The Semileptonic Decay Fraction of $B$-Mesons in the Light of Interfering Amplitudes

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Abstract

Consequences of the interference between spectator amplitudes for the lifetimes and semileptonic decay fractions of $B^0$ and $B^+$ mesons are discussed. Extracting these amplitudes from a fit to 11 exclusive hadronic $B$ decay fractions we find $a_1 = 1.05 \pm 0.03 \pm 0.10$, $a_2 = +0.227 \pm 0.012 \pm 0.022$, an inclusive semileptonic decay fraction of $(11.2 \pm 0.5 \pm 1.7)\%$, and a lifetime ratio $\tau(B^+)/\tau(B^0) = 0.83 \pm 0.01 \pm 0.01$.

Although there has been significant progress in the calculation of QCD corrections in the decays of heavy flavour mesons, there are still some unsolved puzzles. One of the most intriguing is the low semileptonic decay fraction of $B$ mesons [1]. We will show in this paper that the discrepancy between theory and experiment is considerably reduced by the interpretation of recent CLEO results [2] on hadronic decay fractions in the framework of the spectator model with factorization.

The $D^0-D^+$ meson lifetime difference has been satisfactorily reproduced in such a model by Bauer, Stech, and Wirbel [3,4], using interfering amplitudes of spectator diagrams for $D^+$ hadronic two-body decays. The same model predicts a negligible difference between the decay rates for two body modes of $B^0$ and $B^+$ mesons. However, in contrast to the $D$ system, only a small fraction of all hadronic $B$ meson decays is included in the calculation so a conclusive prediction of the lifetime ratio is not possible. Using duality between the quark and the hadron pictures of strong interactions, we will demonstrate

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1 Supported under DOE grant number DE-FG02-91-ER40690.

2 Supported by the German Bundesministerium für Forschung und Technologie, under contract number 056DD11P.
how interfering spectator amplitudes on the quark level may explain the low semileptonic decay fraction for $B$ decays, and at the same time predict the lifetime ratio.

The spectator diagrams for hadronic $B^0$ and $B^+$ decay are shown in Fig. 1. Diagrams (a) and (b) are topologically identical as are (c) and (d), but the quarks are combined into final state hadrons in a different way. In $B^+$ meson decay, diagrams (c) and (d) can lead to the same final state hadrons and hence the corresponding amplitudes interfere. The two diagrams for describing $B^0$ decays do not interfere, since in (a) both final state hadrons are charged, while in (b) both are neutral. In the quantitative description, two modifications of the weak decay amplitude lead to diagrams (a) to (d):

- Owing to the additional exchange of one or more gluons in parallel to the $W$, effective neutral current contributions occur, with the same $V - A$ structure as the charged current matrix element from pure $W$ exchange. These contributions would be the only source for diagrams (b) and (d) in the infinite colour limit $N_c \to \infty$. They have been calculated in next to leading log QCD approximation [5], leading to a coefficient $c_2 \approx -0.26$ at the $b$ mass scale for 4 flavours and $\Lambda_{\text{QCD}} = 250\text{MeV}$ [6].
Table 1 Experimental averages and theoretically predicted decay fractions for hadronic $B$ decays, assuming $|V_{cb}|^2 \cdot \tau_B = 2.35 \cdot 10^{-15}$ s, and $f_D = f_{D^*} = 220$ MeV

