Recent Development of CDCC

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This paper shows a brief review on CDCC and the microscopic reaction theory as a fundamental theory of CDCC. The Kerman-McManus-Thaler theory for nucleon-nucleus scattering is extended to nucleus-nucleus scattering. New development of four-body CDCC is presented. An accurate method of treating inclusive reactions is presented as an extension of CDCC and the Glauber model.

§1. Introduction

The construction of microscopic reaction theory is one of the most important subjects in nuclear physics. It is a goal of the nuclear reaction theory. Furthermore, the construction is essential for many applications. Particularly for the scattering of unstable nuclei, there is no reliable phenomenological optical potential, since measurements of the elastic scattering are not easy. An important theoretical tool of analyzing inclusive reactions is the Glauber model.\(^1\) The theoretical foundation of the model is shown in Ref. 2. The model is based on the eikonal and the adiabatic approximation. It is well known that the adiabatic approximation makes the removal cross section diverge when the Coulomb interaction is included. The Glauber model has thus been applied mainly for lighter targets in which the Coulomb interaction is negligible; see for example Refs. 3–9) and Refs. 10, 11) for Coulomb corrections to the Glauber model.

Meanwhile, the method of continuum discretized coupled channels (CDCC)\(^12,13\) is an accurate method of treating exclusive reactions such as the elastic scattering and the elastic breakup reaction in which the target is not excited. The theoretical foundation of CDCC is shown in Refs. 14–16). Actually, CDCC has succeeded in reproducing data on the scattering of not only stable nuclei but also unstable nuclei; see for example Refs. 17–28) and references therein. The dynamical eikonal approximation\(^29\) is also an accurate method of treating exclusive reactions at intermediate and high incident energies where the eikonal approximation is reliable. The nucleon removal reaction is composed of the exclusive elastic-breakup component and the inclusive nucleon-stripping component. CDCC and the dynamical eikonal approximation can evaluate the elastic-breakup cross section, but not the stripping cross section.

The experimental exploration of halo nuclei is moving from lighter nuclei such as He and C isotopes to relatively heavier nuclei such as Ne isotopes. Very recently, Takechi et al. measured the interaction cross section \(\sigma_I\) for the scattering of \(^{28–32}\)Ne at 240 MeV/nucleon and found that \(\sigma_I\) is quite large particularly for \(^{31}\)Ne.\(^30\) A halo structure of \(^{31}\)Ne was reported with the experiment on the one-neutron removal reaction.\(^31\) This is the heaviest halo nucleus in the present stage suggested experien-
mentally and resides in the “Island-of-inversion” region. Determining the spin-parity of $^{31}\text{Ne}$ is essential to understand the nature of “Island of inversion”.

This paper shows a brief review on recent development of CDCC and the microscopic reaction theory that yields the foundation of CDCC. We present the microscopic reaction theory in §2 and new development of four-body CDCC in §3. We finally propose an accurate method of treating inclusive reactions as an extension of CDCC and the Glauber model in §4.

§2. Microscopic reaction theory

In this section, we present a microscopic reaction theory for nucleus-nucleus scattering. This is an extension of the Kerman-McManus-Thaler formalism\(^{32}\) of the multiple scattering theory\(^{33}\) for nucleon-nucleus scattering to nucleus-nucleus scattering. In principle this reaction theory is applicable for many cases, but we consider the simple case in which the projectile breakup is weak, because it is not easy to perform fully-microscopic calculations including the projectile breakup. In the case, the theory is reduced to the double-folding model with the effective nucleon-nucleon (NN) interaction. The double-folding model is applied to nucleus-nucleus scattering at intermediate energies, particularly the scattering of Ne isotopes. The breakup effect is estimated by reducing the microscopic model to a three-body model and solving the three-body model with CDCC. This section is a brief review ofRefs. 34)–36).

2.1. Model building

Let us consider the scattering of projectile (P) on target (T). The most fundamental equation for this case is the many-body Schrödinger equation with the realistic nucleon-nucleon (NN) interaction \(v_{ij}\). The multiple scattering theory\(^{32},^{33}\) for nucleon-nucleus scattering was extended to nucleus-nucleus scattering.\(^2\) According to the theory, the many-body Schrödinger equation is approximated into

\[
\left( K + h_P + h_T + \sum_{i \in P, j \in T} \tau_{ij} - E \right) \hat{\Psi}^{(+)} = 0 ,
\]

where \(E\) is an energy of the total system, \(K\) is a kinetic energy of the relative motion between P and T, and \(h_P\) (\(h_T\)) is an internal Hamiltonian of P (T). Here \(\tau_{ij}\) is the effective NN interaction in nuclear medium. The Brueckner \(g\)-matrix has commonly been used as \(\tau\) in many applications; see for example Refs. 37)–45). The \(g\)-matrix interaction includes nuclear-medium effects, but not the effect of collective excitations induced by surface vibration and rotation of finite nucleus, since the interaction is evaluated in nuclear matter. The effect of collective excitations is small for intermediate energy scattering, as shown later.

