On the Performance of Multiple Antenna Cooperative Spectrum Sharing Protocol under Nakagami-\(m\) Fading

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Abstract—In a cooperative spectrum sharing (CSS) protocol, two wireless systems operate over the same frequency band albeit with different priorities. The secondary (or cognitive) system which has a lower priority, helps the higher priority primary system to achieve its target rate by acting as a relay and allocating a fraction of its power to forward the primary signal. The secondary system in return is benefited by transmitting its own data on primary system’s spectrum. In this paper, we have analyzed the performance of multiple antenna cooperative spectrum sharing protocol under Nakagami-\(m\) Fading. Closed form expressions for outage probability have been obtained by varying the parameters \(m\) and \(\Omega\) of the Nakagami-\(m\) fading channels. Apart from above, we have shown the impact of power allocation factor (\(\alpha\)) and parameter \(m\) on the region of secondary spectrum access, conventionally defined as critical radius for the secondary system. A comparison between theoretical and simulated results is also presented to corroborate the theoretical results obtained in this paper.

Index Terms—Nakagami-\(m\) fading, cognitive radio, spectrum sharing, decode and forward relaying, cooperative communication.

I. INTRODUCTION

Cooperative spectrum sharing (CSS) have attracted a great deal of attention among researchers in the past few years due to its dual utilization of cooperative diversity for reliable communication and cognitive abilities to utilize the spectrum more efficiently [1], [2]. The concept of CSS, to employ the secondary transmitter (ST) as a relay to forward the information of the primary system and get spectrum access in exchange, can be utilized in cellular and ad-hoc networks [1]-[3].

Considerable work has been done to validate the performance of CSS protocols in Rayleigh faded channels, however to the best of our knowledge very few literature is publicly available to demonstrate the performance of these protocols on Nakagami faded channels. Many experimental works show that Nakagami distribution, as compared to Rayleigh distribution, is often more accurate for modeling the urban multipath channels [4]. Although Rayleigh fading models are frequently utilized in modeling the non light-of-sight channels however it is better fit for the signals propagating within small areas, as it does not gauge for large-scale propagation effects like shadowing by buildings, bridges and other obstructions which are typically encountered in mobile communication channel. Hence, Nakagami fading models are usually preferred in modeling long distance fading effects, specifically with respect to mobile communications [5].

Moreover, by tuning the fading severity parameter, \(m\), Nakagami-\(m\) fading can be used to represent a wider class of fading channel conditions. For instance, \(m=1\) represents Rayleigh fading whereas \(m = 0.5\) represents one-sided Gaussian fading [5].

The authors in [6] have analyzed the performance of classical decode and forward (DF) cooperative communications over Nakagami-\(m\) fading channels. They have measured the performance in terms of symbol error rate (SER) for different modulation schemes. By varying the parameters of the fading channel the authors are able to enhance the cooperation performance between primary and secondary system. In [7], [8] the outage performance of an underlay system with cognitive decode and forward (DF) and amplify and forward (AF) relaying schemes has been investigated. The authors in their system model have used relays for transmitting secondary system’s data. Secondary transmitter and its relay limits their transmit power so that the interference on the primary system do not exceed a certain threshold.

Compared to the previous work proposed in the literature on Nakagami fading channels, in this paper we have considered an overlay model in which there is no limitation on the secondary transmit power. On a contrary, depending on the power allocation factor, \(\alpha\), the performance of the primary and secondary system may increase with an increase in the secondary transmit power. This paper can also be seen as an extension of the work done for Rayleigh fading channels...
channel coefficients $h_1, h_{(21)}, h_{(22)}, \ldots, h_{(2N)}, h_3, h_4$ and $h_5$ respectively. The probability density function (PDF) of a Nakagami random variable $\gamma = |h|^2$ is given by

$$p_\gamma(h) = \frac{2^{m}h^{2m-1}}{\Gamma(m)\Omega^m}e^{-\frac{\gamma\Omega}{m}},$$

where $\Omega = E\{\gamma^2\}$ is the variance of $\gamma$, $m$ is the Nakagami fading figure and $\Gamma(.)$ is the Gamma function. Generally, when $m = 1$ the above PDF reduces to the PDF of well-known Rayleigh fading model. For $0.5 \leq m < 1$, the fading is Nakagami which is more severe than that of Rayleigh fading.

