Soft-rough interval-valued fuzzy matrix
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Abstract
Motivated by the theory of soft-rough fuzzy matrix and interval-valued fuzzy set, our aim in this paper is to introduce the notion of soft-rough interval-valued fuzzy matrix (S-RIVFM). Various properties of S-RIVFM are discussed in this paper.

Keywords
Soft-rough set, soft-rough fuzzy set, interval-valued set, interval-valued fuzzy set.

AMS Subject Classification
03B52, 90B50, 91B06, 91B30.

1. Introduction
Pawlak \textsuperscript{7} took the task of defining uncertainty by means of rough set. He discussed the vagueness by dividing the set into two approximations namely upper and lower approximation. Molodtsov \textsuperscript{5} did a through study on the existing uncertainty theories till 1999 and pointed out the difficulties that arises in those theories. He derived a remarkable theory namely soft sets that gives the solution to the problem of uncertainty prevailed till 1999. Feng et al.\textsuperscript{3} derived a new hybrid structure namely soft-rough set. In the same paper they extended the idea of soft-rough set (SR-set) in fuzzy setting and named it as SRFS.

Membership function concept was initiated by the eminent personality Zadeh \textsuperscript{11}. His idea motivated Thomason \textsuperscript{8} to combine matrix theory together with fuzzy sets. In recent years combining of unpredictability theories and developing new mixed models is the task of several researchers. Vijayabalaji \textsuperscript{9} extended the mixed model namely soft-rough set to matrix setting and termed as SR- matrices. He also provided a decision theory on it. Later Vijayabalaji\textsuperscript{10} extended this idea as generalized SR- matrices.

Mixed theories namely soft-rough fuzzy matrix (SRFM) and interval-valued fuzzy set (IVFS) motivated for the development of this paper. Interesting operation on soft-rough interval-valued fuzzy matrix (in short S-RIVFM) are derived.

Some basic definitions on soft and rough sets are provided in section 2.

The third section exhibits our new approach on soft-rough set and fuzzy set theory namely soft-rough fuzzy matrix. Various interesting operations on this matrix is provided with suitable results and examples.

2. Preliminaries
Essential results needed for the paper are listed below.

Definition 2.1. \textsuperscript{3} Let $U$ be the universe set, $A$ be a set of parameters, $P(U)$ denotes the power set of $U$ and $f : A \rightarrow P(U)$ be a soft set. The soft approximation space is denoted by $S = (U, f)$ with two operations, $A_S(X) = \{ u \in U : \exists a \in A, (u \in f(a), f(a) \subseteq X) \}$, $B_S(X) = \{ u \in U : \exists a \in A, (u \in f(a), f(a) \subseteq \complement X \neq \phi) \}$, for every $X \subseteq U$. Note that $A_S(X)$ and $B_S(X)$ mean the lower and upper SR- approximation of $X$ in $U$. Furthermore $Poss(X) = A_S(X)$ $Neg_S(X) = U - B_S(X)$. 

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We now define a special function $I_{\lambda}$ \cite{3} Full soft set is a soft set $\mathcal{S} = (f, A)$ over $U$ with the condition $\bigcup_{a \in A} f(a) = U$.

Definition 2.3. \cite{3} Suppose a full soft set is defined in the soft approximation space $S = (U, \mathcal{S})$, then for a given fuzzy set $\lambda \in \mathcal{S}(U)$, with $SR_{\text{apr}}(\lambda)$ and $SR_{\text{ap}}(\lambda)$, namely lower and upper SR-approximation with respect to $S$ are have $SR_{\text{apr}}(\lambda)(u) = \bigwedge \{ \lambda(v) : \exists a \in A([u, v] \subseteq f(a)) \}$, and $SR_{\text{ap}}(\lambda)(u) = \bigvee \{ \lambda(v) : \exists a \in A([u, v] \subseteq f(a)) \}$, for all $u \in U$. They are also termed as lower SR-approximation operators and upper SR-approximation operators on $\lambda$. When they coindice $\lambda$ is called as soft definable, if not SRFS.

3. S-RIVFM and its properties

The new notion of S-RIVFM is as follows.

