Behavior of Quantum Correlations under Local Noise

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We characterize the behavior of quantum correlations under the influence of local noisy channels. Intuition suggests that such noise should be detrimental for quantumness. When considering qubit systems, we show for which channel this is indeed the case: the amount of quantum correlations can only decrease under the action of unitary channels. However, non-unital channels (e.g. such as dissipation) can create quantum correlations for some initially classical state. Furthermore, for higher-dimensional systems even unital channels may increase the amount of quantum correlations.

Thus, counterintuitively, local decoherence can generate quantum correlations.

Composite quantum states often reveal puzzling features of nature. Recently, much interest has been devoted to the study of quantum correlations that may arise without entanglement: here, the quantumness of a composite system manifests itself even in a separable state. The fact that such quantum correlations are present in an algorithm for mixed state quantum computing has stimulated intensive investigations into measures for quantum correlations \[2, 14–26\] and their properties and interpretations \[14–26\]. Some studies of the dynamics of quantum correlations have been presented \[27, 28\].

An appeal of mixed state quantum computation lies in the possibility to be run in a noisy environment: pure entangled states are typically fragile, and the resource of entanglement is easily destroyed by noise. For an open system the transition from entangled to separable states is only a matter of time - as the volume of the set of separable states is non-zero \[29\], typically it takes a finite time for entanglement to disappear under noise such as dissipation or decoherence \[30\].

Mixed state quantum computation as suggested in \[3\] already uses separable states, so it is natural to assume that it can be run also in a noisy environment. However, in order to verify or falsify this conjecture, one has to study the behavior of quantum correlations under noisy channels (described by trace-preserving completely positive maps). Here we only consider local noisy channels - as correlated channels may also preserve entanglement (with or even without some degradation, depending on the amount of correlation), see e.g. \[31\]. The goal of this Letter is to answer questions such as: Which types of noisy channels decrease the amount of quantum correlations? Are there any noisy channels that might even increase the amount of quantum correlations? How does dissipation influence quantum correlations, and how are they affected by decoherence? - We point out that our answers to these questions also apply to the situation where one actively performs local operations on a composite quantum system, e.g. with the aim of creating or preserving quantum correlations.

In general, a bipartite quantum state is called fully classically correlated, if it can be written in the form

\[
\rho_{cc} = \sum_{i,j} p_{ij} \ket{ i^A } \bra{ i^A } \otimes \ket{ j^B } \bra{ j^B },
\]

where \( \{ \ket{ i^A } \} \) and \( \{ \ket{ j^B } \} \) are sets of orthogonal states of party A and B, respectively, with nonnegative probabilities \( p_{ij} \) that add up to one. If a state cannot be written as in Eq. \(1\), it is called quantum correlated. These definitions can be extended to any number of parties \[13\]. As a simple example consider the classically correlated state of two qubits

\[
\rho_{cc} = \frac{1}{2} \ket{0^A} \bra{0^A} \otimes \ket{0^B} \bra{0^B} + \frac{1}{2} \ket{1^A} \bra{1^A} \otimes \ket{1^B} \bra{1^B},
\]

Using a local channel on qubit A only (namely a local measurement and subsequent replacement) it is possible to create from the classically correlated state \[2\] the quantum correlated state

\[
\rho = \frac{1}{2} \ket{0^A} \bra{0^A} \otimes \ket{0^B} \bra{0^B} + \frac{1}{2} \ket{+^A} \bra{+^A} \otimes \ket{1^B} \bra{1^B}
\]

with \( \ket{+^A} = \frac{1}{\sqrt{2}} (\ket{0} + \ket{1}) \). The quantum channel that achieves this transformation can be formally written as the completely positive trace-preserving map

\[
\rho = \Lambda_A (\rho_{cc}) = E_1 \rho E_1^\dagger + E_2 \rho E_2^\dagger
\]

with local Kraus operators \( E_1 = \ket{0^A} \bra{0^A} \) and \( E_2 = \ket{+^A} \bra{1^A} \) acting only on qubit A. The state in Eq. \(4\) is not of the form \(1\), i.e. it is quantum correlated.

As will become clear below in this Letter, one reason why the local quantum channel in Eq. \(4\) is able to create quantum correlations lies in its action on the maximally mixed state \(\frac{1}{2} \mathbb{1}_{AA}\). Observe that \(\Lambda_A (\frac{1}{2} \mathbb{1}_{AA}) = \frac{1}{2} \ket{0^A} \bra{0^A} + \frac{1}{2} \ket{+^A} \bra{+^A} \neq \frac{1}{2} \mathbb{1}_{AA}\). This property is also known as non-unitality. A single-qubit quantum channel \(\Lambda\) is called unital if and only if it maps the maximally mixed state onto itself: \(\Lambda \left( \frac{1}{2} \mathbb{1} \right) = \frac{1}{2} \mathbb{1}\), see also Fig. \(1\).

