Analysis of the effect of the physicomechanical properties of composite materials of carrier layers on the stress state of sandwich shells with rectangular cutouts

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Abstract. A model is described for studying the effect of the physicomechanical characteristics of composite carrier layers on the stress and strain state of sandwich cylindrical shells weakened by rectangular cutouts. The finite element model considered allows us to perform the layer-by-layer analysis of the stress in a wide range of physicomechanical characteristics of the layers. To obtain effective approximations leading to a high convergence rate of numerical solutions and to build the finite element of the carrier layers, the equations of the general sell theory are used. To build the finite element model of the filler, the elasticity theory equations for a three-dimensional body and the carrier layer finite element approximations obtained are applied. The effect of the physicomechanical characteristics of composite carrier layers on the stress and strain state for sandwich cylindrical shells with rectangular cutouts is studied.

1. Introduction
The interest in sandwich shells, which are characterized by high rates of weight efficiency, specific stiffness, survivability, durability, thermal insulation, soundproofing, vibration isolation and many other important characteristics, is explained by its increasing use in modern rocket, aerospace, shipbuilding, construction and other industries [1]. The use of sandwich shells is constrained, among other things, by the insufficient development of models for a refined analysis of the effect of the physicomechanical properties of layer materials (especially if they are made of composite materials) on the stress and strain state of structures. Therefore, the development of adequate models that allow us to take into account the possibility of varying the physical and mechanical properties not only along the meridional and circumferential coordinates, but also across the thickness of the sandwich shells and the filler layer, which is currently not taken into account accurately enough, is an urgent scientific problem of important applied significance. Especially important is the study of the effect of the physicomechanical properties of composite carrier layers on the stress state of sandwich shells weakened by rectangular cutouts.

To carry out these studies of sandwich shells with cutouts under various fixing and loading conditions, and with varying properties of the layers, it is rather difficult to use analytical methods and
it is necessary to develop models based on numerical approaches and, first of all, the finite element method (FEM).

The most accurate for building models for sandwich and multilayer shell analysis is the broken line hypothesis [2,3].

A more general and accurate, but less common approach is when hypotheses are applied for each layer [4], and the order of the systems of equations depends on the number of layers, which significantly increases the dimension of the matrices and complicates the solution of problems. However, this approach also leads to simplified models or models for a certain class of problems, which often do not meet modern accuracy requirements [5]. For example, one cannot take into account the heterogeneity of the structure and the variation of physicomechanical properties across the thickness of the filler layer. This makes it urgent to develop refined models based on layer-by-layer analysis [5], which make it possible to solve many problems, including the study of the effect of the physicomechanical properties of composite carrier layers on the stress state of sandwich shells weakened by rectangular cutouts.

Layer-by-layer analysis [5] consists in the fact that the wall of the shell, and, if necessary, its layers, and especially the filler, are divided by thickness into additional layers (refined layer-by-layer analysis), which are then joined together. In layered modeling, obtaining analytical solutions for a large number of topical scientific and practical problems in calculating the stress and strain state of laminated shells encounters insurmountable mathematical difficulties. An effective method for solving these problems, allowing to carry out refined analyses with the necessary degree of detail, is the FEM. The proposed approach of refined layer-by-layer analysis consists in constructing blocks of shell finite elements (FE) of the carrier layers and the necessary (to achieve the required accuracy) number of FEs of the filler.

Usually, in studies that use FEM, elements are used with basic functions based on the approximation of displacement fields by the first and second order polynomials [6]. The use of such approximations to achieve acceptable accuracy in many cases forces one to construct very bulky finite element models, which complicates the work and increases computational errors. This greatly complicates, and often makes impossible, modeling with proper accuracy. This problem arises even more acutely in a refined layer-by-layer analysis [7], when the filler, if necessary, is modeled across the thickness by three-dimensional FEs. Therefore, it is relevant to construct effective finite element approximations, which lead to a high convergence rate of numerical procedures and the results obtained, and consequently to a decrease in the required number of FEs and to a decrease in the dimension of finite element models in solving problems.

The development of effective models using the finite element method and other analytical-numerical and numerical-analytical approaches, as well as a study of the effect of the physicomechanical properties of the filler material on the stress and strain state of the layers of sandwich shells, were considered in [8–10].

