Shapiro delay of asteroids on LISA

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Abstract

In this paper, we examine the Shapiro delay caused by the close approach of an asteroid to the LISA constellation. We find that the probability that such an event occurs at a detectable level during the time interval of the mission is smaller than 1%.

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1. Introduction

Laser Interferometer Space Antenna (LISA) [1], a space experiment devoted to the detection of gravitational waves, is a nearly equilateral triangular constellation of three spacecraft, the centre of mass of which follows an Earth-like orbit. This constellation is located about 20° behind the Earth, and the distances between spacecraft are planned to be of the order of 5 million kilometres. The spacecraft exchange optical laser beams, and the oscillations of the distances between spacecraft are interferometrically monitored. Thanks to this time delay interferometry (TDI) method, gravitational wave signatures are tracked in the $\sim 10^{-4}$, $\sim 10^{-1}$ Hz frequency domain. To use this method, spacecraft inter-distances (and their fluctuations) must be known with a precision such that the gravitational field of the Sun has to be modelled in a relativistic framework [2]. The analysis of gravitational wave signal provides another useful way to explore the universe, giving astrophysical and cosmological information, inaccessible from the electromagnetic window and complementary to it.

The gravitational field of the solar system is generally modelled including the Sun and planets only. However, it is known that the terrestrial orbit is frequently crossed by asteroids [3, 4], referred to as geo-cruisers (GC) in the following. Since LISA is on an Earth-like orbit, close encounters with GCs are expected to occur, and the gravitational field of a GC passing close to the LISA constellation can generate a signal in the data.

In a recent paper, Vinet [5] examined the direct action of a GC on the LISA constellation. The author addressed the shift in position of a station due to the direct action of the asteroid’s
gravitational field. He finds that this effect leads to a measurable signal if the involved GC passes sufficiently close to the spacecraft concerned.

In this paper, we are interested in another aspect of the interaction between asteroids and LISA. If a GC passes close to the segment joining two spacecraft, its gravitational field affects the light distance between them by Shapiro delay on the laser beam used for that very measurement. According to the close encounter parameters, this signal can have a duration such that it falls in the LISA frequency domain. Hence, this signal has to be distinguished from the expected extra-solar system gravitational wave signal to be tracked in the data. The aim of the present study is to examine if such an (impulsive) event is likely to occur. We find that, while the effect is effectively measurable for sufficiently massive GCs passing close to the light beam, the probability of the occurrence of such an event at a detectable level, during the time interval of the mission, is quite negligible.

2. Conditions for an asteroid encounter to cause a relevant Shapiro delay

Let us consider a GC passing close to the light beam linking two LISA spacecraft A and B. The spacetime geometry in which the beam propagates can be formally written as

\[ g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}^{(SS)} + h_{\alpha\beta}^{(ast)} + h_{\alpha\beta}^{(GW)}, \]

where \( \eta_{\alpha\beta} = \text{diag}(-1, +1, +1, +1) \) is the Minkowski metric, \( h_{\alpha\beta}^{(SS)} \) is the part of the gravitational field due to the Sun and planets, \( h_{\alpha\beta}^{(ast)} \) is the part due to the close asteroid and \( h_{\alpha\beta}^{(GW)} \) is the part of the gravitational wave. The gravitational wave term induces a change in the distance between the two stations of the order of

\[ \delta L^{(GW)} \sim hL, \]

where \( h \) is the characteristic amplitude of \( h_{\alpha\beta}^{(GW)} \), and \( L \) is the distance between A and B. On the other hand, \( h_{\alpha\beta}^{(ast)} \) is of the order of \( 2Gm/(rc^2) \), where \( m \) is the asteroid mass. The close approach induces a Shapiro time delay \( \delta t \) in the flight time of the photon, hence a change in the light distance given in [6]:

\[ \delta l \approx c \delta t \approx \frac{4Gm}{c^2} \ln \left( \frac{4r_A r_B}{\Delta^2} \right), \]

