Exclusive production of pions and the pion distribution amplitude

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Considering, as a limit case, an approximately flat pion distribution amplitude, which is determined from the hardest, in momentum space, solution of the Bethe-Salpeter equation for the pion wave function, we compute the pion transition form factor $F_{\gamma \gamma \gamma}(Q^2)$ and the pion form factor $F_{\pi}(Q^2)$, taking into account the LO as well as NLO form of the hard coefficient function entering the leading-twist factorization formula. We also compute the exclusive photoproduction of pions pairs at high energies, $\gamma \gamma \rightarrow \pi^+ \pi^-$, where perturbative QCD can be used to compute the hard scattering matrix elements. We verify that the existent data for exclusive pion production can be reasonably described as a function of such flat distribution amplitude.

I. INTRODUCTION

The hadronic distribution amplitudes (DAs) are an essential ingredient for measuring exclusive processes at large momentum transfer. Most of the recent data about the Standard Model parameters rely on QCD factorization, asymptotic freedom and make use of process-independent hadronic DAs. This fact reveals the importance of knowing the intricacies of quarks and gluons within the hadrons. The pion, being the simplest hadron, should, in principle, be the easiest particle to offer a laboratory to learn about hadronic DAs, although its study is still motive of debates.

Some years ago the BaBar Collaboration [1] published results for the $\gamma \gamma \rightarrow \pi^0$ process, where one of the photons is far off mass shell (large $Q^2$) and the other one is near mass shell ($Q^2 \approx 0$). These measurements of the photon-pion transition form factor $F_{\gamma \gamma \pi}(Q^2)$, taken in single-tagged two-photon $e^+e^- \rightarrow e^+e^-\pi^0$ reaction, was performed in a wide range of momentum transfer squared ($4 - 40 \text{ GeV}^2$). At such high $Q^2$ it is expected that the standard factorization approach can be applied [2].

The surprise with the BaBar result was that it was not in agreement with the expected perturbative QCD behavior, where $Q^2 F_{\pi \gamma \gamma}(Q^2 \rightarrow \infty)$ should be limited to the value $\sqrt{2} f_{\pi} \approx 0.185 \text{ GeV}$, which is known as the BL limit [3], and where $f_{\pi} = 131 \text{ MeV}$ is the pion decay constant. Some time later the Belle Collaboration presented data [4] in the same range of transferred momenta showing that the pion transition form factor may not increase as fast as shown by the BaBar results, although some medium values of Belle data also appear to be in contradiction with the BL limit.

These experiments originated several theoretical papers speculating why the data should (or not) obey the BL limit [5–15]. Some of these and recent proposals also claimed that the pion distribution amplitude (DA) at high momentum transfer was not given by the asymptotic form [16]

$$\varphi^{as}_{\pi}(x) = 6x(1-x),$$

but should be replaced by a broad concave distribution [17, 18] or a flatter one [5, 6, 8, 12, 19, 22]. Available information indicates that the above asymptotic distribution is a poor approximation to the pion distribution amplitude even at large momentum scales [17]. As a consequence, predictions of leading-order, leading-twist formula based on $\varphi^{as}_{\pi}(x)$ should be revisited. Actually, a flat DA is consistent with the BaBar data [5], although a field theoretical support for such possibility is still missing. As claimed in Ref. [23], we may assume that at present there is no definite conclusion on which is the asymptotic form of the pion DA, and it is possible that in the future a combined analysis of data of the processes involving pions will shed light on the pion distribution amplitude [24].

The pion transition form factor is quite dependent on the pion distribution amplitude, and this one is directly related to the pion wave function. Recently some of us proposed a limit on the transition form factor based on the hardest solution (in momentum space) of the Bethe-Salpeter equation (BSE) for the pseudoscalar pion state [25]. This wave function leads to the flattest QCD DA, and a flat DA as argued by Radyushkin [5] can describe the BaBar data. Therefore an almost flat pion DA can be naturally explained within QCD first principles when associated to a particular behavior of the BSE solution. Actually, it is possible that non-perturbative effects change the soft asymptotic behavior of the pion wave function leading to a much broader DA than the one of Eq. [4], and this fact was observed in lattice simulations [20].

