Entropy, String Theory, and our World†

Andrea Gregori¹

Humboldt-Universität, Institut für Physik
D-10115 Berlin, Germany

Abstract

We investigate the consequences of two assumptions for String (or M) Theory, namely that:
1) all coordinates are compact and bound by the horizon of observation, 2) the “dynamics”
of compactification is determined by the “second law of thermodynamics”, i.e. the principle
of entropy. We discuss how this leads to a phenomenologically consistent scenario for our
world, both at the elementary particle’s and at the cosmological level, without any fine
tuning or further “ad hoc” constraint.

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¹e-mail: agregori@physik.hu-berlin.de
1 Introduction

Since the time of its introduction, and subsequent interpretation as a viable quantum theory of gravity and of the other interactions, there have been innumerable attempts to make of string theory a “true” theory, and not just a complicate machinery able to produce whatever kind of spectra through a plethora of allowed compactifications (although never precisely “the” spectrum one would like to find). It seems that one of the main problems is the one of providing this theory with a dynamics. The classical way this problem is phrased and solved is through the introduction of a “potential” in the effective action of the theory. Indeed, as it is, String Theory looks like an “empty” theory, comparable to what could be in quantum mechanics having the kinetic term of the harmonic oscillator, and therefore been able to
solve for the “asymptotic states”, without knowing what are the interactions of these states. In the case of string theory, it is already quite hard to find an appropriate formulation of this problem. The reason is that the usual Lagrangian approach treats space and time as “external parameters”, while in String Theory these are fields themselves. And it is correct that things are like that: as a quantum theory of gravity String Theory must contain also a solution for the cosmological evolution of the Universe. In this perspective, one may worry about whether it does make any sense at all to speak in terms of a potential to minimize, in order to “stabilize” the moduli to the solution corresponding to our present world: what kind of “stabilization” has to be looked for in a theory supposed to describe the evolution of “everything”, namely of the space-time itself? And moreover, what do we have to intend for the “space-time”? Normally, we are used to consider this as the natural framework, the “space of the phases” in which to frame our physical problem. As such, the space-time is necessarily infinitely extended, because it is by definition the space in which the theory is allowed to move. But for string theory space and time are not the frame, they are a set of fields. Where do they move these fields? What is their “phase space”, what is the “parameter” that enables to label their configurations? A first, key observation, is that, if we interpret the string coordinates as the coordinates of our Universe, then, despite the common “lore”, there is no evidence that (at least) four of them are infinitely extended. All what we know is what, today, is contained inside our horizon of observation, namely, the portion of Universe included in a space-time region bounded by its Age. Why should a theory of our Universe include in its description also regions “causally disconnected” from our one? In these regions there can be anything, and whatever there is there, our physics will not be affected. Assuming that four string coordinates are infinitely extended is indeed redundant, it is requiring more than what our experimental observation honestly allows us to say. Therefore, in this work we will be “conservative”, and take the more “cautious” approach of considering them compact, and limited precisely by the horizon of observation. As we will see, this apparently “innocuous” assumption has indeed deep consequences on the way we phrase our physical problems. From this perspective, the evolution of String Theory is the history of our Universe, intended as the space-time and the matter and fields inside it. The fact that, as it is, String Theory allows for an extremely huge number of configurations, a priori all equally viable and valid, suggests us that if there has to be a “frame” in which to “order” this history, this has to be provided by statistics. The “dynamic principle” that seems to us the more logical and natural is the second law of thermodynamics. This provides in fact an ordering in physical phenomena, which can be associated to time evolution, but does not involve directly a “classical” treatment of time. It looks therefore suitable for a theory in which time is a field. The fundamental notion on which to base the ordering is therefore “Entropy”. But here the question: what do we have to intend for it? Usually, this quantity is defined in relation to the amount of disorder, or the number of ways it is possible to give rise to a certain configuration, for a system consisting of many degrees of freedom, normally many particles. In any case, a “macroscopic” definition. In this work we will try to extend, or “adapt”, the definition, in order to include a treatment also of “microscopic” systems, such as the configurations of compact string coordinates. Inspired by the fact that particles can be viewed in some way as sources of singularities of space-time, i.e. of large scale coordinates, we will associate to entropy also the treatment of singularities of coordinates.
at a sub-Planckian scale\footnote{In this work we will identify the (duality-invariant) string scale with the Planck scale.}. The inclusion of both scales is a natural requirement in order to keep consistency with a fundamental notion of String Theory: T-duality.

In this work we discuss how with this “dynamical input” String Theory can be “solved” to give a “history” of our Universe, ordered according to the entropy of the various configurations. In particular, we will see how an expanding Universe is automatically implied. Starting at the “origin” from a “flat” configuration, absolutely democratic in all string coordinates, the system attains the minimum of entropy at the Planck scale. At this scale, the string space is the most singular one. In this configuration, T-duality is broken, and many coordinates are “twisted”. Precisely the breaking of T-duality allows to chose an “arrow” in the time evolution, and speak of our “era” as the one in which statistical phenomena act at a macroscopic level, i.e. at scales larger than the Planck length. Our space-time arises as the only part of the string space whose coordinates are not twisted, and therefore remain free to expand. The spectrum of what we call elementary particles and fields is uniquely determined at the time of minimal entropy, and coincides with the known spectrum of quarks and leptons, and their interactions. However, in this set up masses don’t arise as a consequence of the interaction with a Higgs field. Rather, they are a quantum effect related to finiteness of our horizon of observation, in turn related to the “Age of the Universe”. The relation between masses and this quantity is not simply the one expected by ordinary quantum field theory of a system in a compact space: the space-time of string theory is in fact somehow an “orbifold” of what we call space-time. Different types of particles and fields originate from different “orbifold sectors”, and they “feel” the space-time each one in a different way. In a way reminiscent of the difference of perception of space and rotations between Bosons and Fermions, the space of string theory, made up of an extended and an internal one, provides additional degrees of freedom leading to a further discrimination of matter through a varied spectrum of masses. A consequence of these relations is the decay of particle’s masses with time. Although at the present day this effect is too slow to be directly detected, it is by itself able to explain the accelerated expansion of the Universe. Besides the non-existence of the Higgs field, in fact not needed in order to provide masses in String Theory, another aspect of the scenario arising in our set up is the non-existence of supersymmetry, which turns out to be broken at the Planck scale. Precisely a mass gap between the particles and fields we observe and their supersymmetric partners of the order of the Planck mass produces the value of the cosmological constant we experimentally observe. As we will discuss, this parameter can be interpreted as the manifestation of the Heisenberg’s Uncertainty Principle, at a cosmological scale. Put the other way round, this Principle could be “derived” from the existence of the cosmological constant. This fact supports the idea that indeed String Theory provides a unified description of Gravity and Quantum Mechanics.

Among the strong points of our proposal is the natural way several aspects of our world are explained, and come out necessarily, without any fine tuning: besides the already mentioned cosmological constant and accelerated expansion of the Universe, the correct spectrum of particles and interactions, with the correct “prediction” for the electroweak scale. In particular, the parity violating nature of the weak interactions, as well as the breaking of space and time parities, appear as the effect of a “shift” in the space-time coordinates, necessarily
required by the entropy principle. It is noteworthy that, without this operation on the space-time, the weak interactions would not be chiral, something we find rather logical: it had to be expected that the breaking of parity must be due to an operation on the space-time. Another thing that had in some sense to be expected is that masses, being sources of, and coupled to, gravity, only in the context of a quantum theory of gravity, as string theory is, can receive an appropriate treatment and explanation. In this set up, they are not “ad hoc” parameters, but appear as a stringy quantum effect.

The paper is organized as follows. We first introduce and discuss the basic idea, namely the concept of entropy we consider the most suitable for string theory. We state then the “dynamical principles”, according to which we will let the system to evolve. A proper treatment of these topics would require a rigorous mathematical framework in which to phrase the problem of singularities of the string space, something we don’t have at hand. Throughout all the work we will therefore make a heavy use of the closest thing we know, the language of orbifolds: this approach is justified by the hypothesis that this is a good way of approximating the problem, at least in many cases, that we will discuss. (this however does not mean at all that we believe the present world to correspond to a string orbifold!).

Although, owing to the substantial lack of technical tools, our analysis is in many cases forcedly more qualitative than quantitative, nevertheless it is still possible to obtain also some quantitative results, allowing to test, although with a precision limited to the only order of magnitude, the correctness of certain statements, and also to make, up to a certain extent, predictions. Obtaining the correct order of magnitude with a “rough” computation and without any fine tuning, for scales which are $10^{-17}$ (the electroweak scale) or $10^{-61}$ (the square root of the cosmological constant) times smaller than the Planck scale, having assumed this last as the only fundamental mass scale, is anyway already quite significant and encouraging. After having discussed the issues of supersymmetry breaking and the cosmological constant, we pass in section 3 to explain how our “low energy” world arises. A subsection is devoted to the issue of masses and their evaluation. In particular, a prediction for the neutrino masses, necessarily non-vanishing in this set up, is presented. In the same section we discuss the issue of the fate of the Higgs field, and the unification of the couplings. Finally, based on what suggested by the analysis of the previous sections, in section 4 we comment about some aspects of the possible underlying higher dimensional theory, which could be behind string theory. In the framework of our proposal, the breaking of supersymmetry and quarks confinement in our world appear in a natural way, without having to be “imposed” by hand.

Unfortunately, many topics, and especially this last one, can receive only a qualitative treatment. The actual world corresponds in fact to a situation of string coupling of order one. The analysis we present here should therefore be taken, from many respects, more as suggesting a direction of research, than as really providing the final answer to the problems. Despite its degree of approximation, we found it however worth of been presented. We hope to report on more progress in the future.
2 Expansion/compactification and the second principle of thermodynamics

2.1 Entropy in String Theory

The fundamental notion onto which this work is based is Entropy. We will discuss how this quantity has to be considered as the basic ingredient governing the evolution of the Universe, and at the origin of the existence of our world, as it is. In order to do that, we need to give an appropriate definition of this quantity, suitable for string theory. String theory is in fact a theory in which, owing to T-duality, “microscopic” and “macroscopic” phenomena, i.e. phenomena which are above/below the string scale, are mixed, or better, present at the same time. In a context independent on which particular string construction we want to consider, i.e. in a “string-string duality invariant frame”, namely in a true “quantum gravity” frame, the scale discriminating what are “micro” and “macro” phenomena is indeed the Planck scale. The notion of entropy we have been accustomed to is a “macro” aspect, involving statistical fluctuations occurring at a scale above the Planck length. Namely, it has to deal with multi-particle systems. As is known, particles and fields are sources of space-time singularities. On the other hand, they originate from “internal space” singularities. The “internal space” is a completely string theory notion. The distinction between “internal” and “external” space comes out in string theory as a dynamical consequence: the two spaces start on the other hand “on the same footing”, and a definition of entropy suitable for string theory must include in its domain a treatment of the “internal” space as well. Namely, it has to be able to underly also the theory of singularities of this part of the string space. Essentially, what we need is a definition of entropy compatible with T-duality. But what is the meaning of “statistics” for the sub-Planckian scale? It seems to us reasonable to expect that, as “macroscopic” entropy governs singularities in space-time, by providing a statistical theory of the sources of it, in a similar way “microscopic” entropy should be a statistical theory of “compact space” string singularities\footnote{Although in this work we consider all the string coordinates compact, we uniform here to the common way of intending the compact space as the “non extended one”, namely, the complementary of our space-time.}. In this sense, a flat coordinate is more entropic than one which has been orbifolded. The reason why we expect this is that an orbifold is a space in which the curvature is concentrated at certain points. From a geometrical point of view, it is therefore a highly differentiated, singular space. On the other hand, from a physical point of view the spectrum of string theory on an orbifold is richer than on a flat space. Therefore, also in this sense this space corresponds to a more differentiated, less entropic situation. Along this line, we expect more entropic configurations to be those which are more “smoothed down”.

2.2 The “dynamical principles” of the theory

As we discussed in the introduction, String Theory remains an “empty” theory as long as something that we call dynamical principle, which should play a role analog to that of the potential in a Lagrangian theory, is introduced. In the following we will call our theory either “String Theory”, or also “M-theory”, depending on whether we want or not to emphasize the fact that also the coupling of the theory is a coordinate, a priori on the same footing as
the other ones. From any respect, under the two names we intend anyway the same theory. We make the following “Ansätze” for the evolution of the system.

