Ignorance is bliss: General and robust cancellation of decoherence via no-knowledge quantum feedback

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A “no-knowledge” measurement of an open quantum system yields no information about any system observable; it only returns noise input from the environment. Surprisingly, measuring nothing is most advantageous. We prove that a system undergoing no-knowledge monitoring has reversible noise, which can be cancelled by directly feeding back the measurement signal. We show how no-knowledge feedback control can be used to cancel decoherence in an arbitrary quantum system coupled to a Markovian reservoir. Since no-knowledge feedback does not depend on the system state or Hamiltonian, such decoherence cancellation is guaranteed to be general, robust and can operate in conjunction with any other quantum control protocol. As an application, we show that no-knowledge feedback could be used to improve the performance of dissipative quantum computers subjected to local loss.

“More signal, less noise” is the guiding philosophy of experimental science. Increasing measurement sensitivity is a proven strategy for pushing the frontiers of science and technology, yielding both improved knowledge and improved control over Nature. However, at the quantum scale physics pushes back by imposing a fundamental limit on the signal-to-noise ratio by virtue of Heisenberg’s Uncertainty Principle [1, 2]. Nevertheless, “more signal, less noise” also guides the design of protocols for the measurement and control of quantum systems, such as squeezed state photon [3] and atom [4] interferometry, optimal parameter estimation [5], weak measurement [6], measurement-based feedback control [5, 7], and adaptive measurement [8]. In this article, we take the unorthodox “no signal, only noise” approach, and consider measurements that are pure noise, and therefore give no knowledge of the quantum state whatsoever. From a quantum control perspective, one intuitively expects such no-knowledge measurements to be unworthy of study, since robust feedback control requires at least some (and preferably good) knowledge of the system state. On the contrary, we show that a measurement-based feedback protocol based on no-knowledge monitoring can be used to remove decoherence - the bane of quantum technology - from an arbitrary quantum system coupled to a Markovian environment. Although serious attempts to eliminate or reduce decoherence have resulted in significant successes, for example the development of error correction codes for quantum information science [9,12], dynamical decoupling [13], reservoir engineering [14–15], feedback control [16–20], and the engineering of decoherence-free subspaces [21,22], decoherence has yet to be adequately tamed. In our proposal, decoherence is cancelled by directly feeding the no-knowledge measurement signal back into the system, in effect turning quantum noise against itself. Unlike many other methods of decoherence reduction, this feedback operates with no a priori knowledge of the system, and is consequently both effective, robust and can operate in conjunction with other quantum control protocols. This demonstrates that meaningful feedback control without knowledge is not only possible, but desirable.

No-knowledge measurements. Consider a system governed by Hamiltonian $H$ that interacts with a Markovian reservoir via the coupling operator $L$. The system density operator, $\varrho_{s}$, evolves according to the master equation

$$\partial_{t}\varrho_{s} = -i[H,\varrho_{s}] + D[L]\varrho_{s} \equiv L\varrho_{s}, \quad (1)$$

where $\partial_{t} \equiv d/dt$, $D[Z]\varrho_{s} = Z\varrho_{s}Z^{\dagger} - (Z^{\dagger}Z\varrho_{s} + \varrho_{s}Z^{\dagger}Z)/2$, and we have chosen units such that $\hbar = 1$. In principle, it is always possible to indirectly extract information about the state of the system by performing a projective measurement on the reservoir. In particular, for a homodyne measurement of the environment at an angle $\theta$, the conditional dynamics of the system are described by the Stratonovich stochastic master equation [5, 23,24]

$$\partial_{t}\rho_{s} = L\rho_{s} + \sqrt{\eta}A[Le^{i\theta}]\rho_{s} y_{0}^{\dagger} - \frac{\eta}{2}A^{2}[Le^{i\theta}]\rho_{s}, \quad (2)$$

