Fully Nonlinear Analysis of the Interaction between Water and Free Floating Ship

Hongwei Ma1, Zhen Zhang1, Jiwei Wang2, and Xuan Wang1

1College of Civil Science and Engineering, Yangzhou University, Yangzhou, Jiangsu, 225127, P.R.China
2College of Civil and Transportation Engineering, Hohai University, Nanjing, Jiangsu, 210098, P.R.China
*Corresponding author’s e-mail: 1305908458@qq.com

Abstract. Based on the theory of velocity potential function, the fully nonlinear interaction between water and free floating simplified ship is studied through numerical model of fluid-solid coupling of ship-lift. The nonlinear numerical analysis of the coupling model of the water and the simplified ship body with different sizes, in which the boundary conditions included the free water surface boundary condition, the non-moving boundary condition and the water-solid coupling boundary condition. The numerical results show that the size change of the floating body affected the dynamic mechanical characteristics of the fluid-solid interaction of ship lift system, and affected the stress response of the floating simplified ship body.

1. Introduction
Ship-lift is an important building in hydroelectric junction, and ship and water often coexist in its chamber during the work period. There is interaction between water and the floating ship, and this fluid-solid coupling affects the mechanical property of the ship chamber. Over the years, a lot of studies on the interactions between fluid and solid have been carried out. Grilli et al.[1] and Bai et al.[2] investigated interaction between fluid and solid by using the higher-order boundary element method. Boo[3] analyzed interaction between fluid and solid through a time-domain higher-order boundary element scheme. Yang et al.[4] studied interaction between fluid and solid by adopting an embedded-boundary formulation. In this paper, We analyzed the interaction between fluid and free floating ship through 3-D finite element model under earthquake. In addition, the boundary conditions of ship chamber analysis model included free simplified ship surface, water boundary, and fluid-solid coupling boundary[5]. Numerical results of displacement response of different nodes on the free surface of fluid and floating ship are obtained in z-direction. And the stress responses of the ship body are also analyzed. Based on the analysis of numerical results of fluid-solid interaction under earthquake, some conclusions are given for the interactions between water and the 3-D floating simplified ship.

2. Mathematic analysis model
2.1. Laplace equation
The problem about fluid-ship interaction in the ship chamber can be represented by the velocity potential function \( u(x,y,z) \). Based on the principle of mass conservation, the velocity potential function \( u(x,y,z) \) must satisfy the Laplace equation\(^6,7\) in the fluid domain, which is second order partial differential equations, as follows,
\[
\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0
\]
where \( u \) represents scalar potential function, \((x,y,z)\) is the coordinate of a water point, and \( \nabla^2 \) is Laplacian operator.

2.2 Reynolds transport equation
Reynolds transport equation is a transformation relation between fluid particle system and control body. Reynolds transport equation\(^8,9\) was shown as following,
\[
\frac{d}{dt} \iiint_{CS} \sigma \rho \frac{\partial u}{\partial t} \, dV = \frac{\partial}{\partial t} \iiint_{CS} \sigma \rho \frac{\partial u}{\partial t} \, dA + \iiint_{CS} \sigma \vec{V} \cdot dA
\]
Where \( \sigma \) is unit fluid mass, \( CV \) is the volume of fluid particle system at t time, \( CS \) is the edge interface area of volume, \( \vec{V} \) is velocity vector of fluid particles, and \( \rho \) is fluid density.

2.3 Boundary conditions
The boundary conditions of free water surfaces \(^10,11\) can be expressed as follows,
\[
\frac{\partial \sigma}{\partial n_f} + \rho \frac{\partial u}{\partial t} \cdot n_f = 0
\]
\[
\sigma_y n_y = p n_i
\]
Where \( \sigma_f \) is fluid mass density, \( u \) is solid displacement vector, and \( n_f \) is fluid boundary unit outside normal vector.

2.4 Fluid dynamic equation
Based on the law of conservation of mass, the Newton's second law of motion, and the law of conservation of energy, momentum equation, energy equation, and continuity equation can be gotten. Based on these three equations, the ideal Euler equation\(^12,13,14,15\) for the fluid can be observed, which is shown as follows,
\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = R
\]
where
\[
Q = \begin{pmatrix}
\rho \\
p \\
u \\
p \\
\rho v \\
p w \\
\rho E
\end{pmatrix}, \quad F = \begin{pmatrix}
\rho u \\
\rho u^2 + p - \tau_{xx} \\
\rho u v - \tau_{xy} \\
\rho u w - \tau_{xz} \\
\dot{q}_x + (\rho E + p - \tau_{xx})u - \tau_{xz}w - \tau_{xy}v
\end{pmatrix}, \quad R = \begin{pmatrix}
0 \\
\rho F_x \\
\rho F_y \\
\rho F_z \\
\rho F_k v_k
\end{pmatrix}
\]
\[ G = \begin{pmatrix} \rho v \\ -\tau \nu x + \rho \nu v \\ -\tau _{yy} + \rho \nu ^2 + p \\ \rho \nu \nu w - \tau _{yy} \\ \dot{q}_y + (\rho \nu \nu + p - \tau _{yy})u - \tau _{yy} w - \tau _{yy} v \end{pmatrix}, \quad H = \begin{pmatrix} \rho w \\ -\tau \nu x + \rho \nu w \\ \rho \nu ^2 + p - \tau _{zz} \\ \rho \nu \nu w - \tau _{zy} \\ \dot{q}_z + (\rho \nu \nu + p - \tau _{zz})u - \tau _{zy} w - \tau _{zy} v \end{pmatrix}. \]

