SUPERCOHERENT STATES OF $OSp(8^*|2N)$, CONFORMAL SUPERFIELDS AND THE $AdS_7/CFT_6$ DUALITY

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Abstract

We study the positive energy unitary representations of $2N$ extended superconformal algebras $OSp(8^*|2N)$ in six dimensions. These representations can be formulated in a particle basis or a supercoherent state basis, which are labeled by the superspace coordinates in $d = 6$. We show that the supercoherent states that form the bases of positive energy representations of $OSp(8^*|2N)$ can be identified with conformal superfields in six dimensions. The massless conformal superfields correspond precisely to the ultra short doubleton supermultiplets of $OSp(8^*|2N)$. The other positive energy unitary representations correspond to massive conformal superfields in six dimensions and they can be obtained by tensoring an arbitrary number of doubleton supermultiplets with each other. The supermultiplets obtained by tensoring two copies of the doubletons correspond to massless anti-de Sitter supermultiplets in $d = 7$. 
1 Introduction

AdS/CFT dualities in M/superstring theory have been studied intensively over the last several years. These dualities relate M/superstring theory over the product spaces of $d$-dimensional AdS spaces with compact Einstein manifolds to large $N$ limits of certain conformal field theories in $(d - 1)$-dimensions. The conjecture of Maldacena that started the recent interest on AdS/CFT dualities [1] was formulated in a more precise manner in [2, 3]. It represents the culmination of earlier work on the physics of $N Dp$-branes in the near horizon limit [4] and much earlier work on the construction of the Kaluza-Klein spectra of IIB and eleven dimensional supergravity theories [5]-[10]. The relation of Maldacena’s conjecture to earlier work on Kaluza-Klein supergravity theories has been studied in [11, 12]. For an extensive review of AdS/CFT dualities and the references on the subject, we refer the reader to [13].

In this paper, we study the unitary supermultiplets of $OSp(8^*|2N)$. Our work represents an extension of earlier studies of the representations of this supergroup [9, 14]. For $N = 1, 2$ these supermultiplets are important in the study of $AdS_7/CFT_6$ dualities in M/superstring theory. Our main focus is the formulation of the positive energy unitary representations of $OSp(8^*|2N)$ in a non-compact supercoherent state basis. We show that these non-compact supercoherent states correspond to conformal superfields in $d = 6$. Of these, the massless conformal superfields are described by the ultra short doubleton supermultiplets and the massive conformal superfields are obtained by tensoring an arbitrary number of these doubletons.

The non-compact supergroup $OSp(8^*|2N)$ can also be interpreted as the $2N$ extended AdS supergroup in $d = 7$. In fact, the symmetry superalgebra of M-theory compactified to $AdS_7$ over $S^4$ is $OSp(8^*|4)$ [4, 6]. The general method for the oscillator construction of unitary supermultiplets of $OSp(8^*|4)$ was first given in [6] with emphasis on short supermultiplets that appear in the Kaluza-Klein compactification of eleven dimensional supergravity theory. The entire Kaluza-Klein spectrum of the eleven dimensional supergravity over $AdS_7 \times S^4$ can be obtained by a simple tensoring procedure from the “CPT self-conjugate” doubleton supermultiplet [4]. This “CPT self-conjugate” doubleton supermultiplet is simply the $(2, 0)$ conformal supermultiplet of the dual field theory in six dimensions. The doubleton supermultiplets of $OSp(8^*|2N)$ do not have a Poincaré limit in $d = 7$. By tensoring two copies of these doubletons, one can obtain all the massless supermultiplets of $2N$ extended AdS superalgebra in $d = 7$ [14]. Tensoring more than two copies leads to massive $AdS_7$ supermultiplets. The $AdS_7/CFT_6$ duality has been studied from various points of view more recently [15, 16].

More specifically, in Section 2 we discuss the coherent states associated with positive energy unitary representations of $SO^*(8)$ and show that they correspond to conformal fields in $d = 6$. Of particular interest are the doubleton representations of $SO^*(8)$, which correspond to massless conformal fields in six dimensions.

In Section 3, we discuss the compact versus non-compact bases of the supergroup $OSp(8^*|2N)$. In the compact basis, we work with superoscillators, which transform covariantly under the maximal compact subsupergroup $U(4|N)$ of $OSp(8^*|2N)$. On the other hand, in the non-compact basis, we work with operators which transform covariantly under $SU^*(4) \times USp(2N)$ and that have a definite conformal dimension.

In Section 4 we show how to define a supercoherent state basis for each positive energy unitary irreducible representation of $OSp(8^*|2N)$. These supercoherent states correspond to conformal superfields in six dimensions. As mentioned above, the doubleton supercoherent states lead to massless conformal superfields, a complete list of which is given in Section 4.1. By tensoring doubletons with each other, one obtains massive conformal supermultiplets, of which those that are obtained by tensoring two copies of doubletons correspond to massless supermultiplets in $AdS_7$ space. For
$N = 2$, the shortest such supermultiplet is the massless $AdS$ graviton supermultiplet of $OSp(8^*|4)$. We give the explicit expression for the corresponding supercoherent state in Section 4.2.

We conclude with a discussion of our results.

2 Coherent states of the positive energy unitary representations of the group $SO^*(8)$ and conformal fields in six dimensions

The commutation relations of the generators of the conformal group $SO(6,2)$ in $d = 6$, which is isomorphic to $SO^*(8)$, can be written as

\begin{align}
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}), \\
[P_\mu, M_{\rho\sigma}] &= i(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho), \\
[K_\mu, M_{\rho\sigma}] &= i(\eta_{\mu\rho}K_\sigma - \eta_{\mu\sigma}K_\rho), \\
[D, M_{\mu\nu}] &= [P_\mu, P_\nu] = [K_\mu, K_\nu] = 0, \\
[P_\mu, D] &= iP_\mu, \\
[K_\mu, D] &= -iK_\mu, \\
[P_\mu, K_\nu] &= 2i(\eta_{\mu\nu}D - M_{\mu\nu}),
\end{align}

where $M_{\mu\nu}$ are the generators of the Lorentz subgroup $SO(5,1)$, $D$ is the dilatation generator and $P_\mu$ and $K_\mu$ are the generators of translations and special conformal transformations, respectively ($\mu, \nu, \rho, \sigma = 0, 1, \ldots, 5$). We use the Minkowski metric $\eta_{\mu\nu} = \text{diag}(+, -, -, -, -)$. The covering group of the conformal group $SO(6,2)$ is $Spin(6,2)$ and the covering group of the Lorentz group $SO(5,1)$ is $SU^*(4)$. The rotation subgroup $SO(5)$ (or its covering group $USp(4)$) is generated by $M_{\mu\nu}$ with $\mu, \nu = 1, 2, \ldots, 5$.

The isomorphism of the conformal group to $SO^*(8)$ becomes manifest by defining

\begin{align}
M_{\mu6} := \frac{1}{2}(P_\mu - K_\mu), \quad M_{\mu7} := \frac{1}{2}(P_\mu + K_\mu), \quad M_{67} := -D,
\end{align}

as one finds that together with $M_{\mu\nu}$ they satisfy

\begin{align}
[M_{ab}, M_{cd}] &= i(\eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc}),
\end{align}

where $-\eta_{66} = \eta_{77} = 1$ and $a, b, c, d = 0, 1, \ldots, 7$.

Considered as the isometry group of the seven dimensional anti-de Sitter space $AdS_7$, the generators of $SO^*(8)$ acquire a different physical interpretation. In particular, the rotation group becomes $SO(6)$, with the covering group $SU(4)$, generated by $M_{mn}$ ($m, n = 1, 2, \ldots, 6$). The generator $E \equiv M_{07}$ becomes the $AdS$ energy, generating translations along the timelike Killing vector field of $AdS_7$ and together with $M_{mn}$, it generates the maximal compact subgroup $U(4) = SU(4) \times U(1)_E$ of $SO^*(8)$.

The Lie algebra of the conformal group $SO(6,2)$, $g$ has a 3-graded decomposition with respect to its maximal compact subalgebra $L^0 = SU(4) \times U(1)_E$ :

\begin{align}
g = L^- \oplus L^0 \oplus L^+,
\end{align}

\footnote{A complete list of indices we used in this paper is given in Appendix A.}
where

\[
\begin{align*}
[L^0, L^\pm] &\subseteq L^\pm, \\
[L^+, L^-] &\subseteq L^0, \\
[L^+, L^+] &=[L^-, L^-] = 0, \\
[E, L^\pm] &= \pm L^\pm, \quad [E, L^0] = 0.
\end{align*}
\]

The 3-grading is determined by the \(U(1)_E\) generator \(E = \frac{1}{2}(P_0 + K_0)\), which is simply the conformal Hamiltonian. In the oscillator construction of the positive energy unitary representations of \(SO^*(8)\), one first realizes its generators as bilinears of an arbitrary number \(P\) ("generations" or "colors") of pairs of bosonic annihilation \((a_i, b_j)\) and creation \((a^i, b^j)\) operators \((i, j = 1, 2, 3, 4)\), transforming in the fundamental representation of \(SU(4)\) and its conjugate, respectively [17, 18, 9, 19, 14]:

\[
\begin{align*}
A_{ij} &= a_i \cdot b_j - a_j \cdot b_i, \\
A^{ij} &= a^i \cdot b^j - a^j \cdot b^i, \\
M^i_j &= a^i \cdot a_j + b_j \cdot b^i,
\end{align*}
\]

where \(a_i \cdot b_j := \sum_{r=1}^P a_i(r) b_j(r)\), etc. The bosonic annihilation and creation operators \(a^i(r) = a_i(r)^\dagger\) and \(b^j(r) = b_j(r)^\dagger\) satisfy the usual canonical commutation relations

\[
\begin{align*}
[a_i(r), a^j(s)] &= \delta^j_i \delta_{rs}, \\
[b_i(r), b^j(s)] &= \delta^j_i \delta_{rs},
\end{align*}
\]

where \(i, j = 1, 2, 3, 4\) and \(r, s = 1, 2, \ldots, P\).

\(M^i_j\) are the generators of the maximal compact subgroup \(U(4)\). The trace part, \(M^i_i\) generates the AdS energy given by

\[
Q_B := \frac{1}{2} M^i_i = \frac{1}{2} (N_B + 4P),
\]

where \(N_B \equiv a^i \cdot a_i + b^i \cdot b_i\), which is the bosonic number operator. The energy eigenvalues of \(Q_B\) are denoted as \(E\).

The hermitian linear combinations of \(A_{ij}\) and \(A^{ij}\) are the non-compact generators of \(SO(6,2)\) [17, 18, 14].

Practically in all applications to fundamental physics, the relevant representations of the conformal group (AdS group) are the unitary irreducible representations (UIRs) of the lowest weight type, in which the spectrum of the conformal Hamiltonian (the AdS energy) \(E\) is bounded from below. The natural basis for constructing them is the compact basis in which the lowest weight (positive energy) property as well as unitarity are manifest.

The lowest weight UIRs of \(SO(6,2)\) can be constructed in a simple way by using the oscillator realization of the generators given above. Each lowest weight UIR is uniquely determined by the quantum numbers of a lowest weight vector \(|\Omega\rangle\), provided that \(|\Omega\rangle\) transforms irreducibly under \(SU(4) \times U(1)_E\) and is also annihilated by all the elements of \(L^-\) [3, 14].
A complete list of possible lowest weight vectors for $P = 1$, which are called doubleton representations in this compact basis is \[14\],

\[
\begin{align*}
|0\rangle, \\
|0\rangle, \\
|0\rangle, \\
|0\rangle, \\
|0\rangle, \\
&\vdots \\
|0\rangle, \\
&
\end{align*}
\]

(plus those obtained by interchanging $a$-type oscillators with $b$-type oscillators) and the state

\[
a^{(ij)}|0\rangle = |\begin{array}{c}
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\end{array}|,
\]

(2 - 9)

These lowest weight vectors $|\Omega\rangle$ of the doubleton UIRs of $SO^*(8)$ all transform in the symmetric tensor representations of $SU(4)$.

On the other hand, in $d = 6$ conformal field theories one would like to work with fields that transform covariantly under the Lorentz group $SU^*(4)$ with a definite conformal dimension. The conformal group $SO(6,2)$ has a 3-graded structure with respect to its subgroup $SU^*(4) \times \mathcal{D}$ as well, where $\mathcal{D}$ is the Abelian group of scale transformations \[20, 14\]. We shall refer to this subgroup as the homogeneous Weyl group in $d = 6$.

When $G \equiv SO(6,2)$ acts in the standard way on the (conformal compactification of) six dimensional Minkowski spacetime, the stability group $H$ of the coordinate six-vector $x^\mu = 0$ is simply the semi-direct product $(SU^*(4) \times \mathcal{D}) \circ K_6$, where $K_6$ represents the Abelian subgroup generated by the special conformal generators $K_\mu$. The conformal fields in $d = 6$ live on the coset space $G/H$. These fields are labeled by their transformation properties under the Lorentz group $SU^*(4)$, their conformal dimension $l$ and certain matrices $\kappa_\mu$ that describe their behavior under special conformal transformations $K_\mu$ \[14\]. This is identical to conformal fields in $d = 4$ \[21, 22\].

To establish a dictionary between the compact (Wigner picture) and non-compact (Dirac picture) bases of positive energy representations of $SO(6,2)$, its generators were expressed in terms of bosonic oscillators transforming in the left-handed spinor representation of $SO(6,2)$ in \[14\], which we summarize below.

