Magnetic reconnection is a process for re-structuring magnetic fields and rapidly converting magnetic energy into thermal energy, with important consequences in many laboratory, space and astrophysical plasmas [1–3]. Merging of twisted bundles of magnetic field lines, known as ‘flux-ropes’ [4, 5], through reconnection is widespread, and has been investigated in several purpose-built laboratory experiments e.g. [6–9, 11, 12]. Whenever the magnetic field components along the axes of two merging flux ropes co-align, the re-configuration of magnetic fields, known as magnetic reconnection with a guide field, takes place. Whilst many simulations of magnetic reconnection, particularly with application to the Earth’s magnetosphere, consider reconnection with zero or weak guide field (e.g. [10]), the presence of a guide field can strongly affect the reconnection dynamics, the heating and the particle energisation. Reconnection in tokamaks and similar magnetic-confinement devices is always in the high
guide-field regime. Reconnection of oppositely-directed fields is likely to be rare in the solar corona and other strongly-magnetised astrophysical plasmas (and then only at three-dimensional null points—differing in many ways from their two-dimensional analogues); hence most astrophysical reconnection also involves strong guide fields.

The Mega Amp Spherical Tokamak (MAST) [13]—as well as demonstrating the potential for fusion energy generation of the spherical tokamak concept, and providing insight into many physical processes relevant to conventional tokamaks, has incidentally provided a valuable testbed for guide-field reconnection studies. In the merging-compression start-up scheme (one of several alternative plasma start-up methods), two flux-ropes with parallel toroidal current and toroidal magnetic field move together and then merge, creating a single plasma torus with closed magnetic flux surfaces [14]. This provides start-up and current drive without using a central solenoid, which could be attractive for future fusion devices, as well as allowing an experimental study, with good diagnostics, of reconnection and flux-rope merging [14, 15] in a parameter regime more closely resembling the solar corona and other astrophysical plasmas [16] than other experiments.

The dissipation of stored magnetic energy through reconnection [18]—which is the primary source of energy release in solar flares—provides a strong candidate for resolving the mystery of how solar coronal plasma is heated to temperatures of over 10^6K [19, 20]. Such reconnection may occur through merging of twisted magnetic flux ropes [21]. In the flux-tube tectonics scenario [22], a coronal loop may contain multiple twisted threads as its field lines are rooted in several discrete photospheric flux sources. Adjacent twisted flux tubes may also be created through photospheric motions with multiple vortices within a single flux source.

The main focus of this paper is on energy release and heating arising from magnetic reconnection in merging flux ropes, both in the MAST spherical tokamak and in the solar corona. We use single fluid and Hall-magnetohydrodynamic (MHD) simulations, as well as exploiting relaxation theory to derive semi-analytical results. In the next section, we summarise recent results from single-fluid MHD and Hall-MHD simulations of flux-rope merging in MAST, and present new results based on these simulations which predict the heating of ions and electrons through reconnection. In section 3, we outline how relaxation theory provides a useful tool for calculating the final state and the energy dissipated, and present a new model of relaxation in a tight-aspect-ratio geometry. Implications for heating of the solar corona, mainly based on relaxation theory, are presented in section 4, including a novel coronal heating model involving mergers of multiple twisted flux ropes.

2. Overview of resistive MHD and Hall-MHD simulations

MAST [13] is a tight-aspect-ratio toroidal device, with typical major and minor radii $R = 0.95$ m, $a = 0.60$ m, plasma current $I_p = 400–900$ kA, toroidal field $B_T = 0.40–0.58$ T at $R = 0.7$ m, and peak electron density and temperature $n_{e0} \simeq 3 \times 10^{19}$ m$^{-3}$ and $T_{e0} \simeq 1$ keV. The merging-compression scheme is initiated by the production of two toroidal flux-ropes with parallel toroidal currents around the in-vessel P3 poloidal field coils (see figure 1). As the current in the coils is decreased, the attraction of the ‘like’ induced plasma currents in the two flux-ropes causes them to separate from the coils and move together, subsequently merging through magnetic reconnection into a single flux-rope. Recent experiments using merging-compression have produced spherical tokamak plasmas with currents of up to 0.5 MA. The merging is associated with rapid heating (presumably as a result of reconnection), indicated by measured electron temperatures of up to 1 keV and ion temperatures up to 1.2 keV [14, 23].

