Cosmological PPN Formalism and Non-Machian Gravitational Theories

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1. INTRODUCTION

There appears to be little agreement what Mach’s principle actually is - this was my chief impression from a recent conference on Mach’s principle at Tübingen (Germany). Should we relate it to the requirement that the inertial mass of a body is determined by the remaining bodies in the Universe? Or should we follow one of Barbours suggestions (Barbour 1995), perhaps that theories must be formulated in terms of relative quantities to become Machian, since we observe relative quantities (positions, velocities) only? Mach himself seems to be of little help in this respect since for almost every formulation of a Machian principle one may find an appropriate citation in his publications.

Another item from Mach’s General Store (Brill 1995) is the rather vague statement that distant parts of the Universe should have some influence on the form of local laws of physics. In a non-Machian gravitational theory a local experiment should therefore be independent of the cosmological environment. In particular, no fields of cosmological origin should have an influence on the local motion of matter, ”local” being taken to denote a sufficiently small space region. The purpose of this note is to show that - within a post-Newtonian approximation - metric theories of gravity may be divided according to the requirement of time-dependent potentials for a consistent local description of cosmological models. This kind of dependence on cosmological boundary conditions is used to define Machian and non-Machian theories.

To deal with a fairly general class of metric gravitational theories, the Parametrized Post-Newtonian (PPN-) formalism by Kenneth Nordtvedt and Clifford M. Will will be used. The notation is taken from Misner et al. (1973), otherwise I follow Will’s standard book (Will 1993).

The PPN formalism was clearly not designed to deal with cosmology. On the other hand, one has to face the fact that a Newtonian cosmology was developed decades ago (see, e.g., Heckmann and Schuecking 1955, 1956, 1959, Trautman 1966), with results very similar to those of General Relativity. The relation of this Newtonian cosmology to the Friedman models remained unclear, however. One expects that a Newtonian cosmology should emerge as a first approximation of the general-relativistic theory. Then one may ask for a post-Newtonian cosmology as a second-order approximation. Notice the Newtonian

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cosmology as discussed here is based on a Lorentzian manifold, and hence quite different from the Newton-Cartan formulation of a transition to Newton’s gravity (for the latter see, e.g., Trautman 1956, Ehlers 1981, Lottermoser 1988, Dautcourt 1990, and references therein).

Somewhat surprisingly, the PPN formalism as it stands is already able to some extent to deal with cosmological problems, as will be shown subsequently. The keys are (i) to use differential relations to avoid the Minkowskian boundary conditions at spatial infinity in the usual integral formulation of the PPN formalism, and (ii) to take the ratio distance (measured from the center of a selected spatial region) to Hubble distance $c/H_0$ as the expansion parameter. This leads to a simple local description of cosmological models, which should be valid for small values of the expansion parameter.

Section 2 summarizes basic aspects of Newtonian cosmology. In Section 3 PPN relations as well as the divergence freedom of the matter tensor are used to construct locally isotropic cosmological solutions beyond the Newtonian approximation.

Restricting to homogeneous models in Section 4, an inconsistency is found in generic metric theories (with the exception of the non-Machian class) at the post-Newtonian level. We suggest that the discrepancy might be caused by the neglect of time-dependent components in the Newtonian and post-Newtonian potentials at the origin of the PPN coordinate system.

Section 5 illuminates the problem from a different point of view: Starting from a co-moving cosmological coordinate system, in which the cosmological fluid with a divergence-free matter tensor is homogeneous and expands isotropically, a transformation into the standard PPN coordinate system produces generically time-dependent local potentials. Again the potentials are absent in a small subset of theories, formed by a non-Machian class of gravitational theories.

2. NEWTONIAN COSMOLOGY

The PPN formalism assumes the existence of a coordinate system where the metric tensor can be written as

\begin{align}
g_{00} &= -1 + 2U - 2\beta U^2 + 4\Psi - \zeta A + O(\epsilon^6), \\
g_{0i} &= -\frac{7}{2}\Delta_1 V_i - \frac{1}{2}\Delta_2 W_i + O(\epsilon^5), \\
g_{ik} &= \delta_{ik}(1 + 2\gamma U) + O(\epsilon^4).
\end{align}

