AN ARGUMENT THAT THE DARK MATTER IS AXIONS

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An argument is presented that the dark matter is axions, at least in part. It has three steps. First, axions behave differently from the other forms of cold dark matter because they form a rethermalizing Bose-Einstein condensate (BEC). Second, there is a tool to distinguish axion BEC from the other dark matter candidates on the basis of observation, namely the study of the inner caustics of galactic halos. Third, the observational evidence for caustic rings of dark matter is consistent in every aspect with axion BEC, but not with the other proposed forms of dark matter.

One of the outstanding problems in science today is the identity of the dark matter of the universe\(^1\). The existence of dark matter is implied by a large number of observations, including the dynamics of galaxy clusters, the rotation curves of individual galaxies, the abundances of light elements, gravitational lensing, and the anisotropies of the cosmic microwave background radiation. The energy density fraction of the universe in dark matter is 23%. The dark matter must be non-baryonic, cold and collisionless. Particles with the required properties are referred to as ‘cold dark matter’ (CDM). The leading CDM candidates are weakly interacting massive particles (WIMPs) with mass in the 100 GeV range, axions with mass in the \(10^{-5}\) eV range, and sterile neutrinos with mass in the keV range.

Based on work by my collaborators and I over the past three or four years, I have come to the conclusion that the dark matter is axions, at least in part. The starting point is that cold dark matter axions thermalize as a result of their gravitational self-interactions\(^2\). When they thermalize, they form a Bose-Einstein condensate. It may seem surprising that axions thermalize as a result of their gravitational self-interactions since gravitational interactions among particles are usually thought to be negligible. Gravitational interactions among particles are in fact almost always negligible but cold dark matter axions are an exception because the axions occupy in huge numbers a small number of states (the typical quantum state occupation number is \(10^{61}\)) and those states have enormous correlation lengths (of order parsec to Gpc, today).

Let us call \(\Gamma = 1/\tau\) the axion thermalization rate. On time scales short compared to \(\tau\), cold dark matter axions form a degenerate Bose gas described by a classical field equation. Their behavior is then indistinguishable from that of ordinary CDM except on length scales that are too short (10\(^{14}\) cm or so) to be of observational interest. On times scales large compared to \(\tau\), cold dark matter axions thermalize. The thermalization of a degenerate Bose gas is a quantum-mechanical entropy generating process, not described by classical field equations. On time scales larger than \(\tau\) the axion state, i.e. the state that most axions are in, tracks the lowest energy state available to them. The behaviour of such a rethermalizing axion BEC is different from that of ordinary CDM and the differences are observable.
The thermalization of cold dark matter axions is discussed in detail in ref. 3. It is found there that rethermalization of the axion BEC by gravitational self-interactions is sufficiently fast that the axions that are about to fall into a galactic potential well almost all go to the lowest energy state consistent with the angular momentum they have acquired from tidal torquing. That state is one of net overall rotation, implying \( \nabla \times \vec{v} \neq 0 \) where \( \vec{v}(\vec{x},t) \) is the velocity field of the infalling dark matter. In contrast, ordinary cold dark matter (e.g. WIMPS and sterile neutrinos) falls in with an irrotational velocity field, \( \nabla \times \vec{v} = 0 \). The inner caustics of galactic halos are different in the two cases. If the dark matter falls in with net overall rotation, the inner caustics are rings whose cross-section is a section of the elliptic umbilic \((D_{-4})\) catastrophe, called caustic rings for short 4. If the velocity field of the infalling particles is irrotational, the inner caustics have a ‘tent-like’ structure which is described in detail in ref. 5 and which is quite distinct from that of caustic rings. Evidence was found for caustic rings of dark matter. The evidence is summarized in ref. 6. It is shown in ref. 7 that the evidence for caustic rings of dark matter is precisely and in all respects consistent with the predictions of a rethermalizing axion BEC.

