Simulation of 1D Gaussian Wave Packet and 2D Waves in Square Membrane Using Excel Spreadsheet with Visual Basic for Teaching Activity

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Abstract. The aim of this study is to demonstrate 1D Gaussian wave packet propagation and 2D wave deflection in a square membrane using the Excel interactive spreadsheets with the Visual Basic for Application (VBA) programming. It is based on the analytic solution of Gaussian packet wave propagation in x direction and 2D wave propagation in square membrane. The vibration modes of 2D wave are determined by using differential equation and initial conditions. The simulation based on the spreadsheet functions and facilities has been explored to show the 1D and 2D moving objects in transitional processes. This instrument is very useful both in teaching activity and learning process of wave physics. Visualizing of the vibration of wave in the square membrane show a very clear manner of and vibration modes of the wave. Therefore, in the Gaussian packet waves with the superposition processes between those waves show that the amplitude shifted in a different manner.

Key words: Visual Basic for Application, Vibration Modes, MS Excel, and Gaussian Packet Wave

1. Introduction
When we look at relevant literature, we encounter with an abstract or complex concepts such as waves propagation in membrane or in a different medium. Wave propagation sometimes difficult to understand by student in classroom activity. To demonstrate its propagation need some tools in order to visualize and simulate, so the student have to develop a functional understanding of physics. An teaching instrument demonstrating of wave propagation in the circular membrane waves using Excel interactive spreadsheets with VBA programming can be using to show 3D vibration objects in transitional or rotational processes[1]. Simulations can help teacher in creating or performing the real conditions in front of the class. Simulation can be expected to be an effective tool in helping students to explore their higher levels of learning capabilities such as analysis, synthesis and evaluation. The intervention of computer animation and simulation have been used as an alternative tools in order to help students confront their cognitive constraint simulations in classroom makes the real difference in student learning such as interpretation of verbal and graphical representations as well as equations in thermodynamics[2]

The uses of spreadsheets in electrical engineering appear to be more limited – perhaps can offer a reasonable tradeoff between user-defined programming and specific-purpose software[3], likewise, a
menu-driven spreadsheet for solving electromagnetics problems are presented in a manner that makes no use of macro programming; besides, this paper introduces finite element analysis via spreadsheets. Many authors plead for the use of spreadsheets demonstrating their multiple advantages, describing their facile use in physics lessons with examples of data processing according to tables and their graphic representation [4]. Moreover, an emphasis is placed upon their calculus capacity through the functions available for users, and the rapid feedback when modifying data[5]. Studies have shown that students have a better grasp of the definition of certain measures by dealing with basic calculi, as for instance the notion of velocity, while the instantaneous graphic response of the finite product represents an advantage in acquiring the concepts of physics [6]. In a series of recent papers, Excel spreadsheets have helped to describe the simulation of motion in the case of a projectile motion in a gravitational field[7,8].

Physics as a collection of concepts which explain the natural phenomena, even more abstract, complex and difficult to understand by student, need to visualize and demonstrate in a clear and interactive manner to create a great motivated in learning of physics. In this report, we propose the simulation of the propagation 1D Gaussian Wave Packet and 2D wave on the square membrane waves using the Excel interactive spreadsheets with the Visual Basic for Application (VBA) programming. In 1D case, the superposition processes present clearly with some modulation velocities, whereas in 2D case, the wave deflection on square membrane was analysed theoretically by using the separated variable method and the Fourier series involving show varieties of wave shape in several modes. We have explored a series of facilities available on Excel spreadsheets, such as the 2D graphical surface in deflection of membrane.

2. Methods

2.1 Gaussian Wave Packet

Two gaussian function, \( y = A \exp[\left(-\frac{x^2}{a^2}\right)] \) and \( y(x) = A \exp[\left(-\frac{(x - b)^2}{a^2}\right)] \), functions with amplitude \( A \) and the characteristic of width \( a \), have the same form but separated by \( b \) together move in a simultaneous motion in \( x \) direction. A Gaussian wave packet is a vibration of particle where they propagated in modulation frequencies (carrier wave) and phase frequencies in a simultaneous motion. The equation of wave packet motion describe by [9]

\[
\Psi(x, t) = \frac{A \exp\left\{\frac{-1}{2}(x-v_{ext}-x_{init})^2/\sigma^2\right\}}{\sigma(2\pi)^{1/2}} \cos(kx - \omega t) \tag{1}
\]

2.2 Designed Gaussian wave packet in SpreadsheetsExcel

To make the Gaussian wave packet in spreadsheet, firstly the proponent of wave packet such as amplitude, initial position, width controller and group velocity have to be input on definite cell in spreadsheet excel. Thus, the parametric of wave data according to equation (1) must be calculated in spreadsheet excel formulas. After that, the other parametric of wave can be calculated with several asumptions and displayed in FIGURE 1 following
In order to see the object 1D wave packet in the further way this look like moving object running in time, we explore the using of macro VBA programing in excel facilities. Finally, their wave data plotted using $XY$ scatter and result the form of Gaussian wave packet following.

