More on the open string pair production

Nan Zhang\textsuperscript{a,b,c} and J. X. Lu\textsuperscript{b,c}

\textsuperscript{a}Department of Physics, Liaoning Normal University, Dalian, Liaoning, 116029, China
\textsuperscript{b}Interdisciplinary Center for Theoretical Study
University of Science and Technology of China, Hefei, Anhui 230026, China
\textsuperscript{c}Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China

Abstract

Motivated by the recent work \cite{1} by one of the present authors, we here report that there exist two additional systems, namely, D3/(F, D1) and D3/(D3, (F, D1)), either of which can also give rise to a potentially testable open string pair production rate. Here the D3 is taken as our own (1+3)-dimensional world carrying its laboratory collinear electric and magnetic fluxes while the non-threshold bound state (F, D1) or (D3, (F, D1)) is placed parallel nearby in the directions transverse to both our D3 world and the non-threshold bound state considered.
1 Introduction

In a recent publication by one of the present authors [1], we report an earthbound laboratory potentially testable open string pair production rate for a system of D3 brane, taken as our own $(1 + 3)$-dimensional world and carrying collinear laboratory electric and magnetic fluxes, and a nearby D1 brane, placed parallel at a separation in directions transverse to both branes. This was motivated by a series of publications by this same author and his collaborators [2, 3, 4, 5] in uncovering the existence of open string pair production, in the spirit of Schwinger pair production in QED [6], for a system of two Dp branes, placed parallel at a separation, with each carrying electric and magnetic fluxes in Type II superstring theories [7].

However, to actually realize the laboratory testable rate, as discussed in [1], we do need the underlying system of D3/D1 to meet certain conditions which are: 1) there should exist a D1 nearby our D3 in the directions transverse to both, 2) the separation between the two is $\gtrsim \pi l_s$ with string scale $l_s = \sqrt{\alpha'}$, 3) the detection has to be performed during a very short period of time when the brane separation satisfies the previous condition since the force acting between the two is attractive and after this the tachyon condensation will start to occur and the open string pair production rate computed then ceases to work, and 4) the open string and the anti open string connecting the D3 and the D1 with their respective length $\gtrsim \pi l_s$ should have their two charged ends on our D3, appearing as the charged/anti-charged pair to our D3 brane observer, to fall in the laboratory in our D3 so that the detection is possible and this requires the laboratory to be placed at a location with a transversing axis parallel to the D1 with their separation around $\pi l_s$. Meeting all these simultaneously is certain not an easy task and this makes the detection of the open string pair production still extremely difficult if not possible.

To improve the detectability, we may either look for more possibilities for a testable rate or a testable rate with less requirements. Either of these will enhance our chance for a detection.

In this paper, we will discuss both by considering the D3/(F, D1) system for the former and D3/(D3, (F, D1)) for the latter. In other words, we will replace the D1 mentioned above for the system of D3/D1 by the present (F, D1) or (D3, (F, D1)). Here (F, D1) stands for the non-threshold bound state of fundamental strings and D1-branes [9, 10] with their respective co-prime quantized integral charge $(p', q')$. From the viewpoint of the D1 worldsheet, the F-strings are given by a quantized flux [10, 11, 12, 13]. (D3,

1 The open string pair production for un-oriented bosonic string or un-oriented Type I superstring was discussed a while ago by Bachas and Porrati in [7, 8].
(F, D1)) is the non-threshold bound state of D3-branes and the delocalized non-threshold bound state (F, D1)\cite{14, 15}, characterized by three integers \((n', (p', q'))\) without a common divisor. From the D3-brane worldvolume viewpoint, the delocalized (F, D1) non-threshold bound state is given by collinear quantized electric and magnetic fluxes.

This paper is organized as follows. In the following section, we will focus on the system of D3/(F, D1). In section 3, we will move to discuss the system of D3/(D3, (F, D1)). We will discuss and conclude in section 4.

2 The open string pair production: D3/(F, D1)

For this system, we consider that our D3 carries laboratory collinear electric flux \(\hat{f}\) and magnetic flux \(\hat{g}\) while the D1 carries a quantized electric flux \(\hat{f}'\) as follows

\[
\hat{F}_3 = \begin{pmatrix}
0 & \hat{f} & 0 & 0 \\
-\hat{f} & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{g} \\
0 & 0 & -\hat{g} & 0
\end{pmatrix}, \quad \hat{F}'_1 = \begin{pmatrix}
0 & \hat{f}' \\
-\hat{f}' & 0
\end{pmatrix},
\]

where both \(\hat{F}_3\) and \(\hat{F}'_1\) are dimensionless and the quantized electric flux

\[
\hat{f}' = \frac{p'}{\sqrt{p'^2 + q'^2 / g_s^2}},
\]

with \(p'\) and \(q'\) two co-prime integers. Here \(q'\) denotes the multiplicity of D1-branes and \(p'\) the number of F-strings, and \(g_s\) the string coupling. We denote also \(\Delta_{(p', q')} = p'^2 + q'^2 / g_s^2\) for simplicity. Note that we need to keep \(g_s \ll 1\) such that the D-branes used can be treated as rigid ones at least to the probe distance not much smaller than the string scale \(l_s\) as discussed in \cite{16}. In the following, we take \(g_s = 10^{-2}\) as a sample small coupling for later concrete discussion.

To compute the pair production rate, we first compute the closed string tree-level cylinder amplitude between the D3 and the D1. This can be read from the last equality of the general one given in (136) in \cite{16} for \(p = 3, p' = 1\) as

\[
\Gamma_{3,1} = \frac{2q' V_2 \sqrt{(1 - \hat{f}'^2)(1 - \hat{f}^2)(1 + \hat{g}^2)}(\cos \pi \nu_0 - \cos \pi \nu_1)^2}{8\pi^2 \alpha'} \int_0^\infty \frac{dt}{t^3} e^{-\frac{\tilde{y}^2}{2t^2}} \times \prod_{n=1}^\infty \frac{[1 - 2|z|^{2n} e^{-\pi \nu_0} \cos \pi \nu_1 + e^{-2\pi \nu_0} |z|^{4n}]^2 [1 - 2|z|^{2n} e^{\pi \nu_0} \cos \pi \nu_1 + e^{2\pi \nu_0} |z|^{4n}]^2}{(1 - |z|^{2n})^4 [1 - 2|z|^{2n} \cosh 2\pi \nu_0 + |z|^{4n}] [1 - 2|z|^{2n} \cos 2\pi \nu_1 + |z|^{4n}]},
\]

\(^2\)Note that here \(p'\) stands for the spatial dimensions of Dp' brane, not the previously mentioned quantized integral charge of F-strings.
where $y$ is the brane separation along the directions transverse to both D3 and D1, $|z| = e^{-\pi t} < 1$, $V_2$ the volume of D1 worldvolume, $\alpha'$ the Regge slope parameter, and the so-called electric parameter $\bar{\nu}_0$ and the magnetic parameter $\nu_1$ are determined by the electric fluxes ($\hat{f}, \hat{f}'$) and the magnetic one ($\hat{g}$), respectively, via

$$\tanh \pi \bar{\nu}_0 = \frac{|\hat{f} - \hat{f}'|}{1 - \hat{f}\hat{f}'}, \quad \tan \pi \nu_1 = \frac{1}{\hat{g}},$$

with $\bar{\nu}_0 \in [0, \infty), \nu_1 \in (0, 1)$. In the amplitude (3), we have added an integer factor $q'$, counting the multiplicity of D1-branes.

