Arrow of Time in String Theory

Brett McInnes

National University of Singapore
email: matmcinn@nus.edu.sg

ABSTRACT

Inflation allows the problem of the Arrow of time to be understood as a question about the structure of spacetime: why was the intrinsic curvature of the earliest spatial sections so much better behaved than it might have been? This is really just the complement of a more familiar problem: what mechanism prevents the extrinsic curvature of the earliest spatial sections from diverging, as classical General Relativity suggests? We argue that the stringy version of “creation from nothing”, sketched by Ooguri, Vafa, and Verlinde, solves both of these problems at once. The argument, while very simple, hinges on some of the deepest theorems in global differential geometry. These results imply that when a spatially toral spacetime is created from nothing, the earliest spatial sections are forced to be [quasi-classically] exactly locally isotropic. This local isotropy, in turn, forces the inflaton into its minimal-entropy state. The theory explains why the Arrow does not reverse in black holes or in a cosmic contraction, if any.
1. The Arrow of Time: What Inflation Does For Us

One of the deepest mysteries in physics is the origin of the Arrow of time. The first step towards understanding this mystery was identifying where the explanation is to be looked for: in cosmology. For example, the immediate origin of the low-entropy conditions in our local environment is the Sun; and the Sun’s ability to play this role can readily be traced back to the extremely low total entropy of the Big Bang era. [Excellent surveys of this question have been given by Albrecht [1], Price [2], and Carroll and Chen [3]; see also [4] for relevant general background.] The problem is now in the cosmological domain: why was the entropy so low at that time — what set up the conditions that allowed the nucleosynthesis which provided the raw materials for the Sun, together with the subsequent local contraction which formed it?

It is important to understand that we have no right to expect a single answer to this question. The low entropy of the very earliest universe might well have been stored in a vast variety of ways, as Price [2] emphasises. In fact, however, the theory of Inflation [5] provides a beautifully simple answer. The matter and radiation of the early Big Bang era derived its low entropy from a single source: the inflaton. For it was the inflaton that generated the equilibrium that obtained just before re-heating, and it was of course the inflaton that drove re-heating itself. This is the first service that Inflation performs in our search for the origin of the Arrow of time: it reduces what could have been an enormously complex collection of problems [explaining a whole variety of ways in which the initial low entropy state might have been arranged] to a single one: why was the inflaton itself in a special state [6] in the beginning?

The special initial state of the inflaton had a precise form: the inflaton was in a “potential-dominated” state. The stress-energy-momentum tensor for the inflaton has the form

$$T_{\mu \nu} = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu \nu} [g^{\alpha \beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi)].$$

This expression is dominated by the potential $V(\varphi)$, allowing the inflaton to mimic a cosmological constant, provided that this potential is much larger than the kinetic term, involving the derivatives of the inflaton field. Among all of the vast array of possible excitations of the degrees of freedom of $\varphi$, only an infinitesimal fraction would have satisfied this condition; thus, “low entropy” here means something very specific: that the gradient vector of the inflaton field was essentially zero at the earliest times.

The question as to whether Inflation itself can explain this special initial state of the inflaton has given rise to much debate [7][8]. Recently, however, it has become generally agreed that string theory — which will be the context for this work — always maintains strict unitarity, even under extreme circumstances [9][10][11]. This suggests [see [3] for the details of the argument] that Inflation alone cannot solve the problem of the Arrow of time. Albrecht [11] has given more general reasons in favour of this conclusion. But if Inflation itself is not the solution, it does firmly point towards the direction in which a full account must be sought, as follows.

The question of the origin of the Arrow of time has now been reduced to this question: why was the gradient vector of the inflaton so small in the beginning? The most natural way to explain why a spatial vector field should be zero is to invoke local isotropy at each
point of space. This is based on the simple observation that a vector can be isotropic only if it vanishes.

We propose, then, that local spatial isotropy was the specific geometric property which enforced the vanishing of the initial inflaton gradient. Note that for this argument to work, the isotropy must be very precise — we are not speaking of the usual, approximate isotropy discussed in connection with observational cosmology. Instead, we need a fundamental isotropy which, in fact, is exact at the quasi-classical level. This, again, is an extremely “special” state for the spatial geometry at any time, and, in view of the way anisotropies tend to grow when one traces the history of a spatial section back in time, it is still more so at early times. Leaving aside [for the moment] the question as to why the temporal gradient of the inflaton should have been initially zero, we see that the question has been transferred to the domain of spatial geometry: why were the earliest spatial sections exactly locally isotropic at the quasi-classical level?

The idea that the Arrow of time is ultimately due to the uniformity of the initial spacetime geometry is due to Penrose [12]. Penrose, too, implicitly emphasises the role of local isotropy, by focusing on the Weyl curvature, which is a measure of tidal anisotropy. In our view, Inflation is an extremely natural idea in this context, because a scalar field, unlike the vector or spinor fields which represent all of the known fundamental forms of matter, can be non-trivial even on a perfectly locally isotropic spacetime. An initial geometry of this kind will automatically eliminate all vectors and spinors, and it will automatically put a scalar field into the potential-dominated state. Thus, initial local isotropy explains [a] why a scalar field dominated other possible forms of matter in the earliest Universe, and [b] why that scalar field was in a particular, low-entropy state at that time. That is, Inflation provides an extremely natural way of communicating the initial geometric “specialness” to all other physical fields.

To summarize, then: Inflation itself does not explain the Arrow of time, but it provides a major part of the explanation. First, it enormously simplifies the problem, by reducing it to explaining the low initial entropy of a single object, the inflaton; and second, it allows us to reduce this problem to explaining the local isotropy of the earliest spacetime geometry. Conversely, a physical explanation of the initial local isotropy may well explain why the inflaton was the dominant form of matter in the beginning.

In three dimensions, a vector is dual to a two-form, so a locally isotropic three-dimensional Riemannian manifold has the property that its sectional curvature at each point is independent of direction. By Schur’s theorem [see [13] page 202], this means that the space has the geometry of a space of constant curvature throughout the [connected] region in which it is locally isotropic. From the spacetime point of view, we can say that the geometry near the beginning had the property that the intrinsic curvature tensor of the spacelike slices was maximally well-behaved, in the sense that it has the structure of the curvature tensor of one of the classical spaces of constant curvature: the sphere,

1Local isotropy at a point in a Riemannian manifold is the condition that, given a point p and a pair of unit tangent vectors X, Y, at p, there exists an open neighbourhood of p and an isometry of this neighbourhood which maps X to Y. Note that even if the manifold is everywhere locally isotropic, it need not be globally isotropic.

2It may be helpful, in the case of FRW metrics, to think in terms of the fundamental, (1,3) version of the intrinsic curvature tensor, which is not affected by the variations of the scale factor.
Euclidean space, hyperbolic space, and their non-singular quotients. In order to explain the Arrow of time, we now have a specific task: to explain this good behaviour of the intrinsic curvature of the earliest spatial slices.

Now, in fact, it is generally believed that the extrinsic curvature of the earliest spatial sections was also much better behaved than one has a right to expect. That is, classical General Relativity leads one to expect that the extrinsic curvature should diverge as earlier times are examined, but it is generally believed that string theory intervenes to prevent this from happening. In fact, string theory has given rise to a variety of ideas as to how the misbehaviour of the extrinsic curvature might be removed or otherwise tamed. One recent proposal for using the theory to deal with cosmological singularities was made by Ooguri et al [14] [15], who have embedded the idea of “creation from nothing” in string theory; another suggestion, due to McGreevy and Silverstein [16], involves a pre-inflationary era of tachyon condensation.

Since the mystery of the Arrow of time is just the “intrinsic curvature version” of the “extrinsic curvature problem” — the problem of resolving cosmological singularities — it is natural to demand that any theory that claims to solve one of these problems should solve the other. Indeed, it is clearly pointless to resolve cosmological singularities if the resulting, non-singular spacetime fails to evolve to a Universe like ours. But that will surely be the case unless the theory strictly demands initial local spatial isotropy at each point. Equally, it seems very unlikely that a geometric solution of the problem of the Arrow of time can ignore the singularity problem. In other words, string theory must produce an initial spacetime structure which is extremely uniform not only in the timelike, but also in the spacelike directions: it must account for the good behaviour of extrinsic and intrinsic curvature. This is, of course, an extremely formidable task.

Here we shall argue that the version of string cosmology due to Ooguri et al [14] may in fact be capable of this feat. In this theory, the Universe is “created from nothing”, after the manner of Vilenkin [17] or Hartle and Hawking [18] [though the details are very different]. “Creation from nothing” fits particularly well into our discussion above, because it takes place along a hypersurface of time symmetry; along such a hypersurface, by definition, the extrinsic curvature vanishes, so the problem of a divergent extrinsic curvature is solved by the same mechanism that allows “creation from nothing” in the first place. Furthermore, creation along a hypersurface of time symmetry immediately explains why the initial temporal gradient of the inflaton should vanish, something not ensured by local spatial isotropy.