In this work we will explore in detail the predictions of this extreme BSE solution for the high energy behavior of the pion transition form factor, its form factor and
the two-photon production of a pion pair. In Section II we discuss how the pion distribution amplitude can be obtained from the BSE and advocate in favor of a BSE solution for the pion wave function that decreases slowly with the momentum, which is at the origin of the flat pion DA. In Section III the pion DA obtained in the previous section is used to determine the pion transition form factor. In Sections IV and V we continue with the phenomenological implications of our flat DA respectively in the cases of the pion form factor and hard exclusive two-photon production of a pion pair. Section VI contains our conclusions.

II. THE PION DISTRIBUTION AMPLITUDE FROM THE BSE

The pion distribution amplitude at leading twist, as a function of the quark self-energy and the pion-quark vertex, is given by [27]

\[ \varphi_{\pi}(x) = \frac{N_c}{4\pi^2 F_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty \frac{du}{D(u+\lambda x, u-\lambda x)} F(u+\lambda x, u-\lambda x) \times [\bar{\Sigma}(u+\lambda x) + \Sigma(u-\lambda x)] , \]  

(2)

where the u-variable plays the role of the quark transverse momentum squared, \( \lambda x \) and \( -\lambda x \) are the longitudinal projections of the quark momentum on the light cone directions \( (\vec{x} = (1-x)) \), \( \Sigma(u) \) is the dynamical quark mass,

\[ D(u) = u + \Sigma^2(u) , \]  

(3)

is a function related to the quark propagator, and the function \( F \) is the momentum dependent part of the quark-pion vertex, which can be approximated by

\[ F(p^2, p'^2) = \sqrt{\Sigma(p^2)\Sigma(p'^2)} , \]  

where \( p \) and \( p' \) are the quark and anti-quark momenta. The pion DA at leading twist is normalized as

\[ \int_0^1 dx \varphi_{\pi}(x) = 1 . \]  

(4)

It is known that the dynamical quark self-energy \( (\Sigma(p^2)) \), giving by the Schwinger-Dyson equation is exactly identical to the pseudoscalar BSE at zero momentum transfer \( \Phi^{P}_{BS}(p, q) \rvert_{q\rightarrow 0} \), as demonstrated by Delbourgo and Scadron [28]

\[ \Sigma(p^2) \approx \Phi^{P}_{BS}(p, q) \rvert_{q\rightarrow 0} , \]  

(5)

which is a consequence of the fact that they are related through the Ward-Takahashi identity. The homogeneous BSE can be, in general, written as

\[ \Phi(k, P) = -i \int_q^\infty \frac{d^4 q}{(2\pi)^4} K(k; q, P) S(q_+) \Phi(q; P) S(q_-) , \]  

(6)

where the amplitude depends on the quarks total \( (P) \) and relative \( (q) \) momenta, with \( q_+ = q + \eta P \), \( q_- = q - (1-\eta)P \), and \( 0 \leq \eta \leq 1 \), where \( \eta \) is the momentum fraction parameter. In Eq. (6) \( K \) is the fully amputated quark-antiquark scattering kernel, \( S(q) \) are the dressed quark propagators, and the homogeneous BSE is valid on-shell, i.e. \( P^2 = 0 \) in the pion case. Note that we suppressed all indices (color, etc...) in Eq. (6).

