Universal relations and normal-state properties of a Fermi gas with laser-dressed mixed partial-wave interactions

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In a recent experiment [P. Peng, et al., Phys. Rev. A 97, 012702 (2018)], it has been shown that the p-wave Feshbach resonance can be shifted toward the s-wave Feshbach resonance by a laser field. Based on this experiment, we study the universal relations and the normal-state properties in an ultracold Fermi gas with coexisting s- and p-wave interactions under optical control of a p-wave magnetic Feshbach resonance. Within the operator-product expansion, we derive the high-momentum tail of various observable quantities in terms of contacts. We find that the high-momentum tail becomes anisotropic. Adopting the quantum virial expansion, we calculate the normal-state contacts with and without laser field for 40K atoms using typical experimental parameters. We show that the contacts are dependent on the laser dressing. We also reveal the interplay of laser dressing and different partial-wave interactions on various contacts. Particularly, we demonstrate that the impact of the laser dressing in the p-wave channel can be probed by measuring the s-wave contacts, which is a direct manifestation of few-body effects on the many-body level. Our results can be readily checked experimentally.

I. INTRODUCTION

The interplay of s- and p-wave interactions can introduce interesting many-body physics in ultracold Fermi gases [1–6]. Such a scenario exists in the two-component 40K Fermi gases, where the p-wave Feshbach resonances near 198G are close to the wide s-wave Feshbach resonance near 202G. In the previous studies, it has been shown that mixed partial-wave interactions in such a system can give rise to fermion superfluid with hybridized s- and p-wave pairing [2], as well as the interesting normal-state properties exhibiting the interplay of s- and p-wave interactions [3]. For a low-dimensional two-component 40K Fermi gas, the overlap of s- and p-wave interactions can be tuned by using confinement-induced resonance, which would favor the elusive itinerant ferromagnetism in certain parameter regimes [4–6].

In a recent experiment [1], the p-wave Feshbach resonance with the magnetic quantum number \( m_I = 0 \) is shifted to overlap with the s-wave resonance via laser dressing (see Fig. 1). The experiment thus offers an additional control on mixed partial-wave interactions of the system, which is bound to give rise to the interesting many-body physics. As a first attempt at clarifying the impact of few-body physics on many-body properties of the system, we study the universal relations and normal-state properties in an ultracold Fermi gas with coexisting s- and p-wave interactions near a laser-dressed p-wave Feshbach resonance.

In dilute atomic gases with short-range interaction potentials, it has been shown that universal behaviors emerge in the large-momentum limit. Physically, this is because when two atoms get close, the short-distance many-body wave function reduces to the two-body solution, yielding universal relations, as first studied by Tan for a three-dimensional two-component Fermi gas near an s-wave Feshbach resonance [12–14]. As a result of the universality, observable thermodynamic quantities such as the high-momentum tail of the momentum distribution, the radio-frequency (rf) spectrum, the pressure, and the energy are connected by a set of key parameters called contacts [12–18]. Recently, much effort has been devoted to the study of universal relations under synthetic gauge field [19–21], with Raman-dressed Feshbach resonance [22], in low-dimensional atomic gases [23–29], and in high partial-wave quantum gases [30–42]. Partic-
ularly, the universal relations for the $p$-wave Fermi gases have already been experimentally verified \[37\].

In this work, adopting the operator-product expansion (OPE) approach \[21, 22, 41-52\], we derive universal relations of the system with laser-dressed hybrid interactions. We show that the leading-order terms in high-momentum tails of the momentum distribution and the rf spectrum can be expressed by five contacts, with four laser-field-dependent closed-channel contacts. This is quite different from the previous studies in the absence of the laser dressing. Interestingly, one of the contacts is anisotropic, and the high-momentum tail in the momentum distribution shows anisotropic features. We then calculate the normal-state contacts and spectral function using the quantum virial expansion, both with and without the laser field for $^{40}$K atoms using typical experimental parameters. We find that, with the addition of the laser dressing in the closed channel of $p$-wave interaction, the $s$-wave contact significantly decreases around the $p$-wave Feshbach resonance. Such a behavior is a direct manifestation of few-body effects on the many-body level, and is useful for detecting the impact of dressing lasers on the system. Furthermore, the interplay of laser dressing and $p$-wave interaction leads to a much larger $p$-wave contact than the one without laser. Additionally, we show the $p$-wave contacts decrease much rapidly in the Bose-Einstein system. Furthermore, the interplay of laser dressing and $p$-wave interaction leads to a much larger $p$-wave contact than the one without laser. Additionally, we show the $p$-wave contacts decrease much rapidly in the Bose-Einstein system.

The paper is organized as follows: In Sec. II, we give the model Lagrangian density to describe the two-component ultracold Fermi gas with laser coupling. In Sec. III, we present a brief derivation on the renormalization of bare interactions. In Sec. IV, we calculate the high-momentum distribution of this system within the quantum field method of OPE. In Sec. V, we derive the corresponding universal relations such as high-frequency rf spectroscopy, adiabatic relations, pressure relations, and virial theorem for this system. In Sec. VI, we present the formalism of the quantum virial expansion, and express the contacts and the spectral function in the normal state up to the second order. In Sec. VII, we numerically evaluate the high-temperature contacts and spectral functions. We summarize in Sec. VIII.

