THE NATURE OF DARK MATTER

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Dark matter has been recognized as an essential part of matter for over 70 years now, and many suggestions have been made, what it could be. Most of these ideas have centered on Cold Dark Matter, particles that are expected in extensions of standard particle physics, such as supersymmetry. Here we explore the concept that dark matter is sterile neutrinos, a concept that is commonly referred to as Warm Dark Matter. Such particles have keV masses, and decay over a very long time, much longer than the Hubble time. In their decay they produce X-ray photons which modify the ionization balance in the early universe, increasing the fraction of molecular Hydrogen, and thus help early star formation. Sterile neutrinos may also help to understand the baryon-asymmetry, the pulsar kicks, the early growth of black holes, the minimum mass of dwarf spheroidal galaxies, as well as the shape of dark matter halos. As soon as all these tests have been quantitative in its various parameters, we may focus on the creation mechanism of these particles, and could predict the strength of the sharp X-ray emission line, expected from any large dark matter assembly. A measurement of this X-ray emission line would be definitive proof for the existence of may be called weakly interacting neutrinos, or WINs.

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1. Dark Matter: Introduction

Since the pioneering works of Oort\textsuperscript{1} and Zwicky,\textsuperscript{2,3} we know that there is dark matter in the universe, matter that interacts gravitationally, but not measureably in any other way. Oort argued about the motion and density of stars perpendicular to the Galactic plane, and in this case, Oort’s original hunch proved to be correct, the missing matter turned out to be low luminosity stars. Zwicky argued about the motions and densities of

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galaxies in clusters of galaxies, and to this day clusters of galaxies are prime arguments to determine dark matter, and its properties.

Based on the microwave background fluctuations today we know that the universe is flat geometrically, i.e. the sum of the angles in a cosmic triangle is always 180 degrees, provided we do not pass too close to a black hole. This finding can be translated into stating that the sum of the mass and energy contributions to the critical density of the universe add up to unity, with about 0.04 in baryonic matter, about 0.20 in dark matter, and the rest in dark energy; we note that there is no consensus even where to find all the baryonic matter, but a good guess is warm to hot gas, such as found in groups and clusters of galaxies, and around early Hubble type galaxies.

There are many speculations of what dark matter is; we have three constraints:

1) It interacts almost exclusively by gravitation, and not measurably in any other way; 2) It does not participate in the nuclear reactions in the early universe; 3) It must be able to clump, to help form galaxies, and later clusters of galaxies, and the large scale structure.

Obviously, various extensions in particle physics theory, such as supersymmetry, all provide candidates, like the lightest supersymmetric particle. Here we focus on the concept that it may be a “sterile neutrino”, a right-handed neutrino, that interacts only weakly with other neutrinos, and otherwise only gravitationally. Such particles were first proposed by Pontecorvo and later by Olive & Turner. Sterile neutrinos were further proposed as dark matter candidates. It was then shown how oscillations of normal neutrinos to sterile neutrinos could help explain the very large rectilinear velocities of some pulsars.

Observationally the evidence comes from a variety of arguments: i) Dark matter in a halo like distribution is required to explain the stability of spiral galaxy disks; ii) the flat rotation curves of galaxies; and iii) the containment of hot gas in early Hubble type galaxies. Dark matter is required to explain iv) the structure of clusters of galaxies; v) structure formation, and the flat geometry of the universe. We refer the reader to a recent review on dark matter.

Therefore after more than 70 years we still face the question: “What is dark matter?”
2. Proposal

The existing proposals to explain dark matter mostly focus on very massive particles, such as the lightest supersymmetric particle; all the experimental searches are sensitive for masses above GeV, usually far above such an energy. In the normal approach to structure formation, this implies a spectrum of dark matter clumps extending far down to globular cluster masses and below. It has been a difficulty for some time that there is no evidence for a large number of such entities near our Galaxy. The halo is clumpy in stars, but not so extremely clumpy. If, however, the mass of the dark matter particle were in the keV range, then the lowest mass clumps would be large enough to explain this lack. However, in this case the first star formation would be so extremely delayed that there would be no explanation of the early reionization of the universe, between redshifts 11 and 6, as we now know for sure. Therefore, the conundrum remained.

