Generation of Aharonov-Bohm (AB) phases has achieved a state-of-the-art in mesoscopic systems with manipulation and control of the AB effect. The possibility of transfer information encoded in such systems to light increases the possible scenarios where the information can be manipulated and transferred. In this paper we propose a bit-encoding of AB phases contrasting with the usual codifications using chirality or flux orientation. We propose a quantum transfer of the AB phase to a coherent state superposition, leading to the possibility of transferring AB phases to non-classical states of light and store the bit information encoded in this phase to a light mode field. We also discuss the storage of a string of bits encoded by AB phases in a product state and show that this scheme can be implemented to store a string of bits in high-Q or multimode cavities. Our propose can also be useful to further progress in methods of quantum information associated to modern techniques in synthetic gauge fields.

INTRODUCTION

Aharonov-Bohm (AB) effect [1] is a counterintuitive physical phenomena as viewed from the classical perspective, breaking the classical interpretation of gauge potentials. This effect is generally exemplified by confining a magnetic field into the interior of a solenoid and observing the imprinting of a phase factor on the electron’s wave function that travels in the region outside the solenoid. The main importance of such effect is that a phase factor, the so-called AB phase, can be generated in the presence of a non-zero gauge potential in a region with a null electromagnetic field, modifying the corresponding energy spectrum of the charged particle. The presence of the AB phase represents a direct influence of a vector potential on the particle dynamics and illustrates one of the remarkable possibilities of quantum interference [2–4].

In recent years, several measurements of AB phases have been realized and their role associated to the gauge transformation implemented by the symmetry of the electrodynamics $A_\mu + \partial_\mu \lambda$. Methods of generating AB phases have growth in diverse fields, representing an important role in solid-state interferometers [5, 6], nanotubes [7], transmission microscopy [8], AlSb/InAs heterostructures [9], quantum Hall effect regime [10], Kondo resonance [11] and spin transport [12]. Detection of this effect in photonic states [13], optical induction in mesoscopic systems [14] and interaction of AB ring in a high-Q cavity [15, 16] are examples of the increasing importance of such effect in interacting light-and-matter systems and optical versions of the AB effect [17].

Advances in the domain of artificial gauge fields have given rise new possibilities in exploring AB effect and geometrical phases [18]. Optical Berry phase [19], Aharonov-Casher effect [20, 21] as well as AB effect for neutral particles have also been explored in diverse contexts [22–25] representing important roles likewise the original AB effect.

One important point is the possibility of controlling the AB effect in different scenarios, exchanging the domains where the effect was originally created. Such a realization is particularly important in the domain of quantum information and computation. Gate controlled AB oscillations [26] and topological spin transistors [27] are advances in such direction.

From the information perspective, the control of the generation of AB phases can be used for transmitting a bit of information using the AB effect, although application in coherence control can also emerge [28]. In this paper, we demonstrate how this can be implemented sequentially in order to generate a string of bits of a given length. The flux control of interference phenomena and applications in gate operations imply that a bit encoding can be experimentally realized by means of a gate control of the AB effect.

Other important effect is the quantum transfer that can be used for transport of information. Considering a bit-encoding of AB phases, we can demonstrate that these phases can be quantum transferred for a coherent state superposition. This possibility indirectly generate a AB phase for a non-classical state of light and can also be used to store a string of bits encoded by the AB phases. A quantum transfer of an AB phase to a cavity can bring together the possibility of storage the information of the quantum interference effect.

For quantum transfer of an AB phase to a coherent state superposition, we consider the case in which a charged particle acquires an AB phase by means of an AB device, as a Mach-Zender interferometer, and then crosses dispersively a high-Q cavity initially in a coherent state. Under a post-selection of the electronic state,
the coherent state superposition is projected carrying the AB phase. In particular, we show that such a scheme can be improved to store a string of bits.

This paper is organized as follows: In Sec. II, we discuss the generation and bit-encoding of AB phases by exploring the AB effect. In Sec. III, we propose the quantum transfer of AB phase to a coherent state superposition. In Sec. IV, we discuss the use of quantum transfer to store a string of bits encoded by AB phases. Sec. V is reserved to our concluding remarks.

**GAUGE INVARIANCE AND GENERATION OF AB PHASE**

The gauge invariance is usually illustrated by a phase transformation over a (spinor) field minimally coupled to a gauge field $A_\mu$, described by the lagrangian density ($h = c = 1$), $\mathcal{L} = \bar{\psi}(\partial_\mu - ieA_\mu)\psi$, where the Maxwell term $-\frac{i}{2}F_{\mu\nu}F^{\mu\nu}$ and the mass term $m\psi\bar{\psi}$ are not included, due their automatic gauge and phase invariance, respectively.

