Neutral-naturalness from a holographic \( \text{SO}(6)/\text{SO}(5) \) composite Higgs model

Barry M. Dillon

\textit{E-mail: barry.dillon@plymouth.ac.uk}

We study a holographic realisation of a composite Higgs model with an \( \text{SO}(6)/\text{SO}(5) \) symmetry breaking coset in which the top sector includes colour-neutral twin-partners that reduce the sensitivity of the Higgs mass to the cut-off. Key to this ‘neutral-naturalness’ mechanism is a \( \mathbb{Z}_2 \) symmetry that leaves the Yukawa couplings to the Higgs boson invariant under an exchange of the top quark and twin top quark, but the symmetry structure of the model means that the \( \mathbb{Z}_2 \) symmetry is not present in the gauge boson couplings to the Higgs. Within the calculable framework of holography we construct and study the Higgs potential. We examine the relation between the Higgs mass, top-partner spectra, and the input parameters, finding that the presence of the twin-partners pushes the masses of the lightest coloured top-partners up to \( \sim 1500 \text{ GeV} \) while the decay constant remains \( \lesssim 700 \text{ GeV} \). Interestingly, no additional \( \mathbb{Z}_2 \) breaking terms are required to reproduce the observed masses of the electroweak gauge bosons, Higgs boson, and top quark.
1 Introduction

With the discovery of the Higgs boson at a mass of 125 GeV [1] and the absence of new physics between the electroweak scale and the TeV scale, important questions regarding the naturalness of the Higgs sector must be addressed. Natural scenarios accommodating a light Higgs boson include those with a composite Higgs sector [2]. A central aspect here is that the Higgs degrees of freedom are pseudo-Goldstone bosons of a global symmetry that is spontaneously broken by the condensation of a strongly coupled gauge theory. In recent years these models have received a lot of interest, and many phenomenologically interesting models have been identified [3–6]. The general picture is that a set of fermions with a flavour symmetry $G$ are coupled to a gauge theory whose coupling grows strong near the TeV scale. The confinement of these fermions into bound states then spontaneously breaks $G$ to $H$ with the Higgs degrees of freedom being formed from the Goldstone bosons in the $G/H$ coset. With $SU(2)_L \times U(1)_Y \subset H$, coupling the Standard Model (SM) fields to the strong sector explicitly breaks the global symmetries and generates a potential for the Higgs field, which in turn allows for the breaking of the electroweak symmetry. Assuming that the Yukawa couplings are generated via partial compositeness, the large value for the top Yukawa coupling requires a large mixing between the Right-Handed (RH) top quark and composite top-partner states. This leads to large contributions to the Higgs potential which can only result in a naturally light Higgs boson when there are light top-partners in the spectrum [7]. Much work on the phenomenology of these light top-partner states has been carried out [8, 9]. For a general discussion on the 4D construction of composite Higgs models and little Higgs models see [10–13] and [14], respectively. Recent experimental results put lower bounds on coloured top-partner masses in the region $\sim 1$–1.4 TeV [15], thus there is a need for theoretical models which can explain this absence of light top-partner states.
One broad class of models which elegantly evades the collider bounds on top-partners are the ‘neutral-naturalness’ models. These scenarios contain additional states that suppress the Higgs potential and allow a light Higgs to exist naturally in the spectrum. However the new states are not charged under the QCD gauge group and thus the bounds from the LHC do not apply in their full generality, allowing for new physics at mass scales closer to the electroweak scale. In the composite Higgs framework there are models of neutral-naturalness known as twin Higgs models [16], and in supersymmetry there are the models going by the name folded-supersymmetry [17]. Other models of neutral-naturalness such as the quirky little Higgs [18] or models with completely SM-neutral scalar top-partners [19] have also been proposed. Much work on the collider phenomenology [20] of neutral-naturalness models has been done in recent years.

The twin Higgs mechanism fits well within the composite Higgs paradigm, as it also posits that the Higgs field is formed of pseudo-Goldstone bosons. Central to this mechanism is a $Z_2$ exchange symmetry between the top quark and a ‘twin top quark’ which is neutral under SM gauge symmetries. This leads to a $Z_2$ exchange symmetry in the Higgs potential between $s_h \leftrightarrow c_h$, resulting in a softening of the potential even in the presence of a large mixing between the top quark and the composite sector. In the minimal models the Higgs and twin Higgs degrees emerge from an $SO(8)/SO(7)$ coset. Both the QCD and electroweak gauge sectors also having a twin copy. A minimal neutral-naturalness model based on an $SO(6)/SO(5)$ coset has been proposed [21, 22] which contains a similar $Z_2$ symmetry but does not contain a twin electroweak group. The models studied in these papers have slight differences, although the features that allow the neutral-naturalness mechanism to work are the same. The authors found that the lightest coloured top-partners in these models could easily lay above the bounds set by current LHC analyses, and identified some interesting phenomenological aspects of the models. Many studies on non-minimal composite Higgs models can be found in the literature [23, 24], and a lot of work developing UV completions of the composite Higgs scenario has been carried out [25].

In this paper we study a holographic realisation of the $SO(6)/SO(5)$ model of neutral-naturalness proposed in [21, 22], where the models are referred to as the ‘Brother Higgs’ or ‘Trigonometric Parity’ models, respectively. Holography is a well established and indispensable tool used in building calculable effective field theories for strongly coupled gauge theories. These methods have their origins in the AdS/CFT correspondence [26] and became hugely popular in the Beyond-the-Standard Model (BSM) model-building community after the introduction of the Randall-Sundrum (RS) models [27, 28]. Much work has been done in studying the holographic correspondence between the RS models and strongly interacting field theories [29]. For composite Higgs scenarios there are elegant holographic formulations of partial compositeness [30], through which the SM fermions couple to the strong sector, and of the spontaneous breakdown of the global symmetry, through which the composite Higgs degrees of freedom arise [31]. The one-loop Higgs potential calculated in holography is automatically finite due to 5D locality, negating the need to impose sum rules on the model parameters. Many applications of holography to composite Higgs models have been studied [32], and it has been shown that the 5D volume plays a significant role in determining the relationship between the Higgs mass, Higgs vacuum expectation value, and the top mass [33]. A holographic description of the $SO(8)/SO(7)$ twin Higgs model was presented in [34, 35], where it was found that realistic ElectroWeak Symmetry Breaking (EWSB) can take place through the introduction of additional $Z_2$ breaking terms in the Higgs potential.

The paper is outlined as follows. Section 2 presents the details of the model, beginning with an overview of what we want to achieve, and then outlining the 5D holographic model which
does so. In section 2.1.1 we provide the details on the gauge symmetries in the holographic model. The holographic description of the quark sector is outlined in section 2.1.2, where the origin of the $Z_2$ symmetry in the Yukawa couplings becomes apparent. In section 3 we study the Higgs potential of the model, define the form factors as a function of the 5D input parameters, and highlight important features that arise due to the $Z_2$ symmetry. Section 3.1 presents the results of a numerical scan over the parameter space, and discusses the predicted top-partner spectra and the dependence on the parameters of the Higgs potential.

2 The model

We start with a brief review of the neutral-naturalness mechanism outlined in [21] and [22], and refer the reader to these papers for a more in depth discussion. We work in the composite Higgs framework with the compositeness mass scale assumed to be above a TeV. The strong sector has a global symmetry $G = SO(6)$ which is spontaneously broken to $H = SO(5)$ at the compositeness scale. The Higgs doublet ($H$) and a real singlet ($\eta$) emerge as Goldstone bosons in the $SO(6)/SO(5)$ coset. This is the minimal coset that contains an internal parity leading to the required $Z_2$ exchange symmetry between the top and twin top in the Yukawa sector. The electroweak gauge fields are gauged from the $SU(2)_L$ subgroup of $SO(4)$ and the QCD $SU(3)_c$ gauge group is introduced externally to the global symmetries of the strong sector. In a non-linear sigma description the Goldstone bosons can be written in an $SO(6)$ vector as

$$\Sigma = \frac{\sin \Pi}{f_\pi} \left( \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \cot \frac{\Pi}{f_\pi} \right)\quad (2.1)$$

with $\Pi = \sqrt{\pi_a \pi_a}$ and $f_\pi$ being the decay constant of the Higgs field. The Higgs doublet is formed from $\pi_1, \ldots, \pi_4$ and $\pi_5 = \eta$ is the Goldstone boson of the spontaneously broken $U(1)_\eta$ subgroup of $SO(6)$ (the $SO(6)$ generators are given in Appendix A). In addition to the SM gauge symmetries we also introduce a twin QCD, $SU(3)_c$, and gauge the $U(1)_\eta$ subgroup of the $SO(6)/SO(5)$ coset. In unitary gauge the Goldstone bosons eaten by the $W$ and $Z$ bosons ($\pi_1, \pi_2, \pi_3$) and the Goldstone boson eaten by $U(1)_\eta$ ($\pi_5$) are removed from the spectrum.

