PAYING MORE ATTENTION TO SNAPSHOTs OF ITERATIVE PRUNING: IMPROVING MODEL COMPRESsion VIA ENSEMBLE DISTILLATION

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ABSTRACT

Network pruning is one of the most dominant methods for reducing the heavy inference cost of deep neural networks. Existing methods often iteratively prune networks to attain high compression ratio without incurring significant loss in performance. However, we argue that conventional methods for retraining pruned networks (i.e., using small, fixed learning rate) are inadequate as they completely ignore the benefits from snapshots of iterative pruning. In this work, we show that strong ensembles can be constructed from snapshots of iterative pruning, which achieve competitive performance and vary in network structure. Furthermore, we present simple, general and effective pipeline that generates strong ensembles of networks during pruning with large learning rate restarting, and utilizes knowledge distillation with those ensembles to improve the predictive power of compact models. In standard image classification benchmarks such as CIFAR and Tiny-Imagenet, we advance state-of-the-art pruning ratio of structured pruning by integrating simple $\ell_1$-norm filters pruning into our pipeline. Specifically, we reduce 75-80% of total parameters and 65-70% MACs of numerous variants of ResNet architectures while having comparable or better performance than that of original networks. Code associate with this paper is made publicly available at https://github.com/lehduong/ginp.

1 INTRODUCTION

Motivation

Researchers have extensively exploited deep and wide networks for the sake of achieving superior performance on various tasks. Most of state-of-the-art networks are extremely computationally expensive and require excessive memory. However, real-world applications usually require running deep neural networks on edge devices for various reasons: user privacy, security, real-time analysis, offline capability, reducing cost for server deployment, and so on. Adopting large, cumbersome networks to such resource-constrained environments is challenging due to the restrictions of memory, computational power, energy consumption,...

Background: Network pruning [LeCun et al. (1990); Reed (1993); Han et al. (2015); Li et al. (2016)] reduce a cumbersome, over-parameterized network to compact one by removing unnecessary weights and connections of networks. It is widely believed that small networks pruned from large, over-parameterized networks achieve superior performance than those trained from scratch [Frankle & Carbin (2018); Renda et al. (2020); Li et al. (2016); Luo et al. (2017)]. A plausible explanation to this phenomenon is the lottery ticket hypothesis [Frankle & Carbin (2018)] i.e. large, over-parameterized networks contain many optimal sub-networks i.e. winning tickets. In particular, network pruning could be done in two manners: one-shot pruning - prune network with the desired compression ratio and retrain it only one time, or iterative pruning - only prune small ratio of the original network, retrain and repeat that process until target size is reached. It has been shown that iterative pruning could lead to a greater compression ratio compare to one-shot pruning [Han et al. (2015); Luo et al. (2017); Li et al. (2016); Renda et al. (2020)]. Furthermore, [Frankle & Carbin...
(2018) point out that iteratively-pruned-winning-tickets learn faster and reach higher test accuracy at smaller network size.

On the other hand, ensembles of neural networks are known to be much more robust and accurate than individual networks [Huang et al. (2017); Ashukha et al. (2020); Snoek et al. (2019)]. In spite of their superior performance, the tremendous cost of training and inference of ensembles makes them less attractive in practice. For the purpose of accelerating training time of ensembles, prior works propose methods encouraging models to converge to different local minimums during training [Huang et al. (2017); Garipov et al. (2018); Yang et al. (2019b)]. To reduce inference time of ensembles, one could use a single network to mimic behavior of ensembles as pioneered by born-again tree [Breiman & Shang] and knowledge distillation [Hinton et al. (2015); Balan et al. (2015); Bucilu et al. (2006); Malinin et al. (2019)]. In the above approaches, although small networks can not achieve comparable performance with ensembles of networks, dark knowledge transferred from teachers to student network could bridge the gap between their prediction powers.

Our proposal: While existing methods of iterative pruning is more effective than one-shot pruning, the snapshots at each pruning iteration are mostly overlooked. We consider leveraging the snapshots of iterative pruning to take the performance of compact models to the next level.

