Towards Rapid and Robust Adversarial Training with One-Step Attacks

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Abstract. Adversarial training is the most successful empirical method for increasing the robustness of neural networks against adversarial attacks. However, the most effective approaches, like training with Projected Gradient Descent (PGD) are accompanied by high computational complexity. In this paper, we present two ideas that, in combination, enable adversarial training with the computationally less expensive Fast Gradient Sign Method (FGSM). First, we add uniform noise to the initial data point of the FGSM attack, which creates a wider variety of adversaries, thus prohibiting overfitting to one particular perturbation bound. Further, we add a learnable regularization step prior to the neural network, which we call Pixelwise Noise Injection Layer (PNIL). Inputs propagated through the PNIL are resampled from a learned Gaussian distribution. The regularization induced by the PNIL prevents the model from learning to obfuscate its gradients, a factor that hindered prior approaches from successfully applying one-step methods for adversarial training. We show that noise injection in conjunction with FGSM-based adversarial training achieves comparable results to adversarial training with PGD while being considerably faster. Moreover, we outperform PGD-based adversarial training by combining noise injection and PNIL.

Keywords: Deep Learning · Adversarial Training · Noise

1 Introduction

Deep learning has led to breakthroughs in various fields, such as computer vision [12], language processing [22], and reinforcement learning [20]. However, the deployment of deep learning models in real-world applications is currently limited by their vulnerability to small adversarial perturbations to their input (adversarial examples [29]). These perturbations are produced by special optimization methods, often referred to as adversarial attacks. The magnitude of the perturbations is bounded by a value \( \epsilon \) for a given norm (e.g., \( l_\infty \), \( l_2 \)) [6]. This guarantees that a human would still perceive the perturbed input as belonging to the original class.

Making neural networks robust to adversarial examples is still an unsolved problem. In a constant challenge between new adversarial attacks and defenses, most of the proposed defenses have shown to be ineffective [34, 31]. One particularly successful approach to increase the adversarial robustness of a model is
Fig. 1. Example of an input image $X$ from the MNIST data set, the biases $B$ and weights $W$ learned by our proposed method (PNIL), and the resulting variance $\Sigma$ calculated for $X$. The output image $Y$ is obtained by drawing pixel values from a Gaussian distribution according to Equation 3.

adversarial training [6]. This method augments the training data with adversarial examples until the model learns to classify them correctly. Here, the robustness of the model against adversarial attacks is strongly dependent on the adversarial examples used to train the model. In the past, mainly computationally expensive multi-step adversarial attacks such as Projected Gradient Descent (PGD), have proven effective for adversarial training [17]. Less expensive one-step attacks, such as the Fast Gradient Sign Method (FGSM), have shown to be insufficient on their own [13] as they provide no adversarial robustness against stronger attacks.

In this paper, we propose a method consisting of two successive steps, that together enable fast and stable adversarial training based on the FGSM attack. First, we aim to find stronger perturbations with the FGSM attack by adding noise in the range of the adversarial perturbation to the initial data point. Since the perturbation budget of the adversarial attack is limited around the noisy data point rather than the original data point, a wider variety of adversaries can be generated. In turn, the network is less likely to overfit to the particular perturbation bound. Secondly, we improve the adversarial robustness of a neural network by including a learnable data augmentation process we call Pixelwise Noise Injection Layer (PNIL). The PNIL resamples the input features of a neural network from a multivariate Gaussian distribution before passing them to the next layer. The mean of this distribution is given by the input features, and the variance is calculated by the PNIL. The resampling process weakens the adversarial perturbation, which improves the robustness of the model and increases the stability of FGSM-based training. Furthermore, our experiments show that the additional regularization induced by the noise prevents the network from learning to obfuscate its gradients during FGSM-based training. The PNIL is illustrated in Figure 1. We utilize the reparameterization trick, known from the
Variational Autoencoder [10], to update the parameters of the PNIL through backpropagation along with the other parameters of the neural network.

The contributions of this paper are the following:

– We show that former claims that FGSM-based training can lead to robust neural networks are misleading [32]. Our experiments demonstrate that networks trained with the FGSM-based attacks often learn to disguise their gradients, resulting in strong robustness against gradient-based attacks, but no robustness against gradient-free attacks.

