Encoderless Model Predictive Control of Doubly-Fed
Induction Generators in Variable-Speed Wind
Turbine Systems

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Abstract. In this paper, an encoderless finite-control-set model predictive control (FCS-MPC)
strategy for doubly-fed induction generators (DFIGs) based on variable-speed wind turbine
systems (WTSs) is proposed. According to the FCS-MPC concept, the discrete states of the
power converter are taken into account and the future converter performance is predicted
for each sampling period. Subsequently, the voltage vector that minimizes a predefined cost
function is selected to be applied in the next sampling instant. Furthermore, a model reference
adaptive system (MRAS) observer is used to estimate the rotor speed and position of the DFIG.
Estimation and control performance of the proposed encoderless control method are validated
by simulation results for all operation conditions. Moreover, the performance of the MRAS
observer is tested under variations of the DFIG parameters.

Notation
N, R, C are the sets of natural, real and complex numbers. x ∈ R or x ∈ C is a real or complex scalar. x ∈ R^n
(bold) is a real valued vector with n ∈ N. x^T is the transpose and ||x|| = √x^T x is the Euclidean norm of x.
0_n = (0, . . . , 0)^T is the n-th dimensional zero vector. X ∈ R^{n×m} (capital bold) is a real valued matrix with n ∈ N
rows and m ∈ N columns. O_{n×m} ∈ R^{n×m} is the zero matrix. x^y ∈ R^2 is a space vector of a rotor (r), stator (s) or
filter (f) quantity, i.e. z ∈ {r, s, f}. The space vector is expressed in either phase abc-, stator fixed
s-, rotor fixed r-, or arbitrarily rotating k-coordinate system, i.e. y ∈ {abc, s, r, k}, and may represent voltage u, flux linkage ψ
or current i, i.e. x ∈ {u, ψ, i}.

1. Introduction
The electrical power generation by variable speed wind turbines (WTs) has increased
significantly during the last years contributing to the reduction of carbon dioxide emissions and
to a lower environmental pollution [1]. Among various wind energy conversion systems (WECSs),
WECSs with doubly-fed induction generator (DFIG) have been the dominant technology in the
market since the late 1990s [1]. DFIGs can supply active and reactive power, operate with a
partial-scale power converter (around 30% of the generator rating), and fulfill a certain ride
through capability [2]. Operation above and below synchronous speed is possible [2].
Fig. 1 shows a DFIG mechanically coupled to the wind turbine via a shaft and gear box with
ratio $g_r \geq 1$. The stator windings of the DFIG are directly tied to the grid, whereas the rotor winding is tied via a back-to-back partial-scale voltage source converter (VSC), and a filter to the grid. The grid side converter (GSC) and the rotor side converter (RSC) share a common DC-link. Currently, field oriented control (FOC) and direct torque control (DTC) methods dominate both academic and industrial applications for RSC [3]. For GSC, voltage oriented control (VOC) and direct power control (DPC) are two popular methods [3]. However, with the development of faster and more powerful digital signal processors, the implementation of new control strategies such as fuzzy logic and predictive control is possible [4]. One of the most promising predictive controllers in power converters and electric drives is the finite-control-set model predictive control (FCS-MPC) [4]–[6], which exploits the finite number of switching states of the power converter for solving an optimization problem.

Lately, the interest in encoderless control techniques is increasing due to cost effectiveness and robustness, which means that the controllers must run without the information of mechanical sensors (such as encoders or transducers) mounted on the shaft [3]. The required rotor speed/position must be estimated via the information supplied by electrical (e.g. current/voltage) sensors which are cheap and easier to install than mechanical sensors. Furthermore, mechanical sensors decrease the drive system reliability due to their high failure rate, which means shorter maintenance periods and, consequently, higher costs [3].

Encoderless vector control techniques for DFIGs have been proposed by several researchers [7]–[10]. The encoderless method presented in [7] is open-loop and relies on rotor current estimator in which the estimated and measured currents are compared to get the rotor position. The application of model reference adaptive system (MRAS) observers for encoderless control of DFIGs has been reported in [8], where MRAS observers are diversified with different error variables, e.g. stator and rotor currents and fluxes. The encoderless control approach in [9] relies on signal injection. Another alternative is the use of an extended Kalman filter (EKF) [10]. However, encoderless FCS-MPC is rarely presented in the literature [11].

In this paper, a FCS-MPC strategy for DFIGs based on variable-speed WTSs is proposed. The proposed control system uses a MRAS observer for estimation of the DFIG rotor speed and position. Estimation and control behavior of the proposed encoderless control method are illustrated by simulation results for all operation conditions. Moreover, the behavior of the proposed MRAS observer is investigated under variations of the DFIG parameters.
2. Modeling of the WECS with DFIG

The block diagram of WECS with DFIG is shown in Fig. 1. The RSC and the GSC share a common DC-link with capacitance $C_{dc}$ [As/V] with DC-link voltage $u_{dc}$ [V].

