Electromagnetic Production of Quarkonium
in $Z^0$ decay

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Abstract

The decay $Z^0 \rightarrow Q + \ell^+\ell^-$, where $Q$ is a $J^{PC} = 1^{--}$ quarkonium state, has a very clean final state, which should make it easy to detect. The branching ratio of this mode is greater than $10^{-6}$ for $\rho$, $\phi$, and $\psi$, indicating that these processes may be detectable at LEP.
Well over one million $Z^0$ decay events have been accumulated at LEP, and further improvements are expected. This makes it possible to investigate rare decays of the $Z$-boson with branching ratios as small as $10^{-6}$. Processes involving the production of the charmonium state $J/\psi$ that have been considered previously are $Z^0 \rightarrow \psi + gg$, $Z^0 \rightarrow \psi + c\bar{c}$, and $Z^0 \rightarrow \psi + \gamma$ [1, 2, 3, 4, 5]. The production of a lepton pair and a $\psi$ from a $Z^0$ is another example of such a process. This decay is especially interesting because of its clean final state, and its relatively large branching ratio. In this paper the rates for $Z^0 \rightarrow Q + \ell^+\ell^-$, where $Q$ is a $J^{PC} = 1^{--}$ quarkonium state, are calculated to leading order in $\alpha$ in a model independent way. The branching ratios for $\rho$, $\phi$, and $\psi$ are greater than $10^{-6}$ which indicates that these processes may be detectable at LEP.

The specific decay of $Z^0 \rightarrow \psi + \ell^+\ell^-$ is calculated first, and later generalized to other quarkonium states with $J^{PC} = 1^{--}$. The decay rate is

$$
\Gamma(Z^0 \rightarrow \psi + \ell^+\ell^-) = \frac{1}{2M_Z} \int [dk][dk'][dp] (2\pi)^4 \delta^4(Z - k - k' - p) \frac{1}{3} \sum |A|^2, \quad (1)
$$

where $k$, $k'$, $p$ and $Z$ are the 4-momenta of the $\ell^+$, $\ell^-$, $\psi$, and $Z^0$, and $[dp] = d^3p/(16\pi^3p_0)$ is the Lorentz invariant phase space element. The two Feynman diagrams that contribute to the amplitude $A$ at leading order in $\alpha$ are shown in figure 1. Neglecting lepton mass, squaring the amplitude, averaging over initial spins, and summing over final spins gives

$$
\frac{1}{3} \sum |A|^2 = \frac{16\pi^2}{3} \frac{\alpha^2 g_w^2 g_{\psi}^2}{\cos\theta_w} (C_A^2 + C_Q^2) g(z_1, z_2, \lambda)
$$

$$
g(z_1, z_2, \lambda) = \frac{1}{2} \left( \frac{z_1}{z_2} + \frac{z_2}{z_1} \right) - \frac{\lambda}{2} \left( \frac{1}{z_1^2} + \frac{1}{z_2^2} \right) + (1 + \lambda) \left( \frac{1 + \lambda}{z_1 z_2} - \frac{1}{z_1} - \frac{1}{z_2} \right), \quad (2)
$$

where $\lambda = M_{\psi}^2/M_Z^2$, $C_A = -1 + 4\sin\theta_w$, and $C_V = 1$. This is similar to the calculation done in Ref. [3] for the $e^+e^- \rightarrow q\bar{q}g^*$ cross section for producing a virtual gluon. Here $z_1 = (Z - k)^2/M_Z^2$ and $z_2 = (Z - k')^2/M_Z^2$, $g_w = e/\sin\theta_w$ is the weak coupling constant, and $\theta_w$ is the Weinberg angle. The strength of the $\psi$-photon coupling $g_{\psi}$ is related to the matrix element of the electromagnetic current by

$$
<\psi|\sum_i e_i q_i \gamma_\alpha \bar{q}_i|\not{\gamma}> = g_{\psi} M_{\psi}^2 e_\alpha(p). \quad (3)
$$
The coupling $g_\psi$ can be calculated from the rate for the quarkonium state to decay to $e^+ e^-$:

$$\Gamma_{e^+e^-} = \frac{4\pi}{3} \alpha^2 M_\psi g_\psi^2. \quad (4)$$

Using (2) in (1) the rate reduces to

$$\Gamma(Z^0 \to \psi \ell^+ \ell^-) = 4 \alpha^2 g_\psi^2 \Gamma(Z^0 \to \ell^+ \ell^-) \int_\lambda^1 dz_1 \int_{\frac{\lambda}{z_1}}^{1+\lambda-z_1} dz_2 \ g(z_1, z_2, \lambda). \quad (5)$$

