Source separation and localisation via tensor decomposition for distributed arrays

Yuanbing Cheng\textsuperscript{1} E-mail: chengyb@yeah.net, Yapeng He\textsuperscript{2} \\
\textsuperscript{1}Nanjing Research Institute of Electronics Technology, Nanjing, People’s Republic of China \textsuperscript{2}China Academy of Space Technology, Xi’an Branch, Xi’an, People’s Republic of China

Abstract: This study focuses on the problem of power spectra separation and localisation of multiple sources using distributed arrays. First, the array structure and signal model are discussed. By cross-correlating the multi-channel received signals in time-domain, a third tensor is constructed. Then, utilising the multi-dimensional characteristic, the tensor is decomposed to separate the array manifold matrix and the power spectra matrix through alternating least square (ALS) method. Finally, the sources are located using the relative $x$-$y$ plane relationship between the distributed arrays and the direction of arrival (DOA), which can be estimated by spectrum analysis of each column of the array manifold matrix. The effectiveness and superiority of the proposed method is demonstrated by simulation results.

1 Introduction

Source separation and localisation are widely used to solve the problem of spectrum management in cognitive radio and localise the active emitter in electronic warfare and radar electronic countermeasure [1–4]. Spectrum sensing is the first step towards unique in general; hence, the identification of the mixed spectra received signals in time-domain, a third tensor is constructed. Then, utilising the multi-dimensional characteristic, the tensor is decomposed to separate the array manifold matrix and the power spectra matrix through alternating least square (ALS) method. Finally, the sources are located using the relative $x$-$y$ plane relationship between the distributed arrays and the direction of arrival (DOA), which can be estimated by spectrum analysis of each column of the array manifold matrix. The effectiveness and superiority of the proposed method is demonstrated by simulation results.

There is a lot literature on spectrum separation. A parallel joint power spectra separation and localisation of multiple sources using distributed arrays is constructed by cross-correlating the multi-channel received signals, but in cognitive radio and passive situation awareness. The radio frequency environment can be mapped out to highly efficient reuse in space and time by identifying the power spectra and locations of the active emitters, including enemy radars or jammers.

There is a lot literature on spectrum separation. A parallel joint power spectra separation and localisation of multiple sources using distributed arrays is constructed by cross-correlating the multi-channel received signals, but in cognitive radio and passive situation awareness. The radio frequency environment can be mapped out to highly efficient reuse in space and time by identifying the power spectra and locations of the active emitters, including enemy radars or jammers.

1 Introduction

Source separation and localisation are widely used to solve the problem of spectrum management in cognitive radio and localise the active emitter in electronic warfare and radar electronic countermeasure [1–4]. Spectrum sensing is the first step towards unique in general; hence, the identification of the mixed spectra received signals in time-domain, a third tensor is constructed. Then, utilising the multi-dimensional characteristic, the tensor is decomposed to separate the array manifold matrix and the power spectra matrix through alternating least square (ALS) method. Finally, the sources are located using the relative $x$-$y$ plane relationship between the distributed arrays and the direction of arrival (DOA), which can be estimated by spectrum analysis of each column of the array manifold matrix. The effectiveness and superiority of the proposed method is demonstrated by simulation results.

1 Introduction

Source separation and localisation are widely used to solve the problem of spectrum management in cognitive radio and localise the active emitter in electronic warfare and radar electronic countermeasure [1–4]. Spectrum sensing is the first step towards unique in general; hence, the identification of the mixed spectra received signals in time-domain, a third tensor is constructed. Then, utilising the multi-dimensional characteristic, the tensor is decomposed to separate the array manifold matrix and the power spectra matrix through alternating least square (ALS) method. Finally, the sources are located using the relative $x$-$y$ plane relationship between the distributed arrays and the direction of arrival (DOA), which can be estimated by spectrum analysis of each column of the array manifold matrix. The effectiveness and superiority of the proposed method is demonstrated by simulation results.
station, $\theta_{kn}^p$ is the DOA of the $k$th source to the $n$th sub-array in the $p$th station, $d$ denotes the distance between neighbouring received antennas, $\lambda$ is the wavelength, $s_k(t)$ is narrowband transmitted signal of the $k$th source. $z^p(n,t)$ is the noise term. The $N$ sub-arrays in each station are closed spaced, which means $\theta_{kn}^p = \theta_{kn}^p$, hence $h_{nk}^p$ can be replaced by $h_{kn}^p = [1, e^{-j2\pi \sin(\delta_k/n)}, \ldots, \frac{e^{-j2\pi \sin(\delta_k/n)}}{N}]^T$. Then (1) can be written as

