On the relationship between the noise-induced persistent current and dephasing rate.

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AC noise in disordered conductors causes both dephasing of the electron wave functions and a DC current around a mesoscopic ring. We demonstrate that the dephasing rate \( \tau_{\phi}^{-1} \) in long wires and the DC current, induced by the same noise and averaged over an ensemble of small rings are connected. The ensemble-averaged \( \hbar/2e \) flux harmonic \((I)\) of the current and the dephasing rate caused by the same uniform in space high frequency AC field are related in a remarkably simple way: \( \langle I \rangle \tau_{\phi} = C_\beta \epsilon \). Here \( \epsilon \) is an electron charge, and the constant \( C_\beta \) depends on the Dyson symmetry class. For a pure potential disorder the current \((I)\) is diamagnetic \( C_\beta = -(4/\pi) \) and in the presence of strong spin-orbit scattering it is paramagnetic \( C_\beta = (2/\pi) \). The relationship seems to agree reasonably with experiments. This suggests that the two puzzles: anomalously large persistent current [L.P.Levy et al., Phys.Rev.Lett., 64, 2074 (1990)] and the low-temperature saturation of the dephasing [P.Mohanty et al., Phys.Rev.Lett., 78, 3366 (1997)] may have a common solution.

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Since the discovery of universal conductance fluctuations physics of mesoscopic systems has made tremendous progress. However, few important experimental observations still remain unexplained. One of the long-standing challenges is the anomalously large value of persistent current [1]. Levy et al. [1] studied the magnetic response of an ensemble of \( 10^7 \) mesoscopic copper rings as a function of applied weak magnetic field. The average current per ring \( \langle I_0(\phi) \rangle = I_0 \sin(4\pi e\phi/\hbar c) \) found from such measurements is a \( \hbar/2e \)-periodic function of the magnetic flux \( \phi \) threading each ring. The amplitude \( I_0 \) has been found to be of the order of \( 10^{-5} \) for magnetic fields \( \lesssim 1 \) tesla. Other measurements [2] of persistent current brought up similar results.

On the other hand, assuming that the system is in the equilibrium and that electrons do not interact with each other, one gets [5] the amplitude \( I_{0\text{bear}}^{\text{exp}} \sim (e/h) \Delta = g^{-1} I_0 \), where \( \Delta \) is the electron mean level spacing, and \( g = \tau_D \Delta \) is the dimensionless conductance. The sign of the average persistent current predicted by the existing theories of non-interacting electrons, as well as the sign of the contribution of the electron-electron interaction in non-superconducting systems, is paramagnetic, i.e., \( \partial \langle I_0(\phi) \rangle / \partial \phi > 0 \) at small \( \phi \).

In the experiments Ref. [1,2,5] the dimensionless conductance was large \( g \sim 10^2 \), i.e., the observed persistent current exceeded the theoretical estimation by two orders of magnitude. None of numerous attempts (see e.g. [10] and for discussion [3]) succeeded in explaining the magnitude of the persistent current by electron-electron interaction. Therefore, the amplitude of the persistent current is in striking disagreement with existing theories.

Another major puzzle in mesoscopic physics recently attracted much attention. Mohanty et al. [11] experimentally proved that the saturation of the dephasing rate at low temperatures \( T \) cannot be explained by conventional arguments such as magnetic impurities or heating. On the other hand, the dephasing due to interactions between electrons (or between electrons and phonons) in an equilibrium system is theoretically predicted to disappear at \( T = 0 \). Several attempts have been made to resolve the puzzle resulting from the noise caused by the two-level systems [14], 2-channel Kondo effect [15], or external radiation [16]. In all these explanations electrons interact with an ‘environment’ that displays a real time evolution, e.g., real transitions in two-level systems or a time-dependent electric field. From this point of view, the system of free electrons subject to an external AC field captures all the essential features of dephasing. For instance using the proper correlator of the equilibrium intrinsic AC electric noise one can evaluate [12] the dephasing rate due to \( e-e \) interaction, which is in agreement with the experiment at modestly low temperatures [13].

