Joint Routing and Scheduling for Large-Scale Deterministic IP Networks

Jonatan Krolikowski*, Sébastien Martin*, Paolo Medagliani*, Jérémie Leguay*, Shuang Chen†, Xiaodong Chang†, Xuesong Geng†
*Paris Research Center, †Beijing Research Center, Huawei Technologies Co. Ltd

e-mail addresses: {name.surname}@huawei.com

Abstract—With the advent of 5G and the evolution of the Internet protocols, industrial applications are moving from vertical solutions to general purpose IP-based infrastructures that need to meet deterministic Quality of Service (QoS) requirements. The IETF DetNet working group aims at providing an answer to this need with support for (i) deterministic worst-case latency and jitter, and (ii) zero packet loss for time-sensitive traffic.

In this paper we focus on the joint routing and scheduling problem in large scale deterministic networks using Cycle Specified Queuing and Forwarding (CSQF), an extension of Cyclic Queuing and Forwarding (CQF) with multiple transmission queues and support of segment routing. In this context, we present two centralized algorithms to maximize traffic acceptance for network planning and online flow admission. We propose an effective solution based on column generation and dynamic programming. Thanks to the reinforcement of the model with valid inequalities, we improve the upper bound and the solution. We demonstrate on realistic instances that we reach an optimality gap smaller than 10% in a few seconds. Finally, we also derive an ultra-fast adaptive greedy algorithm to solve the problem at the cost of a small extra gap.

Index Terms—Deterministic Networking, Routing, Scheduling.

I. INTRODUCTION

The 5th generation of networks is paving the road for latency-sensitive network services to enable a wide-range of applications like factory automation, connected vehicles and smart grids [1]. Traditional IP services allow to deliver packets with no loss and no ordering issues. However, they cannot provide strict QoS guarantees. Certain service classes can be given preferential treatment but performance is still statistical. Deterministic performances are now a must to support applications with low and worst-case latency requirements.

A collection of IEEE 802.1 Ethernet standards, known as Time-Sensitive Networking (TSN) [2], has been developed in the past decade to support professional applications over Local Area Networks (LAN) with mechanisms such as priority queuing, preemption, traffic shaping and time-based opening of gates at output ports. While these mechanisms are well suited for static traffic requirements and small networks, they are not enough to support large-scale IP networks. The IETF DetNet (Deterministic Networking) [1] working group is taking a step further by defining Segment Routing (SR) mechanisms so that Layer 3 can dynamically exploit Layer 2 functionalities for queuing and scheduling to support (i) deterministic worst-case latency and jitter, and (ii) zero packet loss for time-sensitive traffic. In particular, the working group is currently specifying Cycle Specified Queuing and Forwarding (CSQF) [3], a promising extension to Cyclic Queuing and Forwarding (CQF, a.k.a. IEEE 802.1Qch) with more than 2 transmission queues in order to relax tight time-synchronization constraints and to schedule, in a more flexible way, transmissions at each hop.

In TSN layer-2 networks, several works have optimized the opening and closing of gates at output ports (IEEE 802.1Qbv, [4], [5], [6]) to meet low latency requirements. However, these solutions suffer from two main limitations: 1) the overall gate schedule has to be modified at network-level every time the traffic characteristics evolve and 2) no queue can be used to dynamically delay packets at nodes. Alternatively, CSQF proposes a scalable solution where transmission cycles at each port repeat periodically thanks to the round-robin opening of multiple queues dynamically selected by IP packets using segment routing identifiers (SIDs), a label stack that determines scheduling and routing at each hop. A network controller decides the proper label stack for each flow by solving a joint scheduling and routing problem. In this context, we propose two centralized control plane algorithms to maximize traffic acceptance both in the offline (i.e., global optimization) and the online (i.e., fast demand acceptance) scenarios. Up to our knowledge, it is the first paper to formulate the joint routing and scheduling problem for DetNet and provide efficient algorithms in large-scale deterministic networks.

We formulate the Deterministic Networking (DN) planning problem to maximize the acceptance of time-triggered traffic and we analyze its NP-hardness. Then, we present an effective solution based on column generation and dynamic programming to solve a relaxed version of the problem which we round afterwards. Furthermore, thanks to the reinforcement of the problem model with valid inequalities, we show that we can drastically improve the upper bound (by up to 30%) to better estimate the optimality gap and enhance the final solution (by up to 5%) on large instances. We demonstrate on realistic instances with hundreds of nodes and links that we can reach a gap smaller than 10% in a few seconds. Finally, as an alternative, we derive an ultra-fast adaptive greedy algorithm (10 μs per demand) at the cost of an extra 5% gap when compared to the advanced solution based on column generation. This algorithm can be used for the quick acceptance of new demands in an online fashion.

More details about CSQF are given in Sec. [II] Relevant
In practice, as the real processing delay of each forwarded packet can be smaller than the worst-case, the DetNet-enabled node introduces an artificial additional delay to ensure that the packet forwarding delay is equal to the worst-case.

In Fig. 1 we show an example of how a packet is propagated from node A to node C through node B. Once the packet is sent from A, it is received at B within a cycle (cycle 2 in the figure). As node B decides for immediate packet forwarding, the packet is transmitted in the next cycle. Finally, node C decides for the scheduling of the packet two cycles later, so that the packet will be transmitted at cycle 6. As the same considerations apply if we consider 0 offset between cycles of different nodes, for the sake of simplicity and without loss of generality, we will consider throughout this paper a 0 time offset such that all cycles are aligned at the different nodes.

According to CSQF, a DetNet-enabled device decides how and when a packet is forwarded by consuming the first Segment Routing ID (SID) available in the label stack of packet headers. As a first step, the receiving node maps the SID into the corresponding output port. As a second step, the device uses the same label to select the queue associated with the intended transmission cycle. The SR label stack can be provided by a centralized network controller that (a) computes a feasible path from source node to destination node, (b) computes the right scheduling within each node traversed by the flow, and (c) distributes the corresponding SR label stack to all the network elements via specific protocols (e.g., PCEP).

