Unifiable Supersymmetric Left-Right Model with $E_6$ Particle Content

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Abstract

A new supersymmetric gauge model is proposed with particle content chosen only from the $27$ and $27^*$ representations of $E_6$. The gauge symmetry $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ is realized at the TeV energy scale and the gauge couplings converge to a single value at around $10^{16}$ GeV. A discrete $Z_4 \times Z_2$ symmetry leads to a generalized definition of lepton number and ensures the absence of tree-level flavor-changing neutral-current interactions at the electroweak energy scale.
Several years ago, in connection with superstring theory, a supersymmetric left-right model was proposed\[1\] which has the interesting property that the SU(2)$_R$ charged gauge boson $W_R$ has a nonzero lepton number, together with new exotic quarks $h$ of charge $-1/3$. Its many unconventional implications have been studied in a number of subsequent publications.[2-16] This model is potentially of great phenomenological interest, but only if the SU(2)$_R$ breaking scale $M_R$ is low enough, say of order a few TeV. However, that poses a problem for the unification of gauge couplings. With the particle content of the original model[1] and the experimentally determined values of the gauge couplings at the electroweak energy scale, it is simply not possible for them to converge to a single value unless $M_R$ is very high. This is a well-known general result for left-right models.[17] Hence new particles are necessary if unification of the gauge symmetry is to be achieved.[18]

To be compatible with its possible superstring antecedent, the particle content of the proposed supersymmetric model in this paper is assumed to consist of only components from the 27 and 27* representations of E$_6$. [The 27 of E$_6$ decomposes into 16 + 10 + 1 of SO(10).] At the SU(3) × SU(2)$_L$ × SU(2)$_R$ × U(1) level, it is proposed that there are three copies of

$$Q = (u, d)_L \sim (3, 2, 1, 1/6), \quad d^c_L \sim (\overline{3}, 1, 1, 1/3)$$

$$Q^c = (h^c, u^c)_L \sim (\overline{3}, 1, 2, -1/6), \quad h_L \sim (3, 1, 1, -1/3),$$

one bidoublet

$$\eta = \begin{pmatrix} \eta^0_1 & \eta^+_2 \\ \eta^-_1 & \eta^0_2 \end{pmatrix} \sim (1, 2, 2, 0),$$

six copies of

$$\Phi_L = (\phi^0_L, \phi^-_L) \sim (1, 2, 1, -1/2),$$

$$\Phi_R = (\phi^+_R, \phi'^0_R) \sim (1, 1, 2, 1/2),$$

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three copies of
\[ \Phi_L \sim (1, 2, 1, 1/2), \quad \Phi_R \sim (1, 1, 2, -1/2), \]
and six singlets
\[ N \sim (1, 1, 1, 0). \tag{7} \]
Under SO(10), Q, Q^c, Φ_L, and Φ_R belong to the \( 16, d, h, \) and η belong to the \( 10, \Phi^c_L \) and \( \Phi^c_R \). Anomaly cancellation at the SU(3) \( \times \) SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) level is assured because of the anomaly-free combinations \( Q + Q^c + \Phi_L + \Phi_R, d^c + h, \Phi_L + \Phi^c_L, \) and \( \Phi_R + \Phi^c_R \). The left-right gauge symmetry is broken spontaneously at \( M_R \) by the nonzero vacuum expectation values of \( \Phi_R \) and \( \Phi^c_R \). The scale of soft supersymmetry breaking is assumed to coincide with \( M_R \). The effective particle content at the electroweak energy scale is assumed to be that of the (nonsupersymmetric) standard SU(3) \( \times \) SU(2)_L \( \times \) U(1)_Y model with two Higgs doublets.

