Determining the cosmological parameters from the linewidths of active galaxies.

C.M. Rudge and D.J. Raine

Astronomy Group, University of Leicester, University Road, Leicester, LE1 7RH, UK.

ABSTRACT

We have previously shown that the linewidth distribution in AGN can be accounted for by an axisymmetric broad emission line region. In this paper we show that the linewidth distribution changes with redshift and that these changes are dependent on \( H_0 \) and \( q_0 \). We show that relatively small samples of AGN at high redshift with measured linewidth at half maximum can be used to distinguish between values of \( H_0 \) and \( q_0 \). Furthermore larger low redshift samples can be used to distinguish between luminosity functions and hence different models of quasar evolution.

Key words: galaxies: active – galaxies: Seyfert – quasars: emission lines – cosmology: theory

1 INTRODUCTION

It is generally expected that the width, \( v \), of the broad emission lines in active galactic nuclei (AGN) will show some dependence on the luminosity, \( L \), of the nucleus as it is reasonable to assume that in higher luminosity systems the emitting gas have higher velocities. It is known from observations that in variable sources linewidths vary in this sense with luminosity, e.g. \( \text{CIV} \) in NGC4151 (Fahey, Michalitsianos & Kazanas 1991). However it is not clear that the same dependence of linewidth upon luminosity can be extended across samples of AGN. Observations of samples (Wills et al. 1993, Puchnarewicz et al. 1997) show some evidence for a \( v-L \) relation, with however a large scatter. Linewidths must therefore depend also on another parameter. It is known from observations that several properties of AGN appear to be dependent upon the line of sight to the observer. In radio loud systems the linewidth shows a clear dependence upon \( R \) (Wills & Brotherton 1993), the ratio of core to lobe radio power, which is an indicator of viewing angle. While it is not clear that the broad emission line regions (BLR) are the same in radio loud and radio quiet systems it is not unreasonable to expect that the linewidths in radio quiet systems will also be viewing angle dependent. While it is possible that other parameters play a role, the minimal hypothesis is that \( v = v(L, i, \lambda) \) with the same functional form for all systems, but different for the lines at various wavelengths \( \lambda \) (since these have different widths). The viewing angle, \( i \), is the angle of the axis of the BLR to the line of sight, i.e. \( i = 0 \) for systems viewed face on. Since it is difficult to measure the angle to the line of sight for individual objects with any confidence, this hypothesis has to be tested statistically. To do so requires an assumption about the angular distribution and the luminosity function for AGN. For the angular dependence it is natural to expand in multipole moments to as high an order as the data justifies.

In a previous paper (Rudge & Raine 1998) we used a random distribution in angle (cut off beyond some angle \( \lambda \)), the luminosity function from Boyle, Shanks & Peterson (1988) and a dipolar dependence on angle to construct a line width distribution function which could be matched to observations. In that paper systems were generally at low redshift and so cosmological effects were not considered. These cosmological effects enter in two ways. First through the luminosity function, and second through the minimum observable luminosity in the sample at each redshift \( z \). Comparing the linewidth distribution at different values of redshift therefore provides, in principle, a method to determine the cosmological parameters \( H_0 \) and \( q_0 \). The aim of this paper is to test the feasibility of the method in terms of the number of linewidth measurements required to determine these parameters. In fact, since we do not have an observational sample to test, we have to construct one; this requires assumptions about the cosmology. We shall therefore consider the related question of the number of linewidth measurements required to distinguish between significantly different cosmologies, between \( H_0 = 50 \), 75 and 100 and between \( q_0 = 0.0 \) and \( q_0 = 0.5 \).

The method depends on knowledge of the luminosity function which itself depends on the cosmological parameters. This raises several issues. First, whether the method is any better than using fits to the luminosity function alone to determine the cosmology. While in principle this would be possible, in practice the dependence is weak and it is not easy to distinguish between different cosmological mod-
Influence of the cosmology on the luminosity function means that very little (if any) iteration is necessary.

