Doppler imaging

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Abstract

In this paper, I present a short review of the history and modern status of Doppler imaging techniques, highlighting their dependence on the knowledge of the fundamental stellar parameters, the quality of stellar atmospheric models and the accuracy of spectral synthesis.

(Some figures in this article are in colour only in the electronic version.)

1. Introduction

The term ‘Doppler Imaging’ (DI) was coined by Steven Vogt in 1987 (Vogt et al 1987), who was the first to apply this method to solar-type stars, but the story of DI started some 25 years earlier when Deutsch (Deutsch 1957) presented the concept of ‘oblique rotator’ to explain periodic and persistent variations of spectral line strength in a subset of A-stars classified as chemically peculiar or CP stars (the term CP stars replaced the originally used designation, Ap-stars, when abnormal chemical abundances were found in stars of other spectral types). CP stars have been known since the beginning of the 20th century (Cannon and Pickering 1912). They have characteristically slow rotation (a few tens of km s$^{-1}$) and abnormal line strengths (compared with ‘normal’ stars), which are also variable. The period of variability is the same for all elements and, as was noticed by Deutsch, also consistent with the rotational period of the star estimated from its rotational velocity. The oblique rotator model assumes that chemical elements are concentrated in spots spread over the stellar surface and variability is caused by the stellar rotation. This prompted Mégnissier (Mégnissier 1975) to try using Fourier analysis in order to recover spot location and contrast but it was soon realised that the solution is not unique. In the 1970s, Goncharsky and Khokhlova have shown that direct methods like Fourier analysis are not applicable to mapping of stellar surfaces because this problem is mathematically represented by a Fredholm equation of the first kind, which belongs to a special case of so-called ill-posed problems. The concept of ill-posed problems was introduced by French mathematician Jacques Hadamard (Hadamard 1902) in the late 19th century, which in the case of Fredholm equations manifests itself as a lack of continuity between the solution and the right-hand side. The method for solving ill-posed problems was developed by the Russian mathematician Andrey Tikhonov (Tikhonov 1943) who proposed the concept of a regularized inverse problem. The collaboration among the students of Tikhonov, Goncharsky and my PhD supervisor Vera Khokhlova lead to the development of the first DI maps of CP stars (Goncharsky et al 1981).

Since then the technique was continuously perfected following the dramatic improvement in the quality of astronomical observations, the growth of computer power and our understanding of the physics of the outer stellar regions including structure formation and dynamics.

In the following sections, I will explain the principles of DI techniques, and present some of the important scientific results obtained through the application of DI to hot and cool stars. I will also present generalizations of DI to the case of eclipsing binaries and to magnetic fields (so-called magnetic Doppler Imaging or MDI).

2. How does the DI work?

DI is a technique to reconstruct the shape, the location and the contrast of stellar surface structures from a time sequence of high-resolution spectra. In this section, I will explain the conventional DI procedure in three steps. First, I will give a qualitative explanation why DI is feasible at all and what are the limitations of DI. Then, I will present a more rigorous mathematical formulation for DI, introduce the concept of regularization and give a short overview of the numerical procedure implementing DI. The last part emphasizes the role of spectral synthesis in producing reliable DI maps. All the parts are rather concise. A much more detailed introduction to DI can be found in Piskunov and Rice (1993).
2.1. Intuitive picture

How is it possible to obtain a map of a stellar surface using a spatially unresolved sequence of spectra? This is possible because a spot (or any inhomogeneity on the surface) creates a perturbation to the shape of the disk integrated line profile at the wavelengths corresponding to the Doppler shift of the spot as illustrated in figure 1. Throughout the rotational period of a star, the spot will appear on the approaching side, pass the subsolar meridian and disappear on the receding side. The corresponding profile perturbation will appear in the blue wing of the line profile and gradually move to the red wing. For the case when the rotational axis is exactly perpendicular to the line of sight, the profile distortion will be visible for exactly half of the rotational period. The largest Doppler shift excursion of the profile perturbation occurs for the equatorial spot. For the more realistic case of a non-90° inclination the picture changes. The spots close to the North Pole remain visible for the major part of the period, whereas spots in the Southern Hemisphere are visible for shorter than half of the period. The relation between the Doppler shift amplitude and the latitude remains the same.