The SM quarks are introduced as chiral states external to the strong sector and are neutral under $SU(3)_c \times U(1)_\eta$. The Left-Handed (LH) top and bottom doublet can be embedded in a 6 of $SO(6)$, while the RH top quark is taken as a singlet. The specific embeddings are discussed in Section 2.1.2. The twin quarks required for the softening of the Higgs potential are also introduced as external chiral states, and are neutral under the SM gauge symmetries but triplets under $SU(3)_c$. The LH twin top is embedded in a 6 of $SO(6)$ such that it is charged under $U(1)_\eta$, while the RH twin top can be taken as a singlet under $SO(6)$ and thus has no $U(1)_\eta$ charge. It has been shown in [21, 22] that with a $Z_2$ exchange symmetry fixing the Yukawa couplings of the Higgs with the top and twin top to be equal, the leading order contribution from the top quark to the Higgs potential is cancelled. The $Z_2$ symmetry should generate Yukawa couplings that schematically look like

$$\mathcal{L} \supset \frac{f_\pi}{\sqrt{2}} y_t \left( s_h t_L t_R + c_h \tilde{t} \tilde{t}_R \right)\quad (2.2)$$

where the top quarks are denoted by $t_{L,R}$, the twin tops by $\tilde{t}_{L,R}$, and $s_h$ and $c_h$ equal $\sin h/f_\pi$ and $\cos h/f_\pi$. This Lagrangian is invariant under $s_h \leftrightarrow c_h$ and $t \leftrightarrow \tilde{t}$ simultaneously. The leading order contribution to the Higgs potential is then $V(h) \sim y_t (s_h^2 + c_h^2) f_\pi^4$, which is independent of the Higgs field. Without the twin top coupling the leading order contribution would not vanish. In this section we will show how this type of model can be realised in a 5D holographic scenario.
2.1 The holographic model

Our starting point for the holographic description is the 5D RS model with the metric

$$ds^2 = \left( \frac{R}{z} \right)^2 \eta_{MN} dx^M dx^N$$

(2.3)

where \( z \equiv x^5 \) is the extra dimensional coordinate. The extra dimensional space is cut-off by two three branes; one in the UV at \( z_0 = R \) and the other in the IR at \( z_1 = R' \sim 1/\text{TeV} \). The RS model is thought to be dual to a strongly coupled gauge theory in 4D, whose conformal invariance is broken at the scale dictated by the position of the IR brane. In modelling a composite Higgs model which renders the Higgs sector natural we expect this scale to be close to 1 TeV. This IR scale is also known as the Kaluza-Klein (KK) scale and the excited modes of fields living in the 5D bulk are known as KK modes. The masses of the lightest KK modes are \( \sim M_{KK} = 1/R' \) with the exact mass being determined by the particles spin and bulk dynamics. Once the IR scale is fixed, the UV scale is then related to the number of colours in the dual strongly interacting gauge theory through

$$\log(\Omega) = \frac{16\pi^2}{g^2} \left( \frac{1}{N} - \frac{1}{N} \right)$$

(2.4)

where \( \Omega = R'/R \) is the 5D volume, \( N \) is the number of colours, and \( g \) is the electroweak coupling. The quantity \( \Omega \) will play an important role in the determination of the Higgs mass and its decay constant later in the paper. Due to the 5D NDA condition for calculability the allowed values of \( N \) are constrained to lay in \( 1 \ll N \lesssim 10 \) [3, 4, 6], and in the work presented here we will allow \( N \) in the range 5 to 10.

2.1.1 The gauge sector

Details on the treatment of 5D gauge fields, including their boundary conditions on the branes, are given in Appendix B.1. We now use \( G \) to label the bulk gauge symmetry in the 5D model, while \( H \) and \( H_G \) label the symmetries preserved on the IR and UV branes, respectively. The generators in \( H \) are those left unbroken by the spontaneous symmetry breaking induced by the strong sector, while the generators in \( H_G \) are those which are gauged. The Goldstone bosons in holographic models arise from the \( A_5(x, z) \) components of the bulk gauge fields. We assume the following symmetry structure

$$
G = SU(7) \times SO(6) \times U(1)_X \\
H = SU(7) \times SO(5) \times U(1)_X \\
H_G = SU(3)_c \times SU(3)_{\tilde{c}} \times SU(2)_L \times U(1)_Y \times U(1)_{\eta},
$$

(2.5)

similar to the gauge structure used for the holographic twin Higgs model in [34]. In \( SU(7) \) there is the subgroup \( SU(3)_c \times SU(3)_{\tilde{c}} \times U(1)_{7} \times U(1)_{\tilde{7}} \), while in \( SO(5) \) we have \( SU(2)_L \times SU(2)_R \). The UV boundary conditions are chosen such that the hypercharge generator is given by the linear combination

$$Y = T_R^3 + X - \frac{4}{3} T_\tilde{7}$$

(2.6)

with \( T_R^3 \) being the diagonal generator of \( SU(2)_R \) and \( T_\tilde{7} \) being the generator of \( U(1)_{\tilde{7}} \subset SU(7) \). We denote the abelian group formed from \( T_R^3 \) as \( U(1)_R \). The reason for this choosing the hypercharge generator to be this particular linear combination will be discussed in the next section, along with another consistent definition. The gauging of \( U(1)_{\eta} \) via UV boundary conditions
ensures that the real singlet $\eta$ is eaten to form the longitudinal component of the massive spin-1 mode. Therefore the only massless scalars here are the four Higgs degrees of freedom arising from the $SO(6) \to SO(5)$ breaking boundary conditions on the IR brane. To find the holographic action for these fields we start by writing down the most general $SU(7) \times SO(6) \times U(1)_X$ effective action to quadratic order for the spin-1 sector,

$$\mathcal{L} = \frac{P_{T}^{\mu \nu}}{2} \left[ \Pi_0 (p^2) \text{Tr} (A_\mu A_\nu) + \Pi_1 (p^2) \Sigma^T A_\mu A_\nu \Sigma + \Pi_0^X (p^2) A_\mu^X A_\nu^X + \Pi_0^\mu (p^2) \text{Tr} (A_\mu^c A_\nu^c) \right]$$

(2.7)

where $A_\mu$ are the $SO(6)$ gauge fields and $A_\mu^X, A_\mu^c$ are the $U(1)_X$ and $SU(7)$ gauge fields, respectively. The Goldstone boson multiplet containing the Higgs field is defined in Eq. 2.1, which in unitary gauge is simply $\Sigma = (0, 0, 0, s_h, 0, c_h)^T$ with $s_h = \sin h/f_\pi$ and $c_h = \cos h/f_\pi$. The task is then to calculate these form factors from a 5D theory. In Appendix B.1 we present a summary of the holographic treatment of 5D non-abelian gauge fields, where form factors $\Pi_\pm$ have been defined with the $\pm$ referring to the IR boundary conditions. The form factors in the model under consideration can be written in terms of $\Pi_\pm$. Matching the action in Eq. 2.7, in the limit of $s(h) \to 0$, to the holographic effective action in the Appendix we find

$$\Pi_0 = \frac{\Pi_+}{g_5^2} \quad \Pi_1 = \frac{\Pi_- - \Pi_+}{g_5^2}$$

$$\Pi_0^X = \frac{\Pi_+}{g_{5,X}^2} \quad \Pi_0^c = \frac{\Pi_+}{g_{5,c}^2}$$

(2.8)

where $g_{5,c}, g_5$, and $g_{5,X}$ are the 5D gauge couplings of the $SU(7)$, $SO(6)$, and $U(1)_X$ gauge groups. These are the only form factors we need to describe the gauge fields and their interactions with the Higgs field at quadratic order. Expanding the action in Eq. 2.7, keeping only the gauged generators, we have

$$\mathcal{L} = \frac{P_{T}^{\mu \nu}}{2} \left\{ W_\mu^+ \left( \Pi_0 + \frac{s_h^2}{4} \Pi_1 \right) W_\nu^- + Z_\mu \left( \Pi_0 + \frac{s_h^2}{4c_\mu^2} \Pi_1 \right) Z_\nu + A_\mu \Pi_0 A_\nu 
+ B_\mu \left( \Pi_0 + \frac{c_h^2}{4} \Pi_1 \right) B_\nu + A_\mu^c \Pi_0^c A_\nu^c + A_\mu^c \Pi_0^0 A_\nu^c \right\}. \quad (2.9)$$

