We revisit an extension of the MSSM by adding a hypercharge-neutral, SU(2)-triplet chiral superfield. Similar to the NMSSM, the triplet gives an additional contribution to the quartic coupling in the Higgs potential, and the mass of the lightest CP-even Higgs boson can be greater than $M_Z$ at tree-level. In addition to discussing the perturbativity, fine-tuning, and decoupling issues of this model, we compute the dominant 1-loop corrections to the mass of the lightest CP-even Higgs boson from the triplet sector. When the Higgs-Higgs-Triplet coupling in the superpotential is comparable to the top Yukawa coupling, we find that the Higgs mass can be as heavy as 140 GeV even without the traditional contributions from the top-s-top sector, and at the same time consistent with the precision electroweak constraints. At the expense of having Landau poles before the GUT scale, this opens up a previously forbidden region in the MSSM parameter space where both s-tops are light. In addition to having relatively small fine-tuning (about one part in 30), this leads to a gluophilic Higgs boson whose production via gluon-gluon fusion at the LHC can be twice as large as the SM prediction.

I. INTRODUCTION

The electroweak sector of the standard model (SM) predicts new physics at sub-TeV scales to unitarize $W$-scattering. With a Higgs boson, the hierarchy problem suggests additional new physics near the TeV scale to stabilize the electroweak scale, and the minimal supersymmetric standard model (MSSM) is one of the leading candidates of such new physics. For reviews of the MSSM, see, for examples, Drees [1], Martin [2], Dine [3], and Peskin [4].

In the MSSM, the mass of the lightest $CP$-even Higgs boson is bounded at tree-level by $M_Z$, because the tree-level quartic couplings are parameterized by gauge couplings, and such a light Higgs boson is ruled out by the CERN LEP searches of the SM Higgs boson [5] [6] [7] [8] [9] that impose $m_{h}^{SM} > 114.4$ GeV. At one-loop level, however, there can be large radiative corrections due to heavy scalar tops ($\tilde{Q}_3$ and $\tilde{U}_3$, superpartners of the top-quark) and/or a large coupling of the trilinear interaction $\tilde{Q}_3 H_u \tilde{U}_3$ [10] [11] [12] [13] [14] [15] [16] [17] [18] [19]. While such radiative corrections can be large enough to satisfy the LEP bounds, they also contribute to the quadratic term of the Higgs potential, leading to the “little hierarchy problem”. The MSSM also suffers from a $\mu$-problem in that its lone dimensionful SUSY-invariant parameter, $\mu$, is phenomenologically required to be of order 100 GeV, while its natural scale can in principle be much larger.

The next-to-minimal supersymmetric standard model (NMSSM) solves the $\mu$-problem and alleviates the little hierarchy problem [20] by extending the MSSM with a singlet chiral superfield $S$. For reviews of the NMSSM, see Balazs et al. [21] and references therein. The Higgs couplings with $S$ lead to additional contributions to the quartic couplings in the Higgs potential, while the $\mu$-term is dynamically generated from the vacuum expectation value (vev) of the scalar component of $S$. With these additional contributions to the quartic couplings, the mass of the lightest $CP$-even Higgs boson may be larger than $M_Z$ at tree-level, and the NMSSM can satisfy the LEP bounds on the Higgs mass with lighter s-tops compared to the MSSM [22] [23] [24] [25] [26] [27] [28].
In this paper, we extend the MSSM with a hypercharge-neutral, $SU(2)$-triplet chiral superfield $T$ and name the model triplet-extended supersymmetric standard model (TESSM). Extensions of this type have been studied extensively by Espinosa and Quiros [29], Felix-Beltran [31], Setzer and Spinner [32], and Diaz-Cruz et al. [33]. While this model does not solve the $\mu$-problem, it is an interesting alternative to the NMSSM, as an economical extension of the MSSM, because it can also achieve a mass of the lightest $CP$-even Higgs boson that is larger than $M_Z$ at tree level. Furthermore, compared to the MSSM and the NMSSM, we expect there to be more radiative corrections to the mass of the lightest $CP$-even Higgs boson due to the additional states in the triplet. To the extent that these triplet-induced radiative corrections are significant, we may further alleviate the little hierarchy problem.

Unfortunately, in both the NMSSM and the TESSM, the respective singlet-induced and triplet-induced radiative corrections are typically small when we demand perturbativity at the scale of grand unified theory (GUT) near $10^{16}$ GeV. This is because perturbativity at the GUT scale imposes the bound $\lambda \lesssim 0.7$ at the weak scale, where $\lambda$ is respectively the Higgs-singlet-Higgs and the Higgs-triplet-Higgs coupling in the superpotential of the NMSSM and TESSM. In both models, while the tree-level mass of the lightest $CP$-even Higgs boson can be as large as 100 GeV, the $O(\lambda^4)$ radiative corrections are not large enough to lift the Higgs mass over the LEP bounds. On the other hand, in the TESSM, when we have $\lambda \sim 0.9$ (so that $\lambda$ is comparable with the top Yukawa coupling) at the weak scale, we find the tree-level mass of the lightest $CP$-even Higgs boson to be close to the LEP bound and the $O(\lambda^4)$ radiative corrections alone can easily lift the Higgs mass over the LEP bound even with small SUSY-breaking in the triplet sector. As the small SUSY-breaking in the triplet sector translate into small fine-tuning, we can solve the little hierarchy problem at the expense of giving up perturbativity at the GUT scale.

Without demanding perturbativity at the GUT scale, we also expect the NMSSM to be a solution to the little hierarchy problem, with the mass of the lightest $CP$-even Higgs boson that satisfies the LEP bounds without significant contribution from the top-s-top sector. However, as an alternative to the NMSSM and a reasonably economical extension of the MSSM, the TESSM and its phenomenology are interesting in their own right. For example, as we show in this paper, the MSSM limit of the TESSM is achieved with $M_T \rightarrow \infty$, where $M_T$ is the SUSY-invariant mass of the triplet, keeping $\lambda$ fixed, whereas in the NMSSM one requires $\lambda \rightarrow 0$ to achieve the MSSM limit. As another example, even though the sub-TeV, electrically-neutral component of the triplet acquires a vev, we can still satisfy the precision electroweak constraints without the extreme fine-tuning noted in the triplet-extended SM [34]. Moreover, there may be other considerations that motivate extending the MSSM by a triplet instead of a singlet. For example, in obtaining neutrino masses through the Type-II [35] and Type-III seesaw mechanisms, the SM is commonly extended with Higgs triplets. Though the Higgs triplets may have nonzero hypercharge, hypercharge-neutral triplets are often present when the models are supersymmetrized and embedded in a unified gauge group [32][36][37].

We organize our paper as follows. In Sec. [I] we lay out the superpotential and the Lagrangian of the TESSM, compare it to the NMSSM, and discuss constraints on its parameter space from electroweak precision tests and the requirement of perturbativity at the GUT scale. In Sec. [III] we numerically evaluate the mass of the lightest, $CP$-even Higgs boson to one-loop, and show that we can satisfy the LEP2 bounds without the contributions from the top-s-top sector when $\lambda$ is comparable with the top Yukawa coupling. We also discuss the gluon-gluon fusion production and diphoton decay of the lightest, $CP$-even Higgs boson in Sec. [III]. Our discussions of the gluon-gluon fusion production rely only on the existence of light s-tops and the minimal color sector of the MSSM, and are therefore applicable to any extensions of the MSSM that solves the little hierarchy problem without invoking additional colored states. In Sec. [IV] we estimate two sources of fine-tuning in this model, and find that we can achieve a small fine-tuning of about one part in 30 in the Higgs sector. Finally, we conclude with Sec. [V] that summarizes our results.
II. TRIPLET-EXTENDED SUPERSYMMETRIC STANDARD MODEL

A. The Model

We extend the MSSM with a hypercharge-neutral, $SU(2)$-triplet $T \equiv \frac{1}{2} \sigma^{A}T^{A}$ with the superpotential

$$W_{TESSM} = \mu H_{d}H_{u} + M_{T} \text{Tr}(TT) + 2\lambda H_{d}TH_{u} + \alpha_{T} \text{Tr}(T) + W_{Yukawa},$$

where $H_{u,d}$ are the Higgs doublets of the MSSM, $\alpha_{T}$ is a Lagrange-multiplier determined from the potential, and $W_{Yukawa}$ is the MSSM superpotential sans the $\mu$-term

$$W_{Yukawa} = y_{t}QH_{u}U + y_{b}QH_{d}D + y_{\tau}LH_{d}E.$$  

Note that, since $T$ is a chiral superfield, its scalar component necessarily contains a complex $SU(2)$-triplet, whereas in non-SUSY extensions of the SM [38][39][40][41][42][34], we can extend the SM with a real $SU(2)$-triplet. In components, we have the fields

$$H_{u} = \begin{pmatrix} H_{u}^{+} \\ H_{u}^{0} \end{pmatrix}, \quad H_{d} = \begin{pmatrix} H_{d}^{0} \\ H_{d}^{-} \end{pmatrix}, \quad T = \frac{1}{2} \begin{pmatrix} T^{0} \\ \sqrt{2}T^{-} \\ -T^{0} \end{pmatrix},$$

and the superpotential (sans the SM Yukawa couplings)

$$W_{TESSM} \supset \mu(H_{u}^{+}H_{d}^{-} - H_{u}^{0}H_{d}^{0}) + \frac{M_{T}}{2} (T^{0}T^{0} + 2T^{-}T^{-}) + \lambda(H_{u}^{0}T^{0}H_{u}^{0} + H_{d}^{-}T^{0}H_{u}^{+}) + \sqrt{2}\lambda(H_{d}^{-}T^{+}H_{u}^{0} - H_{d}^{0}T^{-}H_{u}^{+}).$$

The factor of 2 in front of $\lambda$ in Eq. 2 gives us a coefficient of unity for the term

$$W_{TESSM} \supset \lambda H_{d}^{0}T^{0}H_{u}^{0},$$

as with the case of the NMSSM when $T^{0}$ is replaced by a singlet $S$, and facilitates direct comparisons between TESSM and NMSSM.

