Dust Outburst Dynamics and Hazard Assessment for Close Spacecraft–Comet Encounters

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Abstract

Using the gas drag by sublimating cometary surface ices for the acceleration of dust particles and deceleration by the gravity field of the nucleus combined with basic laws of mechanics, the sizes, velocities, and number densities of escaping particles are calculated and evaluated with respect to the hazard assessment of comet–spacecraft flybys and encounters. We find good agreement between our analytical method and the more elaborate and precise DSMC calculations, but, being simpler, our method can more easily be used to explore a wide range of cometary conditions and can be more easily scaled to specific comets with different nucleus sizes, masses, and gravity potentials and various gas and dust production rates. Our analytical method is applied to outbursts expanding into a cone of \(\sim 60^\circ\), where the gas density falls off with height from the surface rather than radial distance from the center of a uniformly outgassing nucleus. In this scenario, larger dust particles can be ejected and attain ballistic trajectories, go into orbit, or escape from the nucleus, thus being potentially more hazardous to a spacecraft. Sample calculations are carried out for potential dust outbursts for the highly active Centaur/Comet 29P/Schwassmann-Wachmann 1 for various assumed active areas and dust particle size distributions. Particle velocity ranges for ballistic trajectories, orbiting particles, and particles escaping into the coma are presented. These calculations are used to estimate the coma particle number densities during outbursts to get an assessment of the hazards and required mitigation for a flyby or orbiting space mission.

Unified Astronomy Thesaurus concepts: Comae (271); Space probes (1545)

1. Introduction

A flyby or orbiter mission to an active comet requires a careful analysis of the dust environment and the resulting hazards to the spacecraft. Instead of using a complex and computer-intensive direct simulation Monte Carlo (DSMC) model, we present in this paper a relatively simple analytical approach using basic physics and orbital dynamics. Our analysis is designed to calculate dust particle accelerations and velocities as a function of distance from the nucleus and the terminal velocities of the escaping particles. We determine which particles enter ballistic trajectories, go into orbit, and escape, forming the extended cometary dust coma. Using observational input for the activity of a comet and its gas production rate and dust output, we estimate particle densities as a function of distance from the nucleus.

Although our method of calculation does not have the sophistication of a comprehensive DSMC model, it is more straightforward. The DSMC model is essentially a “black box” that attempts to include all important physical processes, such as collisions between gas molecules, dust and gas molecules, and dust particles, using appropriate collision cross sections. With this approach, one must have faith that the particular model incorporates all of these processes and parameters correctly and thus represents a close replica of nature. Our present model does not consider the changing gas-kinetic temperature and the increasing velocity of the gas molecules after release from the nucleus, the transport of dust to the nightside via collisions, or the gradual change from a dayside dust distribution to the more nearly spherical distribution generally observed for comets at tens of thousands of kilometers from the nucleus. Nevertheless, our analytical method is relatively simple and thus readily reproducible and confirmable. It is useful for exploring the conditions for comets of different sizes, masses, gravity, and gas and dust production rates and can also serve as a useful check on the less restricted but more complex results of the DSMC calculations. We compare our numerical results with those of extensive pre-encounter DSMC calculations by Tenishev et al. (2011) for comet 67P/Churyumov-Gerasimenko (67P) in Section 2.3. We find good agreement for both the functional relationship and numerical results when using the same parameters for both models.

We emphasize that we do not engage in a rigorous set of fluid dynamic calculations but instead make a number of simplifications and approximations in our paper to keep our calculations simple and tractable. We consider their effects to be considerably smaller than present observational uncertainties. The observational uncertainties include the amount of dust mass lost by the comet, the unknown particle size distribution over its complete range, the unknown size and number of active areas that are likely to be sporadic, the unknown shapes and densities of the particle and whether they are compact or fluzzy, the unknown shape of any particular comet (which is really known well for only one comet, 67P), etc. Each of these effects can introduce uncertainties of several factors and in sum can add up to sizable errors that far exceed any approximations made in our dust outflow model. We feel that unless all of these
factors can be much better constrained, calculations with an accuracy of a factor of 2 or so are sufficient to make an assessment of the possible hazards encountered by an orbiting or flyby spacecraft.

Our analysis stems from a desire to assess the hazards to a spacecraft from the general dust coma environment and especially from large outburst events. For the present paper, we selected Comet 29P/Schwassmann-Wachmann 1 (SW1) as our test case because it exhibits frequent outbursts, many of which have been observed in detail. Starting with its discovery by A. Schwassmann and A. A. Wachmann in 1927, this comet has exhibited extensive and periodic activity and outbursts despite its considerable distance of ~6 au from the Sun (Richter 1941; and see the excellent summary in Miles et al. 2016). Examination of earlier photographic plates showed that SW1 had previously been observed in 1902 March, so it has now made eight observed orbits around the Sun. During this time, its period shortened from about 16.4 to 14.7 yr, and its eccentricity decreased from about 0.15 to its present near-circular orbit with an eccentricity of ~0.04. Since 1927, it has displayed at least one major outburst per year, during which its brightness increases by about 5 mag. In addition, it exhibits continuous steady-state activity and a large number of lesser outbursts (Miles et al. 2016; Wierzchos & Womack 2020). Recent dynamical calculations by Sarid et al. (2019) show that SW1 is a captured Kuiper Belt object that has not been in its present orbit for very long and that it will likely be ejected into a short-period cometary orbit within the next few thousand years. Spectroscopic investigations have identified CO as its major outgassing species, with production rates of \( \sim 1 \times 10^{28} \) molecules s\(^{-1}\) during its quiescent state but reaching \( \sim 1 \times 10^{29} \) molecules s\(^{-1}\) during outbursts (Womack et al. 2017). The production rate of H\(_2\)O is roughly a factor of 10 lower (Ootsubo et al. 2012), and the production rate of CN is a factor of 1000 or more lower, while upper limits exist for CH\(_4\), CH\(_2\)OH, HCN, and CO\(_2\) (Womack et al. 2017; Wierzchos & Womack 2020). Thus, for the present paper, we will use CO as the major gaseous species, although highly volatile but infrared inactive molecules such as O\(_2\) and N\(_2\), both detected in 67P (Bieler et al. 2015; Rubin et al. 2015), should not be discounted. This may be germane to the observations of Wierzchos & Womack (2020), who found that the dust outbursts did not correlate well with the CO gas outbursts during their observing campaigns.

To aid us in the development of our model, it is of interest to examine and discuss the excellent close-up images of the near-nucleus environment of 67P in Figures 1 and 2. These two figures are taken from the large set of images from the Rosetta spacecraft OSIRIS camera system (Keller et al. 2007; ftp://psa.esac.esa.int/pub/mirror/INTERNATIONAL-ROSETTA-MISSION-OSIRIS). They are part of our unpublished study of the dust evolution and distribution of 67P. Of particular interest for dust investigations, the images show that the dust production for 67P is not homogeneous over the sunlit surface, which was the prevailing assumption used in pre-Rosetta dust evolution models. Instead, the dust production originates from a large number of small active surface areas and jets with intermediate quieter regions. This circumstance is not unique to 67P, as similar features have been observed for 1P/Halley, 19P/Borrelly (Soderblom et al. 2002), and 103P/Hartley (Syl et al. 2013). In addition to the ubiquitous jets, there are also major outbursts where the local gas production rate can be very much higher than the quiescent global production rate. The stochastic nature of outbursts makes them difficult to capture during short-duration spacecraft encounters. However, several minor outbursts were detected at 67P that display consistent jetlike morphology (Vincent et al. 2016). This can be ascribed to both the complex bilobate shape of the nucleus and the nonuniform heating and distribution of volatile ices and active areas. The resulting trajectories and outflow velocities will be different than for a uniformly outgassing comet, allowing much larger particles to be lifted and accelerated to higher velocities. The two images selected for the dust distribution can be taken as quite typical of the angular dust distribution for 67P, as observed by the Rosetta spacecraft. The two annuli surrounding the comet show that, close to the nucleus, the gas and dust are typically concentrated in a cone of roughly 60° centered on the subsolar point. We therefore use this value as a representative cone angle in our simulations. In Figure 2, we also show an image with a relatively narrow jet having a cone angle of \( \sim 25^\circ \) superimposed on the typical 60° dust distribution. While such a narrower jet can be readily modeled with our approach, we do not do this in this paper.