Integrating the function $g(z_1, z_2, \lambda)$ over $z_2$ and changing variables from $z_1$ to $E_\psi$ yields the $\psi$ energy distribution shown in figure 2. Note that the energy distribution in the figure does not go to zero at $\sim M_Z/2$ because the lepton mass has been neglected in the calculation. Including the lepton mass makes the curve go to zero at the endpoint, but is negligible near the peak of the distribution. For $\psi$ energy $E_\psi \gg M_\psi$ the energy distribution has a simple analytic form:

$$\frac{d\Gamma}{dE_\psi}(Z^0 \to \psi(E_\psi) + \ell^+ \ell^-) = \frac{4\alpha^2 g_\psi^2}{M_Z} \Gamma(Z^0 \to \ell^+ \ell^-) \left[ \frac{(z - 1)^2 + 1}{z} \left( 4 \log z - 2 \log \lambda - 4z \right) \right] \quad (6)$$

where $z = 2E_\psi/M_Z$. The coefficient of the logarithm is proportional to the Altarelli-Parisi function for the splitting of a lepton into a photon of momentum fraction $z$. Doing the remaining integral gives the full decay rate

$$\Gamma(Z^0 \to \psi \ell^+ \ell^-) =$$

$$4\alpha^2 g_\psi^2 \Gamma(Z^0 \to \ell^+ \ell^-) \left[ \frac{1}{2}(1 + \lambda)^2 \log^2 \lambda + \frac{1}{2}(3 + 4\lambda + 3\lambda^2) \log \lambda + \frac{5}{2}(1 - \lambda^2) - 2(1 + \lambda)^2 \left[ Li_2(-\lambda) + \log(1 + \lambda) \log \lambda + \frac{\pi^2}{12} \right] \right]. \quad (7)$$

where $Li_2(x)$ is the dilogarithm function. Neglecting the lepton mass introduces an error of order $m_\ell^2/M_Z^2$ in the rate.

It is trivial to extend this calculation to other bound quarkonium states by replacing $M_\psi$ by the quarkonium mass $M_Q$, and using the appropriate leptonic decay rate $\Gamma_{e^+e^-}$ for the
| Onium State | Visible Decay Products | Branching Ratio into Visible Modes | Events per Million | Visible Events per Million |
|-------------|------------------------|-----------------------------------|--------------------|---------------------------|
| $\rho$      | $\pi\pi$               | 100%                              | 27                 | 27                        |
| $\phi$      | $K^+K^-$               | 49%                               | 3.6                | 1.8                       |
| $\psi$      | $e^+e^- + \mu^+\mu^-$  | 12.2%                             | 2.3                | 0.27                      |
| $\psi'$     | $e^+e^- + \mu^+\mu^-$  | 1.7%                              | 0.66               | 0.011                     |
| $\Upsilon$  | $e^+e^- + \mu^+\mu^-$  | 5%                                | 0.06               | 0.003                     |

Table 1: Events and visible events per million $Z^0$ decays for $Z^0 \to \ell^+\ell^- + \text{Onium}$, summed over $\ell = e, \mu, \tau$.

quarkonium to go into two electrons. The energy distribution for the production of the $\rho$ and $\phi$ are also shown in figure 2. Note that all of the curves have peaks at approximately $1.8M_Q$. The rates for the decay of $Z^0$ into $\ell^+\ell^-$ and $\rho$, $\phi$, $\psi$, $\psi'$, $\Upsilon$, summed over all leptons, are given in table 1. The most visible decay modes of the mesons are $\rho \to \pi^+\pi^-$ with a branching ratio of 100%, $\phi \to K^+K^-$ with a branching ratio of 49%, and $\psi$, $\psi'$, $\Upsilon \to e^+e^- + \mu^+\mu^-$ with branching ratios of 12.2%, 1.7%, and 5%. This reduces the number of events that can be seen at LEP to the number in the last column of table 1. If a cutoff on the quarkonium energy of $E_Q = 10$ GeV is introduced to account for the difficulties in detecting low energy particles the visible events are further reduced by about 45%, 50%, 60% for $\rho$, $\phi$, $\psi$.

