SIGNATURES, HEEGAARD FLOER CORRECTION TERMS AND QUASI–ALTERNATING LINKS

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Abstract. Turaev showed that there is a well–defined map assigning to an oriented link \( L \) in the three–sphere a Spin structure \( t_0 \) on \( \Sigma(L) \), the two–fold cover of \( S^3 \) branched along \( L \). We prove, generalizing results of Manolescu–Owens and Donald–Owens, that for an oriented quasi–alternating link \( L \) the signature of \( L \) equals minus four times the Heegaard Floer correction term of \( (\Sigma(L), t_0) \).

1. Introduction

Vladimir Turaev \cite{21} \S 2.2] proved that there is a surjective map which associates to a link \( L \subset S^3 \) decorated with an orientation \( o \) a Spin structure \( t_{(L,o)} \) on \( \Sigma(L) \), the double cover of \( S^3 \) branched along \( L \). Moreover, he showed that the only other orientation on \( L \) which maps to \( t_{(L,o)} \) is \( -o \), the overall reversed orientation. In other words, Turaev described a bijection between the set of quasi–orientations on \( L \) (i.e. orientations up to overall reversal) and the set \( \text{Spin}(\Sigma(L)) \) of Spin structures on \( \Sigma(L) \). Each element \( t \in \text{Spin}(\Sigma(L)) \) can be viewed as a Spin\(^c\) structure on \( \Sigma(L) \), so if \( \Sigma(L) \) is a rational homology sphere, then it makes sense to consider the rational number \( d(\Sigma(L), t) \), where \( d \) is the correction term invariant defined by Ozsváth and Szabó \cite{13}. Under the assumption that \( L \) is nonsplit alternating it was proved — in \cite{10} when \( L \) is a knot and in \cite{3} for any number of components of \( L \) — that

\[
\sigma(L, o) = -4d(\Sigma(L), t_{(L,o)}) \quad \text{for every orientation } o \text{ on } L,
\]

where \( \sigma(L, o) \) is the link signature. For an alternating link associated to a plumbing graph with no bad vertices, this follows from a combination of earlier results of Saveliev \cite{19} and Stipsicz \cite{20}, each of whom showed that one of the quantities in (*) is equal to the Neumann–Siebenmann \( \mu \)-invariant of the plumbing tree. The main purpose of this paper is to prove property (*) for the family of quasi–alternating links introduced in \cite{14}:

Definition 1. The quasi–alternating links are the links in \( S^3 \) with nonzero determinant defined recursively as follows:

1. the unknot is quasi–alternating;

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(2) if $L_0, L_1$ are quasi–alternating, $L \subset S^3$ is a link such that $\det L = \det L_0 + \det L_1$ and $L, L_0, L_1$ differ only inside a 3–ball as illustrated in Figure 1, then $L$ is quasi–alternating.

Figure 1. $L$ and its resolutions $L_0$ and $L_1$.

Quasi–alternating links have recently been the object of considerable attention [1, 2, 6, 11, 16, 17, 22, 23]. Alternating links are quasi–alternating [14, Lemma 3.2], but (as shown in e.g. [1]) there exist infinitely many quasi–alternating, nonalternating links. Our main result is the following:

**Theorem 1.** Let $(L, o)$ be an oriented link. If $L$ is quasi–alternating, then

$$\sigma(L, o) = -4d(\Sigma(L), t_{(L, o)}).$$

The contents of the paper are as follows. In Section 2 we first recall some basic facts on Spin structures and the existence of two natural 4–dimensional cobordisms, one from $\Sigma(L_1)$ to $\Sigma(L)$, the other from $\Sigma(L)$ to $\Sigma(L_0)$. Then, in Proposition 1 we show that for an orientation $o$ on $L$ for which the crossing in Figure 1 is positive, the Spin structure $t_{(L, o)}$ extends to the first cobordism but not to the second one. In Section 3 we use this information together with the Heegaard Floer surgery exact triangle to prove Proposition 2 which relates the value of the correction term $d(\Sigma(L), t_{(L, o)})$ with the value of an analogous correction term for $\Sigma(L_1)$. In Section 4 we restate and prove our main result, Theorem 1. The proof consists of an inductive argument based on Proposition 2 and the known relationship between the signatures of $L$ and $L_1$. The use of Proposition 2 is made possible by the fact that up to mirroring $L$ one may always assume the crossing of Figure 1 to be positive. We close Section 4 with Corollary 3 which uses results of Rustamov and Mullins to relate Turaev’s torsion function for the two–fold branched cover of a quasi–alternating link $L$ with the Jones polynomial of $L$.

