Abstract—In this paper, we investigate the task-space consensus problem for multiple robotic systems with both the uncertain kinematics and dynamics in the case of existence of constant communication delays. We propose an observer-based adaptive controller to achieve the manipulable consensus without relying on the measurement of task-space velocities, and also formalize the concept of manipulability to quantify the degree of adjustability of the consensus value. The proposed new control scheme employs a new distributed observer that does not rely on the joint velocity, and a new kinematic parameter adaptation law with a distributed adaptive kinematic regressor that is driven by both the observation and consensus errors. In addition, it is shown that the proposed controller has the separation property, which yields an adaptive kinematic controller that is applicable to most industrial/commercial robots. The performance of the proposed observer-based adaptive schemes are shown by numerical simulations.

Index Terms—Consensus, manipulability, separation, networked robotic systems, observer, adaptive control.

I. INTRODUCTION

Networked robotic systems have many potential applications such as cooperative manipulation, planet/field exploration, and teleoperation. This motivates the active research on the control of networked robotic systems in recent years (see, e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]). A fundamental control problem for networked robotic systems that is actively studied is consensus, in which case all systems are expected to reach agreement concerning certain variables. The major difficulty involved, as is frequently mentioned in the literature, is the nonlinearity and uncertainty of the system model.

The consensus schemes for networked robotic systems can generally be grouped, in accordance with the interaction graphs among the robotic systems, into two categories. The first category of schemes (e.g., [16], [17], [18], [11], [19]) achieves the consensus of robotic systems on undirected interaction graphs. In the case that there are gravitational torques in the system, the control schemes in [16], [17], [18], [19] require the exact knowledge of the gravitational torques to ensure the consensus. This dependence on knowing the gravitational torques is removed in [3], [11] thanks to the employment of adaptive schemes. The second category of schemes (e.g., [12], [18], [5], [6], [9], [13], [12]) achieves the consensus of the robotic systems on the more general directed graphs. As is described/shown in [20], [5], the adaptive version of the scheme in [5] gives rise to the outcome that all systems’ positions converge to the origin in the presence of gravitational torques. This deficiency has been conquered by the adaptive scheme in [5], and other relevant results appear in [21], [6]. The adaptive scheme in [12], by employing the integral-sliding control action, achieves the (stability guaranteed) consensus of the systems with the final consensus value being explicitly expressed in terms of the initial systems’ positions, and in fact, this adaptive scheme realizes the scaled weighted average consensus of the systems. The case of time-varying communication delays is considered in [12] where a small-gain-based consensus scheme is proposed. However, all the results above only take into account the dynamic uncertainties.

When the robotic system performs tasks given in the Cartesian space, kinematic uncertainties (e.g., the lengths of the robot links may not be accurately known) will possibly occur [22], [23], [24]. Therefore, various adaptive control algorithms are proposed to accommodate the kinematic uncertainties, using the estimated Jacobian matrix [23], [24]. The consensus schemes with consideration of the uncertain kinematics or both the uncertain kinematics and dynamics appear in [25], [26], [27], [28] (with the interaction graph being undirected) and in [9], [15] (with the interaction graph being directed and strongly connected). Motivated by the well-recognized fact that the task-space velocity measurement usually involves too much noise (due to the noisy nature of the task-space position measurement), the work in [29] gives an observer-based adaptive consensus scheme that does not require the task-space velocity measurement, where the observer explicitly relies on the joint velocity. The second consensus scheme in [28] (which can be considered as an extension of [29] to the case of teleoperator systems) avoids the task-space velocity measurement at the expense of overparametrization and under the assumption that the communication delay is absent. While these results are effective in the case of an open torque design interface, in practice, however, most industrial/commercial robots do not provide this design interface and typically only the joint velocity (or position) command can be designed (see, e.g., [30]). Specifically, consider a group of robotic systems with uncertain kinematics and dynamics and with an unmodifiable joint servoing controller [typically PI (proportional-integral) velocity or PID (proportional-integral-derivative) position controller] in terms of the joint-space tracking error, and suppose that the joint servoing process is fast enough. Then we may wonder how to design an adaptive kinematic consensus controller without involving task-space and joint-space velocity measurement and without directly involving task-space position measurement so that the task-space consensus with enhanced robustness can be ensured. The above mentioned approaches cannot resolve this problem appropriately due to the dependence on the modification of the low-level feedback controller structure, and the joint velocity measurement of the kinematic parameter adaptation law. In addition, the issue of manipulability of networked robotic systems, which is important in the scenario involving an external stimuli or human input action, has not been formally studied (has been implicitly used though) in the previous work, especially in the presence of system uncertainties.

In this paper, we propose a new observer-based adaptive scheme to achieve the task-space consensus of the networked robotic systems with both the uncertain kinematics and dynamics in the case of existence of constant communication delays, and also formalize the concept of manipulability to quantify the degree of adjustability of the consensus value. The proposed new scheme employs 1) a new task-space observer that relies on the joint reference velocity rather than joint velocity, in contrast to the joint-velocity-dependent observer in [29], 2) the inverse Jacobian feedback control inspired by the results for a single robotic system [31], [32], unlike most existing task-space consensus schemes (e.g., [9], [29], [27], [31], and 3) a new kinematic parameter adaptation law with a distributed adaptive kinematic regressor, which is driven by both the observation error and consensus error (in contrast with [29] where the kinematic parameter...
adaptation law is driven by the observation error only). These features result in the separation property of the proposed consensus scheme, i.e., the design of the joint reference velocity is separated from and allows the high freedom of that of the joint servoing loop while the existing consensus schemes (e.g., [29], [28]) do not enjoy this property due to the coupling between the kinematic and dynamic loops. The separation property leads us to derive an adaptive kinematic controller that does not directly involve the task-space position (i.e., a reduced version of the proposed dynamic controller) well suited to the case of multiple robotic systems with an unmodifiable joint servoing controller yet admitting the design of the joint velocity (or position) command (e.g., most industrial/commercial robots). The separation property achieved as well as the separation stability analysis can be considered as an extension of that for a single robotic system in [31] to the case of multiple robotic systems without task-space velocity measurement, and this extension is realized by designing a new distributed task-space observer and using a new distributed adaptive kinematic regressor. In addition, our control scheme avoids the overparametrization problem and can conveniently handle the communication delays, in contrast with the second scheme in [28]. Another work given in [33] presents a task-space consensus controller that requires neither the task-space nor joint-space velocity measurement, but this is achieved by requiring that the interaction graph is undirected and the system model is exactly known while (or position) command (e.g., most industrial/commercial robots).

