CONSTRaining Tidal Dissipation in Stars From the Destruction Rates of Exoplanets

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Received 2011 November 23; accepted 2012 March 5; published 2012 May 10

ABSTRACT

We use the distribution of extrasolar planets in circular orbits around stars with surface convective zones detected by ground-based transit searches to constrain how efficiently tides raised by the planet are dissipated on the parent star. We parameterize this efficiency as a tidal quality factor (\(Q_\ast\)). We conclude that the population of currently known planets is inconsistent with \(Q_\ast < 10^7\) at the 99% level. Previous studies show that values of \(Q_\ast\) between \(10^5\) and \(10^7\) are required in order to explain the orbital circularization of main-sequence low-mass binary stars in clusters, suggesting that different dissipation mechanisms might be acting in the two cases, most likely due to the very different tidal forcing frequencies relative to the stellar rotation frequency occurring for star–star versus planet–star systems.

Key words: convection – planet–star interactions – stars: interiors – stars: rotation – stars: winds, outflows – turbulence

Online-only material: color figures

1. INTRODUCTION

Exoplanets with orbital distances \(\leq 0.1\) AU from their host stars, called close-in exoplanets, have presented an especially puzzling challenge to theories of planet formation. The protoplanetary disk is too warm (2000 K) so close to a star to allow the condensation and accumulation of icy and rocky material required to form planets (Lin et al. 1996; Miller et al. 2009; Ibgui & Burrows 2009).

For close-in exoplanetary systems, their mutual tidal gravities induce significant tidal bulges in the planets and stars. Dissipation of the accompanying tidal energy drives obliquities to zero and rotation rates to near synchronous processes that probably take millions of years for planets but billions of years for stars. While its orbit is eccentric, dissipation of tidal energy within a planet can reduce the orbital semimajor axis and eccentricity, as well as warming the planet’s interior with significant consequences for the planet’s thermal evolution (Jackson et al. 2008b; Ibgui & Burrows 2009; Liu et al. 2008; Miller et al. 2009). The majority of planet-hosting stars rotate more slowly than their close-in planets revolt, and so tides raised on these stars also reduce eccentricities and semimajor axes.

Although the effects of tides raised on close-in exoplanets become negligible as eccentricities shrink, tides on host stars continue to reduce semimajor axes long after eccentricities are negligible, as long as the stellar rotation rate is smaller than the orbital mean motion. For the systems for which the rotation of the star is observationally constrained, the stellar spin period is known to be longer than the orbital period. Typically, planet–hosting stars are older than 1 Gyr, and so stars without reported rotation rates likely rotate slower than their planets revolve. Moreover, observational biases favor detection of planets around slow rotators. As a result, the tides on the host star tend to dominate the long-term tidal evolution of close-in planets. Eventually, the planets may cross their Roche limits (0.007 AU for a Jupiter-like planet around a Sun-like star), where they are tidally disrupted.

On the other hand, tidal spin-up may synchronize the stellar rotation to the orbital period, in which case the planet will eventually reach a stable orbit. The total angular momentum of the system determines which scenario occurs (Counselman 1973; Greenberg 1974). Levrard et al. (2009) show that all systems with transiting planets found to date by ground transit search surveys except HAT-P 2 b have insufficient angular momentum to prevent this destruction. Loss of angular momentum through shedding of stellar wind dooms even HAT-P 2 b. Thus, given enough time, the loss of close-in exoplanets through orbital decay is inevitable.

Tidal evolution of an orbit increases rapidly for decreasing semimajor axis, and so the distribution of semimajor axes for observed planets is sensitive to the rate of tidal dissipation. Since the probability for a planet to transit its host star increases for planets nearer their stars, transiting planets are especially susceptible to tidal effects. The rate of orbital decay and frequency of tidal destruction also depends sensitively on the rate of tidal dissipation within the host star. This rate is related to the efficiency parameter \(Q_\ast\) (Goldreich 1963): Larger \(Q_\ast\) corresponds to less tidal dissipation and slower orbital evolution. The origins of tidal dissipation within gaseous planets and stars have been studied extensively, but remain poorly understood, with estimates based on theoretical and observational studies ranging from \(10^5\) to \(10^9\) (cf. Zahn 1966, 1970, 1975, 1977, 1989; Goldreich & Nicholson 1977; Scharlemann 1981, 1982; Goodman & Oh 1997; Papaloizou & Savonije 1997; Savonije & Papaloizou 1997; Papaloizou et al. 1997; Terquem et al. 1998; Ogilvie & Lin 2004, 2007; Wu 2005a, 2005b; Papaloizou & Ivanov 2005; Meibom & Mathieu 2005; Ivanov & Papaloizou 2007; Penev et al. 2007, 2009a, 2009b, 2011; Ogilvie 2009; Penev & Sasselov 2011).

Consequently, the time until a planet crosses its Roche limit and is removed, which we will call a planet’s time left (TL), depends both on \(Q_\ast\) and on its current semimajor axis, among other parameters. For a population of tidally evolving planets, we expect to find few planets with TL much less than the
whole lifetime or current system age. Otherwise, we would conclude that we have caught a large fraction of the planets in the last extremely short moments of their lives, just before they are disrupted by their star. By tuning $Q_*$ until we generate a statistically likely distribution of $T_L$, we can constrain $Q_*$ and the frequency of tidal disruption of exoplanets.