Consider the $d = 6$ gamma matrices $\Gamma_{\mu}$ satisfying

\[
\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu},
\]

with $\Gamma_7 = -\Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5$. Then the matrices

\[
\begin{align*}
\Sigma_{\mu\nu} &:= \frac{i}{4}[\Gamma_\mu, \Gamma_\nu], \\
\Sigma_{\mu6} &:= -\frac{1}{2}\Gamma_\mu\Gamma_7, \\
\Sigma_{\mu7} &:= \frac{1}{2}\Gamma_\mu, \\
\Sigma_{67} &:= \frac{i}{2}\Gamma_7,
\end{align*}
\]

(2 - 12)

generate the eight dimensional left-handed spinor representation of the conformal algebra $SO(6,2)$\[6\].

\[6\] Our choice of gamma matrices and our conventions are outlined in Appendix B.
Let
\[ \Psi(r) := \begin{pmatrix} a_i(r) \\ b^j(r) \end{pmatrix}, \] (2 - 13)
and therefore
\[ \bar{\Psi}(r) = \Psi^\dagger(r) \Gamma_0 = \begin{pmatrix} a^i(r) & -b_j(r) \end{pmatrix}. \] (2 - 14)

If we denote the components of the spinor \( \Psi \) with lower indices \( \Psi_A \) and the components of the Dirac conjugate spinor \( \bar{\Psi} \) with upper indices \( \bar{\Psi}^B \) \((A,B = 1, 2, \ldots, 8)\), they satisfy
\[ \left[ \Psi_A(r), \bar{\Psi}^B(s) \right] = \delta^B_A \delta_{rs}. \] (2 - 15)

The bilinears of these twistorial operators involving the \( 8 \times 8 \) matrices \( \Sigma_{ab} \):
\[ \bar{\Psi} \Sigma_{ab} \Psi := \sum_{r=1}^P \bar{\Psi}(r) \Sigma_{ab} \Psi(r), \] (2 - 16)
satisfy the commutation relations of \( SO(6,2) \),
\[ \left[ \bar{\Psi} \Sigma_{ab} \Psi, \bar{\Psi} \Sigma_{cd} \Psi \right] = \bar{\Psi} \left[ \Sigma_{ab}, \Sigma_{cd} \right] \Psi, \] (2 - 17)
and yield infinite dimensional unitary representations of \( SO(6,2) \) in the Fock space of the oscillators \( a^i(r) \) and \( b^j(r) \) \[7\].

The generators of the conformal algebra in \( d = 6 \) can be written as
\[ M_{\mu\nu} = \frac{i}{4} \bar{\Psi}^B [\Gamma_{\mu}, \Gamma_{\nu}]_B A \Psi_A, \]
\[ D = -\frac{i}{2} \bar{\Psi}^B (\Gamma_7)_B A \Psi_A, \]
\[ P_\mu = \frac{1}{2} \bar{\Psi}^B (\Gamma_\mu (I - \Gamma_7))_B A \Psi_A, \]
\[ K_\mu = \frac{1}{2} \bar{\Psi}^B (\Gamma_\mu (I + \Gamma_7))_B A \Psi_A. \] (2 - 18)

Triality, viz. the existence of left-handed spinor, right-handed spinor and vector representations which are all eight-dimensional, allows one to write the generators of \( SO(6,2) \) as anti-symmetric tensors in the spinor representation which satisfy the commutation relations
\[ [\tilde{M}_{AB}, \tilde{M}_{CD}] = \frac{1}{2} \left( C_{BC} \tilde{M}_{AD} - C_{AC} \tilde{M}_{BD} - C_{BD} \tilde{M}_{AC} + C_{AD} \tilde{M}_{BC} \right), \] (2 - 19)
where
\[ \tilde{M}_{AB} = \frac{1}{2} \left( \Psi^C C_{CA} \Psi_B - \Psi^C C_{CB} \Psi_A \right), \] (2 - 20)
and \( C_{AB} \) is the charge conjugation matrix in six dimensions, which is symmetric.

It should be noted that the hermitian conjugate of \( \tilde{M}_{AB} \) can be expressed as
\[ \left( \tilde{M}_{AB} \right)^\dagger = \tilde{M}^{AB} = C^{AC} C^{BD} \tilde{M}_{CD}. \] (2 - 21)

\[7\] In this paper when we write the generators of a Lie (super)algebra, we shall assume that the color indices are summed over, and drop the summation symbol and the color indices.
The positive energy UIRs of $SO^\ast(8)$ can be identified with conformal fields in $d = 6$, with a definite conformal dimension, transforming covariantly under the six dimensional Lorentz group and with trivial special conformal transformations. To establish this connection, we need to find a mapping from the $SU^\ast(4)$- and $D$- covariant basis to the compact $U(4)$ basis.

For this, one introduces the operator

$$U := e^{\Psi(\Sigma_{06} + i\Sigma_{67})\Psi},$$

which satisfies the following important relations:

$$M_{mn}U = UM_{mn} \quad \text{for } m, n = 1, 2, \ldots, 5,$$

$$iM_{m0}U = U(M_{m6} + L^-),$$

$$iDU = U(E + L^-),$$

$$K_{\mu}U = UL^-,$$

where $L^-$ stands for certain linear combinations of the di-annihilation operators $A_{ij}$, whose explicit form is different for the three equations above. Thus $U$ can be considered as the “intertwiner” between the generators $(M_{\mu\nu}, D)$ of the Lorentz group and dilatations and the generators $(M_{mn}, E)$ of the maximal compact subgroup $SU(4) \times U(1)$. The indices $m, n$ above are the $SO(6)$ vector indices.

In four dimensions, the finite dimensional representations of the Lorentz group $SL(2,C)$ are labeled as $(j_1, j_2)$ of the Wick rotated compact Lorentz group $SU(2) \times SU(2)$. In analogy with four dimensions, we can define a compact Wick rotated Lorentz group, $SU^\ast_c(4)$ whose generators are

$$J_{\hat{m}\hat{n}} = \begin{cases} 
M_{\hat{m}\hat{n}} & \hat{m}, \hat{n} = 1, 2, \ldots, 5, \\
iM_{\hat{m}0} & \hat{m} = 1, 2, \ldots, 5; \hat{n} = 6.
\end{cases}$$

The common subgroup of $SU^\ast_c(4)$ and the compact subgroup $SU(4)$ is the rotation group $USp(4)(\cong SO(5))$.

Acting with $U$ on a lowest weight vector $|\Omega\rangle$ corresponds to a (complex) rotation in the corresponding representation space of $SO^\ast(8)$:

$$U|\Omega\rangle = e^{\Psi(\Sigma_{06} + i\Sigma_{67})\Psi}|\Omega\rangle.$$

For any lowest weight vector $|\Omega\rangle$ in the compact basis, which transforms irreducibly under the compact subgroup $SU(4)$, the state $U|\Omega\rangle$ transforms in the same irreducible representation of $SU^\ast_c(4)$. Now the states $|\Omega\rangle$ are created by the action of $SU(4)$ oscillators $a^i$ and $b^j$ on $|0\rangle$. Correspondingly, one can define $SU^\ast_c(4)$ covariant oscillators $A^i$ and $B^j$ that create the states $U|\Omega\rangle$ by acting on $U|0\rangle$:

$$A^i(r)U|0\rangle \propto Ua^i(r)|0\rangle,$$

$$B^j(r)U|0\rangle \propto Ub^j(r)|0\rangle,$$

(up to a possible normalization constant from (2 - 28)). The respective annihilation operators $A^i$ and $B^j$ are chosen such that,

$$A^i(r)|0\rangle \propto Ua^i(r)|0\rangle = 0,$$

$$B^j(r)|0\rangle \propto Ub^j(r)|0\rangle = 0.$$
We further require them to satisfy the following commutation relations,

\[
\begin{align*}
[A_i(r), A_j^\lambda(s)] &= \delta^\lambda_i \delta_{rs}, \\
[B_j(r), B_i^\lambda(s)] &= \delta_i^\lambda \delta_{rs},
\end{align*}
\]

while all the other commutators among them vanish. These oscillators \(A_i, B_j, A_i^\lambda, B_i^\lambda\) transform covariantly with respect to \(SU^*_c(4)\).

Remarkably, one finds that \(L^-|\Omega\rangle = 0\) implies that \(K_\mu U|\Omega\rangle = 0\) \([14]\). Thus, every unitary lowest weight representation (ULWR) of \(SO^*(8)\) can be identified with a unitary representation of \(SO(6,2)\) induced by a finite dimensional irreducible representation of \(SU^*(4)\) (labeled by \(SU^*_c(4)\) Dynkin labels), with a definite conformal dimension \(l\) and trivially realized \(K_\mu\).

\((SU^*(4) \times D) \circ K_6\) is the stability group of the coordinate vector \(x_\mu = 0\). To generate a state at any other point in spacetime, we need to act with the translation operator as shown below :

\[
e^{ix^\mu P_\mu} U|\Omega\rangle = |\Phi_{(d_1,d_2,d_3)}(x)\rangle,
\]

where \((d_1,d_2,d_3)\) are the Dynkin labels of the irreducible representations of \(SU(4)\) and \(SU^*_c(4)\) under which \(|\Omega\rangle\) and \(U|\Omega\rangle\) transform, respectively. Thus every irreducible ULWR of \(SO(6,2)\) corresponds to a conformal field, that transforms covariantly under \(SU^*(4)\) with a definite conformal dimension \(l = -E\).

We recall that the doubleton representations of \(SO^*(8)\) correspond to taking a single pair \((P = 1)\) of bosonic oscillators and that they do not have a smooth Poincaré limit in \(d = 7\). Consider the Poincaré mass operator

\[
M^2 = P_\mu P^\mu
\]

in \(d = 6\) Minkowski spacetime, where the translation generators \(P_\mu\) have the following realization in terms of the \(SU(4)\) covariant oscillators :

\[
egin{align*}
P_0 &= \frac{1}{2} \left\{ (a^1 - b_3)(a_1 - b^3) + (a^2 + b_4)(a_2 + b^4) + (a^3 + b_1)(a_3 + b^1) + (a^4 - b_2)(a_4 - b^2) \right\}, \\
P_1 &= \frac{1}{2} \left\{ -(a^1 - b_3)(a_1 - b^3) + (a^2 + b_4)(a_2 + b^4) - (a^3 + b_1)(a_3 + b^1) + (a^4 - b_2)(a_4 - b^2) \right\}, \\
P_2 &= \frac{1}{2} \left\{ (a^1 - b_3)(a_2 + b^4) + (a^2 + b_4)(a_1 - b^3) - (a^3 + b_1)(a_4 - b^2) - (a^4 - b_2)(a_3 + b^1) \right\}, \\
P_3 &= \frac{1}{2} \left\{ -(a^1 - b_3)(a_4 - b^2) - (a^2 + b_4)(a_3 + b^1) - (a^3 + b_1)(a_2 + b^4) - (a^4 - b_2)(a_1 - b^3) \right\}, \\
P_4 &= \frac{1}{2} \left\{ (a^1 - b_3)(a_4 - b^2) + (a^2 + b_4)(a_3 + b^1) - (a^3 + b_1)(a_2 + b^4) - (a^4 - b_2)(a_1 - b^3) \right\}, \\
P_5 &= \frac{1}{2} \left\{ -(a^1 - b_3)(a_2 + b^4) + (a^2 + b_4)(a_1 - b^3) - (a^3 + b_1)(a_4 - b^2) + (a^4 - b_2)(a_3 + b^1) \right\}.
\end{align*}
\]

Substituting in the above expressions for \(P_\mu\) one finds that the mass operator \(M^2\) vanishes identically for \(P = 1\) \([14]\). Thus all the doubleton irreducible representations of \(SO^*(8)\) are massless in \(d = 6\). For \(P \neq 1\) the mass operator is non-vanishing and the corresponding ULWRs of \(SO^*(8)\) define massive conformal fields in \(d = 6\). We should stress that this is in complete parallel to the situation in \(d = 4\), where the doubleton representations of \(SO(4,2)\) are all massless \([23, 22]\).

The doubleton irreducible representations of \(SO(6,2)\) and the corresponding conformal fields are listed in Table 1.

\footnote{Our definition of Dynkin labels is such that, the fundamental representation corresponds to \((1,0,0)\).}
| lowest weight vector | $SU^*(4)$ field labels | conformal dimension $l$ |
|----------------------|-------------------------|------------------------|
| $U[0]$               | $|\Phi_{(0,0,0)}(x)\rangle$ | $-2$ |
| $A^i U[0]$           | $|\Phi_{(1,0,0)}(x)\rangle$ | $-\frac{3}{2}$ |
| $A^{(i_1} A^{j_2)} U[0]$ | $|\Phi_{(2,0,0)}(x)\rangle$ | $-3$ |
| $\vdots$             | $\vdots$                | $\vdots$              |
| $A^{(i_1} \ldots A^{i_n)} U[0]$ | $|\Phi_{(n,0,0)}(x)\rangle$ | $-\frac{1}{2}(n + 4)$ |
| $A^{(i_1} B^{j_2)} U[0]$ | $|\Phi_{(2,0,0)}(x)\rangle$ | $-3$ |

Table 1. Possible lowest weight vectors of doubleton representations, corresponding conformal fields and their conformal dimensions.

