High-precision $D^0 \rightarrow V^+ P^-$ - more accurate than $D^0 \rightarrow P^+ P^-$

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Recently we derived a nonlinear U-spin amplitude relation for $D^0 \rightarrow P^+ P^-$, $P = \pi, K$, predicted to hold up to fourth order U-spin breaking terms of order $10^{-3}$. Here we study a similar relation for $D^0 \rightarrow V^+ P^-$, $V = \rho, K^*(892)$, $P = \pi, K$, expected to hold at an even higher accuracy of order $10^{-4}$. We confirm this prediction in spite of a large experimental error of about $20\%$ in the amplitude of $D^0 \rightarrow K^+ \pi^-$. We also comment briefly on U-spin breaking in $D^0 \rightarrow P^+ V^-$. 

U-spin symmetry has been known for a long time to play an important role in obtaining approximate relations among hadronic $D$ meson decay amplitudes. In certain cases these relations were found to be violated by large symmetry breaking corrections. For instance, shortly after the discovery of charm in November 1974, the following zeroth order U-spin relations were shown to hold among amplitudes for $D^0$ decays to pairs involving a charged pion or kaon [1],

$$A(D^0 \rightarrow \pi^+ K^-) : A(D^0 \rightarrow K^+ K^-) : A(D^0 \rightarrow \pi^+ \pi^-) : A(D^0 \rightarrow K^+ \pi^-) = \cos^2 \theta_C : \cos \theta_C \sin \theta_C : - \cos \theta_C \sin \theta_C : - \sin^2 \theta_C,$$

(1)

where $\theta_C$ is the Cabibbo angle [2], $\tan \theta_C = 0.23125 \pm 0.00082$ [3].

One of these relations, $|A(D^0 \rightarrow K^+ K^-)| = |A(D^0 \rightarrow \pi^+ \pi^-)|$ is broken experimentally by about $80\%$. A very early rudimental interpretation of this anomaly was suggested in the framework of flavor SU(3) breaking [4]. Global studies within broken flavor SU(3) of $D$ meson decays to two pseudoscalar mesons and to a pair of vector and pseudoscalar meson have been made in Ref. [5]. A mechanism involving constructive interference of U-spin breaking penguin and tree (current-current) amplitudes has been suggested in Ref. [6] to explain this anomaly in the framework of first order U-spin breaking. See also [7].

Very recently we have applied perturbatively high order U-spin breaking to the relations (1), expanding amplitudes up to fourth order in powers of two distinct U-spin breaking parameters, $\text{Re}\epsilon_1 = 0.05$, $\text{Re}\epsilon_2 = 0.30$ [8]. Confronting predictions for ratios of amplitudes by their measurements we confirmed that second and fourth order U-spin breaking terms obey the required hierarchy with respect to these first order parameters. Namely, second order U-spin breaking corrections are a few times $10^{-2}$ while fourth order terms are of order $10^{-3}$. 

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One particularly interesting high-precision relation was predicted in [8] among four ratios of amplitudes each of which equals one in the U-spin symmetry limit. Denoting

\[ A_1 \equiv |A(\pi^+K^-)|, \quad A_2 \equiv |A(K^+\pi^-)|, \quad A_3 \equiv |A(\pi^+\pi^-)|, \quad A_4 \equiv |A(K^+K^-)|, \quad (2) \]

we defined four ratios,

\[ R_1 \equiv \frac{A_2}{A_1 \tan^2 \theta_C}, \quad R_2 \equiv \frac{A_4}{A_3}, \quad R_3 \equiv \frac{A_3 + A_4}{A_1 \tan \theta_C + A_2 \tan^{-1} \theta_C}, \quad R_4 \equiv \sqrt{\frac{A_3 A_4}{A_1 A_2}}. \quad (3) \]

These ratios were shown to obey the following nonlinear relation up to fourth order U-spin breaking,

\[ \Delta R \equiv R_3 - R_4 + \frac{1}{8} \left[ (\sqrt{2R_1} - 1 - 1)^2 - (\sqrt{2R_2} - 1 - 1)^2 \right] = O(\epsilon^4). \quad (4) \]

The parameter \( \epsilon \) represents generically the above-mentioned U-spin breaking parameters, \( \epsilon_1 \) and \( \epsilon_2 \). Considering also first order isospin breaking, we have noted in [8] that the right hand side may also include a tiny term suppressed by both isospin breaking and U-spin breaking parameters.

