A study on the real-time reliability of on-board equipment of train control system

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Abstract. Real-time reliability evaluation is conducive to establishing a condition based maintenance system for the purpose of guaranteeing continuous train operation. According to the inherent characteristics of the on-board equipment, the connotation of reliability evaluation of on-board equipment is defined and the evaluation index of real-time reliability is provided in this paper. From the perspective of methodology and practical application, the real-time reliability of the on-board equipment is discussed in detail, and the method of evaluating the real-time reliability of on-board equipment at component level based on Hidden Markov Model (HMM) is proposed. In this method the performance degradation data is used directly to realize the accurate perception of the hidden state transition process of on-board equipment, which can achieve a better description of the real-time reliability of the equipment.

1. Introduction
CTCS-3 (Chinese Train Control System Level 3) has been widely used for 300km/h high speed railways in China to ensure train operation safety and efficiency [1]. Real-time reliability evaluation is conducive to establishing a condition based maintenance system for the purpose of guaranteeing continuous train operation. The demand of real-time reliability has brought about changes to the reliability evaluation because the traditional reliability evaluation methods can only reflect the failure distribution of average characteristics of products of the same type under the same operating conditions and is not applicable for evaluating the reliability in real time for a specific product under specific conditions [2]. Some methods have been proposed for the real-time reliability evaluation including classical analysis, state analysis and intelligent assessment analysis [3]. This paper focuses on the method of evaluating the real-time reliability of the CTCS-3 On-Board Equipment (OBE) based on Hidden Markov Model (HMM). It is organized as follows. In section 2 a real-time reliability index system for the on-board equipment is established; in section 3 the general idea of evaluating the real-time reliability of on-board equipment is introduced by analyzing the structure of on-board equipment and a fault tree model is established; in section 4 the method of predicting the performance degradation trend and the process of calculating the reliability of on-board equipment at the component level is described in detail; in section 5 Balise Transmission Module (BTM), a key component of the on-board equipment, is taken as an example to illustrate the calculation process of real-time reliability ; in section 6 some conclusions are given.
2. Establishment of real-time reliability index system

According to the classical definition of reliability, the real-time reliability is defined as during the service period, the ability of the equipment to perform the required function at current time and under current operation condition. In this paper, we extend the classical reliability index system to set up a reliability evaluation index system for CTCS-3 on-board equipment, as shown in Figure 1.

2.1. Probability index

Reliability degree is a quantitative index used to indicate the ability of the equipment to perform required function under stated conditions for a specified period of time, which is denoted as $R(t)$.

$$R(t)=1-F(t)=\int_{0}^{t} g(t)dt$$  \hspace{1cm} (1)

Where $F(t)$ is the accumulative failure, used to indicate the probability of failing to perform the required function under stated conditions for a specified period of time, so $F(t)$ is also called the unreliability degree; $g(t)$ is the failure probability density function.

2.2. Performance index

Equipment failure can be divided into two categories: fatal failure and performance degradation failure. The difference between them is that the state of performance degradation failure can’t be simply represented by two states (normal/failure), instead it depends on a number of indicators. Here the performance degradation degree is used to describe the current degradation degree of the on-board equipment, which is defined as

$$K(t)=\begin{cases} 
0, & |y(t)-y_0(t)| \leq \epsilon \\
1, & |y(t)-y_L(t)| \leq \epsilon \\
\frac{|y(t)-y_0(t)|}{|y_L(t)-y_0(t)|}, & \text{other} 
\end{cases}$$  \hspace{1cm} (2)

Where $K(t)$ is the performance degradation degree, valued between 0 and 1; $y(t)$ is the measured value of performance parameters; $y_0(t)$ is the optimal value of performance parameters; $y_L(t)$ is the failure threshold of performance parameters.

