Abstract

We discuss some social contagion processes to describe the formation and spread of radical opinions in the spirit of previous work by Serge Galam. The dynamics of opinion spread involves local threshold processes as well as mean field effects. We calculate and observe phase transitions in the dynamical variables resulting in a rapidly increasing number of passive supporters. This strongly indicates that military solutions are inappropriate.

Passive Supporters of Terrorism and Phase Transitions

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1 Introduction

In this article we discuss some social contagion processes which may play an important role in the dynamics of radical opinion formation. The prime applications in mind are conflict situations as they are met at the time of writing in Afghanistan, Iraq or Palestine where a highly armed alliance of foreign troops fights against a part of the local population which has been radicalized in a way such that western social classification dubs them terrorists. In the following we will use the word "terrorist" solely to refer to a certain subpopulation (whose interaction features will be described below) in our model environment and do not intend to enter the difficult debate of what constitutes the social essence of terrorism. In terms of our model interaction and shortly speaking one could say that terrorists are those individuals on which counter terrorist throw bombs on.

Besides the radical terrorist groups there is a much larger grey-area of supporters of these terrorists. Support has many faces, ranging from just keeping still about what one knows about terrorists locations or movements up to supporting terrorists by providing various forms of logistic infrastructure. Again, we will avoid specifying exactly what is meant with this notion but use it to describe the potential, predecessor states of an opinion state from which radical groups may (by whatever means and tools) recruit new members.

The notion of passive supporters was for the first time used in the context of mathematical modeling by Serge Galam in [2],[3],[4]. He studied the role of passive supporters in a lattice percolation type model and related terrorism power to the spontaneous formation of random backbones of people who are sympathetic to terrorism but without being directly involved in it. His focus was on the correlation between the number of

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passive supporters, the physical mobility of active terrorists and phase transitions associated with it. In a certain sense the present work is a continuation of Galam's work on the subject. We arrive at similar conclusions as Galam did, namely that for these types of conflicts military solutions are inappropriate.

In this paper we discuss some aspects of the dynamics of passive support of terrorist activities in virtual social networks. Our main interest lies in the study of phase transitions in the number of passive supporters induced by what is euphemistically called collateral damage as is common as a consequence of counter terrorist attacks on terrorists moving around in populated places. Phase transitions in the opinion of large parts of a population are particularly important since they violate the classical "linear" action-reaction view common among military leaders and politicians.

We are not concerned with real terrorist networks and their dynamics, for a recent work on this topic see [1] and the references therein. Our model is based on one paradigm. Counter terrorist strikes lead to collateral damage. In many cases terrorists live or hide among civilians, and civilian casualties in turn are likely to cause an increase in the number of passive supporters and increase the willingness of civilians to become members of radical groups.

Cause and effect are not linearly coupled, there is a phase transition instead. It was widely believed among western observers that in Afghanistan a large part of the population was supporting the Allied forces at the beginning of the operation, in spite of many casualties caused by bombings and other military strikes. Then a change in the public opinion seems to have occurred, the atmosphere inclined to the disadvantage of the allied forces and a transition from Allied-friendly to Taliban-friendly took place, causing a boost in the number of passive supporters.

Passive supporters usually do not reveal their nature towards outsiders and such phase transitions are hardly observable by Allied forces. The number of passive supporters can only be measured indirectly via the degree of cooperation of the civil population. If the fraction of passive supporters in a population becomes large it is likely that counter terrorists face increased difficulties to gain help and support from civilians. Further the recruiting pool for radical organizations can become nearly inexhaustible, as it has been popularized in News reports about the Gaza Strip. Such a situation may result in an absurd and tragic solution to secure public safety and to avoid a downward spiral of violence, like the segregation of people from each other with a fence.