Another contribution of our work is that the proposed graph is undirected and the system model is exactly known while (or position) command (e.g., most industrial/commercial robots). A preliminary version of the paper was presented in [34].

II. Preliminaries

A. Graph Theory

Let us first give a brief introduction of the graph theory [35], [36], [37], [38] in the scenario that \( n \) robotic systems are involved. As is now commonly done, we employ a directed graph \( G = (V, E) \) to describe the interaction topology among the robotic systems where \( V = \{1, 2, \ldots, n\} \) is the vertex set that denotes the collection of the \( n \) systems and \( E \subseteq V \times V \) is the edge set that denotes the information interaction among the \( n \) systems. The set of neighbors of system \( i \) is denoted by \( N_i = \{j | (i, j) \in E\} \). A graph is said to have a directed spanning tree if there is a vertex \( k_0 \in V \) such that any other vertex of the graph has a directed path to \( k_0 \). The weighted adjacency matrix \( W = [w_{ij}] \) associated with the graph \( G \) is defined as \( w_{ij} \) if \( j \in N_i \), and \( w_{ij} = 0 \) otherwise. Additionally, we make the assumption that \( w_{ii} = 0 \), \( \forall i = 1, 2, \ldots, n \). The Laplacian matrix \( L_w = [\ell_{w,ij}] \) associated with the graph \( G \) is defined as \( \ell_{w,ij} = \sum_{k=1}^{n} w_{ik} \) if \( i = j \), and \( \ell_{w,ij} = -w_{ij} \) otherwise. Several basic properties of the Laplacian matrix \( L_w \) can be described by the following lemma.

**Lemma 1** ([29], [37], [38]): If \( L_w \) is associated with a directed graph containing a directed spanning tree, then

1. \( L_w \) has a simple zero eigenvalue, and all other eigenvalues of \( L_w \) have positive real parts;
2. \( L_w \) has a right eigenvector \( 1_n = [1, 1, \ldots, 1]^T \) and a non-negative left eigenvector \( \gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]^T \) satisfying \( \sum_{k=1}^{n} \gamma_k = 1 \) associated with its zero eigenvalue, i.e., \( L_w 1_n = 0 \) and \( \gamma^T L_w = 0 \);
3. the entry \( \gamma_i > 0 \) if and only if vertex \( i \) acts as a root of the graph.

B. Kinematics and Dynamics of Robotic Systems

Denote by \( x_i \in R^m \) the position of the end-effector of the \( i \)-th robotic system in the task space (e.g., Cartesian space), and its relation with the joint position \( q_i \in R^m \) can be written as

\[
x_i = f_i(q_i),
\]

where \( f_i : R^m \to R^m \) denotes a nonlinear mapping.

Differentiating (1) with respect to time gives the relation between the task-space velocity and joint velocity

\[
\dot{x}_i = J_i(q_i) \dot{q}_i
\]

where \( J_i(q_i) \in R^{m \times m} \) is the Jacobian matrix. Since the kinematic parameters are unknown, we cannot obtain the information concerning the task-space position/velocity by the direct kinematics (1) and (2). In this paper, we assume that the task-space position \( x_i \) is available from the task-space sensors (e.g., a camera) while the task-space velocity \( \dot{x}_i \) is not available. The kinematics (2) has the linearity-in-parameters property below [28].

**Property 1:** The kinematics given by (2) depends linearly on a constant kinematic parameter vector \( \theta_i \), which gives rise to

\[
J_i(q_i) \xi = Z_i(q_i, \xi) \theta_i
\]

where \( \xi \in R^m \) is a vector and \( Z_i(q_i, \xi) \) is the kinematic regressor matrix.

The equations of motion of the \( i \)-th robotic system can be written as

\[
M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \tau_i
\]

where \( M_i(q_i) \in R^{m \times m} \) is the inertia matrix, \( C_i(q_i, \dot{q}_i) \in R^{m \times m} \) is the Coriolis and centrifugal matrix, \( g_i(q_i) \in R^m \) is the gravitational torque, and \( \tau_i \in R^m \) is the joint control torque. Three standard properties associated with the dynamic model are that shall be useful for the controller design and stability analysis are listed as follows (see, e.g., [42], [41]).

**Property 2:** The inertia matrix \( M_i(q_i) \) is symmetric and uniformly positive definite.

**Property 3:** The Coriolis and centrifugal matrix \( C_i(q_i, \dot{q}_i) \) can be appropriately chosen such that \( M_i(q_i) - 2C_i(q_i, \dot{q}_i) \) is skew-symmetric.

**Property 4:** The dynamics (4) depends linearly on a constant dynamic parameter vector \( \theta_i \), which gives rise to

\[
M_i(q_i) \dot{\zeta} + C_i(q_i, \dot{q}_i) \zeta + g_i(q_i) = Y_i(q_i, \dot{q}_i, \zeta, \dot{\zeta}) \theta_i
\]

where \( \zeta \in R^m \) is a differentiable vector, \( \dot{\zeta} \) is the time derivative of \( \zeta \), and \( Y_i(q_i, \dot{q}_i, \zeta, \dot{\zeta}) \) is the dynamic regressor matrix.
III. Observer-Based Adaptive Control

In this section, we investigate the adaptive controller design for the task-space consensus problem of the \( n \) robotic systems without involving the task-space velocity measurement, and the control objective is to guarantee that their task-space positions converge to a common value with their task-space velocities converging to zero, i.e., \( x_i - x_j \to 0 \) and \( \dot{x}_i \to 0 \) as \( t \to \infty \), \( \forall i, j = 1, 2, \ldots, n \).

