Fixed-Velocity Chiral Sum Rules for Nuclear Matter

Thomas D. Cohen

Department of Physics, University of Maryland, College Park, MD 20742-4111, USA

Wojciech Broniowski

H. Niewodniczański Institute of Nuclear Physics, PL-31342 Kraków, Poland

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Abstract

Infinite sets of sum rules involving the excitations of infinite nuclear matter are derived using only completeness, the current algebra implicit in QCD, and relativistic covariance. The sum rules can be used for isospin-asymmetric nuclear matter, including neutron matter. They relate the chiral condensate and the isospin density to weighted sums over states with fixed velocity relative to the nuclear matter ground state.
Sum rules have been an important tool in theoretical physics for a very long time. Their importance stems in large measure from the fact that they allow one to make concrete predictions concerning sums of matrix elements even when one does not have, or cannot solve the theory for the dynamics of the system. One interesting and important system for which the underlying dynamics is known but cannot presently be solved is the nuclear medium. While we are quite confident that the underlying dynamics for the strong interactions is QCD, it is quite unlikely that it will prove to be a tractable task to calculate nuclear properties from QCD any time in the foreseeable future. This raises an obvious question, namely whether one can use known properties of QCD, such as current algebra, to derive sum rules for nuclear properties.

In this work we will not discuss sum rules for finite nuclei. Instead, we concentrate on infinite nuclear matter with arbitrary baryon density and arbitrary isospin density. We do this for several reasons. One is practical: if the system is translationally invariant (and hence necessarily infinite) momentum is a good quantum number and can be used to label excitations. A second reason is that interiors of large nuclei are well approximated by nuclear matter and learning about nuclear matter gives a more general insight into nuclear systems than the study of individual nuclei. A third reason is that the cores of neutron stars are essentially isospin-asymmetric nuclear matter (neutron matter). Finally, infinite nuclear matter is theoretically interesting in its own right. There is obviously one major downside to studying infinite nuclear matter: the connection between sum rules for infinite nuclear systems and experimental observables for real nuclei is not immediately clear.

In previous works [1,2] we have studied the chiral properties of nuclear matter. That is, we examined the behavior of modes as the quark mass is formally taken to zero. We showed that certain zero-momentum modes had to go to zero energy as the (current) quark masses were taken to zero. In the process of deriving this result we used current algebra and completeness to derive sum rules for the squares of the matrix elements of the divergence of the axial current between zero momentum states. Analogous sum rules are also implicit in the work of Lutz, Steiner and Weise [3].
Now, in Refs. [1,2] the central stress was on the fact that these sum rules were necessarily saturated by modes whose energy went to zero as the quark masses did. However, it is worth observing that these sum rules in no way depend on the chiral limit. As sum rules, they hold for any value of the quark mass. This is significant for two reasons. The first is that compared to typical nuclear mass scales, the effects of finite up and down quark masses should not be regarded as small. Moreover, there has been considerable interest in kaonic excitations in dense nuclear matter leading, for example, to kaon condensation [4]-[11].

Accordingly, it is useful to study strange excitations, and the strange quark mass is certainly not small on nuclear physics scales. The fact that the sum rules hold for any value of the quark mass was used explicitly in Ref. [12] in which the Nambu–Jona-Lasinio model was studied at mean-field level and in which the contribution to the sum rules coming from the zero-sound mode as well as from pion modes (i.e. modes that map smoothly onto the vacuum pion modes as the density goes to zero) were explicitly calculated.

There is one major drawback to the sum rules derived in [1,2]: they only give information about modes with zero momentum. This is unfortunate because one of the most interesting issues about nuclear matter is the dependence of the energy of a mode on its momentum. The previous sum rules are completely insensitive to such effects. In the present paper, we will show how to exploit relativistic covariance to derive sum rules which are sensitive to states with nonzero momentum. More precisely, they involve sums over states with a fixed velocity, $\beta = \vec{q}/E$, where $\vec{q}$ and $E$ are momentum and energy of the state, measured relative to the ground state of nuclear matter.

