Assessment of Co-Variance for Transmission Flow Series-Parallel Reliability Systems

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Abstract-- A methodical hypothesis of dependability depends on likelihood hypothesis. It is outstanding that the ordinary unwavering quality examination utilizing the probabilities has been observed to be insufficient to deal with the uncertainty of disappointment in information and displaying. The fluffy set hypothesis gives a helpful instrument to supplement customary dependability speculations. This paper proposes the transmission stream framework outline in arrangement parallel structure to assess the most extreme unwavering quality subject to the frame work cost. The co-productive of variation (CV) is acquainted with evaluate every enrollment work. This approach is to discover least CV with high reliability gauge.

Keywords: Reliability Optimization, Multi-stage series-parallel System, Co-Productive of Variation, Estimation of uncertainty, Triangular Fuzzy number.

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1. INTRODUCTION

Reliability is an important aspect in the management, scheduling and design of any engineering product. A methodical theory of reliability is based on probability assumption. It is sound known that the conventional reliability analysis using the probabilities has been found to be insufficient to handle the uncertainty of failure in data and modeling. The fuzzy set theory provides a useful tool to harmonize conservative reliability theories. The percept of fuzzy reliability has been introduced and formulated in Cai [2] due to uncertainties and imprecision of data. Chen [3] offered a method for fuzzy system reliability analysis using Simplified Fuzzy Arithmetic Operations of fuzzy numbers.

Hong et al [5] accessible fuzzy system reliability by the use of t-norm based convolution of fuzzy arithmetic operation. Abdul Razak et al [1] proposed a new method for finding fuzzy system reliability in which evolution fuzzy model is represented by trapezoidal fuzzy number. Basically the problem of series, parallel system reliability may be twisted as a typical non-linear programming with cost constraints in fuzzy environment. Park [11] proposed series system reliability subject to a constraint by fuzzy non-linear programming technique. Ruan and Sun [12] offered the method of cost minimization in series reliability system with multiple components.

Sung et al [14] presented a series system reliability with multiple-choice to maximize the system reliability subject to the system cost. Mahapatra et al [8] offered a series system reliability model through fuzzy parametric geometric programming using max-min and max additive operator. Sardar Donighi et al [13] presented a new approach for series-parallel system reliability in which beta type distribution as its membership function. Liu C.M [7] suggested redundancy – reliability allocation problems in multi-stage series parallel system under uncertain environment. Many researchers have applied special techniques and solutions using Series system, Series-Parallel system [6, 10] in
different environment. In this paper, a non-linear programming problem is firstly suggested and analyzed the transmission flow system. General it is series-parallel construction and the flow is transmitted from left to right and to evaluate the maximum reliability subject to the system cost. The cost function is taken as the interval for triangular fuzzy number. In fuzzy system reliability the variance of the module and system reliability estimate is used as a metric. A new approach co-productive of variation (CV) is introduced in the the system reliability estimate for each interval membership function. The advantage of this approach is to find minimum CV with high reliability estimate. The system reliability estimate of CV is obtained by Hatice Tekiner and Coit [4].

This paper is fragmented as the under alluded to areas. Segment 2 offers the scientific model for arrangement parallel structures, documentations and fluffy science essentials. Area 3 gives the scientific definition in fresh model and feathery rendition. Segment four depicts the numerical assessment and the co-proficient of difference for accumulation Parallel device designs. Area 5 gives the arrangement technique for the gathering - Parallel framework models. Segment 6 shows the development of the transmission float gadget. In stage 7 the most extreme dependability with CV has been finished up.

2. MATHEMATICAL MODEL FOR SERIES - PARALLEL SYSTEMS:

For a series–parallel system, there are m subsystem connected in series and those subsystems consisting of n_j components in parallel for j = 1,2 ......... n_k. Fig.1 shows the diagram for m-unit series-parallel systems.