| Decay       | Exp. Average [%] | Neubert et al. [6] | Deandrea et al. [7] |
|-------------|------------------|---------------------|---------------------|
| $B^+ \rightarrow D^0 \pi^+$ | 0.45 ± 0.04 | $0.265(a_1 + 1.230a_2)^2$ | $0.268(a_1 + 1.16a_2)^2$ |
| $B^+ \rightarrow D^0 \rho^+$ | 1.10 ± 0.18 | $0.622(a_1 + 0.662a_2)^2$ | $0.693(a_1 + 0.46a_2)^2$ |
| $B^+ \rightarrow D_s^0 \pi^+$ | 0.51 ± 0.08 | $0.255(a_1 + 1.292a_2)^2$ | $0.268(a_1 + 1.71a_2)^2$ |
| $B^+ \rightarrow D_s^0 \rho^+$ | 1.32 ± 0.31 | $0.70(a_1^2 + 1.49a_1a_2 + 0.64a_2^2)$. | $0.92(a_1^2 + 1.31a_1a_2 + 0.60a_2^2)$ |
| $B^+ \rightarrow \psi K^+$ | 0.106 ± 0.015 | $1.819a_2^2$ | $1.587a_2^2$ |
| $B^+ \rightarrow \psi K^{*+}$ | 0.17 ± 0.05 | $2.932a_2^2$ | $2.325a_2^2$ |
| $B^0 \rightarrow D^- \pi^+$ | 0.26 ± 0.04 | $0.264a_1^2$ | $0.268a_1^2$ |
| $B^0 \rightarrow D^- \rho^+$ | 0.69 ± 0.14 | $0.621a_1^2$ | $0.693a_1^2$ |
| $B^0 \rightarrow D_s^- \pi^+$ | 0.29 ± 0.04 | $0.254a_1^2$ | $0.268a_1^2$ |
| $B^0 \rightarrow D_s^- \rho^+$ | 0.74 ± 0.16 | $0.702a_1^2$ | $0.917a_1^2$ |
| $B^0 \rightarrow \psi K^0$ | 0.069 ± 0.022 | $1.817a_2^2$ | $1.587a_2^2$ |
| $B^0 \rightarrow \psi K^{*0}$ | 0.146 ± 0.029 | $2.927a_2^2$ | $2.325a_2^2$ |

Diagrams (a) and (c) are enhanced by these QCD effects, leading to a coefficient $c_1 \approx 1.12$ for this amplitude.

- Recombination of the mixed quark antiquark pairs is possible if the colours match accidentally. This introduces a factor $1/N_c$ in the amplitude relative to the diagrams where the quarks are already in a colour singlet state, leading to new coefficients

$$a_1 = c_1 + \frac{1}{N_c}c_2 \approx 1.03$$

$$a_2 = c_2 + \frac{1}{N_c}c_1 \approx 0.11$$

for diagrams (a,c) and (b,d), respectively.

Since a fit to exclusive two-body decay fractions of $D$ mesons yields the experimental results $a_1 \approx c_1$ and $a_2 \approx c_2$ [3], it has been argued that the $1/N_c$ correction should be omitted, corresponding to the limit $N_c \rightarrow \infty$. However, so far no convincing argument has been found to support this proposition. It is also possible that $a_1$ and $a_2$ differ from the QCD expectation because $c_1$ and $c_2$ cannot be reliably calculated perturbatively. The $b$ quark mass scale could be sufficiently large for more reliable predictions.

The distinction between interfering amplitudes for the $B^+$ and non-interfering for the $B^0$ may only be valid for two-body decays. On the other hand, many-body final states will most likely start as two colour singlet quark antiquark pairs, including intermediate massive resonances. Interference between final states via different resonant channels involves strong phases which modify the rate for each individual final state in a random way and disappear in the sum of all states. It seems therefore reasonable to extend the model for exclusive two-body decays to the majority of hadronic final states in an
Table 2 Experimental results and theoretical predictions for ratios of $B^+$ and $B^0$ decay rates, scaled to $f_{D(D^*)} = 220$MeV

| $R_1 = \frac{\Gamma(B^+ \to D^0\pi^+)}{T(B^0 \to D^0\pi^-)}$ | exp. average | Neubert et al. [6] |
| $R_2 = \frac{\Gamma(B^+ \to D^0\rho^+)}{T(B^0 \to D^0\rho^-)}$ | $1.71 \pm 0.38$ | $(1 + 1.23a_2/a_1)^2$ |
| $R_3 = \frac{\Gamma(B^+ \to D^0\pi^+)}{T(B^0 \to D^0\rho^-)}$ | $1.60 \pm 0.46$ | $(1 + 0.66a_2/a_1)^2$ |

inclusive picture at the quark level. We assume that the formation of two colour singlets is the essential step of hadron production, which is taken into account quantitatively by $a_1$ and $a_2$. We neglect modifications by decays into baryon anti-baryon pairs, where our assumption is not valid.