In this work, we propose a simple pipeline for model compression by slightly modifying standard approach. Specifically, we make use of large learning rate restarting at each pruning iteration to retrain pruned networks. Hence, each retraining step could be considered as a cycle of Snapshot ensemble [Huang et al. (2017)]. Utilizing both large learning rate restarting and pruning foster the diversity between snapshots, thus, constructing strong ensembles. Once achieve the desired compression ratio, we then distill the knowledge from the ensembles of snapshots of iterative pruning to the final model. Our method acquires the advantages of network pruning, ensembles learning, and knowledge distillation. To the best of our knowledge, this is the first work attempting to exploit snapshots of iterative pruning to further improve the performance of pruned networks.

Our main contributions: The contributions of our work are summarized as below:

1. We empirically show that fine-tuning with large learning rate restarting can achieve competitive or better results than common strategy i.e. small, fixed learning rate on a range of standard datasets and architectures. Surprisingly, such simple modification can create very strong baselines for both structured and unstructured pruning.
2. We demonstrate that snapshots of iterative pruning could construct strong ensembles.
3. We proposed a simple pipeline to combine knowledge distillation from ensembles and iterative pruning. We empirically show that our approach can achieve state-of-the-art pruning ratio by reducing 75 – 80% of parameters and 65 – 70% MACs on numerous variants of ResNet while having comparable or better results than original networks.

2 Related Works

Knowledge Distillation The approach of training small, efficient student network to mimic behavior of large, over-parameterized network has been proposed for long time [Bucilu et al. (2006) and was recently repopularized in [Hinton et al. (2015); Ba & Caruana (2014)]. Later, knowledge distillation was extended to various aspects, transferring knowledge from intermediate layers [Romero et al. (2014); Zagoruyko & Komodakis (2016)], allowing teachers and students guide each others [Zhang et al. (2018), using teacher and student with same architecture [Furlanello et al. (2018); Yang et al. (2019b); Bagherinezhad et al. (2018), knowledge distillation in multiple steps [Mirzadeh et al. (2019)]. To address the cost of training two networks in knowledge distillation, Zhu et al. (2018); Zhang et al. (2018); Yang et al. (2019b) propose online approaches to train the student and teacher networks in one generation. Furthermore, Anil et al. (2018) adopt knowledge distillation to accelerate the traning of large scale neural networks.

Network Pruning The idea behind network pruning is reducing the redundant weights and connections of original network to achieve compact networks without losing much performance [Han et al. (2015); Li et al. (2016)]. In general, pruning can be divided into two categories: structured pruning and unstructured pruning. Unstructured pruning [Hanson & Pratt (1989); LeCun et al. (1990);
and update the parameters to minimize cross-entropy objective:

\[ \text{average of outputs of all networks: } \bar{y}(x) \]

Consider the classification problem in which we need to determine the correct category for input image \( x \) among \( M \) classes. The probability of class \( m \) for sample \( x_n \) given by neural network \( f \) parameterized by \( \theta \) is computed as:

\[ p_m(x_n; \theta, \tau) = \frac{\exp\left(\frac{f_m(x_n; \theta)}{\tau}\right)}{\sum_{i=1}^{M} \exp\left(\frac{f_i(x_n; \theta)}{\tau}\right)} \]  

(1)

Where \( \tau \) is the temperature of softmax function, higher values of \( \tau \) lead to softer output distribution.

Conventional approaches optimize the parameters \( \theta \) by sampling mini-batches \( B \) from the dataset and update the parameters to minimize cross-entropy objective:

\[ \mathcal{L}_{NCE}(B; \theta) = -\frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{M} y_m \log p_m(x_n; \theta, 1) \]  

(2)

The target distribution of a sample is usually represented by one-hot vector i.e. only the true class is 1 and all other classes are 0. Since input images might differ in term of noise, complexity, and multi-modality, enforcing networks to excessively fit the delta distribution of ground truth for all samples might deteriorate their generalization. Besides that, the similarity between classes provides rich information for learning and potentially prevent overfitting Yang et al. (2019a). Knowledge distillation Bucilu et al. (2006); Hinton et al. (2015) use a trained (teacher) network, which usually has high capacity, to guide the training of other (student) network. Let \( q_m(x_n) \) be the probability of class \( m \) for image \( x_n \) given by the teacher network, which is parameterized by \( \psi \). The objective function of knowledge distillation is defined as:

\[ \mathcal{L}_{KD}(B; \theta, \tau, \psi) = -\frac{\tau^2}{N} \sum_{n=1}^{N} \sum_{m=1}^{M} q_m(x_n; \psi) \log \frac{q_m(x_n; \psi; \tau)}{p_m(x_n; \theta, \tau)} \]  

