Quark Gluon Plasma Diagnostics in a Successive Equilibrium Scenario

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Abstract

The relativistic Fokker Planck equation has been used to study the evolution of the quark distribution in the quark gluon phase expected to be formed in ultra-relativistic heavy ion collisions. The effect of thermal masses for quarks and gluons is incorporated to take account of the in-medium properties. We find that the kinetic equilibrium is achieved before the system reaches the critical temperature of quark hadron phase transition. We find that chemical equilibrium is not achieved during this time. We have evaluated the electromagnetic probes of quark gluon plasma from the non-equilibrated quark gluon phase and compared them with those in completely equilibrated scenario. The hard QCD production rates for the electromagnetic ejectiles as well as the heavy quark production rates are also calculated.

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I. Introduction

The theory of strong interactions - Quantum Chromodynamics (QCD) predicts that under extreme conditions of large baryon density and/or high temperature, the hadronic system would dissolve into their fundamental constituents, the quarks and gluons. It is expected that the temperature and density achievable in nucleus-nucleus collisions at ultra-relativistic energies would be favourable for the formation of such a phase, called the Quark-Gluon-Plasma (QGP). It is of fundamental importance to understand whether thermodynamic equilibrium is achieved in the quark gluon system, so as to justify the concept of “phase”. Ideally, the transport theory,
or the kinetic theory, should provide an appropriate framework to consider systems out of equilibrium. The application of Boltzmann equation to relativistic quantum systems is however laced with major difficulties[1]. The non-abelian nature of QCD has made this problem rather formidable and as a result, the "transport equation" approach to many body QCD has found only limited success to date. Nonetheless, the field is in rapid progress and some important results have already been obtained.

The primary motivation to study the non-equilibrium evolution of the quark-gluon system is driven from the fact that the characteristic time scales for the partonic processes ($q, \bar{q}$ and $g$) are of the order of the lifetime of the putative QGP. Even if the system achieves thermodynamic equilibrium at some point of time, the study of the pre-equilibrium aspects is important to evaluate in the sense that the pollutants from this era may affect the kinematical domains where one looks for the signals of QGP. QGP diagnostics rely quite heavily on the phase space densities and distributions of quarks and gluons. To what extent equilibrium is achieved should obviously affect these signals.

To this end, the mechanisms governing the approach to thermalisation in the quark-gluon system have been a very topical issue of late [2, 3]. Recently a parton cascade picture which purports to study the QCD-based space-time evolution of the partonic system has received a fair amount of attention[2]. The importance of the microscopic approach embodied in such pictures notwithstanding, their applicability to actual QGP diagnostics is still largely unexplored or beset with numerical difficulties. Also, the approximations inherent in the parton-cascade model have been questioned by some authors [4]. In this work, we thus propose to use a physically transparent, semi-classical model to understand the evolution of the many body quark-gluon system towards equilibrium.

The central theme of our approach is to exploit the well-known result[5] that $gg$ cross-section is considerably larger than $qg$ or $qq$ cross-sections, primarily because of the colour factor of gluons. It is therefore reasonable to expect that the gluons
would thermalise among themselves appreciably earlier than the whole system of quarks, antiquarks, and gluons. The proper time \( \tau_g \) at which the gluons equilibrate is thus considerably less than the overall equilibration time \( \tau_0 \); the value of \( \tau_0 \) was proposed to be of the order of 1 fm/c by Bjorken some time ago.

The gluons carry about 50% of the momentum and sea quarks only a tiny fraction. Thus, in very high energy collisions (RHIC or LHC energies), if we confine our attention to the central rapidity region, it is quite natural that from \( \tau_g \) onwards, the equilibrated gluons may provide a thermal heat bath for the sea quarks (antiquarks). This picture is further justified by the fact that the sea quark (antiquark) density is very low compared to that of the gluons in this region. Thus we are left with a system where a relatively small mixture of non-equilibrium degrees of freedom (quarks and antiquarks) interact with some equilibrated degree of freedom (the 'gluonic' bath); such processes are known to give rise to Brownian motion which is governed by the Fokker-Planck (FP) equation. QCD being asymptotically free, hard collisions involving large momentum transfers are suppressed compared to soft interactions and in our picture, thermalisation in the quark-gluon system proceeds through many such soft collisions. The FP equation describes, semi-classically, the evolution of the many body quark-gluon system in a kinetic theory framework. The system under consideration is highly relativistic and presumably at high temperatures. Therefore, account of production and annihilation of \( q\bar{q} \) pairs in the gluonic heat bath must be taken.

The goal of this paper is two-fold. First, we develop the framework of the relativistic FP equation to describe the temporal evolution of the quark distribution function and then use this information to estimate the effect of the pre-equilibrium era on the QGP diagnostics. For this latter purpose, we confine our attention to the electromagnetic probes - photons, photon pairs, and dileptons. It is well known that these probes constitute an especially clean class of QGP signals; by the very nature of their interaction; these probes tend to leave the system of strongly
interacting matter with minimal distortion of their energy/momentum. They thus carry the information from within the reaction zone rather more effectively, not being masked by the details of the evolution process. Obviously, such is not the case for hadronic probes which lose the initial information because of their strong coupling to the rest of the system.

The structure of the paper is as follows. In Section II, we derive the FP equation in a form which is appropriate for our present purpose. In Section III, we discuss the production of photons, diphotons, and dileptons from non-equilibrated scenario. Section IV is devoted to the discussion of our results and we conclude with a summary in Section V.

