Fuzzy inventory model with quadratic demand, linear time dependent holding cost, constant deterioration rate and shortages

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Abstract
Fuzzy inventory model with demand rate is quadratic, holding cost is linearly time dependent, constant deterioration rate and allowing shortages. Trapezoidal fuzzy number is used for holding cost, deterioration cost and shortage cost. For defuzzification purpose Graded Mean Representation Integration method is used. Sensitivity Analysis have done and the Total cost is minimized.

Keywords
Purchase inventory model, Quadratic demand, Shortage, Deterioration, Time dependent holding cost and Trapezoidal fuzzy number.

AMS Subject Classification
90B05, 62E86, 62P30.

1. Introduction

Inventory is an essential resource needed for day-to-day operations and it deals with how much to keep on hand and how frequently to reorder. Raw materials inventory, work-in-process inventory and finished goods inventory are three basic types of inventory. The main purpose of inventory is to maintain a trade-off between the minimization of the total cost and maximization of the customer satisfaction.

In the inventory process, the effect of deterioration is very important. While developing an optimal inventory policy for products such as fruits, vegetables, chemicals etc., deterioration cannot be ignored. In inventory models the demand is either deterministic or probabilistic in nature. In reality both demand and supply are uncertain due to unpredictable events, like change of orders, random capacity of supplies, etc. In the real situations Fuzzy concepts can be used in the inventory models.

Fuzzy set theory originally introduced by [12] Zadeh in 1965, provides a framework for considering parameters that are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. In the development of inventory models, the quantities such as the demand rate, deterioration rate and all the costs related to inventory models are considered as fuzzy numbers.

Pande and Gowtham [8] have developed an inventory control model for fixed deterioration and logarithmic demand rates. They used cost minimization technique to obtain the optimal value of stock, time and total cost. They considered deterministic cases of demand by allowing shortage. They obtained an approximate expression for initial inventory, total number of deteriorated units and total minimum average cost.

Amutha and Chandrasekaran [1] have developed an EOQ model for deteriorating items with quadratic demand and time dependent holding cost. They used the exponential distribution for deterioration. Shortages are not allowed and holding cost is time dependent. Using numerical examples, the model
was solved to minimize the total inventory cost.

Mishra et al [7] have designed an inventory model for deteriorating items with time-dependent demand and time varying holding cost under partial backlogging. They considered a deterministic inventory model with time dependent demand and time varying holding cost where deterioration was time proportional. Shortages were allowed and the demand was partially backlogged. Using numerical examples, the model was solved analytically to minimize the inventory cost.

Venkateswarlu and Mohan [11] have developed an inventory model with quadratic demand, constant deterioration and salvage value. They considered two cases (Retarded Decline Model and Accelerated Decline Model) to calculate the optimum total cost and total order quantity. They found that the retarded decline and accelerated decline have shown good results which will be useful to describe a realistic situation for any product. They also formulated inventory models for constant deterioration rate together with salvage value. They found the existence of retarded decline and accelerated decline models in this case.

Jaggi et al [5] have considered fuzzy inventory model for deteriorating items with time varying demand and shortages. They developed a fuzzy economic order quantity model for deteriorating items in which demand increases with time. Shortages were allowed and fully backlogged. They considered the demand, holding cost, unit cost, shortage constant deterioration cost as trapezoidal fuzzy numbers. They defuzzified the total cost function using graded mean integration representation method, signed distance method and centroid method. Using numerical examples, they found that the graded mean integration representation method gave the minimum cost as compared to signed distance method and centroid method.

Dutta and Kumar [4] have designed an inventory model without shortages have been considered in a fuzzy environment. They determined the optimal total cost and the optimal order quantity. Trapezoidal fuzzy number is used. The computation of economic order quantity (EOQ) is carried out through defuzzification process by using signed distance method. Sensitivity for this model is also studied, which shows a linear relation between demand, EOQ, and total cost.

Ranganathan and Thirunavekarasu [9] have developed an inventory control model for constant deterioration in fuzzy environment with demand rate. Shortages were allowed and fully backlogged. All inventory parameters were assumed to be trapezoidal fuzzy number. They defuzzified the fuzzy model using graded mean one level integration representation method. After analysing the result, they obtained the fuzzy optimal solution to minimize the total cost of the inventory system.

