TRANSVERSE $\Lambda^0$ POLARIZATION IN INCLUSIVE QUASI-REAL
PHOTOPRODUCTION:
QUARK SCATTERING MODEL

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The transverse polarization of $\Lambda^0$ hyperons produced in the inclusive $ep$ reaction at the 27.6 GeV beam energy is assumed to appear mostly via scattering of the strange quark in a color field. Results of application of such an idea to the preliminary data of HERMES are presented. Contributions of $\Sigma^0$, $\Xi$, and $\Sigma'$ resonances to the polarization are taken into account.

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I. INTRODUCTION

Polarization of $\Lambda^0$ hyperons has attracted a lot of experimental as well as theoretical activity almost since the very moment of its discovery. Investigations of the phenomenon received a specially great impetus in 1976 due to the striking experimental results obtained at FERMILAB, where the hyperons produced in $pN$ collisions at a 300 GeV proton beam energy were highly polarized \[\mathbb{1}\]. The polarization was transverse and negative, directed opposite to the unit vector \[\mathbf{n} \propto |\mathbf{p}_b \times \mathbf{p}_\Lambda|\], where \[\mathbf{p}_b\] and \[\mathbf{p}_\Lambda\] are the beam and hyperon momenta, respectively.

Only this direction is allowed by the parity conservation in strong interactions provided the incident particles are unpolarized. The results turned out to be in disagreement with the expected negligible polarization in high energy processes as the helicity is conserved in the limit of massless quarks (hereafter, let us imply under polarization just transverse one).

The polarization has also been observed in a variety of other hadron-hadron reactions at different kinematic regimes. Its features qualitatively coincide in almost all the reactions, for instance, being insensitive to the incident particle energy, exhibiting the roughly linear growth with the hyperon transverse momentum \[p_T\] and being negative. The only known exception is the \[K^-p\] process, where the polarization sign has been found to be positive.

The $\Lambda^0$ wave function facilitates to some extent the theoretical study. The SU(6) symmetry requires the spin-flavor part of the wave function to be combined of \[ud\] diquark in a singlet spin state and strange quark of spin \[1/2\], or formally \[|\Lambda\rangle_{1/2} = |ud\rangle_0|s\rangle_{1/2}\], where the subscripts denote the spin states. Thus, one might entirely attribute the \[\Lambda^0\] polarization to its valence strange quark.

Certainly, there have been proposed many approaches attempting to account for the results (see, for example, reviews \[\mathbb{2}, \mathbb{3}, \mathbb{4}\] and the references therein). However there is still no model, which would be able to describe the complete set of the available measurements.

According to the empirical rules proposed by DeGrand and Miettinen, the polarization sign depends on whether the \[s\] quark is accelerated (increases its energy) or decelerated (decreases its energy) in the \[\Lambda^0\] formation process \[\mathbb{5}\]. To illustrate, there are no valence \[s\] quarks in the initial state of the \[pp\] reaction so that they come from the quark sea to form the final \[\Lambda^0\]. But the sea quarks predominantly populate small \[x\]-states and consequently increase their average energy coming in the valence content of \[\Lambda^0\] (\[x\] is Bjorken variable). Here the polarization is negative. Contrary, incident pseudoscalar kaons of the \[K^-p\] reaction already contain valence strange quarks, which are mostly decelerated in the hadronization process. In this case, the sign is positive. Similar ideas were implemented in flux-tube models with orbital angular momentum \[\mathbb{6}\].

The polarization in photoproduction has been investigated, for example, in experiments on high energy $\gamma N$ scattering performed at SLAC \[\mathbb{7}\] and CERN \[\mathbb{8}\]. However, statistical accuracy of the experiments is indecisive and would hardly enable one to conclude on the magnitude or on the sign of the polarization.

In light of the scarce statistics for the \[\Lambda^0\] photoproduction, the HERMES experiments on the 27.6 GeV positron beam scattering off the nucleon target acquire a particular status providing a good opportunity for observation of the polarization in electroproduction. The collaboration has preliminary measured nonzero positive transverse polarization \[\mathbb{9}\], when most of the intermediate photons are quite close to the mass shell, i.e. \[Q^2 = -(p_{ei} - p_{ef})^2 \approx 0\] GeV$^2$, where \[p_{ei,f}\] are the 4-momenta of the initial and scattered positrons, respectively (quasi-real photoproduction).

Experimental properties of the polarization at HERMES turned out to be reminiscent of those in the \[K^-p\] reaction \[\mathbb{10}\], which have been successfully described by a model assuming the polarization to appear mostly via

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strange quark scattering in a color field \[11, 12\].

