Dirac neutrino mass generation from dark matter

Yasaman Farzan\textsuperscript{1} and Ernest Ma\textsuperscript{2,3}

\textsuperscript{1} School of Physics, Institute for Research in Fundamental Sciences (IPM), P. O. Box 19395-5531, Tehran, Iran

\textsuperscript{2} Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

\textsuperscript{3} Kavli Institute for the Physics and Mathematics of the Universe (IPMU), University of Tokyo, Kashiwa 277-8583, Japan

Abstract

In 2006, a simple extension of the Standard Model was proposed in which neutrinos obtain radiative Majorana masses at one-loop level from their couplings with dark matter, hence the term “scotogenic,” from the Greek “scotos” meaning darkness. Here an analogous mechanism for Dirac neutrino masses is discussed in a minimal model. In different ranges of the parameter space, various candidates for dark matter are possible. In particular, the lightest Dirac fermion which appears in the loop diagram generating neutrino mass can be a viable dark matter candidate. Such a possibility does not exist for the Majorana case. Realistic neutrino mixing in the context of $A_4$ is discussed. A possible supersymmetric extension is also briefly discussed.
Dirac neutrino masses have not received much attention in the literature mainly because of two reasons: (1) In the Standard Model (SM) of particle interactions, there are left-handed lepton doublets $(\nu, l)_L$ and right-handed charged-lepton singlets $l_R$ but no $\nu_R$ because it transforms trivially under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and there is no need for its existence. If it is added in by hand, the neutrino can then obtain a Dirac mass $m_D$ in the same way as all the other fermions (quarks and charged leptons), i.e. from the vacuum expectation value of the scalar Higgs doublet of the Standard Model. However, since $\nu_R$ is a neutral singlet, there is no symmetry which prevents it from having a large Majorana mass $M$. As a result, $\nu_L$ obtains an effective small Majorana mass from the seesaw mechanism \[1\], i.e. $m_\nu \simeq -m_D^2/M$. (2) If a symmetry is imposed in such a way that the lepton number is conserved, the Majorana mass term for $\nu_R$ will be forbidden. In that case, because neutrino masses are known to be of order 1 eV or less, the corresponding Yukawa couplings must be of order $10^{-11}$ or smaller. Such a small value is considered by many to be intrinsically unacceptable.

Nevertheless, up to now, there is not any indisputable evidence for the Majorana nature of the neutrinos from the searches for the neutrinoless double beta decay. Thus, the possibility of Dirac neutrino masses cannot be discounted. To overcome the above theoretical objections, it is proposed in this paper that neutrinos are Dirac fermions, with two important properties. (1) They are protected from becoming Majorana fermions by a $U(1)_{B-L}$ global or gauge symmetry. (2) They are protected from having a tree-level mass by a $Z_2$ symmetry which is identifiable with that of dark matter, as well as another $Z_2$ symmetry which sets them apart from other Dirac fermions. The latter symmetry is broken explicitly by soft terms. It may also be replaced by supersymmetry, but that would require a much larger Higgs content. As a result, neutrinos acquire one-loop radiative masses through their couplings with dark matter, hence the term “scotogenic,” from the Greek “scotos” meaning darkness.
These Dirac neutrino masses can be highly suppressed in the same way that the usual seesaw Majorana neutrino masses are highly suppressed. Their smallness can be also explained by the smallness of the soft terms breaking the $Z_2$ symmetry.

In 2006, it was proposed [2, 3] that neutrinos are massive only because of their couplings with dark matter. This idea connects two of the most important issues in the particle physics and astrophysics. The idea was easily implemented [2] in a simple extension of the Standard Model by adding a second scalar doublet $(\eta^+, \eta^0)$ and three neutral singlet fermions $N_i$ which are odd under an extra exactly conserved discrete $Z_2$ symmetry [4], in analogy to the $R$ parity in supersymmetry. As a result, either $\eta_R = \sqrt{2} Re(\eta^0)$ or the lightest $N$ may be considered a candidate for dark matter. In particular $\eta$ has been called the “inert” scalar doublet in a model proposed [5] after Ref. [2] and studied by many authors since then [6]. Variations of the original idea also abound and have become an active area of research [7, 8, 9, 10, 11, 12, 13].

