Impact-parameter dependence of the generalized parton distribution of the pion in chiral quark models *

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Abstract

We compute the off-forward diagonal (non-skewed) non-singlet generalized parton distribution of the pion in two distinct chiral quark models: the Nambu-Jona-Lasinio model with the Pauli-Villars regulator and the Spectral Quark Model. The analysis is carried out in the impact-parameter space. Leading-order perturbative QCD evolution is carried out via the inverse Mellin transform in the index space. The model predictions agree very reasonably with the recent results from transverse-lattice calculations, reproducing qualitatively both the Bjorken-\(x\) and the impact-parameter dependence of the data.

Key words: off-forward generalized parton distribution of the pion, chiral quark models, perturbative QCD evolution

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1 Introduction

Recently, transverse-lattice calculations have provided first data [1] on the impact-parameter dependent diagonal (non-skewed) non-singlet generalized parton distributions of the pion. Generalized parton distribution (GPD) have been

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a subject of intense studies in recent years [2,3,4,5,6,7] (for a review see, e.g., Ref. [8,9] and references therein) providing a unified framework for numerous high-energy phenomena. The impact-parameter-space formulation has been pursued in Refs. [10,11,12,13,14]. Actually, this is the natural framework for the transverse lattice QCD formulation [15,16,17,1]. In addition, the diagonal (non-skewed) distributions incorporate radiative corrections according to the standard DGLAP evolution equations for not-too-small values of the impact parameter $b$ [18,19]. The results of Ref. [1] may also provide some guidance on the yet unknown low-$b$ evolution of the GPD’s.

In this paper we obtain theoretical predictions for the GPD from two different chiral quark models, i.e., models incorporating the dynamical chiral symmetry breaking: the recently-proposed Spectral Quark Model [20,21] and the Nambu–Jona-Lasinio model with the Pauli-Villars regulator [22,23,24,25,26]. For these models it has already been shown that the $b$-integrated (forward) parton distribution functions agree remarkably well with the phenomenological parameterization at $Q^2 = 4$ GeV$^2$ [27]. Our very simple predictions for the GPD, pertaining to a low scale of about 320 MeV, are then evolved with the help of the standard DGLAP equations to the scales corresponding to the transverse-lattice calculations [15,16,17,1]. After the evolution the results of Sec. 6 are in a good qualitative agreement with the data, showing similar Bjorken-$x$ dependence in the corresponding impact-parameter bins.

2 Definitions

The off-forward ($\Delta_\perp \neq 0$) diagonal ($\xi = 0$) generalized parton distribution of the pion is defined by [12]$^1$

$$
H(x, \xi = 0, -\Delta_\perp^2) = \int d^2b \int \frac{dz^-}{4\pi} e^{i(xp^+z^-+\Delta_\perp \cdot b)}
\times \langle \pi^+(p')|\bar{q}(0, -\frac{z^-}{2}, b)\gamma^+ q(0, \frac{z^-}{2}, b)|\pi^+(p)\rangle, \quad (1)
$$

where $x$ is the Bjorken $x$, and $\Delta_\perp = p' - p$ lies in the transverse plane. This function has the interesting properties,

$$
\int_0^1 dx H(x, 0, -\Delta_\perp^2) = F(-\Delta_\perp^2), \quad H(x, 0, -\Delta_\perp^2 = 0) = q(x), \quad (2)
$$

$^1$ We drop the quark flavour index since, e.g., for a positively charged pion, $\pi^+$, one has $H_u(x, 0, t) = H_d(1 - x, 0, t)$. 

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Fig. 1. The diagram for the evaluation of the generalized parton distribution of the pion in chiral quark models.

relating it to the pion electromagnetic form factor, $F(t)$, and to the pion forward parton distribution, $q(x)$. One can introduce the impact-parameter representation \[12\],

$$q(b, x) = \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-ib \cdot \Delta_\perp} H(x, 0, -\Delta_\perp^2) = \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H(x, 0, -\Delta_\perp^2), \quad (3)$$

where the cylindrical symmetry has been used. The second of Eq. (2) corresponds to $\int d^2 b q(x, b) = q(x)$.