– We propose a solution to this problem in the form of learnable data augmentation called Pixelwise Noise Injection Layer (PNIL), which prevents gradient obfuscation of FGSM-based training and increases the adversarial robustness of neural networks.

2 Preliminaries

In this section, we discuss earlier work in this research area and briefly describe common threat models in the adversarial setting.

2.1 Related Work

With the Fast Gradient Sign Method (FGSM) [6], an efficient way to construct adversarial examples with one gradient step was proposed, and used for the first form of adversarial training. In subsequent work, variants of the FGSM have been developed, which find stronger adversaries by making multiple smaller gradient descent steps [14,17]. Networks trained with FGSM adversarial training proved to be vulnerable to those attacks, which made FGSM-based training insufficient, whereas the multi-step Projected Gradient Descent (PGD) method [17] showed to be an effective way of training robust neural networks against multi-step attacks. However, the computational complexity of PGD training is considerably larger, depending on the number of steps used to find the optimal perturbation.

To circumvent the increased computational complexity of PGD training, recent work tried to make FGSM-based adversarial training feasible. The authors of [13] showed that curvature artifacts around the original data point can mask the true gradient direction, which prevents the FGSM attack from finding a useful perturbation. Their modification of the FGSM called R+FGSM partially solved this problem by first taking a small random step with size $\alpha < \epsilon$ before performing the actual attack. Yet, their method only provided little robustness against multi-step attacks.

This idea was improved by [32], where the authors chose a random starting point within the full perturbation budget. This enabled adversarial training with FGSM to achieve competitive adversarial robustness compared to PGD-based training. Nevertheless, this approach only works with specific step sizes of the FGSM attack and is sensitive to the network architecture. Further, it can lead to a sudden drop in model robustness and therefore requires an early stopping
routine that monitors the accuracy against PGD attacks [32]. In our experiment, we also observed that this approach often leads to models learning to obfuscate their gradients during training, which results in no adversarial robustness against gradient-free attacks. We will call this method RFGSM in the following.

Another approach to accelerate adversarial training is to update the parameters of the model and the adversarial perturbation simultaneously in every backward pass [26]. This combines the gradual adjustment of the perturbation and frequent updates of the model parameters. However, it has been shown that this approach does not work well with training optimization methods (e.g., cyclical learning rate [27], mixed precision training [19]) and is overall slower than the RFGSM approach [32].

In addition to adversarial training, a variety of other defense measures have been proposed. An important branch of these methods uses random perturbations to weaken the effect of adversarial attacks [11]. Most of them are based on noise injection, where noise from an isotropic Gaussian distribution is added to each feature in a layer. The parameters of the distribution were either tunable hyperparameters [16,1] or learned through backpropagation [8]. Intuitively, the presence of noise at inference time makes it more complicated to design an adversarial example, as the attacker does not know the specific noise when generating the adversarial perturbation. This intuition is supported by theoretical work [18,24]. It was shown that injecting noise during the training phase of a neural network reduces the sensitivity of the network to small input perturbations [18]. Furthermore, later work [24] showed that noise injection from an exponential family gives a lower bound on the adversarial robustness of neural networks, dependent on the magnitude of the noise.

2.2 Adversarial Attacks

Constructing an adversarial perturbation \( r \) can be described by the following optimization problem:

\[
\max_{\|r\|_p \leq \epsilon} \mathcal{L}(F_\theta(X + r), Y)
\]  

where \( X \) and \( Y \) are pairs of samples in a classification task, \( \mathcal{L} \) is a suitable loss function (e.g., categorical cross-entropy), and \( F_\theta \) is a neural network parameterized by \( \theta \). The manner in which the perturbation \( r \) is found depends on the specific type of adversarial attack, where attacks can be characterized by the following attributes:

**Norm-based:** The magnitude of the adversarial perturbation is usually bounded for a given norm (\( L_\infty, L_1, L_2 \)) in such a way that any perturbed input would still be perceived as belonging to its original class by humans.

