Signature of the Chiral Anomaly in Ballistic Magneto-Transport

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We compute the magneto-conductance of a short junction made out of a Weyl semi-metal. We show that it displays quantum oscillations at low magnetic field and low temperature, before reaching a universal high-field regime where it increases linearly with the field. Besides identifying a new characterization of the physics of materials with Weyl singularities in their energy spectrum, this ballistic regime corresponds to the simplest setup to study and understand the manifestation of the so-called chiral anomaly of massless relativistic particles in condensed matter. At low fields the algebraic in field magneto-conductance incorporates contributions besides the anomalous chiral current while the linear conductance at higher fields constitutes an unambiguous signature of the chiral anomaly in a Weyl conductor. Finally, we study the dependence of the ballistic magneto-conductance on the chemical potential, and discuss the cross-over towards the diffusive regime when elastic scattering is present.

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I. INTRODUCTION

When relativistic particles in three dimensions become massless, they acquire an additional symmetry: chirality. However, while this chirality is a conserved quantity of the Hamiltonian, it is no longer conserved by the associated field theory. This very intriguing and unanticipated property was called a chiral anomaly, or Adler-Bell-Jackiw anomaly [1, 2]. Much efforts have been devoted to identifying measurable consequences of this anomaly. In particular, it was realized that, in the presence of a magnetic field, a difference of chemical potential between particles with opposite chiralities induces a chiral current: the chiral magnetic effect [3].

In recent years, the prospect of probing consequences of the chiral anomaly, which was initially discussed in the context of high energy physics, through the transport properties of solids has brought the study of this effect in the realm of condensed matter physics. Indeed, in several materials electrons behave as relativistic particles. Examples include Weyl and Dirac semi-metals, in which the Fermi energy lies close to a linear crossing between energy bands. The study of such crossing is not new [4]. However, it has been completely revived by the study of topological properties associated with these band crossings in the fields of Helium physics [5], strongly correlated iridates [6], and, more generally, the topological band theory of materials [7]. The search for manifestations of the chiral anomaly was associated with the prediction of the positive magneto-conductance in materials with linear band crossing. Indeed, the presence of both electric E and magnetic B fields induces a chiral current proportional to E.B and modifies the associated electromagnetic response of the massless relativistic particles. This chiral anomaly was discussed in the context of solids by Nielsen and Ninomiya who considered a lattice realization of a Weyl semi-metal [8] (see also Ref. [9]. They related a positive magneto-conductance along B in the quantum regime to the anomalous Landau level in the presence of inter-cone scattering processes. Recently, the anomalous response of Weyl particles to weak electromagnetic fields, which was analyzed by taking into account Berry curvature corrections in the semiclassical equations of motion, was also attributed to the chiral anomaly [10, 11]. This recent progress is reviewed in Ref. [12]. The above cited works characterized the linear response of bulk electrons submitted to both electric and magnetic fields. Alternatively, recent proposals considered the so-called chiral magnetic effect induced by an oscillating magnetic field or chiral chemical potential in a junction [13, 15].

On the experimental side, the magneto-conductance was studied in various materials identified as candidates for this relativistic physics, including Cd₃As₂ [16], Na₃Bi [17], TaAs [18], NbP [19]. In these compounds, the positive magneto-conductance for a parallel magnetic field, as well as its extreme sensitivity to the orientation of the magnetic field with respect to the electric field, were taken as manifestations of the underlying chiral anomaly of the relativist massless equations of motion. More recently, the prospect of probing the chiral anomaly in the so-called hydrodynamic regime, in which elastic scattering of electrons is dominated by the Coulomb interaction, has led to an experimental study of the magneto-conductance of WP₂ [20]. In this material, an elastic mean free path of the order of 100 μm at 4K, that is, larger than the sample size, was detected. While scattering was claimed to be dominated by interactions between electrons, lowering even further the temperature in such a sample would drive the conductor into the so-called ballistic regime.

