Comments on the classification of orientifolds

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Abstract
The simple current construction of orientifolds based on rational conformal field theories is reviewed. When applied to $SO(16)$ level 1, one can describe all ten-dimensional orientifolds in a unified framework.

1 Introduction
It is well known that M-theory has an infinite number of vacua. As long as we do not know the vacuum selection principle of M-theory, it might be a good strategy to study generic properties of these vacua. A large class of vacua are string compactifications with D-branes and O-planes. Such *orientifolds* have been extensively studied in flat space and recently in curved spaces [1]. In this talk I will present a general prescription [2] to construct orientifolds that are based on rational conformal field theories (RCFT). The common property of these theories is that the RCFT has simple currents [3] which enable us to find universal formulas for D-branes and O-planes.

The outline of this talk is as follows. I will explain what simple currents are and how they can be used to construct string theories. From time to time I will clarify this construction with an example, the bosonic string compactified on the $E_8 \times SO(16)$ root lattice[3]. This example is interesting in its own right, since suitable extensions and truncations of this theory are related to the $d = 10$ fermionic strings [4, 5, 6]. We will see that our prescription describes all known $d = 10$ orientifolds [7] in one unified framework.

2 Closed strings and simple currents
The worldsheet of a closed oriented string must be a conformal field theory and splits in a left- and rightmoving sector. The chiral half of these theories is specified by a (chiral)

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1 Based on a seminar given at the RTN-workshop in Leuven, Belgium, September 2002.
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3 The ‘dummy’ $E_8$ will be dropped in the remainder.
algebra $\mathcal{A}$, that we assume to be rational. Rationality simply means that the number of (chiral) primaries of $\mathcal{A}$, labelled by $m$, is finite. Let $Z_{mn}$ denote how many times the left-moving primary $m$ is combined with the right-moving primary $n$. Then we must have

$$[Z, S] = [Z, T] = 0$$

in order for this theory to be finite. Here $S$ and $T$ are the modular matrices of $\mathcal{A}$. The matrix $Z$ is referred to as the modular invariant. The first step will be to classify these. This can (to a certain extent) be done by simple currents \[3, 8\].

Simple currents are primary fields $J$ whose fusion product with any primary $m$ yields just one primary $J \times m \equiv J m$. An illustrative example is the chiral algebra of free bosons. For this theory, the primaries are labeled by momenta $p$ and the fusion product expresses momentum conservation $p \times q = p + q$. So every primary is a simple current. This example falls strictly speaking not in the class we are considering since the chiral algebra is not rational. However, when compactified on a circle of rational radius, the chiral algebra can be made rational. We can for instance compactify $r$ bosons on the coroot-lattice of a Lie group $G$ of rank $r$. The momenta are $G$ weights, but weights that differ by a root can be grouped into a single primary of an ‘extended’ algebra called $G$ level 1. The number of primaries of this extended algebra therefore equals the number of conjugacy classes of $G$. Since this theory still consists of free bosons, all primaries are simple currents. The fusion rules reflect momentum conservation modulo roots which yields a group structure that is isomorphic to the centre of $G$.

Throughout this talk we will simplify the discussion in two ways. First, the simple current group that generates the modular invariant is $\mathbb{Z}_N$. Second, we assume that these currents do not have fixed points\[4\]. Such a cyclic group defines a modular invariant whose only nonzero entries are $Z_{m,J^\alpha_m} = 1$ and this happens only when

$$h_m + (1 - \alpha) h_J - h_{Jm} \in \mathbb{Z} .$$

Here $J$ is the generator of $\mathbb{Z}_N$. We have $J^N = 0$, the vacuum. The index $\alpha$ runs from 1 to $N$ and $h_m$ is the conformal weight of $m$.

Consider $SO(16)$ level 1. The primaries $(O_{16}, S_{16}, V_{16}, C_{16})$ have conformal weights $(0, 1, 1/2, 1)$ and generate a $\mathbb{Z}_2 \times \mathbb{Z}_2$ simple current group. The $\mathbb{Z}_2$ simple current invariants corresponding to $O_{16}, V_{16}, S_{16}$ are\[5\]:

$$Z_{0B} = O_{16} \bar{O}_{16} + V_{16} \bar{V}_{16} + C_{16} \bar{C}_{16} + S_{16} \bar{S}_{16} ,$$

(2)

$$Z_{0A} = O_{16} \bar{O}_{16} + V_{16} \bar{V}_{16} + C_{16} \bar{S}_{16} + S_{16} \bar{C}_{16} ,$$

