Solving the apparent inconsistency between GSI and RIKEN estimates of $^{11}\text{Be} \ dB(E1)/dE$  

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Abstract. $^{11}\text{Be}$ is the archetypical one-neutron halo nucleus. Due to its short lifetime, one of the only way to infer information about its exotic structure is to study reactions involving that nucleus, like breakup. When performed on a heavy target, like lead, breakup is dominated by the E1 transition from the bound state to the continuum, which is characterized by the $dB(E1)/dE$. This strength has been inferred from two experiments, one performed at 520 A MeV at GSI and the other at 69 A MeV at RIKEN. Strangely the analyses of both experiments provide different E1 strengths. In this work, we reanalyze them using the eikonal approximation to study this discrepancy. In particular, we properly take into account relativistic effects, and include a consistent treatment of both nuclear and Coulomb interactions and their interference at all orders. The description of the $^{11}\text{Be}$ structure is provided by halo effective field theory (Halo-EFT). Our cross sections for the $^{11}\text{Be}$ breakup are in good agreement with both RIKEN and GSI data. The $dB(E1)/dE$ extracted from our $^{11}\text{Be}$ model is in agreement with the RIKEN result and $ab\ initio$ predictions. We can conclude that the discrepancy between GSI and RIKEN $dB(E1)/dE$ arises from the method applied to extract this quantity. From our detailed analysis of the reaction, it seems that the most efficient way to extract the $dB(E1)/dE$ from the breakup cross section is to select the data at small angles, where the reaction is dominated by the Coulomb interaction.

1. Introduction

Halo nuclei are exotic nuclear structures found at the edges of the nuclear chart, far from the valley of stability. They exhibit a much larger matter radius than their isobars and this unusual size is qualitatively explained by a quantum-tunneling effect in which one or two loosely bound valence nucleons have a high probability of presence at a large distance from the other nucleons, far beyond the range of the nuclear interaction. These nucleons hence form a diffuse halo surrounding a compact core [1]. Halos have been found with one or two neutrons. Examples of one-neutron halo nuclei are $^{11}\text{Be}$ and $^{15}\text{C}$, that can be seen as a $^{10}\text{Be}$ or $^{14}\text{C}$ core plus one neutron. $^{11}\text{Li}$ and $^{6}\text{He}$ are examples of two-neutron halo nuclei. Since the half-life of such nuclei is short ($^{11}\text{Be}$ half-life is about 13 s), their structure is mostly studied through indirect techniques, such as reactions. Breakup reaction, for instance, is a useful spectroscopic tool to study halo nuclei: this process consists in the dissociation of the halo nucleus projectile into its constituents, i.e. core and valence neutrons. When performed on a heavy target, like lead, it is dominated by the electric dipole (E1) transition from the bound state to the continuum, which
is characterized by the $dB(E1)/dE$ [2]. This is related to the pure Coulomb cross section for the dissociation of the projectile $d\sigma_C/dE$

$$\frac{d\sigma_C}{dE} \propto \frac{dB(E1)}{dE} \propto \sum_i |\langle \phi_i(E) | Z_{\text{eff}}^{(1)} r Y^{(1)} | \phi_0(E_0) \rangle|^2,$$

where $\phi_0(E_0)$ is the wavefunction of the initial state of the projectile and $\phi_i(E)$ represent the projectile continuum, $Z_{\text{eff}}^{(1)}$ is the effective charge and $Y^{(1)}$ is a spherical harmonics [3].

This strength has been inferred from two experiments, one performed at 520 A MeV at GSI [4] and the other at 69 A MeV at RIKEN [5], measuring the breakup of $^{11}$Be on carbon and lead targets. Strangely the analyses of both experiments provide different E1 strengths [2, 4, 5]. This constitutes an interesting issue currently studied by many groups. In particular an ab initio calculation by Calci et al., that has been recently published [6], shows an E1 strength in agreement with the RIKEN results (see Fig. 5 of Ref. [6]). In the present work, we reanalyze both experiments using only one description for the $^{11}$Be structure, to elucidate this discrepancy.