Let us design a joint reference velocity for the \( i \)-th system as

\[
\dot{q}_{i,r} = \hat{J}_i^{-1}(q_i) \left( -\sum_{j \in N_i} w_{ij} [x_{o,i} - x_{o,j}(t - T_{ij})] - \alpha \int_0^t s_{o,i}(r)dr \right)
\]

where \( \alpha \) is a nonnegative design constant, \( T_{ij} \) is the finite constant communication delay from system \( j \) to system \( i \), \( J_i(q_i) \) is the estimate of \( J_i(q_i) \) (which is obtained by replacing \( \hat{\theta}_i \) in \( J_i(q_i) \) with its estimate \( \hat{\theta}_i \)), \( x_{o,i} \) is the observed quantity of \( x_i \) and is updated by the following observer

\[
\dot{x}_{o,i} = \hat{J}_i(q_i)\dot{q}_{i,r} - \beta (x_{o,i} - x_i)
\]

\[
= - \sum_{j \in N_i} w_{ij} [x_{o,i} - x_{o,j}(t - T_{ij})] - \alpha \int_0^t s_{o,i}(r)dr - \beta (x_{o,i} - x_i) - \lambda \int_0^t [x_{o,i}(r) - x_i(r)]dr
\]

where \( \beta \) is a positive design constant and \( \lambda \) is a nonnegative design constant, and the vector \( s^*_i \) is defined by following (5) as

\[
s_{o,i} = \dot{x}_{o,i} + \sum_{j \in N_i} w_{ij} [x_{o,i} - x_{o,j}(t - T_{ij})].
\]

As is typically done, the signal \( x_{o,j}(t - T_{ij}) \) in (5) and (8) is set as \( x_{o,j}(t - T_{ij}) \equiv 0 \) when \( 0 \leq t < T_{ij} \). Note that the proposed observer (7) is independent of the joint velocity \( \dot{q}_i \), in contrast with the results in [32], [29], and additionally distributed in the sense that it does not rely on any global information; this distributed observer, also unlike the one in [6] that is independent of any physical state information of the system, is coupled to the robotic system by the action \(-\beta(x_{o,i} - x_i) - \lambda \int_0^t [x_{o,i}(r) - x_i(r)]dr\). The incorporation of the integral action of \( s^*_i \) in (8) follows the result in [29], and as will be shown later, this integral action allows us to explicitly derive the final consensus value of the systems in the case that \( \alpha > 0 \).

Define a sliding vector

\[
s_i = \dot{q}_i - \dot{q}_{i,r}.
\]

Premultiplying both sides of the above equation by \( J_i(q_i) \) and using equation (5) and Property 1 gives

\[
J_i(q_i)s_i = \dot{x}_{i} + \sum_{j \in N_i} w_{ij} [x_{o,i} - x_{o,j}(t - T_{ij})] + \alpha \int_0^t s^*_i(r)dr + Z_i(q_i, \dot{q}_{i,r})\Delta \dot{\theta}_i
\]

where \( \Delta \dot{\theta}_i = \dot{\theta}_i - \hat{\theta}_i \) is the kinematic parameter estimation error, and in view of (5), the kinematic regressor matrix \( Z_i(q_i, \dot{q}_{i,r}) \) is both adaptive and distributed in that it depends on the estimated kinematic parameter \( \hat{\theta}_i \) updated by the kinematic parameter adaptation law given later and that it does not use any global information of the network.

Subtracting both sides of the kinematics (2) from those of the observer (7) and using Property 1, we obtain the closed-loop observer dynamics as

\[
\Delta \dot{x}_{o,i} = -\beta \Delta x_{o,i} - \lambda \int_0^t \Delta x_{o,i}(r)dr + Z_i(q_i, \dot{q}_{i,r})\Delta \dot{\theta}_i - J_i(q_i)s_i
\]

where \( \Delta x_{o,i} = x_{o,i} - x_i \) denotes the observation error. Now we propose the control law for the \( i \)-th system as

\[
t_i = -K_i s_i + Y_i(q_i, \dot{q}_{i,r}, \dot{q}_{i,r}, \dot{\theta}_i) \dot{\theta}_i
\]

where \( K_i \) is a symmetric positive definite matrix and \( \dot{\theta}_i \) is the estimate of \( \theta_i \). The adaptation laws for updating the estimated parameters \( \dot{\theta}_i \) and \( \dot{\theta}_i \) are given as

\[
\dot{\hat{\theta}}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_{i,r}, \dot{q}_{i,r}, \dot{\theta}_i) s_i
\]

\[
\hat{\theta}_i = \Lambda_i Z_i^T(q_i, \dot{q}_{i,r}) \Delta x_{o,i}
\]

where \( \Gamma_i \) and \( \Lambda_i \) are both symmetric positive definite matrices. The adaptive control scheme given by (12), (13), and (14) can be considered as an extension of those for a single robotic system in [31], [22] to the case of multiple robotic systems without involving task-space velocity measurement. We note that based on (6), (8), and (9), the relation between \( \Delta x_{o,i} \) and \( s^*_i \) can be expressed as

\[
\beta \Delta x_{o,i} + \lambda \int_0^t \Delta x_{o,i}(r)dr = -\left[ s_{o,i} + \alpha \int_0^t s^*_i(r)dr \right]
\]

which means that the observation error is also a reflection of the consensus error concerning the observed task-space positions.

