The local surface density of disc matter mapped by \textit{Hipparcos}

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ABSTRACT

We determine the surface density of matter in the disc of the Galaxy at the solar position using K giant stars. The local space density and luminosity function of the giants is determined using parallaxes from the \textit{Hipparcos} satellite; for more distant giants, observed in a cone at the South Galactic Pole, distances are determined using intermediate band DDO photometry (which has been calibrated to the \textit{Hipparcos} absolute magnitudes). From this sample, we determine the gravitational potential vertically of the local Galactic disc, by comparing the number of giant stars observed in the cone with the number expected for various models of the matter distribution in the disc. We derive an estimate of the dynamical disc mass surface density of $56 \pm 6\,M_\odot\,pc^{-2}$, which may be compared to an estimate of $53\,M_\odot\,pc^{-2}$ in visible matter. For all gravitating matter (disc + dark halo) we find the total density within 1.1 kpc of the disc midplane to be $74 \pm 6\,M_\odot\,pc^{-2}$. As has been found by a number of studies, including our own, we find no compelling evidence for significant amounts of dark matter in the disc.

Key words: Galaxy: kinematics and dynamics – Galaxy: structure – dark matter

1 INTRODUCTION

The measurement of the kinematics and vertical density falloff of a set of stars in the disc of the Galaxy permit a determination of the dynamical, or gravitating, local volume density and/or local column density of matter. Comparison of these quantities with the amount of visible matter has a long history in astronomy, as any difference between these quantities implies there remain mass components in the disc which have not yet been directly observed, i.e. disc dark matter. The first proposal that there might be a lot of such unobserved matter dates to Oort (1932,1960).

The European Space Agency’s \textit{Hipparcos} satellite ESA (1997) has had a major impact on this question, with the consensus now being that there is no dynamically significant dark matter component in the plane of the disc (Pham 1997; Crézé et al. 1998; Holmberg & Flynn 2000). However, these studies were based on the kinematics and density falloff of nearby A and F stars. Traditionally, these stars were never regarded as the ideal tracer with which to work, because they are rather young and it was thought that they would not be well mixed into the Galactic potential. The studies of A and F stars observed with \textit{Hipparcos} now show this view was probably overly cautious; however, the use of K dwarfs or K giants, being considerably older on average and certainly well mixed, is a very useful check on the disc mass determinations.

Previous determination of the surface mass density are $48 \pm 9\,M_\odot\,pc^{-2}$, (Kuijken & Gilmore 1991), $84^{+29}_{-24}\,M_\odot\,pc^{-2}$, (Bahcall, Flynn, & Gould 1992), $52 \pm 13\,M_\odot\,pc^{-2}$, (Flynn & Fuchs 1994), and $67^{+47}_{-24}\,M_\odot\,pc^{-2}$, (Siebert et al. 2003). These numbers give the total mass of the disc component only, and since the surveys do not extend to infinite heights above the Galactic plane they include some element of extrapolation. To circumvent these two limitations, an alternative way is to estimate the total gravitating mass (disc + dark halo) within some range in height above the Galactic plane. We preferably limit the range to the one actually covered by observations. Kuijken & Gilmore (1991) estimated the total mass within 1.1 kpc to $K_{61.1} = 71 \pm 6\,M_\odot\,pc^{-2}$, and Siebert et al. (2003) with their shallower survey give $K_{60.8} = 76^{+25}_{-12}\,M_\odot\,pc^{-2}$.

In this paper we return to the K giants in the post-\textit{Hipparcos} era, and redetermine the disc surface mass density. The work is a modern revision of the studies by Bahcall et al. (1992), hereafter BFG and Flynn & Fuchs (1994).
hereafter FF, which utilize the kinematics and density falloff of K giants at the South Galactic Pole (SGP) observed by Flynn & Freeman (1993). We update these studies by using Hipparcos to greatly improve the measurements of the local space density and luminosity function of K giants; and also to improve distance measurements of the K giants at the SGP. We find (yet again) that the observational data are very well fit by disc models containing only the known, visible components: no disc dark matter is required.

In section 2 we describe the selection of the sample from Hipparcos and the derivation of a new DDO photometry based absolute magnitude scale, as well as metallicity calibrations for K giants. The cone sample from the SGP is presented in section 3. In section 4 we present our model of the disc mass stratification, and fit the data with and without dark matter in the models, finding that we can fit the data without invoking disc dark matter. We summarize and conclude in section 5.

