Performance Analysis of Multihop AF Relaying Systems in the Presence of Cochannel Interferences

Weifeng Mou, Fang Fang, Xiaoming Xu, Weiwei Yang, and Yueming Cai
College of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China

Correspondence should be addressed to Weiwei Yang; wwyang1981@163.com

Received 26 July 2014; Accepted 16 December 2014

Academic Editor: Shukui Zhang

Copyright © 2015 Weifeng Mou et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates the performance of $N$-hop ($N \geq 2$) amplify-and-forward (AF) relaying systems with both relays and the destination subject to independent and not necessarily identically distributed multiple cochannel interferences (CCIs). Based on a new class of upper bound of the equivalent signal-to-interference ratio (SIR) which is the harmonic mean of the minimum of the first $M \geq 0$ hop SIRs and the minimum of the remaining $N - M$ hop SIRs, new approximate closed-form expressions of the outage probability and the ergodic capacity are derived. Furthermore, we derive the asymptotic expression of outage probability which accurately reveals the achievable coding gain and diversity gain. Finally, numerical results validate the correctness of the derived expressions.

1. Introduction

Multihop wireless relaying technique has recently received significant attention especially in cellular, modern ad-hoc, and wireless sensor networks for its performance benefits, including hotspot throughput improvements and signal coverage enhancements. The transmission characteristics of multihop relaying systems have been widely investigated in [1–4]. The authors have studied the outage performance of Nakagami-m fading channels and proposed the upper bound of the end-to-end signal-to-noise ratio (SNR) by the geometric mean of multihop SNRs in [1, 2]. The work in [3] has analysed the average error probability of variable-gain multihop amplify-and-forward (AF) systems by regarding the minimum SNR of all hops as the equivalent SNR. The result in [4] is of particular interest where a new class of upper bounds for the end-to-end SNR is proposed. The key idea is to partition the multihop systems into two parts and bind the SNR by the harmonic mean of the minimum of the first $M$ hop SNRs and the next $N - M$ hop SNRs, where the parameter $M$ varies from 0 to $N$. Compared with [3], the resulting outage probability lower bound in [4] is tighter. Moreover, most of the aforementioned works have just considered the noise-limited scenario.

Due to the aggressive reuse of frequency channels for high spectrum utilization, cochannel interferences (CCIs) have become an important issue in wireless systems. As well known, CCIs seriously deteriorate the system performances and drastically complicate the analysis. Therefore, consideration of CCIs is indeed necessary. Most of the existing works focus their attention on the impact of CCIs on dual-hop relaying systems. In [5, 6], the performance of dual-hop relaying network considering CCIs has been investigated. For a multihop network with nonregenerative relays, exact closed-form expressions of performance appear to be intractable especially when $N \geq 3$. Recently, a signal-to-interference-plus-noise ratio (SINR) upper bound by using the minimum SINR of all hops has been proposed to analyse the outage performance for multihop relay systems in [7–10]. In [7], the effect of CCIs on the performance has been investigated in a Rayleigh fading environment. In [8–10], the minimum SINR of all hops has been used to study the impact of CCIs on the system performance over Nakagami-$m$ fading channels. However, the performance bound does not approximate well at the low-to-medium SNR areas. Thus it cannot provide an accurate assessment of the system performance, which motivates us to develop new tighter bounds for the multihop relaying systems in the presence of CCIs.
Motivated by these considerations, we investigate the performance of multihop AF relaying systems with both relays and the destination subjected to multiple cochannel interferences. Based on a new class of upper bound of the equivalent signal-to-interference ratio (SIR) which is the harmonic mean of the minimum of the first $M$ hops SIRs and the minimum of the next $N - M$ ($0 \leq M \leq N$) hop SIRs, we research the performance in terms of outage probability and the ergodic capacity. Obviously, when $M = 0$ or $M = N$, new upper bound reduces to the bound using the minimum SIR of all hops named the minimum lower bound entire paper. New approximate closed-form expressions of the outage probability and the ergodic capacity are derived. Furthermore, we obtain the asymptotic expression of outage probability which accurately reveals the achievable coding gain and diversity gain. The results indicate that the number of interferers is related to coding gain, but it does not affect the diversity gain. Monte-Carlo simulation results are presented to verify the validity of theoretical analysis which indicates that our performance bound is tighter than the minimum lower bound in low SIRs area.

2. System Model

As is depicted in Figure 1, an $N$-hop interference-limited wireless relaying system is considered, where the source node $T_0$ communicates with the destination node $T_N$ via $(N - 1)$ relays $T_n$ ($n \in \{1, 2, \ldots, N - 1\}$) employing AF relaying protocol. Each node is equipped with a single antenna and works in the half-duplex mode. Furthermore, it is assumed that only one node is allowed to transmit in each time slot.

