THE TOLMAN SURFACE BRIGHTNESS TEST FOR THE REALITY OF THE EXPANSION. IV. A MEASUREMENT OF THE TOLMAN SIGNAL AND THE LUMINOSITY EVOLUTION OF EARLY-TYPE GALAXIES

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ABSTRACT

We review a sample of the early literature in which the reality of the expansion is discussed. Hubble’s reluctance, even as late as 1953, to accept the expansion as real is explained as due to his use of equations for distances and absolute magnitudes of redshifted galaxies that do not conform to the modern Mattig equations of the standard model. The Tolman surface brightness test, once the only known test for the reality of the expansion, is contrasted with three other modern tests. These are (1) the time dilation in Type Ia supernovae light curves, (2) the temperature of the relic radiation as a function of redshift, and (3) the surface brightness normalization of the Planckian shape of the relic radiation. We search for the Tolman surface brightness depression with redshift using the Hubble Space Telescope (HST) data from Paper III for 34 early-type galaxies from the three clusters Cl 1324 + 3011 (z = 0.76), Cl 1604 + 4304 (z = 0.90), and Cl 1604 + 4321 (z = 0.92). Depressions of the surface brightness relative to the zero-redshift fiducial lines in the mean surface brightness–logarithm of the linear radius diagrams of Paper I are found for all three clusters. Expressed as the exponent, n, in 2.5 log (1 + z)n mag, the value of n averaged over Petrosian radii of n = 1.7 and n = 2.0 for all three clusters is n = 2.59 ± 0.17 in the R band and 3.37 ± 0.13 in the I band for a q0 = 1/2 model. The sensitivity of the result to the assumed value of q0 is shown to be less than 23% between q = 0 and +1. The conclusion is that the exponent on (1 + z) varies from 2.28 to 2.81 (± 0.17) in the R band and 3.06 to 3.55 (± 0.13) in the I band, depending on the value of q0. For a true Tolman signal with n = 4, the luminosity evolution in the look-back time, expressed as the exponent in 2.5 log (1 + z)n mag, must then be between 1.72 to 1.91 (± 0.17) in the R band and 0.94 to 0.45 (± 0.13) in the I band. We show that this is precisely the range expected from the evolutionary models of Bruzual and Charlot and other measurements of the luminosity evolution of early-type galaxies. We conclude that the Tolman surface brightness test is consistent with the reality of the expansion to within the combined errors of the observed <SB> depression and the theoretical correction for luminosity evolution. We have also used the high-redshift HST data to test the “tired light” speculation for a nonexpansion model for the redshift. The HST data rule out the tired light model at a significance level of better than 10 σ.

Key words: cosmology: observations — galaxies: clusters: general

1. INTRODUCTION

1.1. Early Commentaries on the Reality of the Expansion

With the announcement by Hubble (1929a) of a correlation between his estimates of distances to nearby galaxies and their redshifts, observational cosmology came of age, beginning its Long Journey into Night. The redshifts used by Hubble had been measured by Slipher, published by Eddington (1923), and added to by Humason (1929) in his crucial extension to higher values before Hubble’s announcement.

However, the announcement of an expanding universe was such an extraordinary claim that proof of its reality by some independent means seemed essential, even though an expanding universe had been predicted by Friedmann (1922; see Tropp, Frenkel, & Chernin 1993) as one of the solutions of the fundamental Einstein equation of general relativity. Expanding solutions were also detailed later by Lemaître (1927, 1931 [the 1927 paper in English translation]) and Robertson (1928), each of which, acknowledging Friedmann, made advances beyond Friedmann by their adumbrations concerning the available observations of galaxies and their relative distances.

It is not clear whether Hubble knew of the Friedmann (1922) prediction or of the theoretical, cum observational, papers of Lemaître and Robertson in 1927 and 1928, although Robertson told one of us (A. S.) that he had discussed with Hubble the existence of an expanding solution to the Einstein equations before 1929 (e.g., Sandage 1995, footnote 16 to Chapter 5). None of these three principal theoreticians are mentioned in Hubble’s 1929 announcement.

Nevertheless, the decade of the 1920s was not entirely free of observational attempts to test a related cosmological prediction. A year after the publication of Einstein’s field equations, de Sitter (1917a, 1917b) had discovered one curious solution to them. Although his solution was that of a static metric (there are only three such solutions; Tolman 1929), it
remarkably exhibited a redshift. The metric coefficient of the four-space time coordinate was a function of distance from the observer. Clocks that are farther from the observer’s origin in the three-space manifold would appear to tick more slowly than clocks at that origin. This effect would give an apparent redshift that would vary with distance. The formal feature of a redshift in the de Sitter metric was called the de Sitter effect. A curious additional feature was that any test particle put into the de Sitter manifold would exhibit a radial motion (Eddington 1923, § 70; de Sitter 1933; Tolman 1934, § 144–145) even though the spatial metric was static.

The predicted existence of the de Sitter redshift effect with its static metric became well known in the decade of the 1920s. Many attempts were made to find the effect by using astronomical data (distances and velocities) for objects such as stars and globular clusters, thought then to compose the wider universe before Hubble (1925 [NGC 6822], 1926 [M33], 1929b [M31]) proved the existence of external galaxies. Among the most accessible papers concerning the search for the de Sitter effect are those by Silberstein (1924), Stromberg (1925), Wirtz (1925), Lundmark (1925), and undoubtedly many others.

With regard to the de Sitter effect, it is a continuing curiosity as to what Hubble meant by the final sentence of his announcement in 1929, in which he wrote:

The outstanding feature is the possibility that the velocity-distance relation may represent the de Sitter effect …. In this connection it may be emphasized that the linear relation found in the present discussion is a first approximation representing a restricted range in distance (emphasis added).

In other words, a larger range in distance may not show a linear relation, he surmised, since he apparently knew that the first-order de Sitter effect is quadratic in distance rather than linear (e.g., Sandage 1995, eq. [5.10]). Cautiously, Hubble left open the possibility that the data that he had discussed might define a redshift-distance relation that would actually vary as the square of the distance, which, in first approximation, would appear to be linear for short distances near the origin (i.e., the first term of a Taylor series). Hubble also undoubtedly knew of the earlier searches for the de Sitter effect by Lundmark (1925) and Silberstein (1924), in which they attempted parabolic fits to their adopted data.

However, Hubble & Humason (1931) soon proved that the redshift-distance relation was, in fact, linear over the much larger range of redshifts than was available in 1929. For this and other reasons, de Sitter (1933) wrote: “We know now, because of the observed expansion, that the actual universe must correspond to one of the nonstatic models …. The static models are, so to say, only of academic interest.”

1.2. Later Attempts to Counter the Reality of the Expansion

Nevertheless, the concept of an expanding universe seemed so bizarre to many commentators that attempts began already in 1929 to find alternate ways to produce large redshifts other than from a true expansion. These attempts continue to this day.

The first alternate suggestion was made by Zwicky (1929), in which he proposed that photons lose energy on their way to us from a distant source. As a consequence, they would show a redshift in their energy distribution, both in the continuum radiation and in the Fraunhofer lines used to measure the redshifts. To first order, the redshift effect would be linear with distance because the first term in a Taylor expansion of $1 + z = e^{H_0 D}$ is linear in $D$. Here, $H_0$ is the Hubble constant and $D$ is the distance. With this suggestion, Zwicky (1929) introduced the notion of “tired light.”

Even as late as mid–twentieth century, Zwicky (1957) maintained that the hypothesis was viable. However, neither Zwicky nor any other subsequent supporter of the proposition (e.g., La Violette 1986; Pecker & Vigier 1987) gave a convincing physical theory for the tiredness. As critics still point out, any scattering process with energy transfer from the photon beam to the scattering medium, as required for a redshift, must broaden (deflect) the beam. This effect would cause images of distant galaxies to be fuzzier than their local counterparts, which they are not

1.3. Hubble’s Reluctance

Perhaps the most interesting attack on the reality of the expansion was the reluctance of Hubble himself to believe that the redshifts represent a true expansion rather than being “caused by an unknown law of nature.” Much has been written about Hubble’s reluctance, most of which is wrong. Some commentators even suggest philosophical or religious reasons related to a presumed abhorrence of a “creation” event that is implied in some interpretations of a real Friedmann expansion.

However, the fact is that Hubble’s reasons were those of a reductionist bench scientist. He relied (mistakenly, it turns out) solely on the interpretation of his observational data and their accuracy, coupled with a mistaken theory of how redshifts should vary with “distance.” His equation for “distance” has no justification within the modern Mattig (1958) equations. This story and why Hubble’s conclusion would not have been reached using current data analyzed with the modern theoretical equations are set out in detail elsewhere (Sandage 1998).

In outline, Hubble’s argument was as follows. By the mid-1930s, Hubble (1934; 1936a, 1936b) had completed his program of galaxy counts, the goal of which was to measure the curvature of space. He also completed the extension of the Hubble diagram of redshift versus apparent magnitude to the limit of the Mount Wilson 100 inch (2.5 m) reflector (Hubble 1936a, 1936b; Humason 1936). The data for each of these programs had to be corrected for the effects of redshifts on the apparent magnitudes. If the expansion was real, Hubble assumed that the observed bolometric magnitudes (or equivalently, the observed magnitudes corrected for the selective part of the $K$-term) must be made brighter by two factors of 2.5 log $(1 + z)$; however, if the redshift was due to “an unknown law of nature” rather than to the expansion, only one such factor was to be applied to the observed magnitudes.

