Multi-sensor joint target detection, tracking and classification via Bernoulli filter

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Abstract

This paper focuses on joint detection, tracking and classification (JDTC) of a target via multi-sensor fusion. The target can be present or not, can belong to different classes, and depending on its class can behave according to different kinematic modes. Accordingly, it is modeled as a suitably extended Bernoulli random finite set (RFS) uniquely characterized by existence, classification, class-conditioned mode and class\&mode-conditioned state probability distributions. By designing suitable centralized and distributed rules for fusing information on target existence, class, mode and state from different sensor nodes, novel centralized and distributed JDTC Bernoulli filters (C-JDTC-BF and D-JDTC-BF), are proposed. The performance of the proposed JDTC-BF approach is evaluated by means of simulation experiments.

Keywords: Distributed target tracking, target classification, random finite set, Bernoulli filter, information fusion

1. Introduction

Target detection and tracking are crucial tasks in surveillance (e.g., radar \cite{1}, sonar \cite{2}, autonomous driving \cite{3}, mobile robotics \cite{4}) systems. Though a possible approach is to separately deal with detection and tracking as two sequential phases \cite{5}, i.e. by first performing target initialization followed by track maintenance, it has been recognized that joint processing of both tasks can substantially enhance the overall performance \cite{6}. In certain circumstances, it is also desired to perform target classification for high-level applications aiming at situation assessment \cite{7}. Moreover, knowledge of the target class provides valuable information on the possible kinematic behaviors of the target \cite{8} (e.g., a fighter aircraft can perform sharper maneuvers than a cargo aircraft which, in turn, can be profitably exploited to improve tracking performance \cite{9}).

The recent development of random finite set (RFS) methods has produced several interesting contributions to joint detection, tracking and classification (JDTC) of both a single and multiple targets \cite{10,11,12,13,14,15,16} but all based on a single-sensor system.

In many practical scenarios, however, multi-sensor surveillance systems entail significant advantages in terms of enhanced tracking accuracy, target observability \cite{17}, reliability as well as expanded coverage \cite{18}, thus motivating the interest of the present work for multi-sensor JDTC. In this respect, the attention will be devoted to both a centralized configuration wherein a fusion center gathers measurements from all sensors, or a distributed one wherein each sensor updates a local posterior with its own measurements and then fuses it with the posteriors of the neighbors. It is well-recognized that centralized fusion can provide better performance while, on the other hand, distributed fusion is preferable in terms of scalability and fault tolerance \cite{19}.

The goal of this paper is to address single-target multi-sensor JDTC, exploiting both centralized and distributed configurations. In order to account for target appearance/disappearance and the presence of clutter, the target is modeled as a Bernoulli RFS \cite{20}. Then, for classification purposes, different target classes are considered, each being characterized by a different set of possible kinematic modes. Overall, target’s information, to be recursively propagated in time, consists of existence probability (EP), class probability mass function (CPMF), class-conditioned (kinematic) mode PMFs (MPMFs) and class\&mode-conditioned state probability density functions (SPDFs). It is worth to point out that joint estimation of class and mode positively affects both classification and tracking. In fact, kinematic mode information can be exploited for target classification and, inversely, class information can help to define the possible modes of a target, thus improving tracking performance \cite{17}.

For the centralized configuration, the optimal posterior based on measurements from all sensors is obtained at each iteration following the Bayesian approach. For the distributed configuration, on the other hand, the local posterior of each sensor is first obtained with the existing Bernoulli-JDTC method \cite{11}, and then generalized covariance intersection (GCI) \cite{21,22,23,24} is exploited to fuse the local posteriors so as to achieve EP, CPMF, MPMFs,

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and SPDFs of the global posterior. Furthermore, Gaussian mixture (GM) implementation of the proposed method is provided.

Finally, simulation experiments are carried out to comparatively evaluate performance of the proposed algorithms. Summarizing, the main contributions of this paper are listed as follows:

- the problem of multi-sensor JDTC is addressed under both centralized and distributed settings;
- the GM implementation of both algorithms, as opposed to the particle filter implementation adopted in [1] for the single-sensor JDTC Bernoulli filter, is discussed.

Notice that, to the best of our knowledge, this is the first paper addressing distributed JDTC. The rest of this paper is organized as follows. Section 2 formulates the JDTC problem of interest, relative to a single target in a clutterly environment surveilled by multiple sensors. We will first model target dynamics and then multi-sensor measurement generation.

2. Problem formulation

This section formulates the JDTC problem of interest, relative to a single target in a clutterly environment surveilled by multiple sensors. We will first model target dynamics and then multi-sensor measurement generation.

2.1. Target dynamics

To represent the target, let us introduce the following three items: kinematic state \( x \in \mathcal{X} \), \( \mathcal{X} \) being an Euclidean state space; class \( c \in \mathcal{C} \), \( \mathcal{C} = \{c_1, \ldots, c_{|\mathcal{C}|}\} \) being a discrete class set; (kinematic) mode \( m \in \mathcal{M}_c \), \( \mathcal{M}_c = \{m_{c,1}, \ldots, m_{c,|\mathcal{M}_c|}\} \) being a class-dependent discrete mode set, where \(|\cdot|\) denotes cardinality. To summarize target information it is, therefore, convenient to define the augmented state vector \( \mathbf{x} = [x^T, c, m]^T \in \mathcal{X} \times \mathcal{C} \times \mathcal{M}_c \).

Since the target can either exist or not, it is naturally modeled as a Bernoulli RFS which can be either empty or a singleton, in the augmented state space, with some existence probability (EP) \( r \in [0, 1] \). Accordingly, the target set density is defined as

\[
f(\mathcal{X}) = \begin{cases} 
1 - r, & \text{if } \mathcal{X} = \emptyset \\
r s(\mathbf{x}), & \text{if } \mathcal{X} = \{\mathbf{x}\} \\
0, & \text{if } |\mathcal{X}| > 1
\end{cases}
\]  

(1)

where \( s(\mathbf{x}) = s(x, c, m) \) is the augmented state PDF. By the chain rule, such a PDF can be factored as

\[
s(\mathbf{x}) = s(x, c, m) = s(x|c, m) \beta(m|c) \gamma(c)
\]  

(2)

where: \( \gamma(c) \) is the class PMF (CPMF); \( \beta(m|c) \) is the class-conditioned mode PMF (MPMF); \( s(x|c, m) \) is the class-modified-conditioned state PDF (SPDF). Hence, the target is completely characterized by EP \( r \), CPMF \( \gamma(\cdot) \), MPMFs \( \{\beta(\cdot|c)\}_{c \in \mathcal{C}} \) and SPDFs \( \{s(\cdot|c, m)\}_{c \in \mathcal{C}, m \in \mathcal{M}_c} \). For the sake of simplicity, hereafter the Bernoulli set density \( f = \{r, \gamma, \beta, s\} \) will be referred to with the shorthand notation \( f = \{r, \gamma, \beta, s\} \).

