I. INTRODUCTION

The fundamental features of relaxation and photoluminescence (PL) of the excitons in quantum wells (QWs) originate from the quasi-two-dimensionality (quasi-2D) of the system. In the last decade these basic processes in GaAs QWs have attracted continual attention. The formation of QW excitons through LO-phonon cascade emission, LA-phonon and carrier-carrier scattering completes within 20 ps after the initial excitation of electron-hole pairs. The created hot QW excitons then thermalize through low-energy acoustic phonon scattering. This process occurs in a sub-ns time scale. In GaAs QWs the relaxation of hot excitons can be observed through resonant PL from the optically-active bulk modes which refer to a small in-plane momentum. Thus the rise and decay times of excitonic PL relate to the thermalization process. Furthermore, the PL kinetics of long-lifetime indirect excitons in a high-quality GaAs/AlGaAs coupled quantum well (CQW) is now the subject of experimental studies. There has been recent progress in the investigation of thermalization of quasi-2D excitons in ZnSe QWs. In the latter case the picosecond LA-phonon-assisted kinetics of QW excitons is visualized through LO-phonon-assisted PL for all in-plane modes $p_{\parallel}$. While the importance of interface disorder and the localization effects were recognized in the very first experimental and theoretical studies on the relaxation kinetics of QW excitons, the quality of GaAs QWs continuously improves from an inhomogeneous excitonic linewidth of about 7.5 meV toward a homogeneous one between 80 and 120 µeV. In the present work we investigate LA-phonon-assisted relaxation kinetics in a perfect QW emphasizing the importance of Bose-Einstein (BE) statistics of quasi-2D excitons. In particular, we address the density-dependent PL kinetics reported, e.g., in Refs. [4] to nonclassical statistics of QW excitons. In the past, most theoretical modeling of the relaxation kinetics in QWs dealt with a classical gas of Maxwell-Boltzmann distributed excitons. Quantum-statistical effects were included in numerical simulations of the relaxation kinetics of a trapped quasi-2D exciton gas [5] and in the study of exciton-biexciton law of mass action in QWs [6].

Crossover from classical to quantum statistics occurs near the degeneracy temperature $T_0$, given by

$$k_B T_0 = \frac{2\pi}{g} \left( \frac{\hbar^2}{M_x} \right) \rho_{2D},$$

where $M_x$ is the in-plane translational mass of a QW exciton, $\rho_{2D}$ is the 2D concentration of excitons, and $g$ is the spin degeneracy factor of QW excitons. We put $g = 4$, because for GaAs QWs the exchange interaction is rather weak [7]. Furthermore, this interaction is completely suppressed for indirect excitons in CQWs. One has a classical gas of QW excitons at temperatures $T \gg T_0$. BE statistics smoothly develops with decreasing $T \sim T_0$ leading to occupation numbers $N_{p_{\parallel}} \geq 1$ of the low-energy in-plane modes $p_{\parallel}$. We will show that at helium temperatures nonclassical statistics affects the...
thermalization process already at moderate densities of QW excitons \( \rho_{2D} \geq 3 - 5 \times 10^{10} \text{ cm}^{-2} \).

In this paper we develop relaxational thermodynamics and study within this approach how BE statistics influences thermalization and photoluminescence of quasi-2D excitons. Relaxational thermodynamics requires that exciton-exciton interaction is much stronger than exciton-LA-phonon coupling and is appropriate if the concentration \( \rho_{2D} \) of QW excitons is larger than some critical density \( \rho_{2D}^c \). In this case, the the exciton system establishes a quasi-equilibrium temperature \( T \). For GaAs QWs we will get an estimate \( \rho_{2D}^c \simeq 1 - 3 \times 10^9 \text{ cm}^{-2} \).

Equation (1), which is the basic equation of the relaxational thermodynamics of QW excitons, provides us with an unified description of the thermalization process. The thermodynamic approach yields temporal evolution of the effective temperature of QW excitons \( T = T(t) \) from the initial value \( T_i = T(t = 0) \) to the bath temperature \( T_b \). While we study the thermalization process in the both limits, classical and degenerate, of a gas of QW excitons, the most interesting results refer to \( T \leq T_0 \). In this case one finds a density-dependent nonexponential relaxation of quasi-2D excitons. Both acceleration and slowing down of the thermalization kinetics may occur due to BE statistics. However, with the decreasing bath temperature \( T_b \), slowing down of the relaxation starts to dominate in a quantum gas. In particular, for \( T_b \rightarrow 0 \) we derive \( 1/\ln t \) cooling law for QW excitons.

The thermalization kinetics of excitons in GaAs QWs can be observed through resonant PL from the in-plane radiative modes. Therefore we generalize the PL theory [4] to well-developed BE statistics of QW excitons and model numerically the PL process in GaAs QWs. It will be shown that at low temperatures nonclassical statistics changes the law \( \tau_{\text{opt}} \propto T \), where \( \tau_{\text{opt}} \) is the effective radiative lifetime of QW excitons. Furthermore, we can calculate the temperature-independent corrections to the classical linear behavior of \( \tau_{\text{opt}}(T) \). The corrections originate from BE statistics and can be traced even at high temperatures.

Relaxational thermodynamics refers to the phonon-assisted thermalization kinetics of QW excitons. We will analyze the relaxation kinetics due to bulk LA-phonons, assuming the initial distribution of hot QW excitons is below the threshold for optical phonon emission. The bulk LA-phonons are due to a semiconductor substrate of a QW. While the lifetime of long wavelength acoustic phonons are in a sub-\( \mu \)s time scale, the scattering LA-phonons leave and enter the QW area within a time \( \sim L_z / v_s \sim 1 - 10 \text{ ps} \) \( (L_z \) is the thickness of a QW, \( v_s \) is the longitudinal sound velocity). Therefore, we assume the Planck distribution of the LA-phonons interacting with QW excitons. The LA-phonon-assisted kinetics will be treated for an isolated band of ground-state QW excitons. Being applied to heavy-hole excitons in GaAs QWs with \( L_z \leq 100 \) \( \AA \) this assumption means QW exciton energies \( E \leq 10 \text{ meV} \) and temperatures less than 100 \( K \). Only the exciton-LA-phonon deformation potential interaction is included in our model.

The in-plane momentum is conserved in the scattering of QW excitons by bulk LA-phonons. The momentum conservation in the \( z \)-direction (the QW growth direction) is relaxed. As a result, a scattered QW exciton interacts with a continuum spectral band of scattering LA-phonons of a given direction of propagation. In contrast, an exciton in bulk semiconductors couples in Stokes or anti-Stokes scattering only with one phonon mode of a given direction. In Fig. 1 we depict a schematic picture of the states which are involved in the scattering of a QW exciton with in-plane momentum \( p_{\parallel} = 0 \). The energy state \( E = 0 \) couples to the continuum energy states \( E \geq E_0 = 2 M_x v_s^2 \), i.e., to the QW states which lie inside the acoustic cone given by \( E = E(p_{\parallel}) = h v_s p_{\parallel} \). The energy \( E_0 \) is an important parameter of the relaxational thermodynamics of QW excitons. For GaAs QWs with \( M_x = 0.3 \ m_0 \) \( (m_0 \) is the free electron mass) and \( v_s = 3.7 \times 10^5 \text{ cm/s} \) one has \( E_0 = 46.7 \text{ \mu eV} \) and \( E_0/k_B = 0.54 \text{ \text{K}} \).

![FIG. 1. Schematic picture of the energy-momentum conservation in scattering of QW excitons by bulk LA-phonons. The parabolic and conical surfaces refer to the excitonic \( E = \hbar^2 p_{\parallel}^2 / 2 M_x \) and acoustic \( E = hv_s p_{\parallel} \) dispersions, respectively. The bold rings show the states \( \hbar p_{\parallel} \geq 2 M_x v_s \) of QW excitons which couple to the ground-state mode \( p_{\parallel} = 0 \).](image)
\[ + \frac{1}{\sqrt{4\varepsilon^2(1-u^2) - (\varepsilon - 1 - \varepsilon u^2)^2}} \times \left[ N_{\varepsilon} n_{\varepsilon}^{ph} (1 + N_{\varepsilon^+}) - (1 + N_{\varepsilon^+}) (1 + n_{\varepsilon^{ph}}^0) N_{\varepsilon^+} \right] \times \Theta \left( 2\varepsilon^\prime (1-u^2) + \varepsilon - 1 - \varepsilon u^2 \right) \times \Theta \left( 2\varepsilon^\prime (1-u^2) - \varepsilon + 1 + \varepsilon u^2 \right), \tag{2} \]

where the dimensionless energy is \( \varepsilon = E/E_0 = E/(2M_L v_s^2) \), \( n_{\varepsilon}^{ph} = 1/\exp(\varepsilon E_0/k_B T_b) - 1 \) and \( N_{\varepsilon} \) are the distribution functions of bulk LA-phonons and QW excitons, respectively, \( \Theta \) is the Heaviside step-function. The integration variable \( u \) is given by \( u = \cos \theta \), where \( 0 \leq \theta \leq \pi \) is the angle between \( z \)-axis and the wavevector of a scattering bulk LA-phonon. The scattering time is \( \tau_{sc} = (\pi^2 h^3 \rho)/(D^2 M_L^2 v_s) \), where \( \rho \) is the crystal density and \( D \) is the deformation potential. The form-factor \( F_\varepsilon(\chi) = [\sin(\chi)/\chi]|e^{i\chi}/(1 - \varepsilon)^2/\pi^2 | \) refers to an infinite QW confinement potential. This function describes the relaxation of the momentum conservation in the \( z \)-direction and characterizes a spectral band of LA-phonons, which effectively interact with a QW exciton. The dimensionless parameter \( a \sim 1 \) is defined by \( a = (L_z M_L v_s)/h \).

The kinetic Eq. (2) deals with an isotropic in-plane distribution of QW excitons, i.e., the occupation number \( N_{\|} \) of the in-plane mode \( \mathbf{p}_\| \) relates to the distribution function \( N_E \) by \( N_{\|} = N_{E_{\|}} = N_E \). This approximation corresponds to the experimental conditions of Refs. [1-10]. The first (second) term in the figure brackets on the r.h.s. of Eq. (2) describes the phonon-assisted Stokes (anti-Stokes) scattering. In accordance with Eq. (2), the relaxation kinetics into the ground-state mode \( \mathbf{p}_\| = 0 \) is given by

\[
\frac{\partial}{\partial t} N_{E=0} = \frac{2\pi}{\tau_{sc}} \int_1^\infty d\varepsilon \varepsilon \sqrt{\varepsilon - 1} \left| F_\varepsilon \left( a \sqrt{\varepsilon (\varepsilon - 1)} \right) \right|^2 \times \left[ N_{\varepsilon} (1 + n_{\varepsilon}^{ph}) - N_{E=0} (n_{\varepsilon}^{ph} - N_{\varepsilon}) \right]. \tag{3} \]

The integral on the r.h.s. of Eq. (3) characterizes the coupling of the ground-state mode \( E = 0 \) to continuum of the energy states \( E \geq E_0 \) (see Fig. 1).

In Sec. II, we develop the relaxational thermodynamics of QW excitons coupled to bulk LA-phonons. A basic equation for the effective temperature \( T \) of quasi-equilibrium QW excitons is derived from LA-phonon-assisted kinetics given by Eq. (2). The conditions of the validity of the thermodynamic picture are analyzed and tested for excitons in GaAs QWs.

In Sec. III, the thermalization law \( T = T(t) \) is studied for both classical \( (T \gg T_b) \) and well-developed BE statistics \( (T \ll T_b) \) of quasi-equilibrium QW excitons. We demonstrate that BE statistics strongly influences the thermalization process and leads to the density-dependent characteristic thermalization time \( \tau_{th} = \tau_{th}(\rho_{2D}) \) for \( |T - T_b| \ll T_b \) and to the nonequilibrium density-dependent relaxation at \( T - T_b \geq T_b \). In particular, for low bath temperatures \( T_b \leq E_0/k_B \) we find a very slow thermalization \( T(t) \propto \ln t \) of quantum degenerate quasi-2D excitons. The numerical simulations of the relaxational dynamics are given for excitons in GaAs QWs.

In Sec. IV, we develop a theory of resonant PL of statistically degenerate QW excitons. An effective radiative lifetime \( \tau_{opt} \) of a quasi-2D excitonic gas is calculated for BE-distributed QW excitons. We show that the law \( \tau_{opt} \propto T \), which is valid for classical statistics of QW excitons, is violated at low temperatures and at \( T \to 0 \) the effective decay time \( \tau_{opt} \) tends to \( 2\tau_R \), where \( \tau_R \) is the intrinsic radiative lifetime of a QW exciton. The PL kinetics of QW excitons is described in the thermodynamic approach within three coupled equations for \( T(t), \rho_{2D}(t) \), and \( \tau_{opt}(T, \rho_{2D}) \). We give numerical modeling of the \( T \) and \( \rho_{2D} \)-dependent PL kinetics of quantum degenerate quasi-2D excitons.

In Sec. V, we discuss the influence of interface polaritons and QW bie excitons on the relaxation kinetics of statistically degenerate excitons in perfect QWs. Our theory is always appropriate for thermalization of indirect excitons in CQWs, where even at small densities \( \rho_{2D} \) the interface polariton effect is rather weak and bie exciton states are unbound. For single QWs we argue that QW polaritons and bie excitons are considerably weakened at large densities, due to particle-particle scattering. In this case our model is applicable to usual QWs with direct excitons.

In Appendix A, in order to estimate the critical density \( \rho_{2D}^c \) of QW excitons for the development of relaxational thermodynamics we calculate the characteristic equilibration time \( \tau_{x-x} \) due to exciton-exciton scattering. The equilibration rate \( 1/\tau_{x-x} \) is found in both limits of classical and well-developed BE statistics.