(1) As physical observers we have a horizon. Namely, the maximal distance at which we are able to see corresponds to the distance covered by light during a time equal to the age of the Universe. This must also be the maximal distance at which a theory of our ”universe” is able to ”see” at the present time. Moreover, all the “events” falling out of our causal region of Universe are causally disconnected from all what we can see. Outside of our region there could be anything: in any case this would not affect our physics. Therefore, a theory describing our Universe must be self consistent within our causal region, and to not depend at all on, and say nothing about, whatever there could be outside. As a consequence, if we impose that String, or better M- Theory, is going to describe our physics, by consistency we must conclude that all its coordinates (including the string coupling), are compact, and limited, in units of time, by the Age of the Universe.

(2) During its evolution, String Theory is governed by the “second law of thermodynamics”. By this we mean that there is a function $S$ that monotonically increases/decreases along with time. We don’t know the ultimate reason for its existence. Let’s however assume that such a “function” of the string parameters exists. This function is “entropy”, as we defined it in the previous section, consistently with a theory describing at the same time the “Universe” both at large and small scale.

We will now discuss how the above principles uniquely determine the evolution of the theory toward the present configuration of space-time, with the spectrum of particles and fields, and their interactions, as we observe in current experiments. A first observation is that, since we are going to consider all the coordinates compact, at any stage we are allowed to talk about (broken or unbroken) T-duality. Indeed, from the fact that any compactification/decompactification limit of M-theory corresponds to a string vacuum, we deduce that the theory is renormalizable. Compactness provides in fact a natural cut-off, and since by T-duality we get anyway a string limit, we learn that the theory is cut off both in the ultraviolet and in the infrared. In other words, once we understand that talking of eleven dimensions is only an approximation, equivalent to the removal of the natural cut-off, we obtain also that we do not have anymore to worry about “non-renormalizability” of this theory. The “non-renormalizability” is only an artifact due to our arbitrary removal of its natural cut-off. We should better speak of “eleven coordinates”, and talk about eleven dimensions only for certain practical purposes. Moreover, since we consider time of the same order of the spatial coordinates, we are allowed to speak of T-duality also in this coordinate, intending that an inversion of time corresponds to an inversion of all length scales. In this sense, T-duality along the time coordinate $t$ tells us that $S$ has a minimum at $t = 1$ in Planck units. $S(t)$ behaves in fact as $S(1/t)$. Therefore, if for $t > 1 \, \partial_t S \geq 0$, $\partial_t S \leq 0$ for $0 < t < 1$. Let’s make the hypothesis that at $t = 0$, the theory starts in a completely symmetric configuration, in which statistical fluctuations involve both the large and the small scale geometry. As long as

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3Instead of time, we could better speak of the “radius” of the Universe.
Figure 1: The approximate behavior of entropy, $S$, as a function of time, in String Theory.

T-duality is not broken, “macro” and “micro” phenomena, namely phenomena taking place “above” or “below” the time scale, are equivalent. As we will see, at the minimum T-duality is broken, and several string coordinates are twisted. This means that the theory is not exactly symmetric in $t$ and $1/t$; this imposes a choice of “time direction”, with a consequence also on the type of statistical phenomena governing the increase/decrease of entropy. The breaking of T-duality has in fact the effect of making inequivalent the two branches, $t > 1$, and $t' \equiv 1/t > 1$. For obvious reasons, we make the choice that it is for $t > 1$ that the mass scale of the Universe is below the Planck scale. According to this choice, for $t > 1$ the statistical phenomena underlying the modification of entropy work at a “macroscopic scale”, namely at the level of observable particles and extended objects, living in the string coordinates which are not twisted. Only these, as we will see, are free to expand, and give, at sufficiently large ages, the impression of infinitely extended space-time in which we live. If the scale was instead above the Planck scale, the statistical fluctuations leading to a change of entropy would act at the string scale, modifying also the configuration of the twisted coordinates. This is what happens for $t < 1$, where, owing to the generically unbroken T-duality, the statistical fluctuations that lead to a change of entropy act both above and below the string scale.

From the time of minimal entropy onwards, the geometry of the “internal space” cannot be “shaken” again. Therefore, what we observe now are only the “macroscopic” statistical fluctuations, leading to the ordinary definition of entropy. Notice that, since for $t < 1$ the “large” scale is what for $t > 1$ is a small scale, the statistic of different “large scale space-time” configurations for $t < 1$ is what from our point of view ($t > 1$) looks as a motion through “internal space” configurations. On the other hand, statistical phenomena underlying the fluctuations of the “small scale” geometry at $t < 1$ are the primordial of
what, in our “era”, $t > 1$, are fluctuations of the space-time geometry. The latter is nothing else than the statistics of a many-particle system, because precisely fields and particles are the singularities working as sources for the space-time geometry. With reference to figure [1], this means that, by going backwards in time toward the origin, the internal coordinates are progressively un-twisted. The theory starts therefore at $t = 0$ from a flat configuration of all the coordinates.

2.3 The supersymmetry breaking scale

Let’s follow the evolution of “M-Theory”, starting from the origin, $t = 0$. We assume that at the very beginning all the coordinates are flat. This is the smoothest configuration, the one that treats all the coordinates in the most democratic way, and represents therefore the most “entropic” situation. We now allow for some mechanism that generates a “time” evolution, through some fluctuations among the various possible configurations [1]. We start therefore flowing toward the right of the X-axis of figure [1]. For what we said, although we don’t know what is the “real” size of the Universe at the beginning, this question has no relevance for “our” theory, bounded by our horizon. Therefore, not only the “time” coordinate but also all the other ones are at their “zero”. This means that the theory contains also a T-dual part, in which we can consider the space-time infinitely extended. In particular, it is legitimate to treat the theory by a “string” approximation, valid whenever the coordinates below the tenth, seen as coupling from the low dimensional theory, are sufficiently small/large.

All that to ensure us about the existence of a massless excitation that we call “graviton”. This propagates with the speed of light; its existence tells us that, as time goes by, also our horizon expands at the same speed, in all the directions along which the graviton is free to propagate. At the very beginning, this means along all the directions. Statistical fluctuations progressively introduce singularities leading to more “curved” configurations. In particular, as we will see, this would lead to the breaking of T-duality, ultimately allowing us to speak about “expansion”, as really distinguished from the “contraction”, and time evolution in one direction.

In order to analyze the statistical fluctuations and the configurations they produce, it is convenient to use the language of $\mathbb{Z}_2$ orbifolds. The reason is that orbifolds provide a “fine” parametrization of a singular space and can be constructed also for spaces whose coordinates are at the Planck length, therefore out of the range of “classical” geometry. Among them, $\mathbb{Z}_2$ orbifolds are the “finest” ones. Only $\mathbb{Z}_2$ projections are in fact allowed to act “coordinate by coordinate”, and the maximal amount of singular points in a compact $S^n$ space is obtained for $S^n \approx (S^1/\mathbb{Z}_2)^n$. Two remarks are however in order: 1) we are not stating here that the string space is an orbifold. Rather, we are here supposing that $\mathbb{Z}_2$ orbifolds are in some sense “dense” in the space of singular configurations, in a way somehow reminiscent of the fact

\[4\] A first remark is in order: strictly speaking time is not a parameter, but one of the coordinates of the theory. What actually determines the “progress”, or “time” evolution, is the change of entropy by moving to different configurations. The fact that this evolution can be “ordered” in a sequence of steps happening along the string zeroth coordinate is made possible by the fact that the latter is free to expand. This is not at all an obvious fact: for this to happen it is essential that this coordinate does not get “twisted” at a certain point.
that the set of rational numbers is dense in the set of real numbers. In other words, we make
the hypothesis that $Z_2$ orbifolds constitute a good approximation of the parametrization of
singular spaces. 2) In analyzing $Z_2$ orbifolds, we will always rely on the properties of string
orbifolds.

Starting at the “zero” with the flat, most entropic situation, and following the time
evolution, for what we said we should expect to progress toward less and less entropic
configurations, up to the less entropic situation, realized at time “1” in Planck units. As we
said, this latter can be identified as the most singular, most “differentiated” configuration.
This means that at that point the string space is the most singularly curved one. To be
honest, speaking of “curvature” is not much appropriate at the Planck scale. The notion
of “space” is in fact inherited from our classical concept of “geometry”, a notion not quite
applicable at the Planck scale. As we said, this is a further reason why orbifolds are a good
tool for our analysis. In order to understand how the theory looks like at this point, we can
proceed by ordering the string configurations according to their “microscopic” entropy, i.e.
from the point of view of a scale “above” the Planck mass. In terms of $Z_2$ orbifolds, the
sequence can be summarized as:

\[ \mathcal{N}_{11} = 1 \rightarrow \mathcal{N}_{10} = 1 \rightarrow \mathcal{N}_6 = 1 \rightarrow \mathcal{N}_4 = 1 \rightarrow \mathcal{N} = 0 \, . \quad (2.1) \]

There are indications that the last step, from $\mathcal{N}_4 = 1$ to $\mathcal{N} = 0$, is not really an independent
one, the complete breaking of supersymmetry being already realized at the $\mathcal{N}_4 = 1$ level $^5$. According to Ref. $^2$, the reason seems to be that in $\mathcal{N}_4 = 1$ string vacua there is always at
least one sector that goes to the strong coupling $^6$: this leads to supersymmetry breaking
by gaugino condensation. In section 3 we will discuss more in detail the configuration of
the string space at time 1. As we will see, minimization of entropy requires the presence
of shifts besides the orbifold twists. For the moment we are however interested only in the
scale of supersymmetry breaking, for which it is enough to know just the pattern of twists,
Eq. (2.1). According to the principles of evolution we have stated, the last step, the one
of maximal twisting, is attained precisely at the Planck scale. Even though it is possible in
principle to imagine that one or more coordinates, under casual fluctuations, may get twisted
at a “previous” scale, namely, before having expanded up to 1, further fluctuations move
the theory away from this “intermediate” configurations: such fluctuations act in fact on
the “internal”, i.e. microscopical, geometry as long as the scale is above 1. Any twist above
this scale is only temporary, and in the average any coordinate is free to expand until 1 is
reached. When scale one is reached, the system has attained the maximal allowed number of
coordinate twists, and from now on things change dramatically. Owing to the shifts required
by minimal entropy, T-duality is broken. A further expansion brings us into the region
“below” the Planck scale, not anymore equivalent to the one “above”. Below the Planck
scale, statistical fluctuations leading to a change of entropy act on a “macroscopic” way.
Namely, they are not anymore able to move the theory away from the singularities of the

$^5$A discussion can be found in Ref $^1$.

$^6$This is due to the fact that in such configurations the so called “$\mathcal{N} = 2$ gauge beta-functions” are never
vanishing. As discussed in Ref. $^2$, this is a signal of the existence of sectors of the theory at the strong
coupling.
compact space produced up to the scale 1; rather, such fluctuations appear in the way we are used to see them, involving configurations of the degrees of freedom living on the untwisted coordinates, the only ones along which interactions and fields propagate. These are the coordinates that therefore, being continuously expanding with our horizon, we interpret as “space-time”. The other ones, stuck at the Planck length, constitute the “internal” space of the theory.

A cross, indirect check of the correctness of our thermodynamical considerations is provided by the analysis presented in Ref. [2], about the mechanism of non-perturbative supersymmetry breaking. In Ref. [2] we argued in fact that this phenomenon happens at any time there is a non-vanishing gauge beta-function, and that, if generically the coupling of a weakly coupled sector is parametrized by the moduli $X$ and $Y$, such that $\frac{1}{g^2} \sim \text{Im} X + \beta \text{Im} Y f(Y)$, for $\text{Im} X$, $\text{Im} Y > 1$ the coupling of the strongly coupled sector seems to be parametrized inversely: $g'^2 \sim \text{Im} Y f(Y) + \ldots$. The moduli $X$ and $Y$ are related to the coordinates of the compact space, and are measured in the duality-invariant Einstein frame, whose scale is set by the Planck mass. The fact that the term $\sim \beta \text{Im} Y f(Y)$ is at its minimum, $\sim O(1)$, when the moduli are at the Planck scale, indeed suggests that precisely this is the scale at which it is possible to smoothly pass from the weak coupling to $\mathcal{N} = 1$, with the consequent splitting into weakly and strongly coupled sectors. This must therefore be the scale at which supersymmetry is finally completely broken. This scale sets also the mass gap between the observed particles and their supersymmetric partners. Put in other words, in “$\mathcal{N} = 1$” string vacua the masses of the superpartners of the non-perturbatively broken supersymmetry are tuned by fields, such as the one parameterizing the string coupling, which are twisted, and “frozen” at the Planck scale. At $t = 1$ this is the scale of all masses, because all the coordinates of the theory, and therefore all mass parameters, are at the Planck scale. However, when the system further evolves toward lower scales, namely, when the horizon further expands, mass terms depending on the “internal”, twisted coordinates, remain at the Planck scale, while those depending only on the coordinates free to further expand with the scale of the Universe, decrease as the horizon increases. Of this second kind are the masses of all the particles and fields we experimentally observe: they are related to the expanding scale, or, in other words, to the Age of the Universe, through a set of relations that we will discuss in detail in the following sections. What we can say now is that these “low” masses originate from “nothing”, i.e. are a quantum effect, that from the string point of view is produced by shifts in the non-twisted, space-time coordinates, as required by entropy minimization at $t = 1$. These shifts, which also lead to the breaking of C, P, T parities, lift also the masses of the would-have-been superpartners; the mass gap $\Delta m$ inside supermultiplets remains on the other hand the one set at the Planck scale: $\Delta m \sim O(1)$.

