In this work we show how well can the cosmological parameters be constrained using galaxy cluster data. We also show how in the process of fitting the model which attempts to describe the data it is possible to get some information about the cluster scaling relations \( T - M \) and \( L_x - T \). Among other conclusions we found that only low density universes (\( \Omega \approx 0.3 \) with or without \( \Lambda \)) are compatible with recent data sets. These constraints will be improved with future SZ data.

1 Introduction

Galaxy clusters have been widely used as cosmological probes. The strong dependence of the cluster mass function with the cosmological model and more particularly of its evolution with redshift makes the study of the cluster population a powerful tool to discriminate among different structure formation scenarios. Following this fact, many works have tried to constrain the cosmological parameters by, given a cosmological model, comparing the theoretical predictions of the cluster mass function (Press-Schechter formalism (PS) (Press & Schechter 1974) or N-body simulations) with cluster data obtained mainly from the X-ray band.

Those theoretical predictions typically provide the cluster abundance as a function of mass and redshift. Redshift estimates can be obtained from optical observations of the cluster and even from X-ray spectroscopic considerations if the data is good enough. But the situation is different with mass estimates. Galaxy cluster masses are uncertain for most of the clusters and error bars are typically of the order of 20% or higher. For this reason, it is preferable to use another better determined function different from the mass function to trace the population of galaxy clusters. This can be done for instance by using the luminosity function of X-ray clusters or the temperature function. These functions can be easily connected with the theoretical (and model dependent) mass function (e.g. PS formalism) by defining some cluster scaling relations, \( T - M \) and \( L_x - T \),

\[
\frac{dN(T,z)}{dV(z)dT} = \frac{dN(M,z)}{dV(z)dM} \frac{dM}{dT},
\]

where \( dN(T,z)/dV(z)dT \) is the temperature function and \( dN(M,z)/dV(z)dM \) is the mass function given for instance by PS. Therefore, if there is an estimate of the cluster temperature function (or the X-ray cluster luminosity function), it would be possible to compare such an estimate with the predictions given by any model.
In this work we will follow this idea but with two important differences with respect to other previous similar works.

As a first important point we will consider different data sets in our fit and not only one as usual. This is important since the best fitting model will be compatible with different data sets and will reduce the degeneracy in the parameters found when only one data set is used.

Our main second difference is that in building the theoretical temperature, X-ray luminosity and flux functions we will consider that the $T - M$ and $L_x - T$ relations are not fixed relations but they will be part of our fitting model. This point will prevent us of doing wrong assumptions about these relations.

2 Results

We have fitted our model (PS and $T - M$ and $L_x - T$) to the following data sets. The cluster mass function of Bahcall & Cen (1993), the temperature function of Henry & Arnaud (1991), the luminosity function of Ebeling et al. (1997), and the flux function of Rosati et al. (1998) and De Grandi et al. (1999), see fig. 2. The fit of the model to these data sets was performed using a Bayesian estimator discussed in Lahav et al. (1999) and as it was shown in Diego et al. (2000) with this estimator the bias in the estimation of the cosmological parameters is very small. A marginalization of the probability over the cosmological parameters $\Omega$ and $\sigma_8$ has shown that by combining the different data sets it is possible to drastically reduce the degeneracy between these two parameters, see fig. 3. The best model (see table 1) was found to be a low density universe with $\Omega \approx 0.3$ and $\sigma_8 \approx 0.8$. The first three parameters in table 1 are the cosmological ones and they appear in the PS formula for the mass function. The other parameters are for the $T - M$ and $L_x - T$ relations.

$$T_{gas} = T_0 M^{\alpha}_{15}(1 + z)^{\psi},$$  \hspace{1cm} (2)
where $M_{15}$ is the cluster mass in $h^{-1}10^{15}M_\odot$ and,

$$L_{X}^{Bol} = L_0 M_{15}^{\alpha} (1 + z)^{\beta}.$$ 

(3)

The constraints found for these relations are consistent with experimental determinations of these scalings. However we found some discrepancy in the $\alpha$ parameter mostly due to a bias in our estimator. See, however, Diego et al. (2000) for a detailed discussion on these parameters.

Both the flat $\Lambda$CDM and the OCDM ($\Lambda = 0$) models were found to be in good agreement with the data (see fig. 2). In fact both models listed in table [table] are undistinguishable and they reproduce the data with almost the same good fit.

