NUCLEAR STRUCTURE CORRECTIONS IN THE ENERGY SPECTRA OF ELECTRONIC AND MUONIC DEUTERIUM*

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The one-loop nuclear structure corrections of order \((Z\alpha)^5\) to the Lamb shift and hyperfine splitting of the deuterium are calculated. The contribution of the deuteron structure effects to the isotope shift \((\mu_p) - (\mu_d)\) in the interval \(1S \div 2S\) is obtained on the basis of modern experimental data on the deuteron electromagnetic form factors. The comparison with the similar contributions to the Lamb shift for the electronic and muonic hydrogen shows, that the relative contribution due to the nucleus structure increases when passing from the hydrogen to the deuterium.

I. INTRODUCTION

Experimental and theoretical investigation of the Lamb shift and hyperfine structure of muonic hydrogen \((\mu p)\) and muonic deuterium \((\mu d)\) can lead to essential progress in determining important fundamental parameters of the proton and deuteron. A better understanding of the nuclear structure and polarizability effects in these hydrogenic atoms can be gained due to such studies [1–5]. Namely the proton structure and polarizability corrections lead to main theoretical uncertainties on the Lamb shift and in the hyperfine splittings (HFS) of the hydrogen atom. One of the recent studies of these effects in the HFS in electronic and muonic hydrogen carried out in [6] shows that 70 % of the difference \((\Delta E_{\text{HFS}}^{\text{QED}} - \Delta E_{\text{HFS}}^{\text{exp}}) = 0.046 \text{ MHz} (\text{QED contribution } \Delta E_{\text{HFS}}^{\text{QED}} \text{ doesn’t take into account the proton recoil, structure and polarizability corrections})\) in the case of electronic hydrogen can be explained on the basis of effective field theory describing the interaction of baryons with photons and leptons. The measurement of the \(2P \div 2S\) Lamb shift in muonic hydrogen has been carried out during recent years at PSI (Paul Scherrer Institute) [7]. The goal of the experiment is to measure the Lamb shift in muonic hydrogen with 30 ppm precision and to obtain the root mean square (rms) proton charge radius with \(10^{-3}\) relative accuracy that is an order of the magnitude better than in the analysis of the elastic e-p scattering and the Lamb shift in electronic hydrogen. Another important task is connected with the study of the hydrogen - deuterium isotope shift for the interval \(1S \div 2S\) [8–10]. The experimental value of the H-D isotope shift

\[
\Delta E_{\text{H-D}}(1S \div 2S) = 670 994 334.64(15) \text{ kHz}, \quad \delta = 2.2 \cdot 10^{-10}
\]

was obtained with such high precision that the nucleus structure and polarizability effects should be taken into account on close theoretical examination of this quantity. Using the result (1) the numerical value for the difference of the proton and deuteron charge radii can be evaluated as follows [11]:

\[
r_d^2 - r_p^2 = 3.8213(11.7) \text{ fm}^2.
\]

So, when the Lamb shift measurement in the system \(\mu p\) with the precision 30 ppm will be accomplished the deuteron radius \(r_d\) can be derived by means of (2) with the accuracy \(10^{-3}\). Another approach to determine more exact value

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of the deuteron charge radius is related to the measurement of the muonic hydrogen - muonic deuterium isotope shift for the interval 1S \( \pm \) 2S. In this case the precision of the appropriate experiment should be comparable with (1). The possibility of such measurement of the \( \mu p - \mu d \) isotope shift for the interval 1S \( \pm \) 2S is considered to be quite real taking into account the relative error of order 10\(^{-5}\) at PSI for the Lamb shift measurement in muonic hydrogen. Evidently the calculation of all possible corrections to the Lamb shift in muonic hydrogen and deuterium with similar precision must be performed. The total value of the Lamb shift is determined by the sum of quantum electrodynamical (QED) contributions (one-loop, two-loop, three-loop corrections, recoil corrections, radiative recoil corrections) and nuclear corrections. Whereas QED contributions were obtained presently with the relative accuracy 10\(^{-7}\) due to numerous calculations [11] the nuclear structure corrections are known less precisely. The accuracy of the corresponding calculation of the nucleus structure and polarizability effects depends on the experimentally measured nuclear densities of the charge, magnetic moment, ... and on the used theoretical models. Both experiments for the isotope shift (1) and the hydrogen HFS can play the selecting role among numerous nuclear models.

