The alpha decay of deformed superheavy elements.*

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Abstract

The interaction potential between the alpha particle and the deformed parent nucleus was used for description of the decay of superheavy nuclei. It consists of centrifugal, nuclear and Coulomb parts suitably modified for deformed nuclei. The significant effect of various shapes of barriers obtained for deformed parent nuclei on calculated alpha half-life times were shown. The finally calculated half-life times due to the spontaneous alpha decay for superheavy elements were compared with the results obtained from other models and the experimental data.

1 Introduction

Theoretical calculations suggesting existence of deformed superheavy nuclei from the areas of doubly magic $^{270}$Hs were made by Sobiczewski et. al.[1, 2]. Recently there have been calculated some collective properties of nuclei such as: deformation energies with the equilibrium shapes, energies of the first excited state $E_{2+}$ and the branching ratios between $\alpha$-decay to this state $2+$, to that to the ground state $0+$. Possible puzzling comparison of the obtained theoretical predictions with the experimental data promotes and motivates further studies of this problem. Therefore more extensive description of an $\alpha$ particle emission including additional degrees of freedom connected with deformation in ground

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state of emitting nucleus is necessary. Then different position of $\alpha$ particle towards this nucleus is possible.

It is commonly known that the action integral giving probability of $\alpha$ particle tunneling depends strongly, among others, on the potential barrier shapes and even insignificant change of this barrier can change the obtained half-lives by the magnitude of the a few orders. Taking deformation of parent nucleus into account in our model can give more realistic results.

The aim of the paper is calculation of half-lives due to spontaneous $\alpha$ decay for the deformed superheavy nuclei which is a predominant process for them. The barriers were calculated using a suitable model in which the total interaction potential ($V_{TOT}$) consists of the nuclear ($V_N$), Coulomb ($V_C$) and centrifugal barrier ($V_L$) modified for the case of existing the deformation of parent nucleus. Chapter II includes brief description of all terms of the potential. Chapter III presents main formulas relating to the $\alpha$ decay used in this paper. Chapter IV includes the discussion of the results and the influence on half-lives of the various factors. The last chapter includes summing up and conclusions.

2 Interaction potential

2.1 Shape parameterization in the exit channel

The shape of nucleus surface area $R_{P(\alpha)}(\theta_{P(\alpha)}, \phi_{P(\alpha)})$ is parameterized in the standard way in the spherical harmonic base which can be written in the vector notation as follows:

$$\vec{r}_{P(\alpha)} = \begin{cases} R_{P(\alpha)}(\theta_{P(\alpha)}, \phi_{P(\alpha)}) \sin(\theta_{P(\alpha)}) \cos(\phi_{P(\alpha)}) \\ R_{P(\alpha)}(\theta_{P(\alpha)}, \phi_{P(\alpha)}) \sin(\theta_{P(\alpha)}) \sin(\phi_{P(\alpha)}) \\ R_{P(\alpha)}(\theta_{P(\alpha)}, \phi_{P(\alpha)}) \cos(\theta_{P(\alpha)}) \end{cases}$$

(1)

The index $P(\alpha)$ refers to parent nucleus or alpha particle adequately. In the reference system connected with the parent nucleus the centre of the emitted particle is parameterized in the spherical system ($R, \Theta, \Phi$):

$$\vec{R} = \begin{cases} R \sin(\Theta) \cos(\Phi) \\ R \sin(\Theta) \sin(\Phi) \\ R \cos(\Theta) \end{cases}$$

(2)

Figure 1 shows geometry of this system where the shortest distance between the nucleus surface areas of $\alpha$ particle and the parent nucleus is denoted by
s, the distance between the mass centers by $R$ and the distance between the infinitely small volume elements in the nucleus of emitter and emitted particle by $r_{P\alpha}$.

Particle $\alpha$ is treated as a sphere of a radius $R_\alpha = 1.84\, fm$. There were considered only axially symmetric deformed parent nuclei which results in the simplification $-\phi_{P(\alpha)} = 0$ and $\Phi = 0$.