Like in TSN, DetNet traffic with CSQF is time-triggered (TT) and follows a specific pattern that repeats over time. This period is referred to as hypercycle. For each cycle, the application specifies how much data will be sent. To ensure deterministic end-to-end performance, it is necessary to provide a scheduling and routing decision at each hop and guarantee that enough resources are available.

III. RELATED WORK

In the literature, most of the papers are focusing on the scheduling of TSN/IEEE 802.1Qbv gate openings and closings to satisfy a certain traffic matrix. The matrix is composed of TT traffic flows which generate packets at known and repeating time instants. Routing information is generally given by the spanning tree protocol operating at layer 2. In this context, the goal is to find a feasible scheduling while minimizing the
number of queues. In this case, a variant of the flow shop scheduling problem must be solved.

For 802.1Qbv, \cite{4} introduces the problem as an Integer Linear Program (ILP) while \cite{5} uses OMT (Optimization Modulo Theory) to formulate a Satisfiability Problem (SAT). \cite{6} also presents a SAT problem but considering robustness to control worst-case performance in case of uncertain traffic inputs. These papers do not introduce practical and efficient heuristics. The resolution of ILP or SAT models with solvers can only be achieved on very small instances.

In case routing can also be decided, \cite{10} presents an online heuristic for 802.1Qbv. In this case, the end-to-end transmission of a cyclic TT flow must be realized in the same global transmission cycle to minimize the end-to-end latency. In other papers from the same authors, an ILP model is formulated to maximize traffic acceptance for a set of flows \cite{11}. \cite{12} formulates a similar problem by considering constant time shifts between incoming and outgoing transmissions at intermediate nodes (no controllable queuing is allowed). \cite{13} presents a SAT problem formulation of the same problem.

Instead, our work focuses on both deterministic latency and jitter requirements rather than minimum latency. Our solution uses the recent CSQF standard proposal to guarantee worst-case performance at each hop thanks to the use of cyclic transmissions and segment routing for dynamic scheduling. We formulate the joint routing and scheduling problem for DetNet to maximize traffic acceptance as an ILP. We analyze the hardness of the problem and solve it at large scale and with quantifiable optimality.

IV. PROBLEM FORMULATION AND COMPLEXITY

This section introduces our model for the routing and scheduling of traffic in DetNet with CSQF. We formulate its Integer Linear Program (ILP) and analyze the complexity.

A. Topology, Demands and Cycles

Let’s consider a network \( G = (V,A) \). The nodes \( v \in V \) represent DetNet-enabled routers or switches. The nodes are connected with data links represented by the (directed) link set \( A \subseteq V \times V \).

A given set of demands \( D \), i.e., a set of TT flows, needs to be routed through the network. The demands are described by a deterministic pattern of packet arrivals over a hypercycle of size \( C \) in unitary cycles. Indeed, time is partitioned into cycles of equal duration thanks to CSQF, e.g. 10 \( \mu s \). Without loss of generality, we assume that the cycles start at the same time across the network. In cycle \( c \), the source node \( s^d \in V \) of demand \( d \) emits packets for a total of \( bw_c^d \in \mathbb{Z}_+ \) in data units (\( du; \) fixed size in Bytes) that need to be routed to the destination \( t^d \in V \). The pattern is the same at every realization of the hypercycle, i.e. \( bw_c^d = bw_{c\%C}^d \) for any \( c \in \mathbb{Z}_+ \). Each demand \( d \) has a maximum acceptable end-to-end delay (in cycles) denoted by \( \Delta^d \).

Each arc \( a = (u,v) \in A \) induces a delay of \( \Delta_a \) cycles which comprises its propagation delay as well as the processing and queuing delay at node \( v \). Furthermore, each arc \( a \) has a per-cycle capacity \( b_a \) (in data units).

\[ \text{link capacity: } b_{a1} = 4 \quad \Delta_{a1} = 5 \quad b_{a2} = 3 \quad \Delta_{a2} = 2 \]

\[ p1 \]
\[ p2 \]

\[ \text{even/odd cycles cycle shifts} \]

\[ \text{Fig. 2: A demand } d \text{ between source } s^d \text{ and destination } t^d \text{ with two } s\text{-paths } p_1 \text{ and } p_2 \text{ and hypercycle length } C = 2, \text{ together with a second demand } d' \text{ with only one path } p'. \]

B. Scheduled Paths

For a demand \( d \) to be accepted, the central controller needs to assign a unique feasible scheduled path (\( s\)-path). An \( s\)-path \( p \) is a path in \( G \), i.e. a sequence of arcs \( (a_1,\ldots,a_{|p|}) \) where arcs \( a_k = (u_k,v_k) \) are such that \( u_1 = s^d \) and for \( k = 1,\ldots,|p|-1 \), together with an integer sequence \( (r_0^p,\ldots,r_{|p|-1}^p) \) where \( r_k^p \in \mathbb{Z}_{\geq 0} \) indicates the number of cycle shifts at corresponding nodes \( v_k \). A shift is an explicit additional delay (expressed in multiple of cycles) that is introduced at nodes to schedule data transmissions into a specific CSQF queue. If \( c \) is the earliest possible cycle in which a packet may be forwarded from \( v_k \) (recall that processing and queuing delays are included in the arc delay of the preceding arc), a cycle shift of \( r_k \) means that transmission is carried out in cycle \( (c+r_k)\%C \). The maximum number of shifts at a node is \( R = N_{DN} - 2 \) where \( N_{DN} \) is the number of CSQF queues reserved to deterministic traffic. The introduction of cycle shifts allows to accept more traffic, as we will see later.