The Glauber model is based on the eikonal approximation for NN scattering and the eikonal and adiabatic approximations for nucleus-nucleus scattering. The condition for the eikonal approximation to be good is

\[
|v_{NN}(r_{NN})/E_{NN}| \ll 1 , \quad k_{NN}a \gg 1 ,
\]

where \(v_{NN}\) is the NN interaction and \(E_{NN}\) is the NN energy.
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Fig. 1. The on-shell NN scattering amplitude $f_{NN}(q)$ calculated with the bare NN potential AV18 at the laboratory energy $E_{NN} = 300$ MeV in the left panel and with the JLM $g$-matrix\(^{38}\) at the laboratory energy $E_{NN} = 150$ MeV in the right panel. The solid (dashed) and dotted (dash-dotted) lines show, respectively, the real and imaginary parts of $f_{NN}(q)$ of the exact (eikonal) calculation. The left panel is taken from Ref. 2).

where $E_{NN}$ and $k_{NN}$ represent a kinetic energy and a wave number, respectively, of the relative motion between colliding two nucleons, and $a$ is a range of $v_{ij}$. This condition is not well satisfied for the realistic NN potential that has a strong short-ranged repulsive core at small relative distance ($r_{NN}$) between two nucleons; for example, $v_{ij} \sim 2000$ MeV at $r_{NN} = 0$ for AV18.\(^{46}\) Actually, the eikonal approximation is not good for NN scattering due to $v_{ij}$, as shown in the left panel of Fig. 1. To avoid this problem, a slowly-varying function such as the Gaussian form is used in the profile function of the Glauber model.\(^{47}\) This procedure is justified in the following.

In general, the $g$-matrix has much milder $r$ dependence than the bare NN potential $v_{ij}$.\(^2\) For example, the JLM $g$-matrix\(^{38}\) keeps this property. This means that the $g$-matrix is more suitable than $v_{ij}$ as an input of the Glauber model. Actually, the eikonal approximation is quite good for NN scattering due to the JLM $g$-matrix, as shown in the right panel of Fig. 1. The Glauber model\(^1\) is then more applicable to Eq. (2.1) than the original many-body Schrödinger equation with $v_{ij}$. In this case, the input of the Glauber model is not the profile function proposed in Ref. 47) but the $g$-matrix.\(^2\) At higher incident energies where the Glauber model is used, the $g$-matrix is reduced to the $t$-matrix that has no medium effect. The $t$-matrix also has weak $r_{NN}$ dependence.\(^2\) This fact justifies the usage of the profile function with the Gaussian form.

2.2. Application of double-folding model to reaction cross sections for Ne isotopes

We analyze the scattering of Ne isotopes from a $^{12}$C target at 240 MeV/nucleon. In the scattering the projectile breakup is weak, since the target is light and $E$ is large; see §2.3 for the point. The double-folding model becomes reliable in this situation. In the model, the potential $U$ between P and T consists of the direct and exchange parts.\(^{48},^{49}\) The exchange part is non-local, but it can be localized with
Fig. 2. Reaction cross sections for the scattering of Ne isotopes on $^{12}$C at 240 MeV/nucleon. The experimental data for $A = 28 - 32$ are taken from Ref. 30). The data for $^{20}$Ne is deduced from measured $\sigma_I$ at around 1 GeV/nucleon$^{57}$ with the Glauber model.$^{30}$ In the left panel, the solid (dotted) line represents the results of AMD (spherical Gogny-HF). A closed square is the result of AMD with the tail and breakup corrections. In the right panel, the dotted line represents the results of the DWS model, while the solid line corresponds to the results of AMD.

the local semi-classical approximation$^{39}$ in which $P$ is assumed to propagate as a plane wave with the local momentum within a short range of the NN interaction. The validity of this localization is shown in Ref. 50). As the $g$-matrix interaction we take the Melbourne interaction$^{44),51}$ that is constructed from the Bonn-B NN potential.$^{52}$ The projectile densities are constructed by either (I) by antisymmetrized molecular dynamics (AMD)$^{53}$ with the Gogny D1S interaction$^{54),55}$ or by (II) the deformed Woods-Saxon (DWS) model$^{36}$ with the deformation evaluated by AMD. Model I has no adjustable parameter, but the density is inaccurate in the asymptotic region. Model II provides the density with the proper asymptotic form, but the model includes potential parameters. As the potential parameter set, we use the parameter set recently proposed by Wyss.$^{56}$ This set is intended to reproduce spectroscopic properties of high-spin states from light to heavy deformed nuclei, e.g., the quadrupole moment and the moment of inertia, and at the same time the root mean square (RMS) radius crucial for the present analysis.

