P}_U = 2/3 \), \( \mathcal{P}_D = -1/3 \) and \( \mathcal{P}_U = -\mathcal{P}_D = 1 \). The latest choice seems to be in better agreement with the experimental data. It is evident also that the experimental data [19] and Eq. (6) point out increase of \( \mathcal{P}_Q(x) \) with \( x \). Therefore we take the above values of constituent quark polarization as the maximal ones and consider the simplest possible dependencies, e.g. linear and quadratic ones:

\[
\mathcal{P}_Q(x) = \mathcal{P}_Q^{\text{max}} x \quad \text{and} \quad \mathcal{P}_Q(x) = \mathcal{P}_Q^{\text{max}} x^2,
\]

(9)

where \( \mathcal{P}_Q^{\text{max}} = -\mathcal{P}_D^{\text{max}} = 1 \). The curves corresponding to the above parameterizations are presented in Figs. 1 and 2 by dashed lines. The value of \( \langle \mathcal{L}_{\bar{q}q} \rangle \approx 1/3 \) has been taken [3] on the basis of the analysis [1] of the DIS experimental data. As it is seen from Figs. 1 and 2 the mean orbital angular momentum seems to be underestimated and experimental data prefer its greater value. Indeed, another analysis of DIS experiments has been carried out in [19] and the smaller value of the total spin carried by quarks has been obtained \( \Delta \Sigma \approx 0.2 \). This value corresponds to \( \langle \mathcal{L}_{\bar{q}q} \rangle \approx 0.4 \). Using the above value of angular orbital momentum we obtain a good agreement with the data in the case of linear dependence of constituent quark polarization and better but still not good agreement in the case of quadratic dependence of this polarization on \( x \). The corresponding curves are presented in Figs. 1 and 2 by solid lines.

To perform the quantitative description of \( x \)-dependence of inclusive cross-sections of the pion production in the collisions of unpolarized hadrons we should choose according to Eqs. (3), (4) the particular parameterizations of the functions \( \omega_{\bar{q}/p}(x) \) and \( \omega_{\bar{q}/Q}(x) \). As it was noted these functions could be associated with the valence and sea quark distributions respectively. Noting that we use the simplest forms [17] appropriate for fragmentation region:

\[
\omega_U/p \sim (1-x)^3, \quad \omega_D/p \sim (1-x)^4 
\]

(10)

and

\[
\omega_{u/U}(x) \sim \omega_{d/D}(x) \sim (1-x)^5.
\]

(11)

Then for the cross-sections of inclusive \( \pi^+ \)- and \( \pi^- \)- production we have:

\[
x \frac{d\sigma^{\pi^+}}{dx} = C_V^{\pi^+}(1-x)^3 + C_S^{\pi^+}(1-x)^5,
\]

(12)

\[
x \frac{d\sigma^{\pi^-}}{dx} = C_V^{\pi^-}(1-x)^4 + C_S^{\pi^-}(1-x)^5,
\]

(13)

where the factors \( C_{V,S} \) are the constants at fixed energy. Using the available experimental data at \( p_L = 400 \text{ GeV}/c \) [18] we obtain a good agreement of Eqs. (12), (13) with experiment at \( 0.2 < x < 0.7 \) with the following values of the factors \( C_V^{\pi^+} = 20 \text{ mb}, C_V^{\pi^-} = 10 \text{ mb}, C_S^{\pi^+} = 1 \text{ mb}, C_S^{\pi^-} = 3 \text{ mb} \). The corresponding results are represented in Fig. 3.
4 Conclusion

We have specified in this note the $x$-dependence of asymmetry in inclusive pion production in the beam fragmentation region in the framework of the approach formulated in [3]. We assume that the polarization of constituent quark in the polarized proton has a nontrivial $x$–dependence. The predictions for asymmetries given earlier in [3] should be referred to the fragmentation region. The above considerations suggest simple linear dependence of $\mathcal{P}_Q(x)$ at large $x$. The main role in the generation of asymmetry belongs to the orbital angular momentum of current quarks inside the constituent one. Present considerations and the experimental data suggest that $\langle L_{(q\bar{q})} \rangle \simeq 0.4$.

We would like to conclude noting that the result of E143 Collaboration on the measurements of $g_2(x)$ function suggests small twist-3 matrix element [20] and the experimental result of ALEPH Collaboration [21] reveals unexpectedly small polarization of $\Lambda_b$ measured in $e^+e^-$–interactions in the $Z^0$-decay which implies a strong depolarization mechanism acting at the stage of fragmentation. These interesting results could have important impact on mechanisms of generation of spin asymmetries in hadron production. However, further experimental studies are needed to clarify the spin puzzles observed in the measurements of asymmetries in hadronic processes.

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References

[1] J. Ellis and M. Karliner, CERN-TH/95-279, TAUP-2297-95, hep-ph/9510402.