While this proposal automatically solves the extrinsic curvature problem, there is no reason to expect the initial intrinsic curvature to be well-behaved: the initial spatial section could in principle have had an extremely distorted non-singular geometry, and indeed that would have been the generic case. The task now is to show that the wave function of Ooguri et al [14] is extremely sharply peaked around some three-dimensional geometry of constant curvature.

Unfortunately, the “OVV wave function” is not understood in enough detail to check this directly. This difficulty can, however, be circumvented by appealing to topological methods. We shall see that a simple argument — based, however, on very deep theorems

\footnote{From footnote 1, it is clear that all of these quotients [such as $\mathbb{RP}^3$ and the torus $T^3$] are indeed locally isotropic at every point.}
due to Schoen, Yau, Gromov, Lawson, and Bourguignon — shows that, in fact, a spatial section can only be “created from nothing” in the OVV framework if it is, classically, exactly a space of constant curvature: in fact, classically it must be exactly locally flat. This is the origin of the extreme initial geometric “specialness” which, via the inflaton, gives rise to the Arrow of Time.

The Ooguri et al version of string cosmology is in its infancy, and there are many subtle aspects of this argument which remain to be clarified. It is nevertheless of interest to understand one possible way in which string theory might account for the Arrow of time. More generally, it is of interest to exhibit a theory in which the Arrow of time is explained without resorting to vastly improbable random fluctuations, to multiple universes, or to the anthropic principle.

We begin with a brief review of the relevant aspects of the theory of Ooguri et al. We then discuss the particular form which the initial value problem takes in the case of spacetimes-with-boundaries. Finally, we give the [logically extremely simple, but mathematically very deep] argument that OVV cosmology implies an Arrow, and discuss its consequences. We specifically address Price’s [2] concern that beginnings and endings [particularly inside black holes] should be treated symmetrically.

The Arrow of time problem has recently attracted increased attention, giving rise to a variety of interesting ideas: see for example [3][19][20]. In our view, it is not enough to explain why the entropy of the early Universe was low: we must explain why it was low in a particular way, namely, in the form of maximally well-behaved intrinsic curvature. Furthermore, for reasons already explained, we believe that an adequate explanation of the Arrow of time can only arise in connection with an explicit solution of the cosmological singularity problem. For this reason, we shall be concerned only with ideas arising directly from string theory, to which we now turn.

2. String Theory and The Arrow of Time: General Background

We have argued that the problem of the Arrow of time cannot be separated from the cosmological singularity problem. As it is generally believed that the singularity problem must be solved by some feature of string theory, that same feature must give rise to the Arrow of time. In view of this, let us briefly draw attention to some relevant properties of string cosmology. We begin with some general observations which might apply to any string-theoretic account of the Arrow of time, and then focus on the relevant properties of the theory of Ooguri et al.

2.1. String Theory and Spatial Compactness

It is now generally agreed that Inflation is not past-eternal [21]. The natural interpretation of this is that the Universe had a beginning, which may or may not have been singular. Our problem is to understand the properties of the spacelike hypersurface along which the Universe came into existence. In particular, we need to answer a basic question: is space, like [past] time, necessarily finite?

Studies of the earliest Universe in string theory suggest that the spatial sections of our Universe were, at least in that era, topologically compact. [We say, “in that era”, because some have argued that a spatially compact spacetime can, in principle, ultimately generate
spatially non-compact “babies”, as in [22].] For example, in string gas cosmology [23], T-duality is applied to the spatial sections of the Universe, which are assumed to have the topology of a three-dimensional torus. In fact, such spatial topologies are natural in any cosmology involving closed strings; for example, windings of closed strings are crucial in the string tachyon cosmology of McGreevy and Silverstein [16]. Finally, spatial compactness plays an essential role in the “creation from nothing” approach, which has been embedded in string theory by Ooguri et al. Indeed, Ooguri et al [14] are very explicit in this regard: they state that one of the principal objectives of their work is to construct a Hartle-Hawking wave function which determines the [finite] sizes of all of the spatial dimensions, small and large: the sizes are to be regarded as “moduli”. In this approach to string cosmology, the compactness of the early spatial sections is fundamental.

To summarize: if we wish to embed cosmology in string theory, we should assume that the initial spatial sections were compact, though not necessarily with spherical topology.

2.2. Background on OVV: Creation on a Torus

With this preparation, let us turn to the relevant details of the theory of Ooguri et al [henceforth, OVV], which we propose to use here. It was suggested in [24] [see [25][26] for recent developments] that the [modified] elliptic genus of a certain IIB Calabi-Yau black hole is squared norm of the topological string partition function associated with the corresponding “attractor geometry”, the Euclidean space \( \mathbb{H}^2/\mathbb{Z}_2 \times S^2 \times \text{CY} \). Here \( \mathbb{H}^2/\mathbb{Z}_2 \) is a partially compactified version of two-dimensional hyperbolic space, \( S^2 \) is the two-sphere, and \( \text{CY} \) denotes a six-dimensional Calabi-Yau space. OVV propose to put this remarkable development to good use by interpreting the partition function in terms of a Hartle-Hawking wave function describing the creation of a two-dimensional cosmological spacetime obtained by taking a Lorentzian version of \( \mathbb{H}^2/\mathbb{Z}_2 \). [This is the “Entropic Principle”.] That is, instead of beginning, in the usual Hartle-Hawking manner, with a Euclidean sphere, OVV begin with a negatively curved space with topology \( \mathbb{R} \times S^1 \). The metric, with curvature \(-1/L^2\), is

\[
g(\mathbb{H}^2/\mathbb{Z}_2)_{++} = K^2 e^{(2\rho/L)} \, d\tau^2 + d\rho^2,
\]

where \( \tau \) is an angular coordinate on a circle with radius \( K \) at \( \rho = 0 \); when \( \mathbb{R} \times S^1 \) is endowed with this metric we can call it \( \mathbb{H}^2/\mathbb{Z}_2 \).

Ooguri et al. argue that the metric \( g(\mathbb{H}^2/\mathbb{Z}_2)_{++} \) actually defines two Lorentzian metrics. In one way of thinking about this metric [which OVV call the “more traditional” interpretation], \( \tau \) is to be regarded as the usual cyclic Euclidean time, and the \( \tau \) translation generator defines a Witten index associated with the degeneracy of the states of the black hole in the OSV equivalence. But, in the other, one interprets \( \rho \) as Euclidean time, and then \( g(\mathbb{H}^2/\mathbb{Z}_2)_{++} \) is regarded as a sort of Euclidean version of de Sitter space-time, with flat but compact spatial sections parametrized by \( \tau \). The Lorentzian version is obtained by complexifying conformal time. It is this “cosmological interpretation” of \( \mathbb{H}^2/\mathbb{Z}_2 \) that allows OVV to embed “creation from nothing” in string theory. Let us see how this works in the four-dimensional case [15].

We begin with \( \mathbb{H}^4/\mathbb{Z}_3^3 \), obtained by compactifying the sections of \( \mathbb{H}^4 \) when it is foliated by copies of \( \mathbb{R}^3 \) with its flat metric. This gives us the correct four-dimensional version of
\( g(H^2/\mathbb{Z})_{++} \): it is the metric on \( \mathbb{R} \times T^3 \), where \( T^3 \) is the three-torus, given by

\[
\begin{align*}
g(H^4/\mathbb{Z}^3; \Phi)_{+++} &= d\Phi^2 + K^2 e^{-2\Phi/L} [d\theta_1^2 + d\theta_2^2 + d\theta_3^2],
\end{align*}
\]  

(3)

where the curvature is again \(-1/L^2\), where \( \theta_{1,2,3} \) are angular coordinates on a cubic torus with side lengths \( 2\pi K \) at \( \Phi = 0 \), and where \( \Phi \) is a coordinate which runs from \(-\infty \) to \(+\infty \). We now define a “Euclidean conformal time” \( \eta_- \), with values in \((-\infty, 0) \), and the metric becomes

\[
\begin{align*}
g(H^4/\mathbb{Z}^3; \eta_-)_{+++} &= \frac{L^2}{\eta_-^2} [d\eta_-^2 + d\theta_1^2 + d\theta_2^2 + d\theta_3^2].
\end{align*}
\]  

(4)

Complexifying \( \eta_- \to \pm i\eta_+ \), where \( \eta_+ \) takes its values in \((0, \infty) \), we obtain a spatially flat version of Lorentzian de Sitter spacetime, with toral sections, in \((++--\) signature:

\[
\begin{align*}
g(STdS_4)_{++-} &= \frac{L^2}{\eta_+^2} [d\eta_+^2 - d\theta_1^2 - d\theta_2^2 - d\theta_3^2] \\
&= dt^2 - K^2 e^{2t/L} [d\theta_1^2 + d\theta_2^2 + d\theta_3^2].
\end{align*}
\]  

(5)

where \( t \) ranges from \(-\infty \) to \(+\infty \). This is Spatially Toral de Sitter spacetime \[\text{[see } 27\text{]}\].