Remark 1: The use of observed quantities of the task-space positions in the definition of \( \dot{q}_{i,r} \) given by (6) avoids involving the task-space velocities in the derivative of \( \dot{q}_{i,r} \). This makes the control law (12) and the dynamic parameter adaptation law (13) independent of the task-space velocity measurement. Contrary to several observer-based algorithms developed in the context of multiple identical linear systems with the model being exactly known (see, e.g., [44], [45]), our algorithm considers the more challenging (perhaps more practical) case of nonidentical nonlinear robotic systems with uncertainties. As we know, the SPR (strictly positive real) condition (or passivity in the case of nonlinear systems) is typically necessary for applying adaptive control, the robot system, by reducing its order with sliding vectors, becomes one among such typical nonlinear systems. In addition, the task-space observer (7) is coupled to the task-space position of the system [by the term \( -\beta(x_{o,i} - x_i) - \lambda \int_0^t [x_{o,i}(r) - x_i(r)]dr \) in (7)] and thus in contrast with the one suggested in [46, 47, 48] which is independent of the system’s state. Substituting the control law (12) into the dynamics (4) yields

\[
M_i(q_i)s_i + C_i(q_i, \dot{q}_i) s_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{i,r}, \dot{\theta}_i) \Delta \dot{\theta}_i
\]

where \( \Delta \dot{\theta}_i = \dot{\theta}_i - \hat{\theta}_i \) is the dynamic parameter estimation error.

The dynamic behavior of the \( i \)-th robotic system can be described by

\[
\begin{align*}
\dot{x}_{o,i} &= -\sum_{j \in N_i} w_{ij} [x_{o,i} - x_{o,j}(t - T_{ij})] + s_{o,i}^* \\
\dot{s}_{o,i} &= -\alpha \int_0^t s^*_i(r)dr - \beta \Delta x_{o,i} - \lambda \int_0^t \Delta x_{o,i}(r)dr \\
\Delta \dot{x}_{o,i} &= -\beta \Delta x_{o,i} - \lambda \alpha \int_0^t \Delta x_{o,i}(r)dr + Z_i(q_i, \dot{q}_{i,r}) \Delta \dot{\theta}_i - J_i(q_i)s_i \\
\dot{\theta}_i &= -\Lambda_i Z_i^T(q_i, \dot{q}_{i,r}) \Delta x_{o,i} \\
M_i(q_i) s_i + C_i(q_i, \dot{q}_i) s_i &= -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{i,r}, \dot{\theta}_i) \Delta \dot{\theta}_i \\
\dot{\theta}_i &= -\Gamma_i Y_i^T(q_i, \dot{q}_{i,r}, \dot{q}_{i,r}, \dot{\theta}_i) s_i
\end{align*}
\]

where the upper four equations describe the kinematic loop and the lower two equations the dynamic loop. The interaction between the two loops is reflected in the term \(-J_i(q_i)s_i\) in the third equation of (17).

We are presently ready to formulate the following theorem.

Theorem 1: If \( \lambda > 0 \), the control law (12) and the parameter adaptation laws (13) and (14) for the \( n \) robotic systems interacting on directed graphs containing a directed spanning tree and subjected to finite constant communication delays ensure the manipulable
consensus of the n robotic systems, i.e., $\dot{x}_i \to 0$ and $x_i - x_j \to 0$ as $t \to \infty$, $\forall i, j = 1, 2, \ldots, n$ with $1/\alpha$ acting as the manipulability index, where the manipulable consensus means that the final consensus value can be adjusted by the external stimuli/input and the manipulability quantifies the degree of the adjustability of this value. In addition, if $\alpha > 0$, the task-space positions of the n robotic systems converge to the scaled weighted average value $[1/(1 + \sum_{i=1}^n \sum_{k \in K_i} w_k(T_{ik})) \sum_{i=1}^n \gamma_i(L_i(k))].$

Before presenting the proof of Theorem 1, we first state the following proposition concerning the input-output properties of marginally stable linear systems and extends the input-output properties (iBIBO) stability stated in [49] to the more general case.

**Proposition 1:** Consider a marginally stable and strictly proper linear time-invariant system $y = G(u)$ with a simple pole at the origin and all other poles in the open left half plane (LHP), where $u \in R^n$ and $y \in R^n$ denote the input and output, respectively. Then
1. if $\int_0^t u(r)dr \in L_2$, then $\omega(y(0), t) \in L_2$;
2. if $\int_0^t u(r)dr \in L_1$, then $\omega(y(0), t) \in L_1$;
3. if $\int_0^t u(r)dr \in L_\infty$, then $\omega(y(0), t) \in L_\infty$
where $\omega(y(0), t)$ is a bounded function in terms of time and the initial value $y(0)$.

**Proof:** The proof follows similar procedures as in [49]. The representation of the system in frequency domain can be written as $Y(p) = G(p)[U(p) + F(y(0))]$ with $p$ denoting the Laplace variable and $Y(p)$ and $U(p)$ the Laplace transforms of $y$ and $u$, respectively. We now rewrite the system as $Y(p) = G(p)[U(p)/p] + G(p)F(y(0))$ where $G(p)$ is a biproper function [30] and $U(p)/p$ is well known, is the Laplace transform of $\int_0^t u(r)dr$. The second part of $Y(p)$ denoted by $\omega(y(0), t) = G(p)F(y(0))$ is obviously bounded and additionally converges to some constant vector, according to the standard linear system theory. Furthermore, we have that the transfer function $pG(p)$ is exponentially stable since the simple zero pole of $G(p)$ is cancelled by the factor $p$ [51]. Then following similar analysis as in the proof of Corollary 3.3.2 in [50], we obtain that the time-domain counterpart of $pG(p)[U(p)/p]$ (i.e., $y(t) - \omega(y(0), t)$) is square-integrable if $\int_0^t u(r)dr \in L_2$. The conclusions 2) and 3) can be similarly derived.

**Proof of Theorem 1:** Following the standard practice (see, e.g., [52, 53]), we consider the Lyapunov-like function candidate for the fifth and sixth subsystems in (17) $V_i = (1/2)s_i^T M_i(q_i)s_i + (1/2)\Delta \dot{\theta}^T \Gamma^{-1} \Delta \theta$, and by exploiting Property 3, we obtain the time derivative of $V_i$ as $V_i = -\dot{s}_i^T K_i s_i \leq 0$, which yields the result that $s_i \in L_2 \cap L_\infty$ and $\dot{s}_i \in L_\infty$, $\forall i$.

