String and Brane Tensions as Dynamical Degrees of Freedom

Eduardo Guendelman and Alexander Kaganovich

Department of Physics,
Ben-Gurion University, Beer-Sheva, Israel
email: guendel@bgumail.bgu.ac.il, alezk@bgumail.bgu.ac.il

Emil Nissimov and Svetlana Pacheva
Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia, Bulgaria
email: nissimov@inrne.bas.bg, svetlana@inrne.bas.bg

Abstract

We discuss a new class of string and p-brane models where the string/brane tension appears as an additional dynamical degree of freedom instead of being introduced by hand as an ad hoc dimensionfull scale. The latter property turns out to have a significant impact on the string/brane dynamics. The dynamical tension obeys Maxwell (or Yang-Mills) equations of motion (in the string case) or their rank p gauge theory analogues (in the p-brane case), which in particular triggers a simple classical mechanism of ("color") charge confinement.

Keywords: modified string and p-brane actions, reparametrization-covariant integration measures, dynamical generation of string/brane tension, color charge confinement.

1 Introduction

In order to build actions describing dynamics in geometrically motivated field theories (for reviews of string and brane theories, see [1]) we need among other things a consistent generally-covariant integration measure density, i.e., covariant under arbitrary diffeomorphisms (reparametrizations) on the underlying space-time manifold. Usually the natural choice is the standard Riemannian metric density $\sqrt{-g}$ with $g \equiv \det |g_{\mu\nu}|$. However, there are no purely geometric reasons which prevent us from employing an alternative generally-covariant integration measure. For instance, introducing additional $D$ scalar fields $\varphi^i$ ($i = 1, \ldots, D$ where $D$ is the space-time dimension) we may take the following new non-Riemannian measure density $\Phi(\varphi)$:

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D} \varepsilon_{\nu_1 \ldots \nu_D} \partial_{\mu_1} \varphi^{\nu_1} \ldots \partial_{\mu_D} \varphi^{\nu_D}.$$ (1)

Using (1) allows to construct new classes of models involving Gravity called Two-Measure Gravitational Models [2], whose actions are typically of the form:

$$S = \int d^Dx \Phi(\varphi) L_1 + \int d^Dx \sqrt{-g} L_2,$$ (2)

$$L_{1,2} = \varepsilon^{\mu_1 \ldots \mu_D} \left[ -\frac{1}{\kappa} R(g, \Gamma) - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi + (\text{Higgs}) + (\text{fermions}) \right],$$ (3)

where $R(g, \Gamma)$ is the scalar curvature in the first-order formalism (i.e., the affine connection $\Gamma$ is independent of the metric), $\phi$ is the dilaton field, $M_P$ is the Planck mass, etc.. Although naively the additional “measure-density” scalars $\varphi^i$ appear in (2) as pure-gauge degrees of freedom (due to the invariance under arbitrary diffeomorphisms in the $\varphi^i$-target space), there is a remnant – the so called “geometric” field $\zeta(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, which remains an additional dynamical degree of freedom beyond the
standard physical degrees of freedom characteristic to the ordinary gravity models with the standard Riemannian-metric integration measure. The most important property of the “geometric” field $\zeta(x)$ is that its dynamics is determined only through the matter fields locally (i.e., without gravitational interaction). The latter turns out to have a significant impact on the physical properties of the two-measure gravity models which allows them to address various basic problems in cosmology and particle physics phenomenology and provide physically plausible solutions, for instance: (i) the issue of scale invariance and its dynamical breakdown, i.e., spontaneous generation of dimensionfull fundamental scales; (ii) cosmological constant problem; (iii) geometric origin of fermionic families.

For a recent review of two-measure gravity models see the contribution in this volume [3]. In what follows we are going to apply the above ideas to the case of string and $p$-brane models. Part of our exposition is based on earlier works [4]. Furthermore, we will elaborate on various important properties of the modified-measure string and brane models with dynamical string/brane tension.

