Semileptonic $B \to \rho$ and $B \to a_1$ transitions in a quark-meson model

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Abstract
We evaluate the form factors governing the exclusive decays $B \to \rho \ell \nu$, $B \to a_1 \ell \nu$, by using an effective quark-meson lagrangian. The model is based on meson-quark interactions, and the computation of the mesonic transition amplitudes is performed by considering diagrams with heavy mesons attached to loops containing heavy and light constituent quarks. This approach was successfully employed to compute the Isgur-Wise form factors and other hadronic observables for negative and positive parity heavy mesons and is presently used for exclusive heavy-to-light weak transitions. We also evaluate a few strong coupling constants appearing in chiral effective lagrangians for heavy and light mesons.

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We evaluate the form factors governing the exclusive decays $B \to \rho \ell \nu$, $B \to a_1 \ell \nu$, by using an effective quark-meson lagrangian. The model is based on meson-quark interactions, and the computation of the mesonic transition amplitudes is performed by considering diagrams with heavy mesons attached to loops containing heavy and light constituent quarks. This approach was successfully employed to compute the Isgur-Wise form factors and other hadronic observables for negative and positive parity heavy mesons and is presently used for exclusive heavy-to-light weak transitions. We also evaluate a few strong coupling constants appearing in chiral effective lagrangians for heavy and light mesons.

I. INTRODUCTION

The impressive experimental programme for the study of $B$ decays carried out in recent years has improved our knowledge of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and CP violations. In the next few years still more abundant data is to come, especially from the dedicated B-factories Belle [1] and BaBar [2].

One of the most important goals in these investigations will be a more precise determination of the CKM matrix element $V_{ub}$ and, to this end, both exclusive and inclusive $b \to u$ semileptonic transitions will be used. The two methods have their own uncertainties. Using the inclusive reaction implies the need to use perturbative QCD methods in the region near the end-point of the lepton spectrum, where many resonances are present and perturbative methods are less reliable. This difficulty can be avoided by considering exclusive channels and summing them up or taking them separately; however the use of the exclusive channels forces to evaluate the hadronic matrix elements by non perturbative methods that are either approximate or model-dependent. Examples of these approximations are given by non perturbative methods derived from Quantum-Chromo-Dynamics (QCD) first principles, i.e. Lattice QCD and QCD Sum rules. The drawback of these methods is the difficulty to improve the precision and to evaluate reliably the theoretical error, which follows from the nature of the approximations, i.e. the quenched approximation in Lattice QCD and a truncated Operator Product Expansion in QCD Sum rules. Although less fundamental, other approaches can be reliably used to give estimates of the hadronic matrix elements that appear in exclusive $b \to u$ transitions and we refer here to constituent quark models. At the present stage of our understanding of the hadronic interactions from first principles, they offer in our opinion a viable alternative, and the model dependence, which is obvious in this approach, can be used to estimate the overall theoretical uncertainty. In this paper we shall follow this route and use a Constituent Quark Meson (CQM) model, introduced and defined in I (hereafter referred to as I) to compute two semileptonic exclusive heavy to light decays, viz. $B$ to the light vector mesons $\rho$ and $a_1$. The first decay has been recently observed by the CLEO collaboration [4] (see also [5]) that has measured the branching ratio of the semileptonic decay $B \to \rho \ell \nu$:

$$B(B^0 \to \rho^- \ell^+ \nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}.$$  \hspace*{1cm} (1)

This decay will be used as a test of the model of Ref. I, since we do not introduce here any new parameter. On the other hand, $B \to a_1 \ell \nu$ is a prediction of this model yet to be tested by the experiment.

The CQM model developed in I tries to conjugate the simplicity of an effective lagrangian encoding the symmetries of the problem together with some dynamical information coming from the observed facts of confinement and chiral symmetry breaking. In spite of the simple way in which the dynamics is implemented, the results found in I are encouraging. We discussed there the following issues: leptonic constants for heavy mesons, Isgur-Wise form factors...
for heavy mesons of both negative and positive parity, strong coupling constants of heavy mesons and pions, radiative decays of $D^*$, $B^*$; the comparison with data was satisfactory whenever it was available.

The plan of the present paper is as follows. The model is briefly reviewed in Section 2. In Section 3 we compute the direct contribution to the form factors for the $B \to \rho \ell \nu$, $B \to a_1 \ell \nu$ decays, i.e. the contribution arising from diagrams where the weak current directly interacts with the quarks belonging to heavy and light mesons. In Section 4 we compute the strong coupling constants of $\rho$ and $a_1$ to heavy mesons: these couplings are relevant for the calculation of the polar diagrams, i.e. the diagrams where the weak current couples to $B$ and $\rho$ (or $a_1$) via an intermediate particle. These contributions (called polar contributions) are described in Section 5. In Section 6 we present our results and compare them with other approaches and with available data. In Section 7 we draw our conclusions and in the Appendix we collect formulas and integrals used in the calculations.

II. THE CQM MODEL

We begin with a short summary of the CQM model; for a more detailed treatment see I. The model is an effective field theory containing a quark-meson lagrangian:

$$\mathcal{L} = \mathcal{L}_{\ell\ell} + \mathcal{L}_{h\ell}. \tag{2}$$

The first term involves only the light degrees of freedom (a constituent quark model for light quarks and mesons was originally suggested by Georgi and Manohar [6]). To the fields considered in I, i.e. the light quark field $\chi$ and the pseudo-scalar $SU(3)$ octet of mesons $\pi$, we add the vector meson and axial vector octets $\rho_{\mu}$ and $a_{\mu}$. Considering only the kinetic part of the light quarks and mesons as well as the quark-meson interactions at the lowest order, we have for $\mathcal{L}_{\ell\ell}$:

$$\mathcal{L}_{\ell\ell} = \frac{f_\pi^2}{8} \partial_\mu \Sigma \Sigma^\dagger + \frac{1}{2g_\Sigma^2} \text{tr}[F(\rho)_{\mu\nu}F(\rho)^{\mu\nu}] + \frac{1}{2g_a^2} \text{tr}[F(a)_{\mu\nu}F(a)^{\mu\nu}]$$

$$+ \bar{\chi}(iD_{\mu}\gamma_{\mu} - m)\chi$$

$$+ \bar{\chi}(h_{\rho}A^\mu\gamma_5 - ih_{\rho}\gamma_{\mu}\gamma_5 - ih_a\gamma_{\mu}\gamma_5)\chi. \tag{3}$$

