NON-MINIMAL SNEUTRINO INFLATION, PECCEI-QUINN PHASE TRANSITION AND NON- THERMAL LEPTOGENESIS

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ABSTRACT

We consider a phenomenological extension of the minimal supersymmetric standard model which incorporates non-minimal chaotic inflation, driven by a quartic potential associated with the lightest right-handed sneutrino. Inflation is followed by a Peccei-Quinn phase transition based on renormalizable superpotential terms, which resolves the strong CP and \( \mu \) problems of the minimal supersymmetric standard model provided that one related parameter of the superpotential is somewhat small. Baryogenesis occurs via non-thermal leptogenesis, which is realized by the inflaton decay. Confronting our scenario with the current observational data on the inflationary observables, the baryon asymmetry of the universe, the gravitino limit on the reheating temperature and the upper bound on the light neutrino masses, we constrain the effective Yukawa coupling involved in the decay of the inflaton to relatively small values and the inflaton mass to values lower than \( 10^{12} \) GeV.

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1 INTRODUCTION

Recently non-minimal inflation (non-MI) [1], i.e. inflation arising in the presence of a non-minimal coupling between the inflaton field and the Ricci scalar curvature, $R$, has gained a fair amount of attention [2–8]. In particular, it is shown that non-minimal chaotic inflation based on a quartic potential [9] with a quadratic non-minimal coupling to gravity can be realized in both a non-supersymmetric [2, 3, 6] and a supersymmetric (SUSY) framework [7, 8], provided that the inflaton couples strongly enough to $R$. In the latter case, the recently developed [8] superconformal approach to supergravity (SUGRA) greatly facilitates the relevant model building. In most of the models proposed, the inflaton is identified with the Higgs field(s) of the Standard Model (SM) or the next-to-MSSM (Minimal SUSY SM) [7, 8] – see also Ref. [10].

Motivated by the various attractive features of the MSSM [11] – such as the resolution of the hierarchy problem, the achievement of gauge coupling unification and the candidature of the lightest SUSY particle as cold dark matter – we consider it as the starting point of our investigation. Despite its successes, however, the MSSM fails to address a number of important issues. For instance, the strong CP and $\mu$ problems, the generation of the observed baryon asymmetry of the universe (BAU) and the existence of tiny but non-zero neutrino masses are some fundamental issues which remain open within the MSSM. For the resolution of these, it seems imperative to supplement the MSSM with additional superfields, which in the simplest cases are singlets under the SM gauge group, $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, so that gauge coupling unification is not disrupted. Consequently, new candidates (besides the Higgs boson) for driving non-MI arise.

In Ref. [12–14] a resolution to the aforementioned problems of the MSSM was proposed within a framework that implements a Peccei-Quinn symmetry breaking phase transition (PQPT). In those models non-renormalizable superpotential terms are added, involving some singlets that develop vacuum expectation values (VEVs) of the order of the PQ symmetry breaking scale. As a consequence, the $\mu$ and the strong CP problems [15] of the MSSM can be simultaneously solved, and in addition
a new intermediate scale arises which generates Majorana masses for three right-handed (RH) neutrinos, $N^c_i$. The inclusion of $N^c_i$ is necessary so that the smallness of neutrino masses is explained through the well-known see-saw mechanism [16]. These same superfields can play an important role in the generation of the BAU via non-thermal leptogenesis [17–19]. This latter attractive possibility is invalidated, though, in the cases studied in Ref. [12, 13], where the PQPT follows a period of thermal inflation [20] that leads to a very low reheating temperature. An enormous entropy production occurs, diluting any preexisting, non-thermally created, lepton asymmetry. This dilution can be avoided if we adopt the scheme of Ref. [14] but then, the PQ field cannot be zero during inflation – see below.

On the other hand, non-thermal leptogenesis can be enhanced if the scalar component, $\tilde{N}$, of the lightest $N^c_i$ is the inflaton itself as firstly proposed in Ref. [21]. In this case the branching ratio of the inflaton decay (which now triggers leptogenesis) into a lepton plus a Higgs boson is [19] maximized. However, $\tilde{N}$-inflation, in its simplest realization [13,21,22], is of the chaotic type – for other scenarios see Ref. [23,24] – and therefore, trans-Planckian inflaton-field values are typically required to allow for a sufficiently long period of inflation. The implementation of inflation then necessitates the adoption of special types of Kähler potential, as in Ref. [18], so that SUGRA corrections are kept under control – for other proposals related to chaotic inflation with a quadratic potential, see Ref. [25]. Moreover, minimal chaotic inflation driven by a quartic potential seems [26] to be ruled out by the fitting to the seven-year data of the Wilkinson Microwave Anisotropy Probe Satellite (WMAP7), baryon-acoustic-oscillations (BAO) and Hubble constant ($H_0$) data [27].

In this paper we construct a model of non-minimal $\tilde{N}$ inflation (non-M$\tilde{N}$I) retaining the successful ingredients of the picture above. To this aim, $\tilde{N}$ (the lightest RH sneutrino) is coupled to one of the PQ fields, which can be confined to zero during inflation – see Ref. [8]. We then show that the model naturally leads to non-MI within SUGRA, provided that a particular parameter of the superpotential is sufficiently small. Sub-Planckian values of the inflaton field are allowed in a wide range of the parameter space, and the adopted type of Kähler potential is more or less well-motivated. Also the inflationary observables turn out to lie within the current data. The non-M$\tilde{N}$I is followed by a PQPT driven by renormalizable superpotential terms as in Ref. [28, 29], whereas the $\mu$ parameter of the MSSM can be generated from the PQ scale via a non-renormalizable term as in Ref. [12–14]. The reheating temperature is determined exclusively by the decay of $\tilde{N}$ and is high enough ($> 100$ GeV) so that non-perturbative electroweak sphalerons are operative and, consequently, non-thermal [17] leptogenesis and the subsequent generation of the BAU can be realized. As usually in similar models – cf. Ref. [19,22–25,30] – consistency with the constraint on the gravitino ($\tilde{G}$) abundance [31–33] requires a relatively small effective Yukawa coupling constant ($10^{-8} - 10^{-3}$). The smallness of this coupling though may be explained through a broken flavor symmetry [19,30].

Below, we present the basic ingredients of our model (Sec. 2) and describe the inflationary (Sec. 3) and post-inflationary dynamics (Sec. 4). We then restrict the parameters of our model (Sec. 5) and summarize our conclusions (Sec. 6). Details concerning the formulation of non-minimally coupled scalar fields within SUGRA are presented in the Appendix. Throughout the text, we use natural units for Planck’s constant, Boltzmann’s constant and the speed of light ($\hbar = c = k_B = 1$); the subscript of type , $\chi$ denotes derivation with respect to (w.r.t.) the field $\chi$ (e.g., $\partial \chi / \partial \chi^2$); charge conjugation is denoted by a star and log [ln] stands for logarithm with basis 10 [$e$]. Finally, we follow the conventions of Ref. [34] for the quantities related to the gravitational sector of our model.

2 Model Description

We focus on a PQ invariant extension of the MSSM, inspired by Ref. [12,13], which links the generation of intermediate masses for RH neutrinos, $N^c_i$, with a PQPT. Besides the (color) anomalous PQ
Table 1: The representations under $G_{SM}$ and the extra global charges of the superfields of our model.

| SUPERFIELDS | REPRESENTATIONS UNDER $G_{SM}$ | GLOBAL CHARGES |
|-------------|--------------------------------|----------------|
| MATTER FIELDS | | |
| $L_i$ | $(1, 2, -1/2)$ | 0 | $-3$ | 0 |
| $e_i^c$ | $(1, 1, 1)$ | 2 | 1 | 0 |
| $N_i^c$ | $(1, 1, 0)$ | 2 | 1 | 0 |
| $Q_i$ | $(3, 2, 1/6)$ | 1 | $-1$ | 1/3 |
| $u_i^c$ | $(3, 1, -2/3)$ | 1 | $-1$ | $-1/3$ |
| $d_i^c$ | $(3, 1, 1/3)$ | 1 | $-1$ | $-1/3$ |
| HIGGS FIELDS | | |
| $H_d$ | $(1, 2, -1/2)$ | 2 | 2 | 0 |
| $H_u$ | $(1, 2, 1/2)$ | 2 | 2 | 0 |
| $P$ | $(1, 1, 0)$ | 4 | 0 | 0 |
| $X$ | $(1, 1, 0)$ | 0 | 2 | 0 |

symmetry $U(1)_{PQ}$, the model possesses also an anomalous $R$ symmetry $U(1)_R$, and the baryon number symmetry $U(1)_B$. The PQ symmetry $U(1)_{PQ}$ is spontaneously broken at the PQ breaking scale $f_a \sim (10^{10} - 10^{12})$ GeV (which coincides with the axion decay constant – for a review see Ref. [35]) via the VEVs acquired by two $G_{SM}$ singlet left-handed superfields $\tilde{X}$ and $\tilde{X}$. The representations under $G_{SM}$ and the charges under the global symmetries of the various matter and Higgs superfields are listed in Table 1.

In particular, the superpotential, $W$, of our model naturally splits into two parts:

$$W = W_{MSSM} + W_{NPQ}, \quad (2.1)$$

where $W_{MSSM}$ is the part of $W$ which contains the usual terms – except for the $\mu$ term – of the MSSM, supplemented by Yukawa interactions among the left-handed leptons and $N_i^c$:

$$W_{MSSM} = h_{Eij} e_i^c L_j H_d + h_{Dij} d_i^c Q_j H_d + h_{Uij} u_i^c Q_j H_u + h_{Nij} N_i^c L_j H_u. \quad (2.2)$$

Here, the group indices have been suppressed; the $i$-th generation $SU(2)_L$ doublet left-handed quark and lepton superfields are denoted by $Q_i$ and $L_i$, respectively, whereas the $SU(2)_L$ singlet antiquark [antilepton] superfields by $u_i^c$ and $d_i^c$ [$e_i^c$ and $N_i^c$] respectively. The electroweak Higgs superfields, which couple to the up [down] quark superfields, are denoted by $H_u$ [$H_d$].

