A methodology for identifying resiliency in renewable electrical distribution system using complex network

Divyanshi Dwivedi1, 2, Pradeep Kumar Yemula2, Mayukha Pal1, 5

1 ABB Ability Innovation Center, Asea Brown Boveri Company, Hyderabad 500084, India.
2 Department of Electrical Engineering, Indian Institute of Technology Hyderabad, Kandi, Sangareddy, Telangana 502285, India.

5 Corresponding author:
Dr. Mayukha Pal
R&D Principal Program Manager
ABB Ability Innovation Center
10th Floor, Western Aqua, Kondapur
Hyderabad – 500084, TS, India.
Tele: +91-9866161632
Email: mayukhapal@gmail.com

Abstract

Recently, Electrical Distribution Systems are extensively penetrated with the Distributed Energy Resources (DERs) to cater the energy demands with general perception that it enhances the system resiliency. However, it may be adverse for the grid operation due to various factors like its intermittent availability, dynamics in weather condition, introduction of nonlinearity, complexity etc. This needs a detailed understanding of system resiliency that our method proposes here. We introduce a methodology using complex network theory to identify the resiliency of distribution system when incorporated with Solar PV generation under various undesirable configurations. Complex correlated networks for different conditions were obtained and various network parameters were computed for identifying the resiliency of those networks. The proposed methodology identifies the hosting capacity of solar panels in the system while maintaining the resiliency under different unwanted conditions hence helps to obtain an optimal allocation topology for solar panels in the system. The proposed method also identifies the critical nodes that are highly sensitive to the changes and could drive the system into non-resiliency. This framework was demonstrated on IEEE-123 Test Feeder system with time-series data generated using GridLAB-D and variety of analysis were performed using complex network and machine learning models.
Keywords: Complex Network, Electrical Distribution System, Data-Driven Analysis, Percolation Threshold, Resiliency, Solar PV generation, Distributed Energy Resources.

Significance
Electrical distribution system is becoming more complex consistently with higher penetration of distributed energy resources. The complex network theory comes out as a good tool for analysis as it can easily accommodate complexities and identify system’s resiliency when subjected to undesirable conditions like overloading, unbalancing, extreme weather conditions and temperature rise. This paper introduces a methodology for identifying the resiliency of electrical distribution system. Data of IEEE-123 Test Feeder under the considered conditions were generated using GridLAB-D simulation software where of the 123 nodes, we electrical collected data at 40 nodes. The developed positive correlation networks were analyzed between the system operating normally with the system configuration incorporating varying percentage of PV to understand changes in electrical parameters like real power of the system and then evaluated resiliency by analyzing various network parameters like percolation threshold, average degree, clustering coefficient etc. We introduced 15 combinations of PV introduction in the system to identify the optimal allocation topology of DER by maintaining the resiliency of electrical network with identification of the critical nodes responsible for hampering the system resiliency. The data driven approach combining complex network and machine learning enabled us to understand the important network features there by using them to identify the resiliency, optimal topology with various degree of DER penetration.

1. Introduction

In the recent decade, an enormous increase in penetration level of distributed energy resources is perceptible due to world’s shift towards clean energy and it is a general perception that their integration enhances the resiliency of electrical distribution system. With emergence of large Distributed Energy Sources (DERs) in Electrical Distribution System (EDS), infrastructure is becoming more complex, thus resiliency of system is getting challenged. For identifying resiliency and breakdown boundaries for complex and large interconnected Electrical Distribution Network (EDN), complex network theory provides a methodology and is considered as an excellent tool which provides collective behavior of the system considering interaction among various network components.

Over few years for identifying the resiliency of Power Distribution Network, optimal partition technique has been used where distribution network is partitioned into sub-networks. This technique exhibits a practice in which the initially local solutions are obtained from sub-networks and then the requisite information is communicated with neighboring sub-networks to obtain an optimal global solution. An adaptive spectral graph partitioning algorithm is used which is based on node resettling with considering computational load balancing for synchronization, real power balance and sub-networks resiliency [1]. It also ensures that resilient power distribution network
partitions can adjust to abrupt and new operating condition. Laplacian spectrum of power distribution network and self-organizing map algorithm is implemented in combinations for partitioning the network and aims to minimize the real power imbalance with desirable voltage profile [2]. It is also reported that spectral clustering method and mixed-integer programming is implemented for partitioning the network [3]. Some of the other approaches considered for resilient distribution network analysis are scheduling coefficient which aims to maximize the load recovery in minimum recovery time. It is achieved by using multi-objective improved simulated annealing algorithm which helps in identifying the optimal scheduling solution [4]. Clustering based method helps in selecting the critical loads after natural calamity while considering the power loss in distribution network. This method allows identifying the high load density clusters and outliers for microgrids which helps in planning microgrid installations [5]. Many papers have also been projected for resilient distribution network for restoring critical loads [6-8]. There are also other tools available such as state estimation technique, which helps in studying the impact of rising penetration of distributed energy resources [9-12], but these approaches unable to visualize the low observability condition present in distribution system occurring because of heterogenous nature of data and measurements.

Complex Network Analysis overcomes these drawbacks by providing better alternate statistical tool to understand the salient features of a network by visualizing the low observability conditions and give its global prospective [13]. Complex network is used to identify and resolve the issues such as overloading in power system, failures, and blackouts. This has been achieved by computing parameters of complex network and analyzing them to interpret critical nodes in the electrical network [14]. Similarly, there has been focuses on identifying possible vulnerabilities, outages, and blackouts when new complexity gets introduced in system to make it more reliable and secure [15]. Centrality analysis technique from complex network is reportedly used in finding the optimal placement of microgrids, as with increase in DER penetration, system required to ensure its resiliency, voltage stability, minimum power loss and line loading [16].

In this paper, we propose a methodology in which complex network theory with non-linear dynamical parameters and machine learning algorithms are used to study the system resiliency. We considered IEEE-123 Test Feeder system for implementation of proposed framework, simulated using GridLAB-D. Steady state simulation is performed considering the loads profile for 24 hours and then gradually solar panels were introduced in the system to check its effects and identify critical nodes. We also modified the standard IEEE-123 Test Feeder’s loading and overhead line conductor parameters to explore system resiliency under different conditions like the impact of weather, overloading and unbalancing in system. Our approach possibly provides a better resilient distribution system which would be more reliable and stable under normal and resilient conditions for varying DER penetration. We have also exploited the diverse visual methodology analysis to use complex networks for effectively visualizing the correlation between the generated data of the system with non-linearity arising under different analysis conditions. This manuscript is organized with the section 2 discussing about materials and methods while section
3 discusses the step-by-step procedure for implementation of the method on simulated data and its description. Section 4 details the results and discusses the observed characteristics. Section 5 concludes the study with our inferences.

