Frame Dependence of Spin-One Angular Conditions in Light Front Dynamics

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Abstract

We elaborate the frame dependence of the angular conditions for spin-1 form factors. An extra angular condition is found in addition to the usual angular condition relating the four helicity amplitudes. Investigating the frame-dependence of angular conditions, we find that the extra angular condition is in general as complicated as the usual one, although it becomes very simple in the $q^+ = 0$ frame involving only two helicity amplitudes. It is confirmed that the angular conditions are identical in frames that are connected by kinematical transformations. The high $Q^2$ behaviors of the physical form factors and the limiting behaviors in special reference frames are also discussed.
I. INTRODUCTION

Bosons with spin-1 are ubiquitous in modern particle physics. In the Standard Model the fundamental interactions are described by gauge-bosons, such as the photon, $W^\pm$ and $Z$, and the gluon. These particles are considered to be truly elementary, i.e. they occur as quanta of local fields.

In hadron physics many vector mesons composed of a quark and an antiquark are found and understanding their structure is a challenging problem in quantum chromodynamics (QCD), related to the mechanism of confinement and the detailed nature of the interactions between the constituents.

Moreover, the deuteron is an interesting laboratory for the application of QCD to nuclear physics. At large distances the deuteron is evidently well described as a spin-1 composite of two nucleon clusters with binding energy $\sim 2.2$ MeV, together with small admixtures of $\Delta\Delta$ and virtual meson components. However, at short distances, in the region where all six quarks overlap within a distance $R \sim 1/Q$, one can show rigorously that the deuteron state in QCD necessarily has “fractional parentage” $1/9\,(np)$, $4/45\,(\Delta\Delta)$ and $4/5\, "hidden\ color”\ (nonnuclear)\ components$ \[1\]. At any momentum scale, the deuteron cannot be described solely in terms of conventional nuclear physics degrees of freedom, but in principle any dynamical property of the deuteron is modified by the presence of non-abelian “hidden color” components \[2\]. Alternatively one may describe the deuteron structure in terms of uncoulored degrees of freedom only, but then a tower of excited nucleons and $\Delta$’s are involved \[3,4\].

Although these spin-1 systems (e.g. $W^\pm$, the $\rho-$meson and the deuteron) do not seem to share a common internal structure, universality of spin-1 systems \[5\] severely constrains them. According to this universality, the fundamental constraints on the magnetic and quadrupole moments of hadronic and nuclear states imposed by the Compton-scattering sum-rules \[6\] and the behavior of the electromagnetic form factors of composite spin-1 systems \[7\] at large momentum transfer are the same as those of a corresponding elementary particle of the same spin and charge. At $Q^2 = 0$, the charge ($G_C(Q^2)$), magnetic ($G_M(Q^2)$) and quadrupole ($G_Q(Q^2)$) form factors define the charge $e$, the magnetic moment $\mu_1$ and the quadrupole moment $Q_1$, respectively. In the limit of zero radius of the bound states (or large binding energies), whether confined or non-confined, the values of $\mu_1$ and $Q_1$ approach the canonical values \[8\] of a spin-1 object with mass $m$ and charge $e$

$$\mu_1 = \frac{e}{m}, \quad Q_1 = -\frac{e}{m^2}.$$ (1.1)

Universality requires that the values obtained in Eq. (1.1) must be the same as those of the fundamental gauge bosons $W^\pm$ in the tree approximation to the standard model. Also, at large $Q^2$ (in the limit $Q \gg \sqrt{2m\Lambda_{QCD}}$), these form factors are required to approach the universal ratios given by \[9\]

$$G_C(Q^2):G_M(Q^2):G_Q(Q^2) \to \left(1 - \frac{Q^2}{6m^2}\right):2:-1,$$ (1.2)

which were obtained in a light-cone frame with $q^+ = 0$. Eq. (1.2) should hold at large momentum transfers in the case of composite systems such as the $\rho$-meson and the deuteron,
with corrections of order $\Lambda_{QCD}/Q$ and $\Lambda_{QCD}/m$ according to QCD. The ratios are the same as those predicted for the electromagnetic couplings of the $W^{\pm}$ for all $Q^2$ in the standard model at tree level.

Furthermore, there are constraints on the current matrix elements, since there are only three form factors for the spin-1 systems. A constraint well-known from the literature [8] is the angular condition obtained by demanding rotational covariance for the current matrix elements given by

$$G_{h', h} = \langle p' h' | J^\mu | ph \rangle,$$

where $|ph\rangle$ is an eigenstate with momentum $p$ and helicity $h$. For example, in the Drell-Yan-West (DYW) frame and the frames that are connected to DYW by only kinematic transformations, the angular condition is given as [3,11,12]

$$(1 + 2\eta)G_{++}^+ + G_{+-}^+ - \sqrt{8\eta}G_{++0}^+ - G_{00}^+ = 0,$$

where $\eta = Q^2/4m^2$. If the angular condition is not satisfied, an identical extraction of form factors ($G_C$, $G_M$, $G_Q$) from the light-front current matrix elements $G_{h'h}^+$ is not attained. As a consequence, there are indeed different extraction schemes for the spin-1 form factors in the literature [5,10–12]. As an example, $G_C$, $G_M$ and $G_Q$ can be given in terms of $G_{++0}^+$, $G_{00}^+$ and $G_{+-}^+$ in the DYW frame $q^+ = 0$, $q_x = Q$, and $q_y = 0$ as follows [3];

$$G_C = \frac{1}{2p^+ (2\eta + 1)} \left[ \frac{16}{3} \eta G_{++0}^+ - \frac{2\eta - 3}{3} G_{00}^+ + \frac{2}{3} (2\eta - 1) G_{+-}^+ \right],$$

$$G_M = \frac{2}{2p^+ (2\eta + 1)} \left[ (2\eta - 1) \frac{G_{++0}^+}{\sqrt{2\eta}} + G_{00}^+ - G_{+-}^+ \right],$$

$$G_Q = \frac{1}{2p^+ (2\eta + 1)} \left[ 2 \frac{G_{++0}^+}{\sqrt{2\eta}} - G_{00}^+ + \frac{\eta + 1}{\eta} G_{+-}^+ \right].$$

(1.5)

However, other choices of the current matrix elements can be made to express the right-hand-side of Eq. (1.5) and the expression also depends on the reference frame. A few examples of other expressions on the right-hand-side of Eq. (1.5) can be found in Ref. [13]. The angular conditions are also useful in testing the validity of model calculations. Especially, as stressed in the recent literature [14–18], the zero-mode contribution is necessary to get the correct result for the form factors unless the good component of the current is used. Even if the good component of the current is used, it was noted that the zero-mode contribution is necessary for the calculation of spin-1 form factors [19]. Such an observation of zero-mode necessity has been made by checking the angular conditions and the degree of necessity can be quantified by examining the angular conditions.

As discussed above, the constraints from universality and the angular conditions are in principle very useful for model-building and a self-consistency-check of theoretical or phenomenological models for spin-1 objects. However, these constraints do depend on the reference frame. For example, in the Breit frame where $q^+ \neq 0$, a less informative prediction of asymptotic form factors is made [20] instead of Eq. (1.2);

$$G_C(Q^2) : G_Q(Q^2) \rightarrow \frac{Q^2}{6m^2} : 1$$

(1.6)
in the limit \( Q \gg 2m \). Thus, it is important to examine the frame dependence of the constraints that are useful for model-building and phenomenology.

In this work, we analyze the frame dependence of the angular conditions for the spin-1 systems. Interestingly, besides the angular condition given by Eq. (1.4) we find another one. Elaborating the frame-dependence of these angular conditions in the generalized Breit and target rest frames as well as the DYW frame, we confirm the advantage of using the DYW frame in the calculation of exclusive processes. The complexity of each angular condition in general depends on the reference frame. In the DYW frame, the extra angular condition is particularly simple so that most theoretical models are expected to satisfy it without any difficulty. We also substantiate that the angular conditions are identical in reference frames that are connected by kinematical transformations. Such an investigation is also important in analyzing the high \( Q^2 \) behavior of spin-1 form factors. We confirm that the angular conditions are consistent with the high \( Q^2 \) behavior predicted by perturbative QCD (PQCD) for the three physical form factors \([5,8,9]\).

In the next Section (Section II), both the instant-form (IFD) and the front-form (LFD) polarization vectors are presented in arbitrary frames. In Section III, we derive the relation between the current operator and the form factors and starting from general grounds obtain the most general angular conditions. We show that there are indeed two angular conditions and discuss the reason why they should be regarded as consistency conditions. In Section IV, we elaborate the details of the angular conditions in the DYW, generalized Breit and target-rest frames. In Section V, we discuss the large momentum transfer behavior of the form factors in each reference frame. We also consider the limiting behaviors of the form factors in approaching special Breit and target-rest frames. Conclusions follow in Section VI. In the Appendix, explicit representations of front-form boost and helicity operators are summarized.

