Anomalous anomalous scaling?

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Abstract

Motivated by speculations about infrared deviations from the standard behavior of local quantum field theories, we explore the possibility that such effects might show up as an anomalous running of coupling constants. The most sensitive probes are presently given by the anomalous magnetic moments of the electron and the muon, that suggest that $\alpha_{\text{em}}$ runs $1.00047 \pm 0.00018$ times faster than predicted by the Standard Model. The running of $\alpha_{\text{em}}$ and $\alpha_s$ up to the weak scale is confirmed with a precision at the \% level.

1 Introduction

The range of validity of Quantum Field Theory (QFT) may be limited not only in the UltraViolet (UV), $E \lesssim \Lambda_{\text{UV}}$, but also in the InfraRed (IR), $E \gtrsim \Lambda_{\text{IR}} \equiv 1/L$, with a non trivial connection between $\Lambda_{\text{UV}}$ and $\Lambda_{\text{IR}}$. This possibility has attracted interest due to the following reasons.

On the theoretical side, requiring that the entropy associated with the QFT degrees of freedom \~{} ($\Lambda_{\text{UV}}/\Lambda_{\text{IR}})^3$ saturates the Bekenstein entropy [1] \~{} $L^2M_{\text{Pl}}^2$ of a black hole with size $L$ leads to $\Lambda_{\text{IR}} \sim \Lambda_{\text{UV}}^3/M_{\text{Pl}}^2$. Alternatively, it has been suggested that one should require that systems whose size $L$ exceeds their Schwarzschild radius $\sim m/M_{\text{Pl}}^2$ do not appear in QFT. For $m \sim \Lambda_{\text{UV}}^3L^3$, this requirement leads to $\Lambda_{\text{IR}} \sim \Lambda_{\text{UV}}^3/M_{\text{Pl}}[2]$.

On the phenomenological side, ref.s [2, 3] discussed possible connections of these ideas with the cosmological constant and the supersymmetry breaking puzzles. Indeed, in standard QFT the values of the vacuum energy, scalar masses squared and dimensionless couplings are given by their bare Planck-scale values plus a quantum correction proportional to $\Lambda_{\text{UV}}^4$, $\Lambda_{\text{UV}}^2$ and $\ln \Lambda_{\text{UV}}$ respectively. Such non-local effects could change this power-counting, solving or modifying the hierarchy problems associated with massive parameters [2, 3].

We observe here that dimensionless couplings may be similarly affected, leading to an anomalous Renormalization Group (RG) running, and we study how accurately present data test the standard QFT prediction.
2 Speculations

The above speculations about a non-local connection between the IR and UV cutoffs do not have a very precise meaning, and one can debate whether they would lead to any of the effects mentioned above. Rather than arguing in any one direction, we present the uncertain issues.

Firstly, when do these non-local phenomena appear in particle physics? The weakest possibility is only when strong gravity effects, such as those arising from black holes, are directly relevant. This would practically mean never, as black hole phenomena (Hawking radiation, etc.) are quantitatively irrelevant in all processes we can realistically observe (possibly unless the true quantum gravity scale is much below $M_{Pl}$). The strongest possibility, according to which states with energy $E \sim \Lambda_{UV}$ that propagate for more than $L$ do not exist and must be dropped from QFT, contradicts experience. We see TeV $\gamma$ rays from the galactic center, particles with energies up to $10^{20}$ eV from extragalactic sources, etc.

Secondly, what is the precise meaning of $\Lambda_{IR}$? There are various possibilities, and we mention two: i) The IR cutoff can be defined by imposing boundary conditions such that QFT lives “in a box” with size $1/\Lambda_{IR}$. ii) $\Lambda_{IR}$ is the minimal energy scale that appears in loop integrations. In practical cases these choices can lead to very different answers. For example, only in the first case the IR cutoff for the cosmological constant would be the Hubble distance $1/H$ (possibly leading to a small $\Lambda_{UV} \sim \sqrt{M_{Pl}}H \sim eV$ [2]). On the other hand, $H$ nowhere appears in the one-loop correction to the vacuum energy, equal to the value of the potential $V$ at its physical local minimum:

$$V \simeq V_{bare} + \frac{1}{2} \int_{\Lambda_{IR}}^{\Lambda_{UV}} \frac{d^4k}{(2\pi)^4} \text{Str} \ln(k^2 + V''_{bare}).$$