2. Materials and Methods

A. Electrical Distribution System as complex network

Consider an electrical distribution network represented by Graph $G = (\nu, E)$ where $\nu = \{1,2,\ldots,N\}$ is set of nodes/vertices and $E$ is a subset of $\nu \times \nu$ that represents the edges $(m,n) \in \nu$ where $m \neq n$ [21]. There exists a swing node/generator node represented by $N. \, \nu = \{1,2,\ldots,N-1\}$ which denotes all the system nodes except the swing node. In such system, we place photovoltaics (PV) panels at all the nodes except swing node. We can define system model as:

- $PV \subseteq \nu$, set of nodes having PV panels.
- $LD \subseteq \nu$, set of nodes having loads connected.

Considering a node $m \in \nu$ of the electrical network, the real and reactive power at every time instant $t$ is given by:

$$P_{m,t}^l = P_{m,t}^{PV} - P_{m,t}^{LD} = \sum_{j \in \nu} \delta_{mn,t} P_{mn,t} \quad \forall \in \nu$$ (1)

$$Q_{m,t}^l = Q_{m,t}^{PV} - Q_{m,t}^{LD} = \sum_{j \in \nu} \delta_{mn,t} Q_{mn,t} \quad \forall \in \nu$$ (2)

where, $P_{mn,t} = g_{mn}V_{m,t}^2 - V_{m,t}V_{n,t}(g_{mn} \cos \theta_{mn,t} + b_{mn} \sin \theta_{mn,t})$ (3)

$$Q_{mn,t} = V_{m,t}V_{n,t}(g_{mn} \cos \theta_{mn,t} - b_{mn} \sin \theta_{mn,t}) - b_{mn}V_{m,t}^2$$ (4)

Here, $P_{m,t}^{LD}$ and $Q_{m,t}^{LD}$ are the real and reactive power of load connected to node $m \in LD$ at time $t$ respectively. $P_{m,t}^{PV}$ and $Q_{m,t}^{PV}$ are the real and reactive power of PV generation to node $m \in PV$ at time $t$ respectively. $V_{m,t}$ is the voltage magnitude at node $m \in \nu$ at time $t. \theta_{mn,t}$ is the voltage angle difference between nodes $(m,n) \in \nu$ at time $t. P_{mn,t}$ and $Q_{mn,t}$ are the real and reactive power transferred from node $m$ to the network through line $(m,n) \in E$ at any time instant $t. \delta_{mn,t}$ is operation status of line $(m,n) \in E$ at any time instant $t$. For $\delta_{mn,t} = 1$, the lines are operational, $\delta_{mn,t} = 0$ means lines are not operational. $z_{mn}$ denotes impedance of line, and $y_{mn} = z_{mn}^{-1}$ as its admittance. Thus, $y_{mn} = g_{mn} + j b_{mn}$, where $g_{mn}$ is conductance and $b_{mn}$ is susceptance of line $(m,n) \in E$ at time $t$.

The voltage magnitude at node $m \in \nu$ lies within lower and upper bounds as:

$$V_{lb} \leq V_{m,t} \leq V_{ub}, \quad \forall \in \nu$$ (5)

PV panel generated output $P_{m,t}^{PV}$ is intermittent in nature as it depends on meteorological conditions. However, let us consider the presence of solar irradiance, then PV real power produced can be modelled as [21]:

$$P_{m,t}^{PV} = \text{intermittent}$$
\[ p_{m,t}^{PV} = \alpha_{m,1} S_{m,t} + \alpha_{m,2} T_{m,t} + \alpha_{m,3} S_{m,t} T_{m,t}, \quad \forall \in \text{PV} \]  

(6)

where, \( T_{m,t} \) is the temperature and \( S_{m,t} \) is the incident solar irradiance at node \( m \in \text{PV} \) at time \( t \). \( \alpha_{m,1}, \alpha_{m,2}, \) and \( \alpha_{m,3} \) are the PV model parameters.

The reactive power \( Q_{m,t}^{PV} \) depends on power electronics equipment present and we can represent reactive power injections using lower and upper bounds as:

\[ Q_{lb} \leq Q_{m,t} \leq Q_{ub}, \quad \forall \in \text{P} \]  

(7)

B. Resiliency in Electrical Distribution System

With high penetration of DERs in EDS, the system become more reliable and resilient but DER also pertains stochastic nature which could affect the distribution grid operation. Altogether with unwanted events these DERs would severely affect the grid performance and thus EDS must have the capability to cope up with the changes effectively, which is termed as short-term resiliency of the system [31]. Basically, resiliency is a sub-category of vulnerability which has two aspects to investigate, first coping capability and second recovering as depicted in Fig 1. Resilient system should withstand the circumstances arising but incase fails to do so then must have the capability to recover self [17]. In this work, we have evaluated the coping capability of Electrical Distribution System (EDS) when incorporated with Solar PV panels under varying circumstances.

For analyzing the resiliency, we used complex network theory as computational tool. The concept of percolation theory was applied in this study for analyzing the system resiliency of graph network modelled using complex network framework and helps in identifying the critical nodes of the system [18]. Percolation theory seems to be successful as a qualitative guide to the resilience of networks [19].

![Percolation Threshold in Identifying Resiliency](image)

C. Percolation Threshold in Identifying Resiliency

Percolation theory helps identify the ‘phase-transition’ when nodes/edges are removed in a network, thus is effectively utilized as statistical tool for identifying the operational transition occurring in Electrical Distribution System.
During normal operation, having all the operating conditions within their specified limits, probability of operational nodes is $\rho = 1$. If system is introduced with undesired events, then it will affect the nodes of system, although sometime nodes will remain operational and have probability less than 1 ($\rho < 1$) or it can become non-operational with $(1 - \rho)$ as probability. Thus, the threshold value for the probability under these events influence the operational nodes and is known as percolation threshold $\rho_c$, which identify the % of critical nodes in system that could easily damage or break the network under undesired circumstances. When $\rho > \rho_c$, then system is considered as resilient and for $\rho \leq \rho_c$, the system is considered as weak that would not cope up with the raising conditions. It also helps in analyzing the hosting capacity of PV in the system.

