Emergent rewirings for cascades on correlated networks

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Recent studies of attacks on complex networks suggest that small initial breakdowns can lead to global cascades of overload failures in communication, economic trading, and supply-transportation systems, considering the defense methods is very important to reduce the huge damage. The propagation of failures depends on the flow dynamics such as packet routing of physical quantities and on the heterogeneous distribution of load or capacity in many realistic networks which have topological properties of scale-free (SF) and degree-degree correlations. We introduce a defense strategy based on emergent rewirings between neighbors of the attacked node, and investigate the size of cascade on SF networks with controllable correlations. We show the differences of damaged size for the types of correlations and the effective range of tolerance in the defense methods. They can be performed on the current wireless communication or dynamic allocation technologies.

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Cascades of overload failures triggered by small initial failures or attacks are sometimes occurred and propagated to very large damage in real networks such as power grid (blackout), Internet (packet congestion or server down by DoS), economic trading (bankruptcy), traffic system (jamming), and so forth. This phenomenon is more severe than the disconnecting of networks by intentional attacks [1], because a physical quantity (hereafter called packet) can not be transmitted even through the connection if one of the terminal nodes exceeds its load capacity. In other words, not only the topological structure of network but also the heterogeneously distributed load or capacity is deeply related to the intrinsic dynamics of packet flow and to the size of cascade as the consequence of propagation of overload failures.

As an important topological structure, we have found scale-free (SF) property in many complex networks [2]. The highly heterogeneous degree distribution follows a power law $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$. Moreover, recent studies classify SF networks according to types of degree-degree correlations of nodes with their neighbors [3]; social networks tend to have assortative connections between peers with similar degrees [4, 5], while technological or biological networks tend to have disassortative ones between those nodes with high degrees, namely hubs, and those with low degrees [3, 6]. The effect of correlations on cascades of overload failures is not yet studied, although a number of important aspects of cascading failures in complex networks have been discussed [7, 8, 9, 10]; in particular a simple model of cascades of overload failures has been introduced [11] with a proposal of defense strategy based on intentional removals (IR) [12]. The numerical results have shown to reduce the size of cascade by the IR on a sacrifice of the removing a fraction of nodes that mainly contribute to generate load rather than to transmit the packets.

We investigate the cascading failures on SF networks with degree-degree correlations, and propose an alternative defense strategy based on emergent rewire which is realized in distributed manner with additional cost in the neighbors of the initial attack. On the current technologies, our proposed methods are basically applicable to wireless communications, airline flights, logistic networks, and so on in the practically important service domains.

The organization of this paper is as follows. We first describe the basic process of cascading failures, and consider the defense strategies. Then, we introduce SF network models whose types of correlations are controllable. Through numerical simulations, we show the differences of damaged size for the types of correlations and the effect of our proposed defense strategy. Finally, we summarize these results.

For a given (undirected) network, we assume that at each time step a request of communication is generated between every pair of nodes $(i, j)$ and a packet is transmitted along the shortest paths connecting nodes $i$ and $j$. If there is more than one shortest paths connecting two given nodes, the packet is divided evenly at each branching point. In this situation, it is natural that the load $L_k(t)$ of node $k$ at time $t$ is defined through the betweeness centrality $B_k(t)$: the total amount of packets passing through the node per unit of time. Note that the shortest paths may be changed by the propagation of failures. Also as in Ref [11, 12], the capacity $C_k$ of node $k$ is assumed to be proportional to its initial load,

$$C_k \equiv \alpha L_k(0), \quad k = 1, 2, \ldots, N,$$

where $\alpha \geq 1$ is the tolerance parameter, and $N$ is the initial number of nodes. From $\alpha \geq 1$, no node is overload in the initial network. After an initial attack, the damaged node is disconnected. Then, some nodes receive much loads that exceed own capacities by changing paths, and the corresponding nodes with overload collapse. These removals lead to the next redistribution of load among the remaining nodes in the network, and the subsequent overload may occur: the failures are propagated. The cascading process is stop at $T$ with size $N'$
only when the updated load satisfies $L_k(T) \leq C_k$ for all the nodes $k$. The damage caused by a cascade is quantified in terms of the relative size of the largest connected component $GC = N^\prime / N$. As the minimum origin, we focus on load-based intentional attack upon an exhausted node which has the highest load, since global cascade can be triggered by removing only the key node [11].

In addition to the defense strategy based on IR [12], we introduce an alternative strategy based on emergent pair-connections (EP) and emergent ring (ER). The $n$ connections from the initially attacked node are replaced by the rewirings between the neighbors as shown in Fig. 1. Each of these strategies is performed after the initial attack before the propagation.

