Quantum Discrete Fourier Transform with Classical Output for Signal Processing

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Abstract

Discrete Fourier transform (DFT) is the base of modern signal or information processing. 1-Dimensional fast Fourier transform (1D FFT) and 2D FFT have time complexity $O(N \log N)$ and $O(N^2 \log N)$ respectively. Quantum 1D and 2D DFT algorithms with classical output (1D QDFT and 2D QDFT) are presented in this paper. And quantum algorithm for convolution estimation is also presented in this paper. Compared with FFT, QDFT has two advantages at least. One of advantages is that 1D and 2D QDFT has time complexity $O(\sqrt{N})$ and $O(N)$ respectively. The other advantage is that QDFT can process very long signal sequence at a time. QDFT and quantum convolution demonstrate that quantum signal processing with classical output is possible.

Keywords: DFT, QDFT, Convolution

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I. INTRODUCTION

A. Introduction of Discrete Fourier transform (DFT)

DFT is the base of modern signal and information processing. No DFT, no modern signal and information processing [1].

Let

\[
W_N = \frac{1}{\sqrt{N}} \begin{bmatrix}
\omega^0 & \omega^0 & \omega^0 & \cdots & \omega^0 \\
\omega^0 & \omega^1 & \omega^2 & \cdots & \omega^{N-1} \\
\omega^0 & \omega^2 & \omega^4 & \cdots & \omega^{2(N-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\omega^0 & \omega^{N-1} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)(N-1)} \\
\end{bmatrix} = \begin{bmatrix}
\vec{W}_0 \\
\vec{W}_1 \\
\vdots \\
\vec{W}_{N-1} \\
\end{bmatrix}
\]

(1)

, where \( \omega = e^{-i\frac{2\pi}{N}} \) and \( \vec{W}_0, \vec{W}_1, \cdots, \vec{W}_{N-1} \) is the line vector of the matrix. The matrix \( W_N \) defined in Eq. 1 is called Fourier transform matrix [1].

1-Dimensional DFT (1D DFT) [1] is defined as

\[
\vec{c} = W_N \vec{x} = \begin{bmatrix}
\vec{W}_0 \cdot \vec{x} \\
\vec{W}_1 \cdot \vec{x} \\
\vdots \\
\vec{W}_{N-1} \cdot \vec{x} \\
\end{bmatrix}
\]

(2)

, where vector \( \vec{x} = (x_0, x_1, x_2, \cdots, x_{N-1})^T \), \( \vec{c} = (c_0, c_1, c_2, \cdots, c_{N-1})^T \) (T denotes the transpose of vector), and “\( \cdot \)” denotes the inner product between two vectors. Each component of vector \( \vec{c} \) is called Fourier coefficient.

For any input matrix \( F = [f_{ij}]_{N \times N} \), 2D DFT [1] is defined as

\[
C = W_N FW_N
\]

(3)

, where \( C \) denotes the matrix of DFT coefficients.

DFT has two important properties, that will be applied to design quantum algorithm in this paper. One property is that DFT is energy conservation transform (i.e., \( ||\vec{x}||^2 = ||\vec{c}||^2 \), or \( \sum(x_i)^2 = \sum|c_i|^2 \)). The other property is that, typical data sequence, such as digital image, has high redundancy, many of its DFT coefficients have values close to zero, and these coefficients can be discarded without seriously affecting the estimated value of \( ||\vec{x}||^2 \)
Therefore, inverse DFT acting on the few big coefficients retained can restore the original data approximatively. Fig. 1 shows the properties of DFT.

**Fig. 1:** The Illustration of the Two Properties of DFT that Applied to Design Quantum Algorithm in This Paper: Fig. A denotes the original data of the top-left image block with size $4 \times 4$ of image Lena [12]. Fig. B denotes the 2D DFT coefficients of the image block. DFT has two properties. One property is that DFT is energy conservation transform. The other property is that, only few DFT coefficients is *not* close to zero, and the inverse DFT acting on these coefficients can restore the original data approximatively. The two properties are applied to design quantum DFT algorithm in this paper.

