Numerical modeling of light-induced drift in a cloud of interstellar gas

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Abstract. We study velocity distribution of the gas in case of resonance interaction between directed radiation line and the gas in medium of non-resonating buffer gas. We have constructed a 6-parameter model numerically describing the effect of uniform quasi-stationary light-induced drift in a rarefied interstellar gas. Based on numerical calculations, we qualitatively determined the dependence of the light-induced drift velocity on the model parameters in the cases of optically thin and optically thick clouds. We checked that light-induced drift has resonance behavior and requires low collision rate of the gas. Also we showed and explained narrowing and shifting of emission line during propagation in cloud.

1. Introduction
The effect of light-induced drift (LID) is one of the most powerful effects of radiation impact on the translational motion of the particles. Theoretically predicted [1] and observed [2] in 1979. The essence of the effect is the appearance of macroscopic directed flow of particles absorbing radiation close to the resonance and being in mixture with the buffer particles. Kinetic energy and momentum of the flow of impurity gas is taken from thermal energy of the environment. In this case, reduction of the entropy of the gas is compensated by an increase in entropy of the scattered photons [3].

1.1. Problem statement
Consider interstellar gas cloud through which the radiation passes, which is close to the resonance frequency of the impurity gas components. Under the influence of radiation, a excitation of particles of impurity gas occurs. Moreover, due to the frequency shift of radiation to the exact resonance excited gas will have a specific asymmetric velocity distribution corresponding to its translational movement in one direction or another. This leads to the separation of components in the cloud.

1.2. Description of the gas
In this problem, velocity distribution of the impurity gas is most important. Since radiation has dedicated axis, we are interested in the distribution of the projection of the velocity on the axis. Define the distribution of impurity gas in the ground state as \( f(v) \), and in the excited - as \( g(v) \). In addition, one can assume that the resonating gas in the absence of radiation is distributed...
according to the Maxwell-Boltzmann:

\[ f_0(v) = \frac{n_0}{\sqrt{2\pi v_T^2}} \exp\left(-\frac{v^2}{2v_T^2}\right) \]  

(1)

Where \( v_T \) - thermal (and turbulent) speed.

1.3. Description of the radiation

Consider radiation having spectral radiance \( I_\nu(\nu) \) and propagating in a small solid angle \( \Theta \). For easy work with a spectral radiance we must take into account the correlation between the frequency of light and the speed of the molecules. If the frequency \( \nu \) of the incident light differs from the exact resonant frequency of the impurity particles \( \nu_0 \), it will resonate only with particles moving at the speed (2). Therefore, as the argument of the spectral intensity it is possible to use \( v \) instead of \( \nu \).

\[ v = c\nu - \nu_0 \]  

(2)

1.4. Approach

Due to the fact that the task has many parameters, its analytical description is very rough. For a detailed study of this process I have built calculation scheme. The task is assumed stationary and quasi-homogeneous:

\[ f/df/\frac{df}{dt} \propto f/df/\frac{1}{dx} v_T \propto T > \tau \]  

(3)

Where \( T \) - typical time of the drift, and \( \tau \) - relaxation time of velocity distribution of the gas that is determined by the slowest process. Stationary examination is justified by the big drift time in the cloud that much more than Maxwellization time and time of radiative transitions. And quasi-homogeneity condition actually identical to the condition of stationarity.

2. Model

2.1. Processes taken into account in model

Boltzmann equation, which describes the distribution of the gas in the ground and excited states:

\[ \frac{\partial f(v)}{\partial t} + \frac{\partial f(v)}{\partial r} v + \frac{\partial f(v)}{\partial v} F_m \frac{1}{m} = \left(\frac{\partial f(v)}{\partial t}\right)_{int} \]  

(4)

Problem stationary and quasi-uniform: \( \frac{\partial f(v)}{\partial t} = 0 \), \( \frac{\partial f(v)}{\partial r} = 0 \). Force affects the motion of all particles together. So, it does not participate in the effect of light-induced drift and we will assume that force is equal to zero: \( F = 0 \). Thus, the Boltzmann equation looks like:

\[ 0 = \left(\frac{\partial f(v)}{\partial t}\right)_{int} = \left(\frac{\partial f(v)}{\partial t}\right)_{m} + \left(\frac{\partial f(v)}{\partial t}\right)_e + \left(\frac{\partial f(v)}{\partial t}\right)_c \]  

(5)

Where the terms are defined by the following processes:

- Maxwellization of the impurity gas in excited and ground states. I consider the relaxation time approximation and describe the resulting velocity distribution of impurity gas through the parameter \( \Theta \) - the part of gas in the excited state. The equilibrium distribution of the resonating gas in the excited and ground state of the proportional concentration of the gas in this state: \( n_g = \theta n_0 \) for the excited state and \( n_f = (1 - \theta) n_0 \) for the ground state.

\[ \theta = \frac{1}{n_0} \int g(v)dv \]  

(6)
\[
\frac{\partial f(v)}{\partial t} = \frac{f_0(v)(1 - \theta) - f(v)}{\tau_f}
\]
(7)\[
\frac{\partial g(v)}{\partial t} = \frac{\theta f_0(v) - g(v)}{\tau_g}
\]
(8)

Where \(f_0\) is Maxwell distribution of gas emission 1.

