String worldsheet theory in hamiltonian framework and background independence

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Abstract. We analyze exact conformal invariance of string worldsheet theory in non-trivial backgrounds using hamiltonian framework. In the first part of this talk we consider the example of type IIB superstrings in Ramond-Ramond pp-wave background. In particular, we discuss the quantum definition of energy-momentum (EM) tensor and two methods of computing Virasoro algebra. One of the methods uses dynamical supersymmetries and indirectly establishes (partially) conformal invariance when the background is on-shell. We discuss the problem of operator ordering involved in the other method which attempts to compute the algebra directly. This method is supposed to work for off-shell backgrounds and therefore is more useful. In order to understand this method better we attempt a background independent formulation of the problem which is discussed in the second half of the talk. For a bosonic string moving in an arbitrary metric-background such a formulation is obtained by following DeWitt's work (Phys.Rev.85:653-661,1952) in the context of particle quantum mechanics. In particular, we construct certain background independent analogue of quantum Virasoro generators and show that in spin-zero representation they satisfy the Witt algebra with additional anomalous terms that vanish for Ricci-flat backgrounds. We also report on a new result which states that the same algebra holds true in arbitrary tensor representations as well.

1. Introduction and summary
This talk will be based on the papers [1, 2, 3, 4] and certain new results in [5]. In [1, 2, 3] we try to understand exact conformal invariance of string worldsheet theory in certain pp-wave backgrounds using hamiltonian framework\(^1\). Usually backgrounds that are nearly flat are considered for such analysis. The reason is that for flat background the theory is exactly solvable and the relevant vacuum is known. One can then define the quantum EM tensor in the weak background field approximation by normal ordering the interaction terms with respect to the flat-space-vacuum. The pp-wave background on the other hand is not close to flat and therefore the corresponding worldsheet theory is strongly interacting. We will discuss this problem in section 2.

The end result of the analysis in section 2 will show that there exists an operator ordering ambiguity in the computation of the Virasoro anomaly if one tries to calculate them directly using the quantum EM tensor. In order to have a better understanding of the situation we seek a framework where the computations can be done in a background independent way. Such a framework was developed by DeWitt in [10] for particle quantum mechanics. As will be

\(^1\) Two-dimensional non-linear sigma model is usually studied using lagrangian framework [6]. Hamiltonian framework has also been used to some extent earlier in [7, 8] (see also [9]).
discussed in section 3, the classical worldsheet theory can be interpreted to be a theory of a
particle moving in an infinite-dimensional curved background subject to certain potential. It
was shown in [4] that the background independent analogue of the quantum Virasoro generators
constructed in the sense of DeWitt satisfy, in spin-zero representation, the Witt algebra with
additional anomalous terms that vanish for Ricci-flat backgrounds. We also discuss how this
might possibly suggest a resolution of the ordering problem for the pp-wave.

In order to understand the implications of the Ricci-flatness condition mentioned above
various other questions regarding this construction need to be answered. One such question,
namely extending the analysis to higher rank tensor representations, has been considered in [5].
The results show that the same Ricci-flatness condition holds true in general. We will discuss
this in section 3. We end with some final remarks in section 4.

2. Hamiltonian framework for worldsheet theory in pp-wave
Here we will consider a pp-wave background, with or without R-R flux, such that the only
non-trivial component of the metric is

\[ G_{++} = K(\vec{x}) , \tag{2.1} \]

where the vector sign refers to the transverse directions \( x^I, I = 1 \cdots D - 2 \), \( D \) being 26 or 10
depending on bosonic/superstring theory. A plane-wave is a pp-wave for which \( K(\vec{x}) \) is quadratic
in \( x^I \).

Proving conformal invariance for the type IIB R-R plane-wave background [13] using
hamiltonian framework was first attempted by Kazama and Yokoi in [14]. In order to formulate
the worldsheet theory as a CFT the authors considered Green-Schwarz superstring in semi-light-
cone gauge following [15]. The Virasoro algebra was also calculated using a direct method in
local form. In this method one directly calculates the commutators \([T(\sigma), T(\sigma')], [\tilde{T}(\sigma), T(\sigma')]
and \([T(\sigma), \tilde{T}(\sigma')], \) where \( T(\sigma) \) and \( \tilde{T}(\sigma) \) are the right and left moving EM tensor components
and \( \sigma \) is the worldsheet space coordinate. The main result of [14] that is relevant to us may be
summarized as follows\(^3\):

\[ \text{If the EM tensor is ordered according to MNO, then the Virasoro algebra is not}
\text{satisfied due to non-zero anomalous terms. However, this anomaly vanishes if}
\text{the the EM tensor is ordered with a different prescription called phase-space}
\text{normal ordering (PNO).} \tag{2.2} \]

This result poses a puzzle as defining the EM tensor according to PNO would imply that the
theory does not have a smooth flat-space limit. This, as we will explain below, is contradictory
with the universality property discussed in [16, 17].

