A Model for Quantum Simulation of Mass Enhancement in Spontaneous SUSY Breaking

Masao Hirokawa

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Abstract

In this paper, we consider the model of a mass enhancement in quantum mechanics with $N = 2$ supersymmetry (SUSY). This model is so simple that it may be implemented as a quantum simulation of the mass enhancement in spontaneous SUSY breaking. It describes how a 1-mode massive boson coupled with a two-level atom gains a part of its mass from the excitation of another 1-mode boson coupled with the two-level atom. This mass enhancement is caused in the process of the transition from $N = 2$ SUSY to its spontaneous breaking.

I. INTRODUCTION

In 2012 the long-sought Higgs boson is found [1, 2], which establishes the triumph of the Higgs mechanism [3, 4]. This mechanism tells us how no-mass gauge particles gain mass in the standard model, while the gauge particle cannot have its mass due to gauge symmetry. That finding shows the Higgs-particle mass of 125 GeV ($\sim 10^2$ GeV). Physicists need a fine-tuning to obtain the Higgs mass when they consider the interaction of the Higgs particle and an elementary particle in the Planck-scale. Since the Planck-scale mass ($\sim 10^{18}$ GeV) is so much heavier than the Higgs mass, physicists usually employ the fine-tuning to cope with the mass gap with the ratio ($\sim 10^{16}$ GeV); thus, they perform the unnatural, huge cancellation between the bare mass term and the quantum correction to obtain the Higgs mass in the standard model. This is the so-called hierarchy problem. Supersymmetry (SUSY) is among the strong candidates for natural theories to solve the hierarchy problem. However, the Higgs-mass of 125 GeV puzzles physicists because it is too much heavier than the mass predicted in the minimal supersymmetric standard model. This gap between the two masses requires a fine-tuning again. It is expected that this gap is plugged by the spontaneous SUSY breaking [5–9]. Unfortunately, any fingerprint of SUSY and its spontaneous breaking had not been firmly observed in the physical reality even for the quantum mechanics (QM) version [10, 11] until 2022, that is, Cai et al. report its observation.

Some months before the Higgs-boson discovery, actually, the quantum simulation for the Higgs mechanism is succeeded [12]. Quantum simulation is for the study of quantum

*hirokawa@inf.kyushu-u.ac.jp  https://nvespm.net/qstl/; Graduate School of Information Science and Electrical Engineering, Kyushu University.
phenomena, which is implemented on a programmable quantum system consisting of quantum devices specially designed to realize those quantum phenomena. The idea of quantum simulation is based on Feynman’s proposal \[13\] and has been developing lately. Some theoretical models for quantum simulation of SUSY and its spontaneous breaking are proposed \[14–18\]. In particular, a simple, prototype model is given with using the quantum Rabi model, and it has the transition from the \( N = 2 \) SUSY to its spontaneous breaking by tuning some parameters in that model \[14\]. The success in observation of that transition in a trapped ion quantum simulator is reported \[19\]. In this transition we cannot observe any mass enhancement because the Lagrangian of the prototype model does not include any term for the mass enhancement. We are interested in quantum simulation showing a mass-enhancement mechanism. In this paper, therefore, we extend the prototype model by putting a natural mass term in the model such that we can make quantum simulation for the mass enhancement in spontaneous SUSY breaking.

II. MATHEMATICAL MODEL

The state space of the 1-mode boson in our model is given by the boson Fock space \( \mathcal{F}_b \), which is spanned by the boson Fock states. The boson Fock state with \( n \) bosons is denoted by \(|n\rangle\). In particular, thus, \(|0\rangle\) is the Fock vacuum. We use the notation ‘1’ for the identity operator acting on \( \mathcal{F}_b \). The two-level atom in our model is represented by spin. We use the standard notations, \( \sigma_x, \sigma_y, \sigma_z \), for the Pauli matrices, i.e., \( \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \), \( \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \), \( \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \). We use the notation ‘1’ for the 2-by-2 identity matrix, i.e., \( 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) as well as the numerical character 1. The spin-annihilation operator \( \sigma_- \) and spin-creation operator \( \sigma_+ \) are defined by \( \sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y) \). We denote the up-spin state by \(|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \), the down-spin state by \(|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). Throughout this paper, we often omit the symbol ‘\( \otimes \)’ in the vectors of \( \mathbb{C} \otimes \mathcal{F}_b \) and the operators acting in \( \mathbb{C} \otimes \mathcal{F}_b \), where \( \mathbb{C} \) is the one-dimensional unitary space, that is, the set of all complex numbers. We often omit the notation ‘1’ in operators.

