Quantum Phase transitional patterns in the SD-pair shell model

Yanan Luo,1 Yu Zhang,2 Xiangfei Meng,1 Feng Pan,3, 4 and Jerry P. Draayer4

1Department of Physics, Nankai University, Tianjin, 300071, P.R. China
2Department of Physics and State key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, 100871, P.R. China
3Department of Physics, Liaoning Normal University, Dalian 116029, P.R. China
4Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

(Dated: April 24, 2009)

Abstract

Patterns of shape-phase transition in the proton-neutron coupled systems are studied within the SD-pair shell model. The results show that some transitional patterns in the SD-pair shell model are similar to the $U(5) - SU(3), U(5) - SO(6)$ transitions with signatures of the critical point symmetry of the interacting boson model.

PACS numbers: 21.60.Cs
I. INTRODUCTION

Recently, based on the Generalized Wick Theory [1], a nucleon-pair shell model (NPSM) was proposed [2], in which nucleon pairs with various angular momenta are used as building blocks. Since modern computers fail for the calculation in the full shell model space for the medium weight and heavy nuclei, some truncation scheme need to be used.

The tremendous success of the interacting boson model (IBM) [3], suggests that \( S \) and \( D \) pairs play a dominant role in the spectroscopy of low-lying nuclear modes [4, 5, 6]. Therefore, one normally truncates the full shell-model space to the collective \( S-D \) pair subspace in the NPSM. The latter is called the \( SD \)-pair shell model (SDPSM) [2, 7, 8].

A crucial point in the SDPSM is the validity of the \( S-D \) pair truncation. In Ref. [9], shell model foundations of the IBM was reviewed by Iachello, the results seem to indicate that the \( S-D \) pair truncation is a reasonable approximation to the full shell model space. This problem was also studied in [10, 11, 12] with the conclusion that the \( S-D \) pair subspace works well in the vibrational region, but in the deformed region, the inclusion of \( G \) pairs is crucial. But Dr. Zhao’s work [13, 14] show that the essential properties within the full shell model space survive in the \( S-D \) pair subspace. What’s more, if a pure quadrupole-quadrupole interaction and a reasonable collective \( S-D \)-pair were used, the rotational behavior can be produced very well within the \( S-D \) pair subspace. The fact that the SDPSM can describe the collectivity of low-lying states for nuclei around \( A=130 \) [15, 16, 17, 18, 19, 20, 21] also imply that the \( S-D \) pair truncation is a good approximation to the full shell model space.

Nuclei, as a mesoscopic system, have been found to possess interesting geometric shapes, such as spherical (vibrational \( (U(5)) \)), axially deformed \( (SU(3)) \), and \( \gamma \)-soft \( (O(6)) \), which is usually described in terms of the Casten triangle [22]. The search for signatures of transitions among various shapes (phases) of atomic nuclei is an interesting subject in nuclear structure theory. An understanding of such shape (phase) transitions may provide insight into quantum phase transitions in other mesoscopic systems [3].

Theoretical study of shape phase transitions and critical point symmetries in nuclei was mainly carried out [3, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40] in the interacting boson model for identical system (IBM-I) [3]. The investigations on nuclear shape phase transition and critical point symmetry for identical nucleon system have also been carried out with fermionic degrees of freedom in [41, 42, 43, 44, 45, 46]. Recently,
investigations of the shape phase transitions and critical point symmetries in nuclei were also carried out\cite{47, 48} in the proton-neutron interaction boson model(IBM-II)\cite{3}.

Since the SDPSM is also built up from $SD$ pairs, it is expected that the SDPSM can produce similar results to those of the IBM. Our previous work show that the vibrational, rotational, and gamma-soft spectra can be well reproduced\cite{49} similar to the $U(5)$, $SU(3)$ and $SO(6)$ limiting spectra in the IBM. What’s more, the vibrational-rotational phase transition for identical system can also be produced within the framework of the SDPSM with fermionic degrees of freedom\cite{50}. Since nuclei are neutron-proton coupled systems, and a rich phase structure can be obtained in the neutron-proton IBM, it is interesting to see if the phase transitional patterns in the neutron-proton coupled system can be produced in the SDPSM with fermionic degrees of freedom. This is the aim of this paper.