A potential problem is that the luminosity function depends on intrinsic evolution as well as the cosmology. However, we find that this shows up in an identifiable manner in the luminosity distribution, so it can be allowed for. Equivalently, we can use the method to solve simultaneously for the cosmology and to constrain the AGN evolution. We conclude that to distinguish between open and closed cosmologies (even if the Hubble parameter is not taken as fixed from other observations) would require in the range 100 to 500 line profiles at a resolution of 500 km s$^{-1}$ at a redshift of order 2.

2 DEVELOPMENT OF THE $z$ DEPENDENT LINE WIDTH DISTRIBUTION

2.1 Line width distribution

As in our previous paper (Rudge & Raine 1998) we assume a model for the BLR in which the FWHM is dependent primarily upon ionising luminosity and viewing angle alone. While it is accepted that there are other parameters which may have some effect on linewidth we have shown that such a restriction still allows us to account for the linewidth distribution. Furthermore, in an axisymmetric model of the BLR, it is not unreasonable to expect these to be the most important physical parameters affecting linewidth. Thus the FWHM, $v$, of a given broad emission line is taken as a function of the ionising luminosity and the inclination of the system. This function can be expanded in spherical harmonics with luminosity dependent coefficients. In Rudge & Raine (1998) we showed that the distribution of linewidths in low-redshift systems could be reproduced if this function were taken to be axisymmetric and only the first two terms of this series were retained and the coefficients taken to have a common dependence on luminosity. The FWHM of a given emission line is then given by

$$v = (a + b \sin i)L_{44}^\alpha$$

(1)

where the constants $a$, $b$ and $\alpha$ are chosen for each emission line, $i$ is the angle to the line of sight of the axis of the BLR and $L_{44}$ is the luminosity in units of $10^{44}$ erg s$^{-1}$.

Since it is difficult to determine the line of sight angle in a given system, at least with any accuracy, we are led to consider the linewidth distribution rather than the linewidths of individual objects. Assuming that the inclination of AGN is random across the sky, the number of systems per unit velocity range at each $v$ is given for objects at low redshift by

$$N(v) = \int \sin i \Phi(L_{44})dL_{44}$$

(2)

where the luminosity function $\Phi(L_{44})$ gives the luminosity distribution. To extend this to high redshift systems we need to take account of the cosmological and intrinsic evolution in $\Phi$ and the redshift dependence of the range of luminosity covered in flux-limited surveys.

2.2 Luminosity functions

The shape of the predicted linewidth distribution will clearly have some dependence on the shape of the luminosity function. It is known from observations that the quasar population evolves with redshift causing the luminosity function to change with redshift. Thus it is natural to expect that the shape of the predicted linewidth distribution curve must also change with redshift. There is still much debate over how best to model this evolution. Once a model for the evolution has been chosen, the parameters used to fit the data are then also dependent on the choice of values of $H_0$ and $q_0$. Thus we shall first assume a luminosity function and an evolution model, and test for differences when $H_0$ and $q_0$ are changed. We then test for differences when the luminosity function and evolution model are changed.

The quasar population can evolve in a combination of two basic ways. First, by pure luminosity evolution, where only the luminosity of quasars evolves with redshift and the total number remains constant. Second, by pure density evolution, where the total number evolves with redshift. Current work appears to favour pure luminosity evolution but it should be noted (e.g. Boyle et al. 1994) that there is still a great deal of uncertainty and no obvious best choice model.