Here we explore the concept that the dark matter is indeed of a mass in the keV range, but can decay, and so produce in its decay a photon, which ionizes, so modifies the abundance of molecular Hydrogen, and so allows star formation to proceed early. The specific model we explore is of “sterile neutrinos”, right handed neutrinos, which interact only with normal, left-handed neutrinos, and with gravity. Such particles are commonly referred to as “Warm Dark Matter”, as opposed to “Cold Dark Matter”, those very massive particles. For most aspects of cosmology warm dark matter and cold dark matter predict the same; only at the small scales are they significantly different, and of course in their decay.

The mass range we explore is approximately 2 - 25 keV. These sterile neutrinos decay, with a very long lifetime, and in a first channel give three normal neutrinos, and in the second channel, a two-body decay, give a photon and a normal neutrino. The energy of this photon is almost exactly half the mass of the initial sterile neutrino.

What is important is to understand that such particles are not produced from any process in thermal equilibrium, and so their initial phase space distribution is far from thermal; all the current models for their distribution suggest that their momenta are sub-thermal. The measure of how much they are sub-thermal modifies the precise relationship between the dark matter particle mass and the minimum clump mass, which should be visible in the smallest pristine galaxies.

This also entails, that as Fermions they require a Fermi-Dirac distribution, as being far from equilibrium, this distribution implies a chemical potential.
Recent work by many others\textsuperscript{19–30} has shown that these sterile neutrinos can be produced in the right amount to explain dark matter, could explain the baryon asymmetry,\textsuperscript{31} explain the lack of power on small scales (as noted above), and could explain the dark matter distribution in galaxies.\textsuperscript{32–34}

2.1. Our recent work

Pulsars are observed to reach linear space velocities of up to over 1000 km/s, and there are not many options how to explain this; one possibility is to do this through magnetic fields which become important in the explosion.\textsuperscript{35–39} Another possibility is to do this through a conversion of active neutrinos which scatter with a mean free path of about ten cm, into sterile neutrinos, which no longer scatter. If this conversion produces a spatial and directional correlation between the sterile neutrinos and the structure of the highly magnetic and rotating core of the exploding star, then a small part of the momentum of the neutrinos can give an asymmetric momentum to the budding neutron star, ejecting it at a high velocity.\textsuperscript{40} This then could explain such features as the guitar nebula, the bow shock around a high velocity pulsar. This latter model in one approximation requires a sterile neutrino in the mass range 2 to 20 keV. It is remarkable that this neutrino model requires magnetic fields in the upper range of the strengths predicted by the magneto-rotational mechanism to explode massive stars as supernovae.

It was also shown from SDSS data, that some quasars have supermassive black holes already at redshift 6.41, so 800 million years after the big bang.\textsuperscript{41,42} We now know, that this is exactly when galaxies grow the fastest, from 500 to 900 million years after the big bang.\textsuperscript{43,44} Baryonic accretion has trouble feeding a normal black hole to this high mass, $3 \times 10^9$ solar masses so early after the big bang, if the growth were to start with stellar mass black holes.\textsuperscript{45} So either the first black holes are around $10^4$ to $10^6$ solar masses, and there is not much evidence for this, or the early black holes grow from dark matter,\textsuperscript{46–48} until they reach the critical minimum mass to be able to grow very fast and further from baryonic matter, which implies this mass range, $10^4$ to $10^6$ solar masses. This model in the isothermal approximation for galaxy structure implies a sterile neutrino in the mass range between 12 and 450 keV.

When Biermann and Kusenko met at Aspen meeting September 2005, it became apparent, that these two speculative approaches overlap, and so it seemed worth to pursue them further.

