Study of $B_c^+$ decays to the $K^+K^−\pi^+$ final state by using $B_s^0$, $\chi_{c0}$ and $D^0$ resonances and weak annihilation nonresonant topologies

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Abstract

In this research the weak decay of $B_c^+$ decays to the $K^+K^−\pi^+$ final state, which is observed by LHCb collaboration for the first time, is calculated in the quasi-two-body decays which takes into account the $B_s^0$, $\chi_{c0}$ and $D^0$ resonances and weak annihilation nonresonant contributions. In this process, the $B_c^+$ meson decays first into $B_s^0\pi^+$, $\chi_{c0}\pi^+$ and $D^0\pi^+$ intermediate states, and then the $B_s^0$, $\chi_{c0}$ and $D^0$ resonances decay into $K^+K^−$ components, which undergo final state interaction. The mode of the $B_c^+\rightarrow D^0(\rightarrow K^−\pi^+)K^+$ is also associated to the calculation, in this mode the intermediate resonance $D^0$ decays to the $K^−\pi^+$ final mesons. The resonances $B_s^0$, $\chi_{c0}$ and $D^0$ effects in the $B_c^+\rightarrow B_s^0(\rightarrow K^+K^−)\pi^+$, $B_c^+\rightarrow \chi_{c0}(\rightarrow K^+K^−)\pi^+$ and $B_c^+\rightarrow D^0(\rightarrow K^+K^−)\pi^+$, $D^0(\rightarrow K^−\pi^+)K^+$ decays are described in terms of the quasi-two-body modes. There is a weak annihilation nonresonant contribution in which $B_c^+$ decays to the $K^+K^−\pi^+$ directly, so the point-like 3-body matrix element $\langle K^+K^−\pi^+|ud|0\rangle$ is also considered. The decay mode of the $B_c^+\rightarrow K^{*0}(892)K^+$ is contributed to the annihilation contribution. The branching ratios of quasi-two-body decays expand in the range from $1.98 \times 10^{-6}$ to $7.32 \times 10^{-6}$.

1 Introduction

$B_c$ meson is one of the most interesting mesons that can be studied at the Tevatron, the discovery of the $B_c$ was reported by the CDF collaboration in the $B_c \rightarrow J/\psi l^\pm \bar{\nu}_l$ process at Fermilab [1]. After that, the decay mode $B_c^{±} \rightarrow J/\psi \pi^{±}$ has been observed by CDF and D0 collaboration significance of more than $8\sigma$ and $5\sigma$ respectively [2][3]. This decay mode addition $B_c^{±} \rightarrow J/\psi D_s^{±}$ decay have also been observed by the LHCb collaboration at the LHC center-of-mass energy $7$ TeV of proton-proton collisions [4][5]. Studies of $B_c$ properties are important, because it is made of two different heavy quarks, bottom-charm antiquark-quark pair. Each of the quarks can participate in a weak interaction in which other quark participates as a spectator. For this, $B_c$ is also the only meson in which decays of both heavy quarks compete with each other, therefore a wide range of decay channels are possible. However, a significant number of

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these channels has not been observed yet \[6\]. Unlike \(B^0\), \(B^+\) and \(B^0\) mesons, more than 70% of the \(B^+_c\) width is due to c-quark decays, in which \(c \to s\) transition has been observed with \(B^{+}_c \to B^{0}_s \pi^+\) decays \[7\]. Around 20% of its width is due to the b-quark decays \[8\]. In charmless final states, the \(\bar{b}c \to W^+ \to q\bar{q}\) annihilation amplitudes account for only 10% of the \(B^+_c\) width \[9\].

The mass of the \(B_c\) meson has been predicted using a variety of theoretical techniques. Non-relativistic potential models have been used to predict a mass of the \(B_c\) in the range of 6247 - 6286 MeV/c\(^2\) \[10\] \[11\] \[12\], by using a perturbative QCD calculation slightly higher value is found \[13\] and recent \(B_c\) mass prediction of 6304(±18) MeV/c\(^2\) have been provided applying lattice QCD calculations \[14\], in which are heavier than that of other \(B\) mesons, this suggests an expected lifetime much shorter than those \[15\].

The decay of \(B^+_c\) mesons to three light charged hadrons like \(K^+, K^-\) and \(\pi^+\), which is observed by LHCb collaboration for the first time \[9\], provide a good way to study standard model for which has a large available phase space. In this work the decay mode of \(B^+_c \to K^+K^−\pi^+\) is studied in which includes other processes such as \(B^+_c \to B^0_s(\to K^+K^-\pi^-)\pi^+\) decay mediated by \(c \to s\) transition, charmonium mode \(B^+_c \to χ_{c0}(\to K^+K^-\pi^+)\) mediated by the \(b \to \bar{t}\) transition, \(B^+_c \to D^0(\to K^+K^-\pi^+)\pi^+\) mediated by the \(\bar{b} \to \bar{u}\) and \(\bar{b} \to \bar{d}\) transitions and finally the decay of \(B^+_c \to D^0(\to K^-\pi^+)K^+\) mediated by the \(\bar{b} \to \bar{u}\) and \(\bar{b} \to \bar{s}\) transitions. In the standard model, there is another process that can mediate via \(\bar{c}b \to W^+ \to ud\) annihilation topology, in this mode the \(B^0_s\) decays with no charm and beauty particles in the final or intermediate states. In the \(B^+_c\) region \(6.0 < m(K^+K^-\pi^+) < 6.5 \text{ GeV/c}^2\), the signals were fitted by authors in Ref. \[9\] separately for regions of the phase space corresponding to the different expected contributions:

