Mirror Majorana zero modes in spinful superconductors/superfluids,
-Non-Abelian braiding of integer quantum vortices-

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Abstract

It has been widely believed that half quantum vortices are indispensable to realize topological stable Majorana zero modes and non-Abelian anyons in spinful superconductors/superfluids. Contrary to this wisdom, we here demonstrate that integer quantum vortices in spinful superconductors can host topologically stable Majorana zero modes because of the mirror symmetry. The symmetry protected Majorana fermions may exhibit non-Abelian anyon braiding.

Keywords:
Topological Superconductivity, \textsuperscript{3}He-A, Edge States, Majorana Fermions

1. Introduction

Unconventional superconductors/superfluids often support gapless states on the boundaries. These states are called the Andreev bound states, which give unique transport phenomena through the surface \cite{1}. While their properties had been studied traditionally by solving the Bogoliubov de-Gennes equation for each unconventional superconductor/superfluid, a recent progress on condensed matter physics reveals that a non-trivial topology of the ground state is a profound origin of the surface gapless states. The view point of topological superconductors/superfluids gives us a universal description of the Andreev bound states as topologically protected states \cite{1,2,3,4,5}.

Remarkably, the topological protected surface states can be Majorana fermions in topological superconductors/superfluids \cite{6,7}. A Majorana fermion is a Dirac fermion with the self-conjugate condition. For instance, in a vortex core of a spinless $p+ip$ superconductor, there is a Majorana zero-energy mode $\gamma_0$ satisfying $\gamma_0^\dagger = \gamma_0$ \cite{8}. This exotic excitation changes the statistics of the vortices drastically. Indeed, a vortex with a Majorana zero mode obeys the non-Abelian anyon statistics: the braiding of vortices with Majorana zero modes gives rise to the superposition of the degenerate many-body ground states. Owing to this entangled character, non-Abelian anyons can be utilized for the construction of fault-tolerant quantum computers \cite{8}.

In addition to the spinless $p+ip$ superconductor, an $s$-wave superconductor can support the non-Abelian anyon if the spin-orbit interaction is taken into account. This possibility was first considered in the context of high energy physics \cite{9}, but the solid state realization was given in a surface of a topological insulator \cite{10}. Also, a simpler scheme using the Rashba spin-orbit interaction and the Zeeman field has been shown to support Majorana fermions \cite{11,12,13}. The essence of these schemes is an effective realization of spinless systems: In both cases, the spin and the momentum of the normal state is locked by the spin-orbit interaction, and thus a spinless superconductor is realized effectively. The latter scheme has been generalized to one-dimensional nanowire systems \cite{14,15,16,17,18,19}.

In contrast to spinless superconductors, non-Abelian anyons in spinful superconductor have been rarely discussed. An exception is a half quantum vortex in a spinful chiral $p+ip$ superconductor \cite{21,22}, but the configuration is rather unstable \cite{21}, and its realization is a hard task. Note that statistics of vortices having multiple Majorana fermions has also been discussed in Refs. \cite{24,25}.

In this paper, we address the problem of non-Abelian anyons in spinful superconductor/superfluids. We present a general argument that an integer quantum vortex of a spinful superconductor/superfluid can support topologically stable Majorana fermions if the mirror symmetry is preserved. As a concrete example, we show that there exists a pair of Majorana zero modes in an integer quantum vortex of two dimensional $^3$He-A phase under a perpendicular magnetic field. From the Majorana zero modes protected by the mirror symmetry, the integer quantum vortex obeys the non-Abelian anyon statistics. We also discuss briefly Majorana fermions in other spinful unconventional superconductors/superfluids.

2. Mirror Topological Phase in Spinful Superconductors/Superfluids

We begin by discussing the topological property of a thin film of chiral $p$-wave superconductors/superfluids. For simplicity, we ignore the thickness of the film, and treat the film as a purely two-dimensional system. The topological property can be captured by the two dimensional Bogoliubov de-Gennes (BdG) Hamiltonian in the momentum space $\mathbf{k} = (k_x, k_y)$,

\[
\mathcal{H}(k) = \begin{pmatrix} \epsilon(k) & \Delta(k) \\ \Delta^*(k) & -\epsilon(-k) \end{pmatrix}.
\]

Preprint submitted to Physica E
where $\epsilon(k)$ is the single-particle Hamiltonian given by

$$\epsilon(k) = \frac{1}{2m} k^2 - \mu - \hbar v_{\text{rel}} \sigma_\mu,$$

(2)

with the Zeeman field $\hbar v_{\text{rel}}$ and the Pauli matrices $\sigma_\mu$ and $\Delta(k) = \hbar d(k) \cdot \sigma_\mu$ is the gap function. The $d$-vector $d(k)$ of a chiral $p$-wave superconductor/superfluid is given by

$$d(k) = R_{\mu\nu}(\Delta, \hat{k}_x + i \Delta, \hat{k}_y),$$

(3)

where $R_{\mu\nu}$ is an SO(3) rotation matrix in the spin space.

