TWO-LOOP CORRECTIONS
FOR
ELECTROWEAK PROCESSES

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Abstract

Theoretical uncertainties affecting electroweak observables are reviewed and the relevance
of two-loop electroweak radiative corrections for the precision tests of the Standard Model
is discussed.

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The increasing accumulation of electroweak data over the last years have brought to full maturity the program of precision tests of the Standard Model (SM). One of the most remarkable outcome of such a program is the determination of the top quark mass from a fit of the available data. By considering the LEP data alone, one finds \[ m_t = 176 \pm 10^{+17}_{-19} \text{ GeV} \] with a $\chi^2$ per degree of freedom of 8.8/9. It has been known since a long time that the potentially largest electroweak radiative corrections are those related to $m_t$. In the absence of a direct evidence for the top quark, this property of the SM has been exploited to indirectly bound $m_t$. The absolute accuracy reached in constraining the top mass is now impressive and the good quality of the fit provides a strong consistency check of the SM beyond the tree level approximation. In addition, there is a striking agreement between the above quoted value of $m_t$ and the recent results of a direct search of the top quark at the Tevatron collider \[ m_t = 176 \pm 8\text{(stat)} \pm 10\text{(sys)} \text{ GeV} \] \[ m_t = 199_{-21}^{+19}\text{(stat)} \pm 22\text{(sys)} \text{ GeV} \] Such an agreement puts on a very sound basis our understanding and interpretation of the electroweak data within the framework of the SM as a renormalizable quantum field theory and places severe constraints on possible extensions of the model.

A closer look to the data reveals that the achieved experimental precision went by far beyond the initial expectations \[ \text{CDF} \]. As an example, consider the most recent LEP data for the width of $Z$ into charged leptons $\Gamma_l$, the effective Weinberg angle in the leptonic sector $\theta^l_{eff}$ and the $W$ mass $M_W$ \[ \Gamma_l = 83.94 \pm 0.13 \text{ MeV} \] \[ \sin^2 \theta^l_{eff} = 0.2318 \pm 0.0004 \] \[ M_W = 80.26 \pm 0.16 \text{ GeV} \] The relative accuracy in these measurements has reached the level of $1-2$ per mill. It is conceivable that the experimental precision may further improve in the future. The completion of the LEP I phase could lead to an overall amount of $2 \cdot 10^7$ $Z$, whereas the SLAC program aims for a total of $5 \cdot 10^5$ $Z$ by the end of 1998. This could reduce the error on $\sin^2 \theta^l_{eff}$ to 0.0002. On the other hand, the measurement of $M_W$ with an error of 50 MeV is one of the main tasks of the LEP II program.

Given these remarkable achievements and future perspectives, a natural question arises. Does the theoretical error affecting the predictions of the SM match the present/expected experimental precision? This question has been recently addressed and discussed by a Working Group on Precision Calculation organized in 1994 at CERN. Main purpose of this Working Group has been to update the predictions of the SM concerning the observables of interest at LEP and to estimate the theoretical uncertainties of these predictions. Here we will summarize

\footnote{The free parameters in the fit are $m_t$ and $\alpha_s$. The Higgs mass $m_H$ is fixed at 300 GeV. The last error in eq. \ref{eq:1} shows the effect of varying $m_H$ from 60 GeV to 1 TeV. The same fit with the inclusion of all low-energy and SLD data gives $m_t = 179 \pm 9^{+17}_{-19}$.}
very concisely some of the results of the Working Group. For a complete information, we address the reader to the full Report [7].

The possible sources of theoretical uncertainties can be divided into two general classes: parametric uncertainties and intrinsic uncertainties. The former are related to the fact that, within the SM, each quantity of interest is a function of a set of input parameters, which are known with a finite experimental precision. Any variation of the input parameters within the experimentally allowed range gives rise to an uncertainty on the observable considered. On the other hand, the intrinsic uncertainties have to do with the perturbative treatment of the quantum corrections: scheme dependence, ignorance of higher orders in the perturbative expansion and so on.

