Magnetized Quark-Gluon Plasma at the LHC

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Abstract—In QCD, the strengths of the large scale temperature dependent chromomagnetic, $B_s$, and usual magnetic, $H$, fields spontaneously generated in quark-gluon plasma after the deconfinement phase transition (DPT), are estimated. The consistent at high temperature effective potential accounting for the one-loop plus daisy diagrams is used. The heavy ion collisions at the LHC and temperatures $T$ not much higher than the phase transition temperature $T_d$ are considered. The critical temperature for the magnetized plasma is found to be $T_d(H) \sim 110-120$ MeV. This is essentially lower compared to the zero field value $T_d(H = 0) \sim 160-180$ MeV usually discussed in the literature. Due to contribution of quarks, the color magnetic fields act as the sources generating $H$. The strengths of the fields are $B_s(T), B_s(T) \sim 10^{18-10^{19}}$ G, $H(T) \sim 10^{16-10^{17}}$ G for temperatures $T \sim 160-220$ MeV. At temperatures $T < 110-120$ MeV the effective potential minimum value being negative approaches to zero. This is signaling the absence of the background fields and color confinement.

INTRODUCTION

At the LHC experiments, in heavy ion collisions a new matter phase—quark-gluon plasma (QGP)—has to be produced. The deconfinement phase transition temperature is expected to be of order $T_d \sim 180–200$ MeV. In theory, investigation of the DPT and QGP properties were carried out by different method—analytic perturbative and nonperturbative, various numerical methods and Monte-Carlo simulations on a lattice (see, for example, [1–6]). QGP and strong magnetic fields had been existed in the hot Universe [8, 9].

One of distinguishable properties of nonabelian gauge fields at high temperature is a spontaneous vacuum magnetization. It is closely related with asymptotic freedom. In fact, asymptotic freedom at high temperature is always accompanied by the background stable, temperature dependent and long range chromo(magnetic) fields [11]. The magnetization phenomenon was investigated in detail in $SU(3)$ gluodynamics [10] and supersymmetric theories [13, 14] by analytic methods and in $SU(2)$ gluodynamics [16], [18] by the Monte-Carlo simulations on a lattice. In all these cases the spontaneous creation of magnetic fields has been detected. Within application to the early Universe the spontaneous vacuum magnetization in the electroweak sector of the standard model is described in review paper [20].

The case of experiments at the LHC requires a special consideration. This is because of much lower temperature $T_d$ as compared to the electroweak phase transition temperature $T_{ew} \sim 100$ GeV. For temperatures $T_d < T < T_{ew}$ the scalar field condensate, supplying particle masses, screens the magnetic field $H$, which was generated at high temperatures by the $W$ boson loops. At the same time, the color magnetic fields $B_s, B_s$ remain unscreened [20]. Within this scenario the question arises: whether or not there exists a mechanism generating magnetic field $H$ in between critical temperatures $T_d$ and $T_{ew}$?

In a qualitative manner it was considered in our paper [21]. Therein, in particular, we have demonstrated that magnetic field $H$ can be generated due to the vacuum polarization of quark fields by the constant color magnetic fields $B_s$ and $B_s$, existed in the QGP after the DPT. In the effective potential of the external fields the mixing terms of the type $- eH \times (gB_3)^3, - eH \times (gB_3)^3, \text{etc}$, where $e, g$ are electromagnetic and strong interaction couplings, present and act as the sources for $H$. The field $H$ is temperature dependent and occupying a large plasma volume as the fields $B_3$ and $B_3$.

In the present paper, we investigate in detail the creation in QGP of the magnetic fields $B_s, B_s, H$ at temperatures close to the DPT and estimate the field.
strengths. The proper time representation is used. The
one-loop plus daisy diagrams effective potential of
external fields $V(B, B_s, H, T)$ accounting for the
 gluons and $u$, $d$- and $s$-quarks at finite temperature is
calculated. This field configuration is stable due to the
diagram contributions which cancel the imaginary
terms presenting in the one-loop effective potential
of charged gluons $V^{(1)} (B, B_s, T)$. For estimation of
the field strengths the asymptotic high temperature
expansion, derived by Mellin’s transformation tech-
nique, is applied. As corollary of these investigations
we observe that strong color magnetic fields $B_3, B_8$, of
the order $10^{18}$–$10^{19}$ G and usual magnetic field
$H \sim 10^{16}$–$10^{17}$ G are generated for temperatures
$T \sim 160$–$220$ MeV. The spontaneous magnetization
disappears at $T \sim 110$–$120$ MeV. This temperatures is
considered as the deconfinement temperature in the
presence of the fields. It is essentially lower that the
one estimated without magnetic fields.

