Diffractive meson production from virtual photons with odd charge-parity exchange

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Abstract: We calculate the cross section of diffractive charge-parity $C = +1$ neutral meson production in virtual photon proton collision at high energies. Due to the opposite $C$-parities of photon and meson $M^+ (M^+ = \eta_C, \pi^0, a_2)$ this process probes the $t$-channel $C = -1$ odderon exchange which is described here as noninteracting three–gluon exchange. Estimates for the cross section of inelastic diffractive process $\gamma^* p \rightarrow \eta_C X_p$ are presented. The total cross section of diffractive $\eta_C$ meson photoproduction is found to be 47 pb. The cross sections for the diffractive production of light mesons ($\pi^0, a_2$) in $\gamma^* p$ collisions are of the same order if the photon virtuality $Q^2$ is $m^2_C$.

\textsuperscript{1}Supported by German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, grant No. 05 7LP91 P0, and in the framework of the German-Polish agreement on scientific and technological cooperation, grant No. N - 115 - 95.
1 Odd charge-parity exchange

The increase of luminosity at HERA offers the possibility of experimental investigations of diffractive processes of photo- and electroproduction of neutral charge-parity even mesons $M^+ = \pi^0, a_2, \eta_C$ at high energies (see, for example, [1]). Studies of these reactions are of great interest since the opposite charge-parities $C$ of photon ($C = -1$) and $M^+$ meson cause definite $C$-parity $C = -1$ of the $t$-channel exchange. Consequently the pomeron exchange being most important at high energies does not contribute there. Therefore these processes allow for clean investigations of other Regge trajectories which are characterized by negative $C = -1$ parity.

Reggeons having $C = -1$ are the $\omega$ trajectory and the odderon. According to fits to data on total hadron–hadron cross sections the intercept of $\omega$ trajectory is close to 0.5 [2] and its contribution decreases with energy. The odderon is the $C$ odd partner of pomeron with an intercept $\geq 1$ which has been introduced in phenomenology long time ago [3]. It is assumed that, like the pomeron, the odderon is related to gluonic degrees of freedom. The odderon exchange results in a difference between, for example, the total $pp$ and $p\bar{p}$ cross sections which does not decrease with increasing energy. Up to now there is no indication of that difference in present high energy $pp$ and $p\bar{p}$ data. Also odderon effects have not been observed in other soft hadronic collisions.

The reasons for the absence of the odderon exchange in soft hadronic reactions remains to be understood since in perturbative QCD the pomeron and the odderon appear on the equal footing. In Leading Logarithmic Approximation (LLA) of perturbative QCD the pomeron corresponds to the exchange of two interacting reggeized gluons (hard pomeron) whereas the odderon is described by exchange of three interacting reggeized gluons (hard odderon), see [4]. In Born approximation, i.e. considering the pure three–gluon exchange in the $t$-channel, the intercept of the odderon is equal to 1. The effect of gluon interactions is expected to increase of odderon intercept [5]. Therefore the hard odderon should manifest itself in hard diffractive processes at high energies.

The idea to study odderon exchange dominated processes in the diffractive meson production was considered by Schäfer, Mankiewicz and Nachtmann in Ref. [6] where the odderon exchange was treated purely phenomenologically, in analogy to the one-photon exchange. The possibility to probe the hard odderon in processes of light meson production in photon-photon collisions at large $t$ was considered in [7]. Recently exclusive $\eta_C$ photoproduction on protons was studied by Czyżewski et al. in [8] describing the odderon as three-gluon exchange.

In this paper we shall calculate the diffractive production of $C = +1$ mesons in Born approximation, i.e. involving the exchange of three noninteracting gluons in the $t$-channel. We use assumptions similar to the ones made in [8], though our method is different. It is based on the impact factors obtained in [7]. The results for heavy mesons are compared with [8] and discrepancies are discussed. Further-
more we study diffractive production of light mesons $M^+ = \pi^0, a_2, \ldots$ in virtual photon-proton interactions and the process of diffractive meson production at large $t$ which is characterized by proton disintegration.

2 Impact factors and meson wave functions

In the following we consider diffractive meson production processes which can be characterized by at least one hard scale. This scale is either the mass of a heavy quark (for example, in $\eta_C$ production) or the virtuality $Q^2$ of the photon in the production of light mesons.