2.3. Life type index

The reliable life is $t(R)=R^{-1}(R)$, where $R$ is the reliability degree and $R^{-1}(R)$ is the inverse function of $R$. The performance retaining time, denoted as $T(y_L)$, is the time period during which the performance parameters remain within the specified range.
3. Real-time reliability evaluation at system level

3.1. Structure of CTCS-3 on-board equipment
The CTCS-3 on-board equipment is a safety-critical system designed with modular structure and based on fail-safe principle[4]. It consists of CTCS-3 Control Unit(C3_CU), CTCS-2 Control Unit(C2_CU), BTM, Radio Transmission Module(DMI), Driver Machine Interface(DMI), Speed and Distance Processor (SDP), Speed and Distance Unit(SDU), Track Circuit Receiver(TCR), Juridical Recorder Unit (JRU), Train Signalling Gateway(TSG), Safe Transmission Unit(STU-V), Vital Digital I/O (VDX), General Crypt Device(GCD), Compact Antenna Unit(CAU) and so on. Its structure is shown as in figure 2 [1].

3.2. System-level reliability model
The Fault Tree Analysis (FTA) [5] is adopted to evaluate the system-level reliability. Hot-standby redundancy structure is adopted for the on-board equipment to ensure that the failure of a module can only affect the output of the module itself and will not affect the standby module. By analysing the structure and functionality of the on-board equipment, a fault tree model is built, as shown in figure 3. According to the measured performance parameters of the on-board equipment, the performance degradation model can be built to calculate the real-time reliability of the component of the on-board equipment. Finally the system-level reliability can be calculated based on the fault tree model. Therefore the real-time reliability calculation of the on-board equipment is a bottom-up process. The general idea is shown in figure 4.

4. Real-time reliability evaluation at component level

4.1. Matching relation of performance parameters
The Failure Mode and Effect Analysis (FMEA) is a basic reliability analysis method [6]. According to FMEA results and performance parameters related to component failure, a performance index system can be established. Component failure can be divided into two categories: fatal failure and degradation failure, and only the latter one is considered in this paper. The state of a degradation failure depends on a number of parameters, and the performance variation trend of the component is always monotonically increasing or decreasing. When the values of the performance parameters exceed the threshold range, the component will be regarded as in failure state.

The selected performance parameters shall meet the following requirements.
(1) They shall be clearly defined and can be measured in real-time;
(2) They shall be able to reflect the failure status of the component;
(3) With the increasing of operation time and worsening of operation condition, the performance parameters shall have significant trend variation.
In order to reasonably select the performance parameters, FMEA is adopted to match the performance parameters with the failure mode of the component. The selection of the real-time reliability parameters normally depends on component structure, expert knowledge and failure log.

4.2. Prediction of performance degradation trajectory
Auto-Regression Integrated Moving Degradation Average Model (ARIMA) is a short-term time series forecasting model with high accuracy[7], which can quantitatively describe the process of component performance degradation failure. The modelling steps are as follows.

(1) Series stationarization
Stationeriness test shall be carried out for the collected condition monitoring signal. A non-stationary stochastic series $y_{ij}$ shall satisfy the following equation:

$$y_{ij} = f(t) + g(t) + X(t)$$  \hspace{1cm} (3)

Where $y_{ij}$ is the value of $j$-th characteristic value at $i$-th time; $f(t)$ is the non-periodic growth component; $g(t)$ is the periodic component of $y_{ij}$; $X(t)$ is the component of stationary stochastic processes of $y_{ij}$. From equation (3) it can be seen that a series can be converted into stationary stochastic series after the non-periodic and periodic trend are separated. Then AR, MA or ARMA can be adopted to build the model.

(2) Model identification and parameter estimation
Through $d$-th order difference, the nonstationary stochastic series \{y_{ij}\} can be converted into a stationary series \{w_{ij}\}. Taking ARMA for example, the parameter estimation is as follows.

$$\theta(B)w_{ij} = \phi(B)e_{ij}$$  \hspace{1cm} (4)

Figure 4. General idea of reliability evaluation of on-board equipment.
\[ E(e_{ij}) = 0, \text{Var}(e_{ij}) = \sigma^2, E(e_{nm} e_{nj}) = 0, m \neq n \]  
\[ \theta(B) = 1 - \sum_{b=1}^{P} \theta_b B^b, \phi(B) = 1 - \sum_{b=1}^{Q} \phi_b B^b \]

Where \( \theta(B) \) and \( \phi(B) \) are the autoregressive coefficient polynomial of ARMA and the moving average coefficient polynomial respectively; \( \{ e_{ij} \} \) is a zero mean white noise series. The process of model identification and parameter estimation is firstly to determine the preliminary computational model by analyzing the Auto-Correlation Function (ACF) and the Partial Auto-Correlation Function (PACF), and then to obtain the optimization model according to AIC information norm.