Our primary goal is to calculate the dust number particle density as a function of their size and distance from the nucleus. To this end, we develop, in Section 2, the basic equations and necessary definitions for the dust acceleration due to gas drag and the retarding acceleration due to the gravity force of the comet’s nucleus. This leads to a relatively simple generalized expression for the dust particle velocity, which is directly proportional to the square root of the surface gas flux and inversely proportional to the particle size and bulk density. For a relatively large, uniformly outgassing surface area (Section 2.2), the particle acceleration can be integrated, and expressions are presented for the velocity as a function of distance from the nucleus and the terminal velocity of the dust.

In Section 3, we extend our model to include outbursts and develop basic equations for the dust evolution from limited active surface areas. These equations cannot be simply integrated but must be integrated numerically. Adopting a variety of active areas and gas production rates, we calculate the particle size range for ballistic, orbital, and escaping particles and illustrate sample trajectories in several figures. We determine a velocity function of the dust particles (having radius \( s \)) escaping from the nucleus that form the dust coma of SW1. This velocity function follows the relationship of \( v = 1/\sqrt{s} \) over a considerable particle size range, but small particles reach an asymptote equal to the gas velocity while large particles are slower, since the gravitational nuclear deceleration term approaches the acceleration due to gas drag.

In Section 4, we estimate spatial particle number densities as a function of their size and distance from the nucleus. Several particle size distributions and active surface areas are explored. For these calculations, we use the particle sizes and their velocities determined in Section 3 for particles that can escape into the coma. The amount of dust emitted from the comet is determined by an estimate of the dust / gas mass ratio and the observed global gas production rates of CO (Womack et al. 2017).

The potential hazards to spacecraft investigations of SW1 are discussed in Section 5. We find that the major hazard to a spacecraft is posed by the largest particles escaping, which are mainly produced by the most confined jet and outburst events. These particles have a sufficient number density in the vicinity of the nucleus to pose a substantial hazard to a flyby mission.
Figure 1. Typical dust distribution and activity for comet 67P as observed by the Rosetta spacecraft on 2015 February 20 at 11:36. We show a 12 s exposure with the 610 nm Vis filter at 2.27 au with a spacecraft distance to the nucleus of 108 km and a solar phase angle of 65°9. The image shows quite clearly that the dust evolution is not uniform but is dominated by distinct active areas and dust jets. The bottom panel shows the angular dust brightness in two annuli 3.85 km and 7.16 km from the nucleus center. The angles start with 0° along the positive x-axis and proceed counterclockwise. The upturn in brightness at 300°–360° is not real but is caused by a ghost image in the camera. The inner annulus clearly shows a more confined and sharper dust distribution. The half-width of the dust distribution is about 60°, which we use for our modeling of dust distribution in Section 3.
Figure 2. Dust jet for comet 67P as observed by the Rosetta spacecraft on 2015 March 10 at 5:23. We show a 12 s exposure with the 610 nm Vis filter with the spacecraft at 2.13 au from the Sun and a distance to the nucleus of 78 km at a solar phase angle of 48°.3. Besides the strong dust jet, about a dozen smaller jets arising from various active areas can also be seen. The bottom panel shows the dust brightness distribution in two annuli 3.2 and 5.2 km from the nucleus center. The inner annulus clearly shows a sharp jet with a half-width of about 25° superimposed on a broader dust distribution of about 60°.
with a high flyby velocity, \( \sim 5 \text{ km s}^{-1} \). To reduce the risk of a catastrophic collision following a large localized outburst, a large flyby distance (\( \sim 10,000 \text{ km} \)) would be necessary. For an orbiting spacecraft, a slow approach speed of \( \text{m s}^{-1} \) and low velocity reconnaissance orbits \( \sim 1000 \text{ km} \) from the nucleus will provide an important safeguard against any potential serious collision damage.

While not a major purpose of this paper, we briefly discuss in Section 5.1 how the large dust fragments observed during the Rosetta investigation can be well explained by our dust production and outburst model.

2. Particle Acceleration and Velocity

Dust particles on the surface of a comet are lifted off by the gas pressure of the sublimating ices and then accelerated by the outflowing gas. Numerical simulations of this process have been carried out using the method of DSMC techniques (Tenishev et al. 2011). These calculations require considerable computation time on high-speed computers. Thus, the number of distinct cases that can be studied is limited. We will show that many of the features of the dust outflow, terminal velocities, bound and escaping particles, etc. can be calculated from first principles using basic dynamical principles and laws of physics. Our equations are analytically integrated in Section 2.2 for the case where the dust particle velocity is a fraction of the gas velocity (\( v_d \ll 0.1v_g \)) and the activity of the comet extends over a large area, such as the sunlit side. Various scenarios can thus be explored without recourse to the more complex computer- and time-intensive DSMC method. For the case of localized outbursts, the equations of motion can be numerically integrated, which will be carried out in Section 3.

2.1. Basic Gas Drag Equations

The equations of motion for dust particles accelerated by the gas drag and being retarded by the gravitational force but neglecting radiation pressure can be written as (Gombosi et al. 1986; Tenishev et al. 2011)

\[
m_d a_d = \sigma_d \rho_g \frac{C_d}{2} (v_k - v_d)^2 - m_d \frac{G M_n}{r^2}.
\]

This equation assumes that the gravitational attraction and gas drag force are along a radial direction from the nucleus. This is generally the case except for local topographic features, for which we do not carry out an analysis in this paper. The definitions of the individual terms in this expression are as follows:

- \( m_d \): mass of a dust grain (kg)
- \( a_d \): dust acceleration (m s\(^{-2}\))
- \( \sigma_d \): cross section of a dust grain (m\(^2\))
- \( \rho_g \): mass density of the gas (kg m\(^{-3}\))
- \( C_d \): drag coefficient (constant with particle size, \( \simeq 2 \))
- \( v_k \): gas outflow velocity (m s\(^{-1}\))
- \( v_d \): dust velocity (m s\(^{-1}\))
- \( G \): gravitational constant (m\(^3\) kg\(^{-1}\) s\(^{-2}\))
- \( M_n \): mass of the nucleus (kg)
- \( r \): distance to the nucleus center (m)

The acceleration of a dust particle can then be written as

\[
a_d = \frac{C_d}{2} \frac{\sigma_d \rho_g}{m_d} (v_k - v_d)^2 - \frac{G M_n}{r^2}.
\]

The first term on the right is the gas drag, which accelerates the particles. The second term on the right is the gravitational deceleration by the nucleus. The acceleration due to the gas drag is thus directly proportional to the dust particle cross-sectional area and its gas drag coefficient and inversely proportional to its mass. The gas drag coefficient is determined by the shape of the particle, and an anonymous reviewer pointed out that the assumption of \( C_d = 2 \) for all particle sizes makes the acceleration of small particles less efficient. It has been known for some time, and amply demonstrated by the Rosetta mission, that the shape of the dust particles is quite complex (Rotundi et al. 2015; Langevin et al. 2016). We presently do not have a consensus as to what the best ratio for the cross section times the gas drag coefficient divided by the mass of a particle should be. The ratio will clearly be different for different types of particles, i.e., compact versus fluffy. If this ratio can be established, the proper number can be substituted in our equations. In the following discussion, we use a gas drag coefficient \( C_d \) constant with particle size, and adopt spherical particles having a radius, \( s \), and a density, \( \rho_d \), and we get

\[
\frac{C_d}{2} \frac{\sigma_d}{m_d} \frac{\rho_g}{s} = \frac{\pi}{3} \frac{s^2}{\rho_d} = \frac{0.75}{s \rho_d}.
\]

Furthermore, the product of the mass density of the gas times its velocity can be expressed in terms of the surface gas production rate as follows:

\[
\rho_g v_g = z_{gm} = z_{gn} \times MW \times m_u = z_{gn} \times MW \times 1.66 \times 10^{-27},
\]

where

- \( z_{gm} \): mass production rate of the gas at the surface (kg s\(^{-1}\))
- \( z_{gn} \): number density production at the surface (\# of molecules s\(^{-1}\))
- \( MW \): molecular weight of a gas molecule (e.g., 18 for H\(_2\)O, 28 for CO, and 44 for CO\(_2\))
- \( m_u \): atomic mass unit (1.660 \times 10^{-27} \text{ kg})

We thus obtain an equation for the acceleration of a dust particle due to gas drag:

\[
a_d(\text{gas drag}) = 0.75 \times MW \times 1.66 \times 10^{-27} \times \frac{(v_k - v_d)^2}{v_g} \frac{z_{gn}}{\rho_d s}.
\]