The ground state hyperfine splitting in deuterium represents another important quantity where the nuclear structure corrections can be tested experimentally. The experimental value of the deuteron HFS was obtained with high accuracy many years ago [12,11]:

\[
\Delta E_{\text{HFS}}^{\text{exp}}(D) = 327 \, 384.352 \, 521 \, 9(17)(3) \, \text{kHz}, \quad \delta = 5.2 \times 10^{-12}.
\]  

The difference between the experimental (3) and theoretical values for the deuteron HFS accounting for the QED corrections is equal to \( \Delta E_{\text{HFS}}^{\text{exp}}(D) - \Delta E_{\text{HFS}}^{\text{th}}(D) = 45 \, \text{kHz} \). The essential contribution to the theoretical quantity \( \Delta E_{\text{HFS}}^{\text{th}}(D) \) is given also by the nuclear structure corrections. The deuteron is the spin 1 particle and its electromagnetic structure is described by three form factors. The aim of our study consists in the exact consideration of the deuteron electromagnetic form factors: charge monopole, charge dipole and quadrupole and magnetic dipole. We leave aside the polarizability effects mentioned above.

II. CORRECTIONS OF ORDER \((Z\alpha)^5\) TO THE DEUTERIUM LAMB SHIFT

The main nuclear structure dependent contribution of order \((Z\alpha)^4\) to the hydrogen Lamb shift is determined by the one-photon interaction. In the one-photon exchange approximation the amplitude of the scattering process \( ed \to ed \) is just the contraction of the electron and deuteron electromagnetic currents, multiplied by the photon propagator. The parameterization of the deuteron electromagnetic current takes the form [17,18]:

\[
J^\mu_d(p_2, q_2) = \varepsilon^\mu(p_2) \left\{ \frac{(p_2 + q_2)_\mu}{2m_2} g_{\rho\sigma} F_1(k^2) - \frac{(p_2 + q_2)_\mu}{2m_2} g_{\rho\sigma} \frac{k^\nu}{2m_2} F_2(k^2) - \frac{\Sigma_{\rho\sigma}}{2m_2} F_3(k^2) \right\} \varepsilon_\sigma(p_2), \quad (4)
\]

where \( p_2, q_2 \) are four momenta of the deuteron in the initial and final states, \( k = q_2 - p_2 \), \( m_2 \) is the deuteron mass. The spin 1 polarization vectors \( \varepsilon_\mu \) satisfy the following conditions:

\[
\varepsilon^\mu_\sigma(k, \lambda) \varepsilon^\nu(k, \lambda') = -\delta_{\lambda\lambda'}, \quad k_\mu \varepsilon^\mu_\nu(k, \lambda) = 0, \quad \sum_\lambda \varepsilon^\mu_\nu(k, \lambda) \varepsilon_\nu(k, \lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_2^2}, \quad (5)
\]

The generator of the infinitesimal Lorentz transformations

\[
\Sigma^{\mu\nu} = g^{\mu\rho} \varepsilon_{\rho\sigma} - g^{\mu\sigma} \varepsilon_{\rho\nu}. \quad (6)
\]