The purpose of the present work is to study the longitudinal magneto-conductance of ballistic junctions made with Weyl semi-metals in the regime where equilibra-
tion takes place only in the leads. This corresponds to so-called cold electrons, as opposed to hot electrons when energy relaxation occurs within the conductor. We find that two regimes must be distinguished: (i) a regime in which the conductance behaves algebraically at low fields and displays quantum oscillations at higher fields (ii) a quantum regime reached for magnetic fields such that $Ba^2_F \geq \hbar/e$ where $a_F$ is the Fermi wavelength, in which the conductance is linear in field with a universal slope. Owing to the conceptual simplicity of the ballistic regime we argue that only the magneto-conductance in the quantum regime is an unambiguous signature of the chiral anomaly, and can be experimentally accessed for a chemical potential close enough to the band crossing.

In Sec. [I] we determine analytically the ballistic magneto-conductance at zero temperature for the simplest model of two well-separated (in momentum space) Weyl cones with opposite chiralities. We find that the magneto-conductance displays quantum oscillations as the magnetic field increases, similar to Shubnikov-de Haas oscillations, but along the direction of the field. These quantum oscillations evolve into a robust linear magneto-conductance at large field. On the other hand, we discuss how the low-field oscillations are suppressed by the temperature, or in the presence of a broadening of the Landau levels due to their coupling with the leads. In Sec. [II] we compare our predictions with the numerics for a tight-binding model with four pairs of Weyl cones. We obtain a good agreement with the analytics at low chemical potential, when Weyl cones are well separated in momentum space. On the other hand, we find that the slope of the linear magneto-conductance at large magnetic field decreases by a factor two as the chemical potential increases above the saddle-point energy corresponding to the merging of two Weyl points with opposite chirality. We explain this effect as the signature of the asymmetry of the magneto-conductance with the magnetic field for a single pair of Weyl cones, as the chemical potential is tuned away from the band crossing. The symmetry is actually recovered in the specific model with eight cones, due to their distribution in momentum space. We confirm this interpretation with numerics for a tight-binding model with a single pair of Weyl cones. In Sec. [IV] we analyze the magneto-conductance within a ballistic semiclassical theory that allows recovering its linear-in-field dependence in the quantum regime. The quantum oscillations are beyond the semiclassical approximation, and we obtain a quadratic magneto-conductance at small magnetic field, which is distinct from the predictions of Sec. [II] in the presence of level-broadening at low field. Furthermore, we study the stability of the linear magneto-conductance in the presence of elastic intra-cone and inter-cone scattering, we find that it is pushed toward very large fields in strongly disordered systems, in agreement with Ref. [21]. In Sec. [V] we discuss our results in the context of the chiral anomaly, and we argue that only the quantum regime is a clear signature of it, before concluding in Sec. [VI].

**FIG. 1.** Schematic representation of a short junction of a Weyl semi-metal between two metallic leads at chemical potentials $\mu_1, \mu_2$, and submitted to a magnetic field $\mathbf{B}$ parallel to the direction of the current $\mathbf{I}$. $\mathbf{K}$ is the vector in the Brillouin zone relating a pair of Weyl cones of opposite chiralities.

### II. BALLISTIC MAGNETO-TRANSPORT

We consider the transport through a short junction made of a Weyl semi-metal in the presence of an external magnetic field $\mathbf{B}$, represented in Fig. 1. We assume that $\mathbf{B}$ is applied along the direction of the current, unless specified otherwise. In such a setup, the charge current is induced by the bias voltage between the leads on each side of the junction. For a short enough junction, scattering inside the conductor can be neglected. Hence, the energy and momentum relaxation only take place in the leads. This so-called ballistic regime is inherently out of equilibrium, in contrast with the linear response to an electric field, which has been considered so far. The current induced by a small difference of chemical potentials across the junction, $\mu_2 - \mu_1 = \mu \mp \delta\mu/2$, takes the simple form $I = G(\delta\mu/e)$. The conductance of the junction, $G$, scales as its transverse area, $W \times W$, and does not depend on the junction’s length, $L$, in the ballistic regime. This allows us defining a dimensionless and scale independent conductance, $g$, such that

$$G = \frac{e^2}{h} \left( \frac{W}{a} \right)^2 g, \quad (1)$$

where $a$ is an UV cutoff (for instance, the lattice spacing).

