A New Electromagnetic Emission Source Frequency and Factor Identification Approach for Nonlinear Circuits From the Output Spectrum Data

DONGLIN SU\textsuperscript{1,2}, (Senior Member, IEEE), HUI XU\textsuperscript{2}, SHUNCHUAN YANG\textsuperscript{1,2}, (Member, IEEE), AND YAOYAO LI\textsuperscript{1,2}

\textsuperscript{1}Research Institute for Frontier Science, Beihang University, Beijing 100191, China
\textsuperscript{2}School of Electronic and Information Engineering, Beihang University, Beijing 100191, China

Corresponding author: Donglin Su (sdl@buaa.edu.cn)

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ABSTRACT For the practical nonlinear circuits, a number of new frequencies are generated in the output spectrum, which makes electromagnetic interference location quite challenging. To mitigate this problem, an approach is proposed to identify the electromagnetic emission source frequencies and nonlinear factors from the output spectrum of nonlinear circuits. The amplitude of the output spectrum with the input of two sine signals is first calculated based on the power series. Then, the Order Image (OI) is defined to reveal the output spectrum features, and its symmetry characteristic has been comprehensively analyzed. Results show that the maximum symmetry index is observed when the reference frequencies are equal to the excitation frequencies. Based on this observation, the electromagnetic emission source frequencies can be obtained. In addition, a Backward Algorithm (BA) has been introduced to extract nonlinear factors. The proposed approach only requires the output spectrum and the amplitude of one excitation. Compared with other techniques, like the Support Vector Machines and the Recurrent Neural Network, it needs much less prior information. Finally, several examples are carried out to verify the effectiveness of the proposed approach. Results show that the emission source frequencies and nonlinear factors can be accurately extracted. Therefore, the proposed approach shows great potential in electromagnetic interference location and characteristic analysis for nonlinear circuits.

INDEX TERMS Electromagnetic emission sources, identification, nonlinear circuits, order image, power series.

I. INTRODUCTION

The integration intensity of electronic devices on a single platform keeps increasing with the development of new technologies. Meanwhile, threshold voltage of transistors is gradually decreasing, which makes those electronic devices more electromagnetic sensitive [1]. Therefore, the electromagnetic compatibility (EMC) problems, especially in the high density integrated circuits (ICs) and printed circuit boards (PCBs), occur more frequently [1]. In order to reduce those troublesome EMC problems, it is necessary to strictly and precisely control electromagnetic emission in highly sensitive devices. Circuit modules, especially nonlinear circuits in electronic devices, generate these electromagnetic emissions. Therefore, it should pay special attention to accurately identifying and controlling electromagnetic emission sources from nonlinear circuits. There are two aspects needed to be considered. The first is the electromagnetic emission sources identification, and the second is the nonlinear characteristic analysis.

There are many efforts and approaches with possible applications in nonlinear circuits in the last few decades...
to identify electromagnetic emission sources. In [2], [3], the wavelet transform is used to identify possible electromagnetic emission sources by classifying frequency characteristics extracted from electromagnetic emission data. In [4], an improved empirical mode decomposition (EMD) is proposed to analyze the emission signals through finding their appropriate mathematical expansions. These two approaches only extract mathematical features and cannot totally reveal the physical characteristics. Therefore, they only have quite limited applications in the emission source identifications.

The historical electromagnetic failures are used to analyze electromagnetic interference (EMI) problems in the early stage [5]. In recent years, the artificial neural networks gain much attention and are used to find interference sources for nonlinear systems [6]. The Support Vector Machines (SVMs) are used to identify electromagnetic radiation sources for complex electronic systems [7], [8]. In addition, a fault diagnosis approach incorporated with the SVMs is proposed to analyze nonlinear spectrum [9]. In those approaches, spectrum features of electromagnetic emission sources, large quantity of related data and a long-time training procedure are usually required. However, it is quite possible that the spectrum features of the sources and the historical data are not available in the practical applications. These approaches are inapplicable to identify electromagnetic emission sources in such scenarios.

In [10], an approach, named as the Basic Waveform Theory, is proposed for electromagnetic emission source identification from the output spectrum. It claims that the electromagnetic emission sources are divided into several groups, like sinusoidal excitation, trapezoidal wave excitation, ringing signal excitation and others, in which each type corresponds to specific physical systems. It has been successfully applied to electromagnetic emission source identification in linear circuits, like the fundamental and duty cycle extraction in trapezoidal excitations [11], parameters extraction in damping excitation [12]. When it comes to nonlinear circuits, it is still possible to identify the electromagnetic emission sources only from the output spectrum based on the Basic Waveform Theory.

Since electromagnetic emission sources may be identified, the nonlinear characteristics of circuits should be known in order to control electromagnetic emission in nonlinear circuits. There are several fitting models to construct the relationship between the input and the output signals for nonlinear circuits. The first model is the power series model, which is mainly used to model weak nonlinear systems. The output signals are expressed as a high-order polynomial, which is a function of input signals [13]–[15]. The second one is the Volterra series for nonlinear systems with memory [16]–[19]. The third one is the Recurrent Neural Network (RNN) for complex nonlinear circuits [20]–[22].

When the above models are used to predict electromagnetic emission, detailed parameters in the corresponding model should be calculated or found. In [23], [24], the two-tone and three-tone testing approaches are proposed to calculate all parameters in the power series and Volterra series. The polynomial coefficients of the power series can also be found by using the intercept points of radio frequency (RF) power amplifiers [14]. The adaptive gbest-guide artificial bee colony (AGABC) optimization algorithm is used to obtain the kernel of Volterra series [19]. In those approaches, input signals are required to be properly set and many information including frequencies, amplitudes, and phases of output signals should be known. As for the RNN model, a large quantity of data are needed to train the network [20]–[22]. However, for a practical nonlinear circuit, many modules may work simultaneously and the input signals cannot be set arbitrarily. Therefore, we do not have enough data for training. In addition, the phase information may also be missing due to the fundamental functionality limitations of instruments, like spectrum analyzer, in which no phase information is available.