Experimentally, values for $a_1$ and $a_2$ have been obtained from the measured partial rates of the $B$ meson decay modes $D\pi, D\rho, D^{*}\pi, D^{*}\rho, J/\psi K$, and $J/\psi K^*$. Combining the experimental decay fractions measured by the ARGUS [8] and CLEO [2] experiments gives the averages listed in Table 1. We have used $B(D^0 \to K^-\pi^+) = (3.90 \pm 0.16)\%$ [9], $B(D^+ \to K^-\pi^+\pi^+) = (9.1 \pm 1.4)\%$ [10], $B(D^{*+} \to D^0\pi^+) = (68.1 \pm 1.6)\%$ [11], and $B(J/\psi \to l^+l^-) = (5.9 \pm 0.2)\%$ [12]. The partial rates are determined under the assumption of equal decay fractions of the $\Upsilon(4S)$ into $B^+B^-$ and $B^0\overline{B}^0$ pairs, i.e. $f^+/f^{00} = 1$. This quantity is not well measured experimentally; we assume in the following $f^+/f^{00} = 1.00 \pm 0.10$.

To estimate $a_1$ we use $B^0$ decays into $D^-\pi^+, D^-\rho^+, D^{*-}\pi^+$ and $D^{*-}\rho^+$. Using theoretical predictions from the model by Neubert et al. [6] and $|V_{cb}|^2 \cdot \tau_B = 2.35 \cdot 10^{-15}$ s, $f_D = f_{D^*} = 220$MeV we obtain

$$|a_1| = 1.03 \pm 0.05 \pm 0.10 \pm 0.05$$

where the first error is statistical including uncertainties in the $D^0$ and $D^+$ decay fractions, the second is from the error on $V_{cb} \cdot \sqrt{\tau(B)}$, and the third comes from the uncertainty on $f^+/f^{00}$. The model of Deandrea et al. [7] gives a similar answer, $|a_1| = 1.02 \pm 0.05 \pm 0.10 \pm 0.05$.

$B \to J/\psi$ decays can be used to obtain an estimate for $|a_2|$. Combining the experimental results in Table 1 with the model of Neubert et al. and the same factors as above, we find

$$|a_2| = 0.23 \pm 0.011 \pm 0.02 \pm 0.01$$

where the errors are given as for $|a_1|$ above. The model of Deandrea et al. leads to $|a_2| = 0.25 \pm 0.013 \pm 0.02 \pm 0.01$. Within the errors, both models give the same answer. In the following we use the model by Neubert et al.

This procedure using decay modes that are only sensitive to either the $a_1$ or the $a_2$ amplitude does not reveal the relative sign between $a_1$ and $a_2$. The sign can be
obtained from $B^+ \to D^0$ and $B^+ \to D^{*0}$ decays, which have contributions from both amplitudes. A relative plus sign between the $a_1$ and the $a_2$ amplitudes would cause $\Gamma(B^+ \to D^{(*)0}\pi(\rho)^+)/\Gamma(B^0 \to D^{(*)0}\pi(\rho)^+) > 1$, while a minus sign would correspond to ratios below 1. The experimental results and the model predictions for the decay ratios in the modes $D\pi^-$, $D\rho^-$, and $D^{*}\pi^-$ are given in Table 2. They show a clear preference for the positive sign. The theoretical prediction for the decay $B^+ \to D^{*0}\rho^+$ is too uncertain [13] to include this mode in the determination of $a_1$ and $a_2$. Taking ratios of $B^+$ and $B^0$ decays eliminates the uncertainties due to $|V_{cb}|$ but leaves those originating from $\tau(B^+)/\tau(B^0)$ and $f^+/f^{00}$. The main difference between the models are details of the $B \to \pi$ and $B \to \rho$ form factors. The predictions also depend on the $D$ and $D^{*}$ meson decay constants $f_D$ and $f_{D^{*}}$. Following Neubert et al. [6] we assume $f_D = f_{D^{*}} = 220$ MeV. On the experimental side, the error due to the $B^0$ decay fractions cancels in the ratios involving $B \to D^{*}$ decays. A least squares fit with seven $D^{(*)}$ modes from Table 1, excluding only $B^+ \to D^{*0}\rho^+$, gives

$$a_1 = 1.04 \pm 0.05,$$
$$a_2 = 0.24 \pm 0.06,$$

corresponding to the ratio

$$a_2/a_1 = +0.23 \pm 0.06 \pm 0.05 \pm 0.05.$$ 

The first error is the one standard deviation error from the fit, the second error is from $f^{+}/f^{00}$, and the third error comes from the uncertainty in $\tau(B^+)/\tau(B^0) = 1.00 \pm 0.10$. The sign of $a_2/a_1$ turns out to be positive in agreement with QCD and $N_c = 3$. The agreement between the $a_2$ values determined from $J/\psi K^{(*)}$ and $D^{(*)}\pi(\rho)$ decays is not necessary since the spectator quarks (graphs a to d in Figure 1) could have a different influence in each decay mode. However, it supports our basic assumption that all hadronic decays of $B$ mesons via two-body decay modes are determined by the same $a_1$ and $a_2$ coefficients. Therefore, we use all 11 decay fractions in the fits described below.

