On $p_T$-distribution of gluon production rate in constant chromoelectric field

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A complete expression for the $p_T$-distribution of the gluon production rate in the homogeneous chromoelectric field has been obtained. Our result contains a new additional term proportional to the singular function $\delta(p_T^2)$. We demonstrate that the presence of this term is consistent with the dual symmetry of QCD effective action and allows to reproduce the known result for the total imaginary part of the effective action after integration over transverse momentum.

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1. Introduction

Recently the calculation of the soft gluon production rate and its transverse momentum $p_T$-distribution in a constant chromoelectric field has been performed $[1,2,3]$. The motivation for this study is to describe the particle production in the quark-gluon plasma produced in hadron collider experiments $[4]$. The obtained result represents a simplest estimate based on the Schwinger’s mechanism of pair creation in the electric field $[5]$ applied to the standard quantum chromodynamics (QCD).

In the present short Letter we give a complete result for the $p_T$-distribution of the gluon production rate in homogeneous chromoelectric field which includes a singular term in addition to the result obtained recently $[2]$. We demonstrate that our result is consistent with the analytic structure of the full QCD effective action (including both real and imaginary parts) and respects the dual symmetry. Moreover, the additional singular term provides a correct known result for the total imaginary part of the effective action $[6,7,8]$. Possible physical implications of the singular term are discussed.

2. Gluon production rate and $p_T$-distribution

Let us consider the gluon production rate in the constant color electric field. For simplicity, we choose a constant Abelian chromoelectric field directed along the $z-$axis and defined by the potential $A_{\mu}^i = -E^i \delta_{\mu0} z$ (i=3,8). The integral expression for the one-loop effective Lagrangian with the transverse momentum $p_T$-dependence can be written as follows $[2]$

$$\Delta L = \frac{g}{16\pi^3} \lim_{\epsilon \to 0} \sum_p \int_0^\infty d^2 p_T \frac{dt}{t^{1-\epsilon}} \frac{E_p}{\sin(gE_p t)} \left[ \exp(2igE_p t) + \exp(-2igE_p t) \right] \exp(-p_T^2 t),$$

where $\vec{r}_p$ ($p=1,2,3$) are root vectors of $SU(3)$. The integration over proper time in $[11]$ is ill-defined due to the poles. If we choose the contour above the $t$-axis starting from $0 + \epsilon$ as in quantum electrodynamics (QED) $[7,8]$ one finds the recent result obtained in $[2]$

$$\text{Im \ } \mathcal{L}_{\text{eff}} = -\sum_p \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{gE_p}{8\pi^3} \exp(-\frac{\pi np_{\vec{r}_p}^2}{gE_p}).$$

Performing integration over $p_T$ one finds the total imaginary part

$$\text{Im \ } \mathcal{L}_{\text{eff \ tot}} = \sum_p \frac{g^2}{96\pi} E_p^2,$$

which describes the total gluon production rate. This result had been derived first long time ago $[7]$ and it contradicts to results obtained in subsequent papers $[9,10]$. It was shown that the source of this controversy originated from the unphysical concept of the constant color field in QCD $[10]$.

To calculate the $p_T$-distribution of gluon production rate which is consistent with the total imaginary part of QCD effective action $[6,9]$ we will follow the method used in $[8,11]$. This method allows to resolve mainly the ambiguity related with the pole structure in the integral expression $[11]$. Let us start with the functional determinant form of the one-loop correction $\Delta S$ to the effective action $[12,13]$

$$\Delta S = i \sum_p \left\{ \text{Tr} \ln[\text{Det}(-D_p^2 + igE_p)] + \text{Tr} \ln[\text{Det}(-D_p^2 - igE_p)] \right\}.$$

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One can rewrite the last equation in momentum representation following the method in [6]:

\[
\Delta S = -\frac{ig}{8\pi^3} \sum_p \int d^4x d^2p_T E_p \left( \ln[p_T^2 + igE_p] - \ln[p_T^2 - igE_p] + 2 \sum_{n=1}^{\infty} \ln[p_T^2 + i(2n+1)gE_p] \right). 
\] (5)

