A New Approach for Single Transverse-Spin Asymmetries from Twist-3 Soft-Gluon Mechanism

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Abstract. A dominant QCD mechanism for the single transverse-spin asymmetry in hard processes is induced by the twist-3 quark-gluon correlations inside nucleon, combined with the soft-gluonic poles to produce the interfering phase for the associated partonic hard scattering. It is shown that the corresponding interfering amplitude can be calculated entirely in terms of the partonic Born cross section which participates in the twist-2 cross section formula for the spin-averaged process.

Keywords: Single spin asymmetry, Twist-3, Soft-gluonic pole

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We investigate the single-transverse spin asymmetry (SSA), which is observed as the effect proportional to \( \vec{S}_\perp \cdot (\vec{p} \times \vec{q}) \) in the cross section for the scattering of transversely polarized proton with momentum \( p \) and spin \( S_\perp \), off unpolarized particle with momentum \( p' \), producing a particle with momentum \( q \) which is observed in the final state. Famous examples [1] are \( p^\uparrow p \rightarrow \pi X \), where the large asymmetry \( A_N \sim 0.3 \) was observed in the forward direction, and semi-inclusive deep inelastic scattering (SIDIS), \( ep^\uparrow \rightarrow e\pi X \). The Drell-Yan (DY) process, \( p^\uparrow p \rightarrow \ell^+\ell^- X \), and the direct \( \gamma \) production, \( p^\uparrow p \rightarrow \gamma X \), at RHIC, J-PARC, etc. are also expected to play important roles for the study of SSA because those processes are clean in that fragmentation functions do not participate.

The SSA requires, (i) nonzero \( q_\perp \) originating from transverse motion of quark or gluon; (ii) proton helicity flip; and (iii) interfering phase for the cross section. For processes with \( q_\perp \sim \Lambda_{\text{QCD}} \), all (i)-(iii) are generated from nonperturbative QCD mechanism associated with the transverse-momentum-dependent parton distributions such as Sivers function [2, 3]. On the other hand, for large \( q_\perp \gg \Lambda_{\text{QCD}} \), perturbative effects also play essential roles to produce the SSA, in the framework of the collinear factorization with twist-3 distributions: In the DY pair production with \( q_\perp \gg \Lambda_{\text{QCD}} \), for example, the large \( q_\perp \) is provided by hard interaction as the recoil from a hard final-state parton, as illustrated in Fig. 1. Proton helicity flip is provided by the participation of the coherent, nonperturbative gluon from the transversely polarized proton, the lower blob in Fig. 1; the lower part of Fig. 1 represents the twist-3 quark-gluon correlation functions such as \( G_F(x_1, x_2) \) with \( x_1 (x_2) \) the lightcone momentum fraction of the quark leaving from (entering into) the proton [4]. Due to the coupling of this coherent gluon, some parton propagators in the partonic subprocess can be on-shell, and this produces the imaginary phase as the pole contribution using \( 1/(k^2 + ie) = P(1/k^2) - i\pi\delta(k^2) \). Depending on the resulting values of the coherent-gluon’s momentum \( k_g \), these poles are called soft-gluon
pole (SGP) for \( k_g = 0 \), and soft-fermion pole (SFP) and hard pole (HP) for \( k_g \neq 0 \).

Among these three types of poles, the SGP plays somewhat distinct roles compared with the other two, from theoretical as well as phenomenological viewpoint (see e.g. \([4, 7]\)). In particular, the SSA from the SGP contribution is known to obey remarkable simplicity: The partonic hard cross section associated with the “derivative term” \((\propto dG_F(x,x)/dx)\) is identical, up to certain kinematical and color factor, to that for the unpolarized twist-2 cross section as observed for direct \( \gamma \) production \([6]\), DY process \([2]\), and SIDIS \([4]\). In this work we concentrate on the SGP contribution to SSA.

We have developed a systematic diagrammatic manipulation approach, and find that the SGP contributions from many Feynman diagrams are united into a derivative of the Born diagrams without coherent gluon line: The SSA for the DY process can be expressed as \([7]\)

\[
d\sigma_{\text{SGP}}^{\text{tw}-3}[d\omega] = \frac{\pi M_N}{2C_F} e^{\sigma_{\text{p}nS\perp}} \sum_{j=q,g} B_j \int \frac{dx'}{x'} \int \frac{dx}{x} f_j(x') \frac{\partial H_{jq}(x',x)}{\partial (x'p^\sigma)} G_{qF}^q(x,x),
\]

