Cosmic Superstrings Stabilized by Fermions

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\textbf{Abstract}

We show that there exist massive perturbative states of the ten dimensional Green-Schwarz closed superstring that are stabilized against collapse due to presence of fermionic zero modes on its worldsheet. The excited fermionic degrees of freedom backreact on the spacetime motion of the string in the same way as a neutral persistent current would, rendering these string loops stable. We point out that the existence of these states could have important consequences as stable loops of cosmological size as well as long lived states within perturbative string theory.

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1 Introduction

Despite their common origin, the study of fundamental strings and cosmic strings have been traditionally quite differentiated [1, 2, 3]. On the other hand, this distinction may just turn out to be an artifact due to historical reasons. Early estimates of the possible role of fundamental strings as cosmic strings were not too optimistic [4], but recent developments in string theory models of the early universe seem to hint to the possibility that there could exist fundamental strings of cosmological size [5, 6, 7, 8, 9]. These fundamental strings would naturally behave as classical objects rendering them effectively a new type of cosmic strings.

This new connection has been exploited in the literature to suggest that we should actually look for signatures of string theory in the cosmological observations that we are able to perform today. This is undoubtedly a very interesting idea that clearly deserves a detailed study. Most of the work in this direction has focused on the study of the characteristics of these new type of one-dimensional objects, trying to extract distinctive signatures of the network of strings that can conclusively show that we are indeed dealing with string theory models of the early universe.

In this letter we would like to explore the opposite route. We will show that there exist quantum mechanical states of perturbative string theory that mimic solutions of the equations of motion of well known models of classical cosmic strings, namely superconducting cosmic strings [10].

The paper is organized as follows. In Section 2, we introduce the Green-Schwarz formalism for the superstring, mainly following the notation of reference [11]. In Section 3 we discuss the simplest example of a circular superstring loop stabilized by a chiral fermionic current. In Section 4, we show that, actually, the loops could have an arbitrary shape and still be stabilized by the fermionic excitations on the worldsheet. We end with some conclusions and speculations on the relevance of these states.

2 The superstring in the Green-Schwarz formalism

In this section we will briefly review the Green-Schwarz formalism describing the dynamics of the superstring in ten dimensional flat spacetime. The basic idea of this formalism is to extend the usual Polyakov or Nambu-Goto action written in terms of the bosonic target space coordinates of the string, $X^\mu$, to include ten dimensional fermionic degrees of freedom. These new degrees of freedom are two 32 component Majorana-Weyl spinors of Spin(9,1), usually denoted by $\theta^{1A}$ and $\theta^{2A}$ ($A = 1, \cdots, 32$), which have the same or opposite chirality defining type IIB and IIA superstrings respectively.

The action contains two pieces: a kinetic term and a Wess-Zumino term. The kinetic term arises as the simplest generalization of the Polyakov action for the bosonic string by replacing $\partial_a X^\mu$ by the combination

$$\Pi^\mu_a = \partial_a X^\mu - i \bar{\theta}^1 \Gamma^\mu \partial_a \theta^1 - i \bar{\theta}^2 \Gamma^\mu \partial_a \theta^2, \quad (1)$$

The reader familiar with this subject may want to skip ahead to Section 3.
that is invariant under global supersymmetry transformations
\[ \delta \theta^I = \epsilon^I, \quad \delta X^\mu = i \bar{\epsilon}^I \Gamma^\mu \theta^I, \quad I = 1, 2. \]  

(2)

Thus, the kinetic term is given by
\[ S_{\text{Kin}} = -\frac{T}{2\pi} \int d^2 \sigma \sqrt{h} \Pi^\alpha \Pi^\beta, \]  

(3)

where \( T \) is the string tension, \( h^{\alpha \beta} \) is the two dimensional, supersymmetry inert, metric on the string worldsheet with \( \alpha, \beta = 1, 2 = \sigma, \tau \); \( \mu \) denotes the spacetime coordinate index with range \( 0, \ldots, 9 \), and \( \Gamma^\mu \) are the ten dimensional \( 32 \times 32 \) Dirac matrices satisfying
\[ \{ \Gamma^\mu, \Gamma^\nu \} = 2 \eta^{\mu \nu}. \]