To mitigate those problems, an approach is proposed to identify the electromagnetic emission source frequencies and nonlinear factors from the output spectrum of nonlinear circuits. In the proposed approach, weak nonlinear circuits are modeled through the power series models due to its strong capability of revealing physical properties of new spectral components [25], [26]. The amplitude of the output spectrum has been predicted based on the power series. During the output spectrum prediction, the Order Image (OI) is defined to reveal the output spectrum features and the symmetry characteristic. We find that the maximal symmetry index is observed when the reference frequencies are equal to the excitation frequencies. Based on this fact, the electromagnetic emission source frequencies can be obtained. In addition, to extract nonlinear factors, the Backward Algorithm (BA) is introduced. Three examples are used to verify the proposed approach. Meanwhile, the narrowband and broadband interference are added to the output spectrum data to verify the effectiveness of the approach under interference conditions. What is more, for the multi-source input the nonlinear circuits, the order reduced process for emission frequency identification is introduced and has been verified in the fourth example.

This article is organized as follows. In Section II, the basic theory for source frequency identification and nonlinear factor extracted is introduced. In Sections III and IV, several examples are used to verify the effectiveness of the proposed approach under different conditions. We draw some conclusions in the last section.

II. BASIC THEORY OF THE PROPOSED APPROACH

A. THE AMPLITUDE OF THE OUTPUT SPECTRUM

For weak nonlinear circuits, the relationship between the input and the output signals can be expressed as a power series. When a \(N\)-order power series is used, the output signal \(F(x)\) is given by

\[
F(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_N x^N, \tag{1}
\]
where \( x \) denotes the input signal, \( N \) denotes the nonlinear order, and \( c_i \) (\( i = 0, \ldots, N \)) is the nonlinear factor. Obviously, due to the nonlinear effects in the power series, the spectrum of \( F(x) \) includes original frequencies of input signals and some new components. For a single frequency excitation, only harmonics appear in the output spectrum. If a signal with two different frequencies, it may generate multiple new spectrum components through various combinations of the input signal frequencies. The input signal \( x \) can be expressed as

\[
x = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2),
\]

where \( A_1, f_1, \varphi_1, \) and \( A_2, f_2, \varphi_2 \) denote amplitudes, frequencies, and initial phases of the two frequency components, respectively, and \( t \) denotes the time variable. Due to the nonlinear effects, the new generated frequencies can be expressed as

\[
f^{p, \pm q} = |p \cdot f_1 \pm q \cdot f_2| = \rho (p, \pm q) (f_1, f_2),
\]

where \( \rho (p, q) \) denotes the time variable. Due to the nonlinear effects, the new generated frequencies can be expressed as

\[
f^{p, \pm q} = |p \cdot f_1 \pm q \cdot f_2| = |(p, \pm q) \cdot (f_1, f_2)|,
\]

where \( (p, q) \) is the order union vector, \( (f_1, f_2) \) is the frequency union vector, \( f^{p, \pm q} \) is the frequency related to \( (p, \pm q) \) and \( (f_1, f_2) \), and “\( \cdot \)” denotes the product vector. It is obvious that \( f^{p, -q} = f^{-p, q} \). An order union vector corresponds to a single frequency. However, a single frequency may correspond to multiple order union vectors since there is certain correlation between \( f_1 \) and \( f_2 \). If \( f_1 \) is the desired input signal frequency and \( f_2 \) is the interference signal frequency, they should show weak correlation. In this article, we are interested in this situation and will focus on development of a novel approach to calculate the excitation sources and nonlinear factors.

For \( f^{p, q} \), it can be rewritten

\[
f^{p, q} = p \cdot f_1 + q \cdot f_2
\]

where \( m \) and \( n \) are non-negative integers, usually \( 0 \leq m + n \leq 0.5 \times (N - p - q) \). \( f^{p, q} \) represents the \( f^{p, q} \) component generated by the \((p + 2m + q + 2n)^{th}\) order power series term.

By considering the input signal in (2), the \((p + 2m + q + 2n)^{th}\) order power series term can be described as

\[
F^{p+2m+q+2n} = c_{p+2m+q+2n} \left[ A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2) \right]^{p+2m+q+2n}.
\]

Let \( a_1 = A_1^2/2, a_2 = A_2^2/2, 2\pi f_1 t + \varphi_1 = \Theta_1, 2\pi f_2 t + \varphi_2 = \Theta_2 \). From the Euler’s formula, (6) can be rewritten as

\[
F^{p+2m+q+2n} = \left( a_1 e^{i\Theta_1} + a_2 e^{-i\Theta_2} \right) + \left( a_1 e^{-i\Theta_1} + a_2 e^{i\Theta_2} \right). (7)
\]

Using the binomial expansion, it is easy to find that the new generated frequencies can be expressed as

\[
f^{p+2m+q+2n} = c_{p+2m+q+2n} \left[ A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2) \right]^{p+2m+q+2n}.
\]

Using a similar manner, the \(-f^{m, n}\) component can be calculated as

\[
f^{p+2m+q+2n} = c_{p+2m+q+2n} C_{p+2m+q+2n} \cdot \left( A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2) \right)^{p+2m+q+2n}.
\]

The new generated frequencies from (3) can be expressed in the form of generating a new frequency from (11) is the \((p + 2m + q + 2n)^{th}\) order power series term in the power series model, and its amplitude is \( A^{p, q}_{m, n} \). Therefore, by summing all the different order power series terms, the amplitude of the \( f^{p, q} \) is expressed as

\[
A^{p, q}_{m, n} = \sum_{m=0}^{N-p-q} \sum_{n=0}^{q} A^{p, q}_{m, n} = \sum_{m=0}^{N-p-q} \sum_{n=0}^{q} 2c_{p+q+2m+2n} C_{p+q+2m+2n} \cdot \left( A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2) \right)^{p+2m+q+2n}.
\]
to be proved that $A^{p,q} = A^{-p,-q}$, (12) can be used to calculate the amplitude of the new frequencies, which is termed as the Forward Algorithm (FA) in this article.

**B. OI OF SPECTRUM**

For nonlinear circuits, the spectrum of the output signal can be calculated through the proposed FA. The $(p, q)$ component of the output spectrum can be characterized by the frequency $f^{\pm p,q}$ and the amplitude $A^{\pm p,q}$.