2. **Triads and Spin structures**

A Spin structure on an $n$–manifold $M^n$ is a double cover of the oriented frame bundle of $M$ with the added condition that if $n > 1$, then it restricts to the nontrivial double cover on fibres. A Spin structure on a manifold restricts to give a Spin structure on a codimension–one submanifold, or on a framed submanifold of codimension higher than one. As mentioned in the introduction, an orientation $o$ on a link $L$ in $S^3$ induces a Spin structure $t_{(L, o)}$ on the double–branched cover $\Sigma(L)$, as in [21]. Recall also that there are two Spin structures on $S^1 = \partial D^2$: the nontrivial or bounding Spin structure, which is the restriction of the unique Spin structure on $D^2$, and the trivial or Lie Spin structure, which does not extend over the disk. The restriction map from Spin structures on a solid torus to Spin structures on its boundary is injective; thus if two Spin structures on a closed 3–manifold agree
outside a solid torus, then they are the same. For more details on Spin structures see for example [7].

If \( Y \) is a 3–manifold with a Spin structure \( t \) and \( K \) is a knot in \( Y \) with framing \( \lambda \), we may attach a 2–handle to \( K \) giving a surgery cobordism \( W \) from \( Y \) to \( Y_\lambda(K) \). There is a unique Spin structure on \( D^2 \times D^2 \), which restricts to the bounding Spin structure on each framed circle \( \partial D^2 \times \{\text{point}\} \) in \( \partial D^2 \times D^2 \). Thus the Spin structure on \( Y \) extends over \( W \) if and only if its restriction to \( K \), viewed as a framed submanifold via the framing \( \lambda \), is the bounding Spin structure. Note that this is equivalent, symmetrically, to the restriction of \( t \) to the submanifold \( \lambda \)-framed by \( K \) being the bounding Spin structure. Moreover, the extension over \( W \) is unique if it exists.

Let \( L, L_0, L_1 \) be three links in \( S^3 \) differing only in a 3–ball \( B \) as in Figure 1. The double cover of \( B \) branched along the pair of arcs \( B \cap L \) is a solid torus \( \tilde{B} \) with core \( C \). The boundary of a properly embedded disk in \( B \) which separates the two branching arcs lifts to a disjoint pair of meridians of \( \tilde{B} \). The preimage in \( \Sigma(L) \) of the curve \( \lambda_0 \) shown in Figure 2 is a pair of parallel framings for \( C \); denote one of these by \( \tilde{\lambda}_0 \). Similarly, let \( \tilde{\lambda}_1 \) denote one of the components of the preimage in \( \Sigma(L) \) of \( \lambda_1 \). Since \( \lambda_0 \) is homotopic in \( B - L \) to the boundary of a disk separating the two components of \( L_0 \cap B \), we see that \( \Sigma(L_0) \) is obtained from \( \Sigma(L) \) by \( \tilde{\lambda}_0 \)-framed surgery on \( C \). Similarly, \( \lambda_1 \) is homotopic in \( B - L \) to the boundary of a disk separating the two components of \( L_1 \cap B \), and \( \Sigma(L_1) \) is obtained from \( \Sigma(L) \) by \( \tilde{\lambda}_1 \)-framed surgery on \( C \).

The two framings \( \tilde{\lambda}_0 \) and \( \tilde{\lambda}_1 \) differ by a meridian of \( C \). In the terminology from [14], the manifolds \( \Sigma(L) \), \( \Sigma(L_0) \) and \( \Sigma(L_1) \) form a triad and there are surgery cobordisms

\[
V : \Sigma(L_1) \rightarrow \Sigma(L) \quad \text{and} \quad W : \Sigma(L) \rightarrow \Sigma(L_0).
\]

The surgery cobordism \( W \) is built by attaching a 2–handle to \( \Sigma(L) \) along the knot \( C \) with framing \( \tilde{\lambda}_0 \). The cobordism \( V \) is built by attaching a 2–handle to \( \Sigma(L_1) \). Dualising this handle structure, \( V \) is obtained by attaching a 2–handle to \( \Sigma(L) \) along the knot \( C \) with framing \( \tilde{\lambda}_1 \) (and reversing orientation).

