Nonlinear Effects in the Behavior and Fracture of Composite Materials

E V Lomakin¹, B N Fedulov², a and A N Fedorenko²

¹ Lomonosov Moscow State University, GSP-1, Leninskie Gory, Moscow, Russia, 119991
² CDM2, Skolkovo Institute of Science and Technology, Bolshoy Boulevard 30, bld. 1 Moscow, Russia 121205

E-mail: a fedulov.b@mail.ru

Abstract. This paper presents an approach to the formulation of models characterizing nonlinear deformation and damage of composite materials. The research includes the consideration as the simplest model as more complex ones. In this research, a number of material relations to build up a theory for failure prediction in laminated composites are presented. The considered approach deals with the degradation and failure of the material using a phenomenological introduction of damage parameters explicitly influencing on the composite material stiffness. The part of this work is dedicated to the extension of formulated approach to capture and take into account the complex effects such as initial nonlinear shear deformation properties of laminated composites and the influence of high strain rate on the strength characteristics.

1. Introduction
Currently there is no well-established and universally accepted approach to strength modelling of composite materials. The failure analysis of heterogeneous materials plays a major role in engineering applications. For areas where the use of lightweight and high strength structures is essential for product design, as in aerospace industry, the capability to predict failure in composite materials is vital for realization of full potential of a material. Restricting one’s attention to phenomenological models only, there can be found a number of works and researches demonstrating essentially different approaches and interpretations of mechanical behavior of heterogeneous materials. Some of them focus on the study of failure criteria for unidirectional composites [1–9], while others study response of stress concentrators in particular lay-ups [10 –13], one can also find works studying nonlinear relations between stress and strain components [14 –22]. There are a lot of review publications discussing these methods [23 –27]. In this research, the different steps necessary to build a theory for failure analysis are discussed, which then are implemented in the simplest form, using standard composite characteristics, as an example. The part of the research describes the way to extend simple failure model to the formulation of more complex models taking into account different effects such as shear nonlinearity of deformation properties of materials and strain rate dependence of their strength. It gives the possibility to understand the necessity of further refinement of approaches and models for the characterization of different effects in the behavior of composite materials and what experimental studies ought to be conducted to improve the models.

2. Damage parameters
Considering the simplest way of introducing damage characteristics, only two parameters, \( \psi_1 \) and \( \psi_2 \), are chosen, where first parameter corresponds to fiber failure and the second one to matrix failure:
The way of evaluation of damage parameters depends on the failure criteria. Considering the simplest initial state, the following equations:

\[
\begin{align*}
\psi_1 &= 0 \text{ fiber failure, } \psi_1 &= 1 \text{ initial value} \\
\psi_2 &= 0 \text{ matrix failure, } \psi_2 &= 1 \text{ initial value}
\end{align*}
\]

The modified constitutive relations with damage parameters can be formulated as follows [28, 29]:

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{13} \\
\gamma_{23}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\psi_1 E_{11}} & -\frac{\psi_2 v_{11}}{E_{22}} & -\frac{\psi_2 v_{12}}{E_{33}} & 0 & 0 & 0 \\
-\frac{\psi_2 v_{12}}{E_{11}} & -\frac{1}{\psi_2 E_{22}} & -\frac{\psi_2 v_{22}}{E_{33}} & 0 & 0 & 0 \\
-\frac{\psi_2 v_{12}}{E_{11}} & -\frac{\psi_2 v_{22}}{E_{11}} & -\frac{1}{\psi_2 G_{12}} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\psi_2 G_{13}} & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{\psi_2 G_{23}} & 0
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{bmatrix}
\]

At initial state, the damage parameters are equal one \(\{\psi_1, \psi_2\} = \{1, 1\}\), and constitutive equations (2) coincide with classical linear elastic one.

The way of evaluation of damage parameters depends on the failure criteria. Considering the simplest one based on the maximum stresses, the following conditions can be formulated:

\[
X_c \leq \sigma_{11} \leq X_t, Y_c \leq \sigma_{22} \leq Y_t, |\sigma_{12}| \leq S
\]

where:

\(X_c\) – compression failure stress in fiber direction
\(X_t\) – tension failure stress in fiber direction
\(Y_c\) – compression failure stress in transversal direction
\(Y_t\) – tension failure stress in transversal direction
\(S\) – in-plane shear failure stress

