Bridging Maximum Likelihood and Adversarial Learning via $\alpha$-Divergence

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Abstract
Maximum likelihood (ML) and adversarial learning are two popular approaches for training generative models, and from many perspectives these techniques are complementary. ML learning encourages the capture of all data modes, and it is typically characterized by stable training. However, ML learning tends to distribute probability mass diffusely over the data space, e.g., yielding blurry synthetic images. Adversarial learning is well known to synthesize highly realistic natural images, despite practical challenges like mode dropping and delicate training. We propose an $\alpha$-Bridge to unify the advantages of ML and adversarial learning, enabling the smooth transfer from one to the other via the $\alpha$-divergence. We reveal that generalizations of the $\alpha$-Bridge are closely related to approaches developed recently to regularize adversarial learning, providing insights into that prior work, and further understanding of why the $\alpha$-Bridge performs well in practice.

1 Introduction
Given observed data samples, a well-known task concerns fitting a generative model to the unknown underlying data distribution. Two popular approaches for that task are classical maximum likelihood (ML) learning and recently-developed adversarial learning. Both of these approaches are equivalent to minimizing a corresponding divergence between the model distribution and the data distribution (Bishop 2006; Goodfellow et al. 2014).

ML learning seeks to find model parameters that maximize the log-likelihood of the model over the observed data samples, which is equivalent to minimizing the forward Kullback-Leibler (KL) divergence between the model distribution and the data distribution (Bishop 2006; Goodfellow et al. 2014). ML learning tends to associate positive mass with each data sample, forming a zero-avoiding phenomenon (Minka and others 2005). Accordingly, all data modes are “covered” by the model distribution; by contrast, adversarial learning is often characterized by a mode-dropping phenomenon (Srivastava et al. 2017). Another advantage of ML learning (forward KL) is that its training procedure is typically much more stable than that of adversarial learning. The instability of adversarial learning is in part because the mode dropping may vary as a function of learning iteration. On the other hand, the zero-avoiding phenomenon of ML learning may loosely distribute probability mass among data modes. An example consequence is that generative models trained with ML tend to generate blurry images (Goodfellow et al. 2014; Larsen et al. 2015). Adversarially-learned models, by contrast, are capable of synthesizing highly realistic natural images (Goodfellow et al. 2014; Nowozin, Cseke, and Tomioka 2016; Zhang et al. 2018; Gulrajani et al. 2017; Brock, Donahue, and Simonyan 2019).

The original generative adversarial network (GAN) minimizes the Jensen-Shannon (JS) divergence between the model distribution and that of the data (Goodfellow et al. 2014). In (Nowozin, Cseke, and Tomioka 2016) it was shown that learning based on minimizing any $f$-divergence can be formulated as an adversarial learning objective (with the JS divergence as a special case). In this paper, we focus on the reverse KL divergence as in (Li et al. 2019) because (i) it naturally relates to ML learning (by reversing the KL); (ii) (Lucic et al. 2018) showed that most GANs with the same budget can reach similar performance with enough hyperparameter optimization and random restarts; (iii) adversarial learning with the $f$-divergence (Nowozin, Cseke, and Tomioka 2016) reduces to estimating a log-likelihood ratio between the true and model distributions, and the reverse-KL is as good as any other $f$-divergence choice for this purpose (Li et al. 2019); and (iv) forward and reverse KL divergences are two ends of the $\alpha$-Bridge developed in this paper.

It is interesting to note that ML learning (based on the forward KL) and adversarial learning (with the reverse
KL as an important example) seem to have complementary advantages and disadvantages (Nguyen et al. 2017), as shown in Table 1. To unify their advantages, an intuitive approach would directly combine them. However, as stated in (Larsen et al. 2015; Mathieu et al. 2016) and empirically shown in (Zhang et al. 2019), such a naive method does not work well. Another intuitive approach would, for example, directly initialize the reverse-KL-based adversarial learning with the parameters learned from ML learning. For the second approach, empirical results in Figure 1 indicate that catastrophic forgetting (Kirkpatrick et al. 2017; Liang et al. 2018) happens when adversarially finetuning the ML-learned parameters. Appendix F discusses/compares other potential approaches to combine adversarial and ML learning. To unify the advantages from ML and adversarial learning in a principled way, we propose a novel $\alpha$-Bridge, via the $\alpha$-divergence, to smoothly connect the forward and reverse KL, through which one can transfer the advantages from one to the other. In addition to the practical value of the $\alpha$-Bridge, our subsequent analysis on the $\alpha$-divergence is deemed an important methodological perspective on how ML and adversarial learning are related and may be linked.

The main contributions of this paper are as follows. (i) An $\alpha$-Bridge is proposed to connect the forward and reverse KL in a principled manner, which can be interpreted as a novel way to “bridge” the two research fields of ML and adversarial learning. (ii) The gradient of the $\alpha$-divergence is shown to have two equivalent expressions, one that utilizes the gradient information from ML learning (forward KL), while the other uses the gradient information from adversarial learning (reverse KL). (iii) The twin gradients of $\alpha$-divergence have complimentary variance properties, $\alpha$-Bridge elegantly combines the advantages of both and manages a low Monte Carlo variance along the varying of $\alpha$. (iv) Two generalizations of our $\alpha$-Bridge are revealed, that are closely related to CycleGAN (Zhu et al. 2017) and ALICE (Li et al. 2017), two methods for regularizing (stabilizing) adversarial learning. (v) It is demonstrated empirically that the proposed $\alpha$-Bridge is capable of benefiting from the advantages of ML learning, transferring information from ML to adversarial learning, and is capable of transplanting the variational posterior in ML learning into an inference arm for adversarial learning.

### 2 Preliminaries

Given observed data $x$, drawn from unknown underlying data distribution $q(x)$, and a parameterized model distribution $p_\theta(x)$ with parameters $\theta$, the task is to learn $\theta^*$ so that $p_\theta^*(x)$ best fits the observed data, or identically $p_\theta^*(x)$ is closest to $q(x)$. For that task, two popular research fields include ML learning (with “closeness” of $p_\theta^*(x)$ and $q(x)$ quantified via the forward KL) and adversarial learning (with the reverse KL as an important example of how “closeness” is measured).

#### 2.1 Maximum Likelihood Learning (Forward KL)

A classic method to match a model $p_\theta(x)$ to the data distribution $q(x)$ is ML learning (or maximum likelihood estimation), namely,

$$\theta^* = \arg\max_\theta \mathbb{E}_{q(x)}[\log p_\theta(x)] = \arg\min_\theta D_{KL}[q(x) \parallel p_\theta(x)].$$

(1)

where $D_{KL}[q(x) \parallel p_\theta(x)] = \mathbb{E}_{q(x)}[\log q(x) - \log p_\theta(x)]$ is the forward KL. The gradient wrt $\theta$ is

$$\nabla_\theta D_{KL}[q(x) \parallel p_\theta(x)] = \mathbb{E}_{q(x)}[-\nabla_\theta \log p_\theta(x)].$$

(2)

For more modeling capacity, it is often convenient to define $p_\theta(x)$ as the marginal of some parameterized joint distribution $p_\theta(x, z)$, with latent variable $z$. Although $\log p_\theta(x)$ is usually intractable, variational inference (Jordan et al. 1999; Kingma and Welling 2014; Blei, Kucukelbir, and McAuliffe 2017) seeks to solve the ML learning in (1) via maximizing the evidence lower bound (ELBO)

$$\text{ELBO}(\theta, \phi) = \mathbb{E}_{q(x)q_\phi(z|x)}[\log p_\theta(x, z) - \log q_\phi(z|x)].$$

(3)

where $q_\phi(z|x)$ is the variational approximation with parameters $\phi$, and the bound is tight when $q_\phi(z|x) = p_\theta(x, z)$. The gradient wrt $\theta$ becomes

$$\nabla_\theta \text{ELBO}(\theta, \phi) = \mathbb{E}_{q(x)q_\phi(z|x)}[-\nabla_\theta \log p_\theta(x, z)].$$

In practice, $\mathbb{E}_{q(x)}[\cdot]$ are approximated as averages over a finite set of observed samples.

