The two-real-singlet Dark Matter model

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We revisit a Dark Matter model with an extension of the Standard Model with two real singlets $\chi$ and $\eta$ obeying a $\mathbb{Z}_2 \otimes \mathbb{Z}_2'$ symmetry, where $\mathbb{Z}_2'$ is broken spontaneously. In particular we study the viability of this model with respect to the requirement of dark matter self-interactions. While $\chi$ serves as a stable Dark Matter candidate, the real $\eta$ field plays the role of a light mediator. Moreover, the Standard Model Higgs which has a tiny mixing with the $\eta$ field appears to be consistent with LHC data. We find in this rather minimal extension of the Standard Model a natural explanation of the cusp-core anomaly in the density profile of dwarf galaxies. Besides, we shall demonstrate that the mediator $\eta$ decays naturally into Majorana neutrinos not disturbing standard big bang nucleosynthesis.

1. INTRODUCTION

The simplest extension of the Standard Model in order to accommodate Dark Matter is one additional real scalar $\chi$ \cite{1,2}. For a recent overview of this minimal extension, in particular with respect to the current status of detection limits, see \cite{4} and references therein. Even that the Dark Matter candidate $\chi$ is usually expected to be stable it has to manage to have been escaped all direct detection limits. Moreover, the Dark Matter candidate not only has to provide the mass of the halo as indirectly observed in galaxy rotations, but also has to explain the cusp-core anomaly observed in the density profile of dwarf galaxies \cite{5}. This can be achieved if the Dark Matter candidate has some appropriate self-interaction. Let us mention in this context the recent observation \cite{6} of an exceptional case of the NGC1052–DF2 galaxy with a stellar mass of approximately $2 \cdot 10^8$ solar masses and a rotational movement indicating a negligible $M_{\text{halo}}$, that is, without any indirectly detected Dark Matter. This observation clearly disfavors modifications of gravity.

Here we would like to revisit a model which contains the Standard Model Higgs doublet $\varphi$, which will act as a dark matter candidate, and an additional real singlet $\eta$ \cite{7,8} mediating the self interaction of $\chi$. In particular, the mediator $\eta$ has to decay sufficiently fast in order not to stay in contradiction to standard big bang nucleosynthesis. We will follow a similar approach as in \cite{9}, where the scalar $\eta$ couples to neutrinos and not to charged leptons which in contrast would disturb the cosmic microwave background observations \cite{10,11}. We shall show that the two-real-singlet model, with an appropriate assignment of discrete symmetries can provide the expected relic abundance of Dark Matter, accomplish self interactions with a mediator which decays sufficiently fast neither disturbing standard big bang nucleosynthesis nor the measurement of the cosmic microwave background. Eventually, from the mixing of the mediator with the ordinary Higgs boson, enhanced invisible Higgs decays are to be expected at the LHC collider.

2. THE TWO SINGLET MODEL

The potential of the model consisting of one doublet $\varphi$, and two real singlets $\chi$ and $\eta$ is \cite{7,8}

$$V_{\text{singlet}} = \mu_{\varphi}^2 \varphi^\dagger \varphi + \mu_{\chi}^2 \chi^2 + \mu_{\eta}^2 \eta^2 + \lambda_{\varphi} \varphi^\dagger \varphi \varphi^\dagger \varphi + \lambda_{\chi} \chi^4 + \lambda_{\eta} \eta^4 + \lambda_{h\varphi} \varphi^\dagger \varphi \eta^2 + \lambda_{\varphi\chi} \varphi^\dagger \varphi \chi^2 + \lambda_{\varphi\eta} \varphi^\dagger \varphi \eta^2 + \lambda_{\chi\eta} \chi^2 \eta^2 .$$  (2.1)

Apart from the Standard Model symmetries we have an additional symmetry $\mathbb{Z}_2$ with $\chi \rightarrow -\chi$ as well as $\mathbb{Z}_2'$ with $\eta \rightarrow -\eta$ and all other Standard Model fields transforming trivially under these discrete symmetries. Now we assume that the neutral component of the doublet $\varphi$ as well as the singlet $\eta$ get vacuum-expectation values (vev’s), $v_{\varphi}$ and $v_{\eta}$, respectively, and in addition to the electroweak group, $SU(2)_L \times U(1)_Y$ , it is supposed that $\mathbb{Z}_2'$ is spontaneously

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broken. Since $\mathbb{Z}_2$ is kept intact, we get in this way a stable particle $\chi$ which becomes a Dark Matter candidate. On the other hand, trilinear couplings arise from the spontaneously broken $\mathbb{Z}_2'$ which provide self-interacting dark matter with $\eta$ working as a light mediator particle.