We can achieve gauge coupling unification at $M_{GUT}$ by including additional chiral superfields with quantum numbers

$$D \sim (1, 2)\frac{1}{2}, \quad \bar{D} \sim (1, 2)\frac{-1}{2}, \quad G \sim (8, 1)_{0},$$

where the first and second entries inside the parenthesis denote, respectively, the representations under the color $SU(3)_{c}$ and weak $SU(2)_{w}$ gauge groups, and the subscripts denote the charge under hypercharge $U(1)_{Y}$ gauge group. This added content can have both SUSY-invariant and SUSY-breaking masses sufficiently large (say 2 TeV’s) so that they decouple from the electroweak scale physics, while still allowing for gauge coupling unification. The added matter content (triplet plus those in Eq. 7) does not constitute a complete multiplet of $SU(5)$, but can form a complete multiplet of trinification group $SU(3)^{3} \times Z_{3}$ [43][44][45].

In addition to the MSSM soft SUSY-breaking parameters, we also have soft terms involving $T$

$$-\Delta \mathcal{L} = 2m_{T}^{2} \text{Tr}(T^{+}T) + B_{T}(\text{Tr}(TT) + \text{h.c.}) + 2\lambda A_{T}(H_{d}TH_{u} + \text{h.c.}).$$

B. Comparison to the NMSSM

1. Perturbativity

For simplicity, we assume that all couplings and masses in the superpotential are real. The tree-level potential involving the $U(1)_{em}$-neutral Higgs doublets and triplet is then

$$V_{TESSM} = V_{H} + V_{T} + V_{\text{mix}},$$
where

\[ V_H = (\mu^2 + m_{H_u}^2)|H_u^0|^2 + (\mu^2 + m_{H_d}^2)|H_d^0|^2 - B_\mu(H_u^0 H_d^0 + c.c.) \]
\[ + \frac{1}{8}(g_2^2 + g_1^2)(|H_u^0|^2 - |H_d^0|^2)^2 + \lambda^2|H_u^0|^2|H_d^0|^2, \]
\[ V_T = (M_T^2 + m_T^2)|T^0|^2 + \frac{B_T}{2}(T^0 T^0 + c.c.), \]
\[ V_{\text{mix}} = \lambda^2|T^0|^2(|H_u^0|^2 + |H_d^0|^2) \]
\[ + \lambda M_T(H_d^0 T^0 H_u^0 + c.c.) + \lambda A_\lambda(H_u^0 T^0 H_u^0 + c.c.) \]
\[ - \lambda \mu(H_u^0 T^0 H_u^0 + H_d^0 T^0 H_d^0 + c.c.). \]

(10)

(11)

(12)

Compared with the Higgs potential in the MSSM, we have an enhancement in the quartic coupling of the form

\[ V \supset \lambda^2|H_u|^2|H_d|^2, \]

and this in principle allows for a tree-level mass eigenvalue larger than \( M_Z \) after electroweak symmetry breaking (EWSB). This is similar to the case in the NMSSM, where such quartic couplings are also generated from a superpotential of the form

\[ W_{\text{NMSSM}} = \lambda S H_d H_u + \frac{\kappa}{3} S^3. \]

(14)

As with the NMSSM, where the requirement of perturbativity at the GUT scale limits \( \lambda \lesssim 0.7 \) at the TeV-scale, the TESSM also has a bound \( \lambda \lesssim 0.7 \) at the TeV-scale while still preserving perturbativity at the GUT scale. Though the bounds are similar, the details of obtaining such bounds are different and may be important for further model-building where perturbativity at the GUT scale is imposed. We elaborate briefly on some key differences between the two models from the relevant renormalization group equations (RGEs) given by

\[ \beta_\lambda^\text{TESSM} = \frac{\lambda}{16\pi^2} \left( 8\lambda^2 + 3g_1^2 - 9g_2^2 - g_1^2 \right), \]

\[ \beta_\lambda^\text{NMSSM} = \frac{\lambda}{16\pi^2} \left( 4\lambda^2 + 2\kappa^2 + 3g_1^2 - 3g_2^2 - g_1^2 \right), \]

\[ \beta_\kappa^\text{NMSSM} = \frac{\kappa}{16\pi^2} \left( 6\lambda^2 + 6\kappa^2 \right), \]

(15)

(16)

(17)

and note the following points:

- In the NMSSM, there are two possible Landau poles in \( \lambda \) and \( \kappa \). The RGE of \( \kappa \) is such that \( \kappa \) always increases when evolving to higher energies, and \( \kappa \) feeds into the evolution of \( \lambda \). In the TESSM, there is no such contribution because \( \text{Tr}(T^3) = 0 \), but there are now additional contributions to the \( \lambda^3 \) coefficient in \( \beta_\lambda \) in the TESSM.

- In the TESSM, \( \beta_\lambda \) has a larger coefficient for the negative contribution of the form \( \lambda g_2^2 \) because \( T \) is charged under \( SU(2) \). As the coupling \( g_2 \) is non-asymptotically-free in the TESSM (also in the NMSSM), this gives a stronger suppression at higher energies and could potentially delay the appearance of the \( \lambda \) Landau pole.

- The coupling \( g_2 \) also flows to larger values in the TESSM than in the NMSSM because of the added matter content. This again gives a suppression at higher energies and may delay the \( \lambda \) Landau pole. (This can be achieved in the NMSSM with added matter content, for example, in the NMSSM with gauge-mediated SUSY-breaking.)

Thorough studies on the upper bounds of \( \lambda \) in TESSM and its extensions would require examining fixed points from the RGEs, and we leave these investigations for the future. For our work, it suffices to note that perturbativity at the GUT scale imposes \( \lambda \lesssim 0.7 \) at the weak scale, so that \( \lambda \) is of similar strength to the weak gauge coupling. As such, while the tree-level mass of the lightest \( CP \)-even Higgs boson can be 100 GeV (as we later show), we expect the \( \mathcal{O}(\lambda^4) \) radiative corrections to the mass of the lightest \( CP \)-even Higgs boson to be insufficient to lift the
Higgs mass above the LEP bounds. However, motivated by solving the hierarchy problem, we take the view point that the Landau pole we encounter at a higher scale (around $10^{10}$ GeV) is rescued by some other new physics and analyze the Higgs spectra and the phenomenology for larger values of $\lambda$. We take values of $\lambda$ comparable to the top Yukawa coupling, so that the TeV scale physics is still perturbative. As $\lambda$ is now near unity at the TeV-scale in the TESSM and we expect there to be more $\mathcal{O}(\lambda^4)$ radiative corrections to the mass of the lightest $CP$-even Higgs boson compared to the NMSSM, it is worthwhile to investigate these radiative corrections in detail.

In extensions of the NMSSM, such as fat Higgs models [46][47][48][49], $\lambda$ can achieve much larger values and give rise to a very large mass for the lightest $CP$-even Higgs boson. The model-building techniques of fat Higgs models can also be applied to the TESSM, but in this work we focus on the TESSM as a simple extension of the MSSM and an alternative to the NMSSM without imposing the constraint of perturbativity at the GUT scale.

2. The MSSM limit

In the NMSSM, the $\mu$-term and the masses of the singlet(ino) are related by

$$\mu \sim \lambda^{\text{NMSSM}} \langle S \rangle, \quad M_S \sim \kappa^{\text{NMSSM}} \langle S \rangle,$$

and the MSSM limit is $M_S \to \infty$ while keeping $\mu$ fixed. Keeping $\kappa$ perturbative in the MSSM limit then gives $\lambda \to 0$. As $\lambda$ is the only coupling between the singlet and the MSSM sector, the MSSM limit of $\lambda \to 0$ with fixed $\mu$ decouples the singlet.

In the TESSM, the MSSM limit is achieved with $M_T \to \infty$, holding the values of all other masses and couplings at the weak scale fixed, and in particular we do not need $\lambda \to 0$ to achieve the MSSM limit. The decoupling of the additional contribution to the Higgs quartic coupling in Eq. (13) is accomplished by the effective operator obtained by integrating out the heavy triplet fields when $M_T \gg M_Z$. Setting $B_T = 0$ for simplicity, the equation of motion for $T^0$, among other terms, has contributions of the form

$$T^0 = -\frac{\lambda}{M_T^2 + m_T^2} \left( M_T H_u^0 H_d^0 + A H_u^0 H_d^0 - \mu (H_u^0 H_d^0 + H_d^0 H_u^0) \right) + \cdots,$$

and this induces a contribution in the effective Lagrangian

$$-\Delta L_{\text{eff}} = -\lambda^2 \frac{M_T^2}{M_T^2 + m_T^2} |H_d^0|^2 |H_u^0|^2 + \cdots,$$

that cancels the $\lambda^2$ contribution to the quartic in the Higgs potential when $M_T^2 \gg A_T^2, m_T^2, \mu^2$. In terms of Feynman diagrams, this effective operator arises from the diagrams such as the one shown in Fig. 1 with the amplitude (in the limit of large $M_T^2 \gg A_T^2, m_T^2, \mu^2$)

$$iA = \lambda^2 \frac{M_T^2}{p^2 - M_T^2},$$

where $p \sim M_Z$ is the scale of external momenta of the Higgs bosons. In the limit $M_T^2 \gg p^2$, this gives the canceling contribution to the Higgs potential, and the resulting theory is the MSSM.

When we do not explicitly integrate out the heavy triplet sector, the full mass matrices (involving both Higgs doublets and the triplet) provide a seesaw-like mechanism in the limit of $M_T \to \infty$ that seesaws away any $\lambda$ dependence in the Higgs doublets sector. We will demonstrate this in the next section when we compute the mass of the lightest $CP$-even Higgs boson.

