A hybrid small-world network/semi-physical model for predicting wildfire spread in heterogeneous landscapes

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Abstract. This paper presents the latest developments and validation results of a hybrid model which combines a broad-scale stochastic small-world network model with a macroscopic deterministic approach, to simulate the effects of large fires burning in heterogeneous landscapes. In the extended version of the model, vegetation is depicted as an amorphous network of combustible cells, and both radiation and convection from the flaming zone are considered in the preheating process of unburned cells. Examples are given to illustrate small-world effects and fire behavior near the percolation threshold. The model is applied to a Mediterranean fire that occurred in Corsica in 2009 showing a good agreement in terms of rate of spread, and area and shape of the burn. A study, based on a fractional factorial plan, is conducted to evaluate the influence of variations of model parameters on fire propagation.

1. Introduction
Heterogeneous conditions of weather, fuel, and topography are generally encountered during the propagation of large fires. As revealed by satellites maps, this shapes irregular patterns and fire’s spread looks fractals [1]. Among existing wildfire spread models [2], stochastic models seem to be good candidates for studying the erratic behavior of large fires. Moreover they generally provide super-real-time simulations. On the contrary, stochastic models do not involve the physics of the fire spread process from one burning vegetation item to other unburned items, at the macroscopic scale. The hybrid stochastic/deterministic model we developed a few years ago [3] combines the advantages of stochastic models with those of macroscopic physics-based models. In the proposed version of the model, the fuel is preheated to ignition by convective and radiative heat transfer from the flaming zone, radiative losses during preheating are considered, and vegetation items are randomly distributed throughout the landscape (amorphous network).

The paper is organized as follows. We first introduce the general concepts of the model and the physical approach to determine the parameters upon which it depends. Second we compare model results with real fire patterns. Third, a sensitivity study is conducted to investigate the effects of changes in model parameters on the area of post-fire patterns. This last section is partly motivated by the difficulty in accurately determining local weather and fuel conditions.
2. Model overview

Vegetation is modeled as an amorphous network of spherical fuel items (called hereafter vegetation cells). Moreover it is well established that the fire spread is governed by the fine fuel elements (e.g. needles, leaves, or small branches), typically less than 6 mm in diameter [4]. The thermal response of wet fine fuel matter to heating is rather complicated, but the simplest viable representation involves three paths. The early heating is a drying process which gives dry fine fuel and water vapor; it is followed by a pyrolysis reaction whereby flammable vapors (volatiles) are emitted and burned in the gas phase. Once all the pyrolyzates are depleted in the particle, flaming ceases. If oxygen is present, glowing combustion of the char occurs. In the present work, char oxidation, and subsequently ember preheating, is ignored.

2.1. Energy conservation

The physical model used is largely inspired from that of Koo et al. [5]. Consider an elementary volume of the combustible cell as shown in Figure 1. The volume involved in preheating is a layer of thickness \( \delta \) on the top half of the cell. As commonly used in fire models involving fine wildland fuels (grass and foliage of shrubs and trees) [6] the thermally thin assumption is here adopted, which means that the fuel temperature in the control volume is assumed to be constant. Applying conservation of energy to this control volume yields

\[
q''_{r\text{g}} - q''_{r\text{l}} + q''_{c\text{g}} = \begin{cases} 
\alpha_k (\rho_k C_p_k)_{\text{wet}} \frac{dT}{dt} & \text{for } T < 373 \text{ K} \\
-\alpha_k (\rho_k)_{\text{dry}} h_{\text{vap}} \frac{dT}{dt} & \text{at } T = 373 \text{ K} \\
\alpha_k (\rho_k C_p_k)_{\text{dry}} \frac{dT}{dt} & \text{for } T > 373 \text{ K}
\end{cases}
\]

where \( q''_{r\text{g}} \) and \( q''_{c\text{g}} \) are the radiative and convective heat fluxes from the flaming zone, \( q''_{r\text{l}} \) is the radiative heat loss to the ambient, \( T \) is the fuel temperature, \( \alpha_k \) the fuel volume fraction or packing ratio, \( \rho_k \) and \( C_p_k \) are the density and specific heat of the solid phase. The subscripts \( \text{dry} \) or \( \text{wet} \) refer to variables evaluated on a dry or wet basis, \( h_{\text{vap}} \) is the heat of water vaporization and \( FMC \) is the fuel moisture content, defined as the mass of water per mass of dry fuel.

\[\text{Figure 1. Flame spread schematic, with energy-transfer mechanisms indicated.}\]
Equation (1) states that the sum of radiative and convective heat transfers for a fuel cell is equal to the sum of the energy absorbed and used for raising the temperature or evaporating the moisture within the fuel cell. Conductive heat transfer through fuel cell may be neglected.