Consider now the evolution of the gauge couplings to two-loop order. Generically,
\[ \mu \frac{\partial \alpha_i(\mu)}{\partial \mu} = \frac{1}{2\pi} \left( b_i + \frac{b_{ij}}{4\pi} \alpha_j(\mu) \right) \alpha_i^2(\mu), \tag{8} \]
where \( \alpha_i \equiv g_i^2/4\pi \) and \( b_i, b_{ij} \) are constants determined by the particle content contributing to \( \alpha_i \). The initial conditions are set at \( M_Z = 91.187 \pm 0.007 \) GeV by the experimental values \( \alpha^{-1} = 127.9 \pm 0.1, \sin^2 \theta_W = 0.2321 \pm 0.0006, \) and \( \alpha_S = 0.120 \pm 0.006 \pm 0.002 \). Hence
\[ \alpha^{-1}_S(M_Z) = 8.33^{+0.60}_{-0.52}, \quad \alpha^{-1}_L(M_Z) = 29.69 \pm 0.10, \quad \alpha^{-1}_Y(M_Z) = 98.21 \pm 0.15. \tag{9} \]
At \( M_R \), the matching conditions of the gauge couplings are
\[ \alpha^{-1}_L = \alpha^{-1}_R, \quad \alpha^{-1}_Y = \alpha^{-1}_R + \alpha^{-1}_X, \tag{10} \]
where \( \alpha_X \) refers to the U(1) gauge coupling of the left-right symmetry. Above \( M_R \), \( \alpha_L \) and \( \alpha_R \) will evolve together identically.
In the one-loop approximation, below $M_R$,

\[
\begin{align*}
  b_S &= -11 + \frac{4}{3}(3) = -7, \\
  b_L &= -\frac{22}{3} + \frac{4}{3}(3) + \frac{1}{6}(2) = -3, \\
  b_Y &= \frac{20}{9}(3) + \frac{1}{6}(2) = 7,
\end{align*}
\]

whereas above $M_R$,

\[
\begin{align*}
  b_S &= -9 + 2(3) + n_h = 0, \\
  b_{LR} &= -6 + 2(3) + n_{22} + n_\phi = 4, \\
  \frac{3}{2}b_X &= 2(3) + 3n_\phi + n_h = 18,
\end{align*}
\]

where $n_{22} = 1$, $n_h = 3$, and $n_\phi = 3$ refer respectively to the one bidoublet $\eta$, the three copies of $h + d^c$, and the three copies of $\Phi_L + \Phi^c_L + \Phi_R + \Phi^c_R$, and the factor $3/2$ for $b_X$ comes from the normalization of the $U(1)_X$ coupling within SO(10). Assuming that $\alpha^{-1}_S = \alpha^{-1}_{LR} = (3/2)\alpha^{-1}_X$ at the unification scale $M_Z$ and neglecting the two-loop coefficients $b_{ij}$, Eq. (8) can be solved for $M_R$ and $M_U$, i.e.

\[
\ln \frac{M_R}{M_Z} = \frac{\pi}{4} \left[ 3\alpha^{-1}(M_Z)\{1 - 5\sin^2\theta_W(M_Z)\} + 7\alpha^{-1}_S(M_Z) \right] < 1.66,
\]

and

\[
\ln \frac{M_U}{M_Z} = \frac{\pi}{2} \left[ \alpha^{-1}(M_Z)\sin^2\theta_W(M_Z) - \alpha^{-1}_S(M_Z) \right] > 32.45.
\]

Hence $M_R < 480$ GeV and $M_U > 1.1 \times 10^{16}$ GeV. The upper bound of $M_U$ is $1.9 \times 10^{16}$ GeV, corresponding to $M_R = M_Z$. Note that these results are identical to those of a recently proposed extension of the conventional supersymmetric left-right model\cite{18} because each corresponding $b_i$ above $M_R$ differs by the same amount, i.e. one.

The allowed parameter space opens up more in two loops. Using\cite{23}

\[
\begin{align*}
  b_{ij} &= \begin{pmatrix}
  -26 & \frac{9}{2} & \frac{11}{10} \\
  12 & 8 & \frac{6}{5} \\
  \frac{44}{5} & \frac{18}{5} & \frac{104}{25}
  \end{pmatrix}
\end{align*}
\]
for $\alpha_{S}^{-1}$, $\alpha_{L}^{-1}$, and $(3/5)\alpha_{Y}^{-1}$ below $M_R$, and

$$b_{ij} = \begin{pmatrix}
48 & 9 & 3 \\
24 & 49 & \frac{15}{2} \\
24 & \frac{45}{2} & \frac{45}{2}
\end{pmatrix}$$

