Invited Comment

Stabilizing coherence with nested environments: a numerical study using kicked Ising models

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Abstract
We study a tripartite system of coupled spins, where a first set of one or two spins is our central system which is coupled to another set considered, the near environment, in turn coupled to the third set, the far environment. The dynamics considered are those of a generalized kicked spin chain in the regime of quantum chaotic dynamics. This allows us to test recent results that suggest that the presence of a far environment, coupled to the near environment, slows decoherence of the central system. After an extensive numerical study, we confirm previous results for extreme values and special cases. In particular, under a wide variety of circumstances an increasing coupling between near and far environment, slows decoherence, as measured by purity, and protects internal entanglement.

Keywords: decoherence, entanglement, nested environments

(1. Introduction
Conserving quantum coherence for long periods is a challenge for quantum information processing and quantum computation implementations. Mechanisms for quantum coherence protection like decoherence free subspaces and dynamical decoupling are very popular nowadays [1]. In many situations the immediate environment (near environment) surrounding a quantum system can be well controlled in order to isolate the system for unwanted external interactions. What about the rest of the universe, i.e., the environment that is not immediately accessible to the system (far environment)? Can it affect the decoherence process of the system due to the interaction with the near environment? More specifically, can it slow down decoherence (or equivalently, protect coherence) in the system? In this work we investigate these questions in a particular situation in which both environments (near and far) are modelled by a quantum kicked spin system.

Recently there has been an increasing awareness that nested environments can actually be used to control coherence of a central system. While it seems that such effects for some time were known to occur in the context of the Jaynes–Cummings model with leaky cavities [2, 3], i.e., in the setting of Haroche’s celebrated experiment [4, 5], there has been increasing interest in this matter recently [3, 6–10] and there are even specific applications that might take advantage of the effect [11, 12]. In the present paper we plan to broaden the...
work presented in the thesis of one of us [9]. We shall study a situation where a central system of one or two qubits is coupled to a near environment which, in turn, is coupled to a far environment. Direct coupling of the system to a far environment is assumed small enough to be neglected. We furthermore assume that the coupling between the central system and the near environment is weak, in order to obtain a situation that is meaningful for quantum information processes. To fix ideas we may think of an ion quantum computer as developed in Innsbruck [13], with very stable ions affected by some decoherence mainly during the control gates via the motional degrees of freedom. The additional degrees of freedom brought into play by this step, in turn have further decay modes or fluctuation sources, not affecting the central system directly in a significant way. This we can think of as a far environment. Considering previous work, the aim of this paper can be formulated more precisely: we wish to consider as far as possible previous results, which to a large degree concentrates on small couplings and very strong couplings to the far environment. Yet the main purpose is to explore in more detail the intermediate zone with a structured dynamical model, that allows for the necessary flexibility and large scale calculations. This will include the use of dephasing interaction between the central system and near environment for two reasons: First because it relates to a generalized concept of fidelity discussed in [14] and second because dephasing clearly separates our results from similar ones seen for energy transfer [2, 3, 15].

We plan to use the kicked Ising model [16] as a paradigmatic example to construct such a situation, because of its great flexibility, as well as for its numerical efficiency [16–21]. While the original model was concerned with nearest neighbours coupled spin chains, more general situations have been studied. The generalization used in [22] is relevant for this work in order to deal with more complicated configurations necessary to handle the decoherence process.

The program for this paper is to use this very flexible and efficient model to cover a wide range of situations for the couplings. This implies in the first place the coupling strength between central system and near environment (λ) as well as the one between near and far environment (γ). Furthermore, the configuration and numbers of the connections between the three subsystems can be varied and will play a significant role. For the central system on the other hand we take the simplest options, namely a single qubit or two non-interacting qubits, and we focus our attention on the evolution of purity and concurrence.

