Heavy solitons in a fermionic superfluid

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Solitons—solitary waves that maintain their shape as they propagate—occur as water waves in narrow canals, as light pulses in optical fibres and as quantum mechanical matter waves in superfluids and superconductors. Their highly nonlinear and localized nature makes them very sensitive probes of the medium in which they propagate. Here we create long-lived solitons in a strongly interacting superfluid of fermionic atoms and directly observe their motion. As the interactions are tuned from the regime of Bose–Einstein condensation of tightly bound molecules towards the Bardeen–Cooper–Schrieffer limit of long-range Cooper pairs, the solitons’ effective mass increases markedly, to more than 200 times their bare mass, signalling strong quantum fluctuations. This mass enhancement is more than 50 times larger than the theoretically predicted value. Our work provides a benchmark for theories of non-equilibrium dynamics of strongly interacting fermions.

Solitons are described by a complex macroscopic wavefunction that is rigid against twists of its phase. The ground state of the superfluid thus has uniform phase, and small perturbations propagate as sound waves. A nonlinear excitation—the dark soliton—occurs when the phase is twisted substantially over a short range. In the extreme case of a phase jump by 180°, the wavefunction changes sign and crosses zero at the location of the jump, creating a stationary black soliton. In weakly interacting Bose–Einstein condensates (BECs) all bosons reside in the condensate, so the particle density vanishes at a black soliton, and is reduced for a moving dark soliton. Solitons in BECs have been studied extensively both theoretically and experimentally.

In a series of pioneering experiments, dark solitons have been created via phase-imprinting or in the wake of shock waves. Collisions of two dark solitons and soliton oscillations have been observed. Solitons in weakly interacting BECs are well described as solutions to the Gross–Pitaevskii equation, known in other contexts as the cubic nonlinear Schrödinger equation.

In fermionic superfluids, solitons are phase twists in the wavefunction of fermion pairs. For s-wave superfluids, the pair wavefunction is also known as the pairing gap, which in general can depend on the spatial location. By tuning the interactions between fermions, one can access the crossover from Bose–Einstein condensation of molecules to the Bardeen–Cooper–Schrieffer (BCS) state of long-range Cooper pairs. In the limit of tight molecular pairing, interactions between molecules are weak and the molecular condensate is still described by the Gross–Pitaevskii equation. Stationary solitons are thus again devoid of particles. In this limit, the wavefunction for a stationary soliton, shown in Fig. 1a, depends on position along the z axis as $A(z) = A_0 \tanh(z/\xi)$, where $A_0$ is the magnitude of the wavefunction in the bulk, far away from the soliton, and the soliton width $\xi$ is equal to the healing length of the condensate. The repulsive interactions between the molecular bosons can be increased by means of a Feshbach resonance, allowing the study of strongly interacting Bose gases. Strong interactions increase the importance of quantum fluctuations that are present even at zero temperature, leading to a depletion of the condensate. The uncondensed bosons are expected to fill in the soliton notch, the void at the soliton’s position, in order to minimize their repulsive interaction with the condensate. Figure 1a shows the density profile of the bosons localized at the soliton, the so-called anomalous mode that is predicted to be the main contribution to the density inside the soliton notch. Similar soliton filling has been predicted for BECs in optical lattices, where the effect of interactions and thus the role of quantum fluctuations is enhanced by reducing the particles’ kinetic energy.

Description of solitons in the BEC–BCS crossover

When the interaction strength in the pair condensate becomes of the order of the Fermi energy ($E_F$), the composite nature of the molecules is revealed. The fermion pair size is now of the order of the interparticle spacing, and the system is a crossover superfluid in between the BEC and BCS limits of superfluidity. A unified description for solitons in fermionic superfluids throughout the BEC–BCS crossover has been found within mean-field theory via the Bogoliubov-de Gennes (BdG) equation for a spatially varying gap $A(z)$ (refs 19, 26–28):

$$\left\{ -\frac{\hbar^2 \nabla^2}{2m} - \mu + A(z) \right\} \left( \begin{array}{c} u_x \\ v_x \end{array} \right) = E_0 \left( \begin{array}{c} u_x \\ v_x \end{array} \right)$$  \hspace{1cm} (1)

