The Completion of Non-Steady-State Queue Model on The Queue System in Dr. Yap Eye Hospital Yogyakarta

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Abstract. Dr Yap Eye Hospital Yogyakarta is one of the most popular reference eye hospitals in Yogyakarta. There are so many patients coming from other cities and many of them are BPJS (Badan Penyelenggara Jaminan Sosial, Social Security Administrative Bodies) patients. Therefore, it causes numerous BPJS patients were in long queue at counter C of the registration section so that it needs to be analysed using queue system. Queue system analysis aims to give queue model overview and determine its effectiveness measure. The data collecting technique used in this research are by interview and observation. After getting the arrival data and the service data of BPJS patients per 5 minutes, the next steps are investigating steady-state condition, examining the Poisson distribution, determining queue models, and counting the effectiveness measure. Based on the result of data observation on Tuesday, February 16th, 2016, it shows that the queue system at counter C has (M/M/1):(GD/∞/∞) queue model. The analysis result in counter C shows that the queue system is a non-steady-state condition. Three ways to cope a non-steady-state problem on queue system are proposed in this research such as bounding the capacity of queue system, adding the servers, and doing Monte Carlo simulation. The queue system in counter C will reach steady-state if the capacity of patients is not more than 52 BPJS patients or adding one more server. By using Monte Carlo simulation, it shows that the effectiveness measure of the average waiting time for BPJS patients in counter C is 36 minutes 65 seconds. In addition, the average queue length of BPJS patients is 11 patients.

Keywords: queue system, (M/M/1):(GD/∞/∞), non-steady-state, Monte Carlo simulation

1. Introduction
The queue is a waiting line that usually happens when the demand for service is over capacity. In daily life, the queue case can be seen and even felt directly by society especially the queue of public facilities. One of the examples is in Dr. Yap Eye Hospital which is the one and only eye hospital in Yogyakarta. Dr. Yap Eye Hospital has cooperated in medical services especially the eye medical with the government instance. The newest medical service from the government is BPJS (Badan Penyelenggara Jaminan Sosial, Social Security Administrative Bodies). BPJS is an Indonesian government-owned enterprise which is
obliged to provide protection for socio-economic. That corporation will minimize the cost of medicine since it has been guaranteed by BPJS company. Thus there are many BPJS patients coming to Dr. Yap Eye Hospital. Since there are so many BPJS patients which are over capacity, it causes patients hoarding. The patients hoarding that mostly occurs is in the counter C of the registration section. This is the result of arrival rate that exceeds the service rate of BPJS patients. Therefore, it requires a perfect decision to increase the quality of the service for the patients especially the BPJS ones. Queue theory used to calculate systematically, so that it can solve the problems of queue system. This theory is also use to predict the important parameter such as the average waiting time and the average queue length of BPJS patients [1].

Several research that relevant to queue system were done by Ratna [2], William, Lawrence & John [3], Toshiba, Sanjay & Anil [4] and so on. Ratna [2] applied on Jamkesmas patients data in RSUD Dr. Karyadi Semarang. The analysis of non-steady-state queue system, or transient case, has been done by William, Lawrence & John [3] and the result was achieved by developing computation formula from both symbolic and numeric exact where result are checked against Monte Carlo simulation. Another simple queue model (M/M/1/∞) was implemented by Toshiba, Sanjay & Anil [4] in Bank service to improve the optimal service rate.

Complex queue case can be solved by using simulation. Simulation can be used to represent a reality that occurs in the queue system. Simulation can be defined as a process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour of the system [5]. One of the applications of Monte Carlo queueing system can be found in the patient queue [1]. Monte Carlo simulation is a technique that converts uncertainties of input variables in the model into probability distributions [6]. In that simulation, it requires a random number generator to re-form the opportunity distribution [7]. Monte Carlo simulation is a very effective technique, widely accepted for true result. Other ways to solve the problems of complex queue system are by restricting the queue system capacity or adding the number of the servers. Thus, the queue system is able to reach stability or steady-state.

