Lowering the Threshold in the DAMA Dark Matter Search

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The DAMA experiment searches for Weakly Interacting Massive Particle (WIMP) dark matter via its expected but rare interactions within the detector, where the interaction rates will modulate throughout the year due to the orbital motion of the Earth. Over the course of more than 10 years of operation, DAMA has indeed detected a strong modulation in the event rate above the detector threshold of 2 keVee. Under standard assumptions regarding the dark matter halo and WIMP interactions, this signal is consistent with that expected of WIMPs of two different approximate masses: \( \sim 10 \text{ GeV} \) and \( \sim 70 \text{ GeV} \). We examine how a lower threshold, allowed by recent upgrades to the DAMA detector, may shed light on this situation. We find that the lower threshold data should rule out one of the two mass ranges for spin-independent couplings (in the worst case, disfavoring one of the masses by still more than 2.6\( \sigma \)) and is likely, though not certain, to do the same for spin-dependent couplings. Furthermore, the data may indicate whether the interaction is predominantly spin-independent or spin-dependent in some cases. Our findings illustrate the importance of a low threshold in modulation searches.

I. INTRODUCTION

Although there exists an abundance of evidence at many different energy and length scales that the vast majority of matter in our universe is non-baryonic and does not possess significant electromagnetic interactions, the nature of this dark matter remains unknown. Understanding the particle nature of dark matter is one of the most important outstanding problems in both cosmology and particle physics. There are currently numerous direct detection experiments searching for interactions between the dark matter particles and the nuclei of the detectors.

The DAMA experiment is the first direct detection experiment to make a claim of the detection of particle dark matter. For more than a decade, the DAMA/LIBRA collaboration (and its predecessor DAMA/NaI) have claimed detection of an annual modulation that they associate with dark matter interacting with the NaI crystals of their detectors [1, 2]. The DAMA annual modulation is currently reported as nearly a 9\( \sigma \) effect [3], and is consistent with a \(~ 70 \text{ or } ~ 10 \text{ GeV} \) WIMP elastically scattering predominantly off of iodine or sodium, respectively [4–10].

Hints of light dark matter may now be evident at other experiments, as well. In February
of 2010, the CoGeNT collaboration reported an excess of scattering events at low energy [11]. The collaboration subsequently reported 2.8σ evidence for an annual modulation in these excess events [12]. At the same conference, CRESST-II also announced events above their known backgrounds that can be interpreted as evidence for light dark matter [13]. Whether DAMA, CoGeNT, and CRESST are consistent in the low-mass window is still debated [14, 15].

To complicate the matter even further, CDMS has not observed an annual modulation in their germanium detectors [16], although they have not yet released results with an energy threshold low enough to be sensitive to the entire energy range for the DAMA modulation. In addition, CDMS [17, 18] and both XENON10 [19] and XENON100 [20] find null results at low energies that appear to be in conflict with the three experiments that report anomalies.

The very recent development of three events observed in the signal window of the silicon detectors of CDMS [21] has only increased the interest (and confusion) in the low mass region. The CDMS result is even more intriguing given that the mass and cross section consistent with the dark matter interpretation of the three events lands in the same area of parameter space as that indicated by the CoGeNT excess, accounting for the latter’s updated estimates for surface event contamination [14, 22].

The DAMA/LIBRA experiment has recently undergone a significant upgrade, and has been taking data in the DAMA/LIBRA-phase 2 configuration since January 2011 [23]. In November of 2010, all of the photo-multiplier tubes (PMT’s) in the DAMA/LIBRA experiment were replaced by high quantum efficiency PMT’s [24]. One of the most interesting aspects of this detector upgrade is the anticipated lowering of the energy threshold from 2 keVee down to 1 keVee. In this paper we will examine the extent to which DAMA’s lower energy threshold can help to clarify the currently confusing situation for light dark matter. We will start by reviewing the basics of direct detection in Section II, followed by a discussion of the DAMA experiment and results in Section III. We examine implications of this lower threshold in Section IV and summarize our findings in Section V.

II. DARK MATTER DIRECT DETECTION

Dark matter direct detection experiments aim to observe the recoil of a nucleus in a collision with a dark matter particle [25]. After an elastic collision with a WIMP \( \chi \) of mass \( m_\chi \), a nucleus of mass \( M \) recoils with energy \( E_{nr} = (\mu v^2/M)(1 - \cos \theta) \), where \( \mu \equiv m_\chi M/(m_\chi + M) \) is the reduced mass of the WIMP-nucleus system, \( v \) is the speed of the WIMP relative to the nucleus, and \( \theta \) is the scattering angle in the center of mass frame.

The differential recoil rate per unit detector mass is

\[
\frac{dR}{dE_{nr}} = \frac{n_\chi}{M} \left\langle v \frac{d\sigma}{dE_{nr}} \right\rangle = \frac{2\rho_\chi}{m_\chi} \int d^3v v f(v, t) \frac{d\sigma}{dq^2}(q^2, v),
\]

where \( n_\chi = \rho_\chi/m_\chi \) is the local number density of WIMPs and \( \rho_\chi \) is the local dark matter mass density, \( f(v, t) \) is the time-dependent WIMP velocity distribution, and \( \frac{d\sigma}{dq^2}(q^2, v) \) is the velocity-dependent differential cross-section with the momentum exchange in the scatter given by \( q^2 = 2M E_{nr} \). The differential rate is typically given in units of cpd kg\(^{-1}\) keV\(^{-1}\), where cpd is counts per day. Below, we briefly describe the particle physics and astrophysics terms of the equation above. More detailed reviews of the dark matter scattering process and direct detection can be found in Refs. [26–31].
**Particle physics.** Most WIMP candidates have WIMP-quark couplings that give rise to spin-independent (SI) and spin-dependent (SD) WIMP-nucleus interactions with a differential cross-section of the form:

\[
\frac{d\sigma}{dq^2}(q^2, v) = \frac{\sigma_0}{4\mu^2 v^2} F^2(q) \Theta(q_{\text{max}} - q).
\]  

(2)

Here, \(\Theta\) is the Heaviside step function, \(q_{\text{max}} = 2\mu v\) is the maximum momentum transfer in a collision at a relative velocity \(v\), \(\sigma_0\) is the scattering cross-section in the zero-momentum-transfer limit—we shall use \(\sigma_{\text{SI}}\) and \(\sigma_{\text{SD}}\) to represent this term in the SI and SD cases, respectively—and \(F^2(q)\) is a form factor to account for the finite size of the nucleus. The total WIMP-nucleus scattering rate is determined by summing the SI and SD contributions, each with its own value of the form factor. A description of form factors can be found in Refs. [28, 32] (SI) and Refs. [33, 34] (SD).

The WIMP-nucleus cross-section \(\sigma_0\) can be given in terms of WIMP-nucleon couplings, though with different scaling behaviors for the two types of interactions. In the SI case,

\[
\sigma_{\text{SI}} = \frac{4}{\pi} \mu^2 \left[Z f_p + (A - Z) f_n\right]^2,
\]  

(3)

where \(A\) is the atomic mass number, \(Z\) is the number of protons, and \(f_p\) and \(f_n\) are the effective couplings to protons and neutrons, respectively. The common assumption is that WIMPs couple to protons and neutrons with nearly equal strength for SI interactions, leading to an \(A^2\) enhancement to the signal for heavier targets. While isospin-violating interactions, where the SI WIMP-proton and WIMP-neutron couplings can substantially differ, have also been explored in the literature [35–37], we will not examine their effects in this paper.

For the SD case,

\[
\sigma_{\text{SD}} = \frac{32\mu^2}{\pi} G_F^2 J(J + 1) \Lambda^2,
\]  

(4)

where \(G_F\) is the Fermi constant, \(J\) is the spin of the nucleus, and

\[
\Lambda \equiv \frac{1}{J} \left(a_p \langle S_p \rangle + a_n \langle S_n \rangle\right),
\]  

(5)

where \(\langle S_p \rangle\) and \(\langle S_n \rangle\) are the average spin contributions from the proton and neutron groups, respectively, and \(a_p\) (\(a_n\)) are the effective couplings to the proton (neutron). Unlike the SI case, the WIMP-proton and WIMP-neutron couplings are not expected to be nearly equal in the SD case.