On the other hand, this first term of the action by itself does not lead to a ten dimensional supersymmetric spectrum for the string, so one is forced to add the second, Wess-Zumino, term (also invariant under supersymmetry) that allows the possibility of finding a new symmetry of the total action. This is a local fermionic symmetry, Siegel (\( \kappa \)) symmetry, that further restricts the degrees of freedom of the string. The total action then becomes,
\[ S = S_{\text{Kin}} + S_{\text{WZ}}, \]

(4)

where the Wess-Zumino term is given by
\[ S_{\text{WZ}} = \frac{T}{\pi} \int d^2 \sigma \left\{ -i \epsilon^{\alpha \beta} \partial_\alpha X^\mu \left( \bar{\theta}^1 \Gamma^\mu \theta^1 - \bar{\theta}^2 \Gamma^\mu \theta^2 \right) + \epsilon^{\alpha \beta} \bar{\theta}^1 \Gamma^\mu \partial_\alpha \theta^1 \bar{\theta}^2 \Gamma^\mu \partial_\beta \theta^2 \right\}. \]

(5)

The classical equations of motion for the fields present in the action \( (\Pi) \), namely, \( h_{\alpha \beta}, X^\mu, \theta^1 \) and \( \theta^2 \) can be derived easily. Note, however, that in a general gauge, these equations would turn out to be quite complicated as it also happens in the purely bosonic case if we do not choose the right gauge conditions. Just as for the bosonic string \( (\Pi) \), it can be shown that the symmetries of the action allow us to select a gauge (by fixing Weyl, 2D reparametrizations and residual semilocal conformal symmetries) in which
\[ X^+ = X^t + X^9 = x^+ + l^2 p^+ \tau, \]

(6)

\[ C = \Gamma^9 = i \tau_2 \otimes 1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Gamma^i = \tau_1 \otimes \begin{pmatrix} 0 & \gamma^i \\ \gamma^i & 0 \end{pmatrix}, \quad \Gamma^9 = \tau_1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \]

where the 8 \( \times \) 8 matrices \( \gamma^i \) are given by:
\[ \gamma^1 = -i \tau_2 \otimes \tau_2 \otimes \tau_2, \quad \gamma^5 = i \tau_3 \otimes \tau_2 \otimes 1, \]
\[ \gamma^2 = i \tau_1 \otimes \tau_1 \otimes \tau_2, \quad \gamma^6 = i \tau_2 \otimes 1 \otimes \tau_1, \]
\[ \gamma^3 = i \tau_1 \otimes \tau_3 \otimes \tau_2, \quad \gamma^7 = i \tau_2 \otimes 1 \otimes \tau_3, \]
\[ \gamma^4 = i \tau_1 \otimes \tau_2 \otimes 1, \quad \gamma^8 = 1 \otimes 1 \otimes 1, \]

and \( \tau_{1,2,3} \) are the Pauli matrices.
where $l^2 = 1/T$.

On the other hand, we can also completely fix $\kappa$ symmetry by setting,

$$\Gamma^+ \theta^1 = \Gamma^+ \theta^2 = 0,$$

where,

$$\Gamma^+ = \Gamma^0 + \Gamma^9.$$  \hspace{1cm} (8)

In this gauge (light-cone gauge) the equations of motion for the surviving dynamical degrees of freedom are extremely simple, since they become,

$$\left( \frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^i = 0,$$  \hspace{1cm} (9)

$$\left( \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma} \right) S^{1a} = 0,$$  \hspace{1cm} (10)

$$\left( \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma} \right) S^{2a} = 0,$$  \hspace{1cm} (11)

where $X^i$ are the eight transverse degrees of freedom of the position of the string and $S^{1a} = \sqrt{p^i} \theta^1$ and $S^{2a} = \sqrt{p^i} \theta^2$, with $a = 1, \cdots, 8$, are the left and right-moving Majorana-Weyl fermions in the $8_s$ and $8_s$ (or $8_c$) $Spin(8)$ spinor representation respectively in type IIB (or A).