#### 2.2 Adversarial Learning (Reverse KL)

Recent progress has resulted in many techniques for adversarial training of generative models (Goodfellow et al. 2014; Gulrajani et al. 2017; Nowozin, Cseke, and Tomioka 2016; Brock, Donahue, and Simonyan 2019). The original GAN (Goodfellow et al. 2014) seeks to solve

$$\min_\theta \max_\beta \mathbb{E}_{q(x)}[\log \sigma(f_\beta(x))] + \mathbb{E}_{p_\theta(x)}[\log(1 - \sigma(f_\beta(x)))].$$

(4)

where $\sigma(f_\beta(x)) \triangleq D_\beta(x)$ is called the discriminator, $\sigma(a) = 1 / (1 + \exp(-a))$, and samples are drawn from $p_\theta(x)$ by the generative process

$$x \sim \delta(x|G_\theta(z)), z \sim p(z),$$

(5)
where $\delta(x|a)$ is the Dirac delta function located at $a$, $G_\theta(z)$ is
called the generator, and $p(z)$ is an easy-to-sample distribution. It is
shown (Goodfellow et al. 2014) that the optimal $\beta^*$ for (4)
satisfies
\[
f_{\beta^*}(x) = \log q(x) - \log p_\theta(x). \tag{6}
\]
Accordingly, (4) seeks to minimize the Jensen-Shannon (JS)
information divergences for parameters $\theta$ (Goodfellow et al. 2014).
Alternatively, one could also consider a similar GAN objective
based on the reverse KL divergence $D_{KL}[p_\theta(x)||q(x)]$
(Nowozin, Cseke, and Tomioka 2016; Li et al. 2019), on
which we focus in this paper; as discussed in the Introduction,
many GANs are closely related (Nowozin, Cseke, and Tomioka 2016)
and can reach similar performance with the
\[\alpha\]

Twin Gradients of
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\alpha \text{-divergence, named the } \alpha \text{-divergence,}
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which utilize the
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\alpha \text{-divergence is a continuous function of } \alpha.
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3 Connecting Maximum Likelihood and
Adversarial Learning via $\alpha$-Bridge
Maximum likelihood and adversarial learning have many
complementary strengths and weaknesses, motivating develop-
ment of a method that achieves their principled integration.
Toward that end, we propose what we term an $\alpha$-Bridge,
designed using the $\alpha$-divergence (Cichocki and Amari 2010).
The $\alpha$-Bridge smoothly connects the forward and reverse KL
divergences, making it possible to transfer advantages from
one to the other.
Given model distribution $p_\theta(x)$ and the underlying data
distribution $q(x)$, the $\alpha$-divergence measuring the dissimilarity
between these two distributions is defined as
\[
D_\alpha[p_\theta(x)||q(x)] = \frac{1}{\alpha(1-\alpha)} \left[ 1 - \int p_\theta(x)^\alpha q(x)^{1-\alpha} dx \right]. \tag{8}
\]
The $\alpha$-divergence has many attractive properties (Cichocki
and Amari 2010), for example, (i) it is unique (Amari 2009);
(ii) $\lim_{\alpha \to 0} D_\alpha[p_\theta(x)||q(x)] = D_{KL}[q(x)||p_\theta(x)];$
(iii) $\lim_{\alpha \to 1} D_\alpha[p_\theta(x)||q(x)] = D_{KL}[p_\theta(x)||q(x)];$
and (iv) the $\alpha$-divergence is a continuous function of $\alpha$. These
properties motivate development of a smooth “bridge” via the
$\alpha$-divergence, named the $\alpha$-Bridge, to continuously transfer
between forward and reverse KL. Before discussing the proposed
$\alpha$-Bridge in detail, below we first reveal its key foundation,
in the context of this paper: the $\alpha$-divergence has two equivalent
expressions for its gradient, which utilize the gradient information
either from the forward or reverse KL.

3.1 Twin Gradients of $\alpha$-Divergence
Given the $\alpha$-divergence defined in (8), with straightforward
derivation, we have
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] = \frac{1}{1-\alpha} \left[ - \int p_\theta(x)^\alpha q(x)^{1-\alpha} \nabla_\theta p_\theta(x) dx \right]. \tag{9}
\]
An interesting fact of (9) is that one can turn it into an expectation-based expression wrt either the data distribution
$q(x)$ or the model one $p_\theta(x)$, resulting in two different
expressions for the same gradient (see Appendix A for details). By forming expectations wrt $q(x)$, we have
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] = \frac{1}{1-\alpha} \mathbb{E}_{q(x)} \left[ - \left[ \frac{p_\theta(x)}{q(x)} \right]^\alpha \nabla_\theta \log p_\theta(x) \right] \triangleq \nabla_\theta D_\alpha^F,
\]
where $\nabla_\theta D_\alpha^F$ is used for brevity. The gradient information
from the forward KL in (2) serves as a building block for (10). However, it is different from the direct gradient of the forward KL in that $\nabla_\theta D_\alpha^F$ has an adaptive ratio-related weight term
\[
\frac{1}{1-\alpha} \left[ \frac{p_\theta(x)}{q(x)} \right]^\alpha
\]
within the expectation (when $\alpha \to 0^+$ this term vanishes, leading to the gradient of the forward KL in that limit). For the gradient expression related to $p_\theta(x)$
modeled in (5), we have
\[
\nabla_\theta D_{\alpha}[p_\theta(x)||q(x)] \triangleq \nabla_\theta D_\alpha^R = \mathbb{E}_{p(z) \in (x|G_\theta(z))} \left[ \nabla_\theta G_\theta(z) \right] \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \left[ \nabla_\theta \log p_\theta(x) \right]. \tag{11}
\]
Similarly we use $\nabla_\theta D_\alpha^R$ for brevity. Compared to (7), $\nabla_\theta D_\alpha^R$
utilizes the gradient information from the reverse KL, with
another adaptive weighting term $\left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha}$ (which vanishes in the limit $\alpha \to 1^-$, yielding the gradient of the reverse KL
in that limit).
For more general model $p_\theta(x)$ beyond (5), the
GO gradient (Cong et al. 2019) can be utilized to calculate
$\nabla_\theta D_\alpha^R$.

It is important to note that $\nabla_\theta D_\alpha^F$ and $\nabla_\theta D_\alpha^R$ are two equivalent
gradient expressions for the same objective
$D_{\alpha}[p_\theta(x)||q(x)]$, even though they utilize different gradient
information (accordingly different MC variance properties as
detailed below) from the forward and reverse KL, respect-
ively. Thus, we call them the twin gradients of $\alpha$-divergence.
In the limits on $\alpha$, the former is associated with the forward
KL and the latter with the reverse KL, but for $\alpha \in (0, 1)$ the
twin gradients are not associated with either; this explains
why the proposed $\alpha$-Bridge in Sec. 3.2 is different from a
(possibly convex) combination of the forward and reverse KL.

Since $\nabla_\theta D_\alpha^F$ and $\nabla_\theta D_\alpha^R$ are equivalent expressions for
$\nabla_\theta D_\alpha[p_\theta(x)||q(x)]$, any convex combination of them re-
main an unbiased gradient estimator, which may be interpreted
as exploiting the information from one side to regular-
ize the other side. We propose to use an $\alpha$-related dynamic
combination as
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] = (1 - \gamma_\alpha) \nabla_\theta D_\alpha^F + \gamma_\alpha \nabla_\theta D_\alpha^R, \tag{12}
\]
where $\gamma_\alpha$ is specified as a smooth increasing function\(^1\) of $\alpha$
satisfying $\gamma_0 = 0$, $\gamma_1 = 1$, ensuring equation (12) ex-

\(^1\)It is consistent with the instinct that, as smoothly transfor-
ning from the forward to reverse KL, the used information from
the forward/reverse KL should smoothly decrease/increase corre-
respondingly. Appendix E shows a series of experiments demonstrat-
ing several intuitive choices for $\gamma_\alpha$. We empirically find that the
sigmoid-like function $\gamma_\alpha = \frac{\alpha(x+c) - \alpha(d)}{\alpha(c+d) - \alpha(d)}$ (with hyperparameters
which is a distinct combination of two different forms of the variance when α = 3.2. Generalizations of α and leave as future research how to optimally choose γc,d motivated by applications associated with GAN, with a goal based on 100 random trials.

actually recovers the gradient of the forward/reverse KL when α = 0/α = 1. Such a γα is motivated by the smoothness of the α-divergence. When α → 0 the α-divergence smoothly approaches the forward KL with increasingly-similar gradients; intuitively to calculate the gradient ∇αDα[pθ(x)||q(x)], one should prefer ∇αDαF more as it uses the ML gradient information. Similarly, ∇αDαR is preferred when α → 1 as it uses the adversarial gradient information and the α-divergence now smoothly approaches the reverse KL. With a simple example, Figure 2 confirms that intuitively by noticing that the twin gradients ∇αDαF and ∇αDαR have complementary variance properties, the former/latter having lower MC variance when α → 0/α → 1. Figure 2 also shows that combining the twin gradients in (12) unifies the advantages from both sides and presents a better gradient estimator with lower MC variance for α ∈ (0, 1). The twin gradients can be interpreted as control variants to each other. This is the foundation of our paper, which is further exploited in the following to develop our α-Bridge. We are taking the convex combination of two different forms of the same gradient, which is distinct from just taking a convex combination of the different gradients from the forward and reverse KL.

3.2 α-Bridge via Twin Gradients

Based on the twin gradients discussed above, we propose a novel α-Bridge to dynamically transfer between forward KL (ML learning) and reverse KL (adversarial learning), so as to unify the advantages from both ends. In this paper, we are motivated by applications associated with GAN, with a goal c, d works well. Accordingly, we use such γα in our experiments and leave as future research how to optimally choose γα.

Figure 2: Illustration of different MC variance properties of different gradient estimators of ∇1/2μ,σ2Dα[N(x; μ, σ2)||N(x; 0, 1)] for α ∈ (0, 1). 1 MC sample is used to estimate the gradient. The results are based on 100 random trials.

Algorithm 1 α-Bridge (from forward to reverse KL)

Input: Data samples x, ∼ q(x), an implicit model pθ(x)

Output: θ∗ such that pθ( x) is closest to the underlying data distribution q(x)

# Step I: ML learning (forward KL, α = 0)
1: ML learning for the generator parameter θ with the gradient in (14). Maximizing the ELBO in (3) for training the variational parameters φ. Pretrain the discriminator parameters β with the objective in (4).

