Adaptive Fuzzy Vertical Vibration Suppression Control of the Mechanical-Hydraulic Coupling Rolling Mill System With Input Dead-Zone and Output Constraints

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\begin{abstract}
This paper investigates the adaptive fuzzy vertical vibration suppression control problem for the six-high rolling mill system. Firstly, a new vibration model is established with the consideration of the coupling of mechanical and hydraulic systems and the unknown uncertainty on nonlinear rolling force. Then, the adaptive active control strategy is proposed to suppress chatter of the rolling mill under the input dead-zone and output constraints. The adaptive fuzzy logic systems are used to deal with the unknown nonlinear functions and the unknown system parameters. Based on the designed controller, the mechanical-hydraulic coupling rolling system is proven to be stable and the performance of the displacement of work roll is preserved. Finally, the simulation comparison shows the validity and the advantages of the proposed algorithm.
\end{abstract}

\begin{IEEEkeywords}
Vertical vibration suppression, rolling mill system, mechanical-hydraulic coupling, input dead-zone, output constraints.
\end{IEEEkeywords}

\section{INTRODUCTION}
Vibration often occurs in the high speed rolling process of strip steel \cite{1}–\cite{3}. The categories of vibration mainly include torsional vibration \cite{4}, \cite{5} and vertical vibration \cite{6}, \cite{7}. Especially for vertical vibration, a large amount of energy will be accumulated in a short time, which can lead to thickness fluctuations of the strip as well as the stripes on the surface of the rolled steel products, and even cause the damage of the equipment. To avoid the adverse effects caused by the vibration, the emergency method for the rolling mill vibration control is to slow the rolling speed down \cite{8}, \cite{9}. However, the reduction of rolling speed affects the production efficiency and can not fundamentally solve the problem of rolling mill vibration. Thus, the effective suppression vibration of rolling mill are always in demand.

Due to the complexity and concealment of the causes of rolling mill vibration, it is not easy to identify the source of the vibration. With the detection and analysis of vibration signals, some causes of rolling vibration can be confirmed and the reasonable governance strategies can be managed \cite{10}–\cite{12}. Vibrations caused by equipment damages or assembly problems can be controlled by replacing equipment and installing anti-vibration gaskets \cite{13}. For the vibration caused by non-mechanical reasons, vibration suppression can be achieved by adjusting lubrication parameters \cite{14} or optimizing rolling schedule parameters \cite{15}, \cite{16}. The above methods are mainly used to suppress rolling mill vibration from the aspects of machinery and technology.

Control algorithms are widely applied in different engineering fields \cite{17}–\cite{21}. For torsional vibration control, many significant achievements have been developed in \cite{22}–\cite{24}, which have an effective effect on the suppression of torsional vibration. Nevertheless, there are few researches on the control algorithm of rolling mill vertical vibration.
Based on the analysis of the mechanical structure of rolling mill, [25]–[27] established the vibration models under the unsteady lubrication condition and [28], [29] established nonlinear vibration models with nonlinear friction or nonlinear stiffness, however, no further studies have been conducted on vibration suppression control. From the perspective of the moving strip, the strip vibration dynamics equation was established, and the boundary control was designed to deal with the problems of vibration suppression in [30]. For the vibration of rolling mill mechanical systems, [31] studied the linear and nonlinear feedback controllers to suppress the vibration of rolling mill system caused by Hopf bifurcation. Combining mechanical and hydraulic systems, [32] and [33] established linear 2-DOF (degree of freedom) coupling chatter models for 4-high rolling mill system and designed the corresponding robust vibration suppression controllers.

On the other hand, the displacement of roll needs to be limited for the sake of product quality and equipment safety during the rolling process. In addition, dead-zone input nonlinearity is a nonsmooth function that characterizes the certain nonsensitivity for small control inputs which is often encountered in electro-hydraulic servo valve. The output constraints and dead-zone input control methods of nonlinear systems have been investigated in many researches. For example, [34], [35] investigated the output constraints for strict feedback nonlinear systems by choosing the proper nonlinear state transformation, and by introducing the logarithm and tangent barrier Lyapunov function, the output constraints-based adaptive control algorithms were proposed for uncertain nonlinear systems in [36], [37]. By modeling the dead-zone input as a combination of a linear term and a disturbance-like term, [38]–[40] studied the memoryless controllers for the strict feedback nonlinear system with unknown asymmetric dead-zone input. However, these factors have not been considered in the rolling vertical vibration system.

Inspired by the above motivations, the adaptive fuzzy chatter suppression strategy for the mechanical-hydraulic coupling rolling mill system with input dead-zone and output constraints is investigated in this paper. The contributions of this article are as follows:

i). Compared with [32], [33], by considering the displacement of the hydraulic cylinder and the force between the backup roll and the hydraulic cylinder, the mechanical-hydraulic coupling vibration model for 6-high rolling mill is established in this paper, moreover, the system parameters are uncertain, making the rolling chatter model more realistic.

ii). The performance constraints of the displacement of work roll are considered for 6-high rolling mill vertical vibration suppression control to improve the attenuation performance of the rolling chatter. The attenuation rate of the chatter, the allowable displacement of work roll and the steady state error are limited to a certain range.

iii). Based on the design frame of backstepping, an adaptive fuzzy chatter suppression controller is proposed for the non-lower triangular rolling vertical vibration system subject to input dead-zone and output constraints, which can suppress the vibration effectively and ensure the stability of the system.