(a) the annihilation region, \(m(K^-\pi^+) < 1.834 \text{ GeV/c}^2\), they have claimed the \(K^{*0}(892)\) meson can be in this region. The contribution of the mode \(B^+_c \to K^{*0}(892)(\to K^−\pi^+)K^+\) and direct annihilation are obtained separately which the estimate of \(K^{*0}(892) \to K^−\pi^+\) is lower than the direct annihilation calculation, (b) the \(D^0 \to K^−\pi^+\) region, \(1.834 < m(K^−\pi^+) < 1.894 \text{ GeV/c}^2\), (c) the \(B^0_s \to K^+K^-\) region, \(5.3 < m(K^+K^-) < 5.4 \text{ GeV/c}^2\), (d) the \(\chi_{c0} \to K^+K^-\) region, \(3.38 < m(K^+K^-) < 3.46 \text{ GeV/c}^2\). A concentration of events was observed by \[9\] around \(m^2(K^+K^-) \sim 11\text{GeV}^2/c^4\), a one-dimensional projection of \(m(K^+K^-)\) shows clustering near \(3.41 \text{ GeV/c}^2\), close to the mass of the charmonium state \(\chi_{c0}\) where has the highest branching fraction into the \(K^+K^-\) final state \[16\]. The accumulation of events was also observed near \(m^2(K^+K^-) \sim 29\text{GeV}^2/c^4\) close to the mass of the \(B^0_s\) meson. In this research the study of the \(B^+_c \to K^+K^-\pi^+\) via quasi-two-body decay was considered, this decay mode observed in the \(B^0_s \to K^+K^−, \chi_{c0} \to K^+K^-\), \(D^0 \to K^+K^-\) and \(D^0 \to K^-\pi^+\) channels. It is known that in the narrow width approximation, in the models we use to obtain the amplitudes of the decays, the 3-body decay rate obeys the factorization relation \[17\]

\[
B(B^+_c \to RM \to M_1M_2M) = B(B^+_c \to RM) \times B(R \to M_1M_2),
\]

with \(R\) being \(B^0_s, \chi_{c0}\) and \(D^0\) intermediate resonance mesons and \(M, M_1\) and \(M_2\) are \(K^+, K^-\) and \(\pi^+\) final state mesons. The intermediate resonance effects are described in terms of the Breit-Wigner formalism. The Breit-Wigner resonant term associated to quasi two body \(R + M\) state which seems to play an important role as indicated by experiments. We have to calculate the branching ratios of the \(B(B^+_c \to RM)\) and \(B(R \to M_1M_2)\) by using the Feynman quark diagrams.
2 Decay amplitudes and branching fractions

In the standard factorization scheme, the decay amplitude is obtained by applying weak Hamiltonian which is given below:

\[ A(P \rightarrow P_1P_2) = \langle P_1 | J^\mu | 0 \rangle \langle P_2 | J^\dagger_\mu | P \rangle + \langle P_2 | J^\mu | 0 \rangle \langle P_1 | J^\dagger_\mu | P \rangle \]  

(2)

where the weak current \( J_\mu \) is given by:  
\[
\begin{pmatrix}
\bar{d}' \\
\bar{s}' \\
\bar{b}'
\end{pmatrix} \gamma_\mu (1 - \gamma_5)
\begin{pmatrix}
d' \\
s' \\
b'
\end{pmatrix},
\]
here \( d', s' \) and \( b' \) are mixture of the \( d, s \) and \( b \) quarks, as given by the CKM matrix. Current elements are defined as:  
\[
(p + p_{1,2} - q(m^2_\mu - m^2_{p_{1,2}})/q^2) + q_\mu (m^2_\mu - m^2_{p_{1,2}})/q^2 F_0(q^2)
\]
with \( p_\mu = p_{1,2} \) and \( P_1(p_{1,2})|J^\mu(0)|0\rangle = i\bar{f}_p P_{1,2}^\dagger.\) It has been pointed out in the BSW2 model [18] that consistency with the heavy quark symmetry requires certain form factors such as \( F_0 \) and \( F_1 \) to have dipole \( q^2 \) dependence i.e.  
\[
F_{0,1}^{P \rightarrow P_1,2}(q^2) = F_{0,1}^{P \rightarrow P_1,2}(0)/(1 - q^2/m_V^2),
\]
as an example for the \( c \rightarrow b \) transition \( m_V \) is \( m_{B_c^+}.\) In the following, we calculate the amplitudes of all decay modes. Different with \( B_{u,d,s} \) mesons, the \( B_c^+ \) system consists of two heavy quarks \( b \) and \( c,\) which can decay individually. Here we will consider \( b \) decays as \( c \) acts as a spectator except \( B_c^+ \rightarrow B_{s,0}^0 \pi^+ \) decay in which we have \( c \rightarrow s \) transition.

