Neutrino physics and the flavour problem
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Abstract

We consider the problem of trying to understand the recently measured neutrino data simultaneously with understanding the hierarchical form of quark and charged lepton Yukawa matrices. We summarise the data that a successful model of neutrino mass must predict, and then move on to attempting to do so in the context of spontaneously broken ‘family’ symmetries. We consider first an abelian $U(1)$ family symmetry, which appears in the context of a type I string model. Then we consider a model based on a non-abelian $SU(3)_{F}$, which is the maximal family group consistent with an $SO(10)$ GUT. In this case the symmetry is more constraining, and is examined in the context of SUSY field theory.

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\textsuperscript{1} talk given by S.F.King
1 Introduction

Since the publication of results by SNO [1] and KamLAND [2], we now have a reasonably good picture of the neutrino sector of the Standard Model. In fact, by far the best fit to the data is the Large Mixing Angle (LMA) MSW fit [3]. We only have measurements of the mass differences in the neutrino sector:

\[
\Delta m_{21}^2 = (0.008\text{eV})^2 \quad |\Delta m_{32}^2| = (0.05\text{eV})^2
\]  

There are three possible neutrino mass patterns consistent with LMA [4]. The first, “normal”, has \( \Delta m_{32}^2 > 0 \) and \( m_1 \approx 0 \). The second, “inverted”, has \( \Delta m_{32}^2 < 0 \) and \( m_3 \approx 0 \). The third, “quasi-degenerate”, has the neutrino masses at a scale where the mass differences are negligible \( m_1 \approx m_2 \approx m_3 \).

We also have two measurably large neutrino mixing angles, and one small mixing angle:

\[
\theta_{\text{sol}} \equiv \theta_{12} \approx \frac{\pi}{6} \quad \theta_{\text{atm}} \equiv \theta_{23} \approx \frac{\pi}{4} \quad \theta_{\text{CHOOZ}} \equiv \theta_{13} \leq 0.2
\]

In the Standard Model, neutrinos are massless, and neutrinos and anti-neutrinos are distinguished by a total conserved lepton number, \( L \). Since we now know that this is not true, we wish to understand why the neutrino masses are so much smaller than that of the quarks and the charged leptons. Related to this is whether they have a Majorana or Dirac mass term, or both.

2 Seesaw models

Dirac mass terms are just like mass terms for the charged leptons and conserve lepton number. They can follow from a neutrino Yukawa coupling, just like charged lepton masses do in the Standard Model:

\[
m_{LR} \overline{\nu}_L \nu_R \tag{3}
\]

Majorana mass terms violate lepton number:

\[
m_{LL} \overline{\nu}_L \nu_L^{(c)} \tag{4}
\]
\[
M_{RR} \overline{\nu}_R \nu_R^{(c)} \tag{5}
\]

The term \( m_{LL} \) violates the electroweak gauge symmetry, so we would expect it to be exactly zero. However, no symmetry exists which protects \( m_{RR} \), so we would expect that to be very heavy, of the order of \( 10^{16} \) GeV. If we have both Dirac and Majorana mass terms, then we can generate small masses of the order of \( \Delta m^2 \), by a Type I seesaw mechanism.

\[
\left( \begin{array}{c} \overline{\nu}_L \\ \nu_R^{(c)} \end{array} \right) \left( \begin{array}{cc} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{array} \right) \left( \begin{array}{c} \nu_L^{(c)} \\ \nu_R \end{array} \right) \tag{6}
\]
In order to get the physical masses, we must block diagonalise this matrix. Assuming that \( M_{RR} \gg m_{LR} \), we find

\[
m'_{LL} \approx m_{LR}M_{RR}^{-1}m_{LR}^T
\]

\[
M'_{RR} \approx M_{RR}
\]

We can then enumerate the forms of \( m_{LL} \) that are consistent with LMA MSW. We refer to terms with a zero in the 11 element ‘type A’, and those without a zero in the 11 element ‘type B’. There is one possibility with a “normal” hierarchy \( m_1^2 \ll m_2^2 \ll m_3^2 \):

\[
m^\text{HI, A}_{LL} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2} \]

There are two possibilities with an “inverted” hierarchy \( m_1^2 \sim m_2^2 \gg m_3^2 \):

\[
m^\text{HI, A}_{LL} \approx \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{m}{\sqrt{2}} \quad m^\text{HI, B}_{LL} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \]

There are three possibilities with a degenerate mass pattern \( m_1^2 \approx m_2^2 \approx m_3^2 \):

\[
m^\text{DEG, A}_{LL} \approx \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{pmatrix} \frac{m}{2} \quad m^\text{DEG, B1}_{LL} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[
m^\text{DEG, B2}_{LL} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \]

\( m^\text{HI, B}_{LL} \) leads to a large rate for neutrinoless double beta decay.

