Abstract

We consider asymptotically flat static spherically symmetric black hole solutions in $SU(N)$ Einstein-Yang-Mills theory. Embedding the $N$-dimensional representation of $su(2)$ in $su(N)$, the purely magnetic gauge field ansatz contains $N - 1$ functions. When one or more of these gauge field functions are identically zero, magnetically charged EYM black hole solutions emerge, consisting of a neutral and a charged gauge field part, based on non-abelian subalgebras and the Cartan subalgebra of $su(N)$, respectively. We classify these charged black hole solutions in general and present numerical solutions for $SU(5)$ EYM theory.
1 Introduction

SU(2) Einstein-Yang-Mills (EYM) theory possesses non-abelian static spherically symmetric globally regular and black hole solutions \[1, 2\]. These solutions are unstable \[3\] and carry no charge. With non-abelian hair outside their regular event horizon, the black hole solutions represent counterexamples to the “no-hair conjecture”.

The SU(2) solutions, based on a purely magnetic ansatz, are labelled by the node number \(n\) of the gauge field function \(u\). For fixed horizon radius \(x_H\) and increasing node number \(n\) the sequence of neutral SU(2) EYM black hole solutions tends to a limiting magnetically charged solution. When \(x_H \geq 1\), this limiting solution is an embedded Reissner-Nordstrøm (RN) solution \[4\]. It has a vanishing gauge field function \(u\) and the RN metric has magnetic charge \(P = 1\) \[5, 6\].

The non-abelian static spherically symmetric globally regular and black hole solutions of SU(3) Einstein-Yang-Mills (EYM) theory are obtained by embedding the 3-dimensional representation of \(su(2)\) in \(su(3)\). The purely magnetic gauge field ansatz then involves two gauge field functions, \(u_1\) and \(u_2\), and the solutions are labelled by the corresponding node numbers \((n_1, n_2)\) \[7, 8, 6\].

The neutral SU(3) EYM solutions form sequences \((i, i+n)\), with \(i\) fixed. In the limit \(n \to \infty\), these sequences of neutral SU(3) EYM solutions tend to limiting solutions, carrying magnetic charge of norm \(P = \sqrt{3}\) \[8, 6\]. When \(x_H \geq P\), the limiting solutions are magnetically charged non-abelian black hole solutions \[11\], in which one of the two gauge field functions is identically zero. The gauge field of these limiting solutions consists of two parts, a non-abelian \(su(2)\) part and a \(su(3)\) Cartan subalgebra (CSA) part. The field strength tensor component \(F_{\theta\phi}\) of the CSA part of the gauge field does not vanish identically asymptotically, yielding a CSA magnetic charge. In contrast, the field strength tensor of the \(su(2)\) part of the gauge field decays to zero asymptotically, yielding no magnetic charge. When both gauge field functions are identically zero, embedded RN solutions are obtained, carrying magnetic CSA charge of norm \(P = \sqrt{4}\) \[4\].

In SU(2) EYM theory all static spherically symmetric EYM black hole solutions with non-zero charge are embedded RN solutions \[4, 11\]. In contrast, this “non-abelian baldness theorem” no longer holds for SU(3) EYM theory, which does allow for black hole solutions, with magnetic CSA charge and with non-abelian \(su(2)\) gauge field configurations \[11\]. SU(3) EYM theory additionally allows for black hole solutions with both electric and magnetic CSA charge and with non-abelian \(su(2)\) gauge field configurations \[11\], based on a more general ansatz not considered here.

Here we consider static spherically symmetric SU(N) EYM solutions, based on the purely magnetic gauge field ansatz, obtained by embedding the \(N\)-dimensional representation of \(su(2)\) in \(su(N)\). The ansatz contains \(N - 1\) gauge field functions. When one or more of these functions are identically zero, magnetically charged EYM
black hole solutions emerge. Their gauge fields consist in general of non-abelian $su(N)$ ($\subset su(N)$) parts and a $su(N)$ CSA part. Only when all gauge field functions are identically zero, embedded RN solutions emerge. We here present a general analysis of the possible types of these magnetically charged $SU(N)$ EYM black hole solutions. We give the full classification for the example of $SU(5)$ EYM theory, and construct several sequences of magnetically charged non-abelian black hole solutions numerically.