In almost all previous such applications, neutrino masses have always been assumed to be of Majorana type. Suppose they are exactly Dirac. Is the connection between neutrino mass and dark matter still possible? If so, what are the necessary theoretical ingredients for it to happen, and what are the phenomenological consequences? In [14], using scalar singlets, a radiative Dirac neutrino mass is obtained; however, in this mechanism, the dark matter fields do not propagate in the loop. Employing an idea similar to that proposed in Ref. [2], Ref [15] suggests a model both for a dark matter candidate and generation of radiative Dirac neutrino mass. As indicated below, this model shares some features with the model introduced in the present paper. In [13], a model is introduced in which neutrinos obtain a Dirac mass via a one-loop diagram similar to that in [15] and a Majorana mass via two-loop diagrams after spontaneous breaking of the lepton number symmetry. In our model described below, the neutrino mass is purely of the Dirac type.

Consider first the imposition of a conserved additive lepton number to protect the neu-
trino mass from becoming Majorana. We choose to do so by extending the Standard Model to include $B - L$ as either a global or gauged $U(1)$ symmetry. The latter has long been known to be a well-motivated anomaly-free extension which requires the existence of three singlet right-handed neutrinos. Of course, in breaking the gauged $U(1)_{B-L}$, we have to be sure that the global $U(1)_{B-L}$ symmetry of the sector relevant to the present study remains intact. This can be done easily by a scalar field transforming under $U(1)_{B-L}$ but not coupling to other fields with nonzero $B - L$. The second step is to forbid a tree-level Dirac neutrino mass $m_\nu$, and yet allow a tree-level charged-lepton mass $m_l$. To do this, the simplest way is to impose a $Z_2^{(A)}$ symmetry such that $\nu^c$ is odd but all other fermions are even. There is therefore no connection between $\nu$ and $\nu^c$ at the tree level. To make them connect in one loop, new particles are postulated which are odd under an exactly conserved $Z_2^{(B)}$, and the previous $Z_2^{(A)}$ is allowed to be broken by soft terms. Another way is to make the model supersymmetric as well so that $m_l$ comes from $\Phi_1 = (\phi_1^0, \phi_1^-)$ but $m_\nu$ is forbidden to couple to $\Phi_2 = (\phi_2^+, \phi_2^0)$ which is assumed odd under $Z_2^{(B)}$. In either case, we need to add heavy neutral singlet Dirac fermions $(N, N^c)$ of odd $Z_2^{(B)}$ transforming under $U(1)_{B-L}$ and a neutral singlet scalar $\chi^0$ of odd $Z_2^{(B)}$ which is trivial under $U(1)_{B-L}$.

First, let us consider the minimal non-supersymmetric model. It is a simple extension of the Standard Model in the same spirit of Ref. [2]. Its particle content is listed in Table 1. In addition to the usual particles of the Standard Model, we have added three copies of the Weyl spinors $\nu^c$, three copies of the Dirac spinor pairs $(N, N^c)$, one extra scalar doublet $\eta = (\eta^+, \eta^0)$ and one real scalar $\chi^0$. The $B - L$ symmetry prevents $N, N^c$ as well as $\nu^c$ from having a Majorana mass. Note that $Z_2^{(A)}$ is broken softly by the trilinear term $A \chi \Phi^* \eta$, whereas $Z_2^{(B)}$ remains unbroken. $(\Phi = (\phi^+, \phi^0)$ is the standard model Higgs doublet.) The one-loop Dirac neutrino mass is thus generated, as shown in Fig. 1.

Note that $\chi^0$ is essential here for a scotogenic Dirac neutrino mass, whereas the scalar
Table 1: Assignments of the particles of the minimal model under $B - L$, $Z_2^{(A)}$, and $Z_2^{(B)}$.

![Figure 1: One-loop generation of Dirac neutrino mass in the minimal model.](image)

Whereas a scalar singlet was discussed as dark matter by itself many years ago [16, 17, 18], our proposal may be considered a natural justification of its existence.

Let the Yukawa interactions be given by $f_{\alpha k} \nu_\alpha N_k \eta^0$ and $h_{k\beta} N_k \nu_\beta^c \chi^0$. Without loss of generality, the $A$ parameter of the trilinear $A \chi \bar{\phi}^0 \eta^0$ term may always be chosen real, as well as the vacuum expectation value $\langle \phi^0 \rangle = v$. Let $\eta^0 = (\eta_R + i \eta_I)/\sqrt{2}$, then there is a mixing between $\eta_R$ and $\chi^0$, but not between $\eta_I$ and $\chi^0$. Assuming in addition that $\eta_I$ is a mass
eigenstate and denoting the mass eigenstates of the $(\chi^0, \eta_R)$ sector as $\zeta_{1,2}$ with mixing angle $\theta$, the one-loop Dirac neutrino mass matrix is then given by

$$\left( \mathcal{M}_\nu \right)_{\alpha\beta} = \frac{\sin \theta \cos \theta}{16\pi^2\sqrt{2}} \sum_k f_{\alpha k} h_k \beta m_{N_k} \left[ \frac{m_{\zeta_1}^2}{m_{\zeta_1}^2 - m_{N_k}^2} \ln \frac{m_{\zeta_1}^2}{m_{N_k}^2} - \frac{m_{\zeta_2}^2}{m_{\zeta_2}^2 - m_{N_k}^2} \ln \frac{m_{\zeta_2}^2}{m_{N_k}^2} \right].$$ (1)

