A covariant action for the eleven dimensional superstring

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Abstract

We suggest a super Poincaré invariant action for closed eleven dimensional superstring. The sector of physical variables \( x^i, \theta_a, \dot{\theta}_a \), with \( a, \dot{a} = 1...8 \) and \( x^i \) the transverse part of the \( D = 11 \) \( x^\mu \) coordinate is shown to possess free dynamics.

1 Introduction

The classical Green–Schwarz (GS) superstring (with the manifest spacetime supersymmetry and local \( \kappa \)-symmetry) can move in spacetime dimensions 3, 4, 6, and 10 [1]. Thus, the standard approach fails to construct a \( D = 11 \) superstring action, while known results (see, for example, [2–4] and references therein) suggest the existence of a consistent quantum theory incorporating the \( D = 11 \) supergravity. At present moment, the latter can be viewed as either strong coupling limit of the type IIA superstring [5], or as an effective theory of the supermembrane [6], or it may be regarded as a constituent of \( M \)-theory [2, 5, 7]. The purpose of this letter is to present some results in this direction.

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The crucial ingredient in construction of GS superstring action is the \( \Gamma \)-matrix identity
\[
\Gamma_\alpha^{\mu(\beta}(C\Gamma_{\mu})_{\gamma\delta)} = 0. \tag{1}
\]
which provides both global supersymmetry and local \( \kappa \)-symmetry for the superstring action \([8]\). The \( \kappa \)-symmetry, in its turn, ensures free dynamics in the sector of physical variables. To elucidate construction that will be suggested below for \( D = 11 \), let us discuss the problem in the Hamiltonian framework, where one faces with the well known fermionic constraints (see, for example, Refs. \( 1 \) and \( 8 \))
\[
L_\alpha \equiv p_{\theta \alpha} - i(\bar{\theta}(\Gamma_{\mu})_\alpha(p_{\mu} + \Pi_{1\mu}) = 0, \tag{2}
\]
obeys the Poisson bracket
\[
\{L_\alpha, L_\beta\} = 2i(\bar{\theta}(\Gamma_{\mu})_\alpha(p_{\mu} + \Pi_{1\mu}) = 0, \tag{3}
\]
By virtue of Eq. (1) the last term in Eq. (3) vanishes for \( D = 3, 4, 6, 10 \).

The next step is to impose an appropriate gauge. Then the full system (constraints and gauges) looks as follows:
\[
L_\alpha = 0, \tag{4}
\]
\[
\Gamma^{+}\theta = 0, \tag{5}
\]
and is second class (without making use of Eq. (1)).

The situation changes drastically for the \( D = 11 \) case, where instead of Eq. (1), one has \([9, 10]\)
\[
10\Gamma_{\alpha(\beta}(C\Gamma_{\mu})_{\gamma\delta)} + \Gamma_{\alpha(\beta}(C\Gamma^{\mu\nu})_{\gamma\delta)} = 0. \tag{6}
\]
Being appropriate for the construction of the supermembrane action [6], this identity does not allow one to formulate a $D = 11$ superstring with desirable properties. As was shown by Curtright [9], the globally supersymmetric action based on this identity involves additional to $x^i, \theta_a, \bar{\theta}_a$ degrees of freedom in the physical sector. Moreover, it does not possess $\kappa$-symmetry which might provide free dynamics [9, 10].

In this letter we suggest a $D = 11$ super Poincaré invariant action for the classical closed superstring which possesses free dynamics in the physical variables sector. Instead of the standard approach which implies the search for an action with a local $\kappa$-symmetry (or, equivalently, with the corresponding first class constraints), we present a theory in which constraints like Eqs. (4), (5) arise among others. Since they are second class, the $\kappa$-symmetry and the identity (6) are not necessary for the construction. Thus, at the classical level, a superstring of the type described can exist in any spacetime dimensions and the known brane scan [4] can be revised. In particular, being applied to the $D = 10$ case, our construction yield the model in which spectrum of physical states coincide with those of $N = 1, D = 10$ Green-Schwarz superstring. For definiteness, in this letter we discuss the $D = 11$ case only.