In the presence of a magnetic field $\mathbf{B} \parallel z$, the kinetic energy of electrons in the $x, y$-directions freezes into Landau levels, while the motion in the $z$-direction is unaffected. Hence each state in each Landau level provides a conduction channel in the ballistic junction. The number of such channels per Landau level is $W^2/(2\pi l_B^2)$ where $l_B = \sqrt{\hbar/|e|B|}$ is the magnetic length. This yields a dimensionless conductance

$$g(\mu, b) = N(\mu, b) |b|, \quad (2)$$

where $N(\mu, b)$ is the number of Landau levels below the chemical potential $\mu$ and $b$ is the rescaled magnetic flux.
per unit cell (in units of the flux quantum), that is, \( b = \phi/\phi_0 = a^2 Be/h = \text{sign}(B) \frac{a^2}{2\pi l_B^2} \).

Let us now study the evolution of \( N(\mu, b) \) for a ballistic junction built out of Weyl fermions. Weyl valleys necessarily come by pairs of chiral fermions with opposite chiralities. Around each valley, the local Bloch Hamiltonian can be written as

\[
H = cp_x \sigma_x + cp_y \sigma_y + \chi c_z p_z \sigma_z. \tag{3}
\]

with \( \chi = \pm 1 \) the right/left-handed chirality of the valley, \( p_i = \hbar k_i \) are quasimomenta, while \( c \) and \( c_z \) are velocities in \( x, y \)-directions respectively. (For simplicity, we assumed same velocities along \( x \) and \( y \).) In the presence of a magnetic field \( B \parallel z \), the spectrum of each valley consists of a series of bands \( E_{n, \pm} = \pm [n(h\omega_0)^2 + (c_z \hbar k_z)^2]^{1/2} \) with \( n \geq 1 \), which disperse along the \( z \)-direction, and which are separated from each other by gaps at \( k_z = 0 \) of the order \( \hbar \omega_0 = \hbar c v/2l_B = \epsilon_0 \sqrt{|b|} \) with \( \epsilon_0 = \hbar c v/4\pi/a \) (see Sec. I of the supplementary material (SM) [22]). Besides these bands, the energy spectrum admits an additional, linearly dispersing band, whose direction of propagation depends both on the chirality of the valley and the direction of the magnetic field, \( E_0 = -\chi \text{sgn}(B) c_z \hbar k_z \), as shown in Fig. 2.

At large magnetic field, \( |b| \geq \mu/\epsilon_0 \), only the anomalous Landau level \( n = 0 \) contributes to transport, a situation reminiscent to that of Ref. [3] albeit considered here in the ballistic regime. We call this regime the quantum regime, in which \( N(\mu, b) = 1 \), and from Eq. (4) we find \( g = |b| \). At smaller fields, \( |b| \leq \mu/\epsilon_0 \), other Landau bands intercept the Fermi level and contribute to the conductance, which thus oscillates as a function of the magnetic field \( |b| \) or the chemical potential \( \mu \). For an ideal ballistic junction, the number of filled Landau levels is deduced from the expression of Landau levels, \( N(\mu, b) = 1 + 2[(\mu/(\hbar \omega_0))^2 - 1] + 2(\mu^2/(\epsilon_0^2|b|))] \), where \(|x| \) stands for the integer part of \( x \). By introducing the sawtooth function \( \text{sw}(x) = [x] - x + \frac{1}{2} \) we obtain the expression of the conductance of an ideal Weyl junction,

\[
g(\mu, b) = g_{sb}(\mu) + 2|b| \text{sw} \left( \frac{g_{sb}(\mu)}{2|b|} \right), \tag{4}
\]

where \( g_{sb}(\mu) = 2\mu^2/\epsilon_0^2 = a^2 \mu^2/(2\pi(hc)^2) \) corresponds to the Sharvin conductance of the junction, which is determined by the number of conduction channels at \( b = 0 \). Let us note that quantum oscillations of the longitudinal conductance, similar to those we identified in the ballistic regime, were also observed in the different regime of diffusion of hot electrons close to equilibrium within a Kubo formula approach [23].

