Spatial Kramers-Kronig relation and unidirectional light reflection induced by Rydberg dipole-dipole interactions

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Kramers-Kronig (KK) relation between the dispersion and absorption responses of a signal field can be mapped from the frequency domain into the space domain via the dipole-dipole interactions between a homogeneous sample of target atoms and a control atom. This is achieved by establishing an effective two-level configuration for the three-level target atoms in the single-photon far-detuned driving regime while maintaining a high Rydberg excitation for the three-level control atom in the single-photon resonant driving regime. We find in particular that it is viable to realize a dynamically tunable spatial KK relation supporting asymmetric and even unidirectional reflection for appropriate signal frequencies in a controlled range. Taking a periodic lattice of target atoms instead, multiple Bragg scattering can be further incorporated into spatial KK relation to largely enhance the nonzero reflectivity yet without breaking the asymmetric or unidirectional reflection.

I. INTRODUCTION

In recent years, great efforts have been made in the realization and manipulation of asymmetric light reflection and even unidirectional invisibility with artificial optical structures of complex optical potentials. One main motivation lies in that relevant advances are essential for developing one-way optical devices unattainable with natural linear materials of real optical potentials. Reflection and transmission properties are typically bidirectional and symmetric for isotropic linear materials based on the Lorentz reciprocal theorem. This can also be understood in view of information optics, which argues that the Fourier transform of a real optical potential is definitely symmetric so that light propagation in natural linear materials always results in balanced forward and backward modes.

Now it is known that unidirectional reflection and invisibility can be attained at an exceptional point in non-Hermitian media exhibiting, e.g., parity-time (PT) symmetry. These media are, however, very challenging in regard of the experimental implementation because they require elaborate designs of gain and loss. Horsley et al. found in 2015 that electromagnetic waves incident upon an inhomogeneous medium, the real and imaginary parts of whose complex permittivity are related in space via the Kramers-Kronig (KK) relation, can be efficiently absorbed from one side but normally reflected from the other side. Soon afterwards, results in this pioneer work were extended in theory, verified in experiment, and explored to develop new techniques of holographic imaging or anechoic chamber. Such spatial KK media, though requiring no elaborate designs on gain and loss, are typically designed with fixed structures and lack the dynamic tunability. A feasible method for overcoming this difficulty is to consider multi-level driven atomic systems, in which the frequency-to-space mapping of an induced susceptibility can be attained via a dynamic Stark or Zeeman effect.

On the other hand, we note that nonlocal dipole-dipole interactions (DDIs) of Rydberg atoms depend critically on the interatomic distance and can be manipulated on demand by external driving fields. This then motivates us to seek a feasible driving scheme where DDIs can be used to realize the spatial KK relation by establishing a nonlinear dependence of atomic transition frequency on atomic spatial position. To be more specific, DDIs may manifest as either van der Waals (vdW) potentials scaling as $1/R^6$ in the non-resonant regime or Förster-like potentials scaling as $1/R^3$ in the resonant regime. In fact, Rydberg atoms have been well studied as an intriguing platform for realizing quantum information processing, and high-precision field sensing, considering that they also exhibit the features of long radiative lifetimes and large electric dipole moments. Note also that Rydberg atoms have been explored in the regime of electromagnetically induced transparency (EIT) to achieve effective interactions between individual photons, which promise the realization of non-trivial photonic devices like single-photon sources, memorizers, and transistors. To the best of our knowledge, DDIs of Rydberg atoms have not been considered to develop photonic devices supporting asymmetric light propagation behaviors.

