Quantum metrology beyond the classical limit under the effect of dephasing

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Quantum sensors have the potential to outperform their classical counterparts. For classical sensing, the uncertainty of the estimation of the target fields scales inversely with the square root of the measurement time $T$. On the other hand, by using quantum resources, we can reduce this scaling of the uncertainty with time to $1/T$. However, as quantum states are susceptible to dephasing, it has not been clear whether we can achieve sensitivities with a scaling of $1/T$ for a measurement time longer than the coherence time. Here, we propose a scheme that estimates the amplitude of globally applied fields with the uncertainty of $1/T$ for an arbitrary time scale under the effect of dephasing. We use one-way quantum computing based teleportation between qubits to prevent any increase in the correlation between the quantum state and its local environment from building up and have shown that such a teleportation protocol can suppress the local dephasing while the information from the target fields keeps growing. Our method has the potential to realize a quantum sensor with a sensitivity far beyond that of any classical sensor.

It is well known that two-level systems are attractive candidates with which to realize ultrasensitive sensors as the frequency of the qubit can be shifted by coupling it to a target field. Such a frequency shift induces a relative phase between the qubits basis states which can be simply measured in a Ramsey type experiment. This method has been used to measure magnetic fields, electric fields, and temperature. With the typical classical sensor measurement devices (including SQUID’s, Hall sensors, and force sensors), the uncertainty in the estimation of the target fields scales as $1/\sqrt{T}$ with a total measurement time $T$. This scaling is considered classical. With a qubit-based sensor using a Ramsey type measurement, the readout signal is periodic against the amplitude of the target fields. So, unless the range of the target fields is known, the interaction time with the target fields should be limited, which reduces the sensitivity. In this case, the sensitivity decreases as $1/\sqrt{T}$ if fast qubit control is available. Although one could achieve the uncertainty with $1/T$ by setting $t = T$ with the knowledge of the target field range, a dynamic range, which allows us to estimate the fields unambiguously, becomes small due to the periodic structure of the readout signal. Fortunately, there is an ingenious way to improve the dynamic range by using a feedback control of the qubit. Actually, several experimental demonstrations have shown a sensitivity that scales as $1/T$ with the high-dynamic range. However, as quantum states are susceptible to decoherence, it has generally been considered that such a scaling $1/T$ can only be realized if the measurement time $T$ is much shorter than the coherence time $\tau_c$. Recently several approaches have been proposed and demonstrated that use quantum error correction and dynamical decoupling to circumvent this limitation. Using quantum error correction, we can measure the amplitude of the target field with an uncertainty scaling as $1/T$ under the effect of specific decoherence such as bit flip errors, while dynamical decoupling makes it possible to estimate the frequency of time-oscillating fields with sensitivity beyond the classical limit on time scale longer than the coherence time. However, there is currently no known metrological scheme to achieve with an uncertainty of $1/T$ when measuring the amplitude of target fields with dephasing.

In this letter, we propose a scheme for measuring the amplitude of target fields with an uncertainty of $1/T$ under the effect of dephasing. We will use a similar concept to the quantum Zeno effect (QZE) for shorter time scales than the correlation time of the environment $\tau_c$, the interaction with the environment induces a quadratic decay rate that is much slower than the typical exponential decay. Frequent measurements can be used to reset the correlation with the environment and so keep this state in the initial quadratic decay region, which suppresses the decoherence. However, if we naively apply the QZE to quantum metrology, the frequent measurements freeze all the dynamics so that the quantum states cannot acquire any information from the target fields. Instead, we use quantum teleportation (QT) based on concepts taken from one-way quantum computation to reset the correlation between the system and the environment. If we transfer the quantum states to a new site, we can prevent any increase in the correlation between the system and environment in the previous site, and the quantum state are then only affected by a slow quadratic decay due to the local environment in the new site. This noise suppression with a qubit motion using a concept drawn from QT has been proposed and demonstrated by using superconducting qubits. The crucial idea in this paper is to use this one-qubit teleportation-based noise suppression for quantum metrology. Interestingly, although the QT protocol eliminates the deterioration effect caused by the dephasing from the local environment, we can accumulate the phase information from the global target fields during this protocol. We have shown that, as long as nearly perfect QT is available, we can achieve a sensor with the uncertainty scaling $1/T$ with dephasing. More-
over, we have found that, even when the QT is moderately noisy, the sensitivity of our protocol is superior to that of the standard Ramsey measurement.

Noise and its suppression.—Our system and the environment in this situation can be described by a Hamiltonian of the form \( H = H_S + H_I + H_E \), where \( H_S = \sum_{j=1}^{L} \omega_j^{(j)} \sigma_z^{(j)} \otimes I_E \) and \( H_E = \sum_{j=1}^{L} I_S^{(j)} \otimes C_j \) denotes the system (environmental Hamiltonian) while \( H_I = \sum_{j=1}^{L} \lambda \sigma_z^{(j)} \otimes B_j \) denotes the interaction between the system and the environment. Here \( \sigma_z^{(j)} \) is the usual Pauli Z operator of the \( j \)-th qubit with frequency \( \omega \), while \( B_j \) and \( C_j \) denote the environmental operator at that \( j \)-th site. \( I_S^{(j)} \) denotes an identity operator for the system (environment). Furthermore, we set \( \hbar = 1 \).

In an interaction picture, we have \( H_I(t) = \lambda \sum_{j=1}^{L} \sigma_z^{(j)} \otimes B_j(t) \) where \( B_j(t) = e^{iH_E t} B_j e^{-iH_E t} \). The separable initial state is given as \( \rho(0) = \bigotimes_{j=1}^{L} (\rho_S^{(j)}(0) \otimes \rho_E^{(j)}) \). We have assumed \( \rho_E^{(j)} \) is in thermal equilibrium \( (\rho_E^{(j)}, H_E) = 0 \) and our noise is non-biased \( (\text{Tr[}\rho_E^{(j)} B_j) = 0 \) for all \( j \). If the initial state is separable, we consider the first site by tracing out the others. Solving Schrödinger’s equation gives

\[
\rho_S^{(1)}(\tau) \simeq \rho_S^{(1)}(0) - i\lambda \int_0^\tau dt' [\sigma_z^{(1)} \otimes B_1(t')] \rho_S^{(1)}(0)] \quad \text{and} \quad \rho_E^{(1)}(\tau) \simeq \rho_E^{(1)}(0) - \lambda^2 \int_0^\tau dt' dt'' C_{\sigma_z^{(1)} B_1}(t', t'') \rho_E^{(1)}(0) \quad \text{with} \quad C_{\sigma_z^{(1)} B_1}(t', t'') \equiv \frac{1}{2i} \text{Tr} \{ [\hat{B}_1(t') \hat{B}_1(t'')] [\sigma_z^{(1)} \otimes \hat{B}_1(t'')] \rho_E^{(1)}(0) \}.
\]