There exist many percolations process with most used types being bond percolation, explosive percolation, and site percolation which depends on how one considers the analysis in a lattice structure. Site Percolation gives a view of lattice as rectangular array of squares whereas Bond Percolation gives a view of graph as horizontal and vertical edges. For this study, we have considered bond percolation as it gives an analogy of how effectively and strongly nodes are connected through each other.

D. Percolation in Correlated Networks

Real world networks are generally correlated networks. For distribution system, correlated network means how the time-series loading pattern at node-m is correlated with the loading pattern at node-n. Similarly with PV incorporation, it suggests how time-series of PV generation pattern at node-m is correlated with PV generation pattern at node-n. Usually, correlation in networks help in attributing the complex structures [27-28]. Complex network derived out of correlation matrix also provides a great inference especially with explosive percolation [29]. Time-series data when analyzed using correlation matrix provides more robust and reliable solutions even though the data is noisy and ill-framed. To reduce the noises in data, thresholding on correlated network may help in generating robust information from the networks. We verify explosive percolation for correlated network provides similar inference as bond percolation hence, in our work we implemented the bond percolation on correlation networks. The methodology used for constructing correlation network is explained in next section.

E. Complex network parameters for analyzing correlated networks

Correlation matrix represents the relational information of multiple time-series data [20]. Pearson Correlation coefficient helps identify the strength of relationship between the networks. When the correlation coefficient of two time-series data having $n$ number of nodes is calculated, we obtain $n \times n$ coefficient matrix that provides relational information among the nodes. The value of these coefficient varies between -1 and 1. The Pearson correlation coefficient of two time series at nodes m and n is given by:

$$PC = \frac{\sum (p_m(t) - \bar{p}_m)(p_n(t) - \bar{p}_n)}{\sqrt{\sum (p_m(t) - \bar{p}_m)^2 \sum (p_n(t) - \bar{p}_n)^2}}$$  (8)
where, $P_m^i$ and $P_n^i$ are the mean for the real power time series of $P_m^i$ and $P_n^i$ respectively.

This obtained correlation matrix plots the complete graph with all possible edges. To understand connection density among nodes of significance, use of thresholding produces a sparse adjacency matrix that generates desirable graph for the system from the correlation matrix. Correlation networks using variety of such techniques help remove noise from the data while analyzing the system under study [21].

In this work, we correlated the base configuration of EDS with variety of conditions incorporating PV to generate the correlation matrix and then the complex network to compute various network parameters for electrical inferences. We discuss here the computed network parameters:

**Average Degree:** For an undirected graph, it is defined as average number of edges per node. Consider $N$ as number of nodes/vertices and $L$ as number links/edges in network, it is denoted by $\bar{K}_i$ and written as:

$$K_i = \frac{2L}{N} \quad (9)$$

It infers how well the network is connected, where high value of average degree means system is densely connected.

**Clustering Coefficient:** It gives the degree to which neighbors of a given node link to each other. Mathematically, we write it as:

$$C_i = \frac{2L_i}{K_i(K_i-1)} \quad (10)$$

where, $K_i$ is the degree of node $i$ and $L_i$ is the neighbor links of node $i$. It is the coefficient that suggests how a graph tends to cluster together. High values of clustering coefficient indicate the system is strongly connected.

**Minimum Degree:** It is the least degree of a node existing in the network representing connectedness of the network.

**Assortative Coefficient:** It measures the level of homophyly of the graph. It has value ranging from -1 and 1, where for $r = 1$, the network is said to have perfect assortative while for $r = 0$ the network is non-assortative and for $r = -1$ the network is completely disassortative. When the value is 1, it signifies strong nodes tend to connect with strong nodes and weak nodes tend to connect with weak nodes whereas for value -1, it signifies that strong node tend to connect weak nodes or vice-versa. The assortative coefficient is mathematically expressed as [26]:

$$r = \frac{L^{-1} \sum_{i,j} j_i k_i - [L^{-1} \sum_{i} \frac{1}{2}(j_i+k_i)]^2}{L^{-1} \sum_{i} j_i^2 + k_i^2 - [L^{-1} \sum_{i} \frac{1}{2}(j_i+k_i)]^2} \quad (11)$$

where, $j_i, k_i$ are the degrees of the vertices at the ends of the $i^{th}$ edges, with $i = 1,2 \ldots , L$. 

**Power Law**: It shows the relationship between two operating conditions of system and indicates how the system at one operating condition varies as a power of other operating condition without concerning about the system size. Mathematically, power law is expressed as:

\[ f(x) = ax^{-k} \]  \hspace{1cm} (12)

In this paper, we have identified how system parameters are relatively changing with incorporation of PV panels and undesirable conditions. For a complex network, power law degree distribution comes into existence only when probability distribution of degree in any system follows the power law and then only system is considered as resilient. These computed network parameters along with the Percolation Threshold is used for our analysis.

### 3. Data and its Processing

In this work, we first generated the time series data for a standard IEEE-123 Node Test Feeder using GridLAB-D, an open-source software that provides a platform to easily design distribution system with incorporation of DERs. Basically, IEEE 123 Node Test Feeder operates at a nominal voltage of 4.16 KV, comprises of overhead as well as underground lines, loaded with constant current, impedance, and power with four regulators, capacitor banks and switches. This system has all the components of a realistic distribution system thus considered reliable for performing the data driven analysis.

GridLAB-D is a distribution level power system simulator where simulation could be achieved either with event-driven mode or through sub-second simulation mode (delta mode). Reliability of GridLAB-D is well studied in electrical distribution system simulation as it follows an agent-based simulation paradigm and the output obtained is of close resemblance to almost like the data collected from smart grid demonstration project [22]. We simulated data using event-driven mode, considering system to be steady having consistency and coherent characteristics. IEEE 123 Node Test Feeder consists of 85 constant loads, and we collected incoming real power on 40 nodes of the system which are marked red in Fig.2. These nodes are referred as Meter nodes, where we are calculating the incoming real power and other electrical parameters at these specific nodes considering the defined system characteristics.
Non-linearity in standard system is introduced with incorporation of Solar PV panels in the system. We first placed PV on 20% (i.e. 8 meter nodes) out of 40 meters, then gradually increased the PV percentage to 40% (i.e. 16 meter nodes), then 60% (i.e. 24 meter nodes), then 80% (i.e. 32 meter nodes) and finally 100% i.e., placing PV at all meter nodes to analyze the impact and identify PV hosting capacity of the system while maintaining resiliency of the network. Here, meter node 150 is considered as swing node or generator node.

