On RPC Model of Satellite Imagery

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ABSTRACT  The RPC model has recently raised considerable interest in the photogrammetry and remote sensing community. The RPC is a generalized sensor model that is capable of achieving high approximation accuracy. Unfortunately, the computation of the parameters of RPC model is subject to the initial of the parameter in all available literature. An algorithm for computation of parameters of RPC model without initial value is presented and tested on SPOT-5, CBERS-2, ERS-1 imageries. RPC model is suitable for both push-broom and SAR imagery.

KEYWORDS  RPC model; rigorous sensor model; satellite imagery

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Introduction

The rational polynomial coefficient (RPC) model is a generalized sensor model that is used as an alternative solution for the rigorous sensor model for IKONOS of the space imaging. As the number of sensors increases along with greater complexity, and the standard sensor model is necessary, the applicability of the RPC model is increasing. The RPC model has the advantages in being able to substitute for all sensor models, such as the projective, the linear push-broom and the SAR.

Using the RPC to replace the rigorous sensor models has been in practice for over a decade. RPC facilitates the applications of high-resolution satellite imagery due to its high fitting capability and simplicity, and enables photogrammetric interoperability of imagery from different vendors due to its generality. In some literatures published mainly in the last year, the least-squares solution to the nonlinear RPC has been derived and described. The numerical properties and accuracy assessment on the use of RPC for replacing the rigorous sensor model have been reported, but the scholars only study the push-broom imagery and the aerial photographs, no one study the SAR imagery. This paper aimed to generate a RPC model from the rigorous sensor model of the push-broom imagery and SAR imagery. The least square solution is used to estimate the RPC without initial value.

1 RPC model

In RPC model, image pixel coordinates \(d(d_y, d_x)\) are expressed as the ratios of polynomials of ground coordinates \(D(D_y, D_x, D_z)\). In order to improve the numerical stability of equations, the 2D image coordinates and 3D ground coordinates are each offset and scaled to fit the range from \(-1.0 \) to 1.0, that is normalization. The RPC model between the image coordinates \(d\) and the ground coordinates \(D\) for an image can be formulated below:

\[
Y = \frac{N_i(P, L, H)}{D_i(P, L, H)}
\]
The polynomials \( N_s(P, L, H) \), \( D_s(P, L, H) \), \( N_s(P, L, H) \) and \( D_s(P, L, H) \) have the forms:

\[
N_s(P, L, H) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} a_{ijk}P^iL^jH^k
\]

\[
D_s(P, L, H) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} b_{ijk}P^iL^jH^k
\]

\[
N_s(P, L, H) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} c_{ijk}P^iL^jH^k
\]

\[
D_s(P, L, H) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} d_{ijk}P^iL^jH^k
\]

where \( a_{ijk}, b_{ijk}, c_{ijk} \) and \( d_{ijk} \) are polynomial coefficients, the maximum powers of each ground coordinate \( (m_1, m_2, m_3) \) are limited to 3. Furthermore, the total power of all three ground coordinates is limited to 3. That is, the polynomial coefficients are defined to be zero whenever \((i + j + k)>3\).

For RPC model, the latitude, longitude and height ground coordinates are offset and scaled to fit the range from \(-1.0\) to \(+1.0\). The normalized ground coordinates are computed from the un-normalized coordinates with the equations:

\[
P = \frac{D_y - D_{y,off}}{D_{y,Scale}}
\]

\[
L = \frac{D_x - D_{x,off}}{D_{x,Scale}}
\]

\[
H = \frac{D_z - D_{z,off}}{D_{z,Scale}}
\]

where \( P, L \) and \( H \) are normalized ground coordinate values; \( D_r, D_x \) and \( D_z \) are un-normalized ground coordinate values; \( D_{y,off}, D_{x,off} \) and \( D_{z,off} \) are offset values for three ground coordinates; \( D_{y,Scale}, D_{x,Scale} \) and \( D_{z,Scale} \) are scale factor values for three ground coordinates.

The quantities: \( D_z, D_{z,off} \) and \( D_{z,Scale} \) are all in the same units, namely pixel spacing.

The RPC model has nine cases (Table 1).

