P, C and Strong CP in Left-Right Supersymmetric Models

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We systematically study the connection between P, C and strong CP in the context of both non-supersymmetric and supersymmetric left-right theories. We find that the solution to the strong CP problem requires both supersymmetry and parity breaking scales to be around the weak scale.

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A. Introduction There are two possible ways of solving the strong CP problem, such as the first is dynamical relaxation mechanism, as the celebrated Peccei-Quinn symmetry which promotes the strong CP phase into a dynamical variable. The second idea is to utilize some discrete symmetry to make the strong CP phase vanish at the tree level. It then becomes calculable in perturbation theory and a viable solution to the problem requires that these perturbative corrections be below the experimental upper limit. The most appealing candidates for this job are the fundamental spacetime symmetries: parity (P) and time reversal (CP).

Rather natural candidates are left-right (LR) symmetric theories which provide a framework for the spontaneous breakdown of parity. Furthermore, CP can be spontaneously broken even in the minimal version of these theories. In addition they can be embedded in SO(10) grand unified theories, which are the minimal truly unified models of quarks and leptons. In this letter, we focus our attention on these natural candidates for the solution of the strong CP problem, both in ordinary and supersymmetric versions.

It is well known that the strong CP problem contains two aspects, that is the smallness of \( \theta_{\text{QCD}} \), the coefficient of the \( F \bar{F} \) term, and the smallness of \( \theta_{\text{QED}} = \text{ArgDet} M \), where \( M \) is the mass matrix of colored fermions. It is highly suggestive to use parity since it implies both \( \theta_{\text{QCD}} = 0 \) and \( M = M^\dagger \) which in turn gives \( \theta_{\text{QED}} = 0 \). This would be sufficient if parity were an exact symmetry of nature. However, parity must be broken and the real challenge in these theories is to keep \( \theta = \theta_{\text{QCD}} + \theta_{\text{QED}} \) small to all orders in perturbation theory. Without supersymmetry this is an impossible task. Essentially, the problem is that the requirement of weak CP violation destroys the hermiticity of the quark mass matrices already at the tree level which induces in general large \( \theta_{\text{QED}} \).

Another way to see it is to note that the constraint of parity invariance alone allows for complex couplings in the Higgs potential which lead to complex VEVs for the Higgs fields and thereby destroy the hermiticity of the quark mass matrices even at the tree level. Recently it has been argued that making the left-right symmetric model supersymmetric leads to a Higgs potential where all coupling parameters are real thus giving us a CP-conserving vacuum. Furthermore, the perturbative one-loop contributions to \( \theta \) can be shown to be small under certain circumstances.

These observations have inspired us to revisit the SUSY-SYLR model and carefully discuss under what circumstances \( \theta \) in this model is guaranteed to be acceptably small. We find that supersymmetry and parity symmetry by themselves are not sufficient to control the one loop contributions. One needs charge conjugation invariance for the purpose. It then turns out that in general the hermiticity of the quark mass matrices can only be preserved at the expense of weak CP violation thus making the theory unrealistic. We find one exception: low scale of parity breaking \( M_R \) and parity breaking achieved only through nonrenormalizable operators. In this case the smallness of \( \theta \) is achieved by a soft violation of CP and is controlled by the small ratio of \( M_R/M_{\text{Planck}} \). We find it rather interesting that the requirement of smallness of the strong CP phase requires experimentally accessible scale of parity restoration. This is the major new result of our paper.

In order to set the framework for our discussion we first analyze the essential features of parity and charge conjugation and their role in the strong CP problem.

B. No supersymmetry We start with the minimal left-right symmetric theory based on the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \) and the following fermionic content:

\[ Q_L = \begin{pmatrix} u \dagger \\ d \end{pmatrix}_L \ (2, 1, 1/3), \ Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \ (1, 2, 1/3), \]
\[ L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \ (2, 1, -1), \ L_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R \ (1, 2, -1), \]

with gauge quantum numbers spelled out in brackets.

Under parity these fields transform as usual

\[ Q_L \leftrightarrow Q_R; \ L_L \leftrightarrow L_R \]

and similarly under charge conjugation

\[ Q_L \leftrightarrow (Q^c)_L \equiv CQ^c_R; \ L_L \leftrightarrow (L^c)_L \equiv CL^c_R. \]

We will stick to the somewhat conservative assumption that there are no new quarks and leptons. It should be
mentioned that if this assumption is relaxed it is possible to construct viable models based on parity only which predict calculably small theta \[\theta\] \(\leq 10^{-4}\) \(\leq 10^{-12}\). Similarly, with additional fermions, one can use both P and C symmetries and CP can be used to exchange the two SU(2) groups \[SU(2)\] \(\phi\).