Using the Schwinger’s proper time representation

\[
\ln A = -\int_0^{\infty} \frac{dt}{t} \exp[iAt - ct] 
\] (6)

and performing the series summation one can separate \(\Delta S\) into two parts

\[
\Delta S = \frac{ig}{8\pi^3} \sum_p \int d^4x d^2p_T \frac{dt}{t^{1-\epsilon}} E_p (e^{iE_p t} - e^{-iE_p t}) + 2\Delta S_{sc}, 
\] (7)

\[
\Delta S_{sc} = \frac{g}{16\pi^3} \sum_p \int d^4x d^2p_T \frac{dt}{t^{1-\epsilon}} E_p e^{-p_T^2 t} \sin(E_p t), 
\] (8)

where the part \(\Delta S_{sc}\) is identical to the expression for the one-loop effective action of massless scalar QED [5, 6]. The imaginary part of \(\Delta S_{sc}\) is well-known [5].

The calculation of the pole contribution with the Schwinger’s prescription \(E_p + i\epsilon\) leads to the results [5, 6]. However, as we mentioned above, this result is incomplete because of more complex analytic structure of the effective action of QCD. A correct total imaginary part of QCD effective action with a constant chromoelectric field had been obtained long time ago in [6]. One can show that this result can be easily reproduced if one takes into account the contribution from the pole at the origin \(t = 0\). Since the integral in (7) has no pole at \(t = 0\) anymore it might seem that the result (2) is the only imaginary contribution what we have. However, the situation is more subtle, and one should perform a careful analysis of two logarithmic terms in the last line of the Eqn. (5)

\[
\Delta S_1 = -\sum_p \frac{igE_p}{8\pi^3} \int d^4x d^2p_T \left( \ln[p_T^2] + \frac{ig(E_p - i\sigma)}{2} \right) - \ln[p_T^2 - ig(E_p - i\sigma)], 
\] (9)

where, we add an infinitesimal factor \(i\sigma\) to define a proper analytic continuation. To find out a missing part in the \(p_T\)-distribution let us consider the derivation of the total imaginary part in a more detail. Having integrated over \(p_T\) in the last equation with a cut-off parameter \(\Lambda\) one obtains

\[
\Delta S_1 = \sum_p \frac{igE_p}{8\pi^2} \left( \frac{gE_p}{p_T^2 + \sigma} + gE_p \left( \frac{\arctan \left( \frac{-gE_p}{p_T^2 - \sigma} \right) + 2\pi k}{p_T^2 + \sigma} \right) - p_T^2 \ln(p_T^2 + igE_p) 
\]

\[
+ p_T^2 \ln(p_T^2 - igE_p) - igE_p \ln(p_T^2 + g^2 E_p^2) \right) \bigg|_0. 
\] (10)

There is still uncertainty related to the choice of the factors \(2\pi k\) and \(\sigma\) which defines a possible analytic continuation of the logarithmic function and chromoelectric field. Let us consider two choices:

(A) the choice \(\sigma > 0, k = 0\) leads to the imaginary part

\[
Im \mathcal{L}_1 = -\sum_p \frac{g^2 E_p^2}{8\pi^2}. 
\] (11)

Notice, the prescription \(E_p - i\sigma\) is the same as that used in studies of the QCD vacuum stability based on causality requirement [12, 14];
(B) the adopting $\sigma = 0 , k = 1$ at the lower integration limit and neglecting the contribution at the upper limit, i.e., $k = 0$ at $p_T = \Lambda^2$, leads to

$$Im\ L_2 = - \sum_p g^2 E_p^2 \frac{\pi}{4\pi}.$$  \hspace{1cm} (12)

The corresponding expressions for the total imaginary parts for these two cases (including the contribution from $\Delta S_{ce}$) are the following

$$Im\ L_{1tot} = - \sum_p \frac{11 g^2 E_p^2}{96\pi},$$

$$Im\ L_{2tot} = - \sum_p \frac{23 g^2 E_p^2}{96\pi}.$$  \hspace{1cm} (13)

For the case of SU(2) QCD the first result, $Im\ L_{1tot}$ (with no summation over $p$ in the above equations), corresponds to a perturbative contribution which has been obtained by perturbative methods \cite{9,13}, while the second result, $Im\ L_{2tot}$, represents a total contribution derived first by Ambjorn and Hughes \cite{6}. In both results the sign is negative, that means one has gluon pair annihilation instead of gluon creation \cite{9}. Notice that the imaginary part $Im\ L_{1tot}$ is dual symmetric to the vanishing imaginary part for a possible stable chromomagnetic vacuum \cite{12,13}, whereas the Ambjorn-Hughes result is dual symmetric to the magnetic imaginary part of Nielsen-Olesen for a constant external chromomagnetic field, i.e., for a different physical problem. From the dual symmetry consideration we should assign the Ambjorn-Hughes type result $Im\ L_{2tot}$ as a correct one for the case of external constant chromoelectric field.