We begin by stating the result in the rest frame of the medium; a covariant formulation will be given later. Also, for simplicity the notation is for two flavors. The generalization to three flavors is straightforward. Let us denote the ground state of translationally invariant nuclear medium by $|C\rangle$. The state is subject to space-independent constraints which fix its baryon density, $\rho_B$, and isospin density, $\rho_{I=1}$. We are concerned with the spectrum of excitations with quantum numbers of the pion (analogous results apply for the kaon) on top of the nuclear medium. These excited states are denoted by $|j_a, \vec{q}\rangle$, where $j_a$ labels modes.
with isospin equal \((a = 0)\), greater by one unit \((a = +)\), or lower by one unit \((a = -)\) than the isospin of the state \(|C\rangle\), and \(\vec{q}\) is the momentum of the mode. The excitation energy of the mode, \textit{i.e.}, the difference of its energy and the reference energy of the state \(|C\rangle\), is denoted by \(E_{ja}(\vec{q})\). We stress that no assumptions are made as to what the excited states \(|j_a, \vec{q}\rangle\) are; they include collective excitations, one-particle–one-hole continuum, two-particle–two-hole continuum, etc.—in short, all states that have quantum numbers of the pion.

We define the following spectral densities associated with the divergence of the axial current \(A^a_{\mu}\):

\[
\sigma^a_C(E, \vec{q}) = \sum_{ja} \frac{|\langle ja, \vec{q} | \partial \cdot A^a(0) | C \rangle|^2}{2|E_{ja}(\vec{q})|} \delta(E - E_{ja}(\vec{q})) , \quad a = 0, +, - .
\]

In general, the sum is over continuum states and assumes the form of an integral. We will show that three classes of sum rules exist, which relate quark condensates and the isovector density to spectral integrals over \(\sigma^a_C\):

\[
-m_u \langle uu \rangle_C - m_d \langle dd \rangle_C = 2 \int_0^\infty \frac{dE}{E} \sigma^0_C(E, \vec{q} = \vec{\beta}E) , \quad (i)
\]
\[
-(m_u + m_d) \langle uu \rangle_C - (m_u + m_d) \langle dd \rangle_C = 2 \int_{-\infty}^{\infty} \frac{dE}{E} \left( \sigma^+_C(E, \vec{q} = \vec{\beta}E) + \sigma^-_C(E, \vec{q} = \vec{\beta}E) \right) , \quad (ii)
\]
\[
-\rho_{I=1} = \int_{-\infty}^{\infty} \frac{dE}{E^2} \left( \sigma^+_C(E, \vec{q} = \vec{\beta}E) - \sigma^-_C(E, \vec{q} = \vec{\beta}E) \right) , \quad (iii)
\]

Note that the spectral functions \(\sigma^a_C\) in the integrands have the arguments constrained to \(\vec{q}/E = \vec{\beta}\). This means that the modes contributing to the sum rule move (in the rest frame of the medium) with the fixed velocity \(\vec{\beta}\).

Figure \[\] illustrates schematically the described situation. What is shown is a typical result of a nuclear calculation of the charged pionic excitation spectrum in neutron matter up to the one-particle–one-hole level \[\]. Solid lines correspond to the \(\pi^+\) and \(\pi^-\) poles, and the shaded region represents the one-particle–one-hole continuum. In addition, a possible spin-isospin sound mode \(\pi_+^s\) \[\] is plotted. On the vertical axis is the frequency of the mode, \(\omega\). The energy of the excitation which positive (negative) isospin modes the energy is equal to \(+ (-)\) \(\omega\). The dashed line shows the integration path in the sum rules. It corresponds to
\(|q|/E = \beta\), i.e., to modes moving with the fixed velocity \(\beta\) with respect to the medium (which is at rest). Contributions to the sum rules coming from various excitations are denoted by blobs (poles) and the thick line (the particle-hole cut). The dashed line is inclined to the vertical axis at the angle \(\alpha = \arctan \beta\). At various values of the velocity \(\beta\) different regions of the spectral density are sampled. Note that \(\alpha \leq 45^\circ\). One should note that in reality the corresponding figure would be much more complicated. Due to multi-particle–multi-hole continua spanning the whole range of \(\omega\) and \(|q|\), all states have finite widths, and contributions to sum rules are collected from everywhere along the dashed lines.