C. EWSB in TESSM

As we are assuming real couplings and masses for simplicity, there is no mixing between the real and imaginary components of the complex scalar fields $H_{u,d}^0$ and $T^0$ and it is convenient to separate them into real and imaginary
FIG. 1: An example of Feynman diagrams that give the contributions which cancel λ contributions in the Higgs potential when the triplet field, T, decouples.

\[ H_u^0 = \frac{1}{\sqrt{2}}(a_u + ib_u) = \frac{1}{\sqrt{2}}(a'_u + ib_u) + \frac{1}{\sqrt{2}}v_u, \]  
\[ H_d^0 = \frac{1}{\sqrt{2}}(a_d + ib_d) = \frac{1}{\sqrt{2}}(a'_d + ib_d) + \frac{1}{\sqrt{2}}v_d, \]  
\[ T^0 = \frac{1}{\sqrt{2}}(a_t + ib_t) = \frac{1}{\sqrt{2}}(a'_t + ib_t) + \frac{1}{\sqrt{2}}v_t, \]

where we have also shifted the real components \((a_i)\) to the physical modes \((a'_i)\) by the respective vacuum expectation values \((v_i)\). Prior to EWSB, all the vevs vanish and the real components of the Higgses have the mass matrix (in the basis \((a_u, a_d, a_t)\))

\[
M_a^2 = \begin{pmatrix}
    m_{H_u}^2 + \mu^2 - B_\mu & 0 & 0 \\
    -B_\mu & m_{H_d}^2 + \mu^2 & 0 \\
    0 & 0 & M_T^2 + m_T^2 + B_T
\end{pmatrix}.
\]  

The corresponding mass matrix for the imaginary components can be obtained from Eq. 25 by changing the signs of \(B_T\) and \(B_\mu\) in the \((1,2), (2,1), \) and \((3,3)\) elements.

In the MSSM, the conditions for successful EWSB breaking are that (i) the (top-left 2×2) mass matrix in Eq. 25 has one negative eigenvalue, and (ii) the potential is bounded from below along the \(D\)-flat direction \(H_u^0 = H_d^0\). In the TESSM model, the first condition gives us the same condition as the MSSM

\[ B_\mu^2 > (m_{H_u}^2 + \mu^2)(m_{H_d}^2 + \mu^2), \]

while the second condition is automatically satisfied by the presence of the quartic coupling \(\lambda^2|H_d^0|^2|H_u^0|^2\). However, the minimization conditions now demand

\[ v_t = \sqrt{\frac{2}{\lambda}} \left( \frac{\mu - (A_\lambda + M_T)c_\beta s_\beta}{M_T^2 + m_T^2 + B_T + \frac{\lambda}{2}v^2} \right), \]
\[ m_{H_u}^2 + \mu_{\text{eff}}^2 = t_\beta^{-1}B_\mu + \frac{c_\beta^2 M_Z^2}{2} - \frac{1}{2} \lambda^2 c_\beta^2 v^2 - \frac{\lambda v_t}{\sqrt{2}(M_T + A_\lambda)t_\beta^{-1}}, \]
\[ m_{H_d}^2 + \mu_{\text{eff}}^2 = t_\beta B_\mu - \frac{c_\beta^2 M_Z^2}{2} - \frac{1}{2} \lambda^2 s_\beta^2 v^2 - \frac{\lambda v_t}{\sqrt{2}(M_T + A_\lambda)t_\beta}, \]

where we have defined

\[ \mu_{\text{eff}} \equiv \mu - \frac{1}{\sqrt{2}}\lambda v_t, \]
\[ \tan \beta \equiv \frac{v_u}{v_d}, \]
\[ v^2 \equiv v_u^2 + v_d^2, \]
\[ v_u = v \sin \beta, \quad v_d = v \cos \beta, \]

so that the gauge bosons receive masses of

\[ M_Z^2 = \frac{1}{4}(g_2^2 + g_1^2)v^2, \]
\[ M_W^2 = \frac{1}{4}g_2^2 v^2 + g_1^2 v_t^2, \]
where \( g_2 \) and \( g_1 \) are respectively the gauge couplings of the SU(2) and U(1)_Y groups. We have also abbreviated for convenience the trigonometric functions

\[
s_\beta \equiv \sin \beta, \quad c_\beta \equiv \cos \beta, \quad t_\beta \equiv \tan \beta, \\
s_{2\beta} \equiv \sin 2\beta, \quad c_{2\beta} \equiv \cos 2\beta.
\]  

(35)

D. Oblique Corrections

While the condition of successful EWSB in Eq. 26 gives a constraint on the parameters, electroweak precision tests offer a much more stringent constraint. The induced vev \( v_t \) contributes to the oblique parameter \( \alpha_T \) because it contributes to the mass of the charged gauge bosons \( W^\pm \), but not to that of the neutral gauge boson \( Z \). We find

\[
\alpha \Delta T = \frac{\delta M^2_W}{M^2_W} = 4\frac{v_t^2}{v^2} \\
= \frac{\lambda^2 v^2}{2} \left( \frac{2\mu - (A_\lambda + M_T)\sin 2\beta}{M_T^2 + m_T^2 + B_T + \lambda^2 v^2} \right)^2.
\]

(36)

The oblique correction due to the triplet vanishes in the limit of \( M_T \to \infty \) holding all other parameters fixed, as expected. However, even if \( M_T \) is of the same order of \( \mu \) and \( A_\lambda \), \( \Delta T \) can be small due to a partial cancellation between \( \mu \) and \( \sin 2\beta(A_\lambda + M_T) \).

We impose the constraint that \( |\Delta T| < 0.1 \), which in turn translates to an upper bound on \( v_t \)

\[
|v_t| < 3.43 \text{ GeV},
\]

and provides the main constraint on the parameters \( \lambda \) and \( M_T \). In Fig. 2 we plot the allowed regions on \( M_T - \lambda \) plane with \( B_T = m_T^2 = A_\lambda^2 = (200 \text{ GeV})^2 \), for various values of \( \tan \beta \) and \( \mu \). For small values of \( \tan \beta \) and \( \mu \), \( \alpha \Delta T \) is only viable with either small \( \lambda \) or a cancellation in the numerator of Eq. 36. In Fig. 3 we plot the allowed region in \( \mu - \tan \beta \) plane for \( \lambda = 0.9 \), \( B_T = m_T^2 = A_\lambda^2 = (200 \text{ GeV})^2 \), for various values of \( M_T \). As expected, for larger values of \( M_T \), there is a thicker band on the \( \mu - \tan \beta \) plane that is allowed.

We will quantify the degree of fine-tuning in the cancellation for allowed \( \alpha \Delta T \) in Sec. IV. For now, we may estimate the fine-tuning along the ideas of Athron et al. \[50\]. For example, with the parameters that require a fine-tuning in \( M_T \)

\[
\tan \beta = 3, \quad \lambda = 0.9, \quad \mu = 150 \text{ GeV}, \\
m_T^2 = B_T = A_\lambda^2 = (200 \text{ GeV})^2,
\]

(37)

we have viable \( \Delta T \) in the regions

\[
250 \text{ GeV} < M_T < 375 \text{ GeV}, \text{ or } M_T > 3.0 \text{ TeV}.
\]

(38)

For \( M_T \) below 3.0 TeV, we would typically have unacceptably large \( \Delta T \) that violates precision electroweak constraints except in the a small region of \( M_T \) between 250 GeV and 375 GeV because of cancellations in the numerator of Eq. 36. If we sample \( M_T \) at random in the range between 0 and 3.0 TeV, the only region with viable \( \Delta T \) is only 125(=375-250) GeV wide, and we can thus estimate the fine-tuning as

\[
\frac{3.0 \text{ TeV}}{375 \text{ GeV} - 250 \text{ GeV}} = 24,
\]

(39)

so a cancellation of 1 part in 24 is required to have small \( \alpha \Delta T \) for the parameters in Eq. 37.
FIG. 2: Regions allowed by $\Delta T$ (in gray) on $\lambda - M_T$ plane for various values of $\tan \beta$ and $\mu$ as indicated in each plot.

### E. Neutralino and Charginos

After EWSB, the neutralino ($\tilde{N}$) and chargino ($\tilde{C}$) mass matrices are now extended with the triplet sector. The mass matrix for the neutralinos in the basis $(\tilde{b}, \tilde{w}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{T}^0)$ is given by

$$
M_{\tilde{N}} = 
\begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & 0 \\
0 & M_2 & -\frac{1}{2}g_2 v_d & \frac{1}{2}g_2 v_u & 0 \\
\frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu_{\text{eff}} & 0 & \frac{1}{\sqrt{2}}\lambda v_u \\
\frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu_{\text{eff}} & 0 & \frac{1}{\sqrt{2}}\lambda v_u \\
0 & 0 & 0 & \frac{1}{\sqrt{2}}\lambda v_u & M_T
\end{pmatrix},
$$

(40)
where $M_1$ and $M_2$ are respectively the SUSY-breaking bino and wino masses, and $\mu_{\text{eff}}$ is as defined in Eq. 30.

For the charginos, in the basis $\psi^\pm = (\tilde{\psi}^+, \tilde{H}^+_u, \tilde{T}^+, \tilde{\psi}^-, \tilde{H}^-_d, \tilde{T}^-)$, the chargino mass matrix appears in the Lagrangian as

$$\mathcal{L} = -\frac{1}{2} (\psi^\pm)^T \left( \begin{array}{cc} 0 & M_T^C \\ M_C^T & 0 \end{array} \right) \psi^\pm, \quad (41)$$

where

$$M_C = \left( \begin{array}{ccc} M_2 & \frac{\sqrt{2}}{2} g_2 v_d & g_2 v_t \\ \frac{\sqrt{2}}{2} g_2 v_u & \mu_{\text{eff}} + \sqrt{2} \lambda v_t & -\lambda v_d \\ -g_2 v_t & \lambda v_u & M_T \end{array} \right). \quad (42)$$

### III. Lightest CP-Even Higgs Boson in the TESSM

#### A. Tree-Level Mass

The lightest $CP$-even Higgs boson in TESSM is a linear combination of the $CP$-even components of the Higgs doublets $H_u, d$ and the neutral component of the triplet $T^0$. After EWSB, the squared-mass matrix for the neutral scalar has the entries

$$\begin{align*}
(M_a^2)_{11} &= c_\beta M_A^2 + s_\beta^2 M_Z^2 - \frac{\lambda}{\sqrt{2}} t_\beta v_t (M_T + A_\lambda), \\
(M_a^2)_{12} &= -s_\beta c_\beta (M_A^2 + M_Z^2) + c_\beta s_\beta \lambda^2 v^2 + \frac{\lambda}{\sqrt{2}} v_t (M_T + A_\lambda), \\
(M_a^2)_{13} &= \lambda v \left( \frac{1}{\sqrt{2}} (A_\lambda + M_T) c_\beta + (\lambda v_t - \sqrt{2} \mu) s_\beta \right), \\
(M_a^2)_{22} &= s_\beta^2 M_A^2 + c_\beta^2 M_Z^2 - \frac{\lambda}{\sqrt{2}} t_\beta v_t (M_T + A_\lambda), \\
(M_a^2)_{23} &= \lambda v \left( \frac{1}{\sqrt{2}} (A_\lambda + M_T) s_\beta + (\lambda v_t - \sqrt{2} \mu) c_\beta \right), \\
(M_a^2)_{33} &= M_T^2 + m_T^2 + B_T + \frac{1}{2} \lambda^2 v^2,
\end{align*} \quad (43)$$

where $v_t$ should be considered as a function of the input parameters via the minimization condition in Eq. 27 and we define $M_A$ as in the case of the MSSM

$$M_A^2 = \frac{2 B_v}{\sin 2\beta} \quad (44)$$
As in the case of the NMSSM, the lightest mass-squared eigenvalue is bounded by the lightest eigenvalue of top-left 2×2 block of the mass matrix,

\[ m_h^2 \leq M^2_Z \left( \cos 2\beta + \frac{2\lambda^2}{g_2^2 + g_1^2} \sin 2\beta \right). \]  