We assume to have right-handed neutrinos in the model which transform under $\mathbb{Z}_2'$ as $\nu_{R,e} \rightarrow + \nu_{R,e}$, $\nu_{R,\mu} \rightarrow - \nu_{R,\mu}$, and $\nu_{R,\tau} \rightarrow - \nu_{R,\tau}$. In this way we can add to the Lagrangian the Majorana kinetic and mass terms

$$\mathcal{L}_{\nu R} = i\bar{\nu}_{R,i} \gamma^\mu \nu_{R,i} - \left( \frac{\lambda_{ij}}{2} \bar{\nu}_{R,i} \nu_{R,j} + h.c. \right),$$

(2.2)

where $i = e, \mu, \tau$ denote the three flavors of the neutrinos. We note that also a Dirac term is possible for the right-handed electron neutrino but for simplicity we assume its coupling to be negligible here. Let us mention that different assignments of the right-handed neutrinos with respect to $\mathbb{Z}_2'$ could be made. From the charges under $\mathbb{Z}_2'$ we see that the $3 \times 3$ matrix $(\lambda_{ij})$ in (2.2) can only have non-vanishing entries in the first row and column except for the diagonal entry.

Stability considering only the quartic terms of the potential (2.1) yields the requirements $\lambda_h, \lambda_\chi, \lambda_\eta \geq 0$, and $\lambda_h + \lambda_\chi + \lambda_{h\chi} \geq 0$, $\lambda_h + \lambda_\eta + \lambda_{h\eta} \geq 0$, $\lambda_\chi + \lambda_\eta + \lambda_{\chi\eta} \geq 0$, Since we want to have non-vanishing vev’s $v_h$, $v_\eta$, but $\mathbb{Z}_2$ unbroken, we find the constraints

$$\mu^2_h, \mu^2_\eta < 0, \quad \mu^2_\chi \geq 0, \quad \lambda_h, \lambda_\eta > 0, \quad \lambda_\chi > 0, \quad \lambda_h + \lambda_\chi + \lambda_{h\chi} \geq 0, \quad \lambda_h + \lambda_\eta + \lambda_{h\eta} \geq 0, \quad \lambda_\chi + \lambda_\eta + \lambda_{\chi\eta} \geq 0.$$

We note that by a $SU(2)_L \times U(1)_Y$ gauge transformation we can always achieve the form $\varphi = (0, h)^T$ with $h$ real. The tadpole conditions with $(h) \neq 0$, $(\eta) \neq 0$, and $(\chi) = 0$ read

$$\langle h \rangle^2 = \frac{1}{2} v_h^2 = \frac{\lambda_h \mu^2_\eta - 2 \lambda_\eta \mu^2_h}{4 \lambda_h \lambda_\eta - \lambda^2_\eta}, \quad \langle \eta \rangle^2 = \frac{1}{2} v_\eta^2 = \frac{\lambda_h \mu^2_\eta - 2 \lambda_\eta \mu^2_h}{4 \lambda_h \lambda_\eta - \lambda^2_\eta}.$$ 

(2.4)

These relations may be used to fix the parameters $\mu_h$ and $\mu_\eta$ in terms of the vev’s $v_h$ and $v_\eta$. that is,

$$\mu^2_h = -(v_h \lambda_h + \frac{1}{2} v_\eta \lambda_\eta), \quad \mu^2_\eta = -(v_\eta \lambda_\eta + \frac{1}{2} v_h \lambda_\eta).$$

(2.5)

The two Higgs bosons $h$ and $\eta$ mix and form the states $h'$ and $\eta'$. In the basis $(h, \eta)$ the corresponding squared mass matrix reads

$$
\begin{pmatrix}
2 v_h^2 \lambda_h & v_h v_\eta \lambda_\eta \\
v_h v_\eta \lambda_\eta & 2 v_\eta^2 \lambda_\eta
\end{pmatrix},
$$