The proof of (4) for the four amplitudes of \( D^0 \to P^+P^- (P = \pi, K) \) was based on the U-spin vector behavior of the weak Hamiltonian, and on the fact that \( D^0 \) is a U-spin singlet while each of the two pairs \((\pi^-, K^-)\) and \((K^+, -\pi^+)\) is a U-spin doublet. Replacing the second pair by a U-spin doublet pair of vector mesons \((K^{*+}, -\rho^+)\), one may immediately apply this relation to other four decay amplitudes for \( D^0 \to V^+P^- (V = \rho, K^*, P = \pi, K) \). Thus denoting

\[ A_1' \equiv |A(\rho^+K^-)|, \quad A_2' \equiv |A(K^{*+}\pi^-)|, \quad A_3' \equiv |A(\rho^+\pi^-)|, \quad A_4' \equiv |A(K^{*+}K^-)|, \quad (5) \]

and defining

\[ R_1' \equiv \frac{A_2'}{A_1' \tan^2 \theta_C}, \quad R_2' \equiv \frac{A_4'}{A_3'}, \quad R_3' \equiv \frac{A_3' + A_4'}{A_1' \tan \theta_C + A_2' \tan^{-1} \theta_C}, \quad R_4' \equiv \sqrt{\frac{A_3'A_4'}{A_1'A_2'}}, \quad (6) \]

one has

\[ \Delta R' \equiv R_3' - R_4' + \frac{1}{8} \left[ (\sqrt{2R_1'} - 1 - 1)^2 - (\sqrt{2R_2'} - 1 - 1)^2 \right] = O(\epsilon'^4). \quad (7) \]

Here \( \epsilon' \) represents generically two U-spin breaking parameters, \( \epsilon_1' \) and \( \epsilon_2' \), defined for \( D^0 \to V^+P^- \) in analogy to \( \epsilon_1 \) and \( \epsilon_2 \) defined for \( D^0 \to P^+P^- \).

We will now compare the current experimental status of the two predicted amplitude relations (4) and (7), paying special attention to experimental errors. Hadronic decay amplitudes are obtained from measured branching ratios \( B \) by eliminating U-spin breaking phase space factors. For s-wave \( D \to P^+P^- \) decays and for p-wave \( D^0 \to V^+P^- \) decays one has

\[ |A| = M_D \sqrt{\frac{8\pi B}{\tau_D p}}, \quad |A'| = M_D \sqrt{\frac{8\pi B}{\tau_D' p^2}}. \quad (8) \]
limits considerably the precision of \( R \) that this is actually not exactly the case. That is, in spite of a dominant large error in \( \Delta \) prohibit a precise test of this high order U-spin relation using current data. We will show \( A \) originating in Dalitz plot analysis of \( D \) other hand, the relative error in the DCS amplitude \( A \) in CF and SCS amplitudes are reasonably small, between two and three percent. On the disregard a common factor \( M \) meson mass and its lifetime. Since we are only concerned with ratios of amplitudes we will
disregard a common factor \( M_D \sqrt{8\pi/\tau_D} \) which cancels in these ratios.