This is in complete analogy to that of the radiative Majorana seesaw [2], with suppression of the neutrino mass from the usual assumption of very large $m_N$ (now Dirac) as well as the loop factor. In addition, this diagram is only nonzero because of the soft breaking of $Z_2^{(A)}$. Thus, it is natural for the parameter $A$ to be small. In the limit $A = 0$, the mixing angle $\theta$ in the above equation would be zero.

We assume that there are three copies of $(N, N_c)$ so that all three neutrinos obtain scotogenic Dirac masses. If there is only one copy, then two neutrinos will be massless, which is clearly unrealistic. If there are two copies, one will be massless, which is acceptable as far as present neutrino phenomenology is concerned. From Table 1, it can be easily confirmed that with three copies of $\nu^c$, $U(1)_{B-L}$ will be anomaly-free.

In this model, $\Phi$ is the SM Higgs doublet with the usual Higgs boson $H$ as its only physical degree of freedom. It has the usual SM decay modes, except for corrections due to its interactions with $\eta$ and $\chi^0$. For example, $H$ may decay into $\zeta_1 \zeta_1$ if kinematically allowed. If $\zeta_1$ is dark matter, this decay would then be invisible. It would affect the search for the SM Higgs boson, as studied already in Ref. [19]. Another possible effect is that the coupling of $H$ to $\eta^+ \eta^-$ would change the one-loop decay of $H$ to $\gamma \gamma$, thus affecting also the search for the SM Higgs boson via this channel. A third effect is the existence of the quartic $\chi \chi \Phi^i \Phi$ coupling, which may contribute significantly to the effective potential of $H$ and modify its stability condition as a function of mass. It may also induce a one-loop contribution to the $H^3T$ term at finite temperature to cause a first-order phase transition needed for the electroweak baryogenesis.
The couplings $f_{\alpha k}L_{\alpha}N_k\eta$ contribute to radiative lepton flavor violating rare decays:

$$\Gamma(l_\alpha^- \rightarrow l_\beta^- \gamma) = \frac{m_\alpha^3}{16\pi} \frac{\sigma_R^2}{\sigma_R^2},$$

(2)

where

$$\sigma_R = \sum_k e f_{\alpha k} f^\ast_{\beta k} m_\alpha \frac{i}{16\pi^2 m_{\eta^+}^2} \left[ \frac{t \ln t}{2(t-1)^4} + \frac{t^2 - 5t - 2}{12(t-1)^3} \right],$$

(3)

with $t = (m_{N_k}^2/m_{\eta^+}^2)$. For $t \rightarrow 0$, $t \rightarrow \infty$ and $t \rightarrow 1$, the combination in the last parenthesis of Eq. (3) converges respectively to $1/6$, $1/(12t)$ and $1/24$. For $m_{N_k} \gg m_{\eta^+}$, which is the seesaw limit, we find

$$\left( \sum_k \frac{f_{\alpha k} f^\ast_{\beta k}}{m_{N_k}^2} \right)^{1/2} \sim 8 \times 10^{-5} \left( \frac{B(l_\alpha^- \rightarrow l_\beta^- \gamma)}{10^{-12}} \right)^{1/4} \text{GeV}^{-1}.$$ 

(4)

We will consider first this scenario, so that the dark-matter candidate of our model is the lightest of the three exotic neutral scalars: $\zeta_{1,2}$ or $\eta_I$.