A. NOTATIONS

The Series-Parallel System reliability has been urbanized and worked out for the following notations

R_i - Reliability of subsystem i, for i=1,2 ... m

\( r_{ij}^{l} \) - Left interval value reliability of jth component in subsystem i, for i=1,2 ... m, j=1,2,... n_k

\( r_{ij}^{r} \) - Right interval value reliability of jth component in subsystem i, for i=1,2 ... m, j= 1,2,... n_k

\( c_{ij}^{l} \) - Left interval cost value of jth component in subsystem i, for i=1,2 ...m, j=1,2,.... n_k

\( c_{ij}^{r} \) - Right interval cost value of jth component in subsystem i, for i=1,2 ...m, j=1,2,... n_k

R_{SP}(R_1,R_2,... R_m) – Reliability of m subsystem with each reliability R_i, for i=1,2 ... m

C_{SP}(R_1,R_2,... R_m) – Cost of m subsystem with each reliability R_i, for i=1,2 ...m

The reliability for the series system is

\[ R_{s} (R_i) = \prod_{i=1}^{m} R_i \quad i=1,2 ... m \]

The reliability for parallel system is

\[ R_{p} (R_i) = 1 - \prod_{j=1}^{n_k} (1 - r_{ij}) \quad \text{for } j=1,2,..., n_k \]

The reliability for Series-Parallel System has m subsystem with reliability R_i, i=1,2...m is

\[ R_{SP} (R_1,R_2,... R_m) = \prod_{i=1}^{m} R_{p}(R_i) = \prod_{i=1}^{m} \left[ 1 - \prod_{j=1}^{n_k} (1 - r_{ij}) \right] \]
B. Fuzzy Mathematics Prerequisites

**Definition 2.2.1: Triangular Fuzzy Number**

Let $a_1 \leq a_2 \leq a_3$. A Triangular Fuzzy number (TFN) $\tilde{A}$ in $\mathbb{R}$ is a fuzzy number with the membership function $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ defined as follows.

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\
0 & \text{otherwise}
\end{cases}
$$

3. MATHEMATICAL FORMULATION

A. Crisp Model

Consider the reliability of $m$-unit series-parallel system. The maximization of reliability is found to be $R_{sp}(R_1, R_2, ..., R_m)$ have subject to the limited available cost $C_j, j=1,2,... n_k$.

The mathematical forms of $i^{th}$ stage series-parallel system are as follows.

Max $R_{sp}(R_1) = 1 - \sum_{j=1}^{n_k}(1-r_j)$

Sub to $C_{sp}(R_i) = \sum_{j=1}^{n_k} c_{ij} \bar{r}_j \leq C_i$, where $0 \leq R_i < 1, 0 \leq r_j \leq 1$ and $i=1,2,...m, j=1,2,...n_k$ 

(1)

B. Fuzzy Model

Commonly the gadget reliability of fee element and price constraints can be involved in unsure elements. So the constraint of device reliability becomes uncertain in a reliability optimization trouble. Therefore it can be represented as fuzzy non-linear programming fuzzy range. The above problem may be changed as follows.

For $i^{th}$ Stage of given fuzzy model as

Max $R_{sp}(R_1) = 1 - \sum_{j=1}^{n_k}(1-r_j)$

Sub to $C_{sp}(R) = \sum_{j=1}^{n_k} \tilde{c}_{ij} \tilde{r}_j \leq \tilde{C}_i$, where $0 < R_i \leq 1$ and $0 < r_j \leq 1$ for all $i,j$ 

(2)

4. MATHEMATICAL ANALYSIS

Consider a non-linear programming problem having one imbalance imperative of the sort

Maximize $Z = f(x_1, x_2, ..., x_n)$
Subject to \( g(x_1, x_2, ... x_n) \leq b \), \( x_1, x_2 ... x_n \geq 0 \)

The above problem can be uttered as

Maximize \( Z = f(x_1, x_2 ... x_n) \)

Subject to \( h(x) \leq 0 \) where \( h(x) = g(x_1, x_2 ... x_n) - b \), \( h(x) \geq 0 \) \hspace{1cm} (3)

By Kuhn-Tucker condition the vital conditions for the expansion in non-linear programming problem can be condensed as

\[
\frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0
\]

\( \lambda h(x) = 0 \), where \( h(x) \leq 0 \) and \( \lambda \geq 0 \) \hspace{1cm} (4)

The objective function and constraint of (3) into fuzzy non-linear programming problem is

Max \( z = f(x_1,x_2 ... x_n) \)

Sub to \( h(x) \leq 0 \) \hspace{1cm} (5)

A. The Co-Productive of Variance for Series-Parallels System Models

The co-productive of variation (CV) is outstanding measurable device to describe the estimation of vagueness. CV is characterized as the level of standard deviation partitioned by the mean or level of test standard deviation separated by the normal.

