Neutrinos in a gravitational background:
a test for the universality of the gravitational interaction

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Abstract

In this work we propose an extended formulation for the interaction between neutrinos and gravitational fields. It is based on the parametrized post-Newtonian aproach, and includes a violation of the universality of the gravitational interaction which is non diagonal in the weak flavor space. We find new effects that are not considered in the standard scenario for violation of the equivalence principle. They are of the same order as the effects produced by the Newtonian potential, but they are highly directional dependent and could provide a very clean test of that violation. Phenomenological consequences are briefly discussed.

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Despite the great success of general relativity to explain the gravitational interaction, this theory has proved to be very difficult to test in detail. There are several systems where gravitation is important, such as the very early universe, pulsars, quasars, black holes, and gravitational waves. However, gravitational fields in astrophysical systems can be considered as weak, even in the extreme cases of the neighborhood of a neutron star or at few Schwartzchild radii from a black hole. Usually the gravitational effects beyond the Newtonian level are very small and too tightly interwoven with other local physical effects to be clearly observed.

The interest in these effects is twofold. They can shed new light on the character of the gravitational interaction, and they can provide invaluable information on some astrophysical systems, such as supernovas and neutron stars. With respect to the first point, two questions arise naturally: the validity of general relativity as a description of the gravitational interaction, and the universality of this interaction.

In fact, the strongest evidence of the universality of the gravitational interaction involves electrons, protons and neutrons, that is the members of the lightest family of matter fields in the standard model. Tests of the weak equivalence principle for these particles include laboratory experiments of the Eötvös-type, which measure the gravitational acceleration of macroscopic bodies. They state that gravity accelerates all macroscopic objects at the same rate to an accuracy of one part in $10^{12}$ \cite{1}. The experimental limits for the universality between matter fields and gauge fields are weaker. For example, the supernova SN1987A gave the opportunity of a direct comparison between the transit time for photons and neutrinos traversing the same path in a gravitational potential $\phi(\mathbf{r})$, which leads to limits on the violation of the weak equivalence principle by massless particles of the order of $|\gamma_\gamma - \gamma_\nu| \lesssim 10^{-3}$, where $\gamma$ is the PPN (parametrized post-Newtonian formalism) parameter for the scalar potential \cite{2}.

It is more difficult to obtain observational evidence on the gravitational coupling of the
heavier families, except for the case of the kaon system where the bound $|\phi \Delta \gamma| \leq 2 \times 10^{-13}$ has been set. Information of this kind can also be acquired from the propagation of neutrinos in a medium with a gravitational background. As is well known, ordinary matter affects the neutrino propagation in a flavor dependent way and, under favourable conditions, large transformations of one neutrino flavor into another can take place, even for small mixing between the mass eigenstates. The implications of this mechanism in astrophysics and cosmology has been extensively examined during the last years. A similar resonant enhancement could be induced by a non-universality in the gravitational interaction of the neutrinos, even if they are massless. Violations of the equivalence principle of the order of $10^{-20}$ cannot be ruled out in this context, and in fact the observed deficit in the solar and atmospheric neutrino fluxes have already been interpreted as a positive signal of this violation [4,5]. An interpretation in terms of a violation of the Lorentz invariance is also possible [6], but this could be included within the violation of the equivalence principle scheme in the case of constant fields [7]. Note that neutrinos are unequaled as test particles for probing the gravitational field. Because of the smallness of their interactions, the level of accuracy that can be achieved with them is several orders of magnitude better than in any other previous test.

The present work is partially motivated by the above considerations and develops a general framework for analyzing the possible flavor dependence of the gravitational interaction. Our approach is a generalization of the one proposed in Refs. [4,5], where only the effects due to the scalar gravitational potential $\phi$ was considered. In this way, we find new contributions to the oscillations that are of the same order of magnitude than the terms involving the Newtonian potential, and we extend the analysis to the next PPN order, which includes contributions generated by the angular momentum of the gravitational source. In contrast with the scalar potential contribution, this gives place to new highly anisotropic effects, which in principle could be verified in several astrophysical systems. These new effects provide a more precise and characteristic signature for a possible violation of the equivalence principle.
Whereas the integer spin fields can be consistently described in a curved space-time, the half-integer spin fields need to be defined with reference to a locally inertial frame at each point of the space-time. If we are considering several neutrino flavors, we have to extend the usual construction by defining each gravitational flavor in its own inertial frame. Furthermore, we can introduce a possible violation of the equivalence principle and assume that these frames are not necessarily related by Lorentz transformations, and thus could be physically non-equivalent. At each of the orthonormal frames the gravitational field is supposed to have the structure given by the PPN formalism, which provides a general account of the possible deviations from the Einstein theory.