Fast Fourier transforms (FFT) with time complexity $O(N\log N)$ and $O(N^2 \log N)$ for 1D and 2D DFT respectively were presented at 1965 [1]. And the desire for designing more fast DFT algorithm is still very strong. However, there is no more fast algorithm is presented up till now. Classical computation can not improve the efficiency of DFT any more maybe, and the principle of classical computation becomes the bottleneck of designing fast DFT algorithm maybe. Can quantum computation be applied to design more fast DFT algorithm with classical output to break the bottleneck?

**B. Introduction of Quantum Computation**

Introduction of Quantum Fourier Transform (QFT): QFT [2, 3] on an orthonormal basis $|0\rangle, |1\rangle, \cdots, |N-1\rangle$ is defined to be a linear operator $QFT |j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle$. QFT is the key of the famous Shor’s order-finding and factoring algorithm [2]. QFT has time complexity $O(\log^2 N)$. However, QFT is not suitable to signal and information processing.
because the result of DFT defined in Eq.2 (or Eq.3) can not be generated and measured out by this QFT [4, 5].

**Introduction of Grover’s Algorithm:** Grover’s algorithm [6] solves the problem of searching for an element with a unique index $i_0$ in a list of $N$ unsorted elements, similar to searching a database like a telephone directory when we know the number but not the person’s name [3]. Grover’s algorithm has time complexity $O(\sqrt{N})$ [3]. Long proposes a modified Grover’s algorithm, that has the probability of success 100% even for the case that the number of elements is very small [7]. Boyer, Brassard, Høyer, and Tapp present the modified Grover’s algorithm named BBHT algorithm in this paper for the case that the number of solutions is unknown [8]. BBHT algorithm is a very smart algorithm because it saves many quantum circuits.

**Introduction of Quantum Loading Scheme** $U_L$ [9]: Grover’s algorithm has the function that find the index $i_0$ of a special database record $record_{i_0}$ from the index superposition of state $\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle)$. And the record $record_{i_0}$ is the genuine answer wanted by us. However, the corresponding record $record_{i_0}$ can not be measured out unless the 1-1 mapping relationship between index $i$ and the corresponding record $record_{i}$ is bound in the entangled state $\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle |record_{i}\rangle)$. That is, we need a unitary operation $U_L$ to load all records that are stored in a classical database into quantum state. The function of unitary operation $U_L$ can be described as

$$\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle |0\rangle) |ancilla\rangle U_L \frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle |record_{i}\rangle) |ancilla\rangle$$

(4)

, where ancillary state $|ancilla\rangle$ is known.

Pang proposes a design method of the unitary operation $U_L$, that has time complexity $O(\log N)$ (unit time: phase transform and flipping the qubits of registers) [9]. Operator $U_L$ is so fast that its running time can be ignored when analyzing the time complexity of a algorithm.

**Introduction of Quantum Search Algorithm with Complex Computation (i.e., the Method of Rotation at Subspace)** [10, 11, 12, 13]:

Grover’s algorithm can find a database record according to the given index. However, database search is complex in general. E.g., police often hopes to find a mug shot from the database in which many sample photos are stored by the method of matching every sample photo and the photo captured by the vidicon at the entrance of airport real-time. Grover’s
algorithm is invalid for this kind of search case because the coupling between search and other computation (e.g., image matching) is required at this case. Pang et.al. presents a quantum method named "\textbf{rotation at subspace}" \cite{10, 11, 12, 13} to generalize Grover’s algorithm to the search case with arbitrary complex computation, that is derived from the research of quantum image compression \cite{13}. The method of rotation at subspace is described as following briefly:

First, All input datum are stored in classical memory as database records. Assume that total number of records is \(N\). All these records can be loaded into a superposition of state using quantum loading scheme \(U_L\).

