Neural Network-Based Distributed Finite-Time Tracking Control of Uncertain Multi-Agent Systems With Full State Constraints

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ABSTRACT This paper addresses the distributed tracking control problem of pure-feedback multi-agent systems with full state constraints under a directed graph in finite time. By introducing the nonlinear mapping technique, the system with full state constraints is converted into the form without state constraints. Furthermore, by combining fractional dynamic surface and radial basis function neural networks, a novel finite-time adaptive tracking controller is conducted recursively. In light of Lyapunov stability theory, it is proven that all signals of multi-agent systems are semi-globally uniformly ultimately bounded in finite time and the full states satisfy the constraints. Lastly, numerical simulations are supplied to demonstrate the effectiveness of the proposed control strategy.

INDEX TERMS Multi-agent systems, finite-time tracking control, full state constraints, pure-feedback form.

I. INTRODUCTION

In recent years, the multi-agent system (MAS) has attracted considerable attention since it is widely used in lots of applications such as power grids [1], transportation networks [2], and wireless sensor networks [3]. In practical applications, the constraints often appear owing to the physical limitations or the safe operation of systems [4]. For example, velocity and position constraints were considered simultaneously for multi-agent systems in [5]. To deal with constraints, Barrier Lyapunov Functions (BLFs) had been extensively applied [6]–[11]. In [9], the asymmetrical BLFs for strict-feedback nonlinear systems with output constraints were investigated. By introducing an auxiliary signal to the asymmetric time-varying BLF, an adaptive tracking controller for time-varying output-constrained nonlinear systems with input delay was studied in [10]. A novel universal asymmetric BLF was proposed for multiple-input multiple-output (MIMO) nonlinear systems with output constraints or without constraints in [11]. Moreover, the BLF was extended to multi-agent systems [12]–[16].

The output-constrained leaderless and leader-follower consensus protocols for second-order nonlinear MASs were proposed in [13], [14] by applying BLFs. The work in [15] focused on the distributed control of strict-feedback nonlinear systems with state constraints, in which the unknown nonlinear dynamics existing in the systems were approximated by fuzzy logic systems (FLSs). In [16], the BLF was adopted to deal with consensus problems for heterogenous high-order nonlinear MAS with output constraints.

For the BLF-based control method, it is inconvenient to make new designs of controllers to adapt to changes of Lyapunov functions. As a result, a novel nonlinear mapping (NM) methodology was proposed. By introducing the NM method, the output-constrained system in strict-feedback form was converted into a new one without constraints [17]. In [9], the asymmetrical BLFs for strict-feedback nonlinear systems with output constraints were investigated. By introducing an auxiliary signal to the asymmetric time-varying BLF, an adaptive tracking controller for time-varying output-constrained nonlinear systems with input delay was studied in [10]. A novel universal asymmetric BLF was proposed for multiple-input multiple-output (MIMO) nonlinear systems with output constraints or without constraints in [11]. Moreover, the BLF was extended to multi-agent systems [12]–[16].

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NM method was generalized to deal with tracking control problems of full state-constrained strict-feedback systems and pure-feedback systems, respectively. Although control problems with input constraints, output constraints or full state constraints for different categories of nonlinear systems have been extensively studied by employing BLF or NM methods, the tracking control issues of MAS with full state constraints have not been fully investigated.

It is also worth noting that the design of finite-time distributed control laws such that each state reaches consensus in a fast way is very useful for the MAS. Compared with the asymptotical convergence algorithm, the finite-time consensus control has significant advantages such as faster convergence rate, robustness against uncertainties as well as better disturbance rejection [20–22]. In [22], adaptive finite-time consensus control problems were addressed for the first-order MAS, in which the linearly parameterized method was used to estimate the unknown nonlinear dynamics. The works [23] and [24] discussed the finite-time consensus control of second-order and high-order MASs with disturbances, respectively. However, the results presented in those papers did not consider full state constraints. Based on BLFs and the finite-time stability theory, the finite-time tracking controller for strict-feedback nonlinear systems with full state constraints and dead-zone was investigated in [25]. Compared with [25], the unknown nonlinear dynamics were considered in [26], where an adaptive finite-time fuzzy controller was designed to solve the full state-constrained tracking control problem of strict-feedback nonlinear systems by introducing the tan-type BLF into the backstepping procedure. Nevertheless, it is noted that the finite-time tracking control of the uncertain multi-agent system with full state constraints is rarely considered.

Inspired by the above articles, the distributed finite-time tracking control of the pure-feedback MAS with full state constraints under a directed graph is investigated in this paper. Based on the finite-time stability theory, it is proven that all signals of multi-agent systems are semi-globally uniformly ultimately bounded in finite time and the full states satisfy the constraints. The main contributions are summarized as follows:

1) Compared with the works [22], [24]–[26], the pure-feedback MAS with full state constraints is considered. Furthermore, the NM method is extended to the more general multi-agent systems. It is challenging and difficult to analyze the stability of the uncertain multi-agent systems. By applying the NM technique, the full state-constrained MAS in pure-feedback form is firstly converted into a novel one without constraints. Moreover, the proposed control design process is very simple because it can employ traditional Lyapunov functions rather than redesigning the Lyapunov functions.

2) By combining backstepping and fractional dynamic surface control, a novel distributed finite-time tracking strategy is proposed, which is different from the works [17]–[19]. To avoid the explosion of complexity for the pure-feedback systems in backstepping design and achieve the finite-time control, the fractional dynamic surface is introduced and employed. And the adaptive neural networks are used to deal with unknown nonlinearities of the MAS.