Values for measured branching ratios, center-of-mass momenta, and amplitudes defined in the above manner are quoted in Tables I and II for \( D^0 \to \pi^+\pi^- \) and \( D^0 \to \pi^+\pi^- \), respectively. Focusing at this point on experimental errors, we note that we have included no error in the amplitude for the Cabibbo-favored (CF) decay \( D^0 \to \pi^+K^- \). All other three \( D^0 \to \pi^+\pi^- \) branching ratios including errors have been measured relative to this process [3]. The three errors in amplitudes for singly Cabibbo-suppressed (SCS) decays \( D^0 \to \pi^+\pi^- \), \( D^0 \to \pi^+\pi^- \) and for the doubly Cabibbo-suppressed (DCS) decay \( D^0 \to \pi^+\pi^- \) are all around 0.8%, below the level of one percent. The high precision achieved recently in measuring the DCS amplitude is remarkable. It required time-dependent separation between this highly suppressed decay and \( D^0\) mixing followed by the CF decay \( D^0 \to K^+\pi^- \) [9].

Considering current errors in \( D^0 \to V^+P^- \) amplitudes we note that the relative errors in CF and SCS amplitudes are reasonably small, between two and three percent. On the other hand, the relative error in the DCS amplitude \( A(D^0 \to K^+\pi^-) \), obtained through a Dalitz plot analysis of \( D^0 \to K_S\pi^+\pi^- \) [10], is quite large -+26% -15%. This large asymmetric error limits considerably the precision of \( R_1', R_2' \) and \( R_4' \) occurring in Eq. (7). This would seem to prohibit a precise test of this high order U-spin relation using current data. We will show that this is actually not exactly the case. That is, in spite of a dominant large error in \( \Delta R' \) originating in \( A(D^0 \to K^+\pi^-) \), the amplitude relation [7], in which one neglects fourth order U-spin breaking, holds even better than the corresponding relation in \( D^0 \to P^+P^- \).

| Decay mode | Branching fraction | \( p \) (GeV/c) | \( A = \sqrt{\mathcal{B}/p} \) (GeV/c)\(^{-1/2} \) |
|------------|-------------------|----------------|---------------------------------|
| \( D^0 \to \pi^+K^- \) | \( 0.108 \pm 0.007 \) | 0.675 | 0.593 \pm 0.019 |
| \( D^0 \to K^+\pi^- \) | \( 3.42^{+1.80}_{-1.02} \) \times 10^{-4} | 0.711 | 0.0308^{+0.0081}_{-0.0046} |
| \( D^0 \to \rho^+\pi^- \) | \( 9.8 \pm 0.4 \) \times 10^{-3} | 0.764 | 0.148 \pm 0.003 |
| \( D^0 \to K^+K^- \) | \( 4.38 \pm 0.21 \) \times 10^{-3} | 0.610 | 0.1389 \pm 0.0033 |

\( \mathcal{B}_{eK} = (3.88 \pm 0.05) \times 10^{-2} \) \( B_{eK}^1 = 1.0785 \) \( A_1 \)
\( (3.56 \pm 0.06) \times 10^{-3} \) \( B_{eK}^1 = 0.861 \) \( A_2 \)
\( (3.59 \pm 0.06) \times 10^{-2} \) \( B_{eK}^1 = 0.922 \) \( A_3 \)
\( (10.10 \pm 0.16) \times 10^{-2} \) \( B_{eK}^1 = 0.791 \) \( A_4 \)

Table I: Branching fractions and amplitudes for \( D^0 \to P^+P^- \) decays [3]

Table II: Branching fractions and amplitudes for \( D^0 \to V^+P^- \) decays [3]
Using current experimental amplitudes we will now calculate the quantities \( \Delta R \) and \( \Delta R' \) defined in (4) and (7) including their errors.

Taking values of amplitudes given in Tables I and III we first calculate, separately for D\(^0\) \( \rightarrow \) P\(^+\)P\(^-\) and D\(^0\) \( \rightarrow \) V\(^+\)P\(^-\), the four ratios of amplitudes defined in Eqs. (3) and (6). The measured branching ratios, obtained in independent analyses for different two-body P\(^+\)P\(^-\) and three-body V\(^+\)P\(^-\) final states, involve no error correlations. Therefore we compute errors in ratios by adding in quadrature errors due to the relevant amplitudes. We find

\[
R_1 = 1.115 \pm 0.012, \quad R_2 = 1.811 \pm 0.020, \quad R_3 = 1.052 \pm 0.008, \quad R_4 = 1.008 \pm 0.007, \quad (9)
\]

and

\[
R_1' = 0.971^{+0.257}_{-0.148}, \quad R_2' = 0.939 \pm 0.029, \quad R_3' = 1.061^{+0.082}_{-0.140}, \quad R_4' = 1.061^{+0.083}_{-0.142}. \quad (10)
\]