II. POLARIZATION VECTORS IN INSTANT-FORM AND LIGHT-FRONT DYNAMICS

A. Polarization Vectors in Three Dimensions

For the polarization vectors we use the standard spherical tensors for spin-1 \([21]\)

\[
\vec{e}'(0) = (0, 0, 1), \quad \vec{e}'(\pm) = \mp \frac{1}{\sqrt{2}} (1, \pm i, 0).
\]

(2.1)

Complex conjugation gives

\[
\vec{e}'(h)^* = (-1)^h \vec{e}'(-h).
\]

(2.2)

The orthogonality relation is

\[
\vec{e}'(h) \cdot \vec{e}'(h')^* = \delta_{hh'},
\]

(2.3)

which can also be written as

\[
\vec{e}'(h) \cdot \vec{e}'(-h') = (-1)^h \delta_{hh'}.
\]

(2.4)
The closure property can be written as

\[ \sum_h e_i(h) e_j(h)^* = \delta_{ij}. \] (2.5)

**B. Polarization Vectors in Four Dimensions**

It is easy to extend these three polarization vectors to four vectors:

\[ \vec{\epsilon}(h) = (0, \vec{e}(h)). \] (2.6)

Then the orthogonality and closure properties are

\[ \vec{\epsilon}(h) \cdot \vec{\epsilon}(h')^* = -\delta_{hh'}, \] (2.7)

and

\[ \sum_h \vec{\epsilon}_{\mu}(h) \vec{\epsilon}_{\nu}(h)^* = - \left( g_{\mu\nu} - \frac{\vec{p}_{\mu}\vec{p}_{\nu}}{m^2} \right), \] (2.8)

where \( \vec{p}_{\mu} = (m, 0, 0, 0). \)

**C. Polarization Vectors in a Specific Lorentz Frame**

We now consider the polarization vectors given above as belonging to a particle of mass \( m \) in its rest frame. Then the definition Eq. (2.6) reflects the transversality property

\[ p_{\mu} \epsilon^{\mu}(p; h) = 0. \] (2.9)

In order to extend this property to all Lorentz frames, we define the polarization vectors in a specific frame by boosting the vectors (Eq. (2.6)) to the specific frame. The vectors we obtain will depend on the Lorentz transformation. Later on, we shall be interested in the kinematic operators only, the number of which is maximized in LFD. In IFD all boosts are dynamical, so we cannot impose the same limitation. As the instant-form results are for the purpose of illustration only, we shall not worry about this point, but just limit ourselves to pure Lorentz transformations, i.e. rotationless boosts.

**D. Front-Form Polarization Vectors**

In the front form we need the kinematical front-form boosts. They are given in Appendix A.

We note that the LF components we use satisfy the following relations

\[ p_{\perp}^2 = -2p^r p^l, \quad p \cdot q = p^+ q^- + p^- q^+ + p^0 p^l + p^l q^0, \] (2.10)
where we use the spherical components of the three momentum vectors to simplify the notation. They are defined as follows

\[ p^r = -\frac{p_x + ip_y}{\sqrt{2}}, \quad p^l = \frac{p_x - ip_y}{\sqrt{2}}. \] \hspace{1cm} (2.11)

Occasionally we use the notation \( p^h \) with \( h = +1, 0, -1 \) for \( p^r, p_z, \) and \( p^l, \) respectively. Then the usual relation for spherical tensors applies

\[ (p^h)^* = (-1)^h p^{-h}. \] \hspace{1cm} (2.12)

The polarization vectors in the rest system, where the four momentum has the LF components \((p^+, p^1, p^2, p^-) = (m/\sqrt{2}, 0, 0, m/\sqrt{2})\), are

\[ \epsilon^h_{\alpha}(\pm) = (0, \mp 1/\sqrt{2}, -i/\sqrt{2}, 0), \quad \epsilon^0_{\alpha}(0) = (1/\sqrt{2}, 0, 0, -1/\sqrt{2}). \] \hspace{1cm} (2.13)

Upon application of the front-form boost Eq. (A12) we find the polarization vectors

\[
\begin{align*}
\epsilon^h_{\alpha}(p^+, p^1, p^2; +) = \left\{ \begin{array}{c}
(0, \pm \frac{i}{\sqrt{2}}, \pm \frac{p^x}{\sqrt{2}}, \pm \frac{p^y}{\sqrt{2}}) \\
\frac{p^x}{m}, \frac{p^y}{m}, \frac{p^2 - m^2}{2mp}, \frac{p^2}{p^2}
\end{array} \right\}, \\
\epsilon^h_{\alpha}(p^+, p^1, p^2; 0) = \left\{ \begin{array}{c}
(0, \frac{1}{\sqrt{2}}, \pm \frac{i}{\sqrt{2}}, \pm \frac{p^x}{\sqrt{2}}) \\
\frac{p^x}{m}, \frac{p^y}{m}, \frac{p^2 - m^2}{2mp}, \frac{p^2}{p^2}
\end{array} \right\}.
\end{align*}
\] \hspace{1cm} (2.14)

It is easy to check that these are mutually orthogonal, transverse, and satisfy the closure property Eq. (2.8) if one uses the front-form for the metric.

### III. CURRENTS

For a spin-1 particle the current operator has the form

\[ J^\mu_{\alpha\beta}(p', p) = -g_{\alpha\beta}(p' + p)^\mu F_1(q^2) + (g_\mu^\alpha q_\alpha - g_\mu^\alpha q_\beta)F_2(q^2) + \frac{q_\alpha q_\beta(p' + p)^\mu}{2m^2}F_3(q^2), \] \hspace{1cm} (3.1)

where the momenta \( p \) and \( p' \) are the momenta of the particle before and after absorption of a photon with momentum \( q = p' - p \). The coefficient functions \( F_i(Q^2) \) in Eq. (3.1) are given by the physical form factors, i.e.

\[
\begin{align*}
F_1 &= G_C - \frac{2}{3} \eta G_Q, \\
F_2 &= -G_M, \\
F_3 &= \frac{1}{1 + \eta} \left[ -G_C + G_M + (1 + \frac{2}{3} \eta)G_Q \right].
\end{align*}
\] \hspace{1cm} (3.2)

A spin tensor \( G \) is obtained by taking matrix elements with the polarization vectors, viz

\[ G_{\mu h}^\alpha = \epsilon^*_{\alpha}(p'; h') J_{\alpha\beta}^\mu(p; h). \] \hspace{1cm} (3.3)

This form can be derived on very general grounds. First, we write down all tensors of third rank that can be constructed using \( g_{\alpha\beta}, p'^\mu, \) and \( p^\mu \) alone. There are fourteen possible
structures. As the matrix elements are obtained by contracting with the polarization vectors \( \varepsilon^\alpha(p'; h') \) and \( \varepsilon^\beta(p; h) \), the structures containing a factor \( p'_\alpha \) or \( p_\beta \) do not contribute to the matrix element. Therefore, only six remain and we write

\[
J^\mu_{\alpha\beta}(p'; p) = f_1 g_{\alpha\beta} p'^\mu + f_2 g_{\alpha\beta} p^\mu + f_3 g_{\alpha\beta}^p p'_\beta + f_4 p^\mu p_\alpha p'_\beta + f_5 p^\mu p_\alpha p'_\beta + f_6 p^\mu p_\alpha p'_\beta. \tag{3.4}
\]

Secondly, we require current conservation, which means \( q_\mu G^\mu_{h'h}(p', p) = 0 \) for all \( \mu, h' \), and \( h \). Contracting with \( q \) gives

\[
0 = (f_1 - f_2) g_{\alpha\beta}(m^2 - p' \cdot p) + f_3 g_{\alpha\beta} p'_\beta + f_4 q_\beta p_\alpha + (f_5 - f_6) p_\alpha p'_\beta (m^2 - p' \cdot p). \tag{3.5}
\]

We can immediately conclude that \( f_1 = f_2 \) and \( f_5 = f_6 \). In order to reduce the number of terms further, we again contract with the polarization vectors and see that

\[
\varepsilon^*(p'; h') \cdot q = -\varepsilon^*(p'; h') \cdot p, \quad \varepsilon(p; h) \cdot q = -\varepsilon(p; h) \cdot p'. \tag{3.6}
\]

So we are left with the term \((f_1 - f_2)p_\alpha p'_\beta\). This structure is independent of the one containing \((f_5 - f_6)\), because the latter originates from a term that contains the factor \( p'^\mu + p^\mu \) while the former does not. So we conclude that \( f_1 = f_2 \), which means that only three independent form factors remain.

Next we impose hermiticity, i.e.

\[
\langle p'; h' | J^\mu | p; h \rangle = \langle p; h | J^\mu | p'; h' \rangle^*, \tag{3.7}
\]

which gives after some rearrangement

\[
\varepsilon^\alpha(p'; h') J^\mu_{\alpha\beta}(p', p) \varepsilon^\beta(p; h) = \varepsilon^\alpha(p'; h') J^\mu_{\beta\alpha}(p, p') \varepsilon^\beta(p; h). \tag{3.8}
\]

This is an identity for all \( p, p', h, \) and \( h' \), so we find

\[
J^\mu_{\alpha\beta}(p', p) = J^{\mu*}_{\beta\alpha}(p, p'). \tag{3.9}
\]

If we apply this identity to the structures we found, we see that the coefficients of the tensors must be real, which means that \( F_1, F_2, \) and \( F_3 \) in Eq. (3.11) are real.

The symmetry of \( J^\mu_{\alpha\beta}(p', p) \) entails relations between the matrix elements too. If we, in addition, apply Eq. (2.2), which we owe to the fact that the polarization vectors are standard spherical tensors, we can deduce

\[
G^\mu_{h'h}(p', p) = (-1)^{h'+h} G^\mu_{-h'-h}(p', p). \tag{3.10}
\]

The explicit expressions we are writing down in the next sections of course satisfy these identities.

Eq. (3.11) can be split in an obvious way into the pieces \( J(1)F_1, J(2)F_2, \) and \( J(3)F_3 \). Then we find for the polarization tensor \( G = G(1)F_1 + G(2)F_2 + G(3)F_3 \) with the partial tensors

\[1\text{Note that the kinematic region for this discussion is spacelike, i.e. } q^2 < 0.\]
Clearly, we need three simple scalar products which we shall write in the front form only

\[ \varepsilon^e(p'; 0) \cdot \varepsilon(p; 0) = \frac{p'^2 + (\vec{p}^{'2} - m^2) + p^2 + 2(p^{'2} - m^2) - 2p' + p^' \cdot \vec{p}^{'2}}{2m^2p'p^+}, \]

\[ \varepsilon^e(p'; 0) \cdot \varepsilon(p; h) = \frac{p'^{p^h} + p^{p'h} - p^p}{mp^+}, \]

\[ \varepsilon^e(p'; h') \cdot \varepsilon(p; h) = -\frac{1 + h'h}{2}, \]

\[ \varepsilon^e(p'; 0) \cdot p = \frac{p'^{p^-} + p^p\cdot p'^{h-h} - 2p' + p^p' \cdot \vec{p}^{'2}}{2mp^+p^+}, \]

\[ \varepsilon^e(p'; h) \cdot p = \frac{p'^{p^-} - p^p p'^{h-h}}{p^+}. \]  

(3.12)

where we made the obvious identification \( p^{h=+1} \leftrightarrow p^r, \ p^{h=-1} \leftrightarrow p^l \).