The above is the gist of the argument. It is elaborated further in the three sections below. A few comments may be in order. One question is: what fraction of the dark matter must be axions to justify the evidence for caustic rings. We hope to comment on this soon. Another question is: to what extent does the evidence for caustic rings require the dark matter to be QCD axions 8, as opposed to some other kind of axion-like particle(s). The evidence requires that a sizable fraction of the dark matter be identical bosons, whose number is conserved on cosmological time scales, and which are sufficiently cold and thermalize sufficiently fast that they form a BEC. Furthermore the BEC must rethermalize sufficiently fast that the particles go to a state of net overall rotation as they are about to fall into galactic potential wells. It happens that the QCD axion with mass of order \( 10^{-5} \) eV has all these properties and since it solves in addition the strong CP problem of the Standard Model of elementary particles, it is reasonable to assume that the dark matter is in fact QCD axions. However, there are many axion-like particles 9 that can equally well reproduce the evidence for caustics rings. Furthermore, whether or not the particle in question is the QCD axion, the prediction of Bose-Einstein condensation and subsequent caustic ring formation is rather insensitive to the particle mass and therefore does not provide a good guide to it. The axion is being searched for as a constituent of the Milky Way halo 10, as a particle radiated by the Sun 11 and in experiments that convert photons to axions and axions back to photons behind a wall 12.

Finally, many authors have proposed 13,14 that the dark matter is a Bose-Einstein condensate of particles with mass of order \( 10^{-21} \) eV or less. When the mass is that small, the dark matter BEC behaves differently from CDM on scales of observational interest as a result of the tendency of the BEC to delocalize. Due to the Heisenberg uncertainty principle, a BEC has Jeans’ length

\[
\ell_J = (16\pi G \rho m^2)^{-\frac{1}{3}} = 1.02 \times 10^{14} \text{ cm} \left( \frac{10^{-5} \text{ eV}}{m} \right)^{\frac{1}{4}} \left( \frac{10^{-29} \text{ g/cm}^3}{\rho} \right)^{\frac{1}{4}},
\]

where \( \rho \) is the BEC density and \( m \) the constituent particle mass. As mentioned earlier, this length scale is unobservably small in the QCD axion case. In contrast, when \( m \sim 10^{-21} \) eV, the Jeans’ length is of order 3 kpc and has implications for observation. It leads to a suppression of the dark matter density near the galactic center. This may be a remedy for the excessive concentration of dark matter near galactic centers seen in numerical simulators of structure formation with ordinary CDM 17.
1 Bose-Einstein condensation of cold dark matter axions

Shortly after the Standard Model of elementary particles was established, the axion was postulated\(^8\) to explain why the strong interactions conserve the discrete symmetries P and CP. For our purposes the action density for the axion field \(\phi(x)\) may be taken to be

\[
\mathcal{L}_a = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 - \ldots
\]  

(2)

where \(m\) is the axion mass. The self-coupling strength is\(^2\)

\[
\lambda = \frac{m^2}{f^2} \frac{m_d^3 + m_u^3}{(m_d + m_u)^3} \simeq 0.35 \frac{m^2}{f^2}
\]  

(3)

in terms of the axion decay constant \(f\) and the masses \(m_u\) and \(m_d\) of the up and down quarks. In Eq. (2), the dots represent higher order axion self-interactions and interactions of the axion with other particles. All axion couplings and the axion mass

\[
m \simeq 6 \cdot 10^{-6} \text{ eV} \frac{10^{12} \text{ GeV}}{f}
\]  

(4)

are inversely proportional to \(f\). \(f\) was first thought to be of order the electroweak scale, but its value is in fact arbitrary\(^18\). However, the combined limits from unsuccessful searches in particle and nuclear physics experiments and from stellar evolution require \(f \gtrsim 3 \cdot 10^9 \text{ GeV}\)\(^19\).

Furthermore, an upper limit \(f \lesssim 10^{12} \text{ GeV}\) is provided by cosmology because light axions are abundantly produced during the QCD phase transition\(^20\). In spite of their very small mass, these axions are a form of cold dark matter. Indeed, their average momentum at the QCD epoch is not of order the temperature (GeV) but of order the Hubble expansion rate \((3 \cdot 10^{-9} \text{ eV})\) then. In case inflation occurs after the Peccei-Quinn phase transition their average momentum is even smaller because the axion field gets homogenized during inflation. In addition to the cold axion population, there is a thermal axion population with average momentum of order the temperature.