(3)

In case the teacher is an ensemble of \( K \) networks, the target distribution of knowledge distillation is the average of outputs of all networks:

\[ \hat{q}_m(x_n; \psi_1:K, \tau) = \frac{1}{K} \sum_{k=1}^{K} q_m(x_n; \psi_k, \tau) \]

An alternative approach is optimizing the mean of Kullback-Leibler divergence between student and each teacher network:

\[ \mathcal{L}'_{KD}(B; \theta, \tau, \psi_1:K) = -\frac{\tau^2}{KN} \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{k=1}^{K} q_m(x_n; \psi_k; \tau) \log \frac{q_m(x_n; \psi_k, \tau)}{p_m(x_n; \theta, \tau)} \]  

(4)

We experimented with two above objectives but did not observe significant difference in performance of student networks, thus, we only report results of second approach.

4 Snapshots of Iterative Pruning

In contrast to previous works, which mainly focus on the aforementioned usage of iterative pruning (i.e. alleviating the noise of weight’s importance estimation), we exploit the benefits of generating multiple models varying in structure, capacity to construct strong ensembles.
Figure 1: Overview of our approach to combine the advantage of knowledge distillation, ensembles of networks, and network pruning. At the start, we prune the filters/weights according to some criteria ($\ell_1$-norm, Taylor approximation, ...). With KESI, we retrain the pruned networks with large learning rate and minimize the conventional supervised loss function. Once we achieve the desired pruning ratio, we use knowledge distillation to transfer the knowledge from ensembles of snapshots of iterative pruning to the final model.

Inspired by the prior works of Smith (2015); Loshchilov & Hutter (2016) in which the authors show that promising local optimums could be found in a small number of epochs after restarting the learning rate. Furthermore, Huang et al. (2017) demonstrate that utilize large learning rate restarting during training can construct strong ensembles without much additional cost.

Broadly speaking, the accurate of ensembles depends on: the accurate of individual networks and the diversity of them. On the other hand, network pruning generates snapshots varying in structure and achieving competitive performance. Hence, if the pruned networks could achieve minimal loss in predictive power relative to the original networks, the ensembles of them could outperform ensembles of networks having identical architecture (and trained with large learning rate restarting).

Prior works such as Han et al. (2015); Liu et al. (2018); Molchanov et al. (2016) retrain the pruned networks for $T$ more epochs with a fixed learning rate, which is usually the final learning rate of the training. However, this approach might results in multiple snapshots stuck in similar local optimums, thus, leading to very weak ensembles as shown in our experiments. Similar to Huang et al. (2017), we adopt the large learning rate restarting at each every pruning iteration to encourage each snapshot converges to different optimum. For learning rate restarting, we utilize the one-cycle policy Smith & Topin (2019), which is proved to increase convergence speed of several models. Due to the similarity of our proposed method and Snapshot Ensembling Huang et al. (2017), we refer each pruning and retraining step as a cycle. One-cycle policy adjusts learning rate at each mini-batch update and has two phases:

**INCREASING LEARNING RATE** The learning rate and momentum of optimizer will be initialized to $\eta_{initial}$ and $\beta_{initial}$ respectively. During the first $T$ iterations of fine-tuning, learning rate and momentum gradually increase from initial values to $\eta_{max}$, $\beta_{max}$. The learning rate and momentum at $i$-th step with cosine annealing strategy are given by:

$$\eta_i = \eta_{max} + \frac{\eta_{initial} - \eta_{max}}{2} (1 + \cos\left(\frac{i}{T} \cdot \pi\right))$$ (5)

$$\beta_i = \beta_{max} + \frac{\beta_{initial} - \beta_{max}}{2} (1 + \cos\left(\frac{i}{T} \cdot \pi\right))$$ (6)

