A bar model of a temporary wooden support used to remove deflections of buildings

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Abstract. The deflection of buildings located in mining areas is removed by uneven raising of the overground part of the structure by means of supports consisting of a hydraulic piston jack and a stack of underpinning elements. Temporary supports may also consist of stacks of wooden elements only. The article presents a bar model of a temporary support consisting of a stack of elements. The model, which is described by a differential, second-order equation, allows to determine horizontal displacements. It was demonstrated that the unintended eccentricity and the initial curvature of the element have influence on such displacements. In addition, the effect of the load transmitted by the deformed adjacent supports is important. The model presented can be used to determine the limit load capacity of temporary supports due to unintended horizontal displacements of the structure. The model therefore allows to design safely the removal of building structures’ deflections.

1. Introduction
Underground hard coal mining causes land deformations [1], [2]. Those deformations cause damages of residential buildings [3] as well as historical objects [4]. One of such damages is the vertical deflection of buildings [5]. Such damage is removed by installing in the structure walls the temporary supports consisting of a stack of cuboidal elements and hydraulic piston jacks [6]. The structure is detached with the jacks and then its part situated above the detachment plane is lifted unevenly [7]. Using this method vertical deflection of residential building [8] as well as historical tower [9] were removed. There are still a lot of deflected buildings which need remove it’s deflection [10].

The vertical building movement during deflection removal is forced by forced extensions of jack pistons. The jack extension is limited to 200 mm, hence in the case where the structure should be lifted by more than 200 mm, it is necessary to periodically underlay the jacks with stacks of elements (Figure 1a). The stacks are made of wooden elements in case of buildings with up to two floors. For example, when it is necessary to lift a structure unevenly to a height of 1.2 m, the length of the stack of wooden elements under the jack is 1 m.

In many situations, a temporary support of buildings consists of stacks of wooden elements themselves (Figure 1b) - stiffness of such support was analysed in [11] and tested in [12]. This is the case, for example, when the deflection is removed and the jacks are dismantled, when the walls of the building are being restored (Figure 2) or when a local support of the structure is required. The observations made when removing building deflections indicate that if the stack already consists of several wooden elements, second-order effects appear. They manifest themselves as unintended displacements $u_{x1}$ of vertices of supports in a horizontal direction (and hence of the building resting on
them), i.e. in the direction perpendicular to the load $Q_z$ applied (Figure 1b). Moreover, first-order displacements $u_{z,l}$ occur in the direction of the acting force $Q_z$, which were subject to laboratory tests and are not the subject of this article.

![Figure 1. Temporary supports of buildings: a) support consisting of a jack (1) and a stack of wooden elements (2) with cross-section $b/h$ and thickness $g$, b) support consisting of a stack of wooden elements with the marked adopted coordinate system $(x, y, z)$, and marked displacements of the jack vertex ($u_{z,l}$, $u_{x,l}$) and marked forces acting on the support ($Q_z$, $Q_x$).](image)

It can be concluded based on the experience gained during the removal of building deflections that there are three main causes of second-order effects. The first one is occurrence of the unintended eccentricity $e_a$. If the force $Q_z$ is applied to the vertex of the support inaccurately, its axis does not coincide with the building weight vector (Figure 3a). The second is an imperfection resulting from an inaccurately assembled stack of wooden elements and the lack of full mutual adhesion of its elements. Then, even before the starting to remove the deflection, an initial bending of the stack appears with the amplitude $u_{0,x,l}$ (Figure 3b), which increases when the load is applied. The third cause for the occurrence of second-order deformations is the combined action of the vertical load $Q_z$ and the effect of the adjacent supports marked in Figure 3c as $Q_x$.

![Figure 2. Example of using temporary supports when removing building deflections: a) building resting on temporary supports after removing deflection during the restoration of walls, b) temporary wooden support installed in the building bearing wall](image)
The support analysed in the article is a stack of cubic elements with the cross-section $b/h$ and length $l$. The load $Q_z$ is applied to the vertex of the stack, which may be applied on the unintended eccentricity $e_a$, and the support may have an initial bending with the amplitude $u_{0,x,l}$. As a consequence of the load applied this way, the displacements $u_x$ of the support appear in the horizontal direction, and the displacement of the stack vertex is marked as $u_{x,l}$. In addition, adjacent supports may also act on the support, which, together with the effect of the load $Q_z$, also leads to second-order displacements.

