Synthesized spatiotemporal mode-locking and photonic flywheel in multimode mesoresonators

Dissipative Kerr soliton (DKS) frequency combs—also known as microcombs—have arguably created a new field in cavity nonlinear photonics, with a strong cross-fertilization between theoretical, experimental, and technological research. Spatiotemporal mode-locking (STML) not only adds new degrees of freedom to ultrafast laser technology, but also provides new insights for implementing analogue computers and heuristic optimizers with photonics. Here, we combine the principles of DKS and STML to demonstrate the STML DKS by developing an unexplored ultrahigh-quality-factor Fabry–Pérot (FP) mesoresonator based on graded index multimode fiber (GRIN-MMF). Complementing the two-step pumping scheme with a cavity stress tuning method, we can selectively excite either the eigenmode DKS or the STML DKS. Furthermore, we demonstrate an ultralow noise microcomb that enhances the photonic flywheel performance in both the fundamental comb linewidth and DKS timing jitter. The demonstrated fundamental comb linewidth of 400 mHz and DKS timing jitter of 500 attosecond (averaging times up to 25 μs) represent improvements of 25× and 2.5×, respectively, from the state-of-the-art. Our results show the potential of GRIN-MMF FP mesoresonators as an ideal testbed for high-dimensional nonlinear cavity dynamics and photonic flywheel with ultrahigh coherence and ultralow timing jitter.

Due to the low size, weight, power and cost and easy access to large comb spacing in nonconventional spectral ranges,[1,2] Kerr microcomb has emerged as a promising frequency comb source and opens new applications such as highly multiplexed coherent optical communication,[3,4] astrocombs,[5,6] ranging,[7,8] dual-comb spectroscopy,[9,10] integrated frequency synthesizers,[11,12] and optical clockwork.[13,14] Recently, a two-step pumping scheme for dissipative Kerr soliton (DKS) microcomb, utilizing the interplay between stimulated Brillouin laser (SBL) and cavity Kerr nonlinearity, is introduced to fundamentally narrow the comb linewidth and lower the repetition rate phase noise towards the quantum limit.[15] This two-step pumping scheme enables the photonic flywheel demonstration first in a monolithic fiber Fabry–Pérot (FP) cavity platform[15] and later in a silica disk microresonator[16] and a silica wedge microresonator.[17]

Among all Kerr microcomb platforms, fiber FP platform provides a unique opportunity to study the spatiotemporal mode-locking enabled by the use of graded-index multimode fiber (GRIN-MMF). The parabolic core index profile of GRIN-MMF renders its transverse modes to cluster into nearly degenerate mode families whereas the

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intermodal dispersion between two mode families is orders of magnitude smaller than that of a regular step-index MMF or waveguide. Such low modal dispersion enhances the intermodal nonlinear interaction and leads to the observation of new physical phenomena that attract growing interests including spatial beam self-cleaning and spatiotemporal mode-locking. In particular, spatiotemporally mode-locked (STML) lasers not only add new degrees of freedom to applications such as telecommunications, imaging, and ranging, but also provide new insights for implementing analogue computers and heuristic optimizers with photonics.

Using an unexplored GRIN-MMF FP mesoresonator (Fig. 1a), here we leverage the large mode volume and ultrahigh quality factor (Q) of the GRIN-MMF FP mesoresonator to achieve an ultralow-noise Kerr microcomb that enhances the photonic flywheel performance in both the frequency-calibrated transmission spectra (top) and cavity ring-down traces (bottom).

### Results

#### Ultrahigh-Q GRIN-MMF FP mesoresonators

Our ultrahigh-Q GRIN-MMF FP mesoresonator is fabricated through three steps: (i) commercial GRIN-MMF (GIF50E, Thorlabs) is carefully cleaved and encapsulated in a ceramic fiber ferrule; (ii) both fiber ends are coated with optical dielectric Bragg mirror with reflectivity over 99.9% from 1530 to 1570 nm (Fig. 1a). The FP cavity length is 10 mm, and the roundtrip length is 20 mm. Figure 1b shows the parabolic core index profile of the GRIN-MMF and the resulting small cold cavity free spectral range (FSR) difference between the first few linearly polarized (LP) mode families. By coupling a tunable external-cavity diode laser (ECDL, Toptica CTL1550) into the resonator, we observe five specific transverse eigenmodes with good cavity loading as shown in Fig. 1c. All five modes have FSRs in the 10.079 GHz ± 2 MHz range, ready for X-band microwave photonic applications.
SBL, OS A Collimator Pump PNA

