Analysis of cage slip in angular contact ball bearing considering non-Newtonian behavior of elastohydrodynamic lubrication

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Abstract. A dynamic model of angular contact ball bearing considering non-Newtonian behavior and interaction between balls and cage is deduced to predict cage slip. And then the cage slip are investigated and compared with measured results under different rotational speeds and axial loads to verify the proposed dynamic model of bearing. It is concluded that the results based on the non-Newtonian behavior of elastohydrodynamic lubrication is more agreement in trend with the tests than that based on the Newtonian assumption, and so the non-Newtonian behavior of elastohydrodynamic lubrication are needed in the cage slip analysis.

Keywords. Angular contact ball bearing, cage slip, dynamic model, non-Newtonian

1. Introduction

Bearing is one of the important basic parts in the major equipment, such as aero-engine, turbo-machinery and so on [1,2]. Its performance will directly affect the rotor dynamics, even the whole equipment [3,5]. The dynamic characteristics of cage, considered as one of the most critical components in rolling bearings, greatly affect the bearing performance [6]. For example, the cage slip, i.e. the linear velocity of rolling elements and raceway contact differences, often happens at high speed and light load condition, which causes frequent collision between rollers and cage, and abnormal vibration and temperature of bearing abnormal changes, and even cause cage wear and fracture failures [7,8]. The sliding behavior of the cage cannot be obtained by the conventional quasi-static analysis, in which the forces between the rolling bodies and cage and the inertial angular acceleration of the cage are ignored. It is also be faced numerous difficulties by use of the complete dynamic model to investigate...
cage slip behavior, simply because the model involves many degree of freedoms and the complex relationship among parameters needed to determine, and between them. Kannel[9], Boesiger[10], Gupta[11,12], Meeks [13,14], Ghaisas [6]make an outstanding contribution. 
Tu et al. [15] established a dynamic model of the roller bearing with a single degree of freedom considering the contact and friction between the rolling body and the cage, studied the sliding phenomenon of the cage during the acceleration of the bearing cage, and calculated the friction by using Coulomb's friction formula. Jain S[16] based on the multi-body dynamics theory and considering the elastohydrodynamic lubrication Newtonian fluid oil film force and the interaction between cage and rolling body, established the dynamics model of rolling bearing, and carried out under the condition of axial load skid characteristics research. But the contact angle in the model as a fixed value, without considering the change of the load, speed under the condition of contact angle, so the model is only applicable to low speed bearings. Han Qinkai et al. [17],on the basis of Jain S, considered the change of contact angle to establish the angular contact bearing sliding dynamics model, and carried out the influence of fixed radial load and variable radial load on the bearing sliding velocity, etc. However, the model should be based on the raceway control theory hypothesis. Wang Yulong[18] considered the contact and lubrication between the rolling body and the inner and outer rings, as well as the cage, and established the rolling bearing dynamic model with 6 degrees of freedom, and studied the sliding and sliding velocity changes under different working conditions. The above dynamic models of bearing are based on Newtonian fluid model and Newtonian oil film thickness calculation, without considering the non-Newtonian fluid behavior in elastohydrodynamic force, the calculation of elastohydrodynamic friction tends to produce a large error. The shear stress of traditional Newtonian fluid has a linear relationship with shear strain rate, while the shear stress of non-Newtonian fluid has a nonlinear relationship with shear strain rate, which is more approximate with the actual. In order to predict the bearing performance more accurately, the non-Newtonian rheological properties of lubricating oil should be considered.
In this paper, the dynamic model of the rolling bearing is established by introducing the modified Carreau model and modified oil film thickness formula based on the non-Newtonian fluid theory, considering the interaction between the cage and the rolling bodies and the fluid resistance of the cage. And the cage slip measurements are also carried out to verify the proposed model.

2. Dynamic model of angular contact ball bearing

2.1. Differential equations of ball motion
The external load of the bearing is $F = \{F_x, F_y, F_z, M_y, M_z\}^T$. In order to describe the motion of the ball, three coordinate systems are developed as shown in Figure 1. The first coordinate system (OXYZ) is fixed at the bearing centre with X axis coinciding with the bearing axis; the second coordinate system (o′x′y′z′) is a moving coordinate system axis $x'$ axis parallel with bearing axis and the coordinate origin attached to the centre of ball, rotating around x axis with speed $\omega_m$. In this coordinate, the ball has three angular displacement components $\theta_x, \theta_y, \theta_z$ around $x', y', z'$ axis, respectively.
Figure 1. Coordinate system and forces of rolling bearing

(a) x' o' z' (b) y' o' z' (c) x' o' y'

Figure 2. Forces on the ball

Subscript i, o and j represent the parameter of the inner ring, outer ring and jth ball, respectively. Q is the contact force between ball and inner or outer raceway, which can be calculated based on the Hertz in Ref[19]. F_{cbj} is the interaction force acted on the cage by ball. F_{cj} the centrifugal force, which can be expressed as

\[ F_{cj} = \frac{1}{2} m d_{m} \omega_{mj}^2. \]

M is the ball inertia moment can be decomposed to by \( M_{x'j}, M_{y'j}, M_{z'j} \) in x, y and z directions respectively. \( F_{dj} \) and \( M_{e} \) is the viscous drag force and torque by lubricant oil on the ball which can be decomposed to by \( M_{ex'j}, M_{ey'j}, M_{ez'j} \) in x, y and z directions respectively[12,16].