(45)

In Fig. 4 we plot this tree-level upper bound as a function of \( \tan \beta \) for \( \lambda = 0.7, 0.8, \) and 0.9. For \( \lambda = 0.9, \) it is possible to obtain a tree-level Higgs mass larger than 100 GeV for \( \tan \beta \lesssim 6, \) and even satisfy the LEP2 bounds at tree-level for small \( \tan \beta \lesssim 3.5. \)

We can also see the MSSM limit in the mass matrix when the triplet decouples with fixed \( \lambda. \) In the limit \( M_T \to \infty, \) keeping all other parameters fixed, we have

\[ M_T v_t \to -\frac{\lambda v^2}{2\sqrt{2}} \sin 2\beta, \]  

(46)

and the mass matrix has the form

\[ M^2_{\text{a}} \xrightarrow{M_T \to \infty} \begin{pmatrix} M^2_{\text{MSSM}} + \Delta M^2 & \epsilon \\ \epsilon^T & M_T^2 \end{pmatrix}, \]  

(47)

where \( M^2_{\text{MSSM}} \) is the 2×2 MSSM mass matrix for the \( CP \)-even Higgs bosons, and

\[ \Delta M^2 = \frac{\lambda^2 v^2}{2} \begin{pmatrix} c_\beta^2 & c_\beta s_\beta \\ c_\beta s_\beta & s_\beta^2 \end{pmatrix}, \]  

(48)

\[ \epsilon = \frac{\lambda}{\sqrt{2}} M_T v \begin{pmatrix} c_\beta \\ s_\beta \end{pmatrix}. \]  

(49)

Integrating out the third row and column of the mass matrix in Eq. (47) the effective top-left 2×2 sub-matrix becomes

\[ M^2_{\text{eff}} = M^2_{\text{MSSM}} + \Delta M^2 - \epsilon (M_T^2)^{-1} \epsilon^T + O \left( \frac{\epsilon^3}{M_T^2} \right), \]  

(50)

and we recover the MSSM limit as \( M_T \to \infty. \) In Fig. 5 we show this decoupling behavior by plotting the tree-level mass of the lightest Higgs boson as a function of \( M_T \) for various values of \( \tan \beta, \) and see that, for \( M_T \gtrsim 10^4 \) GeV, we recover the MSSM results.
FIG. 5: Tree-level mass of the lightest $CP$-even Higgs boson as a function $M_T$ for $\lambda = 0.8$ (left) and $\lambda = 0.9$ (right). The other parameters are kept fixed as $A_{\lambda} = m_{H_u}^2 = B_T = 0$, $\mu = 200$ GeV, and $M_A = 300$ GeV. The three curves have values of $\tan \beta$ of 10 (solid), 5 (dashed), and 3 (dotted). For each case, we see decoupling in large $M_T$, and the limiting value agrees with the MSSM result.

1. Numerical Results

In this subsection, we numerically evaluate the mass of the lightest $CP$-even Higgs boson at tree-level. With the minimization conditions, we can take as input parameters 

$$ \tan \beta, \mu, M_A, \lambda, M_T, m_T^2, B_T, $$

and fix $m_{H_u}^2$, $m_{H_d}^2$ and $v_t$ by solving the minimization conditions with the experimental inputs of $M_Z = 91.19$ GeV and the gauge couplings $g_2(M_Z) \simeq 0.65$, $g_1(M_Z) \simeq 0.36$ (this fixes $v \simeq 245$ GeV). We discard sets of input parameters that give large $v_t$ inconsistent with electroweak constraints. For all our numerical studies, we analyze two the cases of $\lambda = 0.8$ and $\lambda = 0.9$, and scan the parameter space in the range

$$ 3 \leq \tan \beta \leq 30, \quad 100 \text{ GeV} \leq \mu, M_A \leq 500 \text{ GeV}, \quad 300 \text{ GeV} \leq M_T \leq 1000 \text{ GeV}, \quad -2000 \text{ GeV} \leq A_{\lambda} \leq 2000 \text{ GeV}, \quad -(1000 \text{ GeV})^2 \leq m_{H_d}^2, B_T \leq (1000 \text{ GeV})^2. $$

The range of $\tan \beta$ is chosen so that the bottom Yukawa coupling is smaller than the top Yukawa coupling, and we can neglect the bottom Yukawa coupling when we study the production and decay properties of the lightest $CP$-even Higgs boson.

Since solutions to the minimization conditions only guarantee an extremum, we only keep solutions that give a local minimum by checking that all scalar masses are positive at the desired vev. We also discard any points that give unacceptably large $\alpha \Delta T$ or contain charged scalar particles lighter than 100 GeV.

For the range of parameters listed in Eq. (52), we show the mass of the lightest $CP$-even boson as a function of $\tan \beta$ in Fig. 6. In Fig. 6, we also plot the upper bound of the tree-level Higgs mass given in Eq. (45). The plots show that we can indeed achieve large (greater than $M_Z$) tree-level Higgs mass with large $\lambda$, and we can even satisfy LEP2 bounds at tree-level for small values of $\tan \beta$ ($\tan \beta \lesssim 3.5$) when $\lambda = 0.9$.

B. Mass at the One-Loop Level

Since the lightest $CP$-even Higgs boson is a linear combination of $a'_i$ for $i = u, d, t$, we will construct the Coleman-Weinberg (CW) potential [51] only for the fields $a_i$, and extract the corrections to $m_h^2$ from the CW potential.
Furthermore, we will make the two following assumptions:

- We assume that both the s-top masses are close to the top-quark mass, and the famous $O(y_t^4)$ contributions in the MSSM are small. In other words, we only consider the corrections from the Higgs boson, neutralino, and chargino sectors. These contributions are dominated by the coupling $\lambda$ and the SUSY-breaking parameters in the triplet sector. Our results will show that these contributions are sufficient to satisfy the LEP2 bounds on the Higgs mass, and we do not need large contributions from the top–s-top sector as in the case of the MSSM.

- In the neutralino and chargino mass matrices, we ignore the mixing induced by gauge interactions. This removes dependencies on the bino and wino SUSY-breaking masses in our analysis as we do not include the bino and wino states, and we expect their contributions to be small when $M_{1,2} \sim M_Z$. (If we include the bino and wino states, we would also have to include the corresponding superpartners in the $W$ and $Z$ gauge bosons.)

The Coleman-Weinberg potential is given by

$$V_{CW} = \frac{1}{64\pi^2} \text{STr} \left[ \mathcal{M}^4 \left( \ln \frac{\mathcal{M}^2}{\mu_t^2} - \frac{3}{2} \right) \right],$$

where $\mathcal{M}^2$ are field-dependent mass matrices in which the fields are not replaced with their vev’s, $\mu_t$ is the renormalization scale, and the supertrace includes a factor of $(-1)^J(2J+1)$ so that fermions contribute oppositely to bosons, and the spin degrees of freedoms are appropriately summed over. Since here we are only interested in the CW potential of the fields $a_i$ that always appear in the combination $(a'_i + v_i)$, the field-dependent matrices for the charginos and neutralinos are simply those in Eqs. (40) and (42) with the vevs $v_i$ replaced by the corresponding fields $a_i$.

For the scalars, the naive replace-vev-by-field method fails and we need to distinguish between the contributions from the minimization conditions and those from the replacement of the fields with their corresponding vevs. For example, while the $(11)$-element of the mass matrix of the $CP$-even neutral Higgs boson is

$$(M_a^2)_{11} = c_\beta^2 M_A^2 + s_\beta^2 M_Z^2 - \frac{\lambda}{\sqrt{2}} f^{-1}_i v_i (M_T + A_{\lambda}),$$

$$= B_u \frac{v_d}{v_u} + \frac{1}{4} (g_2^2 + g_1^2) \frac{v_u^2}{v_u} - \frac{\lambda}{\sqrt{2}} v_i (M_T + A_{\lambda}) \frac{v_d}{v_u},$$

it is incorrect to have the field-dependence

$$(M_a^2)_{11} \neq B_u \frac{a_d}{a_u} + \frac{1}{4} (g_2^2 + g_1^2) a_u^2 - \frac{\lambda}{\sqrt{2}} a_i (M_T + A_{\lambda}) \frac{a_d}{a_u},$$
because some of the vev-dependence in Eq. 55 comes from the minimization conditions Eq. 28. The correct field-dependent (11)-element of the $\mathcal{CP}$-even neutral Higgs boson is

$$ (M^2)_{11} = m^2_{H_u} + \mu^2 + g_u^2 + \frac{g^2}{8} (3a_u^2 - a_d^2) + \frac{\lambda^2}{2} (a_d^2 + a_t^2) - \sqrt{2} \lambda \mu a_t, $$  (57)

where $m^2_{H_u}$ is related to the vev’s (but not the fields) through the minimization condition in Eq. 28. In Appendix A, we give the field-dependent mass matrices used in the calculation of the Coleman-Weinberg potential.

