Nonlinear Equations for Symmetric Massless Higher Spin Fields in \((A)dS_d\)

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Abstract

Nonlinear field equations for totally symmetric bosonic massless fields of all spins in any dimension are presented.

1 Introduction

Higher spin (HS) gauge theories are theories of gauge fields of all spins (see e.g. [1] for a review). Because HS gauge symmetries are infinite-dimensional, HS gauge theories may correspond to most symmetric vacua of a theory of fundamental interactions presently identified with superstring theory. The problem is to introduce interactions of HS fields in a way compatible with nonabelian HS gauge symmetries containing diffeomorphisms and Yang-Mills symmetries. Full nonlinear dynamics of HS gauge fields has been elaborated so far at the level of equations of motion for \(d = 4\) [2, 3] which is the simplest nontrivial case since HS gauge fields do not propagate if \(d < 4\). Some lower order interactions of HS fields in the framework of gravity were worked out at the action level for \(d = 4\) [4] and for \(d = 5\) [5]. As a result, it was found out that (i) consistent HS theories contain infinite sets of infinitely increasing spins; (ii) HS gauge interactions contain higher derivatives; (iii) in the framework of gravity, unbroken HS gauge symmetries require a non-zero cosmological constant; (iv) HS symmetry algebras [6] are certain star product algebras [7]. The properties (i) and (ii) were deduced in the earlier work [8] on HS interactions in flat space.
The feature that unbroken HS gauge symmetries require a non-zero cosmological constant is crucial in several respects. It explains why the analysis of HS–gravitational interactions in the framework of the expansion near the flat background led to negative conclusions [9]. The $S$-matrix Coleman-Mandula-type no-go arguments [10] become irrelevant because there is no $S$-matrix in the $AdS$ space. Also it explains why the HS gauge theory phase is not directly observed in the superstring theory prior its full formulation in the $AdS$ background is found, and fits the idea of the correspondence between HS gauge theories in the bulk and the boundary conformal theories [11, 12, 13, 14, 15, 16].

In the context of applications of the HS gauge theory to the superstring theory ($d = 10$) and M theory ($d = 11$), it is important to extend the 4$d$ results on the HS–gravitational interactions to higher dimensions. The aim of this paper is to present the full nonlinear formulation of the field equations for totally symmetric bosonic HS fields in any dimension. The form of the proposed equations is analogous to that of the 4$d$ equations of [3].

## 2 Free higher spin gauge fields

There are two equivalent approaches to description of totally symmetric bosonic massless fields of all spins at the free field level. The approach developed by Fronsdal [17] and de Wit and Freedman [18] is parallel to the metric formulation of gravity. Here an integer spin $s$ massless field is described by a totally symmetric tensor $\varphi_{a_1...a_s}$ ($a, b, c = 0...d - 1$) subject to the double tracelessness condition [17] $\varphi_{bb...a_s} = 0$ which is nontrivial for $s \geq 4$. The free field Abelian HS gauge transformation is

$$\delta \varphi_{a_1...a_s} = \partial_{[a_1} \varepsilon_{a_2...a_s]} , \tag{2.1}$$

where the parameter $\varepsilon_{a_1...a_{s-1}}$ is a totally symmetric rank $(s - 1)$ traceless tensor, $\varepsilon^{b}_{ba_3...a_{s-1}} = 0$. A nonlocal version of the same theory with unrestricted gauge parameters was developed in [19].

An alternative approach operates in terms of a 1-form frame like HS gauge field $e_{a_1...a_{s-1}} = dx^a e^a_{a_1...a_{s-1}}$ [20, 21, 22] which is traceless in the fiber indices,

$$e^b_{ba_3...a_{s-1}} = 0 . \tag{2.2}$$

(We use underlined letters for indices of vector fields and forms.) The Abelian HS gauge transformation law in the flat space-time is

$$\delta e^{a_1...a_{s-1}} = d\varepsilon^{a_1...a_{s-1}} + h_b e^{a_1...a_{s-1}, b} . \tag{2.3}$$
Here $h^a$ is the flat space frame field (i.e., $h^a = dx^a$ in the Cartesian coordinates). $\varepsilon^{a_1...a_{s-1}}$ is a totally symmetric traceless gauge parameter equivalent to that of the Fronsdal’s formulation. The gauge parameter $\varepsilon^{a_1...a_{s-1}}$, is also traceless and satisfies the condition that the symmetrization over all its indices is zero $\varepsilon^{\{a_1...a_{s-1}, a_s\}} = 0$. It is a HS generalization of the parameter of the local Lorentz transformations in gravity ($s = 2$) which is $\varepsilon^{a,b}$. The Lorentz type ambiguity due to $\varepsilon^{a_1...a_{s-1},b}$ can be gauge fixed by requiring the frame type HS gauge field to be totally symmetric

$$e_{n\,a_2...a_s} = \varphi_{na_2...a_s}$$

thus establishing equivalence with the Fronsdal’s formulation. Note that such defined $\varphi_{a_1...a_s}$ is automatically double traceless as a consequence of (2.2).

The Lorentz type HS symmetry with the parameter $\varepsilon^{a_1...a_{s-1}}$, assumes a HS 1-form connection $\omega^{a_1...a_{s-1}}$. From the analysis of its transformation law it follows [21, 22] that, generically, some new gauge connections and symmetry parameters have to be introduced. As a result, the full set of HS connections associated with a spin $s$ massless field consists of the 1-forms $dx^n \omega^{a_1...a_{s-1},b_1...b_t}$ which take values in all irreducible representations of the $d$-dimensional massless Lorentz group $o(d - 1, 1)$ described by the traceless Young tableaux with at most two rows such that the upper row has length $s - 1$

$$\begin{array}{cccccc}
\vline & \vline & \vline & \vline & \vline & \vline \\
| & | & | & | & |
\vline & \vline & \vline & \vline & \vline & \vline \\
\vline & \vline & \vline & \vline & \vline & \vline \\
\vline & \vline & \vline & \vline & \vline & \vline \\
| & | & | & | & |
\vline & \vline & \vline & \vline & \vline & \vline \\
\vline & \vline & \vline & \vline & \vline & \vline \\
\end{array}
$$

(2.5)

In other words, the 1-forms $dx^n \omega^{a_1...a_{s-1},b_1...b_t}$ are symmetric in the Lorentz vector indices $a_i$ and $b_j$ separately and satisfy the relations $\omega^{a_1...a_{s-1},a_s}_{b_2...b_t} = 0$, $\omega^{a_1...a_{s-3},c}_{b_1...b_t} = 0$. (From here it follows that all other traces of the fiber indices are also zero.) For the case $t = 0$, the field $\omega^{a_1...a_{s-1}}$ identifies with the dynamical spin $s$ frame type field $e^{a_1...a_{s-1}}$.