2.1. Wind turbine (WT)

The output mechanical power of a WT is given by \[ p(t) = c_p(\lambda, \beta) \frac{1}{2} \rho \pi r_1^2 v_\infty^3(t) \] (1)

where $\rho > 0$ [kg/m$^3$] is the air density, $r_1 > 0$ [m] is the radius of the wind turbine rotor ($\pi r_1^2$ is the turbine swept area), $c_p \geq 0$ [1] is the power coefficient, and $v_\infty(t) \geq 0$ [m/s] is the wind speed. The power coefficient $c_p$ is an indication for the "efficiency" of the WT. It is a nonlinear function of the tip speed ratio

$$\lambda = \frac{\omega_{bm}(t)r_1}{g_r v_\infty(t)} \geq 0 \quad [1]$$ (2)

and the pitch angle $\beta \geq 0$ [°] of the rotor blades. In reality, the power coefficient ranges from 0.4 to 0.48 [10, 12].

2.2. Doubly-Fed induction generator (DFIG)

The stator and rotor voltage equations of the DFIG can be written as follows [2]:

$$\mathbf{u}_s^{abc}(t) = R_s \mathbf{i}_s^{abc}(t) + \frac{d}{dt} \mathbf{\psi}_s^{abc}(t) \quad \text{and} \quad \mathbf{u}_r^{abc}(t) = R_r \mathbf{i}_r^{abc}(t) + \frac{d}{dt} \mathbf{\psi}_r^{abc}(t)$$ (3)

where (considering linear flux linkage relations)

$$\mathbf{\psi}_s^{abc}(t) = L_s \mathbf{i}_s^{abc}(t) + L_m \mathbf{i}_r^{abc}(t) \quad \text{and} \quad \mathbf{\psi}_r^{abc}(t) = L_r \mathbf{i}_r^{abc}(t) + L_m \mathbf{i}_s^{abc}(t).$$ (4)

Here $\mathbf{u}_s^{abc} = (u_a^s, u_b^s, u_c^s)^\top$ [V], $\mathbf{u}_r^{abc} = (u_a^r, u_b^r, u_c^r)^\top$ [V], $\mathbf{i}_s^{abc} = (i_a^s, i_b^s, i_c^s)^\top$ [A], $\mathbf{i}_r^{abc} = (i_a^r, i_b^r, i_c^r)^\top$ [A], $\mathbf{\psi}_s^{abc} = (\psi_a^s, \psi_b^s, \psi_c^s)^\top$ [Vs], and $\mathbf{\psi}_r^{abc} = (\psi_a^r, \psi_b^r, \psi_c^r)^\top$ [Vs] are the stator and rotor voltages, currents and flux linkages, respectively, all in the abc-reference frame (three-phase system). $L_s$, $L_r$, and $L_m$ [Vs/A] are the stator, rotor and mutual inductances. $R_s$ [0] and $R_r$ [0] are stator and rotor winding resistances. The DFIG rotor rotates with mechanical angular frequency $\omega_m$ [rad/s]. Hence, for a machine with pole pair number $n_p$ [1], the electrical angular frequency of the rotor is given by $\omega_r = n_p \omega_m$ and the rotor reference frame is shifted by the rotor angle $\phi_r(t) = \int_0^t \omega_r(t) \, dt + \phi_0^r, \phi_0^r \in \mathbb{R}$, with respect to the stator reference frame ($\phi_0^r$ is the initial rotor angle). Equation (3) can be written in the stationary/rotating reference frame as follows $\mathbf{x}^k = \mathbf{T}_p(\phi)^{-1} \mathbf{x}^s = \mathbf{T}_p(\phi)^{-1} \mathbf{J}_C \mathbf{x}^{abc}$ by using the Clarke and Park transformation (see, e.g., [12]), respectively, given by (neglecting the zero sequence)

$$\mathbf{x}^s = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \mathbf{x}^{abc} \quad \text{and} \quad \mathbf{x}^k = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \mathbf{x}^s = \mathbf{T}_p(\phi)^{-1} \mathbf{x}^s$$ (5)

where $\mathbf{x}^k = (x^d, x^q)^\top$, and $\mathbf{x}^s = (x^{\alpha s}, x^{\beta s})^\top$. The rotor voltage equation (3) with respect to the stationary reference frame (i.e. $\mathbf{u}_r^s = \mathbf{T}_p(\phi_r)^{-1} \mathbf{J}_C \mathbf{u}_r^{abc}$) can be expressed as

$$\mathbf{u}_s^s(t) = R_s \mathbf{i}_s^s(t) + \frac{d}{dt} \mathbf{\psi}_s^s(t), \quad \text{and} \quad \mathbf{u}_r^s(t) = R_r \mathbf{i}_r^s(t) + \frac{d}{dt} \mathbf{\psi}_r^s(t) - \omega_r(t) \mathbf{J} \mathbf{\psi}_r^s(t),$$ (6)

where $\mathbf{J} := \mathbf{T}_p(\pi/2) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ [12]. The stator voltage orientation (SVO) is realized by aligning the d-axis of the synchronous (rotating) reference frame with the stator voltage vector $\mathbf{u}_s^s$. 


