Towards a fundamental safe theory of composite Higgs and Dark Matter

Giacomo Cacciapaglia and Shahram Vatani
Université de Lyon, F-69622 Lyon, France; Université Lyon 1, Villeurbanne CNRS/IN2P3, UMR5822, Institut de Physique Nucléaire de Lyon.

Teng Ma
CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China.

Yongcheng Wu
Ottawa-Carleton Institute for Physics, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario K1S 5B6, Canada.

We present a novel paradigm that allows to define a composite theory at the EW scale that is well defined all the way up to any energy by means of safety in the UV. The theory flows from a complete UV fixed point to an IR fixed point for the strong dynamics (which gives the desired walking) before generating a mass gap at the TeV scale. The model includes a composite Higgs, Dark Matter and partial compositeness for all SM fermions.

The ultra-violet (UV) behaviour is crucial for a quantum field theory (QFT) to be predictive and fundamental up to high scales [1, 2]. The presence of fixed points in the renormalisation group evolution of gauge and non-gauge couplings plays a central role in this. The prime example is Quantum Chromo-Dynamics (QCD), which features a free fixed point where the gauge coupling vanishes in the UV [3, 4]. The possible existence of an interacting fixed point has been first proposed by S. Weinberg in the context of quantum gravity [5], but prematurely discarded for renormalisable QFTs. Until, F. Sannino and D. Litim [6] found a first example of perturbative interacting UV fixed point in a theory with scalars and gauge-Yukawa couplings. Pure gauge theories with fermions have been obtained by employing resummation techniques for large number of fermionic flavours [7–9]. Recent progress can be found in Refs [10–13].

The least attractive feature of this class of asymptotically safe theories is the presence of a large multiplicity of fermion matter fields, as it can be seen in the attempts to build a safe extension of the Standard Model [16]. This problem is not ruled out experimentally, postulating the presence of tens of new massive fermions at the multi-TeV scale for the sole purpose of giving mass to the top quark only. This problem is absent in theories with scalar fields [20] at the price of reintroducing the hierarchy problem related to elementary scalar masses. In any case, these theories remain underlying descriptions of the composite Higgs dynamics of top partial compositeness, but far from being true UV completions. In fact, the origin of the light fermion masses as well as the source for the couplings generating the partial compositeness remain absent. Our goal is therefore to take a decisive step towards addressing these issues and being able to construct a genuine UV complete theory that can be trusted at arbitrarily high energies, at least up to the Planck scale. In this perspective, providing a Dark Matter candidate becomes a key ingredient.

In this letter we present a new paradigm that allows to define composite Higgs models with underlying fermions up to arbitrary high energies. The large number of fermions needed to give mass to all standard quarks and leptons drives the theory to a complete UV interacting fixed point. The fermions associated with the two light generations are supposed to have a large mass, thus explaining the lightness compared to the electroweak scale. Once integrated out, the remaining degrees of freedom drive the confining gauge interaction towards an Infra-Red (IR) fixed point [21]. The resulting conformal win-

---

* g.cacciapaglia@ipnl.in2p3.fr  
† shahram.vatani@ens-lyon.fr  
‡ mat@itp.ac.cn  
§ ycwu@physics.carleton.ca  
1 Quantum gravity effects may also be able to drive the Standard Model interactions to a safe UV, see for instance Ref. [17].
dow, similar in nature to walking Technicolor [22,23], allows to further split the scale of the heavy fermions where flavour effects also arise, from the condensation and electroweak scales. The exit from the IR fixed point can be driven by integrating out a subset of the remaining light fermions, leaving one of the models of Ref. [18] at low scale. In this framework, fundamental scalar fields can also be added in a natural way, as long as their masses are close to the mass of the heaviest fermions [24]. A Dark Matter candidate can be easily included in this class of theories [25–27]. The new scenario we discuss here, therefore, allows to define composite models that can be as predictive as supersymmetric extensions of the Standard Model.