These assumptions lead us to a generalization of the standard scenario for the violation of the equivalence principle (VEP). With our present theoretical understanding it seems that if we want to keep the spinor structure, the theory does not correspond to a metric one which can be settled on a consistent basis. More precisely, our approach should be considered as a phenomenological one, which allows us to search for possible signatures of flavor-changing effects associated with a violation of the equivalence principle. In the following we do not consider the dynamics of the gravitational fields but we still use a manifold as the space-time framework, i.e., although the metric is not defined we assume that the space-time is well defined.

II. NEUTRINOS IN A GRAVITATIONAL BACKGROUND

Given the frame $V^µ_α$ the equation for a (massless) neutrino is the Dirac equation in a curved space time:

$$\gamma^α V^µ_α (\partial_µ + i \Gamma_µ) \Psi = 0 ,$$

(1)

where the connection is

$$\Gamma_µ = -\frac{1}{2} V^α_µ V^ν_μ V^μ_β \sigma_αβ ,$$

(2)
with $\sigma_{\alpha\beta} = \frac{i}{2} [\gamma_\alpha, \gamma_\beta]$. All covariant derivatives are metric, so we are neglecting small torsion effects \[9\]. Other possibilities for the equation involve generical couplings with the curvature \[10\], but these terms are highly suppressed in the usual astrophysical situations by the small values of the gradients of the gravitational fields as compared to the neutrino momentum. Explicitly, the linearized Dirac equation for a static gravitational field reads \[11\]:

$$
\left( i\gamma^\mu \partial_\mu - \frac{i}{4} \{ h_{00}, \gamma \cdot \nabla \} - \frac{i}{4} \{ h_{ij}, \gamma^j \partial^i \} + \frac{i}{2} \{ h_{0i}, \gamma^0 \partial^i \} + \frac{1}{2} \gamma^0 \epsilon^{ijk} \partial^j h^{0j} s^k \right) \Psi_\nu = 0 , \quad (3)
$$

where the $h^{\mu\nu}$ fields are defined by $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$ and $s^k = \frac{1}{4} \epsilon^{ijk} \sigma^{ij}$. The spatial derivatives of the gravitational potentials are proportional to the inverse of their characteristic variation length, $\partial_i h^{\mu\nu} \propto L^{-1}$. In astrophysical systems the spatial dimensions and the energy of the neutrinos render $L \gg \lambda_\nu$, where $\lambda_\nu \propto p^{-1}$ is the neutrino wavelength. Accordingly, we can neglect the terms with spatial derivatives of the gravitational fields, including the spin contributions. This approximation can be justified on a more general basis by using a geometric optic-like expansion of the Dirac equation in powers of the parameter $\lambda/L$ \[12\]. The main effects of the neglected terms, which include a chirality transition induced by gravity, are independent of the equivalence principle violation, and have been already analyzed in Ref. \[11\]. In the above approximation Eq. (3) reduces to

$$
i \partial_\nu \Psi_\nu = H \Psi_\nu , \quad (4)$$

with the Hamiltonian given by

$$
H = -i \gamma_0 \gamma_i [(1 - \frac{1}{2} h^{00}) \partial_i - \frac{1}{2} h_{ij} \partial_j] - i h_{0i} \partial_i , \quad (5)
$$

where all metric dependent terms are assumed to be slowly varying functions of the position.

In an astrophysical scenario we can work within the framework of the PPN theories \[13\]. The assumptions for constructing the metric in the PPN formalism involve virialized sources such that $\frac{M}{R} \sim w^2$, where the quantities $M, R$, and $w$ represent estimations of the order of magnitude of the mass, distance and characteristic (average) velocity of the source.
The metric is the Minkowskian one plus source dependent perturbations. The latter have the correct tensorial character and dimensions, falling at least like $1/R$ at infinity. This metric is in general given by

$$h_{oo} = 2\gamma' U + O(w^4),$$

$$h_{oi} = -\frac{7}{2}\Delta_1 V_i - \frac{1}{2}\Delta_2 W_i + (\alpha_2 - \frac{1}{2}\alpha_1)v_i U - \alpha_2 v_j U_{ij} + O(w^4),$$