Figure 2 represents the total reaction cross section $\sigma_R$ for the scattering of Ne isotopes on a $^{12}$C target at 240 MeV/nucleon. As shown in the left panel, the AMD calculations (solid line) succeed in reproducing the data,$^{30}$ whereas the spherical Gogny Hartree-Fock (HF) calculation (dotted line) undershoots the data; note that the spherical Gogny-HFB calculation yields the same result as the spherical Gogny-HF calculation within the thickness of line. The nuclei with $A > 30$ are unbound in these spherical calculations. The enhancement from the dotted line to the solid line comes from the deformation of the ground state, since the deformation is a main difference between the two calculations. The deformation increases the $\sigma_R$ by
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at most 5%. The AMD results are consistent with all the data except $^{31}\text{Ne}$. The underestimation of the AMD result for $^{31}\text{Ne}$ comes from the inaccuracy of the AMD density in its tail region.

The tail problem is solved by the following resonating group method (RGM). In principle the ground state $\Phi(31\text{Ne}; 3/2^-)$ of $^{31}\text{Ne}$ can be expanded in terms of the ground and excited states $\Phi(30\text{Ne}; J^\pi_n)$ of $^{30}\text{Ne}$. This means that the ground state of $^{31}\text{Ne}$ is described by the $^{30}\text{Ne}+n$ cluster model with $^{30}\text{Ne}$ excitations. The cluster-model calculation can be done with RGM in which the ground and excited states of $^{30}\text{Ne}$ are constructed by AMD. Here the wave function of $^{30}\text{Ne}$ includes many excited states with positive- and negative-parity below 10 MeV in excitation energy. This AMD+RGM calculation is quite time consuming, but it is done for $^{31}\text{Ne}$. The tail correction to $\sigma_R$ is 35 mb that corresponds to 2.5% of $\sigma_R$. The reaction cross section with the tail correction (a square symbol) well reproduces the experimental data with no adjustable parameter. Thus $^{31}\text{Ne}$ is a halo nucleus with large deformation.

The DWS model well simulates the result of AMD+RGM for $^{31}\text{Ne}$, as shown in the right panel of Fig. 2. This may suggest that the DWS model is a handy way of simulating the AMD calculation with the tail correction. The difference between the AMD model and the DWS model for $^{28}$–$^{32}\text{Ne}$ may show tail corrections to the AMD results.

The same analysis is made for the scattering of $\text{Ne}$ isotopes on a $^{28}\text{Si}$ target at 38–60 MeV/nucleon. As shown in Fig. 3, the deformation effect is significant also for the lower incident energies, and consequently, the double-folding model yields better agreement with the experimental data, where a normalization factor is multiplied in the theoretical results so as to reproduce measured $\sigma_R$ of $^{12}\text{C}+^{12}\text{C}$ scattering at 38–60 MeV/nucleon.

2.3. Breakup, dynamical deformation and reorientation effects

For a weakly bound system such as $^{31}\text{Ne}$, the projectile breakup effect is not perfectly negligible. This effect is estimated by assuming the two-body model for

![Fig. 3. Reaction cross sections for the scattering of $\text{Ne}$ isotopes on a $^{28}\text{Si}$ target at 38–60 MeV/nucleon. The experimental data are taken from Ref. 58). See Fig. 2 for the definition of lines.](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTPS.196.87/1892904)

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the \(^{30}\)Ne+n system and solving the three-body dynamics of the \(^{30}\)Ne+n+\(^{12}\)C system with CDCC. Here the potential between \(^{30}\)Ne and \(^{12}\)C and that between n and \(^{12}\)C are constructed with the double-folding model with the Melbourne \(g\)-matrix, and the potential between \(^{30}\)Ne and n is made with the well-depth method; see Ref. 59) for the potential parameters. The correction is 10 mb corresponding to 0.7\% of \(\sigma_R\).

When the projectile is deformed in the intrinsic frame, the deformation enlarges the radius of the projectile density in the space-fixed frame and eventually enhances the reaction cross section. This static deformation effect has already been included in the double-folding model by making the angular momentum projection. Another effect is the dynamical deformation effect, i.e., the effect of rotational motion of deformed projectile during the scattering. This effect on \(\sigma_R\) is small for intermediate-energy nucleus-nucleus scattering.34) This has been confirmed with the adiabatic approximation to the rotational motion of projectile and the eikonal approximation to the relative motion between projectile and target.34) In this subsection, the effect is estimated with no approximation. For this purpose, we consider the scattering of \(^{30}\)Ne from \(^{12}\)C at 240 MeV/nucleon and do coupled-channel calculations between the \(^0\)\(^+\) ground state and the first \(^2\)\(^+\) state of \(^{30}\)Ne. The projectile density is calculated by the DWS model with the deformation evaluated by AMD. The coupling potentials in the coupled-channel calculations are obtained by the so-called single-folding model. Namely, the nucleon-\(^{12}\)C potential is first evaluated by folding the Melbourne-\(g\)-matrix interaction with the target density and the coupling potentials are obtained by folding the nucleon-\(^{12}\)C potential with the projectile transition densities.