For the position operator \( X \) and the momentum operator \( P \) acting in \( \mathbb{C} \otimes \mathcal{F}_b \), we give the Hamiltonian \( H \) of a harmonic oscillator coupled with spin. This describes the energy operator of a 1-mode massive boson coupled with the two-level atom, and given by

\[
H = 1 \otimes \left( \frac{1}{2}P^2 + \frac{\omega^2}{2}X^2 \right), \tag{1}
\]
where $\hbar \omega_g$ is the boson energy. We note that $H$ acts in the total state space $\mathbb{C} \otimes F_b$. We call this 1-mode massive boson the ‘heavy boson.’

We arbitrarily give a positive parameter $\omega$, a non-negative parameter $C$, and a positive constant $g$ such that $\omega^2 g = \omega^2 + 4C \omega g^2$. We consider another Hamiltonian $H_{ss}$ for the position operator $x$ and the momentum operator $p$ acting in $\mathbb{C} \otimes F_b$. The Hamiltonian $H_{ss}$ is popular in supersymmetric quantum mechanics (SUSYQM) [20] and given by

$$H_{ss} = \frac{1}{2} \left( p^2 + W^2 + \hbar \sigma_z \frac{dW}{dx} \right),$$  

(2)

where $W$ is the superpotential given by $W(x) = \omega x$.

Our spin-boson interaction is based on $\sigma_x x$. We suppose that an extra second-order term $(\sigma_x x)^2 = x^2$, different from the second-order term by $W^2$ in Eq.(2), appears as well as the first-order term $\sigma_x x$. We can meet that second-order term when we consider the minimal coupling for the interaction of an atom and the light field in electrodynamics. Then, the extra second-order term is called $A^2$-term [21], where $A$ denotes the magnetic vector potential, one of gauge fields. Representing $x$ with the photon annihilation and creation operators, then we can regard $x$ as the 1-mode magnetic vector potential (see below in this section). Thus, we prepare an interaction,

$$H_{\text{int}}(r) = g(r) \sqrt{\frac{2\hbar}{\omega}} \sigma_x W + \frac{2C}{\omega} g(r)^2 W^2 + \frac{h g(r)^2}{4C g(r)^2 + \omega} + \frac{1}{2} \hbar \sigma_z \frac{dW_a(r)}{dx}, \quad 0 \leq r \leq 1,$$  

(3)

for some functions, $g(r), W_a(r) = (\omega_a(r) - \omega)x, \omega_a(r)$, of $r$. This interaction $H_{\text{int}}(r)$ is introduced to cause a spontaneous SUSY breaking for the SUSY Hamiltonian $H_{ss}$. Unlike Nambu and Jona-Lasinio’s case [22] and Goldstone’s [23], the interaction $H_{\text{int}}(r)$ does not include the so-called Mexican-hat potential in our case. Thus, we expect that the $A^2$-term in $H_{\text{int}}(r)$ to play a role of the mass enhancement instead of the effect coming from the fourth-order term. We here note that the $A^2$-term naturally appears in quantum electrodynamics (QED) and cavity QED, and moreover, it may be controlled in circuit QED (see Ref. [24] and Methods of Ref. [25]). Our total Hamiltonian reads $H(r) = H_{ss} + H_{\text{int}}(r)$ then. We control the interaction appearance using the functions $g(r)$ and $\omega_a(r)$, where $g(r)$ is a continuous function satisfying $g(0) = 0$ and $g(1) = g$, and $\omega_a(r)$ is also a continuous function satisfying $\omega_a(0) = \omega$ and $\omega_a(1) = 0$. Then, the total Hamiltonian attains the SUSY Hamiltonian at $r = 0$: $H(0) = H_{ss}$.