Under our assumption of duality, the coefficients $a_1$ and $a_2$ can be used to predict hadronic and semileptonic partial widths of the $B^+$ and $B^0$ mesons. Since $b \to u$ and $b \to s$ transitions have been shown to be very small, we may safely neglect them in the following formulae; $b \to u$ contributions are included in the fits using $V_{ub}/V_{cb} = 0.08$. In Table 3, we consider only $b \to c$ spectator decays. The phase space factors for $b \to c q_2 q_3$ are

$$I(r_c, r_2, r_3) = 24 \int \left(1-r_3\right) \left(\xi^2 - r_c^2 - r_2^2\right) \left(1 + r_3^2 - \xi^2\right)$$
$$\sqrt{[\xi^2 - (r_c + r_2)^2][\xi^2 - (r_c - r_2)^2][1 - (r_3 + \xi)^2][1 - (r_3 - \xi)^2]} \frac{d\xi}{\xi}$$

with $r_i = m(q_i)/m(b)$. In Table 3, we give the relative factors $PS = I(r_c, r_2, r_3) /I(r_c, 0, 0)$ which have been calculated using current masses of 0.009, 0.005, 0.18, 1.24,
and 4.65 GeV/c² for d, u, s, c, and b quarks. The perturbative QCD correction for the semileptonic width is [14]

\[ \Gamma(\bar{b} \to \bar{c}e^+\nu) = \Gamma_0 \cdot \left(1 - \frac{2\pi}{3}\alpha_s + \frac{25}{6\pi}\alpha_s\right) \approx 0.86\Gamma_0 \]

for \( A_{\text{QCD}} = 200\text{MeV} \) and 4 flavours.

From the factors in Table 3 we obtain the following total widths, normalized to the lowest order semileptonic width \( \Gamma_0(b \to e^-\bar{\nu}) \)

\[ \Gamma(B^+)/\Gamma_0 = 1.91 + 4.44(a_1^2 + a_2^2) + 5.99a_1a_2, \]
\[ \Gamma(B^0)/\Gamma_0 = 1.91 + 4.44(a_1^2 + a_2^2). \]

Using these widths, we can calculate two important quantities.

- The average semileptonic decay fraction of \( B^0 \) and \( B^+ \),

\[ B(B \to e\nu X) = \frac{1}{2.22 + 5.16(a_1^2 + a_2^2) + 3.49a_1a_2}, \]
Table 3 Contributions from all $b \rightarrow c$ spectator diagrams. Partial widths are obtained as $\Gamma = \Gamma_0(\bar{b} \rightarrow \bar{c}e^+\nu) \cdot \text{CKM} \cdot \text{QCD} \cdot \text{PS}$.

| $B^+ (bu)$ | $\text{CKM}$ | $\text{QCD}$ | $\text{PS}$ | $B^0 (bd)$ | $\text{CKM}$ | $\text{QCD}$ | $\text{PS}$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{c}u$ $e^+\nu$ | 0.86 | 1.00 | $\bar{c}d$ $e^+\nu$ | 0.86 | 1.00 |
| $\bar{c}u$ $\mu^+\nu$ | 0.86 | 0.99 | $\bar{c}d$ $\mu^+\nu$ | 0.86 | 0.99 |
| $\bar{c}u$ $\tau^+\nu$ | 0.86 | 0.23 | $\bar{c}d$ $\tau^+\nu$ | 0.86 | 0.23 |
| $\bar{c}u$ $\bar{d}u$ \footnote{\text{ }} | $|V_{ud}|^2 = 0.951$ | $3(a_1 + a_2)^2$ | $1.00 | \bar{c}d$ $\bar{d}u$ \footnote{\text{ }} | $|V_{ud}|^2$ | $3a_1^2$ | 1.00 |
| $\bar{c}u$ $\bar{s}u$ \footnote{\text{ }} | $|V_{us}|^2 = 0.049$ | $3(a_1 + a_2)^2$ | $0.98 | \bar{c}d$ $\bar{s}u$ \footnote{\text{ }} | $|V_{us}|^2$ | $3a_1^2$ | 0.98 |
| $\bar{c}c$ $\bar{s}c$ \footnote{\text{ }} | $|V_{cs}|^2 = 0.949$ | $3a_1^2$ | $0.48 | \bar{c}d$ $\bar{s}c$ \footnote{\text{ }} | $|V_{cs}|^2$ | $3a_1^2$ | 0.48 |
| $\bar{c}c$ $\bar{s}u$ \footnote{\text{ }} | $|V_{cs}|^2$ | $3a_2^2$ | $0.48 | \bar{c}c$ $\bar{s}d$ \footnote{\text{ }} | $|V_{cs}|^2$ | $3a_2^2$ | 0.48 |
| $\bar{c}c$ $\bar{d}c$ \footnote{\text{ }} | $|V_{cd}|^2 = 0.049$ | $3a_1^2$ | $0.49 | \bar{c}d$ $\bar{d}c$ \footnote{\text{ }} | $|V_{cd}|^2$ | $3a_1^2$ | 0.49 |
| $\bar{c}c$ $\bar{d}u$ \footnote{\text{ }} | $|V_{cd}|^2$ | $3a_2^2$ | $0.49 | \bar{c}c$ $\bar{d}d$ \footnote{\text{ }} | $|V_{cd}|^2$ | $3a_2^2$ | 0.49 |

