Optimization of Wireless Optical Communication System Based on Augmented Lagrange Algorithm

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Abstract. The optimal model for wireless optical communication system with Gaussian pointing loss factor is studied, in which the value of bit error probability (BEP) is prespecified and the optimal system parameters is to be found. For the superiority of augmented Lagrange method, the model considered is solved by using a classical quadratic augmented Lagrange algorithm. The detailed numerical results are reported. Accordingly, the optimal system parameters such as transmitter power, transmitter wavelength, transmitter telescope gain and receiver telescope gain can be established, which provide a scheme for efficient operation of the wireless optical communication system.

1. Introduction

Wireless optical communication (WOC) technology, or free space optical communication (FSO) is a kind of broadband access way, which is the combination of wireless and optical communications product and transmits signal using the beam signal through the atmospheric space, rather than through fiber. For the feasibility for applications in: (i) last-mile access, (ii) service acceleration, (iii) metro network extensions, (iv) enterprise connectivity, (v) fiber backup, and (vi) backhaul, many researchers have been interested in WOC, see e.g. [1], [3], and [6]-[12]. Since the WOC must operate under the condition of line-of-sight, one of the critical affects is the transmitter and receivers sway caused by building sway due to strong wind, weak earthquake, and other reasons, which will make a significant impact on the bit error probability (BEP) of communication. Thus it has been an important issue that how to operate and design of WOC systems under the given BEP value. The model for wireless optical communication system with Gaussian pointing loss factor was studied and the optimal system parameters were reported by solving a system of nonlinear equations in [8]. However, the system of nonlinear equations involves the first-order derivative of BEP with respect to some variable in the model and the second-order derivative of BEP with respect to the same variable is concerned during the numerical implementation, which brings a heavy burden to calculation procedures.

The augmented Lagrange method is one kind of methods for solving optimization problems with equality constraints in mathematical programming, in which the augmented Lagrange function has the same smoothness with the primal objective function and constrained function [3]. Compared with the penalty method, the augmented Lagrange method can obtain the optimal solution without the penalty parameter being infinitely large, but with the penalty parameter being larger than a threshold. And the optimal Lagrange multipliers have special significance in the corresponding practical problem. In this paper, we conduct the analysis on the optimal model for wireless optical communication system with
Gaussian pointing loss factor under the prespecified value of BEP. Based on the superiority of augmented Lagrange method, the optimal solutions to the model are obtained by using a classical quadratic augmented Lagrange algorithm. Then the optimal transmitter power, transmitter wavelength, transmitter telescope gain and receiver telescope gain are determined, which means that the WOC systems can operate well still by appropriately adjusting these system parameters to alleviate the unfavorable factor of building sway.

2. Background

We list the variables and parameters used in this paper in the following.

\( d \): distance between the transmitter and the receiver (meter);

\( D_R / D_T \): receiver/transmitter aperture diameter (meter);

\( G_R / G_T \): receiver/transmitter antenna gain;

\( L_A \): atmospheric loss factor;

\( R_h \): detector’s responsivity factor (ampere/watt/meter);

\( \gamma / \theta \): receiver/transmitter radial pointing error angle (radian);

\( L_R (G_R, \gamma) / L_T (G_T, \theta) \): receiver/transmitter pointing loss factor;

\( P_R / P_T \): received/transmitted optical power (watt);

\( \eta_R / \eta_T \): optics efficiency of the receiver/transmitter;

\( \lambda \): wavelength (meter);

\( \sigma \): azimuth/elevation pointing error deviation (radian);

\( \sigma^2 \): covariance of the additive white Gaussian noise.

From the basic form of received optical power as follows

\[
P_R = P_T G_T G_R \left( \frac{\lambda}{4 \pi d} \right)^2 \eta_T \eta_R L_A L_T (G_T, \theta) L_R (G_R, \gamma),
\]

for the assumptions that \( L_T (G_T, \theta) = \exp \left( -G_T \theta^2 \right) \) in the Gaussian beam case and \( L_R (G_R, \gamma) = 1 \) if the receiver telescope with a sufficiently wide field of view (FOV) is used, the BEP with the Gaussian pointing loss factor was reported in [8]:

\[
BEP = \frac{1}{2} \int_0^\infty \text{erfc} \left[ \alpha_0 \lambda P_T G_T \exp \left( -2 G_T \sigma^2 x \right) \right] \exp \left( -x \right) dx,
\]

where \( x = \frac{\theta^2}{2 \sigma^2}, \alpha_0 = \left( \frac{R_h}{2 \sqrt{2} \sigma} \right) \eta_T \eta_R \left( \frac{\lambda}{4 \pi d} \right)^2 G_R L_A \). Furthermore, for convenience of computation, the normalized transmitter power and the normalized square-root transmitter gain are defined as \( v = \left( \alpha_0 \pi D_T P_T \right) / \sigma, z = \left( \pi D_T \sigma / \lambda \right) \), then the BEP is simplified as

\[
BEP = f \left( v, z \right) = \frac{1}{2} \int_0^\infty \text{erfc} \left[ vz \exp \left( -2xz^2 \right) \right] \exp \left( -x \right) dx.
\]  