As discussed in Section 2, observational biases have important and complex influences on the distribution of calculated $T_L$ values for transiting planets and must be considered in order to produce statistically reliable constraints on $Q_*$. Several previous studies (e.g., Carone & Pätzold 2007) have attempted to place constraints on $Q_*$ using considerations similar to ours but only applied to individual planets. Results from some of those studies are consistent with $Q_* > 10^7$, but inferences based on the orbital evolution of a single planet may not be statistically meaningful.

The outline of this paper is as follows: in Section 2, we describe our methods and assumptions for calculating the orbital evolution of planetary systems; in Section 3, we show the sample of transiting planets and the corresponding parameters which were used in this work; in Section 4, we discuss the various observational and astrophysical biases that affect the sample of systems with transiting planets found by ground-based transit searches and our procedures and assumptions for how to deal with them in the analysis; in Section 5, we outline the procedure we use to derive constraints on the $Q_*$ value; in Section 6, we show our limits to $Q_*$; and in Section 7, we show the apparent discrepancy between our results and estimates of $Q_*$ derived from binary stars in open clusters.

2. ORBITAL EVOLUTION

Tidal decay of close-in planets involves the exchange of angular momentum between a planet’s orbit and its host star’s rotation. For the stars we consider here, several processes influence the stellar rotation, in addition to tidal processes, and accurate modeling of the orbital decay requires consideration of these effects. For this purpose, we solve the following system of ordinary differential equations:

$$\frac{da}{dt} = \text{sign}(\omega_{\text{conv}} - \omega_{\text{orb}}) \frac{9}{2} \sqrt{\frac{G}{a M_\star}} \left( \frac{R_\star}{a} \right) \frac{5 m_p}{Q_\star}, \tag{1}$$

$$\frac{dL_{\text{conv}}}{dt} = \frac{1}{2} m_p M_\star \left( \frac{G}{a(M_\star + m_p)} \right) \frac{da}{dt}, \tag{2}$$

$$\frac{dL_{\text{conv}}}{dt} = -K \omega_{\text{conv}} \min(\omega_{\text{conv}}, \omega_{\text{sat}})^2 \times \left( \frac{R_\star}{R_\odot} \right)^{1/2} \left( \frac{M_\star}{M_\odot} \right)^{-1/2}, \tag{3}$$

$$\frac{dL_{\text{conv}}}{dt} = \Delta L \frac{2}{3} \frac{R_\star^{5/2}}{R_{\text{rad}}^{3/2}} \frac{dM_{\text{rad}}}{dt}, \tag{4}$$

$$\frac{dL_{\text{rad}}}{dt} = -\Delta L \frac{2}{3} \frac{R_\star^{5/2}}{R_{\text{rad}}^{3/2}} \frac{dM_{\text{rad}}}{dt}. \tag{5}$$

where $M_\star$ is the mass of the star; $R_\star$ is the radius of the star; $m_p$ is the mass of the planet; $Q_\star$ is the tidal quality factor of the star; $\text{sign}(\omega_{\text{conv}} - \omega_{\text{orb}})$ takes the value 1 when the stellar convective zone is spinning faster than the planet and -1 when the reverse is true; $K = 0.35 M_\odot R_\odot^2 \text{day}^{-2} \text{Gyr}^{-1}$ is the proportionality constant, parameterizing the strength of the magnetic wind of the star; $\omega_{\text{sat}} = 1.84 \text{day}^{-1} \text{Gyr}^{-1}$ is the wind saturation frequency; $L_{\text{conv}}$ is the moment of inertia of the stellar convective zone; $L_{\text{rad}}$ is the angular momentum of the stellar convective zone; $\tau_c$ is the core–envelope coupling timescale; $M_{\text{rad}}$ is the mass of the stellar radiative core; $R_{\text{rad}}$ is the radius of the radiative–convective boundary in the star; and $\omega_{\text{conv}} \equiv L_{\text{conv}}/I_{\text{conv}}$ is the angular frequency of the stellar convective zone.

We wish to follow a planet–star system as its semimajor axis shrinks under the influence of tidal friction. There are two sources of friction: the tides on the star and those on the planet. However, the latter is only important as long as either the orbit is eccentric and/or the planet is rotating asynchronously. Since the angular momentum stored in the rotation of the planet is quite small compared with the orbital or stellar spin angular momenta, it is safe to assume that the planet spin is synchronized quickly compared to any orbital evolution. Further, we will restrict our sample with only systems with nearly circular orbits. In this case, the evolution of the semimajor axis ($a$) is given by Equation (1) above (Goldreich 1963; Kaula 1968; Jackson et al. 2008a). This expression makes the approximation that the planet’s mass can be neglected compared with the star’s mass, a perfectly reasonable assumption for all the systems we consider; given the uncertainty in the value of $Q_*$.

The angular momentum that is taken away from the orbit by the tidal friction is deposited in the star, acting to spin it up, and while for most of the known transiting planets the orbit does not have enough angular momentum to spin up the star to synchronous rotation, for at least one planet this is not true. Further, since the tidal friction couples the planet with the surface convective zone of the star, it is possible that if the core–envelope coupling is not sufficient, synchronous rotation can be imposed on the envelope only. For this reason we also follow the evolution of the angular momentum of the star ($L_\star$) — Equation (2). Here, we do not make the approximation $M_\star \gg m_p$, like we did for the orbital evolution above, in order to make the final set of equations conserve angular momentum exactly.

