Improved Calculation of Electroweak Radiative Corrections and the Value of $V_{ud}$

William J. Marciano $^1$ and Alberto Sirlin $^2$

$^1$ Brookhaven National Laboratory, Upton, NY 11973
$^2$ New York University, Department of Physics, 4 Washington Place, New York, NY 10003

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Abstract

A new method for computing hadronic effects on electroweak radiative corrections to low-energy weak interaction semileptonic processes is described. It employs high order perturbative QCD results originally derived for the Bjorken sum rule along with a large N QCD-motivated interpolating function that matches long and short-distance loop contributions. Applying this approach to the extraction of the CKM matrix element $V_{ud}$ from superallowed nuclear beta decays reduces the theoretical loop uncertainty by about a factor of 2 and gives $V_{ud} = 0.97377(11)(15)(19)$. Implications for CKM unitarity are briefly discussed.
Precision studies of low-energy semileptonic weak-charged and neutral current processes can be used to test the $SU(3)_c \times SU(2)_L \times U(1)_Y$ Standard Model at the quantum loop level and probe for potential “new physics” effects. Examples for which a fraction of a percent experimental sensitivity has already been achieved include: pion, neutron and nuclear beta decays \cite{1}, as well as atomic parity violation \cite{2}. In those cases, electroweak radiative corrections (RC) have been computed \cite{3,4,5} and found to be significant (of order several percent). They must be included in any meaningful confrontation between theory and experiment.

Of course, inherent to any low-energy semileptonic process are uncertainties due to strong interactions, since quarks are involved. To minimize such effects, one often focuses on weak vector current-induced reactions, where CVC (conserved vector current) protects those amplitudes at tree level from strong interaction corrections in the limit of zero momentum transfer. However, even for those amplitudes, electroweak loop corrections can involve weak axial-vector effects not protected by CVC, which give rise to hadronic (strong interaction) uncertainties in their evaluation \cite{3,4}. In this paper, we focus on the best known and tested examples of that phenomenon, the electroweak radiative corrections to neutron and correspondingly superallowed nuclear beta decays along with their implications for the extraction of the CKM matrix element $V_{ud}$. However, the method we describe is quite general and can be easily applied to other charged and neutral current semileptonic low-energy reactions.

The extraction of $V_{ud}$ (in fact all CKM matrix elements) entails normalizing a semileptonic reaction rate with respect to the muon lifetime, or equivalently the Fermi constant derived from it

$$G_\mu = 1.16637(1) \times 10^{-5} GeV^{-2}$$

(1)

For high precision, electroweak radiative corrections to both processes must be included and hadronic as well as environmental effects (e.g., nuclear structure) must be controlled. Toward that end, super-allowed ($0^+ \rightarrow 0^+$) nuclear beta decay transitions are very special since they only involve the weak vector current at tree level. Small violations of CVC due to the up-down mass difference or non-zero momentum transfer are small $\sim O(10^{-5})$ and can generally be neglected (or incorporated). Such an analysis leads to the very accurate relationship \cite{6,7}

$$|V_{ud}|^2 = \frac{2984.48(5) \text{sec}}{ft(1 + RC)} \quad \text{(Superallowed } \beta - \text{decays)}$$

(2)
where \( ft \) is the product of a phase space statistical decay rate factor \( f \) (which depends on the Q value of a specific nuclear beta decay) and its measured half-life \( t \). RC designates the total effect of all radiative corrections relative to muon decay as well as QED-induced nuclear structure isospin violating effects. It is nucleus dependent, ranging from about +3.1% to +3.6% for the nine best-measured super-allowed decays. So, measuring \( Q \) and \( t \) combined with computing RC determines \( V_{ud} \). A similar formula will be given later for neutron beta decay. In that case, the \( Q \) value = \( m_n - m_p \) is very precisely known, but in addition to the neutron lifetime, \( g_A \equiv G_A/G_V \) must be accurately measured because both weak axial and vector currents contribute at tree level [1, 6].