The $SU^*_c(4)$ covariant oscillators $A_i$, $B_j$, $A^i$, $B^j$ can be expressed in terms of $a_i$, $b_j$, $a^i$, $b^j$ as follows\footnote{Here we omit the color index.}:

\[
\begin{align*}
A_1 &= \frac{1}{\sqrt{2}}(a_1 + b^3) & B_1 &= \frac{1}{\sqrt{2}}(b_1 - a^3) \\
A_2 &= \frac{1}{\sqrt{2}}(a_2 - b^4) & B_2 &= \frac{1}{\sqrt{2}}(b_2 + a^4) \\
A_3 &= \frac{1}{\sqrt{2}}(a_3 - b^1) & B_3 &= \frac{1}{\sqrt{2}}(b_3 + a^1) \\
A_4 &= \frac{1}{\sqrt{2}}(a_4 + b^2) & B_4 &= \frac{1}{\sqrt{2}}(b_1 - a^2) \\
A^1 &= \frac{1}{\sqrt{2}}(a^1 - b_3) & B^1 &= \frac{1}{\sqrt{2}}(b^1 + a_3) \\
A^2 &= \frac{1}{\sqrt{2}}(a^2 + b_4) & B^2 &= \frac{1}{\sqrt{2}}(b^2 - a_4) \\
A^3 &= \frac{1}{\sqrt{2}}(a^3 + b_1) & B^3 &= \frac{1}{\sqrt{2}}(b^3 - a_1) \\
A^4 &= \frac{1}{\sqrt{2}}(a^4 - b_2) & B^4 &= \frac{1}{\sqrt{2}}(b^4 + a_2).
\end{align*}
\] (2 - 31)

Note that for our $SU^*_c(4)$ covariant oscillators, $A^i \neq (A_i)^\dagger$ and $B^j \neq (B_j)^\dagger$ with respect to the standard conjugation $\dagger$ in the Fock space of $SU(4)$ covariant oscillators $a_i$, $b_j$, $a^i$, $b^j$. In fact,

\[
\begin{align*}
(A_1)^\dagger &= B_3 & (A_2)^\dagger &= -B_4 & (A_3)^\dagger &= -B_1 & (A_4)^\dagger &= B_2 \\
(B_1)^\dagger &= A^3 & (B_2)^\dagger &= -A^4 & (B_3)^\dagger &= -A^1 & (B_4)^\dagger &= A^2.
\end{align*}
\] (2 - 32)

The generators of $SO(6,2)$ in the non-compact basis $SU^*(4) \times D$ can then be written in the form

\[
\begin{align*}
M_{\mu\nu} &= \frac{i}{8} \left( \Xi^B \left[ \Gamma_{\mu}, \Gamma_{\nu} \right] B^A \Xi_A + \Xi_B \left[ \Gamma_{\mu}, \Gamma_{\nu} \right] A^B \right), \\
D &= -\frac{i}{4} \left( \Xi^B \left( \Gamma_7 \right) B^A \Xi_A + \Xi_B \left( \Gamma_7 \right) A^B \right), \\
P_{\mu} &= \frac{1}{2} \Xi_B \left( \Gamma_{\mu} \right) A^B \Xi_A, \\
K_{\mu} &= \frac{1}{2} \Xi_B \left( \Gamma_{\mu} \right) A^B \Xi_A.
\end{align*}
\] (2 - 33)
where

\[ \Upsilon_A = \frac{1}{\sqrt{2}} (I + \Gamma_7)^{A}_B \Psi_B = \frac{1}{\sqrt{2}} \begin{pmatrix} a_1 + b_3 \\ a_2 - b_4 \\ a_3 - b_1 \\ a_4 + b_2 \\ -a_3 + b_1 \\ a_4 + b_2 \\ a_1 + b_3 \\ -a_2 + b_4 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ -A_3 \\ A_4 \\ A_1 \\ -A_2 \end{pmatrix}, \quad (2 - 34) \]

\[ \Xi_A = \frac{1}{\sqrt{2}} (I - \Gamma_7)^{A}_B \Psi_B = \frac{1}{\sqrt{2}} \begin{pmatrix} a_1 - b_3 \\ a_2 + b_4 \\ a_3 + b_1 \\ a_4 - b_2 \\ -a_4 + b_1 \\ a_3 + b_1 \\ -a_4 + b_2 \\ -a_1 + b_3 \\ a_2 + b_4 \end{pmatrix} = \begin{pmatrix} -B^3 \\ B^4 \\ B^1 \\ -B^2 \\ B^1 \\ -B^2 \\ B^2 \\ B^3 \\ B^4 \end{pmatrix}, \quad (2 - 35) \]

and hence

\[ \Upsilon^A = \frac{1}{\sqrt{2}} \Psi^B (I - \Gamma_7)^{A}_B \]
\[ = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} a^1 + b_3 & a^2 - b_4 & a^3 - b_1 & a^4 + b_2 \\ B_3 & -B_4 & -B_1 & B_2 \\ -B_1 & B_2 & -B_3 & -B_4 \end{array} \right), \quad (2 - 36) \]

\[ \Xi^A = \frac{1}{\sqrt{2}} \Psi^B (I + \Gamma_7)^{A}_B \]
\[ = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} a^1 - b_3 & a^2 + b_4 & a^3 + b_1 & a^4 - b_2 \\ A^1 & A^2 & A^3 & -A^3 \\ A^4 & A^1 & A^1 & -A^2 \end{array} \right). \quad (2 - 37) \]

These SU_+(4) spinors satisfy the commutation relations,

\[ [\Upsilon_A(r), \Xi_B(s)] = (I + \Gamma_7)^{A}_B \delta_{rs}, \]
\[ [\Xi_A(r), \Upsilon_B(s)] = (I - \Gamma_7)^{A}_B \delta_{rs}, \quad (2 - 38) \]

while all the other commutators vanish.

In terms of these SU_+(4) covariant oscillators, \( P_\mu \) and \( K_\mu \) are purely di-creation and di-annihilation operators, respectively, and as a result, the proof that the conformal fields associated with the doublet irreducible representations are all massless in \( d = 6 \) \([14]\), is greatly simplified when these covariant oscillators are used. The realization of \( P_\mu \) in terms of these oscillators is given below.

\[
\begin{align*}
P_0 &= -A^1 B^3 + A^2 B^4 + A^3 B^1 - A^4 B^2, \\
P_1 &= A^1 B^3 + A^2 B^4 - A^3 B^1 - A^4 B^2, \\
P_2 &= A^1 B^4 - A^2 B^3 + A^3 B^2 - A^4 B^1, \\
P_3 &= A^1 B^2 - A^2 B^1 - A^3 B^4 + A^4 B^3,
\end{align*}
\]
\[ P_4 = i \left( -A^1 B^2 + A^2 B^1 - A^3 B^4 + A^4 B^3 \right), \]
\[ P_5 = i \left( -A^1 B^4 - A^2 B^3 + A^3 B^2 + A^4 B^1 \right). \]  

(2 - 39)

Similarly, one finds the following expressions for special conformal transformations \( K_\mu \) and dilatation generator \( D \) in terms of these \( SU_v(4) \) covariant oscillators:

\[ K_0 = A_1 B_3 - A_2 B_4 - A_3 B_1 + A_4 B_2, \]
\[ K_1 = A_1 B_3 + A_2 B_4 - A_3 B_1 - A_4 B_2, \]
\[ K_2 = A_1 B_4 - A_2 B_3 + A_3 B_2 - A_4 B_1, \]
\[ K_3 = A_1 B_2 - A_2 B_1 - A_3 B_4 + A_4 B_3, \]
\[ K_4 = i (A_1 B_2 - A_2 B_1 + A_3 B_4 - A_4 B_3), \]
\[ K_5 = i (A_1 B_4 + A_2 B_3 - A_3 B_2 - A_4 B_1). \]  

(2 - 40)

\[ D = -\frac{i}{2} \left( A^1 A_1 + A^2 A_2 + A^3 A_3 + A^4 A_4 + B_1 B^1 + B_2 B^2 + B_3 B^3 + B_4 B^4 \right). \]  

(2 - 41)

The massless representations of \( SO^*(8) \), considered as the seven dimensional \( AdS \) group, are obtained by taking \( P = 2 \) pairs of oscillators. However, as representations of the conformal group in \( d = 6 \), they are massive.

In Table 2, we give these irreducible representations and their corresponding conformal fields.

| lowest weight vector | \( SU_v(4) \) field labels | \( l \) |
|----------------------|---------------------------|------|
| \( U|0\)              | \( |\Phi_{(0,0,0)}(x)\rangle \) | -4   |
| \( A^{i_1}(r)U|0\)   | \( |\Phi_{(1,0,0)}(x)\rangle \) | -\( \frac{3}{2} \) |
| \( A^{(i_1}(r)A^{j_2)}(r)U|0\) | \( |\Phi_{(2,0,0)}(x)\rangle \) | -5   |
| \( A^{(i_1}(r)A^{j_2)}(s)U|0\) | \( |\Phi_{(0,1,0)}(x)\rangle \) | -5   |
| \( A^{i_1}(r)A^{j_1}(s)U|0\) | \( |\Phi_{(2,0,0)}(x)\rangle \) | -5   |
| \( A^{i_1}(r)B^{j_1}(s)U|0\) | \( |\Phi_{(0,1,0)}(x)\rangle \) | -5   |
| \( A^{i_1}(r)B^{j_1}(s)U|0\) | \( |\Phi_{(0,1,0)}(x)\rangle \) | -5   |
| \( A^{(i_1}(r)\ldots A^{i_n}(r)U|0\) | \( |\Phi_{(n,0,0)}(x)\rangle \) | -\( \frac{1}{2} \) \( (n + 8) \) |
| \( A^{(i_1}(r)\ldots A^{i_m}(r)A^{j_{m+1}}(s)\ldots A^{i_n}(s)U|0\) | \( |\Phi_{(n,0,0)}(x)\rangle \) | -\( \frac{1}{2} \) \( (n + 8) \) |
| \( A^{(i_1}(r)\ldots A^{i_m}(r)B^{j_{m+1}}(s)\ldots B^{j_n}(s)U|0\) | \( |\Phi_{(n,0,0)}(x)\rangle \) | -\( \frac{1}{2} \) \( (2m + n + 8) \) |
| \( A^{i_1}(r)A^{j_1}(s)\ldots A^{i_m}(r)A^{j_m}(s)A^{j_{m+1}}(r)\ldots A^{i_{m+n}}(r)U|0\) | \( |\Phi_{(n,m,0)}(x)\rangle \) | -\( \frac{1}{2} \) \( (2m + n + 8) \) |
| \( A^{i_1}(r)B^{j_1}(s)\ldots A^{i_m}(r)B^{j_m}(s)A^{j_{m+1}}(r)\ldots A^{i_{m+n}}(r)U|0\) | \( |\Phi_{(n,m,0)}(x)\rangle \) | -\( \frac{1}{2} \) \( (2m + n + 8) \) |

Table 2. Possible lowest weight vectors of \( SO^*(8) \) for \( P = 2 \), corresponding massive conformal fields in \( d = 6 \) and their conformal dimensions. Above, the color indices \( r, s = 1, 2 \) and \( r \neq s \).
Similarly, for $P > 2$ one obtains representations of $SO^*(8)$, which are massive, both as $d = 6$ conformal fields and as $AdS_7$ fields.

For $P = 3$, the possible lowest weight vectors $|\Omega\rangle (U|\Omega\rangle)$ can be in any representation of $SU(4)$ ($SU^*_c(4)$). If $|\Omega\rangle (U|\Omega\rangle)$ transforms in the representation $(d_1, d_2, d_3)_D$, then the corresponding conformal field $|\Phi(d_1, d_2, d_3)(x)\rangle$ has the conformal dimension

$$l = -\frac{1}{2}(d_1 + 2d_2 + 3d_3 + 12). \quad (2 - 42)$$

For $P \geq 4$, there can be $SU(4)$ singlet lowest weight vectors in addition to the vacuum $|0\rangle$ (e.g. $|\Omega\rangle = \epsilon_{ijkl}a^i(r_1)a^j(r_2)a^k(r_3)a^l(r_4)|0\rangle$). In this case, it is convenient to use the Young Tableaux of the lowest weight vectors with respect to $U(4) = SU(4) \times U(1)$. If we denote the Young Tableaux of $U(4)$ by $(m_1, m_2, m_3, m_4)_Y$, then the corresponding conformal fields $|\Phi(d_1, d_2, d_3)(x)\rangle$, transform in the representation $(d_1 = m_1 - m_2, d_2 = m_2 - m_3, d_3 = m_3 - m_4)_D$ of $SU(4)$ and has the conformal dimension

$$l = -\frac{1}{2}(m_1 + m_2 + m_3 + m_4 + 4P). \quad (2 - 43)$$

Since the bosonic oscillators, in terms of which we realized the generators, transform in the spinor representation of $SO^*(8)$, the oscillator construction can be given a dynamical realization in terms of twistors as was done for $SU(2, 2)$ [24, 25].

### 3 Compact versus non-compact bases of the supergroup $OSp(8^*|2N)$

The supergroup $OSp(8^*|4)$ with the even subgroup $SO^*(8) \times USp(4)$ is the symmetry group of M-theory on $AdS_7 \times S^4$. One can interpret $OSp(8^*|4)$ either as the $N = 4$ extended $AdS$ superalgebra in $d = 7$ or as the $(2, 0)$ extended conformal superalgebra in $d = 6$ [4]. The finite dimensional representations of $SO(6, 2)$ ($\cong SO^*(8)$) possess the triality property and the anti-symmetric tensor of any one of left-handed spinor, right-handed spinor and vector representations with itself transforms like the adjoint representation of $SO(6, 2)$. Therefore, there exists three different forms of the $OSp(8^*|2N)$ superalgebra.

