Soft gluon suppression of $1/N_c$ contributions in color suppressed heavy meson decays

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Abstract:
We discuss the non-factorizable terms in color suppressed (Class II) decays. Our emphasis is on the non-perturbative soft gluon exchange mechanism, which has been previously found to be responsible for the rule of discarding $1/N_c$ in the Class I decays. The non-factorizable contribution to the decay $\bar{B}^0 \rightarrow D^0\pi^0$ at the tree level is estimated within the light cone QCD sum rule method which combines the technique of the QCD sum rules with the description of the pion in terms of the set of wave functions of increasing twist. We find that the same soft gluon exchange mechanism tends to cancel the $1/N_c$ term in the factorized amplitude.

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1. There has been raising interest during a last few years in testing the factorization approach to nonleptonic heavy meson decays. The current activity in this direction has been triggered by the observation [1] that the available data for D-meson decays seems to support a rule of discarding \(1/N_c\) terms in factorized hadronic matrix elements [2], which is in contrast to the standard prescription that keeps such terms. This approximate cancellation of \(1/N_c\) terms in the D decays has been explicitly checked [3] within the QCD sum rule approach. Surprisingly, the recent data on B-meson decays [4] signals that discarding \(1/N_c\) terms is not a universal rule for all channels. The \(1/N_c\)-suppression has rather a dynamical character and varies for different channels [5-10]. Still, there exist (both perturbative and non-perturbative) indications that patterns of deviation from the factorization approximation are alike inside each separate class of decays if they are classified according to factorization properties of corresponding effective Hamiltonians. In particular, it has been found in Ref. [5],[6] that the \(1/N_c\) rule is likely to hold in the Class I decays (see (5) below). The effect has a dynamical origin, and is due to non-perturbative gluon effects which have been estimated in [5],[6],[7] by the QCD sum rule method. The present letter is aimed to test an importance of these effects on the so-called color suppressed (Class II) decays. As a particular example, we will study the decay \(\bar{B}^0 \to D^0 \pi^0\) following the method suggested in Ref. [5].

At the tree level, weak hadronic Cabibbo favored B-decays correspond to the quark transitions \(b \to c \bar{s}s\) and \(b \to c \bar{u}d\) and are governed by the effective Hamiltonian

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{cb}V_{cs}^*(C_1(\mu)O_1^c + C_2(\mu)O_2^c) + V_{cb}V_{ud}^*(C_1(\mu)O_1^u + C_2(\mu)O_2^u)]
\] (1)

where \((\Gamma_\mu = \gamma_\mu(1 - \gamma_\gamma))\)

\[
O_1^c = (\bar{c}\Gamma_\mu b)(\bar{d}\Gamma_\mu u) \quad \text{and} \quad O_2^c = (\bar{d}\Gamma_\mu b)(\bar{c}\Gamma_\mu u)
\] (2)

and the operators \(O_{1,2}^c\) are obtained from \(O_{1,2}^u\) by the substitution \((\bar{d}, u) \to (\bar{s}, c)\). The Wilson coefficients \(C_i(\mu)\) are due to the renormalization of the bare Hamiltonian \(H_W \sim O_1\) by hard gluons with virtualities larger than \(\mu^2 = O(m_b^2)\). In the leading-log approximation \(C_{1,2} = \frac{1}{2}(C_+ \pm C_-)\) with [11]

\[
C_\pm(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{\frac{6\gamma_\pm}{n_{fl}}}\] (3)

with \(\gamma_- = -2\gamma_+ = 2\). At \(\mu \approx 5\text{ GeV}\) and \(n_{fl} = 5\), this yields (\(\Lambda_{MS} \approx 200\text{ MeV}\))

\[C_1 = 1.117 \quad \text{and} \quad C_2 = -0.266\] (4)

