Quasi-satellite system for leader-follower spacecraft formation

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Abstract. The quasi-satellite motion is where a celestial body appears to orbit around another body that is also orbiting a common focus. The recent discoveries of asteroids in quasi-satellite motion around the Earth inspired this work. This paper attempts to look into exploiting the quasi-satellite dynamics in nature to provide a new perspective in spacecraft formation flight. We consider a two-spacecraft formation flight in which the follower appears to orbit of the leader in its reference frame. We perform a parametric analysis by varying the orbital elements of leader and follower examine the initial conditions needed to simulate a quasi-satellite formation system. We illustrate the trajectories of the follower spacecraft. This serves two purposes: firstly, to demonstrate formation flight based on the idea of quasi-satellite; and secondly, to gain some insights on the shapes of the resultant “quasi-orbits”.

1. Introduction

The discovery of the Earth’s new “moon” [1] have inspired the work in this paper. Asteroid 2016 HO3 is not technically a moon but one of Earth’s orbital companions called quasi-satellites. It is a term first coined in a paper by Mikkola and Inmanen [2] to define a celestial body that librates around the longitude of its associated planet. A quasi-satellite is different from a true satellite in that its orbit lies outside its planet’s Hill sphere, meaning that its motion is not influenced by the planet’s gravity. The first observed quasi-satellite system was the asteroid 2002 V E68 and Venus system reported in [3].

In their 1997 paper, Mikkola and Inmanen [2] assert several conditions for a formation of a quasi-satellite around a planet: (i) it has to have the same mean motion and mean longitude as its associated planet; and (ii) the distance must be large enough to be outside of the planet’s gravitational influence. Such celestial body will exhibit a retrograde motion around the planet of which orbits are, in general, not stable.

This observation of a natural celestial phenomenon opens up the idea of applying quasi-satellite dynamics in spacecraft formation flight. In fact, Mikkola and Prioroc [4] has very recently introduced the concept of creating a spacecraft formation flight using quasi-satellite dynamics. In their dynamics model, however, it is imposed that the spacecraft to have ion thrusters to provide inverse square-law type of force in order to imitate the gravitational force of celestial bodies like planets.

We point out here that the concept of a quasi-satellite in tandem motion with its planet can be applied in spacecraft formation flying without the need of thrusters. We can relax some of the quasi-satellite conditions proposed in [2] and [4]. The “planet” in the formation can
be a spacecraft, which has no gravitational attraction of its own or any attractive/repulsive propulsion. This simplifies the dynamics of quasi-satellite in our case in that we can reduce the dynamics into a two-body problem.

Inspired by the nature, this paper studies a way to mimic the dynamics of natural quasi-satellites in our solar system using artificial satellites. We start our investigation of this problem by modeling the relative motion of two spacecraft – a leader and a follower – using an unperturbed two-body problem model. The imposed condition for our spacecraft formation is this: the follower is to orbit like a quasi-satellite where the leader acts as the formation’s planet. We conduct a parametric analysis in that we vary the orbital elements of both spacecraft to find and specify the conditions for a leader-follower formation to imitate the quasi-satellite tandem motion. We present some numerical simulations to show the trajectories of the quasi-satellite formation for different variations of orbital elements. We show that tandem flight of two spacecraft akin to a quasi-satellite system can be achieved without attractive or repulsive mutual force.

2. Related Works in Formation Flight
Satellite formation flying enables the concept of having multiple, smaller satellites cooperating to accomplish the objective of a monolithic, more expensive satellite. The type of formation can be many, e.g. a trailing formation (the A-train satellite formation [5]), a rigid arrangement and coordination, (typically for interferometry missions as discussed in [6]), and a constellation (GPS satellites).

Formation flight studies and analyses are mainly originates and based from terminal rendezvous studies. The Clohessy-Wiltshire equations of relative motion [7], or the CW equations, is by far the most popular method in the studies of spacecraft formation flight. There are some limitations in that the CW equations only work if the reference (leader) orbit is strictly circular and the distance between the leader and the follower has to be much less that the leader’s orbit radius. A bounded, periodic solution - to find a bounded, periodic orbit of the follower spacecraft - also requires a strict initial condition which may be impractical in real application. The Tschauner-Hempel [8] equations provide a description of relative motion for an eccentric reference orbit. This model has been used to overcome the eccentricity limitation of the CW equations but its solutions are generally more complex which complicates analysis.

