Continuous-Variable Quantum Teleportation with a Conventional Laser

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We give a description of balanced homodyne detection (BHD) using a conventional laser as a local oscillator (LO), where the laser field outside the cavity is a mixed state whose phase is completely unknown. Our description is based on the standard interpretation of the quantum theory for measurement, and accords with the experimental result in the squeezed state generation scheme. We apply our description of BHD to continuous-variable quantum teleportation (CVQT) with a conventional laser to analyze the CVQT experiment [A. Furusawa et al., Science 282, 706 (1998)], whose validity has been questioned on the ground of intrinsic phase indeterminacy of the laser field [T. Rudolph and B.C. Sanders, Phys. Rev. Lett. 87, 077903 (2001)]. We show that CVQT with a laser is valid only if the unknown phase of the laser field is shared among sender’s LOs, the EPR state, and receiver’s LO. The CVQT experiment is considered valid with the aid of an optical path other than the EPR channel and a classical channel, directly linking between a sender and a receiver. We also propose a method to probabilistically generate a strongly phase-correlated quantum state via continuous measurement of independent lasers, which is applicable to realizing CVQT without the additional optical path.

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Quantum teleportation is a method to move quantum states from a sender “Alice” to a receiver “Bob” by the aid of entanglement. The original protocol [1] has later been extended to continuous-variable quantum teleportation (CVQT) using a two-mode squeezed state and balanced homodyne detection (BHD) [2]. While experimental demonstration of CVQT was reported as the first achievement of unconditional quantum teleportation [2], there has been controversy over its validity on the ground of intrinsic phase indeterminacy of the laser field [3, 4, 5]. The laser field is often assumed to be a coherent state having a fixed phase, but the steady-state solution of the master equation in the quantum theory of the laser shows that the phase of the laser field inside the cavity is completely unknown in operation well above threshold [4, 5, 6, 7].

In this Letter, we first discuss a description of the laser field outside the cavity in line with the previous discussions [4, 5, 6]. We then analyze BHD using a laser as a local oscillator (LO), and give a physically appropriate description of BHD based on the standard interpretation of the quantum theory for measurement. We apply our description of BHD to CVQT with a laser to analyze the CVQT experiment [2]. Finally, we propose a method to probabilistically generate a strongly phase-correlated quantum state via continuous measurement of independent lasers, which is applicable to realizing CVQT without an optical path between Alice and Bob for sharing the same laser field.

In Ref. [3], van Enk and Fuchs claim that the standard description of the laser field used in Ref. [5] is surprisingly insufficient to understand CVQT with a laser. By employing the input-output theory [6], they find that the laser field outside the cavity is a continuous-mode mixed state which is in form identical to the steady-state field inside the cavity. They then go on to express the field state in terms of noncontinuous operators given by them, and discuss its coherency.

The definition of noncontinuous operators is based on the narrow bandwidth approximation (NBA) $B \ll \omega_0$, where $\omega_0$ is the central frequency of the bandwidth $B$. Under NBA, the annihilation operator part of the continuous-mode electric field is approximated as $\hat{E}^+(z,t) \sim i\xi_0 \sum_n \phi_n(t-z/c) \hat{c}_n$, where $\xi_0 \equiv [\hbar \omega_0/(2\varepsilon_0\varepsilon A)]^{1/2}$, $\{\phi_n(t)\}$ are basis functions giving a profile of the fundamental modes, and $\hat{c}_n$ is a noncontinuous annihilation operator [7].

In Ref. [5], a subset of basis functions $\{\psi_n(t)\}$ is specifically chosen as $\psi_n(t) \equiv (T)^{-1/2} \exp(-i\omega_0t) \Pi(t/T-n)$ where $\Pi(t)$ is the rectangle function, and importance of the field expression in terms of noncontinuous operators defined by $\{\psi_n(t)\}$ is emphasized in view of the quantum de Finetti theorem. But this given expression is nothing more than one possible approximation of the field state outside the cavity by NBA, where the exact state obtained by the input-output theory is actually identical to the standard description of the laser field used in Ref. [5].

We will show that, contrary to the claims of Ref. [5], the standard description of the laser field used in Ref. [5] is sufficient to understand CVQT with a laser. Throughout this Letter, we use the standard description of the laser field in Ref. [5] for the continuous mode outside the cavity. We explain coherency of the laser field not by...
interpreting a new approximate expression of the laser field as Ref. [3], but by appropriately formulating measurement process for the laser field.