The quark masses are dynamically generated along with bound state Goldstone bosons (the pions). The BSE, Eq. (6), is an integral equation that can be transformed into a second order differential equation. The two solutions of the differential equation can be found, for example, in Ref. [29, 30] and are characterized by one soft asymptotic solution

\[ \Phi_{\pi}^{R}(p^2) \sim \Sigma(p^2 >> \mu^2) \sim \frac{\mu^3}{p^2} , \]  

(7)

and by the extreme hard high energy asymptotic behavior of a bound state wave function

\[ \Phi_{\pi}^{L}(p^2) \sim \Sigma(p^2 >> \mu^2) \sim \mu \left[ 1 + b g^2 \left( \frac{\mu^2}{\mu^2} \right) \right]^{-\delta} , \]  

(8)

where \( \delta = c/2b \), \( b \) is the first coefficient of the perturbative \( \beta \) function (function of \( N_c = 3 \), the number of colors, and \( n_f \), the number of quark flavors), and \( c = 4/3 \) is the Casimir eigenvalue for quarks in the fundamental representation. In the above equations \( \mu \) is the dynamically generated quark mass at zero momentum.

The asymptotic expression shown in Eq. (8) was determined in the appendix of Ref. [31] and it satisfies the Callan-Symanzik equation. This last solution is constrained by the BSE normalization condition [32], which imply \( n_f > 5 \) [29, 30, 33], otherwise it is not consistent with a possible bound state solution in a \( SU(3) \) non-Abelian gauge theory. We will take \( n_f = 6 \) as will be explained later. This solution is one alternative to the soft one \( (\Sigma(p^2) \sim 1/p^2) \) [31] which leads to the standard DA \( \varphi_{\pi}^{as}(x) = 6x(1-x) \). Nowadays it is known that we may have solutions with a momentum behavior varying between Eq. (7) and Eq. (8) depending on the theory dynamics (particularly as the number of fermions is increased) [33, 34].

It has been argued that Eq. (8) may be a realistic wave function in a scenario where the chiral symmetry breaking is associated to confinement and the gluons have a dynamically generated mass [33, 36, 37]. This solution also appears when using an improved renormalization group approach in QCD, associated to a finite quark condensate [38], and it minimizes the vacuum energy as long as \( n_f > 5 \) [39]. This specific solution is the only one consistent with Regge-pole like solutions [30]. Moreover, recently it has been argued that a logarithmic self-energy, as the one of Eq. (3), may appear naturally in models where all quark masses are generated dynamically [40], although in these cases the power \( \delta \) will depend on the details of the model.

It is interesting to recall that models with origin in the Nambu-Jona-Lasinio model (NJL), like the ones of
Ref. [21, 22], also describe the data with a flat pion DA, what is not surprising since the NJL model naturally lead
to dynamical masses with a behavior similar to the one of
eq (5), which, as shown in the sequence, induce a quite
flat DA.

The important fact to be noticed here is that 
\text{Eq. (5)}
gives the hardest (in momentum space) asymptotic be-

havior allowed for a bound state solution in a non-
Abelian gauge theory, and it is exactly for this reason
that the constraint on \( n_f \) arises from the BSE normal-
ization condition. No matter this solution is realized in
Nature or not, as we shall see, it will lead to the flattest
pion DA, any other flatter distribution than this one can-
not be a realistic BSE wave function, and would not be
consistent with a composite pion. A totally flat DA can
only be related to a fundamental pion. A realistic DA,
in principle, should be related to a solution of the BSE
and should obey a normalization condition peculiar to a
well behaved wave function.

In order to compute the pion DA we will perform an
integral over the wave function in the full range of mo-
menta (i.e. \( p^2 \rightarrow \infty \)), this is why we will consider
\( n_f = 6 \) indicating that we cover all possible thresholds,
meaning also that the sea quark contribution is impor-
tant as we go to extreme \( x \) values. To obtain the extreme
field theoretical limit on the pion DA, we shall also work
with a simple interpolating expression that roughly re-

flects the full behavior of the “hardest” quark self-energy
(or BSE solution), namely [33, 37]

\[
\Sigma(p^2) = \mu \left[ 1 + bg^2 \left( \frac{\mu^2}{p^2} \right)^2 \right].
\]  