The relevant form of $OSp(8^*|4)$ for M-theory on $AdS_7 \times S^4$ is the one for which the supersymmetry generators, $\mathcal{R}_{AI}$ transform as the left-handed spinor representation of $SO(6, 2)$, which decomposes as $(4 + \bar{4})$ with respect to the compact subgroup $SU(4)$ as well as $SU^*_c(4)$.

The supersymmetry generators $\mathcal{R}_{AI}$ of $OSp(8^*|2N)$ satisfy the following anti-commutation relation [26, 14]:

$$\{\mathcal{R}_{AI}, \mathcal{R}_{BJ}\} = -\frac{1}{2} \left( \tilde{M}_{AB} \Omega_{IJ} + C_{AB} U_{IJ} \right),$$

where $A, B = 1, 2, \ldots, 8$ and $I, J = 1, 2, \ldots, 2N$. $\tilde{M}_{AB}$ are the $SO(6, 2)$ generators given in (2 - 20). $U_{IJ} = U_{JI}$ are the $USp(2N)$ generators and $\Omega_{IJ} = -\Omega_{JI}$ is the symplectic invariant tensor [26, 14]. The $USp(2N)$ generators satisfy

$$[U_{IJ}, U_{KL}] = \Omega_{I(K} U_{L)J} + \Omega_{J(K} U_{L)I}. \quad (3 - 2)$$

One can define fermionic annihilation ($\alpha_\kappa$, $\beta_\lambda$) and creation ($\alpha^\kappa$, $\beta^\lambda$) operators transforming in the fundamental representation of $U(N)$ and its conjugate, similar to their bosonic counterparts ($a$, $b$ or $A$, $B$), such that they satisfy the anti-commutation relations

$$\{\alpha_\kappa(r), \alpha^\lambda(s)\} = \delta^\lambda_\kappa \delta_{rs},$$

$$\{\beta_\kappa(r), \beta^\lambda(s)\} = \delta^\lambda_\kappa \delta_{rs}. \quad (3 - 3)$$
where \( \kappa, \lambda = 1, 2, \ldots, N \) and \( r, s = 1, 2, \ldots, P \), while all the other anti-commutators vanish. Then the \( USp(2N) \) generators \( U_{IJ} \) can be realized as

\[
U_{IJ} = \frac{1}{2} \left( \Lambda^K \Omega_{KI} \Lambda_J + \Lambda^K \Omega_{KJ} \Lambda_I \right),
\]

where

\[
\Lambda_I(r) = \begin{pmatrix} \alpha_{\kappa}(r) \\ \beta_{\lambda}(r) \end{pmatrix},
\]

\[
\Lambda^J(r) \equiv (\Lambda_I(r))^\dagger = \begin{pmatrix} \alpha^\kappa(r) \\ \beta^\lambda(r) \end{pmatrix}.
\]

Thus, the supersymmetry generators \( R_{AI} \) of \( OSp(8^*|2N) \) have a realization in terms of spinors \( \Psi \) and \( \Lambda \) as,

\[
R_{AI} = \frac{1}{2} \left( \bar{\Psi} C_{BA} \Lambda_I - \Lambda^J \Omega_{JI} \Psi_A \right).
\]

### 3.1 The 3-grading of the superalgebra \( OSp(8^*|2N) \)

\( OSp(8^*|2N) \) has a 3-grading with respect to its maximal compact subsuperalgebra \( U(4|N) \) as follows:

\[
OSp(8^*|2N) = A_{MN} \oplus M^M_N \oplus A^{MN},
\]

where

\[
A_{MN} = \xi_M \cdot \eta_N - \eta_M \cdot \xi_N = A_{ij} \oplus A_{\kappa\lambda} \oplus R_{\kappa\lambda},
\]

\[
A^{MN} = (A_{MN})^\dagger = \eta^N \cdot \xi^M - \xi^N \cdot \eta^M = A^{ij} \oplus A^{\kappa\lambda} \oplus R^{\kappa\lambda},
\]

\[
M^M_N = \xi^M \cdot \xi_N + (-1)^{(degM)(degN)} \eta_M \cdot \eta^N = M^{ij} \oplus M^{\kappa\lambda} \oplus R^{\iota\kappa} \oplus R_{\iota\kappa},
\]

where \( degM = 0 \) (\( degM = 1 \)) if \( M \) is a bosonic (fermionic) index.

The superoscillators \( \xi_M, \eta_N, \xi^M, \eta^N \), which transform covariantly and contravariantly, respectively, under the \( U(4|N) \) subsupergroup of \( OSp(8^*|2N) \) are defined as

\[
\xi_M(r) = \begin{pmatrix} a_i(r) \\ a^i_{\kappa}(r) \end{pmatrix}, \quad \xi^M(r) = \begin{pmatrix} a^i(r) \\ a^i_{\kappa}(r) \end{pmatrix},
\]

\[
\eta_N(s) = \begin{pmatrix} b_j(s) \\ b^j_{\lambda}(s) \end{pmatrix}, \quad \eta^N(s) = \begin{pmatrix} b^j(s) \\ b^j_{\lambda}(s) \end{pmatrix},
\]

with \( i, j = 1, 2, 3, 4; \kappa, \lambda = 1, \ldots, N \) and \( r, s = 1, 2, \ldots, P \). They satisfy the supercanonical commutation relations

\[
\{ \xi_M(r), \xi^N(s) \} = \delta^N_M \delta_{rs},
\]

\[
\{ \eta_M(r), \eta^N(s) \} = \delta^N_M \delta_{rs},
\]

while all the other commutators/anti-commutators vanish.

The operators \( A_{\kappa\lambda}, A^{\kappa\lambda}, \) and \( M^\kappa_{\iota} \) generate the internal symmetry group \( USp(2N) \).

The odd elements of \( OSp(8^*|2N) \) are of the form \( R_{\iota\kappa}, R^\iota_{\kappa}, R_{\iota\kappa}^\iota \), of which \( R_{\iota\kappa}^\iota \) and \( R^{\iota\kappa} \) involve only di-annihilation and di-creation operators, respectively. The ULWRs of \( OSp(8^*|2N) \) are constructed starting from a lowest weight vector \( \Omega \), which is annihilated by \( A_{MN} \):

\[
A_{MN} |\Omega\rangle = 0.
\]
and transforms irreducibly under $U(4|N)$. Acting on $|\Omega\rangle$ with $A^{MN}$ repeatedly generates an infinite dimensional basis of ULWR of $OSp(8^*|2N)$. The irreducibility of the resulting ULWR follows from the irreducibility of $|\Omega\rangle$ under $U(4|N)$. These lowest weight representations constructed in the compact basis are manifestly unitary [1, 13, 14].

3.2 The 5-grading of the superalgebra $OSp(8^*|2N)$

Recall that $SO(6,2)$, as the $d = 6$ conformal group, has the 3-grading $K_\mu \oplus (M_{\mu\nu} + D) \oplus P_\mu$ with respect to Lorentz group times dilatations. The transition from the compact basis of $OSp(8^*|2N)$ to the non-compact basis requires that we work with its 5-graded structure. The Poincaré supersymmetries $Q_{AI}$ ($A = 1,2, \ldots, 8$ and $I = 1,2, \ldots, 2N$) close into the momentum generators $P_\mu$ under the anti-commutation. Similarly, the special conformal supersymmetries $S_{AI}$ close into $K_\mu$. Hence the superalgebra $OSp(8^*|2N)$ has a 5-graded decomposition with respect to the subalgebra $SU^*(4) \times D \times USp(2N)$:

$$OSp(8^*|2N) = \left(K_\mu \oplus S_{AI} \oplus [M_{\mu\nu} + D + U_{IJ}] \oplus Q_{AI} \oplus P_\mu \right).$$

(3 - 14)

These $Q_{AI}$ and $S_{AI}$ are right-handed (negative chiral) and left-handed (positive chiral) spinor generators with respect to $SU^*(4)$, respectively:

$$(\Gamma_7)_A^B Q_{BI} = -Q_{AI},$$

$$(\Gamma_7)_A^B S_{BI} = S_{AI}. \quad (3 - 15)$$

They can be realized in the following way in terms of the spinors $\Psi_A (\bar{\Psi}^B)$ and $\Lambda_I (\bar{\Lambda}^J)$ introduced before:

$$Q_{AI} = \frac{1}{2} (I - \Gamma_7)_A^B \mathcal{R}_{BI} = \frac{1}{4} (I - \Gamma_7)_A^B \left( \bar{\Psi}^C C_{CBA} \Lambda_I - \Lambda^J \Omega_{JI} \Psi_B \right),$$

$$S_{AI} = \frac{1}{2} (I + \Gamma_7)_A^B \mathcal{R}_{BI} = \frac{1}{4} (I + \Gamma_7)_A^B \left( \bar{\Psi}^C C_{CBA} \Lambda_I - \Lambda^J \Omega_{JI} \Psi_B \right), \quad (3 - 16)$$

or, in the $SU_c^*(4)$ covariant basis,

$$Q_{AI} = \frac{1}{2\sqrt{2}} \left( \Xi^B C_{BA} \Lambda_I - \Xi_A \Lambda^J \Omega_{JI} \right),$$

$$S_{AI} = \frac{1}{2\sqrt{2}} \left( \Upsilon^B C_{BA} \Lambda_I - \Upsilon_A \Lambda^J \Omega_{JI} \right). \quad (3 - 17)$$

It is important to note that in this realization, the only bosonic oscillators in $Q_{AI}$ are $A_i^\dagger$ and $B_j^\dagger$, while the only bosonic oscillators in $S_{AI}$ are $A_i$ and $B_j$. This is consistent with the 5-graded structure of $OSp(8^*|2N)$.

The commutation/anti-commutation relations of $Q_{AI}$ and $S_{AI}$ among themselves and with conformal generators in $d = 6$ are [27]:

$$[Q_{AI}, M_{\mu\nu}] = \frac{i}{4} [\Gamma_\mu, \Gamma_\nu]_A^B Q_{BI},$$

$$[S_{AI}, M_{\mu\nu}] = \frac{i}{4} [\Gamma_\mu, \Gamma_\nu]_A^B S_{BI},$$

$$[Q_{AI}, D] = \frac{i}{2} Q_{AI},.$$
[S_{AI}, D] = -\frac{i}{2} S_{AI}, \\
[Q_{AI}, P_{\mu}] = 0, \\
[S_{AI}, P_{\mu}] = (\Gamma_{\mu})^{B}_{A} Q_{BI}, \\
[Q_{AI}, K_{\mu}] = (\Gamma_{\mu})^{B}_{A} S_{BI}, \\
[S_{AI}, K_{\mu}] = 0, \\
\{Q_{AI}, Q_{BJ}\} = \frac{1}{16} ((I - \Gamma_{7}) \Gamma^{\mu})_{AB} P_{\mu} \Omega_{IJ}, \\
\{S_{AI}, S_{BJ}\} = \frac{1}{16} ((I + \Gamma_{7}) \Gamma^{\mu})_{AB} K_{\mu} \Omega_{IJ}, \\
\{Q_{AI}, S_{BJ}\} = \frac{i}{16} \left\{ \frac{1}{4} ((I - \Gamma_{7}) [\Gamma^{\mu}, \Gamma^{\nu}])_{AB} M_{\mu\nu} - (I - \Gamma_{7})_{AB} D \right\} \Omega_{IJ} - \frac{1}{4} (I - \Gamma_{7})_{AB} U_{IJ}. \tag{3 - 18}

We note that any state that is annihilated by $S_{AI}$ is also annihilated by $K_{\mu}$, but the converse is not necessarily true. We also recall that any lowest weight vector $U|\Omega\rangle$ is annihilated by $K_{\mu}$. On the other hand, the components of $S_{AI}$ are either di-annihilation operators (for $I = 1, \ldots, N$) or involve bilinears of a bosonic annihilation and a fermionic creation operator (for $I = N + 1, \ldots, 2N$). Therefore, for those components of $S_{AI}$ that are di-annihilation operators, we have

$$S_{AI} U|\Omega\rangle = 0 \quad \text{for} \quad I = 1, \ldots, N. \quad (\text{cf. equation (3 - 13)}) \tag{3 - 19}$$

Note that, $U|\Omega\rangle$ is not an irreducible representation of $USp(2N)$. To obtain them, one needs to act on $U|\Omega\rangle$ with the operators $\alpha^{(\alpha \beta \lambda)}$ repeatedly. The resulting irreducible representations of $USp(2N)$ are labeled by the $U(N)$ labels of $U|\Omega\rangle$. If we denote the irreducible representation of $USp(2N)$ defined by the lowest weight vector $|\Omega\rangle$ as $|\Pi\rangle$, then it satisfies

$$K_{\mu} U|\Pi\rangle = 0. \tag{3 - 20}$$

Therefore, $e^{ix_{\mu} P_{\mu}} U|\Pi\rangle$ form the coherent state basis of a ULWR of $SO(6,2)$ transforming in an irreducible representation of $USp(2N)$. We should note that $U|\Pi\rangle$ in general is not annihilated by all $S_{AI}$.