We now pass to the proof of sum rules (1)-(3). We use two facts. Firstly, the chiral current algebra satisfied by QCD yields the following operator identities:

\[
[Q_a^5, [Q_b^5, \mathcal{H}(0)]] = \mathcal{F}(0) \{\tau^a/2, \{\tau^a/2, M\}\} q(0), \quad \text{any } a = 1, 2, 3,
\]

\[
[Q_a^5, A_b^0(0)] = i \epsilon^{abc} V_c^0(0),
\]

where \(V_\mu^a(x) = \mathcal{F}(x) \gamma_\mu \frac{1}{2} \tau^a q(x)\) and \(A_\mu^a(x) = \mathcal{F}(x) \gamma_5 \gamma_\mu \frac{1}{2} \tau^a q(x)\) are vector and axial currents, \(Q_a^5 = \int d^3x A_\mu^a(x)\), \(\mathcal{H}(x)\) is the QCD Hamiltonian density, and \(M = \text{diag}(m_u, m_d)\) is the quark mass matrix.

Secondly, we assume that the medium is translationally invariant. This is true for infinite-volume nuclear matter. The important observation is, however, that the medium need not be at rest. Medium moving with a constant velocity \(-\vec{\beta}\) in a reference frame is also translationally invariant. We denote such a state by \(|C, -\vec{\beta}\rangle\). The sum rules are constructed in the usual way [14]. The identities (2)-(3) are sandwiched by the state \(|C, -\vec{\beta}\rangle\). Inside the LHS we insert covariantly normalized intermediate states [1], using the identity

\[
1 = \sum_j \int \frac{d^3p}{(2\pi)^3 2 |E_j^{(\beta)}(\vec{p})|} |j, \vec{p}\rangle \langle j, \vec{p}|, \quad \text{(4)}
\]

The intermediate states can be labeled by momentum \(\vec{p}\) since \(|C, -\vec{\beta}\rangle\) is translationally invariant. Index \(j\) sums over all additional quantum numbers. The excitation energy \(E_j^{(\beta)}(\vec{p})\) is the difference of the energy of the excited state and the state of the moving medium, i.e., we have
\[ H|C, -\bar{\beta}\rangle = E_C^{(\beta)}|C, -\bar{\beta}\rangle, \quad H|j, \bar{p}\rangle = \left( E_C^{(\beta)} + E_j^{(\beta)}(\bar{p}) \right)|j, \bar{p}\rangle. \quad (5) \]

The momentum \( p \) is defined analogously—it is the momentum relative to the ground state of nuclear matter. Note that \( E_j^{(\beta)}(\bar{p}) \) and \( \bar{p} \) form a Lorentz four-vector. The immediate result of the described construction is the following set of sum rules:

\[
\begin{align*}
-\langle C, -\bar{\beta}|(m_u \bar{u} u + m_d \bar{d} d)|C, -\bar{\beta}\rangle &= \sum_{j_0} \frac{|\langle j_0, \bar{p} = 0|\partial \cdot A^0(0)|C, -\bar{\beta}\rangle|^2}{|E_{j_0}^{(\beta)}(\bar{p} = 0)|^2 E_{j_0}^{(\beta)}(\bar{p} = 0)}, \\
-\langle C, -\bar{\beta}|(m_u + m_d)(\bar{u} u + \bar{d} d)|C, -\bar{\beta}\rangle &= \sum_{j_+} \frac{|\langle j_+ + \bar{p} = 0|\partial \cdot A^+(0)|C, -\bar{\beta}\rangle|^2}{|E_{j_+}^{(\beta)}(\bar{p} = 0)|^2 E_{j_+}^{(\beta)}(\bar{p} = 0)} + \sum_{j_-} \frac{|\langle j_- - \bar{p} = 0|\partial \cdot A^-(0)|C, -\bar{\beta}\rangle|^2}{|E_{j_-}^{(\beta)}(\bar{p} = 0)|^2 E_{j_-}^{(\beta)}(\bar{p} = 0)}, \\
-(1 - \beta^2)^{-1/2} \rho_{I=1} &= \sum_{j_+} \frac{|\langle j_+ + \bar{p} = 0|\partial \cdot A_+^+(0)|C, -\bar{\beta}\rangle|^2}{|E_{j_+}^{(\beta)}(\bar{p} = 0)|^2} - \sum_{j_-} \frac{|\langle j_- - \bar{p} = 0|\partial \cdot A_0^-(0)|C, -\bar{\beta}\rangle|^2}{|E_{j_-}^{(\beta)}(\bar{p} = 0)|^2}. \quad (6) - (8) \end{align*}
\]

Sum rules (6)-(8) are the consequence of Eq. (2), and sum rule (8) follows from Eq. (3). The factor \((1 - \beta^2)^{-1/2}\) is the dilatation factor for the isospin density, which is the time-component of a Lorentz four-vector (our notation is that \( \rho_{I=1} \) is the isospin density in the rest frame on the medium). Note that only states with \( \bar{p} = 0 \) contribute to the above sum rules.

We can now make a boost with velocity \( \bar{\beta} \) to the rest frame of the nuclear matter. This boost transforms Eqs. (6)-(8) into Eqs. (I)-(III). After this boost, the medium is at rest, and the excitations move with the fixed velocity \( \bar{\beta} \). We can write the sum rules in the covariant form. Note there are two Lorentz vectors available for constructing invariants: the four-momentum \( p^\mu \) formed by the excitation energy and the momentum of the excited mode relative to the medium, and the four-velocity of the medium, \( u^\mu \). Introducing \( s = p_\mu p^\mu \) and \( y = p_\mu u^\mu \), we can rewrite (6)-(III) as

\[
\begin{align*}
-m_u \langle \bar{u} u \rangle_C - m_d \langle \bar{d} d \rangle_C &= 2 \int_0^\infty \frac{dy}{y} \sigma_C^0(s = y^2(1 - \beta^2)^2, y), \quad (I) \\
-(m_u + m_d) \langle \bar{u} u + \bar{d} d \rangle_C &= 2 \int_{-\infty}^\infty \frac{dy}{y} \left( \sigma_C^+(s = y^2(1 - \beta^2), y) + \sigma_C^-(s = y^2(1 - \beta^2), y) \right), \quad (II) 
\end{align*}
\]
$$-\langle V_\mu^0(0) \rangle_C = u_\mu \int_{-\infty}^{\infty} dy \frac{dy}{y^2} \left( \sigma_C^+(s = y^2(1 - \beta^2), y) - \sigma_C^-(s = y^2(1 - \beta^2), y) \right),$$  

(III)

Covariant forms of the spectral densities (I) appear in the above equations. However, the arguments are constrained to $s = y^2(1 - \beta^2)$ where $\vec{\beta}$ is the velocity of the modes in the medium’s rest frame. An alternative—and equivalent—covariant formulation would be to use $s$ as the integration variable in (I)-(III), with $y$ constrained. Such a formulation is in some sense more natural in that it has an obvious vacuum limit—in the vacuum the spectral density is independent of $y$ and the sum rules are written in terms of integrals over $s$. However, in the medium this is slightly awkward to do since negative energy states relative to the nuclear matter are possible—e.g., in neutron matter a state with $\pi^+$ quantum numbers may lie below the neutron matter ground state—and hence $s$ can be a multi-valued function of $E$ and the integral must be extended over each branch (see comment (4) below).