Under the prespecified value of BEP, i.e., \( \text{BEP}=B \), the mathematical model to find the optimal system parameters is established in [8]:

\[
\begin{align*}
\min & \quad v \\
\text{s.t.} & \quad g(v, z) = 0,
\end{align*}
\]

where \( g(v, z) = f(v, z) - B \), and \( \alpha_0, D_T, \theta, \sigma \) are taken as constants. The optimal solution to the model (2) was reported in [8] by solving the following system of nonlinear equations
\[ \frac{\partial f(v, z)}{\partial z} = 0, \]
\[ g(v, z) = 0. \]  \hspace{1cm} (3)

And then the optimal system parameters were established.

However, the concrete algorithm for (3) was not given in [8]. Furthermore, during (3) is solved by some algorithm, not only first-order derivative of \( f(v, z) \), in which the integrands concerns \( \text{erfc}(z) \), with respect to the variable \( z \) is involved, i.e., \( \frac{\partial f(v, z)}{\partial z} \), but also \( \frac{\partial^2 f(v, z)}{\partial^2 z} \) need to be calculated, which make the optimal solution difficulty to obtain numerically. For the superiority of augmented Lagrange method, the model (2) will be solved by using a classical quadratic augmented Lagrange algorithm in this paper. The detailed augmented Lagrange algorithm and the numerical results for the model (2) are given in the next section.

3. The Numerical Experiments

We will list the augmented Lagrange algorithm for model (2) and then report the detailed optimal solutions to the model (2) in this section.

Let \( x = (x_1, x_2)^T = (v, z)^T \in R^2 \), then the model (2) is written as the general optimization model with equality constraints

\[ \min x_1 \]
\[ s.t. \ g(x) = 0. \]  \hspace{1cm} (4)

The quadratic augmented Lagrange function corresponding to (4) is constructed as follows

\[ F(x, \lambda, M) = x_1 + u g(x) + \frac{M}{2} g^2(x), \]  \hspace{1cm} (5)

where \( M > 0 \) is the penalty parameter, \( u \) is the Lagrange multiplier of constraint \( g(x) = 0 \).

3.1. The Augmented Lagrange Algorithm

Based on the quadratic augmented Lagrange function (5), the concrete algorithm is listed in the following.

**Step 1** Choose \( M^0 = 1, \ u^{(0)} = 1, \ v = 10^{-6} \) and let \( k = 0 \);

**Step 2** Solve \( x^{(k)} = \arg \min_x F(x, u^{(k)}, M^{(k)}) \);

**Step 3** If \( |g(x^{(k)})| \leq \epsilon \), then stop and \( x^{(k)} \) is the approximate optimal solution to the model (2); else go to Step 4;

**Step 4** Modify the Lagrange multiplier \( u^{(k)} \) and penalty parameter \( M^{(k)} \)

\[ u^{(k+1)} = u^{(k)} + M^{(k)} g(x^{(k)}) \]

\[ M^{(k+1)} = 2^k \]

Let \( k = k + 1 \), go to Step 2.

3.2. Numerical Results

Using the above augmented Lagrange algorithm, we perform the numerical experiments for model (2) in which \( B = 10^{-3}, B = 10^{-4}, B = 10^{-5}, B = 10^{-6}, B = 10^{-7}, B = 10^{-8}, B = 10^{-9} \) and \( B = 10^{-10} \), respectively. The numerical experiments are implemented in MATLAB 7.1 runtime environment. The minimum point \( x^{(k)} \) in Step 2 is obtained by the FMINUNC function from the Optimization Toolbox in MATLAB 7.1. And during this experiment, the BEP function \( f(v, z) \) is not directly implemented in the form of (1) but in the following equivalent expression derived in [8]:

\[ \text{erfc}(z) \]
\[ f(v, z) = \left( \frac{1}{4z^2} \right) \times \left( \frac{1}{v} \right) \left( \frac{1}{2z^2} \times \text{erfc}(z) \right) \times \frac{1}{\sqrt{\pi}} \times \frac{1}{\sqrt{s}} \times \int_{0}^{\infty} \frac{1}{\sqrt{2\pi s}} ds \]  

in order to improve the efficiency of numerical computation.

