Lorentz invariance of effective strings

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Abstract: Starting from a Poincaré invariant field theory of a real scalar field with interactions governed by a double-well potential in 2+1 dimensions, the Lorentz representation induced on the collective coordinates describing low-energy excitations about an effective string background is derived. In this representation, Lorentz transformations are given in terms of an infinite series, in powers of derivatives along the worldsheet. Transformations that act on the direction transverse to the string worldsheet involve a universal dimension $-1$ term. As a consequence, Lorentz invariance holds in this theory of long effective strings due to cancellations in the action between irrelevant terms and the dimension two term that describes free massless scalar fields in two dimensions.

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1. Introduction

The equations of motion of several quantum field theories of physical interest have classical static solutions that can be interpreted as possessing string-like defects. Such solutions spontaneously break translation invariance in the $D - 2$ transverse space dimensions. As a result, there are Nambu-Goldstone massless excitations about such backgrounds, even if the field theory has only massive excitations about homogeneous background configurations. These massless modes have wavefunctions supported in the vicinity of the spacetime sheet swept out by the defect, so the effective field theory describing low energy phenomena about such classical solutions is two dimensional. On general grounds, the leading term in the action for these modes must take the form

$$ S \approx S_o \equiv \text{const.} \int dt dy \left[ (\partial_t f^i)^2 - (\partial_y f^i)^2 \right], $$

where $S$ is the action of the underlying field theory, $S_o$ is the action governing low energy phenomena, $i = 1, \ldots, D - 2$, and we have chosen the $(t, y)$ plane as the plane of the worldsheet.

The derivation of $S_o$ (and higher terms) is standard, and excellent treatments are available in the literature, including careful treatments of the introduction of collective coordinates. We have not, however, found a calculation of the Lorentz transformations starting from the underlying field theory. In this note we do this in a $2 + 1$ dimensional example and show that Lorentz invariance in such theories (of long effective strings) is a consequence of cancellations between $S_o$ and terms that are irrelevant for long-distance physics.

We consider the induced quantization of one string, neglecting overhangs and many string (interacting or not) sectors. Large overhangs produce effects suppressed by exponentials of the form $\exp(-\text{const.} m R)$, where $R$ is a length-scale characterizing the overhang. For small overhangs ($R = O(1/m)$), the curvature ($\propto R^{-2}$) becomes large enough that the interactions of the Goldstone bosons cannot be neglected, or equivalently, the long distance effective field theory description is not appropriate.

Section two is a brief review of the effective action governing $f$. Section three provides a quick derivation of the Lorentz algebra, section four concludes.

The problems afflicting massless fields in two dimensions are not relevant to the discussion. It will be assumed that we are just above the roughening transition, or that appropriate boundary conditions are in place.

2. Review

The specific example is a real scalar field in $2+1$ dimensions, with an action:

$$ S \equiv \frac{1}{2} \int d^3x \left[ \partial_\tau \phi \partial^\tau \phi - \lambda (\phi^2 - \frac{m^2}{\lambda})^2 \right]. $$

* See Wallace and Zia[1], Diehl, Kroll and Wagner[2], and Lüscher[3] for much of the material in this section.
The metric $\eta_{ij}$ has signature $+--$, and the coordinates are $x^r \equiv (t, y, z) \equiv (y^\mu, z)$. The equation of motion is
\[
\partial^2 \phi + 2\lambda (\phi^2 - \frac{m^2}{\lambda}) \phi = 0.
\]
Static configurations invariant under translations in the $y$ direction are solutions of
\[
-\partial_z^2 \phi(z) + 2\lambda \phi(z)^3 - 2m^2 \phi(z) = 0.
\]
The classical solution corresponding to one string (or domain wall) is
\[
\phi_{cl}(x) \equiv \frac{m}{\sqrt{\lambda}} \tanh mz.
\] (1)

