A logistic regression analysis approach for sample survey data based on phi-divergence measures

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Abstract A new family of minimum distance estimators for binary logistic regression models based on $\phi$-divergence measures is introduced. The so called “pseudo minimum phi-divergence estimator” (PM$\phi$E) family is presented as an extension of “minimum phi-divergence estimator” (M$\phi$E) for general sample survey designs and contains, as a particular case, the pseudo maximum likelihood estimator (PMLE) considered in Roberts et al. [8]. Through a simulation study it is shown that some PM$\phi$Es have a better behaviour, in terms of efficiency, than the PMLE.

Keywords and phrases: Logistic regression models, Sample survey data, Phi-divergence measures, Maximum likelihood estimator, Pseudo maximum likelihood estimator.

1 Introduction

Suppose that the population of interest is partitioned into $I$ cells or domains according to the levels of one or more factors. Let $N_i$ ($i = 1, ..., I$) denote the $i$-th domain size, $N = \sum_{i=1}^{I} N_i$ the population domain total and $N_{1i}$, the population counts, out of $N_i$, where the binary response (0 for failure and 1 for success) variable is equal to 1. Since $N_{1i}$ and $N_i$ are fixed but unknown values ($i = 1, ..., I$), $\hat{N}_i$ denotes the survey estimator of the $i$-th domain size

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I had the great honor to have Pedro as a friend; a deeply beloved friend. We have shared not only many scientific interests in connection with Statistical Information Theory but also, in accordance with our frequent conversations, many similarities in our youthful life experiences. The research in this paper, written with two coauthors, is dedicated in memoriam to Professor Pedro Gil.
theory provides an estimator of the covariance matrix of the \( \hat{\beta} \). Another choice is using the logistic regression

\[
\pi(x_i^T \beta) = \frac{\exp\{x_i^T \beta\}}{1 + \exp\{x_i^T \beta\}}, \quad i = 1, ..., I,
\]

(1)

to modelize the population proportion of successful events,

\[
\pi_i = \pi(x_i^T \beta) = \frac{N_{i1}}{N_i},
\]

which is assumed to depend on constants \( x_{ij}, \ j = 1, ..., k \) (\( k < I \) derived from the factor levels, summarized in a \((k + 1)\)-vector of known constants \( x_i = (1, x_{i1}, ..., x_{ik})^T \), and also on a \((k + 1)\)-vector of parameters \( \beta = (\beta_0, \beta_1, ..., \beta_k)^T \).

Under independent binomial sampling in each domain, it is well-known that the maximum likelihood estimator (MLE) of \( \beta, \hat{\beta} \), is obtained through iterative calculations from the following likelihood equations

\[
X^T \text{diag}(n)\pi(\beta) = X^T \text{diag}(n)\tilde{q},
\]

(2)

where \( X = (x_1, ..., x_I)^T \) is a full rank matrix, \( \pi(\beta) = (\pi(x_1^T \beta), ..., \pi(x_I^T \beta))^T \), \( \tilde{q} = (\tilde{q}_1, ..., \tilde{q}_I)^T \) with \( \tilde{q}_i = n_{i1}/n_i \), \( n_i \) being the sample size from the \( i \)-th domain, \( n = \sum_{i=1}^I n_i \) the \( i \)-th sample domain total and \( n_{i1} \) the sample total of successful events the \( i \)-th domain. If we consider the probability vectors

\[
\hat{p}^* = \left( \frac{n_1}{n} \tilde{q}_1, \frac{n_1}{n} (1 - \tilde{q}_1), ..., \frac{n_I}{n} \tilde{q}_I, \frac{n_I}{n} (1 - \tilde{q}_I) \right)^T
\]

\[
= \left( \frac{n_{11}}{n}, \frac{n_1 - n_{11}}{n}, ..., \frac{n_{I1}}{n}, \frac{n_I - n_{I1}}{n} \right)^T
\]

\[
= \left( \frac{n_{11}}{n}, \frac{n_{12}}{n}, ..., \frac{n_{I2}}{n} \right)^T, \quad (n_{i2} = n_i - n_{i1}),
\]