To better visualize the magnitudes of different frequency components, we define a two-dimensional plot, termed as the OI, with the order number $\pm p$ and $q$ as its horizontal and vertical axes, respectively, and the magnitude is $A^{\pm p,q}$ for $f^{\pm p,q}$. The first reference frequency is $f_1$ with $\pm p$ order, and the second reference frequency is $f_2$ with $q$ order. It should be noted that the OI is not uniquely defined and closely related to the reference frequencies. For the practical spectrum whose reference frequencies are not equal to $f_1$ and $f_2$ even for the same output spectrum, different OIs for the same output spectrum show distinct characteristics. For example, the first reference frequency marked as $f_1$ is set to $f_2 - f_1$. Then, $f^{\pm p,q}$ can be expressed in terms of $f_1$ and $f_2$ as

\[
\begin{align*}
    f^{p,q} &= |p \cdot f_1 + q \cdot f_2| \\
    f^{-p,q} &= |-(p+q) \cdot f_1 + q \cdot f_2| \\
    f^{-q,p} &= |-(p+q) \cdot f_1 + q \cdot f_2| \\
    f^{-q,-p} &= |-(p+q) \cdot f_1 + q \cdot f_2|,
\end{align*}
\]

where $p$ and $q$ denote the order number of $f_1$ and $\sim$, respectively. All the frequencies can be included in the OI with the reference frequencies $f_1$ and $f_2 - f_1$. The amplitude of $f^{\pm p,q}$ needs to be placed at the corresponding order union point of $(p, q)$. In other words, the amplitude of $f^{\pm p,q}$ is needed to draw the OI $f^{\pm p,q}(f_1, f_2)$ is given by

\[
\begin{align*}
    f^{p,q}(f_1, f_2) &= |p \cdot f_1 + q \cdot f_2| \\
    f^{-p,q}(f_1, f_2) &= |-(p+q) \cdot f_1 + q \cdot f_2| \\
    f^{-q,p}(f_1, f_2) &= |-(p+q) \cdot f_1 + q \cdot f_2| \\
    f^{-q,-p}(f_1, f_2) &= |-(p+q) \cdot f_1 + q \cdot f_2|,
\end{align*}
\]

The corresponding amplitudes can be expressed as

\[
\begin{align*}
    A^{p,q}(f_1, f_2) &= A^{-p,-q}(f_1, f_2), \\
    A^{-p,q}(f_1, f_2) &= A^{-p,-q}(f_1, f_2),
\end{align*}
\]

where $A^{p,q}(f_1, f_2)$ and $A^{-p,q}(f_1, f_2)$ denote the amplitudes of $f^{p,q}(f_1, f_2)$ and $f^{-p,q}(f_1, f_2)$, respectively.

Since $A^{-p,q} = A^{p,q}$, we have $A^{p,q}(f_1, f_2) = A^{-p,-q}(f_1, f_2) = A^{-p,q}(f_1, f_2)$, which implies $A^{p,q}(f_1, f_2) = A^{-p,q}(f_1, f_2)$. Therefore, the OI with the reference frequencies $f_1$ and $f_2$ is symmetrical upon the order number $p$. However, the OI of the same output spectrum with the reference frequencies $f_1$ and $\sim$ shows no symmetrical properties.

More general, we further analyze the symmetrical characteristics of the OIs with different reference frequencies. Two aspects need to be considered in our analysis: the completeness of frequency representation and the symmetry of amplitudes unions.

First, we analyze the completeness of frequency representation. Two other reference frequencies can be chosen as the superposition of $f_1$ and $f_2$:

\[
\begin{align*}
    \sim &= z_1 f_1 + z_2 f_2, \\
    f_\sim &= z_1 f_1 + z_2 f_2, \\
    f_\sim &= z_1 f_1 + z_2 f_2,
\end{align*}
\]

However, not all the frequencies of $f^{\pm p,q}$ can be expressed with $(f_1, f_2)$. We mark the percentage that the $f^{\pm p,q}$ can be represented with $(f_1, f_2)$ as $P$. It is obvious that $0 \leq P \leq 1$. The $P$ has different values when different reference frequency unions are chosen.

Second, we analyze the symmetry of the amplitudes unions for $(f_1, f_2)$. For $(p, q)$ in (21), the corresponding amplitudes can be expressed as

\[
\begin{align*}
    A^{p,q}(f_1, f_\sim) &= A^{p,q}(f_1, f_\sim) = A^{p,q}(f_1, f_\sim), \\
    A^{-p,q}(f_1, f_\sim) &= A^{-p,q}(f_1, f_\sim), \\
    A^{-p,q}(f_1, f_\sim) &= A^{-p,q}(f_1, f_\sim), \\
    A^{-p,q}(f_1, f_\sim) &= A^{-p,q}(f_1, f_\sim),
\end{align*}
\]

Since $A^{-p,q} = A^{p,q}$, we have $A^{p,q}(f_1, f_\sim) = A^{-p,q}(f_1, f_\sim) = A^{-p,q}(f_1, f_\sim)$, which implies $A^{p,q}(f_1, f_\sim) = A^{-p,q}(f_1, f_\sim)$. Therefore, the OI with the reference frequencies $f_1$ and $f_2$ is symmetrical upon the order number $p$. However, the OI of the same output spectrum with the reference frequencies $f_1$ and $\sim$ shows no symmetrical properties.
Since \( \tilde{p}, \tilde{q}, \tilde{z}_{11}, \) and \( \tilde{z}_{22} \) are integers, if \( |z_{11}| \neq 1 \) or \( |z_{22}| \neq 1 \), not all the frequency unions \( \{f_1, f_2\} \) can be expressed with \( \{\tilde{f}_1, \tilde{f}_2\} \). As a result, \( P \) is strictly less than 1.

The maximal product of \( P, Q \) appears when the reference frequencies are equal to the source frequency union \( \{f_1, f_2\} \), and the following condition must be satisfied

\[
\begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix} = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix},
\] (28)

By considering both the completeness of frequency representation and symmetry of amplitudes unions, if and only if the reference frequency parameters satisfy (28), \( P \times Q = 1 \). Otherwise, \( P \times Q < 1 \).