Let’s consider the example in Fig. 2 with a single demand \( d \) where a path from \( s \) to \( t \) has two hops with an intermediate node \( u \). In all even cycles (\( c\%2 = 0 \)), \( s \) sends 2 \( du \), and in all odd cycles (\( c\%2 = 1 \)), it sends 1 \( du \). The pattern repeats after two cycles (\( C = 2 \)). We consider two possible \( s\)-paths \( p_1 \) and \( p_2 \) that may be used for \( d \). From \( u \), the earliest transmission of the 2 \( du \) is in the odd cycles (\( c = 5\%2 = 1 \)), while the one of the 1 \( du \) is in the even cycles (\( c = (1+5)\%2 = 0 \)), as the delay is \( \Delta_{a1} = 5 \) cycles. The \( s\)-path \( p_1 \) does not introduce any additional shift (recall that the one induced by the delay on \( a_1 \) is mandatory) at node \( u \) \( (r_1^p = 0) \). The \( s\)-path \( p_2 \) has a cycle shift of \( r_2^p = 1 \) at \( u \), so it forwards 2 \( du \) in even cycles and 1 \( du \) in odd cycles. Now, to extend the example and show the need to introduce scheduling (i.e., extra cycle shifts) at intermediary nodes, let’s consider a second demand \( d' \) from \( u \) to \( t \) that has only one available \( s\)-path \( p' \) due to delay constraints. This \( s\)-path uses arc \( a_2 \) such that in even cycles, \( d' \) requires 0 \( du \) on \( a_2 \) and 2 \( du \) in odd cycles. \( s\)-paths \( p_1 \) and \( p' \) together thus require 4 \( du \) in odd cycles, prohibitive with a per-cycle capacity \( b_{a2} = 3 \). However, the additional cycle shift in \( p_2 \) for demand \( d \) allows both \( d \) and \( d' \) to be routed via \( a_2 \).

A single \( s\)-path is feasible for demand \( d \) if the following...
two conditions hold.

1) End-to-end delay: The s-path delay $\Delta(p)$ must not exceed the maximum end-to-end delay $\Delta^d$. $\Delta(p)$ has two aspects: (i) the sum of arc delays $\Delta_a$ and (ii) the sum of cycle shifts $r^p_k$ at the intermediate nodes. In the example in Fig. 2, $\Delta(p_1) = 7$, and $\Delta(p_2) = 8$, as indicated by the arc delays of 5 and 2, respectively. The difference comes from the shift at $u$ on $p_2$.

We denote as $\Delta_{u_k}(p)$ the shift (in cycles) for the data to be transmitted at intermediate node $u_k$. It is easily calculated as

$$\Delta_{u_k}(p) = \sum_{i=1}^{k-1} (\Delta_a + r^p_i)$$

where $\Delta_{u_k}(p) = 0$ since there is no delay at $u_1 = s^d$. The total delay of the s-path is $\Delta(p) = \Delta_{u/p}(p) + \Delta_a[p]$.  

2) Arc-cycle capacity: A demand $d^p$ consumes a certain capacity $bw^d_{a,p}(c)$ on arcs $a$ of the s-path $p$ at cycle $c$. This value is determined by following the cyclic shifts along $p$: on the first arc of $p$, the required capacity during cycle $c$ is $bw^c_a$; the bandwidth emitted by the source $s^d$. As seen above, the delay at intermediate node $u$ is $\Delta_{u}(p)$. Any packet emitted from $s^d$ in cycle $c$ is thus forwarded from $u$ in cycle $(c+\Delta_{u}(p))\%C$. The required bandwidth for demand $d$ during cycle $c$ on arc $a = (u,v)$ within s-path $p$ is therefore given by

$$bw^d_{a,p}(c) = bw^d_{(c+\Delta_{u}(p))\%C}.$$ 

If enough capacity is available on every arc and during every cycle, $p$ can be assigned to $d$.

For convenience of notation, each s-path corresponds to a unique demand $d(p) \in D$. However, two demands with identical sources and destinations may have otherwise identical s-paths. The set of feasible s-paths for $d$ is denoted by $P^d$. $P = \bigcup_d P^d$ is the disjoint union of all s-paths.

Remark 1. The set path $P$ is not given as an input. For each demand $d$, $P^d$ needs to be generated. For general graphs, the cardinality of $P^d$ may be exponential in the input size.

C. Problem Statement

The central controller tries to route each demand $d$ via a unique feasible s-path in $P^d$. This is indicated with the variable $y_p$ which is set to 1 if $p$ is chosen for $d(p)$, 0 otherwise. Uniqueness of the s-path is ensured with the constraint

$$\sum_{p \in P^d} y_p \leq 1 \quad \forall d \in D.$$  

The arc capacities are shared among the routed demands. No more data than its capacity $b_a$ may be sent onto any arc $a$ during any cycle. This condition is expressed by the constraint

$$\sum_{p \in P^d : a \in p} bw^d_{a,p}(c) y_p \leq b_a \quad \forall a \in A, \forall c.$$  

Note that due to the given cyclic structure, it suffices to calculate the bandwidth in the cycles 0 to $C-1$. Note also that cycle shifts $r^p_k > 0$ may allow for otherwise incompatible demands to be transmitted via the same arc.

The aim of the central controller is to accept a subset of demands such that the total accepted bandwidth is maximized. The bandwidth $bw^d$ of demand $d$ is the sum of the bandwidth transmitted over the cycles 0, $\ldots$, $C-1$, i.e.

$$bw^d = \sum_{c=0}^{C-1} bw^d$$

Thus, the Deterministic Networking (DN) problem can be formulated as an ILP in the following way:

$$\text{(DN) max} \sum_{p \in P} bw^d(p)y_p$$

$$\text{s.t.} \sum_{p \in P^d} y_p \leq 1 \quad \forall d,$$

$$\sum_{p \in P^d : a \in p} bw^d_{a,p}(c) y_p \leq b_a \quad \forall a, c,$$

$$y_p \in \{0,1\} \quad \forall p.$$

D. Complexity Analysis

The DN problem is an NP-hard optimization problem. This is due to Theorem 1 that shows NP-completeness for the decision counterpart, called DN.

Theorem 1. The DN problem is NP-complete.

