A MEASUREMENT OF THE ANGULAR POWER SPECTRUM OF THE ANISOTROPY IN THE COSMIC MICROWAVE BACKGROUND

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ABSTRACT

We report on a measurement of the angular power spectrum of the anisotropy in the cosmic microwave background (CMB). The anisotropy is measured in 23 different multipole bands from $l = 54$ ($\approx 3\degree$) to $l = 404$ ($\approx 0\degree.45$) and in six frequency bands from 26 to 46 GHz over three observing seasons. The measurements are consistent from year to year. The frequency spectral index of the fluctuations (measured at low $l$) is consistent with that of the CMB and inconsistent with either dust or Galactic free-free emission. Furthermore, the observations of the MSAM1-92 experiment (Cheng et al. 1994) are repeated and confirmed. The angular spectrum shows a distinct rise from $\delta T_l \equiv [(2l + 1)(a_l^m)^2/4\pi]^{1/2} = 49^{+3}_{-2}$ µK at $l = 87$ to $\delta T_l = 85^{+10}_{-8}$ µK at $l = 237$. These values do not include an overall $\pm 14\%$ ($1\sigma$) calibration uncertainty. The analysis and possible systematic errors are discussed.

Subject headings: cosmic microwave background — cosmology: observations

1. INTRODUCTION

The discovery of the anisotropy in the cosmic microwave background (CMB) by the COBE satellite (Smoot et al. 1992; Bennet et al. 1992) and its subsequent confirmation (Ganga et al. 1993) at large angular scales (greater than 7\degree) has been followed by positive detections of anisotropy in the microwave sky at intermediate angular scales by a number of other experiments (Cheng et al. 1994; De Benardis et al. 1994; Devlin et al. 1994; Dragovan et al. 1994; Gundersen et al. 1995; Gutierrez De La Cruz et al. 1995; Piccirillo & Calisse 1993) and upper limits at smaller angular scales (Meyers, Readhead, & Lawrence 1993; Tucker et al. 1993). In order to reject foreground contamination, most of these experiments observe the sky over a range of frequencies. Additionally, several have successfully repeated their measurements of previous years.

Characterization of the anisotropy at medium angular scales ($2\degree$–$0.2\degree$) can strongly constrain theories of structure formation and cosmological parameters, mainly through the angular power spectrum of the anisotropy (Bond et al. 1994; Crittenden & Turok 1995; Kamionkowski et al. 1994; Jungman et al. 1995. For a recent review of the CMB see White, Scott, & Silk 1994).

We present results from an experiment designed to measure the angular power spectrum of the CMB at medium angular scales. The SK telescope observes from the ground in Saskatoon, Saskatchewan, Canada. The observing scheme has many internal consistency tests that allow checks of the integrity of the measurement. A frequency span of 20 GHz provides discrimination against foreground contaminants. Measurements made by this experiment are consistent from year to year. In addition, we have reproduced the results of the MSAM1-92 (Cheng et al. 1994) experiment. Partial descriptions of the SK instrument are given in Wollack et al. (1993) and Wollack (1994) (SK93); Wollack et al. (1994), Page et al. (1994), and Netterfield (1995) (SK94); and Netterfield et al. (1995) (SK93 to SK94 comparison). The instrument and calibration are described in detail in Wollack et al. (1997). Previous results (SK93 and SK94) are included in this analysis.

2. INSTRUMENT AND CALIBRATION

The SK telescope is comprised of a corrugated feed horn illuminating a parabolic primary followed by a chopping flat with a vertical chopping axis. The telescope is steerable in azimuth but fixed in elevation. Observations are made in $K_u$-band (26–36 GHz) and $Q$-band (36–46 GHz). Observations made in each band are broken up into three frequency sub-bands and two linear polarizations. Total power radiometers based on HEMT amplifiers are used. The supernova remnant Cas A is used to calibrate the telescope and determine the beam size and pointing. Table 1 lists the beam parameters and uncertainties. The beam widths are known to better than 2%. The calibration uncertainty is comprised of a 13% contribution due to uncertainty in the absolute calibration of Cas A, which is constant between years and radiometers, and a 3%–5% contribution due to measurement uncertainty.

The telescope pointing is determined from the position of Cas A. In 1995, the telescope was pointed 0\degree.05 differently than was intended, which was included in the analysis. In addition, there was a 0\degree.03 ($1\sigma$) jitter in the beam position from the chopping plate and base pointing inaccuracy. When convolved with the beam, this can be approximated by adding this uncertainty in quadrature to the nominal beam width. For 1995 this is results in a 1% widening of the effective azimuth beam.

3. OBSERVING STRATEGY: SYNTHESIZED BEAMS

The observing strategy is designed to offer a variety of internal systematic checks, minimize contamination by atmospheric temperature gradients and to simultaneously probe a variety of angular scales. These goals are achieved by sweeping the beam in azimuth on the sky by many beam widths with a large chopping flat and then synthesizing effective antenna patterns in software. For simplicity in the discussion that follows, telescope and observing parameters
for the 1995 season are used. Parameters for all years are listed in Table 1.

As the beam is swept on the sky, the radiometer is sampled 168 times per complete sweep. In analysis the 168 samples from each sweep, $T_i$, are multiplied by a weighting vector, $w_i$, to give $\Delta T_{\text{sweep}} = \sum w_i T_i$. The relative weighting of each spatial point in the sweep is set in software, allowing the synthesis of arbitrary effective antenna sensitivity pat-

| Parameter | $K_{93}$ | $K_{94}$ | $Q_{94}$ | $Q_{95\, \text{Cap}}$ | $Q_{95\, \text{Ring}}$ |
|-----------|---------|---------|---------|----------------|----------------|
| Beam:     |         |         |         |                 |                 |
| $x$ FWHM (deg) | 1.42 ± 0.005 | 1.42 ± 0.005 | 1.004 ± 0.005 | 0.461 ± 0.003 | 0.461 ± 0.003 |
| $y$ FWHM (deg) | 1.42 ± 0.02  | 1.42 ± 0.02  | 1.08 ± 0.02   | 0.513 ± 0.004 | 0.513 ± 0.004 |
| Chopper:  |         |         |         |                 |                 |
| Pattern   | Sine    | Linear  | Linear  | Sine            | Sine            |
| Amplitude on sky (deg) | 4.90 | 7.00   | 7.35    | 7.96            | 3.36            |
| Rate (Hz) | 3.906   | 3.906   | 3.906   | 2.976           | 2.976           |
| Samples/sweep | 16   | 64     | 64      | 168             | 168             |
| Pointing: |         |         |         |                 |                 |
| Azimuth (W:E) (deg) | -8.2:7.8 | -7.15:7.25 | -7.15:7.25 | -7.392:7.288 | 0.14            |
| Jitter (deg) | ...     | 0.02    | 0.02    | 0.03            | 0.03            |
| Elevation (deg) | 52.2 | 52.2    | 52.2    | 52.24           | 60.2            |
| Timing:   |         |         |         |                 |                 |
| Time per base move (s) | 16.4 | 20     | 20      | 40.3            | 2 weeks         |
| File length (minutes) | 17   | 15     | 15      | 20              | 20              |

Fig. 1.—Beam synthesis. Shown is the synthesis of the 9pt beams for Q94 and Q95 using eq. (1). $H(x)$ is shown along the path of the swept beam. The effect of binning in R. A. is neglected here for clarity. In 1994, an approximately linear sweep pattern, $X_i$, was used as shown. This allowed the use of sinusoidal weighting vectors, $w_i$, to produce synthesized beams with equally spaced lobes. The use of a sinusoidal sweep rendered this not optimal in 1995. The Q95 weighting vectors were generated by optimizing the sensitivity of the synthesized beam to a specified band of angular scales. Spatially similar synthesized beams are produced.
Fig. 2.—Synthesized beams. The contour lines are every 3dB. Dashed lines denote negative weighting. The NCP is located at (0, 0). Straight lines from the NCP are lines of constant hour angle. The bottom right plot is of the Ring 4pt synthesized beam, which is centered 8° above the NCP. The remaining plots are of Cap data acquired in the east. The curvature is due to the fact that the beam is swept in azimuth and not declination. The lobes farthest from the NCP are wider in R. A. than those closest because the data are averaged into 24 (3pt to 5pt) or 48 R. A. bins.
terns, \(H(x, y)\), given by

\[
H(x) = \left( \sum_{i} \frac{w_i G(x - X_i) - 2}{\sigma^2} \right)_{\text{bin}},
\]

where \(X_i\) is the position on the sky of the center of the main beam corresponding to weighting vector element \(w_i\) and

\[
G(x, y) = \frac{1}{2\sigma_x \sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right)
\]

is the main beam pattern of the telescope pointed at \((0, 0)\). The azimuth dimension is denoted by \(x\) and the elevation dimension by \(y\). The beam width of the telescope is given by \(\sigma = \text{FWHM}/[8 \ln(2)]^{1/2}\). The effect of the rotation of the Earth during an integration is included in \(H(x)\), as denoted by angle brackets with the subscript "bin."

For example, if the weighting vector \(w_i\) is selected so that data that are acquired when the chopping plate is centered are given a positive weight, and data that are acquired when the chopping plate is offset to the left or right are given negative weight, then a three lobed synthesized beam on the sky is produced (hereafter the 3pt synthesized beam). The weighting vectors are normalized so that \(\int |H(x)| dx = 2\) for a single sweep, not including the rotation of the Earth. The mean of the weighting vector is zero in order to eliminate sensitivity to fluctuations in the mean temperature of the sky. Since the weighting vectors for different effective antenna patterns are made to be nearly orthogonal, all of the effective antenna patterns may be acquired from the same data. Synthesized antenna patterns up to 19pt (\(\approx 60\) Hz) are produced. The number of lobes is limited by the beam size to throw ratio and not by the detectors.

The synthesis of the 9pt beams is demonstrated in Figure 1. Figure 2 displays some of the synthesized beams. In total, including the 3 years, there are 37. The 3pt through 9pt antenna patterns are similar for \(K_{94}, Q94\) and \(Q95\), allowing these data to be compared. Since the sweep amplitude and pointing for \(K_{93}\) was substantially different than for the other years, the \(K_{93}\) 3pt beam is also substantially different. Comparing \(K_{93}\) with \(K_{94}\) requires the use of a specially optimized weighting vector (\(K_{93}\) overlap).

