Sliding Mode Observer and Control for Quadrotor Stabilization

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Abstract. In recent years, the development of unmanned aerial vehicles (UAVs) has brought about brilliant progression in many industrial fields. For UAV motion control, the most fundamental and core factor is the controller design for attitude stability. This paper focuses on the stabilization of a quadrotor. Quadrotor is influenced by external disturbances such as wind gust and meteorological changes. For these reasons, quadrotor has a modeling limitation in precision. In this paper, we propose a control method with the estimated three angles to follow the desired value for quadrotor attitude stabilization. We suggest the sliding mode observer (SMO) based attitude estimation model of a quadrotor. The configured error variable by SMO improves the estimation accuracy of the angles such as roll, pitch, and yaw. Using sliding mode control (SMC), moreover, we obtain the appropriate control input to stabilize the quadrotor attitude with the estimated angle based on SMO. Sliding surface that contains the desired angles is determined to reduce the tracking error. The performance and effectiveness of the proposed system are verified through a simulation result.

1. Introduction
Recently, the interest in UAVs has grown rapidly. The quadrotor is maneuvered or propelled by four motors. The quadrotor is classified as a rotorcraft, as opposed to fixed wing aircraft, because it takes off vertically with four motors. Many researchers have studied on a quadrotor for various purposes, such as video recording, swarm, parcel service, exploration of risk, and vehicle. In diverse studies, the core technology is the attitude control and is to use the proportional-integral-derivative (PID) or SMC as a controller. PID is a widely used control algorithm in controller design. However, PID has a limitation in precision for control. Many researchers have studied on other controller to improve the performance. As a result, SMC has been applied successfully in various design problems [1]. The quadrotor is a system with four inputs, independently and six coordinate outputs. Various control methods have been proposed for quadrotor. Castillo proposed the quadrotor obtained via a Lagrange approach dynamic model and control algorithm based on nested saturation [2], and nonlinear control strategy based on Lyapunov analysis [3]. Bouabdallah proposed nonlinear control techniques to stabilize attitude and altitude by using backstepping and SMC [4]. Davila proposed an observer which estimates only position measurements for mechanical systems[5].

In this paper, we propose the stabilization of quadrotor attitude control with the estimated angle to follow the desired angle. We set up the attitude dynamic model of quadrotor and design the adapted quadrotor attitude dynamics by SMO. We obtain the appropriate input u by SMC with adapted attitude dynamics of a quadrotor by SMO. Input u obtained by SMC helps to follow the desired attitude for the stability of a quadrotor. This paper is composed as follows. We obtain the dynamic model of quadrotor attitude in Section 2. We design the adapted quadrotor dynamic and estimate the
angle of a quadrotor by using SMO in Section 3. The input \( u \) for stabilization of a quadrotor by SMC and Lyapunov stability is obtained in Section 4 with adapted dynamic designed by SMO. We show simulation results of the proposed method in Section 5. Finally, the conclusion appears in Section 6.

2. Quadrotor Dynamics

The quadrotor can take off and land vertically by four motors that generate the propeller forces. Figure 1 shows the earth fixed frame and rigid body model of a quadrotor. Fig.1 describes two pairs of propellers \( (\Omega_1, \Omega_2) \) and \( (\Omega_3, \Omega_4) \). Two diagonal motors \( (\Omega_1, \Omega_3) \) rotate clockwise, and the others \( (\Omega_2, \Omega_4) \) rotate counterclockwise. Quadrotor can change its direction on the spot by two pairs of motors which rotate in the opposite direction.

![Figure 1. Composition of quadrotor.](image)

We can acquire different position and attitude of the quadrotor by changing the velocity of the four propellers. The quadrotor dynamic model can be presented as follows:

\[
\begin{align*}
\dot{\phi} &= \dot{\psi} \left( \frac{I_x - I_z}{I_x} \right) + \dot{\theta} \left( \frac{J_z}{I_x} \right) \Omega + \left( \frac{l}{I_x} \right) u_1 \\
\dot{\theta} &= \dot{\phi} \psi \left( \frac{I_x - I_z}{I_y} \right) + \dot{\phi} \left( \frac{J_z}{I_y} \right) \Omega + \left( \frac{l}{I_y} \right) u_2 \\
\dot{\psi} &= \phi \dot{\theta} \left( \frac{I_x - I_z}{I_z} \right) + \left( \frac{l}{I_z} \right) u_3
\end{align*}
\]