(3) Model prediction

The linear expression of ARIMA is as follows.

\[ y_j = e_j + \psi_1 e_{i-1,j} + \psi_2 e_{i-2,j} + \ldots = \psi(B)e_j \]  

Where \( \psi(B) \) is determined by equation (8),

\[ \theta(B)\nabla^d \psi(B) = \phi(B) \]  

If the predictive step is 1, the predicted value of \( \hat{y}_j \) is,

\[ \hat{y}_j = \left( \psi_1 e_j + \psi_2 e_{i-1,j} + \psi_3 e_{i-2,j} \ldots \right) \]

\[ = \left( e_{i+1,j} + \psi_1 e_{i+2,j} + \psi_2 e_{i+3,j} \ldots \right) \]

(4) Residual and normal distribution test

The residual of performance parameters can be obtained by calculating the difference between the predicted values of performance parameters obtained from the ARIMA model and the measured value of the parameters. Normal distribution test can be conducted for the residual. If the residual complies with the normal distribution, the model is proved to be reasonable.

4.3. Reliability evaluation based on HMM

From the perspective of signal probability, HMM is categorized into Discrete Hidden Markov Model (DHMM) and Continuous Hidden Markov Model (CHMM) [8]. From the perspective of state transition the HMM is categorized into left-right HMM and traversal HMM. Because the performance degradation is an irreversible process, the left-right DHMM is adopted in this paper to describe the process of component performance degradation.

HMM is described as \( \lambda = (N, M, \pi, A, B) \), where \( N \) is the number of hidden states, and the finite set of hidden states is defined as \( S = \{ S_1, S_2, \ldots, S_N \} \); \( M \) is the number of observations, and the finite set of observations is defined as \( V = \{ v_1, v_2, \ldots, v_M \} \); the probabilistic distributions of the initial states is \( \pi = \{ \pi_i \} \); \( A \) is the state transition probability matrix and \( a_{ij} = P(q_i = j | q_{i-1} = i), 1 \leq i, j \leq N \); \( B \) is the observation probability matrix and \( B = (b_{j}(k)), b_{j}(k) = b_{j}(o_t) = (o_t = v_k | q_t = j) \).

4.3.1 Method for engineering application.

The real-time reliability evaluation of component of on-board equipment involves serveral performance degradation parameters. As the traditional HMM can only handle single observation time series for evaluating single parameter, it should be modified to be suitable for handling multiple observation series. The process of the modified HMM multi-parameter estimation is as follows.

In order to use the multiple observation series to train the HMM, the equation of Baum-welch algorithm shall be revised. The set of observation series is defined as \( O = \{ O^{(1)}, O^{(2)}, \ldots, O^{(k)} \} \), and a
observation series is defined as \( O^{(k)} = \{ O^{(1)}, O^{(2)}, \ldots, O^{(K)} \} \), and all the observation series are independent from each other. The probability of multiple observation series is 
\[
P(O | \lambda) = \sum_{k=1}^{K} \omega_k P(O^{(k)} | \lambda).
\]
The model parameter re-estimation equation and combinational weight of observation series are as follows:

\[
\hat{\pi}_i = \sum_{k=1}^{K} \alpha_i^{(k)}(i) \beta_i^{(k)}(i) P(O^{(k)} | \lambda)
\]

\[
\hat{a}_y = \sum_{k=1}^{K} \frac{1}{P_k} \sum_{i=1}^{T-1} \alpha_i^{(k)}(i) \beta_i^{(k)}(i)
\]

\[
\hat{b}_{jm} = \sum_{k=1}^{K} \frac{1}{P_k} \sum_{i=1}^{T} \alpha_i^{(k)}(j) \beta_i^{(k)}(j)
\]

\[
\omega_0 = \frac{1}{K} P(O^{(1)} | O^{(0)} \lambda) \ldots P(O^{(K)} | O^{(0)}, O^{(1)}, \lambda)
\]

\[
\omega_K = \frac{1}{K} P(O^{(K)} | O^{(0)}, O^{(1)}, O^{(2)}, O^{(K-2)}, O^{(0)}, \lambda)
\]

4.3.2 Real-time reliability calculation
The real-time reliability of the component is defined as the probability that the performance parameters are within the threshold range, namely the probability that the component is not in failure state. In order to calculate the probability of component in different states, conditional state probability \( C(i) = \{ C_1(i), C_2(i), \ldots, C_N(i) \} \) is introduced to describe the probability of component degradation state.