# Step II: Transferring from α → 0+ to α → 1−
2: for α gradually increasing from 0+ to 1− do
3: Train θ by minimizing the α-divergence Dα[pθ(x)||q(x)] with the gradient in (15);
4: Train φ by maximizing the ELBO in (3);
5: Train β with the objective in (4);
6: end for

# Step III: Adversarial learning (reverse KL, α = 1)
7: Refine θ, φ, and β with the objectives in (7), (3), and (4), respectively.

of generating realistic samples from our model. Accordingly, we set our α-Bridge to transfer from the forward KL to the reverse KL, in order to gradually transfer the advantages (see Table 1) of ML learning to adversarial learning. Specifically, we propose to train pθ( x) via the α-Bridge with the following three successive steps.

In Step I, we adopt ML learning (forward KL, α = 0) for efficient initialization thanks to its mode-covering and stable-training properties. One can skip this step if pretrained models from ML learning are available. From the perspective of practical implementation, one often need to approximately calculate the gradient in (2), as pθ( x) may be intractable for example for the implicit model in (5). For this issue, we first add small Gaussian noise3 on top of the generative process of pθ( x) to form a semi-implicit surrogate model (Yin and Zhou 2018) µθ(x) : x ∼ N(x; x′, σ2I), x′ ∼ pθ(x′), for which we have ∇α log pθ(x) = limα→0 ∇α log pθ(x). Observing that pθ(x) is equivalent to

µθ(x) : x ∼ N(x; Gθ(z), σ2I), z ∼ p(z),

which has computable joint distribution pθ(x, z), we then use the ELBO technique to get

∇θ log pθ(x) ≈ ∇θ log pθ(x) = Eθ(x)qφ∗(z|x) [∇θ log pθ(x, z)],

with an additional variational inference arm.

In the middle Step II, we continue the training of pθ(x) by gradually changing α from 0+ to 1−, so as to transfer what’s learned during Step I to the next Step III (reverse-KL-based adversarial learning, α = 1). The gradient of the α-divergence in (12) is used during training. The same techniques discussed above is adopted to calculate the ∇θ log pθ(x) term within ∇θDα in (10). To calculate the

3We need not to add the noise if pθ(x) is modeled as (13) in the first place, for example to take into consideration the widely-existing observation noise of data. See Appendix B for details.
density-ratio-related terms in both $\nabla_{\theta}D_{\alpha}^F$ and $\nabla_{\theta}D_{\alpha}^H$ (see (10) and (11)), we follow the common practice in the GAN literature to solve (4) for $f_{\beta^*}(x)$ in (6). Accordingly, we have $\frac{p_\theta(x)}{q_\alpha(x)} = e^{-f_{\beta^*}(x)}$, $\nabla_x \log \frac{p_\theta(x)}{q_\alpha(x)} = -\nabla_x f_{\beta^*}(x)$, and

$$\nabla_{\theta}D_{\alpha}[p_\theta(x)\|q_\alpha(x)] \approx \frac{1}{1-\alpha} \nabla_{\theta} \log \frac{p_\theta(x)}{q_\alpha(x)} - \frac{e^{-\alpha f_{\beta^*}(x)}}{\gamma \theta} \nabla_{\theta} \log q_\alpha(x) + \gamma \alpha \nabla_{\theta} \log q_\alpha(x) \right] - \left[ \nabla_{\theta} \log q_\alpha(x) \right] \right|_{(15)} \right)
$$

which combines the gradient information from ML and adversarial learning with automatic weights related to both $\alpha$ and the GAN discriminator.

Finally in Step III, we use the zero-forcing reverse-KL-based adversarial learning ($\alpha = 1$) to continually refine the generator parameters $\theta$ and the discriminator parameters $\phi$ using (7) and (4), respectively. The corresponding training process is summarized in Algorithm 1.

### 3.3 Connections to Prior GAN-Learning Regularization

Considering the aforementioned twin gradients and the $\alpha$-Bridge, we next present an interpretation of the gradient in (15), with which we reveal two generalizations that are highly related to CycleGAN (Zhu et al. 2017) and ALICE (Li et al. 2017). Details are given in Appendix C. With $\mathbf{x}$ denoting the stop-gradient operator\(^4\), the gradient in (15) can be reformulated as

$$\nabla_{\theta}D_{\alpha}[p_\theta(x)\|q_\alpha(x)] \approx \nabla_{\theta} \left[ \frac{1}{1-\alpha} \mathbb{E}_{\theta} [e^{-\alpha f_{\beta^*}(x)}] \right] - \frac{e^{-\alpha f_{\beta^*}(x)}}{\gamma \theta} \nabla_{\theta} \log q_\alpha(x) + \gamma \alpha \nabla_{\theta} \log q_\alpha(x) \right|_{(16)} \right]
$$

where the first term can be interpreted as weighted half cycle-consistency (Li et al. 2017; Zhu et al. 2017; Kim et al. 2017), and the second one is related to the reverse-KL-based adversarial learning. Based on the interpretation in (16), one can readily verify (see Appendix C) that by generalizing the $\alpha$-Bridge derivations as in (16) to consider both marginals

$$D_{\alpha}[p_\theta(x)\|q_\alpha(x)] + D_{\alpha}[q_\phi(z)\|p_\theta(x)] \approx \nabla_{\theta} \left[ \frac{1}{1-\alpha} \mathbb{E}_{\theta} [e^{-\alpha f_{\beta^*}(x)}] \right] - \frac{e^{-\alpha f_{\beta^*}(x)}}{\gamma \theta} \nabla_{\theta} \log q_\alpha(x) + \gamma \alpha \nabla_{\theta} \log q_\alpha(x) \right|_{(17)} \right]
$$

where $\approx$ means both sides have approximately equal gradients mimicking (16), and $g_{\gamma}(z) = \log p(z) - \log q_\phi(z)$ corresponds to the optimal discriminator in the $z$ space. Similarly, by considering both joint distributions

$$D_{\alpha}[p_\theta(x)\|q_\alpha(x)\|p_\theta(x)] + D_{\alpha}[q_\phi(z)\|p_\theta(x)\|q_\alpha(x)] \approx \nabla_{\theta} \left[ \frac{1}{1-\alpha} \mathbb{E}_{\theta} [e^{-\alpha h_{\beta^*}(x,z)} \log p_\theta(x|z)] \right] + \nabla_{\theta} \left[ \frac{1}{1-\alpha} \mathbb{E}_{\theta} [e^{-\alpha h_{\beta^*}(x,z)} \log q_\phi(z|x)] \right] + \gamma \alpha \nabla_{\theta} \left[ e^{-\alpha h_{\beta^*}(x,z)} q_\phi(z|x) \right]$$

one can generalize the $\alpha$-Bridge to a model very much resembling ALICE (Li et al. 2017). We believe that the connections revealed above, and the techniques developed earlier, may be helpful for constituting a foundation that unifies ML learning, adversarial learning, and intuitive (regularization) properties like cycle-consistency.

### 4 Related Work

Motivated by the complementary properties of ML and adversarial learning, many methods have been considered for combining these two popular research fields, to unify their advantages. A direct combination of the VAE with GAN objectives was considered in (Larsen et al. 2015), only to "observe the devil in the details" during model development and training. Accordingly gradients were heuristically controlled in back-propagation. It is also stated in (Mathieu et al. 2016) that naively combining those two objectives unstabilizes the system and does not lead to perceptually better generation, which is consistent with the empirical results from (Zhang et al. 2019). The principle combination of ML and adversarial learning deserves a thorough exploration. Instead of directly combining their objectives, the $\alpha$-Bridge dynamically transfers (information) between both sides to bypass the unstable problem. Many other works combining ML and adversarial learning were motivated differently. On the one hand, with the target of ML learning unchanged, (Makhzani et al. 2015; Mescheder, Nowozin, and Geiger 2017) exploited GAN techniques to better handle the KL term between the prior and posterior of the latent variables, within the ELBO. On the other hand, keeping the target of adversarial learning, a variational auto-encoder/autoencoder was used as a building block within GAN discriminators, mainly for stabilizing training (Berthelot, Schumm, and Metz 2017; Ulyanov, Vedaldi, and Lempitsky 2018). A symmetric KL divergence was exploited to build objectives (Pu et al. 2017; Chen et al. 2018). Since those methods employed discriminators to estimate/replace the ratios within both the forward and reverse KL, the likelihood (gradient) information from forward KL was ignored. By comparison, the $\alpha$-Bridge has the advantage of benefiting from the gradient information from ML learning. Although combining ML and adversarial learning is enticing, no previous work has achieved this in a principled manner. The proposed $\alpha$-Bridge seeks to fill this gap.

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\(^4\)tf.stopgradient/torch.no_grad in TensorFlow/PyTorch.
We demonstrate the proposed $\alpha$-Bridge from three perspectives. First we show that the $\alpha$-Bridge, dynamically transferring advantages from ML to adversarial learning, exhibits a more stable training with improved robustness to hyperparameters (this is expected because of the aforementioned discussions of control variants interpretation and connections to prior GAN regularization methods). We then show that the $\alpha$-Bridge is capable of smoothly transferring the information learned during ML learning to adversarial learning, circumventing the forgetting issue shown in Figure 1. Finally we highlight the versatility of the $\alpha$-Bridge, by showing its capability in transplanting the variational approximation within ML learning into an inference arm for adversarial learning. See Appendix G for the detailed experimental settings.

