Scaling the Number of Tasks in Continual Learning

Timothee Lesort1, Oleksiy Ostapenko1,2, Diganta Misra1,2, Md Rifat Arefin1,2, Pau Rodríguez3
Laurent Charlin1,4,5, Irina Rish1,2

1Mila - Quebec AI Institute, 2Université de Montréal, 3ServiceNow, 4HEC Montréal, 5Canada CIFAR AI Chair

Abstract

Standard gradient descent algorithms applied to sequences of tasks are known to produce catastrophic forgetting in deep neural networks. When trained on a new task in a sequence, the model updates its parameters on the current task, forgetting past knowledge. This article explores scenarios where we scale the number of tasks in a finite environment. Those scenarios are composed of a long sequence of tasks with reoccurring data. We show that in such setting, stochastic gradient descent can learn, progress, and converge to a solution that according to existing literature needs a continual learning algorithm. In other words, we show that the model performs knowledge retention and accumulation without specific memorization mechanisms. We propose a new experimentation framework, SCoLe (Scaling Continual Learning), to study the knowledge retention and accumulation of algorithms in potentially infinite sequences of tasks. To explore this setting, we performed a large number of experiments on sequences of 1,000 tasks to better understand this new family of settings. We also propose a slight modifications to the vanilla stochastic gradient descent to facilitate continual learning in this setting. The SCoLe framework represents a good simulation of practical training environments with reoccurring situations and allows the study of convergence behavior in long sequences. Our experiments show that previous results on short scenarios cannot always be extrapolated to longer scenarios.

1 Introduction

Continual learning (CL) aims to design algorithms that can learn from non-stationary sequences of tasks. Thereby, each task is usually seen only once, and agents are evaluated by their average performance on all tasks [Lesort et al., 2020, Lange et al., 2019, Belouadah et al., 2021]. While such protocol is useful to evaluate the agents’ ability to retain knowledge, those benchmarks are not made to evaluate the agents’ capabilities to learn through long sequences with re-occurring data.

This type of scenario is relevant for many practical settings. One example is an autonomous robot working in a factory on a classification task for object manipulation. The robot can witness data distribution shifts through time depending on various context factors. In this scenario, the same data can re-occur under various contexts, at fix frequency or randomly. It would then be interesting to design an efficient way to learn continually in such a setting, accumulate knowledge and converge to good accuracy for classifying all objects. Existing continual learning settings, such as class-incremental or domain incremental scenarios [van de Ven and Tolias, 2019, Douillard and Lesort, 2021, Normandin et al., 2021], do evaluate catastrophic forgetting in short sequences of unique tasks but they do not evaluate knowledge accumulation in the long term.

Inspired by this, we propose SCoLe (Scaling Continual Learning), an evaluation framework for CL with a potentially unlimited number of tasks. As visualized in Fig.1 the general idea is to create each new task by gathering the data from a randomly selected subset of all classes. A model is trained for some epochs on this data, until task switch. Each task is created independently from the others.
In this paper, we show a new phenomenon that emerges from such scenario: knowledge accumulation in deep neural networks overcomes catastrophic forgetting. That is, the model learns faster than it forgets. In a long sequence of tasks, a simple optimizer such as vanilla SGD can learn continually and converge to a solution for all tasks. This is in contrast to common results from the CL literature, according to which catastrophic forgetting should prevent convergence to the optimum for all tasks in such setting. We design scenarios with 1,000 tasks, where tasks can re-occur in different contexts and with different periodicity, to study knowledge accumulation and forgetting more precisely. We propose some modifications to SGD by applying a gradient masking strategy at the single-head output layer and removing momentum to enable and improve the knowledge accumulation and retention. This modification helps prevent modification of weights in the last layer that are not related to the current tasks and avoid interference of gradient when tasks change.

Figure 1: Illustration of SCoLe scenario. With 4 classes in total (one per color) and 2 classes per tasks. The data are selected randomly based on their label to build the scenario dynamically, into a potential infinite sequence.

In summary, our contributions are as follows.

- We propose a new experimentation framework SCoLe based on the finite world assumption [Mundt et al. 2020] with a potentially infinite number of tasks. SCoLe scenarios are built to estimate the convergence of learning to a global solution from sequential partial access to the data.
- We show that in such scenarios standard gradient descent optimizers retain and accumulate knowledge without any continual-learning algorithm, i.e. without supplementary memorization mechanism. This result is counter-intuitive given the well know “catastrophic forgetting” in neural networks phenomenon in sequence of tasks.
- We propose to use a simple masking strategy and remove momentum in SGD to boost the continual learning ability of SGD in long sequences of tasks.
- We study the capabilities of such training and the limitations on a wide variety of datasets (MNIST, Fashion-MNIST, KMNIST, CIFAR10, CIFAR100) and scenarios.

The paper is organized as follows: First, in Sec. 2, we present the details of the proposed framework. Then, in Sec. 3 we show how to improve knowledge accumulation by modifying stochastic gradient descent (SGD) algorithm through a set of simple experiments. In Sec. 4, we study how knowledge accumulation behaves depending on the task complexity. In Sec. 5, we study how models learn if the level of non-stationarity in the training stream is increased. Finally, we introduce related works in Sec. 6, discuss our findings in Sec. 7 and conclude in Sec. 8.

2 SCoLe: A General Framework for Long-Term Continual Learning

**General Idea:** We propose a framework that allows the creation of an arbitrarily long sequence of tasks with controlled data distribution shifts. The setting is motivated by the finite-world assumption [Mundt et al. 2020]. This assumption hypothesizes that the world has a finite set of states. Therefore, in a long enough period of time, all states will have been seen and next states will not be new anymore. During each task, the agent has access to a subset of world data to learn from. At each time step, we evaluate the agent’s ability to understand the whole world. Hence, after seeing a long sequence of small tasks, the agent should be able to accumulate knowledge in order to solve a larger and more challenging problem than each of the seen tasks in isolation. The implementation code for the default scenario is provided in the appendix using the continuum library [Douillard and Lesort 2021].

**Framework:** We instantiate this idea in a classification setting as illustrated in Fig. 1. We start from a classification dataset composed of a train and a test set. At each task, a subset of classes are
randomly selected. The agent learns to classify on this subset only and is tested on the full test set with all classes. The framework considers scenarios with varying numbers of tasks $T$, classes per task $N_t$, number of epochs per task, and long-term distribution shifts (cf. Sec. 5). This versatility and adaptability makes SCoLe convenient for studying various phenomena and assessing them as we will show in the experiments.