The non-perturbative QCD effects that give the axion its mass turn on at a temperature of order 1 GeV. The critical time, defined by \(m(t_1) = 1\), is \(t_1 \simeq 2 \cdot 10^{-7} \text{ sec} \ (f/10^{12} \text{ GeV})\)\(^1\). Cold axions are the quanta of oscillation of the axion field that result from the turn on of the axion mass. They have number density

\[
n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{5}{8}} \left( \frac{a(t_1)}{a(t)} \right)^3
\]  

(5)

where \(a(t)\) is the cosmological scale factor. Because the axion momenta are of order \(\frac{1}{t_1}\) at time \(t_1\) and vary with time as \(a(t)^{-1}\), the velocity dispersion of cold axions is

\[
\delta v(t) \sim \frac{1}{mt_1} \frac{a(t_1)}{a(t)}
\]  

(6)

if each axion remains in whatever state it is in, i.e. if axion interactions are negligible. The average state occupation number of cold axions is then

\[
\mathcal{N} \sim n \frac{(2\pi)^3}{3(f \delta v)^3} \sim 10^{61} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{5}{8}}
\]  

(7)

That \(\mathcal{N}\) is much larger than one tells us that the effective temperature of cold axions is much smaller than the critical temperature for Bose-Einstein condensation.
Bose-Einstein condensation may be briefly described as follows: if identical bosonic particles are highly condensed in phase space, if their total number is conserved and if they thermalize, most of them go to the lowest energy available state. The condensing particles do so because, by yielding their energy to the remaining non-condensed particles, the total entropy is increased. Eq. (7) tells us that the first condition is overwhelmingly satisfied. The second condition is also satisfied because all axion number violating processes, such as their decay to two photons, occur on time scales vastly longer than the age of the universe. The only condition for axion BEC that is not manifestly satisfied is thermal equilibrium.

Axions are in thermal equilibrium if their relaxation rate $\Gamma$ is large compared to the Hubble expansion rate $H(t) = \frac{1}{2\pi}$. The relaxation rate $\Gamma$ is given in the particle kinetic regime by

$$\Gamma \sim n \sigma \delta v N$$

where $\sigma$ is the relevant scattering cross-section. The particle kinetic regime is defined by the condition $\Gamma \ll \delta E$ where $\delta E$ is the energy dispersion of the particles. Cold dark matter axions are however mostly in the opposite regime: $\Gamma \gg \delta E$ which we call the condensed regime. Thermalization in the condensed regime is discussed in detail in our recent paper. We find that the relaxation rate of cold axions through their $\lambda \phi^4$ self-interactions is of order 2

$$\Gamma_\lambda \sim \frac{1}{4} \lambda n m^{-2}$$

$\Gamma_\lambda(t)/H(t)$ is of order one at time $t_1$ but decreases as $t a(t)^{-1} \propto a(t)^{-1}$ afterwards, implying that cold axions briefly thermalize as a result of their $\lambda \phi^4$ interactions when they are first produced during the QCD phase transition but, after this brief period of thermalization, the axions are decoupled again.

However the axions rethermalize later as a result of their gravitational self-interactions. Their relaxation rate by gravitational interactions is of order

$$\Gamma_g \sim 4\pi G n m^2 l^2$$

where $l \sim (m \delta v)^{-1}$ is their correlation length. $\Gamma_g(t)/H(t)$ is of order $5 \cdot 10^{-7} (f/10^{12} \text{ GeV})^2$ at time $t_1$ but grows as $t a(t)^{-1} \propto a(t)$. Thus gravitational interactions cause the axions to thermalize and form a BEC when the photon temperature is of order 500 eV $(f/10^{12} \text{ GeV})^2$. Bose-Einstein condensation causes the axion correlation length to grow until it becomes of order the horizon. The growth in the correlation length causes the thermalization to accelerate; see Eq. (10). When $l$ is some fraction of $t$, $\Gamma_g(t)/H(t) \propto a(t)^{-3} t^3$, implying that thermalization occurs on ever shorter time scales compared to the Hubble time. The question is now whether axion BEC has implications for observation.