With the above fermion content the field that provides quark and lepton masses is the Higgs bidoublet

\[
\phi(2, 2, 0),
\]

which under parity transforms as

\[
\phi \leftrightarrow \phi^T.
\]

Keeping in mind eventual SO(10) embedding, we allow a sign ambiguity in the charge conjugation transformation of \(\phi\)

\[
\phi \leftrightarrow \pm \phi^T.
\]

The imposition of parity and the gauge symmetry determine the Yukawa couplings

\[
L_y = h_q \bar{Q}_L \phi Q_R + h_t \bar{L}_L \phi L_R
\]

to be hermitean i.e.

\[
h_q = h_q^\dagger; h_t = h_t^\dagger.
\]

Clearly, since the quark mass matrices are given by

\[
M_q = h_q \langle \phi \rangle,
\]

they will be hermitean if and only if \(\langle \phi \rangle\) is real (\(\langle \phi \rangle\) real obviously preserves parity). But \(\langle \phi \rangle\) can be real only if the Higgs potential is CP conserving. Here lies the crux of the problem. Now, \(h\) is either real or complex. If it is real then \(\langle \phi \rangle\) must be complex in order for CP to be broken, in which case \(M_q\) cannot be hermitean. If on the other hand, \(h\) is complex then unfortunately there are complex couplings in the Higgs potential and \(\langle \phi \rangle\) itself becomes complex, destroying again the hermiticity of \(M_q\).

Let us demonstrate this in some detail. Consider first the minimal case with a single bidoublet due to \(\phi\). It is a simple exercise to show that the potential which depends on \(\phi\) only has all the couplings real due to parity symmetry. However, in order to break \(SU(2)_R\) symmetry at the scale \(M_R >> M_W\) we need other Higgs fields, \(\chi_L\) and \(\chi_R\), which are nontrivial representations under \(SU(2)_L\) and \(SU(2)_R\), respectively. The troublesome couplings in the schematic representation are

\[
(\alpha \chi_L^T \chi_L + \beta \chi_R \chi_R) \phi^T \phi + h.c.
\]

Parity imposes only \(\alpha = \beta^*\) so that \(\alpha\) is in general complex. Only if one imposes charge conjugation on top of parity, are these couplings made real. As we will see, this additional requirement of C-invariance in addition to parity happens also in the supersymmetric version. Now however, \(\langle \phi \rangle\) must be complex in order to have nonvanishing weak CP violation since C-invariance also makes the Yukawa couplings real. The hermiticity of \(M_q\) required for \(\theta\) to vanish is then lost.

One could imagine a possible way out along the lines of Ref. 8. Suppose that there are two bidoublets with opposite transformation properties under C

\[
\phi_1 \rightarrow \phi_1^T, \phi_2 \rightarrow -\phi_2^T.
\]

This implies that \(h_1 = h_1^T\) and real, and \(h_2 = -h_2^T\) and purely imaginary. In the context of SO(10), \(\phi_1\) would belong to 10-dimensional representation and \(\phi_2\) to 20-dimensional one. It is noteworthy that in SUSY one must have at least two bidoublets in order to have nonzero quark mixing angles.

Now the quark mass matrices become

\[
M_q = h_1 \langle \phi_1 \rangle + h_2 \langle \phi_2 \rangle.
\]

Notice that \(\langle \phi_2 \rangle \rightarrow \langle \phi_2 \rangle^*\) under CP, so that real \(\langle \phi_2 \rangle\) breaks CP invariance. Obviously if both \(\langle \phi \rangle\) are real, M is hermitean and complex. This would guarantee weak CP violation without the strong one. At this point all seems well since as before the interaction terms between the \(\phi\)s in the potential are real. However, again there are complex couplings with \(\chi_L\) and \(\chi_R\) fields of the type are

\[
i(\alpha \chi_L^T \chi_L + \beta \chi_R \chi_R) \phi_1 \phi_2 + h.c.
\]

Parity imposes \(\beta = -\alpha^*\), and charge conjugation makes \(\alpha\) real. Obviously, the presence of both real and imaginary couplings in the potential will render the VEVs of the bidoublets complex. This in turn kills the hermiticity of the mass matrices and implies a strong CP phase already at the tree level. We should stress that this problem is generic and does not depend on the choice of \(\chi\) fields, i.e. whether they are doublets, triplets or higher representations.

In supersymmetry it is the superpotential that defines the theory and one might hope that at least at the renormalizable level such dangerous terms may be absent. However, the issue is more subtle and now we discuss it in detail.