and

\[
p^*(\beta) = \left( \frac{n_1}{n} \pi(x_1^T \beta), \frac{n_1}{n} (1 - \pi(x_1^T \beta)), ..., \frac{n_I}{n} \pi(x_I^T \beta), \frac{n_I}{n} (1 - \pi(x_I^T \beta)) \right)^T
\]

the MLE of \( \beta, \hat{\beta} \), can be equivalently defined by

\[
\hat{\beta} = \arg \min_{\beta \in \mathbb{R}^{k+1}} d_{\text{allback}} (\hat{p}^*, p^*(\beta)),
\]
where \( d_{\text{Kullback}} (\hat{p}^*, p^*(\beta)) \) is the Kullback-Leibler divergence between the probability vectors \( \hat{p}^* \) and \( p^*(\beta) \) defined by

\[
d_{\text{Kullback}} (\hat{p}^*, p^*(\beta)) = \sum_{i=1}^{I} \left[ \frac{n_{i1}}{n} \log \frac{n_{i1}}{n_i \pi(x_i^T \beta)} + \frac{n_{i2}}{n} \log \frac{n_{i2}}{n_i (1 - \pi(x_i^T \beta))} \right].
\]

In Pardo et al. [5] the minimum phi-divergence estimator (M\( \phi \)E) was introduced, as a natural extension of the MLE, as

\[
\hat{\beta}_\phi = \arg \min_{\beta \in \mathbb{R}^{k+1}} d_\phi (\hat{p}^*, p^*(\beta)),
\]

where \( d_\phi (\hat{p}^*, p^*(\beta)) \) is the phi-divergence measure between the probability vectors \( \hat{p}^* \) and \( p^*(\beta) \) given by

\[
d_\phi (\hat{p}^*, p^*(\beta)) = \sum_{i=1}^{I} \frac{n_{i}}{n} \left[ \pi(x_i, \beta) \phi \left( \frac{n_{i1}}{n_i \pi(x_i^T \beta)} \right) \right.

\[
+ \left. (1 - \pi(x_i, \beta)) \phi \left( \frac{n_{i2}}{n_i (1 - \pi(x_i^T \beta))} \right) \right],
\]

with \( \phi \in \Phi^* \). By \( \Phi^* \) we are denoting the class of all convex functions, \( \phi(x), x > 0 \), such that at \( x = 1 \), \( \phi(1) = \phi'(1) = 0 \), and at \( x = 0 \), \( 0 \phi(0/0) = 0 \) and \( 0 \phi(p/0) = p \lim_{u \to \infty} \frac{\phi(u)}{u} \). For every \( \phi \in \Phi^* \), differentiable at \( x = 1 \), the function

\[
\Psi(x) = \phi(x) - \phi'(1)(x - 1)
\]

also belongs to \( \Phi^* \). Therefore, we have \( d_\phi (\hat{p}^*, p^*(\beta)) = d_\psi (\hat{p}^*, p^*(\beta)) \) and \( \psi \) has the additional property that \( \psi'(1) = 0 \). Since the two divergence measures are equivalent, we can consider the set \( \Phi^* \) to be equivalent to the set

\[
\Phi = \Phi^* \cap \{ \phi : \phi'(1) = 0 \}.
\]

For more details see Cressie and Pardo [3] and Pardo [7]. In what follows, we give our theoretical results for \( \phi \in \Phi \), but often apply them to choices of functions in \( \Phi^* \).

For general sample survey designs we do not have maximum likelihood estimators due to difficulties in obtaining appropriate likelihood functions. Hence, it is a common practice to use a pseudo maximum likelihood estimator (PMLE) of \( \beta, \hat{\beta}_P \), obtained from (2) by replacing \( n_i/n, i = 1, ..., I \), by the estimated domain relative size \( w_i = \hat{N}_i/\hat{N}, i = 1, ..., I \), and the sample proportions \( \hat{q}_i = n_{i1}/n_i, i = 1, ..., I \), by the ratio estimate \( \hat{p}_i = \hat{N}_{i1}/\hat{N}_i, i = 1, ..., I \),

\[
X^T \text{diag}(w) \pi(\beta) = X^T \text{diag}(w) \hat{p},
\]

where \( w = (w_1, ..., w_I)^T \).
In this paper we extend the concept of MoE by considering the “pseudo minimum phi-divergence estimator” (PMϕE) as a natural extension of the PMLE and we solve some statistical problem for the model considered in (1). In Section 2 we shall introduce the PMϕE for general sample designs and we study its asymptotic behavior. A numerical example is presented in Section 3 and, finally, in Section 4 a simulation study is carried out.