Table 1: Nine cases of RPC model

| Case | Denominator | Order of RPC | Number of RPCs | Min. number of GCPs |
|------|-------------|--------------|----------------|---------------------|
| 1    |             | 1            | 14             | 7                   |
| 2    |             | 2            | 38             | 19                  |
| 3    |             | 3            | 78             | 39                  |
| 4    |             | 1            | 11             | 6                   |
| 5    |             | 2            | 29             | 15                  |
| 6    |             | 3            | 59             | 30                  |
| 7    |             | 1            | 8              | 4                   |
| 8    |             | 2            | 20             | 10                  |
| 9    |             | 3            | 40             | 20                  |

Table 1 shows the number of unknown RPC and the required minimum number of GCPs (ground control points) under these nine cases (three different polynomial orders against three different denominator configurations). It is worth noting that the case of \( D_s(P, L, H) = D_s(P, L, H) = 1 \) is a regular 3D polynomial model, and the case of \( D_s(P, L, H) = D_s(P, L, H) = 1 \) with the first order is the direct linear transformation (DLT) model. So the RPC model is the generalized sensor models.

2 Rigorous sensor models for linear push-broom and SAR

Because of the movement along orbit of satellite, the scan of the camera and the autorotation of the earth, the position of satellite imagery is the combination of the space geometry with chronological order. So the rigorous sensor models of the linear push-broom is as below:
where $m$ is the scale factor; $x_i, y_i$ are the image coordinates of object point; $X, Y, Z$ are the object coordinates of image point in the CIS (celestial inertial system); $R_{\text{hs}} = R_i (\Psi_i) R_j (-\Psi_j)$ is the rotation matrix between the camera system and the body system, $\Psi_i$ and $\Psi_j$ are the cross-track and along-track viewing angles; $R_{\text{hp}} = R_i (-\omega) R_i (\phi) R_i (\kappa)$ is the rotation matrix between the body system and orbit system, $\omega, \phi$ and $\kappa$ are attitudes of the camera; $R_{\text{os}} = \begin{bmatrix} (X_2)_x & (Y_2)_x & (Z_2)_x \\ (X_2)_y & (Y_2)_y & (Z_2)_y \\ (X_2)_z & (Y_2)_z & (Z_2)_z \end{bmatrix}$ is the rotation matrix between orbit system and the CIS system; $Z_2 = \begin{bmatrix} P(t) \\ V(t) \end{bmatrix}$, $X_2 = [X_i, Y_i, Z_i]$, $Y_2 = [X_i, Y_i, Z_i]$, $X_i, Y_i, Z_i$ are the coordinates of the position and velocity of the satellite in the CIS system.

The relationship between image coordinate systems and CIS is based on three rotations using combinations of the basic movement elements, plus three rotations for the additional undefined rotations of the satellite at the time of imaging, and the offnadir viewing angles of the linear array sensor.

SAR, however, being an active radar instrument, provides very accurate information on the range to the target and the Doppler history of the returned signal. Because these quantities can be related to the precise spacecraft and earth surface coordinates, it is possible to solve a set of equations giving the earth location for each image pixel. So the rigorous sensor models of SAR is:

$$\frac{X^2}{A^2} + \frac{Y^2}{B^2} = 1$$

$$f_D = \frac{2}{\lambda R} (R_{k} - R_{r}) \cdot (V_{s} - V_{r})$$

$$R^2 = (X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2$$

where $R_{k} = (X, Y, Z)^T$ and $V_{s} = (X_i, Y_i, Z_i)^T$ are the object coordinates of position and velocity of image point in the CIS system; $A = a + h, B = b - h, h$ is the elevation of revolutionary ellipsoid with local topographic elevation, $a = 6378137.0, b = 6356752.3$; $f_D$ is the frequency shift; $R_{k} = (X_i, Y_i, Z_i)$ and $V_{s} = (X_i, Y_i, Z_i)$ is the coordinates of the position and velocity of the satellite in the CIS; $R$ is the distance between the object and the satellite.

