Effects of the velocity on the reversible-irreversible transition in a periodically sheared vortex system

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Abstract. A reversible-irreversible transition (RIT) is studied using a periodically-driven vortex system in an amorphous film with random pinning that causes local shear, as a function of shear amplitude \(d\). The relaxation time to reach the steady state exhibits a power-law divergence at a threshold value \(d_c\) with critical exponents in agreement with the values predicted for an absorbing phase transition in the two-dimensional (2D) directed-percolation (DP) universality class. In our previous work, the experiment was conducted at relatively high frequency \(f\), giving rise to a large mean vortex velocity \(v\). Here we use lower \(f\) to study the effects of reduced \(v\) and increased dynamic pinning on the RIT. The results show that the critical behavior of RIT stays essentially unchanged, while we find a trend for \(d_c\) to increase with decreasing \(v\). We will propose a possible model to qualitatively explain this unexpected result.

1. Introduction
When many-particle assemblies with disordered configuration are subjected to a periodic shear with a small amplitude \(d\), the particles gradually self-organize to avoid next collisions and transform into an organized configuration. For small \(d\), the particles settle into a reversible state where all the particles return to their initial position after each shear cycle, while they reach an irreversible state for \(d\) above a threshold \(d_c\) [1, 2, 3, 4]. We have studied a reversible-irreversible transition (RIT) using periodically driven vortices in amorphous (\(\alpha\))-Mo\(_x\)Ge\(_{1-x}\) films with random pinning, as a function of \(d\) [5, 6, 7, 8]. The relaxation time \(\tau\) to reach the reversible or irreversible state shows a power-law divergence at \(d_c\). The critical exponents nearly coincide with the values expected for an absorbing phase transition in the two-dimensional (2D) directed-percolation (DP) universality class [9, 10].

In our previous work, experiments have been conducted at relatively high frequency \(f\), giving rise to the large mean velocity of the ac driven vortices [8]. When we use lower \(f\) for a given \(d\) and the vortex velocity is decreased, the pinning force that the moving vortices feel will become more effective and this would increase the local shear and hinder the observation of the reversible behavior. Contrary to the expectation, however, we find a trend for \(d_c\), namely, the reversible regime to increase at small \(v\), while the critical behavior of RIT stays essentially unchanged. We will propose a possible model to qualitatively explain the somewhat counterintuitive result.

2. Experimental
We prepared the \(\alpha\)-Mo\(_x\)Ge\(_{1-x}\) film with thickness of 330 nm by rf sputtering on a silicon substrate mounted on a water-cooled rotating copper stage [5, 6, 11, 12]. The zero-field(\(B = 0\))
superconducting transition temperature where the resistivity decreases to zero is 6.3 K. The ohmic resistivity was measured using a standard four-probe method. The film was directly immersed into the liquid $^4$He. By applying the current in the vortex state, the vortices are driven along the sample width of 0.3 mm. The voltage contacts spaced at intervals of $l = 1.2$ mm were used to measure the voltage $V$ generated by vortex motion. We measured the time-evolution of the voltage $V(t)$ just after applying the ac current $I_{ac}$ of square waveform by means of an oscilloscope [5, 8]. The frequency $f$ of the ac current $I_{ac}$ was fixed to be either 60 or 450 kHz, while the amplitude of $I_{ac}$ was adjusted to generate the steady-state voltage $V^\infty(\equiv |V(t \to \infty)|)$, corresponding to the mean vortex velocity $v = V^\infty/(IB)$ that yields the shear amplitude $d = v/(2f) = V^\infty/(2fB)$ with desired values. Thus, we were able to study the RIT both in the small and large $v$ regimes, using $f = 60$ and 450 kHz, respectively. Specifically, the velocities $v$ studied in this work span the range from 6 to 15 mm/s and from 25 to 86 mm/s for $f = 60$ and 450 kHz, respectively.

3. Results and discussion

Since the experimental results taken at 450 kHz, which provide the first convincing evidence of RIT caused by local shear, have been reported in a recent paper [8], here we mainly show the data measured at 60 kHz, focusing on the effects of the reduced velocity $v$ on RIT. It is already known that in our weak-pinning $\alpha$-Mo$_2$Ge$_{1-x}$ films a vortex-solid state in low and intermediate fields $B$ is a weakly disordered vortex lattice or Bragg glass, while the structural transition from the Bragg glass to vortex glass takes place in a high-field region below the upper critical field associated with the peak effect [13, 14, 15, 16, 17, 18, 19, 20]. All the data in this work were acquired at 4.1 K in 3.5 T, corresponding to the peak-effect regime, where the pinning is effective. From the field value of 3.5 T, the mean intervortex spacing $a_0$ is estimated to be 26 nm.