II. MODEL

In the presence of optical field as shown in Fig. 1, the local Lagrangian density (at coordinate $R$) is given by

$$L = L_A + L_M + L_{AM},$$

where \[1\]

$$L_A = \sum_{\sigma = \uparrow, \downarrow} \psi_\sigma^\dagger \left( i \partial_t + \frac{\nabla^2}{2m} \right) \psi_\sigma - u_s \psi_\uparrow^\dagger \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow, \quad (1)$$

$$L_M = \sum_{m_i} \varphi_{m_i} \left[ i \partial_t + \frac{\nabla^2}{4m} - \nu_{m_i} - \Sigma_{m_i}(R) \right] \varphi_{m_i}, \quad (2)$$

$$L_{AM} = -\sum_{m_i} \frac{g_{m_i}}{\sqrt{2}} \left( \varphi_{m_i}^\dagger \mathcal{Y}_{m_i} + \mathcal{Y}_{m_i}^\dagger \varphi_{m_i} \right). \quad (3)$$

Here the self-energy in coordinate space is

$$\Sigma_{m_i}(R) = \frac{|\Omega_{m_i}|^2}{4 \left( i \partial_t + \frac{\nabla^2}{4m} - \nu_c + \delta_{m_i} \pm \frac{i\pi}{2} \right)}, \quad (4)$$

$$\mathcal{Y}_{m_i} = -\frac{1}{2} \sum_{\alpha} \frac{3}{4\pi} C_{\alpha, m_i} \left[ (i\nabla_\alpha \psi_\uparrow) \psi_\uparrow - \psi_\uparrow (i\nabla_\alpha \psi_\uparrow) \right], \quad (5)$$

$\psi_\sigma (\sigma = \uparrow, \downarrow)$ denotes the open-channel fermionic atom-field operator, $\varphi_{m_i}$ denotes the field operator for the closed-channel molecule in ground state $|g\rangle$ with the magnetic quantum number $m_I = 0, \pm 1$, $\alpha = x, y, z$ denotes the direction of spin polarization. $C_{\alpha, m_i}$ are the coefficients when transforming $k_0/k$ to the $p$-wave spherical harmonics $Y_{1, m_i}(\hat{k})$, which satisfies \[\sum_{\alpha} \sqrt{3/(4\pi)} C_{\alpha, m_i} k_0 = k Y_{1, m_i}(\hat{k})\]. Therefore, $C_{x, 0} = C_{y, 0} = 0, C_{z, 0} = 1; C_{x, \pm 1} = \pm 1/\sqrt{2}, C_{y, \pm 1} = -i/\sqrt{2}, C_{z, \pm 1} = 0$. $R$ is the center-of-mass (CoM) coordinate, $t$ is the time, $m$ is the atom mass, $u_s$ is the $s$-wave bare coupling between two fermionic atoms, $g_{m_i}$ is the $p$-wave bare coupling between two fermionic atoms and a bosonic molecule, and $\nu_{m_i}$ is the bare magnetic detuning. The difference in the energy levels of atoms and excited molecules is denoted by $\nu_c$. $\Omega_{m_i}$ is the strength of the effective laser-induced coupling between the molecular ground state $|g\rangle$ and excited state $|e\rangle$. $\delta_{m_i} \equiv 2\pi(\omega_L - \omega_{e, m_i})$ is the detuning of the laser light with respect to the energy difference between the ground and excited states of molecules. $\omega_L$ is the frequency of the laser light, and $\omega_{e, m_i}$ is the energy difference between the ground and excited states of molecules. The spontaneous decay of the excited molecular state $|e\rangle$ is treated phenomenologically by a decay rate $\gamma_e$. The natural units $h = k_B = 1$ will be used throughout the paper.

Accordingly, we can write the Hamiltonian in momentum space from the Lagrangian by the Legendre and
Fourier transformations

\[ H = \sum_{\sigma = \uparrow, \downarrow} \mu_\sigma N_\sigma = H_A + H_M + H_{AM}, \]

\[ H_A = \sum_{k, \sigma = \uparrow, \downarrow} \left( \frac{k^2}{2m} - \mu_\sigma \right) a^\dagger_{k, \sigma} a_{k, \sigma}, \]

\[ + \frac{\hbar u}{\sqrt{V}} \sum_{Q', \mathbf{k}', \mathbf{k}} \frac{1}{2} a^\dagger_{Q', \uparrow} a^\dagger_{-\mathbf{k}, \downarrow} a_{-\mathbf{k}', \downarrow} a_{\mathbf{k}', \uparrow} + \text{V terms} \]

\[ + \sum_{\mu, m} \left( \frac{g_{m, \mu}}{\sqrt{2V}} k \left( Y_{1, m} (\mathbf{k}) b^\dagger_{Q, m, \mu} a_{\mathbf{k}, \uparrow} + a_{Q, m, \mu}^\dagger a_{\mathbf{k}, \downarrow} b_{Q, m, \mu} \right) + H_{AM} \right), \]

where the self-energy in momentum space is

\[ \Sigma_{\mu, m}(q_0, \mathbf{Q}) = \frac{|\Omega_{m}|^2}{4 \left( q_0 - \frac{Q^2}{4m} - \nu_\mu - \delta_{m, 1} + \frac{i\pi}{2} \right)}, \]

\[ a_{k, \sigma}, a_{k, \sigma}^\dagger \text{ is the annihilation (creation) field operator of Fermi atom in momentum space, } b_{Q, m, \mu}, b_{Q, m, \mu}^\dagger \text{ is the annihilation (creation) field operator of ground-state bosonic molecule in momentum space, } \mathbf{Q} \text{ is the CoM momentum, and } q_0 = \frac{Q^2}{4m} + k^2/m \text{ is the total incoming energy, } V \text{ is the volume of the system, } \mu_\sigma \text{ is the fermionic chemical potential with spin } \sigma, \text{ and the particle numbers are given by } N_\uparrow = \sum_k a_{k, \uparrow}^\dagger a_{k, \uparrow}, \text{ and } N_\downarrow = \sum_k a_{k, \downarrow}^\dagger a_{k, \downarrow}. \]

III. INTERACTION RENORMALIZATION

A. s wave

In s-wave case, we consider zero total momentum for each pairing state, so that an incoming state can be set as \(|I_s\rangle = |k, \uparrow; -k, \downarrow\rangle\) with two fermions of different species having momentum \(k\) and \(-k\) to an outgoing state \(|O_s\rangle = |k', \uparrow; -k', \downarrow\rangle\) with two fermions having momentum \(k'\) and \(-k'\). Therefore, as shown in Fig. 2, the two-body \(T\) matrix for the s-wave interaction is given by

\[ -iT_{k,k'}^{(s)}(k) = \frac{-iu_s}{1 - (-iu_s)\Pi_s(k)}, \]

where the polarization bubble for s wave is

\[ \Pi_s(k) = \int \frac{d^3p}{(2\pi)^3} \frac{i}{k^2/m - p^2/m + i0^+} \]

\[ = \frac{i m}{2\pi} \left( -\frac{i k}{2} - \frac{\Lambda}{\pi} \right). \]

The s-wave scattering length is given by

\[ a_s = \frac{m}{4\pi} T_{k,k'}^{(s)}(k = 0) = \frac{m}{4\pi \frac{1}{u_s} + \frac{mA}{2\pi}}, \]

where \(\Lambda\) is an ultraviolet momentum cutoff.