Our main goal in this paper is to reduce the hadronic uncertainty in the radiative corrections to super-allowed nuclear beta decays and thereby improve the determination of \( V_{ud} \). The need for such an improvement is well illustrated by a survey of \( ft \) values and RC for super-allowed beta decays by Hardy and Towner [7], more recently updated by G. Savard et al. [8], which found

\[
V_{ud} = 0.9736(2)(4)_{EW}
\]

where the first uncertainty stems primarily from nuclear structure corrections (including \( \mathcal{O}(Z^2\alpha^3) \) effects) and very small \( ft \) value errors while the second, dominant error is due to hadronic uncertainties in electroweak loop effects. Although, as we mention later, the first error may currently be an underestimate and the central value of \( V_{ud} \) could shift due to future \( Q \) value updates, it is clear that the hadronic loop uncertainty, which comes from weak axial-current loop effects, currently limits the determination of \( V_{ud} \) and must be improved if further progress is to be made.

Here, we describe a new method for controlling hadronic uncertainties in the radiative corrections to neutron and super-allowed nuclear beta decays. It validates our previous results [4, 6] increasing \( V_{ud} \) by only a small +0.00007, but reduces the loop uncertainty by about a factor of 2, \( (0.0004)_{EW} \rightarrow (0.0002)_{EW} \) as we now demonstrate.

The one-loop electroweak radiative corrections to the neutron (vector current contribution) and super-allowed nuclear beta decays are given by [3, 4, 9]

\[
RC_{EW} = \frac{\alpha}{2\pi} \left\{ \tilde{\gamma}(E_m) + 3\ln \frac{m_Z}{m_p} + \ln \frac{m_Z}{m_A} + A_g + 2C_{Born} \right\}
\]

The first two terms result from loop corrections and bremsstrahlung involving electromagnetic and weak vector current interactions, with \( \tilde{\gamma}(E_m) \) a universal function [9] integrated
over phase space and $3ln \frac{m_Z}{m_p}$ a short-distance loop effect. They are not affected by strong interactions up to $O(\frac{E}{m_p}) \simeq 10^{-5}$ corrections which can be neglected at our present level of accuracy. Higher order leading logs of order $\alpha^n ln^n(m_Z/m_p)$ etc. can be summed via a renormalization group analysis [4] and $O(Z\alpha^2)$ as well as $O(Z^2\alpha^3)$ contributions have been computed for high Z nuclei [10]. They will not be explicitly discussed here, but are included in our final results.

The last three terms in eq. (4) are induced by weak axial-vector current loop effects. Their primary source is the $\gamma W$ box diagram which involves the time-ordered product of the electromagnetic and weak axial-vector currents. That product contains a leading vector current component which contributes to $0^+ \rightarrow 0^+$ nuclear transition elements. Employing the current algebra formulation, one finds [3]

$$Box(\gamma W)_{VA} = \frac{\alpha}{8\pi} \int_0^\infty dQ^2 \frac{m_W^2}{Q^2 + m_W^2} F(Q^2)$$  \hspace{1cm} (5)

where Q is a Euclidean loop momentum integration variable.

Previous estimates of eq. (5) employed the operator product expansion plus lowest order QCD correction to obtain the leading effect [3, 4]

$$F(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{1}{Q^2} \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} \right] + O(\frac{1}{Q^4})$$  \hspace{1cm} (6)

Integrating over the range $m_A^2 \leq Q^2 < \infty$ and combining with smaller vertex corrections and ZW box diagrams, that prescription gave a short-distance amplitude contribution

$$\frac{\alpha}{4\pi} \left[ ln \frac{m_Z}{m_A} + A_g \right], \ A_g \simeq -0.34$$  \hspace{1cm} (7)

In the numerical estimate, the low energy cutoff was chosen to be $m_A = 1.2 GeV$, roughly the mass of the $A_1$ resonance, and the error was estimated by allowing $m_A$ to vary up or down by a factor of 2. Such a heuristic, albeit crude procedure led to a $\pm 0.0004$ uncertainty in $V_{ud}$. For the long-distance $\gamma W$ box diagram contribution, nucleon electromagnetic and axial-vector dipole form factors were used to find for neutron decay [4, 5]

$$C_{Born}(\text{neutron}) \simeq 0.8g_A(\mu_n + \mu_p) \simeq 0.89$$  \hspace{1cm} (8)

where $g_A \simeq 1.27$ and $\mu_n + \mu_p = 0.88$ is the nucleon isoscalar magnetic moment. In the case of superallowed nuclear decays, nuclear quenching modifies $C_{Born}(\text{neutron})$ and nucleon-nucleon electromagnetic effects must be included [11]. Overall, in the case of a neutron,
axial-vector-induced one-loop RC to the decay rate amount to 0.67(8)%. Roughly the same uncertainty ±0.08% applies to superallowed nuclear decays.