The letter is organised as follows: after summarising in Section I the guiding principles behind the construction of underlying theories for composite Higgs, in Section II we show results for one specific example that features a Dark Matter candidate. In Section III we present the phenomenology of the Dark Matter candidate, which proves that the model is feasible, before offering our conclusions.

I. CHOOSING THE MODEL

To define an underlying theory for composite Higgs models, we need to specify a confining hyper-colour (HC) gauge symmetry, $G_{\text{HC}}$, and the irreducible representation (irrep) of the underlying fermions, $\psi_i$. Furthermore, the electroweak (EW) quantum numbers of the $\psi_i$ should be suitably chosen such that a Higgs doublet arises as a pseudo-Nambu Goldstone boson (pNGB) after confinement and chiral symmetry breaking. The partial compositeness paradigm imposes a strong additional constraint: the presence of spin-1/2 bound states that mix with the standard fermions.

Bound states of three $\psi_i$’s are possible for the fundamental irrep of $G_{\text{HC}} = SU(3)$ [15], as long as some of the fermions also carry QCD charges in addition to the EW ones. Another possibility, proposed in Ref. [15], is to sequester QCD charges to a second class of underlying fermions, $\chi_j$, transforming under a different irrep of $G_{\text{HC}}$. The benefit of this choice is that the breaking of the EW symmetry via vacuum misalignment in the $\psi$-sector is decoupled from QCD, which should not be broken. Furthermore, the spin-1/2 bound states that enter partial compositeness arise as chimera baryons [25] made of both species of fermions, in the two alternative forms

$$\mathcal{B} = \langle \psi \psi \chi \rangle \quad \text{or} \quad \langle \psi \chi \chi \rangle. \quad (1)$$

For each HC gauge group, the multiplicity of fermions, and their irreps, are limited by the requirement that the theory remains asymptotically free, i.e. it confines at low energy, and outside the IR conformal window [29,30], i.e. a mass gap is generated. Furthermore, the minimal number of $\psi$’s is given by the requirement of having a Higgs doublet in the coset, as listed in Table I while the minimal $\chi$ sector needs to contain a QCD colour triplet and an anti-triplet in order to generate - at least - the top mass.

For each HC-group, the multiplicity of fermions, and their irreps, are limited by the requirement that the theory remains asymptotically free, i.e. it confines at low energy, and outside the IR conformal window [29,30], i.e. a mass gap is generated. Furthermore, the minimal number of $\psi$’s is given by the requirement of having a Higgs doublet in the coset, as listed in Table I while the minimal $\chi$ sector needs to contain a QCD colour triplet and an anti-triplet in order to generate - at least - the top mass. This leaves only 12 feasible models [19] with minimal Higgs cosets, which we denote M1 to M12, following Ref. [14].

To define a genuine UV completion for these models, the issue of Dark Matter cannot be avoided. The simplest possibility is that one of the additional pNGBs may be stable. The minimal case is offered by the SU(4)²/SU(4) cosets [25,31]. As it was shown in Ref. [31], in the EW sector there is a unique $Z_2$ parity that is conserved by the fermion condensate (if custodial symmetry is preserved) and by the EW gauging, as well as being anomaly free: it is defined in terms of charge conjugation in the $\psi$ sector plus a flavour rotation in the SU(4) space. If the top couplings also respect this parity, the pNGB spectrum will contain several odd scalars, in particular a doublet and a triplet of SU(2)L plus a neutral and a charged singlet. Such states mix, and the lightest neutral one plays the role of Dark Matter candidate (see Ref. [31] for more details on the pNGB structure).