A few remarks are in place:

(1) If $|C\rangle$ is the vacuum, then (I)-(II) become the Gell-Mann–Oakes–Renner sum rule [15]. In the chiral limit and for $m_u = m_d \equiv \bar{m}$ one obtains the familiar relation $\bar{m}\langle \bar{q}q \rangle = F_\pi^2m_\pi^2$. Note, however, that the sum rules are exact for any values of $m_u$ and $m_d$, also far away from the chiral limit.

(2) Sum rule (III) or (III) is trivial for the vacuum, and also for isospin-symmetric nuclear matter. However, it is nontrivial for a medium which breaks the isospin symmetry. Its form is reminiscent of the sum rules of Fubini and Furlan (see e.g. [14]).

(3) In the vacuum the sum rules for various values of $\beta$ are equivalent. This is because the dispersion relations for pionic excitations are fixed by Lorentz invariance, i.e. $E = \sqrt{q^2 + m_\pi^2}$. This is no longer true in the presence of the medium, and sum rules with different $\beta$ are physically distinct. The point is that if one were to apply one of our finite velocity sum rules on a Lorentz-invariant state one would find that the size of the contributions to the sum from any given mode would not depend on $\beta$. In contrast, since the medium breaks Lorentz invariance one finds that the size of the contribution from a given mode to the sum does, in general, depend on the value of $\beta$. In this sense these finite velocity sum
rules provide additional information about the spectrum which is not present in the $\beta = 0$ sum rules of Refs. [1]-[3].

(4) The integration variable in the sum rule for the $a = 0$ excitations, (I) or (I), ranges from 0 to $\infty$, which reflects the fact that the state $|C\rangle$ is the ground state of the matter subject to constraints. Therefore all excitations within the constrained space have to raise the energy of the system. Hence, the integration variables in (I) and (II) are positive. This is not true for excitations with $a = \pm$, which take the system out of the constrained space [1]. In that case, states $|j_\pm, q\rangle$ may have lower energy than $|C\rangle$, and the integration in (II)-(III) and (III)-(IV) has to range from $-\infty$ to $\infty$. A model example of such a behavior is given in [12]. It is for this reason that it is awkward to write the covariant versions of these sum rules over the $s$ variable.

(5) In models where the pion fields satisfy the partially-conserved axial current condition, i.e. where $\partial \cdot A^a = -m_\pi^2 F_{\pi} \pi^a$, the spectral densities (I) are proportional to the imaginary parts of the pion propagator. For such models the sum rules can be written as dispersion relations for the in-medium pion propagator.

(6) The final remark concerns renormalization. Strictly speaking, sum rules (I)-(III) or (I)-(IV) are ill defined on both sides. The left-hand side has an ill-defined composite operator and the right-hand side has a divergent sum. In order to make sense of the sum rules one needs to define some scheme to renormalize the $\overline{q}q$ operator, as well as a subtraction term for the spectral sums. At first blush this seems to suggest that these sum rules are useless. However, the need for renormalization stems from the vacuum sector of the theory. No new divergences are induced from the presence of the medium. Thus, once the renormalization is carried for the vacuum sector, it holds also for the medium. This means, that the sum rules (I)-(III) and (I)-(IV) should be regarded as vacuum-subtracted sum rules, involving the difference of the in-medium and the vacuum values of the quark condensates, $\langle C|\overline{q}q|C\rangle - \langle \text{vac}|\overline{q}q|\text{vac}\rangle$, and accordingly subtracted spectral densities. The third sum rule, (III) or (IV) involves a conserved current, and as such requires no subtractions.