2 THE HIPPARCOS CATALOGUE AND SELECTION OF SAMPLE

Our determination of the surface mass density is made from a local sample of K giants (in a sphere of 100 pc radius around the Sun) and a second sample in a cone, extending to some 1 kpc vertical to the Galactic plane, at the South Galactic Pole (SGP). The technique used is elaborated fully in FF, although we greatly improve on that work by selecting a local sample using the Hipparcos catalogue, improving the determination of the luminosity function and local density of K giants; and further, by using a Hipparcos calibration of the DDO photometric system to determine much more reliable distances to the giants in the cone at the SGP. We also lift the limitations on metallicity used by FF who only used stars in the interval $-0.5 < \text{[Fe/H]} < 0.0$. The basis for the high metallicity cut at $\text{[Fe/H]} = 0.0$ was to exclude young luminous giants from the sample, on the basis of the best understanding of the age-metallicity relation at the time; however, the validity of a clear age-metallicity relation has now been put into question by most modern investigations (Feltzing, Holmberg, & Hurley 2001; Friel et al. 2002; Nordström et al. 2004), as well as already noted by FF; the evidence for the high metallicity cut as a means of excluding the youngest stars has weakened considerably. In any case, with the improved luminosity calibration, contamination from young, luminous giants is greatly alleviated. The low metallicity cut at $\text{[Fe/H]} = -0.5$ was chosen to exclude kinematically hotter stars which did not give information on the disk surface density within the limited vertical range studied; in this new investigation we extend the K giant study from several hundred parsecs to about 1 kpc above the disc midplane, so that also the kinematically hotter stars provide information on the Galactic potential.

2.1 Local K giants

The local K giants are drawn from the Hipparcos Survey, that part of the Hipparcos Catalogue intended to be a complete, magnitude limited stellar sample. For stars of spectral type later than G5 this magnitude limit is $V \leq 7.3 + 1.1 \sin |b|$. Figure 1 shows the giant region of the Hipparcos colour magnitude diagram for stars within 100 pc. The giants are bright ($V < 7.5$) and almost all have intermediate band DDO photometry available. In figure 2 we show the luminosity function (LF) of the giants in the colour range $1.0 < B - V < 1.5$. Note that for stars fainter than $M_V = 2$, the sample distance completeness limit moves closer than 100 pc and we make appropriate corrections to the LF. The absolute magnitudes are of very high quality, with errors of order 0.05 mag, i.e. much less than a bin width. The luminosity function is strongly peaked at $M_V \sim 0.85$, the position of the ‘clump’ (i.e. core helium burning) giants on the giant branch.
BFG and FF adopted the absolute magnitude cuts $0.8 < M_V < 2.2$ in isolating tracer K giants. The motivation for these cuts were to exclude nearby faint subgiants, and to avoid luminous giants for which the absolute magnitudes (which were based on a pre-Hipparcos calibration) were known to be rather poor. The calibration of the DDO photometric system for the absolute magnitudes of K giants adopted here does not suffer from deteriorating accuracy with absolute brightness as the older calibration did, and it is therefore opportune to change the brighter absolute magnitude cut to $M_V = 0.0$. We furthermore change the lower luminosity limit to $M_V = 2.0$. These changes have the very desirable advantage that the absolute magnitude window adopted, $0.0 < M_V < 2.0$ now brackets the clump stars, rather than cutting into them on the brighter side of the peak in the LF. Cutting into the LF in the earlier studies meant that quite careful correction for the Malmquist biases were necessary: whereas the absolute magnitude window adopted here now renders these corrections quite straightforward.

We note that although the luminosity cuts now neatly bracket the red giant clump, the colour cut at $B - V = 1.0$ excludes the bluer clump stars. Ignoring the bluer clump stars potentially introduces bias, as these stars will differ slightly from their redder counterparts in their age and metallicity distribution. Unfortunately, we are not at liberty to change this colour limit; as it is the blue completeness limit of the giants in the cone sample at the SGP, which was constructed in the pre-Hipparcos era when the colour and absolute magnitude extent of the clump was poorly known. However, even in the Hipparcos era, the colour cut turned out to be no real limitation for the present study. As can be seen in Figure 1, the clump to the blue side of $B - V = 1.0$ is not as tightly delineated as it is to the red side. The origin of this effect appears to be the intermixing of young giants, extending both below and especially above the ‘true’ clump (consisting of old giants) (Girardi 1999). The young giants have a much smaller scaleheight than the ordinary giants, and would be very scarce in the SGP cone sample. This is clearly demonstrated by the velocity dispersion of the stars with $0.0 < M_V < 2.0$ and $0.8 < B - V < 1.0$ which is $\sigma_W = 15.7 \pm 0.7$ kms$^{-1}$, significantly smaller than the redder K giants as will be shown below. The observational limitations imposed by the pre-Hipparcos studies that the bluer clump stars must be excluded from the sample seem to have been expedient.

