Strong decays of the explicitly exotic doubly charmed $DDK$ bound state

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(Dated: March 18, 2022)

Nowadays, it is generally accepted that the $DK$ interaction in isospin zero is strongly attractive and the $D_{s0}^*(2317)$ can be described as a $DK$ molecular state. Recent studies show that the three-body $DDK$ system binds as well with a binding energy about $60 - 70$ MeV. The $DDK$ bound state has isospin $1/2$ and spin-parity $0^-$. If discovered either experimentally or in lattice QCD, it will not only provide further support on the molecular nature of the $D_{s0}^*(2317)$, but also provide a way to understand other exotic hadrons expected to be of molecular nature. In the present work, we study its two-body strong decay widths via triangle diagrams. We find that the partial decay width into $DD\pi$ is at the order of $2 - 3$ MeV, which seems to be within the reach of the current experiments such as Belle II. As a result, we strongly recommend this decay channel of the $DDK$ bound state to be searched for experimentally.

PACS numbers:

I. INTRODUCTION

In 2003, the BaBar Collaboration observed a narrow state in the inclusive $D_s^+\pi^0$ invariant mass distribution of the $e^+e^-$ collision at energies near 10.6 GeV [1], i.e., the $D_{s0}^*(2317)$ ($D_{s0}$ for short in the present work), which was later confirmed by the CLEO Collaboration [2] and the Belle Collaboration [3].

Because the low mass, small width, and decay mode of the $D_{s0}$ are quite different from those of a conventional $J^P = 0^+$ $c\bar{s}$ state in the naive quark model, its nature has remained a topic of tremendous theoretical interests ever since its discovery [4-37]. In recent years, the importance of the $DK$ interaction in forming the $D_{s0}$ has been confirmed by lattice QCD simulations [38-42]. See Ref. [43] for a short summary of the theoretical, experimental, and lattice QCD supports for the molecular interpretation of the $D_{s0}$ as a $DK$ bound state.

If the $D_{s0}$ is indeed (dominantly) a $DK$ bound state, a natural question to ask is whether the $DDK$ three-body system is still bound. In Ref. [44], by describing the $D_{s0}DD$ interaction via one kaon exchange (OKE), it was shown that the OKE interaction is strong enough to form a $D_{s0}DD$ molecular state with a binding energy of $50 - 60$ MeV, regardless whether the $D_{s0}DK$ coupling is determined by treating the $D_{s0}$ as a $c\bar{s}$ state or a $DK$ molecule. In Ref. [45], a study was done by explicitly considering the three-body $D(DK - D_s\pi - D_s\eta)$ system and by solving the Faddeev equations using the two-body inputs provided by the unitarized chiral perturbation theory and the local hidden symmetry approach. A three-body bound state was found in this latter work, with a total mass around 4140 MeV, which is an isospin doublet containing two states ($R^{++}, R^+$). In a more recent work [46], using the Gaussian expansion method, the existence of this state has been further confirmed though with a lighter smaller binding energy of $\sim 60 - 70$ MeV and it has been found that even the $DDDK$ or $DDD_{s0}$ system is bound. It is interesting to note that the $DDK$ [47,49], the $DKK$ and $DDK$ [50], as well as the $DKN$ [51] systems bind as well, because of the strong attraction between $D$ and $K$.

As pointed out in Ref. [45], the three-body $DDK$ bound state can decay strongly via diagrams such as those shown in Fig. 1. In the present work, we calculate explicitly the partial decay widths from such processes, aiming to provide further motivation for the experimental search for this state. The present manuscript is organized as follows. The theoretical formalism is explained in Sec. II. The predicted partial decay widths are presented in Sec. III, followed by a short summary in Sec. IV.
II. THEORETICAL FORMLISM

