Hysteresis in a swirling two-fluid flow

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Abstract. This experimental study describes a hysteresis—a vivid manifestation of strongly nonlinear flow physics. A sealed vertical cylindrical container of radius 45 mm and height 90 mm is filled with water and sunflower oil. The rotating lid drives swirl and the meridional circulation of both fluids. As the rotation strength Re increases, the oil-water interface rises near the axis, touches the lid at Re = Re1, and moves toward the container sidewall. Then as Re decreases, the interface returns to the axis and separates from the lid at Re = Re2 < Re1. At each Re from the range, Re2 < Re < Re1, two different stable steady flow states are observed that is typical of hysteresis. The hysteresis only occurs if a volume fraction of oil is small. The hysteresis disappears as the oil fraction exceeds a threshold, which is around 0.4.

1. Introduction

Swirling, two-fluid flows recently attracted the attention of researchers due to applications in aerial vortex bioreactors, a rapidly developing technology [1-3]. These flows have a number of intriguing features of fundamental interest: numerous topological metamorphoses [4], multiple eddies [5], and thin circulation layers [6-8]. Fujimoto and Takeda [9] visualized a flow of silicon oil and water driven by the rotating lid in a sealed vertical cylindrical container. As the rotation strength Re increases, the interface takes shapes named by the authors as “hump”, “cusp”, “Mt. Fuji” and “bell”. Analogous results were obtained for a flow of soybean oil and water by Tsai et al. [10], who also detected vortex breakdown in the oil part of flow.

Our study explores one more strongly nonlinear phenomenon—hysteresis—in a confined two-fluid flow. A hysteresis can be related to an inverse bifurcation, e.g., in a plane channel flow [11]. A hysteresis, composed by fold bifurcations, is described by the analytical solutions [12-14] of the Navier-Stokes equations. These solutions model jump transitions observed in tornadoes [15]. Hysteretic transitions are typical of strongly swirling flows. The experiment [16] revealed a hysteresis in the appearance and disappearance of time oscillations in a single-fluid flow driven by a rotating disk in a sealed container. The experimental [17] and numerical [18] studies described hysteresis in annular swirling jets.

A hysteresis also can be a capillary phenomenon [19]. However, we have found no study of hysteresis in a two-fluid swirling flow in the literature. This work partially fills this gap and describes a hysteresis related to the capillary interaction of the sunflower-oil-water interface with the rotating lid in a vertical sealed cylindrical container.
2. Experimental set up

Figure 1(a) is a schematic of experimental set up showing the cylindrical container (1), CCD camera (2), stepping motor (3), LED 250W lamp (4) and reflecting white screen (5). The apparatus was housed in a darkened room, with lighting provided by a LED 250W lamp reflected off a white sheet of paper. The photos of interface were shot by CCD 6-megapixel camera CASIO EX-F1. The cylindrical container is made of transparent optical glass with the inner radius 45.75 mm and height 126 mm. The lid of radius $R = 45$ mm can be positioned at a varying height $h$ and is set here at $h = 2R = 90$ mm. The container is filled with tap water ($0 < z < h_w$, dark in figure 1) and sunflower oil ($h_w < z < h$, light in figure 1); $z$ is the distance from the container bottom and $h = h_w + h_o$.

At the room temperature $22.6^\circ C$, the fluid densities and kinematic viscosities are $\rho_w = 1000 \text{ kg/m}^3$, $\rho_o = 920 \text{ kg/m}^3$, $\nu_w = 1 \text{ cSt}$, and $\nu_o = 55 \text{ cSt}$. Hereafter subscripts “w” and “o” denote “water” and “oil”.

The oil-water interface tension is $\sigma_{ow} = 26 \text{ mN/m}$. The container is still except the lid which can rotate with angular velocity $\omega$ driven by the stepping motor. The rotation strength is characterized by the Reynolds number $\text{Re} = \omega R^2/\nu_o$; $\text{Re} = 100$ corresponds to about 0.5 RPS. For the Re range here considered, the variation in $\omega$ during a revolution is less than 0.3%.