Further, we get the renormalization relation

\[ \frac{1}{u_s} = \frac{m}{4\pi a_s} - \frac{mA}{2\pi^2}. \]

B. \(p\) wave

We consider an incoming state \(|I_p\rangle = |Q/2 + k, \uparrow; Q/2 - k, \downarrow\rangle\) with two fermions of different species having momentum \(Q/2 + k\) and \(Q/2 - k\) to an outgoing state \(|O_p\rangle = |Q/2 + k', \uparrow; Q/2 - k', \downarrow\rangle\) with two fermions having momentum \(Q/2 + k'\) and \(Q/2 - k'\). As shown in Fig. 3, the two-body \(T\) matrix for \(p\)-wave interaction is given by

\[ T_{k,k'}^{(p)}(k) = \frac{m}{4\pi \frac{1}{u_p} + \frac{mA}{2\pi}}. \]
the polarization bubble is
\[ -T_{k,k'}^{(m)}(k) = 2D_{m}^{(0)}(k) \left( -\frac{ig_{m}}{\sqrt{2}} \right)^2 k^2 Y_{1,m_1}(\hat{k}) Y_{1,m_1}^*(\hat{k}') + 2D_{m}^{(0)}(k) \left( -\frac{ig_{m}}{\sqrt{2}} \right)^4 2\Pi_{m_1}(k) k^2 Y_{1,m_1}(\hat{k}) Y_{1,m_1}^*(\hat{k}') + \cdots \]

where the factor 2 in front of \( D_{m}^{(0)}(k) \) comes from the scattering of two identical fermions \([37][33]\), the bare molecule propagator is
\[ D_{m}^{(0)}(k) = \frac{1}{k^2/m - \nu_{m_1} - \Sigma_{m_1}(k) + i0^+}, \] (16)

the polarization bubble is
\[ \Pi_{m_1}(k) = \int \frac{d^3p}{(2\pi)^3} \frac{i|Y_{1,m_1}(\hat{p})|^2}{k^2/m - p^2/m + i0^+} = \frac{i}{4\pi} \left( -\frac{m\Lambda}{6\pi^2} - \frac{m\Lambda k^2}{2\pi^2} - \frac{imk^3}{4\pi} \right), \] (17)

and the full molecule propagator \( D_{m_1}(k) \) satisfies
\[ D_{m_1}^{-1}(k) = [D_{m}^{(0)}(k)]^{-1} - 2 \left( -\frac{ig_{m}}{\sqrt{2}} \right)^2 \Pi_{m_1}(k). \] (18)

In the presence of optical field, the \( m_1 \) wave effective range is \( \nu_{m_1} \). Further, we have the renormalization relations \([10][11][37]\)
\[ \frac{\nu_{m_1}^2}{g_{m_1}^2} = \frac{\nu_{m_1} + m\Lambda^3}{g_{m_1}^2} = \frac{\nu_{m_1} + m\Lambda^3}{24\pi^3}, \] (20)
\[ \frac{1}{g_{m_1}^2} = \frac{1}{g_{m_1}^2} = \frac{m^2\Lambda}{8\pi^3}, \] (21)

where \( \nu_{m_1} \), \( g_{m_1} \), and \( 1/g_{m_1}^2 \) are renormalized in the form of
\[ \tilde{\nu}_{m_1}/g_{m_1} \] and \( 1/g_{m_1}^2 \) are renormalized in the form of
\[ \tilde{\nu}_{m_1}/g_{m_1}^2 = \frac{m}{16\pi^2\nu_{m_1}}, \] (22)
\[ \frac{1}{g_{m_1}^2} = \frac{m^2}{16\pi^2\nu_{m_1}}. \] (23)

In the presence of optical field, the \( m_1 \) wave scattering volume and \( \tilde{\nu}_{m_1}/g_{m_1}^2 \) are renormalized in the form of
\[ \frac{1}{g_{m_1}^2} = \frac{m^2}{16\pi^2\nu_{m_1}}. \] (24)

and the \( p \)-wave effective range is
\[ \frac{1}{R_{m_1}} = \frac{16\pi^2\nu_{m_1}}{m^2g_{m_1}^2} \left[ 1 + \frac{|\Omega_{m_1}|^2}{4(\nu_{m_1}^2 - \delta_{m_1} - i2\pi/3)^2} \right] + 2\Lambda^3/3\pi. \] (25)

Notice that, in the section of numerical calculations, we use a large detuning in experiment, i.e., \( \nu_{m_1} \ll \delta_{m_1} \). \([11]\)

**IV. MOMENTUM DISTRIBUTION**

In this section, we study the tail of the momentum distribution for fermions with coexisting \( s \) and \( p \) wave interactions near a laser-dressed \( p \) wave Feshbach resonance using the quantum field method of OPE \([21][22][41][52]\).

OPE is an ideal tool to explore short-range physics. Furthermore, OPE is an operator relation that the product of two operators at small separation can be expanded in terms of the separation distance and operators, which can be interpreted as a Taylor expansion for the matrix elements of an operator. Therefore, one can expand the product of two operators as
\[ \psi_{\sigma}^\dagger(R - \frac{r}{2})\psi_{\sigma}(R + \frac{r}{2}) = \sum_n C_n(r)O_n(R), \] (26)

where \( O_n(R) \) are the local operators and \( C_n(r) \) are the short-distance coefficients. \( C_n(r) \) can be determined by calculating the matrix elements of the operators on both sides of Eq. \([26]\) in the two-body state \( |k, \uparrow; -k, \downarrow\rangle \) for \( s \)-wave interaction and \( |Q/2 + k, \uparrow; Q/2 - k, \uparrow\rangle \) for \( p \)-wave interaction.

