1 Introduction

The formal specification of processes can play a crucial role in a wide range of fields, such as factories, biological systems, railway networks, or software. Such a formalism may help to map out the details of any process and to discover bugs, such as deadlocks. They can also be used to validate the behaviour of a system, or even to develop a quick prototype.

Why not apply this notion to the design of a programming language? It is more than common that, behind the surface of its syntax, a programming language hides the process of its execution. A programmer can discover this process by reading the sometimes millions of lines of documentation or simply by trial-and-error. The process is also encoded in the toolchain provided with the language: compilers, virtual machines, interpreters, modules, etc.

In [5] I gave an abstract description of a programming language that is concurrent by nature. By programming state machines that can only communicate through events, one can build programs without the need of a call stack, and with an immediate possibility to use parallel or distributed systems.

One of the possible applications of this programming language is to provide a tool for educational purposes. It could, for instance, be used to provide example implementations for a distributed algorithms course. In order for this to be useful, the language must come with tools that give insight of the workings of such algorithms, for example in the form of Message Sequence Charts (see [8]). This is another reason for formally specifying the language’s semantics; the design of such tools can be based on the specification, instead of being based on an implementation.

By using process algebra (in particular ACP: the Algebra of Communicating Processes; see [4], [2] and [1]) combined with notions from PSF (a Process Specification Formalism; see [9] and [3]) this paper gives a complete and thorough description of the process of purely event-driven programs. This specification reveals the concurrent nature of the language as described in [5] in a formal manner.
2 Purely event-driven programming

The Purely Event-driven Programming (PEP) language is designed for implicit concurrency. This means that the order in which a program’s components are executed is not relevant for its result. Many programming languages support concurrent programming, but this needs to be done explicitly by the programmer. PEP is concurrent by design.

Another aim of PEP is to eliminate the call stack. Programs commonly consist of sequences of function calls. Such calls take memory space to provide a context, to know where to return the results to, and to remember where to continue the program after finishing. PEP uses state machines to get rid of these notions, summarized in [7] as context, coordination and continuation.

A state machine always has the same context, namely its own collection of variables. One method waiting for a called method to finish is called coordination, and remembering where to continue after a method is finished is called continuation. Both these notions are irrelevant when two state machines are used side-by-side. A state machine only needs to know the state it is in, and how to react to its environment. A state never “calls” another state, it merely transitions to it.

To run a state machine, one instantiates it. This way, a state machine can have multiple running instances. Such an instance can emit events, i.e. a message that has a string as type and an optional value. Other instances can be programmed to react to certain event types, or events from specific instances.

Every state machine has a special state in which it deals with incoming events. This state is called “listen”, because it polls for events, i.e. it actively consults a queue to see if any events have come in. Once there is an event, the instance’s reaction settings are checked. If a reaction is set, the instance transitions to the set state. Otherwise, the whole process starts again.

Because of the independent nature of state machines, they are not responsible for the actual execution of their program. The language has a central component called Machine Control which is responsible for starting and running machines and distributing events.

In Appendix D, accompanying a program specification, one can find the code for a sample PEP program that simulates a very simple computer that reads from a hard drive. However, to fully understand the PEP language itself, please consult [5], which contains a thorough description of the language, together with justifications for certain design choices as well as experiments and a Python simulation.

3 The PEP process

The process of a PEP program is complex. It consists of many concurrent subprocesses that can communicate in several ways and all have their own specific purpose. First, I will go into some conventions used in describing the processes. This is followed by the implementation of two important data structures: the queue and lookup table.

Then, I give a detailed description of the process of a single running instance of a state machine. The central part that binds a program together is Machine Control. I finish this section by giving the processes Machine Control consists of.

Because these processes are described as independent subprocesses, it can be difficult to have an idea as to how the system comes together. Therefore, I have included a schematic view of Machine Control and one state machine instance in appendix A.
3.1 Conventions

The following specifications are neither strictly ACP nor strictly PSF. Therefore, there have to be some rules on how to name actions and how to define communications, i.e. conventions. In order to make the specifications readable for anyone, I list here the conventions used.

- All actions are subscripted by an identifier. This identifier can be anything, from a natural number to a string, or a combination of the two.
- Actions for specific machines are subscripted with the machine’s identifier. For example, the action for an outgoing event: out\textsubscript{id}(\ldots).
- If actions apply to some specific internal process, the name of the process is included in the identifier. For example, the action to dequeue an incoming event: deq\textsubscript{in(id)}(\ldots).
- Functions on internal data structures never appear outside regular actions. As they return immediate results on their arguments, they do not need to be subscripted. For example, to continue a queue after dequeuing: Q\textsubscript{id}(\text{tail}(l))
- Many processes communicate. Actions that “send” data after processing, i.e. outbound communication, are seen as the initiator. Therefore, the actions that “receive” data for processing, i.e. inbound communication, have the same name except with a prime. Both sides of the communication are subscripted by the same identifier. A few examples:

| Description       | Outbound | Action | Inbound | Action |
|-------------------|----------|--------|---------|--------|
| Emission of an event | M\textsubscript{out\_id} | out\textsubscript{id}(\ldots) | M\textsubscript{in\_id} | out\textsubscript{id}'(\ldots) |
| Enqueuing of an event | M\textsubscript{prog\_id} | enq\textsubscript{out\_id}(\ldots) | Q\textsubscript{out\_id} | enq\textsubscript{out\_id}'(\ldots) |

- Many specifications contain conditionals. They consist of some test that can be true or false and a question mark contained in square brackets, followed by a colon and an arrow. They are used to distinguish cases that can be determined upon entering the process term. For example, to check whether an event (s, d, t, a) requires an acknowledgment: \[ a = \top \rightarrow \ldots \]
- It is required that every process is designed to be fail-safe: communicating actions can only be used if they are paired with the option to halt the process. This prevents the situation explained below.