(20)

for $\alpha_{S}^{-1}$, $\alpha_{LR}^{-1}$, and $(3/2)\alpha_{X}^{-1}$ above $M_R$, and solving Eq. (8) numerically with the proper boundary conditions at $M_U$:

$$\alpha_U^{-1} - \frac{2}{3\pi} = \alpha_S^{-1} - \frac{1}{4\pi} = \alpha_{LR}^{-1} - \frac{1}{6\pi} = \frac{3}{2}\alpha_{X}^{-1},$$

(21)

it is found that

$$1.0 \times 10^{16} \text{ GeV} < M_U < 2.3 \times 10^{16} \text{ GeV}$$

(22)

with

$$1.3 \text{ TeV} > M_R > M_Z.$$  

(23)

As an example, Fig. 1 shows the case with $M_R = 1 \text{ TeV}$ and $M_U = 1.1 \times 10^{16} \text{ GeV}$ for the values $\alpha^{-1}(M_Z) = 127.9$, $\sin^2 \theta_W(M_Z) = 0.2316$, and $\alpha_S(M_Z) = 0.112$.

In the original model, there are three copies of the 27 representations of $E_6$, i.e. three copies of the 16, 10, and 1 representations of SO(10). To arrive at the present model, two bidoublets (belonging to the 10) are removed, but three copies of $\Phi_L + \Phi_R + \Phi^c_L + \Phi^c_R$ (belonging to the $16 + 16^*$) and three more singlets are added. This assumed modification at low energies is what makes this model unifiable despite having $M_R$ at about 1 TeV. At the unification scale $M_U$, there are presumably at least six copies of 27 and three copies of 27*. The missing components are assumed to be superheavy with masses of order $M_U$.

The interactions of this model at the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ level are assumed to obey a discrete $Z_3 \times Z_2$ symmetry, under which the various superfields transform as given in Table 1. Consider first only those terms in the superpotential involving the quarks. Only three are allowed:

$$QQ^c\eta = dh^c\eta_1^0 - uh^c\eta_1^- + uu^c\eta_2^0 - du^c\eta_2^+,$$

(24)
$$Qd^c\Phi_L^{(1)} = dd^c\phi^0_L - ud^c\phi^-_L, \quad (25)$$
$$hQ^c\Phi_R^{(1)} = hh^c\phi^0_R - hu^c\phi^+_R. \quad (26)$$

This means that the exotic heavy quark $h$ gets its mass from the vacuum expectation value $\langle \phi^0_R \rangle$, the ordinary quarks $u$ and $d$ get their masses from $\langle \eta^0 \rangle$ and $\langle \phi^0_L \rangle$ respectively, and the fermionic components of the SU(2)$_L$ doublet $(\eta^0, \eta^-)$ and singlet $\phi^+_R$ can be identified as ordinary leptons, such as $(\nu, \tau^-)$ and $\tau^+$. Consequently, $h$ picks up a nonzero lepton number $L = 1$ and since $W^-_R$ converts $h^c$ to $u^c$, it also has $L = 1$. This prevents $d - h$ as well as $W_L - W_R$ mixing, and since only one scalar vacuum expectation value contributes to the mass of each quark species, the absence of tree-level flavor-changing neutral-current interactions is also assured.

Consider next the terms involving $\Phi^c_L$ and $\Phi^c_R$. There are three quadratic terms:

$$\Phi^{(4,5,6)}_L \Phi^{(1,2,3)}_L, \quad \Phi^{(4)}_R \Phi^{(1)}_R, \quad \Phi^{(5,6)}_R \Phi^{(2,3)}_R. \quad (27)$$

This means that $\Phi^{(4,5,6)}_L + \Phi^{(1,2,3)}_L + \Phi^{(4,5,6)}_R + \Phi^{(1,2,3)}_R$ can be assumed heavy with masses of order $M_R$. There are also three cubic terms:

$$\Phi^{(2,3)}_L \Phi^{(1,2,3)}_L \eta_L, \quad \Phi^{(1)}_R \Phi^{(1)}_R N_R, \quad \Phi^{(2,3)}_R \Phi^{(2,3)}_R N_R. \quad (28)$$

This means that $N_R$ should have $L = 0$ whereas $\Phi^{(2,3)}_L$ can be assigned $L = 1$, $\phi^+_R(2,3)$ and $N_L$ assigned $L = -1$. However, the quadratic terms $N_L N_L$ and $N_R N_R$ are also allowed. Hence additive lepton number is explicitly violated by the $N_L$ Majorana mass terms, but they are the sole source of this violation, as is often the case when the standard model is extended to include neutral singlet leptons.