Let us consider an on-shell pp-wave background with flat transverse space in string theory.
The following theorem can be proved (see [17]) for the corresponding worldsheet theory,

\[ \text{There exists a universal sector of worldsheet operators whose correlation functions}
\text{are evaluated to be same as that in flat background.} \tag{2.3} \]

\(^2\) In absence of R-R flux, the pp-wave should correspond to an exact CFT whenever it is Ricci-flat (i.e. vanishing
transverse laplacian of \( K \)) [12]. We will consider a particular case of constant R-R flux in type IIB string theory,
in which case conformal invariance should require the Ricci tensor to be proportional to the square of the flux
strength [13].

\(^3\) MNO refers to massless normal ordering [14], the one that is relevant to flat background. The reason for
this nomenclature is that the worldsheet theory for flat background may be understood as the massless limit of
that corresponding to R-R plane-wave in light-cone gauge. PNO was also discussed in the literature earlier (for
example [8]), but we will not go into the technical details of its definition.
This implies that the worldsheet theory corresponding to any pp-wave should admit a smooth flat space limit. In fact it requires that the spectrum of conformal dimensions within the universal sector of Hilbert space (i.e. the states created by universal operators) be universal. This argument implies that PNO cannot be the right way to order the EM tensor.

2.1. Testing universality argument in bosonic string theory
In [1] we considered the same problem in the simpler setting of bosonic string theory and showed that the results are consistent with the above universality argument. It is easy to check that the PNO ordered EM tensor gives rise to a spectrum of negative conformal dimensions in flat background. The physical spectrum was computed for the plane-wave in [1] by explicitly constructing the string field theory (SFT) quadratic term and diagonalizing it. It indeed shows the existence of negative conformal dimensions for the PNO ordering. On the other hand the MNO-ordered EM tensor reproduces the result that is expected from light-cone gauge analysis.

The Virasoro algebra was also calculated for the off-shell pp-wave using the same method as in [14]. In addition to the standard central charge terms, this produces certain additional operator anomaly terms. The calculations have been done using both PNO and MNO ordered EM tensor and in both the cases the operator anomaly terms, ordered accordingly, were shown to be proportional to the equation of motion for the pp-wave.

2.2. A closer look at R-R pp-wave
Although the results of [1] support the universality argument, the case of R-R pp-wave seems to be more subtle. In particular, the above results do not explain the observation in (2.2). This was explained in [2] which we discuss below.

We first write,

\[ T = T^{(0)} + \delta T, \quad \tilde{T} = \tilde{T}^{(0)} + \delta \tilde{T}, \]

where the superscript (0) refers to the flat background, i.e. the free part and \( \delta T \) is the interaction part containing non-trivial background fields. The correct way of defining the quantum EM tensor was shown to be as follows,

\[ \text{The free part of the EM tensor is ordered according to MNO, while the interaction part needs to be ordered according to PNO.} \]

The reason for the PNO ordering for the interaction term is as follows. Given the supergravity background characterized by the function \( K(\tilde{x}) = \int d\tilde{k} \tilde{K}_{\text{sugra}}(\tilde{k}) e^{i\tilde{k}.\tilde{x}} \), the relevant couplings \( \tilde{K}_{\text{ws}}(\tilde{k}) \) in the worldsheet sigma model are in general related to \( \tilde{K}_{\text{sugra}}(\tilde{k}) \) by a field redefinition. The effect of such field redefinition can be absorbed into the normal ordering prescription of the corresponding interaction term. It turns out that if the term is ordered according to PNO then the field redefinition becomes identity. By constructing the SFT quadratic term and diagonalizing it one finds that the definition in (2.5) reproduces the correct physical spectrum as found in light-cone analysis [18].