We will turn this observation into Theorem \(1\) by showing that non-unitality is one property which enables a local channel to create quantum correlations in a multi-qubit system. In Theorem \(2\) we will show that on the other hand local unital quantum channels cannot increase...
to linearity,
\[ \Lambda_{sc}^{A} (\rho_{cc}) = \sum_{i,j} p_{ij} \Lambda_{sc}^{A} (|i^{A}\rangle \langle i^{A}| \otimes |j^{B}\rangle \langle j^{B}|). \]  \(6\)

The definition of a semi-classical channel in Eq. (6) directly implies that \(\Lambda_{sc}^{A} (\rho_{cc})\) is classically correlated.

Now we will show that a local unital channel never creates quantum correlations in a multi-qubit system. A local unital channel \(\Lambda_{u}^{A}\) on the qubit \(A\) takes a classically correlated state to the state
\[ \Lambda_{u}^{A} (\rho_{cc}) = \sum_{i,j} p_{ij} \Lambda_{u}^{A} (|i^{A}\rangle \langle i^{A}| \otimes |j^{B}\rangle \langle j^{B}|). \]  \(7\)

The action of the unital channel on the pure state \(|i^{A}\rangle \langle i^{A}|\) can be studied using the Bloch representation: \(|0^{A}\rangle \langle 0^{A}| = \frac{1}{2} (I_{A} + \sum_{i} r_{i} \sigma_{i}^{A})\), where \(\sigma_{i}^{A}\) are the Pauli operators with \(i \in \{x,y,z\}\), and \(|1^{A}\rangle \langle 1^{A}| = \frac{1}{2} (I_{A} - \sum_{i} r_{i} \sigma_{i}^{A})\). Using linearity and unitality of \(\Lambda_{u}^{A}\) we see that the state \(|0^{A}\rangle \langle 0^{A}|\) is mapped onto the state \(\rho_{u}^{A} = \Lambda_{u}^{A} (|0^{A}\rangle \langle 0^{A}|) = \frac{1}{2} (I_{A} + \sum_{i} r_{i} \Lambda_{u}^{A} (\sigma_{i}^{A}))\). The same procedure for \(|1^{A}\rangle \langle 1^{A}|\) results in \(\rho_{u}^{A} = \Lambda_{u}^{A} (|1^{A}\rangle \langle 1^{A}|) = \frac{1}{2} (I_{A} - \sum_{i} r_{i} \Lambda_{u}^{A} (\sigma_{i}^{A}))\). Note that the Bloch vectors of the states \(\rho_{u}^{A}\) and \(\rho_{u}^{A}\) point into opposite directions, see Fig. 1 for illustration. States with this property can be diagonalized in the same basis. This implies that it is possible to write the state \(\Lambda_{u}^{A} (\rho_{cc})\) in the form (4).

Thus we proved that local unital quantum channels cannot create quantum correlations in a classically correlated multi-qubit state.

In the following we will complete the proof of Theorem 1 by showing that any local quantum channel \(\Lambda_{nu}^{A}\) that is neither unital nor semi-classical can create quantum correlations. By definition \(\Lambda_{nu}^{A}\) maps the maximally mixed state \(\frac{1}{2} I_{A}\) onto some state that is not maximally mixed:
\[ \Lambda_{nu}^{A} \left( \frac{1}{2} I_{A} \right) = \frac{1}{2} \sum_{i} s_{i} \sigma_{i}^{A}, \]  \(8\)

with \(\sum_{i} s_{i}^{2} \neq 0\). Since we demand that the quantum channel is not semi-classical, there exists a state \(|\psi^{A}\rangle\) such that \(\Lambda_{nu}^{A} (|\psi^{A}\rangle \langle \psi^{A}|)\) is not diagonal in the eigenbasis of \(\Lambda_{nu}^{A} \left( \frac{1}{2} I_{A} \right)\). Again we consider the Bloch representation
\[ \Lambda_{nu}^{A} (|\psi^{A}\rangle \langle \psi^{A}|) = \frac{1}{2} \left( I_{A} + \sum_{j} r_{j} \sigma_{j}^{A} \right) \]  \(9\)

