Bottomonia production and polarization in the NRQCD with $k_T$-factorization. III: $\Upsilon(1S)$ and $\chi_b(1P)$ mesons

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Abstract

The $\Upsilon(1S)$ meson production and polarization at high energies is studied in the framework of the $k_T$-factorization approach. Our consideration is based on the non-relativistic QCD formalism for a bound states formation and off-shell production amplitudes for hard partonic subprocesses. The direct production mechanism, feed-down contributions from radiative $\chi_b(mP)$ decays and contributions from $\Upsilon(3S)$ and $\Upsilon(2S)$ decays are taken into account. The transverse momentum dependent (TMD) gluon densities in a proton were derived from the Ciafaloni-Catani-Fiorani-Marchesini evolution equation and the Kimber-Martin-Ryskin prescription. Treating the non-perturbative color octet transitions in terms of multipole radiation theory, we extract the corresponding non-perturbative matrix elements for $\Upsilon(1S)$ and $\chi_b(1P)$ mesons from a combined fit to transverse momenta distributions measured at various LHC experiments. Then we apply the extracted values to investigate the polarization parameters $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$, which determine the $\Upsilon(1S)$ spin density matrix. Our predictions have a reasonably good agreement with the currently available Tevatron and LHC data within the theoretical and experimental uncertainties.

Keywords: bottomonia, non-relativistic QCD, CCFM evolution, TMD gluon density
Since it was first observed, the production of heavy quarkonium states in high energy hadronic collisions remains a subject of considerable theoretical and experimental interest [1, 2]. These processes are sensitive to the interaction dynamics both at small and large distances: the production of heavy (c or b) quarks with a high transverse momentum is followed by the bound states formation with a low relative quark momentum. Accordingly, a theoretical description of these processes involves both perturbative and non-perturbative methods, as it was proposed in the non-relativistic QCD (NRQCD) [3–6]. However, it is known that the NRQCD at the next-to-leading order (NLO) accuracy meets difficulties in a simultaneous description of all the collider data in there entirety (see also discussions [7–12]). In particular, it has a long-standing challenge in the J/ψ and ψ(2S) polarization and provides an inadequate description [13–18] of the recent ηc production data taken by the LHCb Collaboration at the LHC [19]. One of possible solutions of the problems mentioned above, which implies a certain modification of the NRQCD rules, has been proposed recently [20]. As it was shown, the approach [20] allows one to describe well the recent data on the production and polarization of the entire charmonia family. The bottomonia production, namely Υ(nS) and χb(mP) mesons, provides an alternative laboratory for understanding the physics of the hadronization of heavy quark pairs. Due to heavier masses and a smaller quark relative velocity v (in the produced quarkonium rest frame), these processes could be even a more suitable case to apply the double NRQCD expansion in QCD coupling αs and v. The NLO NRQCD predictions for the Υ(nS) production at the LHC were presented [21, 22]. Of course, it is important to apply also the approach [20] to the bottomonia family.

Our present work continues the line started in the previous studies [23, 24]. We have considered there the inclusive production of Υ(3S), Υ(2S), χb(3P) and χb(2P) mesons and now come to Υ(1S) and χb(1P) mesons. The motivation for the whole business has been already given [23, 24]. Below we present a systematic analysis of the CMS [25, 26], ATLAS [27] and LHCb [28, 30] data on the Υ(1S) and χb(1P) production collected at √s = 7, 8 and 13 TeV (including the different relative production rates) and extract from these data the non-perturbative matrix elements (NMEs) for Υ(1S) and χb(1P) mesons. Then we make predictions for polarization parameters λθ, λφ, λθφ (and a frame-independent parameter ˜λ), which determine the Υ(1S) spin density matrix and compare them to the currently available data [31, 32]. As it is known, the feed-down contributions from χb(2P), χb(3P), Υ(2S) and Υ(3S) decays give a significant impact on the Υ(1S) production and polarization, so studies [23, 24] are important and necessary points for our present consideration. Another important issue concerns the relative production rate σ(χb2)/σ(χb1) recently measured by the CMS [33] and LHCb [34] Collaborations. This ratio is sensitive to the color singlet (CS) and color octet (CO) production mechanisms and provide information complementary to the
study of the $S$-wave bottomonia states.

