NLO QCD corrections to $W^+W^-b\bar{b}$ production with leptonic decays in the light of top quark mass and asymmetry measurements

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ABSTRACT: We present the NLO QCD corrections to the processes $pp$ and $p\bar{p} \rightarrow W^+W^-b\bar{b}$ including leptonic decays of the $W$ bosons. Non-resonant contributions as well as diagrams with doubly resonant and singly resonant top quark propagators are fully taken into account. We employ the narrow width approximation to perform the decays of the $W$ bosons; spin correlations are however preserved. We also calculate observables relevant for top quark mass measurements, and study the impact of kinematical requirements and different scale choices on $t\bar{t}$ asymmetries.

KEYWORDS: QCD, NLO Computations, LHC, Top Quark
1 Introduction

The production of top quarks is one of the most important reactions studied at the Tevatron and the LHC. Especially the latter is known for its status as a top quark factory, producing top quark pairs copiously. Up to now more than $5 \times 10^6 \ t\bar{t}$ pairs have been produced. Top quarks play a major role as a background to New Physics searches, and also in precision studies which can provide indirect hints to physics beyond the Standard Model. Currently a lot of effort is put into the precise determination of the top quark pair production cross section and differential distributions such as the pair transverse momentum and the pair invariant mass. Among the interesting observables are also the top quark mass as well as the forward-backward and charge asymmetry \[1–14\] as measured at the Tevatron and LHC, respectively.

To match the experimental precision reached for quantities like the top quark mass, the theory predictions need to go beyond the simple approximation of factorizing top quark production and decay. For example, finite width effects and non-factorizing contributions to observables based on $W$ boson decay products and $b$ jets can have a non-negligible impact on mass measurements. The latest combinations of the top quark mass from the Tevatron and the LHC can be found in Refs. \[15\] and \[16\], respectively. Also, the first combined measurement using ATLAS, CDF, CMS and DØ data has recently been published \[17\]. The contributing measurements relevant to the work of our paper are those utilizing the leptonic decay mode. They are discussed in \[18–23\] and \[24, 25\] based on results obtained at the Tevatron and the LHC, respectively.
The next-to-leading order (NLO) QCD corrections to top quark pair production have been already known for a long time [26–30]. The NLO electroweak corrections were calculated in [31]. Very recently the full NNLO cross section for $t\bar{t}$ production has become available [32]. These calculations treat the top quarks as stable on-shell particles. Decays can then be attached to the top quarks in the narrow width approximation (NWA), where production and decay decouple. In most applications, these decays are calculated only at the leading order. One however makes use of spin density matrix or reweighting techniques to preserve the spin correlations between particle production and decay. This, especially, is the standard in multi-purpose Monte Carlo event generators. At parton level, NLO calculations using the NWA were further improved by promoting the treatment of top quark decays to NLO.\footnote{One recent development presented in [33] concerns the calculation of NLO corrections to polarized top quark decays with an additional jet in the final state.} The complete evaluation of the $O(\alpha_s)$ corrections to $t\bar{t}$ production and decay based on the NWA and in full regard of spin correlations is documented in Refs. [34–36].

The full process $p\bar{p}$ or $pp \to W^+W^-b\bar{b}$ at $O(\alpha_s^2\alpha^2)$, where top quarks are treated as off-shell particles, represents a $2 \to 4$ process which is of much higher complexity. It includes resonant top quark production and decay, but also singly resonant and non-resonant contributions. Using massless $b$ quarks, this process was calculated at NLO in QCD in [37–39]. More recently, as shown in [40, 41], it was also computed in the 4-flavour scheme, i.e. for massive $b$ quarks.

In this paper we calculate the NLO QCD corrections to the $O(\alpha_s^2\alpha^2)$ processes $p\bar{p}$ and $pp \to W^+W^-b\bar{b} \to (e^+\nu_e)(\mu^-\bar{\nu}_\mu)b\bar{b}$ in the 5-flavour scheme, including singly resonant and non-resonant contributions, corresponding to Feynman diagrams containing only one or no top quark propagator that can go on-shell. The impact of non-resonant $W$ boson contributions has been studied in [38] and found to be small. Therefore, non-resonant contributions from $W$ bosons are neglected in our calculation. On the other hand, in contrast to the calculations in [37–39], contributions from (massless) $b$ quarks in the initial state are included in the calculation presented here.

The structure of this paper is as follows: in Section 2, we give details about the calculation and present some numerical results for LHC collisions at 7 TeV, in particular for observables which are sensitive to the non-factorizing contributions. In Section 3, we perform a detailed phenomenological analysis of observables that are of particular interest for precision studies: the top quark mass and observables related to $t\bar{t}$ asymmetries. Finally, we conclude in Section 4.

## 2 Calculational framework and numerical results

For all our perturbative QCD, parton level calculations, we use the GoSam [42] plus Sherpa [43] combined generator package, in short GoSam+Sherpa. For examples of applications, see Refs. [44–47], for a list of pre-generated process packages, see [48].

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multi-purpose Monte Carlo event generator SHERPA is used to provide the Born, real radiation and subtraction term contributions, as well as to accomplish the phase-space integration [43]. The tree-level amplitudes are obtained from both AMEGIC [49] and COMIX [50], which are the SHERPA in-house matrix-element generators, while the dipole subtraction terms are generated with the automated Catani–Seymour procedure [51] as implemented in SHERPA [52]. The code for the evaluation of the virtual corrections has been generated by GOSAM [42] and is linked to SHERPA via the Binoth–Les-Houches interface [53, 54]. GOSAM is an automated one-loop amplitude package, combining automatized diagram generation and algebraic manipulation [55–58] with $d$-dimensional integrand-level reduction as implemented in the libraries SAMURAI [59, 60] and NINJA [61]. Alternatively, the integrand reduction can also proceed via a tensorial decomposition [62] using the library GOLEM95C [63–65]. In certain cases, the latter serves as the rescue route for phase space points yielding insufficient one-loop amplitude precision in the first place.

Our NLO accurate calculations of the $2 \rightarrow 4$ processes $pp$ and $p\bar{p} \rightarrow W^+W^-b\bar{b} \rightarrow (e^+\nu_e)(\mu^-\bar{\nu}_\mu)b\bar{b}$ provide a full description of the final state, which is typically used as a signature for the decay of a $t\bar{t}$ pair with leptonic $W$ boson decays. As mentioned in the introduction, we include singly resonant top quark and non-resonant contributions, see Figure 1. Owing to their small overall effect, diagrams that involve Higgs bosons have been neglected throughout. Our computation relies on the 5-flavour scheme. While the subprocesses with charm and strange quarks in the initial state are equivalent to those of the $u\bar{u}$ and $d\bar{d}$ channels, the $b\bar{b}$ subprocess has to be generated separately because the initial state $b$ quarks can propagate to the final state, thus leading to additional diagrams.