and note that the two Bloch vectors \(\sigma\) and \(s\) are linearly independent. Otherwise the states \(\Lambda_{nu}^{A} (|\psi^{A}\rangle \langle \psi^{A}|)\) and \(\Lambda_{nu}^{A} \left( \frac{1}{2} I_{A} \right)\) could be diagonalized in the same basis, which is in contradiction to the definition of \(|\psi^{A}\rangle\). Consider now the classically correlated state
\[ \rho_{cc} = \frac{1}{2} |\psi^{A}\rangle \langle \psi^{A}| \otimes |0^{B}\rangle \langle 0^{B}| + \frac{1}{2} |\phi^{A}\rangle \langle \phi^{A}| \otimes |1^{B}\rangle \langle 1^{B}|. \]  \(10\)

\begin{figure}[h]
  \centering
  \includegraphics[width=0.8\textwidth]{channel_diagram}
  \caption{Quantum channels on a single qubit: The upper figure shows a unital quantum channel \(\Lambda_{u}\) (green arrow) which maps the maximally mixed state \(\frac{1}{2} I\) onto itself: \(\Lambda_{u} \left( \frac{1}{2} I \right) = \frac{1}{2} I\). Two orthogonal states \(|\psi_{1}\rangle\) and \(|\psi_{2}\rangle\) with collinear Bloch vectors are mapped onto the states \(\rho_{1} = \Lambda_{u} (|\psi_{1}\rangle \langle \psi_{1}|)\) and \(\rho_{2} = \Lambda_{u} (|\psi_{2}\rangle \langle \psi_{2}|)\) with collinear Bloch vectors. The lower figure shows a non-unital quantum channel \(\Lambda_{nu}\) (yellow arrow) which maps the maximally mixed state onto the state \(\sigma = \Lambda_{nu} \left( \frac{1}{2} I \right) \neq \frac{1}{2} I\). The Bloch vectors of \(\sigma_{1} = \Lambda_{nu} (|\psi_{1}\rangle \langle \psi_{1}|)\) and \(\sigma_{2} = \Lambda_{nu} (|\psi_{2}\rangle \langle \psi_{2}|)\) add up to twice the non-zero Bloch vector of \(\sigma\), see main text.
  \end{figure}
with orthogonal states $|\psi^A\rangle|\phi^A\rangle = 0$. We can write the states as $|\psi^A\rangle|\psi^A\rangle = \frac{1}{2}(1_A + \sum_i v_i \sigma_i^A)$, and $|\phi^A\rangle|\phi^A\rangle = \frac{1}{2}(1_A - \sum_i v_i \sigma_i^A)$. We define the vector $w$ such that the equality $\Lambda_{uu}(\sum_i v_i \sigma_i^A) = \sum_i w_i \sigma_i^A$ with $\sum_i w_i^2 \neq 0$ is satisfied. This is always possible, since $\Lambda_{uu}$ is trace-preserving. The action of the channel onto the two states $|\psi^A\rangle$ and $|\phi^A\rangle$ is as follows:

$$\Lambda_{uu}^A(|\psi^A\rangle|\psi^A\rangle) = \frac{1}{2} \left(1_A + \sum_i (s_i + w_i) \sigma_i^A \right), \quad (11)$$

$$\Lambda_{uu}^A(|\phi^A\rangle|\phi^A\rangle) = \frac{1}{2} \left(1_A + \sum_i (s_i - w_i) \sigma_i^A \right). \quad (12)$$

As noted above, the two Bloch vectors $s$ and $r = s + w$ are linearly independent. The same must hold for the vectors $s + w$ and $s - w$. This implies that the two states $\Lambda_{uu}^A(|\psi^A\rangle|\psi^A\rangle)$ and $\Lambda_{uu}^A(|\phi^A\rangle|\phi^A\rangle)$ are not diagonal in the same basis. This completes the proof. \hfill \Box

So far we saw that local unital and local semi-classical channels acting on a single qubit cannot create quantum correlations from a classically correlated multi-qubit state. These results hold independently of the chosen measure for quantum correlations. In the following we will go one step further by showing that these local channels never increase a very general class of measures for quantum correlations in multi-qubit systems. We consider distance-based measures of quantum correlations $Q_D$, which are defined via the minimal distance $D$ to the set of the classically correlated states $CC \[12]$. \hfill (13)

$$Q_D = \min_{\sigma \in CC} D(\rho, \sigma),$$

where $D$ does not necessarily have to be a distance in the mathematical sense. The statement mentioned above will be shown to hold for all distance measures $D$ with the property of being non-increasing under any quantum channel $\Lambda$, i.e.

$$D(\Lambda(\rho), \Lambda(\sigma)) \leq D(\rho, \sigma). \quad (14)$$

This property is also frequently used for defining entanglement measures [32, 33].