In the following, we focus on the doubly charged state $R^{++}$. Due to isospin symmetry, the decay width of its isospin partner $R^+$ can be calculated analogously and only small differences are expected because of the slightly different masses of its molecular components. As mentioned in Ref. [52], though the $R^{++}$ is a bound state of the DDK system or $D_{s0}D$ system, it is possible for such a state to decay strongly. Keeping in mind that the $D_{s0}^*$ is observed in the inclusive $D_s^0 p^0$ invariant mass distribution, which violates isospin, the $R^{++}$ can decay via $R^{++} \rightarrow D_{s0}^* D^*(D_s^0 p^0)$. An alternative process, without involving isospin breaking, is via triangle diagrams such as those shown in Fig. 1. These processes conserve isospin and therefore should be the dominant ones, as compared to the ones that violate isospin. In the following, we explain how to calculate the four diagrams shown in Fig. 1.

$$\Phi(p^2) \equiv \exp(-p_E^2/\Lambda^2)$$

where $\Lambda \sim 1.0$ GeV [52-68] is a size parameter, which characterizes the distribution of the molecular components inside the molecule, and $p_E$ is the Euclidean Jacobi momentum [52-68]. In the present work, we take $\Lambda = 1$ GeV, unless otherwise stated.

The coupling constant $g_{RD_{s0}D}$ in Eq. (1) could be determined by the compositeness condition [52-53], where the renormalization constant of the composite particle should be zero, i.e.,

$$Z_R \equiv 1 - \Sigma_R(m_R^2) = 0,$$

with $\Sigma_R(m_R^2)$ being the derivative of the mass operator of the $R$. The concrete form of the mass operator of the DDK bound state $R$ corresponding to the diagram in Fig. 1 is

$$W_R(k_0) = \frac{g_{RD_{s0}D}}{16\pi^2} \int_0^\infty dx \int_0^\infty d\beta \frac{1}{z^2} \exp[-\frac{1}{\Lambda^2}]
\times \left[-2k_0^2\omega_{D_{s0}}^2 + \alpha m_D^2 + \beta(-k_0^2 + m_D^2) + \frac{\Lambda^2}{4z}\right].$$

where $z = 2 + \alpha + \beta$, $\Delta_M = -4\omega_{D_{s0}}k_0 - 2k_0$, and $k_0^2 = m_R^2$ with $k_0$, $m_D$ denoting the four-momenta and mass of the $R$, respectively. Here, we set $m_R = m_{D_{s0}} + m_D - E_5$ with $E_5$ being the binding energy of $R$, $k_1$, and $m_{D_{s0}}$ are the four-momenta and mass of the $D_{s0}$, and $m_D$ is the mass of the $D$-meson, respectively.

In the present work, we calculate the two-body decay width of the $R$ via the triangle diagrams shown in Fig. 1. To evaluate the diagrams, in addition to the Lagrangian of Eq. (1), the following effective Lagrangian terms, responsible for the interactions between heavy-light pseudoscalar and vector mesons, are needed as well [53]

$$\mathcal{L}_{P\Phi P} = ig(P^\mu u^\nu P^\nu - Pu^\mu P_\mu^\nu),$$

where $P = (D^0, D^+, D_s^+)$ and $P^\nu = (D_{s0}^0, D_{s0}^*, D_{s0}^{*+})$, $u^\mu$ is the axial vector combination of the pseudoscalar-meson fields and their derivatives,

$$u^\mu = (\bar{u} i\gamma^\mu u - i\not{\partial} u^\mu),$$
where $u^2 = U = \exp(i\phi)$, $f_0 = 92.4$ MeV, and the pseudoscalar- meson octet $\phi$ is represented by the $3 \times 3$ matrix

$$
\phi = \sqrt{2}
\begin{pmatrix}
\frac{\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{6}} & \pi^+ \\
\pi^- & -\frac{\eta}{\sqrt{6}} & K^0 \\
-\frac{\eta}{\sqrt{6}} & K^0 & \frac{\eta}{\sqrt{2}}
\end{pmatrix}.
$$

(7)

From Eqs. (5)-(7), one can easily obtain the interaction vertices $\eta DD^*, KDD^*$, and $\pi DD^*$. The coupling constant $g$ can be determined from the strong decay width $\Gamma(D^{++} \to D^0\pi^+) = 56.46 \pm 1.22$ keV, together with the branching ratio $BR(D^{++} \to D^0\pi^+) = (67.7 \pm 0.5)$%. With the help of Eq. (3), the two body decay width $\Gamma(D^{++} \to \pi^+ D^0)$ is related to $g$ via

$$
\Gamma(D^{++} \to \pi^+ D^0) = \frac{g^2}{12\pi} \frac{1}{M^2}
$$

where $\vec{p}_\pi$ is the three-momentum of $\pi^+$ in the rest frame of the decaying vector meson $D^{++}$. Using the corresponding experimental decay width and the masses of the relevant particles given in Table 1, we obtain $g = 1.097 \pm 0.012$ GeV.