Radiative heat transfer can occur in two ways: radiation preheating by the flame front, $q''_{fg}$, and radiative losses, $q'_{rl}$, which is radiative heat transfer from the fuel cell to the ambient (Figure 1). The amount of radiant energy received by a cell depends on flame emission, attenuation by the air layer between the flame and the cell, and absorption by fuel bed.

Overhead flame radiation is calculated by means of the Monte Carlo method where the visible flame is regarded as a uniformly-radiating solid body with a cylindrical shape and with thermal radiation emitted from its surface (solid flame model). The detail of the method can be found in [7]. Flame properties and emissive power, and the flaming residence time of a burning cell, $t_c$, are the inputs of the radiation model. The amount of quanta, launched from cell $i$ that finally reaches cell $j$ is counted $n_{ij}$. The total amount of radiation received by each cell located within the interaction domains of $n$ burning cells is $q_{rg,j} = \sum_{i=1}^{n} n_{ij} q_i$. Every bundle represents some quantity of radiative power ($P'_i$), $q_i = P'_i / n''$ where $n''$ is the number of bundles emitted per unit flame area. The contribution of burning cell $i$ is not taken into account if another burning cell is in between $i$ and $j$ (screening effect).

Whenever the temperature of cell $j$ reaches a fixed value $T_{ign}$, cell $j$ begins to burn.

Radiation from the flame may be attenuated by the atmospheric layer between the flame and the cell. The probability of a quantum launched from a burning cell $i$ in the direction $ij$ being scattered or absorbed is determined from the macroscopic law of attenuation, namely the Beer-Lambert law, using the transmittance of the atmospheric layer $\tau_{ij}$. This coefficient depends on the distance between the flame and the receptive cell $d_{ij}$, on the ambient temperature $T_\infty$, and on the relative humidity ($RH$) of ambient air. $\tau_{ij}$ is calculated using the SNB model of Malkmus [8] for $0 < d < 250$ m, $T_\infty = 300$ K, $RH = [10; 25; 50; 100\%]$ and a source corresponding to a black body at 1210 K. The transmittance is then approximated by a power law, $\tau_{ij} = a + b d_{ij}^c$, where $a$, $b$ and $c$ have been fitted for the four values of relative humidity. For each quantum a random number $R_s$ is generated to determine whether the quantum is absorbed or scattered ($R_s > \tau_{ij}$) or not.

The radiative gain at the top half of the cell $j$, $S_j$, can be written as

$$q''_{rg,j} = a_{fb} \frac{1}{S_j^{\delta}} q_{rg,j}$$

where $a_{fb}$ is the fuel bed absorptivity.

Unburned fuel elements lose heat to the ambient by the radiative heat loss, so that

$$q''_{rl,j} = \frac{1}{\delta} \varepsilon_{fb} \sigma (T_j^4 - T_\infty^4)$$

where $\varepsilon_{fb}$ is the fuel bed emissivity.

The term $q''_{fg}$ corresponds to the convection preheating of the fuel cell by the hot gases coming from the flame region. It can be written as [9]

$$q''_{fg,j} = \frac{h_c}{\delta} (T_f - T_j) e^{-0.3 d_{ij}/L_f}$$

where $L_f$ is the flame length and $h_c$ is a heat transfer coefficient obtained for external turbulent flow over a sphere [10].
2.2. Model parameters

Model parameters are the flaming residence time of burning cells, the height and angle (from the vertical) of flame and its emissive power. The former is determined from laboratory-scale experiments, whereas the others are determined empirically.