To take the top quark decay width into account in a gauge invariant way, the complex mass scheme [66] is used. In our setup, this amounts to replacing the top quark mass everywhere by a complex number $\mu_t$ according to

$$\mu_t^2 = m_t^2 - i m_t \Gamma_t.$$  \hspace{1cm} (2.1)

The weak mixing angle remains real-valued in our calculation, as we neglect non-resonant $W$ and $Z$ boson contributions.

Using this setup, the correctness of the virtual amplitude has been checked by comparing it with the results of [38] for a given phase-space point. Furthermore, the calculation
of the real radiation component was verified by evaluating the cross section for different values of the dipole $\alpha$-parameter \cite{e1}. Employing $\alpha_{dp} = \{0.1, 0.05, 0.01\}$, the results were found to be in agreement within the numerical uncertainty.

2.1 Treatment of top quarks

To investigate top quark finite-width effects, we compare the outcomes of two different types of calculations for the $W^+(e^+\nu_e)W^-(\mu^-\bar{\nu}_\mu)\bar{b}b$ final states:

(I) the full or $WW\bar{b}b$ approach based on the NLO or LO treatment of the $2 \rightarrow 4$ processes where finite width effects of the top quarks and non-resonant contributions are fully taken into account, and

(II) the factorized or $t\bar{t}$ approach based on the narrow width approximation where the production of the top quarks factorizes from their decays. In our case, the higher-order treatment will be limited to the production part: all NLO corrections only apply to the $2 \rightarrow 2$ processes of top quark pair production, i.e. the top quark decays are still described with leading order accuracy. In the NLO context, one commonly refers to this approach as “narrow width approximation with leading-order decays” to distinguish it from the complete NLO treatment combining $t\bar{t}$ production and decay in the narrow width approximation, as accomplished in Ref. \cite{e2}.

The narrow width approximation (NWA) is motivated by the fact that, in the limit $\Gamma_t \rightarrow 0$, the denominator of the top quark propagator can be written as

$$\lim_{\Gamma_t \rightarrow 0} \frac{1}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} = \frac{\pi}{m_t \Gamma_t} \delta(p_t^2 - m_t^2) + O \left( \frac{\Gamma_t}{m_t} \right). \quad (2.2)$$

Since this approximation introduces a factor of $1/\Gamma_t$ for each top quark resonance, singly resonant and non-resonant contributions are suppressed in the $\Gamma_t \rightarrow 0$ limit. Consequently, one only keeps the Feynman diagrams where two top quarks can become resonant, because only those are proportional to $1/\Gamma_t^2$. In the $\Gamma_t \rightarrow 0$ limit, the full process therefore factorizes
into top quark pair production and decay, i.e. $pp$ and $p\bar{p} \to t\bar{t} \to W^+b W^-\bar{b}$. Thus, when working in the NWA, at NLO one also neglects radiative corrections that either connect production and decay, or both decays. Two example Feynman diagrams contributing to the virtual corrections, which are not present in the NWA, are given in Figure 2.

From Eq. (2.2) one recalls that the contributions neglected in the NWA are suppressed by powers of $\Gamma_t/m_t \lesssim 1\%$. While this is true for sufficiently inclusive observables, the corrections can be much larger for observables such as $m_{lb}$, the system invariant mass of the charged lepton and the (associated) $b$ jet. We will discuss this issue in more detail in Section 3.2.

2.2 General input parameters

For the (N)LO calculations, the MSTW2008(N)LO parton distributions [68] were used, relying on the strong coupling constant, $\alpha_s$, and its running as provided by these PDF parametrizations. The electroweak parameters are given in the $G_\mu$ scheme:

$$G_\mu = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2},$$
$$M_W = 80.399 \text{ GeV}, \quad \Gamma_W = 2.0997 \text{ GeV},$$
$$M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.5097 \text{ GeV}.$$ (2.3)

All quarks other than the top quark are taken to be massless. For the top quark mass, we use $m_t = 172.0 \text{ GeV}$. From the parameters given above, it is possible to derive the value of the top quark decay width at LO and NLO using the expressions calculated in [69]. We use the numerical values

$$\Gamma_{LO}^t = 1.4426 \text{ GeV},$$
$$\Gamma_{NLO}^t = 1.3167 \text{ GeV}.$$ (2.4)

2.3 Numerical results for LHC collisions at 7 TeV

Using the full approach, cf. Section 2.1 (I), we now study the impact of the NLO corrections to $W^+W^-b\bar{b}$ production in dilepton final states at the LHC for a collision energy of 7 TeV. To produce these results, we impose the following set of kinematical requirements: all final state partons are clustered into jets with a separation in azimuthal angle ($\phi$) and pseudo-rapidity ($\eta$) space defined by

$$\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} > 0.5,$$ (2.5)

using the anti-$k_T$ jet algorithm [70, 71] implemented in FastJet [72]. Each event is required to contain at least two $b$ jets obeying the conditions

$$p_T,b > 30 \text{ GeV} \quad \text{and} \quad |\eta_b| < 2.5.$$ (2.6)

The requirements on the charged leptons ($l$) and the missing energy are\(^2\)

$$p_T,l > 20 \text{ GeV}, \quad |\eta_l| < 2.5 \quad \text{and} \quad \not{p}_T > 20 \text{ GeV},$$ (2.7)

respectively.

\(^2\)Here, we employ the transverse vector sum of the neutrinos to determine the missing energy.
Figure 3: Scale variation of the LO and NLO cross sections in the full approach (3a), ranging from $x = 1/4$ to $x = 16$ where $x = 2 \mu / \hat{H}_T$ and $\mu = \mu_R = \mu_F$. Transverse momentum distribution of the leading $b$ jet at LO and NLO in the full approach (3b). The bands were obtained by varying $\mu$ by a factor of two around the central scale $\hat{H}_T/2$.

Similarly to the computation presented in Ref. [40], we use $\hat{H}_T$, specified through

$$\hat{H}_T = \sum_i p_{T,i} ,$$

(2.8)

to define our default scale choice, $\mu_R = \mu_F = \mu \equiv \hat{H}_T/2$, in setting the renormalization and factorization scales. Note that the sum in Eq. (2.8) runs over all final state particles, including the neutrino momenta. The central scale is chosen to be $\mu \equiv \hat{H}_T/2$ because this helps minimize the difference between the LO and the NLO cross section as well as the uncertainty induced by scale variations. Concerning the latter fact, this scale uncertainty estimate turns out to be smaller than those obtained from the scale choices $\mu = \hat{H}_T$ or $\mu = m_t$.

