Abstract

Score-based generative models can produce high quality image samples comparable to GANs, without requiring adversarial optimization. However, existing training procedures are limited to images of low resolution (typically below \(32 \times 32\)), and can be unstable under some settings. We provide a new theoretical analysis of learning and sampling from score models in high dimensional spaces, explaining existing failure modes and motivating new solutions that generalize across datasets. To enhance stability, we also propose to maintain an exponential moving average of model weights. With these improvements, we can effortlessly scale score-based generative models to images with unprecedented resolutions ranging from \(64 \times 64\) to \(256 \times 256\). Our score-based models can generate high-fidelity samples that rival best-in-class GANs on various image datasets, including CelebA, FFHQ, and multiple LSUN categories.

1 Introduction

Score-based generative models [1] represent probability distributions through score functions—a vector field pointing in the direction where the likelihood of data increases most rapidly. Remarkably, these score functions can be learned from data without requiring adversarial optimization, and can produce realistic image samples that rival GANs on simple datasets such as CIFAR-10 [2].

Despite this success, existing score-based generative models only work on low resolution images (\(32 \times 32\)) due to several limiting factors. First, the score function is learned via denoising score matching [3, 4, 5]. Intuitively, this means a neural network (named the score network) is trained to denoise images blurred with Gaussian noise. A key insight from [1] is to perturb the data using multiple noise levels so that the score network captures both coarse and fine-grained image features. However, it is an open question how these noise levels should be chosen. The recommended settings in [1] work well for \(32 \times 32\) images, but perform poorly when the resolution gets higher. Second, samples are generated by running Langevin dynamics [6, 7]. This method starts from white noise and progressively denoises it into an image using the score network. This procedure, however, might fail or take an extremely long time to converge when used in high-dimensions and with a necessarily imperfect (learned) score network.

We propose a set of techniques to scale score-based generative models to high resolution images. Based on a new theoretical analysis on a simplified mixture model, we provide a method to analytically compute an effective set of Gaussian noise levels from training data. Additionally, we propose an efficient architecture to amortize the score estimation task across a large (possibly infinite) number of noise levels with a single neural network. Based on a simplified analysis of the convergence properties of the underlying Langevin dynamics sampling procedure, we also derive a technique to approximately optimize its performance as a function of the noise levels. Combining these techniques with an exponential moving average (EMA) of model parameters, we are able to significantly improve...
the sample quality, and successfully scale to images of unprecedented resolutions ranging from 64 × 64 to 256 × 256. As illustrated in Fig. 1, the samples are sharp and diverse.

2 Background

2.1 Langevin dynamics

For any continuously differentiable probability density \( p(x) \), we call \( \nabla_x \log p(x) \) its score function. In many situations the score function is easier to model and estimate than the original density function \([3, 8]\). For example, for an unnormalized density it does not depend on the partition function. Once the score function is known, we can employ Langevin dynamics to sample from the corresponding distribution. Given a step size \( \alpha > 0 \), a total number of iterations \( T \), and an initial sample \( x_0 \) from any prior distribution \( \pi(x) \), Langevin dynamics iteratively evaluate the following

\[
x_t \leftarrow x_{t-1} + \alpha \nabla_x \log p(x_{t-1}) + \sqrt{2\alpha} \, z_t, \quad 1 \leq t \leq T
\]

where \( \forall t : z_t \sim \mathcal{N}(0, I) \). When \( \alpha \) is sufficiently small and \( T \) is sufficiently large, the distribution of \( X_T \) will be close to \( p(x) \) under some regularity conditions \([6, 7]\). Suppose we have a neural network \( s_\theta(x) \) (called the score network) parameterized by \( \theta \), and it has been trained such that \( s_\theta(x) \approx \nabla_x \log p(x) \). We can approximately generate samples from \( p(x) \) using Langevin dynamics by replacing \( \nabla_x \log p(x_{t-1}) \) with \( s_\theta(x_{t-1}) \) in Eq. (1). Note that Eq. (1) can be interpreted as noisy gradient ascent on the log-density \( \log p(x) \).

2.2 Score-based generative modeling

We can estimate the score function from data and generate new samples with Langevin dynamics. This idea was named score-based generative modeling by Song and Ermon [1]. Because the estimated score function is inaccurate in regions without training data, Langevin dynamics may not converge correctly when a sampling trajectory encounters those regions (see more detailed analysis in [1]). As a remedy, Song and Ermon [1] propose to perturb the data with Gaussian noise of different intensities and jointly estimate the scores of all noise-perturbed data distributions. During inference, they combine the information from all noise levels by sampling from each noise-perturbed distribution sequentially with Langevin dynamics.

More specifically, suppose we have an underlying data distribution \( p_{\text{data}}(x) \) and consider a sequence of noise levels \( \{\sigma_i\}_{i=1}^L \) that satisfies \( \sigma_1 > \sigma_2 > \cdots > \sigma_L \). Let \( p_\sigma(\bar{x} \mid x) = \mathcal{N}(\bar{x} \mid x, \sigma^2 I) \), and denote the corresponding perturbed data distribution as \( p_\sigma(x) \triangleq \int p_\sigma(\bar{x} \mid x)p_{\text{data}}(x)dx \). Song and Ermon [1] propose to estimate the score function of each \( p_\sigma(x) \) by training a single neural network

Figure 1: Generated samples on datasets of decreasing resolutions. From left to right: FFHQ 256\(^2\), LSUN bedroom 128\(^2\), LSUN tower 128\(^2\), LSUN church_outdoor 96\(^2\), and CelebA 64\(^2\).
When the range of pixel values is L with less variation under small noise. The initial noise level ϵ (iii) the step size parameter α = ϵ · σ^2 / σ_L^2 for the i-th noise level. Samples from each noise level are used to initialize Langevin dynamics for the next noise level until reaching the smallest one, where it provides final samples for the NCSN.

There are many design choices that are critical to the successful training and inference of NCSNs, including (i) the set of noise levels {σ_i}^L_{i=1}, (ii) the way that s_θ(x, σ) incorporates information of σ, (iii) the step size parameter ϵ and (iv) the number of sampling steps per noise level L in Algorithm 1. Below we provide theoretically motivated ways to configure them without manual tuning, which significantly improve the performance of NCSNs on high resolution images.

3 Choosing noise levels

Noise levels are critical for the success of NCSNs. As shown in [1], score networks trained with a single noise can never produce convincing samples for large images. Intuitively, high noise facilitates the estimation of score functions, but also leads to corrupted samples; while lower noise gives clean samples but makes score functions harder to estimate. One should therefore leverage different noise levels together to get the best of both worlds.