Let us discuss the various terms in this equation. The first three terms refer to pions, light vector and axial vector respectively. We have $\xi = \exp(i\pi/f_\pi)$, $\Sigma = \xi^2$, $f_\pi = 132$ MeV; the $\rho$ and $a_1$ field strengths are given by

$$F(x)_{\mu\nu} = \partial_\mu x_\nu - \partial_\nu x_\mu + [x_\mu, x_\nu], \tag{4}$$

where, consistently with the notations of [6] (see also [8]), we write

$$\rho_{\mu} = i\frac{g_\Sigma}{\sqrt{2}}\hat{\rho}_{\mu}, \quad g_\Sigma = \frac{m_a}{f_\pi} \approx 5.8. \tag{5}$$

By analogy we also write ($m_a \approx 1.26$ GeV is axial vector meson mass):

$$a_\mu = i\frac{g_A}{\sqrt{2}}\hat{a}_{\mu}, \quad g_A = \frac{m_a}{f_\pi} \approx 9.5. \tag{6}$$

Here $\hat{\rho}$, $\hat{a}$ are hermitean $3 \times 3$ matrices of the negative and positive parity light vector mesons. The fourth term in Eq. (3) contains the light quarks, with $D_{\mu} = \partial_{\mu} - i\gamma_{\mu}$ and

$$V_{\mu} = \frac{1}{2}(\xi^\dagger\partial_{\mu}\xi + \xi\partial_{\mu}\xi^\dagger). \tag{7}$$

Therefore it gives both the kinetic term of the light quark and its coupling to an even number of pions. For $m$, in I we took the value $m = 300$ MeV (for non-strange quarks). The last three terms describe further interactions between light quarks and light mesons. The coupling of the light quark to an odd number of pions is mediated by

$$A_{\mu} = i\frac{1}{2}(\xi^\dagger\partial_{\mu}\xi - \xi\partial_{\mu}\xi^\dagger). \tag{8}$$

Moreover, consistently with a low energy theorem for pions, we put $h_{\pi} = 1$. Concerning the interactions of vector particles, we put $h_{\rho} = \frac{\sqrt{2}m^2_{\rho}}{g_{\rho}f_{\rho}}$, $h_a = \frac{\sqrt{2}m^2_a}{g_{A}f_a}$, where $f_{\rho}$ and $f_a$ are the leptonic constants. For the $\rho$ leptonic constant we
use $f_a = 0.152 \text{GeV}^2$, as given by $\rho^0$, $\omega$ decay into $e^+e^-$. For $f_a$, a phenomenological determination using $\tau \rightarrow \nu_\tau \pi^+\pi^-\pi^-$ was obtained in [3], i.e. $f_a = 0.25 \pm 0.02 \text{GeV}^2$, a result which agrees with the one found by QCD sum rules [14]. On the other hand from lattice QCD one obtains $f_a = 0.30 \pm 0.03 \text{GeV}^2$ [3]. Since $1/f_a$ multiplies all the amplitudes involving the $a_1$ meson, the uncertainty on $f_a$ will induce a normalization uncertainty on all the amplitudes involving the light axial-vector meson. We note that our choice for $h_\rho$ and $h_\alpha$ implements the hypothesis of the Vector and Axial-Vector Meson Dominance. Numerically we find:

$$h_\rho \approx h_\alpha \approx 0.95.$$  (9)

We also observe that our choice for the normalization of the light axial vector meson field Eq.(3) is conventional since $g_A$ disappears from the physical quantities (in [3] $g_A = g_V$ is assumed). We also differ from the phenomenological analyses of Ref. [3] since we do not assume the current algebra relations $m_a^2 = 2m_\rho^2$ and $f_a = f_\rho$ that seem to have substantial violations.

Let us now discuss $L_{hL}$, i.e. the part of the lagrangian that contains both light and heavy degrees of freedom, in particular the heavy quark ($Q$) and mesons ($Q\bar{q}$). According to the Heavy Quark Effective Theory (HQET) [12], in the limit $m_Q \rightarrow \infty$, these mesons can be organized in spin-parity multiplets. We shall consider here the negative parity spin doublet ($P, P^*$) (e.g. $B$ and $B^*$) and its extension to $P$-waves, i.e. the doublet containing the $0^+$ and $1^+$ degenerate states ($P_0, P_1^+$). Incidentally, we note that HQET predicts another low-lying multiplet, comprising two degenerate states with $1^+$ and $2^+$ which is of no interest here. In matrix notations these fields can be represented by two $4 \times 4$ Dirac matrices $H$ and $S$, with one spinor index for the heavy quark and the other for the light degrees of freedom. An explicit matrix representation is, for negative parity states:

$$H = \frac{(1 + \gamma^5)\hat{P}^\dagger}{2} [P_\mu^*\gamma^\mu - P\gamma^5]$$  (10)

($\hat{H} = \gamma_0 H^\dagger\gamma_0$), whereas, for positive parity states:

$$S = \frac{(1 + \gamma^5)\hat{P}}{2} [P_\mu^*\gamma^\mu \gamma_5 - P_0] .$$  (11)

In these equations $v$ is the heavy meson velocity, $v^\mu P_\mu^* = v^\mu P_\mu^* = 0$; $P^{*\mu}$, $P^{\mu}$, $P_\mu^*$ and $P_0$ are annihilation operators normalized as follows:

$$\langle 0 | P | Q\bar{q}(0^-) \rangle = \sqrt{M_H}$$  (12)

$$\langle 0 | P^{*\mu} | Q\bar{q}(1^-) \rangle = e^\mu \sqrt{M_H} ,$$  (13)

with similar equations for the positive parity states ($M_H = M_P = M_{P^*}$ is the common mass in the $H$ multiplet). With these notations the heavy-light interaction lagrangian is written as follows:

$$L_{hL} = \bar{Q}_\nu iv \cdot \partial Q_\nu - (\bar{\chi}(H + S)Q_\nu + h.c.)$$

$$+ \frac{1}{2G_3} \text{Tr}[(\hat{H} + \hat{S})(\hat{H} - \hat{S})] ,$$  (14)

where $Q_\nu$ is the effective heavy quark field of HQET and we have assumed that the fields $H$ and $S$ have the same coupling to the quarks, which is a dynamical assumption based on a simplicity criterion (in I we justify it on the basis of a four quark Nambu-Jona Lasinio interaction by partial bosonization [3]). After renormalization of the heavy fields $H$ and $S$ [3] one obtains the kinetic part of the heavy meson lagrangian in a form that is standard for heavy meson effective chiral theories [3]:

$$L_{hL} = \text{Tr} \tilde{H}(iv \cdot \partial - \Delta_H)\tilde{H} + \text{Tr} \tilde{S}(iv \cdot \partial - \Delta_S)\tilde{S} .$$  