By using the Fourier transformation on both sides of Eq. \([26]\), we have the expression of momentum distribution \([46]\)
\[ n_{\sigma}(q) = \int \frac{d^3R}{V} \int d^3re^{-iq\cdot r} \left\langle \psi_{\sigma}^\dagger(R - \frac{r}{2})\psi_{\sigma}(R + \frac{r}{2}) \right\rangle, \] (27)

where \( q \) is the relative momentum.

In the following subsections, we will show the calculations for the momentum distribution \( n_{\sigma}(q) \) for instance.

**A. s-wave channel**

As shown in Figs. \([2]\) (a)-(d), there are four types of diagrams which can be used to denote the operators on the left-hand side of OPE equation \([26]\). However, the
where we average over the direction of \( \mathbf{p} \) as an approxima-

only nonanalyticity comes from the diagram as shown in

Fig. 3 (d). Therefore, we can evaluate the diagram in

Fig. 3 (d) as

\[
\langle O_s | \psi^+_s (\mathbf{R} - \frac{\mathbf{r}}{2}) \psi_s (\mathbf{R} + \frac{\mathbf{r}}{2}) | I_s \rangle_d = \int \frac{d^3 \mathbf{p} d \mathbf{p}_0}{(2\pi)^4} \left[ p_0 - (-\mathbf{p})^2/(2m) + i0^+ \right] \left[ k^2/m - p_0 - \mathbf{p}^2/(2m) + i0^+ \right]^2 \]

\[
\approx \frac{m^2 [T^{(s)}_{k,k'}(k)]^2}{8\pi k} + \mathcal{O}(r^2) + \cdots. \tag{28}
\]

\[
\langle O_s | \psi^+_s (\mathbf{R}) \psi^+_s (\mathbf{R}) \psi_s (\mathbf{R}) \psi_s (\mathbf{R}) | I_s \rangle_d \]

\[
= \sum_{j=a,b,c,d} \langle O_s | \psi^+_s (\mathbf{R}) \psi^+_s (\mathbf{R}) \psi_s (\mathbf{R}) \psi_s (\mathbf{R}) | I_s \rangle_j \]

\[
= [1 - iT^{(s)}_{k,k'}(k) \Pi_s (k)^2]. \tag{29}
\]

Substituting Eq. (11) into (29), we have

\[
\langle O_s | \psi^+_s (\mathbf{R}) \psi^+_s (\mathbf{R}) \psi_s (\mathbf{R}) \psi_s (\mathbf{R}) | I_s \rangle_d = \frac{[T^{(s)}_{k,k'}(k)]^2}{a_s^2}. \tag{30}
\]

**B. \( p \)-wave channel**

Similar to the case of \( s \)-wave interaction, we can evaluate the diagram in Fig. 3 (d) as

\[
\langle O_p | \psi^+_p (\mathbf{R} - \frac{\mathbf{r}}{2}) \psi_s (\mathbf{R} + \frac{\mathbf{r}}{2}) | I_p \rangle_d
\]

\[
= \sum_{m_1} \int \frac{d^3 \mathbf{p} d \mathbf{p}_0}{(2\pi)^4} \left[ p_0 - (\mathbf{Q}/2 - \mathbf{p})^2/(2m) + i0^+ \right] \left[ Q_0 - p_0 - (\mathbf{Q}/2 + \mathbf{p})^2/(2m) + i0^+ \right]^2
\]

\[
\approx \frac{m^2}{4\pi} \sum_{m_1} k^2 Y_{1,m_1} (\hat{k}) Y^*_s (\hat{k}) D^2_{m_1}(k) \left( -ig_{m_1} \right)^4 \]

\[
\times \left[ \frac{1}{r^2} + i \frac{3k}{2} - k^2 r - \frac{Q_0^2 r}{24} + \left( \frac{Q_0}{2} - \frac{3kQ_0}{4} \right) P_1 (\hat{Q} \cdot \hat{r}) - \frac{Q_0^2}{12} P_2 (\hat{Q} \cdot \hat{r}) + \mathcal{O}(r^2) + \cdots \right], \tag{31}
\]

where we average over the direction of \( \mathbf{p} \) as an approxima-

tion, \( Q_0 = k^2/m + Q_0^2/(4m) \) is the total incoming energy,
calculate the expectation values of the molecule operator \( \psi_j^\dagger (\mathbf{R} - \frac{\hat{r}}{2}) \psi_j (\mathbf{R} + \frac{\hat{r}}{2}) \) in \( p \)-wave interacting channel.

\[ \langle \mathbf{R} | \mathbf{Q} | \mathbf{R} \rangle = \sum_{m_1} \left[ C_{Q, m_1} - C_{Q, m_2} + 4 C_{Q, m_3} (\hat{q} \cdot \mathbf{Q})^2 \right] \frac{q^2 V}{q^4 V}, \]  

where the corresponding contacts are defined as

\[ C_a \equiv m^2 a_0^2 \int d^3 \mathbf{R} \langle \psi_j^\dagger (\mathbf{R}) \psi_j^\dagger (\mathbf{R}) \psi_j (\mathbf{R}) \psi_j (\mathbf{R}) \rangle, \]

\[ C_{v, m_1} \equiv m^2 g_{m_1} \int d^3 \mathbf{R} \langle \varphi_{m_1} (\mathbf{R}) \varphi_{m_1} (\mathbf{R}) \rangle, \]

\[ C_{R, m_1} \equiv m^3 g_{m_1} \times \int d^3 \mathbf{R} \langle \varphi_{m_1} (\mathbf{R}) \left( i \partial_t + \frac{\nabla R}{4m} \right) \varphi_{m_1} (\mathbf{R}) \rangle, \]

\[ C_{Q, m_1} \equiv m^2 g_{m_1} \int d^3 \mathbf{R} \langle \varphi_{m_1} (\mathbf{R}) \left( -i \nabla \mathbf{R} \right) \varphi_{m_1} (\mathbf{R}) \rangle, \]

\[ C_{Q, m_2} \equiv m^3 g_{m_1} \int d^3 \mathbf{R} \langle \varphi_{m_1} (\mathbf{R}) \left( -i \nabla \mathbf{R} \right) \varphi_{m_1} (\mathbf{R}) \rangle. \]

Notice that the distribution of \( \mathbf{Q} \) here is anisotropic. Therefore, we find that \( C_{Q, m_1} \) is anisotropic and the \( q^{-3} \) tail and part of the \( q^{-4} \) tail of the momentum distribution Eq. \( (36) \) show anisotropic behaviors of CoM momentum \( \mathbf{Q} \). Especially in the previous studies, it has been shown that the contacts of a similar nature to \( C_{Q, m_1} \) in \( q^{-3} \) tail and \( C_{Q, m_2} \) in \( q^{-4} \) tail can also exist for one-dimensional \( p \)-wave Fermi gases \( [42] \).