## 2 Dark matter caustics

The study of the inner caustics of galactic halos provides a useful tool. An isolated galaxy like our own accretes the dark matter particles surrounding it. Cold collisionless particles falling in and out of a gravitational potential well necessarily form an inner caustic, i.e. a surface of high density, which may be thought of as the envelope of the particle trajectories near their closest approach to the center. The density diverges at caustics in the limit where the velocity dispersion of the dark matter particles vanishes. Because the accreted dark matter falls in and out of the galactic gravitational potential well many times, there is a set of inner caustics. In addition, there is a set of outer caustics, one for each outflow as it reaches its maximum radius before falling back in. We exploit the catastrophe structure and spatial distribution of the inner caustics of isolated disk galaxies.
The catastrophe structure of the inner caustics depends mainly on the angular momentum distribution of the infalling particles. There are two contrasting cases to consider. In the first case, the angular momentum distribution is characterized by 'net overall rotation'; in the second case, by irrotational flow. The archetypical example of net overall rotation is instantaneous rigid rotation on the turnaround sphere. The turnaround sphere is defined as the locus of particles which have zero radial velocity with respect to the galactic center for the first time, their outward Hubble flow having just been arrested by the gravitational pull of the galaxy. The present turnaround radius of the Milky Way is of order 2 Mpc. Net overall rotation implies that the velocity field has a curl, \( \vec{\nabla} \times \vec{v} \neq 0 \). The corresponding inner caustic is a closed tube whose cross-section is a section of the elliptic umbilic \( (D_4) \) catastrophe. We call it a 'caustic ring', or 'tricusp ring' in reference to its shape. In the case of irrotational flow, \( \vec{\nabla} \times \vec{v} = 0 \), the inner caustic has a tent-like structure quite distinct from a caustic ring. Both types of inner caustic are described in detail in ref.

If a galactic halo has net overall rotation and its time evolution is self-similar, the radii of its caustic rings are predicted in terms of a single parameter, called \( j_{\text{max}} \). Self-similarity means that the entire phase space structure of the halo is time independent except for a rescaling of all distances by \( R(t) \), all velocities by \( R(t)/t \) and all densities by \( 1/t^2 \). For definiteness, \( R(t) \) is taken to be the turnaround radius at time \( t \). If the initial overdensity around which the halo forms has a power law profile

\[
\delta M_i / M_i \propto (1/M_i)^{\epsilon},
\]

where \( M_i \) and \( \delta M_i \) are respectively the mass and excess mass within an initial radius \( r_i \), then its subsequent evolution is self-similar with \( R(t) \propto t^{3/2} \). In an average sense, \( \epsilon \) is related to the slope of the evolved power spectrum of density perturbations on galaxy scales. The observed power spectrum implies that \( \epsilon \) is in the range 0.25 to 0.35. The prediction for the caustic ring radii is

\[
a_n \approx \frac{40 \text{kpc}}{n} \left( \frac{v_{\text{rot}}}{220 \text{ km/s}} \right) \left( j_{\text{max}} \right) \left( \frac{0.18}{\epsilon} \right) \]

where \( v_{\text{rot}} \) is the galactic rotation velocity. Eq. (12) is for \( \epsilon = 0.3 \). The \( a_n \) have a small \( \epsilon \) dependence. However, the \( a_n \propto 1/n \) approximate behavior holds for all \( \epsilon \) in the range 0.25 and 0.35.

Observational evidence for caustic rings of dark matter with the radii predicted by Eq. (12) was found in: the statistical distribution of bumps in a set of 32 extended and well-measured galactic rotation curves, the distribution of bumps in the rotation curve of the Milky Way, the appearance of a triangular feature in the IRAS map of the Milky Way in the precise direction tangent to the nearest caustic ring, and the existence of a ring of stars at the location of the second \( (n = 2) \) caustic ring in the Milky Way. The observational evidence for caustic rings of dark matter is summarized in ref. The recent improved measurement of the rotation curve of our nearest large neighbor, the Andromeda galaxy, provides new evidence. The new rotation curve shows three prominent bumps at radii 10 kpc, 15 kpc and 29 kpc, whose ratios accord with Eq. (12).