Proof. DND decides if, for a given threshold $\ell \in \mathbb{R}_+$, there is a feasible solution to DN with objective value $\geq \ell$. The following reduction proof is based on the well-known $k$-Disjoint Paths ($k$DP) problem [14 Theorem 19.7]. We consider the (NP-complete) version of $k$DP which decides if $k$ arc-disjoint paths can be found between nodes $s$ and $t$ in a directed graph $G$. This problem can be reduced to an instance of DND by setting the number of cycles to $C = 1$ and $bw^d = 1$ for $k$ demands that all have source $s$ and destination $t$. The capacity of every arc $a$ is chosen to be $b_a = 1$. Choosing $\ell = k$, DND returns true if and only if there are $k$ arc-disjoint paths in $G$. Since all reduction steps are polynomial in the problem size, the NP-hardness proof is complete. Furthermore, it is clear that DND belongs to NP since the validity of any solution can be checked in polynomial time. Thus, DND is NP-complete. $\Box$

In fact, there are two aspects which induce the “hardness” of DN: the number of cycles $C$ and the routing aspect, i.e. the multitude of available paths per demand.

a) Complexity due to routing: The DN problem generalizes the unsplittable Multi-Commodity Flow problem (uMCF, also called Unsplittable Flow problem, see for example [15]) through the introduction of cycles and delays. However, in general the coefficients in objective function and constraints of uMCF are independent and not related as in DN (recall that the objective in DN is maximization of the bandwidth that is also used in the capacity constraint). Guruvswami et al [15] show that it is NP-hard to approximate uMCF within $|E|^{1/2-\varepsilon}$ for any $\varepsilon > 0$. Their proof, however, can easily be extended to DN (with its related coefficients) with the same result even in the case of $C = 1$.

b) Complexity due to cycles: If $C$ is part of the input, and not a priori bounded, DN cannot even be efficiently approximated in polynomial time (unless P=NP), i.e. there is no polynomial-time approximation scheme (PTAS). This
is true even if the graph \( G \) consists only of one single arc. Then, \( \text{DN} \) is equivalent to the 0-1 Multidimensional Knapsack (01MK) problem (see [16]): a 01MK instance is transformed to a DN instance by multiplying each constraint such that the right-hand side (rhs) is the least common multiple of the given rhs values. If the number of constraints (given by \( C \)) is unbounded, there is no PTAS for 01MK [17].

Note that \( \text{DN} \) becomes weakly NP-hard (and thus solvable in pseudo-polynomial time) if the number of cycles \( C \) and the set of feasible s-paths \( |\mathcal{P}| \) are bounded (and can be computed in polynomial time) since the same is true for 01MK with bounded dimensions [17].

V. SCALABLE GLOBAL ALGORITHM

This section presents a solution to DN based on column generation (CG), a classic approach for intractable ILPs. It solves the linear relaxation (referred to as LDN) to optimality. Firstly, this provides a DN upper bound (UB). Secondly, rounding the LDN solution provides a high-quality feasible DN solution called CG-RR. Strengthening the capacity constraints (see Sec. V-C) allows for an enhanced LDN formulation which helps to improve the CG-RR solution as well as the UB.

A. Solving the Linear Relaxation

LDN relaxes the integrality constraints on the variables \( y_p \). It is well-known that linear programs (LPs) such as LDN can be solved in polynomial time in terms of input size [18]. However, as to Remark 1 the number of variables in DN in general is not polynomial in the input size which poses a problem solving LDN in practice. We overcome this problem by applying column generation [19] to LDN.

1) Column Generation: We start with a restricted LP which contains only a subset of the variables of the so-called master LP LDN. By solving the pricing problem, we decide whether there are variables that are currently not contained in the restricted LP but might improve the objective value. If no such variables can be found, the current subset of variables is guaranteed to be sufficient to solve the master LP optimally. Otherwise, the newly generated variables are added to the restricted LP and the process iterates. This method is based on LP duality (see for example [20]).

In the following, we consider a subset of s-paths \( \mathcal{P}' \subseteq \mathcal{P} \). And for simplicity of notation, we assume that for all \( d \in D \), there is an s-path \( p \in \mathcal{P}' \) such that \( d(p) = d \). The induced restricted relaxation of DN is:

\[
\begin{align*}
\text{(LDN') max} & \quad \sum_{p \in \mathcal{P}'} b_w^{d(p)} y_p \\
\text{s.t.} & \quad y_p \leq 1 \quad \forall d, \\
& \quad \sum_{p \in \mathcal{P}' : d(p) = d} \sum_{p' : a \in p'} b_w^{d(p)}(c) y_{p'} \leq b_a \quad \forall a, c, \\
& \quad y_p \geq 0 \quad \forall p \in \mathcal{P}'.
\end{align*}
\]

(4)

Note that a feasible solution \( y'_p \) to LDN' induces a feasible solution \( y_p \) to LDN by setting \( y_p = y'_p \) for \( p \in \mathcal{P}' \) and \( y_p = 0 \) otherwise. If \( y'_p \) is optimal for LDN', we can determine if the induced solution \( y_p \) is optimal to LDN by considering the dual of LDN' :

\[
\begin{align*}
\text{(D-LDN')} \min & \quad \sum_{d} \lambda_d + \sum_{a} \sum_{c} b_a \mu_{a,c} \\
\text{s.t.} & \quad \lambda_{d(p)} + \sum_{a \in p} \sum_{c} b_w^{d(p)}(c) \mu_{a,c} \geq b_w^{d(p)} \forall p \in \mathcal{P}', \\
& \quad \lambda_d \geq 0 \quad \forall d, \\
& \quad \mu_{a,c} \geq 0 \quad \forall a, c,
\end{align*}
\]

(5)

where the dual variables \( \lambda_d \) relate to primal constraints (4) and dual variables \( \mu_{a,c} \) relate to constraints (5).

Let \( \left( \lambda^{*}_{d}, \mu^{*}_{a,c} \right) \) be an optimal solution for D-LDN'. If there exists a separating s-path \( p \in \mathcal{P} \setminus \mathcal{P}' \) such that

\[
\lambda^{*}_{d(p)} + \sum_{a \in p} \sum_{c} b_w^{d(p)}(c) \mu^{*}_{a,c} < b_w^{d(p)},
\]

(6) then the solution is infeasible to D-LDN, the dual of LDN. The problem D-LDN" with \( \mathcal{P}'' = \mathcal{P} \setminus \{p\} \) constitutes an improved approximation to D-LDN. If no such separating s-path exists, the solution is feasible to D-LDN and also optimal for DLN.