Once the failure criterion is chosen, we can formulate rule for modifying damage parameters using following equations:

\[
\begin{align*}
\psi_1 &= 1, \text{ if } X_c \leq \sigma_{11}^{el} \leq X_t, \text{ or } \psi_1 = 0 \text{ if not} \\
\psi_2 &= \min(\psi_2^{22}, \psi_2^{12}, \psi_2^{13}, \psi_2^{23}),
\end{align*}
\]

where

\[
\psi_2^{22} = \text{solution of equation } \sigma_{22}^{el} = Y_t, \text{ if } \sigma_{22}^{el} > Y_t, \\
\psi_2^{22} = \text{solution of equation } \sigma_{22}^{el} = Y_c, \text{ if } \sigma_{22}^{el} < Y_c,
\]

\[
\begin{align*}
\psi_2^{12} &= S/|\sigma_{12}^{el}| \text{ if } |\sigma_{12}^{el}| > S, \text{ else } \psi_2^{12} = 1, \\
\psi_2^{13} &= S/|\sigma_{13}^{el}| \text{ if } |\sigma_{13}^{el}| > S, \text{ else } \psi_2^{13} = 1, \\
\psi_2^{23} &= S/|\sigma_{23}^{el}| \text{ if } |\sigma_{23}^{el}| > S, \text{ else } \psi_2^{23} = 1, \\
\psi_2^{33} &= \text{solution of equation } \sigma_{33}^{el} = Y_t, \text{ if } \sigma_{33}^{el} > Y_t, \\
\psi_2^{33} &= \text{solution of equation } \sigma_{33}^{el} = Y_c, \text{ if } \sigma_{33}^{el} < Y_c,
\end{align*}
\]

where \(\sigma_{ij}^{el}\) – stress tensor components, defined by equations (4) before the damage parameters are updated.

In this case, for the determination of the value of parameter \(\psi_2\) we have the condition \(\sigma_{22}^{el}(\psi_2^{22}) = Y_t\), which can be represented in the following form:
The minimum positive real root less than one must be chosen in the solution of this cubic equation. If this requirement is not satisfied then $\psi_2^2 = 1$. Typical loading curves for the equations (4) are shown on figure 1.

$$\left[ E_{22} \nu_{13} E_{33} \psi_1^2 \left( ( - \nu_{13} \varepsilon_{22} + \varepsilon_{33} \nu_{12} ) E_{22} + \nu_{23} \left( 2 Y_C \nu_{12} \psi_1 + E_{11} \varepsilon_{11} \right) \right) \right] \psi_2^2 = \left[ \nu_{12} \left( E_{11} \varepsilon_{11} \psi_1 + E_{11} \varepsilon_{11} \right) E_{22}^2 + E_{33} \left( \nu_{13}^2 \psi_1 + E_{11} \varepsilon_{33} \nu_{23} \right) E_{22} + E_{11} E_{33} \nu_{23}^2 \psi_1 \right] \psi_2^2 + \left[ E_{11} E_{22}^2 \varepsilon_{22} \right] \psi_2^2 - E_{11} E_{22} Y_C = 0 \quad (5)$$

3. Nonlinear elasticity

To characterize the effect of different possible response of the composite material to the type of loading, the formalization of stress state is required for the developing of the mathematical model. By means of $\sigma = \sigma_w/3$ which is the hydrostatic stress component, and $\sigma_0 = \sqrt{3/2} S_{ij} S_{ij}$ which is the effective stress, where $S_{ij} = \sigma_{ij} - \sigma \delta_{ij}$ is a deviator stress, it is possible to formulate the parameter $\xi = \sigma / \sigma_0$. This parameter $\xi$ is a good candidate for proposed formalization due to its mechanical sense, invariant nature and scalar simplicity. This introduced parameter can be found in the literature under the name of the stress triaxiality.

3.1. Plane shear formalization

The parameter, which can be used to describe the influence of the degree of shear loading on material properties can be formalized as a shear stress component in the case of plane shear loading and formulated in the invariant form as

$$Q = D_{ij} \sigma_{ij} \quad (6)$$

$$D_{ij} = \begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, it obviously gives in the chosen coordinate system, $Q = \sigma_{12}$.