The limit \( b \to 0 \) of the conductance [4] for an ideal junction is actually ill-defined: as usual, we expect the quantum oscillations described by Eq. (4) to be cut-off at low field by any broadening of the Landau bands [24]. In the ballistic junction that we consider, the finite dwell time of the electrons between the leads, \( \tau_d = L/c_z \) provides an inherent broadening of order \( \eta \approx \hbar/\tau_d = \hbar c_z/L \).
We can phenomenologically incorporate this effect by a standard Lorentz broadening of the energy levels, leading to

\[ g(\mu, b) - g(\mu, 0) = 2\beta \int \frac{d\mu'}{\eta} f \left( \frac{\mu - \mu'}{\eta} \right) \text{sw} \left( \frac{g_{\text{sh}}(\mu')}{2|b|} \right), \]  

with \( f(x) = 1/|\pi(1 + x^2)| \). In the regime where a non-oscillating density of states is recovered, corresponding to \( \hbar \omega_0 = \epsilon_0 |b| < \eta \), we find a non-analytic scaling of the positive magneto-conductance,

\[ g(\mu, b) - g(\mu, 0) \approx \alpha \frac{\epsilon_0}{\eta} f \left( \frac{\mu}{\eta} \right) b^{3/2} \]  

with \( \alpha \approx 1.16 \), see Sec. II in the SM \[22\]. Noteworthy, this scaling is different from the typical \( b^2 \)-behavior predicted in the diffusive regime \[10\]. Similarly if a thermal broadening of the levels supersedes the intrinsic broadening, we can still use Eq. (5), but now with \( f(x) = -f_F(x) \) with \( f_F(x) = 1/(1 + e^x) \). For Weyl fermions, it yields

\[ g(\mu, b) - g(\mu, 0) = |b| f_F \left( \frac{\mu}{k_B T} \right) - |b|^2 \frac{\epsilon_0}{4k_B T} f_F \left( \frac{\mu}{k_B T} \right). \]  

In particular, the anomalous magneto-conductance is exponentially suppressed at \( \mu \gg k_B T \).

Let us contrast this behavior with that for a standard non-relativistic parabolic dispersion relation \( E = \frac{p^2}{2m} \), whose Landau bands read \( E_n = \hbar \omega_0 (n - \frac{1}{2}) + \frac{p_x^2}{2m} \) with \( n \geq 1 \), \( \hbar \omega_0 = \hbar e |B|/m = \epsilon_1 |b| \), and \( \epsilon_1 = h^2/(2\pi m a^2) \). The number of filled Landau bands is now \( N(\mu, b) = [\mu/(2\hbar \omega_0) + \frac{1}{2}] \) corresponding to a conductance \( g(\mu, b) = g_{\text{sh}}(\mu) + |b| \text{sw} (g_{\text{sh}}(\mu)/|b| - \frac{1}{2}) \) with \( g_{\text{sh}}(\mu) = \mu/\epsilon_1 \) (we assumed \( \mu > 0 \)). It yields strikingly different predictions compared with Eq. (4). In particular, the conductance vanishes in the quantum regime, which is reached for fields \( |b| \geq 2g_{\text{sh}} \), as opposed to the positive linear magneto-conductance at high fields. Furthermore, a Lorentz broadening of the Landau levels leads to an exponentially suppressed magneto-conductance at low field, in contrast with Eq. (6) for the Weyl Hamiltonian. The sharp difference between the ballistic magneto-conductance of a Weyl junction with that of a standard material with a non-relativistic dispersion relation is highlighted in Fig. 3 which illustrates the presence vs absence of the positive magneto-conductance in the quantum regime.

