Moduli space of $G$-connections on an elliptic curve

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Abstract Let $X$ be a smooth complex elliptic curve and $G$ a connected reductive affine algebraic group defined over $\mathbb{C}$. Let $\mathcal{M}_X(G)$ denote the moduli space of topologically trivial algebraic $G$-connections on $X$, that is, pairs of the form $(E_G, D)$, where $E_G$ is a topologically trivial algebraic principal $G$-bundle on $X$ and $D$ is an algebraic connection on $E_G$. We prove that $\mathcal{M}_X(G)$ does not admit any nonconstant algebraic function while being biholomorphic to an affine variety.

Keywords Elliptic curve · $G$-connection · Moduli space · Riemann–Hilbert correspondence

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1 Introduction

Take an irreducible smooth complex projective curve $X$ of genus one and also take a connected reductive complex affine algebraic group $G$. The moduli space of topologically trivial algebraic principal $G$-bundles on $X$ is an extensively studied topic (see [3,5–7,9] and references therein). In particular, an explicit description of this moduli space was obtained. Our aim here is to study the moduli space of algebraic $G$-connections on $X$. More precisely, let $\mathcal{M}_X(G)$ be the moduli space pairs of the form $(E_G, D)$, where

- $E_G$ is an algebraic principal $G$-bundle on $X$ such that the underlying topological principal $G$-bundle is trivial, and
- $D$ is an algebraic connection on $E_G$. 

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(The definition of an algebraic connection on $E_G$ is recalled in Sect. 2.2). The Riemann–Hilbert correspondence, which sends any flat connection on $X$ to its monodromy representation, produces a biholomorphism from $\mathcal{M}_X(G)$ to the Betti moduli space $M^B_X(G)$ that parametrizes all equivalence classes of homomorphisms from $\pi_1(X)$ to $G$ that lie in the connected component of the trivial homomorphism (see Sect. 4.1). The moduli space $M^B_X(G)$ is an affine variety. Since $\mathcal{M}_X(G)$ is biholomorphic to $M^B_X(G)$, we conclude that there are many nonconstant holomorphic functions on $\mathcal{M}_X(G)$. This contrasts with the following theorem proved here (see Theorem 4.1):

**Theorem 1.1** The variety $\mathcal{M}_X(G)$ does not admit any nonconstant algebraic function.

Let $C$ be an irreducible smooth complex projective curve and $\mathcal{M}_C(G)$ the moduli space of $G$-connections on $C$ such that the underlying topological principal $G$-bundle is trivial. As before, $\mathcal{M}_C(G)$ is biholomorphic to an affine variety, namely the Betti moduli space. Theorem 1.1 raises the following question:

**Question 1.2** Does $\mathcal{M}_C(G)$ admit any nonconstant algebraic function?

### 2 Algebraic connections on bundles

#### 2.1 Connection on vector bundles

Let $X$ be an irreducible smooth complex projective curve of genus one. The holomorphic cotangent bundle of $X$ will be denoted by $K_X$. We note that $K_X$ is algebraically trivial.

An algebraic connection on an algebraic vector bundle $E \rightarrow X$ is a holomorphic differential operator $D : E \rightarrow E \otimes K_X$ of order one satisfying the Leibniz rule which says that for any locally defined holomorphic section $s$ of $E$ and any locally defined holomorphic function $f$ on $X$,

$$D(f \times s) = fD(s) + s \otimes df.$$  \hspace{1cm} (2.1)

Let $\delta_E : E \rightarrow E \otimes \Omega^{0,1}$ be the Dolbeault operator defining the holomorphic structure on $E$. It is straightforward to check that $D$ is an algebraic connection on $E$ if and only if $D + \delta_E$ is a flat connection on $E$ such that the locally defined (in analytic topology) flat sections of $E$ are holomorphic.

A rank $r$ connection on $X$ is a pair $(E, D)$, where $E$ is an algebraic vector bundle on $X$ of rank $r$ and $D$ is an algebraic connection on $E$.

