A hybrid chaos map with two control parameters to secure image encryption algorithms

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Abstract—In this paper, we introduce a hybrid chaos map for image encryption method with high sensitivity. This new map is sensitive to small changes in the starting point and also in control parameters which result in having more computational complexity. Also, it has uniform distribution that provides resisting of the new system against attacks in security applications. Various tests and plots are demonstrated to show more chaotic behavior of the proposed system. Finally, to show the ability of the generated chaotic map in the existences image cryptography approaches, we further report some results in this area.

1 INTRODUCTION

Due to increasing demand for information security, cryptography is used to transfer information safely. Chaos is known as a natural tool for cryptography applications because of its properties such as unpredictability and initial state sensitivity.

Early chaotic systems such as logistic and tent maps have weaknesses such as limited chaotic area and non uniform distribution. The idea of combining these systems and creating new systems has been developed to solve the mentioned problems [1], [2], [3]. A combined chaotic system has been suggested and applied to color image encryption in [4]. Also, a novel type of uniformly distributed 2D hybrid chaos map using the cellular automata and discrete framelet transform has been developed to transfer images securely by using cryptography and steganography methods [5].

The purpose of this paper is to introduce a new type of uniformly distributed hybrid chaos map using chaos maps introduced in [7, 5, 6] The proposed system depends on the values \( r_1 \), \( r_2 \) and the starting point unlike the existing systems which based on only the value \( r \) and the starting point. This idea leads to have more computational complexity, a large enough positive Lyapunov exponent which provides hyper-chaos map, better sensitivity and high security.

Performance analysis is provided to show that this map has the wider chaotic range and hyperchaotic features than existing chaotic systems. To show its performance in security applications, it is utilized in the image encryption method. The image encryption algorithm introduced in [7] is used to test the features of the new system.

2 HYBRID CHAOS MAP

In this section, the new chaos map is introduced using hybrid chaos and 2D hybrid chaos maps [3, 6]. Fristly, we recall the structure of these maps and then the new map is detailed in [2].

\[ x_{n+1} = s(r, x_n) := r \sin(\pi x_n)/4, \]

and Logistic Tent map is defined as follows:

\[ x_{n+1} = \begin{cases} 
LT_1(r, x_n) := r x_n(1-x_n) + \frac{r \sin(\pi x_n)}{4}, & x_n < 0.5 \\
LT_2(r, x_n) := r x_n(1-x_n) + \frac{r \sin(\pi x_n)}{4}, & x_n \geq 0.5 
\end{cases} \]

By a combination and transformation of these maps, a hybrid chaos map is introduced as follows [6]:

\[ x_{n+1} := HCM1(r, x_n) = \begin{cases} 
r \left( \sin(LT_1(r, x_n)) + \cot(x_n^2) \\
+ S(r, LT_1(r, x_n)) \right), & x_n < \frac{1}{4} \\
(r + 1) \left( \sin(LT_2(r, x_n)) + \cot(x_n) \\
+ 10^5 \sqrt{|x_n|} \right), & x_n \geq \frac{1}{4} 
\end{cases} \]

Also, in [5], 2D Hybrid chaos map is expressed as follows:

\[ x_{n+1} := HCM2x(r, x_n, y_n) = \begin{cases} 
\omega_1 f_1^x \circ F_1^x(r, x_n) + \alpha_1 g_1^x(r, x_n, y_n) \\
+ h_1^x(\beta_1-r) x_n \mod 1, & y_n < 0.5 \\
\omega_2 f_1^x \circ F_2^x(r, x_n) + \alpha_2 g_2^x(r, x_n, y_n) \\
+ h_2^x(\beta_2-r)(1-x_n) \mod 1, & y_n \geq 0.5 
\end{cases} \]

\[ y_{n+1} := HCM2y(r, x_n, y_n) = \begin{cases} 
\omega_1 f_1^y \circ F_1^y(r, y_n) + \alpha_1 g_1^y(r, \zeta, y_n) \\
+ h_1^y(\beta_1-r) \zeta \mod 1, & \zeta < 0.5 \\
\omega_2 f_1^y \circ F_2^y(r, y_n) + \alpha_2 g_2^y(r, \zeta, y_n) \\
+ h_2^y(\beta_2-r)(1-\zeta) \mod 1, & \zeta \geq 0.5 
\end{cases} \]
where

\[ \zeta = x_n \text{ or } x_{n+1}, \]

\((\omega^k, \alpha^k, \beta^k), (f^k, h^k)\) and \(g^k\) for \((k = x, y \& l = 1, 2)\) have been respectively considered as weights, combination maps and transfer maps. Also \(F^k\) have been arbitrary chosen Logistic or Sine maps.

