A testability modeling method based on structure-function-state-test

Peng Wang¹,*¹, Xingxin Li¹, Yongli Yu¹, Chenguang Liu¹ and Xibin Luo²

¹ Army Engineering University of PLA, Shijiazhuang 050003, Hebei, China; ² The Second Test Area of Army Experimental Training Base, Huayin 714200, Shanxi, China

*Corresponding author. Email: wpclever@126.com

Abstract. Aiming at the testability analysis of multi-state system, a testability modeling method based on structure-function-state-test is proposed. Based on the hierarchical structure analysis of the system, taking the function modeling of the structure as the core, the performance, fault and state of the function are described, and the test correlation of the system fault and state is studied. Through the analysis of a typical case, the results show that the model can: 1. Realize the system fault test correlation analysis considering the uncertain factors; 2. Realize the state test correlation analysis of the multi state system; 3. Realize the testability analysis of the multi state system by calculating the fault detection rate, fault isolation rate and state detection rate.

1. Introduction

At present, the common testability models are mainly built on the basis of correlation model, such as logic model, structure model, signal flow model, multi-signal model and hybrid diagnosis model [1]. Among them, multi signal model is a testability model developed by American scholars pattipati and DEB on the basis of in-depth analysis and summary of the advantages and disadvantages of logic model, structure model and information flow model [2]. Because the multi-signal model describes the actual structure and function connection of the system in the form of directed graph, it is simple to model and easy to implement in engineering, so it has been widely used [3-5]. The hybrid diagnosis model is similar to the multi signal model in modeling, but the main difference is that the hybrid diagnosis model considers the hybrid diagnosis reasoning of system fault and function. Multi signal model and hybrid diagnosis model are suitable for describing the relationship between system fault and test qualitatively, which is difficult to describe quantitatively, and both of them take test reliability as the potential assumption. The relationship between fault and test described in the static state of the system can not describe the dynamic process of fault generation and propagation.

To solve these problems, some scholars have studied the modeling and analysis of multi signal model under uncertain information from the aspects of fault propagation uncertainty [6,7] and test uncertainty [7-9]. Although the problem of uncertainty description based on multi signal model has been improved, the multi signal model and hybrid diagnosis model are based on the structure model, and the modeling analysis is carried out through the functional connection between the structure and the structure. However, this modeling method is not reasonable in many cases because the internal functional logic relationship of the system is not the same when it realizes different functions. For example, in a switch system with parallel structure, when the blocking function is realized, its
functional logic should be in series. When the interconnection function is realized, its functional logic should be in parallel, while the multi signal model or hybrid diagnosis model can only express one of them. This is because the multi signal model only models the switch unit itself, ignoring the two different functional relationship models of the switch, and the connection between the units depends on the functional association, which leads to the unreasonable modeling.

Other testability models, such as Dai Jing et al. [10], aiming at the requirements of design for testability (DFT) of aviation electromechanical system, put forward the system testability modeling and analysis method based on object-oriented Bayesian network (OOBN), which can get the importance analysis of a test to the state information quantity of the system, but the test based on the information entropy theory is only the result compared with other tests, although it can get the best single test, but it can not explain the specific state information obtained by the test, so it is difficult to judge the merits of the whole test set. Zhang Yong et al. [11] proposed an integrated model of function fault behavior test environment, which establishes quantitative input-output relationship for each layer of functional units, so as to realize the quantitative analysis of system testability knowledge. It is difficult to describe the relationship between the multi state characteristics of the system and the test due to the neglect of the relationship between the functions and states of different components and the impact on the test. Shi Junyou [12] and Lu Xiaoming [13] have improved the application of the correlation model.

The testability model can describe the relationship between system fault and test, but its test analysis process is based on two-state system, ignoring the multi state characteristics of the system. Multi state system considers one or more degradation states of system and components from normal to fault, and can describe the relationship between state and component state, state and system fault through quantitative transition probability. Multi state information is very important to reveal the potential failure mechanism and rule of complex system, which is helpful to improve the efficiency of system fault diagnosis and prediction and reduce the cost Diagnosis cost [14]. At present, based on the multi state system model, the research mainly focuses on the multi state system reliability modeling and analysis [15-19], reliability optimization design [20-23], maintenance management strategy [24-27], etc. these researches analyze the state transition relationship of the system, and obtain the state distribution of the system combined with the initial state, so as to analyze, optimize and control the reliability of the system. However, the research on test optimization of multi-state system is relatively less.

