Abstract

I summarize the theory of acceleration of non-neutral particle beams by starvation electric fields along the polar magnetic field lines of rotation powered pulsars, including the effect of dragging of inertial frames which dominates the acceleration of a space charge limited beam. I apply these acceleration results to a new calculation of the radio pulsar death line, under the hypotheses that pulsar “death” corresponds to cessation of pair creation over the magnetic poles and that the magnetic field has a locally dipolar topology. While the frame dragging effect in star centered dipole geometry does improve comparison of the theory with observation, an unacceptably large fraction of the observed stars outside the bounds of pair creation theory still persists. Offsetting the dipole improves the correspondence between theory and observation. The result is a “death valley” for pulsars; acceptable comparison of observation and theory occurs if the boundary of death valley corresponds to offsets of the dipole center from the stellar center $\sim (0.7 - 0.8)R_\ast$. I also point out that pulsars are absent for magnetic moments corresponding to star centered polar fields in excess of $\sim 4 \times 10^{13}$ Gauss, and I suggest that this absence is due to pairs forming as bound positronium atoms in such strong fields, creating a neutral, relativistically outflowing gas which cannot participate in low altitude collective radio emission processes in such strongly magnetized objects.

1. Introduction

Most of the scientific community which has an interest in the physics of neutron stars believes that radio emission from Rotation Powered Pulsars (RPPs)
has its origin in the relativistic outflow of electron-positron pairs along the polar magnetic field lines of a dipole magnetic field frozen into the rotating neutron star (e.g., Arons 1992, Meszaros 1992).

The evidence for dipole magnetic fields in RPPs (and in any other neutron star) is indirect, coming primarily from the electromagnetic theory of RPP spindown. The observed increasing pulse periods are readily explained using standard theoretical moments of inertia plus order of magnitude estimates, derivable from dimensional analysis (Dyson 1971, Arons 1979, 1992), of rotational energy loss driven by relativistic electromagnetic spindown torques,

\[ \dot{E}_R = k \frac{\mu^2 \Omega^4}{c^3} = -I \dot{\Omega} \Omega. \]  (1)

Here \( \mu \) is the magnetic moment, \( \Omega \) is the stellar angular velocity with respect to inertial space far from the star, and \( k \) is a function of any other parameters of significance, with magnitude on the order of unity. In the vacuum theory (Deutsch 1955), \( k = (2/3) \sin^2 \theta \), with \( \theta \) the angle between the magnetic moment and the angular velocity. Theoretical work on the torques due to conduction currents stemming back to Goldreich and Julian (1969), coupled to the observation that spindown rates appear to be independent of observationally estimated values of \( \theta \) (Lyne and Manchester 1988), suggest that in reality \( k \) does not substantially depend on \( \theta \). In the subsequent discussion, I assume \( k = 4/9 \), the average of the vacuum value over the sphere. Application of (1) to the observations of RPPs’ periods (\( P = 2\pi/\Omega \)) and period derivatives (\( \dot{P} = -2\pi \dot{\Omega}/\Omega^2 \)) yields \( \mu \sim 10^{30} \) cgs for “normal” RPPs, and \( \mu \sim 10^{27} \) cgs for millisecond RPPs. These results are reasonably firm, the main uncertainty coming from the derived values of \( \mu \) being proportional to \( k^{-1/2} \).

The electromagnetic torque interpretation of pulsar spindown constrains only the exterior dipole moment of the magnetic field. However, not long ago Rankin (1990) presented strong evidence in favor of a low altitude (\( r \approx R_* \)) dipole geometry for the site of the core component of pulsar radio emission. Arons (1993) gave evidence that spun up millisecond pulsars must have a substantially dipolar large scale field at low altitude.

Electron-positron pair creation at low altitude above the polar caps has long been hypothesized to be an essential ingredient of pulsar radio emission, starting with Sturrock’s (1971) pioneering work. If so, all observed pulsars must lie in the region of \( P - \dot{P} \) space where polar cap acceleration has sufficient vigor to lead to copious pair production. Yet, to date, all internally consistent theories of polar cap pair creation have required hypothesizing a large scale (e.g., quadrupole) component of the magnetic field with strength comparable to that of the dipole (Ruderman and Sutherland 1975, Arons and Scharlemann 1979, Barnard and
Arons 1982, Gurevich and Istomin 1985). These non-dipole components were invoked in order to increase the opacity of the magnetic field to pair creating gamma rays. Non-dipole low altitude fields can have magnetic radii of curvature on the order of $R_*$ or less, a factor of 50-100 smaller than the radii of curvature of star centered dipole field lines near the magnetic poles. The resulting increase of optical depth allowed the pair creation models to cover the whole $P, \dot{P}$ diagram. However, such strong magnetic anomalies contradict the evidence in favor of an apparently dipolar low altitude geometry; the alteration of the magnetic geometry also ruins the internal consistency of many models’ electrodynamics.