where $\hbar$ is Planck’s constant divided by $2\pi$, $m$ is the atomic mass, $\mu$ is the chemical potential, $\sigma_{xy} = \tau_1 \tau_2$ are Pauli matrices, and $u_x(z)$ and $v_x(z)$ are the amplitudes describing the particle and hole character of Bogoliubov quasi-particles of energy $E_x$ (we omit spin indices). The order parameter $A(z)$ is related to the quasi-particle amplitudes by the self-consistency relation $A(z) = -g \sum_n n_x(z) v_x(z)$, where $g$ is the coupling strength, tunable via the scattering length between fermions, and $n_x(z)$ denotes complex conjugation. The BdG equations have been shown to reduce to the Gross–Pitaevskii equation for bosonic molecules in the BEC limit, where stationary solitons are devoid of particles. As the interactions are tuned from the BEC to the BCS regime, the BdG equations predict an increasing filling of the soliton. At the
Feshbach resonance, in the unitarity limit where the scattering length diverges, a substantial part of this filling is due to so-called Andreev bound states, localized fermionic states bound to the soliton, also known to reside inside vortex cores\(^2\). Here, the gas density in the vicinity of the soliton is predicted to be suppressed by 80% of the bulk density, as opposed to 100% for solitons in BECs.

In the BCS limit of weak attractive interactions, the BdG equations reduce to the Andreev equation, a Dirac equation where the pairing gap \(A_0\), and density \(n(z)\) of the localized bosonic (fermionic) state versus position \(z\), in the BEC (BCS) regime of the crossover, in units of the BEC healing length (BCS coherence length) \(\xi\). Diagram of the experiment. A phase-imprinting laser beam twists the phase of one-half of the trapped superfluid by approximately \(\pi\). The soliton generally moves at non-zero velocity \(v_{\text{soliton}}\).

The time period is \(T_1 = 12(2)T_F\), much longer than the trapping period of \(T_F = 93.76(5)\) ms, revealing an extreme enhancement of the soliton’s relative effective mass, \(M^*/m\).

Creating solitons in a fermionic superfluid

The creation of solitons in a strongly interacting fermionic superfluid poses several challenges. First, a superfluid with a soliton is not in its ground state, so the temperature of the gas has to be low enough for the soliton not to decay rapidly into thermal excitations. Such dissipation can proceed through collisions of the soliton with sound waves, leading to its acceleration. When the soliton reaches a critical velocity, it is expected to decay into phonons or, in the case of fermionic superfluids, pair excitations\(^27,28,30\). Second, solitons can generally decay into vortices via the so-called snake instability\(^3,13,15,31,32\). In the case of weakly interacting BECs in elongated traps, stability requires the chemical potential \(\mu\) of the condensate to be not much larger than the transverse confinement energy\(^1\). For a Fermi gas, this would require a quasi-one-dimensional geometry where the transverse cloud width is one interparticle spacing. As we show below, this is not necessary. Last, for strongly interacting superfluids, it is a priori not obvious that solitons are stable against quantum fluctuations\(^10,11,21–25,33\).

Here we create and observe long-lived solitons in a strongly interacting fermionic superfluid of \(^6\)Li atoms near a Feshbach resonance. Solitons are created via phase imprinting (see Fig. 1b), a technique successfully employed for weakly interacting Bose condensates\(^13,12,14\).

The superfluid containing typically \(\sim 2 \times 10^5\) atom pairs is prepared in an elongated trap with cylindrical symmetry (axial and radial trapping period respectively \(T_g = 45–210\) ms and \(T_L = 14\) ms) and tunable aspect ratio \(\lambda = T_2/T_L\) (ref. 7). A green laser beam far detuned from the atomic resonance is masked to shine on one half of the superfluid. In a time \(t\), the applied potential \(U\), as experienced by a single fermion, advances the phase of the superfluid order parameter in the exposed region by \(\Delta \phi = 2\pi \mu/\hbar\) relative to the unexposed region. The time \(t = 35\) \(\mu\) s is experimentally adjusted in order to create one high-contrast soliton.

In the strongly interacting regime, the soliton does not cause a density depletion within our resolution. However, it is tied to a phase twist in the pair wavefunction. As in the case of vortices\(^44\), the pair wavefunction can be directly observed via a rapid ramp to the BEC side of the Feshbach resonance. The ramp converts large fermion pairs into tightly bound molecules, empties out the soliton cores and increases the soliton width to the final healing length \(\propto 1/\sqrt{\hbar m_\text{a}}\), where \(m_\text{a}\) is the scattering length at the final magnetic field and \(m\) the density of molecules. The rapid ramp followed by time-of-flight expansion thus enhances the soliton contrast and acts as a magnifying glass (for details, see Supplementary Information).