Steady-state condition is a stable state where the arrival rate is less than the service rate and it can be determined by searching utilization. Utilization refer to the proportion of time that a server (or system of servers) is busy handling customers [8]. Utilization should be strictly less than one for the system to function well (steady-state condition). Utilization is usually represented by the symbol ρ [9]. If ρ > 1 then the arrival rate is faster than it can be served. This shows that the queue length is expected to increase unlimited so it is non-steady-states. Also, if ρ = 1 then the arrival rate is equal to the service rate so that the queue cannot occur. In other words, steady-state will not occur in the queue system.

2. Research Methodology

The research methodology used in this research was descriptive. The descriptive methodology aims to acquire information related to what recently happens and to see the relevancy among the available variables [10]. It has several steps: collecting data, analyzing data, and drawing a conclusion.

2.1 Research Location and Time

This research was done in Dr. Yap Eye Hospital Yogyakarta located in Teuku Cik Ditiro 5 Yogyakarta. The data collection was taken in counter C which is in registration section. This research was taken on Tuesday, February 16th, 2016 at 06.00 am to 13.00 pm.

2.2 Data Collecting Technique

The data used in this research was primary and secondary data. The primary data was gained by interview and observation. The interview resulted in queue system design for BPJS patients while the observation resulted in the arrival data and the service data of BPJS patients. Next, the secondary data was gained by
medical record installation document. The document was in the form of the printout of the BPJS patients queue number in counter C.

2.3 Data Analysis and Non-Steady-state Queue System Problem Solving

2. 3. 1 Data Analysis
Data analysis was performed in counter C of the BPJS patients registration section. The steps to analyze the data were as follows:

a. Categorized the primary data on Tuesday in counter C consisting of the arrival and service data of BPJS patients every 5 minutes. The data grouping taken in this research was the data at 08.00 am to 11.00 am.

b. Sought the average arrival rate (λ) and the average service rate (μ), then investigated the steady-state solution. The queue system will reach the steady-state when ρ = λ/μ < 1. It means the average arrival rate was less than the average service rate or in other words the queue system reached the steady-state in which then was performed Monte Carlo simulation to seek the effectiveness measure.

c. Examined the Poisson distribution of arrival time data and service time data was gained by Kolmogorov-Smirnov test.

With the objective to improve the case BPJS queue system, the following assumptions are considered:

1) The arrival of patients is identically independent and follow the Poisson probability distribution with parameter λ [11] and given by:

\[ P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} ; \quad n = 1, 2, 3 \ldots t > 0 \]

Where \( P_n(t) \) indicates the probability of arriving \( n \) patients in time interval \( t \)

2) The service time of patients is identically independent which meets negative distribution with parameter μ and is given by:

\[ F(t) = 1 - e^{-\mu t}; \quad t > 0 \]

d. Determined the appropriate model using queue system in the counter C of registration section.

e. Counted the effectiveness measure of BPJS patients queue system including counting the average waiting time on the queue and the average queue length.

Queue system analysis can be depicted in the flow chart, Figure 1.
2.3.2 Solution of non-steady-state queue system problems

Characteristic and assumption of queue model in counter C can be summarized into Kendall notation from $(M/M/1)$: $(GD/\infty/\infty)$. In queue model $(M/M/1)$: $(GD/\infty/\infty)$, the inter-arrival times and service times are assumed that they are distributed Exponential [12]. Moreover, queue system in counter C only has one server and unlimited capacity of calling source. Figure 2 is the diagram which shows the model of $(M/M/1)$: $(GD/\infty/\infty)$.