5.1 Stability and Robustness

The 25-Gaussians example from (Tao et al. 2018) is adopted, where the data are generated from a 2D Gaussian mixture model with 25 components, as shown in Figure 3. For direct comparison, reverse-KL-based GANs are chosen as baselines, with recent techniques to stabilize their training, i.e., gradient penalty (GP) (Mescheder, Geiger, and Nowozin 2018) and spectral normalization (SN) (Miyato et al. 2018). Note it is shown in (Lucic et al. 2018) that most GANs with the same budgets can reach similar performance with enough hyperparameter optimization and random restarts. Thus reverse-KL-based GANs further stabilized by GP/SN are considered as fairly good baselines (named as RKL-GP and RKL-SN, respectively). The inception score (IS) (Salimans et al. 2016) and the log-likelihood estimated with kernel density estimation (Parzen 1962) are used to measure the plausibility of generated samples and the data-mode-covering level of learned models, respectively. Baseline methods are carefully tuned with their best settings adopted for fair comparison (see Appendix G.2).

Figure 4 shows the results of the considered methods. It is clear that $\alpha$-Bridge, thanks to its smooth transferring nature, is capable of benefiting from the advantages of ML learning, resulting in more stable training (see Figures 4a and 4c) while keeping most data modes covered (see Figures 4b and 4d). Comparing Figures 4a-4b to Figures 4c-4d shows $\alpha$-Bridge is relatively more robust to hyperparameters than baseline methods (see Appendix G.2 for more details). Figure 3 shows one training curve of the compared methods, highlighting $\alpha$-Bridge’s ability to benefit from the advantages of ML learning. To address the concern of how $\alpha$-Bridge performs on real datasets, we conduct another experiment on CIFAR10 (Krizhevsky and Hinton 2009) and observe an improved performance of $(\text{IS}, \text{FID}(\text{Heusel et al.} .2017))=(7.225, 28.083)$ over $(6.558, 33.707)$ of the vanilla DCGAN baseline (see Appendix G.4 for more results).

5.2 Smooth Transfer of Information from ML to Adversarial Learning

Besides inheriting the advantages of ML learning, another advantage of the $\alpha$-Bridge is a smooth transfer of the information learned during ML to adversarial learning. For an explicit demonstration, we run $\alpha$-Bridge on the MNIST (Le-Cun et al. 1998) and CelebA (Liu et al. 2015) datasets, and present the generated samples along the training process, as shown in Figure 5. ML learning, i.e., Step 1 of Algorithm 1, provides fairly good initialization on both datasets; thanks to the zero-avoiding nature of ML learning, one might anticipate an initialization covering all data modes; similar to the phenomena observed in Figure 3. When it comes to the transfig.
5.3 Transplanting ML Variational Posterior into Inference Arm for Adversarial Learning

In addition to inheriting the advantages and information from ML learning, we find that the smooth dynamical training of Algorithm 1 also enables α-Bridge to transplant the variational approximation within ML learning into an inference arm for adversarial learning. Such a capacity is appealing because it enables exploiting the generative power of GANs for various practical applications. See Appendix G.6 for technical details.

To verify the effectiveness of the transplanted inference arm, Figure 6 (top) shows the encoder-decoder reconstruction for the generated fake images. It is apparent that the reconstructions are fairly good, confirming the effectiveness of the inference arm. One can also exploit that arm for manipulation of GAN generated images, as shown in Figure 6 (bottom). Detailed implementations for reconstruction and manipulation are given in Appendix G.6. It is clear that with this inference arm, one can modify the semantic concepts of the generated images like bangs, hair, gender, etc. Such capacity is valuable for transferring the generative power of GANs to various down-streaming tasks.

6 Conclusions

Motivated by the fact that maximum likelihood (ML) and adversarial learning have complementary characteristics, we have proposed a novel α-Bridge, constituted via the α-divergence, to unify their advantages in a principled manner. Our α-Bridge has as its foundation newly recognized twin gradients of the α-divergence, one of which utilizes the gradient information from the ML (forward KL) perspective, and the other from the adversarial learning (reverse KL) perspective. We also have revealed two generalizations of α-Bridge that closely resemble CycleGAN (Zhu et al. 2017) and ALICE (Li et al. 2017).

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Appendix of
Bridging Maximum Likelihood and Adversarial Learning via \( \alpha \)-Divergence

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A Derivations of the Twin Gradients of \( \alpha \)-Divergence

The \( \alpha \)-divergence between the model distribution \( p_\theta(x) \) and the underlying data distribution \( q(x) \) is defined as

\[
\mathcal{D}_\alpha[p_\theta(x)\|q(x)] = \frac{1}{\alpha(1-\alpha)} \left[ 1 - \int p_\theta(x)^\alpha q(x)^{1-\alpha} dx \right].
\]

(17)

The corresponding gradient wrt \( \theta \) is

\[
\nabla_\theta \mathcal{D}_\alpha[p_\theta(x)\|q(x)] = \frac{1}{1-\alpha} \left[ -\int p_\theta(x)^{\alpha-1} q(x)^{1-\alpha} \nabla_\theta p_\theta(x) dx \right].
\]

(18)

By extracting \( q(x) \) to form an expectation-based expression, we have

\[
\nabla_\theta \mathcal{D}_\alpha[p_\theta(x)\|q(x)] \equiv \nabla_\theta \mathcal{D}_\alpha^F
\]

\[
= \frac{1}{1-\alpha} \mathbb{E}_{q(x)} \left[ -p_\theta(x)^{\alpha-1} q(x)^{-\alpha} \nabla_\theta p_\theta(x) \right].
\]

(19)

Clearly, the gradient information \( \nabla_\theta \log p_\theta(x) \) from maximum likelihood learning (forward KL) is used as a building block.

For the gradient expression related to \( p_\theta(x) \) modeled in (5) we rely on the stop-gradient operator\(^6\) to simplify our

\(^6\)For example, tf.stop_gradient in TensorFlow (Abadi et al. 2015) and torch.no_grad in PyTorch (Paszke et al. 2017). If the stop-gradient operator \( \nabla_x f(\mathbf{x}) \) is applied to a function \( f(x) \), the function value remains the same, i.e., \( f(\mathbf{x}) = f(x) \), but its gradient is removed, i.e., \( \nabla_x f(\mathbf{x}) = 0 \).
derivations as
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] = \frac{1}{1-\alpha} \nabla_\theta D_\alpha^R
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \left[ - \int p_\theta(x)p_\theta(x)^{\alpha-1} q(x)^{1-\alpha} dx \right]
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(x)} \left[ - \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha}
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(z)} \left[ - \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha}_{x=G_\theta(z)}
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_p(z) \left[ - \nabla_\theta G_\theta(z) \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha}_{x=G_\theta(z)}.
\]
Note the \( \nabla_\theta D_\alpha^R \) derived above is a special case of (and thus can be easily verified by) the recently proposed GO gradient (Cong et al. 2019).

### B Derivations for Semi-Implicit \( p_\theta(x) \)

Different from the main manuscript, in this section, we consider a semi-implicitly-defined model
\[
p_\theta(x) : x \sim p_\theta(x) | z = \mathcal{N}(x|G_\theta(z), \Sigma_\theta(z))
\]
(21)
where often \( \Sigma_\theta(z) = \sigma^2 I \). This is reasonable because in practice, the observation noise (usually modeled as Gaussian) exists everywhere. For specific applications, one may want to construct \( \Sigma_\theta(z) \) as neural networks.

The only difference between the generative process of the semi-implicit model in (21) and that of the implicit one in (5) of the main manuscript is that, one shall need an additional add-noise step to generate a fake sample.

Then considering the gradient of interest
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)]
\]
\[
= \frac{1}{1-\alpha} \left[ - \int p_\theta(x)p_\theta(x)^{\alpha-1} q(x)^{1-\alpha} \nabla_\theta p_\theta(x) dx \right]
\]
(22)
we derive the equivalent twin gradients similarly as in the main manuscript as
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] \triangleq \nabla_\theta D_\alpha^F
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_{q(x)} \left[ - \frac{p_\theta(x)}{q(x)} \right]^{\alpha} \nabla_\theta \log p_\theta(x),
\]
(23)
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)]
\]
\[
= \frac{1}{1-\alpha} \left[ - \int \frac{p_\theta(x)}{q(x)} \right]^{1-\alpha} \nabla_\theta p_\theta(x) dx
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(x)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(z)p_\theta(x)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_{p_\theta(x)p_\theta(z)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_{p_\theta(z)p_\theta(x)} \left[ \nabla_\theta \log p_\theta(x) \right]^{1-\alpha} \nabla_\theta \log \frac{p_\theta(x)}{q(x)}
\]
(24)
where the variable-nabla \( \nabla_{\theta} G_\theta(x) \), defined in (Cong et al. 2019), can be interpreted as the gradient of random variables \( x \) wrt \( \theta \) conditional on \( z \).

Accordingly, we have our chosen gradient
\[
\nabla_\theta D_\alpha[p_\theta(x)||q(x)]
\]
\[
= \frac{1}{1-\alpha} \left[ - \int \frac{p_\theta(x)}{q(x)} \right]^{1-\alpha} \nabla_\theta p_\theta(x) dx
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(x)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \nabla_\theta \mathbb{E}_{p_\theta(z)p_\theta(x)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_{p_\theta(x)p_\theta(z)} \left[ - \left[ \frac{q(x)}{p_\theta(x)} \right]^{1-\alpha} \right]
\]
\[
= \frac{1}{1-\alpha} \mathbb{E}_{p_\theta(z)p_\theta(x)} \left[ \nabla_\theta \log p_\theta(x) \right]^{1-\alpha} \nabla_\theta \log \frac{p_\theta(x)}{q(x)}
\]
(25)
For the practical implementation associated with \( \nabla_\theta \log p_\theta(x) \), one could simply resort to the ELBO technique of the classic maximum likelihood learning, namely
\[
\nabla_\theta \log p_\theta(x) = \nabla_\theta \mathbb{E}_{q_\phi^*} \left[ \log \frac{p_\theta(x, z)}{q_\phi^*(x|z)} \right]
\]
(26)
where \( p_\theta(x, z) \) is computable thanks to its semi-implicit definition in (21), different from what’s in the main manuscript for the challenging implicit definition as in (5).