The following table shows the detailed numerical results, \( B \) column denotes the prespecified values of BEP in model (2), \( v^* \) column and \( z^* \) column are the optimal solution to the model (2) with the prespecified value \( B \) respectively, \( u^* \) column is the optimal Lagrange multiplier and \( M^* \) column is the terminal value of \( M \) in the algorithm.

| \( B \) | \( v^* \) | \( z^* \) | \( u^* \) | \( M^* \) |
|-------|--------|--------|--------|--------|
| \( 10^{-10} \) | 10.787100818018 | 0.275174063651 | 1655.42937007216 | 16777216 |
| \( 10^{-9} \) | 14.5945277894 | 0.23732142485 | 16494.660964832 | 1073741824 |
| \( 10^{-8} \) | 18.386612180739 | 0.211654307102 | 159961.191264993 | 34359738368 |
| \( 10^{-7} \) | 20.822819609920 | 0.198940748580 | 539481.109173006 | 68719476736 |
| \( 10^{-6} \) | 25.962724172528 | 0.176462406865 | 1539431.34615309 | 4.5036e+015 |
| \( 10^{-5} \) | 29.7915785238 | 0.16643574064 | 5374812.6071930 | 1.1259e+015 |
| \( 10^{-4} \) | 33.58793484773 | 0.156756326365 | 15236881.1771831 | 1.8014e+016 |
| \( 10^{-3} \) | 37.36576720047 | 0.148572096776 | 25794584.1061635 | 4.5036e+015 |

**Remark 3.1** Table 1 shows that the values of \( v^* \) range from approximately 37.36576720047 at \( B = 10^{-10} \) to about 10.787100818018 at \( B = 10^{-3} \).

**Remark 3.2** Table 1 shows that the values of \( z^* \) range from approximately 0.148572096776 at \( B = 10^{-10} \) to about 0.275174063651 at \( B = 10^{-3} \).

**Remark 3.3** For the transmitter with \( D_T = 10 \text{ cm} \) and \( \sigma = 2 \mu \text{ rad} \), from \( \frac{\pi D_T \sigma}{z^*} = \), when \( B = 10^{-10} \), the optimal wavelength \( \lambda_{\text{opt}} = 4.2290480325 \mu \text{m} \) and when \( B = 10^{-1} \), the optimal wavelength \( \lambda_{\text{opt}} = 2.2833493912 \mu \text{m} \).

**Remark 3.4** For the transmitter with \( D_T = 10 \text{ cm} \), \( \sigma = 2 \mu \text{ rad} \) and \( \alpha_0 = 1 \), from \( P_T = \frac{\sqrt{\pi} \sigma}{\alpha_0 \pi D_T} \), when \( B = 10^{-10} \), the optimal transmitter power \( P_{\text{opt}} = 237.8778623292 \mu \text{w} \) and when \( B = 10^{-1} \), the optimal wavelength \( P_{\text{opt}} = 68.6728167402 \mu \text{w} \).

**Remark 3.5** For the transmitter with \( \sigma = 2 \mu \text{ rad} \), from \( G_T = \frac{\pi r^2}{\sigma} \), when \( B = 10^{-10} \), the optimal transmitter power \( G_{\text{opt}} = 0.0055184170 \) and when \( B = 10^{-1} \), the optimal wavelength \( G_{\text{opt}} = 0.01893019133 \).

**Remark 3.6** For the receiver with \( D_R = 20 \text{ cm} \), and the transmitter \( D_T = 10 \text{ cm} \), \( \sigma = 2 \mu \text{ rad} \), from \( G_R = \left( \frac{D_R z^*}{D_T \sigma} \right)^2 \), when \( B = 10^{-10} \), the optimal transmitter power \( G_{\text{opt}} = 0.022073668 \) and when \( B = 10^{-1} \), the optimal wavelength \( G_{\text{opt}} = 0.0757207653 \).
Remark 3.7 The change relationships between the optimal transmitter power and the optimal wavelength, and the corresponding prespecified BEP value respectively, are revealed in the following Fig. 1, in which the BEP values are put on the abscissa axis, \( \nu^* \) (represented by data1) and \( 1/z^* \) (represented by data1) are put on the ordinate axis.

![Figure 1 Relationships between \( \nu^* \), \( 1/z^* \) and BEP value respectively](image_url)

The above figure shows that the BEP value increases as \( \nu^* \) or \( 1/z^* \) decreases at the optimal points, which implies that under the given \( D_r, \sigma \) and \( \alpha_0 \), if the BEP value is larger, both the optimal transmitter power and the optimal wavelength are less.

4. Conclusions

In this paper, we present the optimal system parameters for the wireless optical communication system with Gaussian pointing loss factor by using the classical quadratic augmented Lagrange algorithm to solve the corresponding mathematical model with the BEP value being prespecified. The detailed numerical results such as transmitter power, transmitter wavelength, transmitter telescope gain and receiver telescope gain are reported. The presented solutions demonstrate that the BEP value decreases as the transmitter power or the wavelength increases at the optimal points.

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