in this work features a 20× larger mode area, and thus the diffraction

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Figures 1d, e show both the frequency-calibrated transmission spectra

detuned pump can compensate the thermal nonlinearity of the red-
detuned comb generating SBL. In our GRIN-MMF FP mesoresonator,
such condition can be satisfied by using the 3rd and the 4th mode
families for the pump and the SBL, respectively. In this configuration,
the offset frequency (10.341 GHz, Fig. 1c) is 2 MHz smaller than the SBS
frequency shift (10.343 GHz, Fig. 1c). Figure 2b shows the optical
spectra, the beam profiles, and the radiofrequency (RF) beat note of
the comb repetition rate when DKS crystals are generated in the SBL
mode family. Single DKS generation (see Supplementary Information
Section III) and comprehensive noise analysis will be discussed in the
latter sections.

The two-step pumping scheme can also be generalized to facilitate the DKS comb generation in the pump mode family as demonstrated in Fig. 2c. We fine-tune the cavity stress to increase the offset frequency between the pump and intermodal SBL (see Supplementary Information Section IV) and make it slightly larger than the SBS frequency shift. As discussed previously, all transverse mode families of the GRIN-MMF exhibit similar anomalous GVD and thus DKS microcomb can also be generated directly by the pump when it is put at the red detuning side of the resonance. In this configuration, the blue-detuned SBL is now used to compensate the thermal nonlinearity of the red-detuned and comb-generating pump. Using this cavity stress tuning method, we can achieve a selective excitation of DKS microcomb in an arbitrary transverse mode family.

The low modal dispersion and the inhomogeneous broadening of the SBS gain spectrum between a great variety of high-Q optical modes and acoustic modes in the GRIN-MMF, greatly enhance the tolerance

Fig. 2 | Selective excitation of multimode DKS microcombs. a Schematic of the microcomb generation and characterization setup. ECDL external-cavity diode laser, EDFA Erbium-doped fiber amplifier, ISO optical isolator, OSA optical spectrum analyzer, PD photodetector, AOFs acousto-optic frequency shifter, PC polarization controller, BPD balanced photodetector, PNA phase noise analyzer. b, c Optical spectra of DKS crystals generated in the SBL mode family (b) and the pump mode family (c). The black dashed lines show the fitted DKS spectral envelope. The vertical purple dashed lines indicate the equidistant comb line positions. Insets are the RF beat note of comb repetition rate (left) indicating the stable mode-locking status and measured output beam profiles (right) indicating the selective excitation of different transverse mode families in Fig. 1c. Of note, comb lines induced by the cross-phase modulation (XPM) contain less than 1% of the overall comb power.

Our previous FP mesoresonators were made of highly nonlinear fibers with small mode area, leading to high diffraction losses in the thick dielectric Bragg mirror coatings at both ends. GRIN-MMF used in this work features a 20× larger mode area, and thus the diffraction loss is significantly mitigated to achieve ultrahigh Qs approaching 4 × 10^9 for all five modes (see Supplementary Information Section II).

Figures 1d, e show both the frequency-calibrated transmission spectra and cavity ring-down traces of the pump and SBL resonances, respectively. At ~1550 nm, the loaded Qs of both pump and SBL resonances are measured to be ~3.8 × 10^9.

Selective excitation of multimode DKS microcombs

Figure 2a shows the microcomb generation and characterization setup schematic. Of note, all five transverse mode families exhibit similar anomalous GVD of ~28 fs^2/mm according to the numerical simulation (see Supplementary Information Section II). Due to the ultrahigh Qs of all modes, the SBL threshold is measured to be as low as 15 mW.

To make the two-step pumping scheme work, the offset frequency between the pump and intermodal SBL should be conditioned slightly smaller than the SBS frequency shift such that the blue-detuned pump can compensate the thermal nonlinearity of the red-detuned comb generating SBL. In our GRIN-MMF FP mesoresonator, such condition can be satisfied by using the 3rd and the 4th mode families for the pump and the SBL, respectively. In this configuration,
to the fabricated FSR variation and the flexibility in locating the proper pump-SBL pair that satisfies the requirement for the two-step pumping scheme. As shown in the Supplementary Information Section V, we achieve DKS generation utilizing either the same pump-SBL mode pair with a different acoustic mode or a different transverse pump-SBL mode pair in two ultrahigh-Q GRIN-MMF FP mesoresonators with FSRs of 10.087 GHz and 10.086 GHz, 8 MHz and 7 MHz larger than the one used in Fig. 2, respectively.