Energy resilient systems could sustain any circumstances arising out of nonlinearity introduction hence we have considered following cases in our study:

Case- I: Normal Operating Conditions - Standard IEEE-123 system. This case is denoted by ‘P’.

Case- II: High Consumption - Increase loading while maintaining system balancing to check system behavior in overloading condition. This case is denoted by ‘HC’.

Case- III: High Resistance –Resistance increases for the overhead line conductor from 0.036ohm to 2ohm. This case is denoted by ‘HR’.

Case- IV: Imbalanced - Load values of PQ type loads increase with load imperfection which causes unbalance in the three-phase voltage. This case is denoted by ‘IB’.
Here, we are considering time series data of real power at the meter nodes. To understand system behavior after incorporation of PV under these scenarios, we computed Pearson correlation coefficient matrices of size 40×40, as this helps in understanding system correlation when these events took place. We further obtained the complex correlated network that gives an analogy how densely the system is interconnected. In this work, correlated network is the measurement of similarity in the dynamics between the system with and without PV and is computed from the considered two time-series data. We considered the positive correlated network as shown in sample Fig. 4, which depicts how two systems in different circumstances behave in a particular direction. We analyzed how load power increases when percentage of PV increase in the system, hence introduced a threshold value $T$ (i.e. we considered values greater than 0) and transformed the correlation matrix to an adjacency matrix by substituting all the values above the threshold $T$ as 1 otherwise 0. It generates a sparse matrix and reduces the complexity in understanding the
correlation in the network. Furthermore, the results are consistent for different values of correlation thresholds thus we can state that our obtained networks are robust and reliable.

Further, various network parameters were computed from the obtained graphs including parameters such as average degree, correlation coefficient, assortative coefficient, minimum degree, power law exponent value and percolation threshold that when analyzed depicts about network interconnection threshold point leading to its collapse which in electrical distribution attributes to system resiliency. We computed bond percolation in the correlation network as it results similar inference as observed in case of explosive percolation. Bond percolation values were computed with 1000 iterations.

Fig. 4 Positively correlated network obtained incorporating 0% and 40% of PV panels under normal operating conditions. a) the adjacency matrix for the correlation network b) the correlation network.

Our analysis also finds out the optimal allocation topology for an example case of 40% PV incorporated IEEE-123 network to understand critical loads in the system. For this analysis, we have considered 15 combinations as mentioned in Table I, and then computed the Percolation Threshold following the process detailed in Fig. 3. Percolation Threshold as a parameter gives inference for the resilient distribution system, optimal topology with incorporation of PV in the system including the critical load.

| Combination Nomenclature | Considered node combinations for the 40% meter nodes incorporated with PV panels |
|--------------------------|---------------------------------------------------------------------------------|
| **C1**                   | 8 21 25 27 36 44 57 78 89 97 105 152 160 197 300 450                            |
| **C2**                   | 3 8 13 14 15 18 21 23 25 54 57 61 93 21 149 152                             |
| **C3**                   | 3 8 13 14 15 23 25 26 27 54 93 18 21 250 149 152                         |
| **C4**                   | 3 8 13 14 15 23 25 26 250 54 93 18 21 61 149 152                         |
| **C5**                   | 13 15 18 21 23 25 40 44 54 57 89 91 93 197 135 152                     |
| **C6**                   | 3 23 27 36 44 57 67 81 89 97 135 197 250 18 300 450                     |
We compute network parameters for each combination which was further subjected to feature selection using random forest regressor machine learning algorithm to understand the network parameters that are highly sensitive to percolation threshold and hence to the system resiliency, thereby identifying the indicator of the weak parameters in the system resiliency. Of the used various machine learning algorithms for feature selection, random forest performed better with higher accuracy.

### 4. Results and Discussion

IEEE-123 Node Test Feeder is considered for implementing the proposed methodology and further analysis of resiliency with the incorporation of PV panels under undesirable circumstances. Time-series data for the system is generated using GridLAB-D and real power of system nodes are taken into consideration for analysis.

#### A. IEEE-123 Node Test Feeder with incorporation of PV panels for various analyzed cases

For normal operating condition i.e., Case- I, with increasing incorporation of PV panels at meter nodes at 20%, 40%, 60%, 80% and 100% of 40-meter nodes; the correlation network is obtained between standard system and PV incorporated system. Percolation Threshold is the deciding parameter that infer the transition of system from resilient to non-resilient. Lower value of percolation threshold is desired when transition in systems’ operation is taken into account [23-24]. On the other hand, we expect our system to be resilient when incorporating more PV panels, thereby operating performance of the system should not get affected with the changes occurring in the system. For resilient electrical distribution system it is desired to have high value of percolation threshold, that means system will not face any transition easily when encountered with the changes, rather if percolation threshold is low it suggests that system will undergo the transition to non-resilient easily and may not persist the changes [18]. It is worth observed that computed percolation threshold for correlated networks are resulting low value as reported in Table II and III for our analysis because generally electrical distribution system is highly correlated and densely clustered thus macroscopic clusters form at low values of occupation probability [25]. Table II, depicts the change of systems’ resiliency in terms of percolation threshold which suggests such
undesired events are hampering the system resiliency. In Table II and Table III the nomenclature used for naming the graph like P_0%-HC_0% suggests that we are finding correlation between case-I denoted by P with 0% PV incorporation and case-II denoted by HC with 0% PV incorporation. This naming convention was used for all the network graphs for different cases under different PV incorporation.

| Graph Name   | Percolation Threshold |
|--------------|-----------------------|
| P_0%-HC_0%  | 0.0346                |
| P_0%-HR_0%  | 0.0342                |
| P_0%-IB_0%  | 0.0339                |

Table II Sample representation of Percolation threshold for the networks under the influence of undesired events.

With various case studies, we analyze and discuss in details the circumstances where the system is experiencing the transition and may not sustain the changes occurred for the considered cases. Table III suggests the distribution system resiliency for various undesirable events with integration of PV.