By default in our experiments, the subset of classes $Y_t$ for a task $t$ is composed of only two classes and the training lasts only one epoch per task. At task $t$, $(x, y)$ data points are sampled from $p(X,Y|T=t_k) = p(X|Y)p(Y|C=t)$. $C$ is the context variable, i.e. the task index here. $p(Y|C=t)$ samples the classes given the task index, by default $p(Y|C=t)$ is uniform among the subset of classes $Y_t$.

We note that, in contrast to e.g. OSAKA/Caccia et al. [2020], the label $y$ of a given $x$ is fixed through time: i.e. $p(Y|X)$ is fixed.

At task $t$, the training objective is: $\theta^*_t = \operatorname{argmin}_\theta \ell_{(x,y)} \in D_t \{f(x; \theta), y\}$. $f(\cdot; \theta)$ is the function realized by the neural network parameterized by $\theta$ and $\ell(\cdot)$ is the cross-entropy loss function. However, the goal is to learn $\theta^*$ that minimizes the loss on the global test set $D_{\text{test}}$, i.e. $\theta^* = \operatorname{argmin}_\theta \ell_{(x,y)} \in D_{\text{test}} \{f(x; \theta), y\}$. $D_{\text{test}}$ represents the full distribution while $D_t$ represents a subset of the distribution.

Goal: The goal of the SCoLe framework is to emulate long sequence of tasks in finite world. It makes it possible to evaluate forgetting but also the knowledge retention and accumulation in a deep neural network of algorithms over long sequences of tasks. If knowledge retention and accumulation are significant enough, they can overcome catastrophic forgetting in DNN and enable performance increase and convergence to a solution of the full scenario. This capability was not previously evaluate in the existing literature (cf. Sec. 6).

With SCoLe, we will show that a simple version of SGD already enables knowledge retention and accumulation in deep neural networks despite catastrophic forgetting. Catastrophic forgetting is expected to prevent continual learning when no specific memorization mechanisms is added by erasing all previously learned knowledge not present in the current task. With SCoLe, we show that some knowledge is retained and accumulated in the model, which, after many tasks, leads the model to converge to a solution that covers the whole scenario test set.

### 2.1 Relationship with IID training

As in most standard CL scenarios, the data in SCoLe is locally identically and independently distributed (IID). However, at the stream level, training distribution is non-IID due to shifting tasks.

By default, the classes for a given task are uniformly sampled over all classes. Still, the overall framework differs from IID training in two ways. First, the batches are sampled only from classes of the current task rather than of the whole dataset. Second, consecutive batches contain the same subset of classes until the task changes. Thus, unlike IID training, the sampling distribution in SCoLe changes through time and depends on the current task.

Formally, in IID training, at each time-step we optimize directly $\ell(f(x; \theta), y)$ with $(x, y) \sim p(X, Y)$. While in SCoLe, we do not have access to the full distribution at once, and, at task $t$, we train from $(x, y) \sim p(X|Y)p(Y|C=t) \neq p(X, Y)$.

In extreme cases, for $N$ classes per task, if the whole task fits in one batch and there is only one epoch, the setting would be close to an IID training setting where the batch size is $N$. For this reason, we assume that for a fixed number of epochs per task, increasing the batch size produces a SCoLe scenario that is closer to an IID training. On the contrary, with fixed batch size, increasing the number of epochs per task produces a scenario further away from an IID training.

### 3 Enabling Knowledge Accumulation in SCoLe

In this section, we show how in SCoLe we can learn a sequence of tasks without a continual learning algorithm such as replay, regularization, or a dynamic architecture.
3.1 Setting

Approaches: The most widespread optimizers in the CL literature are based on stochastic gradient descent (SGD) with momentum [Qian 1999] and Adam [Kingma and Ba 2014]. In this first experiment, we will use these optimizers by default. We also test SGD without momentum. Indeed, in the presence of data distribution drifts, the gradient’s inertia induced by the momentum (or Adam) produces a mixture between the gradient of the previous task and the gradient of the current task. Which can create interference in the training process. Therefore, we tested SGD without momentum to assess if it produces an impact in a SCoLe scenario. For SGD with momentum, we set the momentum to 0.9 since it is the default value in PyTorch [Paszke et al. 2019].

In addition, recent papers show that in masking the gradient in the last layer helps classifiers to learn continually [Caccia et al. 2022], [Zeno et al. 2018] even without a supplementary memorization process [Lesort et al. 2021a]. Therefore, we test whether “group masking” from Lesort et al. [2021a] could also be useful to train a model end-to-end from scratch in a SCoLe scenario. The high-level idea is that when learning in a subset of classes (without replay), we do not get any information about the other classes. Hence, the gradient is applied only to the output of the classes in the current batch. The softmax remains computed on the full output, making the gradient dependent on all class predictions. This is important because we still want the model to be aware of other classes’ predictions.

Data In this first instance of the general framework, we use three simple datasets to evaluate the capabilities of the various gradient descent options, with SGD, Adam, w/ or w/o momentum, and w/ or w/o masking. We create three scenarios on 10-way classification problems with MNIST [LeCun and Cortes 2010], Fashion [Xiao et al. 2017], and KMNIST [Clanuwat et al. 2018]. We run the experiments on a 500 tasks sequence. Each task consists of two classes that are randomly sampled from a uniform distribution over the ten classes of the corresponding dataset. Task classes are independent. The sampled classes in task $i$ do not influence the sampling of the classes in task $i + 1$. We train each task for 1 epoch.

3.2 Results

Initial Experiments: We start by evaluating the scenario with the limited number of tasks on MNIST only. We run the scenario on 20 tasks, which is comparable to the size of most continual learning scenarios, i.e., usually between 5 and 10 tasks. We train a small convolutional neural network in this experiment (details in the Appendix). We use SGD and Adam optimizers, with a default learning rate of 0.01 and a momentum of 0.9 for SGD (we kept the other hyperparameters unchanged from default values). The results in Fig. 2a show no improvement through the number of tasks, with even a performance decrease over time. This experiment can be explained by the catastrophic forgetting phenomena that occur in deep neural networks when trained on a sequence of tasks [French 1999].