Using a second order perturbation expansion in \( \lambda \) \([49]\). Tracing out the environment, we have

\[
\rho_S^{(1)}(\tau) \simeq \rho_S^{(1)}(0) - \lambda^2 \int_0^\tau dt' dt'' C_{\sigma_z^{(1)} B_1}(t', t'') \rho_E^{(1)}(0) \quad \text{with} \quad C_{\sigma_z^{(1)} B_1}(t', t'') \equiv \frac{1}{2i} \text{Tr} \{ [\hat{B}_1(t') \hat{B}_1(t'')] [\sigma_z^{(1)} \otimes \hat{B}_1(t'')] \rho_E^{(1)}(0) \}.
\]

where we define the correlation function of the environment as \( C_{\sigma_z^{(1)} B_1}(t', t'') \equiv \frac{1}{2i} \text{Tr} \{ [\hat{B}_1(t') \hat{B}_1(t'')] [\sigma_z^{(1)} \otimes \hat{B}_1(t'')] \rho_E^{(1)}(0) \} \). If we are interested in a time scale much shorter than the correlation time of the environment, we can approximate the correlation function as \( C_{\sigma_z^{(1)} B_1}(t', t'') \approx C_{\sigma_z^{(1)}}(t') \). For most solid state systems, this is readily satisfied as the environment correlation time is much longer than the coherence time of the qubit \( \approx 54-57 \), and so this condition is readily satisfied for many systems. In such a case \( \rho_S^{(1)}(\tau) \simeq (1 - \epsilon_{\tau} U_{1,\tau}) \rho_S^{(1)}(0) U_{1,\tau}^\dagger + \epsilon_{\tau} \rho_S^{(1)}(0) \) with \( U_{1,\tau} = e^{-i\lambda \tau \sigma_z^{(1)}/2} \).

Quantum metrology with QT.—Here, we focus on using the QT scheme to enable quantum metrology with an uncertainty scaling as \( 1/T \). Consider the situation in which the qubit frequency \( \omega \) is shifted depending on the amplitude of the target fields, and so measurement of the qubit’s frequency shift allows us to infer the amplitude of the target field. Such a qubit frequency shift is estimated from the relative phase between quantum states. The key idea is to use the QT in a ring arrangement with 2L qubits where each qubit has a tuneable
table: Performance of our teleportation based scheme with $L$ qubits for a given time $T$. Except with the general form, we show optimized sensitivity by choosing a suitable interaction time ($t$) and the QT number ($n$). The uncertainty of the standard Ramsey scheme is given as $\delta \omega_T = e^{1/4}/\sqrt{\gamma TL}$. With imperfect quantum teleportation that has an error rate of $p$, we can achieve a sensitivity scaling as $1/T$ using separable states (entangled states with a size $M$) for a short time scale of $T \ll 1/\sqrt{\gamma T}$ ($T \ll 1/\gamma M$). For a longer time scale, if accurate quantum teleportation is available ($p \ll 1$), the sensitivity of our scheme can be still better than the sensitivity of the standard Ramsey scheme. It is worth mentioning that, for the general form, perfect QT, and short T imperfect QT, the sensitivity of the separable sensor can be simply obtained by setting $M = 1$ in that of the entangled sensor.

| Entangled sensor | General form of sensitivity | Perfect QT | Short T imperfect QT | Long T imperfect QT |
|------------------|----------------------------|------------|---------------------|-------------------|
| $\delta \omega \approx \frac{\text{Exp}[[M \gamma^2 t^2/n]]}{(1-p)^{M-1}(1-\gamma/M)}$ | $\delta \omega \approx \frac{\text{Exp}[1/4]}{T \sqrt{M}}$ for $T \ll \sqrt{\gamma T}$ | $\delta \omega \approx 2\gamma/n$ for $T \gg \sqrt{\gamma T}$ |

| Separable sensor | $\delta \omega \approx \frac{\text{Exp}[\gamma t^2/n]}{(1-p)^{M-1}(1-\gamma/M)}$ | $\delta \omega \approx \frac{\text{Exp}[1/4]}{T \sqrt{M}}$ for $T \ll \sqrt{\gamma T}$ | $\delta \omega \approx 2\sqrt{\gamma T}/L$ for $T \gg \sqrt{\gamma T}$ |

Table I: Performance of our teleportation based scheme with $L$ qubits for a given time $T$. Except with the general form, we show optimized sensitivity by choosing a suitable interaction time ($t$) and the QT number ($n$). The uncertainty of the standard Ramsey scheme is given as $\delta \omega_T = e^{1/4}/\sqrt{\gamma TL}$. With imperfect quantum teleportation that has an error rate of $p$, we can achieve a sensitivity scaling as $1/T$ using separable states (entangled states with a size $M$) for a short time scale of $T \ll 1/\sqrt{\gamma T}$ ($T \ll 1/\gamma M$). For a longer time scale, if accurate quantum teleportation is available ($p \ll 1$), the sensitivity of our scheme can be still better than the sensitivity of the standard Ramsey scheme. It is worth mentioning that, for the general form, perfect QT, and short T imperfect QT, the sensitivity of the separable sensor can be simply obtained by setting $M = 1$ in that of the entangled sensor.

interaction with another qubit. Half of the qubits are used to probe the target fields while the remaining qubits are used as an ancilla for QT. The QT is accomplished by implementing a control-phase gate between a probe qubit and an ancilla qubit, followed by a $\sigma_z$ measurement on the probe qubit (and single qubit corrections depending on the measurement result). This QT approach has been widely used in one-way quantum computation [30, 42].

Scheme with entanglement. — Our scheme for measuring the amplitude of the target fields is as follows: First, we prepare GHZ states of $\bigotimes^{k=0}_{k=M} |\psi_k^{\text{(GHZ)}}\rangle$ between the probe qubits where $|\psi_k^{\text{(GHZ)}}\rangle = \frac{1}{\sqrt{M+2kM}} (\sum_{j=1+2kM}^{M+2kM} |0\rangle) + \sum_{j=1+2kM}^{M+2kM} |1\rangle$ for $k = 0, 1, \ldots, (\frac{M}{2}-1)$, while the other qubits (which we call ancillary qubits) are prepared in $|0\rangle$. Second we let the state evolve for time $\tau = t/n$ and then teleport the state of the probe qubit at the site $j$ to another site $j + M$. We assume that our gate operations are much faster than $\tau$. Third, we repeat the second step $(n-1)$ times, while in the fourth step we let the state evolve for time $\tau = \frac{\tau}{n}$, and readout the states. Finally, we repeat these steps $N$ times during time $T$ where $N \approx T/t$ is the repetition number.