As a time-reversal breaking gapped system, the above system is topologically characterized by the Chern number $v_{\text{Ch}}$ like a quantum Hall state: From the negative energy states $|u_n(k)\rangle$ ($n = 1, 2$) of the BdG Hamiltonian Eq.(13), the gauge field $A_\mu(k)$ in the momentum space can be introduced as

$$A_\mu(k) = i \sum_{n=1,2} (u_n(k)[\partial_k u_n(k)],$$

(4)

then the Chern number is defined as

$$v_{\text{Ch}} = \frac{1}{2\pi} \int d{k_x} d{k_y} [\partial_{k_x} A_y(k) - \partial_{k_y} A_x(k)].$$

(5)

Taking into account the spin degrees of freedom, one can show that $|v_{\text{Ch}}| = 2$ for the two dimensional spinful chiral $p$-wave superconductor/superfluid.

Since $v_{\text{Ch}}$ is an even number, the system hosts a pair of gapless fermions at boundaries. In addition, the superconductivity/superfluidity implies that the gapless states should be Majorana fermions. Nevertheless, the doubling of the Majorana fermion obscures the Majorana character. Indeed, a pair of Majorana fermions can be combined into a single Dirac fermion, and thus the physics of the topological phase can be described without using Majorana fermions explicitly.

The doubling problem can be avoided if one consider a special vortex configuration called half quantum vortex. In this configuration, a spinless vortex supporting a single Majorana fermion is realized effectively. As a result, the Majorana character survives. In particular, a half quantum vortex obeys the non-Abelian anyon statistics, where degenerate quantum states can be manipulated by exchange of the vortices [21]. The exotic non-Abelian anyon statistics is of particular interest in the context of realization of topological quantum computation.

While the Majorana character can be sustained in a half quantum vortex, the realization of a half quantum vortex in an actual system is not obvious. Because of twisting in the spin space, there is an attractive force between half quantum vortices. This makes a half quantum vortex unstable. Indeed, due to the attractive force, a pair of half quantum vortices easily collapse into a single integer quantum vortex.

In the following, we discuss another way to circumvent the doubling problem. Our idea is to use symmetry of the system to keep the Majorana character. This idea is somehow similar to that of time-reversal invariant topological superconductors. In a time-reversal invariant superconductor, Majorana fermions appear in a pair, but because of the time-reversal invariance, they form a Kramers pair that are not scattered by each other. As a result, each Majorana fermion can behave independently like a single Majorana, and thus it can maintain most of the Majorana character including the non-Abelian nature [26]. While the same argument cannot apply in our case since the vortex configuration breaks the time-reversal invariance, we can alternatively use the mirror reflection symmetry with respect to the $xy$-plane to sustain the Majorana character, following a recent idea of symmetry protected Majorana fermions [27, 28, 29, 30, 31].

The mirror reflection with respect to the $xy$-plane is defined as the following transformation of the momentum and the spin variables

$$k_x \rightarrow k_x, \quad k_y \rightarrow k_y, \quad k_z \rightarrow -k_z,$$

$$\sigma_x \rightarrow -\sigma_x, \quad \sigma_y \rightarrow -\sigma_y, \quad \sigma_z \rightarrow \sigma_z.$$

(6)

In two dimensions, only the spin variables transforms under the mirror reflection since the system does not depend on $k_z$. The mirror reflection operator in the spin space $M_{\text{xy}}$ is simply given by $M_{\text{xy}} = i\sigma_y$.

