Probing the curvature of the Universe from supernova measurement

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Abstract

We study the possibility to probe the spatial geometry of the Universe by supernova measurement. We illustrate with an accelerating universe model with infinite-volume extra dimensions, for which the 1σ level supernova results indicate that the Universe is closed.

PACS numbers: 04.50.+h, 98.80.Cq
The precision measurements of the Wilkinson Microwave Anisotropy Probe (WMAP) have provided high resolution Cosmic Microwave Background (CMB) data \cite{1,2} and elevated cosmology to a new maturity. Among interesting conclusions that have been reached from these data, the WMAP results indicate that while flatness of the Universe is confirmed to a spectacular precision on all but the largest scales \cite{1}, a closed universe with positively curved space is marginally preferred \cite{3,4,5,6,7}. This tendency of preferring closed universe is not restricted to the WMAP data, it appeared in a suite of CMB experiments before \cite{8,9,10}. The improved precision from WMAP provides further confidence.

In addition to CMB, recently it was argued that the cubic correction to the Hubble law measured with high-redshift supernovae is another cosmological measurement that probes directly the spatial curvature \cite{11}. This is the first non-CMB probe of the spatial geometry, which can provide a cross-check to the result got by CMB. In a toy model, it was already found that a curvature radius is larger than the Hubble distance \cite{11}.

Our Universe is accelerating rather than decelerating. This may be regarded as the evidence for a nonzero but very small cosmological constant (see \cite{12} for a review and related references in \cite{13}). Another possibility is that the phenomenon of accelerated expansion is caused by a breakdown of the standard Friedmann equation due to the extra-dimensional contribution \cite{14,15,16,17}. Studies on this possibility can also be found in \cite{18}. In this work we will consider the accelerated universe model resulted from the gravitational leakage into extra dimensions \cite{16}. We will attempt to extract information from the full redshift data to test the spatial geometry.

Consider the accelerating universe described by the model with infinite-volume extra dimensions \cite{16}, the Friedmann equation is expressed as

\[
H^2 + \frac{k}{a^2} = \left\{ \sqrt{\frac{\rho}{3M_p^2}} + \frac{1}{4r_c^2} + \frac{1}{2r_c} \right\}^2,
\]

where \(\rho\) is the total cosmic fluid energy density and \(r_c\) is the crossover scale. Eq. (1) can also be recasted in terms of the redshift as

\[
H^2 = H_0^2 \left\{ -\Omega_k (1+z)^2 + \left[ \sqrt{\Omega_{r_c}} + \sqrt{\Omega_M} + \Omega_M (1+z)^3 \right]^2 \right\},
\]

where \(\Omega_{r_c} = \frac{\Omega_{r_c}}{3M_p^2 H_0^2}\), \(\Omega_M\) is the non-relativistic matter density. The conservation for energy-momentum tensor of the cosmic fluid is still described by

\[
\dot{\rho} + 3H(\rho + P) = 0.
\]
Using definitions $q_0 = -\frac{\ddot{a}}{a^2}|_0$, $j_0 = \frac{\dddot{a}}{a^2}|_0$ with dot denoting the differentiation with respect to time $t$ for the deceleration parameter and the “jerk”, respectively, we have directly from equation (2)

$$q_0 = -(1 + \Omega_{k0}) + \frac{3\Omega_M \sqrt{1 + \Omega_{k0}}}{2\sqrt{\Omega_{r0} + \Omega_M}},$$

$$j_0 = (1 + \Omega_{k0}) - \frac{9\Omega_M^2 \sqrt{\Omega_{r0}}}{4(\Omega_{r0} + \Omega_M)^{3/2}},$$

where the normalization of (2) at the present epoch

$$\Omega_{r0} = \frac{(1 + \Omega_{k0} - \Omega_M)^2}{4(1 + \Omega_{k0})^2}$$

has been employed. With (6), $q_0$ and $j_0$ are only determined by $\Omega_M$ and $\Omega_{k0}$.