**TABLE 1: Masses of the relevant particles in the present work (in units of MeV) [69].**

| $D^+$ | $D^0$ | $\eta$ | $D^0$ | $D^+$ |
|-------|-------|--------|-------|-------|
| 1869.65 | 1864.83 | 547.862 | 2006.85 | 2010.26 |
| $K^*$ | $D^{*+}$ | $D^{*0}$ | $D^+$ | $K^+$ |
| 497.611 | 2112.2 | 2317.0 | 1968.34 | 493.677 |

In the chiral unitary approaches [20, 21, 33, 70], the $D_{s0}$ is found to be dynamically generated from the $DK$ and $D_{s0}S$-wave interactions. As a result, the vertices $D_{s0}DK$ and $D_{s0}\eta D_s$ can be easily written as

$$
\mathcal{L}_{D_{s0}DK} = g_{D_{s0}DK} D_{s0}DK, \\
\mathcal{L}_{D_{s0}\eta D_s} = g_{D_{s0}\eta D_s} D_{s0}\eta D_s,
$$

(9)

(10)

where the coupling of the $D_{s0}$ to $DK$ and $D_s\eta$ states, $g_{D_{s0}DK}$ and $g_{D_{s0}\eta D_s}$, can be obtained from the coupling constant of the $D_{s0}$ to the $DK$ and $D_s\eta$ channels in isospin zero, which are found to be $g_{D_{s0}DK} = 10.21$ GeV/(10.203 GeV) and $g_{D_{s0}\eta D_s} = 6.40$ GeV(5.876 GeV) in Ref. [21], [20], multiplied by the appropriate Clebsch-Gordan (CG) coefficients, namely, $g_{D^{*+}D^0K^*} = g_{D^{*+}D^0K^*} = -\frac{g_{D_{s0}DK}}{\sqrt{2}}$ and $g_{D^{*0}D^0\eta} = g_{D_{s0}D^0\eta}$.

With the above vertices, the amplitudes of the triangle diagrams of Fig. [1] evaluated in the center of mass frame of final states, are

$$
-i\mathcal{M}_D = g_{R^{*+}D^0}D^0\eta D_{s0}\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+) - k_2\omega_D\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+)
$$

(11)

$$
-i\mathcal{M}_{K^*} = g_{R^{*+}D^0}D^0\eta D_{s0}\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+) - k_2\omega_D\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+)
$$

(12)

$$
-i\mathcal{M}_\eta = g_{R^{*+}D^0}D^0\eta D_{s0}\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+) - k_2\omega_D\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+)
$$

(13)

$$
-i\mathcal{M}_{K^*} = g_{R^{*+}D^0}D^0\eta D_{s0}\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+) - k_2\omega_D\eta \frac{g}{\sqrt{s}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi(k_1\omega_D^+)
$$

(14)

The corresponding partial decay width then reads

$$
d\Gamma[R \to \eta] = \frac{1}{2J+1} \frac{1}{32\sqrt{2}} \frac{1}{m_R^2} |\vec{p}_\eta| |\mathcal{M}|^2 d\Omega,
$$

(15)

where $J = 0$ is the total angular momentum of the initial $R$ state, the overline indicates the sum over the polarization vectors of final hadrons, and $|\vec{p}_\eta|$ is the 3-momenta of the decay products in the rest frame of the $(R^{++}, R^+)$ states. Then the total decay widths of the $(R^{++}, R^+)$ states are

$$
\Gamma_{R^{++}} = \Gamma[R^{++} \to D^+_s D^0] + \Gamma[R^{++} \to D^+ D^{*0}],
$$

(16)

$$
\Gamma_{R^+} = \Gamma[R^+ \to D^+_s D^0] + \Gamma[R^+ \to D^0 D^{*+}],
$$

(17)

**III. RESULTS AND DISCUSSIONS**

To estimate the partial decay widths of the $R$, we first need to determine the coupling constants related to the molecular state and its components.