Karthigeyan et al [6] have expressed a fuzzy optimized production EOQ model with deterioration rate and allowing the shortages. In their continuous production inventory model the demand and production rates along with the holding cost, shortage cost and deteriorating cost are assumed as trapezoidal fuzzy numbers. The deterioration is assumed as exponential distribution. They used graded mean integration method for defuzzification. They obtained the minimum total cost and optimal inventory quantity and optimal shortage quantity.

Balarama Murthy et al [2] have developed fuzzy inventory control problem with Weibull deterioration rate and logarithmic demand rate. They considered holding cost, shortage cost, and deterioration cost and demand rate as a fuzzy trapezoidal number. They defuzzified the fuzzy model using graded mean one level integration representation method. The total inventory cost was minimized.

2. Notations and Assumptions

1. Demand rate \( D(t) = (a + bt + ct^2) \)
   where \( a > 0, b > 0, c > 0 \)
2. \( A \): Ordering cost per order
3. \( I(t) \): Inventory level of the product
4. \( H(t) = (\alpha + \beta t) \), linearly time dependent holding cost.
   where \( \alpha > 0, \beta > 0 \)
5. \( C_s \): shortage cost per unit per time period
6. \( C_d \): deterioration cost per unit per time period
7. \( THC, TDC, TSC \) and \( TIC \) are the total holding cost, total deterioration cost, total shortage cost and total inventory cost respectively
8. Exponential distribution is used for deterioration rate of an item. The probability density function of exponential distribution is
9. Exponential distribution is used for deterioration rate of an item. The probability density function of exponential distribution is
10. \( S \) is the initial inventory
11. \( D \) is deterioration units
12. At time \( t = t_1 \) Inventory level reaches to zero
13. Shortages are in the time period \( (t_1, T) \). Shortages are fully backlogged

3. Fuzzy Concepts

3.1 Fuzzy Number

If a fuzzy set is convex and normalized and its membership function is defined in \( R \) and piecewise continuous is called as fuzzy number. So fuzzy number represents a real number interval whose boundary is fuzzy.

3.2 Trapezoidal fuzzy number

A fuzzy set \( \tilde{A} = (a_1, a_2, a_3, a_4) \) where \( a_1 < a_2 < a_3 < a_4 \) and defined on \( R \), is called the trapezoidal fuzzy number, if the membership function of \( \tilde{A} \) is given by

\[
\mu_{\tilde{A}} = \begin{cases} 
0, & \text{if } x < a_1 \\
\frac{x - a_1}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2 \\
1, & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4 - x}{a_4 - a_3}, & \text{if } a_3 \leq x \leq a_4 \\
0, & \text{if } x > a_4 
\end{cases}
\]
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(4.1)

\[ \mu_\alpha = \max(\min(\frac{x-a_1}{a_2-a_1}, 1, \frac{a_3-x}{a_3-a_2}), 0) \]

3.3 Fuzzy arithmetical operations:

If \( \tilde{A} = (a_1, a_2, a_3, a_4) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4) \) are two trapezoidal fuzzy real numbers, then

\[ \tilde{A} + \tilde{B} = (c_1, c_2, c_3, c_4) \text{ where } T = (a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2), \]

\[ T = (a_2 b_2, a_3 b_1, a_3 b_2, a_4 b_1, a_4 b_2), \]

\[ c_1 = \text{min} T, c_2 = \text{min} T, c_3 = \text{max} T \text{ and } c_4 = \text{max} T \]

If \( a_1, a_2, a_3, a_4, b_1, b_2, b_3 \) and \( b_4 \) are all non-zero positive real numbers,

Then \( \tilde{A} * \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4) \)

\[ b = \sqrt{a_1 a_2 a_3 a_4} = (\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}) \]

\[ -B = (-b_4, -b_3, -b_2, -b_1) \] and \( \tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \)

\[ \frac{1}{b} = b - 1 = 1/b_1, 1/b_2, 1/b_3, 1/b_4 \]

\[ \tilde{A} / \tilde{B} = (a_1 b_1/a_2 b_2/a_3 b_3/a_4 b_4) \]

3.4 Graded mean representation integration method

The method of defuzzication of a generalized trapezoidal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4) \) by its graded mean representation integration was proposed by [3] Chen and Hsieh and is defined by

\[ \text{GMRI} (\tilde{A}) = \int_{0}^{b} \frac{1}{b_2} [a_1 + a_4] h (a_2 - a_1 - a_4 + a_3) dh \]

