Previous research has shown that the peak of the quarkonium entropy at the deconfinement transition would be related to the entropic force which induces the melting of quarkonium. In this article, we study the effect of backreaction on the entropic force in a strongly coupled plasma of adjoint matter. The backreaction covered here comes from the presence of static heavy quarks evenly distributed over such a plasma. It is found that the inclusion of backreaction increases the entropic force thus enhancing the quarkonium dissociation, in accord with the findings of the imaginary potential.

PACS numbers: 11.25.Tq, 11.15.Tk, 11.25-w

I. INTRODUCTION

It is believed that the high energy heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) have created a strongly coupled hot quark-gluon plasma (QGP) [1–3]. One of the main experimental signatures for QGP formation is quarkonium dissociation [4]. It is expected to be produced during the early stages of the collisions and gives us important information about the entire evolution of QGP. e.g., it was predicted that the quarkonium would be suppressed due to the Debye screening induced by the high density of color charges in QGP. But recently, the experimental study on charmonium ($c\bar{c}$) shows a puzzle: the $c\bar{c}$ suppression at the RHIC (lower energy density) seems to be stronger than that at LHC (larger energy density) [5, 6]. Obviously, this is in contradiction with the Debye screening as well as the thermal activation through the impact of gluons [7, 8]. To explain this, some scholars argued [9, 10] that the recombination of the produced charm quarks into charmonium may be a solution. In particular, if a region of deconfined quarks and gluons is formed, the charmonium could be formed from a charm and an anticharm which were originally produced in separate incoherent interactions.

However, D. Kharzeev has recently argued [11] that the puzzle on the suppression of $c\bar{c}$ may be related to the nature of deconfinement. Specifically, the peak of the quarkonium entropy at the deconfinement transition could be related to the entropic force which induces the dissociation of quarkonium. This argument is based on the Lattice results [12–15] showing a large amount of entropy $S$ associated with the heavy quark-antiquark pair ($q\bar{q}$) around the crossover region of QGP. In the proposal of [11], this entropy gives rise to the entropic force

$$F = T \frac{\partial S}{\partial L},$$

where $T$ is the temperature of the plasma and $L$ denotes the interquark distance of $q\bar{q}$. It should be noted that this force does not describe any additional interactions; instead, it is an emergent force that originates from multiple interactions, driving the system towards the state with a larger entropy. The entropic force was introduced for the first time in [16] to explain the elasticity of polymer strands in rubber and recently argued [17] to be responsible for gravity. Here we will not go into detail about these points and restrict our discussion to its application in quarkonium dissociation.

AdS/CFT [18–20], namely the duality between the type IIB superstring theory formulated on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) in four dimensions, provides a new method for studying different aspects of QGP (see [21] for a good review with many phenomenological applications). Using AdS/CFT, K. Hashimoto and D. Kharzeev first studied the entropic force in $\mathcal{N} = 4$ SYM plasma at finite temperature [22]. Since then, this quantity has been discussed in various holographic models, e.g., for moving quarkonium [23], with chemical potential [24] and higher derivative corrections [25]. Other related results can be found in [26–29].

The goal of this paper is to study the effect of backreaction on the entropic force. As is well known, the QGP produced in experiments is comprised of a large amount of free quarks and gluons. That means if one analyzes the dynamics of a quark or quarkonium, the effect of other heavy quarks due to the backreaction of the plasma may have to be considered. However, with rare exceptions, e.g., [30, 31], it remains hard to treat strongly coupled gauge theory
with a large number of flavor quarks. Because of this possibility, in most previous studies of hydrodynamical aspects associated with QGP, the backreaction of the plasma is usually neglected. Recently, S. Chakrabortty proposed a backreacted gravity background, which is parametrized by the mass of the black hole and long string density. In particular, the backreacted geometry is realized as an AdS black hole back reacted in the inclusion of a uniform distribution of large number of fundamental strings. It turns out that this geometry is thermodynamically stable under tensor and vector perturbations (see [33] for a similar study on the stability of the gravity configurations from the free energy calculation). Subsequently, the drag force [32] and jet quenching parameter [34] have been studied in such a model. It is shown that the presence of backreaction increases the drag force and jet quenching parameter thus enhancing the energy loss. More recently, the imaginary potential [35] of a heavy quarkonium was considered in the same model and the results show that the inclusion of backreaction decreases the thermal width thus enhancing the quarkonium dissociation. Inspired by these facts, we wonder how does backreaction affect the entropic force? Does backreaction have the same effect on the quarkonium dissociation associated with the entropic force as with the imaginary potential? We are answering these questions in the present work.

The structure of the paper is as follows. In the next section, we briefly review the backreacted gravity geometry given in [32]. In section 3, we investigate the behavior of the entropic force for this background and discuss how backreaction influences the quarkonium dissociation. Finally, we conclude with a discussion in section 4.