The most general scalar potential consisting of $\Phi$, $\eta$, and $\chi$ is given by

$$V = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \eta^\dagger \eta + \frac{1}{2} \mu_3^2 \chi^2 + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2} \lambda_5 (\Phi^\dagger \eta)^2 + H.c.
+ \frac{1}{4} \lambda_6 \chi^4 + \frac{1}{2} \lambda_7 (\Phi^\dagger \Phi) \chi^2 + \frac{1}{2} \lambda_8 (\eta^\dagger \eta) \chi^2 + A \chi \Phi^\dagger \eta + H.c.$$ 

(5)

This potential preserves $Z_2^{(B)}$ and breaks $Z_2^{(A)}$ softly by the last term. The parameter $A$ may be chosen real by a phase rotation of $\eta$ relative to $\Phi$, but then $\lambda_5$ is in general complex. For simplicity, we choose it to be real so that $\eta_I$ is a mass eigenstate and decouples from the $(\chi^0, \eta_R)$ sector. The resulting mass spectrum is given by

$$m_{H_I}^2 = 2\lambda_1 v^2,$$

(6)

$$m_{\eta^+}^2 = \mu_2^2 + \lambda_3 v^2,$$

(7)

$$m_{\eta_I}^2 = \mu_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2,$$

(8)

$$m_{(\chi,\eta_R)}^2 = \left( \frac{\sqrt{2} A v}{\mu_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2} \right).$$

(9)
This is very similar to previous studies such as Ref. [8] with an important conceptual difference. Since the parameter $A$ breaks $Z_2^{(A)}$, it may be argued that it is small. This suppresses the radiative neutrino Dirac mass as well as the mixing between $\eta_R$ and $\chi$. Hence the dark-matter candidate of this model can be dominantly a singlet and as a result, it can naturally evade the constraints from the electroweak interactions of a doublet. If there is no $Z_2^{(A)}$ symmetry, the mixing between $\eta_R$ and $\chi$ is then arbitrary, as in previous studies. Another difference is that $\eta_I$ is not involved in the one-loop neutrino mass, contrary to the original Majorana case of Ref. [2]. The above possibility has also been discussed in [15]. In the following, we introduce a new possibility for dark matter candidate within the present scenario.

Since $m_{N_k}$ are assumed to be very large in this scenario, the annihilation of the dark-matter scalars in this model do not proceed via their Yukawa interactions, but rather through their gauge or scalar interactions. Examples of the latter have been discussed extensively in the literature [16, 17, 18, 20, 21, 22, 23, 24].

We consider next the lightest $N_k$ (call it $N_1$) as dark matter. As shown previously [25], this is subject to severe phenomenological constraints in the original model [2] of scotogenic Majorana neutrino mass. The reason is as follows. In order for $N_1 \bar{N}_1$ annihilation to account for the correct relic abundance, the $\eta$ masses cannot be too heavy and the Yukawa couplings $f_{\alpha k}$ cannot be too small. However, these values are severely constrained by experimental upper limits on the $\mu \to e\gamma$ rate, as already discussed. It is thus not a viable option, without some detailed fine tuning of parameters. To retain $N_1$ as a natural dark-matter candidate, new interactions involving $N_1$ need to be postulated, such as a singlet scalar [20]. In our present model, the $h_{kj} N_k \nu_j^c \chi^0$ Yukawa couplings are exactly what are required. They are not constrained by flavor-changing charged-lepton radiative decays, so they can be large enough for a realistic $N_1 \bar{N}_1$ annihilation cross section to account for the relic abundance of dark
matter in the Universe today. In this scenario, the $f_{ak}$ Yukawa couplings as well as the $A$ parameter are small and the mass of $\zeta_1$ (which is mostly composed of $\chi$) is not much greater than $m_{N_1}$.

Combining $(\nu, \nu^c)$ and $(N, N^c)$ to form the four-component Dirac fermions $\nu$ and $N$, their Yukawa interactions are given by

$$L_Y = f_{ak} \bar{N}_k \left( \frac{1 - \gamma_5}{2} \right) (\nu_\alpha \eta^0 - l_\alpha \eta^+) + h_{k\beta} \bar{N}_k \left( \frac{1 + \gamma_5}{2} \right) \nu_\beta \chi^0 + H.c. \quad (10)$$

where in this four-component notation, $[(1 + \gamma_5)/2] \nu$ represents $\nu^c$ going backwards. For the dark-matter candidate $N_1$, we assume $h_{1\beta}$ to be dominant, then

$$\sigma(N_1 + \bar{N}_1 \rightarrow \nu_\alpha + \bar{\nu}_\beta) = \sum_{\alpha,\beta} \frac{|h_{1\alpha}^* h_{1\beta}^*|^2}{32\pi v_{rel}} \frac{m_{N_1}^2}{(m_{N_1}^2 + m_{\zeta_1}^2)^2} < \sum_{\alpha,\beta} \frac{|h_{1\alpha}^* h_{1\beta}^*|^2}{128\pi v_{rel} m_{N_1}^2}, \quad (11)$$

where to reach the last inequality we have used $m_{\zeta_1} > m_{N_1}$. Similarly,

$$\sigma(N_1 + N_1 \rightarrow \nu_\alpha + \nu_\beta) = \sum_{\alpha,\beta} \frac{|h_{1\alpha}^* h_{1\beta}^*|^2}{32\pi v_{rel}} \frac{m_{N_1}^2}{(m_{N_1}^2 + m_{\zeta_1}^2)^2} < \sum_{\alpha,\beta} \frac{|h_{1\alpha}^* h_{1\beta}^*|^2}{128\pi v_{rel} m_{N_1}^2} \quad (12)$$