II. MODEL

In the shell model description, the pairing and quadrupole-quadrupole interactions are the most important short-range and long-range correlations. Considering that the Hamiltonian used to study the shape phase transition in the IBM is mainly composed of the monopole pairing and quadrupole-quadrupole interaction(e.g., Ref.\cite{47, 48}), a schematic Hamiltonian is adopted in the SDPSM, which is a combination of the monopole pairing interaction and quadrupole-quadrupole interaction with

$$H_X = \sum_{\sigma = \pi, \nu} \left(-G_\sigma S_\sigma S_\sigma - \kappa_\sigma Q_\sigma^2 - \kappa_{Q\pi} Q_{\pi}^2 \right),$$

$$S_\sigma^\dagger = \sum_a \frac{a}{2} \left(C_\sigma^\dagger \times C_\sigma^\dagger \right),$$

$$Q_{(2)} = \sqrt{16\pi/5} \sum_i r_i^2 Y^2 (\theta_i, \phi_i)$$

where $X$ in $H_X$ is denoted as $U(5)$, $SU(3)$, or $SO(6)$ corresponding vibrational, rotational, or gamma-soft limiting case in the model, $G_\sigma$ and $\kappa_\sigma$ are the pairing and quadrupole-quadrupole interaction strength between identical-nucleons, respectively. $\kappa$ is the quadrupole-quadrupole interaction strength between proton and neutrons. In this paper, we set $G_\pi = G_\nu$ and $\kappa_\pi = \kappa_\nu$.

To study the phase transitional patterns, the Hamiltonian for proton-neutron coupled
system is written as

\[ H = (1 - \alpha) H_U(5) + \alpha H_X, \tag{2} \]

where \( 0 \leq \alpha \leq 1 \) is a control parameter, \( H_X \) is taken as \( H_{SU(3)} \) when we study vibration-rotation transitional patterns, and is taken as \( H_{SO(6)} \) when we study vibration to \( \gamma \)-soft transitional patterns.

The \( E2 \) transition operator adopted is

\[ T(E2) = e_\pi Q^{(2)}_{\pi} + e_\nu Q^{(2)}_{\nu}, \tag{3} \]

where \( e_\pi(e_\nu) \) is the effective charge for proton(neutron).

The collective \( S \)-pair is defined as

\[ S^\dagger = \sum_a y(aa0) (C_a^\dagger \times C_a^\dagger)^0 \tag{4} \]

In this paper, the \( S \)-pair structure coefficient, as an approximation, is fixed to be \( y(aa0) = \hat{a} \sqrt{\frac{N}{\Omega_a - N}} \), where \( \Omega_a \) is defined as \( \Omega_a = a + 1/2 \) and \( N \) is the number of pairs for like-nucleons.

The \( D \)-pair is obtained by using commutator

\[ D^\dagger = \frac{1}{2}[Q^{(2)}, S^\dagger] = \sum_{ab} y(ab2) (C_a^\dagger \times C_b^\dagger)^2. \tag{5} \]

After symmetrization, it is easy to obtain that

\[ y(ab2) = -\frac{1}{2} q(ab2) \left[ \frac{y(aa0)}{\hat{a}} + \frac{y(bb0)}{\hat{b}} \right]. \tag{6} \]

The details of the model can be found in \([2, 7, 8]\).