Both early (Sturrock 1971) and more recent work on polar cap electrodynamics and its implications for the occurrence of pair creation in $P, \dot{P}$ space either employ incomplete (e.g. Sturner et al. 1995) or erroneous (Sturrock 1971, Mestel and Shibata 1994, Bjornsson 1996) theories of polar cap particle acceleration. Most of the internally consistent theories also violate other observational constraints, especially with regard to polar cap heating (Arons 1992), which creates pulsed thermal X-ray emission from hot spots in excess of what is seen (Becker and Trümper 1997, Pavlov and Zavlin 1997). While the Arons and Scharlemann (1979) model does not have this problem, in star centered dipole geometry it dramatically fails to account for pulsar emission over most of the $P - \dot{P}$ diagram and predicts radio polarization variations in contradiction to the observations (Narayan and Vivekanand 1982).

Here I describe a low altitude polar cap acceleration theory which successfully associates pulsar “death” with the cessation of pair creation in an offset dipolar low altitude magnetic field. The basic acceleration physics is that of a space charge limited relativistic particle beam accelerated along the field lines by the starvation electric field, as in the Arons and Scharlemann theory, but with the additional effect of inertial frame dragging, first pointed out by Muslimov and Tsygan (1990, 1992) and by Beskin (1990).

This effect causes the accelerating electric field to be about an order of magnitude larger than that calculated by Arons and Scharlemann for pulsars near the death line, which substantially improves the size of the region in $P, \dot{P}$ space in which polar cap pair creation occurs, but still does not allow the theory to fully account for the observed pulsar distribution, in star centered dipole geometry. If the dipole’s center is offset from the stellar center along a vector parallel to the dipole moment itself, an offset which automatically preserves the symmetries built into the highly successful Radhakrishnan and Cooke (1969) model of polarization swings, the magnetic field at one pole becomes substantially stronger than it would be if the same magnetic dipole were star centered. If the offset is substantial (as much as 80% of the stellar radius turns out to be required), all pulsars can be
accommodated within a single pair creation theory. The location of an individual pulsar’s pair creation death depends on the magnitude of the offset, thus yielding a “death valley” (Chen and Ruderman 1993) for the whole pulsar population.

2. Polar Acceleration

Prior to the work of Muslimov and Tsygan and of Beskin, study of polar cap relativistic particle acceleration in the 1970’s had led to the conclusion that acceleration of a space charge limited particle beam from the stellar surface with energy/particle high enough to emit magnetically convertible curvature gamma rays occurs because of curvature of the magnetic field (Scharlemann et al. 1978, Arons and Scharlemann 1979). With field line curvature, matching of the beam density to $\eta_{\text{R}}$ occurs only at the surface. Along field lines which curve toward the rotation axis (“favorably curved” field lines, $|\eta_{\text{beam}}/\eta_{\text{R}}| < 1$), the beam fails to short out the vacuum, with $|(\eta_{\text{beam}} - \eta_{\text{R}})/\eta_{\text{R}}| \sim R_{\ast}/\rho_{B}$. $R_{\ast} = 10R_{10}$ km is the stellar radius and $\rho_{B}$ is the radius of curvature of the magnetic field lines. Therefore, particles accelerate along $B$ through a potential drop

$$\Delta \Phi_{\parallel} \approx \Phi_{\text{pole}} \left( \frac{R_{\ast}}{\rho_{B}} \right) \sim 10^{-2}P^{-1/2}\Phi_{\text{pole}},$$

where $P$ is the rotation period in seconds, and the numerical value assumes field lines have dipolar radius of curvature. Here

$$\Phi_{\text{pole}} \equiv \frac{Q_{\parallel}^{2} \mu}{c^{2}} = 1.09 \times 10^{13} \left( \frac{I_{45}}{k} \right)^{1/2} \left( \frac{\dot{P}_{15}}{P_{3}} \right)^{1/2} \text{Volts},$$

with $\dot{P}_{15} \equiv \dot{P}/10^{-15}$ s/s and $I_{45} = I/10^{45}$ g-cm$^{2}$. Particles drop through the potential (2) over a length $L_{\parallel} \sim R_{\ast}$ (an electric field of magnitude $\sim 10^{7} - 10^{8}$ Volts/meter, for normal and millisecond pulsars). Note that $\Phi_{\text{pole}}$ is proportional to the magnetic flux contained in the tube of open field lines. Therefore, the total potential of a pole is independent of the magnetic topology, if, and only if, the open field lines map onto a single, more or less round polar cap.

