Transverse mass observables for charged Higgs searches at hadron colliders

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Charged Higgs boson may be produced at hadron colliders via the decay of top quarks. At the LHC, the large production cross section of top quarks pairs will make it one of the earliest channels allowing to search for physics beyond the standard model. Assuming the decay \( H^+ \rightarrow \tau^+ \nu \), previous searches have so far considered only hadronic \( \tau \) decays. Here we show that by using appropriate kinematical variables (transverse mass observables), leptonic \( \tau \) decays can be used as well, for both observing and possibly measuring the mass of the charged Higgs boson. This can increase the overall experimental sensitivity to charged Higgs bosons at hadron colliders.

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I. INTRODUCTION

Charged Higgs bosons arise in models with extended Higgs sector such as two Higgs doublets models (2HDM) and in particular the Minimal Supersymmetric Standard Model (MSSM). If the charged Higgs mass is below the top mass, it is expected to be produced at hadron colliders dominantly via top quark decay, due to the large Yukawa coupling of the top. At the LHC, the large production cross section of top quark pairs means that the existence of charged Higgs bosons could be probed with early data, with a potential of being among the first evidences of physics beyond the standard model. Here, we present methods for observing such a charged Higgs and possibly measuring its mass, assuming that it decays to a \( \tau \) lepton and a neutrino, \( H^+ \rightarrow \tau^+ \nu \), and using the leptonic decay modes of the \( \tau \), either \( \tau^+ \rightarrow e^+ \nu_e \bar{\nu}_e \) or \( \tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\mu \). The decay mode \( H^+ \rightarrow \tau^+ \nu \) dominates in the MSSM for \( \tan \beta > 1 \) and is therefore a favorable search channel. Previous searches at the Tevatron[1,2] considered only hadronic \( \tau \) decays, since those allow the \( \tau \) lepton to be identified and an excess of \( \tau \) leptons over the standard model prediction can provide a direct evidence for a charged Higgs boson. On the other hand, the leptonic decay modes involving an isolated electron or muon have experimental advantages with regard to triggering the event and suppressing backgrounds, and they are expected to be associated with less systematic uncertainties compared to the hadronic mode. This makes these channels particularly important for early stages of the LHC.

The event topology we consider here belongs to a wide class of events where the presence of undetectable particle(s) in the final states prevents full reconstruction of the sought after particle’s mass. It is often possible in such cases to define kinematical variables that are bounded by the unknown mass, and therefore it’s value can be inferred from an edge in the distribution of that variable over many events. We generally refer to such variables as transverse mass observables, following the most known example which is the \( W \) transverse mass in the process \( W \rightarrow \ell \nu \). Many generalizations to more complicated processes have been studied in past years, such as the \( m_{T2} \) variable[3] and others[4][5][6]. In the following sections we apply this approach to charged Higgs searches in \( tt \) events. We consider both semi-leptonic and di-leptonic final states, and show that such observables could be constructed in those cases and provide valuable information for observing and measuring the mass of the charged Higgs boson.

Event simulation

To study the performance of the proposed variables we use MADGRAPH[7] for event generation, interfaced to PYTHIA[8] for showering and hadronization. \( \tau \) decays are handled by TAUOLA[9]. Detector effects and jet clustering are done with the PGS[10] fast simulation package, configured with LHC parameters. All samples are generated for a center of mass energy of 14 TeV.

\[
(m_T^W)^2 = 2P_T^{\ell}P_T^{miss}(1 - \cos \phi_{\ell,miss})
\] (1)

II. SEMI-LEPTONIC \( tt \) EVENTS

In this section we consider events in which one of the top quarks decays to a charged Higgs \( (t \rightarrow bH) \) while the second top quark decays hadronically and is therefore fully reconstructable \( (t \rightarrow bW \rightarrow bijj) \). This is the most useful channel for top reconstruction since the missing energy and isolated lepton together with b-tagged jets provide a good rejection against backgrounds. The main background to this process is the standard model decay \( t \rightarrow Wb \) with a subsequent leptonic decay of the W, either directly \( W \rightarrow \ell \nu \) or via a \( \tau : W \rightarrow \tau \nu \rightarrow \ell \bar{\nu} \nu \). It is natural for this channel to reconstruct the W transverse mass, defined as [11]
where $p^f_T, p^{miss}_T$ are the lepton and missing transverse momenta respectively and $\phi_{f,miss}$ is the azimuthal angle between them. For direct $W$ decays this transverse mass has a Jacobian peak near the $W$ mass, hence it can provide separation between the two $W$ decay modes, as well as charged Higgs decays (see Figure 1).