which rotates with the stator (grid) angular frequency $\omega_s$ (under ideal conditions, i.e. constant grid frequency $f_0 > 0$, it holds that $\omega_s = 2\pi f_0$ is constant). Applying the (inverse) Park transformation with $T_P(\phi_s)^{-1}$ as in (5) with $\phi_s(t) = \int_0^t \omega_s(\tau) d\tau + \phi_0^s$, $\phi_0^s \in \mathbb{R}$, to the voltage equations (6) yields the description in the rotating reference frame

$$u^k_s(t) = R_s i^k_s(t) + \frac{d}{dt} \psi^k_s(t) + \omega_s J \psi^k_s(t), \quad \text{and} \quad u^k_r(t) = R_r i^k_r(t) + \frac{d}{dt} \psi^k_r(t) + \omega_d(t) J \psi^k_r(t), \quad (7)$$

where $\omega_d(t) := \omega_s - \omega_r(t)$ is the slip angular frequency. Since, e.g., $\psi^k_s = T_P(\phi_s)^{-1} \psi^s = T_P(\phi_r)^{-1} T_C \psi^{abc}$, the flux linkages are given by

$$\psi^k_s(t) = L_s i^k_s(t) + L_m \dot{i}^k_s(t) \quad \text{and} \quad \psi^k_r(t) = L_r i^k_r(t) + L_m \dot{i}^k_r(t). \quad (8)$$

For a stiff shaft and a step-up gear with ratio $g_r \geq 1$, the dynamics of the mechanical system are given by

$$\frac{d}{dt} \omega_m(t) = \frac{1}{m_t} \left( m_e(t) - \frac{m_i(t)}{g_r} \right), \quad \omega_m(0) = \omega_m^0 \in \mathbb{R} \quad (9)$$

where

$$m_e(t) = 3 n_p i^r_s(t)^T J \psi^r_s(t) = \frac{3}{2} n_p L_m (i^r_s(t) i^d_r(t) - i^d_s(t) i^d_r(t)). \quad (10)$$

is the electro-magnetic machine torque (moment), $m_t$ [Nm] is the turbine torque produced by the wind (see Sec. 3) and $m_m = \frac{\Theta}{g_r} [\text{Nm}]$ is the mechanical torque acting on the DFIG shaft. $\Theta$ [kg/m^2] is the rotor inertia and $n_p$ [1] is the pole pair number.

### 2.3. Back-to-back converter and DC-Link

As shown in Fig. 1, a balanced generator and grid are assumed in this paper. The output voltage of the RSC and GSC can be calculated as follows [12]:

$$u^abc_s(t) = \frac{1}{3} u_{dc}(t) T^{abc} s^{abc}_s(t) \quad \text{and} \quad u^abc_f(t) = \frac{1}{3} u_{dc}(t) T^{abc} s^{abc}_f(t) \quad (11)$$

where $s^{abc}_s = (s^a_s, s^b_s, s^c_s)^T \in \{0, 1\}$ and $s^{abc}_f = (s^a_f, s^b_f, s^c_f)^T \in \{0, 1\}$ are the switching state vectors of the RSC and GSC, respectively, and $T^{abc}$ is the transformation matrix [12]

$$T^{abc} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (12)$$

describing the relation between switching state vector and output phase voltage vector of the converter. Considering all the possible combinations of the switching state vector $s^{abc}_s$ or $s^{abc}_f$, eight switching states, and consequently, eight voltage vectors are obtained. Note that two different zero voltage vectors are available, see Fig. 2. The DC-link dynamics are given by [12] (neglecting resistive losses)

$$\frac{d}{dt} u_{dc}(t) = \frac{1}{C_{dc}} (I_g(t) - I_m(t)), \quad u_{dc}(0) = 0 \in \mathbb{R} \quad (13)$$

where

$$I_m(t) = i^{abc}_s(t)^T s^{abc}_s(t) \quad \text{and} \quad I_g(t) = i^{abc}_f(t)^T s^{abc}_f(t). \quad (14)$$

are the rotor and grid side DC-link currents (see Fig. 1).
Here be expressed in the rotating reference frame (grid voltage orientation) as follows the grid to the GSC. Invoking Kirchhoff’s voltage law at the AC side of the GSC [12] gives

\[ u_{abc}^{bc}(t) = R_f i_{f}^{abc}(t) + L_f \frac{d}{dt} i_{f}^{abc}(t) + u_{0}^{abc}(t), \quad i_{f}^{abc}(0) = 0 \beta. \] (15)