In the $n$th time slot, the $n$th relay node $T_n$ receives the useful signal just from the immediately transmitting node $T_{n-1}$. In the meantime, it is interfered by $L_{T_n}$ independent and not necessarily identically distributed CCIs.

As far as we know, the effect of noise can be ignored when the SNR and interference-to-noise ratio (INR) are high which is widely used in [11, 12]. Therefore, the received signal at $T_n$ is expressed as

$$y_n = \sqrt{P_{n-1}} h_n y_{n-1} + \sqrt{P_{I_n}} \sum_{j=1}^{L_{T_n}} h_{T_n,j} m_{T_n,j},$$

where $y_{n-1}$ and $P_{n-1}$, respectively, denote the unit-energy signal and energy transmitted from the node $T_{n-1}$, $h_n$ indicates the Rayleigh fading channel coefficient of $T_{n-1} \rightarrow T_n$, which satisfies $E[|h_n|^2] = \sigma_n^2 \propto d_n^{-\alpha}$, and $E[\cdot]$ denotes the expectation operation. $\alpha$ is the path-loss exponent (normally, $\alpha > 2$); $d_n$ is the distance between $T_{n-1}$ and $T_n$. Similarly, $m_{T_n,j}$ and $P_I$ are the unit-energy signal and transmitting energy of the $j$th cochannel interference, respectively. $L_T$ is the total number of interferences. $h_{T_n,j}$ is the Rayleigh fading channel coefficient of the $j$th interference to node $T_n$ which satisfies $E[|h_{T_n,j}|^2] = \sigma_{T_n,j}^2 \propto d_{T_n,j}^{-\alpha}$ and $d_{T_n,j}$ is the distance between the $j$th interference node and the $n$th node.

In the multihop AF relaying systems, the $n$th relay node amplifies the received signal $y_n$ by a gain factor $A_n$ and then forwards the obtained signal to the next node. Therefore, the received signal at the $(n + 1)$th relay node can be expressed as

$$y_{n+1} = y_n \sqrt{P_{n}} A_n h_{n+1} + \sqrt{P_{I_n}} \sum_{j=1}^{L_{T_n}} h_{T_{n+1},j} m_{T_{n+1},j},$$

where the amplification factor at the relay node $T_n$ is

$$A_n = \sqrt{\left( \frac{P_{n-1} |h_n|^2 + P_I \sum_{j=1}^{L_{T_n}} |h_{T_n,j}|^2}{L_{T_n}} \right)^{-1}}.$$ (3)

Therefore, the received signal at the destination node $T_N$ for the $N$-hop AF wireless network in the presence of CCIs is formulated as

$$y_N = \sqrt{P_{N-1}} h_N \prod_{i=1}^{N-1} A_i \sqrt{P_{I_i}} h_{I_i} x_0 + \sqrt{P_{I_N}} \sum_{j=1}^{L_{T_N}} h_{T_N,j} m_{T_N,j}$$

$$+ \sum_{i=1}^{N-1} \prod_{j=1}^{L_{T_i}} A_j \sqrt{P_{I_j}} h_{I_j} m_{T_i,j},$$ (4)

where the first part is the valid signal, the second part is the interference signal received at the destination, and the third part is the accumulatively amplified interference signal from all of the relays.

Based on (4), the equivalent SIR at $T_N$ is derived as

$$y_{end} = \frac{P_{N-1}|h_N|^2 \prod_{i=1}^{N-1} A_i^2 P_{I_i} |h_i|^2}{\sum_{i=1}^{N-1} \prod_{j=1}^{L_{T_i}} A_j^2 P_{I_j} |h_{I_j}|^2 + \prod_{i=1}^{L_{T_N}} |h_{T_N,j}|^2}.$$ (5)
We assume that $y_i = P_{i-1} |h_i|^2$ and $y_i = \sum_{j=1}^{L_{Ti}} P_{ij} |h_{Tij}|^2$ are the signal power and the total interference power at the $i$th relay node, respectively. Then, (5) can be rewritten as

$$\gamma_{end} = \left[ \sum_{i=1}^{N} \left( 1 + \frac{y_i}{y_{th}} \right) - 1 \right]^{-1}. \quad (6)$$