**Spatiotemporally mode-locked DKS microcombs**

Besides the selective excitation of multimode DKS microcombs, the low modal dispersion of GRIN-MMF FP mesoresonator provides another unique opportunity of generating STML DKS with low pulse energy and high repetition rates in the GHz regime. When the offset frequency between the pump and the intermodal SBL is set to be equal to the SBS frequency shift, the operation window within which both pump and SBL are red-detuned and comb-generating are now open and accessible. As discussed previously, all transverse mode families of the GRIN-MMF exhibit similar FSR and thus STML DKS microcombs can be readily generated even without much self-phase modulation (SPM) and intermodal XPM, facilitating the generation of STML DKS with low pulse energy and high repetition rates in the GHz regime.

Figures 3a and 3b show the optical spectrum and the single comb-repetition-rate RF beat note of the demonstrated STML DKS microcomb, respectively. The STML DKS microcomb can be categorized as the STML multimode soliton in MMFs, where multiple transverse modes are locked to share the same repetition rate but not the carrier-envelope offset frequency. This phenomenon is fundamentally different from the previous observations in STML fiber lasers, and it can be attributed to the microcomb’s orders-of-magnitude higher FSR that consequently leads to orders-of-
magnitude higher carrier-envelope offset frequency difference between transverse mode families. Thus, the intermodal nonlinear interaction in microcombs is not sufficient to compensate for the difference in the carrier-envelope offset frequency (fceo) but just the FSR. Such phenomenon is evidenced by the observation of the second RF beat note at ~262 MHz (Fig. 3c), corresponding to the carrier-envelope offset frequency difference (Δfceo) between the STML DKS microcombs in the pump and SBL transverse mode families. With the two-step pumping scheme, the pump DKS and the SBL DKS are intrinsically coherent to each other, and it is key to the successful STML DKS demonstration. As shown in Fig. 3d, the SBS frequency shift (and consequently Δfceo) has a low phase noise at offset frequencies above 10 kHz and the integrated phase noise down to 10 kHz is only 100 mrad. On the other hand, there is an excessive technical noise at low offset frequencies and thus an optical phase lock loop (PLL) is implemented to improve the long-term coherence of the STML DKS (Supplementary Information Section VI). With the PLL, a residual phase noise of only 227 mrad rms from 10 Hz to 1 MHz is achieved, and it has been shown such level of residual phase noise has a negligible effect on the synthesized electric-field waveform32–34.

We also depict the transition from the unlocked two-mode solitons to STML DKS by recording the electrical spectra around the Δfceo while adjusting the pump-cavity detuning (Fig. 3e). Before the transition, several equally spaced RF beat notes around the Δfceo are observed, characteristic of two independent microcombs with a repetition rate difference of ~1.7 MHz. When the pump-cavity detuning is further increased, a transition to STML DKS occurs and only one low-noise Δfceo beat note is observed. The left and right insets in Fig. 3a show the beam profiles measured with a camera and calculated by superposition of the 3rd and the 4th mode families, respectively.

In Fig. 4g, we characterize the repetition rate phase noises of pump DKS and STML DKS, which are limited to ~110 dBc/Hz and ~120 dBc/Hz at 1 MHz, respectively. We attribute the 10-dB phase noise reduction to the synchronization between the pump DKS and the ultralow noise SBL DKS (Fig. 4b). Of note, all DKSs are initiated and self-stabilized over hours without any active control in the lab environment. Deterministic behavior is realized that the DKSs can be repeatedly and reliably generated following the same pump tuning protocol.

Finally, we numerically study the intertwined effects of GVM, XPM, and pump power to delineate the existence condition of STML DKS (Supplementary Information Section VII).
Photonic flywheel with a sub-hertz fundamental linewidth

Figure 4a compares the microcomb line SSB frequency noise spectra of pump and SBL DKSs, measured with an optical frequency discriminator using a fiber-based unbalanced Mach–Zehnder interferometer (UMZI) and a balanced photodetector (Fig. 2a, also see Methods and Supplementary Information Section VIII). Fundamental linewidths are then calculated from the white noise floor of the measured SSB frequency noise spectra. While the pump DKS simply inherits the frequency noise of the pump, yielding a fundamental linewidth of 130 Hz, both the SBL and the SBL DKS exhibit spectral purities that are more than two orders of magnitude better than the pump thanks to the ultrahigh Q of the GRIN-MMF FP mesoresonator. The 53-mHz SBL fundamental linewidth not only outperforms the state-of-the-art on-chip SBLs (Table 1), but also operates at free-running mode without any active detuning stabilization.