Another scenario is considered. When two frequencies are not related to the source frequencies, we can use a combination of signal source frequencies plus a random frequency to represent the reference frequencies as

\[
\begin{cases}
f_1 = z_{11}f_1 + z_{12}f_2 + \Delta f_1 \\
f_2 = z_{21}f_1 + z_{22}f_2 + \Delta f_2,
\end{cases}
\] (29)

where \( \Delta f_1 \) and \( \Delta f_2 \) are random frequencies. With above similar procedure, \( P \) and \( Q \) can hardly be 1 unless \( \Delta f_1 \) and \( \Delta f_2 \) are related to \( f_1 \) and \( f_2 \). Therefore, \( P \times Q \) is strictly less than 1.

Fig. 1 shows the output spectrum of a nonlinear circuit, in the source frequencies \( f_1 \) and \( f_2 \) are equal to 2.3 Hz and 2.71 Hz, respectively. There are other reference frequency unions to describe the same spectrum for the OI, which are listed in Table 1. The order unions \( (P, Q) \), which can be calculated from (21) for \( f^{\pm p-q} \) with the new reference frequencies, are listed in the fourth column. The different OIs are shown in Fig. 2. It is worth mentioning that the spectrum contains valid signals and noise floor. The valid signals after being denoised are used to draw the OI. When the reference frequencies do not belong to \( f^{\pm p-q} \), the OI is chaotic and irregular as shown in Fig. 2(a), in which \( f_1 + 0.1, f_2 + 0.1 \) are chosen as the first and second reference frequencies, respectively. However, when the reference frequencies belong to \( f^{\pm p-q} \), the OI is regular as shown in Fig. 2(b)-(h). The frequency unions \( \{f_1, f_2\}, \{f_1, f_2 - f_1\}, \{f_1, f_2 + f_1\}, \{f_2, f_2 - f_1\}, \{f_2, f_2 + f_1\}, \{f_1, f_1 + 2f_1\}, \) and \( \{f_2, 2f_2 + f_1\} \) are chosen as the reference frequencies for Fig. 2(b), (c), (d), (e), (f), (g), and (h), respectively. When the reference frequencies are exactly the input signal frequencies of nonlinear circuits, the OI does have symmetry as analyzed before and is shown in Fig. 2(b). However, the other OIs have no symmetry.

As shown in Fig. 2, different OIs of the same output spectrum with different reference frequency unions have different symmetrical properties. The symmetry index(SI) \( I_{cal} \) to characterize the OI symmetry is defined as

\[
I_{cal} = \frac{\text{Num}}{M \cdot N},
\] (30)

where \( M \) is the maximal ordinal number of \( f_1 \), \( N \) is the maximal ordinal number of \( f_2 \), and \( \text{Num} \) is a statistical number that calculates the number of “quasi-identical” effective amplitude values in the left and right symmetrical positions in the OI. The “quasi-identical” means that the amplitudes at left and right symmetrical positions only need to be approximately equal. For example, the SIs are 0.0625%, 4.1875%, 0.4063%, 3.0625%, 0.6563%, 2.6875%, 0.0625%, and 0.1563% for Fig. 2 (a), (b), (c), (d), (e), (f), (g) and (h), respectively.

### C. THE PROPOSED APPROACH

Based on the analysis of the OI symmetry, it can be concluded that: the SI of the OI with \( f_1 \) and \( f_2 \) is the largest compared with that of the OI with other reference frequencies. This is a crucial property of the OI for the source frequency extraction. Before the OI and its property are used to identify the source frequencies, we should analyze whether the source frequencies appear in the output signal spectrum. From the FA, it is clear that the odd unions, where \( p + q \) is an odd integer, of the source frequencies in the output signal spectrum result from the odd order of nonlinear factors. The even unions, where \( p + q \) is an even integer, are the results of the even order of the nonlinear factors of the nonlinear system. When there is a certain odd order nonlinear factor, the source frequencies should exist in the output signal spectrum.

The flowchart of the proposed approach to identify the electromagnetic emission source frequency is shown in Fig. (3). First, the valid signals including the frequencies and amplitudes are picked from the output spectrum. Second, different OIs are drawn with different unions of reference frequencies chosen from the valid frequencies. Third, the SIs of different OIs are calculated with (30). Finally, the maximal SI is picked and the corresponding reference frequencies are recognized as the electromagnetic emission source frequencies. We can find that the amplitudes of the electromagnetic...
emission sources are not needed when the proposed approach is used to identify the electromagnetic emission sources frequencies.

For the dual frequency signal is injected into a nonlinear circuit modeled by a power series as shown in (1), the spectrum (both frequencies and amplitudes) of the output signal can be obtained from (12). If the highest order of the power series for the nonlinear circuit is \( N \) series for the nonlinear circuit can be obtained from (12). If the highest order of the power series for the nonlinear circuit is \( N \), the highest and second highest order nonlinear factors of the power series can be calculated from the amplitudes of the highest and second highest order frequencies of \( f^{p,q} \), respectively, as

\[
c_N = \frac{A_{p_1,N-p_1}}{2C_{N}^{p_1}(a_1)p_1(a_2)^{N-p_1}},
\]

\[
c_{N-1} = \frac{A_{N-1-p_2}}{2C_{N-1}^{p_2}(a_1)p_2(a_2)^{N-1-p_2}}.
\]

where \( p_1 \) and \( p_2 \) denote the order of the first frequency component in the highest order unions and the second highest order unions of the dual frequency signal, respectively.