$$h_{ij} = 2\gamma U\delta_{ij} + \Gamma U_{ij} + O(w^4),$$

where the potentials are

$$U = \int \rho(r') \frac{d^3 r'}{|r - r'|},$$

$$U_{ij} = \int \rho(r')(r_i - r'_i)(r_j - r'_j) \frac{d^3 r'}{|r - r'|^3},$$

$$V_j = \int \rho(r') w_j(r') \frac{d^3 r'}{|r - r'|},$$

$$W_j = \int \rho(r')(w(r') \cdot (r - r'))(r_j - r'_j) \frac{d^3 r'}{|r - r'|^3}.$$  

Here $\rho(r)$ is the density of mass and $w(r)$ is the velocity of the source of the gravitational field. We are using a system of unities where $G = \hbar = c = 1$. We keep each term of the same order in the PPN expansion because we are interested in ultrarelativistic neutrinos.

In the particular case of a very confined and distant source, the expressions in Eqs. (9)-(10) can be approximated as follows

$$U \approx \frac{M}{R} + O\left(\frac{1}{R^2}\right), \\
U_{ij} \approx \frac{X_i X_j}{R^4} U + O\left(\frac{1}{R^2}\right),$$

$$V_j \approx w_j U + O\left(\frac{1}{R^2}\right), \\
W_j \approx \frac{w_i X_i X_j}{R^4} U + O\left(\frac{1}{R^2}\right),$$

where $X_i$ are the components of $R$.

Up to order $w^3$ the adimensional parameters of the expansion are $\gamma, \gamma', \Delta_1, \Delta_2, \Gamma, v, \alpha_1$ and $\alpha_2$. In Einstein gravity we have $\alpha_1 = \alpha_2 = \Gamma = 0, \gamma = \gamma' = \Delta_1 = \Delta_2 = 1$ and $v$ is irrelevant. The parameters $\alpha_1$ and $\alpha_2$ are null if the theory is Lorentz covariant, but if there is a preferred reference frame, characterized by a velocity $v$, they should be non null. In general the characteristic frame velocity can be a gravitational flavor dependent quantity.
The parameter $\alpha_1$ can be fixed to be $7\Delta_1 + \Delta_2 - 4\gamma - 4\gamma'$, while $\alpha_2$ is an independent parameter up to this order.

The dispersion relation implied by the Hamiltonian in Eq.(5) is

$$\left(1 - h^{00}\right)p^2 - E^2 + 2Eh_{0i}p_i - h_{ij}p_i p_j = 0,$$

where $E$ is the energy eigenvalue and we keep only first order terms in the metric $h_{\mu\nu}$.

III. NEUTRINO OSCILLATIONS FROM A NON-UNIVERSAL GRAVITATIONAL INTERACTION

The purpose of our work is to state a meaningful phenomenological basis to discuss a possible violation of the equivalence principle. The breakdown of the universality of the gravitational interaction raises the question of the complete consistency of the underlying theory, but we will put aside this issue. Here, we are concerned only with a model that describes the different phase shifts associated to a possible flavor dependence of the gravitational interaction, its consequences, and the possibilities of its detection in an astrophysical framework. We assume that the metric associated with each family are very close to each other and to the Einstein theory predictions. The PPN coefficients depend on the flavor and are assumed to be diagonal matrices in the gravitational flavor basis. The more general case where these coefficients are arbitrary matrices would imply that there exist no local inertial frames where the neutrinos can be defined, and therefore their spinorial nature would not be clear. We do not consider this situation.

Following this approach, each of the $n$ neutrino flavors is defined in a different orthonormal frame $\{V^\alpha_{a\mu}\}$, $a = 1, ..., n$, which form non-equivalent bases for the tangent space. Accordingly, we write the metrics as

$$\eta_{\alpha\beta}V^\alpha_{a\mu}V^\beta_{a\nu} = g^a_{\mu\nu}.$$ (16)

From Eq. (15), the dispersion relation for each neutrino gravitational flavor can be approximated by
\[ E^a = p \left( 1 + h^a_{\alpha\beta} \hat{p}_\alpha - \Gamma^a U_{ij} \hat{p}_i \hat{p}_j - (\gamma^a + \gamma^a)U \right). \] \hspace{1cm} (17)

In the PPN approximation the coordinate system is generally fixed to give \( \Gamma = 0 \) and \( \gamma' = 1 \), (where the last equality is equivalent to the definition of the Newton constant,) but in our case such a coordinate fixing could be done only for one metric at the expense of the others. Therefore, we are setting these parameters at the usual values for the first gravitational family, while the others are left as free parameters. This is an important point as we will see in what follows.