We examine here an effective scheme for the realization of a tunable spatial KK relation in a homogeneous sample of cold target atoms by utilizing their vdw interactions with a control atom. This is done by considering a single-photon resonant driving configuration for the control atom in the dark-state regime while a single-photon far-detuned driving configuration for the target atoms in the EIT regime. Under appropriate conditions, the control atom can be made to exhibit a roughly perfect...
Rydberg excitation via a dark-state manipulation while the target atoms may reduce from a three-level to a two-level configuration by adiabatically eliminating the intermediate state. On this account, it is viable to realize a nonlinear frequency-to-space mapping of the dispersion and absorption responses and hence a well established and modulated spatial KK relation. Consequently, the reflectivity of a signal field incident upon one side is distinct from that upon the other side and may even become vanishing to result in unidirectional reflection. Replacing the homogeneous atomic sample with a periodic atomic lattice, we further show it is viable to improve the asymmetric and unidirectional reflection behaviors, by largely enhancing the nonzero reflectivity yet without activating the vanishing reflectivity, when multiple Bragg scattering is incorporated into spatial KK relation.

II. MODEL AND EQUATIONS

We start by introducing our basic model in Fig. I(a), where a signal field of amplitude (frequency) \(E_s(\omega_s)\) is incident upon a homogeneous sample of cold target atoms from the \(x = 0\) or \(x = L\) side, and the overall optical response of target atoms is modulated by a control atom at \(x = x_0\) via \(vdW\) interactions relevant to a high Rydberg excitation. The control atom is driven by a pumping field of amplitude (frequency) \(E_p(\omega_p)\) on transition \([1] \leftrightarrow [2]\) and an exciting field of amplitude (frequency) \(E_e(\omega_e)\) on transition \([2] \leftrightarrow [3]\) as shown in Fig. I(b), being \(\Omega_p = E_p/2\hbar\) and \(\Omega_e = E_e/2\hbar\) corresponding Rabi frequencies while \(\Delta_p = \omega_p - \omega_{21}\) and \(\Delta_e = \omega_e - \omega_{32}\) corresponding detunings. The target atoms are driven instead by the signal field on transition \([g] \leftrightarrow [e]\) and a coupling field of amplitude (frequency) \(E_c(\omega_c)\) on transition \([e] \leftrightarrow [r]\) as shown in Fig. I(b), being \(\Omega_e = E_e/2\hbar\) and \(\Delta_c = \omega_c - \omega_{re}\) corresponding detunings. Above we have used \(\nu_{\mu}\) and \(\omega_{\mu\nu}\) to denote dipole moments and resonant frequencies, respectively, on transitions \([\mu] \leftrightarrow [\nu]\) with \(\{\nu, \mu\} \in \{1, 2, 3\}\) for the control atom while \(\{\nu, \mu\} \in \{g, e, r\}\) for the target atoms. In addition \(\Delta_p = \Delta_c = 0\) and \(\Delta_e \approx -\Delta_e\) have been considered in Fig. I(b) as an illustration of our interest.

It is worth noting that, the signal and coupling fields have negligible effects on, despite traveling through, the control atom because they are assumed to be far detuned from the \([1] \leftrightarrow [2]\) and \([2] \leftrightarrow [3]\) transitions, respectively. This may be achieved by considering ground states \([1] \equiv [5S_{1/2}, F = 1]\) and \([g] \equiv [5S_{1/2}, F = 2]\), intermediate states \([2] \equiv [5P_{3/2}, F = 0]\) and \([e] \equiv [5P_{3/2}, F = 3]\), and Rydberg states \([3] = [r]\) \(\equiv [90S_{1/2}]\) for the \(^{87}\text{Rb}\) isotope. The pumping and exciting fields, however, don’t travel through the target atoms as arranged in Fig. I(a).