2 Pseudo minimum phi-divergence estimator for general sample designs

For general sample designs, we should consider the kernel of the weighted loglikelihood

\[ \ell_w(\beta) = n \sum_{i=1}^{I} w_i \left[ \hat{p}_i \log \pi(x_i^T \beta) + (1 - \hat{p}_i) \log(1 - \pi(x_i^T \beta)) \right], \]

which is derived from the kernel of the likelihood for \( I \) independent binomial random variables

\[ \ell(\beta) = \sum_{i=1}^{I} n_i \left[ \hat{q}_i \log \pi(x_i^T \beta) + (1 - \hat{q}_i) \log(1 - \pi(x_i^T \beta)) \right] \]

\[ = n \sum_{i=1}^{I} \frac{n_i}{n} \left[ \hat{q}_i \log \pi(x_i^T \beta) + (1 - \hat{q}_i) \log(1 - \pi(x_i^T \beta)) \right], \]

replacing \( \frac{n_i}{n} \) by \( w_i = \hat{N}_i / \hat{N} \), and \( \hat{q}_i = n_{i1} / n_i \) by \( \hat{p}_i = \hat{N}_{i1} / \hat{N}_i, \) \( i = 1, \ldots, I \). If we consider the two probability vectors

\[ \hat{p}_w = (w_1 \hat{p}_1, w_1 (1 - \hat{p}_1), \ldots, w_I \hat{p}_I, w_I (1 - \hat{p}_I))^T \]

and

\[ p_w(\beta) = (w_1 \pi(x_1^T \beta), w_1 (1 - \pi(x_1^T \beta)), \ldots, w_I \pi(x_I^T \beta), w_I (1 - \pi(x_I^T \beta)))^T, \]

we get

\[ \ell_w(\beta) = -nd_{\text{Kullback}}(\hat{p}_w, p_w(\beta)) + k, \]

where \( k \) is a constant not depending on \( \beta \). Therefore the PMLE of \( \beta, \hat{\beta}_p \), presented in (4) can be defined as

\[ \hat{\beta}_p = \arg \max_{\beta \in \mathbb{R}^{k+1}} \ell_w(\beta) = \arg \min_{\beta \in \mathbb{R}^{k+1}} d_{\text{Kullback}}(\hat{p}_w, p_w(\beta)). \]
Based on the previous interpretation of the PMLE, in the following definition we shall present the PMφE.

**Definition 1.** The PMφE in a general sample design for the parameter \( \beta \) in the model considered in (1) is defined as

\[
\hat{\beta}_{\phi,P} = \arg\min_{\beta \in \mathbb{R}^{k+1}} d_{\phi} (\hat{p}_w, p_w (\beta)),
\]

where

\[
d_{\phi} (\hat{p}_w, p_w (\beta)) = \sum_{i=1}^{I} w_i \left[ \pi (x_i, \beta) \phi \left( \frac{\hat{p}_i}{\pi (x_i, \beta)} \right) + (1 - \pi (x_i, \beta)) \phi \left( \frac{1 - \hat{p}_i}{1 - \pi (x_i, \beta)} \right) \right]
\]

is the phi-divergence measure between the probability vectors \( \hat{p}_w \) and \( p_w (\beta) \).

The following result establishes the asymptotic distribution of the PMφE of \( \beta, \hat{\beta}_{\phi,P} \).