The physically reasonable cosmic model has the following requirements [19]: (1) the total density is currently not increasing as a function of time; (2) for causality and stability, the present sound speed $c_s$ of the total system satisfies $0 \leq c_s^2 \leq 1$.

Employing (1) and (3), the variation of the total cosmic fluid energy density and the sound speed of the total cosmic fluid at the present epoch are

$$\dot{\rho}|_0 = -6M_p^2 H_0^3 (1 + q_0 + \Omega_{k0}) \left[1 - \frac{\sqrt{\Omega_{r0}}}{\sqrt{1 + \Omega_{k0}}} \right],$$

$$c_s^2 = \frac{\dot{\rho}}{\rho}|_0 = \frac{(j_0 - 1 - \Omega_{k0})(1 - \sqrt{\Omega_{r0}/\sqrt{1 + \Omega_{k0}}}) + (q_0 + 1 + \Omega_{k0})^2 \sqrt{\Omega_{r0}/(1 + \Omega_{k0})^{3/2}}}{3(1 + q_0 + \Omega_{k0})[1 - \sqrt{\Omega_{r0}/\sqrt{1 + \Omega_{k0}}}]}. \quad (8)$$

The first requirement implies

$$(1 + q_0 + \Omega_{k0})(1 - \frac{\sqrt{\Omega_{r0}}}{\sqrt{1 + \Omega_{k0}}}) \geq 0. \quad (9)$$

Employing (6) and the fact that $|\Omega_{k0}| \leq 0.1$ as a consequence of CMB data, the above requirement reduces to

$$1 + q_0 + \Omega_{k0} \geq 0. \quad (10)$$

Using (4), we see that Eq. (10) can obviously be satisfied.

The second requirement can now be written in a simplified form as

$$f_1 \leq j_0 \leq f_2, \quad (11)$$
where \( f_1 = (1+\Omega_k_0) - \frac{(1+\Omega_k_0-\Omega_M)(q_0+1+\Omega_k_0)^2}{(1+3\Omega_k_0)(1+\Omega_k_0+\Omega_M)} \) and \( f_2 = 4(1+\Omega_M) + 3q_0 - \frac{(1+\Omega_k_0-\Omega_M)(q_0+1+\Omega_k_0)^2}{(1+3\Omega_k_0)(1+\Omega_k_0+\Omega_M)} \).

Substituting Eqs. (4) and (6) into the expression of \( f_1 \), we find that \( j_0 = f_1 \), which means that the sound speed of the total system in this model is exactly zero.

We now turn to determine the cosmological density parameters from the supernova (SN) Ia data compiled by Riess et al. [20]. The likelihood for the parameters \( \Omega_M \) and \( \Omega_k_0 \) can be obtained from a \( \chi^2 \) statistics [20, 21], where

\[
\chi^2(H_0, \Omega_M, \Omega_k_0) = \sum_i \frac{[\mu_{p, i}(z, H_0, \Omega_k_0, \Omega_M) - \mu_{o, i}]^2}{\sigma_i^2}, \tag{12}
\]

\( \mu_p = 5 \log_{10}(d_L/Mpc) + 25 \) and \( \mu_o \) are distance modulus for the model and the observations, respectively. \( d_L \) is the luminosity distance defined for the Friedmann-Robertson-Walker universe model as

\[
d_L = a_0(1 + z)r_1 \tag{13}
\]

\[
= \begin{cases} 
  a_0(1 + z) \sin[\frac{1}{a_0H_0} \int_0^z \frac{dz'}{E(z')}], & \text{closed} \\
  \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, & \text{flat} \\
  a_0(1 + z) \sinh[\frac{1}{a_0H_0} \int_0^z \frac{dz'}{E(z')}], & \text{open}
\end{cases}
\]

for closed, flat and open universes respectively. The function \( E(z) \) quantifies the expansion rate as a function of redshift defined as \( H(z) = H_0 E(z) \). \( \sigma_i \) in (12) is the total uncertainty in the observation. Marginalizing our likelihood function over the nuisance parameter \( H_0 \) by integrating the likelihood function \( L = \exp(-\chi^2/2) \) over all possible values of \( H_0 \) with a flat prior assumption on \( H_0 \), yields the confidence intervals shown in Fig. 1 by combining Eqs. (2), (6), (12) and (13).