From this point on, we focus on the normal hierarchy. In this case, we need to understand why \( m_1^2 \ll m_2^2, \theta_{23} \approx \frac{\pi}{4}, \theta_{12} \approx \frac{\pi}{6} \). The technical requirement for \( m_2^2 \ll m_3^2 \) is for the subdeterminant to be small:

\[
\det \begin{vmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{vmatrix} \ll m^2
\]

We are then led to ask why this sub-determinant is small, and why the solar angle is large. One model which solves this is right-handed neutrino dominance.

If one right-handed neutrino dominates in the see-saw mechanism, and couples equally to the second and third family left-handed neutrinos then \( m_2^2 \ll m_3^2 \) and \( \theta_{23} \approx \frac{\pi}{4} \) \[12\].

Furthermore, if a second right-handed neutrino gives the leading sub-dominant contributions to the see-saw mechanisms and couples equally to all three left-handed generations, then a large solar angle is generated \( \theta_{12} \approx \frac{\pi}{6} \) \[13\].

The corollary of this is that if the dominant right-handed neutrino is the lightest, then there is a link between the neutrino oscillation phase and the phases of leptogenesis and neutrinoless double beta decay \[14\].
3 The flavour problem

There are two parts to the flavour problem. This fist is understanding the origin of the Yukawa couplings, (and heavy Majorana masses for the see-saw mechanism), which lead to low energy quark and lepton mixing angles. In low energy SUSY, we also need to understand why flavour changing and/or CP violating processes induced by SUSY loops are so small. A theory of flavour must address both these problems simultaneously.

Consider, for example the loop in fig. 1. This leads to a rate:

\[ BR(\tau \rightarrow \mu \gamma) \approx \frac{\alpha^3}{G_F^2} f_{32}(M_2, \mu, m_{\tilde{\nu}}) \left| m_{L_{32}}^2 \tan^2 \beta \right| \]

We see that the decay rate depends on off-diagonal slepton masses. There will be two sources of slepton masses. The first is ‘primordial’; this is where there are off-diagonal elements in the SCKM basis at the high-energy scale, generated by the SUSY breaking mechanism. The second is RGE generated.

We can address both problems at the same time by employing a family symmetry. This will be spontaneously by \( \Phi \), a Higgs field for the family symmetry. The idea is that each generation of matter will not be neutral under the symmetry, and so extra powers of the flavon will appear over some UV cutoff. This will lead to effective Yukawa couplings when \( \Phi \) gains a VEV:

\[ \psi^i \tilde{\psi}^j H \left( \frac{\Phi}{M} \right)^{n_{ij}} \rightarrow \psi^i \tilde{\psi}^j H \left( \frac{\langle \Phi \rangle}{M} \right)^{n_{ij}} \left( \frac{\langle \Phi \rangle}{M} \right) \sim 0.1 \]

This gives an explanation of the Yukawa textures:
\[ Y = \left( \begin{array}{ccc}
\frac{\Phi}{\pi}^{n_{11}} & \frac{\Phi}{\pi}^{n_{12}} & \frac{\Phi}{\pi}^{n_{13}} \\
\frac{\Phi}{\pi}^{n_{21}} & \frac{\Phi}{\pi}^{n_{22}} & \frac{\Phi}{\pi}^{n_{23}} \\
\frac{\Phi}{\pi}^{n_{31}} & \frac{\Phi}{\pi}^{n_{32}} & \frac{\Phi}{\pi}^{n_{33}}
\end{array} \right) 
\tag{15}\]

This addresses the first part of the flavour problem. However, it can also make the second part problematic. This is because the new fields in the Yukawa operators can develop F-term VEVs, and contribute to the SUSY breaking F-terms in a non-universal way. This leads to a new and dangerous source of primordial flavour violation \[\Delta A = F_\phi \partial_\phi \ln \Phi^n = F_\phi \frac{n}{\Phi} \tag{16}\]

But the auxiliary field is proportional to the scalar component:

\[F_\phi \propto m_{3/2} \Phi \rightarrow \Delta A \propto nm_{3/2} \tag{17}\]

And example of this with a \(U(1)\) family symmetry is:

\[ Y = \left( \begin{array}{ccc}
\frac{\Phi}{\pi}^{\frac{5}{2}} & \frac{\Phi}{\pi}^{\frac{3}{2}} & \frac{\Phi}{\pi}^{1} \\
\frac{\Phi}{\pi}^{\frac{4}{2}} & \frac{\Phi}{\pi}^{\frac{2}{2}} & \frac{\Phi}{\pi}^{1} \\
\frac{\Phi}{\pi}^{\frac{4}{2}} & \frac{\Phi}{\pi}^{\frac{2}{2}} & \frac{\Phi}{\pi}^{1}
\end{array} \right) \rightarrow \Delta A \sim \frac{m_{3/2}}{2} \begin{pmatrix}
5 & 3 & 1 \\
4 & 2 & 0 \\
4 & 2 & 0
\end{pmatrix} \tag{18}\]

If we take a specific model where we can switch off the new effects, and look at more standard SUGRA flavour violation, we can gauge the relative importance of the new effects \[\text{[8]}\]. In order to do so, we look at three benchmark points. Point A corresponds to minimum flavour violation, where the SUGRA setup is like mSUGRA. In this case the seesaw RGE contributions are the only contributions. Point B corresponds to a ‘standard’ SUGRA FV setup, where non-universal scalar masses generate primordial FV in the SCKM basis. Finally, point C corresponds to the new effects, where \(\Delta A\) generates primordial FV even before switching to the SCKM basis.