Since the analytical results for the mass eigenvalues of the field-dependent matrices are complicated, we will compute the Higgs mass numerically. The one-loop mass matrix can be extracted from the Coleman-Weinberg potential by numerically evaluating the derivatives of the mass eigenvalues with respect to the fields about the vevs [24] (dropping the pre-factor from the supertrace for convenience)

$$ (\Delta M^2_{ij})_{ij} = \left. \frac{\partial^2 V_{CW}(a)}{\partial a_i \partial a_j} \right|_{\text{vev}} - \left. \frac{\delta_{ij} \partial V_{CW}(a)}{\partial a_i} \right|_{\text{vev}} + \sum_k \frac{1}{32\pi^2} m^2_k \ln \frac{m^2_k}{\mu_r^2} \left( \ln \frac{m^2_k}{\mu_r^2} - 1 \right) \left. \partial^2 m^2_k / \partial a_i \partial a_j \right|_{\text{vev}}, $$  (58)

$$ \sum_k \frac{1}{32\pi^2} m^2_k \delta_{ij} \left. \partial m^2_k / \partial a_i \right|_{\text{vev}} \left( \ln \frac{m^2_k}{\mu_r^2} - 1 \right) \left. \partial^2 m^2_k / \partial a_i \partial a_j \right|_{\text{vev}}, $$  (59)

where the second term in Eq. 58 takes into account the shift in the minimization conditions, and $\{m^2_k\}$ is the set of mass eigenvalues that enter the Coleman-Weinberg potential. Our set of $\{m^2_k\}$ includes the eigenvalues of the mass matrices of the $\mathcal{CP}$-even, $\mathcal{CP}$-odd, and charged Higgs bosons, as well as the neutralinos and charginos mass matrices. These field-dependent mass matrices are given in Appendix A.

1. Numerical Results

![FIG. 7: Mass of the lightest $\mathcal{CP}$-even Higgs boson, including one-loop contributions from the triplet sector, as a function $\tan \beta$, scanned over the parameter space as listed in Eq. 52. The plot on the left has $\lambda = 0.8$, and the plot on the right has $\lambda = 0.9$. The input parameters of the individual points are the same as those that give rise to the points shown in the corresponding plot of Fig. 6.](image)

We numerically compute the mass of the lightest $\mathcal{CP}$-even Higgs boson to one-loop using the Coleman-Weinberg potential for the parameter space in Eq. 52. For the same input parameters that give rise to the tree-level results shown in Fig. 6, we show the corresponding Higgs mass computed to one loop in Fig. 7 and the difference between the loop-level and tree-level masses in Fig. 8. We use the value of $M_T$ as the renormalization scale $\mu_r$ that enters the Coleman-Weinberg potential. From these plots, we see that the triplet sector can give a large contribution to
FIG. 8: Difference between the mass of the lightest CP-even Higgs boson with and without the one-loop contribution from the triplet sector for the points shown in the corresponding plots of Figs. [7] and [8]. The plot on the left has $\lambda = 0.8$, and the plot on the right has $\lambda = 0.9$.

the mass of the lightest CP-even Higgs boson, and we can satisfy the LEP bounds without the s-top contributions for all values of $\tan \beta$ in our scanned range.

TABLE I: Sample Higgs spectra. All dimensionful parameters are in units of GeV, except for $m_{T}^2$ and $B_T$, which are in units of $(\text{GeV})^2$. The definitions of fine-tuning $f_T$, $\kappa_T$, and $\kappa'_T$ are given in, respectively, Eqs. [77], [79] and [80]. The value of $f_T$ indicates the percent change in $v^2$ induced by a 1% change in $m_{H_u}^2$ at a fundamental scale of SUSY-breaking, and the value of $\kappa_T$ indicates the percent change in $\Delta T$ induced by a 1% change in $M_T$. The measure $\kappa'_T$ is only applicable to Points 1 and 2, and shows that there is a cancellation of one part in 23 to give a viable value of $\Delta T$. Points 1 and 2 show examples of input parameters that give a viable Higgs mass with a small fine-tuning in the electroweak sector. Points 3 and 4 differ only in $\lambda$, and are samples that give large Higgs masses of about 120 GeV (for $\lambda = 0.8$) and about 135 GeV (for $\lambda = 0.9$). Points 5 and 6 have large $\tan \beta (\geq 20)$ and $m_h \sim M_Z$ at tree-level, but there are large radiative corrections to have viable Higgs masses at one-loop.

| $\tan \beta$ | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 | Point 6 |
|--------------|---------|---------|---------|---------|---------|---------|
| $\mu$        | 270     | 270     | 400     | 400     | 200     | 165     |
| $M_A$        | 430     | 430     | 280     | 280     | 300     | 410     |
| $\lambda$    | 0.8     | 0.9     | 0.8     | 0.9     | 0.8     | 0.9     |
| $M_T$        | 370     | 400     | 400     | 400     | 350     | 330     |
| $m_{T}^2$    | (500)$^2$ | (280)$^2$ | (1970)$^2$ | (1970)$^2$ | (1600)$^2$ | (1500)$^2$ |
| $A_{\lambda}$| 600     | 460     | 1860    | 1860    | 1800    | 1300    |
| $B_T$        | (400)$^2$ | (400)$^2$ | (1730)$^2$ | (1730)$^2$ | (500)$^2$ | (1400)$^2$ |
| $m_{h}^{\text{Tree}}$ | 108     | 113     | 105     | 111     | 88      | 90      |
| $m_{h}^{\text{Tree+Loop}}$ | 114     | 117     | 122     | 137     | 114     | 121     |
| $f_T$        | 33      | 19      | 399     | 505     | 315     | 271     |
| $\kappa_T$  | 33      | 11      | 0.8     | 0.8     | 0.5     | 0.3     |
| $\kappa'_T$ | 4.7     | 6.4     |         |         |         |         |

In Table [I] we give some sample points in our scan. Points 1 and 2 are sample points that have small fine-tuning (as will be defined later in Eq. [77]). The TESSM can achieve small fine-tuning because the Higgs mass can be large at tree-level and does not require large contributions from radiative corrections. Points 3 and 4 are samples of the points with the largest Higgs masses (and therefore fine-tuning) in our scanned range of parameter space, as evident
in the values of SUSY-breaking parameters $m_T^2$, $A_\lambda$, and $B_T$ being near the boundary of the scanned range. Points 5 and 6 are samples of points having a large $\tan\beta(\geq 20)$, where the tree-level Higgs mass is only slightly larger than $M_Z$, and there is a significant one-loop contribution, and, correspondingly, large fine-tuning.

C. Collider Signatures of the Lightest CP-even Higgs Boson in TESSM

With large $\lambda$ in both the TESSM and the NMSSM, we do not require heavy s-tops. In these cases, the gluon-gluon fusion production of the lightest CP-even Higgs boson, $\sigma(gg \to h)$, and its diphoton partial decay width, $\Gamma(h \to \gamma\gamma)$, can be very different from the MSSM because these processes involve s-top loops. In this section we perform a simplified analysis showing that in the TESSM there may be a gluo-philic Higgs boson whose gluon-gluon fusion production cross section can be larger than that of the SM by a factor of 1.8. As stated in the introduction, our discussions of the gluon-gluon fusion production rely only on the existence of light s-tops and the minimal color sector of the MSSM, and are therefore applicable to any extensions of the MSSM that solves the little hierarchy problem without invoking additional colored states. For the diphoton partial decay width, there are several sources of suppression, and we may have a partial decay width that is about 0.8 times that in the SM.

Of course, at the LHC the relevant quantity is the product of the gluon-gluon fusion production cross section and the diphoton branching ratio

$$\sigma(gg \to h)\text{Br}(h \to \gamma\gamma),$$

and a more complete analysis would have to take into account the effects of light s-tops to all the decay channels as well as the large, higher-loop corrections from QCD and large couplings. We leave the complete analysis of the Higgs production and decay for future work.

The well-known formula for the decay width of a real scalar decaying into two photons can be found in Gunion et al. [52]. This formula is also presented in the Appendix [5].

1. Gluo-philic Higgs boson

In the SM, ignoring all the Yukawa couplings except for the top Yukawa coupling, the process $h \to gg$ proceeds only through a top-quark loop. In the MSSM, we have additional contributions from the s-tops (see Fig. 9), as well as all the other s-quarks through $D$-term interactions of the form $h\tilde{q}^*\tilde{q}$ with coupling of the order $M_Z^3$. To simplify our analysis, we will ignore the $D$-term interactions except those in the s-top sector, but we note that these contributions can be important when there are light s-quarks and must be taken into account in a full analysis.

In the MSSM with small s-top mixing, the s-top contributions interfere constructively with the top-quark contribution for the gluon-gluon fusion production cross section [53, 54]. However, with small s-top mixing, the s-tops need to be heavy to satisfy the LEP bounds on the Higgs mass, and the s-top contributions decouple. (With large-stop mixing, it is possible to have s-top contributions that interfere destructively with the top-quark contribution, leading to a gluo-phobic Higgs boson.)

In the TESSM and NMSSM, we can have light s-tops at the expense of perturbativity at the GUT scale, and a large enhancement in the production rate. Assuming large $\tan\beta$ so that $v \approx v_u$ and there is no s-top mixing, and
approximating the lightest $CP$-even Higgs boson $h$ as being dominantly composed of $a'_u$ (the $CP$-even component of $H_u$), we have the interactions

$$-\mathcal{L} \supset \frac{y_t}{\sqrt{2}} h \tilde{t} + \left( \frac{y_t^2 + \frac{1}{12} g_1^2 - \frac{1}{4} g_2^2}{m_{Q_3}^2 + m_{\tilde{t}}^2} \right) v h \tilde{Q}_3 \tilde{Q}_3 + \left( y_t^2 - \frac{1}{3} g_1^2 \right) v h \tilde{U}_3 \tilde{U}_3, \quad (60)$$

where $t$ is the top-quark, and $\tilde{Q}_3$ ($\tilde{U}_3$) is the superpartner to the left-(right-)handed component of the top-quark. From Eqs. 22, 9, and 10, the ratio of the amplitudes $A(gg \rightarrow h)$ due to the s-top s-quarks and top-quark is then

$$r_{gg \rightarrow h} \equiv \frac{A_t}{A_{\tilde{t}}} = \frac{m_t^2 + \frac{1}{4} \left( \frac{1}{6} g_1^2 - \frac{1}{3} g_2^2 \right) v^2 F_{Q_3}}{m_{Q_3}^2 + m_{\tilde{t}}^2} F_t + \frac{m_{\tilde{t}}^2 - \frac{1}{3} g_1^2 v^2 F_{\tilde{U}_3}}{m_{Q_3}^2 + m_{\tilde{t}}^2} F_{\tilde{t}}, \quad (61)$$

where $m_{Q_3}^2$ and $m_{\tilde{t}}^2$ are SUSY-breaking soft masses of the corresponding s-tops, and we have used the relationships $m_t = \left( \sqrt{2} \right)^{-1} y_t v_u$. In Fig. 10 we plot $r_{gg \rightarrow h}$ as a function of a common soft s-top mass $m_{Q_3}^2 = m_{\tilde{t}}^2 = m_{SUSY}^2$, assuming $m_h = 114$ GeV. Since the s-top mass eigenvalues in this simplified analysis are given by

$$m_{\tilde{t}}^2 = m_{SUSY}^2 + m_t^2, \quad (62)$$

and the current searches limit the s-tops masses to be greater than 120 GeV [55], we can have $m_{SUSY} \sim 0$ (so $m_t = m_t$) and $r_{gg \rightarrow h}$ can be as large as 0.48. This gives a gluo-philic Higgs boson whose production cross section via gluon-gluon fusion may be enhanced relative to the SM prediction by a factor of $(1+0.5 \times 0.48)^2 \sim 2.2$.