We reiterate that our analysis is restricted to the situation where the central rapidity region is free of baryon number (complete transparency), a situation which is expected to be achieved in RHIC or LHC energies. In these circumstances, the collective evolution of the system is governed by the scaling hydrodynamics à la Bjorken[7] which we tacitly assume to be the case.

II. The Fokker-Planck (FP) Equation

The Boltzmann equation in the relativistically covariant form can be written as

\[ p^\mu \partial_\mu f(x,p) = C\{f\} \] 

where \( p \) is the four momentum of the test quark and \( f \) is its phase space density. \( C\{f\} \) is the collision term. The left hand side of eq. (1) is known as “streaming term”. In the spirit of boost-invariance incorporated in the scaling hydrodynamics of Bjorken, we assume that the phase space density of the quark is independent of \( \vec{x} \), i.e. the plasma is uniform. Under these conditions the Boltzmann equation reads

\[ \frac{\partial f}{\partial t} + \frac{E}{\partial t} = \frac{\partial}{\partial t} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} \] (2)
We can separate the elastic from the inelastic collision term as follows

$$\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{\text{el}} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{\text{el}} + \left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{\text{inel}}$$  \hspace{1cm} (3)$$

First we consider the elastic collisions of the test quark with other quarks, antiquarks, and gluons present in the system. For the system under study, the gluons are in complete equilibrium, with a density larger than the non-thermal densities of the quarks and antiquarks present in the system. The rate of collisions $w(p, q)$ is given by

$$w(p, q) = \sum_{j=q,\bar{q},g} w^j(p, q)$$  \hspace{1cm} (4)$$

where $w^j$ denotes the rate of collisions of a test quark $q$ with other participant parton $j$, i.e. for the reaction $jq \to jq$. In our case the term $w^g$ of eq. (4) dominates over the other two terms, because of the paucity of quarks and antiquarks in the system.

The expression for $w^j$ can be written as

$$w^j(p, q) = \gamma_j \int \frac{d^3q}{(2\pi)^3} f_j(q) v_{\text{rel}} \sigma^j$$  \hspace{1cm} (5)$$

where $\gamma_j$ is the statistical degeneracy ($2 \times 8$ for gluons) and $f_j(q)$ is the phase space density for the species $j$; $v_{\text{rel}}$ is the relative velocity between the test quark $q$ and the other participating parton $j$ and $\sigma^j$ is the relevant cross section for the elastic collision.

We assume that the transition takes place between two states having momenta, say, $p'$ and $p$, where $p' - p$ is very small. In terms of collision rates this means that the function $w(p, p')$ is sharply peaked at $p \approx p'$. The right hand side of eq. (2) can be written as

$$\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{\text{el}} = \int d^3k [w(p + k, k)f(p + k) - w(p, k)f(p)]$$  \hspace{1cm} (6)$$

Expanding $w(p + k, k)f(p + k)$ by Taylor series we get

$$w(p + k, k)f(p + k) \approx w(p, k)f(p) + \vec{k} \cdot \frac{\partial}{\partial \vec{p}} (wf) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} (wf)$$  \hspace{1cm} (7)$$
Substituting eq. (7) in eq. (6) we get,

\[
\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{el} = \int d^3k \left[ \vec{k} \cdot \frac{\partial}{\partial \vec{p}} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} \right] (wf) \tag{8}
\]

For a one dimensional problem, we can write the above equation as

\[
\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}^{el} = \frac{\partial}{\partial p} [\mu_1(p)f] + \frac{\partial^2}{\partial p^2} [\mu_2(p)f] \tag{9}
\]

where

\[
\mu_1(p) = \int d^3kw(p, k)k = \frac{\langle \delta p \rangle}{\delta t} = \langle F \rangle \tag{10}
\]

\(\langle F \rangle\) is the average force acting on the test particle, and

\[
\mu_2(p) = \frac{1}{2} \int d^3kw(p, k)k^2 = \frac{\langle (\delta p)^2 \rangle}{\delta t} \tag{11}
\]

Combining eq. (2) and eq. (9) we get

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p} [\mu_1(p)f] + \frac{\partial^2}{\partial p^2} [\mu_2(p)f] \tag{12}
\]

This is the celebrated Landau kinetic equation \[\textit{[10]},\] a nonlinear integro - differential equation. The nonlinearity is caused due to the appearance of \(f\) in \(\mu_1(p)\) and \(\mu_2(p)\) through \(w(p, k)\). It arises from the simple fact that we are studying a collision process which involves two particles - it should, therefore, depend on the states of the two participating particles in the collision process and hence on the product of the two distribution functions. As is evident from the derivation, the equation is valid for a weakly coupled system, where the average kinetic energy is large compared to the two particle interaction energy.