According to figure [1] at the quark level corresponding effective Hamiltonian is given by:  
\[
H_{eff} = (G_F/\sqrt{2}) \sum_{p=u,c} \lambda_p (c_1 Q_1^p + c_2 Q_2^p + \sum_{i=3,\ldots,10} c_i Q_i + c_7 q^2 \gamma_7 + c_8 q_{s8})
\]
where \( \lambda_p \) is the CKM matrix elements, \( c_i \) are the Wilson coefficients evaluated at the renormalization scale \( \mu,\) \( Q_{1,2}^p \) are the left-handed current-current operators arising from W-boson exchange, \( Q_{3,\ldots,10} \) are QCD and electroweak penguin operators, and \( Q_7 \) and \( Q_{s8} \) are the electromagnetic and chromomagnetic dipole operators. Because in the tree level diagrams \( a_1 \) coefficient both \( \bar{b} \rightarrow \bar{u} \) and \( \bar{b} \rightarrow \bar{c} \) transitions are available, we have considered to current-current operators \( Q_1^u, Q_2^u \) and \( Q_1^c \) as:  
\[
Q_1^{u,c} = (\bar{b}_u u_a) V^{-A}(\bar{d}_u d_a) V^{-A},
Q_2^{u,c} = (\bar{b}_c c_a) V^{-A}(\bar{d}_c d_a) V^{-A},
\]
and \( Q_1^d = (\bar{b}_u u_a) V^{-A}(\bar{d}_u d_a) V^{-A} \) and \( Q_2^d = (\bar{b}_c c_a) V^{-A}(\bar{d}_c d_a) V^{-A} \). The operators arise from the QCD-penguin diagrams (both \( \bar{b} \rightarrow \bar{d} \) and \( \bar{b} \rightarrow \bar{s} \) transitions) which contribute in the \( a_4 \) coefficient we have considered:  
\[
Q_{1,2} = (\bar{b}_a d_a(s_a) V^{-A}(\bar{u}_b u_b) V^{-A}, Q_{1,2} = (\bar{b}_a d_b(s_b) V^{-A}(\bar{u}_b u_b) V^{-A}.
\]
Here \( a \) and \( c \) are the SU(3) color indices and the subscript \( V^{-A} \) represent the chiral projection \( 1 - \gamma_5.\)

2.1 \( B_c^+ \rightarrow B_{s,0}^0 \pi^+ \) decay

In the following, we calculate the amplitude of the \( B_c^+ \rightarrow B_{s,0}^0 \pi^+ \) decay mediated by \( c \rightarrow s \) transition. The amplitude of the \( B_c^+ \rightarrow B_{s,0}^0 \pi^+ \) decay by using the color-allowed external W-emission tree diagram become:  
\[
(G_F/\sqrt{2}) V_{cb} V_{ud}^* (B_{s,0}^0 V_{cs}) (c)(s)|B_c^+\rangle|\pi^+\rangle|(ud)_{V^{-A}}\rangle 0 F,\]  
where \( F \) denote the factorized hadronic matrix element, which has the same definition as that in the "nonfactorizable" approach. The form factor of the \( (B_{s,0}^0 V_{cs})(c)(s)|B_c^+\rangle|\pi^+\rangle|(ud)_{V^{-A}}\rangle \) can be as follows:  
\[
(p_\mu + p_{1,2} - q_\mu (m^2_{B_c^+} - m^2_{B_s^0})/q^2) F_1^{B_c^+ \rightarrow B_{s,0}^0}(m_\pi^2) + q_\mu (m^2_{B_c^+} - m^2_{B_s^0})/q^2 F_0^{B_c^+ \rightarrow B_{s,0}^0}(m_\pi^2)
\]
and the decay constant of the \( \langle \pi^+|(ud)_{V^{-A}}\rangle \) become:  
\[
i f_\pi p_2^q \]  
where \( q_\mu = p_2 - p_1 \), and \( F_0^{B_c^+ \rightarrow B_{s,0}^0}(m_\pi^2) = F_0^{B_c^+ \rightarrow B_{s,0}^0}(0)/(1 - m^2_\pi/m^2_{D_s^+})^2 \). The expressions of decay amplitude for
Figure 1: Feynman diagrams for $B_c^+ \to K^+K^-\pi^+$ decays using the (a) $B_c^+ \to B_s^0(\to K^+K^-)\pi^+$ channel mediated by $c \to s$ transition; (b) $B_c^+ \to \chi_{c0}(\to K^+K^-)\pi^+$ channel mediated by $\bar{b} \to \bar{c}$ transition; (c) and (d) $B_c^+ \to D^0(\to K^+K^-)\pi^+$ channel mediated by $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{d}$ transitions; (e) and (f) $B_c^+ \to D^0(\to K^-\pi^+)K^+$ channel mediated by $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{s}$ transitions; (g) annihilation process.