When the range of pixel values is [0, 1], the original work on NCSN [1] recommends choosing {σ_i}^L_{i=1} as a geometric sequence where L = 10, σ_1 = 1, and σ_L = 0.01. It is reasonable that the smallest noise level σ_L = 0.01 ≪ 1, because we sample from perturbed distributions with descending noise levels and we want to add low noise at the end. However, some important questions remain unanswered, which turn out to be critical to the success of NCSNs on high resolution images: (i) Is σ_1 = 1 appropriate? If not, how should we adjust σ_1 for different datasets? (ii) Is geometric progression a good choice? (iii) Is L = 10 good across different datasets? If not, how many noise levels are ideal?

Below we provide answers to the above questions, motivated by theoretical analyses on simple mathematical models. Our insights are effective for configuring score-based generative modeling in practice, as corroborated by experimental results in Section 4.3.1 Initial noise level

The algorithm of annealed Langevin dynamics (Algorithm 1) is a refining procedure that starts from generating coarse samples with rich variation under large noise, before converging to fine samples with less variation under small noise. The initial noise level σ_1 largely controls the diversity of the final samples. In order to promote sample diversity, we might want to choose σ_1 to be as large as possible. However, an excessively large σ_1 will require more noise levels (to be discussed in Section 3.2) and make annealed Langevin dynamics more expensive. Below we present an analysis to guide the choice of σ_1 and provide a technique to strike the right balance.

Real-world data distributions are complex and hard to analyze, so we approximate them with empirical distributions. Suppose we have a dataset {x^{(1)}, x^{(2)}, \cdots, x^{(N)}} which is i.i.d. sampled...
from \( p_{\text{data}}(x) \). Assuming \( N \) is sufficiently large, we have \( p_{\text{data}}(x) \approx \hat{p}_{\text{data}}(x) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x = x^{(i)}) \), where \( \delta(\cdot) \) denotes a point mass distribution. When perturbed with \( \mathcal{N}(0, \sigma_i^2 I) \), the empirical distribution becomes \( \hat{p}_{\sigma_i}(x) \approx \frac{1}{N} \sum_{i=1}^{N} \mu_i(x) \), where \( \mu_i(x) \approx \mathcal{N}(x \mid x^{(i)}, \sigma_i^2 I) \). For generating diverse samples regardless of initializations, we naturally expect that Langevin dynamics can explore any component \( p^{(i)}(x) \) when initialized from any other component \( p^{(j)}(x) \), where \( i \neq j \). The performance of Langevin dynamics is governed by the score function \( \nabla_x \log \hat{p}_{\sigma_i}(x) \) (see Eq. (1)).

**Proposition 1.** Let \( \hat{p}_{\sigma_i}(x) \approx \frac{1}{N} \sum_{i=1}^{N} \mu_i(x) \), where \( \mu_i(x) \approx \mathcal{N}(x \mid x^{(i)}, \sigma_i^2 I) \). With \( r^{(i)}(x) \equiv \frac{\sum_{i=1}^{N} \mu_i(x)}{\sum_{i=1}^{N} \mu_i(x)} \), the score function is \( \nabla_x \log \hat{p}_{\sigma_i}(x) = \sum_{i=1}^{N} r^{(i)}(x) \nabla_x \log p^{(i)}(x) \). Moreover,

\[
E_{p^{(i)}(x)}[r^{(j)}(x)] \leq \frac{1}{2} \exp \left( -\frac{\|x^{(i)} - x^{(j)}\|_2^2}{8\sigma_j^2} \right).
\]

In order for Langevin dynamics to transition from \( p^{(i)}(x) \) to \( p^{(j)}(x) \) easily for \( i \neq j \), \( E_{p^{(i)}(x)}[r^{(j)}(x)] \) has to be relatively large, because otherwise \( \nabla_x \log \hat{p}_{\sigma_i}(x) = \sum_{k=1}^{N} r^{(k)}(x) \nabla_x \log p^{(k)}(x) \) will ignore the component \( p^{(j)}(x) \) (on average) when initialized with \( x \sim p^{(i)}(x) \) and in such case Langevin dynamics will act as if \( p^{(j)}(x) \) did not exist. The bound of Eq. (3) indicates that \( E_{p^{(i)}(x)}[r^{(j)}(x)] \) can decay exponentially fast if \( \sigma_i \) is small compared to \( \|x^{(i)} - x^{(j)}\|_2 \). As a result, it is necessary for \( \sigma_i \) to be numerically comparable to the maximum pairwise distances of data to facilitate transitioning of Langevin dynamics and hence improving sample diversity. In particular, we suggest:

**Technique 1 (Initial noise level).** Choose \( \sigma_1 \) to be as large as the maximum Euclidean distance between all pairs of training data points.

Taking CIFAR-10 as an example, the median pairwise distance between all training images is around 18, so \( \sigma_1 = 1 \) as in [1] implies \( E[r(x)] < 10^{-17} \) and is unlikely to produce diverse samples as per our analysis. To test whether choosing \( \sigma_1 \) according to Technique [1] (i.e., \( \sigma_1 = 50 \)) gives significantly more diverse samples than using \( \sigma_1 = 1 \), we run annealed Langevin dynamics to sample from a mixture of Gaussian with 10000 components, where each component is centered at one CIFAR-10 test image. All initial samples are drawn from a uniform distribution over \([0, 1]^{32 \times 32 \times 3}\). This setting allows us to avoid confounders introduced by NCSN training because we use ground truth scores. As shown in Fig. 2b, samples in Fig. 2c using Technique [1] exhibit comparable diversity to ground-truth images (Fig. 2a), and have better variety than Fig. 2b (\( \sigma_1 = 1 \)). Quantitatively, the average pairwise distance of samples in Fig. 2c is 18.65, comparable to data (17.78) but much higher than that of Fig. 2b (10.12).

### 3.2 Other noise levels

After setting \( \sigma_L \) and \( \sigma_1 \), we need to choose the number of noise levels \( L \) and specify the other elements of \( \{\sigma_i\}_{i=1}^{L} \). As analyzed in [1], it is crucial for the success of score-based generative models to ensure that \( p_{\sigma_i}(x) \) generates a sufficient number of training data in high density regions of \( p_{\sigma_{i-1}}(x) \) for all \( 1 < i \leq L \). The intuition is we need reliable gradient signals for \( p_{\sigma_i}(x) \) when initializing Langevin dynamics from \( p_{\sigma_{i-1}}(x) \).