We can attain considerable simplicity if we replace the distribution functions of the collision partners of the test particle by their time independent equilibrium Fermi-Dirac or Bose-Einstein distributions (depending on the statistical nature) in eqs. (10) and (11). Then eq. (12) reduces to a linear partial differential equation - usually referred to as the Fokker-Planck equation\[\textit{[10]}\].
To relate $\mu_1$ and $\mu_2$ with dynamical parameters relevant for the system under study, let us consider the classical equation of motion of a particle executing Brownian motion in a heat bath,

$$\frac{dp}{dt} = F(t) \quad (13)$$

where $F(t) = F_r(t) + F_d(t)$. Here $F_r(t)$ is the rapidly fluctuating part and $F_d(t)$ is the viscous force. Taking the average of the above equation and assuming that the average of $F_r(t)$ over a long interval of time vanishes, i.e.

$$\langle F_r(t) \rangle = 0$$
$$\langle F_d(t) \rangle = -a_p v \quad (14)$$

where $a_p$ is the dynamical friction parameter containing the dynamics of the problem under study (see next section for details), we can, in the relativistic case ($v = p/\sqrt{p^2 + m^2}$), write

$$\mu_1(p) = -a_p v = \frac{-a_p p}{\sqrt{p^2 + m^2}} \quad (15)$$

Assuming that the coupling between the Brownian particle and the bath is weak, we have

$$\mu_2(p) = \frac{\langle (\delta p)^2 \rangle}{\delta t} = 2a_p (vp) \quad (16)$$

For the ultra-relativistic case $v \sim 1$, $p \sim T$, implying

$$\mu_2(p) \approx 2a_p T \equiv 2D_F \quad (17)$$

Using eqs. (12), (15) and (17) we get

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p} \left( \frac{a_p pf}{\sqrt{p^2 + m^2}} \right) + D_F \frac{\partial^2 f}{\partial p^2} \quad (18)$$

This is the Fokker Planck equation describing the evolution of a quark towards equilibrium due to its interaction with the gluonic heat bath (see ref. [12, 13].)

The relativistic FP equation with inelastic collisions can be written as

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial p_z} \left( \frac{a_p p_z f}{\sqrt{p_z^2 + m_T^2}} \right) - D_F \frac{\partial^2 f}{\partial p_z^2} = \left( \frac{\partial f}{\partial t} \right)_{inel} \quad (19)$$
We can linearize the above equation with the relaxation time approximation \[14, 15, 16\] as follows,

\[
\left( \frac{\partial f}{\partial t} \right)_{\text{inel}} = - \frac{f - f_{eq}}{\tau_{\text{relax}}} \tag{20}
\]

where \(f_{eq}\) is the equilibrium distribution and \(\tau_{\text{relax}}\) is the relaxation time estimated from the reactions \(gg \leftrightarrow q\bar{q}\) and \(g \leftrightarrow q\bar{q}\), \(m_T\) is the transverse mass \((= \sqrt{p_T^2 + m_{\text{eff}}^2})\). \(m_{\text{eff}}\) is the effective mass defined as

\[
m_{\text{eff}} = \sqrt{m_{\text{current}}^2 + m_{\text{thermal}}^2} \tag{21}
\]

where \(m_{\text{current}}\) is the current quark mass \((= 10\text{MeV} \text{ for } u \text{ and } d \text{ quarks})\) and \(m_{\text{thermal}}\) is the thermal mass:

\[
m_{\text{thermal}} = \sqrt{g_s^2 T^2/6} \tag{22}
\]

\(g_s\) is the strong coupling constant. In a chemically non-equilibrated scenario, the thermal mass is replaced by \(m_{\text{thermal}}^2 = (1 + r_q/2)g_s^2 T^2/9\) \[17\], where \(r_q\) is the ratio of equilibrium to non-equilibrium density. We have seen \[18\] that the effect of such a change on thermal mass has negligible effects on the final results.

The FP equation reduces to,

\[
\frac{\partial f}{\partial t} - \frac{\partial}{\partial p_z} \left( \frac{a_p p_z f}{\sqrt{p_z^2 + m_T^2}} \right) - D_F \frac{\partial^2 f}{\partial p_z^2} = - \frac{f - f_{eq}}{\tau_{\text{relax}}} \tag{23}
\]

It should be mentioned at this point although several authors \[14, 15, 16\] have used the relaxation time approximation to study the approach to equilibrium in a quark-gluon system, such a treatment is meaningful only for small deviations from the equilibrium configuration. We have dwelt on the relaxation time approach in some detail only to clarify the physical picture. A more consistent way is to evaluate the contribution of the inelastic term through a time-dependent normalization of \(f\), which can be estimated by solving the momentum-integrated Boltzmann equation, taking proper account of the reactions \(g \leftrightarrow q\bar{q}\) and \(gg \leftrightarrow q\bar{q}\). These details have been reported in the literature \[18\]; for the sake of brevity, we do not repeat...
them here but refer the reader to this work. It should be noted that the reactions $gg \leftrightarrow gg...$ etc. do not appear explicitly as the gluons have been assumed to be thermalized so that their density is determined from the temperature of the bath. Also, the thermal mass of the gluons is an essential ingredient; otherwise the reaction $g \leftrightarrow q\bar{q}$ would be forbidden.

IIa. Determination of $a_p$

We assume that the phase space distribution function can be factorised as $f(\vec{p},\tau) = f(p_z,\tau)G(p_T)$, where $\tau$ is the proper time, $G(p_T) = \exp(-p_T^2/\mu^2)/\pi\mu^2$ and $f(p_z,\tau)$ is the solution of FP equation.