**Astrophysics.** The velocity distribution, \(f(v, t)\), and the local dark matter density, \(\rho_\chi\), contain all the necessary information about the local distribution of dark matter to calculate the expected dark matter signal in a detector. The most commonly employed assumption when analyzing direct detection data is that dark matter is uniformly distributed throughout an isothermal sphere. The velocity distribution for this Standard Halo Model (SHM) [38, 39] is

\[
f_{\text{SHM}}(u) = \begin{cases} 
\frac{1}{N v_0^3 \sqrt{\pi^2}} e^{-u^2/v_0^2}, & \text{if } u < v_{\text{esc}} \\
0, & \text{otherwise}
\end{cases}
\]  

(6)

for dark matter particles with velocity \(u\) (in the halo rest frame), Galactic escape velocity \(v_{\text{esc}}\), and with \(N\) being a normalization constant that ensures \(\int d^3 u \, f(u) = 1\). The velocity
dispersion, $v_0$, is the most likely speed for a dark matter particle with this velocity distribution, which is equal to the local circular speed of the sun in the SHM. This function should be thought of as a reasonable, but approximate, parametrization of the dark matter’s true velocity distribution.

There still exists a significant amount of uncertainty in the astrophysical parameters relevant to the SHM. Recent estimates for the local circular speed generally place it at 235 km/s ±10% [40–42], somewhat higher than the historical canonical value of 220 km/s [43], though the latter value remains viable. A sample of high velocity stars from the RAVE survey has been used to place the Galactic escape velocity in the range of 498–608 km/s at the 90% confidence level (CL) with a median likelihood of 544 km/s [44]. While 0.3 GeV/cm$^3$ has long been taken as the canonical value for the local density of the smooth dark matter component, recent estimates tend to favor 0.4 GeV/cm$^3$, though these estimates are model dependent and vary in the literature by as much as a factor of two [45–49]. In this work, we will use $v_0 = 235$ km/s, $v_{esc} = 550$ km/s and $\rho_\chi = 0.4$ GeV/cm$^3$.

The motion of the earth through the Galactic halo will induce a time dependence in a dark matter signal in direct detection experiments [38, 39]. The integral over the velocity distribution in Eqn. (1) must be evaluated for the SHM with $u = v_e + v$, where $u$ is the WIMP velocity (in the halo’s rest frame), $v_e$ is the velocity of the Earth (in the halo’s rest frame) and $v$ is the velocity of the WIMP relative to the nucleus. The Earth’s speed relative to that of the Galactic halo is given by $v_e = v_\odot + v_{orb} \cos \gamma \cos[\omega(t - t_0)]$ where $v_\odot = v_0 + 13$ km/s is the speed of the sun relative to the rest frame of the halo, the orbital speed of Earth about the sun is $v_{orb} = 30$ km/s, $\cos \gamma = 0.49$ accounts for the direction of motion of the Earth relative to that of the halo, $t_0$ is the date at which Earth moves fastest relative to the halo (around June 2), and $\omega = 2\pi$/year. Thus, in the SHM, the Earth’s motion will lead to a sinusoidal modulation of the signal in a dark matter detector.

### III. THE DAMA EXPERIMENT

In this section, we discuss the DAMA experimental setup and current results. We also address two issues affecting the interpretation of these results: the choice of binning for the data and uncertainties in the quenching factor used to calibrate the energy scale of events in the DAMA detectors.

#### A. Overview

The DAMA/LIBRA collaboration uses 25 highly radio-pure NaI(Tl) detectors with a total target mass of ~250 kg. Each detector is instrumented with PMT’s that measure the scintillation light created by a nuclear recoil; these PMT’s have been recently upgraded [24]. The collaboration uses gamma ray sources for calibrating their detectors that produce scintillation light through electron recoils. This means that the energy of a nuclear recoil will be measured in electron equivalent energy, or keVee. The electron equivalent energy, $E_{ee}$, is related to the nuclear recoil energy, $E_{nr}$, via what is termed the “quenching factor.” For the DAMA detectors, the quenching factor, $Q$, is the ratio of the amount of scintillation light created by a recoiling nucleus of some kinetic energy to that created by a recoiling electron of the same kinetic energy, i.e. $E_{ee} = Q E_{nr}$.

Measuring the value of the quenching factor for sodium at the low energies relevant for
light WIMPs is quite difficult. This has led to some uncertainty as to the value and possible energy dependence of the quenching factor (see Ref. [50]). In our work, we adopt the values of the DAMA collaboration, $Q_{Na} = 0.3$ and $Q_{I} = 0.09$ [51] (see Ref. [52] and references therein for quenching factor measurements of NaI and several other scintillators used in direct detection experiments). Broadly speaking, increasing the quenching factor will shift an allowed region in the (mass, cross section) plane towards lower masses while leaving the cross section values relatively unaffected (see Ref. [14]).

Every real detector has both an energy resolution and a detection efficiency for relevant events. We implement these effects for the DAMA/LIBRA detectors using

$$\frac{dR}{dE_{ee}} (E_{ee}, t) = \int_{0}^{\infty} dE_{nr} \varepsilon(QE_{nr}) \phi(E_{nr}, E_{ee}) \frac{dR}{dE_{nr}} (E_{nr}, t).$$  \hspace{1cm} (7)

Here, $\varepsilon(QE_{nr})$ is the event detection efficiency and

$$\phi(E_{nr}, E_{ee}) = \frac{1}{\sqrt{2\pi}\sigma(QE_{nr})} e^{-(E_{ee} - QE_{nr})^{2}/2\sigma^{2}(QE_{nr})}$$  \hspace{1cm} (8)

is the differential response function, defined such that $\phi(E_{nr}, E_{ee}) \Delta E_{ee}$ is the probability that a nuclear recoil of energy $E_{nr}$ will produce a scintillation signal measured between $E_{ee}$ and $E_{ee} + \Delta E_{ee}$ (in the limit of small $\Delta E_{ee}$). The energy resolution for the detectors with the original PMT’s at the energies of interest is given by [2]

$$\sigma(QE_{nr}) = \alpha \sqrt{QE_{nr}} + \beta QE_{nr}$$  \hspace{1cm} (9)

with $\alpha = (0.448 \pm 0.035) \sqrt{\text{keVee}}$ and $\beta = (9.1 \pm 5.1) \times 10^{-3}$. The collaboration presents an efficiency corrected spectrum, so we can safely use $\varepsilon(QE_{nr}) = 1$. Although most of the upgraded detectors do show a small improvement in resolution at higher energies [24] there is no updated information available about their resolution in the low energy region. We will thus use Eqn. [9] to implement the detector resolution.

As discussed in the previous section, due to the changing WIMP velocity distribution at Earth as the Earth orbits the Sun, there is a small ($\sim 1$-10%) variation (or modulation) in the recoil rate throughout the year [31, 38]. The recoil rate can be described as

$$\frac{dR}{dE_{ee}} (E_{ee}, t) = S_{0}(E_{ee}) + S_{m}(E_{ee}) \cos(\omega(t - t_{0}) + \ldots$$  \hspace{1cm} (10)

where $S_{0}$ is the average rate, $S_{m}$ is the modulation amplitude, and higher order terms are often negligible (but see the discussion below). Note that $S_{m}$ can be negative, though this can only occur at low energies.

Most direct detection experiments search for a non-modulating signal of nuclear recoil due to dark matter scattering, i.e. that due to $S_{0}$. These detectors have typically been smaller in fiducial volume relative to DAMA and must be very good at background rejection. The strategy of the DAMA collaboration is unique in that they are looking for the modulation amplitude, $S_{m}$, induced by dark matter scattering. Their detectors have unknown, but presumably non-modulating, backgrounds, providing a natural background rejection strategy. As discussed previously, the modulation amplitude is smaller than $S_{0}$, however, so larger detector masses and exposure are required. Many modulating backgrounds have been put forward as alternative explanations for the DAMA modulation including: detector systematics, muons and fast neutrons produced by muons, environmental neutrons, nuclear decays...
in the detector, and many others. The collaboration has thus far rejected each of these possibilities as described in detail in Ref. [53].