For the log-ratio term \( \log \frac{p_\theta(x)}{q_\phi(x)} \), it is a common practice in GAN literature to solve
\[
\max_\beta \mathbb{E}_{q(x)} \left[ \log \frac{p_\theta(x)}{q(x)} \right] + \mathbb{E}_{p_\theta(x)} \left[ \log (1 - \sigma(f_\beta(x))) \right],
\]
(27)
for \( f_\beta^* (x) = \log \frac{q(x)}{p_\theta(x)} \). The gradient information \( \nabla_x f_\beta^*(x) = \nabla_x \log \frac{q(x)}{p_\theta(x)} \) is then exploited for learning the generative model \( p_\theta(x) \).

Note in this case, we need not to worry about the gap brought by the implicit model as in the main manuscript.
C Derivations for the Two Generalizations of the $\alpha$-Bridge

We first summarize the related parameterized models used in this section.

$$\begin{align*}
\hat{p}_\theta(x) & : x \sim \mathcal{N}(x|G_\theta(z), \sigma^2 I), \quad z \sim \mathcal{N}(z) \\
q_\phi(z) & : z \sim \mathcal{N}(z|E_\phi(x), \sigma^2 I), \quad x \sim q(x)
\end{align*}$$

(28)

Note we use a simplified variational approximation $q_\phi(z|x) = \mathcal{N}(z|E_\phi(x), \sigma^2 I)$ (instead of $q_\phi(z|x) = \mathcal{N}(z|\mu_\phi(x), \sigma^2 I)$) to draw connections to the existing CycleGAN and ALICE. We use $E_\phi(x)$ in place of $\mu_\phi(x)$ to highlight the physical meaning of “encoder” and also to keep consistent with the existing CycleGAN and ALICE.

Revealing Connection to CycleGAN with Marginals:

Start from the practical gradient in (15) of the main manuscript, we first backward-derive the corresponding identical objective (whose gradient is the same as (15)) to reveal the interesting interpretation in (16) of the main manuscript, namely

$$\begin{align*}
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] \\
\approx \nabla_\theta \left[ \frac{1 - \gamma_\alpha E_\phi(x)}{1 - \alpha} [ \frac{e^{-\alpha g_\gamma(z)}}{2\sigma^2} \|x - G_\theta(z)\|^2 ] \\
+ \gamma_\alpha E_{p_\theta(x)} [ - e^{(1-\alpha)g_\gamma(z)} ] \right]
\end{align*}$$

(29)

where $\left[ \mathbf{x} \right]$ is the stop-gradient operator detailed in Appendix A. Similarly to the case in the $x$ space, one can also derive another formula in the $z$ space as

$$\begin{align*}
\nabla_\phi D_\alpha[q_\phi(z)||p(z)] \\
\approx \nabla_\phi \left[ \frac{1 - \gamma_\alpha E_\phi(z)}{1 - \alpha} [ \frac{e^{-\alpha g_\gamma(z)}}{2\sigma^2} \|z - E_\phi(x)\|^2 ] \\
+ \gamma_\alpha E_{p_\phi(x)} [ - e^{(1-\alpha)g_\gamma(z)} ] \right]
\end{align*}$$

(30)

where $g_\gamma(z)$ corresponds to the additional discriminator in the $z$ space, which can be similarly solved via

$$\max_\gamma E_p(z) \log \sigma(g_\gamma(z)) + E_\phi(z) \log (1 - \sigma(g_\gamma(z))).$$

(31)

By combining the above two formula together, we have

$$\begin{align*}
D_\alpha[p_\theta(x)||q(x)] + D_\phi[q_\phi(z)||p(z)] \\
\approx \nabla \left[ \frac{1 - \gamma_\alpha E_\phi(x)}{1 - \alpha} [ \frac{e^{-\alpha g_\gamma(z)}}{2\sigma^2} \|x - G_\theta(z)\|^2 ] \\
+ \frac{1 - \gamma_\alpha E_\phi(z)}{1 - \alpha} [ \frac{e^{-\alpha g_\gamma(z)}}{2\sigma^2} \|z - E_\phi(x)\|^2 ] \\
+ \gamma_\alpha E_{p_\theta(x)} [ - e^{(1-\alpha)g_\gamma(z)} ] \right]
\end{align*}$$

(32)

where $\approx$ means both sides have approximately equal gradients like those in (29) and (30). It is obvious that the right hand side very much resembles the CycleGAN objective (Zhu et al. 2017), with a few differences including

- equation (32) generalized from our $\alpha$-Bridge has the ratio-related weights within its four terms (with the first two associated with cycle-consistency and the last two related to reverse-KL-based adversarial learning); by contrast, CycleGAN employed user-defined hyperparameters to balance its four terms.

- CycleGAN used the original GAN loss (Goodfellow 2016), while equation (32) utilizes the reverse-KL-based GAN objective (the last two terms). In fact, many of the multiple forms of GAN are closely related (Nowozin, Cseke, and Tomioka 2016) and they performs similar with the same computational budget and enough hyperparameter optimization (Kurach et al. 2018; Lucic et al. 2018).

Revealing Connection to ALICE in the Joint Space:

Similar to the above derivations, we first take a step back to re-express the chosen gradient in (12) of the main manuscript as

$$\begin{align*}
\nabla_\theta D_\alpha[p_\theta(x)||q(x)] = (1 - \gamma_\alpha) \nabla_\theta D^F_\alpha + \gamma_\alpha \nabla_\theta D^R_\alpha \\
= \frac{1 - \gamma_\alpha}{1 - \alpha} \frac{E_p(x)}{q(x)} \log q(x) + \\
\gamma_\alpha \frac{E_\phi(z)}{p(z)} \left[ \nabla_\theta G_\theta(z) \right] \log p_\theta(x) + \\
\nabla_\theta \left[ \frac{1 - \gamma_\alpha}{1 - \alpha} \frac{E_q(z)}{q(x)} \log q(x) + \\
\gamma_\alpha E_{p_\phi(x)} \left[ \frac{q(x)}{p(x)} \right] \right]
\end{align*}$$

(33)

By replacing $p_\theta(x)$ and $q(x)$ with $p_\theta(x, z)$ and $q_\phi(x, z)$, respectively, and using another discriminator $h_\eta(x, z) = \log \frac{p_\phi(x, z)}{p_\phi(x, z)}$ in the joint space, we have

$$\begin{align*}
\nabla_\theta D_\alpha[p_\theta(x, z)||q_\phi(x, z)] \\
\approx \nabla_\theta \left[ \frac{1 - \gamma_\alpha}{1 - \alpha} \frac{E_q(z)}{q(x, z)} \log q(x, z) + \\
\gamma_\alpha E_{p_\phi(x, z)} \left[ e^{(1-\alpha)h_\eta(x, z)} \log q(x, z) + \\
\gamma_\alpha E_{q_\phi(x, z)} \left[ e^{(1-\alpha)h_\eta(x, z)} \log q(x, z) + \right]
\end{align*}$$

(34)

By mirroring the above equation, we have

$$\begin{align*}
\nabla_\phi D_\alpha[q_\phi(x, z)||p_\theta(x, z)] \\
\approx \nabla_\phi \left[ \frac{1 - \gamma_\alpha}{1 - \alpha} \frac{E_p(x, z)}{p(x, z)} \log p(x, z) + \\
\gamma_\alpha E_{q_\phi(x, z)} \left[ e^{(1-\alpha)h_\eta(x, z)} \log q(x, z) + \\
\gamma_\alpha E_{q_\phi(x, z)} \left[ e^{(1-\alpha)h_\eta(x, z)} \log q(x, z) + 
\end{align*}$$

(35)

By combining the above two equation together mimicking
Accordingly, we have the right hand side resembling ALICE (Li et al. 2017), as stated in the main manuscript. Despite the ratio-related weights, the first two terms are closely related to the cycle-consistency, which was exploited in ALICE to develop a lower bound for conditional entropy. The last two terms are the reverse-KL-based GAN losses, which are expected to have a similar performance to the original GAN loss used in ALICE (Kurach et al. 2018; Lucic et al. 2018). Similar to (32), parts of the gradient information are removed from the first two cycle-consistency terms.

## D Adversarial Learning initialized by Maximum Likelihood Learning

The details of how we get Figure 1 are given below. The experimental settings and model architectures from Appendix G.1 are used.

First, we follow the classic variational inference to train the model \( p_\theta(x) = \int p_\theta(x|z)p(z)dz \) and the variational posterior \( q_\phi(z|x) \) by maximizing the ELBO in (3). Simultaneously we use the generated fake samples and the true data samples to pretrain the discriminator \( D_\beta(x) \), with the objective in (4). All \( p_\theta(x), q_\phi(z|x) \), and \( D_\beta(x) \) are saved for downstreaming experiments (like the transferring Step of the proposed \( \alpha \)-Bridge).