The rest of the paper is organized as follows. In Section II, the mechanical-hydraulic coupling vibration modeling of the rolling mill is given. The problem description of this paper is given in Section III. Then, the adaptive fuzzy chatter suppression control is designed for rolling mill vertical vibration system in Section IV. In Section V, the simulation shows the effectiveness of the proposed method. Finally, the paper is concluded in Section VI.

II. MATHEMATICAL MODELING

In the rolling process, the pressure of the hydraulic cylinder is controlled by the displacement of the servo valve spool of the hydraulic system, and then the roll gap is adjusted to achieve stable strip rolling. However, the thickness of the strip will fluctuate when the rolling mill is unstable, and its essence reason is the work roll jump up and down during the rolling process. The key to suppress the vertical vibration of the rolling mill is to guarantee the stability of the work roll displacement. In order to facilitate the modeling of the rolling mill vibration, it is assumed that the upper roll system and the lower roll system of the rolling mill are symmetrical along the strip [32], [33]. Therefore, only the upper roll system and the hydraulic system are studied. Thus, the mechanical-hydraulic coupling system of the rolling mill can be simplified as shown in Fig.1, where \( m_1, m_2, m_3 \) are the equivalent mass of work roll and bearings, intermediate roll and bearings, backup roll and bearings, respectively. \( A_1 \) is the area of the piston, \( A_2 \) is the effective working area of the rod chamber. \( P_1 \) is the working pressure of rodless chamber, \( P_2 \) is the working pressure of the rod chamber, \( P_s \) is the supply pressure, \( P_3 \) is the return pressure. With the method of lumped mass, the mechanical-hydraulic coupling vibration system can be simplified as a mass-spring-damping system in Fig.2, where \( k_1, k_2, k_3 \) are the equivalent stiffness between work roll and intermediate roll, intermediate roll and

![Figure 1. Schematic diagram of mechanical-hydraulic coupling system of rolling mill.](image_url)
From the flow characteristics of the servo valve, $k_q$ is satisfied with $0 < k_q \leq \dot{k}_q \leq \ddot{k}_q$ and $k_q, \dot{k}_q, \ddot{k}_q$ are unknown positive constants.

Finally, the mechanical-hydraulic coupling model can be described by the following set of equations:

$$
\begin{align*}
 m_1 \ddot{x}_1 &= F_z (x_1, \dot{x}_1) - k_1 (x_2 - x_1) - c_1 (\dot{x}_2 - \dot{x}_1) \\
 m_2 \ddot{x}_2 &= k_1 (x_2 - x_1) + c_1 (\dot{x}_2 - \dot{x}_1) - k_2 (x_3 - x_2) \\
 &\quad - c_2 (\dot{x}_3 - \dot{x}_2) \\
 m_3 \ddot{x}_3 &= k_2 (x_3 - x_2) + c_2 (\dot{x}_3 - \dot{x}_2) - k_3 (x_4 - x_3) \\
 &\quad - c_3 (\dot{x}_4 - \dot{x}_3) \\
 m_4 \ddot{x}_4 &= P_2 A_2 - P_1 A_1 + k_3 (x_4 - x_3) + c_3 (\dot{x}_4 - \dot{x}_3) \\
 k_q u &= A_1 \dot{x}_4 + C_1 (P_1 - P_2) + \frac{V}{\beta_e} \dot{P}_1
\end{align*}
$$

where $m_4$ is equivalent mass of hydraulic cylinder and piston, $F_z (x_1, \dot{x}_1)$ is the unknown nonlinear rolling force. During the vertical vibration, the nonlinear rolling force can be described as $F_z (x_1, \dot{x}_1) = F_{1z} + F_{2z} (x_1, \dot{x}_1)$, where $F_{1z}$ is the steady rolling force, $F_{2z}$ is the dynamic rolling force.

**Remark 1:** The works [25]–[29] established the vibration model just for the mechanical system. The mechanical-hydraulic coupling vibration models for 4-high rolling mill are established in [32], [33], which considered hydraulic cylinder and backup roll as a whole, the characteristics of their contact interface are ignored. Inspired by the above works, a new mechanical-hydraulic coupling model for 6-high rolling mill is proposed in this paper, which considers the interaction relationship between the hydraulic cylinder and the backup roll, moreover, the unknown nonlinear rolling force and system parameters are also considered, making it more general with physical situation.