In the particle-hole space in the BdG Hamiltonian [13], the mirror operator is naturally extended as

$$M_{\text{xy}} = e^{i\theta} \begin{pmatrix} 0 & -e^{-i\theta} M_{\text{xy}}^\dagger \\ e^{-i\theta} M_{\text{xy}} & 0 \end{pmatrix},$$

(7)

by taking into account the $U(1)$ gauge symmetry $e^{i\theta}$ and the overall phase ambiguity $e^{i\theta}$. The BdG Hamiltonian Eq.(1) is invariant under the mirror reflection, i.e. $M_{\text{xy}} H(k) M_{\text{xy}} = H(k)$, if the following relations hold

$$M_{\text{xy}} \epsilon(k) M_{\text{xy}}^\dagger = \epsilon(k), \quad e^{2i\theta} M_{\text{xy}} \Delta(k) M_{\text{xy}}^\dagger = \Delta(k).$$

(8)

From $M_{\text{xy}}^2 = -1$, the latter equation in Eq.(8) yields $e^{4i\theta} = 1$. Therefore, the BdG Hamiltonian [13] is invariant under the mirror reflection only when $\Delta(k)$ has a definite parity under the mirror reflection as

$$M_{\text{xy}} \Delta(k) M_{\text{xy}}^\dagger = \pm \Delta(k).$$

(9)

Corresponding to the two possible parity of Eq.(2), we have two possible mirror operators $M_{\text{xy}}^\pm$

$$M_{\text{xy}}^\pm = \begin{pmatrix} M_{\text{xy}} & 0 \\ 0 & \pm M_{\text{xy}}^\dagger \end{pmatrix},$$

(10)

where the overall phase $e^{i\theta}$ is chosen so as to be $M_{\text{xy}}^2 = -1$. For example, an $s$-wave gap function, $\Delta(k) = i\hbar \sigma_y$, has even mirror parity as $M_{\text{xy}} \Delta(k) M_{\text{xy}}^\dagger = \Delta(k)$, thus the extended mirror operator is $M_{\text{xy}}^+$. For a two dimensional spin-triplet gap function, the mirror symmetry is preserved either when the $d$-vector is parallel to the $z$-direction or when the $d$-vector is normal to the $z$-direction. The gap function in the former case has even mirror parity, while the gap function in the latter has odd mirror parity. Therefore, the extended mirror operator is given as $M_{\text{xy}}^+$ in the former case, but it is $M_{\text{xy}}^-$ in the latter.

When the BdG Hamiltonian keeps the mirror reflection symmetry, one can introduce a novel topological number called mirror Chern number. Since the mirror invariant BdG Hamiltonian commutes with the mirror operator, it can be block diagonal by

\[
\begin{bmatrix}
H_{\text{even}} & 0 \\
0 & H_{\text{odd}}
\end{bmatrix}
\]
using the eigen values of the mirror operator. For each subsector which has a definite eigen value of \( M_{\text{sym}} \), the gauge field \( \mathcal{A}_\lambda^i \) in the momentum space is defined as

\[
\mathcal{A}_\lambda^i(k) = i \sum_n (u_{n,\lambda}(k) \partial_k u_{n,\lambda}(k)),
\]

where the summation is taken for occupied states \( |u_{n,\lambda}(k)\rangle \) of the BdG Hamiltonian with the eigen value \( \lambda = \pm \) of \( M_{\text{sym}} \). Then, the mirror Chern number \( \nu(\lambda) \) is given by

\[
\nu(\lambda) = \frac{1}{2\pi} \int d^2k \mathcal{F}_i^\lambda,
\]

where \( \mathcal{F}_i^\lambda = \partial_k u_{n,\lambda}^\dagger \partial_k u_{n,\lambda} \) is the field strength of \( \mathcal{A}_\lambda^i \). When the mirror Chern number is non-zero, we have topologically protected gapless edge states in a manner similar to topological crystalline insulators \[32, 33\].

Here we should emphasize that the corresponding gapless edge states can be Majorana only for \( M_{\text{sym}} \) while the mirror Chern number can be defined for both of the two possible mirror symmetries \( M_{\text{sym}}^\dagger \). This is because only the mirror subsector for \( M_{\text{sym}} \) supports its own the particle-hole symmetry. In contrast, for \( M_{\text{sym}}^\dagger \), the mirror subsector does not keep the particle-hole symmetry while the whole system does. From this difference, we find that the former case has a different topological characterization than that of the latter. In terms of the topological table \[34, 35\], the mirror subsector for \( M_{\text{sym}} \) belongs to class D like a spinless chiral superconductors, while the mirror subsector for \( M_{\text{sym}}^\dagger \) belongs to class A like a quantum Hall state. This means that the mirror symmetry protected Majorana fermions are possible only for the former case. In the latter case, only Dirac fermions can be obtained.