Two comments are in order. First, one needs to covariantize Eq. (5), and the simplest possibility is $\Lambda_\mu \Gamma^\mu \theta = 0$, with $\Lambda^2 = 0$. It is assumed that the additional vector variable $\Lambda^\mu$ is introduced in such a way that the gauge $\Lambda^- = 1$ is possible. Second, one can expect that a model with constraints like Eqs. (4) and (5) will possess (if any) off-shell super Poincaré symmetry in a nonstandard realization. Actually, global supersymmetry which does not spoil the equation $\Lambda_\mu \Gamma^\mu \theta = 0$ turns out to be $\delta \theta \sim \Lambda_\mu \Gamma^\mu \epsilon$. On-shell, where $\Lambda^2 = 0$, only half of the supersymmetry parameters $\epsilon^\alpha$ are essential.

It is worth mentioning other motivation for this work. The action for
the super $D$-brane which allows local $\kappa$-symmetry is rather complicated [11, 15]. One can hope, that being applied to that case, our method will lead to a more simple formulation.

The work is organized as follows. In Sect. 2 the action and its local symmetries are presented. In Sec. 3 within the framework of the Hamiltonian approach we prove that the model possesses free dynamics. In Sect. 4 an off-shell realization of the super Poincaré algebra is obtained and discussed.

**Notations.** We use $32 \times 32$ $\Gamma$-matrices in the Majorana representation [16]. They have the properties $(\Gamma^0)^T = -\Gamma^0$, $(\Gamma^i)^T = \Gamma^i$, $(\Gamma^\mu)^* = \Gamma^\mu$ and obey the algebra $\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}$, $\eta^{\mu\nu} = (+, -, \ldots, -)$. Charge conjugate of the Majorana spinor $\theta^\alpha = (\theta_a, \bar{\theta}^\dot{a}, \theta^\dot{a}, \bar{\theta}_a), \alpha = 1, 2, \ldots, 32, a, \dot{a} = 1 \ldots 8$ is $\bar{\theta} = \theta C$ with $C = \Gamma^0$. It will be convenient to use the following light-cone $\Gamma$-matrices:

$$\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^9), \quad \Gamma^i, \quad i = 1, 2, \ldots, 8, 10.$$  

Momenta, conjugate to the configuration space variables $q^i$ are denoted as $p_{qi}$.

## 2 Action and its local symmetries

The action functional to be examined is

$$S = \int d^2 \sigma \left\{ \frac{-g^{ab}}{2\sqrt{-g}} \Pi^\mu_a \Pi^\mu_b - i\varepsilon^{ab} \partial_a x^\mu (\bar{\theta} \Gamma^\mu \partial_b \theta) - i\Lambda^\mu \bar{\psi} \Gamma^\mu \theta - \frac{1}{\phi} \Lambda^\mu A^\mu - \Lambda^\mu \varepsilon^{ab} \partial_a A^\mu_b \right\}, \quad (7)$$

where it was denoted $\Pi^\mu_a \equiv \partial_a x^\mu - i\bar{\theta} \Gamma^\mu \partial_a \theta$. The first two terms are exactly those of the GS superstring action written in eleven dimensions. The origin of the remaining terms is as follows. The third and the fourth terms will
supply the appearance of the equations $\Lambda_\mu \Gamma_\mu \theta = 0$ and $\Lambda^2 = 0$. Hence, the variables $\bar{\psi}^\alpha$ and $\phi$ can be viewed as the Lagrange multipliers enforcing the constraints. The last term was added to suppress the appearance of some undesirable constraints in addition to those mentioned above. The expression of such a kind was successfully used before [17, 18] in a different context.

Note also that the Wess–Zumino term in the $D = 10$ GS action provides the appearance of the local $\kappa$-symmetry [1]. In our model it plays a different role, as shown below.

