Structure of Neutron Star with a Quark Core

G.H. Bordbar ‡ M. Bigdeli and T. Yazdizadeh

Department of Physics, Shiraz University, Shiraz 71454, Iran

and

Research Institute for Astronomy and Astrophysics of Maragha,
P.O. Box 55134-441, Maragha, Iran

Abstract

The equation of state of de-confined quark matter within the MIT bag model is calculated. This equation of state is used to compute the structure of a neutron star with quark core. It is found that the limiting mass of the neutron star is affected considerably by this modification of the equation of state. Calculations are carried out for different choices of the bag constant.

---

*Corresponding author
†E-mail: Bordbar@physics.susc.ac.ir
‡Permanent address
1 Introduction

Neutron stars (NS) are among the densest of massive objects in the universe. They are ideal astrophysical laboratories for testing theories of dense matter physics and provide connections among nuclear physics, particle physics and astrophysics. The maximum mass of a neutron star is a subject that several theoretical astrophysicists have tried to compute it. Below a certain maximum mass, degeneracy pressure prevents an object collapse into a black hole. To calculate the maximum mass, we require enough information about the composition of the star. Different compositions lead to different equations of state (EOS). When nuclear matter is compressed to densities so high that the nucleon cores substantially overlap, one expects the nucleons to merge and undergo a phase transition to de-confined quark matter. Such a system could be realized in two possible ways: (a) complete strange quark matter stars (b) neutron stars with a core of quark matter. Glendenning [1] has shown that a proper construction of the nucleon-quark phase transition inside neutron stars implies the coexistence of nucleon matter and quark matter over a finite range of pressure. This has the effect that a core, or a spherical shell, of a mixed quark-nucleon phase can exist inside neutron stars.

In this work, we calculate the structure of the neutron star with a quark core and also strange star, and compare our results with our previous works in which we investigated the NS structure without a quark core [2].

2 Quark Matter Equation of State

For the de-confined quark phase, within the MIT bag model [3], the total energy density is the sum of a non-perturbative energy shift $B$ (the bag constant) and the kinetic energy for non-interacting massive quarks of flavors $f$ with mass $m_f$ and Fermi momentum
\[ k_F^{(f)} = (\pi^2 \rho_f)^{1/3} \] where \( \rho_f \) is the quarks density of flavor f:

\[
\varepsilon_Q = \frac{3 m^4 c^5}{8 \pi^2 \hbar^3} \left[ x \sqrt{x^2 + 1} (2x^2 + 1) - \sinh^{-1} x \right] + \frac{3 \hbar c}{2 \pi^2} \left( \pi^2 \rho \right)^{1/3} + B, \tag{1}
\]

where \( x = \frac{\hbar k_F}{mc} \), \( \rho_s = \rho_d = \rho_u = \rho \), \( \rho \) is baryon density, and \( \varepsilon_Q = E/V \). We assume in this work that u and d quarks are massless and the s quark has a mass equal to \( m = 150 MeV \). The bag constant \( B \), can be interpreted as the difference between the energy densities of the non-interacting quarks and interacting ones, which has a constant value such as \( B = 55 \) and \( 90 MeV \) in the initial model of MIT. Inclusion of perturbative interaction among quarks introduces additional terms in the thermodynamic potential [4]. We try to determine a range of possible values for \( B \) by using the experimental data obtained at the CERN SPS [5].

According to the analysis of those experiments, the quark-hadron transition takes place at about seven times normal nuclear matter energy density (\( \varepsilon_0 = 156 MeV fm^{-3} \)).

We assume a density dependent \( B \). In the literature there are attempts to understand the density dependence of \( B \) [6, 7, 8]; however, currently the results are highly model dependent and no definite picture has come out yet. Therefore, we attempt to provide effective parameterizations for this density dependence. Our parameterizations are constructed in such a way that at asymptotic densities \( B \) has some finite value \( B_\infty \):

\[
B(\rho) = B_\infty + (B_0 - B_\infty) \exp \left[ -\beta \left( \frac{\rho}{\rho_0} \right)^2 \right]. \tag{2}
\]

The parameter \( B_0 = B(\rho = 0) \) has constant which is assumed to be \( B_0 = 400 \) in this work, and \( \beta \) is numerical parameter usually equal to \( \rho_0 \approx 0.17 fm^{-3} \), the normal nuclear matter density. \( B_\infty \) depends only on the free parameter \( B_0 \). In order to fix \( B_\infty \), we proceed in the following way:

Firstly, we use the equation of state (EOS) of asymmetric hadronic matter characterized by a proton fraction \( x_p = 0.4 \) and the \( UV_{14} + TNI \) potential. By assuming that the
hadron-quark transition takes place at the energy density $\varepsilon = 1100 \text{MeV fm}^{-3}$, we find that hadronic matter baryon density is $\rho_t = 0.98 \text{fm}^{-3}$ (transition density) and at values lower than it the quark matter energy density is higher than that of nuclear matter, while with increasing baryon density the two energy densities become equal at this density and after that the nuclear matter energy density remains always higher. Eq. (1) for quark matter with two flavors u and d reduces to:

$$
\varepsilon_Q = \frac{3hc}{4\pi^\frac{4}{3}} \left[ \rho_u^\frac{4}{3} + \rho_d^\frac{4}{3} \right] + B, \tag{3}
$$

where $\rho_d = 2\rho_u = 2\rho$.