$B_c^+ \to B_s^0\pi^+$ within the factorization framework can be written as

$$A(B_c^+ \to B_s^0\pi^+) = i\frac{G_F}{\sqrt{2}}a_1V_{cs}V_{ud}f_{\pi}(m_{B_c^+}^2 - m_{B_s^0}^2)F_{B_c^+ \to B_s^0}^0(m_{\pi}^2),$$

(3)

where $a_1 = c_1 + c_2/3$. The branching fraction of $B_c^+ \to B_s^0\pi^+$ in $B_c^+$ meson rest frame can be written as

$$B(B_c^+ \to B_s^0\pi^+) = \frac{\tau_{B_c^+} |\vec{p}|}{8\pi m_{B_c^+}^2} |A(B_c^+ \to B_s^0\pi^+)|^2,$$

(4)

in which where $\tau_{B_c^+}$ is the lifetime of the $B_c^+$ meson and $|\vec{p}|$ is the absolute value of the 3-momentum of the $B_s^0$ or $\pi^+$ mesons that can be calculated via:

$$\sqrt{(m_{B_c^+}^2 + m_{B_s^0}^2 - m_{\pi}^2)^2 - 4m_{B_c^+}^2m_{B_s^0}^2/(2m_{B_c^+}^2)}.$$
2.2 $B_c^+ \rightarrow \chi_{c0}\pi^+$ decay

The same calculation, similar to the previous calculation, can also be applied to the $B_c^+ \rightarrow \chi_{c0}\pi^+$ mode, with this difference this state of decay mediated by $b \rightarrow s$ transition so the form factor of the $\langle \chi_{c0}(p_1)|(cb)\overline{A}|B_c^+(p)\rangle$ related to the $F_{0}^{B_c^+\rightarrow \chi_{c0}}(m_{\pi}^2) = F_{0}^{B_c^+\rightarrow \chi_{c0}}(0)/(1-m_{\pi}^2/m_{B_c^+}^2)^2$. The amplitude and branching fraction of the $B_c^+ \rightarrow \chi_{c0}\pi^+$ is the same with the $B_c^+ \rightarrow B_s^0\pi^+$ one which are given by

$$A(B_c^+ \rightarrow \chi_{c0}\pi^+) = \frac{G_F}{\sqrt{2}} a_1 V_{cb} V_{ud}^* f_\pi(m_{B_c^+}^2 - m_{\chi_{c0}}^2) F_0^{B_c^+\rightarrow \chi_{c0}}(m_{\pi}^2),$$

and

$$B(B_c^+ \rightarrow \chi_{c0}\pi^+) = \frac{\tau B_c^+}{8\pi m_{B_c^+}^2} |A(B_c^+ \rightarrow \chi_{c0}\pi^+)|^2,$$

to obtain the absolute value of the 3-momentum of the $\chi_{c0}$ or $\pi^+$ mesons, the $\chi_{c0}$ meson mass should be replaced instead of the $B_s^0$ meson mass in the previous calculation. The branching ratios for other decays $B_c^+ \rightarrow D^0\pi^+$, $B_c^+ \rightarrow D^0K^+$, $B_s^+ \rightarrow K^+K^-$, $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow K^-\pi^+$ have the similar expressions as Eqs. (4) and (6).

2.3 $B_c^+ \rightarrow D^0\pi^+$ and $B_c^+ \rightarrow D^0K^+$ decays

These decays, in addition $b \rightarrow u$ tree level coefficient, involve the $b \rightarrow s$ and $b \rightarrow s$ penguin amplitudes ($a_4$ coefficient) with the QCD penguins participating. The same amplitudes of the $B_c^+ \rightarrow D^0\pi^+$ and $B_c^+ \rightarrow D^0K^+$ decays are given by

$$A(B_c^+ \rightarrow D^0\pi^+(K^+)) = \frac{G_F}{\sqrt{2}} [a_1 V_{ub} V_{us}^* (V_{ub}^* - a_4 V_{tb} V_{td}^* f_{\pi}(F_{0}^{B_c^+\rightarrow D^0}(m_{\pi}^2)),$$

where $a_4 = c_4 + c_3/3$.

