Abstract: We study the neutral Higgs boson production via the gluon fusion process with the $\tau^+\tau^-$ final state at the upgraded Fermilab Tevatron, including a complete simulation of signal channels and leading background processes. For the SM Higgs boson, this $h \rightarrow \tau^+\tau^-$ channel may provide important addition for the Higgs boson discovery in the mass range 120 – 140 GeV. In minimal supersymmetric models, natural enhancement for the signal rate over the SM expectation makes the $h, H, A \rightarrow \tau^+\tau^-$ signal observable for large $\tan\beta$ and low $M_A$, which may lead to full coverage for SUSY Higgs parameters at the Tevatron with a moderate integrated luminosity.

Keywords: Higgs Physics, Hadronic Colliders, Standard Model, Supersymmetric Standard Model
It has been extensively discussed to what extent the Higgs bosons can be discovered at the upgraded Tevatron [1]. The leading contribution to the Higgs boson production at hadron colliders comes from gluon fusion via heavy quark loops, with a typical cross section of one pb at the Tevatron energies. However, a light Higgs boson mainly decays into $b\bar{b}$ and the huge QCD background precludes any hope for finding the Higgs boson in this channel. That is the reason why the most favorable process for finding the Higgs boson at the Tevatron up to 130 GeV is the associated production $W, Z + h$ [2], with a cross section of the order of 0.5 pb. On the other hand, for a SM Higgs boson of mass $140 \text{ GeV} < m_h < 190 \text{ GeV}$, it may be possible to observe the mode $h \rightarrow WW^*$ [3] if higher luminosity of order $10 - 30 \text{ fb}^{-1}$ becomes available.

In this paper we study the feasibility of utilizing the gluon fusion process but with the $\tau^+\tau^-$ final state

$$pp \rightarrow gg \rightarrow h \rightarrow \tau^+\tau^-.$$  \hspace{1cm} (1)

There are several motivations to study this channel. First, it is very difficult at the Tevatron to find a SM Higgs boson signal in the mass range $130 - 140 \text{ GeV}$ at the interplay between $b\bar{b}$ and $WW^*$ final states. It would be desirable to find other potentially useful channels [4] and the $\tau^+\tau^-$ mode is a natural candidate to consider since it has a branching ratio of about 10% in the mass range of interest. Second, this mass range of the current interest is the probable region for $m_h$ in Supersymmetric models (MSSM) and one may expect some possible enhancement for process Eq. (1) due to sparticle loops [3, 5, 6]. Thirdly, it is especially important to determine the relative coupling strength of $h\tau^+\tau^-$ and $hbb$ since many new physics scenarios predict different relations of these Yukawa couplings [8]. Finally, since $\tau$ leptons are a prominent signal for various models of new physics, the final state containing $\tau^\pm$'s has been considered in many recent studies [4], and their experimental identification has become better understood [14]. We find that the channel of Eq. (1) may provide important addition for the SM Higgs boson discovery in the mass range $120 - 140 \text{ GeV}$. Especially in the MSSM, significant improvement at low $M_A$ and high $\tan\beta$ may lead to full coverage for SUSY Higgs parameters at the Tevatron.

In order to perform a complete signal and background simulation, we use the event generator PYTHIA v6.134 [11] for both signal and backgrounds. The effects of initial and final state radiation (IFSR) and hadronization have been taken into account. Final state decays of the polarized $\tau$ leptons have been treated properly by making use of the package TAUOLA v2.6 [2]. CTEQ4M parton distributions [13] have been used. The QCD scale $Q$ for both the factorization and renormalization is set to the average of transverse momenta of the outgoing particles at the parton level. Because of the limited statistics, we consider all possible $\tau$-decay modes, hadronic and leptonic. Detector parameters and energy resolution were chosen the same as in simulation for the Run II Workshop [1]. In particular, the jet energies are smeared according to a Gaussian spread

