Confining potential from interacting magnetic and torsion fields

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Adopting the gauge-invariant but path-dependent variable formalism, we study the coupling of torsion fields with photons in the presence of an external background electromagnetic. We explicitly show that, in the case of a constant electric field strength expectation value, the static potential remains Coulombic, while in the case of a constant magnetic field strength expectation value a confining potential is obtained. This result displays a marked qualitative departure from the usual coupling of axionlike particles with photons in the presence of an external magnetic field.

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I. INTRODUCTION

One of the most actively pursued areas of research in particle physics consists of the investigation of extensions of the Standard Model (SM). This is primarily because the SM does not include a quantum theory of gravitational interactions. We also recall here that the SM has many arbitrary parameters, which may seem too many for a fundamental theory. As is well known, in the search for a more fundamental theory going beyond the SM string theories are the only known candidate for a consistent, ultraviolet finite quantum theory of gravity, unifying all fundamental interactions. It should, however, be noted here that string theories apart from the metric also predict the existence of a scalar field (dilaton) and an antisymmetric tensor field of the third rank which is associated with torsion. This has led to an increasing interest in possible physical effects of these fields. In addition to the string interest, torsion fields have also attracted considerable attention from different viewpoints. Among these, the observed anisotropy of the cosmological electromagnetic propagation, the relativistic and non-relativistic quantum phase acquired by wave function of a neutral spin-1/2 particle with permanent electric and/or magnetic dipole moment in the presence of an electric and magnetic fields, and in connection to the interaction of the light with propagating torsion fields in the presence of an external magnetic field. The advent of the CERN Large Hadronic Collider (LHC) also called attention to test the dynamical torsion parameters and, related to this issue, the production of light gravitons at accelerators justifies the study of dynamical aspects of torsion.

Given the relevance of these studies, it is of interest to improve our understanding of the physical consequences presented by torsion fields. Thus, our purpose here is to further explore the impact of torsion on physical observables, in particular the static potential between two charges, using the gauge-invariant but path-dependent variables formalism along the lines of Ref. [15, 17, 18], which is a physically-based alternative to the usual Wilson loop approach. To this end, we will consider a system consisting of a gauge field interacting with propagating torsion fields when there are nontrivial constant expectation values for the gauge field strength. It is worth recalling at this stage that the phenomenologically relevant part of the torsion tensor is dual to a massive axial vector field, which has a geometric nature. As we shall see, in the case of a constant electric field strength expectation value the static potential remains Coulombic. On the other hand, in the case of a constant magnetic field strength expectation value the potential energy is linear, that is, the confinement between static charges is obtained. Incidentally, the above static potential profile displays a marked departure of a qualitative nature from the results of axionic electrodynamics, where the potential energy is the sum of a Yukawa and a linear potential. One is thus lead to the interesting conclusion that when torsion fields are considered the screening part (encoded in the Yukawa potential) disappears of the static potential profile, describing an exactly confining phase. In such a case the mass of torsion fields contribute to the string tension. What this means in physical terms is that the coupling of torsion fields with photons in the presence of a constant magnetic field strength expectation, behaves like small magnetic dipoles in an external magnetic field.

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II. INTERACTION ENERGY

As we mentioned above, our immediate objective is to calculate explicitly the interaction energy between static point-like sources for the model under consideration. To this end, we will compute the expectation value of the energy operator $\langle H \rangle_{\Phi,\bar{\Phi}}$ in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle \Phi | H | \Phi \rangle$.

The Abelian gauge theory we are considering is governed by the Lagrangian density $[9, 19]$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} S_{\mu\nu}^2 + \frac{1}{2} m^2 S_{\mu}^2 + \frac{g}{4} S^\lambda \partial_\lambda \left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right),$$

(1)

where $m$ is the mass for the torsion field $(S_\mu)$, $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$, and $g$ is a coupling constant with dimension $(-2)$ in mass units.

In expression (1) we have used the unique possible form of the torsion action (in the low-energy sector) $[19]$:

$$\Delta = \frac{g}{8} \varepsilon_{\mu\nu\alpha\lambda} (\partial^\mu \partial^\alpha S_\rho) \left(F^{\nu\kappa} A^\lambda - F^{\nu\lambda} A^\kappa\right),$$

(3)

which renders manifest the coupling of the longitudinal component of $S^\mu$ to the photon spin density tensor.