In the single-channel calculation with no dynamical deformation effect, the resultant reaction cross section is 1469 mb. This result overestimates the corresponding result of the double-folding model by about 10\%, which is accurate enough for the present test. In the coupled-channel calculation, the resulting reaction cross section is 1468 mb. Thus the dynamical rotation effect on the reaction cross section is estimated as less than 0.1\%. The reason why the effect is small for intermediate-energy nucleus-nucleus scattering is shown in Ref. 34). The integrated inelastic cross section to the first \(^2\)\(^+\) state is 2.9 mb. This is 0.2\% of \(\sigma_R\), indicating that \(\sigma_I \approx \sigma_R\).

The folding potential \(U\) is not spherical in general, when the spin of projectile is not zero. This reorientation effect is also tested by the coupled-channel calculation for the scattering of \(^{31}\)Ne(\(3/2^-\)) from \(^{12}\)C at 240 MeV/nucleon, where the single-folding model is used. The resultant reaction cross section is 1512 mb, whereas the corresponding cross section is 1515 mb when the non-spherical part of \(U\) is switched off. The reorientation effect is 0.2\% and hence negligible for intermediate-energy nucleus-nucleus scattering.

2.4. Summary on microscopic reaction theory

We have constructed a microscopic reaction theory for nucleus-nucleus scattering, using the multiple scattering theory. This is an extension of the Kerman-McManus-Thaler theory for nucleon-nucleus scattering. In the theory, nucleus-nucleus scattering is described by a multiple scattering series due to the \(g\)-matrix NN interaction instead of the realistic one. The former has much milder \(r\)-dependence than the latter and therefore the Glauber model is applicable for this theory with
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high accuracy.
In the nucleus-nucleus scattering from lighter targets at intermediate energies, the breakup effect is small and hence the double-folding model becomes reliable. In this situation, we can use the double-folding model in which projectile and target densities are constructed with fully microscopic structure theories such as AMD, HF and HFB. These fully microscopic theories have been applied to measured reaction cross sections for Ne isotopes. In the Island-of-inversion region, the nuclei are strongly deformed. In particular, $^{31}$Ne is a halo nucleus with strong deformation. The dynamical deformation effect and the reorientation effect are also found to be small, so that the interaction cross section is identical with the reaction cross section with high accuracy.

§3. Four-body CDCC

In this section we present a method of treating four-body scattering in which the projectile breaks up into three constituents. This method is called four-body CDCC. In this method, the three-body continuum states of projectile are discretized by diagonalizing the internal Hamiltonian of projectile with the Gaussian basis functions. This discretization is called the pseudo-state method. Recently, we proposed a novel method of calculating continuous breakup cross sections with the complex-scaling method.\cite{60,61} This section is a brief review of Refs. 22, 23, 28).

3.1. Formulation

We consider the scattering of $^6$He as a typical example of four-body breakup reactions. Here we focus our discussion on the differential breakup cross section. The scattering of $^6$He on a target $A$ is described by the four-body Schrödinger equation

$$[H - E_{\text{tot}}]|\psi^{(+)}\rangle = 0 \quad (3.1)$$

with the outgoing boundary condition, where the total energy $E_{\text{tot}}$ satisfies $E_{\text{tot}} = E_{\text{in}}^{CM} + \varepsilon_0$ for the corresponding incident energy $E_{\text{in}}^{CM}$ in the center-of-mass system and the ground-state energy $\varepsilon_0$ of $^6$He. The total Hamiltonian $H$ is defined by

$$H = K_R + h_P + U_{nA} + U_{nA} + U_{\alpha A} + V_{\alpha A}^{\text{Coul}} \quad (3.2)$$

with

$$h_P = K_y + K_r + V_{nn} + V_{n\alpha} + V_{\alpha A}, \quad (3.3)$$

where $h_P$ is the internal Hamiltonian of $^6$He. The relative coordinate between $^6$He and $A$ is denoted by $R$ and the internal coordinates of $^6$He are by a set of Jacobi coordinates, $\xi = (y, r)$. Momenta conjugate to $R$ and $(y, r)$ are represented by $P$ and $(p, k)$, respectively. The kinetic energy operator associated with $R$ ($\xi$) is represented by $K_R$ ($K_\xi$), $V_{xx}$ is a nuclear plus Coulomb interaction between $x$ and $x'$, and $U_{xA}$ and $V_{xA}^{\text{Coul}}$ are nuclear and Coulomb potentials between $x$ and $A$, respectively.

In CDCC with the pseudostate discretization method, the scattering is assumed...
to take place in a modelspace: 20)-23)

\[ \mathcal{P} = \sum_{\gamma} |\Phi_{\gamma}\rangle \langle \Phi_{\gamma}|, \]  

(3.4)

where \( \Phi_{\gamma} \) is the \( \gamma \)-th eigenstate obtained by diagonalizing \( h_P \) with \( L^2 \)-type basis functions. The four-body Schrödinger equation is then solved in the modelspace:

\[ \mathcal{P} [H - E_{\text{tot}}] \mathcal{P} |\Psi^{(+)}_{\text{CDCC}}\rangle = 0. \]  

(3.5)

The modelspace assumption has already been justified by the fact that calculated elastic and breakup cross sections converge with respect to extending the modelspace. 20)-23)