- The lifetime ratio

$$\tau(B^+)/\tau(B^0) = 1 - \frac{a_1a_2}{0.32 + a_1a_2 + 0.74(a_1^2 + a_2^2)}$$

is larger than 1 for negative and smaller than 1 for positive values of $a_2$. The relation between the lifetime ratio and $a_2$ is shown in Fig. 2 as dotted line for fixed $a_1 = 1.05$.

To give consistent results, we determine $a_1$ and $a_2$ in a fit to the 11 decay fractions used above, replacing the assumption of equal $B^+$ and $B^0$ lifetimes with the inclusive prediction in eq. 1 to rescale the theoretical expectations for $B^+$ and $B^0$ decays individually. This fit gives $\chi^2 = 11.6$ for 8 degrees of freedom, and

$$a_1 = 1.05 \pm 0.03 \pm 0.10$$

$$a_2 = 0.227 \pm 0.012 \pm 0.022$$

which implies

$$\mathcal{B}(B \rightarrow e\nu X) = (11.2 \pm 0.5 \pm 1.7)\%$$

$$\tau(B^+)/\tau(B^0) = 0.83 \pm 0.01 \pm 0.01,$$

where the first error is statistical including uncertainties in the $D^0$ and $D^+$ decay fractions, and the second is from the error on $V_{cb} \cdot \sqrt{\tau(B)}$. The uncertainty on $f^+/f^{00}$ yields a negligible error. The semileptonic decay fraction is further reduced if we assume a small contribution of penguin decays. Assuming this fraction to be 2.5% leads to $\chi^2 = 11.3$ and $\mathcal{B}(B \rightarrow e\nu X) = 10.9\%$, while all errors and the values of $a_1$, $a_2$ and $\tau(B^+)/\tau(B^0)$ remain essentially unchanged.
Our fit results agree reasonably well with present experimental values: The average $B^+$ and $B^0$ semileptonic decay fraction is $B(B \to l\nu X) = (10.2 \pm 0.3)\%$. This is the average of all inclusive $e$ and $\mu$ results [15] from data taken on the $\Upsilon(4S)$, where only these two types of $b$-flavoured mesons are produced. The result for all $b$-hadrons obtained at LEP is only slightly higher, $B(b \to l\nu X) = (11.0 \pm 0.5)\%$ [16]. The lifetime ratio from LEP and CDF is [16] $\tau(B^+)/\tau(B^0) = 1.07 \pm 0.12$.

Assuming duality and constructive interference between spectator amplitudes we are able to explain the low experimental value for the semileptonic decay fraction of $B$ mesons. The experimental data on the lifetime ratio are not yet sufficiently precise to either confirm or falsify our prediction that the $B^+$ has a shorter mean life than the $B^0$. However, a small contribution from annihilation diagrams, which enhance $B^0$ decays, could raise the expected value. This would also bring the semileptonic decay fraction even closer to the experimental average.

**Acknowledgements.** We thank V. Rieckert and B. Stech for helpful discussions on hadronic decays, and T. E. Browder for useful comments.

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