On minimal actions of finite groups on Euclidean spaces and spheres

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Abstract. We prove that the minimal dimension of a faithful, smooth action of a finite group on a Euclidean space coincides with the minimal dimension of a faithful, linear action of the group (i.e., with the minimal dimension of a faithful, real, linear representation), for various classes of metacyclic, linear fractional, symmetric and alternating groups. We prove the analogous result also for actions on (homology) spheres.

1. Introduction

We consider faithful, smooth actions of finite groups on homology spheres and on Euclidean spaces. The main problem considered is the following: For which finite groups $G$ does the minimal dimension of a faithful, smooth action of $G$ on a sphere or on a Euclidean space coincide with the minimal dimension of a faithful, linear action (that is, induced from a faithful, real, linear representation of the group).

Our first main result concerns actions on Euclidean spaces $\mathbb{R}^n$.

Theorem 1. The minimal dimension of a faithful, smooth (orientation-preserving) action on a Euclidean space coincides with the minimal dimension of a faithful, linear (orientation-preserving) action on a Euclidean space, for the following finite groups:

i) a metacyclic group (semidirect product) $\mathbb{Z}_p \rtimes \mathbb{Z}_q$, for a prime $p$ and a positive integer $q$, with an effective action of $\mathbb{Z}_q$ on the normal subgroup $\mathbb{Z}_p$;

ii) a linear fractional groups $\text{PSL}(2, p)$, for a prime $p$;

iii) a symmetric group $\text{S}_p$, for a prime $p$;

iv) an alternating group $\text{A}_p$, for a prime $p$ such that $p \equiv 3 \mod 4$.

See section 3 for the various minimal dimensions. At present, we do not know an example of a finite group for which the two minimal dimensions differ.

Next we consider actions on spheres. In the following, a mod $p$ homology sphere is a closed $n$-manifold with the mod $p$-homology of the $n$-sphere (i.e., homology with coefficients in the integers mod $p$).
Theorem 2. For the groups listed in Theorem 1, the minimal dimension of a faithful, smooth (orientation-preserving) action on a mod \( p \) homology sphere coincides with the minimal dimension of a faithful, linear (orientation-preserving) action on a sphere.

By [DH], Theorem 2 is true for finite \( p \)-groups, see also [D] for the case of orientation-preserving actions of finite abelian groups. However, there are finite solvable groups for which the two minimal dimensions do not coincide; specifically, the Milnor groups \( Q(8a, b, c) ([Mn]) \) do not admit faithful, linear actions on \( S^3 \) but by [Mg] some of them admit a free action on a homology 3-sphere. Again we do not know such an example for a faithful, smooth action on a genuine sphere (but see the discussion at the end of section 3 on continuous actions).

The main tool of the proofs of Theorems 1 and 2 will be an analysis of the Borel spectral sequence associated to a group action, generalizing the approach in [Z1]. In section 3, we discuss arbitrary continuous actions of some other finite groups on homology spheres and Euclidean spaces.

We note that the finite simple (and nonsolvable) groups which admit a smooth action on a homology sphere in dimension three or four have been determined in [MZ1-3] and [Z2]: the only simple group which occurs in dimension three is the alternating group \( A_5 \cong \text{PSL}(2,5) \), in dimension four there is in addition the group \( A_6 \cong \text{PSL}(2,9) \). The classification of the simple groups acting in dimension five is still open. In contrast, it is likely that all groups \( \text{PSL}(2, p) \) admit an action already on a mod 2 homology 3-sphere, see [Z3] for a discussion and various examples for small values of \( p \). We note that every finite group admits a free action on a rational homology 3-sphere, see [CL].

2. The Borel spectral sequence associated to a group action

Let \( G \) be a finite group acting on a space \( X \). Let \( EG \) denote a contractible space on which \( G \) acts freely, and \( BG = EG/G \). We consider the twisted product \( X_G = EG \times_G X = (EG \times X)/G \). The "Borel fibering" \( X_G \to BG \), with fiber \( X \), is induced by the projection \( EG \times X \to EG \), and the equivariant cohomology of the \( G \)-space \( X \) is defined as \( H^*(X_G; K) \). Our main tool will be the Leray-Serre spectral sequence \( E(X) \) associated to the Borel fibration \( X_G \to BG \),

\[
E_2^{i,j} = H^i(BG; H^j(X; K)) = H^i(G; H^j(X; K)) \Rightarrow H^{i+j}(X_G; K),
\]

i.e. converging to the graded group associated to a filtration of \( H^*(X_G; K) \) ("Borel spectral sequence", see e.g. [Bd]); here \( K \) denotes any abelian coefficient group or commutative ring.