The interface height at the axis was measured by a laser beam and a precise scale as figure 1(b) illustrates. A semiconductor laser (with power 250 mW and wavelength of 684 nm) was fixed on the coordinate device providing movement along $z$-axis with a step of 0.1 mm. The collimated laser beam was established horizontally. The beam diameter is 0.1 mm at the container axis providing the measuring accuracy about 0.1 mm. As the beam meets the interface it reflects. The laser beam was shifted up starting from the bottom along $z$-axis with a step of 0.1 mm until the reflection of laser beam from the interface terminates and the beam becomes tangent to the interface, touching the interface top.

![Figure 1](image1.png)

**Figure 1.** Schematic of experimental set up: (a) visualization, (b) laser measuring of the interface height at the axis.

3. Visual hysteresis observation

The experimental results are presented here at $h_o/R = 0.1, 0.25, 0.5$ and 0.75. For larger $h_o$, the interface does not reach the rotating lid because the flow becomes time-oscillating, the interface breaks, and water mixes with oil.

At rest, the interface is flat and horizontal. As angular velocity $\omega$ increases, the oil-water interface deforms up near the axis and down near the sidewall as typical of water spouts. Then, as $\omega$ further
increases the interface touches the lid and water spreads over the near-axis part of lid. As $\omega$ further increases, the interface breaks and water mixes with oil. In order to detect hysteresis phenomena, the measurements were repeated by decreasing $Re$. As we started to decrease $\omega$ before the interface breaks, the water separates from the lid at $\omega$ value being less than that at which water touches the lid as $Re$ increases.

The experimental results for the dependence of characteristic $Re$ values on volume fraction of the sunflower oil $h_o/R$ are presented in table 1. The Reynolds number, at which the interface touches the rotating lid at the axis, is denoted as $Re_1$. The Reynolds number, at which the flow becomes unsteady, is denoted as $Re_u$. If Reynolds number decreases before $Re_u$, the Reynolds number, at which the interface detaches from the lid, is denoted as $Re_2$. The hysteresis range is $\Delta Re = Re_1 - Re_2$. In the case, $h_o = 0.75R$, the hysteresis range is small and uncertainty in the difference $\Delta Re$ is around the step in $Re$. The steady state loses its stability for sufficiently high $Re$ except for $h_o \geq 0.75R$ where the flow becomes time-oscillating since $Re_u$ is smaller than $Re_1$ and $Re_2$. For this reasons this experimental investigation focuses on the volume fraction of the sunflower oil $h_o/R \leq 0.75$.

| $h_o/R$ | $Re_1$ | $Re_2$ | $\Delta Re$ | $Re_u$ |
|--------|--------|--------|-------------|--------|
| 0.1    | 170    | 115    | 55          | 440    |
| 0.25   | 266    | 226    | 40          | 456    |
| 0.5    | 465    | 445    | 20          | 570    |
| 0.75   | 675    | 670    | 5           | 660    |

Figure 2 shows photos illustrating the interface shapes at $h_o = 0.75$. As $Re$ increases, first the hump shape develops as figure 2(a) illustrates at $Re = 400$. The bell shape develops as figure 2(b) illustrates at $Re = 575$. This development is similar to that described in the literature [9, 10]. The difference starts at $Re = 675$ as the interface touches the rotating lid at the axis as figure 2(c) illustrates. Then the water spreads over the lid and the interface breaks.

Figure 3 shows photos illustrating the interface shapes at $Re = 300$. Figures 3(a), 3(b) illustrate the hump shape at $h_o = 0.75$ and 0.5. This development is similar to that described in the literature [9, 10]. The difference starts at $h_o = 0.25$ when the interface touches the rotating lid at the axis and the water spreads over the lid as figure 3(c) illustrates. This interface is close to the “Mt. Fuji” shape [9], but here its flattop part is attached to the lid. At $h_o = 0.1R$, the interface shape is close to that at $h_o = 0.5R$. 

![Figure 2](image1)

**Figure 2.** Interface shapes at $h_o = 0.75R$ and $Re = 400$ (a), $Re = 575$ (b), $Re = 675$ (c).

![Figure 3](image2)

**Figure 3.** Interface shapes at $h_o = 0.75R$ and $Re = 300$ (a), $h_o = 0.75R$ (b), $h_o = 0.5R$ (c).
Figure 3. Interface shapes at Re = 300 and h₀ = 0.75 (a), 0.5 (b), 0.25 (c), 0.1 (d).