2 Bosonic Strings with a Modified World-Sheet Integration Measure

We begin by first recalling the standard Polyakov-type action for the bosonic string [5]:

$$S_{Pol} = -T \int d^2 \sigma \frac{1}{2 \sqrt{-\gamma}} \gamma^{\mu\nu} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X).$$  (4)

Here $(\sigma^0, \sigma^1) \equiv (\tau, \sigma); a, b = 0, 1; \mu, \nu = 0, 1, \ldots, D - 1; G_{\mu\nu}$ denotes the Riemannian metric on the embedding space-time; $\gamma_{ab}$ is the intrinsic Riemannian metric on the 1 + 1-dimensional string world-sheet and $\gamma = \det |\gamma_{ab}|; T$ indicates the string tension – a dimensionfull scale introduced ad hoc. The resulting equations of motion w.r.t. $\gamma^{ab}$ and $X^\mu$ read, respectively:

$$T_{ab} \equiv \left( \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) = 0,$$  (5)

$$\frac{1}{\sqrt{-\gamma}} \partial_a \left( \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0,$$  (6)

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa} \left( \partial_b G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\nu G_{b\lambda} \right)$ is the affine connection for the external metric.

Let us now introduce two additional world-sheet scalar fields $\varphi^i (i = 1, 2)$ and replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$ defined in terms of $\varphi^i$:

$$\Phi(\varphi) \equiv \frac{1}{2} \varepsilon_{ij} \varepsilon^{ab} \partial_a \varphi^i \partial_b \varphi^j = \varepsilon_{ij} \varphi^i \partial_j \varphi^j.$$  (7)

However, the naively generalized string action $S_1 = -\frac{1}{2} \int d^2 \sigma \Phi(\varphi) \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$ has a problem: the equations of motion w.r.t. $\gamma^{ab}$ lead to an unacceptable condition $\Phi(\varphi) \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0$, i.e., vanishing of the induced metric on the world-sheet.

To remedy the above situation let us consider topological (total-derivative) terms w.r.t. standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \to \Phi(\varphi)$ the former are not any more topological – they will contribute nontrivially to the equations of motion. For instance:

$$\int d^2 \sigma \sqrt{-\gamma} R \to \int d^2 \sigma \Phi(\varphi) R, \quad R = \frac{\varepsilon^{ab}}{2 \sqrt{-\gamma}} \left( \partial_a \omega_b - \partial_b \omega_a \right),$$  (8)

where $R$ is the scalar curvature w.r.t. $D = 2$ spin-connection $\omega_a^b = \omega_a^c \varepsilon_c^b$ (here $a, b$ denote tangent space indices). The vector field $\omega_a$ behaves as world-sheet Abelian gauge field.

Eq.(8) prompts us to construct the following consistent modified bosonic string action$^1$:

$$S = -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2 \sqrt{-\gamma}} F_{ab}(A) \right],$$  (9)

$^1$In ref.[6] another interesting geometric modification of the standard bosonic string model has been proposed, which is based on dynamical world-sheet metric and torsion.
where $\Phi(\varphi)$ is given by (7) and $F_{ab}(A) \equiv \partial_a A_b - \partial_b A_a$ is the field-strength of an auxiliary Abelian gauge field $A_a$. The action (9) is reparametrization-invariant as its ordinary string analogue (4). Furthermore, (9) is invariant under diffeomorphisms in $\varphi$-target space supplemented with a special conformal transformation of $\gamma_{ab}$:

$$\varphi^i \rightarrow \varphi'^i = \varphi^i(\varphi) \quad , \quad \gamma_{ab} \rightarrow \gamma'_{ab} = J \gamma_{ab} \quad , \quad J \equiv \det \left\| \frac{\partial \varphi'^i}{\partial \varphi^j} \right\| .$$

The latter symmetry, which we will call "$\Phi$-extended Weyl symmetry", is the counterpart of the ordinary Weyl conformal symmetry of the standard string action (4).

The equations of motion w.r.t. $\varphi^i$ resulting from (9):

$$\varepsilon^{ab} \partial_b \varphi^i \partial_a \left( \gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right) = 0$$

imply (provided $\Phi(\varphi) \neq 0$):

$$\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M \left( = \text{const} \right).$$

The equations of motion w.r.t. $\gamma^{ab}$ are:

$$T_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{1}{2} \gamma_{ab} \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0 .$$

Both Eqs.(12)–(13) yield $M = 0$ and $\left( \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) = 0$, which is the same as in standard Polyakov-type formulation (5).

The equations of motion w.r.t. $X^\mu$ read:

$$\partial_a \left( \Phi \gamma^{ab} \partial_b X^\mu \right) + \Phi \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 ,$$

where again $\Gamma^\mu_{\nu\lambda}$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$ as in the standard string case (6).