where \( r_A \) (resp. \( r_B \)) is the distance between the GC and spacecraft A (resp. B) and \( \Delta \) the distance between the GC and the segment joining the spacecraft A and B. Let \( b \) be the impact parameter of the encounter (minimum value of \( \Delta \) during the approach). In the case where \( b \ll L \) (we will find that it is a necessary condition for the signal to be observable), it is easy to see that \( \delta L^{(ast)} \), the maximum possible value for \( \delta l \), satisfies

\[ \delta L^{(ast)} \leq \frac{8Gm}{c^2} \ln \frac{L}{b}. \]

2.1. Conditions on the amplitude

Let \( H_{\text{min}} \) be the smallest value of \( h \) accessible to the experiment. The necessary condition for the GC to generate a gravitational signal with sufficient amplitude to be detectable (see the next sub-section for the necessary condition related to the frequency domain) writes

\[ \frac{8Gm}{c^2} \ln \frac{L}{b} \geq H_{\text{min}} L. \]
Let $\rho$ and $D$ be the density and the (mean) diameter of the considered asteroid, respectively. The above inequality leads to the following condition on the impact parameter:

$$b \lesssim L \exp \left( -\frac{3}{4\pi} \frac{c^2}{G\rho D^2} H_{\text{min}} L \right)$$  \hspace{1cm} (1)

for the asteroid signal to be observable. This gives, numerically,

$$b \lesssim (5 \times 10^6 \text{ km}) \exp \left\{ -8000 \frac{H_{\text{min}}}{10^{-20}} \left( \frac{\rho}{2 \text{ g cm}^{-3}} \right)^{-1} \left( \frac{D}{1 \text{ km}} \right)^{-3} \right\}.$$  \hspace{1cm} (2)

where we have taken $L = 5 \times 10^6$ km, the average inter-distance between spacecraft. Let us consider a GC of 10 km in diameter (resp. 15 km). One finds (with $\rho = 2 \text{ g cm}^{-3}$ and $H_{\text{min}} = 10^{-20}$) $b \lesssim 1700$ km (resp. 470 000 km). We note that for an 8 km diameter asteroid, the impact parameter should be smaller than 1 km, that is smaller than the asteroid radius, which means that the beam would be occulted.

2.2. Conditions for the signal to fall in LISA’s frequency interval

The characteristic time of the encounter is given by $\tau \sim b/V$, where $V$ is the relative velocity of the GC with respect to LISA’s centre of mass. This means that the fundamental frequency, in the Fourier representation of the signal, is of the order of $V/b$. Then, a second necessary condition is that $V/b$ should be inside the frequency interval of LISA for the signal to be detectable. Simulations with fictitious impulsive signals (i.e. signals with limited duration), the duration of which range from 1 to $10^5$ s, confirm that $H_{\text{min}}$ is always $\geq 10^{-20}$ and the detection is not efficient outside the interval $[10 \text{ s}, 10^4 \text{ s}]$ (see appendix A).

Since one should have $\tau$ in the time interval $[10 \text{ s}, 10^4 \text{ s}]$, $b$ has to satisfy the additional condition

$$(10 \text{ s}) \cdot V \lesssim b \lesssim (10^4 \text{ s}) \cdot V$$

besides condition (2). Since $V \sim 15 \text{ km s}^{-1}$, (3) leads to

$$150 \text{ km} \lesssim b \lesssim 150 000 \text{ km},$$

one sees that only a small number of GCs will effectively be relevant to LISA at $H_{\text{min}} = 10^{-20}$, verifying both conditions (2) and (3). Indeed, only asteroids larger than 9 km in diameter can generate a signal at a detectable level with characteristic encounter times in this interval. From astronomical observations, only about 15 GCs are larger than 9 km in diameter [4]. If one takes $\rho = 2.7 \text{ g cm}^{-3}$, this limit in diameter becomes 8 km, and about 20 GCs are larger than 8 km in diameter.

