Discrete symmetries and quantum number conservation

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Abstract
The algebraic formulation of discrete $P$ and $T$ space-time symmetries is related to fermion quantum numbers defined by a $Cl_{3,3}$ sub-algebra of the $Cl_{7,7}$ Clifford Unification algebra. Fermion decays and interactions have been shown to conserve all seven binary quantum numbers defined by $Cl_{7,7}$. The previously formulated Conservation Law, is modified to include the effects of employing distinct F,G quantum numbers in descriptions of fermions with $C=+1$ and $C=-1$. This is relevant in interpreting the results of high energy experiments.

§1 Introduction
Clifford Unification (CU) [1,2] provides a seven binary quantum number description of fermions, related to their physical properties. Each quantum number is defined by one of seven commuting elements of $Cl_{7,7}$, providing unique physical interpretations of all the elements of this algebra. One of the seven quantum numbers was identified as fermion intrinsic parity, leading to the expectation that this is preserved in all fermion interactions. This conflicts with experimental evidence for the non-conservation of parity that is now generally accepted and reported in all the textbooks on particle physics (e.g.[3]). This apparent conflict in the interpretation of experimental results appears to be a consequence of the different algebraic definitions of parity in CU[1,2] and the Standard Model that have been described in [4].

Sections 2 and 3 are concerned with establishing the algebraic relationship between the $Cl_{3,3}$ and classical descriptions of parity and time-reversal symmetries. §2 gives the coordinate descriptions of parity inversion ($P$) and time-reversal ($T$) symmetries in terms of the space-time algebra $Cl_{1,3}$. §3 identifies $P$ and $T$ symmetries with quantum numbers determined by commuting elements of $Cl_{3,3}$. §4 defines the ABCDEFG quantum number description of fermions. It highlights the parity ($C$) dependence of F and G, which distinguish fermion generations, correcting the results in §8 of [2] and earlier versions of this paper. §5 formulates a modified conservation law of all seven quantum numbers in elementary particle decays and interactions. This relates to the table of fermion quantum numbers given in the Appendix.
§2. Algebraic expressions for discrete space-time symmetries in classical physics

Parity inversion and time-reversal \((P\) and \(T\)) are discrete transformations, conventionally defined in terms of sign changes in the space-time coordinates \(x_\mu\{\mu = 0, 1, 2, 3\}\), viz.

\[
P : x_\mu \to -x_\mu \text{ for } \{\mu = 1, 2, 3\}; \quad T : x_0 \to -x_0.
\]  

(2.1)

Changing the sign of a single spatial coordinate also changes the parity of the coordinate system as a rotation of the other two coordinates through \(180^\circ\) changes the sign of both. All rotations of spatial coordinate systems can be expressed as Lorentz transformations, which leave their parity unchanged.

4-vectors describing space-time displacements \(\Delta x_\mu\) of a particle (expressed in terms of an arbitrary Minkowski reference frame) have the algebraic form

\[
\Delta s = \gamma^s \Delta s = \gamma^\mu \Delta x_\mu.
\]

(2.2)

Here the \(\gamma^\mu\) are anti-commuting components of a covariant Minkowski tensor, related to the contravariant components \(\gamma^0 = \gamma_0\), and \(\gamma^\mu = -\gamma_\mu\) if \(\{\mu = 1, 2, 3\}\). Together, the \(\gamma_\mu\) generate a \(Cl_{1,3}\) Clifford algebra, isomorphic to the Dirac matrix algebra, but with a different physical interpretation. \(\gamma^0\) is interpreted as a unit displacement in time, and the \(\gamma^\mu\{\mu = 1, 2, 3\}\) are interpreted as unit displacements in the three orthogonal directions in space. \(\Delta s\) is the change in ‘proper’ time, as measured on a (conceptual) clock at rest with respect to the particle and \(\gamma^s\) (where \((\gamma^s)^2 = 1\)) is the unit proper time. The Lorentz transformation ensures that \(\Delta s\) and \(\Delta x_0\) have the same sign.