### 2.2 Coulomb potential

The electrostatic interaction potential $(V_C)$ between the parent nucleus with the proton $(Z_P)$, mass numbers $(A_P)$ and the emitted $\alpha$ particle is given by integral equation:

$$V_C = 2e \int_{\nu_P} \frac{\rho_P(\vec{r}_P)}{r_{P\alpha}} d\tau_P ,$$  \hspace{1cm} (3)

where $e$ is the charge unit and $\rho_P(\vec{r}_P)$ is a charge distribution with uniform density. It is $\rho_P(\vec{r}_P) = \frac{3Z_P e}{4\pi R_{CP}^3}$ inside the parent nucleus and $\rho_P(\vec{r}_P) = 0$ outside the nucleus. The *equivalent sharp radius* is taken as $R_{CP} = 1.15A_P^{1/3}$. Integration over the parent nucleus volume ($\nu_P$) in the coordinates defined by equation (1) is carried out. In the outside area, when the distance between the mass centers is larger than the distances corresponding to the contact configuration of the parent nucleus and the alpha particle $R > r_P + r_\alpha$, expansion of distance reciprocity $r_{P\alpha}$ is of absolutely convergent series form:

$$\frac{1}{r_{P\alpha}} = \frac{1}{2\sqrt{\pi}} \sum_{l_P} \frac{(1)^{l_P}}{R_{l_P+1}^{l_P+1}} C_{l_P,0}^{0,0} Y_{l_P}^0(\Theta) Y_{l_P}^0(\theta_P) .$$  \hspace{1cm} (4)

After substitution of the expansion into the formula (3) and making simple transformations we obtain suitable multipole expansion for the Coulomb interactions of deformed nuclei:

$$V_C = \frac{2Z_P e^2}{R} + e \sum_{l_P=2}^{\infty} \frac{1}{R_{l_P+1}^{l_P+1}} C_{l_P,0}^{0,0} Y_{l_P}^0(\Theta) Q_{l_P}^0 ,$$  \hspace{1cm} (5)

Here $C_{l_P,0}^{0,0}$ are the Wigner coefficients. The required multipole moments $Q_{l_P}^0$ are calculated using the following definition [6]:

$$Q_{l_P}^0 = \int_{\nu_P} \rho_P(\vec{r}_P)r_P^{l_P}Y_{l_P}^0(\theta_P)d\tau_P .$$  \hspace{1cm} (6)

here $Y_{l_P}^0(\theta_P)$ are corresponding spherical harmonics. Let us note that the first term in the presented multipole expansion corresponds to the case of interaction when both fragments are spherical.
2.3 Nuclear potential

The nuclear part \( V_N \) was calculated using proximity approximation \([7]\). This hypothesis is based on the assumption, that the nuclear interaction takes place between two infinitesimal volume elements located closest for a given configuration, and additionally that the distance between them is of the order of a few fm. The surfaces could be treated then as flat parallel slabs and the proximity energy between two semi-infinite nuclear matters is of a simpler form:

\[
V_N = K \gamma \psi(s),
\]

with the surface tension coefficient \( \gamma = 0.9517(1 - 1.7826I^2) MeV/fm^2 \), where \( I = (N-Z)/A \). It allows for division of nucleus-nucleus interactions into two independent factors: \( K \) connected with the shape of emitting nucleus surface area and the factor expressed by the universal function \( \psi(s) \), independent of the system geometry. The form of universal function given in \([8]\) is used in this paper.

Expansion of this approach for the deformed case consists in suitable re-defining of the geometrical factor. In the case of deformed nuclei it is the geometrical mean of main reduced curvature radii \( k_1 \) and \( k_2 \):

\[
K_{DEF} = 2\pi \sqrt{k_1 \cdot k_2},
\]

calculated on the nucleus surface area in the sites for which with the data \( R_i \) the distance between surfaces is minimal \( s = \min |\vec{R} + \vec{r}_\alpha - \vec{r}_P| \). To calculate the required curvature radii, we calculate fundamental forms of the first and the second orders of the nucleus surface in these points. Then we are able to calculate the Gauss curvature and the mean curvature and finally we obtain the first \( R_{1M(\alpha)} \) and second \( R_{2M(\alpha)} \) main radius curvatures \([9]\). The main reduced curvature radii of the first and second orders \( k_1 \) and \( k_2 \) can be expressed by the relations:

\[
k_1 = \frac{R_{1M}R_{1\alpha}}{R_{1M} + R_{1\alpha}}; \quad k_2 = \frac{R_{2M}R_{2\alpha}}{R_{2M} + R_{2\alpha}}.
\]

Moreover, within small deformations the factor \( K_{DEF} \) passes into the factor \( K \) for the spherical case \( K_{DEF}(\beta_{lp} = 0) - > K \).

