Correlation Heuristics for Constraint Programming

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Abstract—Effective general-purpose search strategies are an important component in Constraint Programming. We introduce a new idea, namely, using correlations between variables to guide search. Variable correlations are measured and maintained by using domain changes during constraint propagation. We propose two variable heuristics based on the correlation matrix, crbs-sum and crbs-max. We evaluate our correlation heuristics with well-known heuristics, namely, dom/wdeg, impact-based search and activity-based search. Experiments on a large set of benchmarks show that our correlation heuristics are competitive with the other heuristics, and can be the fastest on many series.

I. INTRODUCTION

Backtracking search combined with constraint solving is the main approach to solve problems in Constraint Programming (CP). The key to effective search is having a good variable search heuristic to select a variable to branch as the size of the search tree is strongly dependent on the selected variables. In CP, many general purpose variable ordering search heuristics have been proposed and implemented in many CP solvers, such as the conflict-driven heuristic dom/wdeg [1], impact-based search (IBS) heuristic [2], and activity-based search (ABS) heuristic [3]. Search heuristics by their nature are not designed to be optimal search strategies but merely good ones. Thus, our goal in this paper is a new search heuristic which can outperform existing heuristics on some instances across a range of problems.

We propose a new idea which is correlation-based search (CRBS), the search heuristic employs correlations between variables. The correlation of a pair of variables \((x_i, x_j)\) is used to estimate the possibility of a conflict between \(x_i\) and \(x_j\) during search. We maintain a matrix corresponding to the paired variable correlation during search. The correlation matrix is turned into a search strategy by using a function to combine values in the matrix to estimate whether assigning a value to variable \(x_i\) can cause a conflict. Domain changes during constraint propagation are used to measure the correlations between variables. We present two generic and new correlation-based variable heuristics, crbs-sum and crbs-max. Our experiments compare the correlation heuristics with the well known search heuristics dom/wdeg, ABS and IBS on a large set of benchmarks. The results show that correlation heuristics are competitive with the existing heuristics, and can also be the fastest on many problem instances from different problem series. In particular, crbs-sum is shown to be an effective search heuristic.

II. RELATED WORKS

We briefly introduce several well-known general purpose heuristics. One of the simplest heuristics is dom [4] which follows the fail first principle, selecting the variable with smallest domain size. Many general purpose heuristics combine domain size with other information. For example, the well-known heuristics dom/deg [5] and dom/ddeg [6] combine domain sizes with variable degrees, which can be better than dom. The conflict-driven heuristic dom/wdeg [1] associates a weight with each constraint to record conflicts during search. The weight of constraint \(c\) is increased when the constraint solver finds \(c\) to be inconsistent. The dom/wdeg heuristic selects the next variable based on weight degrees and domain sizes, where the weight degree of a variable \(x\) is the sum of the weights of the constraints involving \(x\) and at least another uninstantiated variable. Some variants of dom/wdeg exploit different information to update the weight of constraints such as the explanation-based weight [7] and constraint tightness weight [8].

The impact-based search (IBS) heuristic [2] is motivated by the pseudo-costs used in mixed-integer programming. It uses impact to measure the importance of a variable to the rate of search space reduction. A variant of IBS incorporates variances in reduction [9]. Counting-based search [10] exploits solution counting information to guide search. The activity-based search (ABS) heuristic [3] combines domain sizes with activity for variables where activity is a measure of how often a variable is reduced during search. We remark that it is different from the SAT activity heuristic VSIDS [11] which also records some conflict information during search.

III. BACKGROUND

A constraint satisfaction problem (CSP) instance is a triplet \((C, X, D)\), where \(C = \{c_1, c_2, ... c_n\}\) is a set of \(e\) constraints, \(X = \{x_1, x_2, ... x_n\}\) is a set of \(n\) variables and \(D = \{D(x_1), D(x_2), ... D(x_n)\}\) is the corresponding domains for the variables. \(D(x_i)\) is the initial domain of variable \(x_i\), and \(dom(x_i) \subseteq D(x_i)\) is the current domain of \(x_i\) during search. Every constraint \(c\) consists of a constraint scope scp(c) and a relation \(R(c)\), where scp(c) \(\subseteq X\) and \(R(c) \subseteq \prod_{x_i \in scp(c)} D(x_i)\). A solution of a CSP instance is the set of assignments \(\{(x_1, a_1), ... (x_n, a_n)\}\) which satisfies all constraints in \(C\), where \(a_i \in D(x_i)\). During backtrack search, the search heuristic selects a variable to instantiate at each
search node. The variables which have been instantiated during a path in the search tree are defined as past variables while the variables which have not been instantiated are future variables.