An algebraic vector bundle $V$ on $X$ is called semistable if

$$\frac{\text{degree}(F)}{\text{rank}(F)} \leq \frac{\text{degree}(V)}{\text{rank}(V)}$$

for all algebraic subbundles $F \subset V$ of positive rank.

**Lemma 2.1** Let $(E, D)$ be a rank $r$ connection on $X$. Then, the algebraic vector bundle $E$ is semistable of degree zero.

**Proof** Since $E$ admits a flat connection it follows immediately that degree($E$) = 0.
Assume that $E$ is not semistable. Let $0 \neq F \subsetneq E$
be the first term of the Harder–Narasimhan filtration of $E$, so the algebraic vector bundle $F$
is semistable, and
\[
\frac{\deg(F)}{\rank(F)} > \frac{\deg(V)}{\rank(V)}
\]
for every algebraic subbundle $V \subset E/F$ of positive rank. Now consider the second fundamental form $S_D(F)$ for $F$, which is the following composition:
\[
F \hookrightarrow E \xrightarrow{D} E \otimes K_X \xrightarrow{} (E/F) \otimes K_X.
\]
From (2.1) it follows immediately that $S_D(F)(fs) = f S_D(F)(s)$, where $s$ is any locally defined holomorphic section of $F$ and $f$ is any locally defined holomorphic function on $X$. Therefore, $S_D(F)$ is a holomorphic homomorphism of vector bundles. Since $K_X$ is trivial, from (2.2) it follows that there is no nonzero holomorphic homomorphism of vector bundles from $F$ to $E/F$. So we conclude that $S_D(F) = 0$. Therefore, $D$ induces an algebraic connection on $F$. Since $F$ admits an algebraic connection, we have
\[
\deg(F) = 0.
\]
On the other hand, since $\deg(E) = 0$, from (2.2) it follows that
\[
\deg(F) > 0.
\]
In view of the above contradiction, we conclude that $E$ is semistable. \hfill \Box

**Lemma 2.2** Let $E$ be a semistable vector bundle on $X$ of rank $r$ and degree zero. Then, $E$
admits an algebraic connection.

**Proof** A theorem due to Atiyah and Weil says that an algebraic vector bundle $W$ over an irreducible smooth complex projective curve admits an algebraic connection if and only if the degree of any direct summand of $W$ is zero [15], [4, p. 202, Proposition 17]. Since $E$ is semistable of degree zero, every direct summand of $E$ is clearly of degree zero. Therefore, $E$ admits an algebraic connection. \hfill \Box

**2.2 Criterion for principal bundles**

Let $G$ be a connected reductive affine algebraic group defined over $\mathbb{C}$. The center of $G$ is
denoted by $Z(G)$. Let
\[
Z_0(G) \subset Z(G)
\]
be the connected component containing the identity element. The Lie algebra of $G$ will be
denoted by $\mathfrak{g}$.

An algebraic principal $G$-bundle $E_G$ over the elliptic curve $X$ is called *semistable* if for
every triple of the form $(P, E_P, \chi)$, where
- $P \subset G$ is a parabolic subgroup,
- $E_P \subset E_G$ is an algebraic reduction of structure group of $E_G$ to $P$, and
- $\chi : P \longrightarrow \mathbb{C}^*$ is a character such that $\chi|_{Z_0(G)}$ is trivial and the line bundle on $G/P$
associated to $\chi$ is ample,
the inequality degree\(E_P(\chi)\) \(\geq 0\), where \(E_P(\chi) \to X\) is the algebraic line bundle associated to \(E_P\) for the character \(\chi\) (see [1,11,12]).

For \(G = \text{GL}(r, \mathbb{C})\), the above definition of semistability of \(E_G\) is equivalent to the definition of semistability of the vector bundle of rank \(r\) associated to \(E_G\).

Given an algebraic principal \(G\)-bundle \(p : E_G \to X\), we have the Atiyah exact sequence on \(X\)

\[
0 \to \text{ad}(E_G) := E_G \times^G \mathfrak{g} \to \text{At}(E_G) := (p_*T E_G)^G \to TX \to 0
\]

(see [4]). An algebraic connection on \(E_G\) is a holomorphic (hence algebraic) splitting of the above Atiyah exact sequence.