These arguments inspired us to apply the model to the data preliminary obtained by HERMES. In order to estimate the contributions of \( \Sigma^0, \Xi, \) and \( \Sigma^* \) resonances into account, we have generated the \( ep \) process by PYTHIA 6.2 program \[13\].

II. QUARK SCATTERING MODEL FOR \( \Lambda^0 \)

It has been known that electrons, when scattering off nuclei, are able to get polarization. Theoretically, it can be derived in QED by considering a process of Dirac pointlike particle scattering from static Coulomb potential provided next-to-leading order amplitudes are taken into account \[14\] \[15\]. The corresponding formula reads

\[
P = \frac{2\alpha_{em}mp}{E^2} \frac{\sin \theta/2 \ln[\sin \theta/2]}{[1 - p^2/E^2 \sin^2 \theta/2 \cos \theta/2]} (\mathbf{n} \times \mathbf{p_i} \times \mathbf{p_f}),
\]

where \( P \) is the polarization vector, \( E, p, m \) and \( \theta \) are the energy, momentum by magnitude, mass and scattering angle of the electron, respectively, \( \alpha_{em} \) is the fine structure constant, \( \mathbf{n} \propto [\mathbf{p_i} \times \mathbf{p_f}] \), \( \mathbf{p_i} \) and \( \mathbf{p_f} \) are the vectors of the electron momenta in the initial (\( i \)) and final (\( f \)) states, respectively.

In \[11\], Szwed proposed to consider the \( \Lambda^0 \) polarization as polarization of its valence strange quark using Eq. \( (1) \). In other words, one should perform the following interchanges in Eq. \( (1) \): electron \( \leftrightarrow \) quark, Coulomb potential \( \leftrightarrow \) color field \( \alpha_{em} \leftrightarrow C\alpha_s \), where \( \alpha_s \) is the strong coupling and \( C \) is the color factor.

This approach has been applied to describe the polarization in the \( K^-p \) reaction and successfully reproduced its main features at \( 2C\alpha_s=5.0 \) (while its theoretical value was found to be 2.5) and the \( s \) quark mass \( m_s=0.5 \) GeV \[16\].

We have expressed the model in terms of the variable \( \zeta \), the HERMES data depend on

\[
\zeta(f) = \frac{E_i(f) + p_{z}(f)}{E_b + p_{zb}},
\]

where the index \( b \) refers to the beam, the \( z \) axis defines the beam direction. Note that \( \zeta \) is invariant under Lorentz boosts.

According to recipes given in \[12\], one should move to a frame, where the magnitudes of the initial and final \( s \) quark momenta are the same (originally called \( S \)-frame). It is reached by performing a Lorentz transformation along the proton momentum. For this purpose one can write

\[
(p_i \cdot p_f) = p^2(1 - \cos \theta) + m^2_s,
\]

\[
p_{Tf} = p_T = p \sin \theta,
\]

where \( p_i, f \) are the 4-momenta of the scattering quark, \( p_{Tf} \) is its transverse momentum in the center-of-mass frame of the \( K^-p \) reaction, while \( p = \sqrt{E^2 - m^2_s} \), \( p_T \) and \( \theta \) refer to the \( S \)-frame.

On the other hand, using Eq. \( (2) \) leads to

\[
(p_i \cdot p_f) - m^2_s = \frac{m^2_s (\zeta_i - \zeta_f)^2}{\zeta_i},
\]

\[
\frac{1}{2} \left( \frac{p^2_{Tf} \zeta_i}{\zeta_i} + \frac{p^2_{Tf} \zeta_f}{\zeta_f} \right) + \left( p_{Tf} \cdot p_{Tf} \right),
\]

where \( (p_{Tf} \cdot p_{Tf}) \) denotes the ordinary scalar product of the transverse momentum vectors.

Assuming that \( p_{Tf}=0 \), after some algebra, one can obtain from Eqs. \( (3) \)-(5) that

\[
\cos \theta = \frac{\zeta V^2}{2(1 - \xi^2) + V^2},
\]

\[
V = \frac{(1 - \xi)^2 + V^2}{2\xi \sqrt{(1 - \xi^2)^2 + (1 - \xi)V^2}},
\]

where \( V(V_T) \) and \( \xi \) are defined by

\[
V(T) = \frac{p_T}{m_s}, \quad \xi = \frac{\zeta_f}{\zeta_i}.
\]