We give the matrix elements of the polarization tensors. We define \( \tilde{G}^{+}(1) \) as \( G^{+}(1) = (p^{+} + p^{+}) \tilde{G}^{+}(1) \) and find

\[ \tilde{G}^{+}(1)_{++} = G^{+}(1)_{--} = 1, \quad \tilde{G}^{+}(1)_{+-} = G^{+}(1)_{-+} = 0, \]

\[ \tilde{G}^{+}(1)_{++} = \frac{p^{+} + p^{p^h} - p^p}{mp^+}, \]

\[ \tilde{G}^{+}(1)_{0+} = \frac{p'^{p^h} - p^{p'h}}{mp^+}, \]

\[ \tilde{G}^{+}(1)_{00} = \frac{-p'^2 + p^{p^h} + m^2(p^{p^h} + m^2) - 2p' + p^p \cdot \vec{p}^{'2}}{2m^2p^+ p^+}, \]

\[ \tilde{G}^{+}(1)_{0-} = \frac{p^{p^h} - p^{p'h}}{mp^+}, \]

\[ \tilde{G}^{+}(1)_{-0} = \frac{p^{p^h} - p^{p'h}}{mp^+}. \]  

(3.13)

\[ G^{+}(2)_{++} = G^{+}(2)_{-+} = G^{+}(2)_{-+} = G^{+}(2)_{--} = 0, \]

\[ G^{+}(2)_{++} = \frac{p^{+} + p^{p^h} - p^p}{mp^+}, \]

\[ G^{+}(2)_{0+} = \frac{p'^{p^h} - p^{p'h}}{mp^+}, \]

\[ G^{+}(2)_{00} = \frac{p^{+} + p^+}{2m^2p^+ p^+} \cdot [m^2(p^{p^h} - p^+)^2 - p^{p^h} \vec{p}^{'2} + p^p \cdot \vec{p}^{'2} + 2p' - p^p \cdot \vec{p}^{'2}], \]

\[ G^{+}(2)_{0-} = \frac{p^{p^h} - p^{p'h}}{mp^+}, \]

\[ G^{+}(2)_{-0} = \frac{p^{p^h} - p^{p'h}}{mp^+}. \]  

(3.14)
\( G^+(3) \) also contains an over-all factor, so we define \( G^+(3) = (p^+ + p^+)/ (4m^2p^+p^+) \tilde{G}^+(3) \) with

\[
\tilde{G}^+(3)_{++} = \tilde{G}^+(3)_{--} = p^{+\gamma} p^{\gamma^\prime} (p^{\alpha} p^\alpha - 2p^+ p^+ p^\gamma p^\gamma),
\]

\[
\tilde{G}^+(3)_{0+} = \frac{p^{+\gamma} p^{\gamma^\prime} - p^{\gamma\alpha} p^\alpha}{mp^+} [p^{\gamma\alpha} p^{\alpha^\prime} - m^2(p^{\gamma^\prime})^2 - 2p^+ p^+ p^\gamma' p^\gamma],
\]

\[
\tilde{G}^+(3)_{00} = \frac{-1}{2m^2p^+p^\gamma} [(p^{\gamma\alpha} p^{\alpha^\prime} - m^2(p^{\gamma^\prime})^2 - 2p^+ p^+ p^\gamma' p^\gamma) \times (p^{\gamma\beta} p^{\beta^\prime} - m^2(p^{\gamma^\prime})^2 - 2p^+ p^+ p^\gamma' p^\gamma)],
\]

\[
\tilde{G}^+(3)_{--} = -2(p^+ p^\gamma - p^+ p^\gamma^\prime)^2,
\]

\[
\tilde{G}^+(3)_{00} = \frac{p^{+\gamma} p^{\gamma^\prime} - p^{\gamma\alpha} p^\alpha}{mp^+} [p^{\gamma\alpha} p^{\alpha^\prime} - m^2(p^{\gamma^\prime})^2 - 2p^+ p^+ p^\gamma' p^\gamma].
\]

(3.15)

Hermiticity follows from the simultaneous replacements \( p \leftrightarrow p^\gamma \) and \( p^\gamma \leftrightarrow -p^\gamma^\prime \).

**A. Symmetries of the Polarization Tensor**

The formulae above tell us that the polarization tensor has the following form

\[
G(i) = \begin{pmatrix}
  a_i & c_i & e_i^2 \\
  b_i & d_i & -b_i^* \\
  c_i & -c_i^* & a_i
\end{pmatrix},
\]

(3.16)

which is valid for all three contributions \( G(i), i = 1, 2, 3 \). Using an obvious notation we find for the complete polarization tensor the form

\[
G = \begin{pmatrix}
  a_1 F_1 + a_3 F_3 & c_1 F_1 + c_2 F_2 + c_3 F_3 & e_3 F_3 \\
  b_1 F_1 + b_2 F_2 + b_3 F_3 & d_1 F_1 + d_2 F_2 + d_3 F_3 & -(b_1 F_1 + b_2 F_2 + b_3 F_3)^* \\
  e_3 F_3 & -(c_1 F_1 + c_2 F_2 + c_3 F_3)^* & a_1 F_1 + a_3 F_3
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  G^+_{++} & G^+_{++} & G^+_{++} \\
  G^+_{00} & G^+_{00} & G^+_{00} \\
  G^+_{+-} & G^+_{+-} & G^+_{+-}
\end{pmatrix}.
\]

(3.17)

Apparently, the tensor components we obtain here satisfy an additional identity

\[
G^+_{++} = G^+_{--} = G^+_{++}.
\]

(3.18)

This result is specific for the choice of \( \mu \): it is true for the good current \( J^+ \), but does not apply to the terrible current \( J^- \). The matrix elements \( G^+_{++} \) and \( G^-_{--} \) are not real, but they are complex conjugates.
For later reference we also give the expressions for $a_1, \ldots, e_3$.

\[
\begin{align*}
a_1 &= p'^{+} + p^+, \\
b_1 &= -(p'^{+} + p^+) \frac{p'^{+}p^r - p^+p'^r}{mp^+}, \\
c_1 &= -(p'^{+} + p^+) \frac{p'^{+}p^l - p^+p'^l}{mp'^+}, \\
d_1 &= (p'^{+} + p^+) \frac{m^2(p'^{+} + p^+)^2 + 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l)}{2m^2p'^+p^+}, \\
e_1 &= 0.
\end{align*}
\tag{3.19}
\]

\[
\begin{align*}
a_2 &= 0, \\
b_2 &= -p'^{+} \frac{p'^{+}p^r - p^+p'^r}{mp^+}, \\
c_2 &= -p^+ \frac{p'^{+}p^l - p^+p'^l}{mp'^+}, \\
d_2 &= (p'^{+} + p^+) \frac{m^2(p'^{+} - p^+)^2 + 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l)}{2m^2p'^+p^+}, \\
e_2 &= 0.
\end{align*}
\tag{3.20}
\]

\[
\begin{align*}
a_3 &= -(p'^{+} + p^+) \frac{(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l)}{2m^2p'^{+}p^+}, \\
b_3 &= -(p'^{+} + p^+) \frac{(p'^{+}p^r - p^+p'^r)[m^2(p'^{+} + p^+)^2 - 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l)]}{4m^3p'^{+}p^+}, \\
c_3 &= (p'^{+} + p^+) \frac{(p'^{+}p^l - p^+p'^l)[m^2(p'^{+} + p^+)^2 - 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l)]}{4m^3p'^{+}p^+}, \\
d_3 &= \frac{p'^{+} + p^+}{8m^4p'^{+}p^+ + 2} \left[ m^2(p'^{+} + p^+)^2 - 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l) \right] \\
&\quad \times \left[ m^2(p'^{+} + p^+)^2 - 2(p'^{+}p^r - p^+p'^r)(p'^{+}p^l - p^+p'^l) \right], \\
e_3 &= -(p'^{+} + p^+) \frac{(p'^{+}p^r - p^+p'^r)^2}{2m^2p'^{+}p^+}.
\end{align*}
\tag{3.21}
\]

We see that the nine matrix elements of $G$ have four relations that involve a phase factor only, viz

\[
G^+_{++} = G^+_{--}, \quad G^+_{+-} = G^+_{-+}, \quad G^+_{0-} = -G^+_{0+}, \quad G^+_{+0} = -G^+_{-0}.
\tag{3.22}
\]

We need two more equations that express the fact that there are only three independent form factors. These consistency conditions are the two angular conditions proper. Since we are working only with the $+$ component of the current, we shall use the following short-hand notations

\[
G_a = G^+_{++} = G^+_{--}, \quad G_b = G^+_{0+} = -G^+_{-0}.
\]
\[ G_c = G_{+0}^{-} = -G_{-0}^{+}, \quad G_d = G_{00}^{+}, \quad G_e = G_{-+}^{+} = G_{+-}^{+}. \] (3.23)

We can now solve for \( F_i \) in an obvious way. First we obtain \( F_3 \) from \( G_e \), then \( F_1 \) from \( G_a \) and \( F_3 \). Then we have a choice whether we want to obtain \( F_2 \) from \( G_b, G_c \) or \( G_d \); these solutions we denote by \( F_2^b, F_2^c, \) and \( F_2^d \), respectively. As these results must coincide, the identity of these three results form the angular conditions: \( F_2^b = F_2^c = F_2^d \). We find

\[
F_1 = \frac{1}{a_1} G_a - \frac{a_3}{a_1 e_3} G_e, \\
F_3 = \frac{1}{e_3} G_e, \\
F_2^b = \frac{1}{b_2} \left[ -\frac{b_1}{a_1} G_a + G_b + \frac{a_3 b_1 - a_1 b_3}{a_1 e_3} G_e \right], \\
F_2^c = \frac{1}{c_2} \left[ -\frac{c_1}{a_1} G_a + G_c + \frac{a_3 c_1 - a_1 c_3}{a_1 e_3} G_e \right], \\
F_2^d = \frac{1}{d_2} \left[ -\frac{d_1}{a_1} G_a + G_d + \frac{a_3 d_1 - a_1 d_3}{a_1 e_3} G_e \right].
\] (3.24)

The relations Eq. (3.10) reduce the nine complex elements of the polarization tensor to nine real numbers. As there are only three real independent form factors, we need six linear relations to realize the reduction from nine to three. The equations above serve this purpose. By equating the real and imaginary parts of the two sides of the first three of Eqs. (3.24), we find six relations that must hold for the components of \( G_{+}^{+} \). Having thus achieved the reduction to the minimum number of independent functions, the other equations must be considered to be consistency conditions. As the three equations expressing \( F_2 \) in terms of the tensor components are not independent, but form a system of rank two, only one complex equation, or two real ones remain.

