THEORETICAL ASPECTS OF HEAVY-FLAVOUR ELECTROPRODUCTION

J. SMITH *
Deutsches Elektronen-Synchrotron DESY, Theory Group,
Notkestrasse 85, D-22607 Hamburg, Germany
E-mail: jacksmit@mail.desy.de

ABSTRACT

We discuss three theoretical schemes to describe charm quark electroproduction.

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1. Introduction

The study of charm electroproduction has become an important issue in the extraction of parton densities in the proton. The reason is that the charm content $F_{2,c}(x, Q^2, m^2)$, where $m = m_{charm}$, has grown from around one percent of the contribution to the structure function $F_2(x, Q^2)$ in the $x$ and $Q^2$ region of the EMC experiment to around twenty-five percent in the $x$ and $Q^2$ region of the H1 and ZEUS experiments at HERA. Therefore the analysis of parton densities in the proton can no longer be done without considering c-quark production.

Let us begin with a brief review of some of the technical points in the calculations of c-quark electroproduction in QCD. We consider $F_{2,c}(x, Q^2, m^2)$ which is usually much larger than $F_{L,c}(x, Q^2, m^2)$. We work in a three flavour number scheme (TFNS) where the u, d and s are light mass quarks ($n_f = 3$). The c $\bar{c}$ pair is produced from the gluon by the Bethe-Heitler process (photon-gluon fusion) in leading order (LO). In next-to-leading order (NLO) both the Bethe-Heitler and the Compton processes involve all light mass quarks (and antiquarks) and the gluon. A NLO calculation organizes the contributions from the various Feynman diagrams according to the following formula

$$F_{2,c}(3, x, Q^2, m^2) = x \int_{x}^{z_{\text{max}}} \frac{dz}{z} \left\{ \frac{1}{3} \sum_{k=1}^{3} e_k^c \left[ \Sigma(3, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) L_{2,q}^S(3, z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) \right] + G(3, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) L_{2,g}^S(3, z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) \right\}$$

where $e_c$ is the charge of the c-quark. The variable $z$ is the partonic longitudinal momentum fraction and $z_{\text{max}} = Q^2/(Q^2 + 4m^2)$. The function $\Delta$ is the non-singlet (with respect to the flavour SU(3) group) combination of light-mass parton densities,

*On leave from The Institute for Theoretical Physics, SUNY at Stony Brook, NY 11794-3840, USA
at the scale $\mu$. The function $\Sigma$ is the singlet combination of these densities while $G$ is the density for the gluon. Further $L_{2,k}$ and $H_{2,k}(k=q,g)$ are the singlet and non-singlet heavy-quark coefficient functions at scale $\mu$. Since they contain factors of $\alpha_s$ we have explicitly indicated that they depend on $n_f = 3$. The $L_{2,k}$ describe the reactions where the virtual photon couples to the light quarks (u, d, and s) and the $H_{2,k}$ the reactions where it couples to the $c$-quark.

The NLO contributions to Eq.(1) were originally calculated yielding single-particle inclusive transverse momentum and rapidity distributions of the c-quark. The functions $L_{2,k}$ and $H_{2,k}$ were only available in the form of two-dimensional integrals. To speed up the computation of Eq.(1) we made two-dimensional grids of values for $L_{2,k}$ and $H_{2,k}$ together with an interpolation routine. The NLO calculation was repeated in a completely exclusive fashion so that one can plot all the distributions containing the c-quark, the $\bar{c}$ antiquark and the additional parton. This program incorporates a fragmentation function so that one can calculate distributions for $D^{*\pm}$ mesons. These programs have been used by many authors to discuss the sensitivity of the NLO results to changes in parton densities, renormalization/factorization scales, and the mass of the c-quark. Finally the asymptotic formulae for $L_{2,k}$ and $H_{2,k}$ in the limit $Q^2 \gg m^2$ were calculated.