There are other solutions to the boundary conditions $\phi \to \pm m/\sqrt{\lambda}(z \to \pm \infty)$, but these are multi-string configurations. The spectrum of fluctuations about the kink solution is known and corresponds to eigenvalues of the operator $\Omega = -(\partial_z - 2\sqrt{\lambda} \phi_{cl})(\partial_z + 2\sqrt{\lambda} \phi_{cl})$,

\[
\begin{align*}
\lambda_0 &= 0 : \psi_0 = \sqrt{\frac{3m}{2}} \text{sech}^2(mz) \equiv \sqrt{\frac{3\lambda}{4m^3}} \eta_0(z), \\
\lambda_1 &= 3m^2 : \psi_1 = \sqrt{\frac{3m}{2}} \text{sech}(mz) \tanh(mz), \\
\lambda_k &= k^2 + 4m^2 : \psi_k = \frac{\sqrt{m \exp(ikz)}}{\sqrt{k^4 + 5k^2m^2 + 4m^2}} \left[ 3 \tanh^2(mz) - \frac{3ik}{m} \tanh(mz) - \left( \frac{k^2}{m^2} + 1 \right) \right].
\end{align*}
\]

The physical significance of these modes is as follows:
(a) The zero-mode is the Nambu-Goldstone boson, since $\eta_0 = \partial_z \phi_{cl}$.
(b) The other localized mode has mass $\sqrt{3m}$ and is referred to as the kink ‘excitation’ in the literature. It corresponds to a squeezing of the string. To see this, compute the normalized overlap of $zd\phi_{cl}/dz$ and $\psi_1$. This is $\pi \sqrt{3}/\sqrt{8\pi^2 - 48} \approx 0 \cdot 978$.
(c) A continuum starting at mass $2m$, with $k$ taking arbitrary real values—these modes are the counterparts of the spectrum obtained when expanding about a homogeneous background $\phi_{cl}(x) = \pm m/\sqrt{\lambda}$.

A single classical solution with fixed kink position is not a good ground state to quantize around because of the massless mode corresponding to moving the wall. Instead one introduces a variable describing the position of the wall and integrates over it. The position of the wall is a collective coordinate. It can be introduced in the path integral formulation derived by Gildener and Patrasciou[4] of the implicit collective coordinate method due to Christ and Lee[5]. Writing
\[
\phi(t, y, z) \equiv \phi_{cl}(z - f(t, y)) + \xi(t, y, z - f(t, y)),
\]
where
\[
\xi(t, y, z + \alpha) = a_0(t, y)\psi_0(z + \alpha) + a_1(t, y)\psi_1(z + \alpha) + \int dk a_k(t, y)\psi_k(z + \alpha)
\]

\[= m \lambda \tanh mz. \]
shows the coefficient of the zero mode explicitly. The integral over $k$ is schematic, it is not necessary to be precise since loop effects will not be considered. Inserting $\prod df(t,y) \delta(g(f))[\delta g/\delta f] = 1$ into the path integral, $g(f) = \int dz \partial_z \phi_{cl}(z - f(z)) \phi(z)$, the functional integral becomes:

$$\int Df D\xi \Delta[\xi] \prod_{t,y} \delta \left[ \int dz \eta_0(z') \xi(z') \right] e^{iS}.$$ 

Here

$$\Delta \equiv \prod_{t,y} \int dz' \eta_0(z' - f(t,y)) \partial_z \phi(t,y,z) = \Delta(a)$$

is actually independent of $f$ and $a_0$. The functional integral for $\xi$ is defined, as usual, as an integral over the coefficient fields, $a_i$. The integral over $a_0$ then eliminates the delta function, and all dependence on $a_0$, and the collective coordinate field, $f$, is left in its stead. Thus, $\xi$ can be treated henceforth as if $a_0 = 0$. $\Delta(a)$ enters the action at order $\bar{h}$ and so will be neglected in the following. Terms in the perturbative expansion arising from $\Delta$ need to be regulated. Its field independent term is a constant, so using dimensional regularization $\Delta$ can be set to one$[1]$.

