Study of Minimal String Unification in $Z_8$ Orbifold Models

Hiroshi Kawabe, Tatsuo Kobayashi and Noriyasu Ohtsubo

Department of Physics, Kanazawa University,
Kanazawa, 920-11, Japan
and
*Kanazawa Institute of Technology,
Ishikawa 921, Japan

Abstract

We study the construction of the minimal supersymmetric standard model from the $Z_8$ orbifold models. We use a target-space duality anomaly cancellation and a unification of gauge couplings as constraints. It is shown that some models obtained through a systematical search realize the unification of SU(3) and SU(2) coupling constants.
Superstring theories are promising candidates for unified theories of all the known interactions including gravity. We could write several types of scenarios from the string scale $M_{\text{string}}$ to the low energy scale. Although some of them have intermediate scales such as SU(5) and SO(10) GUT’s, a scenario without the intermediate scales is simplest. We are much more interested in a minimal string vacuum connected at the string scale directly to the minimal supersymmetric standard model (MSSM), which has the SU(3)×SU(2)×U(1) gauge group, three generations, a pair of Higgs particles and their superpartners. Actually, within the framework of the $Z_N$ orbifold models [1] the MSSM massless spectra with extra matter fields have been obtained in $Z_3$ and $Z_7$ models [2-7].

Recent LEP measurements [8-11] indicate that gauge couplings of SU(3), SU(2) and U(1) are simultaneously unified at $M_{\text{GUT}} = 10^{16}$GeV within the framework of the MSSM, while in the string theory all the tree level couplings are identical at $M_{\text{string}} = 5.27 \times g_{\text{string}} \times 10^{17}$GeV [12, 13], where $g_{\text{string}} \simeq 1/\sqrt{2}$ is the universal string coupling constant. The discrepancy between $M_{\text{GUT}}$ and $M_{\text{string}}$ seems to reject the minimal string vacuum. This situation, however, could be changed when we take into account threshold corrections due to towers of massive modes.

Recently, the threshold corrections in the orbifold models have been calculated explicitly in ref. [12-15], where the target-space duality plays an important role. The duality symmetry is the proper ‘stringy’ feature [16, 17]. Here we study duality invariant vacua. In general the six-dimensional orbifolds have SL(2,$\mathbb{Z}$)$^3$ as the duality symmetry [18, 19]. Each complex plane of the orbifold has SL(2,$\mathbb{Z}$) duality symmetry. Loop effects could make this duality symmetry anomalous. In order to obtain consistent field theories, this duality anomaly should be cancelled by the Green-Schwarz (GS) mechanism [20] and loop contributions from the towers of massive modes.

In ref. [21] the constraint of the duality anomaly cancellation was considered systematically and the threshold corrections were estimated explicitly. These analyses have a power to discard a great deal of hopeless models. Actually, it was shown without exhausting the models that all $Z_N$ orbifold models except $Z_6$-II and $Z_8$-I have no candidate of the MSSM possessing the consistent couplings with the measurements and $Z_8$-I orbifold models have a wider range for the promising models than $Z_6$-II ones. In addition, the $Z_8$-I orbifold has a simpler structure than $Z_6$-II, because the former has only two independent Wilson lines (WL’s) of orders two, while the latter has three independent WL’s [22, 23]. Therefore we study the possibility for a minimal
string unification through the threshold effects by examining the $Z_8$-I orbifold models explicitly in this paper.

The content of the paper is as follows. First of all we review the construction of the $Z_8$-I orbifold models and then investigate systematically massless spectra of the orbifold models in order to find the same matter content as the MSSM. In general, models obtained from the string vacua have an anomalous U(1) symmetry. Since such vacua are not stable, the U(1) symmetry breaks through the Higgs mechanism and some matter fields obtain large masses preserving the $N = 1$ supersymmetry \[5, 24\]. After this breaking, the massless spectrum is expected to coincide with the MSSM matter content. Secondly we consider this possibility and we search the models with the MSSM matter content plus some pairs of SU(3) triplets $(3,1)$ and $(\overline{3},1)$ as the string massless spectrum. Next, among the obtained models we investigate the possibility of selecting out vacua without the duality anomaly through the anomalous U(1) breaking. Then we study whether the allowed models are able to unify the gauge couplings of SU(3) and SU(2) so as to be consistent with the measurements through renormalization group (RG) equations.