We derive the sensitivity using imperfect QT and entanglement with general conditions, and subsequently discuss special cases. By letting the GHZ states $|\psi_k^{\text{(GHZ)}}\rangle$ evolve with low-frequency dephasing for time $\tau$, we have

$$\rho_k(\tau) = \frac{1}{2} \left( \frac{M+2kM}{M+2kM} \right) |0\rangle \langle 0| + \sum_{j=1+2kM}^{M+2kM} |1\rangle \langle 1| e^{-iM\omega t - M\gamma t^2}$$

$$\sum_{j=1+2kM}^{M+2kM} |0\rangle \langle 1| e^{-iM\omega t - M\gamma t^2} + \sum_{j=1+2kM}^{M+2kM} |1\rangle \langle 1|$$

for $k = 0, 1, \ldots, (\frac{M}{2}-1)$ where $\gamma$ denotes the dephasing rate for a single qubit. If we use the QT many times, we can suppress the low-frequency dephasing by employing the mechanism that we described before. To readout the GHZ states, we measure a projection operator defined by $\rho_+^{(k)} = |\psi_k^{(\pm)}\rangle \langle \psi_k^{(\pm)}|$ where $|\psi_k^{(\pm)}\rangle = \frac{1}{\sqrt{2}} \left( \sum_{j=1+2kM}^{M+2kM} |0\rangle \langle 0| \pm |1\rangle \langle 1| \right)$ for $k = 0, 1, \ldots, (\frac{M}{2}-1)$. We can then estimate the sensitivity in this situation as

$$\delta \omega_n, T, M = \frac{\sqrt{|d\delta P_\pm/d\omega| \sqrt{N}}}{(1-p)^M(n-1)\sqrt{MT}}$$

where $\delta \hat{P} = \hat{P} - \langle \hat{P} \rangle$ and $N \approx TL/tM$. From this general formula, we can derive many special cases by substituting parameters, which we will describe below. Also, a summary of these results is shown in Table I. By setting $n = 1$, we can reproduce the results discussed in [43, 46] for an entanglement based sensor with low-frequency dephasing.

For the perfect QT ($p = 0$), we achieve the Heisenberg limit $\delta \omega_{n,\text{opt}} = e^{1/4}/\sqrt{\gamma TL}$ when we set $t = T$, $M = cL$, and $n = 4M\gamma^2T^2$ where $c$ denotes a constant number. However, since the entangled state can be teleported to the original site where the entangled state previously interacted with the environment, correlated error may be induced due to the environmental memory effect. This can happen for $n \geq \tilde{n}_\text{en}$ where $\tilde{n}_\text{en}$ denotes the maximum teleportation number of the teleportation without the entangled state being teleported back to the original site. In this case, we have $\tilde{n}_\text{en} = \frac{T}{\gamma M} - 1$. The typical environment has a finite correlation time $\tau_c$. Unless the condition $\tilde{n}_\text{en} \tau_c \gg 1 \Leftrightarrow \gamma M^2 \tau_c \ll 1$ is satisfied, the error could be correlated (See the supplementary materials). Also, to observe the quadratic decay, we need a condition of $\gamma \tau_c \gg t/n$. This means that the correlation time should satisfy these two conflicting conditions. So, although we observe the Heisenberg limit scaling for a small $L$, the correlated error would begin to hinder the Heisenberg limit as we increase the size of the entangled state.

A natural question is what happens if our QT is imperfect and so we consider that here. For short times $T \ll 1/\sqrt{\gamma T} M$, the error due to the QT is negligible, and so we obtain the same results as in the perfect QT case by setting $t = T$ and $n = 4M\gamma^2T^2$. In quantum metrology, another interesting regime that is quite often considered is the scaling law in the limit of long $T$ (much greater than the coherence time of the system). We consider this here. We can minimize the uncertainty with $\tilde{n}_\text{opt} = \frac{N}{2p}\sqrt{\gamma/n}$ to obtain $\delta \omega_{n,\text{opt}} = e^{1/4}/\sqrt{\gamma T\sqrt{MM\gamma T}}$ for $p > 0$ and $n > 1$. Furthermore, with
$M_{\text{opt}} = -1/4 \log(1 - p) \simeq 1/4p$ and $n_{\text{opt}}^{(\text{en})} = 2$, the uncertainty can be minimized as $\delta \omega_{\text{opt}}^{(\text{GHz})} = 2^{1/4} \sqrt{\gamma/2T}$. In this case, the condition for the independent error ($n_{\text{en}} \tau \geq \tau_c$) is written as $Lp^{3/2}/\gamma > \tau_c$, and is satisfied for a large $L$.

![Diagram](image)

**FIG. 1:** Schematic illustration of $2L$ qubits in a ring structure to measure globally applied fields with an uncertainty scaling as $1/T$ when we use separable states. Half of the qubits contain information about the target fields as a probe while the remaining half are used as ancillary qubits for the qubit teleportation. With a controlled phase gate and measurement feedforward operations, we can teleport a quantum state $|\psi\rangle$ from the original site to the right neighboring site [30, 42]. After the teleportation, the measured qubit becomes the new ancilla which we initialize into $|0\rangle$.

**Scheme with separable states.** Now we explore a possibly more practical scheme with separable states, as shown in Fig. 1. We begin by preparing a probe state of $\bigotimes_{j=1}^{L} |+\rangle_{2j-1}$ located at the site $2j-1$ ($j = 1, 2, \ldots, L$). Then we let the state evolve for a time $\tau = t/n$ and teleport the state of the probe qubit to the next site using the ancillary qubit. We repeat this step $(n - 1)$ times before we finish by allowing our state to evolve for time $\tau = t/n$ and reading out the state by measuring $\hat{M}_y = \sum_{j=1}^{L} \delta \omega^{(j)}_{y}$. We repeat these steps $N$ times during the measurement time $T$ where $N \simeq T/t$ is the repetition number. We can calculate the sensitivity for this scheme by substituting $M = 1$ in Eq. 1. For an ideal QT, by setting $t = T$ and $p = 0$, we obtain $\delta \omega_{n,T} \simeq e^{1/4} / (\sqrt{T} \sqrt{L})$ for $n = 4\gamma^2 T^2$, and so we can achieve $1/T$ scaling. For $n \geq \hat{n}$, a correlated error may be induced due to the memory effect where $\hat{n} = 2L - 1$ denotes the maximum number of teleportations without the qubit state being teleported back to the original site. Fortunately, since the typical environment has a finite correlation time $\tau_c$, such a correlation effect becomes negligible for a large number of qubits to satisfy $\hat{n} \tau \gg \tau_c \Leftrightarrow L \gg \gamma^2 T \tau_c$ (See the supplementary materials).