The space-time functions $U, \Psi, A, V_i, W_i$ are defined in terms of the matter density, the pressure $p$ and velocity components $v_i(x, t)$ of the fluid:

\begin{align}
U(x, t) &= \int d{x}' \rho(x', t) / | x - x' |, \\
\Psi(x, t) &= \int d{x}' \rho(x', t) \phi(x', t) / | x - x' |, \\
V_i(x, t) &= \int d{x}' \rho(x', t) v_i(x', t) / | x - x' |,
\end{align}

2
\[ W_i(x, t) = \int d'x' \rho(x', t)[(x - x')(v(x', t))(x^i - x'^i)]/ |x - x'|^3, \]  
(7)

\[ A(x, t) = \int d'x' \rho(x', t)[(x - x')(v(x', t))^2]/ |x - x'|^3, \]  
(8)

where

\[ \phi(x, t) = \beta_1 v^2 + \beta_2 U + \frac{3}{2} \beta_4 p/\rho_0. \]  
(9)

As it stands, the PPN formalism appears not to be applicable to cosmology: If a homogeneous matter density \( \rho(t) \) is introduced, the Newtonian potential \( U \) as calculated from (4) diverges - this is the well-known Seeliger-Neumann paradoxon, seen to be present also in a higher-order correction to the Newtonian potential. A simple way to overcome this difficulty was proposed by Heckmann and Schücking (1959), see also Trautman (1966): In agreement with the equivalence principle, the Newtonian potential and its first derivatives are considered as quantities subject to changes induced by coordinate transformations beyond the Galileo group. The second derivatives only, which are determined by the generating mass density, should have a physical meaning. Thus the function

\[ U = -\frac{2}{3} \pi G \rho(t)r^2, \]  
(10)

was taken as the potential in Newtonian cosmology. It is evidently a solution of the Poisson equation for \( U \),

\[ \Delta U = -4\pi \rho, \]  
(11)

generated by homogeneous matter density. The divergence of \( U \) for large \( r \) can be considered as coordinate singularity, now caused by the use of a local (non-cosmological) coordinate system. The singularity vanishes, if a transformation to cosmological (comoving) coordinates is performed, see below Eq.(19).

The post-Newtonian approximation as applied to objects in the Solar system uses two expansion parameters \( \epsilon = \sqrt{U_m} \) and \( v/c \), where \( U_m \) is the maximal value of the Newtonian potential and \( v \) is the velocity of bodies moving under the influence of gravity. For the bookkeeping of the different approximations both parameters are usually considered as of equal order. This is justified also in the case considered here: The cosmological potential (10) gives \( \epsilon \approx r \sqrt{2\pi \rho_0 G/3} \approx r/L \), where \( L \) is a typical Hubble distance. The radial velocity of galaxies is of equal order as is evident from Hubbles redshift-distance relation. As dimensionless expansion parameter \( \epsilon \) one may therefore take the ratio distance to Hubble distance.

To Newtonian approximation the metric tensor is given by

\[ g_{00} = -1 + 2U + o(\epsilon^4), \]  
(12)

\[ g_{0i} = o(\epsilon^3), \]  
(13)

\[ g_{ik} = \delta_{ik} + o(\epsilon^2), \]  
(14)

and the dynamical equations reduce in the case of dust matter, which will be assumed for simplicity throughout this note, to

\[ \partial \rho/\partial t + \partial (\rho v^i)/\partial x^i = 0, \]  
(15)

\[ \rho(\dot{v}_i + v_{i,k}v_k) = \rho U_{,i}, \]  
(16)
where the potential $U$ is given by the solution (10) of the Poisson equation (11). For an isotropic expansion with a scale factor $R(t)$

$$v^i = x^i f_0(t) = x^i \dot{R}/R,$$

(17) and (16) together with the potential (10) lead to matter conservation and to the Friedman equation:

$$R^3 \rho = \text{const,} \quad \dot{R}^2 = 2GM/R + \text{const.} \quad (18)$$

This similarity between Newtonian and general-relativistic cosmology has sometimes been taken as evidence that Newtonian gravity might have a wide range of applicability in cosmology - in the hope to discard the mathematical apparatus of Einstein’s theory in favour of the much simpler apparatus of Newton’s theory. This is to some degree misleading, and the validity of the Friedman equation within Newtonian cosmology is easily understood: One can introduce comoving cosmological coordinates $\xi^k, \tau$ instead of the local PPN-coordinates $x^k, t$ used so far by requiring

(i) that the space components of the matter 4-velocity vanish and that

(ii) $\hat{g}_{00} = -1, \hat{g}_{0i} = 0$ in cosmological coordinates.