The branching ratio for $Z^0 \to \psi + \ell^+\ell^-$, summed over $\ell = e, \mu, \tau$ is $2.3 \times 10^{-6}$ while the branching ratios for other $\psi$ production mechanisms are: $B( Z^0 \to \psi + c\bar{c} ) = 2.2 \times 10^{-5}$, $B( Z^0 \to \psi + \gamma ) = 5.46 \times 10^{-8}$, and $B( Z^0 \to \psi + gg ) = 2.18 \times 10^{-7}$ \[1,3,5\]. Only the rate for $Z^0 \to \psi + c\bar{c}$ is greater than the rate calculated in this paper. Despite this the process $Z^0 \to \psi + \ell^+\ell^-$ may be more detectable since there is a large background to $Z^0 \to \psi + c\bar{c}$ due to the decay $B \to \psi [4]$. Also note that the rate for the decay $Z^0 \to \psi + \ell^+\ell^-$ is an order of magnitude greater than the rate for $Z^0 \to \psi + \gamma$, even though the former process is of order $\alpha^2$ while the latter process is of order $\alpha$. The reason for this is that $Z^0 \to \psi + \gamma$ is a short distance process with the $c$ and $\bar{c}$ that form the $\psi$ being produced in a region with size of order $1/M_Z$, which suppresses the rate by a factor of $1/M_Z^2$. In the process $Z^0 \to \psi + \ell^+\ell^-$ the dominant mechanism for $\psi$ production is fragmentation of a lepton or a photon. Fragmentation is the process where a high energy parton splits into a collinear...

\[1\] A smaller rate has been reported [3]
ψ with $E \gg M_\psi$ and a parton. In the fragmentation process, the $c$ and $\bar{c}$ that form the $\psi$ are produced in a region with size of order $1/m_c$, so that the rate is only suppressed by a factor of $1/m_c^2$ [7]. Fragmentation is also responsible for the rate for $Z^0 \to \psi + c\bar{c}$ being two orders of magnitude larger than the rate for $Z^0 \to \psi + gg$, in spite of the fact that both rates are the same order in $\alpha_s$. The process $Z^0 \to \psi + gg$ is a short distance process that is suppressed by a factor of $m_c^2/M_Z^2$, while $Z^0 \to \psi + c\bar{c}$ includes a fragmentation contribution that has no such suppression [1].

With over a million $Z^0$ decay events collected at LEP an investigation of rare modes is a realistic prospect. The production of the $\psi$ is particularly interesting since the heavy charmed quarks involved allow calculations to be done using perturbative quantum chromodynamics (QCD). The analysis in Ref. [1] of $Z^0 \to \psi + c\bar{c}$ makes it clear that fragmentation plays an important role in $\psi$ production. Unfortunately there will be a large background to this process from $B$ meson decay. The decay $Z^0 \to \psi + \ell^+\ell^-$ is appealing because of the fragmentation contribution, and its particularly clean final state. The rate is large compared to $Z^0 \to \psi + \gamma$, and may even be detectable at LEP. The rate for the production of the lighter quarkonium states $\rho$ and $\phi$ are even larger and should certainly be detectable at LEP.

This work is supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant DE-FG02-91-ER40684. I wish to thank E. Braaten for many helpful discussions.
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Figure Captions

1. The two Feynman diagrams for $Z^0 \rightarrow \psi + \ell^+\ell^-$ at leading order in $\alpha$.

2. The energy distribution of the decay rate for $\rho$, $\phi$ and $\psi$ production.
dash-dots: $\psi$

solid: $\varphi$

dots: $\rho$