The analog of the lowest weight vector $|\Omega\rangle$ in the compact basis is the “chiral primary state" $|\zeta\rangle$ in the non-compact basis, that is annihilated by all the $S_{AI}$ (I = 1, 2, \ldots, 2N) and that transforms irreducibly under $SU^{*}(4) \times D \times USp(2N)$. By acting on $|\zeta\rangle$ with the translation operator $e^{ix_{\mu} P_{\mu}}$, one generates a coherent state corresponding to the chiral primary field and furthermore, the action of Poincaré supersymmetry generators $Q_{AI}$ on $e^{ix_{\mu} P_{\mu}} |\zeta\rangle$ generates the supermultiplet of conformal fields. The state $|\zeta\rangle$ uniquely defines the supermultiplet and as will become evident later, there exists such a chiral primary state for every ULWR.

4 **Supercoherent states of $OSp(8^{*}|2N)$ and superfields**

Since $\Gamma_{7}$ is not diagonal in our work, to find the components of $\Psi_{A}$ that transform as left-handed and right-handed spinors of $SU^{*}(4)$, we need to act with the projection operators $\frac{1}{\sqrt{2}} (I \pm \Gamma_{7})$ on $\Psi$. One then finds that the first four components of $\Upsilon_{A}$ (equation (2 - 34)) and the last four components of $\Xi_{A}$ (equation (2 - 35)) transform as left-handed and right-handed $SU^{*}(4)$ spinors, respectively.
More explicitly, the $SU^*(4)$ left-handed spinor indices $\alpha, \beta = 1, 2, 3, 4$ and right-handed spinor indices $\dot{\alpha}, \dot{\beta} = 1, 2, 3, 4$ correspond to the $SO^*(8)$ left-handed spinor indices $A, B = 1, 2, 3, 4$ and $A, B = 5, 6, 7, 8$, respectively.

Let

$$u_{\dot{\alpha}} = i \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^4 \end{pmatrix}, \quad s^\beta = i \begin{pmatrix} B^1 \\ B^2 \\ B^3 \\ B^4 \end{pmatrix},$$

$$t_\alpha = -i \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix}, \quad v^{\dot{\beta}} = -i \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix}. \quad (4 - 1)$$

They satisfy

$$[t_\alpha, u_{\dot{\alpha}}] = \delta_{\alpha\dot{\alpha}}, \quad [v^{\dot{\beta}}, s^\beta] = \delta^{\dot{\beta}\beta}, \quad (4 - 2)$$

while all the other commutators are zero.

Then we find that

$$P_\mu = s^\alpha (\Sigma_\mu)^\beta_\alpha u_{\dot{\beta}}, \quad K_\mu = v^{\dot{\alpha}} (\Sigma_\mu)^\beta_\dot{\alpha} t_\beta, \quad (4 - 3)$$

where $\Sigma_\mu = (\Sigma_0, -\Sigma_1, -\Sigma_2, -\Sigma_3, -\Sigma_4, -\Sigma_5)$. These $\Sigma$-matrices in $d = 6$ are the analogs of Pauli matrices $\sigma_\mu$ in $d = 4$.

Note that under the standard hermitian conjugation over the Fock space of $SU(4)$ covariant oscillators, we have

$$(t_\alpha)^\dagger = t_{\dot{\alpha}}, \quad (s^\beta)^\dagger = s^{\dot{\beta}}. \quad (4 - 4)$$

Further, we define

$$\tau_\alpha = c^{\dot{\alpha}}_\alpha t_{\dot{\beta}}, \quad \tau^\beta = c^{\dot{\beta}}_\dot{\alpha} s^{\dot{\alpha}},$$

$$t_{\dot{\alpha}} = t_\beta c^{\dot{\beta}}_\alpha, \quad \tau^{\dot{\beta}} = s^\alpha c^{\dot{\beta}}_\alpha, \quad (4 - 5)$$

where the unitary $c$-matrix [28, 29] is chosen as :

$$c^{\dot{\beta}}_\alpha = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad c^\beta_\dot{\alpha} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad (4 - 6)$$

and satisfy

$$c^{\dot{\beta}}_\alpha c^{\gamma}_\beta = -\delta^{\gamma}_{\dot{\alpha}}, \quad c^\beta_\dot{\alpha} c^{\gamma}_\dot{\beta} = -\delta^\gamma_{\alpha},$$

$$\left(c^{\dot{\beta}}_\alpha\right)^* = c^{\dot{\beta}}_\alpha = -\left(c^{-1}\right)^\beta_\dot{\alpha}, \quad \left(c^\beta_\dot{\alpha}\right)^T = c^\beta_\dot{\alpha} = \left(c^{-1}\right)^\alpha_{\dot{\beta}}. \quad (4 - 7)$$

\[\text{10}\text{The explicit form of the } \Sigma_\mu \text{ matrices is given in Appendix C.}\]
Using the conventions above, it follows that
\[ T_\alpha = v^\alpha, \quad \bar{T}_{\dot{\alpha}} = (T_\alpha)^\dagger = v^\alpha, \]
\[ \bar{s}_\beta = -u_\beta, \quad s_\dot{\beta} = (\bar{s}_\beta)^\dagger = -u_\beta. \] (4 - 8)

Now, one finds
\[ ((I - \Gamma_7)\Gamma^\mu)_{\alpha\beta} P_\mu = 4s_\beta, \quad ((I + \Gamma_7)\Gamma^\mu)_{\alpha\beta} K_\mu = 4t_\beta. \] (4 - 9)

We should note that the \( \Sigma \)-matrices satisfy the identities
\[ (\Sigma_\mu)^\beta_\alpha = (\Sigma_\mu)^\gamma_\alpha c^\beta_\gamma, \]
\[ (\Sigma_\mu)^\beta_\alpha = c^\gamma_\alpha (\Sigma_\mu)^\gamma_\beta, \]
\[ (\Sigma_\mu)^\beta_\dot{\alpha} = (\Sigma_\mu)^\gamma_\dot{\alpha} c^\beta_\gamma, \]
\[ (\Sigma_\mu)^\beta_\dot{\alpha} = (\Sigma_\mu)^\gamma_\dot{\alpha} c^\beta_\gamma. \] (4 - 10)

We now define supercoherent states associated with the ULWR in the non-compact basis as,
\[ e^{i(\xi_\nu P_\nu + \bar{\psi}^A Q_{AI})} |\zeta\rangle = e^{i\xi_\nu P_\nu} e^{i(\bar{\psi}^A Q_{AI})} |\zeta\rangle, \] (4 - 11)
where \( |\zeta\rangle \) is the chiral primary state and \( \theta_{AI} \) are the Grassmann variables obeying,
\[ \bar{\psi}^A = \theta_{BI} (\Gamma_0)_B^A. \] (4 - 12)

Since \( Q_{AI} \) is a right-handed spinor, \( \theta_{AI} \) is chosen as a left-handed spinor, so that \( \bar{\psi}^A Q_{AI} \) is a Lorentz singlet:
\[ \theta_{AI} = (\Gamma_7)_A^B \theta_{BI}. \] (4 - 13)

If one denotes the hermitian conjugate of the supersymmetry generators \( Q_{AI} \) as
\[ Q^A = (Q_{AI})^\dagger, \] (4 - 14)
the Dirac conjugation can be written as
\[ \bar{Q}^A = Q^{BI} (\Gamma_0)_B^A = C^{AB} \Omega^{IJ} Q_{BJ}. \] (4 - 15)

To make \( e^{i\bar{\psi}Q} \) unitary, we require its argument \( \bar{\psi}^A Q_{AI} \) to be hermitian, which imposes the following constraint on \( \theta_{AI} \):
\[ \bar{\psi}^A = C^{AB} \Omega^{IJ} \theta_{BJ}. \] (4 - 16)

Explicitly, \( \bar{\psi}^A Q_{AI} \) is given by
\[ \bar{\psi}^A Q_{AI} = 2 \left( \bar{\psi}^I Q_{1I} + \bar{\psi}^J Q_{2I} + \bar{\psi}^M Q_{3I} + \bar{\psi}^N Q_{4I} \right) \]
\[ = \frac{i}{\sqrt{2}} \left\{ \bar{\psi}^I \left( -s^3 \Lambda_I - s^3 \Lambda^J \Omega_{JI} \right) + \bar{\psi}^J \left( s^4 \Lambda_I + s^4 \Lambda^J \Omega_{JI} \right) \right. \]
\[ + \bar{\psi}^M \left( s^1 \Lambda_I + s^1 \Lambda^J \Omega_{JI} \right) + \bar{\psi}^N \left( -s^2 \Lambda_I - s^2 \Lambda^J \Omega_{JI} \right) \right\}. \] (4 - 17)
Then using equations (4 - 16) and (4 - 13), one obtains

\[
\bar{\theta}^A I Q_{A I} = -\frac{i}{\sqrt{2}} \{ \theta_{I 1} \left( s^1 I^I + \bar{\sigma}^1 \Omega^I \Lambda_J \right) + \theta_{2 I} \left( s^2 I^I + \bar{\sigma}^2 \Omega^I \Lambda_J \right) \\
+ \theta_{3 I} \left( s^3 I^I + \bar{\sigma}^3 \Omega^I \Lambda_J \right) + \theta_{4 I} \left( s^4 I^I + \bar{\sigma}^4 \Omega^I \Lambda_J \right) \}.
\]  

(4 - 18)

Note that the index \( A \) in \( \theta_{A I} \) in the above expression goes from 1 to 4 and therefore from this point onwards, we denote it as \( \alpha = 1, 2, 3, 4 \) (\( SU^*(4) \) indices) and thus write the Grassmann variables as \( \theta_{\alpha I} \). Therefore, we have (cf. equation (3 - 16))

\[
\bar{\theta}^A I Q_{A I} = -\frac{i}{\sqrt{2}} \theta_{\alpha I} \left( \bar{\sigma}^\alpha \Omega^I \Lambda_J + s^\alpha I^I \right).
\]  

(4 - 19)

We restrict ourselves in the rest of the paper to the case \( N = 2 \). For working in the non-compact (5-graded) basis, we define a new Fock vacuum as \( |\bar{0}\rangle = \left\{ \begin{array}{ll}
\beta^1 \beta^2 U |0\rangle & \text{for } P = 1, \\
\beta^1 (1) \beta^2 (1) \beta^2 (2) U |0\rangle & \text{for } P = 2,
\end{array} \right. \) 

(4 - 20)

where \( |0\rangle \) is the “ordinary” Fock vacuum, annihilated by \( a_i, b_j, \alpha_\kappa \) and \( \beta_\lambda \). Note that \( \Lambda_I |\bar{0}\rangle = 0 \).

We also use the notation \( \Lambda^{[K \Lambda L]} \) to denote anti-symmetric symplectic traceless tensors:

\[
\Lambda^{[K \Lambda L]} = \Lambda^{[K \Lambda L]} - \frac{1}{4} \Omega^{K L} \Omega^{K' L'} \Lambda^{K' \Lambda L'}.
\]  

(4 - 21)

In Appendix D, we introduce the compact \( Spin(5)(\cong USp(4)) \) gamma matrices \( \gamma_X (X = 1, 2, \ldots, 5) \), which are \( 4 \times 4 \) matrices with spinor indices \( (\gamma_X)_{I J} \), where \( I, J = 1, 2, 3, 4 \). These matrices are useful in projecting out the irreducible \( Spin(5) \) representations in the Fock space of the fermionic oscillators \( \Lambda_I, \Lambda^I \), e.g. \( (\gamma_X)_{K L} \Lambda^K \Lambda^L \) transforms in the \( 5 \) of \( Spin(5) \) or \( USp(4) \).

### 4.1 Doubleton \((P = 1)\) supercoherent states and the corresponding massless conformal superfields in \( d = 6 \)

The supercoherent state determined by the chiral primary state \( |\zeta\rangle = (\gamma_X)_{K L} \Lambda^K \Lambda^L |\bar{0}\rangle \) is

\[
e^{ix^\mu P_\mu} e^{\bar{\sigma} Q} (\gamma_X)_{K L} \Lambda^K \Lambda^L |\bar{0}\rangle = (\gamma_X)_{K L} \Lambda^K \Lambda^L e^{ix^\mu P_\mu} |\bar{0}\rangle \\
- \frac{1}{\sqrt{2}} \theta_{\alpha I} \left\{ 2 \bar{\sigma}^\alpha (\gamma_X)^I \Lambda^K \\
- s^\alpha (\gamma_X)_{K L} \Lambda^I \Lambda^K \Lambda^L \right\} e^{ix^\mu P_\mu} |\bar{0}\rangle \\
+ \frac{1}{4} \theta_{\alpha I} \theta_{\beta J} \left\{ 2 \bar{\sigma}^{(\alpha \beta)} (\gamma_X)^{I J} \\
+ s^{(\alpha \beta)} (\gamma_X)_{K L} \Lambda^I \Lambda^J \Lambda^K \Lambda^L \\
- \bar{\sigma}^{(\alpha \beta)} \left( 4 (\gamma_X)^K \Lambda^I \Lambda^K + \Omega^{I J} (\gamma_X)_{K L} \Lambda^K \Lambda^L \right) \right\} e^{ix^\mu P_\mu} |\bar{0}\rangle \\
- \theta_{\alpha I} \theta_{\beta J} \bar{\sigma}^{(\alpha \beta)} (\gamma_X)^{I J} \Lambda^K e^{ix^\mu P_\mu} |\bar{0}\rangle \\
+ O(\theta \theta \theta).
\]  

(4 - 22)

We identify \( \bar{\sigma}^{(\alpha \beta)} (\gamma_X)^{I J} \Lambda^K \) and the higher order terms \( O(\theta \theta \theta) \) in the above expansion as “derivative terms” (“excitations” in the language of particle basis) as they are not annihilated
by the special conformal generators $K_{\mu}$. On the other hand, the terms which are annihilated by $K_{\mu}$
correspond to the component fields of the CPT self-conjugate doubleton supermultiplet.