In addition, since we are interested in timescales of order Gyr, we cannot neglect the spin-down of the host star due to its own magnetic wind. The effects of stellar wind shedding on rotation have been extensively studied, but remain poorly understood. Equation (4) represents the current best description of these effects and is motivated by a combination of theory and observation (Stauffer & Hartmann 1987; Kawaler 1988; Barnes & Sofia 1996). Note that we do not introduce any free parameters to describe the stellar wind. The value of $K$ in Equation (4) is determined by the present rotation rate and age of the Sun and the value of the wind saturation frequency $\omega_{\text{sat}}$ comes from fitting the observations of stellar rotation rates in open clusters of different ages. With those values, the expression above matches the rotational evolution of single stars in open clusters at young ages, as well as the rotation rate of the Sun.
and other less well constrained but older stellar populations. Hence, theoretical interpretations aside, it can be viewed as a parameterization of the rotation rate observations over the range of ages we encounter during the orbital evolution of the exoplanet systems in our sample.

Open cluster rotation rates impose one more complication on our model: core–envelope decoupling. In order to explain how stars with a wide spread of rotation rates at young ages end up with similar rotation rates later, it is necessary to allow for quickly rotating cores in the slow surface rotation rate stars, and a re-distribution of the excess angular momentum to the convective zone at a later time (Irwin et al. 2007; Irwin & Bouvier 2009; Denissenkov 2010). The expressions for the separate core and envelope evolution were derived by Allain (1998), and for planet hosts take the form of Equations (4) and (5). We test the sensitivity of our results on the assumed core–envelope coupling by repeating our analysis under the assumption that the core and envelope are perfectly coupled ($\tau_c = 0$).

Finally, Equation (6) for the stellar core–envelope differential rotation was proposed by MacGregor (1991).

Note that in Equation (1) we treat $Q_\star$ as having a fixed magnitude. We feel that more complex assumptions are not justified, since the dependence of $Q_\star$ on frequency is poorly known, and there is substantial disagreement between observational and theoretical estimates as discussed in the introduction.

In this work, we start a planetary system’s evolution from an age of 5 Myr after the (model) birth of the star, with an initial orbital separation that evolves to the observed semimajor axis at the present system age. The initial age is assumed to be early enough so that no significant spin-up of the star due to the tides raised by the planet has occurred, but any non-tidal evolution of the planetary orbit has stopped. This is reasonable if one assumes that hot Jupiters arrive at their extremely close-in orbits through disk migration, since by that time the protoplanetary disk has dissipated (Haisch et al. 2001; Bouwman et al. 2006), but for other migration mechanisms this might not be the case.

The advantage of starting the evolution at 5 Myr is that with these assumptions the initial stellar rotation distribution is relatively well constrained from observations of young open clusters (Irwin & Bouvier 2009, and references therein). However, this age is significantly before the star has arrived at the main sequence, so all stellar parameters, in particular the radius, the masses of the radiative core and convective envelope, and the corresponding moments of inertia all significantly evolve with time. This forces us to allow for stellar evolution along with the orbital and spin evolution of the planet–star systems. In order to follow the evolution of the star, we use evolution tracks calculated using the YREC stellar evolution code (Demarque et al. 2008) with masses 0.4 $M_\odot$, 0.5 $M_\odot$, 0.6 $M_\odot$, 0.7 $M_\odot$, 0.8 $M_\odot$, 0.9 $M_\odot$, 1.0 $M_\odot$, 1.05 $M_\odot$, 1.1 $M_\odot$, and 1.25 $M_\odot$. The evolution of individual stars is determined from cubic spline interpolation within this grid of models. Stars below 0.4 $M_\odot$ and above 1.25 $M_\odot$ are excluded from our analysis (see below).

In our evolution we assume that the tides couple the orbit of the planet only to the convective zone. While, in general, we should split the tidal torque in two parts, one spinning up the convective zone and the other spinning up the core, the coupling to the core is likely to be negligible compared with the convective zone for two reasons: (1) the amplitude of the tidal deformation scales strongly with radius, so it will be much smaller for the core than for the convective zone; (2) currently there is no known mechanism for dissipating the tidal energy in the core, so averaged over an orbit there should be no net angular momentum transfer from the tides to the core.

An additional complication for our models is that the prescription described above is only valid for low stellar masses. For masses larger than approximately 1.2 $M_\odot$ the surface convective zone becomes negligible in mass, so we cannot treat it as the only sink for angular momentum. Further, Wolff & Simon (1997) indicate that the angular momentum loss described above is only valid for low-mass stars ($M_\star \lesssim 1.3 M_\odot$). For more massive stars, presumably the lack of surface convection suppresses the stellar wind, and the loss of angular momentum is much weaker. For reasons of numerical stability, for stars above 1.1 $M_\odot$ we ignore the core–envelope decoupling, treating the star as a solid body and we completely exclude from our analysis stars above 1.25 $M_\odot$, since those stars do not possess a significant surface convection zone and could be subject to a different mode of tidal dissipation, in addition to weaker stellar wind. Consequently, close-in planets around very massive stars may have much larger TL values than around less massive stars, with possible implications for planet surveys of massive stars.

