Uncertainties in the measurements of the Inclusive Jet Cross Section at the Tevatron

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Abstract

The systematic uncertainties of the measurements of the Inclusive Jet Cross Section at the Tevatron and their effect on the $\chi^2$ comparisons between data and theoretical predictions are discussed.

I. INTRODUCTION

The inclusive jet cross section in $\bar{p}p$ collisions has recently been measured by the CDF [1] and DØ [2] collaborations. These measurements are compared with NLO perturbative QCD predictions [3,4]. These experimental measurements have uncertainties that are smaller than the uncertainties of the theoretical predictions, $\sim$30% [5].

The CDF measurement of the inclusive jet cross section showed an excess of jet production at high transverse energy ($E_T$) which could be caused by new physics such as quark compositeness, inaccuracies in the parton distribution functions, or inadequacies in the NLO QCD predictions. The theoretical predictions are in good agreement with the DØ measurement. Both experimental measurements are also in agreement [6]. Our ability to compare quantitatively the theoretical predictions and the measurements depends on a thorough understanding of the systematic uncertainties.

II. SYSTEMATIC UNCERTAINTIES

The major components of the systematic uncertainties of the CDF measurement are depicted in Fig. 1. The dominant uncertainties are due to the jet energy scale correction, the resolution unsmearing, and the integrated luminosity. The uncertainties are divided up into different components (see Fig. 1). Each component is assumed to be 100% correlated as a function of $E_T$ and independent of all other components.

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Similarly, the five largest uncertainties in the DØ measurement are depicted in Fig. 2. The uncertainties are either 100% correlated, partially correlated (correlation lies between −100% to 100%), or uncorrelated as a function of $E_T$.

In general the uncertainties in the jet energy scale, jet energy resolutions, the luminosity, etc., are assumed to be Gaussian. Hence, the uncertainties of the cross section are asymmetric, i.e. the positive and negative errors on the cross section are different. This is a direct result of the steeply falling inclusive jet cross section.

The assumption of Gaussian uncertainties is not always a valid one. One of the major sources of uncertainty in the inclusive jet cross section is the integrated luminosity. The two experiments base their luminosity calculations on different measurements of the total $\bar{p}p$ cross section. CDF uses its own measurement \[7\] while DØ uses a world average cross section \[8\] based on the CDF \[7\] and E710 \[9\] measurements. This leads to a 7% difference between the luminosities quoted by the two experiments. The assumption that the uncertainty due to the luminosity is Gaussian in nature is probably incorrect.

### III. QUANTITATIVE COMPARISONS

DØ has made quantitative comparisons \[2\] between theoretical predictions and their measurement base on a $\chi^2$ test. The $\chi^2$ is given by

$$\chi^2 = \sum_{i,j} \delta_i V_{ij}^{-1} \delta_j$$  \hspace{1cm} (1)$$

where $\delta_i$ is the difference between the data and theory for a given $E_T$ bin, and $V_{ij}$ is element $i, j$ of the covariance matrix.
\[ V_{ij} = \rho_{ij} \cdot \Delta \sigma_i \cdot \Delta \sigma_j. \]  

(2)

where \( \Delta \sigma \) is the sum of the systematic uncertainty and the statistical error added in quadrature if \( i = j \) and the systematic uncertainty if \( i \neq j \). \( \rho_{ij} \) is the correlation between the uncertainties of two \( E_T \) bins.

The construction of the covariance matrix requires that the uncertainties follow a Gaussian distribution. Hence using \( \chi^2 \) to determine the probability that a theoretical prediction agrees with a measurement does not take advantage of all the information available. Additionally it does not take into account boundary conditions (for example you cannot fluctuate a cross section more than 100\% below its value).

Parton distribution fits [10] are now using the measurements of the inclusive jet cross section to constrain the gluon distributions. If the information available in the inclusive jet measurements is to be used to best effect then new methods must be developed to calculate the probability that a theoretical prediction agrees with the data.

**IV. CONCLUSION**

The treatment of the systematic uncertainties of the inclusive jet cross section in \( \bar{p}p \) collisions have been discussed. The \( \chi^2 \) values presented in [2] are based on approximations of the uncertainties and do not use all of the information available.

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