Most importantly, the equations of motion w.r.t. $A_a$ resulting from (9) yield:

$$\varepsilon^{ab} \partial_b \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0 ,$$

which can be integrated to yield a spontaneously induced string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const} \equiv T .$$

Since the modified-measure string model (9) naturally requires the presence of the auxiliary Abelian world-sheet gauge field $A_a$, we may extend it by introducing a coupling of $A_a$ to some world-sheet charge current $j^a$:

$$S = - \int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2 \sqrt{-\gamma}} F_{ab}(A) \right] + \int A_a j^a .$$

In particular, we may take $j^a$ to be the current of point-like charges on the string, so that in the "static" gauge:

$$\int A_a j^a = - \sum_i c_i \int d\tau A_0(\tau, \sigma_i) ,$$

where $\sigma_i \ (0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi)$ are the locations of the charges. Now, the action (16) produces $A_a$-equations of motion:

$$\varepsilon^{ab} \partial_b E + j^a = 0 \quad , \quad E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} .$$

Eqs.(18) look exactly as $D = 2$ Maxwell equations where the variable dynamical string tension $E \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is identified as world-sheet electric field strength, i.e., canonically conjugated momentum.
w.r.t. $A_1$ (the latter fact can be directly verified from the explicit form of the $A$-term in the action (9) or (16)).

**Remark on Canonical Hamiltonian Treatment.** Introducing the canonical momenta resulting from the action (16):

$$\pi_i^\varphi = -\varepsilon_{ij} \partial_j \varphi^i \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu \nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right],$$

$$\pi_{A_i} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}}, \quad P_\mu = -\Phi(\varphi) \left( \gamma^{00} X^\nu + \gamma^{01} \partial_\sigma X^\nu \right) G_{\mu \nu},$$

we obtain the canonical Hamiltonian as a linear combination of first-class constraints only. Part of the latter resemble the constraints in the ordinary string case $\pi_{\gamma ab} = 0$ and

$$T_\pm = \frac{1}{4} G^{\mu \nu} \left( \frac{P_\mu}{E} \pm G_{\mu \nu} \gamma_{\nu \sigma} X^\sigma \right) \left( \frac{P_\nu}{E} \pm G_{\nu \lambda} \gamma_{\lambda \sigma} X^\lambda \right) = 0,$$

where in the last Virasoro constraints the dynamical string tension $E$ appears instead of the \textit{ad hoc} constant tension.

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$ and

$$\partial_\sigma E - \sum_i c_i \delta(\sigma - \sigma_i) = 0,$$

\textit{i.e.}, the $D = 2$ “Gauss law” constraint for the dynamical string tension, which coincides with the 0-th component of Eq.(18). Finally, we have constraints involving only the measure-density fields:

$$\partial_\sigma \varphi^i \pi_i^\varphi = 0, \quad \frac{\pi_i^\varphi}{\partial_\sigma \varphi^i} = 0.$$

The last two constraints span a closed Poisson-bracket algebra:

$$\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma), \partial_\sigma' \varphi^i \pi_i^\varphi(\sigma') \} = 2 \partial_\sigma \varphi^i \pi_i^\varphi(\sigma) \partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma \left( \partial_\sigma \varphi^i \pi_i^\varphi \right) \delta(\sigma - \sigma'),$$

(a centerless Virasoro algebra), and:

$$\left\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma), \frac{\pi_i^\varphi}{\partial_\sigma \varphi^i}(\sigma') \right\} = -\partial_\sigma \left( \frac{\pi_i^\varphi}{\partial_\sigma \varphi^i} \right) \delta(\sigma - \sigma').$$

Therefore, the constraints (22) imply that the measure-density scalars $\varphi^i$ are pure-gauge degrees of freedom.

## 3 Classical Confinement Mechanism of “Color” Charges via Dynamical String Tension

### 3.1 Non-Abelian Generalization

First, let us notice the following identity in $D = 2$ involving Abelian gauge field $A_a$:

$$\frac{1}{2\sqrt{-\gamma}} \varepsilon^{ab} F_{ab}(A) = \sqrt{\frac{1}{2} F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}}.$$

This suggests the proper extension of the modified-measure bosonic string model (9) by introducing a \textit{non-Abelian} (e.g., $SU(N)$) auxiliary gauge field $A_a$ (here we take for simplicity flat external metric $G_{\mu \nu} = \eta_{\mu \nu}$):

$$S = -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \mathrm{Tr}(F_{ab}(A) F_{cd}(A)) \gamma^{ac} \gamma^{bd} \right]$$

$$= -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu - \frac{1}{\sqrt{-\gamma}} \sqrt{\mathrm{Tr}(F_{01}(A) F_{01}(A))} \right],$$

(24)
where \( F_{ab}(A) = \partial_a A_b - \partial_b A_a + i[A_a, A_b] \).