Errors in \( \Delta R \) and \( \Delta R' \) caused by errors in amplitudes are also added in quadrature [rather than using the errors in (9) and (10)]:

\[
\Delta R = (-3.2 \pm 0.4) \times 10^{-3}, \quad \Delta R' = (0.2^{+3.2}_{-5.5}) \times 10^{-4}. \quad (11)
\]

Thus, in spite of its current huge relative error, \( \Delta R' \) is significantly smaller than \( \Delta R \). We have confirmed that the dominant uncertainty in \( \Delta R' \) originates in the large experimental error in \( |A(D^0 \rightarrow K^{*+}\pi^-)| \). Neglecting this error we find \( \Delta R' = (0.2 \pm 0.4) \times 10^{-4} \), which is smaller than \( \Delta R \) by more than an order of magnitude. Can one explain this somewhat unanticipated situation?

As is explicit on the right hand sides of (4) and (7), \( \Delta R \) and \( \Delta R' \) are expected to be tiny but nonzero due to fourth order U-spin breaking terms and possible corrections which break both isospin and U-spin. It is remarkable but not unexpected, we will now show, that these symmetry breaking terms are smaller in D\(^0\) \( \rightarrow \) V\(^+\)P\(^-\) than in D\(^0\) \( \rightarrow \) P\(^+\)P\(^-\).

Using relations derived in Ref. [8] between two U-spin breaking parameters, \( \epsilon^{(1)}_{1,2} \) and the two ratios \( R^{(1)}_{1,2} \), we calculate for D\(^0\) \( \rightarrow \) P\(^+\)P\(^-\) and D\(^0\) \( \rightarrow \) V\(^+\)P\(^-\):

\[
\text{Re} \epsilon_1 = \frac{1}{2} \left( \sqrt{2R_1 - 1} - 1 \right) = 0.054 \pm 0.005, \quad \text{Re} \epsilon_2 = \frac{1}{2} \left( \sqrt{2R_2 - 1} - 1 \right) = 0.310 \pm 0.006, \quad (12)
\]

\[
\text{Re} \epsilon'_1 = \frac{1}{2} \left( \sqrt{2R'_1 - 1} - 1 \right) = -0.015^{+0.118}_{-0.083}, \quad \text{Re} \epsilon'_2 = \frac{1}{2} \left( \sqrt{2R'_2 - 1} - 1 \right) = -0.032 \pm 0.016. \quad (13)
\]

While the U-spin breaking parameter \( \epsilon_2 \) in D\(^0\) \( \rightarrow \) P\(^+\)P\(^-\) is quite large, \( \text{Re} \epsilon_2 \simeq 0.30 \), the corresponding parameter in D\(^0\) \( \rightarrow \) V\(^+\)P\(^-\) is an order of magnitude smaller, \( -\text{Re} \epsilon'_2 \sim 0.01 - 0.05 \). Namely, a U-spin breaking mechanism involving constructive interference of penguin and tree amplitudes, suggested in Ref. [6] to account for the large value of \( R_2 \equiv |A(D^0 \rightarrow K^+K^-)|/|A(D^0 \rightarrow \pi^+\pi^-)| \), is not at work in \( R'_2 \equiv |A(D^0 \rightarrow K^{*+}K^-)|/|A(D^0 \rightarrow \rho^+\pi^-)| \). The ratio \( R'_1 \) does not involve a penguin contribution and is likely to deviate from one by no more than about 20\%, typical for U-spin breaking in tree amplitudes. (This may be
demonstrated, for instance, by a simple model calculation based on factorization, using as input the ratio of $K^*$ to $\rho$ decay constants and the ratio of $D \to \pi$ to $D \to K$ form factors for $q^2 = m_K^2$, and $q^2 = m_\rho^2$, respectively.) Consequently, one expects $|\text{Re } \epsilon_1'| \leq 0.1$, in agreement with [13]. This parameter could be smaller, around 0.05, just like Re $\epsilon_1$. Taking these small values of Re $\epsilon_1'$ and Re $\epsilon_2'$ as typical first order U-spin breaking in $D^0 \to V^+P^-$ amplitudes implies that a fourth order symmetry breaking term in $\Delta R'$ is at most $10^{-4}$. An isospin breaking term suppressed also by U-spin is expected to be a few times $10^{-4}$.

