Transformation optics for the full dielectric electromagnetic cloak and metal–dielectric planar hyperlens

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Abstract. Full dielectric electromagnetic cloaks are studied in this paper for considering the possibility of achieving effective permeability gradients by taking advantage of a magnetic Mie resonance in high-κ ceramics. Extreme performance sensitivity to the dispersion of the complex effective parameters, which could be problematic for practical realization, is pointed out. In a second stage, we apply transformation optics for achieving magnification and high-resolution focusing in highly anisotropic metal–dielectric flat lenses.

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1. Introduction

Moulding the flow of light has already been the main goal of photonic crystal technology [1]. Towards this goal, new degrees of freedom were afforded by the emergence of so-called metamaterial technology based on the structuring of artificial matter on a scale much shorter than the operating wavelength. This permits one to open the effective parameter space, with not only negative values of the effective permittivity and permeability but also the possibility of considering zero-index media or having permittivity and permeability values that vary between zero and one. This latter case corresponds to a superluminal operating condition and hence allows the bending of light in certain regions of the geometrical space by keeping the same time of flight with respect to the corresponding straightforward transmitted waves. The Fermat principle is hence satisfied.

Such a bending of light opens the way to electromagnetically isolating some regions of space, which thus become undetectable by a point or distributed probing source. Any scatterers such as a metal rod or sphere, depending on the dimensionality of the system, placed in these isolated space regions, become invisible or equivalently become transparent for the probing waves. The activities in this research field, termed cloaking, are now mainly focused on the design, fabrication and experimental assessment of transparency cloaks. The references [2]–[10] can be considered as seminal papers in this very recent field which revisits notably the invisibility concept.

So far, most of the experimental works that have been reported for achieving such a high degree of transparency use the idea of space deformation by means of gradient index cloaks. In the microwave spectral region, the index gradients are mainly achieved by an engineering of the effective permeability. Now, it is well known that metal loops, which can be compared with the so-called split-ring resonator (SRR) technology, yield negative values of the effective permeability ($\mu_{\text{eff}}$) between the resonant and the plasma magnetic frequencies. Above the magnetic plasma frequency, the value of $\mu_{\text{eff}}$ can thus be between zero and one. In optics, the engineering of the effective permittivity was found to be preferable and some modelling results have been published on ellipsoidal-shaped radial metal inclusions [6] or non-uniform section bars, in order to induce permittivity gradients. With respect to the first principle experimental demonstrations, a microwave cloak was recently realized [5]. It was based on a reduced set of equations (see [7]), which relax the technological difficulty since an ideal cloak approach requires the engineering simultaneously of the permittivity and permeability parameters. In practice, the cloak was made of concentric dielectric cylindrical-shaped boards with printed edge-coupled C-shaped SRRs.

However, the extension of such studies to higher frequencies, notably in the terahertz spectral region by shrinking the SRR dimensions, appears problematic. The reason for this is the complexity of the realization of such a prototype whose dimensions have to be in the micron scale. In this paper, we show how such a problem can be alleviated by taking advantage of the ground magnetic Mie resonance in high-$\kappa$ dielectric rods. Recently, we showed that the basic principle is similar to that used in SRR, namely an engineering of the magnetic response by size effects [11]. Here, it is shown that the operating frequency can be extended by more than two orders of magnitude with similar prospects in terms of performance. In a more general manner, we also address the problem of the size of the object that we would like to cloak.

On the other hand, transformation optics can be applied to other applications including high-resolution focusing. Among the various possibilities reported in the literature, special
attention was paid here to the concept of a hyperlens whose basic concept was published simultaneously in [12, 13]. An experimental demonstration was published recently in [14, 15] in which the fabrication of a multilayered metal–dielectric microstructure deposited on a curve-etched substrate was reported. In addition, the multilayered stack corresponds to a highly anisotropic system with in-plane values of the permittivity tensor close to zero. From this results a magnification effect without spreading of the propagating beam which can be used to improve the resolution of an imaging system that operates in the far field.

Our contribution in this research field will be to see whether the requirement of a deposition on a pre-etched surface can be avoided by keeping the condition of a highly anisotropic medium. Thus, we demonstrate numerically that a magnification can be achieved with a flat hyperlens. Finally, we report on the preliminary development of an extension of this work to high-resolution focusing systems.