In finding an alternative approach to formation flight dynamics analysis, some researchers have turned into the manipulating the orbital elements. The idea of separating orbital elements to maintain safe spacecraft collocation has been applied for geostationary orbit [9]. In general, it is easier to use orbital elements approach to include orbital perturbations compared to using the state transition matrices approach (like the CW and TH equations). This is the motivation of several publications on the problem formation flight in [10], [11] and [12].

The quasi-satellite formation proposed in this paper is a formation where a spacecraft’s orbit is design in such a way that it appears to orbit another spacecraft closely. We remark that this paper is not proposing any specific space mission but to introduce a type of leader-follower dynamics and present a nature-inspired innovation in formation flying design. The method presented here uses the full nonlinear model and can be applied to all closed Keplerian orbits.

3. Spacecraft Relative Motion Modeling
The analysis assumes the unperturbed two-body motion

\[ \ddot{r} + \frac{\mu}{r^3} r = 0 \]  (1)

which governs the motion of a spacecraft relative to Earth with a gravitational parameter of \( \mu = 398,600 \text{ km}^3/\text{s}^2 \). The vectors \( \dot{r} \) and \( r \) is expressed in the XYZ Earth-centered inertial (ECI)
frame given by the X-, Y-, and Z-axes as shown in Figure 1. The X-axis points in the vernal equinox direction and the Z-axis coincides with Earth’s rotational axis. The Y-axis completes the right-handed triad. We refer to the co-moving \( xyz \) frame, which originates from the spacecraft center of mass at \( L \), as a local vertical local horizontal (LVLH) frame. We define the x-axis of the LVLH frame as pointing along the radial position vector \( \mathbf{r} \) of the leader spacecraft in the ECI frame and the z-axis as pointing along its orbital angular momentum vector \( \mathbf{h} \). The y-axis is aligned along its velocity vector which completes the right-handed triad. The unit vectors are therefore given as

\[
\mathbf{i} = \frac{\mathbf{r}}{r}, \quad \mathbf{j} = \mathbf{k} \times \mathbf{i}, \quad \mathbf{k} = \frac{\mathbf{h}}{h} \tag{2}
\]

Once we have established the systems of coordinate frames of reference, we need to define the two spacecraft: leader and follower. The leader is the spacecraft of which its center of mass is fixed at the origin \( o \) of the LVLH frame. The follower is the spacecraft which trajectory is to be design as to follow in proximity of the leader. One solution to this is, of course, to place the follower exactly in the same orbit as the leader with some distance between one another. The objective here however is to make the follower seen as moving around the leader, therefore making the follower a quasi-satellite of the leader. The quasi-satellite configuration is not intuitive and hard to visualize in the inertial ECI frame. However, if an observer is placed on the leader spacecraft, the follower spacecraft will be seen as "orbiting" the observer.

We model the two spacecraft as having no propulsive force, whether attractive or repulsive. This assumption separates from the previous studies of quasi-satellite dynamics [4], which focus more on massive celestial objects that have their own gravitational pull and satellites with ion thrusters. The apparent orbiting motion therefore is not due a central force of the leader but by a careful orbit placement. We denote \( \mathbf{r}_L \) and \( \mathbf{r}_F \) as the position vectors of the leader and the follower, respectively, as expressed in the ECI frame. The relative position of the follower with respect to the leader is

\[
\mathbf{r}_{rel} = \mathbf{r}_F - \mathbf{r}_L \tag{3}
\]

The velocity and acceleration of the follower relative to the leader can be therefore written as

\[
\dot{\mathbf{r}}_{rel} = (\dot{\mathbf{r}}_F - \dot{\mathbf{r}}_L) - \Omega \times \mathbf{r}_{rel} \tag{4}
\]

\[
\ddot{\mathbf{r}}_{rel} = (\ddot{\mathbf{r}}_F - \ddot{\mathbf{r}}_L) - \Omega \times \mathbf{r}_{rel} - 2\Omega \times (\dot{\mathbf{r}}_F - \dot{\mathbf{r}}_L) - \Omega \times (\Omega \times \mathbf{r}_{rel}) \tag{5}
\]
where the angular velocity $\Omega$ and angular acceleration $\dot{\Omega}$ are

$$\Omega = \frac{\mathbf{r}_L \times \dot{\mathbf{r}}_L}{r_L^2} \quad \text{and} \quad \dot{\Omega} = -2 \frac{\dot{\mathbf{r}}_L \cdot \mathbf{r}_L}{r_L^2} \Omega$$

