A TDOA Estimation Method Through Phase Difference Square Sums (PDSS)

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Abstract. In order to improve the performance of TDOA estimation under multipath, non-line-of-sight and narrow signal bandwidth conditions, a time difference of arrival (TDOA) estimation method by Phase Difference Square Sum (PDSS) is introduced. Within the baseline time-lags of two separate receivers, PDSS between two received samples of the same signal are calculated for per lag; the TDOA is estimated by searching the minimum of PDSS. Under the condition of different signal-to-noise ratio (SNR) and Sample rates to Signal bandwidth ratio, the estimation error distribution of the PDSS are simulated, and are compared with the generalized cross correlation with phase transform (GCC-PHAT). In multipath propagation and non-line-of-sight environment of the metropolis, the estimation success rates are experimented to evaluate PDSS’s performance relative to GCC_PHAT.

1. Introduction

TDOA estimation, whose essence is the relative time delay estimation (TDE) between two received samples of same signal, is a fundamental approach for passive location applications of sound and radio frequency radiation source. Many of the methods devised to estimate the time delay are related through a generalized cross correlation (GCC) approach [1], in which GCC with phase transform (GCC-PHAT) method has been widely used [2–4].

In order to improve the time delay estimation performance under condition of low SNR, non-line-of-sight (NLOS), and reverberation or multipath environment, several methods using phase information have been studied. A Localization Method through Phase Information[5] exploit the phase information captured at multiple receivers and the received antenna beam pattern, find out the ellipses where the antenna beam patterns intersect and search the RF location within the intersect area. A method based on phase difference of arrival (PDAO) [6] estimation is developed to locate sound source. In that method, the principal components of signals are picked out by searching the amplitude spectrum, the phase differences are calculated at the principal frequencies, and TDOA is estimated via phase differences and principal frequencies. A TDOA Estimation method Based on Phase-Voting Cross Correlation and Circular Standard Deviation [7] is proposed, which generates a probability density function (PDF) of TDOA for each frequency bin based on phase delay and kernel function, peak positions of the bin-wise PDFs for the target signal are concentrated at the target time difference.

In engineering practice, there are several reasons for us to explore new methods: 1) the algorithm has acceptable performance in multipath propagation and non-line-of-sight environments; 2) it can work even if the signal bandwidth is narrow; 3) it is easy to implement in engineering. Through engineering experiments, we find that PDSS method has some special behaviors. The working aim of
this paper is to improve the TDE performance via PDSS method.

The rest of the paper is organized as follows: In Section 2, the calculating ways of PDSS are illustrated. In Section 3, estimation error distributions of PDSS and GCC-PHAT are simulated and compared. In Section 4, the estimation success ratios are evaluated with real data captured from the sensor network of an urban district. Section 5 concludes the paper and presents ideas for further work.

2. PDSS method

In the PDSS method, only the phase information is used for estimating the time delay. We express the phases of the two received signal, $\phi_a(t)$ and $\phi_b(t)$, in Eq. (1)

$$
\phi_a(t) = S_a + \phi(t + \tau_a) + n_a(t) \\
\phi_b(t) = S_b + \phi(t + \tau_b) + n_b(t)
$$

Where $\phi(t)$ is original phase of the signal, $\tau_a$ and $\tau_b$ are the propagation time from emission source to two receivers; $S_a$ and $S_b$ is initial phases of received samples; $n_a$ and $n_b$ are noises.

Generally, $\phi_a(t)$ and $\phi_b(t)$ are calculated by mathematical function atan2, the distribution range are within $-\pi \sim \pi$ . Prior to calculating phases difference, we need to first unwrapping the phases, the difference absolute value between adjacent phases is not greater than $\pi$ . Thus, we could get two unwrapping phase date blocks from the receivers, $\psi_a(n)$ and $\psi_b(n)$, with $N$ sample points.

The phase differences square sum (PDSS) can be analyzed and calculated within the baseline time-lags of two receivers with Eq. (2)

$$
D(\mu) = \sum_{n=0}^{N-1} (\psi_a(n + \mu) - \psi_b(n))^2 / N \\
= \sum_{n=0}^{N-1} ((S_a - S_b) + (\phi(n + \tau_a + \mu) - \phi(n + \tau_b)) + (n_a(n + \mu) - n_b(n))^2 / N
$$

Where $\mu$ is time lag, unit is sample. Although the initial phase $S_a$ and $S_b$ of the receivers are unknown, but they are two constants for a sampled data block, we don't need to know its true value. The second term of the equation is difference of the original phases, while $\tau_a + \mu = \tau_b$, it will be 0. The third term is the difference of channel phase noises.

