The Friedmann integrals and physical vacuum in the framework of macroscopic extra dimensions

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The structure and origin of the Friedmann integrals are analyzed within the framework of large extra dimensions proposed by Arkani-Hamed et al. (1998). It is demonstrated that the integrals might emerge from extra-dimension physics and reveal its signature. In the case of two extra dimensions, the integrals are expressed via the product $M_* R$ of the only fundamental energy $M_*$ and the size $R$ of the extra dimensions. The cosmic vacuum density turns out to be $\sim R^4$, in this case. The effective cut-off in the spectrum of the quantum zero-point oscillations at the frequency $\omega_V \sim 1/R$ is assumed to be associated with the epoch when the Hubble radius becomes larger than $R$. This occurs at temperatures $\sim M_*$ in the first three picoseconds of the cosmic expansion.
1 Introduction

The Friedmann cosmology is a successful theory that predicted the cosmic expansion 80 years ago. It is consistent with all the bulk of current observational results, including acceleration of the expansion and the existence of cosmic vacuum recognized recently with the SN type Ia observations [1]. Any generalization or extension of standard cosmology must include the Friedmann theory as a well-working particular or asymptotic case. On the other hand, if the Friedmann theory is really a limit of a more general theory, it may keep some features of such a theory. These features can be mirrored in the free (empirical) parameters of the theory which are the Friedmann integrals.

The Friedmann integrals are associated with the energy content of the Universe and represent four major forms of cosmic energy: cosmic vacuum, dark matter, baryons and radiation. The integrals appear as a result of integration of the Friedmann ‘second’ equation which is essentially the internal energy conservation law for each of the energy forms, and they determine the structure of the Friedmann ‘first’ equation which is the mechanical energy conservation law, if to use the notions of the Newtonian physics.

With the use of the current dataset, the numerical values of the Friedmann integrals can be estimated, and they have proven to be equal to each other, on the order of magnitude. If this is not a purely arithmetical coincidence, the near equality of the Friedmann integrals is a time-independent symmetry relation that puts cosmic vacuum in correspondence with the non-vacuum cosmic energies [2]. Symmetries that do not concern space-time geometry are usually called internal symmetries, in particle physics; we face now cosmic internal symmetry (CISY) that exists as long as the observed forms of cosmic energy are present in nature.

In this note, I discuss the framework of macroscopic extra dimensions proposed in [3] as a multidimensional extension of the Friedmann cosmology; I examine how the Friedmann integrals and CISY may reflect and reveal the basic physics of extra-dimensions, if they really exist. The origin of cosmic vacuum, which is actually the same object as vacuum of particle physics, is one of the major aspects of the problem under consideration.

2 Four integrals

The Friedmann integrals are given by a common relation

\[ A = \left[ (\rho a^{3(1+w)})^{\frac{1}{1+w}} \right], \tag{1} \]

where \( a(t) \) is the 3D curvature radius and/or scale factor; \( \kappa = 8\pi G/3 \) is the Einstein gravitational constant; \( w = p/\rho \) is the pressure-density ratio for each of the energy forms: \( w = -1 \) for vacuum (V), \( w = 0 \) for dark (D) matter and baryons (B), \( w = 1/3 \) for radiation (R). Hereafter \( c = 1 \).
The integrals enter the basic equation for the cosmological expansion in a rather symmetrical way:

$$\dot{a}^2 = \left(\frac{A_V}{a}\right)^2 + A_D/a + A_B/a + (A_R/a)^2 - k.$$  \hspace{1cm} (2)

Here $k = 1, 0, -1$ for close, flat and open models, respectively.

The numerical values of the integrals can be estimated with the use of the observational dataset on the cosmic densities, the age of the Universe and the Hubble constant. The result is as follows [2]:

$$A_V = (\kappa \rho_V)^{-1/2} \sim 10^{61} M_{Pl}^{-1},$$
$$A_D = \kappa \rho_D a^3 \sim 10^{61} M_{Pl}^{-1},$$
$$A_R = \kappa \rho_B a^3 \sim 10^{59} M_{Pl}^{-1},$$
$$A_R = (\kappa \rho_R)^{1/2} a^2 \sim 10^{59} M_{Pl}^{-1}. \hspace{1cm} (3)$$

Here $\rho_V, \rho_D, \rho_B, \rho_R$ are the densities of vacuum, dark matter, baryons, and radiation, correspondingly. The units are used in which $c = \hbar = 1$; $G = M_{Pl}^{-2}$, and the Planck mass $M_{Pl} = 1.2 \times 10^{19}$ GeV.

As we see, the four integrals are equal to each other within two orders of magnitude, and their equality can be treated as a symmetry relation. This symmetry is not exact; CISY is violated at the level of a few percent, on the logarithmic scale. Its physical nature seems to be not too mysterious: it may be understood as a result of the cosmic freeze-out process in the early Universe [2].