At the beginning, we briefly survey the construction of $Z_8$-I orbifold model, whose 6-dim compact space is obtained by dividing $\mathbb{R}^6$ in terms of space group elements $(\theta, e_i)$. Here the vector $e_i$ is placed on an SO(9)$\times$SO(5) lattice and the twist $\theta$ is its automorphism whose eigenvalues are $(1, 2, -3)/8$. The orbifold models have right-moving RNS and left-moving gauge parts. Their momenta, $p^I$ and $P^I$, lie on SO(10) and $E_8\times E_8'$ lattices, respectively. The twist is embedded into the SO(10) lattice as a shift $v^t = (1, 2, -3, 0, 0, 0)/8$. A shift vector $V^I$ and WL’s $a^I_i$ ($I = 1 \sim 16$) on the $E_8\times E_8'$ lattice $\Lambda_{E_8\times E_8'}$ are accompanied with $\theta$ and $e_i$, respectively. Refs. \[22, 23\] show that each lattice of SO(9) and SO(5) allows only one independent WL of order two, i.e.,

$$2a_1 = 2a_6 = 0, \quad a_1 = a_2 = a_3 = a_4, \quad a_5 = 0 \mod \Lambda_{E_8\times E_8'}.$$  \hspace{0.5cm} (1)

The shifts also have to fulfill $8V^I = 0 \mod \Lambda_{E_8\times E_8'}$. The independent shifts of the $Z_8$ orbifold are exhibited at the table VII of ref. \[25\]. The modular invariance requires the following conditions:

$$8 \sum_I (a^I_i)^2 = \text{integer}, \quad 8 \sum_I V^I a^I_i = \text{integer},$$

$$8 \left( \sum_I (V^I)^2 - \frac{7}{32} \right) = \text{even}. \hspace{0.5cm} (2)$$

2
Physical states are classified into untwisted states and twisted ones. The untwisted states are closed on the torus and their $E_8 \times E'_8$ momenta $P^I$ fulfill $P^I a^I_i = \text{integer}$. They contain gauge bosons and untwisted matters. The gauge bosons satisfy $P^I V^I = \text{integer}$ and the matters belong to $P^I V^I = 1/8, 2/8, 5/8$ (mod integer). The twisted states are closed on the orbifold and they are invariant under the $\theta^k$ twist, where $k=1,2,4,5$. They are associated with fixed points, which are represented by the space group elements $(\theta^k, n_i e_i)$. The $k \geq 2$ twisted states attached to each fixed point are not always invariant under the $\theta^k$ twist. In order to obtain $\theta^k$-eigenstates, we must take linear combinations of those states, whose eigenvalues under $\theta^k$ are denoted by $\gamma$ hereafter. The twisted states on the fixed points $(\theta^k, n_i e_i)$ have $E_8 \times E'_8$ momenta $P^I + k V^I + n_i a^I_i$. Massless states with the momenta have to satisfy the following condition,

$$
\frac{1}{2} \sum_I (P^I + k V^I + n_i a^I_i)^2 + N_k - 1 + c_k = 0,
$$

where $N_k$ is a number operator and $c_k$ is obtained as

$$
c_k = \frac{1}{2} \sum_{t=1}^3 \left(|kv^t| - \text{Int}(|kv^t|)\right) \left(1 - |kv^t| + \text{Int}(|kv^t|)\right),
$$

where $\text{Int}(a)$ represents an integer part of $a$. The states with the momenta $(p^I + kv^t, P^I + k V^I + n_i a^I_i)$ have the following GSO phases:

$$
\Delta = P^{(k)}_\gamma \exp\left[2\pi i \left(\frac{1}{2k} \sum_I (k V^I + n_i a^I_i)^2 + \frac{k}{2} \sum_t (v^t)^2 
+ \frac{1}{k} \sum_I (k V^I + n_i a^I_i)(P^I + k V^I + n_i a^I_i) - \sum_t v^t(p^I + k v^t)\right)\right],
$$

where $P^{(k)}_\gamma$ is the $Z_8$ phase factor from oscillator contributions. The physical states should satisfy $\Delta=1$. Degeneracy numbers of the massless states in the twisted sectors are exhibited in refs.[22, 23].

Our aim is to obtain the models which have just the same matter content as the MSSM. We search models with the gauge group $SU(3) \times SU(2) \times U(1)^5$ in the observable sector. Following refs. [27, 4], we fix eight $SU(3) \times SU(2)$ non-zero roots as $P^I = (0, 0, 1, -1, 0, 0, 0, 0)$ and $(1, -1, 0, 0, 0, 0, 0, 0)$, where the underlines represent arbitrary permutations. We investigate the massless spectra (combinations of the

\[1\]See in detail ref. [26, 22, 23].
shift and the WL’s of
· the untwisted matters in the observable sector
  (observed elements of the shifts \( V^I \) and the WL’s \( a^I (I = 1 \sim 8) \)) at Stage 1,
· the twisted matter with vanishing WL’s
  (whole elements of the shifts \( V^I (I = 1 \sim 16) \)) at Stage 2,
· the other twisted matters and the untwisted matters in the hidden sector
  (the whole elements of the shifts \( V^I \) and the WL’s \( a^I (I = 1 \sim 16) \)) at Stage 3.