The sample to 100 pc is volume complete, and allows us to determine the number density of these giants as $1.10 \pm 0.05 \times 10^{-4}$ per cubic parsec (recall that they are selected in the windows $0.0 < M_V < 2.0$ and $1.0 < B - V < 1.5$). The error in the number density is mainly due to Poisson sampling, there being 459 giants in the sample; the parallax errors contribute little. Adopting a typical giant mass of 0.8 $M_\odot$, this implies a mass density of $0.9 \times 10^{-4}$ $M_\odot$ pc$^{-3}$; giants thus represent some 0.2 per cent of the local disc’s stellar content by mass.

Space velocities for the Hipparcos giants have been calculated as described in Feltzing & Holmberg (2000). Radial velocities are available for 438 stars (95 per cent) of our 100 pc volume complete sample. In order to improve the statistical sampling of the local velocity distribution of the K giants, we also make use of a larger magnitude limited (rather than distance limited) sample comprising 1027 stars. The two samples turn out to have very similar velocity distributions. Figure 3 shows the vertical $W$-velocity distribution for the large sample of K giants as well as fits with one and two component Gaussians. The fitted velocity distributions have been corrected for the small contribution from measurement errors in distance, radial velocity and proper motion determination. With a single isothermal component, the velocity dispersion $\sigma_W$ is $18.3 \pm 0.4$ kms$^{-1}$, but a much improved fit is obtained with two components with velocity dispersions of $14.0 \pm 0.8$ and $28.3 \pm 2.9$ kms$^{-1}$ and a relative density from the hot component of $0.23 \pm 0.08$. For the small sample the corresponding numbers are: $18.2 \pm 0.6$, $14.0 \pm 1.1$, $30.0 \pm 4.8$ kms$^{-1}$ and a relative density of $0.19 \pm 0.10$.

The dispersions and relative densities were found using a maximum likelihood decomposition routine, which also give the confidence intervals of the fitted parameters. For the determination of the surface density of matter, the individual components of the velocity dispersion fit matter little; it is their combined distribution which is used as a smooth fit to the observed velocity distribution. The other components of the space velocity distribution are: $\sigma_U = 34$ kms$^{-1}$, $\sigma_V = 23$ kms$^{-1}$, and $V_{lag} = -18$ kms$^{-1}$.

### 2.2 DDO absolute magnitude calibration

Starting from the high quality Hipparcos determinations of absolute magnitude for nearby K giants, we have derived a new calibration of giant luminosity in the DDO intermediate band photometric system. We use all three DDO colours (C4142, C4245 and C4548) to fit for metallicity as well as absolute magnitude in the new calibration. The main difference between this determination and a similar calibration by Hog & Flynn (1998), is that they restricted their study to metal rich giants ([Fe/H] $>-0.5$), whereas our new cali-
bration is valid for the whole metallicity range of our study. DDO measurements were found in Mermilliod & Nitschelm (1989) for 419 stars in our local sample for the luminosity range $-2 < M_V < 3$. For this dataset we fitted a third-order polynomial containing all combinations of the three DDO colours using the downhill simplex amoeba routine (Press et al. 1992). The resulting calibration equation is:

$$M_V = 55.14 - 32.18c_1 + 15.90c_2 - 48.91c_3 - 82.34c_1^2 + 5.05c_1c_2 + 88.82c_2^2 + 20.54c_1c_3 + 87.62c_1c_3 + 12.85c_2c_3 + 49.69c_3^2 + 23.36c_1c_3 + 135.99c_3^2 + 9.52c_1^2c_2 - 79.24c_1^2c_3 - 53.58c_1c_3 - 55.47c_2c_3 - 189.84c_3^2c_1 + 97.40c_2c_3^2 + 84.97c_1c_2c_3$$

where $c_1 = C4548$, $c_2 = C4245$, and $c_3 = C4142$. Figure 4 shows the relation between the fitted DDO absolute magnitudes and the calibrators from *Hipparcos*. The scatter of the fit is 0.35 mag and note that the calibration is valid in the ranges: $1.10 < C4548 < 1.40$, $0.80 < C4245 < 1.30$, $-0.10 < C4142 < 0.40$, and $-2 < M_V < 3$.