As the adiabatic relations shown in the next section, \( C_a, C_{v, m_1} \) and \( C_{R, m_1} \) are associated to the inverse of \( s \)-wave scattering length, the inverse of \( p \)-wave scattering volume, and the inverse of \( p \)-wave effective range. The last two contacts \( C_{Q, m_1} \) and \( C_{Q, m_2} \) are related to the velocity and the kinetic energy of the closed-channel molecules, respectively.

In the similar way, one can have

\[ n_4 (q) = \frac{C_a}{V q^4}. \]  

V. UNIVERSAL RELATIONS

In this section, we derive the corresponding universal relations.

A. High-frequency radio-frequency spectroscopy

The rf spectroscopy can be used as an important experimental tool to detect the contacts \( [30, 55, 61] \). The high-frequency tails of the rf spectroscopy are governed

\[ n_4 (q) = \frac{C_a}{V q^4}. \]
by contacts. The rf with frequency $\omega$ is applied to trans-
fers fermions from the internal spin state $|\sigma\rangle$ ($\sigma = \uparrow, \downarrow$) into a third spin state $|3\rangle$. The resultant number of the atoms transferred to state $|3\rangle$ is proportional to the transition rate, which is given by [59, 60]

$$\Gamma_{rf,\sigma}(\omega) = \Omega_{rf}^2 \text{Im} \int d^3R \int dt e^{i\omega t} \int d^3r \left< T \mathcal{O}_{\sigma 3}^1(R + \frac{r}{2}, t) \mathcal{O}_{\sigma 3}(R - \frac{r}{2}, 0) \right>,$$

where $\Omega_{rf}$ is the rf Rabi frequency determined by the strength of the rf signal, $\mathcal{O}_{\sigma 3}(r, t) \equiv \psi^\dagger_3(r, t) \psi_3(r, t)$, and $T$ is the time ordering operator.

We can evaluate the diagram in Figs. 8 (a) and (b) as

$$\int dt e^{i\omega t} \int d^3r \left< O_s \right| \mathcal{O}_{\sigma 3}^1(R + \frac{r}{2}, t) \mathcal{O}_{\sigma 3}(R - \frac{r}{2}, 0) \left| I_s \right>$$

$$= \int \frac{d^3p dp_0}{(2\pi)^2} \left[ 1 - \langle q_0 - (Q/2 - p)^2/(2m) + i0^+ \rangle \langle q_0 - (Q/2 + p)^2/(2m) + i0^+ \rangle \right]^{\frac{1}{2}}\langle \mathcal{O}_{\sigma 3}(R + \frac{r}{2}, t) \mathcal{O}_{\sigma 3}(R - \frac{r}{2}, 0) \rangle.$$

Matching Eq. (44) and Eq. (45) with Eq. (50) and Eqs. (52)-(55), we have the rf transfer rate from Eq. (43) in high-frequency limit ($1/(mR_{mi}^2) \gg \omega \gg E_F$ with Fermi energy $E_F = k_F^2/(2m)$ and Fermi wave vector $k_F$)

$$\Gamma_{rf,\sigma}(\omega) = \frac{m\Omega_{rf}^2}{4\pi} \left\{ \frac{C_0}{(m\omega)^{3/2}} + \sum_{m_i} \left[ \frac{C_{n,m_i}}{(m\omega)^{3/2}} + \frac{3C_{B,m_i}}{2(m\omega)^{3/2}} \right] \right\},$$

$$\Gamma_{rf,\downarrow}(\omega) = \frac{m\Omega_{rf}^2}{4\pi} \left\{ \frac{C_0}{(m\omega)^{3/2}} \right\},$$

$$\Gamma_{rf,\uparrow}(\omega) = \frac{m\Omega_{rf}^2}{4\pi} \left\{ \frac{C_0}{(m\omega)^{3/2}} \right\}.$$

B. Adiabatic relations

With the Hellmann-Feynmann theorem and Eqs. (37), (38) and (39), we obtain the adiabatic relations

$$\frac{\partial E}{\partial a_s^{-1}} = -\int d^3R \left< \frac{\partial \mathcal{L}}{\partial a_s^{-1}} \right> = -\frac{C_0}{4\pi m},$$

$$\frac{\partial E}{\partial u_{mi}} = -\int d^3R \left< \frac{\partial \mathcal{L}}{\partial u_{mi}} \right> = -\frac{C_{n,m_i}}{4\pi m};$$

$$\frac{\partial E}{\partial R_{mi}} = -\int d^3R \left< \frac{\partial \mathcal{L}}{\partial R_{mi}} \right> = -\frac{C_{B,m_i}}{4\pi m}.$$
where $E$ is the total energy of the many-body system and we have used the relations below

$$\left\langle \frac{\partial \mathcal{L}}{\partial \dot{a}_s} \right\rangle = \left\langle \frac{\partial \mathcal{L}}{\partial a_s} \right\rangle \frac{\partial u_s}{\partial \dot{a}_s},$$

$$= \frac{m \dot{a}_s}{4 \pi} \langle \psi_i^\dagger (R) \psi_j (R) \psi_j^\dagger (R) \psi_i (R) \rangle, \quad (51)$$

$$\left\langle \frac{\partial \mathcal{L}}{\partial \dot{\nu}_m} \right\rangle = \left\langle \frac{\partial \mathcal{L}}{\partial \nu_m} \right\rangle \frac{\partial \nu_m}{\partial \dot{\nu}_m},$$

