Hunting a wandering black hole in M31 halo using GPU cluster

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Abstract. In the hierarchical structure formation scenario, galaxies have experienced many mergers with less massive galaxies and have grown larger and larger. On the other hand, the observations indicate that almost all galaxies have a central massive black hole (MBH) whose mass is $\sim 10^{-3}$ of its spheroidal component. Consequently, MBHs of satellite galaxies are expected to be moving in the halo of their host galaxy after a galaxy collision, although we have not found such MBHs yet. We investigate the current-plausible position of an MBH of the infalling galaxy in the halo of the Andromeda galaxy (M31). Many substructures are found in the M31 halo, and some of them are shown to be remnants of a minor merger about 1 Gyr ago based on theoretical studies using $N$-body simulations. We calculate possible orbits of the MBH within the progenitor dwarf galaxy using $N$-body simulations. Our results show that the MBH is within the halo, about 30 kpc away from the center of M31.

In addition, further simulations are necessary to restrict the area in which the MBH exists, and hence to determine the observational field for the future observational detection. The most uncertainty of the current MBH position is caused by uncertainty about the infalling orbit of the progenitor dwarf galaxy. Therefore, we have performed a large (a few $10^4$ realizations) set of parameter study to constrain the orbit in the six-dimensional phase space. For such purpose, we have already investigated in detail a few ten thousand orbit models using HA-PACS, a recently installed GPU cluster in University of Tsukuba.

1. Astrophysical Background and Motivation

In the context of hierarchical structure formation scenario under the Cold Dark Matter (CDM) universe, large galaxies, such as Milky Way or Andromeda galaxy (M31), have likely experienced many mergers with less massive galaxies and have grown larger and larger. Furthermore, a well known observed correlation between the mass of spheroidal component of galaxies and the mass of central massive black holes (MBHs) in their central region, so called the Magorrian relation [1, 2], suggests the coevolution of galaxies and their central MBHs. However, little is known how the coevolution of galaxies and MBHs proceeds. In the hierarchical structure formation scenario, galaxies collide and merge with each other and subsequently less massive galaxies and their central MBHs drift around the halo region of their host galaxy. In other words, MBHs move in the halo of their host galaxy after galaxy merging events, and finally they sink towards the central region of the host galaxy due to dynamical friction. Therefore, MBHs also locate outside the nucleus of their host galaxy, not only in the central region of galaxies like active galactic nuclei. However, we have not found such MBHs yet. Thus, searching for such MBHs is very hot issue recently [3, 4]. In this study, we investigate the probable position of such an MBH theoretically.
Many cosmological $N$-body simulations of the hierarchical structure formation exhibit a wealth of merger remnants around host galaxies [5]. To test the current cosmology by verifying such theoretical predictions from the CDM scenario, many observational studies have been examined to investigate merger remnants [6, 7]. In such a context, the giant stellar stream was discovered in the south region of the M31 halo [8], which is our neighbor galaxy. Further photometric and spectroscopic observations of spatial distribution [9, 10, 11, 12, 13, 14, 15], radial velocity distribution of red giant stars [16, 17, 18, 19, 20, 21, 22] and metallicity distribution [17, 18, 12, 20, 21, 22] clearly show other substructures near M31. Calculations of the motion of test particles under the gravitational potential of M31 [16, 23] and $N$-body simulations on the interaction between the progenitor of the stream and M31 [24, 25, 26, 27] suggest that the stream, the northeast shell and the west shell are the tidal debris formed in the last pericentric passage of a satellite on a radial orbit. These models reproduce the observed features and successfully constrain the orbit and properties of the progenitor.

The Magorrian relation suggests that the progenitor dwarf galaxy has an MBH whose mass $M_{BH}$ is about $10^{-3}$ of the mass of their host galaxy’s spheroidal component $M_{sph}$ [1, 2]. The similar relation between $M_{BH}$ and host galaxy’s velocity dispersion $\sigma$, $M_{BH} - \sigma$ relation, is held down to $M_{BH} \sim 10^3 M_\odot$ [28, 29]. Therefore, the relation between MBHs and their host galaxies is held down to $M_{sph} \sim 10^8 M_\odot$. Since the dynamical mass of the progenitor is estimated to be an order of $10^9M_\odot$ [25, 26, 27], the progenitor likely has an MBH whose mass is up to an order of $10^6 M_\odot$ if the progenitor consists of spheroidal stellar component alone. If this is true, then an MBH should be now moving in the merger remnants. Finding such an MBH will provide us a hint for understanding the coevolution process of galaxies and MBHs. Thus, we investigate the current position of the MBH using $N$-body simulations, in view of future observational detections.