The exact \( T \)-matrix element to a breakup state with \((p, k)\) can be described by

\[ T_\varepsilon(p, k, P) = \langle \psi_\varepsilon^{(-)}(p, k) \chi_\varepsilon^{(-)}(P) | U - V_{6\text{He}}^{\text{Coul}} | \Psi^{(+)}_{\text{CDCC}} \rangle, \]  

(3.6)

where \( V_{6\text{He}}^{\text{Coul}} \) is the Coulomb interaction between \( ^6\text{He} \) and \( A \). The final-state wave functions, \( |\psi_\varepsilon^{(-)}(p, k)\rangle \) and \( |\chi_\varepsilon^{(-)}(P)\rangle \), with the incoming boundary condition, are defined by

\[ \left[ T_R + V_{6\text{He}}^{\text{Coul}} - (E_{\text{tot}} - \varepsilon) \right] |\chi_\varepsilon^{(-)}(P)\rangle = 0, \]  

\[ [h_P - \varepsilon] |\psi_\varepsilon^{(-)}(p, k)\rangle = 0, \]  

(3.7)

(3.8)

where \( E_{\text{tot}} - \varepsilon = (hP)^2/(2\mu_R) \) and \( \varepsilon = (hp)^2/(2\mu_y) + (hk)^2/(2\mu_r) \) for reduced masses \( \mu_R \) and \( \mu_\xi \) of coordinates \( \mathbf{R} \) and \( \xi \), respectively. Inserting the approximate complete set Eq. (3.4) into Eq. (3.6), one can get the \( T \)-matrix element with high accuracy, 20)-23)

\[ T_\varepsilon(p, k, P) \approx \sum_{\gamma \neq 0} \langle \psi_\varepsilon^{(-)}(p, k) | \Phi_{\gamma} \rangle T_\gamma \]  

(3.9)

with the CDCC \( T \)-matrix element

\[ T_\gamma = \langle \Phi_{\gamma} | \chi_\varepsilon^{(-)}(P_{\gamma}) | U - V_{6\text{He}}^{\text{Coul}} | \Psi^{(+)}_{\text{CDCC}} \rangle \]  

(3.10)

to the \( \gamma \)-th discrete breakup state \( \Phi_{\gamma} \) with eigenenergy \( \varepsilon_{\gamma} \). Here Eq. (3.9) is derived by replacing \( P \) by \( P_{\gamma} \) in \( \chi_\varepsilon^{(-)}(P) \). The \( T_\gamma \) are obtainable with CDCC, but it is quite hard to calculate the smoothing factor \( \langle \psi_\varepsilon^{(-)}(p, k) | \Phi_{\gamma} \rangle \) directly with either numerical integration 24) or the complex-scaling method.25) Hence, we propose a new way of obtaining the differential cross section with respect to \( \varepsilon \) without calculating the smoothing factor.

Using Eq. (3.9), one can rewrite the differential cross section into

\[ \frac{d^2\sigma}{d\varepsilon d\Omega_P} = \int dp' dk' \delta(\varepsilon' - \varepsilon) |T_\varepsilon(p', k', P')|^2 \approx \frac{1}{\pi} \mathcal{R}(\varepsilon, \Omega_P) \]  

(3.11)
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with the generalized response function

\[ R(\varepsilon, \Omega_P) = \text{Im} \left[ \sum_{\gamma, \gamma' \neq 0} T^*_\gamma \langle \Phi_\gamma | G^{(-)} | \Phi_{\gamma'} \rangle T_{\gamma'} \right], \]  

(3.12)

where \( G^{(-)} = \lim_{\eta \to +0} (\varepsilon - h_P - i\eta)^{-1} \). There is no smoothing factor in Eq. (3.12), as expected. The propagator \( G^{(-)} \) operates only on spatially damping functions \( \Phi_\gamma \). This makes the calculation of \( \langle \Phi_\gamma | G^{(-)} | \Phi_{\gamma'} \rangle \) feasible.

In order to calculate \( \langle \Phi_\gamma | G^{(-)} | \Phi_{\gamma'} \rangle \), we use the complex-scaling method in which the scaling transformation operator \( C(\theta) \) and its inverse are defined by

\[ \langle r, y | C(\theta) | f \rangle = e^{3i\theta} f(re^{i\theta}, ye^{i\theta}), \]  

(3.13)

\[ \langle f | C^{-1}(\theta) | r, y \rangle = \{ e^{-3i\theta} f(re^{-i\theta}, ye^{-i\theta}) \}^*. \]  

(3.14)

Using the operators, one can get

\[ \langle \Phi_\gamma | G^{(-)} | \Phi_{\gamma'} \rangle = \langle \Phi_\gamma | C^{-1}(\theta) G^{(-)}(\theta) C(\theta) | \Phi_{\gamma'} \rangle, \]  

(3.15)

where

\[ G^{(-)}_\theta = \lim_{\eta \to +0} \frac{1}{\varepsilon - h_P^\theta - i\eta}. \]  

