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Deterministic Photon Sorting in Waveguide QED Systems

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Sorting quantum fields into different modes according to their Fock-space quantum numbers is a highly desirable quantum operation. In this Letter, we show that a pair of two-level emitters, chirally coupled to a waveguide, may scatter single- and two-photon components of an input pulse into orthogonal temporal modes with a fidelity $\gtrsim 0.9997$. We develop a general theory to characterize and optimize this process and reveal that such a high fidelity is enabled by an interesting two-photon scattering dynamics: while the first emitter gives rise to a complex multimode field, the second emitter recombines the field amplitudes, and the net two-photon scattering induces a self-time reversal of the input pulse mode. The presented scheme can be employed to construct logic elements for propagating photons, such as a deterministic nonlinear-sign gate with a fidelity $\gtrsim 0.9995$.

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Strong nonlinearity at the few-photon level is key to all-optical quantum information processing (QIP) [1]. In the last decade, quantum nonlinear optical phenomena have been demonstrated on various platforms [2], including cavity and waveguide quantum electrodynamics (QED) setups [3], atomic ensembles [4], and optomechanics [5]. Following these experimental achievements, the next step is to develop schemes that can utilize the acquired nonlinearity to perform high-fidelity QIP operations, such as single-photon transistors [6], single-photon subtractors [7], and photonic logic gates [8,9]. Among these quantum devices, a photon sorter which can separate single- and two-photon components of a single-mode input state into orthogonal output modes is particularly useful [10–13]. Reference [11] thus proposed to employ the chiral coupling to a two-level emitter and scatter a single-mode input pulse into a field with the different number states occupying different photonic temporal modes [14].

In this Letter, we establish a systematic approach to evaluate and optimize the performance of photon sorting in temporal-mode space. We find that the sorting by a single emitter, proposed in Ref. [11], is hampered by a small but finite occupation of undesired modes, while, for a suitably optimized input pulse, the subsequent scattering on a second emitter restricts the one- and two-photon states to two orthogonal output modes with a very high fidelity (see Fig. 1). Our theoretical approach identifies a novel self-time-reversal mechanism of two-photon states by pairs of emitters, and we verify that our optimal photon sorting is robust against experimental imperfections. This makes it a promising element in efficient Bell state analysis [10], photonic controlled phase gates [11], as well as measurement-based quantum computing [13].

Model.—We study a waveguide QED system, where an incident pulse interacts with $N_e$ two-level emitters in a unidirectional manner (see Fig. 1 for $N_e = 2$) [15–18]. We focus here on the scattering of a single-photon state $\hat{a}_0^\dagger |0\rangle$ and a two-photon state $(\hat{a}_0^\dagger)^2 |0\rangle/\sqrt{2}$, where $\hat{a}_0^\dagger = \int dk \phi(k) \hat{a}_k^\dagger (k)$ creates a single photon in the input mode $\phi(k)$. We assume a unit propagation speed, and $\hat{a}_k^\dagger (k)$ is the creation operator in wave number (and frequency) space. The output states for the single- and two-photon input are respectively given by $\int d\psi(k) \hat{a}_k^\dagger (k) |0\rangle$ and $\int d\psi(k_1, k_2) \hat{a}_k^\dagger (k_1) \hat{a}_k^\dagger (k_2) |0\rangle/\sqrt{2}$, where the single-photon pulse $\psi(k) = T(k) |\phi(k)|^2$ is modified by a linear transmission coefficient $T(k)$, while the two-photon wave function $\Psi(k_1, k_2) = \int dp_1 dp_2 S(k_1, k_2; p_1, p_2) |\phi(p_1)\phi(p_2)|$ is governed by the scattering matrix $S$ [19–22]. The cascaded feature of the chiral waveguide QED system allows us to obtain $T(k)$ and $S$ from the transmission

![FIG. 1. Sequential scattering of a single-photon pulse $\phi(k)$ and two-photon pulse $\Phi(k_1, k_2) = \phi(k_1)\phi(k_2)$ by a pair of chirally coupled two-level emitters. (a) The linear dispersion of the single photon component. (b) The nonlinear scattering of the two-photon state into multiple modes and back to a single-mode output.](image-url)
To address this question, we first study the sorting performance of a single two-level emitter for a Lorentzian input pulse $\phi(k) \propto 1/(k^2 + \sigma^2)$ with different spectral widths $\sigma$ in units of the coupling strength $\Gamma$ between the emitter and the guided mode. We extract the double and single excitation probabilities $|c_2|^2$ and $|c_1|^2$ by the relations established in Eqs. (3) and (4) as well as by the input-output quantum pulse method [27,28]. As shown in Figs. 2(b) and 2(c), these two methods exhibit excellent agreement with each other, and they both identify perfect zeros of $|c_2(\sigma)|^2$ (dashed lines). They also show unfortunate maxima of $|c_1(\sigma)|^2$ for the same pulses, and this eventually results in an imperfect sorting of Lorentzian input pulses with $|c_1|^2 + |c_2|^2 \geq 0.26$.