The paper is organized as follows. In next section
we adduce the one-loop effective potential of quarks $V^{(1)} (B, B, H, T)$. In sect. 3 we present the one-loop
contributions of gluons $V^{(1)}_{gl}(B, B_s, T)$ calculated in the
high temperature approximation, which is sufficient for the problem under consideration. In sect. 4 the
contribution of daisy diagrams is calculated in brief
and the dimensionless variables used in numeric cal-
culations are introduced. Then in sect. 5 the values of
the field strengths $B_3, B_8(T), H(T)$ are estimated for
a number of temperatures. Discussion of the results
and conclusion are given in the last section. Appendix
A describes the details of calculations of the quark zero
temperature effective potential. Appendix B includes
information about the high temperature expansion of
the one-loop effective potentials $V^{(0)}_q, V^{(0)}_{gl}$.

1. QUARK CONTRIBUTIONS
   TO ONE-LOOP EFFECTIVE POTENTIAL

In what follows, we consider the situation when
temperature of $QGP$ is not much higher than $T_d$. In
this case, according to [10], the color magnetic fields
$B_3$ and $B_8$ are spontaneously created in the gluon sec-
or of QCD because color symmetry is restored. On
the contrary, for the temperature interval $T_q < T < T_w$
the electroweak symmetry is broken and $SU(2)$ con-
stituent of usual magnetic field is screened by the scalar
field condensate. At temperatures $T > T_w$ the
spontaneous generation of this field takes place also
[20]. Having this picture in mind we calculate the one-
loop quark effective potential $V^{(1)}_q(B, B, H, T)$ at the
background of all three fields.

To be in correspondence with the notations of
[10, 21] we present the $SU(3)$, gluon field in the form

$$A^a_\mu = B^a_\mu + Q^a_\mu,$$  \hspace{1cm} (1)

where $B^a_\mu$ is background classical field and $Q^a_\mu$ presents
quantum gluons. We choose the external field potential
in the form $B^a_\mu = \delta_3^{a3} B^3_\mu + \delta_{as} B_\mu^s$, where $B_{3\mu} = H_3 \delta_{\mu2} x^1$
and $B_{8\mu} = H_8 \delta_{\mu2} x^1$ describe constant chromomagnetic
fields directed along third axis in the Euclidean space
and $a = 3$ and $a = 8$ in the color $SU(3)$, space, respec-
tively. The field tensor has the components:

$$F^{ext \mu\nu} = \delta^{a3} F_{3\mu\nu} + \delta_{as} F_{8\mu\nu}, \quad F_{c12} = - F_{c21} = H_c, \quad c = 3, 8.$$  

We direct usual magnetic field also along third axis
and choice its potential in the form: $A^{ext}_\mu = H \delta_{\mu2} x^1.$

We first calculate the quark spectrum in the presence
of all these fields [21]. The corresponding Dirac
equation reads

$$i \gamma^\mu D_\mu + m_f \psi^a = 0,$$  \hspace{1cm} (2)

where $\psi^a$ is a quark wave function, $a$ is color index, $m_f$
is mass of $f$-flavor quark. The covariant derivative
describes the interactions with external magnetic
fields $H$ and $H_3, H_8$:

$$D_\mu = \partial_\mu + iq_f [\gamma^\mu, A^{ext}_\mu + i g(T^3 B^3_\mu + T^8 B_8^8)],$$  \hspace{1cm} (3)

where $T^3 = \frac{\lambda^3}{2}, \quad T^8 = \frac{\lambda^8}{2}$ are the generators of $SU(3)$
group, $\lambda^3, \lambda^8$ are Gell–Mann matrixes. Due to the
choice of the potentials we can present the quark spectrum
as the sum of contributions of the following external field combinations:

$$\mathcal{H}^1_f = q_f [\gamma^\mu H + g \left( \frac{H_3}{2} + \frac{H_8}{2\sqrt{3}} \right)],$$

$$\mathcal{H}^2_f = q_f [\gamma^\mu H + g \left( \frac{H_3}{2\sqrt{3}} - \frac{H_8}{2} \right)],$$

$$\mathcal{H}^3_f = q_f [\gamma^\mu H - g \frac{H_8}{\sqrt{3}}].$$  \hspace{1cm} (4)

Here, $q_f$ [\gamma^\mu$ is electric charge of $f$-quark. Each flavor
energy spectrum is given by the known expression
(see, for instant, [17]):

$$\epsilon^2_{f,p} = m_f^2 + p_z^2 + (2n + 1) \mathcal{H}^i_f - p \mathcal{H}^f_f,$$  \hspace{1cm} (5)

where $p_z$ is momentum along the field direction, $p = \pm 1$.

Vacuum energy is defined as the sum of the modes
having negative energy. At finite temperature, in the
imaginary time formalism for fermions, it is reduced
to the summation over discrete odd imaginary energies
\[ p_\tau = \frac{(2l + 1)\pi}{\beta}, \quad \beta = 1/T \] is inverse temperature [3], [1]. The result yields [11]

\[ V^{(i)}_q(T, H_i) = \frac{1}{8\pi^2} \sum_{j=1}^{6} \sum_{l=1}^{3} \sum_{m=0}^{\infty} (-1)^l \int_0^\infty ds \exp \left( -m^2 s - \frac{B_l^2}{4s} \right) [\tilde{H}_l^i, s \coth(\tilde{H}_l^i, s) - 1]. \] (6)

This is expression of interest. The term with \( l = 0 \) is the vacuum energy \( V^{(i)}_q(H_i) \). Different type asymptotic expansions of (6) are given in [21].

