Orbital evolution with white-dwarf kicks

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ABSTRACT
Recent observations of white dwarfs in globular clusters indicate that these stars may get a velocity kick during their time as giants. This velocity kick could originate naturally if the mass loss while on the asymptotic giant branch is slightly asymmetric. If white dwarfs get a kick comparable to the orbital velocity of the binary, the initial Runge-Lenz vector (eccentricity vector) of the orbit is damped to be replaced by a component pointing toward the cross product of the initial angular momentum and the force. The final eccentricity may be of order unity and if the kick is sufficiently large, the system may be disrupted. These results may have important ramifications for the evolution of binary stars and planetary systems.

Key words: white dwarfs — stars: AGB and post-AGB — binaries – stars: mass loss — stars: winds, outflows

1 INTRODUCTION
Spruit (1998) proposed that white dwarfs can acquire their observed rotation rates from mild kicks generated by asymmetric winds toward the end of their time on the asymptotic giant branch (AGB) (Vassiliadis & Wood 1993). Fellhauer et al. (2003) invoked these mild kicks to explain a putative dearth of white dwarfs in open clusters (e.g. Weidemann 1977; Kalirai et al. 2001). The expected signature of white dwarf kicks has been observed in M4 and NGC 6397 (Davis et al. 2006). They found that young white dwarfs are less centrally concentrated than either their progenitors near the top of the main sequence or older white dwarfs whose velocity distribution has had a chance to relax.

If white-dwarf kicks can have dramatic effects on the distribution of stars in a globular cluster, perhaps they could also affect other bound systems such as binary stars. The case of binary stars is interesting for a second reason as they represent a different physical regime than either white-dwarf kicks in clusters or neutron-star kicks in binaries. Whereas the latter two effects are impulsive, the white dwarf kick accumulates over many orbital periods, so its secular effect is more subtle to calculate.

2 CALCULATIONS
The fate and evolution of binaries with asymmetric mass loss is complicated. Specifically the mass loss turns on and off over many binary orbital periods so its influence on the binary orbit may be adiabatic (in contrast with mass loss during the formation of a neutron star). If the mass loss is symmetric the orbit increases in size in inverse proportion to the decreasing mass of the system. Specifically, the angular momentum of the orbit, the Runge-Lenz vector, and the product of the binding energy and the period of the orbit are adiabatic invariants.

Including the effects of an asymmetric wind requires further analysis. The total kick imparted by the wind is on the order of a few to ten kilometers per second (Davis et al. 2006) over many tens of thousands of years, so the force exerted by the kick is typically much smaller than the gravitational forces in the binary that result in accelerations of tens of kilometers per second per year — the kick is a perturbation. Even in the limit that the kick can not be treated perturbatively but it is still adiabatic, one can use the fact that the Kepler problem with a constant external force is tractable in parabolic coordinates to find that the angular momentum and the Runge-Lenz vector along the direction of the kick and the product of the total mass of the system and the semimajor axis are adiabatic invariants. However, these three quantities are insufficient to determine the final state of the binary, most importantly, its eccentricity.

2.1 Centre of mass motion
It is useful to separate the force that imparts the kick into a force on the centre of mass and a torque about the centre of mass. Using centre of mass coordinates, the Lagrangian of the binary including the kick on the giant star is

\[ L = \frac{1}{2} M \dot{R}^2 + \frac{1}{2} \mu \dot{r}^2 + \frac{G M \mu}{r} + F_1 \cdot \left( R + \frac{M_2}{M} r \right) \]

(1)
where $M$ is the total mass, $\mu$ is the reduced mass, $\mathbf{R}$ is the position of the centre of mass and $\mathbf{r} = r_1 - r_2$. The force exerted on a single giant star by its asymmetric wind is

$$\mathbf{F}_1 = b_{\text{wind}} |\dot{M}|$$

(2)

where $b$ is a dimensionless vector characterising the asymmetry; the magnitude of $b$ can range from zero for a symmetric wind to unity for a wind that blows only along a single direction like a rocket. Therefore, the acceleration of the star is given by

$$\frac{d\mathbf{v}_1}{dt} = \frac{1}{M_1} b_{\text{wind}} |\dot{M}|$$

(3)

and the total velocity kick for the single star is

$$\Delta \mathbf{v}_{\text{single}} = b_{\text{wind}} \ln \frac{M_{W,D,1}}{M_1} |\dot{M}|$$

