Prediction of cavitation performance of a hydrofoil in water at high temperatures

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Abstract. In this paper, the thermodynamic effects of cavitation based on the computational methodology of cavitation mass transfer has been considered in the prediction of cavitation performance of a hydrofoil in water at high temperature. A cavitation mass transfer model to study this phenomenon has been presented. The cavitation model was used to simulate the flow along a NACA0015 hydrofoil at different temperatures. The results were compared with experiments. From the distribution of the pressure coefficient, it can be seen that the simulation results with the improved model have the same trends and values comparing with the experiments. At low temperature, the length of the cavity area is nearly the same with experiment, but at high temperature, the calculated cavity length is slightly longer.

1. Introduction

High temperature water is used in a certain number of specialized pumps. Cavitation characteristics in such conditions are different from that of in water at room temperature, and the cavitation performance (pump suction performance) is better at higher temperature. This is caused by the thermodynamic effects of cavitation.

In water at high temperature, the pressure observed inside cavities are substantially lower than the vapor pressure corresponding to the free-stream temperature. The reason for this is that the temperature of the liquid in the immediate vicinity of the liquid-vapor interface is depressed below the free-stream temperature, because the latent heat of vaporization must be extracted from the bulk liquid. Thus, the growth of cavitation is suppressed and the cavity size is smaller than it would be if there were no temperature depression effects.

The analyses of the thermodynamic effects of cavitation have generally been focused on obtaining correlations with temperature depression as a function of flow conditions and liquid properties. Particular methods of importance include the B-factor method (Stahl and Stepanoff 1956, Moore and Ruggeri 1968), the entrainment theory method (Holl and Wislicenus 1961, Billet and Weir 1975, Billet et al. 1981) and the mass transfer equation model (Utturkar et al. 2005), etc.

In this paper, the thermodynamic effect of cavitation based on the computational methodology of cavitation mass transfer is considered. An improved cavitation mass transfer model to study this phenomenon has been presented. This model has been used to compute the cavitation flow around a foil NACA0015. The simulation results are compared with experiments to verify the model.

2. Nomenclature
c = airfoil chord length, specific heat at constant pressure, empirical constant in evaporation and condensation 
e = total energy, heat of vaporization, condensation rate, evaporation rate 
h = enthalpy 
n = normal direction vector 
p = local static pressure 
Q = volume flux through cavity; primary dependent variable vector 
T = temperature, temperature constant 
u = component of velocity 
U = velocity vector, velocity at inlet, mass fraction 
v = bubble volume, volume fraction of vapor, density 
K = thermal conductivity of fluid 
k = turbulence kinetic energy 
e = turbulence dissipation rate 
μ = viscosity 
σ = cavitation number 
ρ = density 

Subscripts

c = cavitation

3. Governing equations for cavitation flow
In the cavitation flow simulation, the mixture model solves the continuity, momentum and energy equations for the mixture, and the volume fraction equation for the bubble phase. The governing equations are written for the mass, momentum and energy conservation of the mixture fluid, such that:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m U) = 0$$  \hspace{1cm} (1)

$$\frac{\partial}{\partial t} (\rho_m U) + \nabla \cdot [(\rho_m U \times U - (\mu_m + \mu) (\nabla U + (\nabla U)^T - \frac{2}{3} \delta \nabla \cdot U)] = -\nabla p$$  \hspace{1cm} (2)

$$\frac{\partial}{\partial t} (\rho_n h_{n,av}) - \frac{\partial p}{\partial t} + \nabla \cdot \left[ (\rho_n U h_{n,av} - \lambda_n \nabla T) \right]$$  \hspace{1cm} (3)

$$-\nabla \cdot \left[ (\mu_m + \mu_t) U \cdot (\nabla U + (\nabla U)^T - \frac{2}{3} \delta \nabla \cdot U) \right] = (m^+ - m^-) h^c$$  \hspace{1cm} (4)