The action (24) is again invariant under the \( \Phi \)-extended Weyl (conformal) symmetry (10).

Notice that the “square-root” Yang-Mills action (with the regular Riemannian-metric integration measure):

\[
\int d^2 \sigma \sqrt{-\gamma} \frac{1}{2} \text{Tr}(F_{ab}(A)F_{cd}(A)) \gamma^{ac} \gamma^{bd} = \int d^2 \sigma \sqrt{-\text{Tr}(F_{01}(A)F_{01}(A))}
\]

(25)
is a “topological” action similarly to the \( D = 3 \) Chern-Simmons action (i.e., it is metric-independent).

Similarly to the Abelian case (16) we can also add a coupling of the auxiliary non-Abelian gauge field \( A_a \) to an external “color”-charge world-sheet current \( j^a \):

\[
S = - \int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}} \sqrt{\text{Tr}(F_{01}(A)F_{01}(A))} \right] + \int \text{Tr} (A_a j^a).
\]

(26)

In particular, for a current of “color” point-like charges on the world-sheet in the “static” gauge :

\[
\int \text{Tr} (A_a j^a) = - \sum_i \text{Tr} C_i \int d\tau A_0 (\tau, \sigma_i),
\]

(27)

where \( \sigma_i (0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi) \) are the locations of the charges.

The action (26) produces the following equations of motion w.r.t. \( \varphi^i \) and \( \gamma^{ab} \), respectively:

\[
\frac{1}{2} \gamma^{cd} \partial_c X^\mu \partial_d X_\mu - \frac{1}{\sqrt{-\gamma}} \sqrt{\text{Tr}(F_{01}F_{01})} = M (= \text{const}) ,
\]

(28)

\[
T_{ab} \equiv \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}} \gamma^{ab} \sqrt{\text{Tr}(F_{01}F_{01})} = 0 .
\]

(29)

As in the Abelian case the above Eqs.(28)–(29) imply \( M = 0 \) and the Polyakov-type equation (5).

The equations of motion w.r.t. auxiliary gauge field \( A_a \) resulting from (26) resemble, similarly to the Abelian case (18), the \( D = 2 \) non-Abelian Yang-Mills equations:

\[
\varepsilon^{ab} \nabla_a \mathcal{E} + j^a = 0 ,
\]

(30)

where:

\[
\nabla_a \equiv \partial_a + i [A_a, \mathcal{E}] , \quad \mathcal{E} \equiv \pi_{A_1} = \Phi(\varphi) \frac{F_{01}}{\sqrt{-\gamma} \sqrt{\text{Tr}(F_{01}F_{01})}}.
\]

(31)

Here \( \mathcal{E} \) is the non-Abelian electric field-strength – the canonically conjugated momentum \( \pi_{A_1} \) of \( A_1 \), whose norm is the dynamical string tension \( T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma} \).

The equations of motion for the dynamical string tension following from (30) is:

\[
\partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) + \varepsilon_{ab} \frac{\text{Tr} (F_{01} j^b)}{\sqrt{\text{Tr}(F_{01}^2)}} = 0 .
\]

(32)

In particular, the absence of external charges \( (j^a = 0) : T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 \equiv \text{const} \)

Finally, the \( X^\mu \)-equations of motion \( \partial_a (\Phi(\varphi)\gamma^{ab} \partial_b X^\mu) = 0 \) resulting from the action (26) can be rewritten in the conformal gauge \( \sqrt{-\gamma} \gamma^{ab} = \eta^{ab} \) as:

\[
\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \partial^\sigma X_\mu - \tilde{j}^a \partial_a X^\mu = 0 , \quad \text{where} \quad \tilde{j}^a \equiv \frac{\text{Tr} (F_{01} j^a)}{\sqrt{\text{Tr}(F_{01}^2)}} .
\]

(33)