Figure 2. Rate transition diagram for the $(M/M/1)$ queue
Furthermore, for example, $n$ shows the number of customer. Arrival system is assumed as a birth and a departure/service is assumed as a death of the queue system. The average arrival rate ($\lambda$) and the average service rate ($\mu$) do not rely on the number of customer in queue system. Therefore, the queue model $(M/M/1):(GD/\infty/\infty)$ is the process of a birth and a death, where $\lambda_n = \lambda (n \geq 0)$ and $\mu_n = \mu (n \geq 1)$. Based on this process, we have:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

Assumed that $\rho = \frac{\lambda}{\mu}$ , then

$$P_n = (\rho)^n P_0$$

On the queue model $(M/M/1):(GD/\infty/\infty)$, it is obtained the value of $P_0$ and $P_n$:

$$P_0 = 1 - \rho$$

$$P_n = (1 - \rho)\rho^n \quad n \geq 0$$

assumed if $\rho = \frac{\lambda}{\mu} < 1$. In some cases that are found, it shows that the average arrival rate ($\lambda$) is very large, thus $\rho > 1$. This is a non-steady-state case $(M/M/1):(GD/\infty/\infty)$. As a result, the analysis of queue system cannot be solved by analytic model. The solving problem of non-steady-state queue system can be run by some ways, we propose that three ways to cope the problem:

a. Monte Carlo simulation

One of the alternative solutions of the non-steady-state queue system is Monte Carlo simulation. It is caused by non-steady-state queue system that cannot be solved by using analytic model, so that Monte Carlo simulation is the best way. Monte Carlo simulation is an approach for reforming probability distribution based on a choice or a random number. Figure 3 depicts the steps to do Monte Carlo simulation in the flowchart:
b. Bounding the capacity of the queue system
   Besides doing Monte Carlo simulation, another way that can be run is bounding the capacity of queue system. This restriction of the capacity of queue system was carried out by reducing the service data of BPJS patients one by one that has classified per 5 minutes. After it was reduced one by one, then the service data classified again per 5 minutes and counted again until the number of service rate ($\mu$) less than the arrival rate ($\lambda$). Therefore, the average service rate ($\mu$) will be smaller and queue system reach the steady-state condition.

c. Add the number of a server at queue system.
   Besides the two ways described previously, the next way to solve the queue system that is non-steady-state is adding the number of servers. Adding the number of servers can be done until the queue system
reached steady-state condition. Thus, after the queue system reach the steady-state condition then we can count the effectiveness by using an analytical model.

3. Result and Discussion

3.1 Result of research
The results of research which done in counter C of registration section is the average arrival rate and the average service rate of BPJS patients. In addition, the Kolmogorov-Smirnov test using SPSS software can be known that the arrival data and service data of BPJS patients are Poisson distribution. The result of research the average arrival rate and the average service rate after data are grouped per 5 minutes can be shown in Table 1.

| Name     | Steady State |
|----------|--------------|
| Counter C| Not yet      |

Based on Table 1 the queue system at counter C has not reached steady-state conditions because of \( \rho > 1 \). This cause the effectiveness measure cannot be calculated using analytical models. Therefore, to solve the non-steady-state queue system by using Monte Carlo simulation, bounding of capacity queue system and added the servers.

3.2 Discussion

3.2.1 Model of queue system
The discussion of the queue system analysis is obtained appropriate queue model at counter C of registration section that follows \( (M/M/1):(GD/\infty/\infty) \) model. The \( (M/M/1):(GD/\infty/\infty) \) queue model have one server that is a computer and have discipline service called General Discipline or First Come First Served. In addition, the capacity of queue system and calling sources of BPJS patients is unlimited.

3.2.2 The effectiveness measure of non-steady-state queue system
The problem of non-steady-state queue system to calculate the effectiveness measure can be solve in the following way:

a. Monte Carlo simulation
Procedure of Monte Carlo simulation using MS. Excel [13] as follows:

- Determine the inter-arrival time, the cumulative probability, and random number interval.