Hubble believed that the Hubble diagram (logarithm of the redshift vs. apparent magnitude) must be strictly linear; in addition, the radius of curvature of space implied by his “corrected” magnitudes for his galaxy counts must not be
“too small.” Consequently, he became convinced that only one factor of 2.5 log \((1 + z)\) should be applied. If so, the expansion would not be real. Hubble (1936b) wrote:

If the redshifts are not primarily due to velocity shifts ... [then] the velocity-distance relation is linear; the distribution of nebulae is uniform; there is no evidence of expansion, no trace of curvature, no restriction of the time scale .... The unexpected and truly remarkable features are introduced by the additional assumption that redshifts measure recession. The velocity-distance relation deviates from linearity by the exact amount of the postulated recession. The distribution departs from uniformity by the exact amount of the recession. The departures are compensated by curvature which is the exact equivalent of the recession. Unless the coincidences are evidence of an underlying necessary relation between the various factors, they detract materially from the plausibility of the interpretation. The small scale of the expanding model, both in space and time, is a novelty and as such will require rather decisive evidence for its acceptance.

That “rather decisive evidence” is now available from at least three modern experiments that are independent of the Tolman galaxy surface brightness test.

1.4. Other Recent Proofs That the Expansion Is Real

1.4.1. Time Dilation Test

The Tolman (1930) surface brightness \((1 + z)^4\) effect was the only known test for the reality of the expansion until Wilson (1939) suggested that the shape of the light curves of Type Ia supernovae provides a clock. This supposition was based on the uniform shape of the light curve discovered by Baade (1938) and recalled as history by Minkowski (1964). Wilson reasoned that such clocks at different redshifts would measure the special relativity time dilation if the light curve shapes of SNe Ia at high redshifts could be observed. The stretching of the light curves, increasing with redshift, has now been observed. The data give a spectacular confirmation of the time dilation effect (Goldhaber et al. 1997, 2001).

1.4.2. Temperature of the Relic Blackbody Radiation as a Function of Redshift

Blackbody radiation in an expanding cavity remains Planckian in shape but with a decreasing temperature that scales as \(T(z) = T(0)(1 + z)\) with redshift (Tolman 1934, eq. [177.7]; Bahcall & Wolf 1968). The observations of the Boltzmann temperature of interstellar molecules in the spectra of high-redshift galaxies has now apparently been measured in a difficult experiment with the Keck 10 m telescopes (Songaila et al. 1994). An important confirmation of these first results was made in observations by Ge, Bechtold, & Black (1997) and Srianand, Petitjean, & Ledoux (2000). Only upper limits had been achieved before (see Meyer et al. 1986) which, nevertheless, were highly important in pioneering this test.

1.4.3. Measurement of the Chemical Potential: The Alpher-Herman Relic Blackbody Radiation

Although the shape of an initial blackbody spectrum remains Planckian in an expanding cavity, the vertical normalization (i.e., the photon number) remains Planckian only if that normalization is decreased with redshift by \((1 + z)^4\). This fact is derived trivially from the Planck equation as expressed in terms of energy flux per wavelength per unit wavelength interval rather than frequency per unit frequency interval [of course, it is seen that both representations are equivalent by minding the relation between wavelength interval and frequency interval, where \(dv = c \lambda d\lambda/\lambda^2\)].

Hence, because the Planck equation defines a surface brightness, a test of the Tolman surface brightness effect is equivalent to measuring the deviation of the photon number per unit surface area in the sky by comparing the observations with the normalization given by the Planck equation itself. The deviation of the data from the Planck equation is called the “chemical potential.” Among other things, this deviation with wavelength could be due to Compton scattering in the early universe.

No deviation has been found in the observations to within one part in \(10^4\). The perfect Planckian shape of the relic radiation was measured with COBE to within this limit of \(9 \times 10^{-5}\) (Mather et al. 1990; Fixsen et al. 1996). The conclusion to be drawn from this spectacular result is that this seemingly perfect normalization of the spectral energy distribution is a definitive proof of the Tolman surface brightness factor and, therefore, a definitive proof of the reality of the expansion. We shall adopt this assumption later in § 4 where we combine our surface brightness signal with the theoretical \((1 + z)^4\) Tolman signal and interpret the difference to be due to luminosity evolution in the high-redshift look-back time.

1.5. Plan of the Present Paper

In the first three papers of this series (Sandage & Lubin 2001, hereafter Paper I; Lubin & Sandage 2001a, 2001b, hereafter Papers II and III, respectively), we provide the background and observational data required to carry out the Tolman test. Most importantly, we measure the fiducial (zero redshift) relations between the mean surface brightness, absolute magnitude, and linear radius for local early-type galaxies in Paper I. In Paper III, we present the observational data on our high-redshift comparison sample of 34 early-type galaxies in the three clusters used in this program, Cl 1324 + 3011, at \(z = 0.76\), Cl 1604 + 4304, at \(z = 0.90\), and Cl 1604 + 4321, at \(z = 0.92\) (Oke, Postman, & Lubin 1998, hereafter OPL; Postman, Lubin, & Oke 1998, 2001, hereafter PLO98 and PLO01, respectively; Lubin et al. 1998, 2001). To compare accurately the local and high-redshift data, the parameters of each galaxy are measured at discrete values of the Petrosian (1976) \(\eta\) metric radius, which is defined as the difference in magnitude between the mean surface brightness averaged over the area interior to a particular radius and the surface brightness at that radius (see § 2 of Paper I).

In the present paper, we use the results presented in Papers I–III to complete the Tolman test. In § 2 we review the Mattig (1958) cosmological equations with which to calculate the absolute magnitudes and linear radii of galaxies from their apparent magnitudes and angular radii. Explicit equations for the special case of \(q_0 = 1/2\) are given. We use these equations to obtain the total magnitude, \(M\), linear radius, \(R\), and mean surface brightness, \(<SB>\), as functions of five Petrosian \(\eta\) radii of 1.0, 1.3, 1.5, 1.7, and 2.0 mag for the 34 high-redshift early-type galaxies. Tables 2–4 list these values for \(q_0 = 1/2\) and \(H_0 = 50\) km s\(^{-1}\) Mpc\(^{-1}\).
These values are derived from the observational data listed in Paper III. The Tolman test is made in § 3 based on the data in § 2 and the comparison with the surface brightness data at zero redshift from Paper I.

In § 4 we set out the theoretical evolutionary corrections, first, those using the simplest model of passive evolution by main-sequence burn-down in the Hertzsprung-Russell (H-R) diagram as a function of time and, second, those using the sophisticated star formation models of Bruzual & Charlot (1993), following earlier papers by Guiderdoni & Rocca-Volmerange (1987, 1988) and Rocca-Volmerange & Guiderdoni (1988).

In § 5 we show the sensitivity of the observed Tolman signal to the assumed value of $q_0 = 1/2$ used in the data of Tables 2–4. Proof that the tired light assumption does not fit the data by a large factor is given in § 6. In § 7, we discuss the systematic uncertainties in the Tolman test made here and describe plans to strengthen the present test.

2. CALCULATION OF ABSOLUTE MAGNITUDES AND LINEAR RADII FOR THE HIGH-REDSHIFT GALAXIES

Comparison of the high-redshift data in Paper III with the data for local galaxies in Paper I requires knowledge of absolute magnitudes (corrected for K-term) and linear radii. We have two options to calculate these values. (1) If we were to assume that the standard model that leads to the Mattig (1958, 1959) equations (which assume that the expansion is real) did not exist, the natural assumption is that the distance, $D_0$, at the time that light is received is related to redshift by $D_0 = cz/H_0$. This was Hubble’s assumption throughout his work, including the last summary paper in his Darwin lecture (Hubble 1953). (2) Alternately, we can adopt the details of the standard model by choosing a value for the deceleration parameter, $q_0$ (Hoyle & Sandage 1956). We then use the Mattig (1958) equations to calculate the distance moduli, $m - M$, and the linear radii, $R$, from the observed angular radii.$^4$

Because these equations already contain the Tolman ($1 + z)^2$ factor, the test for the Tolman effect becomes one of consistency between the surface brightness observations and the predictions of the standard model. The argument of why this does not lead to a hermeneutical circularity was given in § 5 of Paper I. We adopt option (2) in calculating the distance moduli and the linear radii of the program clusters. We treat only the $q_0 = 1/2$ case of the standard model in this section. The cases for $q_0 = 0$ and $+1$ are treated in § 5 using the correction recipes in Table 8.

2.1. Equations for $q_0 = 1/2$

For the $q_0 = 1/2$ case with $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, the Mattig equations reduce to

$$m - M = 5 \log \left[2(1 + z - \sqrt{1 + z})\right] + 43.89 ,$$

(1)

$$D_1 = \frac{2c}{H_0(1 + z)} \left(1 - \frac{1}{\sqrt{1 + z}}\right) ,$$

(2)

where $D_1$ is the distance (in parsecs) when light left the galaxy at the observed redshift, $z$. The linear radius, $R$ (in parsecs), of a galaxy with angular radius, $\theta$, in radians, is

$$R = \frac{2c(\sqrt{1 + z} - 1)}{H_0(1 + z)^{3/2}} \theta ,$$

(3)

by using the distance $D_1$ from equation (2). By changing the angular radius in radians to arcseconds, equation (3) can be conveniently written as

$$\log R = \log \theta + \log A ,$$

(4)

where $R$ is in parsecs, $\theta$ is in arcseconds, and

$$A = 5.818 \times 10^4 \frac{\sqrt{1 + z} - 1}{(1 + z)^{3/2}} ,$$

(5)

for $q_0 = 1/2$ and $H_0 = 50$. Equations (1)–(3) are derived in Sandage (1961a, 1961b, 1988, 1995), where the standard model is summarized from the point of view of practical cosmology.

Table 1 shows the $m - M$ and $A$ values for the three program clusters for the $q_0 = 1/2$ case using equations (1), (4), and (5). The $K$-terms are those derived in Paper III (see Table 4).