$$\frac{\Delta E}{E} = \frac{0.8}{\sqrt{E \text{ GeV}}}.$$  

Due to the missing energy carried away by the two neutrinos $\nu_\tau\bar{\nu}_\tau$ in the final state, we can only effectively reconstruct the Higgs boson mass from the decay products of $\tau^\pm\tau^-$ if
the tau pairs are not back-to-back in the transverse plane [14]. For this reason we need to consider the Higgs boson production with a finite transverse momentum \( p_T^h \). We adopt the process \( gg(q) \rightarrow hg(q) \) + initial-final state radiation (IFSR) for the signal simulation as implemented in [11]. The cross section of this process is of the order of 0.3 pb for \( m_h = 120 \) GeV with the cut \( p_T^h > 20 \) GeV. With the branching fraction for \( h \rightarrow \tau^+\tau^- \) of roughly 10%, one obtains cross sections for \( p_T^j > 20 \) GeV at \( \sqrt{s} = 2 \) TeV:

\[
\sigma(pp \rightarrow hj \rightarrow \tau^+\tau^-j) = 44, \ 28, \ 15 \ \text{fb for} \ m_h = 120, \ 130, \ 140 \ \text{GeV}, \quad (2)
\]

respectively.

The leading irreducible background to the signal is \( p\bar{p} \rightarrow Zj \rightarrow \tau^+\tau^-j \). The other reducible but huge background comes from QCD jets that can fake a \( \tau \) final state, \( p\bar{p} \rightarrow jjj \rightarrow \tau^+\tau^-j \). Using the acceptance cuts on \( p_T^j \), pseudo-rapidity and separation of the final state jets:

\[
p_T^j > 20 \ \text{GeV}, \ \ |\eta_j| < 2, \ \ \Delta R_{jj} > 0.5, \quad (3)
\]

the cross sections for the backgrounds are:

\[
\sigma(p\bar{p} \rightarrow Zj \rightarrow \tau^+\tau^-j) = 7 \times 10^4 \text{fb}, \quad (4)
\]

\[
\sigma(p\bar{p} \rightarrow jjj) = 2.5 \times 10^8 \text{fb}. \quad (5)
\]

The overwhelming background is formidable and a more efficient set of kinematical cuts is necessary in order to have a chance of extracting the signal.

Although less important, there are other backgrounds that we should comment on. First, the final state \( W^\pm jj \) with \( W^\pm \rightarrow e/\mu/\tau \) and \( j \rightarrow \tau \) can constitute a background to the signal. The production rate with the jet cuts of Eq. (3) is \( \sigma(W^\pm jj) = 300 \) pb. With \( W \) leptonic decay and a jet faking a \( \tau \), this background rate becomes about 200 fb. The next potentially sizeable background is from \( W^+W^-j \) production. This background has a rate 2.0 pb. With \( W \) leptonic decay it becomes \( \sigma(W^+W^-j \rightarrow \ell^+\ell^-j) = 220 \) fb (\( \ell = e, \mu, \tau \)). These background rates of \( Wjj \) and \( WWj \) are still somewhat larger than the signal rate. However, since those backgrounds are continuous ones, they will not be important after the judicial cuts and especially after the Higgs mass reconstruction, as we will discuss next.

To unambiguously identify the Higgs boson signal, one must reconstruct the mass peak \( M_{\tau\tau} \) at \( m_h \). This is also the most efficient way to discriminate against the backgrounds. Due to the fact that the \( \tau^+\tau^- \) from Higgs decay are ultra-relativistic, the jet (or lepton) and the neutrino(s) from the \( \tau \) decay are essentially collinear along \( \vec{p}_\tau \) [14]. We can thus solve for the two neutrino momenta as long as the \( \tau^+\tau^- \) are not collinear. Alternatively, one could consider to make use of the cluster transverse mass variable \( M_{T\tau} \) [15]. This transverse mass variable should reach maximum near \( m_h \), but has a broad tail below \( m_h \).