2. Numerical Modeling of the Interaction between M31 and the Infalling Satellite

The basic equation of $N$-body simulation by direct summation is Newton’s equation of motion expressed as

$$a_i = \sum_{j=0,j\neq i}^{N-1} \frac{G m_j (x_j - x_i)}{\left| (x_j - x_i)^2 + \epsilon^2 \right|^{3/2}},$$

where $G$ is the gravitational constant, $m_i$, $x_i$, and $a_i$ are mass, position, and acceleration of $i$-th particle out of $N$ particles, respectively. The gravitational softening parameter $\epsilon$, introduced to avoid divergence due to division by zero, eliminates self interaction when calculating gravitational force. We use the word $i$-particles, and $j$-particles to denote particles feel gravitational force, and particles cause gravitational force, respectively.

We assume a fixed potential model (Hernquist bulge [30], exponential disk, and NFW halo [31]) for M31 [32, 25], because Mori & Rich [26] analytically and numerically showed the dynamical response of M31’s disk against the collision with the progenitor is negligible. They represented the progenitor and M31 using $N$-body particles and focused on the thickness of the M31’s disk due to the disk heating by dynamical friction after the collision. Their results showed that the effects on the disk thickness and the disk kinematics are negligibly small as far as the dynamical mass of the progenitor is less than $5 \times 10^9 M_\odot$. In this study, we assume a King sphere of $M = 3 \times 10^9 M_\odot$, $c = 0.7$, $r_t = 4.5$ kpc, the best fit model derived by [27], as the progenitor dwarf galaxy.

2.1. Finding the Current-Plausible Position of the MBH

To investigate the current-plausible position of the MBH in the former satellite, we calculate orbital evolution of an MBH particle whose mass is $3 \times 10^6 M_\odot$ and 524,288 $N$-body particles which represent the satellite. We set the gravitational softening length $\epsilon$ to be 13 pc. Following
we adopt an initial position and velocity vector of $(-34.75, 19.37, -13.99)$ kpc and $(67.34, -26.12, 13.50)$ km s$^{-1}$, respectively. We calculate the self gravity of $N$-body particles using Blade-GRAPE on FIRST simulator at CCS, the University of Tsukuba. We use 2nd-order Runge-Kutta integrator and shared, adaptive time step.

2.2. Constraining Uncertainty for the Current Probable Position of the MBH

Further simulations are necessary to restrict the area in which the MBH exists, and hence to determine the observational field for the future observational detection. The most uncertainty of the current MBH position is caused by uncertainty about the infalling orbit of the progenitor dwarf galaxy. Therefore, we have performed a large set of parameter study to constrain the orbit of the infalling satellite which reproduce the observed structures in the six-dimensional phase space. Since the number of dimension for the parameter space, six, is too large to sweep the whole parameter space, the number of dimension is reduced as follows. First, we fix the initial distance of the infalling satellite as 7.63 kpc away from the center of the M31 (corresponds to scale radius of DM halo [25]). In addition, M31 is modeled as an axisymmetric system in this study. Therefore, the resultant number of dimension becomes four; however, the parameter space is still large.

To study such a wide parameter space, we have performed a large set of parameter survey utilizing GPU cluster. In this parameter survey, we represent the infalling satellite with 65536 particles and set the gravitational softening length $\epsilon$ as 50 pc. Numerical simulations of parameter study are performed on HA-PACS at CCS, the University of Tsukuba. We use 2nd-order leap-frog integrator and shared, fixed time step. Algorithm and implementation of our code is explained in Section 4.