### III. PRELIMINARIES

#### A. PROBLEM FORMULATION

To facilitate the control design, by defining $[z_1, z_2, \cdots, z_9] = [x_1, \dot{x}_1, x_2, \dot{x}_2, x_3, \dot{x}_3, x_4, \dot{x}_4, P_1]$. The new dynamic systems can be obtained as:

$$
\begin{align*}
 \dot{z}_1 &= z_2 \\
 \dot{z}_2 &= \frac{1}{m_1} (F_z (z_1, z_2) - k_1 (z_3 - z_1) - c_1 (z_4 - z_2)) \\
 \dot{z}_3 &= z_4 \\
 \dot{z}_4 &= \frac{1}{m_2} (k_1 (z_3 - z_1) + c_1 (z_4 - z_2) - k_2 (z_5 - z_3)) \\
 \dot{z}_5 &= z_6 \\
 \dot{z}_6 &= \frac{1}{m_3} (k_2 (z_5 - z_3) + c_2 (z_6 - z_4) - k_3 (z_7 - z_5)) \\
 &\quad - c_3 (z_8 - z_6) \\
 \dot{z}_7 &= z_8 \\
 \dot{z}_8 &= \frac{1}{m_4} \frac{V}{\beta_e} (k_q u - A_1 z_8 - c_1 (z_9 - P_2)) \\
 \dot{z}_9 &= \frac{1}{m_5} \frac{V}{\beta_e} (k_q u - A_1 z_8 - c_1 (z_9 - P_2))
\end{align*}
$$
where $u$ is the control voltage input with the dead-zone constraint, which can be described by

$$u = \begin{cases} 
\eta(v - b_r), & v \geq b_r \\
0, & -b_l < v < b_r \\
\eta(v + b_l), & v \leq -b_l
\end{cases},$$

(8)

where the unknown parameters $\eta_l, \eta_r$ stand for the right and the left slope of the dead-zone characteristic, $b_r, b_l$ represent the breakpoints of the input nonlinearity. Similarly to [38] and [40], the dead-zone input (8) can be rewritten as follows:

$$u = \eta(t)v + \delta(t)$$

(9)

with

$$\eta(t) = \begin{cases} 
\eta_l, & v < 0 \\
\eta_r, & v \geq 0
\end{cases}, \quad \delta(t) = \begin{cases} 
-\eta_l b_r, & v \geq b_r \\
-\eta_l v, & -b_l < v < b_r \\
\eta_l b_r, & v \leq -b_l
\end{cases}.$$

Then, one knows that there always exist unknown positive scalars $\bar{\eta}$ and $\bar{\delta}$ such that $\bar{\eta} \leq \min\{\eta_l, \eta_r\}$ and $|\delta(t)| \leq \bar{\delta}$. It should be noted that $\eta$ and $\delta$ are not required in implementation proposed control design. They are used only for analytical purposes.

### B. FUZZY LOGIC SYSTEMS

Fuzzy logic systems (FLSs) are found to have a wide range of applications for controllers design of nonlinear systems with precise model unknown in the past years due to the universal approximation capability [41,42]. A fuzzy system is composed of a collection of fuzzy if-then rules:

$$R(i) : \text{If } Z_i \text{ is } A_i^j, \ldots, Z_n \text{ is } A_n^j$$

then $y$ is $C^j$.

With the fuzzy process, the output of the FLSs can be written as

$$y(z) = \frac{\sum_{i=1}^{l} y^j \left( \prod_{i=1}^{n} \mu_{A_i^j}(Z_i) \right)}{\sum_{j=1}^{l} \prod_{i=1}^{n} \mu_{A_i^j}(Z_i)},$$

where $\mu_{A_i^j}(Z_i)$ is the membership function and $y^j$ is the point where $\mu_{C^j}$ obtains the max value.

Introducing the concept of the fuzzy basic function vector $\varphi(Z)$ gives

$$y(Z) = \hat{\theta}^T \varphi(Z),$$

where

$$\varphi(Z) = (\varphi_1(Z), \varphi_2(Z), \ldots, \varphi_l(Z))^T,$$

$$\hat{\theta} = \left( y^1, y^2, \ldots, y^l \right)^T,$$

$$\varphi_j(Z) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(Z_i)}{\sum_{j=1}^{l} \prod_{i=1}^{n} \mu_{A_i^j}(Z_i)}.$$

According to the universal approximation theorem, there exists $\theta = \arg \min_{\theta \in \Omega_{\theta}} \left( \sup_{Z_i \in \Omega_{Z_i}} \left| \hat{\theta}^T \varphi(Z) - G(Z) \right| \right)$ such that $\theta^T \varphi(Z)$ can approximate function $G(Z)$ over a compact set $\Omega_Z$, where $\Omega_Z$ and $\Omega_{\theta}$ represent the sets of the bounds of $Z$ and $\theta$. The minimum approximation error is defined as

$$e_1(Z) = G(Z(\theta)) - G(Z)$$

where $\|e_1(Z)\| \leq e_1^* + e_1^*$ is a positive constant.

### IV. CHATTER SUPPRESSION CONTROLLER DESIGN

In this section, the adaptive fuzzy vertical vibration suppression control design will be proposed for system (6) with input dead-zone and output constraints. For the development of control laws, the following lemmas are proposed at first.