The fact that both models are undistinguishable is because in our fit we have used low redshift data (see Diego et al. 2000 for a brief description of each data set). To distinguish the $\Lambda$CDM and OCDM models it is necessary to explore the cluster distribution as a function of redshift. This will be done with more or less success by undergoing X-ray experiments (CHANDRA,
In Planck data is too large to make the follow up of all of them and consequently the cosmological parameters. In particular we have concentrated on the Planck Surveyor mission which will explore all the sky at different mm frequencies and will detect above 30000 clusters with fluxes above 30 mJy (see Diego et al. 2001 for an explanation of this limiting flux). This huge number of clusters would provide a valuable information about the cluster population. The constraints on the cosmological parameters will increase significantly by fitting the model to Planck observations. However, this information alone will not be enough to distinguish some models like those listed in table [I].

Despite the capabilities of Planck to observe galaxy clusters through the SZE, it will be unable to determine their redshifts and therefore many models will predict the same cluster population as a function of the SZ flux. In fig. [I] we can see that both models would remain undistinguishable if only the information about the fluxes of the clusters is provided.

To distinguish those models it is required some knowledge about the cluster population as a function of redshift. It will be needed, therefore, an optical follow up of some clusters (if their redshifts have not yet been determined in other wavebands). The number of clusters expected in Planck data is too large to make the follow up of all of them and consequently the cosmological studies based on the evolution of the cluster population must be restricted to a small subsample of the whole catalogue. As an application, we have calculated the minimum number of clusters for which we should determine their redshift in order to distinguish the previously undistinguishable models listed in table [I]. In that calculation we have required that the number of clusters above a given $z$ must differ at least $3\sigma$ for both models. The result is shown in figure [I] for three different selection criteria of the clusters. In each one of the 3 lines we show the total number of clusters randomly selected from the catalogue (with the only condition that the total flux must be $S_{mm} > 100$ mJy top, $S_{mm} > 30$ mJy middle and $30$ mJy < $S_{mm} < 40$ mJy bottom) which should be optically observed in order to distinguish (at a $3\sigma$ level) $N^O$ and $N^\Lambda$ at redshift $z$ (i.e. the total number of observed clusters needed to have a $3\sigma$ difference in $N(> z)$ for the two models). As can be seen from the figure, by determining the redshift of about 300 clusters randomly selected from the whole Planck catalogue (fluxes above 30 mJy) it would be possible to distinguish the models listed in table [I] by looking at the different behaviour of the cluster population about redshift $z \approx 0.6$. To illustrate how it is really possible to distinguish them, in fig. [I] we show the behaviour of both models and the comparison with simulated data.
Figure 3: Number of clusters to be observed to distinguish $N(>z)$ for the OCDM and ΛCDM models at a $3\sigma$ level.

(corresponding to the ΛCDM model and for a survey covering a sample of about 300 clusters). It is evident that by looking at the evolution of the cluster population as a function of redshift (for a small subsample of clusters) it would be possible to distinguish both models.

The previous calculation tell us that an analysis based on a small subsample of the whole Planck catalogue (with $z$, fig. 3) could distinguish models which are undistinguishable with recent X-ray data and future Planck data (no $z$, fig. 4).

By looking at figures 4 and 5 we see that a powerful analysis should make full use of both data sets since some models would be excluded by the first data set and others would be excluded by the second one. Following Diego et al. (2000) we combined both data sets in order to see how well could be constrained the cosmological parameters with these future data sets. Also following the same work we have considered that the $T-M$ relation was a free parameter one.

The result of this combination of data sets is shown in fig. 6. As can be seen the recovery of the cosmological parameters is very good. A joint fit to the Planck $N(S)$ curve and an optically identified $N(z)$ curve for a small subsample of clusters of the Planck catalogue will allow an independent test of the cosmological parameters. This result is almost independent of the assumed amplitude $T_0$ and scaling exponent $\alpha$ in the $T-M$ relation as it was discussed in Diego et al. (2001). However the method is sensitive to the choice of the redshift exponent $\psi$.

We can also conclude that the inclusion in the model of the cluster scaling relations ($T-M$ and $L_x-T$) as free parameter ones is important for the modeling of low redshift data since many models can fit the low redshift data and the assumption of one or another scaling in the $T-M$ and/or $L_x-T$ can favor some models to the detriment of others. On the contrary, when fitting data which includes the cluster evolution with redshift, the evo-
Figure 4: $dN/dS$ curve for the $\Lambda$CDM (solid) and OCDM (dotted) models in table.

Figure 5: $dN(S > 30 \text{ mJy}, z)/dz$ (353 GHz) curve for the $\Lambda$CDM (solid) and OCDM (dotted) models in table.
olution of the cluster population with redshift is dominated by the cosmological model and the cluster scaling relations play a secondary role. However, the redshift dependence of these relations (as can be appreciated in fig. 6, \( \psi \) parameter) should be carefully considered in this case.

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