Here \( u_{abc} = (u_{a}^{b}, u_{b}^{b}, u_{c}^{c})^\top [V] \) is the output voltage of the GSC. The voltage equation (15) can be expressed in the rotating reference frame (grid voltage orientation) as follows

\[ u_{k}^{bc}(t) = R_f i_{f}^{k}(t) + L_f \frac{d}{dt} i_{f}^{k}(t) + \omega_n J L_f i_{f}^{k}(t) + u_{0}^{k}(t). \] (16)

### 3. Maximum power point tracking (MPPT)

For wind speeds below the rated wind speed of the WT, maximum power tracking is the required control objective. Consequently, the pitch angle is kept constant at \( \beta = 0 \) and the WT must operate at its optimal tip speed ratio \( \lambda^* \) (a constant) where the power coefficient has its maximum \( c_{p}^{*} := c_p(\lambda^*, 0) = \max_{\lambda} c_p(\lambda, 0) \). Thus, the WT can extract the maximally available power \( p_{c}^{*} := c_{p}^{*} \frac{1}{2} \rho \pi r^2 c_{w}^{*} \) [12]. Maximum power point tracking is realized by the nonlinear speed controller

\[ m_c(t) \approx m_c^*(t) = -k_p^* \dot{\omega}_m(t)^2 \quad \text{with} \quad k_p^* := \frac{g_{m}(t)}{\lambda^*}, \] (17)

which guarantees that the generator angular frequency \( \omega_m(t) \) is adjusted to the actual wind speed \( v_w(t) \) such that \( \frac{\omega_m(t)}{v_w(t)} = \lambda^* \) holds. According to (17) the optimum torque \( m_c^*(t) \) can be calculated from the (estimated) shaft speed \( \dot{\omega}_m(t) = \dot{\omega}_r(t)/n_p \).

### 4. Proposed FCS-MPC

#### 4.1. FCS-MPC for RSC

The stator (grid) voltage orientation (SVO) is achieved by aligning the d-axis of the synchronous reference frame with the stator voltage vector \( u_s^*(t) \). The resultant stator dq-axis voltages are \( u_s^d(t) = \|u_s^*(t)\| \) and \( u_s^q(t) = 0 \) [2]. By substituting the value of \( u_s^k(t) \) from (8) in (7), the rotor voltage \( u_k^r(t) \) can be written as

\[ u_k^r(t) = R_r i_r^k(t) + L_r \frac{d}{dt} i_r^k(t) + M_l \frac{d}{dt} i_s^h(t) + L_m i_r^h(t) + \omega_d(t) L_m i_r^h(t) + \omega_d(t) L_m i_s^h(t). \] (18)
Invoking (8), the stator current \( i_{s}^k(t) \) can be expressed as

\[
i_{s}^k(t) = \frac{1}{L_s} \psi_{s}^k(t) - \frac{L_m}{L_s} i_{r}^k(t)
\]

and substituting (19) in (18) gives

\[
u_{r}^k(t) = R_s i_{r}^k(t) + \sigma L_r \frac{d}{dt} i_{r}^k(t) + \frac{L_m}{L_s} \frac{d}{dt} \psi_{s}^k(t) + \omega_{sl}(t) \sigma L_r J i_{r}^k(t) + \omega_{sl}(t) \frac{L_m}{L_s} J \psi_{s}^k(t)
\]

where \( \sigma = 1 - \frac{L_r}{L_r L_l} \). Substituting \( \frac{d}{dt} \psi_{s}^k(t) \) from (7) and \( \psi_{s}^k(t) \) from (8) in (20) gives

\[
u_{r}^k(t) = R_s i_{r}^k(t) + \sigma L_r \frac{d}{dt} i_{r}^k(t) + (\omega_{sl}(t) L_r - \omega_{s}(t) \frac{L_r}{L_s}) J i_{r}^k(t) - (R_s \frac{L_m}{L_s} + \omega_{r}(t) L_m J) i_{r}^k(t) + \frac{L_m}{L_s} u_{r}^k(t)
\]

Solving (21) for \( \frac{d}{dt} i_{r}^d[k] \) (and writing out both components) yields

\[
\begin{align*}
\frac{d}{dt} i_{r}^d[k] &= \frac{1}{\sigma L_s L_r} \left[ -R_s L_s i_{r}^d[k] + (\omega_{sl}(k) L_r L_s - \omega_{s}(k) L_m^2) i_{r}^d[k] + R_s L_m i_{r}^