In the present note we follow mostly the same steps as in [23,24]. So, to describe the perturbative production of the $\bar{b}b$ pair in the hard scattering subprocess we apply the $k_T$-factorization approach [35,36], which is mainly based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [37] or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) [38] gluon evolution equations. A detailed description and discussion of the different aspects of $k_T$-factorization can be found in reviews [39]. As usual, we see certain advantages in the ease of including into the calculations a large piece of higher order pQCD corrections taking them into account in the form of transverse momentum dependent (TMD) gluon densities in a proton. Our consideration is based on the off-shell gluon-gluon fusion subprocesses representing the true leading order (LO) in QCD:

\[ g^*(k_1) + g^*(k_2) \rightarrow \Upsilon[3S_{1}^{(1)}](p) + g(k), \]  
\[ g^*(k_1) + g^*(k_2) \rightarrow \Upsilon[1S_{0}^{(8)}, 3S_{1}^{(8)}, 3P_{J}^{(8)}](p). \]  
\[ g^*(k_1) + g^*(k_2) \rightarrow \chi_{bJ}(p)[3P_{J}^{(1)}, 3S_{1}^{(8)}] \rightarrow \Upsilon(p_1) + \gamma(p_2), \]  

where $J = 0, 1$ or $2$ and the four-momenta of all particles are given in the parentheses. The color states taken into account are directly indicated. Both initial gluons are off mass shell, that means that they have non-zero transverse four-momenta $k_1^2 = k_{1T}^2 \neq 0$, $k_2^2 = k_{2T}^2 \neq 0$ and an admixture of longitudinal component in the polarization four-vectors (see [35,36] for more information). The corresponding off-shell ($k_T$-dependent) production amplitudes contain projection operators [40] for a spin and color, that guarantee the proper quantum numbers of a final state bottomonia. Following the ideas of [20], to describe the nonperturbative transformations of the color-octet $\bar{b}b$ pairs produced in hard subprocesses into observed final state mesons we employ the classical multipole radiation theory under a key physical assumption that the lifetime of intermediate color-octet states is rather long. Below we apply the gauge invariant expressions obtained earlier [41] and implemented into the Monte-Carlo event generator PEGASUS [42]. The derivation steps are explained in [23,24] in detail.

According to the $k_T$–factorization prescription, to calculate the cross sections of a considered process one has to convolute the partonic cross section $\hat{\sigma}$ (related with an off-shell production amplitude) and TMD gluon densities in a proton $f_g(x, k_T^2, \mu^2)$:

\[ \sigma = \int dx_1dx_2dk_{1T}^2dk_{2T}^2 \hat{\sigma}(x_1, x_2, k_{1T}^2, k_{2T}^2, \mu^2)f_g(x_1, k_{1T}^2, \mu^2)f_g(x_2, k_{2T}^2, \mu^2), \]  

where $x_1$ and $x_2$ are the longitudinal momentum fractions of initial off-shell gluons and $\mu$ is the hard interaction scale. Following [23,24], we have tested several sets of TMD gluon...
densities in a proton. Two of them (A0 \cite{43} and JH’2013 set 1 \cite{44}) were obtained from the CCFM equation where all input parameters were fitted to the proton structure function \( F_2(x, Q^2) \). We have applied a parametrization obtained within the Kimber-Martin-Ryskin (KMR) prescription \cite{45}, which provides a method to construct the TMD quark and gluon densities from conventional (collinear) distributions. For the input, we have applied the recent LO NNPDF3.1 set \cite{46}. The parton level calculations according to \cite{4} were performed using the Monte-Carlo generator PEGASUS. Of course, we take into account the feed-down contributions from \( \chi_b(3P) \), \( \chi_b(2P) \), \( \chi_b(1P) \), \( \Upsilon(3S) \) and \( \Upsilon(2S) \) decays.