For the other order nonlinear factors, they can be recursively calculated through the following formulation as

\[
c_{p+q} = \frac{1}{2C_{p+q}^{q}(a_1)p(a_2)^2} \left[ A_{p,q}^{N-p-q} + \sum_{m=0}^{N-p-q} \sum_{n=0}^{N-2m} C_{p+q+2m+n}^{p+2m}(a_1)^{p+2m}C_{p+q+2m+n}^{q+2n}(a_2)^{q+2n} \right]
\]

We can find that there are \( N + 2 \) unknowns (\( N \) is for the nonlinear factors and the zero-order nonlinear factor \( c_0 \) is not considered, 2 is for the two excitation amplitudes) and \( N \) equations from (31) to (33). When two different orders of the first frequency in the \( N^{th} \) order are chosen as \( p_{11} \) and \( p_{12} \), the same \( c_N \) can be obtained with (31). The two amplitudes of the sources have the following relationship

\[
\frac{A_{p_{11},N-p_{11}}}{2C_{N}^{p_{11}}(a_1)p_{11}(a_2)^{N-p_{11}}} = \frac{A_{p_{12},N-p_{12}}}{2C_{N}^{p_{12}}(a_1)p_{12}(a_2)^{N-p_{12}}}.
\]

The amplitude of one source may be known in the following two situations. The first situation is when the strong and weak signals input the nonlinear circuits, the amplitude of the strong signal can be measured with the online current or voltage measurement method in [27], [28]. Therefore, there are only \( N + 1 \) unknowns. The nonlinear factors can be calculated with the \( N + 1 \) equations through (31), (32), (33), and (34). The maximal amplitude union in the same order unions of \( (p, q) \) is selected as \( A_{p,q}^{N} \) for (31), (32), and (33). In the order unions of \( p + q = N \), the largest and second largest amplitude unions are selected as \( A_{p_{11},N-p_{11}}^{N} \) and \( A_{p_{12},N-p_{12}}^{N} \) for (34).
III. NONLINEAR CHARACTERISTIC ANALYSIS
A. NUMERICAL EXAMPLE ANALYSIS
Let’s consider the following nonlinear system, in which the relationship between the input and the output is described as
\[
F \left[ x (t) \right] = 1 + \sum_{i=1}^{5} \frac{(-1)^{j-1}}{j!} [x (t)]^j, \tag{35}
\]
where \( x (t) \) is the input signal. The output signal spectrum is shown in Fig. 4 with the solid black line. There are both noise and valid signals in the spectrum. The trend components can be removed using the wavelet analysis with Daubechies (db) wavelet basis from the spectrum [29]. Especially, we get rid of 0 Hz from the valid signals. By setting a threshold, the valid signal frequencies can be separated from the output spectrum. The picked frequencies are shown in Fig. 4 with the red marker “*”.

\[
\text{FIGURE 4. The output spectrum for numerical example.}
\]

The SI of OIs with different frequency index unions are calculated and shown in Fig. 5(a). There is a maximal union (5, 6) of 0.4% corresponding to the fifth and sixth marked frequencies from left to right in Fig. 4. They are 2.3 Hz and 2.71 Hz which coincide well with the input frequencies. The OI is shown in Fig. 5(b) and is symmetrical upon the first reference frequency.

\[
\text{FIGURE 5. Results for the OI for the numerical example.}
\]

Each \( A^{\pm p, q} \) may be zero, positive or negative. The modulus value of \( A^{\pm p, q} \) can be directly obtained from the spectrum. However, the sign of \( A^{\pm p, q} \) is unknown. It should be clear when we calculate the nonlinear factors through the BA. \( N \) modulus values can be rewritten into an amplitude matrix as
\[
A = [A_{P_1, q_1}, A_{P_2, q_2}, \ldots, A_{P_N, q_N}], \tag{36}
\]
where \( A_{P_i, q_i} \) is the \( i^{th} \) modulus value of \( A^{\pm p_i, q_i} \) and the sign of each \( A^{\pm p_i, q_i} \) may be positive or negative. The amplitude matrix can be corrected with considering the sign as
\[
\tilde{A} = [B_1 A_{P_1, q_1}, B_2 A_{P_2, q_2}, \ldots, B_N A_{P_N, q_N}], \tag{37}
\]
where the \( B_i (i = 1, \ldots, N) \) is a random of positive or negative one. If the exponent of negative one is used, then
\[
B = [(-1)^{b_1}, (-1)^{b_2}, \ldots, (-1)^{b_N}], \tag{38}
\]
where \( b_i (i = 1, \ldots, N) \) is a binary. It can be seen that different groups of nonlinear factors can be obtained with different \( b = [b_1, b_2, \ldots, b_N] \). The corresponding prediction spectrum will have different matching degree with the original spectrum. The nonlinear factors are achieved when best matching are found. Therefore, the nonlinear factors can be found through an optimization algorithm. Since \( b \) is a binary matrix, the Genetic Algorithm (GA), of which the optimization variables is binary, is selected. The object function is defined as the minimal Root Mean Square Error (RMSE), which can be shown as
\[
\min : \text{error}_{\text{RMSE}} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (A^{p, q}(\text{dB}) - A_0^{p, q}(\text{dB}))^2}, \tag{39}
\]
where \( n \) denotes the set of all the \((p, q)\) corresponding to the valid frequencies, \( n \) is the total number of \((p, q)\), \( A_0^{p, q}(\text{dB}) \) is the output spectrum amplitude in decibels, and \( A^{p, q}(\text{dB}) \) is the corresponding predicting spectrum in decibels obtained from the FA.

In this example, we assume that one of the amplitudes of the two sources is 0.5. The amplitudes of the order unions (2, 3) with \(-61.79 \text{ dB}\) and (4, 1) with \(-67.81 \text{ dB}\) in the OI in Fig. 5(b) is used. The second amplitude is calculated as 0.499 with (34), which has 0.2% deviation from the input signal. Then, after the optimization has been carried out, the nonlinear factors are obtained and shown in Fig. 6 marked with red “*”. Results show that the calculated values agree well with the set values. The maximal deviation is 0.0002 corresponding to the maximal relative deviation 0.41%. The prediction output spectrum is shown in Fig. 7 marked as the red “*”, which shows excellent agreement with the original spectrum. The maximal deviation is 0.094 dB, and the average deviation is 0.015 dB.

B. SIMULATION EXAMPLE ANALYSIS
Two more simulation examples are used to verify the proposed approach. The simulation circuit schematic diagrams are shown in Fig. 8.