The deuteron electromagnetic form factors \( F_1(k^2) \) depend on the square of the photon four momentum. They are related to the deuteron charge \( F_C \), magnetic \( F_M \) and quadrupole \( F_Q \) form factors by the expressions:

\[
F_C = F_1 + \frac{2}{3} \eta [F_1 + (1 + \eta)F_2 - F_3],
\]

\[
F_M = F_3, \quad \eta = -\frac{k^2}{4m_2^2}, \quad (7)
\]
\[ F_Q = F_1 + (1 + \eta)F_2 - F_3. \]

The lepton electromagnetic current has the form:

\[ J_{\mu}^l(p_1, q_1) = \bar{u}(q_1) \left[ \frac{(p_1 + q_1)\mu}{2m_1} - (1 + \kappa_1)\sigma^{\mu\nu} \frac{k_\nu}{2m_1} \right] u(p_1), \] (8)

where \( p_1, q_1 \) are the electron (muon) four momenta in the initial and final states, \( \sigma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2 \), \( \kappa_1 \) is the lepton anomalous magnetic moment, \( m_1 \) is the lepton mass. To obtain the one-photon interaction contribution to the Lamb shift, it is necessary to average the currents (4) and (8) over the electron and deuteron spins. As a result the contribution to the Lamb shift of order \( (Z\alpha)^4 \) is expressed through the deuteron charge radius \( r_d \) as follows:

\[ E^{ls} = \frac{2\mu^3}{3n^3}(Z\alpha)^4 \left[ r_d^2 + F_M(0) - F_C(0) \right], \quad r_d^2 = \frac{6}{F_C(0)} \frac{dF_C(k^2)}{dk^2} |_{k^2=0}, \] (9)

where \( \mu = m_1m_2/(m_1 + m_2) \) is the reduced mass. The numerical value of this expression for the interval \( 1S \to 2S \) in muonic deuterium for the \( r_d = 2.094 \) fm is equal to -186.74 meV. The magnetic quadrupole term of Eq. (9) which is proportional to the difference \( [F_M(0) - F_C(0)] = 2\mu_\alpha - 1 \), coincides with the result of Ref. [19] for the spin \( 1 \) particle. Its contribution to the energy spectrum amounts to 0.2 \%. Consider the two-photon exchange amplitudes shown in fig.1. They give the corrections of order \( (Z\alpha)^5 \) to the deuterium Lamb shift. The spin \( 1 \) particle (deuteron) exists in the intermediate state of the processes of the virtual Compton scattering. The amplitudes of the virtual Compton scattering on the lepton and deuteron can be presented as follows:

\[ M^{(l)}_{\mu\nu} = \bar{u}(q_1) \left[ \gamma_\mu (p_1 + k + m_1)_{\nu} - \gamma_\nu (p_1 - k + m_1)_{\mu} \right] u(p_1), \] (10)

\[ M^{(d)}_{\mu\nu} = \varepsilon^\alpha(q_2) \left[ \frac{(q_2 + p_2 - k)\mu}{2m_2} g_{\rho\lambda} F_1 - \frac{(q_2 + p_2 - k)\nu}{2m_2} k_\rho k_\lambda \frac{F_2 - \Sigma^\mu\alpha k_\alpha F_3}{2m_2} \right] \times \] (11)

\[ \times \frac{g_{\lambda\omega} + (p_2 - k)_\lambda (p_2 - k)_\omega}{(p_2 - k)^2 - m_2^2} \left[ \frac{(p_2 + q_2 - k)\nu}{2m_2} g_{\omega\sigma} F_1 - \frac{(p_2 + q_2 - k)\mu}{2m_2} k_\omega k_\sigma \frac{F_2 + \Sigma^\nu\beta k_\beta F_3}{2m_2} \right] \varepsilon_\sigma(p_2). \]