In real situations, a structure is often supported by several dozen supports, which, as mentioned above, interact with each other forcing the displacement of the analysed support. On the other hand, however, they have a positive effect by reducing the buckling length of the supports on which the structure rests. This beneficial effect will not be considered in this article. This article analyses a support with the length $l$, whose buckling length, due to the adopted static scheme of the support, is $2l$. This situation therefore corresponds to the specific conditions in which all building supports have the same length and imperfections and are equally loaded.

![Figure 3. Deformation of supports caused by the vertical load $Q_z$ and: a) unintended eccentricity $e_a$, b) imperfection with the amplitude $u_{0,x,l}$, c) lateral force with the value $Q_x$.](image)

2. Bar model of the support

A bar model of a temporary support consisting of a stack of wooden elements was adopted. It is characterised by a rectangular cross-section with the height $h$, width $b$ and length $l$. The general situation in which the bar is loaded in the $x$ and $z$ axis directions is assumed. In this model, with omitting the bar deformations caused by lateral forces and the shortening of the $u_{x,l}$ of the bar element axis (Figure 3a), the relationship between the bar curvature and the bending moment is expressed by the formula

$$EI \frac{u''_x(z)}{(1 + (u'_x(z))^2)^2} = -M(z),$$

where $E$ is the modulus of elasticity of the material, $I$ is the moment of inertia of the cross section, $M$ is the bending moment value and $u'_x(z)$ and $u''_x(z)$ is the first and second derivative of the horizontal displacement $u_x$ relative to the variable $z$. Due to the fact that very small displacements are considered, the expression $u''_x(z)$ is disregarded, hence

$$EIu'_x(z) = -M.$$  

Considering that it is possible that the perpendicular load $q$ (Figure 4b) is acting on the supports and considering the commonly known relationships between this load and the lateral force $Q_x$, the differential equation of the bending line can be generally presented as
The analysed stack consists of unconnected wooden elements, therefore it does not transmit positive deformations. Therefore, the adopted model is correct if negative deformations exist in each cross-section of the support normal to its axis. This condition is satisfied if the resultant of the vertical forces is situated in the core of the cross-section, meaning \( u_{\text{tot},x}(z) \leq h/6 \) for each \( z \) (Figure 4c).

The subject of the considerations in the following points is the support model characterised by: the occurrence of unintended eccentricity \( e_a \) of the application of force \( Q_z \), the occurrence of an initial curvature characterised by the amplitude \( u_0 \), at the point where the force \( Q_z \) is applied and the support model whose vertex is simultaneously loaded with the forces \( Q_z \) and \( Q_x \).

\[
EI \frac{d^4 u_x}{dz^4} + Q_z \frac{d^2 u_x}{dz^2} = q. \tag{3}
\]

Figure 4. Support model: a) deformed support bar, b) support element of differential length \( dz \), c) condition concerning the maximum value of the eccentricity \( u_{\text{tot},x} \) of the action of the resultant of vertical force in relation to the element axis

3. Unintended eccentricity \( e_a \)

The analysed model is a support bar whose vertex is loaded with the force \( Q_z \) applied with the unintended eccentricity \( e_a \), as shown in Figure 5a. As a result of this load, the vertex of the bar is displaced by the value \( u_x(l) = u_{x,l} \). A bending moment with the following value exists in any cross-section of the bar

\[
M(z) = Q_z(u_{x,l} + e_a - u_x(z)). \tag{4}
\]

A differential equation of the bar model is obtained after substituting (4) to (2),

\[
EI u_x''(z) = -Q_z[u_{x,l} + e_a - u_x(z)]. \tag{5}
\]

The base of the bar must not move or rotate, thus boundary conditions arise

\[
\begin{align*}
u_x(0) &= 0, \\
u_x'(0) &= 0.
\end{align*} \tag{6}
\]

By assuming the substitution

\[
\zeta = u_{x,l} + e_a - u_x(z) \tag{7}
\]

and the symbol

\[
\beta^2 = \frac{Q_z}{EI} \tag{8}
\]

the following equation is obtained

\[
\zeta'' + \beta^2 \zeta = 0, \tag{9}
\]

whose solution takes the form of
The following is obtained after considering the substitution (7)
\[ u_x(z) = u_{x,l} + e_a - C_1 \sin(\beta z) - C_2 \cos(\beta z). \] (11)

After considering the initial conditions (6), the following is obtained
\[ C_2 = u_{x,l} + e_a, \quad C_1 = 0, \] (12)

hence
\[ u_x(z) = (u_{x,l} + e_a)[1 - \cos(\beta z)]. \] (13)