Let \( V \) be the variance of the system reliability estimate and \( R \) be the system reliability estimate then CV for series-parallel system reliability by Teikner and coit [4] is

\[
(CV)^2 = \frac{V}{R^2} = \frac{\prod_{i=1}^{m} [R_i^2 + V_i] - \prod_{i=1}^{m} R_i}{\prod_{i=1}^{m} R_i^2} = \prod_{i=1}^{m} \left(1 + \frac{V_i}{R_i^2}\right) - 1.
\]

5. SOLUTION PROCEDURE FOR SERIES-PARALLEL MODELS

Step: 1

Let the cost factor be \( \tilde{c}_y = (c_{y1}, c_{y2}, c_{y3}) \) and the cost limitation be \( \tilde{C}_y = (C_{y1}, C_{y2}, C_{y3}) \)

\( i = 1, 2...m \) and \( j = 1, 2...n_k \) are taken as Triangular fuzzy number.

Step: 2

Using \( \alpha \)-cut membership function of cost factors and cost limitations are given by

\[
\tilde{c}_y = [c_{y1} + \alpha(c_{y2} - c_{y1}), c_{y3} - \alpha(c_{y3} - c_{y1})] \text{ for } i = 1, 2, 3,...,m \text{ and } j = 1, 2, 3,...,n_k \text{ and }
\]

\[
\tilde{C}_y = [C_{y1} + \alpha(C_{y2} - C_{y1}), C_{y3} - \alpha(C_{y3} - C_{y1})] \text{ for } i = 1, 2, 3,...,m \text{ respectively.}
\]

Step: 3

Applying the Kuhn-Tucker condition in a fuzzy non-linear programming problem for given models with \( i^{th} \) stage \( i = 1, 2...m \) of left and right interval \( \alpha \)-cut is expressed as
\[
\text{Max } R^L_{\alpha}(R_i) = 1 - \prod_{j=1}^{n_j}(1 - r^L_{ij})
\]

Subject to \[\sum_{j=1}^{n_j} \bar{C}^L_{ij} - \bar{C}^L_i < 0 \quad \text{where } 0 < r_{ij} \leq 1, C^L_{ij} \geq 0 \] and

\[
\text{Max } R^R_{\alpha}(R_i) = 1 - \prod_{j=1}^{n_j}(1 - r^R_{ij})
\]

Subject to \[\sum_{j=1}^{n_j} \bar{C}^R_{ij} - \bar{C}^R_i \leq 0 \quad \text{where } 0 < r_{ij} \leq 1, C^R_{ij} \geq 0 \]

Step: 4

To find the optimal solution of \( R^L_i \) and \( R^R_i \) \( i = 1, 2 \ldots m \) for each membership value of \( \alpha \) from step:2,3 and then to calculate the system reliability \( R^L = \prod_{i=1}^{m} R^L_i \) and \( R^R = \prod_{i=1}^{m} R^R_i \) with their corresponding CV for each value of \( \alpha \).

Step: 5

To find out the maximum reliability and minimum CV from the values of \( \alpha \).

6. NUMERICAL EXAMPLES

A practical repairable system of jet fighter engine design is illustrated as an example. The data’s are secondary and is taken from the journal paper “A knowledge management system for series-parallel availability optimization and design” by Y.S Juang et al. In which the lower bound and upper bound manufacture cost of components (in dollar) are shown as uncertain. Let us consider it as fuzzy and the cost of components are taken as left and right interval value triangular fuzzy number. This system consists of eight components in three main units, with its system frame work in Figure 3.

This system consists of three main units (subsystems) connected in series. The first unit consists of 3 components in parallel, second unit consists of 1 component and third unit consists of 4 components in parallel. The cost components (in dollar) are taken as triangular fuzzy number as follows.

