About the Images and Inverse Images Of Intuitionistic or Vague Fuzzy Subsets

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Abstract. In this paper an exclusive study of some standard (lattice) algebraic properties for Intuitionistic fuzzy (inverse) images of Intuitionistic fuzzy subsets is done. Further as in crisp setup, characterizations for injectivity and surjectivity of maps in terms of some (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images are performed.

1 Introduction

The traditional view in science, especially in mathematics, is to avoid uncertainty at all levels at any cost. Thus, 'being uncertain' is regarded as 'being unscientific'. But unfortunately in real life most of the information that we have to deal with is mostly uncertain.

One of the paradigm shifts in science and mathematics in this century is to accept uncertainty as part of science and the desire to be able to deal with it, as there is very little left out in the practical real world for scientific and mathematical processing without this acceptance!

One of the earliest successful attempts in this direction is the development of the Theories of Probability and Statistics.

However, both of them have their own natural limitations. Another successful attempt again in the same direction is the so called Fuzzy Set Theory, introduced by Lotfi Zadeh.

According to Zadeh, a fuzzy Subset of a set X is a function $\mu$ from X to the closed interval [0,1] of real numbers. The function $\mu_x$ he called, the membership function which assigns to each member x of X its membership value, $\mu_x$ in [0,1].

Fuzzy set theory is one area with large number of applications both in Mathematics and in Computer science. For applications of fuzzy set theory in Mathematics, one can refer to the text books Mordeson and Malik[10] in Fuzzy Algebra and Ying-Ming and Mao-Kang[15] in Fuzzy Topology. For applications of Fuzzy set theory in Computer science, one can refer to the text books Galindo-Urrutia and Piattini[5] in Fuzzy Data Base Management Systems, Tamalika and Aditya[7] in Fuzzy Image Processing, Hiroshi[7] and Morgan[9] in Fuzzy Data Mining, Zhang[14] and Liu and Li[11] in Fuzzy Neural Networks, Valente and Pedrycz[8] in fuzzy clustering.

In 1983, Atanassov[1] generalized the notion of Zadeh fuzzy subset of a set further by introducing an additional function $\nu$, which he called a non-membership function with some natural conditions on $\mu$ and $\nu$, calling these new generalized fuzzy subsets of a set, Intuitionistic fuzzy subsets. Thus according to him an Intuitionistic fuzzy subset of a set X, is a pair $A=(\mu_A, \nu_A)$, where $\mu_A, \nu_A: X \rightarrow [0, 1]$ of real numbers such that for each x in X, $\mu(x) + \nu(x) \leq 1$, where $\mu_A$ is called the membership function of A and $\mu_A$ is called the non-membership function of A.

Interestingly the same notion of Intuitionistic fuzzy subset of a set was also introduced by Gau and Buehrer[6] in 1993 under a different name called Vague subset. Thus whether we called Intuitionistic fuzzy subset of a set or if-subset of a set for short, or vague subset of a set, they are one and the same.

In stead of using long phrases like Intuitionistic fuzzy subset or vague subset, here onwards, we use the phrase if-subset. Obviously, if/\nu-subset only means Intuitionistic fuzzy/vague subset.
Ever since Atanassov[1] introduced the notion of Intuitionistic fuzzy subset of a set, several mathematicians started imposing and studying both algebraic and topological structures on Intuitionistic fuzzy subsets. Looking at several of the papers that are in print and online, one thing which becomes evident is that various (lattice) algebraic properties of images and inverse images of a Intuitionistic fuzzy subset which, incidentally, not only play a crucial role in the study of both Intuitionistic Fuzzy Algebra and Intuitionistic Fuzzy Topology but also are necessary for the individual/exclusive development of Intuitionistic Fuzzy Set Theory, are not yet studied, although these concepts were existing since long. In fact, these concepts of if/v-image and if/v inverse image are dealt in Ming[17], Thakur and Pandey[13], Davvaz, Dudek and Jun[4], Yon, Jun and Kim[16].

However, in this paper, we make an exclusive and somewhat detailed study of these (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images under crisp maps. Further as in crisp setup, we characterize some injectivity and surjectivity of maps in terms of some (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images.

A few of the results in this paper may be available in the literature elsewhere but scattered; however, we presented them here not only collectively but also in a suitable way for further research to the individual/exclusive development of Intuitionistic fuzzy Set Theory.

For any set X, the set of all if-subsets of X be denoted by I(X). By defining, for any pair of if-subsets A = (μA, υA) and B = (μB, υB) of X, A ≤ B iff μA ≤ μB and υB ≤ υA. I(X) becomes a complete infinitely distributive lattice. In this case for any family (A_i)_{i∈I} of if-subsets of X, V_{i∈I}A_i = (V_{i∈I}μA_i, A_{i∈I}υA_i) and A_{i∈I}A_i = ( A_{i∈I}μA_i, V_{i∈I}υA_i), where for λ_i : X → [0,1], (V_{i∈I}λ_i) x = V_{i∈I}λ_i x and (A_{i∈I}λ_i)x = A_{i∈I}λ_i x.

For any set X, one can naturally associate, with X, the if-subset (μX, υX) = (1X, 0X), where 1X is the constant map assuming the value 1 for each x ∈ X and 0X is the constant map assuming the value 0 of for each x ∈ X, which turns out to be the largest element in I(X). Observe that then, the if-empty subset Φ of X gets naturally associated with the if-subset (μΦ, υΦ) = (0, 1), which turns out to be the least element in I(X).