The higher order terms in Eqn. (10) are usually negligible, but may become important in some cases. One case is in the presence of a dark matter cold flow (e.g. a tidal stream), where the modulation can become very non-sinusoidal [54]. Another case is for energies corresponding to the high velocity tail of the SHM, where the modulation becomes more sharply peaked around \( t_0 \). In both cases, the amplitude of higher order terms may become comparable to \( S_m \). However, any significant cold flow would likely change the phase of the modulation from the SHM expected June 2, whereas the DAMA observed modulation (May 26 ± 7 days) is consistent with the SHM. Additionally, the DAMA modulation is also observed over a broad range of energies (\( \sim 2-5 \) keVee), which corresponds to a broad range of \( v_{\text{min}} \) and not just the tail of the distribution. For these reasons, a DAMA analysis can still be safely performed when neglecting higher order terms in Eqn. (10). In any case, the DAMA modulation amplitude determination essentially identifies the first order Fourier expansion coefficient whether or not higher order terms are significant. In this case, a valid analysis can always be performed, even if the modulation is non-sinusoidal, as long as the predicted rates are Fourier expanded using the same phase as the experimental determination (though “modulation amplitude” may lose its meaning).

B. Data, Statistics, and Current Results

DAMA modulation amplitude results are presented in 36 bins, all 0.5 keVee wide, over 2–20 keVee [3]. Measurements over the lower part of this energy range, along with their uncertainties, are shown in Figure 1 (orange boxes). An alternative binning used in this analysis, where some of the original bins have been combined (gray boxes), will be discussed below. These amplitudes have been determined assuming the SHM expected phase (\( t_0 \), the time of year at which Earth moves fastest relative to the dark matter halo) of June 2; the best-fit DAMA phase is May 26 ± 7 days, consistent with this number.

Constraints on WIMP parameters and goodness-of-fit tests are based upon the chi-square,

\[
\chi^2(m_\chi, \sigma) = \sum_k \frac{(S_{m,k} - S^T_{m,k}(m_\chi, \sigma))^2}{\sigma^2_k},
\]

where \( S_{m,k} \) is the modulation amplitude measurement in bin \( k \) (averaged over energy), \( \sigma_k \) is the measurement uncertainty, \( S^T_{m,k}(m_\chi, \sigma) \) is the theoretically expected amplitude for a WIMP mass \( m_\chi \) and scattering cross-section(s) \( \sigma \), and the sum is over all bins. In most cases, we will restrict ourselves to the WIMP parameter space containing only the mass and SI cross-section, though we will also consider the case of SD couplings.

We test the goodness-of-fit of the WIMP framework to the data by minimizing the \( \chi^2 \) over the WIMP parameter space; the resulting \( \chi^2_{\text{min}} \) should follow a \( \chi^2 \) distribution with degrees of freedom (\( \text{dof} \)) equal to the number of data bins less the number of parameters minimized over. In the case of SI-only scattering, the \( \chi^2 \) is minimized at \( m_\chi = 68.3 \) GeV and \( \sigma_{p,\text{SI}} = 1.1 \times 10^{-5} \) pb, with \( \chi^2_{\text{min}}/\text{dof} = 34.5/34 \). This \( \chi^2_{\text{min}} \) has a \( p \)-value of 0.44 in the expected \( \chi^2 \) distribution and, thus, the data are consistent with a WIMP with SI interactions. A second, local \( \chi^2 \) minimum is found at \( m_\chi = 10.0 \) GeV and \( \sigma_{p,\text{SI}} = 1.5 \times 10^{-4} \) pb with \( \chi^2_{\text{min}}/\text{dof} = 37.0/34 \), also a good fit to the data.
FIG. 1: The average modulation amplitude by energy bin as measured by DAMA/LIBRA (orange boxes). Though not all shown here, measurements extend up to 20 keVee. To improve statistical sensitivity, some of the original bins have been combined (gray boxes): 6 original bins from 4–7 keVee have been combined into 3 bins with one final bin extending from 7-20 keVee (see the text for further discussion). The resulting bins are used for analyses in this paper. Also shown for both sets of binning are the modulation amplitude spectra for the WIMP mass and spin-independent (SI) cross-section that provides the global (solid line) and a local (dashed) chi-squared minima.

The global (solid orange) and local (dashed orange) best-fit spectra are shown in Figure 1 (both curves are obscured by black curves of the same style). While both spectra are a reasonable fit to the current data, they exhibit very different behavior below the current threshold of 2 keVee, hence the interest in a lower threshold analysis, which would allow the two cases to be distinguished. The two spectra differ in that, over the current energy range of 2–20 keVee, the 68 GeV spectrum is primarily due to scattering of WIMPs on iodine nuclei, while the 10 GeV spectrum is primarily due to scattering on sodium nuclei. In both cases, there are far more iodine scatters than sodium scatters; however, in the latter case, those iodine recoils are nearly all below threshold. The relatively large number of expected iodine scatters is apparent in the rapid increase in the amplitude below 2 keVee.

Confidence regions are determined using $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$. In a two parameter space, such as the case of a WIMP with SI-only interactions (where $m_\chi$ and $\sigma_\text{p,SI}$ are the two parameters), the confidence regions at a 90%/3$\sigma$/5$\sigma$ confidence level (CL) are defined as the parameters such that $\Delta \chi^2 \leq 4.61/11.83/28.74$.

A brief aside on the use of regions in comparing experimental results is given here. Due to the various potential positive signals seen in some experiments (DAMA [3], CoGeNT [11, 12], CRESST [13], and CDMS silicon [21]) and the lack of signal in others (XENON10 [19], XENON100 [20] and CDMS germanium [16, 18]), the question often arises of whether these experimental results are compatible with each other in particular WIMP frameworks (e.g. SI scattering assuming the SHM). Ideally, such a question can be answered by performing a
FIG. 2: The regions of the SI cross-section vs. mass parameter space with a goodness-of-fit compatible with the DAMA data at the 90% and $3\sigma$ confidence levels (CL) under the assumption of the standard halo model. Regions are shown using both the original 36 DAMA bins (orange) and the reduced set of 8 bins (gray) used in this analysis. The smaller regions in the latter case indicates the improved statistical sensitivity provided by combining bins; see the text for further discussion. Regions above the short-dashed curve are inconsistent at the 90% CL with the total event rate measured by DAMA. Also shown for comparison is the current exclusion constraint from the XENON100 experiment; regions above/right of the long-dashed curve are inconsistent with the XENON100 result at the 90% CL for the type of coupling (SI) and halo model assumed.

An alternative method for examining compatibility is to identify parameter ranges that give a reasonable $\chi^2$. In the case of 36 bins, those regions at the 90%/3$\sigma$/5$\sigma$ CL are defined by $\chi^2 \leq 47.2/64.1/93.2$; see Figure 2 for these regions in the case of SI-only scattering. We will refer to these as “goodness-of-fit” regions and the $\Delta \chi^2$ regions as confidence regions (though, strictly speaking, the former are also technically confidence regions). The latter has better properties desired in confidence regions,\textsuperscript{1} so it is preferable to use when confidence regions

\textsuperscript{1} If a parameter space contains the true hypothesis, the $\Delta \chi^2$ regions tend to be smaller than the $\chi^2$
TABLE I: Average modulation amplitudes observed by DAMA over the given energy bins. Some bins have been combined from the 36 bins of width 0.5 keVee over 2–20 keVee originally given by DAMA [3] in order to improve the sensitivity of statistical tests.

| Energy  | Average $S_m$ |
|---------|---------------|
| [keVee] | [cpd/kg/keVee] |
| 2.0 - 2.5 | 0.0161 ± 0.0040 |
| 2.5 - 3.0 | 0.0260 ± 0.0044 |
| 3.0 - 3.5 | 0.0220 ± 0.0044 |
| 3.5 - 4.0 | 0.0084 ± 0.0041 |
| 4.0 - 5.0 | 0.0080 ± 0.0024 |
| 5.0 - 6.0 | 0.0065 ± 0.0022 |
| 6.0 - 7.0 | 0.0002 ± 0.0021 |
| 7.0 - 20.0 | 0.0005 ± 0.0006 |

are wanted. For issues of compatibility, only the goodness-of-fit regions can be appropriately interpreted in the manner often desired, and overlap of such regions is a better indicator of what the ideal joint likelihood analysis would produce. For a brief review of statistics, see Ref. [55].

