Power-Like Threshold Corrections to
Gauge Unification in Extra Dimensions

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Abstract

One of the much-debated novel features of theories with extra dimensions is the presence of power-like loop corrections to gauge coupling unification, which have the potential of allowing a significant reduction of the unification scale. A recognized problem of such scenarios is the UV sensitivity of the above power corrections. We consider situations where the grand unified group is broken by the vacuum expectation value of a bulk field and find that, because of the softness of this extra-dimensional symmetry breaking mechanism, power-like threshold corrections are calculable and generic in many of the most relevant settings. While the precision is limited by the presence of higher-dimension bulk operators, the most dangerous of these operators are naturally forbidden by symmetries of the bulk theory. Particularly interesting and constrained scenarios arise in the context of higher-dimensional supersymmetry. Our phenomenological exploration of SU(5) models in 5d, linked in particular with more recently discussed orbifold GUT models, shows promising results.
1 Introduction

Grand unified theories provide an elegant explanation of the fermion quantum numbers of the Standard Model (SM) [1] (also [2]). Together with the success of gauge coupling unification [3] in supersymmetric (SUSY) extensions of the SM [4], this has established high-scale grand unification as the standard framework for the discussion of physics above the electroweak scale.

During the last few years, the above paradigm has been challenged by various scenarios with extra dimensions compactified at scales below $M_{\text{GUT}} \sim 10^{16}$ GeV. In particular, Dienes, Dudas and Gherghetta [5] have proposed low-scale gauge unification as a possible consequence of power-like loop corrections to gauge coupling constants [6, 7]. However, objections to this proposal have been raised on the basis that the relevant loop corrections are completely UV dominated and that, as a result, no precise statement about the ratio of low-energy gauge couplings can be made without a UV completion of the higher-dimensional SM-like theory (see, e.g., [8]). The issue of ‘power law running’ was also discussed in connection with ‘deconstruction’ and warped 5d models (see, e.g., [9] and [10,11]). In the present paper we demonstrate that, if the GUT group is softly broken in the weak-coupling regime of the higher-dimensional theory, the resulting power-like threshold corrections can be numerically important, calculable, and of universal nature.

To be specific, let us first consider $d$-dimensional pure Yang-Mills theory, compactified to 4 dimensions on a torus of radius $R$. Scattering processes in the 4d theory at energies near the compactification scale $M_c \sim 1/R$ can be used to define a 4d gauge coupling $\alpha_4(M_c) = g_4^2(M_c)/(4\pi)$.

In the following, this quantity will be considered as the basic physical observable of the low energy effective theory. It is linked to processes at energies far below $M_c$ by conventional 4d logarithmic running. The relation to the coupling constant $\alpha_d(M)$ of the $d$-dimensional theory is given by

$$\alpha_4(M_c)^{-1} \sim \alpha_d(\mu)^{-1} R^d - 4 + f_{\text{1-loop}}(\mu, R) + \text{higher orders,} \quad (1)$$

where $\mu$ characterizes the renormalization point of the higher-dimensional field theory (see, e.g., [12]). For $\mu \gg M_c$, the leading contribution from $f_{\text{1-loop}}$ is $\sim (\mu R)^{d-4}$. It describes the power-divergent loop-correction to the $F^2$ term in the bulk, multiplied by the extra-dimensional volume. Since the l.h. side is $\mu$-independent, we have $\alpha_d(\mu)^{-1} \sim M^{d-4} - \mu^{d-4}$, where $M$ can be considered as the fundamental UV scale of the model, and $O(1)$ numerical coefficients (which depend on the renormalization scheme) have been suppressed. It is convenient to assume $\mu \ll M$, so that $\alpha_d \sim M^{4-d}$.

Next, we assume that the vacuum expectation value (VEV) of a bulk Higgs breaks the simple gauge group $G$ of the fundamental theory to a subgroup $H = H_1 \times \cdots \times H_n$ (which is a direct product of simple groups and U(1) factors). The Higgs breaking is characterized by an energy scale $M_B$, related to the masses of vector bosons and physical scalars. For $M_c \ll M_B \ll \mu \ll M$, the 4d gauge couplings, labelled by the index $i = 1...n$, are now given by

$$\alpha_{4,i}(M_c)^{-1} \sim \alpha_d(\mu)^{-1} R^d - 4 + (\mu R)^{d-4} + f_{\text{1-loop},i}(\mu, R, M_B) + \text{higher orders.} \quad (2)$$
Here we have split the 1-loop correction into a universal \((i\text{-independent})\) piece carrying the leading divergence \(\sim \mu^{d-4}\) and the non-universal piece \(f_{1\text{-loop},i}\). To understand this structure, it is sufficient to observe that, while the bulk theory at energies below \(M_B\) possesses non-universal (with respect to \(i\)) power divergences of degree \(d-4\), such divergences can not be present in the unbroken high-scale theory. Thus, their contribution to the coefficients of \(F_i^2\) is suppressed by \(M_B^2\). To be more specific, the function \(f_{1\text{-loop},i}\) may be considered as arising from differences of one-loop integrals with massive and massless vector bosons,

\[
\int^\mu \frac{d^d k}{(k^2 + M_B^2)^2} - \int^\mu \frac{d^d k}{(k^2)^2} \sim M_B^2 \mu^{d-6},
\]

which demonstrates the structure of the \(M_B\)-suppression. This estimate is, however, only valid for \(d > 6\). For \(d = 5\) this term is finite and calculable, so that Eq. (2) has to be replaced by

\[
\alpha_{4,i}(M_c)^{-1} \sim \alpha_5(\mu)^{-1}R + \mu R + c_i M_B R + \cdots.
\]

This structure was previously discussed in U(1) toy models \[10, 11\]. Except for the non-universal numbers \(c_i\), numerical coefficients have been suppressed in the above estimates. Furthermore, both higher-loop and volume-suppressed terms have been dropped in Eqs. (2) and (4).

For \(d = 6\), the \(M_B\) suppressed term reads \(c_i(M_B R)^2 \ln(\mu/M_B)\). This means that non-universal counterterms (corresponding to higher-dimension operators) have to be present for consistency of the theory. Thus, although an \(\mathcal{O}(1)\) term coming with the log remains undetermined, the coefficients \(c_i\) and therefore the log-enhanced piece is calculable.