C. Supersymmetry \(\equiv\) It is well known that in supersymmetry one needs at least two doublets to get realistic fermion mass matrices so that the above scenario finds here its natural place. There are however new CP problems in supersymmetry: the relevant one for us is that the masses of gauginos are complex in general. Here P and C play again a fundamental role: P makes gluino mass real but not the masses of the left and right winos. At the one loop level, these complex masses lead to a finite but unacceptable contribution to \(\theta\) of order \(\alpha/4\pi\). In Ref. 8, \(\theta\) appears to SO(10) grand unified extension in order to make these masses real. The point is simply that parity and charge conjugation suffice: P makes
ghino mass real, and P and C ensure the same for weak gaugino masses. Thus we impose both of them and study the consequences.

Interestingly enough, we find that complete consistency of the theory requires that the $W_R$ mass must be in the TeV range.

Let us go back to Eq. (11). In the minimal left-right model with the see-saw mechanism the χ fields are taken to be triplets $\Delta$ and $\Delta_c$ under $SU(2)_L$ and $SU(2)_R$ groups respectively [13]. Of course anomaly cancellation in supersymmetry requires the doubling of such fields ($\Delta$ and $\Delta_c$). It is easy to see that at the renormalizable level there are no such dangerous complex couplings in the superpotential. The problem is that in this model there can be no spontaneous breakdown of left-right symmetry and if the theory is augmented by the parity odd gauge singlet, this gets cured at the expense of the breakdown of electromagnetic charge invariance [16].

We find that the condition for the parity breaking and electromagnetic charge invariance [16] is possible to do away with singlets completely. Instead, one can do without the parity-odd singlet [17]. However, in this version of the theory, the splitting of the bidoublets is achieved through the above d=4 terms and thus besides the usual two light doublets of the MSSM above the scale $M^2_R/\tilde{M}_p$ there will appear the other two doublets. It has been shown in Ref. [8] that the running of the Yukawa couplings below $M_R$ quickly generates sizeable $\theta$ when four doublets are present.

Thus one is forced to the low parity breaking scale scenario. One way to get this is to introduce a parity odd singlet superfield $\sigma$ [10]. In this case, in order not to introduce complex couplings in the superpotential an additional parity even singlet $X$ is needed. Namely, a parity odd singlet has imaginary couplings with the bidoublets and thus one must insure that its VEV be imaginary too. This can be achieved if one chooses a superpotential for the singlets of the form

$$W_s = X(\alpha \sigma^2 + M^2)$$

where $\alpha$ and $M^2$ are real by parity.

Those who dislike model building should know that it is possible to do away with singlets completely. Instead, one can obtain a desired pattern of symmetry breaking using only nonrenormalizable operators, as long as the neutrino Yukawa couplings satisfy $f \lesssim 10^{-2} - 10^{-3}$ and $m_S \approx M_R \approx 1$ TeV. Namely, in this case the nonrenormalizable terms should lower the energy at the parity broken extremum to be the minimum of the potential. The couplings $f$, due to running between the scale $M_U$ of assumed universality of soft terms and the scale of right handed neutrinos $M_{\nu_R}$, cause the difference between the VEVs $v = < \Delta_c >$ and $\overline{v} = < \Delta >$

$$v^2 - \overline{v}^2 \approx \frac{f^2}{16\pi^2} m^2_{\nu_R}$$

We find that the condition for the parity breaking and electric charge conserving minimum is

$$\frac{g^2}{2}(v^2 - \overline{v}^2)^2 < \frac{m^2}{\tilde{M}_p} v^2 \overline{v}^2$$

Again $\alpha = -\beta$ is real. Now clearly the complex mixing term between $\phi_1$ and $\phi_2$ is suppressed by $\frac{M_R}{\tilde{M}_p}$. It is easy to see that the relative phase between the $\phi_1$ and $\phi_2$ VEVs can be controlled by the same suppression factor. It is a simple exercise to show that the strong CP phase is of order

$$\theta = \frac{M^2_R}{m_S \tilde{M}_p}$$

Next, it can be shown that $\Omega_i$ and $\Omega_{i\alpha}$ are of the type discussed above and thus $\alpha = -\beta$ real. Next, it can be shown that $\Omega_c$ VEV is real. The terms in the superpotential which are relevant are the couplings of the right handed triplet fields

$$W = m_{\Delta} (Tr(\Delta \Delta_c) + Tr(\Delta \Delta_c))$$

are of the type discussed above and thus $\alpha = -\beta$ real. Next, it can be shown that $\Omega_c$ VEV is real. The terms in the superpotential which are relevant are the couplings of the right handed triplet fields

$$W = m_{\Delta}(Tr(\Delta \Delta_c) + Tr(\Delta \Delta_c))$$

It can be easily seen that C and P render the above couplings real. In the P breaking and electromagnetic charge preserving vacuum $< \Delta > = < \Delta_c > = 0$ and $< \Omega_c > = M_{Rdiag}(1, -1)$ with $M_R$ being a real number

$$M_R = \frac{m_{\Delta}}{\alpha}.\hfill (16)$$

This induces the imaginary $\mu$ type effective mixing term between $\phi_1$ and $\phi_2$, thus making it impossible to keep both bidoublet VEVs real. This just as in the non-supersymmetric case destroys the hermiticity of the quark mass matrices.