We now analyze how imperfect QT affects the performance of our sensing scheme. We can calculate the sensitivity by substituting $M = 1$ and $p > 0$ with Eq. 1. For a short time such as $T \ll 1/\sqrt{\gamma}$, the error due to QT is negligible, and so we obtain the same results as with perfect QT by setting $t = T$ and $n = 4\gamma^2 T^2$, which allows us to achieve uncertainty scaling as $1/T$. We can minimize the uncertainty by setting $t_{\text{opt}} = \sqrt{\pi/2\gamma}$ as long as $T \gg t_{\text{opt}}$ is satisfied. In such a case, $\delta \omega_{n,t_{\text{opt}}} = \sqrt{\pi/4} \sqrt{nT/L}$, which for $n = 1$ gives the standard Ramsey uncertainty $\delta \omega_{\text{Ram}} = \sqrt{1/4L} \sqrt{\gamma}$, where we replace $L$ with $2L$ (because the standard Ramsey scheme can utilize every qubit to probe the target fields without ancillary qubits). For $n > 1$, we can treat $n$ as a continuous variable, and we can analytically minimize the uncertainty as $\delta \omega_{\text{opt}} \sim 2^{1/4} (\gamma/\sqrt{T})$ for $1/16 \gamma^2 T^2 \ll p \ll 1$ where we choose $n_{\text{opt}} = -1/4 \log(1 - p) \simeq 1/4p$. The condition required for the error to be independent ($\hat{n} \tau \geq \tau_c$) is written as $L\sqrt{\gamma}/\gamma \gg \tau_c$, and is satisfied for a large $L$. In this case, we have a constant factor improvement over the standard Ramsey scheme for a longer $T$. In fact, as long as $p < 0.0251$, our scheme is better than that standard Ramsey scheme ($\delta \omega_{\text{Ram}}/\delta \omega_{\text{opt}} > 1$). For $p = 10^{-4}$, we obtain $\delta \omega_{\text{Ram}}/\delta \omega_{\text{opt}} \simeq 3.89$. So our sensor has an advantage with finite errors caused by the imperfect QT.

In conclusion, we have proposed a scheme designed to achieve sensitivity beyond the classical limit and to measure the amplitudes of globally applied fields. We have found that frequent implementations of quantum teleportation provide a suitable circumstance for sensing where the dephasing is suppressed while the information from the target fields is continuously accumulated. If perfect quantum teleportation is available, the uncertainty scales as $1/T$ with our scheme while any classical sensor shows the uncertainty scaled as $1/\sqrt{T}$. Moreover, even when quantum teleportation is moderately noisy, our protocol still realizes superior quantum enhancement to the standard Ramsey scheme. This work was supported by JSPS KAKENHI Grant No. 15K17732 and partly supported by MEXT KAKENHI Grant No. 15H05870. S.C.B. acknowledges support from the EPSRC NQIT Hub, Project No. EP/M013243/1.


**Supplementary materials:** This is a supplementary material of the paper titled “Quantum metrology beyond the classical limit under the effect of dephasing”.

**Error model**

In the main text, we consider a specific noise model with which to calculate the sensitivity of our sensor, and we will explain how we can derive the model from the general expression. We consider the following general Hamiltonian to describe the dephasing.

\[
H = H_S + H_I + H_E
\]

where \( H_S, H_I, \) and \( H_E \) denote a system, an interaction, and an environmental Hamiltonian, respectively. As we describe in the main text, the perturbation theory allows us to solve the Schrödinger equation, and we obtain

\[
\rho_S(\tau) \simeq (1 - \epsilon\tau) U_\tau \rho_S(0) U_\tau^\dagger + \epsilon\tau \hat{\sigma}_z U_\tau \rho_S(0) U_\tau^\dagger \hat{\sigma}_z
\]

(3)

where \( \epsilon = \lambda^2 C_0 \tau^2 \) denotes an error rate for \( \lambda^2 C_0 \tau^2 \ll 1 \) and \( U_\tau = e^{-i\omega \tau \hat{\sigma}_z / 2} \) denotes a unitary operator. We could approximate this expression by

\[
\rho_S(\tau) \simeq \frac{1 + e^{-2\lambda^2 C_0 \tau^2}}{2} U_\tau \rho_S(0) U_\tau^\dagger \hat{\sigma}_z U_\tau \rho_S(0) U_\tau^\dagger \hat{\sigma}_z
\]

(4)

Actually, by performing a Taylor expansion such as \( e^{-2\lambda^2 C_0 \tau^2} \simeq 1 - 2\lambda^2 C_0 \tau^2 \) in Eq. (4), we can reproduce Eq. (3). By defining \( \gamma^2 = 2\lambda^2 C_0 \), we obtain \( \rho_S(\tau) \simeq \frac{1 + e^{-\gamma^2}}{2} U_\tau \rho_S(0) U_\tau^\dagger + \frac{1 - e^{-\gamma^2}}{\gamma^2} \hat{\sigma}_z U_\tau \rho_S(0) U_\tau^\dagger \hat{\sigma}_z. \) In the main text, we use this expression to quantify the effect of the dephasing during the operations in our sensing protocol. It is worth mentioning that, strictly speaking, the form of the Eq. (4) is the same as Eq. (3) only when \( \gamma \tau \ll 1 \). However, typical dephasing models [53-55] actually show the behavior described in Eq. (4) for an even longer time if the correlation time of the environment is much longer than the dephasing time, and this shows the validity of our assumption.

**Independent error accumulation**

In our scheme, the environment interacts with teleported states of the qubit one after another. Here, we consider the decoherence dynamics for this case in more detail than in the main text. Specifically, we will show that, although the environment frequently interacts with the teleported states, the phase error on the qubit can be independent as long as a Born approximation is valid.