**Lemma 1 [37]:** For any constant $\varepsilon_2 > 0$ and any variable $z$, the following relationship holds:

$$0 \leq |z| - \frac{2}{\sqrt{z^2 + \varepsilon_2^2}} \leq \varepsilon_2$$

**Lemma 2 [38]:** For any constant $\varepsilon_3 > 0$ and any variable $v$, the following inequality holds:

$$0 \leq |v| - v \tanh \left( \frac{v}{\varepsilon_3} \right) \leq 0.2785\varepsilon_3$$

By using the backstepping algorithm, the design procedure for the rolling chatter system consists of 9 steps according to (7), where the input voltage of servo valve being deduced at the last step. First, the following coordinate transformation is introduced

$$\xi_1 = z_1, \xi_i = z_i - \alpha_{i-1}$$

where $\alpha_i (i = 1, \ldots, 8)$ are the virtual controllers designed later. Then, the whole system can be rewritten as

$$\begin{align*}
\dot{\xi}_1 &= \xi_2 + \alpha_1 \\
\dot{\xi}_2 &= -\frac{k_1}{m_1} (\xi_3 + \alpha_2) + \frac{1}{m_1} (F_{i_1} + F_{i_2}; (z_1, z_2)) + k_1 z_1 - c_1 (z_4 - z_2) - \hat{\alpha}_1 \\
\dot{\xi}_3 &= \xi_4 + \alpha_3 - \hat{\alpha}_2 \\
\dot{\xi}_4 &= -\frac{k_2}{m_2} (\xi_5 + \alpha_4) + \frac{1}{m_2} (k_1 (z_3 - z_1) + c_1 (z_4 - z_2) + k_2 z_3 - c_2 (z_6 - z_4)) - \hat{\alpha}_3 \\
\dot{\xi}_5 &= \xi_6 + \alpha_5 - \hat{\alpha}_4 \\
\dot{\xi}_6 &= -\frac{k_3}{m_3} (\xi_7 + \alpha_6) + \frac{1}{m_3} (k_2 (z_5 - z_3) + c_2 (z_6 - z_4) + k_3 z_5 - c_3 (z_8 - z_6)) - \hat{\alpha}_5 \\
\dot{\xi}_7 &= \xi_8 + \alpha_7 - \hat{\alpha}_6 \\
\dot{\xi}_8 &= \frac{1}{m_4} (P_{i_2} A_2 + k_3 (z_7 - z_5) + c_3 (z_8 - z_6)) - \frac{A_1}{m_4} (\xi_9 + \alpha_8) - \hat{\alpha}_7 \\
\dot{\xi}_9 &= \frac{\beta_{i_2}}{V} (k_4 \eta (t) v + k_9 \delta (t) - A_1 z_8 - c_1 (z_9 - P_2)) - \hat{\alpha}_8
\end{align*}$$

(10)
Step 1: To stabilize the displacement of work roll $\xi_1$, considering the first Lyapunov function as

\[
V_1 = \frac{\mu^2}{\pi h(\xi_1)} \tan \left( \frac{\pi \xi_1^2 h(\xi_1)}{2 \mu^2} \right)
\]  
(11)

where $h(\xi_1) = \frac{b(\xi_1)}{\xi_1^2} + \frac{1-b(\xi_1)}{\xi_1^2}$, $b(\xi_1) = \begin{cases} 0, & \xi_1 < 0, \delta \text{ and } \hat{\delta} \text{ are positive constants}, \\ 1, & \xi_1 \geq 0 \end{cases}$ and $\delta$ and $\hat{\delta}$ are prescribed positive constants. The virtual control $\alpha_1$ is specified as

\[
\alpha_1 = -\gamma_0 \xi_1 - (2\gamma_0 + a_1) \frac{\mu^2}{2\pi \xi_1 h(\xi_1)} \sin \left( \frac{\pi \xi_1^2 h(\xi_1)}{2 \mu^2} \right) - \frac{1}{2} v_1
\]  
(12)

where $v_1 = \frac{\dot{\xi}_1}{\cos^2 \left( \frac{\pi \xi_1^2 h(\xi_1)}{2 \mu^2} \right)}$, $\gamma_0 = \sqrt{\left( \frac{\hat{\mu}}{\mu} \right)^2 + \epsilon_1}$, $\epsilon_1$ and $a_1$ are designed positive parameters.

Remark 2: By choosing the asymmetric tangent barrier Lyapunov function (11), one can find that the displacement of work roll $\xi_1$ can be strictly constrained within asymmetric upper and lower bounds as follows: $-\delta \mu (t) < \xi_1 < \delta \mu (t)$. Thus, the performance constraints, such as the attenuation rate of chatter, the maximum allowable displacement of work roll, steady state error are all limited to a predetermined range. Moreover, by taking $\hat{\delta} = \delta$ and $\mu (t) \to \infty$, (11) can be simply replaced by the Lyapunov function in quadratic form $V_1 = \frac{1}{2} \dot{\xi}_1^2$ and thus the analysis approach presented in this paper remains the same as the general case without output constraints.

Remark 3: It should be noted that $\lim_{\xi_1 \to 0} \frac{1}{\xi_1^2} \tan \left( \frac{\pi \xi_1^2 h(\xi_1)}{2 \mu^2} \right) \to 0$, thus the term $2(2\gamma_0 + a_1) \frac{\mu^2}{2\pi \xi_1 h(\xi_1)} \sin \left( \frac{\pi \xi_1^2 h(\xi_1)}{2 \mu^2} \right)$ in the controller does not cause the singularity problem. However, in the digital computer, since $(0/0)$ cannot be edited, one can take this term to be 0 when $|\xi_1| < \varrho$ for the small constant $\varrho$.