3.2. Constitutive equations

It is possible to show that the introduction of parameters of stress state $\xi$ and degree of shear loading $Q$ into coefficients of elastic potential

$$\Phi = A_{ijkl}(\xi, Q) \sigma_{ij} \sigma_{kl} \quad (7)$$

gives a robust set of constitutive equations that can capture different effects of elastic nonlinearities exhibited by composite materials. In case of 2D plane stress problem, the constitutive equations have the form:
\[
\begin{align*}
\epsilon_{11} &= A_{1111}(\xi)\sigma_{11} + A_{1122}(\xi)\sigma_{22} + \left[\left(\frac{1}{3\xi} + \frac{3}{2}\xi\right)\sigma - \frac{3}{2}\xi\sigma_{11}\right]\phi_1\sigma_0^{-2}, \\
\epsilon_{22} &= A_{1122}(\xi)\sigma_{11} + A_{2222}(\xi)\sigma_{22} + \left[\left(\frac{1}{3\xi} + \frac{3}{2}\xi\right)\sigma - \frac{3}{2}\xi\sigma_{22}\right]\phi_1\sigma_0^{-2}, \\
\epsilon_{12} &= B(\xi, Q) + \frac{3}{2}\xi\phi_1\sigma_0^{-2},
\end{align*}
\]

where
\[
\phi_1 = \frac{1}{2} \left[ A'_{1111}(\xi)\sigma_{11}^2 + A'_{2222}(\xi)\sigma_{22}^2 + 2A'_{1122}(\xi)\sigma_{11}\sigma_{22} + A'_{1112}(\xi, Q)\sigma_{12}^2\right],
\]
and the case when coefficient for pure shear compliance has no influence of stress traxiality:
\[
B(\xi, Q) = \Gamma(Q).
\]

In spite of the complex form of equation, in the case of proportional loading, the stress state parameter \(\xi\) becomes constant, and all equations remain nonlinearity only due to shear parameter \(Q\). Moreover, for uniaxial tension and uniaxial compression loadings, when \(\xi=\pm 1/3\), the first two equations become similar to classical ones without any nonlinear components. This gives possibility for relatively easy validation of required parameters of the model.

### 3.3. Example and test correlation

Let us consider as an example, the simplest approach with linear dependency on parameter \(\xi\) of compliance coefficients:
\[
\begin{align*}
A_{1111}(\xi) &= a_{11}^0 + c_{11}\xi, \\
A_{2222}(\xi) &= a_{22}^0 + c_{22}\xi, \\
A_{1122}(\xi) &= a_{12}^0 + c_{12}\xi,
\end{align*}
\]

and the case when coefficient for pure shear compliance has no influence of stress traxiality:
\[
B(\xi, Q) = \Gamma(Q).
\]

The last component can be found as \(\Gamma(\sigma_{12}) = \epsilon_{12}/\sigma_{12}\) from the pure in-plane shear test. While coefficients \(a_{11}^0, a_{12}^0\) and \(c_{11}, c_{12}\) can be determined from uniaxial tests. The pictures in Fig. 2 show the possibilities of this model by correspondence between the theoretical dependencies and the results of tests of composite material on the base of fiberglass fabric and polyether matrix for tension, compression and shear conditions tests.
4. Influence on damage
More over such kind of effects have the influence on the damage distribution in strength analysis. The pictures in Figs. 3-5 show the difference in matrix damage distribution for results based on linear elastic model and for nonlinear one presented in this research. Modeling was realized for the compression of graphite fibers and epoxy matrix composite specimen with open-hole and ±45 layup.

Figure 2. Test correlation for fiberglass fabric composite; 1,2 – predicted loading diagrams.

Figure 3. Predicted matrix damage distribution based on linear elastic model

Figure 4. Predicted matrix damage distribution based on nonlinear elastic model
5. Conclusions
An example of failure modeling approach utilizing only standard strength characteristics is demonstrated. It can be seen that proposed failure modeling approach based on formulated assumptions and utilizing only minimum experimental data gives a good correlation with the results of biaxial loading experiments. The approach has a block form and can be modified at all key steps such as elastic relations or first ply failure criterion.

6. Acknowledgement
This research was supported by the Russian Foundation for Basic Research (Grants no. 18-31-20026 and 17-51-52001).