Figure 1c and d report the observation of solitons in a fermionic superfluid prepared at 815 G (close to the 832 G Feshbach resonance) for various hold times following the phase imprint. Here, the interaction parameter at the cloud centre is \(1/k_\text{a}d = 0.30(2)\), where \(a\) is the scattering length and \(k_\text{a} = (3\pi^2n)^{1/3}\) is the Fermi wavevector, related to the total central fermion density \(n\) and the Fermi energy \(E_F = \hbar^2k_\text{a}^2/2m\). Figure 1c shows the optical density in absorption images taken after time of flight and the rapid ramp to \(\sim 580\) G, while Fig. 1d displays residuals obtained by subtracting a smoothed copy of the same absorption image. The optical density contrast of solitons is about 10% (see Supplementary Information). A sequence of radially integrated residuals as a function of time is displayed in Fig. 1e, demonstrating the soliton to be stable for more than 4 s or 100,000 times the microscopic timescale \(\hbar/E_F\), the Fermi time. This establishes that solitons in fermionic superfluids can exist as stable and long-lived excitations that do not decay despite strong quantum fluctuations.

Soliton oscillations

The solitons are observed to undergo oscillations in the harmonically trapped superfluid, demonstrating their emergent particle nature. The motion is to a high degree deterministic, as soliton positions for different realizations of the experiment at varying wait times lie on the same classical sinusoidal trajectory. The force on the soliton is provided by the trapping force experienced by the atoms missing in the soliton, \(N_m \omega_z^2 \equiv N_m \omega^2\), where \(\omega_z = 2\pi/T_z\), \(N_m\) is the number of missing atoms, and \(M = N_m m < 0\) the bare mass of the soliton. \(M\) is negative as the soliton is a density depletion. Introducing the effective, or

Figure 1 | Creation and observation of solitons in a fermionic superfluid. a, Superfluid pairing gap \(A(z)\) for a stationary soliton, normalized by the bulk pairing gap \(A_0\), and density \(n(z)\) of the localized bosonic (fermionic) state versus position \(z\), in the BEC (BCS) regime of the crossover, in units of the BEC healing length (BCS coherence length) \(\xi\). b, Diagram of the experiment. A phase-imprinting laser beam twists the phase of one-half of the trapped superfluid by approximately \(\pi\). The soliton generally moves at non-zero velocity \(v_{\text{soliton}}\). c, Optical density and d, residuals (optical density minus a smoothed copy of the same image) of atom clouds at 815 G, imaged via the rapid ramp method\(^34\), showing solitons at various hold times after creation. One period of soliton oscillation is shown. The in-trap aspect ratio was \(\lambda = 6.5(1)\). e, Radially integrated residuals as a function of time revealing long-lived soliton oscillations. The soliton period is \(T_2 = 12(2)T_F\), much longer than the trapping period of \(T_F = 93.76(5)\) ms, revealing an extreme enhancement of the soliton’s relative effective mass, \(M^*/m\).
inertial mass of the soliton $M^*$, this force causes an acceleration \( \ddot{z} = -\frac{M^*}{M^*} \alpha \dot{z}^2 \). Because we observe oscillations, $M^*$ must be negative as well, implying that the soliton is an effective particle that decreases its kinetic energy as it speeds up. One obtains a direct relation between the relative effective mass $M^*/M$ and the normalized soliton period $T_s/T_z$:

\[
\frac{M^*}{M} = \left( \frac{T_s}{T_z} \right)^2
\]

The observed soliton period of oscillation $T_s$ is about one order of magnitude longer than the trapping period $T_z$ for single atoms. This directly indicates an extreme enhancement of the relative effective mass. In general, the difference between the effective mass $M^*$ and the bare mass $M$ of the soliton arises from the phase slip $\Delta \phi$ across the soliton, which implies a superfluid counterflow. For the soliton to move, an entire sheet of atoms thus has to flow past it. The difference $M - M^*$ is the mass of that sheet, given by the mass density multiplied by the entire soliton volume. In contrast, the soliton’s bare mass $M$ is only due to the mass deficit of $N_i$ atoms and can become much smaller in magnitude than $M^*$ when the soliton is filled. For weakly interacting BECs, where solitons are devoid of particles, the effective mass is still of the same order of the bare mass, $(M^*/M)_{\text{BEC}} = 2$. This leads to an oscillation period that is only $\sqrt{2}$ times longer than $T_z$ (refs 20, 35), as has been observed in experiments (14, 31). In the BCS limit, where only a minute fraction $\Delta p/E_F$ of the gas contributes to Cooper pairing, $|N_i| \propto \Delta p/E_F \propto \exp[-\pi/(2k_F|a|)]$ and thus the soliton’s relative effective mass can be expected to become exponentially large.