Within the factorization approximation one can distinguish between three classes of decays for which the corresponding amplitudes have the following structure [1]

\[A_I \sim a_1(\mu) < O_1 >, \quad A_{II} \sim a_2(\mu) < O_2 >, \quad A_{III} \sim [a_1(\mu) + x a_2(\mu)] < O_1 >\] (5)
Here $<O_i>$ are the (factorized) hadronic matrix elements of the operators $O_i$ and $a_i(\mu)$ are QCD factors related to the coefficients $C_i(\mu)$:

$$a_1(\mu) = C_1(\mu) + \frac{1}{N_c} C_2(\mu) , \quad a_2(\mu) = C_2(\mu) + \frac{1}{N_c} C_1(\mu) \quad (6)$$

An attempt of the global fit of nonleptonic B-decays yields $[12], [13]$

$$a_1 = 1.05 \pm 0.10 \quad \text{and} \quad a_2 = 0.25 \pm 0.05 \quad (7)$$

It should be mentioned that the very possibility of the global fit is rather questionable in view of both experimental and theoretical uncertainties and the expected variation between the naive factorization and the $1/N_c$ rule for different channels [5-10].

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Note that the matrix element $\langle D^+ \pi^- | O_1 | B^0 \rangle$ vanishes in the factorization approximation because of the color conservation. An appealing method to estimate this non-factorizable contribution has been proposed in $[3]$. There it has been shown that the above matrix element can be reduced to a simpler one $(p_\alpha^{(B)} - p_\alpha^{(D)}) \langle D | c \Gamma_\mu g G_{\alpha\mu} | b \rangle$ by virtue of the short distance operator product expansion (OPE), while the latter matrix element is fixed by the heavy flavor symmetry and can be expressed via the experimental number $M_{B^*}^2 - M_B^2 \simeq 0.46 \text{ GeV}^2$. Consequently, the non-factorizable piece is of the same order as the factorizable $(1/N_c)O_1$ part of $O_2$, and has the opposite sign. This observation justifies the (approximate) rule of discarding $1/N_c$ corrections in the class I decays.

Let us now consider the $\bar{B}^0 \to D^0\pi^0$ decay. Then $O_2$ factorizes and

$$\langle D^0\pi^0 | H_{\text{eff}} | \bar{B}^0 \rangle \sim \langle C_2 + \frac{C_1}{N_c} \rangle \langle D | c_\Gamma_\mu u | 0 \rangle \langle \pi | d_\Gamma_\mu b | B \rangle + C_1 \langle D \pi | 2(\bar{c}t^a u)(d_\Gamma_\mu t^a b) | B \rangle \quad (10)$$

Our task is to estimate the non-factorizable contribution to this decay which is given by the second matrix element in (10). Note that within the approach of Ref. $[3]$, this
contribution will be proportional to the matrix element \( p_0\langle \pi|d\tilde{\alpha}_\mu\tilde{G}_{\alpha\mu}b|B\rangle \) (see below), which is not known from general principles and deserves a separate calculation. We will present below an estimate for \( \langle D^0\pi^0|\tilde{O}_1|\tilde{B}^0\rangle \) based on the so-called light cone QCD sum rules method (see [14], [15] and references therein) which combines the traditional technique of the QCD sum rules [16] with a description of an emitted light particle (\( \pi^0 \) in our case) in terms of the wave functions of increasing twists. We will argue that the suppression of \( 1/N_c \) contributions via the soft gluon exchange mechanism [5] holds also for \( \tilde{B}^0 \rightarrow D^0\pi^0 \) and probably for other Class II decays. However, for the color suppressed decays, such cancellation is not what is welcomed by the phenomenology if one assumes the validity of the global fit (7). We will come back to this point later on.