We can therefore write the general solution for a closed superstring loop using the standard decomposition into left and right-movers

$$X^j = X^j_R + X^j_L,$$  \hspace{1cm} (12)

$$X^j_R = \frac{1}{2} x^j + \frac{1}{2} l^2 p^j (\tau - \sigma) + \frac{i}{2} l \sum_{n \neq 0}^{\infty} x_n^{j} e^{-2in(\tau - \sigma)},$$  \hspace{1cm} (13)

$$X^j_L = \frac{1}{2} x^j + \frac{1}{2} l^2 p^j (\tau + \sigma) + \frac{i}{2} l \sum_{n \neq 0}^{\infty} \tilde{x}_n^{j} e^{-2in(\tau + \sigma)},$$  \hspace{1cm} (14)

where $j = 1, \cdots, 8$, and $x^j$ and $p^j$ are the center of mass position and momentum of the loop. Similarly we can decompose the fermionic coordinates as,

$$S^{1a} = \sum_{n = -\infty}^{\infty} S_n^a e^{-2in(\tau - \sigma)},$$

$$S^{2a} = \sum_{n = -\infty}^{\infty} \tilde{S}_n^a e^{-2in(\tau + \sigma)}.$$  \hspace{1cm} (15)

where the coefficients satisfy anti-commutation relations, namely,

$$\{ S_m^a, S_n^b \} = \delta^{ab} \delta_{m+n,0}$$  \hspace{1cm} (16)
This is, of course, not the whole story, since we should also have an equation for the metric \(h_{\alpha\beta}\) coming from the general action. Once we restrict this equation to our particular gauge choice, namely the light-cone gauge, we obtain the following constraint equations,

\[
\alpha_n^{-} = \frac{1}{2lp^+} \sum_{m=Z} \left( \alpha_{n-m}^{i} \alpha_{m}^{i} + \left( m - \frac{n}{2} \right) S_{n-m}^{a} S_{m}^{a} \right)
\]

(17)

\[
\tilde{\alpha}_n^{-} = \frac{1}{2lp^+} \sum_{m=Z} \left( \tilde{\alpha}_{n-m}^{i} \tilde{\alpha}_{m}^{i} + \left( m - \frac{n}{2} \right) \tilde{S}_{n-m}^{a} \tilde{S}_{m}^{a} \right)
\]

(18)

where \(\alpha_n^{-}\) and \(\tilde{\alpha}_n^{-}\) are the expansion coefficients of the light-cone coordinate, \(X^{-}\), namely,

\[
X^{-} = x^{-} + l^2 p^{-} \tau + \frac{i}{2} p^+ \sum_{n \neq 0} \frac{1}{n} \left( \alpha_n^{-} e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^{-} e^{-2in(\tau+\sigma)} \right)
\]

(19)

Finally consistency of these equations requires that,

\[
\left( -l^2 p^+ p^- + l^2 (p^i)^2 + 8 N_{L,R} \right) = 0 ,
\]

(20)

for both left and right movers,

\[
N_L = \sum_{n=1} \alpha_{-n}^{j} \alpha_{n}^{j} + n S_{-n}^{a} S_{n}^{a} ,
\]

\[
N_R = \sum_{n=1} \tilde{\alpha}_{-n}^{j} \tilde{\alpha}_{n}^{j} + n \tilde{S}_{-n}^{a} \tilde{S}_{n}^{a} .
\]

(21)

In the remainder of the paper we work at the quantum level, therefore these constraints should be satisfied on the quantum states.

### 3 A superstring loop

We will consider a quantum state \(|\phi\rangle\) of the Green-Schwarz superstring in the light-cone gauge such that it has no \(X^9\) excitations, in other words we will have that \(p^+ = p^- = p\) which in turn means that,

\[
X^+ = X^- = l^2 p \tau .
\]

(22)

As we explained before in the light-cone gauge, the string is built by acting with several bosonic and fermionic operators on the right and left moving vacuum sectors. In our case, we are interested in studying the state with bosonic excitations only in the left moving sector and fermionic excitations present only in the right moving sector. In particular we can discuss the simplest such a state which we construct in the following way,

\[
|\phi\rangle = e^{v_1 \alpha_1^{x} - \bar{v}_1 \alpha_1^{x}} e^{w_1 \alpha_1^{y} - \bar{w}_1 \alpha_1^{y}} |0\rangle_L \otimes V_{h+1} \otimes V_{h+2} \otimes \cdots \otimes V_{1} |0\rangle_R ,
\]

(23)

where \(v\) and \(w\) are complex parameters and \(h\) is a negative integer and where we have defined

\[
V_n^q = \tilde{S}_n^{2q-1} + i \tilde{S}_n^{2q} ,
\]

\[
\bar{V}_n^q = \tilde{S}_n^{2q-1} - i \tilde{S}_n^{2q} .
\]

(24)
with \( q = 1, \cdots, 4 \).