Definition 3.1. Let $\mathcal{S} = (f, A)$ be a full soft set and $S = (U, \mathcal{S})$ be a soft approximation space. For a interval-valued fuzzy set $\lambda \in \mathcal{S}(U)$ the lower and upper SR-approximations of $\lambda$ with respect to $S$ are denoted by $\lambda_{\text{apr}}$ and $\lambda_{\text{ap}}$ respectively,

$\lambda_{\text{apr}}(u) = \bigwedge \{ \lambda(v) : \exists a \in A([u, v] \subseteq f(a)) \}$

and

$\lambda_{\text{ap}}(u) = \bigvee \{ \lambda(v) : \exists a \in A([u, v] \subseteq f(a)) \}$

Define,

$S_{1} = \{ u | \exists (s, t) \in \lambda_{\text{ap}}(u) = \lambda \}$

$S_{2} = \{ u | \exists (s, t) \in \lambda_{\text{apr}}(u) = \lambda \}$

and

$S_{3} = \{ u | u \notin S_{1} \cup u \notin S_{2} \}$

We now define a special function $I_{SR} : U \rightarrow D[0, 1]$ by

$I_{SR} = \begin{cases} \top, & \text{if } u \in S_{1} \\ \bot, & \text{if } u \in S_{2} \\ \alpha, & \text{if } u \in S_{3}, \text{where } \alpha \in D[0, 1]. \end{cases}$

A soft-rough interval-valued fuzzy matrix is a matrix whose elements are in $S - \text{RIVFM}$.

\[
\begin{pmatrix}
\frac{c_{11}}{\overline{c}_{11}} & \frac{c_{12}}{\overline{c}_{12}} & \ldots & \frac{c_{1n}}{\overline{c}_{1n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{c_{m1}}{\overline{c}_{m1}} & \frac{c_{m2}}{\overline{c}_{m2}} & \ldots & \frac{c_{mn}}{\overline{c}_{mn}}
\end{pmatrix}
\]

where $c_{ij} \in S - \text{RIVFM}$.

Example 3.2. Let $U = \{c_{1}, c_{2}, c_{3}, c_{4}, c_{5}\}$ where $c_{i}(i = 1, 2, \ldots, 5)$ stand for "scorpio", "BMW 3series", "celerio", "econ", "renault duster" be the set of 5 models cars, $E = \{e_{1}, e_{2}, e_{3}, e_{4}, e_{5}\}$ where $e_{i}(i = 1, 2, \ldots, 5)$ stand for "speed", "costly", "branded", "comfort" and "durability", respectively be the set of parameters. Now, let us consider a soft set $(F, E)$ which describes the "speed of the car" that Mr. Z is considering for purchase. Now $F(E_{1})$ means cars (branded) and the set is consisting for all the branded cars in $U$. Let $F(e_{1}) = \{c_{8}\}$; $F(e_{2}) = \{c_{1}, c_{4}\}$; $F(e_{3}) = \{c_{1}, c_{2}, c_{3}\}$ and $F(e_{4}) = \{c_{3}, c_{5}\}$.

Let $A = \{e_{1}, e_{2}, e_{3}, e_{4}\}$.

$S = (U, \mathcal{S})$ be a soft rough approximation space with the following table.

| $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
|--------|--------|--------|--------|--------|
| $e_{1}$ | 0      | 0      | 0      | 1      |
| $e_{2}$ | 1      | 0      | 0      | 1      |
| $e_{3}$ | 1      | 1      | 1      | 0      |
| $e_{4}$ | 0      | 0      | 1      | 0      |

Define an interval-valued fuzzy set $\lambda$ as follows.

$\lambda = \left\{ \begin{array}{c}
\{0.2, 0.4\} & \{0.3, 0.6\} & \{0.4, 0.7\} & \{0.5, 0.7\} & \{0.8, 0.9\}
\end{array} \right\}$

Then

$\lambda_{\text{apr}}(u) = \left\{ \begin{array}{c}
\{0.2, 0.4\} & \{0.2, 0.4\} & \{0.2, 0.4\} & \{0.2, 0.4\} & \{0.4, 0.7\}
\end{array} \right\}$

and

$\lambda_{\text{ap}}(u) = \left\{ \begin{array}{c}
\{0.8, 0.9\} & \{0.4, 0.7\} & \{0.8, 0.9\} & \{0.5, 0.7\} & \{0.8, 0.9\}
\end{array} \right\}$

We have $S_{1} = \{c_{1}, c_{3}\}$

$S_{2} = \{c_{4}, c_{5}\}$

$S_{3} = \{c_{2}\}$.