Now, we will give a description of BHD with a conventional laser. As long as the photon number operator well represents an observable for an efficient photodetector lacking single photon resolution [4, 14], we may regard $\hat{a}_s^\dagger \hat{a}_s + \hat{a}_s^\dagger \hat{a}_s$ as an observable for BHD, where $\hat{a}_s$ and $\hat{a}_s$ are annihilation operators for the LO field and the signal field, respectively [11]. If the signal field $|\psi_s\rangle$ satisfies

$$r \sqrt{\langle \psi | X_s(\theta) | \psi \rangle} \gg \sqrt{\langle \psi | \hat{a}_s^\dagger \hat{a}_s | \psi \rangle},$$

which holds when the intensity of the LO field is extremely larger than that of the signal field, this observable satisfies

$$\langle \hat{a}_s^\dagger \hat{a}_s + \hat{a}_s^\dagger \hat{a}_s \rangle |\psi\rangle = \rho(r) |\psi\rangle,$$

where $X_s(\theta) \equiv \delta e^{-i \theta} + \hat{a}_s^\dagger e^{i \theta}$ and $|\psi\rangle$ is the coherent state in polar coordinates. According to the standard interpretation of the quantum theory [12], Eq. (10) implies if we obtain the measurement outcome $r x$ in one trial of BHD with the prior knowledge of $r$, $|\psi_s\rangle$ instantaneously reduces to $|x, \theta\rangle$ satisfying $X_s(\theta)|x, \theta\rangle = |x, \theta\rangle$.

Since $r$ of the laser field is measurable beforehand, we may define the measurement operator $\hat{M}$ for BHD as [13]

$$\hat{M}(x, r, \theta) \equiv \pi^{-\frac{1}{2}} |x, \theta\rangle \langle x, \theta| \langle x, \theta| \rho |\psi\rangle,$$

where $|x, \theta\rangle$ is the quadrature eigenstate written as

$$|x, \theta\rangle = (2\pi)^{-\frac{1}{4}} e^{-\frac{x^2}{2}} \exp(x e^{i\theta} \hat{a}_s^\dagger - \frac{1}{2} x \hat{a}_s^\dagger \hat{a}_s^\dagger |0\rangle).$$

where $|x, \theta\rangle$ satisfies the orthonormalization condition $\langle x_1, \theta|x_2, \theta\rangle = \delta(x_1 - x_2)$ and the completeness relation $\int_{-\infty}^{+\infty} dx \ |x, \theta\rangle \langle x, \theta| = 1$ on $x$. Since the coherent state also satisfies the completeness relation [6, Eq. (2)], $\langle x_1, \theta_1|x_2, \theta_2\rangle = \delta(x_1 - x_2) (\theta_1 \neq \theta_2)$ by measurement results due to intrinsic phase indeterminacy of the laser field [4, 14], the probability of obtaining the measurement outcome $x = \bar{x}$ with the prior knowledge of $r$ is

$$P(\bar{x}) = \int_{-\infty}^{+\infty} dr \int_{0}^{2\pi} d\theta \ Tr \left\{ \hat{M}(\bar{x}, r, \theta) \hat{\rho}_n \hat{M}_1(\bar{x}, r, \theta) \right\},$$

(4) and the density operator after the measurement is

$$\hat{\rho} = P(\bar{x})^{-1} \int_{0}^{2\pi} d\theta \ \hat{M}(\bar{x}, r, \theta) \hat{\rho}_n \hat{M}_1(\bar{x}, r, \theta),$$

(5) where $\hat{\rho}_n$ is the density operator before the measurement.

We will denote the procedure described above the observable-based projection method (OBPM) in the rest of this Letter. Note that above discussion is not based on the assumption that the laser field is the coherent state ("partition ensemble fallacy" [4, 14]). It is the property of the observable for BHD that approximately projects the strong laser field of the LO mode onto the coherent state after the measurement. On the contrary, the number states in the LO mode cannot be eigenstates of the observable for BHD, because $|n\rangle \neq |n-1\rangle$ even in the limit $n \rightarrow +\infty$ due to their rigid orthogonality.

As an example of BHD, we will calculate $P(\bar{x})$ in the squeezed light generation scheme [15] by OBPM. In the scheme, the same laser source is used for supplying the LO field, and pumping the nonlinear medium to generate the squeezed state. The density operator of the system before the measurement is

$$\hat{\rho}_0 = \int_{0}^{2\pi} d\phi \left| r_o e^{i(\phi + \pi)} \right\rangle \langle x, \phi| s, o e^{i2\phi} \left| s, o e^{i2\phi} \right\rangle \langle x, \phi| s, o e^{i\phi + \pi} |l, 1\rangle,$$

