Correspondences between scalar field and fluid fluctuations in curved spacetime

Seema Satin
Indian Institute for Science Education and Research, Kolkata, India

A correspondence between scalar field fluctuations and generalized fluctuations in a hydrodynamic approximation of fields is obtained. The results presented here are of interest to field-fluid correspondences and form part of theoretical foundations in this area. The intention for such developments is to explore sub-hydro range mesoscopic physics for the relativistic fluids in curved spacetime. The fluid correspondences fall in the classical domain and can replace the quantum fields and fluctuations for scales around the hydrodynamic limits. The present article extends our earlier results with a more elaborate physical insight towards the quantum fluids and retention of partial quantum nature in a stochastic description in bulk of the fluids. This also accounts for non-thermal effects along with thermal and quantum fluctuations for the fields in the hydro limit. Hence the expressions presented here are very general in nature for various applications. The further scope of research that such developments give is discussed in the concluding section.

I. INTRODUCTION

A correspondence between the stress energy tensor for a scalar field and that of a fluid is well known and widely accepted [1, 2]. The field-fluid correspondence is of interest in various aspects and is an active area of research [3–5]. In this article, we show the correspondence between fluctuations of quantum fields and relativistic fluids in a hydrodynamic description. The results presented in this article are of significance for quantum fluids on a spacetime structure, where the quantum nature in the bulk persists in the macroscopic hydro-approximation. This is physically a different situation than that considered in [6] where the approximation is taken after de-coherence of quantum fields takes place. This difference is typically characterised in this article by giving a stochastic nature to the kinetic term (four-velocity), along with the other fluid variables, whereas the four-velocity is considered as a deterministic variable in [6, 7] for the de-coherence limit correspondences. The expressions that we obtain here are full results in the sense of the field-fluid correspondence, which is the focus in this article.

On the other hand, for the theory of semiclassical stochastic gravity [10], the point separated semiclassical noise kernel is of central importance to study structure formation in the early universe [11]. A similar feature in order to probe the sub-hydro mesoscopic scales can be of interest for a different (later) epoch in the evolution of the universe. Our developments given here can also be of significance for the exotic matter fields that the relativistic stars are composed of. This opens up a new direction to explore in the area of massive stars and relativistic fluids. Current efforts towards relativistic fluids for the massive star interiors are being pursued [12, 13] and gaining more attention with the achievements in gravitational waves [14–16] detection. The study of fluctuations of dense matter fluids is hence also of significance to asteroseismology.

Our framework is that of building up a theoretical base to address the new intermediate scales for study in the above two areas of application, along with interest in the field-fluid correspondence, which is the focus in this article.

For modelling the hydro-limit of fields one has to consider the classical relativistic fluid approximation in terms of the stress tensors. The spacetime metric $g_{ab}$ on which a relativistic fluid exists is considered to be deterministic in our work. The generalized stochasticity concept (2) takes into consideration "roughness in physical variables" at mesoscopic scales which are yet unexplored in relativistic fluids. We begin with quantum fields and their fluctuations in terms of the semiclassical noise kernel.
II. FLUCTUATIONS OF QUANTUM FIELDS IN CURVED SPACETIME

In this section we revise the two point semiclassical noise kernel, which defines the fluctuations of quantum scalar fields. Later in this article, we will show its relation and correspondence with the hydro approximation.

\[ \hat{T}^{(\text{field})}_{ab}(x) = \phi_a \phi_b - \frac{1}{2} g_{ab} (\phi^c \phi_c + m^2 \phi^2) + \xi (g_{ab} \Box - \nabla_a \nabla_b + G_{ab}) \phi^2 \]  

(1)

Fluctuations in quantum fields are defined by the tensor which is a two point noise kernel \( N_{abc'd'}(x,x') \) as obtained in the theoretical developments of semiclassical stochastic gravity [9].

\[ 8N_{abc'd'}(x,x') = \langle T_{ab}(x), \hat{T}_{c'd'}(x') \rangle > -2 < \hat{T}_{ab}(x) > < \hat{T}_{c'd'}(x') > \]

(2)

where \( < ... > \) denotes the expectation of the quantum field \( \phi \) on the spacetime background (in this article we denote the quantum expectation with \( < .... > \) and classical averages or expectation with \( E(...) \)), where

\[ N_{abc'd'}(x,x') = \tilde{N}_{abc'd'}(x,x') + g_{ab}(x)\tilde{N}_{c'd'}(x,x') + g_{c'd'}(x')\tilde{N}_{ab}(x,x') + g_{ab}(x)g_{c'd'}(x')\tilde{N}(x,x') \]

