Brane Boxes, Anomalies, Bending and Tadpoles

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Abstract

Certain classes of chiral four-dimensional gauge theories may be obtained as the worldvolume theories of D5-branes suspended between networks of NS5-branes, the so-called brane box models. In this paper, we derive the stringy consistency conditions placed on these models, and show that they are equivalent to anomaly cancellation of the gauge theories. We derive these conditions in the orbifold theories which are T-dual to the elliptic brane box models. Specifically, we show that the expression for tadpoles for unphysical twisted Ramond-Ramond 4-form fields in the orbifold theory are proportional to the gauge anomalies of the brane box theory. Thus string consistency is equivalent to worldvolume gauge anomaly cancellation. Furthermore, we find additional cylinder amplitudes which give the $\beta$-functions of the gauge theory. We show how these correspond to bending of the NS-branes in the brane box theory.

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1 Introduction

Following the work of [1], the study of brane configurations in string theory and M-theory has provided a useful description of supersymmetric field theories. Given the success of this approach, it is desirable to utilize it to confront the interesting questions of four dimensional chiral gauge theories with $N = 1$ SUSY, primarily the question of spontaneous supersymmetry breaking. Certain chiral theories have been constructed in [3], but a general construction has proven to be difficult.

Such a construction of a fairly general class of chiral gauge theories has been introduced in Ref. [4]. The simplest such models (with no orientifold planes) have a product of unitary gauge groups and bifundamental matter. They are constructed as the worldvolume theory of D5 branes that extend in 4 non-compact directions, and are finite in two directions (denoted 4,5). In the directions (4,5) the D5 branes end on infinite NS fivebranes. Such a model is depicted in Figure 1; horizontal and vertical lines denote NS5-branes, while shaded areas represent D5-branes, of which there are $N_{rs}$ in the $(r,s)^{th}$ box. Arrows between boxes represent chiral multiplets, whose $SU(N)$ representation is by convention fundamental for outgoing arrows. A precise description of the matter content and the matter couplings can be found in [4].

![Figure 1: One box of a brane box model, showing chiral multiplets.](image)

\[1\] For a recent review and references see [2].
As given, the brane box represents the configuration at zero string coupling. At a finite string coupling, it is an interesting question as to which brane boxes (given by the “rank matrix” \( \{N_{r,s}\} \)) are consistent. Certainly, we expect that a necessary condition, at the level of the field theory, is gauge anomaly freedom. It is not immediately clear how to derive this condition from string theory. The most imposing problem is the complicated background implied by the presence of the NS5-branes, as well as the fact that \( D5 \)-branes end on them.

Attempts at answering this question have been made. At finite coupling, the NS5-branes generically will bend. Gimon and Gremm\(^5\) have given a condition which they claim was necessary to avoid tearing of the brane configuration. This suggestion has several problems; first, it rules out several clearly consistent models, and second, it has been recently shown\(^6\) that tearing does not occur, as there are sufficient dimensions available into which the branes may distort. Brane bending is in fact connected directly to renormalization group flow (scale anomalies) rather than gauge anomalies.

The question remains then what exactly are the conditions for consistency, and how they are understood in terms of string theory. In this paper, we answer this question in the case of “elliptic” models, where identifications are made across the brane box (i.e. the system is compactified in the 4,5 directions). Since any brane box configuration can be embedded in a periodic configuration (allowing for empty rows and columns), it would seem that this causes no loss of generality. However, since the directions 4,5 are taken to be finite, no gauge group factor can be truly reduced to a flavor symmetry. Therefore one cannot realize gauge theories with anomalous flavor symmetries in the elliptic construction\(^2\).

In the elliptic cases, there is a T-dual configuration \(^7, 8\) of D3-branes at an orbifold singularity. The action of the orbifold group on Chan-Paton factors is given in terms of the rank matrix \( N_{r,s} \). In this orbifold theory, it is a straightforward application of previous work\(^9\) to compute the consistency conditions. We do this by computing certain cylinder amplitudes, and find generically that there are tadpoles for unphysical twisted Ramond-Ramond 4-form potentials. Consistency of string theory in the orbifold background requires the vanishing of all such tadpoles. We show that these conditions are exactly the anomaly freedom conditions in the gauge theory.