Second, construct the general Grover iteration (GGI) \(G_{\text{general}}\) as

\[
G_{\text{general}} = (2|\xi\rangle\langle\xi| - I)(U_L)^\dagger(O_c)^\daggerO_fO_cU_L
\]

, where \(O_c\) denotes computation oracle such as image matching, \(f\) denotes the judge function (i.e., if the output of \(O_c\) satisfies some conditions, let \(f = 1\), else \(f = 0\)), \(O_f\) is the oracle of the judge function, and \(|\xi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle\).

Third, similar to Grover’s algorithm, let unitary operation \(G_{\text{general}}\) act on initial state \(\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle_{\text{registers1}})|0\rangle_{\text{registers2}}\) \(O(\sqrt{N})\) times, we will find the optimal solution.

II. \textbf{1-DIMENSIONAL QUANTUM DFT (1D QDFT)}

First, construct the following data structure (DS) and unitary operations:

\textbf{DS1.} Save DFT matrix \(W\) defined in Eq.\(1\) in classical memory as a database. And each line vector \(\vec{W}_i\) of the matrix is regard as a record, and these records have indices \(0, 1, ..., N-1\).

\textbf{DS2.} Construct six registers that have data format

\[
|\alpha\rangle_{\text{register1}}|\beta\rangle_{\text{register2}}|\vec{x}\rangle_{\text{register3}}|i\rangle_{\text{register4}}|\vec{W}_i\rangle_{\text{register5}}|(\vec{W}_i \cdot \vec{x})^2\rangle_{\text{register6}}
\]

. That is, 1st, 2nd, 3rd, 4th, 5th, and 6th register are used to save input parameter \(\alpha\), input parameter \(\beta\), input vector \(\vec{x}\), index \(i\), line vector \(\vec{W}_i\), and squared inner product respectively.

\textbf{DS3.} Design oracle \(O_{\text{inner}}\) to compute the squared inner product between vector \(\vec{W}_i\) and \(\vec{x}\), i.e.,

\[
|\alpha\rangle|\beta\rangle|\vec{x}\rangle|i\rangle|\vec{W}_i\rangle|0\rangle \xrightarrow{O_{\text{inner}}} |\alpha\rangle|\beta\rangle|\vec{x}\rangle|i\rangle|\vec{W}_i\rangle|(\vec{W}_i \cdot \vec{x})^2\rangle
\]
We can design very simple parallel circuit to calculate squared inner product, and has time complexity $2t_m + \lceil \log_2 N \rceil t_a$, where $t_m$ and $t_a$ denote the unit running time of multiplication and addition respectively. Because addition is more fast than multiplication ($50t_a < t_m$ in general) and $N < 2^{50}$ in general, the running time of $O_{inner}$ can be regarded as few times of multiplication (i.e., $O(1)t_m$).

**DS4.** Design oracle $O_f$:

$$|\alpha\rangle|\beta\rangle|\vec{x}\rangle|i\rangle|0\rangle \xrightarrow{O_f} |\vec{W}_0\rangle |\vec{x}\rangle|\vec{W}_0\cdot \vec{x}\rangle (O_{inner})^{-1} O_f O_{inner} U_L$$

where $f(i) = \begin{cases} 1 & \text{if } \alpha \leq (\vec{W}_0 \cdot \vec{x})^2 \leq \beta \\ 0 & \text{otherwise} \end{cases}$. Oracle $O_f$ is used to mark the DFT coefficients $\vec{W}_0 \cdot \vec{x}$ that satisfy the condition $\alpha \leq (\vec{W}_0 \cdot \vec{x})^2 \leq \beta$.