To reduce the hadronic uncertainty in RC, we have carried out a new analysis of the $\gamma W$ box diagram axial-vector-induced radiative corrections that incorporates the following $F(Q^2)$ improvements [12]:

1) Short Distances $(1.5 GeV)^2 \leq Q^2 < \infty$, a domain where QCD corrections remain perturbative.

$$ F(Q^2) = \frac{1}{Q^2} \left[ 1 - \frac{\alpha_s(Q^2)_{\overline{MS}}}{\pi} - C_2 \left( \frac{\alpha_s(Q^2)_{\overline{MS}}}{\pi} \right)^2 - C_3 \left( \frac{\alpha_s(Q^2)_{\overline{MS}}}{\pi} \right)^3 \right] $$

(9)

$$ C_2 = 4.583 - 0.333 N_F $$

(10)

$$ C_3 = 41.440 - 7.607 N_F + 0.177 N_F^2 $$

(11)

where $N_F$ = number of effective quark flavors.

2) Intermediate Distances $((0.823 GeV)^2 \leq Q^2 < (1.5 GeV)^2)$

$$ F(Q^2) = \frac{-1.490}{Q^2 + m_\rho^2} + \frac{6.855}{Q^2 + m_A^2} - \frac{4.414}{Q^2 + m_\rho' ^2} $$

(12)

$$ m_\rho = 0.776 GeV $$

(13)

$$ m_A = 1.230 GeV $$

(14)

$$ m_\rho' = 1.465 GeV $$

(15)

3) Long Distances: $0 \leq Q^2 \leq (0.823 GeV)^2$

Integrating the long-distance amplitude up to $Q^2 = (0.823 GeV)^2$, where the integrand matches the interpolating function, and using an update of the nucleon electromagnetic and axial-current dipole form factors, we find

$$ C_{\text{Born}}(\text{neutron}) \simeq 0.829 $$

(16)

a reduction from our own previous result in eq. (8), where the integration was carried up to $Q^2 = \infty$. 

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Details of the above calculations will be given in a subsequent publication \[12\]. Here, we briefly discuss the results of the above analysis and its implications.

The QCD corrections to the asymptotic form of $F(Q^2)$ have been given in eq. (9) to $O(\alpha_s^3)$. The additional terms are identical (in the chiral limit) to QCD corrections to the Bjorken sum rule \[13\] for polarized electroproduction and can be read off from well-studied calculations \[14, 15\] for that process. Their validity has been well tested experimentally \[16\].

The interpolating function in eq. (12) is motivated by large $N$ QCD which predicts it should correspond to an infinite sum of vector and axial-vector resonances \[17\]. We impose three conditions that determine the residues: i) The integral of eqs. (5) and (12) should equal that of eqs. (5) and (9) in the asymptotic domain $(1.5 GeV)^2 \leq Q^2 \leq \infty$, which amounts to a matching requirement between domains 1 and 2, ii) In the large $Q^2$ limit, the coefficient of the $1/Q^4$ term in the expansion of eq. (12) should vanish as required by chiral symmetry \[18\], iii) The interpolator should vanish at $Q^2 = 0$ as required by chiral perturbation theory. Three conditions limit us to three resonances.

The $Q^2 = (0.823 GeV)^2$ match between domains 2 and 3 was chosen to be the value at which eq. (12) equals the integrand of the long-distance contribution. Interestingly, that matching occurs near the $\rho$ mass. A novel technical point in the formulation is that in the evaluation of the Feynman diagrams associated with the long-distance contributions the integral over the auxiliary variables is carried out first. This leads to integrands that depend on $Q^2$ and can therefore be matched with eq. (12).

