Recent Status of Neutrino Oscillation Study

OSAMU YASUDA
Department of Physics, Tokyo Metropolitan University
Minami-Osawa, Hachioji, Tokyo 192-0397, Japan
E-mail: yasuda@phys.metro-u.ac.jp

I review briefly the recent status of research on neutrino oscillations, such as three and four flavor analysis of atmospheric neutrinos and solar neutrinos, and prospects of present and future long baseline experiments.

1 Introduction

It has been known that the atmospheric neutrino anomaly (See, e.g., Ref. 1) can be accounted for by dominant $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with almost maximal mixing. According to the most up-to-date result of the two flavor analysis of $\nu_\mu \leftrightarrow \nu_\tau$ with 1289 day SuperKamiokande data, the allowed region of the oscillation parameters at 90%CL is $0.88 < \sin^2 2\theta_{\text{atm}} \leq 1$, $1.6 \times 10^{-3} \text{eV}^2 < \Delta m_{\text{atm}}^2 < 4 \times 10^{-3} \text{eV}^2$. Two flavor analysis of $\nu_\mu \leftrightarrow \nu_s$ has been also done by the SuperKamiokande group using the data of neutral current enriched multi-ring events, high energy partially contained events and upward going $\mu$'s, and they have excluded the two flavor oscillation $\nu_\mu \leftrightarrow \nu_s$ at 99%CL.

It has been also shown that the solar neutrino observations (See, e.g., Ref. 4) suggest neutrino oscillations and the most up-to-date analyses tell us that the large mixing angle (LMA) MSW solution ($\Delta m_{\odot}^2 \simeq 2 \times 10^{-5} \text{eV}^2$, $\sin^2 2\theta \simeq 0.8$) gives the best fit and it is followed by the LOW solution ($\Delta m_{\odot}^2 \simeq 1 \times 10^{-7} \text{eV}^2$, $\sin^2 2\theta \simeq 1.0$). The recent SNO data 9 prefer $\nu_e \leftrightarrow \nu_{\text{active}}$ to $\nu_e \leftrightarrow \nu_s$ oscillations.

On the other hand, it has been claimed by the LSND group that their data suggest neutrino oscillations with $\Delta m_{\text{LSND}}^2 \sim O(1) \text{eV}^2$. If this anomaly as well as the atmospheric and solar neutrino data are to be interpreted as evidence of neutrino oscillations then we would need at least four flavors of neutrinos, since the mass squared differences $\Delta m_{\text{atm}}^2$, $\Delta m_{\odot}^2$ and $\Delta m_{\text{LSND}}^2$ suggested by the atmospheric neutrino anomaly, the solar neutrino deficit and the LSND data have different orders of magnitudes.
2 Neutrino oscillations with three flavors

The flavor eigenstates are related to the mass eigenstates by the $3\times 3$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix:

$$
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
$$

and without loss of generality I assume $|\Delta m_{21}^2| < |\Delta m_{32}^2| < |\Delta m_{31}^2|$ where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, $m_j^2 (j = 1, 2, 3)$ are the mass squared for the mass eigenstates, and the different two mass patterns (Fig. 1(a)) and (Fig. 1(b)) are distinguished by the sign of $\Delta m_{32}^2$. With three flavors of neutrinos, it is impossible to account for the solar neutrino deficit, the atmospheric neutrino anomaly and LSND, so I have to give up an effort to explain LSND and I have to take $\Delta m_{32}^2$ and without loss of generality I assume $s_{13}$ so I have a large hierarchy between $\Delta m_{31}^2$ and $\Delta m_{32}^2$. If $|\Delta m_{32}^2 L/4E| < 1$ then from a hierarchical condition I have the oscillation probability

$$
P(\nu_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 (\Delta m_{32}^2 L/4E),
$$

so if $\Delta m_{32}^2 > 1.5 \times 10^{-3} eV^2$ then the CHOOZ reactor data force us to have either $\theta_{13} \approx 0$ or $\theta_{13} \approx \pi/2$. To account for the solar neutrino deficit $|s_{13}|$ cannot be too large, so by combining the atmospheric neutrino data it follows that $|\theta_{13}| < 1$ and the PMNS mixing matrix $U$ becomes