On the other hand, $W_{NPQ}$ is the part of $W$ which is relevant for non-$\tilde{M}$I, the generation of the Majorana masses for $N_i^c$, the spontaneous breaking of $U(1)_{PQ}$ and the generation of the $\mu$ term of the MSSM. It takes the form

$$W_{NPQ} = \lambda_i X N_i^c N_i^c + \lambda_a P(\tilde{X} X - f_a^2/4) + \lambda_\mu \frac{X^2 H_u H_d}{m_P}, \quad (2.3)$$

where $m_P \simeq 2.44 \cdot 10^{18}$ GeV is the reduced Planck scale and $P$ is a $G_{SM}$ singlet left-handed superfield involved in the breaking of $U(1)_{PQ}$. The parameters $\lambda_a$ and $f_a$ are made positive by field redefinitions.
Moreover, we chose a basis in the $N_i - N_j$ space where the coupling constant matrix $\lambda$ is real and diagonal. In order to produce the CP-violation necessary for leptogenesis, we include three $N_i$ species. Assuming that $N_i$ is strongly hierarchical, i.e. $\lambda_1 = \lambda \ll \lambda_2, \lambda_3$, the scalar components of the two heavier $N_i$ roll to their minima fairly quickly, since their potential is steeper - especially if we further assume that these are minimally coupled to gravity in contrast to the lightest one. Thus the scalar component, $\bar{N}$, of the lightest of the $N_i$’s controls the relevant slow-roll dynamics and it can therefore be identified as the inflaton. On the other hand, the outcome of leptogenesis is governed by the one from $N_i$ with the smallest decay rate. In the following, we concentrate on the case where $\bar{N}$ drives both non-M$\bar{N}$I and leptogenesis. Therefore, the three generation model can be simplified to an effective one-generation model in the $N_i$ sector, with the only remnant of the other two generations being a non-vanishing CP-asymmetry for leptogenesis – see Ref. [21, 23]. Henceforth we suppress family indices.

According to our general discussion in the Appendix – see Eq. (A.10) – the implementation of non-M$\bar{N}$I within SUGRA requires the adoption of a frame function, $\Omega_{\bar{N}I}$, related to the Kähler potential, $K_{\bar{N}I}$, of the following form

$$\Omega_{\bar{N}I} = -3e^{-K_{\bar{N}I}/3m_P^2} - 3 + \frac{|\bar{N}|^2}{m_P^2} + \frac{|P|^2}{m_P^2} + \frac{|\bar{X}|^2}{m_P^2} + \frac{|X|^2}{m_P^2} - k_X \frac{|X|^4}{m_P^4} - \frac{3k_{\bar{N}}}{4m_P^2} \left( \bar{N}^2 + \bar{N}^*2 \right), \quad (2.4)$$

where the complex scalar components of the superfields $P, \bar{X}$ and $X$ are denoted by the same symbol and the coefficients $k_X$ and $k_{\bar{N}}$ are taken, for simplicity, real. Comparing this expression with Eq. (A.9), we remark that we adopt the standard non-minimal coupling for the inflaton, $\bar{N}$, i.e. $F = k_{\bar{N}} \bar{N}^2/4m_P^2$, and we added the sixth term in the RH side (RHS) in order to cure the tachyonic mass problem encountered in similar models [7, 8] – see Sec. 3.1. Note that $F$ breaks explicitly the imposed $R$ and PQ symmetries and causes a dependence of $\Omega_{\bar{N}I}$ on the phase $\theta$ of $\bar{N}$ which can be written as $\bar{N} = \sigma e^{i\theta}/\sqrt{2}$. As we show in Sec. 3.1, the model admits stable inflationary trajectories along which $\theta$ is stabilized at $\theta = 0$. When $\theta \sim 0$, the choice $k_{\bar{N}} > 1$ ensures the positivity of the scale function, $-\Omega_{\bar{N}I}/3$, even for relatively large values of $\sigma$.

In the limit where $m_P$ tends to infinity, the matter sector decouples from gravity, and we can obtain the SUSY limit, $V_{\text{SUSY}}$, of the SUGRA potential, $V_{\bar{N}I}$. This turns out to be

$$V_{\text{SUSY}} = \left(4\lambda_2^2 |\bar{N}|^2 + \lambda_0^2 |P|^2 \right) |X|^2 + \left(4\lambda_0^2 |X|^2 + \lambda_{\bar{N}} P \bar{X} \right)^2 + \lambda_0^2 |X|^2 \bar{X} X - f_a^2/4 \right|^2. \quad (2.5)$$

From the potential in Eq. (2.5), we find that the SUSY vacuum lies at

$$\langle \bar{N} \rangle = 0, \quad \langle P \rangle \simeq 0 \quad \text{and} \quad |\langle \phi_X \rangle| = 2|\langle X \rangle| = 2|\langle \bar{X} \rangle| = f_a, \quad (2.6)$$

where we have introduced the canonically normalized scalar field $\phi_X = 2X = 2\bar{X}$. Note that, since the sum of the arguments of $\langle X \rangle$, $\langle \bar{X} \rangle$ must be 0, $\bar{X}$ and $X$ can be brought to the real axis by an appropriate PQ transformation. Moreover, after including soft SUSY breaking terms, $\langle P \rangle$ can become [28] of order 1 TeV and the minimization of $V_P$ in Eq. (2.5) requires that $\langle \bar{X} \rangle = \langle X \rangle$. Needless to say that for field values greater than $f_a$, the TeV-scale soft SUSY breaking terms can be safely ignored during the cosmological evolution. After the spontaneous breaking of $U_{PQ}(1)$, the first term of the RH side (RHS) of Eq. (2.3) generates intermediate scale masses for the $N_i$’s $M_i \sim \lambda_i f_a$ and, thus, seesaw masses [16] for the light neutrinos, whereas the third of the RHS of Eq. (2.3) leads to the $\mu$ term of the MSSM, with $|\mu| \sim \lambda_\mu |\langle X \rangle|^2/m_P$. Both scales are of the right magnitude if $|\langle X \rangle| = f_a/2 \approx 5 \cdot 10^{11}$ GeV, $\lambda_\mu \sim 1$ and $\lambda_{\bar{N}} \sim (0.001 - 0.01)$.

In conclusion, $W_{NPQ}$ leads to a spontaneous breaking of $U_{PQ}(1)$. The same superpotential $W_{NPQ}$ also gives rise to a stage of non-M$\bar{N}$I and a PQPT, as analyzed in Sec. 3. An indication for such a possibility can be seen by examining $V_{\text{SUSY}}$ in Eq. (2.5), which becomes

$$V_{\text{SUSY}} = \lambda^2 \bar{N}^4 + \lambda_0^2 f_a^2/4 \quad \text{along the direction} \quad X = \bar{X} = 0. \quad (2.7)$$
Clearly, for $\tilde{N} \gg f_\alpha$, $V_{\text{SUSY}}$ tends to a quartic potential. Therefore, $W_{\text{NPQ}}$ can be employed in conjunction with $K_{\tilde{N}I}$ in Eq. (2.4) for the realization of non-M$\tilde{N}$I along the lines of Ref. [8]. Moreover, for lower $\tilde{N}$’s, $V_{\text{SUSY}}$ takes an almost constant value, which can drive a PQPT.

It should be mentioned that the non-minimal gravitational coupling, instanton and soft SUSY breaking effects explicitly break $U(1)_R \times U(1)_{\text{PQ}}$ to a discrete subgroup. It is then important to ensure that this subgroup is not spontaneously broken by $\langle X \rangle$ and $\langle \tilde{X} \rangle$, since otherwise cosmologically disastrous domain walls are produced [36] during the PQPT. Note that $U(1)_R \times U(1)_{\text{PQ}}$ is also broken during non-M$\tilde{N}$I due to the non-zero $\tilde{N}$, but it is restored in the SUSY vacuum. The explicitly unbroken subgroup of $U(1)_R \times U(1)_{\text{PQ}}$ can be deduced from the solutions of the system

$$4r + 2p = 0 \mod 2\pi, \quad 4r = 0 \mod 2\pi \quad \text{and} \quad -12(r + p) = 0 \mod 2\pi,$$

where $r$ and $p$ are the phases of a $U(1)_R$ and $U(1)_{\text{PQ}}$ rotation respectively. Here we took into account that: (a) the $R$ [PQ] charge of the langrangian term caused by the non-minimal gravitational coupling is 4 [2]; (b) the $R$ charge of $W$ and, thus, of all the soft SUSY breaking terms, is 4 and (c) the sum of the $R$ [PQ] charges of the $SU(3)_c$ triplets and antitriplets is $-12$ [–12]. We conclude, therefore, that the explicitly unbroken subgroup is $\mathbb{Z}_4 \times \mathbb{Z}_2$. It is then easy to check that this subgroup is not spontaneously broken by $\langle X \rangle$ and $\langle \tilde{X} \rangle$, since the relevant condition

$$2p = 0 \mod 2\pi,$$

is satisfied automatically as a result of Eq. (2.8). Consequently, cosmologically disastrous domain walls are not produced during the PQPT and so, there is no need to further extend [37] the particle content of the model – cf. Ref. [29].