The soft-rough interval-valued fuzzy matrix is.

$I_{SR} = \begin{pmatrix}
\top & \top & \top & \top & \top \\
\top & \top & \top & \top & \top \\
\top & \top & \top & \top & \top \\
\top & \top & \top & \top & \top \\
\top & \top & \top & \top & \top
\end{pmatrix}$

Definition 3.3. Let $[\delta_{ij}]$ and $[\eta_{ij}] \in S - \text{RIVFM}$. Then the union of $[\delta_{ij}]$ and $[\eta_{ij}]$ is defined by $[\delta_{ij}] \cup [\eta_{ij}] = \max\{[\delta_{ij}], [\eta_{ij}]\}$

Definition 3.4. Let $[\delta_{ij}]$ and $[\eta_{ij}] \in S - \text{RIVFM}$. Then the intersection of $[\delta_{ij}]$ and $[\eta_{ij}]$ is defined by $[\delta_{ij}] \cap [\eta_{ij}] = \min\{[\delta_{ij}], [\eta_{ij}]\}$.

Definition 3.5. The complement of an soft-rough interval-valued fuzzy matrix $S - \text{RIVFM}$ is defined as $\overline{\psi}_{ij} = \top - \psi_{ij}$ for all $\psi_{ij} \in S - \text{RIVFM}$.

Theorem 3.6. Let $[\delta_{ij}]$ and $[\eta_{ij}] \in S - \text{RIVFM}$. Then

(i) $([\delta_{ij}] \cup [\eta_{ij}])^c = [\delta_{ij}]^c \cap [\eta_{ij}]^c$.

(ii) $([\delta_{ij}] \cap [\eta_{ij}])^c = [\delta_{ij}]^c \cup [\eta_{ij}]^c$.

Proof. (i) For all $i$ and $j$.

$([\delta_{ij}] \cup [\eta_{ij}])^c = \max\{[\delta_{ij}], [\eta_{ij}]\}^c$

$= \top - \max\{[\delta_{ij}], [\eta_{ij}]\}$

$= \min\{[\delta_{ij}], [\eta_{ij}]\}$

$= [\delta_{ij}]^c \cap [\eta_{ij}]^c$
(ii) For all \( i \) and \( j \)
\[
(\overline{\delta}_{ij} \cap \eta_{ij})^c = (\min(\overline{\delta}_{ij}, \overline{\eta}_{ij}))^c
\]
\[
= T - \min(\overline{\delta}_{ij}, \overline{\eta}_{ij})
\]
\[
= \max\{T - \overline{\delta}_{ij}, T - \overline{\eta}_{ij}\}
\]
\[
= [\delta_{ij}]^c \cup [\eta_{ij}]^c.
\]

(iii)
\[
[\delta_{ij}] \cap [\eta_{ij}] = \min\{[\delta_{ij}], [\eta_{ij}]\}, \forall i, j
\]
\[
= \min\{[\eta_{ij}], [\delta_{ij}]\}, \forall i, j
\]
\[
= [\eta_{ij}] \cap [\delta_{ij}].
\]

(iv)
\[
[\delta_{ij}] \cup [\eta_{ij}] = \max\{[\delta_{ij}], [\eta_{ij}]\}, \forall i, j
\]
\[
= \max\{[\eta_{ij}], [\delta_{ij}]\}, \forall i, j
\]
\[
= [\eta_{ij}] \cup [\delta_{ij}].
\]

Theorem 3.7. Let \( \overline{\delta}_{ij}, [\overline{\eta}_{ij}] \) and \( [\overline{\psi}_{ij}] \in S - RIVFM \). Then

(i) \( (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}] \cap [\overline{\psi}_{ij}]) = (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}]) \cap ([\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}]) \).