Thus one can write
\[ e^{ix_{\mu}P_{\mu}}e^{\overline{\theta}Q(\gamma X)_{KL}\Lambda^{L}}|\tilde{0}\rangle \cong \left\{ (0,0,0)_{D}, \overline{\mathbf{3}} + (1,0,0)_{D}, \mathbf{1} + (2,0,0)_{D}, \mathbf{1} \right\} + \text{derivative terms.} \tag{4 - 23} \]

This supercoherent state corresponds to Table 1 in [14], which is given below. We draw here
the correspondence between the states in the compact $SU(4) \times SU(2)$ basis and the non-compact
$SU^*(4) \times USp(4)$ basis.

| $SU(4) \times SU(2)$ | $SU^*(4) \times USp(4)$ | $USp(4)$ |
|----------------------|------------------------|----------|
| $|0\rangle$          | $(\gamma X)_{KL}\Lambda^{L}e^{ix_{\mu}P_{\mu}}|0\rangle$ | 5        |
| $|\overline{\mathbf{3}}, \mathbf{1}\rangle$ | $s_{\alpha}^{(\mathbf{3},0)}(\gamma X)_{KL}\Lambda^{L}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle$ | 4        |
| $|\mathbf{1}, \overline{\mathbf{3}}\rangle$ | $s_{\gamma}^{(\mathbf{3},0)}(\gamma X)_{KL}\Lambda^{L}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle$ | 1        |

Table 3. The CPT self-conjugate doubleton supermultiplet in the supercoherent state basis.

Similarly, the supercoherent state determined by the chiral primary state $|\zeta\rangle = \Lambda^{L}|\tilde{0}\rangle$ is
\[ e^{ix_{\mu}P_{\mu}}e^{\overline{\theta}Q\Lambda^{L}}|\tilde{0}\rangle = \Lambda^{L}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ + \frac{1}{\sqrt{2}}\theta_{\alpha I}^{\mu}s_{\alpha}^{I}\Lambda^{L}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ + \frac{1}{\sqrt{2}}\theta_{\alpha I}^{\mu}\left\{ s_{\alpha}^{I}\Omega^{IL} \right. \]
\[ + \frac{1}{4}s_{\alpha}^{I}\Omega^{IL}\Omega^{LJ}\Lambda^{L}\Lambda^{L'} \}
\[ e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ - \frac{1}{12\sqrt{2}}\theta_{\alpha I}^{\mu}\theta_{JL}^{\nu}\theta_{\gamma K}^{\nu}s_{\alpha}^{I}\Lambda^{L}\Lambda^{L}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ - \frac{1}{2}\theta_{\alpha I}^{\mu}\bar{s}_{\beta}^{I}\Omega^{LJ}e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ - \frac{1}{12\sqrt{2}}\theta_{\alpha I}^{\mu}\theta_{JL}^{\nu}\theta_{\gamma K}^{\nu}\left\{ s_{\alpha}^{I}\bar{s}_{\beta}^{I}\gamma \right. \]
\[ \left. \Omega^{IJK} - 2\Omega^{IKJ} \}
\[ + \bar{s}_{\alpha}^{I}\bar{s}_{\beta}^{I}\gamma \left( \Omega^{IJK} - 2\Omega^{IKJ} \right) + \frac{1}{2}\bar{s}_{\alpha}^{I}\bar{s}_{\beta}^{I}\gamma \left( \Omega^{IJK} - 2\Omega^{IKJ} \right) + 3\Omega^{IJK} \}
\[ e^{ix_{\mu}P_{\mu}}|\tilde{0}\rangle \]
\[ + O(\theta\theta\theta) \]
\[ \cong \left\{ (0,0,0)_{D}, \overline{\mathbf{3}} + (1,0,0)_{D}, \mathbf{1} + (2,0,0)_{D}, \mathbf{1} + (3,0,0)_{D}, \mathbf{1} \right\} + \text{derivative terms.} \tag{4 - 24} \]

Above, $s_{\alpha}^{I}, \bar{s}_{\beta}^{I}, \gamma$ and $O(\theta\theta\theta)$ are derivative terms.
This supercoherent state corresponds to Table 2 in [14], which was generated by the lowest weight vector $|\Omega\rangle = |\square\rangle$ in the compact basis.

| $SU(4) \times SU(2)$ | $SU^*(4) \times USp(4)$ | $USp(4)$ |
|----------------------|------------------------|---------|
| $|1, \square\rangle$ | $\Lambda^L e^{ix\mu P_\mu} |0\rangle$ | 4 |
| $|\square, 1\rangle$ | $s^\alpha \Lambda^I \Lambda^L e^{ix\mu P_\mu} |0\rangle$ | 5 |
| $|\square, \square\rangle$ | $s^\alpha \Omega^{IL} e^{ix\mu P_\mu} |0\rangle$ | 1 |
| $|\square, \square, \square\rangle$ | $s^{(\alpha \beta)} \Omega^{IJ} \Lambda^I e^{ix\mu P_\mu} |0\rangle$ | 4 |
| $|\square, \square, \square, \square\rangle$ | $s^{(\alpha \beta \gamma)} \Lambda^I \Lambda^J \Lambda^K \Lambda^L e^{ix\mu P_\mu} |0\rangle$ | 1 |

Table 4. The doubleton supermultiplet defined by the chiral primary state $|\zeta\rangle = \Lambda^L |0\rangle$ in the supercoherent state basis.

The supercoherent state determined by the chiral primary state $|\zeta\rangle = |0\rangle$, which corresponds to Table 3 in [14] (for $j = 1$) is

$$e^{ix\mu P_\mu} e^{ix\mu P_\mu} \tilde{|0\rangle} = e^{ix\mu P_\mu} |0\rangle$$

$$+ \frac{1}{\sqrt{2}} \theta_{ij} \theta_{kl} s^\alpha \Lambda^I e^{ix\mu P_\mu} |0\rangle$$

$$- \frac{1}{4} \theta_{ij} \theta_{kl} \left\{ \frac{1}{4} s^{(\alpha \beta)} \Omega^{IJ} \Omega^{KL} \Lambda^I \Lambda^J \Lambda^K \right\} e^{ix\mu P_\mu} |0\rangle$$

$$- \frac{1}{4} \theta_{ij} \theta_{kl} s^{(\alpha \beta)} \Lambda^I \Lambda^J e^{ix\mu P_\mu} |0\rangle$$

$$+ \frac{1}{8} \delta_{ij} \delta_{kl} s^{(\alpha \beta)} \Lambda^I \Lambda^J \Lambda^K \Lambda^L e^{ix\mu P_\mu} |0\rangle$$

$$- \frac{1}{12 \sqrt{2}} \theta_{ij} \theta_{kl} \theta_{\gamma \delta} s^{(\alpha \beta \gamma \delta)} \left( \Omega^{IJ} \Lambda^K - 2 \Omega^{IK} \Lambda^J \right) e^{ix\mu P_\mu} |0\rangle$$

$$+ \frac{1}{96} \delta_{ij} \delta_{kl} \theta_{\gamma \delta} \left( s^{(\alpha \beta \gamma \delta)} \left( \Omega^{IJ} \Lambda^K - 2 \Omega^{IK} \Lambda^J \right) \right) e^{ix\mu P_\mu} |0\rangle$$

$$+ O(\theta \theta \theta \theta \theta)$$

$$\cong [0, 0, 0]_D, [1, 0, 0]_D, [2, 0, 0]_D, [3, 0, 0]_D + [(4, 0, 0), \square] + \text{derivative terms.}$$

In this expression, $(s^\alpha \beta \gamma \delta \ldots)$, $(\tilde{s}^\alpha \beta \gamma \delta \ldots)$, $(s^\alpha \beta \gamma \delta \ldots)$ and $O(\theta \theta \theta \theta \theta)$ are derivative terms.
Table 5. The doubleton supermultiplet defined by the chiral primary state $|\zeta\rangle = |\tilde{0}\rangle$ in the supercoherent state basis.

Following the same procedure, we give below the supercoherent state obtained, starting from the chiral primary state $|\zeta\rangle = s^\eta|\tilde{0}\rangle$, which corresponds to Table 3 in [4] (for $j = \frac{1}{2}$).

$$e^{ix\nu P_\mu} e^{i\Omega_s \eta}|\tilde{0}\rangle = s^\eta e^{ix\nu P_\mu}|\tilde{0}\rangle$$

$$\begin{align*}
&\quad + \frac{1}{\sqrt{2}} \theta_{\alpha I} s^{(\alpha s^\eta)} \Lambda^I e^{ix\nu P_\mu}|\tilde{0}\rangle \\
&\quad - \frac{1}{16} \theta_{\alpha I} \theta_{\beta J} s^{(\alpha s^\beta s^\eta)} \Omega^{IJ} \Omega^{I' J'} \Lambda^{I'} e^{ix\nu P_\mu}|\tilde{0}\rangle \\
&\quad - \frac{1}{4} \theta_{\alpha I} \theta_{\beta J} s^{(\alpha s^\beta s^\eta)} \Lambda^{[I} \Lambda^{J]} e^{ix\nu P_\mu}|\tilde{0}\rangle \\
&\quad - \frac{1}{12\sqrt{2}} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} s^{(\alpha s^\beta s^\gamma s^\delta s^\eta)} \Lambda^{[I} \Lambda^{J} \Lambda^{K} \Lambda^{L]} e^{ix\nu P_\mu}|\tilde{0}\rangle \\
&\quad - \frac{1}{4} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} \left\{ s^\alpha s^\beta s^\gamma s^\delta s^\eta \left( \Omega^{IJ} \Lambda^{K} - 2\Omega^{IK} \Lambda^{J} \right) e^{ix\nu P_\mu}|\tilde{0}\rangle \\
&\quad + \frac{1}{96} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} \left\{ s^\alpha s^\beta s^\gamma s^\delta s^\eta \left( \Omega^{IL} \Omega^{JK} - 2\Omega^{IK} \Omega^{JL} \right) \\
&\quad + s^\alpha s^\beta s^\gamma s^\delta s^\eta \left( \Omega^{IJ} \Lambda^{K} - 2\Omega^{IK} \Lambda^{J} + 3\Omega^{IL} \Lambda^{J} \Lambda^{K} \right) \right\} e^{ix\nu P_\mu}|\tilde{0}\rangle \right\}
\end{align*}$$

$$\cong [(1,0,0)_D, \underline{1}] + [(2,0,0)_D, \underline{4}] + [(3,0,0)_D, \underline{1}] + [(3,0,0)_D, \underline{2}] + [(4,0,0)_D, \underline{4}] + [(5,0,0)_D, \underline{4}] + \text{derivative terms.} \quad (4 - 26)$$

The derivative terms are, $(\tilde{s}^\alpha s^\beta s^\eta \ldots)$, $(\tilde{s}^\alpha s^\beta s^\gamma s^\eta \ldots)$, $(\tilde{s}^\alpha \tilde{s}^\gamma s^\delta s^\eta \ldots)$, $(\tilde{s}^\alpha s^\beta s^\gamma s^\delta s^\eta \ldots)$ and $\mathcal{O}(\theta\theta\theta\theta\theta)$.
Finally we give the supercoherent state corresponding to the general doubleton supermultiplet, which is determined by the chiral primary state $|\zeta\rangle = s^n|\bar{0}\rangle$ in the supercoherent state basis.

$$e^{ixP}e^{-\theta Q}S^{(\beta_1 \ldots \beta_n)}|\bar{0}\rangle = S^{(\beta_1 \ldots \beta_n)}e^{ixP}|\bar{0}\rangle$$

$$+ \frac{1}{\sqrt{2}} \theta_{a_1i_1} S^{(a_1 \beta_1 \ldots \beta_n)} \Lambda^{i_1} e^{ixP}|\bar{0}\rangle$$

$$- \frac{1}{4} \theta_{a_1i_1} \theta_{a_2i_2} S^{(a_1 a_2 \beta_1 \ldots \beta_n)} \Lambda^{i_1} \Lambda^{i_2} e^{ixP}|\bar{0}\rangle$$

$$+ \frac{1}{12} \theta_{a_1i_1} \theta_{a_2i_2} \theta_{a_3i_3} S^{(a_1 a_2 a_3 \beta_1 \ldots \beta_n)} \Lambda^{i_1} \Lambda^{i_2} \Lambda^{i_3} e^{ixP}|\bar{0}\rangle$$

$$+ \frac{1}{48} \theta_{a_1i_1} \theta_{a_2i_2} \theta_{a_3i_3} S^{(a_1 a_2 a_3 \beta_1 \ldots \beta_n)} \Lambda^{i_1} \Lambda^{i_2} \Lambda^{i_3} e^{ixP}|\bar{0}\rangle$$

$$+ \mathcal{O}(\theta\theta\theta\theta\theta)$$

$$\approx \left[ ([n,0,0]_D, 1] + ([n+1,0,0]_D, 4] + ([n+2,0,0]_D, 5] + ([n+2,0,0]_D, 6] + ([n+3,0,0]_D, 4] + ([n+4,0,0]_D, 1] + \text{derivative terms.} \right)$$

We find that the derivative terms of this expression are, $\left\{ S^{a_1 a_2 a_3 \beta_1 \ldots \beta_n} \right\}$, $\left\{ \tilde{S}^{a_1 a_2 a_3 \beta_1 \ldots \beta_n} \right\}$, $\left\{ S^{a_1 a_2 a_3 \beta_1 \ldots \beta_n} \right\}$, $\left\{ \tilde{S}^{a_1 a_2 a_3 \beta_1 \ldots \beta_n} \right\}$, and $\mathcal{O}(\theta\theta\theta\theta\theta)$.