3 Theory of alpha decay

The emission of alpha particle from the axially symmetric deformed superheavy nuclei is considered. The half-life is given as a function of the position \( \alpha \)
particle:

\[ T_{1/2}(\Theta) = \frac{\ln 2}{\lambda(\Theta)} , \]

(10)

where \( \lambda \) is the decay constant. The \( \alpha \)-decay model for deformed superheavy elements presented here is based on the assumption that forming of \( \alpha \) cluster in the inside of parent nucleus and passing over potential barrier are independent events. It is also assumed that the emission of \( \alpha \) particle does not affect deformation of the nucleus from which it was emitted. Within these approximations the \( \alpha \)-decay tunnelling probability constant can be written as:

\[ \lambda(\Theta) = S \lambda_G(\Theta) , \]

(11)

where \( S \) is the quantum mechanical probability forming of \( \alpha \) particle in the parent nucleus (preformation probability). It is additionally assumed that the deformation and orientation of parent nucleus do not have any influence on the preformation probability. By the \( \lambda_G(\Theta) \) we denoted the tunneling probability (Gamow decay constant), which is a function of the energy of the \( \alpha \) particle and angle \( \Theta \) showing the direction of \( \alpha \) emission. All information about the many body structure is contained in the \( S \) and because of it we sometimes call this constant the spectroscopic factor. This factor can be calculated as the mean value of the suitable projection operator \( \hat{P} \) between parent nucleus wave function \( \Phi_P \) lies in the open channel space \( \Phi_D \otimes \Phi_\alpha \) spread by the daughter nucleus wave function \( \Phi_D \) and \( \alpha \) nucleus wave function \( \Phi_\alpha \).

\[ S = \langle \Phi_P | \hat{P} | \Phi_P \rangle . \]

(12)

This problem was investigated by Blendowske, Fliessbach and Walliser [12] for different exotic spontaneous emissions of clusters and for \( \alpha \) particle needed here. Exact numerical evaluation and fitting procedure lead to the semiempirical formula for spectroscopic factor. In the \( \alpha \) particle case this factor is equal to \( S_\alpha = 6.3 \cdot 10^{-3} \) [12]. The favoured decays are considered here only. The barrier penetration probability \( \lambda_G \) as a function of the angle can be expressed:

\[ \lambda_G(\Theta) = \frac{\omega(\Theta)}{2\pi} e^{I(\Theta)} , \]

(13)

where \( I(\Theta) \) is the action integral:

\[ I(\Theta) = \int_{r_{ex}(\Theta)}^{r_{es}(\Theta)} \sqrt{\frac{2\mu}{\hbar^2} \left( V_{TOT}(R, \Theta) - Q_\alpha \right)} \, dr , \]

(14)
The reduced mass of α particle is denoted by $\mu$, $r_{en}(\Theta)$ and $r_{ex}(\Theta)$ are the classical turning points determined for a given position of α particle by the relation $V_{TOT}(R, \Theta) = Q_\alpha$. The energy released when the nucleus $(Z,N)$ emits an α particle to the state $L^+$ is obtained from the mass or total binding energies:

$$Q_\alpha(Z,N) = E_B(Z - 2, N - 2) - E_B(2,2) - E_B(Z,N) - E_{L^+}(Z,N),$$  \hspace{1cm} (15)

where $E_{L^+}$ is the rotational energy of the $L^+$ state of the nucleus $(Z,N)$. The total potential ($V_{TOT}$) consists of Coulomb ($V_C$), nuclear ($V_N$) and centrifugal ($V_L$) energies:

$$V_{TOT}(R, \Theta) = V_C(R, \Theta) + V_N(R, \Theta) + V_L(R, \Theta),$$  \hspace{1cm} (16)