A process receives an event, which needs to be send to the next process. However, in the meantime another process has decided to halt these two processes. The second process receives the halt request and halts. The first process is now stuck with the event, without option to just quit, resulting in deadlock. If this process would still have the option to receive the halt request (i.e. be fail-safe), the problem would be solved.
3.2 The event data type

An event is a tuple \((s, d, t, a)\) in which

- \(s\) is a natural number identifying the sender of the event;
- \(d\) is the identifier for its destination, or 0 if no destination is given;
- \(t\) is a string representing the event’s type;
- \(a\) is a boolean (\(\top\) for true or \(\bot\) for false) indicating whether the event requires acknowledgment.

Let \(T\) be the set of all event types used in the program to be specified. We define \(Ev\) to be the collection of all possible events.

\[
Ev := \{(s, d, t, a) \mid s \in \mathbb{N}, d \in \mathbb{N}, t \in T, a \in \{\top, \bot\}\}
\]

Given some event \(e \in Ev\), to read, for example, the sender from a tuple, one would need to use a notation such as \(e[0]\). To make the specifications more readable, the event tuples are treated as objects. The interface to extract the event’s parameters is described in the following table.

| Description          | Action |
|----------------------|--------|
| Get the sender       | sndr(e) |
| Get the destination  | dest(e) |
| Get the type         | type(e) |
| Get the acknowledgment boolean | ack(e) |

3.3 Queues

A queue is the process of storing data in a fifo (first in; first out) manner. All queues are the same, except in the type of data they store. Also, every queue should have an identifier so other processes know which queue they are dealing with. In this specification, all queues deal with the same type of data: events.

A queue needs an internal list, which we call \(\ell\). To initialize an empty queue, the process can be started with the empty list \(\epsilon\), as such: \(Q\_id(\epsilon)\). The specification contains a few operations on this list which are purely functional. I.e., they do not alter the list, but they merely return the result of some operation. The following table describes each operation used on some list \(\ell\) with some value \(e\).

| Operation | Description |
|-----------|-------------|
| first(\(\ell\)) | Returns the first item of the list |
| tail(\(\ell\)) | Returns a copy of the list without the first item (i.e. without first(\(\ell\))) |
| \(\ell \land e\) | Returns a copy of the list in which \(e\) is added to the end |
A queue can be stopped by communicating a halt action with it. This particular halt action is subscripted by \( q(id) \), because the queues of instances have the same name as their respective event handlers. Note that this specification is fail-safe, since after every communicating action a halt request can be received, stopping the queue.

\[
Q_{id}(\ell) = [\ell = \epsilon?] \cdot q\text{empty}_{id} \cdot Q_{id}(\ell) \\
+ [\ell \neq \epsilon?] \cdot \text{deq}_{id}(\text{first}(\ell)) \cdot Q_{id}(\text{tail}(\ell)) \\
+ \sum_{e \in Ev} \text{enq}_{id}'(e) \cdot Q_{id}(\ell \land e) \\
+ \text{halt}_{q(id)}
\]

The interface for a queue with id \( id \) is depicted in the following table. Here, \( e \) is any event.

| Description                        | Action       |
|------------------------------------|--------------|
| If the queue is empty, this action can communicate | empty\(_{id}'\) |
| Take the first item from the queue  | deq\(_{id}(e)\) |
| Put an item on the queue           | enq\(_{id}(e)\) |
| Terminate the queue                | halt\(_{q(id)}\) |

3.4 Lookup tables

A lookup table assigns a value to a certain index. In PSF, operations on lookup tables are purely functional. As the tables are only used by the program process, i.e. they are not a shared resource, they don’t need to be implemented concurrently. However, since a state machine’s program has to be able to manipulate and access its tables in every state, it is impractical to use the tables as arguments. Therefore, they are specified as a concurrent process. In this way, the program process manipulates the tables by communicating actions to it, resulting in side-effects, i.e. an updated table.

The variable and reaction tables differ only in the data type of their index and their values. Therefore, in the specification we superscript the process by collections of possible indices (\( I \)) and possible values (\( V \)). The tables themselves use an internal lookup table with purely functional operations, which we call \( tbl \). The functionality of these operations is depicted in the following table for some index \( i \in I \) and value \( v \in V \).

| Operation | Description                                                                 |
|-----------|-----------------------------------------------------------------------------|
| qry\(_{tbl, i}\) | Returns not\(_{\text{fnd}}\) if the table contains no entry for \( i \). If it does, it returns \( \text{fnd}(v) \) for some \( v \in V \). |
| ins\(_{tbl, i, v}\) | Returns a copy of the table with value \( v \) for index \( i \).          |
| dlt\(_{tbl, i}\) | Returns a copy of the table with no entry for index \( i \).              |
A table can be stopped by communicating a halt action with it. Again, this specification is fail-safe, just like for the queue.

\[ T^{I,V}_{id}(tbl) = \sum_{i \in I} (\text{qry}(tbl, i) = \text{not}_{fnd}?) \Rightarrow \text{not}_{set}(i) \cdot T^{I,V}_{id}(tbl) + \sum_{v \in V} (\text{qry}(tbl, i) = \text{fnd}(v)?) \Rightarrow \text{get}_{id}(i, v) \cdot T^{I,V}_{id}(tbl) + \text{set}_{id}(i, v) \cdot T^{I,V}_{id}(\text{ins}(tbl, i, v)) + \text{unset}_{id}(i) \cdot T^{I,V}_{id}(\text{dlt}(tbl, i)) + \text{halt}_{id} \]

The interface for a table with types \( I \) and \( V \) and id \( id \) is depicted in the following table. Here, \( i \) is any index and \( v \) is any value.

| Description                       | Action       |
|-----------------------------------|--------------|
| If \( i : v \) is in the table, this action can communicate | get\(_{id}(i, v)\) |
| If \( i \) is not in the table, this action can communicate | not\(_{set}_{id}(i)\) |
| Set a value                       | set\(_{id}(i, v)\) |
| Remove a value                    | unset\(_{id}(i)\) |
| Terminate the table               | halt\(_{id}\) |

### 3.5 State machines

A PEP program actually runs by creating instances of state machines. Therefore, we define state machines by providing templates. Such a template is subscripted by an identifier. When an instance is made, Machine Control assigns a unique identifier to it. This section describes how to formalise such a template for some state machine with name “M”.