The acceleration of the particles from the surface is determined by the local gas production rate and the outflow gas velocity. The mean velocity of the sublimating molecules can be calculated from the temperature of the sublimating ice. The temperature of the ice is dominated by the equilibrium between the solar radiation and the energy used to sublime the ice and not by its blackbody equilibrium with respect to the solar radiation. As a consequence, neither the surface temperature of the exposed ice nor the velocity of the outflowing gas change greatly as the comet approaches the Sun. Combi (1989) gave gas dynamical calculations for comet 1P/Halley backed up by observed Doppler shifts from microwave observations that show that the typical H\(_2\)O gas outflow velocities near 1 au are \( \sim 1000 \text{ m s}^{-1} \) and decrease to \( \sim 750 \text{ m s}^{-1} \) near 2.5–3.0 au. These outflow speeds include the increase in gas velocity due to its adiabatic expansion into vacuum. Gulkis et al. (2015) reported microwave water vapor measurements of 67P at a
heliocentric distance of 3.5 au with an outflow speed of 680 m s⁻¹. For SW1, microwave gas outflow speeds of about 400 m s⁻¹ were reported by Womack et al. (2017) at a heliocentric distance of ≈6 au. We will therefore use a gas outflow velocity of 700 m s⁻¹ for 67P, which was also used by Tenishev et al. (2011). For SW1, we will use a gas outflow velocity of 500 m s⁻¹, slightly higher than the observed microwave gas velocities but probably more appropriate for velocities from strong outbursts. Compared to the large uncertainties of the local surface gas production rate, discussed below, the range of gas outflow velocities will not introduce major uncertainties in our calculations.

2.2. Dust Velocity for a Large Active Surface Area

In the case of uniform outgassing over a large area of the nucleus (with radius r₀), the gas areal density as a function of distance from the nucleus center can be written as Equation (6). The case of nonuniform outgassing is discussed in Section 3 and at the end of this paragraph:

\[ z_{gn}(r) = z_{gn}(r_0) \times \frac{r_0^2}{r^2}. \]  

(6)

In this case, both the gravitational attraction and the acceleration due to the outflowing gas fall off as \( \frac{1}{r^2} \). The acceleration of a dust particle from Equation (2), the gas drag from Equation (5), and the falloff of the gas areal density from Equation (7) then become

\[
\begin{align*}
a_d &= \left[ 0.75 \ z_{gn} (r_0) \times \frac{MW}{r_0^2} \times \frac{1}{s} \left( \frac{v_g - v_{d}}{v_g} \right)^2 - \frac{GM}{r_0^2} \right] \times \frac{r_0^2}{r^2} \\
&= k_{accel} \times \frac{r_0^2}{r^2}.
\end{align*}
\]

(7)

For most cases, this integration must be carried out numerically. However, it is useful to examine the cases where \( \frac{v_g - v_{d}}{v_g} \approx 0.1 \), since in this paper, we are interested in large particles that can present a hazard to the spacecraft. In this case, \( \frac{v_g - v_{d}}{v_g} \approx v_d \), and the above equation can then be written as

\[
\begin{align*}
a_d &= \left[ 0.75 \ z_{gn} (r_0) \times \frac{MW}{r_0^2} \times \frac{1}{s} \frac{v_g}{r_0^2} \right] \times \frac{r_0^2}{r^2} \\
&= k_{accel} \times \frac{r_0^2}{r^2}.
\end{align*}
\]

(8)

Thus, for particles whose velocity is considerably smaller than the gas velocity, the term in the square brackets only contains the gas velocity and is not dependent on \( v_{d} \); so that the equation can then be integrated analytically. The constant \( k_{accel} \) is the sum of the gas drag acceleration and the gravitational deceleration at the surface of the nucleus and is contained in the square brackets.

Using the basic equations of motion having the acceleration as a function of distance \( x \), we obtain

\[ v^2 = v_0^2 + 2 \ a \ x \ and \ 2 v \ dv = 2 a \ dx. \]

The integration of Equation (8) becomes

\[
\int_0^x v \ dv = \frac{v^2}{2} = \int_{r_0}^r a(x) \ dx
\]

\[
= \int_{r_0}^r k_{accel} \ \frac{r_0^2}{x^2} \ dx = -k_{accel} \ \frac{r_0^2}{r^2} \left( \frac{1}{r} - \frac{1}{r_0} \right)
\]

\[
= k_{accel} \ \frac{r_0^2}{r_0} - \frac{r_0^2}{r_0},
\]

\[
v^2 = 2k_{accel} \ \frac{r_0^2}{r_0} - \frac{r_0^2}{r_0} = 2k_{accel} \ r_0 \left( \frac{r - r_0}{r_0} \right).
\]

(9)

Thus, the particle reaches its terminal velocity within a few nuclear radii, where the gas and dust velocities decouple (in accord with DSMC calculations, as discussed below), and the terminal velocity is

\[ v(\text{terminal}) = \sqrt{2k_{accel}r_0}. \]

(10)

It is interesting to note that in the above case, any particle that can be lifted with \( k_{accel} \) positive will escape from the nucleus. This result is also implicit in the work of Zakharev et al. (2018) and mentioned in the paper by Ivanovski et al. (2017). For situations in which the gas drag falls off more rapidly than \( 1/r^2 \), as in outbursts or activity limited to distinct surface areas, where the gas can expand transversely as well as radially, particles that are lifted can escape, go into orbit, or attain ballistic trajectories. We discuss these cases in detail in Section 3. In practice, a strictly \( 1/r^2 \) falloff represents a special case, because even for a uniformly illuminated hemisphere, the gas production is concentrated near the subsolar point and, as is demonstrated in the figures by Tenishev et al. (2011), falls off faster than \( 1/r^2 \) at midlatitudes and polar regions.

2.3. Comparison with DSMC Calculations

A comprehensive dust analysis was carried out by Tenishev et al. (2011) for comet 67P in preparation for the Rosetta mission. The calculations assumed a spherical nucleus with a radius of 2000 m, a density of 300 kg m⁻³, and a nucleus mass of 1.0 × 10¹³ kg and used a constant gas drag coefficient \( C_d \) of 2.0. Calculations were performed for heliocentric distances of 1.29, 2.0, 2.7, and 3.25 au. The total gas production rate at 1.29 au was 5 × 10⁻²⁷ molecules s⁻¹. More calculations can also be found at the site “Inner Coma Environment Simulator” (ICES: http://ices.engin.umich.edu). We compare our calculations of both particle velocity versus particle size and dust velocity versus surface production rate with DSMC calculations in Figures 3 and 4 and show that our analytical calculations yield the correct functional relationship, as well as quite close numerical values for the resultant particle velocities within the assumptions and uncertainties of both models. Using the parameters for 67P above and Equation (8), the gas drag and gravity terms become

\[
\begin{align*}
a_d(\text{gas drag}) &= 1.57 \times 10^{-23} z_{gn}(r_0); \\
a_d(\text{gravity}) &= 1.64 \times 10^{-4} \ (\text{both in m/s}^2).
\end{align*}
\]

We first examine the relationship between dust velocity and particle size in Figure 3. It shows that the dust velocity approaches the terminal velocity at a distance of about five nuclear radii. The figure also shows the functional relationship of the dust velocity with the particle size \( v = k_{s} s^{-0.5} \) is closely
followed from $\sim 1 \mu m$ to $\sim 1$ cm. For small particles, the curve deviates from this law, since the dust velocity cannot exceed the gas velocity, which is $\sim 700$ m s$^{-1}$. A fit to the DSMC model data gives $v (\text{m s}^{-1}) = 0.142 \times s^{-0.5}$. Our analytical calculations using the parameters above yield $v (\text{m s}^{-1}) = 0.148 s^{-0.5}$.

We next turn to the relationship of the dust velocity with the surface production rate in Figure 4. We select DSMC calculations for comet 67P at heliocentric distances of 1.29, 2.0, 2.7, and 3.25 au, which use thermal modeling of the surface temperatures and DSMC dust and gas outflow computations (Tenishev et al. 2011). We show plots of the calculated dust particle velocities as a function of the subsolar surface production rate $z_{\text{gn}} (r_0)$. The blue points are taken from the model by Tenishev et al. (2011). A fit to the DSMC data yields the relationship $v_d = 7.0 \times 10^{-9} \times z_{\text{gn}}$. The red line represents our ab initio calculations, which result in the relationship of $v_d = 7.9 \times 10^{-9} \times z_{\text{gn}}$.