If the parameters are family dependent, then distinct neutrinos will undergo different phase shifts when passing through the same sector of the space. The phase shift differences become observable when the particle basis that diagonalize the weak and the gravitational interaction are not the same. In this context, the effects of a universality violation at the level of the scalar potential \( U \) has been already considered in Ref. [4], where it is shown that such a violation leads to neutrino oscillations. The factor \( (1 + \gamma^a) \) of their model should be replaced by \( (\gamma'^a + \gamma^a) \) in our case. We extend this model by including the effects of the \( U_{ij} \) potentials that are of the same order of magnitude as \( U \), together with the PPN structure expanded up to \( \omega^3 \), with the corresponding generalized universality violations. In this way, we are taking into account not only the coupling of the neutrinos to the mass of the source of the gravitational field, but also the coupling to its quadrupolar distribution and its angular momentum. As we will show, these last interactions produce highly directional and very characteristic effects.

In what follows we examine the neutrino oscillations induced by a non-universal gravitational coupling. To find out the main features of this phenomena we will consider two neutrino flavors. We assume that the gravitational flavor basis is related to the electroweak basis through a unitary transformation \( U \), characterized by a mixing angle \( \theta_g \):

\[
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
= \begin{pmatrix}
U^\dagger
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix}
\equiv
\begin{pmatrix}
\cos \theta_g & -\sin \theta_g \\
\sin \theta_g & \cos \theta_g
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix}.
\] \hspace{1cm} (18)

The equation for the neutrino evolution is
\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \mathcal{H} \mathcal{U}^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \]

(19)

where \( \mathcal{H} \) is a diagonal matrix in the gravitational flavor basis, whose eigenvalues are given by Eq. (17). After discarding an irrelevant overall phase, we have

\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta_0}{2} \begin{pmatrix} -\cos 2\theta_g & \sin 2\theta_g \\ \sin 2\theta_g & \cos 2\theta_g \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \]

(20)

with

\[ \Delta_0 = E_2 - E_1 = E \left[ \delta h_{0i} \hat{p}_i - \delta \Gamma U_{ij} \hat{p}_i \hat{p}_j - (\delta \gamma' + \delta \gamma)U \right], \]

(21)

where \( E = p \) is the neutrino beam energy, and

\[ \delta \gamma = \gamma^2 - \gamma^1, \quad \delta \gamma' = \gamma'^2 - \gamma'^1, \quad \delta \Gamma = \Gamma^2 - \Gamma^1, \]

(22)

\[ \delta h_{0i} = h_{0i}^2 - h_{0i}^1 \approx -\frac{7}{2} \delta \Delta_1 V_i - \frac{1}{2} \delta \Delta_2 W_i + (\delta \alpha_2 - \frac{1}{2} \delta \alpha_1) v_i U - \delta \alpha_2 v_j U_{ji}. \]

(23)

For a constant gravitational field the survival probability is

\[ P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 (2\theta_g) \sin^2 \left( \frac{\pi L}{\lambda_g} \right), \]

(24)

where \( L = t - t_0 \) is the distance travelled by the neutrino from the production point. This clearly shows that oscillations will appear whenever there exists a non null mixing angle induced by flavor dependent gravitational interactions. These oscillations have a characteristic length given by

\[ \lambda_g = \frac{2\pi}{|\Delta_0|}. \]

(25)

In contrast with vacuum oscillations induced by a mass difference, where \( \lambda_m = \frac{4\pi E}{\delta m^2} \) is proportional to the energy, the effect we are considering has oscillation lengths proportional to \( E^{-1} \), which makes this phenomena suitable to be observed in the case of high energy neutrinos. Note that even though the overall sign of the gravitational potential is irrelevant.
for the oscillations, the relative signs among the parameter differences are very significative. The Eq. (24) leads to the following averaged survival probability

\[ < P(\nu_e \rightarrow \nu_e) > = \frac{1}{2} \left( 1 + \cos^2 2\theta_g \right). \] (26)