Step: 1

For Unit 1,

The cost components are \( c_{11} = (1012452, 1021037, 1029622), c_{12} = (903874, 910864, 917857) \) and \( c_{13} = (908976, 916879, 924,782) \) and the cost constraints is \( C_1 = (2542772, 2585446, 2628119) \).

For Unit 2,

The cost components are \( c_{21} = (1353896, 1363530, 1373164) \) and the cost constraints is \( C_2 = (1313279, 1319191, 1325103) \).

For Unit 3,
The cost components are \( c_{31} = (983849, 987102, 990354), c_{32} = (852,421, 856343, 860265), c_{33} = (677592, 681576, 685559) \) and \( c_{34} = (837584, 842144, 846704) \) and the cost constraint is \( C_3 = (2932515, 2963183, 2993851) \).

**Step: 2** Using \( \alpha \)-cut membership function the cost factors and limitations of left and right interval value are tabulated as follows.

| Unit (i) | Cost components | Cost constraints |
|----------|-----------------|------------------|
| 1        | \( c_{11} = (c_{11}^L, c_{11}^R) = (1012452+8585\alpha, 1029622-8858\alpha) \) | \( C_1 = (C_1^L, C_1^R) = (2542772 + 42674 \alpha, 2628119-42674 \alpha) \) |
|          | \( c_{12} = (c_{12}^L, c_{12}^R) = (903874+6990\alpha, 917857-6990\alpha) \) | |
|          | \( c_{13} = (c_{13}^L, c_{13}^R) = (908976+7903\alpha, 924782-7903\alpha) \) | |
| 2        | \( c_{21} = (c_{21}^L, c_{21}^R) = (1353896+9634\alpha, 1373164-9634\alpha) \) | \( C_2 = (C_2^L, C_2^R) = (1313279 + 5912 \alpha, 1325103-5912\alpha) \) |
| 3        | \( c_{31} = (c_{31}^L, c_{31}^R) = (983849+3253\alpha, 990354-3252\alpha) \) | \( C_3 = (C_3^L, C_3^R) = (2932515 + 30668 \alpha, 2993851-30668\alpha) \) |
|          | \( c_{32} = (c_{32}^L, c_{32}^R) = (852421+3922\alpha, 860265-3922\alpha) \) | |
|          | \( c_{33} = (c_{33}^L, c_{33}^R) = (677592+3984\alpha, 685559-3983\alpha) \) | |
|          | \( c_{34} = (c_{34}^L, c_{34}^R) = (837584+4560\alpha, 846704-4560\alpha) \) | |

**Step: 3** Apply Kuhn-Tucker condition in the optimal solution of a fuzzy non-linear programming problem with \( i^{th} \) unit \( i = 1, 2, 3 \) of left and right \( \alpha \)-cut can be expressed as follows.

In the first Unit,

The optimum solution of left interval \( \alpha \)-cut is

\[
\frac{\partial}{\partial r_{ij}^L}[1 - (1 - r_{ij}^L)(1 - r_{ij}^U)] = \lambda \frac{\partial}{\partial r_{ij}^L}[(\tilde{c}_{ij}^L r_{ij}^L + \tilde{c}_{ij}^U r_{ij}^U) - C_{i}^L] 
\]

(6)

\[
(\tilde{c}_{ij}^L r_{ij}^L + \tilde{c}_{ij}^U r_{ij}^U) - C_{i}^L = 0
\]

(7)

Similarly, the optimum solution of right interval \( \alpha \)-cut is

\[
\frac{\partial}{\partial r_{ij}^U}[1 - (1 - r_{ij}^L)(1 - r_{ij}^U)] = \lambda \frac{\partial}{\partial r_{ij}^U}[((1012452 + 8585\alpha)r_{ij}^L + (903874 + 6990\alpha)r_{ij}^U + (908976 + 7903\alpha)r_{ij}^U) - (2542772 + 42674\alpha)]
\]

(8)

\[
(1012452 + 8585\alpha)r_{ij}^L + (903874 + 6990\alpha)r_{ij}^U + (908976 + 7903\alpha)r_{ij}^U = 2542772 + 42674\alpha
\]