The friction parameter $a_p$ is a very crucial parameter. It contains the dynamics of elastic collisions ($qq, q\bar{q}$ and $gg$). It can be related to the energy loss of a quark in a dense partonic medium [19] in the following way:

$$a_p(p_z,\tau) = \frac{E}{p_z} \left\langle \frac{-dE}{dx} \right\rangle \Rightarrow a_p(\tau) = \left\langle \frac{E}{p_z} \left( -\frac{dE}{dx} \right) \right\rangle$$

(24)

The approach to equilibrium for the different quark species is then determined by eq.(23). In principle, $a_p$ may be determined from kinetic theory formulation of QCD through the application of the fluctuation dissipation theorem[10], but that is indeed far too complex a problem to handle, given the present state of the art. It can, however, be assumed that since the friction constant is expected to be largely determined by the properties of the "bath" and not so much by the nature of the test particle, one may take $a_p(p_z,\tau) \simeq a_p(\tau)$. In this respect, we recall the earlier work of Svetitsky[20] where the classical diffusion and drag coefficients of a nonrelativistic charm quark propagating in a quark gluon plasma were calculated. Although his scenario is somewhat different from ours, the operating equation in both cases happens to be the Fokker Planck. In his dynamical calculations, he found approximate momentum independence of the drag coefficient(fig.2 of [20]), entirely in line with our assumption. It is however not realistic to the values of $a_p$ from [20].
for lighter quarks. We may also remark here that a recent work has appeared in
the literature \cite{21} where a Fokker Planck type equation, including the non-abelian
features of QCD in the collision terms of the transport equation, has been discussed.
The main attraction of this work is in studying the damping of the collective colour
modes, of relevance to jet quenching studies but outside the scope of the present
work. There is also a component which governs diffusion in momentum space, but
the deviation from the abelian case is rather small. The correction is proportional
to the small non-equilibrium deviations and as such can be generally neglected \cite{21}.
It is however noteworthy that these authors also relate the momentum diffusion
(or friction ) constant to the partonic $dE/dx$, as in the present work. Let us also
mention that we have assumed the temperature $T(\tau)$ to arise from the thermal bath
whereas these authors look at non-equilibrium contributions to both $f_g$ and $f_q$ ($f_{\pi}$).
There have nonetheless been some recent developments \cite{22, 23} in connection with
jet quenching studies in QGP which may shed light on this issue. The expression
for energy loss has been calculated by various authors in recent times \cite{22} and for
light quarks, it is given by

$$\frac{-dE}{dx} = 4\pi C_F \alpha_s^2 T^2 \ln \left( \frac{k_{\text{max}}}{k_D} \right) \frac{1}{v^2} \left( v + \frac{v^2 - 1}{2} + \ln \frac{1 + v}{1 - v} \right) \quad (25)$$

where $\alpha_s$ is the strong coupling constant, $C_F$ is the Casimir invariant, $k_{\text{max}}$ is the
maximal momentum ($\sim p$, the momentum of the light quark) and $k_D$ is the Debye
momentum. The value of $\alpha_s$ is calculated from the following parameterisation \cite{24}:

$$\alpha_s = \frac{6\pi}{(33 - 2n_f) \ln(\kappa T/T_c)} \quad \text{For heavy quarks with } E << M_Q^2/T$$

for $dE/dx$ \cite{23} can be written as

$$\frac{-dE}{dx} = \frac{8\pi \alpha_s^2 T^2}{3} \left( 1 + \frac{n_f}{6} \right) \left( \frac{1 - v^2}{2v^2} \ln \frac{1 + v}{1 - v} \right)$$

$$\times \ln \left( \frac{2^{n_f/(6+n_f)} B(v) ET}{m_g M_Q} \right) \quad (26)$$
For $E >> M_Q^2/T$ we have

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left( 1 + \frac{n_f}{6} \right) \ln \left( \frac{2^{n_f/2(6+n_f)} 0.920 (ET)^{1/2}}{m_g} \right)$$

(27)

where $M_Q$ is the mass of the heavy quark, $B(v)$ is a smooth function of velocity (see Ref. [23] for details), $m_g$ is the thermal gluon mass, $m_g = g_s^2 T^2/3(1 + N_f/6)$. In the region of $E \sim M_Q^2/T$, we have used the eqs. (26) and (27) appropriately, as in Ref. [23]. It is important to mention here that we have included the radiative energy loss [24] in $dE/dx \left|_{\text{rad.}} = 2\pi\alpha_s C_2 T^2 \right.$ (modulo log terms)); however, the effect of such processes is rather small. Recently, Baier et al. [26] have calculated the energy loss due to radiative process ($\sim \alpha_s \sqrt{E q_D^2 / l_g} \ln(E/q_D^2 l_g)$) including the rescattering of the radiated gluons which is ignored in [24], $l_g$ is the mean free path of the gluons. Weldon [27] has shown that the energy loss is proportional to $\alpha_s^2$ for radiative process as well as for elastic collisions [22] if one assumes $l_g \sim \alpha_s T$.

\section*{IIb. Cooling of the Gluonic Heat Bath}

The gluonic heat bath is cooling due to expansion and the rate of cooling is determined by the relativistic hydrodynamics. The bulk properties of the system, e.g. the cooling law etc., are governed by the equilibrated degrees of freedom. In our case the cooling law is given by the Bjorken’s scaling law with appropriate modification due to quark production through thermal gluon fusion $gg \rightarrow q\bar{q}$ and thermal gluon decay $g \rightarrow q\bar{q}$. We have obtained the cooling law [18] by solving the hydrodynamic equation which is parameterised as $T = \alpha/\tau^\beta$ where $\alpha = 0.4077$, $\beta = 0.355$ at LHC and $\alpha = 0.33$, $\beta = 0.352$ at RHIC energies respectively. The cooling rate in Bjorken model ($\beta = 1/3$) is slower compared to the present case where the production of quarks cost some energy. We would like to mention here that the effects of transverse expansion on cooling law would be negligibly small for the intial parameters under consideration for RHIC and LHC energies (see ref. [28] for details).
IIc. Solution of the Fokker-Planck Equation

We solve the FP equation with the following initial and boundary conditions for a quark species $j$

$$f_j(p_z, \tau) \xrightarrow{\tau \to \tau_g} \Delta_j \delta(p_z)$$

(28)

and

$$f_j(p_z, \tau) \xrightarrow{|p_z| \to \infty} 0$$

(29)

The parameter $\Delta_j$ is determined from the initial density of the partons. $\delta(p_z)$ is a rather good approximation of the low $x$ structure function. We should also mention here that the final outcome of the model is insensitive to the functional form of the initial distribution function - a typical characteristic of the Markovian process.