In addition to the modulation, DAMA measures the average total (signal plus background) rate in their detector $R = S_0 + B$, where $B$ is the background rate [56]. As the background rate is unknown, DAMA cannot explicitly determine $S_0$; however, an upper limit can be placed: $S_0 \leq R$. The corresponding constraint in the $\sigma_{p,SI}-m_\chi$ plane is shown in Figure 2 (black short-dashed curve); details of this constraint shown can be found in Ref. [10]. While not entirely excluding either of the two mass regions in Figure 2, the constraint placed on $S_0$ does create some tension in the higher mass region as some of this region (including the best-fit point) would result in the signal rate alone exceeding the measured total rate.\footnote{The tension in the higher mass region was somewhat stronger in the first (4 year [56]) DAMA/LIBRA data release [8,10]. However, the additional two years of data added in the most recent release [2] has shifted the center of the high mass region from $\sim$80 GeV to $\sim$70 GeV and to a lower cross-section, relaxing that tension somewhat.}

As a statistical analysis that combines both the DAMA modulation and total rate data is rather difficult (due to the unknown background), we shall consider only the modulation data for the remainder of the paper, though this tension should be kept in mind.

C. Binning

The statistical significance of any $\chi^2$ goodness-of-fit test based on the 36 DAMA bins will be weakened for two reasons: (1) most of the bins are much smaller than the energy resolution of the detector and (2) a WIMP signal at higher energies will be negligible relative to that at lower energies. Both increase the number of degrees of freedom (and, thus, the level of statistical noise in the $\chi^2$) without necessarily increasing the signal-to-noise. An goodness-of-fit regions, i.e. the former statistic is more powerful at rejecting incorrect parameter values.
improved choice of binning is given in Table [1] along with the corresponding modulation amplitude measurements.

To address the first issue (energy resolution), we have combined adjacent bins that were substantially narrower than the energy resolution at those energies. The resulting bins still have similar or smaller widths than the energy resolution [2] (just not substantially smaller). To address the second issue (negligible signal at high energies), the highest energy bins could all be combined into a single bin.\(^3\) To determine the energy \(E_0\) above which all bins should be combined, we scanned over the entire WIMP mass and SI cross-section parameter space, identifying at each point the choice of \(E_0\) that provides the most significant deviation of the typical experimental result from the null hypothesis (i.e. no modulation). We note that the null hypothesis is used only as an example of an alternative hypothesis (see below). We find that \(E_0 \leq 7\) keVee is the optimal choice for any WIMP mass and cross-section that would provide a signal with between a 5\(\sigma\) and 14\(\sigma\) significance, with \(E_0 \leq 6\) keVee preferred for signals with a significance below 5\(\sigma\). For points where the significance is large (\(\gtrsim 14\sigma\)), keeping additional bins above 7 keVee can allow for an improved comparison with the null hypothesis as the signal in this region, while still relatively small, becomes non-negligible. However, when the signal is non-negligible above 7 keVee, the signal at lower energies is extremely large, so the additional high energy bins remain relatively unimportant. For this reason, we choose to combine all bins above 7 keVee into a single bin.

A word of caution is in order here. The alternative binning is meant only to improve the \(\chi^2\) goodness-of-fit test; other statistical tests involving the \(\chi^2\) may not benefit from the new choice of binning. The goodness-of-fit test is improved by increasing the test’s ability to reject a false WIMP hypothesis when the signal may be due to any number of other plausible unknown WIMP scenarios and/or modulating backgrounds.\(^4\) For a known alternative hypothesis, such as the background-only (no modulation) case, a likelihood ratio analysis can be performed to compare the WIMP and null hypotheses. In this case, the alternative binning provides no benefit and, in fact, can weaken the power of the ratio test, though, in practice, the ratio test is not significantly weakened by the alternate binning here.

\(^3\) Alternatively, the highest bins could simply be dropped. However, many potential background sources of modulation would lead to a modulation signal over a broad range of energies; keeping one wide high energy bin allows the goodness-of-fit test to better exclude a WIMP interpretation of a signal that is due to one of these backgrounds.

\(^4\) There is no choice of binning for the goodness-of-fit that can optimally reject a false hypothesis in light of arbitrary unknown alternative hypotheses (one of which may be correct). This choice of binning aims to improve rejection of false hypotheses for the cases where the signal might be due to an unknown WIMP framework (with possibly different couplings and/or a different halo model than considered here), for which the modulation signal would still be expected to be essentially entirely below 7 keVee, or some modulating background that would have only a broad modulation spectrum above 7 keVee. Less plausible background scenarios, such as one where a large modulation amplitude occurs over only a narrow energy window above 7 keVee, would fare worse in this case due to the joining of high-energy bins that would mask such a feature.
Likewise, parameter interval estimates based on a $\Delta \chi^2$ are not improved by combining bins, though again, in practice, there is little difference for the WIMP models considered here. Since there is little difference, to avoid confusion, we will generate confidence regions using the $\Delta \chi^2$ based upon the 8 bins.

The new binning (gray boxes) as well as the corresponding best-fit modulation amplitude spectra in the case of SI-only scattering (black curves) are shown in Figure 1. The best-fit SI-only scattering parameters occur at $m_\chi = 68.4$ GeV and $\sigma_{p,\text{SI}} = 1.1 \times 10^{-5}$ pb with $\chi^2_{\text{min}}/\text{dof} = 7.3/6$, with a second $\chi^2$ minimum occurring at $m_\chi = 10.1$ GeV and $\sigma_{p,\text{SI}} = 1.5 \times 10^{-4}$ pb with $\chi^2_{\text{min}}/\text{dof} = 9.7/6$. The $p$-values are 0.29 and 0.14, respectively, so both fits are reasonably good even under the new binning. The best-fit parameters and corresponding spectra in the two cases are nearly identical to those found with the original binning (the curves in Figure 1 are almost indistinguishable), an indication that the new binning does not significantly weaken the ability to fit WIMP spectra to the data.

The goodness-of-fit regions derived from the new binning are shown in Figure 2 (gray regions). Here, the improvement in the goodness-of-fit test is evident in the significantly reduced area of the regions relative to the original 36 bin case. For comparison, the XENON100 90% CL exclusion limit is also shown (black long-dashed curve); regions to the right of this curve are excluded. While the two experimental results are incompatible in this scenario (SI-only scattering, SHM halo) for either set of binning, the incompatibility is much worse with the new binning.

### D. Quenching Factor

As discussed previously, a recent measurement of the sodium quenching factor indicated a significantly lower value with a strong energy dependence not observed in previous studies [50]. A smaller quenching factor would push the compatible low mass region towards higher masses. Using this energy-dependent quenching factor yields a best fit mass of 11.4 GeV with $\chi^2 = 63.1$ for 34 degrees of freedom ($p = 1.7 \times 10^{-3}$). Utilizing our more optimized binning scheme yields a best fit point at a mass of 11.4 GeV with $\chi^2 = 38.1$ for 6 degrees of freedom ($p = 1.1 \times 10^{-6}$). We thus find that the low mass scenario is excluded at $3\sigma$ with the original binning, and at nearly $5\sigma$ with the optimized binning scheme. This example illustrates the power of our improved binning for the goodness-of-fit test.

The exclusion of the low mass scenario can be attributed to low energy iodine recoils. For the low mass case, the scattering is primarily from the sodium nuclei for energies above $\sim 2$ keVee, but scattering on iodine dominates the signal at lower energies. In fact, it is scattering on iodine that causes the sharp rise at low energies seen in Figure 1. The fact that this steep rise must occur below $\sim 2$ keVee (given the current data) provides an effective upper limit on the WIMP mass. If the sodium quenching factor is decreased, the best fit mass increases, as does the tension with the absence of the steep rise due to iodine scattering above 2 keVee.

A measurement for the iodine quenching factor of $Q_I = 0.04$ is also presented in Ref. [50]. This value is also significantly lower than the $Q_I = 0.09$ found in other studies [51, 52]. If both of the smaller quenching factors from Ref. [50] are used, then the low mass scenario again becomes marginally compatible. In this case, the best fit mass occurs at 17.8 GeV with $\chi^2 = 13.1$ for 6 degrees of freedom ($p = 0.041$).

The reemergence of the low mass scenario can again be understood in terms of the effective mass upper limit imposed by iodine scattering at low energies. If the iodine quenching factor
is reduced, this effectively acts to increase the upper limit on the WIMP mass due to the
absence in the data of a steep rise at low energies, allowing the best fit point to move towards
higher masses. It is important to note, however, the movement of the low mass region
towards increasing masses serves to further strengthen the tension with the XENON100 and
other exclusion limits.

IV. LOW-ENERGY MODELS AND RESULTS

In this section, we examine how additional low-energy modulation data will impact con-
straints placed by the DAMA results. We first examine the details of measuring the modu-
lation, in particular focusing on the uncertainties in those measurements. We then explore
the simple case of a single additional bin over 1.0–2.0 keVee, which allows us to give a qual-
itative description of what additional low-energy data will provide. We finally turn to the
likely case that data for two additional bins, 1.0–1.5 and 1.5–2.0 keVee, will be provided by
DAMA.

