Modeling the Migration of Neptune and the Corresponding Resonant Captures

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Abstract. Due to the angular momentum exchange with planetesimals, Neptune might have migrated outward to the current position, and captured many Kuiper belt objects (KBOs) into resonances. We set up a semi-analytic model to simulate the outward migration of Neptune, and the processes of resonant captures. Our model can naturally explain Neptune’s currently observed semi-major axis and eccentricity. The results show that the current population ratio between 3:2 and 2:1 is mainly due to the original density distribution of KBOs, which might be related to drag-induced inward migrations of proto-KBOs.

1. Introduction

The possibility of planetary adiabatic migrations due to the angular momentum exchange with planetesimals in the Solar System was proposed by Fernandez & Ip (1984). As Jupiter is the innermost gas giant planet and also the most massive one, it is very effective to scatter planetesimals outward. Thus, the Neptune, Uranus, and Saturn will have opportunities to gain angular momentum from those scattered outward planetesimals. Through the gravitational interaction with planetesimals, Jupiter made inward migration and the others, i.e. Neptune, Uranus, and Saturn made outward migrations (Fernandez & Ip 1984). In other words, Jupiter indirectly transferred angular momentum to Neptune, Uranus, and Saturn through planetesimals. However, as these planetesimals are much smaller than giant planets, the amount of transferred angular momentum between planetesimals and a planet is extremely small in each scattering event, and this kind of migration must be a slow adiabatic process.

In order to explain the orbital configuration of Pluto, Malhotra (1993) proposed that Neptune has ever migrated outward and showed that during this migration the mean motion resonances swept through the planetesimal disc and capture Pluto into the 3:2 resonance, leading to an eccentric orbit. Much more details of the above theory have been provided in Malhotra (1995). For example,
explici tly, an exponential orbital migration was suggested as

\[ a(t) = a_f - \Delta a \exp(-t/\tau), \]  

(1)

where \(a(t)\) is Neptune’s semi-major axis at a given time \(t\), \(a_f\) is Neptune’s final semi-major axis, \(\Delta a\) is total change of semi-major axis during migration, and \(\tau\) is the migration timescale. Through that assumption on Neptune’s orbital migration, Malhotra (1995) successfully showed that many particles in the trans-Neptunian region could be captured into resonances. For an orbital migration following Eq.(1), it will be called as Malhotra Migration hereafter in this paper.

Note that Malhotra (1995) also described the heuristic picture that how Neptune could gain angular momentum from planetesimals. The details might be slightly different from what we just mentioned above, but the principle is the same, i.e. Jupiter is very effective in scattering planetesimals outward, and thus Jupiter provides the angular momentum to Neptune indirectly through planetesimals.

Zhou et al.(2002) added a stochastic term on the above exponential orbital migration, and studied the effect of this term on the outcome of resonant captures. They successfully showed that the simulation results can be more consistent with the observational ones, in the sense that less particles are captured into the 2:1 resonance than into the 3:2 resonance for some particular values of the coefficient of this stochastic term. For this model of Neptune’s migration, it is called as Zhou Migration hereafter in this paper.

However, the orbital eccentricity of Neptune is not mentioned in the above two models. In both of the models, the migration was attained by acting an artificial force on the planet and the force was chosen in such a way that the orbital eccentricity of planet is not affected. Thus Neptune holds current eccentricity during the migration. The orbital eccentricity shall be an important parameter during the outward migration. In fact, Neptune does have an eccentricity about 0.00858. It would be a good question whether this eccentricity had affected the results of resonant capture or not. It is even more interesting to study whether this eccentricity can be reproduced or not during Neptune’s outward migration in a good simulation model.

In order to be more realistic, Gomes et al. (2004) employed about 10000 equal mass particles to represent planetesimals, which interact with Neptune and push Neptune to do the outward migration. They simulated and studied the details of Neptune’s outward migration for both high mass (about 100 to 200 \(M_\oplus\)) and low mass (from 40 to 50 \(M_\oplus\)) planetesimal discs. The exponential behavior of outward migrations is successfully produced in these simulations. Recently Li et al (2011) improve this model by including the self-gravitation of the planetesimals and the similar results are obtained. For a model like this, it will be called as Gomes Migration hereafter in this paper.

The main problem of Gomes Migration is that each planetesimal particle’s mass in their simulations are unrealistically larger than the real ones. Moreover, the real planetesimals would not have equal masses, but approximately have masses varying as a power law. We shall note that due to the limitation of computing capacity, it is difficult nowadays to reproduce precisely the interactions between a planet with numerous planetesimals with different sizes and masses, like what happens in the real disc.
On the other hand, in Zhou Migration model, they argued that the stochastic behavior in Neptune’s migration had decreased the capture efficiency of the 2:1 resonance. Such a scenario was confirmed by Levison & Morbidelli (2003). However, it is not exactly known whether this “jumpy” migration, as apparently observed in the numerical simulations by Gomes (2004) and Li et al. (2011), is a physically real behavior or it is just the artificial effects arising from the unrealistic large planetesimals applied in the simulations. In a theoretical model, the parameter such as the maximum radius of the scattering planetesimal, depends on the details of the disc status in the early stage of the Solar System. We will leave these details to be addressed in Jiang et al. (2012).