2.4 The cosmological constant

We come now to the issue of the cosmological constant. We have seen that, in the “string evolution”, the string, or better M-theory coordinates, have to be considered compact. However, we have also seen that, while four of them, that we consider as the space-time, are free to perpetually expand, in our branch, $t = 1$, the other ones are “stuck” at the Planck length.
Namely, when measured in Planck units, the “internal” radii are of length “one”. As we have seen, the Planck scale is also the scale of supersymmetry breaking, and sets the order of magnitude of the mass gap between particles (fields), and s-particles (s-fields). Let’s see what are the implications of this fact for the cosmological constant $\Lambda$. In the past, a high mass of the supersymmetric particles was considered incompatible with the present, extremely small value ($\sim 10^{-122}$ in $M_p^2$ units) of this parameter [3, 4]. More recently, it has also been proposed that the low value of $\Lambda$ could be justified by the fact that, even in the presence of very heavy superparticles, supersymmetry could be near the corner in the gravity sector. In this case the value of $\Lambda$ would mostly depend on the gravitino mass, that experimentally cannot be excluded to be very low. Besides this, a plethora of models able to justify a low value of the cosmological constant has been produced. All these models deal in a way or the other with effective configurations of membranes and fields. We will not review them: here we just want to show that all these arguments are not necessary: the smallness of $\Lambda$ can be seen to be precisely a consequence of the breaking of supersymmetry at the Planck scale. In order to see what is the present value of $\Lambda$ predicted in our set up, we can proceed by mapping the configuration corresponding to the Universe at the present time into a dual description in terms of weakly coupled string theory. This is possible because, owing to the fact that several string coordinates at the present time are very large, we can always trade one extended space-time dimension for the M-theory eleventh coordinate. At least as long as we are interested in just an estimate of the order of magnitude of $\Lambda$, we can compute the vacuum energy in the (dual) string configuration at the weak coupling, in a lower number of dimensions. $\Lambda$ will then correspond to a string partition function:

$$Z \approx \int_{\mathcal{F}} \sum_i (-1)^Q q^{P_L^{(i)}} q^{P_R^{(i)}}, \quad (2.2)$$

where, as usual, $q = e^{2\pi i \tau}$, $\mathcal{F}$ indicates the fundamental domain of integration in $\tau$ and $Q$ indicates the supersymmetry charge. A mass gap between target-space “bosons” and “fermions” of order $\Delta P \sim 1$ leads to a value:

$$Z \sim O(1). \quad (2.3)$$

Although this number is not at all small, it is precisely what is needed in order to match with the experimental value of the cosmological constant. In order to see this, we must consider what we have to compare with what. Namely, consider the usual way the cosmological constant appears in the effective action:

$$S \approx \int d^4x \Lambda + \ldots, \quad (2.4)$$

Now, keep in mind that expression (2.2), although technically computed in a weakly coupled theory in less dimensions, it actually gives the string evaluation of the cosmological constant for the physical string vacuum. In order to compare it with the effective action, we must consider that in the expression (2.2), we have integrated over the ghost contribution, or, in other words, we are in the light-cone gauge, in which only $D - 2$ transverse space-time
coordinates appear. The other two can be considered as "identified" with the string world-sheet coordinates. In order to compare \(2.2\) with \(2.4\), we must therefore integrate over two space-time coordinates also in \(2.4\), or in other words put them at the natural, string-duality-invariant scale, the Planck scale \(^7\). In order to compare with the string result, expression \(2.4\) must therefore be corrected to:

\[
S \rightarrow \tilde{S} = \int d^2x \, V_2 \Lambda, \tag{2.5}
\]

where

\[
V_2 = \left( \int_0^1 \int_0^1 d^2x \right) \sim O(1), \tag{2.6}
\]

in Planck units. Finally, we must take into account that the idea of an infinitely extended space-time is only an approximation, valid in order to describe the local physics in our neighborhood, but has no physical justification: as we said, the theory can not in fact know about what happens in regions causally disconnected from our one, and a fundamental theory of our world, namely, of the world we observe, must be self contained inside our causal region, at our present time. The value of the string coordinates in the target space sets our present day horizon. This means that the integration over the two "transverse" coordinates is performed up to the present-day border of our causal region, namely on a volume determined by the distance covered by a light ray since the "origin" of the Universe. Therefore, what has to be compared with "1", is:

\[
\int_{\text{Age of Universe}} d^2x \, \Lambda \approx T^2 \times \Lambda, \tag{2.7}
\]

where \(T\) is the Age of Universe. In Planck units, \(T = O \left(10^{61} M_p^{-1}\right)\). Comparison of the string prediction, \((2.3)\), with the effective theory parameters in \((2.4)\), requires therefore \(\Lambda\) to be of order \(10^{-122} M_p^2\), as suggested by the most recent measurements \(^3, 4\).

Notice that this value is exactly the one we would obtain by considering that \(\Lambda\) is a measure of the "energy gap" of the Universe within the time of its existence, as derived from the Heisenberg’s Uncertainty Principle:

\[
\Delta E \Delta t \geq 1, \tag{2.8}
\]

if we use \(\Delta t = T\). Namely, since the “time” length of our observable Universe is finite, the vacuum energy cannot vanish: it must have a gap given by \((2.8)\). \(\Lambda\), which has the dimension of an energy squared, is then nothing but the square of this quantity\(^8\). We find this very deep: the existence of the cosmological constant is nothing but the manifestation of the Uncertainty Principle on a cosmological scale. This fact “reconciles” Quantum Mechanics with General Relativity.

\(^7\)This kind of manipulations make sense because indeed we consider also the target space coordinates to be compact, so that we can “rescale” them to the Planck length, and consider them of the same order of the compact, world-sheet coordinates.

\(^8\)In other words, \(\Lambda\) is a pure quantum effect. Since it is not protected by any symmetry, it is naturally generated by “quantum corrections”, and its size is set by the natural cut-off of the Universe.
Notice that, owing to the expansion of the horizon, $\Lambda$ turns out to be a function of time, and decreases as the Age of the Universe increases. This is a simple consequence of (2.3), or, if one wants, of (2.8). In particular, this means that the geometry of our space-time tends to a flat limit. We will come back to this point in section 4.

3 How does the “Standard Model” arise

Let’s consider now what is the low energy world arising in this set up. As we mentioned, the “final” configuration of string theory, namely, down on from the Planck scale, is such that there are no massless matter states. What distinguishes the light, observable particles, from other string massive excitations that we are not anymore allowed to interpret as particles, is that the masses of the first ones originate from the compactness of the space-time, while the others are massive also in “true”, infinitely extended, four dimensions, their mass being related to the internal, twisted coordinates. Therefore, while the first ones have masses related to the space-time scale, the others are “stuck” at the Planck scale.

We start here the analysis of the configuration at $t = 1$ by artificially considering the non-twisted coordinates as infinitely extended. The condition of their compactness will be included later. In the spirit of the usual reference to $Z_2$ orbifolds as to the most convenient situations, what we have to consider are indeed the non-freely acting ones: these are in fact those that lead to the most singular ($\equiv$ less entropic) configuration of the “internal” string space. Orbifolds in which matter and field masses are all lifted by shifts in the internal space, namely the freely acting ones, represent less singular $\equiv$ more entropic configurations, because the configuration of the internal space is more “simple”. For instance, they don’t leave room for a chain of several subsequent rank reductions. They are therefore not favored by the minimal entropy ($\equiv$ maximal singularity) requirement.

3.1 The origin of three generations

The less entropic configuration among non-freely acting, $d = 4$ $Z_2$ orbifolds, is achieved with the so called “semi-freely acting” ones. These orbifolds are those in which, although the projections reducing supersymmetry don’t act freely, the number of fixed points is the minimal one. This condition is achieved by further acting on a non-freely acting orbifold with rank-reducing projections of the type discussed in [5, 6, 7, 2]. They represent therefore the most singular configuration, the one in which acts the highest number of projections, inside this kind of spaces. The presence of massless matter is in this case still such that the gauge beta functions are positive, ensuring thereby “stability” of the vacuum $\equiv$ stability means here that, once we focus on this vacuum, the asymptotic states are the good ones. Namely, the vertex operators indeed describe infrared-free objects.
quantitative aspects, we can suppose the eleventh coordinate to be not exactly at the Planck scale, and then extrapolate the “result” to that scale. Alternatively, it is possible to analyze the structure of singularities by mapping to a dual, weakly coupled description, in which the eleventh coordinate is traded by one space-time coordinate. Although the structure of the spectrum looks there quite different, the two descriptions are equivalent for what matters the counting of singularities, and the counting of orbifold projections can be done in a rigorous way. Let’s therefore speak in terms of orbifolds. After all, this is not so different from what one does when analyzes the strong interactions in terms of free quarks, whose interaction has a coupling with negative beta-function. Starting from the M-theory situation with 32 supercharges, we come, through orbifold projections, to a string-like situations with 16 supercharges and a gauge group of rank 16. Further orbifolding leads then to 8 supercharges and introduces for the first time non-trivial matter states (hypermultiplets). As we have seen in through an analysis of all the three dual string realizations of this vacuum (type II, type I and heterotic), this orbifold possesses three gauge sectors with maximal gauge group of rank 16 in each. The matter states of interest for us are hypermultiplets in bi-fundamental representations: these are in fact those that at the end will describe leptons and quarks (all others are finally projected out). As discussed in, in the simplest case the theory has 256 such degrees of freedom. The most singular situation is however the one in which, owing to the action of further $Z_2$ shifts, the rank is reduced to 4 in each of the three sectors. In this case, the number of bi-charged matter states is also reduced to $4 \times 4 = 16$. These states are indeed the twisted states associated to the fixed points of the projection that reduces the amount of supersymmetry from 16 to 8 supercharges.

The next step in the evolution of string theory is a further reduction to four supercharges (corresponding to $\mathcal{N} = 1$ supersymmetry). If we think in terms of $Z_2$ projections, it is easy to recognize that in this way we generate a configuration in which the previous situation is replicated three times. This cannot be explicitly represented in terms of $Z_2$ orbifolds of the type II string. The best points of view from which to look at this configuration are those of the heterotic and the type I string. There, one of the projections that appeared explicitly on the type II string, the “Hořava–Witten” projection, is “hidden”, i.e. it is fully non-perturbative. On the other hand, the coordinates of the theory that explicitly appear allow us to see the effects of the further projection. This latter generates, together with the previous one, three twisted sectors of the same kind of the one just seen above. In each of these sectors, there is a similar structure of the matter states, that now will however be multiplets of the $\mathcal{N} = 1$ supersymmetry. Therefore, at this stage, the structure of “bi-charged matter” states is triplicated. What is interesting is that, in this case, the so-called “$\mathcal{N} = 2$ gauge beta-functions” are unavoidably non-vanishing. According to the analysis of Ref. , this means that there are hidden sectors at the strong coupling, and therefore that supersymmetry is actually broken due to gaugino condensation. The supersymmetry breaking appears therefore to be non-perturbative, parametrized by the string coupling, or,

\footnotetext[10]{To be honest, the physical situation at $t = 1$ corresponds to the “critical point” in which all the coordinates are of length 1. In this case, there is no perturbative approach. In order to get informations about this point, we have to rely on hypothesis of “continuity” of the solution, that we have under control as soon as at least one coordinate is larger or smaller than one.}

\footnotetext[11]{Notice that these states never appear as explicitly “tri-charged”.