Indeed, as shown in Fig. 2, we find that the soliton period, and hence the relative effective mass, increases dramatically as the interactions are tuned from the limit of Bose–Einstein condensation (Fig. 2a) towards the BCS limit. At 700 G, where $1/k_Fa = 2.6(2)$, the system represents a strongly interacting Bose gas of molecules. The soliton period is $T_s = 4.4(5)T_z$, already three times longer than in the case of a weakly interacting BEC. At the Feshbach resonance (Fig. 2d), we measure a soliton period of $T_s = 14.2(T_z)$, corresponding to a relative effective mass of $M^*/M = 200(50)$. This is more than 50 times larger than the result of mean-field BdG theory in three dimensions (26, 27) that predicts $M^*/M = 3$. Note that the superfluid is fully three-dimensional: on resonance, the chemical potential $\mu = 35ho_\lambda$, where $o_\lambda$ is the radial trapping frequency. Still, for very elongated traps, one expects to reach a universal quasi-one-dimensional regime where the tight radial confinement is irrelevant for propagation along the long axis (37). This prompted us to study the dependence of the soliton period on the aspect ratio of our trap.

Figure 3 summarizes our measurements for the soliton period and the relative effective mass as a function of the interaction parameter $1/k_Fa$ throughout the BEC–BCS crossover, for aspect ratios $\lambda = 3.3$, 6.2 and 15. The strong increase of $M^*/M$ towards the BCS regime is observed for all trap geometries. The normalized soliton period $T_s/T_z$ appears to converge to a limiting value for the most elongated trap: the normalized period changes by only 15% as the aspect ratio is increased by more than a factor of two from 6.2 to 15. This indicates that the soliton dynamics approach a universal quasi-one-dimensional limit. Even in a much less elongated trap with $\lambda = 3.3(1)$, the soliton period is only slightly increased by about 30% compared to $\lambda = 6.2$, accompanied by an increased susceptibility of the soliton towards bending or ‘snaking’ (10, 13, 15) (for examples, see Supplementary Information).

We attribute the large relative effective mass $M^*/M$ in the strongly interacting regime to the filling of the soliton with uncondensed fermion pairs resulting from strong quantum fluctuations. Similar filling with uncondensed particles has been predicted for solitons in strongly interacting Bose condensates (10, 22–25, 35). A substantial filling of the soliton will reduce the number $|N_i|$ of atoms missing inside the soliton, therefore considerably weaken the restoring harmonic force from the trap and strongly increase $M^*/M$. At the Feshbach resonance, our in situ density profiles provide a lower bound on the soliton filling of 90%, compared to the expected 20% from mean-field theory (see Supplementary Information). Mean-field theory for the BEC–BCS crossover heavily underestimates the role of quantum fluctuations already on the BEC side, where it predicts a fraction of uncondensed bosons that scales as $na^2$ instead of the correct $\sqrt{na^2}$ scaling. Our experiment thus directly reveals the importance of beyond mean-field effects for the dynamics of strongly interacting fermionic superfluids. Significant soliton filling was found theoretically in a strongly interacting relativistic superfluid using methods from string theory (38–40). For the resonantly interacting Fermi gas, a theoretical study based on a
density functional approach found solitons with clear filling in the wake of shock waves. The strong increase of the soliton period is reminiscent of the situation for dark-bright solitons in weakly interacting BECs, where a distinguishable atomic species or another spin state resides inside the soliton notch. For fermions, mean-field theory in the strongly interacting regime attributes a substantial part of the soliton filling to Andreev bound states. These are also predicted to carry the dominant fraction of the superfluid flow across the soliton, which can be regarded, in its rest frame, as a Josephson junction of vanishing barrier height. It will be an interesting topic for future experiments to determine the contribution of Andreev states to the soliton filling.

**Temperature dependence**

To demonstrate that the slow soliton oscillations are a truly quantum effect and not due to the finite temperature of our gas, we investigated the soliton motion as a function of temperature for the unitary Fermi gas at the Feshbach resonance (Figs 4 and 5). A measure of temperature is provided by the thermal fraction, the number of uncondensed molecules observed after the rapid ramp. The soliton period is found to be insensitive to changes in temperature within the measurement uncertainty (Fig. 5a).

The stability of solitons is, however, strongly affected by thermal effects. At low temperatures, the soliton oscillation occurs essentially without energy loss, demonstrating dissipationless flow (Fig. 4a). For increasing temperature, we observe anti-damping of soliton oscillations (Fig. 4b). This is characteristic of a particle with negative mass that can lower its energy by accelerating. To our knowledge, such anti-damping of solitons has not been directly observed previously in a quantum gas experiment. The energy loss is likely due to collisions with thermally induced phonons, and we indeed observe a strong increase in the anti-damping time constant as the temperature is raised (Fig. 5b). At even higher temperatures, the soliton’s position becomes less reproducible (Fig. 4c) and its lifetime is strongly reduced (Fig. 5c). Concurrently, we observe increased axial fluctuations in the superfluid (see Fig. 4d–f), some of which appear to have comparable contrast to the imprinted soliton. These additional solitons might be ‘thermal solitons’, predicted to occur even in equilibrium in weakly interacting Bose condensates. Similar to vortex–anti-vortex pairs in two dimensions, soliton–anti-soliton pairs can be expected to spontaneously break in one dimension and proliferate.