2.2. $M$, $R$, and $<SB>$ Data for the Three Program Clusters for the $q_0 = 1/2$ Case

The results for absolute magnitude, $M$, linear radius, $R$, and mean surface brightness, $<SB>$, are listed in Tables 2–4, in which the $<SB>$ values are those in Paper III, corrected for the effects of redshift by the $K$-terms from Table 1. The format of Tables 2–4 is the same as for Tables 5–7 in

| TABLE 1 |
| --- |

**ADDED K-CORRECTIONS, DISTANCE MODULI, AND A FACTORS FOR THE LINEAR RADII USING $q_0 = 1/2$, $H_0 = 50$**

| Cluster | $z$ | $K(R) \pm \Delta K(R)$ (mag) | $K(I) \pm \Delta K(I)$ (mag) | $m - M$ | $\log A$ |
| --- | --- | --- | --- | --- | --- |
| 1324 + 3011 | 0.7565 | ... | 0.71 ± 0.03 | 43.57 | 3.910 |
| 1604 + 4304 | 0.8967 | ... | 0.80 ± 0.07 | 43.97 | 3.924 |
| 1604 + 4321 | 0.9243 | 1.89 ± 0.06 | ... | 44.04 | 3.926 |

$^4$ To avoid confusion with our use of $R$ in this paper to mean the linear radius of a galaxy rather than its use as the time-dependent scale factor in the metric, we have used $D$ here as the distance parameter by replacing $Rr$ with $D$, whereas in the standard notation $R$ is the scale factor that varies with time and $r$ is the dimensionless comoving radial coordinate in the metric. Hence, in our notation here $D_0$ is the distance at the time light is received from a galaxy at redshift $z$. This is given by Mattig (1958; see also Sandage 1988, eq. [30]) as

$$D_0 = \frac{c}{H_0(q_0)(1 + z)} \left[2q_0 + (q_0 - 1)(-1 + \sqrt{4q_0z + 1})\right] .$$
TABLE 2
The $M$, $\langle SB \rangle$, log $R$ data for Cl 1604 + 4321 [R Band; $q_0 = 1/2, H_0 = 50$]

| Galaxy | $\eta = 1.0$ | $\eta = 1.3$ | $\eta = 1.5$ | $\eta = 1.7$ | $\eta = 2.0$ |
|--------|---------------|---------------|---------------|---------------|---------------|
|        | $\langle SB \rangle^a$ | $\log R$ | $\langle SB \rangle^a$ | $\log R$ | $\langle SB \rangle^a$ | $\log R$ | $\langle SB \rangle^a$ | $\log R$ | $\langle SB \rangle^a$ | $\log R$ |
| 23$^b$ | $-23.13 \pm 0.10$ | 18.48 | $3.224$ | $-23.46 \pm 0.09$ | 19.16 | $3.409$ | $-23.57 \pm 0.09$ | 19.49 | $3.493$ | $-23.65 \pm 0.08$ | 19.81 | $3.571$ | $-23.78 \pm 0.08$ | 20.48 | $3.729$ |
| 29$^b$ | $-23.11 \pm 0.10$ | 20.35 | $3.568$ | $-23.28 \pm 0.09$ | 20.71 | $3.674$ | $-23.37 \pm 0.09$ | 20.97 | $3.742$ | $-23.44 \pm 0.08$ | 21.23 | $3.805$ | $-23.56 \pm 0.08$ | 21.84 | $3.955$ |
| 36$^b$ | $-22.46 \pm 0.10$ | 20.24 | $3.420$ | $-22.82 \pm 0.09$ | 20.97 | $3.632$ | $-22.94 \pm 0.08$ | 21.34 | $3.727$ | $-23.05 \pm 0.08$ | 21.74 | $3.809$ | $-23.15 \pm 0.08$ | 22.28 | $3.966$ |
| 38$^b$ | $-22.63 \pm 0.10$ | 20.21 | $3.451$ | $-22.87 \pm 0.09$ | 20.71 | $3.583$ | $-22.97 \pm 0.08$ | 21.02 | $3.666$ | $-23.06 \pm 0.08$ | 21.39 | $3.767$ | $-23.20 \pm 0.08$ | 22.02 | $3.918$ |
| 44$^b$ | $-22.47 \pm 0.09$ | 21.06 | $3.586$ | $-22.84 \pm 0.09$ | 21.76 | $3.790$ | $-22.95 \pm 0.09$ | 22.10 | $3.881$ | $-23.04 \pm 0.08$ | 22.40 | $3.959$ | $-23.12 \pm 0.08$ | 22.82 | $4.058$ |
| 46$^b$ | $-22.30 \pm 0.10$ | 21.20 | $3.581$ | $-22.63 \pm 0.09$ | 21.89 | $3.778$ | $-22.77 \pm 0.08$ | 22.27 | $3.879$ | $-22.86 \pm 0.08$ | 22.62 | $3.966$ | $-$ | $-$ | $-$ |
| 49$^b$ | $-21.95 \pm 0.10$ | 20.07 | $3.303$ | $-22.56 \pm 0.09$ | 21.33 | $3.647$ | $-22.71 \pm 0.09$ | 21.77 | $3.770$ | $-22.80 \pm 0.08$ | 22.10 | $3.853$ | $-22.88 \pm 0.08$ | 22.46 | $3.930$ |
| 57$^b$ | $-22.15 \pm 0.10$ | 21.65 | $3.636$ | $-22.58 \pm 0.09$ | 22.50 | $3.887$ | $-22.79 \pm 0.09$ | 23.05 | $4.029$ | $-22.88 \pm 0.08$ | 23.39 | $4.132$ | $-22.92 \pm 0.08$ | 23.58 | $4.171$ |
| 58$^b$ | $-21.88 \pm 0.10$ | 20.38 | $3.345$ | $-22.24 \pm 0.09$ | 21.11 | $3.547$ | $-22.45 \pm 0.09$ | 21.72 | $3.707$ | $-22.67 \pm 0.08$ | 22.54 | $3.913$ | $-22.82 \pm 0.08$ | 23.25 | $4.083$ |
| 65$^b$ | $-22.10 \pm 0.10$ | 19.83 | $3.278$ | $-22.40 \pm 0.09$ | 20.44 | $3.441$ | $-22.50 \pm 0.08$ | 20.74 | $3.515$ | $-22.57 \pm 0.08$ | 21.01 | $3.590$ | $-22.64 \pm 0.08$ | 21.43 | $3.667$ |
| 78$^b$ | $-21.53 \pm 0.10$ | 20.70 | $3.336$ | $-21.86 \pm 0.09$ | 21.40 | $3.529$ | $-22.00 \pm 0.09$ | 21.80 | $3.643$ | $-22.14 \pm 0.08$ | 22.30 | $3.759$ | $-22.17 \pm 0.08$ | 22.48 | $3.801$ |
| 112$^b$ | $-21.29 \pm 0.10$ | 20.27 | $3.219$ | $-21.60 \pm 0.09$ | 20.94 | $3.383$ | $-21.70 \pm 0.08$ | 21.27 | $3.477$ | $-21.79 \pm 0.08$ | 21.62 | $3.568$ | $-21.89 \pm 0.08$ | 22.12 | $3.667$ |
| 115$^b$ | $-21.16 \pm 0.09$ | 21.09 | $3.325$ | $-21.55 \pm 0.10$ | 21.87 | $3.556$ | $-21.64 \pm 0.08$ | 22.14 | $3.618$ | $-21.71 \pm 0.08$ | 22.44 | $3.718$ | $-21.84 \pm 0.08$ | 22.95 | $3.860$ |
| 144$^b$ | $-21.01 \pm 0.10$ | 20.74 | $3.246$ | $-21.37 \pm 0.09$ | 21.49 | $3.461$ | $-21.48 \pm 0.09$ | 21.82 | $3.536$ | $-21.58 \pm 0.08$ | 22.21 | $3.632$ | $-21.69 \pm 0.08$ | 22.78 | $3.767$ |

Note.—In log $R$, $R$ is in parsecs.

* Values have been corrected with the appropriate $K(R) = 1.89$ correction (see Table 1).