where \( j = q \) and \( g \) represent the “q\(\bar q\)-annihilation” and “qg-scattering” channels, respectively, the sum over all quark and antiquark flavors, \( q = u, \bar u, d, \bar d, \cdots \), is implicit for the index \( q \), and \( f_j(x') \) denotes the twist-2 parton distribution functions for the unpolarized proton. \( M_N \) is the nucleon mass representing nonperturbative scale associated with the twist-3 correlation function \( G_{qF}^q(x,x) \) for the flavor \( q \). \([d\omega] = dQ^2 dy d^2q_{\perp}\) denotes the relevant differential elements with \( Q^2 = q^2 \) and \( y \) the rapidity of the virtual photon. The color factors are introduced as \( B_q = 1/(2N_c) \) and \(-1/(2N_c)\) for quark and antiquark flavors, respectively, \( B_g = N_c/2 \), and \( C_F = (N_c^2 - 1)/(2N_c) \). \( H_{jq}(x',x) \) denote the partonic hard-scattering functions which participate in the unpolarized twist-2 cross section for DY process as

\[
d\sigma_{\text{tw}-2}^{\text{unpol}}[d\omega] = \sum_{j=q,g} \int \frac{dx'}{x'} \int \frac{dx}{x} f_j(x') H_{jq}(x',x) f_q(x).
\]

Namely, in order to obtain the explicit formula for the twist-3 SGP contributions to the SSA, knowledge of the twist-2 unpolarized cross section is sufficient.

A proof of \([11]\) is given in \([7]\), and here we briefly sketch its main points: In the coupling of the coherent-gluon field \( A^\mu \) to the partonic subprocess as in Fig. 1, the scalar-polarized part, whose polarization vector is proportional to the gluon’s momentum \( k_g^\mu \),
and the transversely-polarized part are relevant up to the twist-3 accuracy in the Feynman
gauge. The coupling of the scalar-polarized part can be immediately manipulated using
Ward identity, so that the gluon’s interaction vertex, as well as the SGP from the
parton propagator adjacent to it, is disentangled. On the other hand, the coupling of
the transversely-polarized part to the parton propagator can be decomposed into the
eikonal vertex, combined with the eikonal propagator, and the “contact term” containing
the “special propagator” [6]. After this reduction, the collinear expansion of the relevant
diagrams in terms of the parton transverse momenta is quite straightforward, and various
contributions cancel with the corresponding contributions from the “mirror” diagrams.
The surviving twist-3 contributions are given as the derivative of certain diagrammatic
expression in terms of the parton transverse momenta is quite straightforward, and various
both

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variables \( \hat{s}, \hat{t}, \hat{u} \) can be found in Eqs. (12) and (14) in [2]. The derivative in (1)
can be performed through that for the \( \hat{s} \) and \( \hat{u} \), and this may act on either \( \hat{s} \) or the delta function
of (3), where the latter case produces the derivative term with \( dG_F(x,x)/dx \), as well as
non-derivative term \( \propto G_F(x,x) \), by partial integration with respect to \( x \). We get

\[
\frac{d\sigma_{\text{SGP}, \text{DY}}^{\text{tw-3}}}{dQ^2 dy d^2 q_\perp} = \frac{\alpha^2_\text{em} \alpha e_q^2 \pi N_c}{3\pi N_c s Q^2} \frac{e^{\rho N_s q_\perp}}{C_F} \sum_{j=q,g} B_j \int \frac{dx'}{x'} \int \frac{dx}{x} \delta \left( \hat{s} + \hat{t} + \hat{u} - Q^2 \right) f_j(x')
\]

\[
\times \left\{ \frac{\hat{s}_{j}}{\hat{u}} \frac{dG^q_{F}(x,x)}{dx} + \left[ \frac{\hat{s}_{j}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{u}} - \frac{\hat{t}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{t}} - \frac{\hat{u}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{u}} - \frac{\hat{s}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{s}} \right] G^q_{F}(x,x) \right\} .
\]

Substituting explicit form for \( \hat{s}_{j}(\hat{s},\hat{t},\hat{u}) \), this result completely coincides with that in [2]
with the appropriate substitutions.