2. GLUON CONTRIBUTIONS TO ONE-LOOP EFFECTIVE POTENTIAL

In this section, to give a self-contained presentation, we describe in brief the one-loop contributions of gluons to the effective potential. A detailed calculations are carried out in [10]. The Lagrangian of \( SU(3)_c \) gluodynamics is well known

\[ L = -\frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} + L_{gf} + L_{gh}, \] (7)

where \( F^{a}_{\mu\nu} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} - gf^{abc} A_{\mu}^{b} A_{\nu}^{c} \) is the field strength tensor, \( f^{abc} \) are the group structure constants, \( a = 1, 2, ..., 8 \). In actual calculations, we use the decomposition of the gauge field potential as in (1). The metric is chosen to be Euclidian for introducing the imaginary time formalism. The gauge fixing term in (7) is

\[ L_{gf} = -\frac{1}{2} \partial_{\mu} Q_{\mu}^{a} + gf^{abc} B^{a}_{\mu} Q_{\nu}^{b} Q_{\nu}^{c}, \] (8)

and \( L_{gh} \) represents the ghost field Lagrangian. The components \( Q_{\mu}^{a} \) with \( a = 1, 2, 4, 5, 6, 7 \) correspond to the color charged gluons. In calculations it is convenient to use the “charged basis” of gluons

\[ W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} \left( Q_{\mu}^{a} \pm i Q_{\mu}^{a} \right), \quad W_{\nu}^{\pm} = \frac{1}{\sqrt{2}} \left( Q_{\nu}^{a} \pm i Q_{\nu}^{a} \right), \] (9)

In terms of these fields the spectrum of the charged gluons looks like the spectrum of the spin one massless charged particle with gyromagnetic ratio \( \gamma = 2 \). That is, we have to set in (5) \( m = 0 \) and put \( \rho = \pm 2 \) for gluons and \( \rho = 0 \) for ghosts. As the values of the \( \tilde{H}_l^i \) we have to put \( H = 0 \) and use the combinations of the neutral components \( H_3 \) and \( H_8 \) entering (4).

The detailed calculation of the one-loop effective potential for \( SU(2) \) was carried out in [11, 12]. It corresponds to each \( SU(2) \) subgroup of the \( SU(3)_c \) group.

These subgroups are related with the components \( W_{\mu}^{\pm} \), \( r = 1, 2, 3 \) of the basis (9) and the corresponding combinations of \( H_3 \) and \( H_8 \) (for more details see [10]):

\[ B_{r=1,\mu} = B_{\mu}^{1}, \quad B_{r=2,\mu} = \frac{\sqrt{3}}{\sqrt{2}} B_{\mu}^{3} + \frac{1}{2} B_{\mu}^{2}, \] (10)

\[ B_{r=3,\mu} = \frac{\sqrt{3}}{\sqrt{2}} B_{\mu}^{3} - \frac{1}{2} B_{\mu}^{1}. \]

Just these fields enter the covariant derivatives related with the \( SU(2) \) subgroups, \( D_{\mu}^{1} = \partial_{\mu} + ib_{\mu} \).

The one-loop gluon contribution to the effective potential, presented in the form similar to (6), reads

\[ V^{(i)}_g(T, H_i) = \frac{1}{8\pi^2} \sum_{r=1}^{3} \sum_{l=1}^{\infty} \int_0^\infty ds \exp \left( -s-\frac{B_{l}^2}{4s} \right) \left[ \frac{g H_i \cos(2g H_i s)}{\sin(g H_i s)} - \frac{1}{s} \right], \] (11)

where \( m^2 - i\epsilon, \epsilon \to 0 \) is a parameter playing the role of the normalization point in the field. It is useful for analytic continuations from the weak fields \( g H_i \leq m^2 \) to the fields \( g H_i \geq m^2 \) when an imaginary part of the effective potential is calculated. The term with \( l = 0 \) gives the zero temperature (vacuum) part and other terms describe the statistical part.

In what follows, we take into consideration the gluon contributions in the high temperature limit \( T \gg (g H_i)^{3/2} \geq \mu \), which is sufficient for our problem. The calculation of \( V^{(i)}_g(T, H_i) \) for this case was carried out by Mellin’s transformation technique described in Appendix B for the quark effective potential (6). For gluons it is presented in [10], Eq. (10):

\[ V^{(i)}_g(T, H_3, H_8) = \frac{H_3^2}{2} + \frac{11 g^2}{32 \pi} H_3^2 \log \left[ \frac{T}{\mu} \right] \]

\[ - (g H_8)^{3/2} \frac{H_8^2}{3 \pi} + \frac{11 g^2}{16 \pi^2} H_8^2 \log \left[ \frac{T}{\mu} \right] \]