$$= \frac{m \dot{\nu}_m}{16 \pi^2} \langle \varphi_m^\dagger (R) \varphi_m (R) \rangle, \quad (52)$$

$$\left\langle \frac{\partial \mathcal{L}}{\partial \dot{R}_m} \right\rangle = \left\langle \frac{\partial \mathcal{L}}{\partial R_m} \right\rangle \frac{\partial R_m}{\partial \dot{R}_m} + \left\langle \frac{\partial \mathcal{L}}{\partial \nu_m} \right\rangle \frac{\partial \nu_m}{\partial \dot{R}_m} + \left\langle \frac{\partial \mathcal{L}}{\partial \varphi_m} \right\rangle \frac{\partial \varphi_m}{\partial \dot{R}_m},$$

$$= \frac{m \dot{R}_m}{16 \pi^2} \langle \varphi_m^\dagger (R) \varphi_m (R) \rangle \left( i \dot{\varphi}_m + \frac{\nu_m}{4m} \varphi_m \right). \quad (53)$$

### C. Pressure relation

For a uniform gas, the pressure relation can be derived following the expression of the Helmholtz free energy density $F = F/V$ which can be expressed in terms of [74-76, 93, 94] as

$$5F = \sum_{m} \left( 2T \frac{\partial}{\partial T} + 3n_\uparrow \frac{\partial}{\partial n_\uparrow} + 3n_\downarrow \frac{\partial}{\partial n_\downarrow} - a_s \frac{\partial}{\partial a_s} - 3v_m \frac{\partial}{\partial v_m} + R_m \frac{\partial}{\partial R_m} \right) F. \quad (54)$$

Using the thermodynamic relations and the adiabatic relations [48-50], we can get the pressure relation as

$$P = \frac{2}{3} E + \frac{C_a}{12 \pi m a_s V} + \sum_{m} \left( \frac{C_{\nu,m}}{4 \pi m v_m V} - \frac{C_{R,m}}{12 \pi m R_m V} \right), \quad (55)$$

where $P$ is the pressure density and $E$ is the energy density.

### D. Virial theorem

For an atomic gas in a harmonic potential $V_T = \sum_{ij} \omega_{ij}^2 \frac{x_i^2}{2}$, the total energy can be expressed in terms of [48-50] as

$$E = \sum_{m} \left( \omega_{m} - \frac{1}{2} a_s \frac{\partial}{\partial a_s} \right) - \frac{3}{2} v_m \frac{\partial}{\partial v_m} + \frac{1}{2} R_m \frac{\partial}{\partial R_m} E, \quad (56)$$

which, together with the Feynman-Hellmann theorem and the adiabatic relations [48-50], gives

$$E = 2V_T - \frac{C_a}{8 \pi m a_s} - \sum_{m} \left( \frac{3C_{\nu,m}}{8 \pi m v_m} - \frac{C_{R,m}}{8 \pi m R_m} \right). \quad (57)$$

### VI. QUANTUM VIRIAL EXPANSION

The idea of the quantum virial expansion is to expand the thermodynamic quantities in powers of the fugacity $z_\sigma = e^{\beta m^2}$, where $\beta = 1/T$ and $T$ is the temperature.

In order to investigate the experimental detectable many-body physics of the above system, we calculate the normal-state contacts and spectral function of the system by using the quantum virial expansions [62-64].

#### A. Thermodynamic potential

To further calculate the normal-state contacts of the system, we will first evaluate the thermodynamic potential as follows.

In our model, two fermionic atoms with spin-up species interact with each other by exchanging a bosonic carrier. We find that the total energy can be expressed in terms of the grand thermodynamic potential $\Omega$ [74]

$$\Omega = -T \frac{\int f_{s/2}(z_\uparrow) + f_{s/2}(z_\downarrow)}{\lambda \pi} + 2\tau z_\uparrow \Delta b_{2,s} + 2\tau^2 \Delta b_{2,p}, \quad (58)$$

where $\Delta b_{2,s}$ is the second virial coefficient which includes the $s$-wave two-body interaction shown in Fig. 2 and $\Delta b_{2,p}$ is the second virial coefficient which includes the physical $p$-wave two-body interaction shown in Fig. 3.

$\lambda \equiv \sqrt{2\pi/(mT)}$ is the thermal de Broglie wavelength, and $f_s(z_\sigma) = [1/\Gamma(v)] \int_0^\infty x^{v-1} dx/(z_\sigma^v e^x + 1)$ is the Fermi-Dirac integral with the gamma function $\Gamma(v)$ [82].

#### B. Normal-state contacts

According to the adiabatic relations Eqs. [48-50], the contacts can also be expressed in terms of the grand thermodynamic potential $\Omega$ [74]

$$C_a = -4\pi m \frac{\partial \Omega}{\partial a_s} \bigg|_{T,V,\mu_\uparrow,\mu_\downarrow}, \quad (59)$$

$$C_{\nu,m} = -4\pi m \frac{\partial \Omega}{\partial \nu_m} \bigg|_{T,V,\mu_\uparrow,\mu_\downarrow}, \quad (60)$$

$$C_{R,m} = -4\pi m \frac{\partial \Omega}{\partial R_m} \bigg|_{T,V,\mu_\uparrow,\mu_\downarrow}. \quad (61)$$

#### C. Self-energy

In order to calculate the normal-state self-energy, we first expand the noninteracting fermionic Green’s function in powers of the fugacity $z_\sigma = e^{\beta m}$ [63-67]

$$G^{(0)}_\sigma(k, \tau) = e^{\beta \mu_\sigma} \sum_{n \neq 0} G^{(0),(n)}_\sigma(k, \tau) z_\sigma^n. \quad (62)$$
where $G^{(0,0)}_\sigma(k,\tau) = -\Theta(\tau)e^{i\kappa\tau}$, $G^{(0,n)}_\sigma(k,\tau) = (-1)^n e^{-i\kappa\tau}e^{-2\pi n\beta}$ with $n \geq 1$, $\Theta(\tau)$ is the Heaviside function, $\epsilon_k = k^2/(2m)$, and $\tau$ is the imaginary time.