A. Low-energy Measurements

Of importance in identifying the impact of new low-energy modulation measurements
in DAMA is determining the approximate uncertainties in those measurements. Here, we
discuss how the uncertainties can be approximated.

The modulation signal must be extracted from on top of the large average event rate
\( R = S_0 + B \), where \( B \) is the (possibly unknown) rate per unit energy for some non-modulating
background(s). The size of the error bars in the extracted modulation amplitude should
scale with the square root of this total average rate, i.e. a larger rate in a detector leads to a
larger uncertainty in the modulation amplitude measurement. For a simple estimate of this
uncertainty, we use a two time bin analysis relating the modulation amplitude \( S_m \) to the
average rate in the summer minus the average rate in the winter; see Ref. \[31\] for details of
this type of analysis. In this case, the error bar in a bin of size \( \Delta E_{\text{ee}} \) will be given by

\[
\delta S_m = \frac{\pi}{2} \sqrt{\frac{S_0 + B}{\varepsilon(E_{\text{ee}})MT\Delta E_{\text{ee}}}} = \frac{\pi}{2} \frac{(S_0 + B)}{\sqrt{N_T}} \tag{12}
\]

where \( MT \) is the exposure (in mass×time units), \( \varepsilon(E_{\text{ee}}) \) is again the efficiency for detecting a
nuclear recoil, and \( N_T \) is the total number of detected events. The factor of \( \pi/2 \) comes from
averaging the cosine function over the half year time bin. More detailed analyses than our
simple two bin case would lead to a similar error estimate, though with a different leading
numerical factor than \( \frac{\pi}{2} \).

In our analysis, we assume for the new low-energy bins a similar 6 year running time
as with the first DAMA/LIBRA phase, yielding an exposure of 0.87 tonne-years. Existing
measurements above 2 keVee will have improved precision due to this additional exposure,
but we will neglect changes to the data in this energy range and use only the current results.
Due to the \( \frac{1}{\sqrt{MT}} \) scaling and the 1.17 tonne-years of existing exposure, the uncertainties on
these existing modulation measurements would be expected to drop by only \( \sim 25\% \) if the
new exposure were included. In the region from 1 to 2 keVee, the efficiency of the upgraded
detectors is roughly 0.7 \[24\].
As the uncertainties in the low-energy measurements depend on $S_0$ and the unknown $B$, they can not be generically predicted. However, there are indications that the total event rate in the 1.5–2 keVee bin just below the current 2 keVee threshold is similar to the total rate above it (see Fig. 1 of Ref. [56], though the data below 2 keVee should be treated with caution as it is below the analysis threshold). In this case, one can expect $\delta S_m$ in the 1.5–2 keVee bin to be similar to the $\delta S_m = 0.0040$ cpd/kg/keVee (dru) of the existing 2.0–2.5 keVee bin measurement given the similar exposures and total rate in Eqn. (12). Thus, for our fiducial case, we take $\delta S_m$ to be the same 0.0040 dru in the 1.5–2.0 keVee bin. To be conservative, we allow for the total rate to increase in the 1.0–1.5 keVee bin and take $\delta S_m = \sqrt{2} \times 0.0040$ dru there, corresponding to a doubling in the non-efficiency-corrected count rate. For a single bin over 1–2 keVee, this would correspond to $\delta S_m = 0.0035$ dru, nearly the same as the uncertainty in the lowest existing bin, though in the single bin analysis below we allow $\delta S_m$ to vary.

For the two additional low-energy bins analysis, we will consider a few benchmark models consistent with a particular WIMP candidate, in which case $S_0$ can be calculated. For these models, if $\delta S_m$ as determined from Eqn. (12) is larger than our fiducial case, even assuming $B = 0$, we instead use this larger uncertainty.

**B. Single Bin Analysis**

In this section, we examine the addition to the existing DAMA data of a single new low-energy bin over 1–2 keVee. Though DAMA is likely to provide multiple narrower bins over this energy range, a single bin will allow us to examine the qualitative effect low-energy data will have on WIMP constraints and will also allow us to investigate how the uncertainties in the measurements impact a modulation analysis.

The current DAMA data is consistent with two distinct regions in the cross-section vs. mass plane for spin-independent scattering: a low mass region around $\sim 10$ GeV and a high mass region $\sim 70$ GeV (see Fig. 2). As can be observed from Fig. 1, the modulation behavior of WIMPs in these two regions would be very different in the 1–2 keVee energy range. As discussed in Section III B in the low mass region, where higher energy scatterings occur primarily with sodium nuclei, iodine scatterings begin to dominate the signal for energies below 2 keVee. This leads to the very steep rise in the spectrum just below 2 keVee. In the high mass region, on the other hand, the scattering is dominated by iodine over the entire energy range. This leads to the signal turning over at around 2 keVee, corresponding to a phase reversal where the rate is minimized in the summer rather than maximized (see e.g. Refs. [58, 59] for further discussion of the phase reversal). The high mass scenario thus requires a small or even negative amplitude in the 1–2 keVee energy range opened up by the lower threshold.

The very different behavior by the two scenarios at low energy provides the possibility that the upgraded detector could break the degeneracy between the low mass and high mass regions. In Figure 3 we plot the allowed modulation amplitude assuming one new low-energy bin over 1–2 keVee based on a chi-square goodness of fit. The 90% CL goodness-of-fit region for the high mass scenario is shaded in light blue and bounded by the thin, blue, solid lines and the $3\sigma$ region is bounded by the thick, blue, solid lines. In other words, a measurement with an amplitude within the given area is consistent with at least one set of WIMP parameters with the mass within 25–100 GeV. Similarly, the low mass case (5–20 GeV) is shaded in light red and bounded by red, dashed lines. The upper limit
FIG. 3: The allowed modulation amplitude in the 1–2 keVee energy range based on a chi-square goodness of fit. The 90% CL goodness-of-fit region for the high mass scenario is shaded in light blue and is bounded by the thin, blue, solid lines and the $3\sigma$ region is bounded by the thick, blue, solid lines. The low mass case is shaded in light red and bounded by red, dashed lines. The upper limit for the low mass region would be at $\sim 0.2$ ($\sim 0.35$) cpd/kg/keVee for the 90% ($3\sigma$) regions. The green region indicates points excluded at 90% (but consistent at $3\sigma$) for both mass ranges.

for the low mass region is not indicated on the figure as it is at a much higher scale of 0.2 and 0.35 cpd/kg/keVee for the 90% and $3\sigma$ regions, respectively. The horizontal axis shows the size of the error bar in the energy range from 1–2 keVee relative to the size of the error bar in the current lowest energy bin (2–2.5 keVee). This shows how the limits on the modulation amplitude improve as the overall exposure (and thus sensitivity) increases in the lower threshold region.

As indicated by the absence of an overlap between the red and blue shaded regions, we find at the 90% level that the degeneracy between the low and high mass scenarios can be broken, even with error bars in the lower threshold range up to 3.5 times the size of the current low energy error bars. If we instead expand to the $3\sigma$ goodness-of-fit region, the degeneracy can not be completely broken, as these two regions do overlap. This overlap region is shaded light green in Figure 3. In this range, all points in the cross-section vs. mass plane are excluded at 90%. However, there are both low and high mass WIMPs that are marginally consistent with these measurements at the $3\sigma$ level.

Assuming dark matter is the source of DAMA’s modulation, the improved detectors with lower energy threshold will lead to tighter constraints on the dark matter parameters. In Figure 4, we show the regions allowed by the goodness-of-fit with the 90% CL contours denoted by thin lines and the $3\sigma$ contours shown with thick lines. The grey shaded regions are the ranges using the more optimized binning of the current DAMA data described in Section IIIC (see Figure 2). For all of the other cases, we use an error bar in the 1–2 keVee energy range equal to the current lowest energy error bar.

