Thermal Conductivity as a Probe of Quasi-Particles in the Cuprates.

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In underdoped YBa$_2$Cu$_3$O$_x$ ($x = 6.63$), the low-$T$ thermal conductivity $\kappa_{xx}$ varies steeply with field $B$ at small $B$, and saturates to a nearly field-independent value at high fields. The simple expression $[1 + p(T)|B|]^{-1}$ provides an excellent fit to $\kappa_{xx}(B)$ over a wide range of fields. From the fit, we extract the zero-field mean-free-path, and the low temperature behavior of the $QP$ current. The procedure also allows the $QP$ Hall angle $\theta_{QP}$ to be obtained. We find that $\theta_{QP}$ falls on the $1/T^2$ curve extrapolated from the electrical Hall angle above $T_c$. Moreover, it shares the same $T$ dependence as the field scale $p(T)$ extracted from $\kappa_{xx}$. We discuss implications of these results.

1. Introduction

Thermal conductivity is potentially a very useful probe of the quasi-particle excitations in the superconducting state of the cuprates because it is capable of detecting the quasi-particle ($QP$) current in the bulk. In addition, measurements of its field dependence may yield quantitative information on the $QP$ mean free path. At present, this seems to be the best way to investigate the low-lying excitations of the condensate. However, the task of disentangling the $QP$ current from the larger phonon current in the cuprates poses a difficult problem for experiment.

We report recent experiments in which the direct separation of the $QP$ current is achieved by detailed analysis of the field dependence of the longitudinal conductivity $\kappa_{xx}$. This line of approach was motivated by the observation of plateau features in high-purity Bi$_2$Sr$_2$CaCu$_2$O$_8$ (Bi 2212). The existence of the plateaus at low $T$ (where $\partial \kappa_{xx}/\partial H = 0$) implies that, in the cuprates, vortices are essentially transparent to the phonons. Extensions of these measurements to underdoped YBa$_2$Cu$_3$O$_x$ (YBCO) reveal that this result may be a rather general feature of the cuprates in the clean-limit. This seems to us a significant finding since it allows a direct separation of the $QP$ current by the application of an intense field. In addition, we find that the zero-field mean-free-path $\ell_0$ of the $QP$ may be estimated to within a factor equal to the vortex scattering cross-section $\sigma_{tr}$.

The isolation of the $QP$ current allows more specific information to be extracted from the thermal Hall conductivity $\kappa_{xy}$ (Righi-Leduc effect). With the electronic current independently determined, we may now obtain the Hall angle $\tan \theta_{QP}$. The $QP$ Hall angle uncovers a number of interesting features which we discuss below.

2. Experiment

Measurements of $\kappa_{xx}$ in the mixed state of the cuprates have been reported by several groups. Experiments in intense magnetic fields $H$ are complicated by problems such as cleaving of the crystal by the large torques generated. The most serious problem, however, seems to stem from the field sensitivity of the thermometers. At the resolution needed, the field dependence of the sensors is serious (in thermocouples, moreover, the field sensitivity is also history dependent). Previously, we employed a bridge-balance method to get around the field-sensitivity problem.

![FIG. 1. The $T$ dependence of the in-plane thermal conductivity $\kappa_{xx}(0)$ in zero field (open symbols) in underdoped YBCO with $x = 6.63$ and $T_c = 63$ K. The solid symbols are $\kappa_{xx}$ estimated by the fit of the thermal magnetoconductance to Eq. 5. The $dc$ magnetization of the crystal is shown as open triangles ($H = 5$ Oe).](image-url)