Notice that we can obtain the final equation in \(5\) \[up to an overall sign\] from \(3\) in a purely formal way by complexifying both \( \Phi \) and \( L \). This formal procedure has the advantage of quickly yielding the final answer for more complicated metrics which may be only asymptotically hyperbolic.

In this simple way, we can give concrete form to the intuition that partially compactified hyperbolic space can be used to define a reasonable \[that is, inflationary\] cosmology. “Creation from nothing” in the OVV sense can now be interpreted as follows. We truncate \( H^4/\mathbb{Z}^3 \) so that \( \Phi \) takes values in \((-\infty, 0) \), while Spatially Toral de Sitter, \( STdS_4 \), is truncated so that \( t \) takes values in \([0, \infty) \). The two spaces are thus both truncated along a three-dimensional torus consisting of circles of circumference \( 2\pi K \). We now join the two spaces along this torus, which is the hypersurface where the transition from Euclidean to Lorentzian geometry — that is, “creation from nothing” — takes place. We can summarize the whole construction by

\[
\begin{align*}
g(H^4/\mathbb{Z}^3; -\infty < \Phi \leq 0)_{+++} &\longrightarrow g(STdS_4; 0 \leq t < \infty)_{++-},
\end{align*}
\]  

(6)

where the arrow symbolizes the transition from a Euclidean to a Lorentzian metric, bearing in mind that the Euclidean-to-Lorentzian transition occurs along \( \Phi = t = 0 \) \[\eta_- = -L/K \text{ and } \eta_+ = L/K\].

It turns out \[15\] that this procedure, which allows the OVV wave function to have a cosmological interpretation, \textit{only} works when the spatial sections have toral topology. This topology is therefore a prediction of stringy “creation from nothing”.

This is actually self-consistent, if we recall that one of the avowed objectives of the OVV theory is to \textit{predict} the size of the Universe at its birth; in string theory, this size should be given by the string scale. Note that it is only in the toral case that the initial size is decoupled from the spacetime curvature scale — there are two independent length scales, \( L \) and \( K \), in all of the metrics discussed above. This useful property of the spatially toral version of de Sitter spacetime was applied to quantum cosmology by Zel’dovich and Starobinsky \[28\], and by Linde \[29\]; see also \[30\].
2.3. How Stringy “Creation From Nothing” Evades The Singularity Theorems

“Creation from nothing” clearly avoids an initial singularity. That fact does not absolve us of a duty to explain how the various relevant singularity theorems are circumvented. This will be needed later, when we study the initial value problem for “creation from nothing”.

Technically, “creation from nothing” avoids an initial singularity by happening along a spacelike hypersurface of zero extrinsic curvature. We have to explain how this can be arranged when the initial spatial section has a toral topology.

“Creation from nothing” is formally — not physically — related to “bounce” cosmologies. The spacetimes in the latter case have a hypersurface of zero extrinsic curvature at the bounce, so, by deleting the contracting half of the spacetime and inserting a boundary, one obtains a “creation from nothing” spacetime. Conversely, one can always construct a formal bounce cosmology from the kind of spacetime we are considering here. *This formal equivalence to a bounce spacetime explains how “creation from nothing” spacetimes evade the singularity theorem of Borde et al [21].* For this theorem assumes a preponderance of expansion over contraction up to the present, which is not the case for a bounce spacetime.

However, for spacetimes with toral spatial sections there is a much stronger singularity theorem, and this too must be circumvented. It is well known that FRW bounce cosmologies with flat spatial sections violate the Null Energy Condition if the Einstein equations hold. In fact, this statement has been greatly extended by Andersson and Galloway [31]: one of their results essentially demands a past singularity if [as is the case for any spacetime which evolves towards an inflationary state] a spacetime is non-singular and de Sitter-like to the future, provided only that the Null Ricci Condition is satisfied and that the spatial sections have the topology — not necessarily the canonical geometry — of a torus. [See [32] for further discussions of the physical implications of this remarkable theorem.] We stress that the result holds for any choice of metric on the topological torus.

The Null Ricci Condition [NRC] is the requirement that, for all null vectors $k^\mu$, the Ricci tensor should satisfy

$$ R_{\mu\nu} k^\mu k^\nu \geq 0. \quad (7) $$

The conclusion of this discussion is that the OVV version of “creation from nothing” can only work if the NRC is violated at some time, presumably near to the creation.

The NRC is equivalent to the Null Energy Condition [NEC] provided that the Einstein equations hold exactly. The question as to whether NEC violation is compatible with string theory has recently been a subject of great interest: see for example [33] [34] [35] [36]. The emerging consensus is that NEC violation is acceptable, even in the full string theory, though only under restrictive conditions: for example, the violations of the NEC involved in black hole evaporation are acceptable, as of course are those associated with orientifold planes, and Casimir effects [37] [4] [37] [38] particularly relevant to toral cosmologies can also be acceptable in some circumstances.

On the other hand, in the earliest Universe we do not expect the Einstein equations to hold exactly; if the Einstein equations are indeed modified then it becomes possible to violate the NRC while preserving the NEC. We can call this effective violation of the NEC. Again, we need to ask whether effective violation of the NEC is possible in the
context of the OVV theory — that is, in IIB string theory.

Fortunately, the answer to this is known: the existence of bouncing cosmological solutions in this case was shown explicitly by Kachru and McAllister \[38\], who investigated a brane cosmology in the context of the Klebanov-Strassler conifold in IIB theory. [More generally, the possibility of effective NEC violation in brane world cosmologies has been clearly explained in \[39\]; see also \[40\].] This can be regarded as an effective scalar-tensor theory; it is well known that such theories can easily give rise to effective violations of the NEC. [They are of considerable interest also with regard to the current accelerated expansion: see for example \[41\],\[42\],\[43\].]

Other ways are known of violating the NRC while avoiding the instabilities that often arise from genuinely NEC-violating matter fields; for example, classical constraint fields \[44\], metric-affine theories and so on \[45\]. One particularly interesting possibility \[46\],\[47\] involves ghost-free higher derivative corrections of the Einstein-Hilbert Lagrangian. These arise quite naturally in string theory \[48\].

In all these cases, including the effective theory considered by Kachru and McAllister, the NRC-violating effect can be formally represented by a “fluid” with a negative “energy density” and a causal equation of state. [NEC violation is often associated with causality violation, but that is only the case if the energy density is positive. For a perfect fluid with negative energy density $\rho$ and pressure $p$, there will be no propagation outside the local light cone, for any local observer, provided that $|p| \leq |\rho|$; this, however, automatically entails an effective violation of the NEC. That is, for a fluid with negative energy density, causality automatically ensures that the NEC is violated, not preserved.]

The equation-of-state parameter need not be constant, but in simple cases it is: for example, Casimir effects are represented by a “fluid” with a negative energy density and an equation-of-state parameter equal to $1/3$, while the Gabadadze-Shang constraint field \[44\] can be represented formally as a “fluid” with negative “energy density” and equation-of-state parameter equal to 1. [An alternative way of thinking about these generalized cosmological models is in terms of an effective [classical] potential, as in \[49\]; essentially all of the many models considered in the literature can be described in this way.] We shall assume henceforth that whatever it is that violates the NRC in the OVV theory — thereby permitting the existence of a boundary of zero extrinsic curvature — can be represented formally by a “fluid” [or classical effective potential].