Among the input parameters of the SM, the fine structure constant $\alpha$, the Fermi constant $G_\mu$ and the $Z$ mass $M_Z$ enter the expressions for the electroweak observables already at tree level, and therefore are immediately involved in the discussion of the parametric uncertainties. At one-loop order the dependence is extended to the other parameters, such as the strong coupling $\alpha_s$ and all the masses of the SM particles. By inspecting the experimental accuracy of the values for the three basic parameters

$$\begin{align*}
\alpha^{-1} &= 137.0359895(61) \\
G_\mu &= 1.16639(2) \cdot 10^{-5} \text{ GeV}^{-2} \\
M_Z &= 91.1887 \pm 0.0022 \text{ GeV} ,
\end{align*}$$

one would conclude that the parametric uncertainties they induce are, for each quantity of interest, well below the present and future experimental sensitivity.

However, it is well known that, after the inclusion of quantum corrections, the observables at the $Z$ resonance exhibit an explicit dependence on the fine structure constant $\alpha$ only through the combination $\bar{\alpha}$, which accounts for the running of $\alpha$ from $q^2 = 0$ to $q^2 = M_Z^2$. The effective constant $\bar{\alpha}$ is related to $\alpha$ by the relation:

$$\bar{\alpha} = \frac{\alpha}{1 - \Delta \alpha}$$

The correction $\Delta \alpha$ contains the fermionic contribution to the photon vacuum polarization, usually split into leptonic and hadronic parts:

$$\Delta \alpha = \Delta \alpha_l + \Delta \alpha_h$$

The leptonic part $\Delta \alpha_l$ is computed perturbatively and known with very high precision [7, 8]. On the contrary the hadronic part, involving exchange of gluons at low momenta, cannot be reliably computed in perturbation theory. An estimate of $\Delta \alpha_h$ is then obtained by relating it to the cross-section for $e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}$ via the dispersion relation:

$$\begin{align*}
\Delta \alpha_h &= \frac{\alpha M_Z^2}{3\pi} \int_{4m_e^2}^{\infty} ds \frac{R(s)}{s(M_Z^2 - s)} \\
R(s) &= \frac{\sigma(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons})}{\left(\frac{4\pi \alpha^2}{3s}\right)}
\end{align*}$$

(7)
The low-energy part of the cross-section $\sigma(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons})$ is directly taken from the existing data whose experimental error is propagated to $\Delta \alpha_h$. The estimate of $\Delta \alpha_h$ which has been used up to now in the electroweak libraries is the one of ref. [9]:

$$\Delta \alpha_h = 0.0282 \pm 0.0009$$

which leads to $\alpha^{-1} = 128.87 \pm 0.12$. The sensitivity of the electroweak data to this error can be illustrated by considering the uncertainties induced on $\Gamma_l$, $\sin^2 \theta_{\text{eff}}^l$ and $M_W$. One obtains:

$$\delta \Gamma_l = 0.014 \text{ MeV}$$
$$\delta \sin^2 \theta_{\text{eff}}^l = 0.0003$$
$$\delta M_W = 16 \text{ MeV}$$

Whereas the uncertainties on $\Gamma_l$ and on $M_W$ are negligible, the error on $\sin^2 \theta_{\text{eff}}^l$ is already comparable to the present experimental accuracy. Recent re-evaluations of $\Delta \alpha_h$ [10], reported at this meeting by Burkhardt, reduce somewhat the above quoted uncertainty, but not at the level required by the future experimental sensitivity on $\sin^2 \theta_{\text{eff}}^l$. We refer to ref. [11] for a detailed discussion of this point.

Coming to the intrinsic uncertainties, as mentioned before, they are related to the use of perturbation theory in the evaluation of the quantum corrections. In particular, since the existing electroweak libraries work at least at the one-loop approximation, the intrinsic uncertainties are presently due to the incomplete knowledge of two or higher-loop terms.

One source of error is the dependence on the chosen renormalization scheme. Different renormalization schemes are available for the computation of loop corrections. By working at a fixed order of perturbation theory, e.g. in the one-loop approximation, all the schemes provide consistent results. This implies that the differences among the results obtained in the various schemes are of higher, in this case at least two-loop, order. These differences, although formally negligible at one-loop level, can be in practice numerically important.