Improved Baselines: We then introduce the improvements of removing momentum for SGD and adding gradient masking as discussed in previous section. Fig. 2b shows that these two small improvements result in an improvement in performance even after a few tasks. Recall that there is no other continual learning algorithm during training.
Figure 3: Test acc. with scaling of scenario’s size averaged on MNIST, Fashion-MNIST, KMNIST with 3 seeds. -- - line, represent best IID performance over the same HPs search. SGD+Masking w/t Momentum is the most stable baseline.

Scaling tasks number: In Fig. 2c, we scale the number of tasks to see at which point the accuracy does not improve further. The results show that by scaling the number of tasks, the accuracy of the model can reach an accuracy close to IID training on MNIST. This first experiment proves that catastrophic forgetting does not erase all knowledge of past tasks and that there is knowledge accumulation in the model, which at scale can lead to a good performing model.

HPs search: After the first experiments, we run a small hyperparameter search on the same scenario, with 500 tasks on MNIST, Fashion-MNIST and KMNIST. Fig. 3a shows the average performance on the three datasets with various learning rates. We find that, with a small learning rate, Adam and SGD with momentum can achieve knowledge accumulation and improve their accuracy on the test set when scaling the number of tasks. Interestingly, Fig. 3b shows that SGD without momentum and with masking is a stable baseline. Therefore, we will keep this baseline for further experimentation on the scenario with harder tasks and variations in classes distribution.

Selected Baseline: Fig. 3b shows the learning curve of the selected baseline on MNIST, Fashion-MNIST, and KMNIST with 500 tasks run with 3 seeds. We show that knowledge accumulation occurs in those simple datasets.

3.3 Conclusion

In this section, we showed that SGD without momentum and with gradient masking converges after a high number of tasks to a solution that is close to the IID training accuracy. This shows the limited effect of catastrophic forgetting and the ability of models to retain and accumulate knowledge.

A possible explanation for this behavior is that, globally, the accumulation of knowledge is faster than forgetting in deep neural networks. In classical continual learning, forgetting still happens because the last layer is determinant for prediction and more sensitive to forgetting [Wu et al. [2019], Hou et al. [2019], Zhao et al. [2020], Ramasesh et al. [2021], Bell and Lawrence [2021], Lesort et al. [2021]]. Hence, masking stabilize the last layer weights and when we scale the number of tasks with re-occurring data, knowledge retention and accumulation become visible.

4 Scaling with more complexity

In this section, we experiment with more challenging datasets than in the previous section. We evaluate the scenario to train a ResNet [He et al.] model with CIFAR10 [Krizhevsky et al.] and CIFAR100 [Krizhevsky et al.]. We also study the impact of increasing the complexity of the scenarios: First, by controlling the total number of classes in the scenario and secondly, by reducing the entropy of the class distribution, i.e. increasing the imbalance between probability of sampling classes for each task. In a third experiment, we investigate how the closeness to IID training influences algorithm’s performance.
The goal of the section is to highlight how SCoLe can be modified to create scenarios of various difficulty and to assess that knowledge retention and accumulation still happen when the difficulty of the scenario increases.

4.1 Increasing number of classes

In this experiment, we would like to assess if knowledge accumulation still occurs when we grow the number of classes.

**Setting:** We used the CIFAR100 dataset and experimented with various subsets of classes of the datasets. To create a subset of \( N \) classes, we subsample the total number of classes of the train and the test set. The higher \( N \) is, the higher the difficulty. This is for two reasons: first, because multiclass classification becomes harder with the number of classes. Secondly, given the fixed number of classes per task \( N_t \), increasing \( N \) decreases the probability of sampling each class in a task. The lower probability of sampling classes and the harder complexity of the problem to solve makes the scenario harder when \( N \) increases.

**Results:** In Fig. 4 (left), we see that increasing the number of classes in the scenario significantly increases the difficulty of the scenario. Reaching an accuracy for CIFAR100 close to IID training with 100 classes would presumably require a SCoLe scenario with a significantly larger number of tasks. Alternatively, growing the number of classes per task also helps (cf appendix Fig. 10).

As a conclusion, growing the number of classes slows down knowledge accumulation but it does not prevent it.

4.2 Lowering Class Distribution Entropy

In the previous experiments, the distribution was uniform over classes (balanced). In this experiment, we assess whether knowledge accumulation still happens when the class distribution is unbalanced.

**Setting:** For this experiment, we set the entropy of the scenario’s class distribution and plot the model test accuracy curves. Changing the entropy leads to some classes being more likely to occur than others in the scenario. The class distribution does not change over time in a given scenario. Details in implementation to control entropy and distribution for each entropy value are in appendix F.2

In our experiments, an entropy of 2.3 corresponds to the uniform distribution.

---

\(^1\)In this plot, results have been smoothed with rolling average over 20 tasks for better visual perception.
Results: In Fig. 4 (right), we see that the closer we are to a uniform class distribution for sampling tasks, the better convergence is. An explanation for this is: when the probability of sampling a class gets lower is becomes harder to learn to distinguish it from other classes. Therefore, in the full sequence of tasks, it becomes harder and longer to converge to a solution for the full scenario.

As a conclusion, class imbalance in the class distribution slow down knowledge accumulation but do not prevent it.

4.3 Getting Closer to IID Training

As discussed in Section 2.1, on a fixed number of epochs, increasing the batch size brings the training closer to IID training, while for a fixed batch size, growing the number of epochs per task makes the training further from IID. In this section, we investigate whether scenarios closer to IID training provide better performance. Growing the number of epochs or decreasing the batch size have the same impact: both increase the number of gradient steps within a task. There are, therefore, two forces that oppose each other, learning the current task and forgetting the past ones. On average, the algorithm should learn more about the current task than it forgets about the past ones to expect to be able to converge to a global solution.

Setting: In this setting, we experiment with CIFAR10 with 2 classes per task. Then, we vary (1) the batch sizes and (2) the number of epochs per task to evaluate if a minimum number of epochs with a maximum batch size (the closest to IID training) maximizes performance.

Results: Fig. 6 (bottom) shows that the decrease in batch size during one epoch, i.e., getting further away from the IID training, increases the convergence speed and final performance. We hypothesize that this result is due to a higher number of gradient steps for smaller batch size. If it was the case, experiments with different batch sizes would lead to the same accuracy but with a various number of tasks. However, in Fig. 6 (bottom) a larger batch size leads to lower accuracy, discarding the hypothesis. Hence, in this experiment moving further from IID training performs better, which is counter-intuitive. An explanation for this is that, at the beginning of a task, knowledge accumulation is superior to the knowledge degradation caused by forgetting. Therefore, training longer on each task improves final convergence. In the same way, Fig. 6 (top) shows that growing the number of epochs per tasks accelerates convergence in number tasks seen. However, if we grow the number of epochs to a certain point, convergence slows down. This figure shows that there is a trade off to find in terms of number of gradient steps to converge faster and better.