A local phase transformation $\psi \rightarrow e^{i\theta(x)}\psi$ and a gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \gamma(x)$ imply in the corresponding change of the lagrangian density $\mathcal{L} = \mathcal{L} + i\bar{\psi}\psi\partial_\mu \theta(x) - i\bar{\psi}\psi e\partial_\mu \gamma(x)$. The gauge invariance is immediately satisfied when the phase and the gauge function are related $\theta(x) = -ie\gamma(x)$. This corresponds to a $U(1)$ abelian symmetry in the gauge field $A_\mu$. Even if the electromagnetic tensor $F_{\mu\nu}$ becomes zero in a region with non-zero $A_\mu$, the above expression remains consistent.

This feature remains important when a closed region in the space-time encloses the gauge field

$$-ie\oint A_\mu dx^\mu \rightarrow -ie\oint \left( A_\mu dx^\mu + \frac{i}{c}d\theta(x) \right),$$

(1)

where the phase $d\theta(x) = dx^\mu \partial_\mu \theta(x)$. This imply that the phase transformation imprint a change present in the measure

$$\Delta\Sigma = -ie\oint A_\mu dx^\mu + \oint d\theta(x).$$

(2)

Considering a static field in a closed path under and a gauge fixing, the contribution of this measure is reduced to the known AB phase shift term

$$-i\phi_{AB} = -ie\oint A \cdot dl.$$

(3)

The emergence of this term as a contribution of the problem of a charged particle in a magnetic field

$$\hat{H} = \left( \frac{\mathbf{p} - e\mathbf{A}}{2m} \right)^2 + e\phi,$$

(4)

where $A_\mu = (\phi, \mathbf{A})$, for a singly connected region, where $\mathbf{B} = \nabla \times \mathbf{A} = 0$ and $\phi$ can be null, give rise to the AB effect, with the wave function splitted in two parties with the acquired AB phase term [1]. The interference in the electron states coming from the right and left paths give rise to a superposition state with the acquired AB phase $|\psi\rangle = |L\rangle + e^{i\phi_{AB}}|R\rangle$. This can be understood from an interaction between the source-field and the electron, with the whole state given by

$$|\chi\rangle_I = \frac{1}{\sqrt{2}}\left(|L\rangle e\langle\psi_L|S + |R\rangle e\langle\psi_R|S\right),$$

(5)

collapsing to a final configuration where the left and right states are identical except for the presence of the AB phase, $|\psi_L\rangle_S = |\psi\rangle_S$ and $|\psi_R\rangle_S = e^{i\phi_{AB}}|\psi\rangle_S$, leaving the total state in the following final form

$$|\chi\rangle_F = \frac{1}{\sqrt{2}}|\psi\rangle_S \left(|L\rangle + e^{i\phi_{AB}}|R\rangle\right).$$

(6)

**ENCODING A BIT OF INFORMATION WITH AB PHASES**

Bits are the basic units of information and the core of computation and digital electronics. The emergence of qubits and the whole quantum information brought many promises to the actual scenario of computers. Instead of a linear reduction of computer components, the use of quantum systems in the scenario of computers is another possible path that is not directly restricted to the building of a ‘totally quantum’ computer. One possible use of the generation and control of AB phases is the bit encoding of AB phases using the AB effect.

We can propose such a bit-encoding in the generation of AB phases by means of the flux control in the solenoid, in particular, we can control the imprinting or not of the phase, by absence or not of the potential due to the switch-on or switch-off of the flux in the solenoid. Since the absence of the potential will lead to absence of the AB phase, the off-state can be characterized by a null vector potential, while the on-state can be characterized by the presence of the vector potential. This implies in a bit defined from absence of AB phase as an off-state and the presence of AB phase as the on-state

$$\text{bit} \rightarrow \{0, \phi_{AB}\}.$$

(7)

This can be characterized formally by the binary function dependent of presence of a AB phase or its absence:

$$f(\phi_{AB}) = \begin{cases} 1 & \text{if } \phi_{AB} \neq 0; \\ 0 & \text{if } \phi_{AB} = 0. \end{cases}$$

(8)

A sequential generation of AB phases corresponds to a sequence of the following type

$$\phi_{AB}^{(0)}|f(\phi_{AB})^{(1)}|f(\phi_{AB})^{(2)}|f(\phi_{AB})^{(n)} ... |f(\phi_{AB})^{(n)}|\phi_{AB}^{(n)}.$$