Let us examine an example of the kind of metric that results when the inflaton [represented at the creation by a positive cosmological constant] is combined with such an NRC-violating “fluid”. In \[30\] and \[32\] we argued, on the basis of an analysis of possible instabilities due to pair creation of branes [as in the work of Maldacena and Maoz \[50\]] that the corrected version of spatially toral de Sitter spacetime might resemble the metric

$$g_c(\theta, L_{\text{inf}}) = \frac{dt^2}{\cosh (\frac{t}{L_{\text{inf}}})} - \left(\frac{3}{L_{\text{inf}}} \right)^2 \left( \frac{d\theta_1^2}{1 + \cos \left( \frac{3t}{L_{\text{inf}}} \right)} + d\theta_2^2 + d\theta_3^2 \right);$$

the notation is as above and as in \[32\]; $L_{\text{inf}}$ is the asymptotic inflationary length scale. This metric is obtained by solving the Friedmann equation

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{L_{\text{inf}}^2} - \frac{1}{L_{\text{inf}}^2 a^6};$$

corresponding to a combination of the inflaton with a “fluid” with a negative “energy density” and equation-of-state parameter 1, as in the work of Gabadadze and Shang \[44\].
[The special examples of ghost-free higher curvature models discussed by Biswas et al \[47\] also lead to metrics of this kind.] The metric clearly does have a hypersurface of zero extrinsic curvature, at \( t = 0 \), and it does very rapidly \([\text{with respect to proper time}]\) become indistinguishable from spatially toral de Sitter spacetime as \( t \) increases. It can be smoothly joined to the corresponding Euclidean metric at \( t = 0 \), and that Euclidean metric resembles the metric of \( \mathbb{H}^4 / \mathbb{Z}^3 \) as soon as one moves a short distance away from the transition zone.

To summarize, then: the Andersson-Galloway singularity theorem dictates a violation of the NRC if the Universe is to be created along a torus. We assume that this violation can be formally represented by the presence of a “fluid” which has a negative “energy density” \( \rho_{\text{NRC}} \) and some appropriate equation of state.

### 2.4. Beyond FRW Cosmologies

An obvious shortcoming of our discussion thus far is that it assumes what we wish to prove: the local isotropy of the initial spatial section. A theory which leads to the scale factor in equation (8) has obviously solved the problem of divergent extrinsic curvature, but it has not \([\text{yet}]\) explained why the intrinsic curvature should be so well behaved — the spatial sections are spaces of constant curvature, by assumption. Of course, a three-dimensional space with the topology of a torus need not have this maximally locally isotropic geometry. The generic metric on the torus will be enormously complicated, and the characteristic topological non-triviality of a torus will not be apparent in this generic geometry. \([\text{Think of a very distorted two-torus: the “handle” could be small, and be localized to some isolated region.}]\) It is far from clear that the wave function of the Universe will — or can — choose the maximally symmetric geometry among all these possibilities; even recognising that the topology is non-trivial involves some apparently non-local mechanism.

Thus it is not true, as is sometimes said, that a “creation from nothing” approach automatically solves the problem of the Arrow of time. In fact, Hawking \[51\] and co-workers \[52\] have attempted to establish the existence of an Arrow in the context of the original Hartle-Hawking wave function\[4\]. However, it has been argued convincingly by Kiefer and Zeh \[55\] \([\text{see also [56, 57]}]\) that this whole approach will lead to a reversal of the Arrow of time if the Universe should ultimately begin to contract. This leads to all manner of apparent internal inconsistencies, reviewed for example by Davies and Twamley \[58\]. Until these objections have been fully answered, it seems reasonable to assume that predicting a reversal is a sign that something is seriously wrong with a given attempt to account for the Arrow of time.

In any case, once again, \( \text{all} \) of this work is done in the FRW framework or small perturbations around it. It is doubtful that any firm conclusions can be drawn from this interesting but highly non-representative sample. To explain the Arrow of time in a convincing way, we need to explore \textit{all possible metrics}, satisfying basic physical conditions, defined on compact three-dimensional manifolds of given topology. \([\text{Ultimately, too, we need to understand the consequences of choosing other initial topologies.}]\) That is, we

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\[4\] A much more general argument in this direction has been given by Gibbons and Hartle \[53\]: it depends, however, on imposing a very strong positivity condition on the Ricci tensor of the \textit{entire} spacetime, and it is unlikely \[54\] that this global condition is actually satisfied in our Universe.
need to abandon the FRW assumptions entirely if we are really to explain the Arrow of time.

We can now formulate our “Arrow of time strategy” concretely as follows. We assume, in view of the preceding discussion, that the Universe was created along a three-dimensional hypersurface with the topology, but not necessarily the geometry, of a flat torus. The four-dimensional sector of the Ooguri-Vafa-Verlinde wave function describing the creation will be constructed using Euclidean manifolds-with-boundaries. Near to the boundary, each space will have a Euclidean metric of the form

$$g(F, h) = d\Phi^2 + F^2(\Phi/L, \theta^c) h_{ab} d\theta^a d\theta^b,$$

where $h_{ab}$ is a completely general metric on the three-torus and where $F(\Phi/L, \theta^c)$ is some smooth function such that the extrinsic curvature of the boundary vanishes. Complexifying both $\Phi$ and $L$, we will obtain a Lorentzian metric describing the earliest Universe. The two spaces are joined, as usual, along a common three-torus. The task is then to use this OVV wave function to show that the overwhelmingly most probable spatial geometry, when a quasi-classical spacetime geometry is selected, is an almost perfectly flat metric on this torus.

Unfortunately, the OVV construction is far from being sufficiently well-understood for it to be possible to carry out this programme directly. However, there is an indirect argument which strongly suggests that, if it could be carried out, it would lead to the desired answer. To understand this, let us briefly review the general-relativistic initial value problem, in the special form it takes for “creation from nothing”.

### 3. The Initial Value Problem for “Creation from Nothing”

Albrecht [1] classifies approaches to the Arrow of time problem into two categories: “dynamical” and “based on principles”. The best-known example of the latter is Penrose’s “Weyl Curvature Hypothesis” [12]; the general approach has been ably defended by Wald [60]. Such principles are sometimes criticised on the grounds that they seem to suggest some kind of acausal agency: how do the various parts of the initial hypersurface “know” that they should all have the same local geometry?

In fact, however, such apparently acausal conditions are a standard part of the initial value problem for field theories. For example, Maxwell’s equations do not have solutions for arbitrary initial values of the fields and charge density: $\nabla \cdot E = 4\pi \rho$ and $\nabla \cdot B = 0$ must be satisfied as initial value constraints, relating the initial electric and magnetic fields and the initial charge density $\rho$. To put it another way, given that the equations have physically well-behaved solutions, these constraints are imposed by the mathematical structure of the theory itself. This seems promising: the fact that arbitrary initial conditions are not permitted means that there was something “special” about the beginning of the Universe, and this “special” property has nothing to do with causal processes. It is very natural to conjecture that the Arrow of time was established in this way: what could be more natural than to suppose that the initial state was fixed by the internal mathematical consistency of a fundamental theory? The problem is that initial-value constraints are apparently far

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[5] See [59] for an application of the “entropic principle” of this general kind.

[6] To avoid confusion, we use $(- + + +)$ signature in this Section.
too weak to impose the kind of drastic restrictions we need to obtain an Arrow. But let us examine the question more closely.

3.1. Initial Value Theory for Spacetimes with Boundaries

In the case of the field equations of General Relativity, the initial value problem takes the following form [61]. One begins with a three-dimensional manifold $\Sigma$, on which is given a metric $h_{ab}$, a symmetric tensor $K_{ab}$, a function $\rho$ and a vector field $J^a$, together with appropriate initial data for the “matter fields”. As in the case of electromagnetism, the field equations will have no solutions if these data are given arbitrarily. A solution of the field equations with these initial data will exist — that is, spacetime itself will exist — only if the following equations are satisfied:

$$D^a [K_{ab} - K^c_c h_{ab}] = -8\pi J_b$$
$$R(h) + [K^a_a]^2 - K_{ab} K^{ab} = 16\pi \rho.$$  (11)

Here $D^a$ is the covariant derivative operator, and $R(h)$ is the scalar curvature, defined by $h_{ab}$. This solution will define a unique globally hyperbolic spacetime containing matter fields [evolved forward from suitable data on $\Sigma$] generating a stress-energy-momentum tensor $T_\mu\nu$. This spacetime allows us, retrospectively, to assign interpretations to the “initial” data, as follows. First, $\Sigma$ is a spacelike hypersurface with induced metric $h_{ab}$ and extrinsic curvature $K_{ab}$. Second, we can now speak of a unit timelike vector field $n^\mu$ normal to this hypersurface, and we will find that $J^a$ is the projection into $\Sigma$ of the vector field $-T^\mu_\nu n^\nu$. We will also find that $\rho$ is just

$$\rho = T_\mu^\nu n^\mu n^\nu.$$  (12)

The vector field $-T^\mu_\nu n^\nu$ is the energy-momentum flux vector as seen by a family of observers with unit tangent $n^\mu$. Therefore, $\rho$ is simply the energy density on $\Sigma$.

Now our objective is to determine what happens if we study the initial value problem in the case where $\Sigma$ is the boundary [assumed to be connected, and with the topology of a three-dimensional torus] of a smooth, paracompact manifold-with-boundary. The following simple theorem [see [62], page 200] is now relevant:

**THEOREM [Collar Neighbourhood]**: If $X$ is a smooth paracompact manifold-with-boundary, and $\partial X$ is the boundary, then there exists an open neighbourhood of $\partial X$ in $X$ which is diffeomorphic to the product $\partial X \times [0,1)$.