To conclude, this experiment shows that training in settings that are as close as possible to IID (low number of gradient steps per task) does not always improve the convergence speed and the performance.

4.4 Conclusion

The SCoLe framework is convenient to create scenarios of various difficulty to study knowledge accumulation in depth. In this section, we have shown that knowledge retention and accumulation still happens with increasing difficulty. Nevertheless, by increasing the difficulty of scenarios, knowledge accumulation and convergence become slower. Still, the progressive improvement through tasks never disappears completely. We also found the counter-intuitive results that creating scenarios closer to IID training does not necessarily improve convergence speed and end performance. In the next section, we evaluate how knowledge retention and forgetting happen in SCoLe scenarios.

5 Forgetting in SCoLe

In the experiments presented so-far, the class distributions to build tasks were stationary. While we can build scenario of arbitrary difficulty in this setting, it does not assess how knowledge accumulation happens when there are shifts in the class distribution. In this section, we will show how we can use the SCoLe framework to create scenario with class distribution shift and assess knowledge accumulation with long term shifts. In this experiment, we show that in the long run, the model can still forget, when long-term distribution shift happens. We evaluate the knowledge retention and the interference by designing three different shift patterns in the data distribution. We also evaluate models with increasing wideness similarly to Mirzadeh et al. [2021] to estimate if wider models are
less sensitive to distribution shifts. We use Wide ResNets [Zagoruyko and Komodakis, 2016] (WRN) with different width factors.

5.1 Knowledge retention

In this experiment, we would like to assess the capabilities of deep neural networks to maintain correct prediction in SCoLe scenarios when classes disappear from the scenario. It evaluates knowledge retention in a strict way since we evaluate prediction correctness. Knowledge retention in lower layers is not sufficient in this setting since the model also have to maintain decision boundaries in the last layer.

**Setting:** We train on the original scenario (binary classification with classes sampled uniformly) on CIFAR10. We want to evaluate how the model can memorize classes if we remove them from all tasks mid-training (after 500 tasks). Hence, we start the training with all classes, after 500 tasks we remove half the classes from the class distribution. In contrast to standard CL, where distribution shift is usually caused by adding classes, here we remove classes. In such scenario forgetting behaviour should be smooth. We tested WRN with different layer width similarly to Mirzadeh et al. [2021].

**Results:** Fig. 5 (left), shows that even if no new data is introduced to the learner – i.e. no interfere with old knowledge is possible, the model can still forget when a subset of already learned classes is no longer trained on. Interestingly, in this setup forgetting is very slow and knowledge persists during several hundreds of tasks. This means that even if some classes disappear momentary, forgetting will not be catastrophic. Moreover, we clearly observe that growing the width of the model increases knowledge retention to the point that it looks like the model perfectly memorized removed classes for the maximum model size.

5.2 Interference

Here we investigate a setting with abrupt shift in the class distribution. This allows us to investigate the interference dynamics. This setup is a mixture of class-incremental CL and SCoLe. The goal is to assess if knowledge retention of previous experiments can be useful when new classes replace previous ones.

**Setting:** First, 500 tasks are generated from the first 5 classes of CIFAR10 (first period) and the second 500 tasks are generated from the remaining 5 classes (second period). There is then no overlap between the classes in the first and the second periods. Also here we test model with various widths.

**Results:** The results in Fig. 5 (right) show that this sudden class shift creates perturbation in the training process as in classical CL scenarios. Moreover, in the second period of training, the model struggles more to learn the tasks than during the first period. It could mean that the weights learned during the first period are not a good initialization for later tasks and that the forward transfer is limited in such a regime. Similar to the knowledge retention experiments, we see that wider models deal better with the class shift. Still, this difference looks less clear, and it could be due to the better knowledge retention of first classes.

5.3 Cyclic Shifts

We introduce a cyclic shift in the class distribution in this setting. In addition to the task shifts, there is a high-level shift in the class distribution. At a given time step, the task’s classes are sampled from a predefined subset of all classes that cyclically change through time. This setting aims to assess if the long-term knowledge retention in deep neural networks witnessed in previous experiments allows knowledge accumulation when classes are not seen for a long time.

**Setting:** This subset of $W$ classes shifts progressively through time (i.e. shifting window) and classes are sampled uniformly from it. This shift is cyclic, at every $\lambda$ tasks the class subset repeats, where $\lambda$ is called cycle size. With $N$ the total number of classes, the subset shift by one class every $\frac{N}{\lambda}$ tasks. For example, if the class subset $W_0$ before is $[0, 1, 2]$, after a shift it will become $[1, 2, 3]$. After $\lambda$ tasks, it will be again $[0, 1, 2]$.

We use the full CIFAR100 dataset ($N = 100$) with 5 classes per tasks, the window size is $W = 10$, and the cycle size is $\lambda \in [50, 100, 200, 500]$. We choose

Note that if the cycle size is too low, the shift can be of more than a class per task.
CIFAR100 because it has more classes than with CIFAR10 and create longer shifts over classes. In this experiments, when a class gets out of the subset window it needs $\lambda - W$ tasks to get it back. This scenario evaluates the long term knowledge retention in deep neural networks in SCoLe. In this experiments, we also probe a KNN classifier on top of the feature extractor after each task to estimate knowledge accumulation in the feature extractor. The accuracy can give us a rough estimation of the fast adaptation capability of models trained on SCoLe. The KNN is trained on the full train set with encoded data and $K = 100$.

**Results:** The results of this experiments are displayed in Fig. 7. We can see that the period of shift makes the training harder in Fig. 6a i.e. better learning is achieved if classes re-occur more frequently. Even with large period the model still progresses systematically through time as seen in Figs. 6b and 6c. Consistently with other experiments, also here wider models result in better performance as shown in Figs. 6a and 6c. Something interesting we found through probing a KNN classifier is that the representation space learned just before the classification layer is constantly improving. As seen in Fig. 6c, even if the cyclic effect still appears, forgetting in the representation space decreases with every new cycle. This shows that this training paradigm could be well-suited for continual fast adoption. This observation correlates with the linear probing experiments from [Davari et al., 2022] realized with a multi-head model in a class incremental scenario. Overall this experiment shows that deep neural networks trained with SGD are also capable of long term retention and can still progress even if they do not see some classes for a long period of time.