IV. CORRELATION-BASED SEARCH

Typically the goal of a variable heuristic is to choose variables which can cause backtracking to occur earlier in the search. This suggests to choose variables which can lead to conflicts earlier in the search. In this paper, we propose correlation-based heuristics to achieve this objective. For each pair of variables \((x_i, x_j)\), we define a value \(a_{i,j}\), called the correlation of \((x_i, x_j)\), as a measure of the possibility of having a conflict between \(x_i\) and \(x_j\). During search, a correlation matrix representing all variable pairs \(a_{i,j}\) is maintained, where each value in the matrix represents the correlation of a pair of variables. A special case is \(a_{i,i}\) which estimates the degree of conflict when choosing variable \(x_i\). We propose two functions which use the correlation matrix to estimate the degree of conflict from assigning the variable. Then the heuristic will choose the variable which is estimated to cause more conflicts.

A. Updating the correlation matrix

We maintain the correlation matrix by using domain changes during constraint propagation. Some search heuristics have used the information about domain changes to guide search, such as activity-based search (ABS) [3]. The idea of the ABS heuristic is to select the variable which is the most often updated. It maintains an array \(A\) during search to record the activities of variables. After constraint propagation, if the domain of variable \(x_i\) is updated, then \(A(x_i)\) is increased by 1, otherwise decreased by multiplying with \(\gamma\) where \(0 \leq \gamma \leq 1\). Then the heuristic selects the variable with max \(\frac{A(x_i)}{\text{dom}(x_i)}\).

We use a similar approach. We assume that the more frequent \(\text{dom}(x_i)\) is updated after assigning \(x_i\), the more likely a conflict between \(x_i\) and \(x_j\) can happen. As such, the correlations between variables are updated based on domain changes. After constraint propagation due to variable \(x_i\) being assigned, the remaining variables can be split into two subsets, \(U\) and \(N\):
\[
U = \{\forall x_j \in X' \mid \text{dom}'(x_j) \neq \text{dom}(x_j)\}
\]
\[
N = \{\forall x_j \in X' \mid \text{dom}'(x_j) = \text{dom}(x_j)\}
\]
where \(X' = X \setminus \{x_i\}\) and \(\text{dom}'(x_j)\) is the new domain of \(x_j\) after constraint propagation. The \(U\) variables are those whose domains are updated, while the \(N\) variables are those whose domains are unchanged. If no conflict occurs, then the correlations are updated as follows:
\[
\begin{align*}
\{ a_{i,j} = \hat{a}_{i,j} + 1, a_{j,i} = \hat{a}_{j,i} + 1 \quad \forall x_j \in U \\
a_{i,j} = \hat{a}_{i,j} - 1, a_{j,i} = \hat{a}_{j,i} - 1 \quad \forall x_j \in N \\
a_{i,i} = \hat{a}_{i,i} - 1
\end{align*}
\]
where \(\hat{a}_{i,j}\) is the old correlation value before the update. If \(\text{dom}'(x_j)\) is changed after assigning \(x_i\), then the correlations \(a_{i,j}\) and \(a_{j,i}\) are increased by one. Otherwise, \(a_{i,j}\) and \(a_{j,i}\) are decreased by one. In addition, we decrease the correlation \(a_{i,i}\), this is to make \(a_{i,i}\) small if no conflicts happen after assigning \(x_i\) repeatedly.

Otherwise, if a conflict appears in the constraint propagation after assigning \(x_i\), the correlations of all variables are increased as follows:
\[
\begin{align*}
a_{i,j} = \hat{a}_{i,j} + 1, a_{j,i} = \hat{a}_{j,i} + 1 \quad \forall x_j \in X' \\
a_{i,i} = \hat{a}_{i,i} + 2
\end{align*}
\]
We increase the correlation \(a_{i,j}\) by 2 because the assignment of \(x_i\) causes a conflict. In addition, \(a_{i,j}\) and \(a_{j,i}\) are updated in the same way as before. We see that this definition leads to the correlation matrix being symmetric.

B. Selecting variables using the correlation matrix

We propose two ways of using the correlation matrix with combining functions based on the matrix and problem variables, namely, the \(\text{crbs-sum}\) and \(\text{crbs-max}\) functions which estimate the potential of conflict after assigning variable \(x_i\).