2. To implement the two-step strategy suggested in Ref. [3], we start with the correlation function

\[
A_\mu = i \int dx e^{ipx} \langle \pi^0|\bar{u}(x)\gamma_\mu\gamma_5c(x), \tilde{O}_1(0)|\tilde{B}^0\rangle = i f_D p_\mu \langle D\pi|\tilde{O}_1|\tilde{B}\rangle \frac{1}{m_D^2 - p^2 + \ldots} \tag{11}
\]

where the ellipses stand for higher states contributions. At large Euclidean \( p^2 \) the correlation function (11) can be calculated in QCD. The leading contribution is due to the soft gluon emission from the quark loop. A simple calculation yields the following "sum rule" [6]

\[
if_D \langle D\pi|\tilde{O}_1|\tilde{B}\rangle \frac{1}{m_D^2 - p^2} = \frac{1}{4\pi^2} p_\mu \langle \pi|d\tilde{g}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5b|\tilde{B}\rangle \left[ \frac{1}{-p^2} - \frac{m_c^2}{(-p^2)^2} \ln \frac{m^2 - p^2}{m_c^2} \right] + \ldots \tag{12}
\]

which is to be satisfied in the duality interval \( 1 GeV^2 < -p^2 < 4 GeV^2 \). (In obtaining (12), we have omitted a contribution proportional to the matrix element \( p_\mu \langle \pi|d\tilde{g}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5b|\tilde{B}\rangle \approx p_0 \langle \pi|d\tilde{g}\tilde{G}_{0i}\gamma_i|\tilde{B}\rangle \) as it is down by the inverse heavy quark mass : \( \gamma_i b = O(\frac{1}{m_b}) \). As the ratio of the kinematic factors remains approximately constant and equal 1 in the duality region, we obtain

\[
\langle D^0(p)\pi^0(q)|\tilde{O}_1|\tilde{B}^0(p + q)\rangle \approx -\frac{i}{4\pi^2 f_D} p_\mu \langle \pi(q)|d\tilde{g}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5b|\tilde{B}(p + q)\rangle \tag{13}
\]

(Note that in the case at hand the use of the short distance OPE is justified by the fact that the final c-quark is heavy and its velocity is small in the rest frame of the b-quark [3]). It is important to point out that the formula (13) is obtained at the zero order in \( \alpha_s \), and cannot be considered as the renormalization group covariant one. The answer (13) refers to the normalization point \( \mu = O(m_b) \), the entire \( \mu \)-dependence being implicit. At the one-loop level the operator \( \tilde{O}_1 \) mixes with \( O_2 \), while the operator \( d\tilde{g}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5b \) mixes with the operator \( m_b^2 d\gamma_\mu b \) and operators vanishing on the equations of motion :
\begin{align}
(\bar{d}g\bar{G}_{\mu\nu}\gamma_5 b)^{\mu\nu}_{5} &= (1 - \frac{4}{3}(N_c - \frac{1}{N_c})\frac{\alpha_s}{4\pi}(\frac{g^{2}}{\mu^{2}_{F}})(\bar{d}g\bar{G}_{\mu\nu}\gamma_5 b)^{\mu\nu}_{5} \\
&+ \frac{C_F}{2}\frac{\alpha_s}{4\pi}\ln\frac{\mu^{2}_{F}}{\mu^{2}_{1}} \left( \frac{2}{3} \bar{d}(\tilde{\nabla}\gamma_\mu + \gamma_\mu \tilde{\nabla}) b \right) - \frac{2}{3} \bar{d}(\tilde{\nabla}\gamma_\mu + \gamma_\mu \tilde{\nabla}) b - \frac{2}{3} m_b \bar{d} \sigma_{\mu\nu} \tilde{\nabla}_\nu b - 2im_b \bar{d} \gamma_\mu b)^{\mu\nu}_{5} \\
\end{align}

where we have set \( m_b = 0 \) and \( C_F = \frac{N_c^2 - 1}{2N_c} = 4/3 \). Thus, one has to understand (13) as the tree level relation. For consistency, in what follows we will omit one-loop contributions altogether.