There are several models for the luminosity function available including those of Boyle et al. (1994), Pei (1993), Maccacaro et al. (1991), Hasinger (1998) and Boyle et al. (1998). To begin with we shall concentrate on that of Boyle et al. (1994) because of the large sample size, the detailed information given for different models of evolution and the provision of results for two extreme values of $q_0$ (0.0 and 0.5) spanning the range of possibilities. The luminosity functions of Hasinger (1998) and Boyle et al. (1998) should provide an improvement on this, with Hasinger (1998) using higher quality ROSAT data and Boyle et al. (1998) using ASCA data. However neither of these analyses contain information on the effect of changing $q_0$. The number of objects in the ASCA sample is also small. More information should become available for both of these luminosity functions in the near future. Maccacaro et al. (1991) use only the Einstein EMSS (Extended Medium Sensitivity Survey, Stocke et al. (1991)) objects as used by Boyle et al. (1994) and develop a similar broken power law model of the luminosity function. Perhaps the best alternative to the luminosity function of Boyle et al. (1994), for our purposes, is that of Pei (1993).
Table 1. Luminosity function parameters used for 2 power-law models G and H of Boyle et al. (1994).

| $q_0$ | $\gamma_1$ | $\gamma_2$ | $\log L^*(z = 0)$ | $k$ | $z_{\text{max}}$ | $\Phi^*$ |
|------|------|------|------------------|----|----------------|--------|
| 0.0  | 1.53 | 3.38 | 43.70            | 3.03 | 1.89           | 0.79   |
| 0.5  | 1.36 | 3.37 | 43.57            | 2.90 | 1.73           | 1.45   |

Errors $\pm 0.15 \pm 0.1 \pm 0.2 \pm 0.1 \pm 0.1$

This uses a combined sample of around 1200 sources and tests two different models for the luminosity function. Note however that this is an optical rather than an X-ray luminosity function. While this may seem an advantage in the following work, which focuses on the H$\beta$ optical emission line, Maccacaro et al. (1991) emphasize the incompleteness of optical surveys and question the methods used to correct for this. In developing the redshift dependent luminosity distribution we shall use the luminosity function of Boyle et al. (1994) and use that of Pei (1995) to test for differences in the resulting distribution functions.

The luminosity function given in Boyle et al. (1994) gives several possible fits to the observed data using different evolution models. We have chosen to use models G and H of Boyle et al. (1994) as they are the best pair of fits to the combined ROSAT and EMSS data using the same evolution model but different values of $q_0$. The models are given in the form of a double power law

\[ \Phi(L) = \Phi^* L^\gamma_{\text{opt}} \quad L < L^* \]
\[ \Phi(L) = \frac{L^\gamma_{\text{max}}}{L^*} L^\gamma_{\text{max}} \quad L > L^* . \]

The dependence on $z$ is included by evolution of $L^*$. Luminosity evolution rather than evolution of the density is assumed for the models G and H. Thus, the quasar luminosity evolves by scaling in the following way:

\[ L(z) = L(z=0)(1+z)^k \quad z < z_{\text{max}} \]
\[ L(z) = L(z=0)(1+z_{\text{max}})^k \quad z > z_{\text{max}} . \]

It is important to note that (3) is in the form given in Boyle et al. (1994) and is correct for the de-evolved luminosity function, i.e. the evolution model is applied to give all systems as if they were at $z=0$. To introduce a $z$ dependence into (3), i.e. to replace $\Phi(L)$ by $\Phi(L, z)$, the 2-power law parameterization needs an extra scaling factor of $(1+z)^\gamma k$ to ensure that $\Phi(L^*, z)$ is constant over $z$. Note that $\Phi_{\text{opt}}$ is given in units of $10^{-6}$ Mpc$^{-3}$ (10$^{44}$ ergs s$^{-1}$)$^{-1}$ and thus must be scaled by $L^*(z=0)$ before entering into (3). The parameters for the two models are given in table 1. Fig. 1A shows how this luminosity function evolves with $z$ and how the evolution changes with different values of $q_0$ and $H_0$. Note for later reference that Boyle et al. (1993) suggested that the optical and X-ray luminosities are related in the following way $L_X \propto L^* \propto r_{\text{opt}}^{0.85 \pm 0.08}$.