In our present approach, we adopt a single-heater, two-sensor method in which the temperature difference $\delta T$ between the ends of the sample is detected by two closely matched resistive sensors (cernox). The thermal gradient $-\nabla T$ is applied in the $ab$ plane, and $H$ is parallel to $c$. The temperature is regulated (with a third base-cernox), and measurements are taken after waiting about 10 min. for the field to stabilize. The readings of the two sensors are recorded at three values of the heater current $I = 0, 0.4, 0.6$ mA (typically). The $I = 0$ readings are used to calibrate the field dependence of the two cernox sensors.
sensors, while the values of $\delta T$ determined with $I$ at the two values provide a check on the linearity of the sample response. By testing with a standard material that has no intrinsic field dependence in $\kappa_{xx}$ (nylon), we have found that at temperatures above 8 K this method provides an accurate and highly reproducible determination of the intrinsic conductance to a resolution of 1 in $10^3$. Although the bridge-balance method is capable of higher resolution, the present technique lends itself to full automation. A higher density of points may be obtained, and checks (e.g. for linearity) can be made in situ. A pair of thermocouple junctions are used to detect the $H$-antisymmetric (Hall) gradient to obtain $\kappa_{xy}$.

With the high sensitivity, we have found that, in untwinned, optimally-doped YBa$_2$Cu$_3$O$_6$ ($T_c = 93$ K, $x = 6.95$), $\kappa_{xx}$ in a field $H \parallel c$ becomes increasingly hysteretic below 35 K. Although the hysteresis is small (about 5% of the total $\kappa_{xx}$ at 8 K), it greatly complicates the extraction of the QP current (we discuss this later). In underdoped crystals, however, the hysteresis is unobservable up to 14 tesla (less than $10^{-3}$), and the observed $\kappa_{xx}$ vs. $H$ is a faithful representation of its intrinsic dependence on $B$.

In this report, we discuss data from a twinned, underdoped crystal in which $T_c = 63$ K, and $x = 6.63$. The zero-field temperature profile of $\kappa_{xx}$ is shown in Fig. 1. The relative magnitude of the anomaly in $\kappa_{xx}$ is only about a quarter of that in the 93-K YBCO, but larger than in optimum Bi 2212 and La$_2$-Sr$_x$CuO$_4$ (LSCO). Also shown (solid symbols) is our new estimate of the phonon thermal conductivity ($\kappa_B$). One of our main results is that the entire field dependence of $\kappa_{xx}$ derives from the QP current, while the phonon current is unaffected by $H$.

3. Results and Analysis

Figure 2 displays the field dependence of $\kappa_{xx}$ at selected $T$. With decreasing $T$, the initial slope of $\kappa_{xx}$ increases rapidly. Below 15 K, the rapid decrease crosses over to an almost flat dependence, which recalls the plateau features observed in single-domain Bi 2212. Within our resolution, there is no resolvable hysteresis at the temperatures investigated. The higher precision and larger range of the new data enable us to compare the observed field dependence with various expressions. We found that the field dependence is accurately fitted (Fig. 3) to the expression

$$\kappa_{xx}(B, T) = \frac{\kappa_e(T)}{1 + p(T)|B|} + \kappa_B(T), \quad (1)$$

where the entire $B$ dependence resides in the denominator of the first term, and the term $\kappa_B$ is a field-independent background. At each temperature, the fit yields the two parameters $\kappa_e(T)$ and $p(T)$ associated with the QP current, and the term $\kappa_B$ which we identify with the phonon term, viz. $\kappa_B = \kappa_{ph}$.

In our previous experiments on optimally-doped YBCO and LSCO, fits to Eq. 1 were ambiguous because of strong hystereses (the distortions introduced caused the extracted $p(T)$ to be non-monotonic in $T$). These extraneous effects led us to consider alternate expressions (see below), as well as a possibly field dependent $\kappa_{ph}$. However, the present results have clarified the problem. In addition to the absence of observable hysteresis, the curves for $\kappa_{xx}$ at low $T$ display pronounced curvature in moderate fields, corresponding to a strong attenuation of the QP current. The attenuation uncovers a nearly field-independent background that we identify with the phonon thermal conductivity. In the main panel of Fig. 3, we display the $T$ dependence of the parameters $\kappa_e$ and $p(T)$.

