Condensate Structure of D-particle Induced Flavour Vacuum

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It is argued that four Fermi interactions induced by non-perturbative effects due to scattering of stringy matter from D-particles, D-instantons and more generally bulk gauge fields in models with large extra dimensions have in specific situations condensate structure described by flavour vacua.

I. INTRODUCTION

Recently [1], within the framework of string-inspired quantum space-time foam (D-foam) [2], the existence of flavour mixing and induced oscillations was deduced based on kinematics of scattering and potential energy between D-branes in relative motion. This mixing is over and above the mixing responsible for conventional oscillation phenomena. It was conjectured in [1], and also in [3,4], that the resulting state could be described by a state known as a flavour vacuum introduced initially in [2].

We would like to show here that such a flavour vacuum is a condensate and can arise explicitly through certain four-fermion interactions that can be induced dynamically by the scattering of strings from brane defects in the structure of space-time. Depending on the type of string theory considered, these defects may be either point-like (D0 branes) [2] or compactified Dp-branes, wrapped around p-cycles [6]. Furthermore, rather generically, other non-perturbative interactions induced by D-instantons or Kaluza-Klein modes in brane world scenarios can give rise to similar four fermion interactions. Hence it is possible that flavour vacua may well arise from non-perturbative effects which arise in particle physics beyond the standard model (SM).

II. D-FOAM MODELS AND INDUCED FOUR-FERMION VECTOR INTERACTIONS

In the D-foam models our Universe is represented as a Dirichlet three-brane (D3-brane), perhaps after appropriate compactification of higher-dimensional domain-wall structures of space-time, e.g. as eight-branes in the model of [2]. In such D3 brane Universe conventional particles propagate as open strings. This 3-brane propagates in a 10-dimensional bulk space-time containing orientifold planes, that is punctured by D-particle defects. D-particles cross the D3-brane world as it moves through the bulk. To an observer on the D3-brane, these crossings constitute a realization of ‘space-time foam’ with defects at space-time events due to the D-particles traversing the D3-brane: we term this structure ‘D-foam’. When the open strings encounter D-particles in the foam, their interactions involve energy-momentum exchange that cause the D-particles to recoil due to scattering.

Typical of possibilities beyond SM is the existence of four fermion interactions that arise in string theory. Tree level four-point scattering with matter fields living on D-brane interactions lead to four Fermi interactions [7]. Such a situation arises in models where D3-branes are inside D7−branes [6]. The D3-branes wrap a 3-cycle thus becoming effectively a D-particle. The D7-branes wrap a 4-cycle and form the three-space dimensional world on which the SM particles live. The 3-cycle is taken to be $S^1 \times S^1 \times S^1$. A generic SM particle is represented by an open string with both ends on the D7 brane and is labelled by Chan-Paton indices $a \mathbf{b}$. An open string with one end on the D7−brane and another on the D3−brane will be denoted by ND (denoting Neumann and Dirichlet respectively) [2]. In the D-foam model D-particles are assumed to be uniformly distributed in the Universe. A D-particle in this foam can and another on the D3-brane will be denoted by ND [7]. In the D-foam model D-particles are assumed to be uniformly distributed in the Universe. A D-particle in this foam can capture an $a \mathbf{b}$ string which then splits into two ND strings $a \mathbf{c}$ and $b \mathbf{c}$. These ND strings scatter off the D-particle [6] and leave as $\bar{c}a$ and $\bar{c}b$ states. The calculation of these amplitudes was done for massless fermions [6] (relevant for massless neutrinos) following [2] using string perturbation theory. Such considerations typically lead to four Fermi interactions of the form

$$L_{\text{4-eff}} = \frac{\eta}{M_s^2} V^c \sum_{i,j} \sum_{a,b=L,R} G_{ab} \overline{\psi_i} \gamma^\mu \psi_j a \overline{\psi_j} b \gamma^\mu \psi_i b$$