Where the angles \( (\phi, \theta, \psi) \) obtained from the rotation of quadrotor’s body frame represent roll, pitch, and yaw, respectively. \( I_{x,y,z} \) is the body inertia, \( J_r \) is the rotor inertia, and \( l \) is the lever length. The control inputs are represented by \( u_1, u_2 \) and \( u_3 \), respectively for roll, pitch, and yaw and these are given as:

\[
\begin{align*}
u_1 &= t \left( \Omega_1^2 - \Omega_2^2 \right) \\
u_2 &= t \left( \Omega_2^2 - \Omega_3^2 \right) \\
u_3 &= d \left( \Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2 \right)
\end{align*}
\]

\[\Omega = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3\]
Where $\Omega_i$ is the $i$-th motor rotational velocity, $t$ is the thrust factor, and $d$ is the drag factor. The developed model in equation (1) can be rewritten in a state-space form: 

$$
\dot{x} = f(x, u)
$$

and 

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix}
$$

is the state vector of the system. State vector is composed with 

$$
x_1 = \phi, \quad x_2 = \dot{\phi}, \quad x_3 = \theta, \quad x_4 = \dot{\theta}, \quad x_5 = \psi, \quad \text{and} \quad x_6 = \dot{\psi}.
$$

The developed model in equation (1) can be rewritten in a state-space form: 

$$
c_1 = \frac{(I_y - I_z)}{I_x}, \quad c_2 = \frac{J_x}{I_x}, \quad c_3 = \frac{(I_z - I_x)}{I_y}, \quad c_4 = \frac{J_y}{I_y}, \quad c_5 = \frac{(I_x - I_y)}{I_z},
$$

and 

$$
c_6 = \frac{l}{I_x}, \quad c_7 = \frac{l}{I_y}, \quad c_8 = \frac{l}{I_z}.
$$

(3)

From equations (1) and (3), we obtain the attitude dynamic of the quadrotor: 

$$
\dot{x} = f(x, u) = \begin{bmatrix} x_2 \\ x_4 \\ x_2 x_6 c_1 + x_4 c_2 \Omega + c_6 u_1 \\ x_6 \\ x_2 x_6 c_3 + x_4 c_4 \Omega + c_7 u_2 \\ x_6 x_4 c_5 + c_8 u_4 \end{bmatrix}
$$

(4)

3. Adapted Quadrotor Dynamics with SMO

The purpose of SMO is to estimate the unmeasurable states of a quadrotor system based only on the measured outputs and inputs. Sliding mode observer works by minimizing the error between quadrotor dynamic model and the observer model by using a switching function and can force the output estimation error to converge to zero in finite time. In this section, we design the SMO which estimates a quadrotor rotational angle and rotational velocity, and design the adapted quadrotor dynamics.

In Section 3, we define the state-space form with the state variables $S = \begin{bmatrix} \dot{x}_1, \dot{x}_2, s_1, \dot{x}_3, s_2, \dot{x}_4, s_3, s_4, \dot{x}_5, s_5 \end{bmatrix}^T$. We propose the observer based on the differentiation of the state variables $x_1$ through $x_6$ of the quadrotor attitude dynamics as follows:

$$
\dot{S} = \begin{bmatrix} \dot{x}_2 + z_4 \\ \dot{x}_4 \dot{x}_6 c_1 + \dot{x}_4 c_2 \Omega + c_6 u_2 + s_1 + z_2 \\ \dot{x}_4 + z_4 \\ \dot{x}_2 \dot{x}_6 c_3 + \dot{x}_2 c_4 \Omega + c_7 u_3 + s_2 + z_5 \\ \dot{x}_6 + z_7 \\ \dot{x}_4 \dot{x}_2 c_5 + c_8 u_4 + s_3 + z_8 \\ \dot{x}_2 + z_9 \end{bmatrix}
$$

(5)

In equation (5), the differentiators ($z_i$) are proposed as:
The differentiators \( z_i \) are used in advanced SMO for the estimation of the system state in the presence of unknown external disturbances. In equation (6), \( e_i \) is the error between measurement and estimated value as \( \hat{e}_i = x_i - \hat{x}_i \), \( i = 1, 3, 5 \).