Given that the model parameter \( \lambda' = (A', B', \pi') \) and the observation series of \( t-th \) time \( O_t = \{ q_1, q_2, \ldots, q_t \} \) are known, the conditional state probability of time \( t \) in state \( i \) is defined as 
\[
C_i(t) = P(q_t = i | q_1, q_2, \ldots, q_{t-1}, \lambda') \quad q_t \in \{ S_1, S_2, \ldots, S_N \}, \quad t \geq 0, 1 \leq i \leq N.
\]
When \( t = 0 \), \( C(0) = \pi \).

The proof of the equation of conditional state probability is as follows.
Assume that the observation series of time \( t \) is denoted as \( O_t \) and the observation series of time \( t-1 \) is denoted as \( O_{t-1} \). By definition, it can be known that:

\[
C_i(t) = P(q_t = j | O_t, \lambda)
\]

\[
C_i(t) = \sum_{j=1}^{N} P(q_{t-1} = i, q_t = j | O_{t-1}, O_t, \lambda)
\]
\[
C_j(t) = \sum_{i=1}^{N} \left[ \frac{P(q_{t-1} = i | O_{t-1}, o_i, \lambda)}{P(o_i | O_{t-1}, q_{t-1} = i, \lambda)} \cdots \frac{P(q_{t} = j, o_i | O_{t-1}, q_{t-1} = i, \lambda)}{P(q_{t} = j | O_{t-1}, q_{t-1} = i, \lambda)} \right] 
\]

(16)

\[
P(q_{t} = j, o_i | O_{t-1}, q_{t-1} = i, \lambda) = P(q_{t} = j | O_{t-1}, q_{t-1} = i, \lambda) \times \frac{P(q_{t} = j | o_i | O_{t-1}, q_{t-1} = i, \lambda)}{P(o_i | O_{t-1}, q_{t-1} = i, \lambda)}
\]

(17)

According to equation (14) to (18), we can get

\[
P(o_i | O_{t-1}, \lambda) = \sum_{i=1}^{N} \sum_{h=1}^{N} P(q_{t-1} = i, q_r = h, o_i | O_{t-1}, \lambda)
\]

(19)

In conclusion,

\[
C_j(t) = \sum_{i=1}^{N} \left[ \frac{b_j(o_i) a_j C_i(t-1)}{\sum_{i=1}^{N} \sum_{h=1}^{N} b_h(o_i) a_m C_i(t-1)} \right]
\]

(20)

From equation (20), it can be seen that \(\sum_{j=1}^{N} C_j(t) = 1\). So the conditional state probability introduced in this paper can accurately describe the probability of different component states at any time.

From the C-K equation, it can be derived that \(C(t) = C(t) \cdot A\), where \(A\) is the state transition matrix at time \(t\) and \(C(t) = (C_0(t), C_1(t), \ldots, C_N(t))\) is the conditional state vector. Laplace transform is used to calculate the real-time reliability, which is,

\[
s \cdot \begin{bmatrix} C_0(s) \\ C_1(s) \\ \vdots \\ C_N(s) \end{bmatrix} = \begin{bmatrix} C_0(0) \\ C_1(0) \\ \vdots \\ C_N(0) \end{bmatrix} = \begin{bmatrix} a_{00} & 0 & \cdots & 0 \\ a_{01} & a_{11} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{0k} & a_{1k} & \cdots & a_{kk} \end{bmatrix} \begin{bmatrix} C_0(s) \\ C_1(s) \\ \vdots \\ C_N(s) \end{bmatrix}
\]

(21)
If the component in the initial state is under normal condition, there is the result of \( C(0) \).

\[
C(0) = \left( C_0(0), C_1(0), \ldots, C_k(0) \right) = (1, 0, \ldots, 0)
\]  

(22)

It can be seen from equation (21) that the probability of different component states at time \( t \) is derived by inverse Laplace transformation of \( C(s) \). Therefore, the reliability at time \( t \) is \( R(t) = 1 - C_k(t) \).

