Finite element numerical simulation on thermo-mechanical behavior of steel billet in continuous casting mold

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Received 14 June 1999

Abstract

The thermo-mechanical behavior of a thin, growing shell during the early stages of solidification in a continuous casting mold is very important to the ultimate quality of the final billet. A two-dimensional, transient finite element model has been developed to treat the heat flow and deformation of the solidifying shell in the continuous casting billet mold as a coupled phenomena. The major application of the model is to predict the extent of the gap between the mold and the shell and focus on the influence of mold taper on the thermo-mechanical behavior of the steel billet to help to understand the formation of off-corner cracks and break-outs in the solidifying shell. The calculations indicate that the gap is initially formed at the corner of the billet, where heat transfer is greatly reduced. Insufficient mold taper contributes to a hot spot in the off-corner region, which corresponds to the lowest shell thickness. At the same time, the solidifying front on the diagonal of the billet is subjected to an excessive mechanical strain, which causes the off-corner cracks and even the break-outs. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Continuous casting; Solidification; Stress; Gap; Crack; Finite element method

1. Introduction

The thermo-mechanical behavior of a thin, growing shell during the early stages of solidification in a continuous casting mold is very important to the ultimate quality of the final billet [1]. During the initial solidification, the solidifying shell attempts to shrink away from the mold due to thermal contraction, and a gap is formed between the shell and the mold wall. This causes the rate of heat extraction from the strand to diminish greatly, particularly near to the corner where the gap is largest. Thus, characterizing the gap between the mold and the shell is crucial to understand the behavior of the solidifying shell.

Over the years, several researchers have attempted to predict the shape and size of the gap, but their deficiency was the failure to treat the heat flow and gap formation between the mold and the shell as coupled phenomena [2]. The formation of the gap depends simultaneously on the heat flow condition, the mechanical properties of solid shell and some process variables. Thus, the prediction of gap should be calculated by combining heat flow and stress analysis. Recently, some coupled mathematical models have been used extensively to simulate the thermo-mechanical behavior of the solidifying shell in continuous cast steel billets as well as slabs [2–6]. However, these models neglect some of important factors, such as the plastic strain rate [2–5], the rounded corner of the strand [2,3,5,6]. In the present work, a two-dimensional (2D), transient, heat transfer and stress model has been developed to treat the heat flow and the stress analysis as coupled phenomena, which takes account for the exact geometry of billet rounded corner and the strain rate dependent material properties.

The major application of the model is to predict the extent of gap between the mold and the shell, to simulate the influence of mold taper on the thermo-mechanical behavior of steel billet and to help understanding the formation of off-corner cracks and the breakouts in the solidifying shell.

2. The mathematical model

The mathematical model of the strand has been developed to tracks a transverse slice of a steel billet as it moves down through the mold. It consists of two separate finite element models of heat flow and stress generation that are coupled through the interfacial gap.
2.1. Heat flow model

In the continuous casting of steel, longitudinal conduction can be neglected because the thermal conductivity of steel is small (relative to copper) while the casting speed is high. Thus, the calculation is confined to a plane by employing the transient heat conduction equation

\[
\rho c(\partial \theta / \partial t) - \nabla \cdot (k \nabla \theta) = 0.
\]  

(1)

The model is first discretized in space by employing the classical Galerkin scheme with finite elements. Summing the contribution from the individual elements and boundaries, the following global matrix equation is obtained:

\[
[K] \{ \dot{\theta} \} + [C] \{ \theta \} = \{ F \}.
\]  

(2)

The time function in Eq. (2) was discretized by the Dupont II three-level time-stepping scheme [7]:

\[
[C] \frac{\theta_{n+1} - \theta_n}{\Delta t} + [K] \frac{3\theta_{n+1} + \theta_{n-1}}{4} = \{ F \}.
\]  

(3)

Latent heat in the ‘mushy’ region was treated by the Lemmon technique [8]:

\[
c_l = \left[ \frac{(\partial H/\partial \dot{\alpha})^2 + (\partial H/\partial \dot{\gamma})^2}{(\partial \theta/\partial \dot{\alpha})^2 + (\partial \theta/\partial \dot{\gamma})^2} \right]^{1/2}.
\]  

(4)

At the same time, the temperature-dependent thermal properties were used

\[
c_s = 0.545 + 95.16 \times 10^{-6} \theta
\]  

(5)

\[
\kappa_s = 0.0184 + 9.6 \times 10^{-6} \theta
\]  

(6)

2.2. Stress model

An incremental, two-dimensional, transient, elastic–plastic stress model was developed to determine the internal stress in the shell arising from the changing temperature gradients calculated by the heat flow model.