![FIG. 10: The amplitude $A(gg \rightarrow h)$ through s-top loops normalized with respect to the amplitude through top-quark loop, as a function of a common s-top soft mass $m_t$, assuming no mixing in the s-top sector.](image)

Imposing perturbativity at the GUT scale, we can have a milder gluo-philic Higgs boson when one of the s-top is light (the other is required to be heavy to have a Higgs mass satisfying the LEP2 bounds). However, when only one s-top is light, the enhancement in the gluon-gluon fusion production cross section is only a factor of $(1+0.5 \times 0.48)^2 \sim 1.5$ larger than that of the SM.

2. Diphoton Decay of the Higgs boson

In the SM, the diphoton decay of the Higgs boson proceeds through $W$-boson loop in addition to top-quark loop, and the contribution from the top-quark destructively interferes with the dominant $W$-boson contribution. In the MSSM, we have additional contributions from the s-tops and charginos (the corresponding superpartners of the
top-quark and $W$-boson), and, as in the case of $\Gamma(h \to gg)$, contributions from all the electrically charged $s$-quarks and $s$-leptons through $D$-term interactions.

In the TESSM, we have the additional contributions from the states composed dominantly of the charged triplets, and also new contributions from the MSSM matter content induced by $\lambda$ (see Fig. 11). These contributions may be important when $\lambda$ is as large as the top Yukawa coupling, and so in this subsection we use $\lambda = 0.9$ for our numerical studies. In this work, we will simplify our analysis by ignoring contributions from the $D$-term interactions except those involving the $s$-tops, and, using the same approximations as in the previous subsection of large $\tan \beta$ and $h \simeq a_u'$, we have the interactions and fermion masses

$$-\mathcal{L} \supset 2\lambda^2 v h \left( |T^+|^2 + |H_d|^2 \right) + \lambda (h + v) \left( \overline{H^+} P_L T^+ + \bar{T}^+ P_R \bar{H}^+ \right) + \mu (\overline{H^+} \bar{H}^+) + M_T (\overline{T^+} \bar{T}^+),$$

where $\bar{H}^+$ and $\bar{T}^+$ are Dirac spinors formed from the Higgsinos and the fermionic components of the charged triplet states

$$\bar{H}^+ = \begin{pmatrix} \bar{H}_{+}^+ \\ \bar{H}_{d}^+ \end{pmatrix}, \quad \bar{T}^+ = \begin{pmatrix} \bar{T}_{+}^+ \\ \bar{T}_{d}^+ \end{pmatrix},$$

and $P_{L,R}$ are the projection operators

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$  

Although none of the charged states in Eq. 63 is a mass eigenstate, we approximate the scalar states as mass eigenstates with masses

$$m^2_{T^+} \simeq M_T^2 + m_{\tilde{C}_1}^2, \quad m^2_{H_d^+} \simeq M_{\tilde{A}}^2,$$

so that the contributions of these states to the amplitude $\mathcal{A}(h \to \gamma \gamma)$ have the same form. In the fermionic sector, the contribution to the amplitude $\mathcal{A}(h \to \gamma \gamma)$ comes exclusively from the mixing between $\bar{H}^+$ and $\bar{T}^+$. We can diagonalize the fermionic mass matrix with two unitary transformations

$$\begin{pmatrix} \overline{H^+} \\ \overline{T^+} \end{pmatrix}^T V^\dagger V \begin{pmatrix} \mu v \\ 0 \end{pmatrix} U^\dagger U P_L \begin{pmatrix} \overline{H^+} \\ \overline{T^+} \end{pmatrix} = \begin{pmatrix} \overline{\tilde{C}_1} \\ \overline{\tilde{C}_2} \end{pmatrix}^T \begin{pmatrix} m_{\tilde{C}_1} & 0 \\ 0 & m_{\tilde{C}_2} \end{pmatrix} P_L \begin{pmatrix} \overline{\tilde{C}_1} \\ \overline{\tilde{C}_2} \end{pmatrix},$$

where $\overline{\tilde{C}_1}$ and $\overline{\tilde{C}_2}$ are the mass eigenstates of the charged triplet states. This transformation simplifies the analysis and allows us to focus on the contributions from the charged triplets.
where $U$ and $V$ are respectively parameterized by $\varphi$ and $\varphi'$,
\[ U \equiv \begin{pmatrix} c_{\varphi} & -s_{\varphi} \\ s_{\varphi} & c_{\varphi} \end{pmatrix}, \quad V \equiv \begin{pmatrix} c_{\varphi'} & -s_{\varphi'} \\ s_{\varphi'} & c_{\varphi'} \end{pmatrix}, \tag{68} \]
where $c_{\varphi} = \cos \varphi$ and $s_{\varphi} = \sin \varphi$, and $c_{\varphi'}$ and $s_{\varphi'}$ are similarly defined. These mixing angles are given by
\[ \tan 2\varphi = \frac{2\lambda v}{M_T^2 - \mu^2 + \lambda^2 v^2}, \]
\[ \tan 2\varphi' = \frac{2\lambda v M_T}{M_T^2 - \mu^2 - \lambda^2 v^2}. \tag{69} \]

In terms of the mass eigenstates and mixing angles, the chargino interactions in Eq. 63 take the form
\[ -\mathcal{L} \supset \lambda h \left( -c_{\varphi'} s_{\varphi} \tilde{C}_1 \tilde{C}_1 + c_{\varphi'} s_{\varphi'} \tilde{C}_2 \tilde{C}_2 \right) + \lambda h' \left( c_{\varphi'} c_{\varphi'} \tilde{C}_1 \tilde{C}_1 - s_{\varphi} s_{\varphi'} \tilde{C}_2 \tilde{C}_2 \right). \tag{70} \]

In Figs. 12 and 13 we illustrate the contributions of light s-tops and the additional charged states to the diphoton partial decay width, normalized with respect to the dominant $W$-boson contribution, assuming $m_h = 114$ GeV. In Fig. 12 we show the contributions from the top-quark (constant line), s-tops (solid line), and the charged scalar states (dotted line). For the s-tops (charged scalar states $H_a$ and $T^+$), the horizontal axis should be interpreted as a common soft SUSY-breaking mass $M_{\text{SUSY}}$ (mass of these charged scalar states). In Fig. 13 we show the sum of the fermion contributions as a function of $M_T$ for different values of $\mu$, and see that even for small values of $\mu$ and $M_T$ ($\mu, M_T \lesssim 200$ GeV), these contributions tend to be small. We can partially attribute the smallness to a cancellation between the contributions from the two states $\tilde{C}_1$ and $\tilde{C}_2$, as evident in the relative sign difference between the coefficients of the $h\tilde{C}_1 \tilde{C}_1$ and $h\tilde{C}_2 \tilde{C}_2$ interactions. Though these fermionic contributions are small, it is interesting to note that, while the top-quark contribution interferes destructively with the $W$-boson contribution, the sum of these fermionic contributions interferes constructively. In any case, the additional $\lambda$-induced contributions (both bosonic and fermionic) to the partial decay width $\Gamma(h \rightarrow \gamma\gamma)$ are small compared to the s-top contributions.

Combining all these contributions to $A(h \rightarrow \gamma\gamma)$, the diphoton partial width can be significantly reduced (mostly from the s-top contributions). For example, with $M^2_{\text{SUSY}} = m^2 = 0$, $\mu = 150$ GeV, $M_A = 200$ GeV, and $M_T = 500$ GeV, the amplitude $A(h \rightarrow \gamma\gamma)$ is decreased by (relative to the SM) a factor of
\[ \frac{A_W + A_t + A_{H_a^+} + A_{T^+} + (A_{\tilde{C}_1^+} + A_{\tilde{C}_2^+})}{A_W + A_t} \sim 1 - 0.23 - 0.11 - 0.05 - 0.008 + 0.001 \sim 0.78, \]
and the diphoton decay partial width is decreased, relative to the SM partial decay width, by a factor of \((0.78)^2 \approx 0.6\). We therefore can have a photo-phobic Higgs boson in the TESSM from the contribution of light s-tops.