The above contributions proportional to \(c_i\) provide corrections to \(\alpha_{d,i}^{-1}\) of relative size \((M_B/M)^{d-4}\). At first sight, the phenomenological relevance of these corrections appears to be limited by possible higher-dimensional operators, e.g., \(\text{tr}[F^2, \Phi]\) (where \(\Phi\) is the bulk field developing a symmetry-breaking VEV). In principle, such operators can give rise to non-universal corrections as large as the loop-effects discussed above.\(^1\) However, as we will explain in detail below, in the simplest and most popular higher-dimensional scenarios, the leading dangerous operators are either automatically forbidden or can be forbidden by minimal symmetry assumptions. Furthermore, it turns out that the coefficients \(c_i\) are governed by the basic group theoretic structure of the theory and are therefore fairly model-independent. Thus, we conclude that power-like threshold corrections to gauge unification can and should be taken seriously at a quantitative level.

In Sect. 2 we consider higher-dimensional Yang-Mills theory with a bulk Higgs field \(\Phi\). Given an appropriate bulk potential, \(\Phi\) will develop a symmetry breaking VEV leading to vector bosons with masses \(\sim M_V\) and a number of physical scalars with masses \(\sim M_S\) (the scalar mass spectrum depends on the parameters of the potential). For \(d = 5\), it suffices to forbid operators linear in \(\Phi\) by a \(\mathbb{Z}_2\)-symmetry to obtain potentially sizeable, controlled power-corrections. Furthermore, in sufficiently flat potentials the vector mass will dominate the scalar masses, so that \(M_S < M_B\) and the corrections are independent of the GUT-Higgs potential.

\(^1\)This has been pointed out in \[10\] in the context of a 5d toy model GUT with gauge group \(U(1) \times U(1)'\)
In Sect. 4, we discuss the supersymmetric theory. Higher-dimensional supersymmetry strongly constrains the possibilities of gauge symmetry breaking by a Higgs field and the arising loop corrections. In particular, breaking by an adjoint Higgs hypermultiplet does not lead to power-corrections because the structure of the model is that of \( N = 4 \) super-Yang-Mills theory. However, potentially large and calculable corrections arise from breaking by the adjoint scalar of the 5d vector multiplet.

In Sect. 5, realistic 5d SU(5) versions of the above generic GUT scenario are discussed. Lowering the compactification scale significantly below \( 10^{16} \) GeV potentially leads to fast proton decay which can, however, be avoided by working on an orbifold (\( S^1/Z_2 \) or \( S^1/(Z_2 \times Z_3) \)), where SU(5) is not a good symmetry on at least one of the branes. (This idea, already discussed in the present context in [5], has recently been extensively used in the context of orbifold GUTs [13, 14, 15, 16]. For larger gauge groups and more than 5 dimensions see, e.g., [17].) Given the explicit SU(5) breaking on the brane, it is quite natural that an adjoint bulk scalar develops a VEV in \( U(1)_Y \) direction. The relative size of the resulting power-like contributions to the \( U(1)_Y \), SU(2), and SU(3) couplings is governed by the Casimirs of the respective adjoint representations. Thus, their effect mimics the dominant (pure gauge) part of conventional logarithmic running. Additional bulk fields can, if they acquire non-universal masses because of the symmetry breaking adjoint bulk VEV, contribute further, model-dependent, power law corrections.

Our conclusions are given in Sect. 6.

2 Calculable bulk threshold corrections

Let us begin by considering a \( d \)-dimensional Yang-Mills theory with simple gauge group \( G \) and a Higgs-field \( \Phi \) transforming in some representation of \( G \). The lagrangian reads

\[
\mathcal{L} = -\frac{1}{2g_{d}^{2}} \cdot \text{tr} \left( F_{MN}F^{MN} \right) - (D_{M} \Phi)^\dagger (D^{M} \Phi) - V(\Phi),
\]

where \( F_{MN} \) is the field strength tensor, \( D_{M} \) is the covariant derivative, and the indices \( M, N \) run over \( 0,\ldots,3,5,\ldots,d \). We assume that \( \Phi \) develops a VEV breaking \( G \) to a subgroup \( H = H_{1} \times \cdots \times H_{n} \). Without supersymmetry, this can simply be realized by choosing an appropriate bulk potential \( V(\Phi) \). Higher-dimensional supersymmetry restricts possible bulk interactions and different origins for a bulk VEV have to be considered (cf. Sects. 4 and 5).

At tree level, the couplings \( \alpha_{d,i} \) of the group factors \( H_{i} \) are equal to the coupling \( \alpha_{d} \) of \( G \). At one loop, one has to calculate the contributions of the light and heavy vector bosons and the physical Higgs scalars to the coefficients of the \( F_{i}^{2} \) terms, i.e., to the normalization of the field-strength terms of the unbroken subgroup factors. This calculation was done in the context of 4d GUTs in dimensional regularization \([18, 19]\) (see also [20]), so that the \( d \)-dimensional result can simply be taken from [18]:

\[
\alpha_{d,i}^{-1} = \alpha_{d}^{-1} + \frac{\Gamma(2-d/2)}{6(4\pi)^{d/2-1}} \left[ -(25-d) \sum_{r_{i}} M_{V_{r_{i}}}^{d-4} T_{r_{i}} + \sum_{r_{i}'} M_{S_{r_{i}'}}^{d-4} T_{r_{i}'} + 2s_{d} \sum_{r_{i}''} M_{F_{r_{i}''}}^{d-4} T_{r_{i}''} \right].
\]
Here $r_i, r'_i$ and $r''_i$ label the representations under $H_i$ of the vector, scalar and spinor particles and $M_{V,r_i}, M_{S,r'_i}$ and $M_{F,r''_i}$ stand for the corresponding masses. (Although the minimal setting discussed at the moment has no fermions, we have included a possible fermionic contribution into the above equation for completeness. The number $s_d$ characterizes the dimension of the relevant spinor.) Furthermore, $T_{ri}$ is defined by $\text{tr}[T^{a}T^{b}] = \delta^{ab}T_{ri}$, where $T^{a,b}$ are the generators in the representation $r_i$ (and analogously for $r'_i$ and $r''_i$).

Concerning the structure of Eq. (6), several comments are in order. We have chosen $\alpha^{-1}$ (rather than $\alpha$ or $g$) as our basic quantity because it can be interpreted as the coefficient of the $F^2$ operator and hence the further transition to the 4d theory proceeds simply by multiplication with the volume factor. Of course, this direct relation between Eq. (6) and Eq. (2) works only up to terms suppressed by a volume factor. We will discuss such terms in more detail below. Note furthermore that Eq. (6) does not contain contributions from the gauge bosons of the unbroken subgroup. In our context, the reason for this is the masslessness of these vector bosons. Because of the absence of a mass scale, the corresponding loop integrals have a pure power of the loop momentum in the integrand and therefore vanish in dimensional regularization.