To be able to start new instances of a machine, Machine Control needs access to its process identifier. Therefore, we do not assign unique names for these processes to each machine, but we use one letter \( M \) and superscript it with the state machine’s name. So, for our machine “M”, an instance with id \( id \) would be described by the process \( M^{\text{M}}_{id} \).

When an instance emits an event or wants to react to events, we don’t want it to wait for Machine Control to communicate with it. Therefore, outgoing events are put into a queue and communicated by a separate process \( (M^{\text{M}}_{id})^{\text{out}} \) and incoming events by another \( (M^{\text{M}}_{id})^{\text{in}} \). In this way, the program process \( (M^{\text{M}}_{id})^{\text{prog}} \) can immediately continue after dealing with events.

The identifiers of the event processes are not superscripted with the machine’s name, because they are the same for every machine. However, they do contain the instance’s id. The program process does contain the machine’s name, because it is unique to the machine.
This is not all. A machine needs two lookup tables: one to store how to react to events and one to keep track of the ids of machines it started. So, the definition of an instance includes two tables and two event queues. Let $\text{Vars}^{\text{M}'}$ be the collection of all variable names used in the instance’s program.

The implicit variable name “ctx” is added in the specification. As it will be provided by Machine Control, which starts the instance, the id of its context is a parameter for the process.

The reaction table is started with the default halt reaction, except if the context’s id is 0. In this case, the machine has no context, so a halt reaction with machine id 0 would make the machine halt if any child instance halts, causing all machines to halt. Let $S^{\text{M}'}$ be the set of all states of “M”.

$$
M_{id}^{\text{M}'}(ctxid) = M_{id}^{\text{prog}(\text{M}')} \\
\{ T_{\text{ctx}}^{\text{vars}^{\text{M}'} \cup \{\text{ctx}\} \cup \{(\text{ctx} : \text{ctxid})\}}(\emptyset) \\
| [\text{ctxid} = 0?] \to T_{\text{reactions}(id)}^{\{(m,t) | m \in M, t \in T\} \cdot S^{\text{M}'}\cdot}(\emptyset) \\
| + [\text{ctxid} \neq 0?] \to T_{\text{reactions}(id)}^{\{(m,t) | m \in M, t \in T\} \cdot S^{\text{M}'}\cdot}(\{(\text{ctxid}, \text{“halt”}) : \text{“halt”}\}) \\
\| Q_{\text{in}(id)}(\epsilon) \| Q_{\text{out}(id)}(\epsilon) \\
\| M_{id}^{\text{out}(\epsilon)} \| M_{id}^{\text{in}(\epsilon)}
$$

### 3.5.1 The program process

As this project is about the process of PEP, I will not go into the details of calculating process terms for each state. Instead, I will describe what such a process term should look like. Let “M” be some state machine. For every state $s$ of $S^{\text{M}'}$ and every instance with id $id$ of “M” we define $S_{id}^{\text{M}'}$ to map state names to their corresponding processes.

An instance should always have access to the most recent event it reacted to, to use the event’s value, or to know what instance (e.g. the event’s sender) to return an event to. Although this behaviour is not explained in my thesis, it was indeed used in several experiments, such as Eratosthenes’ Sieve (see [5, Appendix A]). Therefore, I am going to include it in the specification.

To allow every state access to the latest event, $S_{id}^{\text{M}'}$ will take it as a second argument. This argument will only be changed in the “listen” state, where event reaction takes place.

Every branch (summand in process jargon) should be followed by the process of another state. If the original code contains a branch without a state transition, this means there is an implicit transition to the listen state. In such cases, the branch should be changed to be followed by the listen state process.
The listen state process is predefined. The process first dequeues an event and then consults its reaction table in order to decide an action. If the event requires an acknowledgment, an acknowledgment event is enqueued before transitioning. If there is no event or no reaction the process tries again.

If the event queue is empty, the listen state can also take an action and then return to the same state. It makes sure that, just as with enqueuing events instead of communicating directly with Machine Control, an instance’s program can always perform an action.

Because there can be both a machine reaction and a regular reaction to the same type, a priority is defined: first machine reactions, then regular ones. In this process, this is implemented by first trying to get a machine reaction from the table. If there is no such reaction, only the ‘not_set’ action can communicate. After this action, regular reactions can be tried, unless again only ‘not_set’ can communicate.

Because this process is rather complex, I have included a flowchart in Appendix B.

\[
S_{id}^{M^{\text{listen}}}(\text{“listen”}, old_e) = \text{empty}_\text{in(id)} \cdot S_{id}^{M^{\text{listen}}}(\text{“listen”}, old_e) + \sum_{e \in Ev} \text{deq}_\text{in(id)}(e) \cdot \left( \sum_{\text{state} \in S^{M^{\text{get reactions}}}(\text{id})} (\text{sndr(e), type(e)}, \text{state}) \cdot \left( [a = \top?] \rightarrow \text{enq}_{\text{out(id)}}((\text{id}, s, t, \text{“ack”}, \bot)) \cdot S_{id}^{M^{\text{stop}}}(\text{state}, e) + [a = \bot?] \rightarrow S_{id}^{M^{\text{stop}}}(\text{state}, e) \right) \right) + \text{not_set}_{\text{reactions(id)}}((\text{sndr(e), type(e)})) \cdot \left( \sum_{\text{state} \in S^{M^{\text{get reactions}}}(\text{id})} ((0, \text{type(e)}), \text{state}) \cdot \left( [a = \top?] \rightarrow \text{enq}_{\text{out(id)}}((\text{id}, s, t, \text{“ack”}, \bot)) \cdot S_{id}^{M^{\text{stop}}}(\text{state}, e) + [a = \bot?] \rightarrow S_{id}^{M^{\text{stop}}}(\text{state}, e) \right) \right) \right) \right)
\]

The halt state process is also predefined. If an instance is done it should transition to this state. This state tells Machine Control to remove this instance from its administration. Then, it tells all its internal component processes to cease as well. This is done through communicating halt actions subscripted by the id and the name of the process. There should be no particular order in this, so it is done concurrently.