**Targeted/Untargeted:** Adversarial attacks are additionally grouped by their goal: They can either aim at causing any kind of misclassification (untargeted)
our or at causing the model to predict a particular target class (targeted).

**Gradient-Based/Gradient-Free:** Gradient-based attacks utilize the gradient information of the model to find the optimal perturbation (FGSM [6], PGD [17]). These attacks are not effective if the gradient information of the model is obscured (e.g., through a non-differentiable layer in the network). In those cases, a gradient-free method (Simultaneous Perturbation Stochastic Approximation (SPSA) [28]) can evaluate the actual robustness of the model.

**White-box/Black-box:** The attack either utilizes the gradient information off the model itself to create the perturbation (white-box) or utilizes a similar model to find a perturbation which is then transferred to the original model (black-box). Black-box attacks can be used if the model is expected to hide its gradients but are generally weaker than white-box attacks.

**Randomized Models:** Expectation Over Transformation (EOT) [2] is a proven technique for calculating gradients of models with randomized elements. It can be used to calculate gradients with respect to the expectation of the randomized component.

Best practices for evaluating the adversarial robustness of models have been established by prior work [3]. One important practice is to evaluate whether models are obfuscating the gradients. If a model obfuscates its gradient, the typically applied gradient-based attacks fail. These models are however, still vulnerable to gradient-free attacks. Thus, it is important to evaluate with both gradient-free and gradient-based attacks, to get a robustness estimate for the highest risk [31]. Even though black-box attacks can be used to test models against gradient obfuscation, gradient-free attacks are recommended, as the transferability of black-box attacks can not be guaranteed. We will follow these practices in this paper and describe the exact threat model in Section 4.1.

### 3 Proposed Method

In this section, we describe our proposed method that enables fast and stable adversarial training based on the FGSM attack.

#### 3.1 Noise Augmented Adversarial Training

Current research shows that adversarial robustness against strong PGD attacks can be achieved through RFGSM-based training with only small adjustments to the initial FGSM formulation [32]. We argue that the limitation of the FGSM attacks comes from the limited amount of perturbations the attack can generate. By only using the sign of the gradient, the perturbations of the standard FGSM formulation are limited to the corners of the n-dimensional hypercube defined by the perturbation limit. The R+FGSM approach presented by [13] increases the
amount of possible perturbations but still constricts perturbations to lie near the corners of the hypercube. Finally, the FGSM variant presented by [32] increases the amount of possible perturbations by a considerable amount. We follow the idea of increasing the variety of possible perturbations and present a similar approach, wherein we augment the training data with uniform noise before applying the FGSM attack. We call this approach Noise Augmented Fast Gradient Sign Method (NFGSM) and give its details in Algorithm 1. In comparison to the RFGSM method, NFGSM can create a wider variety of adversaries. This is achieved by moving the hypercube away from the data point. Thereby, the whole volume of the hypercube is used to create adversarial examples. In turn, the network is less likely to overfit to the particular perturbation bound used to train the model, which has been observed for prior models [30]. The difference between vanilla FGSM, R+FGSM, RFGSM, and our method is summarized in Figure 2. Keep in mind that NFGSM is only used for training as it produces perturbations that are outside the perturbation bound of the FGSM attack and thus could change the true label of the class for the human perception.

Algorithm 1: NFGSM-based adversarial training with perturbation bound \( \epsilon \), perturbation \( r \), number of Monte Carlo simulations \( L \), number of epochs \( E \), and batches \( B \). An EOT based attack is given by \( L > 1 \).

\[
\text{for } e \text{ in } [1, E] \text{ do} \\
\quad \text{for } (X_b, Y_b) \text{ in } B \text{ do} \\
\quad\quad \text{noise} = \text{Uniform}(\epsilon, \epsilon); \\
\quad\quad X_{bn} = X_b + \text{noise}; \\
\quad\quad r = \epsilon \cdot \text{sign}\left(\sum_{l=1}^{L} \nabla_{X_{bn}} \mathcal{L}(F_{\theta}(X_{bn}), Y_b)\right); \\
\quad\quad \text{gradient} = \nabla_{\theta} \mathcal{L}(F_{\theta}(X_{bn} + r), Y_b); \\
\quad\quad // \text{ Update model weights } \theta \text{ with the given optimizer and calculated gradient} \\
\quad \end{for} \\
\end{for}
\]