In Appendix B, we analyze how a relatively large homogeneous linewidth \( \sim \hbar/\tau_{x-x} \gg \hbar/\tau_{th}, \hbar/\tau_R \) of high-density QW excitons in a perfect QW influences the PL process.

II. RELAXATIONAL THERMODYNAMICS OF QW EXCITONS

In this section we summarize the thermodynamic relations for an ideal two-dimensional gas of bosons and derive the basic equation for relaxational dynamics of QW excitons.

A. Thermodynamic relations for quasi-2D excitons

The thermodynamic equation \( \mu = \mu(T, \rho_{2D}) \) for a quasi-equilibrium gas of QW excitons can be derived from the condition:

\[ \frac{1}{\sqrt{4\varepsilon^2(1-u^2) - (\varepsilon - 1 - \varepsilon u^2)^2}} \times \left[ N_{\varepsilon} n_{\varepsilon}^{ph} (1 + N_{\varepsilon^+}) - (1 + N_{\varepsilon^+}) (1 + n_{\varepsilon^{ph}}^0) N_{\varepsilon^+} \right] \times \Theta \left( 2\varepsilon^\prime (1-u^2) + \varepsilon - 1 - \varepsilon u^2 \right) \times \Theta \left( 2\varepsilon^\prime (1-u^2) - \varepsilon + 1 + \varepsilon u^2 \right), \tag{2} \]
\[ \rho_{2D} = \frac{1}{S} \sum_{\mathbf{p}_i} N_{E=0}^{eq} = \frac{2 g M_S k_B T}{\pi \hbar^2} \int_0^\infty \frac{dz}{e^{-\mu/k_B T} e^z - 1}. \]  
(4)

From Eq. 4 one obtains
\[ \mu = k_B T \ln \left(1 - e^{-T_0/T}\right), \]
where the degeneracy temperature \( T_0 \) is given by Eq. 4 with \( g = 4 \). With the chemical potential \( \mu \) of Eq. 4 the equilibrium distribution function of QW excitons is
\[ N_{E}^{eq} = \frac{1 - e^{-T_{0}/T}}{e^{E/k_B T} + e^{-T_{0}/T} - 1}. \]
(6)

In particular, the occupation number of the ground-state mode is given by \( N_{E=0}^{eq} = N_{E=0}^{eq} = \exp(T_0/T) - 1 \).

Classical, Maxwell-Boltzmann, statistics of QW excitons is realized for \( T \gg T_0 \). In this case Eqs. (5)-(6) reduce to
\[ \mu = -k_B T e^{-T_0/T} \quad \text{and} \quad N_{E=0}^{eq} = e^{T_0/T} \gg 1, \]
(7)
i.e., the occupation numbers of QW modes \( \mathbf{p}_i \) are much less than unity. In the opposite limit \( T < T_0 \) of well-developed BE statistics one has
\[ \mu = -k_B T e^{-T_0/T} \quad \text{and} \quad N_{E=0}^{eq} = e^{T_0/T} \gg 1. \]
(8)

According to Eq. 7, the chemical potential of a degenerate gas of QW excitons approaches zero much faster than the temperature \( T \). While the chemical potential \( \mu < 0 \) for \( T > 0 \) and the BE condensation of the QW exciton is absent within the thermodynamic approach for temperatures above zero, the occupation number of the ground-state mode \( \mathbf{p}_i \) increases exponentially with decreasing \( T \).

Equations (5)-(8) can be applied to an arbitrary two-dimensional quasi-ideal gas of Bose-particles with quadratic dispersion. The specific characteristics of the bosons, like the spin degeneracy factor \( g \) and the translational mass \( M_s \), enter the thermodynamic relationships only through the degeneracy temperature \( T_0 \) defined by Eq. 4.

For GaAs QWs one estimates \( T_0 = (\pi h^2 \rho_{2D})/(2 M_s k_B) = 4.6 K \) \( (k_B T_0 = 399.5 \, \mu eV) \) for \( \rho_{2D} = 10^{11} \, \text{cm}^{-2} \). This density of quasi-2D excitons corresponds to the mean interparticle distance \( \sim 1/\sqrt{\rho_{2D}} \approx 320 \, \text{Å} \) and to the Mott parameter \( \rho_{2D} a_{(SD)}^2 \approx 0.04 \), i.e., the QW excitons are still well-defined quasiparticles. Here \( a_{(SD)}^2 \) is the Bohr radius of a quasi-2D exciton. In next subsection we will use Eqs. (5)-(8) to develop the relaxational dynamics.

**B. Thermalization equation for quasi-equilibrium excitons in QWs**

The thermalization of QW excitons occurs through a sequence of quasi-equilibrium thermodynamic states, which are characterized by an effective temperature \( T = T(t) \) and chemical potential \( \mu = \mu(t) \), provided that the exciton-exciton interaction in a QW is much stronger than the coupling of QW excitons with bulk LA-phonons. The initial interaction is conservative and equilibrates the system of QW excitons without change of total energy. The characteristic equilibration time \( \tau_{x-x} \) depends on the density \( \rho_{2D} \) of QW excitons. For excitons distributed below the threshold for LO-phonon emission, energy (effective temperature) relaxation results from QW exciton - bulk LA-phonon scattering and is characterized by an effective thermalization time \( \tau_{th} \). The hierarchy of interactions means that in a large and interesting range of density we have \( \tau_{x-x} \ll \tau_{th} \). In this case the equilibration and thermalization kinetics can be naturally separated (see, e.g., Ref. 13) and the thermodynamic approach is correct.

The condition \( \tau_{x-x} \ll \tau_{th} \) will hold for \( \rho_{2D} > \rho_{2D}^c \), where \( \rho_{2D}^c \) is some critical density for QW excitons. For a classical distribution of quasi-2D excitons one has \( T_0/T \ll 1 \) and \( N_{E}^{eq} = (T_0/T) \exp(-E/k_B T) \ll 1 \), according to Eq. 4. In this limit the characteristic equilibration time \( \tau_{x-x} \) is estimated by Eq. (A4) of Appendix A, while the thermalization time \( \tau_{th} \) is given by Eq. (15) of section III. The comparison of Eq. (A4) and Eq. (15) yields \( \tau_{x-x} \ll \tau_{th} \) provided that
\[ k_B T_0 > k_B T_0 = \frac{4 h}{\pi} \left(\frac{\mu_x}{M_x}\right)^2 \frac{C_{2D}}{\tau_{sc}}, \]
(9)
where \( \mu_x \) is the reduced mass of a QW exciton and the constant \( C_{2D} = C_{2D}(a) \gg 1 \) is defined by Eq. 13. For a classical gas of excitons in GaAs QW of thickness \( L_z = 100 \, \text{Å} \) we estimate \( \rho_{2D}^c = 1.2 \times 10^9 \, \text{cm}^{-2} \) and the corresponding temperature scale \( T_0^c = 56 \, \text{mK} \). For comparison, the critical density of about \( 4 \times 10^9 \, \text{cm}^{-2} \) has been estimated in experiments with high-quality GaAs QWs of \( L_z = 80 \, \text{Å} \). For well-developed BE statistics one has \( T_0/T \gg 1 \) and the occupation numbers of the energy states \( E > k_B T \exp(-T_0/T) \) are given by the Planck function, i.e., \( N_{E>0}^{eq} = 1/[\exp(E/k_B T)-1] \), according to Eq. 4. In this case \( \tau_{x-x} \) and \( \tau_{th} \) are given by Eq. (A5) of Appendix A and Eq. (15) of section III, respectively. The condition \( \tau_{x-x} \ll \tau_{th} \) reduces to
\[ E_0 \left(\frac{T_0}{T}\right) e^{T_0/T} > \frac{4 h}{(1 - \pi/4)} \left(\frac{\mu_x}{M_x}\right)^2 \frac{C_{2D}}{\tau_{sc}}, \]
(10)
where the constant \( C_{2D} = C_{2D}(a) \gg 1 \) is given by Eq. 13. The inequality (10) always holds for strongly degenerate QW excitons.

The criteria (4) and (10) of the validity of the thermodynamic picture at \( T \gg T_0 \) and \( T \ll T_0 \), respectively, are independent of the scattering length \( a_{(SD)}^2 \) of exciton-exciton interaction. This is due to the quasi-two-dimensionality of QW excitons (for details see Appendix A). In contrast, in a three-dimensional gas the

---

4
The thermalization dynamics of QW excitons is determined by the slowest elementary LA-phonon-assisted relaxation process of the kinetic Eqs. (3) and (4). The joint density of states for Stokes and anti-Stokes scattering from the energy mode E continuously decreases with decreasing E, according to the r.h.s. of Eq. (3) (note, that the density of quasi-2D excitonic states is constant given by \(16\pi^2 M_s / h^2\) for \(g = 4\)). Moreover, the low-energy states \(E \leq E_0 / 4\) are not active in Stokes scattering and couple with the corresponding LA-phonon-separated states \(E \geq E_0\) only through the anti-Stokes process. The above arguments show a pattern of Eq. (4), which describes the relaxation kinetics into the ground-state mode. Furthermore, the ground-state mode \(p_\parallel = 0\) refers to the lowest energy \(E = 0\) and to the largest occupation number \(N_{E=0}\). For well-developed BE statistics of QW excitons one has \(N_{E=0} \gg 1\). As a result, population of the state \(E = 0\) generally requires an additional time in comparison with that for the high-energy states \(E \gg k_B T\) with \(N_E \ll 1\). Thus LA-phonon-assisted occupation of the ground-state mode is the slowest, “bottleneck”, relaxation process, which determines the thermalization dynamics of quasi-equilibrium QW excitons. A similar picture holds for thermalization of excitons in bulk semiconductors.

With the substitution of \(N_E = N_E^0\) given by Eq. (1) in Eq. (2) we derive

\[
\frac{\partial}{\partial t} T = -\frac{2\pi}{\tau_{sc}} \frac{T^2}{T_0} \left(1 - e^{-\tau_b / T}\right) \int_1^\infty \frac{dz}{z} \frac{\varepsilon}{e^{\varepsilon} - 1} \frac{F_z(a\sqrt{z(\varepsilon - 1)})}{(e^{E_0/k_B T} - e^{-E_0/k_B T})(e^{E_0/k_B T} + e^{-E_0/k_B T} - 1)}
\]

Equation (11) describes the thermalization dynamics \(T = T(t)\) of QW excitons from the effective temperature \(T_i = T(t = 0)\) to the bath temperature \(T_b\). A finite lifetime \(\tau'\) of excitons, due to radiative and nonradiative recombination, can be incorporated in Eq. (11) by the degeneracy temperature \(T_0(t) \propto \rho_{2D}(t = 0) \exp(-t/\tau')\). In next section we apply Eq. (11) to the thermalization kinetics of QW excitons in the absence of their recombination (\(\tau' \to \infty\)). This analysis refers to the case when the characteristic thermalization time \(\tau_{th} \ll \tau'\). In section IV we will use Eq. (11) in order to develop a theory of resonant photoluminescence of quantum degenerate excitons in perfect QWs, when \(\tau' = \tau_{pol}\) is determined by the intrinsic radiative lifetime \(\tau_R\) of QW excitons.

### III. THERMALIZATION KINETICS OF QUASI-EQUILIBRIUM QW EXCITONS

In this section the thermalization dynamics of QW excitons is analyzed within Eq. (1) for \(\rho_{2D} = \text{const}\). First we linearize Eq. (1) about some bath temperature \(T_b\) to study the exponential relaxation of QW excitons with the effective temperature \(|T - T_b| \ll T_b\). Linearization is not appropriate for the thermalization kinetics at \(T_b = 0\). Here we show that the thermalization becomes very slow, with \(T(t) \propto 1/\ln t\). In the last subsection, the cooling of hot QW excitons with \(T - T_b \geq T_b\) is treated for both classical and strongly degenerate limits of BE statistics. There are four characteristic temperature scales, \(T, T_0, T_b,\) and \(E_0/k_BT_b\), and the precise form of the relaxation will depend on all four. Note however that if \(T \gg T_0\), the degeneracy temperature is not a relevant parameter, and the most relevant aspects of phonon bottleneck effects are captured by the ratio \(E_0/k_BT_b\). In practice the relaxation dynamics is controlled by two parameters, \(T_0/T\) and \(E_0/k_BT_b\).

#### A. Relaxation kinetics between nearby thermodynamic states

If the effective temperature \(T\) of QW excitons is close to the bath \(T_b\), the basic thermodynamic Eq. (1) reduces to

\[
\frac{\partial}{\partial t} T = -\frac{1}{\tau_{th}} (T - T_b) ,
\]

where the effective thermalization time \(\tau_{th}\) is given by

\[
\frac{1}{\tau_{th}} = \left(\frac{2\pi}{\tau_{sc}}\right) \left(\frac{E_0}{k_BT_0}\right) \left(1 - e^{-T_0/T_b}\right) \int_1^\infty \frac{dz}{z} \frac{e^{\varepsilon}}{e^{\varepsilon} - 1} \frac{F_z(a\sqrt{\varepsilon(\varepsilon - 1)})}{(e^{E_0/k_BT_b} - e^{-E_0/k_BT_b})(e^{E_0/k_BT_b} + e^{-E_0/k_BT_b} - 1)} \times \frac{1}{(e^{E_0/k_BT_b} - 1)} .
\]

The linear approximation of Eq. (1) by Eq. (12) is valid for \(|T - T_b|/T_b < k_BT_b/E_0\) and can be done for any \(T_b > 0\). Equation (12) corresponds to the exponential thermalization law \(T(t) = T_b + \Delta T \exp(-t/\tau_{th})\), where \(\Delta T = T_1 - T_b\), and \(T_1 = T(t = 0)\) is the initial temperature of QW excitons. The thermalization time \(\tau_{th}\) uniquely describes all quasi-equilibrium relaxation processes in a gas of quasi-2D excitons. For example, the relaxation kinetics into the ground-state mode \(E = 0\) is given by \(N_{E=0}(t) = N_{E=0}^f + (N_{E=0}^i - N_{E=0}^f) \exp(-t/\tau_{th})\), where \(N_{E=0}^i = \exp(T_0/T_b) - 1\) and \(N_{E=0}^f = \exp(T_0/T_b) - 1\). We will analyze Eq. (14) in the limit of classical and well-developed BE statistics of QW excitons, respectively.