The paper is organized as follows: in section 2 we introduce both the model and the decoherence and entanglement measures used along the paper. In section 3 we present results for the case of one qubit as a central system. Here, we confirm the basic hypothesis that for weak coupling of the central system to the environment, increasing the coupling of the latter to the far environment will slow down decoherence. Moreover, we shall study in detail the dependence of such behaviour on the parameters of the model. The analysis for the case of two non-interacting qubits is presented in section 4, where one can analyse a similar effect on the internal entanglement of the central system. We finish the paper in section 5 with the conclusions.

2. The model

2.1. The kicked Ising system

We shall follow the spirit of [22] and establish a kicked Ising model (KI) as a tool to analyse numerically the time evolution of coupled quantum systems and their entanglement. The model was developed by T Prosen [16], who showed that this system is amenable to large scale computation and indeed allows the treatment of large numbers of qubits efficiently including packages for the use of GPUs [17].

We can think of our model as a graph of N qubits (spin 1/2 particles) connected by N(N − 1)/2 Ising connections of any strength [22] and a single qubit term, which will be periodically kicked with a magnetic field. This is described by the Hamiltonian

\[ H_{KI} = \sum_{j>k} N J_{j,k} \sigma_j^x \sigma_k^x + K(t) \sum_{j} \hbar j \cdot \vec{\sigma}^j, \]

with \( \sigma_j^x, \sigma_j^y, \sigma_j^z \) being the Pauli matrices of spin \( j \) and \( \vec{\sigma}^j = (\sigma_j^x, \sigma_j^y, \sigma_j^z) \). The time-dependent function \( K(t) = \sum_{n \in Z} \delta(t-n) \) is a train of Dirac-delta functions of period one. The evolution of the KI is described by means of the Floquet operator \( U_{KI} \), which is the evolution operator for one period of time. During the free evolution, the system evolves with the unitary propagator corresponding to the Ising interaction

\[ U_{\text{Ising}} = \exp \left( -i \sum_{j>k} J_{j,k} \sigma_j^x \sigma_k^x \right) = \prod_{j<k} \exp \left( -i J_{j,k} \sigma_j^x \sigma_k^x \right), \]

and the effect of the magnetic kick is obtained with the one-qubit operator

\[ U_{\text{Kick}} = \exp \left( -i \sum_{j=1}^N \hbar j \cdot \vec{\sigma}^j \right) = \prod_{j=1}^N \exp \left( -i \hbar j \cdot \vec{\sigma}^j \right), \]

where we take the units such that \( \hbar = 1 \). Therefore, the Floquet operator for one period of time is

\[ U_{KI} = U_{\text{Kick}} U_{\text{Ising}}. \]

Originally, the KI consists of a ring of \( N \) spins-1/2 (or qubits) interacting via homogeneous nearest-neighbour dimensionless Ising coupling \( J \) and are periodically driven by a uniform dimensionless magnetic field \( \vec{b} \). This system is trivially integrable in the case of magnetic field parallel to the Ising axis. It was shown that integrability still exists if the magnetic field is orthogonal to the Ising interaction [23, 24]. The two-dimensional version of the KI was studied recently using an efficient time evolution for the Floquet operator [17], where evidence of the chaoticity of the transverse case is presented, in the form of statistics of the eigenphases of the corresponding Floquet operator.

Another important family of models is obtained when instead of having a fixed, and small, number of parameters, one allows some of these parameters to be random (though static). In particular, the random two-body interaction
Hamiltonians used in [19] can readily be included in our framework, by considering nearest-neighbour interactions with random strength plus a single particle term (in our case not random, for simplicity).

2.2. Two nested environments in the $\mathcal{K}_I$

Regarding the qubits as nodes, and non-zero Ising couplings as connections, we can regard such a system as a graph. While the analysis of an arbitrary graph will be of interest in the light of recent results for complete quantum graphs in the context of random matrix theory (RMT) [25], such a generalization will not be the subject of the present paper.

If we further allow to have two different types of connections (in our case strong an weak), we can have a collection of connected graphs. To be more precise, we shall work in a Hilbert space structured as

$$\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_e \otimes \mathcal{H}_{e'} \tag{5}$$

where the Hilbert spaces are denoting central system, near environment and far environment, respectively. With respect to the spin environments, we shall consider that within each Hilbert space, there is strong coupling, and from the space of the central system to the near environment weak coupling is assumed, see figure 1. We want to investigate the effect of different coupling regimes from the space of near environment to the far environment on the weakly coupled central system.