3. Probability of a relevant encounter

Let $n(\geq D_0)$ be the mean number density of GCs with a diameter $D \geq D_0$ in the neighbourhood of the Earth orbit. Let $V$ be the mean relative velocity of GCs and the Earth. The number of GCs, of diameter larger than $D_0$, passing at a distance between $b$ and $b + db$ from the segment $[A, B]$ (with $b \ll L$) during a time interval $dr$, is of the order of $2n(\geq D_0) L \, db \, V \, dr$. Let $T_{\text{LISA}}$ be the duration of the LISA mission. From equation (1), the condition of detectability by LISA is $D \geq D_0$, with

$$D_0^3 = \frac{3}{4\pi} \frac{c^2}{G\rho} \frac{HL}{\ln(L/b)}.$$
The number $E$ of events observed during the duration of the mission is then of the order of

$$E \sim 6LVT_{\text{LISA}} \int_{b_{\text{min}}}^{b_{\text{max}}} n(D \geq D_0) \, db,$$

since there are three arms in the LISA configuration. The lower and upper bounds, $b_{\text{min}}$ and $b_{\text{max}}$, are the minimal and maximal values of $b$, related to the LISA frequency sensitivity curve for impulsive events (4). The total number of GCs of diameter $\geq D_0$ is estimated to be [4]

$$N(\geq D_0) \sim 1090 \left( \frac{D_0}{1 \text{ km}} \right)^{-1.95}.$$

To evaluate $E$, only the density number in the vicinity of the Earth is needed, not $N$ the total number of GCs. From (5) and the estimate obtained in the appendix B, the mean number of GCs per unit volume (per (AU)$^3$) in the vicinity of the Earth orbit is

$$n(\geq D_0) \sim 94 \left( \frac{D_0}{1 \text{ km}} \right)^{-1.95}.$$

Then

$$E \sim 0.51 \left( \frac{1 \text{ AU}}{1 \text{ AU}} \right) \int_{b_{\text{min}}}^{b_{\text{max}}} \left[ \frac{\rho}{2 \text{ g cm}^{-3}} \left( \frac{H_{\text{min}}}{10^{-20}} \right)^{-1} \ln \left( \frac{L}{b} \right) \right]^{0.65} \, db.$$

We have taken $V = 15 \text{ km s}^{-1}$ and $T_{\text{LISA}} = 3 \text{ yr}$. The values of $b_{\text{min}}$ and $b_{\text{max}}$ were given in (4) and $\ln(L/b)$ in the integral varies in the interval $[\sim 3.5; \sim 10.4]$. The minimal amplitude $H_{\text{min}}$ depends on the characteristic time of the encounter, hence on $b$, but, as stated before, it can be bounded by $10^{-20}$. Since $\rho$ is always of the order of $2 \text{ g cm}^{-3}$ (for asteroids, it belongs to the interval $[1.3 \text{ g cm}^{-3}; 2.7 \text{ g cm}^{-3}]$), the number of relevant events during the LISA mission is bounded by

$$E \lesssim 1.65 \times 10^{-3},$$

which means that the probability of observing one event caused by a Shapiro delay related to a close GC approach is quite negligible. This number (probability) becomes $2 \times 10^{-3}$ if one takes $\rho = 2.7 \text{ g cm}^{-3}$.

4. Discussion

The present study leads to the conclusion that GCs will not significantly perturb the LISA mission through the related Shapiro effect. While it could appear that this (not very exciting) result is not a surprise, a dedicated study was required. Indeed, we have shown that the close approach of one of the twenty largest geo-cruisers is likely to result in a detectable signal. The low number of relevant encounters comes from the statistical aspect of the problem, but not directly from the physical properties (masses) of asteroids, neither from the geometry of the possible encounters.