Section 2 deals with the experimental demonstration of artificial magnetism in high-κ ferroelectric ceramics. On this basis, we address in section 3 the cloaking issues by considering small-size and large-size cloaked objects. In section 4, the application of transformation optics rules to the synthesis of a flat hyperlens and to a high-resolution focusing system is presented, while conclusions and prospects are given in section 5.

2. Basic principle of Mie magnetic resonance

The basic principle for the achievement of a magnetic dipole by using high dielectric rods can be understood by considering figure 1. This figure shows the displacement current in an infinite dielectric rod whose complex permittivity value is \( \varepsilon = 200 + 5i \) with polarization of the magnetic field parallel to the rods. This typical value of permittivity was chosen in a seminal paper published by O’Brien and Pendry on this topic [16]. On the other hand, the values chosen here are representative of the present ferroelectrics technology notably with respect to the barium strontium titanate (BST) material system for which the real part of the permittivity exceeds 100, whereas the loss tangent can be maintained at values close to \( 10^{-2} \). The large values of the permittivity with respect to the embedding medium (air) ensure a high confinement of the electromagnetic wave on a scale much lower than the wavelength. As a consequence, the
Figure 2. (a) Photograph of high-κ ceramic BST cubes. Source: courtesy of the University of Tsinghua [15]. (b) Illustration of the Mie magnetic resonance.

metamaterial condition can be met despite the fact that we are considering a full dielectric route. The induced circular displacement current induces a magnetic response which can be out of phase (negative value of the permeability) above the resonant frequency. When the rods are organized in arrays, which is the case with the simulation with appropriate boundary conditions in order to mimic an infinite medium, the negative permeability is lost above the magnetic plasma frequency.

These considerations can be quantitatively assessed by means of the retrieval of the effective permeability, which is plotted in figure 1(b) as a function of the normalized angular frequency \( \omega/\omega_0 \). The frequency dependence of the effective permeability follows a Lorentz-type dispersion characteristic with negative values of the permeability between the resonant frequency and the magnetic plasma frequency. Both frequencies can be engineered as for SRRs. In fact, we learnt from the analysis outlined above that the basic principle behind the origin of an artificial magnetic moment is similar with respect to SRR physics. Therefore, the same engineering rules can be applied to tailor the resonant frequency and the magnetic plasma frequency, which depend on the filling factor. As a consequence, two degrees of freedom can be defined with the radius of the rods and the array period. As demonstrated in [11] for cloaking, the values of \( \mu_{\text{eff}} \) above the plasma frequency will be of interest with a radial variation of the rod radius, whereas the period is kept constant. Before considering in further detail the advantages and also the drawbacks associated with such a full dielectric cloak, we report hereafter some experimental verification that a bulk ferroelectric technology is suitable for inducing an artificial magnetic response. This experimental demonstration will be carried out at microwave frequency for the sake of simplicity. However, the underlying principles are valid over a large portion of the electromagnetic spectrum provided that a high value of permittivity can be preserved.

While a cylinder-shaped rod is ideal for achieving circular currents, their fabrication in practice is troublesome at ultra-small dimensions. Recently, the University of Tsinghua succeeded in fabricating BST ceramic cubes, which were subsequently characterized over a wide temperature range around the Curie temperature [17].

Further details of the fabrication techniques and the temperature dependence of the material properties can be found in [17]. In the present paper, special attention will be paid to the experimental derivation of the dispersion characteristics at room temperature. To achieve this aim, BST ceramic cubes were organized into a square lattice embedded in a Teflon film as shown in the photograph in figure 2. Each side of the cube is 0.9 mm long for a permittivity value as

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high as 850. Under this condition, it can be shown that the resonant frequency lies in the X-band (8–12 GHz). The BST unit cell size fulfils the metamaterial condition as can be verified from figure 2(a). The dominant mode in the rectangular hollow waveguide is a $\text{TE}_{10}$ mode with half a wavelength over the aperture of the flange. Figure 2(b) shows the variation versus frequency of the magnitude of the scattering parameter $S_{21}$, which was measured by vectorial network analysis and calculated by means of a full wave analysis (the commercial software High Frequency Simulator from Ansoft). As expected, at the resonant frequency here of about 9 GHz, the frequency dependence $S_{21}$ shows a dip in the transmission, showing that the one-cell layer can be compared with a single negative medium.