With above considerations, we can easily write down the following Hamiltonians by setting \(\hbar = 1\)

\[
H_c = -\Delta_p \sigma_{22} - (\Delta_p + \Delta_e) \sigma_{33} - \Omega_p \sigma_{21} - \Omega_e \sigma_{32} - \Omega_c^* \sigma_{12} - \Omega_e^* \sigma_{23},
\]

\[
H_t = -\Delta_s \sigma_{ee} - (\Delta_s + \Delta_c) \sigma_{rr} - \Omega_s \sigma_{eg} - \Omega_e \sigma_{re} - \Omega_c^* \sigma_{gr} - \Omega_e^* \sigma_{rr} + V_0 \sigma_{33} \sigma_{rr},
\]

for the control and target atoms in order. Here we introduce \(\sigma_{\mu\nu} = [\nu | \mu]\) to denote the transition \((\nu \neq \mu)\) or projection \((\nu = \mu)\) operator, while \(V_0 = C_6/(x-x_0)^6\) represents the \(vdW\) potential of coefficient \(C_6\) for the control atom at \(x_0 < 0\) and a target atom at \(x > 0\).

Dynamic evolution of the control atom is governed by the master equation for density operator \(\rho\)

\[
\partial_t \rho = -i[H_c, \rho] + L_c(\rho),
\]

where \(L_c(\rho) = \sum \Gamma_{\mu\nu} |\sigma_{\nu\mu}\rangle \rho_{\mu\nu} - \frac{1}{2} \{\rho_{\mu\nu} |\sigma_{\nu\mu}\rangle + |\sigma_{\nu\mu}\rangle \rho_{\mu\nu}\}\) describes the dissipation processes contributed by population decay rates \(\Gamma_{32}\) and \(\Gamma_{21}\) on the \([3] \leftrightarrow [2]\) and \([2] \leftrightarrow [1]\) transitions, respectively. Using \(H_c\) and \(L_c(\rho)\), it is easy to expand Eq. (2) into a set of dynamic equations on nine density matrix elements \(\rho_{\mu\nu}\), with \(\{\mu, \nu\} \in \{1, 2, 3\}\). These equations can be solved by setting \(\partial_t \rho_{\mu\nu} = 0\) to attain the dark-state [56, 57] Rydberg population

\[
\rho_{33} \simeq \frac{(\gamma_{21} + \gamma_{31}) \Omega_e^2 \Omega_p^2}{\gamma_{21} \Omega_e^4 + (\gamma_{21} + 3\gamma_{31}) \Omega_e^2 \Omega_p^2 + \gamma_{21}^2 \gamma_{31} \Omega_p^4},
\]
in the limit of $\Delta_p = \Delta_e = 0$ and $\Omega_p \geq \Omega_e > \gamma_{21} \gg \gamma_{31}$ with $\gamma_{31} = \Gamma_{32}/2 + \gamma_{31}^d$ and $\gamma_{21} = \Gamma_{21}/2$. Here $\gamma_{31}^d$ denotes a pure dephasing rate arising from finite laser linewidths and has to be included because $\Gamma_{32}$ is negligible for high Rydberg states. Moreover, keep in mind that $\gamma_{31}$ should be much smaller than other parameters so as to maintain the dark state $|D\rangle = c_1|1\rangle - c_3|3\rangle$ by excluding state $|2\rangle$, hence it is viable to obtain $\rho_{33} = |c_3|^2 \sim \Omega_p^2/(\Omega_p^2 + \Omega_e^2) \rightarrow 1$ of our interest by further requiring $\Omega_p^2 \gg \Omega_e^2$.

With the same strategy, after introducing population decay rates $\Gamma_{re}$ and $\Gamma_{rg}$ as well as dephasing rates $\gamma_{re} = (\Gamma_{re} + \Gamma_{eg})/2$, $\gamma_{rg} = \Gamma_{re}/2 + \gamma_{re}^g$, and $\gamma_{eg} = \Gamma_{eg}/2$, we can write down a new set of dynamic equations on nine density matrix elements $\rho_{\mu\nu}$ with $\{\mu, \nu\} \in \{g, e, r\}$ for the target atoms. These equations can be solved by setting $\partial_t \rho_{\mu\nu} = 0$ and $\rho_{ee} \rightarrow 0$ in the limit of $\Delta_e \simeq -\Delta_c$, $|\Delta_s| \gg \gamma_{eg} \gg \Omega_s$, and $|\Delta_c| \gg \Omega_c \gg \gamma_{re}$ to attain