Numerically, everywhere we set the masses \( m_{\Upsilon(1S)} = 9.4603 \text{ GeV}, m_{\Upsilon(2S)} = 10.02326 \text{ GeV}, m_{\Upsilon(3S)} = 10.3552 \text{ GeV}, m_{\chi_b(3P)} = 10.512 \text{ GeV}, m_{\chi_b(2P)} = 10.522 \text{ GeV}, m_{\chi_b(2P)} = 10.232 \text{ GeV}, m_{\chi_b(2P)} = 10.255 \text{ GeV}, m_{\chi_b(2P)} = 10.268 \text{ GeV}, m_{\chi_b(1P)} = 9.8594 \text{ GeV}, m_{\chi_b(1P)} = 9.8928 \text{ GeV}, m_{\chi_b(1P)} = 9.9122 \text{ GeV} \) and adopt the usual non-relativistic approximation \( m_b = m_Q/2 \) for the beauty quark mass, where \( m_Q \) is the mass of bottomonium \( Q \). We set the necessary branching ratios as they are given in \cite{47}. Note that there is no experimental data for the branching ratios of \( \chi_b(3P) \), so we use the results of assumption \cite{22} that the total decay widths of \( \chi_b(mP) \) are approximately independent on \( m \). So, we have \( B(\chi_b(3P) \rightarrow \Upsilon(1S) + \gamma) = 0.0381 \) and \( B(\chi_b(3P) \rightarrow \Upsilon(1S) + \gamma) = 0.0192 \) \cite{22}.

We use the one-loop formula for the QCD coupling \( \alpha_s \) with \( n_f = 4(5) \) quark flavours at \( \Lambda_{QCD} = 250(167) \text{ MeV} \) for the A0 (KMR) gluon density and two-loop expression for \( \alpha_s \) with \( n_f = 4 \) and \( \Lambda_{QCD} = 200 \text{ MeV} \) for the JH’2013 set 1 one. We set the color-singlet NMEs \( \langle O^{\Upsilon(1S)}[S_1^{(1)}] \rangle = 8.39 \text{ GeV}^3 \) and \( \langle O^{\chi_b(1P)}[P_0^{(1)}] \rangle = 2.30 \text{ GeV}^3 \) as obtained from the potential model calculations \cite{48}. Except, as it was suggested in \cite{49}, the \( \langle O^{\chi_b(1P)}[P_1^{(1)}] \rangle \) and \( \langle O^{\chi_b(1P)}[P_2^{(1)}] \rangle \), which were treated as independent parameters \cite{4}. All the NMEs for \( \Upsilon(2S) \), \( \Upsilon(3S) \), \( \chi_b(2P) \) and \( \chi_b(3P) \) mesons were derived in \cite{23,24}.

To determine the NMEs for both \( \Upsilon(1S) \) and \( \chi_b(1P) \) mesons we have performed a global fit to the \( \Upsilon(1S) \) production data at the LHC. We have included in the fitting procedure the \( \Upsilon(1S) \) transverse momentum distributions measured by the CMS \cite{25,26} and ATLAS \cite{27} Collaborations at \( \sqrt{s} = 7 \) and 13 TeV. We have excluded from our fit a low \( p_T \) region and consider only data at \( p_T > p_T^{cut} = 10 \text{ GeV} \), where the NRQCD formalism is believed to be the most reliable. To determine NMEs for \( \chi_b(1P) \) mesons, we also included into the fit the recent LHCb data \cite{30} on the radiative \( \chi_b(1P) \rightarrow \Upsilon(1S) + \gamma \) decays collected at \( \sqrt{s} = 7 \) and 8 TeV and the recent CMS \cite{33} and LHCb data \cite{34} on the ratio \( \sigma(\chi_b(1P))/(s(\chi_b(1P))) \) collected at \( \sqrt{s} = 8 \text{ TeV} \).

