Supporting Information

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Superhydrophobic Lubrication: Gas–Liquid Bilayer Reduces the Friction Between Two Solids

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Supplementary information

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Detailed results of the oscillating tribometer

The oscillating tribometer measures the three coefficients of equation for general harmonic oscillator with friction. The different parameters depend on the properties of the oscillating object. In this case, the magnetic properties map into spring coefficient, while the viscous and friction properties map into the viscous coefficient and friction force coefficient.

The magnetic properties map to the spring force, since the spring force is based in magnetic potential created by the two neodymium magnets in the oscillating tribometer. The magnetic spring force in the potential well then define the oscillation frequency of the system based on the mass of the slider. The effect of changes in lubricant volume, pressure exerted on lubricant and slider size is shown in Figure S1.

![Figure S1](image)

**Figure S1.** a) Spring coefficient value as a function of lubricating volume. b) Spring coefficient value as a function of pressure exerted on lubricant. c) Spring coefficient value as a function of slider diameter.

The viscous coefficient depends on the lubricating film below the slider. This is done to simplify analysis in the code. The viscous coefficient should only depend on the thickness of...
the lubricating layer and the size of the slider based on the estimation of Couette flow. The effect of changes in lubricant volume, pressure exerted on lubricant and slider size is shown in Figure S2.

**Figure S2.** a) Viscous coefficient value as a function of lubricating volume. b) Viscous coefficient value as a function of pressure exerted on lubricant. c) Viscous coefficient value as a function of slider size.

The contact angle hysteresis force depends only on the length of the three-phase contact-line as in equation

\[ F_{CAH} = \frac{24}{\pi^3} D \gamma (\cos(\theta_{rec}) - \cos(\theta_{adv})). \]  

(1)

This means that only the slider size should affect the hysteresis force while the mass and the lubricating volume should not affect the friction force. The effect of changes in lubricant volume, pressure exerted on lubricant and slider size is shown in Figure S3.
Figure S3. a) Contact angle hysteresis force as a function of lubricating volume. b) Contact angle hysteresis force as a function of pressure exerted on lubricant. c) Contact angle hysteresis force as a function of slider size.

Derivation of the load equation
The load of the model slider can be modelled by a pressure balance between the Laplace pressure and pressure from the mass of the slider. The Laplace pressure $P_L$ comes from the curvature of the liquid-air interphase area, in this case the edge area of the slider. This can be written as

$$P_L = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \gamma \left( \frac{1}{\frac{D}{2}} + \frac{1}{\frac{y}{2}} \right), \tag{2}$$

Where the curvature $\gamma$ is surface tensions of the liquid, $R_1$ corresponds to the radius of the model slider, the $R_2$ to half of the lubricating thickness in the maximal case, $D$ the diameter of the slider and $y$ the thickness of the lubricating layer. The pressure from the mass of the cargo is

$$P_C = \frac{m_{load}g}{\pi \left( \frac{D}{2} \right)}, \tag{3}$$

where $m_{load}$ is the mass of the slider without the lubricating liquid, $g$ gravitational acceleration and $D$ the diameter of a circular slider. When these are set to be equal, the equation for the load can be derived
Location and the velocity of tilted plane measurements

The location curves rotated location data of sliding slider are shown in Figure S5. The rotated data is shown instead of the raw data since it is visually impossible to distinguish the goodness of fit of the unrotated data. However, with the rotated location data, the location data, slope, and confidence intervals can be easily visually examined. The rotation was based on the angle between the camera and the plumb line (\( \phi \)), which is shown in Figure S4. This camera angle was calculated based on the black pixels of the plumb line and the direction of the long axis of a fitted ellipse as documented in MATLAB’s function regionprops. The rotation of the location data was done by rotation matrix multiplied with the x- and y-coordinate data,

\[
\begin{bmatrix}
    x_{\text{rot}} \\
    y_{\text{rot}}
\end{bmatrix} = \begin{bmatrix}
    \cos(\phi) & -\sin(\phi) \\
    \sin(\phi) & \cos(\phi)
\end{bmatrix} \begin{bmatrix}
    x \\
    y
\end{bmatrix}.
\]

where the \( x \) and \( y \) correspond to the original data, the \( x_{\text{rot}} \) and \( y_{\text{rot}} \) correspond to the rotated data and the \( \phi \) corresponds to the angle between the camera and the plumb line. The camera angle was the mean of the orientation of the plumb line, which was consistently near 152.7°, as can be seen from Figure S4.
Figure S4. The tilt angle $\phi$ between the camera and the plumb line during the measurements. The tilt angle $\phi$ stays near the value 152.7° during these measurements. The small variation in the data is due to small swinging of the plumb line.

The location of the rotated data is shown in the Figure S5. The height difference is in the order of 0.5 mm while change in the width direction is in the order of 10 cm. The velocities calculated from the location data is shown in Figure S6. The almost constant velocity and rapid changes at imperfect coating locations implies that the sliders are travelling at terminal velocity. This means that the tilt angle of the surface $\alpha$ can be used to approximate the friction coefficient and if the sliders are moving at velocity slower than terminal velocity, the friction coefficient is smaller than what is reported.
Figure S5. Location data of four different-sized sliders during different measurements.