### 3 The Inflationary Epoch

#### 3.1 Structure of the Inflationary Action

Inserting Eq. (2.4) into Eq. (A.8), we can write the action of our model in the JF (Jordan frame) as follows

$$S_{\tilde{N}I} = \int d^4x \sqrt{-g} \left( \frac{1}{6} m_\pi^2 \Omega_{\tilde{N}I} R + \delta_{\alpha\beta} \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - \Omega_{\tilde{N}I} A_\mu A^\mu / m_P^2 - V_{\tilde{N}I} \right),$$

with $\phi^\alpha = \tilde{N}, P, X$ and $\tilde{X}$. Also $V_{\tilde{N}I} = \Omega^2_{\tilde{N}I} \tilde{V}_{\tilde{N}I}/9$ where $\tilde{V}_{\tilde{N}I}$ is the EF (Einstein frame) $F$–term SUGRA scalar potential, which can be obtained from $W_{\text{NPQ}}$ in Eq. (2.3) – without the last term of the RHS – and $K_{\tilde{N}I}$ in Eq. (2.4) by applying Eq. (A.3). Along the direction $P = X = \tilde{X} = 0$, $\tilde{V}_{\tilde{N}I}$ and $\Omega_{\tilde{N}I} = -3f$ take the forms

$$\tilde{V}_{\tilde{N}10} = m_P^4 \frac{\lambda^2 \sigma^4 + 4 \lambda^2 \sigma M_{\text{NPQ}}^4}{4 f^2} \quad \text{with} \quad f = 1 - \frac{x_\sigma^2}{6} + \left( \frac{1}{6} + c_R \right) x_\sigma^2 \cos 2\theta \quad \text{and} \quad c_R = -\frac{1}{6} + \frac{k_{\tilde{N}}}{4}.$$

Here $M_{\text{NPQ}} = f_\alpha/2m_P$ and $x_\sigma = \sigma / m_P$. Recall also that we set $\tilde{N} = \sigma e^{i\theta}/\sqrt{2}$. From Eq. (3.2), we can easily verify – see also the small fluctuations analysis below – that for given $\sigma$, $\theta = 0$ (modulo $\pi$) minimizes $\tilde{V}_{\tilde{N}I}$. For $\theta = 0$ and $c_R \gg 1$, $S_{\tilde{N}I}$ in Eq. (3.1) takes a form suitable for the realization of non-M$\tilde{N}$I. Then we can set $A_\mu = 0$, and more importantly $\tilde{V}_{\tilde{N}I}$ develops a plateau – note that $M_{\text{NPQ}} \ll 1$. The constant potential energy density $\tilde{V}_{\tilde{N}10}$ and the corresponding Hubble parameter $\tilde{H}_{\tilde{N}10}$ along the trajectory for which non-M$\tilde{N}$I can take place are given by

$$\tilde{V}_{\tilde{N}10} = \frac{\lambda^2 \sigma^4}{4 f^2} \approx \frac{\lambda^2 m_P^4}{4 c_R^2} \quad \text{and} \quad \tilde{H}_{\tilde{N}10} = \frac{\tilde{V}_{\tilde{N}10}^{1/2}}{3 m_P} \approx \frac{\lambda m_P}{2 \sqrt{3 c_R}}.$$


In order to check the stability of the direction $P = X = \hat{X} = \theta = 0$ w.r.t. the fluctuations of the fields $\theta, P, X$ and $\hat{X}$, we expand the latter three in real and imaginary parts as follows

$$X = \frac{x_1 + ix_2}{\sqrt{2}}, \quad \hat{X} = \frac{\bar{x}_1 + i\bar{x}_2}{\sqrt{2}} \quad \text{and} \quad P = \frac{p_1 + ip_2}{\sqrt{2}}, \quad (3.4)$$

Performing a Weyl transformation as described in Eq. (A.4), we obtain [38]

$$S_{\tilde{N}1} = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{1}{2}m_p^2\tilde{R} + \frac{1}{2} \left( \frac{1}{f} + \frac{3f^2}{2f^2}m_p^2 \right) \tilde{g}^{\mu\nu} \left( \partial_\mu \sigma \partial_\nu \sigma + \sigma^2 \partial_\mu \theta \partial_\nu \theta \right) + \frac{1}{2f} \tilde{g}^{\mu\nu} \sum_\chi \partial_\mu \chi \partial_\nu \chi - \tilde{V}_{\tilde{N}1} \right), \quad (3.5)$$

with $\chi = x_1, x_2, \bar{x}_1, \bar{x}_2, p_1, p_2,$ and we also take into account that $f_\chi \ll f_\sigma$ and $f_\theta \ll f_\sigma$ for $\theta \sim 0$. Note that we keep only terms up to quadratic order in the fluctuations $\theta, \chi$ and their derivatives in Eq. (3.5). Along the trajectory $P = X = \hat{X} = \theta = 0$, $f$ becomes a function of $\sigma$, and so we can introduce the EF canonically normalized fields, $\hat{\sigma}, \hat{\theta}$ and $\hat{\chi}$, as follows [5, 8]

$$\left( \frac{d\hat{\sigma}}{d\sigma} \right)^2 = J^2 = \frac{1}{f} + \frac{3}{2}m_p^2 \left( \frac{f_\sigma}{f} \right)^2, \quad \hat{\theta} = J\sigma \theta \quad \text{and} \quad \hat{\chi} = \frac{\chi}{\sqrt{f}}. \quad (3.6)$$

Taking into account the approximate expressions for $\hat{\sigma}$ - where the dot denotes derivation w.r.t. the cosmic time $t$ - $J$ and the slow-roll parameters $\tilde{\epsilon}, \tilde{\eta}$, which are displayed in Sec. 3.2, we can verify that, during a stage of slow-roll non-M$\tilde{N}1$, $\hat{\theta} \approx J\sigma \theta$ since $J\sigma \approx \sqrt{6}m_p$, and $\hat{\chi} \approx \chi/\sqrt{f}$. For the latter, the quantity $\dot{f}/f^{3/2}$, involved in relating $\hat{\chi}$ to $\chi$, turns out to be negligibly small, since $\dot{f}/f^{3/2} = f_\sigma \dot{\sigma}/f^{3/2} = -\lambda\sqrt{\epsilon/\eta}m_p^2/2\sqrt{3}c_R$. Therefore the action in Eq. (3.5) takes the form

$$S_{\tilde{N}1} = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{1}{2}m_p^2\tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \sum_\phi \partial_\mu \phi \partial_\nu \phi - \tilde{V}_{\tilde{N}1} \right), \quad (3.7)$$

where $\phi$ stands for $\sigma, \theta, x_1, x_2, \bar{x}_1, \bar{x}_2, p_1$ and $p_2$.

Along the inflationary path, we can easily check that the first derivatives of $\tilde{V}_{\tilde{N}1}$ w.r.t. $\phi$ are equal to zero. The curvature of $\tilde{V}_{\tilde{N}1}$ w.r.t. $\hat{\theta}$ can be studied separately since $\partial^2 \tilde{V}_{\tilde{N}1}/\partial \hat{\theta} \partial \hat{\chi} = 0$. Moreover the stability of the path $P = X = \hat{X} = \theta = 0$ w.r.t. the fluctuations of $\theta$ is automatic, since the mass squared of $\hat{\theta}, m_{\hat{\theta}}^2$, turns out to be positive. Indeed we find

$$m_{\hat{\theta}}^2 = \frac{\lambda^2m_p^2(1 + 6c_R)}{6f^2f^3} \approx \frac{\lambda^2m_p^2}{3c_R} = 4\hat{H}_{\tilde{N}10}^2. \quad (3.8)$$

The two $3 \times 3$ mass squared matrices $M_A^2 = \left( \partial^2 \tilde{V}_{\tilde{N}1}/\partial \chi_\alpha \partial \chi_\beta \right)$, with $A = 1$ and $\chi_\alpha = \tilde{p}_1, \tilde{x}_1, \tilde{x}_2$ or $A = 2$ and $\chi_\alpha = \tilde{p}_2, \tilde{x}_2, \tilde{x}_2$, have the following eigenvalues

$$m_{\tilde{x}}^2 = \lambda^2m_p^2x_\sigma^2(12 + x_\sigma^2f_1)(6k_Xf - 1) \quad \text{and} \quad m_{y_{\pm}}^2 = \lambda^2m_p^2x_\sigma^2(\lambda \pm 3\lambda_0c_R)x_\sigma^2 \pm 3\lambda_0 \quad \text{for} \quad 0 \leq \lambda_0 \leq 1 \quad (3.9)$$

corresponding to eigenstates $\tilde{x}_1$ (or $\tilde{x}_2$) and $y_{1\pm} = \left( \tilde{p}_1 \pm \tilde{x}_1 \right) / \sqrt{2}$ (or $y_{2\pm} = \left( \tilde{p}_2 \pm \tilde{x}_2 \right) / \sqrt{2}$) respectively. The considered inflationary trajectory, $P = X = \hat{X} = \theta = 0$, is a stable valley of local minima, provided that $m_{\tilde{x}}^2 \geq 0$ and $m_{y_{\pm}}^2 \geq 0$, i.e.

(a) $\sigma \gtrsim \sigma_{1c} = m_p \sqrt{\frac{3\lambda_a}{\lambda - 3\lambda_0c_R}}$ and (b) $\sigma \gtrsim \sigma_{2c} = m_p \sqrt{\frac{3\lambda_a}{\lambda - 3\lambda_0c_R}}$ with $\lambda_a < \lambda/3c_R$. \quad (3.10)
In practice the condition of Eq. (3.10b) is much more restrictive than Eq. (3.10a) for \( k_X \sim 1 \). Indeed, from Eq. (3.9), it is evident that \( k_X \gtrsim 1 \) assists us to achieve \( m_{\tilde{\sigma}}^2 > 0 \) – in accordance with the results of Ref. [8]. On the other hand, given that for \( \sigma < m_P \), we need \( c_R \gg 1 \), Eq. (3.10b) requires a clear hierarchy between \( \lambda \) and \( \lambda_a \), e.g. for \( c_R \sim 10^2 \), we need \( \lambda_a / \lambda \lesssim 10^{-3} \). This ratio can be slightly increased (almost one order of magnitude) if we include additional terms such as \( k_X N |\tilde{N}|^2 |X|^2 \) or \( k_{P, K} |\tilde{N}|^2 |X|^2 \) in the Kähler potential in Eq. (2.4). Since the resulting increase of \( \lambda_a \) has no significant impact on our results, we choose to stick with the most minimal possible Kähler potential needed for the viability of our model, avoiding more complications. We have also numerically verified that \( m_{\tilde{\sigma}} \geq \hat{H}_{\tilde{N}1}, m_{\tilde{\sigma}} \geq \hat{H}_{\tilde{N}1} \) and \( m_{\tilde{\sigma}} \geq \hat{H}_{\tilde{N}1} \), during the last 50 – 60 e-foldings of non-M\( \tilde{N}1 \), and so any inflationary perturbations of the fields \( \theta, \tilde{x}_{1,2} \) and \( \tilde{y}_{1,2\pm} \) are safely eliminated.