### 3 Solution to RPC model

#### 3.1 Solution to RPC model without initial value

Firstly, we can rewrite Eq. (1) as

$$F_X = N_x (P, L, H) - X \times D_x (P, L, H) = 0$$

$$F_Y = N_y (P, L, H) - Y \times D_y (P, L, H) = 0$$

The observation error equations can then be formed as

$$V = Bx - IW$$

where

$$B = \begin{bmatrix} \frac{\partial F_X}{\partial a_i} & \frac{\partial F_X}{\partial b_i} & \frac{\partial F_X}{\partial c_i} & \frac{\partial F_X}{\partial d_i} \\ \frac{\partial F_Y}{\partial a_i} & \frac{\partial F_Y}{\partial b_i} & \frac{\partial F_Y}{\partial c_i} & \frac{\partial F_Y}{\partial d_i} \end{bmatrix}$$

$$I = \begin{bmatrix} -F_x^T \\ -F_y^T \end{bmatrix}, x = [a, b, c, d]^T$$

$W$ is the weight matrix.

With the least square method, the RPC parameters can be obtained as

$$x = (B^TWB)^{-1}B^TWV$$

The observation error equations based on the transformed RPC model is linear equation, so this solution has not the initial values and iteration.

#### 3.2 Flow of solution

The parameters involved in the RPC model can be solved with or without knowing the rigorous sensor models. If the rigorous sensor model is available, the terrain-independent solution can be developed. Otherwise the RPC solution will be highly dependent on the input GPCs from the terrain surface.
A least-squares approach is utilized to determine the RPC model coefficients $a$, $b$, $c$, and $d$, from a 3D grid of points generated with the rigorous sensor models. The 3D grid of object points is generated by intersecting the rays emanating from a 2D grid of image points (computed using the rigorous sensor model) with a number of constant elevation planes. The estimation process only need the rigorous sensor model and the max height and the min height in the image area which can be computed by the world DEM supplied by the USGS. This method includes the following steps and the flow-chart is shown in Fig. 1.

3.2.1 Determination of image grid and establishment of 3D object grid of points

The basic scheme of this algorithm is to build a virtual space reflecting the rigorous sensor model and obtain the RPC that fits to the virtual space. The image of the corresponding object’s points is extracted to build the virtual space.

Initially, $m$ by $n$ (often more than ten) grid points on the image coordinates need to be determined. The world grid points corresponding to each image grid point can then computed. The rigorous sensor model is used to compute the point position of the world coordinates. In addition, about each image point, $k$ world points with different elevations were obtained. $K$ elevation layers are distributed uniformly. The control grid is shown in Fig. 2.

3.2.2 RPC fitting

The unknown parameters of the RPC model are then solved using the corresponding images and object grid points.

3.2.3 Accuracy checking

The obtained RPC model is then used to calculate the image positions of the object grid points. By calculating the difference between the coordinates of the original image grid points and those calculated from the RPC, thus the accuracy of the RPC can be evaluated.

4 Test data

In this study, the linear push-broom image of SPOT-5, CBERS-2 and SAR image of ERS-1 are
used for the test.

4.1 SPOT-5 data

The test image of SPOT-5 is a 60 km × 60 km full scene of Beijing, basic information about the scene is listed in Table 2.

Table 2 Basic information of Beijing's SPOT-5 imagery

| Image size       | 6 000 pixel × 6 000 pixel |
|------------------|---------------------------|
| Resolution       | 10 m                      |
| Band             | 1 pan + 3 muti            |
| Topography       | Flat                      |
| Time of Acquisition | 2002-10-02T11:14 (Beijing time) |

4.2 CBERS-2 data

The test image of CBERS-2 is a 30 km × 30 km full scene of Nanyang, basic information about the scene is listed in Table 3.

Table 3 Basic information of Nanyang's CBERS-2 imagery

| Image size       | 10 000 pixel × 10 002 pixel |
|------------------|-----------------------------|
| Resolution       | 3 m                         |
| Band             | 1 pan                       |
| Topography       | Flat                        |
| Time of Acquisition | 2004-12-07T11:09 (Beijing time) |

4.3 ERS-1 data

Additional data used in this test was attained from ERS-1. It is the full scene of Beijing, a summary of the information is shown in Table.

5 Results and evaluation

5.1 Accuracy of nine cases

In order to compare the accuracy of the nine cases of the RPC model, numerical tests under nine different cases were performed. This results are from the established 3D GCPs consisting of 5 elevation layers each with 15 × 15 grid points, and the 3D check grid consisting of 10 elevation layers each with 30 × 30 grid points and the results are listed in Tables 5-7.