![Figure 2. The loops \( \lambda_0 \) and \( \lambda_1 \).](image)

**Proposition 1.** For any orientation \( o \) on \( L \) such that the crossing shown in Figure 1 is positive, the Spin structure \( t_{(L,o)} \) extends to a unique Spin structure \( s_o \) on the cobordism \( V \) and does not admit an extension over \( W \). The restriction of \( s_o \) to \( \Sigma(L_1) \) is the Spin structure \( t_{(L_1,o_1)} \), where \( o_1 \) is the orientation on \( L_1 \) induced by \( o \).
Proof. Let $\pi : \Sigma(L) \to S^3$ be the branched covering map. The Spin structure $t_{(L,o)}$ is the lift $\tilde{s}$ of the Spin structure restricted from $S^3$ to $S^3 - L$, twisted by $h \in H^1(\Sigma(L) - \pi^{-1}(L); \mathbb{Z}/2\mathbb{Z})$, where the value of $h$ on a curve $\gamma$ is the parity of half the sum of the linking numbers of $\pi \circ \gamma$ about the components of $L$ (following Turaev [21, §2.2]). Suppose that the crossing in Figure 1 is positive as, for example, illustrated in Figure 3, so that the orientation $o$ induces an orientation $o_1$ on $L_1$.

Then, we can compute from Figure 2 that $h(\tilde{\lambda}_1) = 0$ and $h(\tilde{\lambda}_0) = 1$. The Spin structure on $S^3$ restricts to the bounding structure on each of $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ using the 0-framing. The map $\pi$ restricts to a diffeomorphism on neighbourhoods of $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$. Therefore, the restriction of $\tilde{s}$ to each of $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ using the pullback of the 0-framing is also the bounding structure. Also note that the preimage under $\pi$ of a disk bounded by $\lambda_i$ is an annulus with core $C$, so the framing of $\tilde{\lambda}_i$ given by $C$ is the same as the pullback of the 0-framing.

The spin structure $t_{(L,o)}$ is equal to $\tilde{s}$ twisted by $h$. Since $\tilde{s}$ restricts to the bounding spin structure on $\tilde{\lambda}_1$, and $h(\tilde{\lambda}_1) = 0$, we see that $t_{(L,o)}$ restricts to the bounding Spin structure on $\tilde{\lambda}_1$ using the framing given by $C$. On the other hand since $h(\tilde{\lambda}_0) = 1$, $t_{(L,o)}$ restricts to the Lie Spin structure on $\tilde{\lambda}_0$, again using the framing given by $C$. It follows that $t_{(L,o)}$ admits a unique extension $s_o$ over the 2-handle giving the cobordism $V$, and does not extend over the cobordism $W$.

The restriction of $s_o$ to $\Sigma(L_1)$ coincides with $t_{(L_1,o_1)}$ outside of the solid torus $\tilde{B}$, and therefore also on the closed manifold $\Sigma(L_1)$. □

3. Relations between correction terms

By [14 Proposition 2.1] we have the following exact triangle:

$$
\begin{align*}
\hat{HF}(\Sigma(L_1)) & \xrightarrow{F_V} \hat{HF}(\Sigma(L)) \\
& \xrightarrow{F_W} \hat{HF}(\Sigma(L_0))
\end{align*}
$$

where the maps $F_V$ and $F_W$ are induced by the surgery cobordisms of (2). (All the Heegaard Floer groups are taken with $\mathbb{Z}/2\mathbb{Z}$ coefficients.)

By [14 Proposition 3.3] (and notation as in that paper), if $L \subset S^3$ is a quasi-alternating link and $L_0$ and $L_1$ are resolutions of $L$ as in Definition 1 then $\Sigma(L)$, $\Sigma(L_0)$ and $\Sigma(L_1)$ are $L$–spaces. Moreover, by assumption we have

$$
|H^2(\Sigma(L); \mathbb{Z})| = |H^2(\Sigma(L_0); \mathbb{Z})| + |H^2(\Sigma(L_1); \mathbb{Z})|.
$$

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Since for every \( L \)-space \( Y \) we have \( |H^2(Y; \mathbb{Z})| = \dim \widehat{HF}(Y) \), the Heegaard Floer surgery exact triangle reduces to a short exact sequence:

\[
0 \to \widehat{HF}(\Sigma(L_1)) \xrightarrow{F_U} \widehat{HF}(\Sigma(L)) \xrightarrow{F_W} \widehat{HF}(\Sigma(L_0)) \to 0.
\]

The type of argument employed in the proof of the following proposition goes back to [9] and was also used in [20].