(15)

Here $\Delta_H$ and $\Delta_S$ are the mass difference between the meson and the heavy quark at the lowest order; typical values considered in I are $\Delta_H = 0.3 - 0.5 \text{GeV}$. $\Delta_H$ and $\Delta_S$ are related: for example, for $\Delta_H = 0.4 \text{GeV}$ one obtains the value $\Delta_S = 0.590 \text{GeV}$ [3]. These values correspond to a value for the $S-$multiplet mass $m = 2165 \pm 50 \text{MeV}$; these states, called in the literature (for the charmed case) $D_0, D_1^*$, have not been observed yet, since they are expected to be rather broad. $\hat{H}$ and $\hat{S}$ are the renormalized fields and are given in terms of the bare fields $H, S$ by

$$\hat{H} = \frac{H}{\sqrt{Z_H}}$$  (16)

$$\hat{S} = \frac{S}{\sqrt{Z_S}} .$$  (17)

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$Z_H$, $Z_S$ are renormalization constants that have been computed in [3] with the results (the integral $I_3$ can be found in the Appendix):

$$Z_H^{-1} = \left[ (\Delta_H + m) \frac{\partial I_3(\Delta_H)}{\partial \Delta_H} + I_3(\Delta_H) \right],$$

$$Z_S^{-1} = \left[ (\Delta_S - m) \frac{\partial I_3(\Delta_S)}{\partial \Delta_S} + I_3(\Delta_S) \right],$$

where $m$ is the constituent light quark mass.

Let us finally discuss the way to compute the quark-meson loops arising from the previous lagrangian. As we have seen, the CQM model describes the interactions in terms of effective vertices between a light quark, a heavy quark and a heavy meson (Eq. (14)). We describe the heavy quarks and heavy mesons consistently with HQET; for example the heavy quark propagator is given by

$$\frac{i}{\not{v} \cdot k + \Delta},$$

where $\Delta$ is the difference between the heavy meson and heavy quark mass and $k$ is the residual momentum arising from the interaction with the light degrees of freedom.

The light quark momentum is equal to the integrated loop momentum. It is therefore natural to assume an ultraviolet cut-off on the loop momentum of the order of the scale at which the chiral symmetry is broken, i.e. $\Lambda \simeq 1$ GeV (in I we assumed the value $\Lambda = 1.25$ GeV). Since the residual momentum of the heavy quark does not exceed few units of $\Lambda_{QCD}$ in the effective theory, imposing such a cut-off does not cut any significant range of ultraviolet frequencies. We also observe that the value of the ultraviolet cut-off $\Lambda$ is independent of the heavy quark mass, since it does not appear in the effective lagrangian.

Concerning the infrared behavior, the model is not confining and thus its range of validity cannot be extended below energies of the order of $\Lambda_{QCD}$. In order to drop the unknown confinement part of the quark interaction one introduces an infrared cut-off $\mu$. These parameters appear in the regularized amplitudes; as discussed in [3] (see also [4]) we have chosen a proper time regularization; the regularized form for the light quark propagator (including integration over momenta) is

$$\int d^4k \frac{1}{k^2 + m^2} \rightarrow \int d^4k \int_{1/\mu^2}^{1/\Lambda^2} ds e^{-s(k^2 + m^2)},$$

where $\mu$ and $\Lambda$ are the infrared and ultraviolet cut-offs. For $\mu$ in I we assumed the value: $\mu = 300$ MeV. For a different choice of the cut-off prescription in related models see [5], [6].

### III. $B \rightarrow \rho$ AND $B \rightarrow A_1$ FORM FACTORS: EVALUATION OF THE DIRECT CONTRIBUTIONS.

The form factors for the semileptonic decays $B \rightarrow \rho \ell \nu$ and $B \rightarrow A_1 \ell \nu$ can be written as follows ($q = p - p'$):

$$< \rho^+(\epsilon(\lambda), p') | \bar{B}(0) | \not{\rho}(1 - \gamma_5)b > = \frac{2V(q^2)}{m_B + m_\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon^* \nu^\rho \alpha \beta$$

$$- i\epsilon^\mu(m_B + m_\rho) A_1(q^2)$$

$$+ i\epsilon^* \cdot q \frac{(p + p')_\mu}{m_B + m_\rho} A_2(q^2)$$

$$+ i\epsilon^* \cdot q \frac{2m_\rho}{q^2} q^\mu [A_3(q^2) - A_0(q^2)],$$

where

$$A_3(q^2) = \frac{m_B + m_\rho}{2m_\rho} A_1(q^2) - \frac{m_B - m_\rho}{2m_\rho} A_2(q^2),$$

and
\[
< a_1^+ (\epsilon(\lambda), p') | q \gamma_\mu (1 - \gamma_5) b | B(p) > = \frac{2A(q^2)}{m_B + m_a} \epsilon^{\mu \nu \alpha \beta} \epsilon^{* \nu \rho \sigma} p^\rho p^\sigma \\
- i \epsilon^*_\mu (m_B + m_a) V_1(q^2) \\
+ i (\epsilon^* \cdot q) \frac{(p + p')_\mu}{m_B + m_a} V_2(q^2) \\
+ i (\epsilon^* \cdot q) \frac{2m_a}{q^2} q_\mu [V_3(q^2) - V_0(q^2)] ,
\]

where \( m_a \) is the \( a_1 \) mass and

\[
V_3(q^2) = \frac{m_B + m_a}{2m_a} V_1(q^2) - \frac{m_B - m_a}{2m_a} V_2(q^2) ,
\]

We note that, having used this parameterization for the weak matrix elements [17], at \( q^2 = 0 \) the following conditions hold

\[
A_3(0) = A_0(0) \tag{26}
\]

\[
V_3(0) = V_0(0) \tag{27}
\]

The contribution we consider in this section arises from diagrams where the weak current couples directly to the quarks belonging to the light and heavy mesons (see Fig. 1).