$$\rho_{gg} = \frac{\Gamma_{re}[\gamma_{22}^g + (\delta_{eff} + \rho_{gg33})^2]/2 + \gamma_{rg} \Omega_{eff}^2}{\Gamma_{re}[\gamma_{22}^g + (\delta_{eff} + \rho_{gg33})^2]/2 + \gamma_{rg} \Omega_{eff}^2},$$

$$\rho_{rg} = \frac{\Gamma_{re}[\gamma_{21}^g + (\delta_{eff} + \rho_{gg33})^2]/2 + \gamma_{rg} \Omega_{eff}^2}{\Gamma_{re}[\gamma_{21}^g + (\delta_{eff} + \rho_{gg33})^2]/2 + \gamma_{rg} \Omega_{eff}^2},$$

restricted by $\rho_{eg} = -\Omega_e^2 \rho_{gg} + [\Delta_c^2 \rho_{gg}]/\gamma_{re}$, and $\rho_{gg} + \rho_{rr} = 1$. Here $\Omega_{eff} = \Omega_s \Delta_c/\Delta_e$ is an effective two-photon Rabi frequency while $\delta_{eff} = \Delta_s + \Delta_e - \Delta_c$ is an effective two-photon detuning modified by $\Delta_s \equiv \Omega_s^2/\Delta_e$ and $\Delta_c \equiv \Omega_c^2/\Delta_e$.

Further considering $\gamma_{rg} \gg \Omega_{eff}^2$, which is available by enhancing $\gamma_{rg}$ with finite laser linewidths $\gamma_{re}^g$ and $\Gamma_{re}$ via incoherent (downward) pumpings, we can attain with Eq. (4) an induced signal susceptibility

$$\chi_s = \frac{N_0 \rho_{gg33}}{\hbar \omega_0} \left[ \Omega_{eff}^2 \gamma_{21}^g + \Delta_c \Delta_e \gamma_{22}^g + (\delta_{eff} + \rho_{gg33})^2 - \frac{1}{\Delta_s} \right],$$

describing the target atoms reduced to a two-level configuration as shown in Fig. 1(c). It is worth noting that $\chi_s$ is position-dependent in the presence of a finite $\mathbf{D}$ potential $\mathbf{V}_0$ and valid only in the case of $|\delta_{eff}| \ll |\Delta_s| \simeq -\Delta_c|$. We also note that the real ($\chi'_s$) and imaginary ($\chi''_s$) parts of $\chi_s$ describe, respectively, the dispersion and absorption responses and are connected via the KK relation in the frequency domain based on the causality principle and Cauchy’s theorem in the case of $\mathbf{V}_0 = 0$.

The KK relation may also hold in the space domain in the case of $\mathbf{V}_0 = 0$ for appropriate values of $\delta_{eff}$. This is true only if $\chi'_s$ and $\chi''_s$ are related through

$$\chi'_s(\delta_{eff}, x) = \frac{1}{\pi} \int_0^L \frac{\chi''_s(\delta_{eff}, \xi)}{\xi - x} d\xi,$$

where $\mathbf{P}$ denotes a Cauchy’s principle-value integral with respect to atomic position $\xi$. Eq. (6) indicates that $\chi'_s$ and $\chi''_s$ must be spatially out of phase in the case of a perfect spatial KK relation such that the target atoms becomes unidirectional reflectionless to the signal field. This can be understood by considering that, if $\chi'_s$ and $\chi''_s$ are spatially out of phase and meanwhile analytical in the upper half complex plane, their Fourier components contain only positive wavevectors and hence give rise to no backscattering relevant to negative wavevectors. The degree to which the spatial KK relation is violated can be evaluated by a figure of merit defined as

$$D_{kk} = \int_0^L \left[ \chi''_s(\delta_{eff}, x) - \frac{1}{\pi} \int_0^L \frac{\chi''_s(\delta_{eff}, \xi)}{\xi - x} d\xi \right] dx.$$

Consequently, $D_{kk} = 0$ denotes a perfect spatial KK relation in the unbroken regime while a larger $|D_{kk}|$ indicates a greater degree of violation in the broken regime.