Let us briefly comment on the structure of local symmetries for the action (7). Local bosonic symmetries include $d = 2$ reparametrizations, Weyl symmetry, and the following transformations with parameters $\xi^\mu(\sigma^a)$ and $\omega_a(\sigma^b)$:

$$\delta A^\mu_a = \partial_a \xi^\mu + \omega_a \Lambda^\mu, \quad \delta \phi = \frac{1}{2} \phi^2 \varepsilon^{ab} \partial_a \omega_b. \quad (8)$$

These symmetries are reducible because their combination with parameters of special form $\omega_a = \partial_a \omega$, $\xi^\mu = -\omega \Lambda^\mu$ is a trivial symmetry: $\delta A^\mu_a = -\omega \partial_a \Lambda^\mu$, $\delta \phi = 0$ (note that $\partial_a \Lambda^\mu = 0$ is one of the equations of motion). Thus, Eq. (8) includes 12 essential parameters which correspond to the primary first class constraints $p^\mu_0 \approx 0$, $p^\phi \approx 0$ (see below).

There is also a fermionic symmetry with the parameters $\xi^\alpha(\sigma^a)$:

$$\delta \bar{\psi} = \bar{\xi} \Gamma^\mu \Lambda_\mu, \quad \delta \phi = -\phi^2 (\bar{\xi} \theta), \quad (9)$$

from which only 16 are essential on-shell where $\Lambda^2 = 0$. As shown below, reducibility of this symmetry make no special problem for covariant quantization.

Let us present arguments that the action constructed describes a free theory. Equations of motion for the action (7) are

$$\Pi^\mu_a \Pi^\mu_b - \frac{1}{2} g_{ab} (g^{cd} \Pi^\mu_c \Pi^\mu_d) = 0, \quad (10. a)$$
\[
\partial_a \left( \frac{g^{ab}}{\sqrt{-g}} \Pi_b^\mu + i \varepsilon^{ab} \theta \Pi_b^\mu \partial_b \theta \right) = 0, \quad (10.b)
\]

\[
4i \Pi_b^\mu (\Gamma^\mu P^{-ba} \partial_a \theta) \alpha + \varepsilon^{ab} \theta^\beta \partial_a \theta \gamma \partial_b \theta^\delta \Gamma_{\alpha(\beta}^\mu \Pi^\gamma_{\gamma\delta)} + i \Lambda^\mu (\Gamma^\mu \psi) \alpha = 0, \quad (10.c)
\]

\[
\Lambda^\mu \Gamma^\mu \theta = 0, \quad \Lambda^2 = 0, \quad (10.d)
\]

\[
\partial_a \Lambda^\mu = 0, \quad \varepsilon^{ab} \partial_a A^\mu_b + \frac{2}{\phi} \Lambda^\mu + i \bar{\psi} \Gamma^\mu \theta = 0, \quad (10.e)
\]

where

\[
P^{-ba} = \frac{1}{2} \left( \frac{g^{ba}}{\sqrt{-g}} - \varepsilon^{ba} \right).
\]

Multiplying Eq. (10.c) with \( \Lambda^\mu \Gamma^\mu \) one gets

\[
(\Lambda^\mu \Pi_b^\mu) P^{-ba} \partial_a \theta = 0. \quad (11)
\]

In the gauge \( \Lambda^- = 1 \), supplemented by the conformal one, this can be rewritten as

\[
(\partial_0 + \partial_1) \theta = 0. \quad (12)
\]

Hence, any solution \( \theta(\sigma) \) of the system (10) obeys the free equation, which is accompanied with \( \Lambda^\mu \Gamma^\mu \theta = 0 \). The latter reduces to \( \Gamma^+ \theta = 0 \) in the gauge chosen. Thus, Eqs. (10.a–c) for the \( g^{ab}, x^\mu, \theta^\alpha \) variables look like those of the GS superstring. In the result one expects free dynamics in this sector provided that the conformal gauge has been assumed. In the next section we will rigorously prove this fact by direct calculation in the Hamiltonian framework.