The formalism of [21, 22] works both in $(A)dS_d$ and in flat space. In $(A)dS_d$ it allows to build the free HS actions in terms of manifestly gauge invariant linearized (Abelian) field strengths. Explicit form of these linearized curvatures provides the starting point towards determination of nonabelian HS symmetries and HS curvatures. For the case of $d = 4$ this program was realized in [21, 6]. Here we extend these results to the bosonic HS theory in any dimension. To this end it is convenient to use the observation of [5] that the collection of the HS 1-forms $\omega^{a_1...a_{s-1},b_1...b_t}$ with all $0 \leq t \leq s - 1$ can be interpreted as a result of the “dimensional reduction” of a 1-form.
$\omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}$ carrying the irreducible representation of $o(d-1,2)$ or $o(d,1)$ $(A,B = 0,\ldots,d)$ described by the traceless two-row rectangular Young tableau of length $s-1$

$$\omega^{A_1\ldots A_{s-1},A_s}B_2\ldots B_{s-1} = 0, \quad \omega^{A_1\ldots A_{s-3},C}B_1\ldots B_{s-1} = 0. \quad (2.6)$$

Let us first recall how this approach works in the gravity case. $d$ dimensional gravity can be described by a 1-form connection $\omega^{AB} = -\omega^{BA}$ of the $(A)dS$ Lie algebra $(o(d,1)) o(d-1,2)$. The Lorentz subalgebra $o(d-1,1)$ is identified with the stability subalgebra of some vector $V^A$. Since we are discussing local Lorentz symmetry, this vector can be chosen differently in different points of space-time, thus becoming a field $V^A = V^A(x)$. The norm of this vector is convenient to relate to the cosmological constant $\Lambda$ so that $V^A$ has dimension of length

$$V^AV_A = -\Lambda^{-1}. \quad (2.7)$$

$\Lambda$ is supposed to be negative and positive in the AdS and dS cases, respectively (within the mostly minus signature). This allows for a covariant definition of the frame field and Lorentz connection [23, 24]

$$E^A = D(V^A) \equiv dV^A + \omega^{AB}V_B, \quad \omega^{LAB} = \omega^{AB} + \Lambda(E^AV^B - E^BV^A). \quad (2.8)$$

According to these definitions $E^AV_A = 0, D^LV^A = dV^A+\omega^{LAB}V_B \equiv 0$. When the frame $E^A_\underline{m}$ has the maximal rank $d$ it gives rise to a nondegenerate metric tensor $g_{am} = E^A_\underline{m}E^{B}_\underline{n}\eta_{AB}$. The torsion 2-form is $r^A \equiv DE^A \equiv r^{AB}V_B$. The zero-torsion condition $r^A = 0$ expresses the Lorentz connection via (derivatives of) the frame field in a usual manner. The $V^A$ transversal components of the curvature (2.9) $r^{AB}$ identify with the Riemann tensor shifted by the term bilinear in the frame 1-form. As a result, any field $\omega_0$ satisfying the zero-curvature equation

$$r^{AB} = d\omega_0^{AB} + \omega_0^{A,C} \wedge \omega_0^{CB} = 0, \quad (2.9)$$

describes $(A)dS_d$ space-time with the cosmological term $\Lambda$ provided that the metric tensor is nondegenerate.

The Lorentz irreducible HS connections $\omega^{a_1\ldots a_{s-1},b_1\ldots b_t}$ are the $d$-dimensional traceless parts of those components of $\omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}$ which are parallel to $V^A$ in $s-t-1$ indices and transversal in the rest ones. Let some solution of (2.9) which describes the $(A)dS_d$ background be fixed. The linearized HS curvature $R_1$ has the following simple form

$$R^A_{1\ldots A_{s-1},B_1\ldots B_{s-1}} = D_0(\omega^A_{1\ldots A_{s-1},B_1\ldots B_{s-1}}) = d\omega^A_{1\ldots A_{s-1},B_1\ldots B_{s-1}} + \omega_0^{A_1\ldots A_{s-1},C}B_1\ldots B_{s-1} \wedge \omega_0^{A_{s-1},CB_1\ldots B_{s-1}} \quad (2.10)$$
It is manifestly invariant under the linearized HS gauge transformations

$$\delta \omega^{A_1...A_{s-1},B_1...B_{s-1}}(x) = D_0 e^{A_1...A_{s-1},B_1...B_{s-1}}(x)$$  \hfill (2.11)

because $D_0^2 \equiv R(\omega_0) = 0$. The $(A)dS$ covariant form of the free HS action of [22] is [5]

$$S_2^a = \frac{1}{2} \sum_{p=0}^{s-2} a(s,p)e_{A_1...A_{d+1}} \int_{M^d} E_0^A \wedge ... \wedge E_0^{A_d} V^{A_{d+1}} V^{C_1} \cdots V^{C_{2(s-2-p)}}$$

$$\wedge R_1^{A_1B_1...B_{s-2}} A_2 C_1...C_{s-2-p} D_1...D_p \wedge R_1^{A_3B_1...B_{s-2}} A_4 C_{s-1-p}...C_{2(s-2-p)} D_1...D_p,$$ \hfill (2.12)

where $E_0^A = D_0(V^A)$. The coefficients

$$a(s,p) = \tilde{a}(s)(-\Lambda)^{-(s-p-1)} \frac{(d-5+2(s-p-2))!!(s-p-1)}{(s-p-2)!}$$ \hfill (2.13)

are fixed by the condition that the action is independent of all those components of $\omega_1^{A_1...A_{s-1},B_1...B_{s-1}}$ for which $V_{B_1} V_{B_2} \omega_1^{A_1...A_{s-1},B_1...B_{s-1}} \neq 0$. As a result of this condition, the free action (2.12) depends only on the frame type dynamical HS field $e_{A_1...A_{s-1}}$ and the Lorentz connection type auxiliary field $\omega^{a_1...a_{s-1},b}$ expressed in terms of the first derivatives of $e^{a_1...a_{s-1}}$ by virtue of its equation of motion equivalent to the “zero torsion condition”

$$0 = T_{1A_1...A_{s-1}} = R_1^{A_1...A_{s-1},B_1...B_{s-1}} V^{B_1} \cdots V^{B_{s-1}}.$$ \hfill (2.14)