The rest of this paper is organized as follows. Section II highlights the main theoretical concepts including some preliminaries and problem formulation. The design of adaptive controller and the stability analysis are conducted in Section III. Section IV presents some simulation examples. The last Section V concludes this paper.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

A. SOME CONCEPTS OF GRAPH THEORY

Consider a MAS including $M$ agents and the corresponding communication topology is expressed by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_M\}$ is a nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. The set of neighbours of node $v_j$ is denoted as $N_j = \{v_i \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}, j \neq i\}$, where $v_i$ can obtain information from node $v_j$. The adjacency matrix is denoted as $A = [a_{ij}]$ with $a_{ij} > 0$ when $(v_j, v_i) \in \mathcal{E}$ or else $a_{ij} = 0$. Define the in-degree matrix $D = \text{diag}[d_1, \ldots, d_M]$, where $d_i = \sum_{j \in N_i} a_{ij}$ refers to the in-degree of node $v_i$. The Laplacian matrix can be described as $L = D - A$. The graph $\mathcal{G}$ is applied to represent the topology among the $M + 1$ agents with a leader node $v_0$. The Laplacian matrix $\bar{L}$ with regard to $\mathcal{G}$ is expressed as

$$\bar{L} = \begin{bmatrix} L + B & -b \\ 0 & 0 \end{bmatrix},$$

where $b = (b_1, \ldots, b_M)^T$ and $B = \text{diag}(b_1, \ldots, b_M)$ with $b_1 = 1$ if the leader directly connects with the $i$th follower, and $b_i = 0$ otherwise.

B. PROBLEM DESCRIPTION

Consider a MAS comprising the leader $v_0$ and $M$ followers with full state constraints. The dynamics of $M$ followers is modeled by

$$\begin{align*}
\dot{x}_{i,m} &= f_{i,m}(\bar{x}_{i,m}, x_{i,m+1}), & m = 1, \ldots, n - 1 \\
\dot{y}_i &= x_{i,1}, & i = 1, \ldots, M,
\end{align*}$$

(1)

where $\bar{x}_{i,m} = [x_{i,1}, \ldots, x_{i,m}]^T \in \mathbb{R}^m$ is the state vector in $i$th follower, $x_{i,m} \in \mathbb{R}$, $u_i \in \mathbb{R}$, $y_i \in \mathbb{R}$ are system states, control input, and output of the $i$th follower, respectively. $f_{i,m}()$, $m = 1, \ldots, n$ and $g_{i,n}()$ are unknown smooth functions. All states $x_{i,m}$ are expected to always remain in an open predefined set $\Omega_{x_{i,m}} = \{x_{i,m} : -b_{m1} < x_{i,m} < b_{m2}\}$, where $b_{m1}$ and $b_{m2}$ are known positive constants.

An objective of this paper is to design a distributed adaptive consensus protocol for MAS (1) such that the outputs of followers $y_i$ follow the specified desired trajectory $y_d$ in finite time, and all states always remain in the constraints $x_{i,m} \in \Omega_{x_{i,m}}$, $m = 1, \ldots, n$.

Assumption 1: The directed communication topology $\mathcal{G}$ has a directed spanning tree and the leader is the root node.
Assumption 2: The expected trajectory $y_r$ is continuous with $|y_r| < B_1 < \min\{b_{m_1}, b_{m_2}\}$, where $B_1$ is a known positive constant. In addition, the derivative $\dot{|y_r|}$ satisfies $|\dot{y_r}| \leq r < \infty$ with $r$ being an unknown positive constant.

Assumption 3: The function $g_{l,n}(\bar{x}_i,n)$ is bounded, and there exists constant $g_{l,n}$ satisfying $0 < g_{l,n} \leq |g_{l,n}(\bar{x}_i,n)|$.

Without loss of generality, we assume that $0 < g_{l,n} \leq g_{l,n}(\bar{x}_i,n)$ in this paper.

Remark 1: Assumption 1 is generally made in the case of leader-following tracking. Assumption 2 is the description of the expected trajectory, which is widely applied in the design of controllers. These assumptions are frequently used in the existing relevant literature.

C. PRELIMINARIES

Lemma 1 [27]: For $\bar{a}, \bar{b} \in R$, if $\bar{a}, \bar{d} > 0$, then $|\bar{a} \bar{c}| \bar{b} \bar{d} \leq \bar{c}/(\bar{c} + \bar{d}) |\bar{a}| \bar{c} + \bar{b} \bar{d} / (\bar{c} + \bar{d}) |\bar{b}| \bar{c} + \bar{d} / (\bar{c} + \bar{d}) |\bar{b}| \bar{d}$.

Lemma 2 [27]: For $\bar{a}, \bar{b} \in R$, if $0 < h < h_1/h_2 \leq 1$, then $|\bar{d}_h - \bar{b} h| \leq 2^{-1} h |\bar{a} - \bar{b} h|$, where $h_1, h_2$ are positive odd integers.

Lemma 3 [28]: For $\bar{a}_i \in R, i = 1, 2, \ldots, n, 0 < h \leq 1$, then $(\sum_{i=1}^n |\bar{a}_i|)^{\frac{1}{h}} \leq n^{-1} \sum_{i=1}^n |\bar{a}_i|$.\hfill (12)

Lemma 4 [29]: For $\bar{a}, \bar{b} \in R$, and $0 < h < h_1/h_2 \leq 1$, then $\bar{d}^{-1} (\bar{b} - \bar{a}) \leq 1 \bar{d}^{-h} |\bar{a} - \bar{b} h|$, where $h_1, h_2$ are positive odd integers.