(4)

so the force on the single star can be written in terms of the resulting kick

$$\mathbf{F}_1 = \frac{\Delta \mathbf{v}_{\text{single}}}{\ln (M_{W,D,1}/M_1)} |\dot{M}|.$$  

(5)

From Eq. (1), the equation of motion for $\mathbf{R}$ is

$$\mathbf{F}_1 = (M_1 + M_2) \ddot{\mathbf{R}}$$

(6)

and

$$\frac{d\mathbf{v}}{dt} = \frac{1}{M_1 + M_2} \frac{\Delta \mathbf{v}_{\text{single}}}{\ln (M_{W,D,1}/M_1)} |\dot{M}|$$

(7)

so the total kick to the centre of mass is

$$\Delta \mathbf{v}_{\text{binary}}^{(1)} = \frac{\Delta \mathbf{v}_{\text{single}}(M_1)}{\ln (M_{W,D,1}/M_1)} \ln \frac{M_{W,D,1} + M_2}{M_1 + M_2}$$

(8)

where the product of the velocity and asymmetry of the wind is assumed to be constant (the mass-loss rate may vary) as the mass of the giant star decreases and $\Delta \mathbf{v}_{\text{single}}(M_1)$ is the typical velocity kick imparted to a main-sequence star of mass $M_1$. As the secondary becomes a white dwarf, the binary receives a second kick

$$\Delta \mathbf{v}_{\text{binary}}^{(2)} = \frac{\Delta \mathbf{v}_{\text{single}}(M_2)}{\ln (M_{W,D,2}/M_2)} \ln \frac{M_{W,D,1} + M_2}{M_{W,D,1} + M_2}.$$  

(9)

If the mass of the primary and that of the secondary are about equal (due to dynamical biasing, for example, McDonald & Clarke 1993), the direct sum of the two kicks would equal the kick received by a single star. However, the two kicks will generally not be aligned with each other, so they must be added in quadrature yielding a smaller combined kick for the binary. The total change of the velocity of the centre of mass of the binary is typically about 70%-80% of kick imparted to an individual star (using the initial-final mass relation of Iben & Renzini 1983).

### 2.2 Constant Masses

To treat the evolution of the orbit, the influence of the wind on the orbital dynamics must be determined, especially whether or not it is adiabatic. Fortunately, for the reasons mentioned earlier, the dynamics of the orbit with the asymmetric wind are remarkably similar to the Kepler problem. Redmond (1964) found a generalisation of the Runge-Lenz vector in the presence of an external force ($\mathbf{F} = \mathbf{F}_1$). Specifically, the generalised Runge-Lenz vector is given by

$$\mathbf{C} = \dot{\mathbf{r}} + \frac{\mathbf{L} \times \mathbf{p}}{GM\mu^2} - \frac{(\mathbf{r} \times \mathbf{F}) \times \mathbf{r}}{2GM\mu}$$

(10)

where $M$ is the total mass of the binary, $\mu$ is the reduced mass of the binary, $\mathbf{L}$ is the orbital angular momentum, $\mathbf{r}$ is the relative position of the two stars and $\mathbf{p}$ is their relative momentum. The first two terms give the Keplerian Runge-Lenz vector. In this system only the component of $\mathbf{C}$ parallel to the applied force is conserved. The vector evolves according to

$$\dot{\mathbf{C}} = \frac{3}{2GM\mu^2} \mathbf{L} \times \mathbf{F}.$$  

(11)