3. General Purpose computing on Graphics Processing Unit and HA-PACS

Hence performing four dimensional parameter study is a challenging task, we need to accelerate calculation and sweep wide parameter space concurrently to complete the parameter study in realistic time. In recent days, GPU (Graphics Processing Unit) becomes one of the most attractive accelerator due to development of GPGPU (General Purpose computing on GPU). A C/C++ based programming environment named CUDA (Compute Unified Device Architecture) provided by NVIDIA enables programmers to quite easily implementing GPU codes run on NVIDIA’s GPUs [33]. Furthermore, many GPU clusters exhibit on the TOP 500 list [34], such as Titan, Tianhe-1A, Nebulae, TSUBAME 2.0, and HA-PACS. Rapid performance increase of GPUs and development of such GPU clusters support accelerating numerical simulations. This preferred feature of GPU computing strongly encourages us performing four dimensional parameter study about the infalling orbit of the satellite on GPU cluster.

In this study, we have used HA-PACS (Highly Accelerated Parallel Advanced system for Computational Sciences), a newly installed GPU cluster at University of Tsukuba [35]. HA-PACS is equipped with the high-end GPUs and CPUs connected by PCI-express generation 3.0. Each node of HA-PACS consists of two sockets of Intel Sandy Bridge-EP and four boards of NVIDIA Tesla M2090, and the CPUs support full bandwidth connection of the GPUs without any performance bottleneck. The peak performance of HA-PACS is 1.604 PFLOPS in single precision, due to high performance GPUs of 1.427 PFLOPS in single precision. The Table 1 lists other detailed information of HA-PACS.

4. Implementation and Optimization of Single GPGPU Code

Many earlier studies reported that massive parallelization about $i$-particles achieves high performance of GPU [36, 37, 38, 39, 40]. Implementation and optimization technique in this work is based on CUDA SDK [37] and our previous work [40] except for one additional optimization. Implementation of CUDA SDK, based on [37], and our previous work [40] achieve
Table 1. Detailed information of HA-PACS

|                | 268                                      |
|----------------|------------------------------------------|
| Number of nodes|                                         |
| CPU            | Intel Xeon E5-2670                       |
|                | 16 cores per node, 2.6GHz                |
| RAM            | 128 GB (DDR 3, 1,600MHz)                 |
| GPU            | NVIDIA Tesla M2090                       |
|                | 512 CUDA cores, 1.3GHz                   |
|                | 4 boards per node                        |
| Video RAM      | 6 GB (GDDR 5, ECC on) per GPU            |
| C Compiler     | icc 12.1.0                               |
| MPI Library    | Intel MPI 4.0.3.008                      |
| CUDA toolkit   | nvcc 4.1, CUDA SDK 4.1.28                |
| Interconnection| Infiniband QDR ×2 rails                  |

Listing 1. Difference between CUDA SDK and our previous work

```c
/* Implementation of CUDA SDK */
float r2 = rji.x * rji.x + rji.y * rji.y + rji.z * rji.z;
r2 += eps2;

/* Our Previous Work */
float r2 = eps2 + rji.x * rji.x + rji.y * rji.y + rji.z * rji.z;
```

high performance of 930 and 991 GFLOPS in single precision on a board of NVIDIA Tesla M2090.

In both implementation, a block contains 256 threads and the shared memory stores position data of 256 \(j\)-particles to minimize access time to the global memory within the innermost loop. Differences between [37] and [40] are the number of unroll count for the innermost loop, cache configuration, and the number of operations to calculate gravitational interaction. In our implementation, the number of unroll count for the innermost loop is 128 against for 32 of CUDA SDK, and we set ”L1 cache preferred” since experiments exhibit slight performance increase compared with ”shared memory preferred” in most cases. The last difference, the most influential one, is due to calculation process of \(r_{ji}^2 + \epsilon^2\). In both implementation, \(a float3\) type variable \(rji\), a \(float\) type variable \(eps2\), and a \(float\) type variable \(r2\) store the displacement vector \(r_{ji} \equiv x_j - x_i, \epsilon^2\), and result of \(r_{ji}^2 + \epsilon^2\) calculated as Listing 1, respectively. The source codes shown in the Listing 1 look like almost the same, however, generated instruction sets are quite different. For implementation of CUDA SDK, one multiplication and two fused multiply-add (FMA) operations are performed at first, and one addition follows at the next step. Thus, its computational cost corresponds to 4 clock cycles according to CUDA C Programming Guide [33]. On the other hand, only three FMA operations are performed using 3 clock cycles in our implementation. Therefore, our implementation would be faster than CUDA SDK. The most influential point of this optimization is that the innermost loop includes the calculation of \(r^2\); thus, this small care directly increase performance.