We can either solve the inhomogeneous FP equation (eq. (23)) with the initial and boundary conditions given by eqs. (28) and (29), or equivalently, solve the homogeneous FP equation with a time-dependent normalization for $f$ which accounts for the evolution of quark density as a result of the inelastic reactions $g \leftrightarrow q \bar{q}$ and $gg \leftrightarrow q \bar{q}$ \[18\]. Our actual calculations show that there is not much difference between the two approaches, insofar as the electromagnetic signals are considered. We therefore show here the results for the relaxation time approach, as most other authors in this field have worked in this framework.

It is important to mention here that the final outcome depends on the value of $\Delta_i$. There is a lack of consensus about the initial value of the quark density. We take the initial values of quark densities from HIJING \[30, 31\]. The phase-space density of quark is larger in case of parton cascade model \[4\] and also in the work of Shuryak \[8\]. In this sense our work corresponds to a conservative situation. The data from RHIC and LHC should make a distinction among various models.
III. Electromagnetic Probes

IIIa. Single Photons

As mentioned in the introduction, photons and dileptons are the most efficient signals for QGP. However, apart from the photons from QGP, there are other sources of photons, e.g., photons from hadronic reactions, initial hard collisions of partons and hadronic decays. In this section we shall evaluate the photons from a non-equilibrated quark gluon system, thermalised QGP, and from initial hard collisions.

The dominant reaction channel for single photon emission are the annihilation \((q\bar{q} \rightarrow g\gamma)\) and Compton \((q(\bar{q})g \rightarrow q(\bar{q})\gamma)\). The transverse momentum distribution of photons produced in a reaction \((1 + 2 \rightarrow 3 + \gamma)\) is given by

\[
\frac{dN}{d^2p_T dy} = \frac{N}{16(2\pi)^8} \pi R^2 \int \tau d\tau d\eta p_{1T} dp_{1T} d\theta_1 dp_{2T} dy_1 dy_2 \\
\times \frac{|\mathcal{M}|^2}{|p_{1T} \sin(\theta_1 - \theta_2) + p_T \sin \theta_2|_{\theta_2}} \\
\times f(p_{1z}, \tau)G(p_{1T})f(p_{2z}, \tau)G(p_{2T}) (1 \pm f(p_{3z}, \tau)G(p_{3T}))
\]

with

\[
\begin{align*}
\theta_2^0 &= \psi - \cos^{-1} \left( \frac{H}{2Rp_{2T}} \right) \\
R &= (p_{1T}^2 + p_T^2 - 2p_T p_{1T} \cos \theta_1)^{1/2} \\
\psi &= \tan^{-1} \left( \frac{p_{1T} \sin \theta_1}{p_{1T} \cos \theta_1 - p_T} \right)
\end{align*}
\]

where

\[
H = m_1^2 + m_2^2 - m_3^2 + 2p_{1T}p_T \cos \theta_1 + 2m_{1T}M_{2T} \cosh(y_1 - y_2) \\
-2m_{1T}p_T \cosh(y_1 - y) - 2m_{2T}p_T \cosh(y_2 - y)
\]

We have written the total distribution function as \(f(\vec{p}, \tau) = G(p_T)f(p_z, \tau)\) where \(G(p_T) = \exp(-p_T^2/\mu^2)/\pi \mu^2\) with \(\mu = 0.42\) GeV.\[42\]
For the photons from non-equilibrated and equilibrated QGP we take the phase space distribution from the solution of FP equation and Fermi-Dirac distribution, respectively, for the quarks. Gluons are always described by Bose-Einstein distribution.

The hard QCD photon spectra from the Compton process is evaluated [11] by using the following expressions,

\[
\frac{d\sigma^C(y = 0)}{dyd^2p_T} = \frac{\alpha\alpha_s}{3s^2(x_T/2)} \int_{x_{\text{min}}}^{1} \frac{dx_a}{x_a - (x_T/2)}
\times \left[ F_2(x_a; A)G(x_b; B) \frac{x_b^2 + (x_T/2)^2}{x_a^2x_b^2} + (x_a \leftrightarrow x_b; A \leftrightarrow B) \right]
\]

(33)

where

\[
x_b = \frac{x_ax_T}{2x_a - x_T}, x_{\text{min}} = \frac{x_T}{2 - x_T}
\]

(34)

\(F_2(x) = x \sum e_i^2 [q(x) + \bar{q}(x)]\) and \(G(x) = xg(x)\), where \(g(x)\) and \(q(x) (\bar{q}(x))\) are the structure functions for gluons and quarks (antiquarks), respectively.