We shall limit the central system $\mathcal{H}_c$ to simple cases of one or two qubits. The former is obviously the simplest non-trivial system we can have and the latter is the building block for quantum information systems as quantum gates involving pairs of qubits are sufficient to represent a universal quantum computer [26]. Furthermore, as we are not interested in the operation of quantum gates, we shall actually turn off the interaction between these two qubits, i.e., treat a quantum memory, which for weak decoherence can be understood entirely in terms of single-qubit decoherence [27]. We shall separate the rest of the system into two sets of qubits forming $\mathcal{H}_e$ and $\mathcal{H}_{e'}$, respectively. For example, these qubits can be organized in an open chain where $\mathcal{H}_e$ will be connected to the central system at one spin and to the far environment by connecting a spin of this environment to another (or the same) spin of the central system. $\mathcal{H}_{e'}$ will have no connections to $\mathcal{H}_c$.

Within each of these strings we shall consider nearest-neighbour interaction. We will also present some results for additional couplings within each subset, which for quantum chaotic situations result rather irrelevant, as may be expected following universality arguments. For the coupling between the two subsets we will vary from a single coupling to many pairs interacting.

We can reorganize the Hamiltonian (1), and write it as

$$H = H_0 + \lambda V_{ce} + \gamma V_{ee'}, \tag{6}$$

with

$$H_0 = H_c + H_e + H_{e'}, \tag{7}$$

where the indices denote the Hilbert spaces in which the operators act. $\lambda$ and $\gamma$ are real non negative coupling parameters between central system and near environment and between near environment and far environment, respectively. The Hamiltonian $H_0$ represents the internal dynamics of the central system, the near environment and the far environment. The operators $V_{ce}$ and $V_{ee'}$ represent the interaction between each shaded area in figure 1. To be more precise, we can label the set of spins belonging to $\mathcal{H}_e$ as $S_e$ with $\kappa = c, e \text{ or } e'$. Then,

$$H_k = \sum_{j,k \in S_k} t_{jk}^{(\kappa)} \sigma_j^z \sigma_k^z + K(t) \sum_{j \in S_k} b_j^\dagger b_j, \tag{8}$$

and $t_{jk}^{(\kappa)}$ is a matrix of zeros and ones containing the configuration of the subsystem. The open chain system where $t_{jk}^{(e)} = \delta_{j,k+1}$ will be of particular interest. The interaction between central system and near environment and the one between near and far environment are given by the Ising terms

$$V_{ce} = \sum_{j \in S_c, k \in S_e} t_{jk}^{(1)} \sigma_j^x \sigma_k^x, \quad V_{ee'} = \sum_{j \in S_{e'}, k \in S_{e'}} t_{jk}^{(2)} \sigma_j^x \sigma_k^x \tag{9}$$

respectively, where in this case $t_{jk}^{(1,2)}$ are other matrices with zeros or ones, containing the particular configurations of the interactions. Notice that the propagator (2) is periodic (up to a global phase) in $J_{jk}$, as $\exp[i(J_{jk} + \pi)\sigma_j^z \sigma_k^z] = -\exp[iJ_{jk} \sigma_j^z \sigma_k^z]$, so the behaviour of all observables will also be periodic in $\gamma$ and $\lambda$ with period $\pi$. We have also observed a symmetry of the channel induced in the central system with respect to sign changes both in $\gamma$ and $\lambda$, so it will suffice to study, with respect to both parameters, the interval $[0, \pi/2]$. 

**Figure 1.** Example for a tripartite configuration of qubit systems. Each of the Hilbert spaces composing the total space (5), is displayed as a shaded area. The open circles represent the different qubits, thick lines represent strong interactions, and thin ones weak interaction.
We will show however one example for a full period of $\gamma$ to illustrate the effect.