Proof. (i) For all \( i \) and \( j \),
\[
(\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}] \cap [\overline{\psi}_{ij}])
\]
\[
= \max\{\overline{\delta}_{ij}, \min([\overline{\eta}_{ij}], [\overline{\psi}_{ij}])\}
\]
\[
= \max\{\overline{\delta}_{ij}, \min([\overline{\eta}_{ij}], [\overline{\psi}_{ij}])\}
\]
\[
= \min\{\max(\overline{\delta}_{ij}, [\overline{\eta}_{ij}]), \max(\overline{\delta}_{ij}, [\overline{\psi}_{ij}])\}
\]
\[
= (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}]) \cap ([\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}]).
\]

(ii) For all \( i \) and \( j \),
\[
(\overline{\delta}_{ij} \cap ([\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}])
\]
\[
= \min\{\overline{\delta}_{ij}, [\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}])
\]
\[
= \min\{\overline{\delta}_{ij}, \max([\overline{\eta}_{ij}], [\overline{\psi}_{ij}])\}
\]
\[
= \min\{\overline{\delta}_{ij}, \max([\overline{\eta}_{ij}], [\overline{\psi}_{ij}])\}
\]
\[
= (\overline{\delta}_{ij} \cap [\overline{\eta}_{ij}]) \cup ([\overline{\eta}_{ij}] \cap [\overline{\psi}_{ij}]).
\]

Theorem 3.8. Let \( \overline{\delta}_{ij} \) and \( [\overline{\eta}_{ij}] \in S - RIVFM \). Then

(i) \( \overline{\delta}_{ij} \cup \overline{\delta}_{ij} = \overline{\delta}_{ij} \).

(ii) \( \overline{\delta}_{ij} \cap \overline{\delta}_{ij} = \overline{\delta}_{ij} \).

(iii) \( \overline{\delta}_{ij} \cap \overline{\eta}_{ij} = \overline{\eta}_{ij} \cap \overline{\delta}_{ij} \).

(iv) \( \overline{\delta}_{ij} \cup \overline{\eta}_{ij} = [\overline{\eta}_{ij}] \cup [\overline{\delta}_{ij}] \).

Proof. (i) \( \overline{\delta}_{ij} \cup \overline{\delta}_{ij} = \overline{\delta}_{ij} \).
\[
\overline{\delta}_{ij} \cup \overline{\delta}_{ij} = \overline{\delta}_{ij} \cup [\overline{\delta}_{ij}], \forall i, j
\]
\[
= \max\{[\delta_{ij}], [\delta_{ij}]\}, \forall i, j
\]
\[
= [\delta_{ij}].
\]

(ii) \( \overline{\delta}_{ij} \cap \overline{\delta}_{ij} = \overline{\delta}_{ij} \).
\[
\overline{\delta}_{ij} \cap \overline{\delta}_{ij} = \overline{\delta}_{ij} \cap [\overline{\delta}_{ij}], \forall i, j
\]
\[
= \min\{[\delta_{ij}], [\delta_{ij}]\} \forall i, j
\]
\[
= [\delta_{ij}].
\]

Theorem 3.9. Let \( \overline{\delta}_{ij}, [\overline{\eta}_{ij}] \) and \( [\overline{\psi}_{ij}] \in S - RIVFM \). Then

(i) \( (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}]) = \overline{\delta}_{ij} \cup [\overline{\eta}_{ij}] \).

(ii) \( (\overline{\delta}_{ij} \cap [\overline{\eta}_{ij}] \cup [\overline{\psi}_{ij}]) = \overline{\delta}_{ij} \cap [\overline{\eta}_{ij}] \).

Theorem 3.10. Let \( \overline{\delta}_{ij} \) and \( [\overline{\eta}_{ij}] \in S - RIVFM \). Then

(i) \( (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}])^c = T - (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}])^c \).

(ii) \( (\overline{\delta}_{ij} \cap [\overline{\eta}_{ij}])^c = \overline{\delta}_{ij} \cap [\overline{\eta}_{ij}]^c \).

