Design and complexity analysis of an improved adaptive filtering algorithm for non-sparse impulse response

Songlin Sun¹,²*, Tiantian Ran¹,², Na Chen¹,², Guihong Li¹,² and Chenglin Zhao¹,²

Abstract
An improved adaptive filtering (IMPNLMS+) algorithm has been proposed for non-sparse impulse responses by incorporating an adaptive parameter \( \mu \) of the \( \mu \)-law compression into the improved \( \mu \)-law PNLMS (IMPNLMS) algorithm. It not only achieves optimal step-size control factors but also overcomes that the convergence of classical \( \mu \)-law PNLMS (MPNLMS) is even slower than conventional NLMS algorithm for dispersive channels. In this paper, we propose IMPNLMS++ algorithm, where normalized algorithm is analyzed to reduce computational complexity of proposed improved \( \mu \)-law PNLMS (IMPNLMS+) algorithm. The validity has been proved by the simulation results.

Keywords: IMPNLMS + algorithm; Non-sparse impulse response; Computational complexity

1. Introduction
In echo cancelation systems, the adaptive filters are used to identify the impulse responses for echo paths. However, the impulse responses are usually sparse in nature [1-3]. For these systems, the classical normalized least-mean-square (NLMS) algorithm which assigns the same step-size to all filter coefficients converges slowly. Some adaptive algorithms exploiting the sparse nature of the impulse response have been proposed to resolve this problem.

The proportionate NLMS (PNLMS) algorithm [4] presented by Duttweiler converges slowly dramatically after the initial fast period. The \( \mu \)-law PNLMS (MPNLMS) algorithm [5] was proposed to solve this problem. Although it achieves optimal proportionate step size, it converges even slower than the classical NLMS algorithm in dispersive channels. The improved MPNLMS (IMPNLMS) algorithm [6] for non-sparse impulse responses was proposed to improve the performance of MPNLMS with an automatic adjustable parameter. If the convergence speed of IMPNLMS algorithm can be further improved, this algorithm will get a good application in time-varying environment. In [7], we have presented the improvement of the IMPNLMS algorithm using a variable parameter \( \mu \) instead of the constant value. As a consequence, the convergence features of the MPNLMS algorithm are improved significantly.

In this paper, the improved IMPNLMS algorithm, referred to as IMPNLMS + algorithm all over the paper, is reviewed. The computational complexity is then compared with other adaptive filter algorithms. Non-normalized algorithm is analyzed to bring down the computational complexity of the IMPNLMS+; the feasibility is confirmed by numerical situations.

This paper is organized as follows. In Section 2, the IMPNLMS + algorithm is reviewed. In Section 3, in order to lower computational complexity, the non-normalized technology is analyzed. In Section 4, numerical simulations confirm the computational complexity of IMPNLMS + is lower without normalization. Finally, Section 5 presents conclusions.

2. Review of the improved IMPNLMS algorithm
For non-sparse response, we have proposed an IMPNLMS + algorithm [7] by applying time variable parameter \( \mu \), which does not perform worse than...
NLMS even for dispersive channels. It can be applied in time-varying environment as well. In this section, the IMPNLMS + algorithm is recalled.

The steepest descent algorithm with step-size control matrix using $\mu$-law [4] in IMPNLMS + algorithm can be written as:

$$ w^{k+1} = w^k + \beta G(k+1) x(k) e(k) x^T(k) G(k+1) x(k) + \delta $$  \hspace{1cm} (1)

The step-size control matrix ($L \times L$):

$$ G(k+1) = \text{diag}\{g_1(k+1) \quad g_2(k+1) \ldots g_L(k+1)\} $$  \hspace{1cm} (2)

The $l$th coefficient $g_l(k)$ has been presented in [6]:

$$ g_l(k) = \frac{1-\alpha(k)}{2N} + \frac{(1 + \alpha(k))F\left(\frac{|\tilde{w}_l(k)|}{\hat{w}_l(k)}\right)}{2\left\|F\left(\frac{|\tilde{w}_l(k)|}{\hat{w}_l(k)}\right)\right\|_1} + \varepsilon $$  \hspace{1cm} (3)

Where, logarithmic function in IMPNLMS + differs from [6]:

$$ F\left(\left|\tilde{w}_l(k)\right|\right) = \ln\left(1 + \mu(k) \left|\tilde{w}_l(k)\right|\right) $$  \hspace{1cm} (4)

Here, $\mu(k)$ [8] is a time variable parameter instead of constant:

$$ \mu(k) = \frac{1}{e(k)} $$  \hspace{1cm} (5)

$$ e(k) = \sqrt{\frac{\gamma(k)}{L\lambda^2}} $$  \hspace{1cm} (6)

$$ \gamma(k) = \eta(k-1) + (1-\eta)e^2(k-1) $$  \hspace{1cm} (7)

The parameter $\alpha(k)$ in IMPNLMS + can be described as:

$$ \alpha(k) = 2\xi(k)-1 $$  \hspace{1cm} (8)

$$ \xi(k) = (1-\rho)\xi(k-1) + \rho \xi_w(k), \quad 0 < \rho \leq 1 $$  \hspace{1cm} (9)

$$ \xi_w(k) = \frac{L}{L-\sqrt{L}} \left(1 - \frac{\left\|\hat{w}_l(k)\right\|_1}{\sqrt{L} \left\|\hat{w}_l(k)\right\|_2}\right) $$  \hspace{1cm} (10)

In IMPNLMS + algorithm, thanks to the adaptation of $\mu(k)$, the algorithm is more flexible to minimize the MSE related to the time-varying $\mu(k)$. The IMPNLMS + algorithm can achieve better convergence even in time-varying environment where the echo path changes obviously. However, the computation of step-size control matrix with $\mu$-law in IMPNLMS + algorithm is expensive.