### 3 Analysis of dynamics

From the explicit form of the action functional it follows that the variable \( \Lambda^\mu \) can be excluded by making use of its equation of motion. The Hamiltonian analog of the situation is a pair of second class constraints \( p_{\Lambda^\mu} = 0 \), \( p_{A^\mu_1} - \Lambda^\mu = 0 \), which can be omitted after introducing the associated Dirac
bracket. The Dirac brackets for the remaining variables prove to coincide with the Poisson ones. The Hamiltonian looks like

\[ H = \int d\sigma \left\{ -\frac{N}{2}(\dot{p}^2 + \Pi_1^\mu \Pi_1^\mu) - N_1 \dot{\phi} \Pi_1^\mu + p_{1\mu} (\partial_1 A_0^\mu + i\bar{\psi} \Gamma^\mu \theta) + \frac{1}{\phi} (p_1^\mu)^2 + \lambda_\phi \pi_\phi + \lambda_{0\mu} \dot{p}_0^\mu + \lambda^{ab}(\pi_g)_{ab} + \lambda_\psi^\alpha p_{\psi\alpha} + L_\alpha \lambda_\theta^\alpha \right\}, \quad (13) \]

where \( p_\mu, \dot{p}_0^\mu, p_1^\mu \) are momenta conjugate to the variables \( x^\mu, A_0^\mu, A_1^\mu \), respectively, and \( \lambda_\phi \) are the Lagrange multipliers corresponding to the primary constraints. In Eq. (13) we also denoted

\[ N = \frac{\sqrt{-g}}{g^{00}}, \quad N_1 = \frac{g_{01}}{g^{00}}, \quad \dot{p}_1^\mu = p_\mu - i\bar{\theta} \Gamma^\mu \partial_1 \theta, \]

\[ L_\alpha \equiv p_{\theta\alpha} - i(p_\mu + \Pi_1^\mu)(\bar{\theta} \Gamma^\mu)_{\alpha} = 0. \]

Detailed analysis shows that the constraints \((\pi_g)_{ab} = 0\) are first class, which suggests the gauge choice \(g^{ab} = \eta^{ab}\). Then the full set of constraints can be written in the form

\[ \pi_\phi = 0, \quad p_0^\mu = 0; \]

\[ (p_1^\mu)^2 = 0, \quad \partial_1 p_1^\mu = 0, \quad (\dot{p}_1^\mu + \Pi_1^\mu)^2 = 0, \]

\[ L_\alpha = 0, \quad \bar{\theta} \Gamma^\mu p_{1\mu} = 0, \]

\[ p_{\psi\alpha} = 0, \quad S_\alpha \equiv \bar{\psi} \Gamma^\mu p_{1\mu} + (\bar{\theta} \Gamma^\mu)_{\alpha} D_\mu = 0. \]

where

\[ D_\mu \equiv \xi(\dot{p}_1^\mu + \Pi_1^\mu) - \partial_1 p_\mu, \quad \xi \equiv \frac{\partial_1 \dot{p}_1^\mu p_{1\mu}}{(\dot{p}_1^\nu + \Pi_1^\nu) p_{1\nu}}. \quad (16) \]

Besides, some of the Lagrange multipliers have been determined in the process

\[ \bar{\lambda}_\theta = \partial_1 \bar{\theta} + \frac{\xi}{2} \bar{\theta}, \quad \lambda_1^\mu = \partial_1 A_0^\mu + \frac{2}{\phi} A_1^\mu + i\bar{\psi} \Gamma^\mu \theta. \quad (17) \]

