Numerical Study of Droplet Behavior on an Oscillating Plate Using OpenFOAM

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Abstract. The interfacial behaviour of a liquid droplet placed on an oscillating plate was simulated numerically by VOF method for the better understanding of the transition process of the surface fluctuation from the axisymmetric mode to the complicated mode which is leading to the interface burst and spray formation. The target of analysis was a water droplet of 100μL placed on a flat plate. In the place of the plate oscillation, a sinusoidal acceleration-oscillation was applied with the gravitational acceleration. The frequency of the acceleration-oscillation was ranging from 300 Hz to 900 Hz. The range of the acceleration-oscillation amplitude was from 3g to 60g. The computational domain was set to include a quarter of the droplet. In the numerical simulation, axisymmetric concentric waves and superposed azimuthal waves appeared on the droplet surface. The surface fluctuation grew in magnitude with an increase in the acceleration-oscillation amplitude, and the complexity of the interface increased due to the coalescences of the crests of the surface fluctuation.

1. Introduction

The vibration-induced liquid atomization has been applied widely in many industrial processes [1]. A unique cooling method of microelectronic packages was also proposed that utilized a two-phase mist flow generated by the vibration induced atomization [2]. Concerning to these applications, experimental investigations had been made on the interfacial behaviour of a liquid droplet placed on a vertically oscillating plate. It was revealed that an axisymmetric wave appeared on the droplet surface at relatively small oscillation amplitudes, the surface fluctuation grew in magnitude and complexity with an increase in the amplitude, and fine droplets were ejected from the crests of the fluctuation at larger oscillation amplitudes [3]. The experimental investigations had been also made on the critical oscillation amplitude for the fine-droplet ejection (onset of atomization) and the spray characteristics of ejected fine-droplets [4,5]. However, the transition process of the surface fluctuation from the axisymmetric mode to the complicated mode has not been understood well. Further investigation has been required on the surface fluctuation of the oscillated liquid droplet. Numerical simulations should be suitable for the detailed investigation of the surface fluctuation.

In the present study, for the better understanding of the transition process, the interfacial behaviour of a water droplet on an oscillating plate was simulated numerically by VOF method with dynamic adaptive mesh refinement. In the place of the plate oscillation, a sinusoidal acceleration-oscillation with relatively high frequency was added to the gravitational acceleration term. The
interDyMFoam of OpenFOAM [6] was employed for solving the flow equations after the modification to make the acceleration term a function of time. The shape of the fluctuating surface, which was obtained numerically, resembled to the previous experimental observation [3]. It was considered that the flow field and the interface motion were well simulated by the employed computational method. The flow field and the interface motion were simulated numerically in several cases. The features of the surface fluctuations were revealed and the transition process of the surface fluctuation was discussed from the results.

2. Computational Method

The problem to be investigated in this study involves a water droplet of 100μL volume located on a vertically oscillating flat plate in atmospheric air, which is schematically shown in Figure 1 (a). The radius of a volume-equivalent half-sphere to the droplet is 3.63mm. The fluid flow, as well as the motion of the air-water interface, was simulated numerically by the volume of fluid (VOF) method. The computational domain is shown in Figure 1 (b). To reduce the computational cost, the computational domain was chosen as a cubic of 5mm which contained a quarter of the droplet, assuming the fluid motion and the interface shape were symmetrical with respect to the orthogonal cutting planes. In the place of the plate oscillation, a sinusoidal acceleration-oscillation was applied to the field with the gravitational acceleration in the simulation. The contact angle of the gas-liquid interface onto the plate was assumed to be constant. Test fluids, air and water, were both assumed to be incompressible and immiscible.

The governing equations of the incompressible fluid flow are as follows:

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla \cdot \mathbf{\tau} = -\nabla p + \rho \mathbf{G} + \mathbf{F}_s \]  \hspace{1cm} (2)

where \( t \) is the time, \( \mathbf{u} \) denotes the velocity vector, \( \mathbf{\tau} \) is the viscous tensor, \( p \) is the pressure, \( \mathbf{G} \) is the acceleration, and \( \mathbf{F}_s \) is the surface tension force which is modelled by CSF method and acts in the interface region only. The transport properties of mixed fluid density, \( \rho \), and viscosity, \( \mu \), are evaluated from the volume-fraction weighted properties of two fluids:

\[ \rho = \alpha \rho_1 + (1-\alpha) \rho_2, \quad \mu = \alpha \mu_1 + (1-\alpha) \mu_2 \]  \hspace{1cm} (3)

where \( \rho_1 \) and \( \rho_2 \) are the densities of water and air respectively, and \( \mu_1 \) and \( \mu_2 \) are the viscosities of water and air respectively. The void fraction, \( \alpha \), has the value of 0 for air and 1 for water, and the plane of \( \alpha=0.5 \) represents the air-water interface. The transport equations of \( \alpha \) can be written as:

\[ \frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{u} \alpha) + \nabla \cdot [\mathbf{u}, \alpha(1-\alpha)] = 0 \]  \hspace{1cm} (4)

where \( \mathbf{u}_i \) is the artificial interface compression velocity. The last term is introduced and optimized in order to keep the free-surface tracking accurate. The acceleration in \( z \)-direction was oscillated sinusoidally around the gravitational acceleration, \( -g \).

