Scaling laws of turbulence intermittency in the atmospheric boundary layer: the role of stability

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Abstract. Bursting and intermittent behavior is a fundamental feature of turbulence, especially in the vicinity of solid obstacles. This is associated with the dynamics of turbulent energy production and dissipation, which can be described in terms of coherent motion structures. These structures are generated at random times and remain stable for long times, after which they become suddenly unstable and undergo a rapid decay event. This intermittent behavior is described as a birth-death point process of self-organization, i.e., a sequence of critical events. The Inter-Event Time (IET) distribution, associated with intermittent self-organization, is typically a power-law decay, whose power exponent is known as complexity index and characterizes the complexity of the system, i.e., the ability to develop self-organized, metastable motion structures. We use a method, based on diffusion scaling, for the estimation of system’s complexity. The method is applied to turbulence velocity data in the atmospheric boundary layer. A neutral condition is compared with a stable one, finding that the complexity index is lower in the neutral case with respect to the stable one. As a consequence, the crucial birth-death events are more rare in the stable case, and this could be associated with a less efficient transport dynamics.

1. Introduction
The atmospheric boundary layer (ABL) is a complex system characterized by time and spatial scales that can vary by different orders of magnitude. It is characterized by turbulent behavior [15] and by the presence of coherent structures ([13]). Following [7], a coherent motion for turbulent flow can be defined as a three-dimensional region of the flow over which at least one fundamental flow variable (velocity components, density, temperature, humidity, vorticity, other scalars or vector fields) exhibits significant correlation with itself or with another variable.
over a range of space and/or time that is significantly larger than the smallest local scales of the flow. This means that coherent structures are characterized by some kind of self-organization determined by the presence of cooperative dynamics. Coherent structures in the ABL are metastable, i.e., they are characterized by a relatively long life-time after which a rapid decay of memory and spatial coherence occurs. Near the ground, such fast transitions are typically associated with bursting events. In Ref. [3] the authors denote these quasi-instantaneous transitions as critical events and relate them to the intermittent behavior of the system, that is a fundamental feature of turbulent fluid flows [1]. Then, the dynamics of coherent structures is characterized by intermittent events or, in other words, a birth-death process of self-organization. However, the definition of intermittency varies between studies, depending on the examined scale and adopted methods [8]. In Ref. [9] the author distinguishes between small scale or microscale intermittency and global intermittency. In the former, the intermittent behavior arise from overall modulation of the turbulence by the main eddies or in connection with sharp edges of the main eddies. The dissipation of turbulent energy is then confined in small subregions of individual coherent eddies. On the contrary, global intermittency occurs at scales larger than the main coherent eddies [9], as in the case of episodic bursting of turbulence in a strongly stratified boundary layer. In Ref. [30] the terms internal and external intermittency are introduced to distinguish between intermittency originated from the mean turbulent kinetic energy dissipation rate and intermittency resulting from interaction between the flow and the boundary conditions. Being large and small scale strongly coupled, at finer scales external and internal intermittency interact.

Statistics dissimilarity between velocity and scalars within the inertial subrange are dependent on external intermittency. The authors of Refs. [30, 32] investigate the fine-scale intermittency analysing the clustering features of canopy turbulence. The intermittent behavior of coherent structures give a substantial contribution to turbulent transport. Many studies are devoted to investigate the role of coherent structures on turbulent transport of momentum and scalars. In Ref. [31] the authors analyze velocity, air temperature and scalar concentrations fluctuations above different surfaces in order to investigate the role of surface characteristic and atmospheric stratification on intermittency and clustering. Dissimilarity of turbulent transport of momentum and scalars and the relations with the topology and the role of turbulent coherent structures as sweep and ejections is also investigated ([11], [10],[12]) for different stability conditions. The sequence of intermittent bursting events is mainly described by two statistical indicators: the Inter-Event Times (IETs) distribution and the correlation among events and IETs. Regarding the correlation among events, a renewal condition is here assumed, i.e., both events and the IETs are statistically independent random variables. This is in agreement with several findings in literature. In Ref. [4] critical events in the ABL are introduced and the renewal condition is verified. Further, in Refs. [16] the authors developed the so-called surface renewal model. This is a model for the dynamical formation of turbulent structures near the surfaces, given as random intrusions of fluid alternated with a period (residence time) of unsteady Fickian diffusion. More recently, it has been recognized that the rate of surface renewals is not constant, a condition associated with a non-exponential IET distribution [17]. In summary, the renewal condition is a reasonable assumption in the ABL.