**DS5.** Define 1D QDFT iteration $G_{1DQDFT}$:

According to Eq.5 1D QDFT iteration $G_{1DQDFT}$ is

$$G_{1DQDFT} = (2|\xi\rangle\langle\xi| - I)(U_L)^{-1} (O_{inner})^\dagger O_f O_{inner} U_L$$

Second, design the following subroutine 1 to find a coefficient $\vec{W}_0 \cdot \vec{x}$ that satisfies the condition $\alpha \leq (\vec{W}_0 \cdot \vec{x})^2 \leq \beta$:

**subroutine 1:**

**Step1.** Initialize $m = 1$ and set $\lambda = 6/5$. (Any value of $\lambda$ strictly between 1 and $4/3$ would do.)

**Step2.** Choose $j$ uniformly at random among the nonnegative integers smaller than $m$.

**Step3.** Apply $j$ iterations of $G_{1DQDFT}$ acting on initial state $|\psi_0\rangle = \frac{1}{\sqrt{N}} |\alpha\rangle|\beta\rangle|\vec{x}\rangle (\sum_{i=0}^{N-1} |i\rangle)|0\rangle|0\rangle$.

**Step4.** Observe the 4th register: let $i_0$ be the outcome.

**Step5.** Calculate value $\vec{W}_0 \cdot \vec{x}$ using classical computation. If $\alpha \leq (\vec{W}_0 \cdot \vec{x})^2 \leq \beta$, preserve $i_0$, and exit.

**Step6.** Otherwise, set $m$ to $\min(\lambda m, \sqrt{N})$ and go back to step 2.

Subroutine 1 is similar to BBHT algorithm [8], and the main difference between them is that Grover iteration is replaced by 1D QDFT iteration $G_{1DQDFT}$ that realizes the coupling between quantum search and the computation of inner product. Subroutine 1 has time
complexity $O(\sqrt{N/M})$ \[8\], where $M$ denotes the number of coefficients $\vec{W}_i \cdot \vec{x}$ that satisfy the condition $\alpha \leq (\vec{W}_i \cdot \vec{x})^2 \leq \beta$.

Third, design the following 1D QDFT algorithm:

**Step 0.** Let $\Delta E = ||\vec{x}||^2 = \sum_{i=0}^{N-1} (x_i \times x_i)$, $\alpha = \frac{\Delta E}{N}$, $\beta = \Delta E$, $nS = 0$. We can design a very simple parallel circuit to calculate value $\Delta E$, and the parallel circuit has computation complexity $O(1)$ (unit time : multiplication) approximately.

**Step 1.** Generate the initial state $|\psi_0\rangle = \frac{1}{\sqrt{N}}|\alpha\rangle|\beta\rangle|\vec{x}\rangle((\sum_{i=0}^{N-1} |i\rangle)|0\rangle|0\rangle$. This can be achieved in $O(\log_2 N)$ steps using a $\lceil \log_2 N \rceil$-bit Hadamard transform, which is so fast that the running time can be ignored.

**Step 2.** while($\Delta E ||\vec{x}||^2 \geq \varepsilon$), where $\varepsilon$ is the given threshold.

\{ 

**Step 2.1:** Apply subroutine 1 to find a coefficient $c[i_0] = \vec{W}_{i_0} \cdot \vec{x}$ that satisfies the condition $\alpha \leq (c[i_0])^2 \leq \beta$; 

**Step 2.2:** If the DFT coefficient $c[i_0]$ is the result that is not be found previously by this algorithm, preserve it, and let $\Delta E = \Delta E - (c[i_0])^2$, $nS = nS + 1$, $\alpha = \frac{\Delta E}{N-nS}$, and $\beta = \Delta E$, where $\alpha$ denotes the average residual energy per DFT coefficient which is not be obtained still by this algorithm, $\beta$ denotes the total residual energy that is preserved by the DFT coefficients which are not be obtained still by this algorithm, and $nS$ denotes the number of the coefficients which have been obtained by this algorithm.