Given the settings above, we obtain the LO and NLO inclusive cross sections for the full approach, reading

$$\sigma_{LO} [fb] = 638.4^{+38.5\%}_{-24.8\%} \text{ (scale)} \pm 0.03\% \text{ (stat)} ,$$

$$\sigma_{NLO} [fb] = 758.5^{+2.5\%}_{-2.3\%} \text{ (scale)} \pm 0.2\% \text{ (stat)} .$$

(2.9)

This corresponds to a $K$-factor of about 1.2. The scale uncertainties given in Eqs. (2.9) are obtained from varying the central scale $\hat{H}_T/2$ by factors of two, i.e. using the multiplicative factors $x = 1/2$ and $x = 2$ where $x = \mu / \hat{H}_T/2$. We also indicate the statistical uncertainty of our Monte Carlo integrations. As the NLO cross section at the central scale is larger.
Figure 4: Differential distributions of the $\Delta R$ separation (4a) and the relative azimuthal angle between the two charged leptons (4b) in $W^+W^-b\bar{b}$ production at LO and NLO (in the full approach). The bands were obtained by varying the scales by a factor of two around the central scale $\hat{H}_T/2$.

than for both the upwards and downwards scale variation (see Figure 3a), the NLO scale uncertainties given in Eqs. (2.9) can only be negative. Another, perhaps more reasonable way of estimating the scale uncertainties is to consider the highest and lowest cross section within a $\mu$ range determined by factors of two around the central scale. According to this procedure, the maximum (minimum) value for the NLO cross section is $759.6$ fb ($718.3$ fb) occurring at a scale slightly lower than (twice as large as) the central one. Finally, and for an extended $x$ range, the sensitivity of the LO and NLO cross sections to the choice of the renormalization and factorization scales, $\mu_R = \mu_F$, is shown in Figure 3a. Here, the scales have been varied between $\hat{H}_T/8$ and $8\hat{H}_T$ retaining $\mu_R = \mu_F$.

In Figure 3b, we now present a first differential distribution comparing the LO and NLO predictions with their absolute normalizations for the leading $b$ jet transverse momentum. The respective bands have been determined, as before, from scale variations evaluated at $x = 1/2$ and $x = 2$. The “NLO/LO” ratio plot clearly exhibits the reduction of the theory uncertainties, as well as the hardening of the $p_T$ spectrum owing to the generation of real radiation that recoils against the $t\bar{t}$ system. We furthermore notice that shape changes to the $p_{T,b_1}$ distribution only occur at NLO; at LO, the shape is more or less predicted to be constant, as reflected by the uniform envelope around the $p_{T,b_1}$ LO prediction.

Figure 4 shows differential distributions related to the two charged leptons stemming from the $W$ boson decays. Figure 4a displays the $\Delta R$ separation between the leptons, $e^+$ and $\mu^-$, while Figure 4b shows the projection of the relative angle between these two leptons onto the plane transverse to the beam axis, $\phi_{e^+\mu^-}$. These lepton correlations play an important role in the measurement of top quark spin correlations at the LHC. For both distributions, we observe a substantial reduction of the scale uncertainties at NLO. Again,
scale variations by and large do not affect the LO shapes, a description at NLO therefore is much more reliable. The distribution of the azimuthal angle \( \phi_{e^+\mu^-} \) receives the largest NLO corrections of \( \mathcal{O}(30\%) \) in regions where the separation between the two leptons is small. Even for small angles, the \( K \)-factor varies no more than \( \sim 10\% \).

Figures 5a and 5b visualize the system transverse momentum spectra of the two charged leptons and the two \( b \) jets, respectively. These observables receive large NLO corrections with \( K \)-factors growing as large as \( \sim 3 \) in the region of hard \( p_T \). The reason lies in the generation of the real radiation component that recoils against the entire \( WWb\bar{b} \) system. This component, which is absent at LO, leads to a \( p_T \) imbalance between the \( WW \) and \( b\bar{b} \) subsystems, which can be noticed in particular for \( p_{T,b\bar{b}} \gtrsim 2M_W \) where the effect becomes largest. Therefore, it is not surprising that the scale uncertainty band associated with the LO distribution does not contain the NLO result in the tail of this and similar distributions. For the charged lepton pair \( p_T \), the effect is somewhat washed out and smaller – simply because the dileptons do not carry the full information on the \( p_T \) of the \( WW \) system.

3 Phenomenological studies

In the following we will concentrate on two applications of our parton level calculations using both the full and the factorized approach, as described in Section 2.1. Firstly, any shape-based \( m_t \) measurement relies on the precise modelling of the differential distribution whose shape depends on the value of the top quark mass. Shape uncertainties induced by \( \mu_{F,R} \) scale variations will therefore impact the accuracy of \( m_t \) measurements. For the example of the \( m_t \) measurement based on the \( m_{lb} \) observable, we will study this issue in
detail. Secondly, in the context of $t\bar{t}$ asymmetry measurements, it is crucial to understand the relation between top quark and lepton-based asymmetries. We will discuss the strength of their correlation, particularly for Tevatron analyses using the dilepton channel.

### 3.1 Top quark mass measurements

As the top quark mass $m_t$ is not a physical observable, its definition is scheme dependent. The most commonly used mass definitions are the pole mass and the $\overline{\text{MS}}$ mass. The different masses are related by a perturbative series, see e.g. Refs. [73, 74].

The pole mass scheme is a long distance scheme, where implicitly the top quark is considered as a stable particle, with the pole mass being defined as the real part of the pole of the propagator. However, the fact that quarks do not appear as isolated particles implies that non-perturbatively there is no pole in the scattering amplitude due to the quark propagator, and only in perturbation theory the pole mass is properly defined. The pole mass also gets corrections of order $\Lambda_{\text{QCD}}$ from the infinite sum of self energy insertions, which is called the renormalon ambiguity [75, 76].

The most common short distance scheme is the modified minimal subtraction ($\overline{\text{MS}}$) scheme. In contrast to the pole mass, the $\overline{\text{MS}}$ mass is not sensitive to corrections related to the renormalon ambiguity. Despite this fact, a similar convergence behaviour of the top quark mass in both schemes has been observed up to NNLO [77, 78].

Experimental results for the top quark mass are obtained by comparing experimental observables to the prediction from Monte Carlo event generators. The exact relation between the mass parameter $m_{t}^{\text{MC}}$ used in the Monte Carlo program and the pole mass at a given order in perturbation theory is still an open issue [79–82]. The related uncertainty is estimated to be about 1 GeV. A study aiming at disentangling systematically genuine non-perturbative effects from perturbative ones can be found in Ref. [83], see also [78].