2 Description of the Model

In the following we will specify the structure of our model which is inspired by generalized epidemic processes (for more information about this class of processes and some other applications see [5]). Let $G = (V, E)$ be a graph on a finite set of vertices $V$, representing our model population, and a set of edges $E$ encoding the social contacts relevant for transmissions. We start with the description of the state space of the vertices. To keep things simple we distinguish only four states $S = \{0, 1, 2, 3\}$. State 0 encodes the members of the susceptible population which are more or less neutral

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3 at least in the local family and friendship network of a civilian victim this is highly plausible
in their opinion about terrorists. State 1 individuals correspond to the passive supporters of terrorism and state 2 encodes active terrorists. State 3 vertices correspond to vertices, which are isolated from the network and do not interact anymore with other vertices. Although it would be natural, to consider more refined states we can restrict the discussion of phase transitions to this rather simple setting by arguing that the type of phase transition described here is robust to many refinements of the model. There is usually an immune subpopulation which is resistant to any radical opinion influence. Including this group will reduce the size of the susceptible population but does not influence the existence of phase transitions. Also there usually exist dynamics within the terrorist groups in complex hierarchical networks which in turn may have a strong influence on terrorists activities (see [1]). These intrinsic network dynamics within the relatively small subpopulation of terrorists have only marginal influence on the $0 - 1$ transition process (the process of becoming a passive supporter via a social contagion process).

The dynamics will be defined via two transition probabilities $P_{loc}$ and $P_{mean}$, where $P_{loc}$ depends on the state distribution in the neighborhood of a vertex and $P_{mean}$ corresponds to a mean field term depending on the overall prevalence of the state variables at a given time $t$. Although mesoscopic dependencies are in principle possible, we think they are of secondary importance and do not change the dynamical picture in general. With $\chi_i(t)$ denoting the state variable of individual $i$ at time $t$, for the probability of switching the state from $a$ to $b$ we formally have:

$$P\{\chi_i(t+1) = b \mid \chi_i(t) = a\} = P_{loc}(\chi_{B_i}(t), b) \oplus P_{mean}(s(t), a, b) ; a, b \in \{0, 1, 2, 3\}$$
where the sum sign stands for independent superposition\(^4\) of the local and global infection events. The subscript \(B_1(i)\) denotes the neighborhood of \(i\) and \(s(t)\) the total number (or density) of vertices not in a susceptible or immune state at time \(t\).

The local dynamics we use here is inspired by generalized epidemic processes \([5]\). In classical epidemics, the probability of infection is an independent superposition of the single infection events caused by each infected neighbor (for small infection probabilities this means that the probability to get infected is proportional to the number of infected neighbors). In generalized epidemic processes to every vertex \(i\) is assigned a threshold \(\Delta_i\) of infected neighbors, at which the probability of infection suddenly jumps to a high probability. More precisely, let \(N_i(t)\) denote the number of neighbors of a susceptible individual \(i\) at time \(t\) which support or accomplish terrorism. Hence \(N_i(t)\) is the sum of state 1 and state 2 neighbors of \(i\) at time \(t\). Then, the probability that individual \(i\) will become a passive supporter at time \(t + 1\) is given by:

\[
P_{loc}\{\chi_i(t + 1) = 1 \mid \chi_i(t) = 0\} = \begin{cases} \epsilon N_i(t), & N_i(t) < \Delta_i \\ \alpha, & N_i(t) \geq \Delta_i \end{cases}
\]

This type of local dynamics applies with appropriate modifications to many social processes like the spread of rumors, prejudices, knowledge or beliefs. So, whenever one vertex has contact with too many (here \(\Delta\) or more) passive supporters or terrorists, they will sooner or later become passive supporters of the terrorists, either because they get convinced by the ideology or because they get exploited, unintentionally or even unnoticed by themselves. Passive supporters as well as terrorists themselves may become uncooperative or useless, so they discontinue support. We assume therefore that for every vertex there is a chance of switching back from 1 or 2 to 0 with a given probability \(\gamma\).

Also there is a fixed rate \(\kappa\) at which terrorists recruit from passive supporters. Due to the action of external forces like the state or allied troops, or by death or by migration, terrorists can be neutralized and removed from the network with probability \(\rho\). Emotionally involved relatives and friends of victims, who where in state 0 until then, will potentially decide to join the terrorists side to take revenge. Civilians watching the News may be upset and enraged when they observe counter terrorists taking out their fellow citizens and destroying their buildings to capture or to eliminate terrorists.