The target dynamics has to account for appearance (birth), disappearance (death) and motion. In this respect, it can be completely characterized by the transition density \( \Phi_{k|k-1}(\mathcal{X}_+|\mathcal{X}) \) which, expresses, in probabilistic terms, the transition of the target set from \( \mathcal{X} \) at time \( k-1 \) to \( \mathcal{X}_+ \) at time \( k \). Specifically,

\[
\Phi_{k|k-1}(\mathcal{X}_+|\emptyset) = \begin{cases} 
1 - p_B, & \text{if } \mathcal{X}_+ = \emptyset \\
p_B \cdot b(x_+), & \text{if } \mathcal{X}_+ = \{x_+\}
\end{cases}
\]  

(3)

\[
\Phi_{k|k-1}(\mathcal{X}_+|\mathcal{X}) = \begin{cases} 
1 - p_S, & \text{if } \mathcal{X}_+ = \emptyset \\
p_S \cdot \phi_{k|k-1}(x_+|\mathcal{X}), & \text{if } \mathcal{X}_+ = \{x_+\}
\end{cases}
\]  

(4)

where: \( p_B \) and \( p_S \) are the probabilities of appearance of a newborn target and, respectively, of survival of an existing target; \( b(x_+) = \gamma_B(c_+) \beta_B(m_+|c_+) s_B(x_+|c_+, m_+) \) is the a priori state PDF of the potential new target; \( \phi_{k|k-1}(x_+|\mathcal{X}) \) is the transition PDF, in the augmented state space, of the existing target. For ease of presentation, the survival probability \( p_S \) is supposed to be independent from the (augmented) state, but all the ensuing developments can be readily generalized to the case of a state-dependent survival probability. For the target class, mode and state evolution, the following reasonable assumptions are made:

- the target class remains constant over time, i.e. \( c_k = c_{k-1} \);
- the mode transition is governed by a class-dependent homogeneous Markov chain with transition probabilities

\[
\text{Prob}(m_k = m_k|m_{k-1} = m, c) = \pi_c(m|m); 
\]  

(5)

- the target motion is modeled by the state-dependent transition density

\[
\text{Prob}(x_k = x_k|x_{k-1} = x, m_k = m) = \varphi(x_k|x, m). 
\]  

(6)

Taking into account the above assumptions, the transition PDF turns out to be given by

\[
\Phi_{k|k-1}(x_+|\mathcal{X}) = \Phi_{k|k-1}(x_+, c_+, m_+|x, c, m) = \delta_{c_+, c} \pi_c(m|m) \varphi(x_+|x, m_+) 
\]  

(7)

where \( \delta_{c_+, c} \) is the Kronecker delta equal to 1 if \( c_+ = c \) and to zero otherwise.
2.2. Multi-sensor measurement model

The area of interest is monitored by a set of sensors $\mathcal{N} = \{1, \ldots, |\mathcal{N}|\}$. At sampling time $k$, sensor node $i \in \mathcal{N}$ provides the measurement RFS

$$ Z_k^i = T^i(X_k) \cup K_k^i, \quad (8) $$

which is the union of the target-originated RFS $T^i(X_k)$ and the clutter set $K_k^i$. The target-originated RFS takes the form

$$ T^i(X_k) = \begin{cases} \emptyset, & \text{if } X_k = \emptyset \\ \emptyset, & \text{if } X_k = \{x_k\} \text{ with prob. } 1 - p_D^i(x_k, c) \\ z_k^i, & \text{if } X_k = \{x_k\} \text{ with prob. } p_D^i(x_k, c) \end{cases} \quad (9) $$

where $\ell(z|x)$ is the likelihood function associated to the $i$th sensor. The clutter set $K_k^i$ is modeled as Poisson RFS with probability hypothesis density (PHD) $\kappa(z)$ defined over the measurement space. The measurements of different sensors are supposed to be mutually conditionally independent. Notice that, in the considered multi-sensor measurement model, the target class $c \in \mathcal{C}$ only affects detection probabilities $p_D^i(x, c)$ while the target mode $m \in \mathcal{M}_c$ is irrelevant.

3. Centralized JDTC-BF algorithm

This section focuses on the centralized configuration wherein all sensor nodes $i \in \mathcal{N}$ convey their measurement sets $Z_k^i$ to a fusion center that, in principle, should be able to perform optimal multi-sensor fusion, i.e. to provide the Bernoulli set density $f(X_k|\bigcup_{i \in \mathcal{N}} Z_k^i)$ where $Z_k^i$ denotes the sequence of measurements collected by sensor $i$ from time 1 to time $k$. Hereafter, it will be shown how to extend the joint detection and tracking Bernoulli filtering approach of [6, 19, 25] to the JDTC setting of this paper. Specifically, assuming that at time $k - 1$ the augmented Bernoulli density $f_{k-1} = \{r_{k-1}, \gamma_{k-1}, \beta_{k-1}, s_{k-1}\}$ is given and following a Bayesian approach, the aim is to first perform prediction to obtain $f_{k|k-1} = \{r_{k|k-1}, \gamma_{k|k-1}, \beta_{k|k-1}, s_{k|k-1}\}$ by exploiting the target dynamics (3-4), then followed by multi-sensor update to get $f_k = \{r_k, \gamma_k, \beta_k, s_k\}$ by exploiting the measurement model (8).

3.1. JDTC-BF prediction

Prediction of a standard Bernoulli RFS density $f_{k-1} = \{r_{k-1}, s_{k-1}\}$ into $f_{k|k-1} = \{r_{k|k-1}, s_{k|k-1}\}$ can be found in [19] Eqs. (10)-(12)]. The following result provides the extension of such prediction to a Bernoulli RFS density defined over the augmented class-mode-state space.

**Proposition 1.** Given the Bernoulli RFS density $f_{k-1} = \{r_{k-1}, \gamma_{k-1}, \beta_{k-1}, s_{k-1}\}$, the predicted density $f_{k|k-1} = \{r_{k|k-1}, \gamma_{k|k-1}, \beta_{k|k-1}, s_{k|k-1}\}$ is obtained as follows

$$ r_{k|k-1} = p_B(1 - r_{k-1}) + p_S r_{k-1}, \quad (10) $$

$$ \gamma_{k|k-1}(c) = \frac{p_B(1 - r_{k-1})}{r_{k|k-1}} \gamma(c) + \frac{p_S}{r_{k|k-1}} \gamma_{k-1}(c), \quad (11) $$

$$ \beta_{k|k-1}(m|c) = \frac{p_B(1 - r_{k-1})}{r_{k|k-1}} \beta_B(m|c) \gamma_{k|k-1}(c) + \frac{p_S}{r_{k|k-1}} \beta_{k-1}(m|c), \quad (12) $$

$$ s_{k|k-1}(x|m, c) = \int \frac{\phi(x|x', m)}{\kappa(x)} \delta_{k-1}(x'|x, m') dx'. \quad (13) $$

**Proof:** see Appendix A.

3.2. JDTC-BF centralized multi-sensor update

The following result concerns the centralized multi-sensor update of the augmented target Bernoulli density