The remaining allowed terms in the superpotential all involve the bidoublet:

$$\eta \eta^0 N_L = \eta_1^0 \eta_2^0 N_L - \eta^- \eta_2^+ N_L, \quad (29)$$
$$\Phi^{(1)}_L \Phi^{(1)}_R \eta = \phi^-_L(1) \phi^+_R(1) \eta_1^0 - \phi^+_L(1) \phi^-_R(1) \eta_1^- + \phi^0_L(1) \phi^0_R(1) \eta_2^0 - \phi^-_L(1) \phi^0_R(1) \eta_2^0, \quad (30)$$
and
\[ \Phi_{L(1,5,6)}\Phi_{R(2,3)}\eta. \]  

This means that the fermionic component of \( \eta_1^0 \) (i.e. \( \nu_\tau \)) gets a seesaw mass through its coupling to \( N_L \) via \( \langle \phi^0_0 \rangle \) and the \( N_L \) Majorana mass. Since the \( \tau \) lepton is identified as the fermionic components of \( \eta_1^- \) and \( \phi^+_R(1) \), it gets a mass via \( \langle \phi^0_L(1) \rangle \). The particle content of this model under SU(3) \( \times \) SU(2) \( \times \) U(1) \( _Y \) is given in Table 2 together with the baryon number \( B \), lepton number \( L \), and \( R \) parity of the fermions \( R_f = (-1)^{1+3B+L} \). Strictly speaking, because \( N_L \) has a Majorana mass, lepton number is conserved only multiplicatively.

The electron and muon are identified in this model as the fermionic components of \( \phi^-_{L(2,3)} \) and \( \phi^+_R(2,3) \), whereas \( \nu_e \) and \( \nu_\mu \) are equated with the fermionic components of \( \phi^0_{L(2,3)} \). The latter are coupled to \( N_L \) via \( \langle (\phi^0_L)^0 \rangle \), hence they acquire seesaw masses in the same way as \( \nu_\tau \) and all three neutrinos may mix with one another. On the other hand, \( e \) and \( \mu \) are massless at tree level. To see how they acquire radiative masses, note that the soft supersymmetry-breaking term

\[ \Phi_{L(2,3)}\Phi_{R(2,3)}\tilde{\eta} = \phi^-_{L(2,3)}\phi^+_R(2,3)\bar{\eta}_2^0 - \phi^0_{L(2,3)}\phi^+_R(2,3)\eta^-_2 + \phi^0_{L(2,3)}\phi^0_R(2,3)\bar{\eta}_1^0 - \phi^-_{L(2,3)}\phi^0_R(2,3)\eta_1^+ \]  

involving only the scalar fields is allowed by the discrete \( Z_4 \times Z_2 \) symmetry. Since \( \phi^-_{L(2,3)} \) and \( \phi^+_R(2,3) \) are now identified as the scalar supersymmetric partners of \( e \) and \( \mu \), the radiative mechanism\[24, 25\] of gaugino exchange allows \( e \) and \( \mu \) to become massive via \( \langle \eta_2^0 \rangle \). Note that there is mixing between \( \phi^-_{L(2,3)} \) and \( \eta_1^+ \) via \( \langle \phi^0_R(2,3) \rangle \), hence \( e \) and \( \mu \) also mix with \( \tau \) through radiative corrections. In addition, because of the \( \Phi_{L(4,5,6)}\Phi_{R(2,3)}\eta \) term, flavor-changing leptonic processes are possible at the TeV energy scale. Phenomenological details will be given elsewhere.

