Gravitational Lensing of Epoch-of-Reionization Gas

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Abstract

I present a weak lensing sensitivity estimate for upcoming high redshift (epoch of reionization and beyond) 21cm surveys. Instruments such as PAST, LOFAR and SKA should be able to measure the weak lensing power spectrum to precisions far exceeding conceivable optical surveys. Three types of sources are detectable, which include the re-ionization stromgren spheres, large scale structure, and minihalos.

Tomography allows the measurement of the time evolution of the dark matter power spectrum. Raw sensitivities allow measurement of many cosmological parameters, including dark energy, neutrino mass and cosmic equations of state, to percent accuracy. It also has the potential for inflationary gravity wave measurements.

Ultimate limits may be achievable through radio observations of $10^{18}$ minihalos. Inflationary Hubble parameters $H_I$ down to $10^{-9} M_{\text{Planck}}$ can be detected through this effect. Second order effects may also be observable, allowing tests for backreaction and the quantum mechanical origin of perturbations.

Key words: Cosmology-theory-simulation-observation: gravitational lensing, dark matter, large scale structure
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1 Introduction

Weak gravitational lensing has recently been established as a clean tool to probe the distribution of all matter using only its gravitational effects. This can also be modelled from first principles, and the matter power spectrum has already been measured to some accuracy [Pen et al., 2003].

Current measurements of the cosmic microwave background have resulted in power spectra that are accurate at the percent level when appropriately binned.
This constrains some combinations of cosmological parameters to similar percent level accuracies. But there are many degeneracies in standard cosmological parameters arising from purely CMB data. Using the WMAP data alone, even unpopular regimes are allowed, such as a closed model with low Hubble constant \(H_0 = 35\) (Spergel et al., 2003). The authors dismiss such models as “unreasonable”, primarily due to the non-standard required value of \(H_0\). Nevertheless, one can find references in the recent literature which predicted such low Hubble constants (Bartlett et al., 1995).

Some parameters, such as the absolute baryon density \(\Omega_b h^2\) are very well determined by the CMB alone, while many other physical parameters require external data to constrain accurately, and thus have much larger uncertainties. Some such examples include the matter density of the universe, equation of state, or neutrino mass. Precision low redshift measurements are also needed to break these degeneracies. Galaxy distributions have been used successfully as a calibrator (Spergel et al., 2003), but its precision may have fundamental limitations. The external calibrators tend to be limited by systematic errors in the models, whose error bars are difficult to quantify.

Robust low redshift degeneracy breaking measurements have already come from weak gravitational lensing (Contaldi et al., 2003). Future lensing surveys will significantly improve on the statistical accuracy, which is currently at the 10% level. Accuracies greater than 1% are in progress (Van Waerbeke and Mellier, 2003). Much of the most interesting information, such as the equation of state of the universe or neutrino mass (Cooray, 1999; Fukugita et al., 2000), requires even more precise measurements. Multiple redshift measurements would be invaluable. Large physical scales are important to measure the linear power spectrum, which are not affected by baryon feedback. All of these are very challenging for the current weak lensing strategy. The surface densities of galaxies on the sky is limited, and large sky coverage is expensive.

The ideal source for weak lensing maps should be at high redshift, and have structure on small scales. The CMB satisfies the first criterion, but not the second. Faint galaxies partially satisfy both. The epoch of reionization universe is a natural candidate to use as a source screen to measure gravitational lensing. It is far away, emits brightly in the hydrogen hyperfine transition line, has structures on many scales ranging from several arcminutes to less than a milliarcsecond, a high surface density, and redshift information on each source. Several experiments are under construction, including PAST (http://astrophysics.phys.cmu.edu/~jbp/past6.pdf), LOFAR (Kassim et al., 2000) and SKA is in the design phase (van Haarlem, 2000). These instruments will measure this source population to high accuracy. Several more experiments, including T-REX (http://orion.physics.utoronto.ca/sasa/Download/poster/casca_poster.pdf) and CATWALK (ftp://ftp.astro.unm.edu/pub/users/john/AONov03.ppt), may result in earlier detections of re-ionization effects, but at lower signal-to-
noise. The Canadian Large Adaptive Reflector CLAR (Carlson et al., 2000) will also measure this redshift range at lower spatial resolution. In this paper I present sensitivity estimates for these experiments, and continue to estimate the cosmological limits if one exhausted the information.

