The pair production of Charged and Neutral Higgs bosons in W and Z gauge boson fusion process

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Abstract

We study the signatures of a two Higgs doublet model of Davidson and Logan. The model includes an extra Higgs doublet with the vacuum expectation value (VEV) much smaller than the one of the standard model like Higgs. The smaller VEV is related to the origin of the small neutrino mass in the two Higgs doublet model. In the model, a single non-standard model like Higgs production of weak gauge boson fusion is suppressed due to the smallness of the vacuum expectation value. In contrast to the single Higgs production, the cross section of the Higgs pair production due to gauge boson fusion is not suppressed. Using the model, we compute the charged Higgs and neutral Higgs pair production cross section in $W^+ Z$ annihilation channel. In the two Higgs doublet model, the charged Higgs $H^+$ decays into a pair of the charged anti-lepton and right-handed neutrino. The neutral Higgs boson decays into right-handed neutrino and left-handed anti-neutrino pair which is invisible. A single charged anti-lepton and three neutrinos are the products of the subsequent decays of the charged Higgs and the neutral Higgs. $W^+ Z$ pair production gives the background for the signal through the decays $W^+ \rightarrow \nu l$ and $Z \rightarrow \nu \bar{\nu}$.

By multiplying the charged and neutral Higgses production cross section with the lepton flavor specific decay branching fractions of charged Higgs, we define a measurement which characterizes the present model. We numerically compute the measurement and find the sizable deviation from the standard model prediction.

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I. INTRODUCTION

Probing the Higgs sector beyond the standard model is a main subject of new physics search in collider experiments and in flavor factories. Among them, two Higgs doublet models have been studied in their many aspects. Here we study the two Higgs doublet model in which the hierarchy of the neutrino mass and the other fermions mass is explained by two Higgs vacuum expectation values (VEVs) with large hierarchy [1, 2]. In the model, the neutrino mass is protected from the large standard model like Higgs VEV by assigning U(1) charge for right-handed neutrino and the second Higgs. The corresponding U(1) symmetry is softly broken. Moreover the tiny VEV of the second Higgs is stable against the radiative corrections, [3, 4]. In this letter, we study the production of the Higgs bosons in the second doublet in gauge bosons fusion process. A single Higgs boson production of gauge boson fusion is suppressed because the amplitude is proportional to the smaller Higgs VEV. In contrast to the single Higgs boson production, the Higgs boson pair production of gauge boson fusion is not suppressed. Therefore it is worthwhile to study the process in their detail. We study the pair production cross section of the charged Higgs and the neutral Higgs in W and Z boson fusion process. Since the neutral Higgs bosons decay into neutrino and anti-neutrino pair, they are invisible. Therefore the signal of the event is a single charged lepton of the charged Higgs decay. The standard model background of the process is related to W Z scattering process. Invisible decay of Z boson with the W decay into lepton and neutrino pair is a potential background. In this letter, we compute the differential cross section of the charged and neutral Higgses pair production cross section and compare it with W^+ Z pair production in the standard model. Since the charged Higgs coupling to charged lepton depends on the lepton flavor, we expect the strong flavor dependence in the charged Higgs decay while W decay is flavor independent.

The paper is organized as follows. In section (II), we study the single Higgs production. In section (III), the Higgs pair production cross section and the angular distribution of the production cross section are studied and they are compared with the one of W^+ Z scattering. Section (IV), we propose a measurement which identifies the signal and predictions are given. A brief summary is also given.
II. A SINGLE HIGGS PRODUCTION FROM WEAK GAUGE BOSON FUSION

We study the Dirac neutrino model of [1, 2]. In this two Higgs doublet model, besides the standard model like Higgs, there are two extra neutral Higgs and one charged Higgs. We follow the notation of [3].

