Dynamic evolution of ionization and recombination processes and ion front acceleration in the presence of nonthermal and trapped electrons

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Abstract. The dynamic evolution of ionization, three-body and radiative recombination processes in high intensity laser ion acceleration mechanisms, has been studied. For that, the expansion of a collisional thin plasma slab in vacuum is modeled using mixture hydrodynamic fluids equations for ions and neutral atoms, in the presence of fast nonthermal and slow trapped electrons, obeying a Cairns-Gurevich distribution. In addition, the characteristics of ion front acceleration and ion gained energy profiles are obtained, for three types of accelerated ions (H⁺, C⁺ and Al⁺). It is proved that, ionization and recombination processes are responsible for the energy transfer between plasma particles. These processes are also strongly influenced by the impact of electron nonthermal phenomena, generated by the interaction of an intense laser pulse with the target. On the other hand, parametric studies have proved that ion energy profiles, maximum electric fields and ion energies at the ion front acceleration are also significantly affected by these phenomena. This study is useful in applications involving the creation of energetic ion beams, such as protontherapy.

1. Introduction
Recent remarkable advancements in laser charged particle accelerators have sparked great interest in sources of energetic particle beams for use in various applications. The type of application depends on the accelerator energy. These high energy plasma based particle accelerators have been described by several authors, in astrophysics [1], in nuclear physics [2], for high energy density physics research [3] and for medical applications [4]. They allow particles to be accelerated over much shorter distances than in conventional particle accelerators. A plasma-based accelerator is a device containing partially ionized plasma composed of free electrons, ions and neutral atoms, where the ions are accelerated by the ambipolar electric field of separation of the electron and ion charges, neutral atoms are accelerated by the impact of electron-neutral collisional ionization.

In the TNSA (Target Normal Sheath Acceleration) mechanism, many models are proposed for describing the physical phenomena involving plasma expansion and ion front dynamic, where the electrons are modeled by the Maxwell distribution. Most of them have characterized the structure of ion front for collisionless free plasma expansion. Srivastava et al. [5] have predicted...
and localized an ion front in laser-plasma interaction process under the isothermal assumption, they have showed that the tendency of front formation is reduced under the effect of hot electrons. Medvedev [6] has studied the plasma motion near the ion front in expanding collisionless plasma by a self-similar analytical solution. He showed that, at the ion front, the potential has an inflection point and there is a sharp drop in the ion density. In their work, Allen and Perego [7] have characterized the profiles of the plasma boundary at the front of a plasma expansion, to improve the formation of multi-valued ion velocity distributions and wave breaking. Murakami et al. [8, 9] used a hydrodynamic model to study the ion front expansion of laser-plasma with limited mass. They established expressions for the maximum velocity and kinetic energy of the ions according to the parameters of the laser and the target. Beck and Pantellini [10] used the self-similar hydrodynamic model to study the ion front of non-collisional plasma expansion. Using the Murakami model, they showed the appearance of an ion front followed by a steep electron front. They demonstrated an agreement of the established analytic expressions for the ion and electron densities, the electron temperature and the heat flux with ab initio plasma simulations.

Other studies have assumed source terms of ionization and recombination at equilibrium [11] but, this assumption is easily violated for many situations where fast progress phenomena are appearing. Among them, nonlinear effects, generated shock waves and high energy tails of energetic electrons are observed in high-energy and ultra-brief laser-pulses interaction with matter [12, 13, 14, 15]. In the presence of these effects, the dynamic evolution of ionization and recombination during plasma expansion does not have time to reach its equilibrium values. In addition, for expanding hot plasma with an electron temperature above 10 keV, the collision phenomena must be modeled with a non-equilibrium model, using a non-thermal electron distribution function [16].

In that context, Mahboub et al. [17], using the Murakami model, showed the existence of two phases of the ion acceleration process. The first one concerns dense and collisional plasma located near the target, the second is the core of the expansion dominated by the recombination processes, to the position of the front of the expansion, where the electric field of charge separation reached its maximum values.

In this work, using the model of Mahboub et al. [17], most attention has been focused on the analysis of the dynamic competition of source terms in the framework of ion beams acceleration at the front of plasma expansion by ultra-intense laser, to study their effect as function of nonthermal energetic electron proportion, non-linear electrostatic potential well, initial hot electron temperature and for different target materials.