Based on the above problems, this paper proposes a multi state system test model based on structure function state test. In order to make the model close to the real structure of the system, based on the hierarchical structure model of the system, the function of the system unit is quantitatively modeled, and the connection between the functions is established according to the logical relationship between the functions. Through function modeling and performance definition, the model describes the fault and state of the system in a unified way, so as to establish the probability correlation between system fault and test and the correlation between function state and test.

2. Structure-function-state-test modeling

2.1. Modeling considerations

In order to realize the function centered description of the relationship between fault, state and test, the structure-function-state-test model (SFSTM) is constructed. Firstly, it is necessary to make clear the modeling requirements.

Function modeling requirements: a structure may have multiple functions, and multiple structure components may also realize the same function. For the convenience of analysis and description, when a structure contains multiple functions, each function is modeled on the basis of the structure; when multiple structures realize one function, they are not merged, but their function models are still established for each structure. However, when multiple structures realize one function, they are not merged. After that, the functional relationship is modeled and described. The description of function
should reflect its practical application value. For example, the function of motor should be described as the rated torque provided, not as the conversion of electric energy into kinetic energy. This description method is to facilitate the judgment and test of system state. In order to realize the quantitative analysis of the system, the function modeling should try to realize the quantitative description of the input-output relationship.

Structural modeling requirements: the structural model can reflect the actual composition of the product, which is convenient for engineering implementation. Because the function depends on the structure, the structure modeling reflects the hierarchical division requirements of the system as far as possible, which is convenient to analyze the products at different levels.

Test modeling requirements: in order to facilitate the unified test of function status and fault, the test signal should be able to directly or indirectly reflect the function performance index. Therefore, the test point can be set at the output position of the corresponding function to test the performance signal of the function output.

2.2. Description of the model

The system test model can be described as a directed graph \( G = \{C, CF, FM, E, T, D, \Psi\} \). where \( C = \{c_1, \cdots, c_n\} \) is the finite set of component of the system; \( CF = \{c_{f_1}, c_{f_2}, \cdots, c_{f_a}\} \) is the finite set of all functions of the component units of the system. if \( c_x \) has \( c_{f_x} \) functions, then \( a = \sum_{x=1}^{n} c_{f_x} \); \( FM = \{f_{m_1}, \cdots, f_{m_k}\} \) is the finite set of fault modes of the system, \( f_{m_k} \) is the kth fault mode of the function \( c_{f_x} \); \( E = \{e(c_{f_{i}}, c_{f_j}) | e(c_{f_{i}} \rightarrow c_{f_j}) \} \) is the set of directed edges from the function \( c_{f_i} \) to the function \( c_{f_j} \), which describes the flow relationship of the system functions; \( T = \{t_1, \cdots, t_k\} \) represents a limited set of k available test points in the system, which are used to test the output signal of a function; \( \Psi = [\varphi_1, \cdots, \varphi_a] \) represents the relationship between the system functions, Where \( \varphi_x \) is the structure function of system function \( c_{f_x} \), which is determined by the system structure and function attributes. It can be added one by one in the modeling process. Through the structure function, the relationship between the top-level function and the bottom-level function of the system can be obtained layer by layer.

2.3. The construction process of SFSTM

The construction process of the model is described as follows:

1) It is necessary to analyze the design information of the system and establish a hierarchical structure model according to the corresponding agreement level. The lowest level can be established to the component level.

2) To analyze the function of each lowest level unit (which may be component or component, determined by the agreement level), and establishes the operation mechanism model of each function of the unit, that is, the description of the relationship between input and output.

3) According to the product function design, add the flow relationship between functions, and connect them with directional arrows.

4) According to the operation mechanism of functions, to analyze the performance attributes of each function, the purpose is to analyze the relationship between functions and describe them with structure function.

5) According to the function operation mechanism, we analyze the output signal of each function, and establish the mapping relationship between the output signal and function status, fault.