There are certain twist sectors in the cylinder amplitudes which do not

\(^2\)At least without introducing additional branes.
contribute to the unphysical tadpole. Instead, they lead to logarithmic behaviour for certain physical modes. The coefficient of this is proportional to the $\beta$-function coefficients of the gauge theory. By identifying these modes and their T-dual partners, it is possible to relate this to brane bending, at least in simple cases where the geometry is most clear.

The paper is organized as follows. We begin by discussing anomalies in the brane configuration from the field theory point of view. In particular we derive the set of conditions that is necessary and sufficient to ensure anomaly freedom in the gauge theory. We find the general solution of these conditions for “non-degenerate” models (defined to be boxes such that $N_{r,s} \neq 0$ for all $r, s$). In section 3 we discuss the consistency conditions for string theory in the orbifold background, and show that these conditions are identical to those derived from the field theory perspective. In Section 4, we discuss the $\beta$-functions.

## 2 Field Theory Analysis

### 2.1 Non-Degenerate Cases

We begin the discussion by identifying scale and gauge anomalies in the worldvolume theory of a given brane box model. There will be one anomaly for each $SU(N_{rs})$ factor. The gauge anomaly is proportional to

$$A_{(r,s)} = \frac{1}{2}(N_{r-1,s} + N_{r,s+1} + N_{r+1,s-1} - N_{r+1,s} - N_{r,s-1} - N_{r-1,s+1}).$$

(1)

In a consistent gauge theory each of these expressions has to vanish, as long as $N_{r,s} \geq 1$. Obviously, for $N_{r,s} = 0$, there is no corresponding gauge group, and from the field theory perspective there is no resulting constraint. The situation for $N_{r,s} = 1$ is more surprising: our results indicate that even though the $U(1)$ theory decouples in the infra-red, the anomaly constraints of this gauge theory must be imposed for consistency of the string theory.

We start by analyzing configurations for which $N_{r,s} \neq 0$ for all $r, s$; these configurations we will call “non-degenerate.” Later on we extend the analysis to the degenerate cases. In the non-degenerate cases one imposes $A_{r,s} = 0$ for all $r, s$.

The key to comparing these expressions to the tadpoles in an orbifold realization of the gauge theory is to express them in terms of Chan-Paton
matrices which represent the orbifold group action. We do this explicitly for the case of a $\mathbb{Z}_k \times \mathbb{Z}_{k'}$ orbifold. This corresponds to a brane box with trivial identifications. The generalization to the other cases is straightforward.

The correspondence is as follows. An irreducible representation of $\mathbb{Z}_k \times \mathbb{Z}_{k'}$ assigns to the two generators $\alpha, \beta$ the phases $(\omega^r, \omega^s) \equiv (e^{2\pi ir/k}, e^{2\pi is/k'})$. The Chan-Paton matrix representing the element $\alpha^r \beta^{n'}$ is then a diagonal matrix $\gamma_{(n,n')}$. Thus the Chan-Paton matrix satisfies

$$\text{tr} \gamma_{(n,n')} = \sum_{r,s} N_{r,s} \omega^{rn} \omega^{sn'}$$  \hspace{1cm} (2)$$

Note then that T-duality can be thought of as a discrete Fourier transform between the rank matrix $N_{r,s}$ and the Chan-Paton matrix $\gamma_{(n,n')}$, where twist plays the role of momentum. In particular $\text{tr} \gamma_{(0,0)} = \text{tr} 1 = \sum_{r,s} N_{r,s} = n_3$, where $n_3$ is the number of D3-branes in the orbifold theory.