(b) Alternatively, one can work without $\Omega$ fields assuming that there are nonrenormalizable terms in the superpotential to achieve the spontaneous breakdown of parity. Again, one can do without the parity-odd singlet [18]. In this case the analog of the mixing of $\Omega$ and $\phi$ fields [14] is achieved through following d=4 terms in the superpotential

$$i \frac{\alpha}{M_{Pl}} \Delta \Sigma \phi_1 \phi_2 + i \frac{\beta}{M_{Pl}} \Delta \Sigma \phi_1 \phi_2.\hfill (17)$$
for a range of values of parameters that characterize the non-renormalizable terms in the superpotential. The condition on \( f \) noted above follows from this inequality.

Next, in order to break CP we need nonzero (and real) VEVs of both bidoublets. This can only happen if there is a mixing term between \( \phi_1 \) and \( \phi_2 \). This term (real due to parity) breaks C softly. This result is a reflection of a general theorem regarding the impossibility of spontaneous CP violation in the SSM with four Higgs doublets. If one does want to stick to spontaneous violation, this is easily achieved with two singlets as in the above example. In the presence of this soft C-breaking term, one expects finite contributions to such phases. If they arise at the two or higher loop level, their contribution to the \( \Theta \) is \( \propto \alpha^3/(4\pi)^3 \) which is of order \( 10^{-9} \) and is therefore small. Also we repeat that there is a one loop contribution to quark masses due to the soft SUSY breaking terms that has already been evaluated in Ref. 9 and is shown that it can be around \( 10^{-9} \) to \( 10^{-10} \) level.

D. Summary and outlook  The main implication of our work is the low scale of parity breaking, necessary for the solution to the strong CP problem. Let us briefly comment on the implications of this model for neutrino masses. The smallness of the neutrino masses in our model is of course guaranteed by the see-saw mechanism. As far as the values of the neutrino masses are concerned, it depends on the precise model for the Dirac neutrino masses. In order for the neutrino masses to be below the upper experimental bounds, one must assume neutrino Dirac mass terms order of magnitude or so smaller than the charged lepton masses. This in turn implies that \( \nu_\mu \) and \( \nu_\tau \) have to decay and both the \( \nu_\tau \) and \( \nu_\mu \) can decay only through the exchange of the neutral component of the left-handed triplet \( \Delta \) rapidly enough to satisfy necessary cosmological constraints. Although in nonsupersymmetric version of the theory this requires some mild fine tuning since the mass of the left-handed triplet is proportional to \( M_R \) in the supersymmetric version discussed here this does not happen. This scenario is phenomenologically completely consistent and has interesting predictions of rare \( \mu \) decays and \( \mu \to e \gamma \) conversions. Another possibility for being in accord with cosmological limits on neutrino masses is to suppress the neutrino Dirac mass terms as a higher order loop effect. The model in this case has to be supplemented by the addition of extra color triplet fields coupling to quark fields, which do not affect the discussion of the strong CP problem given above.

It is well known that in left right models with low \( M_R \), there are tree level neutral Higgs contributions to the flavor changing neutral current effects. Present observations require that the mass of these neutral Higgs bosons must be more than 5 TeV or so. Since these masses are proportional to \( M_R \), this is consistent with our result that that puts \( M_R \) also in the same TeV range.

Since our results heavily depend on the imposition of charge conjugation on top of parity it is natural to consider the SO(10) GUT extension of LR models. Namely, in SO(10) charge conjugation is an automatic gauge symmetry and furthermore, as we remarked before, our choice of the C-transformation properties of bidoublets would simply imply that \( \phi_1 \) lies in the 10-dimensional, and \( \phi_2 \) lies in the 120-dimensional representation. On the other hand, it is hard, if not impossible, to achieve low \( M_R \) in the supersymmetric SO(10), at least in the minimal version of the theory.

In conclusion, we stress that this is a natural solution to the strong CP problem since low \( M_R \) scale (order \( m_S \)) can be achieved naturally in the process of minimization of the potential. Consistency with the hierarchy problem suggests then \( M_R \) of order few TeVs. We find it rather appealing that the smallness of \( \theta \) in left-right symmetric theories is linked to both supersymmetry and \( M_R \) being at the low scale. This provides to date the strongest theoretical motivation for a low mass \( W_R \) which has long been of great phenomenological and experimental interest.

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