The dynamic equations of the jth ball can be expressed as follows

\[
\begin{align*}
Q_{yj} \sin \alpha_{y} - Q_{oj} \sin \alpha_{oj} + F_{x'j} \cos \alpha_{yj} - F_{x'o} \cos \alpha_{oj} &= 0, \\
Q_{yj} \cos \alpha_{yj} - Q_{oj} \cos \alpha_{oj} - F_{x'j} \sin \alpha_{yj} + F_{x'o} \sin \alpha_{oj} + F_{cj} &= 0 (j = 1 \cdots N) \quad (1a) \\
I \dot{\omega}_{x'} &= r(F_{y'o} \cos \alpha_{o} - F_{y'o} \cos \alpha_{i}) + M_{m} \cos \alpha_{o} + M_{o} \cos \alpha_{i} - M_{ex'}, \\
I \dot{\omega}_{y'} &= r(F_{x'o} - F_{x'o}) - M_{ey'}, \\
I \dot{\omega}_{z'} &= r(F_{y'o} \sin \alpha_{o} - F_{y'i} \sin \alpha_{i}) - M_{so} \sin \alpha_{o} - M_{si} \sin \alpha_{i} - M_{ez'}, \quad (1b) \\
I_{m} \dot{\omega}_{mj} &= \frac{D_{j}}{2} F_{y'j} + \frac{D_{o}}{2} F_{y'o} - 0.5F_{cbj} d_{m} - 0.5F_{dj} d_{m}.
\end{align*}
\]
where \( \alpha \) is the contact angle of the bearing. \( I = \frac{2}{5}mr^2 \) is the rotational inertia of its own axis and is the mass of ball. Besides, \( \omega_{xj}, \omega_{yj}, \omega_{zj} \) denote the \( j \)-th ball rotating angular velocities around a axis of its own. \( \omega_{mj} \) is \( j \)-th ball orbital revolution speed, \( \omega_{mj} = \dot{\theta}_{mj} \). In addition, \( Q \) is the contact force and traction force between the ball and the inner or outer raceway, \( F_x, F_y, M_x, M_y \) is traction force and traction moment between the ball and the inner or outer raceway can be obtained as follows. The traction force between the ball and the inner or outer raceway in Equation (6) can be calculated based on the elastohydrodynamic (EHD) lubrication model referring to Ref[19].

\[
F_{i/o} = -a \int_{-h_{i/o}}^{h_{i/o}} \int_{-b_{i/o}}^{b_{i/o}} \mathbf{t} \cdot d\mathbf{s} 
\]

The traction moment is obtained as follows

\[
M_{i/o} = -\frac{1}{h_{i/o}} \int_{-h_{i/o}}^{h_{i/o}} \int_{-b_{i/o}}^{b_{i/o}} \mathbf{t} \cdot d\mathbf{s} 
\]

where \( a \) and \( b \) are the long and short axes of the contact ellipse, which can be obtained by Hertz contact. \( \tau \) is the shearing stress between the ball and raceway.

The shear stress considering non-Newtonian behaviour of elastohydrodynamic lubrication can be obtained as Carreau[20]

\[
\begin{cases}
\tau = \eta \dot{\gamma}[1 + (\eta \dot{\gamma} / G_p)^{2}]^{(n-1)/2} & \tau < \lambda \rho \\
\tau = \lambda (T) \rho & \tau \geq \lambda \rho
\end{cases}
\]

where \( \eta \) is the oil viscosity at any temperature and pressure, \( G_p \) is the shear modulus of the lubricant and \( n \) is power-law exponent, \( \lambda \) is the ultimate shear, \( \dot{\gamma} = \frac{\Delta \mathbf{v}}{h}, \) in which \( \Delta \mathbf{v} \) is the sliding velocity referring to[19], \( h \) is the oil film thickness between each ball and raceway. \( \rho \) is the contact pressure between each ball and raceway according to the Hertz contact theory. \( \eta \) is the oil viscosity at any temperature and pressure, which can be defined as

\[
\eta = \eta_0(T) \exp^{\beta(T - T_c)}
\]

The oil viscosity at constant temperature is \( \eta_0(T) = \eta_{0i} e^{-\beta(T - T_c)} \), in which \( \beta \) temperature viscosity coefficient, and \( T \) is the lubricant temperature.

