Particle-Antiparticle Metamorphosis of
Massive Majorana Neutrinos and Gauginos

D. V. Ahluwalia-Khalilova

Theoretical Physics Group, Fac. de Fisica de la UAZ, Zacatecas, Ap. Postal C-600, ZAC 98062, Mexico.
Present address: Department of Mathematics, University of Zacatecas, Zacatecas, ZAC 98060, Mexico

Abstract

Recent results on neutrinoless double beta decay, as reported by Klapdor-Klein-grothaus et al., take us for the first time into the realm of Majorana spacetime structure. However, this structure has either been treated as an afterthought to the Dirac construct; or, when it has been attended to in its own right, its physical and mathematical content was never fully unearthed. In this Letter, we undertake to remedy the existing situation. We present a detailed formalism required for the description of the non-trivial spacetime structure underlying the $\nu \leftrightarrow \bar{\nu}$ metamorphosis — where $\nu$ generically represents a massive Majorana neutrino, or a massive gaugino.

Key words: Majorana neutrinos, gauginos, neutrinoless double beta decay, beta decay, supersymmetry.

1 Introduction

Spacetime structures in supersymmetric theories find themselves deeply intertwined with the Majorana realization of fermionic representation spaces.

In its simplest form, the algebra of supersymmetry is generated by fourteen operators. These consist of ten generators of the Poincaré group, and four anticommuting generators. The latter are called “supertranslations,” and are
components of a Majorana spinor [1]. So, it comes about that spacetime structure of supersymmetry finds itself deeply connected with the Majorana aspect.

In this context, recent results of Klapdor-Kleingrothaus et al., [2–5], which provided a first direct evidence for neutrinoless double beta decay, $0\nu\beta\beta$, are of particular interest.

In its simplest interpretation, the $0\nu\beta\beta$ experimental signal arises from the Majorana nature of massive $\nu_e$. Even though the experiment by itself does not necessarily require a supersymmetric framework for its explanation, the indication towards a Majorana spacetime structure suddenly acquires a pivotal importance.

In this Letter we, therefore, undertake an *ab initio* look at the spacetime structure of the massive Majorana particles — whether they be neutrinos or gaugions. In particular, we focus our attention on the $\nu = \bar{\nu}$ metamorphosis.

In the next section, we present a critique of the conventional Majorana construct in the context of $0\nu\beta\beta$. The critique also provides additional motivation for undertaking the reported work. In Sec. 3, we present an *ab initio* formulation Majorana spinors. In Sec. 4, we examine to what extent it is possible to associate them with negative and positive energies. There we find that the particle/antiparticle interpretation of self/antiself conjugate Majorana spinors fails, thus opening the door to the $\nu = \bar{\nu}$ metamorphosis leading to $0\nu2\beta$ decay. In Sec. 5, we show the insensitivity of Majorana self/antiself conjugate spinors to the arrow of time. The explicit construct of the particle-antiparticle metamorphosis is presented in Sec. 6 in terms of the momentum-space Feynman propagator. The Letter closes with some brief remarks and a summary.

## 2 A critique of conventional Majorana construct in $0\nu\beta\beta$

As already noted, in the simplest scenario, the recent results of Klapdor-Kleingrothaus et al. are interpreted as arising from the Majorana nature of neutrinos. The essential physics of interest lies in the following. A virtual $W^-$, which induces a weak $n \rightarrow p$ transition, decays into $e^-$ and $\bar{\nu}_e$,

\[ W^- \rightarrow e^- + \bar{\nu}_e \, , \]  

and the emitted $\bar{\nu}_e$ is reabsorbed as an $\nu_e$,

\[ \nu_e + W^- \rightarrow e^- \, , \]  

(1)
where the second (virtual) $W^-$ takes responsibility for the second $n \rightarrow p$
transformation. The net result is the $0\nu\beta\beta$,

$$A(N, Z) \rightarrow A(N - 2, Z + 2) + 2e^-,$$

(3)
where $A(N, Z)$ represents a nucleus carrying $N$ neutrons, and $Z$ protons. Now, the Majorana nature of $\nu_e$ does not mean ($\times$) that one can naively identify the $\bar{\nu}_e$ of (1) with the $\nu_e$ of (2),

$$\times : \ \bar{\nu}_e = \nu_e.$$

(4)
This is so because a neutrino carries a negative helicity in the $m_{\nu_e}/E_{\nu_e} \rightarrow 0$
limit, and transforms as a $(0, 1/2)$ object; while an antineutrino carries a positive helicity in the $m_{\nu_e}/E_{\nu_e} \rightarrow 0$ limit, and transforms as a $(1/2, 0)$ object. As such there is a $m_{\nu_e}/E_{\nu_e}$ suppressed amplitude for the above identification [6,7]. This result arises in a framework in which the Majorana field [8] is obtained from the Dirac field,