\(^1\)Note that the constraints \((\pi_g)_{ab} = 0\) are not separated from the \( S_\alpha \) in Eq. (12). An appropriate modification is

\[ (\bar{\pi}_g)_{ab} \equiv (\pi_g)_{ab} + \frac{1}{2(\dot{p}_1^\nu + \Pi_1^\nu)p_1} (p_{\psi} \Gamma^\mu \Gamma^\nu \theta)(\dot{p}_1^\mu + \Pi_1^\mu)T^\nu_{ab}, \]

where the coefficients \( T^\nu_{ab} \) can be extracted from the equality \( \{ (\pi_g)_{ab}, S_\alpha \} = T^\nu_{ab} (\bar{\theta} \Gamma^\mu)_{\alpha} \).
It is interesting to note that the constraints $S_\alpha = 0$ appear as tertiary ones in the Dirac–Bergmann algorithm. It is worth mentioning also that the fermionic constraints $L_\alpha = 0$ obey the algebra (3), and being considered on their own (without making use the constraints $\bar{\theta}\Gamma^\mu p_{1\mu} = 0$) form a system which has no definite class (it corresponds to the absence of the $\kappa$-symmetry in the GS action written in eleven dimensions).

To go further, let us impose gauge fixing conditions to the first class constraints (14). The choice consistent with the equations of motion is

$$\phi = 2, \quad A_0^\mu = -i \int_0^\sigma d\sigma' \bar{\psi}\Gamma^\mu \theta.$$  \hspace{1cm} (18)

After that, dynamics for the remaining variables is governed by

$$\partial_0 \psi^\alpha = \lambda^\alpha, \quad \partial_0 p_{\psi\alpha} = 0,$$

$$p_{\psi\alpha} = 0, \quad S_\alpha = 0;$$  \hspace{1cm} (19.a)

$$\partial_0 A_1^\mu = p_1^\mu, \quad \partial_0 \bar{p}_1^\mu = 0,$$

$$(p_1^\mu)^2 = 0, \quad \partial_1 \bar{p}_1^\mu = 0;$$  \hspace{1cm} (19.b)

$$\partial_0 x^\mu = -p^\mu, \quad \partial_0 p^\mu = -\partial_1 \partial_1 x^\mu,$$

$$(\hat{p}^\mu \pm \Pi_1^\mu)^2 = 0;$$  \hspace{1cm} (19.c)

$$\partial_0 \theta = -\partial_1 \theta - \frac{\xi}{2} \theta,$$

$$L_\alpha = 0, \quad (\bar{\theta}\Gamma^\mu)_\alpha p_{1\mu} = 0.$$  \hspace{1cm} (19.d)

The sector (19.a) includes $32 + 16$ independent constraints from which the first class ones can be picked out as follows:

$$(p_{\psi\Gamma^\mu})_\alpha p_{1\mu} = 0.$$  \hspace{1cm} (20)

Let us impose the following covariant (and redundant) gauge fixing conditions to Eq. (20)

$$S^1_\alpha \equiv \frac{1}{(\hat{p} + \Pi_1)p_1} \bar{\psi}\Gamma^\mu (\hat{p}_\mu + \Pi_1^\mu) = 0.$$  \hspace{1cm} (21)
Then the set of equations $S_\alpha = 0, S_1^{\alpha} = 0$ is equivalent to
\[\tilde{S} \equiv \bar{\psi} - \frac{1}{2(p + \Pi_1)p_1} \bar{\theta} \Gamma^\mu D_\mu \Gamma^\nu (\dot{p}_\nu + \Pi_{1\nu}), \tag{22}\]
which forms a nondegenerate Poisson bracket with the constraint $p_\psi = 0$
\[\{p_\psi, \tilde{S}_\beta\} = -C_{\alpha\beta}. \tag{23}\]
After transition to the Dirac bracket associated with the second class functions $p_\psi, \tilde{S}_\alpha$, the variables $\psi, p_\psi$ can be dropped.