Now, we can isolate $N_{r,s}$ by an inverse Fourier transform:

$$N_{r,s} = \frac{1}{kk'} \sum_{n,n'} \omega_{k}^{nr} \omega_{k'}^{-ns} \text{tr} \gamma_{(n,n')}.$$  \hspace{1cm} (3)$$

Using this expression we can rewrite eq. (1)

$$A_{(r,s)} = \frac{i}{kk'} \sum_{n,n'} \omega_{k}^{nr} \omega_{k'}^{-ns} a_{n,n'} \text{tr} \gamma_{(n,n')}$$  \hspace{1cm} (4)$$

where

$$a_{n,n'} = \sin(2\pi z) - \sin(2\pi z') - \sin(2\pi(z-z'))$$

$$\equiv -4 \sin \pi z \sin \pi z' \sin \pi(z-z')$$  \hspace{1cm} (5)$$

and we have introduced the notation $z = n/k$ and $z' = n'/k'$.

Since the above expressions involve a sum over a complete set of Fourier modes, the vanishing of the anomaly requires a vanishing of each coefficient separately. Therefore in an anomaly free theory we must have either $a_{n,n'} = 0$ or $\text{tr} \gamma_{(n,n')} = 0$, for every $n, n'$. These are at most $kk'$ conditions on the rank matrix $N_{r,s}$, which has $kk'$ entries. For every instance where $a_{n,n'} = 0$ we expect then to find a free integer parameter in the solution for $N_{r,s}$.

Similarly, the one-loop coefficient of the $\beta$-function (scale anomaly) is

$$\beta_{(r,s)} = 3N_{r,s} - \frac{1}{2} (N_{r-1,s} + N_{r,s+1} + N_{r+1,s-1} + N_{r+1,s+1} + N_{r,s-1} + N_{r-1,s+1})$$  \hspace{1cm} (6)$$
These may be re-written as

$$\beta_{(r,s)} = \frac{1}{kk'} \sum_{n,n'} \omega_k^{-nr} \omega_{k'}^{-ns} b_{n,n'} \text{ tr } \gamma(n,n')$$  \hspace{1cm} (7)

where

$$b_{n,n'} = \begin{cases} 
3 - \cos(2\pi z) - \cos(2\pi z') - \cos(2\pi(z - z')) & \text{if } n = n' = 0, \\
-2 \left( \sin^2 \pi z + \sin^2 \pi z' + \sin^2 \pi (z - z') \right) & \text{otherwise.}
\end{cases}$$  \hspace{1cm} (8)

We first comment on the general conditions for scale invariance of the gauge theory. We note that $b_{0,0} = 0$, and therefore there is no restriction on $\text{tr } \gamma_{0,0} = n_3$. However, all other traces must vanish since $b_{n,n'}$ is non-zero. This singles out the regular representation of $\mathbb{Z}_k \times \mathbb{Z}_k'$, or any number of copies of it, as the most general solution. The corresponding brane box model has $N_{r,s} = N$ for every $r, s$. The free parameter here is the rank $N$ of each gauge group factor. Such models were considered in [10] and were shown to be finite theories. The vanishing of the beta function was related to the absence of bending in the brane box construction.

More generally, we now discuss the conditions for anomaly freedom of the non-degenerate models. Note that $a_{n,n'} = 0$ for $z = 0, z' = 0$ or $z = z'$ (the latter is only possible when $k, k'$ are not relatively prime). Therefore there is no restriction on $\text{tr } \gamma(n,n')$ in those cases. For all other values of $n, n'$ the corresponding trace has to vanish.

The most general solution to this condition has a simple geometrical interpretation in the brane box picture. We start with the models with $\text{tr } \gamma_{(n,0)} \neq 0$ and all other traces vanishing. The corresponding box configurations are translationally invariant in the vertical direction, and have $N = 2$ SUSY. Similarly the two other cases have horizontal and diagonal symmetry, and have therefore the matter content and interactions of $N = 2$ SUSY theory. The horizontal and vertical models have $k', k$ free integers in $N_{r,s}$, whereas the diagonal ones have $\gcd(k, k') - 1$ such free parameters.