At Stage 1, we select combinations of the shift and the WL’s which induce just eight non-zero roots as expressed above and no extra matter of \((3,2)\) or \((3,1)\) representation of \(SU(3) \times SU(2)\). Under this selection rule, twenty shifts remain and they belong to No.15,16,20,22,23,24,25,26 and 29 of the table VII of ref. [25]. Each shift has 1\sim4 types of WL’s as allowed combinations. At Stage 2 we impose a nonexistence condition of \((3,2)\) and \((3,1)\) matters upon the massless spectra in order to choose the hidden elements of the shifts \( V^I (I = 9 \sim 16) \) corresponding to the observable ones selected in Stage 1. We obtain allowed combinations as follows,

\[
(\text{Shift, WL's})=(I,i),(I,ii),(II,i),(II,ii),(III,iii),(IV,iii), (IV,iv),
\]

where the shifts are given by

\[
\begin{align*}
I & : V = (1, 1, 2, 2, 2, 3, 2, -1; 4, 4, 3, 3, 1, 1, 0, 0)/8, \\
II & : V = (1, 1, 2, 2, 2, 3, 2, -1; 3, 3, 2, 2, 2, 2, 1, -1)/8, \\
III & : V = (2, 2, 2, 2, 2, 2, 1, -1; 3, 3, 3, 3, 2, 2, 2, -2)/8, \\
IV & : V = (2, 2, 2, 2, 2, 3, 2, -1; 3, 3, 3, 2, 2, 2, 2, -1)/8,
\end{align*}
\]

and the observable elements of the WL’s are given by

\[
\begin{align*}
i & : a_1 = (0, 0, 0, 0, 0, 0, 2, 2)/4 \quad a_2 = (1, 1, -1, -1, 1, -1, 1, 1)/4, \\
ni & : a_1 = (0, 0, 2, -2, -2, -2, 0, 0)/4 \quad a_2 = (1, 1, 3, -1, -1, 1, 1, -1)/4, \\
niii & : a_1 = (0, 0, 2, -2, -2, 0, -2, 0)/4 \quad a_2 = (1, 1, 3, -1, -1, -1, 1, 1)/4, \\
niv & : a_1 = (0, 0, 2, -2, -2, 0, 0, 2)/4 \quad a_2 = (1, 1, 3, -1, -1, -1, -1, 1)/4.
\end{align*}
\]
At Stage 3 we have searched the models which have three or more generations and no \((\overline{3},2)\), but we have not been able to obtain such models for all combinations of eq. (6).

By the above result, however, we can not conclude that the minimal string model is not obtained from the \(Z_8\)-I orbifold models. The matters \((\overline{3},2)\) or \((3,1)\) are possible to be included in the massless spectra, because they might couple with extra \((3,2)\) or \((\overline{3},1)\) matters and obtain heavy masses through the anomalous U(1) breaking \([3,24]\).

Now, we permit the existence of some pairs of \((3,1)\) and \((\overline{3},1)\) matters in the twisted sector by way of trial. Let us go back to Stage 2. Then we find each observed shift subjects 4~9 hidden shifts. We have obtained 249 combinations of the shifts \(V^I\) \((I = 1 \sim 16)\) and the observed elements of the WL’s \(a_1^I\) \((I = 1 \sim 8)\).

It is notable that models with larger hidden gauge groups are easy to analyze and models involving smaller ones have the possibility of mixture of observed and hidden matters. Therefore, we pay attention to models with rather large gauge groups. We have carried out Stage 3 under the condition of non-existence of \((3,2)\). The result is as follows. The largest hidden gauge group is \(SO(10)' \times U(1)'^3\), which is realized in two models named Model 1 and Model 2. \(\) (There are also models with hidden gauge groups \(SU(6)'\), \(SU(5)'\) and so on.) Massless contents of the models are obtained as follows,

\[
3[(3, 2) + 2(\overline{3}, 1) + (1, 2) + (1, 1)] + 2(1, 2) + 17[3, 1] + (3, 1)] + 34(1, 2) + 2(10)' + 159(1, 1) \quad \text{for Model 1,}
\]

\[
3[(3, 2) + 2(\overline{3}, 1) + (1, 2) + (1, 1)] + 2(1, 2) + 16[(\overline{3}, 1) + (3, 1)] + 32(1, 2) + 2(10)' + 159(1, 1) \quad \text{for Model 2.}
\]

Model 1 is derived from the following shift and the WL’s:

\[
V = (1, 1, 2, 2, 2, 3, 3, 0; 2, 2, 1, 1, 1, 1, 1, 1)/8
\]

\[
a_1 = (0, 0, 0, 0, 0, 0, 2, 0, 2, 0, -2, 0, 0, 0, 0)/4,
\]

\[
a_2 = (1, 1, -3, 1, 1, -1, 1, -1; 1, -1, -1, 1, 1, 1, 1)/4.
\]