The parameter $\Omega_i = d_{i}^{-k}$ where $k$ is the path loss component and $d_i$ is the normalized distance between the respective transmitters and receivers. This normalization is done with respect to distance between PT and PR, i.e. $d_1 = 1$. Primary and secondary signals are denoted by $x_p$ and $x_s$ respectively, with zero mean and $E\{|x_p|^2\} = 1$, $E\{|x_s|^2\} = 1$. $R_{pt}$ and $R_{st}$ are the target rates for $x_p$ and $x_s$ respectively. We denote the transmit power at PT and ST as $P_p$ and $P_s$, respectively.

The additive white Gaussian noise (AWGN) at each receiver is denoted by $n_{ij} \sim \mathcal{CN}(0, \sigma^2)$ where $i \in \{1, 2\}$ represents the transmission phase and $j \in \{1, 21\}, \{22\}, \ldots, \{2N\}, \ldots, \{5\}$ represents the respective channel link, assumed to have identical variance $\sigma^2$. In the following sections we will analyze the performance of cooperative spectrum sharing based on DF protocol under Nakagami-$m$ fading channels.

III. OUTAGE PERFORMANCE OF PRIMARY SYSTEM

In phase 1, PT will broadcast the signal $x_p$. This signal is overheard by PR, ST$_1$, ST$_2$, ..., ST$_N$ and SR. The received signal at PR is denoted by $y_{pr1}$, which is given by

$$y_{pr1} = \sqrt{P_p}h_1 x_p + n_{11},$$

where, $n_{11} \sim \mathcal{CN}(0, \sigma^2)$. The received signal at ST is denoted by

$$y_{ST} = h_d y_p x_p + n,$$

where.

$$y_{ST} = \begin{bmatrix} y_{st}^{(21)} & y_{st}^{(22)} & \ldots & y_{st}^{(2N)} \end{bmatrix}^T.$$ 

$y_{st}^{(21)}, y_{st}^{(22)}, \ldots, y_{st}^{(2N)}$ denote the signals coming from channel $h_{(21)}, h_{(22)}, \ldots, h_{(2N)}$, respectively. Also,

$$h_d = [\sqrt{P_p}h_{(21)}, \sqrt{P_p}h_{(22)}, \ldots, \sqrt{P_p}h_{(2N)}]^T$$

and $n = [n_{1(21)} n_{1(22)} \ldots, n_{1(2N)}]^T$. The signals thus received at ST is decoded for $x_p$. The rate at ST is given by

$$R_{ST} = \frac{1}{2} \log_2 \left[ 1 + \frac{P_p \left(|h_{(21)}|^2 + |h_{(22)}|^2 + \ldots + |h_{(2N)}|^2\right)}{\sigma^2} \right]$$

$$R_{ST} = \frac{1}{2} \log_2 \left[ 1 + \frac{P_p \left(||h||^2\right)}{\sigma^2} \right].$$

where $||h||^2 = |h_{(21)}|^2 + |h_{(22)}|^2 + \ldots + |h_{(2N)}|^2$, the factor $\frac{1}{2}$ in the above equation accounts for the fact that the transmission is being divided into two phases. In phase 2, if $x_p$ is decoded
successfully, ST will transmit $x_p$ along with its own data $x_s$. The signals received at PR is given by

$$y_{pr2} = g_{df}v_{st} + n_{23},$$

where

$$g_{df} = \left[ \sqrt{\alpha P_s h_3}, \sqrt{(1-\alpha) P_s h_3} \right],$$

$$v_{st} = \left[ x_p, x_s \right]^T$$

and $n_{23} \sim CN(0, \sigma^2)$. The signals in phase 1 and 2, $y_{pr1}$ and $y_{pr2}$, are then combined at PR using MRC. The achieved rate is then derived as in [1] is given by

$$R_p = \frac{1}{2} \log_2 \left( 1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{\alpha P_s \gamma_3}{(1-\alpha) P_s \gamma_3 + \sigma^2} \right).$$

