Imaging focused ultrasound pulses in superfluid helium 4

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Abstract. Focusing of sound pulses emitted by a hemispherical piezo-electric transducer in liquid helium 4 at 1.1 K and 24 bar is studied in detail. Time variations of the density map are recorded by using an optical interferometric method. Numerical integration of elastic wave equations by a finite difference scheme is used for comparison. A good agreement is found between both density maps. The amplification factor from the transducer surface to the focus is deduced.

1. Introduction
Since the early proposal of Greutzmacher[1], focusing ultrasound radiators shaped as a portion of sphere have been much studied[2, 3] and have found multiple applications. At low temperature, spherical and hemispherical transducers have been used to generate intense pressure swings to bring liquid helium in metastable states with respect to vaporisation or solidification[4, 5]. In the first case, the spherical symmetry reduces the modelisation to a 1-D radial differential equation. The non linearity of the propagation equation and of the fluid response are the remaining difficulties[6]. They can be neglected in the small amplitude limit. The hemispherical geometry is more involved : the cylindrical symmetry leaves us with a 2-D partial differential equation for which no analytical solution exists, even in the weak amplitude limit. Two questions are of interest for applications of those transducers : the shape of the wave in the focal region, and the amplification factor to be expected from their wide aperture focusing.

Analytical expressions of the pressure field have been derived from Rayleigh integrals[7, 2]. They yield formulae in closed form for the amplitude of the wave on the axis and in the focal plane. However these formulae are only approximately correct[2], neglecting the rescattering of the partial waves by the emitting surface.

This article reports a detailed imaging of the ultrasound wave emitted by a hemispherical transducer in liquid helium 4. The density maps deduced from these images are compared to a numerical simulation of the sound propagation in this geometry. Apart from small contributions of the non linearities, a good agreement is found, yielding quantitative answers to the above questions. This validates the experimental method for being used with other transducer shapes in anisotropic media such as hcp solid helium 4.
2. Apparatus and density measurement technique

The piezoelectric transducer (PZT) is a hemisphere (internal radius \( R_{\text{PZT}} = 6 \) mm, thickness 2 mm) manufactured by Channel Industries and modified in two ways. First, a 1 mm diameter hole is drilled at the top in which a metallic tube is inserted. It allows the PZT to be suspended in the middle of the cell, and provide also an electrical connection to the inner PZT electrode without any clamp in the lower rim region (see Fig.1). Second, the aperture angle is reduced from \( \theta = 90^\circ \) to \( \theta = 83^\circ \) by cutting 0.7 mm from the lower rim using a diamond saw. This puts the focal region near the center of the hemisphere in direct view while preserving the cylindrical symmetry. The PZT is driven at the frequency \( f_1 = 1.15 \) MHz of the first thickness resonance mode by a RF power amplifier. The excitation signal consists of a four oscillation pulse with an amplitude \( V = 95 \) Volt.

The experimental cell is cooled by a helium 4 fridge in an optical cryostat with four windows. The cell is filled with superfluid helium near the solidification line (\( T=1.12 \) K, \( P=24 \) bar). The speed of sound is \( v_s = 357.6 \) m/s [8].

Interferometric imaging is used to measure the map of helium density modulated by the ultrasound wave. The technique is described in details in reference [9] and may be summarized as follows. An image of a 4 \( \times \) 2.4 mm rectangular region is made on a CCD camera (502 \( \times \) 300 pixels) (see Fig.1). The imaged region is illuminated by a pulsed green laser. Part of the laser pulse is separated by a semi-transparent plate and goes through the cell in a lower region, unperturbed by the sound wave. It is used as a reference beam which is recombined coherently onto the CCD camera. Thus the camera actually records the interference between the image beam and the reference beam. Optical phase shifts \( \varphi(x, z) \) (see Fig.1 for axis orientation) induced by density modulations are extracted from several images taken with different phase of the reference beam. This process is then reproduced for different values of the delay \( t \) between the sound pulse emission and the imaging laser pulse to construct a movie of the acoustically induced phase map.

A movie containing 500 images of \( \varphi(x, z) \) is recorded, covering 25 \( \mu \)s with a 50 ns time step. The 4 mm field of view is smaller than the PZT radius. In order to have a wide field of view, the experiment is repeated three times with the interferometer translated by a few millimeter from the center to the edge of the PZT (see Fig.1). The three resulting movies are then merged to get a complete wide field movie of \( \varphi(x, z, t) \).

Because of the rotational invariance of the sound field around the \( z \)-axis, the variation of the refractive index \( \delta n \) depends only on the coordinate \( z \) and on the distance \( r \) to the \( z \)-axis. The optical phase map is then the integral along the \( y \)-axis of \( \delta n(r, z) \), i.e. its Abel transform. An inverse Abel transform[10] of \( \varphi(x, z) \) provides us the radial profile \( \delta n(r, z) \).
3. Numerical simulation of the focused spherical sound wave

For comparison with the experimental maps, a numerical simulation of the sound wave propagation was performed, neglecting non-linearities. Taking the PZT symmetry axis as the z-axis, only waves invariant around this axis are considered. They obey two coupled second order differential equations for the components $u_r$ and $u_z$ of the displacement vector:

$$\ddot{u}_r = v_s^2 \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) + v_s^2 \frac{\partial^2 u_z}{\partial r \partial z}$$

$$\ddot{u}_z = v_s^2 \left( \frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r} \frac{\partial u_r}{\partial z} \right) + v_s^2 \frac{\partial^2 u_z}{\partial z^2}$$

where $v_s$ is the speed of sound. These equations are integrated using a finite difference method and a staggered leap frog scheme with initial conditions corresponding to a uniform density. $\delta n$ is then computed as $\delta n = (n_0 - 1) \delta V/V = (n_0 - 1)(\frac{\partial n}{\partial r} + \frac{u_r}{r} + \frac{\partial u_r}{\partial z})$, with $n_0 = 1.033$. Some more details are given in reference [11]. At the transducer boundary, the displacement $(u_r, u_z)$ is forced to be that of the transducer. The motion of the PZT inner surface is set to that of a damped oscillator driven at resonance (1.15 MHz) by a 4 oscillation excitation. Its amplitude increases linearly for 4 periods up to $\delta u_0^{\text{sim}}$ and then decreases exponentially.