**5.4 Conclusion**

Forgetting still happening in long sequences of tasks is expected. However, experiments in this section show that our models are capable of long-term knowledge retention, enabling knowledge accumulation even when data is not seen for a long time. Our results are in-line with the findings of [Mirzadeh et al., 2021]: wider models forget less in incremental scenarios. Further, we have also shown that the widest models are capable of almost perfect knowledge retention and that with long-term distribution shifts such as cyclic shifts, deep neural networks can accumulate knowledge. They can memorize and reuse knowledge from classes not seen since more than 200 hundred tasks. The forgetting phenomenon with abrupt distribution drifts of section 5.2 means that SGD with masking and no momentum should be improved for such a scenario.

**6 Related Work**

Most scenarios in the continual learning literature study catastrophic forgetting [van de Ven and Tolias, 2019]. They are composed of a sequence of tasks where each data point is presented in only one task without reoccurring in other tasks, and tasks are seen only once. This setting makes possible to evaluate if models are able to remember tasks they have seen only once. These scenarios are usually built with different data distribution shifts [Lesort et al., 2021b] such as new tasks introduce new classes (class-incremental), new domains (domain-incremental), or new labelization of data [Abdelsalam et al., 2021]. The hard constraint of no reappearance makes those scenarios efficient to evaluate catastrophic; however, they can not evaluate if knowledge accumulation happens in the
model that would lead to convergence in a long sequence of task in a finite environment. \textit{SCoLe} is built to evaluate such behaviour.

The scenario we propose has some similarity with the OSAKA \cite{Caccia2020} framework. However, in our setting, we do not evaluate fast adaptation but the capacity of learning a solution to a general problem from a long sequence of sub-problems. Moreover, there is no real-concept shift in \textit{SCoLe}, i.e., $p(y|x)$ is fixed through time. Our evaluation protocol is also similar to ALMA \cite{Caccia2021} scenario, since we evaluate on a fixed test set. However, in ALMA the data distribution is the same on all tasks, and there are no distribution drifts while in \textit{SCoLe} there are drifts between tasks. A scenario with a long sequence of tasks was proposed by \cite{Wortsman2020}. Their scenario is composed of various permutations for the permut-MNIST scenario. This scenario also makes it possible to scale the number of tasks, but it does not make it possible to highlight the progressive knowledge accumulation witnessed in \textit{SCoLe} scenarios because there is no re-occurrence of past tasks.

In \cite{Lesort2021}, the masking method, referred to as “group masking”, applies gradients only to the outputs of classes within the current batch while training. In particular, the authors showed that this masking is possible while training a linear classifier on top of a frozen pre-trained model. \cite{Caccia2022, Zeno2018} applied a similar method in continual learning respectively in end-to-end training and multi-head training and show that this method was helping mitigate forgetting.

7 Discussion

The purpose of the proposed \textit{SCoLe} framework is to assess knowledge retention and accumulation in long sequences of tasks. We have shown that it can also be used to measure forgetting in the presence of long term distribution shifts. This framework stays complementary to existing scenarios such as class-incremental and domain-incremental which are necessary for better understanding the learning behaviour in tasks that do not reoccur. In the literature, other works used indirect methods such as linear probing to witness that forgetting in deep neural networks is maybe not as catastrophic as expected \cite{Fini2022, Davari2022}. However, \textit{SCoLe} scenarios provide settings to assess this phenomenon directly and study it through different angles as we did in this article. A better understanding of knowledge accumulation should help design better continual learning approaches that can profit from it to be more efficient.

\textit{SCoLe} is also conceived to provide a practical evaluation tool for a long-term continual learning setting, where knowledge accumulation becomes useful in the long run. The proposed SGD with gradient masking is a very light approach that could serve an embodied agent with restricted compute or memory. Note that with replay, the \textit{SCoLe} scenarios could be quite straight-forward to solve. However, the training would be considerably slower and more compute intensive. Moreover, replay or regularization need a lot of practical decision on hyper-parameters selection, validation methods, data selections, etc., while SGD with masking only requires the learning rate to be set. Finally, while this paper investigates how stochastic gradient descent can learn continually in \textit{SCoLe} scenario, a comparison with continual learning approaches in terms of performance, memory and learning speed would be an interesting direction for future research.

Knowledge accumulation is probably a phenomenon that happens in reinforcement learning. Reinforcement learning agents often learn in finite environments. However, while they learn, the training distribution changes, either because of the policy change or because the agent explores new parts of the environment. This leads to local data distribution drifts with still re-occurring data as in \textit{SCoLe} scenarios. In the long run, reinforcement learning agents are still able to learn a policy applicable to the whole environment. The results of this paper correlate with this behavior. The need for the sequence of tasks or episodes to converge would explain why reinforcement learning trainings are time and data-consuming \cite{Raffin2022, Rudin2022, Gupta2022}.

8 Conclusion

\textit{SCoLe} is a framework for creating various continual learning scenarios with long sequences of tasks. It can be adjusted in many ways to choose scenarios’ difficulty or to change the characteristics of the data distribution shits. This versatility makes \textit{SCoLe} a good tool for studying the capabilities of deep neural networks to continually learn. This paper shows that standard gradient descent is capable of
knowledge retention and accumulation when we scale the number of tasks in a finite environment. To the best of our knowledge, this phenomenon has not been investigated in the literature before. Furthermore, we increase the retention capability by applying simple modifications to SGD by masking gradients of unused classes. In the classical continual learning literature the common intuition is that such a simple approach would not be enough to enable knowledge accumulation and overcome catastrophic forgetting. However, our results show that knowledge retention and accumulation is sufficient to overcome catastrophic forgetting in long sequences under finite world assumption. This behavior is still dependent on the difficulty of the tasks and on the evolution of the data distribution through time. Still, this framework looks very promising for further research in this domain. Our experiments with SCoLe show interesting insights on forgetting, knowledge accumulation and knowledge retention.