2.2.1. Thermoelastic–plastic constitutive equation

Considering that the deformation of the strand shell in the mold is subjected to significant variation of temperature and strain rate, the total incremental strain vector may be assumed to consist of the following components:

\[
\{ \Delta \varepsilon \} = \{ \Delta \varepsilon_e \} + \{ \Delta \varepsilon_p \} + \{ \Delta \varepsilon_\theta \} + \{ \Delta \varepsilon_{e,0} \} + \{ \Delta \varepsilon_{e,\dot{\varepsilon}} \},
\]  

(7)

where \{ \Delta \varepsilon_e \}, \{ \Delta \varepsilon_p \} and \{ \Delta \varepsilon_\theta \} are the incremental elastic strain components, plastic strain components and thermal strain components, respectively. \{ \Delta \varepsilon_{e,0} \} and \{ \Delta \varepsilon_{e,\dot{\varepsilon}} \} are the incremental strain components due to temperature dependent and strain-rate dependent material properties, respectively.

At the same time the yield function can no longer be treated as a function of stresses alone, as it has to incorporate the effects of work hardening rates, the temperature and the strain rates. Now it should be expressed as:

\[
\Phi = \Phi(\{ \sigma \}, k, \theta, \dot{\varepsilon}).
\]  

(8)

Employing the theory of plasticity, the thermoelastic–plastic constitutive equation can be derived from the
above equations
\[
\{d\sigma\} = \{D_{ep}\}(\{d\epsilon\} - \{d\epsilon_0\}) - \{d\sigma_0\},
\]  
(9)

where
\[
\{d\epsilon_0\} = \left(\begin{bmatrix} \alpha \end{bmatrix} \right) \frac{\partial [D_{ep}]^{-1}}{\partial \theta} \{\sigma\} d\theta + \left(\begin{bmatrix} \alpha \end{bmatrix} \right) \frac{\partial [D_{ep}]^{-1}}{\partial \hat{e}} \{\sigma\} d\hat{e},
\]
\[
\{d\sigma_0\} = \frac{\{D_c\}}{S} \{\alpha'\} \left(\begin{bmatrix} \frac{\partial \Phi}{\partial \hat{e}} \frac{d\hat{e}}{d\theta} + \frac{\partial \Phi}{\partial \theta} \frac{d\Theta}{d\theta}\end{bmatrix} \right)
\]

2.2.2. Finite element formulation

Employing the theory of incremental plasticity and the Virtual Work Principle, the following finite element equations of the shell stress analysis can be derived from the above equations
\[
[K_R] \{\Delta u\} = \{\Delta R_0\} + \{\Delta R_1\} + \{\Delta R_2\},
\]  
(10)

where
\[
[K_R] = \sum \{K_{R}^e\} = \sum \int_S \{B]\{D_{ep}\}\{B\} \, dV,
\]
\[
\{\Delta R_0\} = \sum \{\Delta R_{0}^e\} = \sum \int_S \{N]\{\Delta \hat{r}\} \, dV,
\]
\[
\{\Delta R_1\} = \sum \{\Delta R_{1}^e\} = \sum \int_S \{B]\{D_{ep}\}\{\Delta \epsilon_0\} \, dV,
\]
\[
\{\Delta R_2\} = \sum \{\Delta R_{2}^e\} = \sum \int_S \{B]\{\Delta \sigma_0\} \, dV.
\]

The calculation of the shell stress field is assumed as a plane-strain problem. The plastic modulus being determined from the stress-strain curves measured by a Gleeble-1500 thermo-mechanical simulator under different strain rates \((10^{-2} - 10^{-4} \, \text{s}^{-1})\) and temperatures \((1000 - 1400^\circ\text{C})\). Temperature-dependent Young’s modulus and Poisson’s ratio were used [3] as follows:
\[
E = 3.146 \times 10^4 - 22.56 \theta + 1.38 \times 10^{-3} \theta^2,
\]  
(11)
\[
\nu = 0.278 + 8.23 \times 10^{-5} \theta.
\]  
(12)

2.3. The coupled thermo-mechanical analysis

The stress model is coupled step-wise with the heat flow model through the interfacial gap, as the solution alternates between the thermal and stress calculations as the slice moves down through the mold in successive time steps. Thus a gap-dependent heat transfer model should be developed.