These diagrams are computed using the rules described in the previous Section. The results of a straightforward, but lengthy calculation are as follows:

\[
V_D(q^2) = - \frac{m_a^2}{f_\rho} \sqrt{\frac{Z_H}{m_B}} (\Omega_1 - mZ) (m_B + m_\rho) \tag{28}
\]

\[
A_1^D(q^2) = \frac{2m_a^2}{f_\rho} \sqrt{\frac{Z_H m_B}{m_B + m_\rho}} \frac{1}{m_B + m_\rho} [ (m^2 + m_\rho \bar{\omega}) Z - \bar{\omega} m_\rho \Omega_1 - m_\rho \Omega_2 - 2 \Omega_3 - \Omega_4 - \Omega_5 - 2 \bar{\omega} \Omega_6 ] \tag{29}
\]

\[
A_2^D(q^2) = \frac{m_a^2}{f_\rho} \sqrt{\frac{Z_H}{m_B}} \left[ \Omega_1 \left( m_\rho \bar{\omega} + 2 m \frac{q^2}{m_B^2} - \frac{r_1}{m_B} \right) + m_\rho \Omega_2 + 2 \Omega_3 + \Omega_4 (1 - \frac{2 q^2}{m_B^2}) + \Omega_5 + 2 \Omega_6 \left( \bar{\omega} - \frac{r_1}{m_B m_\rho} \right) \right] \tag{30}
\]

\[
A_0^D(q^2) = - \frac{m_a^2}{f_\rho} \sqrt{\frac{Z_H}{m_B}} \left[ \Omega_1 \left( m_\rho \bar{\omega} + 2 m \frac{q^2}{m_B^2} - \frac{r_1}{m_B} \right) + m_\rho \Omega_2 + 2 \Omega_3 + \Omega_4 (1 - \frac{2 q^2}{m_B^2}) + \Omega_5 + 2 \Omega_6 \left( \bar{\omega} - \frac{r_1}{m_B m_\rho} \right) \right] \tag{31}
\]

where

\[
\bar{\omega} = \frac{m_B^2 + m_\rho^2 - q^2}{2 m_B m_\rho} , \tag{32}
\]

and

\[
r_1 = \frac{m_B^2 - q^2 - m_\rho^2}{2} \tag{33}
\]

and the functions \( Z, \Omega_j \) are given by the formulae of the Appendix with \( \Delta_1 = \Delta_H, \Delta_2 = \Delta_1 - m_\rho \bar{\omega}, x = m_\rho; m \) is the constituent light quark mass.
The calculation for the $B \to a_1$ transition is similar. The results are:

\[
A^D(q^2) = -\frac{m_a^2}{f_a} \sqrt{\frac{Z_H}{m_B}} \left( \Omega_1 - mZ - \frac{2m}{m_a} \Omega_2 \right) (m_B + m_a)
\]

\[
V_1^D(q^2) = \frac{2m_a^2}{f_a} \sqrt{\frac{Z_H m_B}{m_B + m_a}} \left[ (-m^2 + mm_a \bar{\omega}) Z + 2m \Omega_1 - \bar{\omega} m_a \Omega_1 + + (2 \bar{m} \bar{\omega} - m_a) \Omega_2 - 2 \Omega_3 - \Omega_4 - \Omega_5 - 2 \bar{\omega} \Omega_6 \right]
\]

\[
V_2^D(q^2) = \frac{m_a^2}{f_a} \sqrt{\frac{Z_H}{m_B}} \left[ mZ - \Omega_1 - 2 \frac{\Omega_6}{m_a} + 2 \frac{m}{m_a} \Omega_2 \right] (m_B + m_a)
\]

\[
V_0^D(q^2) = -\frac{m_a}{f_a} \sqrt{\frac{Z_H m_B}{m_B + m_a}} \left[ \Omega_1 \left( m \bar{\omega} + 2m \frac{q^2}{m_B} - r_1' \frac{m}{m_B} - 2m \right) + \right.
\]

\[
\Omega_2 \left( m_a + 2m \frac{r_1'}{m_B m_a} - 2m \bar{\omega} \right) + 2 \Omega_3 + \Omega_4 \left( 1 - \frac{2q^2}{m_B} \right) + \Omega_5 + \frac{2 \Omega_6 \left( \bar{\omega} - \frac{r_1'}{m_B m_a} \right) + Z \left( m^2 + m \frac{r_1'}{m_B} -:mm_a \bar{\omega} \right)}{2} .
\]

where now:

\[
\bar{\omega} = \frac{m_B^2 + m^2 - q^2}{2m_B m_a}
\]

\[
r_1' = \frac{m_B^2 - q^2 - m^2}{2} .
\]

The previous results for the form factors can be used directly for the numerical analysis. In order to allow an easier way of using our results we have fitted these formulas by the simple parameterization:

\[
F^D(q^2) = \frac{F^D(0)}{1 - a_F \left( \frac{q^2}{m_B} \right) + b_F \left( \frac{q^2}{m_B} \right)^2}
\]

for a generic form factor $F^D(q^2)$; $a_F, b_F$ have been fitted by a numerical analysis performed up to $q^2 = 16$ GeV$^2$, both for $\rho$ and $a_1$ mesons. We have collected the fitted values in Table I. We note explicitly that the results for $B \to a_1$ form factors at $q^2 = 0$ are proportional to the factor $(0.25$ GeV$^2/f_a)$. Besides the normalization uncertainty due to $f_a$, we estimate a theoretical error of 15% on these parameters.

\[\text{IV. STRONG COUPLING CONSTANTS}\]