Step 2(i = 2, 4, 6): The coordinate transformation $\xi_2$, $\xi_3$, $\xi_6$ will be considered. Choosing the Lyapunov function candidates for step $i$ as

\[
V_i = \frac{1}{2} \xi_i^2 + \frac{1}{2\sigma_i \rho_i} \dot{\rho}_i^2 + \frac{1}{2\sigma_i} \ddot{\rho}_i^2
\]  
(13)

and then the virtual controllers $\alpha_i$ are proposed as

\[
\alpha_i = \frac{\xi_i \dot{\xi}_i, \rho_i \ddot{\rho}_i, \ddot{\rho}_i^2}{\xi_i^2 \dot{\rho}_i^2, \alpha_i, \rho_i \ddot{\rho}_i, \ddot{\rho}_i^2}
\]  
(14)

and the adaptive laws are selected as

\[
\dot{\rho}_i = \sigma_1 \xi_i, \dot{\alpha}_i - l_1 \dot{\rho}_i,
\]  
(15)

\[
\dot{\xi}_i = \frac{\sigma_2 \xi_i^2}{4\tau \psi_i^2 (\bar{\xi}_i) \phi_i (\bar{\xi}_i)} - l_2 \dot{\xi}_i
\]  
(16)

where $\alpha_i = \xi_i + a_i \xi_i + \frac{\dot{\theta}_i, \dot{\xi}_i}{4\tau \psi_i^2 (\bar{\xi}_i) \phi_i (\bar{\xi}_i)}$, $\alpha_i$, $\tau_i$, $l_i$, $s_j$ ($j = 1, 2$) are designed positive parameters, $\gamma = (\cdot, - (\cdot))$, $\Theta_i = ||\theta_i||^2$, $\rho_2 = \frac{m_2}{\alpha_1}$, $\rho_4 = \frac{m_2}{\alpha_1}$, $\rho_5 = \frac{m_2}{\alpha_1}$, $\theta_i$ and $\phi_i$ are defined in (35-37).

Step 2(2, 3, 5, 7): The dynamics of $\xi_1$, $\xi_5$, $\xi_7$ will be considered in the corresponding steps. Choosing the Lyapunov function candidates as

\[
V_j = \frac{1}{2} \xi_j^2 + \frac{1}{2\sigma_j} \ddot{\theta}_j^2
\]  
(17)

and the virtual controllers $\alpha_j$ are designed as

\[
\alpha_j = -\frac{3}{2} \xi_j - a_j \xi_j - \dot{\theta}_j \psi_j (\bar{\xi}_j),
\]  
(18)

with the adaptive laws are selected as

\[
\dot{\theta}_j = \sigma_j \xi_j \psi_j (\bar{\xi}_j) - l_j \dot{\theta}_j
\]  
(19)

where $\alpha_j$, $l_j$, $\sigma_j$ are designed positive parameters, $\theta_j$ and $\psi_j$ are defined in (40).

Step 2(k = 8, 9): For the coordinate transformation $\xi_k$ and $\xi_9$, the Lyapunov function candidates can be selected as

\[
V_k = \frac{1}{2} \xi_k^2 + \frac{1}{2\sigma_k \rho_k} \dot{\rho}_k^2 + \frac{1}{2\sigma_2} \ddot{\rho}_k
\]  
(20)

then, the virtual control $\alpha_k$ and the voltage input $v$ can be proposed as

\[
\alpha_k = \frac{\xi_k \rho_k \ddot{\rho}_k^2}{\sqrt{\xi_k \rho_k^2 \ddot{\rho}_k^2 + \epsilon_2}}
\]  
(21)

\[
v = \frac{\xi_k \rho_k^2 \ddot{\rho}_k^2}{\sqrt{\xi_k \rho_k^2 \ddot{\rho}_k^2 + \epsilon_2}}
\]  
(22)

with the adaptive laws are designed as

\[
\dot{\rho}_k = \sigma_k \xi_k \rho_k - l_k \dot{\rho}_k
\]  
(23)

\[
\dot{\theta}_k = \sigma_k \xi_k \phi_k (\bar{\xi}_k) - l_k \dot{\theta}_k
\]  
(24)

where $\bar{\alpha}_k = \xi_k + a_k \xi_k + \ddot{\theta}_k \psi_k (\bar{\xi}_k)$, $\alpha_k$, $s_j$, $l_j$ ($j = 1, 2$) are designed positive parameters, $\rho_8 = \frac{m_8}{\alpha_1}$, $\rho_9 = \frac{m_9}{\alpha_1}$, $\theta_k$ and $\phi_k$ are defined in (38-39).

Then, the main result of this paper is as follows.

Theorem 1: Considering the vertical vibration system expressed by (6) subject to the input dead-zone and the output constraints, the dynamic controllers (12), (14), (18) and (21-22) with adaptive laws (15-16), (19), (23-24) render all signals in the vertical vibration system are uniformly ultimately bounded and the displacement of work roll is strictly constrained with asymmetric upper and lower boundaries, thus, the vertical vibration can be suppressed.