Using this approach, we find that at the one-loop electroweak level the last three terms in eq. (4) are effectively replaced by $2.82 \frac{\alpha}{\pi}$ in the case of neutron decay. Comparison with eqs. (4) and (8) in conjunction with $m_A = 1.2 GeV$, $A_g = -0.34$ shows that in the new formulation these corrections are reduced by $1.4 \times 10^{-4}$, which increases $V_{ud}$ by $7 \times 10^{-5}$. The smallness of that shift is a validation of our previous result \[4, 6\].

More important than the small reduction in the radiative corrections, our new method provides a more systematic estimate of the hadronic uncertainties as well as experimental verification of its validity \[16\]. Allowing for a $\pm 10\%$ uncertainty for the $C_{Born}$ correction in eq. (16), a $\pm 100\%$ uncertainty for the interpolator contribution in the $(0.823 GeV)^2 \leq Q^2 < (1.5 GeV)^2$ region and $\pm 0.0001$ uncertainty from neglected higher order effects, we find the total uncertainty in the electroweak radiative corrections is $\simeq \pm 0.00038$ which leads to $\simeq a \pm 0.00019$ uncertainty in $V_{ud}$. That corresponds to more than a factor of 2 reduction in the
loop uncertainty from hadronic effects.

Employing our new analysis, we find the improved relationship between $V_{ud}$, the neutron lifetime and $g_A \equiv G_V/G_A$

$$|V_{ud}|^2 = \frac{4908.7(1.9)\text{sec}}{\tau_n(1 + 3g_A^2)} \text{ (neutron)}$$  \hspace{1cm} (17)

Future precision measurements of $\tau_n$ and $g_A$ used in conjunction with eq. (17) will ultimately be the best way to determine $V_{ud}$, but for now it is not competitive.

In the case of superallowed ($0^+ \rightarrow 0^+$ transitions) nuclear $\beta$-decays, there are a number of corrections, some nucleus dependent, that must be applied to the ft values. They are collectively called RC in eq. (2). To make contact with previous studies, we factorize them as follows:

$$1 + RC = (1 + \delta_R)(1 - \delta_C)(1 + \Delta)$$  \hspace{1cm} (18)

The first two factors are nucleus dependent while $\Delta$ is roughly nucleus independent, coming primarily from short-distance loop effects. The axial-vector contributions discussed above are included in the product $(1 + \delta_R)(1 + \Delta)$. Because we include leading logs from higher orders as well as some next-to-leading logs, the factorization is not exact and $\Delta$ will exhibit some small nucleus dependence. In Table 1, we give the decomposition of RC along with their uncertainties, using the results from Hardy and Towner for nuclear effects. The uncertainty in $1 + \delta_R$ comes from $Z^2\alpha^3$ and nuclear structure contributions while a common $\pm 0.03\%$ error in the Coulomb distortion effect is assigned to $1 - \delta_C$. The entire loop uncertainty described in this paper is assigned to $1 + \Delta$, even though much of it comes from medium and long-distance effects and might better be attributed to $1 + \delta_R$. We have not made such a distribution, since it is common to all decays.

Employing the corrections in Table 1 together with eqs. (2) and (18) leads to the $V_{ud}$ values illustrated in Table 2. One finds for the weighted average

$$V_{ud} = 0.97377(11)(15)(19) \text{ (Superallowed } \beta - \text{decays)}$$  \hspace{1cm} (19)

Comparing with eq. (3) we see that our analysis gives a somewhat larger $V_{ud}$ due to a $\pm 0.00007$ increase from our new prescription along with refinements from ref. which were not included in Savard et al. Also, Savard et al. rounded down in their analysis.
Table 1. Decomposition of the RC for the nine best-measured superallowed nuclear \( \beta \)-decays. Coulomb corrections in \( 1 - \delta_C \) are taken directly from ref. [1, 7], while \( 1 + \delta_R \) has been somewhat modified due to our new results. The short-distance \( 1 + \Delta \) factor is based on the recent update in ref. [8] which includes higher order leading logs and some next-to-leading logs.