where $V^c$ denotes the compactification volume element to four space-time dimensions, in units of the string length $\sqrt{a} = 1/M_s$; the indices $i,j$ refer to fermion species, including flavour, $G_{ab}$ are numerical coefficients depending on the couplings of the particular interactions, and are of order $O(1)$ as far as the string scale $M_s$ is concerned, and $L, R$ denote the appropriate chirality of the spinors $\psi_i, \psi_j$. The constant $\eta$ depends on the details of the model considered. (Although the D3/D7 model allows for four Fermi interactions among charged particles, these are suppressed [8] in comparison to neutral fermions.)
The structure of $G_{ab}$ can be quite involved in more realistic models since it may depend on $i, j$. Serious attempts have been made using complex D-brane configurations and orbifold structures to create low energy effective theory. One such attempt is known as the intersecting brane models [8]; in this context and in relation to flavour changing neutral currents four Fermi interactions have been derived where $G_{ab}$ have additional dependence on $i, j$. The effect is based on a non-perturbative mechanism involving D-instantons. This dependence, at its root, arises because matter fields can be localised at different points in the extra dimensions. The calculations are more technically involved for the case when $a \neq b$ (of special interest to us). However it was noted that the results can be obtained more easily within the brane world field theory where there is typically one brane and a fifth dimension which is the bulk direction [10] associated with a mass scale much smaller than the Planck scale. The explicit dependence in such a model is summarised in the appendix in (A3). $G$ can be both positive and negative depending on the details. We will later investigate a simple model with negative $G$ for detailed consideration.

We wish to know whether any condensates implied by (1) and its variants are compatible with the flavour vacuum. In order to answer this we will first investigate the possible condensates that can arise from Fierz factorisations of the four Fermi interaction. We will then evaluate such condensate order parameters in the flavour vacuum. We will then show that a simple four Fermi interaction in a brane world scenario can be described by such a condensate in the case of neutrinos this would imply the necessity of right-handed sterile neutrinos.

### III. Condensates from Four-Fermi Interactions

Upon a Fierz rearrangement, the four-fermion interactions (1), involving both left- and right-handed flavoured fermions, includes terms:

$$L_{4\text{-eff}} \equiv \frac{\eta G V c}{M_{s}^{2}} \sum_{i,j} \left[ -\langle \bar{\psi}_{i}\psi_{j} \rangle \langle \bar{\psi}_{j}\psi_{i} \rangle + \langle \bar{\psi}_{i}\gamma_{5}\psi_{j} \rangle \langle \bar{\psi}_{j}\gamma_{5}\psi_{i} \rangle + \frac{1}{2} \langle \bar{\psi}_{i}\gamma^\mu\psi_{j} \rangle \langle \bar{\psi}_{j}\gamma^\mu\psi_{i} \rangle + \frac{1}{2} \langle \bar{\psi}_{i}\gamma_{5}\gamma^\mu\psi_{j} \rangle \langle \bar{\psi}_{j}\gamma_{5}\gamma^\mu\psi_{i} \rangle \right],$$

(2)

where for brevity and concreteness we have assumed one type of Non-Standard-Model interaction among neutrinos due to the intermediate string states, and that the spinors have both left and right-handed components, i.e. $\psi_i = \begin{pmatrix} \psi_{L(i)} \\ \psi_{R(i)} \end{pmatrix}, \quad i = 1, \ldots, N$ where $N$ is the number of flavours. The reader should notice that in the Fierz transformation (2) the scalar and pseudoscalar terms have the opposite sign (but the same sign for the vector and axial vector terms). This implies that for fields of the same chirality the scalar and pseudo scalar contributions would cancel. Hence, if this induced four-Fermi interaction was to contribute, it is necessary to have both right and left-handed fermions. In the case of neutrinos this would imply the necessity of right-handed sterile neutrinos.

This has important consequences for the possibility of formation of fermion scalar condensates $\langle \bar{\psi}_{i}\psi_{j} \rangle$. Provided $G$ is negative, from (2), we would obtain attractive interactions in the scalar channel of the form

$$L_{4\text{-eff}} \equiv -\frac{\eta G V c}{M_{s}^{2}} \sum_{i,j} \langle \bar{\psi}_{i}\psi_{j} \rangle \langle \bar{\psi}_{j}\psi_{i} \rangle \psi_{i} \psi_{j}.$$

(3)

The condensates would lead to dynamical mass matrix contributions:

$$m_{ij} = \frac{\eta G V c}{M_{s}^{2}} \langle \bar{\psi}_{i}\psi_{j} \rangle, \quad i, j = 1, \ldots, N.$$

(4)

For models with extra dimensions and flavours, located at different positions in the bulk, it is easy to calculate $G$ [10] and find that it can be negative. This is given in an appendix (cf. (A3)). In principle the other terms in (2) can contribute to a condensate. The nature of the condensates supported by the flavour vacuum will be investigated next.