2 SU(N) EYM Equations of Motion

We consider the $SU(N)$ Einstein-Yang-Mills action

$$S = S_G + S_M = \int L_G \sqrt{-g} d^4x + \int L_M \sqrt{-g} d^4x$$

with

$$L_G = \frac{1}{16\pi G} R, \quad L_M = -\frac{1}{2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}),$$

and with field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$, gauge field $A_\mu = \frac{1}{2}\lambda^a A^a_\mu$ and gauge coupling constant $e$. Variation of the action eq. (1) with respect to the metric $g_{\mu\nu}$ leads to the Einstein equations, and variation with respect to the gauge field $A_\mu$ leads to the matter field equations.

To construct static spherically symmetric black hole solutions we employ Schwarzschild-like coordinates and adopt the spherically symmetric metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -A^2 dt^2 + N^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

with the metric functions $A(r)$ and $N(r) = 1 - \frac{2m(r)}{r}$.

The static spherically symmetric ansätze for the gauge field $A_\mu$ of $SU(N)$ EYM theory are based on the $su(2)$ subalgebras of $su(N)$. Here we do not consider all inequivalent embeddings of $su(2)$ in $su(N)$ but restrict ourselves to the embedding of the $N$-dimensional representation of $su(2)$. The corresponding ansatz is given by

$$A^{(N)}_\mu dx^\mu = \frac{1}{2e} \begin{pmatrix} (N-1) \cos \theta d\phi & \omega_1 \Theta & 0 & \ldots & 0 \\ \omega_1 \Theta & (N-3) \cos \theta d\phi & \omega_2 \Theta & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \omega_{N-1} \Theta & (1-N) \cos \theta d\phi \end{pmatrix}$$

with $\Theta = id\theta + \sin \theta d\phi$, and $A_0 = A_r = 0$. The ansatz contains $N-1$ matter field functions $\omega_j(r), \ j = 1, \ldots, N - 1$, and leads to the field strength tensor components $F_{r\theta} = \partial_r A_\theta, F_{r\phi} = \partial_r A_\phi$ and

$$F_{\theta\phi} = (1/2e) \text{diag}(f_1, \ldots, f_N) \sin \theta$$

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with
\[ f_j = \omega_j^2 - \omega_{j-1}^2 + \delta_j \quad \delta_j = 2j - N - 1 \quad j = 1, \ldots, N \quad (\omega_0 = \omega_N = 0). \quad (6) \]

Let us now introduce the dimensionless coordinate \( x = er/\sqrt{4\pi G} \), the dimensionless mass function
\[ \mu = \frac{em}{\sqrt{4\pi G}}, \quad (7) \]
and the scaled matter field functions \[ u_j = \frac{\omega_j}{\sqrt{\gamma_j}} \quad \gamma_j = j(N-j). \quad (8) \]

The Einstein equations then yield for the metric functions the equations
\[ \mu' = NG + \mathcal{P}, \quad (9) \]
\[ \frac{A'}{A} = \frac{2G}{x}, \quad (10) \]
\( (\text{the prime indicates the derivative with respect to } x) \), where
\[ G = \sum_{j=1}^{N-1} \gamma_j u_j'^2 \quad \mathcal{P} = \frac{1}{4x^2} \sum_{j=1}^{N} f_j^2 \quad (11) \]

and \( f_j = \gamma_j u_j^2 - \gamma_{j-1} u_{j-1}^2 + \delta_j \) (see eq. (6)). The equations for the matter field functions are
\[ (\mathcal{AN} u_j')' + \frac{1}{2x^2} \mathcal{A}(f_{j+1} - f_j) u_j = 0, \quad (12) \]
\( \) where the metric function \( \mathcal{A} \) can be eliminated by means of eq. (10). We note the symmetry of the equations with respect to the transformation \( u_j \to u_{N-j}, \quad j = 1, \ldots, N - 1. \)

### 3 SU(N) EYM Solutions

Our aim here is to classify the charged \( SU(N) \) EYM black hole solutions, which appear as limiting solutions of sequences of neutral non-abelian \( SU(N) \) solutions, based on the \( N \)-dimensional embedding of \( su(2) \) in \( su(N) \), eq. (4).