6) Add test points and tests. In order to obtain the output signal of the function, the logical position of the test point is set at the output position of each function signal. Accordingly, the test signal is the
output signal of the function. If the function has several kinds of output signals, several kinds of tests can be added.

7) Finally, we need to add the common failure mode of the function, and analyze the quantitative impact of the failure mode.

3. Testability analysis based on SFSTM

3.1. Correlation analysis of fault and test

Firstly, the one-step propagation adjacency matrix of the fault is analyzed, and the fault test correlation row matrix of the whole system can be obtained through the one-step propagation adjacency matrix. The value of adjacency matrix in multi signal model is binary, that is, the propagation probability is 1 or 0, which is unreasonable in many cases. Many functional failures may propagate with a certain probability, rather than a simple 0 or 1. From the function structure function, the one-step propagation probability adjacency matrix can be obtained.

$$A = \{a_{ij}\}$$ is the adjacency matrix of fault propagation probability of system unit, $a_{ij}$ is the probability of function j failure caused by function i failure, when function i and function j are not directly related, $a_{ij} = 0$. According to the location of the system test points and the accessibility matrix, the probability accessibility matrix $D = [d_{ij}]$ of the system unit function failure test can be obtained. The corresponding value of $d_{ij}$ represents the probability of function i fault being detected by test j. Then the fault set of the system can be detected as

$$FD = \left\{ \sum_{i \in CF} d_{ij} \neq 0 \right\}$$ (1)

3.2. Correlation analysis of state and test

In order to establish the relationship between test and function state, set the function set related to test point $i_j$ as $RCF_i = \{c_{f_i}|d_{ij} \neq 0, c_{f_i} \in CF\}$. The test value space of system test point is defined here as

$$H(t_i) = \phi(g_i, \cdots g_n)$$ (2)

Where $\phi(*)$ is the test structure function, which represents the mapping relationship between the performance level space of each function and the test value space. Then the state set that the test can detect satisfies the

$$Hs(t_i) = \left\{ s_{\text{st}} | H(t_i|g_{st}) \cap H(t_i|g_{st}) = \phi, \forall c_{f_m} \in RCF_i, g_{st} \in g_{st}, k \neq p \right\}$$ (3)

Where $H(t_i|g_{st})$ represents the value space of test $t_i$ when function $c_{f_m}$ is in state $g_{st}$. When $t_i = r \in H(t_i)$, it may correspond to multiple functional state combinations of the system, and its corresponding functional state combination space is expressed as

$$L_{t_i} = \{s_{st}, \cdots s_{st}\} \phi(c_{f_m}, \cdots g_{st}) = r$$ (4)

If the elements in the function state combination space contain some common states, then these function states can be detected by test. That is to say, the combination of these function states can be determined when the value of test $t_i$ is $r$. then the detectable combination function state set is expressed as

$$Hcs(t_i) = \{L_{t_i}(1) \cap \cdots \cap L_{t_i}(q) \forall r \in H(t_i)\}$$ (5)

Where $q = |L_{t_i}|$ is the number of state combinations in $L_{t_i}$. 

4
In order to describe the relationship between test and test, let $RCF_i \subset RCF_j$ to show that the function tested by $t_j$ is the low-level function of the function tested by $t_i$. At this time, the test value space of the test point can be calculated, namely

$$H(t_i) = \varphi_{i,j} \left( g_{cf_i}, \cdots, g_{cf_j}, H(t_j) \right)$$  \hspace{1cm} (6)

$\varphi_{i,j}(\bullet)$ is the test structure function from test $t_j$ to test $t_i$.

When the test $t_j$ is known, the combined function state determined by the test $t_i$ can be expressed as

$$\text{Hcs}(t_i, t_j) = \left\{ L^{a,b}_{c,s} \mid H(t_i) = a, H(t_j) = b \right\}$$  \hspace{1cm} (7)

$L^{a,b}_{c,s} = \{ s_{c_1}, \cdots, s_{c_p} \} \mid H(t_i) = a, H(t_j) = b$ is the state combination space of $cf_i, \cdots, cf_j \in RCF_i - RCF_j$ when $t_j = b \in H(t_j)$ and $t_i = a \in H(t_i)$. $l = \left| L^{a,b}_{c,s} \right|$ is the number of state combinations in $L^{a,b}_{c,s}$.