The explanation of (2.2) goes as follows. The operator anomaly terms corresponding to the three commutators mentioned above (2.2) have the following structure,

\[ \mathcal{A}^R(\sigma, \sigma') = A^R(\sigma, \sigma') + A_F(\sigma, \sigma'), \]

\(^4\) The field redefinition discussed above involves the transverse laplacian of \( K \) which is proportional to the R-R flux strength when the background is on-shell. Therefore for the bosonic example considered in [1], where there is no R-R flux, the field redefinition becomes trivial on-shell. This is why in the bosonic case the EM tensor, with both the free and interaction parts ordered according to MNO, gives the right spectrum, as mentioned toward the end of subsection 2.1.
\[ A^L(\sigma, \sigma') = A^L(\sigma, \sigma') + A_F(\sigma, \sigma'), \]
\[ A(\sigma, \sigma') = A(\sigma, \sigma') + A_F(\sigma, \sigma'), \]

respectively, where \( A^R, A^L \) and \( A \) are the bosonic and \( A_F \) is the fermionic contribution. The calculations are done by introducing a UV-regulator \( \epsilon \) on the worldsheet. All such terms turn out to have a special structure: \( \mathcal{O}_i(\sigma, \sigma') \delta_i(\sigma - \sigma') \), where \( \mathcal{O}_i(\sigma, \sigma') \) is a composite operator anti-symmetric in \( \sigma \) and \( \sigma' \) and \( \delta_i(\sigma - \sigma') \) is the Poisson kernel representation of Dirac delta function which approaches the latter in the limit \( \epsilon \to 0 \). Notice that at the classical level, where \( \mathcal{O}_i(\sigma, \sigma') \) remains the same under reordering of operators, such a term must vanish as expected.

In the quantum theory this term is well-defined at a finite \( \epsilon \). However, first reordering the term in a different way and then taking \( \epsilon \to 0 \) produces a different answer.

The above result indicates that the direct method of calculating the Virasoro anomaly suffers from ordering ambiguity. It turns out that if the anomaly terms are ordered according to PNO then the correct supergravity equation of motion (EOM) is reproduced. This is the reason for the observation in (2.2). However, there is no good understanding why such an ordering prescription should work.

We will come back to this question in section 3 where a background independent construction will be described. In this construction covariant Ricci-anomaly terms will arise whose technical origin is quite different. This indicates that the observation that PNO-preservation described above gives the right supergravity EOM may be misleading and does not generalize to arbitrary backgrounds.

2.3. Dynamical supersymmetry argument for conformal invariance

The ordering ambiguity described above may be attributed to the fact that the relevant vacuum is non-perturbative and it is not generically known how to define quantum operators in a controlled manner. However, in this particular case the background is supersymmetric (in fact, as much as flat space) which may be used to gain such control. It turns out that in semi-light-cone gauge the following relations hold true in both flat and R-R plane-wave backgrounds [3],

\[ \{ Q_a, Q_b \} = 2\delta_{ab} \int \frac{d\alpha}{2\pi} T_\perp(\sigma), \quad \{ \tilde{Q}_a, \tilde{Q}_b \} = 2\delta_{ab} \int \frac{d\sigma}{2\pi} \tilde{T}_\perp(\sigma), \]

where \( Q_a \) and \( \tilde{Q}_a \) are properly scaled dynamical supercharges and \( T_\perp \) and \( \tilde{T}_\perp \) are the transverse parts of the EM tensor components. As shown in [3], the above relations can be used in certain Jacobi identities to relate the second order susy variations of the transverse EM tensor components and certain integrated forms of the anomaly terms,

\[ \{ [T_\perp(\sigma), Q_a], Q_b \} + \{ [T_\perp(\sigma), Q_b], Q_a \} = 2\delta_{ab} \int \frac{d\sigma'}{2\pi} A^R(\sigma, \sigma') + \cdots, \]
\[ \{ [\tilde{T}_\perp(\sigma), \tilde{Q}_a], \tilde{Q}_b \} + \{ [\tilde{T}_\perp(\sigma), \tilde{Q}_b], \tilde{Q}_a \} = 2\delta_{ab} \int \frac{d\sigma'}{2\pi} A^L(\sigma, \sigma') + \cdots, \]
\[ \{ [\tilde{T}_\perp(\sigma), Q_a], Q_b \} + \{ [\tilde{T}_\perp(\sigma), Q_b], Q_a \} = 2\delta_{ab} \left[ - \int \frac{d\sigma'}{2\pi} A(\sigma', \sigma) + \cdots \right], \]

where the ellipses refer to certain known terms. The idea is to compute the anomalies indirectly by calculating the second order susy variations on the left hand side independently. Although such a computation does not suffer from ordering ambiguity in the sense discussed earlier, the problem arise in the form of divergences as non-(anti)commuting operators appear at the same point. However, this subtlety can be taken care of by considering a point-split definition of the EM tensor. The final results show that all the integrated anomaly terms appearing on the right hand sides of (2.8) are zero.
The above argument establishes conformal invariance only partially as the anomaly terms have been computed with one coordinate integrated over. The complete proof would require one to extend this argument by incorporating local susy currents.

