Sterile Neutrinos as the Warm Dark Matter in the Type II Seesaw Model

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Abstract

In the framework of type II seesaw mechanism we discuss the number of sterile right-handed Majorana neutrinos being the warm dark matter (WDM). When the type II seesaw mass term $M_{\nu}^{II}$ is far less than the type I seesaw mass term $M_{\nu}^{I}$, only one of three sterile neutrinos may be the WDM particle. On the contrary, the WDM particles may contain all sterile neutrinos. If $M_{\nu}^{II} \sim M_{\nu}^{I}$, the allowed number is not more than $N - 1$ for $N$ sterile neutrinos. It is worthwhile to stress that three different types of neutrino mass spectrum are permitted when $M_{\nu}^{II} \gg M_{\nu}^{I}$ and $M_{\nu}^{II} \sim M_{\nu}^{I}$.

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I. INTRODUCTION

Recent solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. The ordinary type I seesaw mechanism [5] gives a very simple and appealing explanation of the smallness of left-handed neutrino masses – it is attributed to the largeness of right-handed neutrino masses. On the other hand, recent cosmological observations have provided convincing evidence in favor of the existence of Dark Matter (DM) [6]. To clarify the identity of the DM remains a prime open problem in particle physics and cosmology. The idea that right-handed Majorana neutrinos may be the Warm Dark Matter (WDM) has been investigated in detail [7]. It has been shown that a sterile neutrino with the mass of a few keV appears to be a viable warm dark matter candidate in the $\nu$ Minimal Standard Model ($\nu$MSM) [8]. This model is very interesting since it can also explain neutrino oscillations, baryon asymmetry [9], inflation [10], the observed velocities of pulsars [11] and the early reionization [12]. In the $\nu$MSM, only one of three right-handed sterile neutrinos can be the WDM particle.

If there is an additional SU(2)$_L$ Higgs triplet, one can derive the so-called type II seesaw mechanism [13]. The neutrino mass matrix $M_\nu$ is composed of two parts. Hence we may relax the WDM constraints on the parameters of the $\nu$MSM. In this note, we shall investigate the WDM in the type II seesaw model. The remaining part of this paper is organized as follows. In Section II, we briefly describe the main features of the type II seesaw model. In Section III, constrains from various cosmological observations are shown for the WDM. In Section IV, we shall discuss the number of sterile neutrinos being the WDM in detail. Finally the summary are given in Section V.

II. THE TYPE II SEESAW MODEL

In the type II seesaw model, the Lagrangian relevant for neutrino masses reads [13, 14]:

$$-\mathcal{L} = \frac{1}{2} N_R M_R N_R + M^2_\Delta \text{Tr}(\Delta^\dagger_L \Delta_L) + \bar{\psi}_L Y_{\nu} N_R H$$

$$+ \psi_L^c Y_{\Delta} i\tau_2 \Delta_L \psi_L - \mu H^T i\tau_2 \Delta_L H + h.c.$$