We bring up the parameter $r$ from $r = 0$ to $r = 1$ in the total Hamiltonian $H(r)$. Following the mathematical methods [14, 26–28], we can show $H(r) \to H(1)$ as $r \to 1$ in
the norm resolvent sense [29]. As shown below, actually, \( H(1) = H \). In the case \( C = 0 \), it can mathematically be proved that this limit produces the transition from the \( N = 2 \) SUSY at \( r = 0 \) to its spontaneous breaking at \( r = 1 \) in the same way as in Ref. [14] and [19]. Cai \textit{et al.} report its two kinds of experimental observations in a trapped ion quantum simulator [19]. The condition \( C = 0 \) means that there is no mass-enhancement term in the interaction \( H_{\text{int}}(r) \), and there is no possibility that the spontaneous SUSY breaking can yields a mass enhancement [14, 28]. In the case \( C > 0 \), however, that possibility is pointed out [28].

We check this proposal in this paper. Thus, we allocate the mass-enhancement role to the second-order term, \( 2C\omega g(r)^2x^2 \) with \( C > 0 \), in our model, and theoretically show that for \( C > 0 \) the mass enhancement takes place in the process of the transition from \( N = 2 \) SUSY to its spontaneous breaking.

According to the proposal [28], we consider the limit, \( H(r) \rightarrow H(1) \) as \( r \rightarrow 1 \). Defining the 1-mode boson annihilation operator \( B \) by \( B = \sqrt{\frac{\omega}{2\hbar}} X + i\sqrt{\frac{1}{2\hbar\omega g}} P \), the Hamiltonian \( H \) of the heavy boson can be rewritten as \( H = \hbar \omega g \left( B^\dagger B + \frac{1}{2} \right) \). Meanwhile, we define the 1-mode boson annihilation operator \( b \) by \( b = \sqrt{\frac{\omega}{2\hbar}} x + i\sqrt{\frac{1}{2\hbar\omega}} p \). We call this 1-mode massive boson the ‘light boson’ compared with the heavy boson. Then, we can rewrite the total Hamiltonian \( H(r) \) of the light boson as \( H(r) = H_{\text{Rabi}}(r) + \hbar C g(r)^2(b + b^\dagger)^2 + \frac{\hbar g(r)^2}{4C g(r)^2 + \omega} \), where \( H_{\text{Rabi}}(r) \) is the Hamiltonian of the quantum Rabi model [30] given by \( H_{\text{Rabi}}(r) = \hbar \omega (b^\dagger b + \frac{1}{2}) + h g(r) (b + b^\dagger) + \frac{\hbar \omega g(r)}{2} \sigma_z \).

Following the results in Refs. [26, 28], there is a unitary operator \( U_1 \) such that \( B = U_1 b U_1^\dagger \) and \( U_1 \sigma_\pm \exp \left( \pm 2\sqrt{\frac{\omega}{\omega_g}} (b^\dagger - b) \right) U_1^* = -\frac{1}{2} (\sigma_z \mp i\sigma_y) \), where \( \sqrt{\frac{\omega}{\omega_g}} = g \sqrt{\frac{\omega}{\omega g}} \). We extend this unitary operator \( U_1 \) to the unitary operator \( U_r \) for \( 0 \leq r \leq 1 \). For every \( r, 0 \leq r \leq 1 \), we obtain a unitary operator \( U_r \), and define a boson annihilation operator \( B_r \) and the spin operators \( D_\pm \) such that

\[
B_r = U_r b U_r^* = (c_1 + c_2) b + (c_1 - c_2) b^\dagger + \frac{\bar{g}(r)}{\omega_g(r)} \sigma_x , \tag{4}
\]

\[
D_\pm = U_r \sigma_\pm \exp \left( \pm 2\sqrt{\frac{\omega}{\omega_g}} (b^\dagger - b) \right) U_r^* = -\frac{1}{2} (\sigma_z \mp i\sigma_y) , \tag{5}
\]

