FIRST MEASUREMENT OF THE INTEFERENCE FRAGMENTATION FUNCTION IN $e^+e^-$ AT BELLE

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Abstract

A first measurement of the di-hadron interference fragmentation function of light quarks in pion pairs with the Belle detector is presented. The chiral odd nature of this fragmentation function allows the use as a quark polarimeter sensitive to the transverse polarization of the fragmenting quark. Therefore it can be used together with data taken at fixed target and collider experiments to extract the quark transversity distribution. A sample consisting of 711 $\times$ $10^6$ di-hadron pairs was extracted from 661 $fb^{-1}$ of data recorded near the $\Upsilon(4S)$ resonance delivered by the KEKB $e^+e^-$ collider.

1 Introduction

The di-hadron interference fragmentation function (IFF), suggested first by Collins, Heppelmann and Ladinsky [1] describes the production of unpolarized hadron pairs in a jet from a transversely polarized quark. The transverse polarization is translated into an azimuthal modulation of the yields of hadron pairs around the jet axis. In addition the IFF is chiral odd and can therefore act as a partner for the likewise chiral odd quark transversity function. The resulting amplitude is chiral even and therefore leads to observable effects in semi deep inclusive scattering off a transversely polarized target [2, 3] or in proton-proton collisions [4] in which one beam is transversely polarized. Besides the fragmentation into two hadrons, the fragmentation of a transversely polarized quark into one unpolarized hadron can be used to extract transversity via the Collins effect. Measurement of this effect at Belle [5] made the first extraction of transversity possible [6]. However as compared with the Collins fragmentation function, using the IFF to extract transversity exhibits a number of advantages. These are connected to the additional degree of freedom provided by the second hadron. It allows to define the azimuthal angle between the two hadrons as an observable in the transverse plane and at the same time integrate over transverse momenta of the quarks and hadrons involved. Because transverse momenta are integrated over collinear schemes in factorization and evolution can be used which are known and which do not need assumptions of the intrinsic transverse momenta [7]. Since the IFF is not a transverse momentum dependent function (TMD) it is universal and therefore directly applicable to SIDIS and proton proton data. From
both types of experiment results are available [2–4]. The results from SIDIS are indicating a non-zero IFF while the first analysis of the IFF effect at PHENIX [4] opens a way to disentangle transverse spin effects in proton-proton collisions. Because the IFF is not a TMD function, extraction of transversity times IFF is not dependent on a model of the transverse momentum dependence, which is the case in the measurement of the Collins effect [6]. Here transverse momentum in the final state originates from a convolution of quark distribution and fragmentation function. This leads also to the Sudakov suppression of the effect [8]. Technically, the extraction of the IFF from electron colliders is also easier, as the signal is not competing with asymmetries from QCD radiation. Also acceptance effects are smaller for the relative azimuthal angles of hadron pairs.

2 Observables in \(e^+e^-\) collisions

The transverse polarization of the fragmenting quark leads to a cosine modulation of the azimuthal angle of the plane spanned by the two hadrons \(h_1, h_2\) which is described by the vector \(\mathbf{R} = \mathbf{P}_{h1} - \mathbf{P}_{h2}\) lying in the plane. Since the electron beams are unpolarized any effect that one would measure in one hemisphere of an event would average out. Instead one can make use of the fact that the spins of quark and anti quark in electron-positron annihilation are 100% correlated. Thus the correlation of the azimuthal angles of the vectors \(\mathbf{R}^\alpha\) in the hemispheres \(\alpha \in \{1, 2\}\) around the thrust axis with regard to the event plane is sensitive to the IFF. The measurement uses the center of mass system and defines the event plane as the plane which contains the beam axis \(\hat{z}\) and thrust axis \(\hat{n}\). Figure 1 shows the coordinate system with the relevant quantities. With these the angles \(\phi_\alpha\) between \(\mathbf{R}_\alpha\) and the event plane can be expressed as

\[
\phi_{\{1,2\}} = \text{sgn} \left[ \hat{n} \cdot (\hat{z} \times \hat{n} \times (\hat{n} \times \mathbf{R}_{1,2})) \right] \\
\times \arccos \left( \frac{\hat{z} \times \hat{n} \times \mathbf{R}_{1,2}}{|\hat{z} \times \hat{n}| \cdot |\hat{n} \times \mathbf{R}_{1,2}|} \right). \tag{1}
\]