where $\rho_{s}$ is the unnormalised conditional density operator for the system, $\eta$ is the detection efficiency, $A[Z]\rho_{s} = Z\rho_{s}Z^{\dagger}$, and $A^{2}[Z]\rho_{s} = Z(A[Z]\rho_{s}) + (A[Z]\rho_{s})Z^{\dagger}$. Conditional expectations of system operators are calculated using $\langle X \rangle_{t} = \text{Tr}[X\rho_{s}] / \text{Tr}[\rho_{s}]$. The first term of Eq. (4) corresponds to the unconditional Lindblad master equation [1], and gives the unitary dynamics due the system Hamiltonian and the decoherence caused by the system-reservoir coupling. The second term is the innovations,
FIG. 1. (a) A two-level system (qubit), with underlying state $\rho_t$, undergoing dephasing and perfect homodyne detection at angle $\theta$. The measurement signal $y_0(t)$ is fed into a filter, which provides the optimal estimate of the system state, $\pi_t$. (b) Plots of Frobenius distance $||\rho_t - \pi_t|| = \sqrt{\text{Tr}[(\rho_t - \pi_t)/(\text{Tr}[\rho_t - \pi_t]^2]}$ for $\Omega/\gamma = 1$ and homodyne angles $\theta = \pi(1 + \epsilon)/2$ with $\epsilon = 10^{-1}$ (red), $10^{-2}$ (blue), $10^{-3}$ (magenta), and 0 (black). The underlying initial state of the qubit, $\rho_0 = [I + (\sigma_x + i\sigma_y)/\sqrt{2}]/2$, is different to the initial estimate of the filter, which is the mixed state $\pi_0 = I/2$. $||\rho_t - \pi_t||$ is averaged over 5000 trajectories, leading to a standard error on the order of the line thickness. The system and filter take a longer time to converge as $\theta$ approaches the no-knowledge quadrature $\theta = \pi/2$. (c) The no-knowledge measurement ($\theta = \pi/2$) is directly fed back into the system via the Hamiltonian $\hat{H} = \Omega\sigma_z + \sqrt{\gamma}\sigma_y y_{\pi/2}(t)$ in order to cancel the dephasing. (d) The system (red, solid line) and filter (blue, dashed line) evolution of the qubit under no-knowledge feedback. Despite the filter’s inaccurate estimate of $\rho_t$, decoherence is removed from the system. This demonstrates that accurate knowledge of the system state is not required for effective no-knowledge feedback.

which conditions the system dynamics based on the homodyne measurement photocurrent

$$y_0(t) = \sqrt{\eta} \langle Le^{i\theta} + L^* e^{-i\theta} \rangle_t + \xi(t), \quad (3)$$

where $\xi(t)$ is a Stratonovich stochastic integral [25, 26].

The final term of Eq. (4) is the Stratonovich correction, which is a deterministic drift that must be included along with any Stratonovich stochastic integrals (see supplemental material, Sec. I). Note that Eq. (1), the unconditional evolution of $\rho_t$, is obtained by averaging Eq. (4) over different realisations of the measurement record, up to a normalisation factor.

Equation (8) shows that the measurement signal is composed of two parts; the first term represents the knowledge obtained about the system from the measurement, whereas the second term is the corrupting (white) noise input from the reservoir. However, there exist choices of $L$ for which the measurement returns no information about the system operators, which we term a no-knowledge measurement. More specifically, when $L$ is Hermitian, homodyne detection of the reservoir at an angle $\theta = \pi/2$ is a no-knowledge measurement, since the measurement signal $y_{\pi/2}(t) = \xi(t)$ returns only noise.