The IET distribution associated with self-organization is typically a power-law decay with an asymptotic power exponent, known as complexity index [3] and characterizing the complexity of the system, i.e., the ability to develop self-organized, metastable motion structures. In Ref. [14] the authors study the IET distribution associated with anomalous diffusion of gusts wind. In the present work we investigate the time intermittency of turbulent transport associated with the birth-death of self-organized coherent structures in the atmospheric boundary layer. The intermittent behavior of ABL is investigated by means of a scaling analysis, the Event-Driven Diffusion Scaling (EDDiS) method, which is applied to the sequence of IETs extracted from a
set of wind data.

In Appendix A the EDDiS method is briefly recalled. In Section 2 we describe the results of the statistical analysis and show the main results about the estimated scaling exponents. In Section 3 we draw some conclusions.

2. Diffusion scaling analysis of turbulence data

Before applying the EDDiS method (see Appendix A), we need to extract the critical events from the turbulence data with an event detection algorithm. Wind velocity data observed in the Atmospheric Boundary Layer (ABL) were considered. The data were collected in a measurement campaign carried out in Lecce (Salentum Peninsula, Apulia region, South-East part of Italy) during March 2004 [33]. The instrumental set-up for the measurement of wind velocity was a ultrasonic anemometer Gill R3, with 100 Hz sampling frequency. This was mounted on a horizontal bar placed at the top of a telescopnic mast, 9.6 m above the ground. 8 hours of collected wind data were approximatively considered for the analysis.

Firstly, a sequence of events was extracted from the experimental time series. Our goal is to identify bursts of air motion in the vicinity of the ground, marking the birth or the death of a coherent structure of fluid motion. As bursting events are associated with a sudden fluid acceleration, the events are defined as abrupt changes in the moduli of the turbulent velocity increments:

\[ |\Delta S(t)| = |S(t + \Delta t) - S(t)| > S_0 , \]

where \( S(t) \) is the generic signal at time \( t \), \( \Delta t \) the sampling time of the time series and \( S_0 \) a threshold defining the abrupt change. In our case, the signal \( S(t) \) is a turbulent velocity fluctuation in the generic direction \((x, y, z)\). The fluctuation is defined in such a way to minimize the non-stationary effects, which are related to large scale meteorological patterns. These long-time behaviors are mainly included in the direction and intensity of the mean wind and also in the turbulence intensity (i.e., kinetic energy). Following the common practice in meteorological studies, the turbulent velocity fluctuations are defined by the linear detrending of the air velocity, evaluated over time windows of 30 minutes [15]. The detrended velocity components were also normalized with the local standard deviation, again computed over time windows of 30 minutes, thus defining the fluctuating signal \( S(t) \) in Eq. (1).

The turbulence features in the ABL depend mainly by the so-called atmospheric stability, which characterizes the local tendency of small air particles to remain in the same vertical position (quote) or not. This depends mainly by the temperature and humidity profiles, which can be combined to derive a stability parameter. In particular, a stable atmosphere means that an air particle, when perturbed, has the tendency to come back to the original quote (attractive bouyancy forces), while, in the unstable case, it escapes far from the starting quote (repulsive bouyancy forces). In the neutral case, there are no vertical forces and the wind dynamics is independent from temperature and humidity. Here, we compared two periods with different stability features: a neutral case and a stable one.

The detection of a bursting event formally depends on the threshold \( S_0 \), but there’s a range in which the scaling is not affected by this choice. This is easily seen in Fig. 1 (Left Panel), where different values of \( S_0 \) are compared.