(6)

The expressions for the relative position, velocity and acceleration in this current form are expressed in the ECI frame. Therefore, the coordinates need to be rotated from the ECI frame to the LVLH frame using a rotation matrix $\text{levh} R_{eci}$, which elements can be found from Equation (2).

For instance, the position of a follower spacecraft expressed in the LVLH frame can be found by

$$\text{levh} [\mathbf{r}_{rel}] = \text{levh} R_{eci} [\mathbf{r}_{rel}]$$

(7)

Defining orbits using the orbital elements provides more intuition of its shape and orientation in space. We employ the standard six classical orbital elements: $a$ (semi-major axis), $e$ (eccentricity), $i$ (inclination), $\omega$ (argument of perigee), $\Omega$ (right ascension of ascending node), and $M$ (mean anomaly). The last one is used interchangeably with $\theta$ (true anomaly). The definitions of these elements can be easily found in any standard reference on orbital mechanics.

Additionally, we will use another quantity, the mean longitude $l$, which is defined as $l = M + \omega + \Omega$ [13]. Mean longitude does not measure an angle between any vectors, but a quantity that shows the mean angular position of an object in an orbit. In our case, an indication of mean longitude is useful in showing how close, in terms of mean angular position, are two spacecraft when they are in two different orbits. Difference between the orbital elements of the leader and the follower will be denoted by the prefix $\delta$. For example, $\delta e \triangleq e_F - e_L$.

### 4. Numerical Simulation

In the previous section, we have set up the referential reference frames, particularly, the Earth-centered Inertial (ECI) frame and the local-vertical local-horizontal (LVLH) frame. We have also shown the relative kinematics between the two frames. The task now is to find the state vectors, $\mathbf{r}$ amd $\mathbf{v}$, of both spacecraft at any later time $\Delta t$ after their initial conditions $\mathbf{r}_0$ amd $\mathbf{v}_0$. The universal Kepler’s equation is employed to solve the two-body problem. This requires the use of universal anomaly $\chi$. Readers may consult Battin [13] for details on universal anomaly and universal variable formulation.

Initial orbital elements of the spacecraft is first converted to state vectors. From the initial state vector $\mathbf{r}$ and $\mathbf{v}$, we define the radial component of $v_{r0} = \frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{r_0}$. Using $\alpha$ to denote the reciprocal of semi-major axis

$$\alpha = \frac{2}{r_0} - \frac{v_0^2}{\mu}$$

(8)

we can estimate the initial universal anomaly $\chi$ as [14]

$$\chi_0 = \sqrt{\mu} |\alpha| \Delta t$$

(9)

At any given step $i$ where $i = 0, 1, 2, ..., $ the subsequent universal anomalies can be obtained from [15]

$$\chi_{i+1} = \chi_i - \frac{f(\chi_i)}{f'(\chi_i)}$$

(10)

where

$$f(\chi) = \frac{r_0 v_{r0}}{\sqrt{\mu}} \chi_i^2 C(z_i) + (1 - \alpha r_0) - \chi_i^3 S(z_i) + r_0 \chi_i - \sqrt{\mu} \Delta t$$

$$f'(\chi) = \frac{r_0 v_{r0}}{\sqrt{\mu}} \chi_i (1 - \alpha \chi_i^2 S(z_i)) + (1 - \alpha r_0) \chi_i^2 C(z_i) + r_0$$

(11)
where $z_i = \alpha \chi_i^2$. The functions $C(z)$ and $S(z)$ are Stumpff functions. For $z > 0$, the functions are defined by

$$S(z) = \frac{\sqrt{z} - \sin \sqrt{z}}{(\sqrt{z})^3}$$
$$C(z) = 1 - \frac{\cos \sqrt{z}}{z}$$

(12)

Note that the Stumpff functions shown in Equation 12 are applicable for elliptic orbit only.