We can examine the effect of a no-knowledge measurement by comparing the dynamical evolution of the underlying system state, $\rho_t$, to that of the quantum filter [27], $\pi_t$, which is the optimal Bayesian estimate of the system density matrix conditioned on the measurement record [28]. The quantum filtering equation for the unnormalised quantum filter $\pi_t$ is [29, 30]

$$\partial_t \pi_t = \mathcal{L}\pi_t + \sqrt{\eta} A[Le^{i\theta}]\pi_t y_0(t) - \frac{\eta}{2} A^2[Le^{i\theta}]\pi_t. \quad (4)$$

Now suppose that $\pi_0 \neq \rho_0$, which encodes an observer’s initial ignorance of the underlying system state. For most measurement choices, information extracted from the measurement signal leads to a better estimate of the system state over time, and so $\pi_t$ will converge to $\rho_t$ in finite time. However, this is not true for a no-knowledge measurement. The absence of knowledge about the system means that the filter is conditioned only on noise, Eqs (4) and (4) decouple, and hence the filter never converges to the system state (we prove this explicitly in Sec. II of the supplemental material). To illustrate this point we consider a resonantly-driven two-level system ($H = \Omega\sigma_z$) undergoing dephasing ($L = \sqrt{\gamma}\sigma_z$) while continuously monitored with perfect homodyne detection ($\eta = 1$) at angle $\theta$ (see Fig. 1(a) and Sec. III of supplemental material). As shown in Fig. 1(b), the filter takes longer to converge to the system as $\theta$ approaches the no-knowledge quadrature $\theta = \pi/2$. When $\theta = \pi/2$ the system and filter fail to converge, indicating that the filter never gives an accurate estimate of the system.

**Cancelling reservoir noise with no knowledge.**

In classical control theory, if the initial state of a system can not be determined from the measurement signal, then the system-observation pair is called unobservable. A system undergoing a no-knowledge measurement is clearly unobservable, as neither the past or present state of the system can be determined from the measurement record. One may expect, therefore, that this lack of
knowledge renders meaningful measurement-based feedback control of the system impossible. This intuition is incorrect. For although a no-knowledge measurement signal has no explicit dependence on any system observable, the fundamental noise that corrupts the measurement signal is correlated with the reservoir noise coupled to the system. A no-knowledge measurement therefore still contains information (in an information-theoretic sense). More interestingly, the information contained in a no-knowledge measurement can be used to cancel the effects of reservoir noise on the system.

Specifically, consider the evolution of the quantum state when \( L \) is Hermitian, and we make a measurement of the no-knowledge quadrature \( \theta = \pi/2 \) with perfect efficiency \( \eta = 1 \). Under these conditions, Eq. (4) takes a particularly simple form:

\[
\partial_t \rho_t = -i \left[ H - L \, y_{\pi/2}(t), \rho_t \right].
\]  

(5)

Note that the system dynamics due to the reservoir noise are trace preserving and coherent in nature. Consequently, their effect is reversible and can be entirely cancelled by directly feeding back the measurement signal. Explicitly, if we make the replacement \( \eta = \phi/2 \), then Eq. (5) reduces to \( \partial_t \rho_t = -i[H, \rho_t] \). Crucially, this feedback only works because the noise that formed the measurement signal was perfectly correlated with the reservoir noise inducing decoherence in the system.

What is particularly interesting about feedback based on a no-knowledge measurement is that it works when the quantum filter and underlying system state are initially very different [see Figs 1(c) and (d)]. This becomes clear when we notice that the feedback is performed without any filtering of the measurement signal; the signal is simply fed back into the system through the Hamiltonian.

Indeed, no-knowledge feedback control can be successfully implemented with almost no a priori knowledge of the underlying system state or dynamics. No-knowledge feedback only requires a correct identification of the no-knowledge quadrature, which depends only on the form of the system-reservoir coupling operator \( L \). A precise description of the system state and its evolution, coherent or otherwise, is not required. This natural robustness gives no-knowledge feedback a key advantage over other quantum-state-dependent methods of decoherence reduction, particularly for systems where the precise dynamics and noise sources cannot be precisely quantified.