Furthermore, we have implemented one additional optimization to hide memory access to the global memory. Since the shared memory stores the position data of \(j\)-particles, synchronization of whole threads within a block is necessary before and after updating information of \(j\)-particles. As far as a streaming multiprocessor contains multiple blocks, memory access time of a block can be hidden by overlapping with calculation of other blocks. However, such overlapping might not be occur since the CUDA schedulers determine which calculation of executable warp proceeds. Therefore, maximizing probability to occur the overlapping between calculation and memory
access can help to achieve high performance. For the purpose, we have separated the load instruction from the global memory and the store instruction to the shared memory by using the two \_syncthreads()\ . By this careful treatment, overlapping of memory access and calculation becomes more effective within a block, and the peak performance of our implementation reaches 1004 GFLOPS in single precision.

5. Results: Where is the Wandering MBH?

Results of Section 2.1 and Section 2.2 are shown in Section 5.1 and Section 5.2, respectively.

5.1. Current-Plausible Position of the MBH

The result of the $N$-body simulation to investigate the current plausible-position of the MBH when the infalling satellite follows Fardal’s orbit [25] is shown in Figure 1. The red curve on the mass distribution map shows the orbit of the MBH from 910 Myr ago to 320 Myr future. The position of the MBH with 99.7% confidence level is shown by a blue curve. The black circle indicates the most probable current position of the MBH, where it is approaching the Milky Way with a (heliocentric) line-of-sight velocity of $-170$ km s\(^{-1}\) (the combination of a receding velocity of $+130$ km s\(^{-1}\) in the M31 rest frame plus the approaching heliocentric velocity of M31 of $-300$ km s\(^{-1}\) [41]). The MBH is close to the apocenter, where it is closer to the Milky Way than the M31 center, which means that the velocity of the MBH is relatively slow, and the uncertainty of the current position is smaller than any other position such as pericenter. The distance of the MBH from the center of M31 is about 30 kpc, so it is far away from the disk of M31.

5.2. Results of Parameter Study

The first results for 34,000 runs (among $\sim 10^5$ in total) of the parameter study to investigate the infalling orbit which reproduce observed structures well are shown in Figure 2. Black circles represent results reproduce the stream structure, shell structure and contrast among the stream and the two shells. On the other hand, crosses are runs which failed to reproduce observed structures. The infalling orbit assumed in earlier studies ([25, 26, 27]) is infalling velocity of $-430$ km s\(^{-1}\), specific angular momentum of 660 kpc km s\(^{-1}\) (corresponds to the result shown in Figure 1). The Figure 1 shows that the maximum initial infalling velocity is about $-300$ km s\(^{-1}\). Furthermore, the escape velocity of the infalling satellite is about $-560$ km s\(^{-1}\) at 7.63 kpc away from the center of the M31. Therefore, the minimum initial infalling velocity is expected to be around $-560$ km s\(^{-1}\).

By completing this parameter survey, the plausible area where the wandering MBH exists will be restricted more tightly.

Figure 1. Mass distribution (column density) maps of the debris of the dwarf galaxy in standard coordinates centered on M31. Global distribution of $N$-body particles are shown as color image while white curves and lines show the disk of M31. White open squares represent observed points of the giant stellar stream [23], and white filled circles show edge of the observed shells [25]. Black circle shows the most probable current position of the MBH. Magenta and blue curve show the position of the MBH when the observed shells are reproduced with 95.4% and 99.7% confidence level, respectively. The red curve shows the orbit of the MBH from 910 Myr ago to 320 Myr future.
6. Summary
We investigate the current position of an MBH moving in the M31 halo. The current-plausible position of the MBH is within the halo, 30 kpc away from the center of M31. To determine the observational field for the future observational detection, we study uncertainty of the infalling orbit of the satellite by performing parameter study. Hence the required parameter space is large, we develop a highly optimized collisionless N-body code runs on GPU cluster, and we have performed a parameter study about the infalling orbit of the progenitor galaxy to determine the observation field to detect the MBH. The tentative results of the parameter study begin to restrict the possible parameter space of the infalling orbit which reproduce the observed structures well.