1. **Classical gas of QW excitons (\(T_b \gg T_0\)).**

In this case Eq. (13) yields...
\[ \frac{1}{\tau_{th}} = \left( \frac{2\pi}{\tau_{sc}} \right) \int_{1}^{\infty} d\varepsilon \frac{\varepsilon^{2}}{\varepsilon^{2} - 1} \sqrt{\varepsilon - 1} \]  
\[ \times \frac{[F_{z}(a \sqrt{\varepsilon - 1})]^{2}}{(\varepsilon E_{0}/k_{B}T_{b} - 1)}. \]  
(14)

From Eq. (14) one concludes, as expected, that the characteristic thermalization time of the Maxwell-Boltzmann distributed QW excitons is indeed independent of their concentration \( \rho_{QW} \). In Fig. 2 we plot \( \tau_{th} = \tau_{th}(T_{b}) \) calculated by Eq. (14) for GaAs QW of thickness \( L_z = 100 \) Å. The ratio \( E_{0}/k_{B}T_{b} \) determines the high- and low-temperature limits of \( \tau_{th} \).

**FIG. 2.** The thermalization time \( \tau_{th} = \tau_{th}(T_{b}) \) of a classical gas of QW excitons. GaAs QW with \( L_z = 100 \) Å and \( \tau_{sc} = 13.9 \) ns [see Eq. (14)]. Inset: the high-temperature thermalization time \( \tau_{H} = \tau_{H}(L_z) \propto L_z^{2} \) of Maxwell-Boltzmann distributed QW excitons (solid line) and the high-temperature thermalization time \( \tau_{H} = \tau_{H}(L_z) \propto L_z \) of strongly statistically degenerate QW excitons ( dot-dashed line). The latter dependence refers to \( T_{b} = (k_{B}T_{b}^{2})/E_{0} = 1 \) (\( T_{H} \gg T_{b} \)).

A. High-temperature limit \( (k_{B}T_{b} \gg E_{0}) \). The characteristic thermalization time, which in this case we will designate by \( \tau_{H} \), is given by

\[ \frac{1}{\tau_{H}} = \frac{C_{2D}}{\tau_{sc}}, \quad \text{where} \]
\[ C_{2D} = 2\pi \int_{1}^{\infty} d\varepsilon \frac{\varepsilon^{2}}{\varepsilon^{2} - 1} \sqrt{\varepsilon - 1} \left[ F_{z}(a \sqrt{\varepsilon - 1}) \right]^{2}. \]  
(15)

In the high-temperature limit of a classical gas of QW excitons the thermalization time \( \tau_{H} \) is independent of the bath temperature \( T_{b} \). The constant \( C_{2D} = C_{2D}(a) \), which completely determines \( \tau_{H} \) in terms of \( \tau_{sc} \), is much larger than unity. For example, for GaAs QW with \( L_z = 100 \) Å the dimensionless parameter \( a = 0.096 \) and the constant \( C_{2D} = 2530 \). The corresponding thermalization time of the QW excitons is \( \tau_{H} = 5.5 \) ps. For comparison, the high-temperature thermalization time of a classical gas of excitons in a bulk semiconductor is given by \( \tau_{H}^{(3D)} = (3/8\pi)\tau_{sc}^{2} \). This estimate yields \( \tau_{H}^{(3D)} = 1.66 \) ns for bulk GaAs, i.e., \( \tau_{H}^{(3D)} \gg \tau_{H}^{(2D)} = \tau_{H} \). The ratio \( \tau_{H}^{(2D)}/\tau_{H}^{(3D)} = (8\pi)/(3C_{2D}) \approx 3.3 \times 10^{-3} \), which refers to bulk GaAs and GaAs QW with \( L_z = 100 \) Å, clearly demonstrates effective cooling of QW excitons in the presence of a bath of bulk phonons. Due to the relaxation of momentum conservation in \( z \)-direction a ground-state QW exciton couples to the continuum states \( E \geq E_{0} \) rather than to the single energy state \( E = E_{0} \) as occurs in bulk materials. With decreasing bath temperature \( T_{b} \) the effective thermalization time \( \tau_{th} \) of QW excitons monotonously increases starting from its high-temperature limit given by \( \tau_{H} \) (see Fig. 2). For \( T_{b} = 5 \) K one finds from Eq. (15) that \( \tau_{th} = 19.1 \) ps in GaAs QW with \( L_z = 100 \) Å.

There is an uncertainty in values of the deformation potential \( D \) of exciton – LA-phonon interaction, published in literature (see, e.g., Refs. [3, 4, 5], from \( D_{min} = 7.0 \) eV to \( D_{max} = 18.1 \) eV. In our numerical evaluations we use \( D = 15.5 \) eV. According to Eq. (13), the deformation potential contributes to \( \tau_{sc} \) only through \( \tau_{sc} \propto 1/D^{2} \). Therefore the numerical calculations of \( \tau_{ph} \) can be straightforwardly re-scaled to another value of \( D \) (we will use this procedure in calculations of Fig. 9a to reproduce qualitatively the experimental data of Ref. [6]). The value \( D = 15.5 \) eV corresponds to \( \tau_{sc} = 13.9 \) ns.

B. Low-temperature limit \( (k_{B}T_{b} \leq E_{0}) \). In this case Eq. (14) reduces to

\[ \frac{1}{\tau_{L}} = \frac{2\pi^{1/2}}{\tau_{sc}} \left( \frac{E_{0}}{k_{B}T_{b}} \right)^{1/2} e^{-E_{0}/k_{B}T_{b}}, \]  
(16)

where we designate the low-temperature limit of \( \tau_{th} \) by \( \tau_{L} \). According to Eq. (14), the thermalization time \( \tau_{L} \) increases exponentially with decreasing bath temperature \( T_{b} \) below \( E_{0}/k_{B} \). The origin of this result can be understood from the initial Eq. (3) of the phonon-assisted relaxation kinetics into the ground-state mode \( E = 0 \). For a classical gas of QW excitons one has \( N_{E} \ll 1 \). As a result, both the spontaneous (independent of \( N_{E=0} \)) and the stimulated (proportional to \( N_{E=0} \)) processes contribute to population of the ground-state mode. The first, Stokes, process is \( \propto N_{E \geq E_{0}}(1 + n_{ph}^{E \leq E_{0}}) \) and increases \( N_{E=0} \), while the second, anti-Stokes, one is \( \propto -N_{E=0}n_{ph}^{E \geq E_{0}} \) and decreases the occupation number of the mode \( E = 0 \). For the high-temperature limit of a classical gas of QW excitons both opposite fluxes are intense because \( n_{ph}^{E \geq E_{0}} = k_{B}T_{b}/E \gg 1 \). In the low-temperature limit \( n_{ph}^{E \geq E_{0}} = \exp(-E/k_{B}T_{b}) \ll 1 \) and both the spontaneous process, which is proportional to \( N_{E \geq E_{0}}(1 + n_{ph}^{E \leq E_{0}}) \approx N_{E \geq E_{0}} = (T_{0}/T)\exp(-E/k_{B}T), \) and the stimulated process, which is proportional to \( -N_{E=0}n_{ph}^{E \geq E_{0}} = -(T_{0}/T)\exp(-E/k_{B}T_{b}) \), are exponentially weak.

The plot \( \tau_{th} = \tau_{th}(T_{b}) \) (see Fig. 2) shows that for excitons in GaAs QWs the high-temperature limit given by
Eq. (15) means $T_b \geq 30 - 50$ K. On the other hand, a strong increase of $\tau_{th}$ occurs already at $T_b \simeq 5 - 10$ K, i.e., at temperatures much above the low-temperature range determined by $T_b \leq E_0/k_b \simeq 0.54$ K and where the approximation of $\tau_{th}$ by Eq. (13) is valid.

2. Quantum gas of QW excitons ($T_0 \gg T_b$).

In this case Eq. (13) reduces to

$$\frac{1}{\tau_{th}} = \frac{2\pi}{\tau_{sc}} \left( \frac{E_0}{k_BT_0} \right) \int_1^\infty d\varepsilon \sqrt{\varepsilon - 1} \left| F_\varepsilon \left( a\sqrt{\varepsilon(\varepsilon - 1)} \right) \right|^2 \times \left[ F_\varepsilon \left( a\sqrt{\varepsilon(\varepsilon - 1)} \right) \right]^2. \quad (17)$$

Equation (17) shows that the thermalization time of quantum degenerate quasi-2D excitons depends on the concentration $\rho_{2D}$, i.e., $\tau_{th} \propto \rho_{2D}$.

A. High-temperature limit ($k_BT_b \gg E_0$). In this limit Eq. (17) yields for $\tau_{th} = \tau_{th}$:

$$\frac{1}{\tau_{th}} = \left( \frac{k_BT_b^2}{T_0E_0} \right) \left( \frac{C_{2D}}{C_{2D}} \right) \frac{1}{\tau_{th}} \quad (18)$$

From Eqs. (17) and (18) we find

$$\frac{1}{\tau_{th}} = \left( \frac{k_BT_b^2}{T_0E_0} \right) \left( \frac{C_{2D}}{C_{2D}} \frac{1}{\tau_{th}} \right). \quad (19)$$

The constant $C_{2D} = C_{2D}(a)$ is much larger than unity, but much less than $C_{2D}$. For example, $C_{2D} = 170$ so that $C_{2D}/C_{2D} = 0.07$ for GaAs QW with $L_z = 100$ A. According to Eq. (19) one has $\tau_{th} < \tau_{th}$ for $T_0 \gg T_b > [T_0E_0C_{2D}]/[k_BT_bC_{2D}]$ and $\tau_{th} > \tau_{th}$ for $T_b < [T_0E_0C_{2D}]/[k_BT_bC_{2D}]$. The acceleration or slowing down of the relaxation kinetics in comparison with that in a classical gas of QW excitons originates from BE statistics. Because for the quasi-equilibrium degenerate QW excitons $N_{E=0} = \exp(T_0/T) \gg N_{E \geq E_0} = 1/[\exp(E_0/k_BT) - 1]$, only the stimulated processes $N_{E=0}(N_{E \geq E_0} - n_{E \geq E_0}^{ph})$ contribute to occupation kinetics of the ground-state mode $E = 0$. From Eqs. (7) and (8) one obtains $T/T_0 = -\delta N_{E=0}/N_{E=0}$ for classical statistics and $T/T_0 = -(T/T_0)\delta N_{E=0}/N_{E=0}$ for well-developed BE statistics of the ground-state mode, i.e., the same relative change of the occupation number $N_{E=0}$ is accompanied at $T \ll T_0$ by the much less relative change of the effective temperature than that at $T \gg T_0$. Therefore, BE occupation of the ground-state mode with $N_{E=0} \gg 1$ slows down the thermalization kinetics of QW excitons. On the other hand, one has $N_{E \geq E_0} - n_{E \geq E_0}^{ph} = n_{E \geq E_0}^{ph}(\delta T/T_b)$ for well-developed BE statistics of the energy modes $E \geq E_0$ and $N_{E \geq E_0} - n_{E \geq E_0}^{ph} = n_{E \geq E_0}^{ph}(E/k_BT_b)(\delta T/T_b)$ for a classical distribution of QW excitons at $E \geq E_0$. As a result, the BE occupation numbers $N_{E \geq E_0} \gg 1$ enhance the relaxation dynamics by the factor $\sim k_BT_b/E_0$, which is much larger than unity in the high-temperature limit. With decreasing bath temperature $T_b$ the slowing down of thermalization, which results from $N_{E=0} \gg 1$, starts to dominate over the acceleration of the relaxation due to BE statistics of the modes $E \geq E_0$.

![Fig. 3](image-url)  
**FIG. 3.** The thermalization time $\tau_{th} = \tau_{th}(T_b)$ of BE-distributed QW excitons of the densities $10^{12}$ cm$^{-2}$ (solid line), $5 \times 10^{11}$ cm$^{-2}$ (long dashed line), $10^{11}$ cm$^{-2}$ (dashed line), and $10^{10}$ cm$^{-2}$ (dotted line). The calculations with Eq. (13) refer to GaAs QW with $L_z = 100$ A and $\tau_{sc} = 13.9$ ns.

The thermalization time $\tau_{th} = \tau_{th}(a)$ is very sensitive to the dimensionless parameter $a \propto L_z$ through the form-factor function $F_\varepsilon$ on the r.h.s. of Eq. (13). With decreasing $L_z$ the spectral width of $F_\varepsilon(a\sqrt{\varepsilon(\varepsilon - 1)})$ increases indicating a stronger relaxation of the momentum conservation in the z-direction. From Eqs. (17) and (18) we conclude that for the infinite square QW confinement potential $\tau_{th} \propto L_z^2$ and $\tau_{th} \propto L_z$, respectively. The dependences $\tau_{th} = \tau_{th}(L_z)$ and $\tau_{th} = \tau_{th}(L_z)$ are plotted in the inset of Fig. 2.