\[
S_{id}^{M^{\text{halt}}}(\text{“halt”}, e) = \text{halt}_{\text{id}} \cdot \left( \text{halt}_{\text{ctx(id)}} || \text{halt}_{\text{reactions(id)}} \right) || \left( \text{halt}_{\text{q(in(id))}} || \text{halt}_{\text{q(out(id))}} \right) || \left( \text{halt}_{\text{in(id)}} || \text{halt}_{\text{out(id)}} \right)
\]

8
Other state processes can be anything, as long as every branch continues in another state process. In this article I do not go into the details of what is possible in a state. However, working with events is relevant, so I have included the following table to show how to encode event actions in process terms. The code syntax is from [5].

| Description                                      | Code                                      |
|--------------------------------------------------|-------------------------------------------|
| Transition to another state                      | $\mathcal{S}_{id}^M$ (“$s'$”, $e$) => $s$ |
| Starting a machine instance                      | $m = \text{ctl.start}(M, \ldots)$         |
|                                                  | $\Rightarrow \text{start}_{id}(\text{"M"}) \cdot \Sigma_{n \in \mathbb{N}}(\text{new}_{id}'(n) \cdot \text{set}_{ctx}(id)(\text{"m"}, n))$ |
| Register an event reaction                       | $\Rightarrow \text{set}_{reactions(id)}((0, "$t"), "$s"))$ |
| Unregister an event reaction                      | $\Rightarrow \text{ignore when "t"}$      |
| Register a machine reaction                       | $\Rightarrow \Sigma_{n \in \mathbb{N}}(\text{get}_{ctx}(id)(\text{"m"}, n) \cdot \text{set}_{reactions(id)}((n, "$t"), "$s"))$ |
| Unregister a machine reaction                     | $\Rightarrow \Sigma_{n \in \mathbb{N}}(\text{get}_{ctx}(id)(\text{"m"}, n) \cdot \text{unset}_{reactions(id)}((n, "$t"))$ |
| Emit an event                                     | $\Rightarrow \text{emit ("t")}$           |
| Emit an event with acknowledgment                 | $\Rightarrow \text{emit ("t")} => s$      |
| Emit a directed event                             | $\Rightarrow \Sigma_{n \in \mathbb{N}}(\text{get}_{ctx}(id)(\text{"m"}, n) \cdot \text{emit}_{out(id)}((id, n, "$t"), \bot))$ |
| Emit a directed event with acknowledgment          | $\Rightarrow \Sigma_{n \in \mathbb{N}}(\text{get}_{ctx}(id)(\text{"m"}, n) \cdot \text{emit}_{out(id)}((id, n, "$t"), \bot) \cdot \text{set}_{reactions(id)}((n, "$t\_ack"), "$s"))$ |
3.5.2 Event handling

The outgoing event handler takes an event from its queue and communicates it with Machine Control. A halt action stops the process. This specification is fail-safe, because if Machine Control’s incoming event handler halts while this process dequeues an event, it still has the option to halt.

\[
M_{id}^{\text{out}} = \sum_{e \in Ev}( \\
\quad \text{deq}_{\text{out}(id)}(e) \cdot ( \\
\quad \quad \quad \text{out}_{id}(e) \cdot M_{id}^{\text{out}} \\
\quad \quad \quad + \text{halt}_{\text{out}(id)}' \\
\quad ) \\
\quad + \text{halt}_{\text{out}(id)}')
\]

Incoming events are put into a queue by the incoming event handler. From there, a machine instance’s listen state dequeues the event and decides what to do with it. A halt action stops the process. This specification is also fail-safe: if the incoming event queue halts while this process receives an event from Machine Control, it still has the option to halt.

\[
M_{id}^{\text{in}} = \sum_{e \in Ev}( \\
\quad \text{distrib}_{id}(e) \cdot ( \\
\quad \quad \quad \text{end}_{\text{in}(id)}(e) \cdot M_{id}^{\text{in}} \\
\quad \quad \quad + \text{halt}_{\text{in}(id)}' \\
\quad ) \\
\quad + \text{halt}_{\text{in}(id)}')
\]

3.6 Machine Control

Machine Control is the central unit responsible for starting state machine instances and the distribution of events. In order to keep track of the ids of instances, it is dependent on a counter and a list. Because most events need to be distributed to all running instances, distribution cannot be separated from the process that keeps these ids. This part of Machine Control is the scheduler (MC_{sched}).

However, the reception of events is completely independent from this. So, in order to be able to distribute events whilst receiving events, this is done by a separate process and a queue, as done in machine instances. This part of Machine Control is the incoming event handler (MC_{in}).
The process for Machine Control is defined with two parameters: $n$, a natural number representing the amount of instances it started (i.e. the highest instance id), and $\ell$, the set of ids of running instances.

\[
\text{MC}(n, \ell) = \text{MC}_{\text{sched}}(n, \ell) \parallel Q_{\text{MC}(\epsilon)} \parallel \text{MC}_{\text{in}}
\]

3.6.1 Scheduler

The scheduling process can do a number of things. These things deal with the starting and halting of instances and the distribution of events. Because halting and starting influence the list of ids $\ell$, and events need to be distributed to all instances in $\ell$, they cannot run concurrently. Therefore, if the scheduler is busy halting a machine, it cannot receive a request to start another, and vice versa.

The communication of halting and starting requests is implemented by a recursive sum, because it’s not possible to sum over the elements of an internal data structure (i.e. over the instance ids in $\ell$). The options to start and to halt for each id $\in \ell$ are implemented in $\text{MC}_{\text{schedsum}}$, which recursively adds these options for the next instance id. The same holds for the distribution of events, which is implemented by a recursive merge in $\text{MC}_{\text{distrib}}$.

If a request to start an instance is communicated, the scheduler communicates the new id to the requesting instance. Then it continues its process with updated $n$ and $\ell$, merged with the new instances’ process.