Now we outline the scheme used for these NLO calculations. The renormalization of the virtual graphs with loops containing light quarks was done in the $\overline{\text{MS}}$-scheme while loops with heavy quarks were subtracted at zero external momentum. This scheme was originally proposed so that the light quark sector remains the same as in a purely massless pQCD calculation. Hence there is a matching condition at the scale $\mu = m$ for both $\alpha_s$ and for the parton densities. To explain this consider QCD with only massless quarks. Then the number of flavours enters via the factors $n_f$ in the $\beta$-function and in the gluon splitting function $P_{gg}$. We solve the equation

$$\frac{\partial}{\partial \ln \mu^2} \left( \frac{\alpha_s}{\pi} \right) = -\beta_0 \left( \frac{\alpha_s}{\pi} \right)^2 - \beta_1 \left( \frac{\alpha_s}{\pi} \right)^3 - \cdots$$

(2)

where

$$\beta_0 = \frac{1}{4} \left( \frac{11}{3} C_A - \frac{4}{3} T_f n_f \right), \quad \beta_1 = \frac{1}{16} \left( \frac{34}{3} C_A^2 - 4 C_F T_f n_f - \frac{20}{3} C_A T_f n_f \right),$$

(3)

by introducing the $\Lambda_{\overline{\text{MS}}}(n_f)$. The solution of Eq.(2) is

$$\frac{\alpha_s(n_f,\mu)}{\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda_{\overline{\text{MS}}}^2)} - \frac{\beta_1}{\beta_0^2} \ln(\ln(\mu^2/\Lambda_{\overline{\text{MS}}}^2)) + \cdots$$

(4)

A matching condition is therefore necessary to go from a TFNS with $n_f = 3$ to a FFNS with $n_f = 4$. We have to change the value of $\Lambda_{\overline{\text{MS}}}$ as we cross the scale $\mu = m$ to keep
the condition that $\alpha_s(4, \mu = m) = \alpha_s(3, \mu = m)$. However heavy particles cannot be ignored in QCD because they exist in vacuum fluctuations. It is well-known that effects from very heavy particles can be absorbed into an equivalent field theory at a smaller scale by an appropriate renormalization of the coupling constants and fields (decoupling of heavy flavours). QCD is an unbroken gauge theory which does not have heavy particle decoupling when renormalized in the $\overline{\text{MS}}$-scheme as all quarks have $m = 0$. However the top quark cannot be considered massless and virtual top-quark loops yield explicit terms in $\ln \mu^2/m^2$ (here $m = m_{\text{top}}$) so there has to be a matching condition between say a five flavour theory and a six flavour theory. This two-loop matching condition on $\alpha_s$ was first worked out in $\overline{\text{MS}}$ and corrected in $\overline{\text{MS}}$. It reads

$$\frac{\alpha_s(6, \mu)}{\pi} = \frac{\alpha_s(5, \mu)}{\pi} + \left(\frac{\alpha_s(5, \mu)}{\pi}\right)^2 \ln \left(\frac{\mu^2}{m^2}\right) + \left(\frac{\alpha_s(5, \mu)}{\pi}\right)^3 \left[\frac{1}{2} T_f \ln^2 \left(\frac{\mu^2}{m^2}\right) + 12 T_f C_A - 6 T_f \frac{C_A}{2} \right] + \ldots \quad (5)$$

Note that $\alpha_s(6, \mu = m) = \alpha_s(5, \mu = m)$ in order $\alpha_s$ but not in order $\alpha_s^2$.