(9)
\[
\frac{\partial}{\partial r_j^L}[1 - (1 - r_j^L)(1 - r_j^L)] = 2\frac{\partial}{\partial r_j^L}(\tilde{c}_j^L + c_{j1}^L + c_{j2}^L)
\]

(10)

\[
(\tilde{c}_j^L + c_{j1}^L + c_{j2}^L) = -C_1^L
\]

(11)

\[
\frac{\partial}{\partial r_j^L}[1 - (1 - r_j^L)(1 - r_j^L)(1 - r_j^L)] = 2\frac{\partial}{\partial r_j^L}(1029622 - 8858\alpha) r_j^L
\]

+ (917857 - 6990\alpha) = (2628119 - 42674\alpha)

(12)

\[
(1029622 - 8858\alpha) r_j^L + (917857 - 6990\alpha) r_j^L + (924782 - 7903\alpha) r_j^L = (2628119 - 42674\alpha)
\]

(13)

**In the Second Unit,**

The optimum solution of left interval \(\alpha\)-cut is

\[
c_{j1}^L r_j^L = C_2^L
\]

(1353896 + 9634\alpha) r_j^L = 1313279 + 5912 \alpha

(14)

The optimum solution of right interval \(\alpha\)-cut is

\[
c_{j1}^R r_j^R = C_2^R
\]

(1373164 - 9634\alpha) r_j^R = 1325103 - 5912\alpha

(15)

**In the third Unit,**

The optimum solution of left interval \(\alpha\)-cut is

\[
\frac{\partial}{\partial r_j^L}[1 - (1 - r_j^L)(1 - r_j^L)(1 - r_j^L)] = 2\frac{\partial}{\partial r_j^L}(\tilde{c}_j^L + c_{j1}^L r_j^L + c_{j2}^L r_j^L + c_{j3}^L r_j^L + c_{j4}^L r_j^L) - C_3^L
\]

(16)

\[
c_{j1}^L r_j^L + c_{j2}^L r_j^L + c_{j3}^L r_j^L + c_{j4}^L r_j^L = C_3^L
\]

(17)

\[
\frac{\partial}{\partial r_j^L}[1 - (1 - r_j^L)(1 - r_j^L)(1 - r_j^L)] = 2\frac{\partial}{\partial r_j^L}((983849 + 3253\alpha) r_j^L + (852421 + 3922\alpha) r_j^L
\]

+ (677592 + 3984\alpha) r_j^L + (837584 + 4560\alpha) r_j^L = (2932515 + 30668\alpha)

(18)

\[
(983849 + 3253\alpha) r_j^L + (852421 + 3922\alpha) r_j^L + (677592 + 3984\alpha) r_j^L + (837584 + 4560\alpha) r_j^L = 2932515 + 30668 \alpha
\]

(19)

Similarly, the optimum solution of right interval \(\alpha\)-cut is
Substitute for $\alpha = 0.0, 0.2, 0.4, 0.6, 0.8$ and $1.0$ respectively in the above equation and solving them then the reliability values are obtained and listed in the table.

**Step: 4**

The Table 1 and Table 3 shows that the left and right interval optimal solutions of jet fighter engine design system reliability through fuzzy parametric non-linear programming respectively.

The Table 2 and Table 4 show that the Co-productive of Variance of left and right interval jet fighter engine design system of reliability respectively.

**Step: 5**

From the Table 1 and Table 3, the maximum reliability is indentified and their corresponding CV of fuzzy membership values of $\alpha$.

7. CONCLUSION

In the industrial system, the cost factor is considered to be the most important factor and the reliability of system is also considered to be important. The existing information about the constituent factors of the system is imprecise, vague and uncertain often; in such a situation most decision makers specify the reliability evaluations be risk averse for which they feel like to have higher reliability and lower estimations. In General, the Series-Parallel system is a non-linear programming problem with system available cost. But cost factor and cost limitation are fuzzy numbers in nature. This paper attempts to provide the assessment of reliability in transmission flow series-parallel system and the cost factor and cost limitation of each subsystem are taken as triangular fuzzy number. Kuhn-Tucker conditions are taken into concern to solve the non-linear programming problem with fuzzy factors to find out the optimal solutions for left and right interval valued membership function of $\alpha$. This can be maximized for the system reliability subject to the existing cost. Table 2 demonstrate that the left interval optimal solution of fuzzy system reliability with CV, which identifies that maximum reliability and minimum CV for $\alpha = 0.0$ is 0.9890. Table 4 demonstrate that the right interval optimal solution of fuzzy system reliability with CV, which also identifies that the maximum reliabilities and minimum CV for $\alpha = 0.8$ is 0.9851. Decision maker can make use of the strategy of system reliability where optimization is involved.