We first consider the single Higgs production with the gauge boson fusion. Denoting the standard model like Higgs as $H$, and the other two neutral Higgs as $A$ (CP odd Higgs) and $h$ (CP even Higgs) respectively, the interaction Hamiltonian for gauge boson fusion to neutral Higgses is given as,

$$L = gM_W(W^+ W^- + \frac{1}{2c_W^2}Z^+ Z^-)(\sin(\beta + \gamma) h + \cos(\beta + \gamma) H),$$  \hspace{1cm} (1)

where $\tan \beta$ is the ratio of the two vacuum expectation values of Higgs and is given as,

$$\frac{v_2}{v_1} = \tan \beta \simeq \frac{m_{12}^2}{M_A^2},$$  \hspace{1cm} (2)

where $m_{12}^2$ is the soft breaking term for U(1) symmetry. Since $v_1$ is standard model like Higgs VEV and $v_2$ is VEV of the Higgs which gives rise to the light neutrino masses, $\tan \beta$ is very small. If one takes $v_2 = 1$ (eV), then $\tan \beta = O(10^{-11})$. $\gamma$ is related to a mixing angle of CP even Higgses. Since the ratio of $\frac{\gamma}{\beta}$ is given as $\frac{\gamma}{\beta} \simeq \frac{M_H^2 - \lambda H M_H^2}{M_H^2 - M_A^2}$, $\gamma$ is as small as $\beta$. From Eq.(1), we note only the CP even Higgses $h$ and $H$ can be produced by the gauge bosons fusion. Since the coupling of the non-standard like Higgs $h$ with gauge boson pairs is proportional to $\sin(\beta + \gamma)$, the single non-standard model like Higgs production is strongly suppressed.

III. HIGGS PAIR PRODUCTION FROM WEAK GAUGE BOSON FUSION

In contrast to the single Higgs production, the pair production of the extra Higgses is not suppressed by the small VEV. Here we consider the charged Higgs ($H^+$) and a neutral Higgs ($h$ or $A$) pair production due to $W^+ Z$ fusion. In the neutrinophilic model [1-4], the non-standard model like Higgs couples to left-handed lepton doublets and right-handed singlet neutrinos. The visible decay product of the pair produced Higgses is a mono charged lepton jet. The other decay products are three (anti-)neutrinos which are invisible. We first study the cross section of $W^+ + Z \rightarrow H^+ + h$ and $W^+ + Z \rightarrow H^+ + A$. The interaction
vertices for the pair production are,

\[ \mathcal{L} = \frac{g^2}{2} s_W (A_\mu - t_W Z_\mu) [(H^+ W^{\mu -} + H^- W^{\mu +})(h \cos(\beta + \gamma) - H \sin(\beta + \gamma)) \]

\[ - i(H^+ W^{\mu -} - H^- W^{\mu +}) A] \]

\[ + i \frac{g \cos 2 \theta_W}{2 \cos \theta_W} Z_\mu (\partial^\mu H^- H^+ - \partial^\mu H^+ H^-) \]

\[ + \frac{g \cos(\beta + \gamma)}{2 \cos \theta_W} (\partial_\mu h A - \partial_\mu A h) Z^\mu \]

\[ + \left\{ i \frac{g}{2} \cos(\beta + \gamma) W^{\mu +} (\partial_\mu h H^- - \partial_\mu h H^+) + \frac{g}{2} W^{\mu +} (H^- A - A \partial_\mu H^-) + h.c. \right\}. \] (3)

Then the amplitude for \( Z + W^+ \rightarrow h + H^+ \) is,

\[ M = \epsilon^\mu_W \epsilon^\nu_Z T_{\mu \nu}, \]

\[ T_{\mu \nu} = \frac{g^2 \cos(\beta + \gamma)}{2 \cos \theta_W} (a g_{\mu \nu} + d q_{H^{+ \mu}} q_{H^{+ \nu}} + b q_{H^{+ \nu}} q_{H^{+ \mu}}), \] (4)

where we compute the four Feynman diagrams corresponding to the s channel \( W^+ \) exchange (Fig. 1), the contact interaction (Fig. 2), u channel charged Higgs exchange (Fig. 3), and t channel CP odd Higgs (A) exchanged diagram (Fig. 4). \( a, b \) and \( d \) in Eq.(4) are given as,

\[ a = -s W - s W \frac{M^2_h - M^2_{H^+} - M^2_W}{s - M^2_W} - c^2_W \frac{t - u + M^2_Z - M^2_W}{s - M^2_W}, \]

\[ b = \frac{2 \cos 2 \theta_W}{u - M^2_{H^+}} + \frac{2(\cos 2 \theta_W + 1)}{s - M^2_W}, \]

\[ d = -\frac{2}{t - M^2_A} - \frac{2(\cos 2 \theta_W + 1)}{s - M^2_W}, \] (5)