By power counting, we expect the one-loop correction to $\alpha^{-1}_{d,i}$ to diverge with the $(d-4)$th power of the cutoff. The fact that this does not show up in Eq. (6) is due to the use of dimensional regularization. However, this does not restrict the validity of our conclusions in any way. On the one hand, this leading power-divergence is $G$-universal (independent of $i$) because of the symmetric structure of the UV theory and can thus be absorbed in a redefinition of $\alpha^{-1}_{d,i}$. On the other hand, the main phenomenological implications depend only on the differences between the inverse gauge couplings of the group factors $H_i$ and are therefore not affected by a $G$-universal correction.

The further analysis depends crucially on two closely related issues: the possible existence of higher-dimension operators that can compete with the corrections on the r.h. side of Eq. (6) and the UV divergences that are present even in these non-$G$-universal loop corrections. To be specific, the lagrangian generically contains terms

$$\sim \frac{1}{M^k} \Phi^n (F_{MN})^2,$$

where we have assumed that the relevant product of representations of $G$ contains a singlet, and $k$ is chosen to ensure the overall mass dimension $d$. When $\Phi$ develops a VEV $v = M_V/g_d$ (vector boson masses are generated as in the familiar 4-dimensional setting), the operator in Eq. (7) can lead to non-universal corrections to $\alpha^{-1}_{d,i}$ at tree-level. Since $g_d^2 \sim M^{4-d}$ (the fundamental UV scale of the theory), the relative size of this correction is given by

$$\frac{\Delta \alpha^{-1}_{d,i}}{\alpha^{-1}_{d,i}} \sim \left( \frac{M_V}{M} \right)^n.$$

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2This argument works only for $d > 4$, where the coupling can be defined at zero external momentum. In 4 dimensions, the relevant loop integrals require the external momentum as IR regulator, and as a result the familiar contribution $\sim \ln(\mu^2/Q^2)$ from massless gauge bosons appears.
This has to be compared to the correction from Eq. (6) which, focusing on the vector boson part, is of relative size \( (M_V/M)^{d-4} \). Given the possibility that \( n = 1 \), this appears to be discouraging. However, it is important to keep in mind that certain values of \( n \) may be forbidden by group theory or other symmetries. For example, for \( d = 5 \) the leading calculable correction is of relative size \( M_V/M (M_V/M \ll 1) \), and a simple \( Z_2 \) symmetry \( \Phi \to -\Phi \) is sufficient to forbid the competing \( n = 1 \) term from Eq. (8). The \( n = 2 \) term is of relative size \( (M_V/M)^2 \) and therefore negligible.

Higher-dimension operators can act as counter terms and are therefore intimately linked to the divergence structure of the non-universal corrections on the r.h. side of Eq. (6). In 5d, the non-universal part of the loop correction is finite, which is consistent with the possibility of forbidding the relevant operator by a \( Z_2 \) symmetry. For \( d \to 6 \), the Gamma function develops a pole, showing that the non-universal term is afflicted by a logarithmic divergence. Although this implies the existence of a higher-dimension operator \( \sim F^2 \Phi^2 \) providing the counter term, predictivity is maintained at the leading logarithmic level. To be specific, we assume that the divergence is cured by a theory of higher symmetry at the scale \( M \) and that there are no anomalously large non-universal threshold effects associated with this transition. In short, we work at leading-log approximation in \( M/M_V \). This logarithm can be extracted from Eq. (20), as is common in 4d, by setting \( d = 6 - 2\epsilon \), introducing appropriate factors \( \mu^{2\epsilon} \) to keep the correct dimensionality, expanding in \( \epsilon \), and letting \( \epsilon \to 0 \) and \( \mu \to M \). Focusing on the vector contribution, the result reads

\[
\alpha^{-1}_{4,i}(M_c) = V \alpha^{-1}_{6} + \frac{1}{3(4\pi)^2} 19 \sum_r (V M^2_{V,r} T_r) \ln \frac{M}{M_{V,r}}.
\]

This should provide a good description if \( M^2 \ll M^2 \ll M^2 \) and scalar and fermion masses are small.

In more than 6 dimensions, power-counting suggests that there are non-universal power-divergences. More specifically, the explicitly calculated logarithmic divergence in \( d = 6 \) suggests a power-divergence of degree \( d - 6 \) in \( d \) dimensions, which would have to come with a factor \( M^2_V \) for dimensional reasons. The corresponding counter term is provided by the operator \( \Phi^2 F^2 \), which should therefore always be included in the lagrangian. The term \( \sim M^{d-4}_V \) in Eq. (6) is subdominant with respect to this operator. Thus, quantitative statements depend on a more detailed knowledge of the UV structure of the theory. However, it is likely that the group-theoretical specification of the VEV of \( \Phi \) and a classification of the singlets contained in \( \Phi^2 F^2 \) will be sufficient to uniquely determine or strongly constrain the way in which \( \Phi^2 F^2 \) terms contribute to gauge coupling differences \( \alpha^{-1}_{4,i}(M_c) - \alpha^{-1}_{4,j}(M_c) \).

Dangerous higher-dimension operators mixing \( \Phi \) and \( F^2 \) may also reside on branes. But in this case their contribution to the observed effective 4d couplings is further suppressed by volume factors. For example, the contributions of operators on branes of co-dimension \( d_c \) are suppressed by the potentially small factor \( (M R)^{-d_c} \), where \( R \) is the compactification radius.

To summarize, a generic and particularly predictive setup can be described as follows. Assume that there are no bulk fermions or at least no non-\( G \)-universal mass splitting of
bulk fermions. Assume furthermore that $M_S \ll M_V$, i.e., the potential stabilizing the VEV of $\Phi$ is relatively flat (this is generic in supersymmetry which, however, will be discussed in more detail below). If dangerous higher-dimension operators are forbidden by appropriate symmetries, the leading power correction is calculable and the resulting 4d gauge couplings are obtained by multiplying Eq. (6) with the volume factor $V$:

$$\alpha_d^{-1}(M_c) = V\alpha_d^{-1} - \frac{\Gamma(2 - d/2)}{6(4\pi)^{d/2-1}}(25 - d) \sum_{r_i} (V M_{V,r_i}^{d-4}) T_{r_i}. \quad (10)$$

Here $\alpha_d^{-1}$ on the r.h. side is defined in dimensional regularization, which makes it independent of the subtraction scale $\mu$ since the coefficient of the relevant $G$-universal power-divergence vanishes. One may think of $\alpha_d$ as the $d$-dimensional gauge coupling defined at zero momentum (in complete analogy with $1/M_P$ in 4d gravity). The $T_{r_i}$ and the relative sizes of the $M_{V,r_i}$ are determined by group theory (the representation of $\Phi$ and the direction of its VEV $v$), so that the power correction is proportional to $V v^{d-4}$. The relative size of this correction is $(M_V/M)^{d-4}$. It has to be small enough so that even higher powers of $M_V/M$ are suppressed. Nevertheless, it can be significantly larger than the usual 4d GUT threshold corrections of relative size $\alpha_{\text{GUT}} \sim 1/25$. Jumping somewhat ahead we would like to note that higher supersymmetry or string theory may forbid or fix all dangerous higher-dimension operators and higher-loop corrections to the $d$-dimensional gauge couplings (cf. [21]), in which case one might hope to go to the region $M_V \sim M$ so that the relative sizes of low-energy gauge couplings are dominantly determined by power-law effects.