One particular effect of including the stellar wind for low-mass stars is that even after the star has synchronized its spin with the orbit, the semimajor axis continues to decay. In fact, a positive feedback loop is created at this point: tides keep the stellar spin synchronized with the planet, so as the stellar wind removes angular momentum, the orbit continues to shrink. This spins up the star, enhancing the stellar wind, which draws more angular momentum from the system and increases the rate of orbital decay. The effect is that after a short amount of time the tidal spin-up fails to keep up with the angular momentum lost to the wind and the spin–orbit lock is lost.

It should be noted that the above tidal evolution equations are only valid under the assumption of good alignment between the orbital and stellar angular momenta. This has been checked only for a subset of the currently known transiting extrasolar planets, through the measurement of the Rossiter–McLaughlin effect, and a non-trivial fraction of misaligned planets is found (Johnson et al. 2009, 2008, 2011; Winn et al. 2006, 2008, 2009, 2011, 2010a, 2010b; Narita et al. 2008, 2007, 2010a, 2010b; Bayliss et al. 2010; Tripathi et al. 2010; Queloz et al. 2000; Bundor Marcy 2000; Snellen 2004, etc.). However, even for significant misalignment, the tidal torques will only change by a factor of order unity, much smaller than the uncertainties on $Q_\star$, which range over several orders of magnitude. In addition, the current measurements of the stellar spin–orbit alignment suggest that the shortest period planets, which are the only ones sensitive to the tidal dissipation, are well aligned with their parent star’s rotation.

Figure 1 shows an example of the evolution of HAT-P-20 b, computed with $Q_\star = 10^5$. The left boundary of all our plots has been placed at 30 Myr in order to show more details in the part of the evolution that is important for this analysis.

From the top right plot one can see that for the first 55 Myr, due to its pre-main sequence contraction, the star spins faster than the planet orbits. As a result the semimajor axis increases (to a degree not noticeable on the plot), and the star spins down, in spite of the fact that it is contracting. During the subsequent approximately 35 Myr, the orbit and the convective zone of the star are tidally locked. This lock is quickly lost due to the star shrinking (bottom left panel), causing the tidal coupling to sharply decrease.

The core and the envelope of the star are clearly decoupled for the first two Gyrs of the evolution. Initially, the core rotates...
slower than the convective zone, due to the fact that its moment of inertia does not change quite so much. However, at around 35 Myr, the situation is reversed due to the stellar wind (and initially the planet) taking angular momentum away from the convective zone, but not the core. Significant differential rotation is maintained to an age of about 2 Gyr by the stellar wind, which is gradually losing strength as the star spins down, until the core and the envelope are completely coupled.

By around 3 Gyr, the orbit has shrunk enough for tidal torques to once again dominate the rotational evolution of the star causing it to spin up to a period of a few days as the planet inspirals.

3. OBSERVATIONAL DATA

From the more than one hundred currently known transiting planets, we based our analysis on fifty-three. Those that were selected orbit stars with masses between 0.25 \( M_\odot \) and 1.25 \( M_\odot \). The lower cutoff was imposed because we do not have reliable stellar models for masses below this range. For masses above the upper limit, the dissipation is likely dominated not by the convective zone, which at this point is next to non-existent, but by some dissipation mechanism in the radiative bulk of the star. Hence, assuming the same \( Q^* \) value applies beyond this point is not reasonable.

In particular, probably the planetary system most often given as an example of a very fast tidal orbital evolution, WASP-18 b, (cf. Hellier et al. 2009, 2011b; Hansen 2010; Lai 2012; Penev & Sasselov 2011; Brown et al. 2011) is not among the planets included in this work, due to the fact that its star lies above the 1.25 \( M_\odot \) cutoff we impose. The planetary orbit in this system, even just by itself, argues strongly against efficient tidal dissipation in the star. However, as discussed above, the mechanism of this dissipation is likely different than for the majority of the exoplanet host stars found by transit searches.

Further, we restrict our sample to only systems which are consistent with having a circular orbit, and age limits quoted in the literature at least partially overlap with our 10 Gyr cutoff. Finally we exclude \textit{Kepler} and \textit{CoRoT} planets, because they are generally subject to much different biases. Table 1 lists the systems and their relevant parameters that were included in our analysis, along with references to where those parameters were published.

4. CORRECTING FOR OBSERVATIONAL BIASES

For each transiting planet system, we need to figure out what the probability is that it would be observed and detected at any moment during its evolution. We split this probability into four parts:

1. \( p_{\,\text{transit}} \). The geometric probability that the planet’s transit is observed from the Earth.
2. \( p_{\,\text{detect}} \). The probability that the orbital phase coverage of a survey allows the detection of the transit.
3. \( p_{\,\text{followup}} \). The probability that the transit candidate will be chosen for follow-up by the survey and confirmed.
4. \( p_{\,\text{age}} \). The distribution of ages of target stars for transiting surveys expressed as a probability.

