One-loop calculations for SUSY processes

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Strategy and results for complete one-loop computations in the Minimal Supersymmetric Standard Model are reviewed, with applications to the calculation of SUSY mass spectra and SUSY-particle processes. Determination of renormalization constants and counterterms are described in the on-shell renormalization scheme, and a translation between DR and on-shell parameters is given. As an example, cross sections for chargino and neutralino pair production in $e^+e^-$ annihilation are presented, complete at the one-loop level.

1. Introduction

Experiments at future high-energy colliders will be able to discover supersymmetric particles and to investigate their properties. A linear electron-positron collider will be the best environment for precision studies of supersymmetric models [1], especially of the minimal supersymmetric standard model (MSSM). From precise measurements of masses and cross sections, the fundamental parameters of the MSSM Lagrangian can be reconstructed [2] to shed light on the mechanism of SUSY breaking. Adequate theoretical predictions matching the experimental accuracy require the proper inclusion of higher-order terms in the calculations of mass spectra, cross sections, decay rates.

In this talk we present a brief overview on complete one-loop calculations for SUSY processes and supersymmetric mass spectra, where all MSSM particles with electroweak couplings are included in the virtual states. As a calculational frame, the on-shell renormalization scheme has been chosen where all particle masses are defined as pole masses, i.e. on-shell quantities. Cross sections are thus directly related to the physical masses of the external particles and of the other particles entering the loops. We outline the theoretical basis of the calculation and show in a few examples the numerical size of the loop effects, where we restrict ourselves to the case of the CP-conserving MSSM with real parameters.

The presentation is based on [3,4], for other approaches see [5,6].

2. Renormalization

2.1. Charginos and neutralinos

The bilinear part of the Lagrangian describing the chargino/neutralino sector of the MSSM involves the $\mu$ parameter, the soft-breaking gaugino-mass parameters $M_1$ and $M_2$, and the Higgs vacua $v_i$, which are related to $\tan \beta = v_2/v_1$ and to the $W$ mass $M_W = g v/2$ with $v = (v_1^2 + v_2^2)^{1/2}$.

Renormalization constants are introduced for the chargino mass matrix $X$ and for the chargino fields $\tilde{\chi}_i^+$ ($i = 1, 2$) by the transformation

$$X \rightarrow X + \delta X,$$

$$\omega_L \tilde{\chi}_i^+ \rightarrow \left( \delta_{ij} + \frac{1}{2} \left[ \delta Z^L_{\tilde{\chi}_x^+} \right]_{ij} \right) \omega_L \tilde{\chi}_j^+,$$

$$\omega_R \tilde{\chi}_i^+ \rightarrow \left( \delta_{ij} + \frac{1}{2} \left[ \delta Z^R_{\tilde{\chi}_x^+} \right]_{ij} \right) \omega_R \tilde{\chi}_j^+. \quad (1)$$

The matrix $\delta X$ consists of the counterterms for the parameters in the mass matrix $X$,

$$\delta X = \begin{pmatrix} \delta M_2 & \sqrt{2} \delta (M_W \sin \beta) \\ \sqrt{2} \delta (M_W \cos \beta) & \delta \mu \end{pmatrix}. \quad (2)$$

The field-renormalization constants $\delta Z^L_{\tilde{\chi}_x^+}$ and $\delta Z^R_{\tilde{\chi}_x^+}$ are general complex $2 \times 2$ matrices. The set of renormalization constants in (1) renders both $S$-matrix elements and Green functions for
charginos finite and allows moreover to get the matrix of renormalized self energies diagonal on the mass shell of each of the chargino mass eigenstates $\tilde{\chi}^0_i$.