#### 3.1 Neutral SU(N) EYM Solutions

We begin by briefly considering the neutral \( SU(N) \) EYM solutions and their boundary conditions. In these globally regular or black hole solutions all \( N - 1 \) gauge field functions are non-trivial. (In the trivial case, where all gauge field functions are identical
to one, Schwarzschild solutions are obtained.) For asymptotically flat solutions the metric functions $A$ and $\mu$ must approach constants at infinity. The time coordinate is fixed by $A(\infty) = 1$, and the mass of the solutions is given by $\mu(\infty)$. The gauge field functions $u_j$ satisfy

$$u_j(\infty) = \pm 1, \quad j = 1, \ldots, N - 1.$$  \hfill (13)

The field strength tensor component $F_{\theta\phi}^{(N)}$ decays like $O(r^{-1})$ asymptotically, whereas the components $F_{r\theta}^{(N)}$ and $F_{r\phi}^{(N)}$ decay like $O(r^{-2})$. Therefore these solutions are magnetically neutral. For globally regular solutions the boundary conditions at the origin are $\mu(0) = 0$ and

$$u_j(0) = \pm 1, \quad j = 1, \ldots, N - 1.$$  \hfill (14)

For black hole solutions with a regular horizon with radius $x_H$, the boundary conditions at the horizon are $\mathcal{N}(x_H) = 0$, i.e. $\mu(x_H) = x_H/2$, and

$$\mathcal{N}' u_j + \frac{1}{2x^2}(f_{j+1} - f_j)u_j \bigg|_{x_H} = 0.$$  \hfill (15)

For $SU(2)$ these neutral globally regular and black hole solutions are discussed in [1, 2], for $SU(3)$ in [4, 5, 6] and for $SU(4)$ in [12]. The solutions are labelled by the node numbers $n_j$ of the functions $u_j$. When the node numbers of one or more gauge field functions tend to infinity, the solutions approach limiting solutions, carrying magnetic CSA charge of norm $P$. Considering black hole solutions with $x_H > P$ (or the exterior part of the solutions with $x_H < P$)\footnote{For globally regular solutions and black hole solutions with $x_H < P$ the limiting solutions consist of two parts: an interior part for $x < P$ and an exterior part for $x > P$.}, we observe that in these limiting solutions the corresponding gauge field functions are identically zero.

### 3.2 Charged SU(N) EYM Solutions

When one or more of the $N - 1$ gauge field functions are identically zero, magnetically charged solutions are obtained. To classify the charged black hole solutions obtained within the ansatz [9], let us first assume that precisely one gauge field function is identically zero, $\omega_{j_1} \equiv 0$. The ansatz for the gauge field then splits into two parts

$$A^{(N)}_{\mu} dx^\mu = \left( \begin{array}{c} A^{(j_1)}_{\mu} dx^\mu \\ A^{(N-j_1)}_{\mu} dx^\mu \end{array} \right) + \mathcal{H}_{j_1}.$$  \hfill (16)
with \( H_{j_1} = \frac{\cos \theta d \phi}{2e} h_{j_1} \) and
\[
h_{j_1} = \begin{pmatrix} (N - j_1) \mathbb{1}_{(j_1)} \\ -j_1 \mathbb{1}_{(N-j_1)} \end{pmatrix}.
\] (17)