Our analysis strategy is the following. First, we found that the \( p_T \) shape of the direct \( \Upsilon[S_1^{(8)}] \) and feed-down \( \chi_b[S_1^{(8)}] \) contributions to \( \Upsilon(1S) \) production is almost the same in all

\footnote{See also discussion in \cite{50}.}
From the known kinematical regions probed at the LHC. Thus, the ratio
\[
\frac{\sum_{J=0}^{2} (2J + 1) \, B(\chi_{bJ}(1P) \rightarrow \Upsilon(1S) + \gamma) \, d\sigma[\chi_{bJ}(1P), 3S_1^{(8)}]/dp_T}{d\sigma[\Upsilon(1S), 3S_1^{(8)}]/dp_T}
\]
can be well approximated by a constant for a wide \(\Upsilon(1S)\) transverse momentum \(p_T\) and rapidity \(y\) range at different energies. For example, we estimate the mean-square average \(r = 1.743 \pm 0.010\) for the A0 set, which is practically the same for the other TMD gluon densities in a proton. So, we construct a linear combination
\[
M_r = \langle O^{\Upsilon(1S)[3S_1^{(8)}]} \rangle + r\langle O^{\chi_{b0}(1P)[3S_1^{(8)}]} \rangle,
\]
which can be only extracted from the measured \(\Upsilon(1S)\) transverse momentum distributions. Secondly, we found that the \(p_T\) shapes of the direct \(3P_j^{(8)}\), feed-down \(\chi_{b}[3P_1^{(1)}]\) and \(\chi_{b}[3P_2^{(1)}]\) contributions to the \(\Upsilon(1S)\) production are also the same in all kinematical regions. So, the ratios
\[
r_1 = \frac{B(\chi_{b2}(1P) \rightarrow \Upsilon(1S) + \gamma) \, d\sigma[\chi_{b2}(1P), 3P_2^{(1)}]/dp_T}{B(\chi_{b1}(1P) \rightarrow \Upsilon(1S) + \gamma) \, d\sigma[\chi_{b1}(1P), 3P_1^{(1)}]/dp_T},
\]
\[
r_2 = \frac{\sum_{J=0}^{2} (2J + 1) \, d\sigma[\Upsilon(1S), 3P_j^{(8)}]/dp_T}{B(\chi_{b1}(1P) \rightarrow \Upsilon(1S) + \gamma) \, d\sigma[\chi_{b1}(1P), 3P_1^{(1)}]/dp_T}
\]
can be approximated by constants for a wide \(\Upsilon(1S)\) transverse momentum \(p_T\) and rapidity \(y\) range at different energies. For example, we estimate the mean-square average \(r_1 = 0.91 \pm 0.02\) and \(r_2 = 104 \pm 2\) for the A0 set. Then we construct a linear combination
\[
M_{r_1r_2} = \langle O^{\chi_{b1}(1P)[3P_1^{(1)}]} \rangle + r_1\langle O^{\chi_{b2}(1P)[3P_2^{(1)}]} \rangle + r_2\langle O^{\Upsilon(1S)[3P_0^{(8)}]} \rangle,
\]
which can be extracted from the measured \(\Upsilon(1S)\) transverse momentum distributions. As a next step, we use the recent LHCb data \([30]\) on the ratio of \(\Upsilon(1S)\) mesons originating from the \(\chi_{b}(1P)\) radiative decays measured at \(\sqrt{s} = 7\) and 8 TeV:
\[
R_{\Upsilon(1S)}^{\chi_{b}(1P)} = \frac{\sum_{J=1}^{2} \, \sigma(pp \rightarrow \chi_{bJ}(1P) + X)}{\sigma(pp \rightarrow \Upsilon(1S) + X)} \times B(\chi_{bJ} \rightarrow \Upsilon(1S) + \gamma).
\]
From the known \(M_r\), \(M_{r_1r_2}\) and \(R_{\Upsilon(1S)}^{\chi_{b}(1P)}\) values one can separately determine the \(\langle O^{\Upsilon(1S)[3S_1^{(8)}]} \rangle\), \(\langle O^{\chi_{b0}(1P)[3S_1^{(8)}]} \rangle\), \(\langle O^{\Upsilon(1S)[3P_0^{(8)}]} \rangle\) and the linear combination \(M_{CS} = \langle O^{\chi_{b1}(1P)[3P_1^{(1)}]} \rangle + r_1\langle O^{\chi_{b2}(1P)[3P_2^{(1)}]} \rangle\). Finally, we use the recent CMS \([33]\) and LHCb data \([34]\)
measured at $\sqrt{s} = 8$ TeV on the ratio