The key point in our derivation of (13) is that the non-vanishing values of the imaginary parts (11,12) originate from the behavior near vicinity of the point $p_T^2 = 0$. In the first case (A) the imaginary part (11) is not zero because the sum of angle functions on the right hand side of Eqn. (10) does not vanish in the infinitesimal region $p_T^2 < \sigma$. In the second case (B) the angle argument $2\pi k$ changes its value from 0 at the upper integration limit to $2\pi$ at the lower limit $p_T^2 = 0$ and must have behavior like a step function since $k$ is integer. This implies that in both cases the corresponding $p_T$-distribution is represented by a singular $\delta$-function. We choose a numeric factor in front of the $\delta$-function which reproduces the result (12) in agreement with Ambjorn-Hughes result (6)

$$Im\ L_2(p_T) = - \delta(p_T^2) \sum_p g^2 E_p^2 \frac{\pi}{4\pi}.$$  \hspace{1cm} (14)

The appearance of the singular function $\delta(p_T^2)$ reflects the low-momentum phenomenon of strong interaction. A simple heuristic argument is that any color field in the confinement phase should be confined, so that the constant field configuration must shrink and be localized in a small space region. One should stress again that the constant color field is not a well-defined physical concept and the imaginary part of the effective action depends on a concrete physical problem \cite{10}. Remind, that the negative imaginary part implies the gluon pair annihilation which assumes the presence of a source supporting the existence of the constant field itself \cite{6}.  

3. Consistence with dual symmetry

An important feature of the effective action is its invariance under the dual transformation $H_p \rightarrow -i E_p$, $E_p \rightarrow i H_p$. For a general constant background the dual symmetry of SU(2) QCD had been established in \cite{14}. The duality provides a powerful tool to check the consistence of obtained results. In particular, since the duality is intrinsically based on the analytic structure of full effective action, the real and imaginary parts must be interrelated in a non-trivial manner. From this point of view, the consistence with duality should imply the existence of a singular term in the chromomagnetic counterpart of the $p_T$-distribution (11). Besides, such a term should reproduce the well-known Nielsen-Olesen imaginary part of the effective action after integration over transverse momentum.

Let us consider a formal expression for a dual symmetric magnetic counter-part to Eqn. (11)

$$\Delta L = \frac{g}{16\pi^2} \sum_p \int_0^\infty d^2 p_T \int \frac{dt}{n-t} \frac{H_p}{\sinh(g H_p t)} \times \left( e^{2 g H_p t} + e^{-2 g H_p t} \right) e^{-\frac{p_T^2}{1}}.$$  \hspace{1cm} (15)

Notice that for the case of magnetic background the parameter $p_T$ should be treated just as a formal variable as it follows from the derivation method of the Eqn. (11) \cite{2,13}. The integral in (15) has a strong infra-red divergence when $p_T^2 < g H_p$ which causes an imaginary part. Using the series expansion

$$\frac{x}{\sinh x} = 1 - \frac{x^2}{6} - \frac{2 x^4}{\pi^2} \sum_{n=1}^\infty \frac{(-1)^n}{n^2} \frac{1}{x^2 + n^2 \pi^2}.$$  \hspace{1cm} (16)

one can perform the integration over proper time with the $\zeta$-function regularization. Finally, using analytical properties of the logarithmic and cosine integral functions, (notice that $H_p$ has an infinitesimal factor $+i$ due to the dual symmetry \cite{14}) one can find that the imaginary part is non-vanishing only when $p_T^2 < g H_p$

$$Im\ \Delta L = \frac{1}{16\pi^2} \sum_p (2 g H_p - p_T^2).$$
\[-\frac{g}{8\pi^3} \sum_{p,n=1}^{\infty} \frac{(-1)^n}{n} H_p \sin \left( \frac{\pi n p_T^2}{g H_p} \right). \]  

(17)