\( A_{\mu}^{(j_1)} \) and \( A_{\mu}^{(N-j_1)} \) denote the non-abelian spherically symmetric ansätze for the \( su(j_1) \) and \( su(N-j_1) \) subalgebras of \( su(N) \) (based on the \( j_1 \) and \( (N-j_1) \)-dimensional embeddings of \( su(2) \), respectively), referred to by \( su(\bar{N}) \) in the following. \( H_{j_1} \) represents the ansatz for the element \( h_{j_1} \) of the CSA of \( su(N) \). The field strength tensor splits accordingly into
\[
F_{\mu \nu}^{(N)} dx^\mu \wedge dx^\nu = \begin{pmatrix} F_{\mu \nu}^{(j_1)} dx^\mu \wedge dx^\nu \\ F_{\mu \nu}^{(N-j_1)} dx^\mu \wedge dx^\nu \end{pmatrix} + F^{(H_{j_1})}
\] (18)
with
\[
F^{(H_{j_1})} = -\frac{\sin \theta}{2e} d\theta \wedge d\phi \ h_{j_1}.
\] (19)

The \( su(\bar{N}) \) parts and the CSA part of the gauge field are coupled via the metric functions. Identifying the non-vanishing functions \( \omega_j \) of the \( su(N) \) ansatz with the corresponding functions \( \bar{\omega}_i \) of the non-abelian \( su(\bar{N}) \) ansätze,
\[
\gamma_i u_i^2 = \bar{\gamma}_j \bar{u}_j^2,
\] (20)
the asymptotic boundary conditions for the functions \( \bar{u}_i \),
\[
\bar{u}_i(\infty) = \pm 1,
\] (21)
yield for the functions \( u_j = \sqrt{\gamma_i/\gamma_j} \bar{u}_i \) the boundary conditions
\[
u_j(\infty) = c_j = \pm \sqrt{\gamma_i/\gamma_j}.
\] (22)

Considering the charge of the solution, we note that as above the \( su(\bar{N}) \) parts of the solutions are neutral, because the components \( F_{r\theta}^{(\bar{N})} \) and \( F_{r\phi}^{(\bar{N})} \) decay like \( O(r^{-2}) \) asymptotically and the component \( F_{\theta\phi}^{(\bar{N})} \) decays like \( O(r^{-1}) \). In contrast to \( F^{(N)} \), \( F^{(H_{j_1})} \) does not depend on \( r \). Therefore the charge of the solutions is carried by the CSA part of the gauge field. A solution based on the element \( h_{j_1} \) of the CSA then carries charge of norm \( P \hat{\mathcal{H}} \),
\[
P^2 = \frac{1}{2} \text{Tr} \ h_{j_1}^2.
\] (23)
By expanding the element $h_{j_1}$ in terms of the basis $\{\lambda_{n^2-1} \mid n = 2, \ldots, N\}$, the charge can also be directly read off the expansion coefficients. A particular convenient expansion involves the charge coefficients $P_{n^2-1} = \sqrt{\frac{n(n-1)}{2}}$

$$h_{j_1} = \sum_{n=2}^{N} d_{j_1}^{n} P_{n^2-1} \lambda_{n^2-1}. \quad (24)$$

The squared norm of the charge, $P^2$, is a gauge invariant quantity [4], which enters the equation for the mass function $\mu$. For the special case $j_1 = N - 1$, the ansatz [4] for the gauge field reduces to the non-abelian spherically symmetric ansatz for the subalgebra $su(N-1)$ together with the ansatz for the element of the CSA $h_{N-1} = P_{N^2-1} \lambda_{N^2-1}$. Consequently the CSA charge has norm $P = P_{N^2-1}$. (The case $j_1 = 1$ is equivalent.)