IV. FINE-TUNINGS IN TESSM

A. Electroweak Sector

Before discussing the fine-tuning in the electroweak sector of the TESSM, we briefly review the little hierarchy problem in the MSSM. In the MSSM with large \(\tan \beta\), the Higgs doublet \(H_u\) is responsible for most of the EWSB since \(v \approx \sqrt{2} \langle H_u \rangle\), and it has the potential

\[
V_{H_u} = (m_{H_u}^2 + \mu^2) |H_u|^2 + \frac{1}{8} (g_2^2 + g_1^2) |H_u|^4.
\]

Minimizing the potential then gives

\[
2\langle H_u^2 \rangle = v_u^2 = -8 \frac{m_{H_u}^2 + \mu^2}{g_2^2 + g_1^2},
\]

so that

\[
m_{H_u}^2 = -\frac{1}{8} (g_2^2 + g_1^2) v_u^2 - \mu^2.
\]

Under radiative corrections, \(m_{H_u}^2\) receives large logarithmic corrections from the s-top sector, and we can use the renormalization group equations to infer the value of \(m_{H_u}^2\) at a fundamental scale \(\Lambda\),

\[
m_{H_u}^2(\Lambda) \simeq m_{H_u}^2(M_Z) + \frac{3y_t^2}{8\pi^2} \left( m_{Q_3}^2 + m_{\overline{Q}_3}^2 + A_t^2 \right) \left( \ln \frac{\Lambda}{M_Z} \right),
\]

where \(m_{Q_3}^2\) and \(m_{\overline{Q}_3}^2\) are the SUSY-breaking s-top masses, \(y_t A_t\) is the coupling of the trilinear interaction \(\tilde{Q}_3 H_u \overline{U}_3\), and \(\Lambda\) can be taken as the scale of SUSY-breaking. The large radiative correction leads to fine-tuning \(f_s\) because the electroweak scale \(v\) depends sensitively on the value of \(m_{H_u}^2\) at the fundamental scale of SUSY-breaking \(\Lambda\). We can
quantify this fine-tuning as \[ f_s \equiv \frac{\delta \ln v^2}{\delta \ln m_{H_u}^2(\Lambda)} \simeq \frac{3\gamma_t^2}{4\pi^2} \left( \frac{m_{Q_3}^2 + m_{\tau_3}^2 + A_t^2}{M_Z^2} \right) \left( \ln \frac{\Lambda}{M_Z} \right). \] 

As a reference of comparison, for \( m_{Q_3}^2 = m_{\tau_3}^2 = A_t = 1 \text{ TeV}, \) and \( \Lambda = 10^3 \text{ TeV}, \) we have \( f_s = 80 \) so that the Higgs sector needs to be fine-tuned to one part in 80. Thus, even though the electroweak scale is no longer quadratically sensitive to the fundamental scale \( \Lambda \) with softly-broken SUSY, it is quadratic sensitive to the s-top masses and trilinear coupling \( A_t \), which are required to be large to have a Higgs mass that satisfies the LEP bounds, and this leads to a fine-tuning in the Higgs sector of about one part in 100. This is the little-hierarchy problem in the MSSM.

We can also define other measures of fine-tuning when given a more fundamental theory (for example, an organizing principle of the soft SUSY-breaking parameters) \[ f_m, f_\mu, f_{\beta} \] \cite{57, 58, 50}. However, in this work we are mainly interested in the low-energy phenomenology of the TESSM without appealing to a particular fundamental theory, and we will simply define fine-tuning as in Eq. \[ f \]

In the TESSM with \( \lambda \) comparable to the top Yukawa coupling, we do not need heavy s-top masses nor significant mixing in the s-top sector for the Higgs mass to satisfy the LEP bound, and as such there is little or no fine-tuning from the s-top sector. On the other hand, \( m_{H_u}^2 \) now receives radiative corrections from the triplet sector as well as the s-top sector

\[ m_{H_u}^2(\Lambda) \simeq m_{H_u}^2(M_Z) + \frac{3\gamma_t^2}{8\pi^2} \left( m_{Q_3}^2 + m_{\tau_3}^2 + A_t^2 \right) \left( \ln \frac{\Lambda}{M_Z} \right) + \frac{3\lambda^2}{8\pi^2} \left( m_T^2 + A_\lambda^2 \right) \left( \ln \frac{\Lambda}{M_Z} \right), \] \[ \text{(76)} \]

and we can follow the same steps and reasoning as before to have an estimate of the fine-tuning due to the triplet sector \( f_T \)

\[ f_T \simeq \frac{3\lambda^2}{4\pi^2} \left( \frac{m_T^2 + A_\lambda^2}{M_Z^2} \right) \left( \ln \frac{\Lambda}{M_Z} \right), \] \[ \text{(77)} \]

so that \( f_T = 40 \), for example, would mean a tuning in \( m_{H_u}^2(\Lambda) \) to one part in 40. The value of \( f_T \) indicates the percent change in \( v^2 \) per a one-percent change in \( m_{H_u}^2 \) at a fundamental scale of SUSY-breaking. Generally, with large \( \lambda \), for a given mass of the lightest \( CP \)-even Higgs boson, the fine-tuning in \( m_{H_u}^2 \) is less in the TESSM than the MSSM. In Fig. \[ f_T \] we plot \( f_T \) for the data points shown in Fig. \[ \text{where we see a rough general trend of increasing fine-tuning with increasing Higgs mass. On the other hand, it is possible to have points with relatively small } \]

so that \( f_T \leq 20 \) that satisfy the LEP2 bound of \( m_H > 114.4 \text{ GeV} \), as demonstrated in Point 1 of Table \[ \text{This is a great improvement over the MSSM, and it is a consequence of the large tree-level mass we can obtain in TESSM, so we do not have to rely on large radiative corrections from } m_T^2 \text{ and } A_\lambda. \]

**B. Triplet Sector**

The vev of \( T^0 \) is induced by the vev’s of the Higgs doublets because the vev’s of the Higgs doublets \( v_{u,d} \) induce a tadpole from the trilinear interactions of the form \( HTH \) in the second line of Eq. \[ \text{We noted earlier that some cancellation between } a \text{ priori unrelated parameters (} \mu \text{ and } M_T \sin 2\beta, \text{ for example) is required to keep } v_t \text{ (and thus } \Delta T) \text{ small and this leads to fine-tuning in the triplet sector. However, it is worth pointing out that } v_t \text{ here does not receive a large radiative correction that requires a fine-tuning as severe as the fine-tuning in the hierarchy problem in the triplet-extended standard model potential analyzed in Chivukula et al. \[ \text{It is easiest to see this in the limit } m_T^2 = B_T = A_\lambda^2 = 0 \text{ (SUSY-limit in the triplet sector) where the triplet vev } v_t \text{ in Eq. \[ \text{takes a particularly simple form} \]

\[ v_t = \frac{\sqrt{2}}{2} (\lambda v^2) \left( \frac{\mu - M_T s_\beta c_\beta}{M_T^2 + \lambda^2 v^2} \right), \] \[ \text{(78)} \]

and the 1-loop corrections to \( v_t \) then involve 1-loop corrections to the parameters \( \lambda, v_{u,d}, \text{ and } M_T \). The parameters \( \lambda, \mu, \text{ and } M_T \) come from the superpotential, and the nonrenormalization theorem dictates that the radiative corrections...
FIG. 14: Fine-tuning (as defined in Eq. 77) as a function of the mass lightest $CP$-even Higgs boson. This is typically less than the fine-tuning of the MSSM (as defined in Eq. 75) and the NMSSM. The plot on the left has $\lambda = 0.8$, and the plot on the right has $\lambda = 0.9$.

to these parameters run only in a logarithmical manner due to wavefunction renormalizations only. Though the loop corrections to $v_{u,d}$ may require a fine-tuning of one part in a few hundreds (this is the little hierarchy problem in the MSSM), this is much more benign than the fine-tuning in the triplet-extended SM studied in Chivukula et al. [34].

On the other hand, there is a source of fine-tuning in $v_t$ because we often require some degree of cancellation to make $\Delta T$ small. We can define a quantitative measure of fine-tuning in $\Delta T$ by

$$\kappa_T \equiv \frac{\delta \ln \Delta T}{\delta \ln M_T} = 2 \frac{\delta \ln v_t}{\delta \ln M_T} = \left( \frac{2M_T}{\sin 2\beta(A_\lambda + M_T) - 2\mu} \right) \left( \frac{4\mu M_T + \sin 2\beta(m_T^2 + B_T - 2A_\lambda M_T - M_T^2 + \frac{\lambda^2}{2} v^2)}{M_T^2 + m_T^2 + B_T + \frac{\lambda^2}{2} v^2} \right),$$

(79)

so that $\kappa_T$ is large when there is a large cancellation in the combination

$$\sin 2\beta(A_\lambda + M_T) - 2\mu,$$

that makes $\Delta T$ unnaturally small.

The definition in Eq. 79 however, may not be satisfactory because it does not take into account the range of allowed $\Delta T$. For example, for the parameters listed in Eq. 37

$$\tan \beta = 3, \quad \lambda = 0.9, \quad \mu = 150 \text{ GeV}, \quad m_T^2 = B_T = A_\lambda^2 = (200 \text{ GeV})^2,$$

we have viable $\Delta T$ in the regions

$$250 \text{ GeV} < M_T < 375 \text{ GeV}, \text{ or } M_T > 3.0 \text{ TeV},$$

and it may be reasonable to expect that any value of $M_T$ in the small range between 250 GeV and 375 GeV is equally fine-tuned. However, Eq. 79 would give different values of $\kappa_T$ for different values of $M_T$, and may even diverge if $M_T$ is such that we have $v_t = 0$. It is true that when $v_t = 0$ we have unnatural, complete cancellation, but in our work we only use $v_t$ in a binary way: to distinguish cases with viable $\Delta T$ from those with unacceptably large $\Delta T$. Once $v_t$ is small enough to have viable $\Delta T$, we do not care whether $v_t = 1 \text{ GeV}$ or $v_t = 0.01 \text{ GeV}$, for example.

As in Section 11 we can also estimate the fine-tuning in $\Delta T$ due to $M_T$ as shown in Athron et al. [50] when there is a cancellation in the numerator of Eq. 36 that makes $\Delta T$ small. With all parameters other than $M_T$ fixed, we first compute $M_T^*$ such that for $M_T > M_T^*$, $\Delta T$ is always viable ($\Delta T < 0.1$), and define fine-tuning as

$$\kappa_T' \equiv \frac{M_T^*}{\text{Range of } M_T(\text{with } M_T < M_T^*)}$$

(80)
This definition of fine-tuning is harder to implement because, given a set of parameters except $M_T$, we first have to find out if regions of $M_T$ allowed by $\Delta T$ comes about because of cancellations, before we can apply Eq. 80. For example, it is possible that $\Delta T$ is always viable for any value of $M_T$ (as are the cases for Points 3 through 6 of Table I), so that we can not apply Eq. 80 as there is no fine-tuning in $\Delta T$. Despite its limited applicability compared to $\kappa_T$, $\kappa'_T$ may be a more reasonable measure of fine-tuning when there is a cancellation that leads to a small value for $\Delta T$. For Point 1(2) in Table I we have $\kappa_T \sim 33(11)$ and $\kappa'_T \sim 4.7(6.4)$, corresponding to a $33(11)$% change in $\Delta T$ per a 1% change in $M_T$, and also cancellation of one part in 4.7(6.4). For the other four points in Table I where $\kappa'_T$ in Eq. 80 is not well-defined, the values of $\kappa_T$ are small, indicating small fine-tuning for these sets of parameters.