If the quantum correction to the minimum of $V$ is naturally small thanks to an anomalous dependence on $\Lambda_{UV}(\Lambda_{IR})$ [2], one could expect a similar anomalous running of the whole potential, and in particular of its coupling constants. This leads to the third issue: what is the precise meaning of $\Lambda_{UV}$? In standard QFT, the one-loop corrections to any dimensionless coupling (e.g. the gauge boson vertex $g$) has the form

$$g(p) = g_{bare} - \frac{\beta g_{bare}^3}{8\pi} \left[ \ln \frac{\Lambda_{UV}^2}{p^2} + \text{finite} \right],$$

where $p$ is (some combination of) the external momenta that sets the IR cutoff in loop integration. In standard QFT the physical coupling $g(p)$ ‘runs’ with the energy $p$ of the process, and the RG coefficient $\beta$ is a number that depends on the particle content of the theory above $p$. (In eq. (2) we assumed that all the masses are negligibly small.)

If instead non-QFT effects produce some physical UV cutoff $\Lambda_{UV}$ that depends on $\Lambda_{IR} \sim p$, one generically obtains an anomalous RG running of $g(p)$. For example, in the one-loop approximation the running is proportional to

$$\beta \rightarrow \beta \left( 1 - \frac{\partial \ln \Lambda_{UV}}{\partial \ln \Lambda_{IR}} \right) \equiv \beta(1 - \delta).$$

Even so, one could argue that no anomalous running $\delta$ needs to appear, because $g_{bare}$ might depend on $\Lambda_{UV}$ in a way that counter-acts the explicit dependence on $\Lambda_{UV}$ in eq. (2) [2]. A physical realization of this mathematical possibility is that the Standard Model (SM) at high energies below the Planck mass gets replaced by some other model where couplings do not run (e.g. an UV-finite theory, or some fixed point of the RG flow).

By using the $\beta$ function of eq. (3) within the dimensional regularization formalism, we get for the one-loop RG running of a gauge coupling $\alpha$

$$\frac{1}{\alpha(\mu^\prime)} - \frac{1}{\alpha(\mu)} \simeq \beta \ln \frac{\mu^\prime \delta}{\mu^{1 - \delta}} + \cdots.$$
\( \mu^{1-\delta} \). In theories with several particle masses and sizable RG corrections, the factor \( \delta \) generically can become some unknown function of the energy. In order to compute it, we would need to know the physics around \( \Lambda_{\text{UV}} \).

In the next sections, we explore the most sensitive experimental probes of anomalous RG running, assuming for definiteness that all Standard Model formulæ get modified as in eq. (2), with a constant \( \delta \) to be extracted from data.

### 3 Running of \( \alpha_{\text{em}} \) from \( m_e \) to \( m_\mu \)

The measurements of the anomalous magnetic moments of the electron \([4]\)
\[
g_e/2 = 1.00115965218085(76)
\]
and of the muon \([5]\)
\[
g_\mu/2 = 1.00116592080(63),
\]
together with the assumption of the validity of the Standard Model, allow us to infer the electromagnetic coupling \( \alpha_{\text{em}}(\mu) \) at the scales \( \mu = m_e \) and \( m_\mu \), in view of the theoretical prediction
\[
g_i = 2 + \alpha_{\text{em}}(m_i)/\pi + \cdots,
\]
where \( \cdots \) denotes higher-order effects. We recall that \( g_e \) gives the most precise determination of \( \alpha_{\text{em}} \), that is consistent with lower-energy probes from atomic physics \([4]\). Assuming the anomalous running
\[
\frac{1}{\alpha_{\text{em}}(m_e)} - \frac{1}{\alpha_{\text{em}}(m_\mu)} = \frac{1-\delta}{3\pi} \ln \frac{m_\mu}{m_e} + \cdots
\]
one gets the presently most precise determination of \( \delta \):
\[
\delta = -(0.047 \pm 0.018) \%.
\]
The central value of \( \delta \) is about \( 3\sigma \) below zero, because, for \( \delta = 0 \), \( g_\mu \) is about \( 3\sigma \) above the SM prediction, \( (g_\mu - g_{\mu}^{\text{SM}})/2 = (23 \pm 9) \cdot 10^{-10} \), with the precise number depending on how one deals with the theoretical uncertainties on higher-order QCD corrections to \( g_\mu \): relying on \( e^-e^+ \) data and/or on \( \tau \)-decay data \([5]\).