Further information about the dispersion of the effective parameters can be achieved by means of retrieval techniques. Figure 3 shows the results extracted by means of a Fresnel inversion technique. Similar results are obtained by using a field summation technique [18] which was used in our previous publication on cloaking. It can be seen that above the resonant frequency the one-cell layer exhibits a negative permeability, whereas the effective permittivity is almost constant with an effective value close to 1.7.

3. High-$\kappa$ ceramic electromagnetic cloak at terahertz frequencies

As outlined in the introduction, the main advantage of a BST cloak is the simplicity of fabrication in the terahertz range. Also, one can imagine extending the operating frequency up...
Figure 4. Steady-state $E_z$ pattern calculated at 0.58 THz for a copper rod without (a) and with (d) a 3D microstructured cloak shown in (b). The plane of observation is located at mid-distance between the bottom and top faces of the simulation domain. The wavefronts are well reconstructed behind the cloak without noticeable backscattering. The metallic particle placed at the centre of the device is nearly ‘invisible’ to a detector located at the output port. (c) Magnified view of the field pattern within the cloak. For clarity, the amplitude within the cloak was magnified.

to the mid- and near-infrared spectral region. However, the main difficulty in achieving this goal stems from the fact that the BST cube exhibits Debye relaxation effects. As a consequence, it becomes more and more difficult to preserve high permittivity values and to fulfil the metamaterial condition, resulting in a high confinement in a sub-wavelength scale.

Let us now consider how to take advantage of the artificial magnetism pointed out above for cloaking. Figure 4 summarizes the main features of a 3D cloak with the rod arrangement, and the mapping of the $E$-field magnitude for an infinite conductivity metal cylinder with and without the cloak. The calculations were here performed by describing all the details of the microstructure and not by considering a multilayered homogeneous structure. In order to illustrate these ab-initio calculations, a magnified view of the cloak region (figure 4(c)) is also displayed showing the localization of the electromagnetic fields on a sub-wavelength. These calculations were performed with a permittivity of the rods equal to $\varepsilon = 200 + 5i$ and for an operating frequency of about 0.5 THz. Further details of the reduced equations and the variation in the geometry of high-κ rods can be found in [11].

Mapping of the EM fields is the first stage in the assessment of an invisibility cloak. However, the calculation of the radar cross section (RCS), which is the cross sectional area of a conductive cylinder that would reflect as much energy as the object in question, can bring further quantitative information about the sensitivity of the performance to the index gradients.
Figure 5. Total scattering cross section normalized to the scattering cross section of a bare metallic cylinder as a function of frequency for the bare metal cylinder (red), the reduced cloak (blue) and the lossless (green) and lossy (black) BST cloaks.

and also the cloaking bandwidth. In the following, we will apply this approach to the case of small-size and large-scale cloaks with respect to the wavelength of the incident wave. For this systematic study, we used a homogeneous multilayered description for saving computational time [7]. In other words, we break the continuous cloaking material into shells along the radial direction. For each shell, the effective parameters are those which can be computed by the field summation technique [18]. This technique, whose results have been compared recently with the Fresnel inversion procedure, is comparable with the averaging technique of Smith and Pendry [19]. In the first stage, it can be verified that the performances of a microstructured and a multi-shell cloak are similar provided that the pitch determined by the number of discrete layers is small with respect to the wavelength. The main difference appears qualitatively on the field maps, which show finer features in the space variations for a microstructured cloak. Let us first consider a small-size cloak in the sense that the scatterer placed inside the cloak is of the order of the incident wavelength.

First we calculated the total RCS of the cylinder-shaped perfect electric conductor and then the RCS of a BST cloak and a reduced cloak. The latter is a single frequency cloak whose effective parameters can be found in [7]. The scattered fields are calculated using the COMSOL Multiphysics finite-element solver. By integrating the scattered field components in all directions, the RCS figure of merit can be deduced and the results are presented in figure 5. With respect to the RCS of 100% for a metal rod, it can be seen that introducing a non-dispersive cloak reduces the RCS to 30%. This reduction is substantial but is still far from the invisibility criterion of an ideal cloak. However, for a BST cloak, namely when we introduce dispersive effective parameters, this partial transparency exhibits a narrow band, which was estimated here to be of the order of 10%. The fact that the frequency dependence of the effective permeability exhibits a Lorentz-type behaviour could explain such a narrow band, while a non-resonant
feature obtained in a Drude-like dispersion characteristic would appear more favourable in terms of bandwidth. However, with respect to these simple arguments, it has to be emphasized that we are far from the frequency range in which the metamaterial shows strong anomalous dispersion. Indeed, in this range the material exhibits considerable absorption close to the resonant frequency which counteracts the purpose of transparency.

