AZIMUTHAL CORRELATION IN DIS

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We introduce a new angular correlation in DIS process and study its differential distribution in the region in which the observable is small. We perform a perturbative resummation at single logarithmic accuracy and estimate leading non-perturbative power corrections.

1 Introduction

Energy-energy correlation was one of the first collinear and infrared safe (CIS) observables studied in QCD. It involves the polar angle between two produced hadrons in $e^+e^-$ annihilation.

An analogous observable in a process with incoming hadrons should involve an azimuthal angle. In DIS we then define the azimuthal correlation

$$H(\chi) = \sum_{a,b} \frac{p_{ta}p_{tb}}{Q^2} \delta(\chi - \chi_{ab}), \quad \chi_{ab} = \pi - |\phi_{ab}|,$$

being $\phi_{ab}$ the angle between $\vec{p}_{ta}$ and $\vec{p}_{tb}$, the transverse momenta of produced hadrons $a$ and $b$ with respect to the photon axis in the Breit frame. The differential distribution in $H(\chi)$ takes its first non-zero contribution at order $\alpha_s^2$, so that its study is better performed in DIS events with two high $p_T$ jets. Such events can be selected for instance by constraining the two-jet resolution variable $y_2$.

2 The observable at parton level

Since the observable is linear in outgoing particle momenta, we can replace the sum over hadrons in eq. with a sum over partons. The Born process is $q P_1 \rightarrow P_2 P_3$, with $q$ the virtual boson, $P_1$ the struck parton inside the proton, $P_2$ and $P_3$ two outgoing hard partons. Since $P_2$ and $P_3$ are in the same plane, we have $P_{t2} = P_{t3} = P_t$ and $\chi_{23} = 0$.

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Beyond Born level the QCD process is \( q p_1 \to p_2 p_3 k_1 \ldots k_n \), with \( k_i \) secondary partons. Outgoing partons \( p_a \ (a = 2, 3) \) are displaced from \( P_a \) by soft recoil components, so that \( \chi_{23} \) no longer vanishes. Furthermore, one has to consider also the correlation of secondary partons \( k_i \) with \( p_2 \) or \( p_3 \).

For small \( \chi \) we can consider \( k_i \) soft and, to first non-trivial order, \( H(\chi) \) can be split into a hard and a soft piece:

\[
H_h(\chi) = \frac{2P_i^2}{Q^2} \delta(\chi - |\phi_x|), \quad \phi_x = \frac{\sum_i k_i^{\text{out}}}{P_t},
\]

\[
H_s(\chi) = \frac{2P_i^2}{Q^2} \sum_i k_i \left( \delta(\chi - |\phi_i - \phi_x|) - |\cos \bar{\phi}_i| \delta(\chi - |\phi_x|) \right).
\]

The hard piece \( H_h(\chi) \) depends on the total out-of-plane recoil of \( p_2 \) or \( p_3 \), which is proportional to the sum of \( k_i^{\text{out}} \), the out-of-event-plane momenta of emitted particles. The soft piece \( H_s(\chi) \) represents the correlation of \( k_i \) with \( p_2 \) and \( p_3 \). The angle \( \phi_i \) is \( \phi_{22} \) (\( \phi_{33} \)) for \( k_i \) near \( p_2 \) (\( p_3 \)). The subtraction term comes from the in-plane recoil of \( p_2 \) and \( p_3 \) and ensures that the observable is CIS.

3 Perturbative resummation

The first order PT contribution to the differential \( H(\chi) \) distribution \( d\Sigma/d\chi \) is given by:

\[
\chi \frac{d\Sigma(\chi)}{d\chi} = \frac{2\alpha_s C_T}{\pi} \ln \frac{1}{\chi} + \ldots, \quad C_T = 2C_F + C_A.
\]

The presence of such a logarithm is due to an incomplete real-virtual cancellation. Moreover, terms of the form \( \alpha_s^m \ln^n \chi \) arise at any order in PT theory. For small \( \chi \) they become large and need to be resummed to give meaning to the perturbative expansion.

Resummation of logarithmic enhanced terms can be achieved by introducing the impact parameter \( b \), the Fourier variable conjugate to \( \phi_x \). We aim at single logarithmic (SL) accuracy, i.e. having under control all double (\( \alpha_s^2 \ln^{n+1} \chi \)) and single (\( \alpha_s^n \ln^n \chi \)) logarithms in \( \ln \Sigma \). We find that only \( H_h(\chi) \) contributes to SL level, while the effect of \( H_s(\chi) \) is subleading.