Now suppose that \( G \) acts on a (open or closed) \( n \)-manifold \( M \); we denote by \( \Sigma \) the singular set of the \( G \)-action (all points in \( M \) with nontrivial stabilizer). Crucial for the
homomorphism. 

The cohomology ring $H^*(M; K)$ is isomorphic to $H^*(\Sigma; K)$.

We use this to prove the following:

**Proposition 1.** In dimensions greater than $n$, inclusion induces isomorphisms

$$H^*(M; K) \cong H^*(\Sigma; K).$$

**Proposition 2.** For an odd prime $p$ and a positive integer $q$, let $G = \mathbb{Z}_p \rtimes \mathbb{Z}_q$ be a semidirect product with an effective action of $\mathbb{Z}_q$ on the normal subgroup $\mathbb{Z}_p$. Suppose that $G$ admits a faithful action on a manifold $M$ with the mod $p$ homology of the $n$-sphere, and that the subgroup $\mathbb{Z}_p$ of $G$ acts freely. Then $n + 1$ is a multiple of $2q$ if all elements of $G$ act as the identity on $H^n(M; \mathbb{Z}_p) \cong \mathbb{Z}_p$ (the "orientation-preserving case"), and an odd multiple of $q$ if some element of $G$ acts as minus identity (the "orientation-reversing case").

**Proof.** We consider first the Borel spectral sequence $E(\Sigma)$ converging to the cohomology of $\Sigma_G$, with $E^2_{i,j} = H^i(G; H^j(\Sigma; Z_p))$. Let $\Sigma_q$ denote the singular set of the subgroup $\mathbb{Z}_q$ of $G$. The singular set $\Sigma$ of $G$ is the disjoint union of the singular sets of the $p$ conjugates of $\mathbb{Z}_q$ in $G$; these fixed point sets are all disjoint since the action of $\mathbb{Z}_q$ on $\mathbb{Z}_p$ is effective and the action of $\mathbb{Z}_p$ is free, by assumption. Now the action of $G$ on the cohomology $H^j(\Sigma; Z_p)$ of the orbit $Z_p(\Sigma_q) = \Sigma$ is induced (or co-induced) from the action of $\mathbb{Z}_q$ on $H^j(\Sigma_q; Z_p)$, and by Shapiro’s Lemma ([Bw, Proposition III.6.2]), $H^i(G; H^j(\Sigma; Z_p))$ is isomorphic to $H^i(\mathbb{Z}_q; H^j(\Sigma_q; Z_p))$ and hence trivial, for $i > 0$. So also $H^*(\Sigma_G; Z_p)$ is trivial, in positive dimensions. By Proposition 1, also $H^*(M_G; Z_p) \cong H^*(\Sigma_G; Z_p)$ is trivial, in sufficiently large dimensions.

Next we analyze the spectral sequence $E(M)$ converging to the cohomology of $M_G$. The $E_2$-terms $E_2^{i,j} = H^i(G; H^j(M; Z_p))$ are concentrated in the two rows $j = 0$ and $j = n$ where they are equal to $Z_p$, with a possibly twisted action of $G$ on $H^n(M; Z_p) \cong Z_p$. In particular, the only possibly nontrivial differentials of $E(M)$ are $d_1^{i,j} : E_2^{i,n} \to E_2^{i+n,0}$, of bidegree $(n + 1, -n)$.