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{m}{2p_0} \sum_{h=\uparrow,\downarrow} \left[ a_h(p) u_h(p) e^{-ip \cdot x} + b_h^\dagger(p) v_h(p) e^{+ip \cdot x} \right],$$

(5)
through the identification,

$$b_h^\dagger(p) = \zeta a_h^\dagger(p),$$

(6)
where $\zeta$ is a "creation phase factor" [9].

As is apparent, the above Fock space construct ceases to place fundamentally neutral fields at the same footing as the intrinsically charged fields. Instead, it patches together the old Dirac spacetime structure of the $(1/2, 0) \oplus (0, 1/2)$ representation space, $u_h(p)$ and $v_h(p)$, with a new Fock-space constraint as suggested by Majorana (6). In other words, the Fock space is treated correctly, via Eq. (6), as is required for a fundamentally neutral field. However, the underlying representation space is still obliged to the Dirac spinors. The Majorana construct, as used by the high-energy physics community, is a hybrid and it asks for the formulation of a framework that is of Majorana type both in spinor- and Fock- spaces.

The question now is, whether it is possible to create a fully symmetric framework for the intrinsically neutral fields that carries an explicit counterpart of Eq. (6) for its $(1/2, 0) \oplus (0, 1/2)$ spinorial structure. In this Letter we provide such a formulation. In the process we shall arrive at a deeper understanding of the spacetime structure underlying the $\nu \equiv \bar{\nu}$ metamorphosis.
The presented formulation is self contained. It carries its interest beyond understanding $0\nu\beta\beta$, in terms of Majorana neutrinos, to theorists and phenomenologists interested in left-right symmetric theories [7] and gauginos.

3 Ab initio formulation of Majorana spinors

The boost generator for the $(1/2, 0)$ representation space is $-i\sigma/2$, and that for $(0, 1/2)$ is $+i\sigma/2$. Consequently, the respective boosts are

$$B_R = \exp \left( +\frac{\sigma}{2} \cdot \varphi \right) = \sqrt{\frac{E + m}{2m}} \left( \begin{pmatrix} 1 & \sigma \cdot p \\ \sigma \cdot p & E + m \end{pmatrix} \right),$$

$$B_L = \exp \left( -\frac{\sigma}{2} \cdot \varphi \right) = \sqrt{\frac{E + m}{2m}} \left( \begin{pmatrix} 1 & \sigma \cdot p \\ \sigma \cdot p & E + m \end{pmatrix} \right),$$

where the boost parameter is defined as:

$$\cosh(\varphi) = \frac{E}{m}, \quad \sinh(\varphi) = \frac{|p|}{m}, \quad \hat{\varphi} = \frac{p}{|p|}.$$  

The boosts take a particle at rest to a particle moving with momentum $p$ in the “boosted frame.” We use the notation in which $1_n$ and $0_n$ represent $n \times n$ identity and null matrices, respectively.

These well-known results allow for the observation:

$$B_{L,R}^{-1} = B_{R,L}^\dagger.$$  

Further, if $\Theta$ is the Wigner’s spin-1/2 time reversal operator, $^1$

$$\Theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

then

$$\Theta [\sigma/2] \Theta^{-1} = -[\sigma/2]^*,$$

$^1$ We refrain from identifying $\Theta$ with $-i\sigma_2$ because such identification does not exist for higher-spin $(j,0) \oplus (0,j)$ representation spaces. The existence of Wigner time reversal operator for all $j$, allows, for fermionic $j$’s, the introduction of $(j,0) \oplus (0,j)$ Majorana objects.
The above observations imply that:

1. If $\phi_L(p)$ transforms as a left handed spinor, then $(\zeta_\lambda \Theta) \phi_L^*(p)$ transforms as a right handed spinor – where, $\zeta_\lambda$ is an unspecified phase.
2. If $\phi_R(p)$ transforms as a right handed spinor, then $(\zeta_\rho \Theta)^* \phi_R^*(p)$ transforms as a left handed spinor – where, $\zeta_\rho$ is an unspecified phase.