(6)

where $|\phi\rangle$ is the unknown phase of the pump field, $\phi$ is the phase delay by a controllable phase shifter, and $|0, \varepsilon\rangle = \hat{S}(\varepsilon)|0\rangle$ is the squeezed vacuum state [6]. The unknown phase of the squeezed state is $2\phi$ instead of $\phi$, because frequency of the pump field is doubled by second harmonic generation before the field enters an optical parametric oscillator. By using Eqs. (10), (11), orthogonality approximation of the coherent state $(\langle \psi | \rho |\psi\rangle)^{2} \sim (\pi/\rho)^{2}\delta(r-r_o)\delta(\theta-\theta_o)$ in the limit $r_o \rightarrow +\infty$ derived from $\lim_{\varepsilon \rightarrow 0} \exp(-\varepsilon^{2}/4\varepsilon) = (2\sqrt{\pi}) = (2\varepsilon)^{-1}$, and the relation

$$\langle x, \theta|0, s e^{i(\phi + \phi')}\rangle = \sum_{n=0}^{\infty} (\pi/\sqrt{2})^{\frac{n}{2}} [2^n \cosh(s)]^{-\frac{n}{2}} \langle x, \theta| s, o e^{i(\phi + \phi')}\rangle \langle s, o e^{i2\phi} |l, 1\rangle,$$

where $H_n(x)$ are Hermite polynomials, we find $P(\bar{x}) = |\langle \bar{x}, \phi|0, s\rangle|^{2}$, which agrees with the experimental result of Ref. [13].

Next, we will apply OBPM to CVQT with a laser [4, 14]. In the measurement step by Alice, the probability of obtaining $\bar{x}_1$ in BHD1 and $\bar{x}_2$ in BHD2 is $P(\bar{x}_1, \bar{x}_2) \equiv \int_{0}^{\infty} r_o dr_1 \int_{0}^{2\pi} d\theta_1 \int_{0}^{\infty} r_o dr_2 \int_{0}^{2\pi} d\theta_2 \ Tr \{ \hat{M}_2\hat{M}_1\hat{M}_1 \hat{M}_1 \}$ and the density operator after the measurement is $\hat{\rho}_1 = P^{-1}(\bar{x}_1, \bar{x}_2) \int_{0}^{\infty} r_o dr_1 \int_{0}^{2\pi} d\theta_1 \int_{0}^{\infty} r_o dr_2 \int_{0}^{2\pi} d\theta_2 \ Tr \{ \hat{M}_2\hat{M}_1\hat{M}_1 \hat{M}_1 \}$, where $\hat{M}_j \equiv \pi^{-1/2} |r_j e^{i\phi_j}|_{l_1} |\bar{x}_j, \theta_j\rangle \langle \bar{x}_j, \theta_j|_{l} |r_j e^{i\phi_j}|_{l_1} |l, j = 1, 2\rangle$. $\hat{\rho}_1$ is the density operator of the total system before the measurement written as

$$\hat{\rho}_1 = \int_{0}^{2\pi} d\phi \ 1/2 \left| r_o e^{i\phi} \right\rangle \langle x, \phi|_{l_1} \left| r_o e^{i\phi} \right\rangle \langle x, \phi|_{l_2} \left| \eta e^{i2\phi} \right\rangle_{1, 2} \left| \eta e^{i2\phi} \right\rangle_{1, 2}$$

(7)\(\otimes\) |r_o e^{i\phi}\rangle_{l_3} \hat{\rho}_{in} \left| r_o e^{i\phi}\right\rangle_{l_3} \left| \eta e^{i2\phi} \right\rangle_{1, 2} \left| \eta e^{i2\phi} \right\rangle_{1, 2}$
\[ \sqrt{1-\eta^2} \exp(\eta e^{i2\phi} a_1^\dagger a_2^\dagger)|0\rangle_1|0\rangle_2 \] is a two-mode squeezed state \[ \mathcal{F} \] as the EPR state. Again, the unknown phase in the modes \( 1,2 \) is \( 2\phi \) instead of \( \phi \). (See Fig. 1 in Ref. [1].)

By using Eq. (3) and \( \hat{a}_s = (\hat{a}_s - \hat{a}_s^\dagger)/\sqrt{2} \), \( \hat{a}_s^2 = (\hat{a}_s + \hat{a}_s^\dagger)/\sqrt{2} \) where the modes \( s, l \) are for the signal field of BHD1,2, the quadrature eigenstates of the modes \( s, l \) are written in the modes \( 1,2 \) as

\[
|\bar{x}_1, \phi\rangle_s (|\bar{x}_2, \phi + i\bar{y}_2\rangle) = |\exp(\frac{-\bar{x}_1}{2\sqrt{2}})\exp(\frac{\gamma \bar{y}_1}{\sqrt{2}})\exp(\frac{\gamma \bar{y}_2}{\sqrt{2}})|0\rangle_1|0\rangle_2.
\]

With orthogonality approximation of the coherent state, we find \( \rho_\text{in} \) includes Eq. (3). Ideal quantum teleportation is possible only when \( \eta = 1 \), where a two-mode squeezed state is maximally entangled. Eq. (2). Ideal quantum teleportation is possible only when \( \rho_\text{in} \) includes Eq. (3). Ideal quantum teleportation is possible only when \( \eta = 1 \), where a two-mode squeezed state is maximally entangled. Eq. (2). Ideal quantum teleportation is possible only when \( \eta = 1 \), where a two-mode squeezed state is maximally entangled.