An interesting additional approximate relationship between the total gas production rate $Q(t)$ of 67P and its subsolar production rate for a uniformly outgassing hemisphere can be deduced from the Tenishev et al. (2011) DSMC calculations. The resulting approximate relationship, depending slightly on the comet’s heliocentric distance and modeled nighttime production, is

$$z_{\text{gn}} (r_0, \text{subsol}) \approx \frac{Q(t)}{\pi \times 0.67 \times r_0^2}.$$  \hspace{1cm} (12)

This relationship is not universal, and it has been shown by Zakharov et al. (2021) that the effects of heliocentric distance, the physical processes of gas production such as sublimation versus diffusion, and the roughness of the surface can modify this result. Nevertheless, considering the large number of undetermined observational parameters, for a reasonably uniformly outgassing spherical comet, this relationship can be used to estimate a surface gas production rate from the global gas production, since for almost all ground-based comet observations, only the total production rate can be measured.

2.4. Maximum Liftable Particle Size and Functional Relationship of Dust Velocity

An important parameter for dust dynamics is an estimate of the largest particle that can be lifted off the surface by gas drag. This size can be estimated from the above equations. Such a particle will have an acceleration and velocity at the surface of zero, so that the gas drag force and the surface gravity balance
each other:

\[ s_{\text{max}} = \frac{3}{8} \frac{C_d z_{\text{gas}} v_k r_0^2}{GM \rho_d} \]

\[ = 0.75 \times \text{MW} \times 1.66 \times 10^{-27} z_{\text{gas}} (r_0) v_k r_0^2 \]  

\]

\[ v_d \sim \sqrt{\frac{z_{\text{gas}} (r_0)}{\rho_d \frac{1}{s}}} . \]  

A similar relationship is given as Equation (18) in Ivanovski et al. (2017), but instead of the local surface gas production rate \( z_{\text{gas}}(r_0) \) uses the global gas production rate, \( Q(t) \). The latter may be a good rule of thumb for uniform outgassing but will fail for discrete active areas and localized outbursts. The relationship \( v \sim s^{-0.5} \) for the terminal velocity of the dust is implied in the work of Gombosi et al. (1986) and has been in fairly common use in the dust literature (e.g., Agarwal et al. 2016).

### 3. Dust Velocities and Trajectories from an Outburst

For an outburst from a limited area, a different set of relationships for the dust acceleration apply. In our model, the gas and dust production are concentrated in a small area, and the number of molecules/unit area of the expanding gas density \( z_{\text{gas}}(h) \) falls off with height above the surface, rather than with distance from the center of the nucleus. These relationships cannot be integrated analytically but must be integrated numerically. Because the activity is isolated, gas can expand transversely, so the gas drag force falls off quite rapidly with height above the surface. Thus, the dust acceleration is confined to a distance close to the surface. The total gas production rate, \( Q(t) \), which is the quantity that is commonly observed, now originates from a limited outburst area rather than from emission over an entire hemisphere, so that for equal observed \( Q(t) \), the local gas production rate per unit area is much higher, and larger particles can be lifted. The diagram given in Figure 5 illustrates this situation.

We assume the outburst produces a total gas production rate of \( Q \) (molecules s\(^{-1}\)) and is confined to an area of diameter \( d_0 \). The surface number density gas production then becomes

\[ z_{\text{gas}}(r_0) = \frac{Q(t)}{dA} = \frac{Q(t)}{\pi d_0^2} . \]  

\]

Figure 4. Dust velocity versus surface gas production rate at the subsolar point for 1.0 μm particles using DSMC calculations and our analytical approach. Blue dots are calculations using DSMC modeling (Tenishev et al. 2011) taken from their Figure 1 for the subsolar surface gas production and their Figure 10 for the dust terminal velocity. The red curve is our analytical calculation. It can be seen that our ab initio calculations and the DSMC model within the assumptions inherent in the two methods are essentially the same.
As mentioned in the introduction for the description of Figure 1, gas and dust close to the nucleus are typically concentrated in a cone angle of roughly 1 sr or $\sim 60^\circ$. We assume that within reasonable distances from the nucleus, no particles leave the cone, so that the mass in the cone is conserved. To simplify our calculations, we will use a half-angle for the cone of 26°, as indicated in our diagram (Figure 5). Other angles for an outburst cone could also be used, but to limit the number of parameters explored, we do not do this here. This results in an expression for the gas density per unit area at height $h$ above the surface:

$$q = z_{\text{gn}}(r_0) \times \frac{d_0^2}{(2 \tan \theta + d_0)^2} \approx \frac{d_0^2}{(h + d_0)^2}. \quad (16)$$

Using Equation (7) and substituting Equation (16), the areal gas density falls off with height, and the acceleration with altitude, $h$, above the surface becomes

$$a_d(h) = 0.75 \times \text{MW} \times 1.66 \times 10^{-27} \times (v_g - v_d)^2 \times \frac{z_{\text{gn}}(r_0)}{v_g} \frac{d_0^2}{\rho_d s} \frac{(h + d_0)^2}{(h + r_e)^2} - \frac{G M_n}{(h + r_e)^2}. \quad (17)$$

The velocity at each height is determined by numerical integration of

$$v \ dv = a \ dh \ \text{or} \ \ v^2 = v_0^2 + 2 a \Delta h. \quad (18)$$

### 3.1. Trajectories and Velocities for Various Outburst Scenarios

Calculations of trajectories and velocities of dust particles that are ejected from the comet’s surface during outbursts are necessary to make estimates of the maximum size of the lifted, ballistic, orbiting, and escaping particles, as well as their escape velocities. Presently, we do not know the exact cause or mechanism of the outbursts observed for SW1. One common explanation is the energy released when ice changes from amorphous to the crystalline state. Other possible mechanisms are listed and discussed in Miles et al. (2016). They also proposed a somewhat complex mechanism in which cometary ices such as CH$_4$ are confined under pressure beneath the crust, begin to melt, and absorb gaseous molecules such as CO and N$_2$, which can then liberate a significant amount of heat, causing an outburst. Expansion velocities observed from dust images, which measure the outer edge of the expanding dust coma and thus are most likely associated with the smaller, faster-moving particles, are 100–500 (Richter 1941) and $\sim 260$ (Miles et al. 2016) m s$^{-1}$. This agrees well with our calculated dust velocities for such particles, carried out below (see Figure 7).

These velocities are much lower than typical explosive ballistic events of $\geq 1000$ m s$^{-1}$. Thus, it does not seem probable that dust outbursts are produced by near-instantaneous explosive events over a short time and limited area. The considerable amount of dust ejected and the observed expansion velocities indicate that they are likely produced over some time period, originating from a sizable area, and propelled by sublimating...
gas. For example, measurements of the rise time of a 2 mag dust outburst on SW1 were consistent with an active period of several hours (Miles et al. 2016). We therefore follow the assumption of a sustained outflow in our analysis using a gas velocity of \( \sim 500 \text{ m s}^{-1} \), discussed in Section 2.1.

We will use the measured global gas production rate to estimate local surface gas production rates \( \dot{Q}(r) \) of \( \sim 1 \times 10^{29} \text{ molecules s}^{-1} \) during an outburst and roughly 1–2 \( \times 10^{28} \) molecules s\(^{-1}\) during the quiescent period. Since we do not presently know the size of the outburst area, the local surface gas production rates are difficult to determine.

Observations of 67P by Rosetta have shown that the small outbursts arise from sources tens of meters in size (Vincent et al. 2016). On the other hand, SW1 routinely exhibits much larger events, ejecting massive amounts of dust that are easily seen from Earth. With an assumed radius of 30 km (instead of an equivalent 67P radius of 1.65 km), SW1 has a size about 18 times larger and a surface area about 330 times larger. Thus, we assume that the SW1 active areas are proportionally larger than those on 67P. Because we have no severe constraints on their actual dimensions, we adopt sources 100 m, 1 km, and 10 km across to explore the conditions over a reasonable range of possible sizes (cf. Table 2). A 1 \( \times \) 1 km outburst area represents only 0.018% of the sunlit hemisphere of SW1, and a 10 \( \times \) 10 km area represents about 1.8% of the sunlit hemisphere. Using the measured total gas production rate results in surface gas production rates of 1 \( \times \) 10\(^{25} \), 1 \( \times \) 10\(^{27} \), and 1 \( \times \) 10\(^{21} \) molecules m\(^{-2}\) s\(^{-1}\), respectively, for the outburst case and 1 \( \times \) 10\(^{22} \) molecules m\(^{-2}\) s\(^{-1}\) for a 1 \( \times \) 1 km area of the quiescent case. Such high surface production rates cannot be applicable to the whole sunlit surface, since they would result in unrealistically high global gas production rates. We also note that, unlike the sustained steady-state case, an outburst must utilize some reservoir of stored gas or energy, since the latent heat of sublimation for CO is 50.4 Calorie g\(^{-1}\) or 1 \( \times \) 10\(^{-20} \) J molecule\(^{-1}\) (Handbook of Chemistry and Physics), and the solar irradiation at \( \sim 6 \) au is about 38 watts m\(^{-2}\), which would only suffice to sublimate maximally \( \sim 4 \times 10^{21} \) molecules s\(^{-1}\). This places a rough limit on the active area during the steady-state activity that must be at least several square kilometers to yield the observed quiescent production rate of 1–2 \( \times \) 10\(^{28} \) molecules s\(^{-1}\).