14
in general, by the “extra” coordinates of “M-theory”.

Let’s summarize the situation. The initial M-theory underwent three projections and now is essentially the following orbifold:

\[ Z_2^{(H.W.)} \times Z_2^{(N_6=1)} \times Z_2^{(N_4=1)}. \]  

(3.1)

The labels are intended to remind that the first projection is essentially the one of Hořava and Witten [8]. For convenience, we distinguished also the other two, although all these projections are absolutely equivalent, and interchangeable. The “twisted sector” of the first projection gives rise to a non-trivial, rank 16 gauge group; the twisted sector of the second leads to the “creation” of one matter family, while after the third projection we have a replication by 3 of this family, from the \( Z_2^{(N_4=1)} \) and \( Z_2^{(N_6=1)} \times Z_2^{(N_4=1)} \) twisted sectors.

Notice that, as it can be seen on the type II side, where only the pairs \( Z_2^{(H.W.)} \times Z_2^{(N_6=1)} \) or \( Z_2^{(H.W.)} \) and \( Z_2^{(N_4=1)} \) can be explicitly realized, the products \( Z_2^{(H.W.)} \times Z_2^{(N_6=1)} \) and \( Z_2^{(H.W.)} \times Z_2^{(N_4=1)} \) don’t lead to new matter families, but rather to doubling of gauge sectors under which the matter states are charged; in other words, the product of these operations leads precisely to the spreading into sectors that at the end of the day separate into weakly and strongly coupled. Precisely this fact will allow to interpret the matter states as quarks. The product (3.1) represents the maximal number of independent twists the theory can accommodate. Further projections are allowed, but no further twists of coordinates. Precisely these twists allow us to distinguish “space-time” and “internal” coordinates. While the first ones (the non-twisted) are free to expand, the twisted ones are stuck at the Planck length. The reason is that the graviton, and as we will see the photon, in a theory with three twists, live in the (four) non-twisted coordinates. Precisely the fact that light propagates along these allows us to perceive them as extended, namely, as our “space-time”. Indeed, the physical horizon starts expanding as soon as the graviton is created. This means, from the very beginning. However, before the point of minimal entropy, the theory possesses “T-duality”[13]: expansion and contraction are therefore essentially undistinguished. When the string space is sufficiently curved, T-duality is broken, because the configuration of minimal entropy involves also shifts along space-time coordinates. This is what happens at \( t = 1 \), when, as we will see, also the photon is created. Since the process of entropy minimization before the Planck scale leads to the maximal twist (3.1), the further expansion of the horizon at length scales larger than the Planck length, namely, in our “era”, takes place only along four coordinates, those in which matter and its interactions live.

Let’s now count the number of degrees of freedom of the matter states. We have three families, that for the moment are absolutely identical: each one contains \( 16 = 4 \times 4 \) chiral fermions (plus their superpartners, that however, as we discussed, are actually massive, with a non-perturbative mass at the Planck scale). These degrees of freedom are suitable to arrange in two doublets of two different \( SU(2) \) subgroups of the symmetry group. Each doublet is therefore like the “up” and “down” of an \( SU(2) \) doublet of the weak interactions, but this time with a multiplicity index 4. As we will see, this 4 will break into 3+1, the 3 corresponding to the three quark flavors, and the 1 (the singlet) to the lepton. The number of

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12 As we will discuss, the leptons show up as singlets inside quark multiplets.
13 This appears for instance through T- and S-dualities of sufficiently “regular” heterotic string vacua.
degrees of freedom seems to be the correct one, because, in order to build up massive particles, we have to couple two chiral fields, left and right say. Their transformation properties seem however to go in the wrong direction: in our case in fact all the fields are charged under an $SU(2)$. If we make on the other hand the hypothesis that for some reason the two $SU(2)$ are identified, we obtain that the weak interactions are not chiral, in evident contradiction with the experiments. Although surprising, the spectrum we find is nevertheless correct; the point is that, as long as we don’t consider the compactness of the string space-time, as we cannot introduce masses for the light particles and fields, we cannot even speak about breaking of parity in the weak interactions. This is what we will discuss in the following section.

3.2 Light masses

Once compactness of space-time is taken into account, the most ($Z_2$-) singular configuration of the system appears to be the one in which orbifold operations act also on the non-twisted coordinates. In this case, the only allowed operations left over are shifts. As we have seen, shifts in the “internal” coordinates lead to masses that, owing to the fact that such coordinates are also twisted, remain for ever stuck at the Planck scale. Of this kind is the “shift” in masses that breaks supersymmetry. Further shifts in the space-time coordinates produce mass shifts that “decay” as they expand. Since the expansion of these coordinates is related to that of the time coordinate, this means that such masses decay with time. The precise relation between the mass scales and the time scale is not simply the one we would expect by the Uncertainty Principle: $m \sim \frac{1}{T}$, the behavior we would expect from a pure field theory analysis. The String Theory space is an “orbifold”, and the low string modes “feel” the space-time in a different way. Since however the orbifold shifts lift positively all masses inside supersymmetry multiplets, the mass gap between particles and “s-particles” remains, for any time, at the Planck scale.

Besides the conceptual problem of viewing as “functions” of space-time what we are used to consider instead as “stable” parameters of our world, relating masses of particles to the age of the Universe seems to pose anyway the problem of explaining the GeV scale, namely a scale “only” $10^{-19}$ times less than the Planck mass but still much higher than the inverse of the time scale. If we make the rough approximation of considering the compactification space as a product of circles, we would expect masses to be given by an expression like:

$$m_{(0)}^D \approx \prod_{i=1}^{D} \frac{1}{R_i},$$

where $R_i$ are the “radii” of the various coordinates. The index $i$ runs then from 1 to the dimensionality $D$ of the theory (e.g. $D = 11$ in the case of M-theory). In the (more realistic?) case of compactification on a curved space, such as a sphere or an ellipsoid, expression (3.2) then simply states that the mass scale is set by the “curvature”:

$$m_{(0)} \sim \frac{1}{R},$$

(3.3)
with $R^D = \prod_{i=1}^D R_i$. Our problem is to understand what is precisely the “correct geometry”. Clearly, since many coordinates are frozen at the Planck scale, $R$ cannot be just the “Age of the Universe”. Moreover, we have to understand how different particles can “live” on different spaces, and therefore have different masses.

A first observation is that, owing to the fact that the string space is an orbifold, the string “age”, $T_{\text{string}}$, does not coincide with what we perceive as the Age of the Universe, $T$: any $Z_2$ shift implies in fact a square-root relation between $T_{\text{string}}$ and $T$. In order to see this, consider the Uncertainty Principle on such a $Z_2$ orbifold: owing to the $Z_2$ identification inside this space, the error $\Delta R$ around the edge of the space (or time) coordinate, set at “$R$”, is half of the error of the torus. We have therefore:

$$\frac{\Delta R}{R}|_{\text{orbifold}} = \frac{1}{2} \frac{\Delta R}{R}|_{\text{flat space}}. \quad (3.4)$$

Integrating both sides, we get:

$$\log R_{\text{orb}} = \frac{1}{2} \log R_{\text{flat,sp}}, \quad (3.5)$$

which precisely implies the square-root relation among the two coordinates.\(^{14}\) In our space-time there is room for two independent shifts, plus a set of Wilson lines. We want now to see their effect on the way the various particles feel space and time. We have seen that, before any shift in the space-time coordinates, any twisted sector gives rise to four chiral matter fermions transforming in the $4$ of a unitary gauge group. The first $Z_2$-shift in the space-time breaks this symmetry, reducing the rank through a “level doubling” projection. This implies that 1) half of the gauge group becomes massive; 2) half of the matter becomes also massive. The initial symmetry is therefore broken to only one $SU(2)\(^{15}\) under which only half of matter transforms. The remaining matter degrees of freedom become massive. Since this stringy phenomenon happens below the Planck scale, it admits a description in terms of field theory; from this, we see that, in order to build up massive fields, these degrees of freedom must combine with those that a priori were left massless by this operation. Therefore, of the initial fourfold degeneracy of (left-handed) massless matter, we make up light massive matter, of which only the left-handed part feels the $SU(2)$ symmetry, namely what survives of the initial symmetry. This group can be identified with the electroweak group\(^{16}\) The chirality of weak interactions comes out therefore as a consequence of a shift in space-time. This had to be expected: the breaking of parity is in fact somehow like a free orbifold projection on the space-time. It is legitimate to ask what is the scale of mass of the gauge bosons of the “missing” $SU(2)$, namely whether there is a scale at which we should expect to observe an enhancement of symmetry. The answer is no, there is no such a scale. The

\(^{14}\)Another way to see this is by considering that a $Z_2$ projection introduces a factor $\frac{1}{2}$ in expressions like (3.16), that give the running of couplings as a function of the space-size: $\beta \log \mu \rightarrow \frac{1}{2} \beta \log \mu = \beta \log \sqrt{\mu}$. This implies that the effective space-time of the projected theory is the square-root of the unprojected one.

\(^{15}\)This is the maximal non-Abelian group. We will discuss in section 3.3.2 about the fate of the $U(1)$ factors.

\(^{16}\)We skip here for simplicity the issue of the mixing with the electromagnetic $U(1)$, of which we will talk later.
reason is that the scale of such bosons is simply T-dual, with respect to the Planck scale, to the scale of the masses of particles. This fact can be understood by mapping this problem to a specific string configuration realizing such effect: in any string realization of such a situation, the gauge bosons originate from the “untwisted” sector, while matter comes from the “twisted” sector. It is clear therefore that a shift on the string lattice lifts the masses of gauge bosons and of matter in a T-dual way. Since the scale of particle masses is several orders of magnitude below the Planck scale, the mass of such bosons is much above the Planck scale; at such scale, we are not anymore allowed to speak about “gauge bosons” or, in general, fields, in the way we normally intend them.

A second $Z_2$ breaks then the symmetry inside each $SU(2)$ multiplet. This results in a non-vanishing mass of the corresponding gauge bosons. We will discuss below how these operations precisely lead to the “standard” spectrum of quarks and leptons, with the unbroken electromagnetic and broken weak group of interactions. For the moment, we are interested in seeing what is the mass scale corresponding to the breaking of this second $SU(2) \approx SU(2)_{w.i.}$. For the gauge bosons of $SU(2)_{w.i.}$ the situation is different from the one of the first shift: in this case, the “shift” associated to the projection that breaks the symmetry inside the multiplets cannot be a further level-doubling rank reduction: the only possibility is that it acts in a way equivalent to a Wilson line. As is known (see for instance ref. 9), a Wilson line acts as a shift in the windings, namely, as in the twisted sector. This is why the electroweak scale is of the same order of the highest particle’s scale, that corresponds to the top-quark mass. Therefore, we expect this scale to roughly correspond to the quartic root of the inverse of the Age of the Universe. This gives $\sim 10^{-15}\text{M}_\text{P}$, i.e. $m_{w.i.} \approx \mathcal{O}(10^4 \text{GeV})$, almost, but not quite exactly, corresponding to what actually it is. The point is that our evaluation at this stage is too naive: for instance, with just the two operations on the space-time sofar considered, we cannot distinguish between the three sectors that give rise to the replication of matter in three families. Two shifts in the space-time directions are not enough to account for the spreading of masses as we observe it: as long as we consider the string space as factorized in “space-time” and “internal space”, these sectors will always appear degenerate. In this naive set up, the masses of our, “observable” world, would be related only to the scale of the extended coordinates, according to the following expression, the same for all particles:

$$m \sim \left(\frac{1}{T_{\text{str.}}^4}\right)^{\frac{1}{4}} \times \left(1^{D-4}\right)^{\frac{1}{D-4}}.$$  \hspace{1cm} (3.6)$$

However, in this way the string space appears as a “trivial” bundle, with a base, the space-time, just multiplied by a fiber, the internal space. All families appear on the same footing. From a very rough point of view, this scheme is acceptable, as long as it is a matter of qualitatively explaining the difference between compact twisted, and extended, expanding coordinates, and why observable particles have masses so small as compared to the Planck scale. However, the condition of “minimal initial entropy” requires the introduction of more operations, that further “curve” and complicate the configuration of the space, so that the

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\[\text{In order to visualize the situation, it is convenient to think in terms of heterotic string. Although a priori all the string constructions are equivalent, this is the one in which this kind of phenomena are more explicitly described.}\]
internal space cannot be so simply “factorized out” from the external one. What is left is actually room for Wilson lines. Generically, a Wilson line is an object that acts on two pieces of the string space: it links an operation on some charges of an “internal space” to the charges, or windings, of other coordinates. In some sense, it provides therefore an “embedding” of the internal space it acts on, onto the space of the other coordinates. In our case, the Wilson lines under consideration provide an “embedding” of the internal coordinates in the external ones. We can roughly account for their effect by saying that, at any time such a Wilson line effectively acts, one (or more) of the $D-4$ internal coordinates, instead of contributing to the second factor, has to be counted as contributing to the first factor of the r.h.s. of (3.6). The result is that the mass expression gets modified to something like:

$$m \sim \left( \frac{1}{\mathcal{T}^{4 \times 1^n}} \right)^{\frac{1}{1+n/2}} \times \left( 1^{D-4-n} \right)^{\frac{1}{D-4-n}},$$

(3.7)

where $n$ is the number of the embedded coordinates, $0 \leq n \leq D-4$ (we will explain below why only $n/2$ enters in the mass expression) and for $\mathcal{T}_{str}$ we intend now $\sqrt{T}$, the square root of the age of the Universe.