The centrifugal term of energy can be calculated:

$$V_L(R, \Theta) = \frac{\hbar^2 L(L+1)}{2\mu R(\Theta)^2}. \hspace{1cm} (17)$$

The knocking frequency $\frac{\omega(\Theta)}{2\pi}$ is a slowly varying function of $E$.

$$\omega(\Theta) = \frac{2\pi}{2\mu} \sqrt{\frac{E}{r^2_{in}(\Theta)}}, \hspace{1cm} (18)$$

In our investigations we accepted the α particle kinetic energy $E = 112.1 \text{ MeV}$ and α particle binding energies $E_B(2,2) = -28.30 \text{ MeV}$ according to [10, 11]. The obtained half-lives were averaged with the surface function in order to compare them to the experimental data. This surface function in the axially symmetric geometry case is given by:

$$S = 2\pi \int_0^\pi dS(\Theta) = 2\pi \int_0^\pi d\Theta R_P^2(\Theta) \sin(\Theta) \sqrt{1 + \frac{1}{R_P^2(\Theta)} \left(\frac{dR_P}{d\Theta}\right)^2}, \hspace{1cm} (19)$$

The mean half-life can be written in the following way:

$$\langle T_{1/2} \rangle = \frac{\int_0^\pi T_{1/2}(\Theta) dS(\Theta)}{\int_0^\pi dS(\Theta)}. \hspace{1cm} (20)$$

### 4 Results

As follows from the formulae presented in the previous chapter, the precise knowledge of the barriers shapes depending on the angle $\Theta$ at which the particle is emitted from the nucleus is necessary for the correct estimation of the
tunnelling probability (formulae 13,14). Figure 2 shown the obtained total bar-
riers \( V_{TOT}(R, \Theta) \) in the function of the distance between the mass centres of \( \alpha \)
particle and the emitter for the deformed of the superheavy element \( ^{264}Hs_{156} \). 
The equilibrium deformations \( \left\{ \beta_2, \beta_4, \beta_6 \right\} \) became taken from [14]. The barri-
ers for different orientations \( \Theta \) of the emitted \( \alpha \) particle in the relation to the 
parent nucleus are shown. Note that position of the barrier maximum and its 
value change with the angle of \( \alpha \) cluster emission as shown in the upper inside 
panel in figure 2. The lowest barrier height in the given example appears at 
the angle \( \Theta \approx 30^\circ \) but the highest one is obtained for the equatorial position 
\( \Theta \approx 90^\circ \). The difference in the barrier maxima is \( \Delta V_{MAX} = 1.7 \ MeV \) for this 
nucleus. It has significant influence on the half-lives as the difference in the 
barrier height being only \( \Delta V_{MAX} = 0.5 \ MeV \) gives approximately the half-life 
differing the order of magnitude.

Another parameter to which emission probability is very sensitive is the 
entrance energy of \( \alpha \) particle in the barrier \( V_{ENT} = V_{ST} + Q_\alpha \) where \( Q_\alpha \) is the 
\( \alpha \)-decay energy calculated from the formula (15) and is equal \( Q_\alpha = 10.57 MeV \) 
for the studied isotope \( ^{264}Hs_{156} \). \( V_{ST} \) is the energy of potential corresponding 
to the configuration in the contact point between \( \alpha \) particle and the parent 
nucleus (sticking configuration). It is functional dependence on the angle \( \Theta \) 
is shown in the bottom panel in Figure 2. Taking into account the presented 
dependences leads to angular anisotropy and the largest difference is \( \Delta V_{ENT} = 2.5 \ MeV \). As previously it can lead to essential differences in the values of 
action integral, the obtained tunnelling probabilities and further in calculated 