In the literature usually only one is given, said to be the angular condition. From our considerations it must be clear that there are indeed two conditions.

**B. Angular Conditions**

The angular conditions can now be formulated succinctly:

\[ F_2^b = F_2^c, \quad F_2^b = F_2^d, \quad F_2^c = F_2^d. \] (3.25)

We shall write these conditions explicitly for unspecified kinematics.

The first one, denoted henceforth by AC 1, is

\[
F_2^b - F_2^c = 0 \\
= \frac{p^+ + p^+}{p^+ + p^+} \left[ G_a + \frac{m^2 (p^+ + p^+)^2}{2 (p^+ + p^+ - p^+ p^+)^2} G_e \right] \\
- m \frac{p^+}{p^+ + p^+ - p^+ p^+} G_b + m \frac{p^+}{p^+ + p^+ - p^+ p^+} G_c.
\] (3.26)
\[ F_2^b - F_2^d = 0 \]

\[ = \left[ -\frac{1}{p^+} + \frac{m^2(p'+2 + p^+2) + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')}{(p'+p^+)^2 + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')} \right] G_a \]

\[ -m\frac{p^+}{p'+p^+ - p+p'} G_b \]

\[ -2m^2\frac{p'+p^+}{p'+p^+ - p+p'} G_d \]

\[ + \left[ -\frac{m^2}{p'+p^+ - p+p'} \right. \]

\[ \left. \frac{m^2}{2(p'+p^+ - p+p')^2} - \frac{m^2}{2(p'+p^+ - p+p')^2} \right] \]

\[ \times \frac{m^2(p'+2 + p^+2) + 2(p'+2 + p^+2)(p'+p^+ - p+p')^2}{(p'+p^+ - p+p')^2 + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')} \right] G_e. \]

The last one is

\[ F_2^c - F_2^d = 0 \]

\[ = \left[ -\frac{1}{p^+} + \frac{m^2(p'+2 + p^+2) + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')}{(p'+p^+)^2 + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')} \right] G_a \]

\[ -m\frac{p^+}{p'+p^+ - p+p'} G_c \]

\[ -2m^2\frac{p'+p^+}{p'+p^+ - p+p'} G_d \]

\[ + \left[ -\frac{m^2}{p'+p^+ - p+p'} \right. \]

\[ \left. \frac{m^2}{2(p'+p^+ - p+p')^2} - \frac{m^2}{2(p'+p^+ - p+p')^2} \right] \]

\[ \times \frac{m^2(p'+2 + p^+2) + 2(p'+2 + p^+2)(p'+p^+ - p+p')^2}{(p'+p^+ - p+p')^2 + 2(p'+p^+ - p+p')^2(p'+p^+ - p+p')} \right] G_e. \]

If we substitute Eq. (3.26) into Eq. (3.27) we see that it is equivalent to Eq. (3.28), as it must be, because these equations are not independent as there are only two independent angular conditions.

Clearly, these conditions are quite complicated. We can simplify them by factoring out some common factors, at the same time avoiding denominators that may vanish. Instead of Eqs. (3.26,3.27) we get the conditions AC 1

\[ 2(p'+p^+)(p'+p^+ - p+p')^2(p'+p^+ - p+p') G_a \]

\[ -2mp^2(p'+p^+ - p+p')(p'+p^+ - p+p') G_b \]

\[ +2mp^2(p'+p^+ - p+p')^2 G_c \]

\[ +m^2(p'+p^+)(p'+p^+ + p^+)^2(p'+p^+ - p+p') G_e = 0. \]

(3.29)

and AC 2

\[ 2(p'+p^+ - p+p')^2[2m^2(p'+2 - 2p'+p^+ - p'+2) + 2(p'+p^+ - p+p')(p'+p^+ - p+p')] G_a \]
+2m(p'+ p')(p'+ p' - p+ p') [m^2(p'+ p')^2 + 2(p'+ p' - p+ p')(p'+ p' - p+ p')] G_b \\
+4m^2(p'+ p' - p+ p')^2 G_d \\
m^2(p'+ p' - p+ p')^2 + 2(p+ p' + p' - p'+ p')(p'+ p' - p+ p')(p'+ p' - p+ p')] G_e = 0.

(3.30)

Clearly, these conditions are minimal, as we cannot eliminate any of the five tensor components to obtain a simpler one.

It is useful to realize the phase relations that occur. Besides the relations expressed in Eq. (3.16, 3.22) we can use the fact that \((p')^* = -p\) and the fact that \(G_a\) and \(G_d\) are real to infer that both angular conditions have the form

\[ C_a G_a + C_b e^{-i\phi} G_b + C_c e^{i\phi} G_c + C_d G_d + C_e e^{-2i\phi} G_e = 0, \]

where the coefficients \(C_a, \ldots C_e\) are real and \(\phi\) is the argument of the complex number \(p'+ p' - p+ p'\), given by

\[ \tan \phi = -\frac{p'_y - p'^y}{p'_x - p'^x}. \]

(3.32)

This angle can be set to zero by a rotation of the reference frame about the z-axis. This rotation being kinematical in LFD we may expect the phase relations to be satisfied always.

It may turn out for some kinematics, that these relations simplify. This happens to be the case in e.g. the DYW-frame, where \(p'+ p' = p+\) and \(\vec{p}_\perp = 0\). Moreover, when \(\vec{q}\) is purely longitudinal, i.e., \(\vec{q}_\perp = 0\), we can rotate the reference frame such that \(\vec{p}_\perp = \vec{p}'_\perp = 0\). Then both angular conditions are identically satisfied, as all coefficients vanish.

IV. SPECIFIC FRAMES

We consider three specific frames: the Drell-Yan-West (DYW), Breit and Target-Rest (TRF) frames. For simplicity, only the kinematics and the angular conditions in the form \(F^{b}_2 - F^{c}_2 = 0\) (AC 1) and \(F^{b}_2 - F^{d}_2 = 0\) (AC 2) are presented in this section and the detailed formulas of the polarization tensors in the form of the coefficients \(a_1, \ldots, e_3\) are summarized in Appendix C.

A. Drell-Yan-West Frame

1. Kinematics

For the DYW frame,

\[ p = (p^+, 0, 0, m^2/(2p^+)) \]
\[ q = (0, q_x, q_y, q^2_\perp/(2p^+)) \]
\[ p' = p + q \]
\[ = (p^+, q_x, q_y, (q^2_\perp + m^2)/(2p^+)), \]

(4.1)
with the identification \( q_x = Q \cos \phi, q_y = Q \sin \phi \) one finds the explicit formulas

\[
p = (p^+, 0, 0, m^2/(2p^+))
q = (0, Q \cos \phi, Q \sin \phi, Q^2/(2p^+))
p' = (p^+, Q \cos \phi, Q \sin \phi, (Q^2 + m^2)/(2p^+))
\tag{4.2}
\]

and

\[
q^r = -\frac{Q}{\sqrt{2}} e^{i\phi}, \quad q^l = \frac{Q}{\sqrt{2}} e^{-i\phi}.
\tag{4.3}
\]

2. Angular Conditions

We write the angular conditions mentioned in the previous section.

AC 1

\[
e^{-i\phi} G_b + e^{i\phi} G_c = 0,
\tag{4.4}
\]

AC 2

\[
(2m^2 + Q^2) G_a + 2\sqrt{2m} Q e^{-i\phi} G_b - 2m^2 G_d + 2m^2 e^{-2i\phi} G_e = 0.
\tag{4.5}
\]

B. Breit Frame

1. Kinematics

We define the quantity \( \beta \) as

\[
\beta = \sqrt{1 + \left(\frac{Q}{2m}\right)^2}.
\tag{4.6}
\]

Then

\[
p = \left(\frac{2m\beta - Q \cos \theta}{2\sqrt{2}}, \frac{-Q \sin \theta \cos \phi}{2}, \frac{-Q \sin \theta \sin \phi}{2}, \frac{2m\beta + Q \cos \theta}{2\sqrt{2}}\right),
p' = \left(\frac{2m\beta + Q \cos \theta}{2\sqrt{2}}, \frac{Q \sin \theta \cos \phi}{2}, \frac{Q \sin \theta \sin \phi}{2}, \frac{2m\beta - Q \cos \theta}{2\sqrt{2}}\right),
q = \left(\frac{Q \cos \theta}{\sqrt{2}}, Q \sin \theta \cos \phi, Q \sin \theta \sin \phi, -\frac{Q \cos \theta}{\sqrt{2}}\right).
\tag{4.7}
\]
2. Angular conditions

By now we give only the two linearly independent conditions. We simplify the expressions as much as possible by dividing out common factors to find the two conditions.

\[ AC \ 1 \]
\[-2 \sqrt{2} \beta Q^2 \cos \theta \sin^2 \theta \ G_a + (2 \beta m - Q \cos \theta)^2 \sin \theta e^{-i\phi} \ G_b + (2 \beta m + Q \cos \theta)^2 \sin \theta e^{i\phi} \ G_c - 8 \sqrt{2} \beta^2 m^2 \cos \theta e^{-2i\phi} \ G_e = 0, \]  
(4.8)

\[ AC \ 2 \]
\[-[4 \beta m Q \cos \theta - Q^2 \cos^2 \theta + 2 \beta^2 (2m^2 + Q^2 \sin^2 \theta)] \sin^2 \theta \ G_a - 4 \sqrt{2} m Q (\beta^2 \sin^2 \theta - \cos^2 \theta) \sin \theta e^{-i\phi} \ G_b + (2 \beta m + Q \cos \theta)^2 \sin^2 \theta \ G_d + [(8m^2 + Q^2 \sin^2 \theta) \cos^2 \theta + 4 \beta m Q \cos \theta \sin^2 \theta - 4 \beta^2 m^2 \sin^2 \theta] e^{-2i\phi} \ G_e = 0. \]  
(4.9)

We note that Eqs. (4.8) and (4.9) are reduced to the results in DYW Eqs. (4.4) and (4.5), respectively, if \( \theta = \pi/2 \) as they should, because the two frames are related by a kinematical transformation in that case and the angular conditions do not change under any kinematical transformation.