The generalised Runge-Lenz vector is no longer perpendicular to the angular momentum,

$$\mathbf{L} \cdot \mathbf{C} = \frac{r^2 \mathbf{F} \cdot \mathbf{L}}{2GM\mu}$$

(12)

and the relative position of the two bodies is constrained by

$$\mathbf{C} \cdot \mathbf{r} = r - \frac{L^2}{2GM\mu} = r - l$$

(13)

where $l$ is the semilatus rectum. With some rearrangement this yields the equation for an ellipse with a focus at the origin in polar coordinates

$$r = \frac{l}{1 - |\mathbf{C}| \cos \theta} = \frac{l}{1 + e \cos \theta}$$

(14)

where $\theta$ is the angle between $\mathbf{r}$ and $\mathbf{C}$. This equation shows that the vector $\mathbf{C}$ has a length equal to the eccentricity of the orbit and points from the pericenter toward the apocentre of the orbit. The external force naturally exerts a torque on the orbit, so the angular momentum is not conserved,

$$\dot{\mathbf{L}} = \mathbf{r} \times \mathbf{F}.$$  

(15)

However, because the external force is taken to be a perturbation, both $\mathbf{C}$ and $\mathbf{L}$ can be taken to be constant over an orbital period ($T$). When averaged over an orbit the only component of $\mathbf{L}$ that remains must be perpendicular to both $\mathbf{F}$ and $\mathbf{C}$ because the secondary spends the same amount of time between pericentre and apocentre as between apocentre and pericentre and consequently the component of $\mathbf{r}$ perpendicular to $\mathbf{C}$ vanishes. The remaining component of the torque is

$$\dot{\mathbf{L}}_{\mathbf{C} \times \mathbf{F}} = \frac{\mathbf{C} \cdot \mathbf{r}}{\mathbf{C} \cdot \mathbf{C} \times \mathbf{F}} \mathbf{C} \times \mathbf{F} = \frac{l}{e^2} \left( \frac{1}{1 + e \cos \theta} - 1 \right) \mathbf{C} \times \mathbf{F}.$$  

(16)

yielding the torque averaged over an orbit

$$\langle \dot{\mathbf{L}} \rangle = \frac{\mathbf{C} \times \mathbf{F}}{\mathbf{C} \cdot \mathbf{C}} \left[ \frac{1}{P e^2} \int_0^{2\pi} \left( \frac{1}{1 + e \cos \theta} - 1 \right) \ d\theta \right]$$

(17)

$$\langle \dot{\mathbf{L}} \rangle = \frac{\mathbf{C} \times \mathbf{F}}{\mathbf{C} \cdot \mathbf{C}} \left[ \frac{l^3 \mu}{P e^2 L} \int_0^{2\pi} \frac{1}{(1 + e \cos \theta)^{3/2}} \ d\theta - \frac{l}{e^2} \right]$$

(18)

Using the definition of the angular momentum, $L = \mu r^2 \dot{\theta}$ gives

$$\langle \dot{\mathbf{L}} \rangle = \frac{\mathbf{C} \times \mathbf{F}}{\mathbf{C} \cdot \mathbf{C}} \left[ \frac{l^3 \mu}{P e^2 L} \int_0^{2\pi} \frac{1}{(1 + e \cos \theta)^{3/2}} \ d\theta - \frac{l}{e^2} \right].$$

(19)

To calculate this integral, the substitution $z = e^\theta$ converts it to a contour integral, so

$$\cos \theta = \frac{1}{2} \left( z + \frac{1}{z} \right), \ d\theta = \frac{dz}{iz}$$

(20)
The angular momentum has a similar solution

\[
\langle \mathbf{L} \rangle = C \times F \left[ \frac{1}{e^2 \mathcal{L}_p} \int \left( \frac{1}{1 + \frac{2}{a} \sqrt{1 - e^2}} \right) \frac{dz}{dz} - \frac{1}{e^2} \right]
\]

(21)

\[
= C \times F \left[ -8i \frac{\mu^3}{e^2 \mathcal{L}_p} \int \left( \frac{z^2}{(z^2 + \frac{2}{a} + 1)^3} \right) dz - \frac{1}{e^2} \right]
\]

(22)

\[
= C \times F \left[ -8i \frac{\mu^3}{e^5 \mathcal{L}_p} \int \left( \frac{z^2 dz}{(z - z_+)^3} (z - z_-)^3 - \frac{1}{e^2} \right) \right]
\]