$$
U_{PMNS} \simeq \begin{pmatrix}
c_{\odot} & s_{\odot} & \epsilon \\
-s_{\odot}/\sqrt{2} & c_{\odot}/\sqrt{2} & 1/\sqrt{2} \\
s_{\odot}/\sqrt{2} & -c_{\odot}/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix},
$$

which indicates that the solar neutrino problem is explained by oscillations half of which is $\nu_e \rightarrow \nu_\mu$ and the other of which is $\nu_e \rightarrow \nu_\tau$, and that the atmospheric neutrino anomaly is accounted for by oscillations of almost 100% $\nu_\mu \rightarrow \nu_\tau$ ($I = |\theta_{13}| < 1$). In (L) $\theta_{\odot} = \theta_{\odot}(LMA)$ or $\theta_{\odot}(LOW)$ at 90%CL.

3 Neutrino oscillations with four flavors

In the case of four neutrino schemes there are two distinct types of mass patterns. One is the so-called $(2+2)$-scheme (Fig. 1(c)) and the other is the $(3+1)$-scheme (Fig. 1(d) or (e)). Depending on the type of the two schemes, phenomenology is different.
\[ \begin{align*}
\theta = \theta_{\odot} (\text{LMA}) & \quad \text{or} \quad \theta = \theta_{\odot} (\text{SMA})
\end{align*} \]

where \( \theta_{\odot} \) is either (LMA) or (SMA). The dominant sterile oscillation gives a good fit to the atmospheric neutrino data because

\[ \sin^2 2\theta_{\odot} = \sin^2 2\theta_{\odot} (\text{SMA}) \sim 10^{-3}. \]

With the SNO result the best fit solution is described by

\[ U_{PMNS} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix} \approx \begin{pmatrix}
c_{\odot} & s_{\odot} & \epsilon & \epsilon \\
\epsilon & \epsilon & 1/\sqrt{2} & 1/\sqrt{2} \\
\epsilon & \epsilon & -1/\sqrt{2} & 1/\sqrt{2} \\
-s_{\odot} & c_{\odot} & \epsilon & \epsilon
\end{pmatrix}, \]

where \( |\epsilon| \ll 1 \). In the solution (3) the ratio of active and sterile oscillations in \( \nu_{\odot} \) is 80% and 20%, respectively, while that of active and sterile oscillations in \( \nu_{\text{atm}} \) is approximately 30% and 70% at the maximum of \( \sin^2 (2\Delta m_{\text{atm}}^2 L/4E) \), respectively. The reason that dominant sterile oscillation gives a good fit to the atmospheric neutrino data is because
the disappearance probability can contain a constant term $B$ which serves
as an extra free parameter: 19

\[ 1 - P(\nu_\mu \rightarrow \nu_\mu) = A \sin^2(\Delta m^2_{\text{atm}} L / 4E) + B \sin^2(\Delta m^2_{\text{LSND}} L / 4E) \rightarrow A \sin(\Delta m^2_{\text{atm}} L / 4E) + B / 2, \]

where I have averaged over rapid oscillations. The goodness of fit for the mixing is 67% ($\chi^2 = 73.8$ for 80 degrees of freedom), which is quite good. On the other hand, the mixing with the SMA MSW solution still gives a good fit even with the SNO result (the goodness of fit is 62%, or $\chi^2 = 75.6$ for 80 degrees of freedom), so that pure sterile oscillations in $\nu_\odot$ plus pure active oscillations in $\nu_{\text{atm}}$ is still an acceptable solution in the four flavor framework despite the SNO data. To exclude the $(2+2)$-scheme, therefore, one needs to improve much more statistics and systematics both in the atmospheric and solar neutrino data.