In order to investigate the performance of the arbitrary $N$-hop AF relaying systems, we need to derive the statistical distributing character of $\gamma_{end}$. Although (6) has accurately described the equivalent SIR, the probability distribution function (PDF) of (6) is not mathematically tractable, particularly for $N \geq 3$. We adopt the similar method proposed in [4] to partition the set of $\gamma_i$ into two groups, where $\gamma_i = y_i/y_{th}$ denoting the equivalent signal-to-interference ratio (SIR) at the $i$th node. Then, the upper bound of the end-to-end equivalent SIR is rewritten as

$$\gamma_{end} \leq y_{up} = \left[ \min_{1 \leq i \leq M} \left( \gamma_i^{eff} \right) + \min_{M+1 \leq i \leq N} \left( \gamma_i^{eff} \right) \right]^{-1}, \quad (7)$$

where $0 \leq M \leq N$. The derived SIR bound $y_{up}$ is related to the harmonic mean of the minimum SIR of the first $M$ hops and the minimum SIR of the next $N-M$ hops. Intuitively, the tightness of the derived bound increases as $M$ gets closer to $N-M$. Denoting $\Gamma_1 = \min_{1 \leq i \leq M} (\gamma_i^{eff})$ and $\Gamma_2 = \min_{M+1 \leq i \leq N} (\gamma_i^{eff})$, $y_{up}$ is expressed as $y_{up} = \Gamma_1 \Gamma_2 / (\Gamma_1 + \Gamma_2)$.

In particular, when $M = 0$ or $M = N$, (7) can be reduced to the minimum lower bound $\gamma_{min} = \min_{1 \leq i \leq N} (\gamma_i^{eff})$ which uses the minimum SIR of all hops as the equivalent SIR [11]. In addition, it is worth noting that (7) with $N = 2$ and $M = 1$ can be reduced to the exact SIR for the case of dual-hop systems [12].

### 3. Outage Probability Analyses

In interference-limited environments, the outage probability is defined as the probability that the received equivalent SIR drops below the threshold $y_{th}$, or mathematically

$$P_{out}(y_{th}) = \Pr(\gamma_{end} \leq y_{th}) = F_{\gamma_{end}}(y_{th}) = F_{\gamma_{up}}(y_{th}). \quad (8)$$

where $\Pr(x)$ denotes the probability and $F_X(x)$ denotes cumulative distribution function (CDF) of $X$.

Based on (7), $F_{\gamma_{up}}(y_{th})$ is expressed as

$$F_{\gamma_{up}}(x) = 1 - \int_0^{\infty} \left[ 1 - F_{\gamma_i} \left( \frac{z+x}{z} \right) \right] f_{\gamma_i}(z+x) \, dz, \quad (9)$$

where $f_X(x)$ is the PDF of $X$.

In order to obtain $F_{\gamma_i}(x)$ and $f_{\gamma_i}(x)$, we need to know the CDF of SIR of every hop $\gamma_i^{eff}$ in $T_{i-1} \rightarrow T_i$ link, which can be expressed as

$$F_{\gamma_i^{eff}}(y) = \int_0^{y} F_{\gamma_i} \gamma x \, f_{\gamma_i}(x) \, dx, \quad (10)$$

where $F_{\gamma_i}(x)$ is the CDF of $\gamma_i$, which follows exponential distribution and can be written as $F_{\gamma_i}(x) = 1 - e^{-x/\gamma_i}$. And $f_{\gamma_i}(y)$ is the PDF of $\gamma_i$, which denotes the sum of the multiple independent and not necessarily identically random variables. And it is written as $f_{\gamma_i}(x) = x^{L_{Ti}-1} e^{-x/\gamma_i} / \Gamma(L_{Ti})$, where $\gamma_i = E[\gamma_i] = P_{i-1} \sigma_i^2$. We assume that the distance between each interference and the relay is equal, so $\gamma_i = E[\gamma_i] = P_{i-1} \sum_{j=1}^{L_{Ti}} \sigma_{Tij}^2$.

Substituting $F_{\gamma_i}(x)$ and $f_{\gamma_i}(x)$ into (10), the CDF of $\gamma_i^{eff}$ can be rewritten as

$$F_{\gamma_i^{eff}}(x) = 1 - \left( \frac{1 + x/\gamma_i^{eff}}{\gamma_i^{eff}} \right)^{-L_{Ti}}, \quad (11)$$

where $\gamma_i^{eff} = \gamma_i / \gamma_i^{up}$ denotes the average SIR at the $i$th relay node.