2. Theoretical modeling of ion front dynamic

To study the evolution of the ion front as a function of the nonthermality parameter of energetic electrons and the trapping induced by the plasma potential wells, a non-equilibrium multi fluid-model for a plasma expansion has been used.

2.1. Non-equilibrium dynamic competition of source terms

In the present model, we will focus on collisions, as the main mechanism that can generate ion and neutral atoms, in order to describe the competition of the non-equilibrium ionization and recombination processes and to study their influence on the ion acceleration process. The ion-neutral collisions are less frequent than electron-neutral collisions, so we well consider that, collisions between electrons and neutrals as the dominant collision mechanism.

2.2. Electron distribution function

The electrons are assumed to be kinetic and nonthermal, modeled by the Cairns-Gurevich equation previously proposed by [18, 19], consisting of two terms describing nonthermal and
trapped phenomena. The mass density that combines these two effects is given by:

\[
\rho_e = \frac{2\rho_{e0}}{3\alpha + 1} \left\{ \left( -2\alpha \psi^2 + 2\alpha \psi - \frac{3\alpha + 1}{2} \right) \exp(\psi) \text{erf}(\sqrt{\psi}) \right. \\
+ \left( 2\alpha \psi^2 - 2\alpha \psi + \frac{3\alpha + 1}{2} \right) \left( 2\alpha \psi^2 - 2\alpha \psi + \frac{3\alpha + 1}{2} \right) \right\} \tag{1}
\]

Where, \( \rho_e = m_e n_e \). \( n_e \), \( m_e \) are the density and mass of electron, respectively, \( \psi \) is the electrostatic potential of charges separation, \( \alpha \) is the parameter of non-thermality designating the proportion of non-thermal energetic electrons in the plasma and \( \rho_{e0} \) is the electron mass density at equilibrium.

If we set \( \alpha = 0 \), that is to say in the absence of non-thermal electrons, we find the electronic density in the case of Gurevich trapping [20].

\[
n_e = \frac{n_e}{n_{e0}} = \exp(\Psi) \text{erf} \left( \frac{\sqrt{\Psi}}{\sqrt{\pi}} \right) + \frac{2\sqrt{\Psi}}{\sqrt{\pi}} \tag{2}
\]

With, \( n_{e0} \) is the electron density at equilibrium.

In the limit, \( \psi \to 0 \) and \( b \neq 0 \), the Cairns-Gurevich equation tends to the nonthermal electron distribution of Cairns [21]. The presence of a non-zero potential shows the effects of electrostatic perturbation on the state of equilibrium.

At the limit \( \alpha = 0 \) and \( \psi \to 0 \), the equation (1) describes the Maxwellian distribution function.

### 2.3. Collisional two-fluids model for ion and neutral atoms

A two-fluids model for the plasma expansion process is used, assuming a collisional partially ionized plasma composed of electrons (\( e \)), ions (\( i \)) and neutral atoms (\( n \)) expanding into vacuum.

At the first stage of the plasma creation and during its expansion, the plasma undergoes ionization and recombination processes included as source terms in the equations of the model. The model investigates ionization dynamics governed by collisional processes as the balance between the electron impact ionization \( S_i^{\text{ion}} \) on the one hand and the radiative and three-body recombination processes \( S_n^{\text{rec}} \) on the other hand, as a function of the plasma temperature and density. We assumed a one-dimensional model for simplicity.

The spatio-temporal \((z, t)\) hydrodynamic equations describing the continuity, motion and the energy of the ions and neutral atoms can be written as [22]:

\[
\frac{\partial \rho_i}{\partial t} + \frac{\partial (\rho_i u_i)}{\partial z} = m_i (S_i^{\text{ion}} - S_n^{\text{rec}}) \tag{3}
\]

\[
\frac{\partial \rho_n}{\partial t} + \frac{\partial (\rho_n u_n)}{\partial z} = -m_n (S_i^{\text{ion}} - S_n^{\text{rec}}) \tag{4}
\]

