Tearing of thin sheets: Cracks interacting through an elastic ridge

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We study the interaction between two cracks propagating quasistatically during the tearing of a thin brittle sheet. We show that they attract each other following a path described by a power law resulting from the competition between elastic and fracture energies. The power law exponent (8/11) is in close agreement with experiments. We also show that a second (asymptotic) regime, with an exponent of 9/8, emerges for small distances between the two crack tips due to the finite transverse curvature of the elastic ridge joining them.

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I. INTRODUCTION

Cracks and fractures are very common phenomena occurring in various contexts [1–4]. They are observed during the desiccation of films made of colloidal suspensions, like bentonite clay or cornstarch [5–11], in sol-gel films [12, 13], in broken objects like windows [14–16] or in sea ice [17, 18] and ice floes collisions [19, 20].

A material fractures when sufficient stress is applied at the level of its elementary constituents to break the bonds that hold them together. This process occurs mainly at the atomic scale near the crack tip, where the energy is dissipated in the sheet. Nevertheless, macroscopic parameters, like work of fracture γ or fracture toughness K, can be defined and measured to describe the progression of cracks when the material properties are uniform without necessarily resorting to microscopic analysis. The classical fracture theories, initially formulated by Griffith and Irwin [21, 22], reliably describe the onset of crack motion but there is no general theory able to predict the path of a crack as it propagates. Understanding and predicting the propagation of a crack in a brittle material is a central challenge in fracture mechanics [23].

There are three ways of applying a force to enable a crack to propagate: in-plane tensile (opening mode) or shear (sliding mode) loading and out-of-plane shear (tearing mode) loading. Thin films offer an efficient setup to study the tearing mode with some practical interests on the material properties and to scale only with the sheet thickness [38]. With a given peeling angle, see Fig. 1. In this configuration and when the film adheres to a flat substrate, the balance between fracture, adhesion and bending energies yields to converging linear crack paths [34]. This system has been used to study mechanical properties of graphene and to show that the curvature of the substrate modifies the crack paths leading even to diverging trajectories [36]. When adhesion is negligible, the crack paths are no longer linear and follow power laws with characteristic exponents: 3/4 in the “peeling” configuration with a peeling angle equals to π and 2/3 in the “trousers” configuration [37, 38]. Surprisingly, in contrast with results obtained for adhesive sheets, the crack path was found to be independent on the material properties and to scale only with the sheet thickness [38].

We revisit this system in the peeling configuration by using the formalism developed in Ref. [34] and by analysing the elastic energy of the film essentially contained in the ridge joining the two cracks. We find that both elastic and fracture energies determine the crack paths.

II. SETUP AND MAIN EQUATIONS

Figure 1 shows pictures and schematics of the system under consideration. A thin film is clamped on a flat plate with narrow adhesive tapes along its borders. There is no significant adhesion between the film and the plate. Two parallel notches, separated by a distance W0, are cut on one of its edges such that a rectangular flap is created. The flap is pulled with a peeling angle equals to π at constant slow speed (in the range 0.05 – 1.5 mm/s [38]) such as the cracks propagate quasistatically. The two crack tips propagate both forwards along the x-axis and inwards (towards y = 0) such that they eventually annihilate and a pointy flap is detached from the film.

The pulling force F applied to the flap deforms the fold joining the flap to the film such that, at the onset of crack motion, a small ridge focussing the elastic energy is formed, see Fig. 1(b). The shape of this ridge is shown in Fig. 1(e)-(g). This ridge possesses two curvatures: one in the longitudinal direction joining the flap to the film and

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We revisit this system in the peeling configuration by using the formalism developed in Ref. [34] and by analysing the elastic energy of the film essentially contained in the ridge joining the two cracks. We find that both elastic and fracture energies determine the crack paths.
another one in the transverse direction along the $y$-axis with a maximal deviation from a straight line denoted $b$ (sag of the ridge), see Fig. 1(e). This second curvature is due to the pulling force $F$ which applies along the entire width of the ridge whereas the resistive fracture force applies only at its edges where the crack tips are located. This leads also to the formation of two pinched edges, see Fig. 1(f). Therefore, the ridge possesses the characteristics of a Lobkovsky-Witten ridge which appears generically between two points of high curvature in thin sheets. Notice that for the tearing of adhesive sheets, the corresponding fold does not possess a transverse curvature because the adhesive force applies along its whole width and prevents any transverse bending. The elastic energy stored in the ridge can be released in two ways: by decreasing the longitudinal curvature of the ridge (advancing the crack in the pulling direction) or by simply reducing the width of the ridge (the cracks move inwards). The actual direction followed by the cracks is a combination of both effects.