However, an extensive grid search on \( \{\sigma_i\}_{i=1}^{L} \) can be very expensive. To give some theoretical guidance on finding good noise levels, we consider a simple case where the dataset contains only one data point, or equivalently, \( \forall 1 \leq i \leq L : p_{\sigma_i}(x) = \mathcal{N}(x \mid 0, \sigma_i^2 I) \). Our first step is to understand the distributions of \( p_{\sigma_i}(x) \) better, especially when \( x \) has high dimensionality. We can decompose \( p_{\sigma_i}(x) \) in hyperspherical coordinates to \( p(\phi)p_{\sigma_i}(r) \), where \( r \) and \( \phi \) denote the radial and angular coordinates of \( x \) respectively. Because \( p_{\sigma_i}(x) \) is an isotropic Gaussian, the angular component \( p(\phi) \) is uniform and shared across all noise levels. As for \( p_{\sigma_i}(r) \), we have the following
Proposition 2. Let $x \in \mathbb{R}^D \sim \mathcal{N}(0, \sigma^2 I)$, and $r = \|x\|_2$. We have
\[
p(r) = \frac{1}{2^{D/2} \sqrt{\pi D/2}} \frac{r^{D-1}}{\sigma^D} \exp \left( -\frac{r^2}{2\sigma^2} \right) \quad \text{and} \quad r - \sqrt{D}\sigma \xrightarrow{d} \mathcal{N}(0, \sigma^2/2) \text{ when } D \to \infty.
\]

In practice, dimensions of image data can range from several thousand to millions, and are typically large enough to warrant $p(r) \approx \mathcal{N}(r/\sqrt{D}\sigma, \sigma^2/2)$ with negligible error. We therefore take $p_{\sigma_i}(r) = \mathcal{N}(r|\mu_i, \sigma_i^2)$ to simplify our analysis, where $\mu_i \equiv \sqrt{D}\sigma$, and $\sigma_i^2 \equiv \sigma^2/2$.

Recall that our goal is to make sure samples from $p_{\sigma_i}(x)$ will cover high density regions of $p_{\sigma_{i-1}}(x)$. Because $p(\phi)$ is shared across all noise levels, $p_{\sigma_i}(x)$ already covers the angular component of $p_{\sigma_{i-1}}(x)$. Therefore, we need the radial components of $p_{\sigma_i}(x)$ and $p_{\sigma_{i-1}}(x)$ to have large overlap.

Since $p_{\sigma_{i-1}}(r)$ has high density in $I_{i-1} \triangleq [m_i-1-3s_i-1, m_i-1+3s_i-1]$ (employing the “three-sigma rule of thumb” [9]), a natural choice is to fix $p_{\sigma_i}(r \in I_{i-1}) = \Phi(\sqrt{2D}(\gamma_i - 1) + 3\gamma_i) - \Phi(\sqrt{2D}(\gamma_i - 1) - 3\gamma_i) = C$ with some moderately large constant $C > 0$ for all $1 < i \leq L$, where $\gamma_i \equiv \sigma_{i-1}/\sigma$, and $\Phi(\cdot)$ is the CDF of standard Gaussian. This choice immediately implies that $\gamma_2 = \gamma_3 = \cdots = \gamma_L$ and thus $\{\gamma_i\}_{i=1}^L$ is a geometric progression.

Ideally, we should choose as many noise levels as possible to make $C \approx 1$. However, having too many noise levels will make sampling very costly, as we need to run Langevin dynamics for each noise level in sequence. On the other hand, $L = 10$ (for $32 \times 32$ images) as in the original setting of [1] is arguably too small, for which $C = 0$ up to numerical precision. To strike a balance, we recommend $C \approx 0.5$ which performs well in our experiments. In summary,

Technique 2 (Other noise levels). Choose $\{\gamma_i\}_{i=1}^L$ as a geometric progression with common ratio $\gamma$, such that $\Phi(\sqrt{2D}(\gamma - 1) + 3\gamma) - \Phi(\sqrt{2D}(\gamma - 1) - 3\gamma) \approx 0.5$.

3.3 Incorporating the noise information

For high resolution images, we need a large $\sigma_1$ and a huge number of noise levels as per Technique 1 and 2. Recall that the NCSN is a single amortized network that takes a noise level and gives the corresponding score. In [1], Song and Ermon use a separate set of scale and bias parameters in normalization layers to incorporate the information from each noise level. However, its memory consumption grows linearly w.r.t. $L$, and it is not applicable when the NCSN has no normalization layers.

We propose an efficient alternative that is easier to implement and more widely applicable. For $p_{\sigma}(x) = \mathcal{N}(x \mid 0, \sigma^2 I)$ analyzed in Section 3.2, we observe that $\mathbb{E}[\|\nabla_x \log p_{\sigma}(x)\|_2] \approx \sqrt{D}/\sigma$. Moreover, as empirically noted in [1], $\|s_{\theta}(x, \sigma)\|_2 \propto 1/\sigma$ for a trained NCSN on real data. Because the norm of score functions scales inverse proportionally to $\sigma$, we can incorporate the noise information by rescaling the output of an unconditional score network $s_{\theta}(x)$ with $1/\sigma$. This motivates our following recommendation

Technique 3 (Noise conditioning). Parameterize the NCSN with $s_{\theta}(x, \sigma) = s_{\theta}(x)/\sigma$, where $s_{\theta}(x)$ is an unconditional score network.

It is typically hard for deep networks to automatically learn this rescaling, because $\sigma_1$ and $\sigma_L$ can differ by several orders of magnitude. This simple choice is easier to implement, and can effortlessly handle a large number of noise levels (even continuous ones). As shown in Fig. 3 (detailed settings in Appendix B), it achieves similar training losses compared to the original noise conditioning approach in [1], and may generate better samples (see Appendix C.3).

4 Configuring annealed Langevin dynamics

In order to sample from an NCSN with annealed Langevin dynamics, we need to specify the number of sampling steps per noise level $T$ and the step size parameter $\epsilon$ in Algorithm 1. Song and Ermon recommend $\epsilon = 2 \times 10^{-5}$ when $T = 100$. It remains unclear how we should change $\epsilon$ and $T$ for different sets of noise levels.
Unfortunately, this often results in an unnecessarily large volatility. In particular, training NCSNs on CIFAR-10 \(32 \times 32\) and CelebA \(64 \times 64\) following the settings of \[1\], we observe that the generated image samples sometimes exhibit unstable visual quality, especially for images of larger resolutions. We empirically demonstrate this fact by training NCSNs on CIFAR-10 \(32 \times 32\) and CelebA \(64 \times 64\) following the settings of \[1\], which exemplifies typical behavior on other image datasets. We report FID scores \[11\] computed on 1000 samples every 5000 iterations. As shown in Fig. 4, the FID scores for the vanilla NCSN often fluctuate significantly during training. Additionally, samples from the vanilla NCSN sometimes exhibit characteristic artifacts: image samples from the same checkpoint have strong tendency to have a common color shift (cf., Fig. 6a). Moreover, samples are shifted towards different colors throughout training. We provide more samples in Appendix C.2 to manifest this artifact.