The parton densities must satisfy similar matching conditions to Eq. (5). In a NLO calculation the collinear singularities are regularized by $n$-dimensional regularization. We set the mass factorization scale equal to the renormalization scale $\mu$ for simplicity. Only the gluon-gluon splitting function

$$P_{gg}(n_f, z) = 2 C_A \left[z \left(\frac{1}{1 - z}\right) + \frac{1 - z}{z} + z(1 - z)\right] + \frac{1}{6} (11 C_A - 4 T_f n_f) \delta(1 - z), \quad (6)$$

depends on the number of light quarks $n_f$. The c-quark is considered heavy with mass $m = m_{\text{charm}}$ and we calculate the $L_{2,k}$ and $H_{2,k}$ in Eq. (1) in the TFNS. There is no charm density at the scale $\mu = m$ but it does exist for $\mu > m$. This means that the light flavour densities $f_k(x, \mu), k = u, d, s, \bar{u}, \bar{d}, \bar{s}$ satisfy

$$f_k(4, x, \mu = m) = f_k(3, x, \mu = m). \quad (7)$$

One can examine the DGLAP equations for the parton densities when one chooses a scale with $\mu > m$. By taking the difference of the solutions with $n_f = 4$ and $n_f = 3$ and using Eq. (6) one finds that only the gluon density changes in order $\alpha_s$, namely

$$f_g(4, x, \mu^2) = f_g(3, x, m^2) \left[1 - \frac{1}{3 \pi} T_f \alpha_s(3, \mu) \mu^2 m^2\right], \quad (8)$$

and there is a c-quark density, proportional to $\alpha_s(3, \mu) \ln(\mu^2/m^2)$, which grows at the expense of the gluon density. The changes in the light flavour densities begin in order $\alpha_s^2$. Hence at scales $\mu \gg m$ we can switch to a FFNS which contains a (massless) c-quark density and a different momentum sum rule. Eq. (8) is a matching condition on the gluon density between the TFNS and the FFNS at the scale $\mu = m$. 
A way to implement a smooth matching of $F_{2,c}(x, Q^2, m^2)$ from the TFNS in Eq. (1) to a charm density description in the FFNS was proposed in [20], and we call it the ACOT scheme. Their scale choice, which we will use in the rest of this article, is

$$\mu^2 = m^2 + kQ^2(1 - m^2/Q^2)^n \quad \text{for} \quad Q^2 > m^2,$$

$$= m^2 \quad \text{for} \quad Q^2 \leq m^2,$$

with $k = 0.5$, $n = 2$ and $m = 1.5 \text{ (GeV/c}^2\text{)}$. They proposed that $F_{2,c}^{ACOT,(1)}(x, Q^2, m^2)$ should consist of three terms. The first one is the TFNS LO process in Eq.(1), which has the correct kinematics near threshold. It contains the convolution of the gluon density in the proton with the LO hard scattering cross section. The third term is a FFNS charm density, i.e., the coefficient function in the proton structure function $F_2(x, Q^2)$ is simply a $\delta$-function, and this should be the best description at $\mu^2 \gg m^2$. Finally the second term is the product of two convolutions. Namely that of the gluon density in the proton with the massless $c$-density in the gluon and with the LO hard scattering term (a $\delta$-function). This second term is taken with a negative sign so that it cancels the FFNS $c$-density term at small scales $\mu^2 \approx m^2$ while it tends to cancel the TFNS term when $\mu^2 \gg m^2$. The sum of these three terms is more stable under a variation of the scale $\mu$ than each term separately. In Fig.1 we show the scale dependence of $F_{2,c}(x, Q^2, m^2)$ for the TFNS LO term from Eq.(1) (labelled EXACT,(1)), the ACOT description (labelled ACOT,(1) and the FFNS charm density (labelled PDF,(1)). We choose the same CTEQ parton densities[21] as in[20]. The monotonic decrease in the EXACT result, which is due to the decrease in $\alpha_s$, is modified in the other two approaches. In Fig.2 we plot the three results for
Figure 2: $F_{2,c}(x = 0.01)$ as a function of $Q$.