(2)

On the other hand if ST is not able to decode $x_p$ in phase 1 then it will not transmit in phase 2. In such a case PR can still receive $x_p$, through a direct link from PT to PR with achievable rate of

$$R_{pd} = \log_2 \left( 1 + \frac{P_p \gamma_1}{\sigma^2} \right).$$

Thus, the outage probability of the primary signal transmission with target rate $R_{pt}$ is given as

$$F_{op} = 1 - P_r \{ R_{ST} > R_{pt} \} P_r \{ R_p > R_{pt} \} - P_r \{ R_{ST} < R_{pt} \} P_r \left( \frac{1}{2} R_{pd} > R_{pt} \right).$$

(3)

Assuming $P_s >> \sigma^2$ as in [1], [2] we obtain

$$P_r \{ R_p > R_{pt} \} = \begin{cases} \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{\Omega_2 p_p} \left( \rho_1 - \frac{\alpha}{1-\alpha} \right) \right) \right] & 0 \leq \alpha < \tilde{\alpha} \\ 1 & \tilde{\alpha} \leq \alpha < 1 \end{cases},$$

(4)

where $\rho_1 = 2^{R_{pt} - 1}, \tilde{\alpha} = \frac{\rho_1 + 1}{\rho_1}, \Gamma(.)$ is the Gamma function and $\Gamma(.,.)$ indicate the incomplete Gamma function.

$$P_r \{ R_{ST} < R_{pt} \} = \frac{1}{\Gamma(Nm)} \left[ \Gamma(Nm) - \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_2 p_p} \rho_1 \right) \right].$$

(5)

$$P_r \{ R_{ST} > R_{pt} \} = \frac{1}{\Gamma(Nm)} \left[ \Gamma(Nm) - \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_2 p_p} \rho_1 \right) \right].$$

(6)

$$P_r \left( \frac{1}{2} R_{pd} > R_{pt} \right) = \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \rho_1 \right) \right].$$

(7)

Substituting (4), (5), (6) and (7) in (3). We get,

$$F_{op} = \begin{cases} F_{op1}, & 0 \leq \alpha < \tilde{\alpha} \\ F_{op2}, & \tilde{\alpha} \leq \alpha < 1 \end{cases}$$

(8)

$$F_{op1} = 1 - \frac{1}{\Gamma(Nm)} \left[ \Gamma(Nm) - \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_2 p_p} \rho_1 \right) \right] - \frac{1}{\Gamma(Nm)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \left( \rho_1 - \frac{\alpha}{1-\alpha} \right) \right) \right].$$

(9)

$$F_{op2} = 1 - \frac{1}{\Gamma(Nm)} \left[ \Gamma(Nm) - \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_2 p_p} \rho_1 \right) \right] - \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \rho_1 \right) \right].$$

(10)

## IV. REGION FOR SECONDARY SPECTRUM ACCESS

In this section, we are going to define the region, within which the secondary system can access primary’s spectrum without compromising the performance of primary system. This region has been conventionally defined as critical radius in [1]. To calculate a critical region for such a system, the outage probability of primary system with cooperation i.e. $F_{op}$, must be less than the outage probability without cooperation, i.e. $F_{op} \leq P_d$. The outage probability of direct transmission (without cooperation) is given as

$$P_d = P_r \{ R_{pd} < R_{pt} \} = \frac{1}{\Gamma(m)} \left[ \Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \rho_2 \right) \right].$$