The formalism we used to describe the system has been introduced in Ref. and is briefly reproduced here for self-containment. The total energy of the system is

$$U = U_E + 2\gamma ts,$$  \hspace{1cm} (1)

where the first term is the elastic energy, which is essentially focussed in the ridge, and the second one is the fracture energy for the two cracks. $t$ is the film thickness, $s$ is the crack length and $\gamma$ is the work of fracture of the film. The position of the crack tips is denoted $\ell$ and the position of the border of the flap where the force is applied is denoted $x$, see Fig. 1(d). The excess of length $2\ell - x = \lambda$ is the length of the ridge, see Fig. 1(c),(g).

As shown in the next section, the elastic energy of the ridge depends only on its width, $W$, and its length, $\lambda$: $U_E = U_E(\lambda = 2\ell - x, W)$.

In order to derive the relevant equations in a simple way, we first neglect the transverse curvature of the ridge ($\varphi = 0$). Assuming that the crack tip moves to a position that minimizes the total energy $dU/ds = 0$, where $ds$ is an infinitesimal increment of crack length, we have from Eq. (1)

$$-2\partial_W U_E \sin \theta + \partial_x U_E \cos \theta + 2\gamma t = 0,$$  \hspace{1cm} (2)

where $d\ell/ds = \cos \theta$ and $dW/ds = -2\sin \theta$ (by convention a positive $\theta$ corresponds to a decrease of $W$ as the crack advances). This equation is simply the balance of forces projected along the crack direction. The force, $F$, needed to make the crack propagate is given by the work theorem as

$$F = \partial_x U_E = -\partial_\theta U_E = -\frac{1}{2} \partial_\theta U_E.$$  \hspace{1cm} (3)

The fracture path is obtained by requiring that the tear follows the direction where the force is minimal, $\partial_\theta F = 0$, which is equivalent to the condition $\partial_\theta (dU/ds) = 0$. The second equation, obtained through a derivation of Eq. (2) with respect to $\theta$ and by using Eq. (3), reads

$$-\partial_W U_E \cos \theta + F \sin \theta = 0,$$  \hspace{1cm} (4)

and represents the balance of force in a direction normal to the crack direction. Instead of using the crack direction as a reference, the forces can be projected along the
x and y-axis to obtain equations equivalent to Eqs. (2)
and (4):

\[ F = \gamma t \cos \theta \quad \text{and} \quad \partial_W U_E = \gamma t \sin \theta. \] (5)

Equations (5) correspond to the projections along the forward and sidewise directions of the forces shown in Fig. (c), (d) when \( \varphi = 0 \). Therefore, a finite transverse curvature of the ridge modifies Eqs. (5) as follow

\[ F + \partial_W U_E \sin \varphi = \gamma t \cos \theta, \]
\[ \partial_W U_E \cos \varphi = \gamma t \sin \theta. \] (6a, 6b)

Notice however that, as shown below, the influence of the angle \( \varphi \) is essentially negligible except in a small region, \( W \ll W_c \), near the tip of the detached flap where the two cracks meet. Equations (6a) and (3) allow to determine the dependence of the elastic energy parameters as a function of the work of fracture and Eq. (6b) gives then the expression of \( \theta \) as a function of the system parameters. Since \( \theta \) is the local angle between the tangent to the crack path and the x-axis, the path is determined from the differential equation \( dW/d\ell = -2 \tan \theta \) with the initial condition \( W(0) = W_0 \). However, it is more convenient to place the point where the two cracks meet at the origin of the coordinates and to consider the increase of the distance \( W \) between the two cracks as a function of the distance to the origin (which we still denote \( \ell \) for simplicity). This is achieved with the differential equation

\[ \frac{dW}{d\ell} = 2 \tan \theta \quad \text{and} \quad W(0) = 0. \] (7)