**DECREASING LEARNING RATE** After $T$ iterations, learning rate and momentum will be gradually decreased from $\eta_{max}$ and $\beta_{max}$ to $\eta_{min}$ and $\beta_{min}$ in $L - T$ iterations where $L$ is total number of iterations for fine-tuning.

$$\eta_i = \eta_{min} + \frac{\eta_{max} - \eta_{min}}{2} (1 + \cos\left(\frac{i - T}{L - T} \cdot \pi\right))$$ (7)
\[
\beta_i = \beta_{\text{initial}} + \beta_{\text{max}} - \beta_{\text{initial}} \frac{1}{2} (1 + \cos \left( \frac{i - T}{L - T} \pi \right)) \tag{8}
\]

It is worth noticing that differ from previous works [Huang et al. (2017); Yang et al. (2019b)], which use cosine annealing schedule, by using one-cycle policy, we also "warm-up" learning rate at the start of each cycle. In our experiments, warming up learning rate is extremely important to achieve high accuracy with deep and large networks.

Surprisingly, retraining with one-cycle policy does not only generate significantly stronger ensembles, but also consistently outperform standard policy for both structured and unstructured pruning in terms of predictive accuracy of individual snapshot. We hypothesize that the (local) optiums of pruned networks are actually far from those of original networks, thus, large learning rate is needed to guarantee the convergence of pruned networks. We leave rigorous evaluation to investigate this phenomenon for future works.

5 EFFECTIVE PIPELINE FOR MODEL COMPRESSION

Since we already obtain strong ensembles during pruning, it is straightforward to distill the knowledge from them to the final pruned network. Our proposed pipeline can be summarized as follow:

1. TRAIN the baseline model to completion.
2. PRUNE redundant weights of the network based on some criteria.
3. RETRAIN the pruned network with large learning rate.
4. REPEAT step 2 and 3 until desired compression ratio is reached.
5. DISTILL knowledge from ensembles of snapshots of pruning.

From now, we refer our pipeline for model compression as Knowledge Distillation from Ensembles of Snapshots of Iterative Pruning (KESI). An overview of our approach is depicted in Figure 1. Our approach is extremely simple, easy to implement and can be adopted with any pruning mechanisms. We discuss the reasons why ensembles of snapshots of pruning are naturally suited for knowledge distillation.

Quality of Teacher In knowledge distillation, student can either learn to jointly optimize the supervised loss (Equation 2) and knowledge distillation loss (Equation 4) or only optimize the distillation objective. In the former case, if the teacher is poorly trained, mathematically speaking, the two objectives will conflict with each other. In the latter case, a poor teacher provides weak supervision (noisy label), making it's harder to learn from the student perspective. Furthermore, ensembles provide more robust predictions on noisy labeled datasets [Lee & Chung (2019)] and out-of-distribution examples [Lakshminarayanan et al. (2017)].

Student and Teacher Gap Although ensembles of snapshots have superior performance than the original network, it is not sufficient to guarantee the improvement in the performance of the student network with Knowledge Distillation. In fact, many works such as [Mirzadeh et al. (2019); Cho & Hariharan (2019); Yang et al. (2018)] show that a powerful teacher might impair its student performance when there is a large gap between their predictive powers. However, ensembles of snapshots of pruning consist of models varying in capacity. Hence, teacher’s predictions of hard-to-learn samples (because of their complexity, multi-modality) will have softer distributions as the small networks could not "remember" those samples and would be more uncertain about them.

In this work, we only investigate knowledge distillation from ensembles of fixed-weights teachers, however, we can also jointly train all models and allow them to guide each other, which is referred to as deep mutual learning [Zhang et al. (2018)].

6 EXPERIMENTS

We conduct experiments on CIFAR-10, CIFAR-100 [Krizhevsky et al. (2009)] and Tiny-Imagenet[1]. We run each experiment 3 times then report the mean and standard deviation. In our experiments, we

https://tiny-imagenet.herokuapp.com
Table 1: Comparing performance of pruned networks with different approaches on CIFAR-10 dataset.

prune all networks in 5 cycles unless otherwise stated. The configurations used for training baselines models are described in supplementary document.