5. BTM real-time reliability calculation

5.1. Selection of BTM performance parameters

BTM is one of the key components of the CTCS-3 on-board equipment, which has relatively high failure rate [9]. Therefore, this paper takes BTM as an example to analyze the real-time reliability. The physical structure of BTM is shown as in figure 5, which consists of BTM host, transmission cable D and BTM antenna (CAU). A redundancy structure of double 2oo2 is adopted to improve the safety and reliability.

According to the BTM fault information collected by the data output system and the analysis of fault data, the performance parameters of BTM can be determined. Table 1 is the performance parameters corresponding to FMEA, which establishes the relationship between the FMEA and performance parameters. According to the requirements for choosing performance parameters described in section 3.1, the BTM self-test temperature, cable attenuation, train-track transmission power carrier frequency and bit error rate are chosen as the parameters for evaluation.

5.2. ARIMA model prediction

The EVIEWS8.0 tool is used to build the ARIMA model. The sampling time is 20h. Sampling interpolation is adopted to improve the accuracy of fitting and prediction. A total of 120 data items are obtained, of which the first 100 data items are used for model training, and the other 20 data items are used for model validation. Figure 6 shows the track of the 4 performance parameters, where the empty circle stands for the measured value and the solid circle for the interpolated value. In the following BTM temperature is taken as an example to illustrate the ARIMA prediction process.

![Figure 5. Structure of BTM.](image-url)

| Functional Unit       | Failure Mode                      | Parameter          | Monitoring Point | Measuring Instrument               |
|-----------------------|-----------------------------------|--------------------|------------------|-----------------------------------|
| Power Board           | Working indicator light           | Voltage value      | Power board pins 1. 3 | Digital Voltmeter                 |
| Amplifier Board       | flashing or go off                |                    |                  |                                    |
| Receiver Board        | BTM fails to complete self-       |                    |                  |                                    |
|                       | test, the receiver indicator      |                    |                  |                                    |
|                       | displays abnormally, Power-on     |                    |                  |                                    |
|                       | Temperature self-test fault code  |                    |                  |                                    |
|                       | 4-128-15-03                        | BTM temperature    | Temperature Sensor |                                    |
|                       |                                   | self-test          |                   |                                    |
| Cable D               | BTM repeats self-test, fault      | Vibration frequency | J3 line and P3 socket | Piezoelectric                     |
| Communication         | code is 3-132-4., 3-132-5         |                    |                  | Acceleration Sensors              |
|                       |                                   | Cable attenuation  | Cable            | Attenuator                        |

Table 1. Parameter Matching of BTM.
Figure 7 is the Augmented-Dickey-Fuller (ADF) test for the original time series. It can be seen that the value of t is -1.95, which is much higher than the threshold value at the significance level of 1% - 10% level. So the original hypothesis is accepted, which means that the time series is non-stationary. After carrying out first order difference on the original time series, the result of ADF test for the obtained time series is shown in figure 8. It can be seen that the value of t is -3.999995, which is lower than the threshold value at the significance level of 1%. So the original hypothesis is rejected, which means the obtained time series is stationary. Figure 8 shows the result of ADF test for the obtained time series.

Figure 6. Measured value & interpolated value of performance parameters.

Figure 7. ADF test for temperature series.

Figure 8. ADF test for temperature series after first order difference.

Figure 9 shows the correlation analysis of the temperature series. It can be seen that all the values of Prob are less than 0.05, which suggests that the time series is correlated. By analyzing the autocorrelation and partial autocorrelation functions, it can be seen that the time series is suitable for the model of ARIMA (p, d, q)
In the ACF and PACF analysis of the BTM temperature series after the first order difference, it can be seen that the 1-2 order autocorrelation coefficients are significant and the 3-25 order coefficients are not significant, so the value of q is initially set to 2; the 1-3 order partial autocorrelation coefficients are significant, so the value of p is initially set to 3. The values of AIC and SC are the most important criteria for choosing the model parameters p & q. In the optimal model, the values of AIC and SC are expected to be as small as possible. The model parameters of ARMA (3, 2) and ARMA (3, 3) are shown in figure 10 and figure 11 respectively. Table 2 is the value of AIC and SC for different p & q values.