The $U(1)_\eta$ boson is denoted by $B_\mu$, the photon by $A_\mu$, and the $SU(3)_c$ and $SU(3)_\varepsilon$ gauge bosons by $A_\mu^c$ and $A_\mu^\varepsilon$, respectively. The $s_W$ and $c_W$ symbols denote sine and cosine functions of the Weinberg angle. There are a few points worth mentioning here. At low energies the $\Pi_\pm$ form factors behave as

$$\Pi_+ (p_E \sim 0) \to \sim p_E^2 R \log \Omega, \quad \Pi_- (p_E \sim 0) \to \frac{-2R}{R^2}. \quad (2.10)$$

Requiring the proper normalisation of Eq. 2.9 implies that $g_5^2 = g^2 R \log \Omega$, $g_5^2 = g^2 R \log \Omega$, and $g_{5,c}^2 = g_c^2 R \log \Omega$, with $g$, $g'$, and $g_c$ being the SM gauge couplings. The SM gauge fields couple to the Higgs with a term $\sim s_h$ while the $U(1)_\eta$ field couples with a term $\sim c_h$, indicating that the electroweak gauge group is unbroken in the $h = 0$ limit while the $U(1)_\eta$ gauge symmetry is broken. The decay constant of the Higgs is identified from the low energy limit of $\Pi_1$ through

$$\Pi_1 (p_E = 0) = -\frac{f_\pi^2}{2} \quad \Rightarrow \quad f_\pi^2 = \frac{4}{g^2 R \log \Omega} \quad = \frac{N}{4\pi^2 R^2}. \quad (2.11)$$

This implies that the SM Higgs vacuum expectation value ($v$) is related to $\sin \theta_W \equiv s(h)$ via $s(h) = \frac{v}{f_\pi}$, and that the mass of the $U(1)_\eta$ gauge boson is

$$m_{B'} = \frac{g}{4} \sqrt{f_\pi^2 - v^2}. \quad (2.12)$$

Page 5
The couplings between the Higgs and the electroweak gauge bosons have the usual corrections one encounters in composite Higgs models, \( g_{hVV} = g_{hVV}^{SM} \sqrt{1 - s_w^2} \). The \( U(1)_\eta \) gauge coupling is the same as the electroweak gauge coupling and the coupling between the Higgs and the \( U(1)_\eta \) boson is given by \( g_{hBB} = -g_{hVV}^{SM} \sqrt{1 - s_w^2} \). The gauge coupling for \( U(1)_\eta \) is not required to be the same as the electroweak gauge coupling, as this could be altered in the 5D theory by adding UV brane kinetic terms for the gauge fields. Although this could have interesting phenomenological consequences we will not consider it here.

### 2.1.2 The top sector

We embed the LH quark doublet and the LH twin top in a \( (7, 6)_{\frac{2}{3}} \) of \( SU(7) \times SO(6) \times U(1)_X \), while the RH top and RH twin top are embedded in a \( (7, 1)_{\frac{1}{6}} \). The \( SU(7) \) subgroups, \( SU(3)_c \times SU(3)_c \times U(1)_Y \times U(1)_X \), are labelled such that the twin quarks are charged under \( SU(3)_c \times U(1)_Y \), and the SM quarks are charged under \( SU(3)_c \times U(1)_X \). Specifically, the SM quarks are in the \((3, 1)_{\frac{1}{2}, 0}\) representation, whereas the twin top quarks are in the \((1, 3)_{0, \frac{1}{2}}\) representation. The \( SO(6) \) vectors containing the LH top quark \((t_L)\) and its twin \((\tilde{t}_L)\) are

\[
q_L = \frac{1}{\sqrt{2}} \begin{pmatrix} ib_L \\ b_L \\ it_L \\ -t_L \\ 0 \\ 0 \end{pmatrix}, \quad \tilde{t}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ it_L \\ \tilde{t}_L \end{pmatrix}. \tag{2.13}
\]

The first four components of these vectors are \( SO(4) \) multiplets and thus carry \( SU(2)_L \) charge, while the last two components are vectors of \( SO(2)_\eta \), i.e. \( U(1)_\eta \). Under the \( SU(2)_L \times U(1)_R \times U(1)_\eta \) subgroup of \( SO(6) \), with \( U(1)_R \) being the diagonal subgroup of \( SU(2)_R \), the above embeddings imply that the SM doublet has charges \( 2 \cdot \frac{2}{3}, 0 \), whereas the twin top has charges \( 1, \frac{1}{2} \).

To summarise, the subgroups of the bulk gauge symmetry relevant for the quark sector are

\[
\mathcal{H}_F = SU(3)_c \times SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_X \times U(1)_Y \times U(1)_\eta \tag{2.14}
\]

under which the SM quarks and twin quarks have charges

\[
(t_L, b_L) : (1, 3, 2)_{\frac{1}{2}, \frac{2}{3}, 0, 0}, \quad t_R : (1, 3, 1)_{\frac{2}{3}, 0, 0}, \quad \tilde{t}_L : (3, 1, 1)_{\frac{2}{3}, \frac{1}{2}, \frac{1}{2}}, \quad \tilde{t}_R : (3, 1, 1)_{\frac{2}{3}, \frac{1}{2}, 0}. \tag{2.15}
\]

With \( Y = T_R^3 + X - \frac{4}{3} T_7^\tau \) from Eq. 2.6, these embeddings will result in the SM quarks having their appropriate hypercharges while the twin quarks will be neutral under SM gauge symmetries. This is the motivation behind choosing hypercharge as the linear combination in Eq. 2.6.

In the 5D holographic model the external chiral fermions arise as massless modes of a bulk 5D Dirac fermion. Thus in the effective theory each chiral fermion is accompanied by a tower of vector-like fermions with the same charges under the bulk gauge symmetries as the chiral fermion. So the SM quarks be accompanied by a tower of coloured vector-like states while the
twin quarks will be accompanied by a tower of uncoloured vector-like states. The $Z_2$ symmetry between the top and twin top is enforced by embedding them in a single representation of the 5D gauge symmetry. We include the SM top and the twin top through two 5D fermion multiplets with the following charge assignments under the bulk $SU(7) \times SO(6) \times U(1)_Y$ gauge symmetry,

$$\xi_{q1} = (7, 6)_{2 \over 3}, \quad \xi_u = (1, 6)_{2 \over 3}. \quad (2.16)$$

The couplings of the quarks and twin quarks are now explicitly related to one another because they arise from the same 5D multiplet. The LH doublet and its twin arise from $\xi_{q1}$ whereas the RH top and its twin arise from $\xi_u$. We can see from here that the Yukawa couplings will feature the $Z_2$ symmetry from Eq. 2.2, since $y_{\ell \ell q} \Sigma^T \Sigma_{qL} = y_{\ell \ell q}^2 \tilde{t}_L t_L/2$ and $y_{\ell \ell q} \Sigma^T \Sigma_{qL} = y_{\ell \ell q}^2 \tilde{t}_L t_L/2$.