As a consequence, the following spinors (in Weyl basis) belong to the $(1/2, 0) \oplus (0, 1/2)$ representation space:

$$\lambda(p) = \begin{pmatrix} (\zeta_\lambda \Theta) \phi_L^*(p) \\ \phi_L(p) \end{pmatrix}, \quad \rho(p) = \begin{pmatrix} \phi_R(p) \\ (\zeta_\rho \Theta)^* \phi_R^*(p) \end{pmatrix}. \quad (13)$$

The operation of charge conjugation is:

$$C = \begin{pmatrix} 0_2 & i \Theta \\ -i \Theta & 0_2 \end{pmatrix} K, \quad (14)$$

where the operator $K$ complex conjugates any object that appears on its right. Demanding $\lambda(p)$ and $\rho(p)$ to be self/anti-self conjugate under $C$,

$$C\lambda(p) = \pm \lambda(p), \quad C\rho(p) = \pm \rho(p), \quad (15)$$

restricts the phases, $\zeta_\lambda$ and $\zeta_\rho$, to two values:

$$\zeta_\lambda = \pm i, \quad \zeta_\rho = \pm i. \quad (16)$$

The plus sign in the above equation yields self conjugate, $\lambda^S(p)$ and $\rho^S(p)$ spinors; while the minus sign results in the anti-self conjugate spinors, $\lambda^A(p)$ and $\rho^A(p)$.

### 3.1 The $\lambda(p)$ spinors

To obtain explicit expressions for $\lambda(p)$, we first write down the rest spinors. These are:

$$\lambda^S(0) = \begin{pmatrix} +i \Theta \phi_L^*(0) \\ \phi_L(0) \end{pmatrix}, \quad \lambda^A(0) = \begin{pmatrix} -i \Theta \phi_L^*(0) \\ \phi_L(0) \end{pmatrix}. \quad (17)$$
Next, we choose the $\phi_L(0)$ to be helicity eigenstates,

$$\sigma \cdot \hat{p} \phi^\pm_L(0) = \pm \phi^\pm_L(0),$$

and concurrently note that

$$\sigma \cdot \hat{p} \Theta \left[ \phi^\pm_L(0) \right]^* = \mp \Theta \left[ \phi^\pm_L(0) \right]^*.$$  \hspace{1cm} (19)

We are thus led to four rest spinors. Two of which are self-conjugate,

$$\lambda^S_\uparrow(0) = \begin{pmatrix} + i \Theta \left[ \phi^+_L(0) \right]^* \\ \phi^+_L(0) \end{pmatrix}, \quad \lambda^S_\downarrow(0) = \begin{pmatrix} + i \Theta \left[ \phi^-_L(0) \right]^* \\ \phi^-_L(0) \end{pmatrix},$$

and the other two, which are anti-self conjugate,

$$\lambda^A_\uparrow(0) = \begin{pmatrix} - i \Theta \left[ \phi^+_L(0) \right]^* \\ \phi^+_L(0) \end{pmatrix}, \quad \lambda^A_\downarrow(0) = \begin{pmatrix} - i \Theta \left[ \phi^-_L(0) \right]^* \\ \phi^-_L(0) \end{pmatrix}.$$  \hspace{1cm} (20)

The boosted spinors are now obtained via the operation:

$$\lambda(p) = \begin{pmatrix} B_R & 0_2 \\ 0_2 & B_L \end{pmatrix} \lambda(0),$$  \hspace{1cm} (22)

Derivation of Eq. (19): Complex conjugating Eq. (18) gives,

$$\sigma^* \cdot \hat{p} \left[ \phi^+_L(0) \right]^* = \pm \left[ \phi^+_L(0) \right]^*.$$  \hspace{1cm} (18)

Substituting for $\sigma^*$ from Eq. (12) then results in,

$$\Theta \sigma \Theta^{-1} \cdot \hat{p} \left[ \phi^+_L(0) \right]^* = \mp \left[ \phi^+_L(0) \right]^*.$$  \hspace{1cm} (18)

But $\Theta^{-1} = -\Theta$. So,

$$-\Theta \sigma \Theta \cdot \hat{p} \left[ \phi^+_L(0) \right]^* = \mp \left[ \phi^+_L(0) \right]^*.$$  \hspace{1cm} (18)