An amplitude measurement of 0.09 keVee (nearly 3.5 times larger than largest current
FIG. 4: The goodness-of-fit regions for an amplitude of 0.09 cpd/kg/keVee (0 cpd/kg/keVee) in the energy range 1–2 keVee which is a good fit in the low (high) mass region is shown in red (blue). An amplitude measurement of 0.035 keVee shown in green, is excluded at 90% CL but marginally allowed at the 3σ level for both low and high masses. The regions with current DAMA data using our more optimized binning are shaded grey. In all cases, the 90% (3σ) contours are shown by thin (thick) lines.

measurement) is the expected value for the low mass best-fit point to the current DAMA data. The resulting allowed regions are shown in red for this low mass case. We see that a measurement near the best fit point in the low mass region would lead to very small region of allowed masses and cross sections. An amplitude measurement of ∼ 0 cpd/kg/keVee would be the expected value for a point that provides a good fit to the current DAMA data in the high mass region. This high mass scenario is indicated in blue, showing the modest improvement in the allowed region by lowering the threshold. If the amplitude is measured to be 0.035 keVee, then the point would fall in the marginally allowed green region of Figure 3. This case is shown here again shaded in light green. As expected, all masses and cross sections for this situation are excluded at the 90% level, but there are allowed regions at both low and high mass at the 3σ level. These regions provide a rather poor fit to the data, however, with the best fit points having χ² = 18 and χ² = 16 (7 degrees of freedom) for the high and low mass regions, respectively.

C. Two Bin Analysis

As discussed previously, the new threshold of the experiment is anticipated to be 1 keVee. If the current binning scheme is continued into the lowered threshold region, then we would expect two additional energy bins: 1.0–1.5 keVee and 1.5–2.0 keVee. The two smaller bins allows for better constraints in the WIMP parameter space relative to the single bin discussed
|                  | Model 1          | Model 2          | Model 3          |
|------------------|------------------|------------------|------------------|
|                  | Average $S_m$ [cpd/kg/keVee] |                  |                  |
| 1.0 - 1.5 keVee  | -0.0042 ± 0.0083 | 0.1484 ± 0.0057  | 0.0219 ± 0.0057  |
| 1.5 - 2.0 keVee  | 0.0062 ± 0.0068  | 0.0258 ± 0.0040  | 0.0260 ± 0.0040  |

**Benchmark WIMP**

|                  | Benchmark WIMP |
|------------------|----------------|
| Mass [GeV]       | 68.4          |
| SI cross-section [pb] | $1.1 \times 10^{-5}$ |

TABLE II: The low-energy (1–2 keVee) pseudo-data for the three models considered in this paper as well as the benchmark WIMP for which the expected spectrum is used to generate the pseudo-data (Models 1 and 2 only). All models also use the 2-20 keVee results given in Table I.

|                  | DAMA       | Model 1      | Model 2      | Model 3      |
|------------------|------------|--------------|--------------|--------------|
| $m_\chi$ [GeV]   | 68.4 (10.1)| 67.3         | 10.2         | 50.8         |
| $\sigma_{p,SI}$ [pb] | $1.1 \times 10^{-5}$ (1.5 $\times 10^{-4}$) | 1.1 $\times 10^{-5}$ | 1.5 $\times 10^{-4}$ | 6.7 $\times 10^{-6}$ |
| $\chi^2_{\text{min}}$/dof | 7.3/6 (9.7/6) | 8.5/8 | 10.2/8 | 14.4/8 |
| spin-dependent, proton-only ($a_n = 0$) |            |              |              |              |
| $m_\chi$ [GeV]   | 10.3 (43.7)| 11.0         | 3.4          | 10.0         |
| $\sigma_{p,SD}$ [pb] | 0.60 (0.43) | 0.50         | 7.1          | 0.62         |
| $\chi^2_{\text{min}}$/dof | 9.5/6 (26.6/6) | 22.8/8 | 91.9/8 | 10.6/8 |
| spin-dependent, neutron-only ($a_p = 0$) |            |              |              |              |
| $m_\chi$ [GeV]   | 10.0 (52.5)| 58.7         | 12.3         | 47.6         |
| $\sigma_{n,SD}$ [pb] | 84. (9.5)  | 10.3         | 77.          | 9.0          |
| $\chi^2_{\text{min}}$/dof | 9.6/6 (10.0/6) | 14.0/8 | 18.0/8 | 11.6/8 |
| spin-dependent, mixed couplings |            |              |              |              |
| $m_\chi$ [GeV]   | 8.3 (52.1)| 58.7         | 9.1          | 9.9          |
| $a_p$            | 12.0 (0.24)| 0.043        | 3.7          | 1.8          |
| $a_n$            | -147. (-6.1)| -5.6        | -60.        | -4.2         |
| $\chi^2_{\text{min}}$/dof | 8.6/5 (9.9/5) | 14.0/7 | 9.6/7 | 10.3/7 |
| spin-independent and spin-dependent |            |              |              |              |
| $m_\chi$ [GeV]   | 67.9 (10.4)| 66.9         | 9.2          | 10.0         |
| $\sigma_{p,SI}$ [pb] | $1.1 \times 10^{-5}$ (0.0) | 1.0 $\times 10^{-5}$ | 0.0 | 0.0 |
| $a_p$            | 0.29 (2.3)| 0.37         | 3.3          | 2.6          |
| $a_n$            | -0.35 (-10.3)| -0.66    | -55.        | -14.         |
| $\chi^2_{\text{min}}$/dof | 7.3/4 (9.5/4) | 8.4/6 | 9.6/6 | 10.3/6 |

TABLE III: The best-fit mass ($m_\chi$), cross-sections ($\sigma_{p,SI}$, $\sigma_{p,SD}$, $\sigma_{n,SD}$), and/or couplings ($a_p$, $a_n$) as well as the minimum chi-square/degrees-of-freedom ($\chi^2_{\text{min}}$/dof) for the various data sets. The “DAMA” column is for the existing DAMA data (see Table I), while the remaining columns are for the three models that extend this existing data down to lower energies (see Table II). In the first case, parameters values at a second (local) $\chi^2$ minimum are given in parentheses.
FIG. 5: (top) The modulation amplitude data for the three models studied: pseudo-data is generated for two new low-energy bins over 1–2 keVee, while actual DAMA results are used for $E_{ee} \geq 2$ keVee. The first bin (1.0–1.5 keVee) of Model 2 is outside the range of this figure. Models 1 (blue) and 2 (red) are consistent with the spectra expected from the high- and low-mass best-fit SI points, respectively, while Model 3 (green) extends the measurements to lower energies assuming a similar modulation amplitude as observed over 2-3 keVee. All pseudo-data includes random fluctuations consistent with the estimated uncertainties. Also shown are the best fit SI spectra in each case (solid lines), as well as the the best-fit spin-dependent (SD) spectra for Model 3 (dashed line). (bottom) The 90% and 3$\sigma$ CL confidence regions in the SI cross-section vs. mass parameter space for the DAMA results (gray) as well as for the three low-energy-extended models shown in the top panel. The standard halo model has been assumed here.
in the previous section. Here, we will examine what those constraints will be for three test cases corresponding to three different behaviors of the low-energy modulation spectrum. We will additionally examine what measurements in these two bins will be consistent with either a light or heavy WIMP.

We define three models representative of the possible low-energy modulation amplitude spectrum behavior: spectra consistent with a heavy WIMP (Model 1) and a light WIMP (Model 2), and a spectrum that assumes the modulation amplitude below the current 2 keVee threshold remains similar to that just above it (Model 3). For the first two models, we choose as our benchmarks WIMPs with masses and SI scattering cross-sections consistent with the two sets of best-fit parameters to the existing DAMA data: the global best fit at a mass of 68 GeV (Model 1) and the local best fit at a mass of 10 GeV (Model 2). The spectra for these two models are the ones previously shown in Figure 1. For Model 3, we assume a purely phenomenological spectra (not based upon any particular WIMP) where \( S_m \) is a constant 0.0210 cpd/kg/keVee below 2 keVee, equal to the average currently observed amplitude over 2.0–3.5 keVee. This is a plausible scenario for many modulating backgrounds, where the amplitude would not necessarily be expected to rise rapidly at low energies nor have its phase reversed.

For each of the three models, we generate pseudo-data in the two new low-energy bins and combine this with the existing measurements above 2 keVee. Pseudo-data is generated by taking the true spectra as described above and adding random fluctuations consistent with the expected measurement uncertainties in each bin. The uncertainties in each bin are as described in Section IV A, noting that the uncertainties in the Model 1 measurements have been increased from the fiducial case due to the larger expected \( S_0 \) in that scenario. This pseudo-data is given in Table II and shown in the upper panel of Figure 5. Note the first bin for Model 2 is outside the range of this figure.