$^b$ Member galaxy selected based on its photometric redshift (see Brunner & Lubin 2000).
| Galaxy | $M^a$ | $\langle SB \rangle^a$ | $\log R$ | $M^a$ | $\langle SB \rangle^a$ | $\log R$ | $M^a$ | $\langle SB \rangle^a$ | $\log R$ | $M^a$ | $\langle SB \rangle^a$ | $\log R$ |
|--------|-------|-----------------|--------|-------|-----------------|--------|-------|-----------------|--------|-------|-----------------|--------|
| 9 ...... | $-23.23 \pm 0.07$ | 19.11 | 3.425 | $-24.03 \pm 0.05$ | 20.75 | 3.905 | $-24.46 \pm 0.04$ | 21.87 | 4.294 | $...$ | $...$ | $...$ |
| 11 ...... | $-23.21 \pm 0.07$ | 19.89 | 3.574 | $-23.87 \pm 0.05$ | 21.21 | 3.964 | $-24.11 \pm 0.04$ | 21.82 | 4.136 | $-24.18 \pm 0.04$ | 22.04 | 4.222 | $...$ | $...$ | $...$ |
| 12 ...... | $-23.31 \pm 0.06$ | 20.27 | 3.667 | $-23.54 \pm 0.05$ | 20.73 | 3.802 | $-23.63 \pm 0.05$ | 20.97 | 3.866 | $-23.68 \pm 0.04$ | 21.15 | 3.915 | $-23.73 \pm 0.04$ | 21.39 | 3.968 |
| 18 ...... | $-22.69 \pm 0.07$ | 19.65 | 3.424 | $-23.05 \pm 0.05$ | 20.39 | 3.638 | $-23.20 \pm 0.05$ | 20.81 | 3.749 | $-23.32 \pm 0.05$ | 21.24 | 3.859 | $-23.46 \pm 0.04$ | 21.95 | 4.029 |
| 21 ...... | $-22.45 \pm 0.08$ | 19.36 | 3.327 | $-22.91 \pm 0.06$ | 20.31 | 3.603 | $-23.08 \pm 0.05$ | 20.81 | 3.734 | $-23.20 \pm 0.05$ | 21.26 | 3.843 | $-23.29 \pm 0.04$ | 21.69 | 3.952 |
| 26 ...... | $-22.24 \pm 0.07$ | 20.20 | 3.446 | $-22.88 \pm 0.05$ | 21.44 | 3.816 | $-23.02 \pm 0.05$ | 21.82 | 3.912 | $-23.12 \pm 0.05$ | 22.16 | 3.995 | $-23.17 \pm 0.04$ | 22.41 | 4.070 |
| 29 ...... | $-22.47 \pm 0.06$ | 20.36 | 3.519 | $-22.83 \pm 0.05$ | 21.08 | 3.742 | $-23.00 \pm 0.04$ | 21.55 | 3.851 | $-23.25 \pm 0.04$ | 22.45 | 4.087 | $-23.32 \pm 0.04$ | 22.81 | 4.174 |
| 30 ...... | $-22.42 \pm 0.07$ | 20.52 | 3.546 | $-22.61 \pm 0.05$ | 20.93 | 3.657 | $-22.72 \pm 0.05$ | 21.22 | 3.740 | $-22.82 \pm 0.04$ | 21.62 | 3.841 | $-22.89 \pm 0.04$ | 21.95 | 3.918 |
| 40 ...... | $-21.78 \pm 0.07$ | 19.42 | 3.229 | $-22.19 \pm 0.05$ | 20.28 | 3.447 | $-22.37 \pm 0.05$ | 20.81 | 3.588 | $-22.49 \pm 0.04$ | 21.28 | 3.702 | $-22.58 \pm 0.04$ | 21.72 | 3.811 |
| 55 ...... | $-21.43 \pm 0.07$ | 20.50 | 3.348 | $-21.69 \pm 0.05$ | 20.54 | 3.502 | $-21.78 \pm 0.05$ | 21.31 | 3.571 | $-21.83 \pm 0.04$ | 21.52 | 3.619 | $-21.91 \pm 0.03$ | 21.94 | 3.710 |
| 59 ...... | $-21.60 \pm 0.08$ | 19.73 | 3.245 | $-21.89 \pm 0.05$ | 20.30 | 3.395 | $-21.99 \pm 0.04$ | 20.57 | 3.465 | $-22.04 \pm 0.04$ | 20.81 | 3.526 | $-22.11 \pm 0.04$ | 21.16 | 3.605 |
| 69 ...... | $-21.48 \pm 0.04$ | 19.24 | 3.109 | $-21.71 \pm 0.06$ | 19.76 | 3.256 | $-21.80 \pm 0.04$ | 20.05 | 3.327 | $-21.86 \pm 0.04$ | 20.29 | 3.382 | $-21.92 \pm 0.03$ | 20.61 | 3.455 |
| 74 ...... | $-21.27 \pm 0.07$ | 21.28 | 3.470 | $-21.57 \pm 0.05$ | 21.91 | 3.648 | $-21.72 \pm 0.05$ | 22.34 | 3.764 | $-21.92 \pm 0.04$ | 23.03 | 3.938 | $-21.99 \pm 0.04$ | 23.35 | 4.016 |

Note.—$\log R$, $R$ is in parsecs.

Values have been corrected with the appropriate $K(I) = 0.71$ correction (see Table 1).

Member galaxy selected based on its photometric redshift (see Brunner & Lubin 2000).

The $M$, $\langle SB \rangle$, $\log R$ data for Cl 1324 + 3011 [I band; $q_0 = 1/2$, $H_0 = 50$].
| Galaxy | η = 1.0 | η = 1.3 | η = 1.5 | η = 1.7 | η = 2.0 |
|--------|---------|---------|---------|---------|---------|
|        | $M^*$   | ⟨SB⟩*  | log $R$ | $M^*$   | ⟨SB⟩*  | log $R$ | $M^*$   | ⟨SB⟩*  | log $R$ | $M^*$   | ⟨SB⟩*  | log $R$ | $M^*$   | ⟨SB⟩*  | log $R$ |
| 9 ...... | -23.24 ± 0.10 | 20.59 | 3.651 | -23.72 ± 0.10 | 21.55 | 3.938 | -23.89 ± 0.09 | 22.03 | 4.062 | -23.98 ± 0.09 | 22.36 | 4.149 | -24.04 ± 0.09 | 22.59 | 4.204 |
| 10$^b$ .... | -23.12 ± 0.10 | 20.90 | 3.687 | -23.59 ± 0.10 | 21.82 | 3.963 | -23.71 ± 0.09 | 22.16 | 4.058 | -23.80 ± 0.09 | 22.46 | 4.131 | ... | ... | ... |
| 13 ...... | -23.17 ± 0.10 | 20.00 | 3.525 | -23.41 ± 0.10 | 20.50 | 3.669 | -23.51 ± 0.09 | 20.80 | 3.746 | -23.59 ± 0.09 | 21.10 | 3.821 | -23.68 ± 0.09 | 21.53 | 3.924 |
| 34$^b$ ...... | -22.06 ± 0.10 | 20.98 | 3.495 | -22.35 ± 0.09 | 21.60 | 3.674 | -22.50 ± 0.09 | 22.03 | 3.785 | -22.58 ± 0.09 | 22.32 | 3.866 | -22.64 ± 0.09 | 22.60 | 3.930 |
| 46$^b$ ...... | -22.01 ± 0.11 | 20.40 | 3.378 | -22.25 ± 0.10 | 20.90 | 3.513 | -22.34 ± 0.09 | 21.17 | 3.589 | -22.42 ± 0.09 | 21.46 | 3.661 | -22.50 ± 0.09 | 21.90 | 3.761 |
| 50 ...... | -21.83 ± 0.11 | 21.11 | 3.479 | -22.18 ± 0.10 | 21.83 | 3.689 | -22.39 ± 0.09 | 22.43 | 3.855 | -22.47 ± 0.09 | 22.73 | 3.919 | -22.60 ± 0.09 | 23.38 | 4.077 |
| 58$^b$ ...... | -21.72 ± 0.10 | 20.65 | 3.371 | -21.94 ± 0.09 | 21.11 | 3.499 | -22.03 ± 0.09 | 21.37 | 3.569 | -22.10 ± 0.09 | 21.63 | 3.632 | -22.18 ± 0.09 | 22.03 | 3.725 |

**NOTE.**—In log $R$, $R$ is in parsecs.

$^a$ Values have been corrected with the appropriate $K(I) = 0.80$ correction (see Table 1).

$^b$ Member galaxy selected based on its photometric redshift (see Brunner & Lubin 2000).
Paper III. Data for the five η values of 1.0, 1.3, 1.5, 1.7, and 2.0 are listed. For each η, the K-corrected absolute magnitude is calculated by using the m − M value from Table 1 and the apparent magnitudes from Paper III. The K-corrected mean surface brightnesses from Paper III are given next. The logarithms of the linear radii (in parsecs) are obtained by adding the logarithm of the angular radii from Tables 5−7 of Paper III to the A values from Table 1, calculated using equations (4) and (5).

3. TOLMAN TEST USING THE HST DATA FOR EARLY-TYPE GALAXIES FROM THE THREE PROGRAM CLUSTERS FOR q₀ = 1/2

The data that are necessary to make the Tolman test are given in Tables 2−4. The search for a Tolman signal, as modified by luminosity evolution, is made by comparing these high-redshift data with the calibrations at zero redshift in Paper I. The comparisons for the R photometric band of cluster Cl 1604 + 4321 (z = 0.9243) can be made directly because the Postman & Lauer (1995) calibrations in Paper I are already in the standard Cape/Cousins R photometric band.

Comparisons in the I band for clusters Cl 1324 + 3011 (z = 0.7565) and Cl 1604 + 4304 (z = 0.8967) are made by converting the calibrations in Paper I to the I photometric band by applying the adopted color of (R − I) = 0.62. This mean R − I color for early-type galaxies at zero redshift was derived by (1) using ⟨B − R⟩ = 1.52 from the photometry of Postman & Lauer (1995), as corrected for K-term and Galactic absorption in Paper I, and (2) using the two-color (B − R, R − I) diagram from the data of Poulain & Nieto (1994) to give ⟨R − I⟩ = 0.62 at ⟨B − R⟩ = 1.52. A color of (R − I) = 0.62 corresponds to a mean stellar spectral type of about K5 (see Fig. 10 of Sandage 1997).

3.1. Tolman Signal in the log R−⟨SB⟩ Correlation Diagram

We approach the problem first by considering the data in the parameter space that contains the smallest evolutionary signal and, therefore, is expected to have the strongest Tolman signal (if it exists). Beyond a doubt, luminosity evolution occurs in a stellar population with no new star formation after the initial star burst. The main-sequence termination point in the H-R diagram burns down to fainter luminosities as the population ages, making the total luminosity of the aggregate fainter with time. Calculation of the burn-down rate, coupled with the luminosity function that determines relative numbers of stars that partake in the evolution, gives a first estimate of the change of total luminosity with time. This is how we make a first estimate of the effect in § 4.

Luminosity evolution affects both the observed surface brightness and the absolute magnitude. However, we expect that it does not affect the radius as a function of time. The consequence is that the correlation of surface brightness versus absolute magnitude has a double dose of evolution because both coordinates are affected, whereas the correlation of surface brightness and linear radius contains only a single dose, on the assumption that the radius does not change. On this basis, the most powerful of the three correlation diagrams from Paper I is the ⟨SB⟩−log R diagram. In this section we examine this correlation, searching for a Tolman signal in each of the three high-redshift clusters.

3.1.1. Data in R for Cl 1604 + 4321 (z = 0.9243)

Figure 1 shows the ⟨SB⟩−log R correlation diagram for Cl 1604 + 4321 in the R photometric band as corrected for K-dimming using Table 1. Large dots represent those galaxies in Table 2 that have directly measured redshifts. Small dots represent assumed cluster members on the basis of their photometric redshifts (for details, see Paper III).