Eqs. (9) and (10) clearly show that in the special case \( \eta = 1 \), \( \hat{T}_{2,in} \) is independent of the unknown phase \( \phi \) where ideal quantum teleportation is realized, while in the usual case \( 0 \leq \eta < 1 \), \( \hat{T}_{2,in} \) is dependent on the unknown phase \( \phi \) where the reconstructed density operator in the mode 2 is distorted from \( \rho_\text{in} \).

We will subsequently discuss generation of a strongly phase-correlated quantum state necessary in CVQT by measuring two independent laser fields

\[
\hat{\rho}_0 = \int \frac{d\theta}{2\pi} \int \frac{d\phi}{2\pi} \frac{\rho_{a,e^{i\phi}\phi_0} \rho_{b,e^{i\phi}\phi_0} \rho_{r,e^{i\phi}\phi_0} \rho_{b,e^{i\phi}\phi_0} \rho_{a,e^{i\phi}\phi_0} \rho_{b,e^{i\phi}\phi_0} \rho_{a,e^{i\phi}\phi_0}}.
\]
where $\rho(0)$ is Eq. (11) with $r_a = r_b (\equiv r_0)$, $e^{-R\hat{d}_+\hat{d}^\dagger(\tau T)} \sim e^{-R\hat{d}_+\hat{d}(\tau T)} \sim 1$, $B(x, y)$ is the beta function, $r_t \equiv r_0e^{-Rt}$, $R \equiv g^2\tau/2$, and $g$ is the atom-field coupling constant $|\alpha\rangle$.

The proposed continuous measurement is valid when $\tau \ll (\sqrt{2}r_0g)^{-1}$. In Eq. (13), we find that atoms simultaneously intersecting the output modes with no absorption (null measurement) damp both laser fields, leaving phase correlation between the fields unchanged. The absorption rate is assumed to be quite high, where $t$ is much smaller than the dynamical time scale of an individual laser.

For $s \gg 1$, the distribution of the phase difference of states in the integrand of Eq. (13) has a peak at $|\phi_a - \phi_b| = \pi$ when $p = s$, or at $|\phi_a - \phi_b| = 0$ when $p = 0$. Since Eq. (12) has peaks at $p = 0, s$, the probability of obtaining Eq. (13) with $p = 0, s$ is not negligible.

The photon number distribution of the mode $c$, $P_c(m) \equiv \langle m|\text{Tr}_{d}\{\hat{\rho}(t; p, q)\}|m\rangle_c$, is found to be

$$P_c(m) = e^{-2r_t^2}\frac{(2r_t^2)^m}{m!}B(m + p + \frac{1}{2}, q + \frac{1}{2})\times_1F_1(q + \frac{1}{2}; m + p + q + 1; 2r_t^2), \quad (14)$$

where $_1F_1(\alpha; \beta; z)$ is the confluent hypergeometric function of the first kind. $P_c(n)$ is easily obtained by replacing $m$ with $n$ and interchanging $p \leftrightarrow q$ in Eq. (14). Fig. 2 is for $P_c(m), P_d(n)$.

When $p = 0, s$ with $s \gg 1$, the generated quantum state is applicable to CVQT as a means to share the unknown phase of the laser field between Alice and Bob, though the phase correlation formed after the continuous measurement will slowly be broken by the phase diffusion effect of lasers.

The famous experiment for interference of two independent lasers by Pfleegor and Mandel \cite{4}, where weak

![FIG. 1: Experimental setup for continuous measurement of two independent laser fields. $\hat{a}, \hat{b}$ and $\hat{c}, \hat{d}$ are annihilation operators for the input and output modes of a 50/50 beamsplitter, respectively, satisfying $\hat{c} = (\hat{a} - \hat{b})/\sqrt{2}$, $\hat{d} = (\hat{a} + \hat{b})/\sqrt{2}$. Two-level atoms resonant with the laser fields are all prepared in the ground state beforehand, and go across the output fields one by one at regular intervals.](image)

![FIG. 2: Photon number distributions from Eq. (14) for $p = s$ with $r_t^2 = 10^4$. Given that $s = 100$, the probability Eq. (14) for $p = 0$ or $p = 100$ is about 11.3%. If the Monte Carlo wave-function procedure \cite{3} is performed, gradual decay of $r_t$ due to null measurement shall be seen besides the above distribution change. $P_c(m)$ approaches a Poisson distribution as $s$ becomes large.](image)

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