From table 2 it can be seen that when the value of p and q are 3, the value of AIC and SC will both be the smallest. In figure 11 the model is ARMA (3, 3) and the R-squared is 0.959, which indicates

**Table 2. ARMA Model Order Determination.**

| Model | AIC   | SC   |
|-------|-------|------|
| Eq03_02 | 1.11  | 1.27 |
| Eq03_03 | 0.98  | 1.17 |
| Eq03_01 | 1.30  | 1.44 |
| Eq04_02 | 1.21  | 1.40 |
| Eq04_03 | 1.22  | 1.43 |
| Eq05_02 | 1.20  | 1.42 |
| Eq05_03 | 1.22  | 1.47 |

**Figure 9.** AC-PAC analysis of temperature series.

**Figure 10.** ARMA (3, 2) model.

**Figure 11.** ARMA (3, 3) model.
that the model is satisfactory. Except for the coefficient of MA (3), the other coefficients are significant. The temperature degradation model of BTM is determined as the ARMA (3, 3) model, which can be expressed as follows,

$$\hat{y}_i = 0.1656y_{t-1} + 0.5718y_{t-4} + 0.4363y_{t-2} - 0.4991y_{t-3} + \epsilon_i + 2.0441\epsilon_{t-1} + 1.0337\epsilon_{t-2} - 0.0655\epsilon_{t-3}$$  (23)

Where $\hat{y}_i$ is the predicted value. White noise test is carried out for the model residuals, as shown in figure 12. It can be seen that the residuals of the autocorrelation and partial autocorrelation coefficients converge within 2 times the standard deviation. All the values of $p$ are greater than 0.05, which suggests that the residuals are white noise process and the model is usable.

The fitting model ARIMA (3, 1, 3) is used for short-term prediction. Figure 13 is the prediction of ARIMA (3, 1, 3). It can be seen that the value of Theil is 0.1106. This value is small and it suggests that this model has an excellent prediction capability and the error ratio of the model is small. The variance proportion is 0.012339. It means that the prediction can simulate the fluctuation of the time series better. The covariance proportion is 0.987091. This value is big and it suggests that the result of prediction is desirable.

Similarly the performance degradation models of other parameters can be established.

1) The performance degradation model of cable attenuation:

$$\hat{y}_i = 0.3241y_{t-1} + 0.5639y_{t-2} + \epsilon_i - 0.9952\epsilon_{t-1} - 0.1513\epsilon_{t-2}$$  (24)

2) The performance degradation model of train-ground transmission power carrier frequency:

$$\hat{y}_i = 0.3205 + 0.7312y_{t-1} - 0.1163y_{t-2} + 0.9622y_{t-3} + \epsilon_i - 0.2458\epsilon_{t-1}$$  (25)

3) The performance degradation model of bit error rate:

$$\hat{y}_i = 1.1438y_{t-1} - 0.1666y_{t-2} + 0.4721y_{t-3} + \epsilon_i + 0.4735\epsilon_{t-1} - 0.1187\epsilon_{t-2}$$  (26)

5.3. Reliability calculation based on DHMM

The future data of the performance parameters can be obtained by the ARIMA model. DHMM is used to identify the probability of state transition. Firstly vector quantization is carried out for the obtained data from ARIMA, by using the performance status threshold matrix $H$ and Lloyd algorithm. The calculation equation of the thresholds of performance parameters at each level is as follows:

$$S_i = S_0 + k_i\sigma$$  (27)
Where $S_i$ is the state thresholds of the $i$-th level. $S_0$ is the factory value of performance parameters. $\sigma$ is the standard deviation of performance parameters and $k_i$ is the correction coefficient. Partition is determined by the performance status threshold matrix and the performance characteristic parameter series is divided into several sub-series by partition. These sub-series correspond with several consecutive integer values which are generally defined starting from 1. The index at different intervals is determined by the following equation:

$$\text{Index}(y_j) = \begin{cases} 1, & y_j \leq \text{partition}(1, j) \\ i, & \text{partition}(i-1, j) \leq y_j \leq \text{partition}(i, j) \\ N, & \text{partition}(N-1, j) \leq y_j \end{cases}$$

(28)

Where $\text{partition}(i, j) = h_{ij}$, $h_{ij}$ is the value of performance status threshold matrix $H$. The matrix is defined as follows:

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1m} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm} \end{bmatrix}$$

(29)