B. Low-temperature limit ($k_BT_b < E_0$). In this case we get from Eq. (17):

$$\frac{1}{\tau_{th}} = \frac{2\pi^{3/2}}{\tau_{sc}} \left( \frac{E_0}{k_BT_0} \right)^{1/2} e^{-E_0/k_BT_0} = \left( \frac{T_b}{T_0} \right) \frac{1}{\tau_{th}}. \quad (20)$$

where we designate the low-temperature thermalization time of degenerate QW excitons by $\tau_{th}$ and the corresponding thermalization time of Maxwell-Boltzmann distributed QW excitons $\tau_L$ is given by Eq. (16). The slowing down of the thermalization process by the factor $T_0/T_b \propto \rho_{2D}$, which is much larger than unity, stems from well-developed BE statistics of the ground
state mode, i.e., from $N_{E=0} \gg 1$. The distribution function of QW excitons at $E \geq E_0$ is classical, i.e., $N_{E \geq E_0} = \exp(-E_0/k_BT) < 1$, and does not enhance the relaxation processes into the ground-state mode $E = 0$.

$$\frac{\partial T}{\partial t} = -\frac{2\pi^{3/2}}{\tau_{sc}} \left( \frac{T^2}{T_0} \right) \left( \frac{k_BT}{E_0} \right)^{1/2} e^{-E_0/k_BT}.$$

In dimensionless time and temperature units $\tau = (2\pi^{3/2}E_0t)/(k_BT_0\tau_{sc})$ and $\tilde{T} = k_BT/E_0$ Eq. (22) takes the canonical form $d\tilde{T}/d\tau = -\tilde{T}^{5/2}\exp(-1/\tilde{T})$. The solution $T = T(t)$ of Eq. (22) is given by the transcendental equation:

$$F(k_BT/E_0) = \frac{2\pi^{3/2}}{\tau_{sc}} \left( \frac{E_0}{k_BT_0} \right) t + A_i,$$

where $F(x) = [1/(\sqrt{x}) - D_0(1/\sqrt{x})]^{1/3}$, $D_0(y) = e^{-y^2} \int_0^y e^{t^2} dt$ is Dawson’s integral, and the integration constant $A_i$ is defined by the initial condition $T_i = T(t = 0) \leq E_0/k_B$, i.e., $A_i = F(k_BT_i/E_0)$. For $t \gg [(k_BT_0)/(2\pi^{3/2}E_0)]\tau_{sc}$ the asymptotic solution of Eq. (24) is

$$k_BT(t) = \frac{E_0}{\ln \left( \frac{2\pi^{3/2}E_0t}{(k_BT_0\tau_{sc})} \right)},$$

i.e., $\tilde{T}(\tau) = 1/\ln(\tau)$ for $\tau \gg 1$.

The asymptotic law Eq. (24) characterizes the nonexponential and extremely slow thermalization kinetics of a quantum gas of QW excitons. Because the phonon occupation numbers $n^b_{\pi} = 0$ at $T_b = 0$, only Stokes scattering of the QW excitons determines the relaxation process. In this case the integrand on the r.h.s. of Eq. (3) is proportional to $N_{E \geq E_0}(N_{E=0} + 1) \simeq N_{E \geq E_0} N_{E=0}$. At $k_BT \leq E_0$ the energy states $E \geq E_0$, which couple with the ground-state mode $E = 0$ [see Eq. (3)], are weakly populated by the QW excitons, while $N_{E=0} \gg 1$. The Maxwell-Boltzmann distribution function $N_{E \geq E_0} = \exp(-E/k_BT)$ gives rise to the factor $\exp(-E_0/k_BT) \ll 1$ on the r.h.s. of Eq. (24). This term together with the need to accumulate a huge number of QW excitons in the ground-state mode $p_0 = 0$, due to BE statistics, are responsible for $1/\ln(\tau)$ thermalization law at $T_b = 0$.

In Fig. 5 we plot the results of numerical evaluation of the relaxation dynamics at $T_b = 0$ from Eq. (24) for various concentrations $\rho_{2D}$ of QW excitons and for an initial effective temperature $T_i = 100 \text{ K}$. The simulations refer to the long-lived quasi-2D excitons in GaAs/AlGaAs CQWs. The first hot relaxation (see the inset of Fig. 5) is completed within $\Delta t \leq 30 - 100 \text{ ps}$. At the end of the transient stage the effective temperature $T$ of QW excitons is still much higher than $E_0/k_B \simeq 0.54 \text{ K}$ so that $N_{E \geq E_0} > 1$ for $T \leq T_b$. As a result, the duration of hot thermalization decreases with increasing concentration $\rho_{2D}$ of QW excitons. Here one has an acceleration of the relaxation kinetics due to BE statistics of energy modes $E \geq E_0$. The further thermalization kinetics of cold QW excitons refers to $t \geq 100 \text{ ps}$ and reveals $1/\ln(\tau)$ law (see Fig. 5). According to Eq. (24), critical slowing down of the phonon-assisted relaxation dynamics develops with

---

**FIG. 4.** The thermalization time $\tau_{th} = \tau_{th}(\rho_{2D})$ of BE-distributed QW excitons at the bath temperatures 3 K (solid line), 5 K (dotted-dashed line), 10 K (long dashed line), 20 K (dash-dotted line), and 50 K (dotted line). The calculations with Eq. (13) deal with GaAs QW with $L_z = 100 \text{ A}$ and $\tau_{sc} = 13.9 \text{ ns}$. Note that numerical evaluations of Figs. 2-4 refer to effective temperatures of QW excitons close to the bath temperature, i.e., $|T - T_b| \ll T_b$.

The influence of BE statistics on the thermalization of QW excitons is demonstrated in Fig. 3 for $\tau_{th} = \tau_{th}(T_b)$, $\rho_{2D} = \text{const.}$, and in Fig. 4 for $\tau_{th} = \tau_{th}(\rho_{2D})$, $T_b = \text{const.}$, respectively. The numerical evaluations of $\tau_{th}$ refer to Eq. (13).

**B. Thermalization of QW excitons at $T_b = 0$**

The linearization of the basic Eq. (12) at $T_b > 0$, developed in previous subsection, does not hold when $|T - T_b| > k_BT_b^2/E_0$ and especially at zero bath temperature. Because $N_{E=0} \to \infty$ at $T_b = 0$, the effective temperature $T$ of QW excitons is a nonanalytic point of the r.h.s. of Eq. (11) at $T_b = 0$. For $T_b = 0$ Eq. (11) reduces to

$$\frac{\partial T}{\partial t} = -\frac{2\pi^{3/2}}{\tau_{sc}} \left( \frac{T^2}{T_0} \right) \left( 1 - e^{-T_b/T} \right) \int_1^\infty d\varepsilon \varepsilon \sqrt{\varepsilon - 1}$$

$$\times \left( \sqrt{\varepsilon} - 1 \right)^2 \frac{\pi^{3/2}}{\tau_{sc}} \left( \frac{T^2}{T_0} \right) \left( \frac{k_BT}{E_0} \right)^{1/2} e^{-E_0/k_BT}.$$

Equation (23), which describes how the QW excitons with effective temperature $T$ cool down towards $T_b = 0$, can be further simplified for $k_BT \leq E_0$:
increasing degeneracy temperature $T_b \propto \rho_{2D}$. An extension of the numerical evaluations presented in Fig. 5 on $\mu$s time scale shows that even at $\Delta t = 1 \mu$s the effective temperature of QW excitons is still a few hundred mK for $\rho_{2D} \geq 10^{11} \text{ cm}^{-2}$, i.e., $T(t = 1 \mu$s) = 89 mK for $\rho_{2D} = 10^{10} \text{ cm}^{-2}$, $T(t = 1 \mu$s) = 129 mK for $\rho_{2D} = 10^{11} \text{ cm}^{-2}$, $T(t = 1 \mu$s) = 181 mK for $\rho_{2D} = 5 \times 10^{11} \text{ cm}^{-2}$, and $T(t = 1 \mu$s) = 213 mK for $\rho_{2D} = 10^{12} \text{ cm}^{-2}$. On a time scale much longer than the duration of the initial hot thermalization the effective temperature $T$ is nearly independent of $T_i = T(t = 0)$, provided that $k_B T_i \gg E_0$. Recently, a strong increase of the thermalization times in a highly degenerate quantum gas of quasi-2D excitons has indeed been observed in GaAs/AlGaAs CQWs.\[\text{FIG. 5. Thermalization dynamics} T = T(t) \text{ of QW excitons at} T_b = 0, \rho_{2D} = 10^{12} \text{ cm}^{-2} \text{ (solid line),} 5 \times 10^{11} \text{ cm}^{-2} \text{ (long dashed line),} 10^{11} \text{ cm}^{-2} \text{ (dashed line), and} 10^{10} \text{ cm}^{-2} \text{ (dotted line). Inset: the initial transient thermalization of hot QW excitons BE-distributed at} t = 0 \text{ with} T_i = 100 \text{ K.}\]

However, our treatment has neglected possible low-temperature collective states due to interactions between QW excitons. At a critical temperature $T_c = \alpha T_0$, where $\alpha = \alpha(\rho_{2D}^{(2D)} a_z^{(2D)})^{1/2} < 1$, a system of quasi-2D excitons may undergo a phase transition to a superfluid state. Our calculations of relaxational thermodynamics become invalid at $T \leq T_c$, i.e., one cannot trace with Eq. (21) the transition to the excitonic superfluid phase and how further a collective ground state of QW excitons arises at $T \to 0$ for $\rho_{2D} [a_z^{(2D)}]^2 < 1$ the ground state can be interpreted in term of BE condensation of QW excitons. While in the dilute limit $\ln((\rho_{2D} [a_z^{(2D)}]^2) \ll 1$ the parameter $\alpha \ll 1$, i.e., $T_c \ll T_0$, there is still no first-principle theory to estimate $\alpha$ for the densities $\rho_{2D} [a_z^{(2D)}]^2 \leq 1$. With the mentioned above restrictions, we find that cooling of high-density QW excitons to very low temperatures is rather difficult. This conclusion may have some bearing on the search for the collective ground state of indirect excitons in GaAs/AlGaAs CQWs.

However, our treatment has neglected possible low-temperature collective states due to interactions between QW excitons. At a critical temperature $T_c = \alpha T_0$, where $\alpha = \alpha(\rho_{2D}^{(2D)} a_z^{(2D)})^{1/2} < 1$, a system of quasi-2D excitons may undergo a phase transition to a superfluid state. Our calculations of relaxational thermodynamics become invalid at $T \leq T_c$, i.e., one cannot trace with Eq. (21) the transition to the excitonic superfluid phase and how further a collective ground state of QW excitons arises at $T \to 0$ for $\rho_{2D} [a_z^{(2D)}]^2 < 1$ the ground state can be interpreted in term of BE condensation of QW excitons. While in the dilute limit $\ln((\rho_{2D} [a_z^{(2D)}]^2) \ll 1$ the parameter $\alpha \ll 1$, i.e., $T_c \ll T_0$, there is still no first-principle theory to estimate $\alpha$ for the densities $\rho_{2D} [a_z^{(2D)}]^2 \leq 1$. With the mentioned above restrictions, we find that cooling of high-density QW excitons to very low temperatures is rather difficult. This conclusion may have some bearing on the search for the collective ground state of indirect excitons in GaAs/AlGaAs CQWs.

C. Thermalization dynamics of hot QW excitons

($T \gg T_b$)

In experiments [1-9] with nonresonant excitation of heavy-hole excitons in GaAs QWs the initial effective temperature $T_i \gg T_b$. While in this case the thermalization kinetics can be analyzed numerically from the basic Eq. (11), we will clarify the various relaxation scenarios by analytic approximations of Eq. (11).

1. Classical gas of QW excitons ($T_i \gg T_b \gg T_0$).

A. High-temperature limit ($k_B T_b \gg E_0$). In this case Eq. (11) reduces to

$$\frac{\partial}{\partial t} T = -\frac{1}{\tau_H} (T - T_b) ,$$

(25)

where the characteristic thermalization time $\tau_H$ is given by Eq. (14). Thus, in the high-temperature limit $k_B T_b \gg E_0$ the exponential law $T(t) = T_i + (T_i - T_b) \exp(-t/\tau_H)$ is valid even for $(T_i - T_b)/T_b \gg 1$.

B. Low-temperature limit ($k_B T_b < E_0$). The initial hot thermalization down to $k_B T \approx E_0$ is approximated by

$$\frac{\partial}{\partial t} T = -\frac{T}{\tau_L} .$$

(26)

This exponential cooling $T(t) = T_i \exp(-t/\tau_L)$ completes at $t' \approx \tau_L \ln(T_i/T_b)$. The hot thermalization refers to the high-temperature limit given by Eq. (25). At $t > t'$ the cold QW excitons relax to the bath temperature according to

$$\frac{\partial}{\partial t} T = -\frac{1}{\tau_L} (T - T_b) ,$$

(27)

where $\tau_L$ is the characteristic relaxation time of the excitonic superfluid.
where the low-temperature thermalization time \( \tau_L \) is defined by Eq. (16). Approximation (27) is valid for \((T - T_b)/T_b < k_BT_b/E_0\), yields the exponential relaxation with \( \tau_L \), and corresponds to the linearization of Eq. (1) at \( T = T_b \).

If the high-temperature limit does not hold, the thermalization kinetics of Maxwell-Boltzmann distributed QW excitons is exponential only locally in time, i.e., \( \tau_{th} = \tau_{th}(t) \). The described above picture of the relaxation in the low-temperature limit in fact deals with a continuous increase of the instant thermalization time \( \tau_{th}(t) \) from \( \tau_H \) to \( \tau_L \). As a result, for \( T_1 - T_b > T_b \) the total relaxation process can show a nonexponential behavior. In Fig. 6 we illustrate how a single-exponential thermalization kinetics with \( \tau_H = \tau_{th} \) of a classical gas of QW excitons initially distributed at \( T_1 \gg T_b \) develops with the decreasing bath temperature towards the nonexponential relaxation.

\[ T(t) = \frac{T_0}{1 + 2(T_0/T_b)^2[(t - t')+\tau_{0H}]}^{1/2}. \quad (29) \]

The nonexponential transient lasts for a time \( \Delta t \approx \tau_{0H}/2 \). The relaxation kinetics from \( t \geq t' \approx \tau_{0H}/2 \) refers to \( T - T_b < T_b \) and recovers the exponential law with \( \tau_{0H} \).