The luminosity function of Pei (1993) is given by

\[ \Phi(L, z) = \frac{\Phi_{\text{opt}}}{L_{\text{opt}}} \left( \frac{L}{L_{\text{opt}}} \right)^{-\beta} e^{-\left( \frac{L}{L_{\text{opt}}} \right)^{1/4}} \]

where the evolution model defines

\[ L_{\text{opt}} = L_{\text{opt}}(1+z)^{(1+\alpha)} e^{-\left( z-\delta^2 / 2\delta^2 \right)} . \]

The parameters for the model fits to the luminosity function are reproduced in table 2. Fig. 1B shows similar information to fig. 1A, but in this case for the exponential fit rather than the broken power law fit of Boyle et al. (1994).

We now have two models for the luminosity function at two different values of $q_0$. These have both been produced under the assumption that $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$. To use other values of $H_0$ requires only an application of simple scaling factors to $L$ and $\Phi(L)$. These are

\[ L(H_0 = 50) = L(H_0) \left( \frac{H_0}{50} \right)^2 . \]
\[ \Phi(L, z, H_0) = \Phi(L, z, H_0 = 50) \left( \frac{H_0}{50} \right)^3 . \]

Figure 1. Broken power law model of the luminosity function by Boyle et al. 1994. Both panels show $0 < z < 3$ with the plot moving to the right with increasing $z$. The upper panel is for $q_0 = 0$, $H_0 = 50$ and the lower panel is for $q_0 = 0.5$, $H_0 = 100$. 

Table 2. Parameters of the Exponential $L^{1/4}$ law luminosity function of Pei (1993). Note that $h = H_0/100$.

| Parameter | $(h, q_0, \alpha)$ | $(h, q_0, \alpha)$ |
|-----------|-------------------|-------------------|
| $\beta$   | $0.93 \pm 0.03$   | $2.73 \pm 0.05$   |
| $z_{\text{max}}$ | $9.87 \pm 0.10$ | $9.78 \pm 0.10$ |
| $\log(L_{\text{opt}}/L_{\odot})$ | $5.33 \pm 0.17$ | $4.34 \pm 0.20$ |

Equation (7) scales the luminosity in a universe with $H_0 \neq 50$ to the value to be used in (8) where $H_0 = 50$. Equation (8)
Figure 2. Pei (1995) exponential model of the luminosity function. Both panels show $0 < z < 3$ with the plot moving to the right with increasing $z$. The upper panel is for $q_0 = 0.1$, $H_0 = 50$ and the lower panel is for $q_0 = 0.5$, $H_0 = 100$.

scales the calculated value of $\Phi(L, z)$ to the value required for the universe where $H_0 \neq 50$. These conversions reflect the fact that a smaller value of $H_0$ would imply a more extended universe, making luminosities greater but thinning out the galaxies, making calculated densities less (Weedman 1986).

2.3 Limits of integration

The redshift dependence of the linewidth distribution arises in just 2 places (assuming that the BLR itself does not change explicitly with $z$). The first is in the luminosity function as detailed above. The second is through the $z$ dependence of the limits of integration in (3). This is partly accounted for in the intrinsic evolution used in the luminosity function e.g. equation (4). However the lower limit of integration is also affected by the flux limit of the observations. At some value of $z$ the faintest source we can see is brighter than the evolved lower limit; in the Boyle et al. model this occurs for $L_{\text{observable}} > L_{\text{min}}(1+z)^8$. So at any value of $z$ the lower luminosity limit is the maximum of the two minima (the observational limit and the evolved lower limit).