or (1),

where $\psi_L = (\nu_L, h_L)^T$ and $H = (H^0, H^-)^T$ denote the left-handed lepton doublet and the Higgs-boson weak isodoublet respectively, $N_R$ stands for the sterile right-handed Majorana
neutrino singlets, and
\[
\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta^+ & \Delta^+ \\ \Delta^0 & -\frac{1}{\sqrt{2}} \Delta^+ \end{pmatrix}
\]
(2)
is the SU(2)_L Higgs triplet. After spontaneous gauge symmetry breaking, we have
\[
-\mathcal{L} = \frac{1}{2} \left( \bar{\nu}_L, N_R \right) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c_L \\ N_R \end{pmatrix} + h.c.
\]
(3)
Then one may obtain the effective (light and left-handed) neutrino mass matrix \( M_\nu \) via the type II seesaw mechanism:
\[
M_\nu = M_L - M_D M_R^{-1} M_D^T = M_{II}^\nu + M_L
\]
(4)
where \( M_D \equiv Y_\nu \langle H \rangle \) with \( \langle H \rangle = v \approx 174 \text{GeV} \) and \( M_L \equiv 2 Y_\Delta \langle \Delta^0 \rangle \) with \( \langle \Delta^0 \rangle \approx \mu^* v^2 / M_\Delta^2 \).
\( M_L^I = -M_D M_R^{-1} M_D^T \) is the ordinary type I seesaw mass term, and the type II seesaw mass term \( M_{II}^\nu = M_L \) arises from the additional Higgs triplet vacuum expectation value.
Without loss of generality, both \( M_R \) and the charged lepton mass matrix \( M_l \) can be taken to be diagonal, real and positive; i.e., \( M_R = \text{Diag}\{M_1, M_2, M_3\} \) with \( M_1 \leq M_2 \leq M_3 \) and \( M_l = \text{Diag}\{m_e, m_\mu, m_\tau\} \). The flavor eigenstates \( \nu_L \) can be expressed as \( \nu_L = K \hat{\nu}_L + R \hat{N}_R^c \), where \( R \approx M_D M_R^{-1} \) and \( K \) is an approximate unitary matrix. \( N = \hat{N}_R + \hat{N}_R^c \) and \( \nu = \hat{\nu}_L + \hat{\nu}_L^c \) are the heavy and light Majorana neutrino mass eigenstates, respectively.

The type I seesaw mass term \( M_L^I \) can be diagonalized as follow:
\[
M_{\nu}^{I_{\text{diag}}} = \text{Diag}\{\bar{m}_1, \bar{m}_2, \bar{m}_3\} = -U^\dagger M_D M_R^{-1} M_D^T U^* = -[S_1 + S_2 + S_3]
\]
(5)
where \( S_I \) denotes a contribution from each sterile neutrino \( N_I \) and is given by \( (S_I)_{ij} = X_{iI} X_{jI} \) with \( X_{iI} = (U^\dagger M_D)_{ii} / \sqrt{M_i} \). When \( M_L^{II} = 0 \), \( M_{\nu}^{I_{\text{diag}}} \) and \( U \) are the diagonal neutrino mass matrix and the Maki-Nakagawa-Sakata (MNS) lepton flavor mixing matrix \([15]\), respectively.
Since \( \text{Det}[M_{\nu}^{I_{\text{diag}}} + S_I] = 0 \), one may arrive at
\[
\bar{m}_1 \bar{m}_2 X_{3i}^2 + \bar{m}_1 \bar{m}_3 X_{2i}^2 + \bar{m}_2 \bar{m}_3 X_{1i}^2 + \bar{m}_1 \bar{m}_2 \bar{m}_3 = 0.
\]
(6)
By taking the trace of both sides in Eq.(5), we find that
\[
\bar{m}_1 + \bar{m}_2 + \bar{m}_3 = \left| -\sum_{i=1}^3 (X_{i1}^2 + X_{i2}^2 + X_{i3}^2) \right| \leq \sum_{i=1}^3 (|X_{i1}|^2 + |X_{i2}|^2 + |X_{i3}|^2).
\]
(7)
III. CONSTRAINTS ON THE WARM DARK MATTER

In this scenario, there are not any stable particles. When the active-sterile mixing matrix \( R \) is sufficiently small, the lifetime of sterile neutrinos \( (M_I \ll m_e) \) will exceed the age of the universe. These sterile neutrinos may be the WDM particles. The production mechanism of sterile neutrinos is due to the active-sterile neutrino oscillations \([7]\). In terms of the correct dark matter density, one can derive \([8]\)

\[
\sum_I \sum_{i=1,2,3} |M_{DiI}| = m_0^2 ,
\]

where \( m_0 = \mathcal{O}(0.1) \text{eV} \) and the summation of \( I \) is taken over the sterile neutrino \( N_I \) being dark matter. The above equation can be reexpressed as

\[
\sum_I \sum_{i=1,2,3} \frac{M_I}{M_1} |X_{iI}| = \frac{m_0^2}{M_1} \equiv m^{dm}_\nu .
\]