B. Low-temperature limit \((k_BT_b < E_0)\). At \( 0 \leq t \leq t' \approx \tau_H \ln(T_i/T_0) \) the first hot thermalization from \( T = T_i \) to \( T \approx T_0 \) is characterized by the exponential law of Eq. (24). The next nonexponential transient relaxation from \( T \approx T_0 \) to \( T \approx E_0/k_BT_b \) is given by Eq. (29) and corresponds to the time interval \( t' \leq t \leq t'' \approx (k_BT_b/E_0)^2(\tau_{0H}/2) \). These two stages are similar to those found in the high-temperature limit. For the thermalization from \( T \approx E_0/k_BT_b \) to \( T \approx T_b \) one recovers Eq. (23) and the corresponding 1/\( \ln(\tau) \) kinetics of Eq. (24) treated in the previous subsection for \( T_b = 0 \). This stage is rather long and completes at \( t'''' \approx (k_BT_bT_{sc}/2\tau_B^3/E_0)^{-1} \exp(E_0/k_BT_bT_{sc}) \). The final exponential relaxation refers to \( (T - T_b)/T_b < k_BT_b/E_0 \) and is given by \( T(t' \geq t''') = T_b + (k_BT_b^2/E_0)\exp[-(t - t''')/\tau_{0L}] \), where the low-temperature thermalization time \( \tau_{0L} \) of the degenerate QW excitons is defined by Eq. (24).

In order to demonstrate how BE statistics can influence the thermalization kinetics of QW excitons at \( T_1 \gg T_b \), in Fig. 7 we plot the numerical solution \( T = T(t) \) of Eq. (11) for \( T_1 = 50 \text{ K}, T_b = 4.2 \text{ K} \), and various values of the concentration \( \rho_{2D} \). The development of the nonexponential relaxation with the increasing degeneracy temperature \( T_0 \propto \rho_{2D} \) is clearly seen. The inset of Fig. 7 illustrates the transient thermalization given by Eq. (24).

IV. RESONANT PHOTOLUMINESCENCE OF QUANTUM DEGENERATE QUASI-2D EXCITONS

In high-quality GaAs QWs the decay of quasi-2D excitons is mainly due to radiative recombination.\[ \text{Moreover, following the first studies of} \] the optical properties of excitons in GaAs/AlGaAs CQWs, very recent experiments\[ \text{show that the radiative recombination channel can also be dominant for the long-lived indirect excitons. In this section we generalize the theory of steady-state resonant PL of a quasi-equilibrium classical gas of QW excitons to well-developed BE statistics and exploit the relaxation thermodynamics of Eq. (11) in order to analyze the PL kinetics of QW excitons.}\]
\( \omega \) is the frequency of light and \( \epsilon_b \) is the background dielectric constant for an exciton line. This means that the radiative zone of QW excitons is given by \( p_l \ll k_0 \), where \( k_0 = k(\omega) \) corresponds to crossover of the photon and exciton dispersions and \( \hbar \omega_e \) is the exciton energy. The intrinsic radiative rates for in-plane transverse (T-polarized) and in-plane longitudinal (L-polarized) dipole-active QW excitons are given by

\[
\Gamma_T(p_l) = \frac{\Gamma_0}{\sqrt{k_0^2 - p_l^2}} \quad \text{and} \quad \Gamma_L(p_l) = \frac{\Gamma_0}{\sqrt{k_0^2 - p_l^2}},
\]

(30)
respectively. The radiative lifetime \( \tau_R \) of a QW exciton is defined by \( \tau_R = 1/\Gamma_0 \), where \( \Gamma_0 = \Gamma_0(L_z) = \Gamma_T(p_l = 0) = \Gamma_L(p_l = 0) \) is the radiative rate for the ground-state mode \( p_l = 0 \). \( Z \)-polarized heavy-hole QW excitons are forbidden. Here we use the polarization notations of Ref. [4].

The total radiative decay rate \( \Gamma_{opt} = 1/\tau_{opt} \) of a gas of quasi-equilibrium BE-distributed QW excitons is

\[
\Gamma_{opt} = \frac{1}{\rho_{2D}} \int_0^{k_0} N_{p_l} \left[ \Gamma_T(p_l) + \Gamma_L(p_l) \right] \frac{dp}{2\pi}, \quad (31)
\]

where the occupation number \( N_{p_l} = N_{E_p} \) is given by Eq. (1). Equation (31) takes into account the equal probabilities of the T- and L-polarizations and two-fold spin degeneracy, \( \sigma^+ \) and \( \sigma^- \), of dipole-active QW excitons. Using Eqs. (1) and (31) we find from Eq. (31):

\[
\Gamma_{opt} = \frac{\Gamma_0}{2} \left[ J_T(T, T_0) + J_L(T, T_0) \right],
\]

\[
J_T(T, T_0) = \left( \frac{E_{k_0}}{k_B T_0} \right) \int_0^1 dz \frac{dz}{A e^{-z E_{k}^{\sigma}/k_B T} - 1},
\]

\[
J_L(T, T_0) = \left( \frac{E_{k_0}}{k_B T_0} \right) \int_0^1 dz \frac{z^2 dz}{A e^{-z E_{k}^{\sigma}/k_B T} - 1}, \quad (32)
\]

where \( A = A(T, T_0) = e^{E_{k_0}/k_B T}/(1 - e^{-E_{k_0}/T}) \) and \( E_{k_0} = h^2 k_0^2/2M_e \) (for GaAs QWs with \( \epsilon_b = 12.9 \) and \( \hbar \omega_e = 1.55 \) eV one has \( E_{k_0} \approx 101 \) \( \mu eV \) and \( E_{k_0}/k_B \approx 1.17 K \)). The contribution of two-fold dipole-inactive (triplet) QW excitons to the total density \( \rho_{2D} \) is included in Eq. (32) through the degeneracy temperature \( T_0 \propto \rho_{2D} \).

For Maxwell-Boltzmann distributed QW excitons at the effective temperature \( T \) much larger than \( T_0 \) and \( E_{k_0}/k_B \) we find from Eq. (32) that \( J_T = 3J_L = (h^2 k_0^2)/(M_e T) \) and

\[
\Gamma_{opt} = \left( \frac{h^2 k_0^2}{3M_e k_B T} \right) \Gamma_0. \quad (33)
\]

In the limit \( T \ll T_0 \) of a strongly degenerate gas of QW excitons one approximates \( J_T = 1+(T/T_0)\ln(4E_{k_0}/k_B T) \) and \( J_L = 1+(T/T_0)\ln(4E_{k_0}/k_B T) - 2 \). In this case \( \Gamma_{opt} \) of Eq. (32) reduces to

\[
\Gamma_{opt}^T = \left[ 1 - \frac{T}{T_0} + \frac{T}{T_0} \ln \left( \frac{2h^2 k_0^2}{M_e k_B T} \right) \right], \quad (34)
\]

The expansion of \( J_L(T, T_0) \), \( J_T(T, T_0) \), and \( \Gamma_{opt} \) in terms of the ratio \( T/T_0 \ll 1 \) is valid for the effective temperature of QW excitons \( T \leq E_{k_0} \exp(-T_0/T) \). The second and third terms in the square brackets on the r.h.s. of Eq. (34) are small corrections, i.e., \( \Gamma_{opt}^T(T \to 0) \to \Gamma_0/2 \). The non-zero limit of \( \Gamma_{opt} \) at \( T \to 0 \), which is completely determined by the intrinsic radiative rate \( \Gamma_0 \) of the ground-state mode \( p_l = 0 \), is due to the effective accumulation of low-energy QW excitons with \( N_E \gg 1 \) in the radiative zone \( p_l \ll k_0 \).

In the limit \( T \gg T_0, E_{k_0} \) of a classical behavior one derives from Eqs. (31)-(32):

\[
\tau_{cl} = \left( \frac{3M_e k_B T}{\hbar^2 k_0^2} \right) \tau_R + \left( \frac{9}{10} - \frac{3M_e k_B T_0}{2\hbar^2 k_0^2} \right) \tau_R, \quad (35)
\]

where the expansion of \( \tau_{cl} = 1/\Gamma_{opt} \) we keep not only the leading term \( 1/\Gamma_{opt} \propto T \) [see Eq. (33)], but also the temperature-independent correction. The next terms of the expansion are proportional to \( 1/T^{n=1} \to 0 \) and can indeed be neglected. The first term on the r.h.s. of Eq. (35), i.e., \( 1/\Gamma_{opt}^T = (3M_e k_B T_0)/\hbar^2 k_0^2 \), is a well-known result of Ref. [4]. The temperature-independent correction, which is given by the second term on the r.h.s. of Eq. (35), originates from the BE distribution function used in Eq. (31) and consists of the density-independent and density-dependent contributions of the opposite signs. The density-dependent contribution can be much larger than \( \tau_R \) if \( k_B T \gg k_B T_0 \gg E_{k_0} \). In the leading term \( 1/\Gamma_{opt} \propto T \) one has that the optical decay time \( \tau_{cl} \gg \tau_R \), because only a small fraction of QW excitons occupy the radiative modes \( p_l \ll k_0 \) at

\( T \gg T_0, E_{k_0} \).

In Fig. 8 we plot the effective decay time of quasi-equilibrium QW excitons \( \tau_{opt} = \tau_{opt}(T) \), calculated numerically with Eq. (32) for various values of the concentration \( \rho_{2D} \). Following Ref. [4] for a GaAs QW with \( L_z = 100 \) \( A \) we use \( \tau_R = 25 \) ps. For \( T \gg T_0, E_{k_0}/k_B \) the linear behavior given by Eq. (35) is clearly seen (dotted, dashed, and long dashed lines in Fig. 8). The reference bold solid line indicates \( 1/\tau_{opt}^T \propto T \) law. Note that the linear asymptotics of \( \tau_{opt} = \tau_{opt}(T, \rho_{2D}) \) from the classical range \( T \gg T_0, E_{k_0}/k_B \) down to low temperatures \( T \to 0 \) reveal nonzero density-dependent values. Thus one concludes that even at high temperatures \( T \gg T_0 \) the influence of nonclassical statistics can be found in the PL of QW excitons. At \( T \to 0 \) the limit \( \tau_{opt} \to \tau_R [9/10 - (3E_{k_0}/4E_{k_0})] \) given by Eq. (35) breaks and all curves approach the double intrinsic radiative lifetime \( 2\tau_R \).

The density-dependent deviation
from the linear classical law of Eq. (35) develops with increasing $\rho_{2D}$. For example, at $\rho_{2D} = 5 \times 10^{11} \text{ cm}^{-2}$ the quantum asymptotics $\tau_{\text{opt}}^q(T \rightarrow 0) = 2\tau_R$ can be traced already for the effective temperature $T \approx 10 \text{ K}$ (see Fig. 8).

According to Eq. (30), the decay rate of T-polarized QW excitons diverges at $p || \rightarrow k_0$ in Appendix B. We show how the homogeneous linewidth of low-energy QW excitons $\hbar \Gamma_{h\text{om}} = \hbar / \tau_{x-x} + \hbar / \tau_{ph} + \hbar / \tau_R \simeq \hbar / \tau_{x-x}$ removes this divergence [see Eq. (B6)]. In the presence of homogeneous broadening, due to dominant exciton-exciton scattering, $\Gamma_T(p ||)$ of Eq. (30) changes only in a very narrow band of QW states given by $0 \leq k_0 - p || \leq \tilde{\gamma} k_0$, where $\tilde{\gamma} = 1 / (\tau_{x-x} \omega_i) \ll 1$. In particular, $\Gamma_T(p || \rightarrow k_0) = \Gamma_{0} / (2 \tilde{\gamma}^{1/2})$ rather than diverges. Furthermore, homogeneous broadening of QW excitons from the radiative zone only slightly renormalizes the PL efficiency: $\Gamma_{\text{opt}}' = \Gamma_{\text{opt}} / [1 - (3/4) \tilde{\gamma}^{1/2}]$ [for details see Appendix B]. The correction is even less for $\Gamma_{\text{opt}}^q$ of Eq. (34).

Therefore we conclude that the corrections to the optical decay rate $\Gamma_{\text{opt}}$ due to the homogeneous linewidth $\Gamma_{h\text{om}}$ can indeed be neglected. Note, however, that inhomogeneous broadening strongly influences the PL process and changes our results obtained for a perfect QW.

The optical decay of QW excitons from the radiative zone $p || \leq k_0$ does not violate the assumptions of relaxational thermodynamics provided that $\tau_{x-x} \ll \tau_R$. In this case the thermal quasi-equilibrium distribution of the finite lifetime QW excitons at $E \leq E_{k0}$ holds due to exciton-exciton scattering. Because both the minimal thermalization time $\tau_{th}$ given by Eq. (13) and the intrinsic radiative time $\tau_R$ are on the same time scale of $5 - 50 \text{ ps}$, the condition $\tau_{x-x} \ll \tau_H$ [see Eq. (3)] of the thermodynamic picture also nearly guarantees $\tau_{x-x} \ll \tau_R$.