The constant tree-level potential energy density in Eq. (3.3) causes SUSY breaking, leading to the generation of one-loop radiative corrections, which can be calculated by employing the well-known Coleman-Weinberg formula [39]. We find

\[
V_{rc} = \frac{1}{64\pi^2} \left( m_{\tilde{\sigma}}^2 + m_{\tilde{\sigma}}^2 \ln \Lambda^2 + 2m_{\tilde{\sigma}}^2 \ln \Lambda^2 + 2m_{\tilde{\sigma}}^2 \ln \Lambda^2 + 2m_{\tilde{\sigma}}^2 \ln \Lambda^2 - 4m_{\tilde{\sigma}}^2 \ln \Lambda^2 \right) \quad (3.11)
\]

where \( \Lambda \) is a renormalization mass scale and \( \tilde{m} = \sqrt{\lambda \Lambda^2 / \sigma} \) is the eigenvalue of the fermion matrices. As we verified numerically, \( V_{rc} \) has no significant effect on the inflationary dynamics. This is because the slope of the inflationary path is generated at the classical level – see the expressions for \( \hat{c} \) and \( \hat{\eta} \) below – and so, the contribution of \( V_{rc} \) to \( \tilde{V}_{\tilde{N}1} \) remains subdominant.

Based on the action of Eq. (3.7) with \( \tilde{V}_{\tilde{N}1} \sim \tilde{V}_{\tilde{N}10} + V_{rc} \), we can proceed to the analysis of non-M\( \tilde{N}1 \) in the EF, using the standard slow-roll approximation [40, 41]. It can be shown [42] that the results calculated this way are the same as if we had calculated them using the non-minimally coupled scalar field in the JF.

### 3.2 The Inflationary Observables

According to our analysis above, when Eq. (3.10) is satisfied, the universe undergoes a period of slow-roll non-M\( \tilde{N}1 \), which is determined by the condition – see e.g. Ref. [40, 41]:

\[
\max \{ \hat{c}(\sigma), \hat{\eta}(\sigma) \} \leq 1, \quad \text{where}
\]

\[
\hat{c} = \frac{m_{\tilde{\sigma}}^2}{2} \left( \frac{\tilde{V}_{\tilde{N}1,\tilde{\sigma}}}{\tilde{V}_{\tilde{N}1}} \right)^2 = \frac{m_{\tilde{\sigma}}^2}{2J^2} \left( \frac{\tilde{V}_{\tilde{N}1,\sigma}}{\tilde{V}_{\tilde{N}1}} \right)^2 \approx \frac{4m_{\tilde{\sigma}}^2}{3C_R\sigma^4} \quad (3.12a)
\]

\[
\hat{\eta} = \frac{m_{\tilde{\sigma}}^2}{2J} \left( \frac{\tilde{V}_{\tilde{N}1,\tilde{\sigma}}}{\tilde{V}_{\tilde{N}1}} \right)^2 = \frac{m_{\tilde{\sigma}}^2}{2J^2} \left( \frac{\tilde{V}_{\tilde{N}1,\sigma}}{\tilde{V}_{\tilde{N}1}} \right) \left( \frac{\tilde{V}_{\tilde{N}1,\sigma} - \tilde{V}_{\tilde{N}1,\sigma}}{\tilde{V}_{\tilde{N}1}} \right) \approx -\frac{4m_{\tilde{\sigma}}^2}{3C_R\sigma^2} \quad (3.12b)
\]

Here we employ Eq. (3.3) and the following approximate relations:

\[
J \approx \sqrt{6}m_P / \sigma, \quad \tilde{V}_{\tilde{N}1,\sigma} \approx \lambda^2 m_P^6 / c_R^3 \sigma^3 \quad \text{and} \quad \tilde{V}_{\tilde{N}1,\sigma} \approx -3\lambda^2 m_P^6 / c_R^3 \sigma^4. \quad (3.13)
\]

The numerical computation reveals that non-M\( \tilde{N}1 \) terminates due to the violation of the \( \hat{c} \) criterion at a value of \( \sigma \) equal to \( \sigma_f \), which is calculated to be

\[
\hat{c} (\sigma_f) = 1 \quad \Rightarrow \quad \sigma_f = (4/3)^{1/4} m_P / \sqrt{c_R}. \quad (3.14)
\]

Note, in passing, that for \( \sigma \geq \sigma_f \) the evolution of \( \tilde{\sigma} \) or \( \sigma \) via Eq. (3.6) – is governed by the equation of motion

\[
3\tilde{H}_{\tilde{N}1} \dot{\tilde{\sigma}} = -\tilde{V}_{\tilde{N}1,\tilde{\sigma}} \Rightarrow 3\tilde{H}_{\tilde{N}1} \dot{\tilde{\sigma}}^2 = -\tilde{V}_{\tilde{N}1,\sigma} \Rightarrow \dot{\tilde{\sigma}} = -\lambda m_P^6 / 3\sqrt{3} c_R^2 \sigma. \quad (3.15)
\]
Using Eqs. (3.12a), (3.12b) and (3.15), we can derive the expression for $\dot{f}/f^{3/2}$ given above Eq. (3.7).

The number of e-foldings, $\hat{N}_s$, that the scale $k_s = 0.002/\text{Mpc}$ suffers during non-M$\tilde{N}$I can be calculated through the relation

$$\hat{N}_s = \frac{1}{m_P^2} \int_{\bar{\sigma}_i}^{\bar{\sigma}_f} d\sigma \frac{\dot{V}_{\tilde{N}I}}{V_{\tilde{N}I,\sigma}} = \frac{1}{m_P^2} \int_{\sigma_i}^{\sigma_f} d\sigma \frac{\dot{V}_{\tilde{N}I}}{V_{\tilde{N}I,\sigma}}, \quad (3.16)$$

where $\sigma_{\hat{s}} [\hat{\sigma}]$ is the value of $\sigma [\hat{\sigma}]$ when $k_s$ crosses the inflationary horizon. Given that $\sigma_i \ll \sigma_{\hat{s}}$, we can write $\sigma_{\hat{s}}$ as a function of $\hat{N}_s$ as follows

$$\hat{N}_s \simeq \frac{3c_R}{4m_P^2} (\sigma_{\hat{s}}^2 - \sigma_{\hat{f}}^2) \Rightarrow \sigma_{\hat{s}} = 2m_P \left( \frac{\hat{N}_s}{3c_R} \right)^{1/2}. \quad (3.17)$$

The power spectrum $P_{\tilde{R}}$ of the curvature perturbations generated by $\sigma$ at the pivot scale $k_s$ is estimated as follows

$$P_{\tilde{R}}^{1/2} = \frac{1}{2\sqrt{3} \pi m_P^3} \left( \frac{\dot{V}_{\tilde{N}I} (\hat{\sigma})}{|\dot{V}_{\tilde{N}I,\hat{\sigma}} (\hat{\sigma})|} \right)^{3/2} = \frac{|J(\sigma)| \dot{V}_{\tilde{N}I,\sigma} (\sigma)}{2\sqrt{3} \pi m_P^3} \lambda \frac{\sigma_{\hat{s}}^2}{8 \sqrt{2} m_P^2} \simeq \frac{\sqrt{2} \lambda \hat{N}_s}{12 \pi c_R}, \quad (3.18)$$

where Eq. (3.17) is employed to derive the last equality of the relation above. At the same pivot scale, we can also calculate the (scalar) spectral index, $n_s$, its running, $\alpha_s$, and the scalar-to-tensor ratio, $r$, via the relations:

$$n_s = 1 - 6\tilde{c}_s + 2\tilde{\eta}_s \simeq 1 - 2/\hat{N}_s, \quad (3.19a)$$

$$\alpha_s = \frac{2}{3} \left( 4\tilde{c}_s^2 - (n_s - 1)^2 \right) - 2\tilde{c}_s \simeq -2/\hat{N}_s^2 \quad (3.19b)$$

and

$$r = 16\tilde{c}_s \simeq 12/\hat{N}_s^2, \quad (3.19c)$$

where $\tilde{c} = m_P^2 \dot{V}_{\tilde{N}I,\hat{\sigma}} / \dot{V}_{\tilde{N}I} = m_P \sqrt{2} \tilde{c}_s J + 2\tilde{\eta} \hat{c}$ and the variables with subscript $\hat{s}$ are evaluated at $\sigma = \sigma_{\hat{s}}$. Comparing the results of this section with the observationally favored values, we constrain the parameters of our model in Sec. 5.

### 4 THE POST-INFLATIONARY EVOLUTION

A complete SUSY inflationary scenario should specify the transition to the radiation dominated era and also explain the origin of the observed BAU consistently with the $\tilde{G}$ constraint. These goals can be accomplished within our set-up, as we describe in this section. The basic features of the post-inflationary era of our model are exhibited in Sec. 4.1. A more precise analysis of the evolution during this era can be obtained by solving numerically the relevant Boltzmann equations, as in Sec. 4.2. Finally, useful analytical expressions reproducing accurately our results are presented in Sec. 4.3.