\(P\) and \(T\) can be expressed algebraically as similarity transformations of all the \(\gamma_\mu\), viz.

\[
P : \gamma^\mu \to \gamma^0 \gamma^\mu (\gamma^0)^{-1} = \gamma^\mu = -\gamma_\mu \text{ for } \{\mu = 1, 2, 3\} \text{ or } +\gamma^\mu \text{ for } \{\mu = 0\},
\]

\[
T : \gamma^\mu \to \nu \gamma^\mu (\nu)^{-1} = +\gamma^\mu \text{ for } \{\mu = 1, 2, 3\} \text{ or } -\gamma^\mu \text{ for } \{\mu = 0\}.
\]

(2.3)

where \(\nu = \gamma^1 \gamma^2 \gamma^3\) is the algebraic expression for unit spatial volumes. Defining \(\gamma^\pi = \gamma^0 \nu = \gamma^0 \gamma^1 \gamma^2 \gamma^3\), the combined \(PT\) transformation is

\[
PT : \gamma^\mu \to \gamma^\pi \gamma^\mu (\gamma^\pi)^{-1} = -\gamma^\mu \text{ all } \mu, \text{ and } \gamma^s \to \gamma^\pi \gamma^s (\gamma^\pi)^{-1} = -\gamma^s,
\]

(2.4)

showing that \(PT\) produces proper time reversal. As the above analysis is necessarily confined to space-time geometry, it does not provide a description of charge conjugation. It is of interest, nevertheless, that electric current 4-vectors \(j = j^\mu \gamma_\mu\) are invariant under the \(PT\) transformation and a change in sign of the charge.
§3. *Cl*\textsubscript{3,3} description of space-time symmetries

In CU discrete symmetries are identified with the eigenvalues of specific commuting elements of the algebra. This provides unique identifications, which are preferable to descriptions based on transformations of wave-functions. The physical interpretation of the *Cl*\textsubscript{3,3} algebra has been given in \[1,2\]. It has six generators: \(\hat{\gamma}^1, \hat{\gamma}^2, \hat{\gamma}^3\) which correspond to the unit elements in the three spatial directions, and \(\hat{\gamma}^0, \hat{\gamma}^7, \hat{\gamma}^8\) which are Lorentz invariants describing fermions and their interactions. Unit elements of time are defined as the Lorentz invariants describing fermions and their interactions. Unit elements of time are defined as the product \(\hat{\gamma}^0 = \hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^6 \hat{\gamma}^7 \hat{\gamma}^8\). Equation (2.2) for a fermion displacement becomes

\[\Delta s = \hat{\gamma}_{s0} \Delta x^0 = \hat{\gamma}_\mu \Delta x^\mu.\]  (3.1)

where \(\hat{\gamma}_{s0}\) is the unit of proper time measured in the fermion rest frame. More generally starred coordinates indicate that unit space-time displacements referred to the fermion rest frames. Ordered products of generators are abbreviated, for example \(\hat{\gamma}^{27} \hat{\gamma}^6 = \hat{\gamma}^{26}\).

*Cl*\textsubscript{3,3} has, at most, seven commuting elements. Of particular interest is the set \(\hat{\gamma}^{13}, \hat{\gamma}^{26}, \hat{\gamma}^{78}, \hat{\gamma}^{*0}, \hat{\gamma}^{078}, \hat{\gamma}^{031}, \hat{\gamma}^{026}\). Any three of these that are not related by multiplication take eigenvalues that distinguish the eight leptons, the simplest physical interpretation being obtained with \(\hat{\gamma}^{13}, \hat{\gamma}^{*0}, \hat{\gamma}^{78}\). \(\hat{\gamma}^{*2}\) is the spatial orientation of the fermion spin \(\hat{\gamma}^{+13}\).