**Theorem 2.** Quantum correlations $Q_D$ in multi-qubit systems do not increase under local unital channels $\Lambda_{lu}$ and local semi-classical channels $\Lambda_{lsc}$:

$$Q_D(\Lambda_{lu}(\rho)) \leq Q_D(\rho), \quad (15)$$

$$Q_D(\Lambda_{lsc}(\rho)) \leq Q_D(\rho). \quad (16)$$

**Proof.** Let $\xi$ be the classically correlated state which minimizes the distance, i.e. $Q_D(\rho) = D(\rho, \xi)$. Using the property (14) of the distance to be nonincreasing under quantum channels we obtain

$$Q_D(\rho) = D(\rho, \xi) \geq D(\Lambda_{lu}(\rho), \Lambda_{lu}(\xi)), \quad (17)$$

$$Q_D(\rho) = D(\rho, \xi) \geq D(\Lambda_{lsc}(\rho), \Lambda_{lsc}(\xi)). \quad (18)$$

Now we use Theorem 1 noting that local unital channels $\Lambda_{lu}$ and local semi-classical channels $\Lambda_{lsc}$ map the classically correlated state $\xi$ onto another classically correlated state $\Lambda(\xi)$ which is not necessarily the one that minimizes the distance to $\Lambda(\rho)$. This observation finishes the proof. \hfill \Box

One example for a measure that satisfies the properties (15) and (16) - and thus Theorem 2 holds - is the geometric measure of quantumness which we define as

$$Q_G(\rho) = \min_{\sigma \in CC} (1 - F(\rho, \sigma)) \quad (19)$$

with the fidelity $F(\rho, \sigma) = (\text{Tr}\sqrt{\sqrt{\rho} \sigma \sqrt{\rho}})^2$. Using the fact that the fidelity is non-decreasing on quantum channels together with Theorem 2 we see that the geometric measure of quantumness does not increase under local unital channels and local semi-classical channels. Alternatively, we can use the quantum relative entropy $S(\rho|\sigma) = -\text{Tr}[\rho \log_2 \sigma] + \text{Tr}[\rho \log_2 \rho]$, which also fulfills the property (19) [32, 33]. From Theorem 2 follows that the resulting measure of quantum correlations $Q_S = \min_{\sigma \in CC} S(\rho|\sigma)$ does not increase under local unital and local semi-classical channels. $Q_S$ was also studied in [12], where it was called relative entropy of quantumness.

So far we considered states consisting of an arbitrary number of qubits. We have shown that local unital and local semi-classical channels acting on a single qubit never increase quantum correlations as defined by $Q_D$ in Eq. (13). On the other hand, any local channel which is non-unital and not semi-classical can in principle create quantum correlations, independently of the considered measure, out of a classically correlated state. An example for such a channel is the amplitude damping channel as a model for dissipation. Thus, dissipation can increase quantum correlations.

At the present stage it is natural to ask the question, for what kind of input states this behavior can or cannot be observed in general. The following theorem shows that pure states are special.

**Theorem 3.** The geometric measure of quantumness of multipartite systems with arbitrary dimension cannot increase under any local quantum channel, if the initial state is pure:

$$Q_G(\Lambda_I(|\psi\rangle\langle\psi|)) \leq Q_G(|\psi\rangle\langle\psi|), \quad (20)$$

where $\Lambda_I$ is an arbitrary local quantum channel.

**Proof.** Let $\xi \in CC$ be defined such that $Q_G(|\psi\rangle\langle\psi|) = 1 - F(|\psi\rangle\langle\psi|, \xi)$. Using the properties of the fidelity $F$ we see that $\xi$ can be chosen to be a pure product state $\xi = |\phi\rangle\langle\phi|$. Moreover $1 - F$ does not increase under the action of any quantum channel, i.e. $1 - F(|\psi\rangle\langle\psi|, |\phi\rangle\langle\phi|) \geq 1 - F(\Lambda_I(|\psi\rangle\langle\psi|), \Lambda_I(|\phi\rangle\langle\phi|))$. Since $|\phi\rangle$ is a product state, $\Lambda_I(|\phi\rangle\langle\phi|)$ is also a product state. This observations completes the proof. \hfill \Box
Note that Theorem 3 does not follow from the fact that for pure states the amount of quantum correlations is equal to the amount of entanglement.