[2] G. Altarelli and G. Ridolfi, in QCD 94, Proceedings of the Conference, Montpellier, France, 1994, ed. S. Narison [Nucl. Phys. B (Proc. Suppl.) 39B , (1995)].

[3] S. M. Troshin and N. E. Tyurin, Phys. Rev. D52, 3862 (1995).

[4] S. M. Troshin and N. E. Tyurin, Phys. Lett. B355, 543 (1995).

[5] A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982);
J. Qiu and G. Sterman, Nucl. Phys. B378, 52 (1992);
S. J. Brodsky, P. Hoyer, A. H. Mueller and W.-K. Tang, Nucl. Phys. B369, 519 (1992).

[6] D Sivers, Phys. Rev. D41, 83 (1990), ibid. D43, 261 (1991);
M. Anselmino, M. E. Boglione and F. Murgia, AIP Conf. Proc. 343, 11th Int. Symp. on High Energy Spin Physics, Bloomington, IN, 1994, eds. K. Heller and S. Smith, p. 446.

[7] J. C. Collins, Nucl. Phys. B396, 161 (1993);
J. C. Collins, S. F. Heppelman and G. A. Ladinsky, ibid. B420 565, (1994).
[8] X. Artru, J. Czyżewski and H. Yabuki, Preprint TPJU 13/95, 1995, hep-ph 9508239.

[9] C. Boros, Liang Zuo-tang and Meng Ta-chung, Phys. Rev. Lett 70, 1751 (1993); M. Doncheski, MAD/PH/778, 1993.

[10] N. E. Tyurin, Talk given at the Meeting on PHENIX Program, Vladimir, Russia, November 1995, Preprint IHEP 95-140, hep-ph 9512233; G. Ladinsky, Talk given at the 2nd Meeting on Possible Measurements of Singly Polarized $p\bar{p}$ and $p\bar{n}$ Collisions at HERA, Zeuthen, Aug.-Sept. 1995; O. Teryaev, ibid.; W.-D. Nowak, Contribution to the Proceedings of the Workshop on the Prospects of Spin Physics at HERA, Zeuthen, August 1995, Report DESY-Zeuthen 95-06.

[11] A. D. Krisch, AIP Conf. Proc. 343, 11th Int. Symp. on High Energy Spin Physics, Bloomington, IN, 1994, eds. K. Heller and S. Smith, p. 3.

[12] R. D. Ball, Intern. Journ. of Mod. Phys. A 5, 4391 (1990); M. M. Islam, Zeit. Phys. C 53, 253 (1992); Foundation of Phys. 24, 419 (1994).

[13] S. M. Troshin and N. E. Tyurin, Phys. Rev. D49, 4427 (1994).

[14] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); V. Bernard, R. L. Jaffe and U.–G. Meissner, Nucl. Phys. B308, 753 (1988); S. Klimt, M. Lutz, V. Vogl and W. Weise, Nucl. Phys. A516, 429 (1990); T. Hatsuda and T. Kunihiro, Nucl. Phys. B387, 715 (1992); K. Steininger and W. Weise, Phys. Lett. B329, 169 (1994).

[15] A.A. Logunov, V.I. Savrin, N.E.Tyurin and O.A.Khrustalev, Teor. Mat. Fiz. 6, 157 (1971).

[16] D. Adams et al. (FNAL E704 Collaboration), Phys. Lett. B261, 201 (1991).

[17] J. Gunion, Phys. Rev. D10, 242 (1974).

[18] M. Aguillar-Benitez et al., (LEBC-EHS Collaboration) Z. Phys. C 50, 405 (1991).

[19] R. Voss, Talk given at the Workshop on the Prospects of Spin Physics at HERA, Zeuthen, August 1995.

[20] K. Abe et al. (The E143 Collaboration), SLAC-PUB-95-6982, 1995, Submitted to Phys. Rev. Lett.

[21] The ALEPH Collaboration, CERN-PPE/95-156, 1995, Submitted to Phys. Lett. B.
Figure Captions

Fig. 1
Asymmetries $A_N$ in $\pi^+$ (positive values) and $\pi^-$ (negative values) production in $pp$–collisions at $p_L = 200$ GeV/c. Curves correspond to linear dependence on $x$ of constituent quark polarization. Solid lines correspond to $\langle L_{\bar{q}q} \rangle \simeq 0.4$ and dashed lines — to $\langle L_{\bar{q}q} \rangle \simeq 0.33$.

Fig. 2
Asymmetries $A_N$ in $\pi^+$ (positive values) and $\pi^-$ (negative values) production in $pp$–collisions at $p_L = 200$ GeV/c. Curves correspond to quadratic dependence on $x$ of constituent quark polarization. Solid lines correspond to $\langle L_{\bar{q}q} \rangle \simeq 0.4$ and dashed lines — to $\langle L_{\bar{q}q} \rangle \simeq 0.33$.

Fig. 3
Inclusive cross–sections of $\pi^+$ (solid line) and $\pi^-$ (dashed line) production in $pp$–collisions at $p_L = 400$ GeV/c.