When the detection efficiency is imperfect, the effectiveness of the no-knowledge feedback is reduced. For in this case the evolution is not purely coherent:

\[
\partial_t \rho_t = -i[H - \sqrt{\eta}L \, y_{\pi/2}(t), \rho_t] + (1 - \eta)D[L] \rho_t,
\]  

(6)

and therefore cannot be entirely cancelled by feeding back the measurement signal. Nevertheless, by choosing the no-knowledge feedback \( H \to H + \sqrt{\eta}L \, y_{\pi/2}(t) \) the rate of decoherence can be reduced by a factor of \((1 - \eta)[c.f. \text{Eq. (1)}] : \)

\[
\partial_t \rho_t = -i[H, \rho_t] + (1 - \eta)D[L] \rho_t.
\]  

(7)

Experiments with imperfect detection efficiency can therefore still enjoy a significant and robust decoherence reduction by employing no-knowledge feedback.

An analogous result can be derived for photodetection, where a unitary \( L \) corresponds to a no-knowledge measurement. Noise is cancelled by applying a unitary gate to the system after the detection of a photon (see Sec. IV of supplemental material for details; a particular example of this approach is in \[33\]).

**Removing decoherence for general \( L \).** As formulated above, a no-knowledge measurement is only possible when the coupling operator is Hermitian \[34\]. Since physical observables are Hermitian, direct no-knowledge measurements are possible for a broad range of physical systems. Examples include dephasing in qubits (\( L = \sigma_z \)), \[32\], optomechanical devices under position measurement (\( L = x \)), \[33\], and non-destructive detection of Bose-Einstein condensates \[18–20, 37\]. However, some common system-reservoir coupling operators are not Hermitian. One example is the annihilation operator for a harmonic oscillator \( a \), since \( a \dagger \neq a \). Fortuitously, we can still remove decoherence for a general \( L \) via a similar measurement-based feedback scheme to that outlined above. Counterintuitively, this is achieved by first adding an extra noise source \[33\] to the system which has a coupling operator \( L^\dagger \), giving the unconditional dynamics

\[
\partial_t \rho_t = -i[H, \rho_t] + D[L] \rho_t + D[L^\dagger] \rho_t.
\]  

(8)

For our purposes, the ‘trick’ is to recognise that \( D[L] \rho_t + D[L^\dagger] \rho_t = D[L^+] \rho_t + D[L^-] \rho_t \), where \( L^\pm = i^{(1+\eta)/2} (L \pm L^\dagger)/\sqrt{2} \) are Hermitian operators. Thus \( L^\pm \) are effective coupling operators that admit possible no-knowledge measurements.

Measurements of \( L^\pm \) are possible by first taking the output channels of both reservoirs, mixing them via

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**FIG. 2.** Diagram illustrating how measurements of \( L_\pm \) are engineered from the outputs of reservoir couplings \( L \) and \( L^\dagger \).
a 50:50 beam splitter, introducing a relative phase shift of $\pi/2$ between the two outputs and subsequently measuring each output with homodyne detection (see Fig. 2). This yields the two measurement signals $y^\pm(t) = 2\sqrt{\eta}\cos\theta(L_x + \xi(t))$, where $\xi(t)$ are independent Stratonovich noises. No-knowledge measurements of $L_x$ are realised by taking a quadrature angle $\theta = \pi/2$. The beam splitting step of the feedback protocol is vital, and it is this step which has no classical analogue, making our result a quantum feedback protocol.

The evolution of the quantum state under these no-knowledge measurements is given by a straightforward generalisation of Eq. (6):

$$
\partial_t \rho_t = -i \left[ H - \sqrt{\eta}(L + y^+_{1/2}(t) + L - y^-_{1/2}(t)), \rho_t \right] + (1 - \eta)(D[L]\rho_t + D[L^\dagger]\rho_t).
$$

The advantages of such dissipative engineering have been examined in the context of quantum computation [38] and entanglement protection [33] for a given choice of $L$, where, despite the absence of feedback that entirely decouples the system from decoherence, the level of entanglement was found to be immune to the resulting unitary conditional evolution.