For static charges \( \tilde{j}^0 = - \sum_i \tilde{e}_i \delta(\sigma - \sigma_i) : \)

\[
T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 + \sum_i \tilde{e}_i \theta(\sigma - \sigma_i) , \quad \tilde{e}_i \equiv \frac{\text{Tr} (F_{01} C_i)}{\sqrt{\text{Tr}(F_{01}^2)}} \bigg|_{\sigma = \sigma_i};
\]

(34)

\[
T \partial^\sigma \partial_a X^\mu + \left( \sum_i \tilde{e}_i \delta(\sigma - \sigma_i) \right) \partial_\sigma X^\mu = 0 \rightarrow \left\{ \begin{array}{l}
\partial^\sigma \partial_a X^\mu = 0 \\
\partial_\sigma X^\mu \bigg|_{\sigma = \sigma_i} = 0 .
\end{array} \right.
\]

(35)
3.2 Classical Confinement Mechanism

Recall that the modified string action (26) yields the $D = 2$ Yang-Mills-like Eqs.(30) whose 0-th component $\partial_\sigma \mathcal{E} + i \left[ A_1, \mathcal{E} \right] + \mathcal{J}^0 = 0$ is the “Gauss law” constraint for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$). For point-like “color” charges and taking the gauge $A_1 = 0$ (i.e., $\mathcal{E} \to \tilde{\mathcal{E}} = G\mathcal{E}G^{-1}$ where $A_1 = -iG^{-1}\partial_\sigma G$), the latter reads:

$$\partial_\sigma \tilde{\mathcal{E}} - \sum_i \tilde{C}_i (\sigma - \sigma_i) = 0 \ , \quad \tilde{C}_i^\dagger \equiv G\mathcal{C}_i G^{-1} \bigg|_{\sigma = \sigma_i} \ .$$

Let us consider the case of closed modified string with positions of the “color” charges at $0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$. Then, integrating the “Gauss law” constraint (36) along the string (at fixed proper time) we obtain:

$$\sum_i \tilde{C}_i = 0 \ , \quad \tilde{E}_{i,i+1} = \tilde{E}_{i-1,i} + \tilde{C}_i \ ,$$

where $\tilde{E}_{i,i+1} = \tilde{E}$ in the interval $\sigma_i < \sigma < \sigma_{i+1}$.

The discussion in this section leads to the following conclusions:

- We see from Eqs.(34)–(35) that the modified-measure (closed) string with $N$ point-like (“color”) charges on it ((16) or (26)) is equivalent to $N$ chain-wise connected regular open string segments obeying Neumann boundary conditions.

- Each of the above open string segments, with end-points at the charges $e_i$ and $e_{i+1}$ (in the Abelian case) or $C_i$ and $C_i+1$ (in the non-Abelian case), has different constant string tension $T_{i,i+1}$ such that $T_{i,i+1} = T_{i-1,i} + \varepsilon_i$ (the non-Abelian $\tilde{e}_i$ are defined in (34)).

- Eq.(37) tells us that the only (classically) admissible configuration of “color” point-like charges coupled to a modified-measure closed bosonic string is the one with zero total “color” charge, i.e., the model (26) provides a classical mechanism of “color” charge confinement.

4 Branes with a Modified World-Volume Integration Measure

Before generalizing our construction from the previous two sections to the case of higher-dimensional $p$-branes, let us recall the standard Polyakov-type formulation of the bosonic $p$-brane action:

$$S = -\frac{T}{2} \int d^{p+1}\sigma \sqrt{-\gamma} \left[ \gamma^{ab} \partial_\sigma X^\mu \partial_\sigma X^\nu G_{\mu\nu}(X) - \Lambda (p-1) \right] \ .$$

Here $\gamma_{ab}$ is the ordinary Riemannian metric on the $p + 1$-dimensional brane world-volume with $\gamma \equiv \text{det}||\gamma_{ab}||$. The world-volume indices $a, b = 0, 1, \ldots, p$; $T$ is the given ad hoc brane tension; the constant $\Lambda$ can be absorbed by rescaling $T$ (see below Eq.(43). The equations of motion w.r.t. $\gamma^{ab}$ and $X^\mu$ read:

$$T_{ab} \equiv \left( \partial_\sigma X^\mu \partial_\sigma X^\nu - \frac{1}{2} \gamma^{cd} \partial_\sigma X^\mu \partial_\sigma X^d X^\nu \right) G_{\mu\nu} + \gamma^{ab} \Lambda \frac{1}{2} (p-1) = 0 \ ,$$

$$\partial_\nu \left( \sqrt{-\gamma} \gamma^{ab} \partial_\sigma X^\mu \right) + \sqrt{-\gamma} \gamma^{ab} \partial_\sigma X^\nu \partial_\sigma X^\lambda \Gamma_{\nu\lambda}^\mu = 0 \ .$$