$$x^p(n,t) = \sum_{k=1}^{K} \beta_{nk}^p h_{kn}^p s_k(t) + z^p(n,t) \tag{2}$$

Our aim is to estimate power spectra and the DOAs of all $K$ sources, denoted by $\{s_k(f), \theta_{kn}^p\}$ with $k = 1, 2, \ldots, K$ and $p = 1, 2, \ldots, P$.

### 3 Proposed approach

#### 3.1 Temporal correlation

To estimate power spectra and the DOAs of all $K$ sources, we propose to formulate the problem in the temporal correlation domain. The $K$ sources are uncorrelated, so the cross-correlation of the $m$th and $n$th antennas of the $n$th sub-array in the $p$th station is

$$R^K_{n,n}(\tau) = E[{x(n, m, t) \cdot x(n, m, t - \tau)}^*] \tag{3}$$

where $q = m - m_0, 1 \leq m_0, m \leq M$. $r_1(\tau) = E[s_k(t) s_k(t - \tau)]$ is auto-correlation of $s_k(t)$, $\delta(x)$ denotes the Kronecker delta function, i.e. $\delta(x) = 1$ when $x = 0$ and $\delta(x) = 0$ otherwise. Assuming that the noise at each antenna is white Gaussian, both temporally and spatially, with zeros mean and variance $\sigma^2$, i.e. $z(n,t) \sim N(0, \sigma^2)$ for all $p$ and $n$. Then, (3) can be expressed as

$$R^K_{n,n}(\tau) = \sum_{k=1}^{K} |\beta_{nk}^p|^2 e^{-j2\pi \sin(\delta_k/n)} R_1(\tau) + \sigma^2 \delta(\tau) \delta(q) \tag{4}$$

Applying discrete time Fourier transform on (4), we have

$$G^K_{n,n}(q,f) = \sum_{k=1}^{K} |\beta_{nk}^p|^2 e^{-j2\pi \frac{q \sin(\delta_k/n)}{N}} \delta_k(f) + \sigma^2 \delta(q) \tag{5}$$

The frequency axis in (5) can be discretised as

$$G^K_{n,n}(q,f) = A^K D^K f^T + Z_n^p \tag{6}$$

where $G^K_{n,n}$ is the received array manifold of the $n$th station, $A^K = [\alpha_1, \alpha_2, \ldots, \alpha_K]$ is the received array manifold of the $p$th station, and $\alpha = e^{j2\pi \sin(\delta_k/n)}$. $D^K$ denotes a diagonal matrix with $d^K = [1, 2, \ldots, K]$. $S$ is the PS matrix, $S_k \in R^{K \times K}$ is the PS of the $k$th source, $\Lambda$ is the noise term. Let $\mathbf{G}_n = [g_{n1}, g_{n2}, \ldots, g_{nP}] \in C^{PQ \times F}$ and $g_n = \text{vec}(\mathbf{G}_n)$, stacking one after another, we have

$$H = [h_1, h_2, \ldots, h_P]^T = (A \oplus S) \mathbf{H}^T + Z \tag{7}$$

where $\mathbf{H} \in C^{PQ \times NP}$, $A = [A^1, A^2, \ldots, A^K] \in C^{PQ \times K}$, $B = [\beta_1, \beta_2, \ldots, \beta_K] \in R^{PQ \times K}$, and $\beta_k = \text{vec}(\mathbf{B}_k)$, $\mathbf{B}_k$ is the received path loss matrix with the $|\beta_{nk}^p|$ on the $n$th row and the $p$th column. $Z \in C^{PQ \times NP}$ is the noise term.