In this simulation, RNG k-ε turbulence model is used. For this turbulent flow, the equations for turbulent kinetic energy and turbulent dissipation rate can be written as:

$$\frac{\partial (\rho_m k)}{\partial t} + \nabla \cdot (\rho_m U k) = \nabla \cdot \left[ (\mu_m + \frac{\mu_t}{\sigma_k}) \nabla k \right] + P_k - \rho_m \varepsilon$$  \hspace{1cm} (5-1)

$$\frac{\partial (\rho_m \varepsilon)}{\partial t} + \nabla \cdot (\rho_m U \varepsilon) = \nabla \cdot \left[ (\mu_m + \frac{\mu_t}{\sigma_\varepsilon}) \nabla \varepsilon \right] + \frac{\varepsilon}{K} \left[ C_{\alpha_k} P_k - C_{\varepsilon_k} \rho_m \varepsilon \right]$$  \hspace{1cm} (5-2)

where the eddy viscosity and production of turbulent energy are:

$$\mu_t = C_{\mu} \rho \frac{k^2}{\varepsilon}, \quad P_k = \mu_t \nabla U \cdot (\nabla U + \nabla U^T) - \frac{2}{3} \nabla \cdot U (\delta \mu_t \nabla \cdot U + \rho \varepsilon) + P_{lb}$$  \hspace{1cm} (5-3)

And the $C_{\mu}, C_{\alpha_k}, C_{\varepsilon_k}, \sigma_k, \sigma_\varepsilon$ are all model constants.

4. Cavitation mass transfer model with thermodynamic effects

4.1. Expression of density and saturation vapour pressure
In this paper, in order to simulate the cavitation phenomenon more accurately, some expressions for fluid physical properties, e.g., $\rho_l, \rho_v, p_v$, as functions of temperature and pressure are formulated. When the temperature increases, the density of water decreases, and the density of vapor and the saturation pressure increases. The variations of the density and the saturation pressure become more significant at the higher temperature. Other properties, such as surface tension, specific heat capacity, thermal conductivity, dynamic viscosity are also related with the temperature.

Since the heat transfer between phases caused by cavitation has been involved in the thermal energy equation, one can simulate cavitation thermodynamic effects only if the fluid properties, such as density, saturation pressure are changed as the function of temperature and pressure. Here, the functions we adopted are in the following forms:

The following equation of state for water by Tamman is employed.

$$\rho_l = \frac{p + p_v}{K_l(T + T_0)}$$  \hspace{1cm} (6)

where $p_v=1944.61\text{MPa}$, $T_0=3837\text{K}$ are the pressure and temperature constants of water. $K_l=472.271/\text{kgK}$ is the liquid constant of water.

Then, the equation of state for vapor can be expressed as follows by using the equation of state for ideal gas:

$$\rho_v = \frac{p}{RT}$$  \hspace{1cm} (7)

where $R_v=461.6 \text{J/kgK}$ is the gas constant of vapor. Based on a local homogeneous assumption, the mixture density $\rho_m$ is defined as

$$\rho_m = \alpha \rho_v + (1-\alpha) \rho_l$$  \hspace{1cm} (8)

where $\alpha$ is the local vapor void fraction.

In the present model, the following relation is satisfied between the local vapor void fraction $\alpha$ and the vapor mass fraction of gas phase $f$

$$\rho_m f = \alpha \rho_v, \rho_m(1-f) = (1-\alpha) \rho_l$$  \hspace{1cm} (9)

Finally, the equation of state for a locally homogeneous vapor–liquid two-phase medium can be expressed as follows by using the mass fraction of vapor phase $f$

$$\rho = f \rho_v R_T + (1-f) \rho_l K_l(T + T_0) \frac{p}{p + p_v}$$  \hspace{1cm} (10)

The saturation vapor pressure of water $p_v^*$ is given by the empirical formula as:

$$p_v^* = p_v \exp \left\{ (1-\frac{T}{T_s}) \left[ a + (b-cT)(T-d)^2 \right] \right\}$$  \hspace{1cm} (11)

$p_v=22.130\text{MPa}$, $T_s=647.31\text{K}$, $a=7.21379$, $b=1.1520\times10^{-5}$, $c=-4.787\times10^{-9}$, $d=4816$.