3.2 Pixelwise Augmentation Layer

To prevent the network from learning to obfuscate its gradients during the FGSM-based training, we introduce a learnable data augmentation process we call Pixelwise Noise Injection Layer (PNIL) that can be included in any neural network. Let us consider \( X_i \in \mathbb{R}^n \) as the \( i^{th} \) sample of the data set \( X \) described by its features \( (x_{i,1}, x_{i,2}, ..., x_{i,J}) \). The PNIL learns weights \( W = (w_1, w_2, ..., w_J) \) and biases \( B = (b_1, b_2, ..., b_J) \) for every input feature and uses them to calculate the variance \( \Sigma_i \) for the respective input according to Equation 2. The output of the PNIL \( X'_i \) is computed by sampling noise for each feature \( x_{i,j} \) from a standard Gaussian distribution \( \mathcal{N}(0, 1) \) which is weighted with the calculated variance \( \Sigma_i \).
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Fig. 2. Simplified illustration of the difference between FGSM, R+FGSM, RFGSM, and the proposed NFGSM. The potential starting point of the attack (blue dot) is given by the light blue region. An arbitrary gradient direction for each FGSM attack is shown by a dotted arrow and the possible adversarial perturbations by a red dot at the adversarial perturbation limit (solid black line). For the RFGSM attack the step size \(1.2 \cdot \epsilon\) is additionally shown by a dashed black line. The NFGSM attack enables more possible perturbations, by utilizing the full area given by the maximum perturbation rather than only a part of the area.

The reparameterization trick enables the network to update the weights and biases of the PNIL by backpropagation \[10\]. The forward pass outlined above is given by Equation 3 and illustrated in Figure 1.

\[
\Sigma_i = X_i \cdot W + B \quad (2)
\]
\[
X'_i = X_i + \mathcal{N}(0, 1) \cdot \exp \frac{\Sigma_i}{2} \quad (3)
\]

Networks trained with one step attacks learn to obfuscate their gradients by converging to a global minimum where small curvature artifacts around the minimum prohibit a one-step attack from finding a strong perturbation \[13\]. Thereby, the networks fail to learn to defend against strong adversarial attacks but instead learn to weaken the FGSM attack. By injecting noise to the individual input features, the PNIL destroys the specific structure that the FGSM-based attack generates and thus prohibits gradient obfuscation. This leads to strong robustness against gradient-free and multi-step attacks, as we show in this paper.

Contrary to other approaches \[1, 16, 8\], we learn a variance for every feature of the input with the PNIL, instead of applying identical Gaussian noise to the features. Depending on the data set, this can lead to different values for \(B\) and \(W\). For simple data sets such as MNIST \[15\], the PNIL learns low biases \(B\) for the central region of the images, as the objects of interest are located there (see figure 1). For more complicated data sets like CIFAR10 \[11\], this approach does not work, since the objects of interest are not always centered in the image. In this case, the PNIL learns uniform biases \(B\) and focuses only on the intensity of the pixels in each color channel using the weights \(W\) as exemplified by Figure 3.

Note that prior work showed that architectures that apply a random transformation to the input during inference time should be evaluated with regard to
the expectation of their output with the EOT method [2]. We will follow this approach in our evaluation.

4 Experiments

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example_image.png}
\caption{Example of an input image from the CIFAR10 data set, the respective variance $\Sigma$ learned by the PNIL for all color channels, and the output image after forwarding the input through the PNIL. The background of the image gets lost to noise, while only the object of interest is still visible.}
\end{figure}