Note that the \( \mu \) factor introduced into the logarithm
denominator leads to the right infrared (IR) behavior
\( \Sigma(p^2 \rightarrow 0) = \mu \), which is the approximate \( \Sigma(p^2) \)
infrared behavior. Eq. (9) is quite dependent on the ultravi-
iolet self-energy, and this is the reason for Eq. (9)
being responsible for a flat DA. A softer self-energy (or
wave function) will not lead to a flat DA. For instance,
the solution of Eq. (7) is the one that leads to Eq. (1). On
the other hand Eq. (11) is the only condition constraining
the infrared \( \Sigma(p^2) \) behavior, which may roughly be
described by Eq. (9).

The coupling constant \( g^2 \) is calculated at the chiral
symmetry breaking scale \( \mu \), and given by

\[
g^2(k^2) = \frac{1}{b \ln((k^2 + 4M^2 G)/\Lambda_{QCD}^2) }, \]

which is an infrared finite coupling determined in QCD
where gluons have an effective dynamical mass \( M_g \),
with an infrared value \( M_g(0) \approx 2\Lambda_{QCD} \), consistent with
the models of Ref. [33, 36, 37, 42]. \( \Lambda_{QCD} \) is the QCD
characteristic scale. There are more sophisticated fits
for this coupling constant which take into account the
running of the gluon mass [43, 44], although the one of
Eq. (10) is sufficient and precise enough for our purposes.

Our pion DA numerical result calculated with Eq. (2),
Eq. (9) and constrained by Eq. (11) can be quite well re-
produced by the normalized form

\[
\varphi(x) = \frac{\Gamma(2 + 2\epsilon)}{\Gamma^2(1 + \epsilon)} x^\epsilon (1 - x)^\epsilon, \]

where

\[
\epsilon \approx 0.0298, \]

which will be used in the following calculations. Note
that, according to Radzuskhin [5], QCD corrections will
barely affect such flat distribution amplitude, where no
dependence with the factorization scale will be assumed.

III. PION TRANSITION FORM FACTOR

At sufficiently high \( Q^2 \) it is expected that the standard
factorization approach can be applied [45, 46] (for a re-
view, see [2]), and the pion transition form factor is given by

\[
F_{\pi \gamma \gamma}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \varphi_\pi(x) T^H_{\gamma \pi}(x, Q^2, \mu'). \]

This equation is obtained assuming factorization of the
pion distribution amplitude \( \varphi_\pi(x) \) and the hard scatter-
ing amplitude \( T^H_{\gamma \pi}(x, Q^2, \mu') \) given by [33, 37]

\[
T^H_{\gamma \pi}(x, Q^2, \mu') = T^H_1(x, Q^2, \mu') + T^H_2(x, Q^2, \mu'),
\]

where \( x = 1 - x \), \( x \) is the longitudinal momentum fraction
carried by the quark in the meson and \( \mu' \) is an arbitrary
momentum scale which separates the hard and soft mo-
menta regions.

The hard-scattering amplitude \( T^H_\gamma(x, Q^2, \mu') \) must be
symmetrized under exchange \( x \leftrightarrow \bar{x} \)

\[
T^H_\gamma(x, Q^2, \mu') = T^H_1(\bar{x}, Q^2, \mu'),
\]

and at the next to leading order \( T^H_1(x, Q^2, \mu') \) is given by [48, 49]

\[
T^H_1(x, Q^2, \mu') = \frac{1}{x Q^2} \left[ 1 + \frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \times A(x, Q^2, \mu') \right],
\]

where

\[
A(x, Q^2, \mu') = \left[ \frac{1}{2} \ln^2 x - \frac{x \ln x}{2\bar{x}} \right] - \frac{9}{2} + \frac{3}{2} + \ln x \ln \left( \frac{Q^2}{\mu'^2} \right). \]

