Remining Useful Life Prediction of Equipment Driven by Multi-Sensor Data

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ABSTRACT: This paper presents a method for predicting the remining useful life (RUL) of equipment by using multi-sensor monitoring data of equipment. Firstly, the performance degradation of equipment is evaluated based on the maximum information coefficient. Then the performance degradation model is built by using Wiener process with non-linear drift. The maximum likelihood is used to estimate the parameters of the model, and the probability density distribution of RUL considering random failure threshold is deduced. Finally, the parameters of the model are updated by improved particle filter, and the RUL of the remaining equipment is predicted.
1. INTRODUCTION

At present, the maintenance strategy of complex equipment is CBM [1]. The condition-based maintenance strategy is conducive to controlling the maintenance cost of equipment, avoiding "out of repair" and "out of repair", improving the utilization rate of maintenance resources, and improving the attendance rate of equipment [2]. Accurate prediction is an important prerequisite for condition-based maintenance. Among them, RUL is the main content of prediction work [3].

The RUL prediction methods of equipment are mainly divided into failure mechanism model and data-driven prediction. The prediction of failure mechanism model has the characteristics of strong pertinence and high prediction accuracy, but it has the shortcomings of difficult modeling and poor model adaptability [5]. Data-driven prediction refers to the prediction of RUL using historical and real-time condition monitoring data of equipment operation. RUL prediction based on artificial intelligence refers to the use of knowledge base composed of historical data and related experience, and certain learning methods (such as support vector machine [7], grey system theory [8], neural network [9, 10], etc.) to form a mapping model between monitoring data and RUL, and through this model and real-time monitoring data, to carry out RUL prediction. The prediction of RUL based on intelligent algorithm relies on fewer assumptions and has a wide range of applications, but the method relies heavily on samples and needs sufficient historical data. In addition, most artificial intelligence estimates the RUL by point, and can't give the probability distribution of RUL, which is not conducive to maintenance decision-making, spare parts optimization and other maintenance support work. The main idea of RUL prediction based on statistical model is to model the performance degradation process by using the monitoring data of the equipment, using probability and statistics theory and stochastic process model, and to predict the RUL of the engine according to the model [11].

RUL prediction based on Wiener process is a commonly used prediction method based on statistical model [12]. Many scholars use Wiener process model to study the non-monotonic degradation process of equipment [13-15]. In the application of RUL prediction of complex equipment, due to the complex structure, bad working environment and large noise of monitoring data, the performance degradation modeling needs to consider non-linear, random effects and observation errors. To solve the above problems, Xu et al. [16] constructed a Wiener process model with measurement errors, and took the observation errors into account in the degradation modeling. Considering measurement error, Si et al. [17] analyzed the influence of random effect on equipment performance degradation process. Ye et al. [18] introduced the non-linear degradation into the Wiener process model.

2. MODELING OF EQUIPMENT PERFORMANCE DEGRADATION

2.1 Modeling of Wiener Process with Nonlinear Drift

Wiener process is a stochastic process. The univariate Wiener process is usually expressed as [12]:

\[ X(t) = \mu t + \sigma W(t) \]  

(1)

Among them, \( \mu \) represents the drift part of Wiener process, \( \mu \) represents the drift coefficient, \( W(t) \) represents Brownian motion, \( \sigma \) represents the uncertainty factor in the random process.

Degradation process of equipment performance has inherent degradation trend, but due to the changes of workload, system state and external environment, under the influence of many factors, it shows non-monotonic characteristics. Wiener process can better reflect the characteristics of equipment performance degradation, and can be used to model its degradation.

Because the degradation process of equipment usually presents strong non-linear characteristics, the drift part of Wiener process can be represented by the non-linear function \( \lambda(t,b) \) of time \( t \), and \( \lambda \) is the drift coefficient. Different non-linear degradation characteristics can be described by different forms of \( \lambda(t,b) \). Thus, the Wiener process model of actual degradation can be established:

\[ X(t) = \lambda(0) + \lambda \lambda(t,b) + \sigma W(t) \]  

(2)
Among them, \( X(0) \) is the initial degradation quantity and \( \sigma_x \) is the diffusion coefficient. Considering the measurement noise:

\[
Y(t) = X(0) + \lambda X(t) + \sigma_x W(t) + \epsilon(t)
\]  

The measurement noise \( \epsilon(t) \sim N(0, \sigma_e^2) \) is independent of \( \lambda \) and \( W(t) \).