We assume a fixed rate \(\beta\) at which a neutralized terrorist induces new passive supporters via the previously described social processes. Thus, for a susceptible vertex in the network, the probability of becoming a passive supporter depends (besides the above describe local infection way) on the number of neutralized terrorist per time step and therefore on the number of active terrorists in the network. In this way, \(\beta\) constitutes the mean field dynamics of the infection process. If \(C(t)\) denotes the number of captured terrorists at time \(t\) we have formally for the probability that a state 0 vertex changes its state into 1 in the next time step:

\[
P_{mean}\{\chi_i(t + 1) = 1 \mid \chi_i(t) = 0\} = 1 - (1 - \beta)^{C(t)}
\]

\(~ \beta C(t) \) for small \(\beta\). \(^2\)

\(^4\)Independent superposition of two probabilities \(A\) and \(B\) is just \(A + B - AB\)
Note that if one ignores mass media effects the total number of new mean field induced passive supporters should stay bounded irrespective of the size of the population (reflecting the fact that it is at most an effect on mesoscopic scale). We therefore scale the $\beta$ value with the population size appropriately.

Table 1 summarizes all parameters and their interpretations.

| parameter | transition | affiliation                        |
|-----------|------------|------------------------------------|
| $\epsilon$ | 0 $\rightarrow$ 1 | local process below the threshold                                        |
| $\alpha$  | 0 $\rightarrow$ 1 | local process above the threshold                                           |
| $\beta$   | 0 $\rightarrow$ 1 | mean field                                                                    |
| $\gamma$  | 1, 2 $\rightarrow$ 0 | changing mind                                                                 |
| $\kappa$  | 1 $\rightarrow$ 2 | procreation rate of new terrorists                                          |
| $\rho$    | /          | neutralization rate of terrorists                                            |

### 3 Phase Transitions in the Local Infection Process

The usage of local threshold dynamics yields additional phase transitions that are different from those in classical epidemics, which are phase transitions in the parameters. Here, in addition to these phase transitions in the parameters there are such in the dynamical variables like the number of passive supporters. In the following we state some analytic results for these phase transitions for the case of the classical Erdős-Rényi random graph $G(n, p)$, the graph with $n$ vertices and independent edge probability $p$ between each pair of vertices.

To investigate the average infection density $b_t$ on a $G(n, p)$ with $p = c/n$ we assume for simplicity $\epsilon = 0, \alpha = 1$ and that the thresholds $\Delta_i$ are distributed uniformly, i.e. $\Delta_i = \Delta$ for all vertices $i$. It is easy to see that the case of very small $\epsilon$ and rather large $\alpha$ gives similar results although the formulas become more complicated. For a detailed mathematical treatment see [5]. There is a recursion for $b_t$, namely

$$b_{t+1} = 1 - (1 - b_0) e^{-\epsilon \alpha} \sum_{k=0}^{\Delta-1} \frac{(\epsilon \alpha)^k}{k!}.$$  

Thus, for $\Delta = 2$ we get for the fixed point equation

$$b^* = 1 - (1 - b_0) e^{-\epsilon \alpha} (1 + \epsilon \alpha).$$  

Note that there can be several solutions one has to take the dynamically stable one (closest to $b_0$). A closer examination shows that, depending on the value of $b_0$ and $c$, there are either 3 fixed points or just one in the domain $[0, 1]$. Since the iteration mapping has no critical points in this interval the smallest of these fixed point is also the attractor for the orbit starting with $b_0$. Three examples are shown below. The phase transition happens when the first two fixed points (in case there are three) join together and form an indifferent (slope one) fixed point, see fig. 3. That means that for $b_0 < b_0^*$,
i.e. if the initial prevalence is smaller than the critical value, the final infection density $b^*$ will be not much larger than $b_0$. If $b_0 > b_0^c$ then $b^*$ will be close to the number of vertices in the $k$-core of the graph. The phase transitions are closely connected with the $k$-cores of a network. If $k \geq \Delta$ then every vertex in the $k$-core succeeds the threshold and gets infected sooner or later.