(3)

$$Z_{1IB} = (O_{16} + S_{16})(\bar{O}_{16} + \bar{S}_{16}) .$$

(4)

Note that we have written $Z = \sum_{mn} \chi_m Z_{mn} \bar{\chi}_n$ where $\chi_m$ is the Virasoro specialized character of primary $m$. We denote the characters of $SO(16)$ by $O_{16} \equiv \chi_{O_{16}}$ etcetera.

\[4\]For the general case including fixed points, see \[8\].

\[5\]The invariant obtained from $C_{16}$ gives after the truncation \[3\] a fermionic spectrum that is equivalent to that of $Z_{1IB}$.
Right-moving characters are barred. The spectra of these bosonic theories can be related to the fermionic string theories as indicated by the subscript. In order to read off the fermionic spectrum, one has to replace the $SO(16)$ characters by $SO(8)$ characters as follows [5]:

$$O_{16} \rightarrow V_8, \quad V_{16} \rightarrow O_8, \quad S_{16} \rightarrow -S_8, \quad C_{16} \rightarrow -C_8.$$  \hspace{1cm} (5)

The $O_{16}, V_{16}$ are in the NS sector and $S_{16}, C_{16}$ are in the R sector. Due to the minus signs in (5), states in the $RNS$ and $NSR$ sectors contribute with a minus sign to the partition function and thus describe fermions. Type OB and OA do not contain fermions, are not space-time supersymmetric and are tachyonic due to the groundstate of $V_{16}\bar{V}_{16}$. Type IIB is supersymmetric since it contains gravitinos from $S_{16}\bar{O}_{16}$ and $\bar{S}_{16}O_{16}$ and is tachyon free.

When we use the full $\mathbb{Z}_2 \times \mathbb{Z}_2$ centre, the resulting invariant describes the type IIA string:

$$Z_{IIA} = (O_{16} + S_{16})(\bar{O}_{16} + \bar{C}_{16}).$$  \hspace{1cm} (6)

Since this theory is not invariant under $\mathcal{O}$, we cannot perform the standard orientifolding. However, this theory is invariant under $\mathcal{T}^{-1}0\mathcal{T}$ where $\mathcal{T}$ interchanges $S_{16}$ and $C_{16}$. Orientifolds of type IIA are beyond the scope of this talk.

From this example we see that all uncompactified string theories are simple current invariants. For a generic compactified string theory this is no longer true, since the ‘internal’ CFT might be interacting (i.e., is not described by free bosons). For interacting CFT’s, not all primaries are simple currents, but many invariants are simple current invariants. Note from our example that simple currents nicely implement the ten dimensional GSO projection; this is true for any dimension (see for instance [9]).

### 3 Orientifolds and simple currents

Next we consider two-dimensional CFT’s on worldsheets with boundaries and crosscaps [10, 11, 12, 13, 14]. A necessary ingredient for calculating the correlation functions on such surfaces are the one-point functions of closed strings on the disc and $RP^2$, i.e., the tadpoles. Let $|\phi_{ij}\rangle$ denote a closed oriented string state whose left/right-mover is (a descendant of) the primary $i/j$. Then the disc and crosscap tadpoles of this field are given by

$$\langle B|\phi_{ij}\rangle, \quad \langle C|\phi_{ij}\rangle.$$  \hspace{1cm} (7)

Here $|B\rangle$ and $|C\rangle$ are boundary and crosscap states whose precise definition is not important for the rest of this talk. These tadpoles are constrained as follows. First note that on a surface with a boundary, the left- and rightmovers on the string are no longer independent. In particular, the left- and rightmoving worldsheet currents are related by a boundary condition. We can for instance choose a diagonal ‘gluing condition’ in which the left- and rightmoving currents are equal. Then, the full $\mathcal{A} \times \bar{\mathcal{A}}$ algebra is broken to a diagonal subalgebra by the boundary. Similar remarks apply for the crosscap. From
this we then conclude that the tadpoles are only defined when the left- and rightmoving representations of $|\phi_{ij}\rangle$ are equal. So

$$
\langle B | \phi_{ij} \rangle \sim \delta_{ij}, \quad \langle C | \phi_{ij} \rangle \sim \delta_{ij} .
$$

(8)