In any case, the losses impact the performance of the cloak in terms of RCS reduction and bandwidth. In order to have a deeper insight into this issue, we have also plotted in figure 5 the frequency dependence of the RCS for the lossless case. It seems that the accuracy of the calculations is a little bit degraded probably due to a poor conditioning of the numerical system but the trends can be analysed. In the vicinity of the cloaking frequency, the frequency behaviour between the two cases is similar except for a narrowing of the bandwidth. As expected, the losses reduce the signature of the object with a concomitant broadening of the cloaking characteristics. As a consequence, there is a trade-off between the bandwidth and efficiency of the cloak, which can be monitored by a reasonable degree of loss notably when partial invisibility is sufficient. Outside the cloaking band, the difference between the two cases is significant, particularly in the lower part of the spectrum, where the losses are pronounced.

Let us now consider an oversized cloak with the following parameter characteristics: $\varepsilon_r = 4$, $\varepsilon_\theta = 1$ and $\mu_r(r) = (1 - a/r)^2$. In order to maintain a reasonable number of layers, the continuous variations of the reduced parameters were sampled into 20 shells for an inner radius of 3 mm, which has to be compared with the wavelength at the operating frequency, namely $\sim 0.6$ THz ($a/\lambda_0 = 6$). Figure 6 illustrates the scattering patterns by the plot of the $E$-field component perpendicular to the 2D cloak ($E_z$) for a dispersionless situation (reduced cloak) and when the dispersion of the BST material is taken into account. The corresponding frequency dependence of the normalized RCS with respect to the bare metal rod is also plotted. The fact that the reduced equation permits us to lower the radar signature of a cloaked perfect electric conductor is confirmed here with a reduction of the RCS down to 0.3–0.25. When the

**Figure 6.** Variation versus frequency of the RCS for a metal rod (red), a reduced cloak (green) and a BST oversized cloak (red).
dispersion of the parameter is introduced in the simulation of this large cloak, it can be seen that the cloaking bandwidth is very narrow with a value close to 1%. Therefore, very close to the vicinity of partial transparency frequency the efficiency of the cloak is rapidly degraded. This very high sensitivity of the bandwidth to the size of the object located in the isolated region of space was confirmed by systematic simulations. It is not a specificity of a BST cloak provided that the dispersion characteristics, which are primarily responsible for the narrowing of the band, are comparable.

4. Transformation optics for a flat hyperlens

The transformation procedure used to design a cloaking device is quite general [3]. In this section, we will use this technique to magnify near-field patterns. Such a function is performed by a class of devices termed hyperlenses [12, 13]. The main idea is to channel and magnify any near-field pattern in order to bring its fine details, originally carried by evanescent waves only, over the Abbe limit. Consequently, at the output of the hyperlens, those details are converted into propagating waves, which can then be probed in the far field by a conventional imaging system.

The already existing hyperlens devices are based on a channelling effect, which can be observed in highly anisotropic metamaterials when their transverse permittivity (for transverse magnetic (TM) polarization) is positive and very close to zero. Such effective parameters can be obtained with nanostructures that alternate metallic and dielectric layers. The magnification effect is then obtained by means of a cylindrical conformation of the layers. In other words, this hyperlens maps the near-field patterns of a small inner cylinder onto a large outer one. The magnification coefficient is simply given by the ratio of the radii. One of the drawbacks of such a structure is that both interfaces are cylinders and, therefore, introduce an important limitation in the lateral extent of the input patterns. In the following, starting from the flat channelling device, we will add the magnification effect by means of transformation optics while keeping both interfaces flat. The main difference between the flat lens proposed in the present paper and that of the work described in [8] stems from the input boundary, which is a flat surface in our case, whereas it is a cylinder in [8]. The flat surface permits one to greatly simplify the interfacing of the lens with the object to be imaged.