Recently, fully nonlinear and compressible 2D/axisymmetric simulations with a strong guide field have been performed, in order to study flux-rope merging in MAST, using both single-fluid MHD and Hall-MHD [15]. The Hall terms in Ohm’s Law are considered because, for typical MAST parameters, the ion skin-depth is a macroscopic length scale (the ratio of the ion skin depth to the typical system size, 1 m, is $d_i = 0.145$) and is much larger than the width of the Sweet–Parker current sheet predicted by resistive MHD.

It should be noted that there are a large number of numerical studies of guide field magnetic reconnection in the semi-collisional and collisionless regimes using Hall-MHD [24, 25], two-fluid [26, 27] and kinetic particle-in-cell (PIC) approaches [28–32]. Many of these studies have shown that fast reconnection in the strong-guide field regime requires the width of the dissipation region falls below the ion-sound Larmor radius. Energy release in MHD and PIC simulations are compared by [33], for the case of a current sheet subject to a transient external perturbation, demonstrating agreement in several aspects—notably a close similarity between the final states, lending support to the relaxation approach. Much of the earlier work done in the context of the tearing modes in a tokamak relied on fluid-based models in the incompressible limit, while more recent gyro-kinetic and PIC simulations are necessarily limited to small system sizes [34–36]. However, such studies focus on spontaneous tearing modes in unstable current sheets, whereas our aim here is to study reconnection driven by force imbalance in merging flux ropes.

A few previous works specifically consider reconnection with the motivation of understanding merging flux ropes. Hall-MHD simulations with a guide field of the MRX device [37] reveal that the reconnection rate decreases with increasing guide field (consistent with results from previous simulations). These simulations have both a lower Lundquist number and lower normalised ion skin depth than the MAST simulations described below. Inoue et al [38] perform 2D PIC simulations of the current sheet region, but neglect the coupling to the flux-ropes and instead drive reconnection by a constant imposed electric field. They find that poloidal magnetic energy is efficiently converted into plasma kinetic energy, with the ions gaining approximately twice the energy of the electrons.

Here, we choose to employ a fully-compressible Hall-MHD model, which captures the essential physics of interest in the merging-compression plasmas. We note that undertaking PIC simulations for global modelling of flux rope merging of
MAST-size systems, with strong guide fields, would currently be very challenging (at least). The dimensionless Hall-MHD equations [2] used for the simulations are as follows:

\( \partial_t (n \mathbf{v}) + \nabla \cdot (n \mathbf{v} \mathbf{v} + p l + \pi) = \mathbf{j} \times \mathbf{B} \)  

\( \mathbf{E} = -\mathbf{v} \times \mathbf{B} - \frac{d_l}{n} \nabla p_l + \eta j - \eta_H \nabla^2 j \)  

\( \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \)  

\( (\gamma - 1)^{-1} [\partial_t p_i + \mathbf{v}_i \cdot \nabla p_i + \gamma p \mathbf{v} \cdot \mathbf{v}_i] = \eta j^2 + \eta_H \nabla j^2 \)  