To get dynamics in the final form, we pass to the light-cone coordinates $x^\mu \to (x^+, x^-, x^i), i = 1, 2, \ldots, 8, 10, \theta^\alpha \to (\theta_a, \bar{\theta}^\dot{a}, \theta_a^\prime, \bar{\theta}^\dot{a}'), a, \dot{a} = 1, \ldots, 8$
and impose the gauge fixing conditions
\[A_1^- = \tau, \quad A_1^+ = A_1^i = 0, \quad x^+ = P^+ \tau, \quad p^+ = -P^+ = \text{const} \tag{24}\]
to the remaining first class constraints from Eqs. (19.b), (19.c). Then it is easy to show that $32 + 16$ constraints $L_\alpha = 0, \bar{\theta} \Gamma^\mu p_1^\mu = 0$ are second class. One gets also that $p_1^- = 1, p_1^+ = p_1^i = 0$, while the equation $\bar{\theta} \Gamma^\mu p_1^\mu = 0$ acquires the form $\Gamma^+ \theta = 0$. The solution is $\theta^\alpha = (\theta_a, 0, 0, \bar{\theta}^\dot{a})$, with $\theta_a$ and $\bar{\theta}^\dot{a}$ the $SO(8)$ spinors of opposite chirality. Note that the condition $p_1^- = 1$ is consistent with the closed string boundary conditions only. It is worth mentioning also that in the gauge chosen the relation $(\dot{p}^\mu + \Pi_1^\mu)p_1^\mu \neq 0$ holds, which correlates with the assumption made above in Eqs. (16), (21). For the remaining variables one gets free equations
\[\partial_0 x^i = -p^i, \quad \partial_0 p^i = -\partial_1 \partial_1 x^i; \tag{25}\]
\[(\partial_0 + \partial_1)\theta_a = 0, \quad (\partial_0 + \partial_1)\bar{\theta}^\dot{a} = 0\]
which look similar to those of $D = 10$ GS superstring. Moreover, $\theta_a$ and $\bar{\theta}^\dot{a}$ form two pairs of selfconjugate variables under the Dirac bracket, associated with the constraints (15)
\[\{\theta_a, \theta_b\} = \frac{i}{\sqrt{8P^+}} \delta_{ab}, \quad \{\bar{\theta}^\dot{a}, \bar{\theta}^\dot{b}\} = \frac{i}{\sqrt{8P^+}} \delta_{\dot{a}\dot{b}}. \tag{26}\]
It is interesting to note that omitting the Wess–Zumino term in Eq. (7) one arrives at the theory which possesses all the properties of the model (7) with the only modification in Eq. (25): $(\partial_0 - c\partial_1)\theta = 0$, with $c$ a constant. Depending on the gauge chosen it can take any value except $c = \pm 1$. So the dynamics is not manifestly $d = 2$ Poincaré covariant, provided that $\theta$ is a $d = 2$ scalar. It is the Wess–Zumino term which corrects this inconsistency.

4 Off-shell realization of the $D = 11$ super-Poincaré algebra

It is convenient first to recall the situation for $D = 10$ GS superstring. Off-shell realization of the super Poincaré algebra for that case includes the Poincaré transformations accompanied with the supersymmetries

$$
\delta \theta^\alpha = \epsilon^\alpha, \quad \delta x^\mu = -i\bar{\theta}\Gamma^\mu \epsilon. \tag{27}
$$

Being considered on their own, these transformations in the gauge $\Gamma^+\theta = 0$ are reduced to trivial shifts in the sector of physical variables

$$
\delta \bar{\theta}_a = \bar{\epsilon}_a, \quad \delta x^i = 0. \tag{28}
$$

To get on-shell realization of the supersymmetry algebra, one needs to consider a combination of the $\epsilon$- and $\kappa$-transformations $\delta_\epsilon + \delta_{\kappa(\epsilon)}$, which does not spoil the gauge $\Gamma^+\theta = 0$. These transformations are (see, for example, Ref. 19)

$$
\delta \bar{\theta}_a = \bar{\epsilon}_a + \frac{1}{P_+} \partial_- x^i \tilde{\gamma}_i \tilde{\alpha}\alpha \epsilon_a, \quad \delta x^i = -i\sqrt{2}(\bar{\theta}\tilde{\gamma}^i \epsilon). \tag{29}
$$

