FPGA Based Tsunami Wave Propagation Calculator

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Abstract. The authors continue to use hardware acceleration to calculate tsunami wave propagation in a few seconds at modern PC. The goal of this work is to develop original implementation the FPGA-based algorithm for nested-grids computations of tsunami wave propagation. The FPGA-based Calculator, for regular grid has been proposed and numerically tested earlier. Use of the nested grid approach makes it possible to calculate wave propagation almost until the shore line. Numerical tests are based on the real digital bathymetry at southern part of Japan. Mesh step variation of the grid is from 180 m to just 7 m. Realistic shape of the initial sea surface displacement at tsunami source was used. The detailed distribution of wave height maxima around and inside the Onahama port have been issued as a result of tsunami generated by a model source located along the Japan deep-water trench. The superfast FPGA calculator provides an opportunity to obtain the calculated wave parameters inside ports and harbours even before arrival of tsunami wave.

1. Introduction

The problem of timely warning about the danger of nearfield tsunami after the strong offshore earthquake is still unresolved, even if the number of publications is substantial, see, for example, [1]. For the coast of Japan it takes nearly 20 minutes for tsunami wave to approach the nearest dry land after offshore seismic event. Robust evaluation of tsunami wave danger should be based on correct process simulation: wave generation, propagation, and inundation to a dry land. There are several tools to calculate the wave propagation over the real digital bathymetry, see [2-6]. Among the most popular in the tsunami researcher community numerical methods for long wave modeling, the Method Of Splitting Tsunami (MOST) should be mentioned first. It was developed in late eighties of the last century in the Novosibirsk Computing Center (Russian Academy of Sciences) by V. Titov [6], and now, after certain upgrades, this software package is used by USA National Oceanic and Atmospheric Administration for the tsunami short-time forecast. We would like to mentioned also the TUNAMI N2 package for the Tsunami Inundation Modeling Exchange (TIME). This software was worked out by Imamura in 1993. The registered copyright holders for this package are professors Imamura, Yalziner, and Synolakis. TUNAMI N2 has been successfully used for analyzing some tsunami, see, for example, [4, 5].

In the previous studies the authors propose and test numerically the Calculator, which is able to solve numerically shallow water system at fixed grid 2500×3000 points, see [7-9]. Due to the Field
Programmable Gates Array (FPGA) microchip it takes a few seconds with modern PC. Precision is similar to well-known MOST software, while performance is much better.

Numerical modeling of tsunami propagation from deep-ocean site up to a shoreline on a sequence of refining grids has also been studied by the authors [10] using MOST software package. Tsunami wave maximal heights from model sources were estimated in a number of ports and harbors along Japanese coast using such an approach.

Rest of the paper is arranged as follows. We first introduce the equivalent form of shallow water system, used in numerical experiments. Then, digital bathymetry for the refined computational grids is described as well as the initial sea surface displacement (tsunami wave source) is introduced. The obtained numerical results show the distribution of tsunami wave height maxima along the coast of northeast part of Japan including the fine resolution area around and inside the Onahama port. The final section gives the discussion and the conclusion derived from this work.

2. Mathematical model and numerical scheme

2.1. Mathematical model

Following [4], we use the following equivalent form of a shallow water system (which does not take into account such external forces as sea bed friction, Coriolis forces and others):

\[ H_t + (uH)_x + (vH)_y = 0 \]
\[ u_t + uu_x + vu_y + gH_x = gD_x \]
\[ v_t + uv_x + vv_y + gH_y = gD_y \]  

(1)

where \( H(x,y,t)=\eta(x,y,t)+D(x,y) \) is the entire height of water column, \( \eta \) being the sea surface disturbance (wave height), \( D(x,y) \) – depth (which is supposed to be known at all grid points), \( u \) and \( v \) components of velocity vector, \( g \) - acceleration of gravity.

The initial conditions are as follows: tsunami wave parameters (ocean surface elevation and components of water flow velocity vector) in the entire computational domain are equal to zero, except of the source area, where the surface elevation is non-zero. On the land-ocean boundaries within the computation domain the full reflection conditions of a wave is realized. On the other boundaries the free boundary conditions are established.

For numerical treatment we use Mac-Cormack scheme. This is a direct difference scheme at three-points stencil of a “cross” type. The calculation process drops in two similar stages. At each stage the unknown values at a certain point depend on the previous time values at the same point and just two adjacent mesh points, see details in [7,8].

2.2. Digital bathymetry

The entire computational domain, presented in figure 1, covers geographic area from 137° up to 143° E and from 34° up to 38° N. The computational grid dimension is of 3000 x 2000 nodes with resolution 0.002 arc degree (180 m in west-east direction and 222.6 m in south-north direction).

Three refining grids, B1, B2, and B3, are considered to implement the nested grid method for numerical experiments. Corners of the 1481×2401 nodes subdomain B2 have the following coordinates in terms of the original mesh nodes: (1780, 380), (2150, 380), (1780, 980), (2150, 980). Count of vertical positions of mesh nodes start from the upper bound of B1. Grid step in B2 subdomain is 4 times less compared to B1, that is 45 m in horizontal direction and 55.6 m in vertical direction.