The product of the quark and anti quark interference fragmentation functions \(H_1^{\mathcal{Q}} \cdot \tilde{H}_1^{\mathcal{Q}}\) is then proportional to the amplitude \(a_{12}\) of the modulation \(\cos(\phi_1 + \phi_2)\) of the di-hadron
pair yields [9]. The di-hadron fragmentation functions are dependent on the kinematic variables \( m_{\text{inv}} \), the invariant mass of the hadron pair, and \( z = \frac{E_h}{Q} \) the normalized energy of the hadron pair. Here \( E_h \) is the energy of the hadron and \( Q \) the absolute energy transferred by the virtual photon. Therefore the normalized azimuthal yield of di-hadron pairs can be described as

\[
N(y, z_1, z_2, m_1, m_2, \phi_1 + \phi_2) \propto a_{12}(y, z_1, z_2, m_1, m_2) \cdot \cos(\phi_1 + \phi_2)
\]

with

\[
a_{12}(y, z_1, z_2, m_1, m_2) \sim B(y) \cdot \frac{\sum_q e_q^2 H_1^s(z_1, m_1) \bar{H}_1^s(z_2, m_2)}{\sum_q e_q^2 D_1(z_1, m_1) \bar{D}_1(z_2, m_2)}.
\]

Here the sum goes over all quark flavors \( q \) and \( B(y) = \frac{y(1-y)}{1-y^2} \) is the kinematic factor describing the transverse polarization of the quark-anti quark with regard to its momentum. The polar angle \( \theta \) is defined between the electron axis and the thrust axis as shown in fig. 1. The yields are dependent on \( z_1, z_2, m_1, m_2 \), the fractional energies and invariant masses of the hadron pairs in the first and second hemisphere, and \( y \). In the following the yields are integrated over \( y \) in the limits of the fiducial cuts on \( \theta \) introduced in section 2.3. The labeling of the hemisphere is at random, and it is experimentally not possible to distinguish between quark and anti quark fragmentation. Further information about the IFF can be learned from the dependence on the decay angle \( \theta_h \) in the CMS of the two hadrons produced. Using a partial wave decomposition to isolate components that contain the interference between waves with one unit difference in angular momentum, one expects a dependence on \( \sin \theta_h \) which gives the interference term between s and p waves. This term is expected to dominate at Belle kinematics and is favored by the acceptance. Since the acceptance is symmetric around \( \theta_h = \frac{\pi}{2} \) the p-p contribution proportional to \( \cos \theta_h \) should average out. Table 3 shows that this is approximately the case.

### 2.1 Models

As described in the previous section the IFF is an interference effect between hadrons, here pions, created in partial waves with a relative angular momentum difference of one. The dominant contribution being from the s-p interference term [10,11]. Therefore information about the IFF can be gained from a partial wave analysis for di-pion production [12]. Here the data from [13] suggest a phase shift around the \( \rho \) mass, leading to a sign change in the fragmentation function. In some models [14] the location of the phase shift around the mass of the \( \rho \) meson is caused by the interference of pion pairs produced in a p wave coming from the decay of the spin one \( \rho \) which interferes with the non-resonant background. Based on this model and estimation of particle yields from simulations, Bacchetta, Ceccopieri, Mukherjee and Radici [15] made model predictions for the magnitude of IFF Asymmetries at Belle. Again, a strong dependence on the invariant mass is predicted, with a maximum around the rho mass. The asymmetries are expected to rise with \( z \) due to the preservation of spin information early in the fragmentation.

### 2.2 The Belle experiment

The Belle detector, located at the KEKB asymmetric energy \( e^+e^- \) collider is described in detail in [16]. For the purpose of this study it is important that a high number of events
is recorded, a good particle identification allows to identify pions up to high values of $z$ and that the detector is hermetic to minimize acceptance effects. Both is well fulfilled by the Belle detector. It is located at the collision point of the 3.5 GeV $e^+$ and 8 GeV $e^-$ beam and is almost symmetric in the center of mass system of the beams. It is a large solid angle magnetic spectrometer consisting of barrel and end caps parts. For a more homogeneous acceptance function only the barrel part was used for this analysis. It comprises a silicon vertex detector, a 50-layer drift chamber and an electromagnetic calorimeter (CsTI) in a magnetic field of 1.5T. Particle identification over a wide range of momenta is done using an array of aerogel Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters and an instrumented iron flux return yoke outside of the coil to detect $K^0_L$ mesons and muons.