The neutralino fields $\tilde{\chi}^0_i$ ($i = 1, \ldots, 4$) and the mass-matrix $Y$ are – in analogy to the chargino case – renormalized by the substitutions

\[ Y \to Y + \delta Y, \]
\[ \omega_L \tilde{\chi}^0_i \to \left( \delta_{ij} + \frac{i}{2} \left[ \delta Z_{\tilde{\chi}^0_j}^* \right]_{ij} \right) \omega_L \tilde{\chi}^0_j, \]
\[ \omega_R \tilde{\chi}^0_i \to \left( \delta_{ij} + \frac{i}{2} \left[ \delta Z_{\tilde{\chi}^0_j}^* \right]_{ij} \right) \omega_R \tilde{\chi}^0_j. \] (3)

Besides those parameter counterterms already present in (2), the counterterm matrix $\delta Y$ contains the counterterms for the soft-breaking gaugino-mass parameter $M_1$ for the $Z$ mass and the electroweak mixing angle, respectively. The matrix-valued renormalization constant $\delta Z_{\tilde{\chi}^0}^0$ is a general complex $4 \times 4$ matrix.

Using the on-shell approach of [3], the pole masses of the two charginos, $m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}$, and of one neutralino, $m_{\tilde{\chi}^0_1}$, are considered as input parameters, to specify the chargino/neutralino Lagrangian in terms of physical quantities. This is equivalent both to the specification of the parameters $\mu, M_1, M_2$, which are related to the input masses in the same way as in lowest order, as a consequence of the on-shell renormalization conditions, and to the definition of the respective counterterms. In that way, the tree-level masses $m_{\tilde{\chi}^+_1}$ and $m_{\tilde{\chi}^0_1}$ as well as the counterterm matrices $\delta X$ and $\delta Y$ are fixed.

Furthermore, one requires for both, charginos as well as neutralinos, that the matrix of the renormalized one-particle-irreducible two-point vertex functions $\Gamma^{(2)}_f$ becomes diagonal for on-shell external momenta. This fixes the non-diagonal entries of the field-renormalization matrices $\delta Z_{\tilde{\chi}^+_1}, \delta Z_{\tilde{\chi}^0_1}$, and $\delta Z_{\tilde{\chi}^-_1}$; their diagonal entries are determined by normalizing the residues of the propagators to unity.

2.2. Sfermions

In the sfermion sector, we introduce renormalization constants for the mass matrices in the $L, R$ basis for each sfermion species, $M^{\tilde{f}}$, and for the mass-eigenstates $f^s$ ($s = 1, 2$ in general, $s = L$ for $f = \nu$) by means of the transformation

\[ M^{\tilde{f}} \to M^{\tilde{f}} + \delta M^{\tilde{f}}, \]
\[ f^s \to \left( \delta m_f + \frac{1}{2} \left[ \delta Z_{f^s}^* \right]_{st} \right) f^t. \] (4)

The matrix $\delta M^{\tilde{f}}$ is made of the counterterms for the parameters in the mass matrix $M^{\tilde{f}}$. In the case of sneutrino fields, the respective mass matrix actually consists of only one single element. The field-renormalization constants $\delta Z^{f_s}$ are general complex $2 \times 2$ matrices, except for the $f = \nu$ case with a single field-renormalization constant $\delta Z^\nu = \delta Z_L$ only.

Applying the renormalization procedure of [4], the sfermion soft-breaking $L, R$ mass parameters for the members of each isodoublet, together with their renormalization constants, are determined via on-shell mass renormalization conditions for three independent pole masses in the case of squarks and two in the case of sleptons. The trilinear couplings $A_{\tilde{f}}$ can be related to the non-diagonal entries of the field-renormalization matrices $\delta Z^{f_s}$, and are determined via the requirement of having diagonal two-point vertex functions for on-shell momenta, as in the case of charginos and neutralinos. The diagonal entries of $\delta Z^{f_s}$ are determined by normalizing the residues of the propagators to unity.

2.3. Other sectors

The formal description of parameter and field renormalization in the Standard-Model-like part of the MSSM is taken over from [7], yielding for the electric charge

\[ e \to Z_e e = (1 + \delta Z_e) e, \] (5)

and for the masses of $W, Z$, and of the fermions

\[ M_{W,Z}^{\tilde{f}} \to M_{W,Z}^{\tilde{f}} + \delta M_{W,Z}^{\tilde{f}}, \]
\[ m_f \to m_f + \delta m_f. \] (6)

The counterterms are determined by the usual on-shell conditions in each case; field renormalization is treated as described in [7].