For simplicity, we consider just two sites for a Hamiltonian of the form \( H = H_S + H_I + H_E \) where \( H_S = \sum_{j=1}^2 \frac{\lambda}{2} \sigma_j^z \) (\( H_E = \sum_{j=1}^2 \rho_{Ej} \otimes C_j \)) denotes the system (environmental) Hamiltonian, \( H_I = \sum_{j=1}^2 \lambda \sigma_j^z \otimes B_j \) denotes an interaction between the system and the environment. \( B_j \) and \( C_j \) denote the environmental operators. In the interaction picture, we have \( H_I(t) = \lambda \sum_{j=1}^2 \sigma_j^z \otimes \tilde{B}_j(t) \) where \( \tilde{B}_j(t) = e^{-iH_Et} B_j e^{iH_E t} \). The initial state is given as \( \rho(0) = \rho_S(0) \otimes \rho_S(0) \otimes \rho_E(1) \otimes \rho_E(2) \). We also assume that \( \rho_E^{(j)} \) is in thermal equilibrium such that \( [\rho_E^{(j)}, H_E] = 0 \) and our noise is non-biased such that \( \text{Tr}[\rho^{(j)} \rho_E^{(j)}] = 0 \). We consider the following case. After the environment interacts with the system qubit at each site for a time \( 0 \leq t \leq \tau \), we perform a quantum teleportation to send the state at site 1 to site 2 (while the state at site 2 is teleported to another site), and the environment interacts with the new state at each site for a time \( \tau \leq t \leq 2\tau \). To describe the interaction between the system and environment before the quantum teleportation, we use the Schrödinger equation

\[
\frac{d\rho(t)}{dt} = -i[H(t), \rho(t)].
\]

(5)

By integrating this, we obtain

\[
\rho(t) = \rho(0) - i \int_0^t dt' [H(t'), \rho(0)]
\]

(6)

Since there are no interactions between site 1 and site 2, we consider that the state at site 1 is separable from the state at the site 2.

\[
\rho_1^{(j)}(\tau) \simeq \rho_1^{(j)}(0) - i \lambda \int_0^\tau dt' \sigma_j^z \otimes \tilde{B}_j(t'), \rho(0)]
\]

(7)

\[
-\lambda^2 \int_0^\tau dt' dt'' [\sigma_j^z \otimes \tilde{B}_j(t'), [\sigma_j^z \otimes \tilde{B}_j(t''), \rho(0)]
\]

(8)

for \( j = 1, 2 \) where we use a second order perturbation expansion in \( \lambda \). We use the Born approximation \( \rho_1^{(j)}(\tau) \simeq \rho_S^{(j)}(\tau) \otimes \rho_E^{(j)} \) [49]. This means that, since the environment has a large degree of freedom, the correlation between the system and the environment is negligible. In fact, it is known that the Born approximation becomes more accurate as the size of the bath is increased [50-51]. By tracing out the environment, we obtain

\[
\rho_S^{(j)}(\tau) \simeq \rho_S^{(j)}(0)
\]

(9)

\[
-\lambda^2 \int_0^\tau dt' dt'' C^{(j)}_{\tau-t} \left[ \sigma_j^z, [\sigma_j^z, \rho_S^{(j)}(0)] \right]
\]

(10)
for \( j = 1, 2 \) where \( C_{r-s}^{(j)} \equiv \frac{1}{2} \text{Tr}[(\hat{B}_j(t')\hat{B}_j(t''))(\rho_E^{(j)})] \) denotes the correlation function of the 

environment at the site \( j \). It is worth mentioning that the 
correlation function does not depend on the state of the system but 
depends on the properties of the environment. By performing 
a quantum teleportation to send the state at site 1 to site 2 
(while the state at site 2 is teleported to another site), we 

obtain 

\[
\rho_S^{(2)}(\tau) \simeq \rho_S(0)
\]

\[
-\lambda^2 \int_0^\tau \int_0^t dt' dt'' C_{t-t''}^{(1)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

at site 2. By allowing this state to interact with the 
environment for a time \( \tau \leq t \leq 2\tau \), we obtain 

\[
\rho_S^{(2)}(2\tau) \simeq \rho_S^{(2)}(\tau) - i\lambda \int_0^{2\tau} dt' \lambda [\sigma_z^2(t') \otimes \hat{B}_2(t'), \rho_S^{(2)}(\tau)]
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' [\sigma_z^2(t') \otimes \hat{B}_2(t''), [\sigma_z^2(t'') \otimes \hat{B}_2(t''), \rho_S^{(2)}(\tau)]
\]

where we consider \( \rho_S^{(2)}(\tau) = \rho_S^{(2)}(\tau) \otimes \rho_E^{(2)} \) as the initial 
state. By tracing out the environment, we obtain 

\[
\rho_S^{(2)}(2\tau) \simeq \rho_S^{(2)}(\tau) \otimes \rho_E^{(2)}
\]

By considering up to the second order term 
of \( \lambda \), we obtain 

\[
\rho_S^{(2)}(2\tau) \simeq \rho_S(0)
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(1)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(2)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

where we use a Born approximation such as 

\( \rho_S^{(2)}(2\tau) \simeq \rho_S^{(2)}(\tau) \otimes \rho_E^{(2)} \). By considering up to the second order term 
of \( \lambda \), we obtain 

\[
\rho_S^{(2)}(2\tau) \simeq \rho_S(0)
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(1)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(2)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

Since \( C_{t-t''}^{(2)} \) does not depend on the absolute value of \( t' \) (or 

\( t'' \)) but depends on the time difference \( t' - t'' \), we obtain 

\[
\rho_S^{(2)}(2\tau) \simeq \rho_S(0)
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(1)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

\[
-\lambda^2 \int_0^{2\tau} \int_0^t dt' dt'' C_{t-t''}^{(2)}(\hat{\sigma}_z^2, [\hat{\sigma}_z^2, \rho_S(0)])
\]

So the interaction between the system and the environment at 
site 1 for a time \( 0 \leq t \leq \tau \) does not induce a 
correlation in the decoherence dynamics between the 

system and environment at site 2. (There is no effect such as an accelerated or 

decelerated noise accumulation of the dephasing at site 2 due 
to past decoherence at site 1.) So the \( \hat{\sigma}_z \) error caused by the 
environment at site 2 acts on the qubit independently of the \( \hat{\sigma}_z \) 
error caused by the environment at site 1.

Although we consider the dynamics at sites 1 and 2 above, 
we can easily generalize this to \( n \) sites. By repeating the above 
calculations, we obtain 

\[
\rho_S^{(n)}(n\tau) \simeq \rho_S(0)
\]

\[
-\sum_{j=1}^n \lambda^2 \int_0^\tau \int_0^{t'} dt'' C_{t''-t'}^{(j)} \left[ \hat{\sigma}_z^{(j)}, [\hat{\sigma}_z^{(j)}, \rho_S(0)] \right]
\]

at site \( n \) after \( n-1 \) teleportation where the \( \hat{\sigma}_z \) error accumu-
lates independently at each site, and this is consistent with the 
calculations in the main text.