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Table 1: Left interval optimal solution of jet fighter engine design system reliability

| $\alpha$ | $r_{11}^L$ | $r_{12}^L$ | $r_{13}^L$ | $R_{11}^L$ | $R_{12}^L$ | $R_{13}^L$ | $r_{11}^R$ | $r_{12}^R$ | $r_{13}^R$ | $R_{11}^R$ | $R_{12}^R$ | $R_{13}^R$ |
|----------|------------|------------|------------|----------|----------|----------|------------|------------|------------|----------|----------|----------|
| 0.0      | 0.90698160 | 0.89580775 | 0.89639257 | 0.99899586 | 0.96999991 | 0.89354794 | 0.87713495 | 0.84543390 | 0.87495851 | 0.99974722 | 0.96878094 |
| 0.2      | 0.90840093 | 0.89738229 | 0.89793595 | 0.99904110 | 0.96949351 | 0.89437750 | 0.87812394 | 0.84671723 | 0.87598592 | 0.99975530 | 0.96832685 |
| 0.4      | 0.90981546 | 0.89891519 | 0.89955765 | 0.99908467 | 0.96988854 | 0.89520597 | 0.87911114 | 0.84799758 | 0.87701111 | 0.99976317 | 0.96787232 |
| 0.6      | 0.91122524 | 0.90051685 | 0.90113198 | 0.99912684 | 0.96848500 | 0.89603356 | 0.88096767 | 0.84927522 | 0.87803432 | 0.99977084 | 0.96741761 |
| 0.8      | 0.91263024 | 0.90207688 | 0.90270086 | 0.99916756 | 0.96798289 | 0.89685964 | 0.88108009 | 0.85054923 | 0.87905483 | 0.99977830 | 0.96696267 |
| 1.0      | 0.91403054 | 0.90363216 | 0.90426436 | 0.99920686 | 0.96748220 | 0.89768484 | 0.88206186 | 0.85182063 | 0.88073366 | 0.99978556 | 0.96650755 |

Table 2: Left interval Co-productive of Variance of jet fighter engine design

| $\alpha$ | $R_{11}^L$ | $R_{12}^L$ | $R_{13}^L$ | $\mu_{11}^L$ | $\mu_{12}^L$ | $\mu_{13}^L$ | $V_{11}^L$ | $V_{12}^L$ | $V_{13}^L$ | $CV$ |
|----------|------------|------------|------------|-------------|-------------|-------------|----------|----------|----------|-----|
| 0.0      | 0.99899586 | 0.96999991 | 0.99974722 | 0.89972731  | 0.87276883  | 0.00002637  | 0.99999991 | 0.96999991 | 0.89354794 | 0.87713495 |
| 0.2      | 0.99904110 | 0.96949351 | 0.99957530 | 0.90125639  | 0.87380115  | 0.00002558  | 0.99999991 | 0.96999991 | 0.89437750 | 0.87812394 |
| 0.4      | 0.99908467 | 0.96898854 | 0.99976317 | 0.90277503  | 0.87483145  | 0.00002484  | 0.99999991 | 0.96988854 | 0.89520597 | 0.87911114 |
| 0.6      | 0.99912684 | 0.96848500 | 0.99977084 | 0.90429136  | 0.87585997  | 0.00002410  | 0.99999991 | 0.96848500 | 0.89685964 | 0.88108009 |
| 0.8      | 0.99916756 | 0.96798289 | 0.99977830 | 0.90580266  | 0.87688596  | 0.00002373  | 0.99999991 | 0.96798289 | 0.89685964 | 0.88108009 |
| 1.0      | 0.99920686 | 0.96748220 | 0.99931019 | 0.93616279  | 0.89388265  | 0.00004804  | 0.99999991 | 0.96748220 | 0.89768484 | 0.88206186 |