It is known [17] that the AC electric field may also cause a random DC current in mesoscopic systems. However, in contrast to classical physics where the rectification exists only in media without inversion center, the Aharonov-Bohm effect makes the disorder-averaged rectified current also possible [18]. This rectified DC current leads to the DC magnetic response similar to the one
which results from the equilibrium persistent current.

We show below that such a noise-induced DC current averaged over an ensemble of small rings of the circumference \( L \ll L_\varphi \) and the dephasing rate induced by the same noise in long wires of the length \( \mathcal{L} \gg L_\varphi \) are related in a remarkably simple way:

\[
I_E \tau_\varphi = C_\beta e, \quad C_\beta = \begin{cases} 
-4/\pi, & \beta = 1 \\
+2/\pi, & \beta = 4 
\end{cases} \quad (1)
\]

where \( e \) is an absolute value of the charge carriers and \( C_\beta \) is a constant that depends on the Dyson symmetry class: \( \beta = 1 \) for the pure potential disorder and \( \beta = 4 \) in the presence of a spin-orbit scattering with the characteristic length \( L_{so} \ll L \). Thus, the important differences between the equilibrium persistent current and the rectified DC current are (1) the magnitude and (2) the sign if \( L_{so} \gg L \). In this orthogonal case the ensemble-averaged DC current \( \langle I_E(\phi) \rangle = I_E \sin(4\pi e\phi/hc) \) is diamagnetic, i.e. \( \partial \langle I_E(\phi) \rangle / \partial \phi < 0 \) at small \( \phi \).

This is the central result of the Letter. The relationship Eq.\((1)\) holds regardless of the nature of the noise, since no single parameter of the system and ‘environment’ enters Eq.\((1)\). The noise could be external or intrinsic and not even necessarily electric (e.g., the phonon wind). The only important condition is that the noise must be non-equilibrium. This suggests (see also Ref.\([19]\)) that the two puzzles: an anomalously large persistent current and an anomalously large temperature-independent dephasing rate maybe closely related.

Actually, the relationship Eq.\((1)\) can be understood by the dimension analysis. Consider as an example of a noise a monochromatic AC electric field with a frequency \( \omega \) and an amplitude \( E_\omega \). Given the diffusion constant \( D \) one can construct a dimensionless combination:

\[
\alpha = \frac{D}{\omega} \left( \frac{e}{h} E_\omega \right)^2 = \left( \frac{L_\omega}{L_E} \right)^2, \quad (2)
\]

where \( L_\omega = \sqrt{D/\omega} \), and the characteristic length \( L_E \) is determined by the equation \( eE_\omega L_E = h\omega \). One can estimate the dephasing rate in long wires at \( T = 0 \) as:

\[
\frac{1}{\tau_\varphi} = \omega f_\varphi(\alpha). \quad (3)
\]

Evaluation of \( f_\varphi(\alpha) \) goes beyond the dimension analysis. As to the nonlinear DC current in mesoscopic rings, its amplitude depends on two parameters:

\[
I_E = e \omega f_1(\alpha, \gamma), \quad (4)
\]

- the parameter \( \alpha \) Eq.\((2)\) and the ‘mesoscopic’ parameter

\[
\gamma = \omega \tau_D = \left( \frac{L}{L_\omega} \right)^2. \quad (5)
\]

In the weak-field limit \( \alpha \to 0 \) both the DC current and the dephasing rate are quadratic in \( E_\omega \), i.e., linear in \( \alpha \):

\[
I_E = e \omega \alpha f_1(\gamma), \quad \tau_\varphi^{-1} = \omega f_\varphi'(0) \alpha \quad (6)
\]

where \( f_1(\gamma) \) is yet an unknown function of \( \gamma \). Provided that this function has a non-zero limit \( f_1(\infty) \) at \( \gamma \gg 1 \), we immediately arrive at:

\[
I_E \tau_\varphi = e C_\beta, \quad C_\beta = f_1(\infty)/f_\varphi'(0), \quad (7)
\]

where \( C_\beta \) is a constant of order 1.