\[ - (\lambda_{\pm}^{3/2} + \rho_{\pm}^{3/2}) \left[ \frac{3}{2} \right] (g H_3)^{3/2} \frac{T}{2 \pi} \]

\[ - i \left[ (g H_3)^{3/2} + \rho_{\pm}^{3/2} \left[ \frac{3}{2} \right] (g H_8)^{3/2} \frac{T}{2 \pi} + O(g^2 H_{3,8}^2) \right], \] (12)

where tree-level terms of \( H_3 \) and \( H_8 \) were added, \( \lambda_\pm = 1 + \frac{1}{\sqrt{6} H_8} \) and \( \mu \) is normalization point. This expression contains the imaginary part related with the lower state of the gluon spectrum \( \epsilon_{m=0,\rho=2}^2 = p_\tau^2 - g H_\mu \). It was realized already, this term is exactly canceled by the imaginary term coming from...
the daisy diagrams for charged gluons [10, 12]. This is the main reason for using this approximation in further analysis.

3. CONTRIBUTION OF DAISY DIAGRAMS

As it is well known [15], at finite temperature along with a one-loop effective potential we have to take into consideration the so-called daisy diagram contributions which account for the long distance correlations. Graphically, this is a series of one-loop gluon diagrams with infinite number of insertions of the polarization tensors taken at zero external momenta, \( \Pi(T, g, H_3, H_4) \). They have the order \( \sim g^{3/2} \) in coupling constant and therefore must be included in the effective potential after the one-loop terms. The two-loop contributions have the order \( \sim g^2 \). This order terms were neglected in (12).

The calculations of the daisy diagram contributions coming from charged gluons are described in [10, 12]. In notation (10) the part given by the unstable modes is

\[
V_{\text{daisy}}^{\text{unst.}} = \frac{T}{2\pi} \sum_{\rho = 1}^{3} (gH_{\rho}) [\Pi'(H_{\rho}, T) - gH_{\rho}]^{1/2} + i \left[ \left( gH_{\rho} \right)^{3/2} + \left( \lambda^{\rho}_{+} \right)^{3/2} \right] \left( T^{3/2} \right) \left( gH_{\rho} \right)^{1/2} T \left( 2\pi \right) .
\] (13)

Here, \( \Pi'(H_{\rho}, T) \) denotes the one-loop polarization tensors of color charged gluons averaged over the ground (unstable) state of the tree-level spectrum taken at \( p_i = 0 \) : \( n = 0, \rho = 2 \). As we see, the imaginary parts in (12) is exactly canceled by the one in (13). Thus, the effective potential \( V_{\text{daisy}}^{\text{unst.}} \) is real if \( [\Pi'(H_{\rho}, T) - gH_{\rho}] > 0 \).

Detailed calculations of the charged gluon polarization tensor have been carried out in [10] (see also review [20]). The most important for us is the temperature and field dependencies of it: \( \Pi'(H_{\rho}, T) - g^2 \sqrt{gHT} \). Hence, the remaining after the cancelations part in the effective potential is of the order \( \sim g^2 g^{3/2} \). It is smaller than the accuracy preserved in (12). We can conclude that the role of the unstable mode daisies consists in the stabilization of the effective potential in the chosen approximation in coupling \( g \). All other terms are negligibly small and have to be dropped. As concerns the contributions of the neutral gauge field daisies, they have the order \( \sim g^{3/2} \) and have to be dropped also [10]. Thus, the consistent effective potential for gluons including the one-loop plus daisies is real.

To carry out numeric calculations we use the dimensionless variables for the effective potential, temperature and fields. We consider the proton mass \( m_p = 938.3 \text{ MeV} \) as a reference parameter and introduce the dimensionless variables:

\[
V^{0}_{q, g} = \frac{V^{0}_{q, g}}{m_p^{4}}, \quad V^{T}_{q, g} = \frac{V^{T}_{q, g}}{m_p^{4}}, \quad \mu_f = \frac{m_f}{m_p}, \quad \beta_f = \frac{m_f}{m_p}, \quad \omega_f = \frac{\mu_f}{\beta_f}.
\] (14)

We also take into consideration three sorts of quarks with the masses and electric charges

\[
m_u = 336 \text{ MeV}, \quad q_u = \frac{2}{3} |\epsilon|,
\]

\[
m_d = 340 \text{ MeV}, \quad q_d = -\frac{1}{3} |\epsilon|,
\]

\[
m_s = 486 \text{ MeV}, \quad q_s = -\frac{1}{3} |\epsilon|,
\]

and the coupling values \( \alpha_s = 1, \quad \alpha_c = \frac{1}{137}, \quad g = \sqrt{4\pi} |\epsilon| \). The dimensionless field strengths are:

\[
x = \frac{|\epsilon| H}{m_p}, \quad x_3 = \frac{gH_3}{m_p^2}, \quad x_8 = \frac{gH_8}{m_p^2}.\]

The field combinations in (4) and (10) should be expressed in term of them. In next section we present the results of the calculations fulfilled for a number of temperatures. Details of calculations are placed in the Appendices.

