Quantum Loops in Non-Local Gravity

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Collaborators

• Tirthabir Biswas
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Other people’s work

• E.T. Tomboulis, arXiv: 9702146 [hep-th]

• L. Modesto, arXiv: 1107.2403 [hep-th]

• D. Anselmi, arXiv: 1302.7100 [gr-qc]
Aim

• Our aim is to construct a UV-finite theory of quantum gravity that is not plagued by pathologies such as ghosts

• Towards that end, we consider a scalar field theory toy model

• Based on that, can we formulate a complete theory of quantum gravity?
Degree of Divergence in GR

- The superficial degree of divergence in d dimensions is \( D = Ld + 2(V - I) \)

- L is the number of loops, V is the number of vertices and I is the number of internal propagators

- Use the topological relation \( L = 1 + I - V \)

- In four dimensions, we get \( D = 2 + 2L \)

- The superficial degree of divergence keeps increasing as L increases
Renormalizability of GR

- Einstein-Hilbert action:
  \[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R \]

- Pure gravity is renormalizable at 1-loop order
- 1 new counterterm required at 2-loop order
Renormalizability of GR

• Stelle (1977) has shown that fourth-order pure gravity is renormalizable!

\[ S = - \int d^4x \sqrt{-g} \left( \alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2 + \kappa^{-2} \gamma R \right) \]

where \( \gamma = 2 \) & \( \kappa^2 = 32\pi G \).

• We do not have to include \( \int d^4x \sqrt{-g} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \)

because of the Gauss-Bonnet topological invariance in four dimensions:

\[ \int d^4x \sqrt{-g} \left( R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu\alpha\beta} R^{\mu\nu} + R^2 \right) \]

vanishes in Minkowski spacetime.
Ghosts

• Unfortunately, Stelle’s theory, as higher-derivative theories generically do, contains ghosts (poles in the propagator with negative residue); specifically, a massive spin-2 ghost

\[ \Pi(p^2) = \frac{1}{p^2(p^2 + m^2)} \sim \frac{1}{p^2} - \frac{1}{p^2 + m^2} \]

where \( m^2 > 0 \)

• Unitarity is violated

• We want to get rid of the ghost
Non-local Higher-derivative Gravity

- Non-local means that we consider an infinite series of higher-derivative terms in the action.

- The most general covariant action up to $\mathcal{O}(h^2)$ (Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. 108 (2012) 031101) is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R_{\mu_2 \nu_2 \lambda_2 \sigma_2} \right]$$

- $\mathcal{O}$ is a differential operator containing covariant derivatives and $\eta_{\mu \nu}$.

- The quadratic curvature part of the action up to $\mathcal{O}(h^2)$ can be written, after many simplifications, as

$$S_q = \int d^4x \left[ R \mathcal{F}_1(\Box) R + R_{\mu \nu} \mathcal{F}_2(\Box) R^{\mu \nu} + R_{\mu \nu} \lambda \sigma \mathcal{F}_3(\Box) R^{\mu \nu} \lambda \sigma \right],$$

since the covariant derivatives take on the Minkowski values.
Non-local Higher-derivative Gravity

- As we shall see later, if we choose \( \mathcal{F}_3(\Box) = 0 \)
  \[ \& \quad \mathcal{F}_1(\Box) = e^{-\frac{\Box}{M^2}} - 1 = -\frac{\mathcal{F}_2(\Box)}{2}, \]
  we obtain the ghost-free action (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* 108 (2012) 031101)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ e^{-\frac{\Box}{M^2}} - 1 \right] R - 2R_{\mu\nu} \left[ e^{-\frac{\Box}{M^2}} - 1 \right] R^\mu_\nu \right]
\]
Linearized Action

- We perturb the metric fluctuations around the Minkowski background: \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \)
- We want to obtain the \( \mathcal{O}(h^2) \) part of the action
- If we perturb the metric fluctuations around the Minkowski background, we get (Biswa, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. 108 (2012) 031101)

\[
S_q = -\int d^4x \left[ \frac{1}{2} h_{\mu\nu} \Box a(\Box) h^{\mu\nu} + h^\sigma_{\mu} b(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} + \frac{1}{2} h \Box d(\Box) h + h^\lambda\sigma \frac{f(\Box)}{\Box} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \right]
\]
Linearized Action