| Inter-Arrival Time (minutes), \( x \) | Frequency, \( f(x) \) | \( P(x) = \frac{f(x)}{\sum_x f(x)} \) | Cumulative Probability | Random Number Interval |
|-----------------|-----------------|-----------------|---------------------|----------------------|
| 0               | 12              | 0.1644          | 0.0000              | 0.0000-0.1643        |
| 1               | 20              | 0.2740          | 0.1644              | 0.1644-0.4383        |
| 2               | 16              | 0.2192          | 0.4384              | 0.4384-0.6574        |
| 3               | 9               | 0.1233          | 0.6575              | 0.6575-0.7807        |
| 4               | 3               | 0.0411          | 0.7808              | 0.7808-0.8218        |
Inter-Arrival Time (minutes), $x$ | Frequency, $f(x)$ | $P(x) = \frac{f(x)}{\sum f(x)}$ | Cumulative Probability | Random Number Interval |
---|---|---|---|---|
5 | 7 | 0.0959 | 0.8219 | 0.8219-0.9177 |
6 | 2 | 0.0274 | 0.9178 | 0.9178-0.9451 |
8 | 1 | 0.0137 | 0.9452 | 0.9452-0.9588 |
10 | 1 | 0.0137 | 0.9589 | 0.9589-0.9725 |
12 | 1 | 0.0137 | 0.9726 | 0.9726-0.9862 |
13 | 1 | 0.0137 | 0.9863 | 0.9863-0.9999 |
$\sum_{=73}$ | $\sum_{=1.0000}$ | $\sum_{=1.0000}$ |

- Determine the service time, cumulative probability and random number interval

| Service Time (minutes), $y$ | Frequency, $f(y)$ | $P(y) = \frac{f(y)}{\sum f(y)}$ | Cumulative Probability | Random Number Interval |
---|---|---|---|---|
0 | 3 | 0.0492 | 0.0000 | 0.0000-0.0491 |
1 | 16 | 0.2623 | 0.0492 | 0.0492-0.3114 |
2 | 14 | 0.2295 | 0.3115 | 0.3115-0.5409 |
3 | 9 | 0.1475 | 0.5410 | 0.5410-0.6884 |
4 | 11 | 0.1803 | 0.6885 | 0.6885-0.8688 |
5 | 2 | 0.0328 | 0.8689 | 0.8689-0.9015 |
6 | 1 | 0.0164 | 0.9016 | 0.9016-0.9179 |
8 | 2 | 0.0328 | 0.9180 | 0.9180-0.9507 |
14 | 2 | 0.0328 | 0.9508 | 0.9508-0.9835 |
23 | 1 | 0.0164 | 0.9836 | 0.9836-0.9999 |
$\sum_{=61}$ | $\sum_{=1.0000}$ |

- After we obtained random number interval for each cumulative probability of arrival and services data of BPJS patients, then we make 73 trial rows of arrival BPJS patients as shown in Table 2.
Table 2. Monte Carlo Simulation

| Patient | Random Number (Arrival Time) | Inter-Arrival Time (Minutes) | Arrival Time | Time to Enter Service | Waiting Time | Queue Length | Random Number (Service Time) | Service Time (Minutes) |
|---------|------------------------------|-----------------------------|--------------|-----------------------|--------------|--------------|------------------------------|------------------------|
| 1       | R1                           | 0                           | 0            | 0                     | 0            | 0            | 0.1484                      | 1                      |
| 2       | 0.5819                       | 2                           | 2            | 2                     | 0            | 0            | 0.5421                      | 3                      |
| 3       | 0.1413                       | 0                           | 2            | 5                     | 3            | 1            | 0.8849                      | 5                      |
| 4       | 0.2517                       | 1                           | 3            | 10                    | 7            | 2            | 0.3384                      | 2                      |
| 5       | 0.2166                       | 1                           | 4            | 12                    | 8            | 3            | 0.9336                      | 8                      |
| 6       | 0.1505                       | 0                           | 4            | 20                    | 16           | 4            | 0.9269                      | 8                      |
| 7       | 0.7623                       | 3                           | 7            | 28                    | 21           | 4            | 0.7954                      | 4                      |
|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 71 | 0.2827 | 1 | 184 | 230 | 46 | 10 | 0.8206 | 4 |
| 72 | 0.7865 | 4 | 188 | 234 | 46 | 10 | 0.1079 | 1 |
| 73 | 0.6873 | 3 | 191 | 235 | 44 | 11 | 0.7825 | 4 |
After doing the computation, the result can be used to calculate the average waiting time in queue \( (W_q) \) and length of queue \( (L_q) \). Here are the results of the calculation of average waiting times and length queues with repetition as much as 10 times:

| Repetition | Average waiting time in queue \( (W_q) \) | Average queue length \( (L_q) \) |
|------------|------------------------------------------|---------------------------------|
| 1          | 32.74                                    | 10.59                           |
| 2          | 32.29                                    | 9.55                            |
| 3          | 33.26                                    | 9.49                            |
| 4          | 38.51                                    | 11.07                           |
| 5          | 31.41                                    | 10.10                           |
| 6          | 42.42                                    | 12.70                           |
| 7          | 40.44                                    | 13.75                           |
| 8          | 30.85                                    | 7.75                            |
| 9          | 34.48                                    | 13.40                           |
| 10         | 42.45                                    | 13.30                           |

Furthermore, we make a graph to find the midpoint of both the average waiting time in queue and the average queue length. The values are currently being sought by using the lower and upper limits. In Table 3 it can be seen that the lower limit value \( W_q \) currently on the 8th repetition and the upper limit is at the 10th repetition. The lower limit value contained in the \( L_q \) replicates the 8th and the upper limit on the repetition 7th. Two graphs in Figure 4 and Figure 5 are the visualization of Table 3:

![Figure 4. Repetition of average waiting time \( (W_q) \)](image-url)
Based on the previous chart, it can be seen that the average waiting time and length of queue are:

\[
W_q = \frac{30.85 + 42.45}{2} = 36.65 \text{ minutes}
\]

\[
L_q = \frac{7.75 + 13.75}{2} = 10.75 \approx 11 \text{ patients}
\]

b. Bounding capacity of the queue system

Bounding capacity of the queue system is the second way to solve a problem of non-steady-state queue system. The following service data of BPJS patients when it was reduced one by one and regrouped per 5 minutes:

**Table 4. Service data of BPJS patients based on the interval per 5 minutes**

| Interval with I services | The number of BPJS patients services at intervals \( K_I \) | The frequency or the number of intervals \( f(I_i) \) | The number of BPJS patients who served during the period \( I_i \) = \( K_i \times f(I_i) \) |
|--------------------------|--------------------------------------------------------|---------------------------------|---------------------------------|
| \( I_0 \)               | 0                                                      | 9                               | 0                               |
| \( I_1 \)               | 1                                                      | 8                               | 8                               |
| \( I_2 \)               | 2                                                      | 13                              | 26                              |
| \( I_3 \)               | 3                                                      | 6                               | 18                              |
| \( I_4 \)               | 4                                                      | 0                               | 0                               |

\[ \sum I = 36 \quad \sum N = 52 \]

Based on the service data of BPJS patients regrouping per 5 minutes, obtained value of the average service rate is less than previous one:

\[
\mu = \frac{\sum N}{\sum I} = \frac{52}{36} = 1.4444 \frac{\text{patients}}{5 \text{ minutes}} = 0.2889 \frac{\text{patients}}{\text{minutes}}
\]
So that utility level of queue system at counter C can be recalculate as follows:

| Name     | $\lambda$ | $\mu$ | $\rho = \frac{\lambda}{\mu}$ | Steady State |
|----------|-----------|-------|-----------------------------|--------------|
| Counter C| 0.2722    | 0.2889| 0.9422                      | Yes          |