2.4 $B_s^0 \rightarrow K^+K^-$, $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow K^-\pi^+$ decays

In the factorization approach the amplitudes of the $B_s^0 \rightarrow K^+K^-$, $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow K^-\pi^+$ decays have the following form

$$A(B_s^0 \rightarrow K^+K^-) = \frac{G_F}{\sqrt{2}} a_1 V_{ub} V_{us}^* f_{K^+}(m_{B_s^0}^2 - m_{K^-}^2) F_0^{B_s^0\rightarrow K^-}(m_{K^+}^2),$$

$$A(D^0 \rightarrow K^+K^-) = \frac{G_F}{\sqrt{2}} a_1 V_{ub} V_{us}^* f_{K^+}(m_{D^0}^2 - m_{K^-}^2) F_0^{D^0\rightarrow K^-}(m_{K^+}^2),$$

$$A(D^0 \rightarrow K^-\pi^+) = \frac{G_F}{\sqrt{2}} a_1 V_{ub} V_{us}^* f_{\pi^+}(m_{D^0}^2 - m_{K^-}^2) F_0^{D^0\rightarrow K^-}(m_{\pi^+}^2).$$
2.5 Annihilation topology

In this subsection we offer two ways for annihilation topology, the first is two body pure annihilation topology for $\chi_{c0} \to K^+K^-$ decay, this decay mode proceeds only through the $W$-annihilation diagram and the second is nonresonant three body $B_c^+ \to K^+K^-\pi^+$ annihilation contribution. The contribution of the mode $B_c^+ \to K^+0(892)K^+$ in the annihilation processes could be also prominent.

2.5.1 Pure annihilation $\chi_{c0} \to K^+K^-$ decay

The $\chi_{c0} \to K^+K^-$ decay is a pure annihilation decay channel, this process only occurs via annihilation between $c$ and $s$ quarks. In the factorization method, Feynman diagram for the $\chi_{c0} \to K^+K^-$ decay is shown in figure[1]. When all the basic building blocks equations are solved, for the case that both mesons are pseudoscalar, it is found that weak annihilation Kernels exhibit endpoint divergence. Divergence terms are determined by $\int_0^1 dx/\bar{x}$ and $\int_0^1 dy/y$. For the liberation of the divergence, a small $\epsilon$ of $\Lambda_{QCD}/m_{\chi_{c0}}$ order was added to the denominator. So the answer to the integral becomes $\ln(1+\epsilon)/\epsilon$ form, which is shown with $X_A$. Specifically, we treat $X_A = (1 + \rho_A e^{i\varphi_A})\ln(m_{\chi_{c0}}/\Lambda_{QCD})$ as a arbitrary parameter obtained by using $\rho_A = 0.5$ and a strong phase $\varphi_A = -55^0$.[19]. The factorization amplitude when both mesons are pseudoscalar is given by

$$A(\chi_{c0} \to K^+K^-) = \frac{iG_F}{\sqrt{2}} b_1 V_{cs} V_{cb}^* f_{\chi_{c0}} f_{K^+} f_{K^-},$$ (11)

where $b_1$ is the building block of the non-singlet annihilation coefficient which is given by:

$$(8/27)\pi\alpha_s c_1 [9(X_A - 4 + \pi^2/3) + r_{K^+} K^- X_A^2].$$

The light-cone expansion implies that only leading-twist distribution amplitudes are needed in the heavy-quark limit. There exist however a number of subleading quark-antiquark distribution amplitudes of twist 3, which have large normalization factors for pseudoscalar mesons, e.g. $r_{K^+}$ for the kaon we use $2m_K^2/((m_c - m_u)(m_u + m_s))$

2.5.2 Effects of $B_c^+ \to \bar{K}^0(892)K^+$ decay to the annihilation processes

According to the panel (g) of the figure[1] the decay mode of the $B_c^+ \to \bar{K}^0(892)K^+$ could be contributed to the annihilation processes. The amplitude of the $B_c^+ \to \bar{K}^0(892)K^+$ annihilation decay reads

$$A(B_c^+ \to \bar{K}^0 K^+) = \frac{iG_F}{\sqrt{2}} b_2 V_{ub} V_{ud}^* f_{B_c^+} f_{\bar{K}^0} f_{K^+},$$ (12)

when one of the final state meson is vector and another is pseudoscalar, the building block of the non-singlet annihilation coefficient $b_2$ become: $(8/9)\pi\alpha_s c_2 [3(X_A - 4 + \pi^2/3) + r_{K^+} r_{K^+}(X_A^2 - 2X_A)]$, the ratio of the $r_{K^0}$ is calculated via: $(2m_{K^0}/m_b)(f_{K^0}/f_{K^+}).$