Let us consider a flat channelling device in the $xy$-plane; $z$ is the direction of the magnetic field (TM polarization), whereas $y$ is the main propagation direction. Consequently, we have to choose a very low value for $\varepsilon_{xx}$. In the following, we will use the lossless parameters $(\varepsilon_{xx}, \varepsilon_{yy}, \mu_{zz}) = (0.001, 2.5, 1)$. The device is positioned between $y = a$ and $y = b$. The magnification factor is $t$. The coordinate transformation used in the procedure is as follows:

$$
\begin{align*}
    x' &= \left[\frac{y - a}{b - a}\right] (t - 1) + 1 \right] x, \\
    y' &= y, \\
    z' &= z.
\end{align*}
$$

This transformation, similar to equations (17)–(19) in [9], is linear with respect to $x$ and $y$. On the input interface ($y = a$), we have an identity between the original and the transformed coordinates. On the output interface ($y = b$), we ‘stretch’ the coordinates along the $x$-direction by a factor of $t(x' = tx)$. It should be noted that such a transformation introduces an
‘optical axis’ since the translation along $x$ is proportional to the distance to $x = 0$ at the input. However, at this point, it is not necessary to limit the device along the $x$-direction.

By following the procedure presented in [4] for this linear transformation, we obtain the local expressions for both the $2 \times 2$ permittivity tensor $\varepsilon$ and the out-of-plane permeability (in our 2D TM case) of the device equivalent to the transformed space. These expressions only depend on the position, the geometrical parameters ($a, b, t$) and the original material parameters ($\varepsilon_{xx}, \varepsilon_{yy}, \mu_{zz}$). They are plotted in figure 7 for $a = 1$ mm, $b = 5$ mm and $t = 5$, as an example.

The next step is to find out whether we can avoid the off-diagonal term $\varepsilon_{xy}$ in the permittivity tensor, which would be the most difficult to implement. This can be done by a proper local rotation $R(x, y)$ of angle $\theta(x, y)$ of the global Cartesian coordinate system $(x, y, z)$. We solve:

$$
\begin{pmatrix}
\varepsilon_{ii} & 0 \\
0 & \varepsilon_{jj}
\end{pmatrix}
= R \bar{\varepsilon} R^{-1} = R
\begin{pmatrix}
\varepsilon_{11} & \varepsilon_{12} \\
\varepsilon_{21} & \varepsilon_{22}
\end{pmatrix}
R^{-1}.
$$

The full expression for the local rotation is

$$
\theta = \frac{1}{2} \tan^{-1}\left(\frac{2\varepsilon_{yy}(b-a)^2(t-1)x \cdot f(y)}{\varepsilon_{xx} f(y)^4 + \varepsilon_{yy}(b-a)^2((t-1)^2x^2 - f(y)^2)}\right)
$$

with $f(y) = (b - at) + (t - 1)y$.

The $\theta(x, y)$, $\varepsilon_{ii}(x, y)$ and $\varepsilon_{jj}(x, y)$ functions are plotted in figure 8 for the same values of $(a, b, t)$ as before. The permeability distribution is unchanged. On the $\theta$ distribution, the existence of a central ‘optical axis’ can be clearly seen. $\varepsilon_{ii}, \varepsilon_{jj}$ and $\mu_{zz}$ show significant gradients over the surface of the lens. However, $\varepsilon_{ii}$ and $\varepsilon_{jj}$ are of the same order of magnitude as $\varepsilon_{xx}$ (positive and close to 0) and $\varepsilon_{yy}$ (greater than 1), respectively. Similarly, $\mu_{zz}$ is comparable with 1. It should be noted that $\varepsilon_{jj}$ diverges for large values of $x$, particularly when $y \to a$, which limits the lateral extent of the input pattern.

We introduced these local parameters as material parameters for a slab into a frequency-domain 2D finite element method solver (namely the commercial software COMSOL).
Figure 8. (a) Spatial distribution of the rotation angle of the local coordinate system. The red arrows show the direction of the local $j$ vector, for a discrete set of points. However, the angle is defined at every point inside the homogeneous slab. (b)–(d) Full set of effective parameters inside the flat hyperlens in the local rotated coordinate system $(i, j, z)$.

The input near-field pattern is simply two very narrow ($\lambda_0/60$) sources of magnetic field centred at $+\lambda_0/20$ and $-\lambda_0/20$ and positioned directly at the bottom interface of the lens. The rest of the bottom boundary is a perfect magnetic conductor. Above the lens is placed an air layer. The top and side boundaries are perfectly matched layers. The wavelength in air was chosen to be 7.5 times larger than the thickness of the lens.