#### 4.1 THE GENERAL SET-UP

When non-M$\tilde{N}$I is over, $\hat{N}$ undergoes a very short period of fast-roll until it reaches its critical value $\sigma_{2c}/\sqrt{2}$. Afterwards, $\hat{N}$ and the PQ system (comprised by $X$, $\tilde{X}$ and $P$) fall to their SUSY minimum values, acquiring masses $m_{\hat{N}}$ and $m_{\text{PQ}}$ respectively, which can be computed from $V_{\text{SUSY}}$ in Eq. (2.5). These are given by

(a) $m_{\hat{N}} = \lambda f_a$ and (b) $m_{\text{PQ}} = \lambda_a f_a/\sqrt{2}$. \quad (4.1)

Note that due to hierarchy between $\lambda$ and $\lambda_a$ established in Eq. (3.10b), $m_{\hat{N}} > m_{\text{PQ}}$. Consequently, the post-inflationary energy density of the universe is dominated by the $\hat{N}$ condensate which undergoes a
phase of damped oscillations about the SUSY vacuum, when $H \simeq m_{\tilde{N}}$, and decays [21] predominantly into $\tilde{H}_u + L$ or $H_u^* + L^*$, via the tree-level couplings derived from the last term in the RHS of Eq. (2.2). The initial energy density of this oscillatory system is estimated by $\rho_{ii} \simeq 3m_{\tilde{N}}^2m_{\tilde{N}}^2$, corresponding to $H_{ii} \simeq m_{\tilde{N}}$. The decay temperature of $\tilde{N}$, $T_{\tilde{N}}$, which coincides with the reheating temperature, $T_{rh}$, in our model – see below – is [44] given by

$$T_{\tilde{N}} = c_T \sqrt{\Gamma_{\tilde{N}}m_{\tilde{N}}} \quad \text{with} \quad c_T = \left( \frac{72}{5\pi^2g_*} \right)^{1/4},$$

(4.2)

where $g_*$ counts the effective number of relativistic degrees of freedom at temperature $T_{\tilde{N}}$. We find $g_* \simeq 240$ for the MSSM spectrum plus the particle content of the superfields $P$, $\bar{N}$ and $X$. Also $\Gamma_{\tilde{N}}$ is the decay width of $\tilde{N}$ given by

$$\Gamma_{\tilde{N}} = \frac{1}{4\pi} h_{\text{eff}}^2 m_{\tilde{N}} \quad \text{where} \quad h_{\text{eff}} = \sqrt{\sum_i |h_{\tilde{N}i}|^2}$$

(4.3)

is an effective Yukawa coupling, linked to the light neutrino masses, and can be considered as a free parameter.

The aforementioned two channels for the $\tilde{N}$ decay have different branching ratios when CP conservation is violated. Interference between tree-level and one-loop diagrams generates a lepton-number asymmetry [19, 34] which, for a normal hierarchical mass spectrum of light neutrinos, reads

$$\varepsilon_L = \frac{3}{8\pi} \frac{m_{\nu_3}}{\langle H_u \rangle^2} \delta_{\text{eff}}.$$  

(4.4)

Here $|\delta_{\text{eff}}| \lesssim 1$, which is treated as a free parameter in our approach, represents the magnitude of CP violation; $m_{\nu_3}$ is the heaviest neutrino mass and we take $\langle H_u \rangle = 174$ GeV (adopting the large $\tan \beta$ regime). If $T_{rh} < m_{\tilde{N}}$, the out-of-equilibrium condition [19] for the implementation of leptogenesis is automatically satisfied. The resulting lepton-number asymmetry after reheating can be partially converted through sphaleron effects into baryon-number asymmetry. However, the required $T_{rh}$ must be compatible with constraints for the $\tilde{G}$ abundance, $Y_{\tilde{G}}$, at the onset of nucleosynthesis.

On the other hand, the system consisting of the two complex scalar fields $P$ and $(\delta \bar{X} + \delta X)/\sqrt{2}$ (where $\delta \bar{X} = \bar{X} - f_a/2$ and $\delta X = X - f_a/2$) enters into an oscillatory phase about the PQ minimum and eventually decays, via the non-renormalizable coupling in the RHS of Eq. (2.3), to Higgses and Higgsinos with a common decay width $\Gamma_{PQ}$ [41] and a corresponding decay temperature $T_{PQ}$ given by

$$T_{PQ} = c_T \sqrt{\Gamma_{PQ}m_{P}} \quad \text{where} \quad \Gamma_{PQ} = \frac{1}{2\pi} \lambda_{\mu}^2 \left( \frac{f_a}{2m_{\tilde{P}}} \right)^2 m_{PQ}.$$  

(4.5)

Note that due to the hierarchy between $\lambda$ and $\lambda_a$ in Eq. (3.10), the decay of $X$ to $N_i^*$’s is kinematically forbidden. Due to the same fact, $T_{PQ}$ turns out to be quite suppressed, and so a possible domination of the PQ oscillatory system could dilute any preexisting $Y_L$ and $Y_{\tilde{G}}$. However, $\langle X \rangle \ll m_{\tilde{P}}$ – in contrast to the VEVs of moduli occurring in superstring theory [20] which are of the order of $m_{\tilde{P}}$ – and therefore, the initial energy density of the universe at the onset of these oscillations, $\rho_{HPQ}$ – cf. Ref. [19, 30]. Namely

$$\rho_{P2i} \simeq m_{PQ}^2 |\langle X \rangle|^2 \ll \rho_{HPQ} = 3m_{P}^2m_{PQ}^2 \quad \text{for} \quad H \simeq m_{PQ}.$$  

(4.6)

This is a crucial point since it assists us to avoid any dilution of the produced $Y_L$ (and $Y_{\tilde{G}}$) at $T = T_{\tilde{N}}$, as we show in the following.
4.2 The Relevant Boltzmann Equations

The energy density, \( \rho_1 [\rho_2] \), of the oscillatory system with decay width \( \Gamma_{N} [\Gamma_{PQ}] \), the energy density of produced radiation, \( \rho_R \), the number density of lepton asymmetry, \( n_L \), and the one of \( G, n_G \), satisfy the following Boltzmann equations – cf. Ref. [21, 29, 33, 34, 43]:

\[
\begin{align*}
\dot{\rho}_1 + 3H\rho_1 + \Gamma_N \rho_1 &= 0, \\
\dot{\rho}_2 + 3H\rho_2 + \Gamma_{PQ} \rho_2 &= 0, \\
\dot{\rho}_R + 4H\rho_R - \Gamma_{N} \rho_1 - \Gamma_{PQ} \rho_2 &= 0, \\
\dot{n}_L + 3Hn_L - \varepsilon_L \Gamma_N \rho_1/m_N &= 0, \\
\dot{n}_G + 3Hn_G - \Gamma_G (n^{eq})^2 &= 0.
\end{align*}
\]

(4.7a)–(4.7e)

Here \( n^{eq} = \zeta (3) T^3 / \pi^2 \) is the equilibrium number density of the bosonic relativistic species; \( \Gamma_G \) is a collision term for \( G \) production which, in the limit of the massless gauginos, turns out to be [32, 33]

\[
C_G = \frac{3\pi}{16\zeta(3)m_P^2} \sum_{i=1}^{3} c_ig_i^2 \ln \left( \frac{k_i}{g_i} \right) \quad \text{where} \quad \begin{cases} (c_i) = (33/5, 27, 72) \\
(k_i) = (1.634, 1.312, 1.271) \end{cases}
\]

(4.8)

and \( g_i \) (with \( i = 1, 2, 3 \)) are the gauge coupling constants of the MSSM calculated as functions of the temperature \( T \). The latter quantity and the entropy density, \( s \), can be obtained through the relations

\[
\rho_R = \frac{\pi^2}{30} g_s T^4 \quad \text{and} \quad s = \frac{2\pi^2}{45} g_s T^3.
\]

(4.9)

Also the Hubble expansion parameter, \( H \), during this period is given by

\[
H = \frac{1}{\sqrt{3m_P^2}} \left( m_{\widetilde{G}} n_{\widetilde{G}} + \rho_1 + \rho_2 + \rho_R \right)^{1/2}.
\]

(4.10)

Clearly, in the limit of massless MSSM gauginos, the \( n_{\widetilde{G}} \) computation is \( m_{\widetilde{G}} \)-independent.

The numerical integration of Eqs. (4.7a)–(4.7e) is facilitated by absorbing the dilution terms. To this end, we find it convenient to define [44] the following dimensionless variables

\[
f_1 = \rho_1 R^3, \quad f_2 = \rho_2 R^3, \quad f_R = \rho_R R^4, \quad f_L = n_L R^3 \quad \text{and} \quad f_{\widetilde{G}} = n_{\widetilde{G}} R^3.
\]

(4.11)

Converting the time derivatives to derivatives w.r.t. \( \tau = \ln \left( R/R_0 \right) \), with \( R_0 \) being the value of the scale factor at the onset of the \( N \) oscillations – the precise value of \( R_0 \) turns out to be numerically irrelevant – Eqs. (4.7a)–(4.7e) become

\[
\begin{align*}
H f'_1 &= -\Gamma_N f_1, \\
H f'_2 &= -\Gamma_{PQ} f_2, \\
H f'_R &= \Gamma_N f_1 R + \Gamma_{PQ} f_2 R, \\
H f'_L &= \varepsilon_L \Gamma_N R^3, \\
H f'_{\widetilde{G}} &= C_G (n^{eq})^2 R^3.
\end{align*}
\]

(4.12a)–(4.12e)

Also \( H \) and \( T \) can be expressed in terms of the variables in Eq. (4.11) as

\[
H = \frac{\sqrt{m_{\widetilde{G}} f_{\widetilde{G}} + f_1 + f_2 + f_R / R}}{\sqrt{3R^3m_P}} \quad \text{and} \quad T = \left( \frac{30 f_R}{\pi^2 g_s R^4} \right)^{1/4}.
\]

(4.13)
The system of Eqs. (4.12a)–(4.12e) can be solved, imposing the following initial conditions (the quantities below are considered functions of the independent variable $\tau$):

\[
\rho_1(0) = \rho_{1i}, \quad \rho_R(0) = n_G(0) = n_L(0) = 0 \quad \text{and} \quad \rho_2(\tau_{\text{HPQ}}) = \rho_{2i},
\]

(4.14)

where $\tau_{\text{HPQ}}$ is the value of $\tau$ corresponding to the temperature $T_{\text{HPQ}}$ which is defined as the solution of the equation $H(T_{\text{HPQ}}) = m_{\text{PQ}}$ and can be found numerically. Needless to say that we set $\rho_2(\tau) = 0$ for $\tau < \tau_{\text{HPQ}}$.