By [Bw, Theorem III.10.3], for $i > 0$

$$H^i(G; H^j(M; Z_p)) \cong H^i(Z_p; H^j(M; Z_p))^Z_q.$$

The cohomology ring $H^*(\mathbb{Z}_p; Z_p)$ is the tensor product of a polynomial algebra $\mathbb{Z}_p[t]$ on a 2-dimensional generator $t$ and an exterior algebra $E(s)$ on a 1-dimensional generator $s$ (see [AM, Corollary II.4.2]); also, $t$ is the image of $s$ under the mod $p$ Bockstein homomorphism.
Suppose first that $\mathbb{Z}_q$ acts trivially on $H^n(M; \mathbb{Z}_p) \cong \mathbb{Z}_p$. Since the action of $\mathbb{Z}_q$ on $\mathbb{Z}_p$ is effective, also the action of $\mathbb{Z}_q$ on $H^1(\mathbb{Z}_p; \mathbb{Z}_p)$ and $H^2(\mathbb{Z}_p; \mathbb{Z}_p)$ is effective: denoting by $\sigma$ the action of a generator of $\mathbb{Z}_q$ on the cohomology $H^i(\mathbb{Z}_p; \mathbb{Z}_p)$, one has $\sigma(s) = ks$ and $\sigma(t) = kt$, for some integer $k$ representing an element of order $q$ in $\mathbb{Z}_p$. Hence $\sigma(t^a) = k^a t^a$ and the only powers of $t$ fixed by $\sigma$ are those divisible by $q$ (in dimensions $i$ which are even multiples of $q$); similarly, $\sigma(st^a) = k^{a+1} t^{a+1}$, so the only elements in odd dimensions fixed by $\sigma$ are the products $st^a$ such that $a+1$ is a multiple of $q$ (in dimensions $i$ such that $i+1$ is an even multiple of $q$). Consequently, $H^i(G; H^j(M; \mathbb{Z}_p))$ is nontrivial exactly for $j = 0$ and $n$ and in dimensions $i$ such that either $i$ or $i+1$ is a multiple of $2q$.

Since $H^*(M_G; \mathbb{Z}_p) \cong H^*(\Sigma G; \mathbb{Z}_p)$ is trivial in sufficiently large dimensions, the differentials $d_{n+1}$ of the spectral sequence $(E^2_{2, j}(M) = H^i(G; H^j(M; \mathbb{Z}_p))$ (concentrated in the rows $j = 0$ and $n$) have to be isomorphisms in large dimensions. This can happen only if $n + 1$ is a multiple of $2q$, which completes the proof of Proposition 2 in the orientation-preserving case.

Now suppose that a generator of $\mathbb{Z}_q$ acts as minus identity on $H^n(M; \mathbb{Z}_p) \cong \mathbb{Z}_p$, in particular $q$ is even. Denoting by $\tilde{\sigma}$ the action of a generator of $\mathbb{Z}_q$ on the cohomology $H^i(\mathbb{Z}_p; H^n(M; \mathbb{Z}_p))$, we now have that $\tilde{\sigma}(t^a) = -\sigma(t^a) = -k^a t^a$, $\tilde{\sigma}(st^a) = -\sigma(st^a) = -k^{a+1} t^{a+1}$. Now the elements of $H^i(\mathbb{Z}_p; H^n(M; \mathbb{Z}_p)) = H^i(\mathbb{Z}_p; \mathbb{Z}_p)$ invariant under $\tilde{\sigma}$ are the powers of $t$ by odd multiples of $q/2$ (in dimensions $i$ which are odd multiples of $q$), and the products $st^a$ such that $a + 1$ is an odd multiple of $q/2$ (in dimensions $i$ such that $i + 1$ is an odd multiple of $q$). Hence $H^i(G; H^j(M; \mathbb{Z}_p))$ is nontrivial exactly in the following situations: either $j = 0$ and $i$ or $i + 1$ is an even multiple of $q$ (since the action of $G$ on $H^0(M; \mathbb{Z}_p)$ is trivial), or if $j = n$ and $i$ or $i + 1$ is an odd multiple of $q$ (with the twisted action of $G$ on $H^n(M; \mathbb{Z}_p)$). Since again $H^*(M_G; \mathbb{Z}_p) \cong H^*(\Sigma G; \mathbb{Z}_p)$ is trivial in large dimensions, the differential $d_{n+1}$ has to be an isomorphism and $n + 1$ an odd multiple of $q$. (Note that, in order to obtain just the lower bound $n \geq q - 1$, one may apply the orientation-preserving case to the subgroup $\mathbb{Z}_p \times \mathbb{Z}_{q/2}$ of $\mathbb{Z}_p \times \mathbb{Z}_q$.)