Proof. (i) For all \( i \) and \( j \),
\[
(\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}])^c
\]
\[
= T - (\overline{\delta}_{ij} \cup [\overline{\eta}_{ij}])
\]
\[
= T - \max(\overline{\delta}_{ij}, [\overline{\eta}_{ij}])
\]
\[
= \min(T - \overline{\delta}_{ij}, T - [\overline{\eta}_{ij}])
\]
\[
= \min([\delta_{ij}]^c, [\overline{\eta}_{ij}]^c)
\]
\[
= [\overline{\psi}_{ij}]^c \cap [\overline{\delta}_{ij}]^c.
\]
(ii) For all \( i \) and \( j \),
\[
(\delta_{ij} \cap \eta_{ij})^c = T - (\delta_{ij} \cap \eta_{ij}) = \{x \in S \mid T - x \in \delta_{ij} \cap \eta_{ij}\}.
\]

\[= \min\{\max(\delta_{ij}, \eta_{ij})\} \cap \min\{\max(\delta_{ij}, \eta_{ij})\} \cap \eta_{ij} \cap \eta_{ij}^c.
\]

Theorem 3.11. Let \( \delta_{ij}, \eta_{ij} \) and \( \psi_{ij} \) be elements of \( S - RIVFM \). Then
(i) \( (\delta_{ij} \cup \eta_{ij}) \cap \psi_{ij} = \max(\delta_{ij}, \eta_{ij}) \cap \psi_{ij} \)
(ii) \( (\delta_{ij} \cap \eta_{ij}) \cup \psi_{ij} = (\delta_{ij} \cap \eta_{ij}) \cup (\eta_{ij} \cap \psi_{ij}) \).

Proof. Let \( \delta_{ij}, \eta_{ij} \) and \( \psi_{ij} \) be elements of \( S - RIVFM \).

(i)
\[
(\delta_{ij} \cup \eta_{ij}) \cap \psi_{ij} = \max\{(\delta_{ij}, \eta_{ij}) \cap \psi_{ij}\} = \min\{\max(\delta_{ij}, \eta_{ij})\} \cup \psi_{ij} = \max\{\min(\delta_{ij}, \eta_{ij}), \min(\eta_{ij}, \psi_{ij})\} = \max\{(\delta_{ij}, \eta_{ij}) \cap \psi_{ij}\} \cup \min\{(\delta_{ij}, \eta_{ij}) \cap \psi_{ij}\}
\]

(ii)
\[
(\delta_{ij} \cap \eta_{ij}) \cup \psi_{ij} = \min\{(\delta_{ij}, \eta_{ij}) \cup \psi_{ij}\} = \max\{\min(\delta_{ij}, \eta_{ij}), \max(\eta_{ij}, \psi_{ij})\} = \max\{(\delta_{ij}, \eta_{ij}) \cup \psi_{ij}\} \cup \min\{(\delta_{ij}, \eta_{ij}) \cup \psi_{ij}\}
\]

Theorem 3.12. Let \( \delta_{ij} \) be an element of \( S - RIVFM \). Then
\( \delta_{ij} \cup \delta_{ij} = \delta_{ij} \cap \delta_{ij} = \delta_{ij} \).

Proof. Follows from Theorem (3.8) in (i) & (ii)

Theorem 3.13. Let \( \delta_{ij} \) and \( \eta_{ij} \) be elements of \( S - RIVFM \). Then
(i) \( (\delta_{ij} \cup \eta_{ij})^c = \delta_{ij} \cap \eta_{ij} \)
(ii) \( (\delta_{ij} \cap \eta_{ij})^c = \delta_{ij} \cap \eta_{ij} \).

Proof. Let \( \delta_{ij} \) and \( \eta_{ij} \) be elements of \( S - RIVFM \).

(i)
\[
(\delta_{ij} \cup (\delta_{ij} \cap \eta_{ij})) = \max\{\delta_{ij}, (\delta_{ij} \cap \eta_{ij})\} = \max\{\delta_{ij}, \min(\delta_{ij}, \eta_{ij})\} = \delta_{ij}
\]

(ii)
\[
(\delta_{ij} \cap (\delta_{ij} \cup \eta_{ij})) = \min\{\delta_{ij}, (\delta_{ij} \cup \eta_{ij})\} = \min\{\delta_{ij}, \max(\delta_{ij}, \eta_{ij})\} = \delta_{ij}
\]

Definition 3.14. Let \( \delta_{ij} \in S - RIVFM \). Then the Transpose of \( \delta_{ij} \) is defined as \( \delta_{ij}^T = \delta_{ji} \).