In Refs. [14, 18], the $R^{++}$ state is found to have a binding energy about $15 \sim 45$ MeV, with respect to the $D^*_s D^*$ threshold. In this mass range, the coupling constant is dependent on the mass of the bound state $R$ as shown in Fig. [3]. One finds that the coupling constant $g_{R^{++}D^0}$ decreases with $m_{R^{++}}$. With a value of the mass $m_{R^{++}} = 4140$ MeV, the corresponding coupling constants is $g_{R^{++}D^0} = 9.02$ GeV.

We show the dependence of the total decay width on the masses of the bound state $R^{++}$ in Fig. [4] One can see that the total decay width increases slightly with the mass of the bound state $R^{++}$ from 4.13 to 4.17 GeV. The predicted total decay width is small and found to be $\Gamma_{R^{++}} = 2.5 \sim 2.6$ MeV.
TABLE II: Partial decay widths of $R^{++} \to D_s^+ D^{*+}$ and $D^* D_{s1}^{*+}$ for different $R^{++}$ masses (in units of MeV).

| Mode                  | $\Lambda$(GeV) | 4130 | 4140 | 4150 | 4160 | 4170 |
|-----------------------|----------------|------|------|------|------|------|
| $R^{++} \to D_s^+ D^{*+}$ | 1.0           | 0.27 | 0.28 | 0.29 | 0.29 | 0.26 |
| $R^{++} \to D^* D_{s1}^{*+}$ | 1.0           | 2.33 | 2.43 | 2.49 | 2.47 | 2.30 |

In Fig. 3, we also show the partial decay widths as a function of the mass of $R^{++}$.

In this work, inspired by the recent series of studies that showed the likely existence of a $DDK$ bound state, we have studied its partial decay widths into $D_s D^*$ and $DD^*_s$. Such a decay involves the treatment of the $R$ state as a quasi-bound state of $D_s^0(2317)D$ and utilizing the Weinberg compositeness condition to determine the corresponding coupling. Our studies find a relative small total decay width, of the order of $2 \sim 3$ MeV, mainly to $DD^*_s$, and the results depends only moderately on the single parameter of the method, the cutoff $\Lambda$.

The predicted decay width seems to suggest that it is possible to observe such a state at Belle or BelleII, e.g., via the inclusive invariant mass distribution $D^* D_{s1}^{*+} \pi^0$, which is quite similar to the experimental discovery of the $D_{s0}^0(2317)$ by BaBar, Belle, and CLEO. On the other hand, its production yields at these experimental setups remain to be studied.

Recent lattice QCD studies of compact tetraquark states, see, e.g., Refs. [71, 72], suggest that a study of the $DDK$ bound state in terms of its minimal quark content $cc\bar{q}\bar{s}$ might be within the reach of the state of the art of lattice QCD simulations, even taking explicitly into account its three-meson molecular nature [73].

IV. SUMMARY

In this work, inspired by the recent series of studies that showed the likely existence of a $DDK$ bound state, we have studied its partial decay widths into $D_s D^*$ and $DD^*_s$. Such a decay involves the treatment of the $R$ state as a quasi-bound state of $D_s^0(2317)D$ and utilizing the Weinberg compositeness condition to determine the corresponding coupling. Our studies find a relative small total decay width, of the order of $2 \sim 3$ MeV, mainly to $DD^*_s$, and the results depends only moderately on the single parameter of the method, the cutoff $\Lambda$.

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Acknowledgements

LSG thanks Chen-Ping Shen and Manuel Pavon Valderrama for some stimulating discussions. This work was partly supported the National Natural Science Foundation of China (NSFC) under Grants Nos. 11975041, 11735003,
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