To find the new matrix element (13), consider another correlation function

\[ T_\alpha(p, q) = i \int \frac{d^4x e^{ipx}}{(2\pi)^4} \langle \pi | \bar{d}(x) g\bar{G}_{\alpha\mu} \gamma_5 b(x) | b(0) \rangle \] 

\[ = \frac{m_B^2 f_B}{m_b} \frac{1}{m_B^2 - (p + q)^2} \left[ p_\alpha f_1(p^2) + (p_\alpha + 2q_\alpha) f_2(p^2) \right] \]

(15)

where \( \langle B|b\gamma_5 d|0 \rangle = m_B^2 f_B/m_b \). At this stage the light cone QCD sum rules method suggests a simple and straightforward way of the calculation. At large Euclidean \( (p + q)^2 \) the leading contribution to (15) is

\[ T_\alpha(p, q) = ig \int \frac{d^4x d^4k}{(2\pi)^4(m_B^2 - k^2)} e^{i(p-k)x} \langle \pi | \bar{d}(x) \bar{G}_{\alpha\mu}(x) \gamma_\mu \gamma_5 (\hat{k} + m_b) \gamma_5 d(0) | 0 \rangle \]

(16)

(here \( \hat{k} = k_\mu \gamma_\mu \)), that can be further evaluated introducing the pion wave function (WF) of twist 3 \( \phi_3 \)

\[ \langle \pi | \bar{d}(x) g\bar{G}_{\mu\nu} (vx) \sigma_{\alpha\beta} \gamma_5 d(0) | 0 \rangle = -\frac{i}{\sqrt{2}} f_{3\pi} \left[ q_\alpha (g_\beta g_{\nu\beta} - q_\nu g_{\beta\mu}) - q_\beta (g_\mu g_{\nu\alpha} - q_\nu g_{\mu\alpha}) \right] \times \int D\alpha_\gamma \phi_3(\alpha_\gamma)e^{iqx(\alpha_\gamma + v\alpha_3)} \]

(17)

(here \( D\alpha_1 = d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1) \) and \( f_{3\pi} \simeq 0.0035 \text{ GeV}^2 \) for \( \mu^2 \simeq 1 \text{ GeV}^2 \) [7]), and the set of WF’s of twist 4:

\[ \langle \pi^0 | \bar{d}(x) \gamma_\mu i g\bar{G}_{\alpha\beta}(vx) d(0) | 0 \rangle = -\frac{f_\pi}{\sqrt{2}} \left[ q_\beta (g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx}) - q_\alpha (g_{\beta\mu} - \frac{x_\beta q_\mu}{qx}) \right] \times \int D\alpha_\gamma \phi_4(\alpha_\gamma)e^{iqx(\alpha_\gamma + v\alpha_3)} \]

\[ -\frac{f_\pi}{\sqrt{2}} \left[ q_\alpha x_\beta - q_\beta x_\alpha \right] \int D\alpha_\gamma \phi_6(\alpha_\gamma)e^{iqx(\alpha_\gamma + v\alpha_3)} \]

(18)

Two more WF’s of twist 4 \( \phi_\perp, \phi_\parallel \) are defined analogously to (17) with the substitution \( (ig\bar{G}_{\alpha\beta}) \rightarrow (\gamma_5 g\bar{G}_{\alpha\beta}) \).