4. ESTIMATE OF THE FIELD STRENGTHS

The total effective potential used in our investigation consists of the one-loop quark contribution (6) including the zero temperature term \( V^0_q \) with \( l = 0 \) and the statistical part \( V^T_q \) with \( l \neq 0 \) and the gluon contributions (12), (13) presented above. The calculation of the zero temperature quark potential is given in Appendix A. To be in correspondence with the gluon sector approximation, we apply the high temperature expansion for quark sector also. The calculations are given in Appendix B. The final expressions read

\[
V^0_q = \frac{x^2}{2e^2} + \frac{x^2_3}{2g^2} + \frac{x^2_8}{2g^2}
\]

\[
+ \frac{1}{8\pi} \sum_{f} \sum_{a=1}^{3} \left[ \frac{1}{3} h^2_{f,a} - 2h^2_{f,a} \right]
\]

\[
\times \left[ 2 \Gamma_1 \left( \frac{\mu^2_f}{2h_{f,a}} + \frac{\mu^2_f}{2h_{f,a}} \ln \frac{\mu^2_f}{2h_{f,a}} + 2\zeta(-1) \right) \right]
\]

\[
+ \frac{1}{3} h^2_{f,a} \ln \frac{\mu^2_f}{2h_{f,a}} + \frac{1}{2} \mu^2_f \ln \frac{\mu^2_f}{2h_{f,a}} - \frac{1}{4} \mu^2_f \right],
\] (16)
and

\[
V_q^{T,\alpha} = \frac{1}{4\pi} \sum_i \sum_{a=\pm1} \left[ \frac{2}{3} h_{i,a}^2 \right] \times \left[ \frac{1}{2} \left( \gamma + \ln \left( \frac{\alpha}{\pi} \right) \right) + \frac{7\zeta(-2)}{4} \omega_f + \frac{31\zeta(-4)}{64} \omega_f \right]
\]

\[
- \frac{h_{i,a}^2}{90\mu_f} \left[ -1 + \frac{31\zeta(-4)}{8} \omega_f \right] \right].
\]

Here, \( \Gamma_i(x) \) is generalized Gamma function, \( \zeta(x) \) is \( \zeta \)-function, the notations are given in (14).

These two expressions plus (12) and (13) are used in the estimation of the magnetic field strengths. For doing so we numerically solve the stationary equations

\[
\frac{\partial V(H, H_3, H_8, T)}{\partial H} = 0,
\]

\[
\frac{\partial V(H, H_3, H_8, T)}{\partial H_{3,8}} = 0,
\]

at a number of fixed temperatures and obtain the roots \( h_{i,\min}^i(T) \). If for a particular set of \( h_{i,\min}^i(T) \) the total effective potential is negative, we have to conclude that these magnetic fields are spontaneously generated. The results of the calculations are presented in Table 1.

In Table 1, in first column we show the temperature. The next tree columns give the values of the dimensionless field strengths, the next one shows the behavior of the dimensionless effective potential. The field strengths [Gauss] are shown in the last three columns.

We have detected the negative values of the effective potential for the stationary field strengths. It means that magnetic \( H \) and chromomagnetic \( H_3, H_8 \) fields have to be generated spontaneously after the DPT in QGP. If the temperature is lower than 110–120 MeV, the effective potential value is close to zero. Hence, within the high temperature approximation adopted, we expect the background fields disappear and confinement is realized. We see from Table 1 that the strength of the magnetic field is two orders of magnitude less than the strength of the colored fields and equals ~ \( 10^{16} \) G at the LHC experiment temperatures.

### DISCUSSION AND CONCLUSIONS

In our calculations, we applied the consistent approximation for the effective potential accounting for the one-loop plus daisy diagrams. It includes the terms of the order \( g^2 \) and \( g^4 \) and makes the potential real due to cancellation of the imaginary terms in Eqs. (12)–(13). That is sufficient at high temperature because for small couplings we can neglect the two-loop contributions having the order \( g^4 \). This simple approximation is convenient also because the Legendry transformed magnetic field in the potential coincides with the one entering the one-loop expressions (6), (12). Thus, the induced in the plasma fields are properly described.

The most interesting observation of the above investigation is two fold. Firstly, with temperature lowering the magnetic field strengths are decreased. Secondly, simultaneously the value of the effective potential in the minimum, being negative, increases and tends to zero. Beginning from the value \( V_{\min} = 0.0024 \) at \( T = 200 \) MeV it equals to \( 0.00024 \) at \( T = 120 \) MeV, that is it increases in one order. Such type behavior detects that the magnetic fields act to decrease the DPT temperature \( T_d \). Due to spontaneous magnetization at finite temperature, QGP must be created at essentially lower temperature as compared to the zero field case. To our knowledge, this fact was not noted in the literature already.