To examine the reflection and transmission spectra, we resort to the transfer matrix method sketched below. First, we partition the atomic sample into a large number ($J \gg 1$) of thin slices labeled by indices $j \in \{1, J\}$, which exhibit an identical thickness $\ell = L/J$ but different susceptibilities $\chi_s(\delta_{eff}, x) \rightarrow \chi_s(\delta_{eff}, j\ell)$.

Second, we establish a $2 \times 2$ unimodular transfer matrix $M_j(\delta_{eff}, \ell)$ with $\chi_s(\delta_{eff}, j\ell)$ to describe the propagation of a signal field of wavelength $\lambda_s$ through the $j$th slice via

$$\begin{pmatrix} E^+_s(\delta_{eff}, j\ell) \\ E^-_s(\delta_{eff}, j\ell) \end{pmatrix} = M_j(\delta_{eff}, \ell) \begin{pmatrix} E^+_s(\delta_{eff}, (j-1)\ell) \\ E^-_s(\delta_{eff}, (j-1)\ell) \end{pmatrix},$$

where $E^+_s$ and $E^-_s$ denote, respectively, the forward and backward components of a scattered signal field. Third, it is straightforward to attain the total transfer matrix $M(\delta_{eff}, L) = M_J(\delta_{eff}, \ell) \cdots M_1(\delta_{eff}, \ell)$ as a sequential multiplication of the individual transfer matrices of all slices of the atomic sample. Finally, we can write down the (asymmetric) reflectivities $R_t \neq R_r$ and (reciprocal) transmissivities $T = T_{l_r}$ in terms of relevant matrix elements $M_{(ij)}(\delta_{eff}, L)$ as given by

$$R_t(\delta_{eff}, L) = |r_l(\delta_{eff}, L)|^2 = \frac{|M_{(22)}(\delta_{eff}, L)|^2}{|M_{(23)}(\delta_{eff}, L)|^2},$$

$$R_r(\delta_{eff}, L) = |r_r(\delta_{eff}, L)|^2 = \frac{|M_{(21)}(\delta_{eff}, L)|^2}{|M_{(22)}(\delta_{eff}, L)|^2},$$

$$T(\delta_{eff}, L) = |t(\delta_{eff}, L)|^2 = \frac{1}{|M_{(22)}(\delta_{eff}, L)|^2},$$

where ‘$l$’ and ‘$r$’ refer to a signal field incident from the left ($x = 0$) and right ($x = L$) sides, respectively.

So far we have been considering a homogeneous sample of cold target atoms. Now we switch to another scenario where target atoms are trapped in an optical lattice of period $\Lambda$ and exhibit a periodic Gaussian density

$$N(x) = \sum_{k=1}^K N_k(x) = \sum_{k=1}^K \frac{N_0}{\sqrt{\pi} \Delta x} e^{-(x-x_k)^2/\Delta x^2}.$$
the dispersion (Fig. e and ∆ MHz, γ eff approximation (Ω − s 32 signal susceptibility χ triubutions of − δ effective detuning x against position x for a homogeneous sample of target atoms. From left to right, the curves in cyan, magenta, red, and blue refer to δeff/2π = −5.0 MHz, −1.2 MHz, −0.4 MHz, and −0.2 MHz in order. Other parameters are N0 = 2.0 × 1013 cm−3, L = 30 µm, and x0 = −10 µm except those specified at the beginning of sect. III.