By applying these considerations again to the subalgebras $su(\bar{N})$ of eq. (16), we obtain the general case for SU($N$) EYM theory. When $M$ gauge field functions are identically zero, $\omega_{j_m} = 0$, $j_m \in \{j_1, j_2, \ldots, j_M\}$, $j_1 < j_2 < \cdots < j_M$, the ansatz (4) reduces to

$$A^{(N)}_{\mu} dx^{\mu} = \begin{pmatrix}
A^{(j_1)}_{\mu} dx^{\mu} \\
A^{(j_2-j_1)}_{\mu} dx^{\mu} \\
\vdots \\
A^{(N-j_M)}_{\mu} dx^{\mu}
\end{pmatrix} + H_{j_1,j_2,\ldots,j_M} \quad (25)$$

with $H_{j_1,j_2,\ldots,j_M} = \frac{\cos \theta d\phi}{2e} h_{j_1,j_2,\ldots,j_M}$ and the element $h_{j_1,j_2,\ldots,j_M}$ of the su($N$) CSA

$$\begin{pmatrix}
(N-j_1)\mathbf{1}_{(j_1)} \\
(N-j_1-j_2)\mathbf{1}_{(j_2-j_1)} \\
\vdots \\
(N-j_1-j_2-j_3)\mathbf{1}_{(j_3-j_2)} \\
\vdots \\
-j_M\mathbf{1}_{(N-j_M)}
\end{pmatrix}. \quad (26)$$

Here $A^{(j_1)}_{\mu}, \ldots, A^{(N-j_M)}_{\mu}$ denote the non-abelian spherically symmetric ansätze for the $su(j_1), \ldots, su(N-j_M)$ subalgebras of su($N$), and $H_{j_1,j_2,\ldots,j_M}$ represents the ansatz for the element $h_{j_1,j_2,\ldots,j_M}$ of the su($N$) CSA. The black hole solutions carry CSA charge
of norm $P$,
\[
P^2 = \frac{1}{2} \sum_{m=1}^{M} (N - j_m)(N - j_{m-1})(j_m - j_{m-1}) \quad (j_0 = 0) .
\] (27)

In the special case that all gauge field functions are identically zero, an embedded RN solution is obtained with CSA charge of norm $P$,
\[
P^2 = \frac{1}{6}(N - 1)N(N + 1) .
\] (28)

For the charged $SU(N)$ EYM black holes considered here, this is the maximal possible norm of the charge.

RN black hole solutions exist only for horizon radius $x_H \geq P$, and the extremal RN solution has $x_H = P$. As first observed for $SU(3)$ EYM theory \cite{11}, the same is true for charged non-abelian black holes. Non-abelian black hole solutions with CSA charge of norm $P$ exist only for horizon radii $x_H \geq P$. For extremal black hole solutions $\mathcal{N}' = 0$, and the coefficient of $u_j'$ in eq. (15) vanishes, yielding the boundary conditions
\[
(f_{j+1} - f_j)u_j|_{x_H} = 0 ,
\] (29)
i.e. $u_j(x_H) = c_j$, corresponding to $\bar{u}_i(x_H) = \pm 1$. There are no globally regular charged solutions.

4 Example: Charged SU(5) EYM Black Holes

We now apply the above general analysis to $SU(5)$ EYM theory and present magnetically charged $SU(5)$ EYM solutions. The classification of the charged $SU(5)$ EYM black hole solutions is presented for all inequivalent cases (within the ansatz (4)) in Table 1.

Like the neutral non-abelian spherically symmetric solutions of $SU(N)$ EYM theory, the charged solutions are classified by the node numbers $n_i$ of their (non-vanishing) gauge field functions $\bar{u}_i$. When the node numbers of one or more gauge field functions tend to infinity, the solutions approach limiting solutions with higher norm of the charge.

In Table 2 we show the mass $\mu(\infty)$ of the lowest solutions of five sequences of black hole solutions, corresponding to the five inequivalent cases of Table 1, together with the mass of their limiting solutions. Shown are case 1a, case 2a with node structure $(n, 0)$, case 2b with node structure and 0, case 3a with node structure $(n, 0)$ and 0, for their extremal horizon $x_H = P$ and for $x_H = \sqrt{20}$. Their limiting solutions are the RN solutions with CSA charge of norm $P = \sqrt{20}$ (1a), $P = \sqrt{19}$ (2a,2b), $P = \sqrt{16}$ (3a) and $P = \sqrt{18}$ (3b), for $x > P$. The generalization to all other cases is straightforward. For instance, for the case 2a the limiting solution for
a sequence \((i + n, i)\), \(i > 0\), is a non-abelian solution with the same norm of the CSA charge, \(P = \sqrt{19}\).