**Case-I:**

From Table III we could observe that the clustering coefficients for all the networks are above 80% which depicts the system is strongly connected and if any node get interrupted, continuity of supply will be maintained as these are well connected networks. The assortativity coefficient values of the networks are ~ -0.70, hence the networks are disassortative having its weak nodes establishing good relationship with the strong nodes. The significance of disassortative in electrical network we explain with an example: when overloading condition occurs at any node whereas other nodes have abundance power then these nodes could support the overloaded node to fulfill the demand without resulting any break out in the system. Analogy of the scenario in complex network, those overloaded nodes are weak nodes whereas the node with abundant power is a strong node. With strong relationship between both the nodes, the electrical system becomes more stronger as it helps maintaining the supply continuity hence resolves unwanted failure. Thus for this case, the networks are strongly connected and withstand the incorporation of PV panels in system. We could also observe that these networks follow power law as detailed in section 2.

**Case-II:**

In this case, the system persistence is evaluated when load consumption increased. From Table III, the clustering coefficients of these networks are ~ 0.75 and compared to Case-I it is less resilient however system is strong enough to cope up the changes. Assortativity coefficient values are also ~ -0.67 hence the networks are disassortative. Network parameters suggest, system here is not as strong as in case of Case-I.

**Case-III:**

For Case-III, extreme weather conditions is modelled considering increase of resistance in overhead lines. From Table III, we observe average degree is high in comparison to other cases.
which implies the system is relatively high clustered. The clustering coefficients are ~0.90 hence the network connections are effectively strong. On the other hand, assortative coefficients are almost zero for all the networks except for the network having 60% of PV incorporated thus networks are non-assortative. A network with a given assortativity comprises its nodes that contribute to the assortativity characteristics [30]. Power law fitting values are high for all the networks which suggest that network disobeys the power law except when incorporated with 60% of PV in this case. From this case study, it is observed that for the only condition of having 60% PV incorporated it may able to deal with the changes as the network is disassortative thus strong nodes connect well with weak nodes. Also this is the only network combination following power law. This observation from our analysis brings out a critical inference in system design for optimal PV incorporation for this topology/configuration.
Fig. 5 Change in Percolation Threshold with increase in PV percentage for all the Cases, (a) system in steady state becoming more resilient with increase in PV as percolation threshold is increasing, (b) system under high loading also becomes more resilient with increase in PV as percolation threshold is increasing, (c) in this case the system has high probability to become non-resilient with increase in PV as percolation threshold is decreasing, (d) here system may sustain imbalancing conditions with lower percentage of PV but with PV percentage increase, system could become non-resilient as percolation threshold is decreasing.

Case- IV:

System balancing is a major issue that distribution systems face. With incorporation of PV, non-linearity is getting introduced in system which interrupt the systems’ desired operation and hinder its performance. The unequal distribution of loads between the three phases of the system causes the flow of unbalanced currents hence the line losses in the system and produces unbalanced voltage drops on the electrical lines. Resiliency is a good alternative to check system behaviour in such conditions. From Table III, we could observe the clustering coefficients are ~ 0.72 which suggest the system is strongly connected but less efficiently connected as in Case-I and Case-II. Assortative coefficients are towards negative side thus the networks are disassortative where weak meters nodes are strongly connected with strong meter nodes and when conditions becomes unfavorable strong nodes could supply the weak nodes to avoid any interruption and maintain the continuity. Power law fitting values are high for all the networks which suggest the network disobeys power law except when incorporated with 40% and 60% of PV in this case.

For all the cases, by computing complex network parameters including the percolation threshold, we could observe from Table III that for case-I and case-II, percolation threshold is increasing with incorporation of PV panels in the system which infers system is becoming more resilient and could easily host PV panels. From Fig. 5(a) and 5(b), we could infer that IEEE-123 test feeder is capable of maintaining resiliency in normal loading as well as in high loading when incorporated with PV panels. On the other hand, for case-III and case-IV when PV percentage increases, percolation threshold is decreasing which suggest that system may more likely transit from resilient to non-resilient. From Fig. 5(c) and 5(d) we observe that when IEEE-123 test feeder is loaded with resistive load and dewatered with imbalancing in loading, with PV percentage increase, the system may not cope up with changes hence becomes non-resilient. Hence we could infer that due to stochastic nature of DER under extreme conditions, the systems’ resiliency got hampered and requires preventive measure to avoid possible grid failure.

| Cases       | Graph Name | Average Degree | Clustering Coefficient | Minimum Degree | Assortative Coefficient | Power Law Fit Value | Percolation Threshold |
|-------------|------------|----------------|-------------------------|----------------|-------------------------|---------------------|----------------------|
| Case-I      | P_0%-P_20%| 18.150         | 0.836                   | 10.000         | -0.704                  | 3.551               | 0.052                |
|             | P_0%-P_40%| 17.750         | 0.849                   | 10.000         | -0.697                  | 3.746               | 0.054                |
|             | P_0%-P_60%| 17.700         | 0.848                   | 10.000         | -0.698                  | 3.755               | 0.054                |
|             | P_0%-P_80%| 17.700         | 0.849                   | 9.000          | -0.701                  | 3.146               | 0.056                |
|             | P_0%-P_100%| 17.650        | 0.849                   | 10.000         | -0.697                  | 3.774               | 0.057                |
| Case  | HC_0%_HC_20% | HC_0%_HC_40% | HC_0%_HC_60% | HC_0%_HC_80% | HC_0%_HC_100% |
|-------|---------------|---------------|---------------|---------------|---------------|
|       | 20.900        | 20.900        | 20.400        | 20.650        | 20.600        |
|       | 0.742         | 0.789         | 0.774         | 0.780         | 0.777         |
|       | 12.000        | 12.000        | 11.000        | 11.000        | 12.000        |
|       | -0.669        | -0.676        | -0.676        | -0.669        | -0.685        |
|       | 3.316         | 3.512         | 3.178         | 3.111         | 3.613         |
|       | 0.045         | 0.048         | 0.049         | 0.051         | 0.051         |