- We have the relations (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* 108 (2012) 031101)

\[
\begin{align*}
a(\square) &= 1 - \frac{1}{2} \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square, \\
b(\square) &= -1 + \frac{1}{2} \mathcal{F}_2(\square)\square + 2\mathcal{F}_3(\square)\square, \\
c(\square) &= 1 + 2\mathcal{F}_1(\square)\square + \frac{1}{2} \mathcal{F}_2(\square)\square, \\
d(\square) &= -1 - 2\mathcal{F}_1(\square)\square - \frac{1}{2} \mathcal{F}_2(\square)\square, \\
f(\square) &= -2\mathcal{F}_1(\square)\square - \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square
\end{align*}
\]

- If \( f(\square) = 0 \Rightarrow a(\square) = c(\square), \) then we observe

\[
2\mathcal{F}_1(\square) + \mathcal{F}_2(\square) + 2\mathcal{F}_3(\square) = 0
\]
Propagator in Non-local Higher-derivative Gravity

- As a consequence of the generalized Bianchi identities, we have
  \[ a + b = 0 \]
  \[ c + d = 0 \]
  \[ b + c + f = 0 \]

- The field equations can be written in the form
  \[ \Pi^{-1}_{\mu\nu} \lambda\sigma h_{\lambda\sigma} = \kappa \mathcal{T}_{\mu\nu} \]

- \( \Pi^{-1}_{\mu\nu} \lambda\sigma \) is the inverse propagator

- The propagator is
  \[ \Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2} \]

- To recover GR in the IR, we must have
  \[ a(0) = c(0) = -b(0) = -d(0) = 1 \]
  As \( k^2 \to 0 \), we obtain the physical graviton propagator
  \[ \lim_{k^2 \to 0} \Pi^{\mu\nu}_{\lambda\sigma} = \frac{P^2}{k^2} - \frac{P_s^0}{2k^2} \]
Ghosts in Non-local Higher-derivative Gravity

• If we apply the assumption \( f = 0 \Rightarrow a = c \), then the propagator becomes

\[
\Pi_{\mu\nu}^{\lambda\sigma} = \frac{1}{k^2 a(-k^2)} \left( P^2 - \frac{1}{2} P_s^0 \right) = \frac{1}{a(-k^2)} \Pi_{GR} a(\Box)
\]

• We are left with a single arbitrary function since \( a = c = -b = -d \)

• Provided \( a(\Box) \) has no zeroes, only the graviton propagator is modified and ghosts are avoided (Biswa, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* 108 (2012) 031101)

• Choosing \( a(-k^2) \) to be a suitable entire function, the ultraviolet behavior of the gravitons can be tamed

• One such choice is \( a(-k^2) = e^{k^2/M^2} \)

• \( M \) is a mass scale at which the non-local modifications become important
Symmetries

- Field equations of GR satisfy the global scaling symmetry

\[ g_{\mu \nu} \rightarrow \lambda g_{\mu \nu} \]

- Quadratic curvature actions of the form

\[ S_q = \int d^4x \sqrt{-g} \left[ R\mathcal{F}_1(\Box) R + R_{\mu \nu} \mathcal{F}_2(\Box) R^{\mu \nu} + R_{\mu \nu \lambda \sigma} \mathcal{F}_3(\Box) R^{\mu \nu \lambda \sigma} \right] , \]

where the \( \mathcal{F}_i \) 's are analytic functions of \( \Box \),

are invariant under the aforementioned symmetry.

- When we expand the action around Minkowski space, the symmetry for \( h_{\mu \nu} \) becomes, infinitesimally,

\[ h_{\mu \nu} \rightarrow (1 + \epsilon) h_{\mu \nu} + \epsilon \eta_{\mu \nu} \]

- Relates the free and interaction parts of the action (not a fundamental symmetry); it is useful to have a theory with propagators and vertices having opposing momentum dependence, which is a key feature of gauge theories

- We arrive at the shift-scaling symmetry

\[ \phi \rightarrow (1 + \epsilon) \phi + \epsilon \]

- We can formulate scalar toy model whose quantum behavior resembles that of the full gravitational theory
Degree of Divergence in Non-local Gravity

- Our modified superficial degree of divergence counting exponents is $E = V - I$
- Use again the topological relation $L = 1 + I - V$
- We obtain $E = 1 - L$
- For $L > 1$, $E$ is negative, implying superficially convergent loop amplitudes
- Clear contrast with GR
Scalar Field Theory Toy Model Action

• Our scalar field theory toy model action is

\[ S = \frac{1}{2} \int d^4x \left( \phi \Box a(\Box) \phi \right) + \frac{1}{M_p} \int d^4x \left( \frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \Box a(\Box) \phi - \frac{1}{4} \phi \partial_\mu \phi \Box a(\Box) \partial^\mu \phi \right) \]

where \( a(\Box) = e^{-\Box/M^2} \)