Where \( Q = \int_{0}^{t_1} (a + bt + ct^2) dt \)

4. Formulation of the Model

\[ I(t) \] denote on-hand inventory at time \( (0, T) \), then the linear first order differential equation which the on-hand inventory \( I(t) \) satisfies in two different parts of the cycle time \( T \) are given by:

\[ \frac{dI(t)}{dt} + (a + b + c) I(t) = - (a + bt + ct^2), 0 \leq t \leq t_1 \]

(4.1)

\[ \frac{dI(t)}{dt} = -(a + bt + ct^2), \quad t_1 \leq t \leq T \]

(4.2)

With initial and boundary conditions:

\[ I(0) = S, I(t_1) = 0, I(T) = Q \]

(4.3)

Solving equations (4.1) and (4.2) with using equation (4.3)

\[ I(t) e^{\theta t} = - (at + (a + b + c) t^2) + \frac{(a + bt + ct^2) t^3}{3} \]

+ \( c \theta^4 t^4 \)

(4.4)

Put \( t = 0, I(0) = S \), then \( S = C_1 \)

\[ I(t) = - (at + (a + b + c) t^2) + \frac{(a + bt + ct^2) t^3}{3} + c \theta^4 t^4 + S e^{\theta t} \]

(4.5)

Put \( t = t_1, I(t_1) = 0 \)

\[ S = \left[ a t_1 + (a + b + c) t^2 + (b \theta + c) \frac{t^3}{3} + c \theta^4 \frac{t^4}{4} \right] \]

(4.6)

Substitute (4.6) in (4.5)

\[ I(t_1) = \left[ a t_1 + (a + b + c) t^2 + (b \theta + c) \frac{t^3}{3} + c \theta^4 \frac{t^4}{4} \right] \]

\[ -(a + b t_1 + c t_1^3 + c \theta^4) \]

\[ -(b - a \theta) \frac{t^2}{2} - (2c - b \theta) \frac{t^3}{6} \]

(4.7)

solving equation (4.2)

\[ I(t) = \left[ a t_1 + \frac{b t^2}{2} + \frac{c t_1^3}{3} \right] + c \theta^4 \]

Put \( I(t_1) = 0 \), then \( t = t_1 \)

Where \( C_2 = a t_1 + \frac{b t^2}{2} + \frac{c t_1^3}{3} \)

\[ I(t) = a(t_1 - t) + \frac{b t^2}{2} (t_1^2 - t^2) + \frac{c}{3} (t_1^3 - t^3) \]

\[ t_1 \leq t \leq T \]

(4.8)

Put \( t = Q, t = T \)

\[ Q = a(t_1 - T) + \frac{b}{2} (t_1^2 - T^2) + \frac{c}{3} (t_1^3 - T^3) \]

The deterioration units \( D = S - \int_{0}^{t_1} (a + bt + ct^2) dt \)

\[ D = \left[ a t_1^2 \frac{t}{2} + \frac{b t_1^3}{6} + \frac{c t_1^4}{4} \right] \]

(4.9)

Deterioration cost \( = C_d \) \( \times \) (number of deterioration units)

\[ = C_3 \left[ a t_1^2 \frac{t}{2} + \frac{b t_1^3}{6} + \frac{c t_1^4}{4} \right] \]

(4.10)

Shortage units \( \int_{t_1}^{T} I(t) dt \)

\[ = \left[ a t_1 T + \frac{b t_1^2 T}{2} + \frac{c t_1^3 T}{3} \right] - \frac{a T^2}{2} \frac{b T^3}{6} \frac{c T^4}{12} - \frac{a t_1^2}{2} \frac{b t_1^3}{6} \frac{c t_1^4}{4} \]

(4.11)

Shortage cost \( = C_s \) \( \times \) (no. of shortage units)

\[ = C_s \left[ a t_1 T + \frac{b t_1^2 T}{2} + \frac{c t_1^3 T}{3} \right] - \frac{a T^2}{2} \frac{b T^3}{6} \frac{c T^4}{12} - \frac{a t_1^2}{2} \frac{b t_1^3}{6} \frac{c t_1^4}{4} \]

(4.12)
The condition for optimality:

\[
\frac{d(T_{\text{avg}})}{dt_1} = \frac{C_d}{T}[a\theta + b\theta t_1 + c\theta t_1^2]
\]

When \(\frac{d(T_{\text{avg}})}{dt_1} = 0\) gives the minimum point.