**Proposition 2.** Given the predicted Bernoulli RFS density $f_{k|k-1} = \{r_{k|k-1}, \gamma_{k|k-1}, \beta_{k|k-1}, s_{k|k-1}\}$ and the measurement RFSs $\{Z_k^i\}_{i \in \mathcal{N}}$, the updated density $f_k = \{r_k, \gamma_k, \beta_k, s_k\}$ is obtained as follows

$$ r_k = \frac{r_{k|k-1} \sum_{c \in \mathcal{C}} \gamma_{k|k-1}(c) \ell(c)}{1 - r_{k|k-1} + r_{k|k-1} \sum_{c \in \mathcal{C}} \gamma_{k|k-1}(c) \ell(c)}, \quad (14) $$

$$ \gamma_k(c) = \frac{\gamma_{k|k-1}(c) \ell(c)}{\sum_{c \in \mathcal{C}} \gamma_{k|k-1}(c) \ell(c)}, \quad (15) $$

$$ \beta_k(m|c) = \frac{\beta_{k|k-1}(m|c) \ell(m|c)}{\sum_{m \in \mathcal{M}_c} \beta_{k|k-1}(m|c) \ell(m|c)}, \quad (16) $$

$$ s_k(x|m, c) = \frac{s_{k|k-1}(x|m, c) \ell(x|m, c)}{\int s_{k|k-1}(x'|m, c) \ell(x'|m, c) dx'}, \quad (17) $$

where

$$ \ell(c) = \sum_{m \in \mathcal{M}_c} \beta_{k|k-1}(m|c) \ell(m|c), \quad (18) $$

$$ \ell(m|c) = \int s_{k|k-1}(x|m, c) \ell(x|m, c) dx, \quad (19) $$

$$ \ell(x|m, c) = \prod_{i \in \mathcal{N}} \left[1 - p_D^i(c) + p_D^i(c) \sum_{z \in Z_k^i} \ell(z|x) \kappa(z)\right]. \quad (20) $$

**Proof:** see Appendix B.
Algorithm C-JDTC-BF (Time $k$)

**Input:** $Z_k = \{ Z_k^i \}_{i \in N}, f_{k-1} = \{ r_{k-1}, \gamma_{k-1}, \beta_{k-1}, s_{k-1} \}$

**Prediction:**
Predicted EP $r_{k|k-1}$, CPMF $\gamma_{k|k-1}(c)$, MPDMFs $\beta_{k|k-1}(m|c)$ and SPDFs $s_{k|k-1}(x|m,c)$ are computed by (10)-(13).

**Multi–sensor update:**
Updated EP $r_k$, CPMF $\gamma_k(c)$, MPDMFs $\beta_k(m|c)$ and SPDFs $s_k(x|m,c)$ are computed by (13)-(20).

**Augmented state extraction:**
if $r_k < 0.5$ no target is detected,
otherwise do

\[
\begin{align*}
\hat{c}_k &= \arg \max_{c \in C} \gamma_k(c) \\
\hat{m}_k &= \arg \max_{m \in M_c} \beta_k(m|c) \hat{c}_k \\
\text{Extract } \hat{x}_k &\text{ from } s_k(\hat{c}_k, \hat{m}_k) \text{ according to either MAP or MMSE criterion} \\
\text{Set } \hat{x}_k &= (\hat{x}_k^T, \hat{c}_k, \hat{m}_k)^T \\
\end{align*}
\]

end

**Output:** $f_k = \{ r_k, \gamma_k, \beta_k, s_k \}$ and $\hat{x}_k$

### 4. Distributed JDTC-BF algorithm

This section deals with the distributed setting wherein each sensor node $i \in N$ computes a local posterior $f^i = \{ r^i, \gamma^i, \beta^i, s^i \}$ and fuses it with those of the in-neighbors. The idea is to approximate the global posterior $f_{\{X_k\} | \bigcup_{i \in N} Z_k^i}$ in each node via repeated fusion (consensus) iterations with the neighboring nodes. In particular, the GCI fusion rule \cite{22} is adopted by which the fused density is nothing but the geometrical average of the fusing ones, i.e.

$$T(X) = \frac{\prod_{i \in N} [f^i(X)]^{-\omega^i}}{\prod_{i \in N} [f^i(X)]^{\omega_i}} dX, \quad (21)$$

with suitably chosen fusion weights $\omega^i \in (0,1)$ such that $\sum_{i \in N} \omega^i = 1$. GCI fusion of Bernoulli RFS densities $f^i = \{ r^i, s^i \}$ into $\hat{f} = \{ \hat{r}, \hat{s} \}$ can be found in \cite{22}, Eqs. (24)-(25).

The next result provides an extension to the JDTC case with augmented Bernoulli RFS densities consisting of EP, CPMF, MPDMFs and SPDFs.

**Proposition 3.** Given local Bernoulli RFS densities $f^i = \{ r^i, \gamma^i, \beta^i, s^i \}$ and fusion weights $\omega^i$ for any $i \in N$, the GCI-fused density in \cite{22} turns out to be a Bernoulli RFS density $\bar{f} = \{ \bar{r}, \bar{\gamma}, \bar{\beta}, \bar{s} \}$ given by

$$\bar{f} = \frac{\bar{r} \sum_{c \in C} \bar{\gamma}(c) \sum_{m \in M_c} \bar{\beta}(m|c) \int \bar{s}(x|m,c) dx}{\zeta + \bar{r} \sum_{c \in C} \bar{\gamma}(c) \sum_{m \in M_c} \beta(m|c) \int \bar{s}(x|m,c) dx}, \quad (22)$$

initialized from $f^i_0(X) = f^i(X)$ and with consensus weights $\omega^{i,j} > 0$, satisfying $\sum_{j \in N^i} \omega^{i,j} = 1$, possibly selected so as to ensure that $f^i_l(X)$ converges to $T(X)$ as $l \to \infty$. The resulting D-JDTC-BF algorithm is summarized in Table 2.

**Remark 1.** It is worth pointing out that the D-JDTC-BF approach of this section significantly extends previous work in \cite{22} on distributed multiple-model Bayesian tracking of a maneuvering target. In fact, D-JDTC-BF allows to perform also target detection and classification, besides tracking, and considers the presence of clutter as well as target appearance/disappearance.

### 5. Gaussian-mixture implementation of JDTC-BF

In this section, the JDTC-BF is implemented by utilizing the Gaussian mixture (GM) approach \cite{27}. For the
approach and linearize the function $h^i$ based on the Taylor expansion at $x_0$

$$h^i(x) \equiv h^i(x_0) + H^i(x_0)(x - x_0), \quad (33)$$

where $H^i$ is the Jacobian matrix. Similarly, the function $f$ is linearized as

$$f(x, m) \equiv f(x_0, m) + F(x_0, m)(x - x_0) \quad (34)$$

where $F(x_0, m)$ is the Jacobian matrix with respect to $x$ for a given $m$.