The sterile neutrino masses \( M_I \) can receive constraints from various cosmological observations and the possible mass range is very restricted as \([16]\)

\[
0.3 \text{ keV} < M_I < 3.5 \text{ keV} ,
\]

where the lower bound is given by the Tremaine-Gunn bound \([17]\), while the upper bound is given by the radiative decays of sterile neutrinos in dark matter halos limited by X-ray observations \([16]\). The stronger constraint coming from the Lyman-\( \alpha \) observations \([18]\) is \( M_I \geq 10 \text{ keV} \) which is inconsistent with Eq.(10). Therefore, if the Lyman-\( \alpha \) constraint is taken for granted, the production of sterile neutrinos due to active-sterile neutrino transitions happens to be too small to account for observed abundance of dark matter. In other words, physics beyond our model is likely to be required to produce dark matter sterile neutrinos \([10, 11]\). Another option is to assume that the universe contained relatively large lepton asymmetries \([19]\).

IV. WARM DARK MATTER IN THE TYPE II SEESAW MODEL

In the type I seesaw model, only one of three right-handed neutrinos can be the WDM particle \([8]\). Since the neutrino mass matrix \( M_\nu \) contains two parts of contributions in our scenario, interesting results can be obtained. In the following parts, we shall discuss the
number of sterile neutrinos being the WDM in terms of Eqs.(9) and (10). When \( M_{\nu}^{II} \ll M_{\nu}^{I} \), we can derive the same conclusions as in Ref.[8]. If \( M_{\nu}^{II} \gg M_{\nu}^{I} \), it is obvious that all right-handed neutrinos may be the WDM particles.

In this section, we shall analyze the \( M_{\nu}^{II} \sim M_{\nu}^{I} \) case in detail. It is worthwhile to stress that all three kinds of neutrino mass spectrum (Normal hierarchy, Inverted hierarchy and Degenerate) are permitted when \( M_{\nu}^{II} \gg M_{\nu}^{I} \) and \( M_{\nu}^{II} \sim M_{\nu}^{I} \). Moreover, one can derive the same consequences for different neutrino mass spectrum. For illustration, we assume that the neutrino mass spectrum is the normal hierarchy case, i.e., \( m_1 < m_2 \ll m_3 \) with \( m_3 \approx \sqrt{\Delta m_{\text{atm}}^2} = 0.05 \text{ eV} \) [20]. When \( M_{\nu}^{II} \sim M_{\nu}^{I} \), one can directly obtain \( \bar{m}_3 \sim m_3 \) where we have assumed \( \bar{m}_1 \leq \bar{m}_2 \leq \bar{m}_3 \). \( \bar{m}_1 \) and \( \bar{m}_2 \) may be equal to zero.

If there are only two heavy Majorana neutrinos \( N_1 \) and \( N_2 \), \( \bar{m}_1 = 0 \) holds [21]. When both sterile neutrinos are assumed to be the dark matter, Eq.(7) becomes

\[
\bar{m}_2 + \bar{m}_3 \leq \sum_{i=1}^{3} (|X_{i1}|^2 + |X_{i2}|^2) \leq m_{\nu}^{dm} ,
\]

where we have used Eq.(9). The above inequality can not be satisfied since \( \bar{m}_3 \sim 0.05 \text{ eV} \) and \( m_{\nu}^{dm} \sim 10^{-5} \text{ eV} \).