In Fig. 1, we illustrate the cosmological evolution of the quantities $\log \rho_i$ with $i = 1$ (dark gray line), $i = 2$ (gray line), $i = R$ (light gray line), $\log \rho_{2i}$ (dashed gray line), $\log Y_L$ (black solid line) and $\log Y_{\tilde{G}}$ (black dashed line) as functions of $\log T$ for values of the parameters which are allowed by all the restrictions described in Sec. 5. In particular we set $\lambda = 0.0071$, $\lambda_0 = 10^{-6}$, $f_a = 10^{12}$ GeV, $\mu = 1$ TeV, $h_{\text{eff}} = 10^{-5}$, $k_X = 1$ and $c_R \simeq 307$, resulting to $Y_{\tilde{G}} = 2 \cdot 10^{-14}$ and $Y_L = 2.5 \cdot 10^{-10}$ for $m_{\nu3} = 0.05$ eV and $\delta_{\text{eff}} = 0.01$. From Fig. 1–(a), we observe that non-$\text{MNI}$ is followed by a matter dominated (MD) era, due to the oscillating and decaying inflaton system, which lasts until $T \simeq T_{\delta}$ given by Eq. (4.2). The completion of the reheating process corresponds to the intersection of $\rho_1$ with $\rho_R$. Afterwards the universe enters the conventional radiation dominated (RD) epoch of standard Big Bang cosmology. This becomes possible thanks to Eq. (4.6), since then the decay of the PQ system commences at $T = T_{\text{HPQ}}$ and is completed at $T \simeq T_{\text{PB}}$ given by Eq. (4.5), before its domination over radiation. So a second episode of reheating does not occur. As we show below, this is a generic feature of our model. Due to this fact, the $L$ and $\tilde{G}$ yields, $Y_L = n_L/s$ and $Y_{\tilde{G}} = n_{\tilde{G}}/s$ respectively, take their actual values ($2.5 \cdot 10^{-10}$ and $2 \cdot 10^{-14}$) for $T \simeq T_{\delta}$ as shown in Fig. 2–(b). Both numerical values are compatible with observational data – see Sec. 5.

### 4.3 Analytical Approach

The numerical findings above can be understood by some simple analytic formulas. Most of them are widely employed in the literature – cf. Ref. [19,22–25,30]. In particular, the $B$ yield can be computed as

\[
\begin{align*}
(a) \quad Y_B & = -0.35 Y_L \quad \text{with} \quad (b) \quad Y_L & = n_L/s = c_L T_{\tilde{N}} \quad \text{where} \quad c_L = -5 \varepsilon_L/4 m_{\tilde{N}}.
\end{align*}
\]

(4.15)
Both Eqs. (4.15) and (4.16) calculate the correct values of the entropy production occurs for $T < T_{\tilde{N}}$. We show in the following that this is the case for our model.

The evolution of the various energy densities involved in the post-inflationary dynamics can be well approximated – see e.g. Ref. [34, 44] – by the expressions

$$
\rho_1 = \rho_1 e^{-3\tau} \quad \text{for} \ T \geq T_{\tilde{N}},
$$

$$
\rho_2 = \rho_2 e^{-3(\tau - \tau_{H_{PQ}})} \quad \text{for} \ T \geq T_{PQ},
$$

$$
\rho_R = \rho_R (T_{\tilde{N}}) (T/T_{\tilde{N}})^4 \quad \text{for} \ T \leq T_{\tilde{N}}.
$$

Possible domination of $\rho_2$ may occur for $T \leq T_{\tilde{N}}$, since $\rho_R$ is steeper than $\rho_2$. The equality between these two energy densities could be attained for $T = T_{eq}$ where

$$
\rho_R (T_{eq}) = \rho_2 (T_{eq}) \Rightarrow T_{eq} \simeq \frac{T_{\tilde{N}} \rho_{2i}}{\rho_{H_{PQ}}} = \frac{M_{PQ}^2}{3} T_{\tilde{N}}.
$$

In deriving the equation above, we use the fact that $\rho_R (T_{\tilde{N}}) = \rho_1 (T_{\tilde{N}})$ given by Eq. (4.17a) and that $e^{-3 \tau} \sim T^{-1}$ for $T_{eq} \leq T \leq T_{\tilde{N}}$ due to the isentropic expansion. Also we assume that for $T \geq T_{H_{PQ}}$ we have a MD era driven by $\rho_1$. This is a natural assumption since the condition of the out-of-equilibrium decay of $\tilde{N}$ gives an upper bound on $h_{\text{eff}}$, which prevents the unlimited increase of $T_{\tilde{N}}$. Indeed

$$
m_{\tilde{N}} \geq T_{\tilde{N}} \Rightarrow h_{\text{eff}} \leq \frac{2}{c_T} \sqrt{\frac{m_{\tilde{N}}}{m_p}} = \frac{2\sqrt{2}\lambda_p}{c_T} \sqrt{M_{PQ}},
$$

where Eqs. (4.2) and (4.1) are employed. The domination of $\rho_2$ over the several energy densities – and therefore, a second reheating process – can be avoided if we impose the condition

$$
T_{PQ} \geq T_{eq} \Rightarrow \lambda_a \geq \lambda h_{\text{eff}}^2 M_{PQ}^2 / 3 \lambda_{\mu}^2.
$$

Combining the last relation with Eq. (3.10), we arrive at $h_{\text{eff}} \leq \sqrt{3/2} \lambda_{\mu} M_{PQ}/c_R$. Taking e.g. $\lambda_{\mu} \simeq 0.01$, $c_R \simeq 10^2$ and $M_{PQ} \simeq 10^{-6}$ the last relation implies $h_{\text{eff}} \lesssim 100$ which is much less restrictive than Eq. (4.19) which gives $h_{\text{eff}} \lesssim 0.01$ for $\lambda \leq 3.5$ and $c_T \simeq 0.3$ – see Sec. 5.2. Therefore, the decay of the PQ system before its domination over radiation can be naturally accommodated within our set-up. Moreover, we remark that the results on $T_{\tilde{N}}$, $Y_L$ and $Y_{\tilde{G}}$ are independent of $\rho_{H_{PQ}}$ and $\rho_{2i}$ or $\mu$, $\lambda_a$ and $f_a$ when Eq. (4.20) holds.

## 5 Constraining the Model Parameters

We exhibit the constraints that we impose on our cosmological set-up in Sec. 5.1, and delineate the allowed parameter space of our model in Sec. 5.2.

### 5.1 Imposed Constraints

Under the assumption that (i) the curvature perturbations generated by $\sigma$ are solely responsible for the observed curvature perturbations and (ii) the violation of Eq. (3.10) occurs after the violation of the slow-roll conditions in Eqs. (3.12a) and (3.12b), the parameters of our model can be restricted once we impose the following requirements:
5.1.1 According to the inflationary paradigm, the horizon and flatness problems of the standard Big Bag cosmology can be successfully resolved provided that $\hat{N}_s$ defined by Eq. (3.16) takes a certain value, which depends on the details of the cosmological scenario. Employing standard methods [5,45], we can easily derive the required $\hat{N}_s$ for our model, consistently with the fact that the PQ oscillatory system remains subdominant during the post-inflationary era. Namely, we obtain

$$\hat{N}_s \simeq 22.5 + 2 \ln \frac{V_{N_1}(\sigma_s)^{1/4}}{1 \text{ GeV}} - \frac{4}{3} \ln \frac{V_{N_1}(\sigma_f)^{1/4}}{1 \text{ GeV}} + \frac{1}{3} \ln \frac{T_{\text{rh}}}{1 \text{ GeV}} + \frac{1}{2} \ln \frac{f(\sigma_f)}{f(\sigma_s)}. \quad (5.1)$$

5.1.2 The inflationary observables derived in Sec. 3.2 are to be consistent with the fitting [27] of the WMAP7, BAO and $H_0$ data. As usually, we adopt the central value of $P_{\text{R}}^{1/2}$, whereas we allow the remaining quantities to vary within the 95% confidence level (c.l.) ranges. Namely,

(a) $P_{\text{R}}^{1/2} \simeq 4.93 \cdot 10^{-5}$, \hspace{1em} (b) $n_s = 0.968 \pm 0.024$, \hspace{1em} (c) $-0.062 \leq a_s \leq 0.018$ and \hspace{1em} (d) $r < 0.24. \quad (5.2)$

5.1.3 For the realization of non-$M\tilde{N}I$, we assume that $c_R$ takes relatively large values – see e.g. Eq. (3.5). This assumption may [4,46] jeopardize the validity of the classical approximation, on which the analysis of the inflationary behavior is based. To avoid this inconsistency – which is rather questionable [8,46] though – we have to check the hierarchy between the ultraviolet cut-off, $\Lambda = m_P/c_R$, of the effective theory and the inflationary scale, which is represented by $\tilde{V}_{N_1}(\sigma_s)^{1/4}$ or, less restrictively, by the corresponding Hubble parameter, $\tilde{H}_s = \tilde{V}_{N_1}(\sigma_s)^{1/2}/\sqrt{3}m_P$. In particular, the validity of the effective theory implies [46]

(a) $\tilde{V}_{N_1}(\sigma_s)^{1/4} < \Lambda$ \hspace{1em} or \hspace{1em} (b) $\tilde{H}_s < \Lambda$ \hspace{1em} for \hspace{1em} (c) $c_R \geq 1. \quad (5.3)$

5.1.4 In agreement with our assumption about hierarchical light neutrino masses and the results of neutrino oscillation experiments [47], $m_{\nu_3}$ – involved in the definition of $\varepsilon_L$ in Eq. (4.4) – can be related to the squared mass difference measured in atmospheric neutrino oscillations, $\Delta m_{\odot}^2$. Taking the central value of the latter quantity, we set

$$m_{\nu_3} \simeq \sqrt{\Delta m_{\odot}^2} = (2.43 \cdot 10^{-3})^{1/2} \text{ eV} \simeq 0.05 \text{ eV}. \quad (5.4)$$

This value is low enough to ensure that the lepton asymmetry is not erased by lepton number violating $2 \to 2$ scatterings [48] at all temperatures between $T_{\text{R}}$ and 100 GeV.

5.1.5 The interpretation of BAU through non-thermal leptogenesis dictates [27] at 95% c.l.

$$Y_B = (8.74 \pm 0.42) \cdot 10^{-11} \Rightarrow 8.32 \leq Y_B/10^{-11} \leq 9.16. \quad (5.5)$$

Given our ignorance about $\delta_{\text{eff}}$ in Eq. (4.4), we impose only the lower bound of the inequality above as an absolute constraint.