Since $J_i(q_i)$ is obviously bounded, we obtain that $J_i(q_i)s_i \in L_2$. As a consequence, there exists a positive constant $l_{M,i}$ such that $\int_0^t s_i^T J_i(q_i)J_i(q_i) s_i(r)dr \leq l_{M,i}$, for all $t \geq 0$, $\forall i$. Then, we consider the following nonnegative function

$$V_i^* = \frac{1}{2} \Delta x_{ao,i}^T \Delta x_{ao,i} + \frac{1}{2} \left[ \int_0^t \Delta x_{ao,i}(r)dr \right]^T \left[ \int_0^t \Delta x_{ao,i}(r)dr \right]\n + \frac{1}{2\beta} \left[ l_{M,i} - \int_0^t s_i^T J_i(q_i)J_i(q_i) s_i(r)dr \right]$$

$$\n + \frac{1}{2\Delta \theta^T \Lambda_i^{-1} \Delta \theta},$$

where the adoption of the last term in $V_i^*$ follows the typical practice (see, e.g., [52, p. 118]) and is for taking into account the interaction between the kinematic and dynamic loops, $\forall i$. The time derivative of $V_i^*$ along the third and fourth subsystems in (17) can be shown to satisfy

$$\dot{V}_i^* \leq -\frac{\beta}{2} \Delta x_{ao,i}^T \Delta x_{ao,i} \leq 0, \forall i$$

where we use the following result that is derived from the standard theory of inequalities

$$\Delta x_{ao,i}^T J_i(q_i) \Delta x_{ao,i} \leq \frac{\beta}{2} \Delta x_{ao,i}^T \Delta x_{ao,i} + \frac{1}{2\beta} s_i^T J_i(q_i)J_i(q_i) s_i.$$

The inequality [19] as well as the definition of $V_i^*$ immediately leads us to obtain that $\Delta x_{ao,i} \in L_2 \cap L_\infty$ and that $\dot{V}_i^* \Delta x_{ao,i}(r)dr \in L_\infty$ if $\alpha > 0$, $\forall i$.

From the second subsystem in (17), we obtain by the Laplace transformation that

$$S_{ao,i}(p) \equiv -(\beta p + \lambda)/(p + \alpha) \Delta x_{ao,i}(p),$$

and this leads us to obtain that $S_{ao,i}^\omega \in L_2 \cap L_\infty$ according to the input-output properties of biproper linear systems [50, p. 82], $\forall i$. Applying Laplace transformation to the first subsystem in (17) yields $\dot{p}x_{ao,i}(p) - pX_{ao,i}(p) = -\sum_{j \in N_i} w_{ij}[X_{ao,i}(p) - e^{-T_{ij}p}X_{ao,i}(p)] + S_{ao,i}^\omega(p).$

(20)

By letting $\Phi_i(p) = pX_{ao,i}(p) - x_i(0)$ denote the Laplace transform of $x_{ao,i}, \forall i$, we have that

$$\Phi_i(p) = -\sum_{j \in N_i} w_{ij} \Phi_i(p) e^{-T_{ij}p} \Phi_j(p)$$

$$\n - \sum_{j \in N_i} \sum_{k \in K_i} w_{ij} \sum_{l \in K_j} w_{jk} x_{ao,k}(0) e^{-T_{ij}p} x_{ao,j}(0) + S_{ao,i}^\omega(p).$$

(21)

Stacking up all the equations like above with further manipulations gives

$$\Phi(p) = [G(p) \otimes I_m]\left[ -[\sum_{i} \Delta x_{ao,i}(p) + pS_{ao,i}^\omega(p)] \right]$$

$$\n \quad - \sum_{i} \sum_{j \in N_i} w_{ij} \sum_{k \in K_i} w_{ik} x_{ao,k}(0) e^{-T_{ij}p} x_{ao,j}(0) + \sum_{i} S_{ao,i}^\omega(p).$$

(22)

where $\sum_{i} \Delta x_{ao,i}(p) \otimes I_m$ being the time-domain counterpart of $\sum_{i} \Delta x_{ao,i}(p)$, the Laplace integral of this function is $\int_0^t \left[ \delta(t) \right] \left[ \delta(t) \right] dt = s_p^\omega(t)$ is uniformly continuous, $\forall i$. Hence, we have that $x_{ao,i} \in L_2 \cap L_\infty$ and $\dot{x}_{ao,i} \in L_\infty$, $\forall i$. From [56], we obtain that $J_i(q_i)$ is nonsingular, and thus $\dot{q}_i \in L_\infty$, $\forall i$. From (22), we obtain that $\dot{x}_i \in L_\infty$, $\forall i$. By the differentiation of $\dot{x}_i$, we obtain that $\dot{x}_{ao,i} \in L_\infty$, and therefore $\dot{x}_{ao,i} \in L_\infty$, which implies that $\dot{x}_{ao,i}$ and $\dot{x}_{ao,i}^\omega$ are both uniformly continuous, $\forall i$. According to the properties of square-integrable and uniformly continuous functions [51, p. 232], we obtain that $\dot{x}_{ao,i} \to 0$ and $\dot{x}_{ao,i}^\omega \to 0$ as $t \to \infty$, $\forall i$. From the first subsystem in (17), we immediately obtain that $\dot{s}_i \in L_\infty$ and $\dot{s}_i \in L_\infty$ implies that $\Delta x_{ao,i} \in L_\infty$, which means that $\Delta x_{ao,i}$ is uniformly continuous, $\forall i$. Then we have that $\dot{x}_{ao,i} \to 0$ as $t \to \infty$, $\forall i$. The result that $\dot{x}_{ao,i} \in L_\infty$ and $\dot{x}_i \in L_\infty$ implies that $\Delta x_{ao,i} \in L_\infty$, which means that $\Delta x_{ao,i}$ is uniformly continuous, $\forall i$. Hence, we have that $x_i - x_j \to 0$ as $t \to \infty$, $\forall i, j$.
and exploiting Property 2, we obtain that $s_i \in \mathcal{L}_\infty$ and further $q_i(0) \in \mathcal{L}_\infty$, $\forall i$. Based on the differentiation of (22), i.e., $\dot{x}_i = J_i(q_i)\dot{q}_i + J_i(q_i)\dot{q}_i$, we have that $\dot{x}_i \in \mathcal{L}_\infty$ and thus $\Delta \dot{x}_i \in \mathcal{L}_\infty$, which implies that $\Delta \dot{x}_i$ is uniformly continuous, $\forall i$. From Barbital’s Lemma [42], we have that $\Delta \dot{x}_i \rightarrow 0$ as $t \rightarrow \infty$ and thus $\dot{x}_i \rightarrow 0$ as $t \rightarrow \infty$, $\forall i$.