(2.6)

which is, as usual, diagonalized by an orthogonal matrix with mixing angle $\theta$ given by

$$\tan(2\theta) = \frac{v_h v_\eta \lambda_\eta}{v_h^2 \lambda_h - v_\eta^2 \lambda_\eta}.$$ 

(2.7)

Since we want to have a light mediator $\eta'$, that is, $v_\eta \ll v_h$, the mixing angle $\theta$ has to be small, that is, the doublet $h'$ has only a small contribution from the singlet $\eta$. Note that due to the $\mathbb{Z}_2 \otimes \mathbb{Z}_2'$ symmetry, the Yukawa interactions of $h'$ are not changed compared to the Standard Model but deviations arise from the small mixing with angle given in (2.7).

From the scalar potential eq. (2.1), the trilinear couplings are given by

$$
\begin{align*}
\mu_{\chi \chi' h'} &= \sqrt{2} (s_\theta v_h \lambda_h + c_\theta v_\eta \lambda_\eta), \\
\mu_{\chi \chi' \eta'} &= \sqrt{2} (c_\theta v_h \lambda_h - s_\theta v_\eta \lambda_\eta), \\
\mu_{h' h' h'} &= \sqrt{2} (2 c_\theta^2 v_h \lambda_h - c_\theta^2 s_\theta v_\eta \lambda_\eta + c_\theta s_\theta v_h \lambda_h - 2 s_\theta^2 v_\eta \lambda_\eta), \\
\mu_{h' h' \eta'} &= \sqrt{2} (2 c_\theta^2 v_h \lambda_h + c_\theta^2 s_\theta v_\eta \lambda_\eta + c_\theta s_\theta v_h \lambda_h + 2 s_\theta^2 v_\eta \lambda_\eta), \\
\mu_{h' \eta' \eta'} &= \sqrt{2} (2 c_\theta s_\theta v_h (3 \lambda_h - \lambda_\eta) - c_\theta s_\theta v_\eta \lambda_\eta + c_\theta v_h \lambda_h + 2 c_\theta^2 s_\theta v_\eta (\lambda_\eta - 3 \lambda_h)), \\
\mu_{h' \eta' h'} &= \sqrt{2} (2 c_\theta s_\theta v_h (3 \lambda_h - \lambda_\eta) + c_\theta v_h \lambda_h + 3 c_\theta v_\eta \lambda_\eta - 2 c_\theta^2 s_\theta v_\eta (\lambda_\eta - 3 \lambda_h)),
\end{align*}
$$

(2.8)

with $s_\theta = \sin(\theta)$ and $c_\theta = \cos(\theta)$, where the mixing angle $\theta$ is given in (2.7). Let us also give the quartic couplings

$$
\begin{align*}
\lambda_{\chi \chi' \eta'} &= s_\theta^2 \lambda_h + c_\theta^2 \lambda_\eta, \\
\lambda_{\chi \chi' h'} &= c_\theta^2 \lambda_h + s_\theta^2 \lambda_\eta, \\
\lambda_{\chi h' \eta'} &= 2 c_\theta s_\theta (\lambda_h - \lambda_\eta).
\end{align*}
$$

(2.9)
Eventually we consider the right-handed neutrinos. Apart from the Yukawa couplings in (2.2) we get from \( \langle \eta \rangle \) spontaneously broken a mass matrix

\[
(M_\nu)_{ij} = \lambda_{ij} \langle \eta \rangle, \quad i, j = e, \mu, \tau.
\]  

(2.10)

3. DARK MATTER PHENOMENOLOGY

With the model set up in the previous section we now study the essential aspects of its phenomenology:

- Relic abundance of Dark Matter.