The relative delay $\hat{\tau}$ is valid only if $D(\hat{\tau})$ is a valley point, that is, $D(\hat{\tau}) < D(\hat{\tau} - 1)$, $D(\hat{\tau}) < D(\hat{\tau} + 1)$. The relative time delay $\hat{\tau}$ could be obtained by searching the Minimum valley value in the phase differences square sum function through Eq. (3)

$$\hat{\tau} = \arg \min_{\mu} D(\mu)$$

Overview the calculation procedure, PDSS method is a cross-correlation algorithm designed according to the characteristics of phase data, which is difficult to be handled by classical cross-correlation algorithm. From the perspective of Engineering implementation, PDSS is not more complicated than classical cross-correlation algorithm.

3. Simulation and Comparison: PDSS vs GCC-PHAT

Because of GCC-PHAT sensitivity to the relation between the sampling rate and the signal bandwidth, sampling rate - signal bandwidth ratio (SR-BW-Ratio) is more suitable for the TDOA performance evaluation in simulation.

We used the MATLAB to simulate BPSK signals with white Gaussian noise. Under different SNR and SR-BW-Ratio, the TDOA estimation error distributions are evaluated with PDSS and GCC-PHAT method. We assume the size of a data block is 16384 sampling points. The time-lag maximums of receivers are assumed as 8 samples. Prior to estimate TDOA, signals are filtered by a low pass fir filter.
Fig. 1. TDOA estimation error distributions of PDSS and GCC-PHAT methods under different simulation conditions.

Fig. 1 shows TDOA estimation error distributions of PDSS and GCC-PHAT methods, respectively at SNR = 15, SR-BW-Ratio = 64, and SNR = 0, SR-BW-Ratio = 16. In the figures, we could observe that:

1) The 0 error probability of PSDD is higher than GCC-PHAT; it is proved that the performance of PSDD method is better than GCC-PHAT;

2) With the rise of SR-BW-Ratio, the both performance will have degenerated. The reason why the sampling rate bandwidth ratio is emphasized is that in practice, we find that increasing the sampling rate may degrade the performance of TDOA when the signal bandwidth is constant. MATLAB simulation also proves this point.

4. Experiment and Comparison

Fig. 2 shows our experimental site. Three RF monitoring stations, R1, R2, R3, are deployed in the urban district, with 1.4~2.0 km baseline length. The RF monitoring stations are timed by GPS. The station R1 elevation is 8m, R2 and R3 is 14m. The antenna elevation of The station R1 is 30m, R2’s and R3’s is 30m. In this area, in addition to dense buildings, there exists a terrain hump of 28 m elevation in the triangular area of R1, R2 and R3.

Fig. 2. Experimental site in an urban district; three RF monitoring stations are deployed with 1.4~2.0 km baseline.

Each monitoring station consists of a radio frequency receiver, a GPS timing module and a data acquisition and transmission computer. A central station computer receives data blocks uploaded by the stations, performs TDOA estimation and location calculation. The signal sample rate of the stations is set to 1 MSPS. Prior to estimate, received signal data are filtered by a low pass fir filter, and are interpolated with 2 times factor. The sample rates for the TDOA estimator grow to 2 MSPS.

Due to the constraints of buildings, terrain and emitter power, it is not easy to find an emission point at which the signal can reach three receiving stations. Four test RF emission points, P1~P4, are selected. We could calculate the true TDOA value between station pairs, the estimation success ratio of PDSS and GCC-PHAT methods could be evaluated. There exist 3 TDOA values among 3 receivers,
we estimate TDOA T12 between R1 and R2, T13 between R1 and R3, T23 between R2 and R3.

| PT | BLK | BW   | LVL | T12  | T13  | T23  |
|----|-----|------|-----|------|------|------|
| P1 | 32  | 100  | -91;85;63 | 38:50 | 22:56 | 6:59 |
| P2 | 48  | 100  | -69;85;86 | 6:6   | 4:15  | 48:88 |
| P3 | 20  | 100  | -85;59;77 | 15:30 | 45:65 | 35:85 |
| P4 | 132 | 100  | -85;85;74 | 30:85 | 46:75 | 43:77 |
| P1 | 48  | 60   | -88;85;62 | 27:46 | 6:21  | 6:77 |
| P2 | 20  | 60   | -87;84;82 | 45:80 | 20:70 | 65:70 |
| P3 | 16  | 60   | -86;59;76 | 0:38  | 13:63 | 0:0  |
| P4 | 28  | 60   | -84;82;69 | 57:75 | 0:0   | 11:11 |

PT = the location of the test points; BLK = the numbers of estimated data blocks; BW = received signal bandwidth, unit is kHz; LVL = received signal level, units is Decibel-milliwatts (dBm), are arranged in sequence of “R1;R2;R3”;

T12 = two estimation success ratio between the receiver 1 and the receiver 2, units is percentage, with difference thresholds ; first item is the success ratio with absolute errors <= 1 sample, second item with absolute errors <= 2 sample; T13 and T23 are similar to T12.

