Elasto-plastic finite element analysis of deformation of soft ground and retaining structures

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ABSTRACT

It seems important to develop a numerical method for analyzing complex interactive action of soils and structures. The finite element used in this study is a pseudo-equilibrium model by one-point integration of a 4-noded iso-parametric element. The rotating failure of a retaining structure with excavation is analyzed. A layered approach based on 8-noded iso-parametric element to reinforced concrete structures is also developed. A material model for a concrete is used similarly by applying Mohr-Coulomb failure criterion and strain softening behavior associated with shear band localization. Then a elasto-plastic strain softening plastic model combined with endochronic theory is developed and applied to a earthquake response analysis of embankment dam.

Keywords: shear behavior, numerical method, reinforced concrete, earthquake response

1. INTRODUCTION

It is shown that the dynamic relaxation method combined with the generalized return mapping algorithm is effective (Tanaka and Kawamoto, 1988; Tanaka, 1992). We analyzed the rotating failure of a retaining structure by excavation using a finite element analysis. A material model for a concrete is developed similarly by applying Mohr-Coulomb failure criterion and strain softening behavior associated with shear band localization. A reinforced concrete structures is modeled by employing layered approach based on 8-noded iso-parametric element. Then a elasto-plastic strain softening plastic model combined with endochronic theory is developed. The constitutive model is applied to cyclic behaviors of element tests and a dynamic progressive failure of embankment dam.

2. RETAINING STRUCTURE

It is normally difficult to obtain the stable limit stress condition by computational method when the rotating failure of the retaining structure occurs (Potts, 2003). There are few effective numerical methods to be able to solve this problem. We developed the elasto-plastic finite element analysis with the implicit-explicit dynamic relaxation method to solve this problem (Okajima et al.,2001). In model experiments (Fig. 1), sand mass consists of Toyoura sand and the wall consists of aluminum plate. The Toyoura sand is $D_{50} = 0.16 \text{ mm}$, the peak internal friction angle $\phi_p = 50^\circ$ and residual internal friction angle $\phi_r = 34^\circ$ by the plane strain compression test. The finite element model for this analysis is shown in Fig. 2.

The results of the finite element analyses and the experiments are shown in Fig. 3 and we can see that the results by calculation agree with experimental results. In Fig. 4 the maximum shear strain contours excavated to 29cm are shown.
3 CONCRETE OPEN CHANNEL

3.1 Concrete Model

Two yield functions are used to represent the concrete behaviour, that is, Mohr-Coulomb function and a quadratic curved function. The Mohr-Coulomb function is the same as the soil model and given by following equation.

\[ f_1 = a I_1 + \frac{J_2^{1/2}}{g(\theta)} - \gamma \kappa = 0 \]  

The quadratic function is given by;

\[ f_2 = \frac{d^2 + I_1}{a^2} + \frac{J_2}{[g(\theta)]^2} = 0 \]  

where \( \alpha = \frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}} \), \( I_1 \) is the first invariant (positive in tension) of stresses and \( J_2 \) is the second invariant of deviatoric stress. With the Mohr-Coulomb model, \( g(\theta_L) \) takes the following form

\[ g(\theta_L) = \frac{3 - \sin \phi}{2 \sqrt{3 \cos \theta_L - 2 \sin \theta_L \sin \phi}} \]

\( \phi \) is the mobilized friction angle and \( \theta_L \) is the Lode angle.