(11)

where $\rho_2 = 2^{R_{pt} - 1}$ and $\Gamma(.,.)$ indicate the incomplete Gamma function. From (9), (10), we can observe that $F_{op}$ not only depends on the $\Omega$ but it also varies with change in the value of $\alpha$. Therefore, there are two cases which describes the successful spectrum access of the secondary system, i.e for $\Omega_2 \leq \tilde{\Omega}_2$ and $\alpha > \tilde{\alpha}$. The theoretical values of $\tilde{\Omega}_2$ after solving $F_{op2} \leq P_d$ is given as below

$$\Omega_2 \leq \tilde{\Omega}_2 = \left[ \frac{P_p}{m \sigma^2} \Gamma^{-1} \left( \left( Nm, \frac{m \sigma^2 \rho_2}{\Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \rho_1 \right) } \right) \right) \right]^{-1},$$

(12)

where $\Gamma^{-1}(.,.)$ indicate the inverse incomplete Gamma function. The $\tilde{\alpha}$ for $P_d \geq F_{op1}$ is given as

$$\alpha \geq \tilde{\alpha} = \frac{\rho_1 - \chi}{1 + \rho_1 - \chi},$$

(13)

where

$$\chi = \frac{\Omega_1 P_p}{m \sigma^2} \Gamma^{-1} \left( m, \varphi \right),$$

and

$$\varphi = \frac{\Gamma \left( m, \frac{m \sigma^2 \rho_2}{\Omega_2 p_p} \right) - \Gamma \left( m, \frac{m \sigma^2}{\Omega_1 p_p} \right) \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_2 p_p} \rho_1 \right)}{1 - \Gamma \left( Nm, \frac{m \sigma^2}{\Omega_1 p_p} \rho_1 \right)}.$$

We can note that for $m = 1$ (12), (13) reduces to the results given in [1], [2] for Rayleigh flat fading.
V. Outage Performance of Secondary System

In phase 1, received signals at secondary receiver is given by

\[ r_{sr1} = \sqrt{P_p h_5} x_p + n_{15}, \]

where \( n_{15} \sim \mathcal{CN}(0, \sigma^2) \). The rate at SR for the direct transmission from PT is given by

\[ R_{sd} = \frac{1}{2} \log_2 \left[ 1 + \frac{P_p \gamma_5}{\sigma^2} \right]. \]  

(14)

At SR, an estimate of \( x_p \) is obtained as

\[ \hat{x}_p = \frac{y_{sr1}}{\sqrt{P_p h_5}} = x_p + \frac{n_{15}}{\sqrt{P_p h_5}}. \]

The achievable rate at ST is given in (1). In phase 2, signal received at SR is given by

\[ y_{sr2} = h_s v_{st} + n_{24}, \]

where

\[ h_s = \left[ \sqrt{\alpha P_p h_4} \sqrt{(1 - \alpha) P_s h_4} \right], \]

and \( n_{24} \sim \mathcal{CN}(0, \sigma^2) \) is the AWGN. The estimate \( \hat{x}_p \) is used to cancel the interference component, to obtain

\[ \hat{y}_{sr2} = \sqrt{(1 - \alpha) P_s h_4} x_p + n_{24}. \]

The achieved rate between ST and SR, conditioned on successful decoding of \( x_p \) at both ST and SR in the first phase, is given by

\[ R_s = \frac{1}{2} \log_2 \left[ 1 + \frac{(1 - \alpha) P_s \gamma_4}{\sigma^2} \right]. \]  

(15)

Outage is declared if ST and SR are not able to decode \( x_p \), and therefore the outage probability of the secondary signal transmission with target rate \( R_{st} \) is given as

\[ F_{os} = \frac{1 - P_r \{ R_{ST} > R_{pt} \} P_r \{ R_{sd} > R_{pt} \}}{P_r \{ R_{sd} > R_{pt} \} P_r \{ R_s > R_{st} \}}. \]  

(16)

\[ P_r \{ R_s > R_{st} \} = \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{4 (1 - \alpha) P_s \rho_1} \right) \right], \]  

(17)

\[ Pr \{ R_{sd} > R_{pt} \} = \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{2 P_p \rho_1} \right) \right], \]  