**Theorem 1.** Let us assume that \( \beta_0 \) is the true value of \( \beta \) and

\[
w \xrightarrow{p} W, \quad W = (W_1, ..., W_I)^T, \quad W_i = \frac{N_i}{N},
\]

\[
\hat{p} \xrightarrow{p} \pi (\beta_0), \quad \sqrt{n}(\hat{p} - \pi (\beta_0)) \xrightarrow{L} N(0, V).
\]

Then, we have

\[
\sqrt{n}(\hat{\beta}_{\phi,P} - \beta_0) \xrightarrow{L} N(0_{k+1}, V (\beta_0)),
\]

where

\[
V (\beta_0) = (X^T \Delta X)^{-1} X^T \text{diag} (W) V \text{diag} (W) X (X^T \Delta X)^{-1},
\]

\[
\Delta = \text{diag} \{ W_i \pi (x_i^T \beta_0) (1 - \pi (x_i^T \beta_0)) \}_{i=1,...,I}.
\]

**Proof.** Based on Theorem 1 in Pardo et al. [5], we have

\[
\hat{\beta}_{\phi,P} = \beta_0 + (X^T \Delta X)^{-1} X^T \text{diag} \{ c_i^T \}_{i=1}^I \text{diag}^{-1/2} (p_w (\beta_0)) (\hat{p}_w - p_w (\beta_0))
\]

\[
+ o \left( \text{diag} \{ c_i^T \}_{i=1}^I \text{diag}^{-1/2} (p_w (\beta_0)) (\hat{p}_w - p_w (\beta_0)) \right)_{1k+1}
\]

with

\[
c_i = \left( w_i \pi (x_i^T \beta_0) (1 - \pi (x_i^T \beta_0)) \right)^{1/2} \left( (1 - \pi (x_i^T \beta_0))^{1/2} - \pi (x_i^T \beta_0)^{1/2} \right), \quad i = 1, .., I.
\]
Since 
\[ \text{diag} \left\{ \mathbf{c}_i^T \right\}_{i=1}^{I \times 1} \mathbf{w}^{-1/2} \left( \mathbf{p}^w (\beta_0) \right) \left( \hat{\mathbf{p}}^w - \mathbf{p}^w (\beta_0) \right) = \text{diag} \left( \mathbf{w} \right) \left( \hat{\mathbf{p}} - \pi (\beta_0) \right), \]
\[ \mathbf{w} \xrightarrow{p}{\mathcal{N}} \mathbf{W} \quad \text{and} \quad \hat{\mathbf{p}} \xrightarrow{p}{\mathcal{N}} \pi (\beta_0), \]
it holds
\[ \sqrt{n} (\hat{\mathbf{p}} - \pi (\beta_0)) = \left( \hat{\mathbf{p}}^w - \mathbf{p}^w (\beta_0) \right) = \text{diag} \left( \mathbf{w} \right) \left( \hat{\mathbf{p}} - \pi (\beta_0) \right) + o_p(1) \]
\[ \xrightarrow{p}{\mathcal{N}} \mathbf{N} (0_{k+1}, \mathbf{V}). \]

From the Shuysky’s theorem and taking into account
\[ \sqrt{n} (\hat{\mathbf{p}} - \pi (\beta_0)) \xrightarrow{L}{\mathcal{N}} \mathbf{N} (0_{k+1}, \mathbf{V}), \]
it follows the desired result. \( \square \)

**Remark 1.** Under independent binomial sampling in each domain, it is well-known that 
\[ \mathbf{V} = \text{diag} \left\{ \pi \left( x^T \beta_0 \right) \left( 1 - \pi \left( x^T \beta_0 \right) \right) \right\}_{i=1,...,I \times 1} \text{diag}^{-1} \left( \mathbf{W} \right) \]
and hence
\[ \mathbf{V} (\beta_0) = \left( \mathbf{X}^T \Delta \mathbf{X} \right)^{-1}, \]
which matches Theorem 2 in Pardo et al. [5].

**Remark 2.** The asymptotic results obtained in the current paper differ from Castilla et al. [2] in the elements tending to infinite, here the total individuals in the whole sample, \( n \), while in the cited paper is the total number of clusters what tends to infinite.