The fermionic content discussed so far implies gauge anomalies from triangle diagrams involving the top twin. However we have not included a complete description of the quark and lepton sector, therefore this is to be expected. Also, the anomalies are only associated with the non-SM gauge symmetries, therefore additional chiral twin-quarks with $U(1)_Y$ charge can be added to the model to cancel the gauge anomalies. The bottom quark has already been introduced in Eq. 2.16 but in order to generate a bottom quark mass from EWSB we add two new 5D multiplets to the model, $\xi_{q2} = (7, 6)_{-1 \over 3}$ and $\xi_d = (1, 6)_{-1 \over 3}$. It should be noted that with the bottom quark embedding $\xi_{q1}$ contains a copy of the SM quark doublet and a twin top, whereas $\xi_{q2}$ contains another copy of the SM quark doublet and a twin bottom. A mass mixing must be introduced on the UV brane such that only one linear combination of the $SU(2)_L$ doublets in $\xi_{q1}$ and $\xi_{q2}$ has a massless mode. This scenario has an interesting CFT dual description in which the SM LH quark doublet couples to two different operators from the composite sector [6]. Due to the fact that the top quark Yukawa coupling is significantly larger than that of the bottom quark, we can safely neglect the effects of the bottom sector when we study the Higgs potential and top-partner spectra. Therefore from this point on we will only consider the $\xi_{q1}$ and $\xi_u$ multiplets in our study. The complete 5D $SO(6)$ multiplets for the top quark and the twin top quark can be written as,

$$\xi_{q1}^c = {1 \over \sqrt{2}} \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ iT + iX_{2/3} \\ -T + X_{2/3} \\ iT_+ - iT_- \\ T_+ + T_- \end{pmatrix}, \quad \xi_{q1}^c = {1 \over \sqrt{2}} \begin{pmatrix} i\tilde{B} - i\tilde{X}_{5/3} \\ \tilde{B} + \tilde{X}_{5/3} \\ i\tilde{T} + i\tilde{X}_{2/3} \\ -\tilde{T} + \tilde{X}_{2/3} \\ i\tilde{T}_+ - i\tilde{T}_- \\ \tilde{T}_+ + \tilde{T}_- \end{pmatrix}. \quad (2.17)$$

The subscripts on the $X$ and $\tilde{X}$ fields label the EM charge, whereas the $T_\pm$ and $\tilde{T}_\pm$ fields have 2/3 EM charges with the $\pm$ referring to $\pm {1 \over 2}$ $U(1)_Y$ charges. The different states in these multiplets have both LH and RH components, and we will use projection operators $(P_{L,R})$ to identify between different chiralities. We identify the LH top quark with the zero mode of $P_L T$, and the LH twin top with the zero mode of $P_L \tilde{T}_+$. By choice we enforce that all other components of these multiplets do not have massless modes, this is done through the introduction of Lagrange multiplier fields on the UV brane which couple linearly to these components [41], and is analogous to the Dirichlet UV boundary conditions for components of the multiplets without zero modes. The RH top quark and its twin are trivially embedded as singlets of $SO(6)$. So the SM quarks and the twin quarks are identified with the massless chiral modes of these 5D multiplets belonging to the following representations of $SU(3)_c \times SU(3)_{\tilde{c}} \times SU(2)_L \times U(1)_Y$,

$$t_L = (3, 1, 2)_{1/6}, \quad \tilde{t}_L = (1, 3, 1)_{0},$$
Although we have used the $U(1)_\eta$ charges to identify some of the quarks in the $SO(6)$ multiplets, this is not a particularly useful labelling in the effective theory where $U(1)_\eta$ is spontaneously broken. When describing the mass spectra of the composite resonances it is better instead to use $\mathcal{P}, m = \frac{1}{\sqrt{2}}(T_+ \pm T_-)$ and $\mathcal{P}, \bar{m} = \frac{1}{\sqrt{2}}(\bar{T}_+ \pm \bar{T}_-)$. At the zero mode level the LH components of $\mathcal{P}, \bar{m}$ reduce to $\mathcal{P}, \bar{m} = \frac{1}{\sqrt{2}}(\mathcal{T}_L \bar{m}_L)$. 

In Appendix B.2 we give a brief discussion of how the holographic technique is applied to 5D fermion fields. In the 5D model the bulk gauge symmetry is broken to $SO(5)$ on the IR brane, therefore we will allow mass mixings ($\bar{m}$) between the multiplets to exist on the IR brane which respect only the $SU(7) \times SO(5) \times U(1)_X$ invariance. Analogously to Eq. B.6, the 5D Lagrangian for this scenario can be written as

$$\mathcal{L} = \int dx^5 \sqrt{|g|} \left\{ \frac{i}{2} \bar{q}_1 \gamma^M \partial_M q_1 - \frac{i}{2} \left( \partial_M \bar{q}_1 \right) \gamma^M q_1 - m_{q_1} \bar{q}_1 q_1 \right\}$$

$$\frac{i}{2} \bar{q}_u \gamma^M \partial_M q_u - \frac{i}{2} \left( \partial_M \bar{q}_u \right) \gamma^M q_u - m_u \bar{q}_u q_u - \delta(z-R')\bar{m} \left( \bar{q}_1 L \Sigma_0 \bar{u}_R + \bar{u}_R \Sigma^T \Sigma q_1 L \right)$$

$$\delta(z-R) \left\{ \frac{1}{2} \left( \bar{q}_1 L \Sigma_0 \bar{u}_R - q_1 L \bar{u}_R \right) - \delta(z-R') \right\}$$

where $\Sigma^T = (0, 0, 0, 0, 1)$. The 5D masses $m_{q_1}$ and $m_u$ can be written in terms of dimensionless quantities $c_{q_1, u} = c_{q_1, u}/R$. The UV boundary conditions for the 5D fields choose a LH source field for $\xi_{q_1}$, and a RH source field for $\xi_u$,

$$\xi_{q_1}(p, R) = \xi_{q_1}(p), \quad \xi_u(p, R) = \xi_u(p).$$

The IR boundary conditions are derived from the mass mixing between the multiplets mediated by $\bar{m}$. These mass mixings are analogous to the partial compositeness mixing commonly used in 4D implementations of composite Higgs models. The goal now is to use this 5D model to calculate the effective action for the Yukawa sector. The most general effective action for the $\xi_{q_1, u}$ multiplets at quadratic order is

$$\mathcal{L} = \bar{q}_1 p \left( \Pi_0^f(p) + \Pi_1^f(p) \Sigma \Sigma^T \right) q_1 + \bar{q}_u p \Pi_0^f(p) q_u + \bar{q}_1 M_1^f(p) \Sigma q_u + \text{h.c.}$$

Utilising the holographic techniques outlined in Appendix B.2 for the model described here, we can match the holographic action in Eq. B.8 to the action in Eq. 2.21 in the limit that $\Sigma = \Sigma_0$. In doing so we find

$$\Pi_0^f = \Pi_1^f (\bar{m} = 0) \quad \Pi_1^f = (\Pi_1^f - \Pi_1^f (\bar{m} = 0))$$

$$\Pi_0^f = \Pi_2^f \quad M_1^f = M^f$$

with the the 5D form factors $\Pi_1^f$ and $M^f$ being given in the Appendix. Expanding the action in Eq. 2.21 with $\langle h \rangle \neq 0$, keeping only the top and twin top, we have

$$\mathcal{L} = \bar{t}_L p \left( \Pi_0^t \frac{1}{2} \bar{s}_h^2 \right) t_L + \bar{t}_L p \left( \Pi_1^t \frac{1}{2} \bar{s}_h^2 \right) t_L$$

$$+ \bar{t}_R p \Pi_0^t t_R + \bar{t}_R p \Pi_1^t t_R - \frac{M_1^t}{\sqrt{2}} \left( \bar{t}_L t_R \bar{s}_h^2 + \bar{t}_L t_R \bar{c}_h^2 \right) + \text{h.c.}$$

with the form factors now determined by 2.22 with the explicit expressions being given in the Appendix B.2. The $Z_2$ ($s_h \leftrightarrow c_h, t_L \leftrightarrow t_L$) symmetry is now explicit in the top Yukawa
couplings, the implications of which we will study in the next chapter. Other crucial features that we should extract from the effective action are the masses of the lightest vector-like top partners, i.e. $T_{(L,R)}, X_{(L,R)}, T_{M(L,R)}$ and $T_{P(L,R)}$ and their twin counterparts. In the limit of $\langle h \rangle = 0$ these masses are given by

$$m_T = \text{zeros} (\Pi_0^2)$$

$$m_{X_{2/3}} = m_{T_M} = \text{poles} (\Pi_0^0)$$

$$m_{T_P} = \text{poles} \left( \Pi_0^0 + \frac{1}{2} \Pi_1^1 \right)$$

(2.24)

where the $\langle h \rangle \neq 0$ effects are small for top-partners at the TeV scale. Note that apart from the chiral modes, the top and twin top sectors have the same spectra in the $\langle h \rangle \rightarrow 0$ limit. The top and twin top masses are given by

$$m_t = \frac{1}{\sqrt{2}} \frac{M_{1}^{(h)}}{\sqrt{\Pi_0^0} \sqrt{\Pi_0^0 + \frac{1}{2} \Pi_1^1}} \left| p^2 \geq 0 \right.$$  

$$m_{\tilde{t}} = m_t \bigg|_{s(h) \rightarrow c(h)} = m_t \sqrt{\frac{f_{\pi}^2 - v^2}{v}}.$$  

(2.25)

These masses are strongly sensitive to the 5D mass parameters $c_{q_1,u}$. In a KK decomposition the localisation of the fermion zero modes actually depends exponentially on these parameters, with $c_{q_1} > 0$ indicating that the SM doublet zero mode is localised away from the IR region of the extra dimension, with the opposite being true for $c_u$ and the singlet zero mode.