• \( M \) is a mass scale at which the nonlocal modifications become important

• Every propagator comes with an exponential suppression and every vertex comes with an exponential enhancement

• The superficial degree of divergence argument for non-local theories of gravity also holds true for the scalar field theory toy model
Propagator

- Our propagator in Euclidean space is

\[ \Pi(k^2) = \frac{-i}{k^2 e^{k^2/M^2}} \]

- The propagator is exponentially suppressed

- As \( k^2 \to 0 \), we obtain the \( k^{-2} \) momentum dependence of the propagator in GR, as it should be in the IR
Vertex Factors

• We have that

\[ V(k_1, k_2, k_3) = iC \left[ 1 - e^{k_1^2/M^2} - e^{k_2^2/M^2} - e^{k_3^2/M^2} \right], \]

where \( C = \frac{1}{4} \left( k_1^2 + k_2^2 + k_3^2 \right) \)

• The momenta are assumed to be incoming and satisfy the conservation law

\[ k_1 + k_2 + k_3 = 0 \]
1-loop, 2-point diagram with external momenta

- Here is the 1-loop, 2-point Feynman diagram with external momenta $p$, $-p$:

\[
\Gamma_{2,1}(p^2) = \frac{i}{2i^2 M_p^2} \int \frac{d^4 k}{(2\pi)^4} \frac{V^2(-p, \frac{p}{2} + k, \frac{p}{2} - k)}{(\frac{p}{2} + k)^2 (\frac{p}{2} - k)^2 e^{(\frac{p}{2} + k)^2/M^2} e^{(\frac{p}{2} - k)^2/M^2}}.
\]
1-loop, 2-point diagram with external momenta

- We have that

\[ \Gamma_{2,1}(p^2) = \frac{iM^4}{M_p^2} f(x), \]

\[ f(x) = \frac{x^4}{256\pi^2} \left( \frac{2}{\epsilon} - \log \left( \frac{x^2}{4\pi} \right) - \gamma + 2 \right) \]

\[ + \frac{e^{-x^2}}{512\pi^2x^2} \left( \left( e^{x^2} - 1 \right) \left( -2 \left( x^4 + 3x^2 + 2 \right) - e^{\frac{x^2}{2}} \left( 2x^4 + 5x^2 + 4 \right) \right) \right. \]

\[ + e^{x^2} \left( e^{x^2} - 1 \right) x^6 \text{Ei} \left( -\frac{x^2}{2} \right) + e^{\frac{3x^2}{2}} \left( 2x^4 + 5x^2 + 4 \right) + 2e^{x^2} \left( 7 \left( x^4 + x^2 \right) + 2 \right) \]

\[ - 2e^{x^2} \left( e^{2x^2} - 1 \right) x^6 \text{Ei} \left( -x^2 \right) \]

\[ + \frac{1}{128\pi} \int_0^1 dr \, e^{(1-2r)x^2} \left[ p(r, x) Y_0 \left( 2\sqrt{r - r^2 x^2} \right) \right. \]

\[ \left. + q(r, x) \sqrt{r - r^2} Y_1 \left( 2\sqrt{r - r^2 x^2} \right) \right] \quad \text{(DR)} \]

and

\[ x = \frac{p}{M} \quad \text{&} \quad p(r, x) = -16x^4 r^4 + (32x^4 + 8x^2)r^3 - (26x^4 + 12x^2)r^2 + (10x^4 + 4x^2)r - 2x^4, \]

\[ q(r, x) = -16x^4 r^3 + (24x^4 + 4x^2)r^2 - (16x^4 + 4x^2 - 8)r + 4x^4 + 3x^2 - 4 \]

The \( \frac{1}{\epsilon} \) pole in DR is equivalent to a \( \Lambda^4 \) divergence if we employ a hard cutoff.
2-loop, 2-point diagram with zero external momenta

For simplicity, we have set the external momenta equal to zero.

\[
\Gamma_{2,2a} = \frac{i^2}{2i^5 M_p^4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{V(k_1, -k_1, 0)V(k_2, -k_2, 0)V^2(k_1, k_2, k_3)}{k_3^2 k_2^4 k_1^4 e^{k_3^2} e^{2k_2^2} e^{2k_1^2}},
\]

where \( k_3 = -k_1 - k_2 \)
The other 2-loop, 2-point diagram

\[ \Gamma_{2,2b} = \frac{i^2}{2\epsilon^5 M_p^4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{V^2(k_1, -k_1, 0))V^2(k_1, -\frac{k_1}{2} + k_2, -\frac{k_1}{2} - k_2)}{k_1^6(k_1/2 + k_2)^2(k_1/2 - k_2)^2 e^{3k_1^2/M^2} e^{(k_1/2 + k_2)^2/M^2} e^{(k_1/2 - k_2)^2/M^2}} \]

