Research on the Calculation Method of Internal Force of Combined Energy Dissipation Shed-tunnel Roof

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Abstract: The paper studied on the internal force calculation process of roof of the combined energy dissipation shed-tunnel through using the theory of thin plates, the theory of structural dynamics and coordinated deformation theory, the paper taked the top slab as one-way slab, used the heavy series theory to study the equation, and got the internal force analytical solution under the action of a rockfall impact. The research results can provide reference for shed-tunnel structure design.

1. Introduction
Dangerous rock collapse is a common geological disaster in mountain and hilly area. It has the characteristics of randomness and suddenness. There are countless traffic accidents caused by the collapse of the crisis every year. At present, passive protection techniques such as shed-tunnel are adopted in the prevention and control of dangerous rock collapse, which make use of plastic deformation of cushion soil to absorb and dissipate the impact energy of rockfall. A lot of researches have been carried out on the shed tunnel at home and abroad. The damping plate structure is proposed by Mougin et al, which can achieve energy dissipation by deflection deformation or local damage [1]. Delhomme et al have found that under the impact of low strength, reinforced concrete slabs would produce severe cracking in the impact area and the edge of the Influence area would produce warping deformation [2][3]. OLSSON studied the plastic effect of the bearing plate under impact loading and the dynamic response of the bearing plate under the impact of sphere [4]. He Siming uses energy method to analyze the shock resistance mechanism of a new type of energy dissipation shed tunnel [5]. Yang Lu has studied the dynamic response of the rolling stone impact load to shed-tunnel structure under different speeds and angles by using the finite element numerical simulation and obtained the relationship between the displacement and the rockfall velocity and the impact angle [6]. Yang Lu has studied the dynamic response of the rolling stone impact load to shed-tunnel structure under different speeds and angles by using the finite element numerical simulation and obtained the relationship between the displacement and the rockfall velocity and the impact angle [7].

In summary, the analysis of the force structure of the rockfall shed tunnel under the impact load mainly depends on the numerical simulation and model test, and the relevant analytic study are less. In particular, there is less research on shed hole with buffer energy dissipation bearing. In view of this, this paper uses the plate theory, structural mechanics and deformation coordination theory to analyze the force characteristics of the shed tunnel with buffer energy dissipation bearing.

2. The Structure Characteristics of Combined Energy Dissipation Shed Tunnel
The object of this study is the energy dissipation shed tunnel structure proposed by Professor Chen
Hongkai [8] (ZL201420649539.5), As shown in Figure 1. The structure from the bottom as follows: two longitudinal beams fixed longitudinally on both sides of the road on the energy dissipating bearing (1), a transverse beam (2) fixed transversely on two longitudinal beams, a protective roof (3) secured to a beam and a energy dissipation bearing is arranged between the connecting plate and the support. The energy dissipation support is arranged at the top of each column, and the crossbeam and the longitudinal beam of the shed hole are combined with the support column, the composite support structure of the rock falling energy dissipation shed cavity is formed by these structures.

![Figure 1. Schematic diagram of the combined energy dissipation shed tunne](image1)

3. **Calculation of Roof Force in Shed Tunnel**

Because the roof and the beam of the combined shed are simply supported and overlapped, the roof can be simplified as a simply supported plate, as shown in figure 2. The overlapping end of unidirectional simply supported plate is b. The width of the free end is a, and the thickness of the plate is t. According to the theory of elasticity, when the ratio of the thickness of the plate t to the minimum characteristic size b in the plate is 1/80~1/5, it is called a thin plate. Referring to the general code for design of highway bridges and culverts (JTG D60-2015), it is known that the roof of tunnel shed belongs to thin plate type.

![Figure 2. Schematic diagram of the simplified roof slabs](image2)

Shed cave is mainly to withstand the impact of falling rocks, when the rockfall hits the roof of the shed tunnel, the impact position is random, and the impact force changes dynamically with time. It is assumed that the impact of rockfall is at one point of the roof and the maximum impact load is \( F(x, y) \), as shown in figure 2.

According to the differential equation of the elastic surface, the expression of the major stress components in the thin plate can be derived:
\[
M_x = -D(\frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2}) \\
M_y = -D(\frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2}) \\
M_{xy} = M_{yx} = -D(1-\nu) \frac{\partial^2 \omega}{\partial x \partial y}
\]

(1)

Furthermore, the secondary stress component can be obtained according to the equilibrium equation and the boundary condition of the simply supported plate:

\[
\tau_{xx} = -\frac{Et^2}{2(1-\nu^2)}(\frac{\partial}{\partial x}(\nabla^2 \omega)) \\
\tau_{yy} = -\frac{Et^2}{2(1-\nu^2)}(\frac{\partial}{\partial y}(\nabla^2 \omega)) \\
\tau_{xy} = -\frac{Et^2}{2(1-\nu^2)}(\frac{\partial}{\partial y}(\nabla^2 \omega)) \\
\tau_{yx} = -\frac{Et^2}{2(1-\nu^2)}(\frac{\partial}{\partial x}(\nabla^2 \omega))
\]

(2)

The deflection of a roof under normal concentrated load \(F(x,y)\) can be calculated by the following formula:

\[
\nabla^4 \omega = \frac{F(x,y)}{D}
\]

(3)

type: \(D\) is the bending rigidity of the plate, \(D=Et^3/12(1-\nu^2)\), \(t\) is the thickness of the plate, and \(\nu, E\) are the Poisson's ratio and the modulus of elasticity of the plate respectively.