As noted above, structure formation arguments lead to an over-
prediction in power at small scales in the dark matter distribution in the case of cold dark matter, and any attempt to solve this with warm dark matter delayed star formation unacceptably. We convinced ourselves that this was the key problem in reconciling warm dark matter (keV particles) with the requirements of large scale structure and reionization. We then showed that the decay of the sterile neutrino could increase the ionization, sufficiently to enhance the formation of molecular hydrogen, which in turn can provide catastrophic cooling early enough to allow star formation as early as required.\textsuperscript{17,18} In our first simple calculation this happens at redshift 80. More refined calculations confirm, that the decay of sterile neutrinos helps increase the fraction of molecular Hydrogen, and so help star formation, as long as this is at redshifts larger than about 20.\textsuperscript{49–51}

3. The tests

3.1. Primordial magnetic fields

In the decay a photon is produced, and this photon ionizes Hydrogen: at the first ionization an energetic electron is produced, which then ionizes much further, enhancing the rate of ionization by a factor of about 100. In the case, however, that there are primordial magnetic fields, this energetic electron could be caught up in wave-particle interaction, and gain energy rather than lose energy. As the cross section for ionization decreases with energy, the entire additional ionization by a factor of order 100 would be lost in this case, and so there basically would be no measurable effect from the dark matter decay. This gives a limit for the strength of the primordial magnetic field, given various models for the irregularity spectrum of the field: In all reasonable models this limit is of order a few to a few tens of picoGauss, recalibrated to today. Recent simulations matched to the magnetic field data of clusters and superclusters, give even more stringent limits, of picoGauss or less.\textsuperscript{52}

It follows that primordial magnetic fields can not disturb the early ionization from the energetic photons, as a result of dark matter decay. It then also follows that the contribution of early magnetic fields from magnetic monopoles, or any other primordial mechanism, is correspondingly weak.\textsuperscript{53}

Stars at all masses are clearly able to produce magnetic fields,\textsuperscript{54–56} but the evolution and consequent dispersal are fastest for the massive stars, almost certainly the first stars. As the magnetic fields may help to drive the wind of these massive stars,\textsuperscript{57} then the wind is just weakly super-Alfvénic, with Alfvénic Machnumbers of order a few. This implies that the massive
stars and their winds already before the final supernova explosion may provide a magnetic field which is at order 10 percent equipartition of the environment; this magnetic field is highly structured. However, even these highly structured magnetic fields will also allow the first cosmic rays to be produced, and distributed, again with about 10 percent of equipartition of the environment.

However, the large scale structure and coherence of the cosmic magnetic fields clearly remain an unsolved problem.\textsuperscript{58–60}

Therefore the first massive stars are critical for the early evolution of the universe: In addition to reionization, magnetic fields and cosmic rays, they provide the first heavy elements. These heavy elements allow in turn dust formation, which can be quite rapid (as seen, e.g., in SN 1987A, already just years after the explosion.\textsuperscript{61} This then enhances the cooling in the dusty regions, allowing the next generation of stars to form much faster.

In combination everywhere one first massive star is formed, we can envisage a runaway in further star formation in its environment.

### 3.2. Galaxies

Galaxies merge, and simulations demonstrate that the inner dark matter distribution attains a power law in density, and a corresponding power law tail in the momentum distribution:\textsuperscript{62–64} Here the central density distribution as a result of the merger is a divergent power law, as a result of energy flowing outwards and mass flowing inwards, rather akin to accretion disks\textsuperscript{65,66} where angular momentum flows outwards and mass also flows inwards; in fact also in galaxy mergers angular momentum needs to be redistributed outwards as such mergers are almost never central.\textsuperscript{67} This then leads to a local escape velocity converging with $r$ to zero also towards zero, and so for fermions the Pauli limit is reached, giving rise to a cap in density, and so a dark matter star or a fermion ball is born;\textsuperscript{46,47} this dark matter star can grow further by dark matter accretion. The physics of fermion balls at galactic centers has been studied in a series of papers.\textsuperscript{68–74} For realistic models an integral over a temperature distribution is required, and a boundary condition has to be used to represent the surface of the dark matter star both in real space as in momentum space. This then allows the mass of this dark matter star to increase; such models resemble in their quantum statistics white dwarf stars or neutron stars; the Pauli pressure upholds the star. For fermions in the keV range the mass of the dark matter star has a mass range of a few thousand to a few million solar masses.

The first stellar black hole can then enter this configuration and eat
the dark matter star from inside, taking particles from the low angular momentum phase space. With phase space continuously refilled through the turmoil of the galaxy merger in its abating stages, or in the next merger, the eating of the dark matter star from inside ends only when all the dark matter star has been eaten up.