Curvature gamma rays have typical energy $\varepsilon_{c} \sim (hc/\rho_{B})(e\Delta \Phi_{\parallel}/mc^{2})^{2} \propto \Phi_{\text{pole}}^{2}/\rho_{B}^{2}$, while the optical depth for pair creation, due to one photon conversion of gamma rays emitted by electrons (or positrons) accelerating through the unshorted potential (3), can be shown to be (Arons and Scharlemann 1979, Luo 1996, Bjornsson 1996)

$$\tau = \Lambda \exp[-a(mc^{2}/\varepsilon_{c})(B_{q}/B_{\ast})(\rho_{B}/R_{\ast})],$$

where $a$ is a pure number (typically $\sim 30$) and $\Lambda$ is a combination of the basic parameters which is quite large ($\ln \Lambda \sim 20$). Pair creation does not short out the acceleration if $\tau \ll 1$; thus, if pairs are important for radio emission, a reasonable theoretical definition
of the death line is $\tau = 1$. A more precise definition requires estimating the number of charges that must be added to restore the total charge density to $\eta_R$ and reduce $E_\parallel$ to zero. This refinement is included in the results shown in Figures 3 and 4. Using $B_\ast = 2(\Phi_{\text{pole}}/R_\ast)(c/\Omega_\ast R_\ast)^2$, the potential (2) and setting $\tau$ equal to unity yields the death line, expressed as $\Phi_{\text{death}}(P)$ such that stars $\Phi_{\text{pole}} < \Phi_{\text{death}}$ do not make pairs. This death line, first found by Arons and Scharlemann (1979), appears as the dashed line in Figure 1, when $\rho_B$ assumes the star centered dipole value $\sim (R_\ast c/\Omega_\ast)^{1/2}$.

The expression for $\tau$ shows that $\Phi_{\text{death}} \propto \rho_B^{5/4}$. Arons and Scharlemann argued, following Ruderman and Sutherland (1975), that if magnetic anomalies reduced $\rho_B$ to be on the order of $R_\ast$, better agreement with the cessation of radio emission might be achieved, for “normal” pulsars, almost all of which have periods between 0.1 and 1 second. Figure 1 shows clearly that the large dynamic range in $\Phi_{\text{pole}}, P$ space made available by the cataloging of millisecond pulsars falsifies even this “fudged” version of the theory - the scaling with period, $\Phi_{\text{death}} \propto P^{3/8}$, flatly disagrees with the shape of the boundary of pulsar radio emission in the $\Phi_{\text{pole}}, P$ diagram. When combined with the more recent arguments in favor of dipolar topology for the low altitude magnetic field, I drew the conclusion either that something else governs the low altitude acceleration which leads to pair creation, or that pair creation is not important to radio emission.

Muslimov and Tsygan (1990, 1992) revivified this subject by uncovering a previously overlooked effect on the acceleration of the non-neutral beam from the stellar surface. Stellar rotation drags the inertial frame into rotation, at the angular velocity $\omega_{\text{LT}} = (2GI/R_\ast^3c^2)\Omega_\ast (R_\ast/r)^3$, where I is the moment of inertia. Therefore, the electric field required to bring a charged particle into corotation is $E_{\text{co}} = -(1/c)[(\Omega_\ast - \omega_{\text{LT}}) \times \mathbf{r}] \times \mathbf{B}$; the rotation of the magnetic field with respect to local inertial space, not inertial space at infinity, determines the electric field which in turn sets a charged particle’s $E \times B$ drift velocity. The charge density required to support this local corotation electric field therefore is

$$\eta_R = -\frac{(\Omega_\ast - \omega_{\text{LT}}) \cdot \mathbf{B}}{2\pi c} = -\frac{\Omega_\ast \cdot \mathbf{B}}{2\pi c} \left[ 1 - \kappa_g \left( \frac{R_\ast}{r^3} \right) \right],$$