Figure 4 presents photos illustrating the moment when the interface touches the rotating lid and the established steady state just after the detachment. Figure 4(a), 4(b) shows the interface at h₀ = 0.5R and Re = 465 as Re increases, Re = 445 as Re decreases. Figure 4(c), 4(d) shows the interface at h₀ = 0.1R and Re = 170 as Re increases, Re = 115 as Re decreases.

Figure 4. Interface shapes at h₀ = 0.5R and Re₁ = 465 (a), Re₂ = 445 (b) and at h₀ = 0.1R and Re₁ = 170 (c), Re₂ = 115 (d).

4. Summarized hysteresis results

Figure 5 characterizes the hysteresis by presenting the dependence of the interface height at the axis, hᵢ, on the rotation speed, Re. First, Re was increased from 0 (the rest state) with an increment ≤ 25. The results are presented by filled symbols in figure 5. Next, Re was decreases with the step down being ≤ 25. The results are presented by empty symbols in figure 5.

Figure 6 depicts the dependence of hysteresis characteristics on the oil volume fraction, h₀/h, where h₀ = h-hₚ is the depth of oil layer at rest; see figure 1. The symbols depict the measured values and the curves are fitting approximations. In figure 6(a), the solid curves show the upper (Re₁) and lower (Re₂) hysteresis values of the Reynolds number while the dashed curve corresponds to the stability boundary: the flow becomes unsteady above this curve. It is important that the stability and hysteresis curves intersects near h₀/h = 0.375 (h₀ = 0.75R) in figure 7(a). For h₀/h > 0.375, Reₐ is smaller than Re₁ and Re₂.
Figure 5. Dependence of interface height $h_i$ on the rotation speed $Re$ as $Re$ increases (filled symbols) and decreases (empty symbols) at $h_o = 0.75R$ (rhombuses), 0.5R (squares), 0.25R (circles) and 0.1R (triangles).

Figure 6(b) depicts the width of hysteresis range $\Delta Re$, where the square symbols present the measurement results and the curve depicts the fitting parabolic relation, $\Delta Re = 132(h_o/h)^2 - 215(h_o/h) + 65$. This relation helps predict important hysteresis characteristics which are not easy to measure: the maximal hysteresis range, $\Delta R_{\text{max}} = 65$, as $h_o/h$ tends to zero, and the oil fraction, $h_o/h = 0.402$ (or $h_o = 0.804R$), at which the hysteresis disappears ($\Delta Re$ diminishes down to zero).

Figure 6. Dependence on the oil volume fraction ($h_o/h$) of (a) the critical ($Re_u$) and the hysteresis upper ($Re_1$) and lower ($Re_2$) values of the Reynolds number and (b) the hysteresis range $\Delta Re$.

5. Conclusion - Physical reason of hysteresis
This experimental study of a hysteresis was performed in a sealed vertical cylindrical container filled with water and sunflower oil. The rotating lid drives swirl and the meridional circulation of both fluids. As the rotation strength $Re$ increases, the oil-water interface rises near the axis, touches the lid and moves toward the container sidewall. Then as $Re$ decreases, the interface returns to the axis and separates from the lid at a smaller $Re$ value. At each $Re$ two different stable steady flow states are observed that is typical of hysteresis. The hysteresis only occurs if a volume fraction of oil is small.
The physical reason of the hysteresis is the capillary effect. To detach the water from the lid requires a remarkable effort similar to the sufficient weight required for detaching water droplets from a wet ceiling. Here the rotating lid generates the centrifugal force, which pushes the oil toward the sidewall near the lid thus developing the oil meridional circulation in addition to the swirl. The oil converges toward the axis near the interface and goes upward near the axis. This oil motion entrains the adjacent water, acts against gravity, and raises the interface near the axis — the development typical of water spouts [6, 20]. As the rotation speeds up, this effect becomes sufficiently strong for the interface to reach the lid at the axis. Then the centrifugal force helps water to spread over the lid.

To detach the interface from the lid in the reversed process, the water wetting requires a weaker rotation and therefore stronger gravity contribution than those in the direct process. This results in the hysteresis. The hysteretic range of rotation strength decreases as the oil volume fraction increases because the oil motion decays as the distance from the lid increases. Therefore, the entrainment effect becomes weaker while the gravity action becomes stronger at the same rotation speed. This explains why the hysteresis disappears as the oil volume fraction exceeds a threshold which is 0.402 here.

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