Using the contour \( \Omega_k_0, \Omega_M \) values, we can get the corresponding \( q_0, j_0 \) and \( f_1 \) as plotted in Fig. 2. Note that the contours shown here are from the gold sample SN Ia data compiled in [20].

Lines added in Fig.2 show the second requirement for a reasonable cosmic model, \( j_0 = f_1 \), with \( \Omega_M \) varying in the range [0.2-0.4] and different \( \Omega_k_0 \) for open, flat and closed universes, respectively. It is clear that in the 2\( \sigma \) level, there are only overlaps with the supernova data for \( \Omega_k_0 > 0 \). This corresponds to say that the data favors the closed universe almost at 2\( \sigma \) level.

To obtain tighter constraints on the parameter space, we also include constrains from
Figure 1: The 1σ, 2σ and 3σ confidence contours for Ω_M and Ω_k0 with the prior Ω_M = 0.3 ± 0.04.

combined WMAP data and SN Ia data. We minimize
\[ \chi^2 = \sum_i \frac{[\mu_{p,i}(z_i, H_0, \Omega_k0, \Omega_M) - \mu_{o,i}(z_i)]^2}{\sigma_i^2} + \frac{[R_p(\Omega_k0, \Omega_M) - R_o]^2}{\sigma_R^2}, \]

where σ_R is the uncertainty in R, the CMB shift parameter \( R \equiv \Omega_M^{1/2} H_0 r_1(z_b) = 1.710 \pm 0.137 \) and \( z_b = 1089 \pm 1 \). The results are shown in Fig.3. The combined constraints give \( \Omega_M = 0.25^{+0.05}_{-0.04} \) and \( \Omega_k0 = 0.01^{+0.09}_{-0.08} \). This shows that in the absence of positive spatial curvature, Ω_M tends to take a smaller value. It implies that from the observed Ω_M around 0.3, we should have the positive curvature.

From the SN Ia data, Ω_r_c is constrained to be 0.23 ([0.18, 0.28] in 1σ region; [0.14, 0.31] in 2σ region and [0.1, 0.33] in 3σ region); combined with CMB, we have tighter constraint, \( \Omega_r_c = 0.14 \) ([0.12, 0.16] in 1σ region; [0.11, 0.17] in 2σ region and [0.10, 0.18] in 3σ region). The corresponding crossover scale \( r_c = 1.04 H_0^{-1} \) from supernova data and \( r_c = 1.34 H_0^{-1} \) from combined CMB and SN Ia data. This constrained parameter is in good agreement with the result comes from lunar laser ranging experiments that monitor the moon's perihelion procession with a great accuracy.

In summary, we have probed the geometry of a specific model describing the accelerating
universe by using the full redshift data in supernova measurements. To almost 2σ level, our result indicates that the universe is closed. This result is also favored by including WMAP data constraint, which agrees to a suite of CMB experiments. The result obtained is consistent with the interpretation from other models, e.g. the matter plus cosmological constant case, that the Riess et al. data show a tendency towards a closed universe. Of course it is too early to draw conclusions just on 2σ level data, and we expect that future supernova measurements can determine the spatial curvature precisely.

ACKNOWLEDGEMENT: This work was partly supported by NNSF, China, Ministry of Science and Technology of China under Grant No. NKBRSFG19990754 and Ministry of Education of China. Y. Gong’s work was supported by Chongqing University of Post and Telecommunication under grant Nos. A2003-54 and A2004-05. R. K. Su would like to acknowledge National Basic Research Program of China 2003CB716300. B. Wang thanks
Figure 3: The $1\sigma$, $2\sigma$ and $3\sigma$ confidence contours for $\Omega_M$ and $\Omega_{k0}$ by combining the CMB data and supernova data with the prior $\Omega_M = 0.3 \pm 0.04$.

helpful discussions with Prof. E. Abdalla.

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