Lemma 5 [30]: Consider the system $\dot{x} = f(x, u)$. Suppose that there exist continuous function $V(x)$, scalars $\alpha > 0$, $0 < \alpha < 1$, $0 < \eta < \infty$, $0 < \theta < 1$, such that $\dot{V} \leq -\alpha V^\eta (x) + \eta$, then for the given initial state $x(0)$, the trajectories of the closed-loop system converge into a bounded set $\Omega = \{x \mid V(x) \leq \eta/(1 - \theta) \alpha\}$ in the finite settling time satisfying $T^* \leq (V^{1-\eta}(x(0)) / \eta \theta (1 - \alpha))$.\hfill (13)

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

A. SYSTEM TRANSFORMATION

To deal with full state constraints, the NM technique [17] is introduced. In this case, the considered dynamics (1) with full state constraints can be converted into the pure-feedback nonlinear system without state constraints. Define

$$M (b_{m_1}, b_{m_2}, x_{i,m}) = \log \left( b_{m_1} + x_{i,m} \right)$$

where $b_{m_1}$ and $b_{m_2}$ are known positive constants. Let $z_{i,m} = M (b_{m_1}, b_{m_2}, x_{i,m})$. From (2), it is easy to get that

$$x_{i,m} = b_{m_2} e^{z_{i,m}} - b_{m_1} e^{z_{i,m}^2} - 1.$$\hfill (14)

Therefore, we obtain

$$\dot{z}_{i,m} = q_{i,m}(z_{i,m}) \dot{x}_{i,m}$$

$$q_{i,m}(z_{i,m}) = \frac{e^{z_{i,m}^2} + e^{-z_{i,m}^2} + 2}{b_{m_1} + b_{m_2}}.$$\hfill (15)

Then, the dynamics (1) can be rewritten as follows:

$$\dot{z}_{i,m} = F_{i,m}(z_{i,m}, \dot{z}_{i,m} + 1) + z_{i,m} + 1$$

$$\dot{z}_{i,n} = F_{i,n}(z_{i,n}) + q_{i,n}(z_{i,n}) G_{i,n}(\hat{z}_{i,n}) u_i$$

$$\dot{y}_{i,m} = y_{i,m}, m = 1, \ldots, n - 1,$$\hfill (16)

where

$$F_{i,m}(\bar{z}_{i,m}, \dot{z}_{i,m}+1) = q_{i,m}(z_{i,m}) \dot{x}_{i,m} + x_{i,m} + 1$$

$$F_{i,n}(\bar{z}_{i,n}) = q_{i,n}(z_{i,n}) \dot{z}_{i,n}$$

$$G_{i,n}(\bar{z}_{i,n}) = g_{i,n}(\bar{x}_i,n).$$\hfill (17)

Remark 2: From (9) and Assumption 3, $G_{i,n}(\bar{z}_{i,n}) \geq g_{i,n}(\bar{x}_i,n) > 0$ can be easily obtained. Therefore, the control gain sign is positive.

Remark 3: After the system transformation, the specific object of this study is changed to solve distributed tracking problems of the transformed MAS (6) so that the output $y_{i,m}$ of the followers will track the reference trajectory $y_d = [\log(b_{l_1} + y_r)/(b_{l_2} - y_r)]$. Besides, from (3) we know that the system states $x_{i,m}, m = 1, \ldots, n$ will keep in the bounded set $\Xi_{i,m}$ if the boundedness of $\hat{z}_{i,m}$ is guaranteed.

In this paper, the radial basis function neural networks (RBNNNs) are employed to estimate unknown continuous functions $\psi_{i,m}(m = 1, \ldots, n)$ as follows

$$\psi_{i,m}(\hat{z}_{i,m}) = W_{i,m}^T \phi_{i,m}(\hat{x}_{i,m}) + \delta_{i,m}(\hat{z}_{i,m}),$$\hfill (18)

where $m = 1, \ldots, n, \phi_{i,m}(\hat{x}_{i,m})$ denotes the basis function vector; $\hat{x}_{i,m}$ denotes the input of neural network; $W_{i,m}^T$ is the ideal unknown weight vector; $\delta_{i,m}(\hat{x}_{i,m})$ is the approximation error such that $|\delta_{i,m}(\hat{x}_{i,m})| \leq \hat{z}_{i,m}$. Meanwhile, let $\chi_{i,m} = g_{i,0}^{-1} \left\| W_{i,m}^T \right\|$, $\chi_{i,m} = x_{i,m} - \hat{x}_{i,m}$ where $\hat{x}_{i,m}$ is the estimation of $x_{i,m}$ and $\hat{x}_{i,m}$ denotes the estimation error with $m = 1, \ldots, n, g_{i,0} = \min[1, g_{i,n}].$

B. CONTROLLER DESIGN

Based on the transformed system (6), the fraction dynamic surface control and the backstepping design technique are employed to design distributed finite-time tracking controllers.

Firstly, local error surface $e_{i,1}, e_{i,m}$ and boundary layer errors $S_{i,m}$ are respectively defined as

$$e_{i,1} = \sum_{j=1}^M a_j(z_{j,1} - \hat{z}_{j,1}) + b_j(z_{j,1} - y_d)$$

$$e_{i,m} = z_{i,m} - \alpha_{i,m}$$

$$S_{i,m} = \alpha_{i,m} - \alpha_{i,m}, m = 2, \ldots, n,$$\hfill (19)

where $\alpha_{i,m}$ refers to the output of the fraction filters as follows.

$$\alpha_{i,m} = \left( \alpha_{i,m} - \alpha_{i,m} \right)^T, \quad \alpha_{i,m}(0) = \alpha_{i,m}(0).$$\hfill (20)

where $0 < \varepsilon_{i,m} < \infty$ is a constant and $l = (2a - 1)/(2a + 1)$ with $a \in Z^+/\{1\}$ denotes the fraction.