In summary, the proposed supersymmetric left-right model of this paper has the following interesting features. (1) Its particle content is chosen from six copies of the 27 and three
copies of the $27^*$ of $E_6$, consistent with the possibility that it is of superstring origin. (2) It is unifiable at around $10^{16}$ GeV even though the $\text{SU}(3) \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$ gauge symmetry is realized at the TeV energy scale with $g_L = g_R$. (3) The problem of requiring two or more scalar bidoublets for realistic quark masses in the conventional left-right model is circumvented because the $\text{SU}(2)_R$ doublet is not $(u, d)_R$, but $(u, h)_R$ where $h$ is heavy and has lepton number $L = 1$. This allows the model to be free of tree-level flavor-changing neutral currents at the electroweak energy scale. (4) The $\tau$ lepton gets a mass spontaneously as usual, but $e$ and $\mu$ acquire radiative masses through gaugino exchange. (5) All three neutrinos have small seesaw masses through their couplings to neutral gauge singlets with large Majorana masses. Additive lepton number $L$ is explicitly violated but multiplicative lepton number $(-1)^L$ is preserved. (6) The structure of this model is naturally maintained with a discrete $Z_4 \times Z_2$ symmetry which is spontaneously broken down to a generalization of $R$ parity. (7) At the TeV energy scale, there will be many unique manifestations of this model. For example, the $W_R$ vector gauge boson here has a nonzero lepton number and negative $R$ parity, hence its final decay product must contain at least a lepton as well as the LSP (lightest supersymmetric particle).

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FIGURE CAPTION

Fig. 1. Evolution of $\alpha_i^{-1}$ with $M_R = 1$ TeV and $M_U = 1.1 \times 10^{16}$ GeV.
| Superfield  | $Z_4$ | $Z_2$ |
|------------|-------|-------|
| $Q$        | 1     | +     |
| $Q^c$      | $-i$  | +     |
| $d^c$      | 1     | +     |
| $h$        | $-1$  | +     |
| $\eta$     | $i$   | +     |
| $\Phi_{L(1)}$ | 1     | +     |
| $\Phi_{L(2,3)}$ | $i$   | −     |
| $\Phi_{L(4,5,6)}$ | $-i$  | −     |
| $\Phi_{R(1)}$ | $-i$  | +     |
| $\Phi_{R(2,3)}$ | 1     | −     |
| $\Phi_{R(4)}$ | $i$   | −     |
| $\Phi_{R(5,6)}$ | $-1$  | +     |
| $\Phi^c_{L(1,2,3)}$ | $i$   | −     |
| $\Phi^c_{R(1)}$ | $-i$  | −     |
| $\Phi^c_{R(2,3)}$ | $-1$  | +     |
| $N_{L(1,2,3)}$ | $-1$  | +     |
| $N_{R(1,2,3)}$ | $-1$  | −     |

Table 1: Transformation properties of the various superfields of this model under $Z_4 \times Z_2$. 
| Superfield | SU(3) × SU(2) × U(1) | B  | L  | R_f |
|------------|------------------------|----|----|-----|
| (u, d)     | (3, 2, 1/6)            | 1/3| 0  | +   |
| d^c        | (3, 1, 1/3)            | -1/3| 0  | +   |
| u^c        | (3, 1, -2/3)           | -1/3| 0  | +   |
| h          | (3, 1, -1/3)           | 1/3| 1  | -   |
| h^c        | (3, 1, 1/3)            | -1/3| -1 | -   |
| (η_1^0, η_1^-) | (1, 2, -1/2)        | 0  | 1  | +   |
| (φ_L(2,3), φ_L(2,3))^- | (1, 2, -1/2) | 0  | 1  | +   |
| φ_R^+      | (1, 1, 1)              | 0  | -1 | +   |
| (φ_R^-)^-  | (1, 1, -1)             | 0  | 1  | +   |
| N_L        | (1, 1, 0)              | 0  | -1 | +   |
| φ_R^0, (φ_R^c)^0, N_R | (1, 1, 0)          | 0  | 0  | -   |
| Φ_L(1,4,5,6)   | (1, 2, -1/2)       | 0  | 0  | -   |
| (η_2^+, η_2^-)^0, Φ_L^c | (1, 2, 1/2)      | 0  | 0  | -   |

Table 2: Transformation properties of the various superfields of this model under SU(3) × SU(2) × U(1), B, L, and R_f.
This figure "fig1-1.png" is available in "png" format from:

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