Some of these instruments do not have finalized designs and the cosmic reionization history is not well known. Therefore we only make rough estimates, which we expect are probably good to a factor of two.

2 Pre-reionization Weak Lensing

We first estimate the strength of gravitational lensing for sources at redshift $z_s = 9$. We use the WMAP cosmological parameters (Spergel et al., 2003) with $\Omega_0 = 0.27$, $\Omega_{\Lambda} = 0.73$, $\sigma_8 = 0.84$. The angular diameter distance to redshift 9 is $2.23c/H_0 = 6.7h^{-1}$ comoving Gpc. One arcsecond is $33h^{-1}$ kpc (comoving).

Figure 1 shows the lensing convergence ($\kappa$) power spectrum computed by integrating the Limber equation over the Peacock and Dodds (1996) nonlinear matter power spectrum. The horizontal axis represents the angular scale, with $\theta \sim 2\pi/l$. The vertical scale is the variance in the $\kappa$ power, which corresponds to the magnification factor of point sources in the weak lensing limit. The plot also shows the Poisson noise from PAST/LOFAR and the second generation PAST+/SKA.

The most prominent feature is the peak at $l \sim 10^4$. This corresponds to an angular scale of two arc minutes. Current lensing surveys have signal to noise less than unity at the peak of the spectrum, which prevents the construction of direct dark matter maps. At these higher redshifts, the lensing power is a few parts in a thousand. The rms amplification or shear is several percent, which makes its measurement more tractable than the one percent level fluctuations in current cosmic shear surveys. The variations are not linear to the statistical accuracy, and strong lensing corrections can be considered. These have been presented, both in perturbative and non-linear form, by Pen (2000).

Before reionization, most of the baryonic matter in the universe is in the form of neutral hydrogen. After $z \ll 30$, the majority of that gas gravitationally collapses with the dark matter into virialized ’mini-halos’. The cosmic microwave background has a temperature $T_{\text{CMB}} = 2.73(1 + z)$. The gas is collisionally heated to the virial equilibrium temperature of the minihalos, which is between $10^2 - 10^4$ K, so typically hotter then the CMB. The average density of a minihalo is 200 times the mean density. At these temperatures and densities, the spin temperature is tightly coupled to the kinetic temperature, and decou-
Fig. 1. The lensing power spectrum for sources at $z = 9$. The solid line is the shear and convergence power spectrum. Images in the source plane are distorted by the square root of the power, which is several percent. The left dotted line is the noise level expected from PAST/LOFAR. The dotted line on the right is the noise expected from the second generation PAST+ and the SKA.

ampled from the CMB (Madau et al., 1997; Iliev et al., 2002, 2003). In the limit of a spin temperature much higher than the CMB temperature, the average surface brightness at mean density in spontaneous emission on the 21cm line is $T_{21}\text{cm} = 23\text{mK}$ (Madau et al., 1997). These minihalos provide a fluctuating source screen with a characteristic scale that can be used to measure the gravitational lensing effect of the intervening dark matter. In the subsequent sections we will discuss the observability of this effect.
3 Measuring the power spectrum

3.1 High $S/N$

Two regimes exist for measuring the weak lensing shear at high redshift. With high angular resolution and signal-to-noise, one expects to resolve individual minihalos. The characteristic mass is $10^5 M_\odot$, and one expects $\sim 10^{18}$ such minihalos on the sky. Each virialized halo is approximately round, which can measure the local shear with a signal-to-noise of order unity. The characteristic size of a halo is 1kpc, corresponding to 30 mas. This requires a baseline of $D = 13000$km, which is resolvable with earth sized baselines. The corresponding spherical harmonic number is $l \sim 10^7$. The temperature of such a halo is $\sim 10^3$K, with a velocity width of about 3km/sec. Virialized halos are about a factor of 200 overdense in 3-D. Along the line of sight the radial velocity dispersion is comparable to the size of the infall region, so one only expects a two dimensional density enhancement, which would be $\sim 40$. The brightness temperature on a halo would then be $\sim 0.1$K.