- Stimulated and spontaneous transitions between the ground and excited states:

\[
\frac{\partial f(v)}{\partial t} = -B \frac{\Omega I(v)}{c} f(v) + B \frac{\Omega I(v)}{c} g(v) + A g(v)
\]
(9)\[
\frac{\partial g(v)}{\partial t} = B \frac{\Omega I(v)}{c} f(v) - B \frac{\Omega I(v)}{c} g(v) - A g(v)
\]
(10)

Where \(A\) and \(B\) - Einstein coefficients for this transition (for simplicity we assume that the statistical weights of the levels are equal: \(g_u = g_d\).

- Transitions caused by collisions. Qualitatively, effect of light-induced drift exists without collisional transitions. However, the speed of drift may significantly increase.

\[
\frac{\partial f(v)}{\partial t} = Z \theta f_0(v)
\]
(11)\[
\frac{\partial g(v)}{\partial t} = -Z g(v)
\]
(12)\[
Z = W_{u \rightarrow d}^{coll}
\]
(13)

Substituting (7-12) into (5), we obtain the equations describing the balance of the gas in the excited and ground state:

\[
\frac{f_0(v)(1 - \theta) - f(v)}{\tau_f} - B \frac{\Omega I(v)}{c} f(v) + B \frac{\Omega I(v)}{c} g(v) + A g(v) + Z \theta f_0(v) = 0
\]
(14)\[
\frac{\theta f_0(v) - g(v)}{\tau_g} + B \frac{\Omega I(v)}{c} f(v) - B \frac{\Omega I(v)}{c} g(v) - A g(v) - Z g(v) = 0
\]
(15)

2.2. Parameters of the problem

For the numerical solution of this problem it is required to introduce dimensionless parameters. We will consider the emission with Gaussian spectral radiance, where \(v\) is related to the detuning of frequency according to (2).

\[
I(v) = I_{max} \exp\left(-\frac{(v - v_r)^2}{2\Delta v^2}\right)
\]
(16)

It is characterized by three parameters: \(I_{max}\) - intensity in the maximum, \(v_r\) - position of the maximum, \(\Delta v\) - the width of the emission line. Thus, the task is determined by 10 parameters: \(n_0, v_T, v_r, \Delta v, \tau_f, \tau_g, A, B, Z, I_{max}\), and \(B\) can be expressed in the following way:

\[
B = A \frac{\pi^2 c^3}{\hbar \omega_0^3}
\]
(17)
It is possible to leave 6 dimensionless parameters affecting the solution:

\[
\begin{align*}
\beta &= \frac{v_r}{v_T} \quad \text{shift of the emission line relative to the absorption line} \\
\delta &= \frac{\Delta v}{v_T} \quad \text{relative width of the emission line} \\
\gamma &= \tau_g A \quad \text{relative time of Maxwellization of excited gas} \\
\eta &= \frac{\tau_f}{\tau_g} = \frac{\sigma_g}{\sigma_f} \quad \text{the ratio of the cross sections of elastic interactions in the gas in the excited and ground states} \\
\epsilon &= \frac{B \Omega I_{\text{max}}}{A c} \quad \text{the ratio of probabilities of spontaneous and induced transitions} \\
\zeta &= \frac{Z}{A} \quad \text{the ratio of probabilities of collisional transitions and spontaneous radiative transitions}
\end{align*}
\]

2.3. Solving method

Program was written in the package Wolfram Mathematica. The essence of the algorithm is as follows. The system of equations (14,15) is resolved with respect to \( f(v) \) and \( g(v) \). The solution has a linear dependence of the parameter \( \Theta \). Then, after the numerical integration of the coefficients under zero and first powers of \( \Theta \) in \( g(v) \), one can solve the equation (6) with respect to \( \Theta \), which finally determines the distribution of \( f(v) \) and \( g(v) \).

3. Results

3.1. Examples of the gas distribution

Qualitatively obtained graphs with distribution of resonating gas (1,2) are described as follows. Since the rate of Maxwellization of impurity gas in the ground state is small (compared with other processes), then gas accumulates in a state with low transitions activity. On the other hand, at velocities where transition activity is high, there is an alignment of the gas concentration in the excited and ground states.