\}

It’s the main idea of step 2.1 that apply subroutine 1 to find a coefficient which energy is bigger than the average residual energy (i.e., value $\alpha$) and smaller than the total residual energy (i.e., value $\beta$). Because DFT is energy conversation transform (i.e., $\sum_{i=0}^{N-1} (x_i)^2 = \sum_{i=0}^{N-1} (c_i)^2$), the stop criterion $\frac{\Delta E}{||\vec{x}||^2} \geq \varepsilon$ shows that the above algorithm will find all big coefficients that preserve nearly all energy and information if threshold $\varepsilon$ is enough small. In addition, almost signal sequences have high redundancy and the main task of DFT is to find and retain big coefficients to eliminate this redundancy (see Fig.11). Therefore, 1D QDFT algorithm realizes 1D DFT computation approximately.

1D QDFT has time complex $O(\sqrt{mN})$ approximately, where $m$ denotes the number of the big coefficients. Because signal sequences have high redundancy in general, the case
$m \ll N$ often happens (see Fig.1). Thus, we regard 1D QDFT has time complexity $O(\sqrt{N})$ (unit: multiplication and addition), while classical 1D FFT has time complexity $O(N \log_2 N)$ (unit: multiplication and addition) and even parallel DFT computation has time complexity $O(N)$. QDFT can process long signal sequence at a time, while classical computer requires that long signal sequence must be divided into many small sections (such as $8 \times 8$ or $16 \times 16$ section) on which DFT acts sequentially to evade the efficiency bottleneck of loading data.

III. 2D QDFT

2-D DFT (see Eq.3) is a separable linear transformation. That is, the result of 2-D DFT may be obtained by first taking transforms along the columns of $F$ and then along the rows of that result $|F|$. That is, $C = W_N F W_N = (W_N F) W_N$. We define

$$G = W_N F = (\overrightarrow{W_i} \cdot \overrightarrow{f_j})_{N \times N}$$

, where $\overrightarrow{W_i}$ is the line vector of matrix $W_N$ and $\overrightarrow{f_j}$ is the column vector of input matrix $F$.

The main task of 2D QDFT is to calculate out matrix $G$ and matrix $GW_N$ (i.e., $C$). 2D QDT is described as following:

First, Construct the following data structures (DS) and unitary operations:

DS1: Store all of line vector $\overrightarrow{W_i}$ in a database, and each index of line vector is denoted by $i$. Store all of column vector $\overrightarrow{f_j}$ of matrix $F$ in the database, and each index of column vector is denoted by $j$.

DS2: Construct seven registers that has data format $|\alpha\rangle|\beta\rangle|i\rangle|j\rangle|0\rangle|0\rangle|0\rangle$.

DS3: Design two loading operation $U_{L1}$ and $U_{L2}$ according to Eq.4 and ref.[9], where $U_{L1}$ and $U_{L2}$ are applied to load vector $\overrightarrow{W_i}$ and $\overrightarrow{f_j}$ into registers from classical database respectively. That is,

$$
\begin{align*}
|\alpha\rangle|\beta\rangle|i\rangle|j\rangle|0\rangle|0\rangle|0\rangle & \xrightarrow{U_{L1}} |\alpha\rangle|\beta\rangle|i\rangle|j\rangle|\overrightarrow{W_i}\rangle|0\rangle|0\rangle \\
|\alpha\rangle|\beta\rangle|i\rangle|j\rangle|0\rangle|0\rangle|0\rangle & \xrightarrow{U_{L2}} |\alpha\rangle|\beta\rangle|i\rangle|j\rangle|0\rangle|0\rangle|0\rangle
\end{align*}
$$

DS4: Design oracle $B_{inner}$ to calculate squared inner product. That is,

$$
|\alpha\rangle|\beta\rangle|i\rangle|j\rangle|\overrightarrow{W_i}\rangle|\overrightarrow{f_j}\rangle|0\rangle \xrightarrow{B_{inner}} |\alpha\rangle|\beta\rangle|i\rangle|j\rangle|\overrightarrow{W_i}\rangle|\overrightarrow{f_j}\rangle|0\rangle|0\rangle|0\rangle
$$
DS5: Design oracle $O'_f$:

$$|\alpha\rangle|\beta\rangle|i\rangle|j\rangle (|\bar{W}_i\rangle|\bar{f}_j\rangle)^{O'_f} \rightarrow (-1)^{f'(i,j)}|\alpha\rangle|\beta\rangle|i\rangle|j\rangle (|\bar{W}_i\rangle|\bar{f}_j\rangle)^2$$

where $f'(i,j) = \begin{cases} 
1 & \text{if } \alpha \leq (\bar{W}_i \cdot \bar{f}_j)^2 \leq \beta \\
0 & \text{otherwise}
\end{cases}$.