This is essentially Eq.\((1)\). The above analysis suggests that Eq.\((1)\) is valid when \( \alpha \) is small, and \( \gamma \) is large. According to Eqs.\((3)\) \( L_\varphi \sim L_\omega \alpha^{-1/2} \sim L(\alpha\gamma)^{-1/2} \). Thus, the mesoscopic condition \( L \ll L_\varphi \) is equivalent to \( \alpha \gamma \ll 1 \), and the above consideration is valid when

\[
1 \ll \gamma \ll \alpha^{-1}, \quad L_\omega \ll L \ll L_\varphi. \quad (8)
\]

One can also write Eq.\((8)\) as \( \omega \gg \max\{\tau_D^{-1}, eE_\omega L/h\} \).

Another condition concerns the space correlation of the field \( E_\varphi \). We neglected space dependence of the field. This can be done if at the length scale of \( L_\omega \) the field is strongly correlated.

An assumption that \( f_1(\gamma) \) has a finite limit at \( \beta \to \infty \) is anything but trivial. It implies that in the quadratic in \( E_\omega \) approximation the DC current flows coherently even at \( L_\omega \ll L \). It was first mentioned in Ref.\([17]\) and further discussed in Refs.\([18,20]\) that the nonlinear DC current is not destroyed at \( L_\omega \ll L \). Note that this conclusion applies only to the DC current and is not correct for the ensemble-averaged second harmonic current \( E_\omega \).

It is intuitively clear that for DC current \( I_E \) to flow, the environment and the electrons should be out of the thermal equilibrium. Indeed, \( I_E \) vanishes identically for the equilibrium electric noise \( f_\varphi \). On the other hand, even the equilibrium electric noise causes dephasing \( f_\varphi \).

Eq.\((1)\) involves \( T \to 0 \) limits of \( 1/\tau_\varphi \) and the DC current, which are their maximal values for a given sample. At finite temperatures, the dephasing rate exceeds \( I_E/C_\beta e \) even at \( L \ll L_\varphi \), due to the T-dependent contribution from the equilibrium part of the noise to \( 1/\tau_\varphi \).

Of course, the arguments presented above cannot substitute an analytic derivation which we proceed with. Consider a quasi-1D system of non-interacting electrons with an external AC field \( E(t) = -\frac{1}{2} \beta A(t) \) where \( A(t) \) is a time-dependent tangential vector-potential with the zero mean value \( \overline{A_t} = 0 \). Here the bar means the time averaging. In contrast with Ref.\([18]\) the field \( A_t \) represents a noise with short-range time-correlations rather than a strictly monochromatic field \( E_\omega \). The correlation function \( A_t A_{t'} \) is supposed to decrease at \( |t - t'| > t_c \sim \omega^{-1} \). For simplicity we consider this field to be constant along a ring though the actual requirement \([18]\) for the scale \( r_c \) of space variation is much weaker \( r_c \gg L_\omega \).

We consider two different geometries: a long wire with the length \( \mathcal{L} \gg L_\varphi \) and a ring with the circumference
$L \ll L_\varphi$. In the latter case we study a DC current that flows when a \textit{time-independent} flux $\phi$ threads the ring.

The weak localization correction ($I_{wl}(t)$) to the disorder-averaged current in such a system is given by the well known cooperon contribution \cite{[12]}:

$$\langle I_{wl}(t) \rangle = \frac{C_\beta e^2 D}{2hL} \int_0^\infty d\tau C_\tau \left( \frac{\tau}{2} \right)^2 \left( \frac{\tau}{2} \right) E(t - \tau). \quad (9)$$

Here $C_\tau = \sum_q C_\tau(q, \tau, \tau')$ is a cooperon at coincident space points, and $C_\tau(q, \tau, \tau')$ is determined by Eq. (12):

$$\frac{\partial C_\tau}{\partial \tau} + D \left( q - \frac{e}{\hbar c} (A_{\tau+\tau} + A_{\tau-\tau}) \right)^2 C_\tau = \delta(\tau - \tau'), \quad (10)$$

where $q$ is a momentum. It is continuous if the wire is long: for a ring $q = (2\pi/L) (m - 2\phi/\hbar c)$, $m = 0, \pm 1, \pm 2 \ldots$. Eqs. (9,10) are valid if the conductance of the system is large $\gg 1$, and the field is weak enough ($e/\hbar c)A_{<}l \ll 1$, ($I$ is the mean free path of the electrons).