with \( s_W^2 = \sin^2 \theta_W, c_W^2 = \cos^2 \theta_W, t = (q_{H^+} - p_W)^2, \) and \( u = (p_W - q_h)^2. \) \( t \) and \( u \) are also written in terms of the center of mass (CM) energy \( \sqrt{s} \) and the scattering angle \( \theta \) between \( W^+ \) and \( H^+ \) in CM frame as,

\[ t = M^2_{H^+} + M^2_{W^+} - \frac{(s + M^2_W - M^2_Z)(s + M^2_{H^+} - M^2_h)}{2s} + 2P_W P_h \cos \theta, \]

\[ u = -s + M^2_Z + M^2_h + \frac{(s + M^2_W - M^2_Z)(s + M^2_{H^+} - M^2_h)}{2s} - 2P_W P_h \cos \theta. \] (6)
Using Eq. (3), one can compute the cross section. The unpolarized cross section of $W^+$ and $Z$ fusion into the charged Higgs ($H^+$) and neutral Higgs ($h$) pair is given as,

$$
\frac{d\sigma_{W^+Z\to H^+h}}{d\cos\theta} = \frac{g^4 \cos(\beta + \gamma)^2}{36c_W^2} \frac{1}{32\pi s} \frac{P_h}{P_{W^+}} \left[ a^2 \left\{ 3 + \frac{(s - (M_{W^+} + M_Z)^2)(s - (M_{W^+} - M_Z)^2)}{4M_Z^2M_{W^+}^2} \right\} 
+ b^2 \left\{ \frac{(M_h^2 + M_W^2 - u)^2}{4M_W^4} - M_h^2 \right\} \left\{ \frac{(M_{H^+}^2 + M_Z^2 - u)^2}{4M_Z^2} - M_{H^+}^2 \right\} \right] 
+ d^2 \left\{ \frac{(M_{H^+}^2 + M_Z^2 - t)^2}{4M_Z^4} - M_{H^+}^2 \right\} \left\{ \frac{(M_{H^+}^2 + M_Z^2 - t)^2}{4M_Z^2} - M_{H^+}^2 \right\} \right] 
+ 2bd \left\{ \frac{s - M_{W^+}^2 - M_{H^+}^2}{2} - \frac{(M_h^2 + M_Z^2 - t)(M_{H^+}^2 + M_Z^2 - u)}{4M_Z^2} \right\} 
\times \left( \frac{s - M_{W^+}^2 - M_{H^+}^2}{2} - \frac{(M_h^2 + M_Z^2 - t)(M_{H^+}^2 + M_Z^2 - u)}{4M_Z^2} \right) \right] 
+ ad \left\{ \frac{-(M_h^2 + M_Z^2 - t)(M_{H^+}^2 + M_Z^2 - t)(M_{W^+}^2 + M_Z^2 - s)}{4M_W^4M_Z^2} + s - M_{H^+}^2 - M_h^2 
- \frac{(M_h^2 + M_Z^2 - u)(M_{H^+}^2 + M_Z^2 - t)}{2M_W^2} \right\} \left( \frac{M_h^2 + M_Z^2 - t}{2M_Z^2} \right) 
+ ab \left\{ \frac{-(M_h^2 + M_Z^2 - u)(M_{H^+}^2 + M_Z^2 - s)}{4M_W^4M_Z^2} + s - M_{H^+}^2 - M_h^2 
- \frac{(M_h^2 + M_Z^2 - u)(M_{H^+}^2 + M_Z^2 - t)}{2M_W^2} \right\} \left( \frac{M_h^2 + M_Z^2 - t}{2M_Z^2} \right) \right] 
\right]. \tag{7}
$$

$P_{W^+}$ and $P_h$ are momentum of $W^+$ and $h$ in the CM frame,

$$
P_h = \sqrt{(s^2 - 2s(M_{H^+}^2 + M_h^2) + (M_{H^+}^2 - M_h^2)^2)} \frac{1}{2\sqrt{s}},
$$

$$
P_{W^+} = \frac{\sqrt{(s^2 - 2s(M_{W^+}^2 + M_Z^2) + (M_{W^+}^2 - M_Z^2)^2)}}{2\sqrt{s}}. \tag{8}
$$

In the same way as that of the CP even Higgs production, the amplitude for CP odd

![FIG. 1: S channel W exchange](image-url)
Higgs and charged Higgs pair production is obtained by computing the Feynman diagrams corresponding to Figs. 1-4.