\[
\frac{\partial (\rho_i u_i)}{\partial t} + \frac{\partial (\rho_i u_i^2)}{\partial z} = -\frac{\partial \pi_i}{\partial z} - \frac{Ze}{m_i} \frac{\partial \psi}{\partial z} + R_i^{\text{in}} + S_i^{\text{ion}} m_i u_n - S_n^{\text{rec}} m_i u_i \tag{5}
\]

\[
\frac{\partial (\rho_n u_n)}{\partial t} + \frac{\partial (\rho_n u_n^2)}{\partial z} = -\frac{\partial \pi_n}{\partial z} + R_n^{\text{in}} - S_i^{\text{ion}} m_n u_n + S_n^{\text{rec}} m_i u_i \tag{6}
\]

\[
\frac{\partial E_i}{\partial t} + \frac{\partial (E_i u_i)}{\partial z} = -\frac{\partial }{\partial z} (\pi_i u_i) - \rho_i u_i \frac{e}{m_i} \frac{\partial \psi}{\partial z} + u_i R_i^{\text{in}} + Q_i^{\text{in}} + \frac{m_i}{m_n} \left( S_i^{\text{ion}} \frac{1}{2} m_n u_n^2 + Q_n^{\text{ion}} \right) - S_n^{\text{rec}} \frac{1}{2} m_i u_i^2 - Q_i^{\text{rec}} \tag{7}
\]
To normalize the plasma parameters, we have based on the Murakami transformation, given by

$$\frac{\partial E_n}{\partial t} + \frac{\partial(E_n u_n)}{\partial z} = - \frac{\partial(u_n \pi_n)}{\partial z} - u_n R_i^{in} + Q_i^{ni} + S_n^{rec} \frac{1}{2} m_i u_i^2 + Q_i^{rec} - (S_i^{ion} \frac{1}{2} m_n u_n^2 + Q_n^{ion}) \quad (8)$$

$m_{i(n)}$ are the ion and neutral atom masses, respectively, and $\pi_{i(n)}$ are the ion and neutral atom pressures, respectively.

$\rho_{i(n)}$ are the ion and neutral atom mass densities, respectively, and $u_{i(n)}$ are the ion and neutral atom velocities, respectively.

$$S_i^{ion} = \frac{\rho_i}{m_i} \nu_{ion} = \frac{\beta_i}{m_i m_n} \rho_e \rho_n \left( \frac{1 + P \sqrt{U_i / \theta_i}}{X + \frac{m_e}{m_i}} \right) \left( \frac{U_i}{\theta_e} \right) \frac{1}{\rho_e} K \exp(-\frac{U_i}{\theta_e})$$

$$S_i^{rec} = \frac{\rho_i}{m_i} \nu^{rec} = \rho_e \rho_n \left( \frac{1}{m_i m_e} + \frac{\beta_{rth} \theta_e}{m_i m_e} \rho_e \frac{1}{\beta_{rth}} \right)$$

$\beta_i$ and $\beta_e = \beta_{rth} + \beta_{rth}^b$ are the rates of ionisation and the recombination, respectively, given by [23], with $\beta_e$ is the sum of radiative $\beta_{rth}^b$ and three-body recombination $\beta_{rth}^b$.

The $P$, $X$ and $K$ are taken from the data values in [24].

$\theta_e$, the electron temperature and $U_i$, the ionization energy of the target material are in eV.

$R_i^{in(n)}$ are the ions momentum transfers with neutral atoms, through the collision process, given by:

$$R_i^{in(n)}(n_{i(n)}) = m_{i(n)} m_i \rho_{i(n)} \nu_{in(n)}(u_{i(n)} - u_{i(n)})$$

Where $\nu_{n(n)}$ are the collision frequencies given by, $\nu_{in(m)} = \rho_{n(m)} \frac{\sigma_{nm}}{m_{n(m)}} \frac{1}{\pi_{m(n)}}$.

$\theta_{in} = \theta_{mi} = \theta_t / \sqrt{2}$, $m_{in} = m_{n} = m_i + m_e$, $\sigma_{in}$ is the cross section, $K_i$ is the Boltzmann constant and $\theta_{i(n)}$ are the ion and neutral temperatures, respectively.