Then, the events were defined as the 90\textsuperscript{th} percentile in the statistical distribution of the increments, corresponding to a threshold of about 0.15: \( \Delta S(t) > 0.15 \). A sequence of occurrence times \( \{t_n\} \) was derived in this way and used to define the discrete variable \( \xi(t) \) for the three walking rules introduced in Appendix A. Then, the diffusion variable \( X(t) \) was computed by applying Eq. (A.1). An example of the DFA function, evaluated from the turbulent bursting events, is reported in Fig. 1 (Right Panel) for the three walking rules. The DFA displays the typical power-law \( F(t) \sim t^H \), valid for monoscaling signals. The SJ rule gives a normal scaling \( H = 0.5 \), which means that \( \mu > 2 \). The SV rule seems to give a normal scaling in the long-time
Stable, DFA, AJ, w: threshold comparison

DFA, Thresh = 0.03

DFA-SJ
DFA-SV
DFA-AJ
H=0.5
H=0.97

Figure 1. Left Panel: threshold dependence of the DFA function applied to the event-driven random walk with the AJ rule (vertical velocity). Right Panel: The DFA function applied to the three walking rules (AJ, SJ and SV). The diffusion scaling is also reported.

|      | $H_{AJ}$ |      | $\mu$ |
|------|----------|------|--------|
| $u$  | 0.97     | Stable| 0.86   |
| $v$  | 0.97     | Neutral| 2.06  |
| $w$  | 0.99     | Stable| 0.87   |

Table 1. Estimated values of $H_{AJ}$, from the AJ rule, and $\mu$ for the three turbulent velocity components.

limit. However, this rule is less accurate as it is affected by the presence of noise, which can be related to the experimental data and instrumentation, but also to the thresholding technique [5]. On the contrary, the AJ rule gives a well-defined anomalous scaling $H \sim 0.9$. The values of the diffusion scaling $H$ and, under the renewal assumption, of the associated complexity index $\mu$ are reported in Table 2. For a given stability condition, all the fluctuating, or turbulent, velocity components have the same $H$ scaling and, consequently, the same complexity index. The diffusion scaling $H$ is greater in the neutral case that in the stable one. In the renewal assumption, this gives a slower power-law decay of the IET distribution in the stable case. This means that the bursting, birth-death, events are more rare, so that the transport dynamics associated with these critical events is less efficient.

3. Conclusions
We have applied a scaling analysis, denoted as EDDiS method, based on the generation of three different random walks to a set of turbulence data in order to characterize the diffusive behavior associated with intermittent bursting events in the ABL turbulence. Two different stability conditions, one neutral and the other stable, have been compared, showing that scaling is affected by the atmospheric stability. We have found that the complexity index associated with intermittency is lower in the neutral case with respect to the stable one. As a consequence, the crucial birth-death events are more rare in the stable case, and this could be associated with a less efficient transport dynamics. Future investigations will carry out to better characterize the dissimilarity of different scalars and the consequences of the different scaling on the modeling of turbulent transport in the ABL.
Figure 2. Scaling $\delta$ and $H$ vs. complexity index $\mu$ for the three walking rules: AJ (continuous line), SJ (dotted-dashed line) and SV (dashed line). Left Panel: scaling $\delta$. Right Panel: scaling $H$.

Appendix A. The diffusion scaling analysis of event-driven random walks

Given an experimental sequence of random events in time, this method is based on the generation of three different random walks and on the estimations of two different scaling exponents, the self-similarity index of the diffusion variable Probability Density Function (PDF) $\delta$ and the second moment scaling $H$. It is worth noting that the three random walks introduced here, and their scaling properties in the case of renewal events, are well known in literature by many years [21, 22, 26, 23, 24, 25]. The calculation of these scaling exponent is based on the analytical treatment of the Continuous Time Random Walk developed by Montroll and co-workers [18, 19, 20]. In particular, these walking rules have been extensively used as a tool of scaling analysis in experimental event sequences [6, 3, 5]. However, the joint use of all these random walks and scaling analyses was carried out in Ref. [6] for the first time as an application to brain data (see also [3] for a brief review). This scaling analysis, which is here denoted as Event-Driven Diffusion Scaling (EDDis) method, allows to evaluate the diffusion scaling exponents $\delta$ and $H$, which are asymptotic features of the signal under consideration. In the renewal condition this allows also to derive a robust estimation of the complexity index $\mu$. When a reliable estimation of all the different scaling exponents is carried out, the EDDis also gives a reasonable justification of the renewal condition itself [6, 3, 5]. In fact, for renewal events, the relationships among the different scaling exponents and of these exponents with the complexity index are theoretically known. Then, it is possible to derive independent estimations of the complexity index. When these evaluations are compatible with each other, the renewal condition is reasonably proved and, further, a robust estimation of the complexity index is obtained.

