C/O white dwarfs of very low mass: 0.33-0.5 M\(_\odot\)

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Abstract. The standard lower limit for the mass of white dwarfs (WDs) with a C/O core is roughly 0.5 M\(_\odot\). In the present work we investigated the possibility to form C/O WDs with mass as low as 0.33 M\(_\odot\). Both the pre-WD and the cooling evolution of such nonstandard models will be described.

1. Introduction

It is by now a well-known and commonly accepted result of the stellar evolution theory that stars of low and intermediate mass will end their lives as white dwarfs (WDs). Since the pioneering papers by Chandrasekhar (1931) and Stoner (1932), it has been known that there must be an upper limit of the mass of WDs. The current value for such a limit, now universally known as Chandrasekhar’s mass, is M\(_{\text{Ch}}\) = 1.456 \((2/\mu_e)^{\frac{5}{2}}\) M\(_\odot\) (where \(\mu_e\) is the electronic mean molecular weight, Hansen, Kawaler & Trimble 2004). Below this critical mass there are essentially three different families of WDs concerning the chemical composition of their cores: the O/Ne/Mg, C/O and He. The upper mass limit for C/O WDs, which is still subject to some debate, since it is the aftermath of the evolution of a star with initial mass slightly lower than M\(_{\text{up}}\), the lowest mass for star that succeeds in igniting carbon burning before the asymptotic giant branch (AGB), it is around 1.05 M\(_\odot\) (Dominguez et al. 1999, Weideman 2000, Prada Moroni & Straniero 2007, Catalan et al. 2008, Meng, Cheng & Han 2008).

On the other hand, the value for the minimum mass of WDs with a C/O core is commonly believed to be around 0.5 M\(_\odot\) (Weideman 2000, Meng et al. 2008). The theory of stellar evolution predicts that an isolated single star with mass lower than about 0.5 M\(_\odot\), the exact value depending on the chemical composition (0.46 and 0.50 for Z=0.04 and Z=0.0001, respectively, Dominguez et al. 1999, Girardi 1999), do not ignite the helium-burning and die as WDs with an helium rich core. For these stars, during the red giant phase the electronic thermal conduction and the emission of neutrinos, which cool the core, prevail on the heating due to the contraction caused by the accretion of fresh helium processed by the hydrogen-burning shell. This is the main reason why WDs of mass less than about 0.5 M\(_\odot\) are commonly believed to have a He-core.

On the other hand, as early as 1985 Iben & Tutukov, in a very beautiful and instructive paper, showed that a star with an initial mass of 3 M\(_\odot\) evolving in a close binary can produce a remnant of 0.4 M\(_\odot\) with a sizeable C/O core. A similar result was obtained 15 years later by Han, Tout & Eggleton (2000), who computed the evolution of close binary systems with several different parameters. They showed that in a binary system with the primary star of
mass $M_1 = 2.51 \, M_\odot$, the mass ratio $q = M_1/M_2 = 2$ and the initial period $P = 2.559 \, \text{d}$, the primary star succeeded to ignite the helium-burning. They did not followed the evolution of the remnant C/O WD, since their code met numerical problems when the primary star was as low as $0.33 \, M_\odot$ with a C/O core of $0.11 \, M_\odot$.

In this paper we will investigate both a possible evolutionary scenario able to produce C/O WDs of low mass, that is $M < 0.5 \, M_\odot$, and their cooling evolution. We will show the results of detailed evolutionary computations performed with a full Henyey code able to follow consistently the evolution of stars from the initial pre-main sequence to the final cooling phase of WDs.

### 2. The red giant phase transition

The stellar models showed in the present work have been computed with an updated version of FRANEC (Prada Moroni & Straniero 2002, Degl’Innocenti et al. 2008), a full Henyey evolutionary code. We adopted a metallicity $Z = 0.04$ and an initial helium abundance $Y = 0.32$ suitable for the very metal-rich stars belonging to some open galactic cluster, such as NGC6791.

![Figure 1](image1.png)

**Figure 1.** Mass of the He-core at the tip of the RGB (diamond and dashed line) and mass of the hydrogen exhausted core at the first thermal pulse on the AGB (circles and solid line) as a function of the initial mass for stars with $Z = 0.04$ $Y = 0.32$.

![Figure 2](image2.png)

**Figure 2.** Mass of the hydrogen exhausted core as a function of the age for a star of $M = 2.3 \, M_\odot$, $Z = 0.04$ $Y = 0.32$. The circles mark: the central hydrogen exhaustion, the base and the tip of the RGB and the central helium exhaustion.

The figure[1] shows the mass of the hydrogen exhausted core $M_H$ at the first thermal pulse on the AGB (upper curve) and the mass of the helium core $M_{He}$ at the tip of the red giant branch (RGB tip, lower curve) as a function of the initial mass. As it is well known, $M_H$ at the first thermal pulse is nearly constant around $0.55 \, M_\odot$, the exact value depending on the chemical composition, for initial masses lower than $3 \, M_\odot$. On the other hand, the behavior of the $M_{He}$ at the tip of RGB as a function of the initial mass is much less smooth than the previous one, in particular a deep and sharp minimum is present around $2.3 \, M_\odot$ for this chemical composition (Castellani, Chieffi, Straniero 1992). Such a behavior is the consequence of the physical conditions present in the helium-core at the onset of the $3\alpha$ nuclear reaction.