Taking into account that both, the free and the kicked part can be decomposed in a simple multiplication of one and two qubit operations, we can see that this model can be numerically evolved efficiently. The memory requirements are set by the size of the state to be evolved ($2^n$, where $n$ is the total number of qubits), and the speed of the algorithm is also linear with respect to the size of the Hilbert space, for each time step to be evolved. We have used efficient tools [17] developed for GPUs (graphics processing unit) to do our numerical experiments.

The dynamics of the central system is obtained tracing over both environments, which leads to the reduced density operator

$$\rho_c(t) = \text{tr}_{\text{e},\text{f}'}[\rho(t)],$$  \hspace{1cm} (10)

where the total density operator evolves unitarily as $\rho(t) = U_{K}\rho(0)U_{K}^\dagger$. We also assume the absence of initial quantum correlations between subsystems, i.e., the initial state of the total system is of the form

$$\rho(0) = \rho_c \otimes \rho_{e} \otimes \rho_{f}.$$  \hspace{1cm} (11)

In all numerical results we will present, two kinds of initial pure states for the central system are used. For a one qubit central system, $\rho(0)$ will be taken as an eigenstate of the operator $\sigma_z$, while for a two qubit central system we shall use the Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$, which gives us the opportunity to study the evolution of internal entanglement within the central system. In order to emulate a high-temperature spin bath, the initial state of the total environment (near and far) is chosen as a product of two random pure states, one for each environment.

2.3. Quantifying decoherence and entanglement

In order to measure the loss of coherence in our central system we use the purity of a density operator defined as

$$P[\rho] = \text{tr} \rho^2.$$  \hspace{1cm} (12)

This quantity varies from $1/\dim(H_c)$ for the totally mixed state to a vale of one for pure states. If $\rho$ is the partial trace of a pure state, as in (10), it measures the entanglement between the two subsystems implied in the partial trace operation. Purity is just one of a large number of convex functions that can describe decoherence. Its main advantage is the simple analytic structure which allows us to compute it without previous diagonalization of the density matrix. Another commonly used convex function is the von Neumann entropy. Partial orders can be obtained using all or complete sets of positive functions [28–30]. Any of these convex functions reveal, in general, different aspects of decoherence. In fact, for a single qubit they are all equivalent and for larger systems, near pure states, they also tend to be equivalent.

On the other hand, a good measure to quantify the entanglement shared between two qubits is the so-called concurrence, which is defined for a general mixed state as [31]

$$C(\rho) = \text{max} \{0, \hat{\lambda}_1 - \hat{\lambda}_2 - \hat{\lambda}_3 - \hat{\lambda}_4\},$$  \hspace{1cm} (13)

where $\hat{\lambda}_i$ are the square roots of the eigenvalues of $\rho^\dagger \rho$ in decreasing order. The operator $\rho^\dagger$ is the result of applying a ‘spin flip’ operation on $\rho$, i.e., $\rho = (\sigma_i \otimes \sigma_j)\rho^\dagger(\sigma_i \otimes \sigma_j)$ and the complex conjugate is taken in the computational basis of two qubits.

Decoherence of one and two qubits measured by purity and the entanglement of the former through concurrence in an environment described by a kicked Ising chain is the main study of [22].

3. One-qubit in a nested environment

We shall first explore the effect of nested environments with the simplest central system possible, namely a single qubit. The coupling between the central system and the near environment will be chosen to be weak, while the coupling of the near environment to the far environment will range from weak to strong but typically stronger that the former. As in previous works we neglect any coupling between the central system and the far environment, but we will test this assumption in a typical situation. Throughout the paper, we shall choose the dimensions as large as we could for both the near and far environments without having excessive computation times. We also set the parameters of the model in a regime where the dynamics of the spin systems is in the quantum chaotic regime [16], so as to mimic universal results [32].