\[
M_A = \epsilon_W^\mu \epsilon_Z^\nu T_{A\mu\nu},
\]

\[
T_{A\mu\nu} = \frac{-ig^2}{2\cos\theta_W} \left( a_A g_{\mu\nu} + d_A q_{\mu\nu} q_{H^+\mu} + b_A q_{H^+\mu} q_{A\mu} \right). \tag{9}
\]

\(a_A, b_A\) and \(d_A\) in Eq. (9) are given as,

\[
a_A = s_W^2 + s_W^2 \frac{M_A^2 - M_{H^+}^2 - M_W^2}{s - M_W^2} + c_W^2 \frac{t_A - u_A + M_Z^2 - M_W^2}{s - M_W^2},
\]

\[
b_A = -\frac{2 \cos 2\theta_W}{u_A - M_{H^+}^2} - \frac{2(\cos 2\theta_W + 1)}{s - M_W^2},
\]

\[
d_A = \frac{2}{t_A - M_h^2} + \frac{2(\cos 2\theta_W + 1)}{s - M_W^2}, \tag{10}
\]

with \(t_A = (q_{H^+} - p_W)^2, u_A = (p_W - q_A)^2\). The kinematical variables \(t_A\) and \(u_A\) for \(H^+\) and \(A\) production can be obtained by replacing \(M_h\) by \(M_A\) in the Mandelstam variables \(t, u\) in Eq. (6),

\[
t_A = M_{H^+}^2 + M_{W^+}^2 - \frac{(s + M_W^2 - M_Z^2)(s + M_{H^+}^2 - M_A^2)}{2s} + 2P_{W^+}P_A \cos \theta,
\]

\[
u_A = -s + M_Z^2 + M_A^2 + \frac{(s + M_W^2 - M_Z^2)(s + M_{H^+}^2 - M_A^2)}{2s} - 2P_{W^+}P_A \cos \theta. \tag{11}
\]

Since \(\beta\) and \(\gamma\) is very small and CP even and CP odd Higgses are almost degenerate, \(M_h \approx M_A\), the production cross sections for \(H^+ + h\) and \(H^+ + A\) are almost identical. Then
the production cross section for charged Higgs and CP odd Higgs ($A$) is given as,

\[
\frac{d\sigma_{W^++Z\to H^++A}}{d\cos\theta} = \frac{g^4}{36c_W^2} \frac{1}{32\pi s} \frac{P_A}{P_W^+} \left[ a^A_2 \left\{ 3 + \frac{(s - (M_{W^+} + M_Z)^2)(s - (M_{W^+} - M_Z)^2)}{4M_Z^2M_W^2} \right\} \right.
\]

\[
\left. + b^A_2 \left\{ \frac{(M^2_A + M^2_W - u_A)^2}{4M_W^2} - M^2_A \right\} \left( \frac{(M^2_{H^+} + M^2_Z - u_A)^2}{4M_Z^2} - M^2_{H^+} \right) \right\}
\]

\[
+ d^A_2 \left\{ \frac{(M^2_{H^+} + M^2_Z - t_A)^2}{4M_Z^2} - M^2_{H^+} \right\} \left( \frac{(M^2_A + M^2_Z - t_A)^2}{4M_Z^2} - M^2_A \right) \right\}
\]

\[
+ b_Ad_A \frac{M^2_A(M^2_{H^+} + 3M^2_Z - t_A) + 2M^2_W(M^2_{H^+} - s) + (M^2_{H^+} + M^2_W - t_A)(M^2_W - u_A)}{8M^4_{H^+}M^2_Z} \times \left\{ \frac{M^2_A(M^2_{H^+} + 3M^2_Z - u_A) + M^2_{H^+}(3M^2_Z - t_A) + M^4_Z - 2M^2_Z s - M^2_Z t_A - M^2_Z u_A + t_A u_A}{8M^4_{H^+}M^2_Z} \right\}
\]

\[
+ a_Ad_A \left\{ \frac{(M^2_A + M^2_W - u_A)(M^2_{H^+} + M^2_W - t_A)}{2M^2_W} - \frac{(M^2_A + M^2_Z - t_A)(M^2_{H^+} + M^2_Z - u_A)}{2M^2_Z} \right\}
\]

\[
+ a_Ad_A \left\{ \frac{(M^2_A + M^2_W - u_A)(M^2_{H^+} + M^2_Z - u_A)}{4M^4_WM^2_Z} - \frac{(M^2_A + M^2_Z - t_A)(M^2_{H^+} + M^2_Z - u_A)}{2M^2_Z} \right\} \right]. \quad (12)
\]