Also, the ratios $R_3'$ and $R_4'$ which involve large experimental errors are predicted to equal one up to second order U-spin breaking corrections [5]. In view of the above small values of Re $\epsilon_1'$ and Re $\epsilon_2'$, we expect these corrections to be at most a few percent.

High-precision relations of the form (11) and (17) hold whenever final states in $D^0$ decays involve pairs of U-spin doublet mesons. We do not study the relation in decays to $P^+V^-$ because no data are available at this time for the DCS decay $D^0 \to K^+\rho^-$. We checked that the magnitude of a corresponding U-spin breaking parameter given by a ratio of two SCS $D^0 \to P^+V^-$ amplitudes is intermediary between corresponding parameters in $D^0 \to P^+P^-$ and $D^0 \to V^+P^-$. Using branching ratios given in Table III, we calculate

$$R''_2 \equiv \frac{|A(D^0 \to K^+K^{-})|}{|A(D^0 \to \pi^+\rho^-)|} = 0.786 \pm 0.036 \text{ implying } \text{Re } \epsilon_2'' = -0.122 \pm 0.024 \ . \ (14)$$

This value of Re $\epsilon_2''$ should be compared with Re $\epsilon_2 = 0.310 \pm 0.006$ and Re $\epsilon_2' = -0.032 \pm 0.016$ in $D^0 \to P^+P^-$ and $D^0 \to V^+P^-$, respectively.

| Decay mode | Branching fraction | $p$ (GeV/c) | $A'' = \sqrt{B/p^3}$ (GeV/c)^{-3/2} |
|------------|-------------------|------------|-----------------------------------|
| $D^0 \to \pi^+\rho^-$ | $(4.96 \pm 0.24) \times 10^{-3}$ | 0.764 | $0.1055 \pm 0.0026$ |
| $D^0 \to K^+K^{-}$ | $(1.56 \pm 0.12) \times 10^{-3}$ | 0.610 | $0.0829 \pm 0.0032$ |

In conclusion, we have studied the current experimental status of a nonlinear precision relation predicted for $D^0 \to V^+P^-$ decay amplitudes, comparing its precision to that of a similar relation in $D^0 \to P^+P^-$ decays. We found that while the latter relation is violated at a very low level of $10^{-3}$, the relation for $D^0 \to V^+P^-$ holds at an even higher precision. We have shown that the correction to this relation, representing a fourth order U-spin breaking term or an isospin breaking term suppressed also by U-spin, is consistent with first order U-spin breaking parameters that are smaller in $D^0 \to V^+P^-$ than in $D^0 \to P^+P^-$. A small uncertainty in the $D^0 \to V^+P^-$ relation, at a level less than $10^{-3}$, is due to the current large experimental error in $B(D^0 \to K^+\pi^-)$. This error is expected to be reduced in future experiments by the LHCb and Belle II collaborations. It will be interesting to watch future improvements in this measurement, providing more precise determinations of $\text{Re } \epsilon_1'$, of second order U-spin breaking terms in $R_3'$, $R_4'$ at a percent level, and of a fourth order
term in $\Delta R'$ at a level of $10^{-4}$. The high-precision relations discussed in this report within the Standard Model provide useful constraints on $|\Delta C| = 1$ new physics operators [11].

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