**Effect of a finite number of the qubits**

In the main text, we did not mention the case where the 
qubits are teleported back to the original sites whose envi-
ronment the qubit initially interacted with. If qubits have the 
chance to interact with the environment with which they pre-
viously interacted, this may induce a correlated error on the 
qubit. However, we will show that we can still avoid the cor-
related error as long as there are a large number of the qubits, 
which is a common assumption in the field of quantum metrol-
ogy..

For simplicity, we consider just a single site for a Hamil-
tonian of the form \( H = H_S + H_1 + H_E \) where \( H_S = \frac{1}{2} \hat{\sigma}_z \) 
\( H_E = 1_S \otimes C_j \) denotes the system (environmental) Hamilto-

nian, and \( H_1 = \lambda f(t) \sigma_z \otimes B_j \) denotes an interaction between 

the system and the environment. \( B \) and \( C \) denote envi-
ronmental operators. We define \( f(t) \) as follows 

\[
f(t) = \begin{cases} 
1 & (0 \leq t \leq \tau) \\
1 & ((m-1)\tau \leq t \leq m\tau) \\
0 & \text{(otherwise)} 
\end{cases}
\]

where \( m \) denotes a natural number. This means that, after the 
qubit interacts with the environment for a time \( 0 \leq t \leq \tau \), the 
qubit is decoupled from the environment for a time \( t < n\tau \), and the qubit interacts with the same environment 
again for a time \( m\tau \leq t \leq (m+1)\tau \). From this calculation, 
we can estimate the way in which the correlated error will be 
induced by multiple interactions with the same environment in 
our scheme. Using a similar calculation to that used in the 
previous section, we obtain 

\[
\rho_S(n\tau) \simeq \rho_S(0)
\]

\[
-\lambda^2 \int_0^{(m-1)\tau} \int_0^t dt'' f(t') f(t'') C_{t''-t'} \left[ \hat{\sigma}_z, [\hat{\sigma}_z, \rho_S(0)] \right]
\]

where we solve the Schrodinger equation for a time \( 0 \leq t \leq 

\]
We define
\[
\rho_S(m\tau) - \rho_S(0) \\
= -\lambda^2 \int_0^\tau dt' \int_0^{t'} dt'' f(t') f(t'') C_{\nu' - \nu''} \rho_S(0)
\]
and the correlated effect of the environment becomes negligible. So the decoherence acts on the qubit independently as long as we have \(m\tau \gg \tau_c\). We define \(\hat{n}\) as the maximum teleportation number without the qubit state being teleported back to the original site. (For example, in the teleportation-based scheme with a ring structure in the main text for separable states, we have \(\hat{n} = 2L - 1\).) We can substitute \(m = \hat{n}\), and the condition needed for the error to be independent is \(\hat{n}\tau \gg \tau_c\).

### High frequency noise

In the main text, we consider the effect of dephasing as decoherence. There are of course other sources of decoherence that cannot be suppressed by the quantum teleportation (QT) protocol, and we consider such an effect. If our quantum systems are affected by high-frequency noise with a short correlation time, the decay is not quadratic in nature but more exponential-like. Energy relaxation in a high temperature environment is known to induce such noise \([61]\). Consider the situation in which an initial state \(|+\rangle\rangle\) evolves under the effect of both low-frequency dephasing and high-frequency noise for a time \(\tau\). In such a case \(\rho_S^{(1)}(\tau) = \frac{1}{2}(|1\rangle\langle 1|) e^{-i\omega\tau - \gamma^2\tau^2 - \Gamma\tau} + |0\rangle\langle 0|) e^{i\omega\tau - \gamma^2\tau^2 - \Gamma\tau}\) where \(\Gamma\) denotes the decay rate associated with the high-frequency noise (\(\Gamma = 0\) gives the same noise model as the one we used previously). It is then straightforward to calculate the uncertainty of the estimation under the effect of this noise with imperfect QT as \(\delta\omega_{n,t} = \frac{1}{(1-p)^{-1/2} \sqrt{LT}}\). Choosing \(t_{opt} = \frac{\sqrt{nm + \sqrt{nm^2 + 1}}}{4(\gamma/\sqrt{n})}\), we minimize this with respect to time as

\[
\delta\omega_{n,t_{opt}} = \frac{2\sqrt{\gamma e^{2 + \frac{1}{nm^2 + 1}}}}{(1-p)^{-1/2} \sqrt{LT}}.
\]

The uncertainty with \(n\) can be numerically minimized as \(\delta\omega_{opt} = \min_n \delta\omega_{n,t_{opt}}\). In Fig. 2 we plot \(\delta\omega_{opt}\) versus \(p\) and \(\Gamma/\gamma\) where \(\delta\omega_{opt}\) is the uncertainty for the standard Ramsey scheme. Our plots shows that our scheme performs better than the standard Ramsey scheme for \(\Gamma/\gamma < 0.130\) and \(p < 0.0123\).

![FIG. 2: Performance of our teleportation based scheme against the standard Ramsey scheme. We plot \(\delta\omega_{QT}/\delta\omega_{opt}\) against \(\frac{1}{\gamma}\) and \(p\). Our scheme performs better than the standard Ramsey scheme for \(\Gamma/\gamma < 0.130\) and \(p < 0.0123\).](image-url)

**Implementation of quantum metrology beyond the classical limit under the effect of dephasing with global control**

In the scheme described in the main text, we suggest the use of quantum teleportation to suppress dephasing. To implement this scheme, the quantum teleportation, which involves projective measurements and feedback operations, should be performed in a much shorter time than the correlation time of the environment. Also, the individual controllability of the
qubit is needed to implement quantum teleportation for every qubit. In the state of the art technology, a gate control with a fidelity of more than 99% has been realized for some systems such as superconducting qubits and ion trap systems [62, 64]. Moreover, many groups aim to realize a scalable quantum computer, and a quantum supremacy with 50 qubits could be demonstrated in the near future [65, 66]. Such a development of quantum technology toward the demonstration of quantum computation also supports the realization of our teleportation-based scheme. So our scheme is within the reach of such a future technology.

However, it is still worth discussing how to realize our scheme with simpler technology and thus improve the practicality. In this section, we suggest an alternative scheme for this purpose, which is useful for our separable states scheme. Here, we use a direct interaction between the qubits. We show that a simple modulation of the Hamiltonian can transfer the information of the target fields as a probe while the remaining half are used as ancilla qubits kept in a ground state. We assume a flip-flop type interaction between nearest neighbor qubits. For the half of the qubits, the frequency is either \( \omega_A \) or \( \omega_B \) where \( \omega_A \) is well detuned from \( \omega_B \), while a frequency of the other half of the qubits is well detuned from both \( \omega_A \) and \( \omega_B \). The modulation of the frequency with a flip-flop type interaction provides us with a way to transfer the state of the probe qubit to the clockwise direction as shown in the Fig. 3.