(3.16)

with \( h_P^\theta = C(\theta) h_P C^{-1}(\theta) \). When \(-\pi < \theta < 0\), the scaled propagator \( \langle \xi | G^{(-)}_\theta | \xi' \rangle \) is a damping function of \( \xi \) and \( \xi' \); note that \( \theta \) is negative since \( G^{(-)} \) has the incoming boundary condition. The scaled propagator can be expanded with \( L^2 \)-type basis functions with high accuracy:

\[ G^{(-)}_\theta \approx \sum_i \frac{|\phi_i^\theta \rangle \langle \tilde{\phi}_i^\theta |}{\varepsilon - \varepsilon_i^\theta}, \]  

(3.17)

where \( \phi_i^\theta \) is the \( i \)-th eigenstate of \( h_P^\theta \) in a modelspace spanned by \( L^2 \)-type basis functions, \( \langle \tilde{\phi}_i^\theta | h_P^\theta | \phi_i^\theta \rangle = \varepsilon_i^\theta \delta_{ii'} \). Inserting Eq. (3.17) into Eq. (3.12) through Eq. (3.15) leads to a useful form of

\[ \frac{d^2 \sigma}{d\varepsilon d\Omega_P} \approx \frac{1}{\pi} \text{Im} \sum_i \frac{T_i^\theta \tilde{T}_i^\theta}{\varepsilon - \varepsilon_i^\theta}, \]  

(3.18)

with

\[ \tilde{T}_i^\theta \equiv \sum_{\gamma'} \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_{\gamma'} \rangle T_{\gamma'}, \quad T_i^\theta \equiv \sum_{\gamma} T^*_\gamma \langle \Phi_\gamma | C^{-1}(\theta) | \phi_i^\theta \rangle. \]  

(3.19)

This method does not require to calculate the exact three-body continuum states \( \psi^{(-)}_\varepsilon(k, p) \). The convergence of this method is shown in Ref. 28).
3.2. **Differential breakup cross section for $^6\text{He}$ scattering**

In Fig. 4, the calculated breakup cross sections are compared with the experimental data\(^{62}\) on $^6\text{He}+^{12}\text{C}$ and $^6\text{He}+^{208}\text{Pb}$ reactions at 240 MeV/A. In the calculation, we take the same potentials as in Ref. 63) for $n-^{208}\text{Pb}$ and $\alpha-^{208}\text{Pb}$ subsystems. The optical potential for $n-^{12}\text{C}$ subsystem is taken from the global nucleon-nucleus potential,\(^{64}\) while the optical potential for $\alpha-^{12}\text{C}$ subsystem is constructed from the $^{12}\text{C}+^{12}\text{C}$ potential at 200 MeV/A\(^{65}\) by changing the radius parameter from $^{12}\text{C}$ to $\alpha$. Nuclear breakup is dominant for $^6\text{He}+^{12}\text{C}$ scattering at 240 MeV/A, while Coulomb breakup to $1^-$ continuum is dominant for $^6\text{He}+^{208}\text{Pb}$ scattering. For $^{12}\text{C}$ target, the present theoretical result is consistent with the experimental data except for the peak of the $2^+$-resonance around $\varepsilon = 1$ MeV. Similar overestimations are also seen in the results of four-body distorted-wave Born approximation (DWBA).\(^{66}\) For $^{208}\text{Pb}$ target, the present method underestimates the experimental data at $\varepsilon \gtrsim 2$ MeV. A possible origin of this underestimation is that the inelastic breakup reactions are not included in the present calculation. It has been reported in Ref. 66) that the inelastic breakup effect is not negligible and thereby the elastic breakup cross section calculated with four-body DWBA underestimates the data.

3.3. **Summary on four-body CDCC**

We have proposed a new version of CDCC for treating four-body breakup. This method is called four-body CDCC. In the method, the three-body continuum of projectile is treated with the complex-scaling method. The validity of this method is checked for not only the elastic scattering but also the breakup reactions of $^6\text{He}$. Clear convergence with respect to expanding the modelspace is seen in both the elastic and the breakup cross section. Four-body CDCC with the complex-scaling method is indispensable to study properties of unstable nuclei with two-neutron halo.
§4. Eikonal reaction theory

In this section, we present an accurate method of treating neutron removal reactions at intermediate incident energies as an extension of CDCC and the Glauber model. This method is referred to as the eikonal reaction theory. This section is a brief review of Refs. 67), 68).