  The relic abundance benchmark value for Dark Matter annihilation is \( \sigma \cdot v_{\text{rel}} \approx 3 \cdot 10^{-26} \text{cm}^3/\text{s} \). In the model considered here the Dark Matter annihilation occurs into particles with even \( \mathbb{Z}_2 \) symmetry, that is, \( \sigma(\chi\chi \to \eta'\eta'/h'h') \). We encounter \( s \) channel contributions with propagators of \( \eta' \) and \( h' \) as well as \( t \) and \( u \) channel contributions with a Dark Matter \( \chi \) propagator. In addition we have to consider the quartic couplings of \( \chi \) with \( \eta' \) and \( h' \). The effective quartic couplings are therefore, with \( s \) denoting the invariant mass squared of the two \( \chi \)'s,

\[
\hat{\lambda}_{\chi\chi\eta'\eta'} = \lambda_{\chi\chi\eta'\eta'} + \frac{\mu_{\eta'\chi\chi} \mu_{\eta'\eta'}}{s - m_{\eta'}^2} + \frac{\mu_{h'\chi\chi} \mu_{h'\eta'}}{s - m_{h'}^2} + \frac{\mu_{\eta'\chi\chi}^2}{s - m_{\chi}^2},
\]

\[
\hat{\lambda}_{\chi\chi h'h'} = \lambda_{\chi\chi h'h'} + \frac{\mu_{h'\chi\chi} \mu_{h'h'}}{s - m_{h'}^2} + \frac{\mu_{\eta'\chi\chi} \mu_{h'h'}}{s - m_{\eta'}^2} + \frac{\mu_{h'\chi\chi}^2}{s - m_{\chi}^2},
\]

\[
\hat{\lambda}_{\chi\chi h'h'} = \lambda_{\chi\chi h'h'} + \frac{\mu_{h'\chi\chi} \mu_{h'h'}}{s - m_{h'}^2} + \frac{\mu_{\eta'\chi\chi} \mu_{h'h'}}{s - m_{\eta'}^2} + \frac{\mu_{h'\chi\chi}^2}{s - m_{\chi}^2}.
\]

(3.1)

We thus get in total,

\[
\sigma(\chi\chi \to \eta'\eta'/h'h') \cdot v_{\text{rel}} = \frac{\hat{\lambda}^2_{\chi\chi\eta'\eta'} + \hat{\lambda}^2_{\chi\chi h'h'} + \hat{\lambda}^2_{\chi\chi h'h'}}{32\pi m_{\chi}^4},
\]

(3.2)

which gives the right value for the relic abundance of Dark Matter \( \chi \) with \( m_{\chi} = 100 \text{ GeV} \) for instance for \( \hat{\lambda}_{\chi\chi\eta'\eta'} = \hat{\lambda}_{\chi\chi h'h'} = \hat{\lambda}_{\chi\chi h'h'} = 0.03 \).

- Self-interacting Dark Matter.

  The benchmark value for self-interacting dark matter is \( \sigma(\chi\chi \to \chi\chi) \approx 1 \text{ cm}^2/\text{g} \). In the two-singlet model, this self interaction arises quite naturally from the mediator \( \eta' \). Explicitly, we compute the cross section for \( \chi + \chi \to \chi + \chi \) near threshold and with the mediator mass \( m_{\eta'} \) supposed to be much smaller than the dark matter mass \( m_{\chi} \) and find in this limit

\[
\sigma(\chi\chi \to \chi\chi) \approx \frac{\mu_{\chi\chi\eta'}^4}{16\pi m_{\chi}^2 m_{\eta'}^2}.
\]

(3.3)

Let us note that the contributions from the quartic \( \chi \) interaction of the potential (2.1) is suppressed by the quartic power of the Dark Matter mass while the contribution from \( s \)-channel \( h' \) exchange is suppressed by \( 1/m_{h'}^4 \) compared to the mediator \( \eta' \) contribution. The Sommerfeld enhancement, taking multiple \( m_{\eta'} \) exchanges into account, is expected to be rather low, similar to the study \( 9 \) where the calculation gives a contribution of the order of 4%. We get the benchmark value easily, for instance, for \( m_{\chi} = 100 \text{ GeV}, m_{\eta'} = 10 \text{ GeV}, \) and \( \mu_{\chi\chi\eta'} = 7 \text{ GeV} \).

- Decay of the mediator \( \eta' \) into neutrinos.