## 3 Brane effects and the KK-mode approach

So far, we have focussed on true bulk effects and completely neglected terms suppressed by powers of the bulk-size $R$. However, it is clear that such contributions are generically present, e.g., on the r.h. side of Eq. (10). One can approach this issue using $d$-dimensional propagators in the full, compactified geometry. However, in the present investigation we find it simpler to discuss these effects using an effective 4d framework and summing KK modes. Clearly, these two methods are equivalent both conceptually and quantitatively.

To be specific, although we are prepared to neglect terms down by full powers of $MR$ (since these terms will in general be sensitive to unknown and largely unconstrained brane operators), we would like to take terms into account that are suppressed by powers of $MR$ but enhanced by $\ln(MR)$. Such terms are known to be important in orbifold GUTs [15,16], where they give rise to the calculable `differential running' [22] above the compactification scale.

For simplicity, we first consider a toy example of one extra dimension compactified on an $S^1$. We start with a theory with one unbroken gauge group $G$ and consider only the contribution of a bulk scalar with mass $M_{S1} \sim M_c$ in a certain representation of $G$.

Further, we compare this to a theory where the scalar mass is shifted to $M_{S2} \gg M_c$. The difference in the scalar contribution to the low-energy gauge couplings in these two
models comes from the difference in log-contributions from the KK towers:

\[
\alpha_4^{-1}(M_c)_{\text{model 2}} - \alpha_4^{-1}(M_c)_{\text{model 1}} = \frac{T_r}{24\pi} \sum_{n=-\infty}^{\infty} \left[ \ln \left( \frac{\mu^2}{(nM_c)^2 + M_{S2}^2} \right) - \ln \left( \frac{\mu^2}{(nM_c)^2 + M_{S1}^2} \right) \right],
\]

(11)

\[
\approx \frac{T_r}{24\pi} \left[ -\ln \frac{M_{S2}^2}{M_c^2} - 2\sum_{n=1}^{\infty} \ln \left( 1 + \frac{N^2}{n^2} \right) \right],
\]

(12)

where \( N = M_{S2}/M_c \) and \( M_{S1} \) has been set to zero everywhere in Eq. (12) except for the zero-mode contribution, where it has been replaced by \( M_c = 1/R \). This introduces only an \( \mathcal{O}(1) \) error. The sum on the r.h. side of Eq. (12) can be estimated as

\[
\sum_{n=1}^{\infty} \ln \left( 1 + \frac{N^2}{n^2} \right) \simeq \pi N - \ln N + \mathcal{O}(1),
\]

(13)

for \( N \gg 1 \), so that the final result reads

\[
\alpha_4^{-1}(M_c)_{\text{model 2}} - \alpha_4^{-1}(M_c)_{\text{model 1}} = -\frac{T_r}{12} \frac{M_{S2}}{M_c}.
\]

(14)

Thus, model 2 differs from model 1 precisely by the power-like contribution \( \sim M_{S2} \), which can also be obtained from Eq. (6) by setting \( d = 5 \). The important point here is that one finds no additional, log-enhanced contribution from the momentum region above \( M_c \). In other words, the zero-mode log merges with the KK logs to give just a pure power.

The situation is different, however, if one compactifies on \( S^1/Z_2 \). In this case, the sum over positive and negative \( n \) in Eq. (11), corresponding to sines and cosines, is replaced by a sum over just positive \( n \), corresponding to cosines only (assuming positive \( Z_2 \) parity of the scalar field). The zero mode still contributes with full strength. As a result, the cancellation of the zero-mode log is incomplete and Eq. (14) is replaced by

\[
\alpha_4^{-1}(M_c)_{\text{model 2}} - \alpha_4^{-1}(M_c)_{\text{model 1}} = -\frac{T_r}{12} \frac{M_{S2}}{M_c} - \frac{T_r}{12\pi} \frac{1}{2} \ln \frac{M_{S2}}{M_c}.
\]

(15)

If, on the other hand, the \( Z_2 \) parity of the scalar field is odd, there is no zero mode and only the sine modes contribute to the KK sum. One then finds

\[
\alpha_4^{-1}(M_c)_{\text{model 2}} - \alpha_4^{-1}(M_c)_{\text{model 1}} = -\frac{T_r}{24} \frac{M_{S2}}{M_c} + \frac{T_r}{12\pi} \frac{1}{2} \ln \frac{M_{S2}}{M_c}.
\]

(16)

This simple calculation allows for the following intuitive interpretation: Without branes, gauge coupling corrections are logarithmic below the compactification scale and purely power-like above it. Introducing 4d boundaries (branes) leads to typical 4d effects even above \( M_c \), i.e., logarithmic corrections. For each brane at which a 5d field is non-zero (Neumann boundary conditions), one finds \((1/4)\) times the usual log from 4d running. For each brane at which a 5d field is zero (Dirichlet boundary conditions), one finds
−(1/4) times this log. It can be easily checked that this rule extends to \(S^1/(Z_2 \times Z_2')\), where a field can be zero at one brane and non-zero at the other.

While the extension of this rule to fermions is straightforward, the case of massive 5d vector fields requires some comments. The rule is that, if \(A_\mu\) (where \(\mu = 0, \ldots, 3\)) is non-zero at a boundary, one finds a scalar log contribution with prefactor \((1/4)(-22)\). The factor \(-22\) can be derived from Eq. (12) recalling that the zero mode (massive vector) has prefactor \(-21\), while the KK tower (massive vectors and \(A_5\)-scalars) has prefactor \(-20\). An intuitive understanding can be obtained if, guided by the scalar case above, one adds the \(A_\mu\) contribution \((1/4)(-21)\) and the \(A_5\) contribution \((-1/4)\) (the \('-'\) arising since \(A_5\) is zero if \(A_\mu\) is non-zero). The rule extends in an obvious way to the case in which \(A_\mu\) is zero at a boundary (orbifold breaking of the gauge group): one finds a scalar log with prefactor \((-1/4)(-22)\). In deriving this, it is important not to forget the \(A_5\) zero-mode. Furthermore, there is a straightforward extension to the case of massless 5d vector fields, where the relevant prefactors of the boundary logs are \((\pm 1/4)(-23)\).

In fact, the above set of rules represents a simple and intuitive way of rederiving the ‘differential running’ in 5d orbifold GUTs above \(M_c\) because it relates 4d logs directly to the boundary conditions of fields (without any reference to the KK mode spectrum).

