On the Rise of the Proton Structure Function $F_2$
Towards Low $x$

H1 Collaboration

Abstract:

A measurement of the derivative $(\partial \ln F_2 / \partial \ln x)_{Q^2} \equiv -\lambda(x, Q^2)$ of the proton structure function $F_2$ is presented in the low $x$ domain of deeply inelastic positron–proton scattering. For $5 \cdot 10^{-5} \leq x \leq 0.01$ and $Q^2 \geq 1.5 \text{GeV}^2$, $\lambda(x, Q^2)$ is found to be independent of $x$ and to increase linearly with $\ln Q^2$.

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The inclusive cross section for deeply inelastic lepton-proton scattering is governed by the proton structure function $F_2(x, Q^2)$. Because of the large centre-of-mass energy squared, $s \simeq 10^5$ GeV$^2$, the $ep$ collider HERA has accessed the region of low Bjorken $x$, $x > Q^2 / s > 10^{-5}$, for four-momentum transfers squared $Q^2 > 1$ GeV$^2$. One of the first observations at HERA was of a substantial rise of $F_2$ with decreasing $x$ [1]. However, this rise may be limited at very low $x$ by unitarity constraints.

Perturbative Quantum Chromodynamics (QCD) provides a rigorous and successful theoretical description of the $Q^2$ dependence of $F_2(x, Q^2)$ in deeply inelastic scattering. In the double asymptotic limit, the DGLAP evolution equations [2] can be solved [3] and $F_2$ is expected to rise approximately as a power of $x$ towards low $x$. A power behaviour is also predicted in BFKL theory [4]. The rise is expected eventually to be limited by gluon self interactions in the nucleon [5].

Recently the H1 Collaboration has presented [6] a new measurement of $F_2(x, Q^2)$ in the kinematic range $3 \cdot 10^{-5} \leq x \leq 0.2$ and $1.5 \leq Q^2 \leq 150$ GeV$^2$ based on data taken in the years 1996/97 with a positron beam energy $E_e = 27.6$ GeV and a proton beam energy $E_p = 820$ GeV. The high accuracy of these data allows the derivative

$$\frac{\partial \ln F_2(x, Q^2)}{\partial \ln x} \bigg|_{Q^2} \equiv -\lambda(x, Q^2) \quad (1)$$

to be measured as a function both of $Q^2$ and of $x$ for the first time. Use of this quantity for investigating the behaviour of $F_2$ at low $x$ was suggested in [7].

Here results are presented of a measurement of this derivative in the full kinematic range available. Data points at adjacent values of $x$ and at fixed $Q^2$ are used [6] taking account of the full error correlations and the spacing between the $x$ values. The results obtained are presented in Table [1]. The sensitivity of the derivative to the uncertainty of the structure function $F_L$ [6] throughout the measured kinematic range is estimated to be much smaller than the total systematic error at the lowest values of $x$ and is negligible elsewhere.

As can be seen in Figure [1], the derivative $\lambda(x, Q^2)$ is independent of $x$ for $x \lesssim 0.01$ to within the experimental accuracy. This implies that the $x$ dependence of $F_2$ at low $x$ is consistent with a power law, $F_2 \propto x^{-\lambda}$, for fixed $Q^2$, and that the rise of $F_2$, i.e. $(\partial F_2 / \partial x)_{Q^2}$, is proportional to $F_2 / x$. There is no experimental evidence that this behaviour changes in the measured kinematic range.

The derivative is well described by the NLO QCD fit to the H1 cross-section data [6], see Figure [1]. In DGLAP QCD, for $Q^2 > 3$ GeV$^2$, the low $x$ behaviour is driven solely by the gluon field, since quark contributions to the scaling violations of $F_2$ are negligible. At larger $x$ the transition to the valence-quark region causes a strong dependence of $\lambda$ on $x$ as indicated by the QCD curves in Figure [1].

Figure [6] shows the measured derivative as a function of $Q^2$ for different $x$ values. The derivative is observed to rise approximately logarithmically with $Q^2$. It can be represented by a function $\lambda(Q^2)$ which is independent of $x$ within the experimental accuracy.

\[1\] Note that derivatives at adjacent $x$ values are thus anti-correlated. The data points at $Q^2 = 150$ GeV$^2$ are obtained from the H1 measurement [6].
The function $\lambda(Q^2)$ is determined from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data, restricted to the region $x \leq 0.01$. The results for $c(Q^2)$ and $\lambda(Q^2)$ are presented in Table 2 and shown in Figure 3. The coefficients $c(Q^2)$ are approximately independent of $Q^2$ with a mean value of 0.18. As can be seen, $\lambda(Q^2)$ rises approximately linearly with $\ln Q^2$. This dependence can be represented as $\lambda(Q^2) = a \cdot \ln[Q^2/\Lambda^2]$, see Figure 3. The coefficients are $a = 0.0481 \pm 0.0013^{\text{(stat)}} \pm 0.0037^{\text{(syst)}}$ and $\Lambda = 292 \pm 20^{\text{(stat)}} \pm 51^{\text{(syst)}}$ MeV, obtained for $Q^2 \geq 3.5$ GeV$^2$. The values of $\lambda(Q^2)$ are more accurate than data hitherto published by the H1 [9] and ZEUS [10] Collaborations.