We turn now to the $D = 11$ case. Off-shell realization of the super Poincaré algebra for the action (7) includes the Poincaré transformations...
in the standard realization and the following supersymmetries with 32-component spinor parameter $\epsilon^a$

\[
\delta \theta = \tilde{\Lambda} \epsilon, \quad \delta x^\mu = -i \bar{\theta} \Gamma^\mu \tilde{\Lambda} \epsilon, \\
\delta A_\mu^a = -2i \epsilon_{ab} \frac{g^{bc}}{\sqrt{-g}} (\bar{\theta} \bar{\Pi}_c \Gamma^\mu \epsilon) - 2i \partial_a x^\nu (\bar{\theta} \Gamma^\nu \Gamma^\mu \epsilon) - 2(\bar{\theta} \epsilon)(\bar{\theta} \Gamma^\mu \partial_a \theta), \\
\delta \bar{\psi} = i \epsilon^{ab} [\bar{\epsilon} \Gamma^\mu (\partial_a \bar{\theta} \Gamma^\mu \partial_b \theta) - 2 \partial_a \bar{\theta} (\partial_b \bar{\theta} \epsilon)], \\
\delta \phi = -i \phi^2 (\bar{\psi} \epsilon),
\]

(30)

where $\tilde{\Lambda} \equiv \Lambda_\mu \Gamma^\mu, \bar{\Pi}_c = \Pi^c \mu \Gamma^\mu$. The action is invariant up to total derivative terms. These transformations are the analog of Eq. (27), since in the physical sector they are reduced to $\delta \theta_a = \sqrt{2} \epsilon_a, \delta \bar{\theta}_a = -\sqrt{2} \epsilon'_a, \delta x^i = 0$.

Global supersymmetries of the action (7), corresponding to Eq. (29) can also be presented. To find them, let us consider the following ansatz:

\[
\delta \theta = \tilde{\Lambda} \bar{\Pi}_c \epsilon^c, \quad \delta \phi = -i \phi^2 (\bar{\psi} \epsilon^c), \\
\delta x^\mu = 4i (\Lambda \Pi_c) (\bar{\theta} \Gamma^\mu \epsilon^c) + 2i (\bar{\theta} \bar{\Pi}_c \epsilon^c) \Lambda^\mu,
\]

(31)

where we denoted

\[
\epsilon^a_\alpha \equiv P^{-ab} \epsilon_{\alpha b}, \quad P^{-ab} = \frac{1}{2} (\frac{g^{ab}}{\sqrt{-g}} - \epsilon^{ab}), \quad (\Lambda \Pi_c) \equiv \Lambda^\mu \Pi^c \mu.
\]

(32)

Variation of the GS part of the action (7) under these transformations looks like

\[
\delta S_{GS} = \epsilon^{ab} [-8 (\bar{\theta} \Gamma^\mu \epsilon^c) (\partial_a \bar{\theta} \Gamma^\mu \Gamma^\mu \partial_b \theta) (\Lambda \Pi_c) - 4 (\bar{\theta} \bar{\Pi}_c \epsilon^c) (\partial_a \bar{\theta} \bar{\Lambda} \partial_b \theta) + \\
+2 (\partial_a \bar{\theta} \Gamma^\mu \bar{\Lambda} \bar{\Pi}_c \epsilon^c) (\bar{\theta} \Gamma^\mu \partial_b \theta) + (\bar{\theta} \Gamma^\mu \Lambda \bar{\Pi}_c \epsilon^c) (\partial_a \bar{\theta} \Gamma^\mu \partial_b \theta)] - \\
-2i P^{-ba} [4 (\bar{\theta} \bar{\Pi}_c \epsilon^c) (\partial_a \Lambda \Pi_b) + 2 (\partial_a \bar{\theta} \Lambda \epsilon^c) (\Pi_b \pi_c) - (\bar{\theta} \Lambda \partial_a \bar{\Pi}_b \bar{\Pi}_c \epsilon^c)].
\]

(33)