\[ \mathbf{G} = \begin{pmatrix} 0 & 0 & -g + A \sin(2\pi ft) \end{pmatrix}^T \]  \hspace{1cm} (5)

where \( A \) and \( f \) are the amplitude and frequency of the acceleration oscillation respectively.

Equations (1)-(5) are solved numerically using interDyMFoam solver of OpenFOAM version 4.0 [6] after the modification to make the acceleration term in equation (2) a function of time. Specific settings of the boundary conditions are listed in Table 1. A hexagonal uniform grid of mesh-width 0.1mm was employed. Local adaptive mesh refinement (AMR) was used to ensure sufficient grid-resolution around the air-water interface. Figure 2 shows typical example of the locally refined grid created by the interDyMFoam. The minimum mesh width in the interface region was 0.025mm.
At the start of the simulation, the water droplet with the shape of one-eighth sphere was placed at the corner of the cuboid among the bottom wall and the symmetry planes. The numerical calculation was advanced for 2 seconds with zero oscillation amplitude and enhanced water viscosity until the droplet shape reached a steady state at which the restoring force by surface-tension balanced with the gravitational deforming force. The calculation was advanced with linearly increasing the acceleration-oscillation amplitude, $A$, to the settling value, $A_s$, within the duration of $10T$, then the calculation was proceeded further with the acceleration-oscillation amplitude of $A_s$. The time variation of the acceleration-oscillation amplitude during the start-up process is shown in Figure 1 (c), where $T$ is the period of the acceleration-oscillation and is equal to $1/f$.

The numerical simulation was performed within the range of the oscillation frequency, $f$, from 300 Hz to 900 Hz, which was much higher than the free-oscillation frequency of a sessile drop [7].

![Figure 1. Problem definition and computational domain.](image1)

| Boundaries       | Velocity          | Pressure                   | VOF($\alpha$)                  |
|------------------|-------------------|----------------------------|---------------------------------|
| Bottom Wall      | noSlip            | fixedFluxPressure          | constantAlphaContactAngle ($\theta_0$ 84) |
| Outer Boundaries | pressureOutletVelocity (uniform (0 0 0)) | totalPressure (uniform 0) | inletOutlet (uniform 0)         |
| Planes of Symmetry | symmetryPlane     |                            |                                 |

![Figure 2. Typical example of local adaptive mesh refinement around air-water interface.](image2)

| Properties | Air | Water |
|------------|-----|-------|
|            |     |       |
The range of the settling value, $A_s$, of the acceleration-oscillation amplitude was from $3g$ to $60g$. The volume of water within the computational domain was $25\mu$L. A static contact angle of $84^\circ$ was employed after the photographic observation of James et al. [3]. The fluid properties used in the simulations are listed in Table 2.

3. Results and discussion

Figure 3 shows typical examples of numerical results. Each image in Figure 3 is top view of the instantaneous shape of the air-water interface. When the acceleration-oscillation amplitude is relatively small, the droplet interface is relatively calm and almost axisymmetric as shown in (a) of the figure. Concentric axisymmetric waves are also observed faintly on the interface. At slightly large oscillation amplitude, azimuthal waves can be observed on the contact-line of droplet as shown in (b). The azimuthal waves are also observed subtly on the crests of the axisymmetric wave inside the contact-line. At larger oscillation amplitude, as the azimuthal waves grow in magnitude, the interfacial fluctuation becomes complicated one and the axial symmetry of the droplet shape is disappeared as shown in (c). The surface fluctuation should transit to the droplet-ejection mode at further large amplitudes. As the shape of fluctuating surface resembled to those of previous experimental observation [3], the flow field and the interface motion were considered to be well simulated by the employed computational method.