Note that for LDN, the latency constraint must be integrated in the pricing problem. To solve the pricing problem, an s-path fulfilling (6) needs to be found if and only if one exists.

2) Generation of Separating s-Paths: Given an optimal solution \( \left( \lambda^{*}_{d}, \mu^{*}_{a,c} \right) \) to D-LDN', an algorithm generating separating s-paths has to determine for each demand \( d \in D \) if separating s-paths \( \mathcal{P}'' \subseteq \mathcal{P} \setminus \mathcal{P}' \) exist. If yes, it should return (a subset of) \( \mathcal{P}'' \), \( \emptyset \) otherwise.

For each demand \( d \in D \), finding the (delay constrained) shortest s-path \( p \) in terms of path weight \( \sum_{a \in p} \sum_{c} b_w^{d(p)}(c) \mu^{*}_{a,c} \) solves the pricing problem. If solved optimally, it guarantees that a path is found if it exists. If the weight of the shortest path is strictly smaller than \( b_w^{d(p)} - \lambda^{*}_{d(p)} \), then we add the column (variable) associated with this path to the problem. If for all demands, no columns can be added, the CG procedure terminates.

In order to compute a shortest s-path, we construct the extended graph \( G^{\text{ext}} = (V^{\text{ext}}, A^{\text{ext}}) \) where \( V^{\text{ext}} = \{u, v \mid (u, v) \in V \setminus \{0, \ldots, C - 1\} \} \). When a path \( p \) in \( G^{\text{ext}} \) contains node \( u \), the respective s-path in \( G \) passes the following arc \((u, v)\) with a cycle shift of \( c \) w.r.t. the source \( s^{d(p)} \) of the respective demand. The arc set \( A^{\text{ext}} \) represents the possible transitions to the following node \( v \). E.g. if s-path

Fig. 3: Constructing the extended graph \( G^{\text{ext}} \) from graph \( G \).
is reached by the algorithm while the current delay $\Delta$ added to the current label are deleted. If destination node $v$ has no additional shift at $w$, then $P$ is set.

As an illustration, Fig. 3 shows three internal nodes $u, v, w$ of some path $P$ for a demand that has demand 2, 1, 0 data units over cycles 0, 1 and 2, respectively ($C = 3$). A maximum of 1 additional shift per node is allowed. Exiting node $u, p$ has a cycle shift of 1, thus it contains $u_1$ in $G^{ext}$. There is no additional shift at $v$, thus the following node in $G^{ext} is $v_{(1+\Delta_{u,v})}\% 3 = v_0$. At the following node there is a shift of one cycle ($r_w = 1$), thus $P$ contains $w_{(0+\Delta_{w,v})}\% 3 = w_2$.

This construction allows setting the arc weights in $G^{ext}$ independently of the specific path as $w_{(u,v,c,v)} = \sum_c w(b_{(c+e)}\%C)\%C_{a,c}$. Thus, finding a separating s-path in $G$ is equivalent to finding a simple path in $G^{ext}$ that respects both the weight and the delay constraint.

In case the end-to-end delay constraint is negligible, shortest path algorithms such as Dijkstra’s may be applied to find the shortest path (in terms of arc weights) in polynomial time. In contrast, finding a shortest path that also meets the weight and the delay constraint.

Our algorithm guarantees to find a separating s-path if one exists. Thus, LDN is solved to optimality and we obtain an upper bound to DN. To efficiently solve this pricing problem, we apply a dynamic programming algorithm (see Algorithm 1) that finds a suboptimal separating s-path for every demand $d$ in case it exists and guarantees to return an empty set in case no separating s-paths exist. The algorithm reduces to a recursive depth-first search (DFS) on the extended graph $G^{ext}$ which can be in practice generated on the fly.

For every node $v \in V$, we maintain a label $(w, \Delta)$ that signifies that $v$ has been reached by an s-path with an accumulated weight at most $w$ and latency at most $\Delta$. At any point during the execution of the algorithm we have $L(v) = L(v_e)$ for all $c, c'$. For every demand $d$, the label sets are initialized by $L(s^d) = \{(0, 0)\}$ and $L(v) = \emptyset$ for $v \in V \setminus \{v\}$. Let the current path at the node $u_c$ have a label $(w, \Delta)$. From $u_c$, the algorithm chooses a neighbor $v_{c'} \in \delta^{ext}_{c'}(u_c)$. The current path label is updated to $(w', \Delta') = (w + w_{(u_c,v_{c'})}, \Delta + \Delta_{(u_c,v_{c'})})$ where $\Delta_{(u_c,v_{c'})}$ is the delay of arc $a'$ including the cycle shift. The new path is rejected if the delay is too high, i.e. $\Delta + \Delta_{(u_c,v_{c'})} > \Delta^d$, or if it is dominated, i.e. if $L(v')$ contains a label $(\bar{w}, \bar{\Delta})$ for which $\bar{w} \leq w'$ and $\bar{\Delta} \leq \Delta'$. In this case, the algorithm goes back to $u_c$. Otherwise, the current label is added to $L(v')$, and all labels in $L(v')$ that are dominated by the current label are deleted. If destination node $t_d^d$ (for any $c$) is reached by the algorithm while the current delay $\Delta$ does not surpass the delay limit $\Delta^d$ and the current weight $w$ is smaller than $bw^d - \lambda^d$, add the corresponding path to the return set.

#### Algorithm 1: generate-s-Paths

```plaintext
\text{Algorithm 1: generate-s-Paths}
\begin{algorithmic}
\State $P^\prime := \emptyset$
\For {each $d \in D$}
  \State $w_{a,c} := \sum_{c'=0}^{c} bw_{(c+e)}\%C^\prime_{a,c'}$ \quad $\forall a, \forall c \in C$
  \State $L(s^d) := \{(0, 0)\}$
  \State $u := s^d, c := 0, w := 0, \Delta := 0$
  \State $P := \text{rec-s-Path}(u, c, w, \Delta, d)$
  \State $P'' := P'' \cup \{P\}$
\EndFor
\end{algorithmic}
```

Finally, return the set of all generated paths.