2. Structure and reaction models

As we have seen, $^{11}$Be can be described as a two-body system composed by a compact $^{10}$Be core and a weakly-bound neutron n forming the halo. This system is described by the internal Hamiltonian

$$H_0 = -\frac{\hbar^2}{2\mu} \Delta + V_{10\text{Be}+n}(r),$$

where $\mu$ is the $^{10}$Be+n reduced mass and $V_{10\text{Be}+n}$ an effective potential describing the interaction between the $^{11}$Be constituents. To obtain a set of wavefunctions $\phi_i(\vec{r},E)$ describing the $^{10}$Be+n system, we need to solve the eigenvalue problem

$$H_0 \phi_i(\vec{r},E) = E \phi_i(\vec{r},E).$$

For the present case, the $V_{10\text{Be}+n}$ potential is provided by halo effective field theory (Halo-EFT) at next to leading order (NLO) (see [7] for a recent review)

$$V_{10\text{Be}+n}(r) = V_0 e^{-\frac{r^2}{2r_0^2}} + V_2 r^2 e^{-\frac{r^2}{2r_0^2}}.$$

The $V_0$ and $V_2$ parameters are fitted in the $s_{1/2}$ and $p_{1/2}$ partial waves to reproduce structure observables, like binding energies and Asymptotic Normalization Constants (ANC) of the bound states or phaseshifts $\delta$ in the continuum; in this case we use the ANC and the phaseshifts predicted by the ab initio calculations [6]. Halo-EFT at NLO only includes $s_{1/2}$ and $p_{1/2}$ bound states, in the other partial waves, the continuum is described by plane waves. The $r_0$ parameter allows us to evaluate the sensitivity of our calculations to short-range physics. For the results presented here we use $r_0 = 1.5$ fm but similar results have been obtained with other values [8, 9], thus meaning that our calculation does not depend on short-range physics.

Using this structure description of $^{11}$Be, we can evaluate its $dB(E1)/dE$. In Fig. 1 we show the RIKEN (blue diamonds) and GSI (black dots) estimates of that E1 strength. The solid red and green dashed lines represent our result before and after the convolution with the GSI experimental resolution: the effect of the GSI experimental resolution is non-negligible and leads to an apparent reduction of the $dB(E1)/dE$. The folding of the solid red line with the RIKEN experimental resolution produces very little change. Thus we can conclude that our structure model presents an E1 strength in agreement with the RIKEN estimate, and that the actual discrepancy between the two experiments is of the order of the difference between the dashed
Figure 1. $dB(E1)/dE$ estimate for $^{11}$Be inferred by GSI [4] and RIKEN [5] experiments (black dots and blue diamonds, respectively), compared to our calculation within Halo-EFT (solid red line). We also provide our result folded with GSI experimental resolution (green dashed line).

green line and the black dots.

To describe the GSI and RIKEN breakup cross sections we employ eikonal-based models [10]. To analyze the RIKEN data we adopt the dynamical eikonal approximation (DEA) [11], which works well at intermediate energies; while at the GSI energy we have developed a model for the high-energy breakup taking into account proper relativistic effects, considering relativistic kinematics and dynamical aspects [9, 12, 14]. Our eikonal model also includes a consistent treatment of both nuclear and Coulomb interactions and their interferences at all orders. As input for $^{11}$Be structure we use Halo-EFT, without considering core excitation.

3. Results

Our cross sections for the $^{11}$Be breakup are in excellent agreement with both RIKEN and GSI data [8, 9]. In Fig. 2 we present the results for $^{11}$Be dissociation at 69A and 67A MeV on (a) lead and (b) carbon targets, respectively (green solid lines); we compare these results with RIKEN data (black diamonds). All the results have been convoluted with the experimental resolution. The result at RIKEN beam energy presents an excellent agreement with the data, especially at low dissociation energies on both targets. The peaks observed in the C data are due to the presence of $d$ resonances in the $^{10}$Be-n continuum spectrum. They are not seen in our calculations, because these states are not included in our Halo-EFT model of $^{11}$Be. On C the effect is more significant than on Pb, because these resonances are populated by quadrupolar transitions that are mostly given by nuclear interaction, dominant in the reaction on carbon target.

The dissociation of $^{11}$Be at 520A MeV is shown in Fig. 3 for (a) Pb and (b) C targets. Our full calculation (red solid line) is compared to the GSI data (black dots) after convolution with the experimental energy resolution. Our estimation of the nuclear component is shown by the magenta dot-dashed line and is lower than the experimental estimate (black stars). The result of our calculation without relativistic corrections is displayed as blue dashed line, and shows that the inclusion of relativistic corrections at these energies is mandatory for a proper analysis of the data. On the carbon target we obtain a good agreement with experiment but, as in the case of the calculation at RIKEN energy, we do not reproduce the peak of the experimental data.
Figure 2. Cross section as a function of the dissociation energy in the continuum for the $^{11}$Be breakup on (a) lead at 69A MeV and (b) carbon at 67A MeV [8], compared to RIKEN measurement [5].

because we do not include the $d$ resonances in our description of $^{11}$Be. On the lead target we find a fine agreement, except at very low dissociation energies where we overestimate the data. However, this is exactly where the experimental uncertainty is the highest.