Using relations \( (6) \) and \( (7) \), one can rewrite Eq. \( (1) \) as

\[
P(\xi, V_T) = \frac{2C\alpha_s V}{1 + V^2 \cos^2 \theta/2} \frac{\sin^3 \theta/2 \ln[\sin \theta/2]}{\cos \theta/2} \text{sign}(\xi-1),
\]

where the rules of DeGrand and Miettinen are expressed by the factor \( \text{sign}(\xi-1) \). It originates from the unit vector \( \mathbf{n} \) in Eq. \( (1) \) as its direction depends on the quark source (e.g., see \[11\]). Actually, \( \xi \) is the longitudinal light cone momentum fraction of the initial quark carried by the scattered one. Therefore, when the quark is decelerated, i.e., \( \xi < 1 \), the polarization defined by Eq. \( (9) \) is positive, while it is negative at \( \xi > 1 \). Note that there is a kinematic restriction in this approach, imposed by Eq. \( (9) \), since \( \cos \theta/2 \leq 1 \). It forbids the variable \( \xi \) to take values in the following interval

\[
1 < \xi < 1 + V^2,
\]

but the situation can be improved by introducing continuous \( \zeta_{i(f)} \) distributions of the quarks, for example.

Plots of the polarization defined by Eq. \( (9) \) versus \( V_T \) (equivalently \( p_T \)) for a few fixed values of \( \xi \) at \( 2C\alpha_s = 5.0 \) are shown in Fig. \[1\]. The linear growth, representative for hadron-hadron reactions, is seen well on the upper panel for both the \( K^-p \)-like (\( \zeta = 0.2 \), solid line) and \( pp \)-like (\( \xi = 10 \), dashed line) events. On the lower panel, we can see that for \( \xi = 0.8 \) (solid line) the polarization reaches a plateau, while for \( \xi = 1.4 \) it grows taking maximal value by magnitude at \( V_T \approx 0.4 \) and then smoothly falls to zero.

A plot of the polarization versus \( \xi \) for \( V_T=0.5 \) at \( 2C\alpha_s = 5.0 \) is demonstrated in Fig. \[2\]. It is also seen

\[\text{Fig. 2.}\]
that for $\xi < 1$ the polarization is positive and linearly grows peaking at $\xi \approx 0.75$, afterwards it steeply falls down to zero. For $\xi > 1.25$, it is negative, quickly growing by magnitude up to $\xi \approx 1.6$, afterwards decreasing very slowly. The restricted area according to Eq. (10) is outlined as well.

III. CALCULATIONS AND RESULTS

The preliminary results of HERMES on the polarization have qualitatively the same properties as those of the $K^-p$ process [9]. It is positive and similarly depends on $p_T$ in both the $K^-p$ and $ep$ reactions suggesting that closely related underlying physics may be responsible for such a picture. These similarities encouraged us to assume that quark degrees of freedom of the positron beam might play significant role in the polarization. Strange quarks may originate from the projectile like the valence those in the incident kaons.

Thus, we have straightforwardly applied the model from [12] to the quasi-real photoproduction. In order to do it, following assumptions were made.

1) Since no information on the momentum of the initial quarks is available in the experiment, it was assumed that the lepton beam provides a collinear quarks with the $\zeta_i$ distribution, the free parameters being $2C_{\alpha s}$ and $m_s$.

2) The final $s$ quark kinematic was determined as

$$\zeta_f = \frac{m_s}{m_A} \zeta, \quad V_T = \frac{p_T}{m_A},$$

(11)

here $\zeta$ and $p_T$ refer to the detected $\Lambda^0$s.

It is evident that $\Lambda^0$s detected in experiments are produced not only via direct processes but may appear indirectly as decay products of heavier hyperons, contributions of the latter to the polarization are presumably considerable. Hence for more adequate describing the process, one should consider such a possibility.

To take possible contributions of $\Sigma^0$, $\Xi$ and $\Sigma^*$ resonances into account, we used events generated by the PYTHIA 6.2 package [13]. For this purpose, we partially reproduced the HERMES acceptance imposing the following limits

$$p_A > 4.35 \text{ GeV}, \quad \left| \frac{p_{T\Lambda}}{p_{z\Lambda}} \right| > 0.15, \quad 0.02 < \left| \frac{p_{y\Lambda}}{p_{z\Lambda}} \right| < 0.14$$

(12)

Additionally, to reflect in the calculations, at least qualitatively, the phenomenological $\xi$ distribution, we selected only events with $\xi > 0.52$.

The indirect process contributions to the polarization were estimated in three steps schematically illustrated in Fig. 3.

First, all the quarks ($q = u, d, s$) originating from the intermediate photon were assumed to get polarization in the scattering process. We calculated the corresponding polarizations using Eqs. (9) - (10).