To illustrate the relevance of the above in the present context, we now give a more complete version of Eq. (10) in 5d. We work on \(S^1/Z_2\) with \(A_\mu\) and \(\Phi\) non-zero at both boundaries. The result, which now includes both power-law and log-enhanced terms, reads

\[
\alpha_{4,i}^{-1}(M_c) = \pi R \alpha_5^{-1} + \frac{1}{24} \left[ 20 \sum_{r_i} (RM_{V,r_i})T_{r_i} - \sum_{r_i'} (RM_{S,r_i'})T_{r_i'} \right] + \frac{1}{12\pi} \left[ \frac{1}{2}(-22) \sum_{r_i} T_{r_i} \ln \frac{M}{M_{V,r_i}} + \frac{1}{2} \sum_{r_i'} T_{r_i'} \ln \frac{M}{M_{S,r_i'}} + \frac{1}{2}(-23)C_i \ln \frac{M}{M_c} \right].
\]

Note, in particular, the appearance of contributions from the vector bosons of the unbroken subgroup \((C_i\) is the adjoint Casimir of \(H_i\)) which, although irrelevant for the power-like terms, contribute to the boundary-driven logarithmic running above \(M_c\). Furthermore, it should be observed that no non-universal logarithmic running occurs above the highest of the scales \(M_{V,r_i}\) and \(M_{S,r_i'}\) since

\[
\sum_{r_i} T_{r_i} + C_i = C_A(G) = i\text{-independent}
\]

and

\[
\sum_{r_i} T_{r_i} + \sum_{r_i'} T_{r_i'} = T_{\Phi-\text{repr.}}(G) = i\text{-independent}.
\]

### 4 The supersymmetric theory

Most of the above extends straightforwardly to supersymmetry. In particular, Eqs. (6) and (17) simply require the inclusion of the additional degrees of freedom (fermions
and scalars) that are present in the relevant supersymmetric multiplets. However, there are also some crucial new points that require a separate discussion. In particular, it is important to understand the possible origin of the bulk VEV and the resulting mass spectrum, both of which are strongly constrained by SUSY.

We first focus on 5 and 6d, where the minimal SUSY corresponds to $N = 2$ in 4d language. This excludes all renormalizable (from the 4d point of view) interactions except those prescribed by gauge symmetry. In particular, the Higgs field $\Phi$, which would have to come from a gauged hypermultiplet, can not have a conventional bulk potential with cubic and quartic terms. Although it appears conceivable that higher-dimension operators, consistent with 5d SUSY, generate a suitable potential $^3$ we chose the simpler option of fixing the bulk Higgs VEV by an appropriate boundary potential. In doing so, we follow the method for breaking $U(1)_\chi$ in the 6d SO(10) model of $^26$. Clearly, we have to rely on the existence of a D-flat direction in the bulk. (Here, by D-flatness we mean that no potential arises from integrating out the SU(2)-R triplet of auxiliary fields of the gauge multiplet. For an explicit component lagrangian of a gauged 5d hypermultiplet see, e.g., $^23$.) In general, such a D-flat direction might not exist. This can, for example, be easily checked in the case of a single U(1) hypermultiplet. We now assume a representation or field content where a flat direction can be found. The non-zero VEV is stabilized only by a brane superpotential which we will not specify at the moment. In the bulk, the VEV will give masses to the whole 5d vector multiplet (in the broken directions) and to a whole hypermultiplet (in the directions corresponding to the would-be Goldstone-bosons). However, we also know that in spontaneous gauge symmetry breaking a single scalar degree of freedom is transferred to the vector field. In the case of 5d SUSY, this is only possible if the masses of the vector multiplet and the hypermultiplet in the broken directions are the same. Let us for the moment assume that these two multiplets exhaust the set of heavy states.

This occurs, in particular if we choose the hypermultiplet to be in the adjoint representation, which makes the model $N = 4$ supersymmetric. We can then imagine the theory to arise via dimensional reduction from a SYM theory in 10d and think of the two complex scalars of the hypermultiplet as $(A_7 + iA_8)$ and $(A_9 + iA_{10})$. It is now clear that flat directions exist (e.g. $A_7 = \text{const.}$) and that the whole hypermultiplet acquires a mass (from terms $\sim [A_7, A_8]^2$ etc.). Furthermore, it is immediately clear from the underlying gauge structure that all scalar and vector masses, and hence also the fermionic masses, corresponding to excitations of the broken directions are identical.

Thus, we have argued that, after spontaneous symmetry breaking driven by bulk hypermultiplets, we find the degrees of freedom of a vector multiplet and a hypermultiplet for every broken direction at the massive level. Simple counting of vector, scalar and fermionic fields according to Eq. (6) shows that no bulk loop correction arises. This does not come as a surprise since we are faced with the field content corresponding to $N = 4$ SUSY.

However, the symmetry-breaking bulk VEV does not have to come from a Higgs.

$^3$A systematic analysis of such operators should be possible using the manifestly gauge-invariant formulation $^26$ of 5d SUSY in terms of 4d superfields $^24, 25$. 
Instead, it is possible that, in a compact geometry, one of the extra-dimensional components of the vector field (e.g., \( A_5 \) in 5d; \( A_5 \) or \( A_6 \) in 6d) develops a VEV. Clearly, only adjoint breaking is possible in this case. However, it is a well-known and difficult problem to stabilize such a VEV. This is probably even more so if we require the \( A_5 \) or \( A_6 \) VEV to be large enough to generate a large power correction.

A closely related and more immediately useful possibility exists in 5d. Consider, for example, a 5d SU(5) model. It is possible that the scalar partner \( \Sigma \) of the gauge fields, which is present in 5d SUSY, develops a bulk VEV in U(1)\(_Y\) direction\(^4\) (cf. \(27\)). Such a scalar VEV can arise in an \( S^1/Z_2 \) model where both boundaries break SU(5) and Fayet-Iliopoulos terms of the U(1)\(_Y\) subgroup are present at both boundaries. As explained in \(28\) (see also \(21,29\)), in the 5d setup this term does not break SUSY or U(1)\(_Y\), but instead drives a non-zero bulk VEV of \( \Sigma \). More generally, whenever we have a 5d orbifold model where the bulk gauge symmetry is broken in such a way that an isolated U(1) factor survives on both branes, Fayet-Iliopoulos terms driving a bulk VEV of \( \Sigma \) can be introduced.