Setting the sum of the two annihilation cross sections times the relative velocity equal to one picobarn, we find

$$m_{N_1} < \left( \frac{\sum_{\alpha,\beta} |h_{1\alpha}^* h_{1\beta}^*|^2}{128\pi v_{rel} m_{N_1}^2} \right)^{1/2} (1.4 \text{ TeV}) \quad (13)$$

For $|h_{1\alpha}| < 1$, we then obtain $m_{N_1} < 4.2 \text{ TeV}$. With such light $N_1$, the seesaw mechanism is not very effective. The smallness of the neutrino masses can be justified by the smallness of the trilinear $A$ term which softly breaks $Z_2^{(A)}$, and the smallness of the $f$ Yukawa couplings. If the $h$ couplings were not available, the cross section must have then come from the $f$ couplings, which are restricted by $\mu \rightarrow e\gamma$, so the annihilation cross section would in general be too small for $N_1$ to be a viable dark-matter candidate. If the $B - L$ symmetry is gauged, there should be another annihilation mode $N + \bar{N} \rightarrow Z' \rightarrow \nu + \bar{\nu}, l + \bar{l}, q + \bar{q}$. This cross section is given by [27]

$$\sigma = \frac{g_{Z'}^4 m_{N_1}^2}{4\pi v_{rel} (4m_{N_1}^2 - m_{Z'}^2)^2} \quad (14)$$
The present lower bound on \( m_{Z'} \) from the Large Hadron Collider (LHC) \[28\] is estimated to be about 2 TeV. For \( g_{Z'} = \sqrt{(5/8)}g_Y = 0.28 \) (i.e. the \( SO(10) \) limit), \( m_{Z'} = 2 \) TeV, and \( \sigma v_{rel} = 1 \) pb, we find \( m_{N_1} = 900 \) GeV. In this case, \( N_1 \bar{N}_1 \) production from \( Z' \) decay at the LHC is possible, as studied previously \[29\], except that \( N_1 \) is now dark matter. It may however be inferred from the increase of the \( Z' \) invisible width on top of the expected \( Z' \rightarrow \nu \bar{\nu} \) mode. As \( N_1 \) is otherwise very difficult to produce, the existence of \( Z' \) seems to be the only realistic chance for it to be observed at the LHC, but still only indirectly. If \( \eta^+ \) is light enough, it can be produced at the LHC. The subsequent decay of \( \eta^+ \) into \( N_1 \) and a charged lepton is a possible signature, as discussed in Ref. \[30\].

As for direct detection of dark matter in underground experiments, if \( B - L \) is not gauged, then \( N_1 \) has no interaction with nuclei. If \( B - L \) is gauged, then the elastic scattering of \( N_1 \) with nuclei may proceed through \( Z' \) exchange. The cross section per nucleon is given by \[27\]

\[
\sigma_0 = \frac{4m_{N_1}^2 \ g_{Z'}^4}{\pi \ m_{Z'}^4}.
\]

For \( g_{Z'} = 0.28 \) and \( m_{Z'} = 2 \) TeV, this implies \( \sigma_0 = 1.7 \times 10^{-7} \) pb, which exceeds the XENON100 bound \[31\] of about \( 7 \times 10^{-8} \) pb for \( m_{N_1} = 900 \) GeV. This means that in this case, \( g_{Z'}/m_{Z'} \) should be reduced by a factor of 1.25 or more.

This minimal model is also very suitable for the implementation of the non-Abelian discrete \( A_4 \) symmetry \[32\] to the neutrino mass matrix \[33\]. In the charged-lepton sector, let \((\nu_i, l_i) \sim \mathbb{3}\) under \( A_4 \), and either \( l_i^c \sim \mathbb{1}, \mathbb{1}', \mathbb{1}'' \) as in Ref. \[32\] or \( l_i^c \sim \mathbb{3} \) as in Ref. \[34\], then with \( \Phi \sim \mathbb{3} \) or \( \mathbb{3} + \mathbb{1} \), and \( A_4 \) breaking to the residual symmetry \( Z_3 \), the charged-lepton mass matrix is diagonalized by the well-known unitary matrix

\[
U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},
\]