Changes in parity are produced by the coordinate transformations

\[\begin{align*}
\hat{\gamma}^{27} \hat{\gamma}^{+1} \hat{\gamma}^{+27} &= \hat{\gamma}^{-1}, & \hat{\gamma}^{27} \hat{\gamma}^{+2} \hat{\gamma}^{+27} &= \hat{\gamma}^{-2}, & \hat{\gamma}^{27} \hat{\gamma}^{+3} \hat{\gamma}^{+27} &= \hat{\gamma}^{-3}, & \hat{\gamma}^{27} \hat{\gamma}^{*27} &= \hat{\gamma}^{*27}, & \hat{\gamma}^{*27} \hat{\gamma}^{*27} &= \hat{\gamma}^{*27}
\end{align*}\]

both of which satisfy \((\hat{\gamma}^{*27})^2 = (\hat{\gamma}^{*28})^2 = 1\). It follows that (3.2) describes similarity transformations changing the sign of \(\hat{\gamma}^{*2}\). The same transformations give

\[\hat{\gamma}^{*27} \hat{\gamma}^{78} \hat{\gamma}^{*27} = -\hat{\gamma}^{78}, & \hat{\gamma}^{*28} \hat{\gamma}^{78} \hat{\gamma}^{*28} = -\hat{\gamma}^{78},\]  (3.3)

changing the sign of \(\hat{\gamma}^{78}\). Equations (3.2) and (3.3) still hold if the coordinate direction \(\hat{\gamma}^{*2}\) is changed throughout into \(\hat{\gamma}^{+1}\) or \(\hat{\gamma}^{+3}\), showing that *any* change in parity produces a change in the sign of \(\hat{\gamma}^{78}\). Furthermore, as equations (3.2) and (3.3) depend only on the multiplication properties of the \(\hat{\gamma}s\), they hold in any Lorentz frame, always producing a change in sign of a single spatial coordinate direction, and a consequent change in the parity of the coordinate system. \(\hat{\gamma}^{78}\) is Lorentz invariant, changing sign for any choice of coordinates, and its eigenvalues therefore determine both coordinate parity and the intrinsic parity of leptons.

Given that the eigenvalues of \(\hat{\gamma}^{*0}\) determine the time direction in the rest frame of the fermion, time reversal must be expressed in terms of an element of the algebra that anti-commutes with \(\hat{\gamma}^{*0}\) and commutes with the unit spatial displacements \(\hat{\gamma}^{+1}, \hat{\gamma}^{+2}, \hat{\gamma}^{+3}\). There are two possibilities, viz. \(\hat{\gamma}^{06}\) and \(\hat{\gamma}^{*0} = \hat{\gamma}^{+1} \hat{\gamma}^{+2} \hat{\gamma}^{+3}\). These satisfy \((\hat{\gamma}^{*0})^2 = -1\) and \((\hat{\gamma}^{*0})^2 = 1\). The most satisfactory choice is \(\hat{\gamma}^{*0}\), which does not involve time-like generators and can be expressed as the similarity transformation

\[\hat{\gamma}^{*0} \to (\hat{\gamma}^{*0})^{-1} \hat{\gamma}^{*0} \hat{\gamma}^{*0} = -\hat{\gamma}^{*0}.\]  (3.4)

The identification of these symmetries with quantum numbers ensures their invariance in physical processes. Nevertheless, while (3.4) holds in any reference frame, only the time direction \(\hat{\gamma}^{*0}\) in the fermion rest frame defines a Lorentz invariant quantum number.
§4. Quantum number description of fermions

It was shown in [1,2] that each of the $2^7$ fermion states is specified by a unique combination of the seven binary quantum numbers $A, B, C, D, E, F, G$ (denoted $\mu_A$, etc. in [1]), which will be referred to as their ‘signatures’. $A=\pm 1$ specifies spin direction, the eigenvalues $B$ of $\hat{\gamma}^{\pm 0}$ distinguish fermions ($B=1$) from antifermions ($B=-1$), so that $A, B$ determine the four states of any fermion. The eigenvalues ($C=\pm 1$) of $\hat{\gamma}^{\pm 8}$ determine their intrinsic parity, distinguishing the two states in all fermion doublets. $D$ and $E$, together, determine the three colours of quarks, and separate quarks and leptons. The $F, G$ quantum numbers, which distinguish fermion generations, also depend on $C=\pm 1$. This is made explicit in the table of fermion signatures given in the Appendix. $F, G$ descriptions of the the $u,c,t$ ($C=1$) quarks and $d,s,b$ ($C=+1$) quarks are related by the CKM matrix (as in the Standard Model). Similarly, descriptions of the $e^-, \mu^-, \tau^-$ ($C=1$) leptons and $\nu_e, \nu_\mu, \nu_\tau$ ($C=1$) neutrinos are related by the PMNS matrix (as in the Standard Model).