In principle, the presence of charged Higgs boson decays will be manifested as an excess of events in the lower region of the transverse mass distribution. In practice however, to observe such an excess would require an accurate modeling of the shape of the distribution which depends on the missing momentum resolution. This may be hard to achieve. A second drawback of this approach is that almost no information about the mass of the charged Higgs boson can be obtained. Still, by requiring $m^W_T$ to be sufficiently below the peak region, one can reject a large part of the background and enhance the signal content of the sample.

It is useful to define the $W$ transverse mass \(^{(1)}\) equivalently as a minimization result, i.e.

$$\begin{aligned}
(m^W_T)^2 &= \min_{\{p^f_T, E^{miss}_T \}} \left( (p^f_T + p^{miss}_T)^2 \right) \\
(2)
\end{aligned}$$

where $p^f_T, p^{miss}_T$ are the lepton and neutrino (missing) four momenta, respectively. It is straightforward to show that the above requirement indeed leads to the expression \(^{(1)}\) (see a similar derivation in appendix A). The definition \(^{(2)}\) makes explicit the fact that $m^W_T \leq m_W$, i.e. $m^W_T$ is bounded by the $W$ mass. Furthermore, it implies that $m^W_T$ is the best bound that can be placed on $m_W$, per event.

For the case of a leptonic $\tau$ decay (either from a $W$ or charged Higgs boson) the constraint used in \(^{(2)}\) is not valid since the missing momentum comes from three neutrinos and therefore $(p^{miss})^2 \neq 0$. However, if one of the two b-jets in the event could be associated with the leptonically decaying top quark then the on-shell constraint for the top quark could be used, and one would define analogously a “Charged Higgs transverse mass” in the following way:

$$\begin{aligned}
(m^H_T)^2 &= \max_{\{p^f_T, E^{miss}_T \}} \left[ (p^f_T + p^{miss}_T)^2 \right] \\
(3)
\end{aligned}$$

note that in this case the transverse mass is defined by maximization of the invariant mass, since it is bounded from above by the top quark mass. The charged Higgs transverse mass by definition therefore satisfies $m^H_T \leq m^H_T \leq m_{top}$, where $m_{H+}$ is the true charged Higgs mass and $m_{top}$ is the nominal value of the top mass used in the constraint. The explicit expression for it can be easily derived and is given by: (see appendix A)

$$\begin{aligned}
(m^H_T)^2 &= \left( \sqrt{m^2_{top} + (p^f_T + p^b_T + p^{miss}_T)^2} - (p^f_T + p^{miss}_T)^2 \right)^2 \\
(4)
\end{aligned}$$

It should be noted that top quark decay can involve gluon radiation \(^{(12)}\). In such case the constraint used in \(^{(3)}\) is not exact. If the top quark emits a gluon and turns off-shell before it’s decay, then the invariant mass of the decay products is smaller than the top pole mass

$$\begin{aligned}
\sqrt{(p^f_T + p^b_T + p^{miss}_T)^2} &= m^*_{top} \leq m_{top} \\
(5)
\end{aligned}$$

however, the charged Higgs transverse mass \(^{(4)}\) is a monotonically increasing function of the input value of $m_{top}$ and therefore the following relation still holds:

$$\begin{aligned}
(m^H_T)^2 &\geq m^H_T (m^*_{top}) \geq m_{H+} \\
(6)
\end{aligned}$$

i.e., $m^H_T$ retains the property of being bounded by the charged Higgs mass also in the presence of gluon radiation. The values of $m_{H+}$ however will be shifted upwards, making the Jacobian peak less pronounced.

