Slepton Flavour Mixing and Neutrino Masses

Eung Jin Chun\textsuperscript{a} and Stefan Pokorski\textsuperscript{b}
\textsuperscript{a}Korea Institute for Advanced Study, Seoul 130-012, Korea
\textsuperscript{b}Institute of Theoretical Physics, Warsaw University, Poland

Abstract

It is pointed out that important contribution to neutrino masses and mixing can come from flavour violating slepton exchange. For low and intermediate \( \tan \beta \) such effects can dominate over the usual tau Yukawa coupling effects included in the renormalization group evolution. The mixing angles satisfy then the relation different from the fixed point solution of the renormalization group equation, and the desired neutrino mass splitting can be generated.

PACS number(s): 14.60.Pq, 14.60.St, 12.60.Jv
Observation of atmospheric [1] and solar neutrinos [2] provides important indications that neutrinos oscillate between different mass eigenstates. If we leave out the not yet confirmed LSND result [3], the atmospheric and solar neutrino oscillations can be explained in terms of three known flavours of neutrinos. We have then

\[ |\Delta m^2| \equiv |m_2^2 - m_1^2| \approx 5 \times 10^{-5}, 5 \times 10^{-6} \text{ or } 10^{-10} \text{ eV}^2 \]

for the large (LAMSW), small angle matter conversion (SAMSW), or vacuum oscillation (VO) solution to the solar neutrino problem, respectively, and

\[ |\Delta M^2| \equiv |m_3^2 - m_2^2| \approx 5 \times 10^{-3} \text{ eV}^2 \]

for the atmospheric neutrino oscillation. Moreover, combining the information on the solar and atmospheric neutrino mixing angles with non-observation of the disappearance of \( \bar{\nu}_e \) in the reactor experiment [4], for each solution (LAMSW, SAMSW, VO) to the solar neutrino problem we can infer the gross pattern of the 3x3 neutrino mixing matrix

\[
U = \begin{pmatrix}
c_2 c_3 & s_3 c_2 & s_2 \\
-c_1 s_3 - s_1 s_2 c_3 & c_1 c_3 - s_1 s_2 s_3 & s_1 c_2 \\
s_1 s_3 - c_1 s_2 c_3 & -s_1 c_3 - c_1 s_2 s_3 & c_1 c_2
\end{pmatrix}
\] (1)

with \( \sin^2 2\theta_1 \approx \sin^2 2\theta_{\text{atm}} > 0.82 \) and \( U_{13}^2 = s_2^2 < 0.2 \). Another important experimental information comes from non-observation of neutrinoless double \( \beta \) decay [5], which provides a bound on the \((ee)\) component of the neutrino mass matrix, that is,

\[ M_{ee} = |m_1 c_2^2 c_3^2 + m_2 c_2^2 s_3^2 + m_3 s_2^2| < 0.2 \text{eV} \]

(2)

The question of theoretical explanation of neutrino masses and mixing has been discussed in a large number of papers [6]. The measured neutrino mass matrix is linked to the fundamental mass generation by two steps. The first one is some flavour violation in the lepton sector which may have origin at a certain fundamental scale. The second step consists of including quantum corrections to obtain physical neutrino masses. The leading effects can be taken into account by the renormalization group (RG) evolution from a fundamental scale \( M \) to the weak scale \( M_Z \). An interesting question is the stability of various mass textures assumed at \( M \) with respect to quantum corrections. Recently, there have been many papers dealing with this issue [7–11] in the context of the see-saw mechanism which generates tiny neutrino masses below the Majorana scale \( M \).

In this letter, we point out that, in addition to the corrections due to lepton Yukawa couplings usually included in the RG evolution and already encoded in the tree-level neutrino mass matrix at the weak scale, important contribution can come from flavour changing slepton exchange. We estimate such effects in one-loop approximation and then discuss their potential impact on the neutrino mass and mixing patterns at the scale \( M_Z \).