(3)

Further on we prescribe the noise kernel expressions for the field as \( N_{abc'd'}^{(\text{field})} \) and for fluid as \( N_{abc'd'}^{(\text{fluid})} \). For the scalar fields then, with non-minimal coupling, the noise kernel is given as

\[ 8\tilde{N}_{abc'd'}^{(\text{field})} = (1 - 2\xi)^2 (G_{c'b}G_{d'a} + G_{c'a}G_{db'}) + 4\xi^2 (G_{c'd'}G_{ab} + G_{ab}G_{c'd'}) - 2\xi (1 - 2\xi)(G_{b}G_{c'd'}G_{a} + G_{a}G_{c'd'}G_{b}) + G_{c'a}G_{abc'} + G_{c'b}G_{ab'd'} + 2\xi (1 - 2\xi)(G_{a}G_{ib}G_{c'd'} + G_{c'}G_{d'}G_{rb}) - 4\xi^2 (G_{a}G_{ib}G_{c'd'} + G_{c'd'}G_{rb})G + 2\xi^2 G_{c'd'}G_{ab}G^2 \]

(4)

\[ 8\tilde{N}_{ab}^{(\text{field})} = 2(1 - 2\xi)G_{p}G_{a}G_{p}G_{b} + \xi (G_{p}G_{a}G_{p'}G_{b'} + G_{b}G_{p}G_{p'}G_{a'}) - 4\xi (2\xi - \frac{1}{2})G_{p}G_{a}G_{p'}G_{b} + \xi(2\xi - \frac{1}{2})G_{p}G_{a}G_{p'}G_{b'} \]

(5)

\[ 8\tilde{N}_{c'd'}^{(\text{field})} = 2(\xi - \frac{1}{2})G_{p}G_{q}G_{p'}G_{q'} + 4\xi^2 (G_{p}G_{q}G_{p'}G_{q'} + G_{p}G_{q}G_{p'}G_{q'}) + 4\xi (2\xi - \frac{1}{2})(G_{p}G_{q}G_{p'q'} + G_{p}G_{q}G_{p'q'}) - (2\xi - \frac{1}{2})(m^2 + \xi R)G_{p}G_{q}G_{p'}G_{q'} - 2\xi (m^2 + \xi R)G_{p}G_{q}G_{p'}G_{q'} + (m^2 + \xi R)G_{p}G_{q}G_{p'}G_{q'}G \]

(6)

\[ 8\tilde{N}_{c'd'}^{(\text{fluid})} = 2(1 - 2\xi)^2 G_{c'd'}G_{a'd'}G_{a}G_{b} + 4\xi^2 (G_{c'd'}G_{a}G_{d'}G_{b} + G_{a}G_{d'}G_{c'd'}G_{b}) - 2\xi (1 - 2\xi)(G_{a}G_{d'}G_{c'd'}G_{b} + G_{a}G_{d'}G_{c'd'}G_{b}) + G_{a}G_{d'}G_{c'd'}G_{a}G_{b} + 2\xi (1 - 2\xi)(G_{a}G_{d'}G_{c'd'}G_{a}G_{b} + G_{a}G_{d'}G_{c'd'}G_{b}) - 4\xi^2 (G_{a}G_{d'}G_{c'd'}G_{a}G_{b} + G_{a}G_{d'}G_{c'd'}G_{b}) \]

(7)

where \( G \equiv G(x,x') \) are the Wightman functions defined by \( < \phi(x)\phi(x') > \). Our aim is to show a correspondence of these fluctuations with the sub-hydro limit fluctuations. These sub-hydro scales that we intend to address here, are expected to lie a little below the classical macroscopic hydrodynamic scales and much above the quantum microscopic scales. We will take the classical limit of the stress tensor for the fluid approximation and show the explicit form of the fluctuations that give access to a new regime between macro and micro scales in a straightforward way.
The quantum fields can be treated as a fluid in the hydrodynamic approximation, such that, the fluid variables associated with the fields are given by [1],