Similarly, the result for the annihilation graph is given by

\[
\frac{d\sigma^A(y = 0)}{dyd^2p_T} = \frac{8\alpha\alpha_s}{9s^2} \int_{x_{\text{min}}}^{1} \frac{dx_a}{x_a - (x_T/2)}
\left[ Q_2(x_a; A)\bar{Q}(x_b; B) \frac{Q_2 + x^2_b}{x_a^2x_b^2} + (x_a \leftrightarrow x_b; A \leftrightarrow B) \right]
\]

(35)

where we have defined \(Q_2(x) = x \sum e_i^2 q(x), \bar{Q}(x) = x \sum \bar{q}(x)\)

The strong "running coupling constant" is given by,

\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda_{QCD}^2)}
\]

(36)

where \(N_f\) is the number of flavours and \(\Lambda_{QCD}\) is the QCD scale parameter.

We have chosen \(Q^2 = p_T^2\) for evaluation of single photon spectra from hard QCD processes.
IIIb. Dileptons

The dilepton spectrum has been considered a promising probe for the QGP diagnostics. In the dilepton mass window $M = 2$–$5$ GeV, the main source is the electromagnetic annihilation of quarks and antiquarks. The invariant dilepton mass spectrum is given by

$$\frac{dN}{dM^2 dy} = \frac{\pi R_A^2}{2(2\pi)^5} \int \tau d\tau d\eta d\theta p_T p_T q_T dq_T \times f(p_0^0, \tau) f(p_0^0, \tau) G(p_T) G(p_T) \Gamma_{q\bar{q} \rightarrow \mu^+\mu^-}$$

(37)

where $\Gamma = |p_0 - \bar{p}_0|/(p_0^0 M_T)$. The basic cross-section for dilepton production due to $q\bar{q}$ annihilation is given by

$$\sigma_{q\bar{q} \rightarrow \mu^+\mu^-} = \frac{80\pi}{9} \alpha^2 s N_c (1 - \frac{4m^2_\mu}{M^2})^{1/2} \left(1 - \frac{4m^2_q}{M^2}\right)^{-1/2} \times \left[1 + \frac{2m^2_\mu}{M^2} + \frac{2m^2_q}{M^2} + \frac{4m^2_\mu m^2_q}{M^4}\right]$$

(38)

We obtain $f(p_z, \tau)$ from the solution of FP equation with appropriate boundary conditions as mentioned above. The dilepton production rate in complete thermal equilibrium (both kinetic and chemical) is obtained by replacing the quark distribution function by Fermi-Dirac distribution function.

The Drell-Yan production has been obtained by using the expression given below \[1\].

$$\frac{d\sigma}{dM^2 dy} = \frac{4\pi\alpha^2}{3sN_c M^2} \sum_{f=1}^{N_f} e_f^2 \left[q_f^B(x) q^A_f(x) + q_f^A(x) q^B_f(x)\right]$$

(39)

The structure functions $q_f(x)$ has been taken from ref. 33.

IIIc. Diphotons

For kinematic purposes, it is naturally preferable and useful to work with a pair of particles in the final state which would allow an invariant mass identification and thus a filter. To this end, working with dileptons or diphotons is advantageous compared to single photons. The disadvantage with diphotons is that it is a $\alpha^2$
process whereas single photon is a $\alpha\alpha_s$ process in the lowest order; consequently a substantial decrease with respect to single photon rates. The possibility of treating diphoton as a signal of QGP was considered in [34, 35]. The basic cross-section for diphoton production is

$$\sigma_{\gamma\gamma}(M) = 2\pi\alpha^2 N_c (2S + 1)^2 \sum_q \frac{e_q^4}{M^2 - 4m_q^2} \times \left[ 1 + \frac{4m_q^2}{M^2} - \frac{8m_q^4}{M^4} \right] \ln \left( \frac{M^2}{2m_q^2} \left[ 1 + \left( 1 - \frac{4m_q^2}{M^2} \right)^{1/2} \right] \right) - 1 \right)$$

The invariant mass distribution for the diphoton is the same as eq. (37), only the dilepton cross-section being replaced by diphoton cross-section. We should however note that the temperature dependence of the two cross-sections through the thermal mass is quite different. For diphoton production from a thermalised QGP and also from the initial hard QCD collisions, the calculation proceeds along the same line as for dilepton production. For the evaluation of dileptons from DY process and hard diphoton we have chosen $Q^2$ to be equal to $M^2$.

**IV Results**

**IVa. Approach to Equilibrium**

As mentioned earlier, in the system under study the quark density changes with proper time due to two mechanisms. The expansion dynamics (flow) dilutes the density and on the other hand, the creation of quarks in the relativistic heat bath enhances the quark density. The gluon density decreases due to expansion only. We calculate the ratio of the width of the distribution in non-equilibrium and equilibrium situations, i.e. $(p_z^2)^{\text{non-eq}}/(p_z^2)^{\text{eq}}$. The advantage of calculating the ratio is that the expansion effects will get cancelled to some extent, though the cooling in the equilibrium and non-equilibrium scenario is different as has been mentioned before.
It is obvious that the cooling in the non-equilibrium situation should be faster than Bjorken boost invariant cooling law: $T \sim 1/\tau^{1/3}$. The reason behind the faster cooling in the present scenario is due to the production of quarks in the gluonic heat bath through the reaction described above, soaking away energy and thus, cooling rapidly.