The transformation

$$\tau = t - r^2 \dot{R}/(2R) + o(t\epsilon^2), \quad \xi^k = x^k/R + o(r\epsilon)$$

leads to

$$\hat{g}_{00} = -1 + o(\epsilon^4), \quad (20)$$

$$\hat{g}^{0k} = o(\epsilon^3), \quad (21)$$

$$\hat{g}^{ik} = \delta_{ik}/R(\tau)^2 + o(\epsilon^2), \quad (22)$$

and satisfies (i) and (ii) to Newtonian order. It is now easy to see why the Newtonian scale factor $R(t)$ must be a solution of the Friedman equation: Consider the Friedman equation for the correct general-relativistic scale factor $S(\tau)$. Writing

$$S(\tau) = S(t - r^2 \dot{R}/(2R)) \approx S(t) - \dot{S}r^2 \dot{R}/(2R)$$

(23)

in this equation and expanding, terms $\sim (r/L)^2$ cancel, and one is left with the Friedman equation for the Newtonian scale factor $R(t) \equiv S(t)$.

Thus it appears that the applicability of the Newtonian approximations (12) - (18) with the potential (10) is confined to a **neighbourhood of the observer, corresponding to distances small compared to the Hubble distance.** As a consequence, the matter density $\rho$ in local coordinates is a pure time function only on the observer world line $x^i = 0$ and changes with $r$ further out just to ensure homogeneity or a Copernican principle. Indeed, any scalar depending only on cosmic time in cosmological coordinates, depends in a local PPN coordinate system also on spatial coordinates (see Eq. (19)).

Many years ago, Callan, Dicke and Peebles (1962) have presented similar ideas concerning the relation between Newtonian mechanics and cosmological expansion.
3. POST-NEWTONIAN EQUATIONS

Beyond the Newtonian approximation we use instead of (5), (6) the corresponding differential formulations ($G = 1$ subsequently, we also write $\Psi = \beta_1 \Psi_1 + \beta_2 \Psi_2$, assuming dust):

\[
\Delta \Psi = -4\pi \rho \phi, \tag{24}
\]
\[
\Delta V_k = -4\pi \rho v_k. \tag{25}
\]

Differential relations also hold for the remaining functions. $W_i$ may be written

\[
W_i = \int d\mathbf{x}' \rho(\mathbf{x}', t)(\mathbf{x} - \mathbf{x}')\mathbf{v}' \frac{\partial}{\partial x^i} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \]
\[
= -x^k V_{i,k} + \frac{\partial}{\partial x^i} \int d\mathbf{x}' \rho(\mathbf{x}', t)\mathbf{x}' \mathbf{v}' / |\mathbf{x} - \mathbf{x}'|,
\]

or after applying the Laplacian

\[
\Delta W_i = -4\pi \rho v_i - 2V_{k,ki}. \tag{26}
\]

Finally we list some further relationships (see Will 1993, chapter 4.1) already in differential form:

\[
V_{k,k} = -U_{,0}, \tag{27}
\]
\[
\Delta \zeta = -2U, \tag{28}
\]
\[
\zeta_{,0} = V_k - W_k, \tag{29}
\]
\[
\zeta_{,00} = \mathcal{A} - \Psi_1. \tag{30}
\]

The differential formulation of the PPN formalism used in this article is based on the (not all independent) relations (24) - (30) and (11). We write down expansions in terms of powers of $r$ similar to (10) for the post-Newtonian potentials $V_i, W_i, \Psi, \mathcal{A}$ as well as for the fluid density and velocity, and look for homogeneous and isotropic purely expanding solutions. The matter density and velocity for isotropically moving dust can be expanded as

\[
\rho = \rho_0 + \rho_2 r^2, \tag{31}
\]
\[
v^k = x^k (f_0 + f_2 r^2), \tag{32}
\]

where $\rho_0, \rho_2, f_0, f_2$ are time functions. Using this power law expansion, a straightforward calculation yields