As illustrated in Fig. 1, with increasing \(n\) the mass \(\mu_n(\infty)\) converges exponentially to the mass \(\mu_{\infty}(\infty)\) of the corresponding limiting solution. The pattern of convergence of the charged solutions is completely analogous to the pattern of convergence of the neutral solutions as observed previously [6], with the extremal charged solutions playing the role of the globally regular neutral solutions. The extremal solutions fall on straight lines, whereas the non-extremal black hole solutions fall on straight lines only beyond some critical \(n\), for which the horizon radius is getting close to the innermost node of the gauge field function with \(n\) nodes, \(\bar{u}_{1,n}\). The slopes are determined by the particular type of sequence. Interestingly, we observe that the slopes of the extremal \(SU(2)\) solutions (1a,2b) agree with the slope of the globally regular \(SU(2)\) solutions, and that their innermost nodes converge with the same exponent towards a limiting value, slightly smaller than the norm of the charge of the limiting solution [5].

In Figs. 2a-c we present the lowest odd solutions of case 3a for the sequence \((n, 0, 0)\) and extremal horizon \(x_H = \sqrt{10}\), also in the interior of the black hole (for \(x < x_H\)). Figs. 2a-b show the functions \(\bar{u}_1\), \(\bar{u}_2\) and \(\bar{u}_3\), and Fig. 2c shows the charge function \(P(x)\), \(P^2(x) = 2x(\mu(\infty) - \mu(x))\) [6]. The charge functions \(P_n(x)\) of this sequence of solutions with CSA charge of norm \(P = \sqrt{10}\) clearly illustrate that the limiting solution carries a CSA charge which has the higher norm \(P = 4\).

5 Inclusion of the Dilaton

Let us end by considering \(SU(N)\) Einstein-Yang-Mills-dilaton (EYMD) theory. Analogously to EYM theory, in \(SU(2)\) and \(SU(3)\) EYMD theory sequences of neutral globally regular and black hole solutions exist [6, 13, 14]. The neutral \(SU(2)\) EYMD black hole solutions converge to magnetically charged Einstein-Maxwell-dilaton (EMD) black hole solutions [14], and the neutral \(SU(3)\) EYMD solutions converge to limiting magnetically charged solutions, in which again one of the two \(SU(3)\) gauge field functions is identically zero [13, 14]. In general, the \(SU(N)\) EYM classification remains valid for \(SU(N)\) EYMD theory. However, the magnetically charged non-abelian black hole solutions now exist for arbitrary horizon radius \(x_H > 0\), completely analogous to their abelian (EMD) counterparts.

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| #  | $u_j, j = 1 - 4$ | $P^2$ | non-abelian subalgebra | Cartan subalgebra $^*$ |
|----|----------------|------|------------------------|------------------------|
| 0  | 0 0 0 0 0       | 20   |                        | $\lambda_3 \quad \lambda_8 \quad \lambda_{15} \quad \lambda_{24}$ |
| 1a | $u_1 \quad 0 \quad 0 \quad 0$ | 19   | $su(2)$                | $P_3 \quad P_8 \quad P_{15} \quad P_{24}$  |
| 2a | $u_1 \quad u_2 \quad 0 \quad 0$ | 16   | $su(3)$                | $0 \quad 0 \quad P_{15} \quad P_{24}$  |
| 2b | $u_1 \quad 0 \quad u_3 \quad 0$ | 18   | $su(2) \oplus su(2)$  | $0 \quad \frac{1}{7} P_3 \quad \frac{2}{7} P_{15} \quad P_{24}$  |
| 3a | $u_1 \quad u_2 \quad u_3 \quad 0$ | 10   | $su(4)$                | $0 \quad 0 \quad 0 \quad P_{24}$  |
| 3b | $u_1 \quad u_2 \quad 0 \quad u_4$ | 15   | $su(3) \oplus su(2)$  | $0 \quad 0 \quad \frac{5}{4} P_{15} \quad \frac{3}{4} P_{24}$  |