The obtained results have interesting interplay with studies of QCD vacuum in applied magnetic fields at zero [26, 27] and finite temperature [23, 28]. These investigations have been stimulated by the presence of strong magnetic fields in either heavy ion collisions or neutron stars. For our problem, the most important conclusion of them is that external magnetic fields lower \( T_d \). Also, in [26] it was observed that usual and color magnetic fields are preferably oriented in parallel to each other. Just this field configuration was chosen in the present paper. Thus, both the zero and high temperature results have detected the \( T_d \) lowering. In this respect, it does not matter how the magnetic fields were generated. The \( T_d \) lowering in external color magnetic fields has been observed also, firstly in [24, 25].

### Table 1. The values of the field strengths spontaneously generated at chosen plasma temperatures

| \( T, \text{MeV} \) | \( x_8 \) | \( x_3 \) | \( x \times 10^{-3} \) | \( V \) | \( H_8 \times 10^{19}, \text{G} \) | \( H_3 \times 10^{18}, \text{G} \) | \( H \times 10^{17}, \text{G} \) |
|-------------------|---------|---------|----------------|-----|----------------|--|----------------|
| 120               | 0.463   | 0.076   | -0.051         | -0.0024 | 0.589         | 0.969         | -0.076         |
| 140               | 0.783   | 0.132   | 0.040          | -0.0063 | 0.996         | 1.674         | 0.060          |
| 160               | 0.901   | 0.152   | 0.249          | -0.0110 | 1.145         | 1.933         | 0.370          |
| 180               | 0.999   | 0.169   | 0.476          | -0.0167 | 1.270         | 2.146         | 0.709          |
| 200               | 1.092   | 0.185   | 0.728          | -0.0235 | 1.388         | 2.349         | 1.083          |
| 220               | 1.182   | 0.200   | 1.005          | -0.0314 | 1.503         | 2.544         | 1.496          |
In particular, in [24] it was found in the lattice calculations that the temperature $T_d$ can even be reduced to zero for sufficiently strong applied color magnetic fields. The more recent studies and the results as well as relevant references on the lattice investigations can be found in [29–31].

In contrast, here we determined similar tendency for the usual and color magnetic fields spontaneously created in QGP. We have to conclude that DPT has to happen at temperatures $\sim 110$–$120$ MeV. For these temperatures, the minimum value of the total effective potential is very close to zero. In the used approach this means the magnetic field screening and color confinement.

To other similarities, we have the essential differences between the external (temperature independent and produced by currents) and the spontaneously created (source less) fields. Really, as we noted already, asymptotic freedom at high temperature has always to be accompanied by temperature dependent background magnetic fields [11]. Screening of them reflects the destroying of the asymptotic freedom regime and color confinement at low temperature. The spontaneously generated temperature dependent macroscopic magnetic fields are intrinsic constituents of QGP and the signals of the DPT. In short, deconfinement is always accompanied by macroscopic magnetic fields. Note ones else that these fields are massless and occupy all the plasma volume [18].

We applied the approximation for the effective potential including the one-loop plus daisies. It is real in the leading order $- O(g^3)$ in coupling constant. Let us mention a number of the mechanisms for the magnetic field stabilization at finite temperature. In detail this problem was investigated by either analytic methods of field theory or simulations on a lattice. It is discussed in [20]. Qualitatively, two factors act to stabilize vacuum. First is a so-called condensate related with the Polyakov loop [22] which appears after the DPT. It enters the gluon spectrum of the type (5) in the form

$$\ldots + (gA_0)^2$$

and acts in favor of eliminating the instability. Second are the radiation corrections forming the magnetic mass of charged gluons

$$\Pi^\perp_{AB}(H, T) = g^2 \sqrt{\frac{Q}{2\pi}} H T$$

and having a large positive real part [12], which also acts to stabilize vacuum. These mechanisms have been proven also in lattice simulations [16, 18]. So, we stress again that the used approximation for the effective potential is consistent and reliable.

Now, we compare the obtained results with that of in [21] where to clarify the role of the quark loop effects the color magnetic fields $H_3(T)$ and $H_8(T)$ were estimated from the effective potential of the gluon fields, only. The comparison shown that the field strength $H(T)$ is $\sim 15$ per cent less in this approximation. For example, at $T = 200$ MeV $x(h) = 6.24 \times 10^{-4}$ whereas from the present results we obtained $x(h) = 7.28 \times 10^{-4}$. But qualitatively this is close.

As it follows from the obtained results, in QGP strong chromo(magnetic) fields of the order $H_3 \sim 10^{-15}$–$10^{-19}$ G and $H \sim 10^{6}$–$10^{7}$ G must be present. This influences all the processes happening and may serve as the distinguishable signals of the DPT. Due to magnetization, in particular, all the initial states of charged particles have to be discrete ones. This could modify the cross sections of particular processes and detected in experiments. Moreover, the fields, as well as the $A_q$ condensate, generate new type processes with $C$-parity violation, which also could be the signals of the plasma formation. Detailed consideration of these problems will be reported elsewhere.

