Hegselmann-Krause model of opinions dynamics of interacting agents with the random noises

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Abstract. We consider the Hegselmann-Krause bounded confidence model of opinion dynamics. We assume that the opinion of an agent is influenced not only by other agents, but also by external random noises. The case of independent normally distributed external noises is considered. We perform computer modeling of deterministic and stochastic models. The properties of the models were analyzed and the difference in their behavior was revealed. We study the dependence of the number of a confidence clusters on the parameters of the problem such as the initial profile of opinions, the level of confidence, the variance of noise.

1. Introduction

During recent decades models of opinion dynamics in groups of interacting agents have become widespread [1]-[7]. The main issues that are studied are reaching consensus (agreement) in the group, clustering opinions [1]-[3], conditions for the existence of consensus [4]-[7], the rate of convergence to consensus [8]-[9]. The opinion dynamics depends on the consensus algorithm, the topology structure of the group of agents and so on. In this paper we study the Hegselmann – Krause bounded confidence model (H-K model). We introduce a stochastic Hegselmann – Krause model, analyze properties of the corresponding stochastic process and compare its behavior with the deterministic H-K model.

2. Hegselmann -- Krause bounded confidence model [2]-[4]

Consider a group of \( m \) agents. Each agent has an opinion which is represented by a real number. Let \( x_i(t) \) be an opinion of the agent \( i \) at the moment \( t \in \mathbb{T} = \{0,1,2, \ldots \} \). We assume that \( x_i(t) \in [0,1] \) for all \( t \) and \( i = 1, \ldots, m \). Denote \( x(t) = (x_1(t), \ldots, x_m(t)) \). The vector \( x(t) \) is called an opinion profile at time \( t \). The agent \( i \) interacts only with those agents whose opinions differ from \( x_i \) not more than a certain confidence level \( \varepsilon \). For each agent \( i \) denote a confidence set by \( D(i, x) \):

\[
D(i, x) = \{1 \leq j \leq m, |x_i - x_j| \leq \varepsilon \} \tag{1}
\]

On the next time step the agent \( i \) replaces its current opinion by the average of the agent opinions that are in \( D(i, t) \): 
\[ x_i(t + 1) = \left| D(i,x(t)) \right|^{-1} \sum_{j \in D(i\cup x(t))} x_j(t), \quad t \in T \]  

(2)

Here \( |D(i,x)| \) is the number of elements of \( D(i,x) \).

An opinion profile \( x \) is a consensus state if \( x_i = \bar{x} \) for all \( i = 1, \ldots, m \). We will call \( \bar{x} \) the consensus value. We say that the model reaches the consensus \( \bar{x} \) if \( x_i(t) \to \bar{x} \) for all \( i \). The main properties of H-K model are the following.

1. The dynamics does not change the order of opinions.
2. If the split between agents \( i \) and \( j \) occurs at the moment \( t_0 \), then this split remains for all \( t \geq t_0 \).
   (If \( |x_i - x_j| > \varepsilon \) then we say that there is a split between agents \( i \) and \( j \).)
3. If agents reach a consensus, then it is reached at the finite time.

3. Stochastic Hegselmann – Krause model

Suppose that the opinions of agents can be influenced not only by other agents, but also by external random factors. Then the dynamics of \( x(t) \) is described by

\[ x_i(t + 1) = g(x_i^*(t + 1)) \]  

(3)

\[ x_i^*(t + 1) = \left| D(i,x(t)) \right|^{-1} \sum_{j \in D(i\cup x(t))} x_j(t) + \sigma \delta_i(t), t \in T \]  

(4)

\[ g(y) = \begin{cases} 
0, & y < 0 \\
0, & 0 \leq y \leq 1 \\
1, & y > 1 
\end{cases} \]  

(5)

We assume that \( \delta_i(t), i = 1, \ldots, m \) are independent identically distributed (i.i.d.) random variables, \( \delta_i(t) \sim N(0,1) \). We use the function \( g(y) \) since opinion values cannot be outside the interval \([0; 1]\).

We are interested which properties of the deterministic H-K model are preserved for the stochastic model. Since \( x(t) \) is a stochastic process, the above items (1) – (3) do not hold. Therefore, we introduce the notion of a confidence cluster.