Renormalization of $\tan \beta$ as the ratio of the VEVs of the two Higgs fields is done in the DR-scheme [8],

\[ \tan \beta = \frac{v_2}{v_1} \to \frac{v_2}{v_1} \left( 1 + \frac{1}{2} \left( \delta Z_{H_2} - \delta Z_{H_1} \right) \right). \] (7)
with the $\overline{\text{DR}}$-singular parts of the Higgs-field renormalization constants $\delta Z_{H_1,2}$.

3. $\overline{\text{DR}}$ versus OS scheme

In the $\overline{\text{DR}}$ scheme, divergent 1-loop quantities are renormalized by adding counterterms that are proportional to the divergent parts,

$$\frac{2}{\epsilon} - \gamma + \log 4\pi,$$

of the 2- and 3-point vertex functions, regularized using the dimensional-reduction method. As a consequence, physical observables depend on the scale $\mu_{\overline{\text{DR}}}$.

Input variables in the $\overline{\text{DR}}$ scheme are a natural choice for GUT-inspired parameter sets (e.g. in SUGRA scenarios).

On the other hand, in the OS scheme the renormalization constants are fixed at physical scales; observables are thus scale independent. The OS scheme is convenient for calculations of cross sections and decay rates, because masses at Born level and in higher order agree (with few exceptions), holding the correct phase-space kinematics already in tree-level calculations.

In order to achieve a translation between $\overline{\text{DR}}$ and OS parameters, the following two steps are performed, specified here for the quantities $\mu$, $M_2$, $M_1$ of the chargino/neutralino sector.

1. Using $\mu$, $M_2$, and $M_1$ in the $\overline{\text{DR}}$ scheme as a starting point, the pole masses of three particles, e.g. both charginos and the lightest neutralino, are calculated at the one-loop level.

2. From those three physical masses the corresponding parameters in the OS scheme are deduced, using tree-level relations (which are left unaltered by construction in the OS scheme).

In the example below we show the values (in GeV) for the various quantities using the SPS1a set [9] of input parameters in the $\overline{\text{DR}}$ scheme, where shifts up to 10 GeV can occur.

\begin{table}
\begin{tabular}{|c|c|c|}
\hline
$\overline{\text{DR}}$ & OS & pole masses \\
\hline
$\mu = 352.4$ & $\mu = 361.6$ & \\
$m_{\tilde{\chi}_1^+} = 184.3$ & $m_{\tilde{\chi}_1^+} = 102.4$ & \\
$M_1 = 99.0$ & $M_1 = 99.5$ & \\
$m_{\tilde{\chi}_2^0} = 387.4$ & $M_2 = 201.0$ & \\
$M_2 = 192.7$ & & \\
\hline
\end{tabular}
\end{table}

4. $e^+e^-$ production cross sections

As a concrete example, we discuss the production of chargino and neutralino pairs in $e^+e^-$ annihilation (see also [10]). For production of sfermion pairs we refer to the literature [11].

4.1. Born amplitudes

At lowest order, the amplitude $M$ for chargino-pair production can be described by $s$-channel photon and $Z$-boson exchange and by $t$-channel exchange of a scalar neutrino $\tilde{\nu}_e$, as displayed in the following Feynman diagrams.

In the case of neutralino pair production, there is no photon exchange at tree level, and $t$-channel exchange is mediated by the two selectrons $\tilde{e}^s$, $s = 1, 2$.

Other diagrams, containing Higgs lines, are always negligible.

4.2. Virtual corrections

The above set of Born diagrams has to be dressed by the corresponding loop contributions containing the full particle spectrum of the MSSM. The renormalization constants determined in section 2 are complete to deliver all counterterms required for propagators and vertices appearing in the amplitudes, and they have become part of the FeynArts package for the MSSM [12]. The one-loop contributions can be classified as follows,
- **self-energy contributions** to the propagators for photon, \(Z\) boson, and \(\tilde{\nu}_e \,(\tilde{e}^\pm)\) neutrino (selectrons);
- **vertex corrections** for \(\gamma\) and \(Z\) in the \(s\) channel and \(\tilde{\nu}_e \,(\tilde{e}^\pm)\) in the \(t\) channel;
- **box-diagram contributions** with double exchange of gauge, Higgs, and SUSY particles in the \(s\) and \(t\) channel.