Data are acquired in two modes: In the first mode (Cap Data), which was used all 3 years, the chopping plate axis is vertical, which places the beam at a constant elevation of 52°2, the elevation of the North Celestial Pole (NCP) in Saskatoon. The beam is swept sinusoidally 12°8 in azimuth (7°35 peak-to-peak on the sky) at 2.976 Hz. The center of the sweep is alternated every 40.3 s between 7°32 east of the NCP and 7°28 west of the NCP. As the earth rotates throughout the day, the celestial polar cap is covered, from the NCP to 82° declination. Data acquired in the east are repeated \(\approx 12\) hr later in the west, giving a very powerful systematic test. These data are multiplied by weighting vectors that produce a family of synthesized beams on the sky from 3pt to 19pt, and are then integrated into 24 (for 3pt to 5pt) or 48 (for 6pt to 19pt) bins in right ascension (R.A.).

In the second mode (Ring Data), which was only used in 1995, the chopping plate is tilted back 4°0, which raises the beam elevation from 8°0 to 60°2. The telescope is pointed so that the center of the sweep is north. The beam is swept sinusoidally with a peak-to-peak amplitude of 3°36 on the sky. In the small angle approximation, the beam is swept in R.A. at a constant declination. As the Earth rotates an entire ring at 82° declination is sampled.

Synthesized beams are generated in the same way as with the Cap data, except that a new weighting vector must be generated every sweep to keep the synthesized beam fixed in R.A. during its integration. The synthesized beams are smaller than the amplitude of the sweep, which allows positions on the sky to be tracked in software. 3pt to 6pt synthesized beams are produced, as well as the effective 3pt beam pattern of the MSAM1-92 experiment. The Ring data lacks the east-to-west comparison test of the Cap data. However, synthesis of the MSAM beam makes it possible to compare directly with a very different experiment.

4. DATA SELECTION AND REDUCTION

Because the SK telescope observes from the ground in Saskatoon, an important part of the analysis is the selection of data where contamination from atmospheric spatial temperature fluctuations do not dominate. The first cut is made by evaluating, for each 20 minute file, the mean deviation of 8 s averages made with a beam synthesized using a 2pt weighting vector that is sensitive to the horizontal component of spatial temperature gradients. This number is large when the spatial gradient is changing and small when it is stable and has been found to be a good indicator of atmospheric contamination. The mean deviation is used because it is less sensitive than the standard deviation to spikes in the data, which are more likely to be due to birds or airplanes than to atmospheric noise. The mean deviation is related to the standard deviation by \(\Delta x = (2/\pi)^{1/2} \sigma_x\) for normal distributions.

A more general expression for the cut levels is given by

\[
\zeta = \sigma \sqrt{\tau/\theta_{\text{eff}}},
\]

where \(\sigma\) is the standard deviation of the 2pt data, \(\tau\) is the integration time, and

\[
\theta_{\text{eff}} = \int \theta_x H(\theta_x, \theta_y) d\theta_x d\theta_y
\]

is the effective sweep amplitude in degrees. The units of \(\zeta\) are mK s\(^{1/2}\) deg\(^{-1}\). After correcting for the radiometric contribution, this measurement of the stability of the atmosphere may be compared from experiment to experiment. The effective sky noise, \(\zeta_n\), due to the receiver NET is given by \(\zeta_n = \kappa \text{NET}/\theta_{\text{eff}}\), where \(\kappa = (N\sum_i w_i^2)^{1/2}\) and \(N\) is the number of samples per sweep.

Figure 3 shows \(\zeta\) as a function of time for each of the observing seasons. Horizontal dashed lines are the nominal cut levels (the upper line) and the receiver noise level, \(\zeta_n\) (the lower line), for each of the observing runs. Each point represents the result for one file of data (see Table 1). Because the throw for the Ring data was significantly smaller than for the Cap data, \(\zeta_n\) is significantly higher. In other words, the Ring observations are significantly less sensitive to the atmospheric gradients than the Cap observations.

It is furthermore required that for a file to be accepted, the files before and after must also pass the atmospheric cuts. Thus, the sky must be stable for three files (45 minutes in 1994 and 60 minutes for 1995) for any data to be accepted at all. Of the data acquired, 27% passes the nominal cut levels. However, the lower cut data for the Ring is comprised of the two long receiver noise limited stretches around day 59 and
day 63 of 1995 rather than being based on a specific sky noise level. The analysis is performed for each of the listed cut levels (see Table 15 below). Decreasing the cut level from the definitive values changes some points in the angular power spectrum by approximately 1σ for the SK95 data (see §13). Because this lower cut level is so close to the receiver noise, good data are cut at random, reducing the length of contiguous stretches of data. This increases sensitivity to long timescale noise and reduces the robustness of error bar calculation. Increasing the cut level has an insignificant effect on the results.

The data are blanked for base moves (e.g., 4 s every 40 s for SK95) and for spikes that may be due to birds, planes or occasional glitches in the data system. An entire sweep of data is blanked if any of the samples in it (64 for SK94, and 168 for SK95) exceed the mean by more than some limit (3.5σ for SK94 and 3.85σ for SK95). The amount of data removed in this step is consistent with the data being normally distributed (15% for SK95).

The data are then multiplied by the weighting vector and binned according to R.A. A weighted mean of the frequency distribution of 15 minute averages is σ_{15 min}/σ_{18 s}. The 15 minute averages are used. “A Offset” is the offset in the vertically polarized channel. “B Offset” is the offset in the horizontally polarized channel. For Q94, the A receiver chain was damaged in shipping and was not used.

### Table 3

| Beam     | σ_{15 min}/σ_{18 s} | A Offset (μK) | B Offset (μK) |
|----------|---------------------|---------------|---------------|
| K_93 3pt | 1.2                 | −60 ± 3       | −360 ± 3      |
| K_94 3pt | 1.1                 | 117 ± 6       | −550 ± 7      |
| 4pt      | 1.3                 | 10 ± 5        | −50 ± 6       |
| 5pt      | 1.0                 | 54 ± 6        | 225 ± 6       |
| 6pt      | 1.0                 | 1 ± 7         | 5 ± 7         |
| 7pt      | 1.0                 | 7 ± 9         | 62 ± 9        |
| 8pt      | 1.0                 | 0 ± 13        | 34 ± 13       |
| 9pt      | 1.0                 | 80 ± 19       | 97 ± 18       |
| Q94 3pt  | 1.2                 | −695 ± 8      |               |
| 4pt      | 1.2                 | 63 ± 7        |               |
| 5pt      | 1.0                 | 281 ± 8       |               |
| 6pt      | 1.0                 | 15 ± 8        |               |
| 7pt      | 1.1                 | 79 ± 9        |               |
| 8pt      | 1.0                 | 1 ± 11        |               |
| 9pt      | 1.0                 | 51 ± 13       |               |
| Q95 3pt  | 1.5                 | −2182 ± 9     |               |
| 4pt      | 1.4                 | 4 ± 11        | 31 ± 7        |
| 5pt      | 1.2                 | 89 ± 10       | 436 ± 6       |
| 6pt      | 1.2                 | 63 ± 9        | 107 ± 6       |
| 7pt      | 1.1                 | 21 ± 9        | 145 ± 6       |
| 8pt      | 1.2                 | −56 ± 9       | −83 ± 6       |
| 9pt      | 1.1                 | 21 ± 9        | 53 ± 6        |
| 10pt     | 1.2                 | −101 ± 10     | −103 ± 6      |
| 11pt     | 1.1                 | 25 ± 10       | 35 ± 6        |
| 12pt     | 1.1                 | 70 ± 10       | 75 ± 6        |
| 13pt     | 1.1                 | −26 ± 11      | 7 ± 7         |
| 14pt     | 1.0                 | −34 ± 12      | −41 ± 7       |
| 15pt     | 1.1                 | −23 ± 12      | −31 ± 8       |
| 16pt     | 1.1                 | −12 ± 13      | −17 ± 8       |
| 17pt     | 1.0                 | 15 ± 14       | 12 ± 9        |
| 18pt     | 1.1                 | −41 ± 15      | −49 ± 9       |
| 19pt     | 1.0                 | −65 ± 17      | −31 ± 10      |

**Note.** — Offsets and long timescale noise. The ratio of error bars based on the distribution of 18 s averages to error bars based on the distribution of 15 minute averages is σ_{15 min}/σ_{18 s}. The 15 minute averages are used. “A Offset” is the offset in the vertically polarized channel. “B Offset” is the offset in the horizontally polarized channel. For Q94, the A receiver chain was damaged in shipping and was not used.

### Table 2

| Beam     | k/θ_{eff} | Cut Level ζ (mK s^{1/2} deg^{-2}) | Amount of Data (hr) |
|----------|-----------|-----------------------------------|---------------------|
| K_93     | 0.60      | 4.5                               | 130                 |
| K_94     | 0.44      | 3.0                               | 79                  |
| Q94      | 0.42      | 2.5                               | 140                 |
| Q95 Cap  | 0.36      | 1.7                               | 107                 |
| Q95 Ring | 0.86      | 6.8                               | 121                 |

**Note.** — Amount of data at different cut levels. The first cut level for each experiment listed is the nominal cut used. The others are used for consistency checks. The value k/θ_{eff} is used to convert ζ to receiver NET (see text).
15 minute bins after removal of corresponding mean signal from the 24 or 48 R.A. bins. For data weighted to synthesize many-lobed beams (6pt or more) the error bars based on 18 s averages agree to 10% with those based on 15 minute averages, but for the 3pt or 4pt data (which is more sensitive to atmospheric noise) the distribution of 15 minute averages predicts error bars as much as 50% larger than would be generated from the distribution of the 18 s averages (see Table 3). For the Ring data, with its substantially smaller sweep that results in less sensitivity to the atmosphere, the error bars are generated directly from the distribution of the 18 s averages.

5. OFFSETS
The design of the SK telescope is intended to minimize the instrumental offset. However, the emission from the large chopping plate is a function of the plate orientation, which theoretically produces a \( \approx -400 \, \mu K \) offset for the horizontally polarized channel and a \( \approx 200 \, \mu K \) offset in the vertically polarized channel in the 3pt data. The measured 3pt offsets are approximately this magnitude for SK93 and SK94. In SK95, however, an additional \( \approx -1 \, mK \) offset was present in both polarizations. The source of this additional offset is not known. See Wollack et al. (1997) for possible explanations.