The usual new-physics interpretation of the \( g_\mu - 2 \) anomaly is that new particles with heavy mass \( M \), like supersymmetric particles, affect \( g_\mu \) giving an extra contribution \( \Delta g_\mu \sim \alpha_2 m_\mu^2/M^2 \). They also affect precision data at higher energies, but have a negligible influence on \( g_e \) in view of \( m_\mu \gg m_e \).

We point out that the relative incompatibility between \( g_\mu \) and \( g_e \) could instead be due to a ‘too fast’ RG running of \( \alpha_{\text{em}} \). We show that \( g_e \) and \( g_\mu \) presently give the most sensitive probes to \( \delta \): this kind of new physics is best seen with higher precision than with higher energy.

### 4 Running of \( \alpha_{\text{em}} \) from \( m_\mu \) to \( M_Z \)

Precision tests at the \( Z \) pole offer another precision determination of the electromagnetic coupling. By performing a global fit within the SM with Higgs mass \( m_h \) \([6]\) we find
\[
\frac{1}{\alpha_{\text{em}}(M_Z)} = 128.92 + 0.23 \ln \frac{m_h}{M_Z} \pm 0.06.
\]
This value can be compared with the RG extrapolation from \( m_e, m_\mu \) up to \( M_Z \) \([7]\)
\[
\frac{1}{\alpha_{\text{em}}(M_Z)} = 128.937 + 8.1\delta \pm 0.028,
\]
where the uncertainty comes from QCD thresholds. So
\[
\delta = \left( -0.2 + 2.9 \ln \frac{m_h}{M_Z} \pm 0.9 \right) \%.
\]
The precise measurement of the muon lifetime does not give another probe of \( \delta \), as the anomalous dimension of the associated Fermi operator
\[
[j\gamma_\mu P_L u_\mu][\bar{\nu}_e \gamma_\mu P_L e]
\]
is zero: indeed the electromagnetic current is not renormalized, and this operator can be related to it, times a neutrino current not affected by electromagnetic interactions.

5 Running of $\alpha_s$ from $m_\tau$ to $M_Z$

Another sensitive probe to $\delta$ comes from the running of the strong coupling $\alpha_s$: in view of its large value, $\alpha_s$ runs fast. The strong coupling constant has been measured at various scales, and the two most precise determinations are at $m_\tau$ and $M_Z$. By performing a global fit of electroweak precision data within the SM with Higgs mass $m_h$ [6] we find

$$\alpha_s(M_Z) = 0.121 + 0.0008 \ln \frac{m_h}{M_Z} \pm 0.0023. \quad (13)$$

On the other hand, the measurement of the strong coupling from $\tau$ decays, $\alpha_s(m_\tau) = 0.334 \pm 0.009$, extrapolated up to $M_Z$ gives [8]

$$\alpha_s(M_Z) = 0.1212 + 0.08 \delta \pm 0.0011. \quad (14)$$

So

$$\delta = \left(-0.4 + 1.1 \ln \frac{m_h}{M_Z} \pm 3.3\right)\%. \quad (15)$$

Finally, flavor-physics observations allow us to test the QCD running of various operators from the weak scale down to the bottom or charm mass. However, the uncertainty on $\delta$ is at the level of several tens of percent.

6 Conclusions

Motivated by possible deviations from the standard QFT predictions for the RG running of couplings, we rescaled $\beta$ functions by $1-\delta$ and studied how data probe the new-physics parameter $\delta$ that parameterizes an anomalous running. Unlike in ordinary new physics, the most sensitive probe to $\delta$ is given by precision experiments at low energies $E \gtrsim m_c$: the measurements of the magnetic moments of the electron and the muon determine $\delta$ with a 0.018% uncertainty, excluding order-one effects. However, the anomaly in the anomalous magnetic moment of the muon indicates a best fit value for $\delta$ which is $3\sigma$ below zero. Running of $\alpha_{em}$ and $\alpha_s$ up to $M_Z$ is confirmed with a 1% and 3% precision respectively.

Acknowledgements  We thank Gino Isidori for comments.

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