The quasi-elastic production of a $M^+$ meson is shown in Fig. 1. The mean transverse distance of the quarks in the upper block of Fig. 1 is $\sim 1/m_c, 1/Q$. The other essential (soft) scale in our problem is the mean transverse separation between the quarks in proton $\sim 1/m_\rho$ (lower part of Fig. 1). With the growth of the transferred momentum $|t| \geq m_\rho^2$ the process with photon disintegration shown in Fig. 2 becomes more important than the elastic one (Fig. 1). In the region of large $t$ ($|t| \gg \Lambda_{QCD}^2$) the cross section for this process can be expressed in terms of the photon-quark cross section and the quark densities inside the proton. In this case there is no soft scale and we deal with a purely hard diffractive process where the perturbative odderon should contribute. In contrast, the production of light mesons by real photons is a typical soft process and is not considered here.

The amplitude determined by three–gluon exchange can be written in impact representation as an integral over the gluon transverse momenta $\vec{\kappa}_i$ [7]

$$\mathcal{M} = \frac{s}{3(2\pi)^4} \int \frac{d^2\kappa_1}{\kappa_1^2} \frac{d^2\kappa_2}{\kappa_2^2} \frac{d^2\kappa_3}{\kappa_3^2} J_{\gamma^* M}(\vec{\kappa}_1, \vec{\kappa}_2; \vec{q}) \cdot J_{pp}(\vec{\kappa}_1, \vec{\kappa}_2; \vec{q}) \cdot (1).$$

The kinematical notations are given in Fig. 1. Here $s = (q + p)^2$ and $t = (p_1 - p_M)^2 = -\vec{q}^2$ is the momentum transfer with $\vec{q} = \vec{\kappa}_1 + \vec{\kappa}_2 + \vec{\kappa}_3$. The impact factors $J_{\gamma^* M}$ and $J_{pp}$ correspond to the upper and lower part of Fig. 1, correspondingly.

The impact factor $J_{\gamma^* M}$ for the $\gamma^* \rightarrow M$ transition can be derived within perturbative QCD. It can be expressed in terms of the amplitude $J_{\gamma^* q\bar{q}}$ for $q\bar{q}$ production which has the form [7]

$$J_{\gamma^* q\bar{q}} = e g^3 Q_0 \left( \frac{d^{abc}}{4N} \right) \bar{u}_1 \left[ \cdots \right] \frac{\hat{p}_2}{s} u_2 \left[ \cdots \right] = mR\hat{e} - 2x(\vec{Q}\hat{e}) - \hat{Q}\hat{e}$$

$$R = \left( \frac{1}{m_0^2 + \vec{q}_1^2} - \sum_{i=1}^{3} \frac{1}{m_0^2 + (\vec{\kappa}_i - \vec{q}_1)^2} \right) - (\vec{q}_1 \leftrightarrow \vec{q}_2)$$

$$\vec{Q} = \left( \frac{\vec{q}_1}{m_0^2 + \vec{q}_1^2} + \sum_{i=1}^{3} \frac{\vec{\kappa}_i - \vec{q}_1}{m_0^2 + (\vec{\kappa}_i - \vec{q}_1)^2} \right) + (\vec{q}_1 \leftrightarrow \vec{q}_2)$$

$$m_0^2 = m^2 + Q^2 x (1 - x)$$

(2)
In the above expression $e^\mu = (0, \vec{e}, 0)$ is the polarization vector of the photon. Furthermore we use the notations $\alpha = e^2/4\pi = 1/137$ and $\alpha_s = g^2/4\pi$. $q_0 \vec{e}$ is the polarization vector of the photon. Furthermore we use the notations $\alpha = e^2/4\pi = 1/137$ and $\alpha_s = g^2/4\pi$. $Q_q e$ is the quark charge and $d_{abc}$ are the totally symmetric structure constants for the $SU(N)$ color group. $\vec{q}_1$ and $\vec{q}_2$ are the transverse momenta of quark and antiquark, $0 < x < 1$ is the fraction of the quark longitudinal momentum. The virtuality of photon is $p_1^2 = -Q^2$, $m$ is the quark mass. The quark spinors are denoted by $u_1 = u(q_1)$, $u_2 = u(q_2)$. It is convenient to introduce the variables

$$\xi = 2x - 1, \quad \nu = \frac{4m^2 + Q^2(1 - \xi^2)}{q^2}.$$ (3)