II. BACKGROUND GEOMETRY

One considers the (n+1)-dimensional gravitational action as follows [32]

$$I = \frac{1}{16\pi G_{n+1}} \int dx^5 \sqrt{-g(R - 2\Lambda)} + S_m,$$

where $G_{n+1}$ is the (n+1)-dimensional Newton constant. $\mathcal{R}$ denotes the Ricci scalar. $\Lambda$ represents the negative cosmological constant. $S_m$ refers to the matter part,

$$S_m = -\frac{1}{2} \sum_i T_i \int d^2\xi \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu},$$

where $h_{\alpha\beta}$ is the world-sheet metric while $g_{\mu\nu}$ denotes the space-time metric, with $\alpha$, $\beta$ the world-sheet coordinates and $\mu$, $\nu$ the space-time directions.

The Einstein’s equations obtained from (2) are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_{n+1} T_{\mu\nu},$$

where $T_{\mu\nu}$ are the stress-energy tensor components.

The ansatz for the geometry can be written as

$$ds^2 = g_{tt}(r) dt^2 + g_{rr}(r) dr^2 + r^2 \delta_{ab} dx^a dx^b,$$

where the strings are supposed to be uniformly distributed over n-1 directions, such that the string cloud density reads

$$a(x) = T \sum_i \delta^{(n-1)}(x - X_i), \quad \text{with} \quad a > 0.$$
Solving Einstein’s equations, one gets

\[ V(r) = K + \frac{r^2}{R^2} - \frac{2m}{r^{n-1}} - \frac{2a}{(n-1)r^{n-3}}, \quad (9) \]

where \( R \) denotes the AdS radius. \( K = 0, -1, 1 \) correspond to the boundary being flat, spherical or hyperbolic, respectively.

In this work, we are mostly interested in the case of \( K = 0, n = 4 \). Given that, the metric becomes

\[ ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2f(r)}dr^2, \quad (10) \]

with

\[ f(r) = 1 - \frac{2mR^2}{r^4} - \frac{2aR^2}{3r^3}, \quad (11) \]

where \( r \) is the 5th dimensional coordinate with \( r = \infty \) the boundary and \( r = r_h \) the horizon, where \( r_h \) satisfies \( f(r_h) = 0 \).

The parameter \( m \) reads

\[ m = \left(1 - \frac{2aR^2}{3r_h^3}\right) \frac{r_h^4}{2R^2}. \quad (12) \]

The temperature of the black hole reads

\[ T = \frac{\sqrt{g^{\tau\tau}}\partial_r\sqrt{|g|}}{2\pi} |_{r=r_h} = \frac{6r_h^3 - aR^2}{6\pi R^2 r_h^2}. \quad (13) \]

As shown in [32], the geometry (10) is thermodynamically stable under tensor and vector perturbations. Moreover, it resembles an AdS Schwarzschild black hole with negative curvature horizon. For more information about it, refer to [32].

### III. ENTROPIC FORCE IN THE BACKREACTIONED GRAVITY BACKGROUND

In this section we follow the prescription in [22] to study the behavior of the entropic force for the background metric (10).

The Nambu-Goto action is

\[ S_{NG} = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \mathcal{L} = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha \beta}}, \quad (14) \]

with

\[ g_{\alpha \beta} = g_{\mu \nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}, \quad (15) \]

where \( g_{\alpha \beta} \) is the induced metric, parametrized by \((\tau, \sigma)\) on the string world-sheet. \( X^\mu \) denotes the target space coordinate.

Taking the static gauge

\[ t = \tau, \quad x_1 = \sigma, \quad (16) \]

and supposing \( r \) depends only on \( \sigma \),

\[ r = r(\sigma), \quad (17) \]

then the Lagrangian density becomes

\[ \mathcal{L} = \sqrt{\frac{r^4f(r)}{R^4} + r^2}, \quad (18) \]
with $\dot{r} \equiv dr/\sigma$.

Since $\mathcal{L}$ does not depend on $\sigma$ explicitly, one has a conserved quantity,

$$\mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = \text{constant.} \quad (19)$$

Imposing the boundary condition at $\sigma = 0$,

$$\dot{r} = 0, \quad r = r_c \quad (r_h < r_c), \quad (20)$$

the conserved quantity turns into

$$\frac{r^4 f(r)/R^4}{\sqrt{\frac{r^4 f(r)}{R^4} + r^2}} = \sqrt{\frac{r^2 f(r_c)}{R^4}}, \quad (21)$$

yielding

$$\dot{r} = \frac{dr}{d\sigma} = \sqrt{\frac{A^2(r) - A(r)A(r_c)}{A(r_c)}}, \quad (22)$$

with

$$A(r) = \frac{r^4 f(r)}{R^4}, \quad A(r_c) = \frac{r^4 f(r_c)}{R^4}, \quad f(r_c) = 1 - \frac{2m R^2}{r_c^4} - \frac{2a R^2}{3r_c^3}. \quad (23)$$