The minimum observable luminosity $L$ corresponding to a given flux limit $F$ in the standard Friedmann cosmology (e.g. Marshall et al. 1984) is obtained from

$$F = \frac{L}{4\pi d_L^2}$$

where the luminosity distance $d_L$ is given by

$$d_L = \frac{c}{H_0} \left[ z q_0 + (q_0 - 1)(\sqrt{1 + 2q_0z} - 1) \right] \quad q_0 > 0$$

$$d_L = \frac{c}{H_0} \left[ 1 + \left(\frac{z}{2}\right) \right] \quad q_0 = 0$$

Thus for a known flux limit and redshift it is possible to calculate the luminosity distance, $d_L$, and hence the minimum observable luminosity at this redshift. For the ROSAT observations used in Boyle et al. (1994), $F \sim 4 \times 10^{-15}$ ergs s$^{-1}$ cm$^{-2}$. In the following work, we assume that the samples generated will be complete down to some flux limit. While any incompleteness, particularly at the low luminosity end, will have an effect on the shape of the luminosity distribution and consequently the application to cosmology, for a sample of a few hundred objects at high redshift it should be possible to generate a luminosity function specific to that sample. It is likely that tailoring the luminosity function to the sample, whether complete or not, will improve the accuracy of the results.

2.4 New form for the linewidth distribution

The linewidth distribution is now redefined to include the redshift dependent luminosity function and integrated over $z$ so that the number of systems, $N$, at each FWHM, $v$, is given by

$$N(v) = \int \int_{L(z)_{\text{min}}}^{L(z)_{\text{max}}} \frac{\sin i}{d\Omega} \Phi(L, z) dL d\Omega dz$$

3 TESTING

We now develop a method to test the way in which the linewidth distribution is affected by changes in $q_0$, $H_0$ and the luminosity function. Any significant differences in the shape of the distribution curve will clearly become more obvious at higher redshift. Any test therefore requires a sample of FWHM from high redshift AGN, say $2.4 < z < 2.6$. As yet there are no published samples with a sizable number of objects and FWHM measurements at high redshift. For example, the RIXOS sample (Puchnarewicz et al. 1997) has only 6 objects in the range $2 < z < 3$. The aim of the current work is to provide a case for carrying out such observations by showing that the size of sample required is not prohibitively large. This problem is approached in the following way.

First we produce a set of model parameters $a$, $b$ and $\alpha$ for a sample of objects at low redshift where differing cosmology and evolution models make little difference. This is done for both luminosity functions. The value of $\chi^2$ between the two model distribution curves is minimized by choice of $a$ and $b$. Then, for a chosen pair of values for $q_0$ and $H_0$ and luminosity function, we use these parameters to produce a set of model distribution curves at high redshift. Once observational data is available this would then be compared with the theoretical distribution curve, and $q_0$ and $H_0$ would be altered in an iterative process until convergence was achieved. However, as this data is not available, at this stage we generate a further model curve for different values of the cosmological parameters. A small random sample is then generated from the first distribution curve and a $\chi^2$ test carried out to find how large this sample needs to be.
so that the second distribution curve differs from the first at the 95 per cent confidence level.

We also consider whether the distribution curve is sensitive to the small changes in the luminosity function model on changing \(q_0\). This is done by generating a distribution curve for say \(q_0 = 0.0\) with the luminosity function as for \(q_0 = 0.5\) and comparing to the true case where \(q_0 = 0.0\) in the luminosity function. The fits of the luminosity function to the observed data currently show no significant improvement by changing \(q_0\). We will show that the linewidth distribution is more sensitive to the value of \(q_0\) than is the luminosity function and thus \(q_0\) can be determined with a smaller data set by this method.

This test of the effect of changing \(H_0\) and \(q_0\) is carried out on the linewidth distribution produced using both luminosity functions. A comparison between the two sets of distributions can then be carried out to find which features of the distribution curve are most sensitive to changes in the luminosity function. We can also test how large a sample would be needed to distinguish between the luminosity functions.