#### Algorithm 2: rec-s-Path$(u, c, w, \Delta, d)$

```plaintext
\text{Algorithm 2: rec-s-Path$(u, c, w, \Delta, d)$}
\begin{algorithmic}
\For {each $v \in \delta^+(u)$}
  \State $w := w + w_{(u,v),c}$ \quad $\triangleright$ iterate over outgoing arcs
  \State $\Delta := \Delta + \Delta_{(u,v)}$ \quad $\triangleright$ update delay
  \If {$v = t_d^d$}
    \State \text{if $(w, \Delta)$ feasible then}
      \State $\text{return} \ {((u, v), \emptyset)}$ \quad $\triangleright$ accept arc, done
    \Else
      \State $\text{reject current path}$
      \EndIf
  \Else\If {$w, \Delta$ feasible and not dominated}
    \State $\text{delete all labels dominated by (w, \Delta)}$
    \State $\text{return} \ {((u, v), r) + \text{rec-s-Path}(v, c, w, \Delta, d)}$ \quad $\triangleright$ accept arc and continue
    \Else
      \State $\text{reject current path}$
      \EndIf
  \EndIf
\EndFor
\end{algorithmic}
```

Note that in the worst case, Algorithm 1 terminates after all $s^d-t^d$-paths have been explored. However, in case of a tight delay bound, the algorithm is very fast. If it can be determined that the delay bound for demand $d$ is very permissive, the Algorithm 1 may be modified by reducing the bound $\Delta^d$ in a first run and, in case no path is found, iteratively increase it until its original value is reached. This procedure may avoid the enumeration of exponentially many paths.

### B. Randomized Rounding

Once the optimal solution $(y^*_p)$ to the linear relaxation LDN has been obtained, a feasible solution $y_p$ to DN is computed by randomized rounding. For a demand $d$ picked at random, we assign a probability of $y^*_p / \sum_{p \in P^d} y^*_p$ to each s-path $p \in P^d$. According to these probabilities, we choose a path $p \in P^d$. If there is sufficient residual capacity in the network, we assign the s-path to demand $d$. Otherwise, delete the path, renormalize the remaining probabilities and iterate until an s-path is assigned or no s-path with positive probability remains. Then, we continue with the next demand. This algorithm is executed several times. The best solution, referred to as CG-RR solution, is selected.
C. Improving the Fractional Solution

In order to improve both the upper bound given by the linear relaxation and the CG-RR solution, we leverage on the fact that in practice for any demand \( d \), the required bandwidth \( bw_c^d \) per cycle \( c \) is a multiple of a packet size \( ps^d \). While the packet sizes may vary among the demands, they are not arbitrarily distributed. If the arc capacities \( b_a \) are not multiples of the packet sizes, we can produce a fractional solution to DN that is closer to its optimal integer solution and thus improve the CG-RR solution as well as the upper bound by tightening the capacity constraints 3.

We denote the largest common divisor of the bandwidth requirements \( bw_{a,p}^d(c) \) of all paths \( p \in P^d \), \( d \in D \) by \( ps_a \), where \( D \) is the set of demands with at least one path through link \( a \) with cycle shift \( c \), \( P^d \) is the respective set of paths for \( d \) and \( P = \bigcup_d P^d \). We assume that \( ps_a \) is not a divisor of the arc capacity \( b_a \). Then capacity constraint is strengthened by division by \( ps_a \) for all \( c \):

\[
\sum_{p \in P:a \in p} \frac{bw_{a,p}^d(c)}{ps_a} y_p \leq \left\lfloor \frac{b_a}{ps_a} \right\rfloor \quad (7)
\]

This constraint is valid since the left hand side is integer, and it is stronger than 3 since \( \frac{b_a}{ps_a} < \frac{b_a}{ps_a} \).

VI. FAST GREEDY ALGORITHM

Alternatively to the CG-RR solution presented in Sec. V a more conventional Greedy approach is to route the demands one-by-one. When a demand \( d \) is next in line, the greedy algorithm tries to find a feasible s-path such that, for all affected arcs and all cycles, the capacity constraint is respected. We call such an s-path \( \bar{P} \)-feasible where \( \bar{P} \) is the set of already assigned paths. If no such s-path can be found, \( d \) is rejected, otherwise it is added to \( \bar{P} \). Such approach encompasses two subproblems: a) paths generation and b) path selection. The order of incoming demands is considered as input, such that the Greedy algorithm can also be used in an online setting.

A. Path Selection

Given a set paths \( P^d \) of \( \bar{P} \)-feasible s-paths for demand \( d \), the simplest approach to path selection is assigning the first (or a random) \( p \in P^d \). However, this can lead to very low traffic acceptance as bottleneck links can quickly appear and partition the network. To address this problem, we use a form of load balancing inspired by competitive online routing algorithms [23]. For two feasible sets of s-paths \( \bar{P}, P' \) for the same subset of demands, we consider a load balancing metric \( lb \) such that \( lb(\bar{P}) > lb(P') \) if solution \( P \) is more balanced.

Given a set of routed demands \( \bar{P} \) and demand \( d \), Greedy selects path \( p \in P^d \) for which \( lb(\bar{P} \cup \{p\}) \) is maximal. Based the idea of proportional fairness (see [23]), we use a load balancing metric \( lb \) that is maximal when the available bandwidth on the arcs \( A \) is fairly distributed:

\[
lb(\bar{P}) = \sum_{a \in A} \log(\alpha_a(\bar{P}) + \varepsilon).
\]

where \( \alpha_a(\bar{P}) \) is the percentage of unused bandwidth on arc \( a \) when paths \( \bar{P} \) are used. The addition of a small \( \varepsilon > 0 \) allows for the case in which exhausting the capacity of some arc cannot be avoided. The percentage of unused bandwidth is defined for the busiest cycle, i.e.

\[
\alpha_a(\bar{P}) = 1 - \max_{c} \sum_{p \in \bar{P}:a \in p} \frac{bw_{a,p}^d(c)y_p}{b_a}.
\]

This definition reflects that bandwidth should be kept available for future demands on all cycles in a fair manner.