In the proposed implementation scheme, the GM is utilized to approximately represent SPDFs of the Bernoulli density. In particular, for a Bernoulli density $f = \{r, \gamma, \beta, s\}$, a GM with $J(c, m)$ Gaussian components (GCs) is employed for the SPDF

$$s(x|c, m) = \sum_{j=1}^{J(c,m)} \alpha_j(c, m) \cdot \mathcal{G}(x; \mu_j(c, m), P_j(c, m)), \quad (35)$$

where $\alpha_j(c, m), \mu_j(c, m)$ and $P_j(c, m)$ are the weight, mean and covariance of $j$-th GC conditioned on the class $c \in \mathcal{C}$ and mode $m \in \mathcal{M}_c$. Thus, the SPDF can be more compactly rewritten as $s = \{\alpha_j, \mu_j, P_j\}_{j=1}^{J(c)}$.

5.1. Prediction

At time $k-1$, given the prior augmented Bernoulli density $f_{k-1} = \{r_{k-1}, \gamma_{k-1}, \beta_{k-1}, s_{k-1}\}$ with class&mode-conditioned SPDFs represented in GM form

$$s_{k-1}(x|c, m) = \sum_{j=1}^{J(c,m)} \alpha_{k-1,j}(c, m) \cdot \mathcal{G}(x; \mu_{k-1,j}(c, m), P_{k-1,j}(c, m))$$

and the augmented birth Bernoulli density $f_B = \{p_B, \gamma_B, \beta_B, s_B\}$ with SPDFs represented in GM form

$$s_B(x|c, m) = \sum_{j=1}^{J_B(c,m)} \alpha_{B,j}(c, m) \mathcal{G}(x; \mu_{B,j}(c, m), P_{B,j}(c, m)),$$

then the class&mode-conditioned SPDF of the augmented predicted density $f_{k|k-1} = \{r_{k|k-1}, \beta_{k|k-1}, \gamma_{k|k-1}, s_{k|k-1}\}$ via (10)–(13) is given by

$$s_{k|k-1}(x|c, m) = \frac{p_B(1 - r_{k-1}) \gamma_B c \beta_B m|c}{r_{k|k-1} \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c)} \cdot \mathcal{G}(x; \mu_{B,j}(c, m), P_{B,j}(c, m)) \times \sum_{j=1}^{J_B(c,m)} \alpha_{B,j}(c, m) \mathcal{G}(x; \mu_{B,j}(c, m), P_{B,j}(c, m)) \cdot$$

Hereafter, in order to approximate the likelihood function of nonlinear measurement [31], we follow the EKF
Accordingly, the updated SPDFs in (17) become as follows

\[
\times \sum_{j=1}^{J_{k-1}(c,m')} \alpha_{k-1,j}(c,m') \times \mathcal{G}(x; \mu_{k|k-1,j}^S(c,m'), P_{k|k-1,j}^S(c,m',m')), \tag{36}
\]

where

\[
\mu_{k|k-1,j}^S(c,m,m') = f(\mu_{k-1,j}(c,m'), m) \tag{37}
\]
\[
P_{k|k-1,j}^S(c,m,m') = F_{k,j} P_{k-1,j}(c,m') F_{k,j}^\top + Q_k(m) \tag{38}
\]
\[
F_{k,j} = \frac{\partial f}{\partial x}(\mu_{k-1,j}(c,m'), m). \tag{39}
\]

Notice that the predicted densities \( s_{k|k-1}(x|c,m) \) are still in GM form, but with an increased number of GCs given by

\[
J_{k|k-1}(c,m) = J_B(c,m) + \sum_{m'=M_c} J_{k-1}(c,m'), \tag{40}
\]

and can, therefore, be rearranged as

\[
s_{k|k-1}(x|c,m) = \sum_{j=1}^{J_{k-1}(c,m')} \alpha_{k|k-1,j}(c,m, m') \times \mathcal{G}(x; \mu_{k|k-1,j}(c,m'), P_{k|k-1,j}(c,m')) \tag{41}
\]

for appropriate weights, means and covariances of the GCs.

5.2. Single-sensor update of local JDTC-BF

Let us now consider single-sensor update with the local measurement set \( Z_k = Z_k^i \), omitting for the sake of simplicity superscript \( i \). Starting from the predicted density \( f_{k|k-1} = \{r_{k|k-1}, \gamma_{k|k-1}, \beta_{k|k-1}, s_{k|k-1}\} \), we can therefore apply (14)–(17) to get \( f_k = \{r_k, \gamma_k, \beta_k, s_k\} \). Exploiting the GM form (11) of \( s_{k|k-1}(x|c,m) \) and (12), the likelihoods (13) take the form

\[
\ell(m|c) = [1 - p_D(c)] + p_D(c) \sum_{z \in Z_k} \sum_{j=1}^{J_k(c,m)} \alpha_{k,j}(c,m,z). \tag{42}
\]

Accordingly, the updated SPDFs in (17) become as follows

\[
s_k(x|c,m) = \frac{1}{\Lambda} \left[ [1 - p_D(x,c)] \sum_{j=1}^{J_{k|k-1}(c,m)} \alpha_{k,j-1}(c,m) \times \mathcal{G}(x; \mu_{k|k-1,j}(c,m), P_{k|k-1,j}(c,m)) + p_D(c) \sum_{z \in Z_k} \sum_{j=1}^{J_k(c,m)} \alpha_{k,j}(c,m,z) \times \mathcal{G}(x; \mu_{k,j}(c,m,z), P_{k,j}(c,m)) \right] \tag{43}
\]
\[
\Lambda = [1 - p_D(c)] + p_D(c) \sum_{z \in Z_k} \sum_{j=1}^{J_{k|k-1}(c,m)} \alpha_{k,j-1}(c,m) \tag{44}
\]

increased by a factor equal to the number of measurement plus one.

Remark 2. JDTC aims to jointly estimate target state, class and mode. In [11], the proposed method solves the problem by exploiting a sequential Monte Carlo (SMC) implementation of the Bernoulli filter. Since typically GM representation of a PDF is more parsimonious than SMC representation and also because JDTC involves multiple SPDFs for all class-mode pairs \((c,m)\), the GM approach seems by far preferable especially in the distributed case wherein posteriors are transmitted and received by each sensor node. Unfortunately, however, the number of GCs increases at each Bayesian step (prediction, update or fusion) so that suitable pruning and/or merging procedures (see [12], Table II) are needed to limit such a number.

5.3. Multi-sensor update of C-JDTC-BF

Proposition 2 provides the update equations based on Bayes-optimal C-JDTC-BF. In the multi-sensor case, for the class\&mode-conditioned SPDF, it is possible to implement centralized fusion by iterating single-sensor Bernoulli filter updates as follows [3, Section IV – B].