### IV. Condensates for the Flavour Vacuum

The well-known Pontecorvo mixing transformation [11] at the level of fields (rather than states) has the form

$$\psi_{\alpha}(x) = T(\theta)_{\alpha j} \psi_{j}$$

(5)

where $T(\theta)_{\alpha j}$ is a c-number (Pontecorvo flavour rotation matrix). For two flavour mixing $T$ is a $2 \times 2$ rotation through an angle $\theta$ (i.e. $SO(2)$) matrix. This is the customary way of dealing with flavour in the SM. The work of Blasone...
and Vitiello (BV) [5] made the ingenious suggestion that (5) is replaced by implementation of mixing via a quantum operator $G_\theta$ rather than a c-number matrix, i.e.

$$\psi_{a,b}(x) = G_\theta^{-1}\psi_{1,2}(x) G_\theta. \quad (6)$$

The operator $G_\theta$ is however formal and takes states from the massive Fock space to another Fock space which in the thermodynamic limit is orthogonal [5] to the massive Fock space. For massive Majorana (Dirac) fields $\psi$, with masses $m_i (i = 1, 2)$, $G_\theta$, for example, is given by

$$G_\theta(t) = \exp\left(\theta \int d^3x \left[ \psi_1^\dagger(\vec{x},t) \psi_2(\vec{x},t) - \psi_2^\dagger(\vec{x},t) \psi_1(\vec{x},t) \right] \right). \quad (7)$$

We will show that it is dynamics that chooses between the inequivalent representations in (5) and (7). As we will show the latter implies a condensate vacuum state whereas the former does not. In terms of the massive Fock vacuum $|0\rangle_{1,2}$, the flavour vacuum $|0\rangle_f$ is given by

$$|0\rangle_f \equiv G_\theta^{-1}|0\rangle_{1,2} \quad (8)$$

The condensate nature of the flavour vacuum can be probed by calculating the expectation values of the various operators that appear in the Fierz decomposition [2]. They can be calculated straightforwardly by writing the usual plane-wave expansion of the massive Majorana fermionic fields [12]

$$\psi_i(x) = \sum_{r=1,2} \int \frac{d^3k}{(2\pi)^{3/2}} \left( a_i^r(\vec{k}) u_i^r(\vec{k}) e^{-i\omega_i(k)t} + v_i^r(\vec{k}) a_i^r(\vec{k}) e^{i\omega_i(k)t} \right) e^{i\vec{k}.\vec{x}} \quad (9)$$

with $\omega_i(k) = \sqrt{k^2 + m_i^2}$ and $k^2 = \vec{k}^2$. The spinors $u_i^r(\vec{k})$ and $v_i^r(\vec{k})$ are determined by

$$\begin{cases} 
\left( \gamma^0 \omega_i(k) + \vec{\gamma}.\vec{k} - m_i \right) u_i^r(\vec{k}) = 0 \\
\gamma^1 u_i^r(\vec{k}) u_i^s(\vec{k}) = \delta^{rs} \\
v_i^r(\vec{k}) = \gamma^0 C u_i^s(\vec{k})^* 
\end{cases}$$

and the charge conjugation operator $C = -i\sigma^1 \otimes \sigma^2$ (where $\sigma^i$ are the Pauli spin matrices). The flavour fields in (6) can be expanded as in (9) but with $a_i^r$ replaced by $a_i^\alpha$ with $\alpha = a, b$. On using (6) and (7) it is then readily shown that we have the Bogolubov relations

$$a^a_i(\vec{k},t) = \cos \theta a_i^r(\vec{k}) - \sin \theta \sum_s \left( W^{rs}(\vec{k},t) a^s_2(\vec{k}) + Y^{rs}(\vec{k},t) a^{s\dagger}_2(\vec{k}) \right) \quad (10)$$

$$a^b_i(\vec{k},t) = \cos \theta a^r_2(\vec{k}) + \sin \theta \sum_s \left( W^{sr}(\vec{k},t) a^s_1(\vec{k}) + Y^{sr}(\vec{k},t) a^{s\dagger}_1(\vec{k}) \right) \quad (11)$$

where

$$W^{rs}(\vec{k},t) = \frac{1}{2} \left( u_{2r}^s(\vec{k}) u_{1}^s(\vec{k}) + v_{2}^s(\vec{k}) v_{1r}^s(\vec{k}) \right) e^{i(\omega_1 - \omega_2)t}$$

and

$$Y^{rs}(\vec{k},t) = \frac{1}{2} \left( u_{2r}^s(\vec{k}) v_{1r}^s(\vec{k}) + u_{1r}^s(\vec{k}) v_{2r}^s(\vec{k}) \right) e^{i(\omega_1 + \omega_2)t}.$$