Normalisation is with respect to typical values in MAST start-up plasmas prior to merging: density \( n_0 = 5 \times 10^{18} \) m\(^{-3} \), length-scale \( l_0 = 1 \) m, magnetic field \( B_0 = 0.5 \) T, and (equal) ion and electron temperatures \( T_0 = 10 \) eV; velocities are normalised by the Alfvén speed \( v_0 = B_0(\mu_0 n_0 m_i)^{-1/2} \) and times by the Alfvén transit time \( (0.29 \mu s) \). Here, \( n \) is the density, \( \mathbf{j} = \nabla \times \mathbf{B} \) the current density, \( \mathbf{v}_i \) the ion velocity (with electron velocity \( \mathbf{v}_e = \mathbf{v}_i - d_l j/m_i \)), \( \mathbf{B} \) the magnetic field, \( p = p_i + p_e \) the total (sum of the ion and electron) thermal pressure and \( \mathbf{E} \) the electric field. The ion stress tensor is \( \pi_i = -\nu_i (\nabla \mathbf{v}_i + \nabla \mathbf{v}_i^T) \), and the heat-flux vector \( \mathbf{q} \) has anisotropic form \( \mathbf{q} = -\kappa_{\parallel} \nabla T - \kappa_{\perp} \nabla \cdot T \) where \( \nabla || = \hat{b} \cdot \nabla \). The coefficients are (all normalised): ion skin-depth, \( d_i \); parallel resistivity \( \eta \); parallel ion viscosity \( \nu_i \) and parallel electron, \( \kappa_{\parallel} \); and perpendicular ion, \( \kappa_{\perp} \); heat conductivities; here, we set the normalised values to be \( \kappa_{\parallel}^i = 10^{-1} \) and \( \kappa_{\perp}^i = 10^{-7} \). These are based on Braginskii values at the pre-merging temperature. The final term on the right-hand side of equation (3) is hyper-resistive diffusion [2], used only the Hall-MHD simulations, which represents anomalous electron viscosity and sets a dissipation scale for Whistler and kinetic Alfvén waves, with normalised hyper-resistivity \( \eta_H \). Normalised Braginskii values of resistivity and parallel viscosity in MAST start-up conditions are \( \eta = 10^{-5} \) and \( \nu_i = 10^{-3} \); but values of \( \eta_i, \nu_i \) and \( \eta_H \) are varied in order to study the scaling effects of collisions.

Simulations were performed in: cartesian geometry, with invariance in the ‘toroidal’ direction, representing an infinite aspect ratio system; and an axisymmetric tight-aspect-ratio toroidal system, invariant in the toroidal \( \phi \) direction. The initial configuration was chosen to represent the instant when the current rings have detached from in-vessel coils, resulting in two localised cylinders/tori of toroidal current. The toroidal field within these flux ropes was calculated to ensure local force balance, but a non-vanishing Lorentz force, associated with the attractive force between the ‘like’ parallel currents, caused the rings to move towards each other and eventually reconnect. In the tight-aspect-ratio case, a confining vertical field was also incorporated. The simulation domain was of rectangular cross-section with rigid, conducting walls (see figure 2).

Results from a Hall-MHD simulation in toroidal geometry are shown in figure 2. The key results of these simulations are as follows:

- The flux-ropes eventually merge into a single flux-rope.

For the simulations in toroidal geometry, the final state
is a single torus with nested flux-surfaces and magnetic field profiles resembling those found in MAST plasmas.

- Oscillations ('sloshing') in the reconnection rate at low resistivity, resulting from magnetic pressure pile-up.
- A much faster reconnection rate in Hall-MHD than in resistive MHD.
- The qualitative nature of the reconnection in Hall-MHD simulations depends on the ratio of the collisional current sheet width—determined by the hyper-resistivity—to the ion-sound Larmor radius, \( \eta \sim \frac{\sqrt{k_B T_e/m_i}}{\ell e B_0} \). At higher collisionality there is a broad current sheet, whilst at intermediate values secondary tearing creates small islands of poloidal flux. At low collisionality, the outflow separatrices open and fast reconnection is attained.
- Toroidal simulations in Hall-MHD show the formation of a double-peaked structure in the radial density profile which closely resembles experimentally-measured profiles.

The role of magnetic reconnection in plasma heating, and the partitioning of the dissipated magnetic energy, is of considerable interest for understanding and predicting start-up using merging-compression, as well as more widely in laboratory and astrophysical plasmas—for example, for solar coronal heating (as discussed further in section 4 below). Previous work [14] demonstrated considerable electron heating in experimental studies of merging in both TS-3 (Tokyo University Spherical Torus Number 3) and MAST, and recent work [23, 39] has provided further insights from experiment, including spatially and temporally resolved information concerning ion temperature during merging. Here, we report on preliminary work to predict the evolution of ion and electron temperatures, using separate equations for ion and electron temperature evolution incorporating, respectively, ion viscous heating and resistive and hyper-resistive electron heating (see also [17]). Temperature equilibration between ions and electrons is ignored in the present simulations, since the equilibration time is of the order 0.2 ms, which is much longer than the merger timescale.