Since the condition for anomaly freedom is linear in the rank matrix $N_{r,s}$, one can superimpose configurations which are separately non-degenerate. This process was called ”sewing” in [7]. We therefore find that the general class of “sewn $N = 2$” models of [7] is the general solution of the anomaly constraints for non-degenerate models. This class of models has $k + k' + \gcd(k, k') - 2$ free integer parameters in the matrix $N_{r,s}$. It includes (but is
not restricted to) the general solution to the conditions suggested by Gimon and Gremm[5].

We note that such “sewing” will in general give an anomalous theory if one of the ingredients is a degenerate configuration. Such a configuration is not required to satisfy $A_{r,s} = 0$ for all boxes, and therefore can violate the necessary anomaly constraints once superimposed on a non-degenerate configuration.

### 2.2 General Case

It is obvious that the above conditions do not yield the most general anomaly free model. An obvious exception is the pure SYM theory that is realized by a box configuration with a single non-vanishing entry in the rank matrix $N_{r,s}$. To allow for such degenerate cases as well we define the quantity:

$$C_{r,s} \equiv N_{r,s} A_{r,s} \quad \text{(no summation)} \quad (9)$$

Clearly, the vanishing of $C_{r,s}$ is exactly equivalent to anomaly freedom of the corresponding gauge theory. This vanishing condition reduces to $A_{r,s} = 0$ for the non-degenerate cases discussed above. We note that this condition is no longer linear in the rank matrix $N_{r,s}$, thus “sewing” models is not allowed in the general case, as expected.

In the next section we derive this general condition from demanding consistency of the string background, which requires the cancellation of certain tadpoles in the orbifold background. To make connection to the tadpole calculation it is convenient to define the quantity

$$C_{n,n'} \equiv \sum_{r,s} \omega_{r,k} \omega_{s,k'}^{n'} N_{r,s} A_{(r,s)} \quad (10)$$

which is simply the Fourier transform of $C_{r,s}$. Using eqs. (3), (4), we can then deduce

$$C_{n,n'} = \frac{i}{kk'} \sum_{m,m'} a_{m,m'} \text{ tr } \gamma_{m,m'} \text{ tr } \gamma_{-n,m'-n}^{-1} \quad (11)$$

The anomaly freedom is thus equivalent to the vanishing of these Fourier modes for every $n, n'$. This imposes at most $kk'$ conditions on the rank matrix $N_{r,s}$. 

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We now turn to derive these conditions as consistency conditions of the orbifold theory. The quantities $C_{n,n'}$ are shown in the next section to correspond to certain cylinder amplitudes that have to vanish to ensure background consistency.

### 3 Tadpoles in the Orbifold picture

Next we consider the T-dual orbifold in detail. This consists of $n_3$ D3-branes at a $\mathbb{Z}_k \times \mathbb{Z}_{k'}$ orbifold singularity. We choose coordinates such that the orbifold action is generated by

$$
\alpha : (z_1, z_2, z_3) \rightarrow (\omega_k z_1, \omega_k^{-1} z_2, z_3) \quad (12)
$$

$$
\beta : (z_1, z_2, z_3) \rightarrow (z_1, \omega_{k'} z_2, \omega_{k'}^{-1} z_3) \quad (13)
$$

Tadpoles may be deduced by factorization on the closed string channel of the cylinder amplitude

$$
\text{tr}_{NS-R} \sum_{n,n'=0}^{k-1,k'-1} \frac{\alpha^n \beta^{n'}}{kk'} \frac{1}{2} \sum_{n,n'} \alpha^n \beta^{n'} 1 + (-)^F e^{-2\pi t L_o} \quad (14)
$$

The amplitude may be deduced by study of Refs. We simplify notation as follows: $\theta_k(z) \equiv \theta_k(z|t)$, defined in the usual way in terms of $q = e^{-\pi t}$.

The factor of $a_{n,n'}$ has appeared in the amplitude through the definition of the $\theta_1$'s. The factor of $W_{n,n'}$ denotes the partition function of winding modes which may be present in some twist sectors (when $z = 0$, or $z' = 0$ or $z = z'$).