As summarized in the Supplementary Information Section I, large mode volume and low Kerr nonlinearity of the GRIN-MMF FP mesoresonator lead to a significantly lower quantum noise and timing jitter (dashed orange line in Fig. 4b). Thus, the photonic flywheel demonstrated here is no longer quantum noise limited as our previous work15, and its DKS timing jitter approaches 500 attosecond on a 25-μs time scale that represents a 2.5× improvement, benefiting applications including coherent optical communications and optical atomic clocks.

Photonic flywheel with a sub-femtosecond timing jitter

Figure 4b plots the SSB phase noise of the SBL DKS repetition rate, measured with the all-fiber reference-free Michelson interferometer (ARMI) setup providing attosecond timing jitter resolution15,38,39 beyond the capability of direct photodetection methods40–42 (see Methods and Supplementary Information Section IX). The measured SSB phase noises at 10 kHz, 100 kHz, and 1 MHz offset frequencies are −125 dBc/Hz, −148 dBc/Hz, and −168 dBc/Hz, respectively. The timing jitter integrated from 12 kHz to 1 MHz is 1 fs, which is only one fifth of the optical cycle at the DKS center wavelength. Table 2 compares the repetition rate phase noise of the state-of-the-art microcombs. Our result outperforms all other DKS microcombs, with or without active control, except the one study in a MgF2 crystalline microresonator40 where a sideband Pound–Drever–Hall lock was implemented to optimize and stabilize the detuning setpoint for the quiet point (QP) operation that requires either avoided mode crossing or large third order dispersion15. Our approach, on the other hand, is completely free running without the need of any active control.

As summarized in the Supplementary Information Section I, large mode volume and low Kerr nonlinearity of the GRIN-MMF FP mesoresonator lead to a significantly lower quantum noise and timing jitter (dashed orange line in Fig. 4b). Thus, the photonic flywheel demonstrated here is no longer quantum noise limited as our previous work15, and its DKS timing jitter approaches 500 attosecond on a 25-μs time scale that represents a 2.5× improvement, benefiting applications including microwave photonics and timing distribution.

To find the dominant factor that limits the current DKS timing jitter, we simultaneously monitor the variations of SBL relative intensity noise (RIN), SBL frequency noise, SBL DKS RIN, and SBL DKS phase noise while we finely change the pump-cavity detunings (see Supplementary Information Section X). SBL RIN is identified as the main limiting factor, and we project the SBL RIN transduced SSB phase noise (blue line) in Fig. 4b with the transduction coefficient calculated from the experimental parameters (see Supplementary Information Section X). A good agreement is achieved with a maximum 6-dB deviation at offset frequencies below 1 kHz where other slow technical noises also start to contribute. The deviation for offset frequencies above 1 MHz is attributed to the pump-SBL interference and the coherent artefacts of the ARMI setup (see Supplementary Information Section IX). With an active control of the SBL RIN15,44, we expect that a quantum-limited phase noise can be achieved which will then be a 20-dB improvement.

Table 1 | Comparison of SBL fundamental linewidth

| Material | Q factor (Million) | Linewidth (mHz) | Reference |
|----------|-------------------|----------------|-----------|
| SiO₂     | 384               | 53             | This work |
| SiO₂     | 269               | 60             | 32        |
| SiO₂     | 140               | 376            | 33        |
| SiO₂     | 96                | 314            | 44        |
| Si₃N₄    | 29.2              | 720            | 47        |

Table 2 | Comparison of DKS microcomb performances

| Fundamental linewidth | Material | Q factor (Million) | Microcomb linewidth (Hz) | Soliton active control | Reference |
|-----------------------|---------|-------------------|--------------------------|------------------------|-----------|
| SiO₂                  | 384     | 0.4               | W/o (SBS)                | This work              |
| SiO₂                  | 44.4    | 24                | W/o (SBS)                | 36                     |
| Si₃N₄                 | 56      | 10                | W/o (SBS)                | 28                     |
| Si₃N₄                 | 11.6    | 1000              | W/                        | 27                     |