\[ \]
Then, using the identity
\[ \int_0^1 du \int D\alpha_i \delta(u - \alpha_1 - \alpha_3) \Phi(\alpha_i) = \int_0^1 du \int_0^u d\alpha_3 \Phi(\alpha_1 = u - \alpha_3, \alpha_2 = \bar{u}, \alpha_3), \]
the answer for the simplest tree diagram can be written as
\[ T_\alpha = \frac{q_\alpha}{\sqrt{2}} \int_0^1 \frac{du}{m_\pi^2 - (p + uq)^2} \int_0^u d\alpha_3 [-2(pq) f_{3\pi} \phi_{3\pi} + f_\pi m_b (\bar{\phi}_\parallel - 2\bar{\phi}_\perp)] (u - \alpha_3, \bar{u}, \alpha_3) \]
A systematic study of higher twist WF’s beyond the asymptotic regime has been done in Ref. [19]. This analysis has been based on the expansion in representations of the so-called collinear conformal group SO(2,1), which is a subgroup of the full conformal group SO(4,2) acting on the light cone. The asymptotic WF’s are defined as contributions of operators with the lowest conformal spin and unambiguously fixed by the group structure. Pre-asymptotic corrections correspond to the operators with the next-to-leading conformal spin, whose numerical values have been calculated by the QCD sum rules method. The result for \( \phi_{3\pi} \) reads [19]:
\[ \phi_{3\pi}(\alpha_i) = 360 \alpha_1 \alpha_2 \alpha_3^2 [1 + \omega_{1,0} \frac{1}{2} (7 \alpha_3 - 3) + \omega_{2,0} (2 - 4 \alpha_1 \alpha_2 - 8 \alpha_3 + 8 \alpha_3^2) + \ldots] \]
where \( \omega_{1,0} = -2.88 \), \( \omega_{2,0} = 10.5 \), \( \omega_{1,1} = 0 \) in a low normalization point [17],[19].

For the twist 4 WF’s the relevant first order formulas are
\[ \bar{\phi}_{\perp}(\alpha_i) = 30 \delta^2 (1 - \alpha_3) \alpha_3^2 \left[ \frac{1}{3} + 2 \varepsilon (1 - 2 \alpha_3) \right] \]
\[ \bar{\phi}_{\parallel}(\alpha_i) = -120 \delta^2 \alpha_1 \alpha_2 \alpha_3 \left[ \frac{1}{3} + \varepsilon (1 - 3 \alpha_3) \right] \]
where the parameter \( \delta^2 \simeq 0.2 \text{ GeV}^2 \) (at \( \mu^2 \simeq 1 \text{ GeV}^2 \)) is defined via
\[ \langle 0 | \bar{d} g \gamma_\mu \gamma_5 u | \pi^+ \rangle = i \delta^2 f_\pi q_\mu \]
and \( \varepsilon \simeq 0.5 \) is the weight of the first conformal spin correction. We will need the above set of parameters renormalized to a higher normalization point \( \mu^2 \simeq \mu_b^2 = \sqrt{m_B^2 - m_\pi^2} \simeq (2.4 \text{ GeV})^2 \) which is the characteristic virtuality of the b-quark in the B-meson [18],[19]. The corresponding anomalous dimensions can be found in [19]. The results read [15] \( f_{3\pi} = 0.0026 \text{ GeV}^2 \), \( \omega_{1,0} = -2.18 \), \( \omega_{1,1} = -2.59 \), \( \omega_{2,0} = 8.12 \), \( \delta^2 \simeq 0.18 \), \( \varepsilon \simeq 0.4 \).

To match the answer (20) with the B-meson contribution to the correlation function (15), we note that (20) can be re-written as the dispersion integral with the expression \( (m_\pi^2 - \bar{u} \bar{p}^2)/u \) being the mass of the intermediate state. The duality prescription tells that this invariant mass has to be restricted from above by the duality threshold \( s_0 \simeq 35 \text{ GeV}^2 \) (this value is obtained from corresponding two-point sum rules). As it is easy to see, this transforms into the effective cut-off from
below in the u-integral \[20\], \[14\]. Finally, we make the standard Borel transformation suppressing both higher states resonances and higher Fock states in the full pion wave function. Under the Borel transformation $-(p + q)^2 \rightarrow M^2$

\[
\frac{1}{m_B^2 - (p + q)^2} \rightarrow \exp \left( -\frac{m_B^2}{M^2} \right)
\]

\[
\frac{1}{m_b^2 - (p + uq)^2} \rightarrow \frac{1}{u} \exp \left( -\frac{m_b^2 - \bar{u}p^2}{uM^2} \right)
\]