### 2.2 Model Parameter Estimation

Maximum likelihood estimation is used to estimate the model parameters. It is assumed that the historical data involve \( n \) devices. For any device \( i \), the degradation at the initial time \( t_0 \) is \( X_{i0} \), the degradation of \( t_1, t_2, \ldots, t_m \) at the time is \( X_{i1}, X_{i2}, \ldots, X_{im} \), and the \( m_i \) is the failure time recorded by the device \( i \). For \( \Delta X_{ij} = X_{ij} - X_{i(j-1)} \), \( \Delta X_{ij} \) is the performance degradation between \( t_{j-1} \) and \( t_j \) of device \( i \). The nature of Wiener process shows that:

\[
\Delta X_{ij} \sim N(\mu \Delta t_{ij}, \sigma^2 \Delta t_{ij})
\]

Where \( \Delta t_{ij} = t_{ij} - t_{(j-1)} \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m_i \).

Similarly, the incremental \( \Delta Y_{ij} = Y_{ij} - Y_{i(j-1)} \) per unit time can be considered to obey the Gauss distribution for the observed values of performance degradation:

\[
\Delta Y_{ij} \sim N(\mu \Delta t_{ij}, \sigma^2 \Delta t_{ij})
\]

Make \( \Delta Y = (\Delta Y_{i1}, \Delta Y_{i2}, \ldots, \Delta Y_{im_i})^T \), then the \( \Delta Y \) obeys multivariate normal distribution, and its expectation and covariance matrices are:

\[
E(\Delta Y) = \mu \Delta T_i
\]

\[
\Sigma = \sigma^2 \Delta T_i \Delta T_i^T + \Omega
\]

Where \( \Delta T_i = [\Delta t_{i1}, \Delta t_{i2}, \ldots, \Delta t_{i(j-1)}, \ldots, \Delta t_{im_i}] \), \( \Delta t_{ij} = \lambda(t_{ij}) - \lambda(t_{(j-1)}) \), \( t_0 = 0 \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m_i \), \( \Omega = \sigma^2 \sigma_0^2 + \sigma^2 \sigma_0 P_i \), \( P_i = \text{diag}(\Delta t_{i1}, \Delta t_{i2}, \ldots, \Delta t_{im_i}) \), \( \Delta t_{ij} = t_{ij} - t_{i(j-1)} \), and \( p_i = (p_{i,1}, \ldots, p_{i,m_i}) \) meet the following conditions:

\[
p_{ij} = \begin{cases} 
1 & i = j = 1 \\
2 & i = j \neq 1 \\
-1 & i = j \pm 1 \\
0 & \text{others}
\end{cases}
\]

Make \( Y = (\Delta Y_1, \Delta Y_2, \ldots, \Delta Y_n) \). All the observed data of performance degradation are expressed in a unified way. The logarithmic likelihood function of \( Y \) is as follows:

\[
\ln L(Y) = -\frac{\ln(2\pi)}{2} \sum_{i=1}^n m_i - \frac{1}{2} \sum_{i=1}^n \ln |\Sigma_i| - \\
\frac{1}{2} \sum_{i=1}^n (\Delta Y_i - \mu_i T_i)^T \Sigma_i^{-1} (\Delta Y_i - \mu_i T_i)
\]

In order to facilitate calculation, the logarithmic likelihood function in equation (9) can be deformed to some extent. Here make \( \sigma^2 = \sigma^2 / \sigma_x^2 \), \( \sigma_0^2 = \sigma^2 / \sigma_x^2 \), \( \Sigma_i = \Sigma / \sigma_x^2 \), then formula (9) is deformed:

\[
\ln L(Y) = -\frac{\ln(2\pi)}{2} \sum_{i=1}^n m_i - \frac{1}{2} \ln \sigma^2 \sum_{i=1}^n m_i - \\
\frac{1}{2\sigma^2} \sum_{i=1}^n (\Delta Y_i - \mu_i T_i)^T \Sigma_i^{-1} (\Delta Y_i - \mu_i T_i) - \frac{1}{2} \sum_{i=1}^n \ln |\Sigma_i|
\]