Secondly, we determine $B_\infty = 8.99$ by putting quark energy density and hadronic energy density equal to each other ($\varepsilon_Q = \varepsilon|_{\rho=\rho_t}$).

Finally, we calculate the EOS for the three flavors quark matter using

$$
P = \rho \frac{\partial \varepsilon_Q}{\partial \rho} - \varepsilon_Q. \tag{4}
$$

3 Mixed Phase

The hadron-quark phase transition takes place within a range of baryon density values. In other words, the fraction of space occupied by quark matter smoothly increases from zero to unity when eventually the last nucleons dissolve into quarks. In this case, we have a mixture of the hadron, quark and electron background in the system. Glendenning’s construction [1] describes a global division of the baryon number between the two phases. The equilibrium conditions, in the case where the geometry of droplets is neglected, are those for bulk systems. The neutron star matter is assumed to be stable and charge neutral. Thermal effects are not expected to play any important role in neutron star cores. We neglect such effects and put the temperature $T = 0$. The equilibrium conditions for the quark matter droplet to coexist with the nucleon medium are that pressure and chemical
potentials in both phases coincide. We choose pressure as an independent variable. The coexistence requires that (Gibbs conditions):

$$\mu^N_n(P) = \mu^N_q(P),$$

(5)

and

$$\mu^P_n(p) = \mu^P_q(P),$$

(6)

where $\mu^N_n$ and $\mu^N_q$ are the neutron chemical potential in the nucleon phase (NP) and the quark phase (QP), respectively. Similarly, $\mu^P_n$ and $\mu^P_q$ are the proton chemical potential in the respective phases. The strange quark and lepton chemical potentials are dictated by the conditions of weak equilibrium

$$\mu_s = \mu_d,$$

(7)

and

$$\mu_d - \mu_u = \mu_l,$$

(8)

where $\mu_l$ is lepton chemical potential.

Using the semi-empirical mass formula, the energy per particle of nuclear matter can be expressed as

$$E(\rho, x) = T(\rho, x) + V_0(\rho) + (1 - 2x)^2V_2(\rho),$$

(9)

where $x = \rho_p/\rho$ is proton fraction. The kinetic energy contribution is

$$T(\rho, x) = \frac{3}{5} \frac{\hbar^2}{2m} \left(3\pi^2 \rho\right)^\frac{2}{3} \left[(1 - x)\frac{x}{2} + x\frac{5}{2}\right].$$

(10)

The functions $V_0$ and $V_2$ represent the interaction energy contributions and we can determine them from the results of symmetric nuclear matter ($x = \frac{1}{2}$) and pure neutron matter ($x = 0$) [9]. Using our results for the nuclear matter with the $UV_{14} + TNI$ potential, we have obtained the following fit for $V_0$ and $V_2$:

$$V_0(\rho) = -559.9\rho^5 + 1695.62312\rho^4 - 1946.86437\rho^3 + 1327.04\rho^2 - 411.57428\rho - 0.30327.$$
From Eqs. (9) and (10), we obtain the chemical potentials of neutrons and protons as:

\[
\mu_N(p) = \frac{\hbar^2}{2m} (3\pi^2 \rho) \frac{\dot{\varphi}}{\varphi} [(1 - x)\frac{\dot{\varphi}}{\varphi} + x(1 - x)\frac{\dot{\varphi}}{\varphi}] + V_0(\rho) + \rho V_0'(\rho)
\]

\[
+ (1 - 2x)^2 \rho V_2'(\rho) + (1 - 4x^2) V_2(\rho) + mc^2,
\]

and

\[
\mu_P(p) = \frac{\hbar^2}{2m} (3\pi^2 \rho) \frac{\dot{\varphi}}{\varphi} [x\frac{\dot{\varphi}}{\varphi} + (1 - x)\frac{\dot{\varphi}}{\varphi}] + V_0(\rho) + \rho V_0'(\rho)
\]

\[
+ (1 - 2x)^2 \rho V_2'(\rho) + (-3 + 8x - 4x^2) V_2(\rho) + mc^2.
\]