Plots of the evolution of both temperature profiles are shown in figure 3. Ions are mainly heated by collisional viscous heating in the reconnection outflow jets, giving a temperature profile which is double-peaked radially, consistent with experiment [23]. The simulations predict ion peak temperatures of the order of 1 keV, comparable with the observations of Ono et al [14], but higher than reported by Tanabe et al [23]; the latter experiments were undertaken at low coil currents. The predicted electron temperature profiles are dominated by hyper-resistive diffusion which is large in the presence of strong gradients of the current density, and so is not co-spatial with the current density; this is also suggested by the experimental profiles. The experimental observation that the electron temperature is sometimes centrally-peaked, and sometimes hollow, may be linked to the ‘stochastic’ appearance and ejection of plasmoids in the simulations. Further work is in progress on simulating the evolution of electron and ion temperature profiles.

Figure 2. Snapshots showing poloidal field (contours) and toroidal current density (colour scale) for Hall-MHD simulation of plasma merging in MAST. Toroidal geometry was used, with \( \eta_H = 10^{-5} \). The plots indicate normalised times, which correspond, respectively, to 2.36 \( \mu \)s, 3.41 \( \mu \)s and 6.27 \( \mu \)s.
Relaxation and self-organisation in merging flux ropes in spherical tokamaks

Relaxation theory, first proposed by Taylor to explain the field configurations in RFP experiments [40], has provided a powerful tool for explaining phenomena in various magnetic confinement devices [41] as well as astrophysical plasmas. It is hypothesised that a disrupted magnetic field in a highly-conducting plasma relaxes to a state of minimum magnetic energy whilst conserving global magnetic helicity

$$\int K \mu V dV.$$

The relaxed state is a linear or constant-$\mu$ field:

$$\nabla \times \mathbf{B} = \mu \mathbf{B},$$

(6)

where $\mu = \mu_0 \mathbf{B} \cdot \mathbf{j}/B^2$ is spatially constant. Note that the relaxed state is a special case of a force-free, satisfying $\mathbf{j} \times \mathbf{B} = \mathbf{0}$.

It was recently proposed [16] that the merging of flux ropes in MAST could be modelled as a helicity-conserving relaxation to a minimum energy state. The model assumes two initial force-free flux ropes separated by a current sheet (a discontinuity in the poloidal field); these merge into a lower energy state, consisting of a single flux rope within the same volume described by (6). In order to permit analytical solutions, it is assumed that each initial flux rope has a linear force-free field described by (6). The initial field has free energy associated with the current sheet between the two flux ropes (this is a delta function layer of reversed current, or negative $\mu$), and hence can release energy through relaxation to a fully constant-$\mu$ state; it is, however, a force-free equilibrium. This represents the field at the moment at which the flux ropes have been brought together by the attractive force, but have not yet commenced reconnection. Browning et al [16] developed this model assuming infinite aspect-ratio i.e. the flux ropes are straight cylinders; a Cartesian coordinate system is used with all quantities independent of the axial coordinate $z$.

Since MAST, and several other laboratory experiments involving flux rope merging, have very low aspect-ratio, it is interesting to extend the relaxation model of [16] to finite aspect-ratio. We therefore model relaxation of two toroidal flux ropes within a cylindrical ‘can’ of a rectangular cross-section (hence still allowing analytical solutions). The spherical tokamak is assumed to be an axisymmetric configuration contained within the volume $0 \leq Z \leq l, a \leq R \leq b$ in cylindrical coordinates $(R, \phi, Z)$. Lengths are normalised with respect to the width of the can, $b - a$ (the minor radius is $(b - a)/2$, and the aspect ratio is $(b + a)/(b - a)$). Similarly to

Figure 3. Ion (top) and electron (bottom) temperature profiles, in eV, at four successive times (normalised with respect to the Alfvén transit time 0.29 $\mu$s), for a Hall-MHD simulation of plasma merging in MAST. The top plot also shows in-plane ion velocity vectors. Toroidal geometry was used, with $n_i = 10^{-8}$.
the infinite-aspect-ratio model, the initial state is given by two constant-μ toroidal flux ropes separated by an annular current sheet at the midplane $Z = 0.5\, l$, where we take $l = 2$ (in dimensionless units), giving each flux-robe a square boundary similar to the earlier infinite-aspect ratio model [16].