In this Section we compute the strong couplings $HH\rho$, $HS\rho$, $HHa_1$, $HSa_1$. As discussed in the introduction they are relevant for the calculation of the polar contribution to the form factors. We parameterize these couplings by considering the following effective Lagrangians (we follow the notations introduced in[3]):

\[
\mathcal{L}_{HH\rho} = i\lambda \text{Tr}(\overline{\Phi} \Phi \sigma^{\mu\nu} F(\rho)_{\mu\nu}) - i\beta \text{Tr}(\overline{\Phi} \Phi \gamma^\mu \rho_{\mu})
\]

\[
\mathcal{L}_{HS\rho} = -i\zeta \text{Tr}(\overline{\Phi} \Phi \gamma^\mu \rho_{\mu}) + i\mu \text{Tr}(\overline{\Phi} \Phi \sigma^{\mu\nu} F(\rho)_{\mu\nu})
\]

\[
\mathcal{L}_{HHa_1} = -i\alpha \text{Tr}(\overline{\Phi} \Phi \gamma^a a_1) + i\mu \text{Tr}(\overline{\Phi} \Phi \sigma^{\mu\nu} F(a)_{\mu\nu})
\]

\[
\mathcal{L}_{HSa_1} = -i\alpha \text{Tr}(\overline{\Phi} \Phi \gamma^a a_1) - i\beta \text{Tr}(\overline{\Phi} \Phi \gamma^a a_1).
\]

The strong couplings can be computed by matching the effective meson lagrangian of Eqs. (42)-(44) with the quark-meson lagrangian (14), i.e. considering triangular quark loops with external legs representing light and heavy mesons. The calculation is similar to the one of the previous section. The results are as follows:

\[
\lambda = \frac{m_B^2}{\sqrt{g}} \frac{Z_H}{f_\rho} (-\Omega_1 + mZ)
\]

\[
\alpha = \sqrt{2} \frac{m_B^2}{g_\rho} Z_H \{ 2m \Omega_1 + m_B \Omega_2 + 2 \Omega_3 - \Omega_4 + \Omega_5 - mB \} .
\]
Here the functions \( Z, \Omega_j \) are given by the formulae of the Appendix with \( \Delta_1 = \Delta_H, x = m_\rho, \omega = m_\rho/(2m_H) \) (we keep here the first \( 1/m_Q \) correction). Moreover

\[
\mu = \frac{m_a^2}{\sqrt{2} g_A f_a} \sqrt{Z_H Z_S (-\Omega_1 - 2 \frac{\Omega_6}{m_\rho} + mZ)}
\]

(47)

\[
\zeta = \frac{1}{\sqrt{2} g_V f_\rho} \sqrt{Z_H Z_S (m_\rho \Omega_2 + 2 \Omega_3 + \Omega_4 + \Omega_5 - m^2 Z)} .
\]

(48)

Here the functions \( Z, \Omega_j \) are given by the formulae of the Appendix with \( \Delta_1 = \Delta_H, \Delta_2 = \Delta_S, x = m_\rho \) and \( \omega = (\Delta_1 - \Delta_2)/m_\rho \). For the axial-vector \( a_1 \) couplings to \( H \) and \( S \) states we find

\[
\lambda_A = \frac{m_a^2}{\sqrt{2} g_A f_a} \sqrt{Z_H Z_S (-\Omega_1 + 2 \Omega_3 - \Omega_4 + \Omega_5 + m^2 Z)}
\]

(49)

\[
\beta_A = \sqrt{2} \frac{m_a^2}{g_A f_a} \sqrt{Z_H Z_S (m_\rho \Omega_2 + 2 \Omega_3 - \Omega_4 + \Omega_5 + m^2 Z)} ,
\]

(50)

where \( Z, \Omega_j \) are given by the formulae of the Appendix with \( \Delta_1 = \Delta_H, \Delta_2 = \Delta_S, x = m_a \) and \( \omega = (\Delta_1 - \Delta_2)/m_a \). Moreover

\[
\mu_A = \frac{m_a^2}{\sqrt{2} g_A f_a} Z_H \left( m \left( Z + 2 \frac{\Omega_3}{m_a} \right) - \Omega_1 - 2 \frac{\Omega_6}{m_a} \right)
\]

(51)

\[
\zeta_A = \sqrt{2} \frac{m_a^2}{g_A f_a} Z_H (-2 m \Omega_1 + m_\rho \Omega_2 + 2 \Omega_3 + \Omega_4 + \Omega_5 + m^2 Z) ,
\]

(52)

where \( \Delta_1 = \Delta_H, x = m_a, \omega = m_a/(2m_H) \) Numerically we get the following results:

\[
\begin{align*}
\lambda &= 0.60 \text{ GeV}^{-1} \\
\beta &= -0.86 \\
\mu &= 0.16 \text{ GeV}^{-1} \\
\zeta &= 0.01
\end{align*}
\]

\[
\begin{align*}
\lambda_A &= 0.85 \times (0.25 \text{ GeV}^2/f_a) \text{ GeV}^{-1} \\
\beta_A &= -0.81 \times (0.25 \text{ GeV}^2/f_a) \\
\mu_A &= 0.23 \times (0.25 \text{ GeV}^2/f_a) \text{ GeV}^{-1} \\
\zeta_A &= 0.15 \times (0.25 \text{ GeV}^2/f_a) .
\end{align*}
\]

A discussion about the theoretical uncertainties of these results is in order. We have explicitly written down the dependence on \( f_a \) of the strong coupling constants involving the light axial-vector meson, since as noted before, this is a major source of theoretical uncertainty for these constants. Another source of spreading in the reported values is the variation of \( \Delta_H \) in the range \( \Delta_H = 0.3 - 0.5 \) GeV (we use \( \Delta_H = 0.4 \) GeV in the calculation). This produces a significant uncertainty only for \( \zeta, \beta_A, \zeta_A \) since we obtain \( \zeta = 0.01 \pm 0.19, \beta_A = -0.81 +0.42 \) and \( \zeta_A = 0.15+0.16 \) while in the other cases only a few percent variation is observed. For the other constants: \( \lambda, \mu, \lambda_A, \mu_A, \) a theoretical uncertainty of \( \pm 15\% \) can be guessed. This estimate follows for example from a different evaluation of the \( \lambda \) parameter performed in I. For other determinations of the coupling constant \( \lambda \) see [18] (QCD sum rules and light cone sum rules) and [19] (this paper uses data from \( D^* \) decays together with vector meson dominance).