We display, in fig. 2 \(BR(\mu \rightarrow e\gamma)\) at the three seesaw points, and in fig. 3 \(BR(\tau \rightarrow \mu\gamma)\) at the three benchmark points.

There is no reason why a family symmetry has to be abelian. Consider a SUSY family GUT \(SO(10)_C \otimes SU(3)_F \text{ [10]}\). \(SU(3)_C\) is the largest family symmetry consistent with a \(SO(10)\) GUT. This model is an example of sequential dominance, gives an excellent description of the quark and lepton masses and mixing angles, and can address the SUSY flavour/CP problems.

In this model, we break the \(SO(10)\) GUT in the Pati-Salam direction by a Wilson line breaking. The Pati-Salam group is also broken to the MSSM group by Wilson line breaking. The \(SU(3)_F\) is broken first to \(SU(2)_F\) by a Higgs field \(\phi_3\). The remnant \(SU(2)_F\) is then broken completely by another Higgs field \(\phi_{23}\).

There are a few global symmetries in the theory to restrict the Yukawa operators allowed. The leading order operator leads to the top Yukawa element
Figure 2: $BR(\mu \to e\gamma)$ for the three benchmark points
Figure 3: $BR(\tau \rightarrow \mu \gamma)$ for the three benchmark points
(τ ≈ 0.15, ε ≈ 0.05):

\[
\frac{1}{M^2} \psi_3 \phi_3 h \rightarrow Y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tau
\]

(19)

The subleading operator leads to the charm Yukawa and the charm-top mixing angles:

\[
\frac{\Sigma}{M^3} \psi_{23} \phi_{23} h \rightarrow Y = \begin{pmatrix} 0 & y \epsilon^2 \\ y \epsilon^2 & y \epsilon^2 \end{pmatrix}
\]

(20)

Setting the \( \mathcal{O}(1) \) coefficients of the operators to get a good fit, we predict the following Yukawa matrices:

\[
Y^u \sim \begin{pmatrix} 0 & 1.2 \epsilon^3 \\ -1.2 \epsilon^3 & -\frac{2}{3} \epsilon^2 \\ -0.9 \epsilon^3 & -\frac{4}{3} \epsilon^2 \\ 1 \end{pmatrix} \tau, \ Y^d \sim \begin{pmatrix} 0 & 1.6 \epsilon^3 \\ -1.6 \epsilon^3 & \epsilon^2 \\ -0.7 \epsilon^3 & \epsilon^2 + \epsilon^3 \\ \epsilon^2 - \epsilon^3 & 1 \end{pmatrix} \tau
\]

(21)

\[
Y^\nu \sim \begin{pmatrix} 0 & 1.2 \epsilon^2 \\ -1.2 \epsilon^2 & -\alpha \epsilon^2 \\ -0.9 \epsilon^3 & -\alpha \epsilon^2 - \epsilon^3 \sqrt{\tau} \\ 1 \end{pmatrix} \tau, \ Y^e \sim \begin{pmatrix} 0 & 0.7 \epsilon^3 \\ 0 & 0.7 \epsilon^3 \\ -1.6 \epsilon^3 & 3 \epsilon^2 \\ 3 \epsilon^2 & 3 \epsilon^2 \\ -0.7 \epsilon^3 & 1 \end{pmatrix} \tau
\]

(22)

\[
M_{RR} \sim \begin{pmatrix} e^{6 \epsilon^3} & e^{6 \epsilon^2} \\ e^{6 \epsilon^3} & 1 \end{pmatrix}
\]

(23)

The first RH neutrino dominates, and we predict \( m_2 / m_3 \sim \tau, \tan \theta_{23} \sim 1.3, \tan \theta_{12} \sim 0.66 \) and \( \theta_{13} \sim \tau \).

This all assumes a canonical Kähler metric, and so we would expect the soft SUSY breaking masses to be universal for a simple SUSY breaking scenario.

4 Conclusions

Small neutrino masses can be elegantly explained by the see-saw mechanism. In that case, sequential dominance then provides a natural explanation of a neutrino mass hierarchy and large mixing angles. If the dominant right-handed neutrino is the lightest one, there is a link between the leptogenesis phase and the CP phase which is measurable at a neutrino factory.

Family symmetries provide a natural way of understanding the hierarchies in the quark and charged lepton masses, and the smallness of the quark mixing angles. In this case, dangerous new sources of flavour changing masses in general arise from Yukawa operators which lead to large off-diagonal soft trilinears. One example is the \( U(1) \) family symmetry. Another is the \( SU(3) \) family symmetry, which provides an excellent description of the fermion spectrum, with SUSY flavour-changing controlled by the family symmetries.
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