Below the deeply inelastic region, for fixed $Q^2 < 1$ GeV$^2$, the simplest Regge phenomenology predicts that $F_2(x, Q^2) \propto x^{-\lambda}$ where $\lambda = \alpha_p(0) - 1 \simeq 0.08$ is given by the Pomeron intercept independently of $x$ and $Q^2$ [11]. When extrapolating the function $\lambda(Q^2)$ into the lower $Q^2$ region it has the value of 0.08 at $Q^2 = 0.45$ GeV$^2$, see also [10].

To summarise, the derivative $(\partial \ln F_2/\partial \ln x)_{Q^2}$ is measured as a function both of $x$ and of $Q^2$ and is observed to be independent of Bjorken $x$ for $x \lesssim 0.01$ and $Q^2$ between 1.5 and 150 GeV$^2$. Thus the behaviour of $F_2$ at low $x$ is consistent with a dependence $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ throughout that region. At low $x$, the exponent $\lambda$ is observed to rise linearly with $\ln Q^2$ and the coefficient $c$ is independent of $Q^2$ to within the experimental accuracy. There is no sign that this behaviour changes within the kinematic range of deeply inelastic scattering explored.

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Figure 1: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the dashed curves represent the extrapolation of the QCD fit below $Q^2 = 3.5$ GeV$^2$. 
Figure 2: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the minimum $Q^2$ value of the data included in this fit is $Q^2 = 3.5$ GeV$^2$. 
Figure 3: Determination of the coefficients $c(Q^2)$ (upper plot) and of the exponents $\lambda(Q^2)$ (lower plot) from fits of the form $F_2(x,Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data [6] for $x \leq 0.01$; the inner error bars illustrate the statistical uncertainties, the full error bars represent the statistical and systematic uncertainties added in quadrature. The straight lines represent the mean coefficient $c$ (upper plot) and a fit of the form $a\ln[Q^2/\Lambda^2]$ (lower plot), respectively, using data for $Q^2 \geq 3.5$ GeV$^2$. 
### Table 1: Measurement of the derivative $\lambda = -(\partial \ln F_2/\partial \ln x)_{Q^2}$ at fixed $Q^2$. For the systematic uncertainties the correlations between adjacent $x$ values are taken into account. The total error is the squared sum of the statistical and systematic uncertainties, given as absolute values.
| $Q^2[GeV^2]$ | $c$     | $\delta^c_{sta}$ | $\delta^c_{tot}$ | $\lambda$ | $\delta^\lambda_{sta}$ | $\delta^\lambda_{tot}$ |
|------------|---------|------------------|------------------|-----------|--------------------------|--------------------------|
| 1.5        | 0.10    | +0.05            | +0.14            | 0.20      | 0.04                     | 0.10                     |
| 2.0        | 0.172   | 0.005            | 0.024            | 0.159     | 0.003                    | 0.015                    |
| 2.5        | 0.167   | 0.003            | 0.012            | 0.169     | 0.002                    | 0.009                    |
| 3.5        | 0.180   | 0.003            | 0.009            | 0.179     | 0.002                    | 0.007                    |
| 5.0        | 0.181   | 0.004            | 0.011            | 0.196     | 0.003                    | 0.008                    |
| 6.5        | 0.190   | 0.005            | 0.014            | 0.202     | 0.004                    | 0.009                    |
| 8.5        | 0.181   | 0.005            | 0.014            | 0.223     | 0.004                    | 0.010                    |
| 12.0       | 0.182   | 0.005            | 0.015            | 0.240     | 0.004                    | 0.011                    |
| 15.0       | 0.184   | 0.003            | 0.013            | 0.250     | 0.002                    | 0.010                    |
| 20.0       | 0.186   | 0.003            | 0.013            | 0.260     | 0.003                    | 0.011                    |
| 25.0       | 0.178   | 0.004            | 0.017            | 0.274     | 0.004                    | 0.014                    |
| 35.0       | 0.180   | 0.005            | 0.018            | 0.286     | 0.005                    | 0.016                    |
| 45.0       | 0.173   | 0.007            | 0.019            | 0.302     | 0.007                    | 0.017                    |
| 60.0       | 0.158   | 0.010            | 0.023            | 0.332     | 0.011                    | 0.024                    |
| 90.0       | 0.197   | +0.025           | +0.045           | 0.304     | 0.022                    | 0.040                    |
| 120.0      | 0.117   | +0.045           | +0.065           | 0.408     | 0.064                    | 0.089                    |
| 150.0      | 0.17    | +0.04            | +0.12            | 0.36      | 0.04                     | 0.11                     |

Table 2: The coefficients $c$ and exponents $\lambda$ from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ using H1 $F_2$ data [9], for $x \leq 0.01$, taking into account the systematic error correlations. Here $\delta_{sta}$ denotes the statistical uncertainty and $\delta_{tot}$ comprises all uncertainties added in quadrature. The uncertainties are given as absolute values. They are symmetric to very good approximation, apart from the uncertainties of the coefficient $c(Q^2)$ at the edges of the $Q^2$ region.