So far we have shown that quantum correlations in multi-qubit systems cannot increase under local unital quantum channels. A prominent example for a unital channel is the phase damping channel, which is a model for decoherence in a quantum system. Under decoherence the quantum state \( \rho = \sum_{i,j} \rho_{ij} |i\rangle \langle j| \) is transformed to the state

\[
\Lambda (\rho) = \sum_i \rho_{ii} |i\rangle \langle i| + (1 - p) \sum_{i\neq j} \rho_{ij} |i\rangle \langle j| \tag{21}
\]

with the damping parameter \( 0 \leq p \leq 1 \). Since \( \Lambda \) is unital, it is not possible to create quantum correlations with local phase damping in a multi-qubit system. Surprisingly, this is not true if the local systems are not qubits: qubits are special. This can be demonstrated via the classically correlated state as input:

\[
\rho_{cc} = \frac{1}{2} |\psi^A\rangle \langle \psi^A| \otimes |0^B\rangle \langle 0^B| + \frac{1}{2} |\phi^A\rangle \langle \phi^A| \otimes |1^B\rangle \langle 1^B| \tag{22}
\]

with the orthogonal single-qutrit states \( |\psi^A\rangle = \frac{1}{\sqrt{2}} (-|0^A\rangle + |1^A\rangle + |2^A\rangle) \) and \( |\phi^A\rangle = \frac{1}{\sqrt{2}} (|0^A\rangle + |1^A\rangle) \).

We will show that a local phase damping channel \( \Lambda \) acting on subsystem \( A \) generates quantum correlations. We consider the action of the channel \( \Lambda \) with the damping parameter \( p = \frac{1}{2} \) on the state \( \rho_{cc} \) in Eq. \( \tag{22} \):

\[
\Lambda_A (\rho_{cc}) = \frac{1}{2} \sum_{i=1}^3 \lambda_i |\psi_i^A\rangle \langle \psi_i^A| \otimes |0^B\rangle \langle 0^B| + \frac{1}{2} \sum_{j=1}^3 \mu_j |\phi_j^A\rangle \langle \phi_j^A| \otimes |1^B\rangle \langle 1^B|, \tag{23}
\]

where the states \( \{ |\psi_i^A\rangle \} \) are the eigenstates of \( \Lambda_A (|\psi^A\rangle \langle \psi^A|) \) with the corresponding eigenvalues \( \lambda_i \). Similarly the states \( \{ |\phi_j^A\rangle \} \) are eigenstates of \( \Lambda_A (|\phi^A\rangle \langle \phi^A|) \) with the eigenvalues \( \mu_j \). One can see as follows that the state \( \Lambda_A (\rho_{cc}) \) is quantum correlated: The eigenvalues of \( \Lambda_A (|\psi^A\rangle \langle \psi^A|) \) are given by \( \lambda_1 = \frac{2}{3} \), \( \lambda_2 = \lambda_3 = \frac{1}{3} \). The eigenstate to the largest eigenvalue \( \lambda_1 \) is given by \( |\psi_1^A\rangle = |\psi^A\rangle \). It is easy to check that \( |\psi^A\rangle \) is not an eigenstate of \( \Lambda_A (|\phi^A\rangle \langle \phi^A|) \), and therefore the state in Eq. \( \tag{23} \) is not classically correlated. Thus we proved that it is possible to create quantum correlations with a local phase damping channel, i.e. via local decoherence.

In conclusion, we have investigated the effect of local noisy channels (i.e. trace-preserving completely positive maps) on quantum correlations. While entanglement can never increase under such local channels, quantum correlations without entanglement may or may not increase, depending on the type of channel and the type of input state. For multi-qubit systems we fully answer the question which local channels can increase quantum correlations: unital and semi-classical local channels cannot enhance quantum correlations, while non-unital and non-semi-classical local channels (e.g., dissipation, corresponding to amplitude damping) can increase quantum correlations. Surprisingly, for higher-dimensional systems, even unital channels such as decoherence, corresponding to phase-damping, can generate quantum correlations from an initially classically correlated state. However, quantum correlations as quantified by the geometric measure of quantumness can become larger under local channels only when the initial state is mixed. - Thus, we have shed some light on the behavior of quantum correlated states in a noisy environment.

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Note added: While finishing this Letter we became aware of related work \cite{14}. There the authors show that the quantum discord can increase under a local amplitude damping channel.

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