### III. RESULTS

To identify shape phase transitions and determine the corresponding patterns, Iachello et al. initiated a study on effective order parameters, which should display different critical behaviors for the phase transitions with different order. Specifically, the quantities related with isomer shifts, defined as \( v_2 = (< 0^+_1 | \hat{n}_d | 0^+_2 > - < 0^+_1 | \hat{n}_d | 0^+_1 >)/N \) and \( v'_2 = (< 2^+_1 | \hat{n}_d | 2^+_1 > - < 0^+_1 | \hat{n}_d | 0^+_1 >)/N \), were proposed as effective-order parameters in \([33]\). Consequently, some other quantities, such as the \( B(E2) \) ratios \( K_1 = B(E2; 4^+_1 \rightarrow 2^+_1)/B(E2; 2^+_1 \rightarrow 0^+_1) \) and \( K_2 = B(E2; 0^+_2 \rightarrow 2^+_1)/B(E2; 2^+_1 \rightarrow 0^+_1) \) \([40]\) as well as the energy ratio \( R_{60} = E_{6^+_1}/E_{0^+_2} \)
were also suggested as the effective order parameters to identify phase transitions and the corresponding orders. Therefore, to study the shape phase transition in the $SD$-pair fermion model space, $\nu_2, \nu'_2$, in which the d-boson number operator $\hat{n}_d$ is replaced by $D$-pair number operator $\hat{N}_D$ in the SDPSM, $K_1, K_2$ and $R_{60}$ will be studied in this paper. Because the importance of $R_{42} = E_{4^+_1}/E_{2^+_1}$ in determining the limiting cases and shape phase transitions\cite{51}, $R_{42}$ is also presented.

\section{vibration-rotation transitional patterns}

We begin by considering the vibration-rotation phase transition. A system with $N_\pi = N_\nu = 3$ in $gds$ shell was studied. By fitting $R_{42} \equiv E_{4^+_1}/E_{2^+_1} = 2$ for vibrational case, the parameters used to produce the vibrational spectra were obtained, and presented in Table I. The detailed discussion about the vibrational spectra can be found in \cite{49}. In the SDPSM, the full shell model space was truncated to the $SD$-pair subspace. The investigation on the validity of the $S-D$ pair truncation in \cite{10,11,12} show that the $S-D$ pair truncation can not produce the rotational spectra. But Dr. Zhao’s work \cite{13,14} and our previous work \cite{49} show that if a pure quaquadrupole-quadrupole interaction and a reasonable collective $S-D$ pair were used, the rotational behaviors can be produced very well. It is found that with $2\kappa_\pi = 2\kappa_\nu = \kappa = 0.2\text{MeV}/r_0^4$ the similar results as the $SU(3)_\pi \times SU(3)_\nu$ limit of the IBM can be produced, the typical energy ratios $E_{4^+_1}/E_{2^+_1}$ and $E_{6^+_1}/E_{2^+_1}$ are 3.33 and 6.96, close to the IBM result 3.33 and 7. The detailed discussion can be found in Ref.\cite{13,49}.

| limit                          | $G_\pi$ | $G_\nu$ | $\kappa_\pi$ | $\kappa_\nu$ | $\kappa$ |
|-------------------------------|---------|---------|---------------|---------------|---------|
| vibration-rotation vibration  | 0.5     | 0.5     | 0             | 0             | 0.01    |
| rotation                      | 0       | 0       | 0.1           | 0.1           | 0.2     |
| vibration-$\gamma$-soft       | 0.5     | 0.5     | 0             | 0             | -0.01   |
| $\gamma$-soft                 | 0.15    | 0.15    | 0             | 0             | -0.015  |

Energy ratios $R_{42}$ and $R_{60}$ against control parameter $\alpha$ are shown in Fig.1. Fig.1a shows...
that the energy ratio $R_{42}$ is 2 (when $\alpha = 0$) and 3.3 (when $\alpha = 1$), typical values of vibrational and rotational spectra in the IBM[3]. It is also shown that the rapid change occurs when $0.3 \leq \alpha \leq 0.6$, which indicates a phase transition occurs in this region.

The energy ratio $R_{60}$ given in Fig.1b shows that similar behavior as that of the IBM for finite number of boson $N_B$ is reproduced. It exhibits a modest peak followed by a sharp decrease across the phase transition, a typical signature of the 1st-order quantum phase transition[52].