References

Timothée Lesort, Vincenzo Lomonaco, Andrei Stoian, Davide Maltoni, David Filliat, and Natalia Díaz-Rodríguez. Continual learning for robotics: Definition, framework, learning strategies, opportunities and challenges. Information Fusion, 58:52 – 68, 2020. ISSN 1566-2535. doi: https://doi.org/10.1016/j.inffus.2019.12.004. URL http://www.sciencedirect.com/science/article/pii/S1566253519307377

Matthias De Lange, Rahaf Aljundi, Marc Masana, Sarah Parisot, Xu Jia, Ales Leonardis, Gregory Slabaugh, and Tinne Tuytelaars. Continual learning: A comparative study on how to defy forgetting in classification tasks. 2019. URL https://arxiv.org/abs/1909.08383

Eden Belouadah, Adrian Popescu, and Ioannis Kanellos. A comprehensive study of class incremental learning algorithms for visual tasks. Neural Networks, 135:38–54, 2021. ISSN 0893-6080. doi: https://doi.org/10.1016/j.neunet.2020.12.003. URL https://www.sciencedirect.com/science/article/pii/S0893608020304202

Gido M van de Ven and Andreas S Tolias. Three scenarios for continual learning. arXiv preprint arXiv:1904.07734, 2019. URL https://arxiv.org/abs/1904.07734

Arthur Douillard and Timothée Lesort. Continuum: Simple management of complex continual learning scenarios. 2021. URL https://arxiv.org/abs/2102.06253

Fabrice Normandin, Florian Golemo, Oleksiy Ostapenko, Matthew Riemer, Pau Rodriguez, Julio Hurtado, Khimya Khetarpal, Timothée Lesort, Laurent Charlin, Irina Rish, and Massimo Caccia. Sequoia - towards a systematic organization of continual learning research. Github repository, 2021. URL https://github.com/lebrice/Sequoia

Robert M. French. Catastrophic forgetting in connectionist networks. Trends in Cognitive Sciences, 3(4):128–135, 1999. ISSN 13646613. doi: 10.1016/S1364-6613(99)01294-2. URL https://www.sciencedirect.com/science/article/abs/pii/S1364661399012942

Martin Mundt, Yong Won Hong, Iulia Pliusch, and Visvanathan Ramesh. A wholistic view of continual learning with deep neural networks: Forgotten lessons and the bridge to active and open world learning. 2020. URL https://arxiv.org/abs/2009.01797

Massimo Caccia, Pau Rodriguez, Oleksiy Ostapenko, Fabrice Normandin, Min Lin, Lucas Caccia, Issam Laradji, Irina Rish, Alexandre Lacoste, David Vazquez, and Laurent Charlin. Online fast adaptation and knowledge accumulation: a new approach to continual learning. NeurIPS, 2020. URL https://arxiv.org/abs/2003.05856

Ning Qian. On the momentum term in gradient descent learning algorithms. Neural networks, 12(1): 145–151, 1999.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980, 2014.

Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy,
Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Garnett, editors, Advances in Neural Information Processing Systems 32, pages 8024–8035. Curran Associates, Inc., 2019. URL https://papers.neurips.cc/paper/9015-pytorch-an-imperative-style-high-performance-deep-learning-library.pdf.

Lucas Caccia, Rahaf Aljundi, Nader Asadi, Tinne Tuytelaars, Joelle Pineau, and Eugene Belilovsky. New insights on reducing abrupt representation change in online continual learning. In International Conference on Learning Representations, 2022. URL https://openreview.net/forum?id=N8MaBz0Z0fb.

Chen Zeno, Itay Golan, Elad Hoffer, and Daniel Soudry. Task agnostic continual learning using online variational bayes. 2018. URL https://arxiv.org/pdf/1803.10123.pdf.

Timothée Lesort, Thomas George, and Irina Rish. Continual learning in deep networks: an analysis of the last layer. arXiv preprint arXiv:2106.01834, 2021a. URL https://arxiv.org/abs/2106.01834.

Yann LeCun and Corinna Cortes. MNIST handwritten digit database. 2010. URL http://yann.lecun.com/exdb/mnist/.

Han Xiao, Kashif Rasul, and Roland Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms, 2017.

Tarin Clanuwat, Mikel Bober-Irizar, Asanobu Kitamoto, Alex Lamb, Kazuaki Yamamoto, and David Ha. Deep learning for classical japanese literature, 2018. URL https://arxiv.org/abs/1812.01718 Comment: To appear at Neural Information Processing Systems 2018 Workshop on Machine Learning for Creativity and Design.

Yue Wu, Yinpeng Chen, Lijuan Wang, Yuancheng Ye, Zicheng Liu, Yandong Guo, and Yun Fu. Large scale incremental learning. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 374–382, 2019. URL https://arxiv.org/abs/1905.13260.

Saihui Hou, Xinyu Pan, Chen Change Loy, Zilei Wang, and Dahua Lin. Learning a unified classifier incrementally via rebalancing. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2019.

Bowen Zhao, Xi Xiao, Guojun Gan, Bin Zhang, and Shu-Tao Xia. Maintaining discrimination and fairness in class incremental learning. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 13208–13217, 2020.

Vinay Venkatesh Ramasesh, Ethan Dyer, and Maithra Raghu. Anatomy of catastrophic forgetting: Hidden representations and task semantics. In International Conference on Learning Representations, 2021. URL https://openreview.net/forum?id=LhY8QdUGSuw.

Samuel J Bell and Neil D. Lawrence. Behavioral experiments for understanding catastrophic forgetting. ArXiv, abs/2110.10570, 2021.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. In Proceedings of 2016 IEEE Conference on Computer Vision and Pattern Recognition. doi: 10.1109/CVPR.2016.90.

Alex Krizhevsky, Vinod Nair, and Geoffrey Hinton. Cifar-10 (canadian institute for advanced research). a. URL http://www.cs.toronto.edu/~kriz/cifar.html.

Alex Krizhevsky, Vinod Nair, and Geoffrey Hinton. Cifar-100 (canadian institute for advanced research). b. URL http://www.cs.toronto.edu/~kriz/cifar.html.

Seyed Iman Mirzadeh, Arslan Chaudhry, Huiyi Hu, Razvan Pascanu, Dilan Gorur, and Mehrdad Farajtabar. Wide neural networks forget less catastrophically. arXiv preprint arXiv:2110.11526, 2021.

Sergey Zagoruyko and Nikos Komodakis. Wide residual networks. arXiv preprint arXiv:1605.07146, 2016.
MohammadReza Davari, Nader Asadi, Sudhir Mudur, Rahaf Aljundi, and Eugene Belilovsky. Probing representation forgetting in supervised and unsupervised continual learning. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 16712–16721, 2022.