Other intrinsic uncertainties are related to the splitting of a given correction into a leading part plus a remainder. To exemplify this, consider the usual parametrization of the $Z$ width $\Gamma_f$ into a fermion pair $f \bar{f}$:

$$\Gamma_f = 4N_c \frac{G_\mu M_Z^3}{24\pi \sqrt{2}} \left[ (g_V^f)^2 + (g_A^f)^2 \right]$$

where final QED and QCD corrections have been removed. The effective vector and axial-vector couplings $g_V^f$ and $g_A^f$ are in turn expressed in terms of the parameters $k_f$ and $\rho_f$ by the relations:

$$\frac{g_V^f}{g_A^f} = 1 - 4|Q_{\text{em}}|^2 s_W^2 k_f, \quad 4(g_A^f)^2 = \rho_f$$

where $s_W^2 = 1 - M_W^2/M_Z^2$. In this way $k_f$ defines an effective weak mixing angle for the flavour $f$, whereas $\rho_f$ measures the overall strength of the neutral current interaction in the $f \bar{f}$ channel.

The $\rho_f$ parameter can be further decomposed into two contributions:

$$\rho_f = \frac{1}{(1 - \Delta \rho)} + \Delta \rho_{f,\text{rem}}$$
The first term in eq. (12) is the so-called leading contribution. The correction $\Delta \rho$ involves all gauge-invariant fermionic contributions to $W$ and $Z$ self-energy diagrams. This correction is universal, that is it does not depend on the flavour $f$, and contains the potentially largest effect, related to positive powers of the top mass:

$$\Delta \rho = N_c x_t \left[ 1 + x_t \Delta \rho^{(2)} \left( \frac{m_H^2}{m_t^2} \right) + c_1 \frac{\alpha_s(M_Z)}{\pi} + c_2 \left( \frac{\alpha_s(M_Z)}{\pi} \right)^2 + \ldots \right]$$

(13)

where $N_c = 3$, $x_t = \frac{G_\mu m_t^2}{8\pi^2 \sqrt{2}}$. After extraction of the common factor $N_c x_t$, the unity represents the well known one-loop contribution [2] which, for values of $m_t$ in the range given in eqs. (1,2) is about 0.01; $\Delta \rho^{(2)}$ is the leading electroweak two-loop result in units of $N_c x_t^2$ [12] and the remaining terms are the known mixed electroweak-strong corrections [13, 14, 15]. These last corrections are relatively large. The $\alpha_s$ contribution [13] is about ten per cent of the one-loop correction $N_c x_t$. Very recently two groups [14, 15] have computed independently the $\alpha_s^2$ term obtaining $c_2 = -\pi^2 (2.56 - 0.18 n_f)$, $n_f$ being the total number of quarks. For $n_f = 6$ one finds $c_2 = -14.6$ and the $\alpha_s^2$ contribution is about 20% of the $\alpha_s$ term. It is reasonable to think that the theoretical errors coming from neglecting higher-orders in the $\alpha_s$ series are below the experimental sensitivity [16]. Finally, notice that in eq. (12) the Dyson series obtained by iterating the correction $\Delta \rho$ has been resummed.

On the contrary, the remainder $\Delta \rho_{f,rem}$ is flavour dependent and of order $G_\mu M_Z^2$. Moreover, since it involves the combination of vertex and self-energy bosonic corrections, which are not separately gauge invariant, there is no simple way to resum iterations of these contributions. To simulate the effect of higher order contributions to $\rho_f$, not explicitly accounted for by the equation (12), some of the existing libraries have considered different ”options”. These options consist in alternative expressions for $\rho_f$ such as:

$$\langle \rho_f \rangle' = \frac{1}{1 - \Delta \rho - \Delta \rho_{f,rem}}$$

$$\langle \rho_f \rangle'' = \frac{1}{1 - \Delta \rho} \left[ 1 + \frac{\Delta \rho_{f,rem}}{1 - \Delta \rho} \right]$$

(14)

The expressions in eqs. (12,14) are equivalent at one-loop level. The differences are of order $\Delta \rho \cdot \Delta \rho_{f,rem} \approx O(G_\mu^2 m_t^2 M_Z^2)$ and can be fixed only by an explicit two-loop computation. In the absence of such a computation one may try to estimate the correspondent theoretical uncertainty by running the code with different options and by looking at the range of values obtained for a given observable.