Eqs.(39) when $p \neq 1$ imply:

$$\Lambda \gamma_{ab} = \partial_\nu X^\mu \partial_\sigma X^\nu G_{\mu\nu} \ ,$$

which in turn allows to rewrite Eq.(39) as:

$$T_{ab} \equiv \left( \partial_\nu X^\mu \partial_\sigma X^\nu - \frac{1}{p+1} \gamma^{cd} \partial_\nu X^\mu \partial_\sigma X^d X^\nu \right) G_{\mu\nu} = 0 \ .$$

Furthermore, using (41) the Polyakov-type brane action (38) becomes on-shell equivalent to the Nambu-Goto-type brane action:

$$S = -T \Lambda^{- \frac{p+1}{2}} \int d^{p+1}\sigma \sqrt{- \det ||\partial_\sigma X^\mu \partial_\sigma X^\nu G_{\mu\nu}||} \ .$$

(43)
4.1 Modified-Measure Brane Actions

Now, similarly to the string case we introduce a modified world-volume integration measure in terms of $p+1$ auxiliary scalar fields $\phi^i$ ($i = 1, \ldots, p+1$):

$$\Phi(\phi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \ldots i_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \phi^i \ldots \partial_{a_{p+1}} \phi^{j_{p+1}},$$

and consider the following modified $p$-brane action:

$$S = - \int d^{p+1} \sigma \Phi(\phi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \frac{1}{\sqrt{-\gamma}} \Omega(A) \right] + \int d^{p+1} \sigma \mathcal{L}(A).$$

The term $\Omega(A)$ indicates a topological density given in terms of some auxiliary gauge (or matter) fields $A^I$ living on the world-volume, “topological” meaning that:

$$\frac{\partial \Omega}{\partial A^I} - \partial_a \left( \frac{\partial \Omega}{\partial \partial_a A^I} \right) = 0 \text{ identically }, \text{ i.e. } \delta \Omega(A) = \partial_a \left( \frac{\partial \Omega}{\partial \partial_a A^I} \delta A^I \right).$$

$\mathcal{L}(A)$ describes possible coupling of the auxiliary fields $A^I$ to external “currents” on the brane world-volume.

The requirement for $\Omega(A)$ to be a topological density is dictated by the requirement that the modified-measure brane action (45) (in the absence of the last gauge/matter term $\int d^{p+1} \sigma \mathcal{L}(A)$) reproduces the ordinary $p$-brane equations of motion apart from the fact that the brane tension $T \equiv \Phi(\phi)/\sqrt{-\gamma}$ is now an additional dynamical degree of freedom.

The simplest example of a topological density $\Omega(A)$ for the auxiliary gauge/matter fields is:

$$\Omega(A) = - \frac{\varepsilon^{a_1 \ldots a_{p+1}}}{p+1} F_{a_1 \ldots a_{p+1}}(A), \quad F_{a_1 \ldots a_{p+1}}(A) = (p+1) \partial_{[a_1} A_{a_2 \ldots a_{p+1}]},$$

where $A_{a_1 \ldots a_p}$ is rank $p$ antisymmetric tensor (Abelian) gauge field on the world-volume$^2$.

More generally, for $p+1 = rs$ we can have:

$$\Omega(A) = \frac{1}{rs} \varepsilon^{a_1 \ldots a_r b_1 \ldots b_r} F_{a_1 \ldots a_r} F_{b_1 \ldots b_r},$$

We may also employ non-Abelian auxiliary gauge fields as in the string case. For instance, when $p = 3$ we may take:

$$\Omega(A) = \frac{1}{4} \varepsilon^{abcd} \text{Tr} \left( F_{ab}(A) F_{cd}(A) \right)$$

or, more generally, for $p+1 = 2q$:

$$\Omega(A) = \frac{1}{2q} \varepsilon^{a_1 b_1 \ldots a_q b_q} \text{Tr} \left( F_{a_1 b_1} \ldots F_{a_q b_q} \right),$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_a + i [A_a, A_b]$.