We have assumed that $\Gamma_1 = \min_{1 \leq i \leq M} (\gamma_i^{eff})$ is the minimum SIR of the first $M$ hops and $\Gamma_2 = \min_{M+1 \leq i \leq N} (\gamma_i^{eff})$ is the minimum SIR of the next $N-M$ hops. Therefore, the CDF of $\Gamma_1$ can be written as

$$F_{\Gamma_1}(x) = 1 - \prod_{i=1}^{M} \left( 1 + x/\gamma_i^{eff} \right)^{-L_{Ti}}. \quad (12)$$

Utilizing decomposition into partial fractions, $F_{\Gamma_1}(x)$ can be rewritten as

$$F_{\Gamma_1}(x) = 1 - \sum_{i=1}^{M} \lambda_{m,i} \left( 1 + x/\gamma_i^{eff} \right)^{-n}, \quad (13)$$

where $\lambda_{m,i}$ is expressed as

$$\lambda_{m,i} = \frac{(\gamma_i^{eff})^{L_{Ti} - n}}{(L_{Ti} - n)!} \times \left\{ \left[ \frac{\partial^{L_{Ti} - m}}{\partial \gamma_i^{eff}^{L_{Ti} - m}} \left( 1 + \frac{\gamma_i^{eff}}{\gamma_i^{eff}} \right) \right] \prod_{i=1}^{M} \left( 1 + \frac{\gamma_i^{eff}}{\gamma_i^{eff}} \right)^{-L_{Ti}} \right\}_{y = \gamma_i^{eff}}. \quad (14)$$

Similarly, the CDF of $\Gamma_2 = \min_{M+1 \leq i \leq N} (\gamma_i^{eff})$ is derived as

$$F_{\Gamma_2}(x) = 1 - \sum_{j=M+1}^{N} \lambda_{m,j} \left( 1 + x/\gamma_j^{eff} \right)^{-m}, \quad (15)$$

where $\gamma_j^{eff} = \gamma_j / \gamma_j^{up}$ denotes the average SIR in the $j$th relay node and $\lambda_{m,j}$ is expressed as

$$\lambda_{m,j} = \frac{(\gamma_j^{eff})^{L_{Tj} - m}}{(L_{Tj} - m)!} \times \left\{ \left[ \frac{\partial^{L_{Tj} - m}}{\partial \gamma_j^{eff}^{L_{Tj} - m}} \left( 1 + \frac{\gamma_j^{eff}}{\gamma_j^{eff}} \right) \right] \prod_{j=M+1}^{N} \left( 1 + \frac{\gamma_j^{eff}}{\gamma_j^{eff}} \right)^{-L_{Tj}} \right\}_{y = \gamma_j^{eff}}. \quad (16)$$
By taking the derivative of $F_{\Gamma_1}(x)$ with respect to $x$, the PDF of $\Gamma_2$ is obtained as
\[
f_{\Gamma_1}(x) = \sum_{j=1}^{L_{\Gamma_1}} \sum_{m=1}^{N} \frac{m \lambda_{m,j}}{T_j} \left(1 + \frac{x}{\gamma_{j}^{\text{eff}}}ight)^{-m-1}.
\] (17)

Substituting (13) and (17) into (9), the integral expression is derived:
\[
F_{\gamma_{eq}}(x) = 1 - \sum_{i=1}^{N} \sum_{j=1}^{L_{\Gamma_1}} \frac{m \lambda_{m,j} \lambda_{n,j} (x + \gamma_{j}^{\text{eff}})^{m+1}}{T_j} (x/T_j + 1)^{m+1} (x^2/T_j)^n
\]
\[
\times \int_0^\infty z^n (z+1)^{-m-1} (az+1)^{-n} dz,
\] (18)

where $\alpha = (x + \gamma_{j}^{\text{eff}})(x + \gamma_{j}^{\text{eff}})/x^2$.

Then, using [13, Eq. 3.197.5], the closed-form expression of CDF of $\gamma_{eq}$ is derived:
\[
F_{\gamma_{eq}}(x) = 1 - \sum_{i=1}^{N} \sum_{j=1}^{L_{\Gamma_1}} \frac{m \lambda_{m,j} \lambda_{n,j} (x + \gamma_{j}^{\text{eff}})^{m+1}}{T_j} (x/T_j + 1)^{m+1} (x^2/T_j)^n
\]
\[
\times B(n+1,m) \frac{1}{2} F_1(n,n+1;n+m+1;1-\alpha),
\] (19)

where $B(x,y)$ is the Beta function [13, Eq. 8.380.1] and $2 F_1(a,b;c;z)$ is the Gauss hypergeometric function [13, Eq. 9.1.11]. It can be computed in many popular software tools, such as MATHEMATICAL and MAPLE.

Thus, the lower bound of the outage probability is expressed as
\[
P_{\text{out}}(\gamma_{\text{th}}) \geq 1 - \sum_{i=1}^{N} \sum_{j=1}^{L_{\Gamma_1}} \frac{m \lambda_{m,j} \lambda_{n,j} (\gamma_{j}^{\text{th}} + \gamma_{j}^{\text{eff}})^{m+1}}{T_j} (\gamma_{j}^{\text{th}}/T_j + 1)^{m+1} (\gamma_{j}^{\text{th}} + \gamma_{j}^{\text{eff}})^n
\]
\[
\times B(n+1,m) \frac{1}{2} F_1(n,n+1;n+m+1;1-\alpha)
\] (20)

It is indicated that the outage probability has much to do with the number and power of cochannel interference and the relationship between $N$ and $M$. However, the exact analysis is too complicated to render insight on the impact of cochannel interferences and the relationship between $N$ and $M$. Therefore, the asymptotic outage probability is investigated at high SIR regime.