where $\kappa_g = 2GI/R_\ast^3c^2 = 0.17(I_{45}/R_{10}^3)$. Relativistic space charge limited flow from the surface has a beam charge density $\eta_b = -(\Omega_\ast \cdot \mathbf{B}_s/2\pi c)(1 - \kappa_g)(B/B_\ast)$. Above the surface, this charge density is too small to short out $E_\parallel$ on all polar field lines, not just the favorably curved part of a polar flux tube, thus restoring the possibility of polar cap acceleration models being in accord with the observed rough symmetry of radio emission with respect to the magnetic axis (e.g., Lyne and Manchester 1988). One can graphically describe this general relativistic origin of electrical starvation simply as the consequence of the field lines rotating faster
with respect to inertial space as the radius increases, at the angular speed $\Omega_s - \omega_{LT}(r) = \Omega_s [1 - \kappa_g (R_s/r)^3]$. The constraint of relativistic flow along $B$ allows the beam to provide only a charge density sufficient to support corotation at the angular speed $\Omega_s (1 - \kappa_g)$. The difference not surprisingly leads to an accelerating potential drop

$$\Delta \Phi \parallel \approx \kappa_g \Phi_{pole} [1 - (R_s/r)^3].$$

(5)

For normal pulsars with dipole fields, $\kappa_g \sim 10 R_s/\rho_B \approx 10 P^{-1/2}$, so that the effect of dragging of inertial frames on the beam’s acceleration can yield curvature gamma ray energies 1000 times greater than occur in the Arons and Scharlemann pair creation theory, for normal pulsars; for MSPs, the theories yield comparable results, although of course the symmetry of the beam with respect to the magnetic axis differs. Expression (5) applies to the polar flux tube at altitudes greater than the width of the polar flux tube; closer to the surface, fringe fields cause the potential to be smaller.

3. Death Lines and Death Valley

When curvature emission is the only source of gamma rays, and curvature emission does not limit an accelerating particle’s energy, the condition that the number of pairs created be just such as to reduce both $E_\parallel$ and $\nabla \cdot E_\parallel$ to zero (which is almost the same as the optical depth unity condition) yields the death line for a star centered dipole (Arons, in preparation):

$$\Phi_{death} = 1.9 \times 10^{13} \left( \frac{P_{10}^4}{I_{45} \cos i} \right)^{3/4} P^{-1/4} \text{ Volts.}$$

(6)

This result appears in Figure [1]. The effects of curvature radiation reaction, important at short periods, are also included.

Dragging of inertial frames clearly improves the agreement between the boundary of pair activity in the $\Phi - P$ diagram and the region where pulsars occur, but the discrepancy is still too large - something else is missing. If the field geometry must be locally dipolar at low altitude, then the only ingredients still not included are 1) offset of the dipole from the stellar center and 2) additional gamma ray emission and absorption processes.

I discuss only the simplest dipole offset here, namely, when the magnetic field is that of a point dipole, with the center of the dipole displaced from the stellar center by an offset vector $\delta$ parallel to $\mu$. This has the effect of increasing the magnetic field at one pole to strength $B_* = 2\mu/(R_* - \delta)^3$, with a resulting drastic increase in the gamma ray opacity, while leaving the accelerating potential unaltered. The result is the death valley shown in Figure [2]. Clearly, dipole offsets
Fig. 1. Pair creation Death Lines for Star Centered Dipoles. Solid line: Standard gamma ray emission and absorption in a star centered dipole, when the beam acceleration model incorporates the effect of inertial frame dragging. Dashed line: Same geometry, gamma ray and pair physics, but with inertial frame dragging neglected in the particle acceleration theory.
Fig. 2. Death Valley for offset dipoles, with magnetic moment \( \mu \) parallel to the offset vector \( \delta \), assuming no inverse Compton gamma rays and one photon magnetic pair creation. Radiation reaction significantly limits the particles’ energies at high voltages and short periods.

This estimate of death valley’s extent assumes curvature emission and magnetic conversion to be the only sources for gamma ray emission and absorption. ROSAT observations have revealed the long sought thermal X-rays from neutron star surfaces (Becker and Trümper 1997). Resonant Compton scattering creates magnetically convertible gamma rays at a spatial rate \((dN_\gamma/ds)_{CC} \propto T_*/\Gamma^2\) (e.g., Luo 1996) where \(T_*\) is the temperature of the cooling neutron star (polar cap heating is unimportant near the death line) and \(\Gamma = e\Phi/\hbar c \) is the Lorentz factor of an electron or positron in the beam. Compton scattering thus can become a significant source of gamma rays in stars with smaller accelerating potentials.