6.1 Experiment setup

Pruning

Structured pruning we use $\ell_1$-norm based filters pruning [Li et al., 2016] for simplicity. In each layer, a fixed number of filters having smallest $\ell_1$-norm will be pruned. Since the bulk of networks tend to be the last layers, we increase the percentage of filters that will be pruned as the layer goes deeper to achieve higher compression ratio.

Unstructured pruning, we exploit (global) magnitude-based weight pruning [Han et al., 2015] i.e. pooling parameters across all layers and pruning weights with lowest magnitude. Specifically, we only prune weights in convolutional layers similar to [Liu et al., 2018].

Retraining

The budget for fine-tuning of each cycle is $T = 40$ and $T = 25$ epochs on CIFAR and Tiny-Imagenet respectively regardless model architectures. In standard policy, the learning rate is set to 0.001 and fixed during retraining. For one-cycle policy, we set the initial learning rate $\eta_{\text{initial}} = 0.01$, gradually increase it to maximum learning rate $\eta_{\text{max}} = 0.1$ in 10% of total (retrain) epochs, then decrease it to minimum learning rate $\eta_{\text{min}} = 0.0001$ for the rest epochs. Other configurations are identical to those of training.

Knowledge Distillation

We use Adam optimizer for ensemble distillation since we find it gives better results than vanilla SGD in general. For knowledge distillation, we also adopt one-cycle policy where we set $\eta_{\text{initial}}, \eta_{\text{max}}, \eta_{\text{min}}$ to $1e^{-4}, 1e^{-3}, 1e^{-6}$ respectively. We do not explicitly use regularization for knowledge distillation. Other configurations e.g. batch size, number of retraining epochs,... are similar to normal retrain. In our experiments, we use temperature $\tau = 5$. The teachers i.e. ensembles of snapshots consist of 6 models including original (unpruned) network and 5 snapshots of pruning.

6.2 Results

6.2.1 Retraining with large learning rate

We conduct experiments to empirically evaluate the performance of pruned networks trained with large learning rate compare to networks fine-tuned with small learning rate. Figure 2 and 3 shows results of pruned networks with different compression ratio for both structured and unstructured pruning. Exhaustive results are reported in supplementary document.

6.2.2 Performance of ensembles of snapshots

We compare the performance of ensembles of snapshots with different approaches: snapshots of pruned networks trained with small learning rate, snapshots of pruned networks trained with large learning rate restarting and snapshots of unpruned networks retrained with large learning rate (i.e.
all snapshots have same architecture as the original network). Figure 4 presents the result of this experiment. We can see that although the capacity are reduced at each cycle, the ensembles of snapshots of iterative pruning achieve competitive or even better than snapshots of networks with same architecture. Detail results of performance of ensembles are reported in supplementary documents.

6.2.3 PERFORMANCE OF COMPACT NETWORKS TRAINED WITH OUR PIPELINE

In this section, we demonstrate that the smaller models trained with our pipeline achieve comparable or even better results than original models. Each final model is iteratively pruned and retrained in 5 cycles with different strategies. Table 2 and 3 present the performance of compact models on CIFAR-10, CIFAR-100 and Tiny-Imagenet. Specifically, we compare the iteratively-pruned-models retrained with small learning rate, large learning rate and our pipeline (i.e. large learning rate + knowledge distillation). Our pipeline consistently outperform standard strategy by a large margin on both structured and unstructured pruning.

Although our approach is general and can be applied to any (iterative) pruning mechanism, we also give a comparison of model trained with our pipeline and conventional approaches in table 6. We conduct experiment to compare performance of student networks trained with single model teacher and ensembles teacher in table 4 for ablation study. The single model teachers are the original.
Table 2: Accuracy (%) of pruned networks on CIFAR-10 and CIFAR-100 dataset trained with different strategies. PFEC (or MWP) are models pruned with \( l_1 \)-norm filters pruning [Li et al., 2016] (or magnitude-based weights pruning [Han et al., 2015]) and fine-tuned with small learning rate. PFEC/MWP+one-cycle are pruned networks retrained with large learning rate restarting. PFEC/MWP+KESI are pruned networks retrained with our pipeline