**Event reconstruction**

To reconstruct the charged Higgs transverse mass, one must correctly associate a b-jet to the leptonic top decay. For the purpose of our study we consider events where exactly two of the jets are b-tagged, such that there are two possible choices. The selection cuts we apply are the following:

- At least four jets with $p_T > 20$ GeV and $|\eta| < 2.5$
- Two of the above jets are b-tagged
- Exactly one isolated lepton ($e$ or $\mu$) with $p_T > 20$

FIG. 1: The W transverse mass distributions for leptonic W decays and charged Higgs boson decays with $m_{H^0} = 130, 150$ GeV. all distributions are normalized to unit area. A minimum cut of 20 GeV for the lepton transverse momentum is applied.
To choose between the two possible b-jet assignments, we first consider the invariant mass \( m^2_{\ell b} = (p_T^\ell + p_T^b)^2 \). For the correct b-jet, this is kinematically bounded by
\[
m^2_{\ell b} \leq m^2_{\text{top}} - m^2_W \tag{7}
\]
ote that this bound holds for both W and charged Higgs boson decays, since we are assuming \( m_{H^+} \geq m_W \). In case that one of the lepton-b pairings violates the above condition, the opposite pairing in selected. In events where both pairings satisfy (7), we further consider the top quark transverse mass :
\[
(m^\text{top}_T)^2 = m^2_{\ell b} + 2(E^\text{th}_T p^\text{miss}_T - p^\text{th}_T \cdot p^\text{miss}_T) \tag{8}
\]
which is required to satisfy \( m^\text{top}_T \leq m_{\text{top}} \). Again, if one of the pairings violates this condition then the opposite one is selected. Finally, if this is not the case, the hadronic top is reconstructed using two of the light (non b-tagged) jets plus one of the b-jets. The b-jet that results in the invariant mass \( m_{jjb} \) being closest to the nominal value of \( m_{\text{top}} \) is associated to the hadronic top, and thus the other b-jet is associated to the leptonic top.

Figure (2) shows the charged Higgs transverse mass distributions after selection and reconstruction. The power of \( m^H_T \) as a discriminant based on the mass difference is clearly observed.

A useful property of \( m^H_T \) is that the background distribution is independent of the W decay mode (direct or \( \tau \)-mediated). This is easily understood from (1), which depends on the lepton and missing transverse momenta only via their sum \( \vec{p}_T^\ell + \vec{p}^\text{miss}_T = \vec{p}^H_T \).

In figure (3) an example pseudo-experiment containing both the main background and a charged Higgs signal with \( m_{H^+} = 130 \) GeV is shown. The number of events correspond to an integrated luminosity of 1 fb\(^{-1}\) at \( \sqrt{s} = 14 \) TeV, and the branching ratio \( Br(t \rightarrow H^+ b) = 15\% \) is assumed to be 15\%, roughly the current upper limit set by the Tevatron searches [2]. The statistical significance of discovery in this case exceeds 5\(\sigma\), demonstrating the potential power of this variable.

To define an analogue to the charged Higgs transverse mass of the previous section, we follow the same procedure, i.e. maximize the invariant mass subject to all available constraints that are given by the known W and top quark masses, as well the conservation of momentum in the transverse plane. We have the following set of six

\[
\text{FIG. 2: The charged Higgs transverse mass distributions for lepton W decays and a charged Higgs decays with } m_{H^+} = 130, 150 \text{ GeV. all distributions are normalized to unit area.}
\]

\[
\text{FIG. 3: Example pseudo-experiments (points with error bars) on top of the signal and background distributions, assuming } Br(t \rightarrow H^+ b) = 15\% \text{ and } m_{H^+} = 130 \text{ GeV, for an integrated luminosity of 1 fb}^{-1}.
\]