Next we load the pretrained \( p_\theta(x) \) and \( D_\beta(x) \) for initialization and start the reverse-KL-based adversarial learning to train the parameters \( \theta \) and \( \beta \) with the reverse KL objective in (7) and the GAN objective in (4), respectively. Note the pretrained discriminator \( D_\beta(x) \) is meant for providing great help for adversarial learning. With a randomly initialized discriminator, worse forgetting is empirically observed for adversarial learning. By contrast, the \( \alpha \)-Bridge can inherit the information from ML even with a randomly initialized discriminator (which is usually the case since in practice one can only expect a pretrained \( p_\theta(x) \) and \( q_\phi(z|x) \) from the classic variational inference, where there is no place for the discriminator \( D_\beta(x) \)). The ML initialization, together with the following fake samples generated after 20, 40, and 60 iterations of adversarial learning, are collected and demonstrated in Figure 7.

### E Empirical Evaluation of the Intuitive Choices of \( \alpha \) and \( \gamma_\alpha \)

The 25-Gaussians experiment (as shown in Section 5.1 and Appendix G.1) is adapted for empirical evaluation of different strategies for \( \alpha \) and \( \gamma_\alpha \) (see Table 2).

#### Table 2: Tested strategies for \( \alpha \) and \( \gamma_\alpha \).

| \( \alpha \)                        | \( \gamma_\alpha \)                |
|-------------------------------------|-----------------------------------|
| Linear: \( (a(t) = 1) \)            | Sigmoid: \( \gamma_\alpha = \sigma(\alpha d) \) |
| X5: \( 16(t - 0.5)^2 + 0.5 \)      | Cosine: \( \gamma_\alpha = 0.5 - 0.5 \cos(\pi \alpha) \) |

Figure 7 shows the experimental results from different combinations of strategies in Table 2. Curves are calculated based on 10 random trials. It seems \( \alpha \)-Sigmoid with \( \gamma \)-Sigmoid and \( \alpha \)-Linear with \( \gamma \)-Sigmoid work better in this experiment.
The Performance of \( \alpha \)-Bridge Compared with The Other potential Combination Methods

Here we discuss and compare \( \alpha \)-Bridge with three other potential approaches to combine adversarial and ML learning. These methods are: (i) FR, directly add up the Forward KL loss and Reverse KL loss; (ii) FRw, weighting average the Forward and Reverse KL loss as \( L = (1- \gamma )L_{FKL} + \gamma L_{RKL} \) with \( \gamma \) varying from 0 to 1; (iii) FRw3, replace the second step of our method as FRw.

In the experiments we try to use a parameterized model \( p_\theta(x) \) to match the real data distribution \( q(x) \). Here \( q(x) \) is a 1-D Gaussian mixture data with three components, each component has equal probability. The experiments are divided into two different cases: (i) the parameterized model \( p_\theta(x) \) is expressive enough (have three components), and (ii) \( p_\theta(x) \) has limited power (only have two components). For the first case, we choose three different ground truth distributions for \( q(x) \) as shown in the first three rows of Table 3. For each distribution, ten trials are conducted and the number of times that \( p_\theta(x) \) perfectly matches \( q(x) \) are recorded in the table. The results for the second case are shown in the last row of Table 3. And we consider \( p_\theta(x) \) matches two modes of \( q(x) \) as successful case.

When \( p_\theta(x) \) is expressive enough, \( \alpha \)-Bridge is comparable with the best of the other method. Obviously, when the parameterized model has limited power, which is often the case in real application, \( \alpha \)-Bridge shows great advantages over the other combination methods. We assume the reason is that, \( \alpha \)-Bridge has its object smoothly varying from FKL to RKL, thus the optimization process is more stable than the other method. Take FRw3 as an example, we show the snapshots of \( p_\theta(x) \) during the optimization process in Figure 9. Apparently, FRw3 has forget the middle mode during training, and randomly catch it by luck, while \( \alpha \)-Bridge doesn’t forget the mode in hand and match them one by one.

Table 3: counts of the trials that \( p_\theta(x) \) matches \( q(x) \) for different combination methods.

| \( q(x) \) | FR | FRw | FRw3 | \( \alpha \)-Bridge |
|---|---|---|---|---|
| \( \mu_1, \mu_2, \mu_3 = (6.0, -1.0, 1.0) \) \( \sigma_1, \sigma_2, \sigma_3 = (1.0, 0.1, 1.0) \) | 9 | 4 | 5 | 10 |
| \( \mu_1, \mu_2, \mu_3 = (6.0, 0.0, -2.0) \) \( \sigma_1, \sigma_2, \sigma_3 = (1.0, 0.1, 0.2) \) | 8 | 8 | 10 | 10 |
| \( \mu_1, \mu_2, \mu_3 = (5.0, -1.0, -3.0) \) \( \sigma_1, \sigma_2, \sigma_3 = (1.0, 0.2, 0.2) \) | 4 | 8 | 8 | 8 |
| \( \mu_1, \mu_2, \mu_3 = (6.0, -1.0, 1.0) \) \( \sigma_1, \sigma_2, \sigma_3 = (0.3, 0.3, 0.3) \) | 0 | 0 | 1 | 10 |

G Experimental Settings

The detailed settings for the experiments are given below.

G.1 25-Gaussians

The data are collected from a 2D Gaussian mixture model with 25 components uniformly distributed on the \( 5 \times 5 \) grid, as shown in Figure 3 of the main manuscript. The covariance matrix for each component is set to \( 0.0002I_{2 \times 2} \). The centroids distance along the grid is set to 2, such that the overall data distribution exhibits severely separated modes, making it a relatively hard task for GAN.

The latent code \( z \) with a dimension \( d_z = 2 \) is sampled from the standard Gaussian. The generator \( G_\theta(z) \) is constructed with Linear layers; the corresponding network architecture is shown in Table 4. The discriminator shares a similar architecture, with the two differences of the last layer having an output size of 1 and different BN settings for different regularizations. When using the spectral normalization (SN) (Miyato et al. 2018) for regularization, we replace the BN layers with SN layers for the discriminator. When using the gradient penalty (GP) (Mescheder, Geiger, and Nowozin 2018), we remove the BN layers. For the variational inference arm used by our \( \alpha \)-Bridge \( q_\phi(z|x) = \mathcal{N}(\mu_\phi(x), \sigma_\phi^2(x)I) \), we choose a similar network architecture as in Table 4, with the...
difference of replacing the last Linear layer with two sub-networks (shown in Table 5) to model $\mu_\phi(x)$ and $\log \sigma_\phi(x)$, respectively.

Table 4: The network architecture used on 25-Gaussians. lReLU for leaky ReLU with slope 0.2.

| Layer   | kernel | output |
|---------|--------|--------|
| Linear, lReLU | —      | 400    |
| Linear, lReLU   | —      | 400    |
| Linear, lReLU   | —      | 400    |
| Linear, lReLU   | —      | 400    |
| Linear         | —      | $d_z = 2$ |

Table 5: The sub-network architecture to model $\mu_\phi(x)$ and $\log \sigma_\phi(x)$ for the variational inference arm $q_\phi(z|x)$ on 25-Gaussians.

The training iterations for Steps $I$, $II$, and $III$ of our $\alpha$-Bridge (see Algorithm 1) are set to 7.5K, 7.5K, and 25K, respectively. All the models are trained using Adam with the batch size 50, the learning rate $lr = 0.0002$, and $\beta_2 = 0.999$.

We use the inception score (IS) (Salimans et al. 2016) and the log-likelihood estimated with kernel density estimation (Parzen 1962) as quantitative metrics, to measure the plausibility of generated samples and the data-mode-covering level of learned models, respectively. Following (Li et al. 2017), we compute $IS = \mathbb{E}_{p(y)}[D_{KL}[p(y)||p(y|x)]]$ with the help of a classifier $p(y|x)$ pre-trained on 10,000 data samples, where $p(y)$ represents the generative model and $p(y)$ is the uniform prior of label $y$. The classifier $p(y|x)$ is a 5-layer neural networks with Linear layers and lReLU activation, pre-trained to yield 100% classification accuracy on the training data. Note this kind of IS evaluation is not related to the inception model trained on ImageNet. To estimate the log-likelihood, we use 5K data samples and choose a Gaussian kernel with the same variance of the data, i.e., $\sigma^2 = 0.0002$. The hyperparameter $\gamma$ of the gradient penalty (GP) is set to $\gamma = 10$ by referring to the original paper (Mescheder, Geiger, and Nowozin 2018). 10 random trials are ran to get the curves shown in Figure 4 of the main manuscript.

G.2 Tuning Baseline Methods on 25-Gaussians
To get competitive baselines, we test several hyperparameter settings and choose among them the best overall settings to build a fair comparison environment. The experimental results are given in Figure 10. It seems the settings $lr = 2 \times 10^{-4}, \beta_1 = 0.1$ work the best overall among the tested settings for the baseline methods. Accordingly, it is chosen.

For parts of the hyperparameter settings, we compare our $\alpha$-Bridge methods to the baseline methods and present the corresponding results in Figure 11. It is clear that our methods are more robust to the tested hyperparameter settings than the baseline methods, especially during the first two steps. Unfortunately for some situations where the baseline methods fail, the $\alpha$-Bridge methods may also fail in Step III sometimes; after all, Step III of the $\alpha$-Bridge is the reverse-KL-based adversarial learning. But we do observe the $\alpha$-Bridge works when the corresponding baseline fails, such as for the settings $(2 \times 10^{-4}, 0.5)$ in Figure 11.