The lines in each plot are the correlations for zero redshift from Tables 2 and 3 of Paper I, based on the photometry by Postman & Lauer (1995) and extended to radii smaller than log R = 4.3 by using the data of Sandage & Perelmuter (1991). Note that Table 3 in Paper I lists the magnitude difference between the best-fit relations of the Sandage & Perelmuter (1991) and the Postman & Lauer (1995) data; therefore, the best-fit lines given in Table 2 of Paper I need to be made fainter by the absolute value of the corrections listed in Table 3 of Paper I. The resulting local lines carry an error in the ⟨SB⟩ vertical position that varies between 0.04 and 0.50 mag, depending on log R (see Table 3 of Paper I).

The effect for which we are searching is clearly seen in Figure 1. It is the depressed surface brightness at a given radius compared with the surface brightness of early-type galaxies at zero redshift. We interpret this depression to be the Tolman signal as diluted by luminosity evolution.

The amount of the depression for each galaxy was calculated relative to the zero-redshift line at a given radius. This is done by using the equations for the zero-redshift lines from Tables 2 and 3 of Paper I and subtracting the observed ⟨SB⟩ values for each galaxy from these fiducial lines read at the same log R value. The subsequent errors on this difference in mean surface brightness include both the measurement error in the ⟨SB⟩ of the galaxy and the uncertainties in the local line (see Tables 2−3 of Paper I). The result for Cl 1604 + 4321 is listed in Table 5 as the mean depressions for the five Petrosian η radii of 1.0, 1.3, 1.5, 1.7, and 2.0, using all the galaxies in Table 2.

Column (2) of Table 5 shows the depression in magni-

| η   | Δ⟨SB⟩ (mag) | n  | ΔM_{avc} (mag) | 4 − n | Number of Galaxies |
|-----|-------------|----|---------------|-------|-------------------|
| 1.0 | 2.44 ± 0.16 | 3.43 ± 0.23 | 0.40 ± 0.16 | 0.57 ± 0.23 | 14                 |
| 1.3 | 2.24 ± 0.16 | 3.15 ± 0.23 | 0.60 ± 0.16 | 0.85 ± 0.23 | 14                 |
| 1.5 | 2.21 ± 0.16 | 3.11 ± 0.23 | 0.63 ± 0.16 | 0.89 ± 0.23 | 14                 |
| 1.7 | 1.96 ± 0.17 | 2.76 ± 0.24 | 0.88 ± 0.17 | 1.24 ± 0.24 | 14                 |
| 2.0 | 1.76 ± 0.18 | 2.48 ± 0.25 | 1.08 ± 0.18 | 1.52 ± 0.25 | 13                 |
FIG. 1.—Surface brightness measured in the $R$ band vs. the logarithm of the linear radius for the galaxies in Cl 1604 + 4321 ($z = 0.9243$) from the data in Table 2, showing the galaxies with directly measured redshifts (large dots) and cluster members inferred for their photometric redshifts (small dots; see Paper III). The mean correlation for zero-redshift early-type galaxies is shown by the lines taken from Tables 2 and 3 of Paper I.

Table 5 shows the values of the mean depression in $\langle SB \rangle$ and, therefore, the exponents $n$ are a strong function of Petrosian radii $\eta$, being measured larger at smaller $\eta$ values. This trend is a direct result of the finite angular

3.1.3. Mean Tolman Signal From the Three Clusters

Tables 5–7 show the values of the mean depression in $\langle SB \rangle$ and, therefore, the exponents $n$ are a strong function of Petrosian radii $\eta$, being measured larger at smaller $\eta$ values. This trend is a direct result of the finite angular...
resolution of the Wide Field Planetary Camera 2 point-spread function described in Paper II. Specifically, for smaller values of $\eta$ we are not accurately measuring the true $\eta(r)$ curve (see §2 of Paper II). As a result, we underestimate the mean surface brightness, $\langle SB(r) \rangle$, and therefore overestimate the signal, i.e., the depression from the zero-redshift relation. We have shown in Paper II that, even for the smallest galaxies, with half-light radii of 0.25, the true $\eta(r)$

### TABLE 6

**SUMMARY OF THE TOLMAN SIGNAL AND THE INFERRED LUMINOSITY EVOLUTION FOR Cl 1324 + 3011 IN THE I BAND USING $q_0 = 1/2$**

| $\eta$ (mag) |  $\Delta\langle SB \rangle$ (mag) |  $n$ (mag) |  $\Delta M_{\text{evol}}$ (mag) |  $4 - n$ (mag) |  Number of Galaxies |
|--------------|-------------------------------|------------|-----------------------------|----------------|-------------------|
| 1.0          | $2.54 \pm 0.15$               | $4.15 \pm 0.25$ | $-0.09 \pm 0.15$          | $-0.15 \pm 0.25$ | 13                |
| 1.3          | $2.33 \pm 0.14$               | $3.81 \pm 0.23$ | $0.12 \pm 0.14$           | $0.19 \pm 0.23$  | 13                |
| 1.5          | $2.33 \pm 0.13$               | $3.81 \pm 0.21$ | $0.12 \pm 0.13$           | $0.19 \pm 0.21$  | 13                |
| 1.7          | $2.13 \pm 0.14$               | $3.48 \pm 0.23$ | $0.12 \pm 0.13$           | $0.19 \pm 0.21$  | 13                |
| 2.0          | $1.99 \pm 0.15$               | $3.25 \pm 0.25$ | $0.46 \pm 0.15$           | $0.75 \pm 0.25$  | 11                |

### TABLE 7

**SUMMARY OF THE TOLMAN SIGNAL AND THE INFERRED LUMINOSITY EVOLUTION FOR Cl 1604 + 4304 IN THE I BAND USING $q_0 = 1/2$**

| $\eta$ (mag) |  $\Delta\langle SB \rangle$ (mag) |  $n$ (mag) |  $\Delta M_{\text{evol}}$ (mag) |  $4 - n$ (mag) |  Number of Galaxies |
|--------------|-------------------------------|------------|-----------------------------|----------------|-------------------|
| 1.0          | $2.92 \pm 0.18$               | $4.20 \pm 0.26$ | $-0.14 \pm 0.18$          | $-0.20 \pm 0.26$ | 7                 |
| 1.3          | $2.73 \pm 0.17$               | $3.93 \pm 0.24$ | $0.05 \pm 0.17$           | $0.07 \pm 0.24$  | 7                 |
| 1.5          | $2.72 \pm 0.17$               | $3.91 \pm 0.24$ | $0.06 \pm 0.17$           | $0.08 \pm 0.24$  | 7                 |
| 1.7          | $2.43 \pm 0.18$               | $3.50 \pm 0.26$ | $0.35 \pm 0.18$           | $0.50 \pm 0.26$  | 7                 |
| 2.0          | $2.29 \pm 0.21$               | $3.29 \pm 0.30$ | $0.49 \pm 0.21$           | $0.71 \pm 0.30$  | 6                 |
curve is reached only for $\eta \gtrsim 1.8$. At these $\eta$ values, the error in the measured mean surface brightness is less than 0.07 mag.

Therefore, to calculate our final numbers, we disregard the data at $\eta$ values of less than 1.7 as undoubtedly unreliable at the greater than 0.1 mag level. Consequently, we calculate the mean value of the exponent $n$ averaged over the two $\eta$ values of 1.7 and 2.0 only. We obtain a final value of the exponent $n$ in the $R$ band by using the data from Cl 1604+4321. For the $I$ band we note that the resulting values for the two clusters observed in the $I$ band, Cl 1324+3011 and Cl 1604+4304 (see col. [3] of Tables 5–7). Therefore, we have combined the data for both clusters for the final measurement in the $I$ band.

As discussed above, we calculate the values for the depression, $\Delta\langle SB \rangle$ (as represented by the exponent $n$), for the $R$ and $I$ bands from a weighted average of the data from $\eta = 1.7$ and 2.0. The results for $q_0 = 1/2$ are

$$\Delta\langle SB \rangle = \begin{cases} 
2.5 \log (1 + z)^{2.59 \pm 0.17} & \text{mag for R band}, \\
2.5 \log (1 + z)^{3.37 \pm 0.13} & \text{mag for I band},
\end{cases}$$

(6)

for the apparent Tolman signal as compromised by luminosity evolution. The required luminosity evolution would then be

$$M_{\text{evol}} = \begin{cases} 
2.5 \log (1 + z)^{1.41 \pm 0.17} & \text{mag for R band}, \\
2.5 \log (1 + z)^{0.63 \pm 0.13} & \text{mag for I band}.
\end{cases}$$

(7)

We adopt equations (6) and (7) as yielding our final values from the current experiment.

We must now test whether equation (7) is reasonable within the independent discipline of luminosity evolution of the stellar content using the precepts of population synthesis. In § 4 we use the 1996 version of the Bruzual & Charlot (1993) models to test for compatibility of equation (7) and the stellar population synthesis models. There is, of course, now a vast literature on population synthesis models. We have used only the Bruzual & Charlot (1993) models here. It will eventually be important to test for compatibility by using different codes, such as earlier papers by Guiderdoni & Rocca-Volmerange (1987, 1988), Rocca-Volmerange & Guiderdoni (1988), and Worthey (1994).

### 3.2. Doubly Diluted Tolman Signal in the $\langle SB \rangle$-$M$ Correlation Diagram

In the last section we chose to analyze the data in the $\langle SB \rangle$, $R$ plane rather than in the $\langle SB \rangle$, $M$ plane as was done in Sandage & Perelmuter (1991). As explained in § 3.1, the present method is preferred because it minimizes the effect of luminosity evolution, which, from the result in the last section, must be present at the level of $2.5 \log (1 + z)^{1.41 \pm 0.17}$ mag in the $R$ band and $2.5 \log (1 + z)^{0.63 \pm 0.13}$ in the $I$ band if the radius does not evolve over the look-back time and if the expansion is real. Nevertheless, it is of interest to examine the data in the $\langle SB \rangle$-$M$ correlation diagram that is doubly degenerate to the effects of luminosity evolution.

Figure 3 shows the data for Cl 1604+4321 in the $R$ photometric band with the same symbols as in Figure 1. The data for the individual galaxies are from Table 2 and therefore refer to $H_0 = 50$ and $q_0 = 1/2$ in calculating $M$ from the redshift. The ordinate of $\langle SB \rangle$, as before, is independent of both $H_0$ and $q_0$ because it is given directly as observed, except that it has been corrected for $K(R)$ dimming via Table 1.