Depending on the \( \tau^\pm \) decay modes, events with \( \tau^+\tau^-j \) signature lead to final states such as: the pencil-like two \( \tau \)-jets + \( j \); or one pencil-like \( \tau \)-jet+ lepton + \( j \); or two leptons + \( j \). All decay channels have at least two missing neutrinos \( \nu, \nu_\tau \), and each charged lepton \( \ell \) will be accompanied by another neutrino \( \nu_\ell \). Among those channels, the leptonic channels lead to a better energy-momentum determination but have a smaller branching fraction.
On the other hand, the hadronic channels have a higher rate but a poor energy resolution for mass reconstruction.

In Fig. 1 we show the normalized reconstructed mass distributions of $M_{\tau\tau}$ for the signal and backgrounds for $m_h = 120$ GeV. Figures (a) is for both $\tau^+\tau^-$ decaying hadronically and Fig. (b) for $\tau^+\tau^-$ decaying leptonically. Although the $jjj$ background rate is a lot higher than that of $Zj$ to begin with, the reconstructed mass spectra are sufficiently different from the peak structure of the signal. This feature is also true for other faked backgrounds $Wjj$ and $WWj$. In contrast, the $Zj$ background naturally presents a peak at the $Z$ mass and leads to a long tail after $M_Z$. This constitutes the major irreducible background as we will see later. This becomes the limiting factor for us to explore a Higgs boson below and near 110 GeV. As anticipated, the leptonic channels have sharper signal mass peaks. Taking into account the kinematical features of the signal and backgrounds discussed above, we devise the following set of kinematical cuts. We first require the $\tau$ identification for the hadronic ($\tau_j$) or leptonic ($\tau_\ell$) decay, Cut I:

\[
\begin{align*}
\tau_j &: \quad p_T^j > 15 \text{ GeV}, |\eta^j| < 2 \\
& \quad \text{one or three tracks in } 10^\circ \text{ cone with no additional tracks in } 30^\circ \text{ cone} \\
& \quad \text{the invariant mass of tracks is less than } 2 \text{ GeV} \\
\tau_\ell &: \quad p_T^\ell > 10 \text{ GeV } |\eta^\ell| < 2; \\
& \quad \text{no additional tracks in } 30^\circ \text{ cone for } jjj \text{ mode: } E_T > 20 \text{ GeV},
\end{align*}
\]

where the missing transverse energy $E_T$ is defined by the imbalance of the observed particles, and it is also smeared according to Gaussian distribution with standard deviation $0.5\sqrt{E_T}$.

![Figure 1](image_url)

**Figure 1:** Normalized reconstructed mass $M_{\tau\tau}$ distributions for $\tau^+\tau^-$ decaying (a) hadronically and (b) leptonically. The solid lines are for $m_h = 120$ GeV, the dashed lines for the $Zj$ background and the dotted lines for $jjj$ background.
The $E_T$ cut is to help triggering the $jjj$ events. For the $jj$ channel we require $2\tau_j$ with no isolated electrons or muons (defined by $\tau_\ell$), for the $j\ell$ channel we require only one $\tau_j$ and one isolated lepton, and for the $\ell\ell$ channel we require no $\tau_j$ in addition to the two isolated leptons.

In Cut I, the cut on the jet invariant mass is essential for reducing the huge QCD background in which a QCD parton jet fakes a $\tau$. The efficiency of $\tau$ ID from the $Z$ decay is in agreement with more realistic studies [10]. The efficiency of more energetic $\tau$’s from Higgs decay has not been previously studied for the Tevatron with the full detector simulation. We find that the efficiency of $\tau$ ID is quite $p_T$ and process-dependent, typically being $60 - 70\%$ for $\tau_j$ and $50 - 60\%$ for $\tau_l$ for $m_h = 120 - 140 \text{ GeV}$. After the $\tau$-pair identification, we then impose the next level of refined kinematical cuts, Cut II:

- $p_T^{\tau_j} > m_h/6$, $p_T^{\tau_e} > m_h/9$,
- additional jet (not $\tau_j$) with $p_T^j > m_h/6$ and $|\eta^j| < 4$.