In particular, when $M = 0$ or $M = N$, (20) is reduced to the minimum lower bound of the outage probability in [11]:
\[
P_{\text{out}}(\gamma_{\text{th}}) \geq 1 - \sum_{i=1}^{N} \sum_{j=1}^{L_{\Gamma_1}} \frac{\lambda_{n,j} (1 + \gamma_{j}^{\text{th}}/T_j)^{-n}}{\gamma_{j}^{\text{eff}}}
\] (21)

When $N = 2$ and $M = 1$, expression (20) is reduced to the accurate outage probability of dual-hop AF relaying systems in the interference-limited environments in [12]:
\[
P_{\text{out}}(\gamma_{\text{th}})
\]
\[
\geq 1 - \sum_{i=1}^{L_{\Gamma_1}} \sum_{j=1}^{L_{\Gamma_2}} \frac{m \lambda_{m,j} \lambda_{n,j} (\gamma_{j}^{\text{th}} + \gamma_{j}^{\text{eff}})^{m+1}}{T_j} (\gamma_{j}^{\text{th}}/T_j + 1)^{m+1} (\gamma_{j}^{\text{th}} + \gamma_{j}^{\text{eff}})^n
\]
\[
\times B(n+1,m) F_1(n,n+1;n+m+1;1-\alpha)
\] (22)

These results in (21) and (22) demonstrate the generality of our analysis.

3.1. Diversity and Coding Gain. In the high SIR regime, the asymptotic outage probability can be expressed as
\[
P_{\text{out}}^{\text{as}} = \Psi \cdot \gamma_{\text{end}}^{-\Phi} + o\left(\gamma_{\text{end}}^{-\Phi}\right),
\] (23)

where $o()$ represents the higher order terms (i.e., we write $f(x) = o(g(x))$ as $x \to x_0$ if $\lim_{x \to x_0} f(x)/g(x) = 0$) and $\gamma_{\text{end}}$ is the average end-to-end equivalent SIR. $\Phi$ and $\Psi$ denote the diversity and coding gain, respectively.

In order to gain some insight about the achievable diversity and coding gain, another upper bound $\gamma_{eq}$ on the end-to-end equivalent SIR is utilized:
\[
\gamma_{eq} = \min(\Gamma_1,\Gamma_2).
\] (24)

Then, the lower bound of the outage probability can be expressed as
\[
F_{\gamma_{eq}}(x) = 1 - \left[1 - F_{\Gamma_1}(x)\right] \left[1 - F_{\Gamma_2}(x)\right].
\] (25)

As $\gamma_{j}^{\text{eff}} = \gamma_{j}/T_j \to \infty$, using $\lim_{x \to 0} (1 + x)^{-j} \approx 1 - jx$, $F_{\gamma_{eq}}(x)$ can be asymptotically written as
\[
F_{\gamma_{eq}}(x) \bigg|_{x \to 0} = \frac{L_{\Gamma_1}}{\gamma_{j}^{\text{eff}}} x + o\left(\gamma_{j}^{\text{eff}}^{-1}\right).
\] (26)

Furthermore, substituting (26) into (13), we obtain the asymptotic expression for $F_{\Gamma_1}(\gamma_{\text{th}})$ as
\[
F_{\Gamma_1}^{\text{as}}(x) = \left(\sum_{i=1}^{M} \frac{L_{\Gamma_1}}{x}\right) \left(\gamma_{j}^{\text{eff}}\right)^{-1} + o\left(\gamma_{j}^{\text{eff}}^{-1}\right).
\] (27)

Similarly, by following the similar steps of $F_{\Gamma_1}(x)$, asymptotic expression for $F_{\Gamma_2}(x)$ is given by
\[
F_{\Gamma_2}^{\text{as}}(x) = \left(\sum_{i=M+1}^{N} \frac{L_{\Gamma_2}}{x}\right) \left(\gamma_{j}^{\text{eff}}\right)^{-1} + o\left(\gamma_{j}^{\text{eff}}^{-1}\right).
\] (28)
Then, substituting (27) and (28) into (25), we derive the asymptotic expression for outage probability as

\[ P_{eq}^{\text{co}}(\gamma_{th}) \approx P_{\Gamma_i}^{\text{co}}(\gamma_{th}) + P_{\Gamma_j}^{\text{co}}(\gamma_{th}). \]  

(29)

Based on (29), we obtain the diversity gain \( \Phi \) which is defined as the expected value of the instantaneous maximum of the asymptotic outage probability curve. And the coding gain \( \Psi \) is derived as

\[ \Psi = \sum_{i=1}^{L_{\Gamma_i}} \frac{N}{N+1} \sum_{j=M+1}^{N} \gamma_{th} \]  

(30)

From (30), we found that the coding gain has much to do with the number of the cochannel interference and the value of the threshold. But these factors do not affect the diversity gain.