Model 2 is obtained by the same shift \(V^I\) and the same WL \(a_1\) as eq. (10) and \(a_2 = (1, 1, -3, 1, 1, -1, 1, -1; -3, -1, 1, -1, -1, -1, -1, -1)/4\). Both the models have the anomalous U(1) symmetry, so the vacua are unstable. We could analyze U(1)
charges following to refs. [5, 24], and discuss the breakings of extra U(1) gauge symmetries to obtain stable vacua. Instead of doing so, we investigate whether the duality invariant vacua with the MSSM matter content can be obtained from the above two models after some type of the U(1) breaking occurs and the extra matters become massive following ref. [21]. One of the reasons for this option is that duality anomaly cancellation condition is powerful enough to discard lots of hopeless models.

Effective field theories derived from the 4-dim orbifold models must be invariant under the following transformation of a moduli $T_i$:

$$T_i \rightarrow a_i T_i - ib_i$$

with $a_i, b_i, c_i, d_i \in \mathbb{Z}$ and $a_i d_i - b_i c_i = 1$. The moduli $T_i$ is associated with one of three complex planes of the orbifold. Under the duality transformation, the matter $A_\alpha$ transforms as

$$A_\alpha \rightarrow A_\alpha \prod_{i=1}^{3} (ic_i T_i + d_i)^{n_\alpha^i},$$

where $n_\alpha^i$ is called modular weight which is associated with the $i$-th plane [28, 21, 29]. For untwisted matters associated with the $i$-th complex plane, the modular weights are $n_\alpha^i = -\delta_{ij}$. The $k=1,2,4,5$ twisted matters without oscillator contributions have the modular weights $n_\alpha = (-7, -6, -3)/8, (-6, -4, -6)/8, (-4, 0, -4)/8, (-3, -6, -7)/8$, respectively. An oscillator $a_i$ of the $i$-th complex plane reduces $i$-th elements of $n_\alpha$ by one.

The duality anomaly is caused by triangle graphs which have Kähler and curvature connections as one of external lines. The anomaly can be cancelled by a combination of two ways. One is GS mechanism, which induces non-trivial duality transformation of the dilaton field. The other way is due to the threshold effects for the gauge coupling terms. The threshold corrections of the gauge coupling constants depend only on the modulus whose complex planes are not rotated in all the twists, because those planes have $N = 2$ supermultiplets as well as $N = 4$ supermultiplets. Since the first and the third planes of $Z_8$-I orbifold are rotated under all the twists, the anomalies associated with the planes have to be cancelled only by the GS mechanism. On the other hand, the second plane concerning with the $\theta$-eigenvalue $2/8$ is fixed under the $\theta^4$ twist, so the threshold correction depends only on $T_2$. Both of the GS mechanism and the threshold effects contribute to the anomaly cancellation about $T_2$.

Actually, anomaly coefficients of the $i$-th plane with respect to SU(3), SU(2) and
SO(10)' are obtained as

\[ b'^i_{SU(3)} = -3 + \sum_{\alpha \in (3,2)} (2n^i_{\alpha} + 1) + \sum_{\alpha \in (\bar{3},1)} (n^i_{\alpha} + \frac{1}{2}), \]

\[ b'^i_{SU(2)} = -2 + \sum_{\alpha \in (3,2)} 3(n^i_{\alpha} + \frac{1}{2}) + \sum_{\alpha \in (1,2)} (n^i_{\alpha} + \frac{1}{2}), \]

\[ b'^i_{SO(10)} = -4 + \sum_{\alpha \in (10)'} (n^i_{\alpha} + \frac{1}{2}). \]  

(13)

Since the GS term is gauge invariant, we obtain the anomaly cancellation condition for completely rotated planes as follows,

\[ b'^i_{SU(3)} = b'^i_{SU(2)} = b'^i_{SO(10)} \quad (i = 1, 3), \]  

(14)

where \( b'^1_{SO(10)} = -39/8 \) and \( b'^3_{SO(10)} = -27/8 \) in both the models.