Or, equivalently,

$$\Theta^{-1} \sigma \Theta \cdot \hat{p} \left[ \phi^+_L(0) \right]^* = \mp \left[ \phi^+_L(0) \right]^*.$$  \hspace{1cm} (18)

Finally, left multiplying both sides of the preceding equation by $\Theta$, and moving $\Theta$ through $\hat{p}$, yields Eq. (19).
where the $B_R$ and $B_L$ are given by Eqs. (7) and (8). In the boost, we replace $\sigma \cdot p$ by $\sigma \cdot \hat{p}|p|$, and then exploit Eq. (19). After simplification, Eq. (22) yields:

$$\lambda_S^+(p) = \sqrt{\frac{E + m}{2m}} \left( 1 - \frac{|p|}{E + m} \right) \lambda_S^+(0),$$

(23)

which, in the massless limit, \textit{identically vanishes}, while

$$\lambda_S^-(p) = \sqrt{\frac{E + m}{2m}} \left( 1 + \frac{|p|}{E + m} \right) \lambda_S^-(0),$$

(24)

does not. We hasten to warn the reader that one should not be tempted to read the two different prefactors to $\lambda_S^+(0)$ in the above expressions as the boost operator that appears in Eq. (22). For one thing, there is only one (not two) boost operator(s) in the $(1/2,0) \oplus (0,1/2)$ representation space. The simplification that appears here is due to a fine interplay between Eq. (19), the boost operator, and the structure of the $\lambda_S^+(0)$. Similarly, the anti-self conjugate set of the boosted spinors reads:

$$\lambda_A^+(p) = \sqrt{\frac{E + m}{2m}} \left( 1 - \frac{|p|}{E + m} \right) \lambda_A^+(0),$$

(25)

$$\lambda_A^-(p) = \sqrt{\frac{E + m}{2m}} \left( 1 + \frac{|p|}{E + m} \right) \lambda_A^-(0).$$

(26)

In the massless limit, the first of these spinors \textit{identically vanishes}, while the second does not. Representing the unit vector along $p$, as,

$$\hat{p} = \begin{pmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{pmatrix},$$

(27)

the $\phi_L^\pm(0)$ take the explicit form:

$$\phi_L^+(0) = \sqrt{me^i\vartheta_1} \begin{pmatrix} \cos(\theta/2)e^{-i\phi/2} \\ \sin(\theta/2)e^{i\phi/2} \\ -\cos(\theta/2)e^{i\phi/2} \end{pmatrix},$$

(28)

$$\phi_L^-(0) = \sqrt{me^i\vartheta_2} \begin{pmatrix} \sin(\theta/2)e^{-i\phi/2} \\ \cos(\theta/2)e^{i\phi/2} \\ \cos(\theta/2)e^{i\phi/2} \end{pmatrix}.$$ 

(29)

On setting $\vartheta_1$ and $\vartheta_2$ to be zero — a fact that we explicitly note — we find the following \textit{bi-orthonormality} relations for the self-conjugate Majorana spinors,
\[ \lambda^S(p) \lambda^S(p) = 0, \quad \lambda^S(p) \lambda^S(p) = +2im, \quad (30) \]
\[ \lambda^S(p) \lambda^S(p) = -2im, \quad \lambda^S(p) \lambda^S(p) = 0. \quad (31) \]

Their counterpart for antiself-conjugate Majorana spinors reads,

\[ \lambda^A(p) \lambda^A(p) = 0, \quad \lambda^A(p) \lambda^A(p) = -2im, \quad (32) \]
\[ \lambda^A(p) \lambda^A(p) = +2im, \quad \lambda^A(p) \lambda^A(p) = 0, \quad (33) \]

while all combinations of the type \( \lambda^A(p) \lambda^A(p) \) and \( \lambda^S(p) \lambda^A(p) \) identically vanish.