The one-loop corrected neutrino mass matrix is

\[ M_{\nu}^{\text{ab}} = m_a \delta_{ab} + \frac{1}{2} (m_a + m_b) I_{ab} \]

(3)

Here \( I_{ab} \)'s (defined in the tree-level mass eigenstate basis) are related to the quantities \( I_{ij}^{\nu} \) (defined in the flavour basis) as follows;

\[ I_{ab} = \sum_{ij} I_{ij}^{\nu} U_{ia} U_{jb} \]

(4)
Note that, in the flavour basis, charged lepton masses are diagonalized but sleptons may have flavour changing masses. We calculate \( I_{ij}^w \) coming from slepton flavour mixing. There are diagrams involving only left-handed (LL) or right-handed (RR) slepton mixing, and left-right mixing (LR). The latter two involve charged lepton Yukawa couplings and thus become sizable for large \( \beta \). For the order of magnitude discussion in this letter, we estimate only (LL) type [see Fig. 1] adopting the mass insertion method as in Ref. [2]. For this, one introduces \( \delta_{ij}^l \equiv \Delta_{ij}/\tilde{m}_i\tilde{m}_j \) for \( i \neq j \) where \( \tilde{m}_i^2 \) and \( \Delta_{ij} \) are the diagonal and off-diagonal components of slepton mass matrix in the flavour basis, respectively. We then have

\[
I_{ii}^w = \frac{g^2}{16\pi^2} B_1(p^2, \tilde{m}_i^2, m_{W-}^2),
\]

\[
I_{ij}^w = \frac{g^2}{16\pi^2} \delta_{ij}^l C(\tilde{m}_i^2, \tilde{m}_j^2, m_{W-}^2),
\]

where \( B_1(p^2, m_i^2, m_j^2) = 1/2\ln - \int_0^1 dx x\ln[\lambda m_i^2 + (1 - x)m_j^2 - x(1 - x)p^2]/Q^2 \), and \( C(\tilde{m}_i^2, \tilde{m}_j^2, m_{W-}^2) = m_1m_2[B_1(p^2, m_1^2, m_2^2) - B_1(p^2, m_2^2, m_1^2)]/[m_1^2 - m_2^2] \). Note that we treated the chargino \( \tilde{W}^- \) as a mass eigenstate for the sake of simplicity and there is also a similar contribution from the zino exchange which we neglect.

The first question we can address is what are the upper bounds on flavour off-diagonal slepton mass matrix elements (i.e. \( \delta_{ij}^l \)) which follow from the requirement that our new contribution is consistent with the observed neutrino mass matrix at the scale \( M_Z \), without the necessity of large cancellations among different contributions. To this end, let us recall that the following low energy mass patterns are consistent with the experimental data:

(i) \( m_1^2 \approx m_2^2 \ll m_3^2 \approx \Delta M^2 \) (hierarchical)

(ii) \( m_1^2 \approx m_2^2 \approx \Delta M^2 \gg m_3^2 \) (inverse hierarchical)

(iii) \( m_1^2 \approx m_2^2 \approx m_3^2 \gtrsim \Delta M^2 \) (full degeneracy)

Here we have taken the patterns with partial or full degeneracy since the fully hierarchical pattern gets essentially no change as can be inferred from Eq. (3). In the leading approximation (up to terms \( \mathcal{O}(I_{ab}^2), a \neq b \), the one-loop correction to the mass eigenvalues is given by \( m_a \to m_a(1 + I_{aa}) \) and thus we get the bounds

\[
|I_{11} - I_{22}| \lesssim \frac{\Delta m^2}{2m_1^2}, \quad |I_{22} - I_{33}| \lesssim \frac{\Delta M^2}{2m_3^2},
\]

where the second inequality applies only for pattern (iii). Taking one insertion \( I_{ij}^w \) at a time, degenerate uncorrected neutrino masses and degenerate diagonal slepton mass matrix elements, and without resorting to cancellations among the mixing elements \( U_{ab} \), Eqs. (4,6) give the bound on the chosen \( I_{ij}^w \). For LAMSW or SAMSWM values of \( \Delta m^2 \), in case (i) with \( 10^{-5}eV^2 \lesssim m_1^2 \lesssim 10^{-4}eV^2 \) we get \( I_{ij}^w \lesssim 1 \). Since \( I_{ij}^w \lesssim (10^{-2} - 10^{-3})\delta_{ij}^l \), we get no bounds on \( \delta_{ij}^l \)'s. Similar conclusion holds in case (ii). In case (iii) we get the bound

\[
\delta_{ij}^l \lesssim (10^{-2} - 10^{-3}) \frac{1}{m_1^2/eV^2}.
\]