**Proposition 2.3** An algebraic principal \(G\)-bundle \(E_G\) over \(X\) admits an algebraic connection if and only if the following two conditions hold:

1. \(E_G\) is semistable, and
2. for every character \(\chi\) of \(G\), the degree of the associated line bundle \(E_G(\chi) \to X\) is zero.

**Proof** We first recall a criterion for \(E_G\) to admit an algebraic connection. The principal \(G\)-bundle \(E_G\) admits an algebraic connection if and only if the following two conditions hold:

1. The adjoint vector bundle \(\text{ad}(E_G)\) admits an algebraic connection, and
2. for every character \(\chi\) of \(G\), the degree of the associated line bundle \(E_G(\chi) \to X\) is zero.

(See [2, p. 444, Theorem 3.1]). On the other hand, the principal \(G\)-bundle \(E_G\) is semistable if and only the vector bundle \(\text{ad}(E_G)\) is semistable [1, p. 214, Proposition 2.10]. Since any algebraic connection on \(E_G\) induces an algebraic connection on \(\text{ad}(E_G)\), the proposition follows by combining these with Lemmas 2.1 and 2.2. \(\square\)

### 3 Moduli space of connections

Let \(\mathcal{M}_X\) denote the moduli space of rank one algebraic connections on \(X\), so \(\mathcal{M}_X\) parameterizes isomorphism classes of pairs of the form \((L, D)\), where \(L\) is an algebraic line bundle on \(X\) of degree zero and \(D\) is an algebraic connection on \(L\). Let

\[
J = J(X) = \text{Pic}^0(X)
\]

be the Jacobian of \(X\). Once we fix a point \(x_0 \in X\), there is the isomorphism \(X \to J\) defined by \(x \mapsto \mathcal{O}_X(x - x_0)\). Let

\[
\varphi : \mathcal{M}_X \to J
\]

be the projection defined by \((L, D) \mapsto L\). This \(\varphi\) is a smooth surjective algebraic morphism. The space of all algebraic connections on an algebraic line bundle \(L\) is an affine space for \(H^0(X, K_X)\). Therefore, \(\varphi\) in (3.1) makes \(\mathcal{M}_X\) a torsor over \(J\) for the trivial vector bundle \(J \times H^0(X, K_X)\). Equivalently, \(\mathcal{M}_X\) is an algebraic principal bundle over \(J\) with structure group \(H^0(X, K_X)\).

Using Serre duality, \(H^0(X, K_X)\) is identified with \(\mathbb{C}\). Therefore, \(\mathcal{M}_X\) is an algebraic principal \(\mathbb{C}\)-bundle over \(J\).

**Lemma 3.1** The algebraic principal \(\mathbb{C}\)-bundle \(\mathcal{M}_X \xrightarrow{\varphi} J\) is nontrivial.
Proof Isomorphism classes of algebraic principal \( \mathbb{C} \)-bundles on \( J \) are parametrized by \( H^1(J, \Omega_J) \). Indeed, given a principal \( \mathbb{C} \)-bundle \( E \) on \( J \), choosing local trivializations (in analytic topology) of \( E \) and taking their differences, the cohomology class in \( H^1(J, \Omega_J) \) corresponding to \( E \) is constructed. We will now recall the Dolbeault analog of this construction.

Let \( \beta : E \to J \) be an algebraic principal \( \mathbb{C} \)-bundle on \( J \). Since \( \mathbb{C} \) is contractible, the fiber bundle \( E \) admits a \( C^\infty \) section. Let

\[
\gamma : J \to E
\]

be a \( C^\infty \) section, meaning \( \beta \circ \gamma = \text{Id}_J \). We note that \( \gamma \) need not be holomorphic; in fact, \( \beta \) admits a holomorphic section if and only if the algebraic principal \( \mathbb{C} \)-bundle \( E \) is trivial. Take a point \( x \in J \). Let

\[
d\gamma(x) : T^R_x J \to T^R_{\gamma(x)} E
\]

be the differential of \( \gamma \) at \( x \), where \( T^R \) denotes the real tangent space. Let \( \mathbb{J}_x \) (respectively, \( \mathbb{J}_{\gamma(x)} \)) be the almost complex structure on \( J \) (respectively, \( E \) at \( x \) (respectively, \( \gamma(x) \)). Now consider the homomorphism

\[
T^R_x J \to T^R_{\gamma(x)} E, \quad v \mapsto d\gamma(x)(\mathbb{J}_x(v)) - \mathbb{J}_{\gamma(x)}(d\gamma(x)(v)).
\]