**Fuzzy Model**

Suppose \(\tilde{C}_d = (\tilde{C}_{d_1}, \tilde{C}_{d_2}, \tilde{C}_{d_3}, \tilde{C}_{d_4})\),
\(\tilde{C}_s = (\tilde{C}_{s_1}, \tilde{C}_{s_2}, \tilde{C}_{s_3}, \tilde{C}_{s_4})\), \(\tilde{C}_h = (\tilde{C}_{h_1}, \tilde{C}_{h_2}, \tilde{C}_{h_3}, \tilde{C}_{h_4})\),
\(\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3, \tilde{\theta}_4)\) are non-negative trapezoidal fuzzy numbers. From the equation (4.15) can be rewritten as

\[
T_{\text{avg}} = \tilde{C}_d \otimes (\tilde{\theta} \otimes P) \oplus \tilde{C}_s \otimes (Q \oplus R) \oplus \tilde{C}_h \otimes ((\tilde{\theta} \otimes L) \oplus M)
\]

\[
\begin{align*}
P &= \left[\frac{a\theta^2}{2} + \frac{b\theta^3}{3} + \frac{c\theta^4}{4}\right] \\
Q &= \left[\frac{a\theta^2}{2} + \frac{b\theta^3}{3} + \frac{c\theta^4}{4}\right] \\
R &= \left[-\frac{a\theta^2}{2} - \frac{b\theta^3}{3} - \frac{c\theta^4}{4}\right] \\
L &= \left[\frac{a\theta^2}{2} - \frac{b\theta^3}{3} + \frac{c\theta^4}{4}\right] \\
M &= \left[\frac{a\theta^2}{2} + \frac{b\theta^3}{3} - \frac{c\theta^4}{4}\right]
\end{align*}
\]

\[
\tilde{T}_{\text{avg}}(t_1) = (\tilde{T}_{\text{avg}}(t_1), \tilde{T}_{\text{avg}}(t_2), \tilde{T}_{\text{avg}}(t_3), \tilde{T}_{\text{avg}}(t_4))
\]

\[
\begin{align*}
T_{\text{avg}}(t_1) &= P\tilde{C}_d + Q\tilde{C}_s + R\tilde{C}_h + L\tilde{T}_{\text{avg}}(t_1) + M\tilde{T}_{\text{avg}}(t_1) \\
T_{\text{avg}}(t_2) &= P\tilde{C}_d + Q\tilde{C}_s + R\tilde{C}_h + L\tilde{T}_{\text{avg}}(t_2) + M\tilde{T}_{\text{avg}}(t_2) \\
T_{\text{avg}}(t_3) &= P\tilde{C}_d + Q\tilde{C}_s + R\tilde{C}_h + L\tilde{T}_{\text{avg}}(t_3) + M\tilde{T}_{\text{avg}}(t_3) \\
T_{\text{avg}}(t_4) &= P\tilde{C}_d + Q\tilde{C}_s + R\tilde{C}_h + L\tilde{T}_{\text{avg}}(t_4) + M\tilde{T}_{\text{avg}}(t_4)
\end{align*}
\]

Defuzzifying the total inventory cost \(T_{\text{avg}}(t_1)\) by GMRI method

\[
\tilde{T}_{\text{avg}}(t_1) = P\tilde{t}_1\tilde{C}_d + Q\tilde{t}_1\tilde{C}_s + R\tilde{t}_1\tilde{C}_h + L\tilde{t}_1\tilde{T}_{\text{avg}}(t_1) + M\tilde{t}_1\tilde{T}_{\text{avg}}(t_1)
\]

To get the optimal value of \(t_1\) can be obtained by solve the
following equation, from this \( t_1 \) can obtain minimized total cost.