3 The Higgs potential

Given the effective actions that we have derived for the gauge and fermion fields, the one-loop Coleman-Weinberg potential for the Higgs field can be written as

$$V(h) = V_G(h) + V_{t,\tilde{t}}(h)$$

$$V_G(h) = \frac{3}{2} \int \frac{d^4 p E}{(2\pi)^4} \left\{ 2 \log \left[ 1 + \frac{s_h^2 \Pi_1^1}{\Pi_0^0} \right] + \log \left[ 1 + \frac{s_h^2 \Pi_1^1}{4c_W \Pi_0^0} \right] + \log \left[ 1 + \frac{c_h^2 \Pi_1^1}{4 \Pi_0^0} \right] \right\}$$

$$V_{t,\tilde{t}}(h) = -2N_c \int \frac{d^4 p E}{(2\pi)^4} \left\{ \log \left[ 1 + \frac{s_h^2 \Pi_1^1}{2 \Pi_0^0} \right] + \log \left[ 1 + \frac{s_h^2 \Pi_1^1 (M_1^1)^2}{2 \Pi_0^0 (\Pi_0^0 + \frac{2}{3} \Pi_1^1)} \right] + (s_h \rightarrow c_h) \right\}. \quad (3.1)$$

In studying potentials of this type one option is to expand the logarithms so that we have a polynomial in $s_h^2$. The leading term in the pre-factor of the $s_h^2$ term in $V_{t,\tilde{t}}$ would then be $\sim (M_1^1)^2$, however due to our $c_h^2$ contribution from the twin top this term vanishes and the leading contributions are $\sim (M_1^1)^4$. A spurious IR divergence enters in this term solely due to the expansion of the logarithm [12]. When the $(M_1^1)^2$ term is present the problem is avoided by introducing an IR cut-off, to which the results are not sensitive. However when this term is not present and the leading contribution is the $(M_1^1)^4$ term, it will be beneficial to use another method for evaluating the Higgs potential which does not introduce IR divergences. To do this we will integrate the whole potential numerically while scanning over values of $s(h)$ to find the minimum of the potential and the Higgs mass.

To begin the analysis we will simply look at the potential as a function of $s_h$. We will fix $R' = 1/M_{KK} = 1/(1500 \text{ GeV}), N = 8$, and $c_q = 0.25$. The mass parameter $\tilde{m}$ is fixed such that
$m_t = \frac{1}{\sqrt{2}} v$ and $c_u$ is varied in the range $[-0.4, 0.4]$. From Figure 1 we can see that for values of $c_u$ closer to $-0.4$ the Higgs mass and vacuum expectation value increase, as does the value of $\tilde{m}$ required to achieve $m_t = \frac{1}{\sqrt{2}} v$. For $M_{KK} = 1500$ GeV the lightest spin-1 resonances are at a mass $\simeq 3.6$ TeV, whereas the masses of the lightest fermionic states depend on the 5D fermion mass parameters and will be the study of the next section. It is noteworthy that there is no tuning required to obtain a light Higgs mass and a small vacuum expectation value, as can be seen through the varying of $c_u$. This is one of the striking results of the $Z_2$ symmetry present in this model, and in twin Higgs models in general. In fact one often finds that the Higgs mass is too light in comparison to the vacuum expectation value leading to the requirement of additional $Z_2$ breaking terms. This is due to the model producing a value of the quartic coupling which is too small, and is a general feature of twin Higgs models. Work has been done in building models in which the Higgs has a quartic interaction at tree-level, but a mass only at loop level [36].

![Figure 1](image-url)  

**Figure 1**: The Higgs potential is plotted as a function of $s_h$ for $c_{q1} = 0.25$, $N = 8$, and $M_{KK} = 1500$ GeV. The parameter $\tilde{m}$ is chosen such that $m_t = \frac{1}{\sqrt{2}} v$, and $c_u$ is varied in the range $[-0.4, 0.4]$. The vertical lines indicate the minimum of each curve, and in the legend we have included the Higgs mass and vacuum expectation value for each potential.

### 3.1 Numerical scan

The free parameters in the model are

$$M_{KK}, \ c_{q1}, \ c_u, \ \tilde{m}, \ N. \quad (3.2)$$

In this section we present the results of a scan over the parameter space, where the brane mass parameter $\tilde{m}$ is fixed to reproduce the top quark mass. The parameter ranges that we have scanned over are $-0.45 \leq c_u \leq 0.45$, $0.15 \leq c_q \leq 0.4$, $1100$ GeV $\leq M_{KK} \leq 4000$ GeV, $5 \leq N \leq 10$. The results are summarised in Fig. 2 and Fig. 3. In Fig. 2 we show how the Higgs mass depends on the number of colours $N$ and on $M_{KK}$. Interestingly we find that in order to reproduce the correct Higgs mass we require the number of colours to be less than approximately 7. This is due to the fact that $m_h$ scales inversely with $f_\pi$, and in models of neutral-naturalness such as the twin Higgs, it is a general feature that Higgs mass is too small. In this scenario we see that this requires us to choose $N$ in a particular range, as opposed to introducing additional sources of $Z_2$ breaking terms as is done in other cases. We also see that in general a Higgs mass of 125 GeV picks out points in parameter space where $M_{KK}$ is $\lesssim 2000$ GeV.
In these plots we show how the Higgs mass depends on the number of colours $N$ and $M_{KK}$. We have scanned over $-0.45 \leq c_u \leq 0.45, 0.15 \leq c_q \leq 0.4, 1100 \text{ GeV} \leq M_{KK} \leq 4000 \text{ GeV}, 5 \leq N \leq 10$. And the top mass and Higgs vacuum expectation value are fixed to their known values.

![Figure 2](image)

In Fig. 3 we have shown the top-partner mass spectra for each of the points in the scan. Interestingly we see that having a Higgs mass of 125 GeV does not require top-partners in the mass ranges excluded by recent LHC analyses, i.e. $\lesssim 1.4 \text{ TeV}$. This happens because the twin top, at a mass of $m_t \cot \langle h \rangle / f_\pi$, cancels the leading order contributions to the Higgs potential. A large constraint on the parameter space comes from the current bound on the decay constant of the composite Higgs, $f_\pi \gtrsim 600 \text{ GeV}$ [37]. From the second plot in Fig. 3 we see that less then half of the viable points at $m_h \approx 125 \text{ GeV}$ pass this constraint. The points which do pass this constraint are those with the larger values of $N$ and $M_{KK}$. The fine-tuning present in obtaining a realistic EWSB can be estimated as $\sim (v/f_\pi)^2$, and in our case this is in the range $\sim 12-17\%$.

![Figure 3](image)

**4 Conclusions**

In this paper we have presented a holographic description of a neutral-natural composite Higgs model based on the $SO(6)/SO(5)$ coset. The model that we have studied is similar to those presented in [21, 22], and the results we have derived through the holographic calculations agree well with those derived in these papers. We studied how the Higgs mass and top-partner spectra depend on the parameters in the model once the Higgs vacuum expectation value and the top quark mass are fixed. We found that the model can easily reproduce the SM observables without predicting top-partners lighter than $\sim 1500 \text{ GeV}$, and without requiring additional sources of $Z_2$ breaking not already present in the model. However we do require a Higgs decay constant $\lesssim 700 \text{ GeV}$, and therefore this scenario could be ruled out with more accurate measurements of the couplings of the Higgs boson to the electroweak gauge bosons. It is worth noting that in [21]...
and no bound on $f_\pi$ was required, however similar features were observed in the holographic twin Higgs model presented in [34], where additional $Z_2$ breaking terms were introduced which increased the allowed range of $f_\pi$.

A phenomenological study of this model is a high priority. The phenomenology of the $U(1)_\eta$ boson will require a detailed study of decay modes both in the twin sector and in the SM sector. Along with this it would be very interesting to study the radion [38] and KK graviton [39] phenomenology in this model and in holographic twin Higgs models. In neutral-naturalness models the decays of the radion and KK graviton states to twin-sector particles could significantly change the phenomenological bounds. This will be done in an upcoming paper by the current author.