Table 3: Right interval optimal solution of jet fighter engine design system reliability

| $\alpha$ | $r_{11}^R$ | $r_{12}^R$ | $r_{13}^R$ | $R_{11}^R$ | $R_{12}^R$ | $R_{13}^R$ | $r_{31}^R$ | $r_{32}^R$ | $r_{33}^R$ | $R_{31}^R$ | $R_{32}^R$ | $R_{33}^R$ |
|----------|------------|------------|------------|----------|----------|----------|------------|------------|------------|----------|----------|----------|
| 0.0      | 0.90981546 | 0.89891519 | 0.89955765 | 0.99908467 | 0.96988854 | 0.89520597 | 0.87911114 | 0.84799758 | 0.87701111 | 0.99976317 | 0.96787232 |
| 0.2      | 0.91122524 | 0.90051685 | 0.90113198 | 0.99912684 | 0.96848500 | 0.89603356 | 0.88096767 | 0.84927522 | 0.87803432 | 0.99977084 | 0.96741761 |
| 0.4      | 0.91263024 | 0.90207688 | 0.90270086 | 0.99916756 | 0.96798289 | 0.89685964 | 0.88108009 | 0.85054923 | 0.87905483 | 0.99977830 | 0.96696267 |
| 0.6      | 0.91403054 | 0.90363216 | 0.90426436 | 0.99920686 | 0.96748220 | 0.89768484 | 0.88206186 | 0.85182063 | 0.88073366 | 0.99978556 | 0.96650755 |
| 0.8      | 0.91543054 | 0.90763216 | 0.90826436 | 0.99924086 | 0.96798220 | 0.89768484 | 0.88206186 | 0.85182063 | 0.88073366 | 0.99978556 | 0.96650755 |
| 1.0      | 0.91683054 | 0.91163216 | 0.91226436 | 0.99927486 | 0.96748220 | 0.89768484 | 0.88206186 | 0.85182063 | 0.88073366 | 0.99978556 | 0.96650755 |
Table 4: Right interval Co-productive of Variance of jet fighter engine design

| α   | $R_1^r$  | $R_2^r$  | $R_3^r$  | $\mu_1^r$ | $\mu_2^r$ | $\mu_3^r$ | $V_1^r$  | $V_2^r$  | $V_3^r$  | CV       |
|-----|----------|----------|----------|-----------|-----------|-----------|----------|----------|----------|----------|
| 0.0 | 0.99938332 | 0.96499770 | 0.99981908 | 0.91476662 | 0.96499770 | 0.88300168 | 0.00001926 | 0.00000000 | 0.00024792 | 0.01634949 |
| 0.2 | 0.99935059 | 0.96549139 | 0.99981274 | 0.91328479 | 0.96549139 | 0.88198738 | 0.00001992 | 0.00000000 | 0.00025288 | 0.01652031 |
| 0.4 | 0.99931661 | 0.96598646 | 0.99980622 | 0.91179810 | 0.96598646 | 0.88097108 | 0.00002058 | 0.00000000 | 0.00025790 | 0.01669189 |
| 0.6 | 0.99928135 | 0.96648292 | 0.99979952 | 0.91030653 | 0.96648292 | 0.87995217 | 0.00002126 | 0.00000000 | 0.00026300 | 0.01686428 |
| 0.8 | 0.99924477 | 0.96698079 | 0.99979264 | 0.90880971 | 0.96698079 | 0.87893248 | 0.00002195 | 0.00000000 | 0.00026817 | 0.01703726 |
| 1.0 | 0.99920685 | 0.96748007 | 0.99978556 | 0.90730866 | 0.96748007 | 0.87791017 | 0.00002266 | 0.00000000 | 0.00027340 | 0.01721105 |
Fig. 1: m-unit Series-Parallel System

Fig. 2: Triangular Fuzzy Number

Fig. 3: Design configuration of Jet fighter engine
Fig. 4: Interval valued reliability of Jet fighter engine design with fuzzy triangular number