The DC component of the current ($I_E(\phi)$) is determined by the time average of Eq. (9). Since this average does not depend on the reference point we can shift $t \rightarrow t + \tau/2$ and express the $n - th$ flux harmonic $I_E^{(n)}$ of the current

$$\langle I_E(\phi) \rangle = \sum_{n=1}^\infty I_E^{(n)} \sin(4\pi n \phi / \hbar c), \quad (11)$$

through the $n - th$ flux harmonic $C_\tau^{(n)}(\tau)$ of the cooperon

$$I_E^{(n)} = -\frac{iC_\beta e^2 D}{\hbar c L} \int_0^\infty d\tau C_\tau^{(n)}(\tau) \frac{\partial A_{\tau+\tau/2}}{\partial t}. \quad (12)$$

Solving Eq. (10) and using the Poisson summation formula, one can find an exact expression for $C_\tau^{(n)}(\tau)$:

$$C_\tau^{(n)}(\tau) = \sqrt{\frac{\pi D}{4\tau}} \left[ -n \right]_0^\infty e^{\frac{\pi^2 t^2}{4\tau^2}} e^{inS_1[A] - \tau S_2[A]} \left( \frac{2\pi i}{\hbar c} \right) e^{i\bar{A}_1(t)} dt_1 \quad (13)$$

Here

$$S_1[A] = \frac{2\pi eL}{\hbar c} \left( \int_0^{t/\tau} A_{t/\tau} dt_1 \right) \equiv \frac{2\pi eL}{\hbar c} \langle A_{t/\tau} \rangle_{t/\tau}, \quad (14)$$

$$S_2[A] = \frac{2\pi^2 D}{\hbar c^2} \left[ \langle A_{t/\tau}^2 \rangle_{t/\tau} + \langle A_{t/\tau} \rangle_{t/\tau}^2 - 2 \langle A_{t/\tau} \rangle_{t/\tau} \right]. \quad (15)$$

According to Eqs. (14,15) the weak-localization correction to the conductance of a long wire equals to

$$\delta \sigma = \frac{C_\beta \sqrt{\pi D e^2}}{2h} \int_0^\infty \frac{dt}{\sqrt{t}} \exp \{-\tau S_2[A]\}. \quad (16)$$

(we substitute a DC field $E_0$ for $E(t - \tau)$ and used the definition Eq. (15)). The form of Eqs. (14,15) suggests that $S_2[A]$ is related with the dephasing rate, while $S_1[A]$ is responsible for the nonlinear DC current.

Now we assume that the correlation time of the AC field is shorter than the relevant time scale $\tau_0$ in the integrals Eqs. (9,10). For the problem of dephasing in a long wire Eq. (16), $\tau_0 \sim \tau_\varphi$, while for the problem of DC current in a ring $\tau_0^{(DC)} \sim > \tau_\varphi$. Under these assumptions one can neglect the second and the third terms in Eq. (13) and identify the average $\langle A_{t/\tau}^2 \rangle_{t/\tau}$ determined in Eq. (4) with the true time-average $A_{t/\tau}^2$. As a result, $S_2[A] = 2D(e^2/\hbar^2 c^2) A_{t/\tau}^2$ becomes independent of $t$ and $\tau$. Using Eq. (16), we identify $S_2[A]$ with the noise-induced dephasing rate:

$$\frac{1}{\tau_\varphi} = 2D(e^2/\hbar^2 c^2) A_{t/\tau}^2. \quad (17)$$

In order to compute the amplitude $I_{E}^{(n)}$ of the DC current we have to evaluate the time-average in Eq. (12):

$$\frac{\partial A_{t-\tau/2}}{\partial t} \exp \left\{ \frac{in}{\tau} \left( \frac{2\pi eL}{\hbar c} \int_{t-\tau/2}^{t+\tau/2} A_{t/\tau} dt_1 \right) \right\} = \exp \left\{ \frac{in}{\tau} \left( \frac{2\pi eL}{\hbar c} \frac{A_{t/\tau}^2}{A_{t/\tau}} \right) \right\} \quad (18)$$