We take note that the bi-orthogonal norms of the Majorana spinors are intrinsically imaginary. The associated completeness relation is:

\[ -\frac{1}{2im} \left( [\lambda^S(p) \lambda^S(p) - \lambda^S(p) \lambda^S(p)] - [\lambda^A(p) \lambda^A(p) - \lambda^A(p) \lambda^A(p)] \right) = 1. \quad (34) \]

3.2 The \( \lambda(p) \) and \( \rho(p) \) spinors

Now, \((1/2, 0) \oplus (0, 1/2)\) is a four dimensional representation space. Therefore, there cannot be more than four independent spinors. Consistent with this observation, we find that the \( \rho(p) \) spinors are related to the \( \lambda(p) \) spinors via the following identities:

\[ \rho^S_i(p) = -i\lambda^A_i(p), \quad \rho^S_i(p) = +i\lambda^A_i(p), \quad (35) \]
\[ \rho^A_i(p) = +i\lambda^S_i(p), \quad \rho^A_i(p) = -i\lambda^S_i(p). \quad (36) \]

Using these identities, one may immediately obtain the bi-orthonormality and completeness relations for the \( \rho(p) \) spinors.

In the massless limit, \( \rho^S_i(p) \) and \( \rho^A_i(p) \) identically vanish.

A particularly simple orthonormality, as opposed to bi-orthonormality, relation exists between the \( \lambda(p) \) and \( \rho(p) \) spinors:

\[ \lambda^S(p) \rho^A(p) = -2m = \lambda^A(p) \rho^S(p), \quad (37) \]
\[ \lambda^S(p) \rho^A(p) = +2m = \lambda^A(p) \rho^S(p). \quad (38) \]

An associated completeness relation also exists, and it reads:
The results of this section are in spirit of Refs. [10–13].

3.3 Reflection-less Majorana spinors

In the massless limit, the Majorana spinors are reflection-less. This can be proved by taking note of the fact that, e.g., under the Parity operation $\lambda_S^S(p)$ transforms to $i\lambda_S^S(-p)$. The latter, in the indicated limit, identically vanishes, as already explained in subsection (3.1). As a consequence, for instance, in the ordinary beta decay

\[ n \rightarrow p + e^- + \bar{\nu}_e , \]

if one identifies $\bar{\nu}_e$ with $\rho^A_S(p)$ [which equals $i\lambda_S^S(p)$, according to the first equation in Eq. (36)], then its Parity transform $-\lambda_S^S(-p)$, identically vanishes in the massless limit. This picture is in accord with the empirical observation that the origin of parity violation in ordinary beta decay lies in the absence of its mirror image. For massive neutrinos, this would be no longer true.

Note a rather subtle aspect of the Majorana framework: Parity violation occurs despite the apparent parity covariance of the Majorana construct.\(^3\) No such kinematic conclusion can be drawn if neutrinos are described by Dirac spinors without explicitly invoking parity violation via the standard projectors $(1/2)(1 \pm \gamma_5)$.

3.4 Wave equations for Majorana spinors

To analyze the time-evolution of the $\lambda(p)$ and $\rho(p)$ Majorana spinors one needs relevant equations of motion. After taking into account the observations of Ref. [16,17], these can be obtained following a text-book procedure similar to that of Ryder [18]. The result is:

\[
\begin{pmatrix}
-\frac{1}{2m} \left( [\lambda_S^S(p)\slashed{p}_S^S(p) + \lambda_A^A(p)\slashed{p}_A^A(p)] + [\lambda_A^A(p)\slashed{p}_S^S(p) + \lambda_S^S(p)\slashed{p}_A^A(p)] \right) = 1_4 .
\end{pmatrix}
\]

(39)

\[\lambda^A(p)\slashed{p}_{\lambda} = 0, \quad \rho^A(p)\slashed{p}_{\rho} = 0. \]

(41)

\(^3\) This circumstance is akin to the violation of special relativity in Rarita-Schwinger framework despite its fully Poincaré covariant structure (see, Refs. [14,15]).
In the above equations we have used the following abbreviations:

\[ O^\pm_\lambda \equiv \zeta_\lambda \exp \left( \pm \sigma \cdot \varphi / 2 \right) \Theta \eta \exp \left( \pm \sigma \cdot \varphi / 2 \right), \quad (42) \]

\[ O^\pm_\rho \equiv \zeta_\rho \exp \left( \pm \sigma \cdot \varphi / 2 \right) \Theta \eta \exp \left( \pm \sigma \cdot \varphi / 2 \right), \quad (43) \]

where

\[ \eta = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}. \quad (44) \]

These wave equations cannot be reduced to the Dirac equation.