The fields (for both initial and final states) are expressed in terms of a flux function $\psi$ as $B = (1/R)(-\partial \psi/\partial Z, \mu \psi, \partial \psi/\partial R)$ and equation (6) leads to the Grad–Shafranov equation

$$\frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} + \mu^2 \psi = 0. \quad (7)$$

The solution closely follows [16], converted from cartesian to cylindrical coordinates. The boundary condition is $\psi = \psi_b = \text{constant}$ on the boundary, where $\psi_b$ (effectively a normalisation constant for the fields) must not be zero. (For $\psi_b = 0$, the only non-trivial solutions to (7) are spheromak-like eigenfunction solutions for discrete eigenvalues of $\mu$ [42]). Following [43] and [42], the solution within a annular shell with height $L$ is expressed as a sum of the eigenfunctions

$$\psi = \psi_b \left[ 1 + \sum_{m,n} a_{mn} R_m(R) \sin \left( \frac{n \pi Z}{L} \right) \right], \quad (8)$$

where the radial eigenfunctions are given in terms of Bessel functions $J_\nu, Y_\nu$ as

$$r_m(x) = Y_\nu(\beta_m b) J_\nu(\beta_m a) - Y_\nu(\beta_m a) J_\nu(\beta_m b), \quad (9)$$

and $\beta_m$ is the $m$th zero of the function $r(R) = Y_\nu(bR) J_\nu(aR) - Y_\nu(aR) J_\nu(bR)$. Note that equation (8) automatically satisfies the required boundary conditions. We normalise so that the toroidal flux $\Phi_t \equiv \int_A B_t dS = \int_0^L \int_a^b \mu \psi dR dZ = 1$ (thereby ensuring conservation of flux during the relaxation process described below): hence $\psi_b(\mu; a, b)$ depends on $\mu$.

Substitution into equation (7) gives

$$\sum_{m,n} \left[ (-\gamma_{mn}^2 + \mu^2) R_m(R)b \sin(n \pi Z/l b) \right] = -\mu^2, \quad (10)$$

where

$$\gamma_{mn}^2 = (\beta_m)^2 + (n \pi l/L)^2. \quad (11)$$

Multiplying by an eigenfunction $r_m(R)b \sin(n \pi Z/l L)$ and integrating over $R$ and $Z$, exploiting the orthogonality of these functions, then yields:

$$a_{mn} = \frac{-\mu^2}{\gamma_{mn}^2 - \mu^2} \frac{4}{b} \frac{1 - j_m b/a}{1 - j_m}, \quad \text{for } m, n \text{ odd}; \quad (12)$$

$$a_{mn} = 0, \quad \text{for } m \text{ or } n \text{ even}; \quad (13)$$

where

$$j_m = J_\nu(\beta_m b) / J_\nu(\beta_m a). \quad (14)$$

The fields from equations (8) and (12) are used both to represent the two individual flux ropes in the initial state (with $L = l/2 = b - a$) and the single relaxed flux-robe (with $L = l = 2(b - a)$). The final value of $\mu, \mu_f$, is determined from the initial value in each flux-robe, $\mu_i$, by the constraint that both helicity, $K$, and total toroidal flux are conserved (where the dimensionless total toroidal flux initially is 2, because of the two flux-ropes). A root-finding process is used to find $\mu_f$ so as to conserve $K/\Phi_t^2$. The helicity is calculated requiring the toroidal component of the vector potential $A_\phi$ to vanish on the boundary so that the gauge-correction term arising from the flux through the torus [44] vanishes. It can be shown that this is related to the total magnetic energy, $W$, by

$$K = 2\mu_i W/\mu - 2\pi \psi_b \Phi_t, \quad (15)$$

where the dimensionless energy is obtained by integration of the fields, after use of Fourier identities for the summation over $\zeta$ and considerable algebra following [42], as

$$W = \frac{\psi_b^2}{\mu_0} \ln \left[ \frac{b + \mu^2}{a} \sum_{\text{value}} \frac{2(1 - j_m b/a)^2}{k_m^2 \beta_m b(1 - j_m^2)} (P_m(\beta_m^2 + \mu^2) + k_m Q_m + S_m) \right] \quad (16)$$

where

$$k_m^2 = \mu^2 - \beta_m^2, \quad (17)$$

$$P_m = \frac{1}{4} (\cos(k_m L) + 1) - 3 \tan(k_m L/2)(\cos(k_m L) + 1), \quad (18)$$

$$Q_m = 2 \tan(k_m L/2)(\cos(k_m L) - 1), \quad (19)$$

$$S_m = (k_m^2 - k_m \sin(k_m L)/L)(\cos(k_m L) + 1), \quad (20)$$

and $\psi_b(\mu; a, b)$ is given by the normalisation $\Phi_t = 1$ and $\mu_0 = 1$ in dimensionless units.