The CKM matrix (e.g. see [3] §14.1) is unitary and has numerical components, viz.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad (4.1)$$

or its inverse

$$V_{\text{CKM}}^* = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix}, \quad (4.2)$$

where the starred components are complex conjugates. These are interpreted in this work as relating the $(F_-, G_-)$ quantum numbers of the $C=1$ $u,c,t$ quarks and the $(F_+, G_+)$ quantum numbers of the $C=+1$ $d,s,b$ quarks.

$$u(1_+1_) = V_{ud}d(1_+1_+) + V_{us}s(1_+1_+) + V_{ub}b(\bar{1}_+1_+)$$
$$c(1_-1_+) = V_{cd}d(1_+1_+) + V_{cs}s(1_+1_+) + V_{cb}b(\bar{1}_+1_+) \quad (4.3)$$
$$t(\bar{1}_-1_+) = V_{td}d(1_+1_+) + V_{ts}s(1_+1_+) + V_{tb}b(\bar{1}_+1_+)$$

The corresponding inverse relations are

$$d(1_+1_+) = V_{ud}^*u(1_-1_-) + V_{cd}^*c(1_-1_-) + V_{td}^*t(\bar{1}_-1_-)$$
$$s(1_-1_+) = V_{us}^*u(1_-1_-) + V_{cs}^*c(1_-1_-) + V_{ts}^*t(\bar{1}_-1_-) \quad (4.4)$$
$$b(\bar{1}_+1_+) = V_{ub}^*u(1_-1_-) + V_{cb}^*c(1_-1_-) + V_{tb}^*t(\bar{1}_-1_-)$$

Magnitudes of the elements of the CKM matrix have been determined experimentally (e.g. §14.3 of [3]):

$$|V|_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix} \quad (4.5)$$

In the following discussion it will be assumed that $V_{\text{CKM}}$ has real components.

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U$ defined in §13.5 of [3], is interpreted in the Standard Model as relating different eigenstates of the neutrino generations. Here it is reinterpreted as relating the eigenstates of negatively charged leptons and neutrinos. This is based on the same $C = \pm 1$ interpretation of states as in the case of quarks, with $C=1$ ($u,c,t$) quarks corresponding to $C=-1$ ($\nu_e, \nu_\mu, \nu_\tau$) neutrinos and $C=1$ ($d,s,b$) quarks corresponding to $C=1$ ($e^-, \mu^-, \tau^-$) leptons. Hence

$$U = \begin{pmatrix} U_{\nu_e e}^* & U_{\nu_e \mu}^* & U_{\nu_e \tau}^* \\ U_{\nu_\mu e} & U_{\nu_\mu \mu} & U_{\nu_\mu \tau} \\ U_{\nu_\tau e} & U_{\nu_\tau \mu} & U_{\nu_\tau \tau} \end{pmatrix}, \quad (4.6)$$

The change in the interpretation of $U$ makes comparisons with neutrino interactions more complicated, which are left for later publications.
\[5.\text{Quantum number conservation}\]

Descriptions of corresponding anti-fermions are obtained by changing the sign of all seven fermion quantum numbers (labelled A B C D E F G). Although fermions can change in interactions, quantum numbers do not change, giving the

\textbf{Conservation Law: All seven quantum numbers are conserved in fermion decays and interactions}

It is convenient to introduce a condensed notation in demonstrating applications of this law, writing $\bar{\mathbb{I}}$ for $-1$ in fermion descriptions. Following the table given in the Appendix, $F_\pm, G_\pm$ quantum numbers for $C=1$ and $C=-1$ fermions are distinguished by attaching $+$ or $-$ suffices to give $1_+, 1_-$ and $1_+, 1_-$. For example, electrons of either spin are described by the set of six quantum numbers $B C D E F G$. Although fermions can change in interactions, quantum numbers do not change, giving the

\[\{d_b : 1 1 1 1 1 \ 1+\} + \{\bar{u}_b : 1 \bar{1} 1 1 1 \ 1-\} + \{e^- : 1 1 \bar{1} 1 1 \ 1+\} + \{\bar{\nu}_e : \bar{1} 1 1 1 \bar{1} \ \bar{1}-\}, \quad (5.1)\]

where the equality can be checked by adding each of the six corresponding numbers in the brackets on the right hand side of the equation. Different quark colours change the signs of $D$ and $E$ in the same way for both $d$ and $u$, leaving the equality the same. The $F,G$ quantum numbers cancel for both $C=1$ and $C=-1$ leptons, so complications produced by the PMNS and CKM matrices do not occur, and the simple form of conservation law (above) is maintained.