It is worth pointing out that, since only objects with a diameter larger than 8 km are relevant, all the corresponding geo-cruisers are known. Hence, if a relevant close encounter with LISA occurs, the corresponding geo-cruiser motion will have been accurately monitored, in such a way that it should be easy to veto the resulting signal. Consequently, it would be useless to make extensive templates of asteroid’s signals, in order to track such events during the whole LISA mission.

In [5], the direct effect of an asteroid on the motion of a LISA spacecraft has been addressed. A statistical analysis of this direct effect would be of interest and remains to be
made. To achieve this study, an analytical expression relating the impact parameter of the encounter and the minimal value of the asteroid’s diameter, for which the direct effect is detectable, is required. Such an expression is not explicitly provided in [5], but it appears from the curves in figure 3 of [5] that asteroids with a size of (say) 100 m lead to a detectable effect for impact parameters in the interval (4). This shows that the direct effect examined in [5] is considerably more important compared to the Shapiro effect examined in the current paper, as the latter requires larger GCs (at least 8 km in diameter).

5. Conclusion

The present study shows that the possibility of detecting an asteroid through Shapiro delay by the LISA mission

• concerns only a small number of geo-cruisers (about 20 at best). An occultation of the laser beam occurs before the detection condition is satisfied for geo-cruisers with a diameter less than ∼8 km;
• has a very low probability to occur during the time interval of the mission, at best of the order of some \(10^{-3}\).

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Appendix A. Minimal detectable amplitude

The signal-to-noise ratio averaged over all sky directions and polarizations can be expressed as [7]

\[
\left( \frac{S}{N} \right)^2 = 2 \int_0^\infty \frac{S_h(v)}{S_{\text{eff}}(v)} \, dv,
\]

with \(S_{\text{eff}}(v)\) is the effective sensitivity of LISA. In our calculations, we adopt the position noise budget for a standard Michelson configuration, including the contribution of the galactic binary WD–WD confusion noise [8].

Let \(h(t) = H f(t)\) be a gravitational signal of duration \(T\) and amplitude \(H\) (the function \(f(t)\) being of amplitude unity). The corresponding spectral density \(S_h(v)\) can be expressed as

\[
S_h(v) = H^2 \left| \tilde{f}(v, T) \right|^2,
\]

where \(\tilde{f}\) is the Fourier transform of \(f\).

Combining the above equations, one obtains for the minimal detectable amplitude

\[
H_{\text{min}} = \frac{(S/N)_{\text{min}}}{2\sqrt{T}},
\]

where

\[
I = \int_0^\infty \frac{dv}{S_{\text{eff}}(v)} \left| \tilde{f}(v, T) \right|^2.
\]

Following the convention adopted in the LISA community, we assumed a detectability threshold of \((S/N)_{\text{min}} = 5\).
For instance, let us consider a (fictitious) signal such that \( f(t) \) is zero outside the interval \([0, T]\), which is unity inside the interval \([t_1, t_2]\), and which is linear in the intervals \([0, t_1]\) and \([t_2, T]\), in such a way that the whole signal is continuous. For this signal, figure A1 exhibits the minimum detectable value \( H_{\text{min}} \) as a function of the global duration \( T \), for \( t_2 = T/2 \) and \( t_1 = T/4, T/8 \), respectively. It confirms that \( H_{\text{min}} \) is always \( \geq 10^{-20} \) and that this conclusion does not depend drastically on the precise signal’s profile. It also shows that the detection is not efficient for a duration outside the interval \([10 \text{ s}, 10^4 \text{ s}]\), i.e. for durations outside the LISA frequency interval. More precisely, the detection is not efficient for a duration \(< 10 \text{ s}\). It can be efficient for a duration \( > 10^4 \text{ s}\), but only for a highly non-symmetric signal, for which the time derivative takes values significantly larger than \( H/T \). However, when a GC passes close to LISA, the relative velocity of the encounter is nearly constant, so that the GC gravitational field varies in a very regular and quasi-symmetric way. The corresponding signal is such that its time derivative is never significantly larger than with \( h_{\text{max}}/\tau \) (\( h_{\text{max}} \) being the maximum value of the signal and \( \tau \) its characteristic duration). Hence the reasonable assumption is that the detection is not efficient when \( \tau \) is outside the interval \([10 \text{ s}, 10^4 \text{ s}]\).