**Table 1**

Classification of the magnetically charged black hole solutions of $SU(5)$ EYM theory. Shown are the non-vanishing gauge field functions (denoted by $u_j$) and the identically vanishing gauge field functions (denoted by zero), the square of the norm of the charge of the black hole solutions, $P^2$, and the subalgebras of the solutions including the non-abelian neutral part and the CSA charged part, with the coefficients (in the given basis $^*$) of the corresponding element of the CSA.
| $n/x_H$ | $\sqrt{19}$ | $\sqrt{16}$ | $\sqrt{18}$ | $\sqrt{10}$ | $\sqrt{15}$ |
|--------|-------------|-------------|-------------|-------------|-------------|
| 1      | 4.44954     | 4.25227     | 4.33571     | 3.69884     | 4.13289     |
| 2      | 4.46834     | 4.33108     | 4.35500     | 3.90097     | 4.21401     |
| 3      | 4.47152     | 4.35195     | 4.35826     | 3.96889     | 4.23549     |
| 4      | 4.47204     | 4.35719     | 4.35880     | 3.99041     | 4.24088     |
| 5      | 4.47212     | 4.35848     | 4.35888     | 3.99706     | 4.24221     |
| $\infty$ | 4.47214 | 4.35890     | 4.35890     | 4.0         | 4.24264     |

$x_H = \sqrt{20}$

| $n/x_H$ | $\sqrt{20}$ | 20 | 21 | 22 | 23 |
|--------|-------------|----|----|----|----|
| 1      | 4.45085     | 4.27291 | 4.34103 | 3.83752 | 4.16591 |
| 2      | 4.46919     | 4.34438 | 4.35861 | 3.98865 | 4.23561 |
| 3      | 4.47182     | 4.35841 | 4.36027 | 4.02026 | 4.24735 |
| 4      | 4.47211     | 4.36018 | 4.36033 | 4.02446 | 4.24845 |
| 5      | 4.47213     | 4.36032 | 4.36033 | 4.02488 | 4.24852 |
| $\infty$ | 4.47214 | 4.36033 | 4.36033 | 4.02492 | 4.24853 |

**Table 2**

The dimensionless mass $\mu(\infty)$ of the lowest black hole solutions of the sequences corresponding to case 1a of Table 1, case 2a with node structure $(n,0)$, case 2b with node structure $n$, 0, case 3a with node structure $(n,0,0)$ and case 3b with node structure $(n,0)$, 0 for their extremal horizons and for $x_H = \sqrt{20}$. For each sequence the corresponding limiting value is shown in the last row (denoted by $\infty$).
Figure 1: The logarithm of the absolute deviation from the limiting solution $\Delta \mu_n = \mu_\infty(\infty) - \mu_n(\infty)$ for the masses of the cases 1a, 2a and 3a of Table 1 as a function of the node number $n$ for the extremal solutions (dashed lines) and the non-extremal solutions (solid lines) with horizon at $x_H = \sqrt{20}$. The dotted lines indicate the asymptotic behaviour of the non-extremal solutions.
Figure 2a:
The matter functions $\bar{u}_{1,n}(x)$ for case 3a of Table 1 with the node structure $(n, 0, 0)$ and extremal event horizon $x_H = \sqrt{10}$. 
Figure 2b:
The same as Fig. 2a for the matter functions $\bar{u}_{2,n}(x)$ and $\bar{u}_{3,n}(x)$. 
Figure 2c:
The same as Fig. 2a for the charge function $P_n(x)$. 