The purpose of this work is to apply the proposed models based on the refined layer-by-layer analysis to study the effect of physicomechanical properties of composite materials of carrier layers on the stress state of sandwich cylindrical shells weakened by rectangular cutouts.

2. Research methods
We will call effective the model, which leads to a high convergence rate of numerical procedures and the results obtained and, therefore, to a decrease in the number of finite elements required for solving problems and to a decrease in the dimension of the model. The development and application of effective models is especially relevant for a detailed layer-by-layer analysis of the effect of the physicomechanical properties of materials of composite layers on the stress state of sandwich shells, especially weakened by rectangular cutouts [7], since the shell wall and, if necessary, the filler layer are not modeled by FEs only along the meridional and circumferential coordinates, but also across the
thickness of the shell and the filler. If the shell is weakened by cutouts, it is necessary to apply a fine mesh of FE partitions near the holes to achieve acceptable accuracy, that is, it forces to build very bulky models. Moreover, since in practice the sandwich shells with thin and rigid facesheets and a relatively thick and less rigid filler are most widely used, the filler is modeled by three-dimensional elements.

To study the effect of the physicomechanical properties of materials of composite carrier layers on the stress state of sandwich shells with rectangular cutouts, it is proposed to use block finite element models of natural curvature based on the refined approach of layer-by-layer analysis and effective finite element approximations. In this case, the general theory of shells is used to obtain effective approximation functions for shell FEAs and to build finite element models of natural curvature (accurately simulating the geometric shape of the objects under study) for carrier layers.

For a filler layer, the original block three-dimensional FEM of natural curvature are constructed using the method of layer-by-layer analysis of the obtained effective finite element approximation and the elasticity theory relations for a three-dimensional body in curvilinear coordinates. These models make it possible to fairly accurately take into account variations in the physicomechanical properties and other parameters not only in the meridional and circumferential coordinates, but also across the thickness of the filler of sandwich shells.

An important requirement for the functions of finite element approximation is to take into account displacements as solids [7], which make it possible to increase the rate of convergence of numerical procedures and, therefore, reduce the number of FE required for solving problems, which is especially important when applying layer-by-layer analysis models.

The functions of displacements as a rigid body are determined by integrating the Cauchy relations that relate deformations \( \varepsilon^i = \{e_1, e_2, \gamma, \chi, \varepsilon_1, \varepsilon_2, \chi_1, \chi_2\}^T \) to displacements \( \delta^i = \{u, v, w\}^T \) at zero deformations (\( i = 1, 3 \) is the layer number starting from the inner surface of the shell).

The FE of the carrier layers (index c) and the filler (index f) are formed by the section of the cylindrical shell by planes perpendicular to its axis and planes that contain this axis. Coordinate systems are located on the median surfaces of the FEs.

The choice of approximating displacement functions is interconnected with the choice of nodal displacements and the number of nodes. The nodal displacements of the FE of the carrier layers are \( u, v, w \) and the rotation angles of the normal to the median surface. The approximating deformation functions and approximating displacement functions will be written using indefinite coefficients, their number is equal to the number of FE degrees of freedom. Of the twenty coefficients, six were used in defining the rigid body displacement.

In contrast to the majority of FEM studies in which approximating displacement functions are used, approximating deformation functions are used in this work. This significantly increased the convergence rate of numerical procedures, and therefore, allowed to reduce the number of FE required for solving problems, which is especially important when applying layer-by-layer analysis models. But with this approach, the approximating deformation functions must satisfy the equations of continuity of deformation. The fourteen indefinite coefficients remaining after recording the rigid body displacement are distributed between strains, into the expressions of which terms are added that are obtained from the condition that the strain continuity equations are satisfied, as a result we will have

\[
\varepsilon^c = \Omega^i \alpha^c
\]

\( \alpha^c = \{\alpha_{10} \ldots \alpha_{20}\}^T \) is the vector of unknown coefficients, \( \Omega^i \) is the approximating deformation functions matrix shown in table 1.
By integrating the Cauchy relations for the obtained strain expressions (1), the approximating displacement functions caused by the deformation of the element are determined. Adding them to the rigid body displacement, we obtain the approximating displacement functions FE of the carrier layers
\[
T^{c}_{i} \alpha^{c}_{i},
\]
\(T^{c}_{i}\) is the approximating displacement functions matrix for carrier layers FE.