We note that on resonance, the fastest solitons we observe move at the exceedingly slow speed of 0.50 mm s⁻¹ or 5% of the independently measured sound of sound on resonance. Their sudden disappearance, observed for example in Fig. 4c, can thus not be related to motion close to the Landau critical speed. Instead, their decay might be tied to inelastic collisions with thermal solitons, as soliton collisions have been found to become increasingly inelastic towards the BCS side in theoretical simulations. Another possibility for their decay at such low speeds is that the soliton’s energy dispersion has a minimum at an unexpectedly small fraction of the critical velocity. One might expect fermion pairs to break at finite temperatures and fill in the soliton, in addition to quantum fluctuations. However, even for the highest thermal fraction where solitons have been observed, the actual temperature is determined to be below $T = 0.10E_F/k_B$ ($k_B$ is the Boltzmann constant), while the bulk pairing gap is about $\Delta_0 = 0.4E_F$ (ref. 45). Pair breaking should thus still be exponentially suppressed, explaining the insensitivity of the soliton period to the thermal fraction.

**Conclusion and outlook**

We have created and observed long-lived solitons in a strongly interacting fermionic superfluid. Their period of oscillation and thus their relative effective mass increases markedly as the interactions are turned from the BEC limit of tightly bound molecules towards the BCS limit of long-range Cooper pairs. This signals strong, beyond mean-field, effects, which are likely to be due to uncondensed fermion pairs filling the soliton, in addition to purely fermionic Andreev bound states. Our study provides an important quantitative benchmark for theories of non-equilibrium dynamics of strongly interacting Fermi gases. An exciting prospect is to directly detect the Andreev bound states spectroscopically. Although they are not topologically protected, their lifetime should equal that of the soliton—many seconds or 100,000 years.
Fermi times—so that they might become a useful quantum resource. In the presence of spin imbalance, the soliton represents a limiting case of the long-sought Fulde–Ferrell–Larkin–Ovchinnikov state of moving Cooper pairs. Indeed, it is energetically favourable for an excess fermion to reside inside a soliton rather than inside the bulk superfluid. Although it is difficult to realize the Fulde–Ferrell–Larkin–Ovchinnikov state in equilibrium, direct engineering of soliton trains might produce a long-lived metastable analogue.

METHODS SUMMARY

Preparation. The atomic gas is composed of a balanced mixture of the two lowest hyperfine states of $^6$Li initially prepared at 760 G (ref. 7). The trapping potential providing tighter radial confinement. The axial periods are $T_a = 210 \text{ ms}$, 95 ms and 45 ms, respectively, for the three aspect ratios considered here.

Phase imprinting. A step-like intensity profile is imprinted on a laser beam (wave-length 532 nm, power 15 W) by means of an opaque mask. A 3-μm-resolution imaging system projects the intensity distribution at the mask location onto the atoms (the beam waist at the atom is 60 μm). A phase twist of $\pi$ corresponds to a pulse time of about 30 μs, much shorter than the timescale $h/\pi$ associated with a typical chemical potential, of the order of few 100 μs. The pulse duration is finely adjusted to yield exactly one soliton with high contrast observed after the rapid ramp. Because we imprint only a phase step but not a density depletion, sound waves must be generated in addition to the soliton. The sound waves are found to die out in a quarter axial trapping period when they have reached the edge of the atom cloud (see Supplementary Information). For the data above 760 G, the soliton is created at 760 G and the magnetic field is subsequently ramped (in about 27 τ) to the final magnetic field where the soliton motion is studied. For final magnetic fields below 760 G, the soliton is created at that field. We found that solitons can be created directly at the Feshbach resonance as well.

Thermometry. Thermometry from fits of density profiles at the Feshbach resonance to the known equation of state yielded an upper limit of $T = 0.05 \text{ K}$ for $<10\text{nK}$, for our lowest temperatures, where $\omega_\perp = 2\pi/T_{\perp}$. At such low temperatures, far below the critical temperature for superfluidity, thermometry via fitting is less sensitive to small changes in temperature. We therefore use the thermal fraction of molecules observed after a rapid ramp to the BEC side of the Feshbach resonance as a robust thermometer.