Since the map \( \beta \) is holomorphic, this homomorphism produces a homomorphism

\[
\omega(x) : T^{0,1}_x J \to \mathbb{C};
\]

using the action of \( \mathbb{C} \) on \( E \), the vertical tangent bundle on \( E \) for the projection \( \beta \) is algebraically identified with the trivial line bundle with fiber \( \mathbb{C} \) (the Lie algebra of the group \( \mathbb{C} \)). Therefore, we get a \((0, 1)\)-form \( \omega \) on \( J \) whose evaluation at any point \( x \in J \) is \( \omega(x) \) defined above. We note that \( \omega \) is the obstruction for the map \( \gamma \) to be holomorphic. The element in \( H^1(J, \Omega_J) \) represented by \( \omega \) is the cohomology class corresponding to the algebraic principal \( \mathbb{C} \)-bundle \( E \).

Now consider the algebraic principal \( \mathbb{C} \)-bundle \( \mathcal{M}_X \) in (3.1). Any algebraic line bundle on \( X \) of degree zero admits a unique unitary flat connection. Let

\[
\gamma : J \to \mathcal{M}_X
\]

be \( C^\infty \) section defined by \( L \mapsto (L, D_L^q) \), where \( D_L^q \) is the unique unitary flat connection on \( L \). Consider the \((0, 1)\)-form \( \omega \) on \( J \) constructed as above from this section \( \gamma \). It is straightforward to check that

- \( \omega \) is invariant under the translations of the group \( J \), and
- \( \omega \) is nonzero.

These two imply that the cohomology class in \( H^1(J, \Omega_J) \) represented by \( \omega \) is nonzero. This completes the proof.

Lemma 3.2 Let \( E \) and \( F \) be two algebraic principal \( \mathbb{C} \)-bundles on \( J \) such that both \( E \) and \( F \) are nontrivial. Then, the total spaces of \( E \) and \( F \) are algebraically isomorphic.

Proof Let \( \eta : E \times \mathbb{C} \to E \) be the action of \( \mathbb{C} \) on the principal \( \mathbb{C} \)-bundle \( E \). Take any nonzero complex number \( \lambda \). Let \( E^\lambda \) be the algebraic principal \( \mathbb{C} \)-bundle on \( J \) whose total space is \( E \) but the action of any \( c \in \mathbb{C} \) on \( E \) is the morphism \( z \mapsto \eta(z, \lambda \times c) \). If \( \omega_0 \in H^1(J, \Omega_J) \) is the cohomology class corresponding to \( E \), then the cohomology class corresponding to \( E^\lambda \) is \( \omega_0/\lambda \). Since \( \dim H^1(J, \Omega_J) = 1 \), it now follows that the total spaces of any two nontrivial algebraic principal \( \mathbb{C} \)-bundles on \( J \) are algebraically isomorphic. □
Let \( 0 \rightarrow \mathcal{O}_J \rightarrow V \overset{\sigma}{\rightarrow} \mathcal{O}_J \rightarrow 0 \) (3.3)
be a nontrivial extension of \( \mathcal{O}_J \) by \( \mathcal{O}_J \); such an extension exists because \( H^1(J, \mathcal{O}_J) \neq 0 \). Let

\[ s_1 : J \rightarrow \mathcal{O}_J \]
be the section given by the constant function 1 on \( J \). Define

\[ Z := \sigma^{-1}(s_1(J)) \subset V, \]
where \( \sigma \) is the homomorphism in (3.3). Using the inclusion \( \mathcal{O}_J \hookrightarrow Z \subset V \) in (3.3) it follows that \( Z \) is an algebraic principal \( \mathbb{C} \)-bundle over \( J \).