References
[1] Hashin Z. Failure Criteria for Unidirectional Fiber Composites. J Appl Mech 1980;47 (2):329–334.
[2] Puck A., Schürmann H. Failure analysis of FRP laminates by means of physically based phenomenological models. Compos. Sci. Technol 1998;58(7):1045–1067.
[3] Reddy J. N., Pandey A. K. A first-ply failure analysis of composite laminates. Computers & Structures 1987; 25(3):371–393.
[4] Rotem A. The Rotem failure criterion: theory and practice. Compos Sci Technol 2002;62 (12–13):1663–1671.
[5] Tsai S. W., Wu E. M. A general theory of strength for anisotropic materials. J Compos Mater 1971;5 (1):58–80.
[6] Hashin Z., Rotem A. A fatigue failure criterion for fiber reinforced materials. J Compos Mater 1973;7 (4):448–464.
[7] Y. Wan, B. Sun, B. Gu, Multi-scale structure modeling of damage behaviors of 3D orthogonal woven composite materials subject to quasi-static and high strain rate compressions, Mechanics of Materials 2016 94: 1–25.
[8] P. Maimi’a, P.P. Camanhob, J.A. Mayugoa, A. Turona, Matrix cracking and delamination in laminated composites. Part I: Ply constitutive law, first ply failure and onset of delamination, Mechanics of Materials, 2011 43 (4): 169–185.
[9] R. M. Christensen, Compressive failure of composites using a matrix-controlled failure criterion with the kink band mechanism, Mechanics of Materials, 2000 32(9): 505–509.
[10] McCarthy C. T., McCarthy M. A., Lawlor V. P. Progressive damage analysis of multi-bolt composite joints with variable bolt-hole clearances. Composites Part B 2005;36 (4):290–305.
[11] Camanho P. P., Matthews F. L., A progressive damage model for mechanically fastened joints in composite laminates. J Compos Mater 1999;33 (24):2248–2280.
[12] Moure M. M., Otero F., García-Castillo S. K., Sánchez-Sáez S., Barbero E., Barbero E.J. Damage evolution in open-hole laminated composite plates subjected to in-plane loads, Composite Structures, 2015; (133): 1048-1057.
[13] Iarve E. V., Pagano N. J. Singular full-field stresses in composite laminates with open holes, International Journal of Solids and Structures, 2001; 38 (1, 5): 1-28
[14] Bogetti T. A., Hoppel C. P. R., Harik V. M., Newill J. F., Burns B. P. Predicting the nonlinear response and progressive failure of composite laminates. Compos Sci Technol 2004; 64(3–4):329–342.
[15] Soden P. D., Hinton M. J., Kaddour A. S. Lamina properties, lay-up configurations and loading conditions for a range of fibre-reinforced composite laminates. Compos Sci Technol 1998;58(7):1011–1022.
[16] Lomakin, E. V. Constitutive Models of Mechanical Behavior of Media with Stress State Dependent Material Properties, Mechanics of Generalized Continua. Advanced Structured Materials, 2011; (7): 339-350.
[17] Lomakin E.V., Fedulov B.N., Nonlinear anisotropic elasticity for laminate composites, Meccanica (2015) 50: 1527.
[18] Petit P. H., Waddoups M. E., A Method of Predicting the Nonlinear Behavior of Laminated Composites. J. Comp Mat 1969;(3) 2.
[19] Chamis, C. C. and Sinclair, J. H. Ten-Deg Off-Axis Shear Properties in Fiber Composites. Experimental Mechanics. 1977; 339-346.
[20] Hahn H. T., Tsai S.W. Nonlinear Elastic Behavior of Unidirectional Composite Laminae Journal of Composite Materials January. 1973 (7): 102-118.
[21] Lomakin E. V., Fedulov B. N. Nonlinear anisotropic elasticity for laminate composites, Meccanica 2015 (50):1527–1535.
[22] Hashin Z. Analysis of Composite Materials—A Survey. J Appl Mech 1983;50(3):481–505.
[23] Hinton M. J., Kaddour A. S., Soden P. D. A comparison of the predictive capabilities of current failure theories for composite laminates, judged against experimental evidence. Compos Sci Technol 2002;62(2–13):1725–1797.
[24] Icardi U., Locatto S., Longo A., Assessment of Recent Theories for Predicting Failures of Composite Laminates. Appl Mech Rev 2007;60 (2):76–86.
[25] Thom H. A review of the biaxial strength of fibre-reinforced plastics. Composites Part A: Appl Sci Manuf 1998;29(8):869–886.
[26] Garnich M. R., Venkata M. K. Akula. Review of degradation models for progressive failure analysis of fiber reinforced polymer composites. Appl Mech Rev 2009;62(1):010801-1–010801-33.
[27] Fedulov B. N., Fedorenko A. N., Kantor M. M. and Lomakin E. V., Failure analysis of laminated composites based on degradation parameters. Meccanica, 2017;53:359–372.
[28] Fedulov B. N., Fedorenko A.N., Safonov A.A. and Lomakin E.V., Nonlinear shear behavior and failure of composite materials under plane stress conditions. Acta Mechanica. 2017 Jun 1;228(6):2033-40.