4. Sliding mode control for attitude stabilization

In this section, we use the SMC to obtain the input \( u_i \) to stabilize the quadrotor attitude with SMO based on quadrotor attitude dynamics. For the first step of SMC controller design, we set up a sliding surface \( (\sigma_i) \) as following:

\[
\sigma_i = (\hat{x}_2 - x_{2,d}) + \lambda_i (\hat{x}_1 - x_{1,d})
\]  

(7)

In equation (7), \( \lambda_i \) is a positive constant. Next we use the Lyapunov function to satisfy the stability condition.

\[
V(\sigma_i) = \frac{1}{2} \sigma_i^2, \quad \dot{V}(\sigma_i) = \sigma_i \dot{\sigma}_i < 0
\]  

(8)

Lyapunov function and sliding surface must satisfy the condition (8). As a result, we derive the time derivative of sliding surface as:

\[
\dot{\sigma}_i = -\mu \text{sat}(\sigma_i) - \mu_\sigma \dot{\sigma}_i
\]

\[
= (\dot{x}_2 - \dot{x}_{2,d}) + \lambda_i ((x_2 - x_{2,d}) + z_1)
\]

(9)

We can obtain the control input \( u_i \) from equation (9) as follows:

\[
u_i = -\frac{1}{c_6} \left[ \hat{x}_1 \dot{x}_6 + \hat{x}_6 c_6 + \hat{x}_4 c_2 \Omega + c_8 u_1 + s_1 + z_2 - \dot{x}_{2,d} + \lambda_i ((\dot{x}_2 - x_{2,d}) + z_1) + \mu \text{sat}(\sigma_i) + \mu_\sigma \sigma_i \right]
\]  

(10)

In equations (9) and (10), we use the saturation function for stabilizing the SMC.

\[
\text{sat}(\sigma, \alpha) = \begin{cases} 
\text{sgn}(\sigma), & |\sigma| > \alpha, \\
\sigma/\alpha, & |\sigma| \leq \alpha 
\end{cases}
\]  

(11)

\( \alpha \) is a width of sliding surface’s \( (\sigma_i) \) boundary. We set up a sliding surface \( (\sigma_2, \sigma_3) \) as follows:
\begin{align*}
\sigma_2 &= (\dot{x}_4 - x_{4d}) + \lambda_2 (\dot{x}_3 - x_{3d}), \\
\sigma_3 &= (\dot{x}_6 - x_{6d}) + \lambda_3 (\dot{x}_5 - x_{5d})
\end{align*}

(12)

We can obtain inputs $u_2$ and $u_3$ following the procedures of $u_i$.

\begin{align*}
u_2 &= -\frac{1}{c_7} [\ddot{x}_2 \dot{x}_4 + \ddot{x}_6 \dot{x}_3 + \dot{x}_2 \dot{x}_3 \Omega + s_2 + z_5 - \dot{x}_{4d} + \lambda_2 ((\dot{x}_4 - x_{4d}) + z_4) + \mu_5 \text{sgn}(\sigma_2) + \mu_4 \sigma_2], \\
u_3 &= -\frac{1}{c_8} [\ddot{x}_4 \dot{x}_5 + s_3 + z_8 - \dot{x}_{6d} + \lambda_3 ((\dot{x}_6 - x_{6d}) + z_7) + \mu_5 \text{sgn}(\sigma_3) + \mu_4 \sigma_3]
\end{align*}

(13)

5. Simulation

In this section, we prove the adapted attitude dynamics of a quadrotor by SMO and the stabilization of quadrotor attitude with SMC. The whole system stabilizes the quadrotor attitude with estimation data. Figure 2 shows the comparison between the attitude estimate values and the desired angles. In figure 2-(a) shows that estimated and stabilized angle ($\phi$) of quadrotor attitude. Fig.2-(b) shows the pitch angle ($\theta$). Pitch angle has a difference between attitude estimate value and the desired angle at initial stage because the initial point of pitch angle is not the same. However, after few seconds SMO estimates the quadrotor attitude correctly and pitch angle is stabilized by SMC.

![Figure 2](image_url)

**Figure 2.** Estimation and stabilization of quadrotor attitude $(\phi, \theta)$. 

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6. Conclusion
In this paper, we proposed a SMC for stabilization and applied to the quadrotor with SMO. At first, we set up the quadrotor attitude dynamic modeling. Secondly we reconfigured the adapted attitude dynamics of quadrotor, the unmeasured states, and their derivatives through the SMO. Finally we obtained the appropriate input $u_i$ by SMC using the adapted attitude dynamics and estimation data. The estimation data, the differentiator, and the saturation function in SMC and SMO were used to improve the performance in stabilization. We confirmed that SMO estimates the quadrotor attitude and SMC interacts with SMO to stabilize the attitude.

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