\section{V. B \rightarrow \rho AND B \rightarrow A_1 FORM FACTORS: EVALUATION OF THE POLAR CONTRIBUTIONS.}

The polar contributions are given by diagrams where the weak current is coupled to \( B \) and to the light vector or axial vector meson by an intermediate heavy meson state (see Fig. 2). These diagrams, because of the heavy meson propagator, produce a typical polar behavior of the form factors, in the form

\[
F_P(q^2) = \frac{F_P(0)}{1 - \frac{q^2}{m_\rho^2}} .
\]

(53)

This behaviour is certainly valid near the pole; we assume its validity for the whole \( q^2 \) range, which can be justified on the basis of the minor numerical role of this polar contribution, as compared to the direct one, in the region of low \( q^2 \), at least for the form factors \( A^P_0, A^P_1 \) (see the numerical results at the end of this Section). The assumption (53) cannot be made for the form factors \( A^P_0(q^2) \) and \( V^P_0(q^2) \), as we discuss below, and is also less reliable for \( A^P(q^2) \) and \( V^P(q^2) \) (see Table [1]).
The values at $q^2 = 0$ in Eq. (53) can be easily computed in term of the strong coupling constants defined in the previous Section and using the leptonic decay constants $\hat{F}$ and $\hat{F}^+$ that give the coupling of the intermediate states to the currents. Neglecting logarithmic corrections, $\hat{F}$ and $\hat{F}^+$ are related, in the infinite heavy quark mass limit, to the leptonic decay constant $f_B$ and $f^+$ defined by

$$
\langle 0|\bar{q}\gamma^\mu\gamma_5b|B(p)\rangle = ip^\mu f_B
$$

$$
\langle 0|\bar{q}\gamma^\mu b|B_0(p)\rangle = p^\mu f^+,
$$

by the relations $f_B = \hat{F}/\sqrt{m_B}$ and $f^+ = \hat{F}^+/\sqrt{m_{B_0}}$ ($B_0$ is the $S$ state with $J^P = 0^+$ and $b\bar{q}$ content). These couplings have been computed in [3] with the results given in Table III for different values of the parameters.

For the values $F^P(0)$ we obtain the following results for the $B \to \rho$ transition.

$$
V^P(0) = -\sqrt{2}g_V\lambda\hat{F}m_B + m_\rho
$$

$$
A^P_1(0) = \frac{\sqrt{2}m_Bg_V\hat{F}^+}{m_{B_0}(m_B + m_\rho)}(\zeta - 2\mu \bar{\omega}m_\rho)
$$

$$
A^P_2(0) = -\sqrt{2}g_V\mu\hat{F}+\sqrt{m_B(m_B + m_\rho)}
$$

where $\bar{\omega} = m_B/(2m_\rho)$. For $A^P_1(q^2)$, we have to implement the condition contained in Eq. (53); for instance a possible choice is

$$
A^P_1(q^2) = A^P_3(0) + g_V\beta\hat{F}\frac{1}{m_\rho\sqrt{2m_Bm^2_B - q^2}}.
$$

For the $B \to a_1$ transition we have:

$$
A^P(0) = -\sqrt{2}g_A\lambda_A\hat{F}^+m_B + m_a
$$

$$
V^P_1(0) = \frac{\sqrt{2}m_Bg_A\hat{F}}{m_B(m_B + m_a)}(\zeta_A - 2\mu_A \bar{\omega}m_a)
$$

$$
V^P_2(0) = -\sqrt{2}g_A\lambda_A\hat{F}+\sqrt{m_B(m_B + m_a)}
$$

where $\bar{\omega} = m_B/(2m_a)$. Similarly to the previous discussion for $A^P_1(q^2)$ we can put for instance:

$$
V^P_0(q^2) = V^P_3(0) + g_A\beta_A\hat{F}+\frac{1}{m_a\sqrt{2m_Bm^2_{B_0} - q^2}}.
$$

We note that Eqs. (59) and (53) have been written down only as an example of a possible behaviour of these form factors satisfying the given constraints. For massless leptons they do not contribute to the semileptonic width and can be neglected.

Numerically we obtain the results in Table I where we have also reported the values of the pole masses for all the form factors except $A^P_1(q^2)$ and $V^P_0(q^2)$ because of Eqs. (59) and (53). Similarly to the previous analyses an overall uncertainty of $\pm 15\%$ can be assumed. In Fig. 3 we plot the form factors $A_1$ and $A_2$ for the semileptonic decay $B \to \rho$. In Fig. 4 are shown the form factors $A$, $V_1$ and $V_2$ for the semileptonic decay $B \to a_1$. Since the behaviour in Eqs. (59) and (53) is only guessed, we have not included the form factors $A^P_1(q^2)$ and $V^P_0(q^2)$ in Fig. 3 and 4; in addition $V(q^2)$ is not reported in Fig. 3 since our prediction is affected by a large error (see the discussion in the next section). Note that the theoretical error is not included in Fig. 3 and 4; one should refer to the numbers in Tables I and II.

VI. BRANCHING RATIOS AND WIDTHS

In this Section we compute the branching ratios and widths for semileptonic decays using the numerical results for form factors reported in Table I.II. Let us first compare our results for the $B \to \rho$ form factors with those obtained
by other methods (see Table IV). These form factors (as well as those concerning the transition $B \to a_1$) are obtained by adding the direct and polar contributions:

$$F(q^2) = F^D(q^2) + F^P(q^2),$$  \hspace{1cm} (64)$$

where $F^D(q^2)$ was introduced in Section III and $F^P(q^2)$ in Section V. Our result for the vector form factor $V^\rho(q^2)$ is affected by a large error since it arises from the sum of two terms opposite in sign and almost equal in absolute value. Apart from this large uncertainty, our results are in relative good agreement with the results of QCD sum rules, but they are in general higher than those obtained by other approaches. For the $B \to \rho \ell\nu$ decay width and branching ratio we obtain (using $V_{ub} = 0.0032$, $\tau_B = 1.56 \times 10^{-12}$ s):

$$B(\bar{B}^0 \to \rho^+ \ell\nu) = (2.5 \pm 0.8) \times 10^{-4}$$

$$\Gamma_0(\bar{B}^0 \to \rho^+ \ell\nu) = (4.4 \pm 1.3) \times 10^7 \text{ s}^{-1}$$

$$\Gamma_+(\bar{B}^0 \to \rho^+ \ell\nu) = (7.1 \pm 4.5) \times 10^7 \text{ s}^{-1}$$

$$\Gamma_-(\bar{B}^0 \to \rho^+ \ell\nu) = (5.5 \pm 3.7) \times 10^7 \text{ s}^{-1}$$

$$(\Gamma_+ + \Gamma_-)(\bar{B}^0 \to \rho^+ \ell\nu) = (1.26 \pm 0.38) \times 10^8 \text{ s}^{-1}$$  \hspace{1cm} (65)$$

where $\Gamma_0$, $\Gamma_+$, $\Gamma_-$ refer to the $\rho$ helicities. The branching ratio for $B \to \rho \ell\nu$ is in agreement with the experimental result quoted in the introduction, Eq. \[\Box\].