Considering that for \( z = l \) the following takes place
\[ u_x(l) = u_{x,l} \] (14)

the following is obtained
\[ u_{x,l} = e_a \frac{1 - \cos(\beta)}{\cos(\beta)}. \] (15)

The total eccentricity \( e_{u_{tot,x,l}} \) of the force in relation to the centre of gravity of the cross-section is the sum of the displacement (15) and eccentricity \( e_a \)
\[ u_{tot,x,l} = e_a \frac{1}{\cos(\beta)}. \] (16)

Calculations according to the dependency (4) - (15) were made for a support with a length of \( l = 1 \) m and a cross-section \( (b/h) \) of 400/300 mm made of wood with a modulus of elasticity \( E = 1 \) GPa and six values of the unintended eccentricity \( e_a \) of: 0, 10 mm, 20 mm, 30 mm, 40 mm and 50 mm. For this data, the critical load value \( Q_{crit} \), determined from the dependency
\[ Q_{crit} = \frac{\pi^2}{2l^2} EI \] (17)
is 2221 kN. A chart showing the displacements \( u_{x,l} \) of the vertex of the support model in the range of the force \( Q_z \) from zero to the critical value \( Q_{crit} \) is shown in Figure 5b. It can be seen that as the load \( Q_z \) increases, the displacement value increases, and \( u_{x,l} \) is aiming for an unlimited value with \( Q_z \) aiming for a critical value. However, this increase is greater for higher values of the eccentricity \( e_a \). The scope of validity of the model analysed here is however limited. On one hand, when designing the removal of building deflections, the force value in stacks of wooden elements is limited to 500 kN, on the other hand, the eccentricity value of the application of the vertical load resultant is limited to \( h/6 \) (Figure 4c). Therefore, Figure 5c shows the dependence \( Q_z - u_{x,l} \) limited to the value \( Q_z = 500 \) kN. However, Figure 5d shows the chart of the dependence \( Q_z - u_{tot,x,l} \). If the value \( u_{tot,x,l} = h/6 \) is reached, the load capacity of the model is exhausted. This is due to the fact that the resultant of the force \( Q_z \) goes beyond the core of the cross-section. The load capacity of the support is lost in a real situation for a real support. It should be noted that this situation would occur already with \( e_a = 40 \) mm and with the loading force of \( Q_z = 350 \) kN (Figure 5d).
Figure 5. Eccentric load of the support: a) support model loaded with the vertical force $Q_z$ applied to the unintended eccentricity $e_a$, b) displacements $u_{x,l}$ of the vertex of the support model in the action range of the force $Q_z$ from zero to the critical value $Q_{crit}$, c) displacements $u_{x,l}$ in the action range of the force $Q_z$ from zero to 500 kN, d) the eccentricity value $u_{tot,x,l} = e_a + u_{x,l}$ with marking the limit value of $h/6$.

4. Support imperfection

As presented above, the real supports consisting of a stack of elements are characterised by the occurrence of an initial curvature resulting from inaccuracies in the laying of cubic wooden elements and from uneven faces of the elements forming the stack. In order to describe the issue mathematically, an initial imperfection of the temporary support described by the following equation was assumed

$$u_{0,x}(z) = u_{0,x,l} \left(1 - \sin \frac{\pi z}{2l}\right).$$

The initially deformed axis of the support (imperfection) is represented by the dashed line shown in Figure 6a. The value of the bending moment existing in the support due to this imperfection is

$$M(z) = Q_z[u_{0,x,l} + u_{x,l} - u_{0,x}(z) - u_x(z)].$$

The model equation is obtained by substituting (19) for (2)

$$EIu_x''(z) = -Q_z[u_{0,x,l} + u_{x,l} - u_{0,x}(z) - u_x(z)].$$

In order to solve the equation (20), the increase in deflection is predicted to have the same shape as the initial bending of the bar, i.e.
By determining \( u''_x(z) \) of the dependence (21)

\[
    u''_x(z) = -u_{x,l} \left( \frac{\pi}{2l} \right)^2 \sin \frac{\pi z}{2l},
\]

and then by substituting the dependencies (21) and (22) to the formula (20), the following is obtained

\[
    -EIu_{x,l} \left( \frac{\pi}{2l} \right)^2 \sin \frac{\pi z}{2l} = -Q \left[ u_{0,x,l} + u_{x,l} - u_{0,x,l} \left( 1 - \sin \frac{\pi z}{2l} \right) \right].
\]