The offsets are dealt with differently between the Cap and Ring data analysis. In the Cap analysis, the weighting \( w_i \) is constant for a given synthesized beam and chopping plate orientation. For this reason the offsets are ignored until the data have been multiplied by the weighting vector and binned in R.A. At that point, the mean value of the 24 or 48 R.A. bins is removed. The error bars are then multiplied by \( (24/23)^{1/2} \) or \( (48/47)^{1/2} \) to compensate for the removed degree of freedom. The levels of the Cap offsets for each synthesized beam are listed in Table 3.

For the Ring data, however, a new weighting vector must be calculated for each sweep, which causes the offset to change as the synthesized beam is tracked over the sky. To deal with this, all of the accepted data are co-added synchronously with the sweep to form a 168 sample mean sweep. This mean sweep, which is essentially the instrumental offset as a function of chopping plate position, is subtracted from the data for each sweep prior to multiplication by the weighting vector. At this point then, the offset for any synthesized Ring beam is zero.

As evidenced by the larger scatter of the 15 minute averages compared to that predicted by the distribution of the 18 s averages in the Cap data (see Table 3), there may be some drift to this offset on longer timescales. Since at least some of offset is due to emission, the excess long timescale noise could be expected to correlate with the temperatures of the optical components. The correlations are not significant however, and regressing out the optical temperatures has no effect on the long timescale noise. Since the gain of the amplifiers is a function of temperature the long timescale noise might be expected to vary with receiver temperature. But, as with the optics temperatures, there is no significant correlation with receiver temperature, and regressing it out has no effect. In the final analysis, nothing is regressed out.

To further investigate the effect of the excess long timescale noise in the Cap data, polynomials in time (linear to 9th degree) were fit out of the data. The maximum slope from fitting a line in time to the data is 7 \( \mu K \) per day for the SK95 3pt data. The typical slope is less than 1 \( \mu K \) per day and consistent with zero. As with regressing out the auxiliary temperatures, removing lines or polynomials has no significant effect on the long timescale noise or on the final angular power spectrum. In the final analysis, only the constant offset is removed.

To test the possibility that the long timescale noise is due to instrumental drifts, data taken in the west can be subtracted from data taken in the east 40 s earlier. Doing this decreases the differences between the short and long timescale-based error bars. This subtraction, however, has no significant effect on the spectrum, beyond increasing the error bars: \( (E - W)/2^{1/2} \) will have the same root mean square (rms) as E or W taken individually, but leaves only one data set to do statistics on, rather than two. It is also possible that the excess long timescale noise is due to atmospheric contamination; this was the interpretation taken by the OVRO Ring experiment (Readhead et al. 1989) for a similar effect.

6. INTERNAL CONSISTENCY CHECKS
The observing scheme used in the SK experiment has a number of internal consistency checks. For the Cap data, the rotation of the Earth causes the same sky as was observed with the telescope pointed to the east of the NCP to be observed \( \approx 12 \) hr later when the telescope is pointed to the west. This test (E - W) gives assurance that the signal that has been observed is truly on the sky: contamination from the atmosphere or ground pickup could be expected to repeat on a 24 solar hour timescale, not a 24 sidereal hour timescale with the signal in the east lagged 12 sidereal hours from the signal in the west.

The data set may also be divided into first half (H1) and second half (H2), and compared. Since there is approximately one month between the centroids of the first and second half, the success of this test gives assurances that the observed signal repeats on sidereal time and not solar time: if the signal repeated on solar time, the signal between halves would be lagged by 4 out of 48 bins. This places limits on contamination by the Sun and any radio frequency interference (RFI) that recurs daily.

The data from the two polarization channels (A is vertical and B is horizontal) may also be compared. This test is useful for detecting contaminants that affect the two receiver chains differently, such as data system pickup or polarized diffraction off of the ground screen.

Since the same region of sky has been observed in 1994 as 1995, the 3pt to 9pt Cap data can also be compared between years. A comparison of the SK93 and SK94 data using a specially prepared weighting vector for SK94 is presented in Netterfield et al. (1995) and repeated here. Since many elements of the telescope, radiometer, and ground shield were changed between years, this test can give assurances that the data have not been contaminated by side lobes, vibrations, or atmospheric gradients.

Finally, weighting vectors may be applied that have the same frequency as sky-sensitive weighting vectors, but with the phase adjusted to have no sensitivity to the sky (quadrature phase). This test is sensitive to synchronous vibrational or electrical signals, which would not necessarily have the same phase as a true sky signal.

The results of this set of internal consistency tests are given in Tables 4, 5, 6, 7, 8, 9, and 10. The reduced \( \chi^2 \) for the difference tests should be distributed around 1.0 with a stan-
For the 12pt data, this results in this case we expect where the case for the H1 lagged correlation is not 100%. For the 12pt data, for instance, the theoretical E - W correlation coefficient (see § 7) is \( \rho_{E,W}^2 = (C_{E,W}^B/C_{E,W}^A)^{1/2} = 0.77 \). For the 3pt data, \( \rho_{E,W}^2 = 0.90 \). In such cases, where the sky signal is not 100% correlated, we find

\[
\frac{\chi^2_{E-W}}{\nu} = \frac{2\rho + (1 - \rho)\chi^2_{E-W}/\nu}{1 + \rho}. \tag{5}
\]

For the 12pt data, this results in \( \chi^2_{E-W}/\nu = 1.16 \pm 0.2 \), which is consistent with 1.29 as determined from the data. Similarly, atmospheric noise correlates the error bars from the A and B polarizations in the large angular scale data. In this case we expect \( \chi^2_{E-W}/\nu = (1 - \rho_D)^2 \), where \( \rho_D^2 = C_{E}^D/C_{E}^A \) is the noise correlation coefficient.

In Tables 4–8, the expected values of \( \chi^2/\nu \) and the probability of exceeding the measured value for the symmetry tests are presented. Rather than using equation (5), the expected distributions of \( \chi^2/\nu \) are generated using Monte Carlo simulations described in § 10.

3 For two correlated zero mean normal deviates \( x \) and \( y \) with \( \sigma^2 = \sigma^2 \), we write \( y = a + \rho x \), where \( a \) is uncorrelated with \( x \) and \( \rho \) is the correlation coefficient between \( x \) and \( y \). One finds that \( \sigma^2_{\text{sys}} = (1 + \rho^2)\sigma^2 \) and \( \sigma^2_{\text{sys}} = (1 - \rho)\sigma^2 \). After identifying \( x \) with the east data and \( y \) with the west data, and noting that \( \chi^2/\nu = 1 + \sigma^2_{\text{sys}}/\sigma^2 \), we find eq. (5).

While the majority of the entries in Tables 4–10 are consistent with the signal being fixed on the sky, there are some potential inconsistencies. The worst entry is for Q95 3pt H1 – H2. The probability of exceeding the measured reduced \( \chi^2 \) of 2.34 is \( 2.8 \times 10^{-4} \), which may indicate trouble (although there is a \( \approx 3\% \) chance for 1 of 124 independent tests to do this poorly). The H1 – H2 residual is dominated by long drifts (\( \approx 12 \) hr), which is consistent with the earlier observation that the 3pt data are especially affected by excess long timescale noise. However, the E - W tests, and more significantly, the year-to-year tests work well for this data set, indicating that if the entire season is used the effect of the drifts is averaged out. A similar effect at a lower level is seen in the Q95 4pt data. Without the ability to compare with the other seasons we would be much less sanguine regarding the quality of the Q95 3pt data.

Taken as a whole, however, the results of the tests are well behaved, with the probabilities of exceeding the measured values of \( \chi^2/\nu (P_\nu) \) for each of the 124 tests distributed evenly between 0 and 1. As well as indicating that the data are largely free of systematic contamination, this indicates that the error bars have been correctly determined. These tests are strong evidence that the measured signal is on the sky.

7. LIKELIHOOD ANALYSIS

In the analysis of these data, two questions are asked: (1) is the spectral index of the fluctuations consistent with that of the CMB, and (2) what is the angular power spectrum of the fluctuations? Comparison of the data with theoretical predictions may be done by comparison with the angular power spectrum. The determination of parameters (such as the amplitude of the fluctuations at various angular scales,
# Table 5

## Internal Consistency Tests II. 6pt to 9pt Cap

| Beam                  | 6pt Cap | 7pt Cap | 8pt Cap | 9pt Cap |
|-----------------------|---------|---------|---------|---------|
|                       | $\text{Exp } (\chi^2/v)$ | Monte Carlo ($\chi^2/v, P_x$) | $\text{Exp } (\chi^2/v)$ | Monte Carlo ($\chi^2/v, P_x$) | $\text{Exp } (\chi^2/v)$ | Monte Carlo ($\chi^2/v, P_x$) | $\text{Exp } (\chi^2/v)$ | Monte Carlo ($\chi^2/v, P_x$) |
| $K_{94}$ Sum          | 1.13    | ...     | 1.40    | ...     | 1.13    | ...     | 1.03    | ...     |
| A – B                 | 0.94    | (1.01 ± 0.20, 0.62) | 0.94    | (1.01 ± 0.20, 0.62) | 0.94    | (1.01 ± 0.20, 0.62) | 1.18    | (1.01 ± 0.20, 0.62) |
| E – W                 | 1.06    | (1.01 ± 0.21, 0.38) | 1.39    | (1.01 ± 0.21, 0.05) | 0.59    | (1.01 ± 0.21, 0.99) |
| Quadrature            | 1.3     | (1.0 ± 0.2, 0.08) | ...     | ...     | ...     | ...     | ...     | ...     |
| Q94 Sum               | 1.19    | ...     | 1.05    | (1.01 ± 0.21, 0.40) | 0.84    | (1.01 ± 0.21, 0.79) |
| E – W                 | 1.05    | (1.01 ± 0.21, 0.35) | 1.05    | (1.01 ± 0.21, 0.40) | 0.84    | (1.01 ± 0.21, 0.79) |
| Quadrature            | 1.4     | (1.0 ± 0.2, 0.04) | ...     | ...     | ...     | ...     | ...     | ...     |
| Q95 Sum               | 2.69    | ...     | 2.69    | ...     | 2.69    | ...     | 2.37    | ...     |
| A – B                 | 0.73    | (0.98 ± 0.20, 0.91) | 1.1     | (1.02 ± 0.21, 0.32) | 1.0     | (1.00 ± 0.20, 0.48) |
| E – W                 | 1.09    | (1.00 ± 0.22, 0.47) | 1.16    | (1.00 ± 0.23, 0.42) | 0.88    | (1.0 ± 0.2, 0.70)  |
| H1 – H2               | 1.06    | (1.0 ± 0.2, 0.36) | 0.88    | (1.0 ± 0.2, 0.70)  | 0.99    | (1.0 ± 0.2, 0.99)  |
| Quadrature            | 2.12    | (1.0 ± 0.2, 0.07) | ...     | ...     | ...     | ...     | ...     | ...     |
| $K_{94}$ + Q94        | 1.16    | ...     | 1.50    | ...     | 1.50    | ...     | 1.58    | ...     |
| $K_{94}$ – Q94        | 0.15    | (1.01 ± 0.20, 0.23) | 0.82    | (1.02 ± 0.20, 0.83) | 1.03    | (1.02 ± 0.20, 0.45) |
| $K_{94}$ + Q95        | 1.50    | ...     | 2.66    | ...     | 2.56    | ...     | 2.56    | ...     |
| $K_{94}$ – Q95        | 1.22    | (1.08 ± 0.22, 0.17) | 1.17    | (1.08 ± 0.22, 0.33) | 0.85    | (1.06 ± 0.22, 0.84) |
| Q94 + Q95             | 2.10    | ...     | 3.10    | ...     | 2.94    | ...     | 2.94    | ...     |
| Q94 – Q95             | 1.48    | (1.04 ± 0.21, 0.42) | 0.78    | (1.05 ± 0.22, 0.90) | 1.04    | (1.05 ± 0.22, 0.53) |