The motivation for Eq. 1 is that, near the vortex core, the steep variation of the pair potential and the circulating superfluid together present a strong scattering potential for an incident QP [10]. Expressing the scattering rate as a transport cross-section $\sigma_{tr}$, we assume additivity of the rates, and write the QP mean-free-path in a field as $\ell(B) = \ell_0/[1 + \ell_0\sigma_{tr}|B|/\phi_0]$, where $\ell_0$ is the zero-field value of the mean-free-path, and $\phi_0$ the flux quantum. Thus, in this model, the QP conductivity is given by Eq. 1 with $\kappa_e$ from $[1]$, $[11]$, $[13]$

$$p(T) = \ell_0\sigma_{tr}/\phi_0. \quad (2)$$

FIG. 2. Variation of $\kappa_{xx}$ with field $H \parallel c$ in underdoped YBa$_2$Cu$_3$O$_{6,63}$ at indicated temperatures. All curves are non-hysteretic to 1 part in $10^3$ (the curves above 15 K are superpositions of sweep-up and sweep-down traces). A fit to the data at 22.5 K is also shown superposed. Curves at 15 K and below (discrete symbols) show the approach of $\kappa_{xx}$ to the $H$-independent value ($\kappa_B$).
In the Boltzmann equation approach, we may write the $QP$ thermal conductivity (in zero field) as

$$\kappa_c(T) = \frac{1}{T} \sum_{k} (\frac{\partial f}{\partial E})E(k)^2\nu_x(k)^2\tau(k),$$

where $E(k)$ and $\tau(k)$ are, respectively, the energy and lifetime of a $QP$ in the state $k$, and $\nu_x(k) = \hbar^{-1}\nabla E(k)$ its group velocity. In a $d$-wave superconductor at low $T$, the excitations are confined to Dirac cones at the nodes where the energy may be parametrized as $E(k_1, k_2) = \hbar\sqrt{(k_1\nu_f)^2 + (k_2\nu_d)^2}$ ($k_1$ and $k_2$ are the components of $k$ normal and parallel to the Fermi Surface), $\kappa_c$ reduces to

$$\kappa_c(T) = \frac{\eta k_B^2 T^2}{\hbar^2} \frac{\ell_0}{v_2} (1 + \frac{v_2}{v_f}^2),$$

where $\ell_0 = v_f\tau_0$ is the mean free path at the nodes, and $\eta = \int_0^\infty dx x^3(-df/dx) \sim 5.41$.

The $T^2$ dependence of $\kappa_c$ in Eq. 3 is masked by the strong $T$ dependence of $\ell_0$. However, in our experiment, the latter is obtained independently from the field dependence (with $p(T)$ given by Eq. 4). We may divide out the $T$ dependence of $\ell_0$, to isolate the quantity $L_c(T) = \kappa_c(T)/p(T)$. Comparing Eqs. 3 and 4, we are left with an expression that contains only two material-specific parameters $v_2$ and $\sigma_{tr}$, viz.

$$L_c(T) = \frac{\eta k_B^2 T^2}{\hbar^2} \frac{\phi_0}{v_2\sigma_{tr}},$$

Figure 3 (inset) reveals that the measured $L_c(T)$ displays a nearly $T^2$ dependence at low $T$, that may be fitted to give $L_c(T) = 6.9 \times 10^{-3}T^2$ WT/mK. Comparing this expression with Eq. 5, we determine from our experiment

$$v_2\sigma_{tr} = 2.11 \times 10^{-4} \text{m}^2/\text{s}.$$ (6)

4. The Hall angle

The field parameter $p(T)$ has been extracted from $\kappa_{xx}$ alone. While its nominally $1/T^2$ variation (Fig. 3) is consistent with the identification $p(T) \sim \ell_0$ (Eq. 3), it is important to see if this is consistent with a separate experiment. We turn next to the Hall conductivity $\kappa_{xy}$. In the previous Hall study on optimal YBCO $\kappa_{xy}$ was analyzed without the benefit of information on the diagonal electronic current. From the analysis above, we may now extract the Hall angle $\tan \theta_{QP}(H) = \kappa_{xy}(H)/\kappa_c(H)$ as a continuous function of $H$ at each temperature. In general, $\tan \theta$ displays strong negative curvature vs. $H$

