Getting to Know BFKL

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Abstract

As a 30th anniversary tribute, I discuss the present and possible future impact of the physics and the physicists of BFKL.

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1 Introduction

The BFKL equation was first derived thirty years ago and at this meeting we have a special session dedicated to the anniversary†. In this talk, I would like to record a personal tribute to the achievement and impact of the physics and the physicists involved. I will do so via my own version of “getting to know BFKL” (which includes well-known happenings). I will also describe a possible long-term significance which is very different from present-day applications of the BFKL equation.

I have known and interacted with BFKL almost since the birth of the equation. In particular, Lev Lipatov has been both a major influence and catalyst for my own research and a good friend, over much of the time period. For many people Lipatov is the obvious descendant of the grand line of russian physicists that passes through Landau, Pomeranchuk and Gribov and, indeed, my first interaction with Lev and his collaborators was during an extended visit with Vlodya Gribov. To present my own perspective on the significance of BFKL, it will be helpful to first talk about my early research and also talk about my interactions with Gribov.

2 Before BFKL

During my early years in physics, immersed in regge theory and S-Matrix theory in Cambridge, I acquired a lasting admiration of russian physics. As graduate students, Peter Goddard and I worked together to master the depths of Toller’s group-theoretic multi-regge formalism. As an off-shoot, we understood how complex helicity should be handled and this led me, as a post-doc, to the remarkable paper by Gribov, Pomeranchuk, and Ter-Martirosyan (GPT) deriving reggeon unitarity. Even though the treatment of complex helicity was a problem, as I had been told, I was astonished by the spectacular leap made from low order field theory calculations to general discontinuity formulae.

At Fermilab, in the Reggeon Field Theory (RFT) group of John Bronzan, Bob Sugar, Henry Abarbanel and Jochen Bartels (a life-long friend and true BFKL aficionado), admiration of russian physics was a communal affair. I came away with the Critical Pomeron heavily imprinted on my psyche, even though I argued against it so strongly at first that I missed the opportunity to be on the historical paper. I went on to Berkeley, where Henry Stapp and I developed the general S-Matrix dispersion theory needed to properly derive the multiparticle complex angular momentum and helicity theory underlying the GPT paper. It has since become apparent to me that

†As is common, for most of my discussion I will not distinguish between the original derivation by Fadin, Kuraev, and Lipatov in a spontaneously-broken gauge theory and the direct application to QCD based on later work of Balitsky and Lipatov that added the B to FKL.
this paper was years (perhaps decades) before it’s time\textsuperscript{†}. Combining the beautiful mesh of multi-regge behavior and multiparticle analyticity properties (that Henry Stapp and I saw) with the GPT paper, convinced me that the unitarity of an S-Matrix must surely involve regge pole behavior in a fundamental manner - with the unitary Critical Pomeron as an essential ingredient.

3 Meeting Gribov

I first met Gribov at the 1977 EPS meeting in Budapest. (His first conference, with significant numbers of western physicists present, in over a decade.) For most of the world it was Gribov’s talk on “Non-abelian Gauge Copies” that made this a historic meeting. For me it was the giddy experience of learning that Gribov endorsed the supercritical RFT that I had developed. This was a major controversy and I was almost alone (as usual) in a minority viewpoint - insisting on reggeon unitarity! It is hard to exaggerate the boost that Gribov’s support gave me. After the meeting was over, I met with Gribov for discussion and began planning a visit to Leningrad as soon as I returned to CERN.

4 Leningrad in 1978 - meeting FKL

I visited Leningrad for six weeks (with a week in Moscow) during the winter of 1978. It was bitterly cold and I could fill a book with stories of my experience of communist russia (part of the time with my wife and young son). I met often with Gribov to discuss supercritical RFT and I gave “Leningrad talks” on this subject and on multiparticle complex angular momentum and dispersion theory. In our discussions, it was hard to divert Gribov from talking (to me) about anomalies. I must have absorbed something because, subsequently, anomalies have become a dominant part of my own thinking.