For initial mass lower than $1.5$-$1.7 \, M_\odot$, an electron-degenerate core develops at the beginning of the red giant phase, and the onset of the $3\alpha$ occurs in a nuclear runaway, the so-called helium-flash (Hoyle & Schwarzschild 1955). For these stars, the $M_{He}$ at the tip of the RGB is almost
constant, about 0.46 M⊙ for metal-rich stars and 0.50 M⊙ for metal-poor ones, and is essentially produced during the hydrogen-shell burning phase.

From this point on, the larger the initial mass, the weaker the degeneracy of the core in the red giant phase, thus the weaker the He-flash and the lower the mass of the helium-core required for the 3α burning. When the initial mass is as high as about 2.3 M⊙, the helium-burning ignites quiescently, without a flash. This explains the steep decrease of the M_{He} at the tip of the RGB between 1.8 and 2.3 M⊙. For higher masses, the core in the red giant phase is not degenerate any more. From this point on, the larger the initial mass, the larger the M_{He} at the tip of the RGB, as the larger the convective core during the previous main sequence phase. Thus, the minimum mass of the helium-core M_{He} at the onset of the 3α occurs in the transition between these two regimes, around 2.3 M⊙, reaching a value of 0.315 M⊙.

Notice that, as previously stated, this deep minimum is almost canceled at the first thermal pulse. In fact, the hydrogen burning shell continues to process hydrogen and move outward, then increasing M_{H}, during the central helium-burning phase. Moreover, it is a well established result of the theory of stellar evolution that the lower the mass of the He-core at the onset of the 3α, the fainter the star and the longer the central helium-burning phase. Thus, as the initial mass of the helium-rich core decreases at the beginning of the red giant evolution, a star in the RGB phase transition, that is around 2.3 M⊙, that undergoes a very strong mass loss during the red giant phase can produce a C/O WD with a mass significative lower than 0.5 M⊙.

3. C/O white dwarfs of very low mass
In order to check such an idea and provide models of C/O WDs with mass lower than 0.5 M⊙, we computed the evolution at constant mass of a star with M=2.3 M⊙ until the red giant phase. Then at about logL/L⊙=1.34, when the He-core was M_{He}= 0.2569 M⊙, we turned on the mass loss, with a constant rate of the order of 10^{-7} M⊙ yr^{-1}. We stopped the mass loss once the desired final mass was obtained and then we followed the next evolution until the final cooling phase.

The figures 3 - 7 show the evolutionary tracks in the HR-diagram for different values of the final mass, from 0.48 M⊙ to 0.33 M⊙, from the red giant to the final WD phase. The model showed in figure 3, with a total mass of 0.48 M⊙, succeeded to ignite He-burning and become a WD with a C/O core whose mass fraction is about 89 %, where we defined the core edge to be the point where the helium abundance became larger than 50%. A very similar evolution is
Figure 3. Evolutionary track of a star of $M=0.48 \, M_\odot$ whose progenitor of $M=2.3 \, M_\odot$ underwent a strong mass loss episode during the red giant phase.

followed by the model with $0.461 \, M_\odot$, as can be seen in figure 4. In this case, the resulting WD has a C/O core of 85% of the total mass.

Figure 4. Same as in figure 3 but for a remnant of mass $M=0.461 \, M_\odot$.

Figure 5. Same as in figure 3 but for a remnant of mass $M=0.43 \, M_\odot$.

The evolution becomes more complex and difficult to compute for lower masses. The figure 5 shows the evolution of the model with $0.43 \, M_\odot$. The star onsets the $3\alpha$ nuclear reaction and after the central He-burning phase it experiences a series of He-shell thermal pulses: during each flash the star describes a loop in the HR diagram. After the end of the thermal pulses the helium-burning shell continues quiescently and the model moves toward higher effective temperatures. Then the star, approaching the cooling track, experiences a few strong hydrogen-shell flashes, more specifically CNO-flashes. During these episodes, the hydrogen rich outermost layer is progressively eroded, until the star can cool down as a WD with a C/O core of 79% of the total mass. The evolution of the model with $0.38 \, M_\odot$, shown in figure 6, presents again both the He-thermal pulses and the CNO-flashes. In addition, after the end of the flashes, when the model moves toward the blue it experiences a late thermal pulse. The resulting WD has a C/O core of 50% of the total mass.

Figure 6. Same as in figure 3 but for a remnant of mass $M=0.38 \, M_\odot$.

Notice that the lower the mass of the remnant, the lower the mass fraction of the C/O core, thus the larger the mass of the helium-rich buffer. In standard C/O WD, the thickness of the
He-rich layer is of the order of a few percent (1-2%), while in the present models of very low mass C/O WDs it can reach the 50% of the total mass.