We also plot in Figure 3 the envelope lines for the zero-redshift calibration. These relations have been calculated from the equations of $\langle SB \rangle$ versus $R$ given in Table 2 of Paper I, as modified by the nonlinearity corrections, given in Table 3 of Paper I, at radii smaller than $R = 4.3$. We use equation (11) of Paper I, which gives $\langle SB \rangle = M + 5 \log R + 22.815$, where $R$ is in parsecs, to calculate the absolute magnitude $M$ from the $\langle SB \rangle$-$R$ local relations. The resulting envelope lines, which are plotted in Figure 3, are the same to within the errors as the linear least-squares fits to the Postman & Lauer (1995) local data given in Table 4 of Paper I over the overlapping linear range.

Figure 4 shows the $I$-band data for Cl 1324+3011 and Cl 1604+4304 as taken from Tables 3 and 4. The envelope lines plotted in this figure are the same as in Figure 3; however, they have been converted to the $I$ band by $\langle R - I \rangle = 0.62$ (see above).

Two features of Figures 3 and 4 are noted. (1) The scatter is larger than in the $\langle SB \rangle$, $R$ plane (Figs. 1–2), and (2) although the mean $\langle SB \rangle$ in both diagrams is depressed relative to the zero-redshift calibration lines, the amount is less in the $\langle SB \rangle$, $M$ plane than in the $\langle SB \rangle$, $R$ plane. The first effect on the larger scatter in Figures 3 and 4 is due to the larger scatter in the $\langle SB \rangle$, $M$ plane even for zero redshift (see Fig. 3 of Paper I). The obvious reason is that errors in $M$ and/or $\langle SB \rangle$ move points perpendicular to the slope of the correlation line. By contrast, the error vectors in the $\langle SB \rangle$, $R$ plane in Figures 1 and 2 are nearly parallel to the correlation lines.

The second effect is due to the double degeneracy to luminosity evolution in the $\langle SB \rangle$, $M$ plane. As said before, luminosity evolution affects both coordinates. Relative to the zero-redshift line, an increase in the luminosity at high redshift due to luminosity evolution moves a point brighter in $M$ and also brighter in $\langle SB \rangle$ relative to that line. Therefore, in an observed $\langle SB \rangle$-$M$ diagram that contains luminosity evolution, the depression from the zero-redshift line is diluted twice, once in $\langle SB \rangle$ and once in $M$.

Because Figures 1 and 2 are more powerful than Figures 3 and 4 for this reason of double degeneracy, we have made the calculations of the signal, as represented by $n$, and the luminosity evolution, as represented by $4 - n$, from only the $\langle SB \rangle$-$R$ data in Figures 1 and 2. Of course, the final answers from the analysis in either representation must be the same because the $\langle SB \rangle$ versus $R$ and the $\langle SB \rangle$ versus $M$ diagrams are simply different representations of the same data, connected by equation (11) of Paper I.

### 3.3. Tautological Absolute Magnitude Signal in the $(\log R, M)$ Plane

Use of the Mattig (1958) equations for linear radius and absolute magnitude guarantees that the combination of $\langle SB \rangle$, $M$, and the linear radius, $R$, are consistent with the variation of $\langle SB \rangle$ with redshift as $2.5 \log (1 + z)^4$ mag. Because the log $R$ and $M$ values in Tables 2–4 are calculated using the Mattig (1958) equations, the deviations of the “observed” $M$, calculated with the Mattig equation,
must be precisely $2.5 \log (1 + z)^{4-n}$. Hence, the observed deviations from the zero-redshift lines prove nothing except to show how much the absolute magnitude, $M$, must change in the look-back time to conform with the relation $\log (1 + z)^{n} + \log (1 + z)^{4-n} = \log (1 + z)^{4}$.

In Figures 5 and 6, we plot the relation between $\log R$ and $M$ for the $R$ band and $I$ band, respectively. The envelope lines are calculated from the zero-redshift relations using the linear equations from Table 2, the nonlinear corrections for small $R$ from Table 3, and equation (11) given in Paper I. These figures do nothing more than demonstrate that $M$ must become brighter with increasing redshift (because of evolution) to make the true depression of $S_{SBT}$ in Figures 1 and 2, corrected for evolution, equal to $(1 + z)^{4}$ if the standard model is correct. From equation (7), we see that the requirements for a precise agreement with the Tolman prediction is that the mean magnitude deviation at constant $\log R$ must be $\langle \Delta M \rangle = 0.39 \pm 0.08$ mag in the $I$ band for Cl 1324+3011 ($z = 0.7565$), $\langle \Delta M \rangle = 0.44 \pm 0.09$ mag in the $I$ band for Cl 1604+4304 ($z = 0.8967$), and $\langle \Delta M \rangle = 1.00 \pm 0.12$ mag in the $R$ band for Cl 1604+4321 ($z = 0.9243$) if the Tolman signal, freed from luminosity evolution, would be precisely $(1 + z)^{4}$. Calculating the average difference at constant $\log R$ between the high-redshift data and the zero-redshift relations plotted in Figures 5 and 6, we measure precisely, by definition, these values. The predictions for luminosity evolution from spectral synthesis models with varying star formation histories are made in the next section to compare with these predictions from the Tolman test.

4. THEORETICAL LUMINOSITY EVOLUTION ESTIMATED USING TWO DIFFERENT METHODS

4.1. Elementary Estimate of the Correction for Passive Luminosity Evolution Due to Main-Sequence Burn-Down

An early order-of-magnitude estimate of the expected change of the luminosities of early-type galaxies in the look-back time was based on the change of the turnover luminosity in the H-R diagram for an old, coeval population in which no new stars are formed after the initial starburst. The method is interesting because of its simplicity. It led early on to the result that the passive luminosity evolution correction was close to $\Delta M_{bol}^* \sim 2.5 \log (1 + z)$. This result is given by the simplistic calculation of direct main-sequence burn-down, including corrections for the main-sequence luminosity function (Sandage 1961b, 1988). In addition, the elaborate population synthesis models by Tinsley (1968, 1972a, 1972b, 1976, 1977, 1980, and references therein) gave nearly the same result.

We show in the next section that nearly the same result is also obtained by fitting the observed spectral energy distribution and the size of the 4000 Å break with the star formation models of Bruzual & Charlot (1993). The look-back times vary, of course, with the assumed value of $q_0$. The calculated ages for galaxies with the observed redshifts of the three clusters must agree with the ages from the Bruzual & Charlot models, at least approximately, if our story is to have coherence. Anticipating the next section, we set out here the "cosmological ages" based on the redshift, the look-back time, and the assumed values of the Hubble
constant, \( H_0 \), and \( q_0 \). The general case of the calculation of the look-back time for all values of \( q_0 \) is solved elsewhere (Sandage 1961b). The special cases of \( q_0 = 0 \) and \( q_0 = 1/2 \) are, of course, trivial. The ages, \( T \), at a given redshift are given by

\[
T = \frac{2H_0^{-1}}{3(1 + z)^{3/2}} \quad \text{for} \quad q_0 = \frac{1}{2} .
\]

(8)

\[
T = \frac{H_0^{-1}}{1 + z} \quad \text{for} \quad q_0 = 1 .
\]

(9)

The case for \( q_0 = 1 \) is more complicated and can be found from the tables in Sandage (1961b).

Equation (8) gives ages of 4.82, 4.30, and 4.21 Gyr at the time light left the three clusters with redshifts of 0.7565, 0.8967, and 0.9243, respectively, for \( q_0 = 1/2 \) and \( H_0 = 58 \) \( \text{km s}^{-1} \text{Mpc}^{-1} \). For these calculations, we must use the real value of \( H_0 \) (Sandage & Tammann 1997; Theureau et al. 1997; Saha et al. 1999; Sandage 1999; Parodi et al. 2000) rather than an arbitrary value as in previous sections because we need real ages in this section.

Equation (9) gives the ages at the time light left of 9.60, 8.89, and 8.76 Gyr, respectively, for the clusters by using \( q_0 = 0 \). The extreme case of \( q_0 = +1 \), in which the age of the universe is only 0.571\( H_0^{-1} \) = 9.63 Gyr for \( H_0 = 58 \), gives ages when light left the three clusters of 3.77, 3.32, and 3.26 Gyr, respectively, by using Table 3 of Sandage (1961b).

It is also interesting to calculate the absolute magnitudes of the main-sequence termination (TO) for these ages. The modern oxygen-enhanced evolutionary models by Bergbush & VandenBerg (1992) for \([\text{Fe/H}] = 0\) give bolometric magnitudes of

\[
M(\text{bol}, \text{TO}) = 2.564 \log T - 21.644 ,
\]

(10)

where \( T \) is in years, as interpolated from their tables (Sandage 1993). Hence, the main-sequence absolute bolometric magnitudes for the \( q_0 = 1/2 \) look-back times for the three clusters are 3.18, 3.06 and 3.03 mag, respectively. The brightest turnover magnitudes are, of course, for the \( q_0 = +1 \) models because the ages when light left are the smallest. These bolometric turnover magnitudes for the \( q_0 = +1 \) case are 2.91, 2.79, and 2.73 mag, respectively.

4.2. Expected Evolution From the Bruzual & Charlot Models

To calculate the expected amount of luminosity brightening with look-back time, we have chosen to use the stellar population synthesis code of Bruzual & Charlot (1993) because, as part of the original photometric and spectral analyses of these clusters PLO98, and PLO01 use the 1996 stellar evolution models of Bruzual & Charlot (hereafter BC96) to study the star formation histories and ages of the cluster galaxies. The authors find that the Bruzual & Charlot models are ideal because they can be used to generate absolute energy distributions over a broad wavelength range, with a spectral resolution comparable to the observed Keck spectra (see OPL; PLO98; PLO01).