An minimal observing strategy would aim to achieve a signal-to-noise of 1 on the typical minihalo. A lower signal to noise cannot identify individual halos. Higher signal-to-noise is expensive to achieve. The actual accuracy on the lensing map reconstruction is boosted by the number of halos along the same line of sight that one averages over. We use a simple model to determine signal-to-noise. A filled aperture telescope of diameter $D$ corresponding to the maximal baseline is used as reference. If the elements are distributed uniformly within this diameter, we can treat the actual sensitivity as just a dilution corresponding to a telescope with a very small aperture efficiency $\eta_A = 4A_{\text{eff}}/(\pi D^2)$ where $A_{\text{eff}}$ is the collective effective area of all the elements. The noise in the map is then $T_{\text{sys}}/(\eta_A \sqrt{t \Delta \nu})$, where we take the integration time $t$ to be one year, and the bandwidth the thermal width of each minihalo of 3km/sec, $\Delta \nu \sim 1.4$khz. To obtain a signal-to-noise of unity on each halo in a year at a system temperature of 200K, an effective aperture of 200,000 km$^2$ is required. As an interferometer, 200 elements with 40 km apertures would suffice. A half wave dipole has an effective area of $A_{\text{eff}} = 3\lambda^2/(8\pi) \sim 0.5$m$^2$, so 400,000km of wire are needed to achieve this aperture. Copper has a resistivity of $1.7 \times 10^{-8}$Ω·m, so a diameter of 0.1mm results in a resistance much less than the impedance. The wire requires 40 tons of copper. At the date of writing, one pound of copper costs US$1.1, which is a negligible cost factor. The amplifiers, interconnects and processing would dominate, but this cost reduces by Moore’s law.
3.2 Low S/N

Initial experiments such as PAST and LOFAR will measure the distribution of gas on linear scales, where the only observable are Gaussian fluctuations, and reionization Stromgren spheres. The simplest way to measure the fluctuations is to construct a three dimensional map, with the redshift as the third dimension. On each point of the angular map, we compute the variance of the density field along the z dimension. One way of achieving that is to Fourier transform the spectrum, and measure the power in each Fourier mode. In practice, some modelling would be needed to achieve appropriate noise weights. There are different ways of reconstructing a lensing map from a 'fuzzy' noisy image. If only one two dimensional image were known, for example in the CMB, this can only be done if we have knowledge of the statistics of the source screen. The CMB, by being Gaussian, allows such reconstruction using the connected four point function which is zero for a stationary Gaussian process, but is induced by lensing\cite{Okamoto and Hu 2003}. If the unlensed properties are not known, it it presumably very difficult to disentangle lensing from the intrinsic statistics, but also see Cooray \cite{2004}.