If a request to halt an instance is communicated, the scheduler enqueues an event announcing this halt. Then it checks whether all instances have halted (i.e. whether its list is empty). If so, it tells its event handler and queue to halt and then stops itself. If not, it continues with the id of the halted instance removed from its list.

If there is an event in Machine Control’s queue, a dequeue action may communicate. After this, the scheduler proceeds to distribute the received event. If the event has a destination, it will only be communicated with that destination. Otherwise, the event will be concurrently communicated with all instances with all running instances (i.e. the ids in $\ell$). This ensures that all running instances receive the event and that this process will not hang trying to communicate with non-existing instances.
Because this process is rather complex, I have included a flowchart in Appendix C.

\[
\begin{align*}
MC^{\text{sched}}(n, \ell) = & MC^{\text{schedsum}}(n, \ell, \text{tail}(\ell), \text{first}(\ell)) \\
& + \sum_{e \in \mathcal{E}_v} \delta'_M(e) \cdot ( \\
& \quad \text{dest}(e) = 0? \rightarrow ( \\
& \quad \quad [\ell \neq (\epsilon \land \text{sndr}(e))?] \rightarrow MC^{\text{distrib}}(e, \text{tail}(\ell), \text{first}(\ell)) \cdot MC^{\text{sched}}(n, \ell) \\
& \quad \quad + [\ell = (\epsilon \land \text{sndr}(e))?] \rightarrow MC^{\text{sched}}(n, \ell) \\
& \quad ) + \text{dest}(e) \neq 0? \rightarrow \text{distrib}_{\text{dest}(e)}(e) \cdot MC^{\text{sched}}(n, \ell)
\end{align*}
\]

\[
\begin{align*}
MC^{\text{schedsum}}(n, \text{orig}\ell, \ell, id) = & [\ell \neq \epsilon?] \rightarrow MC^{\text{schedsum}}(n, \text{orig}\ell, \text{tail}(\ell), \text{first}(\ell)) \\
& \sum_{M \in \mathcal{M} \text{start}_{id}'(M) \cdot \text{new}_{id}(n + 1) \cdot ( \\
& \quad \text{MC}^{\text{sched}}(n + 1, \text{orig}\ell \land (n + 1))|\mathcal{M}^M_{n+1}(id) \\
& \quad + \text{halt}_{id}' \cdot \text{enq}_{MC}((id, 0, \text{"halt"}, \bot)) \cdot ( \\
& \quad \quad [(\text{orig}\ell \land id) = \epsilon?] \rightarrow (\text{halt}_{MC(\text{in})})|\text{halt}_{\text{q}(MC)}) \\
& \quad \quad + [(\text{orig}\ell \land id) \neq \epsilon?] \rightarrow MC^{\text{sched}}(n, \text{orig}\ell \land id) \\
& )
\end{align*}
\]

\[
\begin{align*}
MC^{\text{distrib}}(e, \ell, id) = & [\text{sndr}(e) = id?] \rightarrow MC^{\text{distrib}}(e, \text{tail}(\ell), \text{first}(\ell)) \\
& + [\text{sndr}(e) \neq id?] \rightarrow ( \\
& \quad [\ell = \epsilon?] \rightarrow \text{distrib}_{id}(e) \\
& \quad + [\ell \neq \epsilon?] \rightarrow (\text{distrib}_{id}(e)|MC^{\text{distrib}}(e, \text{tail}(\ell), \text{first}(\ell)))
\end{align*}
\]

### 3.6.2 Incoming event handler

This process is pretty simple and requires no explanation. It is fail-safe, exactly as machine instances’ event handlers are (see Section 3.5.2).

\[
\begin{align*}
MC^{\text{in}} = & \sum_{(s, d, t, a) \in \mathcal{E}_v} ( \\
& \quad \text{out}^t_s(d, t, a) \cdot ( \\
& \quad \quad \text{enq}_{MC}((s, d, t, a)) \cdot MC^{\text{in}} \\
& \quad \quad + \text{halt}'_{MC(\text{in})} \\
& \quad ) + \text{halt}_{MC(\text{in})}
\end{align*}
\]
3.7 The specification of individual programs

With the information in this section, one is able to define all the process specifications necessary to define the specification for a program. In summary, one needs to prepare the following items:

- Define $T$ to be the set of all event types present in the program. This defines the set of all possible events $Ev$ as in Section 3.2. Don’t forget to include “halt” and the necessary acknowledgment event types.

- With $Ev$ defined, define the specification of the queue process as in Section 3.3.

- Define the specification of the lookup table as in Section 3.4.

- Define $M$ to be the set of all state machine types present in the program.

- For each state machine “$M$” of $M$:
  - Define $Vars^{"M"}$ to be the set of all variable names used in the state machine. Don’t forget to add “ctx” to this list.
  - Define $S^{"M"}$ to be the set of all state names in the machine.
  - For each state $s$ of $S^{"M"}$, define $S^{"M"}_{id}(s, e)$ to be the corresponding process specification, with any event $e$ as parameter, as in Section 3.5.1.
  - Assigns the machine’s initial state specification to $M^{prog("M")}$.

- Define $MC(n, \ell)$ as in Section 3.6.

As described in [5, Section 2.2.3], a program is defined by telling Machine Control to start one specific machine. In the spirit of this definition, the specification of a program will take as argument the initial machine to instantiate. It starts Machine Control with one initial id of 1, merged with the initial machine, instantiated with id 1.

[5, Section 2.2.3] also mentions that the context of the first instance is an empty state machine. However, since the context of instances in the specification is only the identifier of another instance, we can safely choose this to be 0.