(18)

where \( \rho_1 = 2^{2R_{st}} - 1 \). Substituting (6), (17) and (18) in (16) we get the outage probability as

\[ F_{os} = 1 - \frac{1}{\Gamma(Nm)} \left[ \Gamma(Nm) - \Gamma \left( Nm, \frac{m \sigma^2}{4 P_p \rho_1} \right) \right] \]

\[ \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{2 P_p \rho_1} \right) \right] \]

\[ \frac{1}{\Gamma(m)} \left[ \Gamma(m) - \Gamma \left( m, \frac{m \sigma^2}{4 (1 - \alpha) P_s \rho_3} \right) \right]. \]  

(19)

VI. Simulation Results and Discussions

In this section, we discuss the performance of a cooperative spectrum sharing protocol for Nakagami-m fading. Target rates of primary as well as secondary systems are chosen to be \( R_{pt} = R_{st} = 1. \) The value of \( m \) is taken as \( m = 0.7 \), which measures the depth of fading envelope. PT, ST, SR, PR nodes are assumed to be collinear as in [1], [10]. The node ST is equipped with N antennas. For simulation, we have taken the value of \( N=2 \) and \( N=4 \). The distance between PT and PR is normalized and taken as \( d_1 = 1 \). The distance between PT and ST is denoted by \( d_2 \) and the respective distances between different nodes is calculated in terms of \( d_2 \). The distance between ST and PR is \( d_3 = |1 - d_2| \), ST to SR and PT to SR is \( d_4 = d_5 = d_2/2 \). We have taken \( P_s = 20 \) dB and \( P_p = 30 \) dB. The \( k = 4 \), is the path loss component.

Fig. 2 shows the outage probability performance of primary system w.r.t. the power allocating factor \( \alpha \) for different values of \( d_2 = \{0.8, 1.5, 3.25\} \) for \( N=4 \) and \( d_2 = \{0.8, 1.5, 2.625\} \) for \( N=2 \). These set values of \( d_2 \) are calculated from \( \Omega_2 = d_2^k \), where the last values in both the sets are the critical values calculated from (12). It can be seen from the figure that as we increase the value of \( \alpha \) the outage probability tends to decrease.

For a particular value of \( d_2 \) between PT and ST, when \( \alpha > \hat{\alpha} \) the outage probability drops below the outage probability of direct link and spectrum access can be achieved by secondary system. As we increase \( \alpha \geq \hat{\alpha} \) then for a particular \( d_2 \) between PT and ST much lower outage probability can be achieved by the primary system. The outage probability performance of the system under Nakagami fading reduces to Rayleigh fading for \( m = 1 \).

Fig. 3 shows reasonably good outage probability of secondary system w.r.t. \( \alpha \). The theoretical results are exactly matching with the simulation results, authenticating the ana-
lytical results obtained for the outage probability of secondary system. We can observe from figure that the outage probability has a constant value for almost all values of $\alpha$ and tends to 1 as $\alpha \to 1$.

![Figure 3. Outage Probability of Secondary System](image)

Fig. 4 shows the outage probability of the primary as well as the secondary system w.r.t the fading coefficient $m$. It can be observed from the figure that as the value of $m$ increases i.e. the fading effect of the channel decreases, the outage probability of the overall system decreases, which is quite obvious from the fact that as there is no fading in the channel the data can be transmitted smoothly and efficiently to the destination. From fig. 3 and fig. 4 it can be inferred that there is good agreement between theoretical and simulating results thus validating the analysis done in this paper.

![Figure 4. Outage Probability of primary and secondary system w.r.t fading parameter m](image)

VII. CONCLUSIONS

In this paper, we analyzed the performance of cooperative spectrum sharing scheme over Nakagami-$m$ fading. A cognitive relay, equipped with multiple antennas decodes the message from primary transmitter and forwards, by means of DF relaying, it to the destination by randomly selecting one antenna in order to achieve the target rate of primary system, getting the spectrum access for secondary system in exchange. It was shown that, even in presence of Nakagami-$m$ fading CSS protocol with multiple antennas at ST can help in considerable improvement in the performance of primary system. From above observations, we can conclude that as the value of $m$ increases the severity of fading decreases and performance of outage probability improves. The excellent agreement between the simulated results and the analytically obtained closed form expressions authenticates the theoretical analysis presented in this paper.

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