III. RESULTS AND DISCUSSION

We now begin to examine the out-of-phase spatial distributions of χ′ and χ′′ as well as the asymmetric spectra of Rl and Re with formulas developed in the last section. To this end, we first specify realistic parameters for the states of 87Rb isotopes mentioned before Eq. 1 with Γ13/2π = 0.5 kHz, Γrr/2π = 40 kHz, Γ21,rr/2π = 6.0 MHz, γ31,rr/2π = 20 kHz, νrg = 2.54 × 10−29 C·m, and C6/2π = 1.68 × 1033 s−1 µm6 [62, 64]. With respect to the applied fields, we may further choose Ωp/2π = 50 MHz, Ωc/2π = 5.0 MHz, and Δp = Δc = 0 to achieve a high enough Rydberg population (ρ33 → 1) for the control atom, while Ωs/2π = 40 kHz, Ωc/2π = 10 MHz, and −Δs/2π ≃ Δc/2π = 200 MHz to justify the two-level approximation (Ωeff/2π = 2.0 kHz, Δ1/2π = 0.5 MHz, and Δ2 → 0) for all target atoms.

For a homogeneous sample of target atoms, we plot in Fig. 2 the dispersion (χ′) and absorption (χ′′) responses against position x by taking a few specific values of effective detuning δeff. It is clear that χ′ and χ′′ exhibit quite narrow spatial profiles and more importantly are out of phase (manifesting as an odd and an even profile, respectively) to a good approximation as δeff is decreased to be less than −5 MHz. It is also clear that the absorption and dispersion profiles tend to be wider in space and become more deviated from their counterparts in the frequency domain as δeff is increased to be larger than −0.2 MHz. Moreover, we note that the dispersion and absorption profiles may move outside of the atomic sample in the case of δeff ≃ −16 MHz or δeff ≳ 0 MHz. These findings can be well understood by looking back at Eq. 5, with which we can determine a common center x0 = x0 + (−C6/δeff)1/6 by setting δeff + ν0ρ33 = 0 while two half-widths δx± = x0 − x ± [−C6/(δeff ± γrg)]1/6 by setting δeff + ν0ρ33 = ±γrg with respect to χ′ and χ′′ in the limit of ρ33 → 1. The nonlinear dependences of x0 and δx± on δeff answer for why the dispersion and absorption profiles move toward the left side (x = 0), become much narrower, and look more symmetric as δeff is decreased, e.g., from −0.2 MHz to −5 MHz.

Above results show that χ′ and χ′′ generally don’t vary in phase with the increase or decrease of position x, hence are expected to satisfy the spatial KK relation if both well contained in the finite atomic sample. The fact is however that an essential part of the dispersion and absorption profiles may extend outside of the finite atomic sample when δeff is either too large or too small, leading to a more or less violation of the spatial KK relation. This has been evaluated by plotting figure of merit Dkk in Fig. 3(a), from which we can see that the spatial KK relation is roughly satisfied with |Dkk| ≤ 0.1 in a wider range between δeff/2π ≃ −0.5 MHz and δeff/2π ≃ −16 MHz, albeit well satisfied with |Dkk| → 0 in a narrower range centered at δeff/2π ≃ −9 MHz. Considering that spatial KK relation is inseparable with unidirectional reflection, we further plot in Fig. 3(b) reflectivities Rl and Re for a weak signal field incident from the left (x = 0) and right (x = L) sides, respectively. It shows that unidirectional reflection with Rl ≠ 0 and Re → 0 occurs in the range between δeff/2π ≤ −0.5 MHz and δeff/2π ≳ −16.5 MHz even if |Dkk| has increased to be larger than 0.1, indicating that the spatial KK relation is not strictly required. It is also worth noting that Re varies with δeff and becomes maximal at δeff/2π ≃ −2.5 MHz, in virtue of a trade-off between the degree of spatial KK relation and the width of real (dispersion) potential χ′.