The detailed design processes are as follows.

Step 1: In what follows, the backstepping method is used to derive the controller. First of all, the Lyapunov function candidate is constructed as

$$V_1 = \sum_{i=1}^M \frac{1}{2 b_i + d_i} e_{i,1}.$$\hfill (21)
The time derivative of $V_1$ can be obtained as below

$$
\dot{V}_1 = \sum_{i=1}^{M} \frac{1}{b_i + d_i} e_i, \dot{e}_i, 1
\]

$$
= \sum_{i=1}^{M} e_i, 1\left(\left(b_i + d_i\right)(F_i, 1 + z_{i, 2})
\right.

\left. - \sum_{j=1}^{M} a_{i,j}(F_j, 1 + z_{j, 2}) - b_i \dot{y}_d\right).

(16)

Let $\psi_i, 1(\xi, 1) = F_i, 1 - \frac{1}{b_i + d_i} \left(\sum_{j=1}^{M} a_{i,j}(F_j, 1 + z_{j, 2}) + b_i \dot{y}_d\right)$.

Then, substituting (12) and (13) into (16) and employing RBFNNs in the general form of (10), we have

$$
V_1 = \sum_{i=1}^{M} e_i, 1\left(W_{i, 1}^{T} \phi_i, 1(\xi, 1) + \delta_i, 1(\xi, 1) + e_i, 2 + \alpha_i, 2 + S_i, 2\right)
\]

(17)

where $|\delta_i, 1(\xi, 1)| \leq \tilde{e}_i, 1$. By employing Young’s inequality, we obtain

$$
e_i, 1W_{i, 1}^{T} \phi_i, 1 \leq \frac{g_{i, 0} X_i, 1}{2 n_i, 1^2} \phi_i, 1^2 \leq \frac{\eta_i^2, 1}{2}
\]

(18)

$$
e_i, 1 \xi_i \leq \left(\frac{I}{1 + k_i, 11}\right)^l \tilde{e}_i, 1^l + \left[\frac{1}{k_i, 11}\right]^l e_i, 1^l
\]

(19)

$$
e_i, 1e_i, 2 \leq \left(\frac{h}{1 + k_i, 12}\right)^l \tilde{e}_i, 2^l + \left[\frac{1}{k_i, 12}\right]^l e_i, 2^l
\]

(20)

$$
e_i, 1S_i, 2 \leq \left(\frac{I}{1 + k_i, 13}\right)^l \tilde{e}_i, 1^l + \left[\frac{1}{k_i, 13}\right]^l e_i, 1^l
\]

(21)

where $\chi_i = \frac{g_{i, 0} X_i, 1}{2 n_i, 1^2}$, $\eta_i$ and $k_{i, 11}$, $k_{i, 12}$, $k_{i, 13}$ are positive constants defined by the designer. Now, we construct the virtual control $\alpha_{i, 2}$ which is described as

$$
\alpha_{i, 2} = -k_{i, 10} e_i, 1 - k_{i, 11} e_i, 1 - \frac{\tilde{e}_i, 1}{2 n_i, 1} \phi_i, 1 e_i, 1
\]

(22)

where $k_{i, 10} > 0, k_{i, 11} > 0$ are design parameters with $k_{i, 1} = \frac{1}{k_{i, 11}} + \frac{1}{k_{i, 12}} + \frac{1}{k_{i, 13}}$.

Substituting (18) - (22) into (17), we obtain

$$
\dot{V}_1 \leq \sum_{i=1}^{M} \left[ -k_{i, 10} e_i, 1^l + \frac{g_{i, 0} X_i, 1}{2 n_i, 1^2} \phi_i, 1 e_i, 1^2 + c_{i, 1}
\right.

\left. + \left(\frac{h}{1 + k_i, 12}\right)^l \tilde{e}_i, 2^l + \left(\frac{I}{1 + k_i, 13}\right)^l \tilde{e}_i, 1^l + \left[\frac{1}{k_i, 12}\right]^l e_i, 2^l + \left[\frac{1}{k_i, 13}\right]^l e_i, 1^l\right]
\]

(23)

with $c_{i, 1} = \frac{\eta_i^2, 1}{2} + \left(\frac{h}{1 + k_i, 12}\right)^l \tilde{e}_i, 2^l + \left(\frac{I}{1 + k_i, 13}\right)^l \tilde{e}_i, 1^l$.