$$R_{\chi_{b1}(1P)}^{(1P)} = \frac{\sigma(\chi_{b2}(1P))}{\sigma(\chi_{b1}(1P))}. \quad (11)$$

From the known $M_{CS}$, $\langle O_{\chi_{b0}(1P)[3S_1]} \rangle$ and $R_{\chi_{b1}(1P)}^{(1P)}$ values one can separately determine the $\langle O_{\chi_{b1}(1P)[3P_1]} \rangle$ and $\langle O_{\chi_{b2}(1P)[3P_1]} \rangle$ values and, therefore, reconstruct full map of the NMEs for both $\Upsilon(1S)$ and $\chi_b(1P)$ mesons.

The fitting procedure described above was separately done in each of the rapidity subdivisions (using the fitting algorithm as implemented in the commonly used gnuplot package [51]) under the requirement that all the NMEs are strictly positive. Then, the mean-square average of the fitted values was taken. The corresponding uncertainties are estimated in the conventional way using Student’s t-distribution at the confidence level $P = 80\%$. The results of our fits are collected in Table 1. For comparison, we also presented there the NMEs obtained in the conventional NLO NRQCD by other authors [21]. The corresponding $\chi^2/d.o.f.$ are listed in Table 2, where we additionally show their dependence on the minimal $\Upsilon(1S)$ transverse momenta involved into the fit $p_T^{cut}$. As one can see, the $\chi^2/d.o.f.$ tends to stay the same or slightly increase when $p_T^{cut}$ grows up and the best fit of the LHC data is achieved with the A0 and KMR gluon, although other gluon densities also return reliable $\chi^2/d.o.f.$ values. We note that including into the fit the latest CMS data [26] taken at $\sqrt{s} = 13$ TeV leads to $2 - 3$ times higher values of $\chi^2/d.o.f.$, as it was with the data on $\Upsilon(2S)$ [24]. We have checked that this is true for both the $k_T$-factorization and collinear approaches and, therefore, it could be a sign of some inconsistency between these CMS data and other measurements. Note that in our calculations we achieved the ratio $\langle O_{\chi_{b2}(1P)[3P_1]} \rangle : \langle O_{\chi_{b1}(1P)[3P_1]} \rangle : \langle O_{\chi_{b0}(1P)[3P_0]} \rangle \sim 2.6 : 4.8 : 1$ for the JH’2013 set 1, $\sim 2.6 : 3.9 : 1$ for the KMR and $\sim 1 : 3 : 1$ for the A0 gluon density, respectively. This is an obvious contradiction with naive expectations based on the number of spin degrees of freedom, $\sim 5 : 3 : 1$ and qualitatively agrees with the suggestions [49,50]. The difference between the predictions for this ratio obtained with the considered TMD gluon densities could be a sign of a sensitivity of the gluon distributions to the ratio $R_{\chi_{b1}(1P)}^{(1P)}$ and/or due to lack of the experimental data.