\[ u_a = \left[ \partial_c(\phi)\delta^c(\phi) \right]^{-1/2} \partial_a \phi \]  
(8)
\[ \epsilon = (1 - \xi \phi^2)^{-1} \left[ \frac{1}{2} \partial_c \phi \partial^c \phi + V(\phi) \right] + \xi \{ \square(\phi^2) - (\partial^c \phi \partial_c \phi)^{-1} \partial^a \phi \partial^b \phi \Delta_a \nabla_b(\phi^2) \} \]  
(9)
\[ q_a = \xi (1 - \xi \phi^2)^{-1} (\partial^c \phi \partial_c \phi)^{-3/2} \partial^c \phi \partial^d \phi \left[ \nabla_c \nabla_d(\phi^2) \partial_a \phi - \nabla_a \nabla_c(\phi^2) \partial_d \phi \right] \]  
(10)
\[ p = (1 - \xi \phi^2)^{-1} \left[ \frac{1}{2} \partial_c \phi \partial^c \phi - V(\phi) - \xi \{ \frac{2}{3} \square(\phi^2) + \frac{1}{3} (\partial_c \phi \partial^c \phi)^{-1} \nabla_a \nabla_b(\phi^2) \partial^a \phi \partial^b \phi \} \right] \]  
(11)
where the fluid stress tensor is of the form,

\[ T_{ab}^{(fluid)} = u_a u_b (\epsilon + p) + g_{ab} p + q_a u_b + u_a q_b + \pi_{ab} \]  
(13)
The variables \( \epsilon, p, u_a, q_a, \pi_{ab} \) denote the energy density, pressure, four-velocity, heat flux and anisotropic stresses respectively in the fluid. The noise kernel in equation (2) in the hydrodynamic approximation then takes an overall classical form in terms of the fluid stress tensor,

\[ 8\pi^{(fluid)}_{abc'd'}(x, x') = 2(E(T_{ab}^{(fluid)}(x)) T_{c'd'}^{(fluid)}(x')) - E(T_{ab}^{(fluid)}(x)) E(T_{c'd'}^{(fluid)}(x')) = 2\text{Cov}[T_{ab}^{(fluid)}(x), T_{c'd'}^{(fluid)}(x')] \]  
(14)
where Cov represents covariance. The two point covariance for the stress tensor \( \text{Cov}[T_{ab}^{(fluid)}(x), T_{c'd'}^{(fluid)}(x')] \) can be worked out easily, and terms arranged in order such that, coefficients of the metric \( g_{ab} \), as in equation (3) can be shown to correspond to the noise kernel for the fields as,

\[ \hat{N}_{abc'd'}^{(fluid)}(x, x') \]  
(relating the terms as :)

\[ (1 - 2\xi)^2(G_{c'd'} G_{d'a} + G_{c'a} G_{d'b}) + 4\xi^2(G_{c'd'} G_{ab} + G_{c'b} G_{ab'}) - 2\xi(1 - 2\xi)(G_{a'b} G_{c'd'} + G_{a'b} G_{c'd'}) \]
\[ + G_{d'a} G_{c'b} + G_{c'a} G_{d'b}) + 2\xi(1 - 2\xi)(G_{a'b} G_{c'd'} + G_{a'b} G_{c'd'}) - 4\xi^2(G_{a'b} G_{c'd'} + G_{a'd'} G_{b'}) \]
\[ + 2\xi^2 R_{c'd'} R_{d'a} \]  
(15)
The notation for terms like \( g_{ab} \) and \( G_{c'd'} \) are not to be mixed up in the upper expressions for the two points \( x \) and \( x' \), as \( a, b \) remain associated with \( x \), while \( c', d' \) remain associated with \( x' \) for all the terms above and the following expressions as well.

Coefficient of \( g_{ab} \)

\[ N_{abc'd'}^{(fluid)} = \frac{1}{2} N_{cd}^{(fluid)} \]  
(16)
Coefficient of $g_{c'd'}$,

$$\tilde{N}^{(field)}_{ab} \rightarrow \tilde{N}^{(fluid)}_{ab}(\text{relating the terms as:})$$

$$2(1 - 2\xi)[\{2\xi - \frac{1}{2}G_{ip'b'G_{a}G_{ip'}} + \xi(G_{ip'}G_{ap'} + G_{ia}G_{ip'b'})\} - 4\xi[\{2\xi - \frac{1}{2}G_{ip'}G_{ap'}\}
+ \xi(G_{ip'}G_{ab} + G_{ip'b'}) - (m^2 + \xi R')(1 - 2\xi)G_{a}G_{b} - 2G\xi G_{ab} + 2\xi[\{2\xi - \frac{1}{2}G_{ip'}G_{ap'}\}
+ 2G G_{ip'}R_{ab} - (m^2 + \xi R')\xi G_{ab}G^2 \rightarrow \frac{1}{2}\{E(u_{a})E(u_{b})\{\text{Cov}[-3(\phi), p(x')] + \text{Cov}[p(x), p(x')]\}\}$$ (17)