| Phase noise of microcomb repetition rate | Material | Configuration | Carrier frequency (GHz) | SSB phase noise (dBc/Hz, scaled to 10 GHz) | Soliton active control | Reference |
|-----------------------------------------|---------|---------------|-------------------------|------------------------------------------|------------------------|-----------|
|                                        | SiO₂    | Bright soliton| 10.08/20.16             | −125/−148                                | W/o (SBS)              | This work |
|                                        | MgF₂    | Bright soliton| 14.09                   | −142/−159                                | W/ (QP)                | 42        |
|                                        | SiO₂    | Bright soliton| 0.945                   | −120/−140                                | W/o (SBS)              | 31        |
|                                        | SiO₂    | Bright soliton| 10.43                   | −125/−144                                | W/o (SBS)              | 31        |
|                                        | SiO₂    | Bright soliton| 11.02                   | −120/−139                                | W/o (SBS)              | 17        |
|                                        | SiO₂    | Dark soliton  | 5.4                     | −108/−134                                | W/o (SIL)              | 29        |
|                                        | MgF₂    | Bright soliton| 9.9                     | −130/−130                                | W/o (SIL)              | 34        |
|                                        | SiO₂    | Bright soliton| 22                      | −111/−147                                | W/ (QP)                | 33        |
|                                        | SiO₂    | Bright soliton| 15.2                    | −117/−143                                | W/ (QP)                | 55        |
|                                        | Si₃N₄   | Bright soliton| 9.78                    | −110/−130                                | W/                     | 57        |

SIL self-injection locking.
over the state-of-the-art thanks to the large mode volume of the GRIN-MMF FP mesoresonator.

**Discussion**

In summary, we develop a compelling DKS platform based on ultrahigh-Q GRIN-MMF FP mesoresonators. Using the cavity stress tuning method, we demonstrate the selective excitation of the previously reported SBL DKS, the pump DKS, and the STML DKS. With the new transverse mode degree of freedom added to microcombs, the GRIN-MMF FP mesoresonator is a promising platform for applications such as space division multiplexing in telecommunications. Leveraging the ultrahigh Q and large mode volume of the GRIN-MMF FP mesoresonator and using the two-step pumping scheme, we achieve an ultralow noise microcomb that enhances the photonic flywheel performance in both the fundamental comb linewidth and DKS timing jitter. The demonstrated fundamental comb linewidth of 400 mHz and DKS timing jitter of 500 attosecond (averaging times up to 25 μs) represent improvements of 25× and 2.5×, respectively, from the state-of-the-art. The intrinsic compatibility between fiber FP mesoresonator and cw fiber laser will facilitate hermetic all-in-fiber packaging to achieve a field-deployable frequency comb. To date, cw fiber laser outperforms all integrated lasers in terms of the output power and the noise performance. Thus, all-in-fiber microcomb is complementary to the existing all-solid-state microcomb and all-in-fiber microcomb provides a promising solution to applications that require high comb power and low comb noise. Our results show the potential of GRIN-MMF FP mesoresonator as an ideal testbed for high-dimensional nonlinear cavity dynamics and photonic flywheel with ultrahigh coherence and ultralow timing jitter.

**Methods**

**Fundamental linewidth measurement**

We measure the frequency noise and fundamental linewidth of the soliton microcombs based on a self-heterodyne frequency discriminator using a fiber-based UMZI and a BPD. As shown in Fig. 2a, one arm of the UMZI is made of 250-m-long single mode fiber, while the other arm consists of an AOFs with frequency shift of 200 MHz and a polarization controller for high-voltage output. The FSR of the UMZI is 0.85 MHz. The two 50:50 outputs of the UMZI are connected to a BPD (Thorlabs PDB570C) with a bandwidth of 400 MHz to reduce the impact of detector intensity fluctuations. The detector output is then analyzed by a phase noise analyzer (RDL NTS-1000A). The relationship between SSB frequency noise PSD $L_{\gamma}(f)$ in the unit of Hz/Hz and PNA output SSB phase noise PSD $L_{\phi}(f)$ in the unit of dBc/Hz, is given by:

$$L_{\gamma}(f) = \frac{f^2}{4 \sin^2(\pi f \tau)} L_{\phi}(f)$$  

where $\tau$ is the delay time (~1.18 μs) of UMZI and $f$ is the frequency offset. The fundamental linewidth of the comb line $\Delta \nu$ in unit of Hz, is given by:

$$\Delta \nu = n L_{\nu \gamma}$$  

where $L_{\nu \gamma}$ is the value of frequency noise where $L_{\nu \gamma}(f)$ is flat, indicating white frequency noise.