In contrast, the spatial rate of curvature emission, \((dN_\gamma/ds)_C \propto \Gamma/\rho_B\), shows that curvature emission dominates gamma ray emission for stars with large voltages. Compton scattering thus may contribute significantly for stars with low overall voltage, just where the theory based solely on curvature emission encounters the most trouble in accounting for the data.
Fig. 3. Death Valley for an offset dipole parallel to the offset vector, with both curvature and resonant inverse Compton emission and stellar temperature kept high by internal heating.

Indeed, this expectation is correct, if internal heating (e.g., Umeda et al. 1993) keeps the surface temperature above $10^5$ K to spindown ages in excess of $10^7.5$ years. In this case, resonant Compton scattering of thermal photons by a polar electron beam does extend death valley to include all the observed pulsars, with somewhat less drastic offsets required, as is shown in Figure 3.

Satisfactory agreement with the observations occurs with the enhancement of polar acceleration discovered by Muslimov and Tsygan, but still requires introducing a special kind of magnetic anomaly (an offset dipole). This kind of anomaly, however, is consistent with the evidence adduced for low altitude dipolar structure in the magnetic fields of rotation powered pulsars. If temperatures decline slower than exponentially with age, resonant Compton scattering eases the magnitude of the required offset, and creates a gap between the observed edge of the pulsar distribution and the theoretical boundary of death valley, as is shown in Figure 4.

4. Positronium Creation in Strongly Magnetized Neutron Stars?

In addition to the edge of death valley shown in Figure 1, I have included a line of constant polar magnetic field equal to the “critical” field, $B_q = 4.4 \times 10^{13}$
Fig. 4. The complete $\Phi_{\text{pole}} - P$ diagram, including a not impossible boundary for death valley (offset $\delta = 0.8R_*$, plus inverse Compton emission with internal reheating controlling the surface temperature at ages greater than $10^6$ years), the usual Hubble and spin-up lines, and the apparent strong polar field boundary $B = 4.4 \times 10^{13}$ Gauss.
Gauss, corresponding to magnetic moments exceeding $\mu_q \sim 2 \times 10^{31}$ cgs. The apparent absence of pulsars with magnetic moments exceeding $\mu_q$ cannot simply be due to rapid spin down and therefore a paucity of observable objects because of this simple evolutionary effect - the empty region would have included stars with ages in excess of $10^5$ years, if such strong field objects exist. One solution (the conventional one) is to assume, for reasons unknown, that stars with magnetic moments in excess of $\mu_q$ simply never form. Another intriguing possibility is that pair creation is suppressed in such strong fields. Positronium formation (Usov and Melrose 1996) is a likely candidate (Arons 1996). Provided the bound pairs are not ionized close to the star (photoionization is the most likely possibility), the pairs form a neutral gas, not a quasi-neutral plasma, which would suppress the low altitude radio emission while still providing the outflow needed to power plerions which appear to be driven by strong field pulsars but don’t show any radiative sign of a central compact source (Helfand, Becker and White 1995). Photon splitting (Baring and Harding 1998) is another means of preventing photons from converting to pairs in strong fields, thus suppressing radio emission. However, as the photons with degraded energy propagate into regions of weaker field above the surface, they will convert to pairs as if the star had had a smaller magnetic moment in the first place.

5. Conclusion

I have shown that polar pair creation based on acceleration of a steadily flowing, space charged limited non-neutral beam in a locally dipolar magnetic geometry at low altitude is consistent with pulsar radio emission throughout the $P - \dot{P}$ diagram, provided 1) the effect of dragging of inertial frames is included in estimates of the starvation electric field; 2) the dipole center is strongly offset from the stellar center, perhaps as much as $0.7 - 0.8R_*$; and 3) inverse Compton emission of thermal photons from a neutron star cooling slower than exponentially at ages in excess of $10^6$ years plays an important role in the emission of magnetically convertible gamma rays. The development of new diagnostics of the low altitude magnetic field, and gamma ray observations sensitive to low altitude emission, will eventually provide tests of these ideas.

6. Acknowledgments

My research on pulsars is supported in part by NSF grant AST 9528271 and NASA grant NAG 5-3073, and in part by the generosity of California’s taxpayers.
7. References

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