$T_s = -47.75404\times\exp(-p/53614.86717) - 37.80143\times\exp(-p/5739.23472) - 110.28134\times\exp(-p/642224.44991) + 475.92627$  \hspace{1cm} (12)

where $T_s$ is the saturation temperature, which changes with pressure.

Other properties, such as surface tension, specific heat capacity, thermal conductivity, dynamic viscosity, have been fitted as the functions of the temperature.

4.2. Cavitation mass transfer model developed

Physically, the cavitation process is governed by thermodynamics and kinetics of the phase change process. The liquid-vapour conversion associated with the cavitation process is modelled through $m^+$ and $m^-$ terms, which represent condensation and evaporation, respectively. These particular phase transformation rates form the basis of the cavitation model.

In the modelling of hydraulic turbine, Wu et al. (2011) used the Zhang and Wu cavitation model (2007), which is based on the theory of evaporation and condensation. The source term was developed using the Kinetic theory of mass transfer.
Mass transfer model for vapour–liquid mixture based on the theory of evaporation/condensation on a plane surface

If $p > p_r$, \[ \dot{m}^+ = C_i \left( \frac{M}{2\pi RT} \right)^{\frac{1}{2}} \left( \frac{p - p_0}{\sqrt{T}} \right) \] \[ (14) \]

If $p < p_r$, \[ \dot{m}^- = \frac{3(1 - \alpha_v - \alpha_s)}{r \rho_\ell} C_i \left( \frac{M}{2\pi RT} \right)^{\frac{1}{2}} \left( \frac{p - p_0}{\sqrt{T}} \right) \] \[ (15) \]

where $C_i = 0.13$ and $C_2 = 0.01$.

Introducing the surface tension of water, and modified to Equation (13), one got

$$ j = \left( \frac{2\sigma}{2 - \sigma} \right) p_r \left( \frac{M}{2\pi RT} \right)^{\frac{1}{2}} $$ \[ (16) \]

Also the vapor volume fraction $\alpha_v$ can be related to the bubble number density, $n$ and radius of bubble $r$ as $\alpha_v = 0.75n\pi r^3$. The value for $n$ is unknown, or is at least very difficult to determine.

For simplicity, the typical bubble size $r$ is taken to be the same as the limiting (maximum possible) bubble size. Then, $r$ is determined by the balance between aerodynamic drag and surface tension forces. A commonly used correlation in the nuclear industry is (Singhal et al. 2002)

$$ r = \frac{0.061W_{ec} \sigma}{2 \rho u_{rel}^2} $$ \[ (17) \]

For bubbly flow regime, $u_{rel}$ is generally fairly small, e.g., 5–10% of liquid velocity. By using various limiting arguments, e.g., $r \rightarrow 0$ as $\alpha \rightarrow 0$, and the fact that per unit volume phase change rates should be proportional to the volume fractions of the donor phase, the source terms can be expressed as

$$ \dot{m}^+ = C_i \frac{3 \alpha_v \rho_\ell \sqrt{k}}{\sigma} \left( \frac{M}{2\pi RT} \right)^{\frac{1}{2}} \left( \frac{p - p_0}{\sqrt{T}} \right) $$ \[ (18) \]

$$ \dot{m}^- = C_2 \frac{3(1 - \alpha_v - \alpha_s) \sqrt{k} \rho_\ell}{\sigma} \left( \frac{M}{2\pi RT} \right)^{\frac{1}{2}} \left( \frac{p - p_0}{\sqrt{T}} \right) $$ \[ (19) \]

5. Simulation results for hydro foil

5.1. Comparing simulation results with experiments

5.1.1. Model parameters for hydrofoil. In this work, the simulation model is a 3D NACA0015 hydrofoil (Stigler and Svozil 2009). Its parameters are shown in Table 1. The computational system around the hydrofoil is shown in Figs. 1 and 2.