In the general heat flow model, the heat extraction in the mold is controlled by the Savage and Pritchard equation [1]:
\[
q_m = 2680 - 335\sqrt{t}.
\]  
(13)

This equation only gives the mean heat flux around the perimeter of the shell at different mold dwell times, which neglects the influence of the gap on the heat extraction from the shell. To overcome this deficiency, a gap-dependent heat transfer model, considering conduction and radiation through the gap, is used to correlate the gap width to the total heat flux
\[
h_g = h_t + \frac{K_g}{d},
\]  
(14)
\[
q_g = h_g(\theta_{surf} - \theta_{amb}).
\]  
(15)

During the computation of the temperature distribution in the billet, Eqs. (14) and (15) were employed to calculate the heat flux through the air gap \(q_g\) at each node. If this heat flux was found to be large than the mean heat flux \(q_m\), \(q_m\) was used in the calculation. This procedure is necessary since the gap width becomes very small and the influence of other contact resistances to the heat flow becomes more important.

During the coupled analysis of heat flow and stress state, the thickness of the gap is calculated at each location and time, knowing the position of the strand surface, calculated by the stress model at the previous time step. Then the new heat flux, calculated by the above heat conduction model of the interfacial gap, is used in the thermal analysis.

The elements that stay completely liquid during a whole time step are assigned zero stiffness in the step. The corresponding nodal displacements are prescribed to zero in that step.

3. Computed results and discussion

The continuous casting of a steel billet under two different mold tapers \((0.6 \text{ and } 0.2\% \, \text{m}^{-1})\), has been simulated with the model. Then, the influence of mold taper on the thermo-mechanical behavior of the solidifying shell is examined. The thermo-physical properties and casting conditions for the calculations are given in Table 1.

3.1. Gap formation and surface temperature

Fig. 1 presents the gap width profile at different mold dwell times under the two different mold tapers. The results indicate that the gap initially forms in the corner, and extends to the mid-face as the mold dwell time increases. It is noted that the largest gap width is always at the top of the corner.

As seen in Fig. 1(a), when the mold taper is 0.6% \(\text{m}^{-1}\), the gap width at the off-corner regions tends to reduce in the lower part of the mold as the mold dwell time increases, so the surface temperature is lower at the off-corner region (Fig. 2(a)) This is most likely a result of the bulging of the thin shell due to ferrostatic pressure. However, when the mold taper is small (0.2% \(\text{m}^{-1}\)), the extent and width of the gap apparently increases, and the gap has been formed around the whole perimeter of the shell even in the upper part of the mold (Fig. 1(b)). This gives rise to a higher
Table 1
Thermo-physical properties and casting conditions used in the calculations

| Property                                      | Value       |
|-----------------------------------------------|-------------|
| Liquidus temperature, \( \theta_i (^{\circ}C) \) | 1513        |
| Solid density, \( \rho_i (\text{kg m}^{-3}) \) | 7400        |
| Liquid specific heat, \( c_l (\text{kJ kg}^{-1} \text{K}^{-1}) \) | 0.842       |
| Carbon content, \( W_c (\%) \)               | 0.16        |
| Billet size, \( L (\text{mm}) \)             | 115 × 115   |
| Radius of billet corner, \( r (\text{m}) \)  | 0.008       |
| Mold length, \( Z (\text{m}) \)              | 0.700       |
| Radiate heat transfer in gap, \( h_r (\text{W m}^{-2} \text{K}^{-1}) \) | 150         |
| Solidus temperature, \( \theta_s (^{\circ}C) \) | 1493        |
| Liquid density, \( \rho (\text{kg m}^{-3}) \) | 7100        |
| Liquid thermal conductivity, \( \kappa (\text{W m}^{-1} \text{K}^{-1}) \) | 3\( \kappa_c \) |
| Casting temperature, \( \theta_c (^{\circ}C) \) | 1520        |
| Casting speed, \( v (\text{m min}^{-1}) \)   | 2.3         |
| Meniscus level, \( Z_m (\text{m}) \)         | 0.100       |
| Mean ambient temperature in gap \( \theta_m (^{\circ}C) \) | 200         |
| Thermal conductivity in gap, \( \kappa_g (\text{W m}^{-1} \text{K}^{-1}) \) | 0.054       |

surface temperature near to the corner of the billet (Fig. 2(b)). It also can be seen that the gap increases slowly in the lower zone of the mold due to the ferrostatic pressure, particularly in the mid-face (Fig. 1(b)).