**IR:** The fraction $f$ of nodes with smallest $\Delta_i \overset{\text{def}}{=} L_i(0) - L_i^\prime$ are removed to avoid the generation of packets from the peripheral nodes, where the total load generated by node $i$ is

$$L_i^g \overset{\text{def}}{=} \sum_j (D_{ij} + 1) = (\bar{D}_i + 1)(N - 1), \quad (2)$$

$$\bar{D}_i = \sum_j D_{ij} / N, \ D_{ij} \text{ is the shortest path length between nodes } i \text{ and } j.$$

**EP:** In the neighbors of the initial attack, pairs of nodes are linked according to the decreasing order of

$$W_{ij} \overset{\text{def}}{=} C_i / k_i + C_j / k_j, \quad (3)$$

where $k_i$ and $k_j$ denotes the degrees of nodes $i$ and $j$. Large $C_i / k_i$ corresponding to large $B_i(0)$ and small $k_i$ means that the node $i$ is a bridge node between subgraphs and that important to construct bypass routes.

**ER:** In the neighbors, a ring is rewired according to the decreasing order of $C_i / k_i$ as shown in Fig. 1 (right).

The variations of IR for selecting removed nodes have almost the same effect [12] because the quantities of smallest closeness centrality, load, and degree have correlations to smallest $\Delta_i$. Similar modifications of EP and ER are also possible as mentioned latter. Here we should remark that the conventional IR is cost-less, but needs the global information to chose the nodes with smallest $\Delta_i$ in the network; while our proposed EP and ER modify local connections by using the capacities and degrees of the neighbors within only two steps, but not cost-less.

Next, we consider SF networks with various types of degree-degree correlations. Unfortunately, we have no general approach to construct SF networks with a given correlations such as the configuration method [13] of random networks for a given degree distribution, in addition there are only a few models to be able to control the correlations between assortative (Ass) and disassortative (Dis) networks. Thus, we consider the following two models. The procedures are repeated until the network reaches to the required size $N$.

**CDD:** coupled duplication-divergence model [14]

![Emergent Pair-connection](image1.png)

**Emergent Pair-connection**

**Emergent Ring**

**FIG. 1:** Defense strategy based on emergent rewirings. The red circle and dashed lines are the initially attacked node and the removed connections, respectively. Blue lines are the rewirings between the neighbors. The number in each circle denotes the order of $C_i / k_i$.

1. At each time step, a new node $i'$ is added.

2. Simultaneously, a node $i$ is randomly chosen, and new connections between all the neighbors $j$ of $i$ and the new node $i'$ are duplicated.

3. With probability $q_v$, a connection between $i$ and $i'$ is established (self-interaction).

4. In the divergence process, each duplicated connection is removed with probability $1 - q_v$.

These local rules are biologically plausible [15] and also suitable for distributed systems. Note that larger $q_v$ enhances the assortativity of network generated by the above rules because the self-interaction means connecting a pair of nodes with similar degrees. In other words, $q_v$ is a control parameter of the correlation. However, the number of total connections may vary because of the duplication processes. Obviously, a network with many bypass routes as the consequence of many connections becomes more tolerant to failures or attacks. Thus, to match the condition, we confirm the variance is bounded in the self-averaging [16]

$$\chi \overset{\text{def}}{=} \sqrt{\langle k^2 \rangle - \langle k \rangle^2} / \langle k \rangle < 0.2, \quad (4)$$

for the set of parameters in Table 1. The randomly generated CDD networks have $\langle k \rangle \approx 4$.

**LPA:** shifted linear preferential attachment model [17]

1. At each time step, a new node is added and linked to old nodes by $m$ new connections.

2. The attached nodes are randomly chosen by the shifted linear preference [17, 18]: a node with degree $k$ is chosen as the terminal of a new connection with probability proportional to $k + w$. 
This is a modification of the BA model [19] including it at \( w = 0 \) as uncorrelated networks (Unc) with \( \langle k \rangle = 2m \). We set the parameter \( w \) as in Table I and \( m = 2 \) to satisfy \( |w| < k_{\text{min}} = m \), where \( k_{\text{min}} \) denotes the minimum degree. The assortativity \( r \) in Ref. [6] is calculated for each type of correlations. Figs. 2(a)(b) show the degree distributions follow power laws with slightly collapsed parts; CDD restricts very low degrees because of the duplication process, and LPA with Dis have a star-like structure with a few hubs of very large degrees. The insets show the three types of correlations: Ass with a positive slope, Unc as almost flat, and Dis with a negative slope.

| Type     | CDD  | LPA  | assortativity |
|----------|------|------|---------------|
| Ass      | 1.0  | 0.26 | 0.19          |
| Unc      | 0.5  | 0.35 | 0.02          |
| Dis      | 0.0  | 0.42 | -0.29         |
| Weak Ass | 1.8  | -1.8 | -0.49         |

TABLE I: A set of parameters for CDD and LPA. The types of correlations Ass, Unc, and Dis denotes assortative, uncorrelated, and disassortative networks, respectively. Unnecessary parameters are blanked. The assortativity \( r \) is measured over the 100 realizations of each network model with \( N = 1,000 \) and \( \langle k \rangle \approx 4 \).