4.1 Evaluation

The objective of the experiments is to evaluate if noisy FGSM-based training leads to robust neural networks and if the PNIL further improves this robustness. Additionally, we investigate whether the PNIL prohibits neural networks from obfuscating their gradients during the FGSM-based adversarial training. This goal was accomplished by training pairs of nearly identical models, the only difference being that the PNIL is added before one model as an additional first layer. All models were trained with the NFGSM approach and with the RFGSM approach [32] for comparison. The robustness of each model was evaluated for two different adversarial attacks (PGD, SPSA). The effective robustness of a neural network was then defined as the lowest accuracy against any of the attacks [31]. Furthermore, to increase the validity of the experiments, we train each model five times following a two-fold cross-validation scheme as proposed in [5] and apply the corrected resampled t-test as introduced in [21], to evaluate if the robustness differences between using the PNIL and not using the PNIL are significant ($\alpha = 0.003$). Every data set used in the experiments has a predefined train and test set, whereby the predefined training sets are used for the cross-validation. For validating the training, we also split the predefined test data into two equally sized folds and assign each of them to one cross-validation fold. We point out that the average accuracy in our experiments may be lower compared to prior work, as less training data is used to train the models when using the cross-validation method. In a preliminary experiment, we show that models trained with the PNIL exhibit lower robustness against adversaries created with the EOT.
method and evaluate how many steps of EOT are needed for reliable evaluation. The PNIL is released alongside this paper to encourage other researchers to evaluate our approach independently (https://github.com/SchwinnL/ML).

4.2 Data

Three different data sets were used to evaluate the adversarial robustness of the different models (MNIST [15], Fashion-MNIST [33], CIFAR10 [11]). All three data sets consist of images and have the goal of classifying the images according to their labels.

**MNIST** consists of greyscale images of handwritten digits each of size 28x28x1 (60,000 training and 10,000 test). The MNIST data set is mainly considered a toy data set since a trivial classifier such as K-Nearest-Neighbour (KNN) can achieve 95% accuracy on the clean data set [15]. However, no adversarially robust neural network architecture has been proposed for the MNIST data set yet. Thus, MNIST can be considered a non-trivial example in the adversarial case [25].

**Fashion-MNIST** consists of greyscale images of 10 different types of clothing, each of size 28x28x1 (60,000 training and 10,000 test). Similar arguments as for the MNIST data set can be made for Fashion-MNIST, however the Fashion-MNIST classification task is slightly more complicated, as it contains more intricate patterns.

**CIFAR10** consists of color images, each of size 32x32x3, with 10 different labels (50,000 training and 10,000 test). CIFAR10 is a subset of the ImageNet data set [4] and the most challenging classification task out of the three. CIFAR10 is included in the experiments, to show that the approach generalized also to a more complicated classification task. Image data from the CIFAR10 data set was normalized to lie between zero and one.

4.3 Adversarial Attack Parameters

White-box attacks lead to stronger adversarial examples and are, in consequence, applied in preference to black-box attacks. Since the transferability of black-box attacks can not be guaranteed, we test the models for gradient obfuscation with the gradient-free SPSA attack. All attacks are untargeted and are considered successful if the input is misclassified. We use the often-used $l_{\infty}$ norm to restrict the magnitude of the adversarial attacks in all experiments. The adversarial perturbation budget was chosen to be 8/255 for MNIST, 4/255 for Fashion-MNIST, and 0.03 for CIFAR following prior experiments in the literature [17]. The RFGSM-based training was done with a step size of $1.2 \cdot \epsilon$ as recommended by the authors [32]. We used 50 gradient steps for the PGD attack and ensured that we could reach every point in the $\epsilon$-ball with a step size of $2 \cdot \epsilon/50$. We use 100 steps for the SPSA attack with a sample size of 2048 based on the results...
We evaluated the SPSA attack on 1000 randomly selected images for each data set, due to the considerable computational overhead of the attack. All adversarial attacks were implemented using the Cleverhans library and were adjusted to work with the EOT method.

4.4 Model and Training Parameters

We trained two different models in our experiments. The first architecture is a small CNN, consisting of two convolutions followed by a fully-connected layer suitable for the simpler MNIST and Fashion-MNIST data sets. In addition, a residual model was used for the more complicated CIFAR10 data set (ResNet11). Residual models, have been used for adversarial benchmarks in prior literature, and hence provide a robust baseline for comparison. A cyclical learning rate was used together with the ADAM optimizer (\(\beta_1 = 0.9, \beta_2 = 0.999\)) to speed up the training. The learning rate bounds were estimated by linearly increasing the learning rate of each individual network for a few epochs, as suggested in the original paper. All models were optimized till convergence, and the checkpoint with the lowest adversarial validation loss was chosen for testing. The batch size was set to 100 for all experiments.