The radiative decay leads to the continuous decrease of the density of QW excitons (or the degeneracy temperature $T_0 \propto \rho_{2D}$) and gives rise to the PL signal according to

$$\frac{\partial}{\partial t} \rho_{2D} = -\Gamma_{\text{opt}}(T_0,T) \rho_{2D} = - \frac{\rho_{2D}}{\tau_{\text{opt}}(\rho_{2D},T)}, \quad (36a)$$

$$I_{PL} = - \hbar \omega_i \frac{\partial \rho_{2D}}{\partial t} = \hbar \omega_i \frac{\rho_{2D}}{\tau_{\text{opt}}(\rho_{2D},T)}, \quad (36b)$$

where $I_{PL} = I_{PL}(t)$ is the photoluminiscence intensity. Thus Eqs. (11), (24), and (36a)-(36b) describe the PL kinetics of QW excitons cooling from the initial effective temperature $T_i$ to the bath $T_0$. For a high-temperature classical gas of QW excitons the optical decay time $\tau_{\text{opt}}^q$ of Eq. (34) is approximated by the density-independent $1 / \Gamma_{\text{opt}}^q$ of Eq. (33) and the solution of Eq. (36a) is simply $\rho_{2D}(t) = \rho_{2D}^{(0)} \exp(- \int_0^t \Gamma_{\text{opt}}^q(t')dt')$, where $\rho_{2D}^{(0)}$ is the initial concentration of QW excitons and $\Gamma_{\text{opt}}^q$ is time-dependent through the effective temperature of QW excitons $T = T(t)$. In the general case, however, Eq. (36a) gives a nonexponential decay of the density. The characteristic times of the both fundamental processes which contribute to the PL kinetics, LA-phonon-assisted thermalization and optical decay, generally depend upon the effective temperature and density of QW excitons, i.e., $\tau_{th} = \tau_{th}(T, T_0)$ and $\tau_{\text{opt}} = \tau_{\text{opt}}(T, T_0)$. As a result, the PL kinetics given by Eqs. (11), (24), and (36a)-(36b) is also $T$- and $\rho_{2D}$-dependent. The thermalization process and radiative decay work in the opposite directions with respect to the quantum-statistical effects. Cooling of QW excitons is accompanied by an increasing number of low-energy particles towards well-developed BE statistics. In the meantime $\tau_{\text{opt}}$ decreases and the optical decay of $\rho_{2D}$ speeds up resulting in decreasing $T_0$. Thus the radiative processes interfere with the development of BE statistics.

For a classical gas of QW excitons in the high-temperature limit $k_B T_0 \gg E_0$ the PL dynamics of Eqs. (11), (24), and (36a)-(36b) can be analyzed analytically. In this case, $T(t) = T_b + (T_i - T_b) \exp(-t / \tau_{th})$, where the temperature-independent $\tau_{th}$ is given by Eq. (13). At the first transient thermalization of hot QW excitons from the initial distribution at effective $T_i \gg T_b$ one has that $\tau_{\text{opt}}^q(T_i) \gg \tau_{th}$, i.e., the optical decay is very slow and practically does not change the concentration of QW excitons. The transient stage lasts a few $\tau_{th}$ and for this time domain Eq. (36a) reduces to

$$I_{PL}(t \leq t_{tr}) = \hbar \omega_i \frac{\hbar^2 k_0^2}{3M x k_BT_i} \rho_{2D}^{(0)} e^{t / \tau_{th}}$$

$$\equiv \hbar \omega_i \rho_{2D}^{(0)} e^{t / \tau_{\text{opt}}^q(T_i)} e^{t / \tau_{th}}, \quad (37)$$

i.e., $I_{PL}(t \leq t_{tr}) \propto \exp(t / \tau_{th})$. Therefore at the hot relaxation stage $t \leq t_{tr}$ the PL intensity exponentially
increases with the rate $1/\tau_H$, due to the population of the radiative zone. The transient stage completes when $T(t_\tau) - T_b < T_b$. The PL intensity gets its maximum at $t = t_\tau$, when the population efficiency of the modes $p_\parallel \leq k_0$ is compensated by the increasing optical decay:

$$
t_0 = \tau_H \ln \left( \frac{3M_e k_B T_i}{\hbar^2 k_0^2} \right) \equiv \tau_H \ln \left( \frac{\tau_{\text{cl}}^{1/2}}{\tau_H} \right). \quad (38)
$$

The corresponding effective temperature of QW excitons is $T(t_0) = T_b [1 + \tau_H/\tau_{\text{cl}}^{1/2}(T_i)]$, i.e., $T(t_0) - T_b \ll T_b$. At $t = t_\tau > t_0$ the PL dynamics becomes steady-state and is determined by the optical decay, because the QW excitons are already thermalized at $T = T_b$. For this time domain the PL kinetics is given by

$$
I_{PL}(t \geq t_\tau) = \hbar \omega_i \left( \frac{h^2 k_0^2}{3M_e k_B T_b} \right) \rho_{2D}^{(s)}(t) e^{-t/\tau_{\text{cl}}^{1/2}(T_b)}
\equiv \hbar \omega_i \frac{\rho_{2D}^{(s)}(t)}{\tau_{\text{cl}}^{1/2}(T_b)} e^{-t/\tau_{\text{cl}}^{1/2}(T_b)}, \quad (39)
$$

where $\rho_{2D}^{(s)} = \rho_{2D}(t_\tau) \simeq \rho_{2D}^{(0)}$. Thus at the steady-state regime $I_{PL}(t \geq t_\tau) \propto \exp[-t/\tau_{\text{cl}}^{1/2}(T_b)]$, i.e., the PL intensity decreases exponentially with the optical decay rate $\tau_{\text{cl}}^{1/2}(T_b)$. Note that because usually $T_\tau \gg T_0$ and $T_0(t \rightarrow 0) = 0$, the very first transient stage and the very last steady-state decay of excitonic PL always follow Eqs. (37) and (39), respectively.

In Fig. 9a we plot numerical simulations of the PL dynamics done with Eqs. (11), (32), and (36a)-(36b) for excitons of various initial densities in GaAs QW with $L_z = 100 \text{ Å}$. In order to model the experimental data on the density-dependent PL kinetics plotted in Fig. 2 of Ref. [4a] we use $T_\tau = 85 \text{ ns}$ and $\tau_R = 40 \text{ ps}$. The change of the PL dynamics with increasing $\rho_{2D}^{(0)}$ shown in Fig. 9a reproduces qualitatively the corresponding experimental observations of Ref. [4a]. The decrease of the rise time $t_0$ of the PL signal with the increasing initial density of QW excitons is mainly due to increase of the optical decay rate at high densities (see the inset of Fig. 9a). This effect originates from BE statistics of QW excitons.

In Fig. 9b we show numerical modeling of PL kinetics in GaAs/AlGaAs CQWs with the intrinsic radiative lifetime of long-lived excitons $\tau_R = 30 \text{ ns}$. Here BE statistics strongly influences the PL temporal behavior in the steady-state regime at $t \geq t_\tau$, but still before a time when indirect excitons become Maxwell-Boltzmann distributed due to continuous decrease of $\rho_{2D}$. At this time domain the thermalization kinetics of high-density CQW excitons undergoes critical slowing down, the effective temperature of CQW excitons is nearly stabilized at $T_0 > T > T_b$, and the occupation numbers of the radiative modes $N_{p_1 \leq k_0} \gg 1$. As a result, with the increasing initial concentration $\rho_{2D}^{(0)}$ the decay rate of the PL signal approaches $\tau_{\text{cl}}^{1/2} = 2\tau_R$, i.e., one has an acceleration of the PL optical decay owing to well-developed BE statistics [see Fig. 9b, where the long dashed and solid reference lines indicate $\ln(I_{PL}/I_0) = (t_0 - t)/2\tau_{\text{cl}}(T_b)$ and $\ln(I_{PL}/I_0) = (t_0 - t)/2\tau_R$, respectively].

![FIG. 9. Photoluminescence dynamics of QW excitons modeled with Eqs. (11), (32), and (36a)-(36b) for GaAs QW with $L_z = 100 \text{ Å}, D = 6.3 \text{ eV}, \tau_{\text{sc}} = 85 \text{ ns}, \tau_R = 40 \text{ ps}, T_b = 5 \text{ K}, \rho_{2D} = 10^9 \text{ cm}^{-2}$ (dashed line), $10^{11} \text{ cm}^{-2}$ (dot-dashed line), and $5 \times 10^{11} \text{ cm}^{-2}$ (solid line). Inset: change of the optical decay time $\tau_{\text{cl}}(T_b) = \tau_{\text{cl}}(T_\tau)$. (b) GaAs/AlGaAs CQWs with $T_{\text{sc}} = 13.9 \text{ ns}, \tau_R = 30 \text{ ns}, T_b = 3 \text{ K}, \rho_{2D} = 10^8 \text{ cm}^{-2}$ (bold dashed line), and $5 \times 10^{11} \text{ cm}^{-2}$ (bold solid line). The long dashed and solid straight lines refer to the exponential kinetics given by $I_{PL}(t) \propto \exp[-t/\tau_{\text{cl}}(T_b)]$ and $I_{PL}(t) \propto \exp[-t/(2\tau_R)]$, respectively. The PL intensities $I_{PL}(t)$ of (a) and (b) are normalized by $I_0 = 248.31 \text{ kW/cm}^2$.](image)
V. DISCUSSION

There are three neglected factors, the interface polariton effect, the low-energy biexciton states, and residual interface disorder, which can affect the developed model.

In perfect QWs, the radiative modes \( p_\parallel < k(\omega) \), which are responsible for the optical decay of QW excitons in bulk photons, are accompanied by confined modes \( p_\parallel > k(\omega) \) located outside the photon cone. These modes give rise to QW interface polaritons. While the QW polaritons cannot be seen in standard optical experiments, which deal with bulk photons, they can modify the quadratic in-plane dispersion of QW excitons and contribute to the total optics in a “hidden” way. Therefore in the general case both conjugated phenomena, the radiative decay into bulk photons and the QW polariton effect, should be treated simultaneously. The dispersion law of \( T \)-polarized QW polaritons is \( \omega^2 \left[ 1 + \left( R_c \sqrt{p_\parallel^2 - \omega \omega_p^2} / (\omega_p^2 + \hbar \omega_p^2 / M_x - i \Gamma_x \omega - \omega_p^2) \right) \right] \), where \( R_c = (\kappa \sigma f_{xy}) / (\kappa \sigma m_0 \omega) \), \( f_{xy} \) is the oscillator strength of exciton-photon coupling per unit area of a QW (the subscript \( \parallel \) refers to the in-plane polarization of the light), and \( \Gamma_x \) is the rate of incoherent scattering of QW excitons. Actually the dispersion equations can be applied to both radiative and confined modes. Within the photon cone the solution \( \omega = \omega(p_\parallel) \) is \( \omega = \omega_\parallel - i \Gamma_T(p_\parallel) / 2 \), where \( \Gamma_T(p_\parallel) \) is given by Eq. (30) with \( \Gamma_0 = (\sqrt{\sigma_0 \kappa / c} \) \( R_c \). The dispersion \( \omega = \omega_T(p_\parallel) \) of \( T \)-polarized QW polaritons is located outside the photon cone, approaches the quadratic dispersion \( \hbar p_\parallel^2 / 2M_x \) at \( p_\parallel \gg k_0 \), and continuously transforms to the photon dispersion \( c p_\parallel / \sqrt{\varepsilon} \) at \( p_\parallel \ll k_0 \). Thus during thermalization the QW excitons can accumulate at the “bottleneck” band of the T-polariton dispersion rather than in the radiative zone given by \( \omega = \omega_\parallel + \hbar p_\parallel^2 / 2M_x \) for \( p_\parallel \ll k_0 \).

The polariton effect will not influence the thermalization and PL kinetics provided that \( E_{k_0} \gg (1/2)h \Omega_{QW}^2 \), where the effective QW polariton parameter \( \Omega_{QW}^2 \) is given by \( (\Omega_{QW}^2)^2 = (\sqrt{\varepsilon / \kappa} R_c \omega_\parallel \). In this case the QW polariton bottleneck band is very narrow, located at energies above \( \hbar \omega_\parallel \), and the mode \( p_\parallel = 0 \) preserves the status of a ground-state mode. For a GaAs QW with \( L_z = 100 \, \text{Å} \) one has \( f_{xy} \approx 5 \times 10^{-4} \) \( \text{Å}^{-1} \). This value of the oscillator strength yields \( \hbar^2 R_c \approx 1.2 \times 10^{-2} \text{eV}^2 \), \( \tau_R = 1 / \Gamma_0 \approx 25 \, \text{ps} \), and \( (1/2)h \Omega_{QW}^2 \approx 3 \, \text{meV} \), i.e., \( (1/2)h \Omega_{QW}^2 > E_{k_0} \approx 0.1 \, \text{meV} \). In perfect QWs the oscillator strength \( f_{xy} \) can be made two or three orders smaller than the used above value. This means that indeed \( E_{k_0} \gg (1/2)h \Omega_{QW}^2 \) and one can apply Eqs. (11), (25), and (34) to the relaxation and PL dynamics of indirect QW excitons.

The exciton-exciton interaction relaxes the polariton effect in perfect QWs and removes it, if \( \Gamma_x = 1 / \tau_{xx-x} \gg \Omega_{QW}^2 \). With the estimate of \( 1 / \tau_{xx-x} \) given by Eq. (A4) of Appendix we conclude that for direct excitons in a single GaAs QW with \( L_z = 100 \, \text{Å} \) the interface polariton effect plays no role at concentrations \( \rho_{2D} > 3 \times 10^{10} \, \text{cm}^{-2} \). Alternatively, in the presence of the well-developed polariton picture with \( E_{k_0} < (1/2)h \Omega_{QW}^2 \), the relaxation kinetics should be reformulated in terms of “QW polariton ± bulk LA-phonon ↔ QW polariton”. Furthermore, in this case relaxational thermodynamics becomes invalid due to the absence of steady-state quasi-equilibrium in a gas of QW excitons (QW polaritons).