Model 1 and Model 2 have extra matters other than the MSSM matter content. Now, we select massless matters remaining in the stable duality invariant vacua in order to get the MSSM model, that is, we pick up six (\( \bar{3},1 \)) and five (1,2) as well as three (3,2) from the matter content of eq. (9) under the condition (14). Table 1~4 express all the matters of Model 1 except (3,1) and (1,1). The first columns show the degeneracy numbers of the states. Types of WL's are found in the fourth columns, where \([n, n']\) denotes \( n = \sum_{i=1}^{4} n_i \) and \( n' = n_6 \) (mod 2). The oscillators \( a_i \) involved in the states are shown in the fifth columns. The last two columns of Table 2 express two types of choice for six (\( \bar{3},1 \)) matters, i.e. I and II. The numerator of the columns means the required number to pick up from the group of matters which belong to the same denominator in a twisted sector, and the denominator is the number of whole matters which have the same modular weights for the first and the third planes. Table 3 expresses three types of choice for five (1,2) matters, i.e. A, B and C, in the same way as Table 2. Making use of these two tables, we can take all the possible combinations with respect to the whole modular weights for deriving the duality invariant vacua. Note that the value of \( b'^2_{SU(2)} \) depends on whether or not we choose oscillators in the types B and C. Alternatively, we consider the case where the two (10)' matters obtain heavy masses. In this case we have \( b'^i_{SO(10)} = -4 \). But, we have found no solution of eq. (14), when we pick up six (\( \bar{3},1 \)) and five (1,2) as well as three (3,2) from the matter content of eq. (9).
Now, we discuss the one-loop running gauge coupling constants including the threshold effects as follows,

\[
\frac{1}{g_a^2(\mu)} = \frac{1}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \log \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2} (b_a^2 - \delta_{GS}^2) \log [(T_2 + \overline{T}_2) |\eta(T_2)|^4],
\]

(15)

where \(\eta(T)\) is the Dedekind function, \(\delta_{GS}^2\) are gauge group independent GS coefficients and \(b_a\) are \(N = 1\) \(\beta\)-function coefficients, i.e., \(b_{\text{SU}(3)} = -3\), \(b_{\text{SU}(2)} = 1\) and \(b_{\text{SO}(10)} = -11\). For the anomaly coefficients of Model 1, the types I and II of SU(3) parts lead to \(b_{\text{SU}(3)}^2 = 1/4\) and \(-7/4\), respectively. The types A, B and C of SU(2) lead to \(b_{\text{SU}(2)}^2 = -7/4, -3/4\) and \(1/4\) for the choice of no oscillators, respectively, while the type B including a \(k = 1\), \(N_2 = 2/8\) oscillator state leads to \(b_{\text{SU}(2)}^2 = -7/4\). Further a choice including a \(k = 5\), \(N_2 = 2/8\) oscillator state in the types B and C reduces the above values of \(b_{\text{SU}(2)}^2\) by one. On the other hand, we obtain \(b_{\text{SO}(10)}^2 = -15/4\) in the case with two \(10'\) matters.

Next, we consider the unification of SU(3) and SU(2) gauge couplings. From eq. (15) the unification mass scale \(M_{\text{GUT}}\) subjects the following equation,

\[
\log \frac{M_{\text{GUT}}}{M_{\text{string}}} = \frac{b_{\text{SU}(2)}^2 - b_{\text{SU}(3)}^2}{2(b_{\text{SU}(3)} - b_{\text{SU}(2)})} \log [(T_2 + \overline{T}_2) |\eta(T_2)|^4].
\]

(16)

We further select the combinations of types of SU(3) and SU(2) by the condition \(M_{\text{GUT}} < M_{\text{string}}\). This condition is equivalent to \(b_{\text{SU}(3)}^2 > b_{\text{SU}(2)}^2\) because the inside of the square brackets in eq. (16) is always smaller than 1. As the results, there are four combinations of the allowed values:

\[
(b_{\text{SU}(3)}^2, b_{\text{SU}(2)}^2) = (1, -7)/4, (1, -3)/4, (1, -11)/4, (-7, -11)/4.
\]

(17)

The similar results are derived in Model 2. Under the condition \(M_{\text{GUT}} \sim M_{\text{string}}/37\), values of the differences \(b_{\text{SU}(3)}^2 - b_{\text{SU}(2)}^2 = 8/3, 4, 8\) lead to \(\text{Re}T_2 \sim 12, 17, 31\), respectively.

Further, we also study the RG flow of the SO(10)' gauge coupling. The gaugino condensation of the hidden group SO(10)' might lead to a realistic SUSY-breaking, although the dynamics of the condensation has never been understood yet. The scale of the condensation \(M_{\text{COND}}\) and that of the observable SUSY-breaking \(M_{\text{SUSY}}\) are related as \(M_{\text{SUSY}} \simeq M_{\text{COND}}^3/M_P^2\), where \(M_P\) is the Planck scale. So the condensation

\[\text{See e.g. [30] and the references therein.}\]
must happen near by \(10^{13}\) GeV in order to derive the SUSY-breaking at 1 TeV. For the allowed four combinations of eq. (17), we impose \(\alpha_{GUT}^{-1} = 25.7\) \((\alpha = g^2/4\pi)\) on eq. (15) in order to draw the RG flow of the SO(10)' coupling constant. We find that the combination \((b_{SU(3)}^2, b_{SU(2)}^2) = (1, -3)/4\) of eq. (17) leads to \(\alpha_{SO(10)}^{-1} = 0\) at \(10^{13}\)GeV. Therefore, in the model with this combination the condensation might happen at a higher energy than \(10^{13}\)GeV. On the other hand, the combination \((b_{SU(3)}^2, b_{SU(2)}^2) = (1, -11)/4\) and the other two lead to \(\alpha_{SO(10)}^{-1} = 5.9\) and 4.4 at \(10^{13}\)GeV, respectively. In these cases, the condensation might happen near by \(10^{13}\)GeV.