Similarly, further theoretical uncertainties arise from the ignorance of two-loop terms in the electroweak expansion concerning the parameters $k_f$ and $\Delta r$. Presently unknown two and higher-loop contributions would also remove ambiguities related to the factorization of QCD and electroweak corrections and to the linearization of expressions containing squares of tree level plus one-loop contributions. To size the effects on electroweak observables, each code has foreseen different options, similar to those described above for the $\rho_f$ parameter. By comparing the results obtained by choosing different options one has an indication about the importance of the higher-order corrections which have not yet been explicitly computed.

As an example, we consider below the values of $\sin^2 \theta_{eff}$ computed in the SM by the existing
electroweak libraries, at the reference point $m_t = 175$ GeV, $m_H = 300$ GeV, $\alpha_s(M_Z) = 0.125$.

$$\sin^2 \theta_{\text{eff}}^L = \begin{pmatrix}
0.23197^{+0.00004}_{-0.00007} & \text{BHM}[17] \\
0.23200^{+0.00008}_{-0.00008} & \text{LEPTOP}[18] \\
0.23199^{+0.00004}_{-0.00007} & \text{TOPAZO}[19] \\
0.23194^{+0.00003}_{-0.00007} & \text{WOH}[20] \\
0.23205^{+0.00004}_{-0.00014} & \text{ZFITTER}[21]
\end{pmatrix}$$

The quoted errors are due to the various options offered by each library for the treatment of higher-orders in the perturbative expansion. Since each code has adopted a different renormalization scheme, a reasonable estimate of the error induced by the scheme dependence can be obtained by comparing the central values for $\sin^2 \theta_{\text{eff}}^L$:

$$\langle \delta \sin^2 \theta_{\text{eff}}^L \rangle_{\text{scheme}} = \frac{0.23205 - 0.23194}{2} = 0.00006$$

On the other hand, unknown higher-order effects may be sized by comparing the absolute maximum and minimum among the values quoted in the previous table.

$$\langle \delta \sin^2 \theta_{\text{eff}}^L \rangle_{\text{higher-orders}} = \frac{0.23209 - 0.23187}{2} = 0.00011$$

This last uncertainty, which shifts to about 0.00015 if one moves away from the reference point, is not far from the future experimental goal.

More than 10 observables have been analyzed in this way in ref. [7]. The qualitative conclusion which one can draw is that the intrinsic theoretical uncertainty on electroweak quantities, mainly due to unknown two-loop terms in the electroweak expansion, is typically small if compared with the present experimental precision and, for $\sin^2 \theta_{\text{eff}}^L$, with the uncertainty induced by the hadronic contribution to the photon vacuum polarization; nevertheless the intrinsic uncertainty is close to the ultimate experimental precision and only the explicit evaluation of the missing two-loop electroweak contribution could reduce it adequately.

As a step towards the realization of this ambitious program, two-loop electroweak corrections of order $O(G_{\mu}^2 m_t^2 M_Z^2)$ have been recently computed for the $\rho$ parameter [22]. These corrections are called sub-leading, since they are suppressed with respect to those explicitly given by the $N_c x_t^2 \Delta \rho^{(2)}$ in eq. (13) by a factor $M_Z^2 / m_t^2$.