Next, by integrating out the $S_\mu$ field in expression (1), one gets an effective Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} S_{\mu\nu}^2 + \frac{1}{2} m^2 S_{\mu}^2.$$

(2)

It should be noted that the torsion mass term is mandatory, since, once torsion becomes dynamical, the gravitational excitations related to the torsion irreducible (irreducibility under Lorentz group) components become massive, as shown in Ref. $[3, 20, 21]$ and $[22]$. We would also like to highlight an important feature of the interacting term that couples the photon to the pseudo-scalar torsion: $F_{\mu\nu} \tilde{F}_{\mu\nu}$, up to a piece that is nothing but the Bianchi identity for the field-strength tensor, is a measure of the spin density tensor for the electromagnetic radiation. Then, the photon-torsion coupling in Eq. (1) may be rewritten as

$$\frac{g}{8} \varepsilon_{\mu\nu\alpha\lambda} (\partial^\mu \partial^\alpha S_\rho) \left(F^{\nu\kappa} A^\lambda - F^{\nu\lambda} A^\kappa\right),$$

(3)

Next, by integrating out the $S_\mu$ field in expression (1), one gets an effective Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{g^2}{8 m^2} \left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right) \Delta \left(F_{\lambda\rho} \tilde{F}^{\lambda\rho}\right),$$

(4)

where $\Delta \equiv \partial^\mu \partial_\mu$. Now, after splitting $F_{\mu\nu}$ in the sum of a classical background $\langle F_{\mu\nu} \rangle$ and a small fluctuation, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, the corresponding Lagrangian density is given by

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{g^2}{8 m^2} \left(v^{\mu\nu} f_{\mu\nu}\right) \Delta \left(v^{\lambda\gamma} f_{\lambda\gamma}\right).$$

(5)

Here, we have simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta} (F_{\mu\nu}) \equiv \varepsilon_{\mu\nu}^{\alpha\beta}$ and $\varepsilon^{\rho\sigma\tau\delta} (F_{\rho\sigma}) \equiv \varepsilon^{\rho\sigma\tau\delta}$. This effective theory thus provides us with a suitable starting point to study the interaction energy. There is now a non-trivial point we should raise: the local form of the 4-photon interaction Lagrangian of Eq. (4), after the $S^\mu$-field has been integrated out. According to the Lagrangian (1) and Eq. (3), it becomes clear that the transverse part of $S^\mu$ decouples from the spin density tensor of the electromagnetic field. So, integrating out the torsion, this transverse mode does not leave any track. The longitudinal mode however does couple to the spin density of the photon, as it is fairly well described in the work of Ref. $[4]$. Then, by virtue of the $\frac{\partial^\sigma \partial_\tau}{m^2}$ piece of the $S^\mu$-field propagator, the 4-photon interaction turns out to be local (upon integration of $S^\mu$), as given above in Eq. (4).

A. Magnetic case

We now proceed to obtain the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows). Using this in (4) we then obtain

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{g^2}{8 m^2} v^{0i} f_{0i} \Delta v^{0k} f_{0k},$$

(6)

where $\mu, \nu = 0, 1, 2, 3$ and $i, j, k, l = 1, 2, 3$. To obtain the corresponding Hamiltonian, we must carry out the quantization of this theory. The Hamiltonian analysis starts with the computation of the canonical momenta $\Pi^\mu = f^{\mu0} + \frac{g^2}{8 m^2} v^{0i} \Delta v^{0k} f_{0k}$, which produces the usual primary constraint $\Pi^0 = 0$ while the momenta are $\Pi_i = D_{ij} E_j$. 


Here $E_i = f_i$ and $D_{ij} = \delta_{ij} + \frac{2}{4m} v_0 \Delta v_j$. Since $D$ is nonsingular, there exists the inverse $D^{-1}$. With this, the electric field can be written as
\[ E_i = \frac{1}{\det D} \left\{ \delta_{ij} \det D - \frac{g^2}{4m^2} v_0 \Delta v_j \right\} \Pi_j. \] (7)
The corresponding canonical Hamiltonian is thus
\[ H = \int d^3x \left\{ -A_0 \partial_i \Pi^i - \frac{M^2}{2} \Pi_i (1 + M^2)^{-1} \Pi^i + \frac{B^2}{2} \right\}, \] (8)with
\[ M^2 = \frac{4m^2}{g^2 v^2} = \frac{m^2}{g^2 B^2}. \] (9)
Here, $B$ and $\mathcal{B}$ stand, respectively, for the fluctuating magnetic field and the classical background magnetic field around which the $A^\mu$-field fluctuates. $B$ is associated to the quantum $A^\mu$-field: $B^i = -\frac{1}{2} \epsilon_{ijk} f^j k$, whereas $B_i$, according to our definition for the background $\langle F_{\mu \nu} \rangle$ in terms of $v_{\mu \nu}$ is given by $B_i = \frac{1}{2} v_{0i}$. Time conservation of the primary constraint yields a secondary constraint. The secondary constraint is therefore the usual Gauss constraint $\Gamma_1 (x) = \partial_i \Pi^i = 0$. Note that the time stability of this constraint does not induce further constraints. Consequently, the extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x \left[ c_0 (x) \Pi_0 (x) + c_1 (x) \Gamma_1 (x) \right]$. Here $c_0 (x)$ and $c_1 (x)$ are arbitrary Lagrange multipliers. It should be noted that $A_0 (x) = [A_0 (x), H] = c_0 (x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither $A_0$ nor $\Pi^0$ are of interest in describing the system and may be discarded from the theory. Thus the Hamiltonian is now given as
\[ H = \int d^3x \left\{ c (x) \partial_i \Pi^i - \frac{M^2}{2} \Pi_i (1 + M^2)^{-1} \Pi^i + \frac{B^2}{2} \right\}, \] (10)where $c (x) = c_1 (x) - A_0 (x)$.