At last we attempt to assign the representations of Model 1 to the matter superfields of the MSSM. Let us take the type I of Table 3 and the type C of Table 2 as an example. Suppose that we assign quarks \((Q_i, U_i, D_i, (i = 1, 2, 3))\), a pair of Higgs particles \((\tilde{H}, H)\) and lepton doublets \((L_i, (i = 1, 2, 3))\) as shown in the tables. Then these particles have correct hypercharges under the following basis:

\[
Y = (2 U_2 - 9 U_3 - 3 U_4 - 3 U_5 + 3 U_1' - 2 U_2' + 16 U_3')/48,
\]

where

\[
\begin{align*}
U_1 &= (1, 1, 0, 0, 0, 0, 0, 0, 0)/2, & U_2 &= (0, 0, 1, 1, 0, 0, 0, 0, 0)/2, \\
U_3 &= (0, 0, 0, 0, 0, 1, 0, 0), & U_4 &= (0, 0, 0, 0, 0, 0, 1, 0), \\
U_5 &= (0, 0, 0, 0, 0, 0, 0, 1), & U_1' &= (1, 1, 0, 0, 0, 0, 0, 0)/2, \\
U_2' &= (1, -1, -2, 0, 0, 0, 0, 0)/2, & U_3' &= (1, -1, 3, 3, 3, 3)/16.
\end{align*}
\]  

All the U(1) invariances and space group selection rules allow renormalizable couplings of \(Q_i H U_j\) and \(Q_i \tilde{H} D_j\), \((i, j = 2, 3)\). If a lepton singlet \(E_3\) is assigned to a singlet in \(k = 2\) sector with WL’s \([1, 1]\) and U(1) charges \((0, 6, -4, -4, 6, 4, 0, 0)\), it couples with the lepton \(L_3\) as \(L_3 HE_3\). The other lepton doublets have no renormalizable coupling with \(H\) and all the singlets. There are 38 candidates for the first and the second generations of lepton singlets \((E_1, E_2)\). Although the above assignment may be consistent with the low energy phenomenology, it conflicts with a theoretical requirement. A sum of the hypercharges of all the massless particles in Model 1 does not vanish \((\sum Y = 21)\).

In this letter, we have studied the \(Z_8\)-I orbifold models with two Wilson lines systematically in order to construct the MSSM. It has been shown that the \(Z_8\)-I orbifold models can not lead to the MSSM as the string massless spectra. We have also examined the models which have matter contents of the MSSM with the extra matters and have obtained two models involving the SO(10)' hidden symmetry. So
as to lead to the MSSM, we have imposed the duality anomaly cancellation condition on the models. The unification of SU(3) and SU(2) gauge couplings has also been investigated. We have found its solutions under some values of $T_2$. The remaining problem is to show what kinds of symmetry breakings induce such vacua. Further the renomalization group flow of SO(10)' was discussed. We have tried to assign representation matters of the Model 1 to the MSSM matters taking into account the hypercharges of them. We could also study other models which have hidden gauge groups SU(6)', SU(5)' and so on. The above approach could be extended to cases with extra matters of the $(\mathbf{3}, 2)$ representation. It is interesting to investigate the $Z_6$-II and $Z_M \times Z_N$ orbifold models similarly.

**Acknowledgement**

The authors would like to thank D. Suematsu and K. Matsubara for useful discussions and S. Kiura for helpful advices of computer analyses.
References

[1] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678; Nucl. Phys. B274 (1986) 285.

[2] L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, Phys.Lett. B191 (1987) 282.

[3] J.A Casas and C. Muñoz, Phys. Lett. B209 (1988) 214; Phys. Lett. B212 (1988) 343; Phys. Lett. B214 (1988) 63.

[4] A. Font, L.E. Ibáñez, H.-P. Nilles and F. Quevedo, Phys. Lett. B210 (1988) 101.

[5] J.A Casas, E.K. Katehou and C. Muñoz, Nucl. Phys. B317 (1989) 171.

[6] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono and K. Tanioka, Nucl. Phys. B341 (1990) 611.

[7] J.A. Casas, A. de la Macorra, M. Mondragón and C. Muñoz, Phys. Lett. B247 (1990) 50.

[8] J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B260 (1990) 131.

[9] U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260 (1991) 447.

[10] P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817.

[11] G.G. Ross and R.G. Roberts, Nucl. Phys. B377 (1992) 571.