This behaviour tells us that the information about surface structures is present in the time sequence of the line profiles. Indeed, the longitude of a spot is directly given by the phase of the perturbation. The latitude affects the visibility time and the amplitude of the Doppler shift excursion, whereas the amplitude of perturbation reflects the contrast of the spot. At this point, it is useful to look at the nature of the ‘spots’. For the cool stars, ‘spot’ is actually a local change of the stellar atmospheric structure, probably caused by magnetic activity, which can be approximately parametrized by a single parameter—local effective temperature. The important point is that the same surface distribution of temperature will affect all spectral lines and thus blends can/should be treated simultaneously while recovering a single surface map. For the CP stars, a ‘spot’ is actually a local change in chemical composition, which is different for different elements. Thus, the only consistent alternative is either to use single non-blended lines (which are hard to find) or to reconstruct multiple maps simultaneously.

This qualitative illustration of how a pattern on the stellar surface is reflected in the variability of the spectrum also gives hints on the limitations of DI. In particular, for inclinations close to 90°, there are no possibilities to differentiate between northern and southern latitudes (mirror degeneracy). Also, the interpretation of the Doppler shifts is only possible if the intrinsic profile is narrower than the disk integrated one, so that the number of spatial elements around the equator is limited by the ratio of the projected rotational velocity on the equator to the intrinsic line width in km s\(^{-1}\). The spatial resolution is then

\[
\Delta_{\text{longitude}} = \frac{360° \Delta \lambda}{v_c \sin i},
\]

where \(\Delta_{\text{longitude}}\) is the smallest achievable resolution element on the stellar equator (degrees) and \(\Delta \lambda\) the intrinsic line width in km s\(^{-1}\) (HWHM).

2.2. Mathematical picture

Now that we know for a fact that the pattern on the stellar surface changes the rotational modulation of the spectrum, we need to establish a realistic physical model for this relation.
Any surface element \( M \) characterized by its latitude \( \theta \) and longitude \( \psi \) at phase zero and the local properties \( X \) (temperature, abundances, etc) radiates towards the observer with an intensity \( I_\lambda \), which can be computed by solving the radiative transfer equation:

\[
\frac{dI_\lambda}{dz} = -\kappa_\lambda(z, X)\rho(z, X)[I_\lambda - S_\lambda(z, X)],
\]

where \( \mu \) is the angle between the local vertical through point \( M \) and the line of sight, \( \kappa_\lambda(z, X) \) the extinction coefficient per unit mass, \( \rho(z, X) \) the local density structure and \( S_\lambda(z, X) \) the source function. Note that these last three functions of geometrical depth also depend on the local properties characterized by \( X \).

We do not resolve the disk of a star, and so we measure the integrated flux at rotational phase \( \phi \) given by the first moment of the intensity as:

\[
F_\lambda(\phi) = \int_{-\pi/2}^{+\pi/2} d\theta \int_0^{2\pi} I_{\lambda+\Delta\lambda_0}(\theta, \psi + 2\pi \cdot \phi, X) \mu(\theta, \psi + 2\pi \cdot \phi) \cos \theta d\psi,
\]

where disk integration takes into account surface inhomogeneities and Doppler shifts, which are functions of surface coordinates and phase. The relation between the Doppler shift and stellar coordinates is assumed to be known analytically and at this stage, we can easily incorporate additional effects like differential rotation and radial–tangential macroturbulence.

The data that are obtained from the reduced high-resolution spectra are actually convolved with the instrumental profile of the spectrometer and continuum normalized. Thus, the simulated observations are given by the expression:

\[
R_\lambda^{\text{calc}}(\phi) = 1 - \frac{\int_{-\infty}^{+\infty} \gamma(\lambda - \ell) F_\ell(\phi) d\ell}{F_\lambda^{\text{cont}}(\phi)},
\]

where \( \gamma(\lambda) \) is the instrumental profile and \( F_\lambda^{\text{cont}}(\phi) \) is the continuum flux computed using equations similar to equations (2) and (3), except that we can ignore Doppler shifts. Note that in the case of imaging temperature spots or high-contrast abundance spots affecting important opacity sources, the continuum flux retains the phase dependence.