The oil film thickness between each ball and raceway can be calculated considering the influence of non-Newtonian shear-thinning and thermal as follows, [21]

\[
h = h_{nc} \phi_i / \phi_{nc}
\]

where \( h_{nc} \) represents the classical evaluated film thickness, i.e., the Newtonian central film thickness. \( \phi_i \) and \( \phi_{nc} \) denote the adjustment factors for the thermal and non-Newtonian shear-thinning, respectively.

The Newtonian central film thickness \( h_{nc} \) in Equation (6) can be obtained by classical Hamrock and Dowson [22]

\[
h_{nc} = 2.69U^{0.67}G^{0.53}W^{-0.067}(1 - 0.61e^{-0.73z})R_e
\]
where the equivalent radius of curvature is defined as $R_i = \frac{1}{2}D(1 + \gamma)$, in which the dimensionless parameter is $\gamma = \frac{D \cos \alpha}{d_m}$ and the “-” and “+” are chosen for the inner and outer raceways respectively. The dimensionless elastic modulus $G$, load $W$ and rotating speed $U$ are

$$
G = \alpha E'
$$
$$
W = Q/\left(E'R_i^2\right)
$$
$$
U = \eta u/(E'R_i)
$$

(8)

Where $\alpha$ denotes the pressure-viscosity coefficient and $E'$ denotes the effective elastic modulus. $u$ is the rolling velocity between ball and raceway.

The adjustment factors for thermal and non-Newtonian shear-thinning in Equation (6) can be evaluated by Gupta [23] and Bair [20] as follows

$$
\phi = \frac{1-13.2\left(\frac{P}{E}\right)L^{0.42}}{1 + 0.213(1 + 2.235^{0.83})L^{0.64}}
$$

(9a)

$$
\phi_i = \left[1 + 4.44\left(-\frac{\eta u}{h_{iso}G_r}\right)^1.69\left(1-n\right)^{1.78}\right]
$$

(9b)

where the thermal loading parameter is defined as $L = \eta \beta \frac{u^2}{4k_f}$, in which $k_f$ is the thermal conductivity.

The slide to roll ratio is defined as $S = \frac{\Delta u}{u}$.

2.2. Equilibrium equations of the inner ring

The equilibrium equations of the inner ring under external loads must be established and solved iteratively combining with the Equation (1a) for unknown the contact parameters including the contact forces, contact areas of the Hertz and contact angles between the balls and the inner or outer raceway. The equilibrium equations of the inner ring are calculated as follows.

$$
F_x - \sum_{j=1}^{N} (Q_y \sin \alpha_y + F_{x,y} \cos \alpha_y) = 0
$$

$$
F_y - \sum_{j=1}^{N} (Q_y \cos \alpha_y - F_{x,y} \sin \alpha_y) \sin \phi_j = 0
$$

$$
F_z - \sum_{j=1}^{N} (Q_y \cos \alpha_y - F_{x,y} \sin \alpha_y) \cos \phi_j = 0
$$

$$
M_y - \sum_{j=1}^{N} r_i (Q_y \sin \alpha_y + F_{x,y}) \cos \phi_j = 0
$$

$$
M_z - \sum_{j=1}^{N} r_i (Q_y \sin \alpha_y + F_{x,y}) \sin \phi_j = 0
$$

(10)

2.3. Interaction between balls and cage
The differential equation governing the rotational motion of the cage is as follow:

\[ I_c \ddot{\theta}_c = \frac{d_m}{2} \sum_{i=1}^{z} F_{cbjerg} - M_{fc} \]  

(11)

where \( I_c \) is the inertia moment of cage about X axis. \( M_{fc} \) is viscous drag moment by lubricant oil on cage.

The interaction force acted on the cage by ball can be expressed as [16]

\[ F_{cbjerg} = \frac{d_m}{2} (k_c \{ \theta_{mj} - \theta_c \} + c_c \{ \omega_{mj} - \omega_c \}) \]  

(12)

where \( k_c \) and \( c_c \) are the interaction stiffness and damping between cage and ball, respectively. \( \theta_{mj} \) and \( \omega_{mj} \) is the position angle and angular velocity of the \( j \)th ball at time \( t \), and \( \theta_c \) is the position angle and angular velocity of cage at time \( t \).