3. A background independent Hamiltonian framework

The susy argument works only for on-shell backgrounds which are supersymmetric. In order to understand background EOM from Virasoro anomaly one needs to start with an off-shell background and compute the Virasoro algebra using the direct method discussed in subsections 2.1 and 2.2. However, as discussed earlier this method leads to ambiguous results for the anomaly terms. In [4] we explore a background independent framework, hereafter called DeWitt-Virasoro (DWV) construction, where we attempt to formulate the problem in a vacuum independent way. Such a framework, if it exists, should define the worldsheet theory, at least formally, over the space of all possible geometrical backgrounds. Starting from this framework it should also be possible to derive all the know properties of the 2-d CFT for a given exact background by specializing to it.

3.1. DeWitt-Virasoro construction

The approach considered in [4] is to describe the string quantum mechanics in a coordinate independent way following DeWitt’s argument [10] in the context of particle quantum mechanics. We consider the simplest case of a bosonic string moving in an arbitrary metric-background and re-write the worldsheet theory in a language where it describes a single particle moving in an infinite-dimensional curved background. The relevant map is given by,

\[ x^i = \int \frac{d\sigma}{2\pi} X^\mu(\sigma) e^{-im\sigma}, \quad g_{ij}(x) = \int \frac{d\sigma}{2\pi} G_{\mu\nu}(X(\sigma)) e^{i(m+n)\sigma}, \quad a^i(x) = \int \frac{d\sigma}{2\pi} \partial X^\mu(\sigma) e^{-im\sigma} \]

(3.9)

where the index identifications are as follows: \( i = \{\mu, m\}, \ j = \{\nu, n\} \) etc. The general coordinate transformation (GCT) in the physical spacetime: \( X^\mu \to X'^\mu(X) \) induces the same in the infinite-dimensional spacetime: \( x^i \to x'^i(x) \) under which \( g_{ij}(x) \) transforms as metric tensor and \( a^i(x) \) as a vector field. The classical worldsheet lagrangian takes the following form in terms of the new set of variables,

\[ L(x, \dot{x}) = \frac{1}{2} g_{ij}(x) \left[ \dot{x}^i \dot{x}^j - a^i(x) a^j(x) \right]. \]

(3.10)

GCT is a point canonical transformation which can be described by a unitary transformation in the quantum theory. DeWitt’s analysis in [10] shows how to construct the quantum mechanics such that general covariance is manifest in its position space representation. As a result it gives generally covariant expressions for all the matrix elements.

Following DeWitt’s method we formally define a background independent notion of the quantum Virasoro generators and algebra, hereafter called DWV generators and algebra. We first write down the classical Virasoro generators using the particle language which then leads to the following natural definition (consistent with the hermiticity property) of the quantum DWV generators,

\[ L_{(i)} = \frac{1}{4} \left[ \hat{\pi}^*_k g^{kl+i}(\hat{x}) \hat{\pi}_l - (\hat{\pi}^*_k a^{k+i}(\hat{x}) + a^{k+i}(\hat{x}) \hat{\pi}_k) + g_{kl}(\hat{x}) a^k(\hat{x}) a^{l+i}(\hat{x}) \right], \]

\[ \tilde{L}_{(i)} = \frac{1}{4} \left[ \hat{\pi}^*_k g^{kl+i}(\hat{x}) \hat{\pi}_l + (\hat{\pi}^*_k a^{k+i}(\hat{x}) \hat{\pi}_k) + g_{kl}(\hat{x}) a^k(\hat{x}) a^{l+i}(\hat{x}) \right], \]  