Plugging the expression for $\omega^{a_1...a_{s-1},b}$ back into (2.12) gives rise to the free HS action expressed entirely (modulo total derivatives) in terms of $e_{a_1...a_{s-1}}$ and its first derivatives. Since the linearized curvatures (2.10) are invariant under the Abelian HS gauge transformations (2.11), the resulting action has required HS gauge symmetries and therefore describes the free field HS dynamics in $(A)dS_d$. In particular, the generalized Lorentz type transformations with the gauge parameter (2.3) guarantee that only the totally symmetric part of the gauge field $e_2^{a_1...a_{s-1}}$ equivalent to $\varphi_{m_1...m_s}$ contributes to the action. Although this action is defined to be independent of the “extra fields” $\omega^{a_1...a_{s-1},b_1...b_t}$ with $t \geq 2$, one has to express the extra fields in terms of the dynamical HS fields because they contribute beyond the linearized approximation. The fields $\omega^{a_1...a_{s-1},b_1...b_t}$ with $t > 0$ express via up to order $t$ derivatives of the dynamical field by virtue of certain constraints [21, 22] analogous to the zero torsion condition in gravity. As a result, the condition that the free action is independent of the extra fields is equivalent to the condition that it does not contain higher derivatives.
### 3 \((A)dS_d\) bosonic higher spin algebra

From the analysis of section 2 it is clear that, to reproduce the correct set of HS gauge fields, one has to find such an algebra \(g\) which contains \(h = o(d - 1, 2)\) or \(h = o(d, 1)\) as a subalgebra and decomposes under the adjoint action of \(h\) in \(g\) into a sum of irreducible finite-dimensional modules over \(h\) described by various two-row rectangular traceless Young tableaux. Such an algebra was described recently by Eastwood in [25] as the algebra of conformal HS symmetries of the free massless Klein-Gordon equation in \(d - 1\) dimensions. Here we give a slightly different definition of this algebra which is more suitable for the analysis of the HS interactions.

Consider oscillators \(Y^A_i\) with \(i = 1, 2\) satisfying the commutation relations

\[
[Y^A_i, Y^B_j] = \varepsilon_{ij} \eta^{AB}, \quad \varepsilon_{ij} = -\varepsilon_{ji}, \quad \varepsilon_{12} = 1,
\]

(3.1)

where \(\eta^{AB}\) is the invariant symmetric form of \(o(n, m)\). For example, one can interpret these oscillators as conjugated coordinates and momenta \(Y^A_i = P^A_i, Y^B_i = Y^B_i\). \(\eta^{AB}\) and \(\varepsilon^{ij}\) will be used to raise and lower indices in the usual manner

\[
A^A_{A} = \eta_{AB} A^B, \quad a^i = \varepsilon^{ij} a_j, \quad a_i = a^j \varepsilon_{ji}.
\]

We use the Weyl (Moyal) star product

\[
(f \ast g)(Y) = \frac{1}{\pi^{(d+1)}} \int \int dSdT f(Y + S)g(Y + T) \exp -2 S^A_i T^i_A.
\]

(3.2)

\([f, g] = f \ast g - g \ast f, \{f, g\} = f \ast g + g \ast f\). The associative algebra of polynomials with the \(\ast\) product law generated via (3.1) is called Weyl algebra \(A_{n+m}\). Its generic element is \(f(Y) = \sum \phi^{i_1 \ldots i_n}_{A_1 \ldots A_n} Y^{A_1}_{i_1} \ldots Y^{A_n}_{i_n}\) or, equivalently,

\[
f(Y) = \sum_{m,n} f_{A_1 \ldots A_m, B_1 \ldots B_n} Y^{A_1}_{i_1} \ldots Y^{A_m}_{i_m} Y^{B_1}_{j_1} \ldots Y^{B_n}_{j_n}
\]

(3.3)

with the coefficients \(f_{A_1 \ldots A_m, B_1 \ldots B_n}\) symmetric in the indices \(A_i\) and \(B_j\).

Various bilinears built from the oscillators \(Y^A_i\) form the Lie algebra \(sp(2(n+m))\) with respect to star commutators. It contains the subalgebra \(o(n, m) \oplus sp(2)\) spanned by the mutually commuting generators

\[
T^{AB} = -T^{BA} = \frac{1}{2} Y^A_i Y^B_i, \quad t_{ij} = t_{ji} = Y^A_i Y^A_j.
\]

(3.4)

Consider the subalgebra \(S\) spanned by the \(sp(2)\) singlets \(f(Y)\)

\[
[t_{ij}, f(Y)] = 0.
\]

(3.5)
Eq. (3.5) is equivalent to \( (Y^{A_i} \frac{\partial}{\partial Y^{A_j}} + Y^{A_j} \frac{\partial}{\partial Y^{A_i}}) f(Y) = 0 \). For the expansion (3.3) this condition implies that the coefficients \( f_{A_1...A_m, B_1...B_n} \) are nonzero only if \( n = m \) and that symmetrization over any \( m + 1 \) indices of \( f_{A_1...A_m, B_1...B_m} \) gives zero, i.e. they have the symmetry properties of a two-row rectangular Young tableau. As a result, the gauge fields of \( S \) are

\[
\omega(Y | x) = \sum_{l=0}^{\infty} \omega_{A_1...A_l, B_1...B_l}(x) Y^{A_1} \cdots Y^{A_l} Y^{B_1} \cdots Y^{B_l}
\]  

(3.6)

with the component gauge fields \( \omega_{A_1...A_l, B_1...B_l}(x) \) taking values in all two-row rectangular Young tableaux of \( o(n + m) \).

The algebra \( S \) is not simple. It contains the two-sided ideal \( I \) spanned by the elements of the form

\[
g = t_{ij} \ast g^{ij},
\]

(3.7)

where \( g^{ij} \) transforms as a symmetric tensor with respect to \( \text{sp}(2) \), i.e., \([t_{ij}, g^{kl}] = \delta^k_i g^{lj} + \delta^j_i g^{lk} + \delta^l_j g^{ik} + \delta^i_j g^{lk} \). (Note that \( t_{ij} \ast g^{ij} = g^{ij} \ast t_{ij} \).) Actually, from (3.5) it follows that \( f \ast g, g \ast f \in I \ \forall f, g \in S, g \in I \). Due to the definition (3.4) of \( t_{ij} \), the ideal \( I \) contains all traces of the two-row Young tableaux. As a result, the algebra \( S/I \) has only traceless two-row tableaux in the expansion (3.6). (Let us note that this factorization is not optional because some of the traces of two-row rectangular tableaux are not themselves two-row rectangular tableaux and may not admit a straightforward interpretation in terms of HS fields.) The algebra \( S/I \) was identified by Eastwood in [25] as conformal HS algebra in \( d-1 \) dimensions.