3 Evaluation in chiral quark models

In chiral quark models the evaluation of $H$ at the leading-$N_c$ (one-loop) level amounts to the calculation of the diagram of Fig. 1, where the solid line denotes the propagator of the quark of mass $\omega$. Formally\(^2\), this yields

$$H(x, 0, -\Delta_\perp^2; \omega) = iN_c \omega^2 \frac{f_\pi^2}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma^+ \frac{1}{k - p' - \omega \gamma^5 k - \omega \gamma^5 k - \bar{p} - \omega} \right] \times \delta \left[ k^+ - (1 - x) p^+ \right], \quad (4)$$

with $f_\pi = 93$ MeV denoting the pion decay constant and $p^2 = p'^2 = m_\pi^2$. The light-cone coordinates are defined as

$$k^+ = k^0 + k^3, \quad k^- = k^0 - k^3, \quad \vec{k}_\perp = (k^1, k^2), \quad dk^0 dk^3 = \frac{1}{2} dk^+ dk^-.$$

\(^2\) The gauge invariant regularizations, allowing to shift the momentum in the integral, will be specified later.
The calculation becomes simplest in the Breit frame, $\Delta^+ = 0$. The Cauchy theorem can be applied for the $k^-$ integration [28], yielding, after integration and in the subsequent chiral limit of $m_\pi \to 0$, the result

$$H(x, 0, -\Delta_\perp^2; \omega) = \frac{N_c \omega^2}{\pi f^2_\pi} \int \frac{d^2 K_\perp}{(2\pi)^2} \frac{1 + \frac{K_\perp \cdot \Delta_\perp (1-x)}{K^2_\perp + \omega^2}}{(K_\perp + (1-x)\Delta_\perp)^2 + \omega^2}, \quad (6)$$

where the relative perpendicular momentum is $K_\perp = (1-x)p_\perp - xk_\perp$.

To proceed further, we need to specify the regularization. First, we consider the recently proposed Spectral Quark Model [20,21]. The approach is successful in describing both the low- and high-energy phenomenology of the pion, and it complies to the chiral symmetry, including the anomalies. The model amounts to supplying the quark loop with an integral over $\omega$ weighted by a quark spectral density $\rho(\omega)$,

$$H(x, 0, -\Delta_\perp^2) = \int_C d\omega \rho(\omega) H(x, 0, -\Delta_\perp^2; \omega), \quad (7)$$

where $C$ is a suitably chosen integration contour in the complex $\omega$ space [21]. Next, we apply the simple techniques described in detail in Ref. [21], use the Feynman trick for the two denominators in Eq. (6), and integrate over $K_\perp$. The result is

$$H(x, 0, -\Delta_\perp^2) = 1 + \frac{N_c}{8\pi^2 f^2_\pi} \int \omega^2 \rho(\omega) d\omega \int_0^1 d\alpha \frac{(1-x)^2 \Delta_\perp^2}{\omega^2 + \alpha(1-\alpha)(1-x)^2 \Delta_\perp^2}, \quad (8)$$

Note the correct normalization condition, $F(0) = 1$. Moreover, the pion electromagnetic ms radius is $\langle r^2 \rangle \equiv -6dF(t)/dt|_{t=0} = N_c/(4\pi^2 f^2_\pi)$.

In the Meson Dominance variant [21] of the Spectral Quark Model the relevant part of the spectral function has the form

$$\rho_V(\omega) = \frac{1}{2\pi i} \frac{3\pi^2 m^3_\rho f^2_\pi}{4 N_c} \frac{1}{\omega (m^2_\rho/4-\omega^2)^{5/2}}, \quad (9)$$

where $m_\rho = 770$ MeV is the mass of the $\rho$ meson\(^3\). The function $\rho_V(\omega)$ has a single pole at the origin and branch cuts starting at $\pm m_\rho/2$. The contour $C$ encircles the branch cuts, i.e., starts at $-\infty + i0$, goes around the branch point at $-m_\rho/2$, and returns to $-\infty - i0$, with the other section obtained by