In fig. 1 we plot these ratios as a function of the proper time $\tau$. At RHIC (LHC) the initial thermalisation time, $\tau_g$ for the gluons is 0.3 fm/c (0.25 fm/c), the temperature $T_g(\tau_g)$ is 500 MeV (660 MeV) and the initial quark density $n_{u/d}$ is 0.7 fm$^{-3}$ (2.8 fm$^{-3}$). At RHIC energy (fig. 1(a)) we observe that the ratio $D$ saturates to a value $\sim 1$, at a proper time $\tau \sim 3$ fm/c, well before the temperature of the system reaches to $T_c$ ($\sim 160$ MeV). In fig. 1(b) we plot $D$ for LHC energies; the thermal equilibration is complete within the proper time $\sim 2$ fm/c. Fig. 2 ((a) and (b)) brings out the same information in greater detail. We plot here the width of the momentum distribution of quarks as well as gluons. The decrease of the width for gluons corresponds to the cooling due to expansion. One can readily see the width for quarks first increases and then starts falling just like the gluons, indicating a clear onset of thermal (kinetic) equilibrium.

We evaluate the density of quarks in the non-equilibrium scenario by integrating the distribution function $f(p_z, \tau)G(p_T)$ over its momentum. The non-equilibrium density $n_q$ has an explicit dependence on $\tau$ and an implicit dependence on $\tau$ through $T(\tau)$. But the equilibrium density $n_{eq}(T)$ has only an implicit dependence on $\tau$ through $T(\tau)$. The ratio $r_q = n(\tau)/n_{eq}(T(\tau))$ thus assumes an universal feature, since the implicit time dependence gets eliminated. The time dependence of the ratio $r_q$ can then be used as a ready marker for chemical equilibrium; the time at which the explicit time dependence of $r_q$ vanishes, simultaneously with $r_q \rightarrow 1$, corresponds to the time for chemical equilibration for the flavour $q$. We observe from fig. (3) that $r_q$ neither saturates nor approaches the value unity before the temperature of the system reaches the value $T_c$ (160 MeV). Therefore, we conclude that the chemical
equilibrium is not achieved in the quark gluon system, although thermal(kinetic) equilibrium is. To show the sensitivity of the evolution of $r_q$ on the initial quark density we include, in fig. (3), the result from one of our previous calculations \[18\] obtained by taking initial quark density from the structure functions \[1, 29\]. For the sake of completeness we also show the cooling law in fig. (3), where $q\bar{q}$ production has been taken into account. We also see in fig. (4) that the production of $s\bar{s}$ and $c\bar{c}$ pairs from $q\bar{q}$ ($q = u/d$), $gg$ fusion and $g$ decay. As expected the results are not drastically altered from the case with $q\bar{q}$ and $gg$ fusion.

**IVb. Electromagnetic Signals**

In the preequilibrium era, the temperature is very high but the quark density is far below its fully equilibrated value. In the standard scenario where one assumes the system is fully equilibrated at a proper time $\tau_0$ ($\sim 1$ fm/c), the temperature is small for a given multiplicity, but the quark density is large. Within the framework of our model, we study the effect of these competing aspects on the photon, dilepton, and diphoton spectra. Clearly the above effect will be maximum for the reactions which involve quark-antiquark annihilation i.e. on dilepton and diphoton spectra.

In fig. 5 we plot the single photon spectrum for LHC energies. The hard QCD photons dominates the spectra for $p_T > 5$ GeV, but these photons are under control through perturbative QCD (pQCD). Photons from non-equilibrated and equilibrated QGP are not distinguishable upto a $p_T$ of 5 GeV. Low $p_T$ photons originate from the late stage of the evolution, when the temperature approaches the critical value $T_c = 160$ MeV. For $p_T > 7$ GeV, the photons from the non-equilibrated QGP system dominate, essentially because of the very high initial temperature though the system is far from chemical equilibrium.

At RHIC\(_s\) (fig .6), the picture is different. Photons from equilibrated QGP dominate the spectra upto $p_T = 8$ GeV. For $p_T > 8$ GeV pre-equilibrium photons dominate because of the high initial temperature. At RHIC energies the system
remains far from chemical equilibrium, \textit{i.e.}, the number of quarks and antiquarks is rarer compared to their equilibrated value till the critical temperature $T_c$. So the photons are less in number in the pre-equilibrium scenario compared to the equilibrium one. The hard photons dominate for $p_T$ above 5 GeV, as in the case of LHC energies. Once again they are under control through pQCD. We have confined our attention entirely to the annihilation and Compton channels for photon production in the quark gluon sector. Even though the number of quarks (antiquarks) is low, these channels still dominate over $gg \rightarrow g\gamma$, as seen from fig. (7).

Figs. (8) and (9) show the dilepton count rates at LHC and RHIC energies respectively. It is readily seen that at LHC (fig. (8)), preequilibrium dominates over equilibrium configurations for all invariant masses. For RHIC (fig. (9)), however, preequilibrium dominates for $M < 2.5 - 3.0$ GeV and equilibrium for $M > 3.0$ GeV. This can be understood in the following way. At LHC, the departure from equilibrium is not so large while at RHIC, it is substantial (fig. (3)). Thus at LHC, the higher initial temperature, together with not too low quark density, results in preequilibrium emission dominating over the equilibrium scenario for all values of $M$. For RHIC energies, however, the higher initial temperature is largely compensated by the very low quark densities. In both cases, Drell-Yan (hard) is seen to dominate over both preequilibrium and equilibrium emissions for $M \geq 2$ GeV but once again, they can be taken care of through pQCD.