According to the Feynman diagram shown in Fig. 9, the lowest order of the self-energies for the s- and p-wave interactions are respectively

$$\Sigma^{(s)}_\sigma(k,\tau) = \int \frac{d^3P}{(2\pi)^3} e^{i\kappa\tau}[G^{(0,1)}_\sigma(k,\tau)z_\alpha]T^{(s)}_{P,P'}(P,\tau), \quad (63)$$

$$\Sigma^{(p)}_{\nu,\lambda}(k,\tau) = \int \frac{d^3P}{(2\pi)^3} e^{i\kappa\tau}[G^{(0,1)}_\sigma(k,\tau)z_\tau]T^{(m1)}_{P,P'}(P,\tau), \quad (64)$$

where

$$\begin{align*}
T^{(s)}_{P,P'}(P,\tau) &= e^{-\frac{z^2}{2\tau}} \int_{-\infty}^{\gamma_{+\infty}} dz \frac{4\pi/m}{\tau^2 - s^2 - m^2}, \\
T^{(m1)}_{P,P'}(P,\tau) &= e^{-\frac{z^2}{2\tau}} \int_{-\infty}^{\gamma_{+\infty}} dz \frac{4\pi/m}{\tau^2 - s^2 - m^2} \\
&\times \left[ \frac{16\pi^2}{\gamma_{+\infty}z} Y_{1,m_z}(P) \gamma_{-\infty} z_\alpha Y_{1,m_z}(P') \right]^{-1} \tau_{m_z}^{-1} + r_{m_z}^{-1} - m_z \tau_{m_z}^{-1}.
\end{align*}$$

with the complex $z = \text{Re}(z) + i\text{Im}(z)$.

Therefore, we derive the retarded self-energy for spin $\sigma$ as follows

$$\Sigma^{(R)}_\sigma(k,\epsilon_\tau) = \Sigma^{(s)}_\sigma(k,\epsilon_\tau) + \sum_{\nu,\lambda} \Sigma^{(p)}_{\nu,\lambda}(k,\epsilon_\tau)$$

$$\Sigma^{(R1)}_\sigma(k,\epsilon_\tau) = \Sigma^{(s)}_\sigma(k,\epsilon_\tau) + \sum_{\nu,\lambda} \Sigma^{(p)}_{\nu,\lambda}(k,\epsilon_\tau)$$

where $E_\sigma = \omega + \mu_\sigma + i\delta_\sigma$, $F^{(s)}(k,\epsilon_\tau)$, $F^{(p)}(k,\epsilon_\tau)$, $H^{(s)}(k,\epsilon_\tau)$, and $H^{(p)}(k,\epsilon_\tau)$ are given in the Appendix.

D. Normal-state spectral function

We calculate the spectral function as follows [64–67]

$$A_\sigma(k,\epsilon_\sigma) = -\frac{1}{\pi} \text{Im}[G^{(R)}_\sigma(k,\epsilon_\sigma)], \quad (69)$$

where the retarded Green’s function is given by

$$G^{(R)}_\sigma(k,\epsilon_\sigma) = \frac{1}{E_\sigma - k^2/(2m) - \Sigma^{(R)}_\sigma(k,\epsilon_\sigma)}. \quad (70)$$

VII. NUMERICAL RESULTS

For the numerical calculations, we take the atom density $n = 1.50 \times 10^{13}$ cm$^{-3}$ [8]. For $^{40}$K atoms, we have the experimental parameters $\delta \mu_{m_z} = 0.134 \mu_B$ with the Bohr magneton $\mu_B$ [7–10].

The s-wave scattering length $a_s$ is given by [7–10]

$$a_s = a_{0g} \left(1 - \frac{\Delta_s}{B - B_{0,s}}\right), \quad (71)$$

where $B_{0,s} = 202.1$ G, $a_{0g} \approx 174 a_0$, $a_0$ is the Bohr radius, and $\Delta_s \approx 8.0$ G.

In absence of the laser field, the p-wave scattering volume can be conveniently calculated using [10–11, 37]

$$\bar{v}_{m_{z}} = \bar{v}_{m_{z}}^{(bg)} \left(1 - \frac{\Delta_{m_{z}}}{B - B_{0,m_{z}}}\right), \quad (72)$$

where $\bar{v}_{m_{z}}^{(bg)} = 101.6 a_0^3$, $\bar{v}_{m_{z} = \pm 1}^{(bg)} = 96.7 a_0^3$, $\Delta_{m_{z} = 0} = 21.95$ G, $\Delta_{m_{z} = \pm 1} = 24.99$ G, $B_{0,m_{z} = 0} = 198.8$ G, and $B_{0,m_{z} = \pm 1} = 198.3$ G.

The p-wave effective range in absence of the laser field is [10–11, 37]

$$\frac{1}{R_{m_{z}}} = \frac{1}{R_{m_{z}}^{(bg)}} \left(1 + \frac{B - B_{0,m_{z}}}{\Delta_{R,m_{z}}}\right), \quad (73)$$

where $R_{m_{z}}^{(bg)} = 47.19 a_0$, $R_{m_{z} = \pm 1}^{(bg)} = 46.22 a_0$, $\Delta_{R,m_{z} = 0} = -18.71$ G, and $\Delta_{R,m_{z} = \pm 1} = -22.46$ G.

In presence of the laser field, we consider the typical experimental values $\gamma_0 = 2\pi \times 6$ MHz, $\delta_0 = -1.55$ GHz, $\delta_{\pm 1} = -2.90$ GHz, $\Omega_0 = 2\pi \times 57.14$ MHz, $\Omega_{\pm 1} = 2\pi \times 32.95$ MHz, $B_{0,m_{z} = 0} = 201.6$ G, and $B_{0,m_{z} = \pm 1} = 198.8$ G [1].

Here, $B_{0,m_{z} = 201.6}$ G is much closer to the s-wave Feshbach resonance $B_{0,s} = 202.1$ G than $B_{0,m_{z} = \pm 1} = 198.8$ G. Accordingly, we calculate the contacts $C_{\nu,m_{z}}$ and $C_{R,m_{z}}$ with $m_{z} = 0$ for instance.