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\(^3\) In this case the relation $m^2_\rho = 24\pi^2 f^2_\pi/N_c$ holds [21].
a reflexion with respect to the origin [21]. In the Meson Dominance model we get from (6) and (9) the explicit result of an appealing simplicity, namely

\[ H(x, 0, -\Delta_\perp^2) = \frac{m_\rho^2(m_\rho^2 - (1 - x)^2\Delta_\perp^2)}{(m_\rho^2 + (1 - x)^2\Delta_\perp^2)^2}. \]  

We check that \( H(x, 0, 0) = 1 \) [21] and \( \int_0^1 dx H(x, 0, t) = m_\rho^2/(m_\rho^2 + t) \), Eq. (2), which is the built-in vector-meson dominance principle. We pass to the impact-parameter space by the Fourier-Bessel transformation (3) and get

\[ q(b, x) = \frac{m_\rho^2}{2\pi(1 - x)^2} \left[ K_0 \left( \frac{bm_\rho}{1 - x} \right) - \frac{bm_\rho}{1 - x} K_1 \left( \frac{bm_\rho}{1 - x} \right) \right]. \]  

In the Nambu–Jona-Lasinio model with the Pauli-Villars regularization one can proceed along similar lines as above to get

\[ H(x, 0, -\Delta_\perp^2) = 1 - \frac{N_c M^2}{8\pi f_\pi^2} \sum_i c_i \int_0^1 d\alpha \frac{(1 - x)^2\Delta_\perp^2}{M^2 + \Lambda_i^2 + \alpha(1 - \alpha)(1 - x)^2\Delta_\perp^2} \]

\[ = 1 + \frac{N_c M^2(1 - x)|\Delta_\perp|}{4\pi^2 f_\pi^2 s_i} \sum_i c_i \log \left( \frac{s_i + (1 - x)|\Delta_\perp|}{s_i - (1 - x)|\Delta_\perp|} \right), \]

\[ s_i = \sqrt{(1 - x)^2\Delta_\perp^2 + 4M^2 + 4\Lambda_i^2}, \]

where \( M \) is the constituent quark mass, \( \Lambda_i \) are the PV regulators, and \( c_i \) are suitable constants. For the twice-subtracted case, explored below, one has, for any regulated function \( F \), the operational definition [25]

\[ \sum_i c_i F(\Lambda_i^2) = F(0) - F(\Lambda^2) + \Lambda^2 dF(\Lambda^2)/d\Lambda^2. \]  

In what follows we use \( M = 280 \) MeV and \( \Lambda = 871 \) MeV, which yields \( f_\pi = 93.3 \) MeV [25].

It is interesting to notice that, quite generally, the chiral quark model results displayed above depend on the momentum \( \Delta_\perp \) and \( x \) only through the combination \( (1 - x)^2\Delta_\perp^2 \). Consequently, in the \( b \) space they depend on the combination \( b^2/(1 - x)^2 \). Due to this property we have

\[ \frac{f \, d^2b \, b^{2n} q(b, x)}{f \, d^2b \, q(b, x)} = \langle b^{2n} \rangle(x) = (1 - x)^{2n} \langle b^{2n} \rangle(0). \]  

\[ \langle b^{2n} \rangle(0). \] (14)
This means, that all the moments except for \( n = 0 \) vanish as \( x \to 1 \), or, in other words, the function becomes an infinitely-narrow \( \delta \) function in this limit. This general prediction of chiral quark models is clearly seen in the lattice data of Ref. [1], cf. Fig. 2(b).

4 Smearing over \( b \)

Our aim is to compare our results, after a suitable QCD evolution, to the transverse-lattice data of Ref. [1]. These data give the non-singlet diagonal parton distribution of the pion at discrete values of the impact parameter \( b \), corresponding to a square lattice with spacing of \( b_0 \approx 2/3 \) fm. It is certainly not obvious how to compare discrete data to a continuum model. Clearly, we cannot achieve the continuum limit on transverse lattices, on the other hand we do not intend, in a simple study as presented here, to put chiral models on the lattice. A simple and reasonable comparison [29] is expected when the model predictions are smeared over square plaquettes, the same ones as in the discrete lattice. The plaquettes are labeled \([i, j]\), which means that they are centered at coordinates \((ib_0, jb_0)\), and have the edge of length \( b_0 = 2/3 \) fm [1]. The smeared GPD is defined as

\[
V(x, [i, j]) \equiv \int_{(i-1/2)b_0}^{(i+1/2)b_0} db_1 \int_{(j-1/2)b_0}^{(j+1/2)b_0} db_2 V(x, \sqrt{b_1^2 + b_2^2}).
\]

Figure 2 shows the results of this smearing. In addition, the degeneracy factor of the number of plaquettes equidistant from the origin is included, i.e., the \([1, 0]\), \([1, 1]\), and \([2, 0]\) plaquettes are multiplied by a factor of four, while \([2, 1]\) would be multiplied by eight.