(23)

where the contour is the unit circle and

\[
z_\pm = \frac{1}{e} \left( \pm \sqrt{1 - e^2} - 1 \right),
\]

(24)

so \(z = z_+\) is a pole within the contour. Using the Cauchy integral formula yields after some algebraic simplifications

\[
\langle \mathbf{L} \rangle = C \times F \left[ \frac{\mu^3}{e^2 \mathcal{L}_p} \int \left( \frac{\pi (2 + e^2)}{2e^2 (1 - e^2)^{3/2}} - \frac{1}{e^2} \right) \right].
\]

(25)

Using the value of the semilatus rectum from Eq. (13) and Kepler’s third law, \( P = \sqrt{GM/\alpha^3/(2\pi)} \) where \( \alpha = l/(1 - e^2) \) is the semimajor axis, yields

\[
\langle \mathbf{L} \rangle = C \times F \left[ \frac{\mu^3}{e^2 \mathcal{L}_p} \frac{2 + e^2}{2e^2 (1 - e^2)^{3/2}} - \frac{1}{e^2} \right] = \frac{3a}{2} C \times F.
\]

(26)

To estimate the timescale of the evolution of the orbital parameters, Eqs. (11) and (26) can be combined to yield

\[
\dot{C} \approx \frac{9}{4GM\mu^2} (C \times F) \times F, \quad \dot{L} = \frac{9}{4GM\mu^2} (L \times F) \times F
\]

(27)

if the masses of the stars and the force are taken to be constant. If the prefactor is constant, these are the equations for a harmonic oscillator with a frequency of zero along the direction of \( F \) and a frequency of

\[
\omega = \frac{3F}{2\mu} \sqrt{\frac{a}{GM}}
\]

(28)

perpendicular to \( F \). The evolutionary timescale is

\[
\tau = \frac{1}{\omega} = \frac{2}{3} \frac{\mu}{P} \sqrt{\frac{GM}{\alpha}} \approx \Delta t_{\text{wind}} \frac{v_{\text{orbital}}}{v_{\text{single}}}
\]

(29)

When the prefactor can be taken to be constant, Eq. (27) can be solved by splitting the Runge-Lenz vector into a component parallel and perpendicular to the applied force, yielding

\[
C_{||} = \text{Constant},
\]

(30)

\[
C_{\perp} = C_{\perp,0} \cos \omega t + \sqrt{1 - e_0^2} \frac{L_\perp \times F}{L_\perp F} \sin \omega t.
\]

(31)

The angular momentum has a similar solution

\[
L_{||} = \text{Constant},
\]

(32)

\[
L_{\perp} = L_{\perp,0} \cos \omega t + \frac{L_0}{\sqrt{1 - e_0^2}} \frac{C_\alpha \times F}{F} \sin \omega t.
\]

(33)

In both cases the component of the vector along the direction of the force is conserved, and the perpendicular components vary harmonically.

### 2.3 Variable Masses

In general the masses in the binary and the force are not constant, so Eq. (11) must be modified to yield

\[
\dot{C} = \frac{3}{2GM\mu^2} L \times F - \frac{\left( r \times F \right) \times r}{2GM\mu} + \frac{d}{dt} \left( \frac{1}{\mathcal{L}^2} \right) \left( L \times p - \frac{P}{\mu} \right) (r \times F) \times r
\]

(34)

If the applied force is written as \( F = (M_2/M) b_{\text{wind}} |M| \) where \( b \) is a dimensionless vector characterising the asymmetry in the wind and the mass ratio gives the force in the centre of mass frame from Eq. (31) Eq. (32) can be expanded in powers of \( M \). If \( v_{\text{wind}} \) and \( b \) are taken to be functions of the mass of the star, the first and third terms are first order in \( M \), and the second and fourth terms are second order in \( M \). The orbit average of the third term vanishes because the orbit is closed to lowest order in \( M \). This leaves only the first term or Eq. (11) at lowest order in \( M \). In taking the orbit average to obtain Eq. (20), it was assumed that \( L \) and \( C \) were constant in time; therefore Eq. (26) is only accurate to first order in \( M \), so it is consistent at lowest order to use Eq. (11) and (26).