Table 4 Basic information of Beijing CBERS-2 imagery

| Image size | 26 454 pixel × 4 900 pixel |
|------------|-----------------------------|
| Resolution | 3 m                         |
| Wave length | 0.056 666 m                      |
| Topography | Flat                        |
| Pulse repetition frequency | 1 679 902 Hz                  |
| Sampling rate | 37 924 936 000 Hz           |
| bandwidth per look in azimuth | 1 378 000 Hz                |
| bandwidth per look in range | 15 550 000 Hz               |
| line spacing | 3 979 m                   |
| sample spacing | 7 904 m           |
| Time of acquisition | 1997-10-18T18:53 (Beijing time) |

Table 5 Accuracy of nine cases on SPOT-5 imagery/pixel

| Denominator Order | Check points | Control points |
|-------------------|--------------|----------------|

Table 6 Accuracy of nine cases on CBERS-2 imagery/pixel

| Denominator Order | Check points | Control points |
|-------------------|--------------|----------------|
From Tables 5, 6 and 7, the following findings are obtained.

1) The high order RPC model may be necessary, because this kind of RPC model resembles the rigorous sensor model very well.

2) The RPC model cases with unequal denominator achieve better accuracy than the cases with equal denominator at check points overall. The RPC model cases with denominator perform better.

3) When we use the RPC model cases with unequal denominator and the 3rd order, the RMS of check points and control points is smaller than 0.06 pixels and the max error is only 0.183 pixels. So for the rest test, we use the RPC model cases with unequal denominator and the 3rd order.

5.2 Accuracy of grid number

To evaluate the effect of the grid number on the RPC accuracy, we establish 3D control grid consisting of 5 elevation layers each with 10×10, 20×20, 30×30, 40×40, 50×50, 60×60 grid points to RPC fit. The results are listed in Table 8, 9 and 10.

From these tables, the RMS of the check points and the control points is reducing with the grid number increasing, that can be seen easily by Fig. 3, 4, and 5.

| Denominator Order | Y     | X     | Planarity | Y     | X     | Planarity |
|-------------------|-------|-------|-----------|-------|-------|-----------|
|                   | max   | RMS   | max   | max   | RMS   | max   |
| 1                 | -39.646 | 12.745 | -102.155 | 32.876 | 109.578 | 35.260 |

| Table 7 | Accuracy of nine cases on ERS-1 imagery/pixel |
|---------|-----------------------------------------------|
| Check points | Control points |
| Y | X | Planarity | Y | X | Planarity |
| max | RMS | max | RMS | max | RMS |
| Y     | X     | Planarity | Y     | X     | Planarity |
| max | RMS | max | RMS | max | RMS |

| Table 8 | Accuracy of grid number cases on SPOT-5 imagery/pixel |
|---------|-----------------------------------------------|
| Grid number | Y | X | Planarity | Y | X | Planarity |
| max | RMS | max | RMS | max | RMS | max | RMS |
| 10     | -0.045  | 0.020 | 0.093  | 0.037 | 0.102  | 0.042 | 0.044  | 0.020 | 0.090  | 0.036 | 0.093  | 0.042 |

| Table 9 | Accuracy of grid number cases on CBERS-2 imagery/pixel |
|---------|-----------------------------------------------|
| Grid number | Y | X | Planarity | Y | X | Planarity |
| max | RMS | max | RMS | max | RMS | max | RMS |
| 10     | -0.104  | 0.033 | -0.132  | 0.055 | 0.144  | 0.065 | -0.104  | 0.033 | -0.132  | 0.054 | 0.144  | 0.064 |

| Table 10 | Accuracy of grid number cases on ERS-1 imagery/pixel |
|---------|-----------------------------------------------|
| Grid number | Y | X | Planarity | Y | X | Planarity |
| max | RMS | max | RMS | max | RMS | max | RMS |
| 10     | -0.078  | 0.031 | -0.234  | 0.072 | 0.237  | 0.079 | 0.076  | 0.030 | -0.234  | 0.072 | 0.237  | 0.078 |

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5.3 Accuracy of height layer

To evaluate the effect of the height layer on the RPC accuracy, we establish 3D control grid consisting of 2, 3, 4, 5, 6, 7, 8, and 9 elevation layers each with $20 \times 20$ grid points to RPC fit. The results are listed Table 11, 12, and 13.