The action now takes the form

$$S = -\frac{m}{\lambda} \int d^3 x \left\{ \eta_0^2 - \frac{1}{2} \partial_\mu f \partial^\mu f [\eta_0 + \partial_z \xi]^2 - \frac{1}{2} \xi \left( -\partial_\mu \partial^\mu + \partial_z^2 - 6\phi_{cl}^2 + 2 \right) \right\}$$

$$+ 2\phi_{cl} \xi^3 + \frac{1}{2} \xi^4 + \partial_\mu \xi \partial^\mu f \partial_z \xi \right\}.$$ 

All the $f$ dependence is explicit and we have rescaled fields and coordinates so that all dependence on the parameters $m$ and $\lambda$ appears, as it must, in the dimensionless combination $m/\lambda$.

It is convenient in the some of the following to work with the components of $\xi$ decomposed in terms of the normalized wavefunctions, $\psi_i, i = 1, \ldots$. Define $\hat{\eta} \equiv (\psi_1, \psi_k)$ and $\hat{a} \equiv (a_1, a_k)$ as vectors, so $\xi = \hat{\xi} \cdot \hat{\eta}$, and let $\hat{\Omega}$ be the mass matrix, which in the $\psi$ basis, is diag$(3, k^2 + 4)$. Integrating out $z$, $S$ is now

$$-\frac{m}{\lambda} \int dt dy \left\{ \langle 0|0 \rangle - \frac{1}{2} \partial_\mu f \partial^\mu f \left[ \langle 0|0 \rangle + 2a_k \langle 0|\partial_z |k \rangle - a_j a_k \langle j|\partial_z^2 |k \rangle \right] + \frac{1}{2} a_i \left( \partial_\mu \partial^\mu + \hat{\Omega} \right) a_i$$

$$+ 2\langle i|\psi_j \phi_{cl} |k \rangle a_i a_j a_k + \frac{1}{2} \langle i|\psi_j \psi_k |l \rangle a_i a_j a_k a_l + \partial_\mu f \partial^\mu a_j \langle j|\partial_z |k \rangle a_k \right\},$$

where $\langle i|g(z) |j \rangle \equiv \int dz \psi_i(z) g(z) \psi_j(z)$. Note that $|0\rangle$ will denote $\eta_0(z)$, which is not normalized, $\int \eta_0^2(z) dz = 4/3$. This exact rewriting of the action in the one string sector is a two dimensional field theory with a single massless field $f(t,y)$, the position of the wall, interacting with an infinite number of massive fields $\{a_i(t,y)\}$.

The equation of motion for $\hat{a}$ is

$$[-\partial_\mu \partial^\mu - \hat{\Omega} - (\partial f)^2 \partial_z^2 + 2\partial_\mu f \partial_\mu \partial_z + (\partial_\mu \partial^\mu f) \partial_z] \xi$$

$$= \partial_z \eta_0 (\partial f)^2 + 6\phi_{cl} \xi^2 + 4\xi^3,$
which can be solved, since $\hat{\Omega}$ is invertible, to obtain $\hat{a}$ as a series in $\partial_{\mu}f$, a useful expansion in the long wavelength limit. One obtains $\xi = \xi^{(2)} + \xi^{(4)} + \xi^{(2,2)} + \ldots$, where

$$
\xi^{(2)} = -\hat{\Omega}^{-1}\partial_z\eta_0(\partial f)^2,
\xi^{(4)} = -\hat{\Omega}^{-1}\left[\partial_z^2(\partial f)^2 + 6\phi_{cl}\xi^{(2)}\right]\xi^{(2)},
\xi^{(2,2)} = \hat{\Omega}^{-2}\partial_z\eta_0\partial_\mu\partial_\mu(\partial f)^2.
$$