$Q_i^{in(n)}$ are the heating transfers of ions with neutral atoms, via the collision process, given by [25],

$$Q_i^{in(n)} = \frac{1}{2} R_i^{in(n)}(u_{i(n)} - u_{i(n)}) + 3 \frac{m_{in}}{m_i^2} \rho_{i(n)} \nu_{in(n)} K_i (\theta_{i(n)} - \theta_{i(n)}) \quad (9)$$

With $E_i(n) = \frac{\rho_{i(n)} u^2_{i(n)}}{2 \pi_{i(n)}}$. $\theta_{i(n)} = Z \mu_{ei} \rho_i$, where $\mu_{ei} = m_e / m_i$.

That is considered within the self-similar approach used in this study.

To investigate the finite plasma slab expansion, we solve the system of equations (3-8), assuming the quasi-neutral condition, $\rho_e \approx Z \mu_{ei} \rho_i$, where $\mu_{ei} = m_e / m_i$.

To normalize the plasma parameters, we have based on the Murakami transformation, given by [26]:

$$\rho_{i(n)} \rho_{i(n)} = \rho_{0(n)} \rho_{0(n)} \left( \frac{R_0}{R} \right)^\nu \tilde{\rho}_{i(n)}(\xi), \quad u_{i(n)} = \tilde{R} U_{i(n)}(\xi), \quad \pi_{i(n)} = \rho_{0(n)} \left( \frac{R_0}{R} \right)^\nu \tilde{R}^2 \tilde{\Pi}_{i(n)}(\xi) \quad (10)$$

In the present model, there are two finite plasma slab typical scale lengths, the plasma size $R(t)$ and the Debye length $\lambda_D(t) = \sqrt{\frac{e^2 \varepsilon_0 n_{i(n)}}{\rho_{i(n)} e^2}}$.

Where, $R_0 = R(0)$ is the initial value radius or the thickness of the plasma slab, at rest [8].

$\nu = 1, 2, 3$ are the plasma slab expansion geometries, corresponding, respectively, to the planar, cylindrical, and spherical cases. Here, $\nu = 1$ for simplicity, $\xi = \frac{2}{R(t)}$ is the dimensionless similarity coordinate.

The derivatives of $z$ and $t$ coordinates as function of $\xi$ become,
\[ \frac{\partial}{\partial z} = \frac{1}{R} \frac{d}{d\xi}, \quad \frac{\partial}{\partial t} = -\xi \hat{R} \frac{d}{d\xi} \] (11)

Using the transformation 10 and the derivatives 11, the coupled equations (3-8), are to be solved numerically starting with initial conditions: ions and neutral atoms are considered cold with \( \theta_i = 100 \text{ eV} \) and \( \theta_n = 50 \text{ eV} \), respectively. The plasma is supposed initially at rest with density, \( n_0 = 10^{18} \text{ cm}^{-3} \), knowing that the quasi-neutrality condition is verified at rest, with \( n_{i0} = n_{e0} = 0.4n_0 \) and \( n_{n0} = 0.2n_0 \). An initial non-negligible arbitrary potential \( \Psi_0 \), which represents the trapping potential well amplitude, is also imposed.

3. Results and discussion

In this work, we are interested in the evolution of the energy and the position of the ion front for three target materials (H, C, and Al), as function of \( b, \Psi_0 \) parameters and the competition of the different source terms of ionization and recombination. The solution of the equations system holds up to a point \( \xi = \xi_{\text{front}} \), where one finds the formation of an ion front. At \( \xi_{\text{front}} \) the energetic \( \epsilon_{i,\text{front}} \) and the electric fields \( E_{i,\text{front}} \) reach their maximum values.

3.1. Dynamic of ionization and recombination processes

In Figure 1, the profiles of the different source term processes are represented as a function of the self-similar variable \( \xi \) for \( H^+ \), in the presence of an initial nonlinear potential well \( \Psi_0 = 20 \), with \( b = 0.5 \) and \( \theta_e = 10 \text{ keV} \).