The combination state that can be determined by a test set includes the combination state that can be determined by a single test and the combination state that can be determined by each test Union in the test set. The combination state determined is only related to the low-level test next to the test, because its low-level test value is the result of lower level test value and related functional state. Therefore, the combined function state set determined by the test set can be expressed as

$$\text{Hcs}(T_i) = \bigcup_{t_i \in T_i} \text{Hcs}(t_i) \bigcup_{t_i, t_j \in T_i} \text{Hcs}(t_i, t_j)$$  \hspace{1cm} (8)

Test structure function describes the relationship between test and function state, and between high-level function test and low-level function test. It can ensure that the low-level functional state of the system is convergent in the process of transferring to the high-level functional state through the structure function, so as to avoid the explosion of the system state space.

### 3.3. Testability analysis of multi state system

Testability analysis of multi state system not only needs to test and analyze the fault of the system, but also needs to test and analyze the state of the system. In order to measure the state detection ability of system test, the concept of state detection rate is proposed, which is the ratio of the state occurrence probability determined by the system to the sum of all state occurrence probabilities of the system. The testability analysis of multi state system is transformed into the solution process of system fault detection rate, fault isolation rate and state detection rate.

Fault detection rate is a testability index to describe the ability of testing to detect system faults, which is expressed by the percentage of the number of faults that can be detected to the number of possible faults. If the system fault test dependency matrix is known, the value of the matrix row represents the probability of each function fault being detected by each test, and the maximum value of each row is taken as the probability of each function fault being detected. Therefore, the fault detection rate is

$$FDR = \frac{\sum_{i=1}^{n} p(i) \text{max}_d}{\sum_{i=1}^{n} p(i)}$$  \hspace{1cm} (9)

Where $FD$ represents the set of detectable faults, $p(i)$ represents the probability that the function $cf_i$ is in fault state, and $\text{max}_d$ represents the maximum value of row $i$ in the dependency matrix.

Fault isolation rate is the ratio of the fault that can be isolated to the detected fault. Whether a fault can be isolated or not can be judged by the fault test reachability matrix. When the position of the non-zero element in this line is different from that in other lines, the fault can be isolated.
Define the test set \( RT = \{ t | d_j = 0, t \in T \} \) related to the probability of function \( cf_i \) in the system, and the function fault set that can be isolated is \( FI \). then for any \( cf_i \in FD \), \( cf_j \in FD \), and \( i \neq j \), \( FI = \{ cf_i | RT \oplus RT \neq 0 \} \) is obtained. The fault isolation rate is

\[
FIR = \frac{\sum_{i \in FD} p(i) \max d_i}{\sum_{i \in RT} p(i) \max d_i}
\]  

(10)

Let the system function have \(|L|\) state combination, and the \( l \)th combination of the system is \( S_l = \{ s_{l1}, \ldots, s_{ln} \} \). In this combination, the function set corresponding to the states that can be determined by the system test is \( S_lD \). The state detection rate (SDR) of SL is defined as the proportion of the number of states that can be determined to the total number of states in SL, i.e. \( S_{lDR} = \frac{|S_lD|}{n} \). Then the state detection rate of the whole system is defined as

\[
SDR = \frac{\sum_{l=1}^{\|L\|} p(S_l) \cdot S_{lDR}}{\sum_{l=1}^{\|L\|} p(S_l)}
\]  

(11)

Where \( p(S_l) \) is the probability that the system is in the state combination \( S_l \).

4. Model validation

4.1. A fuel supply system

As shown in Figure 1, taking the fuel supply system of a certain type of diesel engine as an example, the main fuel supply pipeline of the system includes coarse filter, fuel pump, fine filter, fuel injection pump, high-pressure oil pipe and fuel injector, and the fuel quantity control mechanism includes electronic control actuator and fuel injection gear rod. Among them, the function of the coarse filter is to make the fuel pass through and filter the larger impurities in the fuel; the function of the fuel pump is to deliver the fuel with a certain pressure to the fuel injection pump; the function of the fine filter is to make the fuel pass through and filter the mechanical impurities in the fuel; the function of the fuel injection pump is to provide high-pressure fuel; the function of the high-pressure oil pipe is to deliver high-pressure fuel; the function of the fuel injector is to inject the foggy fuel into the cylinder. The function of the electronic control actuator is to rotate the fuel injection gear bar to the specified position according to the control signal; the function of the fuel injection gear bar is to adjust the fuel injection quantity of the fuel injection pump.