B. Path Generation

In order to generate \( \bar{P} \)-feasible s-paths for any demand \( d \), one can search for a set of \( K \) maximally arc-disjoint paths and hope for a good load balancing. As for the IPRAN scenario described in Sec. VII specific knowledge about the network allows to define sets of bottleneck arcs that should be mutually avoided, the algorithm can become more effective. For this reason, we first identify sets of mutually avoidable arcs \( \{a_1, \ldots, a_k\} \) for which an arc can only belong to one s-path for \( d \). In our scenario, the set is composed by outgoing and incoming arcs respectively at the source and destination nodes. Then, using a shortest path algorithm with the delay as arc length (e.g. Dijkstra’s algorithm) and enforcing the use of exactly one of these arcs generates diversified s-paths \( P^d \).

The runtime of the algorithm depends in large parts on the path generation. Assuming a limitation \( K \) on the number of generated paths per demand and an efficient Dijkstra implementation, the runtime is in \( O(|D| K (|A| + |V| \log(|V|)) \).

VII. NUMERICAL EVALUATION

This section presents results in a realistic 5G scenario computed with a C++ environment on a 40×3.0 GHz machine with 190GB RAM. LPs are solved with IBM CPLEX 12.6.3.

A. Setup

We consider a typical IPRAN (IP Radio Access Network) scenario with 1700 nodes connected via 5200 directed arcs. The topology is divided into 3 layers: access, aggregation, and core. Access layer is composed of 1600 nodes, i.e. 800 BS (Base Station) and 800 CSG (Cell Site Gateway). The aggregation layer is composed of 80 nodes referred to as ASG (Aggregation Site Gateway). In the core layer there are 20 RSG (Radio Service Gateway) nodes connected to the EPC (Evolved Packet Core). We consider that each node is running the CSQF standard and up to \( N_{DN} = 3 \) queues can be used for DetNet traffic. We also study the case in which \( N_{DN} = 2 \) for all nodes, corresponding to a CQF network in which additional shifts are not possible. The capacity of links in the access and aggregation are 10 Gbps and 40 Gbps, respectively. In the core, links have a capacity at either 100 Gbps or 400 Gbps. Each BS has a 1-to-1 mapping with a CSG. Each CSG is connected to a pair of ASG via a direct link. Up to 20 CSG are connected to the same pair of ASG. Groups of ASG are connected via a ring with some additional shortcuts. A group
of connected ASG and their CSG form a domain. There are 10 domains in the network. The core network is fully meshed. The link delay is chosen proportionally to the distance between its nodes; for the access link it is uniformly distributed between 0.2 and 0.8 ms, corresponding to a distance of 10-40 km between elements. In the aggregation, the link delay is uniformly distributed between 0.8 and 1.6 ms, while in the core it is uniformly distributed between 2 and 10 ms. The cycle duration is 10 $\mu$s and the internal processing delay (worst-case) is 30 $\mu$s for each node.

We consider 250 to 2500 demands for each scenario. Each demand has a hypercycle $C = 12$ and a packet size of 500 Bytes. We consider that the traffic pattern is binary: either there is some traffic sent in a cycle or there is no traffic at all. In case there is some traffic, we consider that either 1 or 2 packets are sent per cycle, that corresponds to a max throughput of 200 Mbps. The same number of packets is sent in every cycle with data transmission. We consider three traffic patterns randomly selected: one data transmission every 2, every 3, or every 6 cycles. Demands are shifted at the beginning by a random number of cycles. 60% of demands are directed to a BS which is connected to the same pair of ASG, via the associated CSG nodes, of the source node (labeled as $D_1$), 30% of demands are directed to a BS which is in the same domain of the source node (labeled as $D_2$), and 10% of demands are directed to a BS in a different domain (labeled as $D_3$). The end-to-end delay constraint is using a discrete uniform distribution between 1, 2, and 3 ms for $D_1$ demands, between 4, 5, and 6 ms for $D_2$ demands, and between 40, 50, and 60 ms for $D_3$ demands.

To eliminate statistical fluctuations, results are obtained by averaging on 10 different traffic realizations. In the considered scenario, Greedy computes for each demand at most $K = 4$ disjoint paths when $N_{DN} = 2$ and $K = 8$ paths when $N_{DN} = 3$ to account for each possible time shift at the first CSG.

**B. Results**

Fig. 4a and Fig. 4b show, respectively, the percentage of accepted traffic and the gap to the best upper bound coming from the linear relaxation in CG-RR. We can see that for small amount of demands (i.e., less than 500), both CG-RR and Greedy nearly give the optimal solution. When the traffic increases (around 1000 demands), CG-RR still manages to get a solution equal to the upper bound (i.e., an optimal solution). For larger traffic, instead, both solutions plot an increasing gap because the linear relaxation provides an infeasible solution that accepts more traffic by splitting demands over multiple paths and multiple cycles. However, the real optimum lies in between the best upper bound and the integer solution found by CG-RR. CG-RR allows to provide a gap smaller than 10% for all the considered traffic scenarios, both for 2 and 3 queues.
The gap is slightly better for the scenario with 2 queues as the results provided by the linear relaxation are closer to the integer solution. The use of 3 queues allows to accept more demands as it enables to postpone traffic with non-critical delay constraints.

As shown in Fig. 4c, CG-RR provides a solution within a few seconds, which is quite reasonable for online network planning. While improving the solution by up to 5% (see next paragraph), the reinforcement of the model leads to a marginal increase of the execution time by up to 30% in the case of 3 queues. On the other hand, Greedy, which is paying for a larger gap to the best upper bound, can provide a solution to the planning problem within hundreds of microseconds, making this algorithm very suitable for online and ultra-fast demand acceptance (10 $\mu$s per demand).