For simplicity we set \( \mu' = Q \) and \( T^H_1(x, Q^2, \mu') \) can be written as

\[
T^H_1(x, Q^2, \mu') = \frac{1}{x Q^2} \left[ 1 + \frac{4}{3} \frac{\alpha_s(Q^2)}{4\pi} f(x) \right]. \]
where \( f(x) \) is given by
\[
f(x) = \ln^2 x - \frac{x \ln x}{x} - 9. \tag{19}
\]

As emphasized by Radyushkin \[5\], the finite size \( R \approx 1/M \) of the pion should provide a cut-off for the \( x \) integral. Therefore the \( xQ^2 \) in the denominator of Eq.\((18)\) should be changed as
\[
xQ^2 \to xQ^2 + M^2(xQ^2) . \tag{20}
\]

In principle the factor \( M \) should be related to the dynamical quark mass. It was also proposed by Radyushkin that \( M \) could be treated as an effective gluon mass. Indeed the meson radius may have a deep connection with the effective gluon mass as discussed in \[10\], and in the following we will assume \( M(Q^2) \equiv M_g(Q^2) \). Therefore, no matter we have one case or another, the asymptotic transition form factor will be giving by
\[
F_{\pi\gamma\gamma}(0; Q^2 \to \infty, 0) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \frac{\varphi^*(x)}{xQ^2 + M_g^2}. \tag{21}
\]

\( M_g \), being a dynamical mass, should have a momentum dependence showing the decrease of the mass with the momentum. However when \( xQ^2 \) is small we can safely substitute \( M_g(xQ^2) \) by the infrared \( M_g \) value in Eq.\((21)\), and for large \( xQ^2 \) the value of \( M_g(xQ^2) \) is negligible compared to \( xQ^2 \).

Our result for the pion transition form factor, using Eq.\((11)\) and the hard-scattering amplitude at next-to-leading order is shown in Fig.\((1)\), where it is possible to see a reasonable agreement with the BaBar data. Note that the introduction of the NLO correction is important for this agreement.

\[ \text{FIG. 1. Pion transition form factor calculated with the flat pion distribution of Eq.\((11)\) considering dynamical quark and gluon masses given respectively by 250 and 600 MeV. We also plot the Radyushkin result \[5\] with a 700 MeV gluon mass.} \]

IV. THE PION FORM FACTOR

The pion form factor \( F_\pi(Q^2) \) is also going to be changed if the pion DA is flatter than the usual asymptotic form. As already discussed in Ref.\[51\] the QCD prediction for the form factor is also dependent on the IR non-perturbative behavior of the gluon propagator and of the running coupling constant \[22\]. Therefore we will now compute \( F_\pi(Q^2) \) with the new DA discussed above and also with improved non-perturbative results for the gluon propagator and coupling constant. The asymptotic form factor is predicted by perturbative QCD \[22\ \[53\]. It depends on the internal pion dynamics that is parametrized by the quark distribution amplitude of the pion. The QCD expression for the pion form factor is \[17\]
\[
F_\pi(Q^2) = \frac{f_\pi^2}{12} \int_0^1 dx \int_0^1 dy \varphi^*(y, \bar{Q}_y) \times T_H(x, y, Q^2) \varphi(x, \bar{Q}_x) , \tag{22}
\]

where \( \bar{Q}_x = \text{Min}(x, 1-x)Q \) and \( Q \) is the 4-momentum in Euclidean space transferred by the photon. The function \( \varphi(x, \bar{Q}_x) \) is the momentum dependent pion DA, that gives the amplitude for finding the quark or antiquark within the pion carrying the fractional momentum \( x \) or \( 1-x \), respectively. \( T_H(x, y, Q^2) \) is the hard-scattering amplitude that is obtained by computing the quark-photon scattering diagram as shown in Fig.\(2\).