To avoid these problems, it has been suggested to determine the $\overline{\text{MS}}$ mass by comparing the measured total cross section for top quark pair production with a fixed order calculation performed in the $\overline{\text{MS}}$ scheme [84, 85]. However, this method cannot circumvent the problem completely, as the experimental determination of the $t\bar{t}$ cross section also has to rely on $m_{t}^{\text{MC}}$ [86].

### 3.2 Mass determination using the $m_{lb}$ observable

An observable which has recently been used for a top quark mass determination at the LHC [24] is the invariant mass of a charged lepton and a $b$ jet, $m_{lb}^2 = (p_l + p_b)^2$, where $p_b$ denotes the four-momentum of the $b$ jet. Already in Ref. [35] this observable has been studied in view of top quark mass determinations, however, non-factorizing contributions have not been taken into account by that calculation.

The latest ATLAS result [24] of $m_t$ using the $m_{lb}$ observable is:

$$m_t = 173.09 \pm 0.64 \text{ (stat)} \pm 1.50 \text{ (syst)} \text{ GeV} \ .$$

The systematic uncertainty is dominated by jet energy scale uncertainties, and the theoretical uncertainties assigned to this measurement amount to about 0.8 GeV.
Figure 6: Distribution of $m_{lb}$ at LO (blue lines) and NLO (red lines), including standard scale variations for (6a) the full calculation, and (6b) the factorized calculation. In addition shown are the ratios using the respective LO central predictions as their references. The scale choices differ: they are $\mu = \hat{H}_T/2$ and $\mu = m_t = 172.5$ GeV for the full and the factorized approach, respectively.

One complication arises from the fact that there are two top quarks and therefore two possible $m_{lb}$ values per event. Since experimentally, the charge of the $b$ quark initiating a jet cannot be reconstructed on an event-by-event basis, one needs a criterion to identify a pair of a charged lepton and a $b$ jet as stemming from the same top quark decay. Using events generated with the MC@NLO Monte Carlo program [87], different strategies for this assignment were investigated by ATLAS. Following the procedure given in Ref. [24], the algorithm applied here is to choose the combination, i.e. the ($l^+ b$-jet, $l^- b$-jet$'$) pairing, which minimizes the sum of the two $m_{lb}$ values per event. Finally, the $m_{lb}$ observable used in the analysis is the mean of the two $m_{lb}$ values per event obtained when applying the above procedure.

3.2.1 Parton level $m_{lb}$ predictions at NLO

For our calculations of the $m_{lb}$ distribution, we follow the ATLAS procedure as outlined above. We use $m_t = 172.5$ GeV as our default top quark mass and employ the ATLAS kinematic requirements for 7 TeV LHC $pp$ collisions: we require exactly two oppositely charged leptons (electrons with $p_T > 25$ GeV, and muons with $p_T > 20$ GeV) in the pseudorapidity range $|\eta_l| < 2.5$, and two $b$ jets with $p_{T,b} > 25$ GeV, $|\eta_b| < 2.5$ and $\Delta R > 0.4$, using the anti-$k_T$ algorithm. The leptons have to be isolated from the jets with $\Delta R_{l,j} > 0.4$. Lastly, $H_T$ defined as the sum over the transverse momenta of charged leptons and jets has to be larger than 130 GeV.

For the two types of calculations described in Section 2.1, Figure 6 shows the corresponding $m_{lb}$ distributions at LO and NLO, including their respective scale variation.
bands as well as the ratios taken with respect to the central LO prediction. These results have been obtained from the full calculation evaluated with $\mu = \hat{H}_T/2$ (6a), and from the factorized calculation evaluated at $\mu = m_t$ (6b).

The full and factorized calculations exhibit a few interesting differences. Firstly, the uncertainty bands of the computations in the factorized approach are wider than those of the respective full computations. Secondly, the NLO corrections in the factorized approach (cf. Figure 6b) mainly affect the event rate, while the LO and NLO shapes of the $m_{tb}$ distribution are very similar. The only exception is the region $m_{tb} > 150$ GeV, where the difference is caused by the fact that the LO factorized calculation has a sharp cut-off at $m_{tb} = \sqrt{m_t^2 - m_W^2}$. Even for the full approach, the differences in the tail are found to exceed the estimate from scale variations of the LO calculation.

Measurements of $m_t$ are mostly affected by changes in shape around the peak of the distribution. For the full NLO calculation, there is a significant change in shape over the entire $m_{tb}$ range (cf. Figure 6a). Moreover, this is the only prediction featuring a rather asymmetric uncertainty band. The upward and downward scale variations both lower the cross section in the range $80 - 150$ GeV. This is in contrast to all LO calculations as well as to the NLO factorized calculation, for which the uncertainty bands are fairly symmetric around the respective central predictions. This behaviour is not caused by the choice of a dynamical scale, which has been verified by repeating the full calculations for the fixed scale $\mu = m_t$. In this case, a very similar behaviour was found.

Since the shape variations are most relevant for the $m_t$ measurement, Figure 7 com-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Various normalized LO and NLO predictions of the $m_{tb}$ distribution. Results (solid lines) using the full calculation and the fixed scale choice $\mu = m_t = 172.5$ GeV are compared to (7a) results generated with our default dynamical scale choice of $\mu = \hat{H}_T/2$, and (7b) to results of the factorized approach for the same fixed scale choice, $\mu = m_t$. The prediction of the full calculation at LO utilizing the fixed scale serves as the reference curve in both ratio distributions shown.
}
\end{figure}
compares several normalized \(mb\) distributions. The effect of using different scales, namely \(\mu = \hat{H}_T/2\), and \(\mu = m_t\), is small. This is shown in Figure 7a for the full approach, and separately for the LO and NLO calculations. The evaluation of the \(\hat{H}_T\) scale requires knowledge of the four-momenta of the top quark decay products. This is rather inconvenient when applied to factorized calculations. Therefore, to directly compare the predictions of the full and factorized approach, the fixed scale of \(\mu = m_t\) is used in Figure 7b. Apart from the above discussed differences for \(mb > 150\) GeV, it is the shape of the full NLO prediction that deviates considerably by up to 20% from all other predictions. Given that this difference occurs in a region of large cross section, it will have a visible consequence for the top quark mass measurement discussed below.

Once NLO corrections to the top quark pair production are incorporated, the \(mb\) distribution develops a tail. Already the non-resonant contributions included in the LO full calculation lead to a more pronounced tail, which in addition receives large NLO corrections. Even though the tail of the \(mb\) distribution plays only a minor role in the top quark mass determination, it is important to assess its impact, especially when aiming at a top quark mass measurement with a precision below 1 GeV.