Proof: Consider the overall Lyapunov function for system (10) as

\[
V = \sum_{i=1}^{9} V_i
\]  
(25)
From (11), (13), (17) and (20), the derivative of $V$ yields

\[
\dot{V} = 2\dot{\mu} V_1 - \mu \xi_1 v_1 + v_1 \dot{\xi}_1 + \sum_{i=2}^{9} \xi_i \dot{\xi}_i \\
- \sum_{i=2,4,6,8,9}^{9} \frac{1}{\sigma_1 \rho_1} \dot{\rho}_1 \dot{\rho}_1 - \sum_{i=2,4,6}^{9} \frac{1}{\sigma_2} \dot{\sigma}_2 \\
- \sum_{i=3,5,7,8,9} \frac{1}{\sigma_{12}} \dot{\sigma}_{12} - \sum_{i=2,4,6} \frac{1}{\sigma_1} \dot{\rho}_1 \dot{\rho}_1 \\
\leq 2\gamma_0 V_1 + \rho_0 \dot{\xi}_1 v_1 + v_1 (\dot{\xi}_2 + \alpha_1) \\
+ \sum_{i=3.5,7} \xi_i (\dot{\xi}_{i+1} + \alpha_i - \dot{\alpha}_{i-1}) \\
+ \dot{\xi}_2 \left( -\frac{k_1}{m_1} (\xi_3 + \alpha_2) + \frac{1}{m_1} (F_{1z} + F_{2z}, z_1, z_2) \right) \\
+ k_1 \dot{z}_1 - c_1 (z_4 - z_2) - \dot{\alpha}_1 \\
+ \dot{\xi}_4 \left( -\frac{k_2}{m_2} (\xi_5 + \alpha_4) + \frac{1}{m_2} (k_1 (z_3 - z_1) + c_1 (z_4 - z_2) - \dot{\alpha}_3) \\
+ \dot{\xi}_6 \left( -\frac{k_3}{m_3} (\xi_7 + \alpha_6) + \frac{1}{m_3} (k_2 (z_5 - z_3) + c_2 (z_6 - z_4) - \dot{\alpha}_3) \\
+ \dot{\xi}_8 \left( -\frac{A_i}{m_4} (\xi_9 + \alpha_8) + \frac{1}{m_4} (P_{2A} k_3 (z_7 - z_5) + c_3 (z_8 - z_6) - \dot{\alpha}_3) \\
+ \dot{\xi}_9 \left( \frac{\beta}{V} (k_q \delta v + k_q \delta (v) - A_1 z_8 \right) \\
- c_1 (z_9 - P_2) - \dot{\alpha}_8) \\
- \sum_{i=2,4,6,8,9} \frac{1}{\sigma_1 \rho_1} \dot{\rho}_1 \dot{\rho}_1 - \sum_{i=2,4,6} \frac{1}{\sigma_2} \dot{\sigma}_2 \\
- \sum_{i=3,5,7,8,9} \frac{1}{\sigma_{12}} \dot{\sigma}_{12} - \sum_{i=2,4,6} \frac{1}{\sigma_1} \dot{\rho}_1 \dot{\rho}_1 \right) \tag{26}
\]

Based on Lemma 1, Lemma 2 and Young’s inequality, the following inequalities are all satisfied

\[
v_1 \dot{\xi}_2 \leq \frac{1}{2} v_1^2 + \frac{1}{2} \dot{\xi}_2^2 \\
- \frac{k_1}{m_1} \dot{\xi}_2 \dot{\xi}_{2i+1} \leq \frac{1}{2} \frac{k_1^2}{m_1^2} \dot{\xi}_2^2 + \frac{1}{2} \dot{\xi}_{2i+1}^2, \quad i = 1, 2, 3 \tag{27}
\]

\[
\dot{\xi}_2 F_{1z} \leq |\dot{\xi}_2| F_{1z} \\
\leq |\dot{\xi}_2| F_{1z} \tanh \left( \frac{\dot{\xi}_2}{\dot{\xi}_{23}} \right) + 0.2785 \xi_{12} F_{1z} \tag{29}
\]

\[
-\frac{A_1}{m_4} \dot{\xi}_8 \dot{\xi}_9 \leq \frac{1}{2} \frac{A_1^2}{m_4^2} \dot{\xi}_8^2 + \frac{1}{2} \dot{\xi}_9^2 \tag{30}
\]

\[
\dot{\xi}_i \dot{\xi}_{i+1} \leq \frac{1}{2} \dot{\xi}_i^2 + \frac{1}{2} \dot{\xi}_{i+1}^2, \quad i = 3, 5, 7 \tag{31}
\]

\[
P_{2A} \frac{\dot{\xi}_2}{m_4} \leq |\dot{\xi}_8| \rho_8 \\
\leq |\dot{\xi}_8| \rho_8 \tanh \left( \frac{\dot{\xi}_8}{\dot{\xi}_{83}} \right) + 0.2785 \xi_{83} \rho_8 \tag{32}
\]

\[
\frac{\beta}{V} (k_q \delta (v) + c_1 P_2) \leq |\dot{\xi}_9| \rho_9 \\
\leq |\dot{\xi}_9| \rho_9 \tanh \left( \frac{\dot{\xi}_9}{\dot{\xi}_{93}} \right) + 0.2785 \xi_{93} \rho_9 \tag{33}
\]

where $\rho_9 = \frac{m_2}{P_2 A_2 \rho_3} = \frac{\beta}{V} k_q \delta^2 + \frac{\beta}{V} c_1 P_2$.