For the complex algebra \( S/I \) we will use notation \( hgl(1/\text{sp}(2)[n + m] | \mathbb{C}) \). Its real form corresponding to a unitary HS theory in the \( AdS \) case of \( n = 2 \) will be called \( hu(1/\text{sp}(2)[n, m]) \). The meaning of this notation is as follows. According to [26], \( hgl(p, q|2r) \) is the superalgebra of \((p + q) \times (p + q)\) matrices whose elements are arbitrary even (odd) power polynomials of \( 2r \) pairs of oscillators in the diagonal (off-diagonal) blocks. Because of the \( \text{sp}(2) \) invariance condition (3.5), in our case only even functions of oscillators appear. So we discard the label \( q \) in the notation \( hgl(p, q|2r) \). The label \( \text{sp}(2)[n, m] \) means that the appropriate quotient of the centralizer in \( hgl(1|2(n + m)) \) with respect to the \( \text{sp}(2) \) subalgebra, which commutes with the \( o(n, m) \) spanned by bilinears of oscillators, is taken. \( o(n, m) \) is the subalgebra of \( hu(1/\text{sp}(2)[n, m]) \).

Note that the described construction of the HS algebra is analogous to that of the \( AdS_P \) HS algebra given by Sezgin and Sundell in [27] in terms of spinor oscillators with the symmetric 7d charge conjugation matrix in place of the metric tensor in (3.1). Also let us note that the key role of the algebra...
sp(2) in the analysis of HS dynamics explained below is reminiscent of the role of \(sp(2)\) in the two-time approach developed by Bars [28]. In [29], the \(sp(2)\) invariant technics was applied to the description of interacting massless fields. The important difference is that in our case the \(sp(2)\) invariance condition acts in the fiber space described by polynomials of the auxiliary variables \(Y^A_i\), reducing the set of fields appropriately, while the \(sp(2)\) algebra in the models of [28, 29] acts on the base.

4 Twisted adjoint representation and Central On-Mass-Shell theorem

Now we are in a position to define the twisted adjoint representation which describes the HS Weyl 0-forms. Let a HS algebra admit such an involutive automorphism \(\tau\) (i.e., \(\tau(f * g) = \tau(f) * \tau(g)\), \(\tau^2 = 1\)) that its action on the elements of the \((A)dS_d\) subalgebra is

\[
\tau(P^a) = -P^a, \quad \tau(L^{ab}) = L^{ab}.
\]

(4.1)

Once the Lorentz algebra is singled out by the compensator, the automorphism \(\tau\) describes the reflection with respect to the compensator vector. In particular, for the HS algebra under investigation we set \(\tau(Y^A_i) = \tilde{Y}^A_i\), where

\[
\tilde{A}^A = A^A - \frac{2}{V^2} V^A V_B A^B.
\]

(4.2)

for any vector \(A^A\). Let us introduce notations

\[
A^A_i = \|A^A_i\| + A^A_i, \quad \|A^A_i\| = \frac{1}{V^2} V^A V_B A^B, \quad \perp A^A_i = A^A_i - \frac{1}{V^2} V^A V_B A^B.
\]

(4.3)

We have \(\perp \tilde{A}^A_i = -\|A^A_i\|\) and \(\perp \tilde{A}^A_i = A^A_i\). The action of \(\tau\) on a general element is

\[
\tau(f(Y)) = \tilde{f}(Y) \equiv f(\tilde{Y}) .
\]

(4.4)

It is elementary to see that \(\tau\) is an automorphism of the Weyl algebra. Since \(\tau(t_{ij}) = t_{ij}\), the same is true for the HS algebra. To simplify analysis we will assume that \(\tau\) is \(x\)-independent that requires \(V^A\) to be \(x\)-independent.

Let \(C(Y|x)\) be a 0-form in the HS algebra linear space, i.e. \([t_{ij}, C]_s = 0\) with the ideal \(I\) factored out. The covariant derivative in the twisted adjoint representation is

\[
\tilde{D}(C) = dC + \omega * C - C * \tilde{\omega} ,
\]

(4.5)
Central On-Mass-Shell theorem formulated in [5] in terms of Lorentz components of \( C(Y|x) \) states that the equations for totally symmetric free massless fields in \( (A)dS_d \) can be formulated in the form

\[
R_1(\parallel Y, \perp Y) = \frac{1}{2} E_0^A \wedge E_0^B \frac{\partial^2}{\partial Y_i \partial Y_j} \varepsilon_{ij} C(0, \perp Y) ,
\]

(4.6)

\[
\tilde{D}_0(C) = 0 ,
\]

(4.7)

where \( R_1(Y) = d\omega(Y) + \omega_0 * \omega + \omega * \omega_0 , \tilde{D}_0(C) = dC + \omega_0 * C - C * \omega_0 \), and \( \omega_0 = \omega_0^{AB}(x) T_{AB} \) where \( \omega_0^{AB}(x) \) satisfies (2.9) to describe the \( (A)dS_d \) background.

The components of the expansion of the 0-forms \( C(0, \perp Y) \) on the r.h.s. of (4.6) in powers of \( Y_i^A \) are \( V \) transversal. These are the HS Weyl 0-forms \( c^{a_1...a_s,b_1...b_s} \) described by the traceless two-row rectangular Lorentz Young tableaux of length \( s \). They parametrize those components of the HS field strengths that remain nonvanishing when the field equations and constraints on extra fields are satisfied. For example, the Weyl tensor in gravity \( (s = 2) \) parametrizes the components of the Riemann tensor allowed to be nonvanishing when the zero-torsion constraint and Einstein equations (requiring the Ricci tensor to vanish) are imposed. The equation (4.7) describes the consistency conditions for the HS equations and also dynamical equations for spins 0 and 1. (Dynamics of a massless scalar was described this way in [30].) In addition they express an infinite set of auxiliary fields contained in \( C \) in terms of derivatives of the dynamical HS fields.