Notice that the helium-shell thermal pulses we showed above are not the same that occur in AGB stars. In fact, in that case the thermal instability is the consequence of the accumulation of a critical mass of helium accreted by the quiescent hydrogen-burning shell. While in this case, they are self-excited relaxation oscillations, where the compression is due to the relaxation of the star after a previous contraction. At variance with thermal pulses in AGB, these pulses affect the entire star, with large oscillations in central temperature and density, as early shown in the pioneering paper by Iben et al. (1986).

The evolution of the remnant with mass $M = 0.33 \, M_\odot$ deserves a detailed discussion. In fact, adopting the same mass loss rate along the red giant phase of the progenitor star of 2.3 $M_\odot$ as in the previous cases, the 0.33 $M_\odot$ does not experience $3\alpha$ burning and becomes a WD with a helium core. Figure 7 shows the related evolutionary track on the HR diagram. On the other hand, if the mass loss is turned on later on the RGB phase of the progenitor star, when the mass of the helium core is larger ($\Delta M_{He} \approx 1\%$), the final model with 0.33 $M_\odot$ succeeds to ignite He-burning and eventually evolves to the final WD stage with a C/O core of about the 53% of the total mass. Figure 8 shows its evolutionary track. This is the lowest C/O WD we managed to produce for this chemical composition, in fact the models with mass smaller than 0.33 $M_\odot$ do not ignite the $3\alpha$ and finish their evolution as He-core WDs. The present evolutionary computations prove that it is possible to have C/O WDs with mass as low as 0.33 $M_\odot$, significantly lower than 0.5 $M_\odot$, the classical and commonly accepted lower limit. This means that in the mass range 0.33 - 0.5 $M_\odot$ both He and C/O core WDs can exist. Thus it is interesting to compare the main characteristics of their structure and evolution. A detailed description of the cooling evolution of low mass WDs has been recently published by Panei et al. (2007). Figure 9 shows the comparison between the core chemical profiles of the two remnants of 0.33 $M_\odot$. While in one case the core is constituted by nearly pure helium, in the other a mixture of carbon and oxygen is present. Notice that in the center there is a significant abundance of carbon, $X_{12C} = 0.285$, as in standard C/O WDs, thus these very low mass WDs should not be called simply oxygen WDs. As previously stated, a peculiar feature of these very low mass C/O WDs is that the helium-rich buffer is very thick, up to about 50% of the total mass, while the value of typical C/O WDs is of the order of 1-2 %. The thickness of the hydrogen rich outermost layer is almost the same in the two models, that is $M = 0.0014 \, M_{WD}$ for the WD with a helium rich core and $M = 0.0015 \, M_{WD}$ for that with C/O one.
Figure 9. Abundances in mass of helium, carbon and oxygen as a function of the mass coordinate for the two WDs with M= 0.33 M⊙, the He-core (dashed line) and the C/O core (solid lines).

Figure 10. Comparison between the cooling curves, logL/L⊙ vs. time, of the two WDs of 0.33 M⊙, that with the He-core (dashed line) and with C/O core (solid line).

Figure 11. Radius vs. effective temperature of the two WDs of 0.33 M⊙, that with the He-core (dashed line) and with C/O core (solid line).

Figure 12. Surface gravity vs. effective temperature of the two WDs of 0.33 M⊙, that with the He-core (dashed line) and with C/O core (solid line).

Figure 10 shows the comparison between the cooling curves of the He-core WD (dashed line) and the C/O core (solid line). As expected, the remnant with the helium rich core cools slower than that with a C/O one, as the specific heat per gram of helium is larger than that of carbon and oxygen; thus the thermal content of the He WD is larger than that of a C/O one with the same total mass. Such a difference in the cooling times is slightly counterbalanced at logL/L⊙ ≈ -4 as the C/O core crystallizes, with the concomitant energy release, while the He core does not, in the presently computed range of temperatures and densities.

Figures 11 and 12 show the comparison between the radii and surface gravities, respectively, as a function of the effective temperature for the two WDs of 0.33 M⊙. For a given effective temperature, the He-core WD is more expanded than the other one, difference not negligible at the beginning of the cooling (∼ 8.5%). Figure 13 shows the comparison between the central density ρc as a function of the luminosity for the two WDs of 0.33 M⊙, with the He-core (dashed line) and C/O one (solid line). We did not show the plot with the central temperatures Tc versus luminosity relations for the two WDs because they are nearly identical.
Figure 13. Central density $\rho_c$ vs. luminosity of the two WDs of 0.33 $M_\odot$, that with the He-core (dashed line) and with C/O core (solid line).

4. Conclusion

In conclusion, as already anticipated by Iben & Tutukov (1985), we proved, by means of fully and consistent evolutionary computations, that the minimum mass for a C/O WD is about 0.33 $M_\odot$, near the minimum possible mass of the helium-core at the onset of He-burning, which occurs at the RGB phase transition ($M \approx 2.3 M_\odot$).

As a consequence, in the mass range 0.33-0.5 $M_\odot$ both He and C/O core WDs can exist. As expected and already shown by Panei et al. (2007), the cooling times of these two classes of WDs are quite different, being the He-core remnants significantly slower than the C/O ones. In addition, we showed also that the He WDs are more expanded that their C/O counterparts at a given effective temperature.

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