To determine the trajectories and velocities of various dust particles, we have developed a computer program that integrates Equations (17) and (18), and we show examples of our calculations in Figure 6 and Table 2. The calculations for Figure 6 used the parameters listed in Table 1.

We show in Figure 6 the trajectories of large particles under the above conditions to determine which particles can escape, reach ballistic orbits, and go into orbit. All calculations are performed for particles whose velocity vector is radially outward from the nucleus center. In addition to the particle velocities, we also show the escape velocity as a function of height above the surface as a red line and the minimum orbital velocity as a blue line. As can be seen in Figure 6, particles of radius 6 and 7.5 mm or smaller will escape from the comet. However, larger particles, \( \sim 7.6–11 \) mm, do not reach escape velocity but exceed the minimum orbital velocity. Under our assumption of strictly radial outflow, these particles would simply fall back to the surface as shown. However, given that the gas in the outburst scenario includes a transverse component, it is likely that particles in this size range will have sufficient tangential velocity for their return trajectories to “miss” the nucleus, causing them to go into orbit. Particles 11.4 and 14 mm in this example never reach orbital velocity and will simply fall back to the surface after peaking at \( \sim 60 \) km. Thus, for a given set of outburst conditions, we can expect that there will be a small range of particle sizes that have the potential to end up orbiting the nucleus. Although these are likely to be unstable orbits, they do suggest that there is a population of large grains that can reside in the coma for extended periods of time. For objects that experience frequent outbursts, like SW1, the injection rate of new debris may approach or exceed the loss rate from unstable orbits, resulting in a much more complex and hazardous environment than produced by isolated single events. In this paper, we are mostly concerned with hazards from particles that escape and form the extended coma; thus, we will not examine in detail ballistic particles or particles that potentially go into orbit.

In Table 2, we give a summary of our calculations for the outburst areas of 100 \( \times \) 100 m, 1 \( \times \) 1 km, and 10 \( \times \) 10 km. The outburst area of 100 \( \times \) 100 m yields very high surface gas production rates resulting in a maximum liftable boulder size fragment of 23.4 m, which probably is not realistic. We therefore concentrate on the larger outburst areas of 1 \( \times \) 1 and 10 \( \times \) 10 km. For the outburst area of 1 \( \times \) 1 km, illustrated in Figure 6, the maximum liftable particle size is 233 mm, and particles of size range 233–11.5 mm will enter ballistic orbits, with the 11.5 mm particle reaching a peak velocity of 14 m s\(^{-1}\) and a maximum altitude of \( \sim 60 \) km. It is therefore prudent for a spacecraft to avoid this inner region of the comet during an outburst. Particles that reach orbital velocities have a narrow size range from 11.0 to 7.6 mm and are thus expected to be small in number, though they may accumulate due to repetitive events. Particles smaller than \( \sim 7.6 \) mm will escape and can form a danger to any flyby spacecraft. For an outburst area of 10 \( \times \) 10 km, the maximum liftable particle size is 2.33 mm. Particles that can orbit have an even narrower size range centered around \( \sim 0.775 \) mm and are thus expected to be quite rare. Particles smaller than \( \sim 0.77 \) mm escape to form the extended coma of the comet, and particles larger than 0.78 mm only reach ballistic orbits. This scenario approaches that of a uniformly outgassing hemisphere, discussed in Section 2.2, and for which we have shown that essentially all liftable particles escape.

We use our numerical integration to calculate the velocities of the escaping particles as a function of size for assumed active areas of 1 \( \times \) 1 and 10 \( \times \) 10 km and show these velocities in Figure 7. Over a limited size range, just as in the case of a large uniform outgassing area discussed in Sections 2.3 and 2.4, these velocities follow the relationship \( v = k_p s^{-0.5} \). Actual fits to our numerical integrations yield \( v = 1.40 \) and 0.45 s\(^{-0.5}\) for the outburst areas of 1 \( \times \) 1 and 10 \( \times \) 10 km, respectively. These velocity relationships will be used to calculate the particle densities as a function of particle size and distance from the nucleus in the next section. Since the particle velocities cannot exceed the gas velocity, small micron-sized particles reach an asymptote of our assumed gas velocity of 500 m s\(^{-1}\). Large particles, on the other hand, do not experience sufficient gas drag acceleration compared to their gravitational attraction.
Figure 6. Outburst trajectory calculations of large particles for SW1 using an active area of 1 × 1 km and conditions as explained in the text. The gas drag acceleration occurs within ∼1 km, after which the nucleus gravity dominates the motion. The escape velocity as a function of distance from the comet is shown as a red line, and the orbital velocity limit is shown as blue. The top two particles of size 6 and 7.5 mm escape and show slowing velocity with distance due to the retarding gravitational attraction. The particles of 7.8 and 10 mm do not reach escape velocity. In this plot, with no tangential velocity, they fall back to the surface. Given a small amount of tangential velocity, these particles have the potential to go into orbit around the nucleus. The large particles of 11.5 and 14 mm will only reach a ballistic trajectory with a maximum height of ∼60 km before falling back to the surface.
In the previous section, we introduced the concept of the velocity distribution of dust particles in the coma of a comet and discussed the importance of the velocity distribution for understanding the dust dynamics. We have now arrived at an expression for the velocity distribution as a function of particle size, which we can use to analyze the dust production and mass loss rate of comets.

### 4. Estimate of Coma Dust Particle Density

#### 4.1. Basic Coma Dust Environment Concepts

We require the outflowing particle density \( n(r, s, v) \), (\#/m\(^3\)), at a distance, \( r \), from the center of the nucleus of particles of size, \( s \), having left the comet with terminal velocity, \( v \). To calculate this, we need (1) the source function \( Q(s, t) \) of the number of dust particles of size \( s \) released per unit of time from the surface of a comet, (2) their outflow velocity as a function of size, and (3) the outburst expansion volume or its outburst cone:

\[
n(r, s, v) = \frac{Q(s, t)}{4\pi r^2 v} \text{or} \frac{Q(s, t)}{(s/r)r^2 v}.
\]  

(19)

The number of dust particles of a given size \( s \) is given by the particle size distribution, which generally is expressed by an exponential size distribution given by

\[ Q(s, t) = k_a \ s^{-\alpha}. \]

(20)

In practice, however, only a differential particle size distribution is available, i.e., the number of particles \( dQ(s, t) \) of size \( s \) in each particle size bin \( ds \) ejected from the nucleus:

\[ dQ(s, t) = k_n \ (\alpha) \ s^{-(\alpha+1)} ds. \]

(21)

The differential particle size distribution can usually be measured only over a limited particle size range between a minimum and maximum size, which is then assumed to hold for the whole size range. Unfortunately, there is presently no observational or experimental method to determine the particle size distribution over the complete range of emitted particles. Thus, an extrapolation from a restricted particle size range to a comprehensive particle size distribution can lead to considerable errors for the total number of particles and the total amount of dust mass emitted.

In addition to the distribution of the particle sizes as expressed by the parameter \( \alpha \), we need the important parameter \( k_a \), which governs the total number of particles released. This parameter is difficult to assess, since it governs the total amount of dust in the coma. We will estimate this constant from the observed gas outflow mass-loss rate, which can be measured with good reliability, and the dust-to-gas mass ratio, which has been determined with reasonable accuracy by the Rosetta mission for comet 67P. The relationship of the velocity as a function of particle size was developed in the previous section and is shown in Figure 7 and summarized in Table 2. For the dust expansion volume, ground-based studies of the dust coma generally show a reasonably isotropic dust coma at distances of tens of thousands of kilometers from the nucleus and thus a dust expansion into \( 4\pi \) sr. However, we are concerned with outbursts in the inner coma where the gas and dust outflow is quite concentrated, as shown in Figures 1 and 2. A rough estimate of the dust outflow from those images shows that the dust is typically concentrated in a cone of about 1 sr. This is equivalent to our ~53° dust and gas expansion angle for our outburst velocity calculation in the previous section. In the following analysis, we will therefore use ~1 sr as the expansion volume, as is expressed in the second term of Equation (19).