Knowing the approximating displacement functions, it is easy to write the approximating functions of the rotation angles of the normal through the vector \(\alpha^{c}_{i}\). Substituting the coordinates of the nodes in the approximating displacement functions and the approximating functions of the rotation angles of the normal, we obtain the relation of the nodal displacement vector \(q^{c}_{i}\) to the \(\alpha^{c}_{i}\) vector for the carrier layer FEs
\[
q^{c}_{i} = C^{c}_{i}\alpha^{c}_{i} \quad \text{or} \quad \alpha^{c}_{i} = (C^{c}_{i})^{-1} q^{c}_{i}. \]

Taking into account equation (1), knowing the physical law, the expressions for stresses in the carrier layers are written using \(\alpha^{c}_{i}\). FE stiffness matrices are determined by the condition of minimum total potential energy (the variational Lagrange principle).

When constructing a three-dimensional FEM filler layer, in order to avoid discontinuity of the generalized displacements on the interfaces with the carrier layers, the following approach is proposed. For three-dimensional filler FEs that are joined in thickness with FEs of carrier layers, on the curved surfaces, the same number of nodes is selected as for FEs of carrier layers and the same generalized displacements and approximating displacement functions are used as nodal unknown and approximating functions as for FE carrier layers. Thus, the filler FE will have eight nodes — the outer and inner surfaces contain four nodes each.

Since in the developed approach of refined layer-by-layer analysis the filler is modeled across the thickness by the required number of FEs, a linear approximation of displacements along the radial coordinate can be used.

We apply the Cauchy relation for a three-dimensional body in cylindrical coordinates for the filler.

Substituting approximating displacement functions for the filler layer into the indicated Cauchy relations, we obtain
\[
\varepsilon^{f}_{ij} = \Omega^{f}_{ij}\alpha^{f}_{ij},
\]
where \(\Omega^{f}_{ij}\) is the approximating deformation functions matrix for filler FE.

Taking into account the approximating deformation functions, knowing the physical law, the expressions for the stresses in the FE of the filler layer are written using \(\alpha^{f}_{ij}\). The stiffness matrices of the FE of the filler are determined from the condition of the minimum of the total potential energy (the variational Lagrange principle).

Table 1. \(\Omega^{c}_{i}\) matrix for carrier layers FE

| \(i\) | \(\varphi\) | \(-x'\) | \(-x'/2R\) | \(-x'/6R\) | \(-x'/2R\) | \(-x'/6R\) | \(\varphi\) | \(-x/2R\) | \(-x/6R\) | \(x/2R\) | \(x/6R\) | \(R\varphi\) | \(R\varphi^2/2\) | \(1\) |
|-------|----------|--------|---------|---------|---------|---------|--------|--------|--------|---------|---------|---------|---------|--------|
| 1     | \(\varphi\) | \(-x'/2R\) | \(-x'/6R\) | \(-x'/2R\) | \(-x'/6R\) | \(-x/2R\) | \(-x/6R\) | \(x/2R\) | \(x/6R\) | \(R\varphi\) | \(R\varphi^2/2\) | \(1\) |

\(\Omega^{c}_{i}\) is the approximating deformation functions matrix for carrier layers FE.
If the shell layers have varying properties, then setting them for each FE of the layers, we implement the specified law for changing these properties, including across the thickness of the filler. Blocks are formed from layers FEs by means of which stress and strain state modeling of sandwich shells is carried out.

3. Results and discussion
A study of the effect of the physicomechanical properties of composite materials of carrier layers on the stress state of sandwich cylindrical shells with rectangular cutouts is carried out for the following cases:

1- both carrier layers are made of fiberglass with the following physical and mechanical properties:

\[ E_1^f = 2.1 \times 10^4 \text{ MPa} \]
\[ E_2^f = 1.9 \times 10^4 \text{ MPa} \]
\[ G_{12}^f = 0.36 \times 10^4 \text{ MPa} \]
\[ \mu_{12}^f = \mu_{23}^f = 0.1 \]

2- both carrier layers are made of carbon fiber with the following physical and mechanical properties:

\[ E_1^c = 18.2 \times 10^4 \text{ MPa} \]
\[ E_2^c = 0.93 \times 10^4 \text{ MPa} \]
\[ G_{12}^c = 0.44 \times 10^4 \text{ MPa} \]
\[ \mu_{12}^c = \mu_{23}^c = 0.33 \]

3- the inner carrier layer is made of carbon fiber, and the outer one is made of fiberglass with the physicomechanical properties of composite materials described in paragraphs 1 and 2;

4- the inner carrier layer is made of fiberglass, and the outer one is made of carbon fiber, with the above physical and mechanical properties (shown with a dashed line in the figure).

