Neutrinos, Weak Interactions, and r-process Nucleosynthesis

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Abstract. Two of the key issues in understanding the neutron-to-proton ratio in a core-collapse supernova are discussed. One of these is the behavior of the neutrino-nucleon cross sections as supernova energies. The other issue is the many-body properties of the neutrino gas near the core when both one- and two-body interaction terms are included.

1. Introduction

A good number of the isotopes of nuclei heavier than iron are produced by the rapid neutron-capture process (the so-called r-process) nucleosynthesis, the site of which is not known. R-process nucleosynthesis requires interaction of a large number of neutrons during a relatively short duration, suggesting that its sites are likely to be associated with explosive phenomena. In fact, Burbidge et al originally suggested that the neutrino-rich ejecta outside the collapsed core in a supernova could be an r-process site [1]. Recent observations of metal-poor stars and meteoric data suggest that r-process nuclei may be produced at multiple sites [2]. Possible sites for the r-process nucleosynthesis include the neutrino-driven wind in a core-collapse supernova [3] [4] or binary neutron-star systems [5].

Neutrino interactions play a very crucial role both in the dynamics of core-collapse supernovae and in the r-process nucleosynthesis [6]. Isotopic yields of r-process nucleosynthesis are determined by the neutron-to-proton ratio, n/p [7]. There are two key issues in understanding how this ratio changes: i) Neutrinos and antineutrinos streaming out of the core interact both with nucleons and the seed nuclei. Since these interactions determine the n/p ratio, it is crucial to understand neutrino-nucleon cross sections in considerable detail. ii) Before these neutrinos reach the r-process region they may undergo matter-enhanced neutrino oscillations as well as coherently scatter over other neutrinos. Many-body behavior of this neutrino gas is not understood, but may have significant impact on the r-process nucleosynthesis. Here I briefly discuss our recent work addressing these two issues.

2. Testing Neutrino-Nucleon Interactions

The covariant matrix element for the reaction $\nu_e + p \rightarrow e^+ + n$ is

$$\frac{G_F \cos \theta_C}{\sqrt{2}} \left\{ \bar{u}_n \left[ \gamma_\alpha (f_1 - g_1 \gamma_5) + \sigma_{\alpha\beta} k^\beta (f_2 + g_2 \gamma_5) + k_\alpha (f_3 + g_3 \gamma_5) \right] u_p \right\} \left\{ \bar{\nu}_\alpha (1 - \gamma_5) \nu_e \right\}, \quad (1)$$
where $G_F$ is the Fermi weak coupling constant and $\cos \theta_C = 0.974$ is the Cabibbo angle. The vector $f_1$, the axial-vector $g_1$, the tensor $f_2$ (or weak magnetism), the induced tensor $g_2$, the induced scalar $f_3$ and the induced pseudo-scalar $g_3$ form factors are included in this matrix element. The conserved vector current (CVC) hypothesis states that

$$\lim_{q^2 \to 0} f_1(q^2) = 1; \lim_{q^2 \to 0} f_2(q^2) = \frac{\mu_p - \mu_n}{2m_N}; \ f_3(q^2) = 0,$$

(2)

where $\mu_p - \mu_n = 3.706$ is the difference in the anomalous magnetic moments of the nucleons, and $m_N$ is the mass of the nucleon.

Data for neutrino-nucleon scattering at energies relevant to astrophysics are very scarce. One possibility to measure the cross section for this process is to utilize proposed beta-beam facilities. Beta-beams are pure $\nu_e$ or $\overline{\nu}_e$ beams produced by allowing radioactive ions circulating in a storage ring to decay [10]. The low-energy option [8, 9], where one can vary the Lorentz boost factor $\gamma$ of the stored ions between 7 and 14, is especially suitable to measure the energy dependence of the cross section in the energy range up to 100 MeV. The potential of low-energy beta beams for neutrino-nucleus scattering [11, 12, 13], for electroweak tests of the Standard Model [14, 15], and for core-collapse supernova physics [8, 16] has been previously discussed.

We recently investigated the possibility of measuring antineutrino scattering off protons in a water Cerenkov detector using low-energy beta beams [17]. We analyzed the sensitivity using both the total number of events and the angular distribution of the positrons emitted. Figure 1 shows the expected number of events at the detector over a period of one year as a function of the Lorentz boost factor with (solid-line) and without (dashed-line) the weak magnetism term. One observes that at the highest $\gamma$ value the weak magnetism term suppresses the total number of events by as much as 17 %. We found that the weak magnetism form factor may be determined with better than several percent accuracy using the angular distribution information of the positrons emitted. In addition to providing a test of the CVC hypothesis, measurement of weak magnetism terms in neutrino-nucleon interactions are of direct interest to astrophysics as these terms may play an important role in the dynamic of core-collapse supernovae [18].

3. Many-Body Properties of the Dense Neutrino Gas in Supernovae

Neutrino oscillations in a core-collapse supernova differ from the matter-enhanced neutrino oscillations in the Sun as in the former there are additional effects coming from both neutrino-neutrino scattering [19, 20] and antineutrino flavor transformations [21]. The standard MSW
term describing the interaction of neutrinos with the background electrons is a one-body term for neutrinos. Exact solutions of the full neutrino many-body problem with the one-body MSW term and the two-body neutrino-neutrino forward scattering term have not yet been found.