In order for the VO solution to be realized, severe bounds have to be fulfilled;

\[
\delta_{ij}^l \lesssim (10^{-7} - 10^{-8}) \frac{1}{m_1^2/eV^2}
\]
for (i, ii) or (iii). These bounds have to be contrasted with the present experimental limit on the flavour changing masses coming from $\mu \to e\gamma$ decay which reads $\delta_{12}^l \lesssim 10^{-2}(\tilde{m}/100\text{GeV})^2$ [12–13]. Thus, our considerations put for some of the textures much stronger bounds, which are insensitive to the values of $\tilde{m}_i$.

There are also limits on the splittings among the diagonal masses. For instance, if $\tilde{m}_1 = \tilde{m}_2 \neq \tilde{m}_3$, we have $I_{11}^w = I_{22}^w \neq I_{33}^w$. Taking all $\delta_{ij}^l = 0$, we get $I_{11} - I_{22} = (I_{33}^w - I_{11}^w)(U_{31}^2 - U_{32}^2)$. This puts the same bounds as in Eqs. (7,8) to the quantity,

$$ (U_{31}^2 - U_{32}^2) \ln(\tilde{m}_1/\tilde{m}_3) $$

assuming $\tilde{m}_i \gg m_{\tilde{W}^-}$. Note that this quantity vanishes at the point $U_{31}^2 = U_{32}^2$, like the tau Yukawa coupling contribution $I_{11}^w$. Otherwise, high degeneracy among the diagonal slepton masses is required when the VO solution with a degenerate mass pattern is realized in supersymmetric models.

The next question we ask is whether flavour mixing in the slepton sector can play a positive role in generating the desired neutrino mass splitting and mixing angles with degenerate textures assumed at the scale $M$. From the discussion above one sees that indeed there is some room for that. Such effects come on top of the usual tau Yukawa coupling effects included in the RG evolution, which contribute to the mass-squared difference $\Delta m^2_{ij} \approx O(m^2_\tau \epsilon_\tau)$ where $\epsilon_\tau = h^2_\tau \ln(M/M_Z)/16\pi^2 \approx 10^{-5} \tan^2 \beta$. For $\tan \beta$ in the range $(1 \leq 50)$, $\epsilon_\tau$ is in the range $(10^{-5}, 10^{-2})$ and the equation $\Delta m^2_{ij} \approx O(m^2_\tau I_{ij}^w)$ tells us that, generically, the slepton exchange effects can be comparable or even dominant for $I_{ij}^w \gtrsim 10^{-5}$. For the off-diagonal elements, this requires $\delta_{ij}^l \gtrsim O(10^{-2})$ which is possible for $\delta_{13}^l$ and $\delta_{23}^l$ but excluded for $\delta_{12}^l$ by the bound from $\mu \to e\gamma$ decay. On the other side, since realistically $\delta_{ij}^l < 1$, the flavour changing slepton exchange effects can be important only for $\tan^2 \beta < O(100)$. The diagonal $I_{ii}^w$ can be important even for a 10% violation of universality of the diagonal slepton masses. In addition, those generic orders of magnitude can be modified in special cases when e.g. the leading Yukawa coupling effects cancel out due to special relations between the mixing angles.

Let us now discuss the three mass patterns in turn. For obtaining LAMSW or SAMSW solution with pattern (i) or (ii), the slepton loop corrections could be important only for maximal flavour mixing $\delta_{ij}^l \approx O(1)$. Although not yet excluded for $\delta_{13}^l$ and $\delta_{23}^l$, it looks unrealistic. We expect therefore that in those cases the conclusions based on the usual tau Yukawa coupling effects included in the RG evolution remain unaltered and we focus our discussion on the VO solution and on pattern (iii) in general. For the VO solution with (i) or (ii) we consider two cases.