In the presence of a VEV of \( \Sigma \), all the fields in the 5d vector multiplet corresponding to the broken directions acquire a bulk mass \( M_V \). The formula for threshold corrections relevant to this case reads

\[
\alpha_{4,i}^{-1}(M_c) = V \alpha_d^{-1} + \frac{1}{24\pi} 12 \sum_{r_i} (VM_{V,r_i}) T_{r_i}. \tag{20}
\]

The prefactor 12 can be understood as the sum of 20 for a massive 5d vector and \(-8\) for the spinor. The degree of freedom corresponding to \( \Sigma \) is absorbed in the massive vector field. (As discussed above, it is also immediately clear that a hypermultiplet of mass \( M_V \) would precisely cancel this term.) We can improve the correction by including volume suppressed but log-enhanced terms using the discussion in the previous section. For simplicity, we work on \( S^1/Z_2 \) and assume that both boundaries break the gauge group in the same way as the bulk VEV:

\[
\alpha_{4,i}^{-1}(M_c) = \pi R \alpha_5^{-1} + \frac{1}{24} \left[ 12 \sum_{r_i} (RM_{V,r_i}) T_{r_i} \right] \tag{21}
\]

\[
+ \frac{1}{12\pi} \left[ -\frac{1}{2} (-24) \sum_{r_i} T_{r_i} \ln \frac{M}{M_{V,r_i}} + \frac{1}{2} (-24) C_i \ln \frac{M}{M_c} \right].
\]

Note that fermions do not contribute to the logarithmic terms since the two Weyl fermions contained in the 5d spinor have opposite boundary conditions at every brane.

For \( d = 7 \), the minimal vector multiplet again contains scalar adjoints that could acquire a VEV as \( \Sigma \) in the 5d case above. However, the minimal supersymmetry is \( N = 4 \) in 4d language and we expect no loop corrections to the gauge couplings.

\(^4\)We are indebted to S. Groot Nibbelink for emphasizing this possibility in a very helpful conversation.
5 Towards a realistic SU(5) model

We now turn to a preliminary analysis of phenomenological implications of the power-like threshold corrections calculated above. In this section, we restrict ourselves to 5d SU(5) models following, in essence, the construction principles of the simplest orbifold GUT models [13,14,15,16]. Proton decay is avoided by placing fermions on branes where SU(5) is not a good symmetry. The light SM Higgs doublet(s) can be localized on the same brane, as suggested in the minimal scenario of [16] (which can be dynamically realized using bulk masses as in [30]). Furthermore, as discussed in the previous section, we assume that the scalar partner Σ of the gauge fields, which is present in 5d SUSY, develops a bulk VEV in U(1)Y direction. Such a scalar VEV can arise in an S1/Z2 model where both boundaries break SU(5) and Fayet-Iliopoulos terms are present. We assume that the usual problem with SU(5) models on S1/Z2, namely the existence of massless scalars with quantum numbers of the X,Y gauge bosons, is solved by introducing appropriate non-local interactions giving these fields a mass. More precisely, the zero-modes in question can be understood as a chiral Wilson-line superfield [31] and interactions involving this Wilson-line are naturally generated by integrating out massive degrees of freedom in the bulk [32].

We assume a standard supersymmetric scenario in 4d, in which case the running between the electroweak scale and Mc is the familiar MSSM running. The low-energy data is taken to be α−1i(mZ) = (59.0, 29.6, 8.4) and the effective SUSY breaking scale is set to mZ. In this case, the relation between couplings at mZ and Mc is given by

$$\alpha_{4i}^{-1}(mZ) = \alpha_{4i}^{-1}(Mc) + \frac{1}{12\pi} (-18C_i + 12T_i) \ln \frac{Mc}{mZ} + \text{SM matter contributions} \quad (22)$$

with $C_i = (0, 2, 3)$ (Casimirs of the SM gauge groups) and $T_i = (3/10, 1/2, 0)$ (SM Higgs representation). Furthermore, using the results of the previous sections and working on an S1/Z2, where the Z2 breaks SU(5), we have

$$\alpha_{4i}^{-1}(Mc) = \pi R\alpha_5^{-1} + \frac{1}{24} [12(RM_Y)(5 - C_i)] + \frac{1}{12\pi} \left[ 12T_i \ln \frac{M}{Mc} \right]$$

$$+ \frac{1}{12\pi} \frac{1}{2} \left[ 24(5 - C_i) \ln \frac{M}{M_Y} + (-24)C_i \ln \frac{M}{Mc} \right]$$

$$+ \text{SM matter contributions}.$$ (23)

This follows immediately from Eq. (21), with the brane-localized Higgs contributing even above Mc.

The usual fairly precise MSSM unification is formally obtained in the limit $M = Mc = MV = MGUT$. We can now try to lower Mc and see whether we can maintain gauge unification at the cost of the power law term $\sim MV/Mc$. This is not hopeless because the coefficients $-C_i$ coming with this term represent the main part of the usual MSSM running coefficients. We focus on differences of 4d inverse gauge couplings, $\alpha_{ij} \equiv$
\( \alpha_i^{-1} - \alpha_j^{-1} \). The crucial gauge unification constraint can be characterized by

\[
\frac{\alpha_{12}(m_Z)}{\alpha_{23}(m_Z)} = \frac{59.0 - 29.6}{29.6 - 8.4} = 1.39 .
\] (24)

This has to be compared with the result obtained from combining the above running and threshold formulae:

\[
\alpha_{ij}(m_Z) = \frac{1}{12\pi} (-18C_{ij} + 12T_{ij}) \ln \frac{M_c}{m_Z}
\]

\[+ \frac{1}{24} [-12C_{ij}] \frac{M_V}{M_c} + \frac{1}{12\pi} [12T_{ij} - 12C_{ij}] \ln \frac{M}{M_c} - \frac{1}{12\pi} 12C_{ij} \ln \frac{M}{M_V},
\] (25)

where \( C_{ij} = C_i - C_j \) and \( T_{ij} = T_i - T_j \).

The maximal value of \( M \) suggested by NDA [33] (cf. [16]) can be characterized by \( M/M_c \sim 10^3 \). The validity range of our calculation is \( M_c \ll M_V \ll M \). It is amusing to observe that, if we set \( M_V \simeq \sqrt{M_c M} \) to realize this situation, the logarithmic terms from the energy range above \( M_c \) mimick precisely the MSSM contribution to coupling ratios. Thus, even if \( M_V \) does not have this precise value, the log terms will not affect MSSM-type unification significantly and, given the preliminary character of the present investigation, we now focus on the power term. From Eq. (25) one can read off that just the logarithmic MSSM contribution would give \( \alpha_{12}/\alpha_{23} = 1.4 \) while just the power-like term would give \( \alpha_{12}/\alpha_{23} = 2 \) (cf. Eq. (24)). Thus, to maintain the above field content while lowering the unification scale significantly, one has to sacrifice precision. One can expect to find \( \alpha_{12}/\alpha_{23} = 1.5 \) at \( m_Z \) if about 1/6 of the log-running is traded for the power correction. This lowers the unification scale \( M \) to about \( 10^{14} \) GeV thus allowing, e.g., for a see-saw mechanism based directly on the GUT scale (without the usual mismatch by a factor \( O(10) \)). One could also consider the possibility that there are no right-handed neutrinos and light neutrino masses are based directly on the appropriate higher-dimension operator suppressed by the new GUT scale. However, the price to pay is the extra \( O(1) \) threshold corrections to \( \alpha_i^{-1} \) that are needed for consistency with the low-energy data. Although such corrections are not unnatural, given that a significant log-running continues all the way up to UV-scale \( M \), they are certainly larger than what would be needed in the 4d MSSM.