All the data used in the fits are compared with our predictions in Figs. 1–4. The shaded areas represent the theoretical uncertainties of our calculations, which include the uncertainties coming from the NME fitting procedure and the scale uncertainties. To estimate the latter, the standard variations $\mu_R \rightarrow 2\mu_R$ or $\mu_R \rightarrow \mu_R/2$ were introduced with replacing the A0 and JH’2013 set 1 gluon densities by the A0$^+$ and JH’2013 set 1+, or by the A0$^−$ and

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2 We have used the on-shell production amplitudes for color-octet $2 \to 2$ subprocesses from [4].
JH’2013 set 1—ones. This was done to preserve the intrinsic correspondence between the TMD gluon set and the scale used in the CCFM evolution [43][44]. One can see that we have achieved a reasonably good description of the CMS [25][26] and ATLAS [27] data for the \( \Upsilon(1S) \) transverse momentum distributions in the whole \( p_T \) range within the experimental and theoretical uncertainties. The relative production rates \( R_{\chi_b(1P)}^{\Upsilon(1S)} \) measured by the CMS [33] and LHCb [34] Collaborations and \( R_{\chi_b(1P)}^{\Upsilon(1S)} \) ratios measured by the LHCb Collaboration [30] at \( \sqrt{s} = 7 \) and 8 TeV are also reproduced well. However, our predictions for the \( R_{\chi_b(2P)}^{\Upsilon(1S)} \) and \( R_{\chi_b(3P)}^{\Upsilon(1S)} \) production rates tend to slightly overestimate the LHCb data [30], although they are rather close to the measurements within the uncertainties bands (see Fig. 3). Of course, evaluation of these observables involves the NMEs for \( \Upsilon(2S) \), \( \Upsilon(3S) \), \( \chi_b(2P) \) and \( \chi_b(3P) \) mesons determined previously [23,24]. Note that the NLO NRQCD calculations [21] also overestimate the experimental data for the \( R_{\chi_b(2P)}^{\Upsilon(1S)} \) and \( R_{\chi_b(3P)}^{\Upsilon(1S)} \) rates. In addition, we have checked our results with the data, not included into the fit procedure: namely, rather old CDF data [52] taken at the \( \sqrt{s} = 1.8 \) TeV and the LHCb data [28,29] taken in the forward rapidity region \( 2 < y < 4.5 \) at \( \sqrt{s} = 7, 8 \) and 13 TeV (see Fig. 5). As one can see, we acceptably describe all the data above. Moreover, we find that the KMR gluon distribution is the only one TMD gluon density which is able to reproduce the measurements in the low \( p_T \) region.

Now we turn to the polarization of \( \Upsilon(1S) \) mesons at the LHC conditions. It is well known that the polarization of any vector meson can be described with three parameters \( \lambda_{\theta}, \lambda_{\phi} \) and \( \lambda_{\theta\phi} \), which determine the spin density matrix of a meson decaying into a lepton pair and can be measured experimentally. The double differential angular distribution of the decay leptons can be written as [53]:

\[
\frac{d\sigma}{d\cos \theta^* d\phi^*} \sim \frac{1}{3 + \lambda_{\theta}} (1 + \lambda_{\theta} \cos^2 \theta^* + \lambda_{\phi} \sin^2 \theta^* \cos 2\phi^* + \lambda_{\theta\phi} \sin 2\theta^* \cos \phi^*),
\]

(12)

where \( \theta^* \) and \( \phi^* \) are the polar and azimuthal angles of the decay lepton measured in the meson rest frame. The case of \( (\lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi}) = (0,0,0) \) corresponds to an unpolarized state, while \( (\lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi}) = (1,0,0) \) and \( (\lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi}) = (-1,0,0) \) refer to fully transverse and fully longitudinal polarizations. The CMS Collaboration has measured all of these polarization parameters for \( \Upsilon(1S) \) mesons as functions of their transverse momentum in three complementary frames: the Collins-Soper, helicity and perpendicular helicity ones at \( \sqrt{s} = 7 \) TeV [31]. The CDF Collaboration has measured the polarization parameters in the helicity frame at \( \sqrt{s} = 1.96 \) TeV [32]. The frame-independent parameter \( \tilde{\lambda} = (\lambda_{\theta} + 3\lambda_{\phi})/(1 - \lambda_{\phi}) \) has been additionally studied. As it was done previously [23,24], to estimate \( \lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi} \) and \( \tilde{\lambda} \) we generally follow the experimental procedure. We collect the simulated events in
the kinematical region defined by the experimental setup, generate the decay lepton angular distributions according to the production and decay matrix elements and then apply a three-parametric fit based on (12).