(3.11)

where given the index identification below eqs.(3.9), we have defined: \( \tilde{i} = \{\mu, -m\}, \ (i) = m \) and \( i + j = \{\mu, m + n\} \). The \( \hat{\pi} \)-operators are given by (in \( \hbar = \alpha' = 1 \) unit) [19],

\[ \hat{\pi}_j = \hat{p}_j + \frac{i}{2} \gamma_j(\hat{x}), \quad \hat{\pi}^*_j = \hat{p}_j - \frac{i}{2} \gamma_j(\hat{x}), \]  

(3.12)
where $\gamma_j = \gamma^{jk}_k$ are the contracted Christoffel symbols. DeWitt’s technique in [10] allows us to calculate the matrix elements in spin-zero representation. We find that the DWV algebra in spin-zero representation is given by the Witt algebra with additional anomaly terms that vanish for Ricci-flat backgrounds,

$$
\langle \chi \rangle \left\{ \begin{aligned}
\hat{L}_{(i)} \hat{L}_{(j)} &= (i-j) \hat{L}_{(i+j)} \\
\hat{L}_{(i)} \hat{L}_{(j)} &= (i-j) \hat{L}_{(i+j)} \\
[\hat{L}_{(i)}, \hat{L}_{(j)}] &= \frac{1}{8} \left( \pi^{k+i} r_{k+l}(\hat{x}) a^{l+j}(\hat{x}) - a^{k+i}(\hat{x}) r_{k+l}(\hat{x}) \pi^{l+j} \right)
\end{aligned} \right\} \langle \psi \rangle ,
$$

where $|\chi\rangle$ and $|\psi\rangle$ are two arbitrary spin-zero states and $r_{ij}$ is the Ricci tensor.

The DWV generators constructed above are background independent and are not normal ordered with respect to any particular vacuum. However, as shown in [4], the quantum Virasoro generators in flat and pp-wave backgrounds can be obtained by normal ordering the DWV generators in a suitable way. The resulting quantum Virasoro algebra contains the central charge terms in addition to the operator anomaly terms appearing in (3.13). This enables us to compare (the ambiguous) operator anomaly terms in the bosonic sector in (2.6) with the one in (3.13), at least in the spin-zero representation. This comparison tells us that all the operator anomaly terms $A^R$, $A^L$ and $A$ in (2.6) must vanish. Furthermore, it also implies that the Ricci-term in (3.13) was missing in the previous computation. This apparent discrepancy can be resolved by the following observation. By going to the position space representation it can be argued that the matrix element of the Ricci-term in (3.13) vanishes, though the Ricci tensor itself does not, because of certain restrictions on the index contraction for pp-wave and that the Ricci tensor depends only on the transverse coordinates.

### 3.2. Higher rank tensor generalization

An important question for the construction described above is how to generalize this to higher spin representations which are very relevant in string theory. In particular, one would like to know if the DWV algebra in (3.13) holds true in arbitrary spin representations. We studied this question in [5] and found the answer in affirmative.

In flat background one usually obtains the higher rank tensor states by applying suitable creation operators on the ground state. It turns out that such state-operator mapping is not suitable for the background independent framework. What turn out to be more relevant in this case are the Schrödinger wavefunctions. Such wavefunctions can be obtained either by multiplying a tensor field with a scalar wavefunction or by applying covariant derivatives on it.

In [5] we consider a new framework where an arbitrary tensor state of rank $n$, which is by itself coordinate invariant, is expanded as follows,

$$
|\psi_{(n)}\rangle = \int dw \: \psi^{i_1 i_2 \cdots i_n}(x) |i_1 i_2 \cdots i_n; x\rangle ,
$$

where $\psi^{i_1 i_2 \cdots i_n}(x)$ is the tensor wavefunction under consideration and $|i_1 i_2 \cdots i_n; x\rangle$ are the rank-$n$ position eigenstates which are newly introduced. The problem then reduces to finding the matrix element of the momentum operator in such basis states. This is done by suitably generalizing DeWitt’s argument in [10]. The result is given by,

$$
\langle i_1 \cdots i_n; x | \hat{\pi}_k | j_1 \cdots j_n; \bar{x} \rangle = -i \delta_{n,\bar{n}} \left[ \Delta_{i_1 \cdots i_n; j_1 \cdots j_n}(x, \bar{x}) \partial_k \delta(x, \bar{x}) + F_{i_1 \cdots i_n; j_1 \cdots j_n}(x, \bar{x}) \delta(x, \bar{x}) \right] ,
$$

where $\partial_k \equiv \frac{\partial}{\partial x^k}$,

$$
\delta(x, \bar{x}) = \frac{\delta(x - \bar{x})}{(g(x)g(\bar{x}))^\frac{1}{2}} ,
$$