- $m_1 = m_2$: We recall that this initial condition places us at the scale $M$ in the infrared fixed point $U_{31} = 0$ or $U_{32} = 0$ [14]. At the fixed point, small solar mixing $\theta_3 \ll 1$ requires the smallness of $s_2 \ll 1$, and large mixing is consistent with a rather large value of $s_2^2 \lesssim 0.2$ [15]. Since $\Delta m^2_{12} \approx m^2_1 \epsilon_\tau \approx (10^{-5} \tan^2 \beta)m^2_1$, the VO solution can be obtained only for $10^{-8} \text{eV}^2 \lesssim m^2_1 \lesssim 10^{-5} \text{eV}^2$ (which falls into pattern (i)) and for sizable range of $\tan \beta$, with larger values corresponding to smaller $m^2_1$. Thus, for low $\tan \beta$ and small enough $m^2_1$ there is room for important contribution $\Delta m^2_{12} = 4I_{12}m^2_1$. For instance, for $m^2_1 \approx 10^{-7} \text{eV}^2$ and $\tan^2 \beta \approx O(10)$, $\epsilon_\tau$ is one order of magnitude too small to obtain the VO solution but the latter can be obtained with $\delta_{ij}^l \approx O(0.1)$. 


For such a scenario, it is very interesting to discuss the impact of the slepton loop corrections on the mixing angles which, as we said above, must satisfy one of the relations $U_{31} = 0$ or $U_{32} = 0$ at the scale $M$. For that, we consider the rediagonalization of the one-loop corrected mass matrix $M^\nu$ assuming that, for instance, $I^\nu_{23} \gg \epsilon_r$ and the other $I^\nu_{ij}$ are negligible. Also, we recall that in the considered case, at the tree-level at $M_Z$, $m_1^2 \approx m_2^2 < m_3^2 \approx 10^{-3} eV^2$ and $U_{31} = 0$. We get $I_{12} = 2I^w_{23}U_{12}U_{32}$, $M^\nu_{22} - M^\nu_{11} \approx m_1(I_{22} - I_{11}) \approx 2m_1I^w_{23}U_{22}U_{32}$. Moreover, since $m_{1,2} \ll m_3$, we can make a seesaw diagonalization separating the first 2x2 block and (33) component and treat $M^\nu_{13}, M^\nu_{23}$ as small corrections mixing these two blocks. Then, the effective contribution to the first 2x2 block is of the order $m_3I^w_{13,23}$ which is smaller than the leading term $m_1I_{12}$ as we generically have $m_1 \gtrsim m_3I_{ab}$ for our case of interest. The rediagonalization of the 2x2 block of $M^\nu$ is performed by a rotation $V(\theta'_3)$. The final neutrino mixing matrix reads

$$\bar{U}(\tilde{\theta}_i) = U(\theta_i)V(\theta'_i)$$

where $\tilde{\theta}_1 = \theta_1$, $\tilde{\theta}_2 = \theta_2$, $\tilde{\theta}_3 = \theta_3 + \theta'_3$. We can immediately conclude (or check by explicit calculation remembering that the angles $\theta_i$ satisfy the relation $U_{31} = 0$) that the final angles $\tilde{\theta}_i$ satisfy the relation corresponding to

$$I_{12} = I^w_{ij}(\bar{U}_{i1}\bar{U}_{j2} + \bar{U}_{j1}\bar{U}_{i2}) = 0$$

with $I^w_{23} \neq 0$ and vanishing other $I^w_{ij}$'s. This relation reads

$$\tan 2\tilde{\theta}_3 \tan 2\tilde{\theta}_1 = \frac{2s_2}{1 + s_2^2}. \tag{12}$$

Thus, the angles $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are fixed by the initial conditions $[11]$ and the angle $\tilde{\theta}_3$ is modified by the slepton loop corrections to satisfy Eq. $[12]$. We observe that, similarly to the RG fixed point relation $U_{31} = 0$, the final relation $[12]$ implies very small solar mixing for $s_2^2 \ll 1$.

It is straightforward to extend our consideration to the case of dominant contributions from $I^w_{13}$ (the possibility of a dominant $I^\nu_{12}$ is excluded as discussed). The resulting relations for the angles can be read off from Eq. $[11]$ and are collected in the left column of Table I. One can see that the limit $s_2^2 \rightarrow 0$ is consistent with the maximal solar mixing angle $\sin^22\theta_3 \rightarrow 1$ for dominant $I^w_{13}$. The same results follow for pattern (ii).