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References
[1] Magorrian J, Tremaine S, Richstone D, Bender R, Bower G, Dressler A, Faber S M, Gebhardt K, Green R,

Figure 2. Preliminary results for 34,000 runs of parameter study for the infalling orbit of the satellite. The horizontal axis is the infalling velocity of the satellite at 7.63 kpc away from the center of the M31, and the vertical axis represents the specific angular momentum of the infalling satellite. Filled circles corresponds to results reproduce the stream structure, shell structure and contrast among the stream and the two shells. Crosses represent results which failed to reproduce observed structures.
Grillmair C, Kormendy J and Lauer T 1998 *Astronomical Journal* **115** 2285–2305

[2] Marconi A and Hunt L K 2003 *Astrophysical Journal* **589** L21–L24

[3] Farrell S A, Webb N A, Barret D, Godet O and Rodrigues J M 2009 *Nature* **460** 73–75

[4] Wiersema K, Farrell S A, Webb N A, Servillat M, Maccarone T J, Barret D and Godet O 2010 *Astrophysical Journal* **721** L102–L106

[5] Bullock J S and Johnston K V 2005 *Astrophysical Journal* **635** 931–949

[6] Chiba M, Minezaki T, Kashikawa N, Kataza H and Inoue K T 2005 *Astrophysical Journal* **627** 53–61

[7] Minezaki T, Chiba M, Kashikawa N, Inoue K T and Kataza H 2009 *Astrophysical Journal* **697** 610–618

[8] Ibata R, Irwin M, Lewis G, Ferguson A M N and Tanvir N 2001 *Nature* **412** 49–52

[9] Ferguson A M N, Irwin M J, Ibata R A, Lewis G F and Tanvir N R 2002 *Astronomical Journal* **124** 1452–1463

[10] McConnachie A W, Irwin M J, Ibata R A, Ferguson A M N, Lewis G F and Tanvir N 2003 *Monthly Notices of the Royal Astronomical Society* **343** 1335–1340

[11] Irwin M J, Ferguson A M N, Ibata R A, Lewis G F and Tanvir N R 2005 *Astrophysical Journal* **628** L105–L108

[12] Ibata R, Martin N F, Irwin M, Chapman S, Ferguson A M N, Lewis G F and McConnachie A W 2007 *Astrophysical Journal* **671** 1991–2025

[13] McConnachie A W, Irwin M J, Ibata R A, Dubinski J, Widrow L M, Martin N F, Côté P, Dotter A L, Navarro J F, Ferguson A M N, Puzia T H, Lewis G F, Babul A, Barnby P, Bienaymé O, Chapman S C, Cockcroft R, Collins M L M, Fardal M A, Harris W E, Huxor A, Mackey A D, Peñarrubia J, Rich R M, Richer H B, Siebert A, Tanvir N, Valls-Gabaud D and Venn K A 2009 *Nature* **461** 66–69

[14] Tanaka M, Chiba M, Komiyama Y, Guhathakurta P, Kalirai J S and Iye M 2010 *Astrophysical Journal* **708** 1168–1203

[15] Richardson J C, Irwin M J, McConnachie A W, Martin N F, Dotter A L, Ferguson A M N, Ibata R A, Chapman S C, Lewis G F, Tanvir N R and Rich R M 2011 *Astrophysical Journal* **732** 76

[16] Ibata R, Chapman S, Ferguson A M N, Irwin M, Lewis G and McConnachie A 2004 *Monthly Notices of the Royal Astronomical Society* **351** 117–124

[17] Kalirai J S, Guhathakurta P, Gilbert K M, Reitzel D B, Majewski S R, Rich R M and Cooper M C 2006 *Astrophysical Journal* **641** 268–280

[18] Kalirai J S, Gilbert K M, Guhathakurta P, Majewski S R, Ostheimer J C, Rich R M, Cooper M C, Reitzel D B and Patterson R J 2006 *Astrophysical Journal* **648** 389–404