We will neglect the transverse motion of quarks inside the meson (collinear approximation). Then the $q\bar{q} \rightarrow M$ transition is described by the Eq. (2) in which the quark spinors are substituted by the meson wave function $\varphi_M(\xi)$ (see e.g. [9, 10]). One obtains the substitutions:

(a) for the tensor meson $T$ with helicity $\lambda = 0$

$$Q_q \bar{u}_1 \ldots u_2 \rightarrow \frac{Q_T}{4N} \int_{-1}^{+1} d\xi f_T(\xi) Tr(\cdots \hat{p}_3)$$ (4)

(b) for the pseudoscalar mesons $P$

$$Q_q \bar{u}_1 \ldots u_2 \rightarrow \frac{Q_P}{4N} \int_{-1}^{+1} d\xi f_P(\xi) Tr(\cdots \gamma^5 \hat{p}_3).$$ (5)

The trace is performed over vector and color indices. The quantity $Q_M (M = P, T)$ is the average quark charge in the meson $M$. For instance $|\pi^0>= (|u\bar{u}> - |d\bar{d}>)/\sqrt{2}$, therefore

$$Q_\pi = \frac{1}{\sqrt{2}}(\frac{2}{3} - (\frac{1}{3})) = \frac{1}{\sqrt{2}}.$$ 

It should be noticed that the production of tensor mesons with the helicity $\lambda = \pm 1, 2$ is suppressed in LLA at least by a factor $\sim (\Lambda_{QCD}^2/Q^2)$ [7].

According to Ref. [9] we adopt for numerical calculations the following parametrizations for the wave functions and the coupling constants:

(a) for light tensor mesons $T$

$$\varphi_T(\xi) = \frac{15}{4} \xi (1 - \xi^2); \quad f_T = 85\text{ MeV};$$

$$Q_{a_2} = \frac{1}{\sqrt{2}}, \quad Q_{f_2} = \frac{1}{3\sqrt{2}}, \quad Q_{f_2'} = \frac{1}{3}.$$ (6)

where an ideal mixing is assumed, i.e. $f'$ consists of $s\bar{s}$-states only.
(b) for light pseudoscalar mesons $P$

\[ \varphi_P(\xi) = \begin{cases} \frac{3}{4}(1 - \xi^2) & \text{asymptotical form} \\ \frac{15}{4}\xi^2(1 - \xi^2) & \text{Chernyak-Zhitnitsky form} \end{cases} ; \]

\[ f_\pi = 133 \text{ MeV} , \quad f_\eta = 150 \text{ MeV} , \quad f_{\eta'} = 110 \text{ MeV} ; \]

\[ Q_\pi = \frac{1}{\sqrt{2}} , \quad Q_\eta = 0.38 , \quad Q_{\eta'} = 0.14 \]

where the standard $\eta\eta'$-mixing is taken into account.

c) for the $\eta_C$ meson

\[ \varphi_{\eta_C} = \delta(\xi) , \quad f_{\eta_C} = 400 \text{ MeV} . \]

The photon width of $\eta_C$ is expressed through $f_{\eta_C}$ as \( \Gamma_{\eta_C \to \gamma\gamma} = \frac{e^4 f_{\eta_C}^2}{(4\pi m_{\eta_C})} \approx 7 \text{ keV} \). 

The usage of a delta function for the wave function of heavy mesons is equivalent to the non-relativistic approximation. In the case of light meson production we take into account the nontrivial longitudinal motion of the light quarks.