Figure 9 shows the resulting magnetic field map (a) and the plot of the same field on the input and output boundaries of the lens (b). The distance between the two peaks is multiplied by $t$ (5 here) thanks to the lens. It should be noted that a significant spreading of the peaks can also be observed.

This set of parameters could be implemented by means of individual particles with controlled values of permittivity along two orthogonal directions and a controlled value of permeability along the third. Moreover, each one of them would have to be individually oriented. The structures presented in the first sections of this paper suggest that the realization of such particles is not completely unrealistic. However, the actual importance of the calculated constraints should be checked.

Since we have noted the order of magnitude of the calculated perfect parameters, we can check the effect of dropping all gradients on the local effective parameters in the rotated coordinate system. In other words, we define $\varepsilon_{ii} = 0.001$, $\varepsilon_{jj} = 2.5$ and $\mu_{zz} = 1$ at every point inside the device, with the same $\theta(x, y)$ function as before.

Figures 9(c) and (d) show the same map and plot of the magnetic field as before but with those simplified material parameters. The magnification effect is still present and the distance...
between the peaks is unchanged. This means that when a channelling slab is used, the proper orientation of the anisotropy dominates over its specific values in the transformation procedure.

Additionally, it is possible to explore other $\theta(x, y)$ distributions that could perform different functions. Equation (4) (the variables $a$ and $b$ have the same meaning as before and all $c_i$ are constants), for example, allows point-like sources at the input to be transformed into very large line sources at the output (figures 10(a) and (b)). Such a function could be useful in the field of antennas. Equation (5) leads to a concentration of the field along the transverse direction (figure 11), which could be interesting in detection systems to focus energy onto a probe.

\[
\theta(x, y) = \text{sign}(x) \left[ \tan^{-1} \left( -c_1(|x| + c_2) \frac{b - a}{b - y} \right) \right], \quad (4)
\]

\[
\theta(x, y) = \tan^{-1} \left( \frac{x \left(-y^2 + (a + b)y - ab\right)}{(a - b)^2} \right). \quad (5)
\]

In summary, in order to perform a given operation on the field patterns, one can use either a $\theta(x, y)$ function given by a full transformation procedure or simply search for a suitable function.
Figure 10. (a) Orientation of vector $j$ for the local rotation given by equation (4) (red arrows). The black curves are the limits of the layers normal to $j$ that can be used to implement the required anisotropy. (b) The magnetic field inside the lens for the homogeneous case (b) and the microstructured one (c), using the layer configuration of (a).

directly. The main point to be considered when using the second method is that the propagation direction inside the lens is dominated by the local $j$ vector. It should be noted that this can lead us back to the cylindrical hyperlens already mentioned: we just have to choose two cylindrical interfaces and then a cylindrical $\theta(x, y)$ function is the most straightforward choice to map the field from the inner to the outer interface [14, 15].

At this point, we have only used homogeneous simulations to check the validity of our expressions. However, it is possible to implement a microstructure into our FEM software. In other words, we will replace the locally rotated effective parameters by a metamaterial structure that only uses simple materials arranged into a suitable geometry. Following the previous works on hyperlenses, we can try with a stack of layers of two different materials (of permittivities $\varepsilon_1$ and $\varepsilon_2$). When the layers are of equal thickness, very thin with respect to the wavelength and most importantly normal to the $j$-direction, the effective parameters are directly given by equation (6). For example, in order to obtain $\varepsilon_{ii} = 0.001$ and $\varepsilon_{jj} = 2.5$, we will use layers with permittivities of $\varepsilon_1 = 0.001 - 0.05i$ and $\varepsilon_2 = 0.001 + 0.05i$ (the gain comes from the constraint of zero loss for the effective parameters).

$$
\begin{align*}
\varepsilon_{ii} & = \frac{\varepsilon_1 + \varepsilon_2}{2}, \\
\varepsilon_{jj} & = \frac{2\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}.
\end{align*}
$$

All the possible layer boundaries form a family of curves that can be found by solving the equation of orthogonality with $j(x, y)$. Among these curves, we can choose any subset that corresponds to reasonable thicknesses with respect to the homogenization conditions. Figure 10(a) shows such a set of curves for the lens that follows equation (4). It can be seen that the condition of equal thickness is not globally respected. However, since the gradient on
the thickness is small, we can reasonably consider that we have locally neighbouring layers of approximately equal thickness. The relevance of this assumption is confirmed by the simulation result for the microstructured slab presented in figure 10(c).