4.1. Formulation

Let us assume that a projectile (P) consists of a core nucleus (c) and a neutron (n). The scattering of P on a target (T) is then described by the three-body (c+n+T) Schrödinger equation

\[
-\frac{\hbar^2}{2\mu} \nabla^2 + h + U(c, n) - E \Psi = 0 \quad (4.1)
\]

with the interaction

\[
U = U^{(N)}_n(r_n) + U^{(N)}_c(r_c) + U^{(C)}_c(r_c), \quad (4.2)
\]

where \( h = T_r + V(r) \) is the projectile Hamiltonian, \( R = (b, Z) \) stands for the coordinate between P and T, \( r_x \) \((x = n \text{ or } c)\) represents the coordinate between x and A, and \( U^{(N)}_x \) and \( U^{(C)}_c \) are the nuclear and Coulomb parts of the optical potential between x and T, respectively. Solving (4.1) with the eikonal approximation, one can get the S-matrix operator

\[
S = \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger U \hat{O} \right] \quad (4.3)
\]

for the operator

\[
\hat{O} = \frac{1}{\sqrt{\hbar \hat{v}}} e^{i\hat{K} \cdot Z} \quad (4.4)
\]

with the wave-number operator \( \hat{K} = \sqrt{2\mu(E - h)}/\hbar \) and the velocity operator \( \hat{v} = \hbar \hat{K}/\mu \) of the relative motion between P and T, where \( \mathcal{P} \) is the path ordering operator. In the Glauber model, the adiabatic approximation is made as the secondary approximation in which \( h \) is replaced by the ground-state energy \( \epsilon_0 \), and hence \( \hat{O}^\dagger U \hat{O} \) and \( \mathcal{P} \) in (4.3) are reduced to \( U / (\hbar v_0) \) and 1, respectively, where \( v_0 \) is the velocity of P in the ground state relative to T.

The operator \( \hat{O} \) shows internal motions of c and n during the scattering. The movement is small for short-range nuclear interaction, but not for long-range Coulomb interaction. This means that \( U^{(N)}_n \) is commutable with \( \hat{O} \) with high accuracy:

\[
\hat{O}^\dagger U^{(N)}_n \hat{O} \to U^{(N)}_n / (\hbar v_0). \quad (4.5)
\]

For the scattering of \(^{31}\text{Ne}(1p3/2)\) from a \(^{208}\text{Pb}\) target at 240 MeV/nucleon, the error due to the replacement (4.5) is estimated with CDCC; it is 0.2% for the reaction...
cross section $\sigma_R$, 1.9% for the breakup cross section $\sigma_{bu}$, 4.1% for the stripping cross section $\sigma_{str}$. Using this replacement, one can get an important result

$$ S = S_n S_c $$

(4.6)

with

$$ S_n = \exp \left[ -i \frac{\hbar}{\nu_0} \int_{-\infty}^{\infty} dZ U^{(N)}_n \right] $$

(4.7)

and

$$ S_c = \exp \left[ -iP \int_{-\infty}^{\infty} dZ \hat{O} \left( U^{(N)}_c + U^{(C)}_c \right) \hat{O} \right] $$

(4.8)

Thus $S$ can be separated into the neutron part $S_n$ and the core part $S_c$. One can not calculate $S_c$ directly with Eq. (4.8), because it includes the operators $\hat{O}$ and $P$. However, $S_c$ is the solution to the Schrödinger equation

$$ \left[ -\frac{\hbar^2}{2\mu} \nabla^2_R + h + U^{(N)}(r_c) + U^{(C)}(r_c) - E \right] \psi_c = 0 $$

(4.9)

to the Schrödinger equation when the eikonal approximation is made. One can obtain $S_c$ by solving Eq. (4.9) with eikonal-CDCC$^{19}$ in which the eikonal approximation is made in the framework of CDCC. Non-eikonal corrections to $S_c$ can be easily made by solving Eq. (4.9) with CDCC instead of eikonal-CDCC, although it is not necessary for the present intermediate scattering. As mentioned above, $S_n$ is obtained from Eq. (4.7). We can derive several kinds of cross sections with the product form (4.6), following the formulation on the cross sections in the Glauber model.$^{3,4}$

4.2. One-neutron removal cross section for $^{31}$Ne scattering

The eikonal reaction theory is applied to one-neutron removal reactions for $^{31}$Ne$^{+}$C scattering at 230 MeV/nucleon and $^{31}$Ne$^{+}$Pb scattering at 234 MeV/nucleon. Table I presents several kinds of cross sections and the spectroscopic factor $S = \sigma^{exp}/\sigma^{th}$. Thus $S[1p3/2]$ little depends on the target and less than 1, but $S[0f7/2]$ does not satisfy these conditions. In Ref. 31, the Coulomb component of $\sigma_{-n}[^{208}$Pb] for a $^{208}$Pb target is estimated to be 540 mb from the experimental values of $\sigma_{-n}[^{208}$Pb] and $\sigma_{-n}[^{12}$C]. In the eikonal reaction theory, the Coulomb component of $\sigma_{-n}[^{208}$Pb] agrees with $\sigma_{bu}[^{208}$Pb] with good accuracy. The spectroscopic factor evaluated from the Coulomb component is $S' = 540/\sigma_{bu} = 0.675$ for the 1p3/2 orbit and 7.36 for the 0f7/2 orbit. Thus $S'$ is consistent with $S$ only for the 1p3/2 orbit. Hence, we can infer that the major component of the $^{31}$Ne g.s. wave function is $^{30}$Ne($0^+$$\otimes$1p3/2 ($S \sim 0.69$). We adopt in the following this configuration.