- $m_1 = -m_2$: The mixing angles are stable $[11]$ under all loop corrections as can be inferred from $M^\nu_{12} = \frac{1}{2}(m_1 + m_2)I_{12} = 0$. That is, all mixing angles are determined initially at $M$. Apart from the different behavior of mixing angles, the previous conclusions for $\Delta m^2$ hold here as well. One additional aspect is that for pattern (ii) the $\epsilon_r$ contribution can be suppressed when $U_{31}^2 = U_{32}^2$, a special case of which is the bimaximal mixing solution with $s_2 = 0$. The next-to-leading contribution of $\epsilon_r^2$ is also absent under this condition $[11]$. It is interesting to observe that the desired value for $\Delta m^2$ of the VO solution can, however, come from a very tiny slepton flavour mixing in $I^w_{ij}$.

The most interesting case is the fully degenerate pattern (iii). An attractive feature of this pattern is the possibility of the cosmologically interesting mass range, $m_3 \sim 1eV$. With only the tau Yukawa coupling effects, one cannot explain two distinct mass-squared differences. The slepton exchange contributions read
\[ \Delta m^2 \approx 4m_1^2 I_{ij}^w(U_{ij}U_{j1} - U_{ij}U_{j2}), \]
\[ \Delta M^2 \approx 4m_1^2 I_{ij}^w(U_{ij}U_{j2} - U_{ij}U_{j3}), \]
(13)

and it is interesting to check whether they can help to obtain the correct mass-squared differences. There are two realistic possibilities for \(|m_1| = |m_2| = m_3|.

- \(m_1 = m_2 = -m_3\): The slepton exchange modifies the mixing angles and gives the relation corresponding to Eq. (11), however, it does not modify the RG evolution result \(\Delta m^2 \sim \Delta M^2\).

- \(m_1 = -m_2 = m_3\): This is the case where the mixing angle relation corresponding to \(I_{13} = 0\) [see Table I] is realized. As above, the \(\epsilon_\tau\) or \(I_{ij}^w\) dominance leads generically to \(\Delta m^2 \sim \Delta M^2\). However, there is a texture to accommodate both \(\Delta m^2\) and \(\Delta M^2\). This is the one with bimaximal mixing and the \(I_{23}^w\) dominance. As shown in Table I, this leads to \(s_2 = 0\) at which \(U_{23}^2 = U_{32}^2\) as well as \(U_{21}U_{31} = U_{22}U_{32}\), and thus \(\Delta m^2\) has neither \(\epsilon_\tau\) nor \(I_{23}^w\) contribution. As a consequence, we have

\[ \Delta M^2 \sim m_1^2 I_{23}^w, \quad \Delta m^2 \sim m_1^2 (I_{12}^w, I_{13}^w), \]
(14)

implying that the bimaximal mixing solution can be obtained with \(I_{23}^w \sim 10^{-3} eV^2/m_1^2\), and \(I_{12}^w, I_{13}^w \sim 10^{-5} eV^2/m_1^2\) for the LAMS (VO) solution. Similar conclusions can be drawn with \(-m_1 = m_2 = m_3\). The corresponding mixing angle relations (obtained from \(I_{23} = 0\)) are listed in Table I.

Let us finally mention the case with dominant diagonal \(I_{ij}^w\). If \(I_{33}^w \neq I_{11}^w = I_{22}^w\) as in Eq. (9), there follows the same behavior as with dominant \(\epsilon_\tau\). For instance, one has \(I_{12} = (I_{33}^w - I_{11}^w)U_{31}U_{32}\) and thus the mixing angle relation \(U_{31} = 0\) or \(U_{32} = 0\) fixed by the RG evolution (with \(m_1 = m_2\)) is not altered.