Step m: For $m = 2, \ldots, n - 1$: The Lyapunov function $V_m$ for $m = 2, \ldots, n - 1$ is chosen as

$$
V_m = \sum_{i=1}^{M} e_i, m^2
\]

(24)

According to (10), let $\psi_{i, m}(\dot{\xi}_i, m) = W_{i, m}^{T} \phi_i, m(\xi_i, m) + \delta_i, m(\dot{\xi}_i, m) = F_i, m - \alpha_i, m$. Taking the time derivative of $V_m$ by utilizing (6), (12), (13), we get

$$
\dot{V}_m = \sum_{i=1}^{M} e_i, m \dot{e}_i, m
\]

$$
= \sum_{i=1}^{M} e_i, m \left(F_i, m + \dot{z}_i, m - \alpha_i, m\right)
\]

$$
= \sum_{i=1}^{M} e_i, m \left(W_{i, m}^{T} \phi_i, m(\xi_i, m) + \delta_i, m(\dot{\xi}_i, m) + e_i, m + 1\right)
\]

$$
+ S_{i, m} + \alpha_i, m + 1\right).
\]

(25)

Let $\chi_{i, m} = g_{i, 0} \frac{1}{W_i, m^2}$. Then, utilizing Young’s inequality once more, we get

$$
e_i, mW_{i, m}^{T} \phi_i, m \leq \frac{g_{i, 0} X_i, m}{2 n_i, m^2} \phi_i, m e_i, m^2 + \frac{\eta_i^2, m}{2}
\]

(26)

$$
e_i, m \delta_i, m \leq \frac{\left(\frac{h}{1 + k_i, m1}\right)^l \tilde{e}_i, m}{1 + k_i, m1} + \frac{\eta_i^2, m}{k_i, m1}
\]

(27)

$$
e_i, m e_i, m + 1 \leq \frac{\left(\frac{h}{1 + k_i, m2}\right)^l \tilde{e}_i, m + 1}{1 + k_i, m2} + \frac{\eta_i^2, m}{k_i, m2}
\]

(28)

$$
e_i, m S_i, m \leq \frac{\left(\frac{h}{1 + k_i, m3}\right)^l \tilde{e}_i, m + 1}{1 + k_i, m3} + \frac{\eta_i^2, m}{k_i, m3}
\]

(29)

where $\eta_i, m$, $k_{i, m1}$, $k_{i, m2}$, and $k_{i, m3}$ are positive design constants.

The virtual control law $\alpha_{i, m+1}$ is designed as

$$
\alpha_{i, m+1} = -k_{i, m0} e_i, m - k_{i, m1} \frac{\ddot{\dot{\xi}}_i, m}{2 n_i, m^2} \phi_i, m \phi_i, m e_i, m
\]

(30)

where $k_{i, m0} > 0$, $k_{i, m} > 0$ are design parameters with $k_{i, m} = \frac{1}{k_{i, m1}} + \frac{1}{k_{i, m2}} + \frac{1}{k_{i, m3}}$.

Substituting (30) as well as Young’s inequalities (26)-(29) into (25), it then follows that

$$
\dot{V}_m \leq \sum_{i=1}^{M} \left[ -k_{i, m0} e_i, m + \frac{g_{i, 0} X_i, m}{2 n_i, m^2} \phi_i, m e_i, m^2 + c_{i, m}
\right.

\left. + \frac{\left(\frac{h}{1 + k_i, m2}\right)^l \tilde{e}_i, m}{1 + k_i, m2} + \frac{\left(\frac{h}{1 + k_i, m3}\right)^l \tilde{e}_i, m}{1 + k_i, m3} + \frac{\eta_i^2, m}{1 + k_i, m3}\right]
\]

(31)

with $c_{i, m} = \frac{\eta_i^2, m}{2} + \frac{\left(\frac{h}{1 + k_i, m2}\right)^l \tilde{e}_i, m}{1 + k_i, m2} + \frac{\left(\frac{h}{1 + k_i, m3}\right)^l \tilde{e}_i, m}{1 + k_i, m3}$.

Step m: In this step, actual control laws $u_i$ will be designed. Considering a Lyapunov function as

$$
V_n = \sum_{i=1}^{M} e_i, n^2
\]

(32)
From (12), we have \( e_{i,n} = z_{i,n} - \alpha_{i,n} \). According to (10), let \( \psi_{i,n}(\xi_{i,n}) = W^{t}_{i,n} \phi_{i,n}(\xi_{i,n}) + \delta_{i,n}(\xi_{i,n}) = F_{i,n} - \dot{\alpha}_{i,n} \).

Then, the derivative of \( V_n \) is

\[
\dot{V}_{n} = \sum_{i=1}^{M} e_{i,n} \dot{\eta}_{i,n} = \sum_{i=1}^{M} e_{i,n} (F_{i,n} + q_{i,n} G_{i,n} u_{i} - \dot{\alpha}_{i,n})
\]

\[
= \sum_{i=1}^{M} e_{i,n} (W^{t}_{i,n} \phi_{i,n}(\xi_{i,n}) + \delta_{i,n}(\xi_{i,n}) + q_{i,n} G_{i,n} u_{i}). 
\]

(33)

Let \( \chi_{i,n} = g_{i,0}^{1/2} W^{*}_{i,n} \). By Young’s inequality, we get

\[
e_{i,n} W^{t}_{i,n} \phi_{i,n} \leq \frac{g_{i,0} \chi_{i,n}}{2h_{i,n}^{2}} \phi_{i,n}^{2} + \frac{\eta_{i,n}^{2}}{2} \tag{34}
\]

where \( g_{i,0} \), \( \eta_{i,n} \), and \( k_{i,n1} \) are positive constants.

Construct the actual controller \( u_{i} \) as follows

\[
u_{i} = \frac{1}{q_{i,n}} \left( -k_{i,n0} e_{i,n} - k_{i,n1} e_{i,n} - \frac{\chi_{i,n}}{2h_{i,n}^{2}} \phi_{i,n} e_{i,n} \right), \tag{36}
\]

where \( k_{i,n0} > 0 \), \( k_{i,n1} = \frac{1}{k_{i,n2}} \) and \( \eta_{i,n} > 0 \) are design parameters. Substituting (34)-(36) into (33) yields

\[
\dot{V}_{n} \leq \sum_{i=1}^{M} \left( -k_{i,n0} g_{i,0} e_{i,n}^{2} + \frac{g_{i,0} \chi_{i,n}}{2h_{i,n}^{2}} \phi_{i,n} e_{i,n} + c_{i,n} \right),
\]

(37)

where \( c_{i,n} = \frac{\eta_{i,n}^{2}}{2} + \frac{\left( \frac{k_{i,n2}}{1 + k_{i,n2}} \right)^{2}}{2h_{i,n}^{2}} e_{i,n}^{2} \).