This issue can be easily fixed by exponential moving average (EMA). Specifically, let \(\theta\) denote the parameters of an NCSN after the \(i\)-th training iteration, and \(\theta'\) be an independent copy of the parameters. We follow this guidance to generate all samples in this paper, except for those from the original NCSN. Samples from the vanilla NCSN often have obvious color shifts.

To gain some theoretical insight, we revisit the setting in Section 3.2 where the dataset has one point \(i.e., p_{\sigma_i}(x) = \mathcal{N}(x | 0, \sigma_i^2 I)\). Annealed Langevin dynamics connect two adjacent noise levels \(\sigma_{i-1} > \sigma_i\) by initializing the Langevin dynamics for \(p_{\sigma_i}(x)\) with samples obtained from \(p_{\sigma_{i-1}}(x)\).

When applying Langevin dynamics to \(p_{\sigma_i}(x)\), we have \(x_{t+1} \leftarrow x_t + \alpha \nabla_x \log p_{\sigma_i}(x_t) + \sqrt{2\alpha} \epsilon_t\), where \(\epsilon_t \sim p_{\sigma_{i-1}}(x)\) and \(\epsilon_t \sim \mathcal{N}(0, I)\). The distribution of \(\epsilon_t\) can be computed in closed form:

**Proposition 3.** Let \(\gamma = \frac{\sigma_{i-1}}{\sigma_i}\). For \(\alpha = \epsilon \cdot \frac{\sigma_i^2}{\sigma_{i-1}^2}\) (as in Algorithm 1), we have \(x_T \sim \mathcal{N}(0, \sigma_T^2 I)\), where

\[
\frac{s_T^2}{\sigma_T^2} = \left(1 - \frac{\epsilon}{\sigma_L^2}\right)^{2T} \left(\gamma^2 - \frac{2\epsilon}{\sigma_L^2 - \sigma_T^2 \left(1 - \frac{\epsilon}{\sigma_L^2}\right)^2}\right) + \frac{2\epsilon}{\sigma_L^2 - \sigma_T^2 \left(1 - \frac{\epsilon}{\sigma_L^2}\right)^2}. \tag{4}
\]

When \(\{\sigma_i\}_{i=0}^T\) is a geometric progression as advocated by Technique 2, we immediately see that \(s_T^2/\sigma_T^2\) is identical across all \(1 < i < T\) because of the shared \(\gamma\). Furthermore, the value of \(s_T^2/\sigma_T^2\) has no explicit dependency on the dimensionality \(D\).

For better mixing of annealed Langevin dynamics, we hope \(s_T^2/\sigma_T^2\) approaches 1 across all noise levels, which can be achieved by finding \(\epsilon\) and \(T\) that minimize the difference between Eq. (4) and 1. Unfortunately, this often results in an unnecessarily large \(T\) that makes sampling very expensive for large \(L\). As an alternative, we propose to first choose \(T\) based on a reasonable computing budget (typically \(T \times L\) is several thousand), and subsequently find \(\epsilon\) by making Eq. (4) as close to 1 as possible. In summary:

**Technique 4 (selecting \(T\) and \(\epsilon\)).** Choose \(T\) as large as allowed by a computing budget and then select an \(\epsilon\) that makes Eq. (4) maximally close to 1.

We follow this guidance to generate all samples in this paper, except for those from the original NCSN where we adopt the same settings as in \[1\].

## 5 Improving stability with moving average

Unlike GANs, score-based generative models have one unified objective (Eq. (2)) and require no adversarial training. However, even though the loss function of NCSNs typically decreases steadily over the course of training, we observe that the generated image samples sometimes exhibit unstable visual quality, especially for images of larger resolutions. We empirically demonstrate this fact by training NCSNs on CIFAR-10 \(32 \times 32\) and CelebA \(64 \times 64\) following the settings of \[1\], which exemplifies typical behavior on other image datasets. We report FID scores \[11\] computed on 1000 samples every 5000 iterations. As shown in Fig. 4, the FID scores for the vanilla NCSN often fluctuate significantly during training. Additionally, samples from the vanilla NCSN sometimes exhibit characteristic artifacts: image samples from the same checkpoint have strong tendency to have a common color shift (cf., Fig. 6a). Moreover, samples are shifted towards different colors throughout training. We provide more samples in Appendix C.2 to manifest this artifact.
parameters. We update $\theta'$ with $\theta' \leftarrow m\theta' + (1-m)\theta$, after each optimization step, where $m$ is the momentum parameter and typically $m = 0.999$. When producing samples, we use $s_{\theta'}(x, \sigma)$ instead of $s_{\theta}(x, \sigma)$. As shown in Fig. 4, EMA can effectively stabilize FIDs and remove artifacts (more samples in Appendix C.2). Empirically, we observe the effectiveness of EMA is universal across a large number of different image datasets. As a result, we have the following rule of thumb:

**Technique 5 (EMA). Apply exponential moving average to parameters when sampling.**

6 Combining all techniques together

Employing Technique 1–5, we build NCSNs that can readily work across a large number of different datasets, including high resolution images that were previously out of reach with score-based generative modeling. Our modified model is named NCSNv2. For a complete description on experimental details and more results, please refer to Appendix B and C.

| Model          | HYPE∞ |
|----------------|-------|
| StyleGAN       | 50.7  |
| ProgressiveGAN | 40.3  |
| BEGAN          | 10    |
| WGAN-GP        | 3.8   |
| NCSN           | 19.8  |
| NCSNv2         | 37.3  |

* with truncation tricks

**Quantitative results:** We consider CIFAR-10 $32^2$ and CelebA $64^2$ where NCSN and NCSNv2 both produce reasonable samples. We report FIDs (lower is better) every 5000 iterations of training on 1000 samples. As shown in Fig. 5, we observe that the FID scores of NCSNv2 (with all techniques applied) are on average better than those of NCSN, and have much smaller variance over the course of training. Following [1], we select checkpoints with the smallest FIDs encountered during training, and compute full FID scores on more samples (see Table 5 in Appendix C.1) from them. However, we note that FID scores should be interpreted with caution as they may not correlate with visual quality in the expected way. For example, the samples from NCSNv2 as demonstrated in Fig. 6b have an FID score of 28.9, worse than NCSN (Fig. 6a) whose FID is 26.9, but arguably produce much more visually appealing samples. To compare sample quality more accurately, we use the HYPE∞ score (higher is better), a metric of sample quality based on human evaluation. As Table 1 shows (see the full table in Appendix C.1), our NCSNv2 achieves 37.3 on CelebA $64^2$ which is comparable to ProgressiveGAN [13], whereas NCSN achieves 19.8.