Specifically, we set $\dim(H_c) = 2^6$, $\dim(H_{\text{e}1}) = 2^{10}$, $J = 1$ and $B_c = B_{\text{e}1} = (1, 0, 1)$ in equation (8). This implies that the kicked magnetic field has an angle of $\pi/4$ with respect to the Ising coupling, and the field strength is chosen appropriately. The coupling between the central system and the near environment, shall be fixed to $\lambda = 0.01$ unless
otherwise stated. Similarly, the size of the near and far environment shall be fixed as in figure 2, except when we analyse the effect of the dimensions in the environments. Finally we choose for most of this section a dephasing coupling between near and far environment (i.e. \( \hat{h}_c = 0 \)). This is inspired by the observation [14] that it will lead to a measurable fidelity amplitude for the open near environment using the central qubit as a probe. This corresponds to the original quantum optical proposal [33–35] for the measurement of fidelity decay, but now applied to an open system, which results by adding the far environment. We obtain the dephasing case by dropping the kicked magnetic field for the central qubit, whose Hamiltonian then commutes with the Ising coupling to the near environment. Another advantage of the dephasing case is that it involves no energy transport and thus clearly distinguishes the decoherence behaviour from results for energy transfer. Towards the end of this section we shall lift this restriction.

Our model depends on the configuration of the connections used within each environment, the configuration of the connections between environments and the number of such connections as well as on their strength. The main object of this paper is to study the behaviour of purity on the aforementioned details. The exploration cannot be exhaustive; rather we tested typical changes and we test in each of these cases the behaviour of decoherence.

The configuration we first consider is characterized by including a single connection between both the central system and the near environment (an open chain), and between the latter and the far environment (also an open chain), see figure 2. The decay of the purity of the central system as a function of time for fixed \( \lambda \) and for different values of \( \gamma \) is shown in figure 2. This picture shows the main feature we wish to discuss in this paper: larger values of \( \gamma \) lead to slower decoherence in the central system. At this point we repeat the same calculation as in figure 2, but adding a weak Ising coupling of 0.01\( \lambda \) between one qubit in the far environment and the central qubit. As expected, the resulting plot is indistinguishable, from the aforementioned one, and we thus do not present it.

We shall now proceed to look at which details of our model affect this result.

First we shall explore how the configuration of the internal connections in each of the environments with unchanged connections between the subsystems impact the results. In figure 3 we show the configurations, shown in the upper part, and plot the purity reached at a given time as a function of \( \gamma \) for each of these configurations. We limit \( \gamma \) to the range \([-\pi/2, \pi/2]\) as the function must be periodic with period \( \pi \) (see section 2.1). The figure shows that adding more internal connections in the environments does not qualitatively change the behaviour of purity. As we allow \( \gamma \) to become larger we see that the tendency is reversed due to the reflection symmetry discussed in section 2.1. Note that we see a peak near \( \gamma = 0 \). This is no contradiction, but rather confirms that we need \( \gamma > \lambda \) to assure that we see the effect of improved coherence.

Next we test to what extent more connections between the environments have an effect in the decoherence process. We have found an interesting result: additional connections between the environments appear to have little effect over the decay of purity, except for the implicit strengthening of the coupling. This can be compensated approximately by scaling \( \gamma \) with \( 1/\sqrt{\nu} \), i.e., the decays are very similar for a number \( \nu \) of connections chosen randomly, if simultaneously \( \gamma \) is replaced by \( \gamma' = \gamma/\sqrt{\nu} \), as we show in figure 4. An interesting phenomena appears as we take the connections between environments randomized, while conserving their total number \( \nu \). We found that the configuration of the connections appears to have a definite effect, specially for large values of \( \gamma \). In figure 5 we show how different topologies
affect the decay in different ways. One can notice that, in this case, different topologies cluster around three different behaviours, one with a flat plateau, with respect to $\gamma$, other with a slight, but noticeable maximum around $\gamma = \pi/4$, and finally another one with a skew behaviour. This is not so for other configurations of the internal connections, as can be seen in figure 6. We were not successful in determining the pattern that lead to one or the other behaviour. Additionally, figure 6 shows how for different numbers of $\nu$ it is the configuration of the connections that has an impact on purity decay, and not the number of connections, as long as they are compensated by proper rescaling.