The leading correction $N_c x_t^2 \Delta \rho^{(2)}$ has the property of remaining unchanged in the limit of vanishing gauge coupling constants $g$ and $g'$ of the SM. This fact suggested the possibility of evaluating it in the framework of a Yukawa theory, obtained from the SM by turning the gauge interactions off. Such a theory describes the ideal case of top mass much larger than the vector boson and light fermion masses, with gauge interactions negligible compared to those associated to the top-scalar sector. The explicit computation of $\Delta \rho$ within the Yukawa limit of the SM is made possible by the existence of Ward identities which, in renormalizable gauges, relate vector boson Green functions to Green functions of corresponding unphysical scalars. Indeed, amplitudes for unphysical would-be Goldstone bosons can be defined and worked out in the Yukawa theory. There are various advantages in using this method. There are less diagrams to compute than in the full theory, they are more convergent and all the complications related to gauge theories, such as gauge fixing, ghosts and so on, disappear. This technique has been
successfully used in the last years to evaluate corrections of $O(G_μ^2 m_t^2)$ to the $ρ$ parameter and to the width $Γ_b$ [22], as well as those of $O(G_μ α_s m_t^2)$ to $Γ_b$ [23].

Contrary to the case of the leading corrections, the evaluation of the subleading ones requires the use of the full $SU(2)_L ⊗ U(1)_Y$ gauge theory. Since they involve the computation of vertex and box corrections they are not universal, but process dependent. One of the simplest cases which can be considered is the ratio $ρ$ among neutral and charged current amplitudes in neutrino-lepton scattering at zero momentum transfer [22]. The result can be cast into the following form:

$$ρ = 1 + δρ^{(1)} + N_c x_t δρ^{(1)} + δρ^{(2)}$$

$$≈ \frac{1}{1 - δρ^{(1)} f} (1 + δρ^{(1)} b + δρ^{(2)})$$

(18)

where $δρ^{(1)} = δρ^{(1)} f + δρ^{(1)} b$ is the one-loop result, sum of the separate fermionic and bosonic contributions ($δρ^{(1)} f = N_c x_t$). The two-loop contribution $δρ^{(2)}$, expressed in units of $N_c x_t^2$, is given by $∆ρ^{(2)}$ of eq. (13) plus a subleading term. These terms are explicitly plotted in fig. 1, as a function of $m_t$ for three different values of $m_H$.

We see that the subleading corrections are as large as the leading ones. Actually, for small values of $m_H$, they are even larger, as it could be expected, due to the accidental smallness of the leading result for a vanishing $m_H$ [24]. They have the same sign as the leading ones. For $m_H = 250$ GeV and $m_t = 176$ GeV, the two-loop leading contribution is about $−3 \cdot 10^{-4}$, whereas the sum of leading plus subleading terms gives about $−6 \cdot 10^{-4}$ [24]. Finally, the comparable magnitude of leading and subleading terms raises doubts about the possibility of determining the full two-loop contribution from the first few terms of the expansion in $M_Z^2/m_t^2$.

These results, referring to a specific process at zero momentum transfer, cannot be directly used for LEP observables. It is however tempting to make a naive extrapolation from $q^2 = 0$ to the $q^2 = M_Z^2$. By using the relations:

$$\frac{\delta s^2}{s^2} = - \frac{s^2 c^2}{(c^2 - s^2)} δρ, \quad \frac{\delta M_W}{M_W} = \frac{c^2}{2 (c^2 - s^2)} δρ$$

(19)

where $s$ and $c$ denote the sine and cosine of $θ'_△_{eff}$, one would obtain, for $m_H = 250$ GeV and $m_t = 176$ GeV and by including leading plus subleading terms, the following estimate of the two-loop electroweak corrections on $sin^2 θ'_△_{eff}$ and $M_W$:

$$δ_{EW}^{(2)} (sin^2 θ'_△_{eff}) = +0.0002, \quad δ_{EW}^{(2)} (M_W) = −35$ MeV

(20)

In conclusion, the present level of intrinsic theoretical precision is sufficient to discuss the existing data. Future experimental precisions will however require further improvement in the evaluation of the hadronic contribution $Δα_h$ and the inclusion of two-loop electroweak corrections in the existing libraries.

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Figure Captions:
Fig. 1: \( \delta \rho^{(2)} \) for \( \nu_\mu e \) scattering, in units \( N_c x^2_t \) as a function of \( m_t \) for few values of \( m_H \): including only the \( O(G_\mu^2 m_t^4) \) contribution \( (y) \), and with both the \( O(G_\mu^2 m_t^4) \) and \( O(G_\mu^2 m_t^2 M_Z^2) \) terms \( (g) \).

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