\[ \text{FIG. 2. The leading-order diagrams that contribute to the pion form factor. \( \varphi(x, \bar{Q}_x) \) is the pion wave function, that gives the amplitude for finding the quark or antiquark within the pion carrying the fractional momentum \( x \) or \( 1-x \). The photon transfers the momentum \( q' \) (in Minkowski space), \( Q^2 = -q'^2 \), for the } \bar{q}q \text{ pair of total momentum } P \text{ producing a } q\bar{q} \text{ pair of final momentum } P'. \]

The lowest-order expression of \( T_H(x, y, Q^2) \) is given by
\[
T_H(x, y, Q^2) = \frac{64\pi}{3} \left[ \frac{2}{3} g_s(K^2)D(K^2) + \frac{1}{3} \alpha_s(P^2)D(P^2) \right] . \tag{23}
\]
where \( K^2 = (1 - x)(1 - y)Q^2 \) and \( P^2 = xyQ^2 \). Here \( D(K^2) \) is related to the perturbative QCD gluon propagator that, in the Landau gauge, is given by

\[
D_{\mu\nu}(q^2) = \left( \delta_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) D(q^2), \quad D(q^2) = \frac{1}{q^2}. \tag{24}
\]

In our analysis the perturbative \( D(q^2) = \frac{1}{q^2} \) is now substituted by the non-perturbative (in Euclidean space) expression

\[
D(q^2) = \frac{1}{q^2 + M_g^2(q^2)}, \tag{25}
\]

where \( M_g(q^2) \) is the dynamical gluon mass which is roughly given by \[54, 55\], \( M_g^2(q^2) \approx M_g^2/(q^2 + M_g^2) \). Since this mass decays very fast with the momentum our calculations are not affected if we just assume \( M_g^2(q^2) \approx M_g^2 \), as we took for granted in the previous section.

The inclusion of radiative corrections in the hard-scattering amplitude imply that \( T_H(x, y, Q^2) \) has to be multiplied by the factor \[48\]

\[
[1 - \frac{5 \alpha_s(Q^2)}{6 \pi}]. \tag{26}
\]

Note that in our calculations we are including the radiative corrections in the hard-scattering amplitude, and assume that factorization happens at a scale \( Q^2 > 1 \text{ GeV}^2 \).

The result for the electromagnetic pion form factor is shown in Fig. (3), where it is compared to a simple fit to the experimental data \[56\]:

\[
F_\pi^\ell H(Q^2) = 0.46895 \left( 1 - \frac{0.3009}{Q^2} \right), \tag{27}
\]

although this is a quite naive fit, which does not include one of the highest energy data. It is clear that more data is necessary in order to check the high energy behavior of the pion form factor, but it is quite interesting that the high energy behavior of the electromagnetic form factor seems to be reasonably described by the same factors (pion DA and dynamical masses) that we considered previously.

\section{V. HARD EXCLUSIVE TWO PHOTON PRODUCTION OF A PION PAIR}

The helicity amplitudes for a pion pair production in exclusive two photon collisions at high energies and large center of mass scattering angles \( \theta_{cm} \) is given by

\[
\mathcal{M}^{\lambda \lambda'} = \int_0^1 dx \int_0^1 dy \varphi^*(x, \hat Q_x) \varphi^*(y, \hat Q_y) T_H^{\lambda \lambda'}(x, y, Q^2), \tag{28}
\]

where \( \hat Q_x = \min(x, 1 - x) \sqrt{s} \sin \theta_{cm} \), similarly for \( \hat Q_y \), and \( s = W_{\gamma \gamma}^2 \) is the square of the cm energy of the two-photon system. \( T_H^{\lambda \lambda'}(x, y, Q^2) \) is the helicity dependent perturbative hard scattering amplitude for two pion production. The spin-averaged cross section for producing the pion pair is

\[
\frac{d\sigma}{dz} = \frac{1}{32\pi s} \langle |\mathcal{M}|^2 \rangle, \tag{29}
\]