In order to show the effect of the deformation parameter \( \beta_6 \) on the obtained 
half-lives the calculation for both cases \( \beta_6 > 0 \) and \( \beta_6 < 0 \) (\( |\beta_6| = 0.034 \)) was 
made leaving the deformation parameters \( \left\{ \beta_2, \beta_4 \right\} \) unchanged. Figure 3 show 
the course of variability of tunneling probability function \( I(\Theta) \) in both cases. 
Analyzing the diagrams it can be seen that position of the curve maximum at \( \beta_6 < 0 \) denoted with the solid line correspond to the position of minimum 
for the curve at \( \beta_6 > 0 \) denoted with the broken line. Position of the curve 
maximum for positive \( \beta_6 \) sign is shifted by about 15° towards smaller angles 
compared to the curve minimum with negative sign of \( \beta_6 \). As follows from 
the above analysis half-lives are very sensitive to even smallest changes of 
deformation parameter \( \beta_6 \).

On figure 4 calculated half-lives in the function of angle \( \Theta \) denoted by 
the thick solid line are shown. The value \( \langle T_{1/2} \rangle \) obtained after averaging in 
relation to the surface according to (20) is also denoted (thin solid line). The 
arrow indicates the experimental values [16]. Half-lives differ from each other
depending on the place of $\alpha$ particle emission even by 5 orders of magnitude and the obtained mean value of half-life for this element is close to the experimental value.

The dotted line correspond to the calculus without taking centrifugal potential into consideration (emission from ground to the ground state is considered). The calculated energy of the $2^+$ state for this element is very small being $E_{2^+} = 46.6$ keV see eg. [3]. One can also find that the contribution of the centrifugal barrier is small and it can be neglected.

Systematic calculus of half-life for superheavy elements $Z=108, 110, 112, 114$ is presented in figure 5 where decimal logarithms of the average half-life (dark filled squares) in the function of the mass number $A$ are shown. The obtained results are compared with literature calculations based on the phenomenological formulae given by Viola and Seaborg (V-S) [13, 15] denoted by crosses, on the Generalized Liquid Drop Model (GLDM) [17] marked with blank squares and with available experimental values (EXP) [16] given by full triangles. The calculations were also made not taking deformation into account denoted with open circles. As can been seen not taking deformation into account leads to the lowest half-lives while including deformation degrees of freedom increases significantly the values of half-life. The best agreement with the experimental data is observed for the isotope with $Z=110$. For the element $Z=108$ the obtained results are the closest to the predictions obtained using the Viola-Seaborg formulae [13, 15] and for the element $Z=112$ the largest agreement was obtained with calculations based on the GLDM model. The comparison of our results and those obtained from the Viola and Seaborg model shows that the latter are systematically higher. The results for the isotope $Z=114$ obtained by us lie below the values obtained using other models. Thus for the lately synthesized element $^{289}_{114}$ the phenomenological Viola-Seaborg formulae gives the value of half-life time logarithm $\log(T_{1/2}) = 4.86$ s while that calculated in this paper is $\log(T_{1/2}) = -1.30$ s and the experimental data indicate $\log(T_{1/2}) = 1.32$ s. For the neighbouring isotope $^{288}_{114}$ the experimentally predicted half-life is $\log(T_{1/2}) = 0.26$ s see [18] (and citations in it) but we obtain the value almost three orders of magnitude smaller $\log(T_{1/2}) = -2.86$ s and estimation based on the Viola-Seaborg formulae gives the value $\log(T_{1/2}) = 2.80$ s almost three times as large. This can be explained by small equilibrium deformation of superheavy nuclei under discussion. The results obtained for them are similar to those obtained for the spherical case and as mentioned earlier they are usually lower than the actual ones. Despite some discrepancies the values of the half-time obtained by us not based on any phenomenological formula are similar to the experimental values.
5 Conclusions

The following conclusions can be drawn from our investigation devoted to $\alpha$-decay of deformed superheavy elements:

1. Taking deformation of nuclei into account affects significantly the potential barrier shapes: position of maximum and entrance energies and in consequence, the half-life time.

2. Even small changes of deformation $\beta_6$ modify greatly the values of action integral.

3. The half-life time for superheavy elements of $Z=108 \div 114$ without application of any phenomenological formula but taking into account deformation of nuclei in the ground state reproduce the experimental values well.