According to formula (10), the maximum likelihood estimates of \( \mu_i \) and \( \sigma^2 \) are obtained:
\[ \hat{\mu}_i = \frac{\sum_{i=1}^{\hat{n}} T_i \hat{\Sigma}_i^{-1} \Delta Y_i}{\sum_{i=1}^{\hat{n}} T_i \hat{\Sigma}_i^{-1} T_i} \]  
\[ \sigma^2 = \frac{\sum_{i=1}^{\hat{n}} (\Delta Y_i - \hat{\mu}_i T_i)^\top \hat{\Sigma}_i^{-1} (\Delta Y_i - \hat{\mu}_i T_i)}{\sum_{i=1}^{\hat{n}} m_i} \] 
\[ \lambda = \frac{\sum_{i=1}^{\hat{n}} Y_i T_i (\lambda - \hat{\lambda})}{\sum_{i=1}^{\hat{n}} T_i} \] 

However, in the maximum likelihood estimation of \( \lambda, \mu \), and \( \sigma^2 \), there are still three other parameters, \( \hat{\sigma}^2, b, \hat{\sigma}^2 \), which need to be estimated. It may be advisable to bring the estimated results of the sum of formula (11) and formula (12) into the logarithmic likelihood function of formula (9), and obtain the logarithmic likelihood function for \( \hat{\sigma}^2, b, \hat{\sigma}^2 \) as follows:

\[ \ln L(Y) = -\frac{1+\ln(2\pi) + \ln \hat{\sigma}^2}{2} + \frac{1}{2} \sum_{i=1}^{\hat{n}} \ln \left| \hat{\Sigma}_i \right| \]  

The maximum likelihood estimates of \( \hat{\sigma}^2, b, \hat{\sigma}^2 \) are obtained by calculating the partial derivatives of \( \hat{\sigma}^2, b, \hat{\sigma}^2 \) and making the partial derivatives zero. Then the maximum likelihood estimates of \( \hat{\sigma}^2, b, \hat{\sigma}^2 \) are introduced into the sum of (11) and (12). Finally, the estimates of \( \lambda, \mu \), and \( \sigma^2 \) are obtained.

### 2.3 Probability Density of RUL Considering Random Failure Threshold

According to the degradation model of equipment performance, the RUL of equipment and its probability density function are deduced.

When the measurement error is not considered, the equipment degradation process can be expressed as the non-linear Wiener process shown in equation (3). Assuming that the failure threshold of the equipment is expressed as \( L \), a performance degradation curve may pass through the failure threshold \( L \) several times because the performance degradation model based on the non-linear Wiener process is non-monotonic. The RUL of the equipment can be transformed into:

\[ \text{RUL} = \inf \{ \text{RUL} : X(\text{RUL}) \geq L - x_i, X(0) < L - x_i \} \] 

When the failure threshold is \( L \), the probability density function of the RUL of the equipment can be expressed as:

\[ f_{\text{RUL} \sim \text{L}}(x; \lambda, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2 \text{RUL}}} \left( L - x_i - \lambda \beta(\text{RUL}) \right) \exp \left( -\frac{(L - x_i - \lambda \beta(\text{RUL}))^2}{2\sigma^2 \text{RUL}} \right) \] 

Where \( \beta(\text{RUL}) = \psi(\text{RUL}) - \frac{\psi(\text{RUL})}{d(\text{RUL})} d(\text{RUL}) \).

Considering Measurement Error \( \epsilon(t) \), \( x_i = y_i - \epsilon(t) \), and \( x_i \sim N(y_i, \sigma_i^2) \), there are the following lemmas:

**Lemma 1**: There are two independent normal distributions, \( D_i \sim N(\mu_i, \sigma_i^2) \), \( D_i \sim N(\mu_i, \sigma_i^2) \), and \( \omega \in \mathbb{R}, E \in \mathbb{R}, F \in \mathbb{R}, G \in \mathbb{R}^+ \), then:
According to the formula of total probability, it can be concluded that:

$$f_{RUL_{k},X_{i}}(RUL_{k}|L,Y_{i}) = \frac{1}{\sqrt{2\pi\text{RUL}_{k}^{2}\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2}}} \cdot \exp \left( -\frac{L-Y_{i} - \mu_{k}\psi(RUL_{k})}{2(\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2})} \right) \left[ L-Y_{i} - \mu_{k}\beta(RUL_{k}) \right]$$

(18)

After that, the PDF of equipment RUL considering random failure threshold is derived. The theorem given is as follows:

Theorem 1: if $D \sim \mathcal{N}(\mu, \sigma^2)$, and $E, F \in \mathbb{R}$, then

$$E_{D} \left\{ E_{\theta} \left[ (\omega - D_{k} - ED_{k}) \exp \left( -\frac{(\omega - D_{k} - FD_{k})^{2}}{2G} \right) \right] \right\}$$

$$= \frac{G}{\sqrt{F^{2}\sigma_{\omega}^{2} + \sigma_{\epsilon}^{2} + G}} \cdot \exp \left( -\frac{(\omega - \mu_{k} - F\mu_{k})^{2}}{2(F^{2}\sigma_{\omega}^{2} + \sigma_{\epsilon}^{2} + G)} \right).$$

(16)

$$= E_{\theta} \left\{ E_{\omega} \left[ (\omega - \mu_{k} - F\mu_{k}) \exp \left( -\frac{(\omega - \mu_{k} - F\mu_{k})^{2}}{2(F^{2}\sigma_{\omega}^{2} + \sigma_{\epsilon}^{2} + G)} \right) \right] \right\}$$

According to the formula of total probability, it can be concluded that:

$$f_{RUL_{k},X_{i}}(RUL_{k}|L,Y_{i}) = \frac{1}{\sqrt{2\pi\text{RUL}_{k}^{2}\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2}}} \cdot \exp \left( -\frac{L-Y_{i} - \mu_{k}\psi(RUL_{k})}{2(\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2})} \right) \left[ L-Y_{i} - \mu_{k}\beta(RUL_{k}) \right]$$

(17)

$$= E_{\epsilon} \left\{ E_{\omega} \left[ f_{RUL_{k},X_{i}}(RUL_{k}|L,\tilde{X}_{i}) \right] \right\}$$

Make $X_{i} = D_{i}$, $\tilde{X}_{i} = D_{i}$, $E = \beta(RUL_{i})$, $F = \psi(RUL_{i})$, $G = \sigma_{RUL_{i}}^{2}$, by using Lemma 1, the PDF of the RUL of the equipment under the condition of fixed failure threshold can be obtained as follows:

$$f_{RUL_{k},X_{i}}(RUL_{k}|L,Y_{i}) = \frac{1}{\sqrt{2\pi\text{RUL}_{k}^{2}\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2}}} \cdot \exp \left( -\frac{L-Y_{i} - \mu_{k}\psi(RUL_{k})}{2(\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2})} \right) \left[ L-Y_{i} - \mu_{k}\beta(RUL_{k}) \right]$$

(18)

Based on the total probability formula, on the premise that the monitoring data (or degradation index) $Y_{i}$ is known, the probability density function of equipment RUL considering random failure threshold is as follows:

$$f_{RUL_{k},X_{i}}(RUL_{k}|L,Y_{i}) = \frac{1}{\sqrt{2\pi\text{RUL}_{k}^{2}\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2}}} \cdot \exp \left( -\frac{L-Y_{i} - \mu_{k}\psi(RUL_{k})}{2(\psi(RUL_{k})\sigma_{X_{i}}^{2} + \sigma_{RUL_{k}}^{2} + \sigma_{Y_{i}}^{2})} \right) \left[ L-Y_{i} - \mu_{k}\beta(RUL_{k}) \right]$$

(19)

$$= E_{\omega} \left\{ f_{RUL_{k},X_{i}}(RUL_{k}|L,Y_{i}) \right\}$$

(20)

For the convenience of formula expression, make $H_{1} = \psi(RUL_{i})\sigma_{X_{i}}^{2} + \sigma_{RUL_{i}}^{2}$, $H_{2} = Y_{i} + \mu_{k}\psi(RUL_{i})$.

(21)
\[ H_j = \psi(RUL_j)\gamma^2 + \sigma_j^2 RUL_j - \psi(RUL_j)\sigma_j^2 \]  
\[ H_i = \frac{H_i}{H_i}(y_i + \mu_i + R(RUL_i)) - \psi(RUL_i)\sigma_i^2 \]