To assess the precision of the fitting process, a number of bootstrapped DDO-*Hipparcos* catalogues were created. These catalogues were then used as input to the fitting routine to produce alternative calibration equations. Finally the original catalogue was fed into this ensemble to produce DDO magnitudes which could be compared to the original ones. The resulting mean dispersion of only 0.07 magnitudes shows the high stability of the derived absolute magnitudes to variations of the set of calibration stars.

### 2.3 DDO metallicity calibration

In a similar way to the new DDO $M_V$ calibration, we have also determined a new DDO [Fe/H] calibration. Here we used in addition to the DDO photometry from Mermilliod & Nitschelm (1989) the catalogue of giants with spectroscopic metallicities of Taylor (1999). For 597 stars found in both sources we fitted a relation of the same functional form as the $M_V$ calibration. The resulting calibration equation is:

$$[\text{Fe/H}] = 6.30 - 5.67c_1 + 6.05c_2 - 13.27c_3 - 3.65c_1^2 - 2.42c_1^2c_2 + 4.36c_1c_2 + 9.25c_1c_3 + 1.51c_2c_3 + 2.93c_1^2 + 3.02c_1c_3 - 83.39c_3^2 - 0.46c_2^2c_2 + 7.66c_3^2c_3 - 0.12c_2c_3 - 13.53c_2^2c_3 - 26.76c_3^3c_1 + 58.76c_1^2c_2 - 4.41c_1c_2c_3$$

where $c_1 = C4548$, $c_2 = C4245$, and $c_3 = C4142$. Figure 5 shows the calibration valid in the ranges: $1.14 < C4548 < 1.43$, $0.77 < C4245 < 1.33$, $0.08 < C4142 < 0.45$, and $-0.93 < [\text{Fe/H}] < 0.34$.

### 3 SGP K GIANTS

BFG and FF analyzed the sample of K giants at the SGP of Flynn & Freeman (1993) for which absolute magnitudes and hence distances were determined using DDO photometry, using a calibration which much predated the *Hipparcos* results. In this paper we reanalyze the sample, now with revised K giant absolute magnitudes via the improved, *Hipparcos*-based calibration of giant absolute magnitudes described in the previous section.

The first sample we analyze (the ‘HD sample’) consists of all K giants at the SGP in the Flynn & Freeman (1993) catalogue to a limiting visual magnitude of $V = 9.2$ and for which $0.0 < M_V < 2.0$ and $1.0 < B - V < 1.5$, resulting in 139 K giants in a 430 square degree region.

In addition to the HD stars in the 430 square degree region at the South Galactic Pole, Flynn & Freeman (1993) observed candidate K giants in a 40 square degree region from the catalogue of Eriksson (1978). This catalogue extends to $V = 11.0$. Candidate giants were selected in the colour range $0.95 < B - V < 1.55$ and DDO photometry and radial velocities obtained.

Flynn & Freeman also observed stars with $B - V > 0.7$.
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from their own plate material obtained with a 6 inch Zeiss camera mounted on the 9 inch Oddie refractor at Mount Stromlo, reaching to $V = 11.2$. The Oddie survey covered 100 square degrees.

The giants from these two sources will be referred to as the Eriksson and Oddie samples. Neither of these two samples were included in the disc surface mass density analyses of BFG or FF; this was primarily because the poorer absolute magnitudes of these fainter giants, as determined by DDO photometry, introduced error into the determinations which outweighed the advantage of better Poisson statistics. This expedient, while appropriate at the time, can now be dropped because of two major improvements. Firstly, we now have a greatly improved absolute magnitude calibration of the DDO photometric system for the giants which is tied to the very high precision Hipparcos parallaxes. Secondly, we can utilize the Tycho-2 catalogue (Hog et al. 2000) to directly determine the completeness of these samples (and the HD sample for that matter). The deeper samples improve the disc mass determination by extending the height to which the giants probe the Galactic potential to about 1000 pc above the disc, whereas the HD sample does not probe beyond 500 pc. The Eriksson and Oddie catalogues add 212 K giants to the sample, again selected in the range $0.0 < M_V < 2.0$ and $1.0 < B - V < 1.5$.