2.5.3 Nonresonant annihilation contribution for $B_c^+ \to K^+K^-\pi^+$ decay

As for the three-body nonresonant annihilation matrix element $\langle K^+(p_1)K^-(p_2)\pi^+(p_3)|(ud)\rangle_{V_A}|0\rangle$, we can show that it vanishes in the chiral limit owing to the helicity suppression. To prove
The decay width of a three-body process is given by

\[
\Gamma\left( K^+(p_1)K^-(p_2)\pi^+(p_3)\right) = \frac{G_F^2}{\sqrt{2}} a_1 V_{cb} V_{ud} f_{B^+} \frac{m_K^2 - m_\pi^2}{m_s - m_d} \left( \frac{1}{m_{B^+}^2 - m_K^2} \right),
\]

where \( a_1 \) is the CKM matrix element valid only for low-momentum pseudoscalars. It is easily seen that in the chiral limit the \((K^+(p_1)K^-(p_2)\pi^+(p_3))\left(\langle ud \rangle V_{-A} | 0 \rangle (0 | (c\bar{b}) V_{-A} | B^+ \rangle \right)\) become \((m_K^2 - m_\pi^2)/(m_s - m_d)\)((f_{B^+}^2 m_{B^+}^2)/(f_{\pi^+} m_\pi^2))(1 - (2p_1 \cdot p_3)/(m_{B^+}^2 - m_K^2)) F^{KK\pi}_L(m_{B^+}^2), \) where the form factor \( F^{KK\pi}_L(m_{B^+}^2) \) is needed to accommodate the fact that the final-state pseudoscalars are energetic rather than soft in which is assumed to be \( 1/[(1 - m_{B^+}^2/\Lambda_\chi^2)] \), with \( \Lambda_\chi = 830 \text{ MeV} \) is the chiral-symmetry breaking scale. The direct three body weak annihilation amplitude reads

\[
A(B_c^+ \to K^+ K^- \pi^+)_{\text{ann}} = \frac{G_F}{\sqrt{2}} a_1 V_{cb} V_{ud} f_{B^+} \frac{m_K^2 - m_\pi^2}{m_s - m_d} \left( \frac{1}{m_{B^+}^2 - m_K^2} \right) F^{KK\pi}_L(m_{B^+}^2).
\]

To obtain the \( p_1 \cdot p_3 \), we consider the decays of \( B_c^+ \) meson (with 4-momentum of \( p \) and \( m \) mass) into three \( K^+ \), \( K^- \) and \( \pi^+ \) particles. Denote their masses \( m_1 \), \( m_2 \) and \( m_3 \) and 4-momenta by \( p_1, p_2 \) and \( p_3 \), respectively. Energy-momentum conservation is expressed by \( p = p_1 + p_2 + p_3 \). Define the following invariants \( s_{12} = (p_1 + p_2)^2 = (p - p_3)^2 \), \( s_{13} = (p_1 + p_3)^2 = (p - p_2)^2 \), \( s_{23} = (p_2 + p_3)^2 = (p - p_1)^2 \). The three invariants \( s_{12}, s_{13} \) and \( s_{23} \) are not independent, it follows from their definitions together with 4-momentum conservation that \( s_{12} + s_{13} + s_{23} = m^2 + m_1^2 + m_2^2 + m_3^2 \). We take \( s_{12} = s \) and \( s_{23} = t \), so we have \( s_{13} = m^2 + m_1^2 + m_2^2 + m_3^2 - s - t \). With these definitions, we obtain multiplying of the 4-momentum as: \( p_1 \cdot p_3 = (1/2)(m^2 + m_2^2 - s - t) \).

The decay width of a three-body process is given by

\[
\Gamma(B_c^+ \to K^+ K^- \pi^+) = \frac{1}{(2\pi)^3 32 m_{B_c^+}^3} \int_{s_{\text{min}}}^{s_{\text{max}}} \int_{t_{\text{min}}}^{t_{\text{max}}} |A(B_c^+ \to K^+ K^- \pi^+)|^2 ds dt,
\]

where \( t_{\text{min}}, t_{\text{max}} = m_1^2 + m_2^2 - (1/(2s)) \), \( (m^2 - s - m_2^2)(s - m_2^2 + m_3^2) \neq 0 \), \( \Lambda(s, m^2, m_{B_c^+}^2) \), \( \Lambda(s, m_1^2, m_2^2) \), \( s_{\text{min}} = (m_1 + m_2)^2 \), \( s_{\text{max}} = (m - m_3)^2 \) and \( \lambda(x, y, z) = x^2 + y^2 + z^2 - (2xy + xz + yz) \).

3 Numerical results and conclusion

The theoretical predictions depend on many input parameters such as Wilson coefficients, the CKM matrix elements, masses, lifetimes, decay constants, form factors, and so on. We present all the relevant input parameters as follows:

**Wilson coefficients**, the Wilson coefficients \( c_1, c_2, c_3 \) and \( c_4 \) in the effective weak Hamiltonian have been reliably evaluated to the next-to-leading logarithmic order. To proceed, we use the following numerical values at \( \mu = m_b \) scale, which have been obtained in the NDR scheme \( \text{[13]} \):

\[
c_1 = 1.081, \quad c_2 = -0.190, \quad c_3 = 0.014 \quad \text{and} \quad c_4 = -0.036.
\]