C. Target-Rest Frame

1. Kinematics

Using again \( \beta \), and \( \kappa \), defined as

\[ \kappa = \frac{Q^2}{2m}, \]  
(4.10)

we find

\[ p = \left( \frac{m}{\sqrt{2}}, 0, \frac{m}{\sqrt{2}} \right), \]

\[ q = \left( \frac{\kappa + \beta Q \cos \theta}{\sqrt{2}}, \beta Q \sin \theta \cos \phi, \beta Q \sin \theta \sin \phi, \frac{\kappa - \beta Q \cos \theta}{\sqrt{2}} \right), \]

\[ p' = p + q = \left( \frac{m + \kappa + \beta Q \cos \theta}{\sqrt{2}}, \beta Q \sin \theta \cos \phi, \beta Q \sin \theta \sin \phi, \frac{m + \kappa - \beta Q \cos \theta}{\sqrt{2}} \right). \]  
(4.11)
2. Angular conditions

We give again only the two conditions after simplification by dividing out as many common factors as possible.

AC 1

\[- \beta^2 Q^2 (\kappa + \beta Q \cos \theta) \sin^2 \theta \ G_a + \sqrt{2 \beta m^2 Q} \sin \theta \ e^{-i\phi} \ G_b + \sqrt{2 \beta Q (m + \kappa + \beta Q \cos \theta)^2} \sin \theta \ e^{i\phi} \ G_c - (\kappa + \beta Q \cos \theta) (2m + \kappa + \beta Q \cos \theta)^2 \ e^{-2i\phi} \ G_e = 0,\]

\[\text{(4.12)}\]

AC 2

\[- \beta^2 Q^2 [\kappa^2 + 4\kappa m + 2m^2 + \beta^2 Q^2 + 2\beta (2m + \kappa) Q \cos \theta] \sin^2 \theta \ G_a + \sqrt{2 \beta Q (2m + \kappa + \beta Q \cos \theta) [\kappa^2 + 2\beta \kappa Q \cos \theta + \beta^2 Q^2 \cos 2\theta]} \sin \theta \ e^{-i\phi} \ G_b + 2\beta^2 Q^2 (m + \kappa + \beta Q \cos \theta)^2 \sin^2 \theta \ G_d + [(\kappa + \beta Q \cos \theta)^2 (2m + \kappa + \beta Q \cos \theta)^2 + \beta^2 Q^2 (\kappa^2 - 2m^2 + 2\beta \kappa Q \cos \theta + \beta^2 Q^2 \cos^2 \theta)] \sin^2 \theta \ e^{-2i\phi} \ G_e = 0.\]

\[\text{(4.13)}\]

We note that Eqs. (4.12) and (4.13) are identical to Eqs. (4.4) and (4.5) if \(\beta \sin \theta = 1\).

V. LIMITING CASES

In order to be able to interprete the angular conditions, we studied the dependence on \(Q\) in the limits \(Q \to 0\) and \(Q \to \infty\). We shall use the notation

\[\text{AC 1} \iff R_1^a G_a + R_1^b G_b + R_1^c G_c + R_1^e G_e = 0,\]

\[\text{AC 2} \iff R_2^a G_a + R_2^b G_b + R_2^d G_d + R_2^e G_e = 0.\]

\[\text{(5.1)}\]

A. \(Q \to 0\) Limit

Using the definition of the physical form factors at \(Q^2 = 0\), \textit{i.e.}

\[eG_C(0) = e, \ eG_M(0) = 2m\mu_1, \ eG_Q(0) = m^2 Q_1,\]

we find from Eq. (3.2)

\[F_1(0) = 1, \ F_2(0) = -\frac{2m\mu_1}{e}, \ F_3(0) = -1 + \frac{2m\mu_1}{e} + \frac{m^2 Q_1}{e}.\]

\[\text{(5.3)}\]

According to the universality condition given by Eq. (1.1), in the limit of bound-state radius \(R \to 0\) the form factors \(F_i(0)\) for \(i = 2, 3\) are reduced to
TABLE I. Leading behavior for $Q \to \infty$ the tensor components $G_a \ldots G_e$ and the coefficients $R^1_\alpha \ldots R^2_\varepsilon$ in the different reference frames considered. The BF and TRF are kinematically connected to the DYW frame only in particular angles $\theta_{BF} = \pi/2$ and $\theta_{TRF} = \theta_0 = \sin^{-1}(1/\beta)$, respectively.

|          | DYW                      | Breit                    | $Q \to \infty$            | TRF                       |
|----------|--------------------------|--------------------------|---------------------------|----------------------------|
| $G_a$    | $1$                      | $Q$                      | $Q$                       | $Q^2$                      | $Q^2$                     | $1$                       |
| $G_b$    | $Q$                      | $Q^2$                    | $Q^2$                     | $0$                        | $Q^4$                     | $0$                       |
| $G_c$    | $Q$                      | $Q^2$                    | $Q^2$                     | $0$                        | $Q^2$                     | $Q$                       |
| $G_d$    | $Q^2$                    | $Q^3$                    | $Q^3$                     | $Q^3$                      | $Q^4$                     | $Q^2$                     |
| $G_e$    | $1$                      | $Q$                      | $Q$                       | $0$                        | $Q^2$                     | $0$                       |
| $R^1_\alpha$ | $0$                     | $Q^3$                    | $0$                       | $0$                        | $Q^6$                     | $0$                       |
| $R^1_\beta$ | $1$                     | $Q^2$                    | $Q^2$                     | $0$                        | $Q^2$                     | $Q$                       |
| $R^1_\gamma$ | $1$                     | $Q^2$                    | $Q^2$                     | $0$                        | $Q^6$                     | $0$                       |
| $R^1_\varepsilon$ | $0$                   | $Q$                      | $Q$                       | $0$                        | $Q^6$                     | $Q^6$                     |
| $R^2_\alpha$ | $Q^2$                   | $Q^4$                    | $Q^4$                     | $0$                        | $Q^8$                     | $0$                       |
| $R^2_\beta$ | $Q$                     | $Q^3$                    | $Q^3$                     | $0$                        | $Q^8$                     | $0$                       |
| $R^2_\gamma$ | $1$                     | $Q^2$                    | $Q^2$                     | $0$                        | $Q^8$                     | $0$                       |
| $R^2_\varepsilon$ | $1$                    | $Q^2$                    | $Q^2$                     | $1$                        | $Q^8$                     | $0$                       |

$F_2(0) = -2$, $F_3(0) = 0$. \hfill (5.4)

Since the target is intact in the $Q \to 0$ limit, $p^\mu = p'^\mu$ and thus one can easily see from Eqs. \eqref{3.19}--\eqref{3.21} that $a_i = d_i$ and $b_i = c_i = e_i = 0$ for all $i = 1, 2, 3$. Thus, we find $G_a = G_d$ or $G_{++}^+ = G_{00}^+$ and all other spin-flip amplitudes vanish in this limit regardless of reference frames. This can be understood because the spin would not flip if the target is intact and also the direction of spin wouldn’t matter in this limit. Moreover, all the coefficients ($R^\alpha_\alpha$, etc.) in Eq. \eqref{5.1} vanish in $Q \to 0$ limit and thus both angular conditions, AC 1 and AC 2, are trivially satisfied.

B. Behavior for $Q \to \infty$

Imposing a naturalness condition, namely all three terms in Eq. \eqref{3.1} should be of the same order in $Q$, one can find that the form factors $F_i(Q^2)$ behave as $F_i(Q^2) \sim F_2(Q^2) \sim \frac{Q^2}{m^2} F_3(Q^2)$ in the large $Q^2$ limit. Using this, we can derive high $Q^2$ behaviors of the helicity amplitudes $G_{h'h}$ and the coefficients ($R^\alpha_\alpha$, etc.) of the angular conditions. In table I, we summarize the results.