Typical initial and final field states in a tight-aspect ratio geometry are shown in figure 4. The energy released during relaxation is calculated as $\Delta W = W_i - W_f$ where the initial energy $W_i = 2W(\mu_i; a, b, b - a)$ and the final energy is $W_f = 4W(\mu_f; a, b, 2(b - a))$ (the factors 2 and 4 are to account for, respectively, two initial flux ropes each with $\Phi_t = 1$, and a single final flux rope with $\Phi_t = 2$. In figure 5, the final value of $\mu$ and energy release $\Delta W$ are plotted against the initial value, for both a tight-aspect ratio configuration similar to MAST and a very large-aspect ratio case. Note that the latter graph very closely resembles the infinite-aspect ratio results [16]. Indeed, it can be shown for an aspect-ratio larger than about 10 $\mu$ and $\Delta W$ are almost identical to the infinite-aspect ratio values. However, at lower aspect-ratios, $\Delta W$ significantly decreases; so that the previous infinite-aspect ratio model over-estimates the energy release for MAST geometry by a factor of around 2.

Since the toroidal plasma current is $I_t = \mu \Phi_t$, figure 5 may also be interpreted, re-scaling the axes by the appropriate fluxes, as showing the dependence of final plasma current on initial current in one flux rope. However, the initial state also includes a current sheet, and in order to calculate the total initial plasma current, we must add the (negative) net current in this sheet, which may be simply obtained as

$$I_{c,x} = -(2\mu_i) \int_a^b B_t(r, 0) dr.$$

It can be shown analytically that for small $\mu_i$, $\Delta W \sim \mu_i^2$, $\mu_f \sim \mu_i$. Hence the final plasma current depends linearly on the initial current; whilst the energy release depends quadratically on $\mu$, and hence also on the initial poloidal field. This
agrees with experimental findings in MAST and TS-3 [14].

The dependence of $\mu_f$ on $\mu_i$ shown in figure 5 appears as a straight line, but the scaling is in fact stronger than linear, which becomes evident for larger $\mu_i$ as there is a singularity at the first eigenvalue [42]. Such large values of current are not relevant to spherical tokamaks, but may be found in solar coronal loops (see section 4).

As shown in section 2, the magnetic energy released by reconnection may both heat the plasma and drive plasma flows—but the latter may also be dissipated to provide further heating. We obtain an estimate (strictly an upper bound) on the average temperature increase—here simply within single-fluid MHD—by assuming that the released magnetic energy is converted fully into thermal energy. Taking typical MAST parameters and equating the energy released to the gain in thermal energy, we estimate an average temperature increase of 100 eV, which is compatible with what is experimentally observed—of course, the relaxation model cannot predict any spatial distribution of temperature.

4. Merging flux ropes in the solar corona

The simulations of merging flux ropes described in section 2 may be of some significance in understanding reconnection in the solar corona. The latter is, like MAST, a low $\beta$ plasma, and whilst the Lundquist number in MAST (up to $10^7$) is much lower than the coronal value (around $10^{12-14}$), MAST comes closer to this highly-conducting regime than other laboratory reconnection experiments. Coronal reconnection is widely modelled using MHD, within which framework the classical ‘Sweet–Parker’ model predicts reconnection rates which are far too slow to explain coronal heating or solar flares, suggesting that physics beyond single-fluid MHD may be required [19]. Furthermore, the Sweet–Parker current sheet width in the solar corona is comparable with or smaller than the ion-skin depth, also indicating that Hall physics (at least) must be taken into account [16]. Indeed, analysis of regimes of reconnection suggests that this simple comparison of length-scales may under-estimate the importance of incorporating Hall physics [45]. The features of reconnection dynamics identified through the Hall-MHD simulations described in [15] and in section 2 above should thus be of some relevance to the solar corona. However, it is presently impossible to undertake simulations which encompass the full range of relevant scales in the solar corona.
corona, from the global scales of observable features, to the local scales relevant to dissipation such as the ion gyro-radius and the electron and ion skin depths; thus, the region simulated with Hall-MHD can only represent a localised structure embedded within a larger scale system. Resistive MHD simulations of merging flux ropes in the corona, brought together by attraction of like currents, have been performed by Kondrashov et al [46]; recently, a rather different scenario in which merger is triggered by kink instability in one flux rope has also been demonstrated [47].