The final probability density that we use is the normalized product of these four quantities.
| Planet Name | $M_\ast$ ($M_\odot$) | $R_\ast$ ($R_\odot$) | Stellar age (Gyr) | Planet Mass ($M_{Jup}$) | Semimajor axis (AU) | References |
|-------------|----------------------|----------------------|-------------------|-------------------------|-------------------|------------|
| HAT-P-3 b   | 0.917                | 0.799                | 1.6               | 0.3                     | 4.5               | 0.591      | Chan et al. (2011) |
| HAT-P-5 b   | 1.16                 | 1.167                | 2.6               | 0.8                     | 4.4               | 1.06       | Bakos et al. (2007) |
| HAT-P-10 b  | 0.83                 | 0.79                 | 7.9               | 4.1                     | 11.7              | 0.487      | Bakos et al. (2009b) |
| HAT-P-12 b  | 0.733                | 0.701                | 2.5               | 0.5                     | 4.5               | 0.211      | Hartman et al. (2009a) |
| HAT-P-13 b  | 1.22                 | 1.56                 | 5                 | 4.2                     | 7.5               | 0.851      | Bakos et al. (2009a) |
| HAT-P-16 b  | 1.218                | 1.237                | 2                 | 1.2                     | 2.8               | 4.193      | Winn et al. (2010a) |
| HAT-P-18 b  | 0.77                 | 0.749                | 12.4              | 6                        | 16.8              | 0.197      | Hartman et al. (2011b) |
| HAT-P-19 b  | 0.842                | 0.82                 | 8.8               | 3.6                     | 14                | 0.292      | Hartman et al. (2011b) |
| HAT-P-20 b  | 0.756                | 0.694                | 6.7               | 2.9                     | 12.4              | 7.246      | Bakos et al. (2012) |
| HAT-P-22 b  | 0.916                | 1.04                 | 12.4              | 9.8                     | 15                | 2.147      | Bakos et al. (2012) |
| HAT-P-23 b  | 1.13                 | 1.204                | 4                 | 3                       | 5                 | 2.09       | Bakos et al. (2012) |
| HAT-P-24 b  | 1.191                | 1.317                | 2.8               | 2.2                     | 3.4               | 0.685      | Kipping et al. (2010) |
| HAT-P-25 b  | 1.01                 | 0.959                | 3.2               | 0.9                     | 5.5               | 0.567      | Quinn et al. (2012) |
| HD 189733 b | 0.82                 | 0.78                 | 9                 | 4.1                     | 12                | 0.059      | Hartman et al. (2011a) |
| OGLE-TR-56 b| 1.17                 | 1.32                 | 2.7               | 2.6                     | 2.8               | 1.29       | Pont et al. (2007) |
| OGLE-TR-113 b| 0.78                | 0.77                 | 0.7               | 0.7                     | 10                | 1.32       | Gillon et al. (2006) |
| OGLE-TR-182 b| 1.818               | 1.53                 | 4.3               | 2.4                     | 4.8               | 1.06       | Southworth (2010) |
| Qatar-1 b   | 0.85                 | 0.823                | 6                 | 6                       | 13                | 1.09       | Alsubai et al. (2011) |
| TrES-2      | 0.98                 | 1                    | 5.1               | 2.4                     | 7.8               | 1.253      | Daemgen et al. (2009) |
| WASP-4 b    | 0.92                 | 0.907                | 5.5               | 3.5                     | 8.7               | 1.215      | Wilson et al. (2008) |
| WASP-5 b    | 1                    | 1.084                | 3                 | 1.7                     | 4.4               | 1.637      | Anderson et al. (2008) |
| WASP-10 b   | 0.703                | 0.775                | 0.8               | 0.6                     | 1                 | 3.07       | Southworth (2009a) |
| WASP-13 b   | 1.03                 | 1.34                 | 8.5               | 3.6                     | 14                | 0.46       | Christian et al. (2009) |
| WASP-16 b   | 1.022                | 0.946                | 2.3               | 0.1                     | 8.1               | 0.855      | Skillen et al. (2009) |
| WASP-19 b   | 0.97                 | 0.99                 | 5.5               | 1                       | 14.5              | 1.168      | Lister et al. (2009) |
| WASP-21 b   | 1.01                 | 1.06                 | 12                | 7                       | 17                | 0.3        | Hellier et al. (2011b) |
| WASP-22 b   | 1.11                 | 1.13                 | 3                 | 2                       | 5                 | 0.56       | Bouchy et al. (2010) |
| WASP-24 b   | 1.184                | 1.331                | 3.8               | 2.5                     | 5.1               | 1.071      | Maxted et al. (2010) |
| WASP-25 b   | 1                    | 0.92                 | 0.02              | 0.01                    | 3.98              | 0.58       | Street et al. (2010) |
| WASP-26 b   | 1.12                 | 1.34                 | 6                 | 4                       | 8                 | 1.02       | Southworth et al. (2010a) |
| WASP-28 b   | 1.08                 | 1.05                 | 5                 | 3                       | 8                 | 0.91       | Smalley et al. (2010) |
| WASP-34 b   | 1.01                 | 0.93                 | 6.7               | 2.2                     | 13.6              | 0.59       | West et al. (2010) |
| WASP-35 b   | 1.07                 | 1.09                 | 5.01              | 3.85                    | 6.17              | 0.72       | Smalley et al. (2011) |
| WASP-37 b   | 0.925                | 1.003                | 11                | 7                       | 14                | 1.8        | Simpson et al. (2011) |
| WASP-38 b   | 1.203                | 1.331                | 5                 | 5                       | 14                | 2.691      | Simpson et al. (2011) |
| WASP-39 b   | 0.93                 | 0.895                | 9                 | 5                       | 12                | 0.28       | Fausnaugh et al. (2011) |
| WASP-41 b   | 0.95                 | 1.01                 | 1.8               | ?                       | ?                 | 0.92       | Maxted et al. (2011) |
| WASP-43 b   | 0.58                 | 0.598                | ?                 | 0.3                     | ?                 | 1.78       | Hellier et al. (2011a) |
| WASP-44 b   | 0.951                | 0.927                | 0.9               | 0.3                     | 1.9               | 0.889      | Anderson et al. (2011) |
| WASP-45 b   | 0.909                | 0.945                | 1.4               | 0.4                     | 3.4               | 1.007      | Anderson et al. (2011) |
| WASP-46 b   | 0.956                | 0.917                | 1.4               | 0.8                     | 1.8               | 2.101      | Anderson et al. (2011) |
| XO-2        | 0.98                 | 0.97                 | 5.3               | 4.3                     | 6.3               | 0.57       | Burke et al. (2007) |
| 55 Cnc e    | 0.905                | 0.943                | 10.2              | 7.7                     | 12.7              | 0.027      | Fischer et al. (2008) |