### 3.2.2 Investigation of theoretical uncertainties in the \(m_t\) measurement

The top quark mass measurement presented in Ref. [24] uses a template method. For details of the implementation, see Ref. [88]. In short, in this method, simulated distributions are constructed for different input values of the top quark mass, \(m_t^{\text{in}}\). The distributions (templates) per \(m_t^{\text{in}}\) are then individually fitted to a function. Using templates at different \(m_t^{\text{in}}\), it is verified that all parameters of the function linearly depend on \(m_t = m_t^{\text{in}}\). Consequently, this linearity is imposed in a combined fit to all templates. This fit fixes the theory model (i.e. the parametrization of the theory model or hypothesis) by determining all parameters of the function, except for \(m_t\), which is to be determined from data. Using those parameter values, a likelihood fit of this function to data is performed to obtain the value for \(m_t\) that best describes the data, namely \(m_t^{\text{out}}\), together with its statistical uncertainty. Using different sets of pseudo-data, the same strategy is used to estimate the impact of different theory descriptions. The systematic uncertainties on \(m_t\) stemming from theoretical uncertainties are mostly obtained by changing parameters in the Monte Carlo simulation, and assessing the shift of the fitted value of \(m_t^{\text{out}}\) while keeping the original template fit function.

In experimental analyses, these templates are constructed at the detector level, i.e. mimicking real data. Here, an analogous procedure is employed to assess the impact of the NLO corrections and their theoretical uncertainties on the \(mb\) method used to determine the top quark mass. We follow as closely as possible the procedure detailed in Ref. [24]. More specifically, the goal of this study is to identify the size of mass shifts, which one can expect solely from scale variations of the NLO theory. Therefore, the templates are constructed and analyzed at the parton level. This means that any smearing occurring from the parton level to the detector level, i.e. from the simulated to the observed distributions, cannot be addressed here. However, in this procedure, the top quark mass, \(m_t\), can be identified with the top quark pole mass, since in our investigations, we only rely on parton level pertur-
Figure 8: The normalized parton level $m_{lb}$ distribution calculated at NLO in the full approach for three different top quark masses, utilizing the $\mu = \hat{H}_T/2$ scale. Also shown is a comparison to the LO prediction (dashed line) obtained at the default top quark mass value, which is $m_t = 172.5$ GeV.

To illustrate the sensitivity of $m_{lb}$ to the top quark mass, we show the normalized $m_{lb}$ distributions for three different values of $m_t$ in Figure 8. Comparing the observed differences to Figure 7b reveals why shape changes of the order of 20% will significantly influence the measurement of $m_t$.

In this analysis, the pseudo-data mimicking experimental data (i.e. the data model) are always generated from the NLO predictions simulating a data luminosity of 4.7/fb as was analyzed in Ref. [24]. In contrast, the templates (i.e. the theory hypothesis) are either taken from the NLO or the LO predictions (using the same scale settings), and are referred to as NLO or LO templates, respectively. Because we have found sizeable differences in the predicted shape of the $m_{lb}$ distribution between the full and factorized approach, we have performed our investigations separately, relying on either the full calculations or the factorized ones. For each of these calculational scenarios, we investigate the impact on the top quark mass measurement caused by two aspects, namely the scale variations and the shape modifications arising from NLO corrections. To study the latter aspect, we switch from the NLO to the LO description of our template theory model. The results for the full calculations using $\mu = \hat{H}_T/2$ are summarized in Figure 9, while Figure 10 displays those of the factorized approach at $\mu = m_t$. The two scenarios are discussed in turn.

The points in Figure 9a show a pseudo-data set, i.e. one possible experimental outcome. This pseudo-data set was generated from the NLO prediction of Figure 8 at
Figure 9: Results from pseudo-data sets generated from the NLO calculation in the full approach. In (9a), one NLO pseudo-data set (black points) at $m_t^{\text{in}} = 172.5$ GeV is shown together with its fit (red line) and the underlying NLO template (black histogram). The mean value of $m_b$ for the template is denoted by $\langle m_b \rangle$. The predictions regarding the difference $m_t^{\text{out}} - m_t^{\text{in}}$ (i.e. the $m_t$ offset) are depicted in (9b) for three input values of $m_t$. These results were computed from many pseudo-data sets analyzed with a theory model based on NLO (red) or LO (blue) templates constructed from full calculations, and using $\mu = \hat{H}_T/2$. The points show the observed mean differences in $m_t^{\text{out}} - m_t^{\text{in}}$, together with their statistical uncertainty corresponding to a luminosity of 4.7 fb. The horizontal lines stem from a fit of the three points to a constant, displaying the average offset. The bands indicate the offset observed when replacing the NLO pseudo-data by the ones obtained from the NLO scale variation samples.

$m_t^{\text{in}} = 172.5$ GeV, which in this figure is shown as the (black) histogram. The result of the template fit using the NLO templates is displayed as the (red) line in this figure. Within the sizeable variations of the data points given the data statistics, the fit coincides with the underlying theory hypothesis, demonstrating the internal consistency of the method. Given these uncertainties, with the presently available luminosity, the shape differences of the LO and NLO templates seen in Figure 8 cannot be discriminated from experimental data, but will be compensated for by a different fit-value obtained for $m_t$.

The sensitivity to the theoretical assumptions and their uncertainties is assessed by fits to one thousand pseudo-data sets. For three different values of $m_t^{\text{in}}$, Figure 9b shows the observed difference of $m_t^{\text{out}}$, the mass measured by the procedure, and $m_t^{\text{in}}$, the one used to generate the pseudo-data. The red points correspond to the mean difference observed for all pseudo-data sets that are produced as in Figure 9a and analyzed with the NLO templates. The uncertainty per point is statistical only and corresponds to the expected experimental uncertainty for the assumed data luminosity. The red band corresponds to the scale uncertainty on the measured top quark mass obtained by replacing the pseudo-data with those from the scale variation NLO samples, while keeping the original NLO templates.
Figure 10: Same as Figure 9, but for pseudo-data sets and templates generated from the factorized calculations using $\mu = m_t$. Note that the vertical axes’ ranges of both figures are different from the corresponding ones in Figure 9.

The resulting uncertainty is significantly larger than the statistical precision, and of similar size as the total theoretical systematic uncertainty assigned to the experimental result [24].