As we can see from Table I, the high $Q$ behavior of each helicity amplitude in general depends on the reference frame. This is so because the helicities and the components of the current do mix in general, although the physical form factors are of course identical for any $Q$ regardless of the reference frame. Only in frames connected by a kinematic transformation that keeps the light-front time $\tau = t + z/c (= 0)$ invariant, the helicity amplitudes $G_{h'h}$ are the same \cite{22}. Indeed, our results summarized in Table I are essentially identical in kinematically
connected frames such as DYW, Breit($\theta = \pi/2$) and TRF($\theta = \theta_0$)\textsuperscript{3}. Note that $\theta_0 \to \pi$ in the limit $Q \to \infty$. It is interesting to find that in all cases the behavior of the helicity amplitudes in these frames is consistent with the perturbative QCD predictions obtained in the $q^+ = 0$ frame. Indeed, PQCD predicts \cite{23} that the helicity-zero to helicity zero matrix element $G_{00}^{+}$ (or $G_d$) is the dominant helicity amplitude at large $Q^2$ \cite{5}. For example, in the deuteron form factor \cite{8} calculation using the factorization theorem of PQCD one can show that the five intermediate gluons connecting the six quarks can be arranged in such a way that the gluon polarizations and quark helicities alternate to allow a maximum amplitude when the initial helicity zero state transits to the final helicity zero state. Further, in the $q^+ = 0$ frame, PQCD predicts that the helicity-flip amplitudes $G_{+0}^{+0}(G_c)$ and $G_{+}^{+} (G_e)$ are suppressed by factors of $Q^3$ and $Q^2$, respectively

$$G_c = a \frac{\Lambda_{QCD}}{Q} G_d \quad G_e = b \left( \frac{\Lambda_{QCD}}{Q} \right)^2 G_d$$

(5.5)

where $a$ and $b$ are constants and there are also corrections of order $\Lambda_{QCD}/m$ \cite{5,20}. Our results, based on the naturalness condition, coincide with these PQCD predictions. From the table, we also find that $G_{+}^{+} (G_a)$ should be suppressed by two powers of $Q$ compare to the dominant $G_{00}^{+}$ in the high $Q$ limit. However, neither our analysis nor PQCD can fix the constants $a$ and $b$. Both angular conditions, AC 1 and AC 2, are satisfied independent from $a$ and $b$. Thus, both angular conditions are consistent with the PQCD predictions.

On the other hand, in the frames that are not connected to DYW by a kinematical transformation the results are not consistent with the PQCD predictions as one can see from Table 1. Since there are contributions from embedded states \cite{24} in $q^+ \neq 0$ frames, there are no reasons why they should be consistent with the leading-order PQCD predictions. Nevertheless, it is interesting to note that $G_d$ dominates regardless of the reference frame.

We now discuss some details of AC 1 and AC 2 in each reference frame.

1. Drell-Yan-West Frame

The first angular condition, AC 1, is simple. It reads

$$e^{-i\phi} G_b + e^{i\phi} G_c = 0.$$  

(5.6)

The leading $Q$-behavior of the l.h.s. of AC 1 is

$$\frac{m}{Q} \left( R_b^1 G_b + R_c^1 G_c \right) \overset{Q \to \infty}{\sim} - \frac{p^+}{2\sqrt{2}} \left( 4F_1 + 2F_2 + \frac{m^2}{Q^2} F_3 \right) + \frac{p^+}{2\sqrt{2}} \left( 4F_1 + 2F_2 + \frac{m^2}{Q^2} F_3 \right).$$  

(5.7)

So, if we assume $F_3 \overset{Q \to \infty}{\sim} \frac{Q^2}{m^2} H_3$ and $F_1, F_2$ and $H_3$ are of the same order in $Q^2$ for $Q \to \infty$, then both terms are equal in magnitude.

\textsuperscript{2}The reason for an extra power $Q$ for the Breit($\theta = \pi/2$) and TRF($\theta = \theta_0$) in comparison to DYW can be understood by the kinematic factors in the relation between $(G_C, G_M, G_Q)$ and $G_{h't'h'}$. 

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AC 2 is more involved, but still easy. Its l.h.s. behaves for $Q \to \infty$ to leading order as follows

$$\frac{m^2}{Q^2} (R^2_a G_a + R^2_b G_b + R^2_d G_d + R^2_e G_e)^{Q \to \infty}$$

$$m^2 p^+ \left[ \frac{1}{2} (4F_1 + H_3) - (4F_1 + 2F_2 + H_3) + \frac{1}{2} (4F_1 + 4F_2 + H_3) \right].$$

(5.8)

The term involving $G_e$ does not contribute in leading order.

2. Breit Frame

First AC 1. We multiply with $(m/Q)^4$

$$\frac{m^4}{Q^4} (R^4_a G_a + R^4_b G_b + R^4_c G_c + R^4_e G_e)^{Q \to \infty}$$

$$m^3 \left[ -\frac{1}{4} (4F_1 + H_3) \sin^2 \theta \cos \theta 
- \frac{(4F_1 + 4F_2 + H_3) \cos \theta + \{4F_1 + F_2 (3 + \cos 2\theta) + H_3\}}{8(1 + \cos \theta)^2} \sin^4 \theta 
- \frac{(4F_1 + 4F_2 + H_3) \cos \theta - \{4F_1 + F_2 (3 + \cos \theta) + H_3\}}{8(1 - \cos \theta)^2} \sin^4 \theta \right].$$

(5.9)

Actually, $R^4_e G_e$ is two orders $Q/m$ down compared to the other three terms. The contributions of three terms that remain in leading order will depend on the angle $\theta$. For example, for $\theta = 0$ all vanish identically and we find that then the leading order is lower than $(Q/m)^4$. For $\theta = \pi/2$ only the terms $R^4_b G_b$ and $R^4_c G_c$ survive and cancel each other.

The leading order of AC 2 is $(Q/m)^5$. We find

$$\frac{m^5}{Q^5} (R^2_a G_a + R^2_b G_b + R^2_d G_d + R^2_e G_e)^{Q \to \infty}$$

$$m^3 \left[ -\frac{4F_1 + H_3}{8\sqrt{2}} \sin^4 \theta + \frac{4F_1 + 2F_2 (1 + \cos \theta) + H_3}{4\sqrt{2}(1 - \cos \theta)^2 (1 + \cos \theta)} \sin^6 \theta 
- \frac{(4F_1 + 4F_2 + H_3) (3 - 4 \cos \theta + \cos 2\theta)}{16\sqrt{2}(1 - \cos \theta)^4} \sin^6 \theta \right],$$

(5.10)

and again the term with $G_e$ is not of leading order. For $\theta \to 0$, the first term is of order $\theta^4$ while the two others are of order $\theta^2$ and cancel each other exactly at this order. So, for small $\theta$ the contributions of $G_b$ and $G_d$ dominate. For $\theta = \pi/2$, all three terms are of the same order. This situation corresponds exactly with AC 2 in the DYW frame.

3. Target Rest Frame

Since the leading term in AC 1 is of order $(Q/m)^8$, we multiply it with $(m/Q)^8$ and find
\[
\frac{m^8}{Q^8} (R_a^1 G_a + R_b^1 G_b + R_c^1 G_c + R_e^1 G_e) \xrightarrow{Q \to \infty} \\
\frac{m^4 \sin^2 \theta}{512 \sqrt{2}} \left[ -48 - 64 \cos \theta - 16 \cos 2 \theta \right] F_1 \\
+ \left( -2 + 2 \cos 2 \theta - 2 \cos \theta + 2 \cos \theta \cos 2 \theta \right) H_3 \\
+ \left( 48 + 64 \cos \theta + 16 \cos 2 \theta \right) F_1 \\
+ \left( -12 \cos \theta - 4 \cos \theta \cos 2 \theta - 16 \cos^2 \theta \right) H_3 \\
+ \left( 10 + 15 \cos \theta + 6 \cos 2 \theta + \cos 3 \theta \right) H_3 \right].
\] (5.11)

The contribution from \( G_e \) is not of leading order. The other three terms are comparable in size, but the details depend on the angle \( \theta \).

AC 2 is different, as only \( G_b \) and \( G_d \) contribute in leading order, which is \((Q/m)^{12}\). We find

\[
\frac{m^{12}}{Q^{12}} (R_b^2 G_b + R_d^2 G_d) \xrightarrow{Q \to \infty} \\
\frac{m^5 \sin^2 \theta}{256 \sqrt{2}} \left[ -4F_1 + 4F_2 + H_3 \right] + \left( 4F_1 + 4F_2 + H_3 \right) \\
\times \cos \theta (1 + \cos \theta)(3 + 4 \cos \theta + \cos 2 \theta).
\] (5.12)

VI. CONCLUSIONS

In this work we elaborated the frame dependence of the angular conditions for spin-1 systems. We found that there is an additional angular condition besides the well-known one given by Eq. (1.4). In the \( q^+ = 0 \) frame including DYW, Breit(\( \theta = \pi/2 \)) and TRF(\( \theta = \theta_0 \)), we find that the additional condition is very simple involving only two helicity amplitudes as shown in Eq. (4.4) and most quark models rather easily satisfy it. Thus, it doesn’t seem to provide as strong a constraint as the usual condition Eq. (4.5). However, in \( q^+ \neq 0 \) frames, the additional condition is generally as complicated as the usual one. Since the \( q^+ = 0 \) frame (e.g. DYW) is in principle restricted to the spacelike region of the form factors, it may be useful to impose the additional condition in processes involving the timelike region. Nevertheless, it seems rather clear from our spin-1 form factor discussion that the analysis of exclusive processes is greatly simplified in the DYW frame and in general \( q^+ = 0 \) frames. We note that the angular conditions given by Eqs. (1.4) and (1.5) are identical in any frame connected to the DYW frame by kinematical transformations.

We also find that both angular conditions in the \( q^+ = 0 \) frame are consistent with the PQCD predictions. Our predictions for the \( Q \)-dependence of the helicity amplitudes based on the naturalness condition as well as the angular condition are also consistent with the PQCD predictions given by Eq. (5.3). However, the proportionality constants \( a \) and \( b \) can be fixed neither by our analysis nor by PQCD. Some other inputs such as experimental data are needed to find these values. For example, in the deuteron analysis a value near 5 was obtained for \( a \) [23]. Nevertheless, it is interesting to note that for some particular values of \( a \) and \( b \) the relations among \( F_1, F_2 \) and \( F_3 \) are greatly simplified. For \( a = b = 0 \), we find that
\( F_2 / F_1 = -2 \) and \( F_3 / F_1 = 0 \), which are identical to Eq. (5.4) for a point particle. Since the form factors for a point particle do not depend on \( Q^2 \) at tree level, one can understand this universality result rather easily. Also, for \( a = \sqrt{2} m / \Lambda_{QCD} \) and \( b = 2 m^2 / \Lambda^2_{QCD} \) we find that \( F_2 / F_1 = -1 \) and \( F_3 / F_1 = -1 \). Even though the results are simple for these particular values of \( a \) and \( b \), it is not yet clear what their importance is. In order to analyze the values of \( a \) and \( b \), one may need to have some sort of bound-state information for the spin-1 system. Work along this line, using a simple but exactly solvable mode, is in progress.