For $i = 2, 4, 6, 8$, the following inequalities can be obtained based on Lemma 1

\[
-\frac{1}{\rho_i} |\dot{\xi}_i | \rho_i \leq -\frac{1}{2} \frac{1}{\rho_i} \frac{\xi^2}{\dot{\rho}_i} \tag{34}
\]

then, the similar result can be obtained for terms $\frac{\beta}{V} \xi_9 V$ when $i = 9$.

The following unknown smooth functions can be approximated by the FLSs in each step

\[
-\frac{1}{2} \frac{k_1^2}{m_1^2} \xi_2 + \frac{1}{m_1} \left( F_{1z} \tanh \left( \frac{\dot{\xi}_2}{\dot{\xi}_{23}} \right) + F_{2z}, z_1, z_2 \right) \\
- c_1 (z_4 - z_2) - \dot{\alpha}_1 = \theta \psi_2 (Z_2) + \epsilon_{21} \tag{35}
\]

\[
-\frac{1}{2} \frac{k_2^2}{m_2^2} \xi_4 + \frac{1}{m_2} \left( k_1 (z_3 - z_1) + c_1 (z_4 - z_2) + k_2 z_3 \right) \\
- c_2 (z_6 - z_4) - \dot{\alpha}_3 = \theta \psi_4 (Z_4) + \epsilon_{41} \tag{36}
\]

\[
-\frac{1}{2} \frac{k_3^2}{m_3^2} \xi_6 + \frac{1}{m_3} \left( k_2 (z_5 - z_3) + c_2 (z_6 - z_4) + k_3 z_5 \right) \\
- c_3 (z_8 - z_6) - \dot{\alpha}_5 = \theta \psi_6 (Z_6) + \epsilon_{61} \tag{37}
\]

\[
\frac{1}{2} \frac{A_1^2}{m_4^2} \xi_8 + \frac{1}{m_4} \left( k_3 (z_7 - z_5) + c_3 (z_8 - z_6) \right) \\
+ \rho_8 \tanh \left( \frac{\dot{\xi}_8}{\dot{\xi}_{83}} \right) - \dot{\alpha}_7 = \theta \psi_8 (Z_8) + \epsilon_{81} \tag{38}
\]

\[
\frac{\beta}{V} (\dot{A}_1 z_8 - c_1 z_9) + \rho_9 \tanh \left( \frac{\dot{\xi}_9}{\dot{\xi}_{93}} \right) - \dot{\alpha}_8 \\
= \theta \psi_9 (Z_9) + \epsilon_{91} \tag{39}
\]

\[
-\dot{\alpha}_{i-1} = \theta \psi_i (Z_i) + \epsilon_{i1}, \quad i = 3, 5, 7 \tag{40}
\]

Since $0 < \psi_i^2 (Z_i) \psi_i (Z_i) \leq 1$, by using Young’s inequality, one can obtain the following inequalities for $i = 2, 4, 6, 8$

\[
\xi_i (\theta \psi_i (Z_i) + \epsilon_{i1}) \leq \frac{\theta_i e_{i1}^2}{4 \epsilon_i \psi_i (Z_i) \psi_i (Z_i)} + \frac{1}{2} \dot{\xi}_i^2 + \frac{1}{2} \dot{e}_{i1}^2 + \epsilon_i \tag{41}
\]

Finally, substituting (12), (14-16), (18-19), (21-24), (27-42) into (26) and with $l (\gamma) \leq -\frac{l}{2} \gamma^2 + \frac{1}{2} \sqrt{\gamma}$, the time derivative of $V$ is

\[
\dot{V} \leq -aV + O \tag{43}
\]
TABLE 1. Simulation parameters of mechanical-hydraulic coupling system.

| Parameter | Value     | Parameter | Value     |
|-----------|-----------|-----------|-----------|
| $m_1$     | $4.549 \times 10^5$ kg | $A_2$     | $3.015 \times 10^{-2}$ m$^2$ |
| $m_2$     | $6.898 \times 10^5$ kg | $C_0$     | $0.62$    |
| $m_3$     | $7.791 \times 10^5$ kg | $C_i$     | $5 \times 10^{-16}$ |
| $m_4$     | $1.21 \times 10^5$ kg  | $\rho$    | $872$ kg/m$^3$ |
| $k_1$     | $7.2 \times 10^8$ N/m  | $w$       | $0.119$ m |
| $k_2$     | $6.16 \times 10^8$ N/m | $\beta_1$ | $7.8 \times 10^4$ Pa |
| $k_3$     | $3.03 \times 10^5$ N/m | $V$       | $0.0732$ m$^3$ |
| $c_1$     | $1.2 \times 10^6$ N·s/m | $P_0$     | $1 \times 10^6$ Pa |
| $c_2$     | $3.6 \times 10^6$ N·s/m | $P_0$     | $2 \times 10^5$ Pa |
| $c_3$     | $1.4 \times 10^6$ N·s/m | $K_v$     | $1.25 \times 10^{-4}$ m/v |
| $A_1$     | $19.635 \times 10^{-2}$ m$^2$ | | |