The SDPSM results of $v_2$, $v'_2$, $K_1$ and $K_2$ are given in Fig.2 and Fig.3. The effective charges were fixed with $e_\pi = 3e_\nu = 1.5e$. As argued in [33], $v_2$, $v'_2$ should have wiggling behaviors in the region of the critical point due to the switching of the two coexisting phases for the first order phase transition, then the obvious wiggling behaviors shown by $v_2$, $v'_2$ in Fig.2 further confirm the transition is first order. The results of $B(E2)$ ratio $K_1$ is consistent with those of other effective quantities[33, 40]. The critical behavior of $K_2$ seems to deviate from the character of the first order phase transition.

In the IBM, the critical point symmetry[29] between $U(5)$ and $SU(3)$ is $X(5)$. Since the shape phase transition between vibrational and rotational limit can be reproduced in the SDPSM, it is interesting to see if the properties of the $X(5)$-like symmetry also occurs within the SDPSM. We found that there is indeed a signature with $\alpha = 0.54$ in the SDPSM similar to that of the $X(5)$ in the IBM. A few typical values are given in Table II, from which one can see that typical feature of the $X(5)$ symmetry stated in Ref. [52, 53] indeed occurs in the SDPSM. For example, $R_{42}$, $R_{60}$ and $E_{02^+}/E_{21^+}$ is 2.91, 1.05 and 5.32 in the SDPSM calculation, close to the IBM results 2.91, 1.0 and 5.67, respectively.

**B. vibration-$\gamma$-soft transitional patterns**

The investigation on vibration-$\gamma$-soft shape phase transition in the IBM has been studied in [54], the corresponding quantum phase transition was suggested to be of the 2nd-order. Recently, similar phase transition within the fermion model for identical nucleon system has also been performed[45, 46].

From the periodic chart, one can deduce that nuclei that display an SO(6) spectrum lie close to the end of the shell, at least in the neutron sector. Therefore, to explore whether the transitional patterns between vibration and $\gamma$-soft spectrum can be realized in the SDPSM,
FIG. 1: $R$ vs $\alpha$ for the vibration-rotation transition.

FIG. 2: $v_2$ and $v'_2$ vs $\alpha$ in the vibration-rotation transition.

FIG. 3: B(E2) ratios vs $\alpha$ in the vibration-rotation transition.
TABLE II: Energy and B(E2) ratios at vibrational, rotational limit, and X(5)-like critical point calculated in the SDPSM.

| limit                  | $E_{4^+}^{0^+}$ | $E_{6^+}^{0^+}$ | $E_{6^+}^{4^+}$ | $E_{2^+}^{0^+}$ | $E_{2^+}^{3^+}$ | $E_{2^+}^{6^+}$ | $E_{2^+}^{7^+}$ | $E_{0^+}^{1^+}$ | $E_{0^+}^{2^+}$ | $E_{0^+}^{3^+}$ | $E_{0^+}^{4^+}$ |
|------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| vibrational limit      | 1.99            | 2.97            | 1.47            | 1.49            | 1.48            |                 |                 |                 |                 |                 |                 |
| X(5)-like point        | 2.91            | 5.60            | 1.05            | 1.38            | 1.38            |                 |                 |                 |                 |                 |                 |
| rotational limit       | 3.33            | 6.96            | 0.46            | 1.34            | 1.32            |                 |                 |                 |                 |                 |                 |
| X(5)-like point (0$^+_2$ band) | 5.32          | 2.30            | 5.33            | 0.37            | 0.43            |                 |                 |                 |                 |                 |                 |

we considered a system with $N_\pi = \tilde{N}_\nu = 3$ in the $gds$ shell. Namely, neutron pairs in this case were treated as three neutron-hole pairs and a negative $\kappa$ was used as in [49]. By fitting $R_{42} = 2$ and 2.5 for vibrational and $\gamma$-soft limiting cases, the parameters were fixed, and the results are listed in Table I. The detailed discussion about the two limiting cases in the SDPSM can be found in [49].

The IBM calculation show that the level crossing-repulsion behavior of $0^+_2$ and $0^+_3$ occurs [55] in the critical region of the $U(5)-SO(6)$ transition. The SDPSM results of $0^+_2$ and $0^+_3$ states, given in Fig[4], show the similar behavior of level crossing-repulsion when $\alpha = 0.58$. Therefore, to see the behavior of effective order parameters against the control parameters clearly, the quantities related to $0^+_2$ state were also calculated for the $0^+_3$ state.