3.2. The thermo-mechanical behavior of the shell

Fig. 3 shows the distributions of temperature and the mechanical strain in the solidifying shell at the mold exit under the two kinds of mold taper. With the mold taper of 0.6% m\(^{-1}\), the results show a reasonable temperature distribution with a generally uniform shell thickness (see the dotted line in Fig. 3(a)). This appears to validate that the mold taper of 0.6% m\(^{-1}\) is adequate for the continuous casting process. However, when the mold taper is small (0.2% m\(^{-1}\), the temperature on the inside of the shell and on the surface near to the off-corner region, is higher than at the middle (Figs. 2(b) and 3(b)). 2-D heat transfer at the
corner keeps the corner cold and thick. Thus, hot spots are formed in the off-corner region, which corresponds to the least shell thickness and has been found to coincide with the observed break-out shell section (Fig. 4). The off-corner region that accompanies the thinner shell and higher temperature is believed to be the weakest zone of the shell [2,9], which may be an important factor in the formation of defects, including off-corner cracks and break-outs.

Referring to the distribution of mechanical strain in the solidifying shell (see the solid lines in Fig. 3(a) and (b), the solidifying and shrinking shell generates a compressive stress on its cold outer surface and tension on its hot inner surface near to the off-corner region. The interior tensile regions would contribute to internal cracks when accompanied by metallurgical embrittlement.

As seen in Fig. 3(a) when the mold taper is 0.6% m⁻¹, there is a mechanical strain peak in the inner surface at the off-corner region (Fig. 3(a)), which is close to the critical strain of 0.2% [9]. However, the strain peak is unlikely to cause internal cracks with an adequate steel composition.
without microsegregation. This is because the temperature in the region is about 1200°C, which is lower than the hot tear temperature (\(\theta_c\): 100°C), and the metal has reasonably hot ductility [10].

When the mold taper is small (0.2% m\(^{-1}\)), the solidifying front is subjected to tensile stress at the off-corner region, and there is a mechanical strain peak on the diagonal of the billet, which reaches to the critical strain 0.2% (Fig. 3(b)). This indicates that an internal crack is formed on the diagonal of the billet, and the predicted location of crack coincides with the observed position in the metallographic examination (Fig. 5). This agreement verifies that insufficient mold taper is one of the important factors to cause off-corner cracking. Consequently, the internal crack might spread to the weakest hot spots region in extreme circumstances. Especially as the strand is withdrawn from the mold, the mold wall no longer supports the thin shell, so the bulging of the shell due to the ferrostatic pressure would strengthen the propagation of such internal cracks to the weakest zone in the off-corner region. If the cracks extend

![Fig. 3. Mechanical strain and isotherms in the xy-plane at the mold exit for the two mold tapers: (- - - Temperature (°C); — strain (%)) (a) 0.6% m\(^{-1}\); (b) 0.2% m\(^{-1}\).](image)

![Fig. 4. Break-out of the shell with the lowest thickness at off-corner region.](image)

![Fig. 5. Internal crack at off-corner region.](image)
to the surface, break-outs would occur. This mechanism shows that insufficient mold taper could cause break-outs.

4. Conclusions

1. A 2-D stepwise-coupled transient thermo-mechanical model has been developed to simulate the behavior of the solidifying steel billet in the mold of a continuous casting machine. This model can predict the coupled evolution of temperature, shape, and stress and strain distribution in the shell.

2. The calculations indicate that the gap between the mold and the shell initially forms at the billet corner, and extends to the mid-face of the billet with distance down the strand. The formation of the gap greatly reduces the rate of heat extraction from the shell surface.

3. Insufficient mold taper contributes to hot spots at the off-corner region, which corresponds to the thinnest shell thickness. At the same time the solidifying front on the diagonal of the shell is subjected to an excessive mechanical strain, which gives rise to off-corner cracks and even break-outs.

Acknowledgements

The authors are especially grateful to the National Natural Science Foundation (Grant No. 59734080) for the funding of this work.

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