For each set of data, we determine the best-fit WIMP spectra assuming SI-only couplings. Best-fit parameters are given in Table III for each of the models as well as the original DAMA data; best-fit parameters for the local \( \chi^2 \) minima are shown in parenthesis for this last case. These best-fit spectra are shown as the solid lines in the upper panel of Figure 5 with the color of each line matching that of the low-energy bins they are fitting. For Model 1, the best-fit mass is 67 GeV, very close to the benchmark mass of 68 GeV, with \( \chi^2_{\text{min}}/\text{dof} = 8.5/8 \) (\( p = 0.39 \)), a good fit. The best-fit mass for Model 2 is 10.2 GeV, very close to the benchmark mass of 10.1 GeV, with \( \chi^2_{\text{min}}/\text{dof} = 10.2/8 \) (\( p = 0.25 \)), also good fit. Model 3 has a best-fit mass of 51 GeV, but the \( \chi^2_{\text{min}}/\text{dof} = 14.4/8 \) (\( p = 0.07 \)) indicates the fit is not so good in this case, though not poor enough to exclude an SI-only scattering WIMP interpretation of the modulation.

The confidence regions for each of these three data sets, again assuming SI-only interactions, is shown in the lower panel of Figure 5 with the colors matching the corresponding bins and best-fit spectra in the upper panel. The confidence regions for the existing DAMA data are shown in gray, with a region at both low and high mass showing that the current data is insufficient to distinguish between the two masses. The confidence regions for Model 1 (blue, centered at 70 GeV) and Model 2 (red, centered at 10 GeV) each cover

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5 The low-energy data in Models 1 and 2 were generated from the best-fit spectra to the existing \( \geq 2 \) keVee data, but that existing data is reused again as part of the new data set for these models. This results in a correlation in the data that will lead to a lower \( \chi^2_{\text{min}} \) than expected for a true experimental result (where all bins are independent), but the difference here is small and does not significantly alter our conclusions.
FIG. 6: Modulation amplitude values over 1.0–1.5 keVee and 1.5–2.0 keVee that are compatible with light or heavy WIMPs with SI-only interactions. Amplitudes are given in units of cpd/kg/keVee. Red (blue) contours indicate amplitudes for which at least one light (heavy) WIMP, defined as having a mass within 5–20 GeV (25–100 GeV), would provide a goodness-of-fit consistent with the data at the given CL. Amplitudes outside the contours exclude all light (heavy) WIMPs at at least the given CL. Contours are at the 1/2/3/4/5 σ CL; there is no 1 σ contour for the low mass case. We have assumed relatively conservative error bars of 0.0083 and 0.0068 cpd/kg/keVee for the 1.0–1.5 keVee and 1.5–2.0 keVee bins, respectively.

masses only near their benchmark WIMP masses. The additional low-energy data thus has clearly broken the mass degeneracy. This is as expected since the spectra, and thus data, substantially differ below 2 keVee, even though they were similar above 2 keVee. For these particular models, the opposite mass region (e.g. the light mass region for Model 1, the high mass WIMP case) is excluded at at least the 5σ CL. This suggests that, if the DAMA signal is due to a WIMP, low-energy data will almost certainly be able to unambiguously determine if it is a light or heavy WIMP; we shall shortly provide rigorous proof of this. Our phenomenological Model 3 also identifies only a single mass region, being most consistent with a heavier WIMP mass, though the low-energy data falls somewhere between the high- and low-mass WIMP scenarios.

The previous results, based only on one set of pseudo-data each for three specific spectral models, raises two questions. First, given that Model 3 data was generated from an arbitrary (non-WIMP) spectrum but was still consistent with a WIMP interpretation, are there low-energy measurements that could not be reasonably interpreted as due to WIMPs? Second, as discussed already for the single bin analysis, assuming the data is consistent with a WIMP, will the low-energy measurements always distinguish between a light and heavy WIMP mass as was the case in our three examples?

To address these questions, we have scanned over all possible modulation amplitude measurements in the two low-energy bins. For simplicity, we have taken the measurement
FIG. 7: Modulation amplitude values over 1.0–1.5 keVee and 1.5–2.0 keVee that are compatible with different WIMP couplings at the 90%/3σ CL. Amplitudes are given in units of cpd/kg/keVee. Red (blue) contours indicate amplitudes for which at least one WIMP with proton-only (neutron-only) spin-dependent couplings would provide a goodness-of-fit consistent with the data, while filled purple regions indicate measurements consistent with a WIMP of arbitrary (mixed) SD couplings. For comparison, the SI-allowed regions are also shown (thin black contours); these differ from Figure 6 in that a WIMP of arbitrary mass is allowed.

uncertainties in these two bins to be equal to that of Model 1, the model with the largest uncertainties. For each set of amplitude measurements, we minimize the $\chi^2$ of the full data set (two new low-energy bins plus the existing data) over SI-only scattering WIMPs with either light (5–20 GeV) or heavy (25–100 GeV) masses and determine the goodness-of-fit for the $\chi^2_{\text{min}}$. Results are shown in Figure 6. For each set of amplitude measurements within a contour, there exists at least one set of WIMP parameters within one of the two mass ranges that is not excluded at the given CL by the goodness-of-fit. Outside of the contour, all WIMPs over the corresponding mass range would be excluded by at least the given CL. Contours are shown at the 1/2/3/4/5σ CLs for both mass ranges, except there is no 1σ contour for the light WIMP case as no set of low-energy measurements will be consistent with a light WIMP at the 1σ level. The low-mass compatible contours extend up to $\sim$0.4–0.7 cpd/kg/keVee.

The high mass contours (blue) indicate that the low-energy amplitude measurements can be either positive or negative and still be consistent with a heavier WIMP, though the amplitudes should not be too large. Thus, DAMA will not necessarily see the actual phase reversal over 1–2 keVee even if the signal is due to a heavy WIMP. On the other hand, both measurements must be positive to be compatible with a light WIMP at better than the 3σ level. For the low-mass case, the amplitude of the first bin can be quite large, though the amplitude of the second bin should not be substantially larger than the existing > 2 keVee measurements. Comparison of the two sets of contours shows that there is no set of
measurements for which one of the two mass ranges will not be excluded at least the 2.6σ level, indicating that DAMA will always be able to distinguish between the two mass regions, at least at a moderate level. This is a mild improvement over the minimum exclusion level of 2.3σ for a single 1–2 keVee bin, though there are many two-bin measurements that would be excluded at a far higher level than with only the equivalent single-bin measurement. As we have chosen to use the larger uncertainties expected of a heavy WIMP (Model 1) rather than the smaller uncertainties more likely for the light WIMP, the low-mass compatible regions might be overstated and the expected distinction between high and low mass WIMPs could be even stronger.

Up to this point, we have examined WIMPs with only SI interactions. In Table III, we address the possibilities of SD and mixed SI/SD interactions by showing best-fit parameters to the existing DAMA data and to the pseudo-data of our three models for a variety of coupling scenarios. For the existing DAMA data, one finds good fits in both the low and high mass regions for all the different coupling combinations considered (except for the high mass case with SD proton-only scattering), with the lighter mass slightly preferred for SD couplings and the heavier mass slightly preferred for SI and mixed SI/SD couplings. In other words, the current DAMA modulation results do not prefer any particular coupling type.

The situation is remarkably different when the low-energy data in our models is included. For Models 1 and 2, the fits are substantially poorer (relative to the SI case) in the cases of SD proton-only or SD neutron-only couplings, the two cases often used to show direct detection constraints for SD scattering. When both SD couplings are allowed to vary, Model 2 once again finds a set of WIMP parameters that are consistent with the data. Thus, caution should be used when trying to compare experimental results in the context of SD scattering in the future as the customary SD scattering scenarios (proton-only or neutron-only) may fail to give the full picture. For Model 3, SD scattering gives an improvement in the fits over the SI case. The best-fit mixed SD coupling case gives a spectrum as shown by the green dashed curve in the upper panel of Figure 5. The significant decrease in the χ² suggests that this SD spectrum is preferred to the SI spectrum (solid green), though quantifying the level to which SD is preferred requires a statistical analysis beyond the scope of this paper.