**Proposition 3.3** The variety \( Z \) in (3.4) is isomorphic to \( \mathcal{M}_X \).

**Proof** We will show that the algebraic principal \( \mathbb{C} \)-bundle \( Z \rightarrow J \) is nontrivial. Since local trivializations of \( Z \) give local splittings of the exact sequence in (3.3), the cohomology class in \( H^1(J, \mathcal{O}_J) \) corresponding to the principal \( \mathbb{C} \)-bundle \( Z \) coincides with the cohomology class corresponding to the extension in (3.3). Since the extension in (3.3) is non-split, we conclude that the cohomology class in \( H^1(J, \mathcal{O}_J) \) corresponding to the principal \( \mathbb{C} \)-bundle \( Z \) is nonzero. Now the proposition follows from Lemmas 3.1 and 3.2. \( \square \)

**Proposition 3.4** The variety \( \mathcal{M}_X \) does not admit any nonconstant algebraic function.

**Proof** Let

\[ p : \mathbb{P} := \mathbb{P}(V) \rightarrow J \]
be the projective bundle over \( J \) associated to \( V \) in (3.3). Note that the projection \( \sigma \) in (3.3) defines a section of the projective bundle \( \mathbb{P} \) in (3.5). The image of this section will be denoted by \( S \). We have

\[ \mathbb{P} \setminus S = Z, \]
where \( Z \) is constructed in (3.4). Let \( \mathcal{O}_\mathbb{P}(1) \) denote the tautological quotient line bundle of \( p^*V \). For any \( n \geq 1 \), the tensor product \( \mathcal{O}_\mathbb{P}(1)^{\otimes n} \) will be denoted by \( \mathcal{O}_\mathbb{P}(n) \).

The restrictions of both \( \mathcal{O}_\mathbb{P}(1) \) and \( \mathcal{O}_\mathbb{P}(S) \) to any fiber of \( p \) are of degree one. Therefore, from the see–saw theorem, [10, p. 51, Corollary 6], we know that there is a line bundle \( \xi \) on \( J \) such that

\[ \mathcal{O}_\mathbb{P}(1) \otimes p^*\xi = \mathcal{O}_\mathbb{P}(S). \]

The restriction of \( \mathcal{O}_\mathbb{P}(1) \) to \( S \) is the trivial line bundle because the quotient line bundle in (3.3) is trivial. From the Poincaré adjunction formula we know that the restriction of \( \mathcal{O}_\mathbb{P}(S) \) to \( S \) is the normal bundle to \( S \) (see [8, p. 146] for the Poincaré adjunction formula). The normal bundle to \( S \) is clearly identified with the restriction to \( S \) of the relative tangent bundle for the projection \( p \) in (3.5). From (3.3), we know that the restriction to \( S \) of the relative tangent bundle for \( p \) is \( \text{Hom}(\mathcal{O}_J, \mathcal{O}_J) = \mathcal{O}_J \). Since both \( \mathcal{O}_\mathbb{P}(1)|_S \) and \( \mathcal{O}_\mathbb{P}(S)|_S \) are trivial, from (3.7) we conclude that the line bundle \( \xi \) is trivial. Hence we have

\[ \mathcal{O}_\mathbb{P}(1) = \mathcal{O}_\mathbb{P}(S). \]

So, \( \mathcal{O}_\mathbb{P}(n) = \mathcal{O}_\mathbb{P}(n \times S) \) for all \( n \geq 1 \). Therefore, the projection formula says that