[19] Guhathakurta P, Rich R M, Reitzel D B, Cooper M C, Gilbert K M, Majewski S R, Ostheimer J C, Geha M C, Johnston K V and Patterson R J 2006 *Astronomical Journal* **131** 2497–2513

[20] Gilbert K M, Fardal M, Kalirai J S, Guhathakurta P, Geha M C, Isler J, Majewski S R, Ostheimer J C, Patterson R J, Reitzel D B, Kirby E and Cooper M C 2007 *Astrophysical Journal* **668** 245–267

[21] Koch A, Rich R M, Reitzel D B, Martin N F, Ibata R A, Chapman S C, Majewski S R, Mori M, Loh Y S, Ostheimer J C and Tanaka M 2008 *Astrophysical Journal* **689** 958–982

[22] Gilbert K M, Guhathakurta P, Kollipara P, Beaton R L, Geha M C, Kalirai J S, Kirby E N, Majewski S R and Patterson R J 2009 *Astrophysical Journal* **705** 1275–1297

[23] Font A S, Johnston K V, Guhathakurta P, Majewski S R and Rich R M 2006 *Astronomical Journal* **131** 1436–1444

[24] Fardal M A, Babul A, Geesan J J and Guhathakurta P 2006 *Monthly Notices of the Royal Astronomical Society* **366** 1012–1028

[25] Fardal M A, Guhathakurta P, Babul A and McConnachie A W 2007 *Monthly Notices of the Royal Astronomical Society* **380** 15–32

[26] Mori M and Rich R M 2008 *Astrophysical Journal* **674** L77–L80

[27] Miki Y, Mori M and Rich R M in preparation

[28] Barth A J, Greene J E and Ho L C 2005 *Astrophysical Journal* **619** L151–L154

[29] Xiao T, Barth A J, Greene J E, Ho L C, Bentz M C, Ludwig R R and Jiang Y 2011 *Astrophysical Journal* **739** 28

[30] Hernquist L 1990 *Astrophysical Journal* **356** 359–364

[31] Navarro J F, Frenk C S and White S D M 1996 *Astrophysical Journal* **462** 563

[32] Geesan J J, Fardal M A, Babul A and Guhathakurta P 2006 *Monthly Notices of the Royal Astronomical Society* **366** 996–1011

[33] Nvidia 2011 *NVIDIA CUDA C Programming Guide Version 4.1*

[34] Top500 list URL http://www.top500.org/

[35] HA-PACS Project URL http://www.ccs.tsukuba.ac.jp/CCS/eng/research-activities/projects/ha-pacs

[36] Hamada T and Itakura T 2007 *ArXiv Astrophysics e-prints* (Preprint arXiv:astro-ph/0703100)

[37] Nyland L, Harris M and Prins J 2007 *Fast N-Body Simulation with CUDA*
[38] Hamada T, Narumi T, Yokota R, Yasuoka K, Nitadori K and Taiji M 2009 42 TFlops hierarchical N-body simulations on GPUs with applications in both astrophysics and turbulence Proceedings of the Conference on High Performance Computing Networking, Storage and Analysis SC ’09 (New York, NY, USA: ACM) pp 62:1–62:12 ISBN 978-1-60558-744-8

[39] Hamada T and Nitadori K 2010 190 TFlops Astrophysical N-body Simulation on a Cluster of GPUs Proceedings of the 2010 ACM/IEEE International Conference for High Performance Computing, Networking, Storage and Analysis SC ’10 (Washington, DC, USA: IEEE Computer Society) pp 1–9 ISBN 978-1-4244-7559-9

[40] Miki Y, Takahashi D and Mori M 2012 Procedia Computer Science 9 96 – 105 ISSN 1877-0509 proceedings of the International Conference on Computational Science, ICCS 2012 URL http://www.sciencedirect.com/science/article/pii/S1877050912001329

[41] de Vaucouleurs G, de Vaucouleurs A, Corwin Jr H G, Buta R J, Paturel G and Fouqué P 1991 Third Reference Catalogue of Bright Galaxies. Volume I: Explanations and references. Volume II: Data for galaxies between 0° and 12°. Volume III: Data for galaxies between 12° and 24°.