The transit probability ($p_{\text{transit}}$) is the simplest of the three biases. It is simply proportional to the ratio of the stellar radius to the orbital semimajor axis.

The detection probability ($p_{\text{detect}}$) is a bit more complicated, because the requirement for detecting a transit varies by system. For example, to detect a relatively deep event around a quiet bright star, observing only a few transits might be sufficient, while the detection of a shallower transit around an active faint star might require many transits (we incorporate the dependence of transit detection probability on stellar activity into $p_{\text{followup}}$).
In addition, $p_{\text{detect}}$ is not a smooth function of orbital period, but rapidly oscillates, has sharp local maxima or minima near periods close to an integer multiple of 24 hr, etc. (cf. Collier Cameron et al. 2006; Smith et al. 2006; Burke et al. 2006; Hartman et al. 2009b). Instead of attempting to address all those complications, we will assume a simple smooth dependence of $p_{\text{detect}}$ on the orbital period and present results with two different prescriptions for $p_{\text{detect}}$. The particular dependences of $p_{\text{detect}}$ on orbital period that we use are given in Figure 2. These roughly follow the curves published by various surveys for the recovery probability.

The long period tail of $p_{\text{detect}}$ is not well constrained and is survey dependent (hence the two different prescriptions). However, it is also not particularly important, since planets with long periods are less affected by tides, even for relatively small $Q_*$ values, and so our results are not sensitive to this assumption.

The reason for prescribing shallower dependence of $p_{\text{detect}}$ on period for the HAT survey is that, unlike all other transit surveys, HAT combines observations from two sites (one in Arizona and one in Hawaii), which increases their sensitivity at longer periods.

In addition to orbital period the detection probability will depend on the brightness of the star and on the amplitude and frequency dependence of stellar variability. However, since extrasolar planets are typically found around main-sequence stars, the stellar luminosity does not vary much, and hence the stellar brightness dependence is mostly irrelevant for our purposes, and is ignored in our model. The dependence on stellar variability is generally complicated and difficult to quantify, so we include it as part of the follow-up probability, and only in the general sense that high stellar activity is associated with high stellar spin frequency and hence the probability of a given transiting system being detected drops as the stellar variability is generally complicated and difficult to quantify.

The follow-up probability ($p_{\text{followup}}$) is the most difficult to quantify, since it is subject to non-deterministic human evaluation and limitations due to follow-up resources specific to each project. In this work, we will idealize the situation and assume that it only depends on the rotational period of the star ($P_{\text{rot}}$) or its projected equatorial rotation velocity $v^* \sin i$ ($v^*$ is the equatorial rotation velocity of the star and $i$ is the angle between stellar rotational axis and the line of sight). Stars rotating fast have a smaller probability to be chosen for follow-up because the radial velocity precision will be limited by the rotational broadening of the spectral lines. In addition, a star’s variability and hence a survey’s ability to recognize a transit signal are correlated with the stellar rotation, so there is a bias against detecting transits for quickly rotating stars, which we did not include in $p_{\text{detect}}$.

In this work, we test the dependence of our results on $p_{\text{followup}}$ by assuming two different forms for it: (1) constant and (2) constant up to $v^* \sin i < 20$ km s$^{-1}$, followed by an exponential decay with the follow-up probability reaching half its maximal value at $v^* \sin i = 40$ km s$^{-1}$.

Finally, our prescription for $p_{\text{age}}$ is based on the dotted line of Figure 3 of Takeda et al. (2007). In particular, we use a piecewise linear approximation to their curve (see Figure 3). In addition we also present results with $p_{\text{age}} = \text{const}$.

In the right panel of Figure 1, we show the computed probability density functions $p_{\text{transit}}$, $p_{\text{detect}}$, $p_{\text{followup}}$, and $p_{\text{age}}$ that correspond to the evolution of for HAT-P-20 b presented in the other panels of the same figure. We also show the resulting $P_{\text{older}}$ computed as

$$P_{\text{older}}(\text{age}) = \frac{\int_{\text{age}}^{\text{death}} p_{\text{transit}} p_{\text{detect}} p_{\text{followup}} p_{\text{age}} d(\text{age})}{\int_{0}^{\text{death}} p_{\text{transit}} p_{\text{detect}} p_{\text{followup}} p_{\text{age}} d(\text{age})}.$$  \hfill (7)

In other words, $P_{\text{older}}$ represents the probability to observe a planet at its current age or older. As a planet’s orbit decays, we have less and less chance to observe the planet before it is tidally disrupted.