**Figure 1.** The proponent parametric of Gaussian wave packet

Macro VBA is the tool program in MS. Excel to move data with action button for $m, n, dt, \Delta x, \Delta y, a, b, c$ and $B_n$. The script program of 1D wavepacket showed in following.
2.3 Two Dimensional Wave in Square Membrane

The wave square membrane build in assumption that the wave propagated or vibrated in constant area, thin, homogen, flexible, and the inclination is more small than its deflection. The equation motion of 2D wave in square membrane is described as

\[
\frac{\partial^2 u(x,y,t)}{\partial t^2} = c^2 \left( \frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} \right)
\]

(1)

By using separation variable method \( u(x,y,t) = X(x)Y(y)T(t) \) and Fourier series, the solution of eq.1 is

\[
u_{mn}(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (A_{mn} \sin \omega_{mn} t + B_{mn} \cos \omega_{mn} t) \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \]

\[
\omega_{mn}^2 = c^2 \left( k_x^2 + k_y^2 \right) ; \ \omega_{mn} = c \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}
\]

(2)

\( \omega_{mn} \) is a characteristic frequency of membrane [10]. In boundary condition, the wave move stationary with the tip banded in square membrane \( 0 < x < a, 0 < y < b, t > 0 \), constrain

\( u(0,y,t) = 0, u(x,a,y,t) = 0, u(x,0,t) = 0, u(x,b,t) = 0 \)

The initial deflection \( u(x,y,0) = f(x,y) \) given by [12]

\[
u(x,y,0) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m \pi x}{a} x \sin \frac{n \pi y}{b} \]

(3)
and initial velocity \( g(x, y) = \sum\sum_{m=1}^{\infty} A_{mn} \omega_{mn} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \) derived from relation
\[
\frac{\partial u(x, y, 0)}{\partial t} \bigg|_{t=0} = 0
\]

\( A_{mn} \) and \( B_{mn} \) is Fourier series coefficient represented by [5]
\[
A_{mn} = \frac{4}{ab \omega_{mn}} \int_{0}^{a} \int_{0}^{b} g(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx \, dy
\]
\[
B_{mn} = \frac{4}{ab \omega_{mn}} \int_{0}^{a} \int_{0}^{b} f(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dx \, dy
\]

\[
(4)
\]

3 Designed Square 2D Wave Membrane in Spread sheets Excel

To describe the analytic solution in spreadsheet excel need to define the initial condition of square membrane \( f(x, y) = x(x-1)(y-1) \) where \( g(x, y) = 0 \), so \( A_{mn} = 0 \). Deflection of membrane represented of \( u_{mn}(x, y, t) = \sum\sum_{m=1}^{\infty} (B_{mn} \cos \omega_{mn} t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \). Square membrane

boundaries with \( a = b = 1 \) and \( c = \frac{1}{\pi} \), the initial boundaries
\( u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, b, t) = 0 \), vibrated without initial velocity and shifted in membrane \( 0 < x < 1 \), \( 0 < y < 1 \), and \( t > 0 \). In mathematical calculation, with several values of \( m \) (mode ) and \( n \) (node) [10], \( f_{12}(x, y) = \sin \frac{\pi x}{a} \sin 2\frac{\pi y}{b} \) createin \( y = \frac{1}{2} \), deflection for \( m=1 \) and \( n=2 \) described in \( u_{12}(x, y, t) = A_{12} \sin \omega_{12} t + B_{12} \cos \omega_{12} t \) \( f_{12}(x, y) \). Moreover, \( f_{21}(x, y) = \sin 2\frac{\pi \omega}{a} \sin \frac{\pi y}{b} \) with \( x = \frac{1}{2} \), \( u_{21}(x, y, t) = A_{21} \sin \omega_{21} t + B_{21} \cos \omega_{21} t \) \( f_{21}(x, y) \).