As is well known, neutrino oscillations in matter are qualitatively different from the oscillations in the vacuum. This is because the interaction of the neutrinos with matter modify their dispersion relations. Neutral current interactions are flavor diagonal and can be ignored, as long as we do not consider sterile neutrinos and neutrinos are not part of the medium. In general, this will not be true for the charged current interactions. The forward scattering amplitude is not flavor diagonal in this case, and depends on the leptonic content of the matter. The above gives place to important consequences such as the MSW effect. In general, we expect that a non universal gravity will also affect the electroweak Lagrangian, introducing a number of unknown coefficients, but the combined effect should be of the order \( U G_F \) and therefore is highly suppressed.

If electrons are the only leptons that are present, then the matrix

\[ \frac{b_e(t)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \] (27)

has to be added to the second term of Eq. (24). Here, \( b_e(t) = \sqrt{2} G_F N_e(t) \), with \( N_e(t) \) denoting the electron density. The resulting Hamiltonian \( \mathcal{H}(t) \) can be diagonalized at every moment by introducing the instantaneous flavor basis, defined in an analogous way to Eq. (18), with \( \theta_g \rightarrow \theta_m(t) \) and

\[ \sin 2\theta_m(t) = \frac{\Delta_0 \sin 2\theta_g}{\sqrt{\left( \Delta_0 \cos 2\theta_g - b_e(t) \right)^2 + \left( \Delta_0 \sin 2\theta_g \right)^2}}. \] (28)

In the adiabatic approximation, the average survival probability is given by the following formula

\[ < P(\nu_e \rightarrow \nu_e) > = \frac{1}{2} \left( 1 + \cos 2\theta_g \cos 2\theta_m(t_0) \right). \] (29)
which reduces to Eq. (26) when \( b_e(t) = 0 \), so that \( \theta_m(t) = \theta_g \). The use of the adiabatic approximation is justified whenever

\[
\frac{1}{N_e(t_R)} \left| \frac{dN_e(t)}{dt} \right|_{t_R} \ll |\Delta_0| \frac{\sin^22\theta_g}{\cos2\theta_g}.
\]

There also exists the possibility of a resonant conversion when the diagonal elements of the full Hamiltonian vanish, i.e. when

\[
\sqrt{2}G_FN_e(t_R) = \Delta_0 \cos 2\theta_g.
\]

This mechanism can totally change the flavor, independently of the value of the mixing angle \( \theta_g \), but its efficiency depends on the adiabaticity of the process.

**IV. PHENOMENOLOGICAL EFFECTS**

There is an inherent uncertainty in the potentials in Eqs. (9)-(10), because arbitrary constants can always be added without changing the physics, as far as the effects associated with a violation of the equivalence principle are not involved. In Einstein theory these constants can be eliminated by a coordinate transformation. Instead, in the PPN expansion the coordinate system is fixed, so possible uncertainties arise from the very distant unknown mass distributions. We will restrict ourselves to the most important known near sources for the potentials, leaving aside the problem of the very distant ones. In any case, the distant sources would only produce significant isotropical effects, and thus could only affect the definition of \( U \) and the diagonal part of \( U_{ij} \) (which in the formula for \( \lambda_g \) can be absorbed into \( U \)).

**A. Solar neutrinos**

The sun is subject to a gravitational field that has several sources. The main sources are: our galaxy, the Virgo Cluster, and the Great Attractor. The more important contribution to the potential \( U \) comes from the Great Attractor gravitational field, with small perturbations
due to galactic clusters, and the galactic and solar fields. In consequence, this potential can be approximated by a constant of the order of $10^{-5}$ \cite{3,14}. The effect of this potential regarding a possible universality violation has been already analyzed in Ref. \cite{3}. Resonant flavor oscillations are consistent with the observed deficit of solar and atmospheric neutrinos for $U\delta(\gamma + \gamma') \simeq 10^{-22}$. The effect is isotropic and to account for the more recent data requires a three-neutrino mixing scheme \cite{15}.