The quantization of the theory requires the removal of non-physical variables, which is done by imposing a gauge condition such that the full set of constraints becomes second class. A particularly convenient choice is found to be
\[ \Gamma_2 (x) \equiv \int_{C_{\xi z}} dz^\nu A_\nu (z) \equiv \int_0^1 d\lambda x^i A_i (\lambda x) = 0, \] (11)where $\lambda$ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda (x - \xi)^i$, and $\xi$ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice \textbf{(11)} leads to the Poincaré gauge \cite{23, 24}. As a consequence, the only nonvanishing Dirac bracket for the canonical variables is given by
\[ \{ A_i (x), \Pi^j (y) \}^\ast = \delta_i^j \delta^{(3)} (x - y) - \partial_i^x \int_0^1 d\lambda x^r \delta^{(3)} (\lambda x - y). \] (12)

We have finally assembled the tools to determine the interaction energy for the model under consideration. As mentioned before, in order to accomplish this purpose we will calculate the expectation value of the energy operator $H$ in the physical state $| \Phi \rangle$. Now we recall that the physical state $| \Phi \rangle$ can be written as
\[ | \Phi \rangle \equiv \langle \overline{\Psi} (y) | \Psi (y) \rangle \]
\[ = \overline{\psi} (y) \exp \left( iq \int_{C_{\xi y}} dz^\mu A_\mu (z) \right) | \psi (y) \rangle | 0 \rangle, \] (13)where the line integral is along a spacelike path on a fixed time slice, and $| 0 \rangle$ is the physical vacuum state. The charged matter field together with the electromagnetic cloud (dressing) which surrounds it, is given by $\Psi (y) = \exp \left( -iq \int_{C_{\xi y}} dz^\mu A_\mu (z) \right) \psi (y)$. Thanks to our path choice, this physical fermion then becomes $\Psi (y) = \ldots$. 

\[ \ldots \]
exp \((-iq \int_0^y dz' A_i(z)) \psi(y)\). In other terms, each of the states (|\Phi\rangle) represents a fermion-antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance.

From this and the foregoing Hamiltonian discussion, we then get

\[
\Pi_i(x) \langle \bar{\Psi}(y) \Psi(y') \rangle = \bar{\Psi}(y) \Psi(y') \Pi_i(x) \langle 0 \rangle + q \int_y^{y'} dz_2 \delta(3)(z - x) |\Phi\rangle.
\] (14)

Having made this observation and since the fermions are taken to be infinitely massive (static) we can substitute \(\Delta \) by \(-\nabla^2\) in Eq. (16). Therefore, the expectation value \langle H \rangle_\Phi \ is expressed as

\[
\langle H \rangle_\Phi = \langle H \rangle_0 + \langle H \rangle_\Phi^{(1)} = \langle H \rangle_0 + \frac{M^2}{2} \langle \Phi \rangle \int d^3x \Pi_i \frac{1}{(\nabla^2 - M^2)} \Pi^i |\Phi\rangle,
\] (15)

where \langle H \rangle_0 = \langle 0| H |0 \rangle. Using Eq. (14), the \langle H \rangle_\Phi^{(1)} \ term can be rewritten as

\[
\langle H \rangle_\Phi^{(1)} = \frac{M^2 q^2}{2} \int d^3x \int_y^{y'} dz_3' \delta(3)(x - z') \left(\nabla^2 - M^2\right)_{z'}^{-1} \int_y^{y'} dz^3 \delta(3)(x - z).
\] (16)

Following our earlier procedure [15, 16], we see that the potential for two opposite charges located at \(y\) and \(y'\) takes the form

\[
V = \frac{q^2 m^2}{8\pi g^2 B^2} |y - y'| \ln \left(1 + \frac{\Lambda^2 g^2 B^2}{m^2}\right),
\] (17)

where \(\Lambda\) is a cutoff and \(|y - y'| \equiv L\). Hence we see that the static potential profile displays a confining behavior. Notice that expression (17) is spherically symmetric, although the external fields break the isotropy of the problem in a manifest way. As already pointed out in our comments soon after the action of Eq. (1), the zero mass limit is not allowed here, for torsion, from the very beginning, by virtue of its dynamical character, has to be massive.

Before going ahead, we would like to remark how to give a meaning to the would-be cutoff \(\Lambda\). To do that, we should recall that our effective model for the electromagnetic field is an effective description that comes out upon integration over the torsion, whose excitation is massive. 1/m, the Compton wavelength of this excitation, naturally defines a correlation distance. Physics at distances of the order or lower than 1/m must necessarily take into account a microscopic description of torsion. This means that, if we work with energies of the order or higher than m, our effective description with the integrated effects of \(S^i\) is no longer sensible. So, it is legitimate that, for the sake of our analysis, we identify \(\Lambda\) with \(m\). Then, with this identification, the potential of Eq. (17) takes the form below:

\[
V = \frac{q^2 m^2}{8\pi g^2 B^2} |y - y'| \ln \left(1 + g^2 B^2\right).
\] (18)

### B. Electric case

We now want to extend what we have done to the case \(v^{0i} = 0\) and \(v^{ij} \neq 0\) (referred to as the electric one in what follows). In such a case the Lagrangian reads

\[
\mathcal{L}_{\text{eff}} = \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{g^2}{8m^2} v^{ij} f_{ij} \Delta v^{kl} f_{kl},
\] (19)

with \(\mu, \nu = 0, 1, 2, 3\) and \(i, j, k, l = 1, 2, 3\). Following the same steps employed for obtaining (17), we now carry out a Hamiltonian analysis of this model. First, note that the canonical momenta following from Eq. (19) are \(\Pi^\mu = f^{\mu 0}\), which results in the usual primary constraint \(\Pi^0 = 0\) and \(\Pi^i = f^{0i}\). Defining the electric and magnetic fields by \(E^i = f^{0i}\) and \(B^i = -\frac{1}{2} \epsilon^{ijk} f_{jk}\), respectively, the canonical Hamiltonian takes the form below:

\[
H_C = \int d^3x \left\{-A_0 \partial_t \Pi^i + \frac{1}{2} \Pi^2 + \frac{1}{2} B^2\right\} - \frac{g^2}{8m^2} \int d^3x \left\{\varepsilon_{ijk} \varepsilon_{kln} v^{ij} B^n \Delta v^{kl} B^l\right\}.
\] (20)

Time conservation of the primary constraint leads to the secondary constraint, \(\Gamma_1(x) \equiv \partial_t \Pi^i = 0\), and the time stability of the secondary constraint does not induce more constraints, which are first class. It should be noted
that the constrained structure for the gauge field is identical to the usual Maxwell theory. Thus, the corresponding expectation value is given by

\[ \langle H \rangle_\Phi = \frac{1}{2} \langle \Phi | \int d^3 x \Pi^2 | \Phi \rangle. \] (21)

As we have noted before \[25\], expression (21) becomes

\[ V = -\frac{q^2}{4\pi} \frac{1}{|y - y'|}, \] (22)

which it is just the Coulomb potential.

### III. FINAL REMARKS

In summary, by using the gauge-invariant but path-dependent formalism, we have studied the static potential for a gauge theory which describes the coupling between photons and torsion fields, in the case when there are nontrivial constant expectation values for the gauge field strength, \( F_{\mu\nu} \). While in the case when \( \langle F_{\mu\nu} \rangle \) is electric-like the static potential remains Coulombic, we find that, in the case when \( \langle F_{\mu\nu} \rangle \) is magnetic-like, the result is remarkably different. In fact, when \( \langle F_{\mu\nu} \rangle \) is magnetic-like the potential between static charges displays a confining behavior. We stress here the role played by the torsion field in yielding confinement: its mass contribute to the string tension. Let us also mention here that the singular situation involving the magnetic field is physically justified: torsion, in our proposal, couples to the photon spin density tensor, then it actually probes the magnetic properties of the latter \[20\] and this suggests us to think that torsion is intrinsically associated to the magnetic properties (magnetic dipole moment) of the truly elementary particles.

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