The potential $V$ between c and n is not well known. Hence, $S$ has a theoretical error coming from the potential ambiguity. The error is often estimated by changing the potential parameters by 30%. When the one-neutron separation energy $B_n$ of $^{31}$Ne is 0.33 MeV, $S = 0.693 \pm 0.133 \pm 0.061$ for $^{12}$C target and 0.682 \pm 0.133 \pm 0.062 for $^{208}$Pb target, where the second and third numbers following the mean value stand for the theoretical and experimental uncertainties, respectively. Thus $S$ includes a sizable theoretical error. This situation completely changes if we look at
Table I. Several kinds of cross sections and the spectroscopic factors for \(^{31}\text{Ne}^+^{12}\text{C}\) scattering at 230 MeV/nucleon and \(^{31}\text{Ne}^+^{208}\text{Pb}\) scattering at 234 MeV/nucleon. The cross sections are presented in units of mb and the data are taken from Ref. 31).

| \(p_{3/2}\) | \(f_{7/2}\) | Exp. | \(p_{3/2}\) | \(f_{7/2}\) | Exp. |
|------------|------------|------|------------|------------|------|
| \(\sigma_R\) | 1572.5 | 1489.9 | | 5518.0 | 4589.5 | |
| \(\sigma_{bu}\) | 23.3 | 3.3 | | 799.5 | 73.0 | (540) |
| \(\sigma_R(-n)\) | 1463.5 | 1458.6 | | 5151.5 | 4524.2 | |
| \(\sigma_{bu}(-n)\) | 4.5 | 1.0 | | 677.2 | 60.5 | |
| \(\sigma_{str}\) | 90 | 29 | | 244 | 53 | |
| \(\sigma_{-n}\) | 114 | 32 | 79 | 1044 | 126 | 712 |
| \(S\) | 0.693 | 2.47 | | 0.682 | 5.65 | |

Table II. Integrated cross sections for two-neutron removal of \(^6\text{He}\). The cross sections are presented in units of mb and the experimental data are taken from Ref. 62).

| \(\sigma_{1n\ str}\) | Calc. | Exp. | | \(\sigma_{1n\ str}\) | Calc. | Exp. |
|----------------|-------|------|---|----------------|-------|------|
| 153.4 | 127 ± 14 | | 353.6 | 320 ± 90 | |
| \(\sigma_{2n\ str}\) | 29.0 | 33 ± 23 | | 148.9 | 180 ± 100 | |
| \(\sigma_{-2n}\) | 198.5 | 190 ± 18 | | 1016.6 | 1150 ± 90 | |

Thus we can clearly see the reliability of the eikonal reaction theory for two-neutron removal reactions on both light and heavy targets.

4.3. The eikonal reaction theory for two-neutron removal

The eikonal reaction theory is applicable for two-neutron removal reactions. Here we show the application for the scattering of \(^6\text{He}\) from \(^{12}\text{C}\) and \(^{208}\text{Pb}\) targets at 240 MeV/nucleon. In this case, the projectile is a three-body system and hence four-body CDCC should be used. The potentials for n-target and \(\alpha\)-target subsystems are calculated by the folding procedure shown in §2.2. The wave functions are obtained by spherical Gogny-HF calculations. Table II shows the integrated cross sections for two-neutron removal of \(^6\text{He}\). Our results are almost consistent with the experimental data.\(^{62}\) Thus we can clearly see the reliability of the eikonal reaction theory for two-neutron removal reactions on both light and heavy targets.

4.4. Summary on the eikonal reaction theory

We have presented an accurate method of treating neutron removal reactions at intermediate energies. In the theory, the nuclear and Coulomb breakup processes are accurately and consistently treated by CDCC without making the adiabatic approximation to the latter, so that the removal cross section calculated with the method never diverges even in the presence of the Coulomb interaction. For lower incident energies where the eikonal approximation is not perfectly accurate, one should make non-eikonal corrections to inclusive cross sections. This can be done
easily by using CDCC instead of eikonal-CDCC. 

$C_{\text{ANC}}$ and $S$ of the last neutron in $^{31}$Ne are evaluated from the measured one-neutron removal reaction. $C_{\text{ANC}}$ has a smaller theoretical error and weaker target-dependence than $S$. Thus, $C_{\text{ANC}}$ is determined more accurately than $S$. When the last neutron of $^{31}$Ne is in the $1p3/2$ orbit, $S < 1$ for $B_n \lesssim 0.6$ MeV, and $S$ and $C_{\text{ANC}}$ have weaker target dependence. When the last neutron is in the $1f7/2$ orbit, meanwhile, $S > 1$ and $S$ and $C_{\text{ANC}}$ have stronger target dependence. These results indicate that the last neutron is mainly in the $1p3/2$ orbit. This means that $^{31}$Ne is deformed. This is consistent with our result in §2.2. The accuracy of the Glauber model is systematically investigated for deuteron scattering at 200 MeV/nucleon; see Ref. 68) for the details.

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