The PT answer can then be written as a convolution of the Born matrix element \( M_0^2 \) with a function which resums all double and single logs:

\[
\frac{d\Sigma^{\text{PT}}(\chi)}{d\chi} \sim M_0^2 \otimes \frac{2}{\pi} \int_0^\infty P_t db \cos(bP_t \chi) P(b^{-1}) e^{-R(b)}.
\]

In the above expression, due to coherence of QCD radiation, (virtual) gluons with frequencies below \( b^{-1} \) reconstruct the proper hard scale for the parton density \( P \) of the incoming parton. Gluons with frequencies above \( b^{-1} \) build up the Sudakov exponent \( R(b) \), the 'so-called' radiator. This function, the same occurring in three-jet event shapes, depends on the colour charges of the three hard emitters and on the geometry of the hard underlying event.

The behaviour of \( d\Sigma/d\chi \) near \( \chi = 0 \) can be understood by considering the physical effects which can keep the angle \( \chi \) small. One mechanism is radiation suppression. This is the only one relevant in most event shapes and gives rise to a Sudakov form factor with a characteristic peak. However, since our observable measures radiation only through hard parton recoil, it may happen that \( \chi \) is kept small by successive cancellation of larger out-of-plane momenta. It is this effect which prevails at small \( \chi \), so that \( d\Sigma/d\chi \), unlike event shapes, has no Sudakov peak, but rather approaches a constant for \( \chi \to 0 \).

\( \dagger \)The event plane can be identified for instance by the Breit axis and the thrust-major axis.
4 Non-perturbative power corrections

The soft term $H_s$, although subleading at PT level, gives rise to the leading NP power corrections:

$$\frac{d\Sigma^{NP}(\chi)}{d\chi} \sim M_0^2 \otimes \frac{2}{\pi} \int_0^\infty P_t db \cos(bP_t\chi)P_t(b^{-1})e^{-R(b)}B(b),$$

(6)

Extracting the term of $B(b)$ linear in $b$ we find:

$$B(b) = -b(C_2 + C_3) \int_0^{Q^2} \frac{d\kappa^2}{\kappa^2} \frac{\alpha_s(\kappa)}{\pi}.$$  

(7)

In this expression $C_2$ and $C_3$ are the colour charges of the two outgoing partons $P_2$ and $P_3$, and $\kappa$ is the (invariant) transverse momentum of the emitted gluon with respect to each outgoing parton.

Equation 7 involves the integral of the running coupling in the infrared. Giving sense to that integral requires a genuine NP input which can be provided for instance by the dispersive approach. This includes the following steps:

- extension of the coupling in the infrared via a dispersion relation;
- promoting the gluon to be massive: this allows the gluon to decay inclusively;
- taking into account non-inclusiveness of the observable by multiplying the result by the Milan factor $M$.

The final result thus becomes:

$$B(b) = -b(C_2 + C_3)\lambda^{NP}, \quad \lambda^{NP} = M^4 \frac{\mu_I}{\pi^2} (\alpha_0(\mu_I) + O(\alpha_s)).$$  

(8)

The quantity $\alpha_0$ is the average of the dispersive coupling below the (arbitrary) infrared scale $\mu_I$. This NP parameter has been measured through the analysis of mean values and distributions of two-jet event shapes both in $e^+e^-$ annihilation and DIS.11 The NP contribution to the azimuthal correlation distribution is then:

$$\frac{d\Sigma^{NP}(\chi)}{d\chi} = -\lambda^{NP}(C_2 + C_3)\langle b \rangle, \quad \langle b \rangle_{\chi=0} \sim \frac{1}{\Lambda_{QCD}} \left( \frac{\Lambda_{QCD}}{Q} \right)^\gamma.$$  

(9)

Here $b$ is averaged over the PT distribution in eq. 8. From eq. 8 we see that power corrections scale like a non integer power of $1/Q$ ($\gamma \simeq 0.62$).

5 Results and conclusions

In figure 1a we show the behaviour of azimuthal correlation at HERA energies for $Q^2 = 900 \text{ GeV}^2$, $x_B = 0.1$ and $1.0 < y_2 < 2.5$. We see that the distribution goes to constant for $\chi \ll 1$. The effect of power corrections is to further deplete the distribution by an amount proportional to $\langle b \rangle$. In figure 1b we report also the $\pi^0\pi^0$ azimuthal correlation taken from E706 data. We observe that also in this case the distribution flattens to a constant value thus suggesting the mechanism of cancellation of out-of-plane momenta discussed in section 3. In conclusion, we have now a new observable that can be used not only to provide a further measurement of $\alpha_s$ and to constrain the parton densities, but also to investigate the nature of hadronisation effects in hard QCD processes.
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