![Figure 1 Composition of fuel supply system of diesel engine.](image)

4.2. Test model construction

Firstly, on the basis of the structural relationship model in Figure 1, the function of each unit is modeled. The whole fuel supply system takes the fuel supply as the research object, and the functions of the primary filter, fuel pump, fine filter and fuel injection pump are described as providing a certain flow of fuel, and the fuel supply per unit time is expressed as \( q_1, q_2, q_3 \).

There are two functions of the fuel injection pump: providing a certain flow of fuel \( q_4 \) and providing oil pressure \( p_1 \); the function of the high-pressure oil pipe is described as three: dividing the fuel \( q_4 \) provided by the fuel injection pump into two parts \( q_5 \) and \( q_6 \); then providing fuel for injector 1 and injector 2 respectively; the third function is providing oil pressure \( p_2 \). The functions of the two injectors are the same: providing a certain flow of fuel \( q_7 \) and \( q_8 \); fuel atomization degree \( d_1 \) and \( d_2 \).
The function of the cylinder is to convert the combustion energy into mechanical energy, because only the fuel supply system is considered. Assuming that the intake air is sufficient, the function of the cylinder is to convert the fuel injection quantity provided by the combustion injector into power. Fuel supply system model based on SFSTM is as shown in Figure 2.

Figure 2. Fuel supply system model based on SFSTM.

It can be seen from Figure 2 that \( q_1 \), \( q_2 \), \( q_3 \) and \( q_4 \) are in series. The fuel supply function of injector 1 and injector 2 is in parallel. The performance state space of each function is shown in Table 1.

Table 1. Performance of each function state space.

| Function | \( q_1 \) | \( q_2 \) | \( q_3 \) | \( q_4 \) | \( q_5 \) | \( q_6 \) |
|----------|----------|----------|----------|----------|----------|----------|
| Performance state space | 80,0 | 60,30,0 | 60,0 | 80, 40,0 | 60,30,0 | 60,30,0 |

| Function | \( q_7 \) | \( q_8 \) | \( p_1 \) | \( p_2 \) | \( d_1 \) | \( d_2 \) |
|----------|----------|----------|----------|----------|----------|----------|
| Performance state space | 60,0 | 60,0 | 20,18,0 | 30,15,0 | 1,0 | 1,0 |

The zero performance state of each function is taken as the fault criterion. According to the property of oil transportation function, the structure function composed of primary filter, oil transportation pump, secondary filter and fuel injection pump in series is \( \varphi(q_1,q_2,q_3,q_4) = \min(q_1,q_2,q_3,q_4) \). The structure function composed of high pressure fuel pipe and fuel injector in parallel is \( \varphi(q_5,q_6,q_7,q_8) = \min(q_5,q_6,q_7,q_8) + \min(q_8,q_7) \). Then the structure function of the whole oil supply system is obtained \( q_9 = \min(\min(q_5,q_6,q_7,q_8),\min(q_8,q_7),\min(\min(q_5,q_6,q_7,q_8),\min(q_8,q_7))) \). System test points \( t_1 \), \( t_2 \) and \( t_3 \) are flow test of corresponding points, \( t_4 \) is pressure test of high pressure oil pipe, \( t_5 \) is output power test of cylinder. The fault mode of each function can be quantitatively described as the fault when the function status is 0. For example, the fuel delivery quantity of the fuel injection pump is 0, which means that the fuel injection pump can not deliver oil. The fault causes, whether the fuel injection pump is stuck or the oil circuit is broken, can be uniformly described by the fuel delivery quantity of 0. In addition, this description has another advantage, that is, when the fault identification standard is improved, if the defined oil volume is less than 2, it is a fault, at this time, it can still describe the fault mode with quantitative state. Since there is only one fault in each functional state for the time being, it is unnecessary to add a fault mode to the diagram.