To extract the contribution of the two-photon amplitudes to the Lamb shift we can average the tensors (10) and (11) over the lepton and deuteron spins. Multiplying obtained relations and contracting over the Lorentz indices we can write necessary quasipotential in the form (we used the system FORM [20] for the trace calculations of the Dirac \( \gamma \) matrices and the contraction over the Lorentz indices):

\[ V^{(ed)}_{2S} = \frac{2m_1(Z\alpha)^2}{3m_2^2} \int \frac{id^4k}{\pi^2} \frac{1}{(k^2)^2(k^2 - 4k_0^2m_1^2)(k^2 - 4k_0^2m_2^2)} \left\{ 4F_1^2 m_2^2 k^2(k^2 - k_0^2) \right\}. \] (12)
\[ \times \left[ 3m_2^2 - k^2 + k_0^2 \right] + 4F_1F_2(k^2 - k_0^2)[k^4 - m_2^2(k^2 + 2k_0^2)] + 4F_1F_3m_2^2k^2(k^2 - k_0^2)(k^2 - 2k_0^2) - \\
- F_2^2 \left( \frac{k^2 - k_0^2}{m_2^2} \right) \left[ k^4 - m_2^2(k^2 + 3k_0^2) \right] - 2F_1F_3k^4(k^2 - k_0^2)^2 + F_2^2k^2k_0^2[-3k^4 + 4m_2^2(2k^2 + k_0^2)] \right\}. \]

After rotating the \( k_0 \) contour of the Feynman loop integration we integrate Eq. (12) over the four-dimensional Euclidean space using the relation:

\[ \int d^4k = 4\pi \int_0^\infty k^3dk \int_0^{\pi} \sin^2 \phi \, d\phi, \quad k_0 = k \cos \phi. \] (13)

Then the integration over angle \( \phi \) in Eq.(12) can be done analytically. Calculating the matrix element of obtained potential between Coulomb wave functions we find the following one-dimensional integral representation for the contribution to the energy spectrum:

\[ E_{2\gamma}^{ls} = -\frac{\mu^5(Z\alpha)^5}{6m_1m_2(m_1^2 - m_2^2)\pi n^2} \int_0^\infty \frac{dk}{k} \left\{ F_1^2 \left[ \frac{12m_1^2m_2^4}{k^3} (m_2^2h_1^3 - m_1^2h_2^3) + \frac{1}{k} (m_2^2h_1^5 - m_1^2h_2^5) \right] + \right. \]

\[ + \left. (m_1^2 - m_2^2) (10k^2m_1^2m_2^2 + 12m_1^2m_2^2 + k^4(m_1^2 + m_2^2)) \right] + \frac{F_1F_2}{2m_1m_2} \left[ \frac{1}{k} m_2^2h_1^5 (2m_1m_2^2 + \\
+ k^2(2m_1^2 - m_2^2)) - \frac{1}{k} m_1^2h_2^5 (k^2 + 2m_2^2) + k^2(m_1^2 - m_2^2) (-10m_1^2m_2^2 + k^4(m_1^4 + m_2^4m_1^2 - m_2^2)) + \\
+ 4k^2(3m_1^4m_2^2 - 2m_1^2m_2^4) \right] + \frac{F_2^2}{16m_1^2m_2^2} \left[ -h_2^5km_1^4 + h_2^5km_2^4 (4m_1^2m_2^2 + k^2(4m_1^2 - 3m_2^2)) + \\
+ k^4(m_1^2 - m_2^2) (-50m_1^4m_2^2 + k^4(m_1^4 + m_2^4m_1^2 - 3m_2^2) + 2k^2(7m_1^4m_2^2 - 13m_1^2m_2^4)) \right] + \\
+ F_1F_3 \left[ \frac{4m_1^2m_2^2}{k} (m_1^2h_2^2 - m_2^2h_1^2) - 2k (m_1^2h_1^4 - m_1^4h_2^2) - 2k^2(m_1^4 - m_2^2) (8m_1^2m_2^2 + k^2(m_1^4 + m_2^4)) \right] + \\
+ \frac{F_2F_3}{2m_2} \left[ h_2^5km_1^4 - h_2^5km_2^4 - k^4(10m_1^2m_2^2 + k^2(m_1^4 + m_2^2)) (m_1^2 - m_2^2) \right] + \\
+ F_3^2 \left[ -2h_2^5km_1^4 + h_1^5km_2^2 (8m_1^2m_2^2 + k^2(3m_1^4 - m_2^2)) + k^2(m_1^4 - m_2^2) (6m_1^2m_2^2 + k^2(2m_1^2 - m_2^2)) \right]. \]