The computational area B3 is the 1302×871 nodes domain with grid resolution 0.000075 arc degree in both directions (approximately 8.35 m in the North-South direction and 6.68 m in the West-East direction). It covers the area of Onahama port at the Fukushima prefecture. This gridded bathymetry has been developed by digitizing the paper nautical chart of Onahama issued by the Japan Hydrographic Association (https://www.jha.or.jp/shop/index.php?main_page=product_info_js2&products_id=529).
2.3. Initial sea surface displacement

The goal of this work is to develop the FPGA-based algorithm for nested-grids computations. So, this can be done using the simplified tsunami source model. Accordingly, the initial water surface displacement in our study is presented by ellipsoidal shape, given in figure 2. The distribution of initial elevation in a grid point \((i, j)\) of the source area is established by the formula

\[
H(i, j) = (1 + \cos (\pi \cdot \arg (i, j)))H_0 / 2
\]

where \(H_0\) is the water surface displacement at the central point \((i_0, j_0)\) of the ellipse. The parameter \(\arg(i, j)\) gives the ratio between the distance from the grid-point to the ellipse center and the distance from this center to the ellipse border in this direction

\[
\arg^2(i, j) = \frac{(i - i_0) \cdot \cos(\beta) + (j - j_0) \cdot \sin(\beta))^2}{r_1^2} + \frac{(j - j_0) \cdot \cos(\beta) + (i - i_0) \cdot \sin(\beta))^2}{r_2^2}
\]

Here \(r1\) and \(r2\) are the ellipse axis’ length and \(\beta\) is the long axis azimuth. Figure 2 shows the shape of the 400 cm height and 200 km long ellipsoidal model source with the axis ratio equal to 4. The colour legend in the left part of figure 2 gives the height distribution inside this source. This approach gives the possibility of the numerical simulation of the tsunami waves generated by combination of such sources with a specified location and an initial height.

In the course of numerical calculations the following shape of model tsunami source, presented in figure 2, were used.
2.4. Numerical experiments details
Let us describe the implementation of the nested grid method for the sequence of refining grids, introduced in subsection B, see figure 1. In the original computational area B1 of the 3000 x 2000 size there is an initial displacement of the water surface (tsunami source) located on the western slope of the Japanese Deep Water Trough (figure 1).

On the course of the entire numerical calculation of the tsunami propagation in the B1 area, the values of the flow parameters (elevation of the water surface and flow velocity components) along the internal boundaries of the B2 subarea are saved at each time step in a special boundary data storage file, say “data_b2”. The coordinates of the angular points of B2 subarea inside of B1 computational domain are given in subsection 2.2. After completing the numerical simulation of the tsunami on the rough grid in the B1 area, the data saved in the “data_b2” file, mentioned above, is interpolated onto a new more detailed calculation grid used for calculation in the B2 subarea. At the second stage, the numerical simulation of tsunami propagation continues already in the B2 subarea embedded in the original area B1, cf. figure 1.

At the same time, at the initial moment the sea surface in this subarea is undisturbed, and so, the tsunami wave is transmitted inside the B2 area through the boundary conditions, which are nothing else but the parameters of the wave saved in the “data_b2” file along the boundaries [10]. The calculation of tsunami wave propagation in the B2 area continues until the wave reaches a coast and then it reflects off the coastline located inside the B3 subarea, where the resolution of the calculation grid is just nearly several meters.

3. Numerical experiments
In this section we present visualizations of results of the numerical modeling of tsunami propagation in the sequence of refining grids B1-B2-B2. Figure 3 shows the calculated oceanic surface (tsunami wave) in the entire B1 computational domain, including the B2 subarea, after 1200 seconds of a wave generation.

Figure 3. The sea surface 1200 sec after tsunami generation as the result of the wave propagation modelling in B1 computational domain.

Figure 4 presents the ocean surface in 2100 sec after the start of propagation process. Here the black rectangle indicates the geographic position of B3 computational domain.

Also, as at the first stage, in the course of numerical calculation in subarea B2, the wave parameters (wave height and components of velocity vector) along the internal boundaries of subarea B3 at each time step are saved to specified file, “data_b3”. Then by means of interpolation the values of flow
parameters in nodes of computational grid used in B2 are recalculated into surface elevation and flow velocity components in boundary nodes of more detailed grid of subarea B3.

In addition, we want to propose the distribution of the wave height maxima all around the B3. It is visible from figure 4 that the model ellipsoidal tsunami source taken for numerical experiment radiates wave energy mainly in the B3 subarea direction.

Distribution of tsunami wave maxima along the entire coast line in B2 area is presented in figure 5.

At the last stage of the numerical experiment the calculation is carried out in the area B3 on the original grid linked to the digital bathymetry developed by authors with the smallest spatial grid step length (0.000075 are degree). This is approximately 8.35 m in the North-South direction and 6.68 m in the West-East direction. The final goal of this study is to obtain the distribution of tsunami height maxima inside the B3 sub-area and along the coastline.