where \( c_1 = \frac{1}{2} \sqrt{\frac{\omega_g(r)}{\omega}}, \ c_2 = \frac{1}{2} \sqrt{\frac{\omega_g(r)}{\omega}} \), \( \omega_g(r) = \sqrt{\omega^2 + 4C \omega g(r)^2} \), and \( \bar{g}(r) = 2g(r)c_2 \). Then, of course, we have \( B_1 = B \). We note the canonical commutation relation , \( 1 = [B_r, B_r^\dagger] = [b, b^\dagger] \), canonical anticommutation relation, \( \{D_-, D_+\} = 1 \), and \( \{D_\pm, D_\pm\} = 0 \). In addition, we realize the spin-chiral symmetry, \( [\sigma_x, B_r] = [\sigma_x, B_r^\dagger] = 0 \). Eq.(4) says that the boson
described by $B_r$ and $B_r^\dagger$ consists of the pair of the annihilation and creation of the light boson with the spin-chiral transformation. This pair is produced following the (meson-)pair theory [26, 27, 31]. In particular, since $B_1 = B$, the heavy boson is a quasi-particle of the light boson. Eq. (5) says that the heavy boson cannot see the displacement by the light boson in the spin. Then, we have the equation between the Hamiltonian described by the light boson coupled with the spin and the Hamiltonian described by the heavy boson coupled with the spin,

$$
\hbar \omega_g(r) \left( B_r^\dagger B_r + \frac{1}{2} \right) - \frac{\hbar \omega_a(r)}{2} (D_- + D_+) = H(r).
$$

(6)

How we can construct the unitary operator $U_r$, $0 \leq r \leq 1$, is explained in Refs. [14, 26, 28].

We have $\omega_a(1) = 0$, and $\omega_g(1) = \omega_g$ because $g(1) = g$. Thus, we obtain the limit

$$
H(r) = H_{\text{Itab}}(r) + \hbar C g(r)^2 (b + b^\dagger)^2 + \frac{\hbar g(r)^2}{4C g(r)^2 + \omega} \to H = \hbar \omega_g \left( B^\dagger B + \frac{1}{2} \right)
$$

as $r \to 1$. This limit can be mathematically established in the norm resolvent sense [29] as shown in Refs. [14, 26, and 27]. Therefore, the transition from $H(0) = H_{\text{ss}}$ to $H(1) = H$ is obtained by changing $r$ from $r = 0$ to $r = 1$.

### III. MASS-ENHANCEMENT IN SPONTANEOUS SUSY BREAKING

We introduce the 1-mode Bose field $\Phi_r$ by $\Phi_r = \sqrt{\frac{\hbar}{2\omega_g(r)}} (B_r + B_r^\dagger)$ and its conjugate field $\Pi_r$ by $\Pi_r = -i \sqrt{\frac{\hbar \omega_g(r)}{2}} (B_r - B_r^\dagger)$ for $0 \leq r \leq 1$. Then, we have $[\Phi_r, \Pi_r] = i\hbar$. The Lagrangian $L_r$ corresponding to $H(r)$ is given by $L_r = \frac{1}{2} \Pi_r^2 - \frac{\omega_a(r)^2}{2} \Phi_r^2 + \frac{\hbar \omega_a(r)^2}{2} (D_- + D_+)$. Now, we introduce an auxiliary field $\phi$ and its conjugate field $\pi$ by $\phi = \sqrt{\frac{\hbar}{2\omega}} (b + b^\dagger)$ and $\pi = -i \sqrt{\frac{\hbar \omega}{2}} (b - b^\dagger)$. Taking the limit $r \to 1$, we have $L_r \to L_1$, and $L_1 = \frac{1}{2} \Pi_1^2 - \frac{\omega_g^2}{2} \Phi_1^2$, which is the Lagrangian corresponding to the Hamiltonian $H$ since $B = B_1$ and $B^\dagger = B_1^\dagger$.