with

\[
\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \sum_{\lambda \lambda'} \left| \mathcal{M}^{\lambda \lambda'} \right|^2. \tag{30}
\]

and \( z = \cos \theta_{cm} \). The hard scattering amplitudes (in leading order) for the different helicity structures are given by \[17\]

\[
\begin{align*}
T_H^{00}(++) &= \frac{16\pi\alpha_s}{3s} \frac{32\pi\alpha}{x(1 - x)y(1 - y)} \\
&\times \left[ (e_1 - e_2)^2 a \right] \left( 1 - z^2 \right), \\
T_H^{00}(--) &= \frac{16\pi\alpha_s}{3s} \\
&\times \frac{32\pi\alpha}{x(1 - x)y(1 - y)} \left[ \frac{(e_1 - e_2)^2 a}{1 - z^2} + \frac{e_1 e_2 [x(1 - x) + y(1 - y)]}{a^2 - b^2 z^2} \right],
\end{align*}
\tag{31}
\]

where \( e_i \) are the quark charges (meaning that the pions have charges \( \pm (e_1 - e_2) \)) and

\[
\begin{array}{c}
a \\
b
\end{array} = (1 - x)(1 - y) \pm xy. \tag{33}
\]

In order to restrain the calculation at the perturbative QCD level we can multiply the right side of Eq. (28) by
the following form factor, which smoothly switches off the pQCD contribution at low energies \[ F_{pQCD}(s) = 1 - \exp\left(-\frac{(s - 4m^2)^4}{\Lambda_{pQCD}^8}\right) \] (34)

In Fig. (4) we plot the total cross section for hard exclusive two photon production of a charged pion pair. Again our results seem to be in agreement with the existent data when calculated with the same parameters used in the previous sections.

![Graph](image)

FIG. 4. Total cross section for pion pair exclusive production. Results are also computed with the pQCD contribution suppressed by the form factor given in Eq. (34).

![Graph](image)

FIG. 5. Differential cross section for pion pair exclusive production, compared with experimental data at different energies.

Within the same approach we can compute the differential cross section for exclusive pion pair production. The existent models, the BL one and the one of Ref. [58], are not fully in agreement with the experimental data. This cross section is plotted in Fig. (5) and we verify that at least for large photon pair energy, where we do expect that perturbative QCD can describe the experimental data, our calculation is consistent with the known experimental results. Unfortunately it is still a challenge the full explanation of the experimental data within perturbative QCD, i.e. if we have already arrived at the high energy frontier in this particular case.

VI. DISCUSSION AND CONCLUSIONS

The BaBar results for the pion transition form factor suggested many authors to propose a flat pion distribution amplitude in order to describe the data. In Ref. [25] we proposed that only a very hard BSE solution (in momentum space) for the pion wave function can generate such flat DA. We computed the DA as a function of this type of solution of quark self-energy, which is related to the pion wave function, and our main intention in this work was to verify how this DA describe the experimental data. We stress that, as far as we know, only a very hard (in momentum space) quark self-energy can lead to a natural explanation of a flat pion DA within first QCD principles.

We computed the pion transition form factor, the pion form factor and the exclusive photoproduction of charged pion pairs at high energies with the DA determined in Section II. Following Radyushkin [5] we have assumed that QCD corrections barely affect such flat DA, however the QCD corrections in the hard scattering amplitudes seem to be necessary for a better description of the experimental data. All quantities were computed with the same parameters used to determine the DA, i.e. dynamical quark and gluon masses, providing a consistent picture of pions exclusive production.

In principle we did not may expect that the quark self-energy, or the similar pion wave function, should follow exactly the behavior of Eq. (8), and the results that we could obtain with our “almost” flat DA would just give an extreme limit to the physical quantities that we have computed. However the description of the data is quite reasonable and seems to indicate that the pion wave function may be well approximated at large momentum by the behavior of Eq. (8).

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