The modified $p$-brane action (45) produces the following equations of motion w.r.t. $\phi^i$:

$$\frac{1}{2} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu} + \frac{1}{\sqrt{-\gamma}} \Omega(A) = M \equiv \text{const},$$

and w.r.t. $\gamma^{ab}$ (assuming that $\int d^{p+1} \sigma \mathcal{L}(A)$ does not depend on $\gamma_{ab}$ – true e.g. if it describes coupling of the auxiliary (gauge) fields $A$ to charged lower-dimensional branes):

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \frac{\gamma_{ab}}{\sqrt{-\gamma}} \Omega(A) = 0.$$  

Both Eqs. (51)–(52) imply:

$$\Omega(A) = \frac{2M}{p-1} \sqrt{-\gamma},$$

$^2$A modified $p$-brane model significantly different from (45)–(47) has been proposed in ref.[7]. The latter model also contains world-volume $p$-form gauge fields which, however, appear quadratically in the brane action of [7] and, therefore, they are dynamical rather than auxiliary fields.
\[ \partial_a X^\mu \partial_b X^\nu G_{\mu \nu} = \gamma_{ab} \frac{2M}{p - 1}, \quad \partial_a X^\mu \partial_b X^\nu G_{\mu \nu} - \frac{1}{p + 1} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu \nu} = 0. \] (54)

The last two Eqs. (54) reproduce two of the ordinary brane equations of motion (41)–(42) in the standard Polyakov-type formulation.

We now consider the modified brane (45) equations of motion w.r.t. auxiliary (gauge) fields \( A^I \) – these are the eqs. determining the dynamical brane tension \( T \equiv \Phi(\varphi)/\sqrt{-\gamma} \):

\[ \partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) \frac{\partial \Omega}{\partial a A^I} + j_I = 0, \] (55)

where \( j_I \equiv \frac{\partial \Omega}{\partial a A^I} - \partial_a \left( \frac{\partial \Omega}{\partial a A^I} \right) \) is the corresponding “current” coupled to \( A^I \).

As a physically interesting example let us take the choice (47) for the topological density \( \Omega(A) \) and consider the following natural coupling of the auxiliary \( p \)-form gauge field:

\[ \int d^{p+1}\sigma L(A) = \int d^{p+1}\sigma A_{a_1...a_p} j^{a_1...a_p} \] (56)
to an external world-volume current:

\[ j^{a_1...a_p} = \sum_i e_i \int_{B_i} d^{p}u \frac{1}{p!} \varepsilon^{a_1...a_p} \frac{\partial \sigma^{a_1}_i}{\partial u^{a_1}} \cdots \frac{\partial \sigma^{a_p}_i}{\partial u^{a_p}} \delta^{(p+1)}(\mathbf{a} - \mathbf{a}(u)). \] (57)

Here \( j^{a_1...a_p} \) is a current of charged \((p - 1)\)-sub-branes \( B_i \) embedded into the original \( p \)-brane world-volume \( \varepsilon^a = \sigma^a_0(u) \) with parameters \( u \equiv (u^a)_{a = 0,...,p-1} \). For simplicity we assume that the \( B_i \) sub-branes do not intersect each other. With the choice (56)–(57), Eq. (55) for the dynamical brane tension \( T \equiv \Phi(\varphi)/\sqrt{-\gamma} \) becomes:

\[ \partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) + \sum_i e_i N_a^{(i)} = 0, \] (58)

where \( N_a^{(i)} \) is the normal vector w.r.t. world-hypersurface of the \((p - 1)\)-sub-brane \( B_i \):

\[ N_a^{(i)} = \frac{1}{p!} \varepsilon^{ab_1...b_p} \int_{B_i} d^{p}u \frac{1}{p!} \varepsilon^{a_1...a_p} \frac{\partial \sigma^{a_1}_i}{\partial u^{a_1}} \cdots \frac{\partial \sigma^{a_p}_i}{\partial u^{a_p}} \delta^{(p+1)}(\mathbf{a} - \mathbf{a}(u)). \] (59)

Finally, the modified-brane action (45) yields the \( X^\mu \)-equations of motion \( \partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \gamma^{ab} \partial_b X^\mu \right) = 0 \) (taking for simplicity \( G_{\mu \nu} = \eta_{\mu \nu} \)), which upon using (58) can be rewritten in the form \(^3\):

\[ \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \partial_a \left( \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) - \sum_i e_i N_a^{(i)} \gamma^{ab} \partial_b X^\mu = 0. \] (60)