In the case that $\alpha > 0$, consider the following system

$$\dot{x}_i = -\sum_{j \in N_i} w_{ij} [x_{ij} - x_{ij}(t - T_{ij})] + s_{o,i}^*, \quad i = 1, 2, \ldots, n.$$  

(23)

First, we have from (13) that the system (23) with $s_{o,i}^*$, $i = 1, 2, \ldots, n$ as the input and $x_{ij}, i = 1, 2, \ldots, n$ as the output is integral-bounded-input bounded-output (iBIBO) stable in the sense of (29) and thus we obtain that $x_i \in \mathcal{L}_\infty$ since $\int_0^t s_{o,i}^*(r) dr \in \mathcal{L}_\infty$ from the second subsystem of (17), $\forall i$. This gives rise to the consequence that $x_i \in \mathcal{L}_\infty$ since $\Delta x_i \in \mathcal{L}_\infty$, $\forall i$.

We next illustrate that $1/\alpha$ measures the manipulability of the networked system responding to an external input or stimuli in the sense of certainty equivalence (i.e., assuming that the parameters are exactly known). As is shown above, the final consensus value depends tightly on the integral of $s_{o,i}^*$, $i = 1, \ldots, n$. Suppose that an external input $\tau_{n,i}$ is exerted on system $i$, which gives

$$\begin{align*}
\dot{x}_{n,i} &= -\sum_{j \in N_i} w_{ij} [x_{n,i} - x_{n,j}(t - T_{ij})] + s_{o,i}^*, \\
\ddot{x}_{n,i} &= -\alpha \int_0^t s_{o,i}^*(r) dr - \Delta x_{n,i} - \lambda \int_0^t \Delta \dot{x}_{n,i} dr, \\
\Delta \dot{x}_{n,i} &= -\Delta \dot{x}_{n,i} - \lambda \int_0^t \Delta \dot{x}_{n,i} dr - J_i(q_i)s_i, \\
M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i &= -K_i s_i + \tau_{n,i}.
\end{align*}$$

(24)

This system with $\tau_{n,i}$ as the input and $s_i$ as the output is bounded-input bounded-output stable. From the second and third subsystems of (23), it is obvious that the mapping from $J_i(q_i)s_i$ to $\int_0^t s_{o,i}^*(r) dr$ is bounded, and the scaling factor of this mapping depends tightly on $\alpha$ with a smaller factor, this scaling factor will be smaller, implying a smaller variation of $\int_0^t s_{o,i}^*(r) dr$. On the other hand, the modifiable range of the integral $\int_0^t s_{o,i}^*(r) dr$ measures the manipulability of the system. Therefore, the value of $1/\alpha$ is indeed a qualified measure of the manipulability of the system.

In the case that $\alpha > 0$, we obtain from the third subsystem of (17) that $-\lambda \int_0^t \Delta \dot{x}_{n,i} dr + Z_i(q_i, \dot{q}_i)\Delta \dot{x}_{n,i} \rightarrow 0$ as $t \rightarrow \infty$, $\forall i$. Considering the fact that $\dot{q}_i$ given by 6 can be rewritten as [using (7)]

$$\dot{q}_{r,i} = J_i^{-1}(q_i) \left[ \dot{x}_{r,i} + \beta \Delta x_{n,i} + \lambda \int_0^t \Delta \dot{x}_{n,i}(r) dr \right]$$

(25)

and that $\dot{x}_{r,i} \rightarrow 0$ and $\Delta \dot{x}_{n,i} \rightarrow 0$ as $t \rightarrow \infty$, we obtain that $\lambda J_i(q_i)J_i^{-1}(q_i) \int_0^t \Delta \dot{x}_{n,i}(r) dr \rightarrow 0$ as $t \rightarrow \infty$, $\forall i$. If $J_i(q_i)$ and $J_i(q_i)$ are nonsingular, we obtain that $\int_0^t \Delta \dot{x}_{n,i}(r) dr \rightarrow 0$, $\forall i$. Based on the second subsystem of (17), we can directly obtain by the standard final value theorem that $\int_0^t \Delta \dot{x}_{n,i}(r) dr \rightarrow 0$ as $t \rightarrow \infty$. Using the result in (13), we obtain from (25) that $x_{r,i} \rightarrow [1/(1 + \sum_{k=1}^n \sum_{j \in N_i(q_i) \setminus \{k\}} \omega_{kj} T_{kj})] \sum_{k=1}^n \gamma_k x_{r,k}(0)$ and further that $x_i \rightarrow [1/(1 + \sum_{k=1}^n \sum_{j \in N_i(q_i) \setminus \{k\}} \gamma_k \omega_{kj} T_{kj})] \sum_{k=1}^n \gamma_k x_{r,k}(0)$ as $t \rightarrow \infty$, $\forall i$.

Remark 2: If we set $\lambda = 0$ [i.e., removing the integral action of the observation error in (7) and $\alpha > 0$, it can also be shown that the task-space positions of the robotic systems converge to the scaled weighted average value $[1/(1 + \sum_{k=1}^n \sum_{j \in N_i(q_i) \setminus \{k\}} \omega_{kj} T_{kj})] \sum_{k=1}^n \gamma_k x_{r,k}(0)$, by following the proof of Theorem 1 in (13). Nevertheless, the asymptotic consensus of the systems can possibly no longer be maintained under an external stimuli (e.g., a task-space PD(proportional-derivative)-like input) due to the absence of the integral action of the observation error, and in fact there would generally exist a steady-state observation error. On the other hand, if we set $\lambda = \alpha = 0$, the asymptotic consensus of the systems under an external task-space PD-like stimuli can be recovered, but we can no longer ensure that the task-space positions of the robotic systems converge to a constant value. Furthermore, our result relies on the condition that the estimated Jacobian matrix $J_i(q_i)$ is nonsingular in the kinematic parameter adaptation process, $\forall i$. This can be ensured by the assumption that the manipulator is away from the singular configuration and the use of the parameter projection (see, e.g., 23).