Optimization protocol.—The above results suggest that we need to establish a joint optimization protocol that takes both $c_1$ and $c_2$ into account in the search for the optimal input mode $\phi(k)$. To this end, we first define the sorting error to be $E = |c_1|^2 + |c_2|^2$, and the corresponding sorting fidelity $\mathcal{F} = 1 - E$. With Eqs. (3) and (4), the sorting error $E$ can be expressed as a functional of the input mode function $\phi(k)$ (see the Supplemental Material [29]), i.e., $E(\phi, \phi^*) = \int dp dp' \phi^*(p) \mathcal{H}(p, p') \phi(p')$, where the Hermitian kernel $\mathcal{H}$ is given by

$$\mathcal{H}(p, p') = 2 \int dk_1 \mathcal{L}_1(k_1, p) \mathcal{L}_2(k_1, p') - \mathcal{L}_2^*(p) \mathcal{L}_2^*(p'),$$

with $\mathcal{L}_1(k, p)$ and $\mathcal{L}_2(p)$ defined as

$$\mathcal{L}_1(k, p) = \int dk_1 dp_1 S(k_1, k; p_1, p) \mathcal{T}^*(k_1) \phi^*(k_1) \phi(p_1),$$

$$\mathcal{L}_2(p) = \int dk \mathcal{T}^*(k) \phi^*(k).$$

Now, the optimization problem amounts to finding the wave function that minimizes the error functional $E(\phi, \phi^*)$. 

FIG. 2. (a) Decomposition of the two-photon output wave function using single-photon pulse modes $\psi(k)$ and $\{\theta_n(k)\}$. (b), (c) The probabilities $|c_2|^2$ and $|c_1|^2$ [cf. Eq. (2)], as a function of the linewidth $\sigma$ of an input Lorentzian pulse. The circles and the solid lines are obtained by the quantum pulse method and Eqs. (3) and (4), respectively.
in momentum space and dictates that its time-domain counterpart \( \tilde{\phi}(t) = (1/\sqrt{2\pi}) \int \text{d}k \tilde{\phi}(k)e^{-ikt} \) is a real function, as plotted in the lower inset of Fig. 3(a) (tilde \( \sim \) is used to distinguish the time-domain wave function from the \( k \)-space one throughout this Letter).

As the optimal sorting by a single emitter is far from being deterministic, we then investigate whether an improvement can be made by adding more scatterers to the system. Figure 3(b) shows the sorting performance for two identical emitters. Surprisingly, the optimal sorting fidelity approaches unity (\( F \gtrsim 0.9997 \)) in this case, and the optimal input pulse \( \tilde{\phi}(t) \) is more regular in shape, resembling a Gaussian with a slight asymmetry. To gain more insight in this significant improvement, we analyze the two-photon output wave function for the optimal input pulse based on the Takagi decomposition of the state [Eq. (1)]. For a single two-level emitter, the output state occupies multiple basis functions [see Fig. 3(c)], and the most populated mode \( \tilde{\psi}_1(t) \) [inset of Fig. 3(c)] has a weight \( |a_1|^2 \approx 0.5579 \). An unexpected phenomenon occurs when two emitters are included: the output photons are confined to a single temporal mode \( \tilde{\psi}_1(t) \) with a probability \( |a_1|^2 \approx 0.9911 \) [see Fig. 3(d)], whose shape approaches the time-reversed input pulse, i.e., \( \tilde{\psi}_1(t) \approx \frac{1}{t_d} \tilde{\psi}(t_d - t) \) with \( t_d \) a delay time. We then examine the wave function \( \Psi_m(k_j) \) of the field between the two emitters. As illustrated in Fig. 1(b), the two-photon state is entangled over several temporal modes due to the first scatterer. However, the optimal pulse appears to be a special input mode, which under the action of the single-emitter scattering matrix \( S_0 \) generates an output wave function \( \Psi_m(k_1, k_2) \approx -|\Psi_m(k_1, k_2)|e^{i(k_1+k_2)t_d/2} \). Such a quasi-linear dispersion indicates \( \Psi_m(-k_1, -k_2)e^{i(k_1+k_2)t_d/4} \), which makes the second process a shifted time reversal of the first one, i.e., \( \int \text{d}p_1 \text{d}p_2 S_0(k_1, k_2; p_1, p_2) \Psi_m(p_1, p_2) \approx -\phi(-k_1)\phi(-k_2) \langle \tilde{\psi}_1 \rangle \), resem-
the parity of \( N_e \). For odd \( N_e \), increasing the emitter number can gradually improve the sorting performance. When \( N_e \) is even, the second half of the system can induce an effective time reversal of the two-photon scattering induced by the first half, leading to generally high sorting fidelities (\( F > 0.999 \)). However, larger numbers of emitters do not exceed the already high fidelity of \( N_e = 2 \), and the time-reversal scattering becomes less accurate for \( N_e > 2 \) [see Fig. 4(b)].

