Mutual coupling between two superconducting strip lines horizontally-placed in niobium integrated chips

Yoshinao Mizugaki, Akio Kawai, Ryuta Kashiwa, Masataka Moriya and Tadayuki Kobayashi

Department of Electrical Engineering, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182-8585, Japan
E-mail: mizugaki@ee.uec.ac.jp

Abstract. We evaluate mutual inductance between two horizontally-placed strip lines in niobium integrated chips, by means of both experiments and numerical calculations. In the experiments, one strip line is used as a part of a superconducting quantum interference device (SQUID) loop, and the other is used as a control line for applying external magnetic flux to the SQUID loop. The mutual inductance is obtained from the modulation period of the SQUID. In the numerical calculations, we employ a three-dimensional inductance extraction software (FastHenry). Three types of strip line structures on a lower ground plane (GP) are examined: two horizontally-placed microstrip lines without an upper GP (MSLs), two horizontally-placed strip lines with a floating upper GP (FLSLs), and two strip lines with an upper GP connected to the lower GP (COSLs). We have found that each strip line structure has different dependence of mutual inductance on the line center distance. The mutual inductance between the MSLs exhibits strong dependence on the line center distance, whereas that between FLSLs is almost independent on the distance. Negligible mutual inductance is obtained for the COSLs.

1. Introduction
The state-of-the-art niobium (Nb) integration technology enables us to utilize 10 superconducting (SC) layers with the minimum line width of 1.0 μm [1], which would improve the integration of single flux quantum (SFQ) devices. High-density integration of SC lines will be widely employed in the future multi-layered Nb LSI. Mutual coupling between two SC lines, however, has not fully evaluated for such multi-layered Nb IC, where SC lines are sandwiched by two SC ground planes (GPs).

In this article, we evaluate the mutual inductance between two horizontally-placed SC lines, by means of both experiments and numerical calculations. Dependence of the mutual inductance on the distance between two SC lines is presented for three types of line structures.

2. Methods
2.1. Line Structures
We have examined three types of SC line structures shown in Fig. 1. All of three structures have two SC lines. In this work, one is used as a control line generating magnetic flux, which is referred to as a “CL” line. The other is used as a sense line. In the experiments described below, the sense line is a part of a superconducting quantum interference device (SQUID) loop, and hence, referred to as a “SQ” line.
The first structure among three is the microstrip line (MSL) structure where SC lines are placed over a single SC GP. The second and third structures, FLSLs and COSLs, are strip lines with both lower and upper SC GPs. The upper GP (UGP) for the FLSLs is electrically-floating, whereas that for the COSLs is electrically connected to the lower GP (LGP) at two points along the lines.

2.2. Experiments
Test circuits were fabricated using the SRL Nb standard process [2], in which four Nb layers were available. We use the three layers from the bottom for the LGP, SC lines, and UGP. Their thicknesses are 300, 300, and 400 nm, respectively. The distance between the LGP and SC lines is 300 nm, whereas the distance between the SC lines and UGP is 400 nm. The width of the UGP for the SL structure is 250 μm. The LGP is placed all over a test chip of approximately 5 mm × 5 mm.

In this work, the widths of both CL and SQ lines are fixed at 4.8 μm. The SQUID loop is composed of the SQ line, LGP, and two 2 × 2 μm² Nb/AlOₓ/Nb Josephson junctions placed 300 μm apart. The two contacts between the LGP and UGP in the COSL structure are placed close (within 10 μm) to the two Josephson junctions, respectively.

Hereafter, the distance between the line centers is referred to as “the line distance d.” Four test circuits were designed for each line structure to investigate the dependence of the mutual inductance on the distance. The line distances are 6, 9, 11, and 15 μm. Mutual inductance was obtained from the SQUID modulation period against the control current. At least three chips were measured for each test circuit. Below we present their average values.