| Model   | Method                  | #Params (M) | MACs(G) | Acc   |
|---------|-------------------------|-------------|---------|-------|
| ResNet-18 | baseline                | 11.01       | 1.82    | 67.22 |
|         | FPEC [Li et al., 2016]  | 2.71        | 0.83    | 61.06 ± 0.32 |
|         | FPEC+one-cycle          | 2.71        | 0.83    | 64.70 ± 0.33 |
|         | FPEC+KESI (our)         | 2.71        | 0.83    | 66.87 ± 0.26 |
| ResNet-34 | baseline                | 21.39       | 3.68    | 68.81 |
|         | FPEC [Li et al., 2016]  | 5.40        | 1.57    | 64.93 ± 0.15 |
|         | FPEC+one-cycle          | 5.40        | 1.57    | 67.26 ± 0.21 |
|         | FPEC+KESI (our)         | 5.40        | 1.57    | 70.02 ± 0.43 |

Table 3: Performance of compact models on Tiny-Imagenet

| Model   | Method                  | #Params (M) | C10   | C100  |
|---------|-------------------------|-------------|-------|-------|
| ResNet-56 | baseline                | 0.85        | 93.42 | 71.07 |
|         | single teacher          | 0.28        | 93.13 ± 0.04 | 70.29 ± 0.14 |
|         | ensemble teacher        | 0.28        | 93.34 ± 0.05 | 72.27 ± 0.09 |
| ResNet-110 | baseline               | 1.73        | 94.01 | 72.35 |
|         | single teacher          | 0.39        | 93.48 ± 0.05 | 71.50 ± 0.11 |
|         | ensemble teacher        | 0.39        | 94.01 ± 0.22 | 73.12 ± 0.25 |
| WRN-16-8 | baseline                | 19.96       | 95.02 | 79.71 |
|         | single teacher          | 2.48        | 95.37 ± 0.21 | 78.71 ± 0.24 |
|         | ensemble teacher        | 2.48        | 95.68 ± 0.12 | 79.01 ± 0.20 |

Table 4: Knowledge distillation with ensembles teacher and single model teacher

(unpruned) networks. Clearly, compact model learn from ensembles outperform those learn from single teacher.

7 CONCLUSION

We propose simple pipeline by slightly modifying the standard approach to acquire the advantages of network ensembles, knowledge distillation and network pruning. Our experiments show that small, compact networks trained with our pipeline significantly outperform standard approach and create very strong baselines for model compression. Specifically, our method reduce nearly 80% of parameters and 70% FLOPs of several models by structured pruning without incurring loss in performance.