Even without a detailed knowledge of what precisely the most singular configuration is, we can already have a qualitative insight by taking as usual the approach of parametrizing singularities through $Z_2$ projections. We can therefore think at such Wilson lines as a set of $Z_2$ shifts, acting, as usual for Wilson lines, on the windings. Namely, they generate a set of corresponding “twisted sectors”, from which the various particles arise. As is known, whenever a $Z_2$ projection acts on the coordinates, the effective size of the space is reduced by a factor two. Correspondingly, the mass scale is doubled. In the case of these Wilson lines, mass scales are instead halved, because these projections act only in a “T-dual” way, on the “twisted sector”. Whenever such a Wilson line is active, namely, whenever we look at the corresponding “twisted sector”, the mass scale gets an extra factor 1/2. Taking into account also the rescaling factors on the mass scales due to the three “true” $Z_2$ orbifold twists of (3.1), we obtain the following spectrum of masses:

$$m_n \sim 2^{3-n} \left( \frac{1}{\mathcal{T}^{4 \times 1^n}} \right)^{\frac{1}{1+n/2}} \times \left( 1^{D-4-n} \right)^{\frac{1}{D-4-n}}.$$  

(3.8)

Notice that, while the number of rescaling factors due to the Wilson lines varies from sector to sector, according to whether the corresponding Wilson lines are or not active, the rescaling factor due to the true orbifold projections is always present. This corresponds to the fact that these projections act on all the sectors. We want here to remark that between internal and external coordinates there is an essential difference also for what matters the effect produced by $Z_2$ shifts. As we have seen, when it acts on the internal coordinates, any such shift produces a rescaling by a factor 1/2. When it acts on the space-time coordinates, it leads instead to a “square root law” for the proper time. The reason of this behavior is that, when acting on untwisted coordinates, such a shift produces a “freely acting orbifold”, in which the linear dependence on the string coordinates is suppressed to a logarithmic behavior: $f(X) \sim X \rightarrow f(X) \sim \log X$. In this case, a 1/2 normalization factor for $f(x)$ is then traduced into a square root rescaling for the string coordinate. In the internal space,
all coordinates are twisted, and although untwisted sectors are also present, the dominant behavior is determined by the twisted sectors. In these latter, the twist superposes to, and therefore “hides”, the shift. The reason why only \( n/2 \), namely half of the number of embedded internal coordinates enters in expressions (3.7), (3.8), is that, without Wilson lines, each (space-time transverse) coordinate would be shifted only once, so that its “proper length” would be \( T_{\text{str}} \), the square-root of the Age of the Universe. Once we switch on a Wilson line, the space-time coordinate, in the appropriate “twisted sector” from which the corresponding matter arises, feels a double shift, and therefore follows a quartic root law.

Expression (3.8), derived on the base of “dimensional” considerations, helps to understand how a varied spectrum of masses may arise. It however can by no means be considered to give the precise expression for real masses. In order to get this, we should better understand the details of the “most singular compactification”. Even without that, the above arguments allow us anyway to figure out, at least from a qualitative point of view, how it is possible to arrange masses in a (roughly) exponential sequel. More precisely, we are able to refer the exponential arrangement of families to the increasing number of coordinates embedded into the space-time for the different orbifold twisted sectors, giving rise to matter replication. Inside each twisted sector, the \( SU(2) \)-breaking shift acts then as a “square root law”. This on a different proper length for each family. The mass of the gauge bosons corresponds to the breaking scale, and is related to the highest mass \(^{18}\), which turns out to be of the order of:

\[
m_{\text{e.w.}} \sim 2^{3-n} 10^{-30.5 \times \frac{1}{n+2}} M_p \sim 2^{3-n} 10^{-30.5 \times \frac{1}{n+2}} \times 1.2 \times 10^{19}\text{GeV},
\]

which gives the best fit for \( n = 8 \):

\[
m_{\text{e.w.}} \propto m_8 \approx 200 \text{GeV}.
\]

(For \( n = 7 \), corresponding to \( D = 11 \), we would get \( m \approx 35 \text{ GeV} \)). This fit strongly suggests that the number of internal dimensions is 8, implying that the theory is originally twelve-dimensional. However, it does not necessarily have to be supersymmetric. We will come back to this point in section 4.

3.3 The low-energy spectrum

We have seen how three matter generations arise in String Theory. We have also seen what is the scale corresponding to the heaviest observable particles/fields, and that this necessarily corresponds to the scale at which \( SU(2)_{w.i.} \) is broken. In this section we want to discuss what is the observable spectrum of String Theory. Namely, what are the particles and fields propagating at a scale below the Planck scale. We start by discussing what are the massless states, namely, what are the states that remain strictly massless.

3.3.1 untwisted vs. twisted sector

As we have seen, all the states appearing on the string “twisted sectors”, therefore all the matter states, acquire a mass, that, for each particle, depends on the corresponding (proper)

\(^{18}\)We will come back to the relation between the mass of gauge bosons and matter states in section 3.3.4.
space-time scale. Namely, they would be strictly massless if the observed space-time was infinitely extended. As we discussed, the less entropic situation, the one the theory starts with in our branch, is attained at time $t = 1$. By approaching this point from a time $t < 1$, the next to the final step, before the minimal entropy is reached, is, if we consider the non-twisted coordinates as infinitely extended, a $\mathcal{N}_4 = 1$ vacuum. Once the last projection took place, supersymmetry is not only reduced but necessarily completely broken. This step may appear as perturbative or non-perturbative, depending on the approach one chooses. Anyway, the masses of all the superpartners are lifted to the Planck scale. As seen from a “heterotic point of view”, this breaking is non-perturbative, corresponding to a “shift” along the “dilaton” coordinate, which, as we have seen, is stuck at the Planck scale. The situations of gauge fields is reversed with respect to the one of matter fields: matter states appear in the “twisted sectors”, while gauge multiplets always show up in the “untwisted sector” of the theory. As a consequence, also the supersymmetry-breaking shift acts in a reversed way, by lifting the mass of the fermionic states (gravitino, gauginos) instead of that of the bosonic states, superpartners of the fermionic particles. Therefore, at the point of minimal entropy, the low energy world is made up of light fermionic matter (no scalar fields are anymore present!), and of massless/massive gauge fields. Matter states appear to be charged with respect to bi-fundamental representations. More precisely, they transform in the $(4, 4')$, replicated in three families: $(4, 4'), (4, 4''), (4, 4''')$. We will discuss here how the 4, common to all of them, is the one which gives rise to the ”$U(1)_{\text{e.m.}} \otimes SU(3)_{\text{color}}$“, while the $4'$, $4''$ and $4'''$ are all broken symmetries. Roughly speaking, these correspond to ”$SU(2)_{\text{w.i.}} \times SU(2)$“ (the ”“ symbols are there to recall that the actual $U(1)_{\text{e.m.}}$ and $SU(2)_{\text{w.i.}}$ are mixtures of the two). The fact that the electroweak symmetry appears to be replicated in three copies is only an artifact of our representation of the states. Indeed, for the string point of view, there is no such a symmetry, being all the corresponding bosons massive. The String Theory vacuum knows only about the “($4, \ldots$)”. In Ref. [2] we discussed how, when there is matter bi-charged under two gauge groups, originating from two sectors non-perturbative with respect to each other, like are here the 4 and $4'$, $4''$, $4'''$, then, whenever supersymmetry is reduced to $\mathcal{N}_4 = 1$, either the one or the other sector/group is at the strong coupling. This means that, in a string framework, talking about gauge symmetries as one usually does in a field theory approach is not much appropriate: these symmetries are at least in part already broken, and therefore do not appear in the string vacuum. More than talking about gauge symmetries, what we can do is therefore to just count the matter states/degrees of freedom, and interpret their multiplicities as due to their symmetry transformation properties, as they would appear, in a field theory description, from “above”, before flowing to the strong coupling. This is an artificial, pictorial situation, that does not exist in the actual string realization. That’s fine: the string vacuum indeed describes the world as we observe it, namely, with quarks at the strong coupling. If we look at this configuration from the point

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19Remember that, as required by minimal initial entropy, each “orbifold twisted sector” has a “rank four” group with (bi-charged) matter in the fundamental representation. Of the initial rank 16 per each such sector, a rank 12 part had already been lifted to the Planck scale by “rank-reducing” projections, acting as shifts on the “internal” coordinates.

20From now on, we will refer to the configuration deriving from minimization of entropy at $t = 1$ as to “the” String vacuum.
of view of the $4$, we have a description of an “unstable” vacuum, in which $SU(3)$ has yet to flow to the strong coupling, and we observe that the “quarks” have multiplicities $4'$, $4''$, $4'''$, corresponding to the three twisted sectors they belong to. On the other hand, had we observed the story from the reversed point of view, namely by looking from the point of view of the weakly coupled sector, then we would have said that all the quarks are charged under one common weakly coupled gauge group, and under three “hidden” gauge groups at the strong coupling. Namely, what we count is indeed the number of matter states; we are not allowed to count gauge bosons of broken, or strongly coupled, symmetries. So, either we chose to look from the “weakly coupled $SU(3)$” point of view, and we have the fake impression that $SU(2)_{w.i.}$ is triplicated, with an independent $SU(2)_{w.i.}$ per each family, or we look from the point of view of $SU(2)_{w.i.}$; in the second case we count only one such group and an artificial triplication of $SU(3)$. I stress however that for the string, both these symmetries do not exist: they are only a guideline for our convenience. Notice also that the heterotic realization of the vacuum corresponds to an unstable phase in which the $4$ is realized on the currents. The Wilson line the breaks it further to $SU(3)$ is non-perturbative. The fact that there is no explicit string realization of the vacuum corresponding to our world, in which we can follow the path of single quarks, is somehow unavoidable: it is in fact related to the fact that at low energy only $SU(3)$ singlets exist as free, asymptotic states in Nature.

3.3.2 the splitting of the “untwisted” gauge group: origin of the photon and “$SU(3)$”

We have seen that in the string vacuum at most the $4$ can remain massless. The legitimate question is now whether also this is at the end of the day lifted by some shifts. The answer is that there is no room to accommodate further, independent shifts, able to reduce the rank of this sector: all the internal coordinates have been already used, in order to reduce the rank of the “twisted” sectors to “$4'$”, and the space-time transverse ones to break further this group and its replicas, $4''$, $4'''$, down to the observable, broken weak symmetries of our world. As we already remarked, since at least one coordinate of the theory is extended, the number of allowed orbifold operations can be analyzed within the framework of a perturbative string orbifold. Elementary properties of string orbifolds tell us that with this we exhausted all the possible “rank reducing shifts”, and the only possible operation on the $4$ is a further breaking driven by Wilson lines, the ones that distinguish the matter states transforming in this representation into different mass levels\footnote{Notice that, in a heterotic-like vacuum in which the $4$ is realized on the currents, the double rank reduction of this sector is realized, as observed in Ref.\cite{2}, by shifts that act always, besides the “observable” compact space, also on the “dilaton plane”, a non-perturbative plane for the heterotic string. In dual constructions this plane appears as a normal, two-dimensional orbifold plane, in which it is easy to see that only two independent shifts can be accommodated. These correspond to a maximal double rank reduction, from 16 to 4.}. From: i) the counting of the matter degrees of freedom, ii) knowing that this sector is at the strong coupling as soon as we identify the “$4'$” as the sectors containing the group of weak interactions, and iii) the requirement of consistency with a field theory interpretation of the string states below the Planck scale\footnote{for us, field theory is not something realized below the string scale. It is rather an approximation (that locally works), obtained by artificially considering the space-time coordinates as infinitely extended. As a consequence, the “field theory” scale is in our set up the scale of the compact, but non-twisted, string}. From: i) the counting of the matter degrees of freedom, ii) knowing that this sector is at the strong coupling as soon as we identify the “$4'$” as the sectors containing the group of weak interactions, and iii) the requirement of consistency with a field theory interpretation of the string states below the Planck scale\footnote{not in our set up the scale of the compact, but non-twisted, string.}.
we derive that the only possibility is that this representation is broken to \( U(1) \times SU(3) \). A further breaking would in fact lead to a positive beta-function, in contradiction with point ii). Actually, point iii) tells also us that \( U(1) \) combines in some way with part of the twisted sector’s gauge group to build a non-anomalous \( U(1) \), the only allowed to survive. This gives rise to the photon. Leptons and quarks originate from the splitting of the 4 of \( U(4) \) as \( 3 \oplus 1 \) of \( SU(3) \).