5. TESTING $Q_*$ VALUES

In this section, we outline the procedure we use to determine whether a given value of the $Q_*$ parameter is consistent with the observed set of exoplanet systems.

We begin by finding an initial (5 Myr after stellar birth) semimajor axis for each planetary system, which after following the orbital evolution, according to Equation (1–6) to the present time, results in the observed value of the semimajor axis.

We then continue the evolution until one of the following happens.

1. The semimajor axis of the orbit falls below (1) the radius of the star or (2) the Roche radius for the tidal destruction of the planet, whichever comes first. If only tidal decay drives the orbital evolution, we expect planets to spend very little time between their Roche limit and the stellar surface (when the former is larger), and so for our purposes the distinction between the stellar surface and Roche radius is unimportant: planets that have crossed their Roche radii are as good as gone.
2. The star reaches the end of its main-sequence lifetime.

3. The system reaches an age of $T_{\text{max}} = 10$ Gyr and neither of the above conditions has occurred.

Having the complete time evolution of each system, we use $p_{\text{transit}}$, $p_{\text{detect}}$, $p_{\text{followup}}$, and $p_{\text{age}}$ from Section 4 to calculate the probability that a random observation throughout its lifetime will catch it at any given moment. Integrating this probability density from the present age of the system onward gives us the probability that a random observation throughout its lifetime will catch it at any given moment. Integrating this probability density from the present age of the system onward gives us the probability that a random observation throughout its lifetime will catch it at any given moment.

A plot of the calculated cumulative distribution function of $P_{\text{older}}$ (CDF($P_{\text{older}}$)) appropriately corrected for observational biases and assuming the nominal values for the observed system parameters quoted in the literature for various values of $Q_*$ is presented in Figure 4. The K-S test $p$-values corresponding to the comparison of those curves against a uniform distribution are shown as the red (+) symbols in Figure 5. A plot of the cumulative distribution function of $P_{\text{older}}$ (CDF($P_{\text{older}}$)) appropriately corrected for observational biases and assuming the nominal values for the observed system parameters quoted in the literature for various values of $Q_*$ is presented in Figure 4. The K-S test $p$-values corresponding to the comparison of those curves against a uniform distribution are shown as the red (+) symbols in Figure 5.

There are several observational uncertainties that may affect our results. However, the uncertainty in the stellar ages (often a factor of several) dominates over the uncertainties of all other quantities. Thus, this is the only uncertainty we will account for. To demonstrate the possible impact on our conclusions, we compute cumulative distributions of $P_{\text{older}}$ with two assumptions for the actual ages of stars—the nominal ages and the lower end of the age range given by the corresponding publication. The resulting $p$-values obtained by performing a K-S test against a uniform distribution are presented in Figure 5.

The shift to smaller $Q_*$ values when the exoplanetary systems are assumed systematically younger makes sense. A smaller present age of the system means that the interval between now and when one of the terminal conditions described at the beginning of this section occurs represents a larger fraction of the total lifetime of the system. To offset this, a smaller value of $Q_*$ is needed, shortening the future life of the system.

Because the uncertainty is different for each system and because it depends on factors like the method for determining stellar ages, follow-up instrumentation, stellar models used, etc., we cannot modify the expected distribution to include such an uncertainty. Instead one can show that if we prescribe a distribution for the age of each system, the statistical $p$-value corresponding to a given $Q_*$ is the expectation of the usual K-S $p$-value over the distribution of ages. Since evaluating this expectation requires taking an integral over a space that has as many dimensions as the observed transiting planets, we use a Monte Carlo approach to calculate it. The precise procedure is as follows.

1. Calculate the evolution of each system for a set of present ages covering the observationally allowed range.
2. Draw a random age for each system from some prescribed distribution.
3. Calculate the corresponding $P_{\text{older}}$ and perform a K-S test.
4. Average the results of many such iterations to get the final $p$-value, which incorporates the uncertainty in the system ages.

Since we do not know the appropriate distribution to assume for each system we will consider two options: a uniform distribution over the allowed interval of ages and a normal distribution truncated at the age limits centered on the nominal age, with a standard deviation equal to one quarter of the given age range.

6. RESULTS

The procedure described in the previous section was performed six times with various assumptions for $p_{\text{detect}}$, $p_{\text{followup}}$, and $p_{\text{age}}$ as discussed in Section 4, for the two per system age distributions described above and finally we considered a case where we do not allow the convective and radiative zones to rotate at different frequencies (i.e., $r_c = 0$). Table 2 lists the set of assumptions we consider, and the resulting K-S test $p$-values are plotted in Figure 6.