### III. ELASTIC ENERGY

In order to compute explicitly the crack path, we need to obtain the elastic energy of the system. As seen in Fig. (b), the elastic energy focuses in a small folded region joining the tip of the film between the two crack tips as the applied force increases up to the onset of crack displacement. As mentioned above and seen in Fig. (c)-(g), this folded region, containing essentially all the elastic energy, possesses the characteristics of a Lobkovsky-Witten ridge. We assume that such a ridge describes the elastic energy of our system. The geometry and the elastic energy of this ridge are known \([39–43]\) and are recovered in the Appendix using a simple scaling approach:

\[ U_E = C_R B (W \alpha^7/t)^{1/3}, \] (8a)
\[ \lambda \approx h = C_\lambda (W^2 t/\alpha)^{1/3}, \] (8b)

where \( C_R = R(12(1 - \nu^2))^{1/6} \) with \( R = 1.20 \pm 0.04 \) \([41]\), \( B = Et^3/(12(1 - \nu^2)) \) is the bending modulus, \( E \) is the Young modulus, \( \nu \) is the Poisson ratio and \( h \) is the height of the ridge in the z-direction and is proportional to its length \( \lambda \) (see Fig. (c)). The constant \( C_\lambda \) is unknown and is considered as a free parameter of order 1. The parameter \( \alpha \) is the dihedral angle of the ridge (see Fig. (a)) whose value depends on the applied force \( F \) and thus on the system parameters. The dihedral angle is eliminated between Eqs. (8a) and (8b) to obtain the elastic energy as a function of the width \( W \) and the length \( \lambda \) of the ridge as assumed to derive Eqs. (6):

\[ U_E(\lambda, W) = C_R^2 B t^2 W^5 \lambda^{-7}. \] (9)

Therefore we have

\[ F = -\partial_W U_E = 7C_R^2 B t^2 W^5 \lambda^{-8}, \]
\[ \partial_W U_E = 5C_R^2 B t^2 W^4 \lambda^{-7}. \] (10a, 10b)

The remaining quantity to determine before computing the crack path is the angle \( \varphi \). From Fig. (e), it is expected that \( b/W \ll 1 \) leading to \( \sin \varphi \approx b/W \) and \( \cos \varphi \approx 1 \). When \( W \) decreases as the two cracks get closer, the ratio \( b/W \) could, a priori, increase to reach values of order 1. However, we show in the appendix that \( b \approx \lambda \alpha/4 \) which combined with Eq. (8b) gives

\[ b/W \approx C_\lambda^2 W t/(4 \alpha^2). \] (11)

Therefore, we have to evaluate this quantity a posteriori, once \( \lambda \) is determined, to verify that it is indeed small. We assume \( b/W \ll 1 \) for the moment and we verify below the consistency of this assumption. We thus have

\[ \sin \varphi \approx C_\lambda^2 W t \lambda^{-2} \quad \text{and} \quad \cos \varphi \approx 1. \] (12)

Using Eqs. (6a) and (10) together with the expression of the angle \( \varphi \) we obtain the equation giving the length of the ridge

\[ 7C_R^2 B t^2 W^5 \lambda^{-8} + 5C_\lambda^1 B t^3 W^5 \lambda^{-9} = \gamma t \cos \theta. \] (13)

Depending on which of the two terms of the left-hand side of Eq. (13) dominates, we get different regimes. The first term dominates when

\[ \lambda \gg (5C_\lambda^1/t) t \] (14)

which is expected to be the dominant regime. Physically, \( \lambda \) cannot be smaller than the film thickness. Therefore, the second term never dominates but its influence increases as \( \lambda \) approaches \( t \) and can be estimated by neglecting the first term. The second term encodes the influence of the transverse curvature of the ridge which is thus seen to be essentially negligible.