[Here, we restrict our discussion to the weak-field value $\theta_{QP}(0)$]. In underdoped YBCO, the small $QP$ population generates a weak thermal Hall current, and the uncertainties in determining $\theta_{QP}$ are quite large (compared to optimum YBCO). Nevertheless, we find two interesting features of the Hall angle (see Fig. 4). First, $\theta_{QP}(0)$ (solid triangles) and $p$ (open circles) share the same $T$-dependence from $T_c$ to about 20 K. Secondly, we recall that the normal-state electrical Hall angle $\theta_N$ (open triangles) follows a $1/T^2$ dependence.

![Fig. 3. The $T$ dependence of the zero-field $QP$ conductivity $\kappa_c$ and the field scale $1/p(T)$ extracted from fits to $\kappa_{xx}$ vs. $H$ in underdoped YBCO. The $T^2$ dependence of $1/p(T)$ suggests that it is related to a scattering rate. The solid line is $c_0 + c_1 T + c_2 T^2$, with $c_0 = 0.25$, $c_1 = 0.016$, and $c_2 = 0.0012$ (SI units). The inset plots the $T$ dependence of the ratio $L_c(T) \equiv \kappa_c(T)/p(T)$ (in which $\ell_0$ cancels). In the vortex scattering model, $L_c$ should vary as $T^2$ in a $d$-wave superconductor at low $T$ (Eq. 3).

The new values for $\tan \theta_{QP}(0)$ lie on the curve for $\theta_N$ extrapolated below $T_c$. As shown in Fig. 3, the three quantities $p$, $\tan \theta_{QP}(0)$ and $\tan \theta_N$ fall on the same curve over about 2.5 decades (in the plot $p$ and $\tan \theta_{QP}(0)$ are related by a constant scale factor). Just above $T_c$, $\tan \theta_N$ displays a slight dip associated with fluctuation effects. The similarity between $\theta_{QP}$ and $\theta_N$ has also been pointed out by Zeini et al. [3].

We briefly discuss our interpretation. These results suggest that the $1/T^2$ dependence is, in fact, intrinsic to the $QP$’s below $T_c$. The ubiquitous $T^2$ dependence of $\cot \theta_N$ in the normal state appears to be an extension of the low-temperature behavior into the normal state. The similarity between the $T$-dependences of $p$ and $\theta_{QP}$ imply that the diagonal and the Hall channels relax with the same $T$-dependence, $\sim 1/T^2$, consistent
with simple Drude behavior. Just as in conventional metals, the thermal Hall resistivity in the mixed state, \( W_{xy} \equiv \kappa_{xy}/\kappa_T^2 = \tan \theta_{QP}(0)/\kappa_c \), should provide a measure of the heat capacity of the QP population, as it does (see inset of Fig. 3).

The first feature allows us to fit a much broader range of field scales (as expressed by the dimensionless parameter \( p(T)/B \)). The higher density of measurements also helps. We illustrate this point as follows. In a previous attempt [7] to analyze similar measurements in LSCO, it was found that the \( \kappa_{xx} \) vs. \( H \) curves were equally well-fitted by Eq. 1 and the expression \( G(B) = \psi(1/2 + B_0/B) + \ln(B_0/B) \), with \( \psi(x) \) the digamma function. With the data scatter and the smaller range of reduced fields \( B/B_0 \) in LSCO, the two fits could not be distinguished, and Ong et al. [8] argued the case for adopting \( G(B) \) to describe \( \kappa_{xx} \) in LSCO. However, with the larger field scale \( pB \) here, we find that Eq. 1 provides a much better fit (this is evident if the two fits are compared in a plot versus \( \ln H \)). The physics underlying the two fit expressions is of course quite different. In light of the present work, we now favor adopting Eq. 1 instead of the digamma function fit, for analyzing \( \kappa_{xx} \) vs. \( H \) curves in cuprates.