Surprisingly, using the identity

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n \pi x) \equiv \frac{i}{2} (\ln[1 + e^{i\pi x}] - \ln[1 + e^{-i\pi x}]), \]

one can check that inside the interval \(0 < p_T^2 < g H_p\), all \(p_T^2\)-dependent terms are mutually cancelled, so that, the imaginary part has a very simple \(p_T\) dependence like a step function

\[ Im \Delta L = \begin{cases} \sum_{p} \frac{g H_p}{8\pi^2}, & 0 \leq p_T^2 \leq g H_p, \\ 0, & p_T^2 \geq g H_p. \end{cases} \]

(19)

This result is a dual counter-part to the singular term \(\Delta L\). Performing integration over \(p_T\) we can easily reproduce the total imaginary part of Nielsen-Olesen [11].

4. Conclusion

There has been a lot of controversies on the imaginary part of the effective action with a constant field background [4, 7, 8, 10, 11, 12, 13]. The fact that the concept of constant color field itself is not well-defined and requires a specification of the source [9] and a concrete physical problem [10] leads to the uncertainty in calculating the imaginary part of the effective action. Additional controversies originate from the several sources. The first derivation of the positive imaginary part of the effective action by Yildiz and Cox [7] includes only the pole contributions obtained by using the Schwinger’s prescription for passing poles. The negative value of the imaginary part obtained first by Schanbacher [9] was provided by perturbative calculation. This result has been confirmed later by applying a slightly different perturbative calculation scheme [13]. The physical meaning of the negative imaginary part of the effective action is closely related with the asymptotic freedom and corresponds to gluon pair annihilation [9]. A complete total contribution to the imaginary part of the effective action including all non-perturbative contributions had been calculated by Ambjørn and Hughes [6].

In conclusion, we have obtained a complete strict expression for the \(p_T\)-distribution of gluon production rate in a constant chromoelectric field. We have shown that an additional singular term appears in \(p_T\)-distribution. This result might be of a pure academic interest because it is hardly possible to create a constant chromoelectric field in experiments. Nevertheless, it would be very interesting to find possible physical implications of such singular term, for instance, in quark-gluon plasma, where the concept of constant color field can be adopted in mean field approximation.

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[1] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D20, 179 (1979).
[2] G. C. Nayak and P. van Nieuwenhuizen, Phys.Rev. D71 125001 (2005); G. C. Nayak, Phys.Rev. D72, 125010 (2005).
[3] S.P. Gavrilov, D.M. Gitman and J.L. Tomazelli, hep-th/0612064.
[4] Y. Schutz, J. Phys. G30, S903 (2004); L. McLerran and M. Gyulassy, Nucl. Phys. A750, 30 (2005); B. Muller, nucl-th/0508062.
[5] J. Schwinger, Phys. Rev. 82, 664 (1951).
[6] J. Ambjørn and R. J. Hughes, Phys. Lett. B113, 305 (1982).
[7] A. Yildiz and P.H. Cox, Phys. Rev. D21, 1095 (1980); M. Claudson, A. Yildiz, and P.H. Cox, Phys. Rev. D22, 2022 (1980).
[8] W. Dittrich and M. Reuter, Phys. Lett. B128, 321, (1983); S.L. Adler, Phys. Rev. D23, 2905 (1981).
[9] V. Schanbacher, Phys. Rev. D26, 489 (1982).
[10] J. Ambjørn and R. J. Hughes, Nucl. Phys. B197, 113 (1982).
[11] N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978); H. B. Nielsen and M. Ninomiya, Nucl. Phys. B156, 1 (1979).
[12] Y.M. Cho, H.W. Lee, and D.G. Pak, Phys. Lett. B 525, 347 (2002); Y. M. Cho and D. G. Pak, Phys. Rev. D65, 074027 (2002).
[13] Y.M. Cho, D. G. Pak, and M. Walker, JHEP 05, 073 (2004); Y. M. Cho and M. L. Walker, Mod. Phys. Lett. A19, 2707 (2004).
[14] Y.M. Cho and D. G. Pak, Procs. of TMU-YALE Symp. on "Dynamics of Gauge Fields" (Univ. Acad. Press, Tokyo, 1999); hep-th/0006051.
[15] C. Itzikson and J-B. Zuber, Quantum Field Theory (McGraw-Hill) 1985, p.193.