The updating laws of the unknown parameter \( \chi_{i,m} \) for \( i = 1, \ldots, M \) are designed as

\[
\dot{\chi}_{i,m} = -\sigma_{i,m} \chi_{i,m}^{l} + \frac{\gamma_{i,m}}{2h_{i,m}^{2}} \phi_{i,m}^{2}, \quad m = 1, \ldots, n \tag{38}
\]

with \( \dot{\chi}_{i,m}(0) > 0 \), \( \sigma_{i,m} \) and \( \gamma_{i,m} \) are positive design parameters.

**C. STABILITY ANALYSIS**

From the boundary layer errors \( S_{i,m} \) (13) and the definition of fraction filter for \( \alpha_{i,(m+1)f} \) (14), we have

\[
\dot{\alpha}_{i,(m+1)f} = \frac{(\alpha_{i,m+1} - \alpha_{i,(m+1)f})}{\epsilon_{i,m+1}} = -\frac{S_{i,m+1}^{l}}{\epsilon_{i,m+1}}. \tag{39}
\]

Furthermore,

\[
\dot{S}_{i,m+1} = \frac{S_{i,m+1}^{l} - \dot{\alpha}_{i,m+1}}{\epsilon_{i,m+1}}.
\]

(40)

According to (30), it is obvious that \( \dot{\alpha}_{i,m+1} \) is the function of variables \( e_{i,1}, \ldots, e_{i,m}, \tilde{\chi}_{i,1}, \ldots, \tilde{\chi}_{i,m} \). Thus, from (40),

\[
\dot{S}_{i,m+1} + \frac{S_{i,m+1}^{l}}{\epsilon_{i,m+1}} \leq J_{i,m+1}(e_{i,1}, \ldots, e_{i,m}, \tilde{\chi}_{i,1}, \ldots, \tilde{\chi}_{i,m}),
\]

(41)

where \( J_{i,m+1}(\cdot) \) is a continuous function, \( m = 1, \ldots, n - 1 \).

Construct the overall Lyapunov function candidate as

\[
V = \sum_{m=1}^{n} V_{m} + \sum_{i=1}^{M} \sum_{l=1}^{n} \frac{g_{i,0}^{l} \tilde{\chi}_{i,m}}{2\gamma_{i,m}} + \sum_{m=1}^{n-1} \frac{S_{i,m+1}^{2}}{2}. \tag{42}
\]

(42)

Applying (23), (31), (37), (38), and (41), the time derivative of \( V \) can be obtained as

\[
\dot{V} = \sum_{m=1}^{n} \dot{V}_{m} + \sum_{i=1}^{M} \sum_{m=1}^{n} \frac{g_{i,0}^{l} \tilde{\chi}_{i,m}}{\gamma_{i,m}} \left( -\frac{2}{1 + l} \chi_{i,m} \right)
\]

\[
+ \sum_{i=1}^{M} \sum_{m=1}^{n-1} \left( \frac{1}{1 + l} S_{i,m+1}^{l} \right)
\]

\[
+ \sum_{i=1}^{M} \sum_{m=1}^{n} \left( \frac{1}{1 + l} S_{i,m+1}^{l+1} \right)
\]

\[
+ \sum_{i=1}^{M} \sum_{m=1}^{n-1} \left( \frac{1}{1 + l} S_{i,m+1}^{l+1} \right).
\]

(43)

According to Lemmas 1, 2, and 4, it can be shown that

\[
\dot{\chi}_{i,m}^{l} \leq -\frac{1}{l(1 + l)^{2}} \left( 2^{l-1} - 2^{l-1}(1+l) \right) \chi_{i,m}^{l+1}
\]

\[
+ \frac{1}{1 + l} \left( 1 - 2^{l-1} + \frac{l}{1 + l} + \frac{2^{l-1}}{1 + l} \right) \chi_{i,m}^{l+1}
\]

(44)

From (41), all the variables in the function \( J_{i,m+1}(\cdot) \) are bounded, and thus we assume that there exists an upper bound on \( J_{i,m+1}(\cdot) \) such that \( J_{i,m+1}(\cdot) \leq N_{i,m+1} \). By Young’s inequality, we have that

\[
|S_{i,m+1}| J_{i,m+1} \leq \lambda_{i} S_{i,m+1}^{l+1} + \frac{L_{i,m+1}}{(1 + l)(1 + l)} \chi_{i,m}^{l+1}. \tag{45}
\]

By choosing \( \frac{1}{\epsilon_{i,m+1}} = \lambda_{i} + \frac{1}{1 + l} \chi_{i,m}^{l+1} \), (45) into (43), it follows that

\[
\dot{V} \leq \sum_{i=1}^{M} \sum_{m=1}^{n-1} \left( -\frac{1}{l} \tilde{\chi}_{i,m}^{l} \right)
\]

(41)
\[ k_v = \max \left\{ \frac{1}{2(b_i + d_i)}, \frac{1}{2} \cdot \frac{g_{i,0}}{2\gamma_{i,m}} \right\}. \]  

(53)