3.2 $(3+1)$-scheme

It has been shown in Refs. 20, 21 using the older LSND result that the $(3+1)$-scheme is inconsistent with the Bugey reactor data 22 and the CDHSW disappearance experiment 23 of $\nu_\mu$. However, in the final result 10 the allowed region has shifted to the lower value of $\sin^2 2\theta$ and it was shown 24 that there are four isolated regions $\Delta m^2_{\text{LSND}} \approx 0.3, 0.9, 1.7, 6.0$ eV$^2$ which satisfy all the constraints of Bugey, CDHSW and the LSND data (99%CL). The case of $\Delta m^2_{\text{LSND}}=0.3$ eV$^2$ is excluded by the SuperKamiokande atmospheric neutrino data. For the other three values of $\Delta m^2_{\text{LSND}}$, the best fit solution looks like

\[
U_{\text{PMNS}} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix} \approx \begin{pmatrix}
c_\odot & s_\odot & 0 & \epsilon \\
s_\odot / \sqrt{2} & c_\odot / \sqrt{2} & \frac{1}{\sqrt{2}} & \delta \\
c_\odot / \sqrt{2} & -c_\odot / \sqrt{2} & \frac{\sqrt{2}}{2} & 0 \\
-s_\odot / \sqrt{2} & -\frac{c_\odot}{\sqrt{2}} & \frac{\sqrt{2}}{2} & 0
\end{pmatrix},
\]

(4)

where $|\epsilon|, |\delta| \ll 1, \theta_\odot = \theta_\odot (\text{LMA})$. Since the off diagonal elements are small, the solution gives almost the same phenomenology as that of the three flavor scenario, so that it is difficult to exclude this scheme by atmospheric or solar neutrino experiments. Hence it will remain a viable scheme until the LSND data are disproved by the new experiment MiniBooNE 25.

4 Long baseline experiments

The only long baseline experiment which is already running is K2K 26 and it is expected to give us more precise value of $\Delta m^2_{\text{atm}}$ than the atmospheric neutrino

\[ \text{In Ref.} \ 19 \ \text{the atmospheric data was analyzed in the (2+2)-scheme with CP phase } \delta_1 \text{ which was ignored in Refs. 17, 18, but the results with } \delta_1 = \pi / 4, \pi / 2 \text{ in Ref. 19 are more or less the same as those with } \delta_1 = 0. \]
observations. The next future long baseline experiments such as MINOS\textsuperscript{27}, OPERA\textsuperscript{28} and JHF\textsuperscript{29} will determine more precisely the values of $\Delta m_{23}^2$ and $\sin^2 2\theta_{atm}$, and possibly measure the value of $\sin^2 2\theta_{13}$ by looking at appearance of $\nu_e$ in $\nu_\mu$ beam:

$$P(\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)) = s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta E_{32}}{\Delta E_{32}^{M(\pm)}} \right)^2 \sin^2 \left( \frac{\Delta E_{32}^{M(\pm)} L}{2} \right),$$

where

$$\Delta E_{32}^{M(\pm)} \equiv \sqrt{\left( \Delta E_{32} \cos 2\theta_{13} \pm \sqrt{2} G_F N_e \right)^2 + \left( \Delta E_{32} \sin 2\theta_{13} \right)^2},$$

$\Delta E_{32} \equiv \Delta m_{32}^2/2E$. $N_e$ stands for the electron density of matter, and $-\sqrt{2} G_F N_e$, $+\sqrt{2} G_F N_e$ stands for the matter effect for neutrinos and for anti-neutrinos, respectively. In further future the second stage JHF experiment with 4MW and/or neutrino factory\textsuperscript{30} may be able to measure the value of $\text{sign}(\Delta m_{32}^2)$, which is crucial to determine the mass pattern of three neutrino schemes, and the value of the CP phase from the difference in the appearance probabilities for neutrinos and for anti-neutrinos. Currently there are a lot of issues which are subjects of active research, such as treatment of the uncertainty of the matter effect in measurements of CP violation, each advantage of high energy or low energy option, correlations of errors, etc. Details of recent developments in long baseline experiments and neutrino factories can be found on the web page of nufact’01\textsuperscript{31}.

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