Figure S6. Velocities of four different-sized sliders during different measurements. Sliders have reached terminal velocity around 5 mm s\(^{-1}\), even though some keep accelerating up to 8 mm s\(^{-1}\). The continuous almost step-like changes in velocities are true changes. This means that the small change in velocity reflects a change in coating.

**Tunable superhydrophobic lubrication stage** (Figure S7)

An air lubrication stage was coated with a superhydrophobic coating for the tunable superhydrophobic lubrication demonstration. The air lubrication stage was supplied from Neurotar. By applying pressure on the water reservoir, droplets form at the holes in the
substrate (right) enabling the superhydrophobic lubrication to be switched on. Decreasing the pressure leads to withdrawal of the droplets and switching off of the lubrication.

![Image of lubrication stage with lubrication off and on.]

**Figure S7.** Dynamic lubrication stage with lubrication off and on.

**Oscillating tribometer** (Figure S8)

![Diagram of oscillating tribometer measurement protocol.](image)

**Figure S8.** Oscillating tribometer measurement protocol. a) Schematic of model slider in the oscillating tribometer on a superhydrophobic surface. The model slider is affected by the returning magnetic force $F_k$ based on the displacement $x$, the viscous force $F_\beta$, and contact angle hysteresis force $F_{CAH}$. b) Thresholded black-white side profile of slider from high-speed recording. The red dot represents the centroid of the slider. c) Example harmonic oscillations of a model slider with fitted analytical model ($k = 0.09 \text{ Nm}^{-1}$, $\beta = 0.25 \text{ mNsm}^{-1}$, $F_{CAH} = 5.2 \mu\text{N}$).

**Effect of evaporating the lubrication layer**
The effect time and lubrication layer thickness were explored with a slider (1 cm diameter, mass 122.1 mg) kept on the superhydrophobic surface for 165 minutes with 80 µL of lubricating water. Oscillation tribometry measurements were done every 15 minutes to explore the effect of evaporation to the lubrication (Figure S9). There is only a modest increase in the friction coefficient as the lubricating layer evaporates over time and the lubrication effect is very similar even with a minimal amount of lubricant. The lubricant layer thickness was calculated as the vertical distance between the edge of the cover glass and the baseline (Figure S9b). This distance was measured by hand from the high-speed video. The evaporation is linear over this time scale. The evaporation rate could be reduced by using less volatile liquid with suitable liquid repelling surface.

**Figure S9.** Effect of evaporation on model slider. a) Static and dynamic ($v = 0.1 \text{ ms}^{-1}$) friction coefficients as a function of time. b) The thickness of lubricating layer as a function of time. Each point is a mean of 11 repeat measurement over the 10-oscillation measurement and the error bars are the standard deviation of these measurements. Black line is a linear fit to the data.

**Effect of mass on the wetted lubrication area**

The effect of mass on the wetted lubrication area was explore by photographing the slider through a microscope slide coated with superhydrophobic coating. This way the wetted area could be measured from the photos (Figure S10). The slider masses varied from 135.8 mg to 378.9 mg and the lubricating volume was 40 µL. The change over these slider masses changed the wetted area from 52.33 mm$^2$ to 67.25 mm$^2$, when the whole area of the slider is
78.54 mm$^2$. From this change in area, the lubricating layer thickness can be estimated for the different massed sliders by assuming the lubricating water of being a cylinder with the bottom area of the wetted area and volume of 40 $\mu$L. The estimated change in the lubrication layer thickness was from 0.76 mm to 0.59 mm. In this regime, the mass of the raft does not significantly affect the wetted area (increases 29%) or the lubricant thickness (decreases 22%) as as the mass increases 179%. In contrast, the viscous coefficient increases 167% with the same mass change in slider.

**Figure S10.** Effect of slider mass on the wetted area by lubricant. a) Photograph of the bottoms of a lubricated (40 $\mu$L) and dry slider (diameter 1 cm, m = 145.9 mg) on superhydrophobic surface. Partial circular dashed white line is added to show the estimated wetted area. The sliders were kept still by an iron wire, which touched the cargo portion of the slider. b) Photo of heavier slider (m = 355.4 mg) on superhydrophobic surface. c) Measured wetted area and estimated lubrication layer thickness as the function of slider mass. The black line is the area of the 1 cm diameter cover glass. The linear fits to the corresponding data points exclude the first data point, which was an outlier.
Supplementary Videos:

SV1 – Square slider and cargo carrying: a square PS petri dish slides on a slightly tilted superhydrophobic surface with water lubrication, and a 18 cm slider carrying cargo (81.4 g) is shown to move easily when nudged.

SV2 – 18 cm and 35 cm sliders: Examples of 18 cm and 35 cm sliders moving easily on superhydrophobic surface with water lubrication when nudged.

SV3 – Tunable lubrication: Example of tunable lubrication with microscope glass on a dynamic lubrication stage.

SV4 – Cage with superhydrophobic lubrication: Demo of a 2D mouse treadmill with superhydrophobic lubrication for neuroimaging purposes.