Theorem 3.15. Let \( \delta_{ij} \in S - RIVFM \). Then
(i) \( (\delta_{ij} \cup \delta_{ji})^T = \delta_{ji} \)
(ii) \( (\delta_{ij} \cap \delta_{ji})^T = \delta_{ji} \)
(iii) \( (\delta_{ji}^T)^T = \delta_{ji} \).

Proof. Let \( \delta_{ij} \in S - RIVFM \) and \( \delta_{ji}^T = \delta_{ji} \).

(i) \( (\delta_{ij} \cup \delta_{ji})^T = \max(\delta_{ij}, \delta_{ji})^T = \delta_{ji}^T \)
(ii) \( (\delta_{ij} \cap \delta_{ji})^T = \min(\delta_{ij}, \delta_{ji})^T = \delta_{ji}^T \)
(iii) \( (\delta_{ji}^T)^T = \delta_{ji}^T = \delta_{ji} \).

Theorem 3.16. Let \( \delta_{ij} \in S - RIVFM \). Then
(i) \( (\delta_{ij} \cup \eta_{ij})^T = \delta_{ji} \cup \eta_{ji} \)
(ii) \( (\delta_{ij} \cap \eta_{ij})^T = \delta_{ji} \cap \eta_{ji} \).

Proof. (i) For all \( i,j \)
\[
(\delta_{ij} \cup \eta_{ij})^T = \left(\max(\delta_{ij}, \eta_{ij})\right)^T,
\]
\[
= \max\{\delta_{ij}, \eta_{ij}\}^T = \delta_{ji}^T \cup \eta_{ji}.
\]

(ii) For all \( i,j \)
\[
(\delta_{ij} \cap \eta_{ij})^T = \left(\min(\ delta_{ij}, \eta_{ij})\right)^T,
\]
\[
= \min\{\delta_{ij}, \eta_{ij}\}^T = \delta_{ji}^T \cap \eta_{ji}.
\]

Definition 3.17. Let \( \delta_{ij} \) and \( \eta_{ij} \in S - RIVFM \). Then the
not intersection of \( \delta_{ij} \) and \( \eta_{ij} \) is defined by \( \delta_{ij} \cap \eta_{ij} = \max\{\delta_{ij} \cap \eta_{ij}, \delta_{ij} \cap \eta_{ij}^c\} \).

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**Definition 3.18.** Let $[\delta_{ij}]$ and $[\eta_{ij}] \in S - \text{RIV FM}$. Then the not union of $[\delta_{ij}]$ and $[\eta_{ij}]$ is defined by $[\delta_{ij}] \cup [\eta_{ij}] = \min\{T - \delta_{ij}, T - \eta_{ij}\}$.

**Theorem 3.19.** Let $[\delta_{ij}]$ and $[\eta_{ij}] \in S - \text{RIV FM}$. Then

(i) $([\delta_{ij}] \cap [\eta_{ij}])^c = ([\delta_{ij}]^c \cup [\eta_{ij}]^c)^c$.

(ii) $([\delta_{ij}] \cup [\eta_{ij}])^c = ([\delta_{ij}]^c \cap [\eta_{ij}]^c)^c$.

**Proof.** (i) For all $i$ and $j$,

\[
([\delta_{ij}] \cap [\eta_{ij}])^c = (\max\{T - \delta_{ij}, T - \eta_{ij}\})^c
= T - \max\{T - \delta_{ij}, T - \eta_{ij}\}
= \min\{T - (T - \delta_{ij}), T - (T - \eta_{ij})\}
= \min\{T - [\delta_{ij}]^c, T - [\eta_{ij}]^c\}
= [\delta_{ij}]^c \cup [\eta_{ij}]^c.
\]

(ii) For all $i$ and $j$,

\[
([\delta_{ij}] \cup [\eta_{ij}])^c = (\min\{T - \delta_{ij}, T - \eta_{ij}\})^c
= T - \min\{T - \delta_{ij}, T - \eta_{ij}\}
= \max\{T - (T - \delta_{ij}), T - (T - \eta_{ij})\}
= \max\{T - [\delta_{ij}]^c, T - [\eta_{ij}]^c\}
= [\delta_{ij}]^c \cap [\eta_{ij}]^c.
\]

\qed

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