Taking the single span of the roof of the combined energy dissipation shed as the object of study, it is simplified as simply supported slab. The deflection of Levi solution to shed roof is solved, then the deflection expressions as single rewriting trigonometric series [9]:

\[
\omega = \sum_{m=1}^{\infty} Y_m(y) \sin \frac{m\pi x}{a}
\]

(4)

In the above expression, \(Y_m(y)\) is an undetermined function, and \(M\) is any positive integer.

It can be seen from formula that (4) the deflection single trigonometric series formula should satisfy the following boundary conditions:

\[
\left\{ \begin{array}{l}
\omega\bigg|_{x=0} = 0, \quad \frac{\partial^2 \omega}{\partial x^2}\bigg|_{x=0} = 0 \\
\omega\bigg|_{x=a} = 0, \quad \frac{\partial^2 \omega}{\partial x^2}\bigg|_{x=a} = 0
\end{array} \right.
\]

(5)

To bring (4) into (3):
\[
\sum_{m=1}^{\infty} \left[ Y_m^{(4)} - 2\left(\frac{m\pi}{a}\right)Y_m + \left(\frac{m\pi}{a}\right)^4 Y_m \right] \sin\frac{m\pi x}{a} = \frac{F(x, y)}{D} \quad (6)
\]

Further, the sides of the formula (6) are multiplied by \(\sin\left(M \pi \frac{x}{a}\right)\) at the same time, then the above formula of \(X\) from 0 to \(a\) integral. By using orthogonality of trigonometric functions, we can obtain:

\[
Y_m^{(4)} - 2\left(\frac{m\pi}{a}\right)Y_m + \left(\frac{m\pi}{a}\right)^4 Y_m = 2 \int_0^a \frac{F(x, y)}{D} \sin\frac{m\pi x}{a} \, dx \quad (7)
\]

The upper order is a non-homogeneous linear differential equation of order four, assuming the general solution \(Y_m(y) = Y_m^0(y) + Y_m^*(y)\). Among them, \(Y_m^0(y)\) is the general solution of homogeneous equation, and \(Y_m^*(y)\) is the special solution of non-homogeneous equation.

Equation (7) the general solution of the homogeneous equation on the left \(Y_m^0(y)\) as follow:

\[
Y_m^0(y) = A_m \sinh \frac{m\pi y}{a} + B_m \cosh \frac{m\pi y}{a} + C_m \sin \frac{m\pi y}{a} + D_m \cos \frac{m\pi y}{a} \quad (8)
\]

Therefore, when we obtain the analytic solution of \(Y_m(y)\), we can deduce the deflection equation of the thin plate, then we can get the deflection of the roof of the shed roof under the maximum impact force, so that we can get the internal force of the roof of the shed under the impact force \(F(x, y)\).

For maximum impact load \(F(x, y)\), do the following assumption:

\[
F(x, y) = p\delta(x-x_0, y-y_0) \quad (9)
\]

\[
\delta(x-x_0, y-y_0) = 0, \quad (x \neq x_0, y \neq y_0) \quad (10)
\]

At this point, the expression on the right side of equation (7) is:

\[
\frac{2}{D} \int_0^a \frac{F(x, y)}{D} \sin\frac{m\pi x}{a} \, dx = \frac{2}{D} p\delta(y-y_0) \sin\frac{m\pi x_0}{a} \quad (11)
\]

Transform the formula (11) into series, as follow:

\[
\frac{2p}{Da} \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{i\pi(y-y_0)} \cdot \sin\frac{m\pi x_0}{a} \quad (12)
\]

Among them, \(T\) is the period of Fourier variation. For the particular solution of equation (8), the following assumptions can be made:

\[
Y_m^*(y) = G \sum_{n=-\infty}^{\infty} Z_{mn} e^{-i\pi(y-y_0)} \quad (13)
\]

Among them: \(G = \frac{p}{Da} \cdot \sin \frac{m\pi x_0}{a}\)

The formula (13) is obtained by substitution (7):

\[
Y_m^{(4)} - 2\left(\frac{m\pi}{a}\right)Y_m + \left(\frac{m\pi}{a}\right)^4 Y_m = G \sum_{n=-\infty}^{\infty} Z_{mn} e^{-i\pi(y-y_0)} \left[ \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] \quad (14)
\]
So there: 

\[ Z_{mn} = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \]

\[ Y''(y) = G\sum_{m=1}^{\infty} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] e^{-\frac{n\pi(y-y_0)}{b}} \quad (15) \]

