Manufacture of threads with variable pitch by using noncircular gears

L Slătineanu1, O Dodun1, M Coteață1, I Coman1, G Nagîț1 and I Beșliu2

1Machine Manufacturing Department, “Gheorghe Asachi” Technical University of Iași, Romania
2Department of Mechanics and Technologies, ”Ștefan cel Mare” University of Suceava, Romania

E-mail: slati@tcm.tuiasi.ro

Abstract. There are mechanical equipments in which shafts threaded with variable pitch are included. Such a shaft could be met in the case of worm specific to the double enveloping worm gearing. Over the years, the researchers investigated some possibilities to geometrically define and manufacture the shaft zones characterized by a variable pitch. One of the methods able to facilitate the manufacture of threads with variable pitch is based on the use of noncircular gears in the threading kinematic chain for threading by cutting. In order to design the noncircular gears, the mathematical law of pitch variation has to be known. An analysis of pitch variation based on geometrical considerations was developed in the case of a double enveloping globoid worm. Subsequently, on the bases of a proper situation, a numerical model was determined. In this way, an approximately law of pitch variation was determined and it could be taken into consideration when designing the noncircular gears included in the kinematic chain of the cutting machine tool.

1. Introduction
The threads are considered usually as surfaces obtained by moving a certain profile along a spiral placed on a surface of revolution. The threaded surfaces have a large extent in mechanical equipment, being used in order to clamp together distinct parts, to transform rotation movement in a rectilinear movement or generally to move slides etc. In all such situations, threaded surfaces with constant pitches are frequently used, but there are also cases in which surfaces threaded with variable pitch (figure 1) are necessary. Thus, threads with variable pitch are included in some special pumps, solutions of propellers etc.

In order to manufacture threaded surfaces with variable pitch, some machining methods could be applied. A solution proposed some decades ago is based on some changes in the common kinematic chain of threading by turning or milling, so that the rotation speed of the lead screw changes just during the classical threading method by cutting.

If computer numerical control subsystems are available, there is the possibility to obtain threaded surfaces with variable pitch by using special instructions of numerical control; for example, such a function is G34 [6]. This function ensures a work movement of the cutting tool after a pre-established trajectory so that a variation of the pitch is achievable.

As above mentioned, if the classical kinematic threading chain is examined, a solution of changing the rotation speed of lead screw could be based on the use of a pair of noncircular gears. It is known that the gear ratio depends on the ratio of the instantaneous radiiuses of the gears in the zone of
gearing. If the radii of such zones change, the ratio between the numbers of revolutions per minute of the gears changes also and a variable rotation speed will correspond to the lead screw.

If computer numerical controlled machine tools are not available, the manufacture of threaded surfaces with variable pitch is generally considered as being a difficult technological problem and there are not many solutions proposed in order to solve such a problem.

However, over the years, there are known researchers preoccupations directed to finding solutions for obtaining threaded surfaces with variable pitch by cutting.

Thus, Loveless developed an investigation concerning the design and manufacture of the components specific to a double enveloping worm gear [4]. He used hobbing as a machining method for obtaining the worm gear, but did not discuss the possibility to use a variable pitch for the cutter.

Lunin showed that a globoid gearing is characterized by a driving efficiency higher with 6-10% on a 25:1 ratio and an increase of the loading capacity with about 30% in relation with the traditional worm gearing [5]. He described the so-called wormoid gear, able to improve the performances of the globoid transmissions.

In a short information concerning the globoid worm gearing, the authors of the paper [7] highlighted that the accurate machining of the worm and gear is a difficult technological problem. They showed that during the grinding grooves from the worm, the grinding disc has to continuously change the inclination angle and this generates real difficulties in the industrial practice.

Approaching the problem of efficiency of the globoid gearing and developing an experimental research, Cotetiu noticed a decrease of the efficiency when the worm diameter increases and when inclination angle of the spiral diminishes [1].

![Figure 1. Shaft zone including a threaded surface with variable pitch.](image)

In 1969, Kucher and Ershov published a paper in which they described an improved screwcutting lathe adapted for machining variable pitch screw [3]. Essentially, they changed the speed of the work movement achieved by cutting tool by using a cam rotated, on its hand, by means of a gear found in a rolling movement on the lathe rack.

The objective of this paper was to present the results of an investigation concerning the variation of the pitch in the case of the worm included in the double enveloping worm gearing, in order to facilitate the identification of technological solutions for its manufacturing.

2. Premises for using noncircular gears in the threading kinematic chain of cutting machine tools

The kinematic chains of cutting machine tools include a possibility to ensure an accurate correspondence between the workpiece rotation movement and the rotation speed of the lead screw which moves the slide supporting the cutting tool (figure 2).

If the problem of changing the rotation speed of the lead screw just during the turning process is formulated, a possibility is offered by the introduction of a pair of noncircular gears in the threading kinematic chain (figure 3).

In the case of the kinematic scheme presented in figure 3, one supposed that the variable pitch could need a single rotation of the lathe lead screw and this means that the length of the shaft threaded zone is equal to the lead screw pitch. If the length of the shaft threaded zone is higher than the pitch of the lathe lead screw, the noncircular gears could be placed in the group of changeable gears.
Figure 2. Kinematic chain corresponding to cutting threaded surface with constant pitch on a lathe.

One can mention that the variation of rotation speed corresponding to the lead screw could be achieved in accordance with distinct mathematic laws. For example, a linear variation of the rotation movement speed of the lead screw could be determined by using noncircular gears with a profile corresponding to an Archimedes spiral.

It is known that in the case of such a spiral, the radius $\rho$ increases linearly at the increase of the rotation angle $\omega$ and a linear variation of the gearing ratio and of the lead screw rotation speed could be thus obtained:

$$\rho = k \omega,$$

where $k$ is a proportionality constant.