Note that the dimensionless flux (denoted with a hat above) here is defined via $\hat{F} = 2\pi\alpha'F$ with $F$ the usual dimensionful field strength (without a hat above). So we have $|\hat{g}| \in [0, \infty)$ and $|\hat{f}|, |\hat{f}'| \in [0, 1)$ with unity here as the critical value of the respective electric flux. Unlike what we did in [1] where $\hat{f}' = 0$ is chosen, we here consider $\hat{f}' = p'/\Delta_{(p'q',d)}^{1/2}$ which can be much larger than the laboratory $\hat{f}$ even for small integers $p'$ and $q'$ as well as a small string coupling $g_s$. We will illustrate this numerically later on. In other words, the electric parameter $\bar{\nu}_0$, from (1), can be completely determined by this flux in practice and is much larger than the one considered in [1]. This certainly enhances the open string pair production rate and this will also, to some extent, relax the four conditions mentioned in the Introduction.

Note also that the above amplitude (3) is strictly positive for large $y$ and this remains true even if we turn off our D3 worldvolume fluxes, i.e., by setting $\hat{f} = 0, \hat{g} = 0$ (therefore $\bar{\nu}_0 \neq 0, \nu_1 = 1/2$ from (1)), hence giving an attractive interaction between the D3 and the (F, D1) according to our conventions. This indicates that the underlying system does not preserve any supersymmetry, consistent with the known fact. For large $y$, the dominant contribution to the amplitude comes from the large $t$ integration, due to the exponentially suppressing factor $\text{Exp}[-y^2/(2\pi\alpha't)]$ in the integrand, and the amplitude is therefore positive since every factor in the integrand is positive, so giving an attractive interaction. However, the nature of the amplitude is obscure for small $y$. Now the small $t$ integration becomes important and the factor $[1 - 2|z|^{2n} \cosh 2\pi \bar{\nu}_0 + |z|^{4n}] \approx 2(1 - \cosh 2\pi \bar{\nu}_0)$ in the denominator of the infinite product in the integrand of (3) can be negative for small $t$. Once this happens, the sign of the amplitude is ambiguous since there are an infinite number of such factors in the product. This ambiguity actually indicates a potential new physics to occur and the best way to decipher this is to pass the closed string cylinder amplitude to the corresponding open string one-loop annulus one via a

\[\text{The positive amplitude gives an attractive interaction while a negative one gives a repulsive interaction.}\]
Jacobi transformation\textsuperscript{4} by sending $t \to 1/t$. The resulting annulus amplitude is

$$
\Gamma_{3,1} = \frac{2q'V_2|\hat{f} - \hat{f}'|}{8\pi^2\alpha'} \int_0^\infty \frac{dt}{t} e^{-\frac{x^2t}{2\pi\alpha'}} \frac{\cos \pi\nu_1 t - \cos \pi\nu_0 t}{\sin \pi\nu_0 t \sinh \pi\nu_1 t} \times \prod_{n=1}^\infty \frac{|1 - 2|z|^{2n}e^{i\pi
u_n t} \cosh \pi\nu_1 t + |z|^{4n}e^{-i\pi
u_n t}|^4}{(1 - |z|^{2n})^4(1 - 2z^2 \cosh 2\pi\nu_1 t + |z|^{4n})(1 - 2|z|^{2n} \cosh 2\pi\nu_0 t + |z|^{4n})^4},
$$

(5)

where $|z| = e^{-\pi t}$ continues to hold. Except for the factor $\sin \pi\nu_0 t$, all other factors in the integrand are positive since $1 - 2|z|^{2n} \cosh 2\pi\nu_0 t + |z|^{4n} > 1 - 2|z|^{2n} + |z|^{4n} = (1 - |z|^{2n})^2 > 0$ and $1 - 2|z|^{2n} \cosh 2\pi\nu_1 t + |z|^{4n} = (1 - e^{2\pi\nu_1 t}|z|^{2n})(1 - e^{-2\pi\nu_1 t}|z|^{2n}) > 0$ since $\nu_1 < 1$ and $n \geq 1$. In particular, the integrand gives a potential tachyonic instability when $t \to \infty$ since it blows up

$$
\sim e^{-\frac{x^2t}{2\pi\alpha'}} e^{\pi\nu_1 t} = e^{-2\pi\alpha t \left[\frac{x^2}{(2\pi\alpha')^2} - \frac{m}{2\alpha}\right]},
$$

(6)

when $y < \pi\sqrt{2\nu_1\alpha'}$. This is due to the existence of the so-called tachyonic shift $\nu_1/2$. As we will see, this shift will also give rise to the enhancement of the open string pair production. The aforementioned factor $\sin \pi\nu_0 t$ actually gives rise to an infinite number of simple poles of the integrand at $t_k = k/\nu_0$ with $k = 1, 2, \cdots$ along the positive $t$-axis, reflecting the existence of an imaginary part of the amplitude. This imaginary part indicates the decay of the underlying system via the so-called open string pair production. The decay rate can be computed as the sum of the residues of the integrand in (5) at these poles times $\pi$ per unit worldvolume following [7] as

$$
W = \frac{4q' |\hat{f} - \hat{f}'|}{8\pi^2\alpha'} \sum_{k=1}^\infty (-)^{k-1} \frac{e^{-\frac{kx^2}{2\pi\nu_0}}}{k} \left[\frac{\cosh \frac{\pi\nu_1}{\nu_0} - (-)^k}{\sinh \frac{\pi\nu_1}{\nu_0}}\right] Z_k(\nu_0, \nu_1),
$$

(7)

where

$$
Z_k(\nu_0, \nu_1) = \prod_{n=1}^\infty \left(1 - 2(-)^k |z_k|^{2n} \cosh \frac{\pi\nu_1}{\nu_0} + |z_k|^{4n}\right)^4 \left(1 - 2|z_k|^{2n} \cosh \frac{\pi\nu_1}{\nu_0} + |z_k|^{4n}\right),
$$

(8)

with $|z_k| = e^{-\pi k/\nu_0}$. Following [17], the rate for the open string pair production corresponds just to the leading $k = 1$ term of the above decay rate and it is

$$
\mathcal{W}^{(1)} = \frac{4q' |\hat{f} - \hat{f}'|}{8\pi^2\alpha'} e^{-\frac{x^2}{2\pi\nu_0}} \left[\frac{\cosh \frac{\pi\nu_1}{\nu_0} + 1}{\sinh \frac{\pi\nu_1}{\nu_0}}\right] Z_1(\nu_0, \nu_1),
$$