We note that the smearing has a large effect for the \([0, 0]\) plaquette. This is because in the limit of \( x \to 1 \) the function \( V(x, b) \) becomes a distribution in \( b \), which can be seen immediately from the explicit form of Eq. (11). Thus, the results for \([0, 0]\) are sensitive to the size of \( b_0 \). For lower values of \( b_0 \) the function becomes very sharply peaked at \( x = 1 \).

Figure 2(b) shows the data from the transverse-lattice calculations shown by Dalley in Ref. [1]. These data correspond to the scale \( Q \approx 500 \) MeV, as inferred in Ref. [17] from the analysis of the pion light-cone wave function. Since the scale pertaining to our calculation is much lower, we need to evolve our results upward before comparing to the data of Fig. 2(b).
Fig. 2. Valence impact-parameter dependent diagonal GPD of the pion, $V(x, b)$, plotted as a function of the Bjorken $x$ variable. (a) The results of the chiral quark models at the model scale of $Q = Q_0 = 313$ MeV. Solid lines: the Spectral Quark Model of Ref. [21], dashed lines: the Nambu–Jona-Lasinio model with two Pauli-Villars subtractions. Label all denotes the forward distribution, i.e., the function $V(x, b)$ integrated over the whole $b$-plane. Labels $[i, j]$ denote the function $V(x, b)$ integrated over the square plaquettes centered at coordinates $(ib_0, jb_0)$ of the edge of length $b_0$, times the degeneracy of the plaquette (see the text for details). Following Ref. [1], the value of $b_0$ is taken to be $2/3$ fm. (b) The results for $V(x, b)$ at the scale $Q \sim 500$ MeV, obtained from transverse-lattice calculation of Ref. [1]. Labels as in (a). The model results of (a) can be compared to the data of (b) only after a suitable QCD evolution.

5 QCD evolution

The simple calculation of Sec. 3 has produced distributions corresponding to a low quark model scale, $Q_0$. A priori, the value of $Q_0$ is not known. The way to estimate it is to run the QCD evolution upward from various scales $Q_0$ up to a scale $Q$ where the data can be used. Alternatively, one may use the momentum fraction carried by the quarks at the scale $Q$ and the downward QCD evolution in order to estimate $Q_0$ [22,24,25]. We use the LO evolution with
\[ \alpha(Q) = \left( \frac{4\pi}{\beta_0} \right) \frac{1}{\log(Q^2/\Lambda_{QCD}^2)}, \]  

(16)

where \( \beta_0 = 11C_A/3 - 2N_F/3, \) \( C_A = 3, \) and \( N_F = 3 \) is the number of active flavors. We take \( \Lambda_{QCD} = 226 \) MeV, which for \( Q = 2 \) GeV yields \( \alpha = 0.32 \) [30]. Then one proceeds as follows: The valence contribution to the energy momentum tensor evolves as the first \( x \)-moment of the valence quark distribution,

\[ \frac{V_1(Q)}{V_1(Q_0)} = \left( \frac{\alpha(Q)}{\alpha(Q_0)} \right)^{\gamma_{1NS}/(2\beta_0)}, \]  

(17)

where \( \gamma_{1NS}/(2\beta_0) = 32/81. \) The value of \( V_1(Q) \) has been extracted from the analysis of high-energy experiments. In Ref. [27] it was found that at \( Q = 2 \) GeV the valence quarks carry 47\% of the total momentum of the pion, \( e.g., \) for \( \pi^+ \)

\[ V_1 = \langle x (u_\pi - \bar{u}_\pi + \bar{d}_\pi - d_\pi) \rangle = 0.47 \pm 0.02 \quad \text{at} \quad Q = 2 \text{ GeV}. \]  