To extend the Dark $Z_2$ parity in the case of partial compositeness, we need to make sure that the composite operator $\mathcal{B}$ that mixes with the top has well-defined transformation properties and contains an even state with the same quantum numbers as the top quark fields. As the Dark parity contains charge conjugation in the $\psi$-sector (but not $\chi$), it is crucial that the bound state contains two $\psi$’s: this simple fact rules out the case with HC-charged scalars of Ref. [20]. Furthermore, the $\psi$-bilinear in $\mathcal{B}$ needs to be in a real irrep of $G_{\text{HC}}$ (ruling out M12 and MV, see Table I) and needs to contain both chiralities (ruling out M11). We are therefore left with the model M10, based on SO(10)HC, with the $\psi$ in the

| $\psi$ irrep | coset | pNGBs | pNGB EW charges | models | $G_{\text{HC}}$ | Lattice results |
|-------------|-------|-------|-----------------|--------|--------------|----------------|
| pseudo-real | SU(4)/Sp(4) | 5 | 2$\times$1/2 $\oplus$ 10 | M8-M9 | Sp(4), SO(11) | SU(4), Sp(4) |
| real | SU(5)/SO(5) | 14 | 2$\times$1/2 $\oplus$ 3$\times$1 $\oplus$ 10 | MI-M7 | SU(4), Sp(4), SO(7), SO(9), SO(10) | SU(4), Sp(4) |
| complex | SU(4)/SU(4) | 15 | 2$\times$2$\times$1/2 $\oplus$ 3$\times$1 $\oplus$ 1$\times$1 $\oplus$ 2$\times$10 | M10-M12 MV | SO(10), SU(4), SU(5) | SU(3), SU(4) |

Table I. Minimal cosets with a pNGB Higgs doublet arising from an underlying gauge-fermion theory. The fourth column shows the SU(2)L irrep, with the hypercharge as subscript. The last three columns show some properties of the explicit models, with the nomenclature M1-M12 from Ref. [14], and MV being the model from Ref. [15].
spinoiral irrep and the \( \chi \) in the fundamental. There are several choices for the wave function of the composite operators generating top partners that preserves the \( Z_2 \) symmetry in this model. One of them is

\[
B_{\psi} = \langle (\psi \Gamma_\mu \gamma_5 \chi)^\mu \rangle, \quad B_{\psi'} = \langle (\psi' \Gamma_\mu \gamma_5 \chi')_\mu \rangle,
\]

where the subscript \( l, r \) refer to the chirality, \( c \) indicates the charge conjugate and the brackets means Lorentz scalar. Both top partners will, therefore, transform as a bi-fundamental of \( SU(4) \times SU(4) \), which decomposes into a singlet and an adjoint of the unbroken \( SU(4) \).

We will now study how the model M10 can be UV completed to an asymptotically safe theory.

### II. A FUNDAMENTAL THEORY WITH UV SAFETY

|           | \( SO(10)_{\text{FC}} \) | \( SU(3)_c \) | \( SU(2)_L \) | \( U(1)_Y \) | mass |
|-----------|-------------------------|-------------|-------------|-------------|------|
| \( \psi_L \) | 16                      | 1           | 2           | 0           | \( \pm 1/2 \) \sim 0 |
| \( \psi_R \) | 16                      | 1           | 1           | 2/3         | \( -1/3 \) \sim \Lambda_{\text{HC}} |
| \( \chi_3^3 \) | 10                      | 3           | 1           | 1           | \( -1/3 \) \sim \Lambda_{\text{HC}} |
| \( \chi_1^2 \) | 10                      | 3           | 3           | 1           | \( 2/3 \) |
| \( \chi_1^2 \) | 10                      | 3           | 3           | 1           | \( -1/3 \) \sim \Lambda_{\text{HC}} |
| \( \chi_1^2 \) | 10                      | 3           | 3           | 1           | \( -1/3 \) \sim \Lambda_{\text{HC}} |

TABLE II. Model M10 - all fermions are Dirac spinors.

The model M10 consists of 4 EW-charged \( \psi \)'s and a QCD-colour triplet of \( \chi \)'s, as shown in the upper block of Table II. These states characterise the composite states below the condensation scale \( \Lambda_{\text{HC}} \), including the Higgs, Dark Matter and the top partners.