\( R_\lambda^{\text{calc}}(\phi) \) defined above is to be compared with the observed spectra \( R_\lambda^{\text{obs}}(\phi) \), for example, in the sense of least squares as:

\[
E = \sum_\phi \sum_\lambda \left[ R_\lambda^{\text{calc}}(\phi) - R_\lambda^{\text{obs}}(\phi) \right]^2.
\]

Unfortunately, attempts to solve for the best surface distribution of \( X \) directly or to find the minimum of the discrepancy function \( E \) in the \( X \) space show that, in general, DI belongs to the class of ill-posed problems. Therefore, following Tikhonov, we replace the minimization of the discrepancy function with the minimization of the regularized discrepancy function:

\[
\Xi = E + \Lambda \cdot \Im(X) = \min,
\]

where \( \Im(X) \) is the regularization function that limits the space of possible solutions. If \( \Im \) complies with the requirements for regularization functions, one can find the regularization parameter \( \Lambda \) such that the solution of equation (6) will be unique. One particular form of \( \Im \), named after Tikhonov, is a measure of absolute values of the local gradients in the reconstructed map:

\[
\Im(X) = \sum_M |\vec{\nabla} X|.
\]

Another well-known regularization function is the maximum entropy (Egggermont 1993), which was also actively used in DI applications.

Various numerical techniques have been used to solve equations (2)–(6) in the most efficient and accurate way. The approximate solutions of the radiative transfer were gradually replaced by pre-tabulated local line profiles and nowadays the actual solving is done on the fly to be fully consistent with the surface map of \( X \) in every iteration. The optimization problem, initially solved with conjugate gradient algorithms as one of the least demanding in terms of memory, nowadays is usually handled with the Levenberg–Marquardt algorithm (Marquardt 1963), reducing the number of iterations from hundreds to less than ten. Finally, efficient parallelization techniques were proposed for DI problems (Piskunov 2001). Not only does it scale well with the number of CPUs but also ensures automatic load balancing on heterogeneous or shared systems.

2.3. The importance of spectral synthesis

Considering the rather weak limitations of DI in terms of geometry and resolution, one can expect this technique to perform very well for a wide variety of targets. At this point, I would like to point out that the quality of spectral synthesis is the most critical part of DI. Indeed, the mathematical procedure described above will always produce a map, but how realistic is this map fully depends on the ability of the spectral synthesis to represent the ‘local’ spectrum for various disk locations and for the range of the local characteristics \( X \). Generating such synthetic spectra involves several components: line lists, model atmospheres and the radiative transfer itself. While the last step is sufficiently well established even in the presence of magnetic fields, the reliability of the line lists and model atmospheres remains shaky. Often the best sensitivity of the data to the mapped parameter(s) can be obtained by using several spectral intervals spread across a wide spectral range, but this just aggravates the above-mentioned problem. The remedy for this is to select the best available models that have actually been compared with reliable flux-calibrated observations and to the limb-darkening law of the Sun, and then compare the spectral synthesis for the non-spotted reference star(s) with the high-resolution observations. For the cool stars, the MARCS grid of models (Gustafsson et al 2008) is one example of such carefully tested atmospheric models. The test spectral synthesis also helps in verifying the line lists and oscillator strengths, and in detecting non-identified spectral features. Other fundamental stellar parameters needed for DI, whose determination relies on accurate spectral synthesis, are the projected rotational velocity \( v \sin i \) and the inclination angle \( i \).
3. Examples of applications

As I mentioned before, the first applications of DI were aimed at surface abundance distributions in CP stars. Later, DI was applied to imaging temperature distributions on active solar-type stars. In the first section of this section, I will illustrate the potential of DI when applied to the special case of an active eclipsing binary system.

The dramatic element overabundances and complex spot distributions found on CP stars using DI motivated Georges Michaud to propose a vertical chemical stratification hypothesis (Michaud 1970) produced by element diffusion in the radiation and magnetic fields. While it reconciled the chemical composition of CP stars with the cosmic abundances, Michaud’s theory was in apparent contradiction with the observations as the complex geometry of the chemical spots was inconsistent with the dipolar magnetic fields believed to be typical for CP stars. In the second part of this section, I will show how the generalization of DI to the case of magnetic fields helped in solving this puzzle.

3.1. Application to eclipsing binary system ER Vulpeculae

The interplay between the projected rotational velocity and the equatorial degeneracy for objects with an inclination close to 90° implies that \( i \approx 45° \) is optimal for a single star. Eclipsing binaries have inclinations closer to 90°, but the presence of eclipse breaks the degeneracy (Vincent et al 1993). One of the best results, shown in figure 2, was obtained for a short period system ER Vul. ER Vul consists of G0V and G5V stars on a circular orbit with a radius of only 3.97 \( R_\odot \) seen at an inclination of 67°. Stellar rotation is synchronized with the orbital motion and the period is 0.698 days. Similar radii of the components (1.32 and 1.27 \( R_\odot \)) and eclipses covering nearly half the stellar disks represent an ideal configuration for applying DI.