- Starting from the prior \( s_k^{(0)} = s_{k|k-1} \), first compute \( s_k^{(1)} = \{c_k^{(1)}|\mu_{k|k-1}^{(1)}, P_{k|k-1}^{(1)}|c,m\} \) using measurements and parameters of sensor 1 according to the single-sensor Bernoulli filter update procedure of the previous subsection.
- Next, apply to the prior \( s_k^{(1)} \) the same procedure, with measurements and parameters of sensor 2, to get \( s_k^{(2)} = \{c_k^{(2)}|\mu_{k|k-1}^{(2)}, P_{k|k-1}^{(2)}|c,m\} \).
- Repeat the same step until all sensors have been considered and \( s_k = s_k^{(|K|)} \) is obtained.
The multi-sensor updated Bernoulli density $f_k = \{r_k, \gamma_k, \beta_k, s_k\}$ can be obtained by using (33). Furthermore, the updated SPDFs in (17) are computed by using the above-mentioned iterations

$$s_k(x|m) = \Psi_k^{(1)(c)} \circ \cdots \circ \Psi_k^{(2)}(s_{k|k-1}(x|m))$$

where $\circ$ denotes composition, and

$$\Psi_k^{(i)}[s_{k|k-1}^{(i-1)}(x|m)] = \frac{1}{\Lambda^{(i)}} \left[ \left( 1 - p_D^{(i)}(c) \right) \sum_{j=1}^{J_k^{(i-1)}(c|m)} \alpha_{k,j}^{(i-1)}(c,m) \right. \times \mathcal{G}(x; \mu_{k,j}^{(i-1)}(c,m), P_{k,j}^{(i-1)}(c,m))$$

$$+ \left. p_D^{(i)}(c) \sum_{z \in \mathcal{Z}_k} \sum_{j=1}^{J_k^{(i-1)}(c,m)} \alpha_{k,j}^{(i)}(c,m,z) \times \mathcal{G}(x; \mu_{k,j}^{(i-1)}(c,m,z), P_{k,j}^{(i-1)}(c,m)) \right]$$

where $\alpha_{k,j}^{(i)}(c,m) = \alpha_{k|k-1,j}(c,m), \tilde{\mu}_{k,j}^{(i)}(c,m) = \mu_{k|k-1,j}(c,m), \tilde{P}_{k,j}^{(i)}(c,m) = P_{k|k-1,j}(c,m)$

$$\Lambda^{(i)} = \left( 1 - p_D^{(i)}(c) \right) + p_D^{(i)}(c) \sum_{z \in \mathcal{Z}_k} \sum_{j=1}^{J_k^{(i-1)}(c,m)} \alpha_{k,j}^{(i)}(c,m,z) \times \mathcal{G}(x; \mu_{k,j}^{(i-1)}(c,m,z), P_{k,j}^{(i-1)}(c,m))$$

$$J_k^{(i)}(c,m) = \sum_{j=1}^{J_k^{(i-1)}(c,m)} \mathcal{Z}_k^j$$

$$\alpha_{k,j}^{(i)}(c,m,z) = q^{(i)}(z) \alpha_{k,j}^{(i-1)}(c,m)$$

$$q^{(i)}(z) = \mathcal{G} \left( z; h(\mu_{k,j}^{(i-1)}(c,m)), S_{k,j}^{(i)} \right)$$

$$\mu_{k,j}^{(i)}(c,m,z) = \tilde{\mu}_{k,j}^{(i-1)}(c,m) + K_{k,j}^{(i)} \left( z - h(\tilde{\mu}_{k,j}^{(i-1)}(c,m)) \right)$$

$$H_{k,j}^{(i)} = \frac{\partial h(\tilde{\mu}_{k,j}^{(i-1)}(c,m))}{\partial x}$$

$$P_{k,j}^{(i)}(c,m) = \left( I - K_{k,j}^{(i)} H_{k,j}^{(i)} \right) \tilde{P}_{k,j}^{(i-1)}(c,m)$$

Moreover, the likelihood function of class-based mode in (19) equals the normalization value of the class-mode-conditioned SPDF, thus, $\ell(m|c) = \Lambda^{(i)(c)}$ can be obtained by performing iterations until all sensors have been considered.

At the end of the above multi-sensor update procedure, the resulting posterior SPDFs $s_k(x|m)$ preserve the GM form with a number of GCs given by

$$J_k(c,m) = \prod_{i \in N} \left( \left| Z_k^i \right| + 1 \right) J_{k|k-1}(c,m).$$

### 5.4. Fusion of D-JDTC-BF

This subsection concerns GM implementation of the fusion stage of D-JDTC-BF. For the sake of simplicity, we only consider pairwise fusion of two augmented Bernoulli densities $f_i = \{r_i, \gamma_i, \beta_i, s_i\}$, for $i = 1, 2$, with SPDFs in GM form

$$s_i(x|m) = \sum_{j=1}^{J_i(c,m)} \alpha_{i,j}(c,m) \mathcal{G}(x; \mu_{i,j}(c,m), P_{i,j}(c,m))$$

As well known, the weighted geometric average of GMs

$$\bar{s}(x|m) = \left[ s_1(x|m) \right]^{\omega} \left[ s_2(x|m) \right]^{1-\omega}$$

by due to exponentiation by the fractional exponents $\omega$ and $1-\omega$, is no longer a GM. However, there exist reasonable approximations of (60) in GM form like, e.g., (33)

$$\bar{s}(x|m) \approx \sum_{j_1=1}^{J_1(c,m)} \sum_{j_2=1}^{J_2(c,m)} \alpha_{j_1,j_2}(c,m) \times \mathcal{G}(x; \mu_{j_1,j_2}(c,m), P_{j_1,j_2}(c,m))$$

$$\alpha_{j_1,j_2}(c,m) = (\alpha_{j_1} \omega)(\alpha_{j_2}^{1-\omega}) \varepsilon(\omega, P_{j_1}) \varepsilon(1-\omega, P_{j_2})$$

$$\mathcal{G} \left( \mu_{j_1,j_2}^{1-\omega}, \mu_{j_2}^{\omega}, 0, \frac{P_{j_1}^{1-\omega}}{\omega} + \frac{P_{j_2}^{\omega}}{1-\omega} \right)$$

$$P_{j_1,j_2}(c,m) = [\omega(\alpha_{j_1})^{-1} + (1-\omega)(\alpha_{j_2})^{-1}]^{-1}$$

$$\mu_{j_1,j_2}(c,m) = P_{j_1,j_2}(c,m) \left[ \omega(\alpha_{j_1})^{-1} + (1-\omega)(\alpha_{j_2})^{-1} \right]$$

$$\varepsilon(\omega, P) = \sqrt{\det(2\pi P \omega^{-1}) \det(2\pi P^{-1} \omega)}$$

where, for the sake of brevity, the arguments $c$ and $m$ of $\alpha_i, \mu_i, P_i$ have been omitted. An alternative approximation, more appropriate than (60) whenever there are closely-spaced GCs, can be found in [31]. It is worth to point out that both approximations in [30, 31] produce a GM with a number of GCs given by

$$J(c,m) = J_1(c,m) J_2(c,m).$$

Then, according to Proposition [3] fused EP, MPMFs, CPMMFs and SPDFs are obtained from [22, 25] by setting

$$J(c,m) = J_1(c,m) J_2(c,m).$$

$$\int \bar{s}(x|m) dx = \sum_{j_1=1}^{J_1(c,m)} \sum_{j_2=1}^{J_2(c,m)} \alpha_{j_1,j_2}(c,m) \tag{69}$$
Whenever fusion involves more than two sensor nodes, it is anyway possible to split it, in several ways, into a sequence of pairwise fusion steps to be performed as indicated above. Whatever is the adopted sequence, fusion over \( N \) of Proposition 3 generates GM SPDFs \( \pi(x|c,m) \) with

\[
J(c,m) = \prod_{i \in N} J^i(c,m)
\]

where \( J^i(c,m) \) is the number of GCs of \( s^i(x|c,m) \).