The results for $R_{42}(R_{60})$, $K_1(K_2)$ and $v_2(v'_2)$ are given in Fig[5] Fig[6] and Fig[7], respectively. The effective charges were fixed as $e_\pi = -3e_\nu = 1.5e$ since the neutron pairs were treated as holes.

Fig[5] shows that the typical ratios, $R_{42} = 2$ (when $\alpha = 0$) and 2.47 (when $\alpha = 1$), of vibration and $\gamma$-soft spectra were produced. Interestingly, we found that in comparison with that of the rotation-vibration transitional results, $R_{42}$ in the vibration-$\gamma$-soft transitional region increases with $\alpha$ smoothly.

From Fig[6] one can see that as the IBM results [40] and $R_{42}$ given in Fig[5], the wiggling behavior in $K_1$ is smoothed out in the vibration-$\gamma$-soft transition. One can also see that because the structure of $0^+_2$ and $0^+_3$ exchange at $\alpha \sim 0.58$, the amplitudes of $B(E2; 0^+_2 \rightarrow 2^+_1)$
FIG. 4: Energy levels of $0_2^+$ and $0_3^+$ states vs $\alpha$ in the vibration-$\gamma$-soft transitional region.

and $B(E2; 0_3^+ \rightarrow 2_1^+)$ also exchange at this point.

In [52], the experimental data of Xe and Ba isotopes were analyzed, in which for smaller neutron numbers, $^{134,136}$Ba and $^{128}$Xe, the $0_3^+$ state was taken in the $R_{60}$ if its $B(E2)$ decay was consistent with $\sigma = N - 2$. It was also shown that $[53] \frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} = 0.07$ for $^{196}$Pt, while it is 0.81 for $^{198}$Pt. By considering these results, the $0_3^+$ state were taken in the $R_{60}$, $K_2$ and $v_2'$ when $\alpha > 0.58$. In comparison with those in the vibration-rotation transition, Fig.5b and Fig.6a show that $R_{60}$, $K_1$ and $K_2$ change smoothly with $\alpha$, which are the typical features of the 2nd-order phase transition [40, 52].

Fig.7 shows that as predicted in the IBM and shell model calculation for identical system, the vibration-$\gamma$-soft phase transition takes place and it is the second order phase transition, for which $v_2$ and $v_2'$ change smoothly with $\alpha$, the wiggling behavior changing sign in the region of the critical point are smoothed out.

In the $U(5)$-$SO(6)$ transitional region in the IBM, $E(5)$ is the critical point symmetry [54, 56]. It is interesting to see whether the signature of the $E(5)$-like symmetry can be realized in the SDPSM for proton-neutron coupled system. We found that $E_{4_1^+}/E_{2_1^+} = 2.19$ when $\alpha = 0.54$ corresponding to the typical value of $E(5)$ symmetry in the IBM. Other typical results are listed in Table III in which the IBM results for $N = 5$ are also given [54]. It is seen that except for $E_{0_2^+}/E_{2_1^+} = 2.59$, which is smaller than that of the IBM result for $N = 5$, the properties of $E(5)$ symmetry in the IBM indeed occurs in the SDPSM.
FIG. 5: Energy ratios $R_{42}$ and $R_{60}$ vs $\alpha$ in the vibration-$\gamma$-soft transitional region.

FIG. 6: $B(E2)$ ratios vs $\alpha$ in the vibration-$\gamma$-soft transitional region.