The figure shows the appearance of three phases, separated by the two intersection points of the black and the red curves. At the initial stage of plasma creation, which represents the first phase of plasma expansion, the dynamic of ionization and recombination is characterized by almost a plateau, due to a strongly nonthermal effect of \( b \) and \( \Psi_0 \) on the collisional processes. In the core of plasma expansion situated between the two intersection points, the ionization process reached a maximum value before decreasing suddenly near the expansion front phase, vice-versa for the recombination process. Note that "the expansion front" is a position where the proton energies and the electric fields responsible for proton acceleration reach their maximum values. Here, new protons are created by the process of collisional ionization, which increase the profile of ionization term until a maximum value in the body of the expansion then it decreases quickly at the expansion front phase, where recombination prevails over the ionization process, hence the creation of new neutral atoms.

Note that radiative recombination is weak along the expansion.

In this figure, for any position \( \xi = [0 \ 10] \) along the expansion, the sum of the source terms of recombination values are not equal to the ionization values. Therefore, the non-equilibrium state supposed in the theoretical modeling of the dynamic evolution of source terms is confirmed. The Figures 2(a-b) show the ion energy dynamic as function of \( \xi \), for \( H^+ \) with competition of different source terms (a) and for different ion species in the existence of all source terms (b), with \( \Psi_0 = 10 \), \( b = 0.5 \) and \( \theta_e = 10 \text{ keV} \).

In Figure 2.a, for the case of ionization only, the energy profile is characterized by a weak growth of the energy gained by the protons during the expansion, because of the dominance of the collisional ionization process. In the case where the recombination processes are dominated, the ion acceleration is more enhancing with the presence of nonthermal electrons and initial nonlinear potential well. For the Figure 2.b, the expansion of aluminium ion is slower then the carbon and hydrogen ions, due to an important nonthermal electron population generated from the aluminium atoms with higher ionization energy, which reduces the collision ionization process and improves the ion gained energy.
3.2. Parametric study of ion front energy as function of electron temperature and nonthermal effects

The figures 3(a-c) show the ion energy $E$ evolution as function of $\xi$ for different electron temperatures (a) (with $b = 0.05$, $\Psi_0 = 2$), nonthermal parameters $\Psi_0$ (b) (with $b = 0.05$, $\theta_e = 1 \text{ keV}$) and $b$ (c) (with $\Psi_0 = 2$, $\theta_e = 1 \text{ keV}$), in the presence of recombination and ionization for hydrogen ions. The proton energies are strongly influenced by the electron temperature and the two nonthermality effects ($b$, $\Psi_0$): $\epsilon$ increases rapidly with $\theta_e$, $b$ and $\Psi_0$. As $b$ and $\Psi_0$ increase, the accelerated proton beam reaches its maximum energy values at the front expansion with a smaller value of $\xi_{\text{front}}$, hence the acceleration is more efficient (note that $\xi_{\text{front}}$ is the proton front position). The Figures 4(a-c) show the ion front energy evolution as function of the electron temperature $\theta_e$ for different target materials ($H^+$, $C^+$ and $Al^+$) with nonthermal parameters values ($b = 0.05$, $\Psi_0 = 2$) in the presence of recombination and ionization processes (a), as function of nonthermal parameter $b$, for different competition of source terms with ($\Psi_0 = 2$, $\Psi_0 = 2$).
Figure 3. Ion energy evolution as function of $\xi$ for different electron temperatures (with $b = 0.05$, $\Psi_0 = 2$) (a), nonthermal parameters $\Psi_0$ (with $b = 0.05$, $\theta_e = 1$ keV) (b) and $b$ (with $\Psi_0 = 2$, $\theta_e = 1$ keV), (c), in the presence of recombination and ionization for hydrogen ions.

$\theta_e = 1$ keV) for $H^+$ (b) and as function of $\Psi_0$, for different target materials, with ($b = 0.05$, $\theta_e = 1$ keV) in the presence of recombination and ionization (c).