In order to explain solar coronal heating, it is of primary importance to account for the dissipation of stored magnetic energy and, as first proposed by [48], relaxation theory provides a very useful framework. Browning et al [49] used an approximate relaxation model valid for fields close to potential, to predict heating rates in a set of adjacent twisted flux tubes. The relaxation model of merging flux tubes [16] may thus be adapted to model coronal heating. Since coronal loops have only weak curvature, and are not complete tori, the infinite-aspect-ratio model, in which the initial state comprises adjacent cylindrical twisted flux-ropes, is appropriate. The model has to be adapted for a loop of finite length $l$, line-tied to the photosphere at its ends: but it can be shown that the gauge-invariant definition of helicity used for an infinite-aspect-ratio system (periodic in the axial direction) [16] gives the relative helicity appropriate for a field with finite length and non-zero normal field at the boundaries.

We note that the energy release calculated in [16] may be approximated by

$$\Delta W \sim 0.1 \mu^2 r^4 B^2 l / l_\mu, \quad (21)$$

for a loop of radius $r$, length $l$ and peak axial field $B$. Using the relation $\mu \approx 2\phi/l$ where $\phi$ is the field-line twist-angle $\phi$ (valid near the loop axis) [50], and taking typical coronal values $B = 0.01$ T, $l = 10$ Mm, $r = 1$ Mm, we find for rather weakly-twisted loops with $\phi = \pi$ an energy release of around $10^{19}$ J. Complete conversion of this energy into thermal energy gives a temperature increase of around $10^7$ K for a plasma of density $10^{15}$ m$^{-3}$. This suggests that such flux tube mergers—if occurring sufficiently frequently—could make a significant contribution to coronal heating, and might be observable as microflares.

Twisted flux ropes in the corona may arise either through emergence from below the surface or by twisting through vortical photospheric motions. Both processes allow for the possibility that neighbouring twisted filaments may have twists in opposite directions (a situation not possible in the MAST experiment). We therefore consider the relaxation of two flux ropes with $\mu = \pm \mu_\mu$, hence having the same fields but with twists in opposing directions—this is the counter-helicity case, whereas the case previously discussed is co-helicity [51]. Furthermore, since the solar corona evidently lacks the symmetry of the MAST experiment, we would expect adjacent flux ropes—in general—to have different values of $\mu$; this cannot be accounted for in our simple analytical model (since fields with different magnitudes of $\mu$ would not match across the interface), but studying the case of equal and opposite values of $\mu$ should give a good indication of the expected behaviour for intermediate values.

Figure 6. Energy release as a function of the initial normalised current ($\mu i$) for a pair of flux tubes with twist in the same direction (red) and opposite directions (blue).

It should be noted that for consistency we use the symbol $\mu$ here to represent the ratio of current to magnetic field, see equation (6), but this is usually designated $\alpha$ in the solar literature.
For the case of opposing twists in two flux tubes, the total helicity is zero, and the relaxed state is therefore potential with $\mu = 0$. Hence the free magnetic energy is significantly greater than for two flux tubes with the same twist, as shown in figure 6. However, the initial configuration has no current sheet since the poloidal field is continuous across the interface $z = 0$; thus, reconnection and merging is very much less likely in this configuration. However, it is possible that merger—involving relaxation to the minimum energy state and release of the stored energy—could be initiated by a strong external trigger, although reconnection would proceed at a slower rate. Furthermore, if the magnitudes of $\mu$ are not exactly identical, then there will be some field discontinuity, and reconnection could occur. Indeed, evidence of such counter-helicity merging is observed in some solar flares [52], and the large energy release in such events is consistent with our predictions. As mentioned above, we may also infer from figure 6 the energy release in the case of merger of flux ropes with different magnitudes of twist: evidently, the greater the difference between the initial values of $\mu$, the larger the energy release.