Let us apply our “master formula” (1) to obtain the explicit form for the SGP
contribution to the SSA in the DY process. The hard-scattering functions \( H_{jq}(x',x) \) can be extracted from the twist-2 factorization formula for the unpolarized DY process as

\[
H_{jq}(x',x) = \frac{\alpha^2_\text{em} \alpha e_q^2}{3\pi N_c s Q^2} \sigma_{jq}(\hat{s},\hat{t},\hat{u}) \delta \left( \hat{s} + \hat{t} + \hat{u} - Q^2 \right),
\]

where \( s = (p + p')^2 \), and explicit form of \( \hat{s}_{j}(\hat{s},\hat{t},\hat{u}) \) in terms of the partonic Mandelstam
variables \( \hat{s}, \hat{t} \) and \( \hat{u} \) can be found in Eqs. (12) and (14) in [2]. The derivative in (1)
can be performed through that for the \( \hat{u} \), and this may act on either \( \hat{s} \) or the delta function
of (3), where the latter case produces the derivative term with \( dG_F(x,x)/dx \), as well as
non-derivative term \( \propto G_F(x,x) \), by partial integration with respect to \( x \). We get

\[
\frac{d\sigma_{\text{SGP}, \text{DY}}^{\text{tw-3}}}{dQ^2 dy d^2 q_\perp} = \frac{\alpha^2_\text{em} \alpha e_q^2 \pi N_c}{3\pi N_c s Q^2} \frac{e^{\rho N_s q_\perp}}{C_F} \sum_{j=q,g} B_j \int \frac{dx'}{x'} \int \frac{dx}{x} \delta \left( \hat{s} + \hat{t} + \hat{u} - Q^2 \right) f_j(x')
\]

\[
\times \left\{ \frac{\hat{s}_{j}}{\hat{u}} \frac{dG^q_{F}(x,x)}{dx} + \left[ \frac{\hat{s}_{j}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{u}} - \frac{\hat{t}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{t}} - \frac{\hat{u}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{u}} - \frac{\hat{s}}{\hat{u}} \frac{\partial \hat{s}_{j}}{\partial \hat{s}} \right] G^q_{F}(x,x) \right\} .
\]
our result reveals that the partonic hard-scattering functions associated with the non-derivative term are also completely determined by $\hat{\sigma}_{jq}$. In the real-photon limit, $Q^2 \to 0$, (4) gives the SSA for the direct $\gamma$ production (see [2]).

We now use our master formula (1) for the SIDIS, $ep \uparrow \to e\pi X$. We perform the substitutions corresponding to the crossing transformation discussed above: $p' \to -P_h$, $x' \to 1/z$, $f_q(x') \to D_q(z)$, $f_g(x') \to D_g(z)$, and $q^\mu \to -q^\mu$, where $D_j(z)$ denote the twist-2 parton fragmentation functions for the final-state pion with momentum $P_h$, and the new $q^\mu$ gives $Q^2 = -q^2$. From (1) and (2), we get $(C_q \equiv B_{\bar{q}}, C_g \equiv B_g)$ [7]

$$d\sigma^{\text{SGP}}_{\text{SIDIS}, \text{tw-3}}(d\omega) = \pi M_N e^{pnS_L p_{h\perp}} \frac{\partial}{\partial q_T^2} d\sigma^{\text{unpol,SIDIS}}_{\text{tw-2}}(d\omega) \bigg|_{f_q(x) \to G_q(x,x), D_j(z) \to C_j z D_j(z)},$$

in a frame where the 3-momenta $\vec{q}$ and $\vec{p}$ of the virtual photon and the transversely polarized nucleon are collinear along the $z$ axis. $[d\omega] = dx_{bj}dz_jdq_T^2d\phi$, where, as usual, $x_{bj} = Q^2/(2p\cdot q)$, $z_j = p\cdot P_h/p\cdot q$, $q_T = P_h\perp/z_f$, and $\phi$ is the azimuthal angle between the lepton and hadron planes. The twist-2 unpolarized cross section in the RHS of (5) is known to have several terms corresponding to different dependence on $\phi$, which are proportional to $1, \cos \phi,$ and $\cos^2 \phi$, respectively (see [5]). Performing the derivative in (5) explicitly, the result obeys exactly the same pattern as (4) for both derivative and non-derivative terms. Also, the result completely coincides, for all azimuthal dependence including those beyond the Sivers effect, with the one that has been obtained recently in [5] by direct evaluation of each Feynman diagram.

To summarize, we have discussed the twist-3 mechanism for the SSA arising from the SGP. We have developed a new approach that allows systematic reduction of the coupling of the soft coherent-gluon and the associated pole contribution, using Ward identities and decomposition identities for the interacting parton propagator, and derived the master formula which gives the twist-3 SGP contributions to the SSA entirely in terms of the knowledge of the twist-2 factorization formula for the unpolarized cross section. Our master formula is applicable to a range of processes, DY process, direct $\gamma$ production, SIDIS, and hopefully other processes.

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