The free parameters in the BC96 models include the mass function, the metal abundance, and the star formation rate. Following PLO98 and PLO01, we have chosen models with a Salpeter mass function (Salpeter 1955) and a maximum stellar mass of 125 \( M_\odot \). Comparisons with
models constructed with a Scalo luminosity function (Scalo 1986) show no significant difference at the level of accuracy that can be achieved with low-resolution Keck spectra (PLO98). Based on the metal line equivalent widths and Balmer jump strengths, PLO01 determine that models with assumed metallicities between $Z = 0.004$ (0.2 solar) and $Z = 0.020$ (solar) provide reasonable fits to the data. Because the metallicities of the cluster galaxies are not strongly constrained, we assume a solar abundance for our calculations.

The most significant constraint on the BC96 models is the choice of the star formation history. The simplest model assumes a large, initial burst of star formation after which the galaxy fades in accordance with passive stellar evolution models. These model are called “ssp models” by Bruzual & Charlot. The next simplest models are those in which star formation begins at $t = 0.0$ and decreases exponentially with a fixed time constant. PLO98 and PLO01 refer to these models as tau ($\tau$) models and have considered time constants ranging from 0.2 to 20 Gyr. Any of these models can be used to generate the expected broadband ($BVRI$) AB magnitudes once the galaxy redshift is specified (see, e.g., Fig. 7 of Paper III).

To characterize the observed spectral energy distribution (SED) of each galaxy, PLO01 have used the slope, referred to as $b$, of a linear least-squares fit to the measured AB magnitudes as a function of $\log \nu$ (see § 2.1 of PLO01). The slope, $b$, provides a more robust indicator of the overall broadband SED than any individual color measure, although $b$ is strongly correlated with the usual broadband colors (e.g., $V - R \approx b/13$). The early-type galaxies used in the present experiment are the reddest galaxies in the clusters and, therefore, have the largest values of slope $b$ with $b \gtrsim 10$ (see Tables 2–4 of PLO01). The best-fit BC96 model to each of these galaxies is either the ssp model (which is essentially equivalent to a $\tau = 0.2$ Gyr model) or a tau model with $\tau \lesssim 1.0$ Gyr (see Fig. 6 of PLO01). Therefore, we have chosen these models to measure the range in luminosity evolution that we expect between $z = 0$ and the cluster redshifts of $z = 0.7565, 0.8967, 0.9243$, respectively.

For these calculations, we use the ages for each cluster derived in § 4.1 for different values of $q_0$, and assume that the epoch of star formation occurred at $z_{\text{form}} \gtrsim 2.5$. This high-star formation epoch is consistent with other observations of cluster early-type galaxies (e.g., Aragón-Salamanca et al. 1993; Stanford, Eisenhardt, & Dickinson 1995, 1998; Oke, Gunn, & Hoessel 1996; Ellis et al. 1997; van Dokkum et al. 1998; de Propris et al. 1999).

Using these models to measure the luminosity brightening with look-back time, we find, as expected, that the smallest luminosity evolution is measured with the ssp model because the burst of star formation is the shortest of the models that we are considering; conversely, the largest luminosity evolution is measured with the $\tau = 1.0$ Gyr model because of its extended burst of star formation. For a $\tau = 0.2$ Gyr model with $q_0 = 1/2$, we find that the theoreti-

FIG. 5.—Deviation of the absolute magnitude at a given linear radius in the $R$ photometric band from the zero-redshift correlation line for Cl 1604 + 4321. The solid line represents the zero-redshift calibration derived from the zero-redshift log $R$–$(SB)$ relation in Fig. 1 and eq. (11) of Paper I. The increase in absolute luminosity required to preserve the $(1 + z)^{\eta}$ Tolman factor is the deviation toward brighter magnitudes seen relative to the zero-redshift line. This is clearly due to luminosity evolution. To preserve the Tolman factor requires that the evolutionary factor must be $S^* = M^* - 2.5 \log (1.9243^{1.41} - 1.00)$ mag for the weighted mean of the $\eta$ values of 1.7 and 2.0. Because of the tautology of the argument, this diagram must show this factor, by necessity. The expectation from the theory of passive evolution is set out in § 4.
The tautological deviations in magnitudes from the zero-redshift lines are required to be $2.5 \log (1.7565 - 0.63) - 0.39$ and $2.5 \log (1.8967 - 0.63) - 0.44$ mag, respectively, as the weighted mean of the $\eta$ values of 1.7 and 2.0 if the expansion is real. These required evolutionary corrections are within the errors of the independent calculation of the evolution correction from the spectral synthesis calculations in §4, based on the observed colors.

Clearly, the amount of evolution that we require to make $n = 4$ (see eq. [7]) is fully consistent, within the errors, with the range of theoretical expectations determined above. Therefore, we assert that we have either (1) detected the evolutionary brightening directly from the $S_\lambda$ observations on the assumption that the Tolman effect exists or (2) confirmed that the Tolman test for the reality of the expansion is positive, provided that the theoretical luminosity correction for evolution is real.

Our conclusions are fully consistent with the two previous attempts to perform the Tolman test using primarily ground-based data at more moderate redshifts. Specifically, Sandage & Perelmuter (1991) completed an identical analysis by comparing local elliptical galaxies with the first-ranked elliptical galaxies in clusters at redshifts up to $z \sim 0.6$. Later, Pahre, Djorgovski, & de Carvalho (1996) used the Kormendy relation to compare elliptical galaxies in the Coma Cluster with those in A2390 ($z = 0.23$) and A851 ($z = 0.41$). Both studies found that the data were fully consistent with the universal expansion, assuming simple models of passive evolution of elliptical galaxies.

5. SENSITIVITY OF THE CONCLUSIONS TO THE ASSUMPTIONS THAT $q_0 = 0$ AND $q_0 = \pm 1$

We need now to estimate the sensitivity of the results in §3 to the value of $q_0$. The sensitivity arises from the differences in the values of the linear radii calculated from the angular radii and the redshift for different values of $q_0$. The results of the theoretical calculations agree well with other measures of luminosity evolution for early-type galaxies at $z \sim 0.5$–1. Depending on the particular passband and cosmological parameters, the absolute luminosities of field and cluster early-type galaxies are brighter by approximately $1.0 \pm 0.5$ mag at redshifts of $z \gtrsim 0.5$ (e.g., Im et al. 1996; Oke et al. 1996; Schade et al. 1996, 1999; Smail et al. 1997; van Dokkum et al. 1998; PLO98, PLO01).
TABLE 8

Recipes to Convert the $q_0 = 1/2, H_0 = 50$ Tables for Log $R$ and $m - M$ to the Other Cosmologies

| Cluster      | $z$  | $m - M$ | log $A$ | $\Delta M$ | $\Delta \log R$ | $m - M$ | log $A$ | $\Delta M$ | $\Delta \log R$ | $m - M$ | log $A$ | $\Delta M$ | $\Delta \log R$ |
|--------------|------|---------|---------|-------------|-----------------|---------|---------|-------------|-----------------|---------|---------|-------------|-----------------|
| Cl 1324 + 3011 | 0.7565 | 43.98   | 3.992   | 0.41 brighter | 0.082 larger | 43.28   | 3.853   | 0.29 fainter | 0.057 smaller | 43.26   | 4.215   | 0.31 fainter | 0.305 larger    |
| Cl 1604 + 4304 | 0.8967 | 44.46   | 4.021   | 0.49 brighter | 0.097 larger | 43.66   | 3.860   | 0.31 fainter | 0.064 smaller | 43.62   | 4.270   | 0.35 fainter | 0.346 larger    |
| Cl 1604 + 4321 | 0.9243 | 44.53   | 4.025   | 0.50 brighter | 0.100 larger | 43.73   | 3.861   | 0.32 fainter | 0.065 smaller | 43.68   | 4.280   | 0.36 fainter | 0.354 larger    |
Different linear radii entered into the log $R$–$\langle SB \rangle$ diagnostic diagrams of Figures 1 and 2 will give different displacements from the zero-redshift local envelope lines and therefore a different Tolman signal. We can estimate these differences by calculating the change in the linear radii made by changing the value of $q_0$ and calculating the effect on the Tolman signal by using the zero-redshift envelope lines for different $\eta$ values from Tables 2 and 3 of Paper I.

The Mattig (1958) equations for linear radius $R = f(q_0,z,H_0)$ and absolute magnitude, $M$, are summarized elsewhere (Sandage 1988; 1995, eqs. [3.9] and [4.5]) for any $q_0$ and arbitrarily large $z$ values. They will not be repeated here. Table 8 shows the result of using these equations to calculate the necessary parameters for the $q_0 = 0$ and $+1$ models.

The distance moduli listed in columns (3) and (7) are for $H_0 = 50$. The log $A$ values to convert the logarithm of the angular radii to the logarithm of the linear radii are given in columns (4) and (8) via equations (4) and (5) of Table 2 of Paper I. These slopes vary between 3.0 and 3.5, from columns (6) and (10) of Table 8 by the slopes in other cosmologies.

The opposite sense is required for the case relative to that for $q_0 = 1/2$. That is, there will be a smaller deviation from the local line. The conclusion is that the Tolman prediction is verified and that the expansion is real.

We note that recent studies of high-redshift Type Ia supernovae, the cosmic microwave background, and the statistics of gravitational lenses suggest that the universe is flat with a nonzero cosmological constant, $\Lambda$ (see, e.g., Kochanek 1996; Helbig et al. 1999; de Bernadis et al. 2000; Pryke et al. 2001; Riess et al. 2001 and references therein). Currently, the preferred world model is $\Omega_M \approx 0.35$ and $\Omega_\Lambda \approx 0.65$. In this cosmology, the log $R$ values are almost identical to the empty-universe ($q_0 = 0$) case; they are larger by only 0.01 dex or less at the redshifts of our three clusters. In addition, all flat-universe models with $0 \leq \Omega_\Lambda \leq 0.65$ give log $R$ values that lie within the range that we have calculated for the $\Omega_\Lambda = 0$ cosmologies (see Table 8). Consequently, our conclusions about the universal expansion are still robust for these $\Lambda$ cosmologies.