Our final set of cuts defines the optimal windows in $M_{\tau\tau}$ and $M_{T\tau\tau}$ that will be used to confirm the existence of a signal as well as to estimate the significance for the signal observation, Cut III(A):

- $m_h - 0.5\Delta M < M_{\tau\tau} < m_h + 3\Delta M$,
- $m_h - 4\Delta M < M_{T\tau\tau}^T < m_h + 3\Delta M$,

for pure hadronic decay of both $\tau$’s and Cut III(B):

- $m_h - \Delta M < M_{\tau\tau} < m_h + 3\Delta M$,
- $m_h - 4.5\Delta M < M_{T\tau\tau}^T < m_h + 3\Delta M$,

for others channels of $\tau$ decay, where $\Delta M = \sqrt{m_h/\text{GeV}} \text{ GeV}$.

In the case of the QCD faked background to $\tau$, we do not directly go through the procedure outlined in Cut I as for the signal. Instead, we use the probability for a jet to be misidentified as a $\tau_j$ to be $0.5\%$ [9], and $\tau_l$ to be $0.01\%$. We then apply Cuts II and III. Since the probability for QCD background to fake two leptons is very small, the background to the di-lepton channels is negligible in comparison with the signal, especially with the irreducible $Zj$ background.

With the substantial efforts discussed above, we have effectively suppressed the backgrounds with respect to the signal. The signal efficiencies are at a percentage level. Table 1 shows the final number of events for all signal channels and their corresponding backgrounds at the Tevatron with an integrated luminosity $10 \text{ fb}^{-1}$. Also shown in Table 1 are the signal-to-background ratios (S/B). Because of the overwhelming QCD background, the S/B ratio is the highest for $\ell\ell$ channel of $\tau$ decay ($\sim 6 - 8\%$), intermediate for $j\ell$ channel ($\sim 4 - 5\%$), and the lowest for $jj$ channel ($\sim 3\%$). With respect to $S/\sqrt{B}$ and $S/B$ ratios, the $j\ell$ channel is probably the best one for signal identification. Nevertheless, the rather small $S/B$ ratios render the signal observation systematically challenging.

Our main results, the luminosities required for a 95% CL exclusion or a $5\sigma$ discovery of the SM Higgs boson via $h \to \tau^+\tau^-$ at $\sqrt{s} = 2 \text{ TeV}$ for $m_h = 120, 130$ and $140 \text{ GeV},$
Table 1: Final number of events of signal for all channels from $h \to \tau^+\tau^-$ for representative Higgs boson masses, corresponding backgrounds at $\sqrt{s} = 2$ TeV per 10 fb$^{-1}$ integrated luminosity and S/B ratio.

| $m_h$ (GeV) | 120 GeV | 130 GeV | 140 GeV |
|------------|----------|----------|----------|
| $jj$       | 32       | 20       | 10       |
| $Zj$       | 713      | 281      | 137      |
| $jjj$      | 559      | 346      | 8.5      |
| $S/B$ (%)  | 2.5      | 3.2      | 6.9      |
| $j\mu$     | 18       | 10       | 10       |
| $Zj$       | 430      | 137      | 159      |
| $jjj$      | 13       | 8.5      | 8.4      |
| $S/B$ (%)  | 4.1      | 6.9      | 0.39     |
| $je$       | 1.4      | 0.85     | 0.62     |
| $Zj$       | 338      | 10       | 10       |
| $jjj$      | 13       | 1.7      | 1.5      |
| $S/B$ (%)  | 4.8      | 6.0      | 6.8      |
| $\mu\mu$  | 1.2      | 18       | 15       |
| $Zj$       | 1.4      | 0.17     | 0.78     |
| $jjj$      | 0.26     | 0.17     | 0.11     |
| $S/B$ (%)  | 7.7      | 8.4      | 6.2      |
| $je$       | 2.5      | 15       | 24       |
| $Zj$       | 40       | 0.17     | 0.31     |
| $jjj$      | 0.26     | 0.17     | 0.11     |
| $S/B$ (%)  | 6.2      | 8.4      | 4.9      |