#### 4.2. Dust Mass-loss Rate

As a measure of the amount of dust in the coma of a comet relative to its gas production rate, the quantity \( Af_\rho \) is generally used. It is, however, difficult to relate \( Af_\rho \) to the dust number density or the dust mass released from the comet. Its original definition can be found in A’Hearn et al. (1984), and a more elaborate analysis relating \( Af_\rho \) to the light-scattering properties of the dust in can be found in Fink & Rubin (2012). Because of the complex nature of the light-scattering process, which depends on factors such as the particle size, particle size distribution, complex index of refraction, somewhat uncertain particle shape, density of the dust particles, and scattering phase function, the conversion of an observed \( Af_\rho \) measurement into a dust mass-loss rate or particle ejection rate is associated with large uncertainties that can easily reach a factor of 10. The ratio of \( Af_\rho \) to the production rate of gas varies widely from comet to comet (A’Hearn et al. 1995; Fink & Hicks 1996; Fink 2009; Cochran et al. 2012). Whether this variation is caused mainly by different dust particle scattering properties, such as shape, porosity, density, or size range and distribution, or whether different comets possess inherently different mass dust/gas ratios is presently not known.

For the amount of dust mass ejected, we will therefore use the measured dust/gas mass ratio. While this quantity also has uncertainty, it has been reasonably well constrained for comet 67P from the results of the Rosetta mission. For this comet, the radio science experiment was able to measure a total mass loss during its perihelion passage of 10.5 × 10\(^9\) kg (Pätzold et al. 2019). A comprehensive analysis of the total amount of volatile species lost during 2 yr of the comet’s perihelion passage using the large data set from the Rosina mass spectrometer by Hansen et al. (2016) and Combi et al. (2020) yielded a total volatile loss of 6.1 × 10\(^9\) kg. This results in a refractory/volatile mass ratio of 0.67. A similar analysis by Biver et al. (2019) using the microwave water measurements yielded a ratio of 1.5 ± 0.8, and an analysis by Marschall et al. (2020) gave a value of 0.73 (+1.3, −0.7) for the escaping material. It thus appears that a dust/gas mass ratio of ~1.0 is a reasonable number for comet 67P. This number is considerably smaller than the dust/gas mass ratio estimated at ≥3 by Fulle et al. (2016, 2019). We note, however, that those authors attempted to obtain the dust/gas mass ratio from a limited particle size range and a considerable number of assumptions on the scattering properties of the dust, such as its albedo, density, and extrapolation to the complete particle size range, all of which can give rise to quite large uncertainties.

| Table 1 Parameters Used for SW1 Outburst Trajectories and Velocity Calculations |
|---------------------------------|---------------------------------|
| Outburst area                   | 1 x 1 km                        |
| Mass of nucleus                 | 1.0 × 10\(^{12}\) kg            |
| Radius of nucleus               | 30,000 m                        |
| \( Q \)                         | 1.0 × 10\(^{29}\) molecules s\(^{-1}\) |
| \( \rho_\Delta \)               | 1000 kg m\(^{-3}\)              |
| \( v_\Delta \)                  | 500 m s\(^{-1}\)                |
| \( \Delta h \) (altitude interval) | 10 m                            |
| Number of altitude intervals    | 2,000,000                       |
The major observed outgassing constituent for SW1 is CO, and a good measure of the peak outburst gas production rate of \( \sim 1.0 \times 10^{29} \) CO gas molecules s\(^{-1}\) equivalent to about 5000 kg s\(^{-1}\) has been determined by Wierczchos & Womack (2020). Following our discussion above and using a dust/gas mass ratio of 1 results in a dust mass-loss rate of \( \sim 5000 \) kg s\(^{-1}\). For the quiescent or steady-state case of SW1, Wierczhos & Womack (2020) reported a CO production rate of \( \sim 1.0 \times 10^{28} \) CO molecules s\(^{-1}\), resulting in a mass-loss rate of 500 kg s\(^{-1}\). This number is comparable to the modeled SW1 dust loss range of 300–900 kg s\(^{-1}\) obtained from dust tail modeling by Moreno (2009). Whether the number for the dust/gas mass ratio of 1 determined from 67P is also applicable to other comets is difficult to judge but is presently the best estimate we have. The dust/gas mass ratio could easily vary between about 0.5 and 3, which will translate into a similar uncertainty in our estimated particle density calculations for SW1. Our final dust number densities can readily be scaled to different dust/gas mass ratios when improved data become available.

### 4.3. Dust Particle Number Density from Mass-loss Rate

We will use a canonical power-law distribution with an exponent of \( \alpha = 3 \) (Tenishev et al. 2011; Fulle et al. 2016, 2019) in our sample calculation below, but we will also explore exponents of \( \alpha = 2.5 \) and 3.5. A summary of the results for these latter two equations will be given in Table 3. The constant \( k_n \) is determined by making the total mass-loss rate of the escaping dust particles 5000 kg s\(^{-1}\) and using a maximum escaping particle size of 7.8 mm and a minimum estimated particle size of 1 \( \mu \)m. For the steady-state quiescent case, we will use a mass-loss rate of 500 kg s\(^{-1}\) and a maximum size for the escaping particles of 0.78 mm. Densities are not well constrained but are typically taken between 200 and 2000. We assume that the particles are spherical with a density, \( \rho \), of 1000 kg m\(^{-3}\):

\[
Q(s, t) = k_n s^{-3}; \quad dQ(s, t)/ds = -3 k_n s^{-4.0}.
\] (22)

The mass-loss rate for this particle size distribution is then

\[
\frac{dm}{dt}(s) = -3 k_n s^{-4} \cdot \frac{4}{3} \pi s^3 \rho ds = -3 k_n \frac{4}{3} \pi \rho \frac{1}{s} ds.
\] (23)

The total mass-loss rate for a range of particles is

\[
\frac{dM}{dt} = M = -3 k_n \frac{4}{3} \pi \rho \int_{s1}^{s2} ds s
\]

\[
= 3 k_n \frac{4}{3} \pi \rho (\ln s_2 - \ln s_1).
\] (24)

Using \( s_1 = 1 \times 10^{-06} \) m, \( s_2 = 7.8 \times 10^{-03} \) m, a total mass-loss rate of 5000 kg s\(^{-1}\), and a particle density \( \rho \) of 1000 kg m\(^{-3}\), we get

\[
-5000 = \frac{dM}{dt} = -3 \times k_n \times 4189 (-4.854 - (-13.8155))
\]

\[
= -3 \times k_n \times 4189 \times 8.96,
\] (25)

and \( k_n \) is 0.044.

The differential particle number density (\#/m\(^3\)) of particles of size \( s \) in size bin \( ds \) at a distance \( r \) from the nucleus using our previously determined velocity relationship \( v = kv_s s^{-0.5} \) then becomes

\[
\frac{dn(s, t)}{ds} = \frac{dQ(s, t)}{r^2 v} = -\frac{3}{r^2} k_n s^{-4} \frac{1}{v} ds
\]

\[
= -\frac{3}{r^2} k_n \frac{1}{k_v} s^{-3.5} ds.
\] (26)

The total particle number density (\#/m\(^3\)) within a given size range is

\[
N(s_1 \rightarrow s_2) = -\frac{3}{r^2} \frac{k_n}{k_v} \frac{1}{r^2} \int_{s_1}^{s_2} s^{-3.5} ds
\]

\[
= -\frac{3}{r^2} \frac{k_n}{k_v} \frac{1}{r^2} \left( \frac{1}{(s_2)^{2.5}} - \frac{1}{(s_1)^{2.5}} \right).
\] (27)
The numerical expression for the particle number density for the case considered above with $\alpha = 3$, $k_n = 0.044$, and $k_v = 1.40$ then becomes

$$N(s1 \rightarrow s2) = 0.0377 \frac{1}{r^2} \left( \frac{1}{(s_2)^{2.5}} - \frac{1}{(s_1)^{2.5}} \right). \quad (28)$$

Rather than elaborate the computational cases for $\alpha = 2.5$ and 3.5, we list the derived constant $k_n$ and the expressions for the integrated particle number densities in Table 3. We note that the mass loss using $\alpha = 2.5$ is strongly concentrated at the largest particle sizes. Thus, the largest particles that can escape into the coma have a strong influence on the evaluation of $k_n$. We also give in this table our values for the steady-state quiescent case with $Q = 1 \times 10^{28}$ CO molecules s$^{-1}$ and an active area of $1 \times 1$ km.