$F_{2,c}(x = 0.01, Q^2)$ versus $Q = \sqrt{Q^2}$. One sees that there is a large difference between the EXACT result (TFNS) and the PDF charm density result (FFNS) especially as $Q$ increases. However the charm density result cannot be used at small $Q$ where it is negative. As expected the ACOT result interpolates between the EXACT result at small $Q$ and the charm density PDF result at large $Q$. The ACOT formula has been incorporated into recent parton densities (the set CTEQ4HQ\textsuperscript{22}). Also there are now three light flavour (CTEQ4F3) and four light flavour (CTEQ4F4) global fits to parton densities. The former is basically equivalent to the parton density set in GRV94\textsuperscript{23}. An examination of the CTEQ4HQ set shows that the gluon is diminished from previous sets\textsuperscript{21} and that all the light quark densities have changed, not just the one for charm. This is due to the interplay between the gluon and the light flavour densities in making a global fit to all the experimental data.

As the ACOT description contains algebraic terms in $m$, it is not obvious how to generalize it to higher orders in $\alpha_s$. (One proposal has been made in which we cannot discuss here for lack of space). One would like to retain as much as possible a pQCD description based on massless $\beta$ and $\gamma$ functions in the renormalization group equation, and work in the $\overline{\text{MS}}$ scheme. In we have worked out the two-loop matching conditions on the flavour densities by the following procedure. Analytic formulae for the functions $H_{2,k}$ in Eq.(1) are not known. In the limit that $Q^2 \gg m^2$ they reduce to expressions like $a(z) \ln^2(Q^2/m^2) + b(z) \ln(Q^2/m^2) + c(z)$ (where we call them $H_{2,k}^{\text{ASYMP}}$). If we can find $a(z), b(z)$ and $c(z)$ then we can mass factorize the logarithms into terms involving $\ln(Q^2/\mu^2)$ and $\ln(\mu^2/m^2)$. If we then add the result to the two-loop light-flavour corrections\textsuperscript{25} to the proton structure function $F_2(3, x, Q^2, \mu^2)$ then the terms
in \( \ln(Q^2/\mu^2) \) should recombine to yield the two-loop corrections to \( F_2(4, x, Q^2, \mu^2) \). Note that \( F_{2,c} \) must be totally inclusive so we have to add the light quark vertex corrections where the gluon propagator contains a charm quark loop, which yields terms in \( \delta(1-z) \). When the correct pieces have been identified we are left over with terms in \( \ln^2(\mu^2/m^2) \) and \( \ln(\mu^2/m^2) \) which cannot be absorbed into the known matching conditions on \( \alpha_s \). We can then define two-loop matching conditions for the parton densities to incorporate these pieces, analogous to the two loop relations for \( \alpha_s \) in Eq. (5). We actually derived \( L_{2,k}^{ASYMP}, H_{2,k}^{ASYMP} \) and the relations between the parton densities by calculating all two-loop matrix elements of the operators in the OPE, where one loop contains a massless quark and the other loop a massive quark.

The result is that above the scale \( \mu = m \) we have new FFNS parton densities \( n_f = 4 \) which are calculable from a TFNS set \( n_f = 3 \). All densities are changed. The light quark densities pick up additional terms in order \( \alpha_s^2 \), which are numerically rather small, but are required for the FFNS parton densities to satisfy the momentum sum rule. The FFNS charm density is given in terms of the previous TFNS densities and involves terms in order \( \alpha_s \) and \( \alpha_s^2 \), i.e.,

\[
f_{c+\bar{c}}^{FFNS}(4, x, \mu^2) \equiv f_4(4, x, \mu^2) + f_{\bar{4}}(4, x, \mu^2) = \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \int_x^1 \frac{dz}{z} \sum(3, \frac{x}{z}, \mu^2) \tilde{A}_{cq}^{PS,(2)}(z, \frac{\mu^2}{m_c^2}) + \int_x^1 \frac{dz}{z} G(3, \frac{x}{z}, \mu^2) \left[ \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \tilde{A}_{cg}^{S,(1)}(z, \frac{\mu^2}{m_c^2}) + \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \tilde{A}_{cg}^{S,(2)}(z, \frac{\mu^2}{m_c^2}) \right],
\]  

(10)
Figure 4: $F_{2,c}(x = 0.01)$ as functions of $Q$. 