These relations allow the computation of condensates. For operators normal ordered with respect to the massive vacuum we find:

$$\langle 0 | \bar{\psi}_i \psi_j | 0 \rangle_f = \frac{1}{2} \sin 2\theta \int \frac{dk}{\pi^2} \frac{k^2}{m_i^2 - m_j^2} \quad \text{for } i \neq j. \quad (12)$$
As in treatments of condensates there needs to be input from a more fundamental theory to determine the cut-off \( \Lambda \), necessary to make the integral finite. When \( i = j \) the condensate expectation vanishes. Some arguments in favour of a cut-off

\[
\Lambda = \overline{m} + \mathcal{O}((\delta m)^2/\overline{m}), \quad \overline{m} \equiv \frac{m_1 + m_2}{2}, \quad \delta m \equiv m_1 - m_2,
\]

for two-flavoured systems, that we consider here for simplicity, have been given in \cite{3,4,13} based on particle production characteristics of the flavour vacuum in the context of space-time foam expanding Universes. From \eqref{12}, \eqref{13}, and small \( \delta m \ll \overline{m} \), we then obtain:

\[
f \langle 0 | \overline{\psi}_i \gamma_j | 0 \rangle_f \simeq 2.5 \times 10^{-3} \sin(2\theta) \overline{m} \left( m_1^2 - m_2^2 \right) + \ldots,
\]

where the \ldots indicate higher orders in \( \delta m \).

The other possible condensates implied by \eqref{2} vanish (e.g. \( f \langle 0 | \overline{\psi}_i \gamma^\mu \gamma_j | 0 \rangle_f = 0 \)). Hence the flavour vacuum is a pure mixing condensate. We shall show that in the next section that a simple brane world field theory reproduces the same structure. It should be stressed here that a D-particle foam may have other effects \cite{3,4} beyond those that it shares with other non-perturbative effects such as D-instantons and brane world field theories. This can be seen from the string amplitude considered at the beginning in our D3/D7 discussion. The capture process of the string with both ends on the D7 brane by the D-particle was excluded. This exclusion also ignores interesting effects of particle production and locally non-flat backgrounds. We will come back to this issue in a future publication.

V. MIXING CONDENSATE IN BRANE FIELD THEORY MODEL

We shall consider a simple (toy) field theoretic four dimensional brane model with a large extra dimension denoted by \( z \) where the generation of a mixing condensate due to four fermion interaction can be demonstrated. The brane will be taken to be at \( z = 0 \). The model has two species of fermions located differently with respect to the extra dimension of size \( R \). One species, \( \psi \) is on the brane while the other \( \Psi \) is off the brane. There is no preferred place for us to locate this second fermion and so it will be allowed to explore the bulk dimension \( \overline{m} \). The bulk fermions interact through the exchange of gravitons and gauge bosons (as well as associated Kaluza-Klein modes) and so there are induced four fermion interactions. This model has the virtue of displaying a mixing condensate dynamically and, perhaps even more interestingly, the possibility that the mass scale of the condensate order parameter is much smaller than the inherent mass scale \( R \) (the order of a typical Kaluza-Klein mass). The non-zero condensate starts to appear at a second order phase transition at \( R = R_c \). For \( R < R_c \) a non-zero condensate is not generated dynamically while for \( R \) just above \( R_c \) the condensate mass scale is very small. In order to have a controlled method of calculating the fermion condensate, the fields \( \Psi \) and \( \psi \) come in \( N_f \) different fermion species allowing a calculation to leading order in \( \frac{1}{R} \). It is assumed that the fields \( \Psi \) and \( \psi \) have the same charge \( g \) with respect to a \( U(1) \) gauge field. Just as discussed in the Appendix, on integrating over the heavy Kaluza-Klein modes, the following simplified lagrangian \eqref{14} can be obtained:

\[
L = \overline{\Psi} i \gamma^\mu \partial_\mu \Psi + \overline{\overline{\Psi}} i \gamma^\mu \partial_\mu \overline{\psi} - \overline{\overline{\Psi}} \gamma_\mu \Psi \left( \overline{\psi} \gamma^\mu \psi \right) \delta(z)
\]

where \( \overline{\overline{\Psi}} \propto R g^2 \) and is \( O \left( \frac{1}{N_f} \right) \). We will use our earlier Fierz transformation and make the chiral rotation \( \Psi \rightarrow \exp(i \overline{\Psi} \gamma^5) \Psi \) and \( \psi \rightarrow \exp(i \overline{\psi} \gamma^5) \psi \). Scalar and pseudoscalar composites \( \sigma = \overline{\Psi} \psi \) and \( \varphi = \overline{\overline{\Psi}} \gamma_\mu \partial^\mu \varphi \) are introduced. The usual Kaluza-Klein expansion of the bulk fermion can be made, i.e.