So the general solution \( Y_m(y) \) takes the form of:

\[ Y_m(y) = \left( A_m \cosh \frac{m\pi y}{a} + B_m \sinh \frac{m\pi y}{a} + C_m \sinh \frac{m\pi y}{a} + D_m \cosh \frac{m\pi y}{a} \right) + Y''(y) \quad (16) \]

In the formula: 

\[ Y''(y) = G\sum_{m=1}^{\infty} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] e^{-\frac{n\pi(y-y_0)}{b}} \]

According to the analysis of the structural characteristics of the energy dissipation rock shed cave, the boundary condition of the roof of the shed cave can be expressed as:

\[
\begin{align*}
(M_y)_{y=b} &= 0 \left( \frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2} \right)_{y=b} = 0 \\
(V_y)_{y=b} &= 0 \left( \frac{\partial^3 \omega}{\partial y^3} + (2-\nu) \frac{\partial^3 \omega}{\partial y \partial x^2} \right)_{y=b} = 0 \\
(M_y)_{y=0} &= 0 \left( \frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2} \right)_{y=0} = 0 \\
(V_y)_{y=0} &= 0 \left( \frac{\partial^3 \omega}{\partial y^3} + (2-\nu) \frac{\partial^3 \omega}{\partial y \partial x^2} \right)_{y=0} = 0 
\end{align*}
\quad (17)
\]

Taking the boundary conditions into the type (16), we can get \( A_m, B_m, C_m, D_m \), and then the deflection expression of the roof can be obtained:

\[ \omega = \sum_{m=1}^{\infty} \left[ A_m \cosh \frac{m\pi y}{a} + B_m \sinh \frac{m\pi y}{a} + C_m \sinh \frac{m\pi y}{a} + D_m \cosh \frac{m\pi y}{a} \right] Y''(y) \sin \frac{m\pi x}{a} \quad (18) \]

In the formula (18) substituting formula (1), the internal forces of the roof under the concentrated load \( F(x, y) \) can be obtained.

4. Example Analysis
Xiang Yu line Laohekou East to Ankang section is located in the low mountain of Eastern Qinling Mountains, in which taking a shed hole as an example to calculate the roof internal force of combined energy dissipation shed-tunnel. There are 3 steep cliffs, high 35~50m, almost vertical, there are perilous rock, broken stone and stone particle at the foot of cliff slope, and the size is about 0.2~0.35m. It is a limestone area with a bulk density of 28kN·m⁻³. When the perilous rock collapses, it falls from 18m and hits the roof of the shed. The position of the roof is vertical and horizontal, and the incidence is about 45 degrees, C30 concrete is used in shed tunnel structure, whose elastic modulus is 31.5GPa, and the Poisson’s ratio of concrete plate is 0.2, the spring stiffness coefficient of energy dissipation bearing is 105kN/m. The dimensions of the shed holes are shown in next Figure.
The maximum impact force of rockfall is calculated in JHPC algorithm [9], and the formula is as follows:

$$P = 2.108M^{2/3} \lambda^{2/3} H^{5/2}$$  \hspace{1cm} \text{(19)} $$

Type: \(P\) the maximum rockfall impact force, \(kN\); \(M\) rock mass, \(t\); \(\lambda\) a lame coefficient, generally take \(1000 \text{kN} \cdot \text{m}^{-2}\); \(H\) rockfall free height, \(m\).

Using the above calculation method, the maximum rockfall impact force is 34.145 \(kN\). According to the calculation method of the internal force corresponding to the above energy dissipation shed tunnel structure, the MATLAB software is used to solve the problem, and the shear force and the bending moment of the roof of the shed tunnel under the maximum impact force are obtained.

Due to the impact of rockfall, the roof is in the middle of the roof of the shed, the internal forces of the longitudinal cross section of the plate are shown in the following figure. The maximum shear value is located at the support of the beam at 4.4\(kN\), and the maximum value of the bending moment is 3.56 \(kN \cdot m\) in the midspan.

(5) Conclusion

(1) This paper theoretically analyzed the loading performance of the composite energy dissipation shed tunnel, and obtained the general series expression of roof disturbance of shed under the impact of concentrated load. Make it in accordance with the requirements and conditions through certain mathematical transformation, and then bring into the corresponding boundary condition so as to obtain the general perturbation expression of the roof of shed under the impact of concentrated load which has certain reference significance for the design of the combined energy dissipation shed.

(2) The calculation method of the roof disturbance not only applied to the dynamic response of the plate under concentrated load, but also can be used to obtain the disturbance expression of the roof of the shed tunnel under the linear load by a slight change.

(3) This paper considered only a cross and simplified the shear stress to boundary stress when analyzing the stress of the shed tunnel, which made the calculation results with some errors. Therefore, the following research should consider the influence of adjacent cross plates, in order to obtain a more accurate theoretical results.
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