Our predictions are shown in Figs. 6 – 9. The calculations were done using the A0 gluon density which provides the best description of the measured \( \Upsilon(1S) \) transverse momenta distributions. As one can see, we find only a weak or zero polarization in the all kinematic regions, that perfectly agrees with the CMS and CDF measurements. This agreement shows no fundamental problems in describing the \( \Upsilon(1S) \) polarization data. Moreover, the calculated polarization parameters \( \lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \) and \( \lambda^* \) are stable with respect to variations in the model parameters. In fact, there is no dependence on the strong coupling constant and/or TMD gluon densities in a proton. As it was already pointed out above, our results for \( \lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \) and \( \lambda^* \) are based on the key assumption [20] that the intermediate color octet states are states with a definite total angular momentum \( J \) and its projection \( J_z \), rather than states with definite projections of the spin \( S_z \) and orbital angular momentum \( L_z \). Given that, the transition amplitudes only involve the polarization vector associated with \( J_z \) and not with \( L_z \). As a result, we have no conservation of \( S_z \) in electric dipole transitions. Under this assumption, we have achieved a reasonable simultaneous description for all of the available data for \( \Upsilon(1S) \) and \( \chi_b(1P) \) mesons (the transverse momentum distributions, relative production rates and polarization observables). We have obtained earlier similar results for charmonia \((J/\psi, \psi')\), \( \Upsilon(2S) \) and \( \Upsilon(3S) \) polarizations [23,24,54,55]. Thus, one can conclude that the approach [20] results in a self-consistent and simultaneous description of charmonia and bottomonia data and therefore can be considered as providing an easy and natural solution to the long-standing quarkonia production and polarization puzzle.

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Table 1: The NMEs for $\Upsilon(1S)$ and $\chi_b(1P)$ mesons as determined from our fit at $p_T^{cut} = 10$ GeV. The NMEs obtained in the NLO NRQCD \[21\] are shown for comparison.

|                        | A0          | JH'2013 set 1 | KMR         | NLO NRQCD \[21\] |
|------------------------|-------------|---------------|-------------|------------------|
| $\langle O^{(1S)}[^3S_1(1)] \rangle/\text{GeV}^3$ | 8.39        | 8.39          | 8.39        | 9.28             |
| $\langle O^{(1S)}[^1S_0(8)] \rangle/\text{GeV}^3$ | 0.00        | 0.00          | 0.00        | 0.136 $\pm$ 0.0243 |
| $\langle O^{(1S)}[^3S_1(8)] \rangle/\text{GeV}^3$ | 0.016 $\pm$ 0.006 | 0.0038 $\pm$ 0.0019 | 0.0029 $\pm$ 0.0019 | 0.0061 $\pm$ 0.0024 |
| $\langle O^{(1S)}[^3P_0(8)] \rangle/\text{GeV}^5$ | 0.07 $\pm$ 0.03 | 0.20 $\pm$ 0.10 | 0.18 $\pm$ 0.06 | $-0.0093 \pm 0.005$ |
| $\langle O^{(1P)}[^3F_0(1)] \rangle/\text{GeV}^5$ | 2.30        | 2.30          | 2.30        | 2.03             |
| $\langle O^{(1P)}[^3F_1(1)] \rangle/\text{GeV}^5$ | 7 $\pm$ 3   | 11 $\pm$ 5    | 9 $\pm$ 2   | 6.09             |
| $\langle O^{(1P)}[^3F_2(1)] \rangle/\text{GeV}^5$ | 2.4 $\pm$ 1.9 | 6 $\pm$ 4     | 6 $\pm$ 2   | 10.15            |
| $\langle O^{(1P)}[^3S_1(8)] \rangle/\text{GeV}^3$ | 0.008 $\pm$ 0.002 | 0.0020 $\pm$ 0.0011 | 0.0015 $\pm$ 0.0012 | 0.0094 $\pm$ 0.0006 |