Inserting Eq. (1) into $L_r$, we obtain

$$
L_r = \frac{1}{2} \pi^2 - \phi^2 - g(r) \sqrt{2\hbar \omega} \sigma_z \phi - 2C \omega g(r)^2 \phi^2 - \frac{\hbar g(r)^2}{4C g(r)^2 + \omega} - \frac{\hbar \omega_a(r)}{2} \sigma_z.
$$

$$
\longrightarrow L_1 = 1 \otimes \left\{ \frac{1}{2} \pi^2 - \frac{\omega_g^2}{2} \phi^2 - g \sqrt{2\hbar \omega} \sigma_z \phi - \frac{\hbar g^2}{4C g^2 + \omega}\right\}.
$$

Meanwhile, the free Lagrangian of the light boson coupled with spin is $1 \otimes \left\{ \frac{1}{2} \pi^2 - \frac{\omega_g^2}{2} \phi \right\}$. Thus, the increment of the mass enhancement is included in the factor, $4C \omega g^2$, in $\omega_g$. 

6
Considering the dimension, the mass increment $\Delta m$ is given by $\omega_g = \sqrt{\omega^2 + \frac{(\Delta m)^2}{\hbar^2}}$. We here note that the Lagrangian $L_1$ has the spin-chiral symmetry, $[\sigma_x, L_1] = 0$, though the Lagrangian $L_r$ does not have it, $[\sigma_x, L_r] \neq 0$, for $0 \leq r < 1$ because of the existence of the spin term, $-\frac{\hbar\omega_{r}(r)}{2}\sigma_z$.

As shown in the previous section, the transition from the Hamiltonian $H(0) = H_{ss}$ of the light boson to the Hamiltonian $H(1) = H$ of the heavy boson is obtained:

\[
H(0) = H_{ss} = \hbar\omega \left( b^\dagger b + \frac{1}{2} \right) + \frac{\hbar\omega}{2}\sigma_z \iff H(1) = H = \hbar\omega_g \left( B^\dagger B + \frac{1}{2} \right),
\]

where we omit the $2 \times 2$ identity matrix $1$. It is well known that $H(0) = H_{ss}$ has the following SUSY structure. Its real supercharges, $q_1$ and $q_2$, are given by $q_1 = \frac{1}{2} (W\sigma_x - p\sigma_y) = \sqrt{\frac{\hbar\omega}{2}} (\sigma_+ b + \sigma_- b^\dagger)$ and $q_2 = \frac{1}{2} (W\sigma_y + p\sigma_x) = i\sqrt{\frac{\hbar\omega}{2}} (\sigma_- b^\dagger - \sigma_+ b)$. Then, they satisfy $\{q_k, q_l\} = \delta_{kl}H(0)$, $[q_k, H(0)] = 0$, and $\{q_k, N_F\} = 0$, where $N_F$ is the grading operator defined by $N_F = \sigma_z$. The complex supercharges, $q^+$ and $q^-$, are given by $q^+ = \frac{1}{\sqrt{2}} (q_1 + iq_2) = \sqrt{\hbar\omega} \sigma_x b$ and $q^- = \frac{1}{\sqrt{2}} (q_1 - iq_2) = \sqrt{\hbar\omega} \sigma_x b^\dagger$ such that $H(0) = \{q^+, q^\dagger\}$, $\{q^+, q^\pm\} = 0$, and $[H(0), q^\pm] = 0$. These complex supercharges make the connection between fermion and boson: $q^- | \downarrow \rangle \otimes | n \rangle = q^+ | \uparrow \rangle \otimes | n \rangle = 0$, $| \uparrow \rangle \otimes | n \rangle = \frac{1}{\sqrt{(n+1)\hbar\omega}} q^+ | \downarrow \rangle \otimes | n + 1 \rangle$, $| \downarrow \rangle \otimes | n + 1 \rangle = \frac{1}{\sqrt{(n+1)\hbar\omega}} q^- | \uparrow \rangle \otimes | n \rangle$. Meanwhile, the structure for the spontaneous SUSY breaking of $H(1) = H$ is determined in the following. Its real supercharges, $Q_1$ and $Q_2$, are given by $Q_1 = \sqrt{\frac{\hbar\omega}{2}} B^\dagger B + \frac{1}{2} \sigma_x$ and $Q_2 = \sqrt{\frac{\hbar\omega}{2}} B^\dagger B + \frac{1}{2} \sigma_y$. Then, they satisfy $\{Q_k, Q_l\} = \delta_{kl}H(1)$, $[Q_k, H(1)] = 0$, and $\{Q_k, N_F\} = 0$, where $N_F$ is the grading operator defined by $N_F = \sigma_z$. The complex supercharges, $Q^+$ and $Q^-$, are given by $Q^\pm = \frac{1}{\sqrt{2}} (Q_1 \pm iQ_2) = \sqrt{\hbar\omega_g} (B^\dagger B + \frac{1}{2}) \sigma_\pm$ such that $H(1) = \{Q^+, Q^-\}$, $\{Q^\pm, Q^\mp\} = 0$, and $[H(1), Q^\pm] = 0$. These complex supercharges have the relations, $Q^- | \downarrow \rangle \otimes | n \rangle = Q^+ | \uparrow \rangle \otimes | n \rangle = 0$. However, they do cut the connection with the boson annihilation and creation as $| \uparrow \rangle \otimes | n \rangle = \frac{1}{\sqrt{(n+ \frac{1}{2})\hbar\omega}} Q^+ | \downarrow \rangle \otimes | n \rangle$, $| \downarrow \rangle \otimes | n \rangle = \frac{1}{\sqrt{(n+ \frac{1}{2})\hbar\omega}} Q^- | \uparrow \rangle \otimes | n \rangle$, and then the SUSY is spontaneously broken. Therefore, Eq.(7) says that the transition brings the $N = 2$ SUSY Hamiltonian $H(0)$ to the Hamiltonian $H(1)$ having its spontaneous breaking, and the transition yields the mass enhancement with the increment $\Delta m = 2\sqrt{\omega\omega_g}$, determined by $\omega_g^2 = \omega^2 + \frac{(\Delta m)^2}{\hbar^2}$ coming from the increment of the mass term, $-\frac{\hbar\omega_{r}(r)}{2}\sigma_z$.