### 4.3. Real photons and “QED corrections”

Virtual photons attached to external charged particles give rise to infrared (IR) divergences in the loop diagrams. An IR-finite result is obtained by adding real-photon bremsstrahlung integrated over the photon phase space. The sum of the one-loop contribution to the cross section, \(\sigma^{\text{virt}}\), and the bremsstrahlung cross section, \(\sigma^{\text{brem}}\), is IR-finite. Different from standard-fermion production, these photonic contributions cannot be removed and treated separately as “QED corrections”, because the one-loop result without the virtual photons would not be UV-finite. The presence of the photon is required to cancel the divergence from the photino component of the virtual neutralinos.

For cancellation of the IR divergence, it is convenient to split \(\sigma^{\text{brem}}\) into a (IR-divergent) soft part and a (IR-finite) hard part, both depending on a soft-photon cutoff \(\Delta E\) for the energy of the radiated photon,

\[
\sigma^{\text{brem}} = \sigma^{\text{soft}}(\Delta E) + \sigma^{\text{hard}}(\Delta E) .
\]  

For practical calculations, in order to avoid numerical instabilities, \(\sigma^{\text{hard}}\) is divided into a collinear part, where the photon is within an angle smaller than \(\Delta \theta\) with respect to the radiating particles, and the complementary non-collinear part,

\[
\sigma^{\text{hard}} = \sigma^{\text{coll}}(\Delta \theta) + \sigma^{\text{non-coll}}(\Delta \theta) .
\]  

For \(\sigma^{\text{soft}}\) and \(\sigma^{\text{coll}}\) analytical results are available; \(\sigma^{\text{non-coll}}\) is calculated numerically with the help of DIVONNE and CUBRE, multidimensional numerical integration routines that are part of the CUBA-library [13].

As already pointed out above, there is no diagrammatic way to disentangle QED-like photonic virtual contributions from the MSSM-specific parts. One can, however, isolate the universal and leading QED terms in \(\sigma^{\text{virt}} + \sigma^{\text{soft}}\) resulting from photons collinear to the incoming \(e^\pm\), which contain the large logarithm \(L_e = \log \left( \frac{m_e}{\Delta E} \right)\).

The separation

\[
\sigma^{\text{virt}} + \sigma^{\text{soft}} = \tilde{\sigma} + \sigma_{\text{remainder}} ,
\]  

identifies a one-loop contribution \(\sigma_{\text{remainder}}\) that is IR finite and free of large universal QED terms. \(\sigma_{\text{remainder}}\) contains the MSSM-specific radiative corrections, whereas the subtracted part \(\tilde{\sigma}\) in (10) together with the hard bremsstrahlung part from initial-state radiation can be considered as a contribution of the type “QED corrections”,

\[
\sigma_{\text{QED}} = \sigma^{\text{hard}}_{\text{ISR}} + \tilde{\sigma} .
\]  

It is independent of the auxiliary cut \(\Delta E\), and it contains all large logarithms from virtual, soft, and hard photons.

With this reordering, the complete cross section at the one-loop level can be written as follows,

\[
\sigma_{1\text{-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}} + \sigma^{\text{soft}} + \sigma^{\text{hard}} = \sigma^{\text{Born}} + \sigma^{\text{QED}} + \sigma_{\text{remainder}} + \Delta \sigma .
\]  

The small piece \(\Delta \sigma\) accounts for final-state radiation in case of chargino production, and for photon radiation from the \(t\)-channel selectrons in case of neutralino production.

### 4.4. Results

For illustration, we choose the example of the SPS1a parameter set [9] listed partially in section 3. The \(\overline{\text{DR}}\) parameters are first converted into on-shell quantities and pole masses, from which the cross sections are calculated, both at tree level and at higher order. The following figures contain the integrated cross sections for \(\tilde{\chi}_1^+ \tilde{\chi}^-\) and for \(\tilde{\chi}_1^0 \tilde{\chi}_2^0\) production in \(e^+e^-\) annihilation for unpolarized beams, as functions of the CMS energy. Besides Born cross sections and full one-loop results, also the contributions \(\sigma_{\text{remainder}}\) and \(\sigma_{\text{QED}}\) are shown separately.
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