\[
U = -\frac{2}{3} \pi \rho_0 r^2 - \frac{1}{5} \pi \rho_2 r^4, \tag{33}
\]
\[
\Psi = \frac{1}{15} (-3\pi \beta_1 \rho_0 f_0^2 + 2\beta_2 \pi^2 \rho_0^2) r^4, \tag{34}
\]
\[
V_k = -\frac{2}{5} \pi \rho_0 f_0 r^2 x^k, \tag{35}
\]
\[
W_k = -V_k, \tag{36}
\]
\[
\mathcal{A} = \frac{1}{5} \pi \rho_0 (3f_0^2 + \frac{4}{3} \pi \rho_0) r^4. \tag{37}
\]
Up to post-Newtonian order, the metric tensor can now be written:

\[
g_{00} = -1 + br^2 + cr^4, \quad (38)
g_{0i} = mr^2x^i, \quad (39)
g_{ik} = \delta_{ik}(1 + gr^2). \quad (40)
\]

The time functions \( b, c, m, g \) are determined by the distribution and motion of matter:

\[
b = -\frac{4}{3}\pi \rho_0, \quad (41)
c = -\frac{2}{5}\pi \rho_2 + \frac{4}{45}\pi^2 \rho_0^2(6\beta_2 - 3\zeta - 10\beta) - \frac{1}{5}\pi \rho_0 f_0^2(4\beta_1 + 3\zeta), \quad (42)
m = \frac{1}{5}\pi \rho_0 f_0(7\Delta_1 - \Delta_2), \quad (43)
g = -\frac{4}{3}\pi \rho_0 \gamma. \quad (44)
\]

From the conservation law of the matter tensor, \( T^{\mu\nu} ; \nu = 0 \), one obtains in Newtonian approximation, in agreement with (18),

\[
\dot{f}_0 = -\frac{4}{3}\pi \rho_0 - f_0^2, \quad (45)
\dot{\rho}_0 = -3f_0\rho_0. \quad (46)
\]

The post-Newtonian terms lead to equations for the next coefficients \( f_2 \) and \( \rho_2 \):

\[
\dot{f}_2 = -4f_0 f_2 + \frac{2}{15}k_1 \pi \rho_0 f_0^2 + \frac{4}{45}k_2 \pi^2 \rho_0^2 - \frac{4}{5}\pi \rho_2, \quad (47)
\dot{\rho}_2 = -5f_0 \rho_2 - 5f_2\rho_0 + \frac{2}{3}\pi f_0 \rho_0^2(2 - 3\gamma), \quad (48)
\]

with

\[
k_1 = -12\beta_1 - 9\zeta + 42\Delta_1 - 6\Delta_2 - 20\gamma + 5,
k_2 = -20\beta + 12\beta_2 - 6\zeta + 21\Delta_1 - 3\Delta_2 - 20\gamma.
\]

The system (45) - (48) can be integrated fairly easily: From (45), (46) one obtains a second-order linear differential equation for \( f_0 \) alone, \( \rho_0 \) is determined algebraically upon solution of the \( f_0 \) equation. Similarly, if a solution of (45),(46) is known, (47),(48) can be transformed into a linear second-order differential equation for \( f_2 \), leaving an algebraic equation for \( \rho_2 \).

4. HOMOGENEITY REQUIREMENTS

The models obtained in this way represent - in a post-Newtonian approximation - a large class of locally isotropic, but in general inhomogeneous expanding matter distributions in the neighbourhood of the observer. Restrictions for homogeneity may be
introduced by several methods. Usually one relies on groups of motions. A more intuitive way is to impose homogeneity with a Copernican principle. The origin of the PPN coordinate system may be associated with a freely falling observer \( P \). The worldline of a second observer \( \bar{P} \) at rest with regard to the cosmological substratum in his neighbourhood is given by \( x'^i_0(t) \), with the three functions \( x'^i_0(t) \) as solutions of the differential equations \( \dot{x}_0^i = x_0^i(f_0 + f_2 r_0^2) \). The coordinate transformation leading from the \( P \)-system to the \( \bar{P} \)-system is a generalization of the Chandrasekhar-Contopoulos (1967) transformation, since the second observer moves with acceleration. A Copernican principle should state the equivalence of all these systems. More specific, in the local PPN system of any comoving observer the expansion coefficients of matter density and cosmic velocity (Eqns. (31), (32)) should have the same dependence on the corresponding coordinate time. We may set up a Copernican principle for the matter density and velocity as well as for the metric or derived quantities such as the expansion rate of the matter congruence. It is not obvious whether all these principles lead to the same conclusions.