### III. NUMERICAL STUDY

We now complement the previous arguments based on the simplified low-energy Bloch Hamiltonian \[3\] by a numerical study of transport of two lattice Hamiltonians displaying respectively four pairs and one pair of Weyl cones. First, we compute the conductance of a ballistic junction of size \( W \times W \times L \) using the Kwant numerical software \[20\] applied to a tight-binding two-band model on a cubic lattice with nearest-neighbor couplings.
such that the dispersion relation, $E_\pm^2(k) = t^2(\sin^2 k_x a + \sin^2 k_y a) + [t_z(1 - \cos k_z a) - \Delta^2]$, possesses four pairs of Weyl cones with opposite chiralities at quasi-wavectors $k = (0/\pi, 0/\pi, \pm K_z)$ with $K_z = (2/a) \arcsin \sqrt{\Delta^2/(2t_z^2)}$, assuming $0 < \Delta/(2t_z) < 1$ (see Sec. III in SM [22] for details). Throughout our study, we use $a = 1$ for the lattice spacing, $t = 1$ and $t_z = 1$ for the hopping matrix elements in $(xy)$-plane and along $z$-direction, respectively, and various values of the energy threshold $\Delta$. We study the transport along the $z$-direction as a function of the magnetic field $B \parallel z$ aligned with the junction. Results for $\Delta = 1$ are shown in Fig. 4 in which the conductance of the junction is plotted as a function the dimensionless magnetic flux $b$ threading a lattice unit cell. For all chemical potential of the conductor below the energy threshold between Weyl valleys, $|\mu| < \Delta$, we find a quantitative agreement with the previous analysis. Namely, (i) at low magnetic field, the conductance $g(b)$ oscillates with $b$, with an amplitude of oscillations that increases linearly with $b$, (ii) at high magnetic field, the conductance $g(b)$ reaches a linear regime independent on energies, $g(b) \simeq 4|b|$, where the factor 4 accounts for the presence of 4 pairs of Weyl points in our model. We have checked that both that the conductance is independent of the length $L$ of the junction, and that the above behavior is a contribution of bulk states, while surface states provide a subdominant contribution $\propto W$ to the conductance (see Sec. III in SM [22]). Note that the numerical Landauer technique that we use amounts to introducing semi-infinite systems on both sides of the conducting part, playing the role of leads with perfect contacts, thereby allowing to reach a ballistic regime. In doing so we effectively consider a long ballistic junction, corresponding to an artificially large dwell time for the electrons, which hampers the study of the algebraic low $b$ regime which is relevant experimentally. The thermal broadening of the magneto-conductance is illustrated in Fig. 5, in agreement with Eq. (7) for $\mu \gg kT$.