Timothée Lesort, Massimo Caccia, and Irina Rish. Understanding continual learning settings with data distribution drift analysis. arXiv preprint arXiv:2104.01678, 2021b.

Mohamed Abdelsalam, Mojtaba Faramarzi, Shagun Sodhani, and Sarath Chandar. Iirc: Incremental implicitly-refined classification. CVPR, pages 11038–11047, 2021. URL https://chandar-lab.github.io/IIRC/.

Lucas Caccia, Jing Xu, Myle Ott, Marc’Aurelio Ranzato, and Ludovic Denoyer. On anytime learning at macroscale. arXiv preprint arXiv:2106.09563, 2021.

Mitchell Wortsman, Vivek Ramanujan, Rosanne Liu, Aniruddha Kembhavi, Mohammad Rastegari, Jason Yosinski, and Ali Farhadi. Supermasks in superposition. In Proceedings of the 34th International Conference on Neural Information Processing Systems, NIPS’20, Red Hook, NY, USA, 2020. Curran Associates Inc. ISBN 9781713829546.

Enrico Fini, Victor G Turrisi da Costa, Xavier Alameda-Pineda, Elisa Ricci, Karteek Alahari, and Julien Mairal. Self-supervised models are continual learners. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 9621–9630, 2022.

Antonin Raffin, Jens Kober, and Freek Stulp. Smooth exploration for robotic reinforcement learning. In Conference on Robot Learning, pages 1634–1644. PMLR, 2022.

Nikita Rudin, David Hoeller, Philipp Reist, and Marco Hutter. Learning to walk in minutes using massively parallel deep reinforcement learning. In Conference on Robot Learning, pages 91–100. PMLR, 2022.

Abhishek Gupta, Corey Lynch, Brandon Kinman, Garrett Peake, Sergey Levine, and Karol Hausman. Bootstrapped autonomous practicing via multi-task reinforcement learning. arXiv preprint arXiv:2203.15755, 2022.

Timothée Lesort, Andrei Stoian, and David Filliat. Regularization shortcomings for continual learning. arXiv preprint arXiv:1912.03049, 2019.

Jonathan Krause, Michael Stark, Jia Deng, and Li Fei-Fei. 3d object representations for fine-grained categorization. In 4th International IEEE Workshop on 3D Representation and Recognition (3dRR-13), Sydney, Australia, 2013.

Subhransu Maji, Esa Rahtu, Juho Kannala, Matthew Blaschko, and Andrea Vedaldi. Fine-grained visual classification of aircraft. arXiv preprint arXiv:1306.5151, 2013.

Oleksiy Ostapenko, Timothee Lesort, Pau Rodriguez, Md Rifat Arefin, Arthur Douillard, Irina Rish, and Laurent Charlin. Foundational models for continual learning: An empirical study of latent replay, 2022. URL https://arxiv.org/abs/2205.00329.
A Limitations

In this paper, we investigate the phenomena of knowledge accumulation. We show that knowledge accumulation can lead to convergence to a stable solution for all tasks in a long sequence of tasks within a finite world despite catastrophic forgetting.

We study how increasing the complexity of the task can, however, mitigate this effect and make it longer to reach. Estimating scenario complexity is challenging and possibly impossible in practice. In this condition, it is not clear how long the sequence of tasks needs to be for a given problem. The training time necessary would need to estimate the scenario complexity. It could need to know the future in advance.

Moreover, in robotics, acquiring data can be very long. Even if simple and light, the proposed approach is very data and time-consuming. It might be more efficient to use a more complex and compute costly method to minimize the time and the data needed for convergence.

B Scenario Implementation

The codebase is available under [https://anonymous.4open.science/r/convergence-966C/](https://anonymous.4open.science/r/convergence-966C/).

```python
# num_classes: the total number of classes
# classes_per_tasks: number of tasks per class (2 by default)
# probability: vector defining probability of sampling each class for a task (Uniform by default)
# nb_epochs: epochs of training per tasks (1 by default)

import numpy as np
from continuum.datasets import CIFAR10
from continuum.scenarios import ClassIncremental

scenario = ClassIncremental(CIFAR10(config.data_dir, train=True), nb_tasks=nb_classes)
test_set = CIFAR10(config.data_dir, train=False).to_taskset()

for task_index in range(num_tasks):
    classes = np.random.choice(np.arange(num_classes), p=probability, size=classes_per_tasks, replace=False)
    taskset = scenario[classes]

    for epoch in range(nb_epochs):
        # train the model on "taskset" data
        ...

    # test the model on the full test set
    ...
```

Figure 8: Pseudo-Code to control the distribution imbalance in classes.

The implementation presented in propose a static version of the scenario. However, the “probability” distribution can be modified through the task sequence to create never-ending drifts or cyclic drifts in classes distribution or simply to change the class distribution balance.

C Training IID baselines

The training of the IID baselines has been realized with the same models as the other training processes. They were, however, trained with Adam optimizer for the sake of improving performance.
They were trained with 100 epochs on the full dataset. We experimented with the learning rates of 0.001, 0.01, and 0.1 over 5 seeds and kept only the best performing baselines.

D Additional Experiments

D.1 Architectures

![Figure 9: Comparison on several architectures on default SCoLe scenario with CIFAR10, 2 classes per task.](image)

Setting: We train several architectures (Resnet18, Inception, vit_b_16 and VGG) from torchvision library, and compare them on a default Scole scenario on CIFAR10 with 2 classes per tasks.

Results: Our results (cf Fig. 9) indicates that the resnet model is the best performing on Scole among the 4 models evaluated. Nevertheless, we can also see that knowledge accumulation happens in all architectures.

D.2 Increasing the number of classes per task

![Figure 10: Growing the number of classes per tasks within a task in full CIFAR100 dataset.](image)

Fig. 10, show that our training framework also works when the number of classes per tasks grows. The more classes in the task the faster the learning curve is.

D.3 Making the model deeper

In this experiments, we train on CIFAR10 with binary tasks and classes sampled uniformly. Fig. 11 show that the deepness has low impact in our experiments.
D.4 Structuring the Task Sequence

Figure 11: Growing the number of layers in the resnet model. ———— line represent IID performance with resnet22.