To make things concrete, we will illustrate how our sum rules work out for a simple
toy model \footnote{1}. The model describes the pion moving in an isospin-asymmetric medium and interacting with it only via \( \rho \)-meson exchange. Such a model is obviously quite unrealistic since, among other things, it assumes that the \( \pi \)-N coupling constant, and hence \( g_A \) is strictly zero. We set \( m_u = m_d = m \). In order to ensure chiral symmetry in such a model one must assume universal coupling of the rho meson and the KSFR relation \footnote{10}, \( 2g^2_\rho = m^2_\rho/F^2_\pi \). The inverse-charged pion propagator in the rest frame of the medium has the form

\[
G^{\pm}(q_0, \vec{q}) = \frac{q^2_0 \mp \rho I=1/(2F^2_\pi) + \sqrt{(\rho I=1/(2F^2_\pi))^2 + m^2_\pi + q^2}}{F^2_\pi q^2_0 - \vec{q}^2 - m^2_\pi}. \tag{9}
\]

After some simple algebra, sum rules (i)-(iii) can be cast in the form

\[
-\bar{m}\langle \bar{q}q \rangle_C = F^2_\pi m^2_\pi, \tag{10}
\]

\[
-\bar{m}\langle \bar{q}q \rangle_C = F^2_\pi m^2_\pi \left[ \frac{a_- (\beta)}{a_- (\beta) + a_+ (\beta)} + \frac{a_+ (\beta)}{a_- (\beta) + a_+ (\beta)} \right] \equiv F^2_\pi m^2_\pi, \tag{11}
\]

\[
-\rho_{I=1} = -\rho_{I=1} \left[ \frac{a_- (\beta)^2}{a_- (\beta)^2 - a_+ (\beta)^2} - \frac{a_+ (\beta)^2}{a_- (\beta)^2 - a_+ (\beta)^2} \right] \equiv -\rho_{I=1}, \tag{12}
\]

where \( a_{\pm}(\beta) = \pm \rho_{I=1}/(2F^2_\pi) + \sqrt{(\rho_{I=1}/(2F^2_\pi))^2 + m^2_\pi (1 - \beta^2)} \). Sum rule (i) acquires the trivial form (10). Positive (negative) isospin contributions to the sum rules (ii) and (iii) correspond to first (second) terms in the brackets in Eqs. (11)-(12). The relative contribution of the positive and negative isospin modes depends on the value of \( \beta \). These relative contributions to the sum rules are plotted in Fig. 2. The figure is drawn for negative \( \rho_{I=1} \) (neutron matter). Note that as \( \beta \) changes, the relative weight of the modes changes. This illustrates our central point—for translationally-invariant systems which break Lorentz invariance, these finite velocity sum rules are inequivalent to their zero-velocity counterparts. As \( \beta \to 1 \), the positive isospin mode saturates both sum rules. Also note that the two modes saturate the sum rules (ii)-(iii), since there are no other excitation in this model.
Obviously, the toy model is not realistic, but it illustrates our general statements: distinct sum rules for different velocities $\beta$ and the exactness of the sum rules for any value of $\beta$. A more realistic model would involve at least one-particle–one-hole-excitations. Cuts associated with such excitations contribute to sum rules in addition to poles. An example of such a model can be found in Ref. [12], where it was found that the contribution of cuts to the sum rules (at $\beta = 0$) was very small. In reality, as already mentioned in the discussion of Fig. 1 there are in addition multi-particle–multi-hole continua; hence, all quasiparticles acquire finite widths.

While the discussion heretofore has focused on pions in isospin asymmetric nuclear matter, it should be obvious that the results go over mutis mutandus to the study of excitations with kaonic quantum numbers. The key point is that these sum rules work for any quark mass. Thus one can simply replace up quarks by strange quarks everywhere in the sum rules (or alternatively down quarks by strange quarks). The only change is that instead of the isospin density one will have the u-spin or v-spin density on the left-hand side of sum rule (iii) or (III). This is of significance since there has been considerable interest in kaons in nuclear matter and in the possibility of kaon condensation.

It is worth noting that although these sum rules give information about the spectrum away from $p = 0$, they nevertheless do not make contact with all possible modes. The reason is that these modes are fixed velocity with $\beta = p/E \leq 1$. Now one should notice the $p/E$ is the velocity of the mode and not its group velocity. There is nothing in principle to prevent the existence of modes with $\beta > 1$. Our sum rules however tell us nothing about such modes. Recall, for instance, the p-wave pion condensation. If this were to happen, as one approached the transition density a mode with finite $p$ would approach zero energy. Such modes do not contribute to our sum rules.