![Figure 2. The results of temperature reconstruction on the surface of ER Vul. The phases are shown in various orbital phases. The sizes and the separation between the components are shown to scale. One sees a bright region around the subsolar point on each star associated with the reflection effect. There are also a few cool spots produced by the non-axisymmetric dynamo action. Breaking of the axial symmetry is induced by tidal interaction.](image-url)
Figure 3. The results of magnetic field reconstruction on the magnetic CP stars 53 Cam using our MDI technique. The top row shows the magnetic field strength, whereas the bottom row shows field vector orientation.

Figure 4. The abundance maps of 53 Cam reconstructed self-consistently and simultaneously with the magnetic fields presented in figure 3. Abundances are given as the ratio of specific atom to the total number of atoms.
techniques. Simultaneous reconstruction of the temperature distribution on both stars in addition to the (expected) cool spots, presumably related to the dynamo action, revealed the bright spots at subsolar points. We interpreted the bright spots as the first direct detection of the reflection effect: the heating of the stellar atmosphere by external radiation. Returning to this star during three consecutive seasons showed a persistent pattern of cool spots located at intermediate latitudes (±60°) and shifted by ±90° from the center of the reflection effect. The fact is that we see persistent longitudes of the cool spots points at non-axisymmetric dynamo action in ER Vul, which was investigated by Sokoloff and Piskunov (2002) and Moss et al (2002). These authors demonstrated that the tidal interaction in ER Vul is strong enough to break the axial symmetry of the mean field dynamo. We found a resonant excitation of an oscillatory, strongly non-axisymmetric dynamo driven by the joint action of the α-effect and differential rotation. The results of the numerical simulations predict surface magnetic structures in the form of spots of radial magnetic field. Their form and size are similar to the form and size of cool spots observed on the surfaces of both components of ER Vulpeculae, and their displacements from the α-spot are comparable to the longitude distance between the observed cool spots and the hot spot associated with the reflection effect.

3.2. MDI of 53 Cam

The availability of polarization observations in four Stokes parameters opens a new page in the history of DI. Based on our code for polarized radiation transfer, we developed an MDI code capable of reconstructing the distribution of magnetic vectors on the surface of a star. The first target for MDI was (expectedly) a fast rotating CP star, 53 Cam (figure 3). Modeling time-series of high-resolution spectropolarimetric observations, we constructed maps of magnetic fields and abundances of several chemical elements. The combination of the unique four Stokes parameter data and a state-of-the-art magnetic imaging technique made it possible to infer the stellar magnetic field topology directly from the rotational variability of the Stokes spectra without involving the traditional low multipole assumptions about the field geometry. Thus, the stellar magnetic topology was reconstructed without introducing any global a priori constraints on the field structure. The complex magnetic model of 53 Cam derived with our MDI method achieves an excellent fit to the observed intensity, circular and linear polarization profiles of strong magnetically sensitive Fe II spectral lines used for magnetic reconstruction. Such an agreement between observations and model predictions was not possible with any earlier multipolar magnetic models, based on modeling Stokes I spectra and fitting surface averaged magnetic observables (e.g. longitudinal field, magnetic field modulus, etc). Furthermore, we demonstrated that even the direct inversion of the four Stokes parameters of 53 Cam assuming a low-order multipolar magnetic geometry is incapable of achieving an adequate fit to our spectropolarimetric observations. The main result of our investigation is the demonstration of the high complexity of the magnetic topology in 53 Cam, much more reminiscent of complex geometry of chemical element distribution, which is good news for the magnetic chemical diffusion theory. In addition to the analysis of the magnetic field of 53 Cam, we reconstructed surface abundance distributions of Si, Ca, Ti, Fe and Nd (figure 4). These reconstructions confirmed field maps derived from Fe II and closely reproduced the results of the previous studies of 53 Cam, in particular, a dramatic anti-phase variation of Ca and Ti.

4. Conclusions and future prospects

I have selected two spectacular examples illustrating the capabilities of DI techniques. Nowadays DI is a well understood and widely used tool for studying structure formation and the dynamics of stellar surfaces. The main areas of applications include the study of 3D chemical stratification in rapidly oscillating Ap stars (Kochukhov 2004) and asteroseismology. Here, DI is used not only for mapping long-lived horizontal structures but also to identify pulsation modes by producing velocity and phase maps. Another very new and fast developing area is the application of MDI techniques to very cool stars, in particular, the most active M-dwarfs that seem to show strong and globally organized magnetic fields. The first observations in all Stokes parameters of M-dwarfs are just becoming available and I expect MDI maps to provide crucial hints about the generation of the magnetic field in these fully convective stars.

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