In Figure 4.a, for weak parameters of $\theta_e$, it is the energy of the hydrogen, the lightest element, which is the most important at the front followed by the carbon then the aluminum. The carbon ions and the protons have approximately the same energies at the front of the acceleration and exhibit the same behavior as a function of $\theta_e$. In fact, at the ion front, it is the recombination that governs the ion acceleration [17], which is strongly dependent on the ionization potential, knowing that the ionization potentials values of carbon and hydrogen are very close. For important $\theta_e = [4 9]$, both carbon and hydrogen beams present almost a plateau in energy which means that the beams are quasi-monoenergetic, unlike aluminum which shows a significant growth of the ion front energy, hence a more divergent behavior of the aluminum beam, especially for the large electron temperature values [28].

Fig.4.b shows that for small values of $b = [0 0.3]$, we note that $E_{\text{front}}$ strongly increases with $b$ in the case of recombination alone (black curve), so nonthermal effects dominate the recombination process. Also, the slope is important for recombination alone compared to the three other cases. In the case of ionization alone (blue curve), electron impact ionization produces new electrons during acceleration, this explains the dominance of ionization on the nonthermal effect of energetic electrons. For large values of $b = [0.3 0.75]$, we notice the formation of a plateau with an almost constant proton maximum energy, especially for the case of the presence of ionization alone, while we observe a slight increase of $E_{\text{front}}$ with $b$ for the case of the presence of recombination alone, this can be interpreted by the fact that the recombination
process dominates the effect of nonthermality at the proton front. The main and interesting result is that the effect of electron nonthermality shows a plateau with an almost constant energy $E_{\text{front}}$ of the accelerated proton beam at the front of the expansion, despite the presence of the competition of different source terms. An intense beam of energetic electrons (large $b$) ensures a good quasi-monoenergetic proton acceleration via the electric charge separation field [27].

On the other hand, Figure 4.c shows the effect of the initial potential well $\Psi_0$ on proton energy at the expansion front. It shows that the nonlinear effects strongly influence the $E_{\text{front}}$ energy compared to the influence of nonthermal electrons (Figure 4.b) where we notice a much steeper slope as a function of $\Psi_0$. This is less visible in the case of ionization alone (blue curve) where the energy shows a good trend in function of $\Psi_0$. $E_{\text{front}}$ is strongly influenced by $\Psi_0$ especially in the case of recombination alone (black curve) with respect to ionization. This is due to the fact that the large and deep potential wells created by the laser field interacting with matter, reinforce the collisions of the trapped electrons in these potential wells, hence the dominance of ionization on the nonlinear effects. For a given $\Psi_0$, $E_{\text{front}}$ is larger in the case of recombination alone comparatively to the ionization alone, knowing that it is the recombination that governs the acceleration at the ion front, but the ionization alone favors the energy stability in the presence of non-thermal and nonlinear effects.

To ensure an efficient quasi-monoenergetic acceleration with a large ion beam energies, as it is required by many applications, it is recommended to take into account both ionization and recombination processes in the same model [28].

The study of the competition of the source terms, on the ion energy front as function of the nonthermal effects, nonlinearities and electron temperatures for three target materials enabled us to optimize the good range of the parameters $b$, $\Psi_0$ and $\theta_e$ to have a quasi-monoenergetic ion acceleration which is deduced as $b = [0.3\ 0.5]$, $\Psi_0 = [15\ 30]$ and $\theta_e = [4\ 7] \text{ keV}$.

4. Conclusion
In this work, we have modeled the dynamic evolution of ionization and recombination processes and ion front evolution in order to optimize the electron temperature and nonthermal parameter values that allows to have a quasi-monoenergetic ion acceleration. We have performed a parametric study for the energy of the ion front for three target materials ($H$, $C$ and $Al$), as function of electron temperature, nonthermality of energetic electrons, the trapping induced by the plasma potential wells and the competition of the different source terms of ionization and recombination. The effect of electron nonthermality shows a plateau with an almost constant energy of the accelerated proton beam at the expansion front, despite the presence of the competition of different source terms. An intense beam of energetic electrons ensures a good quasi-monoenergetic proton acceleration via the electric charge separation field. To ensure an efficient quasi-monoenergetic acceleration with a large proton beam energies, as it is required by many applications, it is recommended to take into account both ionization and recombination processes in the same model.