In particular, the wave height in the water area of the port Onahama which is protected by high protective walls separating port area and the open ocean (figure 6). Similarly, using numerical data at B3 boundaries, wave parameters at Onahama port water area are calculated at the fine B3 mesh with approximately 7 m resolution.

Let us discuss in a few words about time required for tsunami propagation modeling on Virtex7-based FPGA card at each stage of the numerical experiment. At the first stage it takes about 23 sec to perform calculation during 6000 time-steps (1800 sec of wave propagation time) in 3000x2000 nodes computational domain B1. The second stage of modeling takes approximately 20 sec for 9000 time-steps (2700 sec of wave propagation time) in 1481x2401 nodes domain B2. In both B1 and B2 areas time step was taken as 0.3 sec. At the last stage the 12000 computing time-steps in 1302x871 nodes area B3 was carried out in 9 sec with time step of 0.15 sec. So, the total calculation time is approximately 52 sec.
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Figure 6. Bathymetry and the coastline of the B3 computational domain.

In order to understand the effectiveness of seawalls surrounding the port entrance let us look at the distribution of the tsunami height maxima inside B3 subarea (figure 7).

Figure 7. Distribution of the tsunami wave height maxima in the B3 computational domain.
The color legend in the right part of figure presents the height-color correspondence. The black spaces along the coastline indicate that no available data just near the coast because of the difference scheme limitations. Method is unable to calculate tsunami parameters when a depth is less than 5 meters.

4. Conclusion
The nested-grids method was used for calculation of tsunami propagation from the deep ocean up to the coastline. Computations in each subarea have been realized at modern PC with FPGA-based hardware acceleration of code execution. Such an approach makes it possible to obtain the approximate tsunami heights along the shoreline under study with high spatial resolution in a short time compared to 1 min only. The detailed distribution of wave height maxima around and inside the Onahama port have been issued as a result of tsunami generated by a model source located along the Japan deep-water trench. The superfast FPGA calculator provides an opportunity to obtain the calculated wave parameters inside ports and harbors even before arrival of tsunami wave. The hardware code acceleration provides the good performance, and nested grids approach gives precision of numerical results.

The simulation also confirmed the effectiveness of the massive protective seawalls around the ports, which in the absence of overflow are able to reduce the tsunami wave height directly at the piers by several times. However, the last major tsunamis show the inability of such protection constructions to resist a catastrophic tsunami. A lot of them had been overturned by the powerful water flow during the Great Tohoku Tsunami 11.03.2011 [11].

5. References
[1] Tsushima H and Yusaku Y 2014 Review on Near-Field Tsunami Forecasting from Offshore Tsunami Data and Onshore GNSS Data for Tsunami Early Warning J. Disaster Res. 9(3) 339–357
[2] Gica E, Spillane M, Titov V, Chamberlin C and Newman J 2008 Development of the forecast propagation database for NOAA’s short-term inundation forecast for tsunamis (SIFT) NOAA Technical Memorandum URL: http://www.ndbc.noaa.gov/dart.shtml
[3] Kensaku H, Vazhenin A P and Marchuk A G 2015 Trans-Boundary Realization of the Nested-Grid Method for Tsunami Propagation Modeling Proc. 25th Ocean /Polar Eng Conf Kona, Big Island, Hawaii, USA 3 741–747
[4] Shuto N, Goto C and Imamura F 1990 Numerical simulation as a means of warning for near field tsunamis Coastal Engineering in Japan 33(2) 173-193
[5] Yalciner A C, Alpar B, Altinok Y, Ozbay I, and Imamura F 2002 Tsunamis in the Sea of Marmara: Historical Documents for the Past, Models for Future Marine Geology 190 445-463
[6] Titov V V and Gonzalez F I 1997 Implementation and testing of the method of splitting tsunami (MOST) model NOAA Technical Memorandum ERL PMEL-112
[7] Lavrentiev M M, Romanenko A A, Oblaukhov K K, Marchuk An G, Lysakov K F and Shadrin M Yu 2017 FPGA Based Solution for Fast Tsunami Wave Propagation Modeling Proc 27th (2017) Int Ocean/Polar Eng Conf, San Francisco, CA, USA, June 25-30, 2017, 924-29.
[8] Lavrentiev M M, Romanenko A A, Oblaukhov K K, Marchuk An G, Lysakov K F and Shadrin M Yu 2017 Implementation of Mac-Cormack scheme for the fast calculation of tsunami wave Propagation Proc. Oceans’17 MTS/IEEE, Aberdeen UK
[9] Lavrentiev M, Lysakov K, Marchuk An, Oblaukhov K, and Shadrin M 2019 Fast evaluation of tsunami waves heights around Kamchanka and Kuril Island Science of Tsunami Hazards 38(1) 1-13
[10] Hayashi K, Marchuk An G and Vazhenin A P 2018 Generating Boundary Conditions for the Calculation of Tsunami Propagation on Nested Grids Numerical Analysis and Applications 11(3) 256-267
[11] Tomita T, Arikawa N and Asai T 2013 Damage in Ports due to the 2011 off the Pacific Coast of Tohoku Earthquake Tsunami J Disaster Research 8 (4) 594-604