According to the ‘flux tube tectonics’ scenario [22], the solar corona may contain many twisted filaments, which might be twisted in mixed directions. In reality, the $\mu$ values of such filaments may cover a range of values, but our analytical relaxation model allows us to model only the case of an array of filaments for which the magnitudes of $\mu$ are all equal (although the signs may vary, allowing for the possibility of
filaments twisted opposing senses). As an initial state, we assume a \( n = N \times N \) square array of flux-ropes each with \( \mu = \pm \mu_i \). First, consider the case of four tubes \( (N = 2) \), so there are three distinct possibilities for the signs of \( \mu_i \): all positive, three positive, one negative; two of each sign. (The relative positions of the different orientations is irrelevant to the energy release in the relaxation model, which is also unaffected by an overall change in sign of twist). As expected, the energy release is greatest in the case when equal numbers of positively and negatively twisted filaments are present, so that the total helicity vanishes and the minimum energy state is possible, see figure 7. As mentioned above, direct merger of the total helicity vanishes and the minimum energy state is attained if (for example) two tubes with \( \mu_i \) merge first, giving a combined tube with a different \( \mu \) which may now reconnect with a tube with \( -\mu_i \). Furthermore, figure 7 also demonstrates the general principle that the greater the initial variation in \( \mu_i \), the larger the energy release.

We then investigate the effect of varying the number of initial filaments, \( n \). The variation of energy release with initial \( \mu_i \) is shown in figure 8. Here, the magnetic flux and the total cross-section are kept constant, so that the width of each initial twisted filament is \( a/N \). In all cases, the final relaxed state consists of a single twisted flux tube; the energy released increases with the number of flux tubes, due mainly to the increasing number of current sheets. The scaling of energy release with number of merging tubes, for a fixed \( \mu_i \) is found to be very close to linear \( \Delta W \sim n^{1.07} \). This exemplifies relaxing an inverse-cascade from small-scale to large-scale structure.

5. Conclusions

We have described magnetic reconnection during the merging of magnetic flux ropes in the MAST spherical tokamak and in the solar corona—in both cases, reconnection leads to the formation of larger-scale magnetic structures and the dissipation of free magnetic energy. Simulations of the merging using 2D resistive MHD and Hall-MHD have been performed, showing in detail how the reconnection rate and dynamics depend on the collisionality. The simulations predict the formation of a spherical tokamak with a single set of closed flux surfaces; the temporal evolution of the density profile matches experimental results. The inclusion of the Hall term in Ohm’s law permits much faster reconnection. Whilst the ion skin depth is relatively much smaller, in comparison with global length scales, in the solar corona, the Hall term is also likely to be significant there, and the simulations are indicative of processes which may be occurring in the corona. Simulations show that the magnetic reconnection is associated with significant heating of both ions and electrons. Future work will model reconnection in merging solar flux ropes, taking account of the Hall term.

An analytical model based on helicity-conserving relaxation to a minimum-energy state allows prediction of the final field configuration and the energy release, which may be easily applied to different configurations and parameters. The average temperature increase can be calculated if it is assumed that all the dissipated magnetic energy is converted to thermal energy. We described how a previous model assuming infinite aspect-ratio can be extended to finite aspect-ratio geometry, with particular relevance to spherical tokamaks. The infinite-aspect ratio model provides a good approximate model, but the energy release for a given initial ratio of current to field is approximately a factor of two lower in the finite aspect-ratio model for MAST parameters.

The infinite-aspect-ratio relaxation model has been adapted to apply to coronal loops. Simple estimates of the temperature increase associated with the merging of two weakly-twisted coronal loops suggest this process could contribute significantly to coronal heating. In the coronal case, adjacent flux tubes may be twisted in the same or in opposite senses—the latter case has significantly larger free energy, but merger is very unlikely if the twists are exactly opposite. However, significant energy release is likely if the twists are opposite, but of different magnitudes. A single coronal loop may consist of multiple twisted threads. Within a given volume, the energy release increases with the number of twisted threads, correlating with the number of current sheets.

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