In components, this amounts to

$$
a_i = -\left\{\langle i|\hat{\Omega}^{-1}\partial_z|0\rangle - \langle i|\hat{\Omega}^{-2}\partial_z|0\rangle\partial^2\right\}(\partial f)^2 + \left\{\langle i|\hat{\Omega}^{-1}\partial_z^2\hat{\Omega}^{-1}\partial_z|0\rangle + 6\langle i|\psi_j\phi_{cl}|k\rangle\langle j|\partial_z|0\rangle\langle k|\partial_z|0\rangle\right\}(\partial f)^4 + \ldots.
$$

Using the identity of [2],

$$
\partial_z = \frac{1}{2}[z,\hat{\Omega}] \Leftrightarrow \langle i|\partial_z|j\rangle = \frac{1}{2}(m_i^2 - m_j^2) < i|z|j >,
$$

and substituting for $\xi$, (i.e., integrating out $\xi$), we find (showing up to $O(\partial^8)$)

$$
S = -\frac{m}{\lambda}\int d^2y\left\{\langle 0|0\rangle \left[1 - \frac{1}{2}(\partial f)^2 - \frac{1}{8}(\partial f)^4 - \frac{1}{16}(\partial f)^6 + \ldots\right]
+ \frac{1}{8}\langle 0|z^2|0\rangle(\partial f)^2\partial_\mu\partial_\mu(\partial f)^2 + \ldots\right\}. \quad (2)
$$

The first four terms, as shown by [2], give the leading terms in the expansion of $\sqrt{1 - (\partial f)^2}$, the Nambu-Goto action, with induced metric $h_{ij} = \eta_{ij} - \partial_i f\partial_j f$. The last term shown can be rewritten partly as the curvature of the induced metric, but there are additional terms as well, which do not appear to have a geometric interpretation. Since the coefficient of this term is non-universal, the appearance of non-geometric terms is not surprising—it does appear to contradict the work of Ref. 6. It may be that different parametrizations of the collective coordinates lead to different non-universal terms.

3. Lorentz transformations

The canonical Lorentz generators are

$$
M_{rs} \equiv \int dydz \left[j_{0r}x_s - j_{0s}x_r\right].
$$

where

$$
j_{rs} \equiv -\eta_{rs}\mathcal{L} + \partial_r\phi\partial_s\phi
$$

are the translation currents.
An arbitrary variation of \( \phi \) can be written as
\[
\delta \phi(z) = -\delta f \partial_z \phi(z) + \delta \hat{a} \cdot \psi(z - f(t, y))
\]
\[
= \delta a_0 \psi_0(z - f(t, y)) + \delta \hat{a}_i \psi_i(z - f(t, y))
\]
since a complete basis \((a_0, \hat{a}_i)\) is dual to the complete set of \(\psi_i\). We then have
\[
\begin{pmatrix}
\delta a_0 \\
\delta \hat{a}_i
\end{pmatrix} = \begin{pmatrix}
- \int dz \partial_z \phi(z) \psi_0(z - f(t, y)) & 0 \\
- \int dz \partial_z \phi(z) \psi_i(z - f(t, y)) & 1
\end{pmatrix} \begin{pmatrix}
\delta f \\
\delta \hat{a}_i
\end{pmatrix}
\]
Inverting this gives \(P_\phi \equiv -i\delta / \delta \phi\), the momentum conjugate to \(\phi\), in terms of the momenta conjugate to \(f\) and \(a_i\),
\[
\frac{\delta}{\delta \phi} = \frac{\delta_0(z - f(t, y))}{\Delta(\hat{a})} \frac{\delta}{\delta f} + \left[ -\frac{\delta_0(z - f(t, y))}{\Delta(\hat{a})} \hat{a}_k \langle i | \partial_z | k \rangle + \psi_i(z - f(t, y)) \right] \frac{\delta}{\delta \hat{a}_i}.
\]
One can verify that \([P_\phi(t, y, z), \phi(t, y', z')] = -i\delta(z - z')\delta(y - y')\).

Ordering ambiguities do not change the commutation relations to leading order in \(\hbar\). In the present context the possible ordering ambiguities which are subleading are also not of concern because it is possible to regulate the theory without violating any relevant symmetry.