Given the experimental sequence of event occurrence times $t_0$, $t_1$, $t_2$, ..., corresponding to the events 0, 1, 2, ..., we have the following walking rules:

(a) Asymmetric Jump (AJ) rule:
the walker makes a positive jump ($\xi(t_n) = 1$) in correspondence of each event $n$, otherwise it stands ($\xi(t) = 0$). In other words, $\xi(t)$ is a sequence of unitary pulses

(b) Symmetric Jump (SJ) rule:
as in the AJ rule, but the walker can make positive or negative jumps in correspondence of an event: $\xi(t_n) = \pm 1$. The sign $\pm$ is randomly chosen.

(c) Symmetric Velocity (SV) rule:
the walker moves with constant velocity towards a given direction, in the time interval between two events, then a new random direction is chosen in correspondence of an event: $\xi(t) = \pm 1$ ; $t_n < t \leq t_{n+1}$.
Each random walk is then defined through a signal \( \xi(t) \), which is a kind of random discontinuous velocity, or with discontinuous derivative, and taking two or three different discrete values (two for AJ and SV and three for SJ). The diffusion variable \( X(t) \) of the random walk is computed by integrating \( \xi(t) \):

\[
X(t) = X_0 + \int_0^t \xi(t') dt'
\]

The scaling properties of these random walks were extensively investigated in several papers [21, 22, 26, 23, 24, 25]. (see [6, 3] for a brief account of the literature). The scaling exponents \( \delta \) and \( H \) are defined in the following way:

\[
P(x, t) = \frac{1}{t^\delta} F \left( \frac{x}{t^\delta} \right),
\]

\[
\sigma^2(t) = \langle (X(t) - \overline{X})^2 \rangle \sim t^{2H},
\]

where \( \overline{X} \) is the mean value of \( X(t) \). These exponents were analytically computed, in the asymptotic long-time limit, for random walks driven by renewal events with asymptotic power-law decay in the IET-PDF: \( \psi(\tau) \sim \tau^\mu \) (this is also denoted as fractal intermittency [3]). The analytical expressions for the diffusion scaling exponents \( \delta \) and \( H \) as a function of \( \mu \) are given by:

\[
\delta_{AJ}(\mu) = \begin{cases} 
\mu - 1 ; & 1 < \mu < 2 \\
1/(\mu - 1) ; & 2 \leq \mu < 3 \\
1/2 ; & \mu \geq 3
\end{cases}
\]

\[
H_{AJ}(\mu) = \begin{cases} 
\mu/2 ; & 1 < \mu < 2 \\
2 - \mu/2 ; & 2 \leq \mu < 3 \\
1/2 ; & \mu \geq 3
\end{cases}
\]

\[
\delta_{SV}(\mu) = \begin{cases} 
1 ; & 1 < \mu < 2 \\
1/(\mu - 1) ; & 2 \leq \mu < 3 \\
1/2 ; & \mu \geq 3
\end{cases}
\]

\[
H_{SV}(\mu) = \begin{cases} 
2 - \mu/2 ; & 1 < \mu < 3 \\
1/2 ; & \mu \geq 3
\end{cases}
\]

\[
\delta_{SJ}(\mu) = H_{SJ}(\mu) = \begin{cases} 
(\mu - 1)/2 ; & 1 < \mu < 2 \\
1/2 ; & \mu \geq 2
\end{cases}
\]

These results are plotted in Fig. 2. All rules give a normal scaling \( \delta = H = 1/2 \) for \( \mu \geq 3 \), corresponding to normal (Gaussian) diffusion. This is a consequence of the generalized limit theorem for Lévy stable distribution [28]. For the SJ rule this is true also in the range \( 2 < \mu < 3 \), while AJ and SV rules are super-diffusive \( (H > 1/2) \) in all the interval \( 1 < \mu < 3 \). On the contrary, the SJ rule is sub-diffusive \( (H < 1/2) \) for \( 1 < \mu < 2 \). For Poisson renewal processes, corresponding to the joint limit \( \mu, T \to \infty \), the values of \( \delta \) and \( H \) are again given by the normal scaling \( 1/2 \) and the diffusion is Gaussian.

The scalings \( \delta \) and \( H \) are evaluated by means of the Diffusion Entropy (DE) method [23, 24] and of the Detrended Fluctuation Analysis (DFA) [27], respectively.
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