(3.16)
\( \delta(x - \tilde{x}) \) being the Dirac delta function, \( g = |\text{det}g_{ij}| \),

\[
F_{i_1 \cdots i_n j_1 \cdots j_n k}(x, \tilde{x}) = -\nabla_k \Delta_{i_1 \cdots i_n j_1 \cdots j_n}(x, \tilde{x}),
\]

where \( \nabla_k \) is the covariant derivative with respect to \( x^k \) and

\[
\Delta_{i_1 \cdots i_n j_1 \cdots j_n}(x, \tilde{x}) = \Delta_{i_1 j_1}(x, \tilde{x}) \cdots \Delta_{i_n j_n}(x, \tilde{x}).
\]

To define \( \Delta_{ij}(x, \tilde{x}) \) we proceed as follows. Let us first consider two points \( X^\mu \) and \( \tilde{X}^\mu \) in the physical spacetime sufficiently close to each other so that a unique geodesic passes through them. Then following [20] we define the bi-vector of geodetic parallel transport \( D_{\mu \nu}(X, \tilde{X}) \). The two vector-indices \( \mu \) and \( \nu \) are associated to the two points \( X \) and \( \tilde{X} \) respectively such that under a GCT the bi-vector transform as a vector at each point separately. Moreover, \( D_{\mu \nu}(X, \tilde{X}) \), contracted with another vector at one of the points, gives the same vector parallel transported to the other point along the geodesic. One generalizes this to a string by considering two string embedding \( X^\mu(\sigma) \) and \( \tilde{X}^\mu(\sigma) \) such that two points belonging to the two strings at the same value of \( \sigma \) are connected by a unique geodesic. This gives us the parallel transport bi-vector \( D_{\mu \nu}(X(\sigma), \tilde{X}(\sigma)) \) on the worldsheet whose infinite-dimensional counterpart is given by \( \Delta_{ij}(x, \tilde{x}) \),

\[
\Delta_{ij}(x, \tilde{x}) = \oint \frac{d\sigma}{2\pi} D_{\mu \nu}(X(\sigma), \tilde{X}(\sigma)) e^{i(m+n)\sigma}.
\]

The above definition can be generalized to any other well-behaved paths which are not necessarily geodesics. However, the expression in (3.15), being sensitive to the expansion of \( \Delta_{ij}(x, \tilde{x}) \) only up to first order in separation, is insensitive to the choice of path.

Although the expression in (3.15) makes the tensorial properties of the relevant matrix manifest, the actual calculations are done using an explicit representation of \( \Delta_{ij}(x, \tilde{x}) \) given in terms of the vielbeins. Given this basic ingredient, one can compute the DWV generators in tensor representations. These expressions contain all the terms that are present in the scalar representation. Moreover, they receive additional contributions involving spin connection.

Finally, following [4] we proceed to compute the DWV algebra. As expected, the computation gets more complicated than the previous one because of the additional contributions. However, we show that all the unwanted contributions cancel in various ways to reproduce the same algebra in (3.13). There arise three kinds of additional terms which depend on Riemann tensor, covariant derivative of spin connection and higher power of spin connection. It turns out that because of the particular definition in (3.11) these three kinds of terms are organized in such a way that they cancel due to various identities in general relativity.

### 4. Final remarks

The fact that the same Ricci-flatness condition is obtained as the condition of vanishing anomaly in the DWV algebra in arbitrary tensor representations is interesting. An important question that arises at this point is how to understand the conventional CFT’s as special cases of the DWV construction. This is understood for flat and pp-wave backgrounds in spin-zero representation. However, because of the new way of describing the tensor states as in (3.14) it is interesting to explore how exactly the conventional CFT computations should be realized in our background independent description. It would also be interesting to understand the central charge terms in the Virasoro algebra in a background independent way. We hope to come back to these questions in future.