\[
\text{FIG. 4: A Feynman diagram illustrating the event topology of a dileptonic } t\bar{t} \text{ event involving a charged Higgs decay.}
\]
constraints, corresponding to the notation of Figure 3:

\[ (p^{H^+} + p_b^b)^2 = m_{top}^2 \]
\[ (p^{e^+} + p_c^c)^2 = m_W^2 \]
\[ (p^{e^+} + p_{\nu}^\nu + p_b^b)^2 = m_{top}^2 \]
\[ (p^{e^+})^2 = 0 \]
\[ \hat{p}_T^{H^+} - \hat{p}_T^{e^+} + \hat{p}_T^{\nu} = \hat{p}_T^{miss} \]

Here \( p^{H^+} \) and \( p^{\nu} \) represent the unknown quantities of the event. Note that we have not specified constraints for the particles that are the decay products of the charged Higgs, since this would not supply us with additional information that can be used to constrain the charged Higgs mass. The system of constraints (9) therefore amounts to two free parameters (compared to only one in the semi-leptonic case) over which we maximize the charged Higgs mass, to obtain the variable \( m_{T2}^{H^+} \):

\[ (m_{T2}^{H^+})^2 = \max_{\{\text{constraints}\}} \left( (p_{T2}^{H^+})^2 \right) \]  (10)

If we choose one of the free parameters to be the \( z \)-component of the charged Higgs momentum, we can immediately perform the maximization over it using the result of the previous section, and so \( m_{T2}^{H^+} \) is equivalent to

\[ m_{T2}^{H^+} = \max_{\{\text{constraints}\}} \left[ m_T^{H^+} \left( \hat{p}_T^{H^+} \right) \right] \]  (11)

with

\[ (m_T^{H^+} \hat{p}_T^{H^+})^2 = \left( \sqrt{m_{top}^2 + (\hat{p}_T^{H^+} + \hat{p}_T^{e^+})^2} - (\hat{p}_T^{e^+})^2 \right)^2 \]  (12)

The maximization over the remaining parameter needs to be performed numerically. A computation procedure which allows for this maximization to be easily performed is given in appendix B.

**Selection and reconstruction**

The selection cuts we apply in this case are similar to those used for the semi-leptonic channel, except that we require an additional isolated lepton with \( p_T > 15 \) GeV. The two leptons and two b-jets need to be paired according to the two corresponding top decays in order for \( m_{T2}^{H^+} \) to be calculated. To do this we calculate the invariant mass \( m_{T2}^{H^+} \) for each possible pair and require it to satisfy the bound (7). We keep only events where this requirement resolves the ambiguity, i.e. events where exactly one pairing satisfies (7). This keeps only about 50% of the events, however the accuracy of the assignment is high, at about 95%. The calculation of \( m_{T2}^{H^+} \) further requires assigning one of the two leptons to the charged Higgs decay. Here we always choose it as the one with lower transverse momentum. The resulting distributions of \( m_{T2}^{H^+} \) are shown in Figure 5. The effect of having additional free parameter clearly worsens the situation compared to the semi-leptonic case, however there is still a clear discrimination between the W and charged Higgs bosons.

![Distribution of \( m_{T2}^{H^+} \) for dileptonic \( tt \) events containing a charged Higgs with \( m_{H^+} = 130, 150 \) GeV (red and green) and background events with a W boson decays (black).](image1)

In figure 6 an example pseudo-experiment containing both the main background and a charged Higgs signal with \( m_{H^+} = 140 \) GeV is shown. Here the number of events correspond to an integrated luminosity of 10 fb\(^{-1} \) at \( \sqrt{s} = 14 \) TeV, and the branching ratio \( Br(t \to H^+ b) \) is assumed to be 15%. In this case, since the kinematic edge is less pronounced, to observe a charged Higgs using the \( m_{T2}^{H^+} \) variable requires a rather accurate knowledge of the background shape. The excess of events above the charged Higgs mass would then provide an indication for the charged Higgs in this channel, complementary to the semi-leptonic one.