In conclusion, we have proposed a consistent holographic description of a neutral-naturalness composite Higgs model based on an $SO(6)/SO(5)$ coset, and determined under what conditions the correct Higgs mass, Higgs vacuum expectation value, and top quark mass can be reproduced without the introduction of additional $Z_2$ breaking terms. The main condition required is that $N \lesssim 7$, in which case $(v/f_\pi)^2$ in the range $\sim 12-17\%$. We then found that the lightest coloured top-partners have masses $\gtrsim 1500$ GeV, above the bounds set by current LHC analyses.

Acknowledgements

The author acknowledges funding from Grant No. EP/P005217/1, and would like to thank Stephan Huber and Aqeel Ahmed for useful comments on the draft.

A $SO(6)/SO(5)$ algebra

The generators of the $SO(6)$ algebra can be written as

$$T_{ij}^a = -\frac{i}{\sqrt{2}} \left( \delta^{\hat{a}i} \delta^{\hat{b}j} - \delta^{\hat{a}j} \delta^{\hat{b}i} \right)$$

$$T_{L,R,i,j}^a = -\frac{i}{2} \left( \frac{1}{2} \epsilon^{abc} (\delta^{\hat{a}i} \delta^{\hat{b}j} - \delta^{\hat{a}j} \delta^{\hat{b}i}) \pm (\delta^{\hat{a}i} \delta^{\hat{4}j} - \delta^{\hat{a}j} \delta^{\hat{4}i}) \right)$$

$$T_{ij}^\alpha = -\frac{i}{\sqrt{2}} \left( \delta^{\hat{\alpha}i} \delta^{\hat{\beta}j} - \delta^{\hat{\alpha}j} \delta^{\hat{\beta}i} \right)$$

(A.1)

where $a = 1, 2, 3$ labels the three generators for the $SU(2)_L$ and $SU(2)_R$ subgroups, $\hat{a} = 1, \ldots, 5$ labels the broken generators in the coset, and the remaining generators are given by $\alpha = 1, \ldots, 4$. The Higgs degrees of freedom are formed from the $\hat{a} = 1, \ldots, 4$ generators while the $U(1)_\eta$ symmetry is generated by $\hat{a} = 5$.

B Holographic form factors

This appendix includes a summary of how 5D gauge fields and fermions are treated holographically, and how their form factors are derived. The 5D scenarios that we present will be simplified versions of the full set-up considered in the main text, however the results arrived at will be used to build the form factors for the $SO(6)/SO(5)$ model. For a full review of holographic techniques on gauge fields and fermion fields we refer the reader to [40] and [41], respectively.
B.1 Gauge fields

To quadratic order, the action for a non-abelian 5D gauge field can be written as

$$ S = \frac{1}{2g_5^2} \int d^5x \sqrt{|g|} \left[ -\frac{1}{2} F_{\mu\nu} A^\mu A^\nu - \frac{1}{2} F_{\mu5A} F^{\mu5A} \right] $$  \hspace{1cm} (B.1)

with \( g \) being the determinant of the metric, and \( A \) labelling the generators of the bulk gauge field. The IR boundary conditions for the unbroken (\( A = a \)) and broken (\( A = \hat{a} \)) generators are Neumann and Dirichlet, respectively, i.e.

$$ \partial_z A^\mu_a(p, z)|_{z=R} = 0, \quad A^\mu_a(p, R') = 0 \quad (\text{B.2}) $$

The UV boundary conditions are used to define a source field, i.e. the 4D degree of freedom with which we will define the effective theory. For the unbroken and broken generators these boundary conditions are

$$ A^\mu_a(p, R) \equiv A_\mu(p), \quad A^\mu_\hat{a}(p, R) \equiv 0. \quad (\text{B.3}) $$

In the Kaluza-Klein method of treating 5D gauge fields this is equivalent to having Neumann and Dirichlet boundary conditions on the UV brane for the unbroken and broken generators, respectively. This in turn implies Dirichlet and Neumann UV boundary conditions for the \( A^\mu_5 \) and \( A^\mu_\hat{5} \) components, respectively, and thus massless modes in the spectrum for the \( A^\mu_a \) and \( A^\mu_\hat{a} \) fields. If we want to impose Dirichlet UV boundary conditions for any of the fields for which we do define a source field we can simply introduce a Higgs mechanism resulting in a large mass term for that field to the UV brane.

Solving the bulk 5D equations of motion for the \( A^\mu_a, A^\mu_\hat{a} \) fields, and inserting these back into the action, allows one to obtain the holographic action. For the gauge fields the effective action is found to be

$$ S_{\text{hol}} = -\frac{1}{2g_5^2} P^\mu_\nu \left( A^\mu_a \Pi_+(p^2) A^\nu_a + A^\mu_\hat{a} \Pi_-(p^2) A^\nu_\hat{a} \right) \quad (\text{B.4}) $$

with \( P^\mu_\nu = \eta^\mu_\nu - \frac{\omega^\mu_\nu}{p^2} \). The form factors are calculated to be

$$ \Pi_-(p_E^2) = \frac{p_E K_1(p_ER') I_0(p_ER) + I_1(p_ER') K_0(p_ER)}{K_1(p_ER') I_1(p_ER) - I_1(p_ER') K_1(p_ER)} $$

$$ \Pi_+(p_E^2) = \frac{p_E K_0(p_ER') I_1(p_ER) - I_0(p_ER') K_0(p_ER)}{K_0(p_ER') I_1(p_ER) + I_0(p_ER') K_1(p_ER)} \quad (\text{B.5}) $$

where \( p_E \) is the Wick rotated momentum. In their original Minkowski space form these form factors have zeros and poles which will be used to determine the mass spectra of the 4D eigenstates in the theory. We call these mass eigenstates Kaluza-Klein modes. With these form factors we can write down all the form factors required in the gauge sector of the \( SO(6)/SO(5) \) model.

B.2 Fermion fields

Take two 5D Dirac fermions living in the bulk of the RS model, \( \Psi_1 \) and \( \Psi_2 \). The UV boundary conditions for these fields choose the \( \Psi_{1L} \) and \( \Psi_{2R} \) Weyl components as the dynamical source fields, i.e. \( \Psi_1(p, R) \equiv \Psi_{1L}(p) \) and \( \Psi_2(p, R) \equiv \Psi_{2R}(p) \). On the IR brane the boundary conditions are determined by dimensionless IR mass mixings (\( \tilde{m} \)) between the two 5D fermions. The action for such a scenario can be written as

$$ \mathcal{L} = \int d^5x \sqrt{|g|} \left[ \frac{i}{2} \bar{\Psi}_1 \gamma^M \partial_M \Psi_1 - \frac{i}{2} (\partial_M \bar{\Psi}_1) \gamma^M \gamma_\mu \Psi_1 \right] $$
\[
\begin{aligned}
\frac{i}{2} \bar{\Psi_2} \gamma^M \partial_M \Psi_2 - \frac{i}{2} (\partial_M \bar{\Psi}_2) \gamma^M \Psi_2 - m_2 \bar{\Psi}_2 \Psi_2 - \delta(z - R') \tilde{m} \left( \bar{\Psi}_1 \Psi_{2R} + \bar{\Psi}_{2R} \Psi_1 \right) \\
\delta(z - R') \frac{1}{2} \left( \bar{\Psi}_1 \Psi_{1R} - \bar{\Psi}_{2L} \Psi_{2R} \right) - \delta(z - R') \frac{1}{2} \left( \bar{\Psi}_1 \Psi_{1R} - \bar{\Psi}_{2L} \Psi_{2R} \right) \right). 
\end{aligned}
\]  

(B.6)

The terms on the last line are necessary additions in order to satisfy the boundary conditions.

The IR boundary conditions following from this are

\[
\Psi_{1R}(R') = -\tilde{m} \Psi_{2R}(R'), \quad \Psi_{2L}(R') = \tilde{m} \Psi_{1L}(R').
\]

(B.7)

In the Kaluza-Klein picture, choosing a LH (RH) source field on the UV brane corresponds to a Neumann boundary condition for the LH (RH) component, with a Dirichlet boundary condition for the other chirality. Taking the \( \tilde{m} \to 0 \) limit on the IR brane we obtain Dirichlet boundary conditions for the \( \Psi_{1R} \) and \( \Psi_{2L} \) components, which implies Neumann boundary conditions for the \( \Psi_{1L} \) and \( \Psi_{2R} \) components. Therefore there exists massless zero modes for \( \Psi_{1L} \) and \( \Psi_{2R} \) in the spectrum, with their localisation in the extra dimension determined by the 5D mass parameter \( m_i \). When \( \tilde{m} \neq 0 \) we still have two massless modes in the model, except now these modes are an admixture of \( \Psi_1 \) and \( \Psi_2 \).