where \( h_i = \sqrt{k^2 + 4m_i^2} \). The analysis of the coefficients with the deuteron form factors in Eq. (14) at small values of the variable \( k \) shows that there are three terms proportional to \( F_1^2, F_1F_2, F_1F_3 \) which contain infrared divergences. For the regularization of these terms it is necessary to supplement the quasipotential of Eq. (14) by the iteration term which gives the following contribution to the energy levels:

\[ \Delta E^{ls}_{2\gamma} = \langle [V_{1\gamma} \times G^4 \times V_{1\gamma}]^{ls} > = \] (15)

\[ = -\frac{16\mu^5(Z\alpha)^5}{\pi n^2} \int_0^\infty \frac{dk}{k^4} \left[ F_1^2(0) + 2F_1(0)F_1'(0)k^2 + \frac{k^2}{3m_2}F_1(0)F_2(0) \right], \]
where \( G^{-1} = (b^2 - p^2)/2\mu_R \) is the inverse free two-particle propagator [21], \( V_{1\gamma} \) is the quasipotential of the one-photon interaction. Furthermore to eliminate the divergence in the part \( \sim F_1(k^2)F_3(k^2) \) we subtract from it the corresponding contribution accounting for the point-like deuteron \( \sim F_1(0)F_3(0) \) (needless to say that the one-loop corrections of order \((Z\alpha)^5\) for the point deuteron in the Lamb shift must be studied independently [22], the relating studies are in progress). To perform the numerical calculations on the basis of Eq. (14) we employ recent experimental data on the deuteron electromagnetic form factors [23] extracted from the tensor polarization data in the elastic electron - deuteron scattering at given values of the four-momentum transfer. Some useful parameterizations exist for the deuteron form factors in the range \( 0 \leq k \leq 1.4 \text{ GeV} \). We used the parameterization from Ref. [24] of the following form:

\[
\begin{pmatrix}
F_C \\
F_Q \\
F_M
\end{pmatrix} = F_D^2 \left( \frac{k^2}{4} \right) \cdot M(\eta) \cdot \begin{pmatrix}
f_0 \\
f_1 \\
f_2
\end{pmatrix},
\]

(16)

where \( F_D \) is the nucleon dipole form factor, \( M(\eta) \) is the matrix of the coefficients, \( \eta = k^2/4m_d^2 \) and

\[
f_m = k^m \sum_{i=1}^{4} \frac{a_{mi}^i}{\alpha^{i}_{mi} + k^2}.
\]

(17)

The parameters \( a_{mi}, \alpha_{mi} \) describing each electromagnetic form factor can be found in Ref. [24] (see also http://www-dapnia.cea.fr/Sphn/T20/Parametrisations). It is necessary to point out that the parameterization (16) leads to the following value of the deuteron charge radius \( r_d = 2.094 \text{ fm} \) [23] which is 2.6 % smaller than the value \( r_d = 2.148 \text{ fm} \), obtained on the basis of Eq. (2) by means of the \( r_p \) from Ref. [11]. The results of the numerical integration in Eq. (14) are presented in Table 1. The momenta which give the main contribution in Eq. (14) have a characteristic nuclear scale. The obtained correction of order \((Z\alpha)^5\) \( 0.53 \text{ kHz} \) for the \( 2S \div 1S \) interval in electronic hydrogen is in agreement with the result \( 0.49 \text{ kHz} \) of Ref. [16]. In the case of the muonic deuterium analogous contribution is equal to \( 2.57 \text{ meV} \). Then the value of the nuclear structure corrections to the isotope shift in muonic (electronic) hydrogen accounting the results of Ref. [25,26] is as follows:

\[
\Delta E_{\text{str}}^{1S}(1S \div 2S) = \begin{cases} 
\mu p - \mu d & 1.41 \text{ meV} \\
ep - ed & 0.497 \text{ kHz}
\end{cases}
\]

(18)

It must be taken into account when the comparison with the experimental data will become available.