Let \( \tilde{c} = k_d/k_v^{\frac{l}{2l}} \). From (50) and (52), it follows that

\[ \dot{V}(t) \leq -\tilde{c} V^\frac{l}{2l}(t) + C. \]  

(54)

In the light of Lemma 5, \( T^* \) satisfies

\[ T^* \leq \frac{V(t_0)^{\frac{l}{2l}}}{(1 - \eta_1)\tilde{c}(1 - \frac{l}{2l})} \]  

with \( 0 < \eta_1 < 1 \). Then, we have

\[ V(t) \leq \left( \frac{C}{\eta_1 \tilde{c}} \right)^{\frac{1}{\frac{l}{2l}}} = \varsigma. \]  

(56)

Let \( E_1 = [e_{1,1}, \ldots, e_{M,1}]^T \). Then, steady-state errors of \( M \) agents are derived as follows.

\[ \|E_1\| \leq \sqrt{2\max_i |b_i + d_i|} V(t) \leq \sqrt{2\max_i |b_i + d_i|} \varsigma \]  

(57)

Let \( E_m = [e_{1,m}, \ldots, e_{M,m}]^T \), \( S_m = [S_{1,m}, \ldots, S_{M,m}]^T \), and \( \tilde{x}_m = [\tilde{x}_{1,m}, \ldots, \tilde{x}_{M,m}]^T \). Then, we have

\[ \|E_m\| \leq \sqrt{2V(t)} \leq \sqrt{2\varsigma}, \quad m = 2, \ldots, n \]  

\[ \|S_m\| \leq \sqrt{2\varsigma}, \quad m = 2, \ldots, n \]  

\[ \|\tilde{x}_m\| \leq \sqrt{\frac{2\gamma_{\text{max}}}{g_{\text{min}}}} V(t) \leq \sqrt{\frac{2\gamma_{\text{max}}}{g_{\text{min}}}} \varsigma, \quad m = 1, \ldots, n. \]  

(58)

where \( \gamma_{\text{max}} = \max \{g_{i,m}\}, g_{\text{min}} = \min [g_{i,0}] \). Under the proposed finite-time control schemes (22), (30), (36), and (38), the convergent region \( \Omega_1 \) of tracking errors is described by

\[ \Omega_1 = \{ \|E_1\| \leq \sqrt{2\max_i |b_i + d_i|} \varsigma \} \]  

Simultaneously, parameter estimation errors converge to the neighborhood \( \Omega_2 \) given by

\[ \Omega_2 = \{ \|\tilde{x}_m\| \leq \sqrt{\frac{2\gamma_{\text{max}}}{g_{\text{min}}}} \varsigma, \quad m = 1, \ldots, n \} \]  

(60)

in the finite time \( T^* \).

It should be noted that there may be a singularity of \( \dot{u}_{i,m+1} \) in (41) due to the existence of \( c_{i,m}(0 < l < 1) \) in the control schemes. To avoid the singularity problem, \( l \) is redefined as follows:

\[ l(e_{i,m}) = \begin{cases} \frac{(2a - 1)}{(2a + 1)}, & e_{i,m} \notin \Omega_0 \\ 1, & \text{otherwise} \end{cases} \]  

(61)

where \( \Omega_0 = \{ e_{i,m} \in \Omega_1 \mid |e_{i,m}| < \varepsilon, \varepsilon > 0 \} \) with \( \varepsilon \) being a small constant.

Remark 4: If \( e_{i,m} \notin \Omega_0 \), \( l(e_{i,m}) = (2a - 1)/(2a + 1) \). Based on the fractional dynamic surface control, the proposed finite-time tracking strategy works, and \( e_{i,m} \) converges to the neighborhood \( \Omega_1 \). Otherwise, \( l(e_{i,m}) = 1 \). That is to say,
\( e_{i,m} \) is practically bounded in \( \Omega_1 \). From (61), it is obvious \( l(e_{i,m}) \) is not always an integer. And thus, the control scheme is essentially a fraction feedback control.

Based on the above analysis, from (56), (57), and (58), they show that \( V(t), e_{i,m}, \tilde{x}_m(m = 1, \ldots, n) \), \( S_i(m = 2, \ldots, n) \) are bounded. Furthermore, \( a_i(m), a_i(m) \) are also uniformly bounded. From (11), we have \( E_1 = (L + B)(z_1 - y_d) \). Then, it is obvious that \( \|z_1\| \leq \|E_1\|/\lambda_{\min}(L + B) + \|y_d\| \) is bounded. Based on (12) and (13), \( z_{i,m}(m = 2, \ldots, n) \) are also bounded. From (3), all system states are remained in the constrains \( x_{i,m} \in \Omega_{i,m} \), \( m = 1, \ldots, n \). In addition, the error sets \( \Omega_1 \) and \( \Omega_2 \) can converge into arbitrary small regions of the origin by selecting suitable design parameters.

Overall, the following conclusion can be drawn from the above analysis.

**Theorem 1:** Consider the multi-agent system (1) under Assumptions 1-3, with virtual control laws (22), (30), actual control laws (36) and adaptive laws given by (38). If the initial conditions satisfy \( x_{i,m}(0) \in \Omega_{i,m} \), \( i \in \mathbb{N} \), \( m = 1, \ldots, n \), all system states \( x_{i,m} \in \Omega_{i,m} \), i.e., the full state constraints are not violated. By selecting suitable design parameters, the local tracking errors converge into arbitrary small regions of the origin in the finite time \( T^* \) given in (55).