6. Simulation Results

Assume that the target can belong to three different classes with corresponding mode sets \( M_1 = \{m_1\}, M_2 = \{m_1, m_2, m_3\} \) and \( M_3 = \{m_1, m_4, m_5\} \). The target kinematic state at time \( k \) is defined as \( x_k = [\xi_k, \hat{\eta}_k, \dot{\eta}_k]^\top \), with Cartesian coordinates of position \( \xi_k, \eta_k \) and, respectively, velocity \( \dot{\xi}_k, \dot{\eta}_k \). For each kinematic mode, the target motion is modeled by (29) with

\[
f(x, m) = F(m)x.
\]

There are five possible modes and their corresponding state transition \( F(m) \) and process noise covariance \( Q(m) \) matrices are as follows.

**Mode \( m_1 \):**

\[
F(m_1) = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
\]

\[
Q(m_1) = \sigma(m_1) \begin{bmatrix}
\frac{T^2}{2} & \frac{T^2}{2} & 0 & 0 \\
\frac{T^2}{2} & \frac{T^2}{2} & 0 & 0 \\
0 & 0 & \frac{T^2}{2} & \frac{T^2}{2} \\
0 & 0 & \frac{T^2}{2} & \frac{T^2}{2} \\
\end{bmatrix},
\]

where \( T = 1[s] \) denotes the sampling interval and \( \sigma(m_1) = 1[\text{m/s}^2] \).

**Mode \( m_2 \):**

\[
F(m_2) = \begin{bmatrix}
\sin(\omega T)/\omega & 0 & (\cos(\omega T) - 1)/\omega \\
0 & \cos(\omega T) & 0 & -\sin(\omega T) \\
0 & (1 - \cos(\omega T))/\omega & 1 & \sin(\omega T)/\omega \\
0 & \sin(\omega T) & 0 & \cos(\omega T) \\
\end{bmatrix},
\]

\[
Q(m_2) = \sigma(m_2) \begin{bmatrix}
\frac{T^2}{3} & \frac{T^2}{3} & 0 & 0 \\
\frac{T^2}{3} & \frac{T^2}{3} & 0 & 0 \\
0 & 0 & \frac{T^2}{3} & \frac{T^2}{3} \\
0 & 0 & \frac{T^2}{3} & \frac{T^2}{3} \\
\end{bmatrix},
\]

where \( \omega = -0.1[\text{rad/s}] \) and \( \sigma(m_2) = 1.4[\text{m/s}^2] \).

**Mode \( m_3 \):** same as mode \( m_2 \) with \( \omega = 0.15[\text{rad/s}] \) and \( \sigma(m_3) = 1.4[\text{m/s}^2] \).

**Mode \( m_4 \):** same as mode \( m_2 \) with \( \omega = 1[\text{rad/s}] \) and \( \sigma(m_4) = 1.4[\text{m/s}^2] \).

**Mode \( m_5 \):** same as mode \( m_2 \) with \( \omega = -1[\text{rad/s}] \) and \( \sigma(m_5) = 1.4[\text{m/s}^2] \).

Transition probability matrices for classes 2 and 3 are

\[
\Pi = \begin{bmatrix}
0.8 & 0.1 & 0.1 \\
0.1 & 0.8 & 0.1 \\
0.1 & 0.1 & 0.8
\end{bmatrix}.
\]

while class 1 has only one mode, i.e. \( \Pi = I \).

This simulation duration is 100[s] for each experiment. The manoeuvring target, belonging to class 2, appears at time \( t_a = 6[\text{s}] \) and disappears at \( t_b = 90[\text{s}] \) in the surveillance region of angle extension \([0, \pi/2]\) and range extension \([0, 5000\sqrt{2}]\). The initial target state is

\[
x = [4786[\text{m}], -8.3[\text{m/s}], 3584[\text{m}], -100.9[\text{m/s}]]^\top.
\]

Its trajectory (see Fig. 1) is a straight line with constant velocity between \( 6[\text{s}] \) and \( 25[\text{s}] \), followed by a clockwise turn \((\omega = -0.10[\text{rad/s}])\) between \( 26[\text{s}] \) and \( 50[\text{s}] \), another straight line with constant velocity between \( 51[\text{s}] \) and \( 60[\text{s}] \), and a final counterclockwise turn \((\omega = 0.15[\text{rad/s}])\) between \( 61[\text{s}] \) and \( 90[\text{s}] \). Target modes over time are reported in Table 3.

A total of \( |\mathcal{N}| = 20 \) sensors are deployed over the surveillance area as shown in Fig. 1. Each sensor \( i \in \mathcal{N} \), of known position \((\xi_i, \eta_i)\), provides a range measurement according to (30) with measurement function

\[
h^i(x) = \sqrt{(\xi - \xi_i)^2 + (\eta - \eta_i)^2},
\]

uniform detection probability \( p^i_d = 0.95 \) and measurement noise variance \( R^i = 25[\text{m}^2] \). Clutter is generated as a Poisson RFS with PHD \( \kappa(z) = \lambda u(z) \), with expected number
of clutter points $\lambda = 5$ and uniform spatial PDF $u(\cdot)$ over the surveillance region.

The JDTC-BF has been tuned with survival probability $p_S = 0.98$ and birth probability $p_B = 0.2$. Further, for target birth we assumed uniform distribution for class and mode as well as class-and-mode independent Gaussian distribution for the state. Specifically, the target birth PDF has been taken as $b(x, c, m) = \gamma_B(c)\beta_B(m|c)s_B(x)$ with

\begin{align}
\gamma_B(c) &= \frac{1}{|C|} \\
\beta_B(m|c) &= \frac{1}{|M_c|} \\
s_B(x) &= \mathcal{G}(x; m_B, P_B)
\end{align}

where $m_B = [4780[m], -8[m/s], 3590[m], -100[m/s]]^\top$ and $P_B = \text{diag}([100[m^2], 100[m^2/s^2], 100[m^2], 100[m^2/s^2]])$.

For D-JDTC-BF, the number of consensus steps has been set to $L = 3$. In particular, the pruning and merging thresholds for GM are set to $T_p = 1 \times 10^{-15}$ and $T_m = 20$, respectively. Moreover, the maximum number of GCs is fixed to 6.

In the simulation experiments, performance will be evaluated in terms of four performance indicators averaged over 100 independent Monte Carlo trials: optimal sub-pattern assignment (OSPA) error \cite{35} (with order $p = 1$ and cutoff $c = 150[m]$), existence probability, mode PMF, and classification probability.

OSPA peaks after each class and/or mode switch: a first peak in the time interval 26–30[s] is due to a simultaneous class and mode switch, while the subsequent two peaks (at time 51[s] and 61[s]) are caused by mode switches.