What this means is that, near to the boundary, the space has the product topology of a globally hyperbolic spacetime [61], pages 208-209]. Thus, the neighbourhood of the boundary has a suitable structure for the initial-value theory. We shall assume, in fact, that the resulting solution can be extended to the entire manifold-with-boundary.

Now we assume, as in Section 2, that the boundary is a hypersurface of zero extrinsic curvature. Then the constraints simplify to the equations

$$R(h) = 16\pi \rho$$
$$J^a = 0.$$  (13)

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7A spacetime is said to be *globally hyperbolic* if it possesses a hypersurface which intersects all inextensible timelike and null curves.
The second equation just means that the total energy-momentum flux vector is parallel or anti-parallel to the unit normal $n^\mu$. If we adopt Gaussian normal coordinates based on $\Sigma$, then this just states that the time-space components of the stress-energy-momentum tensor vanish. That is manifest for the inflaton tensor given in (1), since those components reverse sign with time, and $\Sigma$ is a hypersurface of time symmetry. A similar argument applies to the other contributions to the time-space components.

There is a more interesting way of interpreting the second equation. Take any smooth manifold-with-boundary $X$, and let $m$ be a point on the boundary, $\partial X$. The tangent space to $X$ [not $\partial X$] at $m$ is naturally divided into two subsets, defined as follows [see [62], page 200]. Let $V$ be a tangent vector to $X$, but not to $\partial X$, at $m$. Then $V$ is said to point inwards if $V$ is the tangent vector at $m$ of a smooth curve $\gamma : [0, \epsilon) \to X$ with $\gamma(0) = m$. Vectors pointing outwards are defined analogously, using curves of the form $\gamma : (-\epsilon, 0] \to X$, again with $\gamma(0) = m$.

Let us choose the unit normal $n^\mu$ to point inwards. Then the second equation in (13) just says that the energy-momentum flux vector on $\Sigma$ either points perpendicularly into spacetime or out of it [or is zero]. But clearly, if the classical or quasi-classical Universe is being created from “nothing”, then the energy-momentum flux vector on $\Sigma$ cannot point “outwards” from spacetime. We therefore conclude that, in terms of classical language,

$$\rho = \rho_{inf} + \rho_{NRC} \geq 0,$$

(14)
everywhere on $\Sigma$; here $\rho_{inf}$ is the [positive] energy density of the inflaton, and $\rho_{NRC}$ is the [negative] effective energy density discussed in the previous section.

This is obviously a reasonable condition in the quasi-classical domain, but we ask the reader not to think of it in terms of the familiar energy conditions [such as the dominant or weak energy conditions] — though of course it does have the interpretation that “normal” matter, represented by $\rho_{inf}$, cannot be outweighed by the exotic NRC-violating “fluid” represented formally by $\rho_{NRC}$. Rather, it expresses the idea that, whatever it may do in the interior of spacetime, the energy-momentum flux vector of a quasi-classical spacetime should not, when evaluated on the boundary, point in a “direction” which literally does not exist. In particular, we do not insist that (14) should hold throughout spacetime, away from the boundary. We also do not require it to be true of all of the spaces surveyed by the OVV wave function. Instead, it is to be imposed only on the “initial” boundary of that spacetime which is selected as constituting the quasi-classical world. [The precise way in which this selection is done is of course a deep question which we shall not consider here: see for example [63] and the discussion of the “emergence of classicality” in Section 7 of [1].]

To summarize, then: from the point of view of the initial value problem, the only special condition imposed by “creation from nothing” is that the [total] quasi-classical energy-momentum flux vector should not be allowed to point outwards from the boundary. That imposes (14), which is, or appears to be, a very mild restriction indeed. This is to be fed into the first member of (13), as the only restriction on the geometry of $\Sigma$.

### 3.2. Can The Constraints Always Be Satisfied?

In the case of the initial value problem for electromagnetism, the initial value constraints do not merely restrict the relationships between the initial fields and their sources: they
also forbid certain kinds of situations altogether. To take a relevant example: on a flat torus, the integral version of Gauss’ law shows immediately that the electric charge density cannot be everywhere strictly positive, and the only way it can be everywhere non-negative is by being identically zero. This is still, however, a very weak condition on the electric field, since there are clearly infinitely many distinct vector fields having $\nabla \cdot \mathbf{E} = 0$. The problem, of course, is that the divergence is a sum of terms which need not, individually, have any particular sign, so the total can easily be zero.

In view of this, it would appear that our condition (14), which just requires that the metric on the initial [boundary] hypersurface $\Sigma$ should satisfy

$$R(h) \geq 0,$$  \hfill (15)

does not restrict the geometry of $\Sigma$ very strongly. For the scalar curvature is essentially just the sum of the curvature tensor components, and for a distorted metric on the sphere or the torus one has no grounds for predicting a preponderance of one sign over the other. [Thus for example one can construct metrics on $S^3$ with enough negative curvature components so that the total is negative, although of course some components are positive; see [64]. In fact, since there is no analogue of Gauss’ theorem here, we expect the restriction to be even milder than in the case of electromagnetism. Nevertheless, the relation between the scalar curvature and the metric is very intricate, so we should not jump to the conclusion that (15) imposes no significant restriction on the metric.

Let us consider the case where $\Sigma$ has the topology of a three-dimensional sphere, as in the original Hartle-Hawking construction [18]. The question now is this: given a function $S(x)$ on $S^3$, which is nowhere negative but otherwise arbitrary, can one find a metric $h_{ab}$ on $S^3$ such that the scalar curvature of $h_{ab}$ is precisely equal to $S(x)$?

Intuitively, one might think that the answer should always be positive. We have, after all, all six independent components of $h_{ab}$ at our disposal, and only one condition to be satisfied. If this is so, then of course it means that (14) alone is an exceedingly weak constraint. It would also mean that the round metric on $S^3$, and small perturbations of it, constitute an infinitesimal minority of the possible metrics satisfying (15); one would then conclude that perturbations around the corresponding FRW spacetime probably do not adequately explore the set of all spacetimes with topologically spherical initial data.

Surprisingly, the answer to this question is known.

### 3.3. The Kazdan-Warner Classification

The question we have raised here is a particular case of the following problem. Let $M$ be a smooth compact manifold [without boundary] and let $S(x)$ be a given smooth function on $M$. Let $h_{ab}$ be a metric on $M$ with scalar curvature $R(h)$. Can the second-order partial differential equation

$$R(h) = S(x),$$  \hfill (16)

be solved for $h_{ab}$?

The answer to this question takes a rather strange form. Instead of depending mainly on the details of $S(x)$, it depends primarily on the [differential] topology of $M$, and takes
the form of a classification of smooth compact manifolds [without a fixed metric]. This very remarkable classification is given by the Kazdan-Warner Theorem, as follows \cite{65}:

**THEOREM [Kazdan-Warner]:** All compact manifolds of dimension at least three fall into precisely one of the following three classes:

- **[P]** On these manifolds, every smooth function is the scalar curvature of some Riemannian metric.
- **[Z]** On these manifolds, a smooth function can be a scalar curvature of some Riemannian metric if and only if it either takes a negative value somewhere, or is identically zero.
- **[N]** On these manifolds, a smooth function can be a scalar curvature of some Riemannian metric if and only if it takes a negative value somewhere.

For example, spheres obviously do have a metric of strictly positive scalar curvature; since the theorem classifies all compact manifolds of dimension at least three, it follows that all spheres of dimension at least three lie in KW class P; this means that equation (16) always has a solution for \( S(x) \), no matter how intricate it may be, on \( S^3 \). *Any* initial distribution of energy, positive, negative, or mixed, is compatible with the initial-value constraints: just choose the three-dimensional metric to be the corresponding solution of the scalar curvature equation.

As we feared, requiring that the energy density should be non-negative along the initial hypersurface \( \Sigma \) is an even weaker condition, *when the initial spatial topology is spherical*, than requiring non-negative electric charge density. But the situation is very different in the case of toral topology.

### 4. The Amazing Torus

It is clear that the KW class of \( T^3 \) cannot be N. The question is whether it is P or Z. We remind the reader that intuitions based on the canonical metrics [or on the two-dimensional case, where the Gauss-Bonnet theorem applies] are highly misleading: there certainly are metrics of strictly negative scalar curvature on both \( S^3 \) and \( T^3 \) — indeed, the Kazdan-Warner theorem implies that there are such metrics on *all* compact three-dimensional manifolds.