(18)

The downward LO DGLAP evolution yields at the scale \( Q_0 \)

\[ V_1(Q_0) = 1, \quad G_1(Q_0) + S_1(Q_0) = 0. \]  

(19)

with \( G_1 \) and \( S_1 \) the gluon and sea momentum fractions, respectively. The scale \( Q_0 \) defined with this prescription is called the quark model point, since obviously in effective quark models all the momentum is carried by the quarks. At LO the scale turns out to be [22]

\[ Q_0 = 313^{+20}_{-10} \text{ MeV}. \]  

(20)

This is admittedly a rather low scale, but one can still hope that the typical expansion parameter \( \alpha(Q_0)/(2\pi) \sim 0.34 \pm 0.04 \) makes the perturbation theory meaningful. Actually, the NLO analysis of Ref. [24] supports this assumption. In addition, this is the same scale used in Ref. [31] to compute the pion LC wave function. 4

Following Ref. [18], we apply the DGLAP evolution to the off-forward diagonal distribution function with the evolution kernel that does not depend on \( \Delta_\perp \), or, in the impact-parameter space, on \( b \). Then, at LO the DGLAP evolution in the index space simply reads

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4 An analogous analysis applied to the data of Ref. [1] shows that the momentum fraction carried by the valence quarks is 72\% [29], which at LO would imply the scale of 477 MeV, compatible with the scale of 500 MeV quoted by the authors of Ref. [1].
\[ V_n(Q, b) \equiv \frac{1}{\alpha(Q)} \int_0^1 dx \ x^n V(x, Q, b) = \left( \frac{\alpha(Q)}{\alpha(Q_0)} \right)^{\gamma_n^{NS}/(2\beta_0)} \int_0^1 dx \ x^n V(x, Q_0, b), \quad (21) \]

where the anomalous dimension is

\[ \gamma_n^{NS} = -2C_F \left[ 3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{k=1}^{n+1} \frac{1}{k} \right], \quad (22) \]

with \( C_F = 4/3 \). With \( n \) treated as a complex number, which requires an analytic continuation of both \( V_n(Q_0, b) \) and \( \gamma_n^{NS} \), Eq. (21) can be inverted using the inverse Mellin transform

\[ V(x, Q, b) = \int_{-i\infty}^{i\infty} \frac{dn}{2\pi i} x^{-n-1} V_n(Q, b). \quad (23) \]

The procedure, carried out numerically, is fast and stable. Since the singularity structure of \( V_n(Q, b) \) is the same as for the forward case, we may use the standard Mellin integration contour in Eq. (23).

An interesting feature of the above evolution is the induced suppression at \( x \to 1 \). Thus, using known methods from the \( b \)-integrated case \([32]\), a function which originally behaves as \( V(x, Q_0, b) \to C(b)(1 - x)^N \) evolves into

\[ V(x, Q, b) \to C(b)(1 - x)^{N - \frac{4C_F \log \frac{\alpha(Q)}{\alpha(Q_0)}}{\beta_0}}, \quad x \to 1. \quad (24) \]

In order to compare to the transverse-lattice data of Ref. [1], we apply the evolution to the smeared functions of Eq. (15). Thus, we have explicitly

\[ V(x, Q, [i, j]) = \int_{-i\infty}^{i\infty} \frac{dn}{2\pi i} x^{-n-1} \left( \frac{\alpha(Q)}{\alpha(Q_0)} \right)^{\gamma_n^{NS}/(2\beta_0)} \int_0^1 dy \ y^n V(y, Q_0, [i, j]), \quad (25) \]

where the distribution at the scale \( Q_0 \) is the prediction of either of the two considered chiral quark models.

We also note that in the Spectral Quark Model

\[ V_n(Q_0, b) = \frac{m^2 \Gamma(n+1)}{\pi 2^{n+3}} \left[ b m^2 \frac{G_{2,4}^4}{4} \left( \begin{array}{c|c} n-1, n/2 \\ \frac{1}{2}, \frac{1}{2} \end{array} \right) - G_{2,4}^4 \left( \begin{array}{c|c} n+1, n/2 \\ \frac{1}{2}, 0, 0, 0 \end{array} \right) \right], \quad (26) \]
where $G$ denotes the Meijer $G$ function. This form can be useful for further analytic considerations.