In the PPN approximation the contributions of the $U_{ji}$ potential are in principle considered of the same magnitude as $U$. In the case of a distant source in the $z$ direction, we have $U_{zz} \sim U$. However, the components $U_{xx}$, $U_{yy}$ and $U_{xy}$ are proportional to $(\Delta \theta)^2 U$, where $\Delta \theta$ is the angular size of the source, while $U_{xz}$ and $U_{yz}$ are of the order of $\Delta \theta U$ (in fact, they are proportional to the center of mass distance to the $z$ axis). Considering that the Great Attractor is a rather extended object with an angular size of the order of $10^{-1}$ \cite{16}, we see that in the case of the sun there are only three relevant types of $U_{ji}$ contributions: those coming from our galaxy, which are of the order of $10^{-6}$, a longitudinal component from the Great Attractor, of order of $U_{zz} \simeq U \simeq 10^{-5}$, and transverse-longitudinal components also produced by the Great Attractor, of the same order as the galactic contributions, $U_{xz} \simeq U_{yz} \simeq 10^{-6}$. The contributions of $V_i$ and $W_i$ are roughly proportional to source velocity $\mathbf{w}$ times $U$, which is of the order of $(10^{-3} - 10^{-2})U$. Their directional effect has a dipolar structure and can be assumed to be one or two orders of magnitude smaller than the dominant ones.

Therefore, a violation of the equivalence principle would be characterized by three main effects manifested as flavor oscillations: an isotropic effect ($U \simeq 10^{-5}$), and two additional anisotropic effects ($U_{zz} \simeq 10^{-5}$, $U_{ji} \simeq 10^{-6}$). If we assume that the differences of the PPN parameters due to the flavor dependence are all of the same order, the most significant directional effect is given by a quadrupolar contribution due to $U_{zz}$. This effect could be of the order of the dipolar one originated by the elliptical orbit of the Earth, but the latter only depends on the eccentricity of the orbit (perigee: RA=18:48, DEC=-23:27 in equatorial coordinates), whereas the gravitational one depends on the energy of the neutrinos.
and their direction with respect to the Great Attractor. The approximate position of the Great Attractor center in galactic coordinates is \( l = 325^\circ, b = -7^\circ, v = 4882 \text{ km s}^{-1} \), or in equatorial coordinates is RA=\( a=16:10 \), DEC=\( d=-60^\circ10' \). In ecliptic coordinates the aphelion position is RA=\( 18:48 \) \( (282^\circ) \), DEC=\( d=0^\circ \), whereas the Great Attractor center is at RA=\( A=16:52 \) \( (253^\circ) \), DEC=\( D=-38^\circ17' \). Both axis differ in approximately 30° in RA, and therefore the effects could be discriminated. Taking into account the dominant contributions, due to \( U \) and \( U_{zz} \), and assuming \( U \simeq U_{zz} \), the neutrino wavelength in vacuum becomes:

\[
\lambda_g = \frac{2\pi}{EU |\delta\Gamma \cos^2 \delta_\varpi \cos^2 (\alpha - A) + (\delta\gamma' + \delta\gamma)|}, \tag{32}
\]

where \( \alpha \) is the right ascension of the sun in ecliptic coordinates at a given time.

An easily visible consequence of the gravitational contribution is a breaking of the reflection symmetry of the neutrino flux with respect to the aphelion-perihelion axis of the Earth orbit. This symmetry is characteristic of the scenarios which do not consider the violation of the universality of the gravitational interaction, with the only exception given by a possible interaction between the solar magnetic field and the neutrino magnetic moment [17]. In general, the scenarios usually considered would yield different neutrino fluxes for Earth positions separated by six months. Otherwise, the gravitational contribution has the same sign for these two positions.

### B. Atmospheric neutrinos

The dominant contributions in this case are the same as those already considered in the previous section. The main difference is originated by the Earth rotation which gives place to diurnal neutrino flux variation. This situation can be described more appropriately by means of azimuthal coordinates [18]. In terms of these coordinates the neutrino direction can be written:

\[
\hat{p}_\nu = (\sin \theta_\nu \cos z_\nu, \sin \theta_\nu \sin z_\nu, \cos \theta_\nu), \tag{33}
\]
where $\theta_\nu$ is the zenithal distance and $z_\nu$ is the azimuthal angle of the incident neutrinos. Similarly, for the Great Attractor position we have:

$$\hat{d}_{GA} = (\cos \varphi \sin d - \sin \varphi \cos d \cos \tau, \cos d \sin \tau, \sin d \sin \varphi + \cos d \cos \varphi \cos \tau),\quad (34)$$