(9)

\textsuperscript{4}Certain relations for the Dedekind $\eta$-function and the $\theta_1$-function have also been used, see [16] for detail, for example.
where
\[
Z_1(\bar{\nu}_0, \nu_1) = \prod_{n=1}^{\infty} \frac{\left[ 1 + 2|z_1|^{2n} \cosh \frac{2\pi \nu z}{\nu_0} + |z_1|^{4n} \right]^4}{(1 - |z_1|^{2n})^6 \left[ 1 - 2|z_1|^{2n} \cosh \frac{2\pi \nu z}{\nu_0} + |z_1|^{4n} \right]}. \tag{10}
\]

With the above computed open string pair production rate, we can now discuss the possibility of detecting the pair production. Suppose that the D3 is the $(1 + 3)$-dimensional world we are living in and there exists a nearby $(F, D1)$ in the directions transverse to our D3. For practical purpose, we can only control the sizes of the collinear electric and magnetic fields on our D3, both of which are in general very small compared with the string scale. The possible largest static electric field which can be realized in an earthbound laboratory is $E \sim 10^{10}$ Volt/m which gives $eE \sim 10^{-8} m_e^2 \sim 2.5 \times 10^{-21}$ TeV$^2$ with $m_e$ the electron mass (The largest static magnetic field gives also $eB \sim 10^{-8} m_e^2 \sim 2.5 \times 10^{-21}$ TeV$^2$, see footnote [5]). Note that the lowest experimental bound for the string scale $M_s = 1/l_s$ is a few TeV, see [19], for example. So we have $\hat{f} = 2\pi \alpha' eE \sim 2\pi eE/M_s^2 \leq 10^{-21} \ll 1$ and $\hat{g} = 2\pi \alpha' eB \sim 10^{-21} \ll 1$. So indeed we have both $\hat{f} \ll 1$ and $\hat{g} \ll 1$ in practice.

Given $\hat{g} \leq 10^{-21} \ll 1$, we have $\nu_1 \leq 1/2$ from (1). Note that the quantized electric flux $\hat{f}' < 1$ from (2) since $p', q' \geq 1$ but it is still in general much larger than the applied $\hat{f} \leq 10^{-21}$. So we can completely ignore $\hat{f}$ in general and have from (2)
\[
\tanh \pi \bar{\nu}_0 \approx \hat{f}' = \frac{g_s p'}{\sqrt{q'^2 + g_s^2 p'^2}}. \tag{11}
\]

In general, the larger the electric flux $\hat{f}'$ is, the larger the pair production rate. This can be easily seen from the rate given in [2] since the exponential factor $\text{Exp}[-\frac{q'^2}{2\pi \alpha' \bar{\nu}_0}]$ dominates the rate among other things. It is clear from (11) that the largest possible $\hat{f}'$ can be reached for $q' = 1$. For not too large $p'$ (see [20]), say $p' = 10$ for example, we have $\hat{f}' \approx g_s p' = 0.1$ if $g_s = 10^{-2}$ as given earlier. We can also write $\hat{f}' = 2\pi \alpha' eE' = g_s p'$, giving $eE' = g_s p'/(2\pi \alpha') = M_s^2/(20\pi)$. We have then, from (11), $\pi \bar{\nu}_0 \approx g_s p' \rightarrow \bar{\nu}_0 = g_s p'/\pi = 1/(10\pi)$. With this $\bar{\nu}_0$, we have $|z_1| = e^{-\pi/\bar{\nu}_0} = e^{-10\pi^2} \rightarrow 0$, which implies the factor (10) $Z_1(\bar{\nu}_0, \nu_1) \approx 1$ for the rate (2). In other words, only the lowest-mass modes of the open string connecting the D3 and the D1, with each having mass $m = y/(2\pi \alpha')$, contribute to the rate (2) when $|z_1| \rightarrow 0$.

\footnote{We consulted our experimental colleague Zhengguo Zhao and learned that the current laboratory limit for electric field is on the order of $10^{10}$ Volt/m. The strongest direct-current magnetic field generated is on the order of 50 Tesla, see [18] for example. This gives $eE \sim eB \sim 10^{-8} m_e^2$ with $m_e$ the electron mass. This electric field is still eight orders of magnitude smaller than that required for giving rise to the Schwinger pair production in QED.}
These modes, due to either the open string or the anti open string, are just the eight bosonic modes $8_B$ and eight fermionic modes $8_F$ from the 10 dimensional viewpoint, which are the usual massless ones when $y = 0$. From the D3 brane viewpoint, these modes just give rise to the broken $N = 4$ supersymmetric and broken $U(1)$ massive super Yang-Mills modes, which are five massive scalars, four massive spinors and one vector in 4-dimensions. As discussed in detail in [1], the original $U(2) \rightarrow U(1) \times U(1)$ when $y \neq 0$ and for the two broken generators, one gives the mass $m = y/(2\pi\alpha')$ for the modes associated with the open string and the other gives the same mass for the modes associated with the anti open string. Each of these modes from the open string carries, say, a positive unity charge while each of the modes from the anti open string carries a negative unity charge under the unbroken $U(1)$ associated with our D3. So totally we have 16 pairs of charged/anti-charged modes contribute to the rate (9). Among these, we have five pairs of charged/anti-charged massive scalars, four pairs of charged/anti-charged massive spinors (counting 8 pairs of charged/anti-charged polarizations) and one pair of charged/anti-charged vectors (counting 3 pairs of charged/anti-charged polarizations).

As mentioned earlier and as discussed in [1] for D3/D1 system, the broken supersymmetries are due to the intrinsically quantized electric and magnetic fluxes associated with the (F, D1) even in the absence of the added laboratory small electric field $\hat{f} = 2\pi\alpha'eE \ll 1$ and the small magnetic field $\hat{g} = 2\pi\alpha'eB \ll 1$. As such, these modes actually have different mass splittings as we come to discuss next. As discussed in [1] as well as in [16] [21], so long the amplitude (3) or (5) or the pair production rate (9) is concerned, the D1 appears effectively to our D3 as a stringy scale magnetic flux, giving rise to $\nu_1 = 1/2$ in the absence of $\hat{g}$. This is also consistent with the fact that a co-dimensional 2 D-brane (here D1) can be taken as a constant magnetic flux in the original brane (here D3) [22] [23] [24]. Also as discussed in detail in [1], following [25], an open string with its two ends carrying charges placed in such a magnetic flux will have its mass splittings due to the Landau motion in this magnetic background and the spin coupling with the magnetic flux, just like a charged particle with spin moving in a given magnetic field. In particular, these massive charged modes, with respective to the D3 brane observer, appear as massive charged particles. Because if these splittings, different mode can have its different contribution to the pair production rate.