This completes the proof of Proposition 2.

3. Proofs of Theorems 1 and 2

We start with the

Proof of Theorem 1.

i) We note that the minimal dimension of a faithful, real, linear representation of a group $G = \mathbb{Z}_p \times \mathbb{Z}_q$ as in Theorem 1 i) is equal to $2q$ if $q$ is odd, to $q + 1$ if $q$ is even and $G$ acts orientation-preservingly, and to $q$ is some element of $G$ reverses the orientation.

Suppose that $G = \mathbb{Z}_p \times \mathbb{Z}_q$ admits a faithful, smooth action on Euclidean space $\mathbb{R}^n$. By Smith fixed point theory (see [Bd]), the fixed point set $F$ of the subgroup $\mathbb{Z}_p$ of $G$
is $\mathbb{Z}_p$-acyclic manifold of some dimension $d \geq 0$, in particular non-empty. Note that the action of $G$ extends to a continuous action of $G$ on the sphere $S^n$ and, again by Smith fixed point theory, the fixed point set $\hat{F}$ of the subgroup $\mathbb{Z}_p$ on $S^n$ has the mod $p$ homology of a sphere of dimension $d$. Now by Lefschetz-duality with coefficients in $\mathbb{Z}_p$, applied to the pair $(S^n, \hat{F})$, the complement $\mathbb{R}^n - F = S^n - \hat{F}$ has the mod $p$ homology of a sphere of dimension $n - d - 1$. Hence $G$ admits a faithful, smooth action on the manifold $\mathbb{R}^n - F$, with the mod $p$ homology of a sphere of dimension $m = n - d - 1$, such that the normal subgroup $\mathbb{Z}_p$ of $G$ acts freely. Now Proposition 2 implies that $n - d - 1 \geq 2q - 1$ hence $n \geq 2q$ if $q$ is odd, and $n - d - 1 \geq q - 1$ and $n \geq q$ if $q$ is even.

Suppose that $q$ is even and that $G$ acts orientation-preservingly on $\mathbb{R}^n$. If the fixed point set $F$ of $\mathbb{Z}_p$ has dimension $d = 0$ then $F$ is a single point which hence is a global fixed point of $G$. Now $G$ acts orientation-preservingly on the boundary of an invariant neighbourhood of the fixed point which is a sphere of dimension $n - 1$, and Proposition 2 implies that $n \geq 2q$. On the other hand, if $d > 0$ then, as noted above, $n - d - 1 \geq q - 1$, hence $n \geq q + d \geq q + 1$.

Since these lower bounds for the dimension of a faithful, smooth action of $G = \mathbb{Z}_p \rtimes \mathbb{Z}_q$ on a Euclidean space coincide with the minimal dimension of a faithful, linear action of the group, this completes the proof of part i) of Theorem 1.

ii) The minimal dimension of a faithful, real, linear representation of a linear fractional group $\text{PSL}(2, p)$ is equal to $p - 1$ if $p \equiv 3 \mod 4$, and to $(p + 1)/2$ if $p \equiv 1 \mod 4$.

The subgroup of $\text{PSL}(2, p)$ represented by all upper triangular matrices is a semidirect product $\mathbb{Z}_p \rtimes \mathbb{Z}_q$, with $q = (p - 1)/2$, where $\mathbb{Z}_q$ is the subgroup represented by all diagonal matrices and $\mathbb{Z}_p$ by all matrices with both diagonal entries equal to one; also, $\mathbb{Z}_q$ acts effectively on the normal subgroup $\mathbb{Z}_p$. By i), the minimal dimension of a faithful, smooth, orientation-preserving action of $\mathbb{Z}_p \rtimes \mathbb{Z}_q$ on an Euclidean space is $2q = p - 1$ if $q$ is odd, and to $q + 1 = (p + 1)/2$ if $q$ is even. Since this coincides with the minimal dimension of a faithful, linear action of $\text{PSL}(2, p)$ this proves part ii) of the Theorem.

iii) and iv) The minimal dimension of a faithful, linear action of a symmetric group $S_p$ on a Euclidean space is $p - 1$ (for an arbitrary positive integer $p$), that of an alternating group $A_p$ is $p - 1$, for integers $p > 5$.