So far, we discussed the role of loop corrections due to flavour violation in the slepton sector in quite general terms, based on the fact that current experiments provide no serious constraints on \(I_{ij}^w\) except \(I_{12}^w\). Indeed, our discussion to be more predictive, requires some theoretical framework for the soft sfermion masses departing from universality. As an example we take a supersymmetric SO(10) model with universal soft terms assumed at the Planck scale [12]. In this framework, the large top (neutrino) Yukawa coupling induces sfermion flavour mixing and a splitting of the third generation sfermion masses from the first two, due to RG effects from the Planck scale to the unification scale. This applies also to the left-handed sleptons which have the mass matrix in the flavour basis [12,13];

\[ m_{ij}^2 = m_0^2 \delta_{ij} - V_{3i}^c V_{3j}^e I_G \]
(15)

where \(m_0\) is the universal soft mass at the Planck scale and \(I_G \lesssim m_0^2\) arises due to the large top Yukawa coupling. Here \(V^e\) is related to the CKM matrix by \(V_{3i,32}^e \approx yV_{3i,32}\) and \(V_{33}^e \approx V_{33}\) with \(y \approx 0.7\). Then, again in the limit \(m_0 \gg m_{\tilde{W}^-}\) we have

\[ I_{12}^w \approx \frac{\alpha_2}{8\pi} y^2 \lambda^5 \frac{I_G}{m_0^2}, \quad I_{13}^w \approx \frac{\alpha_2}{8\pi} y \lambda^3 \frac{I_G}{m_0^2}, \]
\[ I_{23}^w \approx \frac{\alpha_2}{8\pi} y \lambda^2 \frac{I_G}{m_0^2}, \quad I_{33}^w - I_{11}^w \approx \frac{\alpha_2}{8\pi} \frac{I_G}{m_0^2}, \]
(16)
where \( \lambda \approx 0.22 \) is the Cabibbo angle. Note that even in such a scenario the flavour diagonal elements \( I_{33}^w - I_{11}^w \) and \( \epsilon_\tau \) (for \( \tan \beta \gtrsim 10 \)) give dominant contributions. However, the departures from universality induced by the running from the Planck scale to the unification scale may still play some role. From the previous discussions, we can read the following effects on each mass pattern set by the seesaw mechanism at \( M_\lambda \): (i) With \( m_1 = m_2 \), the fixed point relation \( U_{31,32} = 0 \) follows from the dominance of \( \epsilon_\tau \) or \( I_{33}^w - I_{11}^w \approx 10^{-3} \). As \( \Delta m^2 \sim m^2_1 \epsilon_\tau \) or \( m^2_1(I_{33}^w - I_{11}^w) \), we cannot have the MSW solutions for reasonable values of \( \tan \beta \). On the other hand, the VO solution can be easily obtained. With \( m_1 = -m_2 \), it is allowed to have \( U_{31}^2 = U_{32}^2 \) for which the contributions from \( I_{33}^w - I_{11}^w \) and \( \epsilon_\tau \) vanish [see Eq. (9)]. Then the VO solution can be obtained as \( \Delta m^2 \sim m^2_1 I_{23}^w \) and \( I_{23}^w \sim 10^{-4} \) for \( m^2_1 \sim 10^{-6} eV^2 \). (ii) Contrary to case (i), we have \( m_1 \sim 10^{-3} eV^2 \) and thus only the MSW solutions can be realized. Note also that the relation \( U_{31}^2 = U_{32}^2 \) with \( m_1 = -m_2 \) yields an undesirable result \( \Delta m^2 \sim m^2_1 I_{23}^w \sim 10^{-7} eV^2 \). (iii) The dominance by \( I_{33}^w - I_{11}^w \) or \( \epsilon_\tau \) leads to the situation, \( \Delta m^2 \sim \Delta M^2 \), with \( |m_1| = |m_2| = m_3 \). Thus it is not possible to generate two distinct mass-squared differences. The only way to realize the cosmologically interesting possibility \( m_\nu \sim 1 eV \) is to have \( \Delta M^2 \) set by an initial texture with \( m_1 = -m_2 \neq m_3 \) and \( s_2 = 0, \sin^2 2\theta_3 = 1 \) (implied by Eq. (4)). Then, we get \( U_{31}^2 = U_{32}^2 \), and \( \Delta m^2 \sim m^2_1 I_{23}^w \) which is consistent with the LAMSW solution. For this we need \( \tan \beta \lesssim 15 \) to suppress the \( \epsilon_\tau \) contribution to \( \Delta M^2 \). Note that the RG running of slepton masses from the the unification scale to the Majorana scale arising from Dirac neutrino Yukawa couplings [14] gives only small corrections to the included effect in Eq. (13).