In fact, the question of the KW class of the torus was settled only relatively recently, by extremely deep results due to Schoen, Yau \cite{66}, Gromov, and Lawson \cite{67}. Let us examine the global geometry of \( T^3 \) more closely.

We can define \( T^3 \) abstractly as the quotient \( \mathbb{R}^3/\mathbb{Z}^3 \), where one can think of \( \mathbb{Z}^3 \) as a *lattice*, that is, the additive group of vectors with integer components relative to some fixed basis; if for example we take this basis to be the canonical orthonormal basis in \( \mathbb{R}^3 \), then we are dealing with a cubic torus. Now let \( n \) be *any* positive integer, and define \( (n \cdot \mathbb{Z})^3 \) in the obvious way. Then \( \mathbb{R}^3/(n \cdot \mathbb{Z})^3 \) is clearly a covering manifold of the original torus; it is an \( n^3 \)-fold covering. Thus we see that we can find *arbitrarily large* coverings of \( T^3 \), since \( n \) can be as large as we please. [By contrast, the real projective space \( \mathbb{R}P^3 \) can be “made larger” by taking a covering, but not *arbitrarily* larger.] This notion can be generalized [by defining “arbitrarily large” in terms of the amount by which the lengths
of tangent vectors are contracted by the covering map]. One says that manifolds like $T^3$ having this property are *enlargeable*. [See [67], page 302, for the formal definition.]

“Enlargeability” has several interesting properties: for example, the connected sum of any manifold with an enlargeable manifold is again enlargeable. Many compact three-manifolds are enlargeable: in particular, any manifold admitting a metric of non-positive sectional curvature is enlargeable; thus, tori [and their non-singular quotients] are very special representatives of this large class.

The work of Schoen, Yau [66], Gromov, and Lawson [67], page 306] can be summarized as follows:

**THEOREM [Schoen-Yau-Gromov-Lawson]:** There is no metric of positive scalar curvature on any compact enlargeable spin manifold.

The proof of this theorem is deep, as is evidenced by the appearance of the “spin” condition; while apparently irrelevant, it is in fact necessary. [In three dimensions, however, it is automatically satisfied.]

It follows from this theorem that compact enlargeable spin manifolds can never be in Kazdan-Warner class P. Now tori are compact, enlargeable, and spin; hence, we conclude that all tori are in KW class Z.

We see now, from the Kazdan-Warner theorem, that the only way for condition (14) to be satisfied is by having

$$\rho = \rho_{\text{nat}} + \rho_{\text{NRC}} = 0,$$  \hspace{1cm} (17)

everywhere on the initial torus. The situation is therefore analogous to the case, discussed earlier, of the Maxwell initial value constraint $\nabla \cdot E = 4\pi \varrho$: we saw that, on a torus, $\varrho \geq 0$ actually implies $\varrho = 0$. As in that case, however, this appears to be a weak constraint, analogous to $\nabla \cdot E = 0$, on the metric: as we have repeatedly emphasised, the scalar curvature is nothing but the total of all of the curvature components, so we expect that it should be easy to find many metrics of vanishing scalar curvature on the initial torus.

This, however, is where the analogy with the electromagnetic case breaks down; and it breaks down spectacularly. For Gromov and Lawson, extending a theorem of Bourguignon, were able to prove [67], page 308] the following theorem.

**THEOREM [Bourguignon-Gromov-Lawson]:** If a metric on a compact enlargeable spin manifold has zero scalar curvature, then that metric must be exactly locally flat, that is, the curvature tensor must vanish identically everywhere on the manifold.

This is an astonishing result: the vanishing of a *sum* of curvature components, the scalar curvature, forces each curvature component to vanish separately on these manifolds.

We now see that, in sharp contrast to the case of “creation from nothing” on $S^3$, the initial value constraints in the case of topologically toral initial data are enormously

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8The connected sum of two manifolds of the same dimension is formed by deleting the interior of a ball in each, and joining the resulting spaces with a cylinder.

9Recall that an orientable Riemannian manifold is said to be *spin* if the $SO(n)$ structure group of its bundle of orthonormal frames can be lifted to $Spin(n)$. 

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powerful: *the only way in which* (14) *can possibly be satisfied is if the initial torus is, classically, perfectly flat.*

We claim that this is the origin of the initial “specialness” from which the Arrow of time derives. The internal mathematical consistency of the OVV theory, in the form of the initial value constraints, forces the Universe to be created in an extremely non-generic initial state: the initial spatial section has to be locally isotropic, with the consequences explained in Section 1.

More generally, we can ask: what happens if we assume that the Universe was created along a compact three-dimensional space with some non-toral topology? This is a question which could not have been fully answered until very recently; but by combining one of the recent celebrated results due to Perelman [68] with the remarkable work of Gromov and Lawson, we can now do so: for these theorems allow us to specify precisely which compact three-dimensional manifolds belong to the respective KW classes. Briefly, the classification runs as follows. [We confine ourselves to the case of orientable spaces; the extension to the non-orientable case is interesting, but raises no issues relevant to our discussion here.]

Milnor’s [69] “prime decomposition” theorem states that any compact orientable 3-manifold $M^3$ can be expressed as a connected sum in the following way:

$$M^3 = P_1 \# P_2 \# \ldots \# (S^1 \times S^2) \# (S^1 \times S^2) \# \ldots \# K_1 \# K_2 \# \ldots, \quad (18)$$

where each $P_n$ is a compact manifold with a finite fundamental group, where $\#$ denotes the connected sum, and where each $K_n$ is an Eilenberg-MacLane space of the form $\tilde{K}(\pi, 1)$. Now it is known [67], page 324 that no $K(\pi, 1)$ can accept a metric of positive scalar curvature, and that the same is true of the connected sum of a $K(\pi, 1)$ with any other compact manifold. [This gives an alternative explanation of the fact that the torus has KW class Z.] Therefore, if $M^3$ has KW class P, there can be no $K_n$ factors in its Milnor decomposition. Furthermore, Perelman’s proof [68] of the elliptization conjecture means that each $P_n$ is diffeomorphic to a manifold of the form $S^3/\Gamma$, where $\Gamma$ is a finite group drawn from a completely known list [given, for example, in 70][71]. Next, it is known [72] that the connected sum of two compact manifolds admitting metrics of positive scalar curvature likewise admits a metric of positive scalar curvature. Finally, it can be shown [67], page 308 that if a manifold is in KW class Z, then any metric of zero scalar curvature on that manifold must in fact be Ricci-flat; in three dimensions, that means that it must be flat.

Combining all these results, we have the following classification. Let $M^3$ be a compact three-dimensional orientable manifold. Then the following statements hold.

- $M^3$ has KW class P if and only if it can be expressed as
  $$M^3 = S^3/\Gamma_1 \# S^3/\Gamma_2 \# \ldots \# (S^1 \times S^2) \# (S^1 \times S^2) \# \ldots, \quad (19)$$

  where each $\Gamma_i$ belongs to an infinite but completely known list of finite groups.

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10A $K(\pi, 1)$ space is just a compact manifold whose only non-trivial homotopy group is its fundamental group.
• $M^3$ has KW class $Z$ if and only if it has the topology of one of the compact orientable platycosms, listed in [70, 73]; these are manifolds of the form $T^3/\Omega$, where $\Omega$ is one of six possible finite groups.

• All other compact orientable 3-dimensional manifolds are in KW class N.

We see that the members of KW class $P$ are very special, while class $Z$ is so special that its members can be given explicitly in a finite [and short] list. In this sense, the generic compact three-dimensional manifold is in KW class $N$. Note that the fact that compact spaces of constant negative curvature are enlargeable, together with known facts about the topology of such spaces, imply that they are in class N; see page 306 of [67].

Now it is easy to see that everything we said earlier regarding $S^3$ applies equally to every manifold in KW class $P$. For all of these spaces, there are spacetimes of arbitrary complexity which can, via the initial value constraints, satisfy condition (14), and so we cannot expect the initial value constraints to yield an Arrow of time if the Universe is created along a space with such a topology.

At the other extreme, we have the enormous collection of three-dimensional manifolds in KW class $N$; but none of these has any metric of non-negative scalar curvature. According to the initial value constraint equations, then, they can never satisfy condition (14), under any circumstances. One can for example take a compact manifold of constant negative curvature, and deform it in an arbitrarily complicated way, but it will never be able to satisfy (14). The apparently innocuous condition (14) has turned out to be very powerful: it rules out all of the many three-dimensional manifolds with topologies putting them in KW class $N$.