After integration by parts, reordering the $\tilde{\Lambda}$ and $\bar{\Pi}$ terms and making use of the identities

\[
P^{-ab} P^{-cd} = P^{-cb} P^{-ad}, \\
(\partial_a \bar{\theta} \Gamma^\mu \Gamma^\mu \partial_b \theta) (\Lambda \Pi_c) = -\frac{1}{2} \partial_a \bar{\theta} \Gamma^\mu \{\tilde{\Lambda}, \bar{\Pi}_c\} \partial_b \theta,
\]

(34)
one can present all the terms in Eq. (33) as either $K \tilde{\Lambda} \theta$ or $\partial_a \Lambda^\mu T^{\mu a}$, with $K$ and $T$ some coefficients. These terms can evidently be cancelled by appropriate variations of the $\bar{\psi}$ and $A_\mu^a$ variables in the action. The final form for those variations is

$$
\delta A_\mu^a = 8(\bar{\theta} \Gamma^\rho \epsilon^c)(\bar{\theta} \Gamma^{\mu \nu} \Pi_c \Gamma^{\nu} \partial_\mu \theta) - 5(\bar{\theta} \Pi_c \epsilon^c)(\bar{\theta} \Gamma^\mu \partial_\mu \theta) - 3(\bar{\theta} \Gamma^\mu \Pi_c \epsilon^c)(\bar{\theta} \Gamma^{\nu} \partial_\nu \theta) - 4i \xi_{ad} P^{-bd}[(\bar{\theta} \Gamma^\mu \epsilon^c)(\Pi_b \Pi_c) - 2(\bar{\theta} \Pi_c \epsilon^c)]
$$

Note that the complicated transformation law for the $\psi$-variable can be predicted, since one of the Lagrangian equations of motion is

$$(\tilde{\Lambda} \psi)_\alpha = -4 \Pi_b P^{-ba} \partial_\alpha \theta + i \xi^{ab} \theta^\beta \partial_\alpha \theta^\gamma \partial_b \theta^\delta \Gamma^\mu_\alpha_\beta (C \Gamma^\mu)_{\gamma \delta} = 0.
$$

Thus, transformation of the $\tilde{\Lambda} \psi$ part of the $\psi$-variable is dictated by this equation and the transformation lows for $x$ and $\theta$ variables.

Being reduced to the physical sector, Eq. (31) look as follows:

$$
\delta \theta_a = -\sqrt{2}(P^+ \epsilon_a - \partial_- x^i \gamma^i_{a \bar{a}} \epsilon^\prime_{a \bar{a}} + \partial_- x^{10} \epsilon^\prime_a),
\delta \bar{\theta}_a = -\sqrt{2}(P^+ \bar{\epsilon}_{\bar{a}} + \partial_- x^i \gamma^i_{\bar{a} a} \epsilon^\prime_a - \partial_- x^{10} \epsilon^\prime_{\bar{a}}),
\delta x^i = 2\sqrt{2}i P^+ (\partial_- x^{10} \epsilon^\prime_{a} - \bar{\theta} \gamma^i \epsilon^\prime_a)
$$

and seems to be an analog of Eqs. (29). Note that these transformations act on the left moving modes only, in contrast to the eleven dimensional superstring considered in Ref. 20. In this respect, the model presented can be viewed as a $D = 11$ analog of $N = 1$, $D = 10$ Green–Schwarz superstring.

To summarize, in this letter we suggested a super Poincaré invariant action for the closed superstring which classically exists in any spacetime
dimension. From Eq. (26) it follows that zero modes of the $\theta_a$, $\bar{\theta}_a$ variables form the Clifford algebra which is also symmetry algebra of a ground state. A representation space is 256 dimensional and corresponds to the spectrum of the $D = 11$ supergravity [19]. Since supersymmetry is realized in the physical subspace, one also gets the corresponding representation in the space of functions on that superspace. This allows one to expect a supersymmetric quantum states spectrum. Analysis of this situation in terms of oscillator variables as well as the critical dimension will be discussed in a separate publication.

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