In general the prefactor is not constant; however the combination \( Ma \) is an adiabatic invariant because the equations of motion of the binary are separable in cylindrical coordinates as discussed in (11). Furthermore, the applied force is proportional to \( M \), so Eq. (27) can be written with the total mass of the system, \( M \), as the dependent variable. This yields

\[
\frac{d^2}{dt^2} C = \frac{9}{4M^2 \mu M_1^2} \frac{Ma^2}{G} v_{\text{wind}} (C \times b) \times b
\]

(35)

and similarly for the angular momentum. This assumes that the value of \( M \) is constant in time. One can argue that the mass-loss rate depends on the mass of the star, so changes in the mass-loss rate would only appear at higher order.

This equation may be rewritten by changing variables to

\[
x = \ln \left( 1 + \frac{M_2}{M_1} \right)
\]

(36)

where star 1 is the star that loses mass and the variable \( x \) increases in time. This yields,

\[
\frac{d^2}{dx^2} C + \left( 1 + 2 \frac{M_2}{M_1} \right) \frac{d}{dx} C = \frac{9}{4M_1^2} \frac{Ma^2}{G} v_{\text{wind}} (C \times b) \times b
\]

(37)

the equation for a damped harmonic oscillator with a damping parameter that decreases in time. The original Eq. (35) yields the solution

\[
C_\perp (M_1) = C_1 \left( \frac{M_1}{M_0} \right)^{\frac{1}{2} + \alpha} \left( \frac{M_1}{M_{1,0}} \right)^{\frac{1}{2} - \alpha} + C_2 \left( \frac{M_1}{M_0} \right)^{\frac{1}{2} + \alpha} \left( \frac{M_1}{M_{1,0}} \right)^{\frac{1}{2} - \alpha}
\]

(38)

where

\[
\alpha = \frac{1}{2} \sqrt{1 - 4e_0^2}
\]

(39)

and the undamped frequency of the oscillator is
\[ \beta_0 = \frac{3}{2} b v_{\text{wind}} \sqrt{\frac{M_0}{G M_2}} \approx 3 \frac{\Delta v_{\text{single}}}{v_{\text{orbital}}} \]  

and the parallel component is constant as before. This frequency is dimensionless because the eccentricity is now written as a dimensionless function of the mass ratios rather than time.

The initial conditions give
\[
C_{1,2} = \frac{1}{2} \left( 1 \mp \frac{M_0 + M_{1,0}}{\sqrt{1 - 4\beta_0^2 M_2}} \right) C_{1,0} \\
  \pm \beta_0 \frac{M_0}{M_{1,0}} \sqrt{1 - C_0^2 L_0 \times b} \]  

If \( \beta_0 < 1/2 \) then the solution is underdamped and Eq. (38) provides a clear definition. On the other hand, if \( \beta_0 > 1/2 \) then the solution is underdamped and oscillatory as
\[
C_{\pm}(M_1) = \left( \frac{M_{1,0}}{M_0 M_{1,0}} \right)^{1/2} \times \\
  \left\{ C_{\pm,0} \cos \left[ \beta \ln \left( \frac{M_1 M_0}{M_{1,0} M_0} \right) \right] + \\
  \left( \frac{\beta_0}{\beta_1 M_0} \frac{M_0}{M_{1,0}} \sqrt{1 - C_0^2 L_0 \times b} - \frac{M_0 + M_{1,0}}{2 \beta M_2} C_{\pm,0} \right) \right\} \sin \left[ \beta \ln \left( \frac{M_1 M_0}{M_{1,0} M_0} \right) \right] 
\]

where \( \beta = \beta_0 \sqrt{1 - 1/(2\beta_0^2)} \).