The final step is to encapsulate the program in order to force communication to take place. We have to define the set $H$ of all actions that can only be performed as communication.

$$H := \{ \text{qempty}_{id}, \text{qempty}'_{id}, \text{deq}_{id}, \text{deq}'_{id} | id \in \{t(n) \mid t \in \{\text{in}, \text{out}\}, n \in \mathbb{N}\} \cup \{\text{MC}\} \}$$

$$\cup \{ \text{get}_{id}, \text{get}'_{id}, \text{not}_{set}_{id}, \text{not}_{set}'_{id}, \text{set}_{id}, \text{set}'_{id}, \text{unset}_{id}, \text{unset}'_{id} | id \in \{t(n) \mid t \in \{\text{ctx}, \text{reactions}\}, n \in \mathbb{N}\} \}$$

$$\cup \{ \text{out}_{id}, \text{out}'_{id}, \text{distrib}_{id}, \text{distrib}'_{id}, \text{start}_{id}, \text{start}'_{id}, \text{new}_{id}, \text{new}_{id}', \text{halt}_{id}, \text{halt}'_{id} | id \in \mathbb{N} \}$$

$$\cup \{ \text{halt}_{id}, \text{halt}'_{id} | id \in \{q(t) \mid t \in \{\text{in}(n), \text{out}(n), \text{MC} \mid n \in \mathbb{N}\} \} \cup \{\text{ctx}(n), \text{reactions}(n), \text{in}(n), \text{out}(n) \mid n \in \mathbb{N}\}$$

$$\cup \{\text{MC}(\text{in})\} \}$$
We also need to define the names of all communicating actions. As mentioned in Section 3.1, any outbound communicating action \( a \) has an inbound counterpart \( a' \). We define the communication of these actions \( \gamma(a, a') \) to be \( a'' \).

\[
\text{Program}(\text{machinetype}) = \partial_H(\text{MC}(1, \{1\}))||M^\text{machinetype}_1(0))
\]

To see the steps in this section in action, take a look at Appendix D.

4 Discussion and conclusion

Firstly, although this specification seems to give the complete picture, it is important to test its actual workings. By implementing individual PEP programs in PSF, it is possible to test the different aspects of the language such as event emission and reaction, state machine instantiation, and (cascading) halting. It also enables us to debug the design, to answer several questions. Does the listen state priority mechanism actually work? Does the design contain a deadlock? How does this implementation compare to the Python simulation (see [6])?

The example program in Appendix D has been implemented in PSF and Go (see [10]). The implementation in PSF allowed me to test some assumptions about what is possible in these process specifications and what is not. As an example, in an earlier version Machine Control’s scheduler was implemented with sums over the internal list of instance IDs. As mentioned in Section 3.6.1, it turned out that a different approach was required. The implementation in Go allowed me to test the workings of the programming language with the scheduling being out of my control, and possible parallelism in the execution. This lead to the discovery of a possible deadlock in an earlier design, which gave rise to the need for fail-safeness, as mentioned in Section 3.1. A short technical report on both implementations can be expected.

Secondly, as the attentive reader might have noticed, there is one important aspect of PEP missing: the ability to attach values to events. Because the language has been designed to keep one instance’s memory private from other instances, the only way to pass values is through events.

However, the notion of value passing between processes is not as simple as the concept of it in a theoretical description of a programming language. For a specification, all the details need to be thought of. What is a value? What data types would be allowed, and how would these data types be implemented in a specification? Will the amount of data be limited?

All of these important decisions need to be made before it is even possible to specify the passing of values. Because the usage of values and variables is necessary for programming anyway, this problem needs to be solved before actually implementing the programming language. Therefore, it would be good to do another project on this detail of PEP alone.

Thirdly, it would be interesting to study PEP programs using the notion of abstraction, such as in [4, Chapter 5]. By applying an abstraction operator to the specification of a program, one can abstract away from internal operations. In this case, it could be used to compress all actions that are concerned with events and state machines to get an external behaviour. This external behaviour can then be compared to the behaviour of another implementation of the program that does not use PEP.
Finally, some remarks about the specification itself. During this project I became more and more convinced that using process algebra to specify the workings of a programming language is very useful. Working out the formulas makes you work out every detail, revealing properties of the language that are new to you.

On top of that, an algebraic specification can help to prevent problems during implementation, one might discover a problem with the language’s design. But after writing a substantial amount of code, programmers usually tend to patch the mistakes, instead of rewriting everything from scratch. A process algebra specification reveals such errors prior to implementation, preventing the source of a compiler or interpreter to be littered with spaghetti code.

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A Process overview schema
B Listen state flowchart

1. Is $Q_{in(id)}$ empty?
   - No: Dequeue an event
   - Yes: Machine reaction?
     - No: Event reaction?
       - No: State transition
       - Yes: Emit acknowledgment
     - Yes: Need to acknowledge?
       - No: State transition
       - Yes: Emit acknowledgment
C  Machine Control schedule state flowchart

- try to communicate
  - start
  - halt
  - deq

- send new id
  - update list
  - start new machine
  - halt MC\textsuperscript{in}

- emit "halt" event
  - update list
  - list empty?
    - yes: halt Q\textsubscript{MC}
    - no: send to destination

- dequeue event
  - destination?
    - yes: send to destination
    - no: send to all machines
D  A program specification example

This example describes how a computer starts two programs A and B. Program B continuously asks the CPU to give it a cycle (what for is not relevant). Program A asks the CPU to lookup something on a hard drive. The CPU delegates this request to the hard drive, which tells its internal head to seek the proper data. Once found, the hard drive sends an interrupt event to the CPU. The CPU then returns the found data to program A, which then shuts down the computer.

This appendix contains this program in the syntax as in [5] (albeit with some slight alterations) and then describes how this program would be specified according to this paper. There is also an implementation for the Python simulator available in [6, harddrive.py].

D.1  Program code

N.B.: Each state contains an argument e, referring to the latest event. This does not appear in [5]. This is discussed in Section 3.5.1.