\[
\Psi(x,z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \Psi_n(x) \chi_n(z)
\]

where

\[
\chi_n(z) = \begin{cases} 
1, & \text{when } n = 0 \\
\sqrt{2} \cos \left( \frac{n \pi}{2} \right), & \text{when } n = 1, 2, \ldots \\
\sqrt{2} \sin \left( \frac{n \pi}{2} \right), & \text{when } n = -1, -2, \ldots
\end{cases}
\]

The Lagrangian then reduces to

\[
L = \overline{\varphi} (M + i \partial^\mu \gamma_\mu) \varphi
\]
where
\[ \Xi_t = (\psi, \Psi_0, \Psi_1, \Psi_{-1}, \Psi_2, \Psi_{-2}, \ldots) \]
and
\[ M = \begin{pmatrix}
0 & m^* & m^* & m^* & m^* & \cdots \\
m & 0 & 0 & 0 & 0 & \cdots \\
0 & \frac{1}{R} & 0 & 0 & 0 & \cdots \\
m & 0 & \frac{2}{R} & 0 & 0 & \cdots \\
m & 0 & 0 & \frac{2}{R} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} \]

with \( m = \sqrt{G/2\pi R\sigma} \). In a \( 1/N_f \) expansion the effective potential for \( \sigma \) (with the customary cut-off \( \Lambda \)) is
\[
V(\sigma) = |\sigma|^2 - \frac{1}{2\pi^2} \int_0^\Lambda dx x^3 \ln \left[ x^2 + |m|^2 (\pi xR) \coth (\pi xR) \right] \]
\[
- \frac{1}{2\pi^2} \sum_{j=1}^{\infty} \int_0^\Lambda dx x^3 \left[ x^2 + \left( \frac{j}{R} \right)^2 \right].
\]
The minimum of \( V(\sigma) \) is given by the solution of
\[
1 - \frac{g^2}{2\pi^2} \int_0^\Lambda dx \frac{x^3}{2x \tanh (\pi xR) + g^2 |\sigma|^2} = 0.
\]
A non-zero solution for \( \sigma \) has been found, numerically, for suitable \( g \) and \( R \) [15]. Furthermore, \( \sigma \) is an order parameter for a second order phase transition (approximate analytic solutions for small \( \sigma \) can be found by deriving a first order differential equation for \( \sigma (\Lambda) \).) Hence, in this case we have explicitly demonstrated a non-zero off-diagonal element of the two-fermion mass matrix. The analysis can be repeated for gravitational Kaluza-Klein modes [16] with an equivalent form for the effective Lagrangian and similar conclusion.

VI. CONCLUSIONS

We have shown that from interactions beyond the standard model fermionic condensates can be formed which may be described by flavour vacua. The existence of the condensates is very much dependent on the details of the model. Hence the flavour vacuum is not a procedure that is a universal panacea when mixing phenomena are involved. The nature of the dynamical interactions responsible for the mixing will be important for describing whether such a discussion is possible. The discussion that we have given in this paper indicates that it may be a way to describe mixing when we have sufficiently large extra dimensions with long range fields in the bulk.

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Appendix A: Brane World Calculation of \( \tilde{G} \)