[12] V.S. Kaplunovsky, Nucl. Phys. B307 (1988) 145.

[13] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145.

[14] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649.

[15] I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B267 (1991) 37.

[16] K. Kikkawa and M. Yamasaki, Phys. Lett. B149 (1984) 357.

[17] N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1986) 692.

[18] R. Djikgraaf, E. Verlinde and H. Verlinde, Commun. Math. Phys. 115 (1988) 649.
[19] A. Shapere and F. Wilczek, Nucl. Phys. B320 (1989) 669.

[20] M.B. Green and J.H. Schwarz, Phys. Lett. B149 (1984) 117.

[21] L.E. Ibáñez and D. Lüst, Nucl. Phys. B382 (1992) 305.

[22] T. Kobayashi and N. Ohtsubo, Phys. Lett. B257 (1991) 56.

[23] T. Kobayashi and N. Ohtsubo, “Geometrical Aspects of $Z_N$ Orbifold Phenomenology”, preprint DPKU-9103 to be published in Int. J. Mod. Phys. A.

[24] A. Font, L.E. Ibáñez, H.-P. Nilles and F. Quevedo, Nucl. Phys. B307 (1988) 109.

[25] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, and K. Tanioka, Prog. Theor. Phys. 82 (1989) 171.

[26] T. Kobayashi and N. Ohtsubo, Phys. Lett. B245 (1990) 441.

[27] J.A. Casas, M. Mondragon and C. Muñoz, Phys. Lett. B230 (1989) 63.

[28] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B329 (1990) 27.

[29] D. Bailin and A. Love, Phys. Lett. B288 (1992) 263.

[30] H.P. Nilles, Int. J. Mod. Phys. A5 (1990) 4199.
Table 1. (3,2) representations

| Deg. | \( k \) | \( P^1 V^1 \) | WL | Osc. | \( U_1 \) | \( U_2 \) | \( U_3 \) | \( U_4 \) | \( U_5 \) | \( U'_1 \) | \( U'_2 \) | \( U'_3 \) |
|------|--------|-------------|-----|------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1    | 0      | 1/8         | -   | -    | -4     | 4      | 0      | 0      | 0      | 0      | 0      | 0      |
| 2    | 2      | [1,1]       | 0   | 1    | 0      | 2      | 4      | 0      | 0      | 0      | 0      | 0      |

\((Q_1)\)

\((Q_2, Q_3)\)

Table 2. (3,1) representations

| Deg. | \( k \) | \( P^1 V^1 \) | WL | Osc. | \( U_1 \) | \( U_2 \) | \( U_3 \) | \( U_4 \) | \( U_5 \) | \( U'_1 \) | \( U'_2 \) | \( U'_3 \) |
|------|--------|-------------|-----|------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1    | 0      | 1/8         | -   | -    | 0      | -4     | 0      | 8      | 0      | 0      | 0      | 0      |
| 1    | 0      | 5/8         | -   | -    | 4      | 2      | -4     | 4      | -4     | 0      | 0      | 0      |
| 1    | 0      | 2/8         | -   | -    | 4      | 2      | -4     | 4      | 4      | 0      | 0      | 0      |
| 1    | 1      | -           | [0,0]| -    | 1      | -1     | -5     | 3      | 0      | 4      | -1     | 1      |
| 1    | 1      | -           | [0,0]| -    | 1      | -1     | 3      | -5     | 0      | 4      | -1     | 1      |
| 1    | 1      | -           | [1,1]| -    | 3      | 2      | -3     | -3     | 2      | 0      | 1      | -1     |
| 1    | 1      | -           | [1,1]| -    | 1      | -1     | 0      | -4     | 1      | 1      | -2     | 0      |
| 2    | 2      | -           | [0,1]| -    | 0      | -1     | 0      | -4     | 2      | 0      | -6     | 0      |
| 2    | 2      | -           | [1,1]| -    | 0      | -1     | -4     | -4     | -2     | 4      | 0      | 0      |
| 2    | 2      | -           | [1,1]| -    | 0      | -1     | 4      | 4      | -2     | 4      | 0      | 0      |
| 1    | 2      | -           | [0,0]| -    | 0      | 2      | 0      | 0      | 0      | 0      | 4      | 8      |
| 2    | 2      | -           | [0,0]| -    | 0      | 2      | 0      | 0      | 4      | 8      | 0      | 0      |
| 2    | 2      | -           | [1,0]| -    | 0      | 2      | -4     | 0      | 0      | -4     | 6      | 0      |
| 2    | 2      | -           | [1,0]| -    | 0      | 2      | -4     | 0      | 0      | 4      | -6     | 0      |

\((U_1)\)

\((U_2, U_3)\)

\((U_1, U_2)\)

\((U_1)\)

\((U_1)\)

\((D_1)\)

\((D_1)\)