### 4.2 Confinement of Charged Lower-Dimensional Branes

Let us consider the solutions for the for the dynamical brane tension Eq. (58). Recalling the definition (59) of \( N_a^{(i)} \) we find from (58) that \( T \equiv \Phi(\varphi)/\sqrt{-\gamma} \) is piece-wise constant on the \( p \)-brane world-volume with jumps when crossing the world-hypersurface of each charged \((p - 1)\)-sub-brane \( B_i \), the corresponding jump being equal to the charge magnitude \( \pm e_i \) (the overall sign depending on the direction of crossing w.r.t. the normal \( N_a^{(i)} \)).

Taking into account the above piece-wise constant solution for \( T \equiv \Phi(\varphi)/\sqrt{-\gamma} \), the \( X^\mu \)-equations of motion (60) for the closed modified brane (45) become equivalent to the following set of equations of motion:

\[ \partial_a \left( \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) = 0, \quad \partial_N X^\mu \bigg|_{B_i} = 0, \] (61)

\(^3\)For a detailed description of techniques for obtaining solutions of equations of motion for standard string and brane systems in non-trivial backgrounds, which can be easily adapted in the present modified-measure string and brane models, see ref.[8].
where $\partial_N$ indicates normal derivative w.r.t. world-hypersurface of the $(p-1)$-sub-brane $B_i$. Therefore, Eqs.(61) together with Eq.(58) describe a set of ordinary open $p$-brane segments with common boundaries, where each open $p$-brane segment possesses different constant brane tension and obeys Neumann boundary conditions.

Integrating Eq.(58) along arbitrary smooth closed curve $C$ on the $p$-brane world-volume which is transversal to (some or all of) the $(p-1)$-sub-brane $B_i$, we obtain the following constraints on the possible sub-brane configurations:

$$\sum_i e_i n_i(C) = 0,$$

where $n_i(C)$ is the sign-weighted total number of $C$ crossing $B_i$. In the present $p \geq 2$-brane case, however, due to the much more complicated topologies of the pertinent world-volumes Eq.(62) may yield various different types of allowed sub-brane configurations.

As a simple illustration, here we will only consider the simplest non-trivial case $p = 2$ and take the static gauge for the $p = 1$ sub-branes (strings), i.e., the proper times of the charged strings coincides with the proper time of the bulk membrane. The latter means that the fixed-time world-volume of the bulk closed membrane is a Riemann surface with some number $g$ of handles and no holes. Further, we will assume the following simple topology of the attached $N$ charged strings $B_i$: upon cutting the membrane surface along these attached strings it splits into $N$ open membranes $M_i$ ($i = 1, \ldots, N$) with Neumann boundary conditions (cf. (61)), each of which being a Riemann surface with $g_i$ handles and 2 holes (boundaries) formed by the strings $B_{i-1}$ and $B_i$, respectively. The brane tension of $M_i$ is a dynamically generated constant $T_i$ where $T_{i+1} = T_i + e_i$. In the present configuration Eq.(62) evidently reduces to the constraint $\sum_i e_i = 0$.

Thus, we conclude that similarly to the string case, modified-measure $p$-brane models describe configurations of charged $(p-1)$-branes with charge confinement. Apart from the latter, in general there exist more complicated configurations allowed by the constraint (62), which will be studied elsewhere.

5 Conclusions

The above discussion shows that there exist natural from physical point of view modifications of world-sheet and world-volume integration measures which may significantly affect string and brane dynamics. Let us summarize the main features of the new modified-measure string and brane models:

- Acceptable dynamics naturally requires the introduction of auxiliary world-sheet gauge field (world-volume $p$-form tensor gauge field).
- The string/brane tension is not anymore a constant scale given ad hoc, but rather appears as an additional dynamical degree of freedom beyond the ordinary string/brane degrees of freedom.
- The dynamical string/brane tension has physical meaning of an electric field strenght for the auxiliary gauge field.
- The dynamical string/brane tension obeys “Gauss law” constraint equation and may be non-trivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the $p$-brane world-volume).
- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like “color” charges or charged lower-dimensional branes due to variable dynamical tension.

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\[ \chi_i = 2 - 2g_i - 2, \text{ so that } \chi = \sum_i \chi_i \text{ or, equivalently, } g = 1 + \sum_i g_i. \]
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