Inventory models for short life cycle clothing products use a logistic growth model

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Abstract. Clothes are products that follow short-life fashion and market demand. Products with a short lifetime occur due to technological developments and/or changes in market tastes. The clothing industry is one of the industries that has a short and obsolete sales period in stages. The excess number of products that accumulate in the warehouse due to obsolescence can cause the company to get a profit that is not optimal. One strategy to increase demand for the product is to apply discounts. There are two types of discounts used, namely a single discount and multiple discounts. The demand function of inventory model is considered analogous to the logistic growth model. The results show profit is maximized to get optimal order quantity and optimal discount price.

1. Introduction

Technological advancements provide change or impact on people's lifestyles and changes in consumer tastes pose challenges for products such as clothing, jewelers-accessories, and electronics facing problems with short-life cycles [1, 2]. At present, the fashion industry market is very competitive to always follow the growing trend in society. Manufacturers are required to be able to maximize profits, one way is to provide the number of products that are in accordance with existing demand. Producers that produce with the make to stock system should well plan production and material purchases which products will be sold later.

Unstable demand causes excess stock or lack of stock. Excess stock causes a buildup of items that gradually become obsolete. While the producer is continuously releasing the latest models following the trends. Stacked goods will be sold at discounted prices or stacked in warehouses until there are consumers who want to buy. Uncertain supply and demand problems must be harmonized to get the optimal order quantity [3].

We discuss the determination of the number of orders and the distribution of discount prices for this type of product clothe gradually. First, the discount time sharing stage has an influence on demand for obsolete products; second, selling price as an exogenous factor influencing demand. The increase in demand for clothing occurs following price changes, with respect to the difference between the two prices, so the selling price and the time function affect demand. The method used for optimal order quantity is the Economic Order Quantity method with the demand model following the logistic growth model.
Discounts can be seen in the sales system of electronic goods such as cellphones. The first time a particular hand phone model was launched the new price of a cellphone will be priced at a normal price. After a few weeks or months, the price of mobile phones will be relatively down because of the presence of competitors to bring up the latest mobile models. The emergence of new models and technologies, old cellphones will experience a decline in buyers. The discount price is done to overcome the accumulation of old products.

The system for selling products such as clothes or shoes has also decreased market appetite. At first, new clothes and shoes will be given a discount that is not too high, for example, 20%. After a few weeks, clothes and shoes that haven't sold well, the discount can reach 20% + 30%. Products with discounts are usually valid when doing a bazaar or special event. In addition, items that have been a little interested will be discounted at a certain price.

There are several studies that discuss short product life cycles, they focus on supply chain paths, but many of them have not considered determining the quantity and importance of discounted prices. Our study has a contribution to discounted pricing decisions, where customers have a choice to buy at the first discount or the next discount. This study is different from previous research, prices and quantity are formed in interval of discount time to fulfill the unstable demand.

2. Preliminaries
The logistic growth model is the development of an exponential population model. The maximum population capacity is $U$. The minimum population capacity is 0, which indicates extinction. Based on the birth and death process, with a short period of time, the average birth is $\beta \Delta t$, and the average death is $\gamma \Delta t$, $\beta > 0, \gamma > 0$ [3].

\[
\frac{N(t + \Delta t) - N(t)}{\Delta t} = (\beta - \gamma)N(t).
\]

A short period of time causes, $\Delta t \to 0$,

\[
\frac{dN(t)}{dt} = r_s N(t).
\]

In the population model, $r_s$ represents the rate of population growth, $r_s$ is constant. The model with population growth rates decreases linearly with increasing population size [4], as follows:

\[
r_s = r_s (N) = r \left(1 - \frac{N(t)}{U}\right).
\]

\[
r_s = r \left(\frac{U - N(t)}{U}\right).
\]

\[
\frac{(U-N(t))}{u}
\]

is part of the maximum amount of unused capacity. The rate of population increase is formulated as follows:

\[
\frac{dN(t)}{dt} = rN(t) \left(\frac{U-N(t)}{U}\right),
\]

\[
\frac{dN(t)}{dt} = \lambda N(t)(U - N(t)), \quad \lambda = \frac{r}{U}
\]

where:

$N(t)$ : number of population

$\lambda$ : positive coefficient

$U$ : maximum population capacity

The logistic growth model can be expressed as a demand model for a product [5]. Economic Order Quantity is a mathematical model that formulates total annual costs or other time periods according to the needs of companies incurred to manage inventory and orders. Total costs are obtained from inventory costs as follows:

\[
B_c = cR, \quad B_A = \frac{AR}{Q}.
\]

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3. Inventory optimization using logistic growth model

Inventory management is needed to meet demand. The demand function is developed from the logistic population model in Equation (2) as follows:

\[
\frac{dD(t)}{dt} = \lambda D_1(t)[U - D_1(t)],
\]

\[
D_1(t,P) = \frac{u}{e^{-\lambda UT(k+1)}}, t \in [0,\mu], k = \frac{U}{D_0} - 1,
\]

\[
\frac{dD_2(t)}{dt} = \lambda D_2(t)[0 - D_2(t)],
\]

\[
D_2(t) = \frac{U}{\delta + t\lambda U}, t \in [\mu, T], \delta = 1 + ke^{-\lambda UT}.
\]

3.1. Single discount

We developed the single discount which occurs at \(t \in [\mu, T]\) that causing the demand function to be:

\[
D_2(t,P) = \frac{u}{\delta + (t-\mu)\lambda U} + a(P_0 - P), t \in [\mu, T].
\]

Inventory models aimed to maximize total profit. The filling material is only valid once. Inventory costs consist of storage costs, ordering costs, and purchasing costs. Storage costs incurred depend on the inventory level.

\[
I_1(t) = Q - \int_0^t \frac{u}{ke^{-\lambda UT} + 1} dt, t \in [0,\mu],
\]

\[
I_1(t) = Q - \frac{1}{\lambda} \ln \frac{(ke^{-\lambda UT(k+1)})}{e^{-\lambda UT(1+k)}}, t \in [0,\mu],
\]

\[
I_2(t) = \int_0^T D_2(t,P) dt, t \in [\mu, T],
\]

\[
I_2(t) = \frac{1}{\lambda} \ln \frac{\lambda(U(T-\mu) + \delta)}{U(t-\mu) + \delta} + a(P_0 - P)(T - t), t \in [\mu, T].
\]

After the inventory level of each time interval is obtained. Furthermore, the storage costs calculated are as follows:

\[
B_h = h \int_0^\mu I_1(t) + h \int_\mu^T I_2(t),
\]
\[ B_h = hQ\mu + \frac{1}{2} ah(P_0 - P)(T - \mu)^2 - \int_0^T \frac{1}{\lambda} \ln \frac{(ke^{-\lambda t}+1)}{e^{-\lambda t}(1+k)} dt + h\int_0^T \frac{1}{\lambda} \ln \frac{\lambda U(T - \mu) + \delta}{\lambda U(T - \mu) + \delta} dt. \] (12)

\( Q \) are the sum of demands per time interval [7] as follows:

\[ Q = \int_0^T D_1(t, P) dt + \int_0^T D_2(t, P) dt, \]

\[ Q = \frac{1}{\lambda} \ln \left( \frac{(1+ke^{-\lambda t})\delta U(T-\mu)}{\delta e^{-\lambda U}(1+k)} \right) + a(P_0 - P)(T - \mu). \] (13)

Purchase costs are:

\[ B_c = cQ \]

\[ B_c = C \left( \frac{1}{\lambda} \ln \frac{(1+ke^{-\lambda t})\delta U(T-\mu)}{\delta e^{-\lambda U}(1+k)} \right) + a(P_0 - P)(T - \mu). \] (14)

Ordering costs are as follows:

\[ B_A = \frac{AR}{Q} = \frac{AQ}{Q}, \]

\[ B_A = A. \] (15)

Based on Equation (7), the total of sales will be determined as follows:

\[ \text{Total of sales} = P_0(Q - l(\mu)) + PI(\mu), \]

\[ = P_0 Q - (P_0 - P)l(\mu). \]