\((D_1)\)

\((D_2, D_3)\)

\((D_2, D_3)\)

\((D_2, D_3)\)
Table 3. (1, 2) representations

| Deg. | k | $P^I V^I$ | WL | Osc. | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_5'$ | $U_3'$ | A | B | C |
|------|---|----------|----|------|-------|-------|-------|-------|-------|-------|-----|---|---|
| 1    | 0 | 5/8      | -   | -    | 0     | -6    | 4     | -4    | -4    | 0     | 0   | 0 | 0 | 1/1 |
| 1    | 0 | 2/8      | -   | -    | -4    | 0     | 0     | 8     | 0     | 0     | 0   | 0 | 0 | 0 |
| 1    | 0 | 2/8      | -   | -    | 0     | -6    | -4    | -4    | 4     | 0     | 0   | 0 | 0 | 0 |
| 1    | 1 | -        | [1,1]| -    | -5    | 0     | 1     | 1     | -2    | 0     | 1   | -1 | 0 | 1/2 |
| 1    | 1 | -        | [1,1]| $a_2$| 3     | 0     | 1     | 1     | -2    | 0     | 1   | -1 | 0 | 1/2 |
| 1    | 1 | -        | [0,0]| $a_1$| 1     | -3    | -1    | -1    | 4     | 4    | -1 | 1 | 0 | 0 |
| 1    | 1 | -        | [1,1]| $(a_1)^2$| 3 | 0 | 1 | 1 | -2 | 0 | 1 | -1 | 0 | 0 |
| 2    | 2 | [1,0]    | -   | -    | 2     | 0     | -2    | 2     | 0     | -4   | -4 | -2 | 1/8 | 1/8 |
| 2    | 2 | [1,0]    | -   | -    | 2     | 0     | -2    | 2     | 0     | -4   | 4  | 2  | 1/8 | 1/8 |
| 2    | 2 | [0,1]    | -   | -    | 0     | -3    | 4     | 0     | -2    | 0     | -6 | 0 | 1/8 | 1/8 |
| 2    | 2 | [1,1]    | -   | -    | 0     | -3    | 0     | 0     | -6    | 4     | 0  | 0 | 1/8 | 1/8 |
| 1    | 4 | -        | [0,0]| -    | 0     | 0     | 4     | 4     | 0     | -8   | 0  | 0 | 1/16 | 0   |
| 1    | 4 | -        | [0,0]| -    | 0     | 0     | 4     | 4     | 0     | 8    | 0  | 0 | 1/16 | 0   |
| 2    | 4 | -        | [0,0]| -    | 0     | 0     | -4    | -4    | 0     | -8   | 0  | 0 | 1/16 | 0   |
| 4    | 4 | -        | [0,0]| -    | 0     | 0     | -4    | -4    | 0     | 8    | 0  | 0 | 1/16 | 0   |
| 2    | 4 | [1,0]    | -   | -    | 0     | 0     | 0     | -4    | 4     | 4    | -6 | 0 | 1/16 | 0   |
| 2    | 4 | [1,0]    | -   | -    | 0     | 0     | 0     | -4    | 4     | 4    | -6 | 0 | 1/16 | 0   |
| 2    | 4 | [1,0]    | -   | -    | 0     | 0     | 0     | 4     | -4    | -4   | 6  | 0 | 1/16 | 0   |
| 2    | 4 | [1,0]    | -   | -    | 0     | 0     | 0     | 4     | -4    | 4    | -6 | 0 | 1/16 | 0   |
| 1    | 5 | [1,0]    | -   | -    | 1     | 3     | 3     | -1    | 4     | 0    | 5  | 1 | 3/3 | 1/3 |
| 1    | 5 | [1,0]    | -   | -    | -1    | 0     | 1     | 5     | 2     | 4    | -5 | -1 | 3/3 | 2/3 |
| 1    | 5 | [0,1]    | -   | -    | -1    | 0     | 1     | 5     | 2     | 4    | -5 | -1 | 3/3 | 2/3 |
| 1    | 5 | [0,0]    | -   | -    | 1     | 3     | -1    | -1    | 0     | -4   | -1 | 1 | 3/3 | 2/3 |
| 1    | 5 | [0,0]    | -   | -    | 1     | 3     | -1    | -1    | 0     | -4   | -1 | 1 | 0  | 0  |

Table 4. Hidden (10)' representations

| Degen. | k | $P^I V^I$ | WL | Osc. | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_5'$ | $U_3'$ |
|--------|---|----------|----|------|-------|-------|-------|-------|-------|-------|-------|
| 1      | 0 | 1/8      | -   | -    | 0     | 0     | 0     | 0     | 8     | -4    | -2    |
| 1      | 1 | -        | [0,1]| -    | -1    | 0     | -3    | 1     | 2     | 4     | -1    | 1     |