The first example is a circuit including a nonlinear element, a diode, as shown in Fig. 8(a). In this example, the threshold voltage of the diode is 0.7 V. There are two sine excitation sources \( V_{S1} \) and \( V_{S2} \) as the input signal. The voltage \( V_0 \) of the load impedance \( R_2 \), which is shown in Fig. 8(a), is taken...
as the output signal for emission source frequencies and nonlinear factors identification. During the simulation, the two excitation sources are set to 2.3 MHz, 500 mV and 2.71 MHz, 500 mV. The output spectrum is shown in Fig. 9 with solid black line. The unions of the source frequencies appear in the output spectrum. In Fig. 9, the reality frequencies are picked marked as red “*”. The diode conducts for signals above the threshold voltage, which has a strong cutting effect and results obviously nonlinear effect. Therefore, there are many interactive tonal components in the output spectrum in Fig. 9 compared with the output spectrum in Fig. 7. Especially, the threshold for spectrum analyzing is set to −150 dBV since further lower amplitude levels are meaningless in reality.

Therefore, the SIs of OIs with different frequency index unions are calculated and shown in Fig. 10(a). In order to show the maximal SI order union, the local results with 50 × 50 is shown in Fig. 10(b). There is a maximal order union (18,21) of 54.48% corresponding to the 18th and 21st marked point frequencies shown in Fig. 9. They are 2.3 MHz and 2.71 MHz which coincide well with the set frequencies. The OI is shown in Fig. 11. It is symmetrical upon the first reference frequency.

Similarly, by supposing one of the amplitudes of the two sources is known as 0.5 V. The amplitudes of order unions (9,10) with −142.87dBV and (10,9) with −142.84 dBV shown in Fig. 11 are used. The second amplitude can be calculated as 0.498 V with the (34), which has 0.4% deviation from the set value. The nonlinear factors can be obtained. In order to verify the nonlinear factors, a new output spectrum is predicted where the input signals are set to 1.71 MHz, 400 mV and 2.1 MHz, 400 mV. The prediction spectrum is shown in Fig. 12 marked as the red “*”. The maximal deviation is 2.951 dB, and the average deviation is 0.185 dB.

The second is a common-emitter amplifier circuit as shown in Fig. 8(b). There is a nonlinear effect and new frequencies are generated when the static operating point of the circuit is not appropriate or the amplitude of the input signal is too large. In this example, the voltage $V_L$ of the load impedance $R_L$, which is shown in Fig. 8(b), is taken as the output signal for emission source frequencies and nonlinear factors identification. There are two sine excitation sources.
\( V_{S1} \) and \( V_{S2} \) as the input signals. During the simulation, the two excitation sources are set to 2.3 kHz, 15 mV and 2.71 kHz, 25 mV. The output spectrum is shown in Fig. 13 with solid black line. The valid frequencies are picked marked as red ‘*‘.

C. INTERFERENCE ANALYSIS

The SIs of OIs with different frequency index unions are calculated and shown in Fig. 14(a). There is a maximal order union (3,4) of 0.12% corresponding to the third and fourth marked frequencies as shown in Fig. 13. They are 2.3 kHz and 2.71 kHz, which agree well with the set frequencies. The OI is shown in Fig. 14(b) with a high degree of symmetry. Similarly, by supposing one of the amplitudes of the two sources is known as 25 mV. The amplitudes of order unions (1,3) with \(-59.36\) dBV and (2,2) with \(-60.26\) dBV in the OI shown in Fig. 14(b) are used. The second amplitude can be calculated as 15.03 mV with (34), which has 0.2% deviation from the set value. Then, the nonlinear factors can be obtained. In order to verify the factor, a new output spectrum is predicted where the input signals are set to 1.3 kHz, 10 mV and 2.11 kHz, 40 mV. The prediction spectrum is shown in Fig.14 marked as the red ‘*‘. The original spectrum is in solid black line. The maximal deviation is 5.12 dB (appear the 6.01 kHz) and the average deviation is 0.9125 dB. If the maximal deviation at 6.01 kHz is removed, the maximal deviation of the remaining frequencies is 1.831 dB and the average deviation is 0.667 dB. It should be noted that there are two points below the simulation noise floor that are not considered in Fig. 15.

The SIs of OIs with different frequency index unions are calculated and shown in Fig. 14(a). There is a maximal order union (3,4) of 0.12% corresponding to the third and fourth marked frequencies as shown in Fig. 13. They are 2.3 kHz and 2.71 kHz, which agree well with the set frequencies. The OI is shown in Fig. 14(b) with a high degree of symmetry. Similarly, by supposing one of the amplitudes of the two sources is known as 25 mV. The amplitudes of order unions (1,3) with \(-59.36\) dBV and (2,2) with \(-60.26\) dBV in the OI shown in Fig. 14(b) are used. The second amplitude can be calculated as 15.03 mV with (34), which has 0.2% deviation from the set value. Then, the nonlinear factors can be obtained. In order to verify the factor, a new output spectrum is predicted where the input signals are set to 1.3 kHz, 10 mV and 2.11 kHz, 40 mV. The prediction spectrum is shown in Fig.14 marked as the red ‘*‘. The original spectrum is in solid black line. The maximal deviation is 5.12 dB (appear the 6.01 kHz) and the average deviation is 0.9125 dB. If the maximal deviation at 6.01 kHz is removed, the maximal deviation of the remaining frequencies is 1.831 dB and the average deviation is 0.667 dB. It should be noted that there are two points below the simulation noise floor that are not considered in Fig. 15.

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C. INTERFERENCE ANALYSIS

The output spectrum data in the above examples are obtained in an ideal condition. In reality, the noise interference should not be ignored. In this section, two kinds of noise interferences are considered to analyze the anti-interference characteristics of the proposed approach. Because the output spectrum is used in this article, we directly add the noise interference to the output spectrum data of the second transistor used circuit shown above.

The first scenario is considered as the narrow-band noise interference. First, the irrelevant narrowband noise interferences are added to the original output spectrum, which are the harmonics of 1 kHz shown in Fig. 16 marked with “Δ”. The original output spectrum data are in the black solid line. The amplitude of the fundamental frequency is 1.752 dBV, which is the maximal amplitude of the original output spectrum. The amplitudes of the rest harmonics decrease by 10 dB till \(-88.248\) dBV. The picked frequencies for proposed approach are marked with “*” . The SIs of OIs with different frequency index unions are calculated and shown in Fig. 17(a). There is a maximal order union (5,6) of 0.16% corresponding to the fifth and sixth marked frequencies shown in Fig. 16. They are 2.3 kHz and 2.71 kHz which agree well with the set frequencies. The OI is shown in Fig. 17(b). Since the noise interference is not related to the original output spectrum. The noise signals either disappear in the OI or far away the main points, just as the two dots in the upper left corner in Fig. 17(b). The amplitudes below the 10 order, which are totally same as the second example in section B shown in Fig. 14(b), are used to extract the nonlinear factors. Therefore, the nonlinear factors are obtained the same as above and the proposed approach is verified.