### 3 A numerical example

In order to obtain the PM\( \phi \)Es, from a practical point of view, we can give an explicit expression for \( \phi \). In this paper we shall focus on the Cressie-Read subfamily
\[ \phi_{\lambda}(x) = \begin{cases} 
\frac{1}{\lambda(1+\lambda)} \left( x^{\lambda+1} - x - \lambda(x-1) \right), & \lambda \in \mathbb{R} - \{-1, 0\} \\
\lim_{\nu \to \lambda} \frac{1}{\nu(1+\nu)} \left[ x^{\nu+1} - \nu(x-1) \right], & \lambda \in \{-1, 0\} 
\end{cases}. \]

We can observe that for \( \lambda = 0 \), we have
\[ \phi_{\lambda=0}(x) = \lim_{\nu \to 0} \frac{1}{\nu(1+\nu)} \left[ x^{\nu+1} - \nu(x-1) \right] = x \log x - x + 1, \]
and the associated phi-divergence, coincides with the Kullback divergence, therefore the PM\( \phi \)Es based on \( \phi_{\lambda}(x) \) contains as special case the PMLE.

We shall consider the example presented in Molina et al. [4]. A random subsample of 50 clusters (primary sampling units) containing 1299 households was selected from the 1975 U.K. Family Expenditure Survey. These households are divided into 12 groups of sizes \( n_1, ..., n_{12} \) by age of head of household (4 levels) and number of persons in the household (3 levels). The binary response is 1 if the household owns the dwelling it occupies and 0
otherwise. The number of households \( r_i \) for which the binary response is 1, together with \( n_i \) are shown in Table 1 of the cited paper.

We denote by \( \beta_1(r) \) the parameter associated to the level \( r \) of the factor “age of head of households”, \( r = 2, 3 \) and \( 4 \) since \( \beta_1(1) = 0 \) and by \( \beta_2(s) \) the parameter associated to the level \( s \) of the factor “number of persons in the households”, \( s = 2, 3 \), since we assume \( \beta_2(1) = 0 \). The parameter vector with unknown values will be denote by

\[
\beta = (\beta_0, \beta_1(2), \beta_1(3), \beta_1(4), \beta_2(2), \beta_2(3))^T.
\]

The design matrix that we are going to consider for the example under consideration is given by

\[
X = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
\end{pmatrix}
\]

and the logistic regression model under consideration is given by

\[
\pi(x_i^T \beta) = \frac{\exp \{x_i^T \beta\}}{1 + \exp \{x_i^T \beta\}}, \quad i = 1, \ldots, 12,
\]
equivalent to

\[
\pi(x_i^T \beta) = \frac{\exp \{\beta_0 + \beta_1(r) + \beta_2(s)\}}{1 + \exp \{\beta_0 + \beta_1(r) + \beta_2(s)\}},
\]

if the \( i \)-th probability is associated with the \( r \)-th level of the first variable (\( r = 1, \ldots, 4 \)) and the \( s \)-th level of the second variable (\( s = 1, \ldots, 3 \)). In the following table we present the pseudo minimum Cressie-Read divergence estimators (PMCREs) of \( \beta, \beta_{\lambda, P} \), for \( \lambda \in \{0, 2/3, 1, 2\} \).

| \( \lambda \) | \( \beta_{0, \lambda, P} \) | \( \beta_{1(2), \lambda, P} \) | \( \beta_{1(3), \lambda, P} \) | \( \beta_{1(4), \lambda, P} \) | \( \beta_{2(2), \lambda, P} \) | \( \beta_{2(3), \lambda, P} \) |
|---|---|---|---|---|---|---|
| 0  | -0.1585 | 0.4403 | -0.1412 | -0.4179 | 0.5042 | 0.4703 |
| 2/3 | -0.1564 | 0.4291 | -0.1436 | -0.4174 | 0.4985 | 0.4735 |
| 1  | -0.1574 | 0.4251 | -0.1438 | -0.4158 | 0.4971 | 0.476  |
| 2  | -0.1663 | 0.4192 | -0.1408 | -0.4075 | 0.4974 | 0.4856 |
4 Simulation Study