**Ablation studies:** We conduct ablation studies to isolate the contributions of different techniques. We partition all techniques into three groups: (i) Technique 5, (ii) Technique 1, 2, 4, and (iii) Technique 3, where different groups can be applied simultaneously. Technique 1 and 2 are grouped together because Technique 1 and 2 collectively determine the set of noise levels, and to sample from NCSNs trained with these noise levels we need Technique 4 to configure annealed Langevin dynamics properly. We test the performance of successively removing groups (iii), (ii), and (i) from NCSNv2, and report results in Fig. 5. All groups of techniques improve over the vanilla NCSN. Although the FID scores are not strictly increasing when removing (iii), (ii), and (i) progressively, we note that FIDs may not always correlate with sample quality well. In fact, we observe decreasing sample quality by visual inspection (see Appendix C.3), and combining all techniques gives the best samples.

**Towards higher resolution:** The original NCSN only succeeds at generating images of low resolution. In fact, [1] only tested it on MNIST $28^2$ and CelebA/CIFAR-10 $32^2$. For slightly larger images such as CelebA $64^2$, NCSN can generate images of consistent global structure, yet with strong color
Figure 7: From top to bottom: FFHQ $256^2$, LSUN bedroom $128^2$, LSUN tower $128^2$, and LSUN church_outdoor $96^2$. Within each group of images: the first row shows uncurated samples from NCSNv2, and the second shows the interpolation results between the leftmost and rightmost samples with NCSNv2. You may zoom in to view more details.

(a) NCSN  
(b) NCSNv2  
(c) NCSN  
(d) NCSNv2

Figure 8: NCSN vs. NCSNv2 samples on LSUN church_outdoor (a)(b) and LSUN bedroom (c)(d).

artifacts that are easily noticeable (compare Fig. 6a with Fig. 6b). For images with resolutions beyond $96^2$, NCSN will completely fail to produce samples with correct structure or color (see Fig. 8).

By combining Technique 1–5, NCSNv2 can effortlessly work on images of much higher resolution. Note that we directly calculated the noise levels for training NCSNs, and computed the step size for annealed Langevin dynamics sampling without manual hyper-parameter tuning. The network architectures are the same across datasets, except that for ones with higher resolution we use more layers and more filters to ensure the receptive field and model capacity are large enough (see details in Appendix B). In Fig. 7 and 8 we show NCSNv2 is capable of generating high-fidelity image samples with resolutions ranging from $96^2$ to $256^2$. To show that this high sample quality is not a result of dataset memorization, we provide the loss curves for training/test, as well as nearest neighbors for samples in Appendix C. In addition, NCSNv2 can produce smooth interpolations between two given samples as in Fig. 7 (details in Appendix B), indicating the ability to learn generalizable image representations.

7 Conclusion

Motivated by both theoretical analyses and empirical observations, we propose a set of techniques to improve score-based generative models. Our techniques significantly improve the training and sampling processes, lead to better sample quality, and enable high-fidelity image generation at high resolutions. Although our techniques work well without manual tuning, we believe that the performance can be improved even more by fine-tuning various hyper-parameters. Future directions include theoretical understandings on the sample quality of score-based generative models, as well as alternative noise distributions to Gaussian perturbations.
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A Proofs

Proposition 1. Let \( \hat{\rho}_\sigma(x) \triangleq \frac{1}{N} \sum_{i=1}^{N} p(i)(x) \), where \( p(i)(x) \triangleq N(x | x^{(i)}, \sigma^2 I) \). With \( r(i)(x) \triangleq \frac{\sum_{i=1}^{D} \nabla x \log p(i)(x)}{\sum_{i=1}^{D} p(i)(x)} \), the score function is \( \nabla_x \log \hat{\rho}_\sigma(x) = \sum_{i=1}^{N} r(i)(x) \nabla_x \log p(i)(x) \). Moreover,

\[
\mathbb{E}_{p(i)(x)}[r(j)(x)] \leq \frac{1}{2} \exp \left( -\frac{\|x^{(i)} - x^{(j)}\|^2}{8\sigma^2_i} \right). \tag{5}
\]

Proof. According to the definition of \( p_\sigma(x) \) and \( r(x) \), we have

\[
\nabla_x \log \hat{\rho}_\sigma(x) = \nabla_x \left( \frac{1}{N} \sum_{i=1}^{N} p(i)(x) \right) = \sum_{i=1}^{N} \frac{\nabla_x p(i)(x)}{\sum_{j=1}^{N} p(j)(x)}
\]

\[
= \sum_{i=1}^{N} r(i)(x) \nabla_x \log p(i)(x).
\]

Next, assuming \( x \in \mathbb{R}^D \), we have

\[
\mathbb{E}_{p(i)(x)}[r(j)(x)] = \int \frac{p(i)(x)p(j)(x)}{\sum_{k=1}^{D} p(k)(x)} \, dx \leq \int \frac{p(i)(x)p(j)(x)}{p(i)(x) + p(j)(x)} \, dx
\]

\[
= \frac{1}{2} \int \frac{1}{p(i)(x)} + \frac{1}{p(j)(x)} \, dx \leq \frac{1}{2} \int \sqrt{\frac{p(i)(x)p(j)(x)}{p(i)(x)}} \, dx
\]

\[
= \frac{1}{2} \left( \frac{1}{2\pi \sigma_i^2} \right)^{D/2} \int \exp \left( -\frac{1}{4\sigma^2_i} \left( \|x - x^{(i)}\|^2 + \|x - x^{(j)}\|^2 \right) \right) \, dx
\]

\[
= \frac{1}{2} \left( \frac{1}{2\pi \sigma_j^2} \right)^{D/2} \int \exp \left( -\frac{1}{4\sigma^2_j} \left( \|x - x^{(i)}\|^2 + \|x - x^{(i)} + x^{(i)} - x^{(j)}\|^2 \right) \right) \, dx
\]

\[
= \frac{1}{2} \left( \frac{1}{2\pi \sigma_i^2} \right)^{D/2} \int \exp \left( -\frac{1}{2\sigma_i^2} \left( \|x - x^{(i)}\|^2 + \|x - x^{(i)} + x^{(i)} - x^{(j)}\|^2 \right) \right) \, dx
\]

\[
= \frac{1}{2} \left( \frac{1}{2\pi \sigma_j^2} \right)^{D/2} \int \exp \left( -\frac{1}{2\sigma_j^2} \left( \|x - x^{(i)}\|^2 + \|x - x^{(i)} + x^{(i)} - x^{(j)}\|^2 \right) \right) \, dx
\]

where (1) is due to the geometric mean–harmonic mean inequality.