#### 3.2 Parameter matrix estimation via tensor decomposition

**Definition 1:** The $n$-mode matrix unfolding of an $N$-order tensor $X \in C^{I_1 \times I_2 \times \cdots I_N}$, denoted by $X_{ni} \in C^{1 \times I_2 \times \cdots I_N}$, contains the $(i_1, i_2, \ldots, i_N)$th element of $X$ at the position $(i_{ni})$, where $j = 1 + \sum_{k=1}^{N} (i_k - 1)I_k$ with $I_k$. On the contrary, given $X_{ni}$, $X$ is constructed. Each third-order tensor can be recognised as a data cube, and its matrix unfoldings consist of the slices of the cube along different directions. Using definition 1, a third-order-tensor $\mathbf{H}$ can be formed by $\mathbf{H}$ in (7) [5]. The signal tensor $\mathbf{H}$ and its matrix unfoldings are given in Fig. 2. The matrix unfolding of $\mathbf{H}$ is rearranged by concatenating $NP$ consecutive $PQ \times F$ matrix slices by fixing the third index to successive values $n = 1, 2, \ldots, NP$. $\mathbf{H}_1$ and $\mathbf{H}_2$ are obtained in similar way.

It is clear that $\mathbf{H}$ is a noisy parallel factor (PARAFAC) model [6], written in the third mode matrix unfolding of the third-order tensor $\mathbf{H} \in C^{NP \times PQ \times F}$. Construct $\mathbf{H}$ using $\mathbf{H}_1$ and decompose $\mathbf{H}$ in the other two modes, we have

$$\begin{align*}
\mathbf{H}_1 &= (B \oplus A)'S^T + Z_1 \\
\mathbf{H}_2 &= (S \oplus B)'A^T + Z_2 \\
\mathbf{H}_3 &= (A \oplus S)'B^T + Z_3
\end{align*} \tag{8}$$

$\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{H}_3$ in (8) are equivalent and merely different formulations of the same model. Given that the number of the sources is known or has been estimated, a third tensor can be fitted to $\mathbf{H}$ by minimisation of the cost functions.
The minimise problem in (9) can be solved via ALS method [7], assuming the other two are known in least squares (LS) principle. Some line search methods can be used to speed up convergence. Let \( A, B, S \), update
\[
\hat{S} = [(\hat{B} \odot \hat{A})]_H \quad \text{T} \tag{10}
\]
Given \( \hat{S}, \hat{B} \), update
\[
\hat{A} = [(\hat{S} \odot \hat{B})]_H \quad \text{T} \tag{11}
\]
Given \( \hat{A}, \hat{S} \), update
\[
\hat{B} = [(\hat{A} \odot \hat{S})]_H \quad \text{T} \tag{12}
\]
In (10)–(12), the pseudo inverse of the Khatri-Rao product is calculated by [5] \((X \odot Y)^{-1} = (X^H Y^{-1})^H (X \odot Y)^{-1}\). It can be seen that the estimated matrix unfolding \( \hat{S} \) contains the power spectra of the sources, and \( \hat{A} \) contains the DOAs.

Remark 1: Using line search to accelerate the convergence of ALS. Mostly, ALS needs a large number of iterations before converging. The slowness in convergence can be due to the large size of the tensor, or to the bad starting values. Line search is an effective solution proposed to cope with the problem of slow convergence. Some line search methods can be used to speed up ALS for searching the global minimum very quickly. Refer to [8, 9] for detail.

3.3 DOA estimation and source localisation
Let \( \hat{a}_k = [\hat{a}_{k1}, \hat{a}_{k2}, \ldots, \hat{a}_{kp}] \) be the \( k \)th column of \( \hat{A} \), where \( \hat{a}_{p,k} = [a_{p,k}^{(M-1)} \ldots , a_{p,k}^{(M-2)} \ldots , a_{p,k}^{(M-1)}]^T \in \mathbb{C}^{q \times 1} \) and \( \hat{a} = e^{i(2\pi \sin(\theta) / \lambda)} \). The DOA of the \( k \)th source respects to the \( p \)th station can be estimated via spectrum analysis of \( \hat{a}_{p,k} \). In this paper, root-MUSIC [10], one of the classical spectrum analysis methods, is used. Calculate the noise subspace \( U \) of the covariance matrix \( R = \hat{a}_{p,k} \hat{a}_{p,k}^H \) via eigen decomposition. Then the \( q \)th coefficient of polynomial equals the sum of the \((q - Q)\)th diagonal of \( G = U U^H \) with \( q = 1, 2, \ldots, (2Q - 1) \). Then we have \( \hat{\theta}_k^p = \arcsin(\hat{\rho}_k/(2\pi)) \), where \( \rho \) is the root of the polynomial that is nearest and inside the unit circle.