Figure 3. Kinematic chain corresponding to obtaining a threaded surface with variable pitch on a lathe by using noncircular gears.

But the mathematical law of the variable pitch and of the lead screw rotation speed could be distinct in relation to the linear variation law and in such a case this mathematical variation law must be known.
3. Hypothesis concerning the pitch variation in the case of a double enveloping globoid worm

The worm gears are gears able to transmit the rotation motion between two shafts whose axes are not coplanar and the projection of the axes in a plane parallel with them highlights an intersection at 90 degrees.

If the shapes of the gear and of the worm are analysed, one can find that there are worm gearings that have neither element throated, that have one element throated (usually the gear), and that have both elements throated (this is the case of double enveloping worm gearing) [4].

The last type of gearing is able to transmit high torsion moments, but its energetic efficiency is relatively low, since a significant part of energy is lost by heating. The specialty literature mentions also that there are technological difficulties in manufacturing the double enveloping worms [2].

If the double enveloping worm gearing is analysed, one can notice that a correct worm could have a variable pitch. Indeed, if the projections of the mean points of the gaps between each two teeth of the gear on the worm axis are examined, one can find that the distance between the projection points is variable and this means that the worm could have a variable pitch.

When the problem of worm manufacture is formulated, the mathematical variation law of the pitch must be determined.

An approximately solution for determining this law could be based on the successive examination of the sizes corresponding to the projections of the mean points placed in gaps between two consecutive teeth found on the pitch circle of the gear on the worm axis (figure 5). If we take into consideration the situation presented in figure 5, one could suppose that the circular pitch on the gear corresponds to an angle $\alpha_1$. In such a case, the pitch of the worm corresponding to the angle $\alpha_1$ is equal to the line segment $AB$. Geometrical conditions valid in the rectangular triangle $CAB$ (figure 5) leads to the following equation for the first pitch $p_1$:

$$p_1 = R_d \cos \frac{2\pi}{z},$$  \hspace{1cm} (2)

where $z$ is the number of gear teeth and $R_d$ is the radius of the gear pitch circle.

In the case of the second gap, the equation will be:

$$p_2 = R_d \cos \left(2\frac{2\pi}{z}\right) - p_1.$$  \hspace{1cm} (3)
Similarly, in the case of the third gap, one can write:

\[ p_3 = R_d \left( \frac{3 \pi}{z} \right) - p_1 - p_2. \]  (4)

By extending this way of writing the relations valid for the gaps, the following general equation could be written:

\[ p_n = R_d \left( \frac{3n \cdot \pi}{z} \right) - p_1 - p_2 - \ldots - p_{n-1}. \]  (5)

If one considers a proper example, let take into consideration a gear for which the diameter of the pitch circle is \( D_d = 183.346 \) mm and the teeth number is \( z = 36 \). In such a case, for the angle \( \alpha \) one has the value \( \alpha = 360/36 = 10^\circ \).

Using the general relation (5), the consecutive values for the pitch \( p_i \) were calculated and inscribed in table 1.

| Pitch No. | 1   | 2   | 3   | 4   | 5   | 6   |
|-----------|-----|-----|-----|-----|-----|-----|
| Pitch size| 15.918 | 15.436 | 14.482 | 13.090 | 11.299 | 9.166 |

**Table 1.** Values for the first five pitch of the worm.

![Figure 6. Decrease of the pitch size on the worm.](image)

By mathematical processing of the information included in table 1 by means of the Excel program, the following regression function was identified as corresponding to these results:

\[ p = -0.2072 i^2 + 0.0941 i + 16.054, \]  (6)

for which the determination coefficient is \( R^2 = 0.9999 \).

The last mathematical relation highlights the variation of the pitch along the worm axis, from its central zone, in accordance with the graphical representation from figure 6.

This relation could be used when the profile of the noncircular gears must be designed.

4. Conclusions

One of the methods of manufacturing threaded surfaces with variable pitch is based on the use of noncircular gears placed in the threading kinematic chain of the classical lathe. In order to design the
profile of the noncircular gears, the mathematical law of pitch variation has to be known. The work zone of the double enveloping globoid worm included in worm gearing could be considered as a threaded surface with variable pitch. A graphical representation and geometrical considerations were used in determining an approximately law of pitch variation in such a case. A proper example was used in order to illustrate the variation of the pitch along the axis of the double enveloping globoid worm and to identify the mathematical law for approximation the variation of the pitch.

5. References
[1] Coteți R 2002 Scientific Bulletin, Fascicle Mechanics, Tribology, Machine Manufacturing Technology, Series C. XVI 53
[2] Elekes C 1985 Tools for worms and worm gears (in Romanian) (București: Publishing house Litera)
[3] Kucher I M and Ershov B A 1969 Equipment Manufacturing Technology Chemical and Petroleum Engineering 5(8) pp 630-633
[4] Loveless W G 1984 Cone Drive Double Enveloping Worm Gearing Design and Manufacturing Gear technology Oct-Nov
[5] Lunin S V 2007 Globoid gear technology from ZAKGEAR, available: http://www.zakgear.com/Wormoid.html, accessed: 10.03.2016
[6] Mach4 CNC Controller Lathe Programming Guide Version 1.0, available: http://www.machsupport.com/wp-content/uploads/2014/05/Mach4%20Lathe%20GCode%20Manual.pdf, accessed: 12.04.2016
[7] Uniprom SRL Globoidale worm gears (in Romanian) 2009, available: http://uniprom.ro/produse_angrenaje_melcute_globoidale.html, accessed: 10.03.2016