Based on Table 5, the queue system at counter C will reach steady-state condition when queue capacity is bounded for 52 $BPJS$ patients. Because it bounded for 52 $BPJS$ patients, so that queue model becomes $M/M/1$; $(GD/52/\infty)$. Then, the effectiveness measure of queue system at counter C can be calculate using analytic formula $(M/M/1)$; $(GD/N/\infty)$ as follow:

$$W_s = \frac{1}{\mu \left(1 - \frac{\lambda}{\mu}\right)}$$

$$= \frac{1}{0.2889 \left(1 - \frac{0.2722}{0.2889}\right)} = 59.8802 \text{ minutes}$$

$$L_s = \frac{\lambda}{\mu \left(1 - \frac{\lambda}{\mu}\right)}$$

$$= \frac{0.2722}{0.2889 \left(1 - \frac{0.2722}{0.2889}\right)} = 16.3010 = 16 \text{ patients}$$

So from capacity bounding of queue system generate the average waiting time is 59.8802 minutes and the average queue length is 16 patients.

c. Adding servers of queue system
The another way to solve non-steady-state queue system problem is adding servers. After adding one server, the queue model is $M/M/C$ and the utility level for each server is $r = \frac{\lambda}{c\mu}$ [12]. The Table 6 bellow is steady-state condition of queue system at counter C:

| Name     | $c$ | $\lambda$ | $\mu$ | $r = \frac{\lambda}{c\mu}$ | Steady State |
|----------|-----|-----------|-------|-----------------------------|--------------|
| Counter C| 2   | 0.4056    | 0.3388| 0.5986                      | Yes          |

Based on the Table 6 the queue system at counter C will reach steady-state condition when adding one more server. Therefore, the model of queue system at counter C become $(M/M/2); (GD/\infty/\infty)$. So that, the effectiveness measure at counter C can be calculate using analytic formula $(M/M/c); (GD/\infty/\infty)$ as follows:

$$\lambda = 0.4056 \ ; \ \mu = 0.3388 \ ; \ c = 2$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.4056}{0.3388} = 1.1972$$
\[
P_0 = \left\{ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left( \frac{1}{1 - \frac{\rho}{c}} \right) \right\}^{-1} \\
= \left\{ \frac{1.1972^n}{n!} + \frac{1.1972^2}{2!} \left( \frac{1}{1 - 1.1972/2} \right) \right\}^{-1} = 0.2511
\]

\[
L_s = \left[ \frac{\rho^{c+1}}{(c-1)! (c-\rho)^2} \right] P_0 + \rho \\
= \left[ \frac{1.1972^{2+1}}{(2-1)! (2-1.1972)^2} \right] 0.2511 + 1.1972 = 1.8657 \approx 2 \text{ patients}
\]

\[
W_s = \frac{\rho^c}{\mu (c-1)! (c-\rho)^2} P_0 + \frac{1}{\mu} \\
= \frac{1.1972^2}{0.3388(1)! (2-1.1972)^2} 0.2511 + \frac{1}{0.3388} = 4.5998 \text{ minutes}
\]

So, the queue system resolution can be done by adding the number of servers results the average queue length is 2 patients and the average waiting time is 4.5998 minutes.

4. Conclusion

Based on the results of this research and discussion of the queue system at the counter C, BPJS patients Dr. Yap Eye Hospital Yogyakarta is following \( (M/M/1):(GD/\infty/\infty) \) model. We obtained that queue system at the counter C was non-steady-state. It means the average arrival rate is more than the average service rate. Therefore the effectiveness measure of the queue system in this research can be solved by three ways such as doing Monte Carlo simulation, bounding the capacity of queue system and adding the number of servers.

First, Monte Carlo simulation works with new trials based on random numbers generated by Ms. Excel and referred to the real data. The average waiting time and length of queue are calculated using the these new trials. By using Monte Carlo simulation, it shows that the average waiting time in queue for BPJS patients at counter C is 36 minutes 65 seconds and the average queue length is 11 patients. Second, bounding the capacity of the new system means we reduced the number of customers one by one until it reaches steady-state condition. By using this way, the arrival capacity of patients is less than 52 BPJS patients. Last, the third solution is by adding the number of server one by one until it reaches steady-state condition. Since the system will follow steady-state condition by using two server, it means that we only need to add one server.

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