To reproduce the evidence for caustic rings of dark matter, the specific angular momentum distribution on the turnaround sphere should be given by

\[
\tilde{\ell} (\hat{n}, t) = j_{\text{max}} \hat{n} \times (\hat{z} \times \hat{n}) \frac{R(t)^2}{t}
\]

where \( \hat{n} \) is the unit vector pointing to a position on the turnaround sphere, \( \hat{z} \) is the axis of rotation and \( j_{\text{max}} \) is the parameter that appears in Eq. (12). Eq. (13) states that the turnaround sphere...
at time $t$ rotates with angular velocity $\omega = \dot{j}_{\text{max}} \hat{z}$. Each property of the angular momentum distribution (13) maps onto an observable property of the inner caustics: net overall rotation causes the inner caustics to be rings, the value of $j_{\text{max}}$ determines their overall size, and the time dependence given in Eq. (13) is responsible for $a_n \propto 1/n$. We now show that each of these three properties follows from the assumption that the infalling dark matter is a rethermalizing axion BEC.

3 Three successes

3.1 Magnitude of angular momentum

We make the standard assumption that the angular momentum of a galaxy is due to the tidal torque applied to it by nearby protogalaxies early on when density perturbations are still small and protogalaxies close to one another [25]. The amount of angular momentum galaxies typically acquire by tidal torquing can be reliably estimated by numerical simulation because it does not depend on any small feature of the mass configuration, so that the resolution of present simulations is not an issue in this case. The dimensionless angular momentum parameter

$$\lambda \equiv \frac{L |E|^{\frac{1}{2}}}{GM^2},$$

(14)

where $G$ is Newton’s gravitational constant, $L$ is the angular momentum of the galaxy, $M$ its mass and $E$ its net mechanical (kinetic plus gravitational potential) energy, was found [26] to have median value 0.05. In the caustic ring model the magnitude of angular momentum is given by $j_{\text{max}}$. The evidence for caustic rings implies that the $j_{\text{max}}$-distribution is peaked at $j_{\text{max}} \approx 0.18$. The relationship between $j_{\text{max}}$ and $\lambda$ is

$$\lambda = \sqrt{\frac{6}{5 - 3\epsilon}} \frac{8}{10 + 3\epsilon} \frac{1}{\pi} j_{\text{max}}.$$

(15)

For $j_{\text{max}} = 0.18$ and $\epsilon$ in the range 0.25 to 0.35, Eq. (15) implies $\lambda = 0.051$. The excellent agreement between $j_{\text{max}}$ and $\lambda$ gives further credence to the caustic ring model. Indeed if the evidence for caustic rings were incorrectly interpreted, there would be no reason for it to produce a value of $j_{\text{max}}$ consistent with $\lambda$.

3.2 Net overall rotation

Next we ask whether net overall rotation is an expected outcome of tidal torquing. The answer is clearly no if the dark matter is ordinary CDM. Indeed, the velocity field of ordinary CDM satisfies

$$\frac{d\vec{v}}{dt}(\vec{r}, t) = \frac{\partial \vec{v}}{\partial t}(\vec{r}, t) + (\vec{v}(\vec{r}, t) \cdot \nabla) \vec{v}(\vec{r}, t) = -\nabla \phi(\vec{r}, t)$$

(16)

where $\phi(\vec{r}, t)$ is the gravitational potential. The initial velocity field is irrotational because the expansion of the universe caused all rotational modes to decay away. Furthermore, it is easy to show [5] that if $\nabla \times \vec{v} = 0$ initially, then Eq. (16) implies $\nabla \times \vec{v} = 0$ at all later times. Since net overall rotation requires $\nabla \times \vec{v} \neq 0$, it is inconsistent with ordinary CDM, such as WIMPs or sterile neutrinos. If WIMPs or sterile neutrinos are the dark matter, the evidence for caustic rings, including the agreement between $j_{\text{max}}$ and $\lambda$ obtained above, is fortuitous.