where $a = \min(2\sigma_1, \ldots, 2\sigma_8, \frac{l_1}{\sigma_1}, \frac{l_2}{\sigma_2}, \frac{l_1}{\sigma_1}, \frac{l_2}{\sigma_2}, i = 2, 4, 6, j = 3, 5, 7, k = 8, 9)$. $O = \sum_{i=2,4,6,8,9} \frac{i^2}{m_i} + \sum_{i=2,4,6} \tau_i + \frac{l_1}{\sigma_1} \rho_i^2 + \frac{l_2}{\sigma_2} \theta_i^2 + \sum_{j=3,5,7} \frac{l_2}{\sigma_2} \theta_j^2 \theta_i + \sum_{k=8,9} \left(0.2785 \varepsilon_{k3} \rho_k^2 + \frac{l_2}{\sigma_2} \theta_k^2 \theta_i + \sum_{j=2} \frac{i^2}{\sigma_1} \rho_k^2 + 0.2785 \varepsilon_{k3} F_{11} \right)$. Then, according to Lyapunov stability criterion, the results of this paper can be obtained and the proof is completed.

V. SIMULATION RESULTS

The vibration suppression algorithm proposed in this paper is applied in the 650mm cold rolling mill to verify the effectiveness. The simulation parameters are shown in Table 1. The input dead-zone of the servo valve voltage can be described by

$$u = \begin{cases} 
(v - 0.2), & v \geq 0.2 \\
0, & -0.5 < v < 0.2 \\
2(v + 0.5), & v \leq -0.5 
\end{cases}$$

The performance constraints functions are selected as $-2(5e^{-2} + 0.1) < z_1 < 2(5e^{-2} + 0.1)$ according to the actual working condition. Based on the control design procedure in section IV, the control laws and the corresponding adaptive laws can be derived as (12), (14-16), (18-19) and (21-25) with $e_1 = 0.1, a_i = 10, e_2 = 0.1, i = 1, \ldots, 9, j = 1, 2$, $\tau_i = 0.5 (i = 2, 4, 6), e_{k3} = 0.05 (k = 2, 8, 9)$. The initial values are selected as $[z_1, \ldots, z_9] = [1, 2, 0, 0, 0, 0, 0, 0, -2]$, $[\hat{\theta}_i (i = 2, 4, 6, 8, 9), \hat{\theta}_j (j = 2, 4, 6), \hat{\theta}_{k3} (k = 3, 5, 7, 8, 9), l = 1, \ldots, 9] = [0, \ldots, 0]$. The fuzzy membership functions are chosen as follows:

$$
\mu_{A_i^1} (z_i) = e^{-\frac{(z_i - 0)^2}{2}}, \quad \mu_{A_i^2} (z_i) = e^{-\frac{(z_i - 0)^2}{2}}, \\
\mu_{A_i^3} (z_i) = e^{-\frac{(z_i - 0)^2}{2}}, \quad \mu_{A_i^4} (z_i) = e^{-\frac{(z_i - 0)^2}{2}}, \\
\mu_{A_i^5} (z_i) = e^{-\frac{(z_i + 0)^2}{2}}, \quad \mu_{A_i^6} (z_i) = e^{-\frac{(z_i + 0)^2}{2}}, \\
\mu_{A_i^7} (z_i) = e^{-\frac{(z_i + 0)^2}{2}}, \quad \mu_{A_i^8} (z_i) = e^{-\frac{(z_i + 0)^2}{2}}, \\
\mu_{A_i^9} (z_i) = e^{-\frac{(z_i + 0)^2}{2}}, \quad \mu_{A_i^9} (z_i) = e^{-\frac{(z_i + 0)^2}{2}},
$$

The simulation results are shown in Figs.3-6. From Fig.3, the displacement of the work roll with performance constraints can be satisfied under the designed controller, and the vertical vibration of the rolling mill can be suppressed effectively. To illustrate the improved performance with the
proposed schemes, the displacement of the work roll under the control of general backstepping method without performance constraints is also plotted in Fig.3. It can be observed from the comparison that the system performance including convergence speed, steady state error, and overshoot are all improved. Figs.4-5 show the responses of the displacements and the vibration speeds of rolls and the hydraulic cylinder, from which one can see that they are all stable. According to Fig.6, it can be found that the oil pressure approaches to a stable value.

VI. CONCLUSION

This paper proposed a 4-DOF mechanical-hydraulic coupling vibration mathematical model and studied the vertical vibration suppression control problem for the six-high rolling mill with input dead-zone and output constraints. With the help of the asymmetric tangent barrier Lyapunov function, the adaptive fuzzy vertical vibration controller was designed via backstepping method. It was proved that the mechanical-hydraulic coupling rolling system was stable and the vertical vibration can be suppressed under the developed control law. At the same time, the performance of chatter attenuation can be guaranteed with the designed controller, including the vibration attenuation rate and the maximum allowable work roll displacement and steady-state error are limited to a given range. The simulation results illustrated the effectiveness of the proposed method to reduce the chatter of the rolling mill.

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