Note that $\bar{\nu}_0 = 1/(10\pi) \ll 1$ as given earlier and further $\nu_1/\bar{\nu}_0 \sim 5\pi \gg 1$ ($\nu_1 \sim 1/2$), the pair production rate (9) becomes now

$$W^{(1)} \approx \frac{eE'}{2\pi} e^{-\pi\frac{m^2}{E'}}.$$  \(12\)
where we have defined an effective mass

\[ m_{\text{eff}}^2 \equiv m^2 - \frac{\nu_1}{2\alpha'}, \]

(13)

with \( m = \gamma/(2\pi\alpha') \). Note also that this effective mass square is nothing but the one appearing in the square bracket of the exponential on the right of (6) when we discuss the tachyonic instability. As discussed in [1], this pair production rate has contribution only from one pair of charged/anti-charged vector polarizations, with each polarization having the lowest energy or effective mass given by (13). All the other 15 pairs of charged/anti-charged polarizations have their masses higher than this one and give almost zero contribution to the pair production rate with respect to this one when \( \nu_1/\bar{\nu}_0 \gg 1 \). The effective mass (13) gives also a tachyonic instability called the Nielsen-Olesen one for the non-abelian gauge theory of the 4-dimensional \( N = 4 \) U(2) super Yang-Mills [26] in the present context. Clearly it is related to the open string tachyonic instability mentioned earlier.

Note that the present rate (12) looks almost the same as that given in [1] for the D3/D1 system except for one important difference. For example, the effective mass in both cases is essentially the same as that given in (13) with \( \nu_1 \approx 1/2 \). However, their sharp difference is the electric field appearing in the rate formula (12). In the present case, the electric field \( E' \) is the one given by the quantized electric flux \( \hat{f}' = 2\pi\alpha' eE' = g_s p \), giving \( eE' = M_s^2/(20\pi) \) which is much greater than the applied laboratory one with its current largest possible value \( eE \sim 10^{-8} m_s^2 \sim 10^{-21} \text{TeV}^2 \) as discussed earlier. As such, the applied laboratory electric field can be completely ignored. However, for the latter rate, the electric field is just the applied laboratory one \( eE \).

In order to detect the pair production, we need an electric field which is large enough to separate the virtual open string and the anti open string (or the charged/anti charged pair viewed from the brane world) once they are created. We can estimate this by setting the work done by the electric force acting on the charged/anti charged pair to separate them over their Compton length \( 1/m_{\text{eff}} \), i.e \( 2eE'/m_{\text{eff}} \), to equal their rest energy \( 2m_{\text{eff}} \). This gives the standard condition for detecting the pair production as

\[ eE' \approx m_{\text{eff}}^2. \]

(14)

Given that \( eE' \) is much larger than the corresponding laboratory \( eE \), we expect that the detection of the present pair production should be much easier than that for the system of

\(^6\)Since the laboratory magnetic flux \( \hat{g} \sim 10^{-21} \ll 1 \), we always have \( \nu_1 \sim 1/2 \) from (4), essentially independent of the laboratory magnetic flux \( \hat{g} \).
D3/D1 reported in [1]. Note that \( \hat{f} = 2\pi\alpha' eE \ll 1 \) and \( \hat{g} = 2\pi\alpha' eB \ll 1 \), the present large rate is basically independent of the applied laboratory electric field \( E \) and the magnetic field \( B \). So we have \( \nu_1 \approx 1/2 \). From (14), we have that the detectable rate occurs at a brane separation \( y_0 \leq y \leq y_0 + 0.1l_s \) with \( y_0 = \pi l_s \) as the brane separation where the tachyonic instability starts to occur. However, for the detectable rate reported in [1] for the system of D3/D1, we need to have \( y_0 \leq y \leq y_0 + 2\pi \times 10^{-8} (m_e/M_s)^2 l_s \approx y_0 + 10^{-21} l_s \) where we have taken the lower bound for the string scale \( M_s = \) a few TeV [19]. This is much more restrictive than the above condition for the present case. In other words, the present consideration is at least much more favorable than the one for the D3/D1 system given in [1] in the sense that the detectable pair production occurs at a brane separation much larger than that for the latter system before the tachyon condensation starts to work. As a result, the four conditions mentioned in the Introduction are all relaxed, to certain extent.

3 The open string pair production: D3/(D3, (F, D1))

We now move to discuss the other system of D3/(D3, (F, D1)) which may give an even better choice for detecting the open string pair production. Here again the D3 is assumed to be our (1 + 3)-dimensional world and we will perform a laboratory detection of the open string pair production, say, by measuring the corresponding electric current produced by the charged/anti-charged ends of the open string pair. The non-threshold bound state (D3, (F, D1)) [14] is placed parallel to our D3 braneworld at a separation \( y \) along the directions transverse to both. Our D3 carries the same worldvolume flux \( \hat{F}_3 \) as given in (1) in the previous section. The non-threshold bound state (D3, (F, D1)) [14, 15] can be viewed as \( n' \) D3 branes carrying their following worldvolume quantized electric and magnetic flux \( \hat{F}_3' \)

\[
\hat{F}_3' = \begin{pmatrix}
0 & \hat{f}' & 0 & 0 \\
-\hat{f}' & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{g}' \\
0 & 0 & -\hat{g}' & 0
\end{pmatrix},
\]

(15)

where the quantized electric flux \( \hat{f}' \) and magnetic flux \( \hat{g}' \) are, respectively, given [15] as

\[
\hat{f}' = \frac{p'}{\sqrt{p'^2 + q'^2 + n'^2}}, \quad \hat{g}' = \frac{q'}{n'},
\]

(16)

The above three integers \( n', p', q' \) without a common divisor count the multiplicity of D3 branes, the quantized electric flux and the quantized magnetic flux, respectively.
The closed string cylinder interaction amplitude between the two D3 branes placed parallel at a separation $y$ and carrying their respective worldvolume flux in the form as given in [11] and [13] has been computed by one of the present authors along with his collaborator(s) in [27, 2, 3, 16] and is given as

$$
\Gamma_{3,3} = \frac{4in'V_4|\hat{f} - \hat{f}'||\hat{g} - \hat{g}'|}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^3} e^{-\frac{y^2}{2\pi\alpha'}} \frac{\theta_2^2 (\frac{i\bar{\nu}_0 - \nu_1}{2} | it) \theta_2^2 (\frac{i\bar{\nu}_0 + \nu_1}{2} | it)}{\eta^6(it)\eta_1(i\bar{\nu}_0 | it)\eta_1(\nu_1 | it)}
$$

$$
= \frac{4n'V_4(\cosh \pi\bar{\nu}_0 - \cos \pi\nu_1)^2}{(8\pi^2\alpha')^2} \sqrt{(1 - \hat{f}^2)(1 - \hat{f}'^2)(1 + \hat{g}^2)(1 + \hat{g}'^2)} \int_0^\infty \frac{dt}{t^3} e^{-\frac{y^2}{2\pi\alpha'}}
$$

$$
\times \prod_{n=1}^\infty \left[ 1 - 2e^{-\pi\bar{\nu}_0}|z|^{2n} \cos \pi\nu_1 + e^{-2\pi\bar{\nu}_0}|z|^{4n} \right] \left[ 1 - 2e^{\pi\bar{\nu}_0}|z|^{2n} \cos \pi\nu_1 + e^{2\pi\bar{\nu}_0}|z|^{4n} \right],
$$