Table 2: Integrated luminosities needed to reach a 95% CL exclusion, 3 and 5σ discovery for a SM Higgs boson at the Tevatron, and the enhancement factor $\kappa$ (at 2 and 15 fb$^{-1}$) needed to reach a 95% CL exclusion and 5σ discovery.

| $m_h$ (GeV) | 120 GeV | 130 GeV | 140 GeV |
|------------|----------|----------|----------|
| 95% CL exclusion $L$(fb$^{-1}$) | 14 | 18 | 32 |
| 3σ discovery $L$(fb$^{-1}$) | 33 | 42 | 77 |
| 5σ discovery $L$(fb$^{-1}$) | 93 | 120 | 210 |
| $\kappa$ for 95% CL (2 fb$^{-1}$) | 2.7 | 3.0 | 4.0 |
| $\kappa$ for 95% CL (15 fb$^{-1}$) | 0.97 | 1.1 | 1.5 |
| $\kappa$ for 3σ (2 fb$^{-1}$) | 4.1 | 4.6 | 6.2 |
| $\kappa$ for 3σ (15 fb$^{-1}$) | 1.5 | 1.7 | 2.3 |
| $\kappa$ for 5σ (2 fb$^{-1}$) | 6.8 | 7.7 | 10 |
| $\kappa$ for 5σ (15 fb$^{-1}$) | 2.5 | 2.8 | 3.8 |

The $\tau^+\tau^-$ channel can provide new addition in combination with the other promising channels such as $Wh, Zh$ and $h \to WW$ \cite{1,2,3} to improve the overall observability of the SM Higgs boson. To illustrate the potential improvement, we estimate the total integrated luminosities needed for the 95%, 3σ, 5σ effects for $m_h = 120 - 140$ GeV by combining the Run II report \cite{4} and our $h \to \tau^+\tau^-$ results, according to a relation

$$L^{-1} = L_1^{-1} + L_2^{-1},$$

as shown in Table 3. We have followed the convention from the Run II report that the numbers in Table 3 correspond to the delivered machine luminosity; while the significance values have been evaluated by combining CDF and D0 (doubling the delivered luminosity).
Especially for the most difficult region $m_h \sim 140$ GeV, the luminosity needed may be reduced by about 40% with the addition of the $h \rightarrow \tau^+ \tau^-$ channel.

| $m_h$ (GeV) | 120  | 130  | 140  |
|-------------|------|------|------|
| $L$ at 95% (fb$^{-1}$) | 1.8 (2.5) | 3.2 (5) | 6.6 (11) |
| $L$ at 3σ (fb$^{-1}$) | 4.4 (6) | 11 (21) | 15 (24) |
| $L$ at 5σ (fb$^{-1}$) | 13 (18) | 20 (31) | 42 (70) |

Table 3: Integrated luminosities needed (for one of the two experiments) to reach a 95% CL exclusion, 3 and 5σ discovery for a SM Higgs boson by combining the current $\tau^+ \tau^-$ results and the Run II report [1]. Numbers in parentheses are from [1].