III. CORRECTIONS OF ORDER \((Z\alpha)^5\) TO THE DEUTERIUM HYPERFINE SPLITTING

To construct the HFS part of the one-loop quasipotential one needs to keep not only the terms of the operator (6) with \( \Sigma_{ij} = 2e_ijklS_j^k \) which are proportional to the deuteron spin \( S_2 \). The terms containing the operator \( \Sigma_{0n} \), which are expressed through the generator of the Lorentz boosts \( \Sigma \) for the spin 1 particle, also must be taken into account. There are two possibilities to determine the HFS part of the interaction operator:

1. To keep consistently all the terms, which can contain the spin - spin interaction operator \( (S_1S_2) \) when calculating the contraction of the amplitudes (10) and (11) over the Lorentz indices.

2. To use the special projection operators on the states of the lepton and deuteron with the total spins 3/2 and 1/2. In this study we carry out covariant construction of the HFS quasipotential introducing the projection operators \( \hat{\pi}_{\mu,3/2} \) and \( \hat{\pi}_{\mu,1/2} \) for the particles in the initial and final states of the following form:

| TABLE I. Nuclear structure corrections of order \((Z\alpha)^4\), \((Z\alpha)^5\) to the Lamb shift in electronic and muonic deuterium. |
|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( E_{\text{Lamb}} \)          | 1S \((Z\alpha)^4\) | 1S \((Z\alpha)^5\) | 2S \((Z\alpha)^4\) | 2S \((Z\alpha)^5\) | 2S \div 1S \((Z\alpha)^5\) | 2S \div 1S \((Z\alpha)^5\) |
| ed (kHz)                       | 6.875 \cdot 10^4 | -0.603          | 0.839 \cdot 10^4 | -0.075          | -6.016 \cdot 10^4          | 0.527          |
| \( \mu d \) (meV)              | 213.42          | -2.94           | 26.68           | -0.37           | -186.74          | 2.57           |


\[ \hat{\pi}_{\mu,3/2} = [u(p_1)\epsilon_\mu(p_2)]_{3/2} = \Psi_\mu(P), \]

\[ \sum_\lambda \Psi_\mu^A \tilde{\Psi}_\lambda^B = \frac{(\hat{P} + M)}{2M} \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2P_\mu P_\nu}{3M^2} + \frac{P_\mu \gamma_\nu - P_\nu \gamma_\mu}{3M} \right), \]

\[ \hat{\pi}_{\mu,1/2} = [u(p_1)\epsilon_\mu(p_2)]_{1/2} = \frac{i}{\sqrt{3}} \gamma_5 \left( \gamma_\mu - \frac{P_\mu}{M} \right) \Psi(P), \]

where the spin-vector \( \Psi_\mu(P) \) and the spinor \( \Psi(P) \) describe the lepton-deuteron states with the total spin 3/2 and 1/2 respectively, \( M = m_1 + m_2 \) is the total mass, \( P = p_1 + p_2 \).