(9)

where $(i)$ corresponds to the $i$-th order of measurement of the $i$-th AB phase term. This results in a string of bits that generates a binary sequence of bits

$$f(\phi_{AB})^{(0)}f(\phi_{AB})^{(1)}f(\phi_{AB})^{(2)}...f(\phi_{AB})^{(n)}.$$

(10)
In particular, this codification can be used to implement any ASCII code or a given quantity of bytes. For instance, the string of bits 10110 corresponds to the sequence

\[ f(0)\langle \phi_{AB} \rangle f(1) (0) f(2) \langle \phi_{AB} \rangle f(3) \langle \phi_{AB} \rangle f(4) (0). \quad (11) \]

As a result, this binary codification using AB phases can be implemented to generate any binary message and consequently can be used in computer devices. It also contrasts with other proposals that consider chirality or flux orientation for bit codification. The components of mesoscopic size can be used with solenoids controlled to generate a given string of bits. The possibility of transfer information encoded to light increases this possibility.

**QUANTUM TRANSFER OF THE AB PHASE TO A COHERENT STATE SUPERPOSITION**

A quantum transfer of the AB phase can be implemented to transmit the bit encoded with AB phases.

We can rewrite this state rearranging the phase terms to the coherent state superpositions

\[ \mid \psi(t) \rangle = \left[ \frac{1}{4} \left( 1 + e^{i\psi_{AB}} \right) e^{-2i\gamma_\beta t} - \frac{1}{4} \left( 1 - e^{i\psi_{AB}} \right) \right] \mid R, \alpha \rangle + \left[ \frac{1}{4} \left( 1 + e^{i\phi_{AB}} \right) e^{-2i\gamma_\beta t} + \frac{1}{4} \left( 1 - e^{i\phi_{AB}} \right) \right] \mid L, \alpha \rangle. \]

We can specify the time of interaction in \( t = \pi/2\beta \), such that we now have

\[ \mid \phi_{\pm} \rangle = \frac{1}{4} \left( 1 + e^{i\phi_{AB}} \right) \mid - \alpha \rangle \pm \frac{1}{4} \left( 1 - e^{i\phi_{AB}} \right) \mid \alpha \rangle. \quad (17) \]

The coefficients in (17) can be rearranged by the action of a Hadamard-type gate operation in the basis of coherent states,

\[ \hat{U}_H = \frac{1}{\sqrt{2}} \left( \mid - \alpha \rangle \langle \alpha \mid - \mid \alpha \rangle \langle - \alpha \mid + \mid - \alpha \rangle \langle \alpha \mid + \mid \alpha \rangle \langle - \alpha \mid \right), \quad (18) \]

considering the projection relations for coherent states \( \langle \alpha | \alpha \rangle = 1 \) and \( \langle \alpha | - \alpha \rangle = e^{-2|\alpha|^2} \). After this gate opera-
tion the state takes the form

\[ |\xi_{\pm}\rangle = \frac{1}{4} (1 + e^{i\phi_{AB}}) | -\alpha \rangle \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) | -\alpha \rangle e^{-2|\alpha|^2} \]

\[ - \frac{1}{4} (1 + e^{i\phi_{AB}}) |\alpha\rangle e^{-2|\alpha|^2} \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) |\alpha\rangle \]

\[ + \frac{1}{4} (1 + e^{i\phi_{AB}}) |\alpha\rangle \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) | -\alpha \rangle e^{-2|\alpha|^2} \]

\[ + \frac{1}{4} (1 + e^{i\phi_{AB}}) | -\alpha \rangle e^{-2|\alpha|^2} \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) | -\alpha \rangle. \]  

(19)

Under the condition of negligible overlap, the coherent states |\alpha\rangle and | -\alpha \rangle can be considered orthonormal, \langle \alpha | -\alpha \rangle \approx 0, being described by a Hilbert space of the two-level system spanned by |\alpha\rangle and | -\alpha \rangle. The resulting states are

\[ |\xi_{\pm}\rangle = \frac{1}{4} (1 + e^{i\phi_{AB}}) | -\alpha \rangle \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) |\alpha\rangle \]

\[ + \frac{1}{4} (1 + e^{i\phi_{AB}}) |\alpha\rangle \pm \frac{1}{4} (1 - e^{i\phi_{AB}}) | -\alpha \rangle, \]  

(20)

Simplifying for each field state, including the normalization factor, we have

\[ |\xi_{+}\rangle = | -\alpha \rangle + e^{i\phi_{AB}} |\alpha\rangle, \]  