![Example pseudo-experiments (points with error bars) on top of the signal and background distributions, assuming \( Br(t \to H^+ b) = 15\% \) and \( m_{H^+} = 140 \) GeV, for an integrated luminosity of 10 fb\(^{-1} \).](image2)
IV. CONCLUSIONS

We have demonstrated how the leptonic decay channel of the $\tau$ could be used as a search channel for a charged Higgs boson at hadron colliders. The charged Higgs transverse mass defined for semi-leptonic $t\bar{t}$ events, as well as the analogous generalization for di-leptonic events, are sensitive to the charged Higgs mass and could therefore be used as discriminating variables that would give evidence to such a particle, and a way to measure its mass. This would provide an important complementary measurement to methods based on hadronic $\tau$ identification, and an enhancement to the overall discovery sensitivity.

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Appendix A: Derivation of $m_T^H$

In this appendix, we derive the expression for the charged Higgs transverse mass [4]. Our goal is to maximize the expression of invariant mass:

$$m_{T+}^2 = (p^\ell + p^{miss})^2$$

(A1)

with respect to the two unknown components of $p^{miss}$, while holding the constraint:

$$m_{top}^2 = (p^{miss} + p^b + p^\nu)^2$$

(A2)

For convenience, we adopt the notation $p_\parallel = (E, p_z)$, satisfying $p_\parallel^2 = E^2 - p_z^2$, and $p_T = (p_x, p_y)$ with $p_T^2 = p_x^2 + p_y^2$. Equation (A.1) and (A.2) can be re-written as:

$$m_{T+}^2 = (p_\parallel + p^{miss}_\parallel)^2 - (p_T + p_T^{miss})^2$$

(A3)

$$m_{top}^2 = (p_\parallel + p^{miss}_\parallel + p^b)^2 - (p_T + p_T^{miss} + p_T^b)^2$$

(A4)

To maximize Eq. (A.3), we introduce a Lagrange multiplier $\lambda$ and differentiate with respect to $p^{miss}_\parallel$:

$$\frac{\partial}{\partial p^{miss}_\parallel} ((p_\parallel + p^{miss}_\parallel)^2 - (p_T + p_T^{miss})^2) = 0$$

(A5)

This gives

$$p^{miss}_\parallel = \frac{\lambda}{1-\lambda} p^\ell - p_\parallel$$

(A6)

plugging this into Eq. (A.4) we obtain for $\lambda$:

$$1 - \lambda = \frac{p_T^b}{\sqrt{m_{top}^2 + (p_T^b + p_T^{miss} + p_T^b)^2}}$$

(A7)

where we have approximated the $b$-quark to be massless. Plugging (A.7) and (A.6) into (A.3) we obtain the final result:

$$(m_T^H)^2 = (\sqrt{m_{top}^2 + (p_T^b + p_T^{miss})^2} - p_T^b)^2 - (p_T^b + p_T^{miss})^2$$

(A8)

It should be noted that exactly the same procedure could be applied to the case of a leptonic W decay $W \rightarrow \ell \nu$, by replacing the constraint (A.2) with $(p^{miss})^2 = 0$. This will result in the usual expression for the W transverse mass.

Appendix B: Computation procedure for $m_T^H$

In this section we describe the computational procedure by which the maximization of Eq. (11) is performed. The transverse component of the charged Higgs momentum $p_T^{H^\pm}$ is constrained by the set of equations

$$\vec{p}_T^{H^\pm} - p_T^{\nu T} = p_T^{miss}$$

$$\begin{cases}
(p^{\ell\ell})^2 = 0 \\
(p^{c\ell} + p^{\nu T})^2 = m_W^2 \\
(p^{c\ell} + p^{\nu T} + p^b)^2 = m_{top}^2
\end{cases}$$

(B1)

We wish to obtain a parametrization of $p_T^{H^\pm}$ as a function of a single variable such that the above constraints are satisfied. To do this we rewrite the last two equations as the following linear matrix equation:

$$ \begin{pmatrix} \mathbf{A}_1 \end{pmatrix} \eta + \begin{pmatrix} \mathbf{A}_2 \end{pmatrix} \xi = m $$