4 Indiscernibility between negative and positive energy solution
for Majorana particles

We note that for self-, as well antiself- conjugate Majorana spinors (i.e., irrespective of whether \( \zeta_\lambda = +i \), or \( \zeta_\lambda = -i \))

\[ \text{Det} \begin{pmatrix} -1_2 & O^+_\lambda \\ O^-_\lambda & -1_2 \end{pmatrix} = (3m^2 + 4Em - \mathbf{p}^2 + E^2)^2 \left( m^2 + \mathbf{p}^2 - E^2 \right)^2. \quad (45) \]

That the determinant must vanish for non-trivial solutions, \( \lambda(\mathbf{p}) \), translates into, \( (m^2 + \mathbf{p}^2 - E^2) = 0 \). The \( \lambda(\mathbf{p}) \) spinors are thus consistent with the causal dispersion relation. To check the hypothesis if the sign of \( \zeta_\lambda \) can determine the sign of energy associated with the \( \lambda(\mathbf{p}) \) spinors, it suffices to look at the rest spinors. The wave equation for the latter, irrespective of the helicity, or the sign of \( \zeta_\lambda \), then reduces to the conditions of the type\(^4\)

\[ \begin{pmatrix} ie^{-i\phi/2} (m^2 - E^2) \sin(\theta/2) \\ -ie^{i\phi/2} (m^2 - E^2) \cos(\theta/2) \\ -e^{-i\phi/2} (m^2 - E^2) \cos(\theta/2) \\ -e^{i\phi/2} (m^2 - E^2) \sin(\theta/2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (46) \]

Clearly, this condition cannot determine the sign of energy associated with the \( \lambda^S_\uparrow(\mathbf{0}) \) to be either plus, or minus. The same remains true for the remaining \( \lambda(\mathbf{0}) \) spinors, and results readily extend to all \( \lambda(\mathbf{p}) \).

\(^4\) Equation below is written for \( \lambda^S_\uparrow(\mathbf{0}) \).
The above discussion is equally valid for the $\rho(p)$ spinors.

5 Insensitivity of Majorana spinors to the direction of time

The plane waves for the $\lambda(x,t)$ and $\rho(x,t)$ are, $\exp(i\epsilon p \cdot x)\lambda(p)$ and $\exp(i\epsilon p \cdot x)\rho(p)$, where $\epsilon = \pm 1$ (depending upon whether the propagation is forward in time, or backward in time). To determine $\epsilon$, it suffices to study the wave equations in the rest frame of the particles. In that frame, the $\lambda(x,t)$ satisfies the following (simplified) differential equation:

$$\begin{pmatrix} \mathcal{O}_a & 0 & 0 \\ 0 & \mathcal{O}_a & \mathcal{O}_c \\ 0 & \mathcal{O}_b & \mathcal{O}_a \end{pmatrix} \lambda(0) e^{-i\epsilon mt} = 0 \quad (47)$$

where

$$\mathcal{O}_a \equiv -2m \left(i\frac{\partial}{\partial t} + m\right), \quad (48)$$
$$\mathcal{O}_b \equiv -\zeta_\lambda e^{-i\phi} \left(m^2 - \frac{\partial^2}{\partial t^2} + 2im\frac{\partial}{\partial t}\right), \quad (49)$$
$$\mathcal{O}_c \equiv \zeta_\lambda e^{i\phi} \left(m^2 - \frac{\partial^2}{\partial t^2} + 2im\frac{\partial}{\partial t}\right). \quad (50)$$

This equation does not fix the sign of $\epsilon$. It only determines $\epsilon^2$ to be unity. That is, Majorana spinors are insensitive to the forward and backward directions in time so important in the Feynman-Stückelberg interpretation of particles and antiparticles. Same result, and similar analysis, holds true for the $\rho$ spinors. The conventional distinction between particles and antiparticles disappears. However, a distinction does reside in the behavior of these spinors as regards the high-energy/massless limit and the associated helicity. In that limit, as is apparent from Eqs. (23) and (24), only the negative helicity $\lambda(p)$ survive. Similarly, Eqs. (25) and (26) show that in the same limit only the positive helicity $\rho(p)$ survive. This circumstance suggests that we identify the $\lambda(p)$ as

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5 To obtain the simplified equation below we first multiplied the momentum-space wave equation by $2m(E + m)$. Then, we exploited the fact that in configuration space, $p = \frac{1}{i}\nabla$. When it acts upon $\lambda(0)$ the resulting eigenvalue vanishes (so we dropped this term), and that $E = i\frac{\partial}{\partial t}$.
the *particle spinors*; and call the \( \rho(p) \) spinors *antiparticle spinors*, in accord with the common terminology prevalent in neutrino physics. However, this identification is very different in spirit from the original notion of particles and antiparticles. As such, it appears advised to introduce a new term that specifies these particles and antiparticles as *Majorana particles/antiparticles*.