Next we consider the formation of quasi-2D biexcitons. The potential for exciton-exciton interaction is repulsive for indirect excitons in CQWs. However, in a single QW two excitons in some spin configurations, e.g., \( \sigma^+ \) and \( \sigma^- \) - spin-polarized, attract each other due to the exchange Coulomb interaction. This interaction may lead to biexciton states which lie below the excitonic energies \( E \geq 0 \). While the biexciton states are absent in CQWs, they are presented in single GaAs QWs at low temperatures and moderate optical excitations. Typical values of the biexciton binding energy \( E_{m}^{(2D)} \) are about \( 1 - 2 \) meV for \( L_z = 200 - 100 \, \text{Å} \). The law of mass action applied to QW excitons and biexcitons, which are Maxwell-Boltzmann distributed at the same effective temperature \( T \), shows that the biexciton states are ionized in unbound excitons at \( k_BT \geq E_{m}^{(2D)} \), i.e., at \( T \geq 10 \, \text{K} \). At low temperatures, with increasing dimensionless parameter \( (\rho_{m,2})^2 \rho_{2D} \) the biexciton binding energy decreases due to screening by quasi-equilibrium QW excitons, which remain well-defined quasi-particles for \( (\rho_{m,2})^2 \rho_{2D} \geq 1 \). Here \( \rho_{m,2}^{(2D)} (\geq \rho_{2D}^{(2D)}) \) is the radius of a QW biexciton. The screening of the biexciton states is mainly due to the repulsive interaction between QW excitons of the identical spin structure, which is much stronger than the attractive potential. The biexciton state is affected by excitons if \( (\rho_{m,2})^2 \rho_{2D} E_{m}^{(2D)} \geq E_{m}^{(2D)} \), where \( E_{m}^{(2D)} \) is the two-dimensional excitonic Rydberg. This estimate refers to low effective temperatures \( T \to 0 \). The squared biexciton radius is approximated by \( (\rho_{m,2})^2 \approx 2(\mu_x / M_x) (E_{m}^{(2D)} / E_{m}^{(2D)}) (\rho_{2D})^2 \approx 2.4(\rho_{2D})^2 \), where \( \rho_{2D} \) is the reduced exciton mass. We estimate that the biexciton state starts to weaken at \( (\rho_{m,2})^2 \rho_{2D} \geq 0.05 - 0.1 \), i.e., at \( \rho_{2D} \approx 10^{11} \, \text{cm}^{-2} \). Certainly in the high-density limit \( \rho_{2D} \approx 2 - 3 \times 10^{12} \, \text{cm}^{-2} \), where the Mott parameter \( \rho_{2D} (\rho_{m,2})^2 \) approaches unity, the thermalization and photoluminescence of strongly degenerate QW excitons occurs in the absence of the biexciton states and can be analyzed with our approach. During the optical decay, at concentrations \( \rho_{2D} \sim 10^{11} \, \text{cm}^{-2} \), the QW biexcitons can contribute to the relaxation and PL processes. For these densities of QW excitons the relaxation thermodynamics given by Eq. (11) can be generalized to include the conversion of QW excitons to QW biexcitons and the LA-phonon-assisted relaxation within the both excitonic and biexcitonic bands. In this case the quasi-equilibrium concentrations of QW excitons and biexcitons are connected by the law of mass action.
The developed thermalization and photoluminescence kinetics refer to a perfect QW. With weak disorder, relaxation can be “disorder-assisted”, whereby momentum and energy conservation are satisfied by LA-phonon emission concomitant with scattering from the disorder potential. This will be relevant for relieving the phonon bottleneck if there are significant Fourier components of the disorder with wavevectors $p_d \geq 2M_s v_s / \hbar = 1.9 \times 10^5 \text{cm}^{-1}$. For the opposite limit of strong interface and alloy disorder, the phonon-assisted relaxation kinetics of localized QW excitons is modeled in Ref. and more recently in Ref. where the PL process is also studied. The optical decay of localized QW excitons is investigated in Ref. Note that the number of localized QW excitons saturates due to Pauli blocking, if the concentration of the residual defect centers $N_d$ is much less than $\rho_{2D}$ (in perfect GaAs QWs $N_d \leq 10^{10} \text{cm}^{-2}$). In this case the localization processes do not affect the BE kinetics of free QW excitons.

At low bath temperatures $T_b \leq E_0$ the large thermalization times $\tau_b = \exp(-E_0/k_b T_b)$ and $\tau_{0L} = (T_0/T_b) \tau_b$ given by Eqs. (44) and (25) for statistically nondegenerate and strongly degenerate QW excitons, respectively, can be changed by two-LA-phonon-assisted relaxation processes “QW exciton ↔ bulk LA-phonon ↔ bulk LA-phonon ↔ QW exciton”. While these processes are of the next order of smallness with respect to the considered QW-exciton – one-phonon scattering, they can be dominant in the presence of drastic slowing down of one-phonon thermalization. The energy-momentum conservation law in two-phonon-assisted relaxation allows for a QW exciton with energy $E < E_0$ to be scattered into the ground-state mode $p_\parallel = 0$ only by first absorbing of a LA-phonon and then re-emitting of another LA-phonon. The rate of the anti-Stokes – Stokes two-phonon scattering into the ground-state mode is given by

$$\frac{1}{\tau_{th}^{(2)}} = \left(\frac{2\pi}{\hbar}\right) \sum_{q,q'} \left| \tilde{M}_{x-\text{ph}}(q) \right|^2 \left| \tilde{M}_{x-\text{ph}}(q') \right|^2 \times \frac{N_q n_q^{\text{ph}} (n_q^{\text{ph}} + 1)}{(E_{p_\parallel} + \hbar \omega_q - E_{p_\parallel + q})^2}\times \delta(E_{p_\parallel} + \hbar \omega_q - \hbar \omega_{p_\parallel + q}) \delta_{p_\parallel - q + q},$$

where $E_{p_\parallel} = \hbar^2 Q_{p_\parallel}^2/2M_s$ and $\hbar \omega_{q} = \hbar v_s q$ are the energies of a QW exciton and a bulk LA-phonon, respectively. The matrix element of QW exciton – bulk LA-phonon interaction is $\tilde{M}_{x-\text{ph}}(q) = [(\hbar D^2 q)/(2v_s \rho V)]^{1/2} F_z(q, L_v/2)$, where $V$ is the volume which refers to bulk LA-phonons. For a quantum gas of excitons Eq. yields the following estimate of the two-phonon-assisted relaxation rate:

$$\frac{1}{\tau_{0L}^{(2)}} = 8\pi \left(\frac{k_B T_b}{E_0}\right)^4 \frac{\hbar}{E_0 T_{sc}}.$$
quantum degenerate gas of quasi-2D excitons, when $T$ is less than the degeneracy temperature $T_0$, BE statistics yields nonexponential and density-dependent thermalization. In particular, for low bath temperatures $T_b \leq E_0/k_B \sim 1$ K the relaxation processes in a degenerate gas of QW excitons slow down dramatically and the thermalization law is given by $T(t) \propto 1/\ln t$.

(ii) The effective radiative lifetime $\tau_{\text{opt}} = \tau_{\text{opt}}(T, T_0)$ of quasi-equilibrium BE-distributed QW excitons is given by Eq. (33). The temperature-independent corrections, given by Eq. (35), to the well-known classical limit $\tau_{\text{opt}} \propto T$ show that BE statistics can be traced in the optical decay of a gas of QW excitons even at high effective temperatures $T$.

Nonclassical statistics strongly influences the optical decay at $T \lesssim T_0$. In particular, at $T \rightarrow 0$ the lifetime $\tau_{\text{opt}}$ approaches $2 \tau_R$ rather than zero as in the classical limit $\tau_{\text{opt}} \propto T$.

(iii) The PL kinetics of QW excitons is modeled by three coupled Eqs. (11), (23), and (66a)-(66l) for the effective temperature $T(t)$, effective radiative time $\tau_{\text{opt}}(T, \rho_{2D}) = 1/T_{\text{opt}}(T, \rho_{2D})$, and concentration $\rho_{2D}(t)$, respectively. This approach describes within the thermodynamic picture both the rise of PL, due to thermalization, and the decay of PL, due to the radiative recombination of thermalized QW excitons. For a classical gas of QW excitons the PL kinetics depends on the effective temperature $T$, but is independent of $\rho_{2D}$. With increasing density $\rho_{2D}$ of QW excitons the quantum-statistical effects builds up and the PL kinetics becomes density-dependent.

ACKNOWLEDGMENTS

We appreciate valuable discussions with P. Stenius, L.V. Butov, and H. Kalt. Support of this work by the EPSRC (UK) and by the DFG-Schwerpunktprogramm “Quantenkohärenz in Halbleitern” (Germany) is gratefully acknowledged.

APPENDIX A: EQUILIBRATION DYNAMICS OF QW EXCITONS

The equilibration kinetics of QW excitons is mainly due to particle-particle scattering and refers to the Boltzmann-Uhlenbeck equation

$$\frac{\partial}{\partial t} N_{k_i} = -\frac{2\pi}{\hbar} \left( \frac{U_0}{S} \right)^2 \sum_{p_i, q_i} \left[ N_{k_i} N_{p_i} (1 + N_{p_i} - q_i) \right]$$

$$\times (1 + N_{k_i} + q_i) - (N_{k_i} + 1)(N_{p_i} + 1) N_{p_i - q_i} N_{k_i + q_i}$$

$$\times \delta (E_{p_i - q_i} + E_{k_i + q_i} - E_{k_i} - E_{p_i}) \tag{A1}$$

where $U_0/S$ is the potential of exciton-exciton Coulombic interaction in a QW, $S$ is the area. For CQWs this potential is repulsive and independent of the spin structure of interacting excitons owing to a rather small contribution of the exchange interaction. For a single QW the potential depends upon the spin state of excitons and can be repulsive (e.g., for the QW excitons with an identical spin structure) as well as attractive (e.g., for $\sigma^-$ and $\sigma^+$ spin polarized excitons)\(^\ddagger\). However, the repulsive potential strongly dominates\(^\ddagger\) and determines the equilibration dynamics of QW excitons at $\rho_{2D} \gtrsim \rho_{2D}^c$.

In order to evaluate a characteristic equilibration time $\tau_{\text{q-x}}$ we analyze within Eq. (A1) the dynamics of a small fluctuation $\delta N_{0}^{(0)} = \delta N_0(t = 0) \ll N_{p_i=0}^{eq}$ of the ground-state mode population, provided that the occupation numbers of all other modes are given by the BE distribution of Eq. (3). The linearization of the r.h.s. of Eq. (A1) with respect to $\delta N_0$ yields an exponential law for the decay of the fluctuation, i.e., $d(\delta N_0)/dt = -\delta N_0/\tau_{\text{q-x}}$ and $\delta N_0(t) = \delta N_0^{(0)} \exp(-t/\tau_{\text{q-x}})$. The decay time of the fluctuation, which we attribute to the characteristic equilibration time, is given by

$$\frac{1}{\tau_{\text{q-x}}} = \left( \frac{U_0 M_x}{2 \pi} \right)^2 \frac{k_B T}{h^5} e^{-T_0/T} \left( 1 - e^{-T_0/T} \right)$$

$$\times \int_0^\infty du \int_0^{2\pi} d\phi \left[ e^{u(1 - \cos \phi)} + e^{-T_0/T - 1} \right]$$

$$\frac{1}{e^{u(1 + \cos \phi)} + e^{-T_0/T - 1} - 1} \left( e^{2u} + e^{-T_0/T - 1} \right). \tag{A2}$$

In order to derive Eq. (A2) from Eq. (A1) we put $k_\parallel = 0$ and introduce new dummy variable $q_i' = q_i - p_i/2$. The dimensionless integration variable $u$ on the r.h.s. of Eq. (A2) is given by $u = (h p_i)^2 / (4k_B M_x T)$, $\phi$ is the angle between $q_i'$ and $p_i$.

While the repulsive potential $U_0$ for QW excitons in an identical spin state can be explicitly written in terms of an integral convolution of the different pair Coulomb potentials between the constituent electrons and holes with the excitonic wavefunctions (see, e.g., Ref. \(^\ddagger\)), here we give a scaling estimate of $U_0$. The potential $U_0$ is determined by $U_0 = \int U_{x-x}(r_\parallel) dr_\parallel$, where $U_{x-x}(r_\parallel)$ is the real-space potential of exciton-exciton interaction and $r_\parallel$ is the distance between two interacting QW excitons. The characteristic energy and length scales for $U_{x-x}(r_\parallel)$ are given by the two-dimensional Rydberg $E_x^{(2D)}$ and the Bohr radius $a_x^{(2D)} = h^2/(2\mu_x e^2)$, where $\mu_x$ is the reduced mass of a QW exciton. For simple model potentials

$U_{x-x}(r_\parallel) = C E_x^{(2D)} \exp(-r_\parallel/a_x^{(2D)})$ and $U_{x-x}(r_\parallel) = C E_x^{(2D)} (a_x^{(2D)} / r_\parallel) \exp(-r_\parallel/a_x^{(2D)})$, where the constant $C \approx 1$, one estimates

\[^\ddagger\]
\[ U_0 = 2\pi C \, E_x^{(2D)}[a_x^{(2D)}]^2 = \pi C \frac{\hbar^2}{\mu_x} \]  

(A3)

A remarkable feature of Eq. (A3) is that while with decreasing confinement in the z-direction (the QW growth direction) \( E_x^{(2D)} \) and \( a_x^{(2D)} \) start to approach the corresponding bulk values, their combination \( E_x^{(2D)}[a_x^{(2D)}]^2 \) does not change. Such behavior of \( U_0 \) is due to the quasi-two-dimensionality of QW excitons. The constant \( C \), which is of the order of unity, depends on the design of a QW, the shape of the model potential \( U_{x-x}(r_x) \), etc. In further analysis we put \( C = 1 \).

For a classical gas of QW excitons, when \( T \gg T_0 \), one obtains from Eqs. (A2)-(A3):

\[ \frac{1}{\tau_{x-x}} = \pi \left( \frac{M_x}{\mu_x} \right)^2 k_BT_0, \]

(A4)

where \( T_0 \) is given by Eq. (3). According to Eq. (A5), the scattering rate due to particle-particle interaction is proportional to the concentration \( n_{p2D} \) of QW excitons. Furthermore, as a signature of the quasi-two-dimensionality of excitons in QWs, the characteristic equilibration time \( \tau_{x-x} \) is independent of the temperature \( T \) (for \( T \gg T_0 \)) and of the scattering length \( \sim a_{x}^{(2D)} \).