The following takes place after simplification

\[
    EIu_{x,l} \left( \frac{\pi}{2l} \right)^2 = Q_z \left[ u_{0,x,l} + u_{x,l} \right],
\]

from where the following is determined

\[
    u_{x,l} = \frac{u_{0,x,l}}{\left( \frac{\pi}{2l} \right)^2 \frac{EI}{Q_z} - 1}.
\]

It should be noted that

\[
    \left( \frac{\pi}{2l} \right)^2 \frac{EI}{Q_z} = Q_{\text{crit}},
\]

therefore

\[
    u_{x,l} = \frac{u_{0,x,l}}{Q_{\text{crit}} - 1}.
\]

The solution to the problem (20) is the function

\[
    u_x(z) = \frac{u_{0,x,l}}{Q_{\text{crit}}} \left( 1 - \sin \frac{\pi z}{2l} \right).
\]

The total eccentricity, being the sum of the imperfection and bending, which occurred after applying the force \( Q_z \), is

\[
    u_{\text{tot},x,l} = u_{x,l} + u_{0,x,l} = \frac{u_{x,0,l}}{1 - \frac{Q_z}{Q_{\text{krit}}}}.
\]

Figure 6b shows the chart of the dependence \( u_x - Q_z \) for six values of the amplitude \( u_{0,x,l} \) of the following imperfections: 0, 10 mm, 20 mm, 30 mm, 40 mm and 50 mm. It can be seen that as the load \( Q_z \) increases, the displacement value \( u_{x,l} \) increases, aiming for an unlimited value, with \( Q_z \) aiming for a critical value. This increase, as expected, is larger for larger values of eccentricity. The same dependence, limited to a value of the force \( Q_z \) of 500 kN, is shown in Figure 6c. Figure 6d shows the chart of the dependence \( Q_z - u_{\text{tot},x,l} \). The value \( u_{\text{tot},x,l} \) is the sum of the amplitude \( u_{0,x} \) and of the displacement of the vertex \( u_{x,l} \). If the value \( u_{\text{tot},x,l} = h/6 \) is exceeded, the load capacity of the model is exceeded. This is due to the fact that the resultant of the force \( Q_z \) goes beyond the core of the cross-section. The load capacity of the support is lost in a real situation for a real support. It should be noted that such failure would take place already with \( e_a = 40 \) mm and with a loading force of \( Q_z = 425 \) kN.

The comparison of the results of the calculations shown in Figures 5 and 6 shows that both, with the increase in the unintended eccentricity \( e_a \), similarly as with the increase in the amplitude of the imperfection \( u_{0,x} \), the displacement value \( u_{x,l} \) of the vertex is rising. However, the same values \( e_a \) correspond to the increase in the displacement \( u_{x,l} \) larger by 19% compared to the same values of the amplitude \( u_{0,x} \) of imperfections.
Figure 6. The deformation of the support characterised by the imperfection described with the dependence $u_0(x) = u_{0,x,l} \left(1 - \sin \frac{\pi x}{l}\right)$: a) support model loaded with the vertical force $Q_z$, b) displacements $u_{x,l}$ of the vertex of the support model in the action range of the force $Q_z$ from zero to the critical value $Q_{crit}$, c) displacements $u_{x,l}$ in the action range of the force $Q_z$ from zero to 500 kN, d) the eccentricity value $u_{tot,x,l} = e_a + u_{x,l}$ with marking the limit value of $h/6$

In real objects, the values of second-order displacements result from the simultaneous action of the unintended eccentricity, imperfections and the acting lateral load. In such a situation, the results of the individual effects add up. In practice, it must be assumed that the adverse effects of the eccentricity and imperfections cause displacements in the same direction, so that the total displacements are the sum of the displacements (16), (29).

$$u_{tot,x,l} = e_a \frac{1}{\cos(\beta)} + u_{0,x,l} \frac{Q_z}{Q_{crit}}$$  \hspace{1cm} (30)

The chart in Figure 7 shows that where $Q_z = 400$ kN and $e_a = u_{0,x,l} = 20$ mm, the load capacity of the support is already exhausted due to the loss of stability.
Figure 7. Total effect of occurrence of eccentricity $e_a$ and imperfections with the amplitude $u_{0,x,l}$ ($e_a = u_{0,x,l}$) with the values equal to 10 mm, 20 mm, 30 mm and 40 mm on the total eccentricity $u_{tot,x,l}$.