with $d$ being the Great Attractor declination angle and $\varphi$ the observatory latitude. The parameter $\tau$ is $a - t_s$, where $a$ is the right ascension of the Great Attractor and $t_s$ is the sidereal time. According to Eqs. (21) and (25) the oscillation wavelength depends on $|\hat{p}_\nu \cdot \hat{d}_{GA}|^2$. If there were no violation in the universality of the gravitational interaction, then the neutrino flux would only depend on the zenithal distance $\theta_\nu$. Here we have an additional dependence on the azimuthal angle $z_\nu$ and the time $\tau$, which implies a diurnal variation of the flux. For instance, if we focus our attention on zenithal neutrinos ($\theta_\nu = 0$), for the Kamiokande site ($\varphi = 36.5^\circ$) we have:

$$\lambda_g = \frac{2\pi}{E U |(-.5 + .4 \cos \tau)^2 \delta \Gamma + (\delta \gamma' + \delta \gamma)|}.\quad (35)$$

This is the effect at a given time $\tau$ on the vacuum wavelength. It can be traced on with the $\theta_\nu$ dependence of the total flux integrated over $\tau$ and $z_\nu$.

C. Neutron star kicks and rotation

Resonant neutrino oscillations influenced by the magnetic field in the early stage of a neutron star have been proposed as a possible mechanism to explain the observed proper motion of pulsars [19]. But to do this excessively strong magnetic fields are required [20]. Furthermore, the origin of the high angular velocity of the pulsars is also not completely clear [21]. In the present discussion, the vectorial potential $h_{0i}$ manifests itself as an anisotropic perturbation in the shape of the resonant surface that is located between the neutrinospheres of the different flavors. This anisotropy could be the source of both the angular and the traslational accelerations, and thus could simultaneously give an explanation for the observed spins and proper motions.
The potential $h_{oi}$ contains two main contributions, both produced by the star. One of them, given by $(\delta\alpha_2 - \frac{1}{2}\delta\alpha_1)v_iU - \delta\alpha_2 v_j U_{ji}$, is relevant when a preferred frame exists, and therefore the coefficients $\alpha$ are non null. Their action is analogous to the one produced by a strong magnetic field [19,22], which can generate a translational kick in the movement of the star during neutrino emission. Taking reasonable values for the potential and a frame velocity of the order $10^{-3}$, the resulting gravitational effect has the correct magnitude to explain the observed kicks. The other contribution to $h_{oi}$ originates from $V_i$ and is caused by the star rotation. It induces an angular acceleration during neutrino emission because the resonant surface depends on the neutrino angular momentum. The sign of this acceleration is determined by the relative sign between $(\delta\gamma' + \delta\gamma)$ and $\delta\Delta_1$.

Since the gravitational fields in the interior of a neutron star are relatively high, $U \lesssim 1$, the PPN approximation is not very good in this case. A more complete treatment should include higher order terms in the expansion, and hence more parameters.

V. CONCLUSIONS

In this work we have formulated a generalized scenario for neutrinos propagating in the presence of gravity. To incorporate the violation of the equivalence principle each neutrino flavor has been defined in a different orthonormal frame at each point of space-time. This is a natural prescription if we are giving up the metric but still want to retain the manifold structure. The violation is parametrized by a generalized PPN expansion.

In this way we have developed a VEP scenario for neutrino oscillations, which leads to several new effects. The most relevant is the one due to the potential $U_{ji}$, which can be of the same order of magnitude as the Newtonian contribution. In the PPN metric theories this potential is irrelevant because it can be set to zero by a coordinate fixing. This is not the case in the present approach, where there is more than one metric. In addition, other potentials could be relevant for the early stages of neutron stars, giving place to angular or translational accelerations.
An interesting feature of the effects we have discussed is that they have a very characteristic directional dependence. These effects should be present in the solar and atmospheric neutrino fluxes if their anomalies are related to a VEP scenario. In such a way, these phenomena could provide relevant information on this violation. Accordingly, the solar neutrino flux must change along the orbit of the Earth in a way that clearly differs in the angular and energy dependence from the geometrical effect due to the orbit eccentricity and its consequences on the ordinary vacuum oscillations. In contrast to the mass-mixing scenario, the seasonal effect we consider could also appear in the case of resonant transitions in the sun, due to the variation of the position of the resonance region. In the case of atmospheric neutrinos, a detailed analysis of the daily and zenithal angle dependence of the flux, could reveal the effects we are interested in. Further analysis along these lines are in progress.

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