To complete the model, we will extend it by adding one appropriate \( \chi \) for each standard fermion that acquires mass via the Higgs mechanism, as shown in Table II. We add the partners for the bottom quark and tau lepton at a scale close to \( \Lambda_{\text{HC}} \), i.e. 4 additional \( \chi \)-flavours. This is enough to push the theory into the conformal window: right above the condensation scale, therefore, the strong sector flows into a conformal phase where the gauge coupling remains strong and slowly walking. This phase ensure that the operators that mix to the light generations acquire a largish anomalous dimension that allows to sufficiently decouple the scale where they are introduced. The \( \chi \) fermions associated to the light generations are, in fact, introduced at a scale \( \Lambda_{\text{FC}} \gg \Lambda_{\text{HC}} \), where flavour effects are also generated. Above \( \Lambda_{\text{FC}} \), the number of fermions is such that the running of all gauge couplings are not asymptotically free any more. We will first study how the theory may flow to a UV safe fixed point.

Following Ref. [22], for each gauge group we define a normalised coupling that takes into account the multiplicity of fermions \( f \) charged under it:

\[
K_i \equiv N_i T_i \frac{\alpha_i}{\pi} = \frac{\alpha_i}{\pi} \sum_f n_f T(r_j); \quad (3)
\]

with \( i = 1, 2, 3, 10 \) labelling the 4 gauge groups, and we fix the normalisation with the fundamental irreps, i.e. \( T_1 = T_{10} = 1 \) and \( T_2 = T_3 = 1/2 \) (for \( U(1) \), replace \( T(r_j) \rightarrow Y_f^2 \)). The multiplicities \( N_i \), different for each group, allow us to define a large “\( N_f \)” counting that we will use in the expansion and resummation. We will assume that formally they are all of the same order. For the UV-complete M10, we find:

\[
N_1 = 93, \quad N_2 = 22, \quad N_3 = 66, \quad N_{10} = 25. \quad (4)
\]

The resummed evolution equations read

\[
\frac{\partial \ln K_i}{\partial \ln \mu} \equiv \beta_i(K_i) = \frac{2 K_i}{3} \left[ 1 + \sum_n \frac{1}{N_f} B_i^{(n)} \right], \quad (5)
\]

with the first-order term equal to

\[
B_i^{(1)} = \frac{1}{N_i} \left[ c_{1,i} H_i(K_i) + \sum_{j \neq i} c_{i,j} F_i(K_j) \right]. \quad (6)
\]

For the \( i = 1 \), the function \( H_1(K_1) \) should be replaced by \( F_1(K_1) \). The coefficients read:

\[
c_{1,1} = \frac{1997}{339}, \quad c_{1,2} = \frac{3}{11}, \quad c_{1,3} = \frac{211}{3}, \quad c_{1,10} = 15; \quad c_{2,1} = \frac{10}{21}, \quad c_{2,2} = \frac{2}{7}, \quad c_{2,3} = \frac{1}{3}, \quad c_{2,10} = \frac{9}{2}; \quad c_{3,1} = \frac{1}{11}, \quad c_{3,2} = \frac{2}{7}, \quad c_{3,3} = \frac{1}{3}, \quad c_{3,10} = \frac{2}{7}; \quad c_{10,1} = \frac{1}{11}, \quad c_{10,2} = \frac{2}{7}, \quad c_{10,3} = \frac{1}{3}, \quad c_{10,10} = \frac{2}{7}. \quad (7)
\]

The functions \( H_1 \) and \( F_1 \) are defined in Ref. [11]. For our discussion, the key property is that the two functions have a pole at negative values for \( K = 3 \) and \( K = 15/2 \) respectively, while the resummation fails for coupling values above the pole. This feature, thus, acts as a barrier for the evolution of the respective coupling towards the UV. In other words, if the value of the coupling at the threshold \( \Lambda_{\text{FC}} \) is below the pole, the evolution towards the UV will stop at that value where the beta function vanishes and the theory approaches a fixed point. The condition for the model to have a UV safe fixed point for all gauge couplings is that their value is below the pole, \( K_i < 3 \) for \( i \neq 1 \) and \( K_1 < 15/2 \). Numerically, in M10 this implies:

\[
\alpha_1(\Lambda_{\text{FC}}) \lesssim 0.25, \quad \alpha_2(\Lambda_{\text{FC}}) \lesssim 0.86, \quad \alpha_3(\Lambda_{\text{FC}}) \lesssim 0.28, \quad \alpha_{10}(\Lambda_{\text{FC}}) \lesssim 0.38. \quad (8)
\]