We now discuss the dependence of the ballistic magneto-conductance on the chemical potential, and show that the number of pairs of Weyl valleys contributing to this regime depends on this chemical potential. In particular, as shown in Fig. 6, we find that the linear regime at large fields survives even above the threshold energy separating the Weyl valleys $\mu > \Delta$ (here taken as $\Delta = 0.7$), though its slope is reduced by a factor two at large $\mu$. To understand this reduction of the slope, we consider the simpler situation of a single pair of Weyl cones separated by an energy threshold $\Delta$. It can be realized with a tight-binding two-band model on a cubic lattice introduced in Ref. [27] where the Weyl points only occur at $k = \pm K/2 = (0, 0, \pm K_z)$ (see SM [22], Sec. IV). The corresponding numerical results for the magneto-conductance are presented in Fig. 7. We observe that above the threshold energy $\Delta$ separating the Weyl valleys, the linear behavior at large field is lost for one direction of the magnetic field oriented along the current with $B \parallel I \parallel \mathbf{K}$ (Figs. 7A and 7B). Moreover, the whole behavior at small magnetic field is now asymmetric in $b$, irrespective of $\mu$. Note that such a simple Weyl semi-metal with only two cones necessarily breaks time-reversal symmetry (TRS). This manifests itself in the breaking of Onsager relation $G(B) \neq G(-B)$, and henceforth an asymmetry of the curves $G(B)$ (or $g(b)$). The amplitude of this TRS breaking and this asymmetry originates from the separation $K$ of the two cones in the...
shown in Fig. A: the regime $g(b) \simeq |b|$ is lost for $b < 0$. Similarly, the asymmetry of the small field regime for any chemical potential $\mu$ is clearly visible for $\Delta = 1.5$, Fig. B. In particular the minimum of $g(b)$ is shifted towards a positive field, reflecting the breaking of time-reversal symmetry for a model with a single pair of Weyl cones located at points $\pm K/2$. For a magnetic field transverse to this direction $K$, a symmetric conductance $g(b) = g(-b)$ is recovered as shown in Fig. C.

![Diagram](image)

**FIG. 8.** Schematic view of the Landau level dispersion for a pair of left- and right-handed Weyl cones (resp. L and R) separated by an energy threshold $\Delta$. The $n = 0$ Landau band depends on the $B \parallel \pm K$. For $B \parallel +K$, this $n = 0$ Landau Level exists for energies $\mu < \Delta$ while for $B \parallel -K$, this $n = 0$ Landau Level exists for energies $\mu > -\Delta$. For a given pair or Weyl points this defines the range of chemical potential, which depends on the orientation of magnetic field, for which a ballistic linear magneto-conductance is observed at magnetic high field.

Brillouin zone, or more precisely on its projection onto the direction of magnetic field. Indeed, when the vector $K$ is aligned perpendicular to both the magnetic field and the junction, no sign of this TRS breaking is observed on magneto-transport and a magneto-conductance symmetric in $b$ is recovered as shown in Fig. C.

The change of linear magneto-conductance in the quantum regime can be understood by considering the simple Bloch Hamiltonian that generalizes Eq. (3) and describes two Weyl valleys separated by an energy saddle point,

$$H = c_p \sigma_x + c_p \sigma_y + \left( \frac{p_z^2}{2m} - \Delta \right) \sigma_z.$$  \hspace{1cm} (8)

For $\Delta > 0$, two Weyl points at $\pm K/2 = (0, 0, \pm \sqrt{2m\Delta}/\hbar)$, and having opposite chirality, are separated by an energy barrier $\Delta$. The chemical potential dependence in the large field regime can now be inferred from the behavior of the Landau levels for the model (8) (see SM [22], Sec. I). Depending on the sign of $b$, i.e., on the orientation of the field with respect to the "chirality vector" $K$ pointing from the left-handed Weyl cone with $\chi = -1$ to the right-handed Weyl cone with $\chi = +1$, the dispersion of the $n = 0$ Landau level changes dramatically. Indeed, it either exists for $\mu$ smaller than $\Delta$, or for $\mu$ larger than $-\Delta$ as illustrated in Fig. B. We expect the larger energy barrier between the two Weyl valleys, neglected in the model (8), to cut-off the energy range of this $n = 0$ Landau band on the other side. Beyond this saddle point energy, the linear magneto-conductance for the pair of Weyl cones is lost. Depending on the sign of $b$, this happens either above the positive or below the negative energy saddle point $\pm \Delta$.