6 Results and conclusions

Figure 2 (a) shows the plaquette-averaged functions $V(x, Q_0, [i, j])$ for the Spectral Quark Model (solid lines) and the NJL model (dashed lines). We note that the predictions of the two models are qualitatively the same, with the NJL curves pushed to somewhat lower values of $x$. For the lack of space, in this paper we display the QCD evolution of the Spectral Quark Model only. The case of the NJL model is qualitatively the same, with the corresponding curves moved to a bit lower values of $x$, simply reflecting the different initial condition of Fig. 2 (a). These results and other details will be presented in a longer paper.

The results of the evolution are shown in Fig. 3 at three values of the reference scale $Q$: 400 MeV (a), 500 MeV (b), and 2 GeV (c). We note a large effect of the evolution on the distribution functions. The lines labeled all correspond to the forward case, i.e., show $\int d^2b V(x, Q, b) = V(x, Q, \Delta_\perp = 0)$. The originally flat distribution of Fig. 2(a) recovers the correct end-point behavior at $x \to 1$ according to Eq. (24). As $Q$ increases, the distribution is pushed towards lower values of $x$, as is well known for the DGLAP evolution. At $Q = 2$ GeV the result agrees very well with the SMRS parameterization of the pion structure function [27], as can be seen from Fig. 3(d) (here we plot for convenience $x V(x, Q)$) by comparing the dashed and solid lines. This result was already obtained in Refs. [22,24].

The results for the plaquette $[0, 0]$ follow, at large $x$, the forward distributions. This is clear from the behavior described at the end of Sec. 3, i.e., from the dependence of the initial function on the variable $b/(1-x)$. Certainly, as $x \to 1$, the integration over the $[0, 0]$ plaquette is the same as the integration over the whole $b$-space. At $Q = 400$ MeV and 500 MeV the values of $V(x, Q, [0, 0])$ reach a maximum at an intermediate value of $x$, and develop a dip at low $x$. This is in qualitative agreement with the transverse-lattice data of Fig. 2(b).

We note that there the dip at low $x$ is lower than in our model calculation, yet, in view of the simple nature of our model and approximations (chiral limit, LO evolution, evolution independent of $b$, uncertainties in the determination of $b_0$ and $Q$ on the lattice) the similarity is quite satisfactory. We have checked that if the value the lattice-spacing parameter, $b_0$, were lowered, an even more quantitative agreement would follow.

The results for non-central plaquettes also qualitatively agree with the lattice measurements. In this case at $x \to 1$ the corresponding functions vanish very
Fig. 3. Results of the LO DGLAP evolution of the impact-parameter dependent diagonal non-singlet generalized parton distribution function of the pion, $V(x, b, [i, j])$, started from the initial condition at $Q = Q_0 = 313$ MeV produced by the Spectral Quark Model (Fig. 2(a), solid lines). Figures (a,b,c) correspond to $Q = 400$ MeV, 500 MeV, and 2 GeV, respectively. Labels as in Fig. 2. Figure (d) shows $xV(x, b, |i,j|)$ for $Q = 2$ GeV, with the dashed line showing the SMRS [27] parameterization of the data for the forward parton distribution function.

fast, in accordance to our model formulas. The difference with the lattice calculation of Fig. 2(b) is that in our case the farther plaquettes naturally bring less and less, and the yield from the $[2,0]$ plaquette is lower than for the $[1,1]$ plaquette. In Fig. 2(b) it is the other way around.

In summary, the obtained agreement of our approach, based on non perturbative chiral quark models in conjunction with perturbative LO DGLAP evolution, with the data from the transverse lattices, is quite remarkable and encouraging, baring in mind the simplicity of the models and the apparently radically different handling of chiral symmetry in both approaches. We also note that the low-energy scale taken for the chiral quark models is consistent with previous analysis based both on the forward parton distribution amplitudes as well as the light cone wave function. Our analysis might be reinforced by extending our calculation to include the NLO perturbative corrections. Such a study is left for a future research.
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