Table 6. The doubleton supermultiplet defined by the chiral primary state $|\zeta\rangle = s^n|\bar{0}\rangle$ in the supercoherent state basis.
As has been discussed extensively in the literature \[9, 14\], the supermultiplets of the chiral primary state \(P = 2\) correspond to massless states. In the 5-graded non-compact basis of \(\text{OSp}(8^*|4)\), the lowest weight vector \(|\alpha\beta\rangle\) is massive. Hence the massless graviton supermultiplet of \(\text{AdS}_5\) is obtained only. One can project each term with \((\mathcal{P}_{XY})_{I_1I_2I_3I_4}\) to obtain the component fields of the massless graviton supermultiplet.

\[
e^{ix\mu P_\mu} e^{\bar{\eta}Q} \Lambda^{I_1(1)\Lambda I_2(1)\Lambda I_3(2)\Lambda I_4(2)} + \sqrt{2} \theta_{\alpha l} \left\{ s^\alpha(1) \Omega^{I_1I_2(1)} \Lambda^{I_3(2)} \Lambda^{I_4(2)} + s^\alpha(2) \Omega^{I_1I_2(1)} \Lambda^{I_3(2)} \Lambda^{I_4(2)} \right\} e^{ix\mu P_\mu} \bar{\eta}
\]

Table 7. The general doubleton supermultiplet defined by the chiral primary state \(|\zeta\rangle = s^{(\beta_1 \ldots s^{(\beta_n)}} |\bar{0}\rangle\) in the supercoherent state basis.

### 4.2 Massless \(\text{AdS}_7\) supermultiplets \((P = 2)\) versus massive conformal superfields in \(d = 6\)

As has been discussed extensively in the literature \[9, 14\], the supermultiplets of \(\text{OSp}(8^*|2N)\) for \(P = 2\) correspond to massless \(\text{AdS}_7\) supermultiplets in \(d = 7\). However, as conformal superfields in \(d = 6\), they are massive. Hence the massless graviton supermultiplet of \(\text{OSp}(8^*|4)\) obtained from the lowest weight vector \(|\Omega\rangle = |\bar{0}\rangle\) for \(P = 2\) corresponds to a massive conformal superfield in \(d = 6\). In the 5-graded non-compact basis of \(\text{OSp}(8^*|4)\), the corresponding conformal superfield is obtained from the chiral primary state

\[
(P_{XY})_{I_1I_2I_3I_4} \Lambda^{I_1(1)\Lambda I_2(1)\Lambda I_3(2)\Lambda I_4(2)} |\bar{0}\rangle
\]

where

\[
(P_{XY})_{I_1I_2I_3I_4} = (\gamma X)_{I_1I_2} (\gamma Y)_{I_3I_4} - \frac{1}{5} \delta_{XY} (\gamma Z)_{I_1I_2} (\gamma Z)_{I_3I_4}.
\]

However, below we give the action of \(e^{ix\mu P_\mu} e^{\bar{\eta}Q} \Lambda^{I_1(1)\Lambda I_2(1)\Lambda I_3(2)\Lambda I_4(2)} |\bar{0}\rangle\) only. One can project each term with \((P_{XY})_{I_1I_2I_3I_4}\) to obtain the component fields of the massless graviton supermultiplet.

\[
e^{ix\mu P_\mu} e^{\bar{\eta}Q} \Lambda^{I_1(1)\Lambda I_2(1)\Lambda I_3(2)\Lambda I_4(2)} + \sqrt{2} \theta_{\alpha l} \left\{ s^\alpha(1) \Omega^{I_1I_2(1)} \Lambda^{I_3(2)} \Lambda^{I_4(2)} + s^\alpha(2) \Omega^{I_1I_2(1)} \Lambda^{I_3(2)} \Lambda^{I_4(2)} \right\} e^{ix\mu P_\mu} \bar{\eta}
\]
\[-\frac{1}{2} \theta_{\alpha I} \theta_{\beta J} \left\{ \bar{\sigma}^{(\alpha)}(1) \bar{\sigma}^{(\beta)}(1) \Omega^{I[I_1 \Omega L_2]J} \Lambda^{I_3}(2) \Lambda^{I_4}(2) + \bar{\sigma}^{(\alpha)}(2) \bar{\sigma}^{(\beta)}(2) \Omega^{I_1 \Omega L_2} J \Lambda^{I_1}(1) \Lambda^{I_2}(1) \\
+ 4s^{(\alpha)}(1) s^{(\beta)}(1) \Lambda^{I_1}(1) \Omega^{I_2}[J \Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{4} \theta_{\alpha I} \theta_{\beta J} \left\{ s^{(\alpha)}(1) s^{(\beta)}(1) \Lambda^{I_1}(1) \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2) + \bar{s}^{(\alpha)}(2) \bar{s}^{(\beta)}(2) \Lambda^{I_1}(1) \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2) \\
+ 2s^{(\alpha)}(1) s^{(\beta)}(1) \Lambda^{I_1}(1) \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{4} \theta_{\alpha I} \theta_{\beta J} \left\{ \bar{s}^{(\alpha)}(1) s^{(\beta)}(1) \left( \Omega^{I I_1 \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2) + 4\Lambda^{I I_1 \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2)} \right) \\
+ \bar{s}^{(\alpha)}(2) \bar{s}^{(\beta)}(2) \left( \Omega^{I I_1 \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2) + 4\Lambda^{I I_1 \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2)} \right) \\
+ 4s^{(\alpha)}(1) s^{(\beta)}(1) \Lambda^{I I_1 \Lambda^{I_2}(1) \Lambda^{I_3}(2) \Lambda^{I_4}(2)} \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-2\theta_{\alpha I} \theta_{\beta J} \bar{s}^{(\alpha)}(1) s^{(\beta)}(1) \Lambda^{I_1}(1) \Omega^{I_2}[I \Omega J][\Lambda^{I_3}(2) \Lambda^{I_4}(2) e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{2} \theta_{\alpha I} \theta_{\beta J} s^{(\alpha)}(1) s^{(\beta)}(2) \Omega^{I I_1 \Omega L_2}(J \Omega K)[\Lambda^{I_3}(2) \Lambda^{I_4}(2) e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{2} \theta_{\alpha I} \theta_{\beta J} \left\{ \bar{s}^{(\alpha)}(1) s^{(\beta)}(1) \Omega^{I I_1 \Omega L_2}[I \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{\sqrt{2}} \theta_{\alpha I} \theta_{\beta J} \theta_{K} \left\{ \bar{\tau}^{(\alpha)}(1) \bar{\tau}^{(\beta)}(1) \bar{\tau}^{(\gamma)}(2) \right\} \Omega^{I_1 \Omega L_2}(K \Omega K)[\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{4 \sqrt{2}} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \left\{ s^{(\alpha)}(1) s^{(\beta)}(1) s^{(\gamma)}(2) \Omega^{I_1 \Omega L_2}[J \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{2 \sqrt{2}} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \left\{ \bar{\sigma}^{(\alpha)}(1) \bar{\sigma}^{(\beta)}(1) \bar{\sigma}^{(\gamma)}(2) \Omega^{I_1 \Omega L_2}[J \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{2 \sqrt{2}} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \left\{ \bar{\sigma}^{(\alpha)}(1) \bar{\sigma}^{(\beta)}(1) \bar{\sigma}^{(\gamma)}(2) \Omega^{I_1 \Omega L_2}[J \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{4} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} \bar{s}^{(\alpha)}(1) s^{(\beta)}(2) \bar{s}^{(\gamma)}(1) s^{(\delta)}(2) \right\} \Omega^{I_1 \Omega L_2}[I \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[\frac{1}{16} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} s^{(\alpha)}(1) s^{(\beta)}(2) s^{(\gamma)}(1) s^{(\delta)}(2) \right\} \Omega^{I_1 \Omega L_2}[I \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

\[-\frac{1}{8} \theta_{\alpha I} \theta_{\beta J} \theta_{\gamma K} \theta_{\delta L} \left\{ s^{(\alpha)}(1) s^{(\beta)}(2) \right\} \bar{s}^{(\gamma)}(1) s^{(\delta)}(2) \right\} \Omega^{I_1 \Omega L_2}[I \Omega K][\Lambda^{I_3}(2) \Lambda^{I_4}(2) \right\} e^{i x^\mu P_\mu} \bar{0}\]

+\text{derivative terms}

+\mathcal{O}(\theta \theta \theta \theta \theta \theta)
\[ (0,0,0)_{D, 14} + (1,0,0)_{D, 16} + (2,0,0)_{D, 5} + (0,1,0)_{D, 10} + (1,1,0)_{D, 4} + (0,2,0)_{D, 1} \]
+ derivative terms. \hfill (4 - 30)

It is important to note that, the terms \((\bar{s}^{(\alpha)(1)s^{\beta}}(2)\ldots, (s^{(\alpha)(1)s^{\beta}}(2)\ldots, (\bar{s}^{(\alpha)(1)s^{\beta}}(2)\ldots, (s^{(\alpha)(1)s^{\beta}}(2)\ldots, \) and \((\bar{s}^{(\alpha)(2)s^{\beta}}(1)\ldots, (s^{(\alpha)(2)s^{\beta}}(1)\ldots, (\bar{s}^{(\alpha)(2)s^{\beta}}(1)\ldots, (s^{(\alpha)(2)s^{\beta}}(1)\ldots) \) on the right hand side above as well as the terms of the form \((\bar{s}^{(\alpha)s^{\beta}s^{\delta}}\ldots, (s^{(\alpha)s^{\beta}s^{\delta}}\ldots) \) (plus those others which are obtained by replacing one or more \(s\) with corresponding \(s\)) vanish when acted upon by \((\mathcal{P}_{XY})_{1,1,2,1,4}\).

In the table below, we give only one term from each type which corresponds to a different lowest weight vector of the graviton supermultiplet.

| \(SU(4) \times SU(2)\) | \(SU^*(4) \times USp(4)\) | \(USp(4)\) |
|-------------------|------------------|---------|
| \(1,1\) | \((\mathcal{P}_{XY})_{1,1,2,1,4} \Lambda_{I_1}^{(1)} \Lambda_{I_2}^{(1)} \Lambda_{I_3}^{(2)} \Lambda_{I_4}^{(2)} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 14 |
| \(1\) \(1\) | \(\bar{s}^{(1)}(1) \mathcal{P}_{XY}^{(1)}_{1,1,2,1,4} \Omega_{I_1}^{[I_1} \Lambda_{I_2}^{I_2] \Lambda_{I_3}^{(2)} \Lambda_{I_4}^{(2)} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 16 |
| \(1\) \(1\) \(1\) | \(\bar{s}^{(1)}(1) \mathcal{P}_{XY}^{(1)}_{1,1,2,1,4} \Omega_{I_1}^{[I_1} \Lambda_{I_2}^{I_2] \Lambda_{I_3}^{I_3] \Lambda_{I_4}^{I_4} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 5 |
| \(1\) \(1\) \(1\) \(1\) | \(\bar{s}^{(1)}(1) \mathcal{P}_{XY}^{(1)}_{1,1,2,1,4} \Omega_{I_1}^{[I_1} \Lambda_{I_2}^{I_2] \Lambda_{I_3}^{I_3] \Lambda_{I_4}^{I_4} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 10 |
| \(1\) \(1\) \(1\) \(1\) \(1\) | \(\bar{s}^{(1)}(1) \mathcal{P}_{XY}^{(1)}_{1,1,2,1,4} \Omega_{I_1}^{[I_1} \Lambda_{I_2}^{I_2] \Lambda_{I_3}^{I_3] \Lambda_{I_4}^{I_4} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 4 |
| \(1\) \(1\) \(1\) \(1\) \(1\) \(1\) | \(\bar{s}^{(1)}(1) \mathcal{P}_{XY}^{(1)}_{1,1,2,1,4} \Omega_{I_1}^{[I_1} \Lambda_{I_2}^{I_2] \Lambda_{I_3}^{I_3] \Lambda_{I_4}^{I_4} e^{ix_{\mu_P} P_\mu} | 0\rangle\) | 1 |

Table 8. The massless graviton supermultiplet in the supercoherent state basis.