2.3. Numerical calculations
Numerical calculations of the mutual inductance were carried out using the three-dimensional inductance extraction program “FastHenry” with superconductor support [3, 4]. For simplicity, all layers were assumed to be flat. Figure 2 shows the cross-sectional view of strip lines. The magnetic penetration depth was set to 91 nm for all Nb layers.
3. Results and Discussion

Figure 3 shows the dependence of the mutual inductance on the line distance. Numerical results presented by curves are in good agreement with the experimental results for the three line structures.

In the MSL structure, the mutual inductance depends on the line distance rather strongly. In the experimental results, the mutual inductance per unit length is 4.77 fH/µm at the line distance of 6 µm. It is reduced to 0.309 fH/µm at the line distance of 15 µm.

On the other hand, in the FLSL and COSL structure, the mutual inductance is almost independent on the line distance. The experimental mutual inductance for the FLSLs is reduced only from 1.26 to 1.19 fH/µm when the line distance increases from 6 to 15 µm. In the COSL structure, the similar dependence on the line distance is confirmed with substantially reduced values of the mutual inductance. The mutual inductance of the FLSLs is as small as 0.11 fH/µm for d = 6 µm.

To investigate dependence of the mutual inductance on the line distance, we numerically calculate magnetic flux densities around the strip lines using the current distribution in the SC layers. The results are demonstrated in Fig. 4. The line distance d is 15µm. In the MSL
structure shown in Fig. 4(a), the magnetic flux density is gradually decreased with increasing the distance from the control line. In the FLSL structure (Fig. 4(b)), on the other hand, the magnetic flux density between the two GPs is almost constant except for the spaces directly over or beneath the lines. Similar result is obtained for the COSL structure, which is shown in Fig. 4(c). Difference between Figs. 4(b) and (c) is the absolute values of the magnetic flux densities.

To confirm the features quantitatively, the absolute values of the magnetic flux densities at \( z = -0.311 \, \mu m \), where is above the upper edge of the LGP \( (z = -0.450 \, \mu m) \) and below the lower edge of the strip lines \( (z = -0.150 \, \mu m) \). The line distance \( d \) is 15 \( \mu m \).

![Figure 5. Numerical results of magnetic flux densities at z = -0.311 \( \mu m \), above the upper edge of the LGP (z = -0.450 \( \mu m \)) and below the lower edge of the strip lines (z = -0.150 \( \mu m \)). The line distance \( d \) is 15 \( \mu m \).](image)

The result for the MSLs demonstrates the flux density decreases rapidly with increasing the distance from the CL line. This is because the SC LGP repels the magnetic flux generated around the CL line. On the other hand, the result for the FLSLs exhibits different dependence of the flux density. The magnetic flux density in the FLSL structure falls down steeply at the edges of the CL line, and keep an approximately-constant value for over dozens of microns. The result for the FLSLs clearly demonstrates that the magnetic flux is confined within the space between two SC GPs. The result for the COSLs is qualitatively same with that for the FLSLs, but the base line is much lower, approximately 10\(^{-4}\) of the base line of the FLSLs.

Also, we have confirmed that the base line of the magnetic flux density depends on the width of the GPs, and on the positions of the contacts between the GPs. For example, a wider GP reduces the base line. Positional relation between the SQ line and the SC loop formed between the GPs with two UGP-LGP contacts is also an important factor [5]. In particular, the mutual inductance is substantially reduced by shielding the SQUID loop with the SC loop between the GPs.

4. Conclusions

We evaluated the mutual inductance between two horizontally-placed SC lines by means of both experimental and numerical methods. Three structures, the MSL, FLSL, and CPSL structures, were investigated. The mutual inductance in the MSL structure strongly depended on the line distance, whereas that in the FLSL structure was almost independent on the line distance. The mutual inductance in the COSL structure exhibited the similar dependence on the line distance as that of the FLSL structure, with substantially reduced values of the mutual inductance. We also calculated the magnetic flux densities around the lines, which explained the dependence of
the mutual inductance on the line distance.

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