  From the Yukawa interaction in (2.2) we have a Higgs boson \( \eta' \) coupling to the right-handed neutrinos \( \nu_R \). Due to the \( \mathbb{Z}_2 \) even properties of the charged leptons, they only couple very weakly to \( \eta' \), that is only through its \( h \) component in contrast to the neutrinos. This absence of \( \eta \) decays into charged leptons avoids any disturbance of the cosmic microwave background. Specifically, the life time of \( \eta \) should be below the benchmark value of one second. Indeed, with the mixing matrix (2.10) we find a rather fast decay of \( \eta' \) into neutrinos

\[
\Gamma_{\eta'} = \frac{m_{\eta} \sum_{ij} |m_{ij}|^2}{32\pi v_{\eta}^2}.
\]

(3.4)
For $\sum_{ij} |m_{ij}|^2 = 10^{-2} \text{eV}^2$, and $m'_{\eta} = 10 \text{MeV}$, $v_{\eta} = 1 \text{GeV}$ we find for the life time $1/\Gamma_{\eta'} \approx 0.7 \text{s}$, that is, below the required limit of one second.

Note that $\eta'$ could also decay through loops to $\eta' \rightarrow gg$ and $\eta' \rightarrow \gamma\gamma$. The couplings of $\eta'$ to the Standard Model particles in the loop are suppressed by the tiny mixing $\theta$, therefore the decays $\eta' \rightarrow gg$ and $\eta' \rightarrow \gamma\gamma$ have both, a couplings suppression as well as loop-factor suppression.

- **Invisible/non-detected decay of the Standard Model-like Higgs boson.**

As we have seen previously, the Standard Model-like Higgs boson $h'$ has a tiny mixing with the $\eta$ field. All the Higgs couplings to Standard Model particles will be modified by an extra factor $\cos(\theta)$ which is close to unity in our case. Therefore, the $h'$ observables in the model considered here are consistent with the LHC measurements. However, one has to worry about the invisible/non-detected decays of the Standard Model-like Higgs boson. Both ATLAS and CMS have performed searches for such invisible/non-detected Higgs decays and set a limit on its branching fraction.

In our model, $\eta'$ couples to Standard Model particles only through the tiny mixing $\sin(\theta)$. All the production mechanisms of $\eta'$ will be suppressed such that they have escaped both at LEP and at LHC. The field $h'$ can decay into one of the following invisible/non-detected final states: $h' \rightarrow \nu\nu$, $h' \rightarrow \chi\chi$, $h' \rightarrow \chi\chi\eta'$ if kinematically allowed, and also $h' \rightarrow \eta'\eta'$. The first decay takes place only through the tiny mixing angle $\theta$ and is therefore suppressed. In our scenario with $m_\chi \approx 100 \text{GeV}$ both $h' \rightarrow \chi\chi$ and $h' \rightarrow \chi\chi\eta'$ are kinematically closed while $h' \rightarrow \eta'\eta'$ is open and its branching fraction should be smaller than 0.24. In the case of a small mixing angle $\theta$ where $m_{h'\eta'\eta'} \approx \sqrt{2} v_\eta \lambda_{\eta\eta}$, this limit on the branching ratio translates into an upper limit on the coupling, that is, $\lambda_{\eta\eta} \leq 1.3 \cdot 10^{-2}$, which can be satisfied in our model.

4. CONCLUDING REMARKS

We have revisited the two-singlet extension of the Standard Model which accommodates Dark Matter. As usual, the Dark Matter particle is stabilized by an unbroken $\mathbb{Z}_2$ symmetry. An additional scalar, odd under a spontaneously broken $\mathbb{Z}_2$ symmetry, is responsible for the generation of a mediator particle. We have shown that it is rather straightforward to achieve the correct relic abundance of Dark Matter which moreover provides self interactions with view on the observed density profiles of dwarf galaxies. Besides, the mediator decays sufficiently fast and therefore does not disturb standard big bang nucleosynthesis. In particular, the mediator decays dominantly into neutrinos; no contradiction with the cosmic microwave background observation is to be expected. Since the mediator singlet $\eta$ mixes with the neutral Higgs-boson doublet component $h$, the model predicts deviations in Higgs decays. This comes from the fact that the singlet component of the mixed state $h'$, (2.6), does not couple to charged leptons in contrast to the Standard Model Higgs boson. Enhanced invisible Higgs decays compared to the Standard Model should appear for instance at the LHC experiment - providing constraints for the parameters of the model.

Eventually let us mention that the two-singlet extension is a rather minimal and therefore attractive extension of the Standard Model, which provides a viable Dark Matter candidate. We leave it for future work to study the phenomenology of the model in detail.

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