To the question of heat and mass transfer in clouds and in engineering

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Abstract. Heat and mass transfer in the nature and engineering with the help of the following physical mechanisms: convective heat transfer, mass transfer, radiant heat transfer (transfer of energy by radiation) are studied in the given work. In various cases each of these physical mechanisms can make major or minor (which may be neglected) contribution to heat and mass transfer. Simplified calculation formulas of heat and mass transfer are received for the following cases: heat amount transmitted by radiant heat transfer is small and may be neglected; convective heat transfer is not significant; mass transfer has poor influence on heat transfer.

Introduction

Phenomena of heat and mass transfer observed in the nature (clouds, precipitation, thunderstorm), and in engineering (heat and refrigeration engineering), as a rule, include all elementary methods of heat and water transfer. Heat and mass transfer between water and air which is a result of the following three physical mechanisms: convective heat transfer, mass transfer and radiant heat transfer (transfer of energy by radiation) is reviewed in the given work. Each of these physical mechanisms in various situations can make significant or insignificant contribution to the whole process. In connection with it, it’s important to modify the general complicated equation of heat and mass transfer by various methods taking into account contribution value of each physical mechanism (convective heat transfer, mass transfer and radiant heat transfer).

Simplified equations of heat and mass transfer for the following cases: when the role (contribution) of radiant constituent of the heat flow $dQ_r$ is not large and can be neglected (equation 16); when the contribution of mass transfer $dQ_m$ is insignificant (equation 22); when the contribution of convective heat transfer $dQ_c$ is not large and can be neglected (equations 29 and 30), are received.

Equation of heat and mass transfer and its particular cases

A considerable number of publications is dedicated to investigation of the processes of heat and mass transfer between water and air, in which the results of as theoretical so experimental analyses are presented.
With immediate contact between air and water surface as heat so mass transfer will be observed. If to distinguish a small area \( dF \), on the water surface, the heat amount \( dQ_\tau \), by which the given mediums exchange for the period \( \tau \), may be presented by the following relation

\[
dQ_\tau = dQ_{c\tau} + dQ_{m\tau} + dQ_{r\tau}
\]

(1)

where \( dQ_{c\tau} = \alpha \cdot \Delta T \cdot dF \cdot \tau \) – is an elementary amount of heat transmitted by convective heat transfer for the period \( \tau \); \( dQ_{m\tau} = \sigma \cdot r(d^* - d) \cdot dF \cdot \tau \) – is an elementary amount of heat transmitted by mass transfer for the period \( \tau \); \( dQ_{r\tau} = \epsilon \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \) \( dF \cdot \tau \) – is an elementary amount of heat transmitted by radiant heat transfer for the period \( \tau \).

For heat flow the equation (1) has the following formula:

\[
dQ = dQ_{c} + dQ_{m} + dQ_{r},
\]

(2)

where \( dQ_{c} = \alpha \cdot \Delta T \cdot dF \) – is an elementary heat flow transmitted by convective heat transfer;

\[
dQ_{m} = \sigma \cdot r(d^* - d) \cdot dF
\]

– is an elementary heat flow transmitted by mass transfer;

\[
dQ_{r} = C \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \ dF
\]

– is an elementary heat flow transmitted by radiation.

For the density of heat flow the equation (2) may be written as following:

\[
q = q_{c} + q_{m} + q_{r},
\]

(3)

where \( q_{c} = \alpha \cdot \Delta T \) – is a convective constituent of the density of heat flow;

\[
q_{r} = C \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \ – a part of the density of heat flow transmitted by radiation;
\]

\[
q_{m} = \sigma \cdot r(d^* - d) \ – a part of the density of heat flow transmitted by mass transfer.
\]

In the equations (1) - (3) the following designations are accepted: \( \alpha \) – is a coefficient of heat transfer; \( \Delta T = T_1 - T_2 \) – is the difference between temperatures of mediums which take part in heat transfer; \( F \) – is a heat emitting surface; \( \sigma \) - is a coefficient of mass transfer; \( d^* \) is a hydrometric content of saturated air; \( r \) – is the heat of vaporization; \( \epsilon = c_0 \cdot \varepsilon \) – is a coefficient of radiation of water surface; \( T_1, T_2 \) – are temperatures of mediums which take part in heat transfer.

Here \( c_0 \) – is a coefficient of radiation of black body; \( \varepsilon \) – is the level of blackness of water surface.

Equation (2) may be modified by using various methods.