The \((x, y)\) location of the \( k \)th source can be estimated using the relative triangular relationship of the \( P \) stations and the DOAs \( \hat{\theta}_k^p \), where \( p = 1, 2, \ldots, P \).

3.4 Steps of the proposed algorithm
The proposed algorithm mainly includes three steps: (i) according to the temporal correlation domain, get the signal matrix \( H \); (ii) Utilising the multi-dimension structure, construct the tensor \( H \), and decompose the tensor by ALS; and (iii) Estimate DOAs from manifold matrix and localise the sources. The outline of the proposed algorithm is summarised in Algorithm 1 (Fig. 3).

4 Simulations
In this section, we use computer simulations to demonstrate the effectiveness of the proposed algorithm.

Case 1: Assuming there are \( P = 3 \) stations in the distributed network configuration, each station has \( N = 6 \) closed spaced sub-arrays, each sub-array is a half-wavelength spaced ULA with

\[
\begin{align*}
\min_{A, B, S} & \quad \epsilon_1 = \left\| H_1 - (B \oplus A) S^T \right\|_F^2 \\
& \quad \epsilon_2 = \left\| H_2 - (S \oplus B) A^T \right\|_F^2 \\
& \quad \epsilon_3 = \left\| H_3 - (A \oplus S) B^T \right\|_F^2 \tag{9}
\end{align*}
\]

\[
\begin{align*}
(14) & \quad \epsilon = \left\| H_1 - (B \oplus A) S^T \right\|_F^2 \\
(11) & \quad \epsilon = \left\| H_2 - (S \oplus B) A^T \right\|_F^2 \\
(10) & \quad \epsilon = \left\| H_3 - (A \oplus S) B^T \right\|_F^2
\end{align*}
\]

\[ M = 8 \] received antennas. The \((x, y)\) locations of the three stations are \((-10, 0)\)Km, \((0, 0)\)Km and \((10, 0)\)Km. The transmit signals of \( K = 3 \) sources are all stationary narrowband Gaussian processes with mean zero. The normalised frequency band zones of three sources are \([0.1, 0.3], [0.4, 0.5], [0.6, 0.7]\). The \((x, y)\) locations of the three sources are \((-15, 52)\)Km, \((15, 45)\)Km, and \((45, 25)\)Km. The band of interest is discretised into \( F = 256 \) frequency bins. The SNRs of the three sources are all \( 10 \) dB.

Fig. 4 shows the estimated power spectra of the sources by the proposed method. The results are obtained by averaging 50 trials, to enable easy visual assessment of estimate variance, including leakage from one source to the others. We see that the proposed method identifies the power spectra of all three sources fairly well.

Case 2: Fig. 5 shows the accuracy of the estimation power spectra and localisation versus SNR using the proposed method under different number of stations \( P = 2, 3, 5 \). Under \( P = 5 \), the \((x, y)\) locations of the stations are \((-20, 0)\)Km, \((-10, 0)\)Km, \((0, 0)\)Km, \((10, 0)\)Km, and \((20, 0)\)Km. Under \( P = 3 \) and \( P = 2 \), the first 3 and 2 stations are used, respectively. The other parameters are the same as Case 1. The number of Monte-Carlo trials is 1000. The root-mean-square error (RMSE) of the estimated power spectra denoted by \( \text{RMSE}_{PS} \) and the RMSE of the estimated \((x, y)\) position denoted by \( \text{RMSE}_{x-y} \) are adopted as the performance measure, defined as

\[
\begin{align*}
\text{RMSE}_{PS} & = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \left\| S_k \right\|_F^2 \left\| \hat{S}_k - S_k \right\|_F^2 } \tag{13} \\
\text{RMSE}_{x-y} & = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \left\| (x_k - \hat{x}_k)^2 + (y_k - \hat{y}_k)^2 \right\| } \tag{14}
\end{align*}
\]