Finally, as above, we can directly feed the measurement signals back via the replacement $H \to H + \sqrt{\eta}(L + y^+_{1/2}(t) + L - y^-_{1/2}(t))$, which gives

$$
\partial_t \rho_t = -i[H, \rho_t] + (1 - \eta)(D[L]\rho_t + D[L^\dagger]\rho_t).
$$

The original decoherence in the system has been suppressed by the factor $(1 - \eta)$, admittedly at the cost of introducing additional decoherence due to $L^\dagger$. However, in the perfect detection efficiency limit, $\eta \to 1$, all decoherence is completely eradicated from the system and only coherent evolution remains.

**Application: dissipative quantum computing.**

It was recently shown that appropriately engineered quasi-local dissipation can be used to perform universal quantum computation (UQC) [39]. Although such dissipative quantum computing (DQC) is robust to decoherence in principle, in practice it is likely to suffer from local errors due to the presence of local loss. For traditional UQC, local errors can be corrected via quantum error correction (QEC) codes. Indeed, the threshold theorem proves that traditional UQC can be scaled to larger numbers of qubits, even when local errors are present, provided QEC is in operation [35]. However, QEC requires precisely timed projective measurement and conditional operations, hence adding this capacity to DQC greatly complicates the engineering of these systems [40].

We provide a much simpler solution. Provided the cause of the local errors is diagnosable, no-knowledge feedback can be used to cancel out local errors. Crucially, the feedback will work concurrently with any given quantum computation. To show this, we consider the effect of local loss on a DQC algorithm designed to generate a linear cluster state [see Fig. 3(a)]. A series of $N$ qubits evolve under the influence of quasi-local dissipators $Q_i = \sqrt{\alpha}(1 + \sigma_i^{-1}\sigma_i^{(1)}\sigma_i^{z(2)})/2$ (with special cases $Q_1 = \sqrt{\alpha}(1 + \sigma_i^{-1}\sigma_i^{z(2)})/2$ and $Q_N = \sqrt{\alpha}(1 + \sigma_i^{-1}\sigma_i^{z(2)})/2$ at the boundaries) and local loss operators $L_i = \sqrt{\gamma}\sigma_i^{z}$. Such that the master equation for the whole system is $\partial_t\rho_t = \sum_{i}\{D[Q_i] + D[L_i]\}\rho_t$. The steady state, $\rho_{ss}$, for the system when there is no local loss ($\gamma = 0$) is a cluster state, $\rho_{ss} = \rho_{cluster}$. However, when local loss is present ($\gamma \neq 0$), the steady state of the system is no longer the target cluster state. As shown in Fig. 3(c), the fidelity $F = \sqrt{\text{Tr}[\rho_{ss}\rho_{cluster}]}$ between the target cluster state and the actual steady state rapidly decreases with system size. However, when no-knowledge feedback is implemented as depicted in Figs 3(b), the decline in the fidelity as a function of system size is arrested. Engineering the additional local dissipator $\sigma_i^{z}$ [33] required for this feedback should be trivial in comparison to engineering the quasi-local dissipators $Q_i$. Figure 3(c) quantifies the effectiveness of the no-knowledge feedback, demonstrating that the fidelity improves as the detection efficiency increases, with the creation of a perfect cluster state possible when $\eta = 1$. In fact, since no-knowledge feedback can operate concurrently to any DQC algorithm, it could be included in addition to QEC. In this case no-knowledge feedback with an imperfect detection efficiency may reduce the error rate to the threshold required for truly scalable DQC.

DQC is just one of many possible quantum technologies that could be improved, or made possible, by the general and robust reduction of decoherence via knowledge measurements-based feedback. Indeed, as this example demonstrates, no-knowledge feedback can operate in conjunction with other quantum control protocols. No-knowledge feedback therefore does not compete with other decoherence reduction methods (e.g. QEC), it compliments them. Furthermore, given the simplicity of no-knowledge feedback, we suspect that the development of no-knowledge coherent feedback control is a strong possibility. The many advantages of no-knowledge feedback strengthen the case for more reliable and robust dissipation engineering, as this is a vital ingredient for the cancellation of general forms of decoherence.