For an odd prime $p$, consider the semidirect product $G = \mathbb{Z}_p \rtimes \mathbb{Z}_{p - 1}$, with an effective action of $\mathbb{Z}_{p - 1}$ on the normal subgroup $\mathbb{Z}_p$. The action by left-multiplication of $G$ on the $p$ left cosets of the subgroup $\mathbb{Z}_{p - 1}$ of $G$ realizes $G$ as a subgroup of the symmetric group $S_p$. By i), the minimal dimension of a faithful, smooth action of $G$ on a Euclidean space is equal to $p - 1$ which is also the minimal dimension of a faithful, real representation of the symmetric group $S_p$, thus proving iii).

For the proof of iv), we consider the subgroup $\mathbb{Z}_p \rtimes \mathbb{Z}_{(p - 1)/2}$ of index two of $\mathbb{Z}_p \rtimes \mathbb{Z}_{p - 1}$ which is realized as a subgroup of the alternating group $A_{p}$. For $p > 5$, the minimal
dimension of a faithful, real representation of $A_p$ is $p − 1$; if $(p − 1)/2$ is odd, by part i) of the Theorem this coincides with the minimal dimension of a faithful, smooth action of $\mathbb{Z}_p \ltimes \mathbb{Z}_{(p−1)/2}$ on an Euclidean space.

This finishes the proof of iv) and of Theorem 1.

Proof of Theorem 2.

We recall that the minimal dimension of a faithful, linear action on a sphere of a group $G = \mathbb{Z}_p \ltimes \mathbb{Z}_q$ as in Theorem 1 is equal to $2q − 1$ if $q$ is odd, to $q$ if $q$ is even and $G$ acts orientation-preservingly, and to $q + 1$ is some element of $G$ reverses the orientation.

Suppose that $G = \mathbb{Z}_p \ltimes \mathbb{Z}_q$ admits a faithful, smooth action on a mod $p$ homology $n$-sphere $M$. If the subgroup $\mathbb{Z}_p$ of $G$ acts freely, Proposition 2 implies that $n + 1$ is a multiple of $2q$ if $G$ acts orientation-preservingly, and an odd multiple of $q$ otherwise. In particular, the minimal possibilities are $n = 2q − 1$ and $n = q − 1$, respectively, which coincide with the minimal dimension of a faithful, linear action on a sphere.

Suppose then that $\mathbb{Z}_p$ has nonempty fixed point set $F$. By Smith fixed point theory, $F$ is a mod $p$ homology sphere of some dimension $d$, $0 ≤ d < n$; by duality, the complement $M − F$ is a $G$-invariant manifold with the mod $p$ homology of a sphere of dimension $n − d − 1$. Proposition 2 implies now that $n − d − 1 ≥ 2q − 1$ hence $n ≥ 2q$ if $q$ is odd, $n − d − 1 ≥ q − 1$ and $n ≥ q$ if $q$ is even.

This completes the proof of Theorem 2 for the groups $\mathbb{Z}_p \ltimes \mathbb{Z}_q$. For all other groups, similar as in the proof of Theorem 1 it is a consequence of the case of $\mathbb{Z}_p \ltimes \mathbb{Z}_q$.

4. Comments on continuous actions of some other finite groups

We present some other finite groups for which the two minimal dimensions coincide, even for the case of arbitrary continuous actions.

Proposition 3. The minimal dimension of a faithful, continuous action on an Euclidean space (or on a homology sphere) coincides with the minimal dimension of a faithful, linear action for the following finite groups:

the unitary and symplectic groups $\text{PSU}(4,2) \cong \text{PSp}(4,3)$ and $\text{PSp}(6,2)$;

the Weyl or Coxeter groups $E_6$, $E_7$ and $E_8$ of the corresponding exceptional Lie algebras.