(17)

where $|z| = e^{-\pi t} < 1$ and the parameters $\bar{\nu}_0 \in [0, \infty)$ and $\nu_1 \in [0, 1)$ are given, respectively,

$$
\tanh \pi\bar{\nu}_0 = \frac{|\hat{f} - \hat{f}'|}{1 - \hat{f}\hat{f}'}, \quad \tan \pi\nu_1 = \frac{|\hat{g} - \hat{g}'|}{1 + \hat{g}\hat{g}'},
$$

(18)

By the same token as we did in the previous section, the present amplitude is also positive for reasonably large brane separation $y$ and therefore the interaction is also attractive. As such, the underlying system does not preserve any supersymmetry. For small $y$ for which the small $t$ integration in the amplitude becomes important, each factor in the infinite product in the integrand can be negative and so the sign of the infinite product becomes indefinite again, signaling new physics to occur. This becomes manifest in terms of the corresponding open string variable. The corresponding open string one-loop annulus amplitude can be obtained once again by a simple Jacobi-transformation via $t \to 1/t$ to the right side of the first equality in (17) in a similar fashion as we did in the previous section and the amplitude is

$$
\Gamma_{3,3} = \frac{4n'V_4|\hat{f} - \hat{f}'||\hat{g} - \hat{g}'|}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^3} \left( \cos \pi\nu_1 t - \cos \pi\bar{\nu}_0 t \right)^2 e^{-\frac{y^2}{2\pi\alpha'}}
$$

$$
\times \prod_{n=1}^\infty \left[ 1 - 2|z|^{2n}e^{-\pi\bar{\nu}_0 t} \cos \pi\nu_1 t + |z|^{4n} \right] \left[ 1 - 2|z|^{2n}e^{\pi\bar{\nu}_0 t} \cos \pi\nu_1 t + |z|^{4n} \right],
$$

(19)

where again $|z| = e^{-\pi t} < 1$. Here again, except for the factor $\sin \pi\bar{\nu}_0 t$ in the denominator of the above integrand, all other factors are positive. For large $t$, the integrand blows up when $y < \pi\sqrt{2\nu_1\alpha'}$ since it behaves

$$
e^{-\frac{y^2}{2\pi\alpha'}e^{\pi\nu_1 t}} = e^{-2\pi\alpha' t \left[ \frac{y^2}{(2\nu_1\alpha')} - \frac{\nu_1}{2\alpha'} \right]},
$$

(20)
where the factor \( \nu_1/2 \) in the second term of the square bracket in the exponential is the tachyonic shift. The quantity \( g^2/(2\pi\alpha')^2 - \nu_1/(2\alpha') \) as before defines the lowest energy square or the effective mass square of the open string, connecting our D3 and the other D3, which we will discuss later on. The aforementioned factor \( \sin \pi\nu_0 t \), as before, gives rise to an infinite number of simple poles of the integrand along the positive t-axis at \( t_k = k/\bar{\nu}_0 \) with \( k = 1, 2, \ldots \), once again signaling the decay of the underlying system via the so-called open string string pair production. The decay rate can be obtained following the procedure given in the previous section as

\[
\mathcal{W} = \frac{8n'|\hat{f} - \hat{f}'|\hat{g} - \hat{g}'|}{(8\pi^2\alpha')^2} \sum_{k=1}^{\infty} \frac{(-)^{k-1}}{k} \left[ \cosh \frac{k\pi\nu_1}{\bar{\nu}_0} - (-)^k \right]^2 \frac{e^{-\frac{k^2}{2\pi\nu_0\alpha'}}}{\sinh \frac{k\pi\nu_1}{\bar{\nu}_0}} Z_k(\bar{\nu}_0, \nu_1),
\]

(21)

where \( Z_k(\bar{\nu}_0, \nu_1) \) is again given by (8). The open string pair production rate is just the \( k = 1 \) leading term of the above rate, following [17], as

\[
\mathcal{W}^{(1)} = \frac{8n'|\hat{f} - \hat{f}'|\hat{g} - \hat{g}'|}{(8\pi^2\alpha')^2} \left[ \cosh \frac{\pi\nu_1}{\bar{\nu}_0} + 1 \right]^2 \frac{e^{-\frac{\nu_1^2}{2\pi^2\alpha'}}}{\sinh \frac{\pi\nu_1}{\bar{\nu}_0}} Z_1(\bar{\nu}_0, \nu_1).
\]

(22)

This rate is the starting point of discussion in the present section. Previously, the rate was discussed for \( \hat{f}' = 0, \hat{g}' = 0 \), for example, in [27, 2, 4]. Here we will focus on the quantized electric flux \( \hat{f}' \) and the quantized magnetic flux \( \hat{g}' \) as given in (16), respectively. There are actually three sub-cases to consider: 1) \( p' = 0, q' \neq 0 \) (\( \hat{f}' = 0, \hat{g}' \neq 0 \)); 2) \( p' \neq 0, q' = 0 \) (\( \hat{f}' \neq 0, \hat{g}' = 0 \)); 3) \( p' \neq 0, q' \neq 0 \) (\( \hat{f}' \neq 0, \hat{g}' \neq 0 \)). Note that once again in practice both the laboratory electric flux \( \hat{f} = 2\pi\alpha'eE \sim 10^{-21} \ll 1 \) and the laboratory magnetic flux \( \hat{g} = 2\pi\alpha'eB \sim 10^{-21} \ll 1 \) as discussed in the previous section. We now discuss each of the sub-cases in order.

**Sub-case 1)** \( p' = 0, q' \neq 0 \): For this sub-case, we have from (16)

\[
\hat{f}' = 0, \quad \hat{g}' = \frac{q'}{n'}.
\]

(23)

So for any finite \( q' \), we have \( \hat{g}' \) being finite since \( n' \geq 1 \). Since \( \hat{f} = 2\pi\alpha'eE \ll 1 \) and \( \hat{g} = 2\pi\alpha'eB \ll 1 \), we have from (18)

\[
\bar{\nu}_0 \approx 2\alpha'eE \sim 10^{-22} \ll 1, \quad \tan \pi\nu_1 = \frac{q'}{n'}.
\]

(24)

where we assume \( q' > 0 \). We therefore have \( Z_1(\bar{\nu}_0, \nu_1) \approx 1 \). Given this and \( \nu_1/\bar{\nu}_0 \gg 1 \) due to \( \nu_1 \) being finite, the rate (22) becomes in the present sub-case as

\[
\mathcal{W}^{(1)} \approx \frac{q'eE}{8\pi^3\alpha'} e^{-\frac{\pi q'^2}{\bar{\nu}_0}}.
\]

(25)
where the effective mass is again given by (13). Just like the case of the D3/D1, the contribution to the present rate comes also from the pair of the massive charged/anti charged vector polarizations with their lowest effective mass mentioned above while the other 15 pairs of massive charged/anti charged polarizations have each a vanishing contribution. This rate looks almost the same as that for the D3/D1 system reported in [1] except for one minor difference and one important difference. For the minor one, we have there $\nu_1 \rightarrow 1/2$ while for the present case we have a finite $\nu_1$ in the range of $(0, 1/2)$ for any finite $q'$ along with $n' \geq 1$. The larger $q'$ and $\nu_1$ are, the larger the rate is. The most efficient is to take $n' = 1$ and this will give the largest $\nu_1$ for given $q'$. Note that the significance of the present rate is more or less the same as that reported in [1] for the system of D3/D1 as briefly summarized in the Introduction except for the important difference to which we come now. The first three conditions mentioned in the Introduction for the D3/D1 continue to apply here the fourth one becomes now trivial. In other words, unlike the case of D3/D1, our laboratory in the present context can be placed in any place in our D3 brane and there is no requirement for where the laboratory is placed. So this serves as the first example of the second scenario mentioned in the Introduction.