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### A Detail Results

| Model               | #Param | % MACs(G1) | C10-SLR | C10-LLR | C100-SLR | C100-LLR |
|---------------------|--------|------------|---------|---------|----------|----------|
| Resnet-110 (baseline) | 1.73M  | 0.00       | 94.01   | 94.01   | 72.35    | 72.35    |
| Resnet-110 #1        | 1.26M  | 23.08      | 93.51±0.00 | 93.58±0.05 | 69.70±0.17 | 71.67±0.03 |
| Resnet-110 #2        | 0.93M  | 38.46      | 92.91±0.02 | 93.49±0.05 | 68.47±0.20 | 70.97±0.43 |
| Resnet-110 #3        | 0.70M  | 50.00      | 92.65±0.02 | 93.53±0.12 | 66.89±0.14 | 70.59±0.19 |
| Resnet-110 #4        | 0.52M  | 57.69      | 92.11±0.01 | 93.62±0.03 | 66.19±0.07 | 70.26±0.26 |
| Resnet-110 #5        | 0.39M  | 65.38      | 91.51±0.08 | 93.24±0.16 | 65.44±0.04 | 69.54±0.07 |
| Resnet-110 Ensemble  | -      | -          | 93.82    | 94.01±0.22 | 73.32    | 75.33±0.14 |
| Resnet-56 (baseline) | 0.85M  | 0.00       | 93.42    | 93.42    | 71.07    | 71.07    |
| Resnet-56 #1         | 0.66M  | 23.07      | 92.73±0.06 | 93.23±0.18 | 69.10±0.15 | 69.83±0.09 |
| Resnet-56 #2         | 0.52M  | 30.77      | 92.24±0.17 | 93.21±0.00 | 67.92±0.05 | 69.58±0.13 |
| Resnet-56 #3         | 0.42M  | 46.15      | 91.64±0.19 | 92.90±0.16 | 66.76±0.03 | 69.50±0.25 |
| Resnet-56 #4         | 0.35M  | 53.85      | 91.10±0.27 | 92.74±0.22 | 65.20±0.05 | 69.49±0.08 |
| Resnet-56 #5         | 0.29M  | 61.54      | 90.35±0.36 | 92.31±0.26 | 64.91±0.14 | 69.03±0.24 |
| Resnet-56 Ensemble   | -      | -          | 93.25    | 94.29±0.02 | 71.37    | 74.23±0.21 |
| VGG-16 (baseline)    | 14.99M | 0.00       | 94.23    | 94.23    | 73.24    | 73.24    |
| VGG-16 #1            | 9.46M  | 0.00       | 94.13±0.09 | 93.95±0.02 | 71.39±0.06 | 72.38±0.11 |
| VGG-16 #2            | 6.27M  | 0.00       | 94.09±0.13 | 93.90±0.03 | 70.48±0.07 | 72.10±0.1 |
| VGG-16 #3            | 4.43M  | 0.00       | 94.09±0.04 | 93.93±0.04 | 69.73±0.06 | 72.28±0.11 |
| VGG-16 #4            | 3.36M  | 0.00       | 94.03±0.13 | 93.89±0.10 | 69.09±0.05 | 72.22±0.19 |
| VGG-16 #5            | 2.71M  | 0.00       | 93.88±0.12 | 94.10±0.09 | 68.37±0.09 | 71.95±0.04 |
| VGG-16 Ensemble      | -      | -          | 94.29    | 95.04±0.07 | 72.86±0.02 | 75.93±0.06 |
| PreResNet-164 (baseline) | 1.7M  | 0.00      | 95.06    | 95.06    | 76.35    | 76.35    |
| PreResNet-164 #1     | 1.09M  | 26.92      | 94.43±0.06 | 94.92±0.05 | 74.65±0.04 | 76.20±0.07 |
| PreResNet-164 #2     | 0.74M  | 46.15      | 93.73±0.07 | 94.74±0.14 | 73.17±0.03 | 75.87±0.03 |
| PreResNet-164 #3     | 0.54M  | 57.69      | 93.50±0.01 | 94.66±0.19 | 71.89±0.01 | 75.15±0.16 |
| PreResNet-164 #4     | 0.4M   | 65.38      | 92.61±0.02 | 94.69±0.17 | 70.50±0.09 | 74.03±0.68 |
| PreResNet-164 #5     | 0.31M  | 69.23      | 92.06±0.11 | 94.15±0.06 | 69.20±0.04 | 73.99±0.06 |
| PreResNet-164 Ensemble | -     | -          | 95.60    | 95.04±0.04 | -        | 79.19±0.07 |
| WideResNet-16-8 (baseline) | 11.01M | 0.00    | 95.82    | 95.82    | 79.57    | 79.57    |
| WideResNet-16-8 #1   | 8.01M  | 20.00      | 95.30±0.01 | 95.18±0.1 | 78.52±0.04 | 78.19±0.19 |
| WideResNet-16-8 #2   | 5.89M  | 35.48      | 95.20±0.02 | 95.25±0.14 | 77.46±0.14 | 77.81±0.2 |
| WideResNet-16-8 #3   | 4.38M  | 47.74      | 94.95±0.01 | 95.08±0.16 | 76.29±0.02 | 77.43±0.36 |
| WideResNet-16-8 #4   | 3.28M  | 57.42      | 94.97±0.01 | 95.08±0.08 | 74.73±0.03 | 76.95±0.21 |
| WideResNet-16-8 #5   | 2.48M  | 64.52      | 94.61±0.09 | 94.91±0.04 | 73.82±0.1 | 76.46±0.27 |
| WideResNet-16-8 Ensemble | -   | -          | 95.63    | 95.79±0.08 | 79.22±0.03 | 80.45±0.14 |

Table 5: Results of iterative Filter Pruning on CIFAR-10 and CIFAR-100 dataset. The SLR column presents the result of the pruned networks trained with small, fixed learning rate while LLR column shows the results of same networks trained with large learning rate.