Figure 3. Forces on the cage

Figure 4. Interaction between balls and cage

2.4. Computation method and its flowchart
Figure 5. Flowchart for calculation of the dynamic model of bearing

Figure 5 shows the brief flowchart for the calculation. In the dynamic model, the input parameters include the geometric, lubrication parameters and operating conditions of the bearing for. The contact parameters between the ball and the raceway with the Newton–Raphson method are calculated based on the combined equilibrium equations of balls and ring. Based on the obtained results, the traction forces and the interactions of balls-cage are introduced in the dynamic model of balls. The dynamic equations of the cage and balls are solved numerically by using the Newmark-β method. Meanwhile, the computed results at the former step apply for the next initial values for the equations. The calculating process will stop when the time exceeds the setting time.

3. Model validation

3.1. Test cage slip of bearing

A test rig is developed to measure the cage motions of bearing and more details for test rig can be described in Ref. [24]. An eddy current sensor is fixed on a subpanel to measure the axial cage motion, as shown in Figure 6. The test results of axial cage motion are illustrated in Figure 7. The measured cage motions are analyzed by FFT and the cage rotational frequency \( f_c \) is obtained to calculate cage slip.
Figure 6. Bearing test rig

Figure 7. Axial cage motion

3.2. Model validation
Firstly, the oil film thickness of the bearing taking account of the influences of non-Newtonian shear-thinning have been validated in Ref[25]. Next, the skidding behaviors are measured and calculated under different rotational speeds and axial loads to validate the proposed model.

Table 1. Geometrical and calculated parameters for an angular contact ball bearing of 7013AC

| Parameter                  | Symbols | Value | Unit  |
|----------------------------|---------|-------|-------|
| Inner curvature factor     | $r_i$   | 0.515 | mm    |
| Outer curvature factor     | $r_o$   | 0.515 | mm    |
| all diameter               | $D$     | 11.105| mm    |
| Pitch diameter             | $d_m$   | 85.25 | mm    |
| Initial contact angle      | $\alpha$| 25    | deg   |
| Number of balls            | $Z$     | 18    |       |
| Cage inertia moment        | $I_c$   | 3.4e-4| Kg.m$^2$|
| Interaction stiffness      | $k_c$   | 1e8   | N/m   |
| Interaction damping        | $c_c$   | 1e6   | N/(m/s)|

An angular contact ball bearing of 7013AC is chosen here, and its cage is made by 45# steel. Additionally, its geometrical and calculated parameters are listed in Table 1. Furthermore, the lubricant oil (46#) is used and its parameters are provided in Table 2.
Table 2. Parameters of oil lubrication

| Parameters                              | Symbols | Value | Unit  |
|-----------------------------------------|---------|-------|-------|
| Lubricant viscosity at 40°C             | \( \eta \) | 0.03956 | Pa.s  |
| Temperature-viscosity Coefficient       | \( \beta \) | 0.0368 | K\(^{-1}\) |
| Thermal conductivity                    | \( k_f \) | 0.14 | W/m.K |
| Tested lubricant temperature            | \( T \) | 36 | °C |
| Dielectric constant                     | \( \varepsilon_0 \) | 2.2 | - |
| Relative permittivity                   | \( \varepsilon_r \) | 8.85 | - |
| Density of the lubricant                | \( \rho_b \) | 860 | Kg/m\(^3\) |
| Pressure-viscosity coefficient          | \( \alpha \) | 1.57e-8 | 1/Pa |
| Shear modulus of the Lubricant          | \( G_r \) | 5.4e4 | Pa |
| Power-law exponent                      | \( n \) | 0.54 |       |

Figure 8 shows the comparison of cage slip under different axial loads between numerical results and the experimental results at the bearing speed of 2400 r/min and 3000 r/min, respectively. As shown in Figure 8, it obviously indicates that cage slip gradually increases with the increment of the rotating speeds. The behavior of lubrication have effect on the analysis of cage slip in angular contact ball bearing, and the calculated results obtained from present model is more agreement in trend with the test findings than the using the Newtonian of elastohydrodynamic, which verifies the developed model. It needs to consider the non-Newtonian behavior of elastohydrodynamic lubrication in cage slip analysis for angular contact ball bearing.

![Figure 8](image_url)

**Figure 8.** Comparison of simulation results with experimental data

4. Conclusions

A dynamic model of angular contact ball bearing taking account of the influences of non-Newtonian shear-thinning and interaction between balls and cage is deduced to predict cage slip. And then the cage slip is calculated and the measurements are compared to verify the dynamic model of bearing. The behavior of elastohydrodynamic lubrication has effect on the cage slip in angular contact ball bearing. The results based on the non-Newtonian theory of elastohydrodynamic lubrication are more agreement in trend with the tests than that based on the Newtonian assumption, and so the non-Newtonian behavior of elastohydrodynamic lubrication is needed in the cage slip analysis.

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Conflict of interest
The Authors declare that there is no conflict of interest.

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