Based on Equation (8), the total profit is:

\[ TP = (P_0 - h\mu - c)Q - P_0 \frac{1}{\lambda} \ln \frac{\lambda U(T-\mu) + \delta}{\delta} + P_0 \frac{1}{2\lambda} \ln \frac{\lambda U(T-\mu) + \delta}{\delta} - a(P_0 - P)^2(T - \mu) + \]

\[ h \int_0^T \frac{1}{\lambda} \ln \frac{(ke^{-\lambda t})\delta U(T-\mu)}{e^{-\lambda U}(1+k)} dt - h \int_0^T \frac{1}{\lambda} \ln \frac{\lambda U(T-\mu) + \delta}{\delta} dt - \frac{1}{2} ahP_0(T - \mu)^2 + \frac{1}{2} ahP(T - \mu)^2 - A. \]

Optimal discount prices are obtained from \( \frac{dTP}{dP} = 0. \)

\[ P^* = -\frac{1}{2} (P_0 - h\mu - c) + \frac{1}{2\lambda(T-\mu)} \ln \frac{\lambda U(T-\mu) + \delta}{\delta} + P_0 + \frac{1}{4} h(T - \mu). \] (16)

Substitute Equation (15) to Equation (13) to get the optimal order quantity as follows:

\[ Q^* = \frac{1}{\lambda} \ln \frac{(1+ke^{-\lambda t})\delta U(T-\mu)}{\delta e^{-\lambda U}(1+k)} - \frac{1}{2\lambda} \ln \frac{\lambda U(T-\mu) + \delta}{\delta} + \frac{a}{2} (P_0 - h\mu - c)(T - \mu) - \frac{ah}{4} (T - \mu)^2. \] (17)

### 3.2. Multiple Discount

The demand model at time intervals \([0, \mu]\) uses Equation (9) with the price \( P_0 \). We developed on multiple discounts over time intervals \([\mu, T]\). The first step is to divide the time interval \([\mu, T]\), into \( n \) subintervals with the range of each subinterval is \([0, L]\). Equation (10) becomes:

\[ D_2(t, P_i) = \frac{U}{\delta + ((t-T)\mu + L)} + aP_{i-1} - aP_i, 0 < t \leq L, i = 1, 2, 3, \ldots, n \] (18)

Inventory control model with multiple discounts is the development of inventory cost models with a single discount. At the time interval \([0, \mu]\), the inventory level at this interval is the same as Equation (11). The time interval \([\mu, T]\), consists of several subintervals.

\[ I_1(t) = I_1(0^+) - \int_0^t \frac{U}{\delta + ((t-T)\mu + L)} dt - aP_{i-1} - P_i t. \]

Boundary condition \( I_n(L) = 0, \)

\[ I_n(L) = I_n(0^+) - \left( \frac{1}{\lambda} \ln \frac{\lambda U((n-1)L+\mu) + \delta}{\delta} + a(P_{n-1} - P_n) L \right), \]

\[ I_n(0^+) = \frac{1}{\lambda} \ln \frac{\lambda U((n-1)L+\mu) + \delta}{\delta} + a(P_{n-1} - P_n) L, \]

\[ I_1(0^+) = \frac{1}{\lambda} \ln \frac{\lambda U\mu + \delta}{\delta} + a(P_{i-1} - P_i) L. \] (20)

Substitute Equation (20) to Equation (19)

\[ I_1(t) = \frac{1}{\lambda} \left( \ln \left( \frac{\lambda U\mu + \delta}{\delta} \right) \right) + a(P_{i-1} - P_i) L - a(P_{i-1} - P_i) t. \] (21)
Holding costs incurred are:

\[
H = h \int_0^t I_0(t) dt + h \sum_{i=1}^n \int_0^L i_i(t) dt, \\
H = hQ\mu - h \int_0^\mu \left( \frac{1}{\lambda} \ln \frac{1+ke^{-\lambda t}}{(1+k)e^{-\lambda t}} \right) dt + hL \sum_{i=1}^n I_i(0^+) - h \sum_{i=1}^n \int_0^L \left( \frac{1}{\lambda} \ln \frac{\lambda U((i-1)L+t) + \delta}{\lambda U((i-1)L+\delta)} \right) dt - \frac{h\alpha L^2}{2} (P_0 - P_n). \tag{22}
\]