### 3.3.3 about the Higgs field

If the origin of masses is a purely Stringy phenomenon, what about the Higgs field? The point is simply that in String Theory there is no need of introducing such a field: its “raison d’être” was justified in a field theory context, in which, once realized the renormalizability of gauge theories, one advocates the principle of “spontaneous breaking” of such a symmetry: in spontaneously broken gauge theories renormalizability is then ensured. But here, renormalizability of the theory holds just because it is string theory, which is not only renormalizable but finite. Masses are the consequence of the “microscopic” singularity of the string space. Precisely thanks to the existence of masses, the microscopic singularity generates in turn space-time singularities. String Theory provides therefore a sort of “micro/macro unification”: microscopic phenomena are mirrored by macroscopic ones.

Even though the origin of masses does not rely on the existence of a Higgs field, something of the old idea of Higgs particle and potential nevertheless survives, although in another form. It is known that, in non-supersymmetric string vacua, at certain points in the moduli space tachyons appear \(^{[10]}\). Such tachyons play precisely the role of Higgs fields. Namely, they have a “mass” term with the wrong sign. Moreover, since the first massive state to come down to negative mass as we move out from the physical region of parameters is of course the “superpartner” of the lightest observed particle \(^{[23]}\), we can think at it as a field that, coming from a multiplet of broken supersymmetry, feels also the interactions of the observed particle. In particular, if we represent these degrees of freedom and their couplings in terms of fields, we see that cubic and quartic terms are automatically generated by interactions. All that to say that the “effective action” for such a state is precisely a Higgs-like mass plus potential term. The physical vacuum of string theory is the one in which this tachyon becomes massive. It is therefore not anymore perceived as a tachyon, but rather as a massive state, with a mass at the Planck scale. However, the fact that it comes from a Higgs-like state tells also us that its “superpartner”, the lightest observed particle, must be electrically neutral. Otherwise, also \( U(1)_{\text{e.m.}} \) would be broken, with a massive photon, something we have seen it does not happen in the vacuum. The idea of the Higgs field as the superpartner of an observed particle was already considered in the past. It was discarded because of the phenomenological non viability of certain interactions it would then had. However, here we are not stating that the Higgs is the superpartner of the neutrino. We are saying that coordinates. In order to keep contact with the usual approach and technology, what we have to do is to send to infinity the size of space-time, and consider the massless spectrum of four dimensional string vacua.

\(^{23}\)Remember that all masses inside multiplets of the broken supersymmetry are positively lifted of the same amount by a space-time shift.
the Higgs does not exist at all \[24\]. It is just the interpretation of certain string states in an unphysical region that resembles what we have in mind for the Higgs. And precisely this technical fact forces the lightest observed particle to be electrically neutral, namely, the neutrino to exist.

3.3.4 mass levels

Once realized that the lightest particle must be electrically neutral, all other charge assignments follow automatically from the requirement of anomaly cancellation of the light spectrum, the one that admits a representation in terms of field theory \[25\]. The particle spectrum of the Standard Model is therefore automatically implied by “minimal initial entropy”, and by the finiteness, at any time of observation, of our horizon. Owing to the “replication” of the spectrum into three families, consequence of the replication of orbifold sectors, we expect the above described “Higgs condensation” argument to be valid for the three families. Therefore, we expect the three neutral particles to be the lightest ones.

According to (3.8), the lightest mass, which should correspond to the lightest neutrino, is:

\[
m_0 \sim 2^3 \times 10^{-30.5} \times 1, 2 \times 10^{19} \text{ GeV} \sim 3 \times 10^{-1} \text{ eV}
\] (3.11)

which is below the experimental bound for the mass of the electron’s neutrino. The following two mass steps, according to (3.8), are: \[m_1 \sim 2^2 \times 10^{-30.5 \times 8/9} \text{ MP} \sim 30 \text{ eV}\] and \[m_2 \sim 2 \times 10^{-30.5 \times 4/5} \text{ MP} \sim 10 \text{ KeV},\] which are also below the actual experimental bounds for \(\nu_\mu\) and \(\nu_\tau\). The fourth mass step, that we expect to correspond to the first excitation of charged matter, is \[m_3 \sim 10^{-30.5 \times 8/11} \text{ MP} \sim 0.55 \text{ MeV},\] quite close to the electron’s mass. This agreement, although not simply a matter of coincidence, has on the other hand to be taken as just an indication. The mass steps contained in expression (3.8) are in fact not enough to account for the difference between all particles and the W and Z bosons. This means that our way of parameterizing the singularity of the space is too simple, and has to be taken only as a rough approximation, more suitable for a qualitative than for a quantitative analysis.

For instance, (3.8) does not account for the effect of all the “Wilson lines” present in the theory. We know in fact that all the three “twisted sectors” are distinguished from each other, and that, inside each of them, Wilson lines distinguish matter by breaking the \(SU(2)\) multiplets, and the 4 into 3 + 1. However, by just counting the coordinates we see only nine steps. What is lacking, and we would need, is a description of the “non-linear” effects produced by all these operations, which most probably require a treatment of the string space in which the coordinates are not simply factorized. Another strong reason why our arguments have to be considered rather qualitative is that, from a rigorous point of view, it is not possible to construct a perturbative “\(Z_2\) string orbifold” in which all the operations we have mentioned are explicitly realized. Our discussion is just intended to give the flavor of what is likely to happen in a curved geometry, a sort of “ellipsoid”, in which the coordinates, not factorizable, are in part very large and in part of length one.

\[24\] or, if one prefers, it exists but its mass is at the Planck scale, therefore it can be considered as decoupled.

\[25\] Notice that the correspondence with field theory is stated at the massless level, for massless fields and particles. As we have seen, the inclusion of masses goes beyond the domain of field theory.
Even with our big simplifications, we can however understand some mass relations, whose explanation is due to pure stringy effects. For instance, String Theory tells us why the mass of the $SU(2)_{w.i.}$ bosons, $(W, Z)$, is roughly the average between the top and bottom masses: $m_{W,Z} \approx \frac{m_t + m_b}{2}$. In order to see this, let’s analyze more in detail the structure of matter states. As we said, each family appears from states that, were not for the compactness of space-time, would be strictly massless, and transform in a $(4, 4')$, bi-fundamental representation. $4'$ is then broken by shifts in the space-time, while the $4$, as we discussed, gives rise to the photon and the strong interactions. Let’s concentrate on the $4'$. In the following, we will look at the problem by either 1) considering matter as appearing in the twisted sector of a heterotic realization; in this case the $4'$ will not appear explicitly, but only through its discrete subgroup of the multiplicity of states in the twisted sectors $[\mathbb{Z}_2]$ or 2) mapping the problem into a description in which the $4'$ appears in the currents of a heterotic construction. For reasons that we will explain in a moment, these two representations are equivalent, and the choice of 1) or 2) is dictated by convenience, depending on which phenomena we want to observe explicitly. Let’s start with the picture 1): the string configuration with minimal entropy requires the action of a double shift in space-time. As we have seen, a consequence of this is the breaking of Parity at the Planck scale $[\mathbb{Z}_2]$ and the breaking of the surviving $SU(2)_{w.i.}$ by a “$Z_2$-Wilson line”, that splits masses inside matter doublets. This is the stringy realization of the Higgs phenomenon. As discussed above, the order of magnitude of such Wilson line essentially sets the “top-quark mass”. In order to better investigate how matter and boson masses are related, we can pass to picture 2), namely, we map the problem to a description in which both matter and gauge states are realized as part of the currents of a heterotic vacuum. This is possible because Wilson lines act on the windings, treating gauge bosons as if they had the same origin as matter, on the “twisted sector”. Therefore, the states that in picture 1) appeared in the twisted sector, matter and “inexplicit” gauge, in picture 2) appear in the currents, with the quarks corresponding to matter in the untwisted sector. In this second picture, matter transforms in the fundamental of a gauge group, with an internal index (the “color index”) on a “twisted sector” gauge group. Under orbifold shifts, it receives therefore the same kind of mass lift as the gauge bosons. In this representation, the situation corresponding to our physical problem appears as follows: one starts with a symmetry, realized now in the untwisted sector, whose gauge bosons arise as states of the kind $A_\mu = \overline{X}_\mu \otimes \phi_i \phi_j$, in terms of left and right moving vertex operators. Let’s restrict to the case of four possible values for $i$ and $j$. In this case we have a symmetry $U(2)$, that we can consider as the one surviving after the first shift in the space-time. For simplicity, let’s neglect here the fact that in the physical situation the anomalous $U(2)$ is broken to a non-anomalous $SU(2)_{w.i.}$. Since we want to consider the breaking of the $2$, let’s group the four indices into two groups, 1 and 2. Our $W$ bosons are represented in terms of vertex operators by $\overline{X}_\mu \otimes \phi_1 \phi_2$ and its conjugate. Now consider a shift (the “Wilson line” under consideration) that lifts the mass of the oscillators with index 2, with the property that all the oscillators $\phi_2$ are eigenvectors of the “right moving” energy-momentum tensor with mass $m$, while the non-shifted states, $\phi_1$, remain massless. The mass of extra-diagonal states, such as the $W$ bosons, and of linear combinations of “diagonal” bosons, $\overline{X}_\mu \otimes \frac{1}{2} (\phi_1 \phi_1 \pm \phi_2 \phi_2)$, 

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26 This is the point of view we used when discussing the origin of the photon and the gluons.

27 More precisely, above it.
results then to be half of the mass of the states built entirely of oscillators with index “2”. Since we are considering a symmetry breaking due to a Wilson lines that splits the mass inside matter doublets, and not a lift in mass due to an ordinary rank reducing shift, the mass of such vectors, $2m$, is the same as the mass of the lifted matter states, namely of the quarks “up” (in our case, the heaviest one, the top quark). Therefore, $m_{W,Z} \sim \frac{1}{2}m_t$.