The quantity on the second line of course vanishes (abstrusely). The $t \to 0$ limit corresponds to a long cylinder, and thus the amplitude will factorize.

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3 We simplify notation as follows: $\theta_k(z) \equiv \theta_k(z|t)$, defined in the usual way in terms of $q = e^{-\pi t}$.

4 We have defined $v_4 = V_4/(4\pi^2 \alpha')^2$, where $V_4$ is the regulated volume of spacetime.
into a product of tadpoles. In particular, there are tadpoles for Ramond-Ramond forms which are unphysical; in a consistent theory, these must all vanish separately. The expression above implies that in sectors \( m, m' \neq 0 \), for which \( W_{n,n'} = 1 \) (no winding modes), there is a tadpole for twisted RR 4-form potentials. Indeed, in the factorization limit (\( \ell = 1/t \to 0 \)), those sectors give

\[
A_{0,0}^{(4)} \sim (1 - 1) \int d\ell \, V_4 \frac{i}{kk'} \sum_{m,m' \neq 0} a_{m,m'} \text{tr} \gamma_{m,m'} \text{tr} \gamma^{-1}_{m,m'}
\]

This tadpole must be cancelled since the field is supported in the orbifold singularity; there are no transverse directions to propagate in.

There are \( kk' \) tadpoles that must be cancelled. One could isolate the tadpoles themselves, but it is more convenient to do the following formal trick to isolate the \( kk' \) quantities in a convenient fashion. Define additional cylinder amplitudes, with the insertion of a twist operator, as indicated in Figure 2. This corresponds to an injection of twist "momentum" into the vacuum-vacuum amplitude, and in the factorization limit, we obtain

\[
A_{n,n'}^{(4)} = (1 - 1) \int d\ell \, V_4 \frac{i}{kk'} \sum_{m,m' \neq 0} a_{m,m'} \text{tr} \gamma_{m,m'} \text{tr} \gamma^{-1}_{m,m'}
\]

\[\text{(17)}\]

\[5\text{Purely numerical overall factors will be dropped.}\]
We obtain $kk'$ amplitudes which are bilinear in the $kk'$ tadpoles and therefore must vanish. Note that these amplitudes are precisely the quantities $C_{n,n'}$, defined in the last section in terms of $N_{r,s}$ and the field theory anomalies $A_{r,s}$.

Thus we conclude that the cancellation of unphysical tadpoles is precisely equivalent to anomaly cancellation. Note as well that the string theory is smart enough to require cancellation of only those anomalies ($N_{r,s} \geq 1$) which must be cancelled, again, because the amplitude is proportional to $N_{r,s}A_{r,s}$.

4 The $\beta$-Functions

We have seen that the twist sectors for which $m, m' \neq 0$ give rise to unphysical tadpoles. There are also the twist sectors with $z = 0, z' = 0$ or $z = z'$, for which $W_{n,n'} \neq 1$. These will produce tadpoles for additional, partially twisted, RR potentials. These sectors are special, in that they effectively reduce to an A-type orbifold singularity. Consequently, in the four-dimensional theory there is effectively $N = 2$ supersymmetry, when restricted to one of these sectors.

The cylinder amplitude for these sectors is of the form:

$$A_{0,0'}^{(6)} = (1 - 1) \int \frac{d\ell}{\ell} \frac{i}{kk'} v_4$$

$$\times \left( \sin^2 \pi z W_{z,0} \operatorname{tr} \gamma_{z,0} \operatorname{tr} \gamma_{z,0}^{-1} + \sin^2 \pi z' W_{0,z'} \operatorname{tr} \gamma_{0,z'} \operatorname{tr} \gamma_{0,z'}^{-1} \\
+ \sin^2 \pi z W_{z,z} \operatorname{tr} \gamma_{z,z} \operatorname{tr} \gamma_{z,z}^{-1} \right)$$