Then we examine two possibilities of dynamically mod-
FIG. 4: (Color online) (a) Reflectivity $R_i$ against effective detuning $\delta_{\text{eff}}$ and Rabi frequency $\Omega_p$ for a homogeneous sample of target atoms with the same parameters as in Fig. 3. (b) 1D cuts of 2D plots in (a) with $\Omega_p/2\pi = 50$ MHz, 8.0 MHz, 5.0 MHz, and 3.0 MHz from left to right in order.

FIG. 5: (Color online) (a) Reflectivity $R_i$ against signal detuning $\Delta_s$ and coupling detuning $\Delta_c$ for a homogeneous sample of target atoms with the same parameters as in Fig. 3. (b) 1D cuts of 2D plots in (a) with $\Delta_c/2\pi = 206$ MHz, 204 MHz, 202 MHz, and 200 MHz from left to right in order.

FIG. 6: (Color online) (a) Real and (b) imaginary parts of signal susceptibility $\chi_s$ against position $x$ for a periodic lattice of target atoms with $\delta_{\text{eff}}/2\pi = -1.2$ MHz (left) or $-0.2$ MHz (right). Relevant parameters are the same as in Fig. 2 except $\lambda_s = 780$ nm, $\Delta = 400$ nm, and $\delta x = \lambda/6$.

ululating unidirectional reflection behaviors based on non-local vdW interactions between the control and target Rydberg atoms. One possibility is shown in Fig. 4 where the pumping field $\Omega_p$ is used as a remote ‘knob’ to control the range of $\delta_{\text{eff}}$ for observing unidirectional reflection. It is clear that this range tends to be saturated in the case of $\Omega_p \gtrsim 20$ MHz, but shrinks evidently from the side of larger $|\delta_{\text{eff}}|$ as $\Omega_p$ gradually deviates from the saturation regime. This can be attributed to the fact that a decrease of $\Omega_p$ will result in a decrease of $\rho_{32}$ and thus a decrease of $x_c$ for a given $\delta_{\text{eff}}$, equivalent to a decrease of the maximal $|\delta_{\text{eff}}|$ referring to $x_c = 0$ and denoting a boundary of the well satisfied spatial KK relation. The other possibility is shown in Fig. 5 where reflectivity $R_i$ is plotted against $\Delta_s$ instead of $\delta_{\text{eff}}$, being $\Delta_c$ an alternative control ‘knob’. It is easy to see that we can move the range of $\Delta_s$ for observing unidirectional reflection as a whole, without shrinking or expanding in terms of both $\Delta_s$ and $R_i$, by modulating $\Delta_c$ in the limit of $\Delta_s \approx -\Delta_c \gg \Omega_p$. This fine tunability relies on the fact that susceptibility $\chi_s$ in Eq. 6 refers to an reduced two-level system where effective detuning $\delta_{\text{eff}}$ is mainly contributed by the sum of signal ($\Delta_s$) and coupling ($\Delta_c$) detunings. A reversed unidirectional reflection with $R_i = 0$ and $R_r \neq 0$ can be attained by driving a second control atom at $x = L - x_0$ into its Rydberg dark state while leaving the first control atom at $x = x_0$ free of excitation (not shown).

So far we have shown that unidirectional reflection can be realized and modulated for appropriate effective ($\delta_{\text{eff}}$) or signal ($\Delta_s$) detunings. However, the nonzero reflectivity $R_i < 0.1$ is obviously small because both real ($\chi_s'$) and imaginary ($\chi_s''$) potentials are rather weak (i.e., less than unit in magnitudes). In order to enhance $\chi_s$ and thus increase $R_i$, we can choose larger atomic density $N_0$ and/or smaller dephasing rate $\gamma_{cg}$ as can be seen from Eq. 6. Unfortunately, the former choice goes beyond the current experimental technologies of cold atoms, while the latter choice is restricted by the residual Doppler broadening of cold atoms ($\sim 20$ kHz at the temperature of $T = 1$ µK). This motivates us to consider another scenario where the homogeneous atomic sample is replaced by a periodic atomic lattice described by Eq. 10 so as to enhance the nonzero reflectivity by incorporating multiple Bragg scattering into spatial KK relation.