| Case  | HR_0%_HR_20% | HR_0%_HR_40% | HR_0%_HR_60% | HR_0%_HR_80% | HR_0%_HR_100% |
|-------|---------------|---------------|---------------|---------------|---------------|
|       | 30.050        | 28.350        | 27.550        | 30.250        | 31.800        |
|       | 0.906         | 0.893         | 0.805         | 0.907         | 0.905         |
|       | 19.000        | 19.000        | 18.000        | 19.000        | 19.000        |
|       | 0.031         | 0.047         | -0.529        | -0.005        | -0.007        |
|       | 11.326        | 10.336        | 3.863         | 8.564         | 12.403        |
|       | 0.043         | 0.042         | 0.040         | 0.040         | 0.038         |

| Case  | IB_0%_IB_20% | IB_0%_IB_40% | IB_0%_IB_60% | IB_0%_IB_80% | IB_0%_IB_100% |
|-------|---------------|---------------|---------------|---------------|---------------|
|       | 22.250        | 22.100        | 19.350        | 21.200        | 24.600        |
|       | 0.723         | 0.726         | 0.739         | 0.712         | 0.693         |
|       | 13.000        | 13.000        | 11.000        | 13.000        | 15.000        |
|       | -0.643        | -0.661        | -0.702        | -0.692        | -0.582        |
|       | 13.213        | 3.477         | 3.517         | 16.220        | 14.969        |
|       | 0.049         | 0.048         | 0.047         | 0.045         | 0.045         |

Table III Network Parameters for different cases with incorporation of PV

When PV is introduced in system, some nodes become source node among the selected ones used for our analysis. These nodes are 61, 151, 250, 300, 450 and 610. It is worth emphasizing that node number 151, 250, 300 and 450 in their network combination have the maximum degree hence could be considered as Hub Nodes in the network. We demonstrated the variations in power in these selected nodes with PV incorporation as shown in Fig.6. For 0% PV, real power consumed at all these meter nodes are 0 except 61 and 610 nodes. For 20% PV, except node 450 all nodes consumed power. Here the meter node 450 still continued as source node. With 40%PV in the system, meter nodes 300 and 450 becomes source node while nodes 61 and 610 becomes load. For 60%, 80% and 100% of PV incorporation, meter nodes 61, 250, 300 and 450 becomes source node and started supplying power to other loads in the system. Similarly, meter node 151 becomes source node with PV percentage increased more than 80%. Fig. 7 demonstrates system’s real powers variation at all the meter nodes when 40% of PV is introduced. As observed, at all other nodes for this PV percentage, real power is following similar pattern except for these selected nodes.
Fig. 6 Real power at these selected meter nodes with increase in PV percentage in the system eventually becomes source node.

Fig. 7 Real power in Watts at all meter nodes when incorporated with 40% PV in the system

B. Detailed analysis for system incorporated with 40% PV
In this section, we discussed in details the system behaviour with 40% of meter nodes incorporated with PV panels considering different combinations of PV placement. The combinations of PV placement is detailed in Table I. Different preselected combinations for placing PV panels at 16 meter nodes are considered for analysis to understand system behaviour when PVs are placed in combinations like: cluster form (C2-C5), far-distance from each other (C13 and C14), sections controlled by switches (C7-C12), randomly scattered in system (C1 and C6) and centrally placed combination (C15).

In Table IV, computed network parameters are tabulated for all combinations where we observe the combination C8 is at critical state as percolation threshold is among the lowest hence transition from resilient to non-resilient system is more likely to occur and the system have higher probability to break down easily. Average degree for C8 is 33.85, which implies system is densely connected whereas assortativity is comparatively towards positive side hence suggest strong nodes trying to connect with strong nodes and maintain weak relationship with weak nodes. Thus network pertaining to the combination C8, has higher probability of breaking down. Similarly, as observed C12 and C15 are the next probable non-resilient system topology. From Table I, it is worth emphasizing that these critical combinations (i.e., C8, C12, and C15) have a PV placed at meter node 610 in common, which we may infer as the critical node for the placement of PV.

| Graph Name | Average Degree | Clustering Coefficient | Minimum Degree | Assortative Coefficient | Power Law Fit Value | Percolation Threshold |
|------------|----------------|------------------------|----------------|-------------------------|---------------------|----------------------|
| P_40%-C1  | 17.750         | 0.849                  | 10.000         | -0.697                  | 3.746               | 0.054                |
| P_40%-C2  | 17.500         | 0.849                  | 9.000          | -0.695                  | 3.204               | 0.054                |
| P_40%-C3  | 17.300         | 0.848                  | 9.000          | -0.687                  | 3.266               | 0.055                |
| P_40%-C4  | 17.750         | 0.849                  | 10.000         | -0.698                  | 3.747               | 0.054                |
| P_40%-C5  | 17.350         | 0.848                  | 9.000          | -0.686                  | 3.250               | 0.058                |
| P_40%-C6  | 17.950         | 0.848                  | 10.000         | -0.698                  | 2.248               | 0.050                |
| P_40%-C7  | 18.200         | 0.817                  | 9.000          | -0.715                  | 2.979               | 0.049                |
| P_40%-C8  | 33.850         | 0.851                  | 24.000         | -0.158                  | 35.218              | 0.034                |
| P_40%-C9  | 17.700         | 0.849                  | 10.000         | -0.700                  | 3.756               | 0.051                |
| P_40%-C10 | 18.500         | 0.824                  | 10.000         | -0.718                  | 3.405               | 0.051                |
| P_40%-C11 | 18.400         | 0.809                  | 10.000         | -0.717                  | 3.444               | 0.054                |
| P_40%-C12 | 32.300         | 0.809                  | 22.000         | -0.109                  | 21.773              | 0.036                |
| P_40%-C13 | 18.250         | 0.817                  | 10.000         | -0.715                  | 3.495               | 0.052                |
| P_40%-C14 | 17.950         | 0.824                  | 10.000         | -0.699                  | 2.248               | 0.050                |
| P_40%-C15 | 31.900         | 0.795                  | 24.000         | -0.212                  | 26.223              | 0.039                |

Table IV Network Parameters for different combinations under incorporation of 40% PV
We could also observe from Table IV that C5 is the optimal topology for placing the PV among all considered combinations as it has highest percolation threshold among all inferring the system will remain resilient. Fig. 8 demonstrates a complete view of the percolation threshold variation for all different combinations. Thus, we could conclude from our analysis that this framework may be used to provide multi-dimensional solution, as it identifies the critical nodes and also the optimal allocation topology for incorporating PV while maintaining system resiliency.

Fig. 8 Percolation Threshold for different combinations showing C5 as the optimal allocation topology for placing 40% of PV in system and the combinations C8, C12, C15 makes system non-resilient.