5 Results

The results of the experiments described in Section 4.1 are reported below. First, a preliminary experiment estimates the effect of the EOT method against the PNIL. Afterward, the impact of noisy FGSM training and the PNIL on the adversarial robustness of neural networks is presented.

5.1 Evaluating EOT Parameters

The steps necessary to perform a strong EOT-based attack were estimated on the CIFAR10 data set against models that have a PNIL as their first layer. The architectures and hyperparameters were chosen according to section 4.1. A two-fold cross-validation was performed five times, and the mean accuracy was reported in Figure 4. Only a marginal decline in robustness against attacks generated with ten or more Monte Carlo simulations could be observed for all data sets. Consequently, we conclude that 100 Monte Carlo simulations are more than sufficient to get a reliable estimate of the real adversarial risk of models, which include the PNIL.

5.2 Adversarial Robustness

Initial experiments conducted on MNIST, where the PNIL was used not only before the first layer but also multiple times throughout the network, yielded comparable results. Since using the PNIL only at the beginning of the neural
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**# Monte Carlo Simulations**

| CIFAR10 - ResNet11 |
|-------------------|
| **Accuracy [%]** |
| NFGSM training    |
| RFGSM training    |

**Fig. 4.** Mean accuracy of the adversarially trained CIFAR10 models with the PNIL against PGD adversaries calculated with different amounts of Monte Carlo simulations. The accuracy is given for NFGSM and RFGSM-based training.

The network has the lowest computational complexity. Therefore we only reported results following this approach.

The evaluation of the adversarial robustness for all configurations is given in Table I. Within each training approach (RFGSM-based, NFGSM-based), the effect of using the PNIL as a first layer is analyzed.

For both approaches, merely training with RFGSM or NFGSM often resulted in low robustness against gradient-free attacks. Those models exhibit no real adversarial robustness and can be exploited by gradient-free attacks such as SPSA. Networks, which included the PNIL were not affected by gradient obfuscation, as seen by their high robustness against the SPSA attack. Furthermore, higher effective adversarial robustness was achieved in five out of six cases when using the PNIL, with significant differences in four cases. The two FGSM training procedures achieved comparable results against the predefined perturbation bound.

To have a better comparison of our approach to results in the prior literature, an additional experiment was performed on the CIFAR10 data set. In this experiment, we compared the robustness achieved through PGD training in [17] (46% accuracy) to our approach and achieved a mean accuracy of 59% over ten runs. For this experiment, we used the same predefined train-test split of the CIFAR10 data set. Note that their approach used a weaker PGD attack with 20 instead of 50 steps with the same perturbation budget $\epsilon$.

**Generalization:** A comparison of both methods against the PGD adversary defined in Section 4.3 with varying perturbation bounds, shows that the NFGSM method leads to slightly higher robustness against perturbation bounds on which
Table 1. Mean accuracy and standard deviation (±) in percent for various configurations, with and without the PNIL as the first layer. The perturbation strength was set to $\epsilon = 8/255$ for MNIST, $\epsilon = 4/255$ for Fashion-MNIST, and $\epsilon = 0.03$ for CIFAR10. Data from CIFAR10 are normalized between zero and one. The effective adversarial robustness of a model (lowest accuracy against any of the attacks) is shown with a bold font in the Min column. Significant differences ($\alpha = 0.003$) between the effective robustness with and without the PNIL are highlighted by a star symbol (*). Adversarial robustness without the PNIL is in some cases achieved through gradient obfuscation as seen by the low robustness against SPSA attacks.