Definition. The set of agents \( I = \{i_1, ..., i_s\} \) form a confidence cluster at time \( t \), if the following conditions are satisfied:

1. \( \forall k \in I \exists l \in I, i_l \neq i_k: |x_{i_k}(t) - x_{i_l}(t)| < \varepsilon \)
2. \( \forall k \in I, \forall j \in \mathbb{N}\setminus I: |x_{i_k}(t) - x_{j}(t)| > \varepsilon \)

4. Simulations

We performed computer simulations using the Python language to reveal the properties of the stochastic H-K model and to compare the deterministic model with the stochastic model. We simulate the stochastic and deterministic H-K models for \( m = 30 \). Random initial opinions are taken uniformly from \([0,1]\). Simulations allow us to formulate the following properties of the stochastic H-K model:

1. Opinions of agents can change their orders; therefore, agents who were in the same cluster at the initial moment may occur in different clusters (see Figure 1).
2. Clusters can exchange agents.
3. The number of clusters depends on the choice of initial opinions. Figure 2 shows that for different initial opinions we obtain the different effects. The number of clusters in the first case equals 5, in the second case equals 4.

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5. The number of clusters varies during the evolution (Fig.2)

6. The number of clusters depends on the choice of the concrete realization of the random noise. Figure 3 shows two different realizations for $m = 30$, the fixed initial opinion profile, $\epsilon = 0.041$ and $\sigma = 0.014$.
Figure 3. The number of clusters depends on the choice of the concrete realization of the random noise. (a) and (b) represent two different realizations of the noises.

In the stochastic model agents don’t reach a consensus, but their opinions can be divided into the confidence clusters. We compare the number of groups of opinions in the model without noise with the number of confidence clusters in the noisy model.

The simulations show that noise can affect the opinions of agents in such a way, that the number of clusters (for some parameter $\gamma_2 \exists \gamma_2 \gamma_3 \exists \gamma_3 \exists \gamma_3$) will decrease compared to the number of opinions groups in the model without noise (for a fixed parameter $\varepsilon$, array $\delta$ and a fixed initial opinions $x_i(0)$). As an example, consider the initial opinions uniformly distributed in the interval $[0; 1]$.

Fix the value $\gamma_2 = 0, 0, 1, \gamma_2 = 0.03, \gamma_1 = 50$. Figure 4 shows the fragmentations for the model without and with noise:

Figure 4. (Left) opinion clustering in the model without noise and (right) opinion clustering in the model with noise.

We have four opinion subgroups in the deterministic H-K models and two confidence clusters in the model with the noise. The number of clusters is less than the groups of opinions in the model without noise.

For a fixed set of initial opinions of $m$ agents $x_0 = [x_0^0, x_0^1, \ldots, x_0^{m-1}]$ and a set of random noises $\delta$ in the Hegselmann-Krause model it is possible to choose the parameters $T, \sigma, \varepsilon$ in such a way, that the number of clusters in the model with random noises will be less than the number agents of groups in the model without random noises, and vice versa.

We change the parameter $\varepsilon$ from 0.001 to 0.191 with a step of 0.1, and calculate the number of opinion groups for H-K model without noise (Fig.5 blue line). For stochastic H-K model for each $\varepsilon$ we change the parameter $\sigma$ from 0 to 0.1 with a step 0.001, and look for the minimum number of confidence clusters by the parameter $\sigma$. The algorithm was done several times for different realizations of noises $\delta_1, \delta_2, \delta_3$, which were set randomly, following the formula (4) (on Fig.5 different realizations are marked with colored lines – red, grey and yellow). Figure 5 shows the dependence of the number of confidence clusters (in the case of a model with the noise of the minimum number of confidence clusters by $\sigma$) on the parameter $\varepsilon$.
Figure 5. The dependence of the number of confidence clusters (in the case of a model with the noise of the minimum number of confidence clusters by $\sigma$) on the parameter; blue line – simulation without noise, red line - with the set of noise $\delta_1$, grey line - with the set of noise $\delta_2$, yellow line - with the set of noise $\delta_3$.

5. Conclusions

We consider the Hegselmann-Krause bounded confidence model with noise. Using computer modeling we analyze the dependence of the opinion dynamics on the parameters of the problem such as the initial profile of opinions, the level of confidence, the variance of noise.

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