Proposition 2. Let \( x \in \mathbb{R}^D \sim N(0, \sigma^2 I) \), and \( r = \|x\|_2 \). We have

\[
p(r) = \frac{1}{2^{D/2-1}\Gamma(D/2)} \frac{r^{D-1}}{\sigma^D} \exp \left( -\frac{r^2}{2\sigma^2} \right) \quad \text{and} \quad r = \sqrt{D} \sigma \overset{d}{\rightarrow} N(0, \sigma^2 / 2) \ \text{when} \ D \rightarrow \infty.
\]

Proof. Since \( x \sim N(0, \sigma^2 I) \), we have \( s \triangleq \|x\|_2 / \sigma^2 \sim \chi^2_D \), i.e.,

\[
p_s(s) = \frac{1}{2^{D/2-1}\Gamma(D/2)} s^{D/2-1} e^{-s/2}.
\]

Because \( r = \|x\|_2 = \sigma \sqrt{s} \), we can use the change of variables formula to get

\[
p(r) = \frac{2r}{\sigma^2} p_s(s) = \frac{2r}{\sigma^2} \frac{r^2}{\sigma^2} = \frac{1}{2^{D/2-1}\Gamma(D/2)} \frac{r^{D-1}}{\sigma^D} \exp \left( -\frac{r^2}{2\sigma^2} \right),
\]

11
which proves our first result. Next, we notice that if \( x \sim \mathcal{N}(0, \sigma^2) \), we have \( x^2/\sigma^2 \sim \chi^2_1 \) and thus \( \mathbb{E}[x] = \sigma^2 \), \( \text{Var}[x] = 2\sigma^4 \). As a result, if \( x_1, x_2, \ldots, x_D \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2) \), the law of large numbers and the central limit theorem will imply that as \( D \to \infty \), both of the following hold:
\[
\frac{x_1^2 + x_2^2 + \cdots + x_D^2}{D} \to \sigma^2
\]
\[
\sqrt{D} \left( \frac{x_1^2 + x_2^2 + \cdots + x_D^2}{D} - \sigma^2 \right) \to \mathcal{N}(0, 2\sigma^4).
\]
Equivalently,
\[
\sqrt{D} \left( \frac{r^2}{D} - \sigma^2 \right) \to \mathcal{N}(0, 2\sigma^4).
\]
Applying the delta method, we obtain
\[
\sqrt{D} \left( \frac{r^2}{D} - \sigma^2 \right) \to \mathcal{N}(0, \sigma^2/2),
\]
and therefore \( r - \sqrt{D}\sigma \to \mathcal{N}(0, \sigma^2/2) \). \( \square \)

**Proposition 3.** Let \( \gamma = \frac{\gamma_1 - \gamma_0}{\sigma} \). For \( \alpha = \epsilon \cdot \frac{\sigma^2}{\sigma_L} \) (as in Algorithm 1), we have \( x_T \sim \mathcal{N}(0, s_T^2 I) \), where
\[
\frac{s_T^2}{\sigma_L^2} = \left( 1 - \frac{\epsilon}{\sigma_L^2} \right)^{2T} \left( \frac{\gamma^2 - \frac{2\epsilon}{\sigma_L^2}}{\sigma_L^2 - \sigma_L^2 \left( 1 - \frac{\epsilon}{\sigma_L^2} \right)^2} \right) + \frac{2\epsilon}{\sigma_L^2 - \sigma_L^2 \left( 1 - \frac{\epsilon}{\sigma_L^2} \right)^2}.
\]

**Proof.** First, the conditions we know are
\[
\begin{align*}
x_0 & \sim p_{\sigma_{t-1}}(x) = \mathcal{N}(0, \sigma_{t-1}^2 I), \\
x_{t+1} & \leftarrow x_t + \alpha \nabla_{\chi} \log p_{\sigma_t}(x_t) + \sqrt{2\alpha} z_t = x_t - \alpha \frac{x_t}{\sigma_t^2} + \sqrt{2\alpha} z_t,
\end{align*}
\]
where \( z_t \sim \mathcal{N}(0, I) \). Therefore, the variance of \( x_t \) satisfies
\[
\text{Var}[x_t] = \begin{cases} 
  \sigma_{t-1}^2 I & \text{if } t = 0 \\
  \left( 1 - \frac{\alpha}{\sigma_t^2} \right)^2 \text{Var}[x_{t-1}] + 2\alpha I & \text{otherwise}.
\end{cases}
\]
Now let \( \nu \triangleq \frac{2\alpha}{1 - (1 - \frac{\alpha}{\sigma_t^2})^2} I \), we have
\[
\text{Var}[x_t] - \nu = \left( 1 - \frac{\alpha}{\sigma_t^2} \right)^2 (\text{Var}[x_{t-1}] - \nu).
\]
Therefore,
\[
\text{Var}[x_T] - \nu = \left( 1 - \frac{\alpha}{\sigma_T^2} \right)^{2T} (\text{Var}[x_0] - \nu)
\]
\[
\Rightarrow \text{Var}[x_T] = \left( 1 - \frac{\alpha}{\sigma_T^2} \right)^{2T} (\text{Var}[x_0] - \nu) + \nu
\]
\[
\Rightarrow s_T^2 = \left( 1 - \frac{\alpha}{\sigma_T^2} \right)^{2T} \left( \sigma_{t-1}^2 - \frac{2\alpha}{1 - (1 - \frac{\alpha}{\sigma_t^2})^2} \right) + \frac{2\alpha}{1 - (1 - \frac{\alpha}{\sigma_t^2})^2}.
\]
Substituting \( \epsilon \cdot \frac{\sigma^2}{\sigma_L^2} \) for \( \alpha \) in Eq. (7), we immediately obtain Eq. (6). \( \square \)
B Experimental details

B.1 Network architectures and hyperparameters

The original NCSN in [1] uses a network structure based on RefineNet [17]—a classical architecture for semantic segmentation. There are three major modifications to the original RefineNet in NCSN:

(i) adding an enhanced version of conditional instance normalization (designed in [1] and named CondInstanceNorm++) for every convolutional layer;
(ii) replacing max pooling with average pooling in RefineNet blocks; and
(iii) using dilated convolutions in the ResNet backend of RefineNet.

We use exactly the same architecture for NCSN experiments, but for NCSNv2 or any other architecture implementing Technique 3, we apply the following modifications: (i) setting the number of classes in CondInstanceNorm++ to 1 (which we name as InstanceNorm++); (ii) changing average pooling back to max pooling; and (iii) removing all normalization layers in RefineNet blocks. Here (ii) and (iii) do not affect the results much, but they are included because we hope to minimize the number of unnecessary changes to the standard RefineNet architecture (the original RefineNet blocks in [17] use max pooling and have no normalization layers). We name a ResNet block (with InstanceNorm++ instead of BatchNorm) “ResBlock”, and a RefineNet block “RefineBlock”. When CondInstanceNorm++ is added, we name them “CondResBlock” and “CondRefineBlock” respectively. We use the ELU activation function [18] throughout all architectures.