The conditions of the collinearity of quark momenta are

\[ \vec{q}_1 = \frac{1}{2}(1 + \xi)\vec{q}, \quad \vec{q}_2 = \frac{1}{2}(1 - \xi)\vec{q} . \]

Introducing the dimensionless vectors $\vec{r}_i$

\[ \vec{r}_i = \frac{1}{2}(\vec{r}_i + \vec{n}) \quad |\vec{q}| , \quad \vec{n} = \vec{q}/|\vec{q}| \]

the impact-factors (11,12) read

\[ J_{\gamma^* M} (\vec{k}_{1,2}, \vec{q}) = e g^3 \left( \frac{d^{abc}}{4N} \right) Q^M f_M |q| \left\{ \begin{aligned} (\vec{e} \cdot \vec{F}_T) & \quad \text{for} \quad M = T \\ [\vec{e} \times \vec{F}_P] & \quad \text{for} \quad M = P \end{aligned} \right\} \]

\[ \vec{F}_M = \int_{-1}^{+1} d\xi \left\{ \frac{\vec{n}(1 + \xi)}{\nu + (1 + \xi)^2} + \sum_{i=1}^{3} \frac{\vec{r}_i - \vec{n}\xi}{\nu + (\vec{r}_i - \vec{n}\xi)^2} \right\} \]

\[ \left\{ \begin{aligned} \xi \varphi_T(\xi) & \quad \text{for} \quad M = T \\ \varphi_P(\xi) & \quad \text{for} \quad M = P \end{aligned} \right\} \]
It should be noted that $\vec{F}_M(\vec{r}_i, -\vec{n}) \to 0$ at $\vec{r}_i \to -\vec{n}$.

The impact factor $J_{pp}$ which describes the lower part shown in Fig. 1 cannot be calculated in the region of small momentum transfers ($|t| \leq \Lambda^2_{QCD}$) within perturbative QCD since the mean transverse distance between the quarks inside the proton is large. Therefore, a phenomenological ansatz is used for this quantity. There are several restrictions on this impact factor imposed by gauge invariance:

$J_{pp}$ must vanish when $\vec{\kappa}_i (i = 1, 2, 3)$ tends to zero. This property ensures the convergence of the integral in Eq. (1). Bose symmetry demands that $J_{pp}$ should be symmetric under interchange of $t$-channel gluons. Of course, both impact factors $J_{\gamma q}$ and $J_{\gamma M}$ discussed above satisfy these requirements.

For $J_{pp}$ we use the expression derived by Fukugita and Kwieciński within the eikonal model [12]. In our conventions it reads

$$J_{pp}(\vec{\kappa}_1, \vec{\kappa}_2, \vec{\kappa}_3) = \bar{g}^3 \left(\frac{d^{abc}}{4N}\right) \cdot 3 \cdot \left[ F(\vec{q}, 0, 0) - \sum_{i=1}^{3} F(\vec{\kappa}_i, \vec{q} - \vec{\kappa}_i, 0) + 2F(\vec{\kappa}_1, \vec{\kappa}_2, \vec{\kappa}_3) \right] (15)$$

where $\vec{q} = \vec{\kappa}_1 + \vec{\kappa}_2 + \vec{\kappa}_3$ and

$$F(\vec{\kappa}_1, \vec{\kappa}_2, \vec{\kappa}_3) = \frac{A^2}{A^2 + \frac{1}{2}[(\vec{\kappa}_1 - \vec{\kappa}_2)^2 + (\vec{\kappa}_2 - \vec{\kappa}_3)^2 + (\vec{\kappa}_3 - \vec{\kappa}_1)^2]} (16)$$

with $\bar{g}^2/(4\pi) \approx 1$ and $A = m_\rho/2$. This eikonal model describes correctly the values of total hadronic cross sections [13, 12]. The normalization of Eq. (15) can be understood in the following way. The three-gluon impact factor of point like fermions does not depend on the gluon momenta. Hence this coupling is given by [1]

$$J^{3G}_{qq} = g^3 \left(\frac{d^{abc}}{4N}\right). (17)$$

The first term in brackets in Eq. (13) results from diagrams where all the three $t$-channel gluons couple to the same quark line. When $q$ is equal to zero the interaction with the $t$-channel gluons does not disturb quark motion and, due to the normalization of proton wave function, this part of the proton impact factor is equal to $3 \cdot g^3 \left(\frac{d^{abc}}{4N}\right)$ where the factor 3 counts the number of constituent quarks inside the proton. In the case of the proton electromagnetic form factor this coefficient 3 will be changed to 1 due to the fractional electric charges of quarks $1 = 2/3 + 2/3 - 1/3$ and the gauge group factor is replaced by 1. The other terms in brackets in Eq. (15) describe the contribution of the other diagrams where gluons couple to different quark lines. The whole expression (13) is gauge invariant and obeys Bose symmetry. In terms of dimensionless variables the proton impact factor reads