### 3 RESULTS

The form of the evolution generally depends on the value of \( \beta_0 \) specifically whether it exceeds one-half, and the initial and final masses of the objects in the binary. It is useful to obtain a more accurate value of \( \beta_0 \) for various situations. Specifically, the rocket equation, Eq. (4) gives
\[
\Delta v_{\text{single}} = 2 b v_{\text{wind}} \ln \frac{M_{1,0}}{M_{1,W,D}} 
\]

where \( \Delta v_{\text{single}} \) is the magnitude of the velocity kick that the star would have received if it were solitary so
\[
\beta_0 = \frac{3}{2} \frac{\Delta v_{\text{single}}}{v_{\text{orbital}}} \sqrt{\frac{M_0}{G M_2}} 
\]

\[ \approx 0.1 \frac{\Delta v_{\text{single}}}{1 \text{ km s}^{-1}} \left( \frac{M}{2 M_\odot \text{ 1 AU}} \right)^{1/2} \left( \frac{M_2}{M_\odot} \right)^{-1} \]  

\[ \approx 80 \frac{\Delta v_{\text{single}}}{1 \text{ km s}^{-1}} \left( \frac{M}{2 M_\odot \text{ 1 AU}} \right)^{1/2} \left( \frac{M_2}{M_\odot} \right)^{-1} \]  

where \( M_1 \) is the mass of Jupiter and
\[
\ln \frac{M_1}{M_{1,W,D}} = \ln \frac{M_{1,0}}{0.38 M_\odot + 0.15 M_1} 
\]

was taken to be 0.63, the value appropriate for one solar mass using the initial-final mass relation of Iben & Renzini (1983).

#### 3.1 Weak winds

Although it might not be obvious from Eq. (43), in the limit where the wind asymmetry \( b \) vanishes, the eccentricity vector is constant. Specifically, in the limit of \( \beta_0 \ll 1/2 \), Eq. (43) through (45) become
\[
C_{\pm}(M_1) \approx C_{\pm,0} - \beta_0 \left( 1 - \frac{M_1}{M_{1,0}} \right) \frac{M_2}{M_{1,0}} \sqrt{1 - \frac{C_0^2 L_0 \times b}{L_0 b}} \]  

so in the limit where the asymmetry of the wind vanishes, the eccentricity vector remains constant. In general the eccentricity will either increase or decrease slightly, but if one considers that the wind will propel the giant star in a random direction compared to the original, orbital angular momentum, the final eccentricity is on average the sum in quadrature of the initial eccentricity and an increment due to the wind, therefore, slightly larger than the initial eccentricity. Furthermore, because on average the two sum in quadrature, the change in the eccentricity decreases as the initial eccentricity increases, so it would be difficult for such a weak wind to unbind a binary.

#### 3.2 Planetary systems

\[ \times \] In the case of a planetary companion, the value of \( \beta_0 \) is large and approximately equal to \( \beta \) and \( M \approx M_1 \) which allows further approximations. An important quantity in this analysis is the product of the undamped frequency and the mass of the secondary
\[
\beta_0 M_2 = 0.08 M_\odot \frac{\Delta v_{\text{single}}}{1 \text{ km s}^{-1}} \left( \frac{M}{2 M_\odot \text{ 1 AU}} \right)^{1/2} 
\]

which is small compared to the mass lost if the velocity kick is small. An expansion in \( \beta_0 M_2 \) for \( M_2 \ll M_1 \), yields
\[
C_{\pm}(M_1) \approx C_{\pm,0} - \beta_0 \left( 1 - \frac{M_1}{M_{1,0}} \right) \frac{M_2}{M_{1,0}} \sqrt{1 - \frac{C_0^2 L_0 \times b}{L_0 b}} \]  

which is identical to the weak wind case. Eq. (45) even though the undamped frequency is large. In both cases, the important factor \( \beta_0 M_2 \) is small yielding a similar expansion. Specifically if the initial eccentricity is low, one would expect for planetary systems, the final eccentricity of the system will increase linearly with the strength of the wind and the amount of mass loss, resulting in final eccentricities of a few percent. In general the initial eccentricity and the increment due to the asymmetric wind will add in quadrature, resulting in a modest gain in eccentricity due to the wind.