### 2.2 The new chaotic system

By taking advantages of the above hybrid chaos maps, we aim to develop a new type of chaotic map. The process of creating proposed hybrid system is shown in Fig. 1. To create the proposed hybrid chaos system, as we can see from the figure, two hybrid chaos maps are used to produce the inputs of third composite box. Within this box, numerical values are entered and by using the combination and transfer operations, the output value is generated. This map is structured as follows:

\[
\begin{align*}
\rho_n &= HCM1(r_1, x_n), \\
\rho_{n+1} &= HCM1(r_1, \rho_n), \\
\tau_n^1 &= HCM2x(r_2, \rho_n, \rho_{n+1}), \\
\tau_n^2 &= HCM2y(r_2, \rho_n, \rho_{n+1}), \\
x_{n+1} &= \gamma \varphi_1 \alpha_2 (\tau_n^1, \tau_n^2),
\end{align*}
\]

where \(\gamma\) is a constant and \(\varphi_1\) and \(\varphi_2\) are appropriate continuous functions.

The histogram of the proposed system for 140,000 points is shown in Fig. 2. By using this figure, it is can be seen that the histogram is almost flat. Another suitable diagram to investigate the behavior of a chaos system is the cobweb plot. Cobweb plots for different values of \(r_1\) and \(r_2\) are given in Fig. 3. The obtained results show that the output states does not converge to a special point.

### 3 Performance analysis of proposed chaotic map

The proposed chaotic map exhibit complex chaotic behaviors. To demonstrate this property, we estimate the chaos performance of chaotic map produced above. The tests are performed in terms of initial state sensitivity, bifurcation diagrams and Lyapunov exponents.

#### 3.1 Initial state sensitivity

To check the system sensitivity to initial points, very small changes as \(+10^{-16}\) have been made to the input data. The output values for different initial points are shown in Fig. 4. It can be seen that the output values are changed by a small change in the initial value.

#### 3.2 Bifurcation diagram

The bifurcation diagram of a dynamical system provides a visualized method to exhibit the chaotic system behaviors. Fig. 5 presents the bifurcation diagrams of the proposed map for two initial values \(x_0 = 0.2\) and \(x_0 = 0.5\). The
existing maps, namely the Logistic, Sine, and Tent maps, have fixed or periodic points in most parameter settings. Moreover, their output states are only distributed in a small area. However, the generated chaotic map exhibit complicated behaviors in all parameter ranges, and its output states are randomly distributed in the entire plane, indicating that it has robust chaotic behaviors and its outputs are more random.

3.3 Lyapunov exponent analysis

Another tool for studying chaos system is Lyapunov exponent. The relationship between Lyapunov exponent and chaotic system can be considered in the following theorem.

**Theorem** If at least one of the average Lyapunov exponents is positive, then the system is chaotic; if the average Lyapunov exponent is negative, then the orbit is periodic and when the average Lyapunov exponent is zero, a bifurcation occurs [8].

Thus, a dynamical system is considered to exhibit chaotic behaviors if it can obtain a positive Lyapunov exponent. A larger Lyapunov exponent means that the trajectories diverge faster, indicating superior chaos performance. Fig. 6 shows that the numerical values for lyapunov exponent are positive then this test indicates that neighboring trajectories diverge.

4 Encryption results

To prove resistance and sensitivity of the new system in security algorithms, we apply outputs of chaos sequence in the image encryption algorithm suggested in [7]. The proposed map is replaced on the encryption algorithm. The obtained results for the encryption algorithm utilizing the generated chaotic map are given in Fig. 7.

The resistance of an encrypted image against differential attacks can be measured by the Number of Pixels Change Rate (NPCR) and the Unified Average Changing Intensity (UACI). When these values are close to 100% and 33.33% respectively, then encryption algorithm has high sensitivity of changing of plain image. In order to test the proposed algorithm, the results obtained for NPCR and UACI have been compared with the algorithm [7] in Table 1.
(a) $x_0 = 0.5$

(b) $x_0 = 0.2$

Fig. 5. Bifurcation diagrams of the proposed system for different values of $x_0$.

Fig. 6. Lyapunov exponent results for proposed hybrid chaotic system with $x_0 = 0.5$.

5 CONCLUSION

This paper has proposed a new chaotic map. It was derived from the hybrid chaos and 2D hybrid chaos systems utilizing the Sine and Logistic maps. Several assessment methods, including the Lyapunov exponents, initial states sensitivity, bifurcation diagram and etc, have been used to evaluate the chaotic performance of the chaos map. Analysis and evaluation results have shown that this map has the wider chaotic range, better ergodicity and hyperchaotic property, and that it has better chaotic performance than existing chaotic maps.

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