In addition to the time-marching gradient method, we also develop an iterative optimization scheme [see Fig. 4(c)], which only requires the output state instead of the full knowledge of the scattering matrix and thus makes it more suitable for experimental implementation. The iteration is composed of a forward scattering and its time-reversal process interspersed by two filters, where the first one filters out the undesired wave function \( \Psi_s \), and the second filter extracts the time-reversed most populated mode \( \Phi'(k_1, k_2) \) from the output wave function \( \Psi(k_1, k_2) \). Iteration by this filter is reminiscent of the one used in optimal quantum storage [32–34], except that our state mapping \( \phi_{n+1} = \mathcal{M}\phi_n \) is nonlinear. It is straightforward to verify that a perfect sorting should be a fixed point of the iteration, but a rigorous proof of convergence to the optimum is difficult as the successive nonlinear mapping \( \mathcal{M} \) cannot be interpreted as a power iteration as in the linear storage problem [32]. Nonetheless, for even values of \( N_e \) we have always found convergence and an equally high fidelity as by the time-marching method [see inset of Fig. 4(a)].

**Experimental considerations.**—Since incorporating more emitters to the system is experimentally challenging and does not improve the fidelity for \( N_e > 2 \), we recommend use of the emitter-pair based photon sorter. To assess if our scheme is feasible for state-of-the-art implementations of the waveguide QED platform, we now consider some realistic imperfections.

First, we investigate the sensitivity of our scheme to deviations between the properties of the two emitters. In particular, we consider a difference in the emitter-photon coupling strengths \( \Gamma_1, \Gamma_2 \) and a detuning \( \Delta = \nu_2 - \nu_1 \) between their resonance frequencies. To that end, we perform the input mode optimization for a range of different values of \( \Gamma_2/\Gamma_1 \) and \( \Delta/\Gamma_1 \). As shown in Fig. 5(a), the optimal fidelity \( F \) remains high for a broad range of parameters assuring that fabrication issues may not be an impediment to the scheme. We do find that the order of emitters plays a role, and for a small imbalance \( \Gamma_2/\Gamma_1 \lesssim 1 \), the optimal fidelity can even be higher than in the case of identical emitters, e.g., \( F > 0.9999 \) can be obtained at \( \Delta = 0, \Gamma_2/\Gamma_1 = 0.95 \). When the coupling strengths are largely imbalanced, \( \Gamma_2 > \Gamma_1 \) is preferable near \( \Delta = 0 \). The order of emitters is not important when the coupling strengths are identical, because the optimal fidelity is a symmetric function of the detunings \( [F(\Delta) = F(-\Delta)] \).

Second, we address the case of imperfect emitter-photon couplings, for which photon losses into free space are assumed and the directional efficiency factor \( \beta = \Gamma/\Gamma_{tot} \) is introduced to quantify the fraction of the total emitter decay that leads to light emission in the desired waveguide mode. When \( \beta \) is less than unity, the single- and two-photon output wave functions \( \Psi(k) \) and \( \Psi(k_1, k_2) \) become unnormalized, and we need to calculate their respective survival probabilities \( N_1 = \int dk |\Psi(k)|^2 \) and \( N_2 = \int dk_1dk_2|\Psi(k_1, k_2)|^2 \). As a result, the Hermitian kernel in Eq. (6) is modified into the following form:

\[
\mathcal{H} = \frac{2}{N_1} \int dk L_1(k, p)L_1(k, p') - \frac{1}{N_1} \int L_2^*(p)L_2(p').
\]

The fidelity to be optimized can be defined in different ways, e.g., the total fidelity \( F_t = N_2 - E \) gives the norm of the
desired two-photon component $\Psi_f(k_1, k_2)$, and the conditional fidelity $F_c = F_f / N_2 = 1 - E / N_2$ characterizes the sorting performance conditioned on survival of both photons. To maximize $F_f$ and $F_c$, we must minimize $E(\phi, \phi^\dagger) - N_2(\phi, \phi^\dagger)$ and $E(\phi, \phi^\dagger) / N_2(\phi, \phi^\dagger)$, respectively. As shown in Fig. 5(b), the fidelity optimized for a given $\beta$ based on the above modified scheme outperforms the one merely using the same mode as was optimized for $\beta = 1$. We also note that while the photon loss causes a proportional reduction in $F_f$, a very high conditional fidelity $F_c > 0.9997$ can be maintained for $\beta < 1$, and hence a heralded photon sorter can be constructed with lossy emitters.

In conclusion, we have presented a theoretical analysis of the sorting of Fock states into orthogonal modes by their coupling to two-level emitters. For the chiral waveguide QED system considered in this Letter, a near-university sorting of one- and two-photon states can be achieved by two or a higher even number of emitters. The proposed optimal photon sorter, combined with single-qubit operations (e.g., temporal mode extraction [23–25] and pulse time reversal [35–37]) can be used to build advanced quantum photonic devices [11,13], such as deterministic Bell state analyzers and nonlinear-sign gates [38] with fidelity $\geq 0.9995$ (see the Supplemental Material [29]). It is also possible to optimize the scheme against the degradation of fidelity due to pure dephasing of the emitter [29]. By applying multilevel [39,40] or driven [41] systems, it may be possible to further improve the sorting performance, and generalizations to nonchiral systems [42,43] as well as multiphoton regime for high-dimensional sorting [44] present attractive topics for further exploration.

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