DS6: Design 2D QDFT iteration $G_{2DQDFT}$:

$$G_{2DQDFT} = (2|\xi'\rangle\langle\xi'|-I)(U_{L1})^\dagger(U_{L2})^\dagger (B_{inner})^\dagger O'_f B_{inner} U_{L2} U_{L1}$$

where $|\xi'\rangle = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} |i\rangle|j\rangle$.

Second, Generate the initial state $|\Psi_0\rangle = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} |\alpha\rangle|\beta\rangle|i\rangle|j\rangle|0\rangle|0\rangle|0\rangle$. And apply the method of 1D QDFT, we will calculate out all elements of matrix $G$. Apply the same method again, we can calculate out all elements of matrix $C$.

2D QDFT has time complexity $O(N)$, while classical 2D FFT has time complexity $O(N^2 \log_2 N)$.

IV. USING 1D QDFT TO CALCULATE CONVOLUTION

For the given two $N$-dimensional vectors $\vec{u} = (u_0, u_1, ..., u_{N-1})$ and $\vec{v} = (v_0, v_1, ..., v_{N-1})$, let $w_k = \sum_{j=0}^{N-1} u_j v_{(k-j) \text{mod} N}$, where $0 \leq k < N$. Then vector $\vec{w} = (w_0, w_1, ..., w_{N-1})$ is called the periodic convolution of vectors $\vec{u}$ and $\vec{v}$. We often denote the convolution by symbol $\vec{w} = \vec{u} \ast \vec{v}$. If $\vec{u}$ and $\vec{v}$ have different dimensions, add zero components to each vector such that both vectors have the same dimension. The periodic convolution of the new vectors can be regarded as the result. Algebraically, convolution is the same operation as multiplying the polynomials whose coefficients are the elements of $\vec{u}$ and $\vec{v}$.

Holding the view of signal processing, we can regard vector $\vec{u}$ as input signal sequence and vector $\vec{v}$ is the function of a physical apparatus. After vector $\vec{u}$ passing through the apparatus, convolution $\vec{w}$ is came out. Furthermore, the output sequence $\vec{w}$ becomes smoother than the input sequence $\vec{u}$ in general. Therefore, convolution is very important for signal or information processing.

The convolution theorem says, roughly, that convolving two sequences is the same as
multiplying their Fourier transforms. That is [1],

\[ DFT(\vec{u} \ast \vec{v}) = DFT(\vec{u})DFT(\vec{v}) \] (7)

In general, the big coefficients of \( DFT(\vec{v}) \) are few and other coefficients have little contribution to the result. That is, we can calculate only big coefficients of \( DFT(\vec{u}) \) and \( DFT(\vec{v}) \) and discard small coefficients to speedup the computation of convolution. According to this property, quantum algorithm for convolution estimation is described as following:

**Step1.** Calculate \( DFT(\vec{u}) \) and \( DFT(\vec{v}) \) using the 1D QDFT.

**Step2.** Calculate \( DFT(\vec{u} \ast \vec{v}) \) according to Eq.7.

**Step3.** Calculate the inverse of \( DFT(\vec{u} \ast \vec{v}) \) to obtain convolution \( \vec{u} \ast \vec{v} \).

It should be noticed that the efficiency of the above algorithm depends on the redundancy (or smooth property) of the signal sequences. The more higher the redundancy is, the more fast the quantum algorithm runs. Fortunately, many digital signal sequences have high redundancy, that is just the existence reason of modern signal processing technique.

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