4. The centrifugal term affects the obtained results insignificantly.
References

[1] Z. Patyk, and A. Sobiczewski, Nucl. Phys. A533, 132 (1991).
[2] Z. Patyk, and A. Sobiczewski, Phys. Lett. B256, 307 (1991).
[3] I. Muntian, and A. Sobiczewski, Acta. Phys. Pol. B32, 629 (2001).
[4] I. Muntian, Z. Patyk, and A. Sobiczewski, Phys. Rev. C60, 041302(R) (1999).
[5] A. Sobiczewski, I. Muntian, and Z. Patyk, Phys. Rev. C63, 034306 (2001).
[6] R. W. Hasse and W. D. Myers, "Geometrical Relationships of Macroscopic Nuclear Physics" Springer-Verlag, Berlin, (1988).
[7] J. Bloch, et al., Ann. Phys. (N.Y.) 105, 427 (1977).
[8] W. D. Myers, W. J. Światecki, Phys. Rev. C62, 044610 (2000).
[9] I. N. Bronstein, K. A. Semenjajev, G. Musiol and H. Mühling, Verlag Harri Deutsch, ISBN 3-8171-2004-4.
[10] B. S. Pudliner et al., Phys. Rev. C56, 1720 (1997).
[11] A. Nogga, H. Kamada, and W. Glöcke Phys. Rev. Lett. 85, 944 (2000).
[12] R. Blendowske, T. Fliessbach, H. Walliser, Nuclear Decay Modes ed. by D. N. Poenaru, IOP Publishing, Bristol (1996).
[13] V. E. Viola, Jr and G. T. Seaborg, J. Inorg, Nucl. Chem. 28, 741 (1966).
[14] P. Möller, J. R. Nix, W. D. Myers, W. J. Światecki, Atomic Data Nucl. Data Tables 59, 185 (1995).
[15] P. Möller, J. R. Nix, and K.-L. Kratz, Atomic Data Nucl. Data Tables 66, 131 (1997).
[16] M. S. Antony, Nuclide Chart 2002, Strasbourg, France, Association Européenne contre les Leucodistrophies.
[17] W. D. Myers and W. J. Światecki, Nucl. Phys. A601, 141 (1996).
[18] Yu.Ts. Oganessian, M. G. Itkis, A.G. Popeko, V.K. Utyonkov, and A. V. Yeremin, Nucl. Phys. A682 108c (2001).
Figure 1: An illustration of the geometry in the exit channel and the parameters employed for describing emission process.
Figure 2: The total potential ($V_{TOT}$) energy barriers obtained within the presented model (solid lines) dependent on the distance $R$ between the centers of the $\alpha$ particle and the daughter nucleus for different orientations. The upper panel correspond to the maximum of the barrier ($V_{MAX}$) depending on $\Theta$, The bottom panel gives the entrance energy ($V_{ENT}$) during the emission process. All presented results are predicted for the exemplary deformed superheavy $^{264}Hs_{156}$ nucleus.
Figure 3: The influence of emitted nucleus curvature on the obtained results is presented. The penetration probability as a function of the position of $\alpha$ particle for curvature with $\beta_6 = -0.034$ (solid lines) and with $\beta_6 = 0.034$ (dashed lines) for the deformed $^{264}Hs_{156}$ is shown.
Figure 4: The logarithm of the α-decay half-life as an orientation function (thick lines) is shown. Solid lines indicate that the centrifugal barrier is included in calculation but the dotted ones that it is not. The mean half-life $\langle T_{1/2} \rangle$ is marked by the thin lines. The solid line is related to the calculation with the centrifugal term and the dotted ones to that without this term. Comparison with the experimental data (EXP) is indicated by the arrow [16].
Figure 5: Logarithm of the α-decay mean half-lives of the isotopes with atomic number $Z = 108, 110, 112, 114$ as a function of the mass number $A$. Full squares indicate the calculated and averaged half-lives $\langle T_{1/2} \rangle$. Open circles are related to our calculation without deformation (SPHERICAL), cross symbols denote the calculation with the semi phenomenological Viola-Seaborg (V-S) formula [13] with a new set of parameters given in [15]. The open squares were calculated using the GLDM model [17]. The triangles indicate the experimental data [16] (EXP).