In the next section, we show that the predictions of the tired light speculation is not verified at the definitive level of better than $10\sigma$.

6. TIRED LIGHT MODEL COMPARED WITH THE OBSERVATIONS

In contrast with the standard model with the Mattig equations, there is no metric theory of how distances and magnitudes are measured in a tired light model. Therefore, we must guess at a reasonable equation for distance. We adopt a discussion that is given elsewhere (Sandage 1995, § 4.3) and use an equation for “coordinate” distance in a flat space, which is

$$D = \frac{c}{H_0} \ln(1 + z) .$$

Because the universe is not expanding in the model, the distance “now” in equation (11) is also the distance when light left. This, of course, is the crucial point. In the expanding case, the distance at the present epoch must be divided by $1 + z$ to give the distance when light was emitted. It is this latter distance in the expanding case that, when multiplied by the angular radius, fixes the linear radius.

With $H_0 = 50$, the linear radius (in parsecs) that corresponds to an observed angular radius (in arcsec) in the tired light case is

$$\log R = 4.464 + \log \left[ \ln(1 + z) \right] + \log \theta ,$$

by using equation (11). Hence, the $A$-term, as defined in equation (4), becomes

$$\log A = 4.464 + \log \left[ \ln(1 + z) \right] .$$

The log $A$ values calculated in this way are listed in column (12) of Table 8 for the three high-redshift clusters. These values, compared with the log $A$ values for the $q_0 = 1/2$ case, give the increase in the log $R$ values that must be
entered in the \(\langle SB\rangle - \log R\) diagnostic diagram. For example, for Cl 1324 + 3011 (\(z = 0.7565\)) the \(\log R\) values must be made larger by 0.305 dex relative to the \(q_0 = 1/2\) values listed in Table 3.

The magnitudes must also be changed because the distance moduli are different than those in the fiducial \(q_0 = 1/2\) case. The absolute magnitude calculation for the tired light model follows from the expected theoretical relation that

\[
l = \frac{L}{4\pi D^2 (1+z)},
\]

where only one power of \(1+z\) for the “energy effect” is required, rather than two powers of \(1+z\) in the Roberston (1938) equation in the standard theory (see Sandage 1975, eq. [2.1]). Hence

\[
m - M = 2.5 \log (1+z) + 5 \log [\ln (1+z)] + 43.89
\]

(15)

for \(H_0 = 50\).

It can be shown from equation (15) that the magnitude-redshift relation for tired light is the same to within a few hundredths of a magnitude as that in the case for \(q_0 = +1\) using the Mattig equation. This is true even for redshifts as large as \(z = 1\). Column (13) of Table 8 confirms this statement, seen by the fact that the entries in column (9) for the \(q_0 = +1\) case are very close to those in column (13) for tired light. However, these magnitude changes are academic here in using the \(\langle SB\rangle - \log R\) diagnostic diagram because no corrections for the magnitude differences need to be applied to the \(\langle SB\rangle\) values. These are observed \(\langle SB\rangle\) values (only corrected for K-dimming); hence, as emphasized before, they are independent of all cosmologies.

With the changes to Tables 2–4 for \(\log R\) that are listed in column (14) of Table 8, we can enter Figures 1 and 2 for the tired light case to measure the depression from the local upper envelope calibration. The result, not shown but which the reader can recover using the \(\Delta \log R\) values given in Table 8 together with Tables 2–4, is that a measurable \(\langle SB\rangle\) depression at these larger \(\log R\) values is again present. It is, of course, smaller than that for the expanding case because of the larger \(\log R\) values. But by how much? The crucial question is whether it is so much smaller as conform to a depression of only 2.5 \(\log (1+z)\) mag when the correction for luminosity evolution is also applied.

We have analyzed the data for the three clusters in the same way as in § 3; however, we now use the correct \(\log R\) values required by the tired light conjecture, using the recipes in Table 8. Expressing the result of the \(\langle SB\rangle\) depression from the zero-redshift fiducial line in the \(\langle SB\rangle - \log R\) diagram as the exponent, \(n\), in 2.5 \(\log (1+z)^p\) gives the weighted mean of \(n = 1.61 \pm 0.13\) for the \(R\) band and \(n = 2.27 \pm 0.12\) for the \(I\) band. As described in § 3, we have used the results from the most reliable \(\eta\) values, \(\eta = 1.7\) and 2.0. The resulting exponents are too large by approximately 5 and 10 \(\sigma\), respectively, compared with the \(n = 1\) prediction of the tired light scenario. Consequently, to produce coherence with the tired light model, we require negative luminosity evolution in the look-back time, i.e., galaxies must be fainter in the past. No feasible model of stellar evolution can produce such luminosity evolution with time.

In fact, just as in the expanding case, positive luminosity evolution must have occurred in the look-back time because there is no way in the tired light model to prevent the stellar content of galaxies from evolving during the look-back time.

A static model in which the redshift is not due to expansion is not the same as a steady state model in which the mean parameters of galaxies, averaged over an ensemble of galaxies, are required to be the same at all distances and at all times. For a steady state to exist, despite the evolution of the stellar content of individual galaxies, requires that there must be young and old galaxies in every volume of space and at every cosmic time such that the mean age is the same at all distances and times. This requires continuous galaxy formation at the same rate at all cosmic times to maintain a constant mean age everywhere and always.

However, such steady state models are not the same as static models in which the redshift is due to an unknown physical cause, either at the source or in the intervening light path from the source to the observer, as originally postulated by Zwicky (1929). In fact, the steady state models proposed by Bondi, Gold, and Hoyle are truly expanding models in which the redshift is due to the expansion. They are not static models. Furthermore, an early proof that steady state models cannot be correct was demonstrated by the failure of the predicted steady state color distribution, with its required mixture of ages, to match the observed color distribution of early-type galaxies (Sandage 1973).

Hence, as in the present paper, evolutionary corrections to magnitudes must be applied to both the expanding and the static models in making the present test. The supposed degeneracy of the Tolman test due to the identity claimed by Moles et al. (1998) of the surface brightness effect in both the expanding and a static (tired light) case is not correct. Their error is due to a category mistake made by confusing static and steady state models. What Moles et al. have done is to combine a static model with a steady state model. This is a higher order departure from the standard expanding model than we have considered here and is not the test we have made. In any case, as stated above, a steady state model can again be disproved by the color argument (Sandage 1973). Hence, even the higher order model proposed by Moles et al. cannot be correct on this ground alone.

The result of the present paper is that a static model, in which the redshift is due to an unknown physical cause, fails the surface brightness test by a large factor. Such a model requires luminosity evolution in the look-back time, just as in the expanding case. Based on the analysis in § 4, we find that a good approximation for the amount of increase in luminosity at the epoch of light emission is 2.5 \(\log (1+z)^p\) mag, where \(p > 0.7\). Applying this correction to the tired light analysis gives the intrinsic tired light prediction for the corrected exponents of greater than 2.31 (± 0.13) and greater than 2.87 (± 0.12) for the \(R\) and \(I\) bands, respectively. Each value is more than 10 \(\sigma\) from the required exponent of 1.0 if the tired light scenario were correct. We take this to be a definitive proof that the hypothesis of nonexpansion does not fit the surface brightness data.

7. SYSTEMATIC UNCERTAINTIES IN THE EXPERIMENT: HOW CAN THE PRESENT RESULT BE IMPROVED?

There are two systematic uncertainties in the present experiment. Although neither of them are severe enough to jeopardize the results presented in § 3 and § 5 that the expansion is real, each can be overcome by more data with
expanded boundary conditions on the parameters compared to the data used here.

First, the principal uncertainty at small radii (log \( R < 4.0 \)) is the position of the zero-redshift fiducial line in the \( \langle SB \rangle \)-log \( R \) diagram, relative to which the \( \langle SB \rangle \) depressions for high-redshift galaxies are compared. The Postman & Lauer (1995) data (Fig. 2 of Paper I) do not extend to radii smaller than log \( R = 4.0 \), where \( R \) is in parsecs. Their data are confined to the first-ranked cluster early-type galaxies. They do not sample the luminosity function of each cluster to provide data for smaller galaxies. We have extended the data to smaller radii with the sample in Sandage & Perelmuter (1991) to generate a nonlinear correction at small radii to the best-fit linear equations to the Postman & Lauer (1995) data (see Table 3 of Paper I). However, the Sandage & Perelmuter (1991) data are also only for the first few brightest cluster galaxies, again not going far into the fainter part of the luminosity function. Hence, although Table 3 of Paper I gives our adopted extension to radii as small as log \( R = 3.3 \), where \( R \) is in parsecs, the uncertainties are large and can be reduced by a more complete study of the \( \langle SB \rangle \)-log \( R \) relation for fainter and smaller galaxies at low redshift.

Second, the three clusters studied here are near the faint end of the distribution of absolute magnitude of first-ranked galaxies in, for example, the sample of first-ranked cluster galaxies whose data are listed by Kristian, Sandage, & Westphal (1978). The mean of the distribution of absolute magnitude in the \( R \) band for the Kristian et al. sample is \( \langle M_R \rangle = -24.5 \), whereas the first-ranked early-type galaxies in the three clusters studied have absolute magnitudes of \( M_R = -23.7 \) for CI 1604+4321, -23.7 for CI 1604+4304, and -24.2 for CI 1324+3011 (see PLO01; note again that the system of \( R \) magnitudes used by Kristian et al. is 0.25 mag brighter than the Cape \( R \) system used here). The galaxies in the three clusters studied here have fainter absolute magnitudes and smaller radii than the average local clusters, exacerbating the problem described above. Richer high-redshift clusters with brighter and therefore larger first-ranked galaxies are known. A study of the \( HST \) data from such clusters will improve the present Tolman test. We suspect that the present experiment is only the beginning of similar work that will be done in the coming years with \( HST \) on such clusters as they are discovered and observed.

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