The first method of transformation of the equation (2):

- instead of \( dQ_{c} \) and \( dQ_{m} \) we insert their values

\[
dQ = \alpha \cdot \Delta T \cdot dF + \sigma \cdot r(d^* - d) \ dF + dQ_{r},
\]

(4)

We transfer the right part of the equation (4), then we factor \( \sigma \) and \( dF \) out of brackets from the first and second terms.

\[
dQ = \sigma \left[ \frac{\alpha}{\sigma} \Delta T + r(d^* - d) \right] dF + dQ_{r}
\]

(5)

We will consider that, while observing the given phenomena, triple analogy is observed. This process is described by the law of Lewis. Then
\[ \frac{\alpha}{\sigma} = c_p', \]  

(6)

where \( c_p' = c_{p_r} + c_{p_v} \cdot d \) is a mass isobar heat capacity of the air.

Here \( c_{p_r} \) is the isobar heat capacity of the dry part of the air; \( c_{p_v} \) is the isobar heat capacity of water vapor.

Considering the correlation (6), we write the equation (5) as following:

\[ dQ = \sigma \left( [i'' - i] - (d'' - d) \right) c_w \cdot t_w \, dF + dQ_r, \]

(7)

Thermo-dynamic parameters of the air are interconnected with the following formulas:

\[ r = r_0 \left( c_w - c_{p_v} \right) t_w, \]

(8)

\[ i = c_{p_r} \cdot t_g + r_0 \cdot d + c_{p_v} \cdot t_g \cdot d, \]

(9)

\[ i'' = c_{p_r} \cdot t_w + r_0 \cdot d'' + c_{p_v} \cdot t_w \cdot d'', \]

(10)

where \( r_0 = 2500 \) kilojoules/kg is a heat of water transformation at \( t = 0^\circ C \); \( i \) is an enthalpy of unsaturated air; \( i'' \) is an enthalpy of saturated air; \( c_w \) is the mass heat capacity of the water; \( t_g \) is a temperature of gas beyond the limits of heat and diffused boundary layer; \( t_w \) is a temperature of water.

Considering the equations (8) - (10), the correlation (7) may be written as following:

\[ dQ = \sigma \left( i'' - i \right) dF + dQ_r, \]

(11)

or

\[ dQ = \sigma \left( i'' - i \right) dF - c_w \cdot t_w \cdot dM + dQ_r, \]

(12)

where

\[ dM = \sigma (d'' - d) dF. \]

Equation (12) may be transformed into

\[ dQ = A \sigma (i'' - i) dF + dQ_r, \]

(13)

where

\[ A = 1 - \frac{c_w \cdot t_w \cdot dM}{(i'' - i) dF}. \]

According to the data [1, 2, 4] we may consider that for air-cooling devices \( A = 1 \) the equation (13) will be modified into

\[ dQ = \sigma (i'' - i) dF + dQ_r, \]

(14)

Referential values \( A \) recommended for calculation of data [2] are given below in the form of Table 1:

| \( t_w[^\circ C] \) | 10 | 15 | 20 | 25 | 30 | 35 |
|-------------------|----|----|----|----|----|----|
| \( A \)           | 0.99| 0.98| 0.97| 0.96| 0.95| 0.94|

If we extrapolate the recent data towards the decrease of temperature \( t_w \), then at \( t_w = 5 \, ^\circ C \) \( A = 1 \).

At constancy of parameters and values which characterize the process of heat and mass transfer between water and air, the integral of the equation (14) will be equal to

\[ Q = \sigma (i'' - i) F + Q_r \]

(15)

If the role of radiant constituent of heat flow is not large, it may not be considered in some cases. Then the equation (15) will be transformed into

\[ \]
\[ Q = \sigma (i^* - i)F \]  

The equation (2) may be modified in other way, such as

\[ dQ = \alpha \cdot \Delta T \, dF + dQ_m + c \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] dF \]  

or

\[ dQ = \alpha_{tot} \cdot \Delta T \, dF + dQ_m, \]  

where

\[ \alpha_{tot} = \alpha + \alpha_r. \]  

Here

\[ \alpha_r = \frac{c \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]}{\Delta T}. \]  

Attractiveness of the form of equations (17)-(20) consists in the fact that numerical values of the formula \( \frac{\left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4}{\Delta T} \) are given in tables.

At the constancy of parameters and values which characterize heat and mass transfer the equation (18) may be modified in the following way

\[ Q = \alpha_{tot} \cdot \Delta T \cdot F + Q_m \]  

If the constituent of heat flow conditioned by mass transfer is not large and it may not be taken into account within the limits of accuracy of used dependencies, so the equation (21) will take the following form

\[ Q = \alpha_{tot} \cdot \Delta T \cdot F \]  

While considering the same task in relation to mass transfer complicated by heat exchange, there will be another solution.

We cut out elementary parallelepiped with sides \( dx, dy, dz \) from the volume of air located within the limits of diffused boundary layer, and orient it in rectangular system of coordinates (refer with: Figure 1).

We consider the case of molecular diffusion and thermo-diffusion. There is no pressure diffusion. Vapor concentration on the left edge (1–2–3–4) is \( C \), and the temperature is \( t \).