For a given closed oriented string spectrum described by a modular invariant $Z_{ij}$, we only need to know the tadpoles for a subset of primaries, namely those that couple diagonally. A primary $i$ for which $Z_{ii} \neq 0$ is called an Ishibashi label. Note that Ishibashi labels are degenerate when $Z_{ii} \geq 2$. For simple current invariants, this can only happen when the currents have fixed points, so we will ignore this degeneracy problem (and its solution [2]) in this talk. A second constraint only applies for the boundaries and is referred to as completeness [13, 14]. It states: the number of inequivalent boundaries of a given gluing condition equals the number of Ishibashi labels. We then write

$$
\langle B | \phi_{ii} \rangle = \sum_a N_a \langle B_a | \phi_{ii} \rangle
$$

(9)

where $|B_a\rangle$ is the boundary state of type $a$. The numbers $N_a$ are Chan-Paton factors and count how many times ‘brane’ $a$ exists in the theory. It is convenient to define the boundary and crosscap coefficients

$$
B_{ia} = \langle B_a | \phi_{ii} \rangle , \quad \Gamma_i = \langle C | \phi_{ii} \rangle .
$$

(10)

A second constraint on the tadpoles is integrality and arises as follows. It is well known that the spectrum of a string theory is encoded in the one-loop partition function. For a string theory including worldsheet boundaries and crosscaps, the total one-loop partition function has three additional contributions besides the torus $Z$: Klein bottle $K$, annulus $A$ and Möbius strip $M$. In terms of characters we can write

$$
K = \sum_m K_m \chi_m , \quad A = \sum_{mab} N_a N_b A_{mab} \chi_m , \quad M = \sum_{ma} N_a M_{ma} \hat{\chi}_m .
$$

(11)

where the definition of $\hat{\chi}$ is such that even/odd level descendants of $m$ are symmetrized/antisymmetrized when $M_{ma} > 0$ and vice versa when $M_{ma} < 0$. The Klein bottle (anti)symmetrizes sector $Z_{mm}$ when $K_m$ is $(-)1$. In order to have nonnegative state multiplicities in both the open and closed sector it is sufficient to have

$$
\frac{1}{2}(Z_{mm} + K_m) \in \mathbb{Z}^+ , \quad \frac{1}{2}(A_{maa} + M_{ma}) \in \mathbb{Z}^+ , \quad A_{mab} = A_{mba} \in \mathbb{Z}^+ .
$$

(12)

where $\mathbb{Z}^+$ denote the positive integers including 0. It is intuitively clear that integrality constrains the tadpoles, since $A$ and $K$ contains two boundaries and two crosscaps respectively and $M$ contains a boundary and a crosscap. We require [10, 13]:

$$
K_m = \sum_i S_{im} \Gamma_i \Gamma_i , \quad A_{mab} = \sum_i S_{im} B_{ia} B_{ib} , \quad M_{ma} = \sum_i P_{im} \Gamma_i B_{ia} .
$$

(13)

More precisely, since left-/right-movers can be viewed as in/outcoming states, the tadpoles for diagonal gluing are only defined for representations that are each others charge conjugate. Alternatively, one can also consider tadpoles for charge conjugation gluing conditions. Then the tadpoles are defined for diagonal closed strings. It is the latter formulation that we are considering in this talk.
where \( P = \sqrt{T}ST^2S\sqrt{T} \) [17] and \( S, T \) are the modular matrices of the chiral algebra under consideration. The third constraint is tadpole cancellation. All one-loop diagrams are finite when
\[
\sum_a N_a B_{0a} = 2d/2 \Gamma_0 \quad , \quad \sum_a N_a B_{ia} = -2d/2 \Gamma_i
\]
for all Ishibashi primaries \( i \) with \( h_i = 1 \). Here \( d \) is the uncompactified dimension of the fermionic string. Tadpole cancellation determines the gauge group in the open sector. We will require that \( A_{0ab} \) is an involution on the boundary labels [13]. This gauge group depends on the Möbius coefficient \( M_0a \) as follows. When \( M_0a = +1 \) the gauge group is \( SO(\mathcal{N}) \), when \( M_0a = -1 \) the gauge group is \( Sp(\mathcal{N}) \). Unitary gauge groups arise as follows. When \( M_0a = 0 \), integrality in the open sector implies \( A_{0aa} = 0 \) as well. Therefore, there must be a conjugate boundary label \( \bar{a} \neq a \) for which \( A_{0a\bar{a}} = 1 \). The conjugate pair represents a \( U(\mathcal{N}_a = \mathcal{N}_\bar{a}) \) gauge group.