**Proposition 2.** Let \( L \) be a quasi–alternating link and let \( L_0, L_1 \) be resolutions of \( L \) as in Definition [1]. Let \( o \) be an orientation on \( L \) for which the crossing of Figure [1] is positive, and let \( o_1 \) be the induced orientation on \( L_1 \). Then, the following holds:

\[
-4d(\Sigma(L), \mathfrak{t}_{(L,o)}) = -4d(\Sigma(L_1), \mathfrak{t}_{(L_1,o_1)}) - 1.
\]

**Proof.** Since \( \Sigma(L), \Sigma(L_1) \) and \( \Sigma(L_0) \) are \( L \)-spaces, we may think of the Spin\(^c\) structures on these spaces as generators of their \( \widehat{HF} \)-groups, and we shall abuse our notation accordingly. Let \( V : \Sigma(L_1) \to \Sigma(L) \) be the surgery cobordism of \([2]\), and let \( s_o \) be the unique Spin structure on \( V \) which extends \( t_{(L,o)} \) as in Proposition [1].

Recall that, by definition, the map \( F_U \) associated to a cobordism \( U : Y_1 \to Y_2 \) is given by

\[
F_U = \sum_{s \in \text{Spin}^c(U)} F_{U,s},
\]

where \( F_{U,s} : \widehat{HF}(Y_1, \mathfrak{t}_1) \to \widehat{HF}(Y_2, \mathfrak{t}_2) \) and \( \mathfrak{t}_i = s|_{Y_i} \) for \( i = 1, 2 \). We claim that

\[
(5)
F_{V,s_o}(\mathfrak{t}_{(L_1, o_1)}) = \mathfrak{t}_{(L,o)}.
\]

The Heegaard Floer \( \widehat{HF} \)-groups admit a natural involution, usually denoted by \( J \). The maps induced by cobordisms are equivariant with respect to the \( \mathbb{Z}/2\mathbb{Z} \)-actions associated to conjugation on Spin\(^c\) structures and the \( J \)-map on the Heegaard Floer groups, in the sense that, if \( \overline{x} := J(x) \) for an element \( x \), we have

\[
(6)
F_{W, \overline{s}}(\overline{x}) = \overline{F_{W,s}(x)}
\]

for each \( s \in \text{Spin}^c(W) \). Since by Proposition [1] there are no Spin structures on the surgery cobordism \( W : \Sigma(L) \to \Sigma(L_0) \) of \([2]\) which restrict to \( t_{(L,o)} \), the element \( F_W(t_{(L,o)}) \in \widehat{HF}(\Sigma(L_0)) \) has no Spin component. In fact, since \( t_{(L,o)} \) is fixed under conjugation and we are working over \( \mathbb{Z}/2\mathbb{Z} \), \([6]\) implies that the contribution of each non–Spin \( s \in \text{Spin}^c(W) \) to a Spin component of \( F_W(t_{(L,o)}) \) is cancelled by the contribution of \( \overline{s} \) to the same component. Therefore we may write

\[
F_W(t_{(L,o)}) = x + \overline{x}
\]

for some \( x \in \widehat{HF}(\Sigma(L_0)) \). By the surjectivity of \( F_W \) there is some \( y \in \widehat{HF}(\Sigma(L)) \) with \( F_W(y) = x \), therefore \( F_W(t_{(L,o)} + y + \overline{y}) = 0 \), and by the exactness of \([4]\) we have \( t_{(L,o)} + y + \overline{y} = F_V(z) \) for some \( z \in \widehat{HF}(\Sigma(L_0)) \). Since \( F_V(\overline{z}) = \overline{F_V(z)} = F_V(z) \), the injectivity of \( F_V \) implies \( z = \overline{z} \). Moreover, \( z \) must have some nonzero Spin component, otherwise we could write \( z = u + \overline{u} \) and

\[
F_V(u + \overline{u}) = F_V(u) + F_V(\overline{u}) = F_V(u) + F_V(\overline{u})
\]

could not have the Spin component \( t_{(L,o)} \). This shows that there is a Spin structure \( t \in \widehat{HF}(\Sigma(L_1)) \) such that \( F_V(t) = t_{(L,o)} \). But, as we argued before for \( F_W(t_{(L,o)}) \), in order for \( F_V(t) \) to have a Spin component it must be the case that there is some Spin structure \( s \) on \( V \) such that \( F_{V,s}(t) = t_{(L,o)} \). Applying Proposition [1] we conclude \( s = s_o \) and therefore \( t = t_{(L_1,o_1)} \). This establishes claim \([5]\).
Using equation (3) and the fact that \( \det(L_1) > 0 \) it is easy to check that \( V \) is negative definite. The statement follows immediately from equation (5) and the degree–shift formula in Heegaard Floer theory [15, Theorem 7.1] using the fact that \( c_1(s_o) = 0, \sigma(V) = -1 \) and \( \chi(V) = 1 \).