4. Comparison between experimental and numerical results

Figure 2 shows the experimental refractive index modulation $\delta n(0,0,t)$ at the acoustic focal point versus the time of flight of the sound wave. The sound wave reaches the acoustic focus at $t \simeq 17 \mu s$ (the predicted value is $R_{PZT}/v_s = 16.78 \mu s$).

Nonlinear behavior is conspicuous. In order to separate the density variations at resonance frequency $f_1$, $\delta n_1(r,z,t)$, from those at the first harmonic frequency $f_2 = 2f_1$, $\delta n_2(r,z,t)$, the signal is filtered numerically with two pass-band filters with a 1 MHz bandwidth, centered respectively at $f_1$ and $f_2$. This also improves the signal to noise ratio. As shown in Fig.2, harmonic amplitude can be neglected only during the first swings.

Since the numerical simulation uses linearized elastic equations, the comparison with experiment will be done only the first swings of $\delta n_1(r,z,t)$. The only unknown parameter is the experimental amplitude of the PZT surface oscillation $\delta u_0^{\exp}$. So matching numerical data $\delta n_1^{\text{sim}}$ to experimental $\delta n_1$ yields a scaling factor $\alpha$ which represents $\delta u_0^{\text{sim}}/\alpha = \delta u_0^{\exp}$. It has been verified that the scaling factor is valid for the first oscillations of the sound pulse for its propagation from the PZT surface to the focal point. Indeed the oscillation amplitudes of $\delta n_1^{\text{sim}}/\alpha$ and $\delta n_1$ differ by less than 10%, although the amplitudes themselves vary by more than two orders of magnitude.

This comparison provides also two useful informations: the transducer displacement $\delta u_0^{\exp} = \delta u_0^{\text{sim}}/\alpha \simeq 20$ nm, and the sound amplification factor $g = 104 \pm 5$ from the PZT surface to the

Figure 2. Experimental refractive index modulation (black) at the focus versus the time elapsed since the beginning of the PZT excitation. The amplitudes of the fundamental frequency and of the second harmonic (both gray), separated by time filtering, are also shown. In the following, only the beginning of the pulse is considered, for which second harmonic can be neglected.
Figure 3. Comparison between the experimental sound field $\delta n_1$ at time $t = 17.95 \mu$s (top) and simulated one $\delta n^{\text{sim}}/\alpha$ (bottom). Acoustical focus is indicated by a + symbol. PZT Lower rim is 0.7 mm above the focus. Profiles along the z-axis and r-axis and passing through the acoustical focus are also shown. The gray line corresponds to $\delta n_1$ and the dashed line to $\delta n^{\text{sim}}/\alpha$.

focus. Note that application of the Rayleigh formula gives $g = 2\pi(1 - \cos \theta)R_{\text{PZT}} f_1/v_s = 105$ in fair agreement with the experimental value.

More extended comparisons can be made. Figure 3 shows with the same gray scale maps of the focal region for $\delta n^{\text{sim}}/\alpha$ and $\delta n_1$. The similarity is indeed striking. From those maps, the focal profiles along the radial and axial directions can be extracted and are also shown.

5. Conclusion

We have demonstrated that an interferometric imaging technique can be used in a cryogenic environment to investigate ultrasound waves in material with small refractive index variations such as in liquid helium. It provides quantitative measurements of density over extended regions of space and time. The method have also some limitations. Cylindrical symmetry of the sound field is mandatory for using the inverse Abel transform. Time evolution can be investigated only for repetitive phenomena, allowing image sampling by the pulsed laser. Nevertheless it appears to be a valuable quantitative method, which can be used in the more complicated case of hcp solid helium 4 [to be published].

References

[1] Greutzmacher J 1935 Zeit.f.Physik. 96 342
[2] O’Neil H T 1949 J. Acoust. Soc. Am. 21 516
[3] Willard G W 1949 J. Acoust. Soc. Am. 21 360
[4] Nissen J A, Bogedom E, Brodie L C and Semura J S 1989 Phys. Rev. B 40 6617
[5] Herbert E, Balibar S and Caupin F 2006 Phys. Rev. E 74 041603
[6] Appert C 2003 The Eur. Phys. J. B 35 531–549
[7] Rayleigh J W S 1945 The Theory of Sound, Vol. 2 (Secs. 278, 302), 1896 (reprinted by Dover (New York))
[8] Maynard J 1976 Phys. Rev. B 14 3868–3891
[9] Souris F, Grucker J, Dupont-Roc J, Jacquier P, Arvengas A and Caupin F 2010 Applied Optics 49 6127
[10] Deutsch M and Beniamy I 1983 J. Appl. Phys. 54 137
[11] Souris F, Grucker J, Dupont-Roc J and Jacquier P 2011 J. of Low Temp. Phys. 162 412–420