Evidently, only the assumption about how stellar ages are distributed over their observationally allowed ranges makes a noticeable difference. This is not altogether surprising considering that of all the assumed probability distributions this is
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Table 2

| Label                  | $p_{\text{detect}}$         | $p_{\text{followup}}$         | $p^{\text{age}}$       | Age Distribution | $\tau_c$ (Myr) |
|------------------------|-----------------------------|-------------------------------|------------------------|------------------|-----------------|
| Default $p_{\text{detect}}$ | Red (solid) curve in Figure 2 | Decreasing for $v^* \sin i > 20$ km s$^{-1}$ | As plotted in Figure 3 | Uniform          | 5               |
| Mod $p_{\text{detect}}$       | Blue (dashed) curve in Figure 2 | Decreasing for $v^* \sin i > 20$ km s$^{-1}$ | As plotted in Figure 3 | Uniform          | 5               |
| Uniform $p_{\text{followup}}$              | Red (solid) curve in Figure 2 | Uniform                        | As plotted in Figure 3 | Uniform          | 5               |
| Uniform $p^{\text{age}}$           | Red (solid) curve in Figure 2 | Decreasing for $v^* \sin i > 20$ km s$^{-1}$ | As plotted in Figure 3 | Uniform          | 5               |
| Normal age distance          | Red (solid) curve in Figure 2 | Decreasing for $v^* \sin i > 20$ km s$^{-1}$ | As plotted in Figure 3 | Normal           | 5               |
| Coupled                     | Red (solid) curve in Figure 2 | Decreasing for $v^* \sin i > 20$ km s$^{-1}$ | As plotted in Figure 3 | Uniform          | 0               |

Figure 6. K-S test $p$-values as a function of $Q_*$, for the range of assumptions detailed in Table 2.

(A color version of this figure is available in the online journal.)

The most poorly constrained observationally. The core–envelope coupling timescale also has little effect, since allowing differential rotation or not never results in the star rotating faster than the planet except during a very short period during the pre-main sequence phase (like in Figure 1).

All assumptions considered lead to very similar conclusions: At the 1% level, $\log_{10} Q_* > 7$. The different assumptions, fortunately, lead to differences only in the low $p$-value range of $Q_*$.  

7. DISCUSSION

The constraints on the stellar dissipation parameter $Q_*$ we derive based on the transiting planet systems detected by ground-based transit surveys ($Q_* > 10^7$) are inconsistent with constraints derived from observing the circularization of binary stellar systems in open clusters. Zahn (1989) shows that, in order for tides to suppress the eccentricity in binary stars up to the observed circularization cutoff period, $Q_* \sim 10^7$ is required during the pre-main-sequence phase, which corresponds to a $p$-value of 1% in our analysis. One can somewhat circumvent this marginal contradiction by assuming an evolution of $Q_*$ as the stellar structure changes. However, Meibom & Mathieu (2005) find that in order to explain the observed rate of circularization during the main-sequence phase even smaller values are required—$Q_* \sim 10^6$.  

One way to reconcile this apparent inconsistency is to note that the ratio of tidal frequency to stellar rotation frequency is very different for binary star circularization and tidal inspiral of a planet onto its star. For binary stars, the two components of the system are synchronized on a very short timescale (compared with circularization), so the tidal frequency is exactly twice the stellar rotational frequency. In the case of an exoplanet inspiral, the tidal frequency is much higher than the stellar rotation.

This different frequency might lead to two different mechanisms dominating the dissipation in the two cases. Ogilvie & Lin (2004, 2007) and Wu (2005a, 2005b) point out that if the tidal frequency is within a factor of two of the stellar rotational frequency, inertial modes are resonantly excited in the star, which could lead to strongly enhanced shear and hence dissipation. Since the inertial mode frequencies are restricted to lie between $-2\omega_*$ and $2\omega_*$, this mechanism cannot operate in the case of exoplanet systems. The currently favored mechanism of dissipation for low-mass stars in the frequency regime of exoplanetary tides is turbulent dissipation (Zahn 1966, 1989; Goldreich & Nicholson 1977; Goldreich & Keeley 1977; Goodman & Oh 1997; Penev et al. 2007, 2009a, 2009b, 2011; Penev & Sasselov 2011). This less efficient mechanism could result in dissipation efficiencies consistent with the constraints derived in this paper. In particular Penev & Sasselov (2011), based on direct simulations of turbulent dissipation, find $Q_* \sim few \times 10^6$ to $10^7$, consistent with our results here.  

Finally, Schlaufman et al. (2010) argue that the sample of Kepler planets favors $10^6 < Q_* < 10^7$, apparently outside our range. However, their analysis does not even consider $Q_*$ values above $10^7$ (other than infinity), and they do not derive the statistical significance of their limits, or the sensitivity of their result on the various assumptions included in their model (e.g., the conversion of mass to radius and the exoplanet population synthesis models they use). All this makes it difficult to make firm statements about the (in)consistency of the two results. The best way to address this would be to repeat our analysis for the sample of Kepler planets, but this is clearly outside the scope of this article.

Our model suggests that the earliest stages of a close-in planet’s dynamical history may be more complicated than widely considered. The top right panel of Figure 1 shows that, early on, HAT-P-20 rotated more quickly than its planet revolved and only after 35 Myr did the situation reverse. Consequently, the tidal torque exerted by the star switched signs at this point. If, for example, the planet were brought close-in through gas disk migration during its first tens of Myrs, then presumably there was a competition between torques from the gas disk driving the planet in and tidal torques from the star driving the planet out. Lin et al. (1996) pointed out the role that such tidal torques might play in stopping 51 Peg b’s inward gas disk migration but favored clearing of the gas disk very near the star for halting the inward migration. With so many more exoplanets in our sample now, many with much shorter orbital periods than 51 Peg b, we plan to revisit this topic.