It can be seen in Fig. 5, the proposed method can jointly separate PS and localise the \((x, y)\) position of multiple sources. \( \text{RMSE}_{PS} \) and \( \text{RMSE}_{x-y} \) both reduce as the SNR increases. The proposed method yields reasonable \( \text{RMSE}_{PS} \) of PS even under lower SNR, e.g. <
−10 dB under SNR = 0 dB. One can see that the RMSE \( x-y \) is improved significantly when more stations are used.

5 Conclusion

The problem of joint power spectra separation and localisation of multiple sources for distributed arrays has been considered in this paper. Utilising the temporal correlation domain, this problem is formulated as a third-order tensor decomposition problem. The tensor and its matrix unfoldings are discussed. ALS method is used to solve the tensor, which results the manifold of each array and the power spectra matrix. The DOAs of each source respected to all arrays can be estimated by imposing root-MUSIC method on the estimated array manifold. Simulations illustrated the accuracy and efficacy of the proposed techniques.

6 References

[1] Axell, E., Leus, G., Larsson, F.G.: 'Spectrum sensing for cognitive radio: state-of-the-art and recent advances', IEEE Signal Process. Mag., 2012, 29, (3), pp. 101–116
[2] Fanzi, Z., Li, C., Tian, Z.: ‘Distributed compressive spectrum sensing in cooperative multiphop cognitive networks’, IEEE J. Sel. Top. Signal Process., 2011, 5, (1), pp. 37–48
[3] Bazerque, J.A., Giannakis, G.B.: ‘Distributed spectrum sensing for cognitive radio networks by exploiting sparsity’, IEEE Trans. Signal Process., 2010, 58, (3), pp. 1847–1862
[4] Hu, Z., Ranganathan, R., Zhang, C., et al.: ‘Robust non-negative matrix factorization for joint spectrum sensing and primary user localization in cognitive radio networks’, IEEE Conf. Waveform Diversity and Design, Kauai, HI, USA, January 2012, pp. 22–27
[5] Liu, S.Z., Trenkler, G.: ‘Hadamard, khatri-rao, kronecker and other matrix products’, Int. J. Inf. Syst. Sci., 2008, 4, (1), pp. 160–177
[6] Sidirooulos, N., Lathauwer, L.D., Fu, X., et al.: ‘Tensor decomposition for signal processing and machine learning’, IEEE Trans. Signal Process., 2017, 65, (13), pp. 3551–3582
[7] Kolda, T.G., Bader, B.W.: ‘Tensor decomposition and application’, SIAM Rev., 2009, 51, (3), pp. 455–500
[8] Nion, D., Lathauwer, L.D.: ‘An enriched line search scheme for complex-valued tensor decompositions. Application in DS-CDMA’, Signal Process., 2008, 88, (3), pp. 749–755
[9] Sorber, L., Domanov, I., Barel, M.V., et al.: ‘Exact line and plane search for tensor optimization’, Comput. Optim. Appl., 2010, 46, (1), pp. 121–142
[10] Ren, Q.S., Willis, A.J.: ‘Fast root-MUSIC algorithm’, IEEE Electr. Lett., 1997, 33, (6), pp. 450–451

Fig. 4 Estimated power spectra (PS) of three sources by the proposed approach averaging 50 trials
(a) PS of the 1st source, (b) PS of the 2nd source, (c) PS of the 3rd source

Fig. 5 RMSEs of the estimated \((x, y)\) location and power spectrum under various SNRs with \(P = 2, 3, 5\)
(a) RMSE of the \((x, y)\) location, (b) RMSE of the power spectrum

J. Eng., 2019, Vol. 2019 Iss. 20, pp. 6616-6619
This is an open access article published by the IET under the Creative Commons Attribution License
(http://creativecommons.org/licenses/by/3.0/)