In this setting, we numerically investigate the size of cascades and the effect of defense strategies IR, EP, and ER for the SF networks. Since the removal of the most central nodes can trigger global cascades into a drastically reduced size, the defense strategies are based on the conventional IR: the restriction of load generation by removing a fraction \( f \) of nodes with most negative \( \Delta_i \), and on the proposed EP and ER: emergent rewiring to compensate the bypass between bridge nodes with large \( C_i/k_i \) (with relatively high centrality).

Figs. 3 show the ratio GC as a function of the tolerance parameter \( \alpha \) for CDD. The parts of dashed lines above the red line for no defense (NO) show that the corresponding strategies are effective. We first compare the effect in the types of correlations, and then discuss the difference in the defense strategies. The damage caused by the intrinsic cascades with NO is larger as the correlation is Ass as shown in Fig. 3(a) rather than Dis as shown in Fig. 3(c). For the damage, the IR with the nearly best fraction \( f = 0.1 \) or 0.2 [12] remains a larger GC for Dis, while there is no such difference for the types of correlations in the EP and ER. For the tolerance parameter, the effective range is contrast: small \( \alpha < 1.5 \) in the IR, however large \( \alpha > 1.5 \) in the EP and ER. It is common for all types of correlations. In particular, the size GC in the IR are strongly saturated for increasing \( \alpha \), while that in the EP or ER is significantly larger. For example, Fig. 3(a) shows the ratio GC is nearly 0.4 in the IR (or NO), while it is improved to 0.6 \( \sim 0.8 \) in the EP or ER at \( \alpha \) = 1.5.

Figs. 4 show the ratio GC as a function of the tolerance parameter \( \alpha \) for LPA. In contrast to the results for CDD, the damage with NO is larger as the correlation is Dis as shown in Fig. 4(c) rather than Ass as shown in Fig. 4(a). In particular, it is annihilative for small \( \alpha < 1.2 \). Thus, LPA is very vulnerable, we have demonstrated [20] there is no effect in the modified EP and ER by applying load or degree instead of \( C_i/k_i \). For the damage, the IR remains a larger GC for Ass (but almost no effect by the saturation in \( \alpha > 1.6 \)). While the ER (marked by diamond) reduces the damage for Ass and Unc, the EP (marked by rectangle) do for Dis in all the range \( 1 \leq \alpha \leq 2 \). On the other hand, as similar to the results for CDD, the IR is effective in small \( \alpha < 1.3 \), but the EP or ER is in large \( \alpha > 1.4 \).

In summary, we have investigated the cascades of overload failures triggered from a load-based attack in SF networks. Such phenomena is widely observed in social and technological SF networks with degree-degree correlations. We have shown that the size of cascades can be reduced by our defense strategy based on emergent rewirings between bridge nodes with large \( C_i/k_i \), and that the effect is slightly different for the types of
FIG. 3: Ratio GC as a function of the tolerance parameter $\alpha$ for CDD with (a) Ass, (b) Unc, and (c) Dis. The solid red line marked by asterisk corresponds to NO: no defense. The dashed green lines correspond to the conventional IR with $f = 0.1$ (plus) and $f = 0.2$ (cross). The dashed blue lines correspond to EP (rectangle) and ER (diamond). They are the averages over 100 realizations.

FIG. 4: Ratio GC as a function of the tolerance parameter $\alpha$ for LPA with (a) Ass, (b) Unc, and (c) Dis. The lines and marks correspond to the same as the previous. They are the averages over 100 realizations.
correlations. With the tolerance parameter $\alpha$, they are more complicated: the conventional IR works better in tight capacity ($\alpha < 1.5$ for CDD with Dis and for LPA), but the proposed EP or ER do in reasonable capacity ($\alpha > 1.5$ for CDD with Ass and LPA with Dis). We have confirmed similar results [21] for the capacity in the other SF network models based on the connecting nearest neighbors [14] and on the triangulation [22]. Thus, rewiring is not always best, however the effect is remarkable in the reasonable capacity. The ad hoc rewirings will be useful for protecting and sustaining our social organizations or communication infrastructures in real systems from huge damage triggered by small attacks against the vulnerable parts.

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