Note that for the \(\chi^2\) test the distribution curves are transformed into histograms with bins of width 500 km s\(^{-1}\) to reflect the expected resolution of observations. The random sample is generated from this by a Monte Carlo type method to produce a similar histogram. The resulting histograms are then normalized to contain the same number of total objects. Thus the \(\chi^2\) test is not sensitive to the absolute value of \(N(v)\) only to the shape of the curve. The test is not then sensitive to the evolution assumed in the luminosity function when only a small range of \(z\) is used. However it will be seen that plots of the peak height of the distribution against \(z\) do show very clear differences when the evolution model is changed. Discarding evolution at this stage of the testing is not unreasonable as the absolute value of \(N(v)\) is difficult to find observationally with a high degree of accuracy, since this requires a high level of completeness in the sample. It will be seen that the evolution model is testable separately by consideration of the predicted dependence of \(N(v)\) upon \(z\).

4 RESULTS

4.1 Low redshift distribution

For a low redshift sample we have selected those objects in the RIXOS sample (Puchnarewicz et al. 1997) with \(z < 0.5\) and measured FWHM H\(\beta\). This provides a sample of 54 objects including some for which the FWHM has considerable uncertainty. Fig. 3 shows the linewidth distribution for these objects overlaid with the model curve for \(a = 2500\) km s\(^{-1}\), \(b = 7000\) km s\(^{-1}\) and \(\alpha = 0.35\) using the luminosity function of Boyle et al. (1994). A similar fit is produced with the Pei (1995) luminosity function using \(a = 1000\), \(b = 10000\) and \(\alpha = 0.31\). Note that \(i_\ast\), the angle beyond which the BLR is totally obscured to view, is fixed at 60° for all calculations.

4.2 Evolution with \(z\)

While the greatest differences in the distribution are naturally expected to be seen at the highest redshifts, in order to make the possibility of verification by observation realistic we have chosen to use values for \(z\) in the range \(2.4 < z < 2.6\) (lower) for \(q_0 = 0.5\) (solid) and \(q_0 = 0.0\) (dashed). All curves produced using the Boyle et al. 1994 luminosity function.

Fig. 4 shows how the distribution around \(z = 2.5\)
Figure 5. Change in the distribution of linewidths with $z$ for $q_0 = 0.5$ (solid) and $q_0 = 0.0$ (dashed) produced using the Boyle et al. 1994 luminosity function. The upper panel shows the FWHM of the distribution, the middle panel shows the linewidth at maximum $N(v)$ and the lower panel shows the value of $N(v)$ at the peak.

Changes between $q_0 = 0.0$ and $q_0 = 0.5$. At high redshift there is a clear difference between the two distributions with that for $q_0 = 0.0$ having a peak at higher FWHM, but a smaller number of systems at that value due to a broader distribution. However at low redshift the distribution for $q_0 = 0.0$ has a narrower width and hence a higher peak. Fig. 5 shows how three key features of the shape of the distribution change with redshift.

(i) The full width at half maximum of the model curve.
(ii) The linewid th at the peak value
(iii) The value of $N(v)$ at this peak

Fig. 5 shows the same information as fig. 3 except that the shape of the luminosity function is fixed as that for $q_0 = 0.5$ to test whether the changes in the linewidth distribution with $q_0$ are caused primarily by the changes in the luminosity function. Both figs. 3 and 5 are produced using the Boyle et al. (1994) luminosity function.

Fig. 5 shows the change with redshift of the distribution width, peak position and peak height produced with the different luminosity functions, with $q_0 = 1.0$ and $H_0 = 50$. We have carried out a similar $\chi^2$ test on the distributions produced at $2.4 < z < 2.6$ to find how large a sample would be needed to rule out one luminosity function assuming that the other is correct. As the $\chi^2$ test is not symmetrical the results are as follows. If the Boyle luminosity function is correct then a sample of about 150 objects would be needed to rule out that by Pei. Conversely, if Pei has the correct function then about 250 objects would be needed to rule out the Boyle form.

These first results show that changes in the value of $H_0$ and $q_0$ do significantly affect the shape of the linewidth distribution. However we need the results of the $\chi^2$ test to show whether these differences can be observed with a reasonable size sample. For each pair of $q_0$, $H_0$ values we have generated a random sample of linewidths and tested this against other pairs of values. Table 3 shows the minimum sample sizes to reject the second pair of values in each case at the 95 per cent confidence level using the Boyle et al. (1994) luminosity function.