4.3. Model based testability analysis

According to the statistical data of each component failure, the number of failures of fuel injector and fuel injection pump is more, followed by high-pressure oil pipe and fuel delivery pump, and the failure rate of other components is similar. Let the probability of each function in the corresponding state in Table 1 be as shown in Table 2.
Table 2. Probability of each functional state.

| Function | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ |
|----------|-------|-------|-------|-------|-------|-------|
| State probability | 0.99, 0.01 | 0.95, 0.03, 0.02 | 0.99, 0.01 | 0.9, 0.07, 0.03 | 0.95, 0.03, 0.02 | 0.95, 0.03, 0.02 |

Table 3. Function fault and test probability correlation matrix.

| Function | $q_7$ | $q_8$ | $p_1$ | $p_2$ | $d_1$ | $d_2$ |
|----------|-------|-------|-------|-------|-------|-------|
| State probability | 0.95, 0.05 | 0.95, 0.05 | 0.95, 0.03, 0.02 | 0.95, 0.03, 0.02 | 0.90.1 | 0.9,0.1 |

Because there is no functional redundancy in this example, the propagation of each functional fault is deterministic. The uncertainty is mainly reflected in the uncertainty of test, that is, the reliability of test equipment. Suppose that the failure rates of the six tests obey exponential distribution and are all 0.01, then the failure test probability correlation matrix of the system can be obtained, as shown in Table 3.

Table 3. Function fault and test probability correlation matrix.

| Function | Test | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ |
|----------|------|-------|-------|-------|-------|-------|
| $q_1$    | 0.99 | 0.99  | 0.99  | 0     | 0     | 0.99  |
| $q_2$    | 0.99 | 0.99  | 0.99  | 0     | 0     | 0.99  |
| $q_3$    | 0.99 | 0.99  | 0.99  | 0     | 0.99  | 0.99  |
| $q_4$    | 0    | 0.99  | 0.99  | 0.99  | 0     | 0.99  |
| $q_5$    | 0    | 0.99  | 0     | 0     | 0.99  | 0.99  |
| $q_6$    | 0    | 0     | 0.99  | 0     | 0.99  | 0.99  |
| $q_7$    | 0    | 0     | 0     | 0     | 0.99  | 0.99  |
| $q_8$    | 0    | 0     | 0     | 0     | 0.99  | 0.99  |
| $p_1$    | 0    | 0     | 0     | 0.99  | 0.99  | 0     |
| $p_2$    | 0    | 0     | 0     | 0.99  | 0.99  | 0     |
| $d_1$    | 0    | 0     | 0     | 0     | 0.99  | 0     |
| $d_2$    | 0    | 0     | 0     | 0     | 0.99  | 0     |

According to Table 3, we can know the probability correlation between the system fault and the test, and all the functional faults can be detected. Combined with equation (9), we can get the fault detection rate of the system as $FDR = 0.99$. The function fault set that can be isolated is $FI = [q_4, q_5, q_6]$. Then the fault isolation rate of the system is $FIR=0.17$ from equation (10). The results show that the fault detection rate of the system is high and the fault isolation rate is low, which indicates that the system can find the fault in time, but it is likely to need more tests to determine the fault location.

By the test structure function, we can know the possible output of the fine filter, that is, the value space of test $t_1$ is $t_1 = [60, 30, 0]$. The corresponding functional state equivalence set is shown in Table 4. The three elements in the equivalence set represent the states of $q_1$, $q_2$, and $q_3$ in turn.

It can be seen from table 1 that when the test values are 30 and 60, the states of $q_1$, $q_2$, and $q_3$ can be determined. When the test value is 0, the states of the three functions cannot be determined. From the system transfer function, according to the value space of $t_1$ and the state space of $t_1$, the fuel output space can be obtained as

$$\varphi(q_1, q_2, q_4) = min(q_1, q_2, q_4) = min(t_1, q_4) = [40, 30, 20, 0]$$