Analogous equations also hold for second and third generation $\beta$ decays, viz.

\[\{s_b : 1 1 1 1 1 \ 1+\} = \{c_b : 1 \bar{1} 1 1 1 \ \bar{1}-\} + \{\mu^- : 1 1 \bar{1} 1 1 \ 1+\} + \{\bar{\nu}_\mu : \bar{1} 1 1 1 1 \ \bar{1}-\}, \quad (5.2)\]

\[\{b_b : 1 1 1 1 1 \ 1+\} = \{t_b : 1 \bar{1} 1 1 1 \ \bar{1}-\} + \{\tau^- : 1 1 \bar{1} 1 1 \ 1+\} + \{\bar{\nu}_\tau : \bar{1} 1 1 1 1 \ \bar{1}-\}. \quad (5.3)\]

Corresponding scattering processes is described by moving one set of fermion quantum numbers to the other side equations of equations (5.1) and (5.2). For example, (5.1) becomes

\[\{d_b : 1 1 1 1 1 \ 1+\} + \{\nu_e : 1 \bar{1} 1 1 1 \ \bar{1}-\} = \{u_b : 1 \bar{1} 1 1 1 \ \bar{1}-\} + \{e^- : 1 1 \bar{1} 1 1 \ 1+\}, \quad (5.4)\]

which describes the scattering of the $\nu_e$ neutrino by a $d$ quark to produce a $u$ quark and an electron.

The same relation also describes the $\beta$ decay of quarks into leptons, e.g.

\[\{d_b : 1 1 1 1 1 \ 1+\} + \{\bar{u}_b : 1 \bar{1} 1 1 1 \ \bar{1}-\} + \{e^- : 1 1 \bar{1} 1 1 \ 1+\} + \{\bar{\nu}_e : \bar{1} 1 1 1 1 \ \bar{1}-\}, \quad (5.5)\]

where the charge difference between the $d$ and $u$ quarks (determined by their values of $C$) corresponds to the charge difference between electrons and anti-neutrinos. This charge is carried by the weak bosons $\hat{\gamma}^\pm$, with the $Cl_{3,3}$ B,C description, given in §6 of [1] and §2 of [2],

\[\{\hat{\gamma}^+ : 0 \ 2\} = \frac{1}{2}(\hat{\gamma}^{\pi^7} + i\hat{\gamma}^{\pi^8}), \quad \{\hat{\gamma}^- : 0 \ 2\} = \frac{1}{2}(\hat{\gamma}^{\pi^7} - i\hat{\gamma}^{\pi^8}). \quad (5.6)\]

\[\textbf{§6. Conclusions}\]

Following the discussion of parity and time-reversal in §2, §3 develops new algebraic definitions, based on the CU formalism developed in [1,2]. This shows that coordinate parity and time direction correspond, respectively, to the quantum numbers $C$ and $B$. Furthermore, $C$ determines the intrinsic parity of fermions. §4 relates the Standard Model interpretation CKM and PMNS matrices to the C dependence of the FG quantum numbers. This leads, in §5, to modification of the Conservation Law, that states that all seven quantum numbers, provided by the commuting elements of the $Cl_{7,7}$ algebra, are conserved in fermion decay processes and interactions. This law has direct applications in the prediction of processes that have yet to be observed. It should be of value revised analyses of existing experimental results and in the design of new experiments.
Appendix