Our final sum rule takes the form

\[
f_1(p^2) = -f_2(p^2) = -\frac{1}{\sqrt{2}} \frac{m_b}{2m_B^2 f_B} \int_0^1 du \int_0^{u3} d\alpha_3 \exp \left( \frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2} \right) \Theta(u - \frac{m_b^2 - p^2}{s_0 - p^2})
\]

\[
\times \left[ -f_3 \frac{m_b^2 - p^2}{u^2} \phi_3 + f_\pi \frac{m_b}{u} (\tilde{\phi}_\parallel - 2\tilde{\phi}_\perp) \right] (\alpha_i)
\]

(25)

3. Now we turn to numerical estimates. In evaluating (25), we have used the following set of parameters: $m_b = 4.7 \text{ GeV}$, $m_B = 5.28 \text{ GeV}$, $s_0 \simeq 35 \text{ GeV}^2$, $f_B \simeq 135 \text{ MeV}$ \[20\], \[15\]. The Borel mass $M^2$ has been varied in the interval from 8 to 20 $\text{GeV}^2$. We have found that within the variation of $M^2$ in this region, the result changes no more than by 10% and yields for $p^2 \simeq m_D^2$

\[
f_1(m_D^2) \simeq -f_2(m_D^2) \simeq \frac{1}{\sqrt{2}} \times 0.08 \text{ GeV}^2
\]

(26)

Then for the matrix element of interest we obtain

\[
p_\alpha \langle \pi | \bar{d}G_{\alpha \mu} \gamma_5 b | B \rangle \simeq M_B^2 f_1(m_B^2) + M_B^2 f_2(m_B^2) \simeq \frac{1}{\sqrt{2}} \times 1.9 \text{ GeV}^4
\]

(27)

For the factorizable amplitude due to the operator $(1/N_c)O_2$ we have

\[
M_f = \frac{1}{N_c} i f_D p_\mu \langle \pi(q) | \bar{\gamma}_\mu b(p + q) \rangle
\]

\[
= \frac{i}{N_c} f_D p_\mu [2q_\mu f_\pi^+(p^2) + p_\mu (f_\pi^+(p^2) + f_\pi^-(p^2))] \]

(28)

The value of the form factor $f_\pi^+(m_D^2)$ can be read off the results of Ref. \[20\] where this quantity has been calculated (for the charged pion) by virtue of the light cone QCD sum rules method. The answer is

\[
f_\pi^+(m_D^2) \simeq -\frac{1}{\sqrt{2}} \times 0.3
\]

(29)

where the number $(-1/\sqrt{2})$ is due to the different isospin structure in our case. For the second form factor we use the model \[1\]

\[
f_\pi(p^2) = -f_\pi(p^2) \frac{m_B - m_\pi}{m_B + m_\pi} \simeq -f_\pi(p^2)
\]

(30)
and therefore neglect the second term in (28). In the nomenclature of Ref. [5], we thus obtain the following estimate for the ratio of the non-factorizable to the factorizable $1/N_c$ amplitudes:

$$r \equiv \frac{M_{nf}}{M_f} \approx -\frac{N_c}{4\pi^2 f_D^2 (m_B^2 - m_D^2)} f_\pi (m_B^2) \approx -0.7 \quad (31)$$

where we have used the value $f_D \simeq 170 \text{ MeV}$ corresponding to omitted $\alpha_s$-corrections in the relevant two-point sum rule [20, 15]. Thus, our final result (31) suggests that the non-factorizable contribution tends to cancel the factorizable $1/N_c$ amplitude due to the operator $(1/N_c)\mathcal{O}_2$ (see (10)), in agreement with expectations of Ref. [5].