**Note:** Reduced $\chi^2$ for internal symmetry tests of 6pt to 9pt 1994 and 1995 Cap data. For each entry, $v = 48$ bins.
| BEAM          | 10pt Cap |          | 11pt Cap |          | 12pt Cap |          | 13pt Cap |          |
|--------------|---------|----------|---------|----------|---------|----------|---------|----------|
|              | Exp ($\chi^2/v$) | Monte Carlo ($\chi^2/v$, $P_\alpha$) | Exp ($\chi^2/v$) | Monte Carlo ($\chi^2/v$, $P_\alpha$) | Exp ($\chi^2/v$) | Monte Carlo ($\chi^2/v$, $P_\alpha$) | Exp ($\chi^2/v$) | Monte Carlo ($\chi^2/v$, $P_\alpha$) |
| Q95 Sum ...... | 1.95    | ...      | 2.12    | ...      | 2.29    | ...      | 2.30    | ...      |
| A – B .......... | 1.01    | (1.02 ± 0.21, 0.49) | 1.36    | (1.02 ± 0.21, 0.06) | 0.78    | (1.01 ± 0.21, 0.88) | 0.85    | (1.03 ± 0.21, 0.81) |
| E – W .......... | 0.85    | (1.07 ± 0.22, 0.84) | 0.81    | (1.09 ± 0.22, 0.90) | 1.29    | (1.13 ± 0.23, 0.23) | 1.42    | (1.14 ± 0.24, 0.12) |
| H1 – H2 ...... | 0.91    | (1.0 ± 0.2, 0.65)  | 1.01    | (1.0 ± 0.2, 0.45)  | 1.29    | (1.0 ± 0.2, 0.09)  | 0.91    | (1.0 ± 0.2, 0.65)  |
| Quadrature.... | 0.97    | (1.0 ± 0.2, 0.53)  | 0.90    | (1.0 ± 0.2, 0.67)  | 1.22    | (1.0 ± 0.2, 0.14)  | 1.05    | (1.0 ± 0.2, 0.38)  |

**Note.**—Reduced $\chi^2$ for internal symmetry tests of 10pt to 13pt 1995 Cap data. For each entry, $v = 48$ bins.
and their spectral indices) are made by maximizing the likelihood.

The Likelihood (or the probability density of the data, assuming a theory) is defined as
\[
L = \exp \left( -\frac{1}{2} t^T M^{-1} t \right)
\] (6)
where \( t \) is a data set and \( M = C_D + C_T \) is a corresponding covariance matrix. \( C_D \) is the data covariance matrix, and \( C_T \) is the pixel-to-pixel theory covariance matrix, which is a function of the model parameters. The model parameters are adjusted to maximize L. For the Saskatoon analysis, the elements of the data vector \( t_i \) correspond to data taken looking at a specified R.A. bin on the sky, with a specified synthesized antenna pattern, in a given frequency and polarization channel.

Confidence intervals on a parameter \( T \) are found from the distribution of \( L(T) \). The best value for the parameter is determined by finding \( T_{\text{max}} \), where \( L(T_{\text{max}}) \) is maximized. For positive definite quantities (such as the amplitude of the fluctuations) a nonzero detection is claimed when \( L(0)/L(T_{\text{max}}) < 0.15 \). In this case the error bars are found by finding \( T_{\pm} \) such that
\[
\int_{0}^{T_{\pm}} L(T) dT = 0.1587
\] (7)
for the lower limit, and finding \( T_{\pm} \) so
\[
\int_{0}^{T_{\pm}} L(T) dT = 0.8413
\] (8)
for the upper limit. When \( L(0)/L(T_{\text{max}}) > 0.15 \), the 95% upper limit is found by finding \( T' \) such that
\[
\int_{0}^{T'} L(T) dT = 0.95
\] (9)
For nonpositive definite quantities (such as spectral indices) the technique is similar, except that the lower limits of the above integrals are \(-\infty \) rather than 0.

The data covariance matrix, \( C_D \), describes the signal due to instrumental noise in the data. The diagonal matrix elements are simply the variance of the data derived from the distribution of the 15 minute averages as described in § 4. The off-diagonal elements, summarized in Table 11, describe other instrumental correlations. For instance, the noise between frequency channels in the same radiometer chain is correlated by gain fluctuations in the HEMT amplifiers (Jarosik 1996). Similarly, atmospheric noise introduces correlations between simultaneously acquired channels. Since the weighting vectors for SK95 are not strictly orthogonal in time, there will be noise correlations between,
for example, the 13pt data and the 15pt data. The noise from data acquired in the east is also correlated with the noise from data acquired in the west 20 s earlier. This is related to the excess long timescale noise discussed previously and may be due to the atmosphere or drifting instrumental offsets. The noise correlation coefficients are generated from the distribution of 18 s synthesized beam averages for all simultaneously acquired data. All known correlations are included in the analysis.

The theory covariance matrix, $C_T$, describes the signal due to a hypothesized sky signal (Bond 1995b, references therein; Peebles 1994; White & Scott 1994). With the theoretical temperature fluctuations on the sky expressed in spherical harmonics as

$$T(\hat{x}) = \sum_{l,m} a_l^m Y_l^m(\hat{x}),$$

the theory covariance matrix can be expressed as

$$C_T^{ij} = \frac{1}{4\pi} \sum_{l} (2l+1)c_l W_l^{ij},$$

where $c_l \equiv \langle |a_l^1|^2 \rangle$ and

$$W_l^{ij} \equiv \int d\hat{x}_1 \int d\hat{x}_2 H(\hat{x}_1)H(\hat{x}_2)P(\hat{x}_1 \cdot \hat{x}_2)$$

is the window function. $H(\hat{x})$ is the effective antenna pattern from equation (1), and $P(\hat{x}_1 \cdot \hat{x}_2)$ are the Legendre polynomials. The window functions associated with the Saskatchewan experiment are presented in Figure 4.

Rather than expressing the sky fluctuations in terms of $c_l$, we use

$$\delta T_l \equiv \sqrt{\frac{l(l+1)}{4\pi}} c_l. \quad (13)$$

In terms of $\delta T_l$ the theory covariance matrix becomes

$$C_T^{ij} = \sum_l \delta T_l^i \frac{W_l^{ij}}{l}. \quad (14)$$

So $\delta T_l^2$ is the variance per logarithmic interval of the angular spectrum of the CMB.

### 8. The Window Function

The angular scale to which a data set is sensitive ($l_e$) is determined from the diagonal components of the window function, $W_l^{ii}$ as follows. The expected rms amplitude of the fluctuations, $\Delta'$, predicted by a theory for data acquired with a specified beam, is given by the theoretical correlation matrix (eq. [14]) as

$$\Delta' = \sqrt{C_T^{ii}} = \sqrt{\sum_l \delta T_l^i \frac{W_l^{ii}}{i}}, \quad (15)$$

or the sum of the theoretical angular spectrum, weighted by $W_l^{ii}/l$. Under the assumption of a flat spectrum in $\delta T_l$, this may be simplified to

$$\Delta' = \frac{\Delta}{l_e} \sqrt{I(W_l^{ii})}, \quad (16)$$

where

$$I(W_l) \equiv \sum_i W_l^{ii}. \quad (17)$$

This provides a convenient conversion between the rms amplitude of the fluctuations, $\Delta$, and the flat-band power estimation, $\Delta T_e$. The effective $l$ for a single window function is the centroid of $W_l^{ii}/l$, or

$$l_e = \frac{I(W_l^{ii})}{I(W_l^{ii})}. \quad (18)$$

The band power estimate, $\delta T_{l_e}^2$, over a window function, $W_l^{ii}$, can be generated directly from the angular spectrum, $\delta T_l$, by

$$\delta T_{l_e}^2 = \frac{I(\delta T_l^i W_l^{ii})}{I(W_l^{ii})}. \quad (19)$$

When combining the results of several measurements with differing window functions it is appropriate to find the

---

**TABLE 11**

| Between          | Source                                | Magnitude [C_T^{ii}/C_T^{ii}C_T^{ii}] |
|------------------|---------------------------------------|---------------------------------------|
| Frequency channels | HEMT correlations, atmosphere         | 0.2–0.7                               |
| Polarization channels | Atmosphere                           | 0.0–0.3                               |
| East/west         | Atmosphere, offset drift              | 0.0–0.2                               |

**Note.**—Summary of the noise correlations included in the analysis and their sources.
weighted mean of $\delta T_{1i}^2$, since these are proportional to the variances, which are normally distributed for large degrees of freedom. Thus,

$$\bar{\delta T}_{1i}^2 = \frac{\sum \delta T_{1i}^2 / \sigma_{\delta T_{1i}}^2}{\sum 1/\sigma_{\delta T_{1i}}^2}.$$  

(20)

Inserting equation (19) into equation (20) we find

$$\bar{\delta T}_{1i}^2 = \sum \delta T_{1i}^2 / l,$$  

(21)

where we have introduced the definition

$$W_l \equiv \frac{\sum W_i^d / [I(W_i^d) \sigma_{\delta T_i}^2]}{\sum 1/\sigma_{\delta T_i}^2}.$$  

(22)

The quantity $W_l$ is the effective window function that results from combining band power measurements with different window functions. The quantity $\sigma_{\delta T_i}^2$ is the measurement uncertainty on $\delta T_{1i}^2$. Note that $I(W_i) \equiv 1$ with this definition.