Following Putnam [11], the flame height under the influence of wind is found to be proportional to the square root of the Froude number based on the local wind speed \( u \) and flame height with no wind, \( H_{f0} \): 

\[
H_f = H_{f0} \left[ 1 + 4 \frac{u^2}{(g H_{f0})} \right]^{-0.5}.
\]

In this equation \( H_{f0} \) is deduced from the correlation proposed by Heskestad [12] in terms of heat release rate, \( \dot{Q} \), and fire source diameter, \( D \), as: 

\[
H_{f0} = 0.0148 \frac{\dot{Q}^{2/5}}{D} - 1.02 D.
\]

Since flaming combustion corresponds to the chemical reaction of volatiles with air, the heat release rate may be expressed as: 

\[
\dot{Q} = \dot{m}_{\text{volatiles}} \Delta h_c = (1 - a_{\text{char}}) \dot{m}_{\text{pyr}} \Delta h_c,
\]

where \( \dot{m}_{\text{volatiles}} \) and \( \dot{m}_{\text{pyr}} \) are the burning rates of volatiles and dry fine fuel, \( a_{\text{char}} \) is the initial char mass content of dry fine fuel, and \( \Delta h_c \) is the heat of combustion of volatiles (18.62 \( \times 10^6 \) J kg\(^{-1}\)). A crude estimate of \( \dot{m}_{\text{pyr}} \) may be provided assuming that dry fine fuel burns at a constant rate for \( t_c \), leading to 

\[
\dot{m}_{\text{pyr}} = \frac{m_{\text{pyr}}}{t_c},
\]

where \( m_{\text{pyr}} \) is the mass of dry fine fuel.

In the modeling work by Albini [13], the flame angle is found to be proportional to the square root of the Froude number based on the flame height as: 

\[
\tan \alpha = 1.22 F_r^{0.5} \quad \text{where} \quad F_r = \frac{u^2}{(g H_f)}.
\]

Assuming that the flame is a gray body at temperature \( T_f \), its emissive power per unit flame area may be expressed as: 

\[
\varepsilon_f = \left[ 1 - \exp(-\kappa D) \right] \sigma T_f^4,
\]

where \( \kappa \) is the extinction coefficient.

2.3. Illustrative examples

Figure 2 (a) shows predicted post-fire patterns, on flat terrain and wind speed of 12 m.s\(^{-1}\). Vegetation is composed of \textit{Arbutus unedo}, with a recovery rate of 0.33 (near the critical threshold). The other model inputs are given in Table 1. The emissive power of flames is \( P_f = 84.9 \) kW.m\(^{-2}\). Small-world network effects, i.e. clustering, fire fingers, and lacunarity, appear clearly in Figure 2 (a).

| Table 1. Input variables for fire spread model calculations. |
|-------------------------------------------------------------|
| Scale | Description | Symbol [units] | Value used | Source |
|---|---|---|---|---|
| Fine fuel element | Char content | \( a_{\text{char}} \) [-] | 0.3 | [14] |
| | Surface-to-volume ratio | \( \sigma_k \) [m\(^{-1}\)] | 5544 | [15] |
| | Density | \((\rho_k)_{\text{dry}}\) [kg.m\(^{-3}\)] | 720 | [14] |
| | Heat capacity | \((C_p_k)_{\text{dry}}\) [J.kg\(^{-1}.K\(^{-1}\)] | 1470 | [14] |
| | Initial moisture content | \( FMC_0 \) [-] | 0.2 | [15] |
| Fuel cell | Cell size | \( D \) [m] | 3 | [17] |
| | Thickness | \( \delta \) [m] | 0.36 | ps |
| | Packing ratio | \( \alpha_k \) [-] | 0.002 | ps |
| | Absorptivity | \( a_{fb} \) | 0.9 | [16] |
| | Emissivity | \( \varepsilon_{fb} \) | 0.9 | [9] |
| | Ignition temperature | \( T_{ig} \) [K] | 500 | [13] |
| Flame | Residence time | \( t_c \) [s] | 30 | ps |
| | Temperature | \( T_f \) [K] | 1210 | ps |
| | Extinction coeff. | \( \kappa \) [m\(^{-1}\)] | 0.4 | [17] |
| Network | Average wind speed | \( U \) [m.s\(^{-1}\)] | 6 | [15] |
| | Relative hum. of air | \( RH \) [-] | 0.3 | [15] |
| | Vegetation recovery rate | \( p \) [-] | 0.33 | [15] |

* present study.
Figure 2 (b) shows the evolution of dimensionless burned area (i.e., number of burned cells/total number of cells) and propagation time as a function of vegetation recovery rate for various wind speeds. Whatever the wind speed, the same behavior is observed near the critical threshold. Phase transition and critical behavior of fire spread have been discussed in a companion paper [18].