$$J_{pp} = 3\bar{g}^3 \left(\frac{d^{abc}}{4N}\right) F_p (18)$$
\[ F_p = \left[ \frac{1}{1 + 4z} - \sum_{i=1}^{3} \frac{1}{1 + z(3r_i^2 + 1)} + \frac{2}{1 + \frac{2}{3}[(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_2 - \vec{r}_3)^2 + (\vec{r}_3 - \vec{r}_1)^2]} \right] \]  
\((19)\)

with \(z = \frac{q_2^2}{m_p^2}, \vec{r}_3 = -\vec{r}_1 - \vec{r}_2 - \vec{n} \).

Substituting the impact factors \((14, 18)\) into Eq. \((1)\) the following expression for the amplitude is obtained

\[ M = g^3(\mu^2)g^3Q_M \frac{s f_M}{18\pi^2 |t|^{5/2}} F \cdot \left\{ \begin{array}{ll} \vec{e} \times \vec{n} & \text{for } M = P \\ (\vec{e} \cdot \vec{n}) & \text{for } M = T \end{array} \right. \]  
\((20)\)

where

\[ I = \frac{1}{12\pi^2} \int \frac{d^2r_1 d^2r_2}{(\vec{r}_1 + \vec{n})^2(\vec{r}_2 + \vec{n})^2(\vec{r}_1 + \vec{r}_2)^2} F_M F_p \]  
\((21)\)

In the case of \(\eta_C\) production Eq. \((20)\) can be compared with the results reported in [8]. Our amplitude \((21)\) is 3 times smaller. We have given here our arguments about the normalizing coefficients in all details in order to show that our normalization is the correct one.

As already mentioned in the introduction, at large \(t\) the diffractive process with proton dissociation \(\gamma p \rightarrow MX\) dominates over the elastic one. In this case the cross section can be expressed in terms of the cross section of photon-quark scattering and the quark densities inside proton (Fig. 2)

\[ \frac{d\sigma}{dt \, dx} = \sum_{q, \bar{q}} \frac{d\hat{\sigma}_{\gamma q \rightarrow Mq}}{dt}[q_i(x, t) + \bar{q}_i(x, t)] \]  
\((22)\)

It is important to notice that the three-gluon \(C\)-odd odderon does not couple to gluons in the proton [14].

### 3 Results on diffractive meson production

The results which are presented below have been obtained by numerical evaluation of the integral \(I\) in Eq. \((20)\) and of a similar integral for hard diffractive scattering on quarks. The hard cross section \(d\hat{\sigma}_{\gamma q \rightarrow Mq}/dt\) was calculated from the amplitude \((20)\) with the replacement \(g^3g^3 \rightarrow g^6(\mu^2)\), where \(\mu\) is the scale for the evaluation of the running strong coupling constant.

Let us start with the discussion of the results for elastic \(\eta_C\) production. In the calculations we use the scale \(\mu^2 = m_c^2\), \(m_c = 1.4\) GeV and \(\Lambda_{QCD}^{(4)} = 0.2\) GeV (for 4 flavors). Our results for the unpolarized differential cross section for the process \(\gamma^* p \rightarrow \eta_C p\) are given in Fig. 3. This cross section vanishes at \(t = 0\), it reaches the maximum at \(|t| \approx 0.5\) GeV and decreases for larger \(|t|\). The differential
cross section decreases rapidly with the growth of photon virtuality. In Fig. 3 we show also the result for $Q^2 = 0$ reported in Ref. [3] which is approximately 4 times larger than ours. The reasons for this discrepancy are the following: (i) as discussed above in detail, the amplitude (20) turns out to be smaller in normalization by a factor of 3 compared to the result quoted in [3], and (ii) we use $\alpha_s(m^2_c) \approx 0.387$ instead of 0.3 as done in [3].

In Fig. 4 we present the cross section integrated over the region $0 < |t| < t_0$ as a function of $t_0$. We see that the main part of total cross section originates from the region of small $|t|$. For real photoproduction the total cross section is about 47 pb, it decreases with increasing $Q^2$.