\[ p_*\mathcal{O}_\mathbb{P}(n \times S) = p_*(\mathcal{O}_\mathbb{P}(n)) = \text{Sym}^n(V). \]
This implies that
\[ H^0(P, \mathcal{O}_P(n \times S)) = H^0(J, f_*(\mathcal{O}_P(n \times S))) = H^0(J, \text{Sym}^n(V)). \] (3.9)
The vector bundle \( \text{Sym}^n(V) \) is isomorphic to the vector bundle \( F_{n+1} \) in [3, p. 432, Theorem 5]. It is shown in [3] that
\[ \dim H^0(J, F_n) = 1 \]
(this is proved in [3, p. 430, Lemma 15]; see also the penultimate line in [3, p. 432]). Therefore, from (3.9) we conclude that
\[ H^0(P, \mathcal{O}_P(n \times S)) = C \quad \forall n > 0. \]
Now from (3.6) it follows that
\[ H^0(Z, \mathcal{O}_Z) = C. \]
In view of this, the proposition follows from Proposition 3.3.

4 Functions of moduli space of G-connections

Let \( G \) be a connected reductive affine algebraic group over \( \mathbb{C} \). Let \( \mathcal{M}_X(G) \) denote the moduli space of topologically trivial algebraic \( G \)-connections on \( X \), meaning pairs of the form \( (E_G, D) \), where \( E_G \) is an algebraic principal \( G \)-bundle on \( X \) such that \( E_G \) is topologically trivial, and \( D \) is an algebraic connection on \( E_G \).

Theorem 4.1 The variety \( \mathcal{M}_X(G) \) does not admit any nonconstant algebraic function.

Proof We saw in Proposition 3.4 that the variety \( \mathcal{M}_X \) does not admit any nonconstant algebraic function. Therefore, for any positive integer \( d \), the Cartesian product \( (\mathcal{M}_X)^d \) does not admit any nonconstant algebraic function.

Fix a maximal torus \( T \subset G \). Let \( \mathcal{M}_X(T) \) be the moduli space of \( T \)-connections on \( X \), meaning pairs of the form \( (E_T, D) \), where \( E_T \) is an algebraic principal \( T \)-bundle on \( X \) and \( D \) is an algebraic connection on \( E_T \). We note that since \( T \) is a product of copies of the multiplicative group \( \mathbb{C}^* = \mathbb{C}\{0\} \), if an algebraic principal \( T \)-bundle \( E_T \) admits an algebraic connection, then \( E_T \) is topologically trivial. The inclusion of \( T \) in \( G \) produces a morphism
\[ \psi : \mathcal{M}_X(T) \longrightarrow \mathcal{M}_X(G). \]
This morphism \( \psi \) is known to be surjective [14, p. 1177, Theorem 4.1].

Let \( \delta \) denote the dimension of \( T \), so \( T \) is isomorphic to \( (\mathbb{C}^*)^\delta \). Therefore, \( \mathcal{M}_X(T) \) is isomorphic to the variety \( (\mathcal{M}_X)^\delta \). Since \( (\mathcal{M}_X)^\delta \) does not admit any nonconstant algebraic function, and \( \psi \) is surjective, we conclude that \( \mathcal{M}_X(G) \) does not admit any nonconstant algebraic function.

4.1 The Betti moduli space

We now recall the definition of the Betti moduli space \( M_B^G(G) \) associated to the pair \( (X, G) \) [13]. The identity element of \( G \) will be denoted by \( e \). We will identify \( \pi_1(X) \) with \( \mathbb{Z} \oplus \mathbb{Z} \). Consider the morphism
\[ f : G \times G \longrightarrow G, \quad (x, y) \longmapsto xyx^{-1}y^{-1}. \]
Let $\mathcal{R}_X(G)$ denote the connected component, containing $(e, e)$, of $f^{-1}(e)$. It is an affine variety because $G$ is so. The simultaneous conjugation action of $G$ on $G \times G$ preserves $\mathcal{R}_X(G)$. The geometric invariant theoretic quotient $\mathcal{R}_X(G)/G$ is the Betti moduli space $M^B_X(G)$. It is an affine variety. The Riemann–Hilbert correspondence produces a biholomorphism between $\mathcal{M}_X(G)$ and $M^B_X(G)$; it sends an algebraic connection to the monodromy of the corresponding flat connection.

In contrast with Theorem 4.1, the variety $\mathcal{M}_X(G)$ has plenty of holomorphic functions because it is biholomorphic to an affine variety.

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