The fact that we have an upper bound close to the solution provided by CG-RR and Greedy mainly depends on the reinforcement of constraints presented in Sec. V-C. Fig. 4d shows that for low traffic there is no significant improvement as the optimal solution is already provided. However, for larger traffic scenarios the improvement can be up to 30% as the linear relaxation is closer to the integer solution. The reinforcement of constraints allows to produce a solution for the linear relaxation that is closer to the integer optimum. As the use of 2 queues reduces the possibility of splitting traffic, a better upper bound is found.

Fig. 4d finally presents the improvement of CG-RR with modified constraints over the case with original constraints in terms of accepted traffic. We can see that the improvement is up to 10% if 2 queues are considered and up to 5% for 3 queues. The improvement comes from the fact that the reinforcement model gets a linear relaxation closer to the integer solution. As before, the smaller improvement for $N_{\text{DN}} = 3$ is due to the split of traffic in the linear relaxation over multiple paths and multiple cycles.

VIII. CONCLUSION

In this paper we presented two algorithms for the joint routing and scheduling problem of time-triggered flows in large scale deterministic networks using CSQF. We formulated the problem as an extension of a multi-commodity flow problem and analyzed its NP-hardness. We proposed an effective solution based on column generation and dynamic programming. Thanks to the reinforcement of the model with valid inequalities, we improved the upper bound and the solution. On realistic IPRAN instances, we demonstrated that we reach an optimality gap smaller than 10% in a few seconds. Finally, we also derived an ultra-fast adaptive greedy algorithm (10 $\mu$s per demand) that can be used online flow admission at the cost of an extra 5% gap when compared to our advanced solution based on column generation.

REFERENCES

[1] E. Grossman, “Deterministic Networking Use Cases,” RFC 8578, May 2019. [Online]. Available: https://rfc-editor.org/rfc/rfc8578.txt
[2] A. Nasrallah, V. Balasubramanian, A. S. Thyagarajan, M. Reisslein, and H. Elbakoury, “Cyclic queuing and forwarding for large scale deterministic networks: A survey,” CoRR, vol. abs/1905.08478, 2019.
[3] M. Chen, X. Geng, and Z. Li, “Segment Routing (SR) Based Bounded Latency,” Internet Engineering Task Force, Internet-Draft draft-chent-detnet-sr-based-bounded-latency-00, Oct. 2018.
[4] P. Pop, M. L. Raagaard, S. S. Craciunas, and W. Steiner, “Design optimisation of cyber-physical distributed systems using ieee time-sensitive networks,” IET CPS: Theory Applications, 2016.
[5] S. S. Craciunas, R. S. Oliver, M. Chmelik, and W. Steiner, “Scheduling Real-Time Communication in IEEE 802.1Qbv Time Sensitive Networks,” in Proc. of RTNS, 2016.
[6] R. Mahfouzi, A. Amirnilar, S. Samii, A. Rezine, P. Elees, and Z. Peng, “Stability-aware integrated routing and scheduling for control applications in ethernet networks,” in Proc. of DATE, 2018.
[7] “IEEE Standard for Local and metropolitan area networks: Cyclic Queuing and Forwarding,” IEEE 802.1Qch-2017, pp. 1–30, June 2017.
[8] N. Finn, J.-Y. L. Boudec, E. Mohammadpour, J. Zhang, B. Varga, and J. Parkas, “DetNet Bounded Latency,” Internet Engineering Task Force, Internet-Draft draft-finn-detnet-bounded-latency-04, 2019.
[9] L. Quang, X. Geng, B. Liu, T. Eckert, and L. Geng, “Large-Scale Deterministic IP Network,” Internet Engineering Task Force, Internet-Draft draft-quang-detnet-large-scale-detnet-04, 2019.
[10] N. G. Nayak, F. Dür, and K. Rothermel, “Incremental flow scheduling and routing in time-sensitive software-defined networks,” IEEE Transactions on Industrial Informatics, 2018.
[11] N. G. Nayak, F. Dür, and K. Rothermel, “Time-sensitive Software-defined Network (TSSDN) for Real-time Applications,” in Proc. of RTNS, 2016.
[12] J. Falk, F. Dür, and K. Rothermel, “Exploring Practical Limitations of Joint Routing and Scheduling for TSN with ILP,” in Proc. of IEEE RTCSA, 2018.
[13] F. Smirnov, M. Gla, F. Reimann, and J. Teich, “Optimizing message routing and scheduling in automotive mixed-criticality time-triggered networks,” in Proc. of ACM/EDAC/IEEE I2AC, 2017.
[14] B. Korte and J. Vygen, Combinatorial Optimization: Theory and Algorithms, ser. Algorithms and Combinatorics. Springer Berlin Heidelberg, 2007.
[15] V. Gurumanni, S. Khanna, R. Rajaraman, B. Shepherd, and M. Yannakakis, “Near-optimal hardness results and approximation algorithms for edge-disjoint paths and related problems,” Journal of Computer and System Sciences, vol. 67, no. 3, pp. 473–496, 2003.
[16] K. Kaparis and A. N. Letchford, “Local and global lifted cover inequalities for the 0–1 multidimensional knapsack problem,” European journal of operational research, vol. 186, no. 1, pp. 91–103, 2008.
[17] B. Korte and R. Schrader, “On the existence of fast approximation schemes,” in Nonlinear Programming 4. Elsevier, 1981, pp. 415–437.
[18] L. Khachiyan, “A polynomial algorithm in linear programming,” Soviet Mathematics, vol. 20, pp. 191–194, 1979.
[19] G. Desaulniers, J. Desrosiers, and M. M. Solomon, Column generation. Springer Science & Business Media, 2006.
[20] A. Schrijver, Combinatorial optimization: polyhedra and efficiency. Springer Science & Business Media, 2003.
[21] A. Juttner, B. Sviatovski, I. Mecs, and Z. Rajko, “Lagrangian relaxation based method for the QoS routing problem,” in Proc. of IEEE INFOCOM, 2001.
[22] B. Awerbuch, Y. Azar, and S. Plotkin, “Throughput-competitive online routing,” in Proc. of SFOCS, 1993.
[23] F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, “Rate control for communication networks: shadow prices, proportional fairness and stability,” Journal of the OR Society, vol. 49, no. 3, pp. 237–252, Mar 1998.