3. **Finite element equation of water-solid coupling**

In this paper, the water in ship chamber was simulated by potential fluid element. According to nonlinear water-solid interaction and fluid dynamic equations, the finite element equation of water-solid interaction[16] in ship chamber can be expressed as:

\[ \begin{bmatrix} M_{SS} \\ M_{FF} \end{bmatrix} \begin{bmatrix} \dot{U}_{SS} \\ \dot{U}_{FF} \end{bmatrix} + \begin{bmatrix} C_{SS} & C_{SF} \\ C_{FS} & C_{FF} \end{bmatrix} \begin{bmatrix} U_{SS} \\ U_{FF} \end{bmatrix} + \begin{bmatrix} K_{SS}+(K_{UU})_S \\ -K_{FF} \end{bmatrix} \begin{bmatrix} U_{SS} \\ U_{FF} \end{bmatrix} = \begin{bmatrix} F_{UB} \\ F_{SF} \end{bmatrix} \quad (6) \]

Where \( M_{SS}, C_{SS}, K_{SS}, U_{SS}, F_{SS} \) are matrix, damping, stiffness, displacement and load of mass solid respectively, \( M_{FF}, C_{FF}, K_{FF}, U_{FF}, F_{FF} \) are respectively water mass matrix, damping, stiffness, displacement and load, and \( (F_{UB})_S, (K_{UU})_S \) denotes stiffness matrix.

4. **Numerical results**

In this paper, the analysis model of a chamber of ship-lift was built by finite element method. The coupling model of ship chamber including water and simplified ship was investigated, as shown in figure 1. Meanwhile, four nodes were chose from the coupling model, whose purpose is for dynamic response analysis. The first node is in front of the ship body, the second node is behind the ship body, the third node is near the lock, and the fourth node is near the floating ship. The Taft seismic waveform was chose, shown in figure 2. Finally, the dynamic response of fluid-solid coupling models of ship chamber with different sizes was analyzed.

![Figure 1. fluid-structure finite element model](image1)

![Figure 2. TAFT seismic waveform](image2)

The z-displacement response curves of the four nodes of ship chamber are shown in figure 3. From these response curves, the displacement dynamic responses in z-direction were similar, but the amplitude of the small simplified ship body is obviously larger than that of the large simplified ship body with the increase of the positive side area. The wave amplitude decreases rapidly and the value of wave amplitude is largest because of wave diffraction in the ship chamber, as shown in figure 3(d). Therefore, the wave diffraction under earthquake has obvious affect the z-displacement dynamic response.
The stress dynamic responses of the floating ship body is also need to pay attention, which were shown in table 1, because the stress can reflect force condition of the floating ship body. The maximum and minimum stress of calculation results of the floating ship body could be seen in this table. The value of maximum and minimum stress in the floating ship body with different sizes are significantly different. Moreover, the maximum value of stress of the node 26491 is the largest, and the minimum value of stress of the node 1193 is the smallest, which is in the large floating ship body. And the maximum change rate of stress of the floating bodies is big. Therefore, the stress responses of the floating simplified ship bodies are very sensitive to the change of ship body sizes.