We are now ready to understand the reduction of the slope illustrated in Fig. B for the model with four pairs of Weyl cones. Indeed, due to the symmetrical distribution of the pairs of Weyl cones in momentum space in that model, above the saddle point energy, the linear magneto-conductance contribution vanishes for the half of Weyl pairs whose chirality vector is parallel with the magnetic field, while it persists for the other half having an anti-parallel chirality vector with respect to $b$. Furthermore, the absence of an asymmetry of the magneto-
density along the $z$-direction follows:

$$ j_z = -e \sum_{\chi, \hat{k}} (w_{\chi} \cdot \hat{z}) f_{\chi}(x, \epsilon, \hat{k}), \quad (10) $$

where the sum runs over constant energy contours and $\hat{k} = k/|k|$. The distribution function satisfies the stationary Boltzmann equation (dropping the energy dependence of $f$)

$$ (w_{\chi} \cdot \hat{z}) \partial_z f_{\chi}(z, \hat{k}) = \frac{1}{\tau} \sum_{k'} \left[ f_{\chi}(z, \hat{k'}) - f_{\chi}(z, \hat{k}) \right] + \frac{1}{\tau'} \sum_{k'} \left[ f_{-\chi}(z, \hat{k'}) - f_{-\chi}(z, \hat{k}) \right]. \quad (11) $$

Here $\tau$ and $\tau'$ are the intra-cone and inter-cone elastic scattering times, respectively, which are assumed to be larger than the dwell time in the ballistic regime. We solve Eq. (11) with an ansatz for $f$ that satisfies boundary conditions at the contacts [29],

$$ f = f_0 (\epsilon_k - \mu_1) \Theta [w_{\chi} \cdot \hat{z}] + f_0 (\epsilon_k - \mu_2) \Theta [-w_{\chi} \cdot \hat{z}] . \quad (12) $$

Solving these equations in the ballistic regime, $\tau, \tau' \to \infty$, we find

$$ g_{cl} = \left\{ \begin{array}{ll} g_{bh} \left[ 1 + b^2/(2g_{bh})^2 \right] & \text{for } |b| < 2g_{bh}, \\
|b| & \text{for } |b| \geq 2g_{bh}. \end{array} \right. \quad (13) $$

Quite remarkably, incorporating the Berry curvature effect into the semi-classical description of the ballistic transport allows to describe the anomalous linear magneto-conductance in the quantum regime at large field, as well as the cross-over towards an anomalous regime at low field. However, the semi-classical description is unable to accurately describe the $|b|^{3/2}$-dependence of the magneto-conductance, see Eq. (11), and predicts a $b^2$-behavior, similar to the diffusive regime [10]. Such a diffusive regime is indeed recovered in the present situation at finite scattering times $\tau, \tau'$. However it corresponds to a different situation from the one considered by Son et Spivak [10], as inelastic scattering occurs in the leads in our case, and not in the junction, thus yielding a qualitatively different result. Solving this problem, we find an expression for the conductance (see Sec. VI in SM [22]), which identifies with that previously derived via a topological non-linear sigma field theory [21]. As expected, when intra-cone disorder is increased a diffusive regime is reached at small magnetic field when $\ell = c\tau L$, with a conductance now scaling as $G \propto W^2/L$. The correction at small field in this regime remains quadratic in $b$. At high magnetic field, the previous ballistic quantum magneto-conductance regime $g(b) = |b|$ is affected neither by intra-cone, nor by inter-cone disorder. Moreover as long as $\ell' \gg L^2$, including the situation when only the intra-cone disorder is relevant $\ell \leq L \leq c\tau',$ this ballistic linear regime is reached at smaller magnetic fields, $|b| \geq g_{bh}\ell/L$ [22]. On the other
hand, when $L^2 \gg \ell \ell'$, or $L \geq \ell \ell'$, inter-cone disorder pushes this ballistic regime to high magnetic field (possibly outside of the experimental regime) for $|b| \geq g_n L / \ell'$. The magneto-conductance corresponding to these different regimes is obtained by numerically solving the semi-classical diffusive equation (see [22], Sec. VI), and the results are represented in Fig. 9.