Appendix B. From total asteroid distribution to volumic distribution near the Earth orbit

The distribution of GCs with respect to orbital elements \((a, e, i)\) is given in [3]. Since one is interested in an order of magnitude estimate rather than in precise results, let us make the following assumptions and simplifications:

- the diameter distribution of GCs is independent of the orbital-element distributions;
- the distribution in inclination is limited to the interval \([0, i_{\text{max}}]\), in which it is uniform.

Let us consider an asteroid with orbital elements \(a\) and \(e\). The probability that this asteroid is at a distance from the Sun in the interval \([r, r + dr]\) at an arbitrary time, is given by \( dP(r, r + dr) = 2 \pi r \frac{dr}{(\theta/\tau)} \), where \(\theta\) is the period. Then, \(r\) is given by the energy integral, and one finds

\[
dP(r, r + dr) = \frac{dr}{\pi a \sqrt{(e^2 \frac{r}{a})^2 - (1 - \frac{e^2}{2})^2}}.
\]
Let $p(a, e)$ be the density distribution in $a$ and $e$, so that $p(a, e) da de$ is the probability that a GC, arbitrarily chosen in the population, has its semi-major axis and eccentricity in the intervals $[a, a + da]$ and $[e, e + de]$, respectively. Using the assumption on inclination distribution, one finds that the number density of GCs on an Earth orbit is related to the total population $N$ by

$$n = \frac{N}{4\pi^2 \sin i_{\text{max}}} \int_{[a,e]} p(a, e) da de \quad \frac{a}{\sqrt{e^2 a^2 - (1 - a)^2}}$$

where one has replaced $r$ by unity ($1 \text{ AU}$). In this expression, $a$ is expressed in AU, $n$ in $(\text{AU})^{-3}$ and $[a, e]$ is the integration domain in the $a$–$e$ plane. From [3], let us consider that the integration domain is bounded by $0.5 < a < 3$ and $0.2 < e < 1$. Besides, for an asteroid to be a GC, one necessarily has $a(1 - e) < 1 < a(1 + e)$. Since one is only interested in an order of magnitude, let us replace $p(a, e)$ by its mean value $\langle p(a, e) \rangle = (1 + \ln 1.44)^{-1} \sim 0.7328$ in the integral. One finds

$$n = \frac{N}{4\pi^2 \sin i_{\text{max}}} \langle p \rangle K$$

where $K = 3\pi/10 + \arcsin(2/3) + 2/3 \ln[(3 + \sqrt{5})/2] \sim 2.3138$. From [3], a reasonable value for $i_{\text{max}}$ is $30^\circ$. This leads to

$$n(D_0) \sim 0.086 N(D_0).$$

This is the mean density (per $(\text{AU})^3$) of GCs with diameter $D \geq D_0$, at one astronomical unit from the Sun.

References

[1] Bender P et al 2000 LISA: a cornerstone mission for the observation of gravitational waves ESA-SCI(2000)11 System and Technology Study Report
[2] Chauvineau B, Regimbau T, Vinet J-Y and Pireaux S 2005 Phys. Rev. D 72 122003
[3] Raymond S N et al 2004 Astron. J. 127 2978
[4] Stuart J S and Binzel R P 2004 Icarus 170 295
[5] Vinet J-Y 2006 Class. Quantum Gravity 23 4939
[6] Weinberg S 1972 Gravitation and Cosmology (New York: Wiley)
[7] de Freitas Pacheco J A, Filloux C and Regimbau T 2006 Phys. Rev. D 74 023001
[8] Larson S L Online sensitivity curve generator available at http://www.srl.caltech.edu/shane/sensitivity

Larson S L, Hiscock W A and Hellings R W 2000 Phys. Rev. D 62 062001