For a quantum gas of QW excitons \( (T \ll T_0) \) we find from Eqs. (A2)-(A3):

\[ \frac{1}{\tau_{x-x}} = \frac{\pi}{\hbar} \left( 1 - \frac{\pi}{4} \right) \left( \frac{M_x}{\mu_x} \right)^2 k_BT e^{T_0/T}. \]

(A5)

The considerable increase of the scattering rate given by Eq. (A5) in comparison with that of Eq. (A4) is due to the low-energy QW excitons with \( N_{eq}^{(2)} \gg 1 \) at \( E \leq k_BT \).

**APPENDIX B: HOMOGENEOUS BROADENING IN THE RADIATIVE DECAY OF QW EXCITONS**

The joint density of states \( J(p_{||}) \) for the resonant optical decay of a QW exciton with \( p_{||} \leq k_0 \) into a bulk photon is given by

\[ J(p_{||}) = \left( \frac{1}{2\pi^2} \right) \int_{-\infty}^{+\infty} dk_{\perp} \times \frac{\Gamma_{hom}}{\Gamma_{hom}^2 + \left[ \omega_x(p_{||}) - \omega_{\perp}(p_{||}, k_{\perp}) \right]^2}, \]

(B1)

where \( \omega_x(p_{||}) = \omega_0 + h^2 p_{||}^2/2M_x \) and \( \omega_{\perp}(p_{||}, k_{\perp}) = (e/\sqrt{\epsilon_0})\sqrt{p_{||}^2 + k_{\perp}^2} \) are the dispersions of QW excitons and bulk photons, respectively, \( k_{\perp} \) is the z-component of the wavevector of bulk photons, which resonantly interact with a QW exciton with in-plane wavevector \( p_{||} \), and \( \Gamma_{hom} \) is the homogeneous linewidth of QW excitons. If \( \partial \omega_{\perp}(p_{||}, k_{\perp})/\partial k_{\perp} \neq 0 \) at \( k_{\perp} \) determined by the equation \( \omega_x(p_{||}) = \omega_{\perp}(p_{||}, k_{\perp}) \), the integrand on the r.h.s. of Eq. (B1) preserves its Lorentzian shape in wavevector coordinates and the joint density of states \( J(p_{||}) \) is independent of \( \Gamma_{hom} \). This is a regular case, which does not hold for \( p_{||} \rightarrow k_0 \) \( (p_{||} \leq k_0) \). In this case the solution of \( \omega_x(k_0) = \omega_{\perp}(k_0, k_{\perp}) \) is \( k_{\perp} = 0 \) and \( \partial \omega_{\perp}(p_{||} = k_0, k_{\perp})/\partial k_{\perp} = 0 \), indicating 1D van Hove singularity in the joint density of states. As a result, \( J(p_{||}) \) becomes \( \Gamma_{hom} \)-dependent in a close vicinity of \( p_{||} = k_0 \). This is the only way that the homogeneous linewidth influences the optical decay of quasi-equilibrium excitons in ideal QWs.

With substitution \( k_{\perp} = p_{||} \tan \phi \), where \( \phi \in [-\pi/2, \pi/2] \), Eq. (B1) reduces to

\[ J(p_{||}) = \frac{1}{\pi^2} \left[ \frac{\omega_{\perp}(p_{||})}{\omega_x(p_{||})} \right] \int_0^{\pi/2} du \times \frac{\tilde{\gamma}}{(1/4)u^4 - (\delta^2/2)u^2 + (\delta^4/4 + \delta^2\tilde{\gamma}^2)}, \]

(B2)

where \( \tilde{\gamma} = \Gamma_{hom}/\omega_x, \delta = \cos \phi_0 = p_{||}/k_0 = (\epsilon p_{||})/(\epsilon_0\omega_x) \), \( \delta = \sin \phi_0 = \sqrt{1 - \delta^2} \), and the integration variable \( u = \phi - \phi_0 + \sin \phi_0 \). Straightforward integration of Eq. (B2) yields the joint density of states:

\[ J(p_{||} < k_0 - \tilde{\gamma}k_0) = \frac{1}{\pi} \left[ \frac{\epsilon \omega}{\epsilon_0 p_{||}^2} \right] \frac{\omega_{\perp}}{\sqrt{k_0^2 - p_{||}^2}}. \]

(B3)

For \( \delta \gg \sqrt{2\gamma} \), which is identical to the condition \( k_0 - \tilde{\gamma}k_0 \gg k_0, \) Eq. (B3) reduces to the standard formula:

\[ J(p_{||} = k_0) = \frac{1}{2\pi} \frac{k_0}{(\Gamma_{hom}^2\omega_x)^{1/2}}. \]

(B5)

Equation (B5) shows how the homogeneous linewidth \( \Gamma_{hom} \) removes the van Hove singularity at \( p_{||} = k_0 \). For \( \delta \leq \sqrt{2\gamma} \) the joint density of states is \( \Gamma_{hom} \)-dependent.

With Eq. (B3) we derive the final expression for the intrinsic radiative rate of \( T \)-polarized QW excitons in the presence of homogeneous broadening:

\[ \Gamma_T(p_{||}) = \frac{\left( \sqrt{2k_0^2p_{||}} \right) \Gamma_0}{\left[ (k_0^2 - p_{||}^2)^2 + 4\gamma^2k_0^2p_{||}^2 \right]^{1/2}} \times \frac{1}{\left[ (k_0^2 - p_{||}^2)^2 + 4\gamma^2k_0^2p_{||}^2 \right]^{1/2}}. \]

(B6)
For $k_0 - p \parallel \gg \gamma k_0$, Eq. (B4) is identical to Eq. (B6). The homogeneous linewidth $\Gamma_{\text{hom}}$ changes Eq. (B6) for $\Gamma_T (p \parallel)$ only at the band of states given by $\gamma \approx k_0 - p \parallel \geq 0$ and, in particular, $\Gamma_T (p \parallel = k_0) = \gamma / (2 \gamma^{1/2})$ rather than divergent. Note that the band of states is rather narrow because $\gamma \simeq 1/\hbar^2 \omega_k \sim 10^{-3}$. The 1D van Hove singularity at $p \parallel = k_0$ is absent for the L-polarized QW excitons.

Homogeneous broadening practically does not change the optical decay rate of a gas of QW excitons. The use of Eq. (B6) for calculation of $\Gamma_{\text{opt}}$ by Eq. (B3) shows that for Maxwell-Boltzmann distributed QW excitons one obtains only a small renormalization of $J_T$ of Eq. (B2), i.e., $J_T \rightarrow J_T (1 - \gamma^{1/2})$. As a result, the optical decay rate of a classical gas of QW excitons changes from $\Gamma_{\text{opt}}$ to $\Gamma_{\text{opt}} [1 - (3/4) \gamma^{1/2}]$. This correction due to the homogeneous linewidth is very small and can indeed be neglected. The corresponding correction for statistically degenerate QW excitons is even smaller because the low-energy excitons tend to populate the ground-state mode $p \parallel = 0$ rather than vicinity of the energy states $\varepsilon \approx E_{k_0}$.

\begin{enumerate}
\item Y. Masumoto, S. Shionoya, and H. Kawaguchi, Phys. Rev. B 29, 2324 (1984).
\item J. Lee, E.S. Koteles, and M.O. Vassell, Phys. Rev. B 33, 5512 (1986).
\item J. Feldmann, G. Peter, E.O. Göbel, P. Dawson, K. Moore, C. Foxon, and R.J. Elliott, Phys. Rev. Lett. 59, 2337 (1987).
\item T.C. Damen, J. Shah, D.Y. Oberli, D.S. Chemla, J.E. Cunningham, and J.M. Kuo, Phys. Rev. B 42, 7434 (1990); P.W.M. Blom, F.J. van Hall, C. Smit, J.P. Cuypers, and J.H. Wolter, Phys. Rev. Lett. 71, 3878 (1993).
\item B. Deveaud, F. Clérot, N. Roy, K. Satzke, B. Stermage, and D.S. Katzer, Phys. Rev. Lett. 67, 2355 (1991).
\item M. Gurioli, A. Vinattieri, M. Colocci, C. Deparis, J. Massies, G. Neu, A. Bossachi, and S. Franchi, Phys. Rev. B 44, 3115 (1991).
\item R. Eccleston, R. Strobel, W.W. Rühle, J. Kuhl, B.F. Feuerbacher, and K. Ploog, Phys. Rev. B 44, 1395 (1991); R. Eccleston, B.F. Feuerbacher, J. Kuhl, W.W. Rühle, and K. Ploog, Phys. Rev. B 45, 11403 (1992).
\item V. Srinivas, J. Hryniewicz, Y.J. Chen, and C.C.E. Wood, Phys. Rev. B 46, 10193 (1992).
\item J. Martinez-Pastor, A. Vinattieri, L. Carraresi, M. Colocci, Ph. Roussignol, and G. Weimann, Phys. Rev. B 47, 10456 (1993).
\item A. Vinattieri, J. Shah, T.C. Damen, D.S. Kim, L.N. Pfeiffer, M.Z. Maiale, and L.J. Sham, Phys. Rev. B 50, 10868 (1994).
\item J. Shah, Ultrafast Spectroscopy of Semiconductors and Semiconductor Nano-structures, Springer Series in Solid-State Sciences, Vol. 115 (Springer, Berlin, 1996).
\item S. Grosse, R. Arnold, G. von Plessen, M. Koch, J. Feldmann, V.M. Axt, T. Kuhn, R. Rettig, and W. Stolz, Phys. Status Solidi B 204, 147 (1997); M. Gulia, F. Rossi, E. Molinari, P.E. Selbmann, and P. Lugli, Phys. Rev. B 55, R16049 (1997).
\item L.V. Butov, A. Imamoglu, A.V. Mintsev, K.L. Campman, and A.C. Gossard, preprint (submitted to Phys. Rev. Lett., 1998).
\item M. Uulmauff, J. Hoffmann, H. Kalt, W. Langbein, J.M. Hvam, M. Scholl, J. Sölner, M. Heukon, B. Jobst, and D. Hommel, Phys. Rev. B 57, 1390 (1998); J. Hoffmann, M. Uulmauff, H. Kalt, W. Langbein, and J.M. Hvam, Status Solidi B 204, 195 (1997).
\item T. Takagahara, Phys. Rev. B 31, 6552 (1985).
\item L.C. Andreani, F. Tassone, and F. Bassani, Solid State Commun. 77, 641 (1991).
\item W. Zhao, P. Stenius, and A. Imamoglu, Phys. Rev. B 56, 5306 (1997).
\item J.C. Kim and J.P. Wolfe, Phys. Rev. B 57, 9861 (1998).
\item L.C. Andreani and F. Bassani, Phys. Rev. B 41, 7536 (1990).
\item P. Stenius and A.L. Ivanov, Solid State Commun. 108, in print (1998); P. Stenius, PhD thesis, University of California, Santa Barbara, 1998.
\item U. Bockelmann, Phys. Rev. B 50, 17271 (1994).
\item E. Levich and V. Yakhot, Phys. Rev. 15, 243 (1977).
\item E.M. Lifshitz and L.P. Pitaevskii, Course of Theoretical Physics (Pergamon, Oxford, 1980), Vol. 9, Part II, Sec. 6 and Sec. 25.
\item A.L. Ivanov, C. Ell, and H. Haug, Phys. Rev. E 55, 6363 (1997).
\item A.L. Ivanov, C. Ell, and H. Haug, Phys. Status Solidi B 206, 355 (1998); C. Ell, A.L. Ivanov, and H. Haug, Phys. Rev. B 57, 9663 (1998).
\item L.V. Butov, private communication. The preliminary result is that a high-density gas of excitons in GaAs/AlGaAs CQWs does not cool below 500 mK within the lifetime of indirect QW excitons. In these experiments, the bath temperature is $T_b \simeq 50 mK$, the initial effective temperature of excitons is $T_i \simeq 1 K$, and the lifetime is about 30 – 100 ns.
\item X. Zhu, P.B. Littlewood, M. Hybertsen, and T. Rice, Phys. Rev. Lett. 74, 1633 (1995).
\item P.B. Littlewood and X.J. Zhu, Physica Scripta T68, 56 (1996).
\item V.N. Popov, Theor. Math. Phys. 11, 565 (1972).
\item T. Fukuzawa, E.E. Mendez, and J. Hong, Phys. Rev. Lett. 64, 3066 (1990); L.V. Butov, A. Zrenner, G. Abstreiter, G. Böhm, and G. Weimann, Phys. Rev. Lett. 73, 304 (1994); L.V. Butov and A.I. Filin, Phys. Rev. B 58, 1980 (1998). The experimental results of the work by T. Fukuzawa et al. were later re-interpreted, see J.A. Kash, M. Zachau, E.E. Mendez, J.M. Hong, and T. Fukuzawa, Phys. Rev. Lett. 66, 2247 (1991).
\item J.E. Golub, K. Kash, J.P. Harbison, and L.T. Florez, Phys. Rev. B 41, 8564 (1990); A. Alexandrou, J.A. Kash, E.E. Mendez, M. Zachau, J.M. Hong, T. Fukuzawa, and Y. Hase, Phys. Rev. B 42, 9225 (1990).
\item M. Nakayama, Solid State Commun. 55, 1053 (1985).
\item A.L. Ivanov, H. Wang, J. Shah, T.C. Damen, L.V. Keldysh, H. Haug, and L.N. Pfeiffer, Phys. Rev. B 56, 3941 (1997).
\item A.L. Ivanov, L.V. Keldysh, and V.V. Panashchenko, Zh.
\end{enumerate}
Eksp. Teor. Fiz 99, 641 (1991) [Sov. Phys. JETP 72, 359 (1991)].

35. L.V. Keldysh, Solid. State Commun. 84, 37 (1992); Phys. Stat. Solidi B 173, 119 (1992).

36. R. Zimmermann and E. Runge, Phys. Status Solidi A 164, 511 (1997); E. Runge and R. Zimmermann, Phys. Status Solidi B 206, 167 (1998).

37. D.S. Citrin, Phys. Rev. B 47, 3832 (1993).

38. H. Haug and S. Schmitt-Rink, Progr. Quantum Electron. 9, 3 (1984).