Figure 10: Empirical evaluation of different hyperparameter settings for the baseline methods, i.e., the reverse-KL-based adversarial learning regularized by GP (RKL-GP) (a) and that regularized by SN (RKL-SN) (b).

Figure 11: Comparing the $\alpha$-Bridge methods with the baseline methods on different hyperparameter settings. The notation following methods denotes (learning rate, $\beta_1$). Two vertical dashed lines are used to indicate the three steps of the $\alpha$-Bridge.

G.3 MNIST
For the experiments on MNIST, we use the DCGAN (Radford, Metz, and Chintala 2015) architectures, as shown in Tables 6 and 7, for the generator and discriminator, respectively. The variational inference arm $q_\phi(z|x) = \mathcal{N}(\mu_\phi(x), \sigma^2_\phi(x)I)$ has a similar network architecture as that of the discriminator, with the difference of replacing the last Linear layer with two shallow sub-networks to model $\mu_\phi(x)$ and $\log \sigma_\phi(x)$, respectively. The prior of the latent code $z$ is the standard Gaussian. $z$ has a dimension $d_z = 64$. We train each step (see Algorithm 1) of our $\alpha$-Bridge for 5, 5, 40 epochs, respectively, on the MNIST training data. The batch size is set to 100. We use GP to regularize the discriminator in this experiment. The GP hyperparameter $\gamma$ is set to $\gamma = 10$. 
Table 6: The generator architecture on MNIST. BN stands for batch normalization (Ioffe and Szegedy 2015). The kernel size is described in format $h_{filter} \times w_{filter} \times stride$. The output shape is $channels \times h \times w$.

| Layer | Kernel | Output          |
|-------|--------|-----------------|
| Linear, BN, iReLU | -     | $256 \times 4 \times 4$ |
| Deconv, BN, iReLU | $4 \times 4 \times 2$ | $128 \times 7 \times 7$ |
| Deconv, BN, iReLU | $5 \times 5 \times 2$ | $64 \times 14 \times 14$ |
| Deconv, BN, iReLU | $5 \times 5 \times 2$ | $32 \times 28 \times 28$ |
| Deconv, Tanh | $5 \times 5 \times 1$ | $1 \times 28 \times 28$ |

Table 7: The discriminator architecture on MNIST.

| Layer | Kernel | Output          |
|-------|--------|-----------------|
| Conv, iReLU | $3 \times 3 \times 2$ | $32 \times 14 \times 14$ |
| Conv, iReLU | $3 \times 3 \times 2$ | $64 \times 7 \times 7$ |
| Conv, iReLU | $3 \times 3 \times 2$ | $128 \times 4 \times 4$ |
| Linear | -     | 1               |

Table 8: The sub-network architecture to model $\mu_\phi(x)$ and $\log{\sigma_\phi(x)}$ for the variational inference arm $q_\phi(z|x)$ on MNIST.

| Layer | Kernel | Output          |
|-------|--------|-----------------|
| Linear, BN, iReLU | -     | 1000            |
| Linear | -     | $d_z = 64$      |

G.4 CIFAR10

Table 9: The generator architecture on CIFAR10.

| Layer | Kernel | Output          |
|-------|--------|-----------------|
| Linear, BN, iReLU | -     | $128 \times 4 \times 4$ |
| Deconv, BN, iReLU | $4 \times 4 \times 2$ | $256 \times 8 \times 8$ |
| Deconv, BN, iReLU | $4 \times 4 \times 2$ | $128 \times 16 \times 16$ |
| Deconv, BN, iReLU | $4 \times 4 \times 2$ | $64 \times 32 \times 32$ |
| Deconv, Tanh | $3 \times 3 \times 1$ | $3 \times 32 \times 32$ |

Table 10: The discriminator architecture on CIFAR10.

| Layer | Kernel | Output          |
|-------|--------|-----------------|
| Conv, BN, iReLU | $3 \times 3 \times 1$ | $32 \times 32 \times 32$ |
| Conv, BN, iReLU | $4 \times 4 \times 2$ | $64 \times 16 \times 16$ |
| Conv, BN, iReLU | $4 \times 4 \times 2$ | $128 \times 8 \times 8$ |
| Conv, BN, iReLU | $4 \times 4 \times 2$ | $256 \times 4 \times 4$ |
| Conv, BN, iReLU | $4 \times 4 \times 2$ | $512 \times 2 \times 2$ |
| Linear | -     | 1               |

Table 11: The sub-network architecture to model $\mu_\phi(x)$ and $\log{\sigma_\phi(x)}$ on CIFAR10.

| Layer | Kernel | output          |
|-------|--------|-----------------|
| Linear, BN, iReLU | -     | $512$            |
| Linear | -     | $d_z = 128$      |

To address the concern of practical benefits of the proposed $\alpha$-Bridge, we conduct another experiment on CIFAR10 (Krizhevsky and Hinton 2009) for quantitative evaluations on real datasets. We implement the experiment based on the codebase from https://github.com/pfnet-research/chainer-gan-lib. For the vanilla DCGAN baseline, we used the implementation therein to report the results shown in the main manuscript (for fair comparison, we replace the original GAN loss with the reverse KL loss). The used network architectures are shown in Tables 9-11, respectively. For our $\alpha$-Bridge, we kept the network architectures and hyperparameters unchanged, only to add another inference arm and modify the loss function following Algorithm 1 of the main manuscript. The inference arm is constructed similar to the discriminator, with the only difference of replacing the last layer with two sub-networks (see Table 11) to output the mean $\mu_\phi(x)$ and the log of the standard deviation $\log{\sigma_\phi(x)}$ of the inference arm, respectively. Roughly speaking, the difference between the baseline method and the $\alpha$-Bridge one is that the baseline method only runs Step III of Algorithm 1, while the $\alpha$-Bridge method runs the whole Algorithm 1.

Figure 12 shows the IS and FID curves of the compared methods as a function of training iterations. It is apparent that with the $\alpha$-Bridge to benefit from ML learning, one observes improved performance both in IS and FID. Note in the beginning, since the $\alpha$-Bridge is running Step I of Algorithm 1 (the ML initialization), it focuses on the general information to cover the data modes as shown in Figure 13b; accordingly, a worse IS and FID are observed. This is expected since after all both IS and FID are proposed for GANs to measure the image quality instead the data-mode-covering-level. However, we do observe that the $\alpha$-Bridge does be able to benefit from the mode-covering of ML learning to get a better final performance.

More comparison results with other GAN models on CIFAR10 based on the chainer-gan-lib codebase are summarized in Table 12, which highlights the priority of the presented $\alpha$-Bridge. Note for fair comparisons, we use the exact experimental settings from that codebase, without adding GP or SN. We attribute the success of our $\alpha$-Bridge to two main reasons. (i) Among the three advantages that $\alpha$-Bridge transfers from MLE to adversarial learning (see Table 1), the mode covering property is expected to mitigate the mode dropping of adversarial learning, and accordingly benefits a better final performance, like a better IS/FID. (ii) Transferring from MLE could efficiently initialize a better manifold for adversarial learning to start with.
Table 12: The FID and IS of compared methods on CIFAR10 based on the chainer-gan-lib codebase https://github.com/pfnet-research/chainer-gan-lib.

| Methods   | FID  | IS   |
|-----------|------|------|
| BEGAN     | 84.0 | 5.4  |
| DRAGAN    | 31.5 | 7.1  |
| WGAN-GP   | 28.2 | 6.8  |
| RKL-DCGAN | 33.7 | 6.6  |
| α-Bridge  | 28.1 | 7.2  |

Table 14: The discriminator architecture on CelebA with image size 64 × 64.

| Layer     | Kernel        | Output         |
|-----------|---------------|----------------|
| Conv, SN, IReLU | 3 × 3 × 1 | 64 × 64 × 64   |
| Conv, SN, IReLU | 3 × 3 × 2 | 128 × 32 × 32  |
| Conv, SN, IReLU | 3 × 3 × 2 | 256 × 16 × 16  |
| Conv, SN, IReLU | 3 × 3 × 2 | 512 × 8 × 8    |
| Conv, SN, IReLU | 3 × 3 × 2 | 1024 × 4 × 4   |
| Linear, SN   | —             | 1              |

For the experiments on CelebA, we use the DCGAN (Radford, Metz, and Chintala 2015) architectures. For the reconstruction and manipulation experiments shown in Figure 6 of the main manuscript, we use the data images of size 64 × 64; accordingly, the network architectures shown in Tables 13-14 are used. To demonstrate the transfer process more clearly i.e., Figure 5, we used the CelebA images of size 160 × 160 and the architectures in Tables 15-16. The variational inference arm \( \phi(x|z) = N(\mu_\phi(x), \sigma^2_\phi(x)1) \) has a similar network architecture as that of the discriminator, with the differences of replacing the last Linear layer with two shallow sub-networks to model \( \mu_\phi(x) \) and \( \log(\sigma_\phi(x)) \), respectively. The prior of the latent code \( z \) is the standard Gaussian. \( z \) has a dimension \( d_z = 128 \). We train each step (see Algorithm 1) of our α-Bridge for 5, 5, and 40 epochs on CelebA. The batch size is set to 64 due to the computational budget. SN is used to regularize the discriminator in this experiment. More examples on the transfer process of our method are shown in Figure 14.