(21)

\[ |f(\phi_{AB})f(\phi_{AB})...f(\phi_{AB})\rangle = \left( |0\rangle_0 + e^{i\phi_{AB}} |1\rangle_0 \right) \otimes \left( |0\rangle_1 + e^{i\phi_{AB}} |1\rangle_1 \right) \otimes \left( |0\rangle_n + e^{i\phi_{AB}} |1\rangle_n \right). \]  

(23)

This state corresponds to a string of bits stored in parallelized high-Q cavities. This quantum transfer can be implemented for transfer in other systems, as it is expected in a portable computation. The cavities can be used to store sequentially the bit information with the generation of a product state. In the case of the previous section, the string of bits is stored in superpositions of coherent states

\[ |f(\phi_{AB})f(\phi_{AB})...f(\phi_{AB})\rangle = \left( | -\alpha \rangle + e^{i\phi_{AB}} |\alpha\rangle \right) \otimes \left( | -\alpha \rangle + e^{i\phi_{AB}} |\alpha\rangle \right) ... \left( | -\alpha \rangle + e^{i\phi_{AB}} |\alpha\rangle \right). \]  

(24)

For instance, the string of bits 10110 encoded in (11)

\[ |f(0)\phi_{AB}f(1)\phi_{AB}f(2)\phi_{AB}f(3)\phi_{AB}f(4)\phi_{AB}\rangle = \left( |0\rangle_0 + e^{i\phi_{AB}} |1\rangle_0 \right) \otimes \left( |0\rangle_1 + |1\rangle_1 \right) \]

\[ \otimes \left( |0\rangle_2 + e^{i\phi_{AB}} |1\rangle_2 \right) \otimes \left( |0\rangle_3 + e^{i\phi_{AB}} |1\rangle_3 \right) \]

\[ \otimes \left( |0\rangle_4 + |1\rangle_4 \right). \]  

(25)

The information encoded in the string of bits can be retransferred to other systems or detected by measurement that |\xi_{+}\rangle corresponds to a selective measurement of the right-handed electron state |R\rangle and

\[ |\xi_{-}\rangle = e^{i\phi_{AB}} | -\alpha \rangle + |\alpha\rangle \]  

(22)

to left-handed electron state |L\rangle. In both cases, the AB phase is transferred to the coherent state superposition.

**STORAGE OF BITS ENCODED BY QUANTUM TRANSFERED AB PHASES**

The quantum transfer of AB phases as described in the previous section can have several possible utilities. The bit encoding of AB phases imply that such a quantum transfer corresponds to a method of transfer a bit information to a state of light in a coherent state superposition. If we have a parallel setup with parallelized high-Q cavities or multimode cavities the transfer can also be implemented to store of a complete string of bits that can be identified to a broader information transfer, for instance a byte transfer.

In order to consider this scheme, the bit encoded AB phases have now a correspondence with the following product state
different methods, imprinting the AB phase in parallel AB devices, and realizing the corresponding protocols of quantum transfer, or modulating the AB phases in a controlled AB device, modulated to generate by quantum transfer the string of bits in a sequential form, storing a bit in each high-Q cavity or in each mode of a multimode cavity.

CONCLUSIONS

In summary, we have proposed a bit-encoding of AB phases and a quantum transfer of AB phases to a non-classical state of light, a superposition of coherent states. We showed that this scheme can be implemented to store a string of bits encoded by AB phases.

Given the experimental advances in the generation and control of AB effect, this binary codification can be experimentally realized for implementing any binary message, for instance realizing a ASCII code, and can be included in computer devices and quantum circuits. Our propose contrasts with other schemes where bit-encoding is implemented by chirality or flux orientation. The advances in mesoscopic size systems where the AB effect is realized and light-matter interaction can be used to develop this scheme. The possibility of transferring information encoded to light increases the possible scenarios where the information can be manipulated and transferred.

We can take advantage of quantum information methods for control and manipulation of quantum interference phenomena. In our propose a quantum state transfer protocol is used to transfer AB phases and can be used for manipulating and control quantum interference associated to AB effect. The bit encoding can be used in parallel to protocols of quantum information theory with many potential applications. In particular, the bit encoding with AB phases imply that the protocol of quantum transfer of AB phases can be implemented as method of transfer a bit of information to a state of light in a coherent state superposition. Applying this to parallel setups of high-Q cavities or multimode cavities the transfer can be implemented to store of a complete string of bits.

Our propose can also be useful to make further progress in methods of quantum information associated to AB effect. The bit encoding can be used for manipulating and control quantum interference as associated to AB effect. The bit encoding can be used to develop this scheme. The possibility of transferring information encoded to light increases the possible scenarios where the information can be manipulated and transferred.

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