However, also the “down” quarks are massive. All masses are then further shifted, and this relation is shifted to:

$$m_{W,Z} \sim \frac{1}{2} (m_t + m_b).$$

(3.12)

### 3.4 Time-dependent masses?

Although very approximate, the above qualitative analysis confirms that the relation between electroweak masses and the time scale of the Universe we argued is essentially correct. Since we are used to think at masses as at “stable” parameters, certainly not quantities that vary with time, this is somehow surprising. However, it is not at all unreasonable. It is in fact quite natural to think that the Universe knows only about one fundamental mass/length scale, the Planck scale. From this point of view, all other scales appears as pure “accidents”, related to the actual point in the evolution of the Universe we live at (and make measurements). The reason why we are used to think at all mass scales as constant, is that their variation with time is quite slow. When we insert a mass parameter in an effective action (like, e.g., the Standard Model Lagrangian), we are always thinking in terms of an infinitely extended space time. As long as we are only interested in “local” phenomena, this assumption is reasonable, because the actual age of the Universe is big enough to allow approximating our finite-size horizon with an infinite space-time. However, when we push our field of investigation to the borders of space-time, or of the domain of energy/momenta, we cannot anymore neglect the “boundary effects” due to the fact that the portion of Universe we can measure is finite. Considering the space-time as infinite corresponds to artificially removing the natural cut-off. This produces the infinities, and the (artificial) distinction between “bare” mass, which is divergent, “physical” mass, finite, and also “running” mass, related to the cut-off by the renormalization procedure. In some sense, here we have only a “running mass”, although this running has to be intended in a string/cosmological framework. Actually, the introduction of the horizon as a natural cut-off would lead to a time-dependence of masses even without considering the output of the string theory analysis. Indeed, what we are here proposing is that masses are a consequence of the existence of this cut-off. In some sense, they “measure” the size of the Universe. From a purely field theoretical point of view, mass terms are naturally created, from “nothing”, for all particles and fields whose (zero) mass is not protected by some symmetry. In this case, only the photon \footnote{and the gluons} has a protected vanishing mass. Since masses are not a consequence of “contact”, or interaction, with heavier particles or fields (no Grand Unification! \footnote{“Unification” takes place at the Planck scale, where we are not anymore allowed to speak in terms of fields and elementary particles.}), we don’t have to worry about hierarchy problems. The electroweak scale is trivially protected because it is the only scale of the effective theory.
Remarkably, what string theory tells us, is that the time/cut-off dependence of masses is different for different types of particles. In some way, they “live” on space-times with different scales. Namely, each type of particle has its own “Uncertainty Principle” relation, leading to a mass related to the inverse of its “proper” time. Although unexpected this may seem, the idea that different particles “live” on different spaces, namely, that they feel in a different way space and time, is after all not that new in Physics: the difference between Bosons and Fermions, namely between particles with integer and half integer spin, is an example. In that case we have in fact that, despite our macroscopic perception of space and rotations inside it, elementary particles live on a more complicated space. We have seen that the space of String Theory is somehow an “orbifold” of what we call space-time. Different particles live on different orbifold sectors, and they have each one a specific perception of it. It is difficult, if not impossible, to give an explanation of these facts in terms of ordinary, four-dimensional field theory: masses are additional degrees of freedom that, being sources of, and charges of, gravity, only in a String Theory framework can be consistently treated. They can well be introduced in a field theory framework, but only at the level of an effective theory, in which they appear as “ad hoc”, external inputs, and at the price of neglecting an entire part of the quantum theory/interactions they feel, i.e. gravity. This behavior of masses goes anyway so much in the opposite direction of our common belief that it deserves some “physical” explanation. Therefore, let’s try to see whether it is possible to make sense of it also with our intuition. Indeed, this phenomenon can be seen to be related to the analogous behavior of the cosmological constant. Our most common way of intending masses is through the concept of “inertia”. The mass of a particle tells us how much energy we have to spend in order to accelerate it of a certain amount. The existence of a cosmological constant tells us that the Universe has a non-vanishing vacuum energy. Since energy, as masses, is source of gravity, this means that in some sense the background in which particles “live” is not flat, but experiences a “background gravitational force”. Accelerating a particle is therefore equivalent to move it from a geodesic to another one. If the cosmological constant decreases with time, this means that as time goes by, the less and less is the energy we have to spend to accelerate the particle, because we have to work against a smaller “background force”. Therefore, what in practice we observe is that the inertia of the particle decreases with time. We must however stress that this is only a “rough” argument, not able to make sense of the fact that different particles have a different time dependence. The true explanation comes from String Theory, and goes beyond our “classical” intuition.

Before discussing the implications of the time-dependence of masses, let’s first stress that, as it is predicted by string theory, at the present day the rate of change of masses with time is very small. Namely, the experimental uncertainty of mass values, $\Delta m/m$, is larger than the expected change in such values due to the time evolution, during the period since we talk about high-energy physics and related experiments. Therefore, this effect is not ruled out if we consider the other side of the argument: the fact that there exist different elementary particle’s masses, is a signal of the existence of internal dimensions.

Even when they are introduced through a Higgs mechanism, still they remain external inputs: simply the problem is rephrased into that of explaining the Yukawa couplings of the Higgs field.

Our “intuition” is based on the observations we make, and is related to the perception of space-time of photons and gravitons, which behave as “truly”, “elementary” four dimensional objects.
out by today’s direct experimental informations about particle’s masses. According to the string theory analysis, during the evolution of the Universe, the particles, that at the Planck scale started all with a mass of the same order (the Planck mass is the minimal mass allowed at such scale by the Uncertainty Principle), become progressively lighter. This process is not uniform: any type of particle behaves differently, as required by the principle of minimal initial entropy. i.e. of maximal initial differentiation. Asymptotically, all particles will anyway meet at infinity, where all their masses will vanish, as it will the average energy of the Universe per unit of volume.

Although not directly detected by current experiments, the decrease of mass of particles can be traced in the very fact that the Universe expands \[33\]. The fact that masses decrease is a consequence of the expansion of the horizon, something a priori independent from the expansion of the Universe itself. However, the fact that particles become lighter implies a decrease of the attractive force, and an increase of the kinetic energy. This leads to an accelerated expansion, with a decreasing rate of acceleration. In order to see this, let’s consider a simple “model” of the Universe. We can make the hypothesis that, at the Planck scale, the Universe, or better, the portion of Universe relevant for us, i.e. the one enclosed in a ball with radius one in Planck units, is at “equilibrium”. By analogy with solid matter, we can imagine that the gravitational attractive force is balanced, say, by some contact, repulsive “electric” force, that forbids all matter to collapse into a point (the details of this force are not important, this is just an artifact to help fixing the ideas on something concrete). The expansion of the horizon, due to the existence of massless modes propagating in the “space-time” coordinates, is equivalent to a perturbation that leads to a decrease of masses during time evolution. Owing to this, all particles (galaxies) feel a nett repulsive force, due to a decrease of gravitational attraction. Let’s consider a particle (a galaxy) of mass \( m \) located on the hypersurface at the border of the ball of radius \( r \). This would experience a force given by:

\[
F = \frac{mM}{r^2} - \frac{qQ}{r^n} = m\ddot{r}, \quad n > 2, \tag{3.13}
\]

where \( M \) stays for the mass enclosed in the ball, \( Q \) its nett charge and \( q \) the charge of the particle. The “initial condition” is \( F = 0 \) at \( t = r = 1 \). For \( t > 1 \), it would move according to:

\[
\ddot{r} = \frac{1}{r^2} \left( \frac{qQ}{mr^{n-2}} - M \right) \propto \frac{1}{r^2} \left( \frac{1}{t^q} - \frac{t^q}{r^{n-2}} \right), \tag{3.14}
\]

where we choose an “average” time-dependence for masses, \( m \sim \frac{1}{t^q} \) (for the lowest mass, \( q = \frac{1}{2} \)). One can easily realize that the motion we obtain, namely the expansion of the ball, is accelerated, and the acceleration decreases with time. For \( r \gg 1, t \gg 1 \), the dominant piece on the r.h.s. is the first term in the brackets, because the “contact force” decays more rapidly (it is reasonable to assume that \( n \) is sufficiently large). Therefore, although the acceleration is positive, its derivative is negative (remember that \( q \gtrsim \frac{1}{2} \)). For an “average” time-dependence of masses, \( \sim t^{q=1/3} \), we can solve (3.14) with the Ansatz \( r \approx t^p \), obtaining \( p = \frac{2}{3} \), a value quite close to \( r \sim t^{2/3} \), the behavior expected from the usual estimates for

\[33\] Another indirect confirmation is provided by the analysis of the emission spectra of quasars.\[11\]. We devote Ref. [12] to this issue.
a matter dominated Universe. This also implies that the red shift due to the expansion rate, \( z_{r.s.} \approx \frac{\Delta R}{R} \sim \frac{2}{3} t^{-1} \), dominates over the violet-shift \( z_{v.s.} \sim \frac{\Delta m}{m} \approx -\frac{1}{3} t^{-1} \), induced by the negative derivative of mass, that raises all atomic energy levels. What we observe is therefore still a red-shift. At this point, it should be clear that the introduction of the “balancing” force \( F \sim \frac{2m}{t} \) was just an artifact. The true point is the increase of “repulsive” force, or, better, of kinetic energy, due to the decrease of masses.

\[ P \text{ and } T \text{ parities violation.} \]

As we have seen, left-handiness of the weak interactions is a consequence of the stringy origin of masses of particles. Namely, only left handed states feel the weak interactions gauge group, while their right handed counterparts would feel a group which is however broken at the (above the) Planck scale. Electromagnetic interactions are on the other hand non chiral: the photon arises in fact as the non-anomalous, surviving massless combination of the original gauge group bosons, under which transform both the “left” and “left-conjugate = right” degrees of freedom, corresponding to the \( 2 \) and \( 2' \) of the \( 4 \). The breaking of parity results therefore as the effect of shifts in the space-time. This had to be expected: parity is in fact defined as the symmetry under space reflections: \( x \rightarrow -x \). If the coordinate \( x \) is shifted by a certain amount, \( a : x \rightarrow x' = x + a \), under \( x \rightarrow -x \) we obtain \( x' \rightarrow x'' = -x + a \). If the space was infinitely extended, this shift would have no effect. However, since the space is compact, it is not invariant under translations, and parity gets broken. As we have seen, the order of magnitude of the parity breaking is T-dual, with respect to the Planck scale, to that of the shift under consideration, related to the electroweak scale.

Finiteness of the horizon implies on the other hand the breaking of the symmetry under time reversal. The reason is that it introduces an energy gap, measured by the cosmological constant, equivalent to a shift in the time coordinate. We expect therefore the order of breaking of this symmetry to be of the order of the cosmological constant. This for what matters direct violations of time reversal symmetry. However, as is known, violations of this symmetry can be detected also in an indirect way, through mixings in meson decays. The “CP” violation parameters are then referable to a phase in the mass matrix. This means that this effect is of the same order of the electro-weak scale. The reason is that not only the cosmological constant, but also the masses of particles = energies at zero momentum, are variables conjugate to time. Their introduction is therefore also somehow equivalent to a shift in time, the “specific” time of each kind of particle. A noteworthy peculiarity of String Theory is that the breaking of time and space parities, while not immediately related in field theory, are here instead two aspects of the same phenomenon: as it can be seen by passing through an Euclidean intermediate step, shifts in the compact space or time coordinates are essentially equivalent, and interchangeable.

\[ ^{34} \text{Although evidence of the acceleration in the rate of expansion of the Universe is provided by astronomical observations [13, 14], the elaboration of data and their exact interpretation is somehow model dependent. At the present stage, we consider premature, in the framework of our proposal, a detailed discussion of issues like the possible very early era of “decelerated expansion”, something suggested for instance in Ref. [15] (just to give an example, it could be that this effect disappears once the appropriate rescaling of masses and emission spectra is taken into account).} \]
3.5 Gauge couplings and unification

Gauge couplings receive contributions both from “internal” and “space-time” moduli. We have seen that the “internal” moduli, that we generically indicate by \( X \), are bound at the Planck scale; their contribution, approximately given as usual by functions of the type:

\[
\frac{1}{g^2}(X) \approx \log \Im X |F(X)|^4 + \mathcal{O}(e^{-\Im X}) ,
\]  

(3.15)

with \( \Im X \sim 1, |F(1)|^4 \sim 1 \), is therefore of order 1. This is the order of the couplings at the “unification” scale. What then distinguishes among the different couplings is their running below the Planck scale, namely the part that depends on the space-time scale. This is accounted for by the so-called “infrared” logarithmic behavior, encoded in the term:

\[
\beta \log \mu ,
\]  

(3.16)

where \( \mu \) \(^{35}\), traditionally the infrared cut-off \[^9\], is precisely related to the space-time scale. In string theory, this term is computed through a regularization procedure, according to which \( \mu \) is the inverse of the “curvature” of our extended space \[^9\]. Namely, it comes out by artificially compactifying the physical space on a conformal background roughly equivalent to a sphere. This approximation, whose choice was dictated in Ref. \[^9\] by technical convenience, although not exactly corresponding to the actual configuration of our space-time, it captures nevertheless a fundamental feature, namely the fact that the cut-off is related to the scale, or “size”, of space-time, as are fields and particles masses. In order to really discuss the running of couplings, a full control of the non-perturbative corrections would be in order: as we have seen, our Universe corresponds in fact to a strongly coupled string vacuum. From a qualitative point of view, we must however observe that, since the string approach which is the closest to the description of the actual configuration is a perturbatively \( \mathcal{N}_4 = 1 \) vacuum, the running of the gauge couplings, unless non-perturbative effects deeply affect the term \(^{3.16}\) may well be the one of a supersymmetric vacuum. Therefore, a heavy breaking of supersymmetry, at the Planck scale, may be compatible with the “nice” rescaling of gauge couplings allowing them to meet \[^{10}\]. We must however stress that, being these considerations very qualitative, one should look at these arguments more as giving the “flavor” of a possible explanation, than as providing a framework in which to perform detailed computations and matching numbers. This is why we have been very “floppy”, and in \(^{3.15}\) we omitted normalization and integration constants. In a quantitative analysis, these cannot be neglected. Here however their inclusion would not change the approximate nature of this discussion.