Since the time-average of the total time-derivative is zero, we can transfer the differentiation to the exponent. In the limit $(L/L_\varphi)^2 (t_c/\tau_D) \ll 1$, one can differentiate only the lower limit of the integral and set $\langle A_{t/\tau} \rangle_{t/\tau} = A_{t/\tau} = 0$ in the exponent after the differentiation. Substituting the result in Eq. (12) we arrive at an integral over $\tau$, which can be evaluated exactly. Finally, we use Eq. (13) to express $A_{t/\tau}^2$ in terms of the dephasing rate and obtain the amplitude of the $n - th$ flux harmonic of the DC current averaged over the ensemble of mesoscopic rings Eq. (11):

$$I_{E}^{(n)} = C_\beta \left( \frac{e}{\tau_\varphi} \right) \exp \left\{ -n \frac{L}{L_\varphi} \right\}. \quad (19)$$

Eq. (19) for the principal $h/2e$- periodic component $I_{E}^{(1)}$ is just in the limit of Eq. (18) at $L \ll L_\varphi$.

Unlike other theories \cite{[10]} of persistent current, the relationship Eq. (19) gives a correct magnitude of the DC current. Indeed, in a given sample at $T = 0$ the current as a function of the noise intensity reaches its maximal value when $L_\varphi$ becomes comparable to the sample size $L$ (further increase of the intensity would suppress the DC current exponentially $\sim \exp(-L/L_\varphi)$). This condition can be rewritten as $\tau_\varphi \sim \tau_D$. Using Eq. (17) we find that the maximal value of the current is of the order of

$$I_{E}^{max} \sim e/\tau_D. \quad (20)$$

This is the order of magnitude of the current which was observed in all experiments.
The ensemble-averaged current observed in copper rings by Levy et al. was about 0.3 nA. We can estimate $2e/\pi\tau_\varphi < 0.9$ nA. In Ref. [4] an ensemble of $10^5$ GaAs/GaAlAs rings have been studied. In this case the estimation gives $4e/\pi\tau_\varphi < 1.2$ nA, while the observed ensemble-averaged current was about 1.5 nA. In both cases there is a great deal of uncertainty: the saturated value of $\tau_\varphi$ has not been measured, and for estimation we used values of $\tau_\varphi$ measured in similar structures at $T \approx 1$ K and $T \approx 50$ mK, respectively. Nevertheless the estimations of the DC current based on Eq. (1) are much closer to the experimental values than the predictions of the theories Ref. [9], which assume thermal equilibrium.

Recently Mohanty et al. measured the low-temperature dephasing and the "persistent" current in the same set up. The dephasing time in long gold wires saturated at $\tau_\varphi \approx 4$ ns. The "persistent" current has been obtained from the magnetization of 30 gold rings fabricated in the same way as the wires. The amplitude of the $h/2e$ DC current was found to be $\sim 0.06$ nA, while $2e/\pi\tau_\varphi \approx 0.03$ nA. Therefore, in all three experiments Refs. [4,7,6,19] Eq.(1) was satisfied up to a factor $\sim 2$, though the magnitude of the persistent current var-

In conclusion, we have derived a relationship, Eq. (1) between the averaged DC current generated by a non-equilibrium AC noise in an ensemble of mesoscopic rings and the dephasing rate caused by the same noise. It provides a much better fit for the magnitude of low-temperature ring magnetization than other existing theories, which assume the equilibrium. More experimental work is needed to confirm the role of the non-equilibrium noise. However, there are reasons to suspect that currently we deal with substantially non-equilibrium mesoscopic systems.

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