We solve the equations of motion for the 5D fermions such that \( \Psi(p, z) \sim G(p, z)\Psi(p) \), where \( G(p, z) \) is some holographic profile and \( \Psi(p) \) is the holographic source field defined by the UV boundary condition. The UV and IR boundary conditions are satisfied by fixing integration constants in \( G(p, z) \). With these holographic profiles the bulk dynamics can be integrated out and we obtain the following effective action for the source fields,

\[
\mathcal{L} = \bar{\Psi}_{1L} \Pi_1^f(p) \Psi_{1L} + \bar{\Psi}_{2R} \Pi_2^f(p) \Psi_{2R} - \bar{\Psi}_{1L} M^f(p) \Psi_{2R}
\]

(B.8)

where the form factors encode the mass spectra and mass mixings of the fields, analogously to those calculated in the case of gauge fields. In terms of the 5D parameters these can be expressed as

\[
\Pi_1^f(p, c_1, c_2, \tilde{m}) = \frac{1}{p} \frac{G_p^+(-c_2)G_p^+(c_1) + \tilde{m}^2 G_p^-(c_2)G_p^-(c_1)}{G_p^+(c_1)G_p^+(c_2) - \tilde{m}^2 G_p^-(c_1)G_p^-(c_2)}
\]

\[
\Pi_2^f(p, c_1, c_2, \tilde{m}) = \frac{1}{p} \frac{G_p^-(c_2)G_p^+(c_1) + \tilde{m}^2 G_p^+(c_2)G_p^+(c_1)}{G_p^+(c_1)G_p^+(c_2) - \tilde{m}^2 G_p^-(c_1)G_p^-(c_2)}
\]

\[
M^f(p, c_1, c_2, \tilde{m}) = \frac{\tilde{m}}{2} \frac{G_p^+(c_2)G_p^+(c_1) + G_p^-(c_2)G_p^-(c_1) + G_p^+(c_1)G_p^+(c_2) + G_p^-(c_1)G_p^-(c_2)}{G_p^+(c_1)G_p^+(c_2) - \tilde{m}^2 G_p^-(c_1)G_p^-(c_2)}
\]

(B.9)

where the \( G_p^\pm \) are the 5D holographic functions derived from the equations of motion in the bulk, and the parameters \( c_{1,2} \) are the dimensionless mass parameters defined by \( c_i = m_i R \). After a Wick rotation these holographic functions can be written as

\[
G_p^+(p_E, c, z) = -\frac{2i}{\pi} \sqrt{T} \left( K_{c=\frac{1}{2}}(p_E R') I_{c=\frac{1}{2}}(p_E z) + I_{c=\frac{1}{2}}(p_E R') K_{c=\frac{1}{2}}(p_E z) \right)
\]

\[
G_p^-(p_E, c, z) = -\frac{2}{\pi} \sqrt{T} \left( K_{c=\frac{1}{2}}(p_E R') I_{c=\frac{1}{2}}(p_E z) - I_{c=\frac{1}{2}}(p_E R') K_{c=\frac{1}{2}}(p_E z) \right).
\]

(B.10)

We present the Wick rotated result because this is what we will use in calculating the Higgs potential, however it is trivial to obtain the original result with \( p_E \to -ip \).
References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[2] D. B. Kaplan and H. Georgi, Phys. Lett. 136B (1984) 183. doi:10.1016/0370-2693(84)91177-8 ; D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. 136B (1984) 187. doi:10.1016/0370-2693(84)91178-X ; H. Georgi and D. B. Kaplan, Phys. Lett. 145B (1984) 216. doi:10.1016/0370-2693(84)90341-1 ; M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B 254 (1985) 299. doi:10.1016/0550-3213(85)90221-4.

[3] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B 671 (2003) 148 doi:10.1016/j.nuclphysb.2003.08.027 [hep-ph/0306259].

[4] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719 (2005) 165 doi:10.1016/j.nuclphysb.2005.04.035 [hep-ph/0412089].

[5] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B 641 (2006) 62 doi:10.1016/j.physletb.2006.08.005 [hep-ph/0605341].

[6] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D 75 (2007) 055014 doi:10.1103/PhysRevD.75.055014 [hep-ph/0612048].

[7] A. Pomarol and F. Riva, JHEP 1208 (2012) 135 doi:10.1007/JHEP08(2012)135 [arXiv:1205.6434 [hep-ph]]; O. Matsedonskyi, G. Panico and A. Wulzer, JHEP 1301 (2013) 164 doi:10.1007/JHEP01(2013)164 [arXiv:1204.6333 [hep-ph]]; A. Carmona and F. Goertz, JHEP 1505 (2015) 002 doi:10.1007/JHEP05(2015)002 [arXiv:1410.8555 [hep-ph]].

[8] A. De Simone, O. Matsedonskyi, R. Rattazzi and A. Wulzer, JHEP 1304 (2013) 004 doi:10.1007/JHEP04(2013)004 [arXiv:1211.5663 [hep-ph]].

[9] C. Grojean, O. Matsedonskyi and G. Panico, JHEP 1310 (2013) 160 doi:10.1007/JHEP10(2013)160 [arXiv:1306.4655 [hep-ph]].

[10] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706 (2007) 045 doi:10.1088/1126-6708/2007/06/045 [hep-ph/0703164].

[11] C. Anastasiou, E. Furlan and J. Santiago, Phys. Rev. D 79 (2009) 075003 doi:10.1103/PhysRevD.79.075003 [arXiv:0901.2117 [hep-ph]].

[12] D. Marzocca, M. Serone and J. Shu, JHEP 1208 (2012) 013 doi:10.1007/JHEP08(2012)013 [arXiv:1205.0770 [hep-ph]].

[13] S. De Curtis, M. Redi and A. Tesi, JHEP 1204 (2012) 042 doi:10.1007/JHEP04(2012)042 [arXiv:1110.1613 [hep-ph]].

[14] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207 (2002) 034 doi:10.1088/1126-6708/2002/07/034 [hep-ph/0206021]; N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 0208 (2002) 021 doi:10.1088/1126-6708/2002/08/021 [hep-ph/0206020]; D. E. Kaplan and M. Schmaltz, JHEP 0310 (2003) 039 doi:10.1088/1126-6708/2003/10/039 [hep-ph/0302049].
[15] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-104; The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-101; The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-102; M. Aaboud et al. [ATLAS Collaboration], arXiv:1803.09678 [hep-ex].

[16] Z. Chacko, H. S. Goh and R. Harnik, Phys. Rev. Lett. 96 (2006) 231802 doi:10.1103/PhysRevLett.96.231802 [hep-ph/0506256]; Z. Chacko, Y. Nomura, M. Papucci and G. Perez, JHEP 0601 (2006) 126 doi:10.1088/1126-6708/2006/01/126 [hep-ph/0510273]; Z. Chacko, H. S. Goh and R. Harnik, JHEP 0601 (2006) 108 doi:10.1088/1126-6708/2006/01/108 [hep-ph/0512088].

[17] G. Burdman, Z. Chacko, H. S. Goh and R. Harnik, JHEP 0702 (2007) 009 doi:10.1088/1126-6708/2007/02/009 [hep-ph/0609152].

[18] H. Cai, H. C. Cheng and J. Terning, JHEP 0905 (2009) 045 doi:10.1088/1126-6708/2009/05/045 [arXiv:0812.0843 [hep-ph]].

[19] T. Cohen, N. Craig, G. F. Giudice and M. Mccullough, JHEP 1805 (2018) 091 doi:10.1007/JHEP05(2018)091 [arXiv:1803.03647 [hep-ph]]; H. C. Cheng, L. Li, E. Salvioni and C. B. Verhaaren, JHEP 1805 (2018) 057 doi:10.1007/JHEP05(2018)057 [arXiv:1803.03651 [hep-ph]].