The right-moving part of the state is reminiscent of a \( q \)-vacuum as constructed in [12] for a general \( b - c \) system. Starting from a vacuum \(^5\)

\[
\tilde{S}^a_n |0\rangle_R = 0, \quad n > 0,
\]

we can build this part of the state as in (23),

\[
|h\rangle_R = V^1_{h+1} V^1_{h+2} \cdots V^1_0 |0\rangle_R,
\]

which has the property that for negative \( h \) it satisfies

\[
V^1_n |h\rangle_R = 0, \quad n > h,
\]

\[
\tilde{V}^1_n |h\rangle_R = 0, \quad n \geq -h.
\]

(27)

Using (16) we can also show that the new operators \( V \) and \( \tilde{V} \) satisfy the following anti-commutation relations,

\[
\{ \tilde{V}^p_n, V^q_m \} = 2 \delta_{m+n,0} \delta^{pq}.
\]

(28)

We will now show that it is indeed possible for this state to fulfill all the physical constraints described in the previous section by fixing the parameters in the state construction, namely, \( v, w, h \).

From the constraint (20) for the left-movers we obtain the condition

\[
-l^2 p^2 + 8 (|v|^2 + |w|^2) = 0.
\]

(29)

Taking into account that,

\[
\sum_{n=1}^\infty n \tilde{S}^a_{-n} \tilde{S}^a_n |h\rangle_R = \frac{h(h+1)}{2} |h\rangle_R,
\]

(30)

we see that the Hamiltonian constraint (20) for the right-movers imposes the following relation,

\[
-l^2 p^2 + 4h(h+1) = 0.
\]

(31)

Also, consistency of the Virasoro condition (17) with eq. (22) implies (only \( n = 2 \) gives a nontrivial equation):

\[
v^2 + w^2 = 0.
\]

(32)

Finally, it is not difficult to check that eq. (18) does not give any other condition for our state. Choosing, therefore, the parameters \( v, w, h \) fulfilling eqns. (29), (31) and (32) would completely fix the state we are looking for.

\(^5\)The vacuum states of the GS superstring in each sector (\( R \) and \( L \)) are a spanned by a set of eight bosonic states \(|i\rangle \) in the vector representation of \( \text{Spin}(8) \) and eight fermions \(|a\rangle \) in the 8s (or 8c) representation. In order to build the state \(|\phi\rangle \) we can choose any of these as our vacuum.
Having done that, the position of the string in this state is given by:

\[
\langle X^t \rangle = t^2 p \tau \\
\langle X^x \rangle = \frac{i}{2} \left( -v^* e^{2i(\tau - \sigma)} + ve^{-2i(\tau - \sigma)} \right) \\
\langle X^y \rangle = \frac{i}{2} \left( -w^* e^{2i(\tau - \sigma)} + we^{-2i(\tau - \sigma)} \right).
\]

(33) (34) (35)

Choosing real \( lp = 4v = 2\sqrt{h(h + 1)} \) and \( w = -iv \) we satisfy the constraints and the string has spacetime parametrization given by

\[
\langle X^\mu \rangle = (4lv\tau, lv \sin 2(\tau - \sigma), lv \cos 2(\tau - \sigma), 0, \ldots).
\]

(36)

A fixed circle of radius \( lv \), which for large enough values of the parameter \( v \) can be of cosmological size. We will address the issue of the stability of this loop in section 5.

4 Loops of arbitrary shape

The loop considered on the previous section had a circular shape but we will now show that this can be easily generalized to states with arbitrary shape. Let us consider a loop constructed from a general coherent state of the form:

\[
|\phi\rangle = e^A |0\rangle_L \otimes |h\rangle_R ,
\]

(37)

where

\[
A = \sum_{n=1}^{N} C_n^i \alpha_{-n}^i - \text{ h. c. },
\]

(38)

and \( C_n^i \) are complex numbers satisfying,

\[
C_n^i = \left( C_{-n}^i \right)^*. \tag{39}
\]

In order for this state to parametrize a static loop in spacetime we should impose the constraint equations described by (17), which in our state reduce to,

\[
\alpha_{-n}^i |\phi\rangle = \frac{1}{2lp^+} \sum_{m=Z} \alpha_{-m}^i \alpha_m^i |\phi\rangle = 0 .
\]

(40)

This in turn means that the coefficients \( C_n^i \) should obey the following relations,

\[
\sum_{m=n-N}^{N} C_n^i C_m^i = 0 ,
\]