2.3 Data selection and Asymmetry extraction

From the 661 fb$^{-1}$ data sample roughly 589 fb$^{-1}$ were taken on the Υ(4S) resonance and 73 fb$^{-1}$ taken in the continuum 60 MeV below. Because the thrust cut used to select events with a two jet topology containing light and charm quarks also rejects events in which B mesons were produced, on-resonance and continuum data can be combined. The thrust is defined as $T = \frac{\sum_i |p_i \cdot \hat{n}|}{\sum_i |p_i|}$ and $\hat{n}$ is direction of the thrust chosen such that $T$ is maximal. B meson events produced on the Υ resonance have a spherical shape, since their high mass does not allow for large kinetic energy. On the other side, light quark anti quark pairs have a more two jet like topology.. An applied thrust cut of $T > 0.8$ reduces the contamination with B events to an order of 2% [5]. The inversion of the thrust cut selects events that don’t have a clear two jet topology and that are contaminated by Υ decays into B mesons. This leads to a decrease of the asymmetry.

As described earlier, only the barrel region of the detector is used in the analysis. Therefore the thrust axis is required to lie in a region well contained within it. This translates into a cut on the $z$ component of the thrust axis $|n_z| < 0.75$, which also allows for the particles of the jet around the axis to be reconstructed in the barrel. Further cuts on the event level are a reconstructed energy of at least 7 GeV to reject $e^+e^- \rightarrow \tau^+\tau^-$ and to reliably reconstruct the thrust axis. The later is computed using all charged tracks and photons passing some minimum energy cuts. A mean deviation of 135 mrad with a RMS of 90 mrad of the thrust axis from the real quark-anti quark axis is computed from simulations.

Only events were selected which satisfy a vertex cut of 2 cm in the radial and 4 cm in the beam direction. On the track level the fiducial cuts to reduce acceptance effects are a constraint to the barrel region of the detector using a cut on the polar angle $\theta$ in the laboratory system of $-0.6 < \cos(\theta) < 0.9$ which translates to an almost symmetric cut in the CMS. To make sure that the azimuthal range of tracks around the thrust axis is not biased, only tracks are chosen that have at least 80% of their energy along the thrust axis. This restricts tracks to be within a cone that is entirely contained in the acceptance and reduces false asymmetries considerably as shown later in sec. 2.4. No false asymmetries from this cut is expected and since most of the energy of the jet is contained within the jet no significant dilution of the asymmetries either. Only tracks above a minimal fractional momentum $z > 0.1$ are considered. They have to be positively identified as pions. These tracks are then sorted into two hemispheres according to the sign of their momentum
projection on the thrust axis. All possible pairs of $\pi^+\pi^-$ in the same hemisphere are selected and the angle $\phi_\alpha, \alpha \in \{1, 2\}$ computed. To this end the vector $\mathbf{R}_\alpha = \mathbf{P}_1 - \mathbf{P}_2$ for each pion pair in hemisphere $\alpha$ is formed and the azimuthal angle around the thrust axis with respect to the event plane computed according to eq. \ref{eq:1}. Charge ordering of $h_1, h_2$ in the computation of $\mathbf{R}$ is always the same so that the effect is not averaged out. A weighting of the hadron momentum vector with the inverse fractional energy $z$ as suggested by \cite{17} only led to differences in the thrust axis within numerical uncertainties. For the computation of the yields in a specific $\phi_1 + \phi_2$ bin all combinations of pairs in the two hemispheres are considered. Due to the cone cut described earlier, the number of hadrons with a false hemisphere assignment is negligible. From Monte Carlo studies a signal purity for di-pion pairs (4 particles) of better than 90\% over the whole kinematic range is obtained.

