Numerical study on the effect of geometry on mechanical behavior of triply periodic minimal surfaces

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Abstract. Metallic cellular structures with triply periodic minimal surfaces (TPMS) is a great approach to enhance multifunctionality for several engineering disciplines. In the present study, we aimed to generate metal-based Neovious, Schoen I-WP and Schwarz Primitive structures, leveraging on TPMS based design methods. Neovious, Schoen I-WP and Schwarz Primitive structures are mathematically formulated. Geometric factors like wall thickness, number of cells and the length of unit cell can affect the mechanical response namely elastic modulus and compressive strength. Herein, Neovious, I-WP and Primitive based materials are fabricated through Selective Laser Melting (SLM) to evaluate the difference between the aforementioned geometrical factors both experimentally and through simulations. The Taguchi method was implemented to explore the effect of geometric factors on structural response. These results were further extended to the structures to optimize the elastic modulus and compressive strength.

1. Introduction
High performance of cellular structures related to excellent energy absorption, lightweight, high strength-weight ratio and excellent thermal, acoustic, mechanical and electrical properties are gaining increasing interest in their specific design. Cellular materials have a wide range of applications in biomedicine, energy absorption and aerospace [1,2]. Recently, many researches have been focused on controllable topology and design of shell-based structures of minimal surfaces which are known as Triply Periodic Minimal Surfaces (TPMS) [2-7]. Triply periodic demonstrates a repeated structure in 3D which is devoid of any self-intersections. TPMS are specified as surfaces with a zero mean curvature at each point on the surface [8]. Nowadays, the fabrication of complex architectures has been perceived due to development of Additive Manufacturing (AM) approaches. Conventional ways to produce cellular materials have many limitations about using material and simplicity in design which can be addressed by application of AM technologies to mitigate the challenges. Selective laser melting (SLM) as a kind of AM process has a great potential to fabricate complex structures [9]. Traditional structures such as a strut-based lattice fabricated by SLM with a lack of continuous transition on their surface and sharp edges demonstrated a poor manufacturability in the AM process [10,11]. It is notable that the absence of nodes and other stress concentration regions in TPMS make them valuable structures with improved strength and toughness [12-15]. The absence of joints and struts in TPMS lattice structure (in
comparison with strut-based architectures) is a considerable benefit in terms of processability [16]. SLM as an AM method is known an appropriate technique to printing metallic structures. Variety of metal powders, such as Ti6Al4V, 316 L stainless steel, Fe–Mn and Zn can be produced with design in TPMS lattice architecture with the help of SLM techniques [17, 18].

The geometric parameters such as wall thickness, sample size, number of surface periods, and the associated iso-value can affect the mechanical response of gyroid structure fabricated of Ti6Al4V by SLM [19]. To quantify the influence of mentioned geometric factors on structural response, OFAT (One Factor at a Time) and Taguchi methods were used and it was shown that an increase in the number of cells and wall thickness strongly effect the elastic modulus and compressive strength whilst a decrease in the sample size and iso-value can lead to improved mechanical response.

This research focuses on finding optimum design and studying the effect of wall thickness and number of cells on a structural response of Neovius (Neo), Schoen I-WP (IWP) and Schwarz Primitive (Pri) structures by implementing Taguchi method. To identify the effect of mentioned factors on mechanical properties, numerical and experimental procedures are required. Validity has been verified by the experimental data of compression test performed on the SLM printed structures of Ti-6Al-4V.

![Figure 1. TPMS unit cells and designed specimens of a) IWP, b) Neovius and c) Primitive](image)

2. Material and method

2.1. Modeling and computer-aided design of cellular lattice

In x-y-z spatial coordinates, the TPMS can be approximated using implicit methods and introduced by a level-set function \( \varphi = c \), which is defined for Neovius (Neo), Schoen I-WP (IWP) and Schwarz Primitive(Pri) surfaces, respectively in equation (1) [20]:

\[
\begin{align*}
\varphi_{\text{Neo}} &= 3(\cos kx + \cos kz + \cos ky) + 4kx \cos kx \cos ky = c \\
\varphi_{\text{IWP}} &= 2(\cos (kx) \cos (ky) + \cos (kx) \cos (kz) + \cos (kz) \cos Kx) - (\cos 2kx + \cos 2ky + \cos 2kz) = c \\
\varphi_{\text{Pri}} &= \cos kx + \cos ky + \cos kz = c
\end{align*}
\]

Where \( k = 2\pi q/l \). \( q \) defines the number of cells in each direction, and \( l \) is the length of the unit cell. The value of \( c \) controls the shape of surfaces and is known as iso-value parameter. In this research, the value of \( c \) is assumed to be zero. MATLAB software was used to generate the initial surface of TPMS and SolidWorks followed by a CAD software which was used to thicken and create 3D models of TPMS. Labeling of structures has been shown in figure 1. For example, the label of “Neo050-2” describes the Neovius structure with wall thickness of 0.5 mm and 2 repeated cells at each edge. Designed models are shown in figure 1.