In the minimal supersymmetric models, there are two more neutral Higgs bosons $H, A$ to contribute to the $\tau^+ \tau^-$ mode. In addition, there are new squark-loop contributions to Higgs boson production through gluon fusion. In our analyses, we use HIGLU program [16] for the calculation of the cross section of Higgs production process in the MSSM. For simplicity, we fix the squark masses to 1 TeV. We find that left-right squark mixing is not a crucial factor in our analysis. This simplification allows us to illustrate the potential of the Tevatron search by the Higgs sector parameters $M_A - \tan \beta$ in the conventional way in a two dimensional plane. In Fig. 2(a) we plot contours of the enhancement factor over the SM expectation which is defined to be the rate at $m_H^{SM} = M_A$. The large enhancement above

![Figure 2](image_url)

Figure 2: (a) Contours (above the curves) in $(M_A, \tan \beta)$ plane for the enhancement factor above the SM Higgs boson signal rate as labeled. The contributing Higgs states are also indicated by $A, h, H$ separated by the dashed lines; (b) contours (above the curves) for 95% CL exclusion and 5σ discovery for $p\bar{p} \rightarrow (A, h) + j \rightarrow \tau^+ \tau^- j$ process within MSSM at the Tevatron with 2 and 15 fb$^{-1}$. 
the SM mainly comes from the new channels from $H, A$. The contributing Higgs states are also indicated by the dashed lines in Fig. 2(a) with the explicit labels $A, h, H$. Also, large tan $\beta$ will enhance the effects from bottom/sbottom loops. On the other hand, the decay branching fractions to $\tau^+\tau^-$ may be enhanced by about 30% at most for various choices of the SUSY parameters. For $M_A \sim 150 - 180$ GeV, $H, A \rightarrow \tau^+\tau^-$ is still observable; while the SM mode dies away due to the opening of the dominant $WW, ZZ$ channels. When combing the cross sections from channels of $A$ and $h$ production, we have used Gaussian combination criteria. In Fig. 2(b) we show the 95% CL exclusion and 5$\sigma$ discovery contours in MSSM for the $\tau^+\tau^-$ mode at the Tevatron. The direct searches at LEP2 have put a lower bound on the parameters as indicated by the dotted curve. The shaded region is the uncovered 5$\sigma$ hole at Run II with 15 fb$^{-1}$ without considering the $\tau^+\tau^-$ mode. We see that the addition of the $\tau^+\tau^-$ mode could reach 5$\sigma$ (2$\sigma$) full coverage for SUSY Higgs parameters with 15 fb$^{-1}$ (2 fb$^{-1}$). As a final remark, we note that in some region of the parameter space $hhb\bar{b}$ Yukawa coupling is accidentally suppressed due to radiative effects. This could lead to a substantial increase of the $h \rightarrow \tau\tau$ branching fraction. However, we found that this gain in branching fraction is mostly balanced by the reduction in the Higgs production via the $hhb\bar{b}$ coupling.

In summary, we have studied the potential of the upgraded Tevatron for searches of the neutral Higgs boson in the SM and in the MSSM via the $\tau\tau + j$ channel. We found that with this single channel alone, the total integrated luminosity needed for a 95% CL exclusion is $14 - 32$ fb$^{-1}$ for $m_h = 120 - 140$ GeV. The addition of the $h \rightarrow \tau^+\tau^-$ channel could improve the SM Higgs observation, as presented in Table 3. For the most difficult region $m_h \sim 140$ GeV, the luminosity needed for a 5$\sigma$ signal could be reduced by about 40%. However, we must note that due to the rather small signal-to-background ratios, the search for $\tau^+\tau^-$ channels is systematically challenging in the Standard Model case.

For the case of MSSM, the signal cross sections can be enhanced by a significant factor due to the addition of $H, A$, and due to high tan $\beta$ or squark-loop contributions. We found 5$\sigma$ (2$\sigma$) full coverage for the SUSY Higgs parameters with 15 fb$^{-1}$ (2 fb$^{-1}$) by including the $\tau^+\tau^-$ channel. Our analyses is applicable for other generic neutral scalar or pseudo-scalar via gluon fusion production and with substantial branching fraction to $\tau^+\tau^-$. More details of our analyses will be presented in an extended version of this work.

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