As regards their properties under the charge and charge conjugation operators, there are thus two type of particles in Nature. The \( e^\pm, \mu^\pm, \tau^\pm \), and the known quarks, are all of the first — Dirac — type. They are eigenstates of the charge operator. The Klapdor-Kleingrothaus et al. [2,3] experiment addresses itself to \( \nu_e \) and \( \bar{\nu}_e \); and finds them to be of Majorana type. This second type of particles are eigenstates of the charge conjugation operator. So beyond pinning down the masses of \( \nu_e \) and \( \bar{\nu}_e \), discovering lepton number violation, the Klapdor-Kleingrothaus et al. experiment for the first time takes us into a realm of spacetime structure which so far had remained only in the sphere of theoretical constructs. That newly unearthed structure, as we are making the case here, must be treated in its own right, and not as an afterthought to the celebrated Dirac construct.

6 Metamorphosis of Majorana \( \bar{\nu} \) to \( \nu \): Propagator for the Majorana field

The completeness relation (34) suggests that we define the Majorana field operator as:

\[
\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p_0} \sum_h \left( \lambda_h^S(p)a_h(p)e^{-ip \cdot x} + \lambda_h^A(p)a_h^\dagger(p)e^{ip \cdot x} \right).
\] (51)

Parenthetically, we take note of the fact that, using Eqs. (35), it may be re-written entirely in terms of the self-conjugate \( \lambda^S(p) \) and \( \rho^S(p) \) spinors. Any relative signs that different helicity Majorana spinors may carry in the indicated sum are implicitly assumed to be absorbed in the annihilation, \( a_h(p) \), and creation, \( a_h^\dagger(p) \), operators. The latter are assumed to satisfy the bi-orthogonality respecting fermionic anticommutators,

\[
\{a_h(p), a_{h'}^\dagger(p')\} = (2\pi)^3 2p_0 \delta^3(p-p') \delta_{h,-h'}.
\] (52)

By postulating this field-theoretic structure we hope to avoid negative norm states, while preserving the essential bi-orthonormality of the underlying \((1/2,0) \oplus (0,1/2)\) Majorana representation space. In equation (52), if \( h = \dagger \), then \(-h\) represents \( \downarrow \), and if \( h = \downarrow \), then \(-h\) represents \( \dagger \). With these defi-
nitions in mind, the configuration-space Feynman-Dyson (FD) propagator is obtained as

\[ \langle x|S_{FD}|y \rangle = \langle \text{vac}|T [\mu(x)\bar{\mu}(y)]|\text{vac} \rangle, \]

in the standard contextual meaning of the used symbols. A straightforward calculation yields:

\[ \langle x|S_{FD}|y \rangle = \sum_h \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p_0} \left[ \lambda^S_h(p)\overline{\lambda}^{S}_{(-h)}(p) e^{-ip\cdot(x-y)} \theta \left( x^0 - y^0 \right) 
- \lambda^A_h(p)\overline{\lambda}^{A}_{(-h)}(p) e^{ip\cdot(x-y)} \theta \left( y^0 - x^0 \right) \right]. \]  

(53)

As a consequence, the momentum-space propagator is obtained to be,

\[ \langle p'|S_{FD}|p \rangle = \int \frac{d^4 x}{(2\pi)^4} \int \frac{d^4 y}{(2\pi)^4} e^{ip\cdot x - ip'\cdot y} \langle x|S_F|y \rangle \]

\[ = i\delta^4(p' - p) \sum_h \left( \frac{\lambda^S_h(p)\overline{\lambda}^{S}_{(-h)}(p)}{p_0 - E(p) + i\epsilon} - \frac{\lambda^A_h(p)\overline{\lambda}^{A}_{(-h)}(-p)}{p_0 + E(p) - i\epsilon} \right), \]