Table 1 The computational complexity of each filter coefficient update equation of NLMS

| Algorithms | Addition | Multiplication | Division | Comparison | Logarithm | Prescribing |
|------------|----------|----------------|----------|------------|-----------|-------------|
| NLMS       | $L$      | $2L+1$         | $1$      | $0$        | $0$       | $0$         |
| PNLMS      | $2L-1$   | $4L+1$         | $2$      | $2L$       | $0$       | $0$         |
| IPNLMS     | $3L$     | $4L+1$         | $2$      | $0$        | $0$       | $0$         |
| MPNLMS     | $2L-1$   | $4L+2$         | $2$      | $2L$       | $L$       | $0$         |
| IMPNLMS    | $5L+2$   | $5L+5$         | $3$      | $0$        | $L$       | $1$         |
| IMPNLMS+   | $5L+3$   | $5L+8$         | $4$      | $0$        | $L$       | $2$         |

Figure 1 Gaussian input with sparseness degree 0.80.
In the next section, we analyze the computational complexity based on IMPNLMS+ algorithm. Non-normalized technology is discussed to reduce the computational complexity of IMPNLMS+, which can be also applied to all proportionate NLMS algorithms.

3. The analytical solution of computational complexity

In general, the computational complexity of adaptive algorithms can be visualized as the number of additions, multiplications logarithm calculations, etc. per iteration. In Table 1, we compare the computational complexity of each filter coefficient update equation. The computation of the filter coefficient update equation in IMPNLMS+ algorithm is expensive. It adds three \( L + four \) additions, \( L + six \) multiplications, two divisions, and two prescribing per iteration in addition to the computational load of MPNLMS. In this section, we improve the IMPNLMS+ algorithm, termed IMPNLMS++, by reducing normalization. The possibility of lowering IMPNLMS+ computational complexity by reducing normalization is discussed.

In [4], the denominator of coefficient update equation in PNLMS is \( x^T(k)x(k) \). Duttweiler normalized the step-size control factors \( g_l(k) \) to avoid direct influence of target impulse response amplitude (i.e., \( |h_m| \)) on efficient step-size parameter \( \beta g_l(k) \). Thus, the value of final efficient step-size assigned to filters is only proportional to parameter \( \beta \). However, when the denominator is \( x^T(k)G(k+1)x(k) \), normalization can be skipped. The analysis can be described as follows.

For the coefficient update equation whose denominator is \( x^T(k)G(k+1)x(k) \), molecular and denominator...
are multiplied by the non-zero real number $c$ at the same time:

$$\frac{\beta G(k + 1)x(k)e(k)}{x^T(k)G(k + 1)x(k) + \delta} = \frac{\beta cG(k)x(k)e(k)}{x^T(k)cG(k + 1)x(k) + c\delta}$$  \hspace{1cm} (11)

Where $\delta$ is a small positive number only to prevent the algorithm from stalling when denominator equals zero. Hence, in general, $x^T(k)cG(k + 1)x(k) \gg c\delta$, the operation (11) which is just the same as normalization does not affect the coefficient update item dramatically. Therefore, the absence of normalization has little effect on adaptive algorithms, but also reduces L divisions. This thesis is applicable to MPNLMS and IMPNLMS as well.

4. Numerical simulations

In this section, the proportionate algorithm IMPNLMS+ is simulated to confirm the little influence on algorithms without normalization. The input of the simulation system, which is similar as [7], is described as follows.

The coefficients of the unknown system $w^0$ and the adaptive filter $\hat{w}$ are multiplied by the non-zero real number $c$ at the same time:

$$\frac{\beta G(k + 1)x(k)e(k)}{x^T(k)G(k + 1)x(k) + \delta} = \frac{\beta cG(k)x(k)e(k)}{x^T(k)cG(k + 1)x(k) + c\delta}$$  \hspace{1cm} (11)

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5. Conclusions

In this paper, we first recall the improved IMPNLMS algorithm for non-sparse impulse response. The complexity of each filter coefficient update equation is then computed. To reduce computational complexity of the IMPNLMS+ algorithm, the possibility of employing the non-normalization technique is verified through theoretical derivations. Simulation results have proven the effectiveness and feasibility of the non-normalized IMPNLMS+ algorithm.

Competing interests

The authors declare that they have no competing interests.

Acknowledgment

This work is supported in part by NSFC 61143008, National High Technology Research and Development Program of China (no. 2011AA01A04), the Fundamental Research Funds for the Central Universities.

Received: 21 October 2013 Accepted: 13 January 2014

Published: 22 January 2014

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Cite this article as: Sun et al.: Design and complexity analysis of an improved adaptive filtering algorithm for non-sparse impulse response. EURASIP Journal on Wireless Communications and Networking 2014, 2014:14.