Our reconstruction will use the intrinsic three dimensional information that is available from redshifted 21cm structures. This gains us two kinds of observables: the intrinsic correlations of matter are presumably statistically isotropic. Lensing will change that anisotropically. Sources that are distant along the line of sight but close in projection pass through the same gravitational lensing screen, and will have induced cross correlations. It is this latter effect that we exploit. We observe the structures at a constant angular scale. Weak lensing changes the mapping of angular to physical scale slightly. Properties of the source generically depend on the physical scales at which they are measured, and thus their statistics will also experience a slight change. This is true for all objects along the same line of sight, even the distant ones. The variance in the beam, for example, will generally increase on small scales. This occurs in a correlated fashion for objects over a range in source redshift. The procedure now is to take a point in the map, smooth the data along the redshift space axis, square that, and compute the mean variance. Now we can ask what the optimal smoothing window would be. The smoothing procedure is a convolution, which is the same as Fourier transforming the line, squaring each Fourier mode, and summing them weighted by a window. Each of the Fourier modes is uncorrelated because of stationarity (independent of Gaussianity), so we do not need to worry about covariances between fourier modes. We presumably want the window which maximizes sensitivity to gravitational lensing in the presence of noise. If the data is noise dominated, we weight each point by the ratio of signal to noise variance. One needs to know the expected variance as a function of redshift, which one can measure from the angular maps.
This results in a two dimensional map $\sigma^2(\theta_x, \theta_y)$ whose entries are projected variances in the density of neutral hydrogen. We denote the three dimensional matter power spectrum by $\Delta^2(k) \equiv k^3P(k)/(4\pi)$. It is shown in figure 2. A pixel size $\theta$ maps to a spherical harmonic number $l \sim 2\pi/\theta$. One can think of the variance as a measure of $\Delta^2(l)$ shown in figure 2. Patchy re-ionization can significantly boost the 21cm power by modulating the neutral fraction. Figure 2 shows a model of the boosted power, using a $b = 4$ bias model on large scales (Santos et al., 2003). Since the neutral fraction cannot be modulated by more than unity, we limited the biased power to the greater of unity and the actual matter power.
Gravitational lensing has two effects on the variance. It changes the area of each pixel in the source plane. Each pixel is a fixed solid angle on the sky. The variance measures the power at the pixel scale. As the physical size corresponding to the pixel is increased or decreased by gravitational lensing, the same pixel probes a larger or smaller angular scale. From Figure 2 we see that moving to larger scales (smaller $l$) decreases the variance in each pixel.

This variance map is related to the strength of lensing convergence. The fractional change in the variance by a fractional change in area is half the logarithmic derivative of the power spectrum:

$$\frac{\sigma^2(x)}{\langle \sigma^2 \rangle} - 1 = \frac{\kappa(x)}{2} \frac{\partial \log \Delta^2(k)}{\partial \log k}. \quad (1)$$

We expect to evaluate the derivative at the physical scale $k$ corresponding to angular scale $l$ of the pixel on the sky. From Figure 1 we see that the logarithmic derivative is of order unity.

The other observable from gravitational lensing is the gravitational shear. If one takes a collection of pixels on the sky, one can construct a three dimensional two-point correlation function. Projecting the correlation function along the radial direction, we can examine the anisotropy of the two point correlation function. In the absence of lensing, the 2-D two-point correlation function is statistically rotation invariant on the sky. Isovariance lines of the correlation function are circles. Gravitational shearing will distort the correlation function on each patch to appear anisotropic. The ellipticity is a measure of the gravitational reduced shear. The signal-to-noise of the two methods is comparable. The benefit of the shear procedure is that no prior knowledge of the power spectrum is required.

3.3 LOFAR/PAST

We examine a rough sensitivity estimate for PAST and an idealized LOFAR configuration. For this purpose, we measure the gas power spectrum at $z \sim 9$. We assume a system temperature $T_{sys} = 400$K which is dominated by the galaxy, and a baseline of 2km as is planned for PAST. The 21cm line becomes 2.1m, and the telescope has an effective resolution of 1/1000 radians if it were a single dish. We can approximate this to be a spherical harmonic number of 3000. The angular scale corresponds to three arcminutes, and 7 $h^{-1}$ Mpc comoving. The redshift difference for the same radial separation is $\Delta z = 0.04$ (Pen 1999), which is a bandwidth of 500 khz. Initial PAST will have an effective aperture of $40,000 \, m^2 \left(140 \, MHz/\nu\right)^2$, which corresponds to an aperture dilution $\eta_A$ of about 1/100. This results in a noise level after a year’s operation of $T_{sys}/(\eta_A \sqrt{\Delta \nu}) \sim 10$mK per synthesized beam of three arc
minutes. Current models place the size of ionization spheres at similar scales as this resolution, so one expect of order unity signal to noise. Radially, the resolution is much better, so one could realistically expect 100 independent radial modes in each synthesized beam across the epoch of reionization. If each radial mode represents a signal to noise of order unity, our sensitivity is about $\Delta \kappa \sim 0.1$. The square is the lensing noise shown in Figure 1 at the corresponding angular scale $l \sim 3000$, and scales as white noise.