FIG. 7: $v_2$ and $v'_2$ vs $\alpha$ in the vibration-$\gamma$-soft transitional region.
TABLE III: The SDPSM results for $E(5)$-like symmetry. The corresponding results with $N = 5$ in the IBM are also given.

| limit     | $E_{4^+}/E_{2^+}$ | $E_{0^+}/E_{0^+}$ | $E_{0^+}/E_{2^+}$ |
|-----------|--------------------|--------------------|--------------------|
| SDPSM     | 2.19               | 0.99               | 2.59               |
| IBM       | 2.19               | 1.04               | 3.68               |
| $4^+-2^+$ | $2^+-2^+$          | $0^+-2^+$          |
| SDPSM     | 1.36               | 1.29               | 0.53               |
| IBM       | 1.38               | 1.39               | 0.51               |
| $0^+-2^+$ | $0^+-2^+$          |
| SDPSM     | 0.06               | 0.03               |
| IBM       | 0                  | 0                  |

IV. SUMMARY

In summary, the shape phase transition patterns for proton-neutron coupled system were studied within the framework of the SD-pair shell model. The results show that patterns of vibration-rotation and vibration-$\gamma$-soft shape phase transitions are indeed similar to the corresponding results obtained from the IBM previously. The signatures of the critical point symmetry in the SD-pair shell model are also close to those shown in the IBM. The procedure may be extended to study quantum phase transitions in other fermion systems.

This work was supported in part by the Natural Science Foundation of China (10675063; 10775064), the U.S. National Science Foundation (0140300; 0500291), the Education Department of Liaoning Province (20060464), and the LSU–LNNU joint research program (LSU-9961).

[1] J. Q. Chen, Nucl. Phys. A562(1993)218.
[2] J. Q. Chen, Nucl. Phys. A 626 (1997) 686.
[3] F. Iachello and A. Arima, The Interacting Boson Model, Cambridge University Press, Cam-
bridge New York, 1987.