2.4 Systematics studies

In order to determine the systematic error on the measurement, simulation and real data was used to determine the contribution of detector effects and competing physical processes to false asymmetries or a dilution of the measurement. The studies that lead to the biggest contribution to the systematic error are the study of false asymmetries in simulation and real data in which no asymmetry is expected due to a wrong assignment of the thrust axis or the hemispheres of the particle pairs. Any false asymmetries, together with their statistical errors were added to the systematic error. Since the IFF effect is not included in the Pythia event generator used, checking for the asymmetries in fully simulated data allows to estimate effects of the detector acceptance and efficiencies on the asymmetry. Table \ref{table:1} shows the false asymmetries extracted from a full simulation of the detector using GEANT.

Another source for false asymmetries are events reconstructed from real data in which the asymmetries are averaged out. This is the case for mixed events in which the angles $\phi_1$ and $\phi_2$ are taken from different events or events in which the angels are computed for pion pairs in the same hemisphere. The later case leads to asymmetries due to phase space restrictions, which could be reproduced, the former to asymmetries in the order of one per-mille, shown in table \ref{table:2}, which has been added to the systematic error. Contributions from higher harmonics in the fits to the cosine modulation are under one per-mille.

Due to the smearing of the thrust axis reconstruction with respect to the true quark-anti quark axis, the extracted asymmetries are diluted. Since this dilution could be reproduced in weighted Monte Carlo using the observed smearing of the thrust axis, it was corrected for.

More physics motivated systematic checks concern the dependence on the various kinematic factors. The dependence on the kinematic factor $\frac{\sin^2 \theta}{1 + \cos^2 \theta}$ should be linear, as should be the dependence on $\sin \theta_{h}$, if the effect is dominated by the s-p interference term. Both could be validated.

Even though the Belle detector is very stable over time, checks have been done to determine the compatibility of data taking periods and data taken on and off the $\Upsilon$ resonance. To this end, the $\chi^2$ of each fit has been computed and all values have been fitted by an appropriate $\chi^2$ distribution. The result of these tests show very good compatibility.

We checked for correlations in the data that might lead to false error estimates by breaking up weighted Monte Carlo data in 500 small chunks and comparing the mean
error of the extracted asymmetries to their variation. The results are compatible, so no systematic error was assigned.

For the interpretation of the results the respective fraction of processes contributing to the asymmetries are very important. These have been estimated from Monte Carlo simulations and are shown in fig. 2. It is evident that the contribution from $\Upsilon$ decays has been almost eliminated by the thrust cut. There are also marginal contributions from $\tau$ pairs. For these the false asymmetries are compatible with zero and they have been added with their statistical error to the overall systematic error. Besides the contribution from light quark pairs there is a considerable contribution from charm quarks. This decay can also cause the invariant mass dependence namely a general decrease with higher masses. However, the highest bin shows a very high charm contribution, in some bins more than half of the events, which is not yet understood. The asymmetry results for these bins indicate that the IFF for charm quarks is non-vanishing and of similar magnitude as that of lighter quarks.

![Fig. 2](image)

**Figure 2.** Relative process contributions from light quark-anti quark events (red), charm events (green), charged B meson pairs (blue), neutral B meson pairs (yellow) and $\tau$ pairs (purple) as a function of $z_2$ for all $z_1$ bins (a) and as a function of $m_2$ for all $m_1$ bins (b)

### 2.5 Results

The results obtained for the $a_{12}$ asymmetry as defined in eq. 2 are shown in figs. 3 binned in the invariant masses $m_1, m_2$ of the hadrons pairs in the first and second hemisphere and their fractional energies $z_1, z_2$, respectively. Table 3 shows the integrated asymmetries and the averaged kinematic observables. The extracted asymmetries are large, especially when considering that a product of the IFF for quark and anti quarks is measured. As expected the magnitude of the effect rises with $z$. However, the invariant mass behavior does not match model predictions from [15]. But these model predictions are only available in leading order and heavily dependent on simulations which were tuned for the SIDIS experiment HERMES at a center of mass energy of roughly 7 GeV. The asymmetries rise up to around the mass of the $\rho$ but then plateau instead of decreasing again. A
sign change of the IFF can therefore not be confirmed. However bins with high invariant masses receive also considerable contributions from charm quarks as shown before.