| Nucleus | \( 1 + \delta_R \)       | \( 1 - \delta_C \)     | \( 1 + \Delta \)   |
|---------|--------------------------|-------------------------|---------------------|
| \( ^{10}\text{C} \) | 1.01298(5)(35)           | 0.9983(3)               | 1.02389(38)         |
| \( ^{14}\text{O} \)  | 1.01274(8)(50)           | 0.9976(3)               | 1.02385(38)         |
| \( ^{26}\text{Al} \) | 1.01468(21)(20)          | 0.9971(3)               | 1.02380(38)         |
| \( ^{34}\text{Cl} \)  | 1.01343(34)(15)          | 0.9939(3)               | 1.02379(38)         |
| \( ^{38}\text{K} \)   | 1.01322(41)(15)          | 0.9939(3)               | 1.02378(38)         |
| \( ^{42}\text{Sc} \)  | 1.01469(49)(20)          | 0.9954(3)               | 1.02377(38)         |
| \( ^{46}\text{V} \)   | 1.01392(57)(7)           | 0.9959(3)               | 1.02377(38)         |
| \( ^{50}\text{Mn} \)  | 1.01394(65)(7)           | 0.9957(3)               | 1.02376(38)         |
| \( ^{54}\text{Co} \)  | 1.01398(73)(7)           | 0.9947(3)               | 1.02376(38)         |
Table 2. Values of $V_{ud}$ implied by various precisely measured superallowed nuclear beta decays. The ft values are taken from Savard et al. [8]. Uncertainties in $V_{ud}$ correspond to: 1) nuclear structure and $Z^2\alpha^3$ uncertainties added in quadrature with the ft error [10, 11], 2) a common error assigned to nuclear coulomb distortion effects [11], and 3) a common uncertainty from quantum loop effects. Only the first error is used to obtain the weighted average.

| Nucleus | ft(sec)   | 1+RC     | $V_{ud}$       |
|---------|----------|----------|----------------|
| $^{10}$C | 3039.5(47) | 1.03542(36)(30)(38) | 0.97381(77)(15)(19) |
| $^{14}$O | 3043.3(19) | 1.03441(52)(30)(38) | 0.97368(39)(15)(19) |
| $^{26}$Al | 3036.8(11) | 1.03582(30)(30)(38) | 0.97406(23)(15)(19) |
| $^{34}$Cl | 3050.0(12) | 1.03121(38)(30)(38) | 0.97412(26)(15)(19) |
| $^{38}$K | 3051.1(10) | 1.03099(44)(30)(38) | 0.97404(26)(15)(19) |
| $^{42}$Sc | 3046.8(12) | 1.03403(54)(30)(38) | 0.97330(32)(15)(19) |
| $^{46}$V | 3050.7(12) | 1.03376(59)(30)(38) | 0.97280(34)(15)(19) |
| $^{50}$Mn | 3045.8(16) | 1.03357(67)(30)(38) | 0.97367(41)(15)(19) |
| $^{54}$Co | 3048.4(11) | 1.03257(75)(30)(38) | 0.97373(40)(15)(19) |

Weighted Average 0.97377(11)(15)(19)
We note that $^{46}\text{V}$ gives a somewhat low value for $V_{ud}$. It differs from the average by 2.7 sigma. That particular nucleus recently underwent a Q value revision which lowered its $V_{ud}$. It may be indicating problems with other Q values. If the other nuclear Q values follow the lead of $^{46}\text{V}$, we could see a fairly significant reduction in the weighted average for $V_{ud}$. Clearly, remeasurements of Q values and half-lives of the superallowed decays are highly warranted.

Employing the value of $V_{ud}$ in eq. (19) and the $K_{l3}$ average for $V_{us}$

$$V_{us} = 0.2257(9)(0.961/f_+(0)), \text{ } K_{l3} \text{ average} \quad (20)$$

with $f_+(0) = 0.961(8)$ leads to the unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992(5)V_{ud}(4)\nu_{us}(8)f_+(0) \quad (21)$$

Good agreement with unitarity is found, with the dominant uncertainty coming now from the theory error in the form factor $f_+(0)$. Eq. (21) provides an important test of the standard model at the quantum loop level and a constraint on new physics beyond the standard model at the ±0.09% level. We note, however, that some other calculations of $f_+(0)$ and studies of other strangeness changing decays suggest a lower $V_{us}$ value. Combined with further Q value revisions possibly leading to a smaller $V_{ud}$, they could cause a significant reduction in eq. (21). A future violation of unitarity is still possible. However, for it to be significant, the theoretical uncertainty in $f_+(0)$ must be further reduced.

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