While the previous examined SD scattering for only three particular pseudo results, the general SD case is shown in Figure 7. The red (blue) contours indicate measurements for which at least one WIMP mass and SD proton-only (neutron-only) cross-section is compatible with the data at the 90%/3σ CL, again assuming the same uncertainties as in Model 1. The filled purple regions indicate the same compatibility, but allowing for an arbitrary set of mixed SD couplings. For comparison, the SI-only case is shown by the thin black contours; these contours allow for an arbitrary WIMP mass, so they contain both the low-mass and high-mass regions shown separately in Figure 6. All four 90% CL regions overlap in an area of the figure where the new measurements are positive but comparable or smaller in magnitude than the current 2–3 keVee measurement of ∼ 0.02 cpd/kg/keVee. For measurements in this area, DAMA will be unable to exclude any type of SI or SD coupling. However, there are measurements, notably for some negative values, for which SI scattering is consistent, but all SD scattering is excluded; measurements here will thus be able to identify the type of coupling. On the other hand, measurements consistent with SD proton-only or SD neutron-only scattering will generally also be consistent with SI scattering. Only the SD mixed coupling case allows for measurements inconsistent with SI scattering, primarily at very high amplitudes for the 1.0–1.5 keVee bin with small, positive amplitudes for the 1.5–2.0 keVee bin. One interesting feature of this plot is that the SD proton-only contours
encompass only a relatively small region, meaning this particular coupling scenario could very easily be fully excluded by future low-energy DAMA measurements.

It is clear that the lower threshold may allow DAMA to distinguish between SI or SD interactions as the source of their modulation, although not to a high significance level. The reason why this may be possible is that the relative contributions to the scattering from sodium and iodine are different under the different couplings. This can be seen by comparing the Model 2 SI spectrum with the Model 3 SD spectrum in Figure 5. These two spectra occur at nearly identical WIMP masses (10.2 vs. 9.9 GeV). The shape of each nucleus’ contribution to the modulation spectrum is similar in both cases, but the amplitudes greatly differ. Most of the events above 2 keVee are due to sodium recoils and, thus, the spectra at those energies are similar. The iodine recoils occur mainly below 2 keVee but, having fixed the appropriate cross-sections to get the sodium scatters to match the data above 2 keVee, the total iodine scattering rates differ by $O(100)$ in the two cases, with SI interactions giving a much larger iodine scattering rate. The result is that the contribution from iodine dominates over that from sodium at $\sim 2$ keVee in the SI case, but not until a much lower $\sim 1$ keVee in the SD case.

While there are regions in Figure 7 that indicate only one of SI or SD couplings is compatible, in which case the data will clearly identify which coupling type is responsible for the DAMA modulation, there are regions where both SI and SD couplings are compatible, in which case the type of coupling producing the signal may be less apparent. However, even if a measurement is compatible with both coupling types, a hypothesis test may still indicate a strong preference for one coupling type over another. As neither of the SI and SD parameter spaces is a subset of the other, the simple likelihood ratio hypothesis test is inapplicable here. Hypothesis tests appropriate for this analysis are more difficult to implement and computationally intensive and, as such, are not performed here.

Figure 7 shows that there are low-energy DAMA measurements for which neither SI nor SD couplings would provide a good fit. Thus, new measurements could rule out the dark matter interpretation of the DAMA modulation, at least for the standard case considered here (non-standard WIMPs, such as velocity-dependent or momentum-dependent WIMPs, might still be of interest; see e.g. Refs. [60, 61]). However, one or both new low-energy bin amplitude measurements must substantially differ from the existing $> 2$ keVee measurements. A background that provides a modulation amplitude that does not rapidly change as the threshold is lowered is unlikely to produce results incompatible with a dark matter interpretation.

Finally, we have shown that the low and high mass WIMP cases will be clearly distinguished for SI scattering. Does the same hold true for SD interactions? Figure 7 does not directly answer that question as contours are not shown separately for the two mass ranges as was the case for SI scattering in Figure 6. There is some indication from the SD neutron-only 90% CL contour that the two mass possibilities may be distinguished in the SD case as well: this contour is nearly separated into two regions, with the left region measurements that are best fit by light WIMPs and the right region measurements that are best fit by heavy WIMPs (here, again, light and heavy refer to $\sim 10$ and $\sim 70$ GeV). By performing separate low and high mass scans, we find that, for SD neutron-only interactions, at least one of the two mass ranges will be excluded at minimum at the 89% CL. The SD proton-only case has already broken the mass degeneracy: even without new low-energy measurements, the high mass scenario is excluded at the 4σ CL, while a low mass WIMP is a good fit to the existing data. That remains unchanged with a lower threshold, which is why the SD
proton-only region is so small compared to other couplings in Figure 7. The two masses cannot be generically expected to be distinguished in the SD mixed coupling case as there are measurements for which both masses provide very good fits. Keep in mind, however, that this is a worst case scenario. While it is possible to have measurements that are good fits to both mass ranges, it is likely that the measurements will result in a poor fit for one of the two masses. For example, the measurements that provide the best low (high) mass fit will exclude the high (low) mass region at the $3\sigma$ ($2.9\sigma$) CL. Furthermore, recall that these calculations use conservative error estimates, so the ability to distinguish the low and high mass regions may be stronger.

V. CONCLUSIONS

Recent interest in light dark matter has increased significantly due to excess events observed in the CoGeNT, CRESST, and CDMS experiments. The anomalies in each of these experiments have been interpreted as evidence for spin-independent, elastic scattering of low mass WIMPs. Although the cross section implied by the DAMA modulation is larger than that indicated by the other experiments, it is very intriguing that they are all pointing towards a similar mass range. Assuming similar sized error bars, we find this low mass dark matter interpretation of the DAMA modulation would require the amplitude in a single energy bin from 1–2 keVee to be above 0.026 cpd/kg/keVee at $3\sigma$, which is larger than the signal measured in any bin thus far. At the 90% level, this lower limit would grow to 0.046 cpd/kg/keVee, or nearly twice the size of any of the values measured thus far.

Additionally, even with only a single bin from 1–2 keVee, the degeneracy between the low mass and high mass regions will be removed at a minimum 90% confidence level no matter what value is measured. If the measured value is closer to one of the best-fit points in the low or high mass region, then the exclusion of the other region would grow considerably, to greater than $3\sigma$. If the data is reported using 2 bins from 1–2 keVee (as expected by the current binning scheme), even with conservative estimates for the uncertainties, this degeneracy will by removed at by at least the $2.6\sigma$ level. Again, if the measured values are closer to one of the best-fit points in the low or high mass regions, then the exclusion level of the other region would strengthen to greater than $5\sigma$. The ability to discriminate so strongly between the two masses has been made possible by selecting a binning scheme that improves the DAMA $\chi^2$ goodness-of-fit test, as discussed in Section IIIIC.

We used the SHM in our analysis but our results are qualitatively robust against different assumptions regarding the velocity distribution of the diffuse dark matter halo. Any (plausible) halo distribution must be able fit the existing DAMA modulation results, which is possible in two cases: a heavier WIMP where iodine recoils dominate the 2–5 keVee signal region, and a lighter WIMP where sodium recoils dominate this signal region. In the latter case, a large iodine signal is unavoidably present somewhere below the 2 keVee threshold, particularly with the $A^2$ enhancement in SI scattering. This iodine signal should become apparent when the DAMA threshold is lowered, providing a means to discriminate between the two mass regions. This qualitative picture should hold for any reasonable background halo distribution; only the quantitative picture should differ (though the two mass regions are unlikely to stray far from the $\sim 10$ GeV and $\sim 70$ GeV regions of the SHM).

Although this work focused on the DAMA/LIBRA data, several features are very relevant to any direct detection experiment seeking to measure the annual modulation. For a detector with a single element, lowering the threshold into the region where the modulation
spectrum turns over and becomes negative would be very strong evidence for the dark matter interpretation of the data. It is quite difficult to imagine a background with a spectrum that undergoes such a phase reversal. For a detector with more than one element, there will always be a degeneracy at higher energies between lighter WIMPs primarily scattering from the lighter element(s) and heavier WIMPs primarily scattering from the heavier element(s). This effect can be observed with the unmodulated signal (as in the CRESST regions) or in the modulation signal (as in the DAMA/LIBRA regions). Once the energy threshold is lowered far enough, however, these two scenarios make very different predictions about the behavior of the annual modulation signal. The annual modulation signal thus provides the opportunity to break the degeneracy between the low and high mass regions. Furthermore, since the relative scattering contribution from each nucleus differs between SI and SD couplings, the modulation signal might also discriminate between these two coupling possibilities with a sufficiently low threshold.

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