9. THE THEORETICAL POWER SPECTRUM

If the angular spectrum varies slowly over the angular scales to which the experiment is sensitive it is reasonable to expand $\delta T_i$ as a power law in both angular scale, $l$, and observing frequency, $v$, as

$$\delta T_i = \delta T_{1i} \left( \frac{l}{l_e} \right)^m \left( \frac{v}{v_o} \right)^\beta,$$  

(23)

where $\delta T_{1i}$ is the amplitude of the fluctuations at angular scale $l_e$ and frequency $v_o$. The angular spectral index is $m$ and the frequency spectral index is $\beta$. With $l_e$ chosen using equation (18) and $v_o$, chosen as the center frequency of the observations (i.e., 36 GHz) the three parameters are roughly orthogonal for $m$ and $\beta$ near 0. Therefore, neither the amplitude of the fluctuations, $\delta T_{1i}$, nor the frequency spectral index, $\beta$, will be a function of choice of angular spectral index, $m$.

Determination of the frequency spectral index, $\beta$, is useful for discriminating the source of the fluctuations. With $\delta T_i$ in thermodynamic units, $\beta \equiv 0$ for the CMB, independent of frequency, while in the Rayleigh-Jeans region, where antenna temperature approximates thermodynamic temperature, $\beta = -2.1$ for Galactic free-free emission and $\beta = 1.7$ for Galactic dust.

By combining data from multiple window functions the angular spectral index, $m$, may be determined. Since the limits on this parameter are often uninterestingly large, it is often simply set to some fixed value. Choosing $m = 0$ assumes constant $\delta T_{1i}$ over the region where the data are sensitive. The determination of $\delta T_{1i}$ for $m = 0$ is known as a flat-band power estimate of the angular spectrum. The choice of $m = 1$ reproduces the “delta function” correlation function used in Wollack et al. (1993) and Netterfield et al. (1995).

The procedure for finding the angular spectrum, $\delta T_i$, is to group data sensitive to a particular range of angular scales (e.g., the Spt data are sensitive to angular scales around $l = 100$) and then find the maximum value of the likelihood by varying the amplitude, $\delta T_{1i}$, and the spectral indices, $\beta$ and $m$, at each angular scale. Where there is insufficient leverage to determine $\beta$ and $m$, they are fixed at 0.

Of mainly historical interest is the Gaussian autocorrelation function,

$$C_{y}(\theta_{12}) = C(0) \int \int dx_1 \int dx_2 H(x_1)H(x_2) \exp \left( -\frac{\theta_{12}^2}{2\theta_c^2} \right),$$  

(24)

where $\theta_{12} = \cos (\hat{x}_1 \cdot \hat{x}_2)$. The angular spectrum associated with this is

$$\delta T_i = \sqrt{C(0) \int \frac{(2l + 1)}{2l_e^2} \exp \left( -\frac{l^2}{2l_e^2} \right)},$$  

(25)

with $l_e = 1/\theta_c$. The coherence angle of the theory is $\theta_c$, and is often varied to maximize overlap with the experiment. The free parameter, which is varied to maximize the likelihood, is $C(0)$. This spectrum has no particular theoretical motivation. However, since it changes only slowly over the window function of most experiments, $C(0)$ can be converted to a band power, $\delta T_{1i}$, by (Bond 1995a)

$$\delta T_{1i} = \sqrt{C(0) \int u^2 \exp \left( -u^2/2W_l \right) I(W_l)},$$  

(26)

where

$$u = \frac{l + 1/2}{l_e + 1/2}.$$  

(27)

10. MONTE CARLO SIMULATIONS AND THE ROOT MEAN SQUARE AMPLITUDE ANALYSIS

It is sometimes convenient to characterize the fluctuations by their rms amplitude, $\Delta_{sky}$ given by $\Delta_{sky}^2 = \Delta_{tot}^2 - \Delta_{inst}^2$. The raw rms of the data including instrument noise is $\Delta_{tot} = (\sum \Delta T_{1i}^2/N)^{1/2}$, and the estimated contribution due to instrument noise is $\Delta_{inst}$.

Monte Carlo simulations using $C_D$ and $C_T$ are used to determine the instrument noise, $\Delta_{inst}$, and the uncertainties in $\Delta_{sky}$ due to the instrument noise ($\sigma_{\Delta_{Inst}}$) and sample variance ($\sigma_{\Delta_{sky}}$). These same simulations are used to find the expected distributions of $x^2/y$ for the symmetry tests presented in § 6 and Tables 4–8. The theoretical correlation matrices used are from a flat-band power model with amplitudes from Table 14 below.

The instrument noise, $\Delta_{inst} \pm \sigma_{\Delta_{inst}}$, is generated from the data covariance matrix, $C_D$; 4096 examples of data sets described by $C_D$ are generated by

$$t = C_D^{1/2} N,$$  

(28)

where $N$ is a vector of the same dimension as $t$ comprised of samples drawn from a unit normal distribution. The quantities $\Delta_{inst}$ and $\sigma_{\Delta_{inst}}$ are found from the distribution of the variances of the fake data sets. The uncertainty in $\Delta_{sky}$ due to measurement noise is given by

$$\sigma_{\Delta_{sky}} = \frac{\Delta_{sky}}{\Delta_{sky} \sigma_{\Delta_{inst}}}. $$  

(29)

This determination of $\Delta_{sky} \pm \sigma_{\Delta_{sky}}$ gives the amplitude (and uncertainty) of the fluctuations only over the region of sky that was measured. This is appropriate for comparing different measurements of the same sky but does not include sample variance.

The uncertainty in $\Delta_{sky}$ due to sample variance ($\sigma_{\Delta_{sky}}$) is generated from a theory covariance matrix, $C_T$, in the same
way as $\sigma_{\text{host}}$ was generated from $C_D$; 4096 fake data sets described by $C_T$, are generated by

$$t = C_T^{1/2} N.$$  \hspace{1cm} (30)

The uncertainty $\sigma_{\text{host}}$ is found from the distribution of the variances of the fake data sets. The uncertainties $\sigma_{\text{host}}$ and $\sigma_{\text{inst}}$ are added in quadrature to estimate the total uncertainty in $\Delta_{\text{sky}}$. The rms amplitude, $\Delta_{\text{sky}}$, and its uncertainties may be converted to $\delta T/I$ using equation (16).

The expected distribution of the reduced $\chi^2$ for the internal symmetry tests described in §6 are determined by generating, for each test, 4096 fake data sets using

$$t = C_T^{1/2} N_1 + C_D^{1/2} N_2.$$ \hspace{1cm} (31)

The given symmetry test is performed on each of the 4096 fake data sets and statistics on the resulting values of the reduced $\chi^2$ are evaluated.

11. DETERMINATION OF THE FREQUENCY SPECTRAL INDEX $\beta$

The first issue addressed in the analysis of the reduced data is whether the signal is due to the CMB or is due to some other foreground contaminant. As noted, the spectral index of most foregrounds is expected to deviate from that of the CMB, $\beta = 0$. Since data are acquired in six frequency bins between 26 and 46 GHz, $\beta$ for the fluctuations may be evaluated using the likelihood analysis described. Since the data acquired with the $K_\nu$-band radiometer only has adequate sensitivity for the 3pt, 4pt, and 5pt synthesis, this may only be done internally for angular scales below $l \approx 100$.

The frequency span given by the $Q$-band data alone is inadequate to determine $\beta$.

To evaluate $\beta$ the data from $K_{\nu}$ 3pt and SK94 and SK95 3pt to 5pt are used. The covariance matrix, $M$ in equation (6), has dimension $4^{1222} \times 1422$. For the theoretical spectrum, equation (23) is used with $m = 0$. The likelihood analysis yields $\delta T/I = 47 \pm 2 \mu K$ and $\beta = 0.2^{+0.3}_{-0.2}$ for $l = 73$. Repeating the analysis with $m = 1$ yields $\beta = 0.2^{+0.3}_{-0.2}$ and $m = 1$ yields $\beta = 0.1^{+0.3}_{-0.2}$. Thus, the limits on $\beta$ are independent of choice of angular spectral index, $m$.

The channel-to-channel calibration uncertainty is 2%–5%. This causes an additional uncertainty in $\beta$ of $\pm 0.1$, which is to be added in quadrature to the quoted errors.

The spectral index, $\beta$, of the fluctuations is consistent with that of the CMB ($\beta = 0$), and inconsistent with known potential foreground contaminants. In particular, contamination by Galactic synchrotron ($\beta = -2.8$), Galactic free-free emission ($\beta = -2.1$), and Galactic dust ($\beta = 1.7$) in this data set are ruled out at large angular scales. Foreground contamination is discussed in detail in §14.

12. COMPARISON WITH MSAM 1-92

Perhaps the most powerful systematic check available with the Saskatoon experiment is the comparison with the MSAM1-92 experiment. MSAM1-92 is a balloon-borne experiment that observed the sky with a 3pt beam at a declination of 82° in $\approx 81$ fields between 14.4 and 20.4 hr R.A. (Cheng et al. 1994). While the Saskatoon experiment observes the sky between 26 and 46 GHz, the MSAM1-92 experiment has observing bands sensitive to the CMB at 180 and 240 GHz, providing very wide frequency coverage between the two experiments. The comparison of the two experiments of very different nature places stringent limits on systematic errors or foreground contamination.

As mentioned in §3, data from 1995 that were acquired in the Ring mode (at a declination of 82°/05) were multiplied by a weighting vector that closely synthesized the MSAM 3pt beam. To generate this weighting vector, the elements $w_i$ are adjusted to minimize the variance between the synthesized SK and MSAM beam patterns, assuming that the declination of the two experiments is equal. In reality, the mean pointing of the two experiments differs by 0°14, and the MSAM1-92 experiment did not track in fixed elevation. This discrepancy is ignored in the in the analysis of $\Lambda$ but included in the Likelihood analysis. Once a SK weighting vector has been produced that maximizes spatial overlap, it is scaled so that $\sqrt{\langle I(W) \rangle^{1/2}} = 1.15$, the value for the MSAM window function. To check the level of overlap between the two experiments, the window function of the difference of the synthesized beams is formed. For the difference beam $\sqrt{\langle I(W) \rangle^{1/2}} = 0.25$. Given this overlap, it is expected that over 90% of the sky signal will be in common between the two experiments. The window functions for the MSAM overlap are given in Figure 5.