5 Conclusion

In this paper, we have studied the positive energy unitary representations of 2N extended superconformal algebras in six dimensions in a supercoherent state basis. These supercoherent states represent conformal superfields in \(d = 6\). The ultra short doubleton supermultiplets of \(OSp(8^*|2N)\) correspond to the massless conformal superfields in \(d = 6\). The massive conformal superfields are defined by those representations of \(OSp(8^*|2N)\) that are obtainable by tensoring these doubleton supermultiplets. Those massive superfields obtained by tensoring two copies of the doubletons correspond to massless \(AdS_7\) supermultiplets. For \(N = 2\), the CPT self-conjugate supermultiplet obtained by tensoring two copies of CPT self-conjugate doubleton supermultiplets is simply the graviton supermultiplet of \(AdS_5\) supergroup \(OSp(8^*|4)\). We give explicitly the supercoherent state basis of the graviton supermultiplet and the corresponding superfield. Even though the doubleton representations of \(OSp(8^*|2N)\) do not have a Poincaré limit the representations obtained by tensoring doubletons with each other have a Poincaré limit in \(d = 7\).

The supermultiplets that are “shortened” (i.e. short or of intermediate length) correspond to BPS supermultiplets preserving various amounts of supersymmetry. Using the results of this paper one can study BPS supermultiplets in a supercoherent state basis.

Our results can also be used to write down explicitly infinite spin anti-de Sitter superalgebras as suggested in [30]. These infinite spin superalgebras have been studied by many authors, in particular by M.A. Vasiliev, et.al. [31]. The oscillator realization of \(N = 8\) \(AdS_5\) superalgebra \(SU(2,2|4)\) [3, 22], was used recently to write down infinite spin superalgebras in \(d = 5\) [12]. One
can use the realization of $OSp(8^*|2N)$ in the non-compact basis to write down infinite spin $AdS_7$ superalgebras in a covariant basis and study its unitary representations.

Finally, we should stress that one can define supercoherent state bases for all non-compact supergroups that admit positive energy unitary representations [33].

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Appendix

A Notations

Here we give a list of indices we used in this paper and their ranges:

- $\mu, \nu, \rho, \sigma = 0, 1, \ldots, 5$ six dimensional Minkowski spacetime indices
- $a, b, c, d = 0, 1, \ldots, 7$ $SO^*(8)$ ($\cong SO(6, 2)$) vector indices
- $m, n = 1, 2, \ldots, 6$ $SO(6)$ vector indices
- $\hat{m}, \hat{n} = 1, 2, \ldots, 6$ $SO_c(5, 1)$ vector indices
- $i, j, k, l = 1, 2, 3, 4$ $SU(4)$ indices
- $\hat{i}, \hat{j}, \hat{k}, \hat{l} = 1, 2, 3, 4$ $SU^c(4)$ spinor indices
- $A, B, C, D = 1, 2, \ldots, 8$ $SO^*(8)$ ($\cong SO(6, 2)$) left-handed spinor indices
- $I, J, K, L = 1, 2, \ldots, 2N$ $USp(2N)$ indices in the fundamental representation
- $\kappa, \lambda = 1, 2, \ldots, N$ $SU(N)$ indices in the fundamental representation
- $M, N = 1, 2, 3, 4|1, \ldots, N$ $SU(4|N)$ indices in the fundamental representation
- $\alpha, \beta, \gamma, \delta = 1, 2, 3, 4$ $SU^*(4)$ left-handed spinor indices
- $\dot{\alpha}, \dot{\beta}, \dot{\gamma}, \dot{\delta} = 1, 2, 3, 4$ $SU^c(4)$ right-handed spinor indices
- $X, Y, Z = 1, 2, \ldots, 5$ Internal $SO(5)(\cong USp(4))$ vector indices
- $r, s, t, u = 1, 2, \ldots, P$ color indices

The $USp(2N)$ symplectic invariant tensor is taken to be

$$\Omega_{IJ} = \Omega^IJ = \begin{pmatrix} O & I \\ -I & O \end{pmatrix}_{2N \times 2N}, \quad \text{(A.1)}$$

so that

$$\Omega_{IJ} \Omega^{JK} = -\delta^K_I, \quad \text{(A.2)}$$

and is used to lower/raise the $USp(2N)$ spinor indices as follows:

$$\lambda_I = \lambda^J \Omega_{JI}, \quad \lambda^I = \Omega^{IJ} \lambda_J. \quad \text{(A.3)}$$
B Six dimensional \( \Gamma \) matrices

Our choice of \( \Gamma \)-matrices is given below:

\[
\begin{align*}
\Gamma_0 &= \sigma_3 \otimes I_2 \otimes I_2, \\
\Gamma_1 &= i\sigma_1 \otimes \sigma_2 \otimes I_2, \\
\Gamma_2 &= i\sigma_1 \otimes \sigma_1 \otimes \sigma_2, \\
\Gamma_3 &= i\sigma_1 \otimes \sigma_3 \otimes \sigma_2, \\
\Gamma_4 &= i\sigma_2 \otimes I_2 \otimes \sigma_2, \\
\Gamma_5 &= i\sigma_2 \otimes \sigma_2 \otimes \sigma_1, \\
\end{align*}
\]  

(B.1)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the Pauli matrices and \( I_2 \) is the \( 2 \times 2 \) identity matrix.

Therefore,

\[
\Gamma_7 = -\Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 = -\sigma_2 \otimes \sigma_2 \otimes \sigma_3. \quad \text{(B.2)}
\]

All these matrices have the index structure \((\Gamma_\mu)^B_A\). We raise or lower these \( SO^*(8) \) left-handed spinor indices using the charge conjugation matrix \( C \) as shown below:

\[
\begin{align*}
(\Gamma_\mu)^{AB} &= (\Gamma_\mu)^C_A C_{CB}, \\
(\Gamma_\mu)^{AB} &= C^{AC} (\Gamma_\mu)^B_C, \quad \text{(B.3)}
\end{align*}
\]

where

\[
C_{AB} = C^{AB} = \begin{pmatrix} O & I \\ I & O \end{pmatrix}_{8 \times 8}
\]

\[
= -\Gamma_1 \Gamma_2 \Gamma_3 \\
= i\Gamma_6 \Gamma_4 \Gamma_5 \Gamma_7. \quad \text{(B.4)}
\]

C \( SU^*(4) \) \( \Sigma \) matrices

The \( \Sigma \) matrices in \( d = 6 \), the analogs of Pauli matrices \( \sigma_\mu \) in \( d = 4 \), which satisfy the equation \((\Gamma_\mu)^B_A \) are given below.

\[
\begin{align*}
(\Sigma_0)_\alpha^\beta &= -i\sigma_2 \otimes \sigma_3, \\
(\Sigma_1)_\alpha^\beta &= i\sigma_2 \otimes I_2, \\
(\Sigma_2)_\alpha^\beta &= i\sigma_1 \otimes \sigma_2, \\
(\Sigma_3)_\alpha^\beta &= i\sigma_3 \otimes \sigma_2, \\
(\Sigma_4)_\alpha^\beta &= I_2 \otimes \sigma_2, \\
(\Sigma_5)_\alpha^\beta &= \sigma_2 \otimes \sigma_1, \\
(\Sigma_0)_\dot{\alpha}^\dot{\beta} &= i\sigma_2 \otimes \sigma_3, \\
(\Sigma_1)_\dot{\alpha}^\dot{\beta} &= -i\sigma_2 \otimes I_2, \\
(\Sigma_2)_\dot{\alpha}^\dot{\beta} &= -i\sigma_1 \otimes \sigma_2, \\
(\Sigma_3)_\dot{\alpha}^\dot{\beta} &= -i\sigma_3 \otimes \sigma_2, \\
(\Sigma_4)_\dot{\alpha}^\dot{\beta} &= I_2 \otimes \sigma_2, \\
(\Sigma_5)_\dot{\alpha}^\dot{\beta} &= \sigma_2 \otimes \sigma_1. \\
\end{align*}
\]  

(C.1)

Equations \((4 - 10)\) lead one to the following matrices:

\[
\begin{align*}
(\Sigma_0)_\alpha^\beta &= -I_2 \otimes I_2, \\
(\Sigma_1)_\alpha^\beta &= I_2 \otimes \sigma_3, \\
(\Sigma_2)_\alpha^\beta &= -\sigma_3 \otimes \sigma_1, \\
(\Sigma_3)_\alpha^\beta &= \sigma_1 \otimes \sigma_1, \\
(\Sigma_4)_\alpha^\beta &= \sigma_2 \otimes \sigma_1, \\
(\Sigma_5)_\alpha^\beta &= -I_2 \otimes \sigma_2, \\
(\Sigma_0)_\dot{\alpha}^\dot{\beta} &= -I_2 \otimes I_2, \\
(\Sigma_1)_\dot{\alpha}^\dot{\beta} &= I_2 \otimes \sigma_3, \\
(\Sigma_2)_\dot{\alpha}^\dot{\beta} &= -\sigma_3 \otimes \sigma_1, \\
(\Sigma_3)_\dot{\alpha}^\dot{\beta} &= \sigma_1 \otimes \sigma_1, \\
(\Sigma_4)_\dot{\alpha}^\dot{\beta} &= -\sigma_2 \otimes \sigma_1, \\
(\Sigma_5)_\dot{\alpha}^\dot{\beta} &= I_2 \otimes \sigma_2. \\
\end{align*}
\]  

(C.2)
D  $SO(5)$ $\gamma$ matrices

In order for one to express the lowest weight vectors in the massless (in AdS$_7$ sense) graviton supermultiplet in a $USp(4)$ covariant form, we introduce the $4 \times 4$ gamma matrices of $Spin(5) (\cong SO(5))$ $(\gamma_X)_I^J$ where $X = 1, 2, \ldots, 5$ are the vector indices and $I, J = 1, 2, 3, 4$ are the spinor indices.

These gamma matrices satisfy the Clifford algebra
\begin{equation}
\{ \gamma_X, \gamma_Y \} = 2\delta_{XY},
\end{equation}
and are symplectic traceless:
\begin{equation}
\Omega^{IJ}(\gamma_X)_{IJ} = (\gamma_X)_I^J = 0.
\end{equation}

The matrices
\begin{equation}
\Sigma(M_{XY}) = \frac{i}{4}[\gamma_X, \gamma_Y]
\end{equation}
generate a four dimensional spinor representation of the algebra of $Spin(5)$,
\begin{equation}
[M_{XY}, M_{X'Y'}] = i(\delta_{XY'M}M_{X'Y'} - \delta_{X'Y'M}M_{XY'} - \delta_{Y'Y'M}M_{X'X} - \delta_{X'X'M}M_{Y'Y'}).\end{equation}

It is worth noting that the $USp(4)$ spinor indices of these $\gamma$-matrices are lowered (raised) by use of the symplectic invariant tensor $\Omega^{IJ}$ introduced in the paper. In particular, we have
\begin{equation}
(\gamma_X)^{IJ} = \Omega^{IK} \Omega^{LJ}(\gamma_X)_{KL},
\end{equation}
\begin{equation}
(\gamma_X)_{IJ} = (\gamma_X)^{KL} \Omega_{KI} \Omega_{LI}.
\end{equation}

Moreover, these gamma matrices can be chosen so that the type (1,1) $(\gamma_X)_I^J$ form are pure anti-hermitian and the type (0,2) $(\gamma_x)_{IJ}$ and type (2,0) $(\gamma_x)^{IJ}$ forms are anti-symmetric, viz.,
\begin{equation}
(\gamma_x)_I^J \Omega_{IK} = (\gamma_x)_{IK} = -(\gamma_x)_{KI},
\end{equation}
and
\begin{equation}
((\gamma_x)_I^J) \dagger = (\gamma_x)^{IJ} = -(\gamma_x)_I^J.
\end{equation}

Now from [14], we know that the lowest weight vectors in the massless graviton supermultiplet transform under one of the following irreducible representations of $USp(4)$: 1, 4, 5, 10, 14 or 16. We consider each of the states in the massless graviton supermultiplet separately. We let the color indices $r, s = 1, 2$.

- $|\tilde{0}\rangle$ is a 1 of $USp(4)$.
- $\Lambda^I(r)|\tilde{0}\rangle$ is a 4 of $USp(4)$.
- $\Lambda^I(r)\Lambda^J(s)|\tilde{0}\rangle$ is reducible under $USp(4)$. One finds the following decomposition into irreducible components:
\begin{equation}
\Lambda^I(r)\Lambda^J(s)|\tilde{0}\rangle \rightarrow \Lambda^{(I}(r)\Lambda^{J)}(s)\epsilon_{rs}|\tilde{0}\rangle \oplus (\gamma_X)_{IJ}\Lambda^I(r)\Lambda^J(s)|\tilde{0}\rangle \\
\oplus \Lambda^I(r)\Lambda^J(s)\Omega_{IJ}|\tilde{0}\rangle \\
= 10 \oplus 5 \oplus 1.
\end{equation}
\( \Lambda^I(r) \Lambda^J(s) \Lambda^K(t) |\bar{0}\rangle \) is reducible. It can be decomposed into irreducible parts as follows:

\[
\Lambda^I(r) \Lambda^J(s) \Lambda^K(t) |\bar{0}\rangle \rightarrow \Lambda^I(r) \Lambda^J(s) \Lambda^K(t) |\bar{0}\rangle \oplus \tilde{\psi}_X^K |\bar{0}\rangle
\]

\[= 4 \oplus 16. \tag{D.9} \]

where we have defined

\[
\psi_X^K = (\gamma_X)_IJ \Lambda^I(r) \Lambda^J(s) \Lambda^K(t)
\]

\[
\tilde{\psi}_X^K = \psi_X^K - \frac{1}{5} (\gamma_Y)_K^J \left[ (\gamma_Y)^J_I \psi_Y^I \right]. \tag{D.10} \]

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