We [2] have found a universal formula for the boundary and crosscap coefficients in case the closed oriented string spectrum is described by an arbitrary symmetric simple current invariant for any rational CFT. In case of a \( \mathbb{Z}_N \) simple current group without fixed points we have for the boundary [14, 18, 19] and crosscap coefficients [17, 20, 21]
\[
B_{ij} = \sqrt{N} \frac{S_{ij}}{S_{iK}} \quad , \quad \Gamma_i = \frac{\sqrt{N} [\sigma(0)P_{iK} + \sigma(1)P_{iJK}]}{\sqrt{S_{iK}}}
\]
where the Ishibashi label \( i \) is such that
\[
Q_J(i) \equiv h_i + h_J - h_{Ji} \in \mathbb{Z}
\]
and the boundary labels \( [j] = \{ k = J^\alpha j | \alpha = 1, ..., N \} \) are given by simple current ‘orbits’. From [3, 22]
\[
S_{iJj} = e^{2\pi Q_J(i) S_{ij}}
\]

it follows that the boundary coefficient does not depend on the orbit representative. It is an easy exercise to show that the boundary coefficient has a left- and right inverse, thus satisfying completeness. The crosscap coefficient contains two signs \( \sigma(0) \) and \( \sigma(1) \). When \( N \) is odd, these signs must be such that \( \Gamma_i = \sqrt{N} \sigma(0) P_{iK} \). When \( N \) is even, these signs are independent. The primary field \( K \) is a simple current called Klein bottle current [23]. We can always choose \( K = 0 \). When nontrivial, it is must be local with all order two currents \( J \in \mathbb{Z}_N \), i.e. \( Q_K(J) = 0 \). To yield inequivalent theories, it is necessary that \( K \notin \mathbb{Z}_N \) and \( K^2 = 0 \).

The solutions [13] reproduce all known ten-dimensional orientifolds [7, 24] when applied to \( SO(16) \) level 1. The \( S \) and \( P \) matrices are given by
\[
S_{ij} = \frac{1}{2} e^{2\pi Q_J(i)} \quad , \quad P_{ij} = \delta_{ij}
\]
where \( i, j = O_{16}, V_{16}, S_{16}, C_{16} \). The role of the Klein bottle current can be nicely illustrated in the OB theory. The simple current group that defines \( Z_{0B} \) is trivial, so all primaries
can be used as a Klein bottle current. The Klein bottle partition functions are given by
\[
\begin{align*}
K_{0B}^{[O]} &= O_{16} + V_{16} + S_{16} + C_{16} , & K_{0B}^{[V]} &= O_{16} + V_{16} - S_{16} - C_{16} , \\
K_{0B}^{[S]} &= O_{16} - V_{16} + S_{16} - C_{16} , & K_{0B}^{[C]} &= O_{16} - V_{16} - S_{16} + C_{16} .
\end{align*}
\]

By completeness, there are four boundary labels denoted by \(O, V, S, C\). The annulus and Möbius for the trivial Klein bottle current are
\[
\begin{align*}
\mathcal{A}_{0B}^{[0]} &= (N_O^2 + N_V^2 + N_S^2 + N_C^2) O_{16} + 2 (N_O N_V + N_S N_C) V_{16} \\
&\quad + 2 (N_O N_S + N_V N_C) S_{16} + 2 (N_O N_C + N_V N_S) C_{16} , \\
\mathcal{M}_{0B}^{[0]} &= \sigma(0) (N_O + N_V + N_S + N_C) \hat{O}_{16} \\
&\quad + \sigma(1) (N_O - N_V - N_S - N_C) \hat{V}_{16} .
\end{align*}
\]

Tadpole cancellation (24) has to be imposed on \(i = O_{16}, C_{16}, S_{16}\) and requires \(\sigma(0) = 1\) and an \([SO(N) \times SO(32 - N)]^2\) gauge group. The effect of a nontrivial Klein bottle current \(K\) in the open sector can be summarized as follows:
\[
A_{mab}^{[K]} = A_{K m, ab}^{[0]} , \quad M_{K a}^{[K]} = e^{2\pi i Q_K(a)} M_{0 a}^{[0]} ,
\]
where we only displayed the nonzero Möbius coefficient. The only field that flows in the Möbius is the Klein bottle current. From (3), we see that space-time vectors come from \(O_{16}\), and therefore a non-trivial Klein bottle leads to unitary gauge groups. The tadpole condition only has non-trivial solutions when we relax the condition for \(O_{16}\) (dilaton tadpole). For \(K = V_{16}\) the gauge group is \(U(N) \times U(M)\). For the \(K = S_{16}\) case we impose in addition
\[
N_O = N_S , \quad N_V = N_C
\]
since they are conjugate. Tadpole cancellation then allows a \(U(N) \times U(N - 32)\) gauge group where \(N \geq 32\). Strictly speaking, we have two such theories, labelled by the free sign \(\sigma(0)\).