**Remark 5:** By introducing the asymmetric NM technique, the problems of MASs with full state constraints are more easily addressed as compared with the BLF-based control method because the NM-based control transforms the constrained system into a new system without state constraints and then the design process can employ traditional Lyapunov functions. In comparison, it is needed to redesign the Lyapunov function if the Barrier Lyapunov Functions are applied. In the work [16], the communication topology is undirected. However, the communication topology of this paper is directed which is different from [16]. On the other hand, the fraction dynamic surface based on RBFNNs is employed to design the finite-time tracking strategy of the uncertain multi-agent system. Compared with the non-finite-time algorithm, the proposed control laws have many advantages including higher accuracy and faster convergence rate.

**IV. SIMULATION RESULTS**

In this section, numerical simulations are conducted to show the effectiveness of the proposed control.

**Example 1:** Consider a group of multi-agent systems (four followers and one leader) with the following dynamics.

\[
\begin{align*}
\dot{x}_{i,1} &= x_{i,2} + 0.05 \sin(x_{i,2}) \\
\dot{x}_{i,2} &= (0.9 + 0.05e^{-x_{i,1}^2}) u_i + 0.1 \sin(x_{i,1})
\end{align*}
\]

(62)

The communication topology is shown in Fig. 1, where the leader node is labeled by 0. The desired trajectory \( y_r \) is given as \( y_r = \sin(0.5t) \). The initial conditions of the followers are set as \( x_{i,1}(0) = [0.5, 1, 0.6, 1.3]^T \), \( x_{i,2}(0) = [0, 0, 0, 0]^T \). The design parameters are selected as \( s = 8, k_{i,10} = 10, k_{i,1} = 5, k_{i,22} = 6, k_{i,2} = 6, \eta_{i,1} = 2, \eta_{i,2} = 2, \)

Fig. 2 shows the tracking performance. By applying the proposed control method, it shows that the outputs of the followers can track the desired trajectory and always remain in \( \Omega_{x_{i,1}} = \{x_{i,1} : -1.1 < x_{i,1} < 1.8\} \). Fig. 3 shows
that the states $x_{i,2}$ of the followers also remain in $\Omega_{x_{i,2}} = \{x_{i,2} : -1.8 < x_{i,2} < 1.8\}$. The tracking errors $e_{i,1}$ and $e_{i,2}$ are illustrated in Fig. 4. It is clear that the tracking errors decline fast and remain in a small bound around zero. Fig. 5 is the estimations of the unknown parameters $\chi_{i,1}, \chi_{i,2}$. Under the same communication topology and the same dynamics of the multi-agent system with the same design parameters mentioned above, the results of the proposed NM-based control schemes are compared with the conventional control scheme without the NM method. As Fig. 6 shows, there is a significant difference between system states $x_{i,2}$ with or without constraints. As we see, it is apparent that the proposed control based on the NM method can guarantee that the system states always stay in the constraints $\Omega_{x_{i,2}} = \{x_{i,2} : -1.8 < x_{i,2} < 1.8\}$.

Example 2: Another example is carried out to further demonstrate the effectiveness of the proposed algorithm. In this example, the communication topology is identical to Example 1. The dynamic model is presented as

$$\begin{align*}
\dot{x}_{i,1} &= x_{i,2} \\
\dot{x}_{i,2} &= -\sin(x_{i,1}) - 0.25x_{i,2} + 1.5\cos(2.5t) + u_t
\end{align*}$$

(63)

The trajectory is given as $y_r = 0.5\sin(0.5t) + 0.5\sin(t)$. And the initial conditions are set as $x_{i,1}(0) = [0.5, 1, 0.6, 1.3]^T$, $x_{i,2}(0) = [1, 0.5, 1.1, 0.3]^T$. The design parameters are selected as $s = 8, k_{i,10} = 9, k_{i,1} = 1, k_{i,22} = 9, k_{i,2} = 1, n_{i,1} = 2, n_{i,2} = 2, \gamma_{i,1} = 2, \gamma_{i,2} = 2, \sigma_{i,1} = 0.5, \sigma_{i,2} = 0.1, \epsilon_{i,2} = 0.001, b_{11} = 4.8, b_{12} = 4.3, b_{21} = b_{22} = 5.1$. The other parameters are the same as Example 1.

Fig. 7 displays the outputs of all agents. It can be observed that the desired tracking performance with state constraints is obtained. Fig. 8 shows the constrained states $x_{i,2}$ of the
finite-time control enjoys a stronger stability performance than the nonfinite-time scheme. In addition, the effect of interference is considered. Fig. 10 demonstrates the robustness properties of the proposed algorithm under the interference with the amplitude of 10 and the duration of 2 seconds at 10 s. It shows that the proposed control schemes ensure the good robustness of the system. Based on the above simulation results, we get that the good performance of proposed controllers is achieved and the local tracking errors converge into the small regions of the origin in the finite time while full state constraints are not violated.

V. CONCLUSION

In this paper, the distributed finite-time tracking problem is discussed for a class of pure-feedback uncertain multi-agent systems with full state constraints. By applying the NM method, the full state-constrained MAS is converted into a novel pure-feedback one without state constraints. Based on fractional dynamic surface control as well as backstepping, a distributed adaptive finite-time tracking control strategy is proposed recursively and the proposed scheme is very simple. It is proved that all the signals are semi-globally uniformly ultimately bounded in finite time, and all the state constraints are always guaranteed. In future studies, we will consider extending the fractional dynamic surface control to a class of nonlinear multi-agent systems with input constraints as well as full state constraints. In addition, further research with switching topologies will be undertaken.

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