The only survivors are the six [orientable] members of KW class $Z$. These are, in Conway’s [prescient] “cosmic” terminology [73], the torus or torocosm $T^3$, the dicosm $T^3/\mathbb{Z}_2$, the tricosm $T^3/\mathbb{Z}_3$, the tetracosm $T^3/\mathbb{Z}_4$, the hexacosm $T^3/\mathbb{Z}_6$, and the didicosm or Hantzsche-Wendt space $T^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. These three-dimensional compact orientable spaces, and these alone, can lead to an Arrow of time in the way we have discussed here.

5. Some Consequences

The solution of the Arrow of Time problem advocated here differs very greatly from previous proposals, and this in several ways. Some of these are obvious: the non-reliance on rare fluctuations, the reliance on deep topological-geometric properties of the torus and its descendants, and so on. Others are less obvious, and will be briefly discussed here.

5.1. Remarks on Time Orientation

We have asserted that the crucial condition (14) [which in fact reduces to equation (17)] does not represent an energy condition, and in fact we only insist that it should hold on the initial hypersurface. Actually, however, the term $\rho_{\text{NRC}}$ in equation (17) decays away very rapidly with the expansion: it was argued in [32] that this rapid decay is demanded as a by-product of conditions ensuring “stringy” non-perturbative stability, such as those discussed in [64]. In fact, the decay is probably as rapid as causality allows; if we assume
an approximately constant equation-of-state parameter, the decay is according to the
inverse sixth power of the scale factor. The specific metric obtained in that case is in
fact precisely \( g_c(6, K, L_{\text{inf}})_{++--} \), given in equation (8), as one can see from equation (9).
This metric gives an approximate description of the spacetime geometry during the pre-
inflationary era; we remind the reader that it rapidly approaches the metric of Spatially
Toral de Sitter spacetime, that is, the inflationary metric.

It should be noted that the total energy density in this spacetime is given by the simple expression
\[
\rho = \frac{3}{8\pi L_{\text{inf}}^2} \tanh^2 \left( \frac{3}{L_{\text{inf}}} \right) \geq 0.
\] (20)
The inequality will remain valid even if we do not model the inflaton by a cosmological
constant, since the inflaton energy density will certainly decay much more slowly than the negative term. Thus, while (14) is not itself an energy condition, it does imply a [very weak] condition of this sort: the total energy density as seen by the fundamental observers is always non-negative. That is, for these observers, the total energy-momentum flux vector points, for all \( t > 0 \), towards [what we may now justly call] the future. This vector field establishes a time-orientation for the early Universe.

In short, the theory advanced here establishes a well-behaved flow of time throughout
the earliest era of the Universe, not just at \( t = 0 \) itself. The importance of establishing this time orientation in any theory of the Arrow of time is discussed in detail by Aiello et al [74].

5.2. The Shape of the Earliest Universe

By using the methods of global differential geometry explained in the preceding section, we have shown that the only way to satisfy condition (14) is for the Universe to be “created from nothing” along a boundary which is perfectly isotropic at every point. The standard arguments of classical cosmology now imply that the spacetime metric is, initially, an exact FRW metric. Thus, the FRW family of metrics with flat [but compact] spatial sections are good models of spacetime at very early, as well as at very late times. In particular, the spacetime with the metric \( g_c(6, K, L_{\text{inf}})_{++--} \) we discussed earlier may be a more accurate representation of the earliest spacetime geometry than we originally supposed. The details of this geometry are therefore of some interest; let us mention a few unusual features.

The most important question in this regard is the value of the parameter \( K \), which fixes the size of the Universe at its birth. In a string theory formulated on a torus, T-duality implies that by far the most natural initial length scale would be the string length scale. This is normally taken to be considerably larger than the Planck scale. Let us assume that \( K \) is indeed given by the string scale.

This scale is independent of the inflationary length scale, \( L_{\text{inf}} \), which is roughly two orders of magnitude larger than \( K \). By computing the full extent of conformal time [30], which we denote by \( \Omega \), one can show that the Penrose diagram [Figure 1] will be a rectangle which is roughly 100 times as high as it is wide. [The width of the diagram is \( \pi \), corresponding to any one of the angular coordinates on the torus.] The horizontal line at the bottom of Figure 1 represents the creation of this universe at proper time \( t = 0 \),
while the upper horizontal line is future conformal infinity, that is, physically, the end of
the inflationary era.

\[ g_c(6, 0.01 L_{\text{inf}}, L_{\text{inf}}) + \ldots, t \geq 0. \]

Figure 1: Penrose diagram corresponding to \( g_c(6, 0.01 L_{\text{inf}}, L_{\text{inf}}) + \ldots, t \geq 0. \)

The principal consequence of this is that the early Universe was small, in the technical
sense that circumnavigations were easily performed. That is, all parts of the spatial
sections are in causal contact [symbolized in the diagram by the worldline of a photon
circumnavigating the torus] until nearly the top of the diagram. This means, as Linde
points out [29], that chaotic mixing [75] will prevent the growth of perturbations until
such circumnavigations cease to be possible. This happens at about \( \pi \) units of conformal
time below the top of the the diagram; at that time, the spatial sections will, because
of chaotic mixing, still be extremely regular. Thus, the FRW structure continues to be
an excellent approximation throughout this period. One might prefer to regard the point
which is \( \pi \) units below the top of the diagram as the “start” of Inflation, since by that
time the Universe has expanded to a size comparable to \( L_{\text{inf}} \); so that the earlier period
is the “pre-inflationary” era during which the conditions for Inflation to begin, in the
conventional sense, were prepared.

In summary, the theory of the Arrow presented here leads to a definite picture of the
geometry of the earliest Universe. It is a FRW geometry with an unusual causal structure.
This causal structure is such that the non-trivial topology of the spatial sections would
have been very much in evidence in the pre-inflationary era. If the sections have non-trivial
holonomy groups\textsuperscript{11}, for example, this could have physical consequences. This observation may ultimately allow us to distinguish physically between the various “platycosms” in KW class Z [all of which, apart from the torus, have finite non-trivial holonomy groups].

5.3. The Weyl Curvature Hypothesis

We argued above that a FRW metric like $g_c(6, K, L_{\text{inf}})$, given in (8), yields a good approximation of the earliest spacetime geometry. Now $g_c(6, K, L_{\text{inf}})$ can be expressed in the following form:

$$g_c(6, K, L_{\text{inf}}) = L_{\text{inf}}^{2} G(\eta) [d\eta^2 + d\theta_1^2 - d\theta_2^2 - d\theta_3^2],$$  

where $\eta$ is conformal time, defined on a certain finite interval [of length fixed by $L_{\text{inf}}/K$], and where $G(\eta)$ is a certain function which diverges at a finite value of $\eta$, which represents the infinite future in proper time. The metric is manifestly conformally flat.

Penrose \textsuperscript{12} has suggested that the Arrow of time is due to the vanishing of the Weyl tensor at initial, and only at initial, cosmological singularities. The present work may be regarded as an attempt to provide a rationale for the Weyl Curvature Hypothesis [adapted to the case of “creation from nothing”] by embedding it in string theory. We should note, however, that our version is much stronger than Penrose’s. For since all FRW metrics have zero Weyl curvature, the latter would allow \textit{any} FRW metric near the creation. Here we have been led to FRW models with necessarily flat spatial sections, with metrics which are \textit{globally} conformal to [spatially compactified] Minkowski spacetime.

It may be of interest at this point to recall that the Euclidean-to-Lorentzian transition is performed here by complexifying \textit{conformal} time: so, up to a global conformal factor, the complexification here is just the usual Wick rotation of flat-spacetime quantum field theory. Perhaps this will answer some or all of the objections to performing complexifications of time coordinates on curved spacetimes.

To summarize: we argue that Penrose’s “Weyl Curvature Hypothesis” can be naturally embedded in, and justified by, the OVV version of “creation from nothing”. However, so far we have only explained why the Weyl curvature should be expected to vanish at the \textit{beginning} of time. Penrose claims that we should not expect it to vanish at the end [if any]. We now explain how this claim, too, can be justified by our approach.

5.4. Pleading Innocent to Cosmic Hypocrisy

Price \textsuperscript{2} has argued very persuasively that most accounts of the Arrow of time are guilty of a “double standard”: they make use of arguments about the beginning of time that seem to apply equally to its \textit{end}. This leads to conclusions that few can accept. A well-known example is Inflation: if it is “natural” [in the sense of “requiring no specific mechanism”] for it to occur at the beginning, then it must be equally natural for “deflation” to occur should the Universe eventually contract. Most authors find this hard to believe.

Here we have argued that Inflation is not “natural” in this sense: it only occurs because of the demands of the internal mathematical consistency of string theory, as revealed by

\textsuperscript{11}These are the groups generated by parallel transport around closed loops. Even for flat manifolds, parallel transport around non-contractible loops can be non-trivial.
the Bourguignon-Gromov-Lawson theorem. Whether “deflation” will occur if the Universe eventually contracts is a matter to be settled in the same way.