For gravitational lensing, one optimizes throughput by using the scale where the signal-to-noise is of order unity. Our procedure is to square the density map at that scale, and use the changes in this variance (square) to estimate the lensing strength $\kappa$. The power spectrum of $\kappa$ is now the two point correlation of our density variance, which is related to a four point function of the underlying density field. Because of our large radial average of squares, we assume that the contribution from the intrinsic spatial connected four point function is negligible. With this procedure our lensing noise power scales $\propto l^2$.

The actual sensitivity of the telescope to neutral hydrogen density structures as a function of scale in the source plane is a different calculation. It depends on the actual layout of the elements, and the number of elements at a given separation. If we keep the collecting area, and just shrink the baselines, the aperture dilution $\eta_A$ decreases as the square of the separation, while the radial scale is proportionate to the angular scale. The noise scales as $\theta^{-2.5}$. At double the angular scale, corresponding to six arc minutes, this corresponds to a noise level of 13 mK $\sqrt{\text{week}/t}$, so PAST can map out larger Stromgren spheres directly. It has been proposed that the ionization spheres up to 50 Mpc have been observed in existing QSO absorption line data \cite{Withe and Loeb, 2004}. These would be observable at high signal to noise with PAST. LOFAR has a similar effective aperture, and we would expect a similar sensitivity. The second generation PAST (which we call PAST+ in this paper) is expected to have ten times the resolution and one hundred times the effective aperture. This is four times larger than the SKA, so for estimation purpose we clump these two instruments into the same class, as shown by the more sensitive noise lines in Figure 1. To estimate the total sensitivity on the full field power spectrum, we assumed that the PAST array instantaneously measures 100 square degrees of the sky, and we measure a 30% redshift depth. For PAST+/SKA we used 10 square degrees.

The problem can be broken into two parts: one must measure the power spectrum of the 21cm emission, and then look for spatial variations in the power. Figure 2 shows the expected matter power spectrum. We show the noise of PAST/LOFAR as the dashed line. We also plotted the statistical accuracy of the power spectrum measurement as dotted lines. The matter power spectrum itself can of course also provide cosmological information. Unfortunately, this will again be limited by the complication of gastrophysical processes such as
radiative transfer and star formation, which will modulate the spatial distribution of neutral hydrogen.

4 Processing Challenges

Several problems arise in the endeavor to map gravitational lensing to such an ambitious scale. Oh and Mack (2003) have shown that fluctuations in synchrotron emission from ionized gases can outshine the fluctuations from the 21 cm emission. As pointed out by Furlanetto et al. (2004), this can probably be addressed by removing all power-law spectrum spatial fluctuations. A similar requirement holds for the modelling of radio point sources. These sources are valuable to calibrate the point-spread function. A promising strategy is to use the brightest point sources in the map to calibrate the spectrum. The radio sources are emit synchrotron radiation from relativistic electrons, which has no spectral structure. The task of looking for spectral features now consists of comparing the spectrum of points on the map with that of nearby bright sources. This can be done to high accuracy.

Computationally, a brute force correlation of a very large number of dipoles can be expensive. A direct correlation for a full sky synthesis with \( N \) dipoles with involves \( O(N^2) \) correlations. Some designs allow beam forming within each synthesis node. For nodes with \( n \) dipoles placed on regularly spaced patterns, the cost to form the \( n \) independent beams can be reduced to \( O(n \log n) \), so the total correlation cost is \( O((N^2/n) \log n) \). Measuring the all-sky actual power spectra can also be computationally challenging, especially if corrections to the weak lensing approximation is desired (Pen, 2000). The general theoretical framework is well understood (Okamoto and Hu, 2003), but a detailed computational pipeline will require significant software development (Pen, 2003).

Since the array is sparse, not all projected baselines are measured simultaneously. Earth rotation fills in the map. To perform statistics on \( 10^{18} \) sources of course will require a correspondingly large catalog. The information is measured in the exabytes. With today’s hard drives, this would cost billions of dollars. But storage and computation both follow Moore’s law, and will become affordable in the foreseeable future. Certainly many challenges must be overcome to implement the ultimate array.
5 Cosmological Constraints

We discuss some of the cosmological constraints achievable with pre-reionization 21cm emission. The cosmic shear can be measured to unity signal-to-noise for \( l < 500 \) for PAST/LOFAR, \( l < 10000 \) using PAST+/SKA and \( l < 10^7 \) using the ultimate detector.