Due to the availability of the Tycho-2 catalogue, it is now possible to complement the HD, Eriksson and Oddie samples with proper motions to obtain full space velocities. The Tycho-2 catalogue consists of $\sim 2.5$ million stars, and is 99 per cent complete to $V = 11$. This is well borne out by our sample: where 350 stars (99.7 per cent) have measured proper motions in the Tycho-2 catalogue. Together with the radial velocities from Flynn & Freeman (1993) we obtain full space velocities for 264 stars in our cone sample. In general the velocity dispersions ($\sigma_U, \sigma_V, \sigma_W, V_{hel}$) = $(41, 32, 25, -23)$ kms$^{-1}$ of the cone sample are larger than the corresponding ones in the local sample in accordance with the increasing contribution of the thick disc above the Galactic plane. Figure 6 shows the $W$ velocities as a function of absolute magnitude for the DDO cone sample compared to the Hipparcos local sample. Figure 7 shows the corresponding distribution of $W$ velocities as a function of metallicity for the DDO cone sample compared to the DDO local sample. We also use the Tycho-2 magnitudes and colours to check the completeness of the SGP cone sample. When using this information from Tycho-2 it is very important that the full transformations (rather than the first order approximations) from the Tycho instrumental $V_T$ and $B_T$ colours into the standard Johnson $V$ and $B - V$ are used. This approach which was used in obtaining the $B - V$ colours given in the Hipparcos catalogue is as described in section 1.3 and Appendix 4 of Volume 1 of the catalogue. We note that use of the first order relations can lead to biases of up to 0.05 in the $B - V$ colour, which is insufficiently accurate for testing sample completeness. For the HD catalogue, which is reported by Flynn & Freeman (1993) as complete to $V = 9.2$ and extending to about $V = 10.5$ with decreasing completeness, we confirm that the sample is indeed complete to $V = 9.2$. For the Eriksson and Oddie catalogues, we find that they are indeed complete to $V = 11.0$.

4 SURFACE MASS DENSITY DETERMINATION

We adopt a disc model similar to the one used in Holmberg & Flynn (2000) to analyze the data. The basic model is shown in Table 1 (and described in section 3) of Holmberg & Flynn (2000): it is of the type introduced by Bahcall (1984a,b,c), in which the disc is represented by a set of massive, kinematically isothermal components, tracing young stars, old stars, stellar remnants and gas. The Poisson-Boltzmann equations are solved simultaneously and the density falloff of each component computed. The difference between the model used here and the old one is that a thick disc component is ex-
The $K_z$ force law resulting from the basic mass model.

Figure 9. Velocity distribution of the SGP cone sample of giants (histogram), compared to the expected velocity distribution (dotted line). This is calculated from the mass model and the local velocity distribution for the same height distribution as the observed sample.

Figure 10. Comparison between the local *Hipparcos* luminosity function (LF) (normalized to the number of stars in the cone sample, dotted line) and the one from the DDO cone sample (full line). When the local *Hipparcos* LF is convolved by the DDO cone sample measurement uncertainties and selection effects, the result is the dashed histogram. The local LF is thus found to be a good match to the LF of the cone giants.

Figure 9 shows a comparison between the measured velocity distribution in the cone sample and the expected distribution, for which $\sigma_W = 26\,\text{km}\,\text{s}^{-1}$. The expected distribution is computed from the combination of the mass model and the velocity distribution in the plane of the disc (as outlined in detail in FF). The good agreement between the measured and calculated distributions is an indication that the local and the cone sample are members of the same tracer population and well suited for the determination of the mass density.

Figure 10 shows a comparison between the local and the cone luminosity functions (LFs). The luminosity function of the cone K giants appears to be quite consistent with being drawn from the same LF as the local K giants. This is in consideration of the much larger typical error in the absolute magnitudes ($0.35$ mag in the cone compared to $0.05$ mag for the local giants), and the fact that the cone sample is magnitude rather than volume limited. Under the assumption that the LFs are the same, then from measured local density of the K giants and the predicted density falloff of the giants in a particular model of the disc mass, we can compute the number of giants which should appear in the cone survey. This can be compared to the actual number of observed giants and the model evaluated. In the manner of FF we ran a series of Monte Carlo simulations of observations of stars in the cone samples, taking into account observational uncertainties and selection effects, in order to compute the expected number of giants in the cone for various mass models of the disc. The error sources include the uncertainty in the local density of the tracer stars and the uncertainty in the fitted velocity distribution of the local sample. This was determined by making a series of bootstrapped velocity catalogues, and then fitting a two-component Gaussian in the

\[ \rho_0 = 0.102 \, \text{M}_\odot \, \text{pc}^{-3}, \quad \Sigma = 52.8 \, \text{M}_\odot \, \text{pc}^{-2}, \quad \Sigma_{1.1} = 70.6 \, \text{M}_\odot \, \text{pc}^{-2}. \]
same way as for the observed velocity sample. Finally the goodness of the fit to the observations were determined using a standard $\chi^2$ statistic.