Each data block is processed by both methods for comparisons. The emitted signal frequency is about 150MHz, and modulation mode is frequency modulation (FM). Three signal bandwidth are selected, 100 kHz, 60 kHz, 8 kHz, the corresponding SR-BW-Ratios were 20, 33, 250. The antenna of the emitter is placed on the roof of the car, the height less than 2 meters. There are multipath and NLOS problems in this experimental site.

| PT | BLK | BW   | LVL | T12 | T13 | T23 |
|----|-----|------|-----|-----|-----|-----|
| P1 | 32  | 100  | -91;85;63 | 63;100 | 16:56 | 28:96 |
| P2 | 48  | 100  | -69;85;86 | 2:2   | 0:6   | 52:75 |
| P3 | 20  | 100  | -87;59;77 | 0:0   | 60:60 | 35:35 |
| P4 | 132 | 100  | -85;85;74 | 39:75 | 64:85 | 64:88 |
| P1 | 48  | 60   | -88;85;62 | 19:56 | 15:85 | 46:94 |
| P2 | 20  | 60   | -67;84;82 | 0:0   | 0:0   | 0:0   |
| P3 | 16  | 60   | -86;59;76 | 56:93 | 9:3   | 75:100|
| P4 | 28  | 60   | -84;82;69 | 43:79 | 43:64 | 39:89 |

The definition of column parameters in Table 2 is the same as that in Table 1.

The experimental results of GCC-PHAT method are shown in Table I, PDSS’s results are shown in
Table II. The experimental results are taken from the application software developed by us; the block size is 4608 samples. Success ratio of TDOA estimation method are show in the columns of T12, T13, and T23; the success ratio on the left of the semicolon is statistical result with threshold of estimated absolute error value less than or equal to 1 sample; the right one is statistical result with threshold of estimated absolute error less than or equal to 2 samples.

1) For the signal of 100 kHz bandwidth with 232 sampled data blocks and 3 time difference values, each estimator output 696 results. With the criterion of absolute TDOA errors less than or equal to 1 sample, GCC-PHAT got 28% success ratio, and PDSS got 35% success ratio. With the criterion of absolute TDOA errors less than or equal to 2 samples, GCC-PHAT got 57% success ratio, and PDSS got 57% success ratio.

2) For the signal of 60 kHz bandwidth with 112 blocks and 3 time difference values, each estimator output 336 results. With the criterion of absolute TDOA errors less than or equal to 1 sample, GCC-PHAT got 21% success ratio, and PDSS got 28% success ratio. With the criterion of absolute TDOA errors less than or equal to 2 samples, GCC-PHAT got 46% success ratio, and PDSS got 63% success ratio.

3) For the 8 kHz signal, 387 blocks, 3 time difference values, each estimator output 1161 results. With the criterion of absolute TDOA errors less than or equal to 1 sample, GCC-PHAT got 0.1% success ratio, and PDSS got 13% success ratio. With the criterion of absolute TDOA errors less than or equal to 2 samples, GCC-PHAT got 0.4% success ratio, and PDSS got 21% success ratio.

4) The experimental results show that, PDSS methods has higher success ratio than GCC-PHAT, especially with the increase of the SR-BW-Ratio to 2MSPS/8kHz = 250, the GCC-PHAT method has almost no qualified output.

5) The experimental results of 100 kHz and 60 kHz signals show that the PDSS and GCC-PHAT methods have complementarity. For example of 60 kHz bandwidth signal, in the test point P2, the success ratio of PDSS is zero, but GCC-PHAT has considerable success ratio; in the test point P3, PDSS has considerable success ratio, but GCC-PHAT has poor success ratio.

5. Conclusions
A time difference of arrival (TDOA) estimation method by Phase Difference Square Sum (PDSS) is introduced. In This method, only phase information of the signal is used, the phase difference square sums are calculated; the delay time between the received signals is estimated by searching minimum value of phase difference square sums function. PDSS method is easy to achieve in practice.

The simulation, and experiment in the real site, both proved that PDSS method has higher estimation success ratio than GCC-PHAT being widely used. In the multipath and NLOS environment, the results of GCC and PDSS both have uncertainties with the change of the emission point location. This is the challenge facing TDOA methods. The real experimental results also show that both methods of PDSS and GCC-PHAT are complementary, which means that the fusion of both methods is likely to further improve the performance of the TDOA estimator.

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