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shown in (b ii), for each qubit a no-knowledge measurement
the general no-knowledge feedback protocol on each qubit. As
severely for a larger number of qubits. (b i) The errors intro-

FIG. 3. (a) Diagram depicting a DQC setup with an N qubit
chain coupled to a set of quasi-local operators $Q_i$ and local loss
operators $L_i$. As demonstrated in (c), a loss rate of $\gamma/\alpha = 10$
decreases the fidelity between the target cluster state and the
system steady state (green triangles), and decreases it more
severely for a larger number of qubits. (b i) The errors intro-
duced by the local loss in the DQC are corrected by applying the
general no-knowledge feedback protocol on each qubit. As
shown in (b ii), for each qubit a no-knowledge measurement
is constructed by coupling an additional reservoir $\sqrt{\gamma} \sigma_z$
and measuring $\sigma_+^i$ and $\sigma_-^i$ at a homodyne angle of $\pi/2$ according to
the setup summarised in Fig. 2. Decoherence is cancelled by
feeding back $H = \sqrt{\gamma} \sum_i [\sigma_z^i \hat{\rho}_i^{(+)}(t) + \sigma_+^i \hat{\rho}_i^{(-)}(t)]$. (c) The fidelity as a function of system size for no feedback (green triangles), and no-knowledge feedback with detection efficiency
$\eta = 0.9$ (yellow diamonds), $\eta = 0.99$ (red squares) and
$\eta = 1$ (blue circles).

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There is a philosophical subtlety here. This separation of the system and filter assumes that an underlying quantum state exists independent of an observer’s knowledge of the system, which is by no means uncontroversial. Should the reader ascribe to a purely epistemic interpretation of quantum theory, then it is best to conceptualise $\rho_t$ and $\pi_t$ as optimal Bayesian inferences on the same system by two independent observers who are free to choose different initial conditions.
SUPPLEMENTARY MATERIAL

I. Stochastic master equation in Stratonovich form

The effect of a no-knowledge measurement is best understood by examining the Stratonovich version of the conditional master equation. However, the literature far more frequently presents conditional master equations with Ito stochastic integrals. Here we present the connection between the two.

To begin, consider an $n \times 1$ vector of stochastic variables $x_t$ that obeys the following linear Ito stochastic differential equation:

$$dx_t = A(x, t)x_t dt + B(x, t)x_t dw(t),$$

where $A$ and $B$ are $n \times n$ matrices, and $dw(t)$ is an Ito Wiener increment satisfying $dw(t)dw(t) = dt$. Furthermore, terms with $dw(t)$ average to zero, making Ito SDEs the convenient choice for most analytic work. Although the differential shorthand is convenient, Eq. (1) is strictly interpreted as an integral equation. In order to express Eq. (1) in terms of the Stratonovich noise $\xi(t)$, we must add the deterministic correction $-B^2x_t/2$ [20]:

$$\partial_t x_t = A(x, t)x_t - \frac{1}{2} [B(x, t)]^2 x_t + B(x, t)x_t \xi(t).$$

Since Stratonovich SDEs satisfy the rules of deterministic calculus, we choose not to use the differential shorthand, which also allows Ito and Stratonovich SDEs to be quickly distinguished. Nevertheless, Eq. (2) should also be strictly interpreted as an integral equation.

Let us return to the stochastic master equation for the conditional evolution of the unnormalised density operator, $\rho_t$. This is commonly written in the Ito form

$$d\rho_t = L\rho_t dt + \sqrt{\eta} A[L e^{i\theta}] \rho_t dy_\theta(t),$$

where $L\rho_t = -i[H, \rho_t] + D[L]\rho_t$, $D[Z]\rho_t = Z\rho_t Z^\dagger - (Z^\dagger Z\rho_t + \rho_t Z^\dagger Z)/2$, $A[Z]\rho_t = Z\rho_t + \rho_t Z^\dagger$ and $dy_\theta(t) = \sqrt{\eta} (L \exp(i\theta) + L^\dagger \exp(-i\theta)) dt + dw(t)$. Since $dy_\theta(t)^2 = dt$ and $D[Z]\rho_t$ and $A[L e^{i\theta}] \rho_t$ are linear superoperators, Eq. (3) is a linear SDE of the form (1). The Stratonovich version is therefore of the same structure as Eq. (2):