The sign of the effect can also be compared with the estimate within the QCD sum rules method done in Ref. [7] for the weak decay $B \to J/\Psi K$, which also belongs to the class of color suppressed decays. The authors of [7] have found that the power corrections due to the gluon condensate partly cancel the $1/N_c$ factorizable amplitude, i.e. their sign of $r$ is negative, too. On the other hand, within the approach of Ref. [5] the non-factorizable amplitude for this decay can be approximately expressed via the matrix element $p_\alpha \langle K|\bar{s}\tilde{G}_{\alpha\mu}\gamma_\mu\gamma_5 b|B\rangle$, which can be extracted from (27) assuming the SU(3) limit. One has to bear in mind, however, that the approach of Ref. [5] cannot be literally applied to the decay $B \to J/\Psi K$. The poor stability of the corresponding sum rule implies that perturbative corrections or operators of higher dimensions must be important there.

4. In this letter we have estimated the soft gluon exchange mechanism contribution to the non-factorizable amplitude of the $B^0 \to D^0\pi^0$ decay and found the tendency for the cancellation of $1/N_c$, in seeming contradiction with (7). One should emphasize, however, that our result does not mean the actual contradiction of the theory with the available data [4]. One possible source of the disagreement may be the unjustified use of the global fit leading to the particular numbers (7). Such fit implies that the factorization properties are alike in all non-leptonic two-body B-decays. The validity of this assumption has to be examined in QCD. Probably the more important origin of disagreement with (7) are large perturbative $O(\alpha_s)$-corrections which are not taken into account in our calculation, and correspond to different contributions to non-factorizable amplitudes. Estimates made in Ref. [7] indicate that radiative corrections are important for a complete evaluation of non-factorizable amplitudes for the Class II decays. As we have mentioned, in a calculation including radiative corrections one needs an accurate separation of $O(\alpha_s)$ corrections related to the matrix element itself and $O(\alpha_s)$ terms due to the mixing with the two-particle operators, cf. (14).

In the recent paper [11] it has been stressed that non-factorizable amplitudes are very important in one more aspect. The point is that the coefficient $a_2$ becomes strongly $\mu$- and scheme- dependent beyond the leading-log approximation. Depending on the renormalization scheme, $a_2$ can scale crudely from 0.1 to 0.2. This is in strong contrast with $a_1$ which exhibits very weak $\mu$— and scheme- dependence. The importance of higher order QCD corrections for an accurate calculation of $a_2$ can
be intuitively understood as a consequence of the fact that \( a_2 \) is the difference of the nearly equal numbers, and thus is very sensitive to their precise values. Since the factorized amplitudes are \( \mu_- \) and scheme-independent, only the non-factorizable contributions can remove this scheme (and \( \mu_- \)) dependence in the physical amplitudes. Unfortunately, we have nothing to add in respect to this important problem in view of our neglecting perturbative \( O(\alpha_s) \) corrections. At the same time, the observation of Ref. [10] suggests that the perturbative corrections to the matrix elements are presumably more important for the Class II decays than for those of the Class I. The main question is whether the account for hard gluon loops is able to change the sign of the ratio (31). A calculation of the matrix element \( \langle D^0 \pi^0 | \hat{O}_1 | \bar{B}^0 \rangle \) including the radiative corrections can be done either by the methods of Ref. [3, 7], or directly by the light cone QCD sum rules method [14]. In the latter approach, one has to calculate the three-point correlation function of the D- and B-meson currents and the effective Hamiltonian between the vacuum and the pion states. We have explicitly checked that in this case the leading \( O(\alpha_s^0) \) contribution is again given by a combination of the three-particle WF’s of twists 3 and 4. However, the corresponding estimate based on retaining only these terms manifests a poor stability, that indicates the importance of radiative corrections or/and higher twist effects in the corresponding sum rule. This instability does not occur in the above sum rule, that formally justifies our neglecting the radiative and higher twist correction. Undoubtedly, the complete evaluation of perturbative gluon effects on hadronic matrix elements of interest is needed before any comparison with the data.

To our knowledge, this work is currently in progress [7].

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