The second feature (no observable hysteresis) relates to the issue of remanence and vortex pinning in cuprates. It is known that the relaxation of nonequilibrium flux distributions may produce a slow drift in \( \kappa_{xx} \) if it is observed a few seconds after a change in \( H \) is made [8]. Further, vortex pinning effects at low \( T \) can lead to step-like jumps in \( \kappa_{xx} \) when the field sweep direction is changed. In optimum YBCO and in strongly overdoped Bi 2212, we find that \( \kappa_{xx} \) increases step-wise when the sweep direction of \( H \) is reversed from up to down. Recently, however, a hysteretic loop of the opposite sign has been reported by Aubin et al. [8] in Bi 2212 (\( \kappa_{xx} \) decreases step-wise when \( H \) is swept down). At present, the origin of the hystereses in \( \kappa_{xx} \) is not understood (especially the existence of hystereses with different signs). We note that the magnitude of the hystereses in is much smaller than that observed in the magnetization \( M vs. H \). In our measurements on underdoped YBCO, no hysteresis in \( \kappa_{xx} \) is observed for fields as large as 14 T at temperatures down to 6 K, even though hystereses are sizeable in the \( M vs. H \) curves. In particular, the magnitude of \( \kappa_{xx} \) at the plateau-like region is not hysteretic. The absence of hysteresis implies that the magnetization is too small to influence the measured \( \kappa_{xx} \), so we may assume that \( B = \mu_0 H \), as tacitly assumed in the fits.

By contrast, hysteretic effects cannot be neglected in optimally doped YBCO (as discussed above). Stronger vortex pinning is clearly responsible for the larger hysteresis in the 93-K crystal. Below 35 K in this crystal, the hysteresis steadily increases to about 5% at 8 K. Although the hysteresis is small, the remanence produces in the trace of \( \kappa_{xx} \) vs. \( H \) both a broadening at small \( H \) and an asymmetry about \( H = 0 \) that strongly distort the fit to Eq. 1. The distortions preclude a meaningful extraction of below about 35 K. Thus, of the various phases of the cuprates we investigated (YBCO, Bi 2212 and LSCO), the underdoped phase of YBCO appears to be the most suitable for our purpose of isolating the QP current from the total thermal current.

**FIG. 4.** The temperature dependences of the field scale \( p(T) \) (open circles) and the weak-field QP Hall angle below \( T_c \) (closed triangles) and the electrical Hall angle \( \tan \theta_N \) above \( T_c \) (open triangles). The close similarity of \( T \)-dependences of \( p \) and \( \tan \theta_{QP} \) strongly suggests that \( p \) is proportional to the QP lifetime. The broken line shows that the QP weak-field Hall angle is numerically equal to the extrapolation of \( \tan \theta_N \).

This conventional behavior is abruptly altered when we cross \( T_c \) into the normal state. The Hall angle continues to relax at the same numerical rate. By contrast, the scattering rate in the diagonal conductivity undergoes a dramatic change [4]. The transport mean-free-path \( \ell_0 \) decreases abruptly by a factor of 4-6 (10 in optimal crystals) across \( T_c \). This sharp decrease, followed by a nominally \( T \)-linear scattering rate, is responsible for the anomalously strong \( T \)-dependence of the Hall coefficient in the normal state. Thus, the anomalous channel responsible for most of the strange-metal properties is the diagonal conductivity. The Hall channel appears to be quite conventional. A detailed discussion of these results including optimal crystals will appear elsewhere [13].