Encouragingly many of the values in the table are of order 100 which is not prohibitively large; hopefully samples of this size will exist in the near future. However this is still beyond the scope of published samples. Table 4 shows the same information as table 3 except we use the Pei luminosity function and $q_0 = 0.1$ in place of $q_0 = 0.0$. 

Figure 6. As for fig. 5 but with $q_0$ fixed at 0.5 in the luminosity function. Comparison with fig. 5 shows that the dependence of the distribution on $z$ is therefore not driven by changes in the luminosity function.
5 DISCUSSION

These results show clearly that the linewidth distribution changes with redshift. McIntosh et al. (1998) find some observational evidence for an increase in FWHM Hβ between their sample at $2 < z < 2.5$ and that of Boroson and Green (1992) at low redshift, $z < 0.5$. This dependence of FWHM upon $z$ may be biased by selection of only higher luminosity objects in the high redshift sample. More importantly this work shows that the evolution depends significantly on the values of $H_0$ and $q_0$ and also on the choice of luminosity function. The $\chi^2$ test shows that we only need samples of $\sim 100$ systems at $z = 2.5$ to be able to distinguish between extreme values of the cosmological parameters for a given luminosity function. While it is encouraging that these sample sizes are not prohibitively large, the amount of FWHM data required is still beyond that currently published. There is however hope that the required data will be available in the near future. Searches for clusters of quasars at high redshift, such as those by Boyle et al. (1997) and Newman et al. (1997), may well provide the necessary information.

Perhaps the most significant result is that the major dependence on $q_0$ is not within the luminosity function, as can be inferred from fig. 3. The X-ray luminosity function of Boyle et al. (1994) uses a large sample ($\sim 500$) of objects but cannot distinguish successfully between values of $q_0$. Using a linewidth distribution model should enable us to distinguish between values of $q_0$ with samples of around 100 objects. Current observational efforts to produce large samples of quasars (e.g. Boyle et al. 1997) will provide us with a good source of candidates to observe further to obtain high resolution spectra from which FWHM measurements can be made.

Comparison of luminosity functions is perhaps easier than testing for $q_0$ and $H_0$ as initially only low redshift samples are needed. These samples do need to be much larger ($\sim 500$) but there is much more available data at low redshift. It may be that with a sample of this size, it is not possible to get a good fit to the linewidth distribution when certain luminosity functions are used. The distribution when using the Pei function was noticeably narrower than that for $q_0$. Using a linewidth distribution model should enable us to distinguish between values of $q_0$ with samples of around 100 objects. Current observational efforts to produce large samples of quasars (e.g. Boyle et al. 1997) will provide us with a good source of candidates to observe further to obtain high resolution spectra from which FWHM measurements can be made.

Table 4. As for table 3 except in this case the distributions were generated using the Pei (1995) luminosity function.

| $q_0$ | 0.1 | 0.1 | 0.1 | 0.5 | 0.5 | 0.5 |
|-------|-----|-----|-----|-----|-----|-----|
| $H_0$ | 50  | 75  | 100 | 50  | 75  | 100 |
| 0.1   | 50  | 126 | 57  | 98  | 51  | 43  |
| 0.1   | 75  | 201 | 176 | 3466| 94  | 57  |
| 0.1   | 100 | 71  | 407 | 407 | 98  |
| 0.5   | 50  | 167 | 3199| 176 | 94  | 57  |
| 0.5   | 75  | 53  | 166 | 2332| 166 | 131 |
| 0.5   | 100 | 46  | 77  | 167 | 408 | 447 |

Figure 7. Comparison of changes in linewidth distribution with $z$ using the Pei (1995) luminosity function (solid) and the Boyle et al. (1994) luminosity function (dashed). Both distributions produced using $H_0 = 50$ and $q_0 = 0.5$. Individual panels are as for fig. 3.