Before we begin, let us note that it is far from certain that string theory even permits eventual contraction. If the current cosmic acceleration is due to a true cosmological constant, there will be no contraction; and some have argued \[76\] that string theory favours a cosmological constant as the explanation of the acceleration. However, let us leave this to one side, and consider universes which do re-contract, since this will throw some light on basic questions regarding the Arrow. In particular, let us consider cosmological models which, like those discussed by Maldacena and Maoz \[50\], actually have a small negative cosmological constant.

A negative cosmological constant, however small in magnitude, will eventually halt the expansion; for all other fields, including \[especially\] the NRC-violating “fluid”, will be insignificant when the Universe is extremely large. The subsequent contraction, however, will eventually revive those fields, and the repulsion generated by the “fluid” will finally dominate and halt the contraction. That is, a final singularity will be averted in the same way as the initial singularity. The result will be a “bounce”, as in the work of Kachru and McAllister \[38\] \[see also \[45\]\], along a spacelike hypersurface of zero extrinsic curvature.

We now claim that the irregularities which developed during the expansion will grow rapidly as the Universe contracts; so the geometry at the bounce, while not singular, should be extremely irregular. That is, entropy \[of all kinds\] will continue to increase up to, and beyond, the bounce.

The question Price would raise at this point is this: can this claim be internally consistent? Why does the mechanism which enforced low entropy at the creation not enforce it at the bounce?

The answer has two parts. First, the initial boundary is very different from the bounce hypersurface: \[the latter has both a past and a future\]. Therefore we have no motive, in this case, for enforcing the requirement that the total energy-momentum flux vector should point in any particular direction, when evaluated on that hypersurface. Since the bounce itself is due to the dominance of the negative “energy density” \(\rho_{\text{NRC}}\), there is no contradiction in assuming that the total flux vector points in one direction at some points on the hypersurface, and in the opposite direction at others: that is, the total density need not be non-negative everywhere on the hypersurface.

However, this argument seems to suggest that the asymmetry here is based on a claim that the “end” of the Universe is objectively different from its beginning. There is such a difference — the “initial” hypersurface does not have a past — but this is not the real crux of the matter. To understand what is happening here, let us proceed as follows. We saw that the contraction is ultimately halted, along a hypersurface of zero extrinsic curvature, by the dominance of the negative quantity \(\rho_{\text{NRC}}\). Let us assume, in order to maintain an exact logical symmetry with the situation at the beginning of time, that the total density is non-positive everywhere at the bounce.

This brings us to the real point: we know that non-negative energy density on a topologically toral hypersurface of zero extrinsic curvature has drastic consequences for the intrinsic geometry. What are the consequences of non-positive energy density? A contradiction will indeed arise if, as one might expect on symmetry grounds, non-positive energy density has similar consequences to non-negative energy density.
Since we are dealing, once again, with a hypersurface of zero extrinsic curvature, the initial value constraints again reduce this to a question about scalar curvature. The answer to our question is then given by the Kazdan-Warner theorem: since the torus and its quotients are in KW class Z, every smooth non-positive function, no matter how convoluted, can be the scalar curvature of a [similarly convoluted] metric on these spaces. [In fact, the same conclusion holds even if there are some points where the function is positive.] It follows that there is no contradiction in assuming that the spatial sections have increasingly distorted geometry as the bounce is approached.

Ultimately, then, the reason that the Universe can have a low-entropy beginning and a high-entropy “end” [the bounce] can be traced back to the asymmetry in the set of all possible metrics on the torus. The torus — in sharp contrast to the sphere, on which “anything goes” — favours one sign of scalar curvature over the other. This is how it is possible for one “end” of the Universe to be more irregular than the other. It is very striking that this extremely deep property of the set of all possible toral geometries accounts for such a basic property of our Universe.

The case of the interior of black holes is also instructive. If the Arrow of time is somehow related to the increase or decrease of the volume of regions of space, then one is entitled to ask, as Price does [2], whether the Arrow of time reverses inside the black hole event horizon. The answer here is clear: the Arrow does not reverse. Consider a black hole, formed from the collapse of a star, subject to Cosmic Censorship. That is, the singular region [or whatever replaces it in string theory] is spacelike, and the black hole does not affect the topology of the spatial sections. Now one hopes that string-theoretic effects somehow resolve the singularity, and this may well impose special conditions on the parts of spatial sections which are inside the event horizon. Whatever these conditions may be, however, they do not control the geometry of those parts of a spatial section which lie far outside the horizon; hence there is, here, simply no analogue of the global vanishing of the extrinsic curvature, or of the condition (14) which we imposed globally on the initial hypersurface. Hence there is no reason to think that the Arrow of time anywhere, including the interior of the event horizon, is affected by the unusual geometry near to [whatever replaces] the classical singularity.

Price’s [2] discussion of these questions is valuable, because it makes it very clear that no local mechanism can give an account of an Arrow of time which never reverses. As Price insists, whatever caused the initial state of the Universe to be so regular might well act in the same way on a black hole singularity; and his argument has the more force in this case, in that the absence of any evidence for this possibility is, for obvious reasons, not surprising. The claim is that conditions near to the singularity of a [Cosmically Censored] black hole are similar to those near to a generic cosmological singularity. This is reasonable, but it does not allow us to conclude that the Arrow reverses inside a black hole, because the Arrow is, we claim, a truly global property of the Universe. It is only when we consider the geometry of an entire compact space that we find strange phenomena such as the extreme asymmetry of the space of all metrics on the three-torus.

In summary: in the present theory, the Arrow of time is correlated with the cosmic expansion only in the sense that both have a common origin, the condition (14) which just expresses the idea that the energy-momentum flux vector should point inwards at the boundary of spacetime — that is, the idea that the Universe was created from nothing. [As
we explained above, this condition implies (17). While the total energy density vanishes initially, the total pressure does not; with a causal equation of state for the NRC-violating “fluid”, the initial total pressure must have been negative. The Raychaudhuri equation shows that this “primordial pressure” forces the Universe to begin expanding as soon as it is born.] In the unlikely event that the Universe should begin to contract, that correlation will be broken: the Arrow of time never reverses.

6. Conclusion

The existence of an Arrow of time is the most wildly improbable feature of our Universe. It is therefore the feature most urgently in need of explanation. We have proposed an explanation here, one based ultimately on the extraordinarily atypical and asymmetrical structure of the space of Riemannian metrics on the torus. These special properties of the torus are communicated to the inflaton, putting it into its lowest-entropy state, by constraints imposed by the internal mathematical consistency of the theory. Much remains to be done to complete this explanation: in particular, of course, we need a much better understanding of the OVV wave function. Let us conclude by mentioning some less apparent issues.

First, an essential step in our argument exploits the detailed structure of the initial value problem for gravity. Note that this is still by no means a closed subject; work continues on understanding it, particularly in the most physically interesting case [scalar fields interacting with gravity]; see for example [77]. Further study of this problem may well be of great interest in connection with the Arrow.

Secondly, our main technical resource in this work has been the global differential geometry of the scalar curvature invariant. This is a well-developed subject, which, however, is also an area of active research; see [78] for mathematical references and a possible physical application. Understanding the ways in which topology constrains geometry is the problem at the core of modern global differential geometry, and it is likely to have further physical implications.

Finally, the following speculation is suggested by this work.

It has often been claimed [22] that “baby universes” can branch off from a given Universe. If these babies can be sufficiently similar to our observed Universe, then we could be living in one of the babies.

It is one thing to produce a baby; quite another, however, to ensure that the baby will behave in the way that one wishes. In this work, we have suggested that the Arrow of time is a consequence of the extremely special conditions that arise when a [toral] Universe is created from nothing. Baby universes, by definition, are not created from nothing; they arise in quite a different way. We therefore conjecture that there is no Arrow of time in any baby universe.

Evidence for this conjecture will be presented elsewhere. Meanwhile, we leave the reader with the following observation. While the conditions for the existence of sentient life are notoriously controversial, surely all can agree that it does not exist in conditions of thermal equilibrium.\footnote{Here we are neglecting the possible existence of “Boltzmann brains”. See for example [79][80][81] for recent interesting discussions of this question.}
to establish an Arrow of time in a baby universe, then the Landscape might be very drastically depopulated. If, for example, baby universes never have an Arrow, then we, the inhabitants of this Universe, may perhaps be alone in the “multiverse”. Whether or not this is so, the following general point should be clear: nothing can be said as to whether a “multiverse” can be populated until the Arrow of time is thoroughly understood. It is to be hoped that this realization will spur further interest in this perplexing problem.

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