Let us now discuss the theoretical uncertainty of these results. The large error of $V^\rho(0)$ affects significantly the values of $\Gamma_+$ and $\Gamma_-$, whose errors are correlated; it has however no effect on $\Gamma_0$ and a small effect on the branching ratio, which increases at most by 8%. The theoretical uncertainties on $A^\rho_1(0)$ and $A^\rho_2(0)$ are likely to be related. To get the theoretical error on the widths we have added in quadrature the error induced by $V^\rho(0)$ and a common $\pm 15\%$ error on $A^\rho_1(0)$ and $A^\rho_2(0)$.

Having used the decay $B \to \rho \ell\nu$ as a test of the CQM model, we can now consider the $B \to a_1 \ell\nu$ semileptonic decay. The results obtained are:

$$B(\bar{B}^0 \to a_1^{+} \ell\nu) = (8.4 \pm 1.6) \times 10^{-4}$$

$$\Gamma_0(\bar{B}^0 \to a_1^{+} \ell\nu) = (4.0 \pm 0.7) \times 10^8 \text{ s}^{-1}$$

$$\Gamma_+(\bar{B}^0 \to a_1^{+} \ell\nu) = (4.6 \pm 0.9) \times 10^7 \text{ s}^{-1}$$

$$\Gamma_-(\bar{B}^0 \to a_1^{+} \ell\nu) = (0.98 \pm 0.18) \times 10^8 \text{ s}^{-1}$$  \hspace{1cm} (66)$$

where $\Gamma_0$, $\Gamma_+$, $\Gamma_-$ refer to the $a_1$ helicities. We have included in the determination of these decay widths only the normalization uncertainty arising from $f_\alpha$; the lower values correspond to $f_\alpha = 0.30$ GeV$^2$ while the higher values to $f_\alpha = 0.25$ GeV$^2$. One should also take into account the theoretical errors arising from the values of the form factors at $q^2 = 0$; they are more difficult to estimate reliably and are not included here. In any case the over-all theoretical uncertainty is larger (presumably by a factor of two) than the one reported in the previous formula.

VII. CONCLUSIONS

The main conclusion of this paper can be read from eqs.\[\Box\]. We predict a branching ratio for the decay $\bar{B}^0 \to a_1^{+} \ell\nu$ significantly larger than the branching ratio for $\bar{B}^0 \to \rho^+ \ell\nu$; in spite of the theoretical uncertainties inherent to the CQM model, which we have discussed in the previous Sections, this is a remarkable outcome. A consequence of this result is that the $B \to a_1 \ell\nu$ decay channel might account for around 50% of the semileptonic $B \to Xs \ell\nu$ decay channel (evaluated, for example, by the parton model), whereas the $B \to \rho \ell\nu$ decay channel adds another 15%; given the relevance of these results for the determination of $V_{ub}$, it would be interesting to test these predictions in the future by other theoretical methods and, hopefully, by experimental data.

VIII. APPENDIX

In the paper we have introduced several integrals and parameters that we list in this Appendix.
\[ I_0(\Delta) = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(v \cdot k + \Delta + i \epsilon)} \]

\[ = \frac{N_c}{16 \pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} \frac{e^{-s(m^2 - \Delta^2)}}{s} \left( \frac{3}{2s} + m^2 - \Delta^2 \right) \left[ 1 + \text{erf}(\sqrt{s}) \right] \]

\[ - \frac{\Delta m^2}{16 \pi^2} \Gamma(-1, \frac{m^2 - \Delta^2}{\Lambda^2}, \mu^2) \]  

(67)

\[ I_1 = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)} = \frac{N_c m^2}{16 \pi^2} \Gamma(-1, \frac{m^2 - \Delta^2}{\Lambda^2}, \mu^2) \]  

(68)

\[ I'_1 = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)^2} \]  

(69)

\[ I_2 = -\frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)^2} = \frac{N_c}{16 \pi^2} \Gamma(0, \frac{m^2 - \Delta^2}{\Lambda^2}, \mu^2) \]  

(70)

\[ I_3(\Delta) = -\frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta + i \epsilon)} \]  

(71)

\[ I_4(\Delta) = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)^2(v \cdot k + \Delta + i \epsilon)} \]  

(72)

where \( \Gamma \) is the generalized incomplete gamma function and \( \text{erf} \) is the error function. Moreover, having defined:

\[ \sigma(x, \Delta_1, \Delta_2, \omega) = \frac{\Delta_1 (1 - x) + \Delta_2 x}{\sqrt{1 + 2(\omega - 1) x + 2(1 - \omega) x^2}}, \]  

(73)

we have:

\[ I_5(\Delta_1, \Delta_2, \omega) = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta_1 + i \epsilon)(v' \cdot k + \Delta_2 + i \epsilon)} \]

\[ = \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times \left[ \frac{6}{16 \pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \, e^{-s(m^2 - \sigma^2)} \sigma^{-1/2} \left( 1 + \text{erf}(\sqrt{s}) \right) + \frac{6}{16 \pi^2} \int_{1/\Lambda^2}^{1/\mu^2} ds \, e^{-s(m^2 - 2\sigma^2)} \sigma^{-1} \right]. \]  

(74)

We also define, if \( q' = xv', \omega = v \cdot v', \Delta_2 = \Delta_1 - x \omega, \) the formula:

\[ Z = \frac{i N_c}{16 \pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)[(k + q)^2 - m^2]} \]

\[ = \frac{I_5(\Delta_1, x/2, \omega) - I_5(\Delta_2, -x/2, \omega)}{2x}. \]  

(75)