At the high beam energy of 520A MeV interference and higher order effects are expected to

be smaller respect to what is found at lower beam energies [13]. We decided to make a more detailed analysis to investigate the magnitude of the interference between nuclear and Coulomb components in the process measured on the lead target. For this purpose we display in Fig. 4 the GSI result (black dots) and the GSI nuclear component (black stars), as well as our full

Figure 3. Cross section as a function of the dissociation energy in the continuum for the $^{11}$Be breakup at 520 MeV/nucleon on (a) lead and (b) carbon targets [9], compared to GSI measurement [4]. We also show a result without relativistic corrections.
calculation (solid red line) and our nuclear component (magenta dotted line); we also represent
our Coulomb component calculated at first order (E1) with a minimum impact parameters of
\(b_{\text{min}} = 10.4 \text{ fm}\) (blue dot-dashed line). The \(b_{\text{min}}\) corresponds to the inflection point of the S-
matrix of the pure nuclear elastic scattering process. By comparing the incoherent sum of the
nuclear and Coulomb components (green dashed line) to the full calculation, we find that the
latter is lower at the peak, and the two are in agreement on the tail. On the contrary, the full
calculation is of the same order as the Coulomb component at low excitation energies and it
becomes higher as the dissociation energy increases. So, we could conclude that at low excitation
energies the nuclear interaction brings mostly absorption and does not contribute much to the
breakup, while at higher energies both interactions act in constructive interference.

As we have seen, the \(dB(E1)/dE\) extracted from our \(^{11}\text{Be}\) Halo-EFT model is in agreement
with the RIKEN result, as well as the ab initio prediction [6]. We can conclude that the
discrepancy between GSI and RIKEN \(dB(E1)/dE\) arises from the method applied to extract
this quantity. Usually, to extract the E1 strength from data the relativistic Coulomb excitation
theory is applied [14, 15]. This makes the use of the number of equivalent photons \(N_{E1}(E,b_{\text{min}})\)
to connect the Coulomb breakup cross section to the \(dB(E1)/dE\)

\[
\frac{d\sigma_C}{dE} = \frac{16\pi^3}{9hc} N_{E1}(E,b_{\text{min}}) \frac{dB(E1)}{dE}.
\] (5)

To estimate the Coulomb breakup cross section in the GSI experiment, it was applied the method
for which the estimate of the nuclear component (usually taken as the breakup cross section on
carbon target) is subtracted to the breakup cross section on lead target

\[
\frac{d\sigma_C}{dE} = \frac{d\sigma_{\text{Pb}}}{dE} - \Gamma \frac{d\sigma_{\text{N}}}{dE},
\] (6)

where the \(\Gamma\) factor is determined following geometrical considerations [16, 4]. This subtraction
method could be useless if not dangerous, as already suggested by other groups, e.g. [17], if
interferences and high order effects are not properly taken into account. In this case we have
found that at low energies the nuclear component mostly contributes to absorption and also
that it has been overestimated in the data analysis, these two effects could affect the subtraction method. To avoid this issue, we suggest to follow what was done by RIKEN, i.e., to restrict the measurement at forward angles, where the nuclear component is negligible.

4. Summary and conclusions
Using only one Halo-EFT description of the $^{11}$Be projectile [7], related to one E1 strength in agreement with ab initio predictions, we obtain quite good an agreement with both RIKEN and GSI measurements of the $^{11}$Be breakup [4, 5, 9, 8]. The apparent discrepancy between RIKEN and GSI estimates of $dB(E1)/dE$ could be explained as a combination of different effects: the small destructive nuclear and Coulomb interference as well as the subtraction method to determine the Coulomb cross section, which together bring an underestimation of the final E1 strength, and the fact that the different experimental resolutions were not taken into account in comparing the two $dB(E1)/dE$ estimates. We believe that to measure at small angles is a safer choice to avoid this issue. However we should not neglect the disagreement at very small excitation energies between the reaction model and GSI data (where data uncertainty is higher). As support in favor of our result we would like to mention that we tested this reaction model also for the case of the breakup of $^{15}$C at 605A MeV measured by GSI [18], obtaining excellent results [19].

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