Given the high degree of overlap expected, a naive test is adequate to verify that the two experiments see the same signal. Figure 6 shows the two data sets, their mean and difference. The reduced $\chi^2$ for the sum is 3.43, while the reduced $\chi^2$ for the difference is 1.05, neglecting the anomalous third sample at 14.58 hr in the MSAM1-92 data set. It is clear that both experiments see the same signal on the sky.

To find the consistency of the amplitudes measured by the two experiments, an analysis of $\Delta$ as described in §10 is performed. The data covariance matrix for the MSAM1-92 data ($C_{\text{Data}}$) is taken to be diagonal, neglecting correlations that were introduced by the removal of offset drifts in the data reduction. For the SK data, $C_{\text{Data}}$ is made from the distribution of the 18 s averages and includes all known correlations. For analysis of the combined MSAM1-92/ SK95 data, the weighted mean is used, and the off diagonal

\[4\text{K}93\text{ has three frequency channels, 21 R.A. bins, and both east and west data (126 rows), K94, Q94 and Q95 each have three frequency channels, 24 R.A. bins, and east and west data for 3pt, 4pt, and 5pt data (1296 rows).} \]

\[\text{Fig. 5.—MSAM1-92 and SK95/MSAM Overlap window functions. The window functions are slightly different due to unequal beam size. The window function for the difference is also presented.} \]
The relative calibration uncertainty of 17%.

Adding these in quadrature yields a larger normalization for MSAM but is still consistent with the absolute calibrations used. For Saskatoon it is 14%, and for SK the 1\sigma confidence interval. Reanalyzing the MSAM1-92 data, we find 55 \mu K for MSAM1-92 and SK, respectively.

A determination of the angular power spectrum is presented in three ways. The first is to find \delta T_i for the data set associated with each of the 37 synthesized beams, H_a described in § 9. The results of this are presented in Table 13 and Figure 7. Also included in

The flat-band power is found by a full likelihood analysis using the angular spectrum in equation (23). These results are presented in Table 12. The results for the two experiments are fully consistent. Note that calibration uncertainties have not been included in quoted errors.

The original MSAM1-92 analysis is done in terms of the GACF, equation (24). This analysis is repeated here. For MSAM1-92, we find \((C(0))^{1/2} = 70.8 \pm 2.5 \mu K\), using \(\theta_0 = 0.3\), which is consistent with the analysis quoted in Cheng et al. (1994). For SK we find \((C(0))^{1/2} = 81.2_{-15}^{+26} \mu K\). Converted to band power estimates using equation (26) we find 53 and 58 \mu K for MSAM1-92 and SK, respectively.

A determination of the angular power spectrum is presented in three ways. The first is to find \(\delta T_i\) for the data set associated with each of the 37 synthesized beams, \(H_a\) described in § 9. The results of this are presented in Table 13 and Figure 7. Also included in

Table 12: MSAM1-92 and SK Compared

| Parameter          | MSAM1-92 | SK      | Mean   | Difference |
|--------------------|----------|---------|--------|------------|
| \(\chi^2/v\)       | 2.66     | 1.82    | 3.43   | 1.05       |
| \(\Delta_{\text{int}}\) (\mu K) | 88.9     | 104.4   | 76.6   | 52.2       |
| \(\Delta_{\text{int}} + \sigma_{\text{int}}\) (\mu K) | 70.5 \pm 5.3 | 79.5 \pm 7.9 | 49.3 \pm 3.9 | 53.2 \pm 4.3 |
| \(\Delta_{\text{sky}}\) (\mu K) | 53.5 \pm 7.0 | 65.9 \pm 10.2 | 58.6 \pm 3.3 | ... |
| \(\delta T(A_{\text{sky}})\) (\mu K) | 46.5     | 57.3    | 51     | ...        |
| \(\delta T(\text{Full likelihood})\) (\mu K) | 52^{+11}_{-6} | 61^{+19}_{-15} | 50.9^{+10}_{-10} | ... |

Note.—Comparison of the MSAM1-92 and SK95/MSAM Overlap data. The reduced \(\chi^2\) entries are for the 80 bins in Fig. 6, neglecting the anomalous third sample from MSAM1-92. The uncertainties on \(\Delta_{\text{sky}}\) do not include sample variance. The uncertainties on \(\delta T\) do. \(\delta T(\Delta_{\text{sky}})\) is \(\delta T\) inferred from \(\Delta_{\text{sky}}\) using eq. (16). Calibration uncertainties are not included.

References:

1. Cheng et al. (1994) find 50 \mu K < \([C(0)]^{1/2} < 90\) \mu K for the 5%-95% confidence interval. Reanalyzing the MSAM1-92 data, we find 55 \mu K < \([C(0)]^{1/2} < 99\) \mu K.

2. Figure 6.—MSAM1-92 and SK95/MSAM Overlap compared. The first panel gives the MSAM1-92 3pt data. The second panel gives SK95/MSAM Overlap data for the same region. The third shows the mean and the fourth the difference. The data are in 85 equally spaced R.A. bins (only 81 have data), and are given in thermodynamic temperature units. The anomalous third bin in the MSAM data set is dropped in the analysis of the comparison. The 0.5 FWHM beam spans 0.24 hr of R.A.

3. Elements on the covariance matrix are generated using

\[
C_{ij}^f = C_{ijk}^l \frac{w_{SK}^i w_{MS}^j}{(w_{SK}^i + w_{MS}^j)(w_{SK}^j + w_{MS}^i)},
\]

where \(w_{SK}^i\) and \(w_{MS}^j\) are the statistical weights of the SK and MSAM1-92 data, respectively.

The results of this analysis are presented in Table 12. The errors on \(\Delta\) include only \(\sigma_{\text{sky}}\) and do not include sample variance. This is appropriate for comparing the consistency of two measurements of the same sky. The amplitudes are consistent, with SK favoring a higher value than MSAM at the 1\sigma level.

This analysis has not included calibration uncertainty in the two instruments. For Saskatoon it is 14%, and for MSAM1-92 it is 10%. Adding these in quadrature yields a relative calibration uncertainty of 17%. A least-squares fit to the best relative calibration between the two data sets finds \(N_{\text{SK}}/N_{\text{MSAM}} = 0.82 \pm 0.16\). The relative calibration based on the data favors a smaller normalization for SK or a larger normalization for MSAM but is still consistent with the absolute calibrations used.
Table 13 are results from an analysis of $\Lambda$ as described in § 10. Note that for SK95 3pt through 10pt, the estimated contribution to the uncertainties due to sample variance, $\sigma_T^2$, is approximately equal to the contribution due to instrument noise, $\sigma_{\text{inst}}^2$. Consequently, to significantly reduce the error bars, both better sensitivity and more sky coverage will be required. This representation has significant spatial correlations between entries. For example, the 3pt beams for Q95, Q94, and K94 all observe the same sky and at a similar angular scales. It is therefore inappropriate to combine entries simply by taking the weighted mean without taking into account the spatial correlations.

The second way the data are presented is to combine data from spatially correlated synthesized beams between years. To do this, a covariance matrix $C_{\text{D}}$ is made that includes all of the data corresponding to a given spatial symmetry. For example, the K94, Q94, and Q95 3pt data forms a $144 \times 144$ matrix. The band power is then evaluated by maximizing the likelihood. An effective window function is generated using equation (22). The K93 3pt data has largest spatial overlap with the 94/95 4pt data and is included there. These results are presented in Table 14 and Figure 8. The tendency for a rising spectrum is evident here though the scatter in the points, particularly at high $l$, is still considerable.

Finally, the data are combined into 5 angular scale bins and presented in Table 15 and Figure 9. To produce each of these bins, all of the data that are sensitive to a given range of angular scales is combined using the full correlation matrix (see § 7). For instance, the first bin is comprised of the 3pt to 6pt $\text{Cap}$ data from all 3 years. The effective window functions for each bin using equation (22) are presented in Figure 10.

Also presented in Table 15 are values for the angular spectral index $m$ (see § 9). The negative spectral index around $l \approx 80$, which was reported in Netterfield (1995) with the 1994 data alone, is again seen in the lowest angular bin. The lowest four points in Figure 8 also shows this locally falling spectrum. However, this trend is dominated by the 3pt data being high. Additionally, for each of the five angular bins, including the first one, the slope is consistent with 0. If the angular spectrum is locally falling, it appears to be only doing so at the 1 $\sigma$ level based on the determination of $m$. This issue will be resolved by extended sky coverage to reduce sample variance.

The CDM, $\Lambda + \text{CDM}$, and open model have been normalized to the COBE 2 yr data. The two free parameters in the PPI model have been adjusted to fit large-scale galaxy distributions while approximating measured COBE scale CMB anisotropies. The others have been normalized here to have the same value of $\sigma_T$ as the CDM model at $l = 10$. To accurately compare each of these spectra with the data, they must be convolved with the window functions in Fig. 10. The 14% overall calibration uncertainty is not included in the error bars. This affects the normalization of the spectrum, but not the shape.
While all known correlations have been taken into account in generating the likelihoods for each of the combined groups, there are still some correlations between groups. They are small (<0.2), however, and may be ignored for most analysis.