4 Comments

We have seen how assuming a (thermo)-dynamical principle, that of Entropy, as the dynamical “input” of string theory, leads to a consistent picture of its evolution, able to justify both the “macroscopic”, cosmological evolution of our Universe, and the “microscopic” details of

\[^{35}\] We always work in Planck units, for which \( M_{\text{Planck}} = 1 \). Therefore, \( \mu = 1 \) at the Planck scale.
space-time, namely, the spectrum of elementary particles and their interactions. An essential point for this is the “physical” assumption that a theory of our world at the present time must be self-contained within our horizon of observation, namely, that it is not supposed to describe phenomena causally disconnected, or belong to our “future”. This implies that String Theory, in which time is not an external parameter, but one of the coordinates of the theory, must always be considered as moving, at any time of its evolution, in a compact space-time. Once implemented with this “phenomenological principle”, the “thermodynamical principle” can be seen to imply also the Heisenberg’s Uncertainty Principle. This appears first at the cosmological level, through the existence of the so called “cosmological constant”. The dynamical principle contains in his “program” also the breaking of T-duality and space-and time-reversal symmetries, something that makes microscopic and macroscopic phenomena to be not exactly the same, and allows us to select an arrow in the time evolution/space expansion. Another dynamical effect is the separation of the string space into “internal” and “extended” one. The internal space is however not simply factorized out. This leads to an intriguing relation between the curvature of the “internal” space, at whose singularities light matter appears, and the curvature of the “space-time”, generated by such a matter: massive objects are “singularities” that generate the space-time curvature. Unfortunately, here we have been able to present only a rather qualitative analysis of these relations. In order to go further and obtain results more precise than those presented in this work, it is absolutely necessary to make computations in a well defined, mathematical scheme. Although over-simplified, our analysis strongly supports the hypothesis that the fundamental theory is essentially twelve-dimensional; in order to embed the problem into a precise mathematical framework, we would need a deeper understanding of this basic theory. In absence of that, we can try at least to collect some informations “from below”. What we can learn from our analysis, is that there is no evidence that this higher dimensional theory should be supersymmetric. All what we know is also compatible with the interpretation of the twelfth coordinate as the order parameter for the breaking of supersymmetry. The only fact that we know are that 1) when we can go to the limit $R_{12} \to 0$, we obtain the so-called “M-theory”, which presents supersymmetry; 2) once the theory has attained the configuration of maximal twisting, supersymmetry is necessarily broken. The twelve dimensional theory may be supersymmetric, but it could also be that it is not, and that M-theory is the first term in an expansion of a non-supersymmetric theory around a supersymmetric vacuum. It would look then supersymmetric only because precisely obtained by an expansion around the zero value of the supersymmetry-breaking parameter. In this case, as long as the $R_{12} \to 0$ (de-)compactification limit can be taken, it is possible to go to a supersymmetric limit, otherwise, supersymmetry would be broken.

**a possible scenario**

The fact that under compactification/decompactification it is possible to go to a supersymmetric limit in eleven dimensions suggests that a naive picture of the situation could be the following: the theory lives on a twelve-dimensional ball, whose boundary is an eleven dimensional sphere: this is fine, since we know that anyway the theory must have a horizon. Supersymmetry is then trivially broken owing to the compactness of this space. Whether the
space is really twelve dimensional, or we have rather to deal with just an eleven dimensional curved space, it does not make at this stage a big difference: for all what we know, it could well be that the twelfth coordinate is just a curvature parameter. From a mathematical point of view, it is always possible to embed a curved hypersurface into a (fictitious) higher dimensional flat space. Although not necessarily twelve dimensional from the very beginning, the theory would then become anyway “effectively” twelve dimensional as soon as its space gets sufficiently “curved” during the process of “singularization” produced by entropy evolution. The supersymmetric limit is just the one in which we send the radius (related therefore to the curvature) to zero/infinity. There, we recover an eleven dimensional, locally flat surface, on which there is (at least locally), supersymmetry. When however this hyperspace is sufficiently curved ($\mathcal{N}_4 = 1$ situations), consistency of the theory implies that also the “radial” coordinate is twisted. This forbids taking a true decompactification limit, and supersymmetry is necessarily broken. This phenomenon does not appear “locally” in the hypersphere: from the point of view of the hypersphere supersymmetry is perturbatively preserved, because perturbation is defined as an expansion around the zero value of the “radial” coordinate. The breaking of supersymmetry is therefore non-perturbative at that stage. However, one can also chose to make a different identification of the string (or M-theory) coordinates inside the twelve-dimensional ball, thereby having the possibility of constructing explicitly non-supersymmetric theories.

**the origin of confinement**

Under decompactification of the radial coordinate, half of the theory is “washed out”. Namely, we obtain a limit in which the theory can be described entirely in terms of weakly coupled states: the part which is T-dual under inversion of the radial coordinate, that would therefore be strongly coupled with respect to the one we keep, is decoupled. When instead the “radial” coordinate is twisted, we cannot take a limit in which we decouple one of the two mutually strongly coupled parts. Therefore, the theory cannot be entirely described in terms of weakly coupled states: in this case there are sectors which are “S-dual” with respect to each other: we have at the same time the presence of weakly and strongly coupled sectors. From this point of view, we understand now why in $\mathcal{N}_4 = 1$ vacua supersymmetry is always non-perturbatively broken due to gaugino condensation in a strongly coupled sector, as suggested in Ref. [2]: in these vacua this coordinate is necessarily twisted. This is also the reason why in our World there is quark confinement.

**a note on space-time geometry**

We have seen that the existence of a non-vanishing cosmological constant is a truly quantum effect, due to the finiteness of our horizon. Asymptotically, namely, in a space in which the horizon is at infinity, the cosmological constant vanishes, and the space-time becomes flat. In this framework, we find inappropriate the search of supergravity configurations, or more in general for classical geometric configurations, reproducing the characteristics of a non-compact space with positive cosmological constant. This does not correspond in fact to the actual physical situation, which better corresponds instead to a “quantum geometry”.

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Moreover, as we have seen, the string coupling, i.e. the coupling of the quantum theory of our world, is of order 1. We are therefore out of the supergravity domain (supergravity, even with broken supersymmetry, is just a first approximation, valid in certain cases at the weak coupling), and we are not allowed to expect that the real physical situation should find a description in that framework.

5 Conclusions

In this work, we presented a proposal for a set up in which String Theory is implemented by a dynamical principle, a “stringy” version of the second law of thermodynamics, that provides us with a “rationale” through the plethora of possible string configurations. This allows us to “solve” the theory during its evolution, and identify the configuration corresponding to the world as we observed it as the only consistent solution at the present time. Although a mathematical framework in which to properly set the problem and make rigorous computations has still to be worked out, nevertheless, by using different types of approximations, we derived phenomenological predictions, both qualitative and up to a certain extent also quantitative. Among these, the expected order of magnitude of neutrino masses. Among the most striking ones, the non-existence of the Higgs particle, in fact not needed in order to generate masses in this set up, and the non-existence of supersymmetric partners of the observed particles and fields. More precisely, “Higgs” and supersymmetric modes are to be found at the Planck scale, a scale at which we are perhaps not anymore allowed to speak in terms of “particles” and “fields”. Precisely a mass gap of the order of the Planck scale between the observed particles and their superpartners is what produces the observed value of the cosmological constant. The latter turns out to be nothing but a quantum effect, “measuring” our horizon of observations. It therefore decreases as space-time expands. Perhaps the most unexpected result is the fact that the Universe knows only about one fundamental scale, the Planck scale; all mass scales below the Planck mass depend on the point in the time evolution of the Universe: light masses precisely arise as a consequence of the existence of a horizon to our observation; they are related to the “time” coordinate, and decrease as the observable horizon increases. This effect, although extremely slow at the present day, is by itself able to explain the accelerated expansion of the Universe, and its direct observation (or non-observation) would be a key test of our proposal, both in order to support it or to rule it definitely out.

Inspection of various perturbative and non-perturbative properties of string vacua lead then us to make some guess about the theory underlying string theory. In our framework, the breaking of supersymmetry and quarks confinement receive a natural explanation, as the effect of a coordinate “twisting” necessarily implied by the second law of thermodynamics.

This work does not have to be considered as conclusive, but rather on the contrary, as a first step toward a “definition” of the theory. To this purpose, absolute priority has to be given to the setting of the correct mathematical tools enabling a rigorous and quantitative investigation of what we called “the most singular configuration” of the string space. If we decide that the history of our Universe starts at the Planck time/scale, this can be regarded as the true “starting point”, or initial condition. After all, our interpretation of the string
path through the various possible vacua to reach this configuration can be questionable, and the arguments be refined. It remains however that all the subsequent history seems to originate from there.

Bibliography

The following list of references is largely incomplete. Correctly quoting all the works related to what I have presented would be a hard task. I therefore decided to list only those references which are immediately needed in order to complete the discussion. I apologize for the many omissions.

References

[1] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, Nucl. Phys. **B593** (2001) 99–126, [hep-th/0003198](https://arxiv.org/abs/hep-th/0003198).

[2] A. Gregori, *String-String triality for d=4, Z_2 orbifolds*, JHEP **06** (2002) 041, [hep-th/0110201](https://arxiv.org/abs/hep-th/0110201).

[3] Boomerang Collaboration, P. de Bernardis et al., *First results from the BOOMERanG experiment*, [astro-ph/0011469](https://arxiv.org/abs/astro-ph/0011469).

[4] Boomerang Collaboration, A. Melchiorri et al., *A measurement of Omega from the North American test flight of BOOMERANG*, Astrophys. J. **536** (2000) L63–L66, [astro-ph/9911445](https://arxiv.org/abs/astro-ph/9911445).

[5] R. Dijkgraaf, E. Verlinde, and H. Verlinde, *C=1 conformal fields theories on Riemann surfaces*, Comm. Math. Phys. **115** (1988) 2264.

[6] A. Gregori et al., *R**2 corrections and non-perturbative dualities of N = 4 string ground states*, Nucl. Phys. **B510** (1998) 423–476, [hep-th/9708062](https://arxiv.org/abs/hep-th/9708062).

[7] A. Gregori, C. Kounnas, and J. Rizos, *Classification of the N = 2, Z(2) x Z(2)-symmetric type II orbifolds and their type II asymmetric duals*, Nucl. Phys. **B549** (1999) 16–62, [hep-th/9901123](https://arxiv.org/abs/hep-th/9901123).

[8] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, Nucl. Phys. **B460** (1996) 506–524, [hep-th/9510209](https://arxiv.org/abs/hep-th/9510209).

[9] E. Kiritsis and C. Kounnas, *Infrared regularization of superstring theory and the one loop calculation of coupling constants*, Nucl. Phys. **B442** (1995) 472–493, [hep-th/9501020](https://arxiv.org/abs/hep-th/9501020).
[10] I. Antoniadis, J. P. Derendinger, and C. Kounnas, *Non-perturbative temperature instabilities in N = 4 strings*, Nucl. Phys. **B551** (1999) 41–77, hep-th/9902032.

[11] A. V. Ivanchik, E. Rodriguez, P. Petitjean, and D. A. Varshalovich, *Do the fundamental constants vary in the course of the cosmological evolution?*, Astron. Lett. **28** (2002) 423, astro-ph/0112323.

[12] A. Gregori, *On the Time Dependence of Fundamental Constants*, hep-ph/0209296.

[13] **Supernova Search Team** Collaboration, A. G. Riess et al., *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, Astron. J. **116** (1998) 1009–1038, astro-ph/9805201.

[14] **Supernova Cosmology Project** Collaboration, S. Perlmutter et al., *Measurements of Omega and Lambda from 42 High-Redshift Supernovae*, Astrophys. J. **517** (1999) 565–586, astro-ph/9812133.

[15] M. S. Turner and A. Riess, *Do SNe Ia Provide Direct Evidence for Past Deceleration of the Universe?*, astro-ph/0106051.

[16] S. Dimopoulos, S. Raby, and F. Wilczek, *SUPERSYMMETRY AND THE SCALE OF UNIFICATION*, Phys. Rev. **D24** (1981) 1681–1683.