We test our optical frequency discriminator with a known sub-hertz level reference laser at 1535 nm. The measured result of 0.3 Hz (see Supplementary Information Section VIII) indicates the reliability of our linewidth measurement setup for sub-hertz fundamental linewidth. We also measure the noise floor of the setup with SBL before soliton generation by a balanced MZI with identical 250-m-long fiber spool for each arm. The noise floor $L_{\nu \gamma}$ is given by:

$$L_{\nu \gamma}(f) = L_{\phi}(f) / 4 \pi^2 f^2$$  

and is shown by the shaded area in Fig. 4a. Large contrast of >20 dB between the setup noise floor and the measured frequency noise, indicates the reliability of the sub-hertz linewidth results. In order to measure the linewidth of comb teeth, a tunable optical filter with 10 GHz bandwidth is employed to extract the desired comb tooth.

**Measurement of comb repetition rate phase noise and timing jitter**

The RF beat notes of comb repetition rate in Fig. 2 are realized by injecting the filtered comb lines into a RF-modulated electro-optic intensity modulator (EOIM), detecting by a fast PD (>10 GHz bandwidth) and analyzing the down-shifted beat note by an electrical spectrum analyzer. However, the EOIM will induce extra noise to the phase noise of the measured comb repetition rate. Therefore, for precise measurement of pump soliton and the STML soliton, we inject the comb lines into the fast PD, electrically divide the 10.079 GHz RF signal by 8 times (with a low noise floor) to 1.26 GHz and then analyze the divided signal with a PNA (see Supplementary Information Section IX).

Since the soliton phase noise measurement based on fast PDs is limited not only by the shot noise but also the available electronics operating at high frequency, we introduce ARMI setup for precise SBL soliton phase noise. Of note, the ARMI setup is not suitable for relatively high phase noise measurement, which invalidates the linear approximation during the process of converting phase noise to intensity noise. The details of ARMI setup can be found in the Supplementary Information Section IX.

**Quantum-limited timing jitter**

The quantum-limited SSB phase noise PSD of soliton repetition rate is given by:

$$L_{\phi}(f) = \frac{\sqrt{2\pi}}{\sqrt{\Delta \nu D_\phi^2}} g \left[ 1 - \frac{\nu \Delta D_\phi^2 f^2}{C_0^2} + 24 \left( \frac{1}{\sqrt{2}} \right)^2 - \frac{\nu \Delta D_\phi^2 f^2}{C_0^2} \right]$$  

where $2 \nu = 2 \pi n_s (0.5 \times 10^6 \text{ rad/s})$ is the FWHM resonance linewidth, $\Delta \nu(=2\nu = 2 \times 10^6 \text{ rad/s})$ is the soliton detuning, $\nu = 1.513 \times 10^9 \text{ rad/s}$ is the frequency shift of a resonant mode per photon, $D_\phi(=0.0115)$ is the normalized GVD of soliton mode, and $\eta(=1)$ is the quantum efficiency of the detector. As the ARMI system is not limited by the shot noise of PD, the shot-noise term is removed from Eq. 68 of ref. 50. According to Eq. (4), quantum noise can be reduced by smaller $g$ and $D$. Since $g = h \omega_0 c n_s / V n_0$, where $h$ is the Planck constant, $\omega_0$ is the DKS center frequency, $c$ is the speed of light, $n_s = 3.6 \times 10^{25} \text{ m}^2/\text{W}$ is the nonlinear Kerr parameter, $V = 5 \times 10^{-18} \text{ m}^3$ is the mode volume and $n_0 = 1.4682$ is the refractive index of the core fiber, the feasible way to lower the quantum limit is to generate soliton comb in a microresonator with small nonlinearity and large mode volume, which requires loss reduction and Q factors enhancement for low threshold.

**Data availability**

The data that support the plots within this paper is available on Zenodo (https://zenodo.org/record/7097706#.Yyn4G3bMI2w). All other data used in this study are available from the corresponding author upon reasonable request.

**Code availability**

The analysis codes will be available on reasonable request.
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Author contributions
M.N., S.W.H., K.J., and Z.X. conceived the idea of the experiment. M.N., Y.X., and S.W.H. performed the simulation and experiment, while K.J., S.Z., and Z.X. designed and fabricated the devices. M.N. and S.W.H. conducted the data analysis and wrote the manuscript. All authors contributed to the discussion and revision of the manuscript.

Competing interests
The authors declare no competing interests.

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