The following simulation study has been designed by following the previous example. Since in the logistic regression model there are two factors, the first one with 4 categories and the second one with 3 categories, in total \( I = 12 \) domains are taken into account. Let

\[
p(\beta) = \left( \frac{N}{N_i} \pi (x_i^T \beta), \frac{N}{N_i} (1 - \pi (x_i^T \beta)) \right), \ldots, \frac{N}{N_i} \pi (x_i^T \beta), \frac{N}{N_i} (1 - \pi (x_i^T \beta)) \right)^T
\]

be the theoretical probability vector in the logistic regression with complex sampling. The values of the components of \( p(\beta) \) are given in Table 2. In total \( n = 1299 \) individuals are taken from the primary units of the sample, \( J = 50 \) clusters, of size \( m(j) = 26, j = 1, \ldots, 49, m(50) = 25 \) (\( \sum_{j=1}^{50} m(j) = n \)). Since the clusters are mutually independent and there is (possibly) correlation inside each cluster, we consider three possible distributions for

\[
(n_{11(j)}, n_{1(j)} - n_{11(j)}, n_{21(j)}, n_{2(j)} - n_{21(j)}), \ldots, n_{12,1(j)}, n_{12(j)} - n_{12,1(j)})^T
\]

corresponding to the \( j \)-th cluster (column, in Table 3), \( j = 1, \ldots, J = 50 \):

- Dirichlet-multinomial with parameters \((m(j); \rho, p(\beta))\), with \( \rho \in \{ \frac{1}{10} (i - 1) \}_{i=1}^{10} \);
- Random-clumped with parameters \((m(j); \rho, p(\beta))\), with \( \rho \in \{ \frac{1}{10} (i - 1) \}_{i=1}^{10} \);
- \( m(j) \)-inflated with parameters \((m(j); \rho, p(\beta))\), with \( \rho \in \{ \frac{1}{10} (i - 1) \}_{i=1}^{10} \).

For details about these distributions see Alonso et al. [1]. The values of interest for the sample are

\[
n_{i1} = \sum_{j=1}^{50} n_{11(j)}, \quad i = 1, \ldots, I \quad \text{and} \quad n_i = \sum_{j=1}^{50} (n_{11(j)} + n_{12(j)}), \quad i = 1, \ldots, I.
\]

\[
\begin{array}{ccccccccccccc}
\text{Table 2} & \text{Theoretical values of } p(\beta) \text{ in the simulation study.} \\
\hline
i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & I = 12 \\
\hline
\frac{N}{N_i} \pi (x_i^T \beta) & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 0.2 & 0.3 & 0.4 \\
\frac{N}{N_i} (1 - \pi (x_i^T \beta)) & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 0.2 & 0.3 & 0.4 \\
\hline
\end{array}
\]

Notice that the assumptions of Theorem 1 are held. In addition:

- If \( \rho = 0 \) (multinomial distribution within each cluster), then \( \mathbf{V} \) is a diagonal matrix since the elements of \( \hat{p} \) are uncorrelated. In this case, we obtain MLEs and M\( \phi \)Es.
- If \( \rho > 0 \), then \( \mathbf{V} \) is not a diagonal matrix since the elements of \( \hat{p} \) are correlated. In this case, we obtain PMLEs and PM\( \phi \)Es.
In these scenarios, the root of the mean square error (RMSE) for the PM-CREs of \( \beta \) are studied, considering different values of the tuning parameter \( \lambda \in \{0, 2/3, 1, 2\} \). Note that when \( \lambda = 0 \), the corresponding PMCRE of \( \beta \) is equal to the PMLE.

Results of the simulation study with 2,000 samples are shown in Figure 1. As expected from a theoretical point of view, the RMSE increases as \( \rho \) increases. With independence to the distribution considered, estimators corresponding to \( \lambda \in \{2/3, 1, 2\} \) present a better performance than the PMLE (\( \lambda = 0 \)). This difference becomes more considerable for large values of \( \rho \).

### 5 Concluding remarks

In this paper we have considered the problem of estimating the parameters of the logistic regression model for sample survey data, introducing the family of the PM\( \phi \)Es that contains as a particular case the PMLE. A simulation study is carried out in order to see that there are PM\( \phi \)Es that have a better behaviour than the PMLE in relation to the mean square error.
Fig. 1 RMSEs for PMCREs of $\beta$ with Dirichlet-multinomial (above), Random-clumped (middle) and m-inflated (below) distributions.
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