Table 15: The generator architecture on CelebA with image size 160 × 160.

| Layer     | Kernel        | Output         |
|-----------|---------------|----------------|
| Linear, BN, IReLU | —             | 1024 × 5 × 5   |
| Deconv, BN, IReLU | 5 × 5 × 2 | 512 × 10 × 10  |
| Deconv, BN, IReLU | 5 × 5 × 2 | 256 × 20 × 20  |
| Deconv, BN, IReLU | 5 × 5 × 2 | 128 × 40 × 40  |
| Deconv, BN, IReLU | 5 × 5 × 2 | 64 × 80 × 80   |
| Deconv, Tanh    | 5 × 5 × 2    | 3 × 160 × 160  |

Table 16: The discriminator architecture on CelebA with image size 160 × 160.

| Layer     | Kernel        | Output         |
|-----------|---------------|----------------|
| Conv, SN, IReLU | 3 × 3 × 2 | 64 × 80 × 80   |
| Conv, SN, IReLU | 3 × 3 × 2 | 128 × 40 × 40  |
| Conv, SN, IReLU | 3 × 3 × 2 | 256 × 20 × 20  |
| Conv, SN, IReLU | 3 × 3 × 2 | 512 × 10 × 10  |
| Conv, SN, IReLU | 3 × 3 × 2 | 1024 × 5 × 5   |
| Linear     | —             | 1              |

G.5 CelebA

Table 13: The generator architecture on CelebA with image size 64 × 64.

| Layer     | Kernel        | Output         |
|-----------|---------------|----------------|
| Linear, BN, IReLU | —             | 1024 × 4 × 4   |
| Deconv, BN, IReLU | 4 × 4 × 2 | 512 × 8 × 8    |
| Deconv, BN, IReLU | 4 × 4 × 2 | 256 × 16 × 16  |
| Deconv, BN, IReLU | 4 × 4 × 2 | 128 × 32 × 32  |
| Deconv, BN, IReLU | 4 × 4 × 2 | 64 × 64 × 64   |
| Deconv, Tanh    | 4 × 4 × 2    | 3 × 64 × 64    |
In theory, the posterior in (37) is a Dirac delta function for common situations where $x$ has a higher dimension than that of $z$; thus for $q_\phi(z|x)$ to be identical to $p_\theta(z|x)$, $\sigma_\phi^2(x)$ must be zero, leading to $q_\phi(z|x) = \delta(x|\mu_\phi(x))$;

2. The first reason can be alternatively understood from the perspective of variational inference. To maximize the ELBO w.r.t. $\phi$

$$
\text{ELBO}(\phi) = \mathbb{E}_{q_\phi(z|x)} \left[ \log p_\theta(x|z) + \log \frac{p(z)}{q_\phi(z|x)} \right]
$$

$$
= \mathbb{E}_{q_\phi(z|x)} \left[ \log \lim_{\sigma^2 \to 0} \mathcal{N}(x; G_{\phi}(z), \sigma^2 I) + \log \frac{p(z)}{q_\phi(z|x)} \right]
$$

$$
= \lim_{\sigma^2 \to 0} \mathbb{E}_{q_\phi(z|x)} \left[ -\frac{\|x - G_{\phi}(z)\|^2}{2\sigma^2} + \log \frac{p(z)}{q_\phi(z|x)} \right] \tag{39}
$$

for the model in (37) is equivalent to $\min_\phi \mathbb{E}_{q_\phi(z|x)} \left[ \|x - G_{\phi}(z)\|^2 \right]$ For situations where $x$ has a higher dimension than that of $z$, one would expect one and only one $z$ for each $x$, satisfying $x = G_{\phi}(z)$. Accordingly, the optimal $q_\phi(z|x)$ should be a deterministic function with zero variance $\sigma_\phi^2(x) = 0$, i.e., a Dirac delta function $q_\phi(z|x) = \delta(x|\mu_\phi(x))$.

How we got the reconstruction part of Figure 6? Please see Figures 15 and 16.

Figure 15: Demonstration on the inference arm transplanted with $\alpha$-Bridge. $\phi$ and $\theta$ denote the inference arm $q_\phi(z|x)$ (or $\mu_\phi(x)$) and the generator $G_{\theta}(z)$, respectively. (a) Generated fake images and the reconstructions. (b) Real data images and the reconstructions.
Figures 15 and 16 clearly demonstrate the effectiveness of the transplanted inference arm. For the generated fake sample, the inference arm is capable of recovering the latent code that almost regenerates the fake sample itself. While for the real data sample, one observes a decent regeneration with high mutual information.

**How we got the manipulation part of Figure 6?** By referring to a similar procedure proposed in (Larsen et al. 2015), we manipulate the generated images via the following two steps:

1. **Extract visual attribute vectors.** We first use the transplanted inference arm $\mu_\phi(x)$ (or $q_\phi(z|x) = \delta(z|\mu_\phi(x))$) to encode all the training data samples to their latent codes. Then with the binary attributes of the dataset at hand, for each attribute, we collect the latent codes of the data samples with that attribute and compute their mean; similarly we compute the mean of those without that attribute; the difference between these two means is the visual attribute vector for that attribute. As mentioned in (Larsen et al. 2015), this simple method will have problems with highly correlated visual attributes. We leave that issue for future research and use this simple method for demonstration in this paper.

2. **Manipulate fake images.** Generating fake images from the generator is straight-forward. For each generated image $x$, we use the inference arm $\mu_\phi(x)$ to encode it to its latent code $z$, add one visual attribute vector to the code to get the manipulated code $\tilde{z}$, and then regenerate with the generator $G_\theta(\tilde{z})$ to get the manipulated image $\tilde{x}$. Of course if no visual attribute vector is added, we get the reconstruction. Accordingly, we get the manipulation part of Figure 6.

More results on manipulating generated images are given below.
**H Discussions on Other Potential Generalizations of $\alpha$-Bridge**

Here, we provide additional discussions to shed light on other potential generalizations of the proposed $\alpha$-Bridge.

**$\alpha$-Bridge with a specific $\alpha$.** With a proper prior knowledge on the task, one can choose a specific $\alpha$ to form a proper $\alpha$-divergence objective (which might be problem dependent (Hernández-Lobato et al. 2016; Li and Turner 2016)). By learning with an unbiased gradient $(1 - \rho)\nabla_{\theta} D_{\alpha}^F + \rho \nabla_{\theta} D_{\alpha}^R$ with some $\rho$ (which exploits the gradient information from both the forward and reverse KL), one can combine the advantage of the twin gradients and enjoy a gradient estimation with low MC variance.

**Forward and backward transfer of $\alpha$-Bridge.** Both ML and adversarial learning are developed for matching a model distribution to a data distribution. As mentioned in the main manuscript, $\alpha$-Bridge is proposed to unify the advantage of ML and adversarial learning and enable the smooth transfer from one to the other. Thus, there are two possible directions to transfer: transferring from ML to adversarial learning, or transferring from adversarial learning to ML. How to choose a proper transfer direction is problem dependent.

On the one hand, if one assumes that the model has enough capability and that the optimization is powerful enough to reach the optimum, it ideally does not matter which transfer direction is chosen, as long as the transfer collects all advantages from both sides to match the two distributions. On the other hand, those assumptions might be violated in practice, where we believe both transfer directions make sense for specific applications. In this paper, we focused on transferring from ML to adversarial learning because (i) the task of interest ultimately favors realistic generation (higher priority) more than keeping all data modes (lower priority); (ii) it’s highly possible that ML-pretrained models are already available. As for the other transfer direction from adversarial to ML learning, it may be preferred for situations where (i) losing data modes leads to prohibitive regrets, with realistic generation a relatively lower priority and/or (ii) one may already have adversarial-pretrained models. Intuitively, such transfer is expected to preserve tight fitting/realistic generation on most data modes initialized by adversarial learning, with a few of them modified to cover the data modes missed by adversarial learning. Accordingly, one may expect a smaller mass distributed among modes compared to pure ML learning.

**Is it possible to combining ML and adversarial learning via $f$-GAN?** Motivated by our paper, one may also consider to develop some other potential ways to bridge MLE and adversarial learning, such as,

(i) combining MLE and an FKL $f$-GAN. As both the combined terms aim at minimizing FKL, ideally they have the same optimal solution that may place mass among modes (thus blurry generation); in practice however, there is a gap between them as $f$-GAN optimizes a lower bound of FKL and exhibits adversarial characteristics (realistic generation). It is shown in (Li et al. 2019) that although an FKL $f$-GAN seems to be learning based on FKL, based on the bound used there, in reality what is done is actually adversarial learning. In fact, each form of the $f$-GAN (for all valid $f$ functions) actually does similar adversarial learning with different functions acting on the same likelihood ratio (see Sec. 2 of (Li et al. 2019) for details). So, there is a potentially large gap between FKL (MLE) and FKL $f$-GAN. Because of that gap, it may be tricky (if not impossible) to combine MLE and an FKL $f$-GAN.

(ii) combining an FKL $f$-GAN and an RKL $f$-GAN. Similarly, due to the above mentioned gap, the mode covering of MLE is not preserved in an FKL $f$-GAN (adversarial learning). Accordingly, one cannot combine an FKL $f$-GAN and an RKL $f$-GAN to bridge MLE and adversarial learning.