3.1. Concentric wave (Case 1)

A detailed investigation was made on the interfacial fluctuation appeared in the case of relatively small oscillation amplitude. Figure 4 shows the radial distribution of the time-averaged interface height and the radial variations of the vertical interface displacement. The droplet resembles the overturned bowl in shape as shown in the figure. There is a small-amplitude fluctuation on the interface. As the radial position of the fluctuation crests do not move, the interfacial fluctuation should be the standing wave. The fluctuation amplitude is large near the central axis of droplet and becomes small with an increase of the radial position, except near the circumference of the droplet where the interface is inclined and the vertical displacement is magnified. Figure 5 shows the time variation of the interface height at three points 1mm apart from the central axis (identified by up-arrow in Figure 4). The azimuths of these points are $22.5^\circ$ apart as illustrated in the right-side of the figure. The fluctuation waveforms are sinusoidal and in the same phase, although the amplitudes of fluctuation are slightly different one another. The period, $T$, of the fluctuation corresponds to the reciprocal of the acceleration-oscillation frequency, $f$. That is, the fluctuating frequency of the interface is identical to the acceleration-oscillation.
These features of the surface fluctuation are similar to the surface wave observed during the surface-tension measurement by the capillary-wave method [8]. The surface fluctuation should be the concentric capillary standing wave. The fluctuation should not be able to grow greatly because the frequency of the acceleration-oscillation is much higher than the free-oscillation of droplet.

Figure 6 shows the distributions of the vertical displacement of interface from the time-averaged height at several representative moments. The almost axisymmetric concentric waves can be clearly seen. The azimuthal waves of minute amplitude are also observed. The azimuthal waves are appeared to be superposed on the concentric waves. Because of the azimuthal waves, the amplitudes of the interface height fluctuations of Figure 5 should be slightly different depending on the azimuths. As the displacement distribution at $t=t_0$ shown in (a) is very similar to that at $t=t_0+T$ shown in (c), the azimuthal waves and the concentric waves should fluctuate in the same frequency, in this case.

Figure 4. Radial distributions of time-averaged interface height and vertical displacements of interface.

Figure 5. Time-variation of interface-height at representative points 1 mm apart from central axis.
3.2. Concentric wave with azimuthal wave (Case 2)
A detailed investigation was made next on the interfacial wave appeared in the case of the intermediate oscillating amplitude. Figure 7 shows the distributions of the vertical interfacial displacement from the time-averaged interface height at several representative moments. The azimuthal waves, as well as the concentric waves, can be observed on the droplet interface. The azimuthal waves are appeared to be superposed on the concentric waves. The amplitude of the azimuthal waves is large near the circumference of droplet and is not so large near the central axis. The azimuthal waves at $t=t_0$ shown in (a) of the figure are in the same phase to those at $t=t_0+2T$ shown in (c) but in the opposite phase to those at $t=t_0+T$ shown in (c). The crests of the azimuthal waves do not move with time. The azimuthal waves should be the standing wave, but the fluctuating frequency of the azimuthal waves may be different from the concentric waves which are synchronized with the acceleration-oscillation.
Figure 7. Vertical displacement of interface from time-averaged interface height at several moments.

The time-variations of the droplet radius on $y=0$ and the results of FFT analysis are respectively shown in (a) and (b) of Figure 8. Although the waveform of the droplet-radius distribution is sinusoidal and only a peak can be found on the FFT result at the acceleration-oscillation frequency in the case of relatively small oscillation amplitude (Case 1) as shown in (I) of the figure, the waveform of the droplet-radius distribution is distorted and two peaks can be found on the FFT result at the acceleration-oscillation frequency and half of the frequency in this case as shown in (II). These results suggest that the large-amplitude azimuthal waves near the circumference of droplet should be the subharmonic waves of the order 1/2.

The characteristics of the azimuthal waves were investigated in more detail. Table 3 lists the number of waves within a quarter of droplet, the wavelength, $\lambda_n$, and the fluctuating frequency, $f_n$, of the azimuthal wave superposed on each concentric wave. The estimated wave-velocities, $f_n\lambda_n$, and the theoretical velocity, $c$, of the surface-tension wave with wavelength of $\lambda_n$, which is obtained from equation (6), are also listed in the table. The wave-velocities were estimated assuming that each azimuthal wave was a standing wave and composed of right-handed and left-handed waves.

\[
c = \frac{2\pi \sigma}{\lambda_n \rho_l}
\]  

(6)

The azimuthal wave on the 1st concentric wave fluctuates at the frequency of the acceleration-oscillation, which is identical to the fluctuating frequency of the concentric wave. However, the azimuthal waves on other concentric waves fluctuate at half of the acceleration-oscillation frequency. The estimated wave-velocities of all azimuthal waves almost agree with the velocity of the surface-tension wave. The azimuthal waves should be the standing wave of the surface-tension wave whose integer multiple of the half-wavelength matches the circumferential crest-length of each concentric wave.
Figure 8. Time variations of droplet radius and results of frequency analysis.