Upon redefinition of the momenta, the two 2-loop diagrams give exactly the same result.
2-loop, 2-point diagrams with zero external momenta

- We have that

\[ \Gamma_{2,2a} = \Gamma_{2,2b} = \frac{M^2}{4096 M_p^4 \pi^4} \left[ - M^4 \left( -12 \log \left( \frac{\Lambda}{M} \right) + 43 + 9i\pi + 3\log(3) \right) + 15\Lambda^2 M^2 + 6\Lambda^4 \right] \]

using a hard cutoff \( \Lambda \)

- We have a \( \Lambda^4 \) divergence

- We observe that \( \Gamma_{2,1} \sim \Gamma_{2,2} \sim \Lambda^4 \)

- The degree of divergence stays the same
Summary of Feynman diagram computations

• At 1-loop, the degree of divergence is $\Lambda^4$ (hard cutoff)
• At 2-loop, the degree of divergence also stays $\Lambda^4$
• Hence, we do not get higher divergences as we proceed from 1-loop to 2-loop
• Gives hope towards renormalizability
Dressed Propagators

• If we sum the infinite geometric series of loop corrections to the propagator, we obtain the dressed propagator

\[ \tilde{\Pi}(p^2) = \frac{\Pi(p^2)}{1 - \Pi(p^2)\Gamma_{2,1\text{PI}}(p^2)} \]

where \( \Gamma_{2,1\text{PI}}(p^2) \) is the renormalized 1-loop, 2-point function

• We have that \( \tilde{\Pi}(p^2) \rightarrow \Gamma_{2,1\text{PI}}^{-1}(p^2) \sim e^{-\frac{3p^2}{2M^2}} \) in the UV
Dressed Propagators

- We observe that the dressed propagator is more exponentially suppressed than the bare one.
- If we replace the bare propagators with the dressed propagators, convergence of Feynman integrals is improved.
- Higher-point 1-loop graphs & 2-loop graphs become finite in the UV.
- Only 1-loop, 2-point function diverges.
- Once we remove the aforementioned divergence, the theory at the 1-loop level is renormalized.
- We believe that higher loops remain finite.
Heuristic argument for 2-point & 3-point diagrams

• We consider 2-point & 3-point diagrams which can be constructed out of lower-loop 2-point & 3-point ones

• Since \( \tilde{\Pi}(k^2) \xrightarrow{UV} e^{-\frac{3k^2}{2M^2}} \) & \( \Gamma_3 \xrightarrow{UV} \sum_{\alpha,\beta,\gamma} e^{\alpha \frac{k_1^2}{M^2} + \beta \frac{k_2^2}{M^2} + \gamma \frac{k_3^2}{M^2}} \),

where \( \Gamma_3 \) is the 3-point function & \( \alpha \geq \beta \geq \gamma \), we have that the most divergent UV part of the 2-point diagram for zero external momenta is

\[
\Gamma_{2,n} \xrightarrow{UV} \int \frac{d^4 k}{(2\pi)^4} \frac{e^{(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \frac{k^2}{M^2}}}{e^{\frac{3k^2}{M^2}}} \]
Heuristic argument for 2-point & 3-point diagrams

• Similarly, for the 3-point diagram,

\[
\Gamma_{3,n} \xrightarrow{UV} \int \frac{d^4 k}{(2\pi)^4} e^{(\alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + \beta_3)} \frac{k^2}{M^2} \frac{e^{\frac{9k^2}{2M^2}}}{2M^2}
\]

• We observe that both the 2- & 3-point diagrams become finite if \( \alpha_i + \beta_i < \frac{3}{2} \)

• Even when one includes non-zero external momenta, finiteness is assured

• One can recursively check that \( \alpha_i + \beta_i < \frac{3}{2} \) for higher loops, which is as would be expected since the exponential suppression coming from the propagators is now stronger than the exponential enhancement originating from the vertices
Conclusions

• Nonlocal gravity possesses many novel features
• Ghosts are avoided
• The degree of divergence stays the same as we proceed from 1-loop to 2-loop
• Dressed propagators improve the convergence at all loop orders
• Once we renormalize the 1-loop graphs, higher-loop graphs do not produce new divergences
• A renormalizable & ghost-free theory of quantum gravity may be within reach