2.2. Material, Machine, and Methods

The Ti6Al4V powder of particle size 20 – 63 \( \mu m \) was printed by Realizer SLM50 metal printer. The associated laser parameters were applied for fabrication with laser power of 84 W, layer thickness of 30 \( \mu m \), hatch distance of 120 \( \mu m \), at a scanning speed of 750 mm/s, rotate hatch at 60 degree and 35 degree with y axis was selected to print structure. According to these parameters, the calculated laser energy density is 31.11 J/mm\(^3\).
2.3. Uniaxial compression test and finite element analysis

Pri030-2 specimen with the length size of 16.67 mm in x, y and z direction was chosen for experimental study, which was performed by a servo-hydraulic model 8500 universal testing machine (Instron Ltd., UK). A computational finite element (FE) analysis was performed to evaluate mechanical performance with experimental compression test (figure 3). For static compression at a room temperature, a strain rate of 0.1 mm/s was applied with compressive load and displacement recorded at each 0.1 s intervals during testing. It is notable that in all simulations from now on, the coefficient of the friction was defined as 0.1 to prevent sliding during applied load. The ANSYS 2019 software was used to simulate compression tests.

2.4. Taguchi method

To investigate the effect of individual geometric factors on mechanical properties, statistical Taguchi method was implemented to improve the quality of the outcomes. According to full factorial approach, with three significant factors considering three level of each TPMS surface, the designed experiments could reach up to 81 which henceforth is very time-consuming. Therefore, in order to decrease the number of tests without any loss in accuracy, it is recommended to implement Taguchi approach. In total 9 test were designed, which were assigned to three factors such as type of structures; surface wall thickness and number of cells (L/I). For designing, a cube, as a bulk sample with constant length 25 mm in x, y and z direction was assumed. Bulk size can be defined by multiple of cell size to the number of cells on each direction. It is noted that with changing cell number, cell size can be altered. This study represents the importance of above-mentioned factors on mechanical properties with Taguchi method. All designed topologies and calculated modulus of elasticity with maximum compressive strength at 4% strain was based on Taguchi orthogonal L9 matrix henceforth represented in Table 1.

| Taguchi L9 Matrix | Factors | Results | Porosity (%) |
|-------------------|---------|---------|--------------|
| Topology          | Structures | Thickness (mm) | Number of cells (L/I) | Elastic Modulus (GPa) | Max. Stress (MPa) |  |
| IWP050-2          | I-WP     | 0.5      | 2             | 2.804                  | 34.336               | 85.87           |
| IWP040-3          | I-WP     | 0.4      | 3             | 3.730                  | 45.718               | 82.93           |
| IWP030-4          | I-WP     | 0.3      | 4             | 3.839                  | 44.084               | 83.09           |
| Neo050-3          | Neoivus  | 0.5      | 3             | 6.839                  | 70.121               | 79.12           |
| Neo040-4          | Neoivus  | 0.4      | 4             | 7.561                  | 73.060               | 77.71           |
| Neo030-2          | Neoivus  | 0.3      | 2             | 2.109                  | 22.318               | 91.50           |
| Pri050-4          | Primitive| 0.5      | 4             | 2.557                  | 39.633               | 81.33           |
| Pri040-2          | Primitive| 0.4      | 2             | 0.536                  | 10.030               | 92.50           |
| Pri030-3          | Primitive| 0.3      | 3             | 0.744                  | 14.108               | 91.56           |

3. Results

3.1. Evaluation compression test result

FE model was performed to validate the result with experiment compression test of Ti6Al4V SLM printed Pri030-2 with 91% porosity and figure 2 represent force-displacement result of both FE analysis and experimental data. The outcomes of elastic modulus and maximum stress indicate 3% and 1.9% deviation in the result of the numerical and experiment which is acceptable and shows strong agreement with the physically obtained result.

3.2. SEM Analysis

The fabricated lattice sample Pri030-2 with its SEM analysis figure 3 which is done for top surface of printed structure to investigate dimensional variations between the designed and 3D-printed structures. The as-fabricated lattice structure clearly followed the CAD model. Residual particles were found
attached to the fabricated structures as shown figure 3(a-c). Residual particles can cause the thickness variations.

**Figure 2.** Compression test of Pri030-2 at 6% strain at t=10 s in experiment and simulation, contour of equivalent (Von-Mises) stress with undeformed wireframe, force-displacement curve

**Figure 3.** SEM image of top surface of printed Pri030-2

3.3. **Taguchi Method**

Based on Taguchi method, a signal to noise ratio (SNR) defines the ratio of calculated data to the error. The accuracy of measured value and calculated data for each test were determined by SNR. In statistical analysis, main effect is the influence of different levels of each factor on outcomes when they are contrary. If SNR and main effect shows the same trend, the obtained data and designed data are correct.