**APPENDIX A**

The term with $l = 0$ in (6) is the vacuum energy $\mathcal{V}^{(1)}_{vac, f}(H, H_3, H_8)$. The well known expression for it reads [17]

$$8\pi^2 \mathcal{V}^{(1)}_{vac, f}(H, H_3, H_8) = \frac{1}{3} \left( \frac{\mathcal{H}_{f}^{\perp}}{\zeta} \right)$$

Calculation of this integral can be done by using the $\Gamma$- and $\zeta$-functions:

$$\int_0^\infty x^{-1} e^{-\alpha x} dx = \Gamma(x), \quad \text{Re} x > 0, \quad \text{Re} \alpha > 0;$$

$$\int_0^\infty x^{-1} e^{-\alpha x} (1 - e^{-\beta x})^{-1} dx = \Gamma(s) \zeta(s, \nu), \quad \text{Re} s > 0, \quad \text{Re} \nu > 0$$

Let us set in denominator of (19) $s^3 \to s^{3-\varepsilon}$ and consider the limit $\varepsilon \to 0$. We present (19) as the sum of four integrals and rewrite them using (20). In this way we get

$$8\pi^2 \mathcal{V}^{(1)}_{vac, f}(H, H_3, H_8) = - (2\mathcal{H}_{f}^{\perp})^2 \left( \frac{m_f^2}{2\mathcal{H}_{f}^{\perp}} \right)^{2-\varepsilon} \times \Gamma(\varepsilon - 2) \left( \frac{\mathcal{H}_{f}^{\perp}}{\zeta} \right)$$

$$+ 2(\mathcal{H}_{f}^{\perp})^2 \zeta \left( \varepsilon - 1; \frac{m_f^2}{2\mathcal{H}_{f}^{\perp}} + 1 \right) \Gamma(\varepsilon - 1).$$
Then we make an expansion in series over $\varepsilon$ and use $\Gamma_1$-function for calculation of the derivative
\[
\zeta'(-1; x) = \left. \frac{d\zeta(x-1, x)}{dx} \right|_{x=0} = \Gamma_1(x) + \zeta'(-1),
\]
\[
\Gamma_1(x) = \int_0^x \ln \Gamma(y) dy + \frac{1}{2} x(x - 1) - \frac{1}{2} x \ln(2\pi).
\]
As a result, we obtain Eq. (16) for vacuum energy.

**APPENDIX B**

The general method for calculation of the high temperature asymptotic used in Secs. 2–3 has been developed in [11, 19].

For the high temperature expansion of the one-loop quark effective potential we perform the following steps. First, the expression in the brackets in (6) we expand in series over $s$ near the point $s = 0$ and insert into the integral over $s$. First two terms of interest are
\[
4\pi^2 V_{s/g} = \sum_{l=1}^{\infty} (-1)^l \int_0^{\infty} ds e^{-m_i^2 s} \frac{b^l}{4s}
\]
\[
\times \left\{ \frac{(\partial \tilde{E}^i)}{3s} - \frac{1}{45} (\partial \tilde{E}^i)^3 s \right\}.
\]
Then we integrate over $s$ by means of the well-known formula for $K$-function
\[
\int_0^{\infty} ds s^{n-1} e^{-as} = 2 \left( \frac{a}{b} \right)^n K_n(2\sqrt{ab}).
\]
As a result, we can separate the sum over $l$ from the fields $\partial \tilde{E}^i_f$
\[
4\pi^2 V_{s/g} = \frac{2}{3} (\partial \tilde{E}^i)^3 \sum_{l=1}^{\infty} (-1)^l K_0(m_i^2 b^l l)
\]
\[
- \frac{(\partial \tilde{E}^i)^4}{90m_i^4} \sum_{l=1}^{\infty} (-1)^l (m_i^2 b^l l)^2 K_2(m_i^2 b^l l).
\]
For the next sums, we can calculate the asymptotic expressions by using Mellin’s transformation [11, 19]
\[
\sum_{n=1}^{\infty} (-1)^n K_0(\omega n) = \frac{1}{2} \left( \gamma + \ln \left( \frac{\omega}{\pi} \right) \right)
\]
\[
+ \sum_{n=1}^{\infty} \frac{(2n+1)\omega^{2n+1} \zeta(-2n)}{2^{2n}(n!)^2}
\]
\[
- \sum_{n=1}^{\infty} (-1)^n (\omega n)^2 K_2(\omega n) = -1
\]
\[
+ \sum_{n=1}^{\infty} \frac{(\omega n)^2}{2^n (n^2 - 1)} \zeta(-2n).
\]
As a result, we obtain the high temperature expansion of the effective potential (17). The high temperature expansion of the gluon one-loop effective potential and daisies has been done in [10].