Second, an interrelated narrowband noise interference signal is added to the original output spectrum. Its harmonics are shown in Fig. 18 marked with “Δ”. The fundamental frequency 0.41 kHz, which is the union of 2.71 kHZ and 2.3 kHz. The amplitude of the fundamental frequency is
1.752 dBV which is the maximal amplitude of the original output spectrum. The amplitudes of the rest harmonics decrease by 10 dB till −88.248 dBV. The picked frequencies for the proposed approach are marked with "*". The SIs of OIs with different frequency index unions are calculated and shown in Fig. 19(a). There is a maximal order union (7, 9) of 0.12% corresponding to the seventh and ninth marked frequencies shown in Fig. 18. They are 2.3 kHz and 2.71 kHz which agree well with the set frequencies. The OI is shown in Fig. 19(b). Since the noise interference is related to the original output spectrum, the noise signals are closely to the original output spectrum data and have effects upon the nonlinear factors. The amplitudes of order unions (−2, 2) with −8.248 dBV and (−1, 3) with −56.62 dBV in the OI shown in Fig. 19(b) are used. The second amplitude is calculated as 4369.7 mV which is far away from the set value. As a result, the extracted nonlinear factors are totally incorrect.

From the above results, we can find that the excitation source frequencies can be correctly extracted with the proposed approach for both interrelated and irrelevant narrowband noise interference, and the nonlinear factors can be correctly extracted in the situation of irrelevant narrowband noise interference. However, the nonlinear factors cannot be correctly extracted in the situation of interrelated narrowband noise interference.

The second scenario is considered as broadband noise interference. In this situation, the positive and negative random perturbations with different amplitudes from 0 dB to 10 dB are added to the original output spectrum. The spectrum added 10 dB of positive and negative random perturbations is shown in Fig. 20. The original spectrum data are in the black solid line. The final spectrum added with random perturbations is marked with red “.". The picked frequencies for the proposed approach are marked with "*". Since the interference signals are randomly added to each frequency, the effectiveness of the proposed approach cannot be proved during only once interference process. Thus, we repeated 100 times at each amplitude (from 0 dB to 10 dB). The proportion of times that the excitation source frequencies are correctly obtained under the corresponding amplitude is used to reflect the effectiveness of the proposed approach for frequency obtaining. Similarly, the average of the maximal fitting errors at each interference process of each amplitude (from 0 dB to 10 dB) for new spectrum prediction with extracted nonlinear factor are calculated to reflect the effectiveness of the proposed approach for nonlinear factor extracted. The results are shown in Fig. 21 and Fig. 22.

We can find that when the amplitude of the positive and negative random perturbations is within 2 dB, the proposed approach can still extract the excitation source frequencies with 100% correct rate. As the amplitude increases, the correct rate decreases. Especially, the correct rate is higher than 91% within 6 dB. For the nonlinear factors extraction, we can find that when the amplitude of the positive and negative random perturbations is within 4 dB, the average maximal errors are within 10 dB. When the amplitude of the positive and negative random perturbations exceeds 4 dB, the fitting error has risen significantly. The above results show that the proposed approach has certain anti-interference ability.

**IV. ORDER REDUCED PROCESSING FOR MULTI-SOURCE**

Under the situation of multi-source input the nonlinear system, the output signal spectrum may be the unions of the
multi-source frequencies as

\[ f(p_1, p_2, \ldots, p_R) = |p_1f_1 \pm p_2f_2 \pm \ldots \pm p_R|, \]

where \( R \) denoted the total number of the input signal frequencies, \( f_i(i = 1, 2, \ldots, R) \) denotes the \( i^{th} \) frequency, and \( p_i(i = 1, 2, \ldots, R) \) denotes the corresponding order for \( f_i \).

When the two-dimensional OI shown before is expanded to higher dimension, it should be symmetrical. If the source frequency unions are chosen as the reference frequencies, the high-dimensional OI has the maximal SI, which can be also used to extract the source frequency. However, if the number of valid frequencies is \( N_f \), the analysis complexity increases dramatically since it is approximately \( O(N_f^R) \). As the number of source frequencies increases, it will take a lot of time. The order reduced process is necessary.

The output frequencies can be classified \( R \) groups. For example, there is only one source frequency used for intermodulation (that is harmonics), there are two source frequencies used for intermodulation, and there are \( R \) source frequencies used for intermodulation and so on, as

\[
\begin{align*}
    f^1 &= \{f_1 \} \\
    f^2 &= \{f_1, f_2 \} \\
    &\quad \ldots \\
    f^R &= \{f_1, f_2, \ldots, f_R \},
\end{align*}
\]

where \( f^i(i = 1, 2, \ldots, R) \) denotes the set of the union sets of \( i \) excitation source frequencies. The followings are some specific examples for \( f^1, f^2, \) and \( f^R \).

\[
\begin{align*}
    \{f_1\} &= |nf_1|, \quad n = 1, 2, \ldots, N \\
    \{f_1, f_2\} &= |p \cdot f_1 \pm q \cdot f_2| \quad \{p = 1, 2, \ldots, N, q = 1, 2, \ldots, N; p + q \leq N \\
    \{f_1, f_2, \ldots, f_R\} &= |q_1 \cdot f_1 \pm q_2 \cdot f_2 \pm \ldots \pm q_R \cdot f_R| \\
    &\quad \{q_1 = 1, 2, \ldots, N \\
    &\quad q_2 = 1, 2, \ldots, N \\
    &\quad \ldots \\
    &\quad q_R = 1, 2, \ldots, N, \\
    &\quad q_1 + q_2 + \ldots + q_R \leq N \}
\end{align*}
\]

The two-dimensional OI is also symmetrical when only two of all the source frequencies are chosen as the reference frequencies. Two of the source frequencies can be extracted based on the symmetrical characteristics. From (41), we can find that there are many union sets of two excitation source frequencies in \( f^2 \) for the output spectrum. When one union set of some two excitation source frequencies in \( f^2 \) is removed from the output spectrum, there are still other union sets of two excitation source frequencies in the output spectrum. The OI of the remaining spectrum can still be symmetrical. Thus, we can extract new excitation source frequencies by analyzing the symmetrical characteristics of the remaining spectrum after removing the union sets of two extracted excitation source frequencies. This procedure can be repeated until there is no new excitation source frequency extracted.