$$\partial_t \rho_t = L\rho_t + \sqrt{\eta} A[L e^{i\theta}] \rho_t y_\theta(t) - \frac{\eta}{2} A^2[L e^{i\theta}] \rho_t,$$

where $A^2[Z]\rho_t = Z(A[Z]\rho_t) + (A[Z]\rho_t)Z^\dagger$.

II. No-knowledge measurements and convergence

In this section we prove that the quantum filter does not in general converge to the underlying system state if the system is undergoing a no-knowledge homodyne measurement. As stated in Eq. (5) of the main text, when the system undergoes no-knowledge monitoring (i.e. homodyne detection at an angle $\theta = \pi/2$), the dynamics of the underlying conditional state are given by

$$\partial_t \rho_t = -i [H - Ly\pi/2(t), \rho_t].$$

We also assume that an observer makes some optimal Bayesian estimate of the system state, $\pi_t$, conditioned on the measurement signal $y\pi/2$. For a no-knowledge measurement, both $\rho_t$ and $\pi_t$ obey Eq. (6). However, in general the initial conditions differ (i.e. $\pi_0 \neq \rho_0$).

Define $\Delta_t \equiv \rho_t - \pi_t$, which clearly satisfies Eq. (5), and will therefore retain its initial normalisation. We now quantify the difference between the system and filter via the Frobenius distance $||\Delta_t|| = \sqrt{\text{Tr}[\Delta_t^2]}$. Due to the form of Eq. (5), the Frobenius distance is constant in time:

$$\partial_t ||\Delta_t|| = \frac{\text{Tr}[\Delta_t (\partial_t \Delta_t) + (\partial_t \Delta_t) \Delta_t]}{2||\Delta_t||} = -i \text{Tr}[\Delta_t (H - L y\pi/2(t), \Delta_t)] ||\Delta_t|| = 0.$$

Thus $\rho_t$ and $\pi_t$ remain the same distance apart from each other for all time. This shows that under no-knowledge monitoring, it is impossible for an experimenter to refine their estimate of the system state.

III. Equations of motion for a qubit undergoing dephasing

Consider the Stratonovich stochastic master equations

$$\partial_t \rho_t = -i[\Omega, \rho_t] + \gamma D[\sigma_x]\rho_t + \sqrt{\eta} A[\sigma_x e^{i\theta}] \rho_t y_\theta(t)$$

$$- \frac{\gamma}{2} A^2[\sigma_x e^{i\theta}] \rho_t,$$ (7a)

$$\partial_t \pi_t = -i[\Omega, \pi_t] + \gamma D[\sigma_x]\pi_t + \sqrt{\eta} A[\sigma_x e^{i\theta}] \pi_t y_\theta(t) - \frac{\gamma}{2} A^2[\sigma_x e^{i\theta}] \pi_t,$$ (7b)

which correspond to the physical setup depicted in Fig. 1(a) of the main text. Although equations (7a) and (7b) look similar, it is important to recognise that they are coupled via the same measurement signal

$$y_\theta(t) = 2\sqrt{\gamma} \cos \theta (\text{Tr}[\sigma_z \rho_t]/\text{Tr}[\rho_t]) + \xi(t).$$ (8)