Nominally, this results in an accuracy of projected dark matter power spectrum determinations to better than \( 10^{-4} \) for PAST+/SKA, and better than \( 10^{-7} \) in principle. If the only uncertainty came from the neutrino mass, we can apply equation (6) from Cooray (1999) and derive a nominal accuracy on the neutrino mass of 0.1meV for PAST+/SKA and \( \mu \) eV as the ultimate limit. This is already significantly more accurate than the mass differences between neutrinos, and it should be possible to measure the individual masses of each of the neutrino generations separately. These masses can then be compared to experiment.

Over a fiducial range of source redshifts \( 8 < z < 10 \), the angular diameter distance varies by 5%. For a lens half way between us and the source \( z \sim 1.6 \), the lensing strength changes by 3%, which is easily measurable. Similarly, tomography allows the reconstruction of the power spectrum at two different redshifts. For a lens screen 1/3 and 2/3 to the source screen (redshifts 0.9 and 2.7 respectively), the relative amplitudes are only 90% degenerate, losing only a factor of ten in accuracy for determining the difference in amplitude. The change in power spectrum can be measured to an accuracy of \( 10^{-3} \), which allows very accurate dynamical constraints on the equation of state of the universe (Hu, 2002). Most of the proposed cosmological constraints by precision measurement of the dark matter power spectrum can be implemented in this precision tomographic survey. Baryon oscillations have been suggested as an example of a standard ruler (Blake and Glazebrook, 2003).

Inflation generically predicts the existence of gravity waves produced by Hawking radiation from the de Sitter horizon at the time of inflation. The amplitude at the quadrupole is \( C_l \sim (H_I/M_{\text{planck}})^2 \), where \( H_I \) is the Hubble parameter during inflation. Using \( l^3 \sim 10^{12} \) effectively independent sources in PAST+/SKA, we can constrain \( H_I < 10^{-6} M_{\text{planck}} \) (Dodelson et al., 2003). The 21cm fluctuations should be at least 2 orders of magnitude more sensitive than the galaxy statistics used in (Dodelson et al., 2003). For the ultimate survey, we expect to see \( 10^{18} \) minihalos on the sky. If one resolves each, and measures weak lensing, we can measure inflationary expansions all the way to a Hubble parameter at inflation of \( H_I \sim 10^{-9} M_{\text{planck}} \).

Such a precise measurement of the power spectrum would also unveil second order effects. Perturbations at inflation have amplitudes of order \( O(10^{-5}) \). Var-
ious second order effects enter at an amplitude of $10^{-10}$, and any measurement of the power spectrum more accurate than one part in $10^5$ will depend on the second order initial conditions.

These estimates are all idealized, and significant work is needed both in the instrument design and the processing stages to achieve these sensitivities.

6 Conclusions

I have presented sensitivity estimates for epoch of reionization 21cm weak lensing surveys. We find that a precise mapping of the projected dark matter structure on the sky can be achieved with several projects under construction and planning, including PAST, its second generation PAST+, LOFAR, and SKA. In principle, it is also possible to image up to $10^{18}$ minihalos with an extremely large dipole array. Such a future step would require an effective aperture of $\gtrsim 10^5$ km$^2$. The overwhelming statistical accuracies would allow accurate measurement or stringent upper bounds on primordial gravity waves, and second order effects from the epoch of inflation.

Even with the upcoming experiments, exquisite maps of dark matter and its evolution is possible, at much higher precision than any other planned or proposed procedure. The two dimensional dark matter distribution can be measured to a signal-to-noise of better than unity at scales down to $l \lesssim 10000$. With the leverage of a redshift range $8 < z < 10$, the evolution of the power spectrum, dark energy and neutrino mass can be measured to better than percent accuracies.

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