Make \( \omega = D, H_4 = E, H_2 = F, H_1 = G \), the probability density function of the RUL of the equipment at \( t_i \) can be obtained by introducing it into equation (29):

\[ f_{Y_i | \omega, p_i, (RUL_i | Y_i)} = \frac{H_i}{2\pi H_i \Phi(\mu_i / \sigma_i) RUL_i \exp \left( -\frac{(\mu_i - H_i)^2}{2(H_i + \sigma_i^2)} \right)} \]

\[ \frac{\sqrt{H_i \sigma_i^2}}{H_i + \sigma_i^2} \exp \left( -\frac{(H_i \sigma_i^2 + H_i \mu_i)^2}{2(H_i + \sigma_i^2) H_i \sigma_i^2} \right) + \frac{2\pi}{\sqrt{H_i + \sigma_i^2}} \left( \frac{H_i \sigma_i^2 + H_i \mu_i}{H_i + \sigma_i^2} - H_i \right) \Phi \left( \frac{H_i \sigma_i^2 + H_i \mu_i}{\sqrt{(H_i + \sigma_i^2) H_i \sigma_i^2}} \right) \]  

3. SIMULATION AND ANALYSIS

This paper makes use of the common data set of turbofan engine performance degradation, to carry out the simulation experiment of equipment RUL prediction.

Firstly, the cumulative contribution rate of each sensor is obtained by using CMAPSS data set as shown in Table 1. Referring to the principal component analysis method, the threshold chosen here is 0.8, and the sensor serial number after dimensionality reduction is 4, 11, 2, 13, 3, 7, 14, 21.

Table 1 MIC values and cumulative contribution rates of sensor monitoring values and RUL

| sensor | MIC   | Contribution rate | Cumulative contribution rate |
|--------|-------|-------------------|------------------------------|
| 4      | 0.5091| 0.1178            | 0.117844                     |
| 11     | 0.4893| 0.1133            | 0.231106                     |
| 2      | 0.4692| 0.1086            | 0.339714                     |
| 13     | 0.4428| 0.1025            | 0.442212                     |
| 3      | 0.442 | 0.1023            | 0.544524                     |
| 7      | 0.4307| 0.0997            | 0.644221                     |
| 14     | 0.4212| 0.0975            | 0.741719                     |
| 21     | 0.3922| 0.0908            | 0.832504                     |
| 20     | 0.2868| 0.0664            | 0.898891                     |
| 12     | 0.2306| 0.0534            | 0.95227                      |
| 15     | 0.2062| 0.0477            | 1                            |

The performance degradation index curve is shown in Figure 1.

Fig. 1 Offline Degradation Index of 100 Engines in CMAPSS Training Data Set
In summary, the process of RUL prediction of aeroengine using C-MAPSS data set can be summarized as Figure 3.

Then, the parameters of the probability density distribution function of the random failure threshold are estimated by using the degradation index of the engine at the failure time in the training data set. The parameters of probability density distribution function for stochastic failure threshold are $\hat{\mu} = 0.5371$, $\hat{\sigma}^2 = 0.0484$.

The maximum likelihood estimation of the parameters of the equipment performance degradation model is carried out by using the engine off-line degradation index obtained. The estimated values of the parameters of the model are shown in Table 2.

The model parameters obtained by maximum likelihood estimation reflect the degradation characteristics of the same kind of equipment. When predicting the RUL of the target equipment, the stochastic parameters need to be updated to better reflect the individual degradation characteristics of the target equipment. In this paper, the improved particle filter based on particle swarm optimization is used to update the parameters. Reference [26] on the principle of improved particle filter.

For aeroengine RUL prediction, the penalty function of RUL prediction deviation is given in reference [25]:

$$\text{penalty}(i) = \begin{cases} \exp(-e_i/10) - 1 & e_i > 0 \\ \exp(e_i/13) - 1 & e_i \leq 0 \end{cases}$$ (26)
is the prediction error of engine . For aero-engines, the penalty value of late prediction is higher when the absolute error is the same, because it is easy to cause safety problems. The cumulative value of all engine penalty functions is taken as the score of the prediction method:

\[
\text{score} = \sum_{i} \text{penalty}(i)
\]  

(27)

The predicted result score of this method is 768.86, and that of fixed failure threshold is 1028.31.

4. CONCLUSION

This paper presents a method of RUL prediction driven by multi-sensor data. The method proposed in this paper is simulated and validated by using C-MAPSS dataset, and the following conclusions are drawn:

1. The equipment performance degradation model based on Wiener process with noise and nonlinear drift can reflect the non-linear and non-monotonous degradation process, and has achieved good prediction results.

2. Compared with the fixed failure threshold prediction method, the RUL prediction method considering random failure threshold has higher accuracy.

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