To realize random organization by ac drive $I_{ac}$ [21, 22, 23], first, we prepared disordered initial vortex configuration in which many vortices are pinned by random pinning sites. This has been attained by driving the vortices with a small ac current of 1.2 kHz yielding $V^\infty = 20 \mu V$ for a long time to reach the steady state. This is based on the knowledge that 20 $\mu V$ corresponds to a disordered plastic-flow state dominated by pinning where dc current-voltage characteristics exhibit strong nonlinearity [5, 7, 11, 12, 13]. The disordered initial vortex configuration thus prepared is then subjected to the ac current $I_{ac}$ of fixed $f = 60$ kHz with various amplitudes $I_{ac}$. Figures 1(a) and (b) representatively show the voltage responses $V(t)/V^\infty$ of the system to the 60 kHz ac drive yielding $d = 49.0$, 50.1, and 57.1 nm from top to bottom and $d = 61.4$, 71.5, and 87.8 nm from bottom to top, respectively. For clarity, vertical lines of the individual voltage pulses are removed and only the amplitude of the pulse, $|V(t)|$, is shown. Furthermore, a large voltage region, $|V(t)|/V^\infty > 0.75$, is enlarged to easily see the difference in the relaxation time $\tau$.

It is commonly observed that $|V(t)|/V^\infty$ grows monotonically and relaxes toward a steady-state value. This result is explained as follows: An initial vortex flow is a disordered flow dominated by pinning and collisions between the vortices, where the vortices cannot move easily. By repeating the collisions, the distribution of the vortices becomes rearranged to avoid next collisions, and in the final steady state the vortices can move more easily than in the initial state [7, 24]. It is seen from figure 1(a) that the relaxation is longer for larger $d$, while in figure 1(b) it is longer for smaller $d$, indicating the occurrence of a peak in the relaxation time $\tau(d)$ at around $d = 57-61$ nm.

To determine the relaxation time $\tau$ precisely, the amplitude of the transient voltage, $|V(t)|$, is fitted using the following relaxation function presented in [3, 25]:

$$|V(t)| = V^\infty - (V^\infty - V^0)\exp(-t/\tau)/t^\alpha.$$  

(1)
Here, $V^0$ and $V^\infty$ are the initial and final voltage amplitudes, respectively, and $\tau$ is the characteristic time at which the relaxation crosses over from a power-law decay with an exponent $a$ to an exponential decay. The best fit is obtained with $a = 0.45 \pm 0.05$. We replotted all the data (not presented here) shown in Figures 1(a) and (b) as $\log \left( \frac{V^\infty - |V(t)|}{V^\infty - V^0} \right)$ versus $t$ on a double logarithmic scale. It is found that as $d$ approaches closer to 57-61 nm, the replotted data of log falls on a straight line over a wide range with a slope of $-a = -0.45$, as shown in figures 1(c) and (d) of Ref.[8] where the data at $f = 450$ kHz are presented. The value of $a = 0.45 \pm 0.05$ thus obtained is consistent with the theory for the DP universality class in 2D [9, 10], which predicts that the fraction of colliding particles exhibits a power-law $t$ dependence with an exponent $a \approx 0.45$.

Shown with the full lines in figures 1(a) and (b) are the results of the fits of $|V(t)|/V^\infty$ to Eq. (1). In figure 2, the values of $\tau$ obtained from the fits are plotted against $d$ with blue filled squares and blue open circles for $d \leq 57$ nm and $d \geq 61$ nm, respectively. They show a power-law divergence at 57.9$\pm$0.3 nm ($\equiv d_c$) from both sides, as marked with a vertical dashed line. Also plotted with dark red filled squares and dark red open circles in figure 2 are the $\tau$ values for $d \leq 45$ nm and $d \geq 47$ nm, respectively, taken at $f = 450$ kHz, which are collected from the data in figure 2(a) of Ref.[8]. They again exhibit a power-law divergence from both sides. Note, however, that the threshold shear amplitude at which $\tau$ diverges is $d_c = 45.2 \pm 0.2$ nm, as indicated with a vertical dotted line, which is clearly smaller than $d_c = 57.9 \pm 0.3$ nm for $f = 60$ kHz. The threshold velocities $v_c$ at RIT ($d = d_c$) are estimated to be $v_c = 41$ and 7 mm/s for $f = 450$ and 60 kHz, respectively.

In the inset of figure 2 we plot all the data of $\tau$ against $|d - d_c|/d_c$ on a double logarithmic
Figure 2. Dark red filled squares and open circles denote $\tau$ for $d \leq 45$ nm and $d \geq 47$ nm, respectively, measured at 450 kHz. They are collected from the data in figure 2(a) of Ref.[8]. Blue filled squares and open circles represent $\tau$ for $d \leq 57$ nm and $d \geq 61$ nm, respectively, taken at 60 kHz. Vertical dashed and dotted lines mark the location of $d_c$ at which a power-law divergence of $\tau(d)$ occurs at 60 and 450 kHz, respectively. Inset: $\tau$ plotted against $|d-d_c|/d_c$ on a double logarithmic scale, where symbols are the same as in the main panel. Both the blue and dark red lines in the main panel and inset indicate the power-law fits by $\tau \propto |d-d_c|^{-\nu}$ with $\nu = 1.33 \pm 0.07$ and $1.38 \pm 0.08$, respectively. These values of $\nu$ are, within error bars, again in agreement with the theoretical value $\nu = 1.295 \pm 0.006$ expected for the absorbing phase transition in the DP universality class in 2D [9, 10]. Thus, the present results with respect to the critical exponents of $\alpha$ and $\nu$ indicate that the critical behavior of RIT is independent of the velocity $v$ or the pinning strength in the $v$ range studied.