It has been shown in Figures 6a and 8 that, at the same top quark mass, the predicted $m_{lb}$ distributions calculated in the full approach at LO and NLO are significantly different. The flatness of the leading order scale variation band in Figure 6a shows that the LO scale variations – although strongly affecting the cross section – introduce only small shape distortions into the $m_{lb}$ distribution. Given that the shape changes observed at NLO are significant, determining shape dependent observables assuming LO predictions as theory model will inevitably suffer from this shortcoming of the LO prediction. Nevertheless, to assess the size of the effect for full calculations, we also performed a determination of the top quark mass using the LO templates, still based on the NLO pseudo-data sets. This mimics the situation in which LO templates are used to measure the top quark mass from data that actually resemble the NLO prediction. The result of this is shown as blue points in Figure 9b. In this parton level investigation, the difference $m_t^{\text{out}} - m_t^{\text{in}}$ turns out to be about $-1.9$ GeV. Consequently, a sizeable (but different) offset is expected when using LO predictions for experimental top quark mass measurements. In this situation, the data would also suffer from the scale variation uncertainties, as can be seen from the blue band, obtained by generating the pseudo-data sets in the same way as for the red band, but keeping the LO templates as theory model. Clearly, although not properly assessable within the LO description of the theory model, the effect would be present in data.

Using exactly the same strategy, the results for the factorized approach are shown in Figure 10. Again, the implications of scale variations and template modelling at different orders in $\alpha_s$ are discussed in turn. For the factorized calculation, the size of the red band reflecting the scale variation uncertainty is about $\pm 0.2$ GeV, which is significantly smaller than the one of the full calculation, where it amounts to about $\pm 0.6$ GeV. This
is a direct consequence of the distinct sizes of the observed shape differences in Figures 6 and 7. The $m_{lb}$ shape variations predicted by the factorized calculations at NLO are very small. However, this only happens because important effects from non-factorizing contributions and higher-order corrections to the top quark decays are not captured by this approximation. Consequently, the larger uncertainty on $m_t$ observed for the full calculation is certainly more realistic than the small one predicted when using the factorized approach. The mean values $\langle m_{lb} \rangle$ of the NLO templates shown in Figures 9a and 10a, which are obtained for the same $m_t^{\text{in}}$, are different by almost 2 GeV. This is a manifestation of the sizeable differences in the predicted $m_{lb}$ distributions for the two NLO calculations. In addition, the predicted cross section of the factorized approach is about 20% higher than the one from the full approach.

To investigate the differences between the NLO and LO description in the factorized approach, we follow the same procedure as for the full calculation, and use NLO pseudo-data together with LO templates. The results are presented in Figure 10b, and as before, they are shown in blue. For the factorized approach, we observe a much smaller scale variation band than for the full approach shown in Figure 9b. In addition, a much reduced $m_t$ offset is found from analyzing NLO pseudo-data with LO templates. It amounts to only about 0.5 GeV. Both effects are caused by the behaviour of the $m_{lb}$ distributions presented in Figures 6b and 7b. In the region of largest sensitivity, the NLO corrections alter the $m_{lb}$ shape as given at LO to a much smaller extent than what is observed for the full calculation. For the factorized approach, sizeable shape differences only occur in the high mass tail of the $m_{lb}$ distribution, whose impact is largely suppressed compared to the peak region due to the small cross section.

### 3.3 Top quark asymmetries

Having both the full and factorized NLO computation (cf. Section 2.1, item (I) and (II), respectively) for the production of the $W^+W^-b\bar{b}$ final state at hand, we are in a convenient position to take a closer look at how finite width effects and non-factorizing contributions impact top quark asymmetries as measured at the LHC and at the Tevatron [1–14]. We treat the leptonic decays of the $W$ bosons in a way such that the spin correlations are preserved. This allows us to study to what extent the lepton-based asymmetries inherit the effects on top quark asymmetries.

The symmetry of the LHC’s $p\bar{p}$ initial state makes it impossible to write down a forward-backward asymmetry variable as known from the Tevatron experiments. The difference in the broadness of the respective rapidity distributions for the top quark ($y_t$) and the antitop quark ($y_{\bar{t}}$) can however be exploited to define the (commonly used) charge asymmetry:

$$A_C^{\bar{t}t} = \frac{\sigma(\Delta |y| > 0) - \sigma(\Delta |y| < 0)}{\sigma(\Delta |y| > 0) + \sigma(\Delta |y| < 0)}.$$  

(3.2)

Using $\Delta |y| = |y_t| - |y_{\bar{t}}|$, this asymmetry is accessible to the LHC experiments. In fact it has been measured by the ATLAS [8, 10] and CMS [11, 12, 14] collaborations. Based on the current results, the asymmetry is found to be in agreement with the Standard
Model predictions. The leptonic charge asymmetry in the dilepton channel, $A^{C}_{ll}$, is defined analogously, where one replaces $|\Delta y|$ in Eq. (3.2) by $|\Delta \eta| = |\eta_l^+ - |\eta_l^-|$. 

Using the kinematic constraints detailed in Section 2.3, we summarize our predictions based on the full approach for the top quark and leptonic charge asymmetry below, reading

$$A^{C}_{t\bar{t}} = 0.008 \pm 0.003 \quad \text{and} \quad A^{C}_{ll} = 0.005 \pm 0.003,$$

respectively. The theoretical uncertainties of these asymmetry values have been estimated from standard scale variations. We found good agreement with the results stated in Ref. [38]. The currently measured experimental values in the dilepton channel are $A^{C}_{t\bar{t}} = 0.057 \pm 0.028$ and $A^{C}_{ll} = 0.023 \pm 0.014$ (ATLAS, 7 TeV) [9] as well as $A^{C}_{t\bar{t}} = 0.050 \pm 0.043^{+0.010}_{-0.039}$ and $A^{C}_{ll} = 0.010 \pm 0.016$ (CMS, 7 TeV) [13]. Note that the comparison to these values can only be of qualitative nature, owing to the different kinematical selections used in our study as opposed to the experiments.

The relation between top quark and leptonic asymmetries has also been studied at the Tevatron where, owing to the $p\bar{p}$ initial state, forward-backward quantities are very meaningful [1, 2, 4, 6, 7]. In fact, these variables are more sensitive to the underlying asymmetry effect. The top quark forward-backward asymmetry $A^{FB}_{t\bar{t}}$ is defined as

$$A^{FB}_{t\bar{t}} = \frac{\sigma (\Delta y > 0) - \sigma (\Delta y < 0)}{\sigma (\Delta y > 0) + \sigma (\Delta y < 0)}$$

where $\Delta y = y_t - y_{\bar{t}}$ and $y_t$ again denotes the rapidity of the top quark. For $t\bar{t}$ production calculated at leading order in QCD, this asymmetry vanishes. The first non-zero contribution to $A^{FB}_{t\bar{t}}$ appears at NLO. Measurements of $A^{FB}_{t\bar{t}}$ at the Tevatron [1–5] give significantly larger values than the Standard Model prediction [89–92]. To help resolve the discrepancy, several suggestions have been made with the aim to obtain additional handles in the measurements, cf. for example Refs. [45, 93–99].