However, it is unknown that how many sources there are. Therefore, during each procedure, it is needed to check whether the new extracted frequencies are the true source frequencies. The reference frequencies at each process come from the original output spectrum including the source frequencies and the unions of the source frequencies. The unions of the source frequencies may be extracted during the processes. If the new extracted frequencies at the \( i^{th} \) calculation step can be combined with the previous source frequencies analyzed before the \( i^{th} \) calculation step, they do not belong to the source frequencies and stop the calculation step. Therefore, the stop criterion is that all the new extracted frequencies are the unions of the previous extracted source frequencies in the earlier calculation processes. The flowchart of the order reduced progress is summarized in Fig. 23.

An example with four signal sources is used to verify the proposed reduced process, which is shown in Fig. 24. In this example, the voltage \( V_L \) of the load impedance \( R_L \),
FIGURE 23. The flow chart of the order reduced processing.

FIGURE 24. The simulation schematic diagram for four signal sources.

which is shown in Fig. 24, is taken as the output signal for emission source frequencies and nonlinear factors identification. The four input sine signals V_{S1}, V_{S2}, V_{S3}, and V_{S4} are set to 1.03 kHz, 10 mV, 2.3 kHz, 25 mV, 2.71 kHz, 4 mV, and 4.22 kHz, 40 mV, respectively. The output spectrum is shown in Fig. 25 with solid black line. The valid frequencies are picked marked as red ‘*’. From the output spectrum, we can find that many new frequencies appear. Especially, the new frequencies may have larger amplitude compared with the input signal frequencies. For example, the amplitude of 1.92 kHz is -18.92 dBV, which is the union of 2.3 kHz and 4.22 kHz. However, the amplitude of the input signal frequency 2.71 kHz is -19.92 dBV.

With the order reduced process, four times of calculation processes are needed in this example.

First Step: the picked frequencies marked as red ‘*’ in Fig. 26 (a) are used. The SIs of OIs are shown in Fig. 27 (a). There is a maximal order union (19, 36) of 0.2% corresponding to the 19\textsuperscript{th} and 36\textsuperscript{th} marked point frequencies from left to right in Fig. 26 (a). The extracted frequencies are 2.3 kHz and 4.22 kHz. Since no source frequencies have been identified, the extracted frequencies are new frequencies at present step and taken as the known source frequencies for the second step.

Second Step: the unions of 2.3 kHz and 4.22 kHz marked as red ‘*’ in Fig. 26 (b) are removed, and the remaining frequencies marked as blue ‘*’ in Fig. 26 (b) are used. The SIs of OIs are shown in Fig. 27 (b). There is a maximal order union (6, 28) of 0.16% corresponding to the sixth and 28\textsuperscript{th} marked point frequencies from left to right in Fig. 26 (b). The extracted frequencies are 1.03 kHz and 4.22 kHz. Since the known source frequencies after the first step are 2.3 kHz and 4.22 kHz, the new frequency at present step is 1.03 kHz which has no correlation with those known source frequencies. Thus, 1.03 kHz is expanded into the source frequencies for the third step.

Third Step: the unions of 1.03 kHz and 4.22 kHz marked as blue ‘*’ in Fig. 26 (c) are removed, and the remaining frequencies marked as pink ‘*’ in Fig. 26 (c) are used. The SIs of OIs are shown in Fig. 27 (c). There is a maximal order union (16, 25) of 0.12% corresponding to the 16\textsuperscript{th} and 25\textsuperscript{th} marked point frequencies from left to right in Fig. 26 (c). The extracted frequencies are 2.71 kHz and 4.22 kHz. Since the known source frequencies after the second step are 1.03 kHz, 2.3 kHz and 4.22 kHz, the new frequency at present step is 2.71 kHz which has no correlation with the those known source frequencies. Thus, 2.71 kHz is expanded into the source frequencies for the fourth step.

Fourth Step: the unions of 2.71 kHz and 4.22 kHz marked as pink ‘*’ in Fig. 26 (d) are removed, and the remaining frequencies marked as green ‘*’ in Fig. 26 (d) are used. The SIs of OIs are shown in Fig. 27 (d). There are three maximal order unions (1, 20), (5, 20), and (8, 20) of 0.08% corresponding to the first, fifth, eighth, and 20\textsuperscript{th} marked point frequencies...
the output signal spectrum with two different frequency signals inputting is analyzed and the BA is proposed to obtain the nonlinear factors. During the output signal spectrum prediction process, the OI is proposed and its symmetry is analyzed, which can be used for the source frequency extraction. When extracting the nonlinear factors, the GA is used for the sign matrix selection. One numerical example and two simulation examples are applied to verify the proposed approach. Based on the third example, the proposed approach has been proved to be effective for excitation source frequencies and nonlinear factors identification under narrowband and broadband interference. For the multi-source inputting the nonlinear circuits, the order reduced process for emission frequency identification is introduced and has been verified in the fourth example.

The four examples show that the source frequencies can be obtained with the SIs of different OIs. The nonlinear factors also can be extracted after the source frequencies have been extracted and if the amplitude of one source can be obtained. The proposed approach can be applied to the interference source location, the analysis of nonlinear system characteristics, and the improved design for electromagnetic emission. In the future, we will analyze the relationship within the amplitudes of the excitation sources, and the nonlinear factors extraction method for the multi-source input system. The measurement may also be carried out to verify the proposed approach, in which the measurement noise will be considered.

V. CONCLUSION
An approach to extract the electromagnetic emission source frequency and nonlinear factors is proposed and comprehensively analyzed in this article. Based on the power series, the total computational complexity of this example is less than \(R^2\). Therefore, for \(R\) sources the calculation complexity is less than \(O(R^2)\).

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