The critical density $b_0^c$, depending on the edge density $c$ of the graph, is given by

$$b_0^c = 1 - \frac{2 \exp\left(-\frac{1}{2}(1 - c + \sqrt{c^2 - 2c - 3})\right)}{c(-1 + c - \sqrt{c^2 - 2c - 3})}$$

Figure 2 shows the function of the right hand side of eq. 3.

The mean field dynamics described in section 2 induces now an effective slow increase of the initial density for the local spreading process until the critical density for the local dynamics is reached. Above the phase transition there remains then a nearly inexhaustible pool for recruiting new terrorists and any attempt to resolve the situation by military means has to fail. We close this section with a short calculation for the pure mean field dynamics. Let $\kappa$ be the probability that a passive supporter becomes a terrorist (for the simulations this is taken to be 0.1 percent). Assume that $n$ is large and $\beta n = B$. Then the branching process approximation to the pure mean field process

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3 A subgraph $G'$ of a graph $G$ is called $k$-core of $G$ if every vertex in $G'$ has at least $k$ neighbors.
(which is valid for small numbers of initially infected) becomes overcritical if

\[ \frac{\kappa \rho B}{(\gamma + \kappa(1 - \gamma))(\gamma + \rho(1 - \gamma))} > 1. \]  

(4)

Note that in case the critical threshold for the local infection process is very small even in the subcritical case the mean field dynamics can due to stochastic fluctuations trigger the dynamics above to the phase transition.

4 Simulation Results

The simulations are run on an Erdős-Rényi random graph \( G(n, p) \) in discrete time and synchronous updates. Throughout all simulations the parameters of the local infection process are chosen to be \( \alpha = 1, \epsilon = 0 \). Note that the difference to a process with transmission probability \( \alpha = 0.5 \) would be just a small time delay. The remaining parameters are chosen to be plausible if one interprets the time step to be one week with the situation in Afghanistan in mind.

On average, one percent of the passive supporters change back to being susceptible, \( \gamma = 0.01 \). The probability of becoming an active terrorist is zero for all susceptible vertices and 0.1 percent for all passive supporters. Thus, on average 1000 passive supporters generate one single terrorist.

On each picture the abscissa is time, where we - as was mentioned above - interpret one time step \( \Delta t = 1 \) to be one week. There are other associations possible of course, but in the context of social networks in regions, which are struck by terror, associating one time step with one week, giving people the opportunity to act and communicate, seems to be a natural choice.

The y-axis is the density of vertices in state 1 or 2, thus having values running from 0 to 1. There are three graphs shown, the dark blue one being the number of active terrorists, the dark green one being the number of passive supporters and the turquoise one being the sum of these two.

Figure 3 shows the case of an Erdős-Rényi random graph \( G(n, p) \), \( p = c/n \) with \( n = 200\,000 \) and an average degree of \( c = 4 \). The threshold for the local dynamics is set to \( \Delta = 2 \), i.e. as soon as one vertex has two passive supporters in its neighborhood it turns to a passive supporter as well. The initial density of passive supporters is set to \( b_0 = 0.04 \), i.e. there are 8000 passive supporters from the start. The mean field parameter \( \beta \), which is the rate at which one captured active terrorist generates passive supporters, is set to \( \beta = 0.00001 \), i.e. one captured active terrorist generates on average 2 passive supporters.

The process is run for a span of \( t = 500 \) steps, corresponding to about 10 years of time. Choosing \( \rho = 0.01 \) (left) yields a phase transition in the number of passive supporters and a sudden growth of the number of active terrorists, while choosing \( \rho = 0.001 \) (right) yields no phase transition. While in both cases the number of passive supporters increases directly from the start to a level, which is slightly decreasing for one third of the time, the higher value of the capturing rate triggers the mean field process to produce enough passive supporters to lift the local process to an overcritical level,
Figure 3: Densities of active terrorists and passive supporters on an Erdős-Renyi random graph $G(n, p) = c/n$ with $n = 200'000$ and $c = 4$, threshold $\Delta = 2$, initial density of passive supporters $b_0 = 0.04$, mean field $\beta = 0.00001$, in a time span of $t = 500$ steps. Choosing $\rho = 0.01$ (left) yields a phase transition in the number of passive supporters and a sudden growth of the number of active terrorists, while choosing $\rho = 0.001$ (right) yields no phase transition or a phase transition outside of the time scale under consideration.

where the calamitous phase transition occurs, resulting in an irreversible increase of active terrorists.