The generic four-Fermi interactions obtained from a string instanton analysis is readily obtained in a field theoretic brane world setting [10]. We will consider for simplicity a five-dimensional model with an Abelian gauge field \( A_M (M = 0, \ldots, 4) \) interacting with four dimensional fields \( \psi_j \) located at \( z = z_j \) (\( z \) being the co-ordinate in the bulk) and \( j \) is a
flavour index (with now a geometrical interpretation). The dimension represented by $z$ is compact and of length $L$. This situation can arise from a D4–brane which wraps around a circle of diameter $\frac{2\pi}{n}$. The lagrangian $\mathcal{L}$ is given as

$$\mathcal{L} = -\frac{1}{4} F_{MN}^{\mu} F_{MN}^{\mu} + i \bar{\psi} j^{\mu} D_{\mu} \psi \delta (z - z)$$

where $F_{MN} = \partial_{M} A_{N} - \partial_{M} A_{N} \cdot D_{M} = \partial_{M} + i g \sqrt{L} A_{M}$ and $\mu, \nu = 0, \ldots , 3$. (The abelian nature of the field is not essential and has been chosen for simplicity.) Now considering a Fourier series in $z$ we have

$$A_{\mu} (x, z) = \frac{1}{\sqrt{L}} A_{\mu}^{(0)} (x) + \sum_{n=1}^{\infty} \left( \cos \frac{2\pi n z}{L} A_{\mu}^{(n)} (x) + \sin \frac{2\pi n z}{L} \tilde{A}_{\mu}^{(n)} (x) \right).$$

(A1)

We have ignored $A_{4}$ since it does not couple to the fermions which we have confined to the brane. The fields $A_{\mu}^{(n)}$ and $\tilde{A}_{\mu}^{(n)}$, $n \geq 1$, are the $n$-th level massive KK fields with mass $M_{n} (\equiv \frac{2\pi n}{L})$. On integrating over $z$ in the action we have an effective Lagrangian $\mathcal{L}$ given by

$$\mathcal{L} = -\frac{1}{4} \left( F_{\mu \nu}^{(0)2} + \sum_{n=1}^{\infty} \left( F_{\mu \nu}^{(n)2} + F_{\mu \nu}^{(n)2} \right) \right) + \frac{1}{2} \sum_{n=1}^{\infty} M_{n}^{2} \left( A_{\mu}^{(n)2} + \tilde{A}_{\mu}^{(n)2} \right)$$

$$+ i \bar{\psi} j^{\mu} \left[ \partial_{\mu} + i g A_{\mu}^{(0)} + i g \sqrt{2} \sum_{n=1}^{\infty} \left( A_{\mu}^{(n)} \cos \frac{2\pi n z}{L} + \tilde{A}_{\mu}^{(n)} \sin \frac{2\pi n z}{L} \right) \right] \psi$$

(A2)

where the argument $x$ has been suppressed and the effective 4-dimensional coupling $g$ is given in terms of the five dimensional coupling $g_{5}$ by $g = g_{5} / \sqrt{L}$. There is a flavour dependence in the interactions through $z_{j}$ (i.e. the couplings with $A_{\mu}^{(n)}$ and $\tilde{A}_{\mu}^{(n)}$ are $\sqrt{2} g \cos (M_{n} z_{j})$ and $\sqrt{2} g \sin (M_{n} z_{j})$ respectively); the KK contribution to the interaction has thus the form

$$\mathcal{L}_{n} = A_{\mu}^{(n)} J^{(n)\mu} + \tilde{A}_{\mu}^{(n)} \tilde{J}^{(n)\mu}$$

where $J^{(n)\mu} = \sqrt{2} g \sum_{j} \cos (M_{n} z_{j}) \bar{\psi}_{j} \gamma^{\nu} \psi_{j}$ and $\tilde{J}^{(n)\mu} = \sqrt{2} g \sum_{j} \sin (M_{n} z_{j}) \bar{\psi}_{j} \gamma^{\nu} \psi_{j}$. Integration over the $A_{\mu}^{(n)}$ and $\tilde{A}_{\mu}^{(n)}$ leads to a current-current (i.e. a four-Fermi) interaction (which is not diagonal in flavour)

$$-\frac{1}{2} \sum_{n} \frac{J^{(n)\mu} J^{(n)\mu}}{M_{n}^{2}}$$

where $M_{n} = \frac{2\pi n}{L}$. The amplitudes have the form

$$c_{ik} (\bar{\psi}_{iL} \gamma^{\mu} \psi_{iL}) (\bar{\psi}_{kL} \gamma_{\mu} \psi_{kL})$$

(A3)

where

$$c_{ik} = 2 g^{2} \sum_{n=1}^{\infty} \frac{\cos (M_{n} (z_{i} - z_{k}))}{M_{n}^{2}}$$

and $\text{Li}_{n} (z) = \sum_{k=1}^{\infty} \frac{z^{k}}{k^{n}}$. If a higher dimensional bulk space had been adopted then instead of $n$ we would require a vector of integers $\vec{n}$.

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