For important nonthermal and nonlinear phenomena, $H^+$ and $C^+$ present quasi-monoenergetic beams, unlike $Al^+$ which shows a more divergent beam, especially for nonlinear effects and does not reach a quasi-monoenergetic behavior. The ion energy evolution of $H$ and $C^+$ are almost the same as function of electron temperature. They show a more efficient acceleration compared to $Al^+$ beam which has larger ion front position values, hence its acceleration is slower. This work is motivated by the need of optimized nonthermal parameter, initial potential well and electron temperature values in a non-equilibrium collisional plasma model, allowing convergent ion beams with large energies and low ion front positions, as it is required by many applications.
Figure 4. Ion front energy evolution as function of: $\theta_e$ for different target materials with ($b = 0.05$, $\Psi_0 = 2$) in the presence of recombination and ionization (a), as function of $b$ for different source terms with ($\Psi_0 = 2$, $\theta_e = 1$ keV) for $H^+$ (b) and as function of $\Psi_0$ with ($b = 0.05$, $\theta_e = 1$ keV) for $H^+$ (c).

References

[1] Blumenfeld Y, Nilsson T and Van Duppen P 2013 Phys. Scr. 152 014023.
[2] Durante M et al 2019 Phys. Scr. 94 033001.
[3] Hoffmann D H H et al 2007 Eur. Phys. J. D 44 293-300.
[4] Bulanov S V, Esirkepov T Zh, Khoroshkov V S, Kuznetsov A V and Pegoraro F 2002 Phys. Lett. A 299 240-7.
[5] Srivastava M K, Sinha B K and Lawande S V 1988 Phys. Fluids 31 394-409.
[6] Medvedev Yu V 2011 Plasma Phys. Control. Fusion 53 125007.
[7] Allen J E and Perego M 2014 Phys. Plasmas 21 034504.
[8] Murakami M, Kang Y G, Nishihara K, Fujioka S and Nishimura H 2005 Phys. Plasmas 12 062706.
[9] Murakami M and Basco M 2006 Phys. Plasmas 13 012105.
[10] Beck A and Pantellini F 2009 Plasma Phys. Control. Fusion 51 015004.
[11] Bradshaw S J and Raymond J 2013 Space Sci. Rev. 178 271-306.
[12] Arefiev A V, Khudik V N and Schollmeier M 2014 J. Phys. Plasmas 21 033104.
[13] Kiefer T, Schlege T and Kaluza M C 2013 Phys. Rev. E 87 043110.
[14] Sarri G, Dieckmann M E, Kourakis I and Borghesi M 2010 Phys. Plasmas 17 082305.
[15] Ballai I 2019 Front. Astron. Space Sci. 6 39.
[16] Chung H-K, Chen M H, Morgan W L, Ralchenko Y and Lee R W 2005 HEDP 1 3-12.
[17] Mahboub M F, Bara D, Bennaceur-Doumaz D and Djebli M 2019 Phys. Plasmas 26 023101.
[18] Bara D, Djebli M and Bennaceur-Doumaz D 2014 Las. Part. Beams 32 391-8.
[19] Annou K, Bara D and Bennaceur-Doumaz D 2015 J. Plasma Phys. 81 905810318.
[20] Landau L D and Lifshitz E M 1981 Course of Theoretical Physics: Physical Kinetics vol.10 (Oxford:Pergamon).
[21] Cairns R A, Manum A A, Bingham R, Boström R, Dendy R O, Nairn C M C and Shukla P K 1995 Geophys. Res. Lett. 22 2709-12.
[22] Meier E T and Shumlak U 2012 Phys. Plasmas 19 072508.
[23] Ni L, Lukin V S, Murphy N A and Lin J 2018 Astrophys. J. 852 95-105.
[24] Voronov G S 1997 At. Data Nucl. Data Tables 65 1-35.
[25] Leake J E, Lukin V S, Linton M G and Meier E T 2012 Astrophys. J. 760 109-20.
[26] Kumar N and Pukhov A 2008 Phys. Plasmas 15 053103.
[27] Nedelea T, Briehl B and Urbassek H M 2005 J. Plasma Phys. 71 589-609.
[28] Tayyab M, Bagchi S, Chakera J A, Avasthi D K, Ramis R, Upadhyay A, Ramakrishna B, Mandal T and Naik P A 2018 Phys. Plasmas 25 123102.