When \( I_1 = 0 \) \( (g(\theta)=1) \), we obtain the following relation.

\[ J_2^{1/2} = \frac{d}{\gamma} = \gamma(\kappa), \quad a = \frac{d}{\gamma} \]

Then the Eq. (2) is transformed to;

\[ f_2 = -\frac{1}{a^2} I_1 + \frac{J_2}{[g(\theta)]^2} - \frac{d^2}{a^2} = 0 \]  

(3)

The plastic potential is given by;

\[ \Phi_2 = -\frac{1}{a^2} I_1 + J_2 - \frac{d^2}{a^2} = 0 \]  

(4)

Following the procedure of plasticity, we obtain the necessary equations for the coding for finite element program. In order to see the behavior of concrete, simulation of tensile test and compression test was carried out. In the analyses, Young’s modulus is 10,000 kN/m², Poisson’s ratio is 0.3, peak friction angle 40.0°, peak cohesion is 196.0 kN/m², residual cohesion is 0.0 kN/m². In the following figures, the compressive stress is defined plus. The localization parameter that is similar to shear band thickness in soil model is 0.3 cm. Fig. 6 shows the relation between uni-axial tensile stress and displacement. Fig. 7 shows the relation between uni-axial compressive stress and displacement.
3.2 Layered Element

We employ a layered approach based on the iso-parametric 8-noded element to a reinforced concrete. Layers are numbered sequentially starting at the bottom and layers of different thickness can be employed.

In the formulation of stiffness matrix, strain matrix $B$ is calculated at the mid-surface of each layer. The element stiffness matrix $K^e$ and internal force vector $f^e$ are defined as follows.

$$K^e = \int \left[ \int \sigma_d \xi \eta \frac{d\xi d\eta}{J} \right]$$  \hspace{1cm} (5)

$$f^e = \int \left[ \int \sigma \xi \eta \frac{d\xi d\eta}{J} \right]$$  \hspace{1cm} (6)

where $\xi$ is a linear coordinate in the thickness direction and $\eta$ are curvilinear coordinate of iso-parametric element, $D$ is the elasticity matrix and $J$ is the determinant of the Jacobian matrix.

3.3 Analysis of Reinforced Concrete Open Channel

The reinforced concrete layer (gray colored layer: 25cm thickness in Fig. 9) is divided to 8 layers and 2 steel layers are added as double reinforcement. Young’s modulus of concrete is 2,100,000 kN/m$^2$, Poisson’s ratio is 0.3, peak friction angle is 50.0$^\circ$, residual friction angle is 35.0$^\circ$, peak cohesion is 1500.0 kN/m$^2$ and residual cohesion is 0.0 kN/m$^2$. In the case of reinforced steel, Young’s modulus is 210,000,000kN/m$^2$, layer thickness is expressed in the normalized coordinate: one is 0.0122 and another is 0.006, steel yield stress is 100,000 kN/cm$^2$, and position of steel: one is –0.723, another is 0.723. The constitutive model of soil layer is simple elasto-perfectly plastic Mohr-Coulomb model. Young’s modulus is 50,000 kN/m$^2$, Poisson’s ratio is 0.3, friction angle is 40.0$^\circ$, cohesion is 10kN/m$^2$.

A pressure load is applied incrementally with 50 cm width. Fig. 10-11 show the development of maximum shear strain distribution. The shear failure occurred in concrete when the applied pressure is 400 kN/m$^2$ (see Fig. 12). No yield occurred in the reinforcement steel and the base of concrete channel.
Fig. 12. Ellipse shows the part of shear failure of reinforced concrete (applied pressure is 400 kN/m²)

4 CYCLIC ELASTO-PLASTIC MODEL OF SAND

4.2 Endochronic and Strain Softening Plastic Model

The densification of sand due to cyclic shear strains has been modeled by endochronic theory (Tanaka et al., 1986). The theory is combined with the elastic and strain softening plastic model to obtain the behavior of saturated sand. The strain softening is accompanied with shear banding. The frictional softening $\alpha$, dilatancy angle $\phi$, and cohesion softening $K$ are given by the following equations.

$$\alpha(k) = \alpha_x + (\alpha_p - \alpha_x) \exp \left\{ -\left( \frac{\sigma}{C} \right)^2 \right\} \tag{7}$$

$$\phi = \phi_s [1 - \beta \exp \left\{ -\left( \frac{K}{\varepsilon_d} \right)^2 \right\}] \tag{8}$$

$$K(k) = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)} \exp \left\{ -\left( \frac{K}{D} \right)^2 \right\} \tag{9}$$