A. Contacts

Figures 10 (a)-(c) show the contacts of $^{40}$K atoms as functions of the magnetic field magnitude changing from 191 G to 205 G across the laser-dressed p-wave resonance.
at a given temperature $T = 6T_F \simeq 2.1\mu K$, spin polarization $P = 0.1$, and $m_I = 0$. The red lines are calculated under the laser dressing, while the blue lines are calculated in the absence of the laser. The solid lines denote the results with coupling of $s$- and $p$-wave, the dashed lines denote the results of pure $p$-wave, and the dot-dashed black line denotes the result of pure $s$-wave. Parameters used for the plots are given in the main text.

According to the laser-dressed $p$-wave interaction, the $s$-wave contact $C_a$ with laser dressing significantly decreases around the $p$-wave Feshbach resonance 198G as shown in Fig. 10 (a). Such a behavior is a direct manifestation of few-body effects on the many-body level, and is useful for detecting the impact of dressing lasers on the system.

Secondly, Figs. 10 (b) and (c) show that the magnetic field points for the maximum values of $C_{\nu,0}$ and $|C_{R,0}|$ with laser are closer to the laser-dressed $p$-wave resonance 201.6G than the corresponding results without laser, and the maximum values of $C_{\nu,0}$ and $|C_{R,0}|$ with laser are much larger than the corresponding results without laser. This is according to the strong interplay of laser dressing and $p$-wave interaction.

Thirdly, it is indicated from Figs. 10 (b) and (c) that the $p$-wave contacts $C_{\nu,0}$ and $|C_{R,0}|$ decrease more rapidly in the BEC limit under the influence of $s$-wave interaction, which is due to the interplay of $s$- and $p$-wave interactions on the many-body level.

**B. Spectral function**

Figure 11 shows the spectral function of $^{40}$K atoms versus the frequency at a given temperature $T = 6T_F$, magnetic field magnitude $B = 201G$, spin polarization $P = 0.1$, and $m_I = 0$.

Similar to the contacts, the spectral function shows a very obvious laser-dressing effect on the many-body level.

**VIII. SUMMARY**

We have shown that, in a three-dimensional Fermi gas with laser-dressed mixed $s$- and $p$-wave interactions, the high-momentum tail of the density distribution can be characterized by a series of contacts which depend on the laser dressing. In particular, we find that the contact related to the velocity of the closed-channel molecules is anisotropic and the high-momentum tail of the momentum distribution show anisotropic behaviors of CoM momentum. We then derive the universal relations, and numerically estimate the high-temperature contacts and spectral function which show the interplay of laser dressing and different partial-wave interactions on the many-body level. Particularly, the laser-dressing effect on the
contacts and spectral function is visualized. The results here can be verified in current cold atom experiments.

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Appendix A: Functions in self-energy

The functions in the self-energy of Eqs. (67) and (68) are given by

\[ F^{(s)}(k, E_\sigma) = \int \frac{d^3p}{(2\pi)^3} e^{-\beta \varepsilon_{p-k}} f^{(s)} \left( E_\sigma + \varepsilon_{p-k} - \frac{p^2}{4m} \right), \]  

(\text{A1})

\[ H^{(s)}(k, E_\sigma) = -\int \frac{d^3p}{(2\pi)^3} e^{-\beta \varepsilon_{p-k}} \times h^{(s)} \left( E_\sigma + \varepsilon_{p-k} - \frac{p^2}{4m} \right), \]  

(\text{A2})

\[ f^{(s)}(z) = \Theta(a^{-1}_s)8\pi e^{-\beta E_{b,s}} \frac{m^2 a_s(z - E_{b,s})}{m^2 a_s(z - E_{b,s})} + 4 \int_0^\infty \frac{\sqrt{x}dx}{(x - E_{b,s})(z - x)} \equiv \Theta(a^{-1}_s)8\pi e^{-\beta E_{b,s}} \frac{m^2 a_s(z - E_{b,s})}{m^2 a_s(z - E_{b,s})} + 4 \int_0^\infty \frac{\sqrt{x}dx}{(x - E_{b,s})(z - x)}, \]  

(\text{A3})

\[ h^{(s)}(z) = \Theta(a^{-1}_s)8\pi e^{-\beta E_{b,s}} \frac{m^2 a_s(z - E_{b,s})}{m^2 a_s(z - E_{b,s})} + 4 \int_0^\infty \frac{\sqrt{x}dx}{(x - E_{b,s})(z - x)}, \]  

(\text{A4})

\[ F^{(p)}(k, E_\sigma) = \sum_{m_l} 16Y_{1,m_l}(\hat{p})Y_{1,m_l}^*(\hat{p}') \times \int \frac{d^3P}{(2\pi)^3} e^{-\beta \varepsilon_{p-k}} f^{(m_l)} \left( E_\sigma + \varepsilon_{p-k} - \frac{P^2}{4m} \right), \]  

\[ H^{(p)}(k, E_\sigma) = -\sum_{m_l} 16Y_{1,m_l}(\hat{p})Y_{1,m_l}^*(\hat{p}') \times \int \frac{d^3P}{(2\pi)^3} e^{-\beta \varepsilon_{p-k}} h^{(m_l)} \left( E_\sigma + \varepsilon_{p-k} - \frac{P^2}{4m} \right), \]  

(\text{A5})

\[ f^{(m_l)}(z) = \frac{-\Theta(v^{-1}_m)\pi}{(m r_{m_l})^2} \left( z - E_{b,m_l} \right) + \int_0^\infty \frac{\sqrt{x}dx}{[(v^{-1}_{m_l} + r_m z)^2 + (m z)^3](z - x)}, \]  

(\text{A6})

\[ h^{(m_l)}(z) = \frac{-\Theta(v^{-1}_m)\pi e^{-\beta E_{b,m_l}}}{(m r_{m_l})^2} \left( z - E_{b,m_l} \right) + \int_0^\infty \frac{\sqrt{x}dx}{[(v^{-1}_{m_l} + r_m z)^2 + (m z)^3](z - x)}. \]  

(\text{A7})

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