Microscopic calculation of the spin-dependent neutron scattering lengths on $^3$He

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Abstract

We report on the spin-dependent neutron scattering length on $^3$He from a microscopic calculation of $p-^3$H, $n-^3$He, and $d-^2$H scattering employing the Argonne $v_{18}$ nucleon-nucleon potential with and without additional three-nucleon force. The results and that of a comprehensive $R$-matrix analysis are compared to a recent measurement. The overall agreement for the scattering lengths is quite good. The imaginary parts of the scattering lengths are very sensitive to the inclusion of three-nucleon forces, whereas the real parts are almost insensitive.
Introduction

The scattering length is an easy way to compare low energy scattering data with calculations. Recently the spin-dependent scattering lengths for neutrons on tritons were calculated with the correlated hyperspherical harmonics technique. The calculations displayed a weak dependence on the various nucleon-nucleon (NN) and three-nucleon (NNN) potentials used. Afterwards an resonating group model (RGM) calculation gave for a modest model space for the triton very close results for the Argonne $v_{18}$ (AV18) and AV18 + Urbana IX (UIX) potentials. Neutron scattering on $^3$He is much more difficult to handle. Since at the n-$^3$He threshold the charge exchange channel is already open by about 700keV, the neutron absorption cross section is much higher than the elastic one for low energies. Therefore the scattering lengths become complex. The imaginary parts are rather well determined from the experiments by $^5$. The real parts of the spin-dependent neutron scattering lengths of $^3$He were recently measured $^6$ with much higher precision than before $^7$. These new results could only be compared to rather old theoretical approaches. Almost 30 years ago Kharchenko and Lebashew $^8$ calculated the real parts of the scattering lengths $a_0 = 7.52$ fm and $a_1 = 3.07$ fm, neglecting the Coulomb force and using a simple separable S-wave potential. Sears and Khanna $^9$ gave a Breit-Wigner estimate of the same values.

We organize the paper in the following way: The next section contains a brief discussion of the Resonating Group Model (RGM) calculation and the model spaces used. Then we compare $R$-matrix and RGM results of the neutron scattering length for various interactions with the data and discuss the effect of NNN-forces.

I. RGM AND MODEL SPACE

We use the Resonating Group Model $^{10, 11, 12}$ to compute the scattering in the $^4$He system using the Kohn-Hulthén variational principle $^{13}$. The main technical problem is the evaluation of the many-body matrix elements in coordinate space. The restriction to a Gaussian basis for the radial dependencies of the wave function allows for a fast and efficient calculation of the individual matrix elements $^{10, 12}$. However, to use these techniques the potentials must also be given in terms of Gaussians. In this work we use suitably parametrized versions of the AV18 $^{3}$ NN potential and the UIX $^{4}$ and $V_3^*$ proposed in
and used in NNN potentials.

In the $^4$He system we use a model space with six two-fragment channels, namely the $p-^3$H, the $n-^3$He, the $d-^2$H, the $d-^2$H(S=0), the $d$-resonance, the $d$- $d$and the $(pp) - (nn)$ channels. The last three are an approximation to the three- and four-body breakup channels that cannot in practice be treated within the RGM. The $^4$He is treated as four clusters in the framework of the RGM to allow for the required internal orbital angular momenta of $^3$H, $^3$He or $^2$H.

For the scattering calculation we include all $S$, $P$ and $D$ wave contributions to the $J^\pi = 0^+, 1^+, 2^+, 0^-, 1^-$ and $2^-$ channels. From the $R$-matrix analysis these channels are known to reproduce the low-energy experimental data. The full wave function for these channels contains over 100 different spin and orbital angular momentum configurations, hence it is too complicated to be given in detail. We started with the 29-dimensional model space for $^3$H/$^3$He as described in [2], increased it to dimension 35 by adding components to the wave function with two $D$-waves on the internal coordinates of the triton, optimized for AV18 and UIX together. By this modest increase of the model space, we gained 650 keV binding energy. Since this change in model space resulted in noticeable effects on observables [15] we aimed at an almost converged model space. Using a genetic algorithm [16] for AV18 and UIX together allowing for $S$, $P$ and $D$ waves on all internal coordinates we found a triton binding energy of -8.460 MeV for dimension 70. This result compares favourably with the numerically exact one of Nogga [17] of -8.478 MeV. Since the Gaussian width parameters were optimized for $NN$ and $NNN$-interaction together, the agreement for the AV18 alone is only -7.57 MeV, compared to the exact one of -7.62 MeV. For the deuteron we used 5 width parameters for the $S$-wave and 3 for the $D$-wave, yielding -2.213 MeV, just 10 keV short of the experimental value. The binding energies and relative thresholds for the various potentials are given in table I. For $NN$ and $NNN$ together the experimental binding energies and thresholds are very well reproduced.