Satisfying the above constraints will provide an upper bound on \( \Lambda_{\text{FC}} \) due to the fact that some of the gauge couplings increase towards the UV above \( \Lambda_{\text{FC}} \). This is in particular true for the \( U(1) \) gauge coupling. On the other hand, an indirect lower bound derives from flavour physics, which gives \( \Lambda \gtrsim 10^3 \) TeV for generic flavour violating effects.
In Fig. 1 we show the running of the 4 gauge couplings above the EW scale for the model in Table I and assuming $\Lambda_{HC} = 10$ TeV. In this example, it is the QCD coupling $\alpha_3$ that first crosses the upper limit in Eq. (8) (shown by the thin horizontal lines) at $10^9$ GeV, thus giving an upper limit to the value of $\Lambda_{F1}$. In dashes lines we show how the running changes when we add the light generation fermions at $\Lambda_{F1} = 10^{8.5}$ GeV: we see that all couplings run to the fixed UV fixed point (for SU(2) it is not shown in the plot). The behaviour of $\alpha_3$ after U(1) saturates the asymptotic value is a numerical artefact, nevertheless it shows the impact of the U(1) running on the other gauge couplings as a large $F_1(\alpha_1)$ will affect all the $\beta$-functions in Eq. (6). This example illustrates that a completely UV-safe composite Higgs and Dark Matter model with partial compositeness is indeed feasible.

In the upper panel we illustrate the running of $\alpha_{10}$ in the strong coupling regime, which features a walking region between $10^4$ and $10^7$ GeV. This part of the plot, being non-perturbative, can only be confirmed by lattice calculations along the lines of Refs [28, 33, 35].

![Strong coupling regime](image)

**FIG. 1.** Renormalisation group running of the gauge couplings $\alpha_i$. The dashed lines show the effect of the large-$N_f$ resummation above $\Lambda_{F1} = 10^{8.5}$ GeV. The upper panel shows a cartoon of the running at strong coupling. Note that $\alpha_2$ also runs towards the UV fixed point $\alpha_2 = 0.86$ (not shown in the plot).

**III. DARK MATTER PHENOMENOLOGY**

In this model, the pNGBs that are odd under the Dark Matter parity are a triplet of SU(2)$_L$, the second Higgs doublet and a charged and neutral singlet (forming a triplet of the custodial SU(2)$_R$ symmetry). Some of the composite fermions and vector mesons are also odd under the Dark parity. However, since the pNGB potential is generated at loop order, it is natural to expect that the pNGBs are lighter than other resonances. Thus, it is the lightest odd pNGB that plays the role of Dark Matter candidate [31].

The mass mixing is determined by the pNGB potential, which is generated by the EW gauge interactions, by a mass of the underlying fermions $\psi$, and by the partial compositeness couplings. The first two contributions have already been computed in Ref. [31]. Since the UV completion is fermionic, the Weinberg sum rules are automatically satisfied and the Higgs potential from the EW gauge sector is finite. In the top sector, we impose the maximal symmetry [30] in order to keep Higgs potential from top loops finite and calculable.

The nature of the lightest neutral stable scalar crucially depends on the mass difference between the two $\psi$’s, i.e. $\delta \equiv (m_{\psi_L} - m_{\psi_R})/(m_{\psi_L} + m_{\psi_R})$. For $\delta < 0$, it mostly coincides with the SU(2)$_L$ triplet, while for $\delta > 0$ it has maximal overlap with the singlets. For $\delta \sim 0$, maximal mixing with the doublet is active. This mixing pattern determines the annihilation rates of the Dark Matter candidate, which is dominated by the final states in two EW gauge bosons and two tops. The annihilation cross section is thus larger for $\delta < 0$, leading to larger Dark Matter masses.