4. Ergodic Capacity Analyses

Besides the outage probability, the ergodic capacity is another important performance measure. The ergodic capacity is defined as the expected value of the instantaneous maximum mutual information. Thus, the end-to-end ergodic capacity of multihop AF relaying systems with CCI can be expressed as

\[ C_{\text{AF}} = \frac{1}{N} \int_0^{\gamma_{\text{eq}}} \log_2 (1 + x) f_{\gamma_{\text{eq}}}(x) \, dx. \]  

(31)

In order to derive the ergodic capacity, the PDF of \( \gamma_{\text{eq}} \) is needed which is found by taking the derivative of \( F_{\gamma_{\text{eq}}}(x) \) in (19) with respect to \( x \):

\[ f_{\gamma_{\text{eq}}}(x) = -\sum_{i=1}^{L_{\Gamma_i}} \sum_{j=1}^{N} \sum_{n=1}^{N+1} \sum_{m=1}^{L_{\Gamma_j}} m \lambda_{\Gamma_i} m_{\Gamma_j} \left( \frac{1}{y_{\text{eq}}} \right)^n B(n + 1, m) \]

\[ \times \left[ \frac{(x + y_{\text{eq}})^n}{(x/y_{\text{eff}} + 1)^{n+1}} x^{2n+1} \right] \]

\[ \times \left[ x^2 (n - m + 1) \left( \frac{1}{y_{\text{eff}}} \right)^{m+1} \right] \]

\[ \times \left[ \frac{(x + y_{\text{eq}})^n}{(x/y_{\text{eff}} + 1)^{n+1}} x^{2n+1} \right] \]

\[ \times \frac{\left( n + 1 \right) \left( x + y_{\text{eq}} \right)^{n+1}}{(n + m + 1) \left( x/y_{\text{eff}} + 1 \right)^{n+1}} \]

\[ \times \left( x^2 / y_{\text{eff}}^2 \right)^n x^3 \]

\[ \times \left( y_{\text{eq}} + y_{\text{eff}} \right) \]

\[ \times \left( y_{\text{eq}}^2 + y_{\text{eff}}^2 \right) \].  

(32)

4.1. Approximate Analysis. Although the expression \( F_{\gamma_{eq}}(x) \) in (19) offers an efficient way to evaluate the outage performance, it is too difficult to derive the closed-form expression of the ergodic capacity. Therefore, we derived the closed-form expression of the approximate close analysis for the ergodic capacity when \( M = 0 \) or \( M = N \).

Substituting (13) and (15) to (25) and by the mathematical calculation, the CDF of \( \gamma_{eq} \) is formulated as

\[ F_{\gamma_{eq}}(x) = 1 - \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{n=1}^{N+1} \sum_{m=1}^{L_{\Gamma_j}} \lambda_{\Gamma_i} \lambda_{\Gamma_j} \left( \frac{1}{y_{\text{eq}}} \right)^n \left( \frac{1}{y_{\text{eff}}} \right)^m \]

\[ \times \left[ \frac{n}{(y_{\text{eff}} + x)^{n+1}} \right] \]

\[ + \left[ \frac{m}{(y_{\text{eff}} + x)^{m+1}} \right] \].  

(33)

Consequently, the PDF of \( \gamma_{eq} \) is obtained by taking the derivation of \( F_{\gamma_{eq}}(x) \) with respect to \( x \):

\[ f_{\gamma_{eq}}(x) = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{n=1}^{N+1} \sum_{m=1}^{L_{\Gamma_j}} \lambda_{\Gamma_i} \lambda_{\Gamma_j} \left( \frac{1}{y_{\text{eq}}} \right)^n \left( \frac{1}{y_{\text{eff}}} \right)^m \]

\[ \times \left[ \frac{n}{(y_{\text{eff}} + x)^{n+1}} \right] \]

\[ + \left[ \frac{m}{(y_{\text{eff}} + x)^{m+1}} \right] \].  