The upper indices for the composite material physicomechanical properties and stresses, and the lower ones for R, h are as follows: 1 and 3 for internal and external carrier layers, respectively, f for filler.

This study of the effect of the physicomechanical properties of composite carrier layers is carried out for sandwich shells with the following parameters:

\[ L/R_0 = 2 \]
\[ H/R_0 = 0.0667 \]
\[ \bar{h}_f = h_f / H = 0.85 \]
\[ \bar{h}_1 = h_1 / H = 0.05 \]
\[ \bar{h}_3 = h_3 / H = 0.05 \]
\[ \bar{t}_a = l_a / R_0 = 0.267 \]
\[ \bar{t}_b = h_b / R_0 = 0.1335 \]
\[ R_0 \] is the shell inner radius, H is the package thickness; \( \bar{t}_a \) and \( \bar{t}_b \) are the axial and circumferential dimensions of the cutout. A sandwich cylindrical shell with fixed ends (axial displacements are allowed) with three identical symmetrically located rectangular cuts is considered. The shell is loaded with internal pressure \( p = 0.1 \text{ MPa} \).

The filler is made of foam with the following physical and mechanical properties:

\[ E_1^f = E_{22}^f = E_{33}^f = 24 \text{ MPa} \]
\[ G_{11}^f = G_{22}^f = G_{12}^f = 10 \text{ MPa} \]

For clarity, we introduce \( E_{24}^f = (E_{22}^f + E_{33}^f) / 2 \). This will make it possible to compare not only the indicated, but also other variants of the physicomechanical properties of the layers. Then case 1 corresponds to \( E_{24}^f = E_{22}^c = 1.9 \times 10^4 \text{ MPa} \); case 2 to \( E_{24}^f = E_{22}^f = 0.9319 \times 10^4 \text{ MPa} \); cases 3 and 4 to \( E_{24}^f = E_{24}^g = E_{24}^c = 1.4159 \times 10^4 \text{ MPa} \). Here the upper indices correspond to: the fiberglass carrier layers (G), the carbon fiber carrier layers (C), the fiberglass inner carrier layer and the carbon fiber outer one (GC), the carbon fiber inner carrier layer and the fiberglass outer one (CG).

Using the developed approach and the considered FEs, the effect of the physicomechanical properties of carrier layer composite materials on the stress state of sandwich cylindrical shells with rectangular cutouts for the above mentioned cases was studied.

The calculation results are shown in the graphs of figure 1.
The effect of the average elastic modulus $E_{22}^{AV}$ on the meridional stresses in the carrier layers for sandwich shells with rectangular cutouts is shown. Stresses in the carrier layers $\sigma_{mn}^{sl}$ relate to $\sigma_{22}^{SL}$ (circumferential stresses in the inner carrier layer of sandwich shells with fiberglass carrier layers with no cutouts) as $\sigma_{mn}^{sl} = \sigma_{mn}^{sl} / \sigma_{22}^{SL}$ $(m = 1, 2; n = 1, 2)$, where $i = 1, 3$ (indices 1 and 3 correspond to the inner and outer carrier layer, respectively); $t$ corresponds to: $(N)$ membrane, $(M)$ moment, $(\Sigma)$ total (on the surface of the layers) stresses. The signs $\Delta$ and $-$ indicate the place in which they are given, in the region of the corner and the middle of the straight edge of the cutout, respectively.

The maximum deflection $w_{\Delta}$ ($w_{\Delta} = w_i / w_0$) ($w_0$ is the deflection of the inner facasheet layer of the shell with fiberglass carrier layers with no cutouts) and the stresses, except for the transverse shear stresses $t$ in the filler layer $\tau_{13\Delta}$, will be the smallest for the case 1. For the case 4 the deflection $w_{\Delta}$ will be increased compared to the case 1 ~ by 10%, $\sigma_{22\Delta}^{11} \sim$ by 4%, but significantly $\tau_{13\Delta}$ will decrease. Unlike the cases 1 and 4, where $\sigma_{22\Delta}^{11}$ are decisive, for the cases 2 and 3 $\sigma_{11-}^{11}$ will be the largest.

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