The key fact is that the equations (4.6) and (4.7) are consistent, i.e., the application of the covariant derivative to the l.h.s. of (4.6) and (4.7) does not lead to new conditions. The only nontrivial point is to check that

\[
\varepsilon_{ij} D_0 \left( E_0^A \wedge E_0^B \frac{\partial^2}{\partial Y_i \partial Y_j} C(0, \perp Y) \right) = 0
\]

(4.8)
as a consequence of (4.7). It is convenient to use the following forms of the covariant derivatives in the adjoint and twisted adjoint representations

\[
D_0 = D_0^L - \Lambda E_0^A V^B (\perp Y_A^i \frac{\partial}{\partial Y_B^i} - \parallel Y_B^i \frac{\partial}{\partial Y_A^i}) ,
\]

(4.9)

\[
\tilde{D}_0 = D_0^L - 2\Lambda E_0^A V^B (\perp Y_A^i \parallel Y_B^i - \frac{1}{4} \varepsilon^{ij} \frac{\partial}{\partial Y_A^i \partial Y_B^j} ) ,
\]

(4.10)
where the Lorentz covariant derivative is
\[ D^L_0 = d + \omega^L_{AB} \nabla^L_0 \partial^B \partial^C \partial_Y Y^A, \]
where \( \omega^L_{AB} \) is the Lorentz connection. One observes that \( D^L_0 \) commutes with \( E^A \wedge E^B \partial^C \partial_Y Y^A \). As a result, it remains to check the frame dependent terms. These all vanish either because the r.h.s. of (4.6) is \( \|Y_i\)-independent or because of the identity
\[ E^A \wedge E^B \partial^C \partial_Y Y^A \partial_Y Y^B \partial_Y Y^C Y \equiv 0 \]
valid as a consequence of the total antisymmetrization over the indices \( i, j \) and \( k \) which take two values (equivalently, because \( C(Y) \) does not contain Young tableaux with more than two rows).

The adjoint covariant derivative (4.9) and twisted adjoint covariant derivative (4.10) commute with the operators \( N^{ad} \) and \( N^{tw} \), respectively,
\[ N^{ad} = Y^A_i \partial_Y Y^A_i, \quad N^{tw} = Y^A_i \partial_Y Y^A_i - \|Y_i\| \partial_Y Y^A_i. \]  
(4.11)
This means that the free field equations (4.6) and (4.7) decompose into independent subsystems for the sets of fields satisfying
\[ N^{ad} \omega = 2(s-1)\omega, \quad N^{tw} C = 2sC, \quad s \geq 0. \]  
(4.12)
(Note that the operator \( N^{tw} \) does not have negative eigenvalues when acting on tensors with the symmetry property of a two-row rectangular Young tableau, because having more than a half of vector indices aligned along \( V^A \) would imply symmetrization over more than a half of indices, thus giving zero.)

The set of fields singled out by (4.12) describes a massless spin \( s \). As expected, the massless scalar field is described only in terms of the 0-form \( C(Y|x) \), having no associated gauge field. In terms of Lorentz irreducible components, the spin \( s \) gauge connections take values in the representations (2.5) with various \( 0 \leq t \leq s-1 \) while the spin \( s \) Weyl tensors take values in the Lorentz representations with various \( p \geq s \). (Note that the missed cells compared to the rectangular diagram of the length of the upper row correspond to the Lorentz invariant direction along \( V^A \).) We see that the twisted adjoint action of the \((A)dS_d\) algebra on the HS algebra decomposes into an infinite set of infinite-dimensional submodules associated with different spins, while its adjoint action decomposes into an infinite set of finite-dimensional submodules. This fits the fact (see e.g. [31]), that the space of physical states is described by the 0-form sector which therefore has to form an infinite module over a space-time symmetry for any dynamical system with infinite number of degrees of freedom.
5 Nonlinear equations

Following to the standard approach in HS theory [32] we will look for a nonlinear deformation of the free field equations (4.6) and (4.7) in the form of a free differential algebra \( dW^\alpha = F^\alpha(W) \), satisfying the consistency condition \( F^\beta \frac{\delta F^\alpha}{\delta W^\sigma} \equiv 0 \), where \( W^\alpha \) describes all dynamical variables in the system (i.e. all 1-forms \( \omega \) and 0-forms \( C \) in our case). The equations (4.6) and (4.7) acquire nonlinear corrections once the fields \( \omega_0 \) and \( \omega \) are interpreted, respectively, as the vacuum and fluctuational parts of some dynamical field. In this case one has to replace the background covariant derivatives by the full ones that gives rise to nontrivial Bianchi relations because \( D^2 \) is proportional to the generalized Weyl tensors (in particular, to the usual Weyl tensor in the spin 2 sector). As shown in [1] the key principle that fixes a form of the 4d nonlinear field equations found in [3] is the condition that the local Lorentz symmetry remains a proper subalgebra of the infinite-dimensional HS algebra. This requirement guarantees that the nonlinear theory can be interpreted in terms of usual (finite-component) tensor fields. Otherwise, a deformation of the algebra might affect the form of the commutator of the Lorentz generators which may get admixture of infinity of HS generators with field-dependent coefficients. The same principle was used for the derivation of 3d equations [33]. The observation that allows us to build bosonic HS equations in any \( d \) is that in this case one has to require the \( sp(2) \) algebra, which singles out the HS algebra spanned by two-row rectangular tensor elements, to remain undeformed at the nonlinear level. Otherwise, the condition (3.5) would not allow a meaningful extension beyond the free field level, i.e., the resulting system may admit no interpretation in terms of the original HS tensor fields described by two-row rectangular Young tableaux, allowing, for example, nonlinear corrections in sectors of tensors which are absent at the linearized level.