We find that the basic disc mass-model gives a very good description of the vertical distribution of stars in the Galactic mid-plane. The no-disc-dark-matter model is an excellent fit to the data in all parts of the survey.

Figure 11. Predicted (solid curve) versus observed numbers of giants in the cone at the SGP for the HD and Eriksson+Oddie samples. The predictions are for the disc model containing visible matter only, shown as a function of $Z$ height above the Galactic mid-plane. The no-disc-dark-matter model is an excellent fit to the data as well.

5 CONCLUSIONS

We have reanalyzed the sample of K giants studied by Flynn & Freeman (1993), Bahcall, Flynn & Gould (1992) and Flynn & Fuchs (1994) for determining the mass density and surface mass density of the local Galactic disc. The reanalysis incorporates the absolute magnitudes for nearby K giants measured by the Hipparcos satellite, and a calibration of absolute magnitudes for distant giants. Our results confirm that there is no significant missing component in the mass inventory of the local Galactic disc. We derive a surface mass density of material at the Solar position of $K_{2,1} = 74 \pm 6 \, M_\odot \, pc^{-2}$ within 1.1 kpc of the mid-plane, with $56 \pm 6 \, M_\odot \, pc^{-2}$ as the total disc contribution, compared to 53 $M_\odot \, pc^{-2}$ in visible material. This is in excellent agreement with the analysis of K dwarfs, for which Kuijken & Gilmore (1991) estimated the total mass within 1.1 kpc as $K_{2,1} = 71 \pm 6 \, M_\odot \, pc^{-2}$.

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REFERENCES

Bahcall J. N., 1984a, ApJ, 276, 156
Bahcall J. N., 1984b, ApJ, 276, 169
Bahcall J. N., 1984c, ApJ, 287, 926
Bahcall J. N., Flynn C., Gould A., 1992, ApJ, 389, 234 (BFG)
Crezé M., Chereul E., Bienaymé O., Pichon C., 1998, A&A, 329, 920
Eriksson P-I.W., 1978, Uppsala Astr. Obs. Report No 11
ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200
Feltzing S., Holmberg J., 2000, A&A, 357, 153
Feltzing S., Holmberg J., Hurley, J.R., 2001, A&A, 377, 911
Flynn C., Freeman K.C., 1993, A&AS, 97, 835
Flynn C., Fuchs B., 1994, MNRAS, 270, 471 (FF)
Friel E.D., Janes K.A., Tavarez M., Scott J., Katsanis, R., Lotz J., Hong L., Miller N., 2002, AJ, 124, 2693
Girardi L., 1999, MNRAS, 308, 818
Hag, E., Flynn C., 1998, MNRAS, 294, 28
Hag E. et al., 2000, MNRAS, 355, L27
Holmberg, J., Flynn, C., 2000, MNRAS, 313, 209
Korchagin, V.I., Girard, T.M., Borkova, T.V., Dinescu, D.I., van Altena, W.F., 2003, AJ, 126, 2896
Kuijken K., Gilmore G., 1991, ApJ, 367, L9
Mermilliod J.C., Nitschelm C., 1989, A&AS, 81, 401
Nordström B. et al., 2004, A&A, 418, 989
Oort J. H., 1932, Bull. Astron. Inst. Netherlands, 6, 249
Oort J. H., 1960, Bull. Astron. Inst. Netherlands, 15, 45
Pham H. A., 1997, in Perryman M. A. C., Bernacca P. L., eds, HIPPARCOS Venice ’97, ESA SP-402, p. 559
Press, W.H., Teukolsky, S.A., Vetterling, W.T., & Flannery B.P., 1992, Numerical Recipes in Fortran
Siebert A., Bienaymé O., & Soubiran C., 2003, A&A, 399, 531
Taylor B.J., 1999, A&AS, 134, 523