The \(\text{crbs-sum}\) function is a linear function of the relevant entries in the correlation matrix. First, we define two auxiliary functions, \(P_c(x_i)\) and \(F_c(x_i)\) on variable \(x_i\):
\[
P_c(x_i) = \sum_{x_j \in P} a_{i,j} \quad F_c(x_i) = \sum_{x_j \in F} a_{i,j}
\]
where \(P\) is a set of past variables and \(F\) is a set of future variables. The variable to be considered, \(x_i\), is part of the set \(F\) of future variables. The idea is that \(P_c(x_i)\) (past correlation) is the sum of correlations of past variables with respect to \(x_i\), and \(F_c(x_i)\) (future correlation) is similar but for the future variables. The \(\text{crbs-sum}\) function for variable \(x_i\) is defined as:
\[
\text{crbs-sum}(x_i) = P_c(x_i) + \theta \times F_c(x_i)
\]
A parameter \(0 \leq \theta \leq 1\) is used to control the combination of the past and future variable correlation. In particular, future variables are used when \(\theta > 0\), otherwise, we consider only past variables when \(\theta = 0\).

We propose another simple combining function, the \(\text{crbs-max}\) function, defined as follows:
\[
\text{crbs-max}(x_i) = \max_{x_j \in P}(a_{i,j})
\]
The idea for \(\text{crbs-max}\) is to choose a future variable which has the largest estimated correlation with the past variables. We also experimented with a variant of the max function on all variables (past and future), i.e. \(\max(x_{i,j})\). Initial experiments found Equation 4 to give better results. In the rest of the paper, we use the max function as defined in Equation 4.

V. EXPERIMENTS

We evaluate the correlation-based heuristics, \(\text{crbs-sum}\) and \(\text{crbs-max}\), with well known, successful and commonly used heuristics: weighted degree (\(\text{dom/\text{wdeg}}\)), activity (\(\text{ABS}\)) and impact (\(\text{IBS}\)). Experiments are run on a 3.40 GHz Intel core
| series               | mean time (s) | nodes | dom/wdeg | ABS | IBS | crbs-sum | crbs-max |
|---------------------|---------------|-------|----------|-----|-----|----------|----------|
| Ortholatin total (4) solved by all (3) | 107.84 | 1 TO | 1 TO | 31.75 | 515K | - | 87K      |
| TSP total (30) | 5.48 | 13.12 | 12.55 | 3.96 | 7.93 | 44K | 228K | 253K | 32K | 74K |
| Latin Square total (30) | 4.48 | 13.12 | 12.55 | 3.96 | 7.93 | 44K | 228K | 253K | 32K | 74K |
| Dubois total (11) solved by all (7) | 4.10 | 4.10 | 5.10 | 17.58 | 71.14 | - | 2M | 3M |
| Magic Square total (11) | 4.10 | 5.10 | 4.10 | 172.16 | 1 TO | - | - | 566K |
| Costas Array total (9) solved by all (7) | 3.30 | 3.44 | 8.62 | 1.54 | 5.65 | 5K | 5K | 18K | 2K | 8K |
| Social Golfer total (4) solved by all (0) | 1.17 | 6.09 | 59.54 | 2.25 | 1.70 | - | - | - | 101K |
| u total (41) solved by all (38) | 1.36 | 1.37 | 1.73 | 1.40 | 1.42 | 389 | 140 | 6K | 94 | 139 |
| Nonogram total (176) solved by all (172) | 4.10 | 1.63 | 1.59 | 1.61 | 4.89 | - | 564 | 181 | 102 | 177 |
| Crt total (18) solved by all (6) | 2.10 | 2.10 | 2.55 | 3.36 | 23.62 | - | - | 82K | 120K | 1M |
| Black hole total (39) solved by all (20) | 19 TO | 27.72 | 15.92 | 34.04 | 1 TO | - | 3M | 1M | 4M |
| Mycel total (12) solved by all (11) | 12.36 | 8.71 | 5.53 | 6.65 | 1.10 | 429K | 234K | 134K | 194K |
| Queen Knights total (11) solved by all (8) | 2.10 | 4.10 | 76.81 | 1.10 | 3 TO | - | - | 4K |
| AllInterval total (9) solved by all (11) | 1.10 | 1 TO | 17.86 | 71.34 | 48.28 | - | - | 470K | 1M | 1M |
| cc total (11) solved by all (11) | 1.40 | 3.06 | 0.91 | 4.61 | 1.21 | 79K | 1M | 19K | 110K | 22K |
| Open Shop total (49) solved by all (43) | 60.95 | 1 TO | 51.00 | 2 TO | 5 TO | - | - | 118K |
| Coloring total (22) solved by all (22) | 1.87 | 0.63 | 0.62 | 0.99 | 7.88 | 144K | 34K | 20K | 55K | 493K |
| Mug total (8) solved by all (4) | 4 TO | 31.57 | 3 TO | 173.65 | 3 TO | - | 7M | - | 28M |
| Knights total (18) solved by all (7) | 41.85 | 29.40 | 28.62 | 33.52 | 35.52 | 1K | 934 | 934 | 934 | 934 |
| Covering Array total (9) solved by all (5) | 4 TO | 2.71 | 1 TO | 3.77 | 3 TO | - | - | 3K |
| Insertion total (21) solved by all (14) | 1.30 | 1.46 | 3.86 | 1.48 | 1.58 | 3K | 823 | 61K | 823 | 5K |
| Radar total (62) solved by all (58) | 8.77 | 23.96 | 4 TO | 43.09 | 38.63 | 107 | 631 | 3K | 1K | 583 |
| Queen Attack total (5) solved by all (3) | 4.54 | 10.90 | 45.95 | 24.38 | 24.66 | 60 | 523 | 1K | 618 | 397 |
| scen11 total (10) solved by all (6) | 45.24 | 4 TO | 1 TO | 23.64 | 3 TO | - | - | 579K |
| Crossword total (140) solved by all (128) | 2.08 | 7.63 | 12 TO | 3.77 | 2.88 | 12K | 73K | - | 25K | 16K |
| Golomb Ruler total (25) solved by all (18) | 1.17 | 6.09 | 59.54 | 2.25 | 1.70 | 10K | 71K | 643K | 22K | 13K |
| Schur Lemma total (9) solved by all (7) | 52.85 | 1 TO | 2 TO | 81.77 | 1 TO | - | - | 144K | - | 416K |
| Total total (807) solved by all (692) | 56 TO | 43 TO | 48 TO | 15 TO | 47 TO | 1M | 364K | 549K | 30K | 108K |