For purpose of this note we can avoid the construction of a generalized Chandrasekhar-Contopoulos transformation by referring to Eq.(19): Homogeneity for scalars \( \Phi \) can be defined by a dependence on the cosmological time \( \tau \) only, hence \( \Phi(\tau) = \Phi(t - r^2 f_0/2) \approx \Phi(t) - r^2 f_0 \Phi(t)/2 \), and the local expansion coefficients \( \Phi_0 \) and \( \Phi_2 \) are related by \( \Phi_2 = -f_0 \Phi_0/2 \). Applying this to the matter density gives with (46)

\[
\rho_2 = \frac{3}{2} f_2 \rho_0.
\]

(49)

It is easy to check that (49) is compatible with the differential equations (45)-(48) if and only if the conditions

\[
6\zeta - 12\beta_1 + 20\beta + 3(\Delta_2 - 7\Delta_1) + 26\gamma - 16 = 0, \quad (50)
\]

\[
9\zeta + 12\beta_1 + 6(\Delta_2 - 7\Delta_1) + 20\gamma + 4 = 0. \quad (51)
\]

for the PPN coefficients are satisfied. Moreover, \( f_2 \) is given by

\[
f_2 = \frac{2}{15} \pi \rho_0 f_0 (8 - 3\gamma). \quad (52)
\]

(49) - (52) also ensure the homogeneity of the fluid’s expansion rate up to post-Newtonian order via a relation similar to (49).

The problem is that (50) and (51) hold only for a fairly small class of gravitational theories. As one easily checks, General Relativity (\( \zeta = 0 \), all other coefficients = 1) belongs to this class. Needless to say that reasonable cosmological solutions are known for many metric theories, e.g. for the often discussed Brans-Dicke theory with \( \gamma = (1 + \omega)/(2 + \omega), \beta_1 = (3 + 2\omega)/(4 + 2\omega), \beta_2 = (1 + 2\omega)/(4 + 2\omega), \Delta_1 = (10 + 7\omega)/(14 + 7\omega) \) (other parameters as in GR), which do not satisfy (50) and (51).

Obviously, the framework used so far is not adequate. To deal with a possible cosmological time evolution in the Newtonian dynamics of compact bodies, K. Nordtvedt (1993) has introduced time-depending PPN parameters (as well as time-dependent coupling constants and masses). It is perhaps not necessary to go so far. Evidently the cosmological
solution (19) of the Poisson equation is incomplete, one could have added an arbitrary
time function $U_0(t)$ to the potential. This also applies to the post-Newtonian potentials,
giving additional time functions $a(t), l(t), d(t)$ in the metric tensor:

\begin{align*}
g_{00} &= -1 + a + br^2 + cr^4, \quad (53) \\
g_{0i} &= x^i(l + mv), \quad (54) \\
g_{ik} &= \delta_{ik}(1 + d + gr^2). \quad (55)
\end{align*}

If the added time function vary on scales not smaller than the Hubble time they have
small or negligible influence on the motion of local bodies. The equations (53) – (55) should
allow to work out a consistent post-Newtonian approximation scheme, as will be discussed
elsewhere.

5. COSMOLOGICAL COORDINATES

To obtain different look at the problem, we may search for solutions in cosmological
coordinates, using symmetries and matter conservation but no other field equations. In a
second step we transform into a local PPN coordinate system and compare the resulting
metric with the local one following from (1)-(3) and (33)-(37).

Purely expanding (or contracting) dust matter satisfies

\begin{align*}
T^\mu{}^\nu &= \mu u^\mu u^\nu, \quad (56) \\
u_{\mu;\nu} &= \Theta(g_{\mu\nu} + u_\mu u_\nu), \quad (57)
\end{align*}

as well as the divergence relation

\begin{equation}
T^\mu{}_{;\nu} = 0. \quad (58)
\end{equation}

One easily verifies the existence of a comoving ($u^\mu = \delta^\mu_0$) coordinate system $(\tau, \xi^i)$, in
which the line element can be written

\begin{equation}
ds^2 = -d\tau^2 + S(\tau, \xi^i)2\gamma_{ik}(\xi^k)d\xi^id\xi^k. \quad (59)
\end{equation}