| Data set     | Model     | RFGSM Training |            | NFGSM Training |            |
|--------------|-----------|----------------|------------|----------------|------------|
|              |           | Clean | SPSA | PGD | Min | Clean | SPSA | PGD | Min |
| CIFAR10      | Baseline  | 75±0  | 58±3 | 52±0 | 52±0* | 77±1  | 53±3 | 50±0 | 50±0* |
|              | + PNIL    | 68±0  | 63±1 | 57±0 | 57±0* | 66±0  | 65±2 | 57±0 | 57±0* |
| Fashion-MNIST| Baseline  | 83±1  | 2±1  | 58±0 | 2±1  | 85±0  | 3±2  | 64±8 | 3±2  |
|              | + PNIL    | 80±0  | 74±1 | 70±0 | 70±0* | 82±0  | 78±1 | 67±0 | 67±0* |
| MNIST        | Baseline  | 98±0  | 46±28| 61±25| 46±28| 98±0  | 23±36| 54±11| 23±36|
|              | + PNIL    | 98±0  | 70±1 | 55±0 | 55±0* | 98±0  | 77±1 | 60±4 | 60±4 |

the classifier was not trained on. The results are given for a baseline model trained on the CIFAR10 data set in Figure 5.

Working mechanism: Figures 1 and 3 exemplify how the adversarial robustness is improved. Models trained with the PNIL learn weights that lead to small variances in foreground regions and high variances in background regions. Thus, the PNIL effectively defends against adversarial attacks that are targeting image regions, which should be irrelevant for the classification.

6 Discussion

Adversarial training provides a reliable method to make neural networks more robust against adversarial attacks. Unfortunately, this approach has been limited by the additional computational effort involved in training with strong adversarial attacks like PGD. We have shown that training with FGSM and additional input noise can lead to similar robustness. Depending on the step count of the PGD attack, this can reduce the training time by orders of magnitude. The proposed NFGSM approach led to a small performance improvement against adversarial attacks with varying perturbation budget compared to RFGSM-based training. Additionally, our method has the benefit that it works with a standard implementation of the FGSM attack and is, therefore, easier to implement. Still, both methods can lead to low robustness against gradient-free attacks such as SPSA. We proposed a solution to this problem by introducing the PNIL and achieved significantly higher adversarial robustness by using the PNIL as the first layer of our neural networks. We argue that the additional resampling of
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Fig. 5. Mean accuracy of the adversarially trained CIFAR10 models without the PNIL against PGD adversaries with varying perturbation bounds. The accuracy is given for NFGSM and RFGSM-based training. It can be seen that NFGSM-based training leads to slightly higher robustness for different perturbation strengths, especially for weak attacks.

the input during training increases the variety of the input and makes the network less likely to obfuscate its gradients. Furthermore, during inference, the random resampling makes it harder for the attacker to construct an adversarial sample since the precise outcome of the network is not known in advance.

Contrary to variational autoencoders, no Kullback-Leibler-divergence loss is used to push the variances computed by the PNIL in a certain direction. Intuitively, the PNIL could learn high negative weights and biases for all features, which would lead to a zero variance in the output. As a result, the PNIL would not change its inputs during the resampling step, and the network would be able to minimize the training loss. We argue that most pixels in an image do not contribute to the classification. Thus, only small gradients are obtained for the corresponding features in the PNIL, and their values remain relatively unchanged. The regions where the object of interest is located contribute primarily to the classification loss and produce larger gradient values. We observe this behavior in our experiments, as seen in Figures 1 and 3. In return, the network must focus on the crucial aspects of each image, since background information is partially lost due to noise generated by the PNIL.

The parameters of the PNIL in Figures 1 and 3 are determined by the fact that the objects of interest are located in the middle of the image. Consequently, the learned biases would be rendered ineffective if the relative position of an object were to change. Nevertheless, in many data sets, such as the ones used in this paper or the popular ImageNet data set [4], the objects of interest are most often found in the center of the image. To circumvent depending on the
position, the variance could be calculated with another neural network. This way, the context of the pixels could be taken into account.

Our experiments indicate that our approach enables stable FGSM-based adversarial training, and can even outperform PGD-based training. We assume that the increase in performance and stability will make it easier to deploy robust neural networks in industrial applications. Furthermore, we hope that researchers who rely on adversarial training will be able to use the increased performance and stability of our method to accelerate their research.

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