This spontaneous SUSY breaking induces the spontaneous breaking with respect to the spin-chiral symmetry; namely, $H(1)$ satisfies $[\sigma_x, H(1)] = 0$ and has the two degenerate ground states. Actually, the ground states are $| \uparrow \rangle \otimes | 0 \rangle$ and $| \downarrow \rangle \otimes | 0 \rangle$. Combining Eq.(2)
of Ref. [28] and the result of Ref. [32], we know that $H(r)$ has a unique ground state for $0 \leq r < 1$, and then, $[\sigma_x, H(r)] = -i\hbar \omega \sigma_y$, $0 \leq r < 1$.

Since each energy level of $H(r)$ is guaranteed for its convergence as $r \to 1$ by the limit in the norm resolvent sense (see Theorem VIII.24 of [29]), we are interested in the energy spectrum of $H(r)$ for every $r$, $0 \leq r \leq 1$. Fig. 1 shows its four examples by numerical analysis with QuTiP [33, 34].

(a) 

(b) 

(c) 

(d) 

FIG. 1. Mass Enhancement in Energy Spectrum of $H(r)$ with $\omega = 6.2832$ and $g = 6.2832$: A ground state energy and seven excited state energies from the bottom are shown in the graph. The parts (a), (b), (c), and (d) show the energy spectrum for $C = 0$, $C = 0.063$, $C = 0.314$, and $C = 0.628$, respectively. In these numerical calculations, $\omega_a(r) = (1 - r)\omega$ is employed.

IV. CONCLUSION

We have proposed a mathematical model, though very simple, for quantum simulation of a mass enhancement in spontaneous SUSY breaking. This model shows a transition from
the $N = 2$ SUSY to its spontaneous breaking. The transition is described in terms of the quantum Rabi model with $A^2$-term, and obtained by the change in its Hamiltonian $H(r)$ from $r = 0$ to $r = 1$. In the case without the $A^2$-term, that transition is experimentally observed in a trapped ion quantum simulator by Cai et al. [19]. Thus, a future experimental problem would be whether $A^2$-term can be added to their experimental set-ups in a quantum simulator, and a similar experimental observation of the energy spectrum can be performed.

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