In Fig. 5, we show the differential cross section $\frac{d\sigma}{d\cos\theta} \bigg|_{\sqrt{s}=600\text{(GeV)}}$ for the neutral Higgs ($h$ or $A$) and charged Higgs pair production. The numerical values for $(M_W, M_Z, \sin^2\theta_W)$ are (80(GeV), 91(GeV), 0.23). We show the cross sections for three cases for Higgs spectrum in GeV unit as $(M_X, M_{H^+}) = (200, 200), (200, 300)$, and $(300, 200)$ with $X = A$ or $h$. The differential cross sections for non-degenerate masses of charged Higgs and neutral Higgs is much larger than the case that they are degenerate. We also study the dependence of the total cross section on CM energy $\sqrt{s}$ in Fig. 6. The total cross section decreases as $\sqrt{s}$ increases. For degenerate case, the total cross section at $\sqrt{s} = 600$ (GeV) is about 0.02 pb. When they are non-degenerate, the cross section is much larger than that of the degenerate case as shown in Fig. 6. The cross section for non-degenerate case is as large as 0.8 pb at $\sqrt{s} = 600$ (GeV).
The production cross section \( \frac{d\sigma}{d\cos\theta} \) of a pair of neutral Higgs \( X = A, h \) and charged Higgs \( H^+ \) for \( \sqrt{s} = 600\text{(GeV)} \). The solid line corresponds to \( (M_X, M_{H^+}) = (200, 200)(\text{GeV}) \). The dashed line corresponds to \( (M_X, M_{H^+}) = (200, 300)(\text{GeV}) \). The dotted line corresponds to \( (M_X, M_{H^+}) = (300, 200)(\text{GeV}) \).

The signals of the neutral Higgs production associated with a single charged Higgs production are large missing momentum and energies. The charged Higgs boson can be identified from their decays into a pair of charged lepton and neutrino. On the other hand, the neutral Higgs \( A \) or \( h \) decays into a pair of neutrino and antineutrino. Therefore, one can not see the decay products of neutral Higgs which leads to the large missing momentum in CM frame of the vector bosons fusion. The signal for the charged Higgs and the neutral Higgs

FIG. 6: \( \sigma(W + Z \rightarrow H^+ + X) \ (X = A, h) \) as a function of \( \sqrt{s} \). The dashed line corresponds to \( (M_X, M_{H^+}) = (200, 300)(\text{GeV}) \). The dotted line corresponds to \( (M_X, M_{H^+}) = (300, 200)\ (\text{GeV}) \). The solid line corresponds to \( (M_X, M_{H^+}) = (200, 200)\ (\text{GeV}) \).

IV. SIGNAL OF THE PAIR PRODUCTION OF THE NEUTRAL HIGGS AND CHARGED HIGGS, AND STANDARD MODEL BACKGROUND

The signals of the neutral Higgs production associated with a single charged Higgs production are large missing momentum and energies. The charged Higgs boson can be identified from their decays into a pair of charged lepton and neutrino. On the other hand, the neutral Higgs \( A \) or \( h \) decays into a pair of neutrino and antineutrino. Therefore, one can not see the decay products of neutral Higgs which leads to the large missing momentum in CM frame of the vector bosons fusion. The signal for the charged Higgs and the neutral Higgs
pair production of the present model is the single charged lepton from the charged Higgs decay. The signal event occurs through the subsequent decays of the charged Higgs and the neutral Higgs into $l^+\nu$ and $\bar{\nu}\nu$ respectively. Within the standard model, the event which is similar to the signal can occur through the process, $W^+ + Z \rightarrow W^+ + Z \rightarrow l^+\nu\bar{\nu}\nu\bar{\nu}$. In the following, we propose a measurement which characterizes the present model and estimate it using the production cross section computed in the previous section. We define the ratio of the numbers of the two events, one is the number of events with a single charged lepton with three neutrinos and the other is the number of the events with a single charged lepton and a $b\bar{b}$ pair,

$$R_l = \frac{\sum_k N(W^+ + Z \rightarrow \nu_k\bar{\nu}_k l^+\nu_l)}{N(W^+ + Z \rightarrow bbl^+\nu_l)},$$  \hspace{1cm} (13)