Table 2: The dependence of the $\chi^2/d.o.f.$ achieved in the fit procedure on the choice of $p_T^{cut}$ at only $\sqrt{s} = 7$ TeV and at $7$ and $13$ TeV combined.
Figure 1: Transverse momentum distribution of the inclusive $\Upsilon(1S)$ production calculated at $\sqrt{s} = 7$ TeV in the different rapidity regions. The red, green and blue histograms correspond to the predictions obtained with the A0, KMR and JH’2013 set 1 gluon densities. Shaded bands represent the total uncertainties of our calculations, as it is described in the text. The experimental data are from ATLAS [27].
Figure 2: Transverse momentum distribution of the inclusive $\Upsilon(1S)$ production calculated at $\sqrt{s} = 7$ TeV (upper histograms) and $\sqrt{s} = 13$ TeV (lower histograms, divided by 100) in the different rapidity regions. Notation of all histograms is the same as in Fig. 1. The experimental data are from CMS [25,26].
Figure 3: The ratio $R_{\Upsilon(1S)}^b(m_P)$ calculated as a function of the $\Upsilon(1S)$ transverse momentum calculated at $\sqrt{s} = 7$ and 8 TeV. Notation of all histograms is the same as in Fig. 1. The experimental data are from LHCb [30].
Figure 4: The ratio $R_{\chi\chi_{1S}}^{(1P)}$ calculated as a function of the $\Upsilon(1S)$ transverse momentum calculated at $\sqrt{s} = 8$ TeV. Notation of all histograms is the same as in Fig. 1. The experimental data are from CMS [33] and LHCb [34].
Figure 5: Transverse momentum distribution of inclusive $\Upsilon(1S)$ production calculated at $\sqrt{s} = 1.8, 7, 8$ and 13 TeV in the different rapidity regions. Notation of all histograms is the same as in Fig. 1. The experimental data are from CDF [52] and LHCb [28, 29].
Figure 6: The polarization parameters $\lambda_\theta$, $\lambda_\phi$, $\lambda_{\theta\phi}$ and $\tilde{\lambda}$ of $\Upsilon(1S)$ mesons calculated in the CS frame as functions of its transverse momentum at $\sqrt{s} = 7$ TeV. The A0 gluon density is used. The blue and red histograms correspond to the predictions obtained at $|y| < 0.6$ and $0.6 < |y| < 1.2$, respectively. The experimental data are from CMS [31].
Figure 7: The polarization parameters $\lambda_\theta$, $\lambda_\phi$, $\lambda_{\theta\phi}$ and $\tilde{\lambda}$ of $\Upsilon(1S)$ mesons calculated in the helicity frame as functions of its transverse momentum at $\sqrt{s} = 7$ TeV. Notation of all histograms is the same as in Fig. 6. The experimental data are from CMS [31].
Figure 8: The polarization parameters $\lambda_\theta$, $\lambda_\phi$, $\lambda_{\theta\phi}$ and $\tilde{\lambda}$ of $\Upsilon(1S)$ mesons calculated in the perpendicular helicity frame as functions of its transverse momentum at $\sqrt{s} = 7$ TeV. Notation of all histograms is the same as in Fig. 6. The experimental data are from CMS [31].
Figure 9: The polarization parameters $\lambda_\theta$ and $\tilde{\lambda}$ of $\Upsilon(1S)$ mesons calculated in the helicity frame as functions of its transverse momentum at $\sqrt{s} = 1.96$ TeV. Notation of all histograms is the same as in Fig. [6]. The experimental data are from CDF [32].