On the process to make wave shaped in square membrane using spreadsheet excel, firstly input the mathematic formulation in spread sheet that showed in figure 1 following

![Figure 3. The proponentparametric of square membrane in spreadsheet excel display](image)

After that, the other parametric of wave can be calculated with several assumptions. In order to make deflection the object of 2D wave, the square membrane construct with the boundary condition
and initial condition and wave vibrated in the further way this look like moving object, this illustrated performing using VBA programming. Finally, their data plotted and displayed in spreadsheet excel in Surface Chart following

**WAVE 2D SIMULATION IN SQUARE MEMBRANE**

| Parameter | Action script | Value |
|-----------|---------------|-------|
| \( dt \) | Range(“B2”) = dt . Value * 0.001 |       |
| \( \Delta x \) | Range(“B3”) = dx . Value * 0.01 |       |
| \( \Delta y \) | Range(“B4”) = dy . Value * 0.01 |       |
| \( a \) | Range(“E3”) = a . Value * 1 |       |
| \( b \) | Range(“E4”) = b . Value * 1 |       |
| \( c \) | Range(“E2”) = contant . Value * 1 |       |
| \( m \) | Range(“H3”) = m . Value * 1 |       |
| \( n \) | Range(“H4”) = n . Value * 1 |       |
| \( Bmn \) | Range(“K2”) = deflection . Value * 0.5 |       |

**Figure 4.** The frame of 2D wave in square membrane and its parametric respectively in spreadsheet excel display

Macro Visual Basic for Application (VBA) is the tool program in MS. Excel to move data with action button for \( m, n, dt, \Delta x, \Delta y, a, b, c \) and \( B_{mn} \). The scriptprogram of 2D wave in square membrane showed in following.

4. Results

In wave packet Gaussian, which propagate in transversal manner show that packets of wave, two Gaussian wave separated with a different position and definite amplitude, can involving with superposition processes in phase velocity. The history of propagation of two Gaussian wave packet in opposite direction can be running in time with VBA programming presented in table following.
Furthermore, considering the superposition processes of wave group velocity, simulation of 1D wave packet Gaussian which they propagate in transversal manner show that the packet wave amplitude increased in time, which in \( t = 60 \text{s} \) the value of amplitude superposition rise two times more high than \( t = 0 \text{s} \). The history of propagation of two wave packet in opposite direction can be running in time with VBA programming displayed in table following.

In the case of 2D wave motion, wave propagation in square membrane will be considering with mode of vibration characterized by a nodal frequency and a mode shape. In system 2D, all modes have different amplitudes (with lower modes having higher amplitudes) and different mode shapes whereas nodes become lines, each mode is entirely independent of all other modes. Some of the simulations of 2D wave in circular membrane using spread sheet excel with several values of \( m \) (mode) and \( n \) (node) represent in table following.
The amplitude of vibration 2D wave in square membrane in this research was counting in Figure 7. Simulations of 2D wave in circular membrane number waves constant, show that the more the number of wave can created the lower of the amplitude. The way to prove this simulation is using MATLAB simulation (Nakhle H. A., 2005) which the vibration of membran have using zero initial velocity, where some value of wave look in the same shape with in interactive spreadsheet simulation.

5. Discussion

In case of 1D wave motion, represented by Gaussian wave packet, propagation of wave in x direction show a clear superposition processes in some times interval where it was running with VBA application. The history of wave propagation in time can be displayed interactive, sophysics students may benefit strongly from such an experience through interactive media using spreadsheet excel and of course will raising their interest in motion of carrier wave (modulation) and phase velocity. On the other hand, 2D wave motion in square membrane, deflection with zero in boundary membrane and zero ininitial velocity, the vibration showed clearly and interactively in spreadsheet excel with n, m mode and the amplitude $B_{mn}$ value depend on each modes. The higher the modes (n,m high) the lower the amplitude value. We can construct some combination of modes shapes interactively which each modes were entirely independent of all other modes. Physics student can visualize an abstract phenomena such as vibration 2D wave in interactive simulation. From two case of motion of wave, student will construct their functional concept in a deep understanding on how 1D and 2D wave propagated using spread sheet excel.
Conclusion

Simulation of wave motion in 1D Gaussian wave packet and 2D wave in square membrane can be created using spreadsheets VBA in Ms Excel. The solution of analytic 2D wave in circular membrane preserve initial deflection and zero initial velocity and the vibration modes in this simulation have the same shape with MATLAB simulation. From the history of 1D wave propagation in time, physics students may benefit strongly to understand the superposition processes on motion of carrier wave (modulation) and phase velocity, whereas in 2D wave vibrated in square membrane they can visualize an abstract phenomena wave in interactive simulation using spreadsheet excel and macro VBA programing.

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