Figure 1. Simulation model and flow conditions
Table 1. Model parameters

| Test section       | 3D NACA0015                          |
|--------------------|--------------------------------------|
| The chord length of the foil | 115 mm                              |
| The span length of the foil    | 80mm                                 |
| Incidence angles           | 8 degree, 5 degree                   |
| Fluid material           | water at 25°C, 50°C and 100°C       |
| Inlet velocity          | 8m/s                                 |
| Cavitation number       | 1.5                                  |

The cavitation number is shown in the following equation:

\[
\sigma = \frac{p_v - p_e}{1/2 \rho_o u_{out}^2}
\]  \hspace{1cm} (20)

where \(p_v\) is saturation pressure, \(p_e\) is the inlet pressure.

The definition for the pressure coefficient can be written as:

\[
C_p = \frac{p - p_v}{1/2 \rho_o u_e^2}
\]  \hspace{1cm} (21)

where \(p_v\) and \(u_e\) are reference pressure and velocity, respectively. In this paper, they are the inlet pressure and outlet velocity of the flow channel respectively.

5.1.2. Comparing simulation results of the model with experiments. Figure 3 shows the pressure coefficient \((C_p)\) distribution at 25°C. It is indicated that the cavity length is the same with the experiments, but the \(-C_p\) values along the cavity edge are larger than experiments.

![Figure 3. Pressure coefficient distribution at 25°C](image-url)
Figure 4 shows the pressure coefficient distribution at 50°C, which has the same trend and length with the experiments.

![Figure 4](image)

**Figure 4.** Pressure coefficient distribution at 50°C

5.2. Comparing between different cavitation models at different temperatures

5.2.1. *Simulation results at different temperatures.* From Figure 5, one finds that the cavity length decreases with the temperature increases, which is the same with the traditional theory considering the thermodynamic effects (Holl et al. 1975).

In order to evaluate the cavitation model, we calculated the NACA 0015 at different cavitation numbers, different attack angles and different inlet velocities. At the cavitation number 1.5, the figures show the same trend. But the cavity length is smaller than those by Tani and Nagashima (2002). At lower cavitation number 1.0, the cavity length increases.

![Figure 5](image)

**Figure 5.** Pressure coefficient distribution at different temperatures
Figure 6. Distribution of pressure coefficient at 25°C, compared with other simulation results (Tani and Nagashima 2002)

(a) 25°C  (b) 50°C  (c) 70°C  (d) 100°C  (e) 150°C

Figure 7. Volume fraction of water vapor at different temperatures
Figure 7 gives the results of volume fraction of vapor at different temperatures.

(a) 150 °C  
(b) 100 °C  
(c) 70 °C

Figure 8. Temperature change at different temperatures

Figure 8 gives the temperature distributions around the foil at different temperature condition. Figure 9 shows temperature difference distributions along the foil surface at 25°C, 100°C and 150°C. One can draw a conclusion that at 70 °C the temperature changes only slightly, so the thermodynamic effects may be neglected in the calculation. At 100°C, the temperature changes about 1.1°C, and at 150°C, the temperature change about 0.5°C. But at the higher temperature, the saturation pressure changes greater than that at the lower temperature.

Figure 9. Temperature difference distributions at 25°C, 100°C and 150°C
6. Conclusions
(1) An improved cavitation model with thermodynamic effects based on evaporation/condensation has been proposed. This model is derived from bubble dynamics and thermal dynamics to considering the thermal effects of cavitation flow.
(2) The expression of density of water and vapor, saturation pressure and saturation temperature are included in the cavitation model.
(3) The cavitation model was used to simulate the flow distribution along a NACA0015 hydrofoil at different temperatures. The results are compared with experiments. From the distribution of pressure coefficient, it can be seen that the simulation results with the improved models have the same trends and values with the experiments. At low temperature, the cavity area length is nearly the same with experiment, but at high temperature, the cavity length is slightly longer.
(4) At 70 °C the temperature changes only slightly, so the thermodynamic effects may be neglected in the calculation. At 100°C, the temperature changes about 1.1°C, and at 150°C, the temperature change about 0.5°C. But at the higher temperature, the saturation pressure changes greater than that at the low temperature.

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