where \( \omega = \exp(2\pi i/3) \). In the neutrino sector, let \( \nu_i^c \sim \mathbb{3}, \eta \sim \mathbb{1}, \) and \( \chi \sim \mathbb{1} + \mathbb{3} \), with the
soft scalar trilinear $\chi \Phi^i \eta$ terms to break $A_4$, the neutrino mass matrix becomes \[33\]

$$M_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix}. \quad (17)$$

If $e = f = 0$, then neutrino mixing is tribimaximal, i.e. $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$, $\theta_{13} = 0$. This was known to be a good approximation of the measured neutrino mixing angles. However, two recent experiments have measured $\theta_{13}$ to be definitely nonzero, i.e.

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}) \quad (18)$$

from the Daya Bay Collaboration \[35\], and

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}) \quad (19)$$

from the RENO Collaboration \[36\]. In that case, $e$ and $f$ should be nonzero. Let

$$\epsilon \equiv e - f \quad \text{and} \quad \delta \equiv e + f. \quad (20)$$

The parameters $a, d, e, f$ are complex, and for small $e, f$, the eigenvalues of $M_\nu$ are $a + d$, $a$, and $a - d$. We can always choose $a$ to be real, the phase of $d$ is then determined by the absolute values of the three masses \[37\]. For the small values of $e$ and $f$, we find

$$\theta_{13} = -\frac{\epsilon}{\sqrt{3}}, \quad \tan^2 \theta_{12} = \frac{1}{2} \left[ \frac{(1 - \sqrt{2} Re\delta)^2 + 2(Im\delta)^2}{(1 + Re\delta/\sqrt{2})^2 + (Im\delta)^2/2} \right]. \quad (21)$$

Thus, a nonzero $\theta_{13}$ and a value of $\tan^2 \theta_{12}$ smaller than 0.5 can be obtained. More precisely, the neutrino mass matrix in the tribimaximal basis is now of the form

$$M_{\nu}^{(1,2,3)} = \begin{pmatrix} m_1 & m_6 & 0 \\ m_6 & m_2 & m_5 \\ 0 & m_5 & m_3 \end{pmatrix} = \begin{pmatrix} a + d & \delta d & 0 \\ \delta d & a & ed \\ 0 & ed & a - d \end{pmatrix}. \quad (22)$$

If $\delta = \epsilon = 0$, the tribimaximal mixing is then recovered. This differs from the originally proposed deviation \[33\] for $A_4$, which was updated recently \[38\], i.e.

$$M_{\nu}^{(1,2,3)} = \begin{pmatrix} m_1 & 0 & m_4 \\ 0 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix} = \begin{pmatrix} a + d - (b + c)/2 & 0 & i\sqrt{3}/2(c - b) \\ 0 & a + b + c & \sqrt{2}e \\ i\sqrt{3}/2(c - b) & \sqrt{2}e & a - d - (b + c)/2 \end{pmatrix}. \quad (23)$$
Given that \( m_4 = 0 \) in Eq. (22), we obtain the approximate relationship

\[
\sin^2 2\theta_{23} \simeq 1 - 8|\text{Re}(U_{e3})|^2.
\] (24)

Using the experimental bound \( \sin^2 2\theta_{23} > 0.92 \), we find \( |\text{Re}(U_{e3})| < 0.1 \). If we take the central value of \( |U_{e3}| \) to be 0.16 (corresponding to \( \sin^2 2\theta_{13} = 0.1 \)), we then obtain \( |\tan \delta_{CP}| > 1.3 \) in this model. Details are given elsewhere [39].

Below we also mention briefly how a supersymmetric model of scotogenic neutrino mass may be constructed. Consider the superfield content listed in Table 2. There are two

| superfields | \( SU(3)_C \) | \( SU(2)_L \) | \( U(1)_Y \) | \( U(1)_{B-L} \) | \( Z_2^{(A)} \) | \( Z_2^{(B)} \) |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \((u, d)\)   | 3              | 2              | 1/6            | 1/3            | +              | +              |
| \(u^c\)     | 3              | 1              | -2/3           | -1/3           | +              | +              |
| \(d^c\)     | 3              | 1              | 1/3            | -1/3           | +              | +              |
| \((\nu, e)\) | 1              | 2              | -1/2           | -1             | +              | +              |
| \(e^c\)     | 1              | 1              | 1              | 1              | -              | +              |
| \(\nu^c\)   | 1              | 1              | 0              | 1              | -              | +              |
| \((\phi^0_0, \phi^-_1)\) | 1       | 2              | -1/2           | 0              | -              | +              |
| \((\phi^+_2, \phi^0_2)\) | 1       | 2              | 1/2            | 0              | +              | -              |
| \((\phi^0_3, \phi^-_3)\) | 1       | 2              | -1/2           | 0              | +              | +              |
| \((\phi^+_4, \phi^0_4)\) | 1       | 2              | 1/2            | 0              | +              | +              |
| \(\chi^0_1\) | 1              | 1              | 1              | 0              | -              | +              |
| \(\chi^0_1\) | 1              | 1              | 0              | 0              | -              | +              |
| \(\chi^0_2\) | 1              | 1              | 0              | 0              | -              | -              |
| \(\chi^-\) | 1              | 1              | -1             | 0              | +              | -              |
| \(N\)       | 1              | 1              | 0              | -1             | +              | -              |
| \(N^c\)     | 1              | 1              | 0              | 1              | +              | -              |