![OSPA error](image)

**Fig. 2.** OSPA error

The OSPA error is plotted in Fig. 2 where it can be seen how C-JDTC-BF clearly provides lower OSPA than D-JDTC-BF as well as a smoother behavior during class and/or mode switches. Specifically, D-JDTC-BF exhibits OSPA peaks after each class and/or mode switch: a first peak in the time interval 26–30[s] is due to a simultaneous class and mode switch, while the subsequent two peaks (at time 51[s] and 61[s]) are caused by mode switches.

![Existence probability](image)

**Fig. 3.** Existence probability

As well known, the OSPA error simultaneously captures detection and tracking performance. A more clear-cut assessment of detection performance is provided by Fig. 3, plotting the estimated existence probability. It can be seen that C-JDTC-BF has better detection capability than D-JDTC-BF which exhibits a decrease of the existence probability in the time interval 26–30[s] due to class switching.

![Mode probabilities](image)

**Fig. 4.** Mode probabilities for class 2

Estimated mode PMFs for classes 2 and 3 versus time are plotted in Figs. 4 and 5 (recall that class 1 has just a single mode). As expected, both C-JDTC-BF and D-JDTC-BF switch mode whenever the target turning rate...
switching to mode 2 occurs at time $t$.

The problem has been formulated in multi-sensor fusion. The problem has been formulated in single-target immersed in clutter has been addressed by changes. Since all three classes include mode 1, during the time interval $6 - 25 [s]$ mode 1 is estimated; then a switching to mode 2 occurs at time $t = 26 [s]$ (see Fig. 4).

At time $t$, we assume that the prior density $f_{k-1}(\mathcal{X})$ is given as

$$f_{k-1}(\mathcal{X}) = \begin{cases} 1 - r_{k-1}, & \text{if } \mathcal{X} = \emptyset \\ r_{k-1}s_{k-1}(\mathcal{X}), & \text{if } \mathcal{X} = \{x\} \\ 0, & \text{if } |\mathcal{X}| > 1 \end{cases} \quad (A.1)$$

According to [11, Eqs. 49-51], (3), (4) and (7), if $X$ is labeled multi-Bernoulli (MBF) settings, and a Gaussian-mixture implementation of both filters has been presented. Simulation experiments have demonstrated the effectiveness of the proposed multi-sensor JDTC-BF approach. Future work will extend multi-sensor JDTC to multiple maneuvering targets by exploiting labeled multi-Bernoulli (LMB) filtering.

Appendix A. Proof of Proposition 1

At time $t$, we assume that the prior density $f_{k-1}(\mathcal{X})$ is given as

$$f_{k-1}(\mathcal{X}) = \begin{cases} 1 - r_{k-1}, & \text{if } \mathcal{X} = \emptyset \\ r_{k-1}s_{k-1}(\mathcal{X}), & \text{if } \mathcal{X} = \{x\} \\ 0, & \text{if } |\mathcal{X}| > 1 \end{cases} \quad (A.1)$$

According to [11, Eqs. 49-51], (3), (4) and (7), if $X = \emptyset$, we get

$$f_{k|k-1}(\emptyset) = (1 - p_B)(1 - r_{k-1})$$

Since $f_{k|k-1}(\emptyset) = 1 - r_{k|k-1}$, the predicted EP is given by

$$r_{k|k-1} = p_B(1 - r_{k-1})$$

$$+ p_S r_{k-1} \sum_{c' \in C} \sum_{m \in M_c} s_{k-1}(x, c, m) dx$$

$$= p_B(1 - r_{k-1})$$

$$+ p_S r_{k-1} \sum_{c' \in C} \sum_{m \in M_c} \gamma_{k-1}(c') \sum_{m \in M_c} \beta_{k-1}(c|m) \int s_{k-1}(x'|c, m) dx.$$  \quad (A.3)

In a similar way, if $\mathcal{X} = \{x\}$, the predicted density can be obtained as

$$f_{k|k-1}(\mathcal{X}) = p_B(1 - r_{k-1}) \sum_{c \in C} \sum_{m \in M_c} s_B(x, c, m)$$

$$+ p_S r_{k-1} \sum_{c' \in C} \sum_{m \in M_c} \Phi_{k|k-1}(x, c, m|x', c, m') \times s_{k-1}(x'|c, m) dx'.$$ \quad (A.4)

Since the predicted augmented SPDF can be written as $s_{k|k-1}(x, c, m) = \gamma_{k|k-1}(c)\beta_{k|k-1}(m|c)s_{k|k-1}(x|c, m)$, we can get \[A.5\], from which the predicted class&mode-conditioned SPDF $s_{k|k-1}(x|c, m)$ is derived in \[A.6\].

The joint class&mode PMF $\beta_{k|k-1}(m|c)\gamma(c)$ is then derived as

$$\gamma_{k|k-1}(c)\beta_{k|k-1}(m|c) = \int s_{k|k-1}(x, m, c) dx$$

of target class, mode and state. Then, the posed augmented Bernoulli filtering (BF) problem has been solved, in both centralized (C-JDTC-BF) and distributed (D-JDTC-BF) settings, and a Gaussian-mixture implementation of both filters has been presented. Simulation experiments have demonstrated the effectiveness of the proposed multi-sensor JDTC-BF approach. Future work will extend multi-sensor JDTC to multiple maneuvering targets by exploiting labeled multi-Bernoulli (LMB) filtering.

7. Conclusion

Joint detection, tracking and classification (JDT) of a single-target immersed in clutter has been addressed by multi-sensor fusion. The problem has been formulated in a Bayesian framework by introduction of a suitably augmented Bernoulli density describing the joint distribution.
Appendix B. Proof of Proposition 2

At time $k$, starting from

$$f_{k|k-1}(x|m,c) = \gamma_{k|k-1}(c)\beta_{k|k-1}(m|c)s_{k|k-1}(x|m,c)$$

and using marginalization with respect to $c$, we have

$$f_{k|k-1}(x|m) = \frac{1}{r_{k|k-1}} \left( p_B(1-r_{k-1}) \gamma_B(c) \beta_B(m|c)s_B(x|m) ight)$$

while $\gamma_{k|k-1}(c)$ is derived as

$$\gamma_{k|k-1}(c) = \sum_{m\in\mathcal{M}_c} \beta_{k|k-1}(m|c)$$

by marginalization with respect to $c$, i.e. summation over $\mathcal{M}_c$.