For two neutrino flavors, i.e. the electron neutrino, $\nu_e$, and a combination of muon and tau neutrinos, called $\nu_x$, the one-body term in the neutrino many-body Hamiltonian is [22]

$$H_\nu = \int \frac{d^3p}{2p} \frac{\delta m^2}{2p} \left[ \cos 2\theta J_0(p) + \frac{1}{2} \sin 2\theta (J_+(p) + J_-(p)) \right] - \sqrt{2} G_F \int d^3p \frac{d}{dp} N_e J_0(p), \quad (3)$$

where $N_e = n_{e^-} - n_{e^+}$ is the net electron density; $\theta$ is the vacuum mixing angle, and $\delta m^2 = m_2^2 - m_1^2$. In writing Eq. (3) we introduced the operators

$$J_+(p) = a_+^\dagger(p) a_+(p), \quad J_-(p) = a_-^\dagger(p) a_-(p), \quad J_0(p) = \frac{1}{2} \left( a_+^\dagger(p) a_x(p) - a_+^\dagger(p) a_x(p) \right), \quad (4)$$

where we used the creation and annihilation operators for $\nu_e$ and $\nu_x$. The operators in Eq. (4) satisfy the commutation relations

$$[J_+(p), J_-(q)] = 2 \delta^3(p - q) J_0(p), \quad [J_0(p), J_\pm(q)] = \pm \delta^3(p - q) J_\pm(p). \quad (5)$$

These operators describe $N$ mutually commuting SU(2) algebras, where $N$ is the number of allowed values of neutrino momenta. In fact, the evolution operator for the standard MSW problem of neutrinos, mixing with each other and interacting with background electrons, can be written down exactly using this algebraic ansatz. In addition, there is a two-body term in the Hamiltonian describing neutrino-neutrino forward scattering:

$$H_{\nu\nu} = \frac{\sqrt{2} G_F}{V} \int d^3p d^3q \left( 1 - \cos \theta_{pq} \right) J(p) \cdot J(q). \quad (6)$$

where $V$ is the quantization volume and $\theta_{pq}$ is the angle between neutrino three-momenta $p$ and $q$.

One can calculate the path integral for the many-body problem using the SU(2) coherent states

$$|z(t)\rangle = \exp \left( \int d^3p z(p,t) J_+(p) \right) \prod_p a_+\dagger(p) |0\rangle. \quad (7)$$

Path integral representation of the matrix element of the evolution operator calculated with $H_\nu + H_{\nu\nu}$ is given by

$$\langle z'(t_f)|U|z(t_i)\rangle = \int D[z, z^*] e^{iS[z, z^*]}, \quad (8)$$

where the action functional is

$$S[z, z^*] = \int_{t_i}^{t_f} dt \langle z'(t)|i \frac{\partial}{\partial t} - H_\nu - H_{\nu\nu}|z(t)\rangle - i \log \langle z'(t_f)|z(t_f)\rangle. \quad (9)$$

The path integral given in Eq. (8), to date, has not been evaluated exactly. However, it is possible to evaluate it using the stationary path approximation and calculate the stationary path $z_{sp}(p,t)$ that minimizes the action functional [22]. By interpreting this stationary path as the ratio of the one-body neutrino wavefunctions, i.e.

$$z_{sp}(p,t) = \frac{\psi_x(p,t)}{\psi_e(p,t)} \quad (10)$$
and imposing the normalization condition

\[ |\psi_e|^2 + |\psi_x|^2 = 1, \]  

one can relate results obtained using stationary path approximation to the commonly utilized Schrödinger-type equation such as the one used in Refs. [23, 24, 25]. Numerical solutions of this approximate Schrödinger-type nonlinear equation are still not easy to obtain as discussed in Refs. [26, 27, 28, 29].

A description of neutrino physics with two flavors is exact only when the third mixing angle, \( \theta_{13} \), is zero [30]. However, it is possible to incorporate both the antineutrinos (which are crucial to nucleosynthesis in core-collapse supernovae) and the presence of three flavors in this algebraic formalism [22].

4. Conclusions

It should be emphasized that neutrino oscillations significantly impact r-process nucleosynthesis only if different flavors initially have well-pronounced energy differences. The precise energy hierarchy of the neutrinos depends on the microphysics of their production [31, 32]. This microphysics is dominated by the inelastic neutrino-nucleon interactions.

In a core-collapse supernova, as the alpha particle mass fraction increases, free nucleons get bound in alphas and, because of the large binding energy of the alpha particle, cease interacting with neutrinos. (This is called “alpha effect” [33]). Electron neutrinos radiated from the proto-neutron stars are typically too energetic to prevent the alpha effect (and the following demise of r-process nucleosynthesis) in most cases. One way to get around this issue is to convert active electron neutrinos into sterile ones [34, 35, 36]. Inclusion of sterile neutrinos in the algebraic approach described above will be discussed elsewhere.

Finally, it should be pointed out that the neutrino gas is not necessarily present in all cases. If, instead of a proto-neutron star, a black-hole is formed, then the neutrino flux emission may be truncated [37]. Note that it may be possible to find signatures of such a neutrino-flux truncation in the fossil record of the isotopes produced in the r-process nucleosynthesis [38].

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