To ensure sufficient capacity and receptive fields, the network structures for images of different resolutions have different numbers of layers and filters. We summarize the architectures in Table 2 and Table 3.

| (a) NCSN 32²–64² | (b) NCSN 96²–128² |
|------------------|------------------|
| 3x3 Conv2D, 128  | 3x3 Conv2D, 128  |
| CondResBlock, 128| CondResBlock, 128|
| CondResBlock, 256| CondResBlock, 256|
| CondRefineBlock, 256| CondRefineBlock, 256|
| CondRefineBlock, 128| CondRefineBlock, 128|
| 3x3 Conv2D, 3    | 3x3 Conv2D, 3    |
Table 3: The architectures of NCSNv2 for images of various resolutions.

| Model       | Dataset                | $\sigma_1$ | $L$  | $T$  | $\epsilon$ | Batch size | Training iterations |
|-------------|------------------------|------------|------|------|------------|------------|---------------------|
| NCSN        | CIFAR-10 32$^2$        | 1          | 10   | 100  | 2e-5       | 128        | 300k                |
| NCSN        | CelebA 64$^2$         | 1          | 10   | 100  | 2e-5       | 128        | 210k                |
| NCSN        | LSUN church_outdoor 96$^2$ | 1       | 10   | 100  | 2e-5       | 128        | 200k                |
| NCSN        | LSUN bedroom 128$^2$  | 1          | 10   | 100  | 2e-5       | 64         | 150k                |
| NCSNv2      | CIFAR-10 32$^2$       | 50         | 232  | 5    | 6.2e-6     | 128        | 300k                |
| NCSNv2      | CelebA 64$^2$         | 90         | 500  | 5    | 3.3e-6     | 128        | 210k                |
| NCSNv2      | LSUN church_outdoor 96$^2$ | 140      | 788  | 4    | 4.9e-6     | 128        | 200k                |
| NCSNv2      | LSUN bedroom/tower 128$^2$ | 190     | 1086 | 3    | 1.8e-6     | 128        | 150k                |
| NCSNv2      | FFHQ 256$^2$          | 348        | 2311 | 2    | 1.8e-6     | 32         | 80k                 |

We use the Adam optimizer [19] for all models. When Technique 3 is not in effect, we choose the learning rate 0.001; otherwise we use a learning rate 0.0001 to avoid loss explosion. We set the $\epsilon$ parameter of Adam to $10^{-3}$ for FFHQ and $10^{-5}$ otherwise. We provide other hyperparameters in Table 4, where $\sigma_1$, $L$, $T$, and $\epsilon$ of NCSNv2 are all chosen in accordance with our proposed techniques. When the number of training data is larger than 60000, we randomly sample 10000 of them and compute the maximum pairwise distance, which is set as $\sigma_1$ for NCSNv2.
B.2 Additional settings

Datasets: We use the following datasets in our experiments: CIFAR-10 [10], CelebA [10], LSUN [20], and FFHQ [12]. CIFAR-10 contains 60000 training images and 10000 test images, all of resolution $32 \times 32$. CelebA contains 162770 training images and 19962 test images with various resolutions. For preprocessing, we first center crop them to size $140 \times 140$, and then resize them to $64 \times 64$. We choose the church_outdoor, bedroom and tower categories in the LSUN dataset. They contain 126227, 3033042, and 708264 training images respectively, and all have 300 validation images. For preprocessing, we first resize them so that the smallest dimension of images is 96 (for church_outdoor) or 128 (for bedroom and tower), and then center crop them to equalize their lengths and heights. Finally, the FFHQ dataset consists of 70000 high-quality facial images at resolution $1024 \times 1024$. We resize them to $256 \times 256$ in our experiments. Because FFHQ does not have an official test dataset, we randomly select 63000 images for training and the remaining 7000 as the test dataset. In addition, we apply random horizontal flip as data augmentation in all cases.

Metrics: We use FID [11] and HYPE$_\infty$ [16] scores for quantitative comparison of results. When computing FIDs on CIFAR-10 $32 \times 32$, we measure the distance between the statistics of samples and training data. When computing FIDs on CelebA $64 \times 64$, we follow the settings in [21] where the distance is measured between samples and the test dataset. We use the official website https://hype.stanford.edu for computing HYPE$_\infty$ scores. Regarding model selection, we follow the settings in [1], where we compute FID scores on 1000 samples every 5000 training iterations and choose the checkpoint with the smallest FID for computing both full FID scores (with more samples from it) and the HYPE$_\infty$ scores.

Training: We use the Adam [19] optimizer with default hyperparameters. The learning rates and batch sizes are provided in Appendix B.1 and Table 4. We observe that for images at resolution $128 \times 128$ or $256 \times 256$, training can be unstable when the loss is near convergence. We note, however, this is a well-known problem of the Adam optimizer, and can be mitigated by techniques such as AMSGrad [22]. We trained all models on Nvidia Tesla V100 GPUs.

Settings for Section 3.3: The loss curves in Fig. 3 are results of two settings: (i) Technique 1, 2, 4 and 5 are in effect, but the model architecture is the same as the original NCSN (i.e., Table 2a); and (ii) all techniques are in effect, i.e., the model is the same as NCSNv2 depicted in Table 3a. We apply EMA with momentum 0.9 to smooth the curves in Fig. 3. We observe that despite being simpler to implement, the new noise conditioning method proposed in Technique 3 performs as well as the original and arguably more complex one in [1] in terms of the training loss. See the ablation studies in Section 5 and Appendix C.3 for additional results.

Interpolation: We can interpolate between two different samples from NCSN/NCSNv2 via interpolating the Gaussian random noise injected by annealed Langevin dynamics. Specifically, suppose we have a total of $L$ noise levels, and for each noise level we run $T$ steps of Langevin dynamics. Let $\{x_{ij}\}_{1 \leq i \leq L, 1 \leq j \leq T}$ denote the set of all Gaussian noise used in this procedure, where $z_{ij}$ is the noise injected at the $j$-th iteration of Langevin dynamics corresponding to the $i$-th noise level. Next, suppose we have two samples $x^{(1)}$ and $x^{(2)}$ with the same initialization $x_0$, and denote the corresponding set of Gaussian noise as $\{z_{ij}^{(1)}\}_{1 \leq i \leq L, 1 \leq j \leq T}$ and $\{z_{ij}^{(2)}\}_{1 \leq i \leq L, 1 \leq j \leq T}$ respectively. We can generate $N$ interpolated samples between $x^{(1)}$ and $x^{(2)}$, where for the $k$-th interpolated sample we use Gaussian noise $\{\cos\left(\frac{k\pi}{2(N+1)}\right)z_{ij}^{(1)} + \sin\left(\frac{k\pi}{2(N+1)}\right)z_{ij}^{(2)}\}_{1 \leq i \leq L, 1 \leq j \leq T}$ and initialization $x_0$.