Since a complete analysis of fine-tuning in the triplet sector in the TESSM is outside the scope of this work, we will conclude this section noting that in an extreme case (Eq. 37), $\kappa'_T \sim 24$, so we suspect that the typical fine-tuning in the triplet sector be less than one part in 24.

V. CONCLUSIONS

In this work we have revisited a very simple extension to the MSSM by adding a hypercharge-neutral, $SU(2)$ triplet chiral superfield. We considered this model as a reasonably economical extension of the MSSM and an alternative to the NMSSM, and extended the phenomenological studies in several directions. In addition to discussing the decoupling behavior of the triplets and comparing it to the decoupling behavior of the singlet of the NMSSM, we have computed the mass of the lightest $CP$-even Higgs boson to one-loop in the large quartic coupling $\lambda$. With $\lambda$, the Higgs-triplet-Higgs coupling in the superpotential, being comparable with the top Yukawa coupling, we find that the model is able to satisfy LEP2 bounds on the Higgs mass without contributions from the s-top sector. At the expense of perturbativity at the GUT scale, we have checked that the model can give much smaller fine-tuning in the electroweak sector than the MSSM. In the triplet sector, there may be fine-tuning in having small oblique corrections, but we estimate this fine-tuning to be no worse than about one part in 30.

With large $\lambda$, the TESSM opens up previously forbidden regions of parameters in the MSSM. In particular, both s-tops can be light in the TESSM. The light s-tops can then lead to phenomenology that is very different from the MSSM with important implications for the LHC, such as a Higgs boson that is both gluo-philic and photo-phobic.

Our simple analysis here can be extended in many directions, and these further studies must be done if the model is going to make precise predictions at the LHC. With large $\lambda$, there can be important higher-loop effects to the mass of the lightest, $CP$-even Higgs boson. Furthermore, important higher-loop QCD effects must also be included to properly study the gluon-gluon fusion production and the diphoton decay of the Higgs boson. We leave these open projects for the future and hope they may add to the already-rich possibilities of phenomenology that will be seen at the LHC.

APPENDIX A: FIELD-DEPENDENT MASS MATRICES

In this appendix, we list the field-dependent matrices that enter into the Coleman-Weinberg potential in Eq. 55. We have five mass matrices, one for each set of particles: the $CP$-even Higgs bosons ($M_a$), the $CP$-odd Higgs bosons ($M_b$), the charged Higgs bosons ($M_c$), the neutralinos ($M_\tilde{\chi}$), and the charginos ($M_{\tilde{c}}$). We first list the elements
of the Higgs bosons.

\[
(M^2_\alpha)_{11} = m^2_{H_u} + \mu^2 + \frac{1}{8} (g_1^2 + g_2^2) (3 a_u^2 - a_d^2) + \frac{\lambda^2}{2} (a_t^2 + a_d^2) - \sqrt{2} \lambda \mu a_t,
\]

\[
(M^2_\alpha)_{12} = -B_\mu - \frac{1}{4} (g_1^2 + g_2^2) a_u a_d + \lambda^2 a_u a_d + \frac{\lambda}{\sqrt{2}} (A_\lambda + M_T) a_t,
\]

\[
(M^2_\alpha)_{13} = \lambda^2 a_u a_t - \sqrt{2} \lambda \mu a_u + \frac{\lambda}{\sqrt{2}} (A_\lambda + M_T) a_d,
\]

\[
(M^2_\alpha)_{22} = m^2_{H_d} + \mu^2 + \frac{\lambda^2}{2} (a_t^2 + a_d^2) + \frac{1}{8} (g_1^2 + g_2^2) (3 a_d^2 - a_u^2) - \sqrt{2} \lambda \mu a_t,
\]

\[
(M^2_\alpha)_{23} = \lambda^2 a_d a_t - \sqrt{2} \lambda \mu a_d + \frac{\lambda}{\sqrt{2}} (A_\lambda + M_T) a_u,
\]

\[
(M^2_\alpha)_{33} = M_T^2 + m^2_T + B_T + \frac{\lambda^2}{2} (a_d^2 + a_u^2),
\]

\[
(M^2_c)_{11} = m^2_{H_u} + \mu^2 + \left( \lambda^2 - \frac{g_1^2 - g_2^2}{8} \right) a_d^2 + \frac{1}{8} (g_1^2 + g_2^2) a_u^2 + \sqrt{2} \lambda \mu a_t + \frac{\lambda^2}{2} a_t^2,
\]

\[
(M^2_c)_{12} = B_\mu + \frac{1}{2} \left( \lambda^2 + \frac{g_2^2}{2} \right) a_d a_u + \frac{\lambda}{\sqrt{2}} (M_T + A_\lambda) a_t,
\]

\[
(M^2_c)_{13} = \lambda \mu a_u + \frac{1}{\sqrt{2}} \left( \lambda^2 - \frac{g_2^2}{2} \right) a_u a_t - \lambda M_T a_d,
\]

\[
(M^2_c)_{14} = \lambda \mu a_u - \frac{1}{\sqrt{2}} \left( \lambda^2 - \frac{g_2^2}{2} \right) a_u a_t - \lambda A_\lambda a_d,
\]

\[
(M^2_c)_{22} = m^2_{H_d} + \mu^2 + \left( \lambda^2 - \frac{g_1^2 - g_2^2}{8} \right) a_u^2 + \frac{1}{8} (g_1^2 + g_2^2) a_d^2 + \sqrt{2} \lambda \mu a_t + \frac{\lambda^2}{2} a_t^2,
\]

\[
(M^2_c)_{23} = -\lambda \mu a_d + \frac{1}{\sqrt{2}} \left( \lambda^2 - \frac{g_2^2}{2} \right) a_d a_t + \lambda A_\lambda a_u,
\]

\[
(M^2_c)_{24} = -\lambda \mu a_d - \frac{1}{\sqrt{2}} \left( \lambda^2 - \frac{g_2^2}{2} \right) a_d a_t + \lambda M_T a_u,
\]

\[
(M^2_c)_{33} = M_T^2 + m^2_T + \frac{g_2^2}{4} (a_d^2 + 2 a_t^2 - a_u^2) + \lambda^2 a_u^2,
\]

\[
(M^2_c)_{34} = B_T - \frac{g_2^2}{2} a_t^2,
\]

\[
(M^2_c)_{44} = M_T^2 + m^2_T + \frac{g_2^2}{4} (a_u^2 + 2 a_t^2 - a_d^2) + \lambda^2 a_d^2,
\]

where \(m^2_{H_u,d}\) satisfy the minimization conditions Eqs. 28 and 29.
For the neutralino and charginos, since we do not take into account mixing with the gauginos, we have reduced matrices compared to those in Eqs. 40 and 42, and here we can simply replace the vevs by the corresponding particle

\[ M_{\tilde{N}} = \begin{pmatrix} 0 & -\mu + \frac{\lambda}{\sqrt{2}} a_t \frac{1}{\sqrt{2}} \lambda a_u \\ -\mu + \frac{\lambda}{\sqrt{2}} a_t & 0 \end{pmatrix}, \]

\[ M_{\tilde{\chi}} = \begin{pmatrix} \mu + \frac{\lambda}{\sqrt{2}} a_t & -\lambda a_d \\ \lambda a_u & M_T \end{pmatrix}. \]

(\text{A23})

\[ (\text{A24}) \]

APPENDIX B: DIPHOTON DECAY WIDTH OF A REAL SCALAR

In this appendix, we review the formula for the decay width of a real scalar \( \phi^0 \) (with mass \( m_\phi \)) decaying into two photons \( \Gamma(\phi^0 \to \gamma \gamma) \) \[52\]. Generally, given the interactions

\[ L \supset -A_s s^+ s^- \phi^0 - \frac{A_\psi}{2} \phi^0 \bar{\psi} \psi + A_W W^+ W^- \phi^0, \]

(\text{B1})

where \( s^\pm (\psi) \{ W^\pm \} \) is a charged scalar (fermion) \{gauge boson\} with mass \( m_s \) \( \{ m_\psi \} \{ m_W \} \) and electric charge \( Q_s \) \( \{ Q_\psi \} \{ Q_W \} \), the diphoton partial decay width is given by

\[ \Gamma(\phi^0 \to \gamma \gamma) = \frac{\alpha^2}{1024 \pi^3} m_\phi \left| N_\psi A_\psi Q_\psi^2 \frac{m_\phi}{m_\psi} F_\psi + N_s A_s Q_s^2 \frac{m_\phi}{m_s} F_s + N_W A_W Q_W^2 \frac{m_\phi}{m_W} F_W \right|^2, \]

(\text{B2})

where \( N_i \) are factors to account for additional degrees of freedom (such as color) and

\[ F_s = \tau_s [1 - \tau_s f(\tau_s)], \]

(\text{B3})

\[ F_\psi = -2 \tau_\psi [1 + (1 - \tau_\psi) f(\tau_\psi)], \]

(\text{B4})

\[ F_W = 2 + 3 \tau_W + 3 \tau_W (2 - \tau_W) f(\tau_W), \]

(\text{B5})

where

\[ \tau_i \equiv 4 \frac{m_i^2}{m_\phi^2}, \quad \text{for} \; i = s, \psi, W, \]

(\text{B6})

\[ f(\tau) = \begin{cases} \left( \arcsin \left( \frac{\sqrt{\frac{1}{\tau}}}{2} \right) \right)^2 & \text{if} \; \tau > 1, \\ -\frac{1}{4} \left( \ln \frac{\eta_+}{\eta_-} - i \pi \right)^2 & \text{if} \; \tau < 1, \end{cases} \]

(\text{B7})

\[ \eta_\pm \equiv 1 \pm \sqrt{1 - \tau}. \]

(\text{B8})

In the case of colored particles, we can make the replacement

\[ N Q^4_\psi \alpha^2_{em} \rightarrow 2 \alpha_s^2 \]

(\text{B9})

to compute the di-gluon decay width \( \Gamma(\phi^0 \to gg) \), which is related to the gluon-gluon fusion production cross section by

\[ \sigma(gg \to \phi^0) = \frac{\pi^2}{8 m_\phi^4} \Gamma(\phi^0 \to gg). \]

(\text{B10})

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