\[
\frac{d(T_{\text{avg}}(t))}{dt_1} = \frac{P'}{6}(\theta_1 C_{d1} + 2\theta_2 C_{d2} + \theta_3 C_{d3} + \theta_4 C_{d4}) \\
+ \frac{Q}{6}(C_{x1} + 2C_{x2} + 2C_{x3} + C_{x4}) \\
+ \frac{R}{6}(C_{i1} + 2C_{i2} + 2C_{i3} + C_{i4}) \\
+ \frac{L}{6}(\theta_1 C_{h1} + 2\theta_2 C_{h2} + \theta_3 C_{h3} + \theta_4 C_{h4}) \\
+ \frac{M}{6}(C_{c1} + 2C_{c2} + 2C_{c3} + C_{c4}) = 0
\]

\[
\frac{d^2T_{\text{avg}}(t)}{dt_1^2} = \frac{P''}{6}(\theta_1 C_{d1} + 2\theta_2 C_{d2} + \theta_3 C_{d3} + \theta_4 C_{d4}) \\
+ \frac{Q''}{6}(C_{x1} + 2C_{x2} + 2C_{x3} + C_{x4}) \\
+ \frac{R''}{6}(C_{i1} + 2C_{i2} + 2C_{i3} + C_{i4}) \\
+ \frac{L''}{6}(\theta_1 C_{h1} + 2\theta_2 C_{h2} + \theta_3 C_{h3} + \theta_4 C_{h4}) \\
+ \frac{M''}{6}(C_{c1} + 2C_{c2} + 2C_{c3} + C_{c4}) > 0
\]

Optimal initial inventory after fulfilling backorder \( S \) denoted by \( S^* \) by GMRI method is

\[
S^* = \frac{S_1^* + 2S_2^* + 2S_3^* + S_4^*}{6}
\]

where

\[
S_1^* = \frac{(a t_1 + (a \theta_1 + b) t_2^2 + (b \theta_1 + c) t_3^2 + c \theta_1 t_4^2)}{4}
\]

\[
S_2^* = \frac{(a t_2 + (a \theta_2 + b) t_3^2 + (b \theta_2 + c) t_4^2 + c \theta_2 t_4^2)}{4}
\]

\[
S_3^* = \frac{(a t_3 + (a \theta_3 + b) t_4^2 + (b \theta_3 + c) t_4^2 + c \theta_3 t_4^2)}{4}
\]

\[
S_4^* = \frac{(a t_4 + (a \theta_4 + b) t_4^2 + (b \theta_4 + c) t_4^2 + c \theta_4 t_4^2)}{4}
\]

Optimal quantity of the unit deteriorated units \( D \) denoted by \( D^* \) by GMRI method is

\[
D^* = \frac{D_1^* + 2D_2^* + 2D_3^* + D_4^*}{6}
\]

where

\[
D_1^* = \frac{(a \theta_1 t_1^2 + b \theta_1 t_3^2 + c \theta_1 t_4^2)}{4}
\]

\[
D_2^* = \frac{(a \theta_2 t_2^2 + b \theta_2 t_3^2 + c \theta_2 t_4^2)}{4}
\]

\[
D_3^* = \frac{(a \theta_3 t_3^2 + b \theta_3 t_3^2 + c \theta_3 t_4^2)}{4}
\]

\[
D_4^* = \frac{(a \theta_4 t_4^2 + b \theta_4 t_4^2 + c \theta_4 t_4^2)}{4}
\]

Optimal value of the total cost \( T_{\text{avg}}^*(t) \) denoted by \( T_{\text{avg}}^*(t) \) by GMRI method is

\[
T_{\text{avg}}^*(t) = A + \frac{1}{6}(P\theta_1 C_{d1} + QC_{x1} + RC_{i1} + L \theta_1 C_{h1} + MC_{c1})
\]

\[
+ \frac{1}{3}(P\theta_2 C_{d2} + QC_{x2} + RC_{i2} + L \theta_2 C_{h2} + MC_{c2})
\]

\[
+ \frac{1}{3}(P\theta_3 C_{d3} + QC_{x3} + RC_{i3} + L \theta_3 C_{h3} + MC_{c3})
\]

\[
+ \frac{1}{6}(P\theta_4 C_{d4} + QC_{x4} + RC_{i4} + L \theta_4 C_{h4} + MC_{c4})
\]

5. Analysis and Interpretation

All the cost related to inventory (holding cost, shortage cost and deterioration cost) and parameter are as a trapezoidal fuzzy number and graded mean representation integration method is used for defuzzification. MATHCAD is used for calculation.

Numerical Example

The given values are \( A=1000, \ C_d = 1.5, \ C_s = 2, \ C_h = \alpha + \beta t = 0.504, \ t_1 = 0.4, \ T = 1, \alpha = 0.5 \) and \( \beta = 0.08 \) are assumed.