Now let us discuss the results for large momentum transfer $t$ when the proton disintegrates. In this case the coupling constant has been evaluated at the scale $\mu^2 = |t|$. The cross section of photon-quark scattering was convoluted with the GRV parametrization [15] of the quark densities in the proton, see Eqs. (22). The results are given in Figs. 5 and 6. Comparing these figures with the corresponding cross sections for elastic $\eta_C$ production (Figs. 3, 4) one can see that the process $\gamma^* p \rightarrow \eta_C X p$ is not considerably suppressed. For example, its cross section is 11 pb for $|t| > 3$ GeV$^2$ and $x \geq 0.1$.

Calculating light meson production we have an additional nontrivial integration over the variable $\xi$, describing the longitudinal motion of light quarks. The results for the cross sections of $\pi^0$ and $a_2$ meson production are shown in Figs. 7 and 8. The cross sections for the production of other mesons can be readily obtained by changing corresponding coupling constants given in Eqs. (6,8). The strong coupling constant has been evaluated at the scale $\mu^2 = Q^2 + |t|$. In the case of $\pi^0$ production the asymptotic form of the wave function (7) has been used. Comparing Eqs. (14) and (5,6) it can be seen that the formula for the amplitude of tensor meson production is similar to one for the production of pseudoscalar mesons described by the Chernyak-Zhitnitsky wave function. They differ only in the meson coupling constants and in the polarization dependence. The unpolarized cross sections differ by the coupling constants only, e.g. $d\sigma_{\pi} = (f_\pi/f_{a_2})^2 d\sigma_{a_2}$. Comparing the Figs. for $\pi^0$ and $a_2$ production it can be seen that the shape of differential cross sections depends strongly on the form of the meson wave function. The position of the dip is very sensitive to the form of the wave function of the light meson. It gets shifted to larger $|t|$ with increasing photon virtuality $Q^2$.

In Tab. 1 the cross sections of diffractive meson production are compared. The cross sections of $\pi^0$ and $a_2$ meson production are comparable to the cross section of $\eta_C$ production for $Q^2 \approx m^2_c$. They fall off very rapidly with increasing $Q^2$.

Performing the calculations for the diffractive production of light mesons we have to take care careful about the lower limit of the photon virtuality $Q^2$ which to ensure the self-consistency of our perturbative approach. To get an idea about this limit the numerical calculations of the cross sections have been done keeping in our formulae a non zero quark mass $m$ which can be considered as the measure
Table 1:  
Table of cross sections for diffractive meson production (for light mesons $|t| > t_0 = 3$ GeV$^2$). The cross sections are given in pb.

| meson / $Q^2$ [GeV$^2$] | 0   | 1   | 2   | 5   | 10  |
|-------------------------|-----|-----|-----|-----|-----|
| $\eta_c$                | 47  | 33  | 24  | 11  | 4.2 |
| $\pi^0$                 | -   | -   | 100 | 8.4 | 1.1 |
| $a_2$                   | -   | -   | 31  | 3.6 | 0.68|

of non-perturbative effects. We have chosen two typical values of the constituent quark masses $m = 0.2$ GeV and $m = 0.3$ GeV. In the Table 2 the results for the ratio of the total cross sections $\sigma(m^2)/\sigma(m^2 = 0)$ for tensor meson production are given for some values of photon virtualities $Q^2$. It can be concluded that

Table 2:  
Ratio of the total cross sections $\sigma(m^2)/\sigma(m^2 = 0)$ for elastic tensor meson production for different photon virtualities.

| quark mass / $Q^2$ [GeV$^2$] | 1    | 2    | 5    | 10   |
|-------------------------------|------|------|------|------|
| $m = 200$ MeV                 | 0.69 | 0.90 | 0.98 | 0.99 |
| $m = 300$ MeV                 | 0.50 | 0.74 | 0.92 | 0.95 |

perturbative calculations should be reliable for $Q^2 \gtrsim 2$ GeV$^2$.

4 Discussion

Summarizing we would like to stress that the numerical results presented here are based on amplitudes the essential part of which have been derived from perturbative QCD. In particular these are the impact representation (1) and the virtual photon impact factor (2). As further inputs we need the meson wave functions and couplings which are well known from phenomenology.