3. Model validation

Validation concerns an arson fire that occurred in Favone in Corsica, in 2009. It was extensively studied and documented by Santoni et al. [15]. Geographical data include a digital elevation model and vegetation data at a resolution of 25 m. The dominant species is Arbutus unedo. The weather conditions at the time of fire were: an average wind speed of 6 m.s\(^{-1}\) at 10 m above the ground, an average direction of 270° (from the West), and a dry bulb temperature of 30°C. The CFD program Flowstar\(^{\text{©}}\) was used to calculate local wind direction and speed from the average wind speed, taking into account the effects of topography and surface roughness of the land site. Model inputs are summarized in Table 1.

Comparison between predicted and real fire contours shows that the rate of spread of the head fire front is well predicted by the model (Figure 3). On the contrary, discrepancies are observed in the lateral extension of the fire, especially towards the south. They are mainly due to the fire crew intervention that was not introduced into the model because of lack of information on the exact location and nature of the firefighting task force deployed. After 29 min of fire propagation (point B in Figure 3), the average rate of spread of the head fire front is well estimated (16.8 vs. 17.0 m.min\(^{-1}\)). In the downslope part of the terrain, between points B and C, the model overestimates the fire spread rate, with 13.3 vs. 16.1 m.min\(^{-1}\).
4. Sensitivity study

The purpose of this section is to hierarchize model parameters using a sensitivity study based on a fractional factorial plan of experiment. This study involves eight input parameters or factors $X_i$ (Table 2) and analyzes the effects of these factors on the area burned by the fire. In this study each parameter takes two levels, a minimum and a maximum values. The full factorial design which would required $2^8 = 256$ runs is replaced by a fractional factorial design in which three factors are aliased, leading to a fractional factorial plan of $2^8/3 = 32$ runs. In this resolution design, main effects are confounded with interactions of third-order and higher, which facilitates the interpretation of results. Table 2 presents the eight factors and their variation range.

Table 2. Factors of the sensitivity analysis and range of variations. Reference values are given in Table 1.

| Variable | Description                          | Symbol, [units] | Variation | Min       | Max       |
|----------|--------------------------------------|-----------------|-----------|-----------|-----------|
| $X_1$    | Solid-phase volume fraction          | $a_k [-]$       | $\pm 25\%$ | $1.5 \times 10^{-3}$ | $2.5 \times 10^{-3}$ |
| $X_2$    | Flame residence time                 | $t_c [s]$       | $\pm 17\%$ | 25        | 35        |
| $X_3$    | Initial fuel moisture content        | $FM_{C0} [-]$   | $\pm 25\%$ | 0.15      | 0.25      |
| $X_4$    | Emissive power                       | $P'' [kW.m^{-2}]$ | $\pm 6\%$ | 80        | 90        |
| $X_5$    | Vegetation recovery rate             | $p [-]$         | $\pm 10\%$ | 0.45      | 0.55      |
| $X_6$    | Relative humidity of air             | $RH [-]$        | $\pm 33\%$ | 0.2       | 0.4       |
| $X_7$    | Average wind speed                  | $U [m.s^{-1}]$  | $\pm 33\%$ | 4         | 8         |
| $X_8$    | Solid-phase ignition temperature     | $T_{ig} [K]$    | $\pm 10\%$ | 450       | 550       |

As expected, it appears that the most influential parameters are the fuel moisture content and the emissive power of the flame (4), with negative and positive effects, respectively. As also expected, the area burned by the fire depends on factors (1) and (2) with a positive effect, whereas the factor (6) has
a negative effect. Increasing wind speed (7) leads to a strong anisotropy of radiation, which in turn induces a decrease in burned area. More surprisingly, the impact of ignition temperature and vegetation recovery rate is weak. For the latter, this means that, far from the percolation threshold, vegetation recovery rate does not significantly affect the burned area.

![Figure 4. Bar histogram of the sensitivity analysis.](image)

5. Conclusion

The latest improvements of a hybrid stochastic/deterministic model are presented. The fire model from now on depicts vegetation as an amorphous network of combustible cells and, at the macroscopic scale, takes into account convective and radiative heat transfer from the flaming zone and radiative loss during preheating. Model validation is achieved to an acceptable degree through comparison of results with real fire data. A sensitivity analysis is performed using a fractional factorial plan of experiment showing how sensitive the area burned by the fire is to small variations of model inputs. The role of the fuel moisture content and the flame emissive power is clearly demonstrated, whereas ignition temperature and vegetation coverage, far from the percolation threshold, seem to have no influence. The next step is to extend the analysis to other model outputs, particularly to the rate of spread and fractal properties of fire patterns.

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