Sub-case 2 $p' \neq 0, q' = 0$: For this sub-case, we have from (16)

$$\hat{f}' = \frac{g_s p'}{\sqrt{(g_s p')^2 + n'^2}}, \quad \hat{g}' = 0. \tag{26}$$

For not too large $p'$, say $p' = 10$, the smaller $n'$ is, the larger $\hat{f}'$. So we take $n' = 1$ for this sub-case. We will have $\hat{f}' \approx g_s p' = 0.1$ if we take $g_s = 10^{-2}$ as before. So we have $\hat{f}' \gg \hat{f} \sim 10^{-21}$ in practice. With this, we have from (18)

$$\tanh \pi \bar{\nu}_0 = \hat{f}' = g_s p' = 0.1, \quad \nu_1 = \frac{\bar{g}}{\pi} = 2\alpha' eB \sim 10^{-22} \ll 1. \tag{27}$$

From the above, we can have $\bar{\nu}_0 \approx g_s p' / \pi = 1/(10\pi) \ll 1$ and $\pi \nu_1 / \bar{\nu}_0 \approx 10^{-20} \ll 1$. The former implies $Z_1(\bar{\nu}_0, \nu_1) \approx 1$, indicating only the lowest mass modes of the open string and the anti open string connecting our D3 and the other D3 to contribute possibly to the pair production. These are the eight bosons ($8_B$) and eight fermions ($8_F$) for either the open string or the anti open string, giving a total of 16 pairs of charged/anti-charged modes. To the brane observer, the above $8_B$ and $8_F$ modes of the open string correspond to one of the two broken generators of the original 4-dimensional $N = 4$ U(2) super Yang-Mills when $y = 0$ as $U(2) \rightarrow U(1) \times U(1)$ when $y \neq 0$. The $8_B$ and $8_F$ modes of the anti open string correspond to the other broken generator and each carries the opposite unity charge with respective to those modes of the open string under the unbroken $U(1)$
associated with our own D3. Note that the underlying supersymmetries are also broken by the presence of fluxes, in particular the quantized flux \( \hat{f}' \). Further with \( \nu_1/\bar{\nu}_0 \sim 10^{-21} \), the pair production rate (22) for the present sub-case becomes

\[
\mathcal{W}^{(1)} \approx \frac{2(eE')^2}{\pi^3} e^{-\frac{m^2}{eE'}},
\]

where we have used \( \hat{f}' = 2\pi\alpha' eE' \) with \( eE' = g_s p'/(2\pi\alpha') = M_s^2/(20\pi) \) and \( m = y/(2\pi\alpha') \) the lowest mass for the above \( 8_B \) and \( 8_F \) modes of the open string or the anti open string. In the presence of the applied laboratory magnetic flux \( \hat{g} \), we expect the mass splittings for these charged modes [25] but these splittings, unlike the previous sub-case and the case discussed in the previous section, are insignificant, as indicated in the above rate, because of \( \nu_1/\bar{\nu}_0 \ll 1 \). So we have here all 16 pairs of charged/anti-charged massive modes contributing to the pair production rate (28).

Note also that the rate itself is independent of the laboratory electric and magnetic fields in practice, similar to the case discussed in the previous section. The present rate is new and its detection, by the same token as discussed in the previous section, requires \( eE' = m^2 \), which gives the brane separation \( y = l_s\sqrt{\pi/5} \). In other words, when the D3 carrying the quantized electric flux \( \hat{f}' = g_s p' = 2\pi\alpha' eE' \) comes close to our D3 at the brane separation \( y \leq l_s\sqrt{\pi/5} \approx 0.79 l_s \), the open string pair production can in principle be large enough to be detectable, say, as the electric current detected by our brane observer, up to the brane separation \( y_0 = \pi\sqrt{2\nu_1} l_s \sim 10^{-10} l_s \) at which the tachyon condensation starts to work as discussed around (20). Note that we don’t see the signal of this tachyon condensation from the pair production rate (28) but it appears in the decay rate for large \( k \) as expected from the discussion around (20). Whenever this happens, our computations for the amplitude, say (19), cease to work and so do the computations for the rates. So we need to have \( y > y_0 = \pi\sqrt{2\nu_1} l_s \).

The present rate (28) has advantages over the rate (12) given in the previous section in terms of detectability even though the two cases have the similar electric flux \( \hat{f}' \). For example, the present distance between the brane separation determined by \( eE' \approx m^2 \) and the brane separation \( y_0 \) for which the tachyon condensation starts to work is almost 8 times larger than the corresponding one for the case given in the previous section even though the attractive forces as given in (19) and in (3) look comparable. So the first three conditions for the present sub-case are further relaxed even in comparison with the case discussed in the previous section. In addition, we don’t have the constraint 4) mentioned in the Introduction and our laboratory can be in any place on our D3 brane just like the previous sub-case. Hence this looks like the best case so far so long the detectability is concerned.
Sub-case 3 $p' \neq 0, q' \neq 0$: This appears to be the most interesting sub-case for which we have, from (16), both $\hat{f}' \neq 0$ and $\hat{g}' \neq 0$. We again take not too large $p'$, say, $p' = 10$. With $g_s = 10^{-2}$, noting $q' \geq 1, n' \geq 1$, we have from (16)

$$\hat{f}' = \frac{g_sq_p'}{\sqrt{n'^2 + q'^2}} < 0.1.$$  \hspace{1cm} (29)

We have then from (18)

$$\bar{\nu}_0 \approx \frac{\hat{f}'}{\pi} < \frac{1}{10\pi}, \hspace{1cm} \tan \pi \nu_1 \approx \hat{g}' \approx \frac{q'}{n'},$$  \hspace{1cm} (30)

noting $\hat{f} \sim \hat{g} \sim 10^{-21} \ll 1$. Since $\bar{\nu}_0 < 1/(10\pi)$ and $|z_1| = e^{-\pi/\bar{\nu}_0} < e^{-104\pi^2} \to 0$, we have $Z_1(\bar{\nu}, \nu_1) \approx 1$, once again indicating that at most the $8_B$ and $8_F$ modes from the open string and the same modes from the anti open string contribute to the pair production.

