Are the Laws of Thermodynamics Consequences of a Fractal Properties of Universe?

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Why in our Universe the laws of thermodynamics are valid? In the paper is demonstrated the reason of it: if the time and the space are multifractal and the Universe is in an equilibrium state the laws of the thermodynamics are consequences of it’s multifractal structure.

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I. INTRODUCTION

It is well known, that the multifractal sets have the characteristics very similar to the characteristics of a physical quantities (a free energy, an entropy, a temperature etc) with which these characteristics can be formally compared. The connections between the characteristics of multifractal set and characteristics of physical quantities formally correspond to connections between the thermodynamic quantities. This surprising correspondence till now is completely inexplicable. In the present paper the multifractal analysis advanced in by Mandelbrot [1], [2], Renyi [3], Halsy [4], etc. (see for example, Rudolf [5]) was used for a substantiation of thermodynamics laws on the base of the supposition that the space and the time are multifractal sets. If our Universe state is the state of equilibrium (or the state nearly equilibrium) the connections between the global characteristics of Universe as a whole (multifractal time and space) and their local fractal characteristics will be the same that thermodynamic relations. From the point of view of the fractal theory of time and space the thermodynamics relations (as well as thermodynamics in whole) are consequences of multifractal structure (structure of time and space) of our Universe.

In the theory [1] - [3] the time and the space are treated as a real physical fields. These fields consist of small multifractal subsets of time and space ("elements" of time and space), in turn, approximately treated as "points". The multifractal sets of time and space defined on a set of the carrier of a measure $R^n$, and contain all characteristics of the real world by reflected it’s in their fractional dimensions. The fractal dimensions $d_t$, $d_r$, and $d_o$ in small neighborhood of points $t$, $r$, and $o$ (belonging the sets of $t$, $r$, and $o$) are global dimensions. At the same time for all space - time continuum these dimensions are local fractal dimensions (Gelder exponents). The purpose of the paper is the establishment of connection between the global and the local characteristics of multifractal space and time on the basis of the multifractal analysis. We suppose that the state of the Universe (consisting from multifractal time and spatial sets) may be described as the state close to a thermodynamic equilibrium. The establishment of such connections enables on to view the new reason of origin of thermodynamic relations existing in our world, reducing it to presence of the fractal properties at time and space. We shall show, that the thermodynamic relations used in physics are a natural consequence of known mathematical connections between the multifractal characteristics of the Universe (Universe is considered as multifractal space - time set described within the framework of the fractal theory of time and space [1] - [3]). Thus thermodynamics can be considered as a natural consequence of multifractal characteristics of time and space of the world in which we live.

II. CONNECTION BETWEEN THE PHYSICAL AND MULTIFRACTAL CHARACTERISTICS IN THE MULTIFRACTAL UNIVERSE

Let’s consider the Universe as a dynamic system at the state that close to a thermodynamic equilibrium (at the present stage of it’s development), defined on a multifractal set $X$. Let the state of the Universe is characterized by fractal dimensions of a space - time continuum and by mean values of an internal energy, a free energy, an entropy, a temperature. If the state of Universe is close to the thermodynamic equilibrium, it’s characteristics is possible to describe by a free energy $F$, entropy $S$, internal energy $E$ and temperature $T$. Let the Universe has a multifractal nature stipulated by fractional dimensions of time and space (accoding to the fractal theory of time and space [1] - [3]) and is characterized by multifractal set $X(r,t)$ it’s space - time points. The multifractal set $X$ is defined on the carrier of a measure (set $R^n$ with topological dimensions), i.e. $X isthesubsetofR^n$. Let the set of the carrier of a measure is characterized by the temperature $T_0$, by the internal energy $E_0 = T_0$ (in the system in which the Boltzmann constant is equal to unity), by a free energy $F_0$. Let’s define a measure $\mu$ on the set $X$ and consider connection between invariant scaling characteristics of the multifractal Universe with a measure $\mu$ on the basis of the theory [1] - [3] and hypothesis about the origin of the Universe as a result of
explosion (big bang). Because of multifractality of space-time sets, the scaling transformations (at measuring volume of the Universe with the help of covering by four-dimension orbs (or cubes) with radius \( \delta \), for example, for mean values of probability of the casual mass distribution \(< p^q >\) (or the random distributions of densities of energy of physical fields), will look like (see, for example, [2] - [3])

\[
< p^q > \sim \delta^{d(q+1)}
\]  

where \( q \) is scale factor bound with \( q \)-dimensions Renyi \( dim_B^q(X) \) by relation

\[
dim_B^q(X) = \frac{\tau(q)}{q-1}
\]  

The dimension Renyi characterizes global scaling characteristics of the Universe. For definition of it’s physical sense we shall consider local properties of the Universe near to the point \( r, t \). The local fractal dimensions in this point (Gelder’s exponent) according to [2] - [3] looks like

\[
\alpha(x) \equiv d_{i,t}(\vec{r}(t), t) = 4 + \sum_i \beta_i L_{i,t}(\vec{r}(t), t)
\]  

where \( L_{i,t}(\vec{r}) \) are densities of energy of physical fields in this point and characterized by the densities of Lagrangians. The quantity \( p_i \) in a neighborhood of a point \((r, t)\) is transformed as

\[
p_i \sim \delta^{d_{i,t}(\vec{r}(t), t)}
\]  

From definition of \( q \)-dimensions Renyi

\[
dim_B^q(\vec{r}, t) = \frac{1}{q-1} \lim_{\delta \to 0} \frac{\log \sum_i p_i^q}{\log \delta}
\]  

follows

\[
dim_B^q(\vec{r}, t) = \frac{qd(\vec{r}(t), t)}{q-1}
\]  