Figure 12: Comparison on several sequence of tasks with a default structure modified by random flip of classes at each task. Scenario created with CIFAR10, 2 classes per task.

Setting: In this experiment, we want to evaluate the role of randomization of classes within tasks. In other words, we try to answer the question: is it important that classes are randomly sampled when building tasks? For this, we start from a fixed sequence of binary classification tasks. This sequence is built such as all possible pairs of classes exist and happen only once. By default, the training is then achieved by training repeatably on this fixed sequence of tasks until the end of the full task sequence. We compare this baseline with the same sequence of tasks, but at each task, we set a probability \( p \) that each class is randomly flipped to another class. The classes of the initial sequence of tasks is \([0, 1] \rightarrow [0, 2] \rightarrow […] \rightarrow [1, 2] \rightarrow [1, 3] \rightarrow [8, 9] \rightarrow [0, 1] \rightarrow […]\). The values of \( p \) are 0, 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 1. We can note that \( p = 1 \) is the same as the default scenario.

Results: The results presented in Fig. 12 show that having a fixed sequence of tasks instead of a randomized one does not reduce knowledge accumulation in our setting.

D.5 Limiting the Possible Pairs of Classes in Tasks

Setting: In this experiment, we want to evaluate how important it is that all possible pairs of classes exist within the full sequence of tasks. For this, we start from the list of all possible tasks and select only a subset of them. When building the task sequence, we only select pairs of classes from the selected list.
Figure 13: Comparison of SCoLe scenario when selecting only a subset of all possible pairs of classes within tasks. We vary the proportion of pairs kept and plot test accuracy.

**Results:** The results presented in Fig. [13] show us that the presence of all possible pairs of classes within the sequence of tasks plays an important role. Indeed, without replay to learn discriminative features between two classes, the classes need to be in the same task [Lesort et al. 2019].

**E Scaling from Pre-trained Models**

In this section we study how ImageNet1K pretrained ResNet-18 and ResNet-101 behave in the SCoLe setting. The goal is to know if pre-training is useful in such scenario with simply gradient descent training.

Figure 14: Fine-tuning of ImageNet1K pretrained (ft.) vs. from scratch ResNet18 and ResNet101 models on tasks sampled from an ID CIFAR100/50 and OOD Cars196/50, FGVC-Aircraft/50 and CUB200/50 datasets (1 epoch per task, SGD optimizer without momentum). We include ImageNet21K pretrained ViT fine-tuning in Fig. [15]

Figure 15: Fine-tuning ViT (ImageNet21K pretrianed) on CIFAR100/50 and Cars196/50 with 1 epoch per task.
Setting: To this end 1000 downstream tasks are samples from either out-of-distribution (OOD) datasets, i.e. Cars196 [Krause et al., 2013] and FGCV Aircraft [Maji et al., 2013], or in-distribution (ID) dataset CIFAR100 as in Ostapenko et al. [2022]. The ID vs. OOD distinction is made based on the relation of the downstream classes to the classes used for pre-training, e.g. CIFAR100 classes are known to be a subset of ImageNet1K classes, while the fine-grained OOD datasets are not. For all downstream datasets we use only a subset of 50 first classes from which we create tasks by randomly sampling 2 classes per task. We also compare to training from scratch. We use SGD optimizer without momentum and learning rate of 0.01. We did not use any data augmentation techniques.

Results: As expected, using a pretrained models generally results in a faster convergence to a higher accuracy on both ID and OOD streams as compared to training from scratch (cf Fig.14). Similarly, ID stream converges to a higher accuracy than the OOD streams. Interestingly, the accuracy on OOD streams when trained from scratched stayed at the chance level, which can be attributed to the fact that no data augmentation was used as well as small amount of samples per class in both OOD streams.

F Increasing Difficulty

F.1 Regresssions

Figure 16: Regression of test accuracy over last 10 tasks on three seeds with entropy on CIFAR10 (left) and number of classes on CIFAR100 (right). More details of the experiments in Sec.4

F.2 Modification of Distribution Entropy

To change the entropy of the class distribution, we start with a uniform vector of probabilities \( u \). For each class, \( u \) gives the probability of this class to be samples for a task. To create imbalance in class probabilities, we slightly modify this vector using \( u' = u - \frac{1}{C} \times \text{numpy.arange}(C) \times \lambda \), with \( C \) the number of classes and \( \lambda \) and the hyperparameter that decide how much the distribution is modified. We choose empirically \( \lambda = \frac{1}{2\sigma^2} \) for a slight imbalance. To increase the imbalance in distribution, we multiply \( u' \) by itself \( d \) times. Complete experimentation tests the values of 0, 1, 2, 5 and 10. Note that \( d = 0 \) means that the distribution is uniform (cf. Appendix Sec. F.2 for the Python implementation and the probability vectors for each \( d \)).

Probability vectors for ten classes and \( \lambda = 0.05 \) with different entropy_decrease parameters (rounded with 3 decimals).

- \( \text{entropy_decrease} = 0 \rightarrow [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1] \)
  entropy=2.303
- \( \text{entropy_decrease} = 1 \rightarrow [0.129, 0.123, 0.116, 0.11, 0.103, 0.097, 0.09, 0.084, 0.077, 0.071] \)
  entropy=2.285
- \( \text{entropy_decrease} = 2 \rightarrow [0.161, 0.145, 0.13, 0.116, 0.103, 0.091, 0.079, 0.068, 0.058, 0.049] \)
  entropy=2.237
- \( \text{entropy_decrease} = 5 \rightarrow [0.264, 0.204, 0.156, 0.117, 0.087, 0.063, 0.044, 0.031, 0.021, 0.013] \)
  entropy=1.985
# num_classes: the total number of classes
# lambda: hyper-parameter = 1/(2*C)
# entropy_decrease: parameter that control the scale of the imbalance

import numpy as np
prob_vec = np.ones(num_classes) / num_classes
# introduction of slight imbalance
prob_vec = prob_vec - (1/num_classes) * np.arange(num_classes) * lambda
prob_vec /= prob_vec.sum()
prob_vec = prob_vec**entropy_decrease / (prob_vec**entropy_decrease).sum()

# we shuffle the vector so for each experiments the imbalance is not the same
np.random.seed(config.seed)
np.random.shuffle(prob_vec)

for task_indef in range(num_tasks):
    selected_classes = np.random.choice(np.arange(10), p=prob_vec, size=2, replace=False)

    # the we can create the task and train the model
    [...]