Here \( C \) – is the volume concentration of relation of the mass of water vapor \( (M_v) \) to the capacity taken by it \( (V) \), \( C = \frac{M_v}{V} \).
Figure 1. To the derivation of differential equation of mass transfer complicated by heat exchange

According to the first law of A.O. Fick [3] and correlations determined by thermo-diffusion through the left edge of parallelepiped towards \( x \) axis the mass comes out to the given figure for the period \( d\tau \)

\[
\delta M_x = -D \frac{\partial C}{\partial x} dy dz d\tau - \rho \frac{D_T}{T} \frac{\partial t}{\partial x} dy dz d\tau,
\]

where \( D \) – is a coefficient of molecular diffusion; \( \rho \) – is density; \( D_T \) – is a coefficient of thermo-diffusion.

On the right edge of the parallelepiped (5–6–7–8) the temperature is \( t + \frac{\partial t}{\partial x} dx \), and the concentration is \( C + \frac{\partial C}{\partial x} dx \).

The mass will pass through the right edge of the given figure

\[
\delta M_{x+dx} = -D \left( \frac{C + \frac{\partial C}{\partial x} dx}{\partial x} \right) dy dz d\tau - D_T \frac{\partial t}{T} \left( \frac{t + \frac{\partial t}{\partial x} dx}{\partial x} \right) dy dz d\tau,
\]

where \( D \) and \( D_T \) are constant, i.e. we consider that in the given figure there is no deformation of geometric dimensions.

Using the equations (23) and (24) we will receive the relation for vapor mass in parallelepiped.

\[
dM_x = \delta M_x - \delta M_{x+dx} = -D \frac{\partial C}{\partial x} dy dz d\tau - \rho \frac{D_T}{T} \frac{\partial t}{\partial x} dy dz d\tau +
\]

\[
+ D \left( \frac{C + \frac{\partial C}{\partial x} dx}{\partial x} \right) dy dz d\tau + D_T \frac{\partial t}{T} \left( \frac{t + \frac{\partial t}{\partial x} dx}{\partial x} \right) dy dz d\tau
\]

or

\[
dM_x = D \frac{\partial^2 C}{\partial x^2} dx dy dz d\tau + D_T \frac{\rho}{T} \frac{\partial^2 t}{\partial x^2} dx dy dz d\tau
\]

Reasoning in the same way we will receive constituents of mass amount came to the given figure by axes \( y \) and \( z \).
\[ dM_x = D \frac{\partial^2 C}{\partial y^2} dx dy dz d\tau + D_r \frac{\partial^2 t}{\partial y^2} dx dy dz d\tau \]  
(27)

\[ dM_z = D \frac{\partial^2 C}{\partial z^2} dx dy dz d\tau + D_r \frac{\partial^2 t}{\partial z^2} dx dy dz d\tau \]  
(28)

The total amount of mass came to the given figure may be received by summing the equations (26)–(28)

\[ dM = dM_x + dM_y + dM_z \]

or

\[ dM = \left( D \nabla^2 C + D_r \frac{P}{T} \nabla^2 t \right) dv d\tau . \]  
(29)

where

\[ \nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} ; \]

\[ \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} ; \]

\[ dv = dx \cdot dy \cdot dz . \]

While receiving the equation (29) the radiant heat transfer was not presupposed.

Reasoning in the same way it is possible to receive equation for the mass amount with pressure diffusion available:

\[ dM = \left( D \nabla^2 C + D_r \frac{P}{T} \nabla^2 t + D_p \frac{P}{P} \nabla^2 P \right) dv d\tau . \]  
(30)

where \( D_p \) – is a coefficient of pressure diffusion; \( P \) – is total mixture pressure;

\[ \nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} . \]

While receiving the equation (30) the constancy of coefficient of pressure diffusion was presupposed and all those assumptions which were revealed earlier, i.e. while receiving the equation (29), were observed.

**Summary**

The general formula of calculation of heat and mass transfer between water and air in the nature (clouds, precipitation, thunderstorms) and in engineering (heat and refrigeration engineering) is presented in the given work. The general formula includes three physical mechanisms of water and heat exchange: convective heat transfer, mass transfer and radiant heat transfer.

Simplified particular solutions of the general formula of calculation of heat and mass transfer for the three physical mechanisms (convective heat transfer, mass transfer and radiant heat transfer) are received. If in general heat and mass transfer the radiant heat transfer \( dQ_r \) is not significant and may not be considered, the general formula of calculation has simple form (16). And if the amount of heat transmitted by mass transfer \( dQ_m \) is not large and in the limits of accuracy of used dependencies it may not be considered, then simplified equation will take the following form (22). And in case when the amount of heat transmitted by convective heat transfer is small, the general formula of calculation simplifies significantly and takes the form of equation (29, 30).

**References**

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