**REFERENCES**

1. H. Satz, “The fireball paradigm,” Lect. Notes Phys. 841, 1 (2012).
2. J. Greensite, “An introduction to the confinement problem,” Lect. Notes Phys. 821 (2011).
3. O. K. Kalashnikov, “QCD at finite temperature,” Fortsch. Phys. 32, 525 (1984).
4. G. S. Bali et al., “The QCD transition in external magnetic fields,” PoS(ConfinementX), 197 (2012).
5. L. Levkova and C. DeTar, “Quark-gluon plasma in an external magnetic field,” Phys. Rev. Lett. 112, 012002 (2014).
6. K. Szabo, “QCD at non-zero temperature and external magnetic fields,” PoS(LATTICE2013), 014 (2014).
7. G. S. Bali, F. Bruckmann, G. Endrodi, and A. Schäfer, “Magnetization and pressures at nonzero magnetic fields in QCD,” PoS(LATTICE2013), 182 (2014).
8. D. Grasso and H. R. Rubinstein, “Magnetic fields in the early universe,” Phys. Rep. 348, 163–266 (2001).
9. E. Elizalde and V. Skalozub, “Spontaneous magnetization of the vacuum and the strength of the magnetic field in the hot universe,” Eur. Phys. J. C 72, 1968 (2012).
10. V. V. Skalozub and A. V. Strelechenko, “On the generation of abelian magnetic fields in SU(3) gluodynamics at high temperature,” Eur. Phys. J. C 33, 105 (2004).
11. V. Skalozub, “Effective coupling constants in gauge theories at high temperature,” Int. J. Mod. Phys. A 11, 5643 (1996).
12. V. Skalozub and M. Bordag, “Colour ferromagnetic vacuum state at finite temperature,” Nucl. Phys. B 576, 430 (2000).
13. M. D. Pollock, “Magnetic fields and vacuum polarization at the Planck era,” Int. J. Mod. Phys. D 12, 1289 (2003).
14. V. I. Demchik and V. V. Skalozub, “The spontaneous generation of magnetic fields at high temperature in a supersymmetric theory,” Eur. Phys. J. C 27, 601–607 (2003).
15. J. I. Kapusta, *Finite-Temperature Field Theory* (Cambridge Univ. Press, Cambridge, 1989).
16. V. Demchik and V. Skalozub, “The spontaneous creation of a chromomagnetic field and A0-condensate at high temperature on a lattice,” J. Phys. A 41, 16405 (2008).
17. A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics* (Nauka, Moscow, 1969; Wiley, New York, 1965).
18. S. Antropov, M. Bordag, V. Demchik and V. Skalozub, “Long range chromomagnetic fields at high temperature,” Int. J. Mod. Phys. A 26, 4831–4843 (2011).
19. H. E. Haber and H. A. Weldon, “On the relativistic Bose-Einstein integrals,” J. Math. Phys. 23, 1852 (1982).
20. V. Demchik and V. Skalozub, “Spontaneous magnetization of a vacuum in the hot universe and intergalactic magnetic fields,” Phys. Part. Nucl. 46, 1–23 (2015).
21. V. Skalozub and P. Minaiev, “On magnetization of quark-gluon plasma at the LHC experiment energies,” Visn. Dniprop. Univ., Fiz., Radioelektron. 24, 25 (2016); arXiv:1612.00216 [hep-ph].

22. A. O. Starinets, A. S. Vshivtsev, and V. Ch. Zhukovsky, “Colour ferromagnetic state in SU(2) gauge theory at finite temperature,” Phys. Lett. B 322, 40 (1994).

23. N. O. Agasian and S. M. Fedorov, “Quark-hadron phase transition in a magnetic field,” Phys. Lett. B 663, 445 (2008).

24. P. Cea, L. Cosmai, and M. D’Elia, “The QCD phase diagram for external magnetic fields,” J. High Energy Phys. 0712, 097 (2007).

25. G. S. Bali et al., “The QCD phase diagram for external magnetic fields,” J. High Energy Phys. 1202, 044 (2012).

26. B. V. Galilo and S. N. Nedelko, “Impact of strong electromagnetic field on the QCD effective potential for homogeneous abelian gluon field configurations,” Phys. Rev. D: Part. Fields 84, 094017 (2011).

27. S. Ozaki, “QCD effective potential with strong U(1)lem magnetic fields,” Phys. Rev. D: Part. Fields 89, 054022 (2014).

28. V. D. Orlovsky and Yu. A. Simonov, “The quark-hadron thermodynamics in magnetic field,” Phys. Rev. D: Part. Fields 89, 054012 (2014).

29. C. Bonati, M. D’Elia, M. Mariti, M. Mesiti, F. Negro, A. Rucci, and F. Sanfilippo, “Magnetic field effects on the static quark potential at zero and finite temperature,” Phys. Rev. D: Part. Fields 94, 094007 (2016).

30. M. D’Elia, E. Meggiolaro, M. Mesiti, and F. Negro, “Gauge-invariant field-strength correlators for QCD in a magnetic background,” Phys. Rev. D: Part. Fields 93, 054017 (2016).

31. C. Bonati, M. D’Elia, M. Mariti, M. Mesiti, F. Negro, and F. Sanfilippo, “Anisotropy of the quark-antiquark potential in a magnetic field,” Phys. Rev. D: Part. Fields 89, 114502 (2014).