We have not yet evaluated the winding factors. They are, for example, of the form:

$$W_{z,0} = \prod_{i=8,9} \sum_w e^{-2\pi tw^2 R_i^2}$$

Strictly speaking, we are working at an isolated orbifold point, and it is necessary to take the $v \to \infty$ limit. This should be done before taking the $\ell \to \infty$ limit (and hence Poisson resummation is unnecessary). Thus, at

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6 We continue to write a contribution from the $z = z'$ sector; as noted, it exists only when $k$ and $k'$ are not relatively prime.
infinite volume, we get

\[ A_{0,0}^{(6)} = \frac{iv_4}{kk'} (1 - 1) \int \frac{d\ell}{\ell} \]

\[ \times \left( \sin^2 \pi z \text{tr} \gamma_{z,0}^{1} + \sin^2 \pi z' \text{tr} \gamma_{0,z'}^{1} + \sin^2 \pi z \text{tr} \gamma_{z,z}^{1} \right) \]

We now compare this expression with the $\beta$-function of the gauge theory, (7),(8), or more precisely to $N_{r,s} \beta_{r,s}$. Given cancellation of unphysical tadpoles, the expression for $N_{r,s} \beta_{r,s}$ simplifies dramatically. As a result, we find

\[ A_{0,0}^{(6)} = v_4 (1 - 1) \int \frac{d\ell}{\ell} \sum_{r,s} N_{r,s} \beta_{r,s} \]

(21)

As in the previous case, one could put in a twist in the cylinder amplitude to reproduce all Fourier modes of $N_{r,s} \beta_{r,s}$.

The logarithmic divergence $d\ell/\ell$ in eq. (21) has a clear physical interpretation. Indeed, the propagator of a two-dimensional field of mass $\epsilon$, at zero spacetime separation, may be written as

\[ \int \frac{dt}{t} e^{-t \epsilon^2}. \]

(22)

Therefore we interpret $\epsilon$ as an infrared cutoff in spacetime.\footnote{There is a similar discussion in Ref. \cite{11} with $N = 2$ supersymmetry. In that case, one could regulate by moving on the Coulomb branch (separating the 3-branes). In this $N = 1$ case, the 3-branes cannot be moved off of the orbifold. The precise regularization is immaterial to our discussion.}

The divergence is caused by the propagation of the partially twisted forms in two transverse directions.

We wish to absorb this divergence in the redefinition of the couplings of the gauge theory on the D3-branes, in the general spirit of renormalization theory, similar to the Fischler-Susskind mechanism\cite{12}. The general Wess-Zumino action of the gauge theory is: \cite{13}

\[ S = \sum_{r,s} \int C^{(r,s)} \wedge Tr(e^{Fr,s}) \]

(23)

where $F_{r,s}$ is the field strength of $SU(N_{r,s})$, and $C^{(r,s)}$ is a formal sum of couplings related to background values of RR forms in spacetime.\footnote{Specifically, $C^{(r,s)}$ is the Fourier transform of $C_{n,n'}$, the background value of the RR form in the $(n,n')$ twisted sector.} The trace is taken in the fundamental representation \cite{13}.
Working in a particular \((r, s)\) sector, the first term in this coupling is:

\[
\int C_{(4)}^{(r,s)} \text{Tr}(1) = \int C_{(4)}^{(r,s)} N_{r,s}.
\] (24)

We see therefore that the regulated amplitude leads to a renormalization of the coupling \(C_{(4)}^{(r,s)}\) as given by

\[
\delta C_{(4)}^{(r,s)} \sim \beta_{r,s} \ln \epsilon.
\] (25)

Another coupling in (24) is the term \(C_{(0)}^{(0)} \wedge F \wedge F\), with the same coefficient (since the couplings are related by T-duality). This identifies \(C_{(0)}^{(r,s)}\) as the gauge coupling of the \((r, s)^{th}\) gauge group. We conclude therefore that the cylinder amplitudes presented above indeed reproduce the correct running of the gauge couplings.