| time |   | small floating ship body |   | large floating ship body |
|------|---|--------------------------|---|--------------------------|
|      |   | maximum | node | minimum | node | maximum | node | minimum | node |
| 1    |   | 14430   | 14187 | 3783    | 3994 | 28442   | 26491 | 3769    | 3982 |
| 2    |   | 14158   | 13894 | 3819    | 4032 | 28057   | 26112 | 3828    | 4044 |
| 3    |   | 13938   | 13761 | 3799    | 4011 | 21322   | 19832 | 3800    | 4015 |
| 4    |   | 14318   | 14121 | 3766    | 3977 | 15086   | 14024 | 3747    | 3959 |
| 5    |   | 14409   | 14197 | 3741    | 3950 | 13894   | 13233 | 3678    | 3886 |
| 6    |   | 14353   | 14153 | 3741    | 3950 | 17965   | 17879 | 1124    | 1193 |
| 7    |   | 14257   | 14064 | 3742    | 3951 | 14354   | 14279 | 3718    | 3928 |
| 8    |   | 14175   | 13988 | 3745    | 3954 | 22218   | 21881 | 1214    | 1397 |
| 9    |   | 14107   | 13925 | 3750    | 3958 | 23609   | 23114 | 1564    | 1849 |
| 10   |   | 14228   | 13519 | 3739    | 3948 | 21713   | 21373 | 1284    | 1462 |
| 11   |   | 14371   | 13626 | 3734    | 3941 | 16687   | 16556 | 2271    | 2357 |
| 12   |   | 14549   | 13762 | 3729    | 3937 | 20424   | 18998 | 3783    | 3997 |
| 13   |   | 14269   | 14024 | 3754    | 3963 | 20430   | 19015 | 3754    | 3966 |
| 14   |   | 14255   | 14021 | 3720    | 3925 | 20430   | 19015 | 3754    | 3966 |
| 15   |   | 14047   | 13819 | 3715    | 3920 | 18044   | 16801 | 3774    | 3986 |

Figure 3. z-displacement dynamic responses of the four nodes
5. Conclusion
Through fully nonlinear analysis of the interaction between water and free floating simplified ship body with different sizes, the numerical results of the z-displacement dynamic responses and the stress dynamic responses of the free floating ship body under earthquake are obtained. The results show that the dimensional change of the floating ship body are more sensitive to the stress responses than to the z-displacement responses. In additional, the wave diffraction under earthquake has obvious affect the z-displacement dynamic response and can directly affect the dynamic mechanical performance of the whole ship lift system.

References
[1] Grilli S T, Guyenne P, Dias F. (2001) A fully nonlinear model for three-dimensional overturning waves over an arbitrary bottom. International Journal for Numerical Methods in Engineering 35, 829-867.
[2] W. Bai, R. Eatock Taylor. (2009) Fully nonlinear simulation of wave interaction with fixed and floating flared structures. Ocean Engineering 36, 223-236.
[3] Boo S.Y. (2002) Linear and nonlinear irregular waves and forces in a numerical wave tank. Ocean Engineering 29, 475-493.
[4] Yang J, Preidikman S, Balaras E. (2008) Astrongly coupled, embedded-boundary method for fluid-structure interactions of elastically mounted rigid bodies. Journal of Fluids and Structures 24, 167-182.
[5] Rugonyi S, Bathe K.J. (2001) On finite element analysis of fluid flows fully coupled with structural interactions. 2(2):195-212.
[6] Messelmi, F. (2017) An evolution infinity Laplace equation modelling dynamic elasto-plastic torsion. Analysis & Mathematical Physics, 7(4):1-11
[7] Kajikiya R, Tanaka M, Tanaka S. (2017) Bifurcation of positive solutions for the one-dimensional (p,q)-Laplace equation. Electronic Journal of Differential Equations, 2017.
[8] Hamba F. (2017) History effect on the Reynolds stress in turbulent swirling flow. Physics of Fluids, 29(2):537.
[9] Purkayastha S, Kumar B. (2018) Analytical solution of the one-dimensional contaminant transport equation in groundwater with time-varying boundary conditions. Ish Journal of Hydraulic Engineering, 4:1-6.
[10] Struchtrup H . (2017) Maxwell boundary condition and velocity dependent accommodation coefficient. Physics of Fluids, 7(2):219-251.
[11] Alberti G.S, Bal G, Cristo M.D. (2017) Critical Points for Elliptic Equations with Prescribed Boundary Conditions. Archive for Rational Mechanics & Analysis, 226(1):117-141.
[12] Ping Z, Zhang Z. (2010) On the free boundary problem of three-dimensional incompressible Euler equations. Communications on Pure & Applied Mathematics, 61(7):877-940.
[13] Song S.N, Fan K, Sciences S.O. (2017) Exact Solutions for IBq Equation with Fluid Dynamic Damping. Journal of Northeastern University, 38(10):1516-1520.
[14] Sun Y, Tian B, Xie X.Y. (2017) Rogue waves and lump solitons for a-dimensional B-type Kadomtsev–Petviashvili equation in fluid dynamics. Waves in Random & Complex Media, 28(3):1-9.
[15] Cyr J, Tang S, Temam R. (2018) The Euler equations of an inviscid incompressible fluid driven by a Lévy noise. Nonlinear Analysis Real World Applications, 44:173-222.
[16] Bao X, Liu J. (2017) Dynamic Finite Element Analysis Methods for Liquid Container Considering Fluid-Structure Interaction. Nuclear Power Engineering, 38(2):111-114.