![Fig. 3: Schematic illustration of our SWAP-based sensing where we arrange 2L qubits in a ring structure. Similar to the teleportation-based scheme described in the main text, half of the qubits contain an information of the target fields as a probe while the remaining half are used as ancilla qubits kept in a ground state. We assume a flip-flop type interaction between nearest neighbor qubits.](image)

First, we arrange qubits in a ring structure, and we prepare a state of \( \otimes^L_{j=1} |+\rangle_{2j-1} \) located at site \( 2j-1 \) \( (j = 1, 2, \ldots, L) \) for probe qubits, while we prepare \( \otimes^L_{j=1} |0\rangle_{2j} \) located at site \( 2j \) \( (j = 1, 2, \ldots, L) \) for the ancillary qubits. For simplicity, we assume that \( L \) is an even number. Second, we then let the state evolve for a time \( \tau = t/n \) and then perform a SWAP gate between the probe qubit and the ancillary qubit at the neighboring site on the right (we assume that our gate operations are much faster than \( \tau \)). Third, we repeat the second step \( (n-1) \) times while in the fourth step we allow this state to evolve for a time \( \tau = \frac{t}{N} \), and readout the state by measuring \( \hat{M}_y = \sum_{j=1}^L \hat{\sigma}_y^{(j)} \). Finally, we repeat these steps \( N \) times during the measurement time \( T \) where \( N \sim T/t \) is the repetition number.

![Fig. 4: Modulation of the frequency of the qubit to implement our SWAP-based scheme. The qubit frequency is either \( \omega_A \), \( \omega_B \), or \( \omega_C \) where these frequencies are well detuned from each other. Also, \( g \) denotes the coupling strength between the qubits and \( \tau = \frac{t}{n} \) denotes an interaction time with the fields. Here, we assume \( \tau \gg \frac{1}{\gamma} \). The modulation of the frequency with a flip-flop type interaction provides us with a way to transfer the state of the probe qubit to the clockwise direction as shown in the Fig. 3.](image)

Importantly, we can implement the SWAP gate by using a direct interaction between the qubits, and we can turn on/off the interaction by modulating the frequency of the qubit. We consider the following flip-flop type Hamiltonian.

\[
H = \sum_{j=1}^{2L} \frac{\omega_j + \omega_{j+1}}{2} \hat{\sigma}_z^{(j)} + g (\hat{\sigma}_+^{(j)} \hat{\sigma}_-^{(j+1)} + \hat{\sigma}_-^{(j)} \hat{\sigma}_+^{(j+1)})
\]

where \( \omega_j \) denotes the frequency of the qubit at the \( j \)th site. 

The main difference from the scheme described in the main text is that, instead of quantum teleportation, we use a SWAP operation between the probe qubit at a site and an ancillary qubit at the neighboring site on the right, as shown in Fig. 4.
We also consider a periodic condition such as $\hat{\sigma}_\pm^{(2L+1)} = \hat{\sigma}_\pm^{(1)}$. We set the frequency of the qubit as

$$\omega_j = \begin{cases} 
\hat{\omega}(t) & j \equiv 1 \pmod{4} \\
\omega_A & j \equiv 2 \pmod{4} \\
\hat{\omega}'(t) & j \equiv 3 \pmod{4} \\
\omega_B & j \equiv 4 \pmod{4}
\end{cases}$$

where $\hat{\omega}(t)$ and $\hat{\omega}'(t)$ denote a tunable time-dependent frequency, as shown in Fig. 4. In addition, we assume a large detuning between $\omega_A$ and $\omega_B$. Here, we focus on two adjacent qubits, and the Hamiltonian is given as

$$H^{(k,k+1)} = H_0^{(k,k+1)} + H_1^{(k,k+1)}$$

$$H_0^{(k,k+1)} = \omega + \hat{\omega}_k \sigma_z^{(k)} + \omega' + \hat{\omega}_{k+1} \sigma_z^{(k+1)}$$

$$H_1^{(k,k+1)} = g(\hat{\sigma}_+^{(k)} \sigma_-^{(k+1)} + \hat{\sigma}_-^{(k)} \sigma_+^{(k+1)})$$

where one of the qubits is a probe and the other qubit is an ancilla. In an interaction picture, we have

$$H_I^{(k,k+1)} = \frac{\hat{\omega}_k - \hat{\omega}_{k+1}}{2} \sigma_z^{(k)} + g(\hat{\sigma}_+^{(k)} \sigma_-^{(k+1)} + \hat{\sigma}_-^{(k)} \sigma_+^{(k+1)})$$

If we have large detuning between these two qubits such as $|\hat{\omega}_k - \hat{\omega}_{k+1}| \gg g$, we obtain

$$H_I^{(k,k+1)} \approx \frac{\hat{\omega}_k - \hat{\omega}_{k+1}}{2} \sigma_z^{(k)}$$

where the coupling is effectively turned off. On the other hand, if we have a resonant condition such as $\hat{\omega}_k = \hat{\omega}_{k+1}$

$$H_I^{(k,k+1)} = g(\hat{\sigma}_+^{(k)} \sigma_-^{(k+1)} + \hat{\sigma}_-^{(k)} \sigma_+^{(k+1)})$$

In this case, the interaction is turned on, and a unitary operation $U = e^{-i H_I^{(k,k+1)} \hat{\tau}}$ provides us with a SWAP gate between the probe qubit and ancillary qubit up to local operations. This means that, if we set $\hat{\omega}(t) = \omega_A$ ($\hat{\omega}(t) = \omega_B$), a probe qubit and ancillary qubit with a frequency of $\omega_A$ ($\omega_B$) start the interaction under a resonant condition, while these qubits do not interact with the other qubits with a frequency of $\omega_B$ ($\omega_A$). On the other hand, if we set $\hat{\omega}(t) = \omega_C$ where $|\omega_A - \omega_C| \gg g$ and $|\omega_B - \omega_C| \gg g$, the qubit does not interact with any other qubits due to the large detuning. Therefore, the simple modulation of the qubit frequency (described in Fig. 4) realizes qubit motion in the same manner as that described in the main text with quantum teleportation. This is much more practical than the teleportation-based scheme, because neither individual addressability nor a rapid measurement is required for the SWAP-based scheme.