The above formula is for a FFNS renormalized in the $\overline{\text{MS}}$ prescription. Using the FFNS one can now calculate $F_{2,c}\text{PDF}(4, x, Q^2, m^2)$ in the region where $Q^2 \gg m^2$ and include the higher order QCD effects. We expect this description to be valid in the region $Q^2 \gg m^2$ where the TFNS formula in Eq. (1) is dominated by the mass factorization logarithms. We have checked numerically that the asymptotic result for the NLO formulae, which we call $F_{2,c}\text{ASYMP}(3, x, Q^2, m^2)$ is a good approximation to the exact NLO result in Eq.(1), which we call $F_{2,c}\text{EXACT}(3, x, Q^2, m^2)$ when $Q^2 \approx 10 m^2$. Hence
Figure 5: The functions $F_{2,c}^{PDF,2}(4, x, Q^2)$ dashed line, $F_{2,c}^{VFNS,2}(x, Q^2, m^2)$ solid line and $F_{2,c}^{EXACT,2}(3, x, Q^2, m^2)$ dotted line, plotted as a functions of $Q^2$ for fixed $x$. The data points are from the EMC, H1 and ZEUS experiments.
above \( Q^2 \approx 10 m^2 \) we have the option of retaining the TFNS description for charm or to switching to the FFNS which sums large logarithms in \( \ln(Q^2/\mu^2) \).

It now remains to find a formula which allows us to interpolate between the TFNS and the FFNS and is valid in all orders in \( \alpha_s \). In\cite{13} and\cite{14} we have proposed the variable flavour number scheme (VFNS) formula

\[
F_{2,c}^{\text{VFNS}}(x, Q^2, m^2) = F_{2,c}^{\text{EXACT}}(3, x, Q^2, m^2) - F_{2,c}^{\text{ASYMP}}(3, x, Q^2, m^2) + F_{2,c}^{\text{PDF}}(4, x, Q^2). \tag{12}
\]

This formula is designed like the ACOT one. We have all the pieces to test it in order \( \alpha_s^2 \). (Full details are available in\cite{15}). The TFNS ASYMP term cancels the TFNS EXACT term at large \( \mu \) leaving the FFNS charm density PDF term. However neither the ASYMP term nor the charm density PDF term have the correct kinematics (they also have different \( n_f \) and electric charges) to cancel each other exactly at small \( \mu \) (near threshold). Using as input the same CTEQ densities\cite{21} as in\cite{15}, we have calculated (1) \( F_{2,c}^{\text{EXACT}} \), the TFNS formulae in Eq.(1) (2) \( F_{2,c}^{\text{VFNS}} \), the VFNS formula in Eq.(12) and (3) \( F_{2,c}^{\text{PDF}} \), the massless FFNS PDF result. In (3) we have evaluated the two convolutions to go from a TFNS to a FFNS and then to add the higher order corrections\cite{25}. In Figs. 3 and 4 we show the analogous results to those in Figs. 1 and 2. One sees that the scale sensitivity is mostly reduced for the TFNS EXACT result. The PDF result is worse at small \( Q \), which is a consequence of the fact that some of the higher order contributions in QCD are negative. The VFNS result Eq. (12) interpolates nicely between the other two. Finally in Fig. 5 we compare the three descriptions for \( F_{2,c} \) with the data from the EMC, H1 and ZEUS experiments. Here we use the GRV92\cite{26} parton densities. One sees that the VFNS interpolation is quite smooth as compared to that in\cite{24}. The massless FFNS results fall dramatically at small \( Q \), due to the choice of \( \mu \) in Eq. (9). All three curves are within the present experimental errors of the H1 and ZEUS experiments. Therefore we will have to wait for more precise data before we can make a firm statement about which theoretical approach is the best.

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