**The CKM matrix elements**, the Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3 \times 3 unitary matrix, the elements of this matrix can be parameterized by three mixing angles \( \theta \), \( \rho \) and a CP-violating phase \( \eta \) \( \text{[15]} \): \( V_{ud} = 1 - \lambda^2/2, \quad V_{us} = \lambda, \quad V_{cb} = A\lambda(\rho - i\eta), \quad V_{cd} = -\lambda, \quad V_{cs} = 1 - \lambda^2/2, \quad V_{cb} = A\lambda^2, \quad V_{td} = A\lambda^3(1 - \rho - i\eta), \quad V_{ts} = -A\lambda^2 \) and \( V_{tb} = 1 \). The results for
the Wolfenstein parameters are \( \lambda = 0.22537 \pm 0.00061, \ A = 0.814^{+0.023}_{-0.024}, \ \bar{\rho} = 0.117 \pm 0.021 \) and \( \bar{\eta} = 0.353 \pm 0.013 \). We use the values of the Wolfenstein parameters and obtain \( V_{ud} = 0.97427 \pm 0.00014, \ V_{us} = 0.22536 \pm 0.00061, \ V_{ub} = 0.00355 \pm 0.00015, \ V_{cd} = 0.22522 \pm 0.00061, \ V_{cs} = 0.97343 \pm 0.00015, \ V_{cb} = 0.0414 \pm 0.0012, \ V_{td} = 0.00886^{+0.00033}_{-0.00032}, \ V_{ts} = 0.0405^{+0.0011}_{-0.0012} \) and \( V_{tb} = 0.99914 \pm 0.00005 \).

**Masses and decay constants** (in units of MeV), the meson masses and decay constants needed in our calculations are taken as [16]: \( m_{B_s^0} = 5415.4^{+1.8}_{-1.5}, m_{B_d^0} = 5366.79 \pm 0.23, m_{\chi_{c0}} = 3414.75 \pm 0.31, m_{D_{s}^{*+}} = 2112.1 \pm 0.4, m_{D_{s}^{0}} = 1864.84 \pm 0.05, m_{K^{*+}} = 493.677 \pm 0.016, m_{\pi^+} = 139.57018 \pm 0.00035, m_b = 4180 \pm 30, m_c = 1275 \pm 25, m_s = 95 \pm 5, m_d = 4.8^{+0.5}_{-0.7}, m_u = 2.3^{+0.7}_{-0.5}, f_{B_s^{+}} = 489 \pm 4, f_K = 159.8 \pm 1.84, f_{K^*} = 217 \pm 5, f_{\bar{K}^*} = 185 \pm 10, f_\pi = 130.70 \pm 0.46 \) and \( \Lambda_{QCD} = 225 \).

**Form factors**, for the parameters \( F^{P \rightarrow M}_0 (0) \) used in transition weak form factors we take [20] [21] [22] [23]: \( F^{B_s^+ \rightarrow B^0}_0 (0) = 0.55 \pm 0.02, F^{B^+_c \rightarrow \chi_{c0}}_0 (0) = 0.58 \pm 0.02, F^{B^+_c \rightarrow D^0}_0 (0) = 0.69, F^{B^+_c \rightarrow K^0}_0 (0) = 0.31 \) and \( F^{D^0 \rightarrow K^*}_0 (0) = 0.78 \).

For the modes \( B_s^+ \rightarrow B^0 \pi^+ \) and \( B_s^+ \rightarrow \chi_{c0} \pi^+ \) we obtain the following values for the branching ratios \( B(B_s^+ \rightarrow B^0 \pi^+) = 9.15\% \) and \( B(B_s^+ \rightarrow \chi_{c0} \pi^+) = 1.22 \times 10^{-3} \), these results should be compared to the experimental measurement, but only \( (\sigma(B_s^+) / \sigma(B^+) ) \times B(B_s^+ \rightarrow B^0 \pi^+) = (2.37 \pm 0.31 \text{stat}) \pm 0.11 \text{syst} \times 10^{-3} \) and \( (\sigma(B_s^+) / \sigma(B^+) ) \times B(B_s^+ \rightarrow \chi_{c0} \pi^+) = (9.8^{+3.4}_{-3.0} \text{stat}) \pm 0.8 \times 10^{-6} \) are available in the experience [7] [9]. The ratio of the production cross-sections of the \( B^+ \) and \( B^0 \) mesons, \( \sigma(B^+) / \sigma(B^0) \), can be get from the measurement involving another charmonium mode, \( (\sigma(B_s^+) / \sigma(B^+) ) \times B(B_s^+ \rightarrow J/\psi \pi^+) = (7.0 \pm 0.3) \times 10^{-6} \) obtained from Ref. [24]. Using the predictions listed in Ref. [25] for \( B(B_s^+ \rightarrow J/\psi \pi^+) \), which span the range \( (0.34 \sim 2.9) \times 10^{-3} \), we obtain \( \sigma(B_s^+) / \sigma(B^+) = (0.23 \sim 2.15)\% \), so the experimental branching fractions become: \( B(B_s^+ \rightarrow B^0 \pi^+) = (8.46 \sim 128.69)\% \) and \( B(B_s^+ \rightarrow \chi_{c0} \pi^+) = (0.30 \sim 6.09) \times 10^{-3} \) which in very good agreement with our prediction.