The fractal dimensions \( d(r, t) \) in [3] are the dimensionless internal energies (after multiplication on \( E_0 \) the relation [3] and correspond an internal energies of Universe in a point with coordinates \((r, t)\) and so, for \( q >> 1 \), follows

\[
dim_B^q(\vec{r}, t) \approx d(\vec{r}(t), t)
\]  

Therefore the \( dim_B^q(X) \) should has sense of an energies. For describing of a thermodynamic equilibrium of the Universe there are only two energies (internal \( E \) and free \( F \)) and \( E_0 \) is bound with \( d_{i,t}(r, t) \), therefore the dimensions Renyi there corresponds to a free energy of the Universe \((F \div E_0)\) in the \( q \)-state. Let’s define now \( q \)-state. From [3] follows, as the Universe cools down and also it’s temperature is decrease and it’s volume grows, that \( q \) must depends on temperature and will increase with Universe cooling. The simplest dimensionless function satisfying to this requirement is the function

\[
T = T_0/T
\]

Now it is necessary to define a function of state of the Universe - the entropy \( S \). Let’s consider subsets \( S'(\alpha) \) (of the set \( X \)) with identical Gelder’s exponents \( d_{i,t} = \alpha \) (in our case it corresponds to a selection of an isoteric sets of the “internal energy” of the Universe). In this case joining of subsets \( S'(\alpha) \), stratifying original set \( X \), will coincide with the original set. Let’s introduce a spectrum of fractal dimensions \( f(\alpha) \). The joining of all such subsets makes set \( X \). Let’s the fractal dimensions of set \( S(\alpha(q)) \) (obtained as a result of such stratifying) is \( f(\alpha(q)) \) (spectrum of singularities). For each value of \( q \) the state of the Universe is determined as a single-valued state and at alteration \( q \) (that is decreasing of energy of the Universe because of decreasing of it’s temperature) and expansion of the Universe function \( f(\alpha(q)) \), describing scaling properties of set \( S(\alpha(q)) \), will grows. Such behavior corresponds to behavior of an entropy (a \( q \)-entropy) which we shall designate by \( S \). Hence, to the every state of Universe there corresponds a spectrum of singularities \( f(\alpha(q)) \) equal to an \( q \)-entropy \( S \).

III. CONNECTION OF A FREE ENERGY AND AN ENTROPY AS A CONSEQUENCE OF A MULTIFRACTAL NATURE OF THE UNIVERSE

We use now the known relation of the multifractal analysis between \( q \)-dimensions Renyi, spectrum of a singularity \( f(\alpha(q)) \) and local fractal dimensions \( q \), (see, for example, [3])

\[
(q-1)dim_B^q(X) = q\alpha(q) - f(\alpha(q))
\]

For \( T << T_0 \), substituting in [3] instead of dimensions Renyi the spectrum of singularities \( f(\alpha(q)) \), the local fractal dimensions \( \alpha(q) \) and the scaling factor \( q \) their physical values (that we have received earlier) the relation reads

\[
F = E - TS
\]

The relation [3] is the basic relation of the thermodynamics. As relation [3] is fulfilled for the Universe as a whole, it will be fulfilled and for it's parts with the state of thermodynamically equilibrium. Therefore in the Universe with the multifractal time and space the realization of the laws of thermodynamics is a simple consequence of it’s structure.

The analysis of connections of global dimensions and local fractal characteristics of the fractal space-time carried out above allows to make the following statements, that are true for a case of equilibrium (or nearly so by equilibrium) of the state of the Universe:
a) The free energy of the Universe $F$ can be viewed as fractal $q$-dimensions Renyi ($q = T_0/T$) of space-time set $X$ that consist the Universe

$$\dim_{B} \frac{T_0}{T}(\vec{r}, t) = \frac{T}{T_0 - T} \lim \frac{\log \sum \mu_i^{T_0/T}}{\log \delta} = F \quad (11)$$

where $\mu_i$ a measure of $i$-th of four-dimensional element of space-time;

b) The inverse temperature of the Universe $T_0/T$ corresponds to the $q$-characteristics of the scaling transformation of multifractal space-time;

c) The entropy of Universe $S$ corresponds the spectrum of fractal dimensions $f(\dim_{B}(T_0/T))$, defined by dependencies of space-time of dimensions Renyi $\dim_{B} \frac{T_0}{T}(\vec{r}, t)$, mean temperature $T_0/T$ and local fractal dimensions of space-time sets with identical energy $\dim_{B}(T_0/T)$;

d) The knowledge of the fractal spectrum and dimensions $\dim_{B}(q)$ allows to find dimensions Renyi from (11). If the dimensions Renyi is known, the differentiation (***6.10) with respect to $q$ gives in the equation

$$d_{\tau}(q) = \frac{d}{dq}[(q - 1) \dim_{B} \frac{T_0}{T}(\vec{r}, t)] \quad (12)$$

It is possible to find, using (12), the entropy (i.e. the spectrum of fractal dimensions $f(T_0/T) = f(q)$);

e) The thermodynamics in viewed model is a consequence of the multifractality of space-time continuum.

IV. CONCLUSION

The problem of a substantiation of the thermodynamics within the framework of the fractal theory of time and space presented in this paper, (as well as a substantiation of irreversibility of time and spatial events (see [2]) is reduced to a postulating of multifractal properties of space and time. If model of fractal time and spaces [3] - [6] is correct, the Universe is open system and exchange it's energy with the carrier of a measure $R^n$ (or with the alien Universe of inflationary model [7]) or model [11].