(54)

where \( E(p) = +\sqrt{m^2 + p^2} \). As the above propagator explicitly exhibits, an exchanged Majorana particle is emitted with one helicity, at one vertex, and is absorbed with opposite helicity, at the second vertex. This is the spacetime picture of \( \nu \leftrightarrow \nu \) metamorphosis for Majorana particles. Using Eqs. (35) and (36), the preceding statement can be made more transparent. Exploiting the indicated identities, one may re-express \( \langle p'|S_{FD}|p \rangle \) by replacing the \( \overline{\lambda}^{S}_{(-h)}(p) \) and \( \overline{\lambda}^{A}_{(-h)}(-p) \) in terms of \( \overline{\rho}^S_{(h)}(p) \) and \( \overline{\rho}^A_{(h)}(-p) \), respectively,

\[ \langle p'|S_{FD}|p \rangle = \delta^4(p' - p) \sum_h \left( \frac{-\lambda^S_h(p)\overline{\rho}^A_{(h)}(p) + \lambda^S_h(p)\overline{\rho}^A_{(-h)}(p)}{p_0 - E(p) + i\epsilon} 
- \frac{\lambda^A_h(p)\overline{\rho}^S_{(-h)}(-p) - \lambda^A_h(p)\overline{\rho}^S_{(h)}(-p)}{p_0 + E(p) - i\epsilon} \right), \]

(55)

In the massless limit, depending on \( h \), either \( \lambda_h(p) \), or \( \rho_h(p) \), identically vanishes. Thus, each of the \( \lambda_h(p)\overline{\rho}_h(p) \) terms contained in \( \langle p'|S_{FD}|p \rangle \) also vanishes. In the massless limit, this results in a non-existent (i.e., identically vanishing) propagator. This implication is in accord with the conventional framework which also predicts that the amplitude for \( 0\nu\beta\beta \) vanishes for massless neutrinos. The new propagator, in addition, manifestly shows how the \( \nu \leftrightarrow \overline{\nu} \) metamorphosis arises from the underlying spacetime structure.
For comparison, the standard Dirac propagator can be cast \[19\] into a form similar to (55):

\[
\langle p' | S_{FD} | p \rangle = i \delta^4(p' - p) \sum_h \left( \frac{u_h(p) \bar{u}_h(p)}{p_0 - E(p) + i\epsilon} - \frac{v_h(-p) \bar{v}_h(-p)}{p_0 + E(p) - i\epsilon} \right). \tag{57}
\]

7 Concluding remarks

Klapdor-Kleingrothaus et al. experiment provides first evidence towards the Majorana nature of massive $\nu_e$, and the associated $\nu_e \equiv \bar{\nu}_e$ metamorphosis. It is an encouraging development towards discovering new spacetime structures beyond the time-honored spacetime constructs. Of these supersymmetry is the most natural and sought after.\(^6\)

Here, we defined Majorana spinors as self/antiself conjugate eigenspinors of the charge conjugation operator. These spinors have null norm; and their biorthonormal norm is necessarily imaginary. We obtained the associated completeness relation, and the relevant wave equation. We considered the notion of particle and antiparticle for Majorana particles, and examined the latter aspect in the context of Dirac and Feynman-Stückelberg frameworks. Finally, we presented the Majorana field as a quantum object, and obtained the Feynman-Dyson propagator. The latter explicitly exhibits the $\nu \equiv \bar{\nu}$ metamorphosis for Majorana particles, whether they be neutrinos, or massive gaugions.

Figure 1 presents the $\nu_e \equiv \bar{\nu}_e$ metamorphosis as it emerges from the formalism presented in this Letter. In presenting this spacetime picture we do not necessarily assume the neutrinos to be left-handed objects; thus leaving room for physics beyond the standard model. Generalization of this spacetime picture to supersymmetric frameworks, and left-right symmetric theories, is implicit in the presented formalism.

Acknowledgments

This work is supported by Consejo Nacional de Ciencia y Tecnología (CONACyT, Mexico) under grant number 32067-E, and by an affiliate agreement with

\(^6\) Besides predictions of supersymmetry, discovery of Wigner-type particles \[20\], where a massive spin one particle and its antiparticle carry opposite relative intrinsic parities, would be a significant benchmark for new type of matter fields in nature. Equally important would be confirmation of the new spacetime structures associated with massive gravitino \[15\] and the massive gauge fields \[21\].
Fig. 1. $\nu_e \equiv \bar{\nu}_e$ metamorphosis in $0\nu\beta\beta$ as seen from the vantage point of the formalism presented in this Letter.

the Los Alamos National Laboratory. Thanks are extended to T. Goldman, Y. Liu, and U. Sarkar for useful remarks on an earlier draft of this work.

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