N.B.2: In HDHead’s state seek you’ll notice an or-construct. This is also not a part of [5], but merely a very general way to describe non-deterministic choice. It is not really necessary for this example to actually write code to move the arm of a hard drive; I just want to demonstrate the possibility that seeking takes a while.

```plaintext
machine CPU {
    hd = null;
    prog_a = null;
    prog_b = null;
    hd_reader = null;

    init state setup(e) {
        hd = ctl.start(HD, null);
        prog_a = ctl.start(ProgA, null);
        prog_b = ctl.start(ProgB, null);

        when "cycle" => cycle;
        when "read" => hd_read;
        when "shutdown" => halt;
    }

    state cycle(e) {
        <cycle>;
    }

    state hd_read(e) {
        hd_reader = e.emitter;
        emit ("read") to hd;
        when hd emits "interrupt" => hd_interrupt;
    }

    state hd_interrupt(e) {
        emit ("return") to hd_reader;
        ignore when hd emits "interrupt";
    }
}
```
machine HD {
   .hd_head = null;

    init state setup(e) {
        .hd_head = ctl.start(HDHead, null);
        when ctx emits "read" => seek;
    }

    state seek(e) {
        ignore when "ctx" emits "read";
        emit ("seek") to .hd_head;
        when .hd_head emits "found_data" => found_data;
    }

    state found_data(e) {
        ignore when .hd_head emits "found_data";
        when "read" => seek;
        emit ("interrupt") to ctx;
    }
}

machine HDHead {
    init state setup(e) {
        when ctx emits "seek" => seek;
    }

    state seek(e) {
        { emit ("found_data") to ctx; }
        or {
            => seek;
        }
    }
}

machine ProgA {
    init state program(e) {
        emit ("read") to ctx;
        when ctx emits "return" => finish;
    }

    state finish(e) {
        ignore when ctx emits "return";
        emit ("shutdown") to ctx;
    }
}

machine ProgB {
    init state cycle(e) {
        emit ("cycle") to ctx => cycle;
    }
}

ctl.run(CPU);
D.2 Specification

We follow the steps in Section 3.7. First we define the set of all event types by going through the code and putting all used event types in a set.

\[ T := \{ \text{"halt"}, \text{"cycle"}, \text{"cycle_ack"}, \text{"read"}, \text{"shutdown"}, \text{"interrupt"}, \text{"seek"}, \text{"found_data"}, \text{"return"} \} \]

This defines the set of all possible events \( E_T \) as well as the specifications for the queue and the lookup table, which can be directly copied. Next, we define the set of all state machine types.

\[ M := \{ \text{"CPU"}, \text{"HD"}, \text{"HDHead"}, \text{"ProgA"}, \text{"ProgB"} \} \]

For each machine, we need to define the respective set of variable names and state names. As each state machine always has a “listen” and a “halt” state, we can define a default set of states \( S_{\text{default}} := \{ \text{"listen"}, \text{"halt"} \} \). We also need to define the state process specifications. The specifications for “listen” and “halt” are omitted because they can be directly copied with the right machine type as superscript. We also define the machine’s program to equal its initial state (with empty event parameters).

N.B. Some specifications slightly deviate from the table in Section 3.5.1, for example in \( S_{\text{id}} \text{"CPU"} (\text{"hd_read"}, e) \). This is because the same variable name is used multiple times in a row, so the id of the relevant instance needs to be looked up just once.

\[ Vars_{\text{CPU}} := \{ \text{"ctx"}, \text{"hd"}, \text{"prog.a"}, \text{"prog.b"}, \text{"hd_reader"} \} \]

\[ S_{\text{CPU}} := S_{\text{default}} \cup \{ \text{"setup"}, \text{"cycle"}, \text{"hd_read"}, \text{"hd_interrupt"} \} \]

\[ S_{id} \text{"CPU"} (\text{"setup"}, e) = \text{start}_id(\text{"HD"}) \cdot \Sigma_{n \in \mathbb{N}} (\text{new}_{id}(n) \cdot \text{set}_{ctx}(id)(\text{"hd"}, n)) \]

\[ \cdot \text{start}_id(\text{"ProgA"}) \cdot \Sigma_{n \in \mathbb{N}} (\text{new}_{id}(n) \cdot \text{set}_{ctx}(id)(\text{"prog.a"}, n)) \]

\[ \cdot \text{start}_id(\text{"ProgB"}) \cdot \Sigma_{n \in \mathbb{N}} (\text{new}_{id}(n) \cdot \text{set}_{ctx}(id)(\text{"prog.b"}, n)) \]

\[ \cdot \text{set}_{\text{reactions}}(id)((0, \text{"cycle"}), \text{"cycle"}) \]

\[ \cdot \text{set}_{\text{reactions}}(id)((0, \text{"read"}), \text{"hd_read"}) \]

\[ \cdot \text{set}_{\text{reactions}}(id)((0, \text{"shutdown"}), \text{"halt"}) \]

\[ S_{id} \text{"CPU"} (\text{"listen"}, e) \]

\[ S_{id} \text{"CPU"} (\text{"cycle"}, e) = \langle \text{cycle} \rangle \cdot S_{id} \text{"CPU"} (\text{"listen"}, e) \]

\[ S_{id} \text{"CPU"} (\text{"hd_read"}, e) = \text{set}_{ctx}(id)(\text{"hd_reader"}, \text{sndr}(e)) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get}^t_{ctx}(id)(\text{"hd"}, n) \cdot \text{enq}_{out}(id)(\langle id, n, \text{"read"}, \bot \rangle)) \]

\[ \cdot \text{set}_{\text{reactions}}(id)((n, \text{"interrupt"}), \text{"hd_interrupt"})) \]

\[ S_{id} \text{"CPU"} (\text{"listen"}, e) \]

\[ S_{id} \text{"CPU"} (\text{"hd_interrupt"}, e) = \Sigma_{n \in \mathbb{N}} (\text{get}^t_{ctx}(id)(\text{"hd_reader"}, n) \cdot \text{enq}_{out}(id)(\langle id, n, \text{"return"}, \bot \rangle))) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get}^t_{ctx}(id)(\text{"hd"}, n) \cdot \text{unset}_{\text{reactions}}(id)((n, \text{"interrupt"})) \]

\[ S_{id} \text{"CPU"} (\text{"listen"}, e) \]

\[ M^\text{prog}_{id}(\text{"CPU"}) = S_{id} \text{"CPU"} (\text{"setup"}, (0, 0, \infty, \bot)) \]
\[ Vars^{HD} := \{\text{"ctx"}, \text{"hd\_head"} \} \]

\[ S^{HD} := S^{default} \cup \{\text{"setup"}, \text{"seek"}, \text{"found\_data"} \} \]