[20] N. Craig, A. Katz, M. Strassler and R. Sundrum, JHEP 1507 (2015) 105 doi:10.1007/JHEP07(2015)105 [arXiv:1501.05310 [hep-ph]]; G. Burdman, Z. Chacko, R. Harnik, L. de Lima and C. B. Verhaaren, Phys. Rev. D 91 (2015) no.5, 055007 doi:10.1103/PhysRevD.91.055007 [arXiv:1411.3310 [hep-ph]]; N. Craig, S. Knapen, P. Longhi and M. Strassler, JHEP 1607 (2016) 002 doi:10.1007/JHEP07(2016)002 [arXiv:1601.07181 [hep-ph]]; R. Barbieri, L. J. Hall and K. Harigaya, JHEP 1611 (2016) 172 doi:10.1007/JHEP11(2016)172 [arXiv:1609.05589 [hep-ph]]; D. Curtin and C. B. Verhaaren, JHEP 1512 (2015) 072 doi:10.1007/JHEP12(2015)072 [arXiv:1506.06141 [hep-ph]]; Z. Chacko, D. Curtin and C. B. Verhaaren, Phys. Rev. D 94 (2016) no.1, 011504 doi:10.1103/PhysRevD.94.011504 [arXiv:1512.05782 [hep-ph]]; A. Ahmed, JHEP 1802 (2018) 048 doi:10.1007/JHEP02(2018)048 [arXiv:1711.03107 [hep-ph]]; Z. Chacko, C. Kilic, S. Najjari and C. B. Verhaaren, Phys. Rev. D 97 (2018) no.5, 055031 doi:10.1103/PhysRevD.97.055031 [arXiv:1711.05300 [hep-ph]].

[21] J. Serra and R. Torre, Phys. Rev. D 97 (2018) no.3, 035017 doi:10.1103/PhysRevD.97.035017 [arXiv:1709.05399 [hep-ph]].

[22] C. Cski, T. Ma and J. Shu, arXiv:1709.08636 [hep-ph].

[23] B. Gripaios, A. Pomarol, F. Riva and J. Serra, JHEP 0904 (2009) 070 doi:10.1088/1126-6708/2009/04/070 [arXiv:0902.1483 [hep-ph]]; J. R. Espinosa, B. Gripaios, T. Konstandin and F. Riva, JCAP 1201 (2012) 012 doi:10.1088/1475-7516/2012/01/012 [arXiv:1110.2876 [hep-ph]]; J. Serra, JHEP 1509 (2015) 176 doi:10.1007/JHEP09(2015)176 [arXiv:1506.05110 [hep-ph]]; G. Cacciapaglia, H. Cai, A. Deandrea, T. Flacke, S. J. Lee and A. Parolini, JHEP 1511 (2015) 201 doi:10.1007/JHEP11(2015)201 [arXiv:1507.02283 [hep-ph]].

[24] M. Frigerio, J. Serra and A. Varagnolo, JHEP 1106 (2011) 029 doi:10.1007/JHEP06(2011)029 [arXiv:1103.2997 [hep-ph]]; J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra and A. Wulzer, Nucl. Phys. B 853 (2011) 1
[25] F. Caracciolo, A. Parolini and M. Serone, JHEP 1302 (2013) 066 doi:10.1007/JHEP02(2013)066 [arXiv:1211.7290 [hep-ph]]; J. Barnard, T. Gherghetta and T. S. Ray, JHEP 1402 (2014) 002 doi:10.1007/JHEP02(2014)002 [arXiv:1311.6562 [hep-ph]]; G. Ferretti and D. Karateev, JHEP 1403 (2014) 077 doi:10.1007/JHEP03(2014)077 [arXiv:1312.5330 [hep-ph]]; G. Cacciapaglia and F. Sannino, JHEP 1404 (2014) 111 doi:10.1007/JHEP04(2014)111 [arXiv:1402.0233 [hep-ph]]; G. Ferretti, JHEP 1406 (2014) 142 doi:10.1007/JHEP06(2014)142 [arXiv:1404.7137 [hep-ph]]; T. Ma and G. Cacciapaglia, JHEP 1603 (2016) 211 doi:10.1007/JHEP03(2016)211 [arXiv:1508.07014 [hep-ph]].

[26] J. M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231] doi:10.1023/A:1026654312961, 10.4310/ATMP.1998.v2.n2.a1 [hep-th/9711200]; E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253 doi:10.4310/ATMP.1998.v2.n2.a2 [hep-th/9802150].

[27] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370 doi:10.1103/PhysRevLett.83.3370 [hep-ph/9905221].

[28] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690 doi:10.1103/PhysRevLett.83.4690 [hep-th/9906064].

[29] R. Rattazzi and A. Zaffaroni, JHEP 0104 (2001) 021 doi:10.1088/1126-6708/2001/04/021 [hep-th/0012248].

[30] S. J. Huber and Q. Shafi, Phys. Lett. B 498 (2001) 256 doi:10.1016/S0370-2693(00)01399-X [hep-ph/0010195]; S. J. Huber, Nucl. Phys. B 666 (2003) 269 doi:10.1016/S0550-3213(03)00502-9 [hep-ph/0303183].

[31] Y. Hosotani, Phys. Lett. 126B (1983) 309. doi:10.1016/0370-2693(83)90170-3; Y. Hosotani, Phys. Lett. 129B (1983) 193. doi:10.1016/0370-2693(83)90841-9.

[32] Y. Hosotani and M. Mabe, Phys. Lett. B 615 (2005) 257 doi:10.1016/j.physletb.2005.04.039 [hep-ph/0503020]; K. y. Oda and A. Weiler, Phys. Lett. B 606 (2005) 408 doi:10.1016/j.physletb.2004.12.007 [hep-ph/0410061].

[33] D. Croon, B. M. Dillon, S. J. Huber and V. Sanz, JHEP 1607 (2016) 072 doi:10.1007/JHEP07(2016)072 [arXiv:1510.08482 [hep-ph]].

[34] M. Geller and O. Telem, Phys. Rev. Lett. 114 (2015) 191801 doi:10.1103/PhysRevLett.114.191801 [arXiv:1411.2974 [hep-ph]].

[35] C. Csaki, M. Geller, O. Telem and A. Weiler, JHEP 1609 (2016) 146 doi:10.1007/JHEP09(2016)146 [arXiv:1512.03427 [hep-ph]].

[36] C. Csaki, M. Geller and O. Telem, JHEP 1805 (2018) 134 doi:10.1007/JHEP05(2018)134 [arXiv:1710.08921 [hep-ph]].
[37] V. Sanz and J. Setford, Adv. High Energy Phys. 2018 (2018) 7168480
doi:10.1155/2018/7168480 [arXiv:1703.10190 [hep-ph]].

[38] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. 83 (1999) 4922
doi:10.1103/PhysRevLett.83.4922 [hep-ph/9907447]; C. Csaki, J. Hubisz and S. J. Lee, Phys.
Rev. D 76 (2007) 125015 doi:10.1103/PhysRevD.76.125015 [arXiv:0705.3844 [hep-ph]]; A.
Chakraborty, U. Maitra, S. Raychaudhuri and T. Samui, Nucl. Phys. B 922 (2017) 41
doi:10.1016/j.nuclphysb.2017.06.006 [arXiv:1701.07471 [hep-ph]]; A. Ahmed, B. M. Dillon,
B. Grzadkowski, J. F. Gunion and Y. Jiang, Phys. Rev. D 95 (2017) no.9, 095019
doi:10.1103/PhysRevD.95.095019 [arXiv:1512.05771 [hep-ph]]; B. M. Dillon, D. P. George
and K. L. McDonald, Phys. Rev. D 94 (2016) no.6, 064045 doi:10.1103/PhysRevD.94.064045
[arXiv:1605.03087 [hep-ph]].

[39] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. 84 (2000) 2080
doi:10.1103/PhysRevLett.84.2080 [hep-ph/9909255]; K. Agashe, H. Davoudiasl, G. Perez and
A. Soni, Phys. Rev. D 76 (2007) 036006 doi:10.1103/PhysRevD.76.036006 [hep-ph/0701186];
A. L. Fitzpatrick, J. Kaplan, L. Randall and L. T. Wang, JHEP 0709 (2007) 013
doi:10.1088/1126-6708/2007/09/013 [hep-ph/0701150]; B. M. Dillon and V. Sanz, Phys.
Rev. D 96 (2017) no.3, 035008 doi:10.1103/PhysRevD.96.035008 [arXiv:1603.09550 [hep-
ph]]; B. M. Dillon, C. Han, H. M. Lee and M. Park, Int. J. Mod. Phys. A 32 (2017) no.33,
1745006 doi:10.1142/S0217751X17450063 [arXiv:1606.07171 [hep-ph]].

[40] R. Contino, arXiv:1005.4269 [hep-ph].

[41] R. Contino and A. Pomarol, JHEP 0411 (2004) 058 doi:10.1088/1126-6708/2004/11/058
[hep-th/0406257].