where the charged lepton flavor is specified as $l$. One can write the ratio as,

$$R_l = \sum_{X=h,A} \left( \frac{\sigma(W^+ + Z \rightarrow H^+ + X)}{\sigma_{SM}(W^+ + Z \rightarrow W^+ + Z)} \frac{Br(X \rightarrow \nu\bar{\nu})}{Br(W^+ \rightarrow l^+\nu_l)} \right) \frac{Br(H^+ \rightarrow l^+\nu_l)}{Br(Z \rightarrow bbl^+\nu_l)} + \frac{Br(Z \rightarrow \nu\bar{\nu})}{Br(Z \rightarrow bb)},$$  \hspace{1cm} (14)

where we used the shorthand notation, $Br(X \rightarrow \nu\bar{\nu}) = \sum_k Br(X \rightarrow \nu_k\bar{\nu}_k)$ for $X = h, A, Z$. Below we compute $R_l$. To compute $R_l$, one needs the ratio of the cross sections for Higgs pair production and vector boson pair production. The cross section of $WZ$ scattering is calculated \[5, 6\] and the ratio of the cross section,

$$\frac{2\sigma(W^+ + Z \rightarrow H^+ + X)}{\sigma_{SM}(W^+ + Z \rightarrow W^+ + Z)},$$  \hspace{1cm} (15)

is shown in Fig. 7. Since Higgs pair production cross section is at most 0.8 pb while gauge bosons scattering cross section is about 200 pb, the ratio can be as large as a few times $10^{-3}$. In what follows, we use $6.0 \times 10^{-3}$ as a numerical value for the ratio of the cross sections of Eq.(15). The other branching fractions which appear in Eq.(14) are quoted from Particle Data Group (PDG) \[7\],

$$Br(W^+ \rightarrow \tau^+\nu) = 11.25 \pm 0.20\%,$$  \hspace{1cm} (16)

$$Br(W^+ \rightarrow \mu^+\nu) = 10.57 \pm 0.15\%,$$  \hspace{1cm} (17)

$$Br(W^+ \rightarrow e^+\nu) = 10.75 \pm 0.13\%,$$  \hspace{1cm} (18)

$$Br(Z \rightarrow b\bar{b}) = 15.12 \pm 0.05\%,$$  \hspace{1cm} (19)

$$Br(Z \rightarrow \nu\bar{\nu}) = 20.00 \pm 0.06\%.$$  \hspace{1cm} (20)
FIG. 7: The ratio of the cross sections \( \frac{2 \sigma(W + Z \rightarrow H^+ + X)}{\sigma_{SM}(W + Z \rightarrow W + Z)} \) as a function of \( \sqrt{s} \) (GeV) for non-degenerate charged Higgs and neutral Higgs. We choose their masses as \( M_{H^+} = 300 \) (GeV), \( M_X = 200 \) (GeV) \( X = h, A \).

For the new physics side, the neutral Higgses can only decay into neutrino and anti-neutrino pairs. Therefore, \( Br(h \rightarrow \nu \bar{\nu}) = Br(A \rightarrow \nu \bar{\nu}) = 100\% \). Using the numerical values, one can write \( R_l (l = e, \mu, \tau) \) as,

\[
R_e = 1.32 + 0.61 \times Br(H^+ \rightarrow e^+ \nu) \frac{2 \sigma(W^+ + Z \rightarrow H^+ + X)}{\sigma_{SM}(W^+ + Z \rightarrow W^+ + Z)},
\]

\[
R_\mu = 1.32 + 0.63 \times Br(H^+ \rightarrow \mu^+ \nu) \frac{2 \sigma(W^+ + Z \rightarrow H^+ + X)}{\sigma_{SM}(W^+ + Z \rightarrow W^+ + Z)},
\]

\[
R_\tau = 1.32 + 0.59 \times Br(H^+ \rightarrow \tau^+ \nu) \frac{2 \sigma(W^+ + Z \rightarrow H^+ + X)}{\sigma_{SM}(W^+ + Z \rightarrow W^+ + Z)},
\]

(21)

where \( Br(H^+ \rightarrow l \nu) \) in \% unit should be substituted. The charged Higgs can decay into charged leptons and neutrino. In contrast to the leptonic decay of W boson, the branching fractions are dependent on the lepton flavor as, \( Br(H^+ \rightarrow l^+ \nu_l) = \frac{\sum_{i=1}^{3} m_i^2 |V_{li}|^2}{\sum_{i=1}^{3} m_i^2} \) [1]. We update the branching fraction to each lepton flavor mode using the recent results on \( |V_{e3}| \).