The results are shown in Table 5, in which the first element represents the value of $t_1$ and the second element represents the value of $q_4$. By replacing the value of $t_1$ with the equivalent set in Table 4, the relationship between the output space of $q_4$ and the states of $q_1$, $q_2$, $q_3$, and $q_4$ can be obtained. It can be seen that when the output value of $q_4$ is 20, the performance state of $q_4$ can be determined to be 20, and the value of $t_1$ is 30 or 60. Combined with the results in Table 4, it can be further determined that the values of $q_1$ and $q_3$ in the functional state are 80 and 60 respectively, and the performance
states of $t_1$ and $q_4$ can be determined to be 30 and 40 respectively when the value is 30. Combined with the results in Table 4, it can be further obtained that the values of $q_4$, $q_4$, $q_5$, and $q_6$ are 80, 30 and 60 respectively. Similarly, the values of $q_1$, $q_4$, $q_5$, and $q_6$ are 80, 60, 60 and 40. From the above analysis, it can be seen that according to the system transfer function, the results of low-level state analysis can be reused by high-level state analysis, which can reduce the amount of calculation in the process of system testability analysis, and it can be seen that in the process of state transfer, due to the existence of equivalent set, the system state converges step by step, and will not cause the explosion of state space. Similarly, according to the system structure function, the output state value space of $q_5$, $q_6$, $q_7$, $q_8$ and the corresponding equivalent set can be obtained. According to the analysis of Table 4 and Table 5, when the function output status is better, it is easier to determine the status of each function in the system, so the system condition monitoring should be more inclined to determine the system fault free detection, which is just complementary to fault diagnosis.

| Table 4. Function state equivalence set corresponding to value of $t_1$ |
|---|---|---|
| The value of $t_1$ | 0 | 30 | 60 |
| Equivalent set | (0,0,0),(0,0,60),(0,30,60),(80,30,60) | (0,60,60),(80,0,60) | (80,60,60) |

| Table 5. The set of state identities corresponding to the output of $q_4$ |
|---|---|---|
| The value of $q_4$ | 0 | 20 | 30 | 40 |
| Equivalent set | (0,0),(0,20),(0,40) | (30,20) | (30,40) | (60,40) |

Through the corresponding output state value space and equivalent set of each test, the detectable combined function state set $H_{cs}(t_i)$ of each test $t_i$ can be calculated. Similarly, the joint detection state set $H_{ucs}(t_i,t_j)$ of any two tests can be obtained by equation (7). Then according to equations (8) and (11), combined with the probability of each function in the corresponding state in Table 2, the state detection rate of the system is calculated to be 0.42. The calculation process of state detection rate shows that when the system is in a good state, it is easier to infer the state of each function through testing, and when the system is in a fault, it is more difficult to test the state of the system, which is consistent with the result that the fault isolation rate is lower. State detection rate is the result of weighted calculation of state occurrence probability, which reflects the probability of determiniing system functional state under existing tests. The result of the state detection rate calculated in this example shows that the current state can not be completely determined by the system test probability, and appropriate tests should be added to the system function.

To sum up, although the occurrence probability of each functional state is assumed, it does not prevent that the testability modeling method based on function state test can realize the calculation of system fault detection rate, fault isolation rate and state detection rate. After the improvement of multi state statistical data, the system testability can be analyzed more accurately. The state detection can determine the intact state of each function, form complementary with the fault diagnosis results of the system, and jointly monitor the system state. The state detection can predict the system reliability according to the multi state reliability theory, so as to prevent possible faults. In addition, because this model takes the function as the modeling object, it can more clearly obtain the relationship between the test signal and the function signal, which is convenient for the design of system test point and test scheme.

5. Conclusion
This paper presents a testability model of multi-state system based on function-structure-state-test. In order to make the model more close to the actual structure, the model construction is clear. Based on the hierarchical modeling of the system, the model takes the function as the modeling object, the
logical relationship between functions as the connection, and the performance index of functions as the test signal to realize the unified modeling of system structure, function, performance, fault, state and test. In order to quantitatively describe the relationship between fault test and state test, the reachability matrix of fault test probability and the solution method of detectable state set are analyzed respectively. The concept of state detection rate is proposed, and the testability analysis of multi state system is realized by combining fault detection rate and fault isolation rate. The results show that the model can not only realize the testability analysis of fault correlation in two-state system, but also realize the testability analysis of state correlation in multi-state system. However, there are still some shortcomings in this paper. For example, in order to accurately calculate the state detection rate, each state of the system needs to be weighted, which may lead to NP problems when applied to large complex systems. Therefore, how to simplify the calculation process of state detection rate or reasonably estimate it is the focus of the next step of this paper.

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