This is a consequence of the following well-known result from Smith fixed point theory ([S], see also [BV]; iii) follows from the fact that the fixed point set of an orientation-preserving involution on a manifold is a mod 2 homology manifold of codimension least two, then one applies ii)).
Proposition 4. i) For an odd prime number \( p \), the minimal dimension of a faithful, continuous action of the group \((\mathbb{Z}_p)^k\) on a mod \( p \) acyclic (mod \( p \) homology-) manifold is \( 2k \) (and \( 2k - 1 \) for the case of a mod \( p \) homology sphere).

ii) The minimal dimension of a faithful, continuous action of \((\mathbb{Z}_2)^k\) on a mod 2 acyclic (mod 2 homology-) manifold is \( k \) (and \( k - 1 \) for the case of a mod 2 homology sphere).

iii) The minimal dimension of a faithful, continuous, orientation-preserving action of \((\mathbb{Z}_2)^k\) on a mod 2 acyclic manifold is \( k + 1 \) (and \( k \) for the case of a mod 2 homology sphere).

Proof of Proposition 3. We refer to [Co] for information about the subgroup structure and the character tables of the finite simple groups. The unitary group PSU(4, 2) has a maximal subgroup \((\mathbb{Z}_3)^3\) and a faithful, linear action on \( \mathbb{R}^6 \), so Proposition 4 i) implies that the two minimal dimensions coincide. Also, PSU(4, 2) is a subgroup of index two in the Weyl group of type \( E_6 \) which has also a linear action on \( \mathbb{R}^6 \).

The Weyl group of type \( E_8 \) has a subgroup \((\mathbb{Z}_3)^4\) and a linear action on \( \mathbb{R}^8 \) (see the closely related orthogonal group \( O_{\mathbb{Q}}(2) \) in [Co]), and the result follows again from Proposition 4 i).

The symplectic group \( \text{PSp}(6, 2) \) is a subgroup of index 2 in the Weyl group of type \( E_7 \), and both act linearly on \( \mathbb{R}^7 \). Since \( \text{PSp}(6, 2) \) has a subgroup \((\mathbb{Z}_2)^6\), Proposition 4 iii) applies.

This completes the proof of Proposition 3.

We do not have an example of a finite group \( G \) such that the minimal dimension of a faithful, smooth or continuous action of \( G \) on a Euclidean space is strictly smaller than the minimal dimension of a faithful, real representation. Interesting examples of continuous actions not conjugate to smooth actions can be obtained as follows.

We consider the Milnor groups \( Q(8a, b, c) \) ([Mn]); these groups have periodic cohomology of period four but do not admit a faithful, free, linear action on the 3-sphere. For odd, coprime integers \( a > b > c \geq 1 \), the Milnor group \( Q(8a, b, c) \) is a semidirect product \( \mathbb{Z}_{abc} \rtimes Q(8) \) of a normal cyclic subgroup \( \mathbb{Z}_a \times \mathbb{Z}_b \times \mathbb{Z}_c \cong \mathbb{Z}_{abc} \) by the quaternion group \( Q(8) = \{ \pm 1, \pm i, \pm j, \pm k \} \) of order eight, and \( i, j \) and \( k \) acts trivially on \( \mathbb{Z}_a, \mathbb{Z}_b \) and \( \mathbb{Z}_c \), respectively, and in a dihedral way on the other two.

It has been shown by Milgram [Mg] that some of the Milnor groups \( Q(8a, b, c) \), for odd, coprime integers \( a > b > c \geq 1 \), admit a faithful, free action on a homology 3-sphere; let \( Q \) be a Milnor group which admits such an action on a homology 3-sphere \( M \). By the double suspension theorem (see e.g. [Ca]), the double suspension \( M \ast S^1 \) of \( M \) (the join with the 1-sphere) is homeomorphic to the 5-sphere. Letting \( Q \) act trivially on \( S^1 \), the actions of \( Q \) on \( M \) and \( S^1 \) induce a faithful, continuous, orientation-preserving action
of $Q$ on the 5-sphere with fixed point set $S^1$, and hence also on $\mathbb{R}^5$ (the complement of a fixed point). At present we do not know the minimal dimension of a faithful, smooth (orientation-preserving) action of such a Milnor group $Q$ on an Euclidean space, or the minimal dimension of a faithful, real, linear (orientation-preserving) representation of $Q$.

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