In contrast, we observe a clear increase in $d_c$ from 45 to 58 nm by decreasing the vortex velocity $v_c$ from 41 to 7 mm/s. The increase in $d_c$ corresponds to an increase in the threshold vortex number $n_c$ at RIT, as defined as $n_c = d_c/a_0$, from 1.7 to 2.2, where $a_0$ is the mean intervortex spacing. The results are somewhat contrary to the following physical intuition: The dynamical pinning effect the moving vortices feel is stronger for smaller $v$, which will strengthen local shear. This leads to the suppression of the reversible phase and that of the threshold values of $d_c$ and $n_c$.

We propose a possible model to qualitatively explain the unexpected finding. In figures 3 (a) and (b) we schematically illustrate the vortex motion during a cycle of ac drive with large and small $v$, respectively. Colored dashed and solid arrows represent the trajectories of vortex motion during the former and latter halves of the ac cycle, respectively. These illustrations are drawn considering the following facts: The local shear exerted to moving vortices originates from two forces with opposite signs: a repulsive force from pinned vortices and an attractive force scale, namely, log $\tau$ versus log $(|d-d_c|/d_c)$, where symbols are the same as in the main panel. Both the blue and dark red lines in the main panel and inset indicate the power-law fits by $\tau \propto |d-d_c|^{-\nu}$ with $\nu = 1.33 \pm 0.07$ and $1.38 \pm 0.08$, respectively. These values of $\nu$ are, within error bars, again in agreement with the theoretical value $\nu = 1.295 \pm 0.006$ expected for the absorbing phase transition in the DP universality class in 2D [9, 10]. Thus, the present results with respect to the critical exponents of $\alpha$ and $\nu$ indicate that the critical behavior of RIT is independent of the velocity $v$ or the pinning strength in the $v$ range studied.

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Figure 3. (a) Schematic illustration of the vortex motion during a cycle of ac drive yielding a shear amplitude $d$ with large and (b) small velocities. Circles and crosses indicate the vortices and random pinning centers, respectively. Colored dashed and solid arrows denote the trajectories of vortex motion during the former and latter halves of the ac cycle, respectively.

from pinning centers. When the number density of the pinned vortices is large as a result of small $v$, as shown in figure 3(b), they will act as a guide rail for the driven vortices that prevents the transverse motion. This can assist individual vortices to return to their initial position after each ac cycle, thus facilitating a reversible flow. This will qualitatively account for the observed increase in $d_c$ at small $v$. We consider that this second mechanism proposed here in figure 3(b), which assumes the guided vortex motion by pinned vortices, competes with the first, simpler and more intuitive, mechanism that the enhanced local shear due to the increased number of the pinned vortices causes the suppression of the reversible phase. In the $v$ range studied the second mechanism overcomes the first one.

From the experiment on RIT in different $v$ regimes, we find that the threshold velocity at RIT for $f = 60$ kHz, $v_c = 7$ mm/s, is smaller than $v_c = 41$ mm/s for $f = 450$ kHz, indicating that reversibility is lost at smaller $v$ in the periodically sheared vortices in the smaller $v$ region with lower $I_{ac}$ and $f$. When we interpret the RIT in terms of $v_c$, this result seems to be intuitively explained by the increased dynamic pinning effect in the small $v$ region. However, RIT has been understood in terms of the shear amplitude $d$ both theoretically and experimentally, and it is not evident how RIT is described by the shear velocity rather than the spatial shear amplitude. This is an interesting issue that should be explored in future work.

4. Conclusions
Using a vortex system of the $\alpha$-Mo$_4$Ge$_{1-x}$ film with weak random pinning, we study the RIT at lower frequency $f$ than before [8] to clarify the effects of the reduced velocity $v$ and the increased dynamic pinning on RIT. We observe the critical behavior of RIT, which is essentially the same as that found at higher $f$ [8], while we find a trend for the reversible regime to increase at smaller $v$. This is contrary to the simple expectation that the increased effective pinning would suppress the reversible behavior and $d_c$ through the enhanced local shear. To qualitatively explain the somewhat counterintuitive finding, we propose a possible model that the pinned vortices act as
a guide preventing the transverse vortex motion.

Acknowledgments
This work was supported by a Grant-in-Aid for Scientific Research B (KAKENHI Grant No. 17H02919), Scientific Research on Innovative Areas (KAKENHI Grant No. 20H05266), and JSPS Fellows (KAKENHI Grant No. 20J21425) from the Japan Society for the Promotion of Science.

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