4.2 Element Test Simulation

The material constants of sand used for calculation of simple strain softening model are as follows: $\phi_p = 40.0^\circ$, $\phi_s = 32.0^\circ$, $\beta = 0.0$, $C = 0.05$, $E = 0.05$, cohesion = 0.0, shear band thickness = 0.3cm. The pore water pressure build-up is computed by the endochronic theory. Fig. 13 shows the computed stress strain relation and Fig. 14 shows the computed mean stress and stress difference relation of plane strain test using one iso-parametric element. Fig. 15 shows computed stress strain relation by changing the parameter $\beta$ in Eq. (8).

5 DYNAMIC RESPONSE ANALYSIS OF ROCKFILL DAM

Mega Cities have to be protected from tidal waves or a slide of large masses of rock, snow or mud down a mountain. The embankment dams are useful for protecting cities from such events. Especially, in case of tsunami, embankments have to withstand strong earthquakes.

The simple elasto-plastic constitutive equation and endochronic theory are applied to the impounded rockfill
dam. Fig. 16 shows Aratozawa Rockfill Dam model and Fig. 17 shows finite element mesh used for dynamic response analysis. The Aratozawa Dam is a rockfill type dam used for irrigation, flood control and hydro-electric power generation completed 1998. It is 74.4 m high and 413.7 m long, impounding up to 14,130,000 m$^3$ of water. On June 14 2008, the Iwate-Miyagi inland earthquake that had a magnitude of 7.2 hit the dam. The seismic peak acceleration of measured at the inspection gallery of Aratozawa Dam was greater than 1000 gal.

The core material is assumed undrained and perfectly elasto-plastic constitutive model is applied. The cohesion is 60 kPa and the friction angle is 20°. The rock and transition zones are assumed drained and the material constants of these zones used for simple strain softening model are as follows: $\phi_p = 42.0^\circ$, $\phi_r = 34.0^\circ$, $\beta = 0.3$, $C = 0.1$, $D = 0.7$, $E = 0.05$, cohesion = 20 kPa, shear band thickness is 14 cm. The endochronic parameters are determined by relative density 90.0 %. Fig. 18 shows the input acceleration applied to the base of the dam foundation. This acceleration is adjusted to give almost same acceleration measured at the inspection gallery of the dam. Fig. 19 shows computed acceleration and Fig. 20 shows computed displacement at the crest of dam. The computed acceleration at the crest of dam is rather higher than measured one and may include some spikes because of numerical process by the sudden change from the elastic to plastic behavior. The measured crest settlement of this dam is about 30cm, so we can say that comparable result is obtained.
5. CONCLUSIONS

In a problem for a retaining structure with excavation, active earth pressure and passive earth pressure are co-exist. We analyzed the rotating failure of a retaining structure with excavation by the finite element analysis with implicit-explicit dynamic relaxation method.

It seems important to develop a numerical method for analyzing complex interactive action of soils and reinforced concrete structures. A layered approach based on the iso-parametric element to reinforced concrete structures is employed. Layers are numbered sequentially and layer of different thickness can be applied. The reinforcement is modeled using steel layer with elastic perfectly plastic material. A material model for a concrete is developed similarly by applying Mohr-Coulomb failure criterion and strain softening behavior with shear band localization. The tension cut-off behavior of concrete is also taken into account by using a quadratic function. A reinforced concrete open channel is analyzed.

A simple elasto-plastic constitutive equation for geomaterials are developed. A simulation of pore water build up for plane strain tests is carried out by the finite element method using one element. The accelerations and settlements of rockfill dam were computed by the earthquake. The elasto-plastic constitutive equations are shown to be robust and promising for the prediction of behavior of the embankment dam.

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