We next study the effect of the dimensions of the environments on purity decay. In figure 7 we show for a single direct connection [see figure 2] how, as the dimension increases, the maxima for purity becomes ever broader but the increase is slow. Maybe this is even hinting at the possibility of this happening for all non-zero values of $\gamma$ in the case where the dimensions of the environments go to infinite, but we shall later see, that there is strong indication that we must also relax the dephasing condition in order to reach that limit.

As a final test we have to look at the $\lambda$ dependence of the effect; as the interaction of the central system with the near environment is unavoidable, we have to check how small it must be to actually obtain an improvement of coherence if we increase the interaction of the near environment with the far environment. For this purpose we show the purity as a function of $\lambda$ in figure 8.
The effect indeed must disappear as $\lambda$ reaches identity. The fact that the central system does not directly interact with the far environment becomes irrelevant as any initial state will soon mix the excitations of the degrees of freedom of the central system with those of the near environment.

This article revolves around the phenomenon that increasing coupling between near and far environment slows decoherence. The case of small couplings is discussed extensively in the perturbative regime in [7, 14], while the case of strong absorption is discussed in some generality in [6]. The intermediate region is, at this moment, essentially accessible only numerically except for very simple integrable models [3]. We have given a survey of many options for the intermediate region, and we have consistently found the effect under discussion. Yet we have not approached the decoherence free limit. This may be related to the finite size we have to use, and which is not implicit in the rate equation approach used by Campos and Zanardi. We have restricted our studies to two-body interactions whose convergence for large spaces is known to be as $1/\ln N$ and thus for numerical purposes inadequate. Another important point is the restriction to dephasing we used. We can lift this restriction by adding the magnetic field kicks to the central system. In figure 9 we show with parameters otherwise the same as in figure 2, that this slows decoherence much more than dephasing. Thus, while our findings do not confirm the decoupling in the strong coupling limit between near and far environment, they certainly are not contradicting it.

To finish this section, we act upon a suggestion by a referee: fit an exponential, of the form

$$P_{\text{fit}}(t) = \frac{1}{2} \left[ 1 + \exp\left( -a \frac{\gamma t}{\sqrt{2}} \right) \right]$$

(14)

with $a$ a fitting parameter. We consider the same conditions studied for figure 2, where $\nu = 1$. As can be seen from figure 10, our modified exponential fits well for larger values of $\gamma$. However, for smaller ones, there is an important deviation, following possibly from the fact that we have effectively a small environment, whereas for larger $\gamma$s, we have an effective larger environment. Thus, non-Markovian effects take over. The different values of $a$ might correspond to an effective density of states (see section 5). With the experience acquired in [36], it appears natural to add a quadratic term in the exponential, to account for perturbative behaviour. This results in a better fit (almost by an order of magnitude) at the cost of an additional parameter.

4. Two qubits as central system

As we discussed above, a central system composed of two qubits is of great relevance because it is the building block for universal quantum computation and other important protocols [26]. Decoherence and the entanglement shared in a two-qubit system is the subject of many interesting papers, mainly in the context of cavity quantum electrodynamics using Markovian master equations in Lindblad form [3, 10, 15, 37].

The aim of this section is to study the evolution of the internal entanglement in the central system and how it is affected by the presence of a nested environment. If we have two qubits we can, in a natural way, think of them as being coupled to the same or different environments and also of the two couplings being of equal or different strength; in the latter case we have the option that of one of the two is uncoupled. Such a situation we call a spectator model [22]. We will focus our attention in this particular configuration in which one of the qubits plays the role of an observer. Furthermore, we assume that the two qubits are non-interacting, avoiding the influence of this internal coupling on the entanglement evolution in the central system. We use as entanglement measure the concurrence, defined in equation (13). In figure 11 we show the evolution of concurrence for an initial Bell pair in a nested environment for same parameters and

Figure 9. Purity for a fixed time, when adding an internal magnetic field $\vec{B}$ at a $\pi/4$ angle with respect to the Ising interaction. The effect is preserved, i.e., larger couplings imply larger values of purities for the central system, as long as the periodicity in this parameter is not coming to bear. The sensitivity on the magnetic field is large and thus the non-dephasing terms are important.