Finally, it should be noted that most of the calculations were performed for lossless cases. The introduction of losses can significantly deteriorate the performance of hyperlenses. However, the deterioration is strongly dependent on the length of the path inside the device. For instance, the deterioration is very strong when we follow equation (4) (figure 10 shows a lossless case only), whereas it is reasonable for the field concentrator with equation (5) (figure 11 shows a lossy case).

5. Conclusion

A full dielectric approach taking advantage of the recent advances in ferroelectric technology seems suitable for fabricating a partial transparency cloak at terahertz frequency. The underlying physics principles that permit one to tailor the effective permeability values are similar to those used for an SRR cloak operating at microwave frequency. As a consequence, a high-κ cloak also suffers from the same frequency limitations by showing comparable Lorentz-type dispersion characteristics. However, the technological challenges are dramatically relaxed when the upper part of the electromagnetic spectrum is targeted. The concept of magnetic Mie resonance

Figure 11. (a) Orientation of vector $j$ for the local rotation given by equation (5) (red arrows). Magnetic field inside the lens and in the air above (c) for the homogeneous case. (b) Corresponding field plot on both interfaces of the lens, showing concentration of the field along the transverse direction. Unlike the previous cases, this simulation includes losses (loss tangent of 21 and 0.03 for $\varepsilon_{ii}$ and $\varepsilon_{jj}$, respectively).
resulting from the light confinement is quite general and could be extended in principle to optics where metal nanowire arrays were proposed to satisfy the localization [20]. Beyond the search for invisibility, which is an ultimate goal in the control of light, other applications could be addressed through transformation optics such as lenses. In particular, we demonstrated that hyperlens or high focusing devices can be designed by this procedure having in mind the technological constraints and notably flat interfaces. Such a condition is a requirement in planar deposition techniques such as those used in full semiconductor technology [21].

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References

[1] Joannopoulos J D, Meade R D and Winn J N 1995 Photonic Crystals: Molding the Flow of Light (Princeton, NJ: Princeton University Press)
[2]Leonhardt U 2006 Science 312 1777–80
[3]Pendry J B, Shurig D and Smith D R 2006 Science 312 1780–82
[4]Schurig D, Pendry J B and Smith D R 2006 Opt. Express 14 9794–804
[5]Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 Science 314 977–80
[6]Cai W, Chettiar U K, Kildishev A V and Shalaev V M 2007 Nat. Photonics 1 224–7
[7]Cummer S A, Popa B I, Schurig D, Smith D R and Pendry J 2006 Phys. Rev. E 74 36621
[8]Kildishev A V and Shalaev V M 2008 Opt. Lett. 33 43
[9]Rahm M, Roberts D A, Pendry J B and Smith D R 2008 Opt. Express 16 11555
[10]Rahm M, Shurig D, Roberts D A, Cummer S A, Smith D R and Prendy J B 2008 Photon. Nanostruct. Fundam. Appl. 6 87
[11]Gaillot D P, Croënne C and Lippens D 2008 Opt. Express 16 3986
[12]Salandrino A and Engheta N 2006 Phys. Rev. B 74 075103
[13]Jacob Z, Alekseyev I V and Narimanov E 2006 Opt. Express 14 8247
[14]Liu Z, Lee H, Xiong Y, Sun C and Zhang X 2007 Science 315 686
[15]Lee H, Liu Z, Xiong Y, Sun C and Zhang X 2007 Opt. Express 15 15886
[16]O’Brien S and Pendry J B 2002 J. Phys.: Condens. Matter 14 4035
[17]Zhao Q, Du B, Kang L, Zhao H, Xie Q, Li B, Zhang X, Zhou J, Li L and Meng Y 2008 Appl. Phys. Lett. 92 051106
[18]Acher O, Lerat J-M and Malléjac N 2007 Opt. Express 15 1096–106
[19]Smith D R and Pendry J B 2006 J. Opt. Soc. Am. B 23 391–403
[20]Wu Qi and Park W 2008 Appl. Phys. Lett. 92 153114
[21]Hoffman J, Alekseyev L, Howard S S, Franz K J, Wasserman D, Podolskiy V, Narimanov E E, Sivco D L and Gmachl C 2007 Nat. Lett. 6 946–50