The above three migration models give very good implications on the possible dynamical evolution of Neptune and other related members in that region. Both Zhou Migration and Gomes Migration are valuable methods to improve the theoretical modeling. In order to find a physically meaningful way to model close encounters without the trouble of massive n-body numerical simulations of real particles, we here develop a semi-analytic model to simulate Neptune’s outward migration. The driving mechanism is the angular momentum exchange through encounters between Neptune and planetesimals, which is the same as previous models. We use a semi-analytic method to treat the encounters, so that the related physical parameters can be meaningfully and reasonably controlled in an easy way. We will investigate how the migration will be for different sizes or distributions of planetesimals. Moreover, Neptune’s initial orbital eccentricity is set to be zero and it will evolve naturally. Whether the final eccentricity is close to the current observed value will be a criterion to determine favored models. Further, the outcome of resonant captures through outward migration of Neptune in our semi-analytic models will also be investigated.

We would present our model in §2. The results and conclusions are in §3.

2. The Model
The general picture is that Neptune moves inside a sea of planetesimals at the outer Solar System. The encounters between Neptune and planetesimals occur along any possible relative directions randomly. Statistically, there are more scattered out-going planetesimals than inward ones, so the net effect is that Neptune gains angular momentum. Thus, encounters in our model are assumed to be along Neptune’s orbit and to increase Neptune’s angular momentum.

We integrate Neptune’s orbit numerically, subject to the Sun’s gravitational force and the kick force from planetesimals during encounters. Planetesimals are not modeled dynamically. They are considered as a reservoir of particles with fixed size and density distributions. Through a Monte Carlo process, Neptune gets kicked by planetesimals occasionally. The probability of encounters with a particular kind of planetesimals will depend on the assumed size distribution. Moreover, the encounter probability is certainly smaller when Neptune moves to outer region where the number density of planetesimals is smaller.

Many physical quantities of this planetesimal disc could influence the overall outcome of random encounters. For example, the size distribution of planetesimals, the density distribution of the planetesimal disc, and velocity distributions of planetesimals.

Naturally, the possible close encounters between Neptune and planetesimals
could occur along any relative directions in a three-dimensional space. However, because Jupiter scatters out many planetesimals effectively, as a net effect, Neptune will gain angular momentum from these out-going planetesimals. This process is due to a statistical average of the inward-outward unbalance of planetesimals. In order to simplify the model, we assume that the net effect of close encounters is dominated by the tangential close approach along Neptune’s orbit. In such an statistical unbalance, the more out-going planetesimals than inward ones, the more tangential close approach shall occur in the model. Thus, it is this inward-outward unbalance to increase Neptune’s angular momentum.

3. The Results and Concluding Remarks
A semi-analytic model is employed to investigate the possible outward migrations of Neptune. The driving mechanism for this outward migration is the angular momentum exchange between planetesimals and Neptune. Thus, the planetesimal sizes, velocities, and inward-outward flux differences play important roles for Neptune’s migration histories. Different from the previous work, we are able to link the exponential migration path with the physical parameters of planetesimals in a clear picture. The evolution of Neptune’s orbital eccentricity is well addressed and a final value close to the current observed one is obtained.

With the physically meaningful exponential paths of outward migrations, the 3:2 and 2:1 resonant captures by Neptune are studied. We find that for any exponential outward migrations, the probabilities of resonant capture into 3:2 and 2:1 are the same. When proto-KBOs are uniformly distributed in $R$ space initially, we get results which are consistent with the results in Hahn and Malhotra (2005): more proto-KBOs are captured into 2:1 than 3:2 resonance. However, in contrast, when proto-KBOs distribute as a decay function of $R$, say $1/R^2$ or $1/R$, there are more proto-KBOs captured into 3:2 than 2:1 resonances. This is more consistent, though still different, with observations. According to data on the IAU Minor Planets Center (http://www.minorplanetcenter.net/iau/lists/TNOs.html), the number of KBOs trapped in the 3:2 resonance is over ten times more than the number in the 2:1 resonance. The differences between simulations and observations are huge, so the problem remains unsolved. It is likely that the current population ratio between 3:2 and 2:1 is mainly due to the original density distribution of KBOs, which might be related to drag-induced inward migrations of proto-KBOs proposed in Jiang & Yeh (2004, 2007).

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