The disadvantage of the top quark asymmetry is that it cannot be measured directly. As the top quarks have to be reconstructed from their decay products and the missing transverse momentum, all experimental results on $A^{FB}_{t\bar{t}}$ depend on the respective reconstruction method. To avoid any potential bias introduced as a result of the kinematic procedure employed to reconstruct the top quark momenta, several lepton-based asymmetries were designed which only depend on the object selection. The drawbacks of the leptonic asymmetries are their decay-channel specific definition and lower sensitivity to detect the actual asymmetry. The effect as seen in $A^{FB}_{t\bar{t}}$ will be washed out. However, as for example pointed out in Ref. [95], this becomes less of an issue once the top quarks are sufficiently boosted such that the leptons mostly follow the respective top quark directions. As shown in [95], a convenient quantity to tune this correlation between $A^{FB}_{t\bar{t}}$ and lepton-based asymmetries is given by the lepton transverse momentum, $p_{T,l}$.

Our calculations concern the dilepton channel, for which we can employ a commonly used leptonic asymmetry that reads

$$A^{FB}_{ll} = \frac{\sigma (\Delta \eta > 0) - \sigma (\Delta \eta < 0)}{\sigma (\Delta \eta > 0) + \sigma (\Delta \eta < 0)}$$

(3.5)
Figure 11: Double differential cross section $\frac{1}{\sigma} \frac{d\sigma}{d\Delta y d\Delta \eta}$ at the Tevatron in dependence on the rapidity differences $\Delta y$ and $\Delta \eta$. Dilepton channel predictions for $W^+W^-b\bar{b}$ production in the full approach normalized by the respective total cross section are shown at LO (11a) and NLO (11b). The difference between the individual NLO and LO two-dimensional shapes is also visualized (11c).

It is based on defining $\Delta \eta = \eta_{l^+} - \eta_{l^-}$ where $\eta_{l^\pm}$ denotes the pseudo-rapidity of the charged leptons. To get a better understanding of the relation between $A_{FB}^{t\bar{t}}$ and $A_{FB}^{ll}$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, we first consider the normalized double differential cross section depending on both rapidity difference measures, i.e. $\frac{1}{\sigma} \frac{d\sigma}{d\Delta y d\Delta \eta}$. We use the fixed scale choice $\mu = m_t$, with $m_t = 172.0$ GeV, and a $b$-flavour sensitive anti-$k_T$ jet algorithm that allows us to keep track of the total $b$ charge contained in the jet, as suggested in Refs. [100, 101], and impose kinematical requirements reading:

$$
\Delta R > 0.4, \quad |\eta_b| < 2.5, \quad |\eta| < 2.5, \quad \not{p}_T > 25 \text{ GeV}, \quad p_{T,b} > 20 \text{ GeV}, \quad p_{T,l} > 20 \text{ GeV}.
$$

To reconstruct the top quarks in our parton level simulation, we first recombine the four final state leptons into the two $W$ bosons according to the Monte Carlo information. We then employ the $b$ jet charge information, which we get from the $b$-flavour specific jet algorithm, to assign exactly one $b$ jet to each $W$ boson. Events that cannot be analyzed this way are disregarded. Additional jets arising from real radiation are tested kinematically whether they belong to the pseudo-top or pseudo-antitop quark candidate. The results which we obtained this way at LO and NLO in the full approach are shown in Figure 11, which is also used to visualize the relative difference between these normalized distributions (cf. Figure 11c). Based on Figure 11, the impact of the NLO corrections on $\Delta y$ and $\Delta \eta$ can be assessed in a convenient manner. We observe that the NLO corrections to the total cross section cause a shift of both $\Delta y$ and $\Delta \eta$ to larger values, where slightly stronger shifts are seen for $\Delta y$.

---

3 Other procedures neglecting $b$-jet truth information were tested, including one based on the sole kinematic reconstruction of the top quark objects. In this parton level analysis, we however did not observe any significant changes in our results.
Figure 12: Fractions of $W^+(e^+\nu_e)W^-(\mu^-\bar{\nu}_\mu)b\bar{b}$ events in four different kinematic areas defined by combinations of positive and negative $\Delta y$ and $\Delta \eta$ regions (see text for the details). Tevatron parton level predictions of NLO calculations in the full approach are shown for two different scale choices, $\mu = m_t$ and $\mu = m_{t\bar{t}}/2$.

Figure 11 emphasizes the importance of an accurate description of final states containing $b$ jets, two oppositely charged leptons and missing energy. Any statement based on how the different asymmetries, $A_{t\bar{t}}^{FB}$ and $A_{ll}^{FB}$, are correlated heavily relies on the robustness of the Standard Model prediction for this final state, whose major contributor is $W^+W^-b\bar{b}$ production including the leptonic decays. This motivated us to investigate the behaviour of the $A_{t\bar{t}}^{FB} - A_{ll}^{FB}$ correlation under different minimal transverse momentum constraints, $p_{T,l}^{\text{min}}$, applied to the lepton momenta, thereby re-visiting part of the ideas of Ref. [95]. Starting from Figure 11, we now simplify the parametrization of the $\Delta y - \Delta \eta$ space by partitioning it into four kinematic regions, which we label $++$, $+-$, $-+$ and $--$ according to $\Delta y > 0 \& \Delta \eta > 0$, $\Delta y > 0 \& \Delta \eta < 0$, $\Delta y < 0 \& \Delta \eta > 0$ and $\Delta y < 0 \& \Delta \eta < 0$, respectively. The $\pm\mp$ bins may suffer from the lower statistics, but the decomposition is simply a compromise between experimental accessibility and theoretical detail whose verification will be of great value for the experiments. Figure 12 displays the fractions of (unweighted) events populating these four bins for five different values of $p_{T,l}^{\text{min}}$. We show the results obtained for two different scale choices in the full NLO approach. Both predictions – the one using the fixed scale $\mu = m_t$ and the one using the dynamical scale $\mu = m_{t\bar{t}}/2$ where $m_{t\bar{t}}$ denotes the pair’s invariant mass – give very similar results.\footnote{The evaluation of the $m_{t\bar{t}}$ scale proceeds via the identification of the two leading $b$ jets and combining}
increasing $p_{T,l}^{\text{min}}$, the relative weight of the ++ and −− bins rises further.