A very memorable event occurred in the middle of my stay. Lev Lipatov suggested we meet for a discussion. Subsequently, three very burly and physically overwhelming russians arrived to meet me for lunch in my hotel restaurant. Victor Fadin, Eduard Kuraev and Lev Lipatov were doing me the honor of coming, in full force, to explain the BFKL equation to me. The downing of glasses of vodka that preceded lunch was very unfamiliar to me, as was the physics I heard afterwards. With my narrow upbringing in S-Matrix theory (and vodka inexperience), I was ill-equipped to

\textsuperscript{†}It is barely mentioned in “The Analytic S-Matrix” (the Cambridge bible of the time) with the reggeon unitarity formula referred to as heuristic.
properly appreciate what I was being told. Only later would I understand the funda-
mental and deeply insightful use of unitarity and analyticity that would, ultimately,
draw me to BFKL.

5 Beginning to Study BFKL

Shortly after returning from Leningrad I realized that my supercritical RFT contained
a vector reggeon (exchange-degenerate with the pomeron), suggesting it should be re-
lated to a spontaneously-broken gauge theory. I wanted to learn about regge behavior
in gauge theories - rapidly! From the outset, I was not content with finite-order per-
turbative calculations but rather wanted to see the impact of unitarity directly. Most
of all, of course, I wanted to locate the Critical Pomeron via my supercritical RFT.

I liked the Grisaru and Schnitzer argument that reggeization is prod-
uced by a non-abelian symmetry group and t-channel elastic unitarity. Also impressive, were
the Bronzan and Sugar papers demonstrating that, in agreement with GPT, the
Cheng and Lo results could be written in terms of reggeon diagrams (with the eighth
and tenth order results, in effect, predicted by the sixth-order results). However,
the Cheng and Wu technique of tracking large light-cone momenta through diagrams
seemed complicated to utilise and lacking insight into the role of unitarity. Fortu-
nately, Jochen Bartels had arrived at CERN and was developing his own program,
based entirely on unitarity. He was an invaluable consultant and guide to the, often
less accessible, russian literature.

My appreciation of the unitarity plus dispersion relation methods utilized by
BFKL (explained to me, presumably, in Leningrad) was immediate. The simplicity
of the born level calculation of gluon-gluon scattering was stunning (here “gluons”
have a mass generated by the Higgs mechanism.) Renormalizability (or, more in-
directly, unitarity boundedness) implies that the leading high-energy term satisfies
an unsubtracted t-channel dispersion relation. The only singularity in the t-channel
comes from the pole diagram. Consequently, the full result is obtained from a sin-
gle diagram. The summation of many diagrams is completely by-passed by a simple
exploitation of unitarity and analyticity!

The calculation of higher orders is equally simple. Multi-regge tree amplitudes
can also be calculated by using unsubtracted dispersion relations in the t-variables
and renormalizability bounds. Using s-channel unitarity and another dispersion rela-
tion then gives the leading log amplitude to all orders. Hundreds of feynman diagrams
are effectively summed and yet the result, by now extremely well known, is extraordi-
narily simple, i.e gluons (and quarks) become regge poles with a calculated trajectory
function§ Continuing the argument, BFKL show that when exchanged gluons are replaced by reggeized gluons the unitarity calculation again gives back the reggeized gluon as the leading result. This remarkable “bootstrap” shows that, in leading logs, reggeized gluon exchange satisfies full s-channel unitarity and elastic t-channel unitarity¶. This is, essentially, the “multiperipheral bootstrap” that many people hoped the pomeron would satisfy.

The non-leading amplitudes contain the exchange of two reggeized gluons. At first sight, reggeon unitarity is satisfied with the (massive) BFKL kernel as the two reggeon interaction and a beautiful solution of both s-channel unitarity and reggeon unitarity is emerging. Unfortunately, reggeon unitarity is surely spoiled by the vacuum channel appearance of the BFKL pomeron, due to the large transverse momentum scaling of the kernel. Reggeon unitarity requires the fundamental j-plane singularities to be regge poles. If they are not, it is hard to see how multiparticle t-channel unitarity can be satisfied. Although the large transverse momentum scaling is essential for the appearance of the BFKL pomeron in QCD, reggeon unitarity would be saved if this scaling instead produced an infra-red divergence related to a supercritical condensate. How this relates to QCD is described in my companion talk. It would be some time before I was able to explain to Lev Lipatov the relationship between his pomeron and my condensate.