[4] J. B. McGrory, Phys. Rev. Lett. 41, 533 (1978).
[5] T. Otsuka, Nucl. Phys. 368, 244 (1981).
[6] P. Halse, L. Jaqua and B. R. Barret, Phys. Rev. C 40, 968 (1989).
[7] J. Q. Chen and Y. A. Luo, Nucl. Phys. A 639 (1998) 615.
[8] Y. M. Zhao, N. Yoshinaga, S. Yamaji, J. Q. Chen and A. Arima, Phys. Rev. C62, 014304(2000).
[9] F. Iachello and I. Talmi, Rev. Mod. Phys. 59(1987)339.
[10] N. Yoshinaga, T. Mizusaki, A. Arima, Y.D. Devi, Prog. Theor. Phys. Suppl. 125(1996)65.
[11] N. Yoshinaga, D.M. Brink, Nucl. Phys. A515(1990)1.
[12] T. Mizusaki, T. Otsuka, Prog. Theor. Phys. Suppl. 125(1996)97.
[13] Y. M. Zhao, N. Yashinaga, S. Yamji and A. Arima, Phys. Rev. C62(2000)014316.
[14] Y. M. Zhao, S. Pittel, R. Bijker, A. Frank and A. Arima, Phys. Rev. C66(2002)041301(R).
[15] Y. A. Luo, J. Q. Chen and J. P. Draayer, Nucl. Phys. A669, 101(2000).
[16] X. F. Meng, F. R. Wang, Y. A. Luo, F. Pan and J. P. Draayer, Phys. Rev. C77(2008)047304.
[17] Y. M. Zhao, N. Yoshinaga, S. Yamaji, and A. Arima, Phys. Rev. C62, 014316(2000).
[18] Y. M. Zhao, S. Yamaji, N. Yoshinaga, and A. Arima, Phys. Rev. C62, 014315(2000).
[19] Y. M. Zhao, N. Yoshinaga, S. Yamaji, and A. Arima, Phys. Rev. C62, 024322(2000).
[20] L. Y. Jia, H. Zhang and Y. M. Zhao, Phys. Rev. C 75, 034307 (2007).
[21] N. Yoshinaga and K. Higashiyama, Phys. Rev. C69, 054309(2004).
[22] R. F. Casten, in: F. Iachello (Ed.), Interacting Bose-Fermi System, Plenum, New York, 1981.
[23] J. N. Ginocchio and M. W. Kirson, Phys. Rev. Lett.44(1980)1744.
[24] A. E. L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. 44 (1980) 1747.
[25] D. H. Feng, R. Gilmore, and S. R. Deans, Phys. Rev. 23 (1981) 1254.
[26] P. Van Isacker and J. Q. Chen, Phys. Rev. C24(1981)684.
[27] F. Iachello, N. V. Zamfir and R. F. Casten, Phys. Rev. Lett.81 (1998) 1191.
[28] R. F. Casten and N. V. Zamfir, Phys. Rev. Lett. 85(2000)3584.
[29] F. Iachello, Phys. Rev. Lett. 87(2001)052502.
[30] J. Jolie, P. Cejnar, R. C. casten, S. Heinze, A. Linnemann and V. Werner, Phys. Rev. lett. 89(2002)182502.
[31] J. Jolie, R. C. casten, P. von Brentano and V. Werner, Phys. Rev. lett. 87(2001)162501.
[32] D. Warner, Nature 420(2002)614.
[33] F. Iachello and N. V. Zamfir, Phys. Rev. Lett. 92(2004)212501.
[34] D. J. Rowe, Phys. Rev. Lett. 93(2004)122502.
[35] D. J. Rowe, P. S. Turner and G. Rosenstell, Phys. Rev. Lett. 93(2004)232502.
[36] P. Cejnar, S. Heinze and J. Dobeš, Phys. Rev. C71(2005)011304R.
[37] Y. X. Liu, L. Z. Mu and H. Wei, Phys. Lett. B633(2006)49; Y. Zhao, Y. Liu, L. Z. Mu and Y. X. Liu, Int. J. Mod. Phys. E15(2006)1711.
[38] F. Pan, J. P. Draayer, and Y. A. Luo, Phys. Lett. B 576 (2003) 297.
[39] A. leviatan, Phys. Rev. Lett. 77(1996)818; A. leviatan and P. Van Isacker Phys. Rev. Lett. 89(2002)222501; A.leviatan, Phys. Rev. Lett. 98(2007)242502.
[40] Y. Zhang, Z. F. Hou and Y. X. Liu, Phys. Rev. C76(2007)011305(R).
[41] W. M. Zhang, D. H. Feng and J. N. Ginocchio, Phys. Rev. Lett. 59(1987)2032.
[42] W. M. Zhang, D. H. Feng and J. N. Ginocchio, Phys. Rev. C37(1988)1281.
[43] D. J. Rowe, C. Bahri and W. Wijesundera, Phys. Rev. Lett. 80(1998) 4394.
[44] C. Bahri, D. J. Rowe and W. Wijesundera, Phys. Rev. C58 (1998)1539.
[45] Y. X. Liu, Z. F. Hou, Y. Zhang abd H. Wei, arXiv:nucl-th/0611035v1.
[46] J. N. Ginocchio, Phys. Rev. C71(2005)064325.
[47] J. M. Arias, J. E. García-Ramos and J. Dukelsky, Phys. Rev. Lett. 93(2004)212501 and the reference cited in this paper.
[48] M. A. Caprio and F. Iachello, Phys. Rev. Lett. 93 (2004)242502.
[49] Y. A. Luo, Feng Pan, Chairul Bahri, and J. P. Draayer, Phys. Rev. C 71 (2005) 044304.
[50] Y. A. Luo, F. Pan, T. Wang, P. Z. Ning and J. P. Draayer, Phys. Rev. C73(2006)044323.
[51] V. Werner, P. von Brentano, R. F. Casten and J. Jolie, Phys. Lett. B527(2002)55.
[52] D. Bonatsos, E. A. McCutchan, R. F. Casten and R. J. Casperson, Phys. Rev. Lett. 100(2008)142501.
[53] R. M. Clark, et. al., Phys. Rev. C68(2003)037301.
[54] F. Iachello, Phys. Rev. Lett. 85(2000)3580.
[55] F. Pan, T. wang, Y. S Huo and J. P. Draayer, Int. J. Mod. Phys. E15(2006)1723.
[56] A. Leviatan and J. N. Ginocchio, Phys. Rev. Lett. 90(2003)212501.