For \( M_{\text{sym}} \), we also find that a conventional integer quantum vortex can support a Majorana zero mode if the mirror Chern number is odd. Because the spin degrees of freedom in each mirror subsector is locked as an eigenstate of the mirror operator, the mirror subsectors realize spinless systems effectively. This means that an integer quantum vortex can support Majorana zero mode as is the case of spinless chiral \( p \)-wave superconductors.

For the two dimensional \( ^3 \)He-A phase, by applying the Zeeman magnetic field in the \( z \)-direction, we can align the \( d \)-vector normal to the \( z \)-direction. Under this situation, the gap function is odd under the mirror reflection, and thus the BdG Hamiltonian has the mirror symmetry \( M_{\text{sym}} \). It is also found that the mirror Chern number of the \( \text{\(^3\)He-A} \) phase is odd. Thus the above argument implies that an integer vortex supports a Majorana zero mode in each mirror subsector. This result will be confirmed numerically in Sec. 3.

### 3. Application to \(^3\)He-A Phase

In this section, we clarify low-lying quasiparticles in an integer quantum vortex state of \( ^3 \)He-A under a magnetic field. Quasiparticles with the wave function \( \varphi_{\text{\(^3\)He-A}} = (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})^T \) and the energy \( E_n \) is described by the BdG equation \[36, 37, 22\],

\[
\int dr_2 \left( \begin{array}{cc} \epsilon(r_1, r_2) & \Delta(r_1, r_2) \\ -\Delta^*(r_1, r_2) & -\epsilon(r_1, r_2) \end{array} \right) \varphi_n(r_2) = E_n \varphi_n(r_1).
\]

Here, the single-particle Hamiltonian \( \epsilon(r_1, r_2) \) is given by replacing \( k \) in Eq. \( 2 \) to \( -\nabla \).

The pair potential for an axisymmetric vortex state of the superfluid \( ^3 \)He-A is expressed in terms of the \( d \)-vector as \( \Delta(k, r) = \iota c_{\mu,\uparrow} c_{\mu,\downarrow} d_{\mu}(k, r) \equiv \int d^3r_1 \Delta(r_1, r_2) e^{i k_1 \cdot r_1} \), where we introduce the relative and center-of-mass coordinates, \( r_{12} = r_1 - r_2 \) and \( r = (r_1 + r_2)/2 \). The \( d \)-vector for an integer vortex state with a winding number \( w \in \mathbb{Z} \) is given in the cylindrical coordinate \( r = (\rho, \phi, z) \) as

\[
d_{\mu}(k, r) = e^{i w \phi} R_{\mu 3}(\Delta_s(\rho) \hat{\kappa}_z + i z \Delta_s(\rho) \hat{\kappa}_x).
\]

Within the orbit parameter ansatz having an axisymmetric and straight vortex line described in Eq. \[14\], the quasiparticle wave function \( \varphi_{\mu}(r) \) is expressed in terms of the eigenstates of the angular momentum \( \ell \) and axial momentum \( q \),

\[
\varphi_{\mu}(r) = e^{i\ell \phi} e^{i q z} v_{\mu,\ell,q}(\rho),
\]

where \( \ell \in \mathbb{Z} \) and \( \sigma = \pm 1 \).

Note that the BdG equation \[13\] holds the particle-hole symmetry. Within the ansatz in Eq. \[14\], the positive energy eigenstates of Eq. \[13\] with \( \varphi_{\mu,\ell,q}(\rho) \) and \( E_{\mu,\ell,q} \) correspond to the negative energy eigenstates with \( \tau_{\nu} \varphi_{\mu_{-\ell-\nu},-q}(\rho) \) and \( -E_{\mu_{-\ell-\nu},-q} \). Here, \( \tau_{\nu} \) denotes the Pauli matrix in the particle-hole space. Hence, the particle-hole symmetry gives the condition on zero energy solutions that a pair of zero energy solutions may exist at \( \ell = (w + 1)/2 \) when the vortex winding number \( w \) is odd. The wave function must satisfy the relation, \( u_{n,\ell,q}(\rho) = v_{n,\ell,-q}^{(\sigma)}(\rho) \), implying that the zero energy quasiparticle is composed of the equivalent contribution from the particle and hole components.