#### 3.3 Strong winds

In the case of a strong wind (a large value of \( \Delta v_{\text{single}} \) compared to the typical orbital velocities), the value of \( \beta_0 M_2 \) is large, as is the value of \( \beta_0 \). Again \( \beta_0 \) is approximately equal to \( \beta \), but the two terms that multiply the sine function in Eq. (12) are similar in magnitude. The results of Davis et al. (2008) and Heyl (2007) indicate that the kicks that white dwarfs receive as giants could be as large as 5-10 km/s, so \( \beta_0 M_2 \approx 1 M_\odot \), so the approximations in 3.1 and 3.2 do not apply.

As before the final Runge-Lenz vector is a linear combination of the initial Runge-Lenz vector and a vector of
length about unity in the direction of the cross-product of the initial angular momentum and the asymmetry. In general the results depicted in Fig. 1 are rather complicated. The final Runge-Lenz vector is given by

\[ C_{\parallel} = C_{\parallel,0} + A_{\perp} C_{\perp,0} + A_{L_0 \times \mathbf{b}} \frac{L_0 \times \mathbf{b}}{L_0 \cdot \mathbf{b}} \]  

(51)

where the two coefficients are depicted in Fig. 1. The two contributions are about ninety degrees out of phase, so the final eccentricity may have little memory of its initial value. Furthermore, even if the initial eccentricity was small, the second term is of order unity, so the final eccentricity may be large. For a particular white dwarf binary one can infer the initial mass of the white dwarf from its current mass using, for example, Eq. (47) and backtrack to obtain the initial semimajor axis and estimate the contribution to the final eccentricity from kicks of different magnitudes using Eq. (42).

### 3.4 Disruption

In Fig. 1 the length of the final eccentricity vector may exceed unity; this implies that the binary has become unbound. To estimate the velocity kick required to unbind binaries with various initial properties the maximum eccentricity of an initially circular orbit is determined. For small velocity kicks, this typically occurs at the end of the mass loss, but for larger kicks the peak eccentricity according to Eq. (12) may be reached while the mass loss is ongoing. Fig. 2 depicts the velocity kick that the primary would receive if it were single that is large enough to unbind the system. For all of the primary masses, the required velocity kick increases with decreasing mass of the secondary. This is simply due to the translation into the centre of mass frame of the system. For large primary masses, the primary loses approximately 85% of its mass independent of its mass, so the results become nearly scale-free in this regime, the required velocity kick is about \( \sqrt{GM_0/(2a_0)} \) for equal mass binaries and increases in inverse proportion to the mass of the secondary. The dependence becomes shallower when the final mass of the primary exceeds that of the secondary.

### 4 CONCLUSIONS

The possibility of asymmetric winds in asymptotic giant stars opens a range of new phenomena in the evolution of binary stars. The presence of an asymmetry in the wind typically increases the eccentricity of the binary, so a comparison of the properties of a binaries containing a white dwarf to binaries of main-sequence stars could provide an independent probe of the importance of white dwarf kicks. In equal mass binaries if the wind is sufficiently asymmetric to induce a kick on the order of the orbital velocity of a single star, the kick is likely to unbind the system, in contrast with the symmetric mass loss that only results in an increase in the binary separation, so the binary fraction of main-sequence stars and white dwarfs may differ providing an additional probe of this process. The effects of white dwarf kicks in planetary systems are more subtle. Even large kicks do not typically unbind the system; however, kicks may pump the eccentricity of the orbits to high enough values to change
Figure 2. The required velocity kick imparted to a single star required to disrupt an initially circular binary as a function of the masses of the two stars in the system. The curves are labeled with the mass of the primary. The required velocity kick increases as the mass of the secondary decreases because the contribution of the wind to the equation of motion of the binary is proportional to $M_2$ from Eq. (1). This is a result of the separation of the system into the motion of the centre of mass and the motion about the centre of mass.

the resonance structure of multiple planet systems (see also Debes & Sigurdsson 2002), possibly resulting in the ejection of smaller planets from the system. Finally the white dwarf kicks could play an important role in the binary evolution and especially double degenerate binaries, yielding systems that would otherwise be difficult to form.

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