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[1] D. Budker and M. Romalis, Nature Physics 3, 227 (2007).
[2] G. Balasubramanian and et al, Nature 455, 648 (2008).
[3] J. Maze and et al, Nature 455, 644 (2008), ISSN 0028-0836.
[4] C. Degen, F. Reinhard, and P. Cappellaro, arXiv preprint arXiv:1611.02427 (2016).
[5] J. Simon, Advances in Physics 48, 449 (1999).
[6] A. Chang, H. Hallen, L. Harriott, H. Hess, H. Kao, J. Kwo, R. Miller, R. Wolfe, J. Van der Ziel, and T. Chang, Applied physics letters 61, 1974 (1992).
[7] M. Poggio and C. Degen, Nanotechnology 21, 342001 (2010).
[8] S. Huelga, C. Macchiavello, T. Pellizzari, A. Ekert, M. Plenio, and J. Cirac, Phys. Rev. Lett. 79, 3865 (1997).
[9] R. Said, D. Berry, and J. Twamley, Physical Review B 83, 125410 (2011).
[10] B. L. Higgins, D. W. Berry, S. D. Bartlett, H. M. Wiseman, and G. J. Pryde, Nature 450, 393 (2007).
[11] E. Demkowicz-Dobrzański, J. Kolodyński, and M. Guta, Nature Communications 3 (2012).
[12] D. Gottesman, Proceedings of Symposia in Applied Mathematics 68, 13 (2009).
[13] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. 82, 2417 (1999).
[14] J. Taylor, P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. Hemmer, A. Yacoby, R. Walsworth, and M. Lukin, Nature Physics 4, 810 (2008).
[15] G. De Lange, D. Ristè, V. Dobrovitski, and R. Hanson, Phys. Rev. Lett. 106, 080802 (2011).
[16] W. Dür, M. Skotiniotis, F. Froewis, and B. Kraus, Phys. Rev. Lett. 112, 080801 (2014).
[17] D. A. Herrera-Martí, T. Gegen, D. Aharonov, N. Katz, and A. Retzker, Phys. Rev. Lett. 115, 205001 (2015).
[18] G. Arrad, Y. Vinkler, D. Aharonov, and A. Retzker, Phys. Rev. Lett. 112, 150801 (2014).
[19] E. M. Kessler, I. Lovchinsky, A. O. Sushkov, and M. D. Lukin, Phys. Rev. Lett. 112, 150802 (2014).
[20] L. Cohen, Y. Pilnyak, D. Istrati, A. Retzker, and H. Eisenberg, Phys. Rev. A 94, 012324 (2016).
[21] T. Unden, P. Balasubramanian, D. Louzon, Y. Vinkler, M. B. Plenio, M. Markham, D. Twitchen, A. Stacey, I. Lovchinsky, A. O. Sushkov, et al., Phys. Rev. Lett. 116, 230502 (2016).
[22] Z. Sisi and et al, arXiv preprint arXiv:1706.02445 (2017).
[23] S. Schmitt and et al, Science 356, 832 (2017).
[24] J. Boss, K. Cuija, J. Zopes, and C. Degen, Science 356, 837 (2017).
[25] B. Misra and E. G. Sudarshan, Journal of Mathematical Physics 18, 756 (1977).
[26] W. Itano, D. Heinzen, J. Bollinger, and D. Wineland, Phys. Rev. A 41, 2295 (1990).
[27] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. 86, 2699 (2001).
[28] H. Nakazato, M. Namiki, and S. Pascazio, Int. J. Mod. Phys. B 10, 247 (1996).
[29] R. Raussendorf and H. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[30] J. Barjkatarevic, R. McKenzie, J. Links, and G. Milburn, Phys. Rev. Lett. 95, 230501 (2005).
[31] M. Silva, V. Danos, E. Kashefi, and H. Ollivier, New Journal of Physics 9, 192 (2007).
[32] S. Olsmschenk, D. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan, and C. Monroe, Science 323, 486 (2009).
[33] M. Baur, A. Fedorov, L. Steffen, S. Filipp, M. Da Silva, and A. Wallraff, Phys. Rev. Lett. 108, 040502 (2012).
[34] D. Averin, K. Xu, Y. Zhong, C. Song, H. Wang, and S. Han,
Phys. Rev. Lett. 116, 010501 (2016).

[49] K. Hornberger, in Entanglement and Decoherence (Springer, 2009), pp. 221–276.

[50] G. De Lange, Z. Wang, D. Riste, V. Dobrovitski, and R. Hanson, Science 330, 60 (2010).

[51] F. Yoshihara, K. Harrabi, A. Niskanen, and Y. Nakamura, Phys. Rev. Lett. 97, 167001 (2006).

[52] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, Phys. Rev. Lett. 98, 047004 (2007).

[53] Y. Kondo, Y. Matsuzaki, K. Matsushima, and J. G. Filgueiras, New Journal of Physics 18, 013033 (2016).

[54] G. M. Palma, K. A. Suominen, and A. K. Ekert, Proc. R. Soc. London. Ser.A 452, 567 (1996).

[55] G. De Lange, Z. Wang, D. Riste, V. Dobrovitski, and R. Hanson, Science 330, 60 (2010).

[56] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, Phys. Rev. Lett. 98, 047004 (2007).

[57] Y. Yoshihara, K. Harrabi, A. Niskanen, and Y. Nakamura, Phys. Rev. Lett. 97, 167001 (2006).

[58] Y. Kondo, Y. Matsuzaki, K. Matsushima, and J. G. Filgueiras, New Journal of Physics 18, 013033 (2016).

[59] F. Wilhelm, M. Storcz, U. Hartmann, and M. R. Geller, in Manipulating Quantum Coherence in Solid State Systems (Springer, 2007), pp. 195–232.

[60] L. T. Hall, J. H. Cole, and L. C. Hollenberg, Physical Review B 90, 075201 (2014).

[61] C. W. Gardiner and P. Zoller, Quantum Noise (Springer, Berlin, 2004).

[62] R. Barends, J. Kelly, A. Megrant, A. Vitiia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, et al., Nature 508, 500 (2014).

[63] C. Ballance, T. Harty, N. Linke, M. Sepiol, and D. Lucas, Phys. Rev. Lett. 117, 060504 (2016).

[64] R. Blume-Kohout, J. K. Gamble, E. Nielsen, K. Rudinger, J. Mizrahi, K. Fortier, and P. Maunz, Nature Communications 8 (2017).

[65] C. Neill, P. Roushan, K. Kchedzhii, S. Boixo, S. Isakov, V. Smelyanskiy, R. Barends, B. Burkett, Y. Chen, Z. Chen, et al., arXiv preprint [arXiv:1709.06678] (2017).

[66] G. Popkin, Science 354, 1090 (2016).