6 The Fall of the Iron Curtain and HERA

During most of the 80’s BFKL, like most other russian physicists, remained locked in the Soviet Union. Bartels and I were amongst the few western physicists referencing BFKL and the authors were not directly working on the subject. After Leningrad, I did not meet Lipatov again until the 1987 Protvino meeting (which was vodka-free - thanks to Gorbachev). Shortly afterwards the collapse of the Soviet Union began and Russian physicists started to appear everywhere in the west. Lev Lipatov visited CERN in 1988 and in 1989 Victor Fadin (and Misha Ryskin) came to the Blois conference in Chicago that I organized with Marty Block. Even before HERA began, Jochen Bartels had the foresight to see how important BFKL would become. He began organizing workshops at which BFKL would appear (in various combinations) and later, as HERA was well underway, extended BFKL visits to DESY would become the norm. (Lev Lipatov would be the first to tell me that H1 had a structure function looking very like my pomeron!)

§McCoy and Wu require several volumes of the Physical Review to arrive at the simple conclusion that, in QED, the leading log result, up to twelfth-order, is that the electron reggeizes.

¶Bartels has also extended the bootstrap to various multiparticle amplitudes.
7 The 90’s and NLO BFKL

Following Jochen’s success at HERA, I started a series of Argonne/Fermilab workshops, with Mike Albrow, at which BFKL would also appear in various combinations. At one workshop we were supposed to have B, F & L but, due to an airport mix-up in Moscow, Fadin did not arrive and Lipatov was forced to give five lectures. Lev also made several extended visits to Argonne, much to my delight. BFKL became an established part of extended perturbative QCD. Although there were qualifications, it became generally accepted that the BFKL pomeron should appear in a variety of small-x processes and it was a constant source of experimental comparisons, even if a distinctive, clear, sighting proved elusive. Many theorists and phenomenologists worked on the subject and citations accumulated rapidly, as they continue to do.

For Victor Fadin and Lev Lipatov the 90’s were a highly productive period. They were often able to work together in institutions where they were well supported and had the freedom to work intensely. The results were an achievement of major proportions. They were able to complete the calculation of the NLO BFKL kernel and demonstrate that it retained all the attractive properties of the leading-order kernel. In addition it was shown that reggeization persists and (more recently and also, perhaps, even more remarkably) that the bootstrap condition is still satisfied. Since unitarity, in all possible channels, and sophisticated dispersion relation techniques were used, it would be very difficult to determine just how large the enormous number of feynman diagrams involved actually is. In fact for some field theorists the techniques of Fadin and Lipatov are sufficiently bewildering that they find the result difficult to accept. With my background in unitarity and analyticity, the results are overwhelmingly impressive.

In the midst of all the BFKL excitement I (temporarily) abandoned my supercritical pomeron focus to see how much of BFKL I could derive from multiparticle j-plane unitarity in the t-channel. I succeeded in deriving the leading-order BFKL kernel, the (so-called) triple pomeron vertex, and also a NLO kernel. Working with Claudio Coriano, I was able to prove that this NLO kernel was conformally invariant and to derive the eigenvalue spectrum. This kernel has subsequently been identified by Kirschner as part of the full BFKL kernel and it has also been shown to fit experiment particularly well. Nevertheless, I have not tried to determine explicitly how my derivation relates to the Fadin and Lipatov derivation. With Mark Wusthoff and Claudio, we were able to express the conformal invariance of my kernel in a compact logarithmic form. Mark also found analogous, rather beautiful, much higher-order kernels, that he never published.
8 The future

Going significantly beyond the NLO BFKL equation, with the aim of obtaining a fully unitary theory, is an extremely difficult challenge. Lev Lipatov and Ian Balitsky have separately developed very different effective action approaches. Unfortunately, neither approach seems to offer any hope for finding critical behavior involving a finite number of interactions or degrees of freedom. Without a transition to pomeron regge pole degrees of freedom, via some form of confinement, gluon infra-red behavior inevitably makes arbitrarily high-order interactions equally important.

In my companion talk, I have described how, in a special version of QCD (and a very special SU(5) theory), massless fermion chiral anomalies within the bound-state S-Matrix create an additional divergence which produces confinement and a regge pole pomeron. I use supercritical RFT to argue that there is a critical phenomenon involving dynamical infra-red chirality transitions and infra-red scaling gluon reggeon kernels that produces the unitary Critical Pomeron. The bound-state S-Matrix is obtained from multi-regge amplitudes that require the full armory of BFKL reggeon diagrams and reggeon unitarity for their construction.

If LHC results were to steer physics in my direction, the calculation of bound-state amplitudes via reggeon diagrams would, very likely, become the focus of much of particle physics. In this case, the fundamental establishment of the reggeon diagram formalism, by BFKL, would have a long term significance in a manner, and in applications, very different from it’s current success.