(34)

Then, substituting (34) to (31) and integral, we derive the approximate ergodic capacity expression in

\[ C_{\text{AF}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{n=1}^{N+1} \sum_{m=1}^{L_{\Gamma_j}} \lambda_{\Gamma_i} \lambda_{\Gamma_j} \left( \frac{1}{y_{\text{eq}}} \right)^n \left( \frac{1}{y_{\text{eff}}} \right)^m \]

\[ \times \left[ \frac{n}{(y_{\text{eff}} + x)^{n+1}} \right] \]

\[ + \left[ \frac{m}{(y_{\text{eff}} + x)^{m+1}} \right] \] \]  

(35)

where

\[ A_s = \frac{1}{(n-s)!} \left[ \frac{1}{(x + y_{\text{eff}})^{m+1}} \right] \]

\[ B_t = \frac{1}{(m-t)!} \left[ \frac{1}{(x + y_{\text{eff}})^{m+1}} \right] \] \]  

(36)
\[ C_u = \frac{1}{(n-u)!} \int \frac{1}{(x + \gamma_j)^m} \, dx \quad x = -\gamma_j \]
\[ D_v = \frac{1}{(m+1-v)!} \int \frac{1}{(x + \gamma_j)^v} \, dx \quad x = -\gamma_j \]

(36)

To calculate (35), using [14, Equation (10, 11, 21)] and after some manipulations, the closed-form expression of ergodic capacity is derived as

\[ C_{AF} = \sum_{i=1}^{M} \sum_{n=1}^{L_i} \sum_{m=1}^{N} \frac{\lambda_i \lambda_m \ln 2}{\ln 2} \left( \frac{V_j}{v_j} \right)^n \left( \frac{V_i}{v_i} \right)^m \]
\[ \times \left\{ \begin{array}{c}
\frac{nA_s}{\Gamma(s)} \left( \frac{V_i}{v_i} \right)^{\frac{s-1}{2}} \left[ \frac{1}{V_i} \right]^0, -1, -1, -1 \\
\frac{nB_t}{\Gamma(t)} \left( \frac{V_j}{v_j} \right)^{\frac{t-1}{2}} \left[ \frac{1}{V_j} \right]^0, -1, -1, -1 \\
\frac{mC_u}{\Gamma(u)} \left( \frac{V_j}{v_j} \right)^{\frac{u-1}{2}} \left[ \frac{1}{V_j} \right]^0, -1, -1, -1 \\
\frac{mD_v}{\Gamma(v)} \left( \frac{V_i}{v_i} \right)^{\frac{v-1}{2}} \left[ \frac{1}{V_i} \right]^0, -1, -1, -1 \\
\end{array} \right\} \]

(37)

where \( G(a, b; c) \) is the Meijer G-function [13, Equation 9.301] that is readily available in the standard mathematical packages such as MATHEMATICA, MATLAB, and MAPLE.

5. Numerical and Simulation Results

In this section, numerical and Monte-Carlo simulation results are provided to verify the accuracy of our analytical results. And we have compared our performance bound with the minimum lower bound [11] and the exact simulations. In all scenarios, the outage threshold is selected as \( y_{th} = 0 \) dB, and the path-loss factor \( \alpha = 4 \). We assume that the distance of \( T_{i-1} \rightarrow T_i \) is \( d_{i-1} = \{0.8, 0.8, 0.7, 0.7, 0.6\} \) and the distance among interference to relay nodes is \( d_{L_{i-1}} = \{1.5, 1.6, 1.7\} \). In addition, the signal transmit power at node \( T_{i-1} \) is expressed as \( P_{i-1} = \{P_0, 1.1P_0, 0.9P_0, 0.8P_0, 0.7P_0\} \) and the transmit power of the CCIs is normalized, for example, \( \{P_j\}_{j=1}^{N} = 1 \). We find excellent agreements between the simulated and analytical results. The asymptotic results accurately provide valuable insights in terms of the diversity order and coding gain of the systems which confirm the validities of our analysis.

In Figure 2, the outage probability of a three-hop relaying system versus the number of CCIs at each hop is shown. We assume that the distances between different relay nodes are identical: \( d_{i-1} = \{0.8, 0.8, 0.7\} \), and the distance between interference and relay node is \( d_{L_{i-1}} = \{1.5, 1.6, 1.7\} \); the number of CCIs at each hop is equal and each interference has the same transmission power, and the relaying node \( T_{i-1}^2 \) has a different transmission power. We set \( M = 2 \) that is to regard the first two hops as the one section and the third hop as the other section. As expected, we observe that the outage performance decays with the increasing of the number of CCIs at each relay and destination. Furthermore, our performance bound is tighter than minimum lower bound at the low-to-medium SIR areas and in accordance with the exact simulations.

In Figure 3, the proposed lower bounds of the outage probability for \( N = 3, 4, 5 \) hops relaying systems are plotted. We assume that the number of interferers is fixed at \( L_{th} = 2 \). We find that, as the number of hops increases, the outage performance deteriorates significantly and both classes of the lower bounds turn to be more deviant from the exact simulations. What is more, compared with the minimum lower bound, our performance bound outperforms well and is more approximate to the exact simulations. Furthermore, it is observed that an increase in the number of hops brings about an obvious decline in outage probability. This can be explained by the fact that the diversity order is \( \Phi = 1 \).