**TABLE I**

Mean results of 5 heuristics. For the Super-JobShop series, crbs-sum is highlighted as the best it has the smallest total runtime on solving the 20 non-timeout instances compared with dom/wdeg and IBS.
i7 CPU on Linux. The existing heuristics are the AbsCon solver implementations of dom/wdeg, ABS and IBS. For the ABS and IBS heuristics, we use the default parameter settings in Abscon.

For the crbs-sum heuristic, we use $\theta = 0.1$, chosen as a value for $\theta$ which we found to work well on many instances (see Section V-B). The initial values in the correlation matrix of CRBS are set to 0.

All heuristics break ties lexicographically, and use the lexical value order heuristic. In all cases, a geometric restart search policy (the initial $\text{cutoff}' = 10$ and $\rho = 1.1$) was used, where $\text{cutoff}'$ is the maximum number of failures before restart and $\rho$ controls the growth of the value of $\text{cutoff}'$ after restart.

We apply the binary search branch strategy. The time-out is set to 1200s for all instances. We have used a large and varied set of well-known CSP benchmarks. In total, there are 807 problem instances which come from the following 30 series:

- All Interval Series (AllInterval), Black Hole, Chessboard Coloration (cc), Coloring, Costs Array, Covering Array, Nonogram, Crl, Crossword, Dubois, Golomb Ruler, ii, insertion, Open Shop (os-taillard), Knights, Latin Square, Schurr’s lemma, Magic Square, Mug, Myciel, Orthogonal Latin Squares, Quasi Group, Queen Attacking, Queen Knights, Radar Surveillance (Radar), Register, RLFAP-scen11 (scen11), Social Golfers, Super-Jobshop, Travelling Salesman Problem (TSP).

We include all instances from each series except those which are not solved by all the heuristics used within timeout.

A. Comparing heuristics

Figure 1 shows a runtime distribution of the benchmark instances solved using the different heuristics. The y-axis is the CPU time (in seconds (s)) and the x-axis is the number of solved instances within the time limit. In the graph, instances which are too fast are ignored, namely, 397 instances where the average time needed by all heuristics is less than 1 second have not been plotted. Thus, there are 410 instances plotted in Figure 1. Note that in this graph, the best performance is towards the lower right corner.

The best runtime distribution result is given by the $\text{crbs-sum}$ heuristic which also solves the most instances. In particular, with the time limit of 1200s, $\text{crbs-sum}$ can solve 395/410 instances, which is better than $\text{dom/wdeg}$, $\text{ABS}$, $\text{IBS}$ and $\text{crbs-max}$ with respectively 354/410, 367/410, 362/410 and 363/410 instances.

Table II gives the mean results of all five heuristics on each series. The row “total (n)” gives the average CPU times and number of search nodes for all instances in a series, where $n$ is the number of instances. The row “solved by all (n)” is the mean results on instances solved by all heuristics. “n TO” denotes that the heuristic time-outs on $n$ instances. The bold numbers in Table II highlight the best result for each series. Furthermore, for the Super-jobShop series, $\text{crbs-sum}$ has both the smallest total time and smallest number of time-outs. The last two rows, labelled as “Total” give the average results on all series for each heuristic with $\text{crbs-sum}$ giving the best overall results.