Not included in the quoted error bars are the effects of calibration and beam uncertainties. As discussed, the calibration uncertainty is 14% (1σ), mainly due to uncertainty in the temperature scale of Cas A. This has very little effect on the uncertainty of the spectral index, β, and no effect on

| Band | Bins | \( \Delta_{\text{inst}} \) (\( \mu K \)) | \( \Delta_{\text{sky}} \) (\( \mu K \)) | \( \Delta_{\text{tot}} \) (\( \mu K \)) | \( \sqrt{\langle W \rangle} \) | \( \delta T_l(\Delta_{\text{sky}}) \) (\( \mu K \)) | \( \sigma_{\text{inst}}^2 \) (\( \mu K^2 \)) | \( \sigma_{\text{sky}}^2 \) (\( \mu K^2 \)) | \( l_p \) | \( \delta T_l \) (\( \mu K \)) |
|------|------|---------------------------------|---------------------------------|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 3pt | 24 | 49 | 57 | 1.12 | 47 | 7 | 14 | 59±10 | 41±12 |
| 4pt | 24 | 85 | 53 | 0.93 | 57 | 15 | 8 | 81±12 | 12, <76 |
| 5pt | 24 | 55 | 22 | 0.77 | 28 | 12 | 9 | 100±13 | 17, <58 |
| 6pt | 48 | 75 | 2 | 0.67 | 0 | 24 | 9 | 108±13 | 25, <62 |
| 7pt | 48 | 109 | 45 | 0.59 | 77 | 33 | 10 | 135±14 | <76 |
| 8pt | 48 | 112 | 47 | 0.53 | 61 | 37 | 13 | 157±13 | 12, <120 |
| 9pt | 48 | 136 | 44 | 0.48 | 91 | 57 | 14 | 176±13 | <120 |
| 3pt | 24 | 101 | 78 | 63 | 1.29 | 49 | 7 | 13 | 64±12 | 65±14 |
| 4pt | 24 | 65 | 33 | 1.02 | 32 | 9 | 7 | 83±13 | 38±16 |
| 5pt | 24 | 62 | 44 | 0.81 | 55 | 7 | 9 | 108±13 | 54±10 |
| 6pt | 48 | 76 | 49 | 0.76 | 64 | 10 | 7 | 135±14 | 64±14 |
| 7pt | 48 | 74 | 52 | 0.67 | 78 | 8 | 9 | 158±13 | 72±12 |
| 8pt | 48 | 76 | 50 | 0.59 | 84 | 9 | 11 | 178±13 | 82±12 |
| 9pt | 48 | 69 | 43 | 0.54 | 79 | 9 | 12 | 197±13 | <76 |
| 10pt | 48 | 70 | 63 | 0.50 | 74 | 15 | 10 | 217±13 | <76 |
| 11pt | 48 | 63 | 31 | 0.46 | 66 | 14 | 11 | 237±13 | 83±15 |
| 12pt | 48 | 80 | 53 | 0.42 | 126 | 14 | 14 | 257±13 | <120 |
| 13pt | 48 | 78 | 50 | 0.40 | 127 | 15 | 14 | 277±13 | 115±10 |
| 14pt | 48 | 74 | 38 | 0.37 | 102 | 32 | 9 | 297±13 | 74±14 |
| 15pt | 48 | 78 | 28 | 0.35 | 79 | 29 | 14 | 316±13 | 124±14 |
| 16pt | 48 | 96 | 63 | 0.33 | 190 | 34 | 13 | 333±13 | 110±13 |
| 17pt | 48 | 95 | 78 | 33 | 0.31 | 172 | 44 | 12 | 357±13 | <197 |
| 18pt | 48 | 97 | 48 | 0.29 | 166 | 74 | 10 | 382±14 | <197 |
| 19pt | 48 | 99 | 40 | 0.27 | 150 | 40 | 10 | 404±14 | <197 |

Note.—Results from each synthesized beam from SK93, SK94, and SK95. The SK93 entries are from Netterfeld et al. 1995. The K94 and Q94 entries differ slightly from those found in Netterfeld 1994 due to the inclusion of east-to-west noise correlations in the analysis. \( \Delta_{\text{inst}} \) is the raw rms of the binned data. \( \Delta_{\text{sky}} \) is the rms predicted by the data covariance matrix, \( C_p \). \( \Delta_{\text{tot}} = \Delta_{\text{inst}} - \Delta_{\text{sky}} \) estimates the sky rms. \( \delta T_l(\Delta_{\text{sky}}) \) gives the band power inferred from \( \Delta_{\text{sky}} \) using eq. (16). \( \sigma_{\text{inst}}^2 \) estimates the contribution to the error bars due to the instrument noise and \( \sigma_{\text{sky}}^2 \) estimates the contribution to the error bar due to sample variance. In the Monte Carlo determination of \( \sigma_{\text{inst}}^2 \) and \( \sigma_{\text{sky}}^2 \), the amplitudes of the angular spectra given in Table 14 are used. The limits on \( l_p \) are for where the window function falls to \( e^{-1} \) of the peak value. The final column lists \( \delta T_l \) from the full likelihood analysis. The overall 14% calibration uncertainty is not included in the errors.
the shape of the angular spectrum. Rather this has only an
effect on the overall temperature scale. This large normalization uncertainty must be included in any comparison with theoretical predictions.

Beam uncertainties, as well as contributing to calibration errors, also produce uncertainties in the angular spectra. For uncertainties in the beam width along the elevation axis, the only effect is in the calibration (misestimating the beam solid angle in predicting the signal from Cas A) and has been included in that error bar. For uncertainties in the azimuth beam width the effect is a function of the weighting vectors used. For the 4pt synthesis, where the physical beam width is small compared to the width of the synthesized lobes, only the temperature scale is affected, as with the elevation beam. For the 13pt synthesis, however, the additional effect of misstating the beam overlap between points

TABLE 14
THE ANGULAR SPECTRUM GROUPED BY SPATIAL SYMMETRY

| Band    | $l_c$ | $\delta T_I$ (\mu K) |
|---------|-------|---------------------|
| 3pt Cap | 60    | 59.1 \pm 1.7        |
| 4pt     | 78    | 38.7 \pm 1.4        |
| 5pt     | 100   | 46.7 \pm 1.4        |
| 6pt     | 125   | 47.7 \pm 0.4        |
| 7pt     | 154   | 62.9 \pm 1.3        |
| 3pt Ring| 171   | 59.1 \pm 1.1        |
| 8pt Cap | 175   | 79.1 \pm 1.1        |
| 9pt     | 197   | 89.2 \pm 0.3        |
| 10pt    | 216   | 76.2 \pm 0.5        |
| 4pt Ring| 234   | 78.1 \pm 0.2        |
| 11pt Cap| 236   | 83.2 \pm 0.5        |
| 12pt    | 257   | 112.2 \pm 0.8       |
| 13pt    | 277   | 115.2 \pm 0.9       |
| 14pt    | 297   | 74.2 \pm 2.4        |
| 5pt Ring| 286   | 69.2 \pm 2.3        |
| 15pt Cap| 316   | 124.2 \pm 2.6       |
| 16pt    | 333   | 110.3 \pm 3.8       |
| 17pt    | 357   | 110.3 \pm 1.9       |
| 6pt Ring| 332   | 19.8, <90.0        |
| 18pt Cap| 382   | 88.8, <191.0       |
| 19pt    | 404   | 0.0, <163.0        |

Note.—Angular spectrum grouped by spatial symmetry. The 14% overall calibration uncertainty is not included in the errors.

Fig. 10.—Five-bin effective window functions. These have been generated from the window functions of the contributing effective antenna patterns using eq. (22).

very nearly cancels the effect on the calibration. The result of this is that the 2% azimuth beam uncertainty yields an additional 2% relative uncertainty between the 4pt and 13pt synthesized beam data. This effect is small for this experiment given the error bars on the spectrum and is ignored.

14. FOREGROUND CONTAMINATION

The angular spectra presented in the previous section are for the microwave sky at 40 GHz. There are, however, potential sources of foreground contamination. Two major classes are diffuse Galactic emission, and unresolved point sources. Limits may be placed on the level of both of these potential contaminants.

Potential sources of diffuse Galactic contamination include dust, synchrotron, and free-free emission. The level of dust contamination at 40 GHz and $l = 100$ has been estimated to be less than 2 $\mu K$ (Tegmark & Efstathiou 1995), which is far smaller than the signals measured here. Similarly, recent measurements of the amplitude of diffuse free-free emission based on Hz emission have been made for the Saskatoon observing region (Gaustad et al. 1995; Simonetti, Dennison, & Topsana 1996). These measurements place an upper limit of 6 $\mu K$ at 30 GHz for the SK93 3pt beam. Once again this is too small to account for the observed signals. An analysis of high latitude Galactic emission in the COBE 2 yr data places similar limits on both foregrounds at large angular scales (Kogut et al. 1996). An estimate of the amplitude of synchrotron emission can be made by extrapolating radio maps at 408 MHz (Haslam et al. 1982) and 1.4 GHz (Reich & Reich 1986) to 30 GHz assuming a temperature spectral index of $\beta = -2.8$. This predicts a signal of less than 5 $\mu K$ in $K_{93}$, which is much smaller than the measured signals. However, if the spectral index of the synchrotron emission varies spatially by $\delta \beta \approx 0.05$, then 30 $\mu K$ signals could be expected. The spectral index of these fluctuations would still be characterized by $\beta \approx -2.8$.

The most powerful discriminant against foreground contamination is based on the frequency spectrum of the fluc-
tations ($\beta$). In §11, for $l \approx 73$, $\beta$ was found to be $0.2 \pm 0.3$, which is inconsistent with the spectral index of dust ($\beta = 1.7$), free-free emission ($\beta = -2.1$), or synchrotron emission ($\beta = -2.8$). Additionally, since the angular spectrum for galactic emission falls as $\delta T \propto l^{-1/2}$ (Tegmark & Efstathiou 1995), it is very unlikely that the galaxy is a significant contaminant at any angular scale probed by this experiment.

Of more concern for several reasons is potential contamination by unresolved point sources. Firstly, the angular spectrum of a family of point sources is expected to rise as $\delta T \propto l$, so the $\beta$ limits placed at large angular scales are weaker at smaller angular scales. Additionally, since the mechanism for radiation varies between individual sources, the expected $\beta$ also varies, and in many cases is not accurately described by a simple power law. This is of particular concern since no flux-limited survey of the SK observing region has been performed over our frequencies (26–45 GHz). However, it is still possible to make some estimates of the level of contamination via comparison with MSAM1-92, source counts from the OVRO Ring experiment (Meyers, Readhead, & Lawrence 1993) at smaller scales and an extrapolation of existing point source surveys.

While $\beta$ varies from source to source, it is expected that over a sufficiently wide frequency range no foreground source will have a spectrum consistent with $\beta = 0$. Consequently, our comparison of SK95 with MSAM1-92, which yields a limit on $\beta$ of $0.1 \pm 0.2$ with a frequency baseline from 40 to 240 GHz, assures that the sky covered by both experiments is devoid of significant point contamination. However, this field only represents 8 deg$^2$ of the total of 200 deg$^2$ observed by SK95.

A second approach is to use results from other experiments and point source surveys. The OVRO Ring experiment has observed 96 fields at 20 GHz. They find $\delta T \approx 6 \pm 100 \mu K$ at $l = 2017$. Here we make the assumption that the signal is dominated by Poisson distributed unresolved point sources. Extrapolating in angular scale by $\delta T \propto l^2$ to $l = 404$, the angular scale of the 19pt data set, one arrives at an expected contribution of 21 $\mu K$. Since OVRO Ring was performed at 20 GHz, this number needs to also be scaled in frequency. A conservative estimate of the “typical” $\beta$ for a point source is that its flux is constant in frequency, (i.e., flat spectrum or $\beta = -2$). Thus, an estimate of the point source contribution at the smallest scales of the SK experiment based on OVRO Ring source counts is 10 $\mu K$ at 40 GHz.