Figure 4 compares the ergodic capacity performance of multihop relay networks for different numbers of hops. Observing the results in the figure, we can see that the more
the number of hops is, the worse the capacity performance is. The approximated ergodic capacity obtained in terms of the Meijer-G function is illustrated along with the upper bound derived in (30) and the gap is about 1.5 dB. And the gap between the exact and the upper bound capacity is just 0.5 dB.

6. Conclusions

This paper investigates the performance of the interference-limited $N$-hop ($N \geq 2$) AF relaying systems. Based on a new class of upper bounds on the equivalent SIR which is the harmonic mean of the minimum of the first $M$ hop SIRs and the minimum of the remaining $N-M$ hop SIRs, we derive new approximate closed-form expressions of the outage probability and the ergodic capacity. Compared with minimum lower bound [11], our performance bound of outage probability is tighter at the low-to-medium SIR areas. Further, the diversity and coding gain are derived from the asymptotic outage probability at the high SIR regime. We find that the number of CCIs can affect coding gain but cannot affect diversity gain. And the closed-form expression of approximate ergodic capacity is used to research the system performance. Finally, numerical results are presented to validate our analysis.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (no. 61371122 and no. 61471393) and Jiangsu provincial National Science Foundation (BK2013105).

References

[1] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, “Bounds for multihop relayed communications in Nakagami-m fading,” IEEE Transactions on Communications, vol. 54, no. 1, pp. 18–22, 2006.
[2] G. K. Karagiannidis, “Performance bounds of multihop wireless communications with blind relays over generalized fading channels,” IEEE Transactions on Wireless Communications, vol. 5, no. 3, pp. 498–503, 2006.
[3] M. O. Hasna and M.-S. Alouini, “Outage probability of multi-hop transmission over Nakagami fading channels,” IEEE Communications Letters, vol. 7, no. 5, pp. 216–218, 2003.
[4] G. Amarasuriya, C. Tellambura, and M. Ardakani, “Asymptotically-exact performance bounds of AF multi-hop relaying over Nakagami fading,” IEEE Transactions on Communications, vol. 59, no. 4, pp. 962–967, 2011.
[5] S. S. Ikki and S. Aissa, “Performance analysis of dual-hop relaying systems in the presence of co-channel interference,” in Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM ’10), pp. 1–5, Miami, FL, USA, December 2010.
[6] D. B. da Costa and M. D. Yacoub, ”Outage performance of two hop AF relaying systems with co-channel interferers over Nakagami-m fading,” IEEE Communications Letters, vol. 15, no. 9, pp. 980–982, 2011.
[7] S. S. Ikki and S. Aïssa, “Multihop wireless relaying systems in the presence of cochannel interferences: performance analysis and design optimization,” *IEEE Transactions on Vehicular Technology*, vol. 61, no. 2, pp. 566–573, 2012.

[8] R. Mesleh, S. S. Ikki, O. Amin, and S. Boussakta, “Analysis and optimization of AF multi-hop over Nakagami-m fading channels in the presence of CCI,” in *Proceedings of the IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC ’13)*, pp. 2021–2026, London, UK, September 2013.

[9] R. Mesleh, S. S. Ikki, and O. Amin, “Multi-hop relaying systems in the presence of co-channel interference over Nakagami-m fading channels,” *IET Communications*, vol. 8, no. 4, pp. 483–491, 2014.

[10] M. Xia and S. Aïssa, “Impact of co-channel interference on the performance of multi-hop relaying over Nakagami-m fading channels,” *IEEE Wireless Communications Letters*, vol. 3, no. 2, pp. 133–136, 2014.

[11] T. Soithong, V. A. Aalo, G. P. Efthymoglou, and C. Chayawan, “Performance of multihop relay systems with co-channel interference in rayleigh fading channels,” *IEEE Communications Letters*, vol. 15, no. 8, pp. 836–838, 2011.

[12] D. Lee and J. H. Lee, “Outage probability for dual-hop relaying systems with multiple interferers over Rayleigh fading channels,” *IEEE Transactions on Vehicular Technology*, vol. 60, no. 1, pp. 333–338, 2011.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, San Diego, Calif, USA, 7th edition, 2000.

[14] V. S. Adamchik and O. I. Marichev, “The algorithm for calculating integrals of hypergeometric type functions and its realization in reduce system,” in *Proceedings of the International Symposium on Symbolic and Algebraic Computation (ISSAC ’90)*, pp. 212–224, August 1990.