For a qubit, the density matrix for the underlying system takes the simple form

$$\rho_t = \frac{1}{2} \begin{pmatrix} 1 + x_{\rho}(t) & y_{\rho}(t) & z_{\rho}(t) \\ y_{\rho}(t)^* & 1 + y_{\rho}(t) & z_{\rho}(t) \\ z_{\rho}(t)^* & z_{\rho}(t)^* & 1 + z_{\rho}(t) \end{pmatrix},$$ (9)

where, for example, $x_{\rho}(t) = \text{Tr}[\sigma_x \rho_t]/\text{Tr}[\rho_t]$, which implies that $(x_{\rho}(t), y_{\rho}(t), z_{\rho}(t))$ are the co-ordinates defining
the Bloch vector. Similarly, \( \pi_t = (I + x_\pi(t) \sigma_x + y_\pi(t) \sigma_y + z_\pi(t) \sigma_z)/2 \). The equations of motion \(^7\) therefore reduce to the following set of Stratonovich SDEs:

\[
\begin{align*}
    dx_\rho &= 2\sqrt{\gamma} (y_\rho \sin \theta - x_\rho z_\rho \cos \theta) y_\theta(t), \\
    dy_\rho &= -\Omega x_\rho - 2\sqrt{\gamma} (x_\rho \sin \theta + y_\rho z_\rho \cos \theta) y_\theta(t), \quad (10a) \\
    dz_\rho &= \Omega y_\rho + 2\sqrt{\gamma}(1 - z_\rho^2) \cos \theta y_\theta(t), \quad (10c) \\
    dx_\pi &= 2\sqrt{\gamma} (y_\pi \sin \theta - x_\pi z_\pi \cos \theta) y_\theta(t), \quad (10d) \\
    dy_\pi &= -\Omega x_\pi - 2\sqrt{\gamma} (x_\pi \sin \theta + y_\pi z_\pi \cos \theta) y_\theta(t), \quad (10e) \\
    dz_\pi &= \Omega y_\pi + 2\sqrt{\gamma}(1 - z_\pi^2) \cos \theta y_\theta(t). \quad (10f)
\end{align*}
\]

IV. No-knowledge measurement and feedback for photodetection

Consider again the open quantum system described by Eq. (1) of the main text. By directly measuring the number of reservoir quanta, the system dynamics can be conditioned according to the stochastic master equation \(^7\)

\[
d\omega_t = -i[H, \omega_t]dt - \frac{1}{2} A[L^\dagger L] \omega_t dt \\
    + (L\omega_t L^\dagger - \omega_t) dj(t), \quad (11)
\]

where \( \omega_t \) is the conditional density operator and \( j(t) \) is the measurement record, which is a Poissonian process with an average jump rate of \( \langle L^\dagger L \rangle \). Since this stochastic master equation commonly describes the direct detection of photons emitted from a system, we call such monitoring photodetection.

Knowledge about the system is contained in the rate at which jumps occur. However, when \( L = U \) for unitary \( U \) (i.e. \( U^\dagger U =UU^\dagger = 1 \)), the jump rate is always unity, and thus the measurement signal gives no-knowledge of the system dynamics. In this case, Eq. (11) reduces to

\[
d\omega_t = -i[H, \omega_t]dt - (U\omega_t U^\dagger - \omega_t) dj(t). \quad (12)
\]

As for the homodyne case, the underlying system \( \omega_t \) and the quantum filter \( \tilde{\omega}_t \) will never converge under no-knowledge photodetection. This can be shown explicitly by examining \( \Delta_t = \omega_t - \tilde{\omega}_t \), which for \( L = U \) satisfies Eq. (12). The evolution of the Frobenius distance is therefore (c.f. Sec. )

\[
d||\Delta_t|| = -i Tr [\Delta_t [H, \Delta_t]] dt \\
    + (||U\Delta_t U^\dagger|| - ||\Delta_t||) dj(t) \\
    = 0, \quad (13)
\]

implying that \( \omega_t \) and \( \tilde{\omega}_t \) remain an equal distance apart for all time.

Under the evolution (12), decoherence can be entirely removed from the system by simply applying the unitary operator \( U^\dagger \) to the system whenever a jump occurs. For after each jump, the state then becomes \( \omega_{t+dt} = U^\dagger (U \omega_t U^\dagger) U = \omega_t \), implying that only the coherent evolution \( d\omega_t = -i[H, \omega_t]dt \) remains.