C. Feature Selection among network parameters

Further to identify the network parameters that are highly sensitive to system resiliency, feature selection and feature importance methodology is employed using random forest regressor. From Fig. 9 we observe that average degree, minimum degree and power law exponents are highly sensitive to percolation threshold. When average degree, minimum degree and power law exponent are high then percolation threshold drops and vice versa. Thus, we could infer that these network parameters are aftermath for system resiliency. On the other hand, clustering and assortativity coefficients even though are important parameters of these networks but are of less significant features in driving percolation threshold as reported in Table V and Table VI.
Fig. 9 Bar plot depicting scores for complex network features with its importance on percolation threshold from parameters average degree, minimum degree and power law value.

| Features                      | Score | Rank |
|-------------------------------|-------|------|
| Average Degree – Feature -0   | 0.320 | 1    |
| Clustering Coefficient- Feature -1 | 0.111 | 5    |
| Minimum Degree- Feature -2    | 0.229 | 2    |
| Assortativity Coefficient- Feature-3 | 0.153 | 4    |
| Power Law Exponent- Feature -4 | 0.188 | 3    |

Table V Ranking Order of features for different cases plotted in Fig 9(a)

Similarly, we had also performed the feature selection for different combinations as mentioned in Table III, which had similar observations as shown in Table V and Fig 9(b).

| Features                      | Score | Rank |
|-------------------------------|-------|------|
| Average Degree – Feature -0   | 0.294 | 1    |
| Clustering Coefficient- Feature -1 | 0.106 | 4    |
| Minimum Degree- Feature -2    | 0.247 | 3    |
| Assortativity Coefficient- Feature-3 | 0.077 | 5    |
| Power Law Exponent- Feature -4 | 0.273 | 2    |

Table VI Ranking Order of features for different combinations

5. Conclusion

To understand the resiliency of electrical distribution system with incorporated DER, we propose a hybrid data driven methodology using complex network and machine learning when the system undergoes undesirable conditions. Our proposed analytical methodology found to be an efficient technique to check the system resiliency with use of various network parameters. With gradual increase in PV incorporation in the IEEE-123 Node Test Feeder, we identified the PV hosting capacity while maintaining the system resiliency. From the analysis, it is observed that in normal condition with 100% PV incorporation, the system becomes more resilient and self-sufficient in generating required power for the loads. For undesirable conditions such as increase in load, increase in line resistance, and imbalance in loading with PV incorporation, we observe different changing behavior of system from the computed network parameters that affects the system resiliency.

Our proposed methodology is found to be effective in identifying the optimal allocation topology for PV in system while maintaining its resiliency. The method also identifies critical nodes of the system from the analysis that may not be suitable for placing PV as the system transit to non-resiliency. We also demonstrate effectiveness of various computed network parameters obtained from a highly correlated and dense network as it effectively identifies the phase transition of Electrical Distribution System from resilient to non-resilient under several considered
circumstances when introduced with DER. Furthermore, feature selection using random forest regressor was used that identified average degree, minimum degree, and power law exponent network parameters as important features driving changes in percolation threshold for the system that decides system resiliency.

**Acknowledgement**
The Authors would like to thank Mr. Siddharth Patwardhan, Dr. P. Manimaran for their useful discussion on various complex network topics and its significance, and Dr. Alok Kumar Bharati for discussion on GridLAB-D.

**Authors’ contributions**
Divyanshi Dwivedi participated in idea generation, implemented, and evaluated the ideas, performed data analysis, and wrote the manuscript. Pradeep Kumar Yemula contributed to discussion, guidance, and review of the manuscript. Mayukha Pal conceived the idea and conceptualized it, prepared analysis methodology, mentored in results analysis and discussion, project guidance, wrote and reviewed the manuscript.

**Declaration of Competing Interest**
The authors declare that they have no known competing financial interest or personal relationships that could have appeared to influence the work reported in this paper.

**References**
[1]. C. Shah and R. Wies, "Adaptive Day-Ahead Prediction of Resilient Power Distribution Network Partitions", IEEE Green Technologies Conference (GreenTech), (2021), pp. 477-483, Doi:10.1109/GreenTech48523.2021.00080.

[2]. Y. Jia, Z. Xu, L. L. Lai and K. P. Wong, "A Novel Network Partitioning Approach in Smart Grid Environment", IEEE International Conference on Systems, Man, and Cybernetics, (2015), pp. 641-646, Doi:10.1109/SMC.2015.122.

[3]. M. Kyesswa, A. Murray, P. Schmurr, H. Çakmak, U. Kühnapfel, V. Hagenmeyer, “Impact of Grid Partitioning Algorithms on Combined Distributed AC Optimal Power Flow and Parallel Dynamic Power Grid Simulation”, IET Generation Transmission & Distribution, (2020), Doi:14. 10.1049/iet-gtd.2020.1393.

[4]. J. Wang et al., "Disaster recovery strategy of resilient distribution network based on scheduling coefficient", 8th Renewable Power Generation Conference, (2019), pp. 1-8, Doi:10.1049/cp.2019.0684.

[5]. H. Ren and N. N. Schulz, "A Clustering-based Microgrid Planning for Resilient Restoration in Power Distribution System", IEEE/PES Transmission and Distribution Conference and Exposition (T&D), (2020) pp. 1-5, Doi:10.1109/TD39804.2020.9299978.

[6]. A. Dubey and S. Poudel, "A robust approach to restoring critical loads in a resilient power distribution system", IEEE Power & Energy Society General Meeting, (2017), pp. 1-5, Doi:10.1109/PESG.2017.8274597.
[7]. H. Gao, Y. Chen, Y. Xu and C. C. Liu, "Resilience-Oriented Critical Load Restoration Using Microgrids in Distribution Systems", IEEE Transactions on Smart Grid, (2016), vol. 7, no. 6, pp. 2837-2848, Doi:10.1109/TSG.2016.2550625.

[8]. M. Jafarian, A. Soroudi and A. Keane, "Resilient Identification of Distribution Network Topology", IEEE Transactions on Power Delivery, (2020), vol. 36, no. 4, pp. 2332-2342, Doi:10.1109/TPWRD.2020.3037639.