5. Deformation caused by lateral force

The gravity force of the building whose deflection is removed, has the direction coinciding with the direction of the axis $z$ of the support. However, horizontal forces resulting from deformation of adjacent supports may act on the support. An example of such a deformation is the rotation of the base of the adjacent support by the angle $\theta$ (Figure 8a). Therefore, the support model shown in Figure 8b is considered. This is a bar to which two forces, $Q_z$ and $Q_x$, were applied at the vertex. As a result of this load, the vertex of the support is displaced by the value $u_{x,l}$.

Figure 8. Horizontal load of the temporary support: a) the load $Q_x$ as a result of rotation by the angle $\theta$ of the adjacent support, b) the support model loaded with the vertical force $Q_z$ and horizontal force $Q_x$.

The change of the bending moment of the support loaded with the forces $Q_x$ and $Q_z$ is described by the relationship

$$M(z) = Q_x(l - z) + Q_z(u_{x,l} - u_x).$$

(31)

Considering a differential equation (2) of the deformed axis

$$E I u_x''(z) = Q_x(l - z) + Q_z(u_{x,l} - u_x)$$

(32)

and by introducing
the following equation is obtained
\[ EIu''(z) = rQ_x(l - z) + Q_z(u_{x,l} - u_x). \]
After substituting (8), the following is obtained
\[ u''(z) + \frac{r}{r} u_x = [u_{x,l} + r(l - z)]. \]
The solution to this equation is the function
\[ u_x(z) = \frac{Q_x}{Q_z} [-z + \frac{\sin \alpha - \sin \beta(l - z)}{\beta \cos \beta}]. \]
The greatest deflection is obtained for \( z = l \)
\[ u_{x,l} = \frac{Q_x}{Q_z} \left[ \frac{\tan \alpha}{\beta} - l \right]. \]

The dependencies presenting the effect of the force value \( Q_x \) on the displacements of the vertex of the support are shown in Figure 9a for six values of the force \( Q_x \) (0, 10 kN, 20, kN, 30 kN, 40 kN, 50 kN) and the value of the force \( Q_z \) changing from zero to the critical value. In turn, Figure 9b shows the values \( u_{x,l} \) for the force \( Q_x \) ranging from 0 to 100 kN and six values of the force \( Q_z \) of: 10 kN, 100 kN, 200 kN, 300 kN, 400 kN and 500 kN. As predicted, for the critical load, the values \( u_{x,l} \) reach unlimited values. In addition, as shown in (37) and in Figure 9b, the values of displacements \( u_{x,l} \) are proportional to the values of the horizontal displacement \( Q_x \).

![Figure 9](image_url)

**Figure 9.** The displacements of the vertex of the support loaded with the vertical force \( Q_z \) and horizontal force \( Q_x \): a) displacements \( u_{x,l} \) of the vertex of the support model in the action range of the force \( Q_x \) from zero to the critical value \( Q_{crit} \), b) displacements \( u_{x,l} \) for constant values of forces \( Q_z \) and variable values of horizontal forces \( Q_x \) in the range of 0 to 100 kN

6. **Conclusions**

Second-order deformations of supports occur when removing the deflection of structures located in mining areas due to inaccurate assembly of temporary supports. Such deformations are characterised by the occurrence of unintended horizontal displacements, perpendicular to the load applied. The article presents a bar model of temporary supports of buildings allowing to determine horizontal displacements of supports in the horizontal direction. The model is described by a differential, second-order equation. After setting the boundary conditions and load values, second-order displacements can be determined. It was demonstrated that the value of the unintended eccentricity and the value of the
initial curvature of the element have influence on such displacements. In addition, the effect of the horizontal load transmitted by the deformed adjacent supports is significant.

In real objects, the values of second-order displacements result from the simultaneous action of the unintended eccentricity, imperfections and the acting lateral load. In such a situation, the results of the individual effects add up. In practice, it must be assumed that the adverse effects of the eccentricity, imperfections and lateral load cause displacements in the same direction. if the support load of $Q_z = 400 \text{ kN}$ exists and at the same time the unintended eccentricity with the value $e_a = 20 \text{ mm}$ exists and the imperfection with the amplitude of $u_{0,x,t} = 20 \text{ mm}$, the load capacity of the support is already exhausted due to loss of stability. For this reason, the installation of temporary supports requires particular precision and experience of the people working on the removal of building deflections.

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