From (30), $\nu_1$ is in general less than $1/2$ but they are on the same order in general. Combining all these, we have the present pair production rate from (22) as

$$W^{(1)} \approx \frac{q'eE'}{8\pi^3\alpha'} e^{-\frac{\pi m_{\text{eff}}^2}{\pi\nu_1}},$$  \hspace{1cm} (31)

where $m_{\text{eff}}$ is given by eq.(13) but in the present context and we have also used $\hat{f}' = 2\pi\alpha'eE'$ in the above. In obtaining the above rate from (22), we have taken $\pi \nu_1/\bar{\nu}_0 \gg 1$ such that we can replace both $\cosh \pi \nu_1/\bar{\nu}_0$ and $\sinh \pi \nu_1/\bar{\nu}_0$ by the exponential factor $e^{\pi \nu_1/\bar{\nu}_0}$ in the rate formulae. This remains true indeed given that $\nu_1$ is in general on the order of $1/2$. For example, for the sample case considered in the following, we have $\pi \nu_1/\bar{\nu}_0 = 5\pi^2/\sqrt{2} \approx 35$. So, just like the sub-case 1) and the case discussed in the previous section, only the pair of the charged/anti-charged vector polarizations with the above lowest effective mass $m_{\text{eff}}$ contributes to the above rate while the other 15 pairs of charged/anti-charged polarizations have their vanishing contributions to this rate.

To detect this rate, we want a possible large $\hat{f}'$ or $eE'$ while also keeping $\nu_1$ as large as possible. A sample choice of both from (29) and (30) is to take $q' = n' = 1$ and this gives $\hat{f}' = g_sq_p'/\sqrt{2} = 1/(10\sqrt{2})$ and $\nu_1 = 1/4$. So we have $eE' = \hat{f}'/(2\pi\alpha') = M_s^2/(20\pi\sqrt{2})$. To detect the rate, we need to have $eE' \approx m_{\text{eff}}^2$ which gives the brane separation, from (13) and with $\nu_1 = 1/4$, as $y = y_0 + 0.1\ l_s$ with $y_0 = (\pi/\sqrt{2})\ l_s$ the brane separation at which the tachyon condensation starts to work. In other words, we expect to be able to detect the pair production once the brane separation falls in the range of $y_0 < y \leq y_0 + 0.1\ l_s$.

Note that the present rate (31) looks essentially the same as that (12) given in the previous section for the system of D3/(F, D1). Both of the cases have the same range of $\Delta y = 0.1\ l_s$ ($y_0 < y \leq y_0 + 0.1\ l_s$) for which both the rates can be detected potentially.
However, there is a big difference between the two. The case for the D3/(F, D1) system still needs the four conditions mentioned in the Introduction, though all of them are relaxed as discussed in the previous section. The present case, however, only needs the first three relaxed conditions as for D3/(F, D1) and the fourth one will be satisfied trivially as the previous two sub-cases discussed in this section. In other words, we don’t need to put a constraint on where our laboratory is placed.

In summary, given the above discussion of all three sub-cases, each of them needs only to satisfy the first three mentioned in the Introduction (or the corresponding three relaxed conditions) but does not need the fourth one. So we fulfill what is promised in the Introduction.

4 Discussion and Conclusion

In this paper, we have considered two new systems, namely, D3/(F, D1) and D3/(D3, (F, D1)), for the purpose of seeking more and better possibilities to improve the detectability of the underlying open string pair production over the previously studied system of D3/D1 reported in [1] by one of the present authors. So long the detectability and/or the associated requirements are concerned, either of these two new systems appears to be in a better position than the D3/D1 one.

For the D3/(F, D1) system, the quantized electric flux gives a much larger electric field \( eE' \sim M_s^2/(20\pi) \) with \( M_s \) the string scale and being larger than a few TeV, see [19], for example. While for the D3/D1 system, the corresponding electric field \( eE \) is a much smaller earthbound laboratory one whose present largest value is only on the order of \( 10^{-8} m_e^2 \) with \( m_e = 5.1 \times 10^{-7} \) TeV, the electron mass. As such, the former gives us a much larger chance of detecting the open string pair production which has the contribution in both cases only from the pair of charged/anti-charged vector polarizations with their effective mass given by (13) with \( \nu_1 = 1/2 \) and \( m = y/(2\pi\alpha') \). To detect the pair production, we need to set \( eE' = m_{\text{eff}}^2 \) for the former and \( eE = m_{\text{eff}}^2 \) for the latter. The former gives the brane separation in the range of \( y_0 < y \leq y_0 + 0.1 l_s \) while the latter gives \( y_0 < y \leq y_0 + 10^{-21} l_s \) for which we take \( M_s \sim \) a few TeV and here \( y_0 = \pi l_s \) the brane separation at which tachyon condensation occurs. So for the former \( \Delta y = 0.1 l_s \) while for the latter \( \Delta y = 10^{-21} l_s \), this great difference of range relaxes the four conditions mentioned in the Introduction for the former against the latter.

For the D3/(D3, (F, D1)) system, we have considered three sub-cases in section 3. The sub-case 1) looks almost identical to that of the D3/D1 system but with an important difference. It does not need the fourth condition obtained from the requirement of a
laboratory detection of the rate for the latter system as mentioned in the Introduction. In other words, for detecting the rate for this sub-case, the corresponding laboratory can be placed in any place in our D3 so long the brane separation meets the first three conditions. This remains true for all the three sub-cases considered for this system. Apart from this, the sub-case 3) looks also almost the same as that for the D3/(F, D1) system. For each of the cases/sub-cases considered in [1] and in this paper, except for the sub-case 2 considered in the previous section, the corresponding pair production rate has the only contribution from the pair of the charged/anti-charged vector polarizations with the lowest mass \( m_{\text{eff}} \) given in (13) with \( \nu_1 \) the same order of \( 1/2 \) in general. However, for the sub-case 2), the story is different. All 16 pairs of charged/anti-charged polarizations, including 5 pairs of charged/anti-charged massive scalars, 4 pairs of charged/anti-charged massive spinors (each spinor has two polarizations) and one pair of charged/anti-charged massive vectors (each massive vector has three polarizations), contribute to the pair production rate. Each of these modes or polarizations has basically the same mass \( m = y/(2\pi\alpha') \).