\[ S^{HD} (\text{"setup"}, e) = \text{start\_id}(\text{"HDHead"}) \cdot \Sigma_{n \in \mathbb{N}} (\text{new\_id}(n) \cdot \text{set\_ctx}(\text{id} \cdot (\text{"hd\_head"}, n)) \cdot \text{set\_reactions}(\text{id} \cdot (n, \text{“read”}, \text{“seek”}))) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{"ctx"}, n)) \cdot \text{set\_reactions}(\text{id} \cdot ((n, \text{"read"}), \text{“seek”}))) \]

\[ \cdot S_{id}^{HD} (\text{“listen”}, e) \]

\[ S^{HD} (\text{“seek”}, e) = \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{"ctx"}, n)) \cdot \text{set\_reactions}(\text{id} \cdot (n, \text{“read”}), \text{“seek”}))) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{"hd\_head"}, n)) \cdot \text{enq\_out}(\text{id} \cdot ((id, n, \text{“found\_data”}, \text{“found\_data”}))) \]

\[ \cdot S_{id}^{HD} (\text{“listen”}, e) \]

\[ S_{id}^{HD} (\text{“found\_data”}, e) = \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{"hd\_head"}, n)) \cdot \text{set\_reactions}(\text{id} \cdot (n, \text{“found\_data”}))) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{“ctx”}, n)) \cdot \text{set\_reactions}(\text{id} \cdot (n, \text{“read”}), \text{“seek”}))) \]

\[ \cdot \Sigma_{n \in \mathbb{N}} (\text{get\_out}(\text{id} \cdot (id, n, \text{“interrupt”}, \bot))) \]

\[ \cdot S_{id}^{HD} (\text{“listen”}, e) \]

\[ M_{id}^{prog(\text{“HD”})} = S_{id}^{HD} (\text{“setup”}, (0, 0, \text{“”}, \bot)) \]

\[ Vars^{\text{HDHead}} := \{\text{“ctx”} \} \]

\[ S^{\text{HDHead}} := S^{default} \cup \{\text{“setup”}, \text{“seek”} \} \]

\[ S_{id}^{\text{HDHead}} (\text{“setup”}, e) = \Sigma_{n \in \mathbb{N}} (\text{get\_ctx}(\text{id} \cdot (\text{“ctx”}, n)) \cdot \text{set\_reactions}(\text{id} \cdot (n, \text{“seek”}, \text{“seek”}))) \]

\[ \cdot S_{id}^{\text{HDHead}} (\text{“listen”}, e) \]

\[ S_{id}^{\text{HDHead}} (\text{“seek”}, e) = (\text{new\_id}(n) \cdot \text{enq\_out}(\text{id} \cdot ((id, n, \text{“found\_data”}, \bot))) \]

\[ \cdot S_{id}^{\text{HDHead}} (\text{“listen”}, e) \]

\[ + (\text{get\_ctx}(\text{id} \cdot (\text{“ctx”}, n)) \cdot \text{enq\_out}(\text{id} \cdot ((id, n, \text{“found\_data”}, \bot))) \]

\[ \cdot S_{id}^{\text{HDHead}} (\text{“listen”}, e) \]

\[ \cdot \langle \text{seek} \rangle \cdot S_{id}^{\text{HDHead}} (\text{“seek”}, e) \]

\[ M_{id}^{prog(\text{“HDHead”})} = S_{id}^{\text{HDHead}} (\text{“setup”}, (0, 0, \text{“”}, \bot)) \]
Vars\(^{\text{ProgA}}\):=\{“ctx”\}

\[ \mathcal{S}^{\text{ProgA}} := \mathcal{S}_{\text{default}} \cup \{ \text{“program”, “finish”} \} \]

\[ \mathcal{S}^{\text{ProgA}} (\text{“program”, } e) = \sum_{n \in \mathbb{N}} (\text{get}^\prime_{\text{ctx}(id)} (\text{“ctx”}, n) \cdot \text{enq}_{\text{out}(id)} ((id, n, \text{“read”}, \bot))) \]

\[ \cdot \mathcal{S}^{\text{ProgA}} (\text{“listen”}, e) \]

\[ \mathcal{S}^{\text{ProgA}} (\text{“finish”, } e) = \sum_{n \in \mathbb{N}} (\text{get}^\prime_{\text{ctx}(id)} (\text{“ctx”}, n) \cdot \text{unset}_{\text{reactions}(id)} ((n, \text{“return”}))) \]

\[ \cdot \mathcal{S}^{\text{ProgA}} (\text{“listen”}, e) \]

\[ M^{\text{prog}}(\text{“ProgA”}) = \mathcal{S}^{\text{ProgA}} (\text{“program”, } (0, 0, \omega, \bot)) \]

Vars\(^{\text{ProgB}}\):=\{“ctx”\}

\[ \mathcal{S}^{\text{ProgB}} := \mathcal{S}_{\text{default}} \cup \{ \text{“cycle”} \} \]

\[ \mathcal{S}^{\text{ProgB}} (\text{“cycle”, } e) = \sum_{n \in \mathbb{N}} (\text{get}^\prime_{\text{ctx}(id)} (\text{“ctx”}, n) \cdot \text{enq}_{\text{out}(id)} ((id, n, \text{“cycle”}, \top))) \]

\[ \cdot \mathcal{S}^{\text{ProgB}} (\text{“listen”}, e) \]

\[ M^{\text{prog}}(\text{“ProgB”}) = \mathcal{S}^{\text{ProgB}} (\text{“cycle”, } (0, 0, \omega, \bot)) \]

These are the ingredients to define the instance process for each state machine. These definitions are mostly the same, so I only give the one for “CPU”.

\[ M^{\text{CPU}}(\text{ctxid}) = (M^{\text{prog}}(\text{“CPU”})) \]

Next, we copy the definition for \( \text{MC}(n, \ell) \). Finally, we define the program.

\[ \text{harddisk} = \text{Program(“CPU”) = } \partial_H(\text{MC}(1, \{1\}) || M^{\text{CPU}}(0)) \]