The formulation of nonlinear HS equations in any dimension is nearly identical to that of 4d and 3d HS field equations given in [3, 33] in terms of spinors. First, we double a number of oscillators \( Y^A_i \rightarrow (Z^A_i, Y^A_i) \). The space of functions \( f(Z, Y) \) is endowed with the star product

\[
(f \ast g)(Z, Y) = \frac{1}{\pi^{2(d+1)}} \int dSdT f(Z + S, Y + S)g(Z - T, Y + T) \exp -2S^A_i T^A_i,
\]

which is associative, normalized so that \( 1 \ast f = f \ast 1 = f \) and gives rise to the commutation relations

\[
[Y^A_i, Y^B_j]_\ast = \varepsilon_{ij} \eta^{AB}, \quad [Z^A_i, Z^B_j]_\ast = -\varepsilon_{ij} \eta^{AB}, \quad [Y^A_i, Z^B_j]_\ast = 0. \quad (5.2)
\]
The star product (5.1) describes a normal-ordered basis in $A_{2(n+m)}$ with respect to creation and annihilation operators $Z - Y$ and $Y + Z$, respectively. For $Z$ independent elements (5.1) coincides with (3.2). The following useful formulae are true

\[ Y_i^A* = Y_i^A + \frac{1}{2} \left( \frac{\partial}{\partial Y_i^A} - \frac{\partial}{\partial Z_i^A} \right), \quad Z_i^A* = Z_i^A + \frac{1}{2} \left( \frac{\partial}{\partial Y_i^A} - \frac{\partial}{\partial Z_i^A} \right), \] (5.3)

\[ *Y_i^A = Y_i^A - \frac{1}{2} \left( \frac{\partial}{\partial Y_i^A} + \frac{\partial}{\partial Z_i^A} \right), \quad *Z_i^A = Z_i^A + \frac{1}{2} \left( \frac{\partial}{\partial Y_i^A} + \frac{\partial}{\partial Z_i^A} \right). \] (5.4)

Important property of the star product (5.1) is that it admits inner Klein operators. Indeed, it is elementary to see with the aid of (5.1) that the element

\[ K = \exp \left( -2z_i y^i \right), \] (5.5)

where

\[ y_i = \frac{1}{\sqrt{V^2}} V_B y_i^B, \quad z_i = \frac{1}{\sqrt{V^2}} V_B z_i^B \] (5.6)

has the properties

\[ K * f = \tilde{f} * K, \quad K * K = 1, \] (5.7)

where $\tilde{f}(Z, Y) = f(\tilde{Z}, \tilde{Y})$. This follows from the following formulae

\[ K * f = \exp \left( -2z_i y^i f(Z_i^A - \frac{1}{V^2} A^B V_B (Z_i^B - Y_i^B), Y_i^A + \frac{1}{V^2} A^B V_B (Z_i^B - Y_i^B)) \right), \] (5.8)

\[ f * K = \exp \left( -2z_i y^i f(Z_i^A - \frac{1}{V^2} A^B V_B (Z_i^B + Y_i^B), Y_i^A - \frac{1}{V^2} A^B V_B (Z_i^B + Y_i^B)) \right). \] (5.9)

We introduce the fields $W(Z, Y|x)$, $B(Z, Y|x)$ and $S(Z, Y|x)$, where $B(Z, Y|x)$ is a 0-form while $W(Z, Y|x)$ and $S(Z, Y|x)$ are connection 1-forms in space-time and auxiliary $Z_i^A$ directions, respectively

\[ W(Z, Y|x) = dx^\omega W_\omega(Z, Y|x), \quad S(Z, Y|x) = dZ_i^A S_i^A(Z, Y|x). \] (5.10)

The fields $\omega$ and $C$ are identified with the “initial data” for the evolution in $Z$ variables

\[ \omega(Y|x) = W(0, Y|x), \quad C(Y|x) = B(0, Y|x). \] (5.11)

The $Z$-connection $S$ will be determined modulo gauge ambiguity in terms of $B$. The differentials satisfy the standard anticommutation relations $dx^\omega dx^\nu = -dx^\omega dx^\nu$, $dZ_i^A dZ_j^B = -dZ_j^B dZ_i^A$, $dx^\omega dZ_j^B = -dZ_j^B dx^\omega$ and commute to all other variables (from now on we discard the wedge symbol).
The full nonlinear system of HS equations is
\begin{align*}
  dW + W \ast W &= 0, \\
  dS + W \ast S + S \ast W &= 0, \\
  dB + W \ast B - B \ast \tilde{W} &= 0, \\
  S \ast B &= B \ast \tilde{S}, \\
  S \ast S &= -\frac{1}{2} (dZ^A_i dZ^i_A + 4\Lambda^{-1} dz_i dz^i B \ast K),
\end{align*}
(5.12)
(5.13)
(5.14)
(5.15)
(5.16)

where \( \tilde{S}(dZ, Z, Y) = S(\tilde{dZ}, \tilde{Z}, \tilde{Y}) \) and \( dz_i = \frac{1}{\sqrt{V^2 V^B}} V_B dZ^B_i \). In terms of a noncommutative connection \( W = d + W + S \) the system (5.12)-(5.16) reads
\begin{align*}
  W \ast W &= -\frac{1}{2} (dZ^A_i dZ^i_A + 4\Lambda^{-1} dz_i dz^i B \ast K), \\
  W \ast B &= B \ast \tilde{W}.
\end{align*}
(5.17)

We see that \( dz_idz^i B \ast K \) is the only nonzero component of the curvature. Note that \( B \) has dimension \( cm^{-2} \) to match the Central On-Mas-Shell theorem (4.6) upon identification of \( B \) with \( C \) in the lowest order. So, \( \Lambda^{-1} B \) is dimensionless. Since the \( B \) dependent part of the equation (5.16) is responsible for interactions, this indicates that taking the flat limit may be difficult in the interacting theory if there is no other contributions to the cosmological constant due to some condensates which break the HS gauge symmetries.

The system is formally consistent in the sense that the associativity relations
\begin{align*}
  W \ast (W \ast W) &= (W \ast W) \ast W \\
  (W \ast W) \ast B &= B \ast (W \ast W),
\end{align*}
equivalent to Bianchi identities, are respected by the equations (5.12)-(5.16). The only nontrivial part of this property might be that for the relationship
\( (S \ast S) \ast S = S \ast (S \ast S) \) in the sector of \( (dz_i)^3 \) due to the second term on the r.h.s. of (5.16) since \( B \ast K \) commutes with everything except \( dz_i \) to which it anticommutes. However, this does not break the consistency of the system because \( (dz_i)^3 \equiv 0 \). As a result, the equations (5.12)-(5.16) are consistent as “differential” equations with respect to \( x \) and \( Z \) variables. A related statement is that the equations (5.12)-(5.16) are invariant under the gauge transformations
\begin{align*}
  \delta W &= [\varepsilon, W]_\ast, \\
  \delta B &= \varepsilon \ast B - B \ast \tilde{\varepsilon}
\end{align*}
(5.18)

with an arbitrary gauge parameter \( \varepsilon(Z, Y|x) \).