The quark chemical potential with flavor \( f \) is

\[
\mu_f = \left[ m_f^2 c^4 + \frac{\hbar^2}{2m} c^2 (\pi^2 \rho f)^\frac{\dot{\varphi}}{\varphi} \right] \frac{\dot{\varphi}}{\varphi}.
\]

From Eqs. (1), (4), (7) and (15), we obtain the chemical potential of quark matter:

\[
\mu_u = \left( 4\pi^2 \hbar^3 c^3 \left[ P + B + D - \rho \left( \frac{\partial D}{\partial \rho} + \frac{\partial B}{\partial \rho} \right) \right] - \mu_d^4 \right) \frac{\dot{\varphi}}{\varphi},
\]

where \( D = \frac{3 m_d^4 c^5}{8 \pi^2 \hbar^3} \left[ x \sqrt{x^2 + 1} (2x^2 + 1) - \sinh^{-1} x \right] \). We also have

\[
\mu_N^q = 2\mu_d + \mu_u,
\]

and

\[
\mu_P^q = 2\mu_u + \mu_d.
\]

By plotting \( \mu_P \) versus \( \mu_N \) for both nucleon and quark phases, we can identify the cross point of two curves that satisfy the Gibbs condition. As the chemical potentials determine
the charge densities of the two phases, the volume fraction occupied by quark matter, $\chi$, can be obtained by exploiting the requirement of global charge neutrality:

$$\chi \rho_q^e + (1 - \chi) \rho_p - \rho_e = 0, \quad (19)$$

where $\rho_q^e$ is the quark charge density. The total energy density and density of mixed phase (MP) are given by:

$$\epsilon_{MP} = \chi \epsilon_{QP} + (1 - \chi) \epsilon_{NP}, \quad (20)$$

and

$$\rho_{MP} = \chi \rho_{QP} + (1 - \chi) \rho_{NP}. \quad (21)$$

We plot pressure versus baryon density for hadron, mixed and quark phases in Figures 1 and 2, for bag constants $B = 90$ and density dependent $B$, respectively. It is seen that there is a mixed phase at a range of densities. A pure quark phase is presented at densities beyond this range and a pure hadronic phase is presented at densities below it.

## 4 Structure of Neutron Star with a Quark Core

We calculate the structure of a neutron star for various values of the central mass density, $\epsilon_c$, by using the equation of state and numerically integrating the general relativistic equation of hydrostatic equilibrium, Tolman-Oppenheimer-Volkoff (TOV) equation[10]. The derivation of TOV-equation can be found in standard textbooks [11, 12, 13, 14].

For a neutron star with a quark core, we use the following equations of state:

- Below the density $0.05 f m^{-3}$, we use the equation of state calculated by Baym et al. [15].

- From this density up to the beginning point of the mixed phase, we use the equation of state which is calculated with the $UV_{14} + TNI$ potential [2].
• For the range of densities in the mixed phase, we use the equation of state which was calculated in the previous section.

• Beyond the end point of the mixed phase, we use the equation of state of pure quark matter which was calculated in section 2.

Calculations are done both for constant $B = 90$, and density dependent $B$. Based on these EOS’s, we calculate the mass and radius of the NS with quark core. The calculations are also repeated for the strange star (i.e. pure quark matter). We plot the NS mass versus central mass energy density for $B=90$ and density-dependent $B$, in Figures 3 and 4, respectively. The NS mass versus radius for quark core NS and strange star are plotted for $B=90$ and density-dependent $B$ in Figures 5 and 6, respectively. For the sake of comparison, we have also plotted our previous results of the neutron star structure without quark core, in these figures. It is seen that there is a profound difference between the new results for NS with a quark core and those of NS without a quark core.

The extracted maximum mass of a NS and the corresponding radius and central mass density for both cases $B=90$ and density dependent $B$ are presented in Tables 1 and 2, respectively. It is seen that the inclusion of the quark core leads to a considerable reduction of the maximum mass, while the radius is not affected appreciably. Note that the maximum mass for the NS with quark core and $B=90$ is quite near to the observed maximum mass of neutron stars [16].

The maximum mass energy density versus the radial coordinate for NS without core, NS with a quark core and strange star are plotted in Figures 7 and 8 for $B=90$ and density-dependent $B$, respectively. It can be seen that a major part of the core is composed of pure quark matter (about 8 Km). A layer of mixed phase (thickness about 1.5 Km) exists between the core and a thin crust.
5 Summary

As we go from the center toward the surface of a neutron star, the state of baryonic matter changes from the de-confined quark-gluon to a mixed state of quark matter and hadronic matter, and thin crust of hadronic matter. The transition between these states occurs in a smooth way. In order to calculate the structure and the mass limit of neutron stars, it is important to have a fairly accurate physical description of these states.