(41)
for \( n = 1, \cdots 2N \). Similarly, we should impose the Hamiltonian constraint to this state which in this case becomes,

\[
\sum_{m=n-N}^{N} C_i^m C_m^i = \text{const} .
\]  

(42)

In order to see what restrictions these constraints have on its shape, we calculate the expectation value of the position of the string at a fixed time, \( X^j(\sigma, \tau = 0) \),

\[
\langle X^j \rangle = l \sum_{n=-N}^{N} \frac{1}{2in} C_n^j e^{2in\sigma} .
\]  

(43)

This shows that the constraints on the coefficients \( C_n^i \) are just imposing that \( \sigma \) parametrizes the position of the string such that

\[
\left| \frac{d \langle X^j \rangle}{d\sigma} \right| = \text{const} .
\]  

(44)

Since for any given function we can always find a parametrization of the position of the string that satisfies (44), we conclude that we will always be able to describe such a string with a coherent state like the one presented in (37).

5 Concluding remarks

We have shown that there are states of the perturbative superstring spectrum which can be identified as string loops stabilized by the presence of fermionic excitations on the string worldsheet. The energy momentum tensor associated with the fermions creates a mechanical backreaction that allows these strings to remain static in spacetime. This is exactly the same effect that occurs in field theory models of cosmic strings where the fermionic degrees of freedom on the string worldsheet come from zero modes trapped on the vortex solution [10].

On the other hand, field theory cosmic strings can also have bosonic zero modes associated with the phase of a condensate living on the string. This degree of freedom can form neutral superconducting loops, the so called vortons [13]. It is therefore natural to wonder whether there are similar configurations in string theory.

In this case, the situation is a little bit more complicated, since there are no bosonic condensates coupled to the superstring. It is nevertheless still possible to find stable string configurations by considering the effect on the loop motion of the excitations of the position of the string along some perpendicular direction. These excitations can be thought of as a bosonic current on the loop worldsheet due to the Goldstone bosons associated with the breaking of translational invariance along those directions, either along a plane perpendicular to the string in ten dimensional spacetime [14] or along some compactified direction [15].
On the other hand, we have also shown that the states we discuss in this paper are characterized by their fermionic content and that the string shape in space is totally arbitrary in the plane perpendicular to $X^9$. This is another characteristic shared with their classical superconducting string counterparts [16, 17], a reflection of the fact that we are basically dealing with the same physical effect.

The existence of these arbitrary shaped loops of fundamental strings could have important cosmological consequences in the recently discussed models of brane inflation, where loops of fundamental strings are expected to be produced at cosmological scales [8, 9]. Some of these loops will be produced with some fermionic excitations and could end up as the states we have been discussing. We note, in this regard, that we certainly expect large loops to be very stable since they preserve some fraction of supersymmetry in the straight string limit.\(^6\)

One can see this by looking at the supersymmetry transformations (2), which in the lightcone gauge split in two classes. Those with $\epsilon$ parameter such that the gauge choice is preserved, which give rise to eight supersymmetries in each sector ($L$ and $R$), of the form

$$\delta S^a = \sqrt{2p^+}\eta^a, \quad \delta X^i = 0,$$

and those that need a compensating $\kappa$ transformation which are of the form

$$\delta S^a = -i\rho \cdot \partial X^i \gamma^i a \epsilon^a \sqrt{2p^+}, \quad \delta X^i = \sqrt{\frac{2}{p^+}} \gamma^i a \epsilon^a S^a.$$

One immediately notices that in the absence of bosonic excitations in the $i$ directions (i.e., an infinite straight string), one is free to put right-moving fermionic excitations and still preserve the 8 supersymmetries [16] on the left-moving sector. These are 1/4 BPS states which support fermionic excitations. Thus, one would expect that at least a string loop of huge size and large curvature radius should be long-lived. This has also been realized recently in the context of fermionic zero modes on supersymmetric cosmic strings in [18].

Finally, we focused in this paper in states of type $IIA$ and $IIB$ superstrings. But the generalization to other models is expected. The states we discussed will also be present in Heterotic strings (just taking the fermions in the supersymmetric side), and a generalization to $D1$ brane states should not be difficult to achieve. Also, this construction (neglecting issues of quantization) should hold in spacetimes with dimensionality 3, 4 and 6 where a Green-Schwarz superstring action can be constructed.

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