Figs. (10) and (11) show the diphoton count rates at LHC and RHIC, respectively. They behave qualitatively in the same way as dileptons. This is to be expected, since both dileptons and diphotons come from the $q\bar{q}$ channels and as such, they only differ in the elementary cross-sections (including the effects of thermal masses).
V. Summary and Conclusions

We have analysed the approach to thermal and chemical equilibrium in a quark gluon system within the framework of a semi-classical, physically transparent model. A fundamental consequence of this picture is that while thermal (kinetic) equilibrium is probable, chemical equilibrium is not, even for LHC energies. Even the kinetic equilibrium is achieved through a succession of time scales. The central issue of this work is to explore the possibility of testing the equilibrium or pre-equilibrium scenario through the so-called signals of quark - gluon plasma.

We have therefore calculated the various electromagnetic probes for such a successive equilibration picture. It is seen that both at RHIC and LHC energies, emission from the preequilibrium phase does affect the electromagnetic signals of the quark-gluon-plasma at the kinematic windows thought to be appropriate for such studies. If the goal of heavy ion studies is to look for fully equilibrated quark-gluon-plasma, then dileptons and photons at $M \geq 2.5$ GeV appear to have a better chance of being successful at RHIC. Curiously, even though the system appears to approach an equilibrium configuration much more closely at LHC than at RHIC, the dilepton or diphoton signals seem to perform better at RHIC energies, at least for the above purpose. It should however be noted that while dilepton and diphoton spectra behave very similarly, a single photon spectrum has a markedly different behaviour. This obviously is due to the fact that the single photon spectrum has contribution also from the Compton ($q\bar{q} \rightarrow q\gamma$) channel, in addition to annihilation ($q\bar{q} \rightarrow q\gamma$) where as dilepton or diphoton are sensitive only to annihilation. Thus a correlated measurement of $\gamma$ and $\gamma\gamma$ would shed light on the early evolution of the gluon density while simultaneous measurement of $\mu^+\mu^-$ and $\gamma\gamma$ would test the validity of the mechanism visualised here to study the approach to equilibrium.

Finally the present calculation $dN_{c\bar{c}}/dy$ at RHIC energies ($T_g = 500$ MeV) turns out to be $\sim 0.2$ which compares favourably with that of Shuryak ($\sim 0.3$).
Inclusion of thermal masses suppresses the heavy quark production in the present case compared to that of Shuryak’s calculation where the thermal quark masses have been neglected.

We note that in Ref. [17] the authors observe photons from chemically non-equilibrated partonic gas is less than that from equilibrated plasma by a factor of $10^{-2}(10^{-1})$ at RHIC (LHC) energies for $1 < p_T < 3$ GeV, whereas Shuryak [6] finds an enhancement in both photon and dilepton productions over the equilibrium scenario at RHIC energies. In Ref. [6, 17] the kinetic equilibrium is assumed throughout the evolution history unlike the present case. However, in the present case at LHC photons from non-equilibrated quark-gluon system and fully equilibrated QGP are not distinguishable, whereas at RHIC the equilibrium photon yield is order of magnitude larger than that of non-equilibrated scenario. This is largely because of the fact that at LHC the initial partonic gas has more chance to get closer to chemical equilibrium compared to RHIC. The difference in photon/dilepton spectra from all these calculations could be largely attributed to the initial conditions where much work needs to be done [4]. The data from RHIC and LHC will make distinction among various initial conditions.

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Figure Captions

Figure 1. Ratio ($D$) of the width of the non-equilibrium to the equilibrium momentum distribution for $u$ and $d$ quarks as a function of proper time $\tau$ (a) at RHIC energies, (b) at LHC energies.

Figure 2. Width of the momentum distributions of quarks ($u/d$) and gluons as a function of proper time $\tau$ (a) at RHIC energies, (b) at LHC energies.

Figure 3. The evolution of non-equilibrium density normalised to equilibrium density, and temperature as a function of proper time $\tau$ (a) at RHIC energies, (b) at LHC energies.

Figure 4. Heavy quark production at RHIC and LHC energies as a function of temperature at $y = 0$.

Figure 5. Transverse momentum distribution of single photons at LHC energies. The initial parameters for the equilibrium case are taken as $T_i = 290$ MeV and $\tau_i = 1$ fm/c.

Figure 6. Same as fig. 5 at RHIC energies. The initial parameters for the equilibrium case are taken as $T_i = 250$ MeV and $\tau_i = 1$ fm/c.

Figure 7. Transverse momentum distribution of single photons from the channel $gg \rightarrow g\gamma$ compared to $q\bar{q} \rightarrow g\gamma$ at RHIC and LHC energies.

Figure 8. Invariant mass distribution of muon pairs at LHC energies with initial parameters same as fig. (5).

Figure 9. Same as fig. 8 for RHIC energies with initial parameters same as fig. (6).

Figure 10. Invariant mass distribution of photon pairs at LHC energies with initial parameters same as fig. (5).

Figure 11. Same as fig. 10 for RHIC energies with initial parameters same as fig. (6).
\[ D = \frac{\langle p_z^2 \rangle}{\langle p_z^2 \rangle_{\text{eq}}} \]

(u/d quark)

RHIC
\[ D = \frac{\langle p_z^2 \rangle}{\langle p_z^2 \rangle_{\text{eq}}} \]

\[ \tau \text{ (fm/c)} \]

(b) u/d quark

LHC