Let A = (μA, υA) be an if-subset of X. Then it turns out that (υA, μA) is also an if-subset of X, thus for any if-subset A = (μA, υA) the if-complement of A, denoted by A^c is defined by (υA, μA). Observe that A^c = X - A = X ∧ A^c. Further for any pair A, B of if/subsets of X, we define B - A to be B ∧ A^c. In other words, for if/v-subsets B = (μB, υB) and A = (μA, υA) of X, B - A = (μB ∧ μA, υB ∧ υA).

Throughout this paper the capital letters X, Y, Z stand for arbitrary but fixed (crisp) sets, the small letters f, g stand for arbitrary but fixed (crisp) maps f: X→Y and g: Y→Z, the capital letters A, B, C, D, E, F together with their suffixes stand for if/v-subsets and the capital letters I and J stand for the index sets. In general whenever P is an if/v-subset of a set X, always μP and υP denote the membership and non membership function of the if/v-subset P respectively. Also we frequently use the standard convention that VΦ = 0 and AΦ = 1.

2 Main Results

In this section the notions and properties of Intuitionistic fuzzy/vague image and Intuitionistic fuzzy/vague inverse image for an Intuitionistic fuzzy/vague subset of a set under a crisp map are recalled and are shown to be well defined.

2.1 if-Images and if-Inverse Images

Let X, Y be a pair of sets and let f: X → Y be a map. Let A = (μA, υA) and B = (μB, υB) be if/v-subsets of X and Y respectively.
Definitions and Statements 2.1.1: (1) Let μ_D, ν_D: Y → [0,1] be defined by μ_{DY} = μ_{AY} f^{-1}_y and ν_{DY} = ΔD f^{-1}_y ∀ y ∈ Y. Then D is a well defined if/v-subset of Y as follows:
Let y ∈ Y. It is enough to show that μ_D y + ν_D y ≤ 1. If f^{-1}y is empty, we are done because VΦ = 0 and ΔΦ = 1. Therefore, let f^{-1}y ≠ Φ. Then μ_{AX} x ≤ 1 - ν_{AX} x ≤ 1 - ΔA f^{-1}_y for all x in f^{-1}y which implies V_{x∈f^{-1}y} μ_{AX} x ≤ 1 - ΔA f^{-1}_y or V_{μA f^{-1}_y} + ΔA f^{-1}_y ≤ 1.
(2) Let X, Y, f and A be as above. Then the if/v-subset D of Y as defined as in (1) above, called the Intuitionistic fuzzy/vague image of A under f or simply the If/image of A under f, is denoted by f^{-1}A.
(3) Let μ_C, ν_C: Y → [0,1] be defined by μ_{CX} = μ_{ fx} and ν_{CX} = ν_{ bx}, ∀ x ∈ X. Then C is a well defined if/v-subset of X as follows:
Let x ∈ X. It is enough to show that μ_{CX} + ν_{CX} ≤ 1 or μ_{ fx} + ν_{ bx} ≤ 1 which is always there because B is a well defined if/v-subset of Y.
(4) Let X, Y, f and B be as above. Then the if/v-subset C of X defined as in (3) above, called the Intuitionistic fuzzy/vague inverse image of B under f or simply the if/v-inverse image of B under f, is denoted by f^{-1}A.

2.2 Mapping Properties of if-Images and if-Inverse Images
In this section we show that several of the mapping properties that hold good for Zadeh fuzzy subsets are also held good for the Intuitionistic fuzzy subsets.

2.2.1 Theorem Let X, Y be a pair of sets and let f: X→Y be a map. Let A, Ai and B, Bi be if-subsets of X and Y respectively. Then the following are true:
1. A ≤ C implies fA ≤ fC
2. B ≤ D implies f^{-1}B ≤ f^{-1}D
3. always A ≤ f^{-1}fA
4. A = f^{-1}fA ⇔ f is 1-1
5. always f f^{-1}B ≤ B
6. f f^{-1}B = B ⇔ if f is onto
7. f(V_{i∈i}A_i) = V_{i∈i}fA_i
8. f(A_{i∈i}A_i) ≤ A_{i∈i}fA_i and the equality is true whenever f is 1-1.
9. f^{-1}(V_{i∈i}B_i) = V_{i∈i}f^{-1}B_i
10. f^{-1}(A_{i∈i}B_i) = A_{i∈i}f^{-1}B_i
11. fA = φ ⇔ A = φ, in particular, f φ = φ, and when f is 1-1, f A = fX ⇔ A = X
12. f^{-1}B = X ⇔ B ≥ fX, in particular, f^{-1}fX = X and f^{-1}B = φ ⇔ B ≤ Y - fX, in particular, f^{-1}φ = φ
13. fX - fA ≤ f(X-A) and the equality holds whenever f is 1-1
14. f^{-1}(Y - B) = X - (f^{-1}B)
15. ft^{-1}B = B△fX and hence always ft^{-1}B ≤ B
16. f^{-1}B = f^{-1}(B△fX)
17. fA ≤ B ⇔ A ≤ f^{-1}B
18. (i) ft^{-1}A = fA (ii) f^{-1}ft^{-1}B = f^{-1}B

2.2.2 Theorem Let X, Y and Z be three sets and let f: X→Y and g: Y→Z be a pair of maps. Then the following are true:
1. (gf)(A) = gf(A) for all A ≤ X
2. (gf)^{-1}(C) = f^{-1}(g^{-1}C) for all C ≤ Z

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