As a final estimate of the level of point source contamination, the expected amplitude of known point sources in the observing region is extrapolated. Table 16 lists the

### Table 16

#### Bright Point Sources

| Source         | $v_s$ (GHz) | Flux at $v_s$ (Jy) | Spectral Index $\alpha$ | Flux at 40 GHz (Jy) | Reference |
|----------------|-------------|--------------------|-------------------------|---------------------|-----------|
| 0014 + 81      | 10.7        | 0.73               | 0.36                    | 1.2                 | S5        |
| 0210 + 860 (3C 61.1) | 30.0        | 1.0                | 0.2                     | 1.1                 | HR        |
| 0454 + 844     | 10.0        | 1.5                | (0.0)                   | 1.5                 | K         |
| 0615 + 820     | 10.7        | 0.86               | (0.0)                   | 0.86                | E, K      |
| 0740 + 82      | 10.7        | 0.65               | -0.47                   | 0.35                | HR, S5    |
| 1003 + 83      | 10.7        | 0.66               | -0.10                   | 0.58                | S5        |
| 1039 + 811     | 90.0        | 0.80               | (0.0)                   | 0.80                | IRAMc     |
| 1053 + 815     | 20.0        | 0.4                | 0.43                    | 0.54                | E, VLBIc  |
| 1050 + 812     | 10.7        | 1.1                | -0.1                    | 0.96                | K         |
| 1221 + 809     | 10.7        | 0.6                | +0.2                    | 0.78                | S5, VLBIc |
| 1637 + 8239 (NGC 6251) | 10.7      | 0.8                | -0.4                    | 0.47                | HR        |
| 1637 + 826     | 10.7        | 0.73               | -0.4                    | 0.43                | S5, VLBIc |
| 2342 + 821     | 30.0        | 0.8                | 0.2                     | 0.85                | HR, S5    |

**Note:** Listed are parameters for the brightest point sources used in the estimate of the level of point source contamination. The flux at 40 GHz is extrapolated from the measurement at $v_s$ by $S(v) = S(v_0^*)/v_0^{\alpha}$, where $\alpha = \beta + 2$. The measured value from the literature for $S(v_0^*)$ is chosen.

![Figure 11](image-url)  
*Fig. 11.—Estimated levels of foreground contamination at 40 GHz. The five-bin data from this experiment are given as asterisks (*). The estimated angular spectra for the various foreground sources are from Tegmark & Efstathiou (1995). The free-free angular spectrum has been renormalized for our observing region using a limit based on Hz emission (Gaustad et al. 1995; Simonetti et al. 1996). An upper limit for the normalization of the synchrotron spectrum has been estimated by extrapolating the 408 MHz Haslam map to 40 GHz for our observing region (Haslam et al. 1982; Wollack et al. 1993). The square boxes represent contributions to our data estimated by extrapolating existing point source surveys to 40 GHz (see text). Also indicated is an upper limit on sources at smaller scales based on the OVRO Ring experiment (Meyers et al. 1993) extrapolated from 20 GHz.*
brightest of these. In addition to the sources listed in Table 16, all of the sources in the S5 catalog (Kühr et al. 1981b) are used. Based on an analysis of IRAS faint sources, it is considered unlikely that a new population of sources exists within our observing frequencies (Condon, Anderson, & Broderick 1995). The spectral index from the literature is used to extrapolate to 40 GHz whenever possible. If a measured frequency spectral index is not known, the source is assumed to be of constant flux with frequency. A source map of the observing area is made, and the SK observing strategy simulated on it. The largest estimated contribution is to the 19pt data, and is \( \delta T_l = 35 \mu K \), which could be significant. However, from Table 13, the 19pt data yields a 163 \( \mu K \) upper limit with a maximum likelihood at 0 \( \mu K \). The estimated contribution to the 17pt data is 23 \( \mu K \), which would imply a 4% contribution to our 69 \( \mu K \) detection at these scales. The predicted signal is dominated by sources in a very large frequency range placed by the successful comparison with the MSAM1-92 experiment further reduce the limits on reionization history of the universe and, within a given theoretical framework, begin to place limits on cosmological parameters. A quantitative comparison with theory will be presented elsewhere.

Given the relative complexity of this analysis, it is important to mention that much of the analysis has been repeated with completely different programs by two of the authors to verify accuracy. The consistency of the naive A-based tests with the full likelihood analysis further strengthens our confidence in our programs and techniques. In addition, Andrew Jaffe and J. R. Bond at CIT have repeated and extended our analysis of the SK93 and SK94 data, again confirming our analysis.

To improve the discriminating power of this experiment aνer several things are needed besides reducing the size of the error bars. First of all, it must be confirmed at high \( l \). Additionally, a wider frequency coverage at smaller scales will improve discrimination against point sources. A better calibration will improve comparison with COBE, effectively increasing the range of angular scales over which a theory can be tested. It is also possible to make a map of the CMB anisotropy from these data. A map of an extended region of the sky with better signal to noise will be useful to distinguish between Gaussian and non-Gaussian models.

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The data, synthesis vectors, and covariance matrices are publicly available at http://pupgg.princeton.edu/~cmb/welcome.html.

REFERENCES

Bennett, C. L., et al. 1992, ApJ, 396, L7
Bond, J. R. 1995a, Astrophys. Lett. Comm., 2
Condon, J. J., Anderson, E., & Broderick, J. J. 1995, ApJ, 455, 2318
Crittenden, R. G., & Turok, N. H. 1995, Phys. Rev. Lett., 75, 14
De Benardis, P., et al. 1994, ApJ, 422, 33
Devin, M. J., et al. 1994, ApJ, 430, 1
Dragovan Vuk, M., Ruhl, J. E., Novak, G., Platt, S. R., Crone, B., Pernic, R., & Peterson, J. B. 1994, ApJ, 427, 67
Eckart, A., Witzel, A., Biermann, P., Johnston, K. J., & Simon, R. 1986, A&A, 168, 17
Edelson, R. A. 1987, AJ, 94, 1150 (VLBI Calibrators)
Ganga, R. A. 1995, private communication
Haslam, C. G. T., Salter, C. J., Stoffel, H., & Wilson, W. E. 1982, A&A, 47, 1
Herbig, T., & Readhead, A. 1992, ApJS, 81, 83
Jungman, G., Kamionkowski, M., Kosowsky, A., & Spergel, D. N. 1995, astro-ph/9507080
Kamionkowski, M., Spergel, D. N., & Sugiyama, N. 1994, ApJ, 426, L57
Kogut, A., et al. 1996, ApJ, 460, 1
Kühr, H., Pauliny-Toth, I. I. K., Witzel, A., & Schmidt, J. 1991b, AJ, 86, 854 (The SK survey)
Kühr, H., Wiltz, A., Pauliny-Toth, I. I. K., & Nauber, U. 1981a, A&A, 45, 367
Kühr, H., Pauliny-Toth, I. I. K., & Nauber, U. 1981a, A&A, 45, 367
Meyers, S. T., Readhead, A. C. S., & Lawrence, C. R. 1993, ApJ, 405, 8
Netterfield, C. B. 1995, Ph.D. thesis, Princeton Univ.
Netterfield, C. B., Rusor, N. C., Page, L. A., Wilkinson, D., & Wollack, E. J. 1995, ApJ, 455, L69
Page, L. A., et al. 1994, in CMB Anisotropies Two Years After COBE: Observations, Theory, and the Future, ed. L. Krauss (Singapore: World Scientific), 57
Peebles, P. J. E. 1994, ApJ, 419, L49
Peterson, J. B. 1994, ApJ, 427, 67
Proctor, J. R. 1995a, Astrophys. Lett. Comm., 2
Rao, J. S., et al. 1995, AJ, 109, 27, 823
S. T., Readhead, A. C. S., & Lawrence, C. R. 1993, ApJ, 405, 8
Netterfield, C. B., Rusor, N. C., Page, L. A., Wilkinson, D., & Wollack, E. J. 1995, ApJ, 455, L69
Page, L. A., et al. 1994, in CMB Anisotropies Two Years After COBE: Observations, Theory, and the Future, ed. L. Krauss (Singapore: World Scientific), 57
Peebles, P. J. E. 1994, ApJ, 419, L49
Richardson, D. M., et al. 1994, ApJ, 422, L37
Spergel, D. N., et al. 1995, ApJ, 426, L57
Steinhardt, P. J. E. 1994, ApJ, 419, L49
Perley, R. 1982, AJ, 87, 859 (VLA Calibrators)
Piccirillo, L., & Calisse, P. 1993, ApJ, 411, 529
Ratra, B., Banday, A. J., Górski, K. M., & Sugiyama, N. 1997, ApJL, submitted
Ratra, B., & Sugiyama, N. 1997, ApJL, submitted
Readhead, A. C. S., Lawrence, C. R., Meyers, S. T., Sargent, W. L. W., Hardebeck, H. E., & Moffet, A. T. 1989, ApJ, 346, 566
Reich, P., & Reich, W. 1986, A&AS, 63, 205
Simonetti, J. H., Dennison, B., & Topasna, G. A. 1996, ApJ, 458, L1
Smoot, G. F., et al. 1992, ApJ, 396, L1
Steppe, H., Liechti, S., Mauersberger, R., Koempe, C., Brunswig, W., & Ruiz-Moreno, M. 1992, A&AS, 96, 441 (IRAM Pointing Calibration List)
Sugiyama, N. 1995, ApJS, 100, 281
Tegmark, M., & Efstathiou, G. 1995, MNRAS, 281, 1297
Tucker, G. S., Griffin, G. S., Nguyen, H. T., & Peterson, J. B. 1993, ApJ, 419, L45
White, M., & Scott, D. 1994, in CMB Anisotropies Two Years After COBE: Observations, Theory, and the Future, ed. L. Krauss (Singapore: World Scientific), 254
White, M., Scott, D., & Silk, J. 1994, ARA&A, 319
Wollack, E. J. 1994, Ph.D. thesis, Princeton Univ.
Wollack, E. J., Devlin, M. J., Jarosik, N. C., Netterfield, C. B., Page, L. A., & Wilkinson, D. 1997, ApJ, submitted
Wollack, E. J., Jarosik, N. C., Netterfield, C. B., Page, L. A., & Wilkinson, D. 1993, ApJ, 419, L49
———. 1994, Astrophys. Lett. Comm., 35, 217