Let us show that the condition (3.5) admits a proper deformation to the full nonlinear theory, i.e. that there is a proper deformation \( t_{ij}^{int} \) of \( t_{ij} \) that allows us to impose the conditions
\begin{align*}
  D(t_{ij}^{int}) &= 0 \\
  [S, t_{ij}^{int}]_\ast &= 0 \\
  B \ast \tilde{t}_{ij}^{int} - t_{ij}^{int} \ast B &= 0
\end{align*}
(5.19)
which amount to the original conditions $[t_{ij}, W]_* = [t_{ij}, B]_* = 0$ in the free field limit. Indeed, let us introduce the generators of the diagonal $sp^{tot}(2)$ algebra

$$t^{tot}_{ij} = Y_i^A Y_{Aj} - Z_i^A Z_{jA} .$$

(5.20)

In any $sp(2)$ covariant gauge (in which $S_A^i$ is expressed in terms of $B$ with no external $sp(2)$ noninvariant parameters - see below) $sp^{tot}(2)$ acts on $S_A^i$ as $[t^{tot}_{ij}, S^A_n]_* = \varepsilon_{in} S^A_j + \varepsilon_{jn} S^A_i$ provided that $B$ is a $sp(2)$ singlet.

Setting $S = dZ_i^A S_A^i$, the equation (5.16) gets the form $[S_A^i, S_B^j]_* = -\varepsilon_{ij} (\eta_{AB} - 4V_A V_B B*K)$. From (5.15) and (5.7) it follows that

$$S_A^i * B*K = B*K * (S_A^i - \frac{2}{V^2} V^A V^B S_B^i) .$$

(5.21)

As a result, we get

$$[s^i, s^j]_* = -\varepsilon_{ij} (1+4\Lambda^{-1} B*K) , \quad s^i * B*K = -B*K * s^i , \quad s^i = \frac{1}{\sqrt{V^2}} V^A S_A^i$$

and $[^* S_A^i, s^j]_* = 0$, $[^* S_A^i, ^* S_B^j]_* = -\varepsilon_{ij} \eta_{AB}$. The commutation relations (5.22) have a form of the deformed oscillator algebra [34] originally found by Wigner in [35] in a particular representation. Its key property is that the operators $t_{ij} = -\frac{1}{2} \{S_A^i, S_A^j\}$ form $sp(2)$ and $[t_{ij}, S_A^i]_*$ is $\varepsilon_{in} S^A_j + \varepsilon_{jn} S^A_i$.

As a result, the operators

$$t^{int}_{ij} = t^{tot}_{ij} - t_{ij}$$

(5.23)

form $sp(2)$ and commute with $S_A^i$. The conditions (5.19) are equivalent to the usual $sp(2)$ invariance conditions for the fields $W$ and $B$ and are identically satisfied on $S_A^i$, which essentially means that nonlinear corrections due to the evolution along $Z$-directions in the noncommutative space do not affect $sp(2)$. In the free field limit with $S_A^i = Z_A^i$, $t^{int}_{ij}$ coincides with (3.4). One uses $t^{int}_{ij}$ in the nonlinear model the same way as $t_{ij}$ (3.4) in the free one to impose (5.19) and to factor out the respective ideal $I^{int}$. This guarantees that the nonlinear field equations described by Eqs.(5.12)-(5.16) and (5.19) make sense for the HS fields associated with $hu(1/sp(2)[n, m])$.

6 Perturbative analysis

The perturbative analysis of the equations (5.12)-(5.16) is analogous to that carried out in spinor notations in [3] for the 4d case. Let us set

$$W = W_0 + W_1 , \quad S = S_0 + S_1 , \quad B = B_0 + B_1$$

(6.1)
with the vacuum solution

\[ B_0 = 0, \quad S_0 = dZ^i A_i, \quad W_0 = \frac{1}{2} \omega_0^{AB}(x) Y^i_A Y_{iB}, \quad (6.2) \]

where \( \omega_0^{AB}(x) \) satisfies the zero curvature conditions to describe \((A)dS_d\). From (5.15) and (6.2) we get

\[ B_1 = C(Y|x). \quad (6.3) \]

Let us consider the equation (5.16). First of all we observe that using the gauge ambiguity (5.18) we can set all components of \(dS_1^A\) equal to zero, i.e. to set \(S_1 = dz_i s^i_1(z, Y|x)\). The leftover gauge symmetry parameters are \(Z\) independent. The equations (5.13) and (5.15) then require the field \(W\) also to be \(Z\) independent. So, the dependence on \(Z\) now enters only through \(z_i\). As a result, (5.16) amounts to the first equation in (5.22). With the help of (5.9) one obtains in the first order

\[ \partial^i s^j_1 - \partial^j s^i_1 = -4\Lambda^{-1} \varepsilon^{ij} C(-\parallel Z, \perp Y) \exp -2z_k y^k, \quad (6.4) \]

where \(\partial^i = \frac{\partial}{\partial z_i}\). The generic solution of this equation is

\[ s^i_1 = \partial^j \varepsilon_1 + 2\Lambda^{-1} z^i \int_0^1 dt t C(-t\parallel Z, \perp Y) \exp -2tz_i y^i. \quad (6.5) \]

The ambiguity in the function \(\varepsilon_1 = \varepsilon_1(Z, Y|x)\) manifests invariance under the gauge transformations (5.18). It is convenient to fix a gauge by requiring \(\partial^i \varepsilon_1 = 0\) in (6.5). (This gauge is covariant because it involves no external parameters carrying nontrivial representations of \(sp(2)\).) This gauge fixing is not complete as it does not fix the gauge transformations with \(Z\) independent parameters

\[ \varepsilon_1(Z, Y|x) = \varepsilon_1(Y|x). \quad (6.6) \]

As a result, the field \(S\) is expressed in terms of \(B\). It is not surprising of course that the noncommutative gauge connection \(S\) is reconstructed in terms of the noncommutative curvature \(B\) modulo gauge transformations. The leftover gauge transformations with the parameter (6.6) identify with the HS gauge transformations acting on the physical HS fields.