The proton impact factor in the quasi-elastic diffraction which we have adopted following [12] can be motivated by perturbative calculations. However there is no hard scale in that part of the process and the form of the proton impact factor enters as an additional assumption. The form of $J_{pp}$ (proposed by Fukugita and Kwiecinski [12]) exhibits the following features: it is gauge invariant, Bose symmetric and incorporates the parameter $\tilde{g}$ (frozen strong coupling) and a parameter related to the size of the proton $A \approx m_p/2$. We suppose that the uncertainty related to its form does not affect seriously our value of the differential cross section in the region of small momentum transfers and also not the value of total cross section since it is saturated mostly by the region of small $t$ (see Figs. 3, 4).
But it might be that at large momentum transfers $|t| \gg \Lambda_{QCD}^2$ this impact factor has to be changed since it does not obey the dimensional counting rules.

In inelastic diffraction being characterized by proton disintegration the quark distributions in the proton take the role played by the proton impact factor before. This non-perturbative input is, of course, much better known. Due to that this process has less theoretical uncertainties, but its cross section in the region $t_0 < |t|, t_0 = 2 \div 3 \text{ GeV}$ is smaller than the total cross section for the elastic process.

Our calculations are done in the Born approximation. One can try to estimate roughly the effects related to the interaction of $t$-channel gluons by multiplying the formulae for cross sections by the Regge factor $(s/\Lambda^2)^{2(\lambda_{odd}-1)}$. Using a value for the odderon intercept of $\lambda_{odd} = 1.1$ and $\Lambda^2 \sim m_{\eta_c}^2, Q^2$ this factor can be as large as 5 at HERA energies.

The value of the total $\eta_c$ photoproduction cross section in Born approximation is 47 pb. Similar cross sections are obtained for the production of light mesons by virtual photons, if the photon virtuality $Q^2$ is of the order $m_{\eta_c}^2$. It might be difficult to observe reactions with such cross sections at HERA. On the other hand the situation is much better if the odderon intercept is greater than 1 resulting in cross sections being a few times larger. Therefore we expect that experimental searches for diffractive production of $C = +1$ mesons at HERA can bring new interesting results relevant for the longstanding and controversial questions about the odderon.

**Acknowledgments**

D.Yu I. and L.Sz. would like to acknowledge the warm hospitality extended to them at University of Leipzig. One of the authors (R.E.) was supported in parts by the Deutsche Forschungsgemeinschaft under contract No Schi 422/1-2.
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Figures

Figure 1: The kinematics of elastic diffractive process $\gamma^* p \rightarrow M p$. 
Figure 2: The hard diffractive process $\gamma^* p \rightarrow MX_p$. $X_p$ are the hadrons originating from proton disintegration.
Figure 3: Differential cross section for $\gamma^* p \rightarrow \eta p$. The strong coupling has been evaluated with the scale $\mu^2 = m_c^2$ and with $\Lambda^{(4)}_{QCD} = 0.2$ GeV.
Figure 4: Total cross section for the process $\gamma^* p \rightarrow \eta_C p$ integrated over the region $0 < |t| < |t_0|$ as a function of $|t_0|$. 
Figure 5: Differential cross section for inelastic process $\gamma^* p \rightarrow \eta C X_p$. The strong coupling has been evaluated with the scale $\mu^2 = |t|$ and $\Lambda_{QCD}^{(4)} = 0.2$ GeV. The integral over the quark densities in the proton has been restricted to $x \geq 0.1$. 
Figure 6:  Total cross section for the process $\gamma^* p \rightarrow \eta_{C} X_p$ integrated over the region $|t_0| < |t|$. The integral over the quark densities in the proton has been restricted to $x \geq 0.1$. 
Figure 7: The differential cross section for the process $\gamma^* p \to \pi^0 p$. The pion wave function has been approximated by the asymptotical form. The strong coupling constant has been evaluated with the scale $\mu^2 = Q^2 + |t|$ and with $\Lambda_{\text{QCD}}^{(4)} = 0.2$ GeV.
Figure 8: The differential cross section for the process $\gamma^* p \rightarrow a_2 p$. The strong coupling has been evaluated with the scale $\mu^2 = Q^2 + |t|$ and with $\Lambda_{\text{QCD}}^{(4)} = 0.2$ GeV.