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APPENDIX A: LORENTZ TRANSFORMATIONS

Most of the formulas given here can be found in or are based on the paper by Leutwyler and Stern [26](See also a recent literature [27].).

1. Instant Form

If we write \( E = \sqrt{p^2 + m^2} \), then a pure boost from the rest frame to frame where the four momentum is \((E, \vec{p})\) is given by

\[
L^\mu_\nu = \begin{pmatrix}
\frac{E}{m} & \frac{p_x}{m m(E+m)} & \frac{p_y}{m m(E+m)} & \frac{p_z}{m m(E+m)} \\
\frac{p_x}{m m(E+m)} & 1 + \frac{p_x p_y}{m m(E+m)} & \frac{p_y^2}{m m(E+m)} & \frac{p_z p_y}{m m(E+m)} \\
\frac{p_y}{m m(E+m)} & \frac{p_x p_y}{m m(E+m)} & 1 + \frac{p_y^2}{m m(E+m)} & \frac{p_x p_z}{m m(E+m)} \\
\frac{p_z}{m m(E+m)} & \frac{p_x p_z}{m m(E+m)} & \frac{p_y p_z}{m m(E+m)} & 1 + \frac{p_z^2}{m m(E+m)}
\end{pmatrix}.
\]  

(A1)

2. Front Form

In order to facilitate the derivation we define the connection between the usual four-vector components \( p^\mu = (p^0, p^1, p^2, p^3) \) and the front-form components \( p^\mu_{ff} = (p^+, p^1, p^2, p^-) \) with the definition

\[
p^\pm = \frac{p^0 \pm p^3}{\sqrt{2}},
\]

(A2)

which can be written with the help of the matrix
Using $\eta$ we can write the relations Eq. (A2) as

$$p_{\mu}^\prime = \eta_{\nu}^\prime p^\nu.$$  

(A4)

The matrix $\eta$ has the nice property that it is idempotent. We can use it to define the components of any tensor. As an example we transform the metric tensor $g$ to $g_{\text{ff}}$

$$g_{\text{ff}} = \eta g \eta = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$  

(A5)

We could write the pure boost in front-form coordinates, if we wanted to, but we don’t, because we want to use the kinematical front-form boost, which we write next.

The kinematical front-form boost is given by

$$L_{\text{ff}}(\vec{v}_\perp; \chi) = \exp(-i\sqrt{2} \vec{v}_\perp \cdot \vec{E}_\perp) \exp(-i\chi K^3),$$  

(A6)

where $K^3 = M^{-+}$ is the third component of the boost generator and the generators $E^1$ and $E^2$ are given by

$$E^1 = M^{++} = \frac{1}{\sqrt{2}} (K^1 + J^2), \quad E^2 = M^{+-} = \frac{1}{\sqrt{2}} (K^2 - J^1).$$  

(A7)

By taking specific values for the transverse velocity $\vec{v}_\perp$ and the hyperbolic angle $\chi$ we obtain the front-form boost from the rest system where the momentum has components $p^\mu = (m/\sqrt{2}, 0, 0, m/\sqrt{2})$ to the frame where it has components $(p^+, p^1, p^2, p^-)$. We must account for the fact that the dispersion relation in the front form is

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{2p^+},$$  

(A8)

where we introduced the obvious notation

$$\vec{p}_\perp = (p^1, p^2).$$  

(A9)

The connection we need is

$$e^{\chi} = \sqrt{2} p^+/m, \quad \vec{v}_\perp = \vec{p}_\perp/\sqrt{2} p^+.$$  

(A10)

The generators $E^1$ and $E^2$ are nilpotent and $K^3$ has also a simple form, viz

$$K_{\text{ff}}^3 = i \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad E^1_{\text{ff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad E^2_{\text{ff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$  

(A11)
Thus, the explicit form for the boost is not difficult to determine. It is

\[
L_{\text{ff}}(\vec{v}_\perp; \chi) = \begin{pmatrix}
\sqrt{2p^+}/m & 0 & 0 & 0 \\
\sqrt{2p^1}/m & 1 & 0 & 0 \\
\sqrt{2p^2}/m & 0 & 1 & 0 \\
\vec{p}_\perp^2/(m\sqrt{2p^+}) & p^1/p^+ & p^2/p^+ & m/(\sqrt{2p^+})
\end{pmatrix}.
\] (A12)

Indeed, if we act with \( L_{\text{ff}} \) on \((m/\sqrt{2}, 0, 0, m/\sqrt{2})\) we find for \( p^\mu \): \((p^+, p^1, p^2, p^-)\) with \( p^- \) given by Eq. (A8). To this simple check we can add the defining property of the Lorentz transformations \( L \), \textit{i.e.}

\[
g^{\mu\nu} = L^\nu_\alpha L^\mu_\beta g_{\alpha\beta},
\] (A13)

which can be translated into matrix form as follows

\[
g = L^\dagger g L.
\] (A14)

This relation can be interpreted as a quasi-orthogonality condition on the rows of the transformation symbol \( L \). Needless to say that the orthogonality condition must be implemented with the right form of the metric \( g \).

3. Helicity

The operator

\[
L_{\text{ff}}(0; \chi) = \exp(-i\chi K^3) = \begin{pmatrix}
\sqrt{2p^+}/m & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & m/(\sqrt{2p^+})
\end{pmatrix}
\] (A15)

commutes with \( J^3 \) because \( K^3 \) does. Therefore, the polarization vectors

\[
L_{\text{ff}}(0; \chi) \varepsilon(h)
\] (A16)

are eigenvectors of \( J^3 \). If we next apply the front-form combinations of rotations and boosts \( \exp(-i\sqrt{2} \vec{v}_\perp \cdot \vec{E}_\perp) \) to move the vector \((p^+, 0, 0, m^2/(2p^+))\) to \((p^+, p_x, p_y, (p^1^2 + m^2)/(2p^+))\) and use the full LF boost Eq. (A12) to obtain the boosted polarization vectors, we see that we can use the operator

\[
h_{\text{ff}} = \exp(-i\sqrt{2} \vec{v}_\perp \cdot \vec{E}_\perp) J^3 \exp(i\sqrt{2} \vec{v}_\perp \cdot \vec{E}_\perp).
\] (A17)

This operator, which we call the \textit{LF helicity}, has the eigenvectors \( \varepsilon_{\text{ff}}(h) \) with \( h = 0, \pm 1 \), Eq. (2.14), and a fourth eigenvector \((0, 0, 0, 1)\). The latter does not correspond to a genuine polarization vector. It has only a minus component, which means that it is orthogonal to all four vectors with \( p^- = 0 \), \textit{i.e.} \( p^+ \to \infty \).

The explicit form of \( h_{\text{ff}} \) is
\[ h_{\text{ff}} = i \begin{pmatrix} 0 & 0 & 0 & 0 \\ p^2/p^+ & 0 & -1 & 0 \\ -p^1/p^+ & 1 & 0 & 0 \\ 0 & p^2/p^+ & -p^1/p^+ & 0 \end{pmatrix}. \]  \tag{A18}

One can write it in operator form as
\[ h_{\text{ff}} = \frac{W^+}{P^+} = J^3 - \frac{P^1 E^2 - P^2 E^1}{P^+}. \]  \tag{A19}

This operator is clearly a kinematic one, as \( J^3, P^1, P^2, E^1, \) and \( E^2 \) all belong to the stability group of \( x^+ = 0 \).

**APPENDIX B: SYMMETRIES OF FRAMES AND RELATIONS BETWEEN DIFFERENT FRAMES**

In this section we give the kinematical Lorentz transformations that connect the different frames in specific cases. We stress that the frames can be transformed into each other by general Lorentz transformation, but only in special cases can this be done using elements from the kinematical subgroup alone.

The kinematical group is generated by \( J^3, K^3 \) and \( E^1 \) and \( E^2 \). As all frames are invariant under rotations about the \( z \)-axis, we shall not discuss \( J^3 \). We can use this kinematical rotation to remove the \( \phi \)-dependence of the angular conditions. The interesting transformations are \( L_{\text{ff}}(0; \chi) \) and \( L_{\text{ff}}(\vec{v}_\perp; 0) \).

### 1. Symmetries of frames

**a. Boosts along the \( z \)-axis**

\( L_{\text{ff}}(0; \chi) \) is a symmetry of the Drell-Yan-West frame, but not of the Breit frame or target rest frame.

**b. Transverse Boosts**

We write \( L_{\text{ff}}(\vec{v}_\perp; 0) \) explicitly
\[ L_{\text{ff}}(\vec{v}_\perp; 0) = \begin{pmatrix} 1 & 0 & 0 \\ \sqrt{2} \vec{v}_\perp & 1 & 0 \\ \vec{v}_\perp^2 & \sqrt{2} \vec{v}_\perp & 1 \end{pmatrix}. \]  \tag{B1}

The transverse boosts are not symmetries of the target rest frame. If we apply it to the DYW momentum transfer we find
\[ L_{\text{ff}}(\vec{v}_\perp; 0) q_{\text{DYW}} = \left( 0, Q \hat{n}, \frac{Q^2}{2p^+} + \sqrt{2} Q \hat{n} \cdot \vec{v}_\perp \right). \]  \tag{B2}
If one generalizes the definition of the DYW frame to $q^+ = 0$, then this transformation is a symmetry of this frame, if one allows for a perpendicular momentum in the initial state

$$\vec{p}_\perp = \sqrt{2} p^+ \vec{v}_\perp,$$

(B3)

otherwise, insisting on $\vec{p}_\perp = 0$ in the DYW frame, it is not.