For Crisp Model:- By substituting the above numerical values in equations (4.6), (4.9) and (4.17) results obtained as

\[
\frac{d^2T_{\text{avg}}}{dt_1^2} > 0 \Rightarrow S^* = S = 13.865 \quad D^* = D = 0.236 \quad T_{\text{avg}}^* = 989.96
\]

For Fuzzy Model:—

\[
\theta = (0.077, 0.079, 0.081, 0.083), \ C_d = (1.47, 1.49, 1.51, 1.53),
\]

\[
C_s = (1.7, 1.95, 2.1, 2.3), \ C_h = (0.47, 0.49, 0.51, 0.53),
\]

\[
t = (0.37, 0.39, 0.41, 0.43) T = 1, \alpha = 0.5 \text{and} \beta = 0.011
\]

We have

\[
S_1^* = 12.517, \ S_2^* = 13.409, \ S_3^* = 14.328, \ S_4^* = 15.272
\]

\[
S^* = 13.877
\]

\[
D_1^* = 0.191, \ D_2^* = 0.222, \ D_3^* = 0.256, \ D_4^* = 0.294
\]

\[
\frac{D_1^* + 2D_2^* + 2D_3^* + D_4^*}{6} = 0.24
\]

Where \( P=2.981, R=2.981, Q=7.207, L=0.208, M=3.69 \)

\[
T_{\text{avg}}^*(t) = A + \frac{1}{6}(P\theta_1 C_{d1} + QC_{x1} + RC_{i1} + L \theta_1 C_{h1} + MC_{c1})
\]

\[
+ \frac{1}{3}(P\theta_2 C_{d2} + QC_{x2} + RC_{i2} + L \theta_2 C_{h2} + MC_{c2})
\]

\[
+ \frac{1}{3}(P\theta_3 C_{d3} + QC_{x3} + RC_{i3} + L \theta_3 C_{h3} + MC_{c3})
\]

\[
+ \frac{1}{6}(P\theta_4 C_{d4} + QC_{x4} + RC_{i4} + L \theta_4 C_{h4} + MC_{c4}) = 978.145
\]

Sensitive analysis

Suppose \( C_d = (C_{d1}, C_{d2}, C_{d3}, C_{d4}), \)

\( C_s = (C_{s1}, C_{s2}, C_{s3}, C_{s4}), \)

\( C_h = (C_{h1}, C_{h2}, C_{h3}, C_{h4}), \)

on the S, Q, TDC, TSC, THC and TIC. In the analysis one
Changes in parameter is changing by –50%, –25%, 25%, and 50% remaining parameters are constant (unchanged).

| Parameter | Changes in C_d
|-----------|----------------|
| Changes in parameter | Changes in TDC | Changes in TIC |
| -50% | -0.501 | -0.00002 |
| -25% | -0.249 | 0.000009 |
| +25% | 0.249 | 0.000009 |
| +50% | 0.4976 | 0.00018 |

Table 1 shows that: Percentage of changes in the TDC and TIC when changes in \( \tilde{C}_{d_1}, \tilde{C}_{d_2}, \tilde{C}_{d_3}, \tilde{C}_{d_4} \)

| Parameter | Changes in C_s
|-----------|----------------|
| Changes in parameter | Changes in TSC | Changes in TIC |
| -50% | -0.4867 | -0.0104 |
| -25% | -0.25 | -0.0052 |
| +25% | 0.2499 | 0.0052 |
| +50% | 0.4997 | 0.0104 |

Table 2 shows that: Percentage of changes in the TSC and TIC when changes in \( \tilde{C}_{s_1}, \tilde{C}_{s_2}, \tilde{C}_{s_3}, \tilde{C}_{s_4} \)

6. Conclusion

EOQ model for quadratic demand, time dependent holding cost with deteriorating items and allowing shortages can be summarized as follows:

1. Taking the deterioration cost, holding cost, shortage cost and parameter are considered as crisp and for a given cycle, deteriorated units, initial inventory after fulfilling back order and minimum total cost are determined.

2. Considering the deterioration rate, holding cost, deterioration cost and shortage cost are considered as trapezoidal fuzzy number. Defuzzyfied by Graded Mean Integration Representation method and for a given cycle and obtained deteriorated units, initial inventory after fulfilling back order. Total inventory cost is minimized.

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