(58) reduces to

\begin{equation}
\mu S^3 = \text{const}. \quad (60)
\end{equation}

No field equation has been used so far. To restrict to homogeneous models, it is assumed
additionally to (56)-(58) that the scale factor $S$ depends on the cosmological time $\tau$ only
and that

\begin{equation}
\gamma_{ik} = \delta_{ik}/(1 - k\xi^2/4)^2, \quad \xi^2 = \xi^k\xi^k \quad (61)
\end{equation}
is the metric of a 3-space of constant curvature. To relate the cosmological coordinate system
to the standard PPN gauge up to post-Newtonian order, we extend the transformation (19) by writing

\begin{align*}
\tau &= u(t) + v(t)r^2 + w(t)r^4, \quad (62) \\
\xi^i &= x^i(1/F(t) + p(t)r^2). \quad (63)
\end{align*}
The function \( u(t) \) accounts for the possibility that the local PPN time differs from the cosmological time even at the origin of the PPN system. Also, the transformation function \( F \) could differ from the scale factor \( R(t) = S(u[t]) \). A PPN coordinate system is defined by the condition, that the quantities entering the metric \((1) - (3)\) must be solutions of the PPN differential relations. One has furthermore to ensure that up to and including the order \( \epsilon^2 \) the spatial components \( g_{ik} \) attain no non-diagonal term. This latter condition restricts the function \( p(t) \) in (63) to

\[
p(t) = F v^2 / R^2.
\]

Comparing now the local and transformed metric on obtains to zero order in \( r \) from the \( g_{00} \) and \( g_{ik} \) components

\[
\dot{u}^2 = 1 - 2U_0 + 2\beta U_0^2 - 4\Psi_0 + \zeta \mathcal{A}_0,
\]

\[
F = R / \sqrt{1 + 2\gamma U_0}.
\]

These relations cast some light on the meaning of local potentials. The relation between comoving and local coordinates is determined by the local scale factor \( F(t) \), which differs from the cosmological scale factor \( R(t) \), if a local potential \( U_0(t) \) is present. A time dependent potential in \( g_{00} \) comes not only from \( U_0 \), but also from the \( \mathcal{A} \) and \( \Psi \) terms, as seen in (65). If the combination of local potentials on the rhs of (65) differs from zero, the PPN coordinate time is not equal to the proper time measured by an observer on the center world line \( x^k = 0 \) of the PPN system (this world line is still a geodesic, however).

In order to check the conditions for the PPN parameters found above we exclude all local components in \( g_{\mu\nu} \). Furthermore, the relation (60) as well as \( u^\mu = \delta^\mu_0 \) can be transformed into the local coordinate system up to post-Newtonian order. We then compare the transformed cosmological solution with the local metric specified by (1)-(3) and (10), (33)-(37). Additionally to the determination of the transformation functions, all these relations give the following restrictions for the PPN parameters:

\[
\gamma = 1,
\]

\[
27(7\Delta_1 - \Delta_2) - 48\beta_1 + 12\beta_2 - 20\beta - 106 = 0.
\]

The domains in the PPN parameter space covered by (50), (51) on the one side and by (67), (68) on the other do not coincide. (Note however, if we assume \( \gamma = 1 \) in (50), (51), Eq. (68) follows from (50), (51)). But coincidence cannot be expected. In deriving (50), (51) we have assumed a restricted formulation for the Copernican principle (homogeneity for the matter density). Other formulations for this principle may lead to different conditions for the PPN parameters. On the other side, the discussion in this section is complete with regard to symmetries but lacks full dynamics.

It is open if a full consideration of both symmetry conditions and dynamics would confine the set of theories which are free from local potentials to General Relativity alone.

6. CONCLUSION

We have discussed the question, how cosmological boundary conditions may influence the motion of local matter. There is no influence for General Relativity (and a small
number of additional theories of gravity) up to post-Newtonian order. For the majority of other gravitational theories there exists an influence at the post-Newtonian level of approximation. We have called this different behaviour of gravity theories non-Machian and Machian, respectively. In a post-post-Newtonian approximation the local motion of matter may depend on cosmological boundary conditions also in General Relativity, so that every metric theory of gravity would become Machian according to the definition given here. Only cosmological background fields have been used in this article. A study of perturbations up to post-Newtonian order may give a chance to determine some PPN parameters also in a cosmological setting.

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