To study a concrete example, we computed the top and gauge boson loop potential, and limited the parameter space by fixing the value of the top (173 GeV) and Higgs (125 GeV) masses at the minimum of the potential. We then scanned the remaining parameter space and computed relic abundance and spin-independent cross section of nuclei by using the micrOMEGAs [40] package. In Fig. 2 we show the results of our scan in the plane of the Dark Matter mass versus the cross section rescaled by the actual relic abundance. In this way, all points can be compared to the Direct Detection exclusion, shown by the solid black, blue and red lines. Points that saturate the relic abundance lie on the green line. We see that the model can explain the Dark Matter abundance without being excluded for masses ranging from few hundred GeV (for $\delta > 0$) to 1.5 $\sim$ 2 TeV (for $\delta < 0$). We remark...
FIG. 2. Direct detection constraints [36–38] for $\delta = 0.5$ (left) and $\delta = -0.5$ (right). The colour encodes the relic density for each parameter points. The green curve saturates the measured relic density value, with points having warmer colour excluded by over-density.

that those masses have reasonable values compared to the typical compositeness scale at the TeV.

IV. CONCLUSIONS AND OUTLOOK

We have presented a new paradigm that allows to define composite Higgs models with partial compositeness for the top quark up to arbitrarily high scales. This allows for the first time to endow composite models with predictivity power. Based on gauge-fermion underlying description of the low energy physics, we use the need for a large multiplicity of fermions, related to the large number of fermions and generations in the standard model, to predict the presence of UV safe fixed points for the complete theory.

We apply this paradigm to a model that also features a composite scalar Dark Matter candidate. We show that the gauge couplings, which include the coupling of the confining SO(10)$_{HC}$ group, can develop a UV interacting fixed points while also allowing for an IR conformal window and a sufficient hierarchy between the scale of flavour physics generation and the EW scale. Furthermore, the resulting unique model predicts a Dark Matter candidate in a consistent mass ballpark, which can also saturate the relic abundance while evading direct detection bounds.

While opening a very promising road, this model is still not a truly UV completion, as it does not include a dynamical origin for the couplings between the standard fermions and the strong sector. Such four-fermion interactions may be generated by scalar mediators without loosing naturalness, as long as the scalar masses are close to the heaviest fermion masses in the theory. However, it has been observed that there is a tension between the running of scalar quartics and U(1) gauge couplings [11]. A possible solution to this issue is to unify the U(1) with any other gauge group in a non-abelian envelope [11]. This also leaves open the possibility that the four-fermion interactions are generated by vector mediators, à la Extended Technicolor. We leave the investigation of this point for further work.

The results presented in this letter are a stepping stone towards complete composite Higgs models, where the origin of the standard fermion masses can be finally addressed. Some crucial ingredients, like the presence of an IR window where large anomalous dimensions are generated, need input from lattice calculations, possible as a detailed underlying model is on the table. The collider phenomenology of composite models is also affected, as non-minimal cosets are the norm in this scenario, thus predicting additional charged and neutral light scalars that can be searched for at the LHC.

ACKNOWLEDGEMENTS

Y. Wu is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). T.M. is supported in part by project Y6Y2581B11 supported by 2016 National Postdoctoral Program for Innovative Talents. G.C. and S.V. acknowledge partial support from the China-France LIA FCPPL and the Labex Lyon Institute of the Origins - LIO.

[1] K. G. Wilson, “Renormalization group and critical phenomena. 1. Renormalization group and the Kadanoff scaling picture,” Phys. Rev. B4 (1971) 3174–3183
[2] K. G. Wilson, “Renormalization group and critical phenomena. 2. Phase space cell analysis of critical behavior,” Phys. Rev. B4 (1971) 3184–3205