For the branching fraction of the \( B_s^+ \rightarrow D^0 K^+ \) and \( B_s^+ \rightarrow D^0 \pi^+ \) decays our calculations become from \( 10^{-4} \) order, as have been obtained in [20]. The experimental result available for \( f_c / f_u \times \sigma(B_s^+ \rightarrow D^0 K^+) \) is \( (9.3^{+2.5}_{-2.5} \pm 0.6) \times 10^{-7} \) and for \( f_c / f_u \times \sigma(B_s^+ \rightarrow D^0 \pi^+) \) is less than \( 3.9 \times 10^{-7} \) [27], which implies \( f_c / f_u \) values in the range \( 0.004-0.012 \), the experimental branching fractions of them become from \( 10^{-4} \) order in which our result a thousand times smaller than experimental one. Maybe for this reason for the mode \( B_s^+ \rightarrow D^0 \rightarrow K^- \pi^+) K^+ \), no significant deviation from the background-only hypothesis is observed [9].

For the \( B_s^0 \rightarrow K^+ K^- \) mode, the decay amplitude is calculated at leading power in \( \Lambda_{QCD} / m_b \) and at next-to-leading order in \( a_1 \) using the QCD factorization approach. The calculation of the relevant hard-scattering kernels is completed. Important classes of power corrections, including "chirally-enhanced" terms and weak annihilation contributions, are estimated and included in the phenomenological analysis as given in Refs. [28] [29]. Applying other contributions from the Feynman graph to this decay (such as the contribution of tree level \( a_2 \) in addition to the \( a_3 \) mentioned in the text, penguin diagrams \( a_3 \) and \( a_4 \) and weak annihilation of \( b_1 \) ) we obtain \( B(B_s^0 \rightarrow K^+ K^-) = 2.16 \times 10^{-5} \), this is while its experimental value is \( B(B_s^0 \rightarrow K^+ K^-) = (2.50 \pm 0.17) \times 10^{-5} \) [16].

For the branching ratios of the \( D^0 \rightarrow K^+ K^- \) and \( D^0 \rightarrow K^- \pi^+ \) decays we obtain \( B(D^0 \rightarrow K^+ K^-) = 3.78 \times 10^{-3} \) and \( B(D^0 \rightarrow K^- \pi^-) = 4.56\% \) where the experimental results of them
are \((4.01 \pm 0.07) \times 10^{-3}\) and \((3.93 \pm 0.04)\% \) \cite{10}.

For the pure annihilation mode of the \(\chi_{c0} \rightarrow K^+K^-\) theoretical calculation via factorization approach become from \(10^{-11}\) order, while the experimental value of that is from \(10^{-3}\) order. This is a strange result because the \(\chi_{c0}\) meson has very small lifetime of \(10^{-23}s\). The fact is that the branching fraction of the pure annihilation decays by using the factorization approaches become smaller than experimental one \cite{50, 51}, from the theoretical calculation it seems that in the decay of \(\chi_{c0} \rightarrow K^+K^-\), before the \(K^+\) and \(K^-\) mesons are produced in the final states, the pure mesons such as \(D_s^+, D_s^-\) and \(D^0, \bar{D}^0\) are produced in the intermediate state so the final state interaction effects are needed.

For effects of \(B_c^+ \rightarrow \bar{K}^0(892)K^+\) decay to the annihilation processes we get \(\mathcal{B}(B_c^+ \rightarrow \bar{K}^0K^+) = 9.61 \times 10^{-7}\) as has been recently predicted to be \((10.0^{+8.1}_{-4.8}) \times 10^{-7}\) \cite{32}.

For weak annihilation region the result has been measured by LHCb \cite{9} is \(R_{\text{ann};KK\pi} = (8.0^{+4.4}_{-3.8}(\text{stat}) \pm 0.6(\text{syst})) \times 10^{-8}\), by taking into account the \(\sigma(B_c^+)/\sigma(B^+) = (0.23 \sim 2.15)\%\) range, the measured annihilation branching ratio to be: \(\mathcal{B}(B_c^+ \rightarrow K^+K^-\pi^+)_{\text{ann}} = (1.67 \sim 56.5) \times 10^{-6}\) for which is in good agreement with the result of our calculation: \(\mathcal{B}(B_c^+ \rightarrow K^+K^-\pi^+)_{\text{ann}} = 10.69 \times 10^{-6}\).

Finally, by using the obtained theoretical branching ratios the results for the quasi-two-body decay of \(B_c^+ \rightarrow K^+K^-\pi^+\) become from \(1.98 \times 10^{-6}\) to \(7.32 \times 10^{-6}\).

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