CDEF$\pm$G signatures for all four generations of fermions, with B=1

| quark | C   | D   | E   | F$_\pm$ | G$_\pm$ | QB  | QC  | Q   |
|-------|-----|-----|-----|---------|---------|-----|-----|-----|
| $u_b$ | −1  | 1   | −1  | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $u_r$ | −1  | 1   | −1  | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $u_g$ | −1  | −1  | 1   | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $d_b$ | 1   | 1   | 1   | 1      | 1+      | 1/6 | −1/2| −1/3|
| $d_r$ | 1   | 1   | −1  | 1      | 1+      | 1/6 | −1/2| −1/3|
| $d_g$ | 1   | −1  | 1   | 1      | 1+      | 1/6 | −1/2| −1/3|
| $c_b$ | −1  | 1   | 1   | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $c_r$ | −1  | 1   | −1  | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $c_g$ | −1  | −1  | 1   | 1      | −1      | 1/6 | 1/2 | 2/3 |
| $s_b$ | 1   | 1   | 1   | 1      | −1      | 1/6 | −1/2| −1/3|
| $s_r$ | 1   | 1   | −1  | 1      | −1      | 1/6 | −1/2| −1/3|
| $s_g$ | 1   | −1  | 1   | 1      | −1      | 1/6 | −1/2| −1/3|
| $t_b$ | −1  | 1   | 1   | −1     | 1      | 1/6 | 1/2 | 2/3 |
| $t_r$ | −1  | 1   | −1  | −1     | 1      | 1/6 | 1/2 | 2/3 |
| $t_g$ | −1  | −1  | 1   | −1     | 1      | 1/6 | 1/2 | 2/3 |
| $b_b$ | 1   | 1   | 1   | −1     | 1      | 1/6 | −1/2| −1/3|
| $b_r$ | 1   | 1   | −1  | −1     | 1      | 1/6 | −1/2| −1/3|
| $b_g$ | 1   | −1  | 1   | −1     | 1      | 1/6 | −1/2| −1/3|
| $\nu_e$ | −1  | −1  | −1  | 1      | −1      | −1/2| 1/2 | 0   |
| $\nu_\mu$ | −1  | −1  | −1  | 1      | −1      | −1/2| 1/2 | 0   |
| $\nu_\tau$ | −1  | −1  | −1  | 1      | −1      | −1/2| 1/2 | 0   |
| $e^-$ | 1   | −1  | −1  | 1      | 1      | −1/2| −1/2| −1  |
| $\mu^-$ | 1   | −1  | −1  | 1      | −1      | −1/2| −1/2| −1  |
| $\tau^-$ | 1   | −1  | −1  | −1     | 1      | −1/2| −1/2| −1  |
| $q_b(-4/3)$ | −1  | 1   | 1   | −1     | −1      | 1/6 | −3/2| −4/3|
| $q_r(-4/3)$ | −1  | 1   | −1  | −1     | −1      | 1/6 | −3/2| −4/3|
| $q_g(-4/3)$ | −1  | −1  | 1   | −1     | −1      | 1/6 | −3/2| −4/3|
| $q_b(5/3)$ | 1   | 1   | 1   | −1     | −1      | 1/6 | 3/2 | 5/3 |
| $q_r(5/3)$ | 1   | 1   | −1  | −1     | −1      | 1/6 | 3/2 | 5/3 |
| $q_g(5/3)$ | 1   | −1  | 1   | −1     | −1      | 1/6 | 3/2 | 5/3 |
| $l(-2)$ | −1  | −1  | −1  | −1     | −1      | −1/2| −3/2| −2   |
| $l(1)$  | 1   | −1  | −1  | −1     | −1      | −1/2| 3/2 | 1    |

Quantum numbers describing fourth generation fermions included in this table are interpreted elsewhere [5].

Fermion electric charges are the sum of $Q_B = \frac{1}{6}(D+E - BDE)$ and $Q_C = -\frac{1}{2}((F+G - BFG)BC$, giving total charges $Q \times$(the magnitude of the electronic charge) with

$$Q = Q_B + Q_C = \frac{1}{6}(D + E - BDE) - \frac{1}{2}(F + G - BFG)BC.$$  \hspace{1cm} (A1)
This formula holds for both definitions of the F,G quantum numbers. Corresponding anti-fermion signatures and charges are obtained by reversing the signs of all quantum numbers A to G. Quantum number conservation for decays and interactions of fermions in the first three generations automatically conserve charge.

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