Next, we want to consider the possibility that power-like threshold corrections beyond those driven by the 5d gauge multiplet arise. This would not be possible if the bulk breaking was realized by a bulk Higgs field since 5d SUSY forbids the necessary coupling of this Higgs with other hypermultiplets. However, since we consider gauge breaking by the scalar adjoint, this possibility exists. If we add a bulk hypermultiplet, say in the 5 of SU(5), then the doublet and triplet part of it acquire different bulk masses due to the coupling to \( \Sigma \). The ratio of \( M_V \) and these two masses \( M_d \) and \( M_t \) is prescribed by elementary group theory:

\[
M_V : M_d : M_t \sim 5 : 3 : 2 .
\] (26)

The power-like threshold corrections arising in this situation read

\[
\Delta \alpha_{4, i}(M_c) = \frac{1}{24} \frac{M_V}{M_c} \left[ 12(5 - C_i) - 12(\frac{3}{5}T_i + \frac{2}{5}T'_i) \right],
\] (27)
where \( T'_i = (2/10, 0, 1/2) \) characterize the Higgs triplet representation. On the basis of just this power-law contribution one would have \( \alpha_{12}/\alpha_{23} \simeq 2.27 \), i.e., a situation worse than without the bulk 5.

However, this effect can be turned to its opposite by also introducing an SU(5)-invariant bulk mass \( M_f \) for the 5-hypermultiplet. In the presence of such a mass, quantified by \( \xi = M_f/M_V \), with \( \xi \simeq 0.4 \) and with the sign chosen such that it almost compensates the \( \Sigma \)-driven doublet mass, Eq. (27) is transformed into

\[
\Delta \alpha_{4, i}^{-1}(M_c) = \frac{1}{24} \frac{M_V}{M_c} \left[ 12(5 - C_i) - 12 \left( \left| \frac{3}{5} - \xi \right| T_i + \left| \frac{2}{5} + \xi \right| T'_i \right) \right],
\]

leading to \( \alpha_{12}/\alpha_{23} \simeq 1.44 \) just from the power-like correction. Now power-like threshold corrections can replace a significant part of the MSSM log-running without loss of precision of unification, but at the cost of tuning \( M_f \). (This tuning can, of course, also be used to achieve perfect unification, including even the brane-driven log-running above \( M_c \).) Dangerous additional terms can come from higher-dimension operators. In particular, an operator \( \sim F^2 \Sigma^2 \) can contribute to \( \alpha_{4, i}^{-1} \) at the level \( M_V^2/(M_c M) \). If we require this term not to be larger than \( \mathcal{O}(1) \) and take \( M \sim 10^3 M_c \), we find the constraint \( M_V \lesssim 30 M_c \).

(More optimistically, one could assume that this term is forbidden or at least uniquely specified in its structure by \( N = 2 \) SUSY.) From Eq. (28) we can now read off that about half of the low-energy value of, say, \( \alpha_{12} \) can be due to power-like term, so that \( M \) and \( M_c \) can be lowered to \( \sim 10^9 \) GeV and \( \sim 10^6 \) GeV respectively. Given that our very crude estimates have produced this quite impressive result, a more detailed numerical study, including two-loop running and considering appropriate NDA factors, appears to be warranted.

Going further, one might even consider that an exact or almost exact cancellation, based on some yet unknown symmetry reason, leads to vanishing bulk doublet mass \( (\xi = 3/5) \). Equation (28) then implies \( \alpha_{12}/\alpha_{23} \simeq 1.20 \), which is close enough to the desired value 1.39 to have considerable power-law effects without a significant loss of unification precision. In fact, it is this specific scenario that comes closest to original proposal of [5]. The price for this is the re-emergence of the familiar SU(5) problem of tuning the doublet mass to zero. Furthermore, one gets only a single Higgs-doublet at the zero mode level from the one bulk hypermultiplet. Two bulk hypermultiplets would, unfortunately, lead to a much stronger deviation from the desired low-energy coupling ratio.

Let us, however, note that both MSSM Higgs doublets can come from the same hypermultiplet as the two boundary-localized massive modes with exponentially suppressed 4d mass. This requires the bulk doublet mass to be sufficiently large rather than small. With the above favoured value \( \xi \simeq 0.4 \) and the maximal allowed vector mass

\[^5\]In contrast to the case where the VEV comes from a hypermultiplet, the \( \Sigma \)-VEV can also couple linearly, \( \sim F^2 \Sigma \), as in the super-Chern-Simons term discussed in [24]. Here we assume that this term is either forbidden or small. However, even if this term is required (e.g., to cancel anomalies at the boundary), its presence does not destroy the predictivity of the scenario because its contribution to low-energy gauge coupling differences is prescribed by simple group theory.
From $M_V \simeq 30 M_c$, we find a bulk doublet mass $M_d \simeq 6 M_c$. The effective 4d Dirac mass linking the two boundary modes is $m_d \simeq 2 M_d \exp(-M_d \pi R) \sim 10^{-8} M_d$ (see, e.g., 28,30), which may be acceptable in settings with very low unification scale.

It certainly would be interesting to extend the above preliminary analysis to various other proposals involving gauge unification in extra dimensions where power-law effects can be important (see, for example, the discussion in 34 and 35). However, this is beyond the scope of the present paper.

Before closing, we now turn to the possible role of power-like threshold corrections in non-supersymmetric 5d SU(5) models. For this purpose, we choose to accept an ad-hoc fine-tuning solution of the well-known problem of quadratically divergent Higgs mass corrections and focus exclusively on the precision of gauge coupling unification.