One can demonstrate this more explicitly by considering the two-point function of \(F\) on the cylinder. It is convenient to perform this calculation instead as the vacuum-vacuum cylinder diagram with a background \(F\) turned on. The \(F^2\) term in the effective action is then obtained by expansion for small \(F\) \(^{[14]}\). It is sufficient to consider the cylinder amplitude with a magnetic field turned on in the 2,3-direction, \(F_{23} = BQ\), where \(Q\) is an element of the Cartan subalgebra. This has a trivial effect on the computation; the effect of the magnetic field \(^{[15]}\) is to shift the oscillator frequencies for the 2,3-modes, by a factor \(\epsilon = \frac{1}{\pi} (\tan^{-1} \pi q_i B + \tan^{-1} \pi q_j B)\). As a result, the main effect is to modify one of the \(\theta\)-functions, and thus the cylinder amplitude becomes

\[
A_{0,0'}(F) = \frac{V_4}{2kk'} \int \frac{dt}{t} (8\pi^2 \alpha' t)^{-1} \sum_{i,j} \frac{(q_i + q_j)B}{2\pi} \sum_{n,n'} W_{n,n'} a_{n,n'} (\gamma_{(n,n')} i) i (\gamma_{(n,n')}^{-1}) j j \\
\times \left[ \theta_3(i\epsilon t) \theta_3(z) \theta_3(z - z') \theta_3(z') - (3 \leftrightarrow 2) - (3 \leftrightarrow 4) + (3 \leftrightarrow 1) \right]
\] (26)

Expansion of this formula for small \(\epsilon\) gives us

\[
A_{0,0'}(F) \sim \frac{v_4}{kk'} \int \frac{d\ell}{\ell} \left( \sin^2 \pi z \text{ tr } \gamma_{(z,0)}^{-1} \text{ tr } \gamma_{(z,0)} F^2 + \ldots \right) + \ldots \] (27)

where the first ellipsis denotes the other twist sectors which contribute. Note that we may write \(\text{tr } \gamma_{(z,0)}^{-1} F^2 = \sum_{r,s} e^{-2\pi i z r} \text{ tr } F_{r,s}^2\), and so the amplitude reduces to

\[
A_{0,0'}(F) \sim v_4 \int \frac{d\ell}{\ell} \sum_{r,s} \beta_{r,s} \text{ tr } F_{r,s}^2 + \ldots
\] (28)
This result clearly demonstrates the gauge coupling renormalization, and is in agreement with the above discussion.

### 4.1 Bending

We may now relate the above effect directly to bending of the NS-branes in the brane box theory. First note that one may think of the 3-branes of the orbifold model as higher dimensional D-branes wrapped on shrunken cycles at the orbifold points. In the present case, we do not have a K3 orbifold, but rather a Calabi-Yau orbifold, and so there are D5-branes wrapped on shrunken 2-cycles, as well as D7-branes wrapped on shrunken 4-cycles. In this general case, the corresponding effects discussed in the previous section are not entirely geometric in the brane box picture.

The situation simplifies however in the “N=2 limits”, (see also discussion in Ref. [17]) corresponding to confining ourselves to a given twist sector, say \((n,0)\). In this case, we effectively have an A-type singularity (times a \(T^2\)). In this case, we have

\[
\int_{\mathbb{R}^4} C^{(4)} \equiv \int_{\mathbb{R}^4 \times S^2} C^{(6)}
\]

(29)

and

\[
\int_{\mathbb{R}^4} C^{(0)} \wedge F \wedge F \equiv \int_{\mathbb{R}^4 \times S^2} C^{(2)} \wedge F \wedge F
\]

(30)

Here, \(C^{(2)} = B_{NS} + i B_{RR}\). In this simplified case, these Wilson lines are T-dual to position of the NS5-branes. Therefore the \(C^{(z,0)}\) and \(C^{(0,z')}\) have a geometric interpretation in the brane box theory as bending of horizontal and vertical NS5-branes. However, in general the \(\beta\)-functions \(\beta_{r,s}\) depend on all the Fourier modes \(C^{(n,n')}\), most of which are unrelated to simple bending in the brane box picture.

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