Finally, we should discuss how our sum rules might be useful. Since they are not based on finite nuclei, it is hard to see what the sum rules can tell us about experiments directly. On the other hand, the sum rules do provide very strong constraints on model building. After all, these sum rules were derived from very basic properties known to be satisfied by
QCD—notably current algebra. Thus, any model which satisfies the various chiral Ward identities must satisfy our sum rules. Of course, it is very simple to construct models which satisfy the Ward identities. For example, mean-field models based on chiral langrangians will. However, such models typically exclude much essential nuclear phenomenology such as the effects of short-range correlations and the need to put in form factors to cut off spurious high-momentum physics. A formalism including such effects may well violate the Ward identities—that is to say, the approximations used to make the calculation tractable while including these effects will in general not be symmetry-conserving. One obvious use of our sum rules is to test how badly the symmetries are violated.

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REFERENCES

[1] T. D. Cohen and W. Broniowski, Phys. Lett. B348, 12 (1995).

[2] T. D. Cohen and W. Broniowski, Phys. Lett. B342, 25 (1995).

[3] M. Lutz, A. Steiner, and W. Weise, Nucl. Phys. A574, 755 (1994).

[4] D. B. Kaplan and A. E. Nelson, Phys. Lett. B175, 57 (1986); B192, 193 (1987).

[5] H. D. Politzer and M. B. Wise, Phys. Lett. B273, 156 (1991).

[6] D. Montano, H. D. Politzer and M. B. Wise, Nucl. Phys. B375, 507 (1992).

[7] G. E. Brown, K. Kubodera, M. Rho, V. Thorsson, Phys. Lett. B291, 355 (1992).

[8] G. E. Brown, C.-H. Lee, M. Rho, V. Thorsson, Nucl. Phys. A567, 937 (1994).

[9] H. Yabu, F. Myhrer, K. Kubodera, Phys. Rev. D50, 3549 (1994).

[10] V. R. Pandharipande, Phys. Rev. Lett. 75, 4567 (1995).

[11] T. Waas, M. Rho and W. Weise, Tech. U. Munich preprint NUCLTH-9610031, Oct. 1996, nucl-th/9610031.

[12] W. Broniowski and B. Hiller, U. of Coimbra preprint 950826, August 1996, nucl-th/9609053.

[13] For a review see Mesons and Nuclei, eds. M. Rho and D. H. Wilkinson (North Holland, 1979).

[14] See for example S. L. Adler and R. F. Dashen, Current algebras and applications to particle physics (Benjamin, New York, 1968).

[15] M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

[16] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966); Riazuddin and Fayazuddin, Phys. Rev. 147, 1071 (1966).
FIGURE CAPTIONS

FIG. 1: Schematic plot of the excitation spectrum of neutron matter, involving $\pi^+$ and $\pi^-$ poles, one-particle–one-hole continuum, and possible spin-isospin sound mode $\pi^+_s$. Positive (negative) isospin modes have the excitation energy $E$ equal to $+ (-) \omega$. The dashed line shows the integration path in the sum rules, and contributions from various excitations are denoted by blobs (poles) and the thick line (cut). The dashed line is inclined to the vertical axis at the angle $\alpha = \arctg \beta$. At various values of the velocity $\beta$ different regions of the spectral density are sampled. Note, that $\alpha \leq 45^\circ$. In reality, due to multi-particle–multi-hole continua all states have finite widths, and contributions to sum rules are collected from everywhere along the dashed lines.

FIG. 2: Toy model. Relative contribution to the sum rules as a function of velocity $\beta$. The isovector density is set arbitrarily to $\rho_{t=1} = -F^2_{\pi} m_\pi$.  

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FIG. 1.
FIG. 2.