Table 2: Assignments of the particles of the supersymmetric model under \( B - L \), \( Z_2^{(A)} \) and \( Z_2^{(B)} \).

one-loop diagrams contributing to the Dirac neutrino mass as shown in Fig. 2. Note that supersymmetry is broken by the soft scalar trilinear \( \chi^0_2 \phi^0_1 \phi^0_2 \) and bilinear \( \bar{N} \tilde{N}^c \) terms. There
are now many particles of odd $Z_2^{(B)}$ as well as superpartners of odd $R$ parity. There are thus at least two dark-matter candidates [40]. Obviously the details of the dark sector are much more complicated. We will not study them further in this paper.

In conclusion, we have studied a minimal model of radiative Dirac neutrino mass induced by dark matter. In order for the scotogenic Dirac neutrino mass to occur in one loop, we need to introduce a scalar singlet $\chi^0$ which mixes with the neutral component of a new electroweak scalar doublet $(\eta^+, \eta^0)$. It is thus a good theoretical justification for the existence of $\chi^0$. In addition to the possibility of direct production at the LHC, the presence of $\eta^+$ can modify the Higgs decay mode to $\gamma\gamma$. As shown in [41], if the $\lambda_3$ coupling in Eq. (5) is negative, it can lead to the enhancement of $\text{Br}(H \rightarrow \gamma\gamma)$ in conformity of the recent observation at the LHC [42]. Moreover, the quartic coupling of $\chi^0$ with Higgs can stabilize its potential against radiative corrections.

This minimal model also requires three heavy neutral Dirac fermions $N_i$. Depending on the mass spectrum, the dark matter might be either the lightest Dirac fermion $N_1$ or one of the neutral scalars; i.e. the imaginary component $\eta_I$ of $\eta^0$ or a linear combination of the real component $\eta_R$ of $\eta^0$ and $\chi^0$. In the latter case, depending on the mixing between $\eta_R$ and $\chi^0$, which should be small because of the soft breaking of $Z_2^{(A)}$, the annihilation rate due to the electroweak interactions can be made equal to about 1 pb which is a value dictated by the
dark matter abundance in the thermal dark matter scenario.

If $N_1$ is the dark-matter candidate, its annihilation can proceed via its Yukawa coupling with the right-handed neutrinos and $\chi^0$. This is a possibility that does not exist within the scotogenic Majorana neutrino mass model because in that case the bounds from the $\mu \to e\gamma$ constraints restrict the annihilation cross section of the $N_1$ pair below the required value. At the LHC, $N_1$ can then be produced via the decay of $\eta^+$ and $\eta^-$ along with a charged lepton [30].

The $B - L$ symmetry used to maintain the conservation of lepton number can be gauged. In that case, the present LHC lower bound on $m_{Z'}$ is about 2 TeV. The interaction with the $Z'$ boson provides another route for the annihilation of the $N_1$ pair as well as a portal for the interaction with quarks and hence direct detection. The bound from the XENON100 experiment already constrains the parameter space.

This minimal model is also suitable for implementing an $A_4$ symmetry in such a way that nonzero $\theta_{13}$ and large $\delta_{CP}$ may be obtained. We have also briefly mentioned how a supersymmetric extension can be constructed.

The work of E.M. is supported in part by the U. S. Department of Energy under Grant No. DE-AC02-06CH11357. The authors thank Galileo Galilei Institute for Theoretical Physics for its hospitality. YF acknowledges partial support from the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN- GA-2011- 289442). She is also grateful to ICTP for partial financial support and hospitality.

References

[1] T. Yanagida, in Proc. of the Workshop on Unified Theories and Baryon Number in the Universe (KEK, Tsukuba, Japan, 1979), edited by O. Sawada and A. Sugamoto,
p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980); P. Minkowski, Phys. Lett. 67B, 421 (1977).