The updated density of the Bernoulli RFS $mathcal{X}$ can be obtained by exploiting the Bayes rule

$$f_{k|k}(x) = \prod_{i\in\mathcal{N}} \ell_k(Z_k^i|\mathcal{X})$$

where $\ell_k(Z_k^i|\mathcal{X})$ is the local likelihood function of node $i$. According to [11], Eqs. 7.52-60] and [11], we can obtain the normalization constant as

$$\ell_i(x|m,c) = 1 - p_D^i(c) + \sum_{z\in\mathcal{Z}_k^i} \frac{K(\mathcal{Z}_k^i\{z\})}{K(\mathcal{Z}_k^i)} \cdot p_D^i(c) \ell_i(z|x)$$

and $K(\cdot)$ is the clutter Poisson PDF defined as

$$K(\mathcal{Z}_k^i) = e^\lambda \prod_{z\in\mathcal{Z}_k^i} \kappa(z)$$

where $\lambda$ is the expected number of clutter (false) measurements. Thus, it is immediate to see that the ratio

$$K(\mathcal{Z}_k^i\{z\})/K(\mathcal{Z}_k^i)$$

can be simplified as $1/\kappa(z)$. Hence, (B.5) can be rewritten as

$$\ell_i(x|m,c) = 1 - p_D^i(c) + \sum_{z\in\mathcal{Z}_k^i} \frac{\ell_i(z|x)}{\kappa(z)}$$

where $\ell_i(z|x)$ is given by (B.2).
\[ f_k(Z_k, Z_{1:k-1}) = \int \ell_k(Z_k|X) f_{k|k-1}(X) \delta X \]

\[ = \ell_k(Z_k|\emptyset) f_{k|k-1}(\emptyset) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \sum_{m \in M_c} \beta_{k|k-1}(m|c) \int \ell_k(Z_k|\{x\}) f_{k|k-1}(\{x\}|Z_{1:k-1}) \, dx \]

\[ = K(Z_k) \left[ (1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \sum_{m \in M_c} \beta_{k|k-1}(m|c) \int s_{k|k-1}(x|c, m) \ell(x|c, m) \, dx \right] \]

\[ = \prod_{i \in \mathcal{N}} K(Z_k') \left[ (1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \sum_{m \in M_c} \beta_{k|k-1}(m|c) \int s_{k|k-1}(x|c, m) \prod_{i \in \mathcal{N}} \ell(x|c, m) \, dx \right] \]

(B.4)

By substituting Eqs. 7, A.3, A.6, A.8 and (A.6) into (B.2) and taking \( X = \emptyset \), we get

\[ f_k(\emptyset) = \frac{\ell_k(Z_k|\emptyset) f_{k|k-1}(\emptyset)}{\prod_{i \in \mathcal{N}} K(Z_k')} \left[ (1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c) \right] \]

\[ = \prod_{i \in \mathcal{N}} K(Z_k') \left[ (1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c) \right] \]

where

\[ \ell(c) = \sum_{m \in M_c} \beta_{k|k-1}(m|c) \ell(m|c) \]

(B.8)

\[ \ell(m|c) = \int s_{k|k-1}(x|c, m) \ell(x|c, m) \, dx \]

(B.9)

\[ \ell(x|c, m) = \prod_{i \in \mathcal{N}} \ell_i(x|c, m) \]

(B.10)

Since \( f_k(\emptyset) = 1 - r_k \), the updated EP is given by

\[ r_k = \frac{r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)}{(1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)} \]

(B.11)

Conversely, when \( X = \{x\} \) we have

\[ f_k(\{x\}) = \frac{\ell_k(Z_k|\{x\}) f_{k|k-1}(\{x\})}{f_k(Z_k, Z_{1:k-1})} \]

\[ = \prod_{i \in \mathcal{N}} K(Z_k') \prod_{i \in \mathcal{N}} \left[ (1 - p_D(c) + p'_D(c) \sum_{x \in Z_k'} \ell(z|x) K(z) \right] \]

\[ \times \frac{r_{k|k-1} \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m)}{r_{k|k-1} \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m) \prod_{i \in \mathcal{N}} \ell_i(x|c, m)} \]

\[ = \prod_{i \in \mathcal{N}} K(Z_k') \left[ (1 - r_{k|k-1}) + r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c) \right] \]

\[ = \frac{1}{r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)} \sum_{n \in M_c} \int \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m) \ell(x|c, m) \, dx \]

(B.12)

Considering that

\[ r_{k|k-1} \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m) \prod_{i \in \mathcal{N}} \ell_i(x|c, m) \]

we can obtain

\[ \gamma_k(c) \beta_k(m|c) s_k(x|c, m) \]

\[ = \frac{\gamma_k(c) \beta_k(m|c) s_k(x|c, m) \prod_{i \in \mathcal{N}} \ell_i(x|c, m)}{r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)} \]

(B.13)

Then, via integration of (B.13) with respect to \( x \), the joint class\&mode PMF is given by

\[ \gamma_k(c) \beta_k(m|c) \]

\[ = \frac{\int \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m) \prod_{i \in \mathcal{N}} \ell_i(x|c, m) \, dx}{r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)} \]

(B.14)

Next, by substituting (B.14) in (B.13), the updated SPDFs turn out to be given by

\[ s_k(x|c, m) = \frac{s_{k|k-1}(x|c, m) \prod_{i \in \mathcal{N}} \ell_i(x|c, m)}{\int s_{k|k-1}(x'|c, m) \prod_{i \in \mathcal{N}} \ell_i(x'|c, m) \, dx'} \]

(B.15)

Moreover, the CPMF can be calculated by

\[ \gamma_k(c) = \frac{1}{r_{k|k-1} \sum_{c \in C} \gamma_{k|k-1}(c) \ell(c)} \sum_{n \in M_c} \int \gamma_{k|k-1}(c) \beta_{k|k-1}(m|c) s_{k|k-1}(x|c, m) \ell(x|c, m) \, dx \]
Clearly, the fused augmented SPDF $\tilde{s}(x, c, m)$ can be factored as the product of $\bar{\pi}(c), \bar{\beta}(m|c)$ and $\bar{\pi}(x|m,c)$.

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\[ f = \frac{\prod_{i \in N} (r^i)^{\omega^i} \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}_c} \int \prod_{i \in N} \left[ s^t_i(x|m,\gamma^i) \right] \sum_{m \in \mathcal{M}_c} \left[ \beta^i(m) \right] dx}{\prod_{i \in N} (1 - r^i)^{\omega^i} + \prod_{i \in N} (r^i)^{\omega^i} \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}_c} \int \prod_{i \in N} \left[ s^t_i(x|m,\gamma^i) \right] \sum_{m \in \mathcal{M}_c} \left[ \beta^i(m) \right] dx} \]

\[ = \frac{\prod_{i \in N} (r^i)^{\omega^i} \sum_{c \in \mathcal{C}} \left\{ \gamma^i(c) \right\} \sum_{m \in \mathcal{M}_c} \left[ \beta^i(m) \right] \int \prod_{i \in N} \left[ s^t_i(x|m) \right] dx}{\prod_{i \in N} (1 - r^i)^{\omega^i} + \prod_{i \in N} (r^i)^{\omega^i} \sum_{c \in \mathcal{C}} \left\{ \gamma^i(c) \right\} \sum_{m \in \mathcal{M}_c} \left[ \beta^i(m) \right] \int \prod_{i \in N} \left[ s^t_i(x|m) \right] dx} \]

\[ = \frac{\tilde{f} \sum_{c \in \mathcal{C}} \gamma^i(c) \sum_{m \in \mathcal{M}_c} \beta^i(m) \int \tilde{s}(x|m) dx}{\tilde{f} + \tilde{r} \sum_{c \in \mathcal{C}} \gamma^i(c) \sum_{m \in \mathcal{M}_c} \beta^i(m) \int \tilde{s}(x|m) dx} \]

(C.2)