With the new $\chi_i$, one must now obtain the Lagrange $f$ and $g$ coefficients

$$f = 1 - \frac{\chi^2}{r_0} C(\alpha \chi^2)$$
$$g = \Delta t - \frac{1}{\sqrt{\mu}} \chi^3 S(\alpha \chi^2)$$

(13)

to find the new position after time $\Delta t$ using

$$\mathbf{r} = f \mathbf{r}_0 + g \mathbf{v}_0$$

(14)

Next, from the new position we can find the new velocity. The first derivatives of the Lagrange coefficients are obtained

$$\dot{f} = \frac{\sqrt{\mu}}{rr_0} (\alpha \chi^3 S(\alpha \chi^2) - \chi)$$
$$\dot{g} = 1 - \frac{\chi^2}{r} C(\alpha \chi^2)$$

(15)

to solve for $\mathbf{v}$

$$\mathbf{v} = \dot{f} \mathbf{r}_0 + \dot{g} \mathbf{v}_0$$

(16)

This step-by-step algorithm is programmed to compute $\mathbf{r}(t)$ and $\mathbf{v}(t)$ given the initial orbital elements.

5. Quasi-satellite Formation

To simulate the orbital trajectories of the leader-follower formation, we start with a simple formation where the leader spacecraft is in a circular orbit around the Earth. For all simulations, the follower spacecraft assumes a condition that it must have the same period as the leader’s orbit. This so that it has a 1:1 orbital resonance with the leader (the follower will complete one revolution around the leader in one orbital period). For this condition to be met, only the semi-major axes of both orbits need to match. In this case we consider $a = 10,000$ km for both orbits.

**Difference in eccentricity**

We fixed the eccentricity of the leader at zero and vary the follower’s orbit eccentricity from 0.01 to 0.55. Figure 2 shows the follower trajectories in the LVLH frame when the leader is at the origin.

In Figure 3, we set the leader’s eccentricity at $e = 0.1$ to and vary the follower’s eccentricity from 0.01 to 0.2, with an increment of $\delta e = 0.05$. This is to see if an elliptical leader orbit would have any effect on the quasi-satellite formation. The trajectories when $e_F < e_L$ is shown in red.

All orbits are shown in the LVLH frame.
Figure 2: Quasi-satellite trajectory for increasing follower’s eccentricity.

Figure 3: Negative $\delta e$ orbits are indicated by red trajectories which shows that the follower’s starting point is on the opposite direction.

Difference in $\omega$, $\Omega$, $i$, and $\theta$

We start with the leader’s orbit having an inclination $i_L = 10^\circ$ and $\delta l_L = \omega_L = \Omega_L = M_L(0) = 0$. The leader is in $e = 0$ orbit and the follower is in $e = 0.2$ orbit for all cases. We vary the follower’s orbital parameters $-\omega_F$, $\Omega_F$, $i_F$, and $M_F$ – one at a time to see the effects of modifying the orbital elements of the follower in relation to the leader.

Figure 4 shows the effect of increasing the followers $\omega_F$ while the leaders $\omega_L$ is fixed. Figure 5 shows similar comparison of different $\delta \Omega$ while keeping other elements constant. The inclination $\delta i$ effect is shown in Figure 6 and the $\delta \theta$ effect in Figure 7. The angles ranges from $0^\circ$ to $110^\circ$ for all cases.

All orbits are shown in the LVLH frame with a globe representing Earth for illustration.
Figure 4: $i_F = 10^\circ$; $\Omega_F = M_F(0) = 0$.
Follower’s argument of perigee is varied \( \omega_f = [0^\circ, 10^\circ, 20^\circ, 45^\circ, 70^\circ, 90^\circ, 110^\circ] \).
Black trajectory are when the follower is in an orbit around the leader.

Figure 5: $i_F = 10^\circ$; $\omega_F = M_F(0) = 0$.
Follower’s right ascension node is varied \( \Omega_f = [0^\circ, 10^\circ, 20^\circ, 45^\circ, 70^\circ, 90^\circ, 110^\circ] \).
Black trajectory are when the follower is in an orbit around the leader.

Figure 6: $\Omega_F = \omega_F = M_F(0) = 0$.
Follower’s inclination angle is varied \( i_f = [0^\circ, 10^\circ, 20^\circ, 45^\circ, 70^\circ, 90^\circ, 110^\circ] \).
Follower orbits the leader for all inclination.