4. The main result and a corollary

**Theorem 1.** Let \((L, o)\) be an oriented link. If \( L \) is quasi–alternating, then

\[
\sigma(L, o) = -4d(\Sigma(L), t_{(L, o)}).
\]

**Proof.** The statement trivially holds for the unknot, because the unknot has zero signature and the two–fold cover of \( S^3 \) branched along the unknot is \( S^3 \), whose only correction term vanishes. If \( L \) is not the unknot and \( L \) is quasi–alternating, there are quasi–alternating links \( L_0 \) and \( L_1 \) such that \( \det(L) = \det(L_0) + \det(L_1) \) and \( L, L_0 \) and \( L_1 \) are related as in Figure 1. To prove the theorem it suffices to show that if the statement holds for \( L_0 \) and \( L_1 \), then it holds for \( L \) as well.

Denote by \( L^m \) the mirror image of \( L \), and by \( o^m \) the orientation on \( L^m \) naturally induced by an orientation \( o \) on \( L \). The orientation–reversing diffeomorphism from \( S^3 \) to itself taking \( L \) to \( L^m \) lifts to one from \( \Sigma(L) \) to \( \Sigma(L^m) \) sending \( t_{(L, o)} \) to \( t_{(L^m, o^m)} \). Thus by [8, Theorem 8.10] and [13, Proposition 4.2] we have

\[
\sigma(L^m, o^m) = -\sigma(L, o)
\]

and

\[
4d(\Sigma(L^m), t_{(L^m, o^m)}) = 4d(-\Sigma(L), t_{(L, o)}) = -4d(\Sigma(L), t_{(L, o)}),
\]

therefore equation (1) holds for \((L, o)\) if and only if it holds for \((L^m, o^m)\). Hence, without loss of generality we may now fix an orientation \( o \) on \( L \) so that the crossing appearing in Figure 1 is positive.

Denote by \( o_1 \) the orientation on \( L_1 \) naturally induced by \( o \). By [11, Lemma 2.1]

\[
\sigma(L, o) = \sigma(L_1, o_1) - 1.
\]

Since we are assuming that the statement holds for \( L_1 \), we have

\[
\sigma(L_1, o_1) = -4d(\Sigma(L_1), t_{(L_1, o_1)}).
\]

Equations (7) and (8) together with Proposition 2 immediately imply equation (1).

**Corollary 3.** Let \((L, o)\) be an oriented, quasi–alternating link. Then,

\[
\tau(\Sigma(L), t_{(L, o)}) = -\frac{1}{12} \frac{V'_{(L, o)}(-1)}{V_{(L, o)}(-1)},
\]

where \( \tau \) is Turaev’s torsion function and \( V_{(L, o)}(t) \) is the Jones polynomial of \((L, o)\).

**Proof.** By [18, Theorem 3.4] we have

\[
d(\Sigma(L), t_{(L, o)}) = 2\chi(\text{HF}^+_\text{red}(\Sigma(L))) + 2\tau(\Sigma(L), t_{(L, o)}) - \lambda(\Sigma(L)),
\]

where \( \lambda \) denotes the Casson–Walker invariant, normalized so that it takes value \(-2\) on the Poincaré sphere oriented as the boundary of the negative \( E_8 \) plumbing. Moreover, since \( L \) is quasi–alternating \( \Sigma(L) \) is an \( L \)–space; therefore the first
summand on the right–hand side of (9) vanishes. By [12, Theorem 5.1], when \( \det(L) > 0 \) we have

\[
\chi(S(L)) = -\frac{1}{6} V'(L,o)(-1) + \frac{1}{4} \sigma(L,o).
\]

Therefore, when \((L,o)\) is an oriented quasi–alternating link, Theorem 1 together with equations (9) and (10) yield the statement. \(\square\)

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