(24), (56).

Remark 3: The proposed scheme requires the communication between the robotic systems. In this context, one may attempt to employ the existing distributed-observer-based schemes (e.g., 46, 48) where the consensus of the communicated quantities is completely separated from the dynamics of the robotic systems and the objective of each robotic system is to unidirectionally track the corresponding communicated quantity. The main limitations of this strategy may lie in the fact that each robotic system is unidirectionally coupled to the virtual consensus system, giving rise to the consequence that the virtual consensus system is independent of the robotic systems and that the robotic systems are actually not coupled with each other. This renders the consensus behavior unresponsive to any external physical command (e.g., in the scenario that one robotic system is tuned to a static position by a human operator, the other systems, however, will not yield any tendency of trying to achieve consensus with this system since each of them is actually only coupled to the artificial communicated quantity). Furthermore, this strategy cannot naturally give rise to the separation of the kinematic and dynamic loops of the robotic systems and the avoidance of the task-space velocity measurement. The proposed distributed-observer-based adaptive control, by introducing a feedback coupling action $-\beta [x_{n,i} - x_i] - \lambda \int_0^t [x_{r,i} - x_i(r)] dr$, results in the manipulability of the robotic systems (i.e., the consensus behavior is responsive to external physical manipulation), in contrast with the unresponsive behavior of the existing distributed-observer-based algorithms mentioned above, and simultaneously avoids the task-space velocity measurement. An important application of the manipulability consensus is the manipulation of two manipulators, i.e., the master and slave where the master is typically manipulated by a human operator and the slave aims to maintain consensus with the master (see, e.g., 57).

Remark 4: Most leader-based schemes and general consensus schemes without using the distributed observers as in 45, 0. 48 in the literature can also achieve the manipulable consensus introduced here, e.g., the consensus schemes for identical single-integrator systems in 10 and the consensus schemes without or with the integral action of the sliding vector (defined as the weighted sum of the velocity and neighbor-to-neighbor position consensus errors) for uncertain nonidentical Euler-Lagrange systems or robotic systems (e.g., 5, 13, 9). The main contribution of the study here is to formalize the concept of manipulability and show that the gain of the integral action concerning the sliding vector acts as a qualified manipulability measure of the closed-loop system, and to address the case of no task-space velocity measurement by developing a new task-space observer which is more robust than the existing ones.

IV. ADAPTIVE KINEMATIC CONTROL

In most industrial/commercial robots, the available design input is the joint velocity (or position) rather than the joint torque and in fact the torque control is hidden from the user (see, e.g., 30). The existing task-space consensus controllers (e.g., 29, 38) cannot be applied to this category of robots due to the dependence on the modification of the low-level feedback controller architecture. The separation property of the proposed dynamic controller in Sec. III
allows us to conveniently obtain an adaptive kinematic controller that is applicable to robots having an unmodifiable joint servoing module yet admitting the design of the joint velocity (or position) command. Specifically, we have the following theorem.

**Theorem 2:** Suppose that the low-level joint servoing controllers for the $n$ robotic systems can ensure that the joint velocity tracking error $s_i = \dot{q}_i - \dot{q}_i, i \in \mathcal{L}\cap\mathcal{L}_H$, $\forall i = 1, 2, \ldots, n$ and $\lambda > 0$ and that there is no task-space and joint-space velocity measurement. Then the adaptive kinematic controller given by (14) for the $n$ robotic systems on directed graphs containing a directed spanning tree ensures the manipulable consensus of the $n$ robotic systems, i.e., $\dot{x}_i \to 0$ and $x_i - x_j \to 0$ as $t \to \infty$, $\forall i, j = 1, 2, \ldots, n$ with $1/\alpha$ acting as the manipulability index. In addition, if $\alpha > 0$, the task-space positions of the $n$ robotic systems converge to the scaled weighted average value $[1/(1 + \sum_{k=1}^{\infty} \sum_{\ell \in \mathcal{N}_k} \gamma_k w_{k\ell}(T_k))] \sum_{k=1}^{\infty} \gamma_k x_{o,k}(0)$.

**Proof:** The proof can be completed by following similar steps as in the proof of Theorem 1. In fact, choose $V^*_i$ in (13) as the quasi-Lyapunov function candidate and it can be directly shown that the derivative of $V^*_i$ along the trajectories of the third and fourth subsystems in (17) satisfies the inequality $\dot{V}^*_i \leq -((\beta/2)\Delta x^2_{i,j}, \Delta x_{o,i} \leq 0$. Then, it can be shown that the manipulable consensus of the $n$ robotic systems is indeed realized, using a procedure similar to that in the proof of Theorem 1.

**Remark 5:** The result of Theorem 2 demonstrates that for robotic systems having an unmodifiable joint servoing controller yet allowing the design of the joint velocity (or position) command (e.g., most industrial/commercial robots), it is also possible to achieve consensus in the presence of kinematic uncertainties and absence of task-space and joint-space velocity measurement and without directly involving task-space position measurement (implying enhanced robustness since $x_{o,i}$ can be considered as a filtered quantity of $x_i$ and $\theta_i$ as a quantity yielded by an integration concerning $x_i$). The adaptive kinematic controller here, in contrast with (11), does not directly involve the task-space position due to the introduced new task-space observer. In addition, we see certain module-like properties of the proposed adaptive kinematic controller in the sense that the joint servoing controller is merely demanded to guarantee the square-integrability and boundedness of the joint velocity tracking error (i.e., $s_i \in \mathcal{L}\cap\mathcal{L}_H, \forall i$). Obviously, the adaptive joint servoing controller given by (14) and (13) is only a special case. The requirement that the joint velocity tracking error is square-integrable may also be interpreted as “fast enough” servoing in the engineering sense.