V. DISCUSSION

In this paper, we have shown that two regimes have to be distinguished when discussing the ballistic conductance of a Weyl junction in parallel magnetic field: (i) a weak magnetic field regime, in which the magneto-conductance behaves algebraically with the magnetic field, whose details are non-universal. Furthermore at higher fields the magneto-conductance displays quantum oscillations when the broadening of Landau levels is weak (such as in long ballistic junctions) and at low temperature, (ii) a linear regime at high fields with a universal slope.

Let us now discuss the relation between these different regimes and the underlying chiral anomaly. In the ballistic regime addressed in this work, there exists an equilibrium chiral current density $j_3 = j_R - j_L \propto |b|$ for non vanishing magnetic fields: the presence of the $n = 0$ Landau band implies that, irrespective of the chemical potential $\mu$, there is an excess of Weyl electrons of, e.g., left-chirality moving to the right of the junction, and electrons of right-chirality moving to the left. This equilibrium chiral current is obviously uncored-related with a charge density current $j = j_R + j_L$, as the later vanishes in equilibrium. On the other hand, in the presence of a chemical potential bias, $\mu_1 - \mu_2 = \delta \mu$, the excess of chiral current, or non-equilibrium chiral current, can manifest itself in a charge current, as first discussed in Ref. [8]. This non-equilibrium chiral current is still entirely due to the $n = 0$ Landau band and reads $e^2 / h (W/a)^2 |b| (\delta \mu / e)$. At weak magnetic field only part of the conductance $2$ is related to the $n = 0$ Landau band contribution, and thus can be related to the chiral anomaly. Hence the anomalous positive ballistic magneto-conductance in the low field regime is not a unique signature of the chiral anomaly. This is in contrast with the situation of "hot electrons", close to equilibrium, considered in Ref. [10]. There, the chiral current driven by the $n = 0$ Landau level leads to a chiral chemical potential $\mu_R - \mu_L$ between left- and right-handed Weyl valleys, and a magneto-conductance via the chiral magnetic effect. Hence attributing the anomalous magneto-conductance at low fields to the chiral anomaly requires first to identify the relevant regime of transport, and the observation of a positive algebraic behavior is not sufficient.

In contrast to the low field regime, the quantum regime of a linear magneto-conductance $G = W^2 (e^2 / h^2) |B|$ at high field is a unique contribution of the $n = 0$ Landau level. Thus it can be unambiguously associated with the chiral anomaly. This regime is reached for magnetic fields satisfying $B \lambda_F^2 \geq h / e$ where $\lambda_F$ is the Fermi wavelength. Thus it can be reached experimentally for chemical potentials sufficiently close to the band crossing. Moreover this regime is robust, and persists even in the presence of disorder: only its domain of existence is affected by elastic scattering. Furthermore, when energy relaxation occurs within the conductor the above ballistic conductance at high fields is replaced by $G = W^2 (l' / L) (e^2 / h^2) |B|$, also linear in magnetic field [8, 9, 23]. In this regime the slope of the linear regime now depends explicitly on the amplitude of inter-cone scattering. Hence the study of this linear regime should provide an unambiguous determination of the regime of transport. In the exact same regime of transport a linear magneto-resistance in transverse magnetic field was also predicted by Abrikosov in Refs. [30, 31] (see also [32]). We believe that this quantum regime of transport in Weyl materials is of high experimental interest. Quite remarkably a linear-in-field longitudinal magnetic conductance has already been observed in narrow wires of NbP, a Weyl semi-metal [19], hence validating that this regime in within experimental reach, although it hasn’t been thoroughly studied.

VI. CONCLUSION

In this paper, we have studied the conductance of a junction of Weyl material in the presence of a parallel magnetic field, and in the ballistic regime. We have shown that the low-field magneto-conductance displays low-temperature quantum oscillations, whose broadening results in an algebraic behavior at vanishing field. At high fields, the magneto-conductance becomes linear in the field. Besides its experimental relevance, this ballistic regime allows discussing in details the relation between this conductance and the chiral anomaly of Weyl fermions. This allows to unambiguously identify the large field regime as a signature of the chiral anomaly.

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