Axions do not obey Eq. (16) because they form a rethermalizing BEC [3]. By rethermalizing we mean that the thermalization rate is larger than the Hubble rate so that the axion state tracks the lowest energy available state. The compressional (scalar) modes of the axion field are unstable and grow as for ordinary CDM, except on length scales too small to be of observational
interest. Unlike ordinary CDM, however, the rotational (vector) modes of the axion field exchange angular momentum by gravitational interaction. Most axions condense into the state of lowest energy consistent with the total angular momentum, say $\vec{L} = L \hat{z}$, acquired by tidal torquing at a given time. To find this state we may use the WKB approximation because the angular momentum quantum numbers are very large, of order $10^{20}$ for a typical galaxy. The WKB approximation maps the axion wavefunction onto a flow of classical particles with the same energy and momentum densities. It is easy to show that for given total angular momentum the lowest energy is achieved when the angular motion is rigid rotation. Rigid rotation is therefore a prediction of tidal torque theory if the dark matter is axions.

3.3 Self-similarity

Because the axion BEC rethermalization rate is larger then the Hubble rate, most axions go to the lowest energy state consistent with the total angular momentum acquired from nearby inhomogeneities by tidal torquing. The time dependence of the specific angular momentum distribution on the turnaround sphere is then predicted. Is it consistent with Eq. (13)? In particular, is the axis of rotation constant in time?

Consider a comoving sphere of radius $S(t) = Sa(t)$ centered on the protogalaxy. As before, $a(t)$ is the cosmological scale factor. $S$ is taken to be of order but smaller than half the distance to the nearest protogalaxy of comparable size, say one third of that distance. The total torque applied to the volume $V$ of the sphere is

$$\vec{\tau}(t) = \int_{V(t)} d^3r \, \delta \rho(\vec{r}, t) \, \vec{r} \times (-\vec{\nabla} \phi(\vec{r}, t))$$

where $\delta \rho(\vec{r}, t) = \rho(\vec{r}, t) - \rho_0(t)$ is the density perturbation. $\rho_0(t)$ is the unperturbed density. In the linear regime of evolution of density perturbations, the gravitational potential does not depend on time when expressed in terms of comoving coordinates, i.e. $\phi(\vec{r} = a(t)\vec{x}, t) = \phi(\vec{x})$. Moreover $\delta(\vec{r}, t) \equiv \delta \rho(\vec{r}, t)/\rho_0(t)$ has the form $\delta(\vec{r} = a(t)\vec{x}, t) = a(t)\delta(\vec{x})$. Hence

$$\vec{\tau}(t) = \rho_0(t)a(t)^4 \int_V d^3x \, \delta(\vec{x}) \, \vec{x} \times (-\vec{\nabla}_x \phi(\vec{x}))$$

(18)

Eq. (18) shows that the direction of the torque is time independent. Hence the rotation axis is time independent, as in the caustic ring model. Furthermore, since $\rho_0(t) \propto a(t)^{-3}$, $\tau(t) \propto a(t) \propto t^\frac{4}{3}$ and hence $\ell(t) \propto L(t) \propto t^\frac{5}{3}$. Since $R(t) \propto t^\frac{5}{3+\epsilon}$, tidal torque theory predicts the time dependence of Eq. (13) provided $\epsilon = 0.33$. This value of $\epsilon$ is in the range, $0.25 < \epsilon < 0.35$, predicted by the evolved spectrum of density perturbations and supported by the evidence for caustic rings. So the time dependence of the angular momentum distribution on the turnaround sphere is also consistent with the caustic ring model.

In conclusion, the phase space structure of galactic halos predicted by tidal torque theory, if the dark matter is axions, is precisely and in all respects that of the caustic ring model proposed earlier on the basis of observations. The other dark matter candidates predict a different, more chaotic phase space structure for galactic halos. Although the QCD axion is best motivated, a broader class of axion-like particles behaves in the manner described here.

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