In this paper, we calculated the equation of state of the de-confined quark phase within the MIT bag model. We then calculated the mixed phase of nucleons and quarks. The equilibrium volume fractions of nucleon and quark matter were obtained by applying the Gibbs condition. Curves were presented which showed the dependence of pressure on the baryon density.

Our results for the equation of state were then used to calculate the structure of a neutron star with a quark core. As usual, the Tolman-Oppenheimer-Volkoff equation were integrated from the center to the surface of the neutron star where the density drops to zero. Calculations were carried out both for $B=90$ and a density-dependent $B$.

The maximum mass, radius, and central mass density of neutron stars with a quark core and strange star were calculated and compared with the traditional neutron star. It was found that the limiting mass decreases when the quark core is taken into account. This brings the maximum mass closer to the observational limits.

Acknowledgements

This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha, and Shiraz University Research Council.
References

[1] N. K. Glendenning, Phys. Rev. D46 (1992) 1274.

[2] G. H. Bordbar, and M. Hayati, Int. J. Mod. Phys. A21 (2006) in press.

[3] A. Chodos, R. L. Jaff, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D9 (1974) 3471.

[4] E. Fahri, and R. L. Jeff, Phys. Rev. D30 (1984) 2379.

[5] U. Heinz, and M. Jacobs, nucl-th/0002042
   U. Heinz, hep-ph/0009170

[6] C. Adami, and G. E. Brown, Phys. Rep. 234 (1993) 1.

[7] xue-min Jin and B. K. Jenning, Phys. Rev. C55 (1997) 1567.

[8] D. Blaschke,H. Grigorian, G. Poghosyan, C. D. Roberts, and S. Schmidt, Phys. Lett. B450 (1999) 207.

[9] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys A369 (1981) 470.

[10] S. Shapiro and S. Teukolsky, Blak Holes, White Dwarfs and Neutron Stars, (Wiley, New york, 1983).

[11] N. K. Glendenning, Compact Stars-Nuclear Physics, Particle Physics, and general Relativity, (Springer, New York, 2000).

[12] F. Weber, Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics, (Institute of Physics, Bristol, 1999).

[13] R. Adler, M. Bazin, and M. Schiffer, Introduction to General Relativity (McGraw-Hill, New York, 1965).
[14] C. W. Misner, K. S. Theorine, and J. A. Wheeler, Gravitation (W. H. Freeman Company, New York, 1973).

[15] G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170 (1971) 299.

[16] S.E. Thorsett and D. Chakrabarty, Astrophys. J. 512, 288 (1999).
Table 1: Maximum gravitational mass \( M_{\text{max}} \), corresponding radius(R) and central mass density\( \varepsilon_c \) for \( B = 90 \).

| star             | \( M_{\text{max}}(M_\odot) \) | R(Km) | \( \varepsilon_c(10^{14}\text{gr/cm}^3) \) |
|------------------|-------------------------------|-------|---------------------------------|
| NS               | 1.98                         | 9.81  | 27.17                           |
| NS+quark core    | 1.57                         | 9.73  | 33.27                           |
| stange star      | 1.34                         | 7.77  | 34.81                           |

Table 2: Maximum gravitational mass \( M_{\text{max}} \), corresponding radius(R) and central mass density\( \varepsilon_c \) for density dependent \( B \).

| star             | \( M_{\text{max}}(M_\odot) \) | R(Km) | \( \varepsilon_c(10^{14}\text{gr/cm}^3) \) |
|------------------|-------------------------------|-------|---------------------------------|
| NS               | 1.98                         | 9.81  | 27.17                           |
| NS+quark core    | 1.75                         | 9.66  | 28.92                           |
| stange star      | 1.63                         | 8.2   | 28.92                           |
Figure 1: The pressure versus baryon density for Hadron Phase (solid line), Mixed Phase (dashed line) and Quark Phase (dotted line) for $B=90$. 
Figure 2: The pressure versus baryon density for Hadron Phase (solid line), Mixed Phase (dashed line) and Quark Phase (dotted line) for density dependent B.
Figure 3: The gravitational mass versus central mass density for different cases with bag constant $B=90$. 
Figure 4: The gravitational mass versus central mass density for different cases with density dependent $B$. 
Figure 5: The mass-radius relation for different cases with bag constant $B=90$.

Figure 6: The mass-radius relation for different cases with density dependent $B$. 

Figure 7: Mass density as a function of radial coordinate for neutron star (dotted line) neutron star with quark core (solid curve) and strange star (dashed curve) with $B=90$. 
Figure 8: Mass density as a function of radial coordinate for neutron star (dotted line) neutron star with quark core (solid curve) and strange star (dashed curve) with density dependent B.