Integrating $(22)$, the interdistance of $q\bar{q}$ is obtained as

$$L = 2 \int_{r_c}^{\infty} \frac{d\sigma}{dr} dr = 2 \int_{r_c}^{\infty} dr \sqrt{\frac{A(r_c)}{A^2(r) - A(r)A(r_c)}}, \quad (24)$$

In fig. 1, we plot $LT$ as a function of $\varepsilon$ for different values of $a$, where $\varepsilon \equiv r_h/r_c$ and $0 < \varepsilon < 1$. From these figures, one can see that increasing $a$ leads to decreasing $LT$, indicating the quark and antiquark get closer under the influence of backreaction.

The next step is to calculate the entropy $S$, which takes the form

$$S = -\frac{\partial F}{\partial T}, \quad (25)$$
where $F$ represents the free energy of $\eta \bar{q}$ (see [34–38] for the original calculation of the rectangular Wilson loop associated with free energy in the vacuum of strongly coupled $\mathcal{N} = 4$ SYM and its generalization to finite temperature).

In general, there are two situations:

1. If $L > \frac{c}{T}$ (with $c$ the maximum value of $LT$), the quarks are completely screened. For this case, one needs to consider some new configurations [39] such that the choice of the $F$ is not unique [40]. Here we choose a configuration of two disconnected trailing drag strings [41, 42]. At this point, the free energy is

$$F^{(2)} = \frac{1}{\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{A(r)}{A(r) - A(r_c)}}.$$  

yielding

$$S^{(2)} = \sqrt{\lambda} \theta(L - \frac{c}{T}),$$

where $\theta(L - \frac{c}{T})$ refers to the Heaviside step function.

2. If $x < \frac{c}{T}$, the fundamental string is connected. For this case, the free energy equals the on-shell action of the fundamental string in the dual geometry,

$$F^{(2)} = \frac{1}{\pi \alpha'} \int_{r_c}^{\infty} dr \sqrt{\frac{A(r)}{A(r) - A(r_c)}}.$$  

As a result, one gets

$$S^{(2)} = \frac{1}{\pi \alpha'} \int_{r_c}^{\infty} dr \sqrt{\frac{A(r)}{A(r) - A(r_c)}}.$$  

where the derivatives are with respect to $r_h$. Note that by plugging $a = 0$ in Eq. (30), the result of SYM [22] is reproduced.

Now we analyze how backreaction modifies the entropic force. To that end, we plot $S^{(2)}/\sqrt{\lambda}$ as a function of $LT$ for different values of $a$ in fig.2, where we have used the relation $\alpha' = 1/\sqrt{\lambda}$. From these figures, it is seen that increasing $a$ leads to larger entropy at small distances. As you know, the entropic force is found to grow as a function of the distance (see Eq.(11)) and responsible for melting the quarkonium. Consequently, one concludes that the inclusion of backreaction increases the entropic force thus enhancing the quarkonium dissociation, in agreement with the findings of the imaginary potential [35].

However, it should be pointed out that the temperature depends on $r_h$ and $a$ (see Eq.(13)), thus, fig.2 actually involves the contributions of temperature variation as well. However, the existing research, e.g., [28], shows that as $T$ increases the entropic force increases. On that basis one reasons as follows: the overall result is that increasing $a$ leads to increasing the entropic force, but at the same time increasing $a$ leads to decreasing $T$ thus decreasing the entropic force. Therefore, one infers that with fixed $T$, increasing $a$ necessarily increases the entropic force. The physical meaning of the results will be discussed in the next section.

**IV. CONCLUSION**

Recent research has suggested that the entropic force may represent a mechanism for dissociating the heavy quarkonium. In this paper, we studied the effect of backreaction on the entropic force in a strongly coupled plasma of adjoint matter. This backreaction is thought to come from the presence of static heavy quarks evenly distributed over such
FIG. 2: $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for different values of $a$. From top to bottom $a = 2, 1, 0$, respectively.

a plasma. It turns out that the inclusion of backreaction increases the entropic force thus enhancing the quarkonium dissociation, consistently with the findings of the imaginary potential.

However, there are many places worth further improvement and in-depth study. For example, how to analyze the backreaction effect associated with the interaction between the heavy quarks in the cloud and the backreaction to the spacetime geometry? (Of course this case is more complicated.) Also, how to study such effect with respect to nonuniform distributed quarks? (It is quite possible that the quarks won’t be evenly distributed in the experiments.) Moreover, how to consider such effect in nonconformal systems? We hope to tackle these issues in the near future.

V. ACKNOWLEDGMENTS

This work is supported by the NSFC under Grant No. 11705166 and the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan) (No. CUGL180402).

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