C Additional experimental results

C.1 Additional tables

After training finished, we follow the model selection process described in [1], which amounts to picking checkpoints with the smallest FID values computed on 1000 samples every 5000 training steps. We then compute full FID scores on more samples from these checkpoints (60000 for CIFAR-10 $32 \times 32$ and 10000 for CelebA $64 \times 64$), and report results in Table 5. Although commonly adopted for evaluating GANs (cf., [13, 12]), this procedure may overestimate the performance when FID values have very large variance across different checkpoints. Indeed, our reproduction of NCSN results on CIFAR-10 gives an FID score of 30.93, while the original paper [1] reported 25.32. This
gap is because the original NCSN can be sensitive to randomness in training (which may result from different GPUs, PyTorch versions, CUDA versions, etc.), and the large variance of FIDs (see the discussion in Section 5) causes the selection of a completely different checkpoint. As discussed before in Section 6, we note that FID scores should be interpreted with caution as they may not correlate with visual quality in the expected manner, so we also report HYPE∞ scores, a metric based on human evaluation, on CelebA 64 × 64. Aside from Table 1, we provide extra results on HYPE∞ scores in Table 6.

Table 5: Full FID scores. *Reported results in the original paper [1]. †Results of a different run on more powerful GPUs and newer PyTorch version.

| Model       | NCSN* (CIFAR-10) | NCSN† (CIFAR-10) | NCSNv2 (CIFAR-10) | NCSN (CelebA) | NCSNv2 (CelebA) |
|-------------|------------------|------------------|-------------------|---------------|-----------------|
| FID         | 25.32            | 30.93            | 31.75             | 26.89         | 28.86           |

Table 6: Extended results of HYPE∞ scores on CelebA 64 × 64. *With truncation tricks.

| Model       | HYPE∞(%) | Fakes Error(%) | Reals Error(%) | Std. |
|-------------|----------|----------------|----------------|------|
| StyleGAN*   | 50.7     | 62.2           | 39.3           | 1.3  |
| ProgressiveGAN | 40.3    | 46.2           | 34.4           | 0.9  |
| BEGAN       | 10       | 6.2            | 13.8           | 1.6  |
| WGAN-GP     | 3.8      | 1.7            | 5.9            | 0.6  |
| NCSN        | 19.8     | 22.3           | 17.3           | 0.4  |
| NCSNv2      | 37.3     | 49.8           | 24.8           | 0.5  |

C.2 Color shifts

Figure 9: EMA reduces undesirable color shifts on CIFAR-10. We show samples from NCSN and NCSN with EMA at the 50k/100k/200k-th training iteration.
Figure 10: EMA reduces undesirable color shifts on CelebA-10. We show samples from NCSN and NCSN with EMA at the 50k/100k/150k-th training iteration.

C.3 Additional results on ablation studies

As discussed in Section 6, we partition all techniques into three groups: (i) Technique 5, (ii) Technique 1, 2, 4, and (iii) Technique 3, and investigate the performance of models after successively removing (iii), (ii), and (i) from NCSNv2. Aside from the FID curves in Fig. 5, we also provide samples from different models for visual inspection in Fig. 11. To generate these samples, we compute the FID scores on 1000 samples every 5000 training iterations for each considered model, and sample from the checkpoint of the smallest FID (the same setting as in Fig. 5). From samples in Fig. 11, we easily observe that removing any group of techniques leads to worse samples.
Figure 11: Samples from models with different groups of techniques applied. NCSN is the original model in [1] and does not use any of the newly proposed techniques. Subscripts of “NCSN” denote the IDs of techniques in effect. NCSN_5 only applies EMA. NCSN_{1,2,4,5} applies both EMA and technique group (ii). NCSNv2 is the result of all techniques combined.
C.4 Generalization

C.4.1 Loss curves

First, we demonstrate that our NCSNv2 does not overfit to the training dataset by showing the curves of training/test loss in Fig. 12. Since the loss on the test dataset is always close to the loss on the training dataset during the course of training, we do not believe that our model performs well by simply memorizing training data.

![Loss curves of NCSNv2](image)

Figure 12: Training vs. test loss curves of NCSNv2.

C.4.2 Nearest neighbors

Starting from this section, all samples are from NCSNv2 at the last training iteration. For each generated sample, we show the nearest neighbors from the training dataset, measured by $\ell_2$ distance in the feature space of a pre-trained InceptionV3 network. Since we apply random horizontal flip when training, we also take this into consideration when computing nearest neighbors, so that we can detect cases in which NCSNv2 memorizes a flipped training data point.

![Nearest neighbors on CIFAR-10](image)

Figure 13: Nearest neighbors on CIFAR-10. NCSNv2 samples are on the left side of the red vertical line. Corresponding nearest neighbors are on the right side in the same row.
Figure 14: Nearest neighbors on CelebA $64 \times 64$.

Figure 15: Nearest neighbors on LSUN church _outdoor $96 \times 96$.

Figure 16: Nearest neighbors on FFHQ $256 \times 256$. 

C.4.3 Additional interpolation results

We generate samples from NCSNv2 and interpolate between them using the method described in Appendix B.2.

Figure 17: NCSNv2 interpolation results on CelebA 64 × 64.

Figure 18: NCSNv2 interpolation results on LSUN church_outdoor 96 × 96.

Figure 19: NCSNv2 interpolation results on LSUN bedroom 128 × 128.
Figure 20: NCSNv2 interpolation results on LSUN tower $128 \times 128$.

Figure 21: NCSNv2 interpolation results on FFHQ $256 \times 256$. 
C.5 Additional uncurated samples

Figure 22: Uncurated CIFAR-10 $32 \times 32$ samples from NCSNv2.
Figure 23: Uncurated CelebA $64 \times 64$ samples from NCSNv2.
Figure 24: Uncurated LSUN church_outdoor 96 × 96 samples from NCSNv2.
Figure 25: Uncurated LSUN bedroom $128 \times 128$ samples from NCSNv2.
Figure 26: Uncurated LSUN tower $128 \times 128$ samples from NCSNv2.
Figure 27: Uncurated FFHQ $256 \times 256$ samples from NCSNv2.