With these expressions at hand, it is straightforward to evaluate the action of the Lorentz generators on \(f\) and \(a_i\):
\[
[M_{0y}, f] = i(t \partial_y f + y \partial_0 f)
\]
\[
[M_{0y}, a_j] = i(t \partial_y a_j + y \partial_0 a_j)
\]
\[
[M_{\mu z}, f] = i \left[ -y_\mu + f \partial_\mu f + \frac{\partial_\mu f}{\Delta(\hat{a})} a_j \langle 0 | z \partial_z | j \rangle - \frac{\partial_\mu a_j}{\Delta(\hat{a})} \langle 0 | z | j \rangle \right]
\]
\[
[M_{\mu z}, a_j] = i \left[ \partial_\mu a_k \langle j | z | k \rangle - \partial_\mu a_i a_k \frac{\langle 0 | z | i \rangle \langle j | z | k \rangle}{\Delta(\hat{a})} - \partial_\mu f \left( \langle j | z | 0 \rangle + \langle j | z \partial_z | k \rangle a_k - a_i a_k \frac{\langle 0 | z \partial_z | i \rangle \langle j | z | i \rangle}{\Delta(\hat{a})} \right) + f \partial_\mu a_j \right]
\]
These are valid to leading order in \(\hbar\). One can substitute \(\xi = \xi(f)\), as derived in section two, to obtain the complete nonlinear transformations that leave the effective action governing \(f\) (eq. 2) invariant. This substitution only affects Lorentz transformations of \(f\) beginning at order \(\partial^3\). The first two terms in the transformation of \(f\) are independent of the details of the wavefunctions and of the potential, and are thus universal (if the kinetic term is canonical, for other possibilities see [7]). They also leave the measure invariant, even though they are nonlinear. The supersymmetric version of the universal part of the transformations was found in [8], using the Volkov-Akulov formalism[9].

4. Concluding remarks

The computations given above are entirely straightforward, and nothing untoward or unexpected was found. The result is the complete, albeit intractable and impractical, form
of the Lorentz transformations, to all orders in the derivative expansion, and leading order in $\hbar$.

This differs from the Lorentz algebra of the fundamental string in light cone gauge\(^\dagger\). In that case, $S_0$ is the full action after solving the constraints. Upon quantizing, Lorentz transformations require compensating conformal transformations to close unless $d = 2, 3$. These conformal transformations are not symmetries in the quantized theory unless $d = 26$. The coordinates used in our example, $X^r = (t, y, z + f(t, y))$ do not obey the light cone conditions $(X \pm X')^2 = 0$, the underlying field theory was not quantized in light cone gauge. A direct comparison of the algebra induced from the field theory with that of fundamental strings involves the quantization of an interacting scalar field theory in light-cone coordinates, a difficult task (aside from integrable theories in two dimensions, which are in some sense free).

Here no conformal invariance is assumed or used,\(^\ddagger\) The variations of irrelevant terms cancel the variation of $S_0$, because the nontrivial Lorentz transformations of $f$ start with an inhomogenous universal term of dimension $-1$. The light cone gauge fundamental string can also be described by $S_0$, but a fixed point theory describes the long (or short) distance behaviour of an entire universality class of theories—it does not follow that a global symmetry, e.g. Lorentz invariance, that appears in a given element of the universality class is common to every other element.

To consistently look at higher orders in $\hbar$ requires summing over loops in the underlying field theory, finding a stringlike solution of the effective action (rather than potential) and then repeating the procedure of eliminating the massive fields in the derivative expansion. (Repeating from earlier, there will also be contributions from the collective coordinate Jacobian at higher orders, which depend on the regulator chosen. These contributions vanish in dimensional regularization.) All the terms obtained by using the equations of motion to eliminate massive excitations are explicitly local. It is not clear to us that the Polyakov–Liouville term formed out of the induced metric can be made local in the gauge choice inherited from the underlying field theory.