### 3 Fundamental energy scales

The standard freeze-out model for the particle-antiparticle annihilation assumes that the dark matter particles are thermal relics of the hot initial stages of the cosmological expansion. With the use of a fairly transparent version of this model [4], one can show that the near equality of the Friedmann integrals for vacuum, dark matter and radiation is a direct outcome of the freeze-out process at the first three picoseconds [2]. One can also find with the freeze-out model that the Friedmann integrals are expressed in the terms of two fundamental energy scales which are the Planck mass and the electroweak mass $M_{EW} \sim 1$ TeV:

$$A_V \sim A_D \sim A_R \sim (\bar{M}_{Pl}/M_{EW})^4 M_{Pl}^4 \sim g^8(M_{Pl}/M_{EW})^4 M_{Pl}^4 \sim 10^{61\pm1} M_{Pl}^{-1}. \hspace{1cm} (4)$$

Here $\bar{M}_{Pl} = gM_{Pl}$ is the reduced Planck mass, $g \simeq 0.1 - 0.3$.

With the same accuracy and in accordance with Eqs.(1,3) one has for the constant vacuum density (cf. also [4]):

$$\rho_V \sim g^8(M_{Pl}/M_{EW})^8 M_{Pl}^4 \sim 10^{-122\pm2} M_{Pl}^4. \hspace{1cm} (5)$$

The Friedmann integral for baryons is not included in the freeze-out model; perhaps it may be obtained with the current models of electroweak baryogenesis.
(this topic is reviewed in [5]). Accidentally or not, the Baryonic Number can also be expressed numerically in the terms of the two fundamental energy scales:

$$B \sim (\bar{M}_{Pl}/M_{EW})^{2/3} \sim 10^{10}. \quad (6)$$

The big dimensionless ratio $\bar{M}_{Pl}/M_{EW} \sim 10^{15}$ proves to be the key quantity that determines the Friedmann integrals and the density of cosmic vacuum, according Eqs.(4,5). In particle physics, the nature of the huge gap between $M_{Pl}$ and $M_{EW}$ is not explained and known as the hierarchy problem. We see now that cosmology is highly sensitive to this problem as well.

4 Extra dimensions

The framework of large extra dimensions [3] provides a new prospective for the hierarchy problem. It may also suggest a new understanding of the structure of the Friedmann integrals.

It is assumed in [3] that there is one and only one fundamental energy scale in nature, and this scale $M_*$ is close to the electroweak scale $M_{EW}$. As for the Planck mass, it is due to the extra dimensions of space:

$$M_{Pl} \sim (M_* R)^{n/2} M_* \quad (7)$$

Here $n, R$ are the number of the spatial extra dimensions and their size, which is proposed to be the same for all of them. It is also argued in [3] that the case $n = 2$ is the most promising; in this case the size of two extra dimensions

$$R \sim 0.1 \text{ cm}, \quad n = 2. \quad (8)$$

In accordance with Eq.(7), the big dimensionless ratio $M_{Pl}/M_{EW}$ is replaced now with the product $M_* R$. Treating the extra dimension framework as a multidimensional extension of the Friedmann theory, one may obtain the Friedmann integrals in the terms of the new fundamental energy and the size of extra dimensions:

$$A \sim (M_{EW} R)^{3/2} M_{EW}^{-1} \quad (9)$$

In the case of two extra dimensions, one has from this:

$$A \sim (M_{EW} R)^2 R, \quad n = 2. \quad (10)$$

Then one finds for vacuum density a general relation

$$\rho_V \sim (M_{EW} R)^{-2n} M_{EW}^{4n} \quad (11)$$

and a particular relation in the case of two extra dimensions:

$$\rho_V \sim R^{-4}; \quad n = 2. \quad (12)$$
Finally, one finds for the Baryonic Number:

\[ B \sim (M_{EW}R)^{n/3} \]  

(13)

Only two extra dimensions provide the correct value of \( B \):

\[ B \sim (M_{EW}R)^{2/3} \sim 10^{10}; \quad n = 2 \]

(14)

5 Discussion

As we see, the multi-dimensional extension of the standard cosmology sheds new light on the origin of the free parameters of the Friedmann theory. These parameters – the Friedmann integrals – might emerge from physics of extra dimensions and carry their signature. According to Eqs.(9-14), this physics is explicitly imprinted in the structure of the Friedmann integrals: the integrals turn out to be expressed in the terms of two basic quantities of extra-dimension physics – the fundamental energy scale and the size of extra dimensions. The two quantities appear in a dimensionless product \( M_{EW}R \), which is of the order of \( 10^{16} \) for the case of two extra dimensions. It is not surprising that this new dimensionless number is similar to the mass ratio \( M_{Pl}/M_{EW} \) involved in the cosmological freeze out (see Sec.2). Actually, we have now a reformulated hierarchy problem: instead of one of the two energy scales, a new spatial scale appears in the framework [3].