Figure 7: $i_F = 10^\circ$; $\omega_F = \Omega_F = 0$.
Follower’s initial mean anomaly is varied \( M_f = [0^\circ, 10^\circ, 20^\circ, 45^\circ, 70^\circ, 90^\circ, 110^\circ] \).
Black trajectory are when the follower is in an orbit around the leader.

6. Discussion Based on the Visualized Trajectories

Our numerical simulation provides several insights in creating a quasi-satellite system for spacecraft

(i) Increasing the eccentricity difference between the leader and the follower increases the average radial distance (in the LVLH frame) of the follower from the leader. Increasing the eccentricity also evolves the followers trajectory from an oval to a kidney-shaped orbits, which are seen in horizontal Lyapunov orbit family around L2 libration point (Reader may refer to [16] for illustrations). This is more prominent when \( \delta e \) is large.

(ii) The follower revolves around the leader in a retrograde motion. In general (general here means if every other orbital elements, except \( e \), is set to be the same), one can observe
that the follower is the fastest when it is on the inside (negative x direction in the LVLH frame) and the slowest when it is outside. This is more intuitive if one can imagine they are the perigee and apogee points of the follower’s orbit. If $e_F < e_L$, the starting point of the follower is changed 180°.

(iii) To get the follower to fly around the leader in the quasi-satellite fashion, the starting mean longitude difference $\delta l = \delta M + \delta \omega + \delta \Omega$ have to be close to zero. This can be seen in Figure 4, Figure 5, and Figure 7, where increasing the angular difference beyond 20° separates the leader spacecraft from the quasi-orbit. It is suggested here that for a quasi-satellite system to work, both spacecraft has to be almost co-planar. This is a good consideration especially when we incorporate perturbations later. This is consistent with the findings of [4] which had included $J_2$ perturbation in their analysis.

(iv) Varying inclination radically changes the shape of the follower’s trajectory but still maintains the follower’s orbit around the leader. We suggest here that, if an out-of-plane motion is desired, $\delta i$ is to be kept small to avoid excessive maneuver to correct orbit drifting due to gravitational perturbation.

(v) The unperturbed model used here suggests that quasi-satellite dynamics is possible without having to use propulsive forces. In the presence of perturbations, however, artificial quasi-satellites cannot be achieved without control to overcome the orbital deviations. We suggest, that in order to keep the quasi-satellite configuration, the almost co-planar condition (small $\delta l$ and $\delta i$) must be adhered as to minimize propulsive correction maneuver to maintain the orbit.

(vi) Recommendation based on our numerical experiment using an unperturbed two-body model is presented as such

| Follower’s Orbit Element | Recommendation Based on Numerical Simulation |
|-------------------------|---------------------------------------------|
| semi-major axis         | $\delta a \approx 0$; for a 1:1 orbital resonance |
| eccentricity            | $\delta e < 0.05$; for close proximity formation |
| inclination             | $\delta i$ small |
| argument of perigee     | $\delta \omega < 20^\circ$ |
| right ascension of ascending node | $\delta \Omega < 20^\circ$ |
| Mean anomaly            | $\delta M < 20^\circ$ |

7. Conclusion

The discovery of quasi-satellite configuration in our solar system shows that formation flight occurs in nature. Inspired by the mechanics of natural celestial bodies, we show that artificial satellites are able mimic the motion of quasi-satellites without any correcting maneuver using propulsion. We show the possible orbit shapes according to different eccentricities and, orbital orientation and position according to different orbital elements. We proposed several conditions to form a quasi-satellite for a leader-follower formation 1) both semi major axes need to be the same as to keep a 1:1 orbital resonance between the two spacecraft 2) the orbital elements relating to orientation, e.g. $\delta i$, $\delta \omega$, $\delta \Omega$ and $\delta M$, must be kept small to ensure a quasi-satellite system. To design a formation using nonlinear equations is a laborious process. This paper provides a parametric analysis based off the idea of natural quasi-satellites, like asteroids, to
provide another perspective to look at the formation flight problem. The logical next step is to include perturbation to this existing model to investigate deeper the quasi-satellite formation dynamics.

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