Two typical examples on periodically modulated dispersion and absorption responses are shown in Fig. 6 with $\delta_{\text{eff}}/2\pi = -0.2$ MHz and $\delta_{\text{eff}}/2\pi = -1.2$ MHz, respec-
where unbalanced reflectivities are out of phase, to different extents depending on $\delta_{eff}$, in their overall profiles similar to their counterparts in Fig. 2. But it is also obvious that they exhibit comb-like fine structures under the not-in-phase overall profiles as a result of the periodic Gaussian density $N(x)$ in Eq. (10). A signal field incident upon the finite atomic lattice are expected to experience an enhanced reflection in the presence of both spatial KK relation contributed by the overall profiles of $\chi'_{s}$ and $\chi''_{s}$ and multiple Bragg scattering contributed by the fine structures of $\chi'_{s}$ and $\chi''_{s}$. This is exactly what we observe in Fig. 7(a) where unbalanced reflectivities $R_{l}$ and $R_{r}$ are plotted against effective detuning $\delta_{eff}$.

We can see from Fig. 7(a) that one reflectivity is largely enhanced albeit in an asymmetric manner and exhibits a maximum $R_{l} \rightarrow 0.7$ at $\delta_{eff} \simeq 0$, while the other reflectivity remains to be $R_{r} \rightarrow 0$ for $\delta_{eff} \lesssim -0.5$ MHz. Moreover, it is worth noting that $R_{r}$ is also largely enhanced for $\delta_{eff} \gtrsim 0$ and may even be equivalent to $R_{l}$, indicating a fully destroyed spatial KK relation therein. The underlying physics should be that Bragg scattering occurs around $\delta_{eff} \simeq 0$ where $\chi'_{s}$ and $\chi''_{s}$ exhibit very wide but not too low spatial profiles on one hand and tend to vary in phase on the other hand. Fig. 7(b) further shows that the asymmetric enhancement of $R_{l,r}$ holds for a smaller atomic density and more importantly the maximal value $R_{l} \approx 0.85$ in a periodic atomic lattice could be equivalent to that in a homogeneous atomic sample with a ten-times larger density $N_{0}$, see Fig. 7(b). These results confirm that multiple Bragg scattering is a valid tool for improving the asymmetric or unidirectional reflection behaviors arising from spatial KK relation, which is unattainable yet by inserting a homogeneous atomic sample into a Fabry-Perot cavity (not shown).

IV. CONCLUSIONS

In summary, we have proposed an efficient scheme for realizing the dynamically tunable spatial KK relation by exploiting nonlocal $vdW$ interactions of Rydberg atoms. One control atom in a Rydberg dark state is used to map the dispersion and absorption responses of a homogenous sample or a periodic lattice of target atoms from the frequency domain to the space domain. This is attained as all target atoms are driven in the EIT regime to an effective two-level configuration by a signal and a coupling field kept near resonance on one two-photon transition but far-detuned from two single-photon transitions. Our numerical results show that the spatial dispersion and absorption responses generally don’t vary in phase and more importantly could well satisfy the spatial KK relation, hence supporting unidirectional ($R_{l} \neq 0$ and $R_{r} = 0$) reflection behaviors. Note also that periodic atomic lattices seem more appealing than homogenous atomic samples in that they promise an obvious enhancement of the nonzero reflection due to a positive interplay of multiple Bragg scattering and spatial KK relation. Our findings should be instructive on combining non-Hermitian quantum optics and coherent manipulation of Rydberg atoms, e.g., to develop one-way optical devices and explore new applications with long-range $vdW$ interactions.

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