Coefficient of $g_{ab}g_{c'd'}$,

$$\tilde{N}^{(field)}_{ab} \rightarrow \frac{1}{2}\tilde{N}^{(fluid)}_{ab}(\text{relating the terms as:})$$

$$2(2\xi - \frac{1}{2})\{2\xi^2 (G_{ip'}G_{ap'} + G_{ip'G_{aj}G_{aj}} + 2\xi G_{ip'}G_{ap'}\}
+ 4\xi[\{2\xi - \frac{1}{2}(G_{ip'}G_{ap'} + G_{ip'}G_{aq}) - (m^2 + \xi R')G_{ip'}G_{ap'} - 2\xi[\{m^2 + \xi R')G_{ip'}G_{ap'} - (m^2 + \xi R')G_{ip'}G_{ap'}\}
+ \frac{1}{2}(m^2 + \xi R')(m^2 + \xi R')G^2 \rightarrow \frac{1}{2}\text{Cov}[p(x)p(x')]$$ (18)

In the above, we can see that the two point covariances of the heat flux, anisotropic stresses, as well as for the four-velocity appear only in the part $\tilde{N}_{abc'd'}$ for the noise kernel of the fluid. For the rest of parts, the two point covariances of the pressure and energy density suffice to characterize them. Given the complex nature of the above expressions, it is operationally difficult to obtain reverse equations with one to one correspondence between each of the the two point fluid variables and that of the fields.

A. Perfect fluid case

For the perfect fluid case, where the heat flux $q_{a}$ and anisotropic stresses $\eta_{ab}$ vanish, which can be related to the vanishing of the the non-minimal coupling factor $\xi$, for the scalar field stress tensor, one has

$$u_{a} = [\partial_{a}(-\phi)\partial_{a}\phi]^{-1/2}\partial_{a}\phi$$
$$\epsilon = \frac{1}{2}[\partial_{a}\phi\partial_{a}\phi + V(\phi)]$$
$$p = \frac{1}{2}[\partial_{a}\phi\partial_{a}\phi - V(\phi)]$$ (19)

$$\tilde{N}^{(fluid)}_{ab} \rightarrow \frac{1}{2}\tilde{N}^{(fluid)}_{ab}(\text{relating the corresponding terms as :})$$

$$\{G_{;c'b;d'a} + G_{;c'a;G_{d'b}}\} \rightarrow \frac{1}{2}\{\{E(\epsilon(x))E(\epsilon(x')) + E(\epsilon(x))E(p(x')) + E(p(x))E(\epsilon(x'))}\}$$ (20)

$$\{4E(u_{a})E(u_{c'})Cov[u_{b}], u_{d'}\} + 2\text{Cov}[u_{a}, u_{c'}]Cov[u_{b}, u_{d'}]\} + \{\text{Cov}[\epsilon(x), \epsilon(x')] + \text{Cov}[\epsilon(x), p(x')] + \text{Cov}[p(x), \epsilon(x')]\}
+ \text{Cov}[p(x), p(x')]\{E(u_{a})E(u_{c'})E(u_{d'}) + 4\text{E}[u_{a}]E(u_{c'})\text{Cov}[u_{b}], u_{d'}\} + 2\text{Cov}[u_{a}, u_{c'}]\text{Cov}[u_{b}, u_{d'}]\} +
4\{E[p(x)]', + E(\epsilon(x'))E(u_{a})E(u_{c'}) + (E[p(x')] + E(\epsilon(x'))E(u_{a})E(u_{c'})\}Cov[u_{b}], u_{d'}\}$$

$$\tilde{N}^{(fluid)}_{ab} \rightarrow \frac{1}{2}\tilde{N}^{(fluid)}_{ab}(\text{relating the corresponding terms as :})$$

$$\{-G_{;ip';a} + m^2G_{a}G_{,b}\} \rightarrow \frac{1}{2}\{E(u_{a})E(u_{b})\{\text{Cov}[\epsilon(x), p(x')] + \text{Cov}[p(x), p(x')]\}\}$$ (21)