Now, let us analyze the equation (5.13). In the first order, one gets

\[ \partial^i W_1 = ds^i_1 + W_0 * s^i_1 - s^i_1 * W_0. \quad (6.7) \]

Using that generic solution of the equation \(\frac{\partial}{\partial z_i} \varphi(z) = \chi^i(z)\) has the form \(\varphi(z) = const + \int_0^1 dt z_i \chi^i(tz)\) provided that \(\frac{\partial}{\partial z_i} \chi^i(z) \equiv 0\) and \(i = 1, 2\), one finds

\[ W_1(Z, Y) = \omega(Y) - Z^A V^A \int_0^1 dt (1-t) e^{-2tz_i y^i E^B B} \frac{\partial}{\partial Y^{ZB}} C(-t\parallel Z, Y\perp) \quad (6.8) \]
(note that the terms with $z_idS^i$ vanish because $z_i z_i \equiv 0$). Since, perturbatively, the system as a whole is a consistent system of differential equations with respect to $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial x}$ it is enough to analyze the equations (5.12) and (5.14) at $Z = 0$. Thus, to derive dynamical HS equations, it remains to insert (6.8) into (5.12) and (6.3) into (5.14), interpreting $\omega(Y|x)$ and $C(Y|x)$ as generating functions for the HS fields. The elementary analysis of (5.12) at $Z = 0$ with the help of (5.3) and (5.4) gives (4.6). For $B = C$, the equation (5.14) amounts to (4.7) in the lowest order. Thus it is shown that the linearized part of the HS equations (5.12)-(5.16) reproduces the Central On-Mass-Shell theorem for symmetric massless fields. The system (5.12)-(5.16) allows one to derive systematically all higher-order corrections to the free equations.

7 Discussion

The system of gauge invariant nonlinear dynamical equations for totally symmetric massless fields of all spins in $AdS_d$ presented in this paper can be generalized to a class of models with the Yang-Mills groups $U(p)$, $USp(p)$ or $O(p)$. This results from the observation that, analogously to the case of $d = 4$ \cite{32, 26}, the system (5.12)-(5.16) remains consistent for matrix valued fields $W \to W_{\alpha \beta}, S \to S_{\alpha \beta}$ and $B \to B_{\alpha \beta}, \alpha, \beta = 1 \ldots p$. Upon imposing the reality conditions

$$W^i(Z, Y|x) = -W(-iz, iY|x), \quad S^i(Z, Y|x) = -S(-iz, iY|x), \quad (7.1)$$

$$B^i(Z, Y|x) = -\bar{B}(-iz, iY|x) \quad (7.2)$$

this gives rise to a system with the global HS symmetry algebra $hu(p/sp(2)[n, m])$. Here all fields, including the spin 1 fields which correspond to the $Z,Y$-independent part of $W_{\alpha \beta}(Z, Y|x)$, take values in $u(p)$ which is the Yang-Mills algebra of the theory.

Combining the antiautomorphism of the star product algebra $\rho(f(Z, Y)) = f(-iz, iY)$ with some antiautomorphism of the matrix algebra generated by a nondegenerate form $\rho_{\alpha \beta}$ one can impose the conditions

$$W_{\alpha \beta}(Z, Y|x) = -\rho^{\beta \gamma} \rho_{\delta \alpha} W_{\gamma \delta}(-iz, iY|x), \quad S_{\alpha \beta}(Z, Y|x) = -\rho^{\beta \gamma} \rho_{\delta \alpha} S_{\gamma \delta}(-iz, iY|x), \quad (7.3)$$

$$B_{\alpha \beta}(Z, Y|x) = -\rho^{\beta \gamma} \rho_{\delta \alpha} \tilde{B}_{\gamma \delta}(-iz, iY|x), \quad (7.4)$$

which truncate the original system to the one with the Yang-Mills gauge group $USp(p)$ or $O(p)$ depending on whether the form $\rho_{\alpha \beta}$ is antisymmetric or symmetric, respectively. The corresponding global HS symmetry algebras are

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called \( hsp(p/sp(2)[n, m]) \) and \( ho(p/sp(2)[n, m]) \), respectively. In this case all fields of odd spins take values in the adjoint representation of the Yang-Mills group while fields of even spins take values in the opposite symmetry second rank representation (i.e., symmetric for \( O(p) \) and antisymmetric for \( USp(p) \)) which contains singlet. The graviton is always the singlet spin 2 particle in the theory. Color spin 2 particles are also included for general \( p \) however.\(^1\)

The minimal HS theory is based on the algebra \( ho(1/sp(2)[n, m]) \). It describes even spin particles, each in one copy. (Odd spins do not appear because the adjoint representation of \( o(1) \) is trivial.)

All HS models have essentially one dimensionless coupling constant \( g^2 = |\Lambda| \frac{d-2}{2} \kappa^2 \), where \( \kappa^2 \) is the gravitational constant. At the level of field equations it can be rescaled away by a field redefinition. The Yang-Mills coupling constant is \( g_{YM}^2 = |\Lambda| \kappa^2 \). Unlikely to the 4d case of [3], for general \( d \) we do not see a way for a consistent nontrivial modification of the system (5.12)-(5.16) that cannot be compensated by a field redefinition. This probably means that the interaction ambiguity in one function found in [3] is due to the possibility to treat independently the selfdual and anti-selfdual parts of the HS curvatures, that makes no sense for totally symmetric fields beyond \( d = 4 \).

The results of this paper indicate that there may exist a large class of consistent HS theories in \((A)dS_d\) with different spin spectra. An important problem for the future is therefore to extend the obtained results beyond the class of totally symmetric gauge fields. A formulation of arbitrary symmetry free massless fields in the flat space-time is fairly well understood by now (see e.g., [38]). The general covariant formulation in \((A)dS_d\) is still lacking however, although a number of important contributions has been made. In particular, the light cone formulation of the equations of motion of generic massless fields in \( AdS_d \) [39] and actions for generic massless fields in \( AdS_5 \) [40] were constructed by Metsaev. In [41] an approach to covariant description of an arbitrary representation of \( AdS_d \) algebra \( o(d - 1, 2) \) was developed in the framework of the radial reduction technique. The unfolded formulation of a 5d HS theory with mixed symmetry fields was studied in [42] in the sector of Weyl 0-forms. The phenomenon of partial masslessness of HS fields in \((A)dS\) is another specificity of the dynamics in \((A)dS_d\) [43]. Extension of flat space results to \((A)dS_d\) is not straightforward for mixed symmetry fields because irreducible massless systems in \( AdS_d \) reduce in the flat limit to a collection of independent flat space massless fields [44].

\(^1\)Let us note that this does not contradict to the no-go results of [36, 37] because the theory under consideration does not allow a flat limit with unbroken HS symmetries and color spin 2 symmetries.
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