Figure 3. Results for the $a_{12}$ modulations in a symmetric $8 \times 8$ binning in $m_1, m_2$ for $m_{1,2}$ between 0.25 GeV and 2 GeV (a) and for a symmetric $9 \times 9$ binning in $z_1, z_2$ for $z_{1,2}$ between 0.2 and 1 (b). The statistical error is shown in blue, the systematic error in green.
2.6 Summary and Outlook

The first direct measurement of the interference fragmentation function using 661 \( fb^{-1} \) of data recorded at the Belle experiment has been presented. The asymmetries are large, up to 10\%, which would correspond to an IFF contribution of over 30\%. Models predicting a sign change or a decrease for invariant masses higher than the \( \rho \) meson’s could not be confirmed. Charm quarks play a significant role at high invariant masses. They seem to introduce an IFF asymmetry of similar magnitude as light quarks and further studies are under way to determine the charm quark contribution to the asymmetries. The analysis presented here should enable a combined analysis to extract the transversity distribution of data taken in SIDIS, proton-proton and \( e^+e^- \). This is very desirable due to the various advantages as compared with the extraction via the Collins effect and its complementarity. In proton-proton collisions the use of the IFF effect to access transversity is especially helpful, since it can help disentangle different contributions to the measured transverse single spin asymmetries \( A_N \). Our plans for the future contain also an extraction of other particle combinations, namely those including neutral pions and charged kaons. Furthermore, an extraction of the unpolarized yields is planned to facilitate the extraction of the IFF.

| sample          | species | \( z_1, z_2 \)-Asymmetries | \( \langle a_{12} \rangle \) | \( \langle a_{12R} \rangle \) |
|------------------|---------|-----------------------------|-----------------------------|-----------------------------|
| No opening cut   |         |                             |                             |                             |
| uds 4\( \pi \)   | \( \pi \pi \) | -0.089 ± 0.008 | -0.108 ± 0.008 |
| uds acceptance   | \( \pi \pi \) | -0.488 ± 0.011 | -0.490 ± 0.011 |
| uds MC rec.      | \( \pi \pi \) | -0.394 ± 0.013 | -0.418 ± 0.013 |
| charm rec.       | \( \pi \pi \) | -0.446 ± 0.041 | -0.388 ± 0.044 |
| With opening cut of 0.8 |         |                             |                             |                             |
| uds 4\( \pi \)   | \( \pi \pi \) | -0.038 ± 0.013 | -0.035 ± 0.013 |
| uds acceptance   | \( \pi \pi \) | -0.112 ± 0.016 | -0.113 ± 0.016 |
| uds MC rec.      | \( \pi \pi \) | 0.012 ± 0.019 | 0.008 ± 0.019 |
| charm rec.       | \( \pi \pi \) | 0.006 ± 0.040 | 0.027 ± 0.040 |

Table 1. MC results averaged over all \( z \) bins in \%.

| sample          | \( z_1, z_2 \)-Asymmetries | \( \langle a_{12} \rangle \) | \( \langle a_{12R} \rangle \) |
|------------------|-----------------------------|-----------------------------|-----------------------------|
| uds 4\( \pi \)   | 0.070 ± 0.013 | 0.030 ± 0.013 |
| uds acceptance   | 0.020 ± 0.016 | -0.021 ± 0.016 |
| uds rec.         | 0.091 ± 0.019 | 0.087 ± 0.019 |
| charm rec.       | -0.024 ± 0.040 | -0.017 ± 0.040 |
| Data             | -0.019 ± 0.017 | -0.012 ± 0.017 |

Table 2. Mixing results averaged over the \( z \) binning in \%. The results integrated over other binnings are nearly identical.
Table 3. Integrated asymmetries and average kinematics.

|                          |       |
|--------------------------|-------|
| $\langle z_1 \rangle$, $\langle z_2 \rangle$ | 0.4313|
| $\langle m_1 \rangle$, $\langle m_2 \rangle$ | 0.6186|
| $\langle \sin^2 \theta/(1 + \cos^2 \theta) \rangle$ | 0.7636|
| $\langle \sin \theta_h \rangle$ | 0.9246|
| $\langle \cos \theta_h \rangle$ | 0.0013|
| $a_{12}$ | $-0.0199 \pm 0.0002 \pm 0.0009$ |

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