$$\tilde{N}^{(fluid)}_{c'd'} \rightarrow \frac{1}{2}\tilde{N}^{(fluid)}_{c'd'}(\text{relating the corresponding terms as :})$$

$$\{\frac{1}{2}G_{;ip'q'G_{a}G_{ap'}} + \frac{1}{2}m^2[G_{ip'}G_{ap'} + G_{ip'}G_{ap'} + m^2G^2] \} \rightarrow \frac{1}{2}\{\text{Cov}[p(x), p(x')]\}$$ (22)

$$\tilde{N}^{(fluid)}_{c'd'} \rightarrow \frac{1}{2}\tilde{N}^{(fluid)}_{c'd'}(\text{relating the corresponding terms as :})$$

$$2G_{;c'd'} \rightarrow \frac{1}{2}\{E(u_{a})E(u_{d'})\{\text{Cov}[p(x), p(x')] + \text{Cov}[p(x), \epsilon(x')]\}\}$$ (23)
One can compare this, with the expressions for perfect fluid in [6, 7] and observe the difference. As we have considered the four-velocity also as a random variable in the present article, the expressions with the expectation $E(u_a)$ as well as covariances of four-velocity vectors appear in the results. This makes it difficult to obtain reverse one to one correspondences as in [3]. The results obtained here, are relevant for the quantum fluids with the stochastic effects showing up in terms of the kinetic as well as bulk variables of the fluid. Also one can assign thermal fluctuations in the quantum fluids with this prescription. We emphasize that our scales of interest are with the sub-hydro mesoscopic range physics, but these expressions may also be used for large scale hydrodynamic description, if one considers fluctuations w.r.t time in a conventional stochastic description rather than the generalized stochastic description. Another interesting possible direction of investigations with these results can be about quantum potential of scalar fields [17], which is responsible for pressure in the fluid approximation for dark matter. Hence it would be interesting to explore and connect the exact mechanism in this regard, of how the fluctuations of the scalar fields characterize the pressure fluctuations in the quantum fluids.

**IV. CONCLUDING REMARKS**

In this article we have obtained a relation between the two point fluctuations of a quantum scalar field and that in the hydrodynamic fluid approximation. These results indicate that, fluctuations of quantum fields can induce or can be approximated with the sub-hydro mesoscopic effects in the fluid description of matter. The "generalized covariances" (or variances) of pressure, energy density, heat flux etc describe them in terms of fluid variables. These results can be applicable for the perturbative theory in general relativity as the noise or source of perturbations. The significance also lies in realising their importance for compact astrophysical objects which are coupled to (say) thermal fields and are of interest to collapsing clouds, towards critical phases and end states of collapse where fluctuations can play a critical role. Thus our results can be used to analyse properties of dense compact matter with the relativistic fluid models in strong gravity regions, and to study their dynamics at intermediate length scales which become interesting with such a stochastic analysis. The extended (point separated) structure and properties given in terms of two point statistical covariances of matter fields is the key feature in this article. We have shown the correspondences between the quantum field fluctuations and fluid fluctuations as a first principles approach, theoretically. One can progress on these lines of investigation to explore more features which connect microscopic effects in a coarse grained description to see the effects that filter out to the mesoscopic range. These will characterize the physics at the intermediate scales in the quantum fluids.

The significance of our correspondences lies in realizing that, these expressions are the starting point for a range of directions as applications. Some of the further questions that arise with this and are open for reflections at the very fundamental level in research are,

- How do the quantum field fluctuations leak or tunnel out into intermediate scales in quantum fluids in terms of fluctuations of the fluid variables. Here we have just shown the correspondence between the two cases from theoretical considerations, but not addressed how the physical correspondence is achieved. This can be compared with decoherence, where the classical results are obtained from quantum states, but the phenomena of decoherence is in itself a different investigation altogether.

- As one may expect, that the stochastic fluctuations averaged out at a given point in spacetime would be vanishing, we have defined in terms of the noise kernel two point correlations of the fluctuations, which are meaningful. These expressions address not just the equilibrium configuration of the system but also the non-equilibrium configurations can be analysed. Thus one can explore various statistical properties with such a prescription, which are otherwise inaccessible with the purely quantum and classical studies. One of the main questions that one can ask here is, how does the quantum fluid behave at intermediate mesoscopic length scales, can these two point or higher correlations of the fluctuations inform us of the global or extended nature of the fluid. What are the length scales in different types of relativistic fluids which can be correlated in such a fashion for mesoscopic studies. Do these correlations extend to macroscopic range in the fluids, and if yes, then which interesting new results can be obtained from them for the hydrodynamic macroscopic scales.