Table 3. Minimum sample size of quasar linewidths for which the pair of $q_0$, $H_0$ values in the first column can be rejected when the sample is generated using the pair of values in the first row of the table. Distributions were generated using the Boyle et al. 1994 luminosity function.

| $q_0$ | 0.0 | 0.0 | 0.0 | 0.5 | 0.5 | 0.5 |
|-------|-----|-----|-----|-----|-----|-----|
| $H_0$ | 50  | 75  | 100 | 50  | 75  | 100 |
| 0.0   | 50  | 73  | 51  | 72  | 43  | 31  |
| 0.0   | 75  | 142 | 106 | 769 | 82  | 51  |
| 0.0   | 100 | 52  | 270 | 444 | 200 | 82  |
| 0.5   | 50  | 121 | 5201| 176 | 94  | 57  |
| 0.5   | 75  | 52  | 142 | 1200| 168 | 135 |
| 0.5   | 100 | 34  | 65  | 165 | 71  | 447 |
sitive to this. Both of the luminosity functions used here are modelled with a specific low luminosity cut-off.

In this work, as in Rudge & Raine (1993), we have assumed that the distribution of $\sin i$ is uniform and that the cut off angle $i_*$ is constant. In reality this is probably not the case. It is reasonable to expect that $i_*$ increases with luminosity, i.e. the opening angle of AGN is greater in higher luminosity systems. In Rudge & Raine 1999 (unpublished) we have found in the RIXOS sample [Puchnarewicz et al. 1997] that this is in fact the case, while the distribution of $\sin i$ is uniform in each luminosity bin. As a result the distribution of $\sin i$ will be biased toward face-on objects. While the effects that this has on the cosmological predictions are not clear as yet, any such effect will be reduced by removing the lower luminosity objects from samples. This, to some extent, is done naturally at high redshift as we cannot observe the lower luminosity sources assumed to be present. However reducing the range of luminosity considered will probably also result in an increase in the sample size needed.

With the exception of the recently developed supernova observations, all methods for obtaining $q_0$ suffer from uncertain evolutionary effects which introduce a scatter comparable to the magnitude of the effect being measured. Our method is no exception. What appears to be surprising, however, is that not only can the data be used to model the source evolution, but that this can be done in many cases with a relatively modest number of observations.

While our work has concentrated on modelling the spread of linewidths by an axisymmetric BLR other authors have suggested other parameters than viewing angle to account for the scatter in the linewidth–luminosity relation, e.g. $v = v(L, \alpha_X)$ model of Wandel & Boller (1998) and the model of Robinson (1999) which has linewidth changing with profile curvature.

In fact our results do not depend on the assumption made here that the scatter in the $v-L$ relation is a viewing angle effect: any analysis which attributes this scatter to a single additional parameter will give similar results. Thus, while this method of obtaining the cosmological parameters is unlikely ever to achieve the accuracy or robustness of the supernova method, we believe it to be worth pursuing as a viable supplementary approach.

6 CONCLUSION

In this paper we have shown that in an axisymmetric broad line region, the linewidth distribution exhibits a strong dependence on $H_0$ and $q_0$. More importantly we have shown that these dependencies should be testable in the near future with observed samples of $\sim 100$ objects at high redshift. We have also shown that the changes are not primarily a result of changes in the luminosity function due to uncertainties in the value of $q_0$. It should also be possible to distinguish between model luminosity functions using large, low redshift samples or small, but complete, high redshift samples. This provides the theoretical basis for a new method of determining the values of $q_0$ and $H_0$ using observations of AGN. This statistical method makes no attempt to find the distance and luminosity of individual objects.

7 ACKNOWLEDGEMENTS

CMR acknowledges the support of PPARC, in the form of a research studentship. The authors wish to thank Gordon Stewart and Adam Blair for discussion of the luminosity function.

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