In conclusion, we have discussed the effects of the flavour violating slepton exchange on the neutrino masses and mixing patterns. The requirement that this new contribution is consistent with phenomenologically acceptable mass matrices without necessity of large cancellation among different contributions gives for some mass patterns interesting upper bounds on flavour violation in the slepton sector. On the other hand, we find that for low and intermediate \( \tan \beta \) values the slepton exchange contribution can be more important than the usual tau Yukawa coupling effect included in RG evolution from the Majorana scale \( M \) to the weak scale and can be responsible for the observed neutrino masses and mixing. For degenerate or partially degenerate mass textures at the Majorana scale, the dominance of slepton exchange contribution gives new relations among mixing angles, different from the RG fixed point solutions [11]. Finally we have discussed supersymmetric SO(10) unification as an example of predictive frameworks for the slepton flavour structure.
REFERENCES

[1] Super-Kamiokande Coll., Y. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998).
[2] Homestake Coll., B.T. Cleveland et al., Astrophys. J. 496, 505 (1998); Kamiokande Coll., K.S. Hirata et al., Phys. Rev. Lett. 77, 1683 (1996); GALEX Coll., W. Hampel et al., Phys. Lett. B388, 384 (1996); SAGE Coll., D.N. Abdurashitov et al., Phys. Rev. Lett. 77, 4708 (1996); Super-Kamiokande Coll., Y. Fukuda et al., Phys. Rev. Lett. 82, 2430 (1999).
[3] LSND Coll., C. Athanassopoulos et al., Phys. Rev. Lett. 77, 3082 (1996).
[4] CHOOZ Coll., A. Apollonio et al., Phys. Lett. B420, 397 (1998).
[5] L. Baudis et al., hep-ex/9902014.
[6] For reviews, see A.Yu. Smirnov, hep-ph/9901208; G. Altarelli and F. Feruglio, hep-ph/9905536; R.N. Mohapatra, hep-ph/9910363.
[7] P.H. Chankowski and Z. Pluciennik, Phys. Lett. B316, 312 (1993); K. Babu et al., Phys. Lett. B319, 191 (1993).
[8] M. Tanimoto, Phys. Lett. B360, 41 (1995); J. Ellis and S. Lola, Phys. Lett. B458, 310 (1999); N. Haba et al., Eur. Phys. J. C10, 677 (1999); hep-ph/9906481; hep-ph/9911481.
[9] E. Ma, J. Phys. G25, 97 (1999).
[10] R. Barbieri, G.G. Ross and A. Strumia, hep-ph/9906470.
[11] P.H. Chankowski, W. Krolikowski and S. Pokorski, hep-ph/9910231.
[12] F. Gabbiani et al., Nucl. Phys. B447, 321 (1995).
[13] R. Barbieri, L.J. Hall and A. Strumia, Nucl. Phys. B445, 219 (1995).
[14] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986).

FIG. 1. A neutrino self-energy diagram at one-loop involving left-handed slepton mixing.

TABLE I. Mixing angle relations obtained for dominant $I_{33}^{w} - I_{11}^{w}$ or $I_{ij}^{w}$ ($i \neq j$) and tree-level mass degeneracies.

|   | $I_{12} = 0 \ (m_1 = m_2)$ | $I_{13} = 0 \ (m_1 = m_3)$ | $I_{23} = 0 \ (m_2 = m_3)$ |
|---|-----------------|-----------------|-----------------|
| $I_{33}^{w} - I_{11}^{w}$ | $\sin^2 2\theta_3 = \sin^2 2\theta_1 s_2^2 / (s_1^2 + c_1^2 s_2^2)^2$ | | |
| $I_{13}^{w}$ | $s_2^2 = \tan^2 \theta_1 \cot^2 2\theta_3$ | $s_2^2 = \cot^2 \theta_1 \tan^2 \theta_3$ | $s_2^2 = \cot^2 \theta_1 \tan^2 \theta_3$ |
| $I_{23}^{w}$ | $\frac{4s_2^2}{(1+s_2^2)^2} = \tan^2 \theta_1 \cot^2 2\theta_3$ | $s_2^2 = \cot^2 \theta_1 \tan^2 \theta_3$ | $s_2^2 = \cot^2 \theta_1 \tan^2 \theta_3$ |