The type OA theory is a \(\mathbb{Z}_2\) simple current invariant based on the half-integer spin current \(V_{16}\). There are no Klein bottle currents. By completeness, we have two boundaries \([O]\) and \([S]\). The partition functions of this orientifold are
\[
\begin{align*}
\mathcal{K}_{0A} &= O_{16} + V_{16} , \\
\mathcal{A}_{0A} &= (N_O^2 + N_S^2) (O_{16} + V_{16}) + 2 N_O N_S (S_{16} + C_{16}) , \\
\mathcal{M}_{0A} &= \sigma(0) (N_O + N_S) \hat{O}_{16} + \sigma(1) (N_O - N_S) \hat{V}_{16} .
\end{align*}
\]

The dilaton tadpole is cancelled when \(\sigma(0) = 1\) and the gauge group equals \(SO(N) \times SO(32 - N)\). The sign \(\sigma(1)\) is unconstrained.

Note that the OA orientifold has tachyons in the open sector. Indeed, from the truncation rules (3), we see that \(V_{16}\) contains the tachyon and there is no tadpole free choice of \(N\) to get rid of it. This happens also for the type OB orientifolds, except when the Klein bottle current \(K = S_{16}\) and the gauge group is \(U(32)\). From the Klein bottle partition function it also follows that the closed string tadpole is projected out [4, 25].
The type IIB string is a $\mathbb{Z}_2$ simple current invariant based on $S_{16}$. The partition functions are

$$ K_{IIB} = O_{16} + S_{16} \quad , $$  

$$ A_{IIB} = (N_{[O]}^2 + N_{[V]}^2) (O_{16} + S_{16}) + 2N_{[O]}N_{[V]} (V_{16} + C_{16}) \quad , $$  

$$ M_{IIB} = \sigma(0) (N_{[O]} + N_{[V]}) \hat{O}_{16} + \sigma(1) (N_{[O]} - N_{[V]}) \hat{S}_{16} . $$  

Note that these expressions are the same as those of the type OA descendant when $S$ and $V$ are interchanged. The tadpole cancellation conditions for the dilaton and (unphysical) axion are

$$ N_{[O]} + N_{[V]} = \sigma(0)32 \quad , \quad N_{[O]} - N_{[V]} = -\sigma(1)32 . $$  

When we insist on dilaton tadpole cancellation, we must choose $\sigma(0) = 1$. When $\sigma(1) = 1$ as well, we have a $SO(32)$ gauge group from 32 branes of type $[V]$. For $\sigma(1) = -1$ we have a $SO(32)$ gauge group from the $[O]$ branes. We can identify the signs $-\sigma(0)$ and $\sigma(1)$ with the tension and RR charges of an O-plane. Similarly, we can identify the $[O]$-boundary states as D-branes and the $[V]$-boundary states as anti-D-branes. Note also that when we relax the dilaton tadpole condition, we can choose $\sigma(0) = -1$ and thus allow symplectic gauge groups $Sp(N) \times Sp(32 - N)$. For $Sp(32)$ the open string tachyon is removed from the spectrum [24].

### 4 Conclusions

We have rediscovered all ten dimensional orientifolds using simple currents. These theories can be supersymmetric or nonsupersymmetric, may have dilaton tadpoles and/or tachyon instabilities. It is intriguing to realize that the fermionic string theories in ten dimensions are closely related to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ centre of $SO(16)$. (See also the contribution of Laurent Houart to these proceedings [26].)

We have found universal formulas for boundary and crosscap coefficients for all symmetric simple current invariants for any rational chiral algebra. These formulas basically contain the $S$ and $P$ matrices of a chiral algebra and a few signs and phases, which are known for many RCFT’s, like WZW models and cosets. Our construction can therefore be used to compute the spectra, brane tensions etcetera for a huge number of string theories [27].

We believe that the classification of D-branes and O-planes in rational CFT’s is a first step in understanding the complete spectrum of branes and planes at arbitrary points in the moduli space of string theories. There has been a lot of interest in the comparison of D-branes at rational and large volume points in moduli spaces of Calabi-Yau compactifications [28]. The hope is that one can derive from this the brane spectrum at arbitrary modulus. A powerful tool is mirror symmetry. Under mirror symmetry, so-called A-type branes and B-type branes are interchanged and each can be used to probe a different part.

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\(^7\)The tension is $-\sigma(0)$ since the ten-dimensional dilaton appears at the first excited level of $O_{16}O_{16}$. 
of the moduli space. In this respect it is interesting to note that the distinction between
mirror compactifications and therefore A- and B-type branes is another application of
simple currents (see for instance [9]).

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