### IV. SCALING FOR \( W \gg W_c \)

Neglecting the second term of the left-hand side of Eq. (13), we obtain

\[ \lambda = \left[7C_R^2 B t W^5 / (\gamma \cos \theta)\right]^{1/8}. \] (15)

Even if we have neglected the term containing \( \sin \varphi \) we still have to consider the condition ensuring the validity...
of Eq. (12), namely $b/W \ll 1$, because we have to verify
the validity of the condition (14) involving both terms. The
condition $b/W \ll 1$ is verified explicitly by using Eqs. (11) and (15) and
leads to the equivalent condition $W \gg W^* \sim \gamma/E \sim 1 \mu m$[38] where we used some typical values for bidirectional polypropylene films used in the
experiments [34 38]. The domain of validity of the approxima-
tion consisting in neglecting the second term of Eq. (13) is made explicit by using Eqs. (14) and (15):

$$W \gg W_c \sim t \left( \frac{\gamma}{Et} \right)^{1/5}.$$  \hspace{1cm} (16)

The length $W^*$ fixes the domain of validity of Eq. (13)
whereas $W_c$ fixes the domain of validity of the approxi-

The equation governing the crack path is obtained by combining Eq. (6b) with Eqs. (10b), (12) and (15)

$$\tan \theta[\cos \theta]^{\frac{8}{11}} = 5 \left[ \frac{C_\lambda}{\gamma} \right]^{\frac{7}{8}} \left[ \frac{C_R \gamma}{Et} \right]^{\frac{1}{8}} \left[ \frac{t}{W} \right]^{\frac{1}{8}} \left[ \frac{\ell}{t} \right]^{\frac{1}{8}}.$$ \hspace{1cm} (17)

For the typical value of $\theta$ reported in the experiments [20 38], the left-hand side can be approximated by $\tan \theta$. We can now solve Eq. (7) to obtain the crack path equation

$$\frac{W(\ell)}{t} = 1.56 \left[ \frac{C_\lambda^2 C_R}{\gamma(1-\nu^2)} \right]^{\frac{1}{11}} \left[ \frac{Et}{\gamma} \right]^{\frac{1}{11}} \left[ \frac{\ell}{t} \right]^{\frac{1}{11}} \left[ \frac{W}{t} \right]^{\frac{1}{11}}.$$ \hspace{1cm} (18)

The exponent $8/11 \simeq 0.73$ is very close to the exponent measured experimentally and fits quite well the data, see Fig. 2. The prefactor $Et/\gamma$ depends on the material parameters and reflects the competition between elastic and fracture energies as expected. The fracture energy favours straight crack paths with $\theta = 0$ to minimize the crack length whereas the elastic energy favours $\theta = \pi/2$ in order to reduce the size of the ridge as “quickly” as possible. Equation (17) shows these tendencies with $\theta \to 0$ as $\gamma \to \infty$ and $\theta \to \pi/2$ when $Et \to \infty$. The small value of the prefactor exponent explains why a simple rescaling by the film thickness leads nevertheless to a good collapse of the data [38].

V. SCALING FOR $W \ll W_c$

As mentioned above, this regime is never fully reached since the length of the ridge cannot be smaller than the film thickness. The exponent derived here may thus be viewed as an asymptotic limit such that the crack path exponent near the tip of the detached flap should approach this limit. This regime is described by neglecting the first term of the left-hand side of Eq. (13) such that

$$\lambda = \left[ 5C_\lambda^3 C_R \right]^{1/9} \frac{Et^2 W^5}{(\gamma \cos \theta)^{1/9}}.$$ \hspace{1cm} (19)

The condition $b/W \ll 1$ is verified by using Eqs. (11) and (17) and leads to the equivalent condition $W \gg W^* \sim t [\gamma/(Et)]^{2} \sim 10^{-2} \mu m$ for $t \sim 50 \mu m$. The domain of validity of the approximation consisting in neglecting the first term of Eq. (13) is obtained by using Eq. (14) with the reverse inequality sign and Eq. (19). We obtain $W \ll W_c$ where $W_c$ is exactly the same, prefactor included, than the one obtained in Eq. (16) as it should. The necessary condition $W^* \ll W_c$ sets the same limit on the film thickness than in the previous regime: $t \gg W^*$. The equation governing the crack path is obtained by combining Eq. (6b) with Eqs. (10b), (12) and (19)

$$\tan \theta[\cos \theta]^{\frac{8}{11}} = C_\lambda^{\frac{7}{8}} \left[ \frac{5C_R}{12(1-\nu^2)} \right]^{\frac{1}{8}} \left[ \frac{Et}{\gamma} \right]^{\frac{1}{8}} \left[ \frac{W^*}{t} \right]^{\frac{1}{8}}.$$ \hspace{1cm} (20)