Given the large quantized electric flux \( eE' = M_s^2/(20\pi) \), as discussed in the previous section, this seems to provide the best scenario for the rate detection. This is very different from what has been considered, for example, in [4], for two D3 placed parallel at a separation \( y \) with our own D3 carrying only the small laboratory collinear electric and magnetic fields. For the latter, that the 16 lowest mass modes of \( 8_B \) and \( 8_F \) of the open string connecting the two D3 all share this same mass \( m = y/(2\pi\alpha') \) is due to the unbroken 16 supersymmetries of the system in the absence of the applied laboratory fluxes. So this open string scale \( m = y/(2\pi\alpha') \) should be no less than TeV due to that the Large Hadron Collider (LHC) at CERN has not found any supersymmetry yet at this scale. Because of this and the available laboratory electric flux \( eE \sim 10^{-21} \text{TeV}^2 \) being too small, it becomes impossible to detect this pair production as discussed in [4] unless there is some other good reason. However, this will not pose a problem for the present consideration. Note that the quantized electric flux \( eE' = M_s^2/(20\pi) \) can be large enough to be comparable to the scale \( m^2 = (y/(2\pi\alpha'))^2 \) even if \( m > \text{TeV} \). This can be easily realized if the string scale \( M_s \) is no less than a few TeV. So this does appear to give us the hope for the detection of the underlying pair production. If we take \( m = y/(2\pi\alpha') > \) a few TeV for all the other cases considered in this paper, similar discussion or the one following [1] for the D3/D1 can also be given.

Given what has been said, we do have an issue here on the actual detection. Unlike the system of two D3 placed parallel at a separation but carrying no quantized flux(s), we have here an attractive interaction between our D3 and the D1 (or (F, D1) or (D3, (F, D1))). As such we need the underlying system to meet the four or the first three conditions
mentioned in the Introduction (or the corresponding relaxed ones). This implies that the pair production, if it occurs indeed, must be in a short period of time (and still transient even with the relaxed conditions). So we have to be lucky enough to catch this short time window to make the detection when the detectable pair production occurs. This surely gives an uneasy task for the detection even if all the conditions are met ideally.

Our above discussion provides also a useful way to determine the lower bound for the string scale $M_s$. For example, setting $eE' = m^2$ for the pair detection, we have $M_s = (20\pi)^{1/2} m$. From this and $m > \text{a few TeV}$, we have the string scale $M_s > \text{a few TeV}$.

**Acknowledgments**

We acknowledge the support by grants from the NNSF of China with Grant No: 11235010 and 11947301.

**References**

[1] J. X. Lu, “A note on the open string pair production of the D3/D1 system,” JHEP **1910**, 238 (2019) [arXiv:1907.12637 [hep-th]].

[2] J. X. Lu, “Magnetically-enhanced open string pair production,” JHEP **1712**, 076 (2017) doi:10.1007/JHEP12(2017)076 [arXiv:1710.02660 [hep-th]].

[3] J. X. Lu, “Some aspects of interaction amplitudes of D branes carrying worldvolume fluxes,” Nucl. Phys. B **934**, 39 (2018) [arXiv:1801.03411 [hep-th]].

[4] J. X. Lu, “A possible signature of extra-dimensions: The enhanced open string pair production,” Phys. Lett. B **788**, 480 (2019) [arXiv:1808.04950 [hep-th]].

[5] Q. Jia and J. X. Lu, “Remark on the open string pair production enhancement,” Phys. Lett. B **789**, 568 (2019) [arXiv:1809.03806 [hep-th]].

[6] J. S. Schwinger, “On gauge invariance and vacuum polarization,” Phys. Rev. **82**, 664 (1951).

[7] C. Bachas and M. Porrati, “Pair creation of open strings in an electric field,” Phys. Lett. B **296**, 77 (1992) [arXiv:hep-th/9209032].
[8] M. Porrati, “Open strings in constant electric and magnetic fields,” arXiv:hep-th/9309114.

[9] J. H. Schwarz, “An SL(2,Z) multiplet of type IIB superstrings,” Phys. Lett. B 360, 13 (1995) Erratum: [Phys. Lett. B 364, 252 (1995)] doi:10.1016/0370-2693(95)01405-5, 10.1016/0370-2693(95)01138-G [hep-th/9508143].

[10] E. Witten, “Bound states of strings and p-branes,” Nucl. Phys. B 460, 335 (1996) doi:10.1016/0550-3213(95)00610-9 [hep-th/9510135].

[11] C. G. Callan and J. M. Maldacena, “Brane death and dynamics from the Born-Infeld action,” Nucl. Phys. B 513, 198 (1998) [hep-th/9708147].

[12] P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, “(F,D(p)) bound states from the boundary state,” Nucl. Phys. B 565, 397 (2000) [hep-th/9906214].

[13] J. X. Lu, B. Ning, R. Wei and S. S. Xu, “Interaction between two non-threshold bound states,” Phys. Rev. D 79, 126002 (2009) [arXiv:0902.1716 [hep-th]].

[14] J. X. Lu and S. Roy, “((F, D1), D3) bound state and its T dual daughters,” JHEP 0001, 034 (2000) [hep-th/9905014].

[15] J. X. Lu, S. Roy and H. Singh, “((F, D1), D3) bound state, S duality and noncommutative open string / Yang-Mills theory,” JHEP 0009, 020 (2000) [hep-th/0006193].

[16] Q. Jia, J. X. Lu, Z. Wu and X. Zhu, “On D-brane interaction & its related properties,” Nucl. Phys. B 953, 114947 (2020) [arXiv:1904.12480 [hep-th]].

[17] A. I. Nikishov, Sov. Phys. JETP 30, 660 (1970); Nucl. Phys. B21, 346 (1970).

[18] S. Hahn, Kwanglok Kim, Kwangmin, Kim, X. Hu, T. Painter, I. Dixon, S. Kim, K. R. Bhattarai, S. Noguchi, J. Jaroszynski and D. C. Larbalestier, “45.5-tesla direct-current magnetic field generated with a high-temperature superconducting magnet,” Nature 570, 496-499 (2019).

[19] D. Berenstein, “TeV-Scale strings,” Ann. Rev. Nucl. Part. Sci. 64, 197 (2014) [arXiv:1401.4491 [hep-th]].

[20] E. J. Copeland, R. C. Myers and J. Polchinski, JHEP 0406, 013 (2004) doi:10.1088/1126-6708/2004/06/013 [hep-th/0312067].

18
[21] J. X. Lu and S. S. Xu, “Remarks on D(p) and D(p-2) with each carrying a flux,” Phys. Lett. B 680, 387 (2009) [arXiv:0906.0679 [hep-th]].

[22] J. C. Breckenridge, G. Michaud and R. C. Myers, Phys. Rev. D 55, 6438 (1997) [arXiv:hep-th/9611174].

[23] M. S. Costa and G. Papadopoulos, “Superstring dualities and p-brane bound states,” Nucl. Phys. B 510, 217 (1998) [arXiv:hep-th/9612204].

[24] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, “Classical p-branes from boundary state,” Nucl. Phys. B 507, 259 (1997) [arXiv:hep-th/9707068].

[25] S. Ferrara and M. Porrati, “String phase transitions in a strong magnetic field,” Mod. Phys. Lett. A 8, 2497 (1993) [hep-th/9306048].

[26] N. K. Nielsen and P. Olesen, “An Unstable Yang-Mills Field Mode,” Nucl. Phys. B 144, 376 (1978).

[27] J. X. Lu and S. S. Xu, “The Open string pair-production rate enhancement by a magnetic flux,” JHEP 0909, 093 (2009) [arXiv:0904.4112 [hep-th]].