To numerically solve the BdG equation \[13\] under the ansatz in Eq. \[14\], the quasiparticle wave function \( \varphi_{\mu,\ell,q}(\rho) \) is expanded with the orthonormal functions associated with the Bessel function \[38, 39\]. The Bessel function expansion imposes the rigid boundary condition on the wave function, \( \varphi_{\mu,\ell,q}(\rho = R) = 0 \) with the radius of the system \( R \). Therefore, low-lying quasiparticles bound at the circumference (edge) of the cylinder may exist in addition to core-bounded states \[36, 37, 40, 41, 42, 43, 44, 45\].

In our numerical calculation, we set \( R = 10\xi \) and \( \mu = E_{\xi} \), where \( \xi = v_F/\hbar \) and \( E_{\xi} \) denote the coherence length and the Fermi energy, respectively.

Figure \ref{fig:1} shows the quasiparticle energy spectra with respect to the azimuthal quantum number \( \ell \) in the integer vortex state, where the energy eigenstates with only \( q = 0 \) are displayed. Throughout this section, we fix the vortex winding number to be \( w = -1 \), implying that the vorticity is anti-parallel to the chirality of Cooper pairs. The magnetic field is applied along the vortex line, that is, the \( z \)-axis, \( \hbar = (0, 0, h) \), where \( h = 0.1\Delta_0 \) is fixed. We here introduce the relative angle \( \theta_0 \) between the applied magnetic field \( \hbar \parallel z \) and the orientation of the \( d \)-vector, which is associated with the SO(3) matrix \( R_{\mu 3} \) in Eq. \[14\].

The quasiparticle spectrum for \( \theta_0 = \pi/2 \), which corresponds to the situation of \( \hbar \parallel z \perp d \), is displayed in Fig. \ref{fig:1}a. The low-lying spectrum is composed of the edge- and core-bounded states, where the former gives rise to the spontaneous mass flow.
In Fig. 2, we display the local density of states \( \mathcal{N}(\rho, E) \) at the core \( \rho = 0 \) of the integer vortex state for various \( \theta_d \)’s, where \( h = 0.1\Delta_0 \).

4. Non-Abelian Braiding of Integer Quantum Vortices

In the previous sections, we found that an integer quantum vortex may support a pair of Majorana zero modes in the core. Now consider physical consequences of the Majorana zero modes.

An immediate consequence of the Majorana zero modes is the non-Abelian statistics. This is easily understood if we consider the system as a set of mirror subsectors. As was shown in the previous section, each mirror subsector effectively realizes a spinless system that supports a single Majorana zero mode in a vortex. Therefore, the integer vortices in each mirror subsector obey the non-Abelian anyon statistics like vortices in a spinless chiral superconductor. Here note that no interference between the mirror subsectors occurs during a vortex exchange process since this process does not break the mirror symmetry. Therefore, even when we put the mirror subsectors together and consider the whole of the system, the integer quantum vortices remain to obey the non-Abelian anyon statistics.

Now we show the non-Abelian anyon statistics more concretely. Consider \( 2N \) integer quantum vortices. For the \( i \)-th integer quantum vortex, we have two Majorana zero modes \( \gamma^i_1 \) corresponding to two possible eigenvalues \( \lambda = \pm i \) of the mirror operator. The Majorana zero modes satisfy the self-conjugate condition \( \gamma^i_1 \gamma^j_1 = -\delta_{ij} \). In a manner similar to vortices in a spinless chiral superconductor [21], when the \( i \)-th vortex and the \( i+1 \) vortex are exchanged, the zero modes behave as

\[
\gamma^i_1 \rightarrow \gamma^i_{i+1}, \quad \gamma^i_{i+1} \rightarrow -\gamma^i_1.
\]
The above transformation is realized by the unitary operator,
\[ \tau_i = \exp \left( \frac{\pi i}{4} \sum_j \gamma^i_j \gamma^j_i \right) = \frac{1}{2} \prod_j (1 + \gamma^i_j \gamma^j_i), \] (21)
which leads
\[ \tau_i \gamma^i_j \tau^{-1}_j = \gamma^i_j, \quad \tau_j \gamma^j_i \tau^{-1}_i = -\gamma^j_i, \] (22)
One can easily find that the exchange operators \( \tau_i \) and \( \tau_j \) do not commute with each other when \( |i - j| = 1 \). This implies the non-Abelian anyon statistics of the integer quantum vortices.