[9]. P. L. Donti, Y. Liu, A. J. Schmitt, A. Bernstein, R. Yang and Y. Zhang, "Matrix Completion for Low-Observability Voltage Estimation", IEEE Transactions on Smart Grid, 2020, vol. 11, no. 3, pp. 2520-2530, Doi: 10.1109/TSG.2019.2956906.

[10]. J. Liu, R. Singh and B. C. Pal, "Distribution System State Estimation with High Penetration of Demand Response Enabled Loads," IEEE Transactions on Power Systems, (2021), vol. 36, no. 4, pp. 3093-3104, Doi: 10.1109/TPWRS.2020.3047269.

[11]. J. Watitwa and K. Awodele, "A Review on Active Distribution System State Estimation," Southern African Universities Power Engineering Conference/Robotics and Mechatronics/Pattern Recognition Association of South Africa (SAUPEC/RobMech/PRASA), (2019), pp. 726-731, Doi: 10.1109/RoboMech.2019.8704833.

[12]. K. Dehghanpour, Z. Wang, J. Wang, Y. Yuan and F. Bu, "A Survey on State Estimation Techniques and Challenges in Smart Distribution Systems", IEEE Transactions on Smart Grid, 2019, vol. 10, no. 2, pp. 2312-2322, March (2019), Doi: 10.1109/TSG.2018.2870600.

[13]. Giuliano Andrea Pagani, Marco Aiello, “The Power Grid as a complex network: A survey”, Physica A: Statistical Mechanics and its Applications, Volume 392, Issue 11, (2013), Pages 2688-2700, ISSN 0378-4371, Doi:10.1016/j.physa.2013.01.023.

[14]. E. P. R. Coelho, J. C. Thomazelli, M. H. M. Paiva and M. E. V. Segatto, "A complex network analysis of the Brazilian Power Test System", IEEE PES Innovative Smart Grid Technologies Latin America (ISGT LATAM), (2015), pp. 113-118, Doi: 10.1109/ISGT-LA.2015.7381138.

[15]. S. F. Myhre, O. Bjarte Fosso, P. E. Heegaard, O. Gjerde and G. H. Kjølle, "Modeling Interdependencies with Complex Network Theory in a Combined Electrical Power and ICT System", International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), (2020), pp. 1-6, Doi:10.1109/PMAPS47429.2020.9183667.

[16]. M. Saleh, Y. Esa, N. Onuorah and A. A. Mohamed, "Optimal microgrids placement in electric distribution systems using complex network framework", IEEE 6th International Conference on Renewable Energy Research and Applications(ICRERA), (2017),pp.1036-1040, Doi:10.1109/ICRERA.2017.8191215.

[17]. Shakeri Kahnamouei, Ali & ghanizadeh bolandi, Tohid & Haghifam, M.-R., “The conceptual framework of resilience and its measurement approaches in electrical power systems”, IET International Conference on Resilience of Transmission and Distribution Networks RTDN, (2017).

[18]. Chowdhury T, Chanda CK, Chakrabarti A, “Resiliency improvement for a part of south Indian power transmission network”, Australasian universities power engineering conference (AUPEC). IEEE, Christchurch, pp 1–5, (2017).

[19]. Newman, Mark, Networks: An Introduction: Part V- Processes on networks- 16 Percolation and network resilience. United Kingdom: OUP Oxford, (2010).
[20]. Shuang Han, Hongbin Dong, Xuyang Teng, Xiaohui Li, Xiaowei Wang, “Correlational graph attention-based Long Short-Term Memory network for multivariate time series prediction”, Applied Soft Computing, (2021), Volume 106, 107377, ISSN 1568-4946, https://doi.org/10.1016/j.asoc.2021.107377.

[21]. Branimir Novoselnik, Mato Baotić, “Dynamic Reconfiguration of Electrical Power Distribution Systems with Distributed Generation and Storage”, European Commission’s FP7-ICT project DYMASOS and Croatian Science Foundation, IFAC-PapersOnLine, (2015), Volume 48, Issue 23, Pages 136-141, ISSN 2405-8963, Doi:10.1016/j.ifacol.2015.11.273.

[22]. David P. Chassin, Jason C. Fuller, and Ned Djilali, “GridLAB-D: An Agent-Based Simulation Framework for Smart Grids”, Journal of Applied Mathematics, Hindawi, (2014). Doi:10.1155/2014/492320.

[23]. B.T.S. Ramanujam, S. Radhakrishnan, “Conducting polymer-graphite binary and hybrid composites: structure, properties, and applications”, Trends and Applications in Advanced Polymeric Materials, Scrivener Publishers, USA (2017), pp. 127-143.

[24]. Mostafizur Rahaman, Rajesh Theravalappil, Subhendu Bhandari, Lalatendu Nayak, Purabi Bhagabati, “Electrical conductivity of polymer- graphene composites”, Polymer Nanocomposites Containing Graphene, 2022.

[25]. Filippo Radicchi, “Predicting percolation thresholds in networks”, PHYSICAL REVIEW E 91, 010801(R) (2015), DOI: 10.1103/PhysRevE.91.010801

[26]. M. E. J. Newman, “Assortative Mixing in Networks”, Phys. Rev. Lett. 89, 208701, (2002), DOI: https://doi.org/10.1103/PhysRevLett.89.208701

[27]. A. V. Goltsev, S. N. Dorogovtsev, and J. F. F. Mendes, “Percolation on correlated networks”, PHYSICAL REVIEW E 78, 051105, (2008), DOI: 10.1103/PhysRevE.78.051105

[28]. Jae Dong Noh, “Percolation transition in networks with degree-degree correlation”, PHYSICAL REVIEW E 76, 026116, (2007), DOI: 10.1103/PhysRevE.76.026116

[29]. D. Remondini, “Explosive percolation in correlation-based networks”, Communications: SIF Congress, IL NUOVO CIMENTO Vol. 34 C, N. 5, (2010), DOI: 10.1393/ncc/i2011-10903-1

[30]. Noldus, R., & Van Mieghem, P, “Assortativity in complex networks”. Journal of Complex Networks, (2015), 3(4), 507–542. DOI:10.1093/comnet/cnv005

[31]. V. B. Venkateswaran, D. K. Saini and M. Sharma, ”Approaches for optimal planning of energy storage units in distribution network and their impacts on system resiliency,” in CSEE Journal of Power and Energy Systems, vol. 6, no. 4, pp. 816-833, (2020), DOI: 10.17775/CSEEJPES.2019.01280.