Table II highlights how many series are solved faster by a particular heuristic from the overall results in Table II. The row “Faster than $\text{dom/wdeg}$” (ABS or IBS) gives the number of series on which the heuristic is better than $\text{dom/wdeg}/\text{ABS}/\text{IBS}$ respectively. The row “Fastest (Second fastest)” is the number of series on which the heuristic is the best (second best) respectively. Overall, $\text{crbs-sum}$ solves more series.

The exact performance of the heuristics vary on different series. $\text{crbs-sum}$ is the fastest on many series. For example, on the dubois series, $\text{crbs-sum}$ solve 11 instances in 193 seconds, but $\text{dom/wdeg}$, $\text{IBS}$ and $\text{ABS}$ time-out on some instances. In total, $\text{crbs-sum}$, $\text{crbs-max}$, $\text{dom/wdeg}$, IBS and ABS are the fastest on 10/30, 1/30, 6/30, 9/30 and 4/30 series respectively. $\text{crbs-sum}$ is also competitive or better with the other general purpose variable heuristics. On 19 series, $\text{crbs-sum}$ is either the fastest or the second fastest heuristic. Overall, $\text{crbs-sum}$ is faster than $\text{dom/wdeg}$, IBS and ABS on 21, 20 and 19 series respectively. For the Super-jobshop series, the mean times of $\text{crbs-sum}$ and $\text{dom/wdeg}$ are respectively 5.61s and 94.99s on the instances solved by the two heuristics. The mean CPU time of $\text{crbs-sum}$ on all series is also less than that of other heuristics.

Between $\text{crbs-sum}$ and $\text{crbs-max}$, our experiments show that the sum of correlations is more useful than the maximal correlation—$\text{crbs-sum}$ is faster than $\text{crbs-max}$ on many series. On most series, the trend of mean times correlates with

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1 We used the AbsCon solver [https://www.cril.univ-artois.fr/~lecoutre/software.html].

2 The value of $\text{cutoff}'$ is updated using $\text{cutoff}' = \text{cutoff}' + \text{init}_\text{cutoff}' \times \rho^k$, where $\text{cutoff}'$ is the cutoff of the last restart, $\text{init}_\text{cutoff}'$ is the initial cutoff with value 10, and $k$ is the number of encountered restarts.

3 Benchmarks are from the 2009 CSP competition website [http://www.cril.univ-artois.fr/CSC09/] and the XCSP3.0 website [http://www.xcsp.org/].
the trend on the number of nodes. We observe that for the RLFAP-scn11 series, the number of search nodes of crbs-sum is less than that of dom/wdeg, but crbs-sum is slower than dom/wdeg, thus, the cost of maintaining crbs-sum may be more expensive than dom/wdeg. Possibly, our implementation could be optimized further.

B. Choosing the crbs-sum parameter

The parameter $\theta$ used in equation 3 affects the performance of the crbs-sum heuristic. In this section, we explore the effect of different choices of $\theta$ on two problem series. Figure 2a gives the results on TSP series, where the y-axis is the mean solving times and the x-axis is the values of $\theta$. Correspondingly, Figure 2b gives the results on the Quasi Group series.

Overall, we found that low values for the $\theta$ parameter generally give better results than higher values. For example, the mean times of solving TSP series is only 3.89s when $\theta = 0.1$, which is 2 times faster than that of $\theta = 0.9$. For extreme values of $\theta$, when $\theta = 0$, the mean time on the Quasi Group series is 9 times faster than that of $\theta = 1$. This suggests that the correlations between $x_j$ and past variables is more important for the crbs-sum heuristic than correlations with future variables. However, we also should not ignore the future variables, for example, when $\theta = 0.1$, the mean times on TSP and Quasi Group are faster than with $\theta = 0$.

VI. CONCLUSION

In this paper, we propose a new idea, measuring correlations between variables, which leads to various correlation-based heuristics. We measure and update the correlation matrix by using domain changes during constraint propagation. We propose two correlation heuristics, crbs-sum and crbs-max, which employ different strategies to estimate the potential of conflict for a variable based on the correlation matrix. The experiments show that correlation heuristics are promising. They are competitive with the state-of-the-art heuristics dom/wdeg, ABS and IBS on a large set of benchmarks. Furthermore, the correlation heuristics can also be the fastest on many problem series. In the future, we will explore more accurate or efficient methods to update the correlations between variables, and design improved correlation heuristics.

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