What is surprising is that the new expression (Eq.12) for the vacuum density is free from any signs of hierarchy problem: the density is a power of the extra-dimension size only, \( \rho_V \sim R^{-4} \), in case of two extra dimensions.

One may assume, as it has been done earlier not once, that the finite value of the vacuum density is a result of an effective cut-off in the frequency spectrum of the zero-point oscillations of quantum fields. If so, the cut-off frequency is \( \omega_V \sim 1/R \), in the case of two extra dimensions. It means that the vacuum energy on the brane is due to the low-frequency band of the oscillations only, \( \omega \leq \omega_V \sim 1/R \). The wave lengths of zero-point oscillations are all larger than the size of the extra dimensions: \( \lambda_V \geq 1/\omega_V \sim R \sim 1 \) mm.

It seems instructive that the energy density in the Casimir effect is the same power function of a size: \( \rho_C \sim d^{-4} \). Here \( d \) is the distance between two parallel plates, or concentric spheres, etc. which is much smaller than the other sizes \( L \) in the experiment. Recent measurements of the Casimir force between a cantilever and a plate see in [6] (the separation was between 0.5 and 3 microns; the accuracy 15%). This analogy, especially in the case of two plates, suggests that the frequency cut-off and the origin of the finite energy density can really be due to a small ratio of the sizes involved, like \( d/L \ll 1 \).

In the context of the early Universe, a small size ratio appears when the the Hubble radius exceeds the size \( R \sim 1 \) mm of extra dimensions. It occurs in the era when the cosmic temperature is about \( M_\ast \sim 1 \) TeV, and the cosmic age is
a few picoseconds. According to the consideration above, this event might lead to the effective cut-off in the spectrum of zero-point quantum oscillations. One can assume, that since that epoch, the vacuum density on the brane is finite and constant, and it keeps then its value the same forever. The Friedmann cosmology with its basic parameters and internal symmetry becomes valid since that era.

Three other almost simultaneous events might happen at the same era of a few picoseconds: electroweak symmetry breaking, electroweak baryogenesis, and freeze out of the dark matter annihilation. It may hardly be a simple coincidence, and one common physical mechanism is rather behind all the four cosmic events. It seems remarkable that the quantitative characteristics of the four events are directly associated with the fundamental energy scale $M_*$ and the size of extra dimensions $R$, according to Eqs.(9-14).

The earlier pre-Friedmann evolution of the Universe is essentially multi-dimensional, and the framework [3] provides promising grounds for the study of this evolution. In its turn, this earlier evolution might start at the epoch when the cosmic age was $t_M \sim 1/M_* \sim 10^{-27}$ sec. This is 16 orders of magnitude larger than the Planck time.

Might it be that the history of the Universe and the very time take start at $t_M$? Indeed, if energies considerably larger than $M_*$ are not possible in nature at all, no temporal structures finer than $\sim 1/M_*$ can physically be resolved. An answer to the question can be expected from the coming experiments at the new big colliders. If it occurs that any considerable excess of the particle energy above $\sim M_*$ is prevented by the generation of new particles, black holes, gravitational waves, etc., it will mean that the principal upper energy limit and the principal lower time limit do exist in nature.

On the other hand, further submillimeter laboratory experiments with the Casimir effect, deviations from the gravity inverse-square law, etc. may identify the size of spatial extra dimensions or at least their upper limit. As is discussed in [7], not 1 TeV, but rather 10 - 30 TeV is an appropriate value for $M_*$ in the case of two dimensions; if so, a constant dimensionless factor $q = M_*/M_{EW}$ must be introduced to the relations above, where $1 \leq q \leq 30$. Then one will have (with the account also of the reduced Planck mass): $A \sim (gq^2M_{EW}R)^3M_{EW}^4$ and $\rho_V \sim g^{-4}q^8R^{-4}$, where $R$ is about 1 mm as above. The larger value of the fundamental mass corresponds to the size of extra-dimensions about 1 - 10 microns, which is, by the way, near the spatial separation in the experiment [6].

Note finally that the framework of large extra dimensions [3] offers a simple explanation to the phenomenon of non-exact symmetries (like, for instance, chiral symmetry) and small symmetry breaking. This explanation addresses the existence of other branes in the bulk world: if symmetry breaking originates on a distant brane, it should be small on our brane, because what happens far away affects our brane only weakly. In this way, one may speculate that CISY is non-exact due to the weak influence of a distant brane or a separate fold of our own brane.
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