We use in the text the following combinations of the previous integrals:

\[ K_1 = m^2 Z - I_3(\Delta_2) \]

\[ K_2 = \Delta_1^2 Z - \frac{I_3(x/2) - I_3(-x/2)}{4x} [\omega x + 2\Delta_1] \]

\[ K_3 = \frac{x^2}{4} Z + \frac{I_3(\Delta_1) - 3I_3(\Delta_2)}{4} + \frac{\omega}{4} [\Delta_1 I_3(\Delta_1) - \Delta_2 I_3(\Delta_2)] \]
\[ K_4 = \frac{x \Delta_1 Z}{2} + \frac{\Delta_1 [I_3(\Delta_1) - I_3(\Delta_2)]}{2x} + \frac{I_3(x/2) - I_3(-x/2)}{4} \]
\[ \Omega_1 = \frac{I_3(-x/2) - I_3(x/2) + \omega[I_3(\Delta_1) - I_3(\Delta_2)]}{2x(1 - \omega^2)} - \frac{[\Delta_1 - \omega x/2]Z}{1 - \omega^2} \]
\[ \Omega_2 = \frac{-I_3(\Delta_1) + I_3(\Delta_2) - \omega[I_3(-x/2) - I_3(x/2)]}{2x(1 - \omega^2)} - \frac{[x/2 - \Delta_1 \omega]Z}{1 - \omega^2} \]
\[ \Omega_3 = \frac{K_1}{2} + \frac{2\omega K_4 - K_2 - K_3}{2(1 - \omega^2)} \]
\[ \Omega_4 = \frac{-K_1}{2(1 - \omega^2)} + \frac{3K_2 - 6\omega K_4 + K_3(2\omega^2 + 1)}{2(1 - \omega^2)^2} \]
\[ \Omega_5 = \frac{-K_1}{2(1 - \omega^2)} + \frac{3K_3 - 6\omega K_4 + K_2(2\omega^2 + 1)}{2(1 - \omega^2)^2} \]
\[ \Omega_6 = \frac{K_1 \omega}{2(1 - \omega^2)} + \frac{2K_4(2\omega^2 + 1) - 3\omega(K_2 + K_3)}{2(1 - \omega^2)^2} . \]
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Fig. 1 - Diagram for the direct contribution to the form factor $B \to \rho$ and $B \to a_1$.

Fig. 2 - Diagram for the polar contribution to the form factor $B \to \rho$ and $B \to a_1$. 
Fig. 3 - Form factors $A_1$ (continuous line) and $A_2$ (dashed line) for the semileptonic decay $B \rightarrow \rho \ell \nu$.

Fig. 4 - Form factors $A$ (continuous line), $V_1$ (dashed line) and $V_2$ (dot-dashed line) for the semileptonic decay $B \rightarrow a_1 \ell \nu$. 
TABLE I. Parameters of the direct contribution to the various $B$ form factors for $B \to \rho$ and $B \to a_1$ decays. The values $F_D(0)$ for the $B \to a_1$ transition (last four columns) should be multiplied by the factor $0.25 \text{ GeV}^2/f_a$. The theoretical uncertainty is ±15%.

|       | $V_D^0$ | $A_1^D$ | $A_2^D$ | $A_3^D$ | $A_4^D$ | $V_1^D$ | $V_2^D$ | $V_3^D$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $F_D(0)$ | 0.83    | 0.69    | 0.81    | 0.33    | 1.62    | 1.13    | 1.13    | 1.13    |
| $a_F$  | 0.93    | 0       | 0.87    | 2.9     | 1.13    | 0.18    | 1.3     | 1.9     |
| $b_F$  | 0.02    | 0       | -0.17   | 2.6     | 0.12    | 0.04    | 3.8     | 0.93    |

TABLE II. Parameters of the polar contribution to the various $B$ form factors for $B \to \rho$ and $B \to a_1$ decays. Pole masses in GeV. The values $F_P(0)$ for the $B \to a_1$ transition (last four columns) should be multiplied by the factor $(0.25 \text{ GeV}^2/f_a)$. The theoretical uncertainty is ±15%.

|       | $V_P^0$ | $A_1^P$ | $A_2^P$ | $A_3^P$ | $A_4^P$ | $V_1^P$ | $V_2^P$ | $V_3^P$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $F_P(0)$ | -0.84   | -0.11   | -0.15   | -0.019  | -1.48   | -0.32   | -0.57   | 0.07    |
| $m_P$  | 5.3     | 5.5     | 5.5     | -       | 5.5     | 5.3     | 5.3     | -       |

TABLE III. $\hat{F}$ and $\hat{F}^+$ for various values of $\Delta_H$. $\Delta_H$ in GeV, leptonic constants in GeV$^{3/2}$.

| $\Delta_H$ | $\hat{F}$ | $\hat{F}^+$ |
|-------------|-----------|------------|
| 0.3         | 0.33      | 0.22       |
| 0.4         | 0.34      | 0.24       |
| 0.5         | 0.37      | 0.27       |

TABLE IV. Form factors for the transition $B \to \rho$ at $q^2 = 0$. Our results are compared with the outcome of other theoretical calculations: potential models, light cone sum rules (LCSR), QCD sum rules (SR), calculations involving both lattice and light cone sum rules. The large error of $V^\rho(0)$ in our approach is due to the large cancellation between the direct and polar contribution.

|       | This work | Potential Model $^{[20]}$ | LCSR $^{[21]}$ | SR $^{[22]}$ | Latt. + LCSR $^{[23]}$ |
|-------|-----------|---------------------------|---------------|-------------|-------------------------|
| $V^\rho(0)$ | $-0.01 \pm 0.25$ | $0.45 \pm 0.11$ | $0.34 \pm 0.05$ | $0.6 \pm 0.2$ | $0.35_{-0.30}^{+0.06}$ |
| $A_1^\rho(0)$ | $0.58 \pm 0.10$ | $0.27 \pm 0.06$ | $0.26 \pm 0.04$ | $0.5 \pm 0.1$ | $0.27_{-0.04}^{+0.05}$ |
| $A_2^\rho(0)$ | $0.66 \pm 0.12$ | $0.26 \pm 0.05$ | $0.22 \pm 0.03$ | $0.4 \pm 0.2$ | $0.26_{-0.06}^{+0.03}$ |
| $A_3^\rho(0)$ | $0.33 \pm 0.05$ | $0.29 \pm 0.09$ | $0.24 \pm 0.02$ | $0.30_{-0.06}^{+0.04}$ |