To quantify this in terms of the asymmetries stated in Eqs. (3.4) and (3.5), we respectively evaluate the dependence of $A_{tt}^{FB}$ and $A_{ll}^{FB}$ on the imposed minimal transverse momentum of the charged leptons. The results are depicted in Figure 13 for five different values of $p_{T,l}^{\text{min}}$. This time we have included the predictions from the factorized approach to enable direct comparison between the two calculational approaches, hence estimating the effects missed by the factorized description. The scale choices utilized to produce the results are, as before, $\mu = m_t$ and $\mu = m_{tt}/2$. We observe, in accordance with the findings above (and within the Monte Carlo statistics achieved) that the difference between $A_{tt}^{FB}$ and $A_{ll}^{FB}$ decreases with increasing $p_{T,l}^{\text{min}}$. It also becomes clear that the absolute change of $A_{tt}^{FB}$ and $A_{ll}^{FB}$ with $p_{T,l}^{\text{min}}$ rather strongly depends on the scale choice. As can be seen, increasing the $p_T$ threshold for the charged leptons causes the asymmetries to rise faster once we rely on the fixed scale ($\mu = m_t$) instead of the dynamical ones ($\mu = m_{tt}/2$). Almost no rise can only be found for the two top quark asymmetry predictions obtained with the dynamical scale choice. These turn out to be rather constant over the $p_{T,l}^{\text{min}}$ range investigated here, but differ in that the factorized prediction has dropped by $\sim 15\%$ below the full one. This trend is more general; in comparison to the NLO $t\bar{t}$ approach, the full $WWb\bar{b}$ treatment is found to generate systematically larger asymmetries. Yet, the difference is not them with the four leptons to the invariant mass of a $t\bar{t}$-like system.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13.png}
\caption{Dependence of the $t\bar{t}$ and leptonic forward-backward asymmetries on the kinematical requirement concerning the minimal charged lepton transverse momentum $p_{T,l}^{\text{min}}$. Different lines belong to different scale choices; the solid and dashed lines respectively correspond to the full ($WWb\bar{b}$) and factorized ($t\bar{t}$) approach in calculating $W^+W^-b\bar{b}$ production at NLO in the dilepton channel at the Tevatron. Both types of calculations are explained in Section 2.1. The vertical bars denote the Monte Carlo statistical uncertainties.}
\end{figure}
Figure 14: Ratio between the leptonic and top quark forward-backward asymmetries at the Tevatron, as a function of $p_{T,l}^{\text{min}}$. The solid (dashed) lines represent the outcomes of the full (factorized) QCD NLO corrections to $W^+W^-b\bar{b}$ final states contributing to the dilepton channel at $\mathcal{O}(\alpha_s^2\alpha^2)$. The vertical bars denote the Monte Carlo statistical uncertainties. Note that the predictions associated with the highest $p_{T,l}^{\text{min}}$ requirement suffer from low statistics. For better visibility of the individual results, these points therefore have been slightly shifted along the horizontal axis.

sufficient to reconcile the theory predictions with the current experimental measurements. Taking the result for the lowest $p_{T,l}^{\text{min}}$ cut and the $m_t$ scale choice, we note good agreement with the results stated in Ref. [91], although slightly tighter pseudo-rapidity constraints (namely $|\eta_{b,l}| \leq 2.0$, cf. Eqs. (3.6)) were used in this work. For the experimental status, see Refs. [3, 6, 7].

The ratio $A_{FB}^{ll}/A_{FB}^{t\bar{t}}$ is known to be less affected by scale choices and uncertainties. We therefore show these ratios for all our different choices in Figure 14. Indeed we find these ratio predictions to be more robust, and conclude that this quantity can be predicted more reliably than the absolute behaviour of the asymmetries. Again, the trend of increasing correlation between the two types of asymmetries can be seen for events containing more strongly boosted leptons.

4 Conclusions

We have calculated the NLO QCD corrections to the processes $pp (p\bar{p}) \rightarrow W^+W^-b\bar{b} \rightarrow (e^+\nu_e)(\mu^-\bar{\nu}_\mu)b\bar{b}$ in the 5-flavour scheme, including non-resonant diagrams and singly resonant top quark contributions, using the automated one-loop generator GoSam in com-
bination with the Monte Carlo program SHERPA. We also performed an NLO calculation of top quark pair production in the narrow width approximation supplemented by LO top quark decays, enabling us to assess the impact of the non-factorizing contributions at NLO. We found a reduction of the scale dependence of the total cross section from about 30% at LO to about 5% at NLO, and a significant impact of the non-factorizing contributions on the shape of the distribution for certain observables, for example the invariant mass of a lepton and a $b$ jet, $m_{lb}$.

We also presented a detailed study of NLO effects in $m_t$ measurements based on the $m_{lb}$ observable, making contact to a recent ATLAS analysis, which uses a template method [24]. Using the factorized calculation, we observed only small shape distortions in the $m_{lb}$ distribution originating from NLO corrections. The situation changes for the full approach where such distortions turn out to be substantial. The size of the resulting parton level prediction for the $m_t$ offset, when using LO templates as the theory model, was investigated using a pseudo-data parton level analysis closely following the ATLAS strategy. For the full approach, the offset was found to be non-negligible, amounting to about 1.9 GeV. This has to be contrasted with a considerably smaller offset of about 0.5 GeV in the factorized approach. In addition, we estimated the uncertainty on $m_t$ resulting from standard NLO scale variations. For the full approach, it was found to be of the order of 1 GeV. This is much larger than the small uncertainty of about 0.2 GeV, which we evaluated for the factorized approach. Clearly, further investigations using fully simulated events are needed to properly assess the corresponding uncertainty on $m_t$ within the experimental analysis. However, given our findings, and considering the fact that presently, the experimental analyses are based on the factorized approach, a larger uncertainty is likely to be found.

Finally, we focused on a study of top quark asymmetries where we investigated the impact of the NLO corrections to $W^+W^-b\bar{b}$ production on both the top quark and leptonic charge asymmetry at the LHC, and the top quark and leptonic forward-backward asymmetry at the Tevatron. Our study centered on a more detailed investigation of the correlation between the top quark and the leptonic forward-backward asymmetries. In particular, we showed how this correlation changes as a function of the kinematic requirement on the minimum transverse momentum of the charged leptons, $p_{T,l}^\text{min}$, quantifying its sensitivity to different scale choices. The difference between the top quark and leptonic asymmetry is observed to decrease for increasing $p_{T,l}^\text{min}$. While the individual absolute asymmetries depend rather strongly on the employed scale choice, the ratio $A_T^p/A_{Tl}^p$ is found to be less sensitive under such variations. Invoking the factorized approach, the NLO corrections to $W^+W^-$ production in association with two $b$ jets yield smaller asymmetry values throughout. This reduction is not seen for the asymmetry ratios. The effect drops out and we obtain predictions similar to those of the full approach.

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