5.1.6 In order to avoid spoiling the success of the SBB nucleosynthesis, an upper bound on $Y_G$ is to be imposed depending on the $\tilde{G}$ mass, $m_{\tilde{G}}$, and the dominant $\tilde{G}$ decay mode. For the conservative case where $\tilde{G}$ decays with a tiny hadronic branching ratio, we have [33]

$$Y_G \lesssim \begin{cases} 10^{-15} & \text{for } m_{\tilde{G}} \simeq 0.45 \text{ TeV} \\ 10^{-14} & \text{0.69 TeV} \\ 10^{-13} & \text{10.6 TeV} \\ 10^{-12} & \text{13.5 TeV} \end{cases} \quad (5.6)$$

The bound above can be somehow relaxed in the case of a stable $\tilde{G}$. However, it is achievable in our model, as we see below.
5 Constraining the Model Parameters

As can be easily seen from the relevant expressions above, our cosmological set-up depends on the following parameters:

\[ \lambda, \lambda_a, f_a, \lambda_\mu, k_X, c_R, h_{\text{eff}}, \delta_{\text{eff}}. \]

Our results are independent of \( \lambda_a \) and \( k_X \), provided that Eqs. (3.10) and (4.20) are satisfied. With these conditions, the contribution of \( V_{\text{tc}} \) to \( \tilde{V}_{N_1} \) remains subdominant and the PQ system decays before radiation domination. We therefore set \( \lambda_a = 10^{-6} \) and \( k_X = 1 \) throughout our calculation. The chosen \( \lambda_a \) is close to its largest value allowed by Eq. (3.10b), whereas \( k_X \) is fixed to a natural value. Also \( \delta_{\text{eff}} \) affects exclusively the \( Y_L \) calculation through Eqs. (4.4) and (4.15). We take \( \delta_{\text{eff}} = 1 \), which allows us to obtain via Eq. (4.4) the maximal [49] possible lepton asymmetry. This choice in conjunction with the imposition of the lower bound on \( Y_B \) in Eq. (5.5) provides the most conservative restriction on our parameters.

Also, we set \( \lambda_\mu = 0.01 \) [\( \lambda_\mu = 1 \)] so as to obtain \( \mu \sim 1 \) TeV with \( f_a = 10^{12} \) GeV [\( f_a = 10^{13} \) GeV] – evidently, the generation of the \( \mu \) term of the MSSM through the PQ symmetry breaking does not favor lower values for \( f_a \). As we show below, the selected values for the above quantities give us a wide and natural allowed region for the remaining fundamental parameters (\( \lambda, c_R \) and \( h_{\text{eff}} \)) of our model. In our numerical code, we use as input parameters \( \sigma_*, h_{\text{eff}}, f_a \) and \( c_R \). For every chosen \( c_R \geq 1 \) and \( h_{\text{eff}} \), we restrict \( \lambda \) and \( \sigma_* \) so as the conditions Eq. (5.1) – with \( T_{\text{th}} \) evaluated consistently using Eq. (4.2) – and (5.2a) are satisfied. Let us also remark that in our numerical calculations, we use the complete formulas for the slow-roll parameters and \( P_{\delta}^{1/2} \) in Eqs. (3.12a), (3.12b) and (3.18) and not the approximate relations, which are listed in Sec. 3.2 for the sake of presentation.

Our results are presented in Fig. 2, where we draw the allowed values of \( \lambda \) (solid line) and \( T_{\text{th}} \) (dashed line) \[ \sigma_1 \] (solid line) and \( \sigma_* \) (dashed line) versus \( c_R \) (a) [(b)] for \( h_{\text{eff}} = 10^{-5} \) and \( f_a = 10^{12} \) GeV. In Fig. 2-(b) we also draw \( \sigma_{2c} \) – derived from Eq. (3.10b) – as a function of \( c_R \). The upper [lower] bound on \( c_R \) comes from the saturation of the inequality in Eq. (5.3b) [Eq. (5.3c)]. On the other hand, Eq. (5.3a) is valid along the gray and light gray segments of the various curves. Along the light gray segments, though, we obtain \( \sigma_* \geq m_P \). The latter regions of parameter space, although can be considered as less favored, are not necessarily excluded [26], since the energy density of the inflaton

**Figure 2**: The allowed by Eqs. (5.1), (5.2a), (5.3b) and (5.3c) values of \( \lambda \) (solid line) and \( T_{\text{th}} \) – given by Eq. (4.2) – (dashed line) \[ \sigma_1 \] (solid line) and \( \sigma_* \) (dashed line) versus \( c_R \) (a) [(b)] for \( \lambda_a = 10^{-6}, k_X = 1 \) and \( h_{\text{eff}} = 10^{-5} \). Also, \( \sigma_{2c} \) (dot-dashed line) given by Eq. (3.10b) as function of \( c_R \) is shown. The light gray and gray segments denote values of the various quantities satisfying Eq. (5.3a) too, whereas along the light gray segments we obtain \( \sigma_* \geq m_P \).
remains sub-Planckian and so, corrections from quantum gravity can be assumed to be small. In all, we obtain

\[ 1 \lesssim c_R \lesssim 1.56 \cdot 10^5 \quad \text{and} \quad 2.6 \cdot 10^{-5} \lesssim \lambda \lesssim 3.5 \quad \text{for} \quad 52.5 \lesssim \tilde{N}_s \lesssim 54.6. \quad (5.7) \]

From Fig. 2-(a), we observe that \( \lambda \) depends on \( c_R \) almost linearly. This can be understood by combining Eq. (3.18) and Eq. (5.2a). The resulting relations reveal that \( \lambda \) is to be proportional to \( c_R \), so as Eq. (5.2a) is satisfied with almost constant \( \tilde{N}_s \). Indeed we find

\[ \lambda = 3 \cdot 10^{-4} \pi c_R / \tilde{N}_s \Rightarrow c_R = 41850\lambda \quad \text{for} \quad \tilde{N}_s \simeq 55. \quad (5.8) \]

On the other hand, the variation of \( \sigma_f \) and \( \sigma_s \) as a function of \( c_R \) drawn in Fig. 2-(b) is consistent with Eqs. (3.17) and (3.14). If \( c_R \) varies within its allowed region as given in Eq. (5.7), we obtain

\[ 0.963 \lesssim \sigma_f \lesssim 0.965, \quad -6.8 \lesssim \frac{\alpha_s}{10^{-5}} \lesssim -6.1 \quad \text{and} \quad 4.4 \gtrsim \frac{r}{10^{-5}} \gtrsim 3.4. \quad (5.9) \]

Clearly, the predicted \( n_s \) and \( r \) lie within the allowed ranges given in Eq. (5.2b) and Eq. (5.2c) respectively, whereas \( \alpha_s \) remains quite small. These findings depend very weakly on \( \sigma_f \), since this controls the value of \( T_{rh} \) via Eq. (4.2) – because \( T_{rh} \) appears in Eq. (5.1) through the one third of its logarithm, and consequently its variation upon some orders of magnitude has a minor impact on the required value of \( \tilde{N}_s \).

On the contrary, \( h_{\text{eff}} \) plays a key role in simultaneously satisfying Eqs. (4.19), (5.5) and (5.6) – see Eqs. (4.15) and (4.16). For this reason we display in Fig. 3-(a) [Fig. 3-(b)] the allowed regions by all imposed constraints in the \( \lambda - h_{\text{eff}} \) plane for \( f_a = 10^{12} \) GeV \([f_a = 10^{11} \) GeV] – cf. Ref. [23]. The restrictions on the parameters arising from the inflationary epoch are denoted by dotted and double-dotted dashed lines, whereas the ones originating from the post-inflationary era are depicted by solid, dashed and dot-dashed lines. In particular, the double-dotted dashed [dotted] lines come from the bounds of Eq. (5.3b) [Eq. (5.3c)]. In the horizontally lined regions Eq. (5.3a) holds, whereas in the vertically hatched region we get \( \sigma_s \gtrsim m_T \). On the other hand, the solid [dashed] lines correspond to the lower [most conservative upper] bound on \( Y_B \) \([Y_G]\) in Eq. (5.5) [Eq. (5.6)]. Since we use \( |\delta_{\text{eff}}| = 1 \), it is clear from Eqs. (4.4) and (4.15) that values of \( h_{\text{eff}} \) above the solid line are compatible with the current data in Eq. (5.5) for conveniently adjoining \( |\delta_{\text{eff}}| < 1 \). However the strength of \( h_{\text{eff}} \) can be restricted by the bounds of Eq. (5.6), which can be translated into bounds on \( T_{rh} \) via Eq. (4.16). Specifically we obtain \( Y_G \simeq (0.1 - 1) \cdot 10^{-12} \) or \( T_{rh} \simeq (0.53 - 5.3) \cdot 10^9 \) GeV (gray area), \( Y_G \simeq (0.1 - 1) \cdot 10^{-14} \) or \( T_{rh} \simeq (0.53 - 5.3) \cdot 10^7 \) GeV (yellow area) and \( Y_G \simeq (0.1 - 1) \cdot 10^{-15} \) or \( T_{rh} \simeq (0.53 - 5.3) \cdot 10^6 \) GeV (light gray area). The competition of the two restrictions above can be presented also analytically. Indeed, plugging \( T_{rh} \) via Eq. (4.2) into Eqs. (4.15) and (4.16), setting \( Y_L = Y_L^{\text{min}} \) and \( Y_G = Y_G^{\text{max}} \) – the exact numerical values can be extracted by Eqs. (5.5) and (5.6) respectively – and solving the resulting equations w.r.t. \( h_{\text{eff}} \), we obtain the following inequilities:

\[ (a) \quad h_{\text{eff}} \geq \frac{2\sqrt{3}}{\sqrt{M_{\text{Pl}}^2} \cdot m_T \cdot |c_L|^{|c_T|}} \quad \text{and} \quad (b) \quad h_{\text{eff}} \leq \frac{2\sqrt{3}}{\sqrt{M_{\text{Pl}}^2} \cdot c_G \cdot |c_T|}. \quad (5.10) \]

These reproduce quit accurately the behavior seen in Fig. 3. On the other hand, the out-of-equilibrium condition depicted by a dashed line – see Eq. (4.19) – puts the upper bound on \( h_{\text{eff}} \) in a minor portion of the parameter space.