From these tables, the RMS of the check point and the control point is reducing with the height layer increasing, that can be seen easily by Fig. 6, 7, and 8.

### 6 Conclusions

The RPC model has recently raised considerable interest in the photogrammetry and remote sensing community. The RPC is a generalized sensor model that is capable to achieve high approximation accuracy. Unfortunately, the computation of the parameters of RPC model is subject to the initial of the parameter in all available literature. In this paper, an algorithm of compu...

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**Table 11 Accuracy of height layer on SPOT-5 imagery/pixel**

| Height layer | Check points | Control points |
|--------------|--------------|----------------|
|              | Y RMS        | X RMS          | Planarity RMS | Y RMS        | X RMS          | Planarity RMS |
| 2            | 0.341        | 0.180          | 0.237         | 0.609        | 0.393          | 0.193          |
| 3            | 0.039        | -0.078         | 0.028         | 0.079        | 0.033          | 0.038          |
| 4            | 0.038        | -0.077         | 0.027         | 0.078        | 0.032          | 0.037          |
| 5            | 0.038        | -0.077         | 0.026         | 0.077        | 0.032          | 0.037          |
| 6            | 0.037        | -0.076         | 0.026         | 0.076        | 0.031          | 0.035          |
| 7            | 0.037        | -0.075         | 0.026         | 0.076        | 0.031          | 0.035          |

**Table 12 Accuracy of height layer on CBERS-2 imagery/pixel**

| Height layer | Check points | Control points |
|--------------|--------------|----------------|
|              | Y RMS        | X RMS          | Planarity RMS | Y RMS        | X RMS          | Planarity RMS |
| 2            | -1.253       | 0.676          | 0.239         | 0.085        | 0.193          | 0.118          |
| 3            | 0.088        | -0.025         | -0.110        | 0.044        | 0.118          | 0.051          |
| 4            | 0.081        | -0.020         | -0.105        | 0.039        | 0.110          | 0.044          |
| 5            | 0.076        | -0.017         | -0.100        | 0.036        | 0.105          | 0.040          |
| 6            | 0.071        | -0.015         | -0.097        | 0.034        | 0.101          | 0.037          |
| 7            | 0.069        | -0.014         | -0.095        | 0.032        | 0.097          | 0.035          |

**Table 13 Accuracy of height layer on ERS-1 imagery/pixel**

| Height layer | Check points | Control points |
|--------------|--------------|----------------|
|              | Y RMS        | X RMS          | Planarity RMS | Y RMS        | X RMS          | Planarity RMS |
| 2            | 0.814        | 0.177          | 0.145         | 0.041        | 0.814          | 0.179          |
| 3            | 0.042        | 0.015          | 0.115         | 0.031        | 0.040          | 0.014          |
| 4            | -0.033       | 0.012          | -0.095        | 0.025        | -0.033         | 0.012          |
| 5            | -0.028       | 0.010          | -0.081        | 0.020        | -0.027         | 0.010          |
| 6            | -0.024       | 0.009          | -0.071        | 0.017        | -0.024         | 0.009          |
| 7            | 0.022        | 0.008          | -0.063        | 0.015        | -0.021         | 0.008          |

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Fig. 3 Accuracy of check points with different grid number on SPOT-5 imagery

Fig. 4 Accuracy of check points with different grid number on CBERS-2 imagery
tation of parameters of RPC model without initial value is put forward. On the basis of the numerous tests, the following conclusions can be draw.

1) The RPC model can achieve an approximation accuracy that is extremely high for satellite data. The results support that the RPC model can be used as a replacement sensor model for photogrammetric restitution.

2) The high order RPC model may be necessary when dealing with satellite data sets, because the RPC model resembles the rigorous sensor model very well.

3) The RPC model cases with unequal denominator achieve better accuracy than the cases with equal denominator at check points overall. The RPC model cases with denominator perform better.

4) For satellite imagery, when the image grid contains 20 × 20 point and the number of elevation layers is three, the accuracy and the efficiency is balance.

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