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5 Under a GCT, \( \hat{x}^i \rightarrow \hat{x}'^i(\hat{x}) \): \( \hat{\lambda}_k(\hat{x}) \rightarrow \hat{\lambda}'_k(\hat{x}) \), where \( \hat{\lambda}_k(\hat{x}) \) is the inverse of the Jacobian matrix for the transformation. This implies that the matrix element on the left hand side of (3.15) has a covariant vector index \( k \) at \( x \). This is obviously true for the second term on the right hand side. It is also true for the first term as \( \delta(x, \tilde{x}) \) is a bi-scalar.
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References
[1] Mukhopadhyay P 2008 JHEP 0811, 034 (2008) (Preprint arXiv:0807.0923 [hep-th]).
[2] Mukhopadhyay P 2009 JHEP 0905, 059 (2009) (Preprint arXiv:0902.3750 [hep-th]).
[3] Mukhopadhyay P 2009 Phys. Rev. D 80, 126007 (2009) (Preprint arXiv:0907.2155 [hep-th]).
[4] Mukhopadhyay P 2009 (Preprint arXiv:0912.3987 [hep-th]).
[5] Mukhopadhyay P 2010 DeWitt-Virasoro construction in tensor representations (to appear).
[6] Friedan D 1980 Phys. Rev. Lett. 45, 1057
Friedan D 1985 Annals Phys. 163, 318
Alvarez-Gaume L, Freedman D Z and Mukhi S 1981 Annals Phys. 134, 85
Fradkin E S and Tseytlin A A, 1985 Phys. Lett. B 160, 69
Fradkin E S and Tseytlin A A 1985 Nucl. Phys. B 261 1
Sen A 1985 Phys. Rev. D 32, 2102
Sen A 1985 Phys. Rev. Lett. 55, 1846
Callan C G, Martinec E J, Perry M J and Friedan D 1985 Nucl. Phys. B 262, 593
Callan C G, Klebanov I R and Perry M J 1986 Nucl. Phys. B 278, 78
Fridling B E and Jevicki A 1986 Phys. Lett. B 174, 75
[7] Maharana J and Veneziano G 1987 Nucl. Phys. B 283, 126
Akhoury R and Okada Y 1987 Phys. Rev. D 35, 1917
Akhoury R and Okada Y 1989 Nucl. Phys. B 318, 176
Das A K and Roy S 1987 Z. Phys. C 36, 317
Das A K, Maharana J and Roy S 1989 Phys. Rev. D 40, 4037
Das A K, Maharana J and Roy S 1990 Nucl. Phys. B 331, 573
Fubini S, Maharana J, Roncadelli M and Veneziano G 1989 Nucl. Phys. B 316, 36
Alam S 1989 Phys. Rev. D 40, 4047
[8] Diakonou M, Farakos K, Koutsoumbas G and Papantonopoulos E 1990 Phys. Lett. B 247, 273
Diakonou M, Farakos K, Koutsoumbas G and Papantonopoulos E 1990 Phys. Lett. B 240 (1990) 351;
Jain S, Mandal G and Wadia S R 1987 Phys. Rev. D 35, 778
Jain S, Mandal G and Wadia S R 1987 Phys. Rev. D 35, 3116
[9] DeWitt B S 1952 Phys. Rev. 85, 653
DeWitt B S 1957 Rev. Mod. Phys. 29, 377
[10] Lawrence A E and Martinec E J 1996 Class. Quant. Grav. 13, 63 (Preprint hep-th/9509149).
[11] Amati D and Klimcik C 1988 Phys. Lett. B 210, 92
Amati D and Klimcik C 1989 Phys. Lett. B 219, 443
Horowitz G T and Steif A R 1990 Phys. Rev. Lett. 64, 260
[12] Blau M, Figueroa-O’Farrill J, Hull C and Papadopoulos G 2002 JHEP 0201, 047 (Preprint [arXiv:hep-th/0110242])
[13] Kaezama Y and Yokoi N 2008 JHEP 0803, 057 (Preprint arXiv:0801.1561 [hep-th])
[14] Berkovits N and Marchioro D Z 2005 JHEP 0501, 018 (Preprint arXiv:hep-th/0412198)
[15] Mukhopadhyay P 2006 (Preprint arXiv:hep-th/0611138)
[16] Mukhopadhyay P 2007 JHEP 0706, 061 (Preprint arXiv:0704.0085 [hep-th])
[17] Metsaev R R 2002 Nucl. Phys. B 625, 70 (Preprint hep-th/0112044)
Metsaev R R and Tseytlin A A 2002 Phys. Rev. D 65, 126004 (Preprint hep-th/0202109)
[18] Omote M and Sato H 1972 Prog. Theor. Phys. 47, 1367
[19] DeWitt B S and Brehme R W 1960 Annals Phys. 9, 220