In Figure 8, we give numerical values for the estimated dust particle densities. This figure uses an outburst area of $1 \times 1$ km and expansion of the dust and gas into a cone of 1 sr. Because of the large range of particle sizes involved, the figure employs logarithmic particle size intervals with six intervals per decade or size interval ratios $(s_2/s_1)$ of 1.468. Each plotted point represents the total number of particles per square meter within the interval from the smaller to the next larger size. For the case of $\alpha = 3.0$, each of our 24 logarithmic intervals from 1 $\mu$m to 10 mm has a mass loss of 214 kg s$^{-1}$ for a total of 5140 kg s$^{-1}$. For the particle range from 14.7 $\mu$m to $\sim$2.0 mm, we used the dust velocity versus size relationship of $v = 1.40 s^{-0.5}$ as shown in Figure 7. For particles smaller than $\sim$10 $\mu$m, we used a constant velocity of 490 m s$^{-1}$. For the largest size that can escape into the coma, we used terminal velocity values calculated in our numerical outburst model (see Figures 6 and 7). Thus, the number densities for the largest particles turn up slightly in our plot because their velocities are slower.

Because the largest particles pose the most severe hazard to the spacecraft, we also list in Table 3 the sum of the particle number densities at a distance of 1000 km from the nucleus for the size ranges 0.1–1.0 and 1.0–10 mm. Because of the steep particle size distribution, these sums are strongly dominated by the smallest particles within the sum. It can be seen that, for the cases considered, the calculated number density sum from 0.1 to 1.0 mm is roughly equal and $\sim$1–3 $\times$ 10$^{-4}$ particles m$^{-3}$. For 1 mm particles, the steep particle size distribution of $\alpha = 3.5$
gives roughly a factor of 10 lower particle density. Both the table and the figure show that the particle density for 1 mm particles is expected to be about a factor of 100 lower than that for 0.10 mm particles.

5. Spacecraft Hazard Assessment

The dust environment poses a hazard to a spacecraft due to the potential damage that could be caused in a collision. In evaluating this hazard, we consider two primary properties that factor into the risk analysis: the grain’s size and its relative velocity. Large grains and high speeds will inflict more damage than small grains and low speeds, so we are interested in exploring the distribution of large grains and their characteristic velocities throughout the coma. We use SW1 as an example for this hazard assessment because this comet exhibits recurring strong outbursts that are well documented. For other comets, this exercise will have to be modified depending on the nature of the comet’s activity.

First, we consider the largest grains that can be lifted off the surface. For the case of a severe outburst confined to a small active area, larger boulders could be lifted off the surface, but they are limited to low-altitude ballistic trajectories with quite short lifetimes (Table 2). For the case of an outburst area of 1 × 1 km on SW1, particles up to ~23 cm can be lifted off the surface, but they will follow ballistic trajectories and fall back onto the nucleus (cf. Figure 6). Thus, a spacecraft at SW1 can minimize the hazard from these largest particles by avoiding the region within ~50 km of the surface.

Dust that orbits the nucleus poses more of a hazard, though still somewhat limited. As shown in Section 3.1, particles that achieve orbit must fall within a narrow range of properties (velocity just slightly below the escape velocity, apex at a characteristic altitude, a slight tangential velocity, etc.), which suggests that there will be relatively few of these particles. They will also be smaller than the ballistic grains (7.6–11.0 mm for the above case), but they have the potential for significantly longer lifetimes in the coma and can accumulate if multiple outburst events occur. As with the ballistic grains, a spacecraft can avoid the orbital region (altitudes up to a few hundred kilometers) to minimize the risk of encountering these grains. Furthermore, orbital velocities are low in this region (m s⁻¹ range), so if an orbiting spacecraft ventured into this region, it would be expected that any collisions with orbiting grains would be low-speed events with ΔV < 10 m s⁻¹.

Our final consideration is the hazard to a spacecraft posed by the largest particles that can escape into the coma. These particles tend to have very low escape speeds; thus, in any collision, it is the velocity of the spacecraft that determines the hazard posed rather than the particle velocity. In Table 4, we present a brief calculation of the severity of any hazards encountered by a spacecraft. Assuming a dust cloud ejection into a cone of 60° for particle sizes escaping into the coma as calculated in Section 3, we find for the quiescent or steady state that the largest particles that can escape are of the order of 1 mm in radius, while for the outburst case, they are of the order of 1 cm (cf. Table 2). Given the escape speeds of ~1 m s⁻¹, the kinetic energy in a collision would not pose a threat to a slowly approaching or orbiting spacecraft.

Even though a spacecraft faces little risk from collision while in orbit, there is a hazard in transitioning from interplanetary cruise to its orbital motion. The spacecraft will need to undergo substantial braking as it nears the comet target (as exemplified for the Rosetta approach, discussed in Section 5.1). A typical approach to this transition is to reduce speed in stages, balancing the transit time to the comet with the increasing particle density closer to the nucleus. An example of this staged approach would have the spacecraft slowing from its interplanetary travel speed of ~5 km s⁻¹ to ~500 m s⁻¹ at a distance of 10⁶ km, ~100 m s⁻¹ at 10⁷ km, and ~10 m s⁻¹ at 10⁸ km from the target comet. For this final stage, the kinetic energy of any impacting particle is reduced by a factor of 250,000, and, while not negligible, the danger of any impacts can be mitigated by proper spacecraft design, such as incorporating redundancy and adding a modest amount of shielding for any vital parts.

It will additionally be prudent to follow a gradual approach to the comet with a series of successively smaller reconnaissance or assessment orbits. For example, with the present estimated nucleus mass for SW1 of 1 × 10¹⁹ kg, the period of an assessment orbit at 1000 km is ~28 days, and the spacecraft orbital velocity is 2.6 m s⁻¹. At 250 km, the period is ~3.5 days, and the orbital velocity is about 5.2 m s⁻¹. Such assessment orbits would allow a fairly thorough reconnaissance by the spacecraft’s suite of cameras of near-comet large particle debris and development of a mission strategy that will minimize any possible hazards.

For a flyby spacecraft with a speed of ~5 km s⁻¹ or higher, escaping grains of 1–10 mm would pose a considerable hazard. The kinetic energy of a collision with a 1 mm particle is roughly the same as being hit by a high-velocity 0.22 caliber rifle bullet having a muzzle velocity of 380 m s⁻¹. Using our
canonical particle size distribution of $n(s) \sim s^{-3}$, we estimate particle densities at 1000 km from the nucleus from Figure 8 to be $\sim 10^{-6}$ m$^{-3}$ for 1 mm particles and $\sim 10^{-8}$ m$^{-3}$ for 1 cm particles. The frequency of being hit by such a particle at a flyby distance of 1000 km and thus flying through a relatively high particle density path length of $\geqslant1000$ km for an outburst concentrated in roughly 1 sr is about 1–2 hits m$^{-2}$ of spacecraft cross-sectional area. Additionally, even particles as small as 10 $\mu$m could pose a danger to high-speed flyby spacecraft, since their number density is $\sim 10^6$ times higher.

Assuming that a spacecraft with solar cells has a cross section of 100 m$^2$ and flies through a path of 1000 km, it would suffer about 100 hits from $\sim$1 mm particles and have a roughly even chance of getting hit by a 1 cm particle. Such a collision would have a kinetic energy of about 35 times that of a 0.22 caliber bullet and could be catastrophic. Since the fly-through path length at a distance, $r$, to the comet increases by $r$ but the particle density falls off as $1/r^2$, the probability of getting hit decreases by $1/r$. In order to reduce the chances of being hit by a 1 mm particle to $\sim$1.0 (or the chances of a hit by a 1 cm particle to 0.01), the flyby distance would have to increase to $10^5$ km or larger, which would severely impact the investigative capabilities of the onboard instruments.

Figure 8. Estimated particle number density vs. particle size at 1000 km from the nucleus center for SW1 for an outburst of area $1 \times 1$ km and three different particle size distributions. The total dust mass-loss rate is estimated to be 5000 kg s$^{-1}$. For large particles, the curves turn up slightly because of their slower velocities. Particles smaller than 10 $\mu$m have a velocity of 490 m s$^{-1}$ because they cannot exceed the gas velocity of 500 m s$^{-1}$ used in our calculations. A curve for the steady-state continuous activity where the maximum escaping particle is 0.78 mm is also shown (see also Tables 3 and 4).