The failure process of BTM can be regarded as a multi-state transition process. The state of BTM can be categorized into 4 types: normal state $\{0\}$, slight degradation state $\{1\}$, serious degradation state $\{2\}$ and failure state $\{3\}$. According to experience and history data of parameters, the performance status threshold matrix of BTM is given by,

$$H = \begin{bmatrix} 55 & 0.4 & 27.095 & 0.005 \\ 60 & 0.6 & 25.095 & 0.05 \\ 70 & 0.8 & 22.095 & 0.1 \end{bmatrix}$$

(30)

The vector quantization set is the input observation series which need to be trained in DHMM. Baum Welch and EM algorithm is used to adjust the model parameters $\lambda$ until the probability of observing the series $P(O | \lambda)$ is maximized, and then the required model is obtained. Set the initial model $\pi_0 = [1, 0, 0, 0]^T$. The state transition probability matrix $A_0$ and the observation probability matrix $B_0$ are set as follows.

$$A_0 = \begin{bmatrix} 0.8 & 0.1 & 0.06 & 0.04 \\ 0 & 0.65 & 0.23 & 0.12 \\ 0 & 0 & 0.63 & 0.37 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(31)

$$B_0 = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

(32)

The iterative training process is shown in figure 14. It can be seen that the log probability increases with the training times. After training for 10 times, the model tends to saturation. The parameters of HMM are as follows:
The real-time reliability computational equation can be derived from equation (21). The probability of BTM in different states is $C(t) = (C_0(t), C_1(t), C_2(t), C_3(t))$.

$$C_0(t) = \exp(0.9641*t)$$  \hspace{1cm} (35)

$$C_1(t) = 0.901\exp(0.9641*t) - 0.901\exp(0.9266*t)$$  \hspace{1cm} (36)

$$C_2(t) = 2.5321\exp(0.9641*t) - 5.7340\exp(0.9266*t) + 3.2019\exp(0.8976*t)$$  \hspace{1cm} (37)

$$C_3(t) = 1.1795\exp(0.8976*t) - 2.1477\exp(0.9266*t) + 0.9682\exp(0.9641*t)$$  \hspace{1cm} (38)

The BTM real-time reliability is $R(t) = 1 - C_3(t)$, as shown in figure 15.

6. Conclusion

On the basis of the traditional reliability analysis, this paper adopts the performance degradation parameters to estimate the real-time reliability of component of on-board equipment. ARIMA model and HMM are used to predict the trend of performance parameters and to calculate the real-time reliability. This method of real-time reliability evaluation can provide the support for conditioning based maintenance of on-board equipment of train control system.

7. References

[1] Transportation Bureau of the Ministry of Railways 2008 *The General Technical Scheme of CTCS-3 Train Control System* (Beijing: China Railway Press)

[2] Transportation Bureau of the Ministry of Railways 2009. *System Requirements Specification for CTCS-3 Train Control System (v1.0)* (Beijing: China Railway Press)

[3] Nagi G, Mark L, LI R and Jennifer R 2005 Residual-life distributions from component degradation signals: a Bayesian approach *Iie Trans.* 37 pp 543-57

[4] European Committee for Electrotechnical Standardization 2002 *EN 50129 Railway applications-Communication, signaling and processing systems safety related electronic
systems for signaling.

[5] Shalev D M and Tiran J 2007 Condition-based fault tree analysis (CBFTA): a new method for improved fault tree analysis (FTA), reliability and safety calculations Reliability Engineering & System Safety 92 pp 1231-41

[6] Gawand H L, Murali N and Swaminathan P 2010 Reliability analysis by FMEA method for object oriented distributed digital control system design model of nuclear power plant Conf. on Reliability, Safety and Hazard (ICRESH) 37 pp 489-92

[7] Che J and Wang J 2010 Short-term electricity prices forecasting based on support vector regression and auto-regressive integrated moving average modeling Energy Conversion & Management 51 pp 1911-17

[8] Schuster-Böckler B and Bateman A 2007 An introduction to hidden Markov models Current Protocols in Bioinformatics pp 4-16

[9] Gao Z, Ma Z and Liang J 2014 Research on anti-electromagnet interference on BTM on-board equipment of CTCS3-300T train control system Railway Signaling & Communication Engineering 11 pp 1-6

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