Second, $q \to H$, $H = \Theta^0$, $\Xi$, $\Sigma^*$, i.e., having been polarized, the quarks combined into the heavier resonances...
TABLE I: Fragmentation spin transfer factors within the framework of the SU(6) symmetry.

| q  | Λ^0 | Σ^0 | Ξ^0 | Ξ^- | Σ^- |
|----|-----|-----|-----|-----|-----|
| u  | 0   | 2/3 | 0   | -1/3| 5/9 |
| d  | 0   | 2/3 | -1/3| 0   | 5/9 |
| s  | -1/3| 2/3 | 2/3 | 5/9 |

transferring them some fraction of the initial polarization. The fractions are defined by the SU(6) fragmentation spin transfer factors \( t_{H,q}^F \) given in Tab. 1 [11].

Third, \( H \to \Lambda^0 \), i.e., the polarized resonances decay into \( \Lambda^0 \) transferring, in turn, to the final hyperon some fraction of their polarization. The fractions are defined by the decay spin transfer factors \( t_{\Lambda,H}^D \), which are taken to be for \( \Sigma^0 \to \Lambda^0 \gamma \), \( t_{\Lambda,\Sigma^0}^D = -1/3 \), for \( \Xi \to \Lambda \pi \), \( t_{\Lambda,\Xi}^D = 0.91 \) and for \( \Sigma^* \to \Lambda \pi \), \( t_{\Lambda,\Sigma^*}^D = 0.93 \) [17, 18]. Of course, in general, there must be included another decay modes, such as \( \Sigma^* \to \Sigma^0 \pi \to \Lambda \gamma \pi \), \( \Omega^- \to \Lambda K^- \), \( \Omega^- \to \Xi \pi \to \Lambda \pi \pi \), but they give very small contribution, which was neglected.

Putting together the three points above, the total contribution of the indirect processes can be formally written as

\[
P = \sum_{q,H} t_{H,q}^F t_{\Lambda,H}^D P_q, \tag{13}
\]

where \( P_q \) is the polarization of a quark of flavor \( q \) to be determined by Eqs. (11-13). A detailed explanation of similar calculations can be found in [18], for instance.

Having chosen the model parameters as \( 2C\alpha_s = 2.5 \), \( m_u = m_d = 0.33 \) GeV and \( m_s = 0.5 \) GeV, we carried out the calculations. The polarization dependence on \( \zeta \) calculated including the contributions of \( \Sigma^0, \Xi \) and \( \Sigma^* \) (scattered plot) is shown in Fig. 4. We can see a reasonable reproduction of the HERMES data (solid points).

The \( p_T \) dependence of the polarization calculated including the contributions of \( \Sigma^0, \Xi, \Sigma^* \) (scattered plot in comparison with the preliminary HERMES data (solid points) is shown in Fig. 5. Here the numerical results sufficiently reproduce the experiment as well.

IV. CONCLUSION

In this paper, we have calculated transverse \( \Lambda^0 \) polarization in the \( ep \) reaction at the 27.6 GeV lepton beam energy within an approach, which has been successfully applied to the \( K^-p \) reaction [12]. As the preliminary HERMES data [9] turned out to be qualitatively similar with those of \( K^-p \) [10], we assumed that the underlying polarization mechanisms of both the \( ep \) and \( K^-p \) reactions could be similar as well. Thus we supposed the \( \Lambda^0 \) polarization to originate mostly in scattering of quarks coming from the positron projectile. Recalling that the quasi-real photon exchange dominated in the HERMES experiment, the said above might suggest quark degrees of freedom of the photon to play considerable role in the polarization process.

The quark scattering model [11, 12] has been expressed in terms of the light cone variable \( \xi \) the HERMES data depend on. On the other hand, a kinematic restriction for values of \( \xi \) defined by Eq. (10) was imposed. One can avoid the problem introducing a continuous \( \xi(f) \) distribution as it has been done by PYTHIA. A sufficient reproduction of the HERMES data has been reached, the contributions from \( \Sigma^0, \Xi \) and \( \Sigma^* \) have been taken into account. For this purpose, we also used the PYTHIA program. All our results should be regarded only as qualitative.
The largest difficulty of this model is the parameter $2\alpha_s$ since the strong coupling is running, additionally it was impossible to derive $\alpha_s$ from the HERMES data.

As a further development, another sources, such as gluon-gluon fusion etc. [19], can be included and treated in the same way. It would be also interesting to compare our results with those recently obtained in the framework of the quark recombination approach [20], they show negative polarization when $p_T$ varies from about 0.2 GeV up to about 0.6 GeV for $\zeta > 0.25$.

When this work had been already completed, the HERMES collaboration published new data on the $\Lambda^0$ polarization [21], which are also in well qualitative agreement with the calculations presented herein.

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