We consider an $S^1/(Z_2 \times Z_2')$ model with SM fermions and Higgs doublet on the SU(5)-breaking brane. The bulk Higgs field $\Phi$ is in the adjoint of SU(5) and develops a VEV in $U(1)_Y$ direction. The 4d running below $M_c$ gives $\alpha_{12}/\alpha_{23} = 1.90$, in significant disagreement with data. To correct this, we consider power-like threshold corrections from the bulk, introducing a set of fundamental fermions of SU(5) coupled to the adjoint Higgs by a standard Yukawa coupling $\sim \bar{\psi} \Phi \psi$. Since these fermions can also have an SU(5) symmetric bulk mass, we can treat the resulting doublet and triplet masses $M_{\psi,t}$ and $M_{\psi,d}$ as essentially independent parameters. Assuming that $\Phi$ has no or only a very small bulk mass, the non-SUSY analogue of Eq. (25) reads

$$\alpha_{ij}(m_Z) = \frac{1}{12\pi} (-22 C_{ij} + 2 T_{ij}) \ln \frac{M_c}{m_Z}$$

For simplicity, let us assume $M_{\psi,d} \ll M_{\psi,t}$ so that the power-like contribution from the doublet can be neglected. Further, we lower the compactification scale as far as possible according to the naive estimate based on the higher-dimension operator discussed above, $M_c \simeq 10^6$ GeV with $M_V \simeq 20 M_c$ and $M \simeq 10^3 M_c$. One now finds that the moderate value $M_{\psi,t} \simeq 3 M_V$ gives $\alpha_{12}(m_Z) \simeq 29.4$ and $\alpha_{23}(m_Z) \simeq 21.2$, in reasonable agreement with the data. Although, given the ad-hoc choice of several parameters, this certainly does not challenge the numerical superiority of the minimal SUSY framework, it is nevertheless interesting to see how easily a non-SUSY SU(5) unification can be achieved with the help of large power-like thresholds.

Given that the above exploratory study has shown the possibility of very low compactification scales $\sim 10^6$ GeV, it is tempting to speculate that further work and a better understanding of the UV theory will reveal viable scenarios with TeV scale precision unification.
6 Conclusions

In this paper, we have analyzed the role of loop corrections to gauge coupling constants in grand unified theories with more than 4 dimensions. Since such theories are non-renormalizable, these corrections are, in general, UV-dominated. However, if the higher-dimensional theory respects a certain large gauge symmetry, which is softly broken in the perturbative domain, the differences of the gauge couplings of the surviving subgroups can be calculable and independent of the UV completion. This is obvious in cases where the relevant counterterm is forbidden by the symmetries of the fundamental theory and can also be checked by an explicit analysis of the relevant loop integrals.

More specifically, in 5d gauge theories softly broken at a scale $M_B$, differences of inverse low-energy gauge couplings receive finite corrections $\sim M_B/M$ (where $M$ is the UV scale set by the dimensionful 5d gauge coupling). If the relevant Higgs field can not appear linearly in the lagrangian for symmetry reasons, higher-dimensional operators can not compete with these loop corrections. In 6d, the corresponding correction is $\sim M_B^2/M^2 \ln \Lambda$, where the $\Lambda$ is the UV cutoff. This logarithmic divergence demands the existence of an appropriate higher-dimension operator (which has to be quadratic in the Higgs field). Nevertheless, at least at the leading logarithmic level, calculability is not lost. In seven and more dimensions, non-universal (with respect to the low-energy gauge groups) power-divergent corrections are expected to arise in non-supersymmetric models. Thus, quantitative statements depend on a more detailed knowledge of the UV structure of the theory. However, group theoretical constraints on the relevant higher-dimension operators may be sufficient to characterize the way in which these terms can contribute to low-energy gauge coupling differences.

A crucial feature of the supersymmetric theory is the strong restriction that the higher supersymmetry places on the bulk Higgs potential. A non-trivial bulk Higgs VEV can, however, be enforced by an appropriate brane superpotential. For adjoint Higgs breaking, power-like threshold corrections do not arise because, even in 5 and 6d, the massive field content is that of $N = 4$ super-Yang-Mills theory. However, if the breaking is due to a bulk VEV of the scalar adjoint from the vector multiplet, potentially large power corrections arise. Further important issues, which have not been discussed in the present paper (see, however, [21]) are the $N = 2$ SUSY restrictions on higher-loop terms and the possibility of a non-trivial UV fixed point (see [36] for the UV fixed point structure in non-SUSY gauge theories in higher dimensions). In particular, it is possible that SUSY constraints on higher-dimension operators and control of higher-loop corrections will allow a significant extension of the calculability range ($M_V \ll M$) assumed in this paper. Such a more detailed knowledge can, of course, also arise from a successful embedding of the GUT scenario in string or M theory (see [37] for a very recent investigation), where threshold effects are known to be calculable, and lead to large and highly-predictive power-law effects.

In the phenomenological part of the present paper, we have focussed on supersymmetric SU(5) models in 5d. In the simplest setting, where only the bulk gauge multiplet contributes power-like thresholds, the deviation from MSSM running is considerable and
the compactification scale $M_c$ can only be lowered moderately (say, down to a phenomenologically favoured neutrino see-saw scale). Since the symmetry breaking bulk VEV does not come from a Higgs hypermultiplet but from the scalar adjoint of the gauge multiplet, gauged bulk matter can contribute to the power-law effects. In this case, we find models that are numerically very close to the original proposal of [5]. This can be understood as follows: in the low-energy approach, one sums KK modes of light fields; in our UV based approach, one finds loop corrections proportional to the mass of heavy 5d fields. However, in an SU(5) symmetric bulk these two sets of fields combine to full SU(5) multiplets and the relevant group theoretical coefficients complement each other in such a way that the effect on low-energy gauge coupling differences is consistent in both approaches. Finally, we have found that it is possible to achieve non-SUSY unification using extra matter content above $M_c$ and to lower the unification scale significantly both in the SUSY and non-SUSY case.

To summarize, we believe that the presented calculations strongly encourage the further quantitative study of power-like threshold corrections in various phenomenologically relevant models. However, it is crucial, both at a qualitative and at a numerical level, to start from a bulk theory with manifest unified gauge symmetry and to specify the details of the soft higher-dimensional breaking mechanism as well as the field content at the high scale.

While this paper was being typed, Ref. [38] appeared, which considers warped 5d models and has some overlap with our results.

Note added: In [39][40], loop corrections to 5d SYM theories were calculated within the prepotential formalism of the corresponding 4d $N = 2$ theory. Since the prepotential is only corrected at one loop and higher-dimension terms are forbidden by 5d gauge invariance, a quantum-exact prepotential could be obtained. From this quantity, the power-like threshold corrections considered in this paper can be extracted. This represents an alternative to our component analysis on the basis of early non-SUSY GUT threshold calculations. More importantly, the quantum-exactness of the one-loop prepotential implies that our analysis is, in fact, complete and can therefore be taken to the strong coupling region. Thus, TeV-scale precision unification becomes a realistic possibility. We are indebted to Erich Poppitz for drawing our attention to Refs. [39][40] some time after this paper was published.

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