An important consideration is that the threat posed by an escaping particle impact is strongly dependent on the timing of an ejection event. Unlike the debris that accumulates into temporary orbits, these particles escape the inner coma on timescales of less than a day. While the quiescent background particle distribution is continually replenished, a stochastic injection of debris from an outburst will increase both the density and size distribution of the escaping material. For an object like SW1 that experiences multiple outbursts per year releasing many times the background production rate, a safe flyby encounter will require careful planning to avoid this.

5.1. Comparison with Comet Missions

Any hazard assessment also needs to avail itself of past spacecraft mission experience with objects with potential debris clouds. For this purpose, the mission with the most detailed data is probably Rosetta, which had extended time periods of orbits closer than 100 km from its target object, 67P. Using the methodology developed in the previous sections, we carry out a brief analysis of large fragments that can be lifted off the surface and escape into the coma. We then compare them to Rosetta observations of large dust fragments observed in the
Table 4
Spacecraft Hazard Assessment

| Particle | Estimated S/C flyby | 5000 m s⁻¹ at 1000 km | Particle Number Density |
|----------|---------------------|-----------------------|------------------------|
|          | Radius (mm) | Mass (kg) | velocity (m s⁻¹) | Quiescent (#/m³) | Outburst (#/m³) | Quiescent (#/m²) | Outburst (#/m²) |
| 0.1      | 1.49 x 10⁻⁹ | 5000 | 0.05 | 1.0 x 10⁻⁴ | 1.0 x 10⁻⁴ | 200 | 200 |
| 0.5      | 5.24 x 10⁻⁷ | 5000 | 6.54 | 3.0 x 10⁻⁶ | 5.0 x 10⁻⁶ | 6 | 10 |
| 1        | 4.19 x 10⁻⁶ | 5000 | 52.4 | 1.0 x 10⁻⁶ | 6.0 x 10⁻⁷ | 2 | 1.2 |
| 5        | 5.24 x 10⁻⁴ | 5000 | 6545 | ... | 3.0 x 10⁻⁸ | 0.06 |
| 10       | 4.19 x 10⁻³ | 5000 | 52,360 | ... | 1.0 x 10⁻⁸ | 0.02 |

Bullet 0.22 Caliber

2.8 0.002 5 380 180.5

Table 5
Estimates of Maximum Liftable and Escaping Particle Sizes for the Rosetta Encounter with 67P

| Distance from Sun (au) | −3.9 | −3.5 | −2.0 | −1.5 | 1.24 | 1.5 | 2.0 |
|-----------------------|------|------|------|------|------|-----|-----|
| Total gas production (molecules s⁻¹) | ≈1 x 10²⁵ | 4.0 x 10²⁵ | 5.0 x 10²⁶ | 2.5 x 10²⁷ | 2.0 x 10²⁸ | 4.0 x 10²⁷ | 1.0 x 10²⁸ |
| Avg. surface gas production $z_{gm}$ (molecules s⁻¹ m⁻²) | 1.8 x 10¹⁸ | 7.0 x 10¹⁸ | 8.7 x 10¹⁹ | 4.4 x 10²⁰ | 3.5 x 10²¹ | 7.0 x 10²⁰ | 1.75 x 10²² |
| Maximum liftable particle size (cm) | 0.02 | 0.08 | 1 | 5.2 | 42 | 8.4 | 2.1 |
| Maximum escaping particle size (cm) at 0.6 m s⁻¹ | 0.016 | 0.066 | 0.08 | 4.1 | 33 | 6.6 | 1.6 |

com. We note, however, that this is not the main purpose of this paper and so represents only a brief summary, leaving a detailed analysis to a possible future investigation.

Our calculations are summarized in Table 5. The total gas production for different heliocentric distances is taken from the detailed analyses of the gas production over the whole mission by Hansen et al. (2016) and Combi et al. (2020). From this total gas production, we estimate the subsolar or maximum surface gas production, $z_{gm}$, using our formulation in Section 2.3 and the maximum liftable size using our equation given in Section 2.4. The parameters used for this calculation are as follows: nucleus mass $1 \times 10^{11}$ kg, average or equivalent radius 1650 m, density of large fragments the same as the nucleus or 535 kg m⁻³ (Pätzold et al. 2019), and outflow velocity of the gas 700 m s⁻¹ as measured by the MIRO instrument (Gulkis et al. 2015). This paper also reported the first gas detection in the coma of 67P as $\sim 1 \times 10^{25}$ molecules s⁻¹ in early 2014 June when the comet was 3.92 au from the Sun. The maximum gas production occurred at perihelion at 1.24 au and is $2-3 \times 10^{28}$ molecules s⁻¹. Using our equations developed in Section 2.2, we also calculate the size of fragments that could reach a terminal escape velocity of $\sim 0.6$ m s⁻¹, neglecting for this simple calculation the centrifugal rotational force of the nucleus, which could be up to $\sim 0.2$ m s⁻¹, thus allowing larger fragments to escape or go into orbit. The maximum liftable and escaping particle sizes are very close, since, as discussed in Section 2.2, for a uniform outgassing model, any particle that can be lifted essentially escapes.

During the approach phase of Rosetta ($r_h = 3.7$ au), Bertini et al. (2015) searched for large fragments orbiting the nucleus that yielded upper limits of the order of 1 m for distances between 20 and 110 km from the nucleus, showing that such large fragments were not lofted into orbit. With Rosetta closing in fast, Davidsson et al. (2015), at 28 km from the nucleus and $r_h = 3.4$ au, obtained orbits for four particles with estimated sizes of 15–50 cm. Our table shows that such large fragments could not be lifted at 3.5 au. Three of the fragments resulted in elliptical orbits and are likely left over from the comet’s previous perihelion passage, for which our table shows that the activity level is high enough to inject 40 cm objects into orbit. Closer to perihelion in early 2015 June at $r_h = 1.48$ au, Güttler et al. (2017) found 109 coma dust particles from millimeter to centimeter size with the Rosetta spacecraft being 200 km from the nucleus. Our table shows that such centimeter-sized fragments are commensurate with the activity of the comet at that time and can readily be lifted and escape. Agarwal et al. (2016) analyzed 76 particle tracks after perihelion, $r_h = 2.06$ au, the Rosetta spacecraft being 86 km from the nucleus. Their sizes clustered around 10 cm. Our calculations show that a uniform activity is unlikely to eject such large grains, but localized outbursts, which have been amply cataloged and described by Vincent et al. (2016), are quite capable of producing such larger fragments.

The Rosetta spacecraft had an approach speed of $\sim 1$ km s⁻¹ at a distance of $5 \times 10^6$ km from the comet, which was reduced to $\sim 500$ m s⁻¹ at a distance of $5 \times 10^8$ km, 100 m s⁻¹ at a distance of 100,000 km, and 10 m s⁻¹ at a distance of 10,000 km from the nucleus. The spacecraft spent roughly 80% of its time...
in an orbit below 200 km from the nucleus and roughly 42% at or below 100 km (where its orbital velocity was $\sim 0.10 \text{ m s}^{-1}$). Several times, it dipped as close as 10 km from the nucleus. It suffered no damage during the $\sim 38$ month time period of its close environment to the nucleus. The maximum gas production rate for this comet near perihelion was $3 \times 10^{28}$ H$_2$O molecules s$^{-1}$ (Hansen et al. 2016; Combi et al. 2020). We note that this perihelion production rate of H$_2$O is very close to the “quiescent” production rate of CO for SW1 of $\sim 1.0 - 2.0 \times 10^{28}$ molecules s$^{-1}$. For 67P, the largest fragments observed close to the nucleus in ballistic trajectories were of the order of centimeter to decimeter size, but these were small in number. The nucleus of comet SW1 has a surface gravity about the Earth. Despite having protective dust impact shields of 1 mm aluminum and 12 cm kevlar, the impact disabled a spacecraft spinning and caused a loss of communication with the Earth. Despite having protective dust impact shields of 1 mm aluminum and 12 cm kevlar, the impact disabled a number of instruments, including the camera. The spacecraft thrusters were able to stabilize the spacecraft after $\sim 32$ minutes, but by then the opportunity to obtain high-resolution data of the nucleus at closest approach was lost, as was the chance to take images of comet 26P/Grigg-Skjellerup in the extended Giotto mission (ESA Giotto overview).

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