**Figure 1.** Gas distribution in the ground and excited states. Parameters \( \beta = -1.5, \delta = 1, \gamma = 10, \eta = 10, \epsilon = 10, \zeta = 0.1 \).

**Figure 2.** Gas distribution in the ground and excited states in case of narrow emission line. Parameters \( \beta = -1, \delta = 1/4, \gamma = 10, \eta = 10, \epsilon = 1, \zeta = 0.1 \).
3.2. The speed of light-induced drift

Here is some typical dependences of the average velocity of the gas on the parameters. The average speed of the impurity gas defined as follows:

\[
\bar{v} = \frac{1}{n_0} \int (f(v) + g(v))vdv
\]  

(18)

As we can see, drift velocity is comparable to thermal velocity. There is a maximum in dependence of drift velocity on the shift of the emission line relative to the absorption line \( \beta \) (figure 3), which is depends on the other parameters. Indeed, the dependence of the average velocity of the impurity gas from the shift of the emission line is antisymmetric. Therefore, when \( \beta = 0 \) there is no drift. Also, for large shift parameter, the activity of stimulated transitions is reduced by a Gaussian function, which is why the average speed of the impurity gas also tends to 0. Figure 4 also shows a maximum in dependence of drift velocity on the width of the emission line \( \delta \). Figure 5 shows that the drift occurs only when Maxwellization in ground state is slower than the radiative transitions. Figure 6 shows that it is also possible to change the direction of flow by varying the ratio of the cross section.

![Figure 3](image1.png)  
**Figure 3.** Graph of drift velocity of the impurity gas on the shift of the emission line relative to the absorption line.

![Figure 4](image2.png)  
**Figure 4.** Graph of drift velocity of the gas on the emission line width.

![Figure 5](image3.png)  
**Figure 5.** Graph of drift velocity of the gas on the relative Maxwellization time.

![Figure 6](image4.png)  
**Figure 6.** Graph of drift velocity of the gas on the ratio of the cross sections of elastic interactions.

All graphs here are for three sets of parameters:
A) the typical set of parameters: \( \beta = -1.5, \delta = 1, \gamma = 10, \eta = 10, \epsilon = 1, \zeta = 0.1 \)
B) case of narrow emission line: \( \beta = -0.5, \delta = 0.25, \gamma = 10, \eta = 10, \epsilon = 1, \zeta = 0.1 \)
C) case of strong radiation: \( \beta = -1.5, \delta = 1, \gamma = 10, \eta = 10, \epsilon = 10, \zeta = 0.1 \)
3.3. The absorption of radiation in the optically thick cloud

The resulting solution for the distribution of the impurity gas can be generalized to the case of optically thick cloud, where there is a significant absorption of radiation. The derivative of the spectral intensity is determined according to:

$$\frac{dI(v)}{dx} = -hBI(v)(f(v) - g(v))$$  \hspace{1cm} (19)

Formula $x = z(\frac{v_T}{n_0 Bh}) = z(\frac{8\pi v_T}{A\lambda^3 n_0})$ for penetration depth is written under the assumption that the density of the gas $n_0$ is constant and can be generalized.

$$\int_{x_0}^{x} \frac{n_0(x')Bh}{v_T} dx' = \int_{x_0}^{x} n_0(x')\lambda^3 A dx' = z$$  \hspace{1cm} (20)

Due to the peculiarities of the velocity distribution of the impurity gas, absorption of radiation is considerably decreased. Strong radiation forms a transparency window in gas distribution. Therefore, the absorption is faster at frequencies where the spectral emission intensity is smaller. Thus, emission line is narrowing and moving away from resonance frequency while spreading in cloud (figure 7).

![Figure 7](image7.png)

**Figure 7.** Dependence of the spectral radiance on the penetration depth in the cloud

Parameters $\beta = -1.5$, $\delta = 1$, $\gamma = 10$, $\eta = 10$, $\epsilon = 1$, $\zeta = 0.1$. Profiles are given for the depth of penetration $x = 0, 6, 12, 25, 50, 100, 200(\frac{v_T}{n_0 Bh}) = (\frac{8\pi v_T}{A\lambda^3 n_0})$

![Figure 8](image8.png)

**Figure 8.** Dependence of the velocity distribution of the gas $f(v) + g(v)$ on the penetration depth in the cloud

4. Conclusions

Based on calculation results of 6-parameter model we showed that velocity distribution of the gas can be significantly affected by radiation line close to its resonance. Then we analyzed dependence of the drift velocity on model parameters that shows resonance behavior of the LID effect. Also we found that Maxwellization of the gas should be slower than radiative transition for drift to occur. In case of optically thick cloud we got narrowing and shifting of the emission line spectral radiance in process of propagation in cloud.

References

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