The impact of changing the capturing parameter $\rho$ is shown in Figure 4 on an Erdős-Renyi random graph $G(n, p) = c/n$ with $n = 50'000$ vertices and average degree $c = 4$, threshold $\Delta = 2$, initial density of passive supporters $b_0 = 0.01$, i.e. 500 passive supporters from the start. The mean field parameter is $\beta = 0.001$, i.e. one captured active terrorist generates 50 passive supporters here. Choosing $\rho = 0.004$ (left) or $\rho = 0.0025$ (right) only alters the moment when the phase transition happens.

Fig 5 shows the simulation run on a sample of the anonymized StudiVz network, including members associated with scholars of Bielefeld and their friends. The network has a size of approximately 400'000 vertices with an average degree of about 7.2, for more details see [6]. With an initial density of passive supporters of $b_0 = 0.00005$, corresponding to an average of 20 passive supporters from the start, and with capturing rate $\beta = 0.00001$, corresponding to an average of 40 citizens getting converted to passive supporters, we observe the phase transition occurring as soon as one or two active terrorists get caught.

5 Conclusions and Perspectives

The main observation is the existence of a phase transition in the number of passive supporters of terroristic activities. Whenever counter terrorist activities lead to collateral damages, the likelihood of outraging civilians rises. A high number of passive supporters provides a steady pool to recruit active terrorists, so the number of active terrorists and their attacks increases, as fig. 4 suggests and as resembled by the graph for the number of active terrorists in figures 3, 4, and 5.
Figure 4: Densities of active terrorists and passive supporters on an Erdős-Rényi random graph $G(n,p)$, $p = c/n$ with $n = 50'000$ and $c = 4$, threshold $\Delta = 2$, initial density of passive supporters $b_0 = 0.01$, mean field $\beta = 0.001$, in a time span of $t = 1500$ steps. Choosing $\rho = 0.004$ (left) or $\rho = 0.0025$ (right) only alters the moment when the phase transition happens.

Figure 5: Densities of active terrorists and passive supporters on a sample of the StudiVz network. The threshold is $\Delta = 2$, initial density of passive supporters $b_0 = 0.00005$, mean field $\beta = 0.0001$, in a time span of $t = 500$ steps. The phase transition occurs as soon as one or two active terrorists get captured.
Our results not only suggest lowering of the rate $\rho$ of removal of active terrorists to avoid the phase transition. The interplay of the mean field term $\beta$, which is the rate at which removed active terrorists generate passive supporters, and $\rho$ has to be taken into account. Avoidable failures resulting in casualties, high collateral damage, pictures and videos of humiliated inmates in Allied prisons, are factors which increase the probability that the civil population will join the terrorist side instead of fighting against it.

If the Allied forces want to avoid the phase transition in the number of passive supporters to not gain a stable number of active terrorist, capturing or removing active terrorists from the network would make sense therefore only if this happened practically without casualties, fatalities, applying torture or committing terroristic acts against the local population.

If this is not possible - and evidence is pointing towards this - our results strongly indicate that there is no military solution to fight terrorism, so only political solutions are available, in strict coincidence with former results obtained by Galam, [3],[4].

A refinement of the model may use weighted graphs, where the vertices have properties like credibility or importance, to include intra-organization dynamics, e.g. to model the emergence of terror cells. In the same fashion the influence of local warlords and tribal conflicts may be described. Finally, it will be important to extend the model to allow studying also the countereffects of the terrorists themselves in case their actions produce more and more civilian victims.

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