For normal hierarchy case, the branching fractions are written as,

\[
Br(H^+ \rightarrow l^+ \nu_l) = \frac{m_1^2 + \Delta m^2_{sol} |V_{12}|^2 + (\Delta m^2_{sol} + \Delta m^2_{atm}) |V_{13}|^2}{3m_1^2 + 2\Delta m^2_{sol} + \Delta m^2_{atm}}.
\]

(22)

In the formulae of Eq.(22), \( m_1 \) denotes the lightest neutrino mass. For inverted hierarchical case, they are written as,

\[
Br(H^+ \rightarrow l^+ \nu_l) = \frac{m_3^2 + \Delta m^2_{atm} (|V_{11}|^2 + |V_{12}|^2) - \Delta m^2_{sol} |V_{11}|^2}{3m_3^2 + 2\Delta m^2_{atm} - \Delta m^2_{sol}},
\]

(23)

where \( m_3 \) denotes the lightest neutrino mass. We have used the following value for the mixing angles [7], \( \sin^2 \theta_{12} = 0.306 \), \( \sin^2 \theta_{23} = 0.42 \), \( \sin^2 \theta_{13} = 0.021 \). We also use \( m^2_{atm} = \)
$2.35 \times 10^{-3} \text{(eV}^2\text{)}$ and $m_{\text{sol}}^2 = 7.58 \times 10^{-5} \text{ (eV}^2\text{)}$ for mass squared differences for neutrino mass eigenstates. In Fig. 8, we have shown $R_l \ (l = e, \mu, \tau)$ for normal hierarchical case as functions of the lightest neutrino mass $m_1$. In Fig. 9, we have shown $R_l$ for inverted hierarchical case as functions for the lightest neutrino mass $m_3$. The shaded bands in Figs. 8,9, show the standard model expectation and errors given as $1.322 \pm 0.006$.

![Graph showing $R_l(l = e, \mu, \tau)$ for normal hierarchical case as functions of the lightest neutrino mass $m_1$.](image1)

![Graph showing $R_l(l = e, \mu, \tau)$ for inverted hierarchical case as functions of the lightest neutrino mass $m_3$.](image2)

FIG. 8: $R_l \ (l = e, \mu, \tau)$ for normal hierarchical case as functions of the lightest neutrino mass $m_1$ (eV). The dotted line corresponds to $R_e$, the dashed line corresponds to $R_\mu$ and the solid line corresponds to $R_\tau$ respectively.

FIG. 9: $R_l \ (l = e, \mu, \tau)$ for inverted hierarchical case as functions of the lightest neutrino mass $m_3$ (eV). The dotted line corresponds to $R_e$, the dashed line corresponds to $R_\mu$ and the solid line corresponds to $R_\tau$ respectively.

As we have seen from Fig. 8 and Fig. 9, we can expect the sizable deviation from the standard model expectation on the a single charged lepton jet production rate with large missing momentum in the neutrinoophilic two Higgs doublet model. The ratio $R_l$ can deviate about 10\% level. The sizable flavor dependence comes from the following reasons.

1. The non-suppressed charged Higgs and neutral Higgs production cross section for non-degenerate charged and neutral Higgses as shown in Fig. 7.
2. The flavor specific charged Higgs decay into charged lepton strongly depends on two different patterns of hierarchy of neutrino mass spectrum corresponding to the normal or inverted. It also depends on the Maki Nakagawa Sakata (MNS) matrix.

Therefore we conclude that $R_l$ is a key measurement which distinguishes the present model from the standard model.

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[1] S. M. Davidson and H. E. Logan, Phys. Rev. D 80, 095008 (2009) [arXiv:0906.3335 [hep-ph]].
[2] S. M. Davidson and H. E. Logan, Phys. Rev. D 82, 115031 (2010) [arXiv:1009.4413 [hep-ph]].
[3] T. Morozumi, H. Takata and K. Tamai, Phys. Rev. D 85 (2012) 055002 [arXiv:1107.1026 [hep-ph]].
[4] Naoyuki Haba and Masaki Hirotsu. TeV-scale seesaw from a multi-Higgs model. Eur. Phys. J., C69:481–492, 2010.
[5] T. Bahnik, [hep-ph/9710265]
[6] D. Green, [hep-ex/0309031]
[7] J. Beringer et. al. Phys. Rev. D 86 (2012) 01001.