The purchase cost depends on the quantity to be purchased. \( Q \) is the sum of requests per time interval as follows:

\[
Q = I_0(\mu) + I_1(0^+), \\
Q = \frac{1}{\lambda} \ln (1+k) e^{-\lambda \mu} + a(P_0 - P_n) L. \tag{23}
\]

The purchase costs incurred are as follows:

\[
B_c = c \left( \frac{1}{\lambda} \ln (1+k) e^{-\lambda \mu} + a(P_0 - P_n) L \right). \tag{24}
\]

The cost of ordering at multiple discounts is the same as the cost of the order at the time of a single discount as follows:

\[
B_A = A. \tag{25}
\]

The total sales are determined by:

\[
Total \ of \ sales = P_0(Q - I_0(\mu)) + \sum_{i=1}^n P_i (I_i(0^+) - I_i(L)). \\
Total \ of \ sales = P_0 \left( \frac{1}{\lambda} \ln (1+k) e^{-\lambda \mu} \right) + \sum_{i=1}^n P_i \left( \frac{1}{\lambda} \ln \frac{\lambda U((i-1)L+\delta)}{\lambda U((i-1)L+\delta)} \right) + \sum_{i=1}^n P_i a(P_{i-1} - P_i) L. \tag{26}
\]

Based on Equation (8), total profit as follows:

\[
TP(P_1,P_2,P_3,...,P_n) = P_0 \left( \frac{1}{\lambda} \ln (1+k) e^{-\lambda \mu} \right) + \sum_{i=1}^n P_i \left( \frac{1}{\lambda} \ln \frac{\lambda U((i-1)L+\delta)}{\lambda U((i-1)L+\delta)} \right) + \sum_{i=1}^n P_i a(P_{i-1} - P_i) L - \\
hQ\mu + h \int_0^\mu \left( \frac{1}{\lambda} \ln \frac{1+ke^{-\lambda t}}{(1+k)e^{-\lambda t}} \right) dt - hL \sum_{i=1}^n I_i(0^+) + h \sum_{i=1}^n \int_0^L \left( \frac{1}{\lambda} \ln \frac{\lambda U((i-1)L+t) + \delta}{\lambda U((i-1)L+\delta)} \right) dt + \frac{h\alpha L^2}{2} (P_0 - P_n) - A - cQ. \tag{27}
\]

Optimal discount prices are obtained using \( \frac{\partial TP}{\partial P_i} = 0. \)

\[
0 = \frac{1}{\lambda} \ln \frac{\lambda U((i-1)L+\delta)}{\lambda U((i-1)L+\delta)} + aL(P_{i-1} - 2P_i + P_{i+1}) - h\alpha L^2, i = 1,2,3,\ldots,n-1, \tag{28}
\]

\[
0 = \frac{1}{\lambda} \ln \frac{\lambda U((n-1)L+\delta)}{\lambda U((n-1)L+\delta)} + aL(P_{n-1} - 2P_n) + h\alpha L^2 + c aL. \tag{29}
\]

We obtain \( P^*_1 \) or \( P^*_n \) from Equation (9), and substitute \( P^*_1 \) or \( P^*_n \) to Equation (23) to get the optimal order quantity as follows:

\[
Q^* = \frac{1}{\lambda} \ln (1+k) e^{-\lambda \mu} + a(P_0 - P_n) L \tag{30}
\]

4. Conclusions

In inventory management, many uncertainties can be found. This uncertainty reflects our inability to predict the behaviour of the whole system with certainty. The impact of uncertainty on inventory management is quite large. For example, due to demand uncertainty, many problems produce a negative impact on company profits. We develop inventory models for types of clothing products that follow the human lifestyle that has a short life cycle. First, we propose a single discount to get the optimal quantity and maximum profit. Secondly, we propose multiple discounts. Multiple discounts are discount prices that are determined at certain intervals during the time of sale. The logistic growth model used to approach the demand function provides a solution to determine product quantity.

5. References

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