In the Breit frame we find for the transformed momentum transfer

$$L_{\perp}(\vec{v}_\perp; 0) q_{\text{Breit}} = (Q \cos \theta / \sqrt{2}, Q(\sin \theta \hat{n} + \cos \theta \vec{v}_\perp), Q(- \cos \theta + 2 \sin \theta \hat{n} \cdot \vec{v}_\perp + \cos \theta \vec{v}_\perp^2)).$$

(B4)

If we require this vector to have the form

$$q_{\text{Breit}}' = (Q \cos \theta / \sqrt{2}, Q \sin \theta \hat{n}', -Q \cos \theta / \sqrt{2}),$$

(B5)

then we must find a vector $\vec{v}_\perp$ that satisfies

$$(\sin \theta \hat{n} + \cos \theta \vec{v}_\perp)^2 = \sin^2 \theta.$$

(B6)

There are two classes of solutions: either $\cos \theta = 0$ and $\hat{n} \cdot \vec{v}_\perp = 0$, or $\cos \theta \vec{v}_\perp^2 + 2 \sin \theta \hat{n} \cdot \vec{v}_\perp = 0$. In the latter case the length of the velocity vector is correlated with its direction through the relation

$$v = -2 \tan \theta \hat{n} \cdot \vec{v}_\perp.$$

(B7)

If we denote the azimuthal angles of $\hat{n}$ and $\vec{v}_\perp$ by $\phi$ and $\psi$ respectively then the vector $\hat{n}'$ in Eq. (B3) is given by

$$\hat{n}' = (- \cos(2\psi - \phi), - \sin(2\psi - \phi)).$$

(B8)

We conclude that there is a class of transverse boosts that leaves the Breit frame invariant.

2. Relations between different frames

If we want two reference frames to be connected by a Lorentz transformation, we need to verify that both the initial momenta ($p$) and the momentum transfers ($q$) are related by the same transformation.

In the case of TRF and DYW the two are identical if $p^+ = m/\sqrt{2}$ and in addition $\beta \sin \theta = 1$. The corresponding angle we denote by $\theta_0$. The latter condition ensures that the momentum transfer in the TRF has vanishing plus-component. Clearly, for every value of $Q$ there is an angle, $\theta_0$, for which the TRF and the DYW are kinematically connected.

If we try the same for the TRF and the Breit frame, we find that they are kinematically related for all $Q$ at $\theta = 0$.

The DYW and the Breit frame can only be related for $\theta = \pi/2$. Then the momentum transfer in the Breit frame has the form

$$q_{\text{Breit}} = (0, Q \hat{n}, 0).$$

(B9)
We now try to find the transformation that transforms the momentum transfer in the DYW frame into this special vector. If we write \( \vec{v} = \hat{v} \vec{v} \), then we find the parameters

\[
\hat{v} = -\hat{n}, \quad v = \frac{Q}{2m\beta}, \quad e^x = \frac{m\beta}{\sqrt{2p^+}}. \tag{B10}
\]

We see that for any value of \( Q \) we can connect the DYW frame to the Breit frame with \( \theta = \pi/2 \).

The main conclusion from this exercise is that the three frames considered here are only in special cases related by kinematical Lorentz transformations. In these cases the angular conditions are the same. In all other cases we find non-equivalent angular conditions.

**APPENDIX C: POLARIZATION TENSORS**

1. **DYW**

\( G^+(1) \)

\[
a_1 = 2p^+, \quad b_1 = -\frac{\sqrt{2}p^+Q}{m} e^{i\phi}, \quad c_1 = \frac{\sqrt{2}p^+Q}{m} e^{-i\phi}, \quad d_1 = p^+ \left( 2 - \frac{Q^2}{m^2} \right), \quad e_1 = 0, \tag{C1}
\]

\( G^+(2) \)

\[
a_2 = 0, \quad b_2 = -\frac{p^+Q}{\sqrt{2m}} e^{i\phi}, \quad c_2 = \frac{p^+Q}{\sqrt{2m}} e^{-i\phi}, \quad d_2 = -\frac{p^+Q^2}{m^2}, \quad e_2 = 0, \tag{C2}
\]

\( G^+(3) \)

\[
a_3 = \frac{p^+Q^2}{2m^2}, \quad b_3 = -\frac{p^+Q^3}{2\sqrt{2m^3}} e^{i\phi}, \quad c_3 = \frac{p^+Q^3}{2\sqrt{2m^3}} e^{-i\phi},
\]

\[
d_3 = -\frac{p^+Q^4}{4m^4}, \quad e_3 = -\frac{p^+Q^2}{2m^2} e^{2i\phi}. \tag{C3}
\]

2. **Breit Frame**

\( G^+(1) \)

\[
a_1 = \sqrt{2m\beta}, \quad b_1 = \frac{2m\beta^2Q \sin \theta}{2m\beta - Q \cos \theta} e^{i\phi}, \quad c_1 = \frac{2m\beta^2Q \sin \theta}{2m\beta + Q \cos \theta} e^{-i\phi}, \quad d_1 = \frac{\sqrt{2}m[Q^2 \cos^2 \theta + 2\beta^2(2m^2 - Q^2 \sin^2 \theta)]}{4\beta^2m^2 - Q^2 \cos^2 \theta}, \quad e_1 = 0. \tag{C4}
\]
\( G^+ (2) \)

\[
a_2 = 0, \\
b_2 = -\frac{\beta Q (2m\beta + Q \cos \theta) \sin \theta}{2(2m\beta - Q \cos \theta)} e^{i\phi}, \\
c_2 = \frac{\beta Q (2m\beta - Q \cos \theta) \sin \theta}{2(2m\beta + Q \cos \theta)} e^{-i\phi}, \\
d_2 = \frac{\sqrt{2\beta m^2 Q^2} [1 - \beta^2 + (1 + \beta^2) \cos 2\theta]}{4\beta^2 m^2 - Q^2 \cos^2 \theta}, \\
e_2 = 0. \quad (C5)
\]

\( G^+ (3) \)

\[
a_3 = \frac{\sqrt{2m\beta^3 Q^2 \sin^2 \theta}}{4\beta^2 m^2 - Q^2 \cos^2 \theta}, \\
b_3 = -\frac{2m\beta^3 Q^2 (2m\cos \theta + \beta Q \sin^2 \theta) \sin \theta}{(2m\beta - Q \cos \theta)(2m\beta + Q \cos \theta)} e^{i\phi}, \\
c_3 = -\frac{2m\beta^3 Q^2 (2m\cos \theta - \beta Q \sin^2 \theta) \sin \theta}{(2m\beta + Q \cos \theta)(2m\beta - Q \cos \theta)} e^{-i\phi}, \\
d_3 = \frac{2\sqrt{2m\beta^3 Q^2 (4m^2 \cos^2 \theta - \beta^2 Q^2 \sin^4 \theta)}}{(4\beta^2 m^2 - Q^2 \cos^2 \theta)^2}, \\
e_3 = -\frac{\sqrt{2m\beta^3 Q^2 \sin^2 \theta}}{4\beta^2 m^2 - Q^2 \cos^2 \theta} e^{2i\phi}, \quad (C6)
\]

3. TRF

\( G(1) \)

\[
a_1 = \frac{2m + \kappa + \beta Q \cos \theta}{\sqrt{2}}, \\
b_1 = -\frac{\beta Q (2m + \kappa + \beta Q \cos \theta) \sin \theta}{2m} e^{i\phi}, \\
c_1 = \frac{\beta Q (2m + \kappa + \beta Q \cos \theta) \sin \theta}{2(m + \kappa + \beta Q \cos \theta)} e^{-i\phi}, \\
d_1 = \frac{(2m + \kappa + \beta Q \cos \theta)[2m^2 + 2m\kappa + \kappa^2 + 2\beta(m + \kappa)Q \cos \theta + \beta^2 Q^2 \cos 2\theta]}{2\sqrt{2m(m + \kappa + \beta Q \cos \theta)}}, \\
e_1 = 0. \quad (C7)
\]

\( G(2) \)

\[
a_2 = 0, \\
b_2 = -\frac{\beta Q (m + \kappa + \beta Q \cos \theta) \sin \theta}{2m} e^{i\phi}, \\
\]

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\[ c_2 = \frac{\beta m Q \sin \theta}{2(m + \kappa + \beta Q \cos \theta)} e^{-i\phi}, \]
\[ d_2 = \frac{(2m + \kappa + \beta Q \cos \theta)(\kappa^2 + 2\beta \kappa Q \cos \theta + \beta^2 Q^2 \cos 2\theta)}{2\sqrt{2m(m + \kappa + \beta Q \cos \theta)}}, \]
\[ e_2 = 0. \quad (C8) \]

\[ G(3) \]
\[ a_3 = \frac{\beta^2 Q^2(2m + \kappa + \beta Q \cos \theta) \sin^2 \theta}{4\sqrt{2m(m + \kappa + \beta Q \cos \theta)}}, \]
\[ b_3 = -\frac{\beta Q(2m + \kappa + \beta Q \cos \theta)\[\kappa^2 + 2m \kappa + \beta^2 Q^2 + 2\beta(m + \kappa)Q \cos \theta\] \sin \theta}{8m^2(m + \kappa + \beta Q \cos \theta)} e^{i\phi}, \]
\[ c_3 = -\frac{\beta Q(2m + \kappa + \beta Q \cos \theta)\[\kappa(2m + \kappa) + 2\beta(m + \kappa)Q \cos \theta + \beta^2 Q^2 \cos 2\theta\] \sin \theta}{8m(m + \kappa + \beta Q \cos \theta)^2} e^{-i\phi}, \]
\[ d_3 = \frac{(2m + \kappa + \beta Q \cos \theta)}{8\sqrt{2m^2(m + \kappa + \beta Q \cos \theta)^2} \[\kappa^2 + 2m \kappa + \beta^2 Q^2 + 2\beta(m + \kappa)Q \cos \theta\] \times \[\kappa(2m + \kappa) + 2\beta(m + \kappa)Q \cos \theta + \beta^2 Q^2 \cos 2\theta\]}, \]
\[ e_3 = -\frac{\beta^2 Q^2(2m + \kappa + \beta Q \cos \theta) \sin^2 \theta}{4\sqrt{2m(m + \kappa + \beta Q \cos \theta)}} e^{2i\phi}. \quad (C9) \]
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