**Figure 4.** SNR and mean values for compressive stress
The criteria ‘large is better’ was chosen to maximize the modulus of elasticity and compressive strength. Figure 4 and Figure 5 illustrate the main effect and SNR for both modulus of elasticity and compressive strength. As mentioned above, they have the same trend indicating the results to be correct. Table 2 shows the ranking of three factors for compressive strength based on ‘large is better’ criteria. First of all, the average response for all parameters is determined. Then the delta (the difference between minimum and maximum average) is calculated. Finally, all factors are categorized from largest to smallest. The result for the means reveals the type of structure as the best type for maximizing compressive strength with the first rank. The factor number of cells (L/l) and wall thickness are identified as the second and third effective factors.

**Table 2. SNR and mean values for compressive strength, ‘Larger is better’**

| Level | Type of structure | Wall thickness | L/l | Level | Type of structure | Wall thickness | L/l |
|-------|------------------|----------------|-----|-------|------------------|----------------|-----|
| 1     | 4.86             | 33.20          | 25.90| 1     | 41.38           | 48.03          | 22.23|
| 2     | 33.72            | 30.17          | 31.04| 2     | 55.17           | 42.94          | 43.32|
| 3     | 24.99            | 27.62          | 34.04| 3     | 21.26           | 26.84          | 52.26|
| Delta | 8.73             | 5.58           | 8.14 | Delta | 33.91          | 21.19          | 30.03|
| Rank  | 1                | 3              | 2   | Rank  | 1               | 3              | 2   |

Table 3 shows the ranking of three factors for elastic modulus based on ‘large is better’ method. Similar to compressive strength according to the means result, the type of structures is the first effective factor to maximize elastic modulus, with number of cell (L/l) is in the second ranking. Based on the main effects, the wall thickness of 0.5 and 0.4 mm shows that there is no significant difference for maximizing elastic modulus. Also, the ranking of the SNR represents the same result for all the three factors.

**Table 3. SNR and mean values for elastic modulus, ‘Larger is better’**

| Level | Type of structure | Wall thickness | L/l | Level | Type of structure | Wall thickness | L/l |
|-------|------------------|----------------|-----|-------|------------------|----------------|-----|
| 1     | 10.6929          | 11.2709        | 3.3448| 1     | 3.458           | 4.067           | 1.817|
| 2     | 13.5856          | 7.8666         | 8.5252| 2     | 5.504           | 3.943           | 3.771|
| 3     | 0.0626           | 5.2035         | 12.4711| 3     | 1.279           | 2.231           | 4.653|
| Delta | 13.5229         | 6.0674         | 9.1263| Delta | 4.224          | 1.836           | 2.836|
| Rank  | 1                | 3              | 2    | Rank  | 1               | 3              | 2    |

These results revealed that the Neovious structure has the most influential effect on maximizing both elastic modulus and compressive strength. Increasing number of cells and wall thickness can decrease...
the volume fraction which means less densification with less porosity that results heavier structure and higher elastic modulus and compressive strength.

4. Case study
Regarding the Taguchi results, among different levels of selected factors, the optimum values were chosen. As result represents, the most influential effect on mechanical behavior is related to the type of structure, maximum cell number and wall thickness. A custom designed Neo050-4 with defined parameter length of 25 mm, with Neovius structure, wall thickness of 0.5 mm and 4 unit cells in each direction and porosity of 72% was developed using Ti6Al4V material. The outcome represents a simulated yield stress of 93.01 MPa and ultimate strength of 140 MPa and elastic modulus of 9.7555 GPa.

Figure. 6 Mechanical response of Neo050-4 – FE analysis

5. Conclusions
This study has demonstrated the effect of geometric parameters on mechanical response of Neovius (Neo), Schoen I-WP (IWP) and Schwarz Primitive (P) complex structures based on a statistical approach. Triply Periodic Minimal Surfaces (TPMS) were produced from Ti6Al4V alloy using SLM. A numerical model has been defined to validate with experimental data which shows a close agreement. Based on the results, Neovius (Neo) has shown to outperform other structures in terms of strength and stiffness, which can lead to significant effect on elastic modulus and compressive strength. The number of cells and wall thickness are the second and third important factors which can affect structural response directly. A candidate structure of Neovius structure with wall thickness of 0.5 mm and cell number of 4 with the porosity of 72% was developed to validate the result and FE analysis of structural response indicated high elastic modulus and compressive yield strength respectively (9.75 GPa) and (93.01 MPa).

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