(B2)

where

$$ \begin{pmatrix} \mathbf{A}_1 \\
\mathbf{A}_2 \end{pmatrix} = \begin{pmatrix}
\begin{pmatrix} E^\ell - p^\ell_x \\ E^{c\ell} - p^{c\ell}_x \\
E^{\nu T} - p^{\nu T}_x \\
E^{\nu T} - p^{\nu T}_x \\
E^{\nu T} - p^{\nu T}_x \\
\end{pmatrix} \\
\begin{pmatrix} -p^\ell_x \\
- p^{c\ell}_x \\
- p^{\nu T}_x \\
- p^{\nu T}_x \\
- p^{\nu T}_x \end{pmatrix} \\
\end{pmatrix}$$

$$ \begin{pmatrix} \eta \\
\xi \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} E^\ell \\
E^{c\ell} \\
E^{\nu T} \\
E^{\nu T} \\
E^{\nu T} \end{pmatrix} \\
\begin{pmatrix} p^\ell_x \\
p^{c\ell}_x \\
p^{\nu T}_x \\
p^{\nu T}_x \\
p^{\nu T}_x \end{pmatrix} \end{pmatrix}$$

$$ m = \frac{1}{2} \begin{pmatrix} (m_{top}^2 - m_W^2 - m_{\bar{b}b}) \\
- m_W^2 \end{pmatrix}$$

and

$$m_{bh}^2 = (p^b + p^{\ell\ell})^2$$

(B3)

The zero-mass constraint of the neutrino is given in this notation by

$$\eta^T g \eta - \xi^T \xi = 0$$

(B4)

where g is the 1+1 Lorentz metric, $g = \begin{pmatrix} 1 & 0 \\
0 & -1 \end{pmatrix}$. Using (B2) this can be written as:

$$\xi^T (1 - \mathbf{A}^T g \mathbf{A}) \xi + 2 \xi^T \mathbf{A}^T g \bar{m} \xi = 0$$

(B5)
where $A \equiv A_1^{-1}A_2$ and $\tilde{m} \equiv A_1^{-1}m$.

The matrix $1 - A^T g A$ is symmetric and can be shown to be positive-definite. It can therefore be decomposed in the following way:

$$1 - A^T g A = L^T L$$

(B6)

e.g. via a Cholesky decomposition. Using this we obtain

$$|L\xi + \omega| = \sqrt{\omega^T \omega + \tilde{m}^T g \tilde{m}} \equiv \sqrt{Q}$$

(B7)

where $\omega = (L^T)^{-1} A^T g \tilde{m}$.

This is solved by

$$\xi = L^{-1}(\sqrt{Q}\hat{r}(\phi) - \omega)$$

(B8)

where $\hat{r}$ is the unit vector in the transverse plane, $\hat{r}(\phi) = (\sin \phi, \cos \phi)^T$.

We can therefore express $p_T^{H^+}$ as a function of $\phi$:

$$p_T^{H^+} = \tilde{p}_T^{miss} + \tilde{p}_T^{\ell^+} - \xi(\phi)$$

(B9)

such that

$$m_T^{H_2} = \max_{\phi} \left[ m_T^{H}(\phi) \right]$$

(B10)

The maximization over $\phi$ can now be performed by any standard function minimization algorithm, or by scanning over the range $0 < \phi < 2\pi$. The result is the generalized charged Higgs transverse mass.

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In the MSSM, the charged Higgs boson is generally produced in association with a top quark, $tH^+$, and its decay in the transverse plane is described by

$$\hat{r}(\phi) = (\sin \phi, \cos \phi)^T$$

where $\phi$ is the angle between the charged Higgs boson and the top quark. The combined LEP limits for the charged Higgs boson of any type-II 2HDM is about 80 GeV, See: DELPHI Collaboration, Eur. Phys. J. C 34, 399 (2004); L3 Collaboration, Phys. Lett. B 575, 208 (2003); ALEPH Collaboration, Phys. Lett. B 543, 1 (2002).