This equation predicts a vanishing value of $\theta$ for a vanishing value of $W$. Therefore, since we are considering very small value of $W$ in this section, we can again approximate the left-hand side by $\tan \theta$. The crack path equation is obtained by solving Eq. (7) using Eq. (20):

$$\frac{W(\ell)}{t} = 1.53 \left[ \frac{C_\lambda^{\frac{7}{8}} C_R}{(1-\nu^2)} \right]^{\frac{1}{11}} \left[ \frac{Et}{\gamma} \right]^{\frac{1}{11}} \left[ \frac{\ell}{t} \right]^{\frac{1}{11}}.$$ \hspace{1cm} (21)

The exponent of the crack path increases from $8/11$ to $9/8$ as the distance $W$ between the two crack tips tends to zero in reasonable agreement with data, see Fig. 2.
VI. COMPARISON WITH DATA

Equations (18) and (21) giving the crack path in the two identified regimes are rescaled as follows

\[ W = \left( \frac{Et}{\gamma} \right)^{\frac{1}{2}} W \]  

and \[ \ell = \left( \frac{Et}{\gamma} \right)^{\frac{1}{2}} \ell, \]  

(22)

to obtain

\[ W = C_1 \ell \tilde{\theta}^\nu \quad \text{for} \quad W \gg W_c, \]  

(23a)

\[ W = C_2 \ell^{\frac{3}{2}} \quad \text{for} \quad W \ll W_c, \]  

(23b)

where

\[ C_1 = 1.56 \left( \frac{C_R^2}{1 - \nu^2} \right)^{\frac{1}{7}}, \quad C_2 = 1.53 \left( \frac{C_R^2}{1 - \nu^2} \right)^{\frac{1}{3}}, \]

(24)

\[ W_c = 0.65 \left( [1 - \nu^2] C\lambda^7 C_R^{-1} \right)^{\frac{1}{7}}, \quad C_R = \mathcal{R} [12(1 - \nu^2)]^{\frac{1}{7}}, \]

where \( \mathcal{R} = 1.20 \pm 0.04 \) [11]. Figure 2 shows a nice collapse of the data rescaled with Eqs. (22) together with a good agreement with Eqs. (23). The value \( C_1 = 2.1 \) is obtained from a fit of the data for large \( \ell \) which implies, from Eq. (24), \( C_\lambda \simeq 1.45 \) for \( \nu = 0.3 \) [15]. As expected, the free parameter is of order 1. The parameter \( C_2 \simeq 1.3 \) is then computed from Eq. (24).

In order to obtain the evolution of \( W \) when it is of order 1, we need to solve the problem numerically because both terms of the left-hand side of Eq. (13) have the same order of magnitude. For this purpose, we rescale \( W \) using Eq. (22) and define \( \lambda = \lambda / t \) such that Eq. (13) becomes

\[ 7A W^5 \lambda^{10} + 5AC_R^4 W^5 \lambda^{-9} = 1, \]

(25)

where \( A = C_R^2 C_R / [12(1 - \nu^2)] \) and where we set \( \cos \theta = 1 \) since \( \lambda \) is always small as seen above. Equation (25) together with Eqs. (7), (10b), (12) gives the following differential equation

\[ \frac{dW}{d\ell} = 5A W^4 \lambda^{-7}, \]

(26)

where \( \ell \) has been rescaled using Eq. (22) and \( \sin \theta \) has been replaced by \( \tan \theta \). The differential equation (26) is thus supplemented by an algebraic constraint (25). This semi-explicit differential-algebraic equation is easily solved numerically using, for example, Mathematica. The resulting crack path is reported in Fig. 2 and describes well the data.

The quantities \( W^*, W^* \) and \( t^* \) set limits for the mathematical consistency of the model; they are all satisfied for the experiments we consider. The quantity \( W^* \) is a limit separating the two identified regimes; one of them being only asymptotic. Physically, the length of the ridge is expected to be limited by the film thickness such that \( \lambda \gtrsim 1 \). Equation (25) imposes then \( W \gtrsim W_p = \left[ A(t + 5C_R^2) \right]^{-1/5} \), which provides a physical limit of this model, see Fig. 2.