When only one of two sterile neutrinos, say \( N_1 \), is assumed to be the dark matter, one can directly derive \( |X_{11}|^2 + |X_{21}|^2 + |X_{31}|^2 = m_{\nu}^{dm} \) from Eq.(9). Since \( \bar{m}_1 = 0 \), Eq.(6) induces \( X_{11}^2 \bar{m}_2 \bar{m}_3 = X_{12}^2 \bar{m}_2 \bar{m}_3 = 0 \) which implies \( \bar{m}_2 = 0 \) and (or) \( X_{11} = X_{12} = 0 \). For the \( \bar{m}_2 = 0 \) case, one may deduce

\[
M_{D,i1} \propto M_{D,i2} (i = 1,2,3)
\]

from Eq.(5). When \( X_{11} = X_{12} = 0 \), the first row and column of \( S_1 \) and \( S_2 \) vanish. Then Eq.(5) is reduced to that for \( 2 \times 2 \) matrices:

\[
\text{Diag}\{\bar{m}_2, \bar{m}_3\} + X_{i1}X_{j1} = -X_{i2}X_{j2} (i, j = 2,3) .
\]

The vanishing determinant leads to \( \bar{m}_2 \bar{m}_3 + \bar{m}_2 X_{31}^2 + \bar{m}_3 X_{21}^2 = 0 \). The upper bound of \( \bar{m}_2 \) turn out to be

\[
\bar{m}_2 = \left| X_{21}^2 + \frac{\bar{m}_2}{\bar{m}_3} X_{31}^2 \right| \leq |X_{21}|^2 + |X_{31}|^2 \leq m_{\nu}^{dm} .
\]

Therefore, one of two sterile neutrinos may be the dark matter in the type II seesaw model.
Now, let us discuss the case including three heavy neutrinos. It is obvious that all three sterile neutrinos cannot simultaneously be the WDM from Eqs. (7) and (9). If two of three sterile neutrinos, say $N_1$ and $N_2$, are the WDM particles. Making use of the $(2,3)$ block of Eq. (5), we have

$$\left|(\bar{m}_2 + X_{21}^2 + X_{22}^2)\bar{m}_3\right| \approx |(X_{21}X_{31} + X_{22}X_{32})|^2 \leq \frac{1}{4} m_{\nu}^{dm}.$$  

Hence we can derive $\bar{m}_2 \lesssim m_{\nu}^{dm}$ since $|X_{21}|^2 + |X_{22}|^2 \leq m_{\nu}^{dm}$ and $\bar{m}_3 \sim 0.05 \text{eV}$. When $\bar{m}_1 = \bar{m}_2 = 0$, one may also arrive at

$$M_{D_{i1}} \propto M_{D_{i2}} \propto M_{D_{i3}} \ (i = 1, 2, 3).$$  

For the $\bar{m}_2 \neq 0$ case, we can obtain

$$\bar{m}_1 = \left|X_{11}^2 + \frac{\bar{m}_1}{\bar{m}_2} X_{21}^2 + \frac{\bar{m}_1}{\bar{m}_3} X_{31}^2\right| \leq |X_{11}|^2 + |X_{21}|^2 + |X_{31}|^2 \leq m_{\nu}^{dm}$$  

with the help of Eqs. (6) and (9).

Finally, we consider the remaining possibility that only one sterile neutrino, say $N_1$, plays a dark matter particle. For the $\bar{m}_1 = \bar{m}_2 = 0$ case, the Eq. (16) can also be obtained. When $\bar{m}_1 = 0$, Eq. (6) induces $X_{11} = X_{12} = X_{13} = 0$. If $\bar{m}_1 \neq \bar{m}_2 \neq 0$, we can derive the same conclusion as in Ref. [8]: $\bar{m}_1 \leq m_{\nu}^{dm}$.

V. SUMMARY

We have analyzed the number of sterile neutrinos that can explain the warm dark matter in the type II seesaw model. When $M_{\nu}^{II} \ll M_{\nu}^I$, only one of three right-handed sterile neutrinos may be the WDM particle [8]. If $M_{\nu}^{II} \gg M_{\nu}^I$, the WDM particles may contain all sterile neutrinos. In this note, the $M_{\nu}^{II} \sim M_{\nu}^I$ case is detailed discussed. We find that the allowed number is not more than $N - 1$ for $N$ sterile neutrinos. It is worthwhile to stress that three different types of neutrino mass spectrum are permitted when $M_{\nu}^{II} \gg M_{\nu}^I$ and $M_{\nu}^{II} \sim M_{\nu}^I$.

Acknowledgments

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