Multiplying amplitudes (10) and (11) and taking into consideration the relations (19)-(21) we can represent the necessary HFS quasipotential by the following expression:

\[ V_{2\gamma}^{HFS} = (Z\alpha)^2 \int \frac{d^4k}{\pi^2} \left\{ \frac{1}{(k^2)^2} \frac{1}{k^4 - 4k^2m_1^2} \frac{1}{k^4 - 4k^2m_2^2} \right\} \times \]

\[ \times \left\{ 4F_1F_3 \left[ 2(k_0^2 + k^2) - \frac{k^4}{m_2^2} k^2 + 2F_2F_3 \frac{k^1 k^2}{m_2^2} \left( \frac{k^4}{m_2^2} - 4k_0^2 + k^2 \right) + 2F_3^2 k^1 k^2 \left( k_0^2 + \frac{k^4}{m_2^2} \right) \right] \right\}. \]

The number of the form factor terms was reduced by half in comparison with Eq. (14) because the spin-dependent terms of the second particle are proportional to the form factor \( F_3 \). The expression (22) is less singular than the operator (14). The sole infrared divergence of Eq. (22) when \( k \to 0 \) is connected with the term \( \sim F_1F_3k^2 \). It can be eliminated entirely by subtracting the iteration of the quasipotential which can be obtained using Eqs. (4) and (8):

\[ \Delta V_{iter}^{HFS} = [V_{1\gamma} \times G^f \times V_{1\gamma}]^{HFS} = \frac{32\mu(Z\alpha)^2}{3m_1m_2} (S_1S_2) \int_0^\infty \frac{dk}{k^2} F_1F_3. \]

Subtracting (23) from (22) we can make the analytical integration over the angle variables in the Euclidean momentum space. The averaging of the obtained expression over the Coulomb wave functions will lead to the appearance of the factor \( |\psi(0)|^2 \). As a result the contribution of the two-photon amplitudes to the deuterium HFS can be written as the one-dimensional integral:

\[ E_{2\gamma}^{HFS} = E_D^F \cdot \delta_{HFS}^{iter} = E_D^F \frac{2(Z\alpha)}{\pi \hbar^3} \int_0^\infty \frac{dk}{k^2} \left[ \frac{F_3}{3F_3(0)} \left[ 4F_1 + \frac{k^2}{m_2^2} (2F_1 - F_2 - F_3) + \frac{k^4}{m_2^2} F_2 \right] \right] \times \]

\[ \times \left[ \frac{m_1^2 m_2}{m_1^2 - m_2^2} \left( 1 + \frac{k^2}{4m_1^2} \right)^{3/2} - \frac{m_1 m_2^2}{m_1^2 - m_2^2} \left( 1 + \frac{k^2}{4m_2^2} \right)^{3/2} + \frac{m_1}{8m_1 m_2} \right] - \frac{F_3 k^2}{F_3(0)} \left[ F_1 + \frac{F_3}{4} + \frac{3F_2}{4} \frac{k^2}{m_2^2} \right] \times \]

\[ \times \left[ \frac{m_2}{m_1^2 - m_2^2} \left( 1 + \frac{k^2}{4m_1^2} \right)^{3/2} - \frac{m_1^2 m_2}{m_1^2 - m_2^2} \left( 1 + \frac{k^2}{4m_2^2} \right)^{3/2} + \frac{m_1^2}{8m_1 m_2} \right] - 4\mu, \]

where the Fermi energy of the ground state hyperfine splitting in deuterium

\[ E_D^F = 2\mu_d \alpha^4 \mu^3 \frac{m_e m_p}{m_n} = 326 967.678(4) \text{ kHz}, \]

\[ \mu_d = 0.857 438 228 4(94) \text{ is the deuteron magnetic moment in the nuclear magnetons, } \mu \text{ is the reduced mass of the deuteron atom, } m_p \text{ is the proton mass. The numerical integration of Eq. (24) was also done by means of the deuteron form factor parameterization (16). As a result the contribution of Eq. (24) to the HFS of the electronic deuterium} \]

\[ E_{2\gamma}^{HFS}(ed) = -34.72 \text{ kHz}. \]
The calculation of the deuteron structure corrections to the HFS of the ground state was performed previously in the analytical form in the zero radius approximation for the deuteron form factors. In this approximation the wave function of the deuteron D-state was omitted and the S-state wave function was written in the asymptotic form $\beta e^{-\beta r}$. The contribution obtained in Ref. [15] is the following:

$$\Delta E_{\text{HFS}}^{\text{(ed)}} = -E_F \cdot \left[ \frac{\alpha m_e}{3\kappa} (1 + 2 \ln 2) - \frac{3\alpha m_e}{8\pi m_p} \ln \frac{\kappa}{m_e} \left( \mu_d - 2 - \frac{3}{\mu_d} \right) + \right.$$  

$$+ \frac{3\alpha m_e}{4\pi m_p} \ln \frac{\kappa}{m_p \mu_d} \left( \mu_p^2 - 2\mu_p - 3 + \mu_n^2 \right) \right] = -21.31 \text{ kHz}. \quad (27)$$

where $\kappa = 45.7$ MeV is inverse deuteron size. The essential growth of the nuclear structure correction of order $Z^{1/5}$ to the deuterium HFS in our case as compared with Ref. [15] can be explained by removing the restriction of the zero radius approximation. The value $E_{\text{HFS}}^{\text{(ed)}}$ obtained here leads to significant growth of the difference between the theory and experiment which amounts to 45 kHz without consideration of the deuteron nuclear structure and polarizability contributions. In the case of muonic deuterium the corresponding contribution

$$E_{\text{HFS}}^{\text{(md)}} = -0.925 \text{ meV}. \quad (28)$$

The theoretical uncertainty of the results (18), (26), (28) and presented in Table 1 is determined by the errors of the experimental data for the deuteron electromagnetic form factors which amounts to 5 % in the most important interval $0 \leq k \leq 0.5$ Gev. So, the theoretical error of the results obtained in this work may amount to 10 %. It may be useful to compare the relative values of the corrections connected with the nuclear structure in the energy spectra of light and heavy hydrogen. The main one-loop contribution to the HFS of electronic (muonic) hydrogen is determined by the following expression (the Zemach correction) [27,28]:

$$\Delta E = E_F \cdot \delta_Z = E_F \frac{2\alpha \mu}{\pi^2} \int \frac{dp}{(p^2 + b^2)^2} \left[ G_E(-p^2)G_M(-p^2) \right] \left[ \frac{1}{1 + \kappa} - 1 \right] = E_F (-2\alpha \mu)R_p, \quad b = \alpha \mu, \quad (29)$$

where $R_p$ is the Zemach radius. In the coordinate representation the Zemach correction is determined by the magnetic moment density $\rho_M(r)$ and by the charge density $\rho_E(r)$. The value $R_p$ can be obtained in the numerical form by using the parameterization for the proton electromagnetic form factors obtained at Mainz 20 years ago from the analysis of elastic electron - proton scattering [29]. The relative contributions of the Zemach correction to the hydrogen HFS are as follows:

- **Electronic hydrogen**: $R_p = 1.067 \text{ fm}$, $\delta_Z = -40.3$ ppm,

- **Muonic hydrogen**: $R_p = 1.064 \text{ fm}$, $\delta_Z = -74.7 \cdot 10^{-4}$.

The relative deuteron structure contributions obtained in this study have the following numerical values:

- **Electronic deuterium** $\delta_{\text{str}}^{\text{HFS}} = -106.19$ ppm

- **Muonic deuterium** $\delta_{\text{str}}^{\text{HFS}} = -188.37 \cdot 10^{-4}$. \quad (33)

Three times growth of the contributions (32), (33) as compared with the expressions (30), (31) can be caused by increasing the distribution region of the deuteron electric charge and magnetic moment. The numerical results obtained in this work for the muonic deuterium must be taken into consideration when both extracting the deuteron charge radius in future experiments on the isotope shift $(\mu p) - (\mu d)$, and comparing theoretical predictions with measurements of the HFS in deuterium.

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