On the contrary to a half quantum vortex, one can introduce a Dirac operator localized on an integer quantum vortex in our system. Indeed, since the integer quantum vortex supports a pair of Majorana zero modes, a Dirac operator \( \psi_i \) localized in the \( i \)-th vortex is defined as
\[ \psi_i = \frac{1}{2} (\gamma^i_j + i \gamma^j_i)^{-i-j}, \] (23)
which satisfies \( \langle \psi_i | \psi_j \rangle = \delta_{ij} \). As was discussed in Refs. 24, 25, the Dirac operators give another expression of the non-Abelian exchange operator \( \tau_i \) as
\[ \tau_i = 1 + \psi_{i+1} \psi_i^\dagger + \psi_{i+1}^\dagger \psi_i - \psi_{i+1}^\dagger \psi_{i+1} \psi_i^\dagger \psi_i + 2 \psi_{i+1} \psi_{i+1}^\dagger \psi_{i+1}^\dagger \psi_i. \] (24)

The above expression implies that the vortex exchange process preserves the fermion number \( N = \sum_i \psi_i^\dagger \psi_i \). We find that the conservation of the fermion number gives alternative and simple interpretation of the non-Abelian anyon statistics for integer quantum vortices: For the Fock vacuum \( |0\rangle \) of the Dirac operators, a vortex \( i \) with the Dirac zero mode, \( \psi_i |0\rangle \equiv |1\rangle \), has non zero fermion number while a vortex \( i \) without the Dirac zero mode, \( |0\rangle \) does not. This means that we can distinguish these two vortex states, \( |1\rangle \) and \( |0\rangle \), by the fermion number. Considering them as different particles, we have the non-Abelian anyon statistics naturally. For example, let us consider a four vortex state \(|1100\rangle\) where the first and the second vortices are accompanied by the Dirac zero modes, while the third and the fourth are not. Up to a phase factor, this state changes under \( \tau_1 \) and \( \tau_2 \) as
\[ |1100\rangle \xrightarrow{\tau_1} |1100\rangle \xrightarrow{\tau_2} |1010\rangle, \] (25)
while it changes under \( \tau_3 \) and \( \tau_1 \) as
\[ |1100\rangle \xrightarrow{\tau_3} |1010\rangle \xrightarrow{\tau_1} |0110\rangle. \] (26)
Since \( |1\rangle \) and \( |0\rangle \) can be considered as different particles, these final states are different from each other. Therefore, we have \( \tau_2 \tau_1 \neq \tau_1 \tau_2 \) naturally.

In real systems, the mirror symmetry is easily broken locally by disorders or ripples. However, recent studies have suggested that the symmetry protection is rather robust if the symmetry is preserved macroscopically 47, 48, 49, 50. Indeed, we can argue that the non-Abelian anyon statistics persists if the local breaking is weak and the mirror symmetry is preserved on average: Although the local breaking effects may lift locally the degeneracy between two possible vortex states \( |0\rangle \) and \( |1\rangle \), the degeneracy is recovered on average. More importantly, because the fermion parity is preserved in a superconductor/superfluid, no transition between \( |0\rangle \) and \( |1\rangle \) occurs unless a bulk quasiparticle is excited or cores of vortices are overlapped. Therefore, the above argument of the non-Abelian anyon braiding works as far as the mirror symmetry is preserved on average.

5. Summary

In this paper, we gave argued how integer quantum vortices in spinful superconductors support Majorana zero modes due to the mirror symmetry. As a concrete example, we have calculated quasiparticle states localized on an integer quantum vortex for a two dimensional \({}^3\text{He}-\text{A}\) phase, and have found that a pair of Majorana zero modes exist when the \( d \)-vector is parallel to the two dimensional surface. Due to the Majorana zero modes, the integer quantum vortices obey the non-Abelian anyon statistics.

The arguments given in this paper is applicable to many unconventional superconductors/superfluids such as \({}^3\text{He}-\text{B}\) phase, \text{Cu}_{x}\text{Bi}_{y}\text{Se}_{z}, \text{Sr}_{x}\text{RuO}_{y}, \text{UPt}_{3}, \) and so on. In particular, even if the gap function preserves the time-reversal invariance, the mirror Chern number can be nonzero. Actually, the above unconventional superconductors/superfluids have non-zero mirror Chern numbers, and thus they support Majorana fermions protected by the mirror symmetry. We will report these results elsewhere.

6. Acknowledgements

The authors are grateful to Takuto Kawakami for fruitful discussions and comments. This work was supported by Grant-in-Aid for Scientific Research from MEXT/JSPS of Japan, “Topological Quantum Phenomena” No. 22103005, No. 25287085, and No. 25800199.

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