**5. Discussion**

We have discussed how the field dependence of \( \kappa_{xx} \) in underdoped YBCO may be analyzed to extract electronic parameters, such as \( \ell_0 \) and \( \sigma_{tr} \). The analysis derives from two striking features of \( \kappa_{xx} \) intrinsic to high-purity 60-K YBCO crystals at low \( T \), namely the steep decrease of \( \kappa_{xx} \) in weak field followed by a saturation at high field, and the absence of resolvable hysteresis.
In high-purity single-domain crystals of Bi 2212, the field dependence of $\kappa_{xx}$ displays a distinct break in slope in $\kappa_{xx}$ at a characteristic field $H_k$, followed by a plateau region in which $\kappa_{xx}$ is nearly independent of $H$. The field $H_k$, which varies approximately as $T^2$, was interpreted as a field-induced phase transition, possibly involving a new order parameter. We compare the present results with the two findings in Bi 2212, i.e. the kink feature at $H_k$ and the existence of the plateau. As shown in Fig. 2, $\kappa_{xx}$ in 60-K YBCO smoothly crosses over into the field-independent region at low $T$, instead of displaying a sharp kink. While the plateau regime is similar in the two systems, the kink feature signalling a phase transition is absent in the YBCO crystal. A difference between the two systems is the electronic anisotropy. From the resistivity anisotropy $\rho_{xx}/\rho_{ab}$ ($\sim 10^5$ compared with $10^3$), Bi 2212 is much closer to the 2D limit than underdoped YBCO. Whether this is a significant factor is a subject for future investigation.

In their experiment Aubin et al. observed the value of $\kappa_{xx}$ in Bi 2212 at the plateau (at 8 K) to be 1% higher in the field sweep-up direction than in sweep-down. In rough analogy with the magnetization profile in the Bean model, they raise the issue that the plateau may be associated with, or reflect a specific state of the vortex system. In our response, we pointed out that the hysteresis in their sample is about 5 times larger (at 8 K) than in the two crystals used by Krishana et al. At higher temperatures, 15-20 K, where the plateau is just as prominent, the hysteresis is almost unresolved. The hysteresis is an extrinsic effect possibly associated with stronger flux pinning in a more disordered crystal. The present results lend a fresh perspective to the question whether the plateau is intrinsic to the QP system or the product of a particular state of the vortex system. The YBCO results show that, whenever the QP current is very strongly suppressed by the available field, the thermal conductivity that remains is indeed field-independent and non-hysteretic. This shows that, in YBCO, a plateau regime definitely exists; the lack of observable hysteresis shows that it has nothing to do with a particular state of the vortex system. However, to access it, it may be necessary to work in the underdoped regime and to use high-purity crystals with weak pinning (as discussed above, we are unable to access the plateau region in 90-K YBCO).

In principle, the analysis described yields quantitative information on the quasi-particles. Having both the diagonal and off-diagonal conductivities available reduces the uncertainties in identifying the measured parameters, as well as provides consistency checks. The weak-field Hall angle may be expressed as a ‘Hall’ mean-free-path $\ell_H \equiv \theta_{QP}/h_kF/e$. The value of $\theta_{QP}$ at 10 K gives $\ell_H \approx 4.200 \text{Å}$. The similarity of the $T$ dependences in $\theta_{QP}$ and $\rho$ implies that $\ell_H$ is proportional to $\ell_0$. If the proportionality constant is 1, we may use $\ell_H$ in Eq. (3) to find that $\sigma_{tr} \sim 90 \text{Å}$, about 1.7 times the diameter of the vortex core ($2\xi$) ($\xi \sim 26 \text{Å}$ if the upper critical field $H_{c2} \sim 50 \text{T}$). Using this value of $\sigma_{tr}$ in Eq. (2) we obtain the velocity $v_2 \sim 2.3 \times 10^6 \text{cm/s}$. From the penetration depth variation $\Delta\lambda = 4.3 \ A/K$ [14], Wen and Lee [15] obtain the velocity anisotropy $v_f/v_2 \sim 7.6$. With our estimate for $v_2$, we find that the Fermi velocity $v_f \sim 1.8 \times 10^7 \text{cm/s}$.

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