Table 3. Characteristics of azimuthal wave superposed on each concentric wave.

| ID of Concentric Waves | Radius, r (mm) | Num. of Waves in Domain | Wavelength, $\lambda_a$ (mm) | Freq. of Wave, $f_a$ (Hz) | Vel. of Wave, $v_a$ (m/s) | Vel. of Surface-tension Wave, $c$ (m/s) |
|------------------------|----------------|-------------------------|-----------------------------|---------------------------|---------------------------|----------------------------------------|
| #1                     | 0.60           | 1.0                     | 0.94                        | 700                       | 0.660                     | 0.691                                  |
| #2                     | 1.53           | 1.5                     | 1.60                        | 350                       | 0.559                     | 0.531                                  |
| #3                     | 2.38           | 2.5                     | 1.49                        | 350                       | 0.522                     | 0.549                                  |
| #4                     | 3.28           | 3.5                     | 1.47                        | 350                       | 0.514                     | 0.553                                  |
| #5                     | 3.83           | 4.5                     | 1.34                        | 350                       | 0.467                     | 0.580                                  |
| #6                     | 4.2            | 4.5                     | 1.47                        | 350                       | 0.513                     | 0.54                                  |
Figure 9. Vertical displacement of interface from time-averaged interface height showing coalescence of fluctuation crests.

Figure 10. Change of droplet interface in shape over time.
3.3. Complicated wave (Case 3)
The interfacial fluctuation at larger acceleration-oscillation amplitude was investigated next. Figure 9 shows the instantaneous distribution of the vertical interfacial displacement from the time-averaged interface height. The amplitude of the azimuthal waves is larger than that of Case 2 as shown in the figure. The amplitude of the azimuthal waves is large not only near the circumference of droplet but also in the range of intermediate radius. However, the amplitude is not so large near the central axis of the droplet. Due to the large amplitude of the azimuthal waves, the crests of the interface fluctuation become high and the bases of the crests spreads in the radial direction at intermediate radius. As the results, the adjacent crests of the azimuthal waves coalesced and merged into one, as indicated by the arrows in the figure. This should be the reason why the interfacial fluctuation becomes complicated one at relatively large acceleration-oscillation amplitudes.

Figure 10 shows the change of the droplet interface in shape over time. The azimuthal wave can be observed obviously on the contact-line of droplet. The interface fluctuates in complex manner especially in the range of relatively large radius. Although the asymmetry of the interface shape disappears, the periodicity of the fluctuation still remains in this case, as can be seen from the figure.

3.4. Map of fluctuation modes
The surface fluctuations were simulated in several cases. Figure 11 shows the map of observed fluctuation modes. The acceleration threshold for the droplet ejection of Vukasinovic et al. [4] is also shown in the figure. Within the range of relatively small acceleration-oscillation amplitude, almost concentric waves were observed. The azimuthal waves superposed on the concentric waves become obvious in the range of acceleration-oscillation amplitude larger than the dashed line in the figure. The surface fluctuation is appeared to transit to the complicated one at the acceleration-oscillation amplitudes above the dotted line in the figure. It is considered from these results that the fine-droplets would be ejected from the crests of the fluctuating interface at further large acceleration-oscillation amplitudes.

Figure 11. Map of interfacial fluctuation modes.

4. Conclusions
The interfacial behavior of water droplet on an oscillating plate was simulated numerically by VOF method with dynamic adaptive mesh refinement for the better understanding of the transition process of the surface fluctuation from the axisymmetric mode to the complicated mode. In the place of the plate oscillation, a sinusoidal acceleration-oscillation at relatively high frequency was applied with the gravitational acceleration. The interDyMFoam of OpenFOAM was employed for solving the flow equations after the modification to make the acceleration term a function of time. Following conclusions were deduced:
Numerically obtained shape and motion of the droplet surface resembled to those of previous experimental observation. It was considered that the flow field and the interface motion were well simulated by the employed computational method.

At relatively small acceleration-oscillation amplitude, almost axisymmetric concentric waves appeared on the droplet interface. The concentric waves were standing waves and the frequency of surface fluctuation was identical to the acceleration-oscillation.

At slightly large acceleration-oscillation amplitude, azimuthal waves also appeared on the interface. The azimuthal waves were standing waves and superposed on the axisymmetric concentric waves. Near the droplet circumference, the azimuthal waves had large amplitude and showed subharmonic oscillation of the 1/2 order. The azimuthal waves were the standing wave of the surface-tension wave whose integer multiple of the half-wavelength matched the circumferential crest-length of each concentric wave.

At larger acceleration-oscillation amplitude, the azimuthal waves became obvious not only near the circumference but also in the range of intermediate radius. As the result, the axial symmetry of the droplet shape is disappeared, and the adjacent crests of the azimuthal waves coalesced into one. This should be the reason why the surface shape becomes complicated one at relatively large acceleration-oscillation amplitudes.

5. References
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Acknowledgments
This study was assisted by graduate school students of Toyohashi University of Technology. Authors would like to express special thanks to Miss Kumada S for her great support.