Comparing Fig. 3-(a) and Fig. 3-(b), we conclude that \( m_{\tilde{N}} \) given by Eq. (4.1a) is reduced for \( f_a = 10^{11} \) GeV w.r.t. its value for \( f_a = 10^{12} \) GeV, and so the condition of Eq. (4.19) cuts a larger slide of the available parameter space. Letting \( \lambda \) vary within its allowed region in Eq. (5.7), we obtain

\[ 4.3 \cdot 10^{-6} \left[1.3 \cdot 10^{-5}\right] \lesssim h_{\text{eff}} / 10^{-3} \lesssim 0.58 \quad \text{for} \quad f_a = 10^{12} \text{ GeV} \quad [f_a = 10^{11} \text{ GeV}] \quad (5.11) \]
Figure 3: Allowed (shaded) regions as determined by Eqs. (4.19), (5.3b), (5.3c), (5.5) and (5.6) in the $\lambda - h_{\text{eff}}$ plane, for $\lambda_a = 10^{-6}$, $k_X = 1$, $\mu = 1$ TeV and $f_a = 10^{12}$ GeV (a) or $f_a = 10^{11}$ GeV (b). The conventions adopted for the various lines and shaded or hatched regions are also shown.

where the overall minimal [maximal] $h_{\text{eff}}$ can be found in the upper, almost central [lower right] corner of the allowed region. As we see above and can be induced by Eqs. (4.19) and (5.10b), the maximum allowed $h_{\text{eff}}$ is $f_a$ independent and it is obtained for $\lambda \simeq 0.005$ [$\lambda \simeq 0.05$] and $f_a = 10^{12}$ GeV [$f_a = 10^{11}$ GeV]. This point gives also a lower bound on $|\delta_{\text{eff}}|$, $|\delta_{\text{eff}}| \gtrsim 2 \cdot 10^{-4}$ – which is obviously also $f_a$ independent. Note finally that in both cases the resulting $m_{\tilde{N}}$’s can be much lower than those obtained within the simplest model of sneutrino inflation with a quadratic potential [21, 22]. On the other hand, $h_{\text{eff}}$ turns out to be comparable with the one obtained in those models.

6 Conclusions

In this paper we attempted to embed within a realistic cosmological setting one of the recently formulated [8] SUSY models of chaotic inflation with non-minimal coupling to gravity. We concentrated on a moderate extension of the MSSM augmented by three RH neutrino superfields and three other singlet superfields, which lead to a PQPT tied to renormalizable superpotential terms. The coupling between the RH neutrinos and one of the fields associated with the PQPT plays a crucial role for the implementation of our scenario. We showed that the model non only supports non-MI driven by the lightest RH sneutrino, but it also resolves the strong CP and the $\mu$ problems of the MSSM and, even
more, it leads to the production of the required by the observations BAU via non-thermal leptogenesis, which accompanies the inflaton’s decay. Moreover the $\tilde{G}$ abundance becomes observationally safe for $\tilde{G}$ masses even lower than $10^{12}$ GeV. An important prerequisite for all these is that the parameter of the superpotential related to the PQPT, $\lambda_\alpha$, is adequately small. Imposing a number of observational constraints arising from the data on the inflationary observables, the BAU, the concentration of the unstable $\tilde{G}$ at the onset of nucleosynthesis and the mass of the heaviest light neutrino, we restrict the effective Yukawa coupling, involved in the decay of the inflaton, to relatively small values, and the inflaton mass to values lower than $10^{12}$ GeV.

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**APPENDIX: NON-MINIMALLY CURVATURE-COUPLED SCALARS IN SUGRA**

Non-MI can be realized by a scalar field with a non-minimal coupling to the Ricci scalar curvature. The formulation of a such theory within SUGRA is described below. Recall that we follow the conventions of Ref. [34] for the quantities related to the gravitational sector of our set-up.

In contrast to the non-SUSY case – see e.g. Ref. [1,2,5,9] – we find it convenient to start our analysis with the general *Einstein-frame* (EF) action for the scalar fields $\phi^\alpha$ plus gravity in four dimensional, $N = 1$ SUGRA [7, 8]:

$$S = \int d^4x \sqrt{-\hat{g}} \left( -\frac{1}{2} \hat{m}_F^2 \hat{R} + K_{\alpha\beta} \hat{g}^{\mu\nu} D_\mu \phi^\alpha D_\nu \phi^{\beta} - \hat{V} \right),$$  

(A.1)

where hat is used to denote quantities defined in the EF; $\hat{g}$ is the determinant of the Friedmann-Robertson-Walker background metric [34];

$$K_{\alpha\beta} = \frac{\partial^2 K}{\partial \phi^\alpha \partial \phi^\beta} > 0 \quad \text{and} \quad D_\mu \phi^\alpha = \partial_\mu \phi^\alpha - A^A_\mu k^\alpha_A$$  

(A.2)

are the covariant derivatives for scalar fields $\phi^\alpha$. Here $A^A_\mu$ stand for the vector gauge fields and $k^\alpha_A$ is the Killing vector, defining the gauge transformations of the scalars [8]. Assuming that the $D$-terms of $\phi^\alpha$ vanish – as for the singlet scalars $\tilde{N}, \tilde{P}, \tilde{X}$ and $\tilde{X}$ in our model – the EF scalar potential, $\hat{V}$, is given in terms of the Kähler potential, $K$, and the superpotential, $W$, by

$$\hat{V} = e^{K/m_P^2} \left( K_{\alpha\beta} F_\alpha F_\beta - 3 \frac{|W|^2}{m_P^2} \right),$$  

(A.3)

with $K_{\alpha\beta} K_{\alpha\beta} = \delta^\beta_\beta$ and $F_\alpha = W_{\phi^\alpha} + K_{\phi^\alpha} W/m_P^2$.

The action in Eq. (A.1) can be brought the *Jordan frame* (JF) by performing a conformal transformation [38]. Indeed, if we define the JF metric, $g_{\mu\nu}$, through the relation

$$\hat{g}_{\mu\nu} = -\frac{\Omega}{3} g_{\mu\nu} \Rightarrow \left\{ \begin{array}{l}
\sqrt{-\hat{g}} = \Omega^2 \sqrt{-g}/9 \quad \text{and} \quad \hat{g}^{\mu\nu} = -3 g^{\mu\nu}/\Omega,
\hat{R} = -3 \left( R - \Box \ln \Omega + 3 g^{\mu\nu} \partial_\mu \Omega \partial_\nu \Omega / 2 \Omega^2 \right) / \Omega
\end{array} \right.$$

(A.4)

– where $\Box = (-g)^{-1/2} \partial_\mu (\sqrt{-g} \partial^\mu)$ – we obtain the action in the JF as follows

$$S = \int d^4x \sqrt{-g} \left( \frac{m_F^2}{6} \Omega R + \frac{m_P^2}{4\Omega} \partial_\mu \Omega \partial^\mu \Omega - \frac{1}{3} \Omega K_{\alpha\beta} D_\mu \phi^\alpha D_\nu \phi^{\beta} - V \right)$$

with $V = \frac{\Omega^2}{9} \hat{V}$.  

(A.5)
Taking into account that $\partial_\mu \Omega = D_\mu \Omega$ – since $\Omega$ is gauge invariant – and that the purely bosonic part, $A_\mu$, of the on-shell value of the auxiliary field $A_\mu$ is given by

$$ A_\mu = -\frac{i}{2\Omega} m_P^2 \left( D_\mu \phi^\alpha \Omega_\alpha - D_\mu \phi^{*\alpha} \Omega^*_\alpha \right) $$

(A.6)

with $\Omega_\alpha = \Omega_{\phi^\alpha}$ and $\Omega^{*}_\alpha = \Omega_{\phi^{*\alpha}}$ and for the choice

$$ \Omega = -3e^{-K/3m_P^2} \Rightarrow K = -3m_P^2 \ln (-\Omega/3), $$

(A.7)

we arrive at the following action

$$ S = \int d^4x \sqrt{-g} \left( \frac{m_P^2}{6} \Omega \mathcal{R} + m_P^2 \Omega_{\alpha\beta} D_\mu \phi^\alpha D^\mu \phi^{*\beta} - \Omega A_\mu A^\mu / m_P^2 - V \right). $$

(A.8)

It is clear from the first term of the RHS of this expression that the resulting $S$ exhibits non-minimal couplings of the $\phi^\alpha$’s to $\mathcal{R}$. However, $\Omega$ enters in the kinetic terms of the $\phi^\alpha$’s too. In order to get canonical kinetic terms, we need $\Omega_{\alpha\beta} = \delta_{\alpha\beta}$ and $A_\mu = 0$. The first condition is satisfied [8] by the choice

$$ \Omega = -3 + \delta_{\alpha\beta} \frac{\phi^\alpha \phi^{*\beta}}{m_P^2} - 3 \left( F(\phi^\alpha) + F^*(\phi^{*\alpha}) \right), $$

(A.9)

where $F$ is a dimensionless, holomorphic function, which expresses the non-minimal coupling to gravity. Note that even when $F(\phi^\alpha) = 0$ for some $\alpha$, the $\phi^\alpha$’s are conformally coupled to gravity due to the second term of the RHS of the expression above. This choice for the frame function leads via Eq. (A.7) to the following Kähler potential

$$ K = -3m_P^2 \ln \left( 1 - \frac{1}{3m_P^2} \delta_{\alpha\beta} \phi^\alpha \phi^{*\beta} + (F(\phi^\alpha) + F^*(\phi^{*\alpha})) \right). $$

(A.10)

On the other hand, $A_\mu = 0$ when the dynamics of the $\phi^\alpha$’s is dominated only by the real moduli $|\phi^\alpha|$. Therefore, it is possible to get a SUGRA realization of the inflationary models with non-minimal coupling.

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