[2] E. Ma, Phys. Rev. D73, 077301 (2006).

[3] C. Boehm, Y. Farzan, T. Hambye, S. Palomares-Ruiz and S. Pascoli, Phys. Rev. D 77, 043516 (2008) [hep-ph/0612228].

[4] N. G. Deshpande and E. Ma, Phys. Rev. D18, 2574 (1978).

[5] R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. D74, 015007 (2006).

[6] See for example L. Lopez Honorez and C. E. Yaguna, JCAP 1101, 002 (2011) and references therein.

[7] P.-H. Gu and U. Sarkar, Phys. Rev. D78, 073012 (2008).

[8] Y. Farzan, Phys. Rev. D80, 073009 (2009).

[9] E. Ma and D. Suematsu, Mod. Phys. Lett. A24, 583 (2009).

[10] E. Ma, Phys. Rev. D80, 013013 (2009).

[11] M. K. Parida, Phys. Lett. B704, 206 (2011).

[12] D. Suematsu and T. Toma, Nucl. Phys. B847, 567 (2011).

[13] S. Kanemura, T. Nabeshima, and H. Sugiyama, Phys. Rev. D85, 033004 (2012).

[14] S. Kanemura, T. Nabeshima, and H. Sugiyama, Phys. Lett. B703, 66 (2011).

[15] P.-H. Gu and U. Sarkar, Phys. Rev. D77, 105031 (2008).
[16] V. Silveira and A. Zee, Phys. Lett. **161B**, 136 (1985).

[17] J. McDonald, Phys. Rev. **D50**, 3637 (1994).

[18] C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619**, 709 (2001) [hep-ph/0011335].

[19] Q.-H. Cao, E. Ma, and G. Rajasekaran, Phys. Rev. **D76**, 095011 (2007).

[20] B. Patt and F. Wilczek, [hep-ph/0605188].

[21] V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf and G. Shaughnessy, Phys. Rev. D **77**, 035005 (2008) [arXiv:0706.4311 [hep-ph]].

[22] S. Andreas, T. Hambye and M. H. G. Tytgat, JCAP **0810**, 034 (2008) [arXiv:0808.0255 [hep-ph]].

[23] S. Andreas, C. Arina, T. Hambye, F. -S. Ling and M. H. G. Tytgat, Phys. Rev. D **82**, 043522 (2010) [arXiv:1003.2595 [hep-ph]].

[24] Y. Farzan, S. Pascoli and M. A. Schmidt, JHEP **1010**, 111 (2010) [arXiv:1005.5323 [hep-ph]].

[25] J. Kubo, E. Ma, and D. Suematsu, Phys. Lett. **B642**, 18 (2006).

[26] K. S. Babu and E. Ma, Int. J. Mod. Phys. **A23**, 1813 (2008).

[27] S. Khalil, H.-S. Lee, and E. Ma, Phys. Rev. **D81**, 051702(R) (2010).

[28] G. Aad et al. [ATLAS Collaboration], Phys. Rev. Lett. **107**, 272002 (2011) [arXiv:1108.1582 [hep-ex]].

[29] K. Huitu, S. Khalil, H. Okada, and S. K. Rai, Phys. Rev. Lett. **101**, 181802 (2008).
[30] Y. Farzan and M. Hashemi, JHEP 1011, 029 (2010) [arXiv:1009.0829 [hep-ph]].

[31] E. Aprile et al., Phys. Rev. Lett. 107, 131302 (2011).

[32] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001).

[33] E. Ma, Phys. Rev. D70, 031901(R) (2004).

[34] E. Ma, Mod. Phys. Lett. A21, 2931 (2006).

[35] Daya Bay Collaboration: F. P. An et al., Phys. Rev. Lett. 108, 171803 (2012) [arXiv:1203.1669 [hep-ex]].

[36] J. K. Ahn et al. [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012) [arXiv:1204.0626 [hep-ex]].

[37] E. Ma, Phys. Rev. D72, 073301 (2005).

[38] E. Ma and D. Wegman, Phys. Rev. Lett. 107, 061803 (2011).

[39] H. Ishimori and E. Ma, arXiv:1205.0075 [hep-ph].

[40] Q.-H. Cao, E. Ma, J. Wudka, and C.-P. Yuan, arXiv:0711.3881 [hep-ph].

[41] M. Carena, I. Low and C. E. M. Wagner, arXiv:1206.1082 [hep-ph].

[42] CMS collaboration, in preparation; ATLAS collaboration, in preparation.