**V. SIMULATION RESULTS**

In this section, we provide numerical simulation results to demonstrate the performance of the proposed adaptive schemes using six standard two-DOF (degree-of-freedom) planar robots moving in the horizontal X-Y plane. The interaction graph among the robots is shown in Fig. 1. The sampling period is chosen as 5 ms.

**A. Dynamic Controller**

We first consider the dynamic controller given in Sec. III. The entries of the weighted adjacency matrix $W$ are chosen as $w_{ij} = 0.5$ if $j \in \mathcal{N}_i$, and $w_{ij} = 0$ otherwise. The communication delays among the robots, for simplicity, are set to be $T_{ij} = 0.5$ s, $\forall j \in \mathcal{N}_i, \forall i = 1, 2, \ldots, 6$. The controller parameters $K_i$, $\Gamma_i$, $\Lambda_i$, $\alpha$, and $\beta$ are chosen as $K_i = 30.0I_2$, $\Gamma_i = 10.0I_3$, $\Lambda_i = 10.0I_2$, $\alpha = 10.0$, $\beta = 10.0$, and $\lambda = 25.0$, respectively, $i = 1, 2, \ldots, 6$. The initial dynamic parameter estimates are determined as $\hat{\theta}_i(0) = [0, 0, 0]^T$, $\forall i = 1, 2, \ldots, 6$. The initial kinematic parameter estimates are determined as $\hat{\theta}_1(0) = [1.5, 2.5]^T$, $\hat{\theta}_2(0) = [3.2, 3.2]^T$, $\hat{\theta}_3(0) = [2.6, 2.8]^T$, $\hat{\theta}_4(0) = [3.2, 2.7]^T$, $\hat{\theta}_5(0) = [3.5, 2.9]^T$, and $\hat{\theta}_6(0) = [1.3, 2.8]^T$. The initial values of the observed quantities are set as $x_{o,i}(0) = x_i(0) + 0.02$, $i = 1, 2, \ldots, 6$.

**B. Dynamic Controller With an External Input/Stimuli**

Suppose that the 1st robot is subjected to an external input $\tau_{0,1} = J_i^T(q_1)[-15\dot{x}_1 - 30(x_1 - x_h)]$ after $t = 10$ s with $x_h = [2.6, 0.9]^T$ denoting the desired position. Under the same context and with the same controller parameters, the task-space positions of the robotic...
systems are shown in Fig. 4 (X-axis) and their X-axis positions finally stays at 2.1111 rather than 2.6. To increase the system manipulability, we decrease $\alpha$ from $\alpha = 10.0$ to $\alpha = 0.05$. The corresponding simulation results are shown in Fig. 5 and the X-axis positions of the robots finally stays at 2.5327, which is much closer to 2.6 in comparison with the case $\alpha = 10$. In the extreme case $\alpha = 0$, the responses of the task-space positions of the robots are shown in Fig. 6 and the final positions of the robots (2.5991) are very close to the the desired one $x_h^{(1)} = 2.6$.

### C. Kinematic Controller

In this last scenario, we suppose that a PI (proportional-integral) velocity controller is embedded in the six robots (similar to most industrial robots) with their PI gains being set as $K_P = 60.0I_3$ and $K_I = 10.0I_3$. Under the proposed kinematic controller in Sec. IV with the parameters being chosen to be the same as those in Sec. V-A except that $\alpha = 0$, the simulation results are shown in Fig. 7, and the X-axis task-space positions of the robots finally stays at 2.3759. This difference from the case of using a dynamic controller (2.5991) is due to some nonlinear terms which are not considered in the kinematic controller.

### VI. Conclusion

In this paper, we have studied the task-space consensus problem for multiple robotic systems with kinematic and dynamic uncertainties in the case of existence of constant communication delays. We propose an observer-based adaptive consensus scheme to achieve the manipulable consensus objective without relying on the task-space velocity measurement. The main new features of our work are that 1) the observer relies on the joint reference velocity rather than the joint velocity, 2) the kinematic parameter adaptation law uses a distributed adaptive kinematic regressor matrix and is driven by both the observation and consensus errors, 3) the separation of the kinematic and dynamic subsystems is achieved, and 4) the concept of manipulability is formalized and the manipulable consensus of the robotic systems is ensured. The performance of the proposed adaptive schemes is shown by numerical simulation results.

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### References

[1] A. Rodriguez-Angeles and H. Nijmeijer, “Mutual synchronization of robots via estimated state feedback: A cooperative approach,” IEEE Transactions on Control Systems Technology, vol. 12, no. 4, pp. 542–554, Jul. 2004.

[2] N. Chopra and M. W. Spong, “Passivity-based control of multi-agent systems,” in Advances in Robot Control: From Everyday Physics to Human-Like Movements, S. Kawamura and M. Svinin, Eds. Berlin, Germany: Springer-Verlag, 2006, pp. 107–134.

[3] L. Cheng, Z.-G. Hou, and M. Tan, “Decentralized adaptive consensus control for multi-manipulator system with uncertain dynamics,” in Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics, Singapore, 2008, pp. 2712–2717.
[54] R. Lozano, B. Brogliato, O. Egeland, and B. Maschke, Dissipative Systems Analysis and Control: Theory and Applications. London, U.K.: Springer-Verlag, 2000.

[55] J. W. Brewer, “Kronecker products and matrix calculus in system theory,” IEEE Transactions on Circuits and Systems, vol. CAS-25, no. 9, pp. 772–781, Sep. 1978.

[56] W. E. Dixon, “Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics,” IEEE Transactions on Automatic Control, vol. 52, no. 3, pp. 488–493, Mar. 2007.

[57] P. F. Hokayem and M. W. Spong, “Bilateral teleoperation: An historical survey,” Automatica, vol. 42, no. 12, pp. 2035–2057, Dec. 2006.