Classical solutions depend on the parameters in the action, in the case above on $m$ and $\lambda$. Modifying these parameters alters the width of the string and its string tension. A question of interest is: are there values of these parameters that lead to the decoupling of all fluctuations, other than the Goldstone mode? In other words, when does the quantum theory, expanded about a classical solution with a string defect, exhibit the characteristics of a structureless fundamental string? Nielsen and Olesen\(^{10}\) addressed this question and argued that the string is effectively of zero width when the length scale for the energy levels for excitations of the string (set by the string tension $\alpha'$) is much greater than the length scales characterizing the width of the string (the penetration depth $\lambda$ and the correlation length $\xi$ in the Abelian Higgs model), i.e. $\sqrt{\alpha'} \gg \lambda, \xi$. In the Abelian Higgs model, this

\(^{\dagger}\) For the particular case of 2+1 dimensions, the light-cone Lorentz algebra is nonanomalous since it has only one nonlinear generator. Inconsistencies appear at the level of interactions.

\(^{\ddagger}\) While this work was being prepared for publication, A. Shapere pointed Ref. [8] out to us where some of these observations are independently made, cited as J. Polchinski (unpublished).
requirement translated into the electric charge, $e \gg 1$. (For flux tubes with more flux the effective charge decreases.) This implies that the physical paradigm for their model, flux tubes in strongly type II superconductors, are not well described by structureless fundamental strings either[11]. Chromoelectric flux tubes in lattice QCD have $\sqrt{\alpha'}$ of about the same order of magnitude as the transverse width, $0.5 \text{ fm}$[12].

The example discussed in this paper can be studied with the same criterion, putting back in the original $\lambda, m$ dependence of the fields and coordinates. The classical string solution in equation (1) has a characteristic width $m^{-1}$, of the order of the mass of fluctuations about homogeneous backgrounds. The other scale is the energy density of the classical solution which sets the energy scale for excitations of the string:

$$\frac{1}{\alpha'} \propto \int dz (\partial_z \phi_{cl}(z))^2 \propto \frac{m^3}{\lambda}.$$ 

The string is ‘thin’ when fluctuations of the string will not excite the internal modes, i.e.

$$m \gg \frac{1}{\sqrt{\alpha'}} \propto \frac{m^{3/2}}{\sqrt{\lambda}} \Leftrightarrow \lambda \gg m.$$ 

So in this case as well, the thin string condition is the strong coupling limit. As Nielsen and Olesen pointed out, this is the $\hbar \to \infty$ limit, which makes ‘classical field considerations very doubtful’. For the example studied here, this limit (in Euclidean space) is in the universality class of the high temperature ferromagnetic Ising model, where entropic considerations dominate over energetic considerations. Thus although an isolated sector of the theory with a single string is an energetically preferred classical configuration, the fluctuations around it are so large (‘$1/\hbar$’ is small) that it is not a good approximation to the most probable state of the system$^*$. 

The above discussion gives little insight into the quantization of the Nambu-Goto area action in dimensions other than 26. There are various suggestions in the literature, such as including the effects of ‘kinks’[13] (making the string massive, quantizing, and then taking the massless limit), or adding to $S_0$ a non-polynomial term in the $f^i$’s, and then using conformal invariance to fix its coefficient[14]. For a further study of the proposal in [14], see [15]. As was first argued by Nielsen and Olesen[10], and also seen in the example studied here, the thin string limit (where an area interpretation of the action might apply) corresponds to strong coupling. In this limit the Lorentz transformations here are renormalized, but beyond this it is hard to make definitive statements. Although problems in quantizing the Nambu-Goto string outside of the critical dimension may manifest themselves as failures of Lorentz invariance, there is no inconsistency in the Lorentz transformation properties of effective strings in any regime where their existence can be reliably assumed.

$^*$ In Ref. [2] it is argued that one can neglect higher loops in the strong coupling limit. This seems to be incorrect; no details are given, however, so this may be due to misunderstanding on our part.
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