- The quantum fields have a correspondence with fluids is known, we have shown here how the fluctuations have a correspondence each other. This is certainly a quantum to classical correspondence description, as the fluid fluctuations are given in terms of classical physical variables. The question one can ask is, to what extent does the quantum nature of the field fluctuations tunnel through or is retained in the fluid variables fluctuations. Quantum fluids are certainly bulk systems, hence they are interesting from the point of view of exploring how quantum nature of matter behaves in the bulk and which unique properties show up. Scalar field models for dark matter [17, 18] are another interesting application for such correspondence and exploring the fluctuations on our lines of thought can have interesting applications for theoretical consideration in this area.

- Thermal and quantum mechanical fluctuations
are widely understood as the two basic types of stochastic fluctuations due to thermal effects and quantum mechanical origin respectively, in physical systems. However the origin of fluctuations in physical quantities and natural systems can have other causes, like in purely mechanical systems coupled to each other, the dynamics or mechanical effects can give rise to stochastic vibrations in the system. For example in massive gravitating bodies, random oscillations due to any external or internal mechanical effects can be realized in certain physical parameters. Whether the correspondence obtained here in terms of fluid variables can be related to only the thermal fluctuations or we can extend this to non-thermal and purely mechanical effects due to the dynamical motion of fluid particles of the relativistic matter under the influence of strong gravity, is an interesting query.

- Dissipative relativistic fluids are an active area of research and modelling dissipation in the hydrodynamic approximation is non-trivial. Yet models of relativistic fluids with dissipation leave provisions for including fluctuations from the first principles. Though fluctuation-dissipation relations have been obtained for relativistic fluids, there is still a large scope to construct basic models where the fluid variable fluctuations can be related with the dissipative effects. One can ask the question, if probed through mesoscopic scales in relativistic fluids, can one define and relate fluctuations in a few of the fluid variables, like that of of pressure, energy density and four velocity present at mesoscopic scales to dissipative parameters in the hydrodynamic description. This calls for including the theory of generalized fluctuations in relativistic hydrodynamics from the first principles, and not just as an ad-hoc assumption.

Some of the applications oriented directions in research with these developing ideas are as following,

- The correspondence of the two point fluctuations given here can be used in the semiclassical Einstein-Langevin equation for cosmological spacetime, where the inflaton field can be replaced by the fluid approximation. The fluid fluctuations would then represent the thermal states as equivalence of quantum fluctuations in a statistical description. Further on these lines, one will have to cast the semiclassical Einstein-Langevin equation with matter sector represented in terms of fluid equivalence of the quantum fields. Thus the next step would be to obtain the dissipation kernel in the semiclassical Einstein-Langevin equation in terms of the fluid variables.

- For dense matter fluids that compose the relativistic stars, one can explore the perturbations that can be induced due to these internal generalized stochastic fluctuations. It is the cumulative effect of the fluctuations that one would be interested in, to see the perturbations of the trajectories of the dense matter in compact stars. Such ideas have just begun with the proposal and formalism of a classical Einstein-Langevin equation, and make way for further scope in asteroseismology. The noise kernel presented here, acts as a source for perturbations in relativistic stars, and thus its characterization is specific to the given configuration that one is interested in exploring.

An interesting direction can be seen to emerge from this work for studying microscopic structure in matter fields and its connections with kinetic theory in curved spacetime. Such an endeavour can begin by trying to consider these generalized fluctuations (including roughness of physical variables) of matter fields as more fundamental than trying to define particles in a curved spacetime. We know that a global definition for particles and that for vacuum in a curved spacetime background is not unique. One may then attempt to formulate a kinetic theory using the field fluctuations and its generalization as the basic entity. This approach may find its way through the four-velocity which represents the kinetic term and its generalized stochastic fluctuations as given in this article. With the framework of two point or higher correlations of fluctuations of matter fields, a tool to study non-local and extended structure of matter in the curved spacetime arises in an interesting way. Thus a kinetic theory of matter in curved spacetime can be based on these fluctuations and their interaction, rather than on the ambiguous localized particles.

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