VII. CONCLUSIONS

We have shown how the energies focussed in the crack tip and in the elastic ridge joining them act together in a non trivial way to produce characteristic crack paths described by a power law with an exponent \( 8/11 \) and a prefactor reflecting the competition between elastic and fracture energies, see Eq. (18). The close agreement with experiments is shown in Fig. 2. In addition, a second regime, induced by the transverse curvature of the ridge, occurs for small distances between the crack tips. This regime is only asymptotic but slightly modifies the crack path such that the exponent of the power law increases to reach values close to \( 9/8 \), see Eq. (21) and Fig. 2. A global rescaling has been found and leads to Eqs. (22)-(24). The governing equation has also been solved numerically to obtain the complete crack path, beside its asymptotic scalings, with a good agreement with experiments using only one free parameter of order 1.

Finally, we expect that a Lobkovsky-Witten ridge emerges only for a peeling angle close to \( \pi \) such that it possesses a transverse curvature and pinched edges. For smaller peeling angles, the ridge should be similar to the one occurring for adhesive sheets [34, 36]. With such a ridge and without adhesion, the crack path is described by a power law with an exponent \( 2/3 \) [26, 36]. Experiments for various peeling angles are needed to test this scaling.

Appendix A: Lobkovsky-Witten ridge

We derive Eqs. (8) reported in the main text in the limit of small dihedral angle \( \alpha \) where the curved parts of the ridge shown in Fig. 3(a,c) can be approximated by arc of circles. Notice that the scalings (8), obtained from a boundary layer analysis, are not restricted to small values of \( \alpha \) [11].

From the triangles \( ABD \), \( BCD \) and \( ABC \) of Fig. 3(b), we have respectively \( \cos \alpha = R_1 / (R_1 + b) \), \( \cos \alpha = (R_1 - a) / R_1 \) and \( \sin \alpha \simeq \alpha = 2(b + a) / h \). The two first relations imply \( a = b \cos \alpha \simeq b \) and \( R_1 = b \cos \alpha (1 - \cos \alpha) \simeq 2b / \alpha^2 \). We thus obtain \( \alpha \simeq 4b / h \). In addition, we have \( \lambda = 2a R_1 \) which is equivalent to \( b \simeq \lambda a / 4 \) as mentioned in the main text. Using the expression of \( \alpha \), we also obtain a relation between the length of the ridge and its height: \( \lambda \simeq h \).

The energy, \( U \), of the ridge is composed essentially of a bending energy, \( U_b \), in the longitudinal direction along its length \( \lambda \) (z-axis) and a stretching energy, \( U_s \), in the transverse direction along its width \( W \) (y-axis), see Fig. 3(a). These energies are localized in a region of area \( S \sim \lambda W \). From Fig. 3(b), the longitudinal curvature is given by \( \kappa \sim a / h^2 \sim b / \lambda^2 \). Therefore, the bending energy reads

\[ U_b \sim \frac{Et^3 (b / \lambda^2)^2}{2} \sim \frac{Et^3 a^2 W}{\lambda}. \]

(A1)

The stretching is due to the sag of the ridge inducing an
increase in length along its width of order \((b/W)^2\). The stretching energy thus reads

\[ U_s \sim Et(b/W)^4 S \sim Et\alpha^4\lambda^5/W^3. \quad (A2) \]

Upon minimization of the total energy \(U = U_b + U_s\) with respect to \(\lambda (\partial U/\partial \lambda = 0)\), we obtain the scalings \(\alpha\) of the main text.

The angle \(\varphi\) is computed from Fig. 3 where we have \(R_2 = W^2/(8b) + b/2\) and \(\sin \varphi = W(2R_2) \approx 4b/W\) at the first order in \(b/W\). Using the expression of \(b\) obtained above, we have \(\sin \varphi \approx \lambda\alpha/W\). Using the expression \(\lambda\alpha/W\) of \(\alpha\), we obtain Eq. \(\alpha\).

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