Figure 10. Purity, for the same conditions reported in figure 2, compared to a fit of the form (14). Symbols correspond to the spin model, and lines to the fit. Notice that errors for larger times seem larger, in this semilog plot. We can see that larger values of $\gamma$ result in better coincidence with our ansatz.
configuration of the environments as in figure 2. We see that the behaviour is indeed quite similar, which strengthens our point, that this mechanism may actually be appropriate in very general terms to improve conditions for quantum information processing and quantum computing. The phenomenon of entanglement sudden death is present in all the curves so we can actually use the coupling $\gamma$ in order to delay it. The inset plot also shows the evolution of purity over time for the Bell pair. The results are similar to the case of one qubit in figure 2.

As in the case of a single qubit, it is interesting to see the evolution of the concurrence for fixed couplings varying the number of connections $\nu$ between near and far environment as shown in figure 12. The connections have been varied in the same manner as in figure 2. Notice that the scaling of $\gamma$ is still valid for this configuration, showing the robustness of the effect and essentially the same behaviour for the evolution of concurrence.

4.1. C-P diagram

A useful tool to characterize the decoherence processes of a two-qubit system is the so-called concurrence-purity (C-P) diagram [38]. One point on this diagram represents the value of entanglement ($C$) and mixedness ($P$) simultaneously. Those quantum states that for a definite value of purity can reach the maximum degree of entanglement are known as maximally entangled mixed states [39]. Looking at the behaviour on the C-P plane of an initial pure maximally entangled state under a local quantum channel (which is the case of the spectator configuration) we can learn about the nature of the corresponding quantum operation acting on the system [38]. Under the action of unital channels, Bell states are mapped to the region between the Werner states and the one corresponding to the action of dephasing channels. In figure 13 we see typical C-P planes. The curve corresponding to the maximally entangled mixed states limits the region corresponding to valid quantum states (light grey area). The thick lines represent the curves for Werner states with $P \in [1/4, 1.0]$ and the one for a Bell state under a dephasing channel with $P \in [1/2, 1.0]$.

It is worth exploring the behaviour of our system in this plane, when an internal magnetic field is applied (otherwise, we would simply have dephasing, and thus we would lie in the lower curve).

In figure 13 we show a pair of typical C-P diagrams with fixed $\lambda$ and varying $\gamma$. The first observation is that the quantum channel induced by the KI for the parameters we have used behaves as unital, all the curves are in the region of unital channels. The top figure shows that the lines tend to follow the Werner state behaviour quite closely as we increase the strength of the interaction between near and far environment (though we should remember, that actually they might not be Werner states). From the bottom figure it is clear that for a sufficiently large number of connections between environments, the increase of $\gamma$ is no longer effective to improve the coherence in the central system. The saturation is reached faster when there is enough connectivity of the environments.

5. Conclusions and perspectives

We have numerically explored various aspects of the effect of nested environments on a central system using the paradigmatic kicked Ising model. Taking advantage of its map structure we have performed simple calculations on relatively large size systems. Our departing point was the growing evidence, that for a situation with a central system coupled weakly to a near environment and no (or negligible) direct coupling to a far environment increasing the coupling
between near and far environment slows decoherence of the central system. This effect was confirmed over a wide range of situations for a one-qubit central system with dephasing or more general coupling to the near environment as well as for a two-qubit system in a Bell state with one of the qubits being in (a non-interacting) spectator situation. We demonstrate a similar behaviour for concurrence, which is an essential point for the usefulness of the encountered effect in the context of quantum information.

The decoherence of quantum systems is often analyzed from the perspective of Fermi’s golden rule, which states that the decay rate between two states is proportional to the density of states in the environment. In this work, we are between the Fermi Golden Rule regime (exponential decay), and the perturbative regime (Gaussian decay). An analysis of how an effective density of states of the near environment changes due to its interaction with the far environment might shed light on general aspects of the effects reported in this paper, as was pointed out by one of the referees. This lies outside of the scope of this paper, but is indeed a promising direction for future research.

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