Given a good description of the dark matter star boundary conditions in real and in momentum phase space, and an observation of the stellar velocity dispersion close to the final black hole, but outside its immediate radial range of influence, we should be able to determine a limit to the dark matter particle mass. If the entire black hole in the Galactic Center has grown from dark matter alone, then we obtain a real number.

This concept suggests that it might be worthwhile to consider the smallest of all black holes in galactic centers. In a plot of black hole mass versus central stellar velocity dispersion $\sigma$ there is a clump above the relation $M_{BH} \sim \sigma^4$, at low black hole masses, suggesting that perhaps we reach a limiting relationship with a flatter slope for all those black holes which grow only from dark matter; for a simple isothermal approach this flatter slope is found to be $3/2$.

3.3. Dwarf spheroidal galaxies

All detected dwarf spheroidal galaxies fit a simple limiting relationship of a common dark matter mass of $5 \times 10^7$ solar masses, suggesting that this is perhaps the smallest dark matter clump mass in the initial cosmological dark matter clump spectrum. This clump mass is of course a lower limit to the true original mass of the pristine dwarf spheroidal galaxy. Given a physical concept for the production of the dark matter particles in the early universe, we would have their initial momentum, probably subthermal, and so the connection between the dark matter particle mass and minimum clump mass is modified. This is very strong support for the Warm Dark Matter concept.

One intriguing aspect of dwarf spheroidal galaxies is that almost all of them show the effect of tidal distortion in their outer regions, and at least one of them has been distended to two, perhaps even three circumferential rings around our Galaxy. To extend so far around our Galaxy must have taken many orbits, and so a some fraction of the age of our Galaxy. The simple observation that these streamers still exist separately, and can be distinguished in the sky, after many rotations around our Galaxy, implies that the dark matter halo is extremely smooth, and also nearly spherically symmetric. Given that the stellar halo is quite clumpy this implies once
more that the dark matter is much more massive than the baryonic matter in our halo.

3.4. **Lyman alpha forest**

In the early structure formation the large number of linear perturbations in density do not lead to galaxies, but just too small enhancements of Hydrogen density, visible in absorption against a background quasar. This so-called Lyman alpha forest tests the section of the perturbation scales which is linear and so much easier to understand, and it should in principle allow a test for the smallest clumps. Unfortunately, systematics make this test still difficult, and with the expected sub-thermal phase space distribution of the dark matter particles we may lack yet the sensitivity to determine the mass of the smallest clumps.

3.5. **The X-ray test**

When the sterile neutrinos decay, they give off a photon with almost exactly half their mass in energy. Our nearby dwarf spheroidal galaxies, our own inner Galaxy, nearby massive galaxies like M31, the next clusters of galaxies like the Virgo cluster, and other clusters further away, all should show a sharp X-ray emission line.\textsuperscript{19,76–80}

The universal X-ray background should show such a sharp emission line as a wedge, integrating to high redshift.

With major effort this line or wedge be detectable with the current Japanese, American or European X-ray satellites: Large field high spectral resolution spectroscopy is required.

4. **Outlook**

The potential of these right handed neutrinos is impressive, but in all cases we have argued, there is a way out, in each case there is an alternative way to interpret the data set. E.g., for the pulsar kick with the help of neutrinos strong magnetic fields are required, but the MHD simulations suggest that perhaps magnetic fields can do it by themselves, even without the weakly interacting neutrinos.\textsuperscript{35,38,39} The dwarf spheroidal galaxies can in some models be explained without any dark matter at all.\textsuperscript{81,82} The early growth of black holes can also be fueled by other black holes, as long as there are enough in number and their angular momentum can be removed. So many alternatives may replace the sterile neutrino concept.
However, the right handed, sterile neutrinos weakly interacting with the normal left handed neutrinos provide a unifying simple hypothesis, which offers a unique explanation of a large number of phenomena, so by Occam’s razor, it seems quite convincing at present.\cite{83} So, given what sterile neutrinos may effect, we may have to call them Weakly Interacting Neutrinos, or soon WINs.

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