This representation of $^3$H/$^3$He, deuteron and the unbound $NN$ systems form the model space of the $^4$He scattering system. We get for the different $J^\pi$ values 5 to 10 physical channels, insufficient to find reasonable results. So-called distortion or pseudo-inelastic channels [12] without an asymptotic part have to be added to improve the description of the wave function within the interaction region. For this purpose all the configurations calculated for the physical channels but one per channel can be reused, keeping only those width parame-
TABLE I: Comparison of experimental and calculated total binding energies and relative thresholds (in MeV) for the various potential models used

| potential                  | $E_{bin}$ $^3$H | $E_{bin}$ $^3$He | $E_{thres}$ $^3$He − $^3$H | $E_{thres}$ d − d |
|----------------------------|-----------------|------------------|--------------------------|-----------------|
| AV18                       | -7.572          | -6.857           | 0.715                    | 3.145           |
| AV18 + UIX                 | -8.460          | -7.713           | 0.747                    | 4.033           |
| AV18 + UIX + $V_3^*$       | -8.452          | -7.705           | 0.747                    | 4.025           |
| exp.                       | -8.481          | -7.718           | 0.763                    | 4.033           |

ters which describe the internal region. Recently Fonseca [18] pointed out that states having a negative parity $J_3^-$ in the three-nucleon fragment increase the $n−^3$H cross section notably. Contrary to the neutron-triton system we found in the $^4$He system in the preliminary small model space calculations that the inclusion of such distortion states gave minor effects compared to adding UIX. Therefore in the converged calculation we did not allow for such states, in order to save computational resources, as we had anyhow to deal with sometimes more than a thousand channels.

II. R-MATRIX ANALYSIS

The charge-independent R-matrix analysis of the $^4$He system from which the $n+^3$He scattering lengths are obtained in this paper is similar to the one described in Section 3 of our previous publication [19]. The isospin-1 R-matrix parameters were determined separately from an analysis of $p+^3$He scattering data, checked by limited comparisons with $n+^3$H data, and used essentially fixed in the analysis of the $^4$He system data in which the isospin-0 parameters were allowed to vary. New data have been added in most of the reactions, but those relevant for determining the $n+^3$He scattering lengths include the neutron total cross sections of refs. [5, 20, 21, 22], the elastic scattering cross sections of [23], and the $t(p, n)$ reaction cross-section measurements of [24, 25]. Charge independence relates the reduced-width amplitudes in the $p + t$ and $n+^3$He channels, imposing additional constraints on the neutron scattering lengths from the $p + t$ scattering data at proton energies near 1 MeV.
III. DETERMINATION OF THE SCATTERING LENGTH

The standard approach to the scattering length starts from the partial wave expansion of the scattering amplitude

\[ f(\Theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell + 1)(\exp(2i\delta_\ell) - 1) P_\ell(\cos \Theta) \]  

with \( k \) the neutron wave-vector. For thermal neutrons only the S-wave survives so that

\[ f(\Theta) = \frac{1}{2ik}(\exp(2i\delta_0) - 1) = \frac{1}{2ik}(S_0 - 1) = (k \cot \delta_0 - ik)^{-1} \]  

(2)

Since \( \delta_0 \) is an odd function of \( k \), one can expand

\[ k \cot \delta_0 = -1/a + 1/2r_e k^2 + O(k^4) \]  

(3)

in which \( a \) is the scattering length and \( r_e \) the effective range parameter. Suppressing from now on the subscript on \( \delta_0 \), we obtain in lowest order that \( a = -\tan \delta/k \).

For neutron - \(^3\)He scattering the proton - triton channel is already open, with a large neutron absorption cross section; hence, \( a \) has to be complex. For thermal neutron scattering \( (ka \ll 1) \), the total scattering and absorption cross section are given (for example in ref. [26]) by \( \sigma_s = 4\pi|a|^2 \) and \( \sigma_a = 4\pi a''/k \) with \( a'' \) the negative imaginary part of \( a \). Unfortunately, due to numerical limitations, the RGM approach cannot be used at energies low enough that terms of order \( ka \) can be neglected, and the expressions above need to be modified. Neglecting the effective range and higher order contributions in eq. (3), we can write eq. (2) as

\[ (S - 1)/(2ik) = (-1/a - ik)^{-1}. \]  

(4)

Solving for \( a \) we find \( a = (1 - S)/(1 + S)/(ik) \) from which the real and imaginary part can be easily evaluated as

\[ \Re a = -|S| \sin(2\delta) \left( \frac{1}{2k} \right) \frac{1}{4(1 - |S|)^2 + |S| \cos^2 \delta} \]  

(5)

and

\[ \Im a = \frac{(|S|^2 - 1)/k}{4(1 - |S|)^2 + |S| \cos^2 \delta} \]  

(6)
Note that for $|S| = 1$ the above expressions go to $-\tan \delta/k$ and zero, respectively, as they should. In the case of the R-matrix analysis, however, the scattering lengths are obtained directly from the zero-energy limit of eq. (4),

$$a = \lim_{k \to 0} \frac{1 - S}{2i k}. \quad (7)$$

FIG. 1: Comparison of the standard neutron cross section of $^3$He (crosses) and various calculations, AV18 alone (av-conv), AV18 + Urbana IX (au-conv), and AV18 + Urbana IX + $V_3^*$ (auv-conv).

Before we discuss the calculation of the scattering length in the actual case, let us first compare the standard neutron cross section $^3$He(n,p)$^3$H. In fig. 1 the evaluated standard cross section is compared to various calculations. This standard total neutron cross section is a bit over-predicted by the AV18 NN-force alone, a bit on the lower side for AV18 + UIX and severely under-predicted by AV18 + Urbana IX + $V_3^*$, see fig. 1. At the lowest energy calculated the S-matrix elements are used to determine the scattering lengths according to eq. (5) and (6).

For n - $^3$He scattering S-waves occur as singlet in $J^\pi = 0^+$ and as triplet in the $J^\pi = 1^+$ channels. At low neutron energies the $0^+$ channel is dominated by the well known resonance, leading to a strong coupling between the two charge conjugate channels. At 5 keV neutron energy this coupling S-matrix element is about 0.5, the triplet and the P-wave $0^-$ ones are about a factor 20 smaller, all others at least another order of magnitude smaller,
TABLE II: Comparison of experimental and calculated real and imaginary scattering lengths (in fm) for the various potential models used

| potential                  | $a_0$  | $a_1$  | $a_0$  | $a_1$  |
|----------------------------|--------|--------|--------|--------|
|                           | $\Re$  | $\Im$  | $\Re$  | $\Im$  |
| AV18                      | 7.81(2) | -4.96(2) | 3.468(1) | -0.0067(1) |
| AV18 + UIX                | 7.62(1) | -4.07(3) | 3.333(1) | -0.0052(1) |
| AV18 + UIX + $V_3^*$      | 7.57(5) | -3.42(1) | 3.310(1) | -0.0049(1) |
| R-matrix                  | 7.400(3) | -4.449(1) | 3.281(2) | -0.0013(2) |
| exp.                      | 7.370(58) | -4.448(5) | 3.278(53) | -0.001(2) |

except for the $^1P_1$ matrix element, which is only another factor of 4 smaller. Therefore we can for every $J^p$ consider only a two-channel S-matrix. We calculated according to eqs. (5) and (6) the real and imaginary parts of $a_0$ and $a_1$ for various potentials. In table II the results are compared to data and the results of the R-matrix analysis.

The numbers in brackets on the calculated values indicate the uncertainty of the scattering lengths, due to higher order effects. We calculate $a$ for a center-of-mass energy $E_0$ of a few keV. Since for this energy $ka$ is of the order of a few percent, higher order contributions might yield changes in $a$ also of this order. By calculating $a$ also at $E_0 +5$ keV and $E_0 +10$ keV we estimate the uncertainty.

The real parts of $a$ agree within 5 percent with the data and are rather insensitive to the changes in the potentials. The imaginary parts are very sensitive to these changes, so they are means to learn about the NNN-forces. Comparing the table of scattering lengths with the standard neutron cross section we find the clear relation, that the imaginary parts of $a_0$ just mirror the ratio of calculated versus evaluated cross sections. When the curve in fig. I is above the data, also the scattering length is larger than the data and vice versa. Since we are always dealing with effective two channel systems, unitarity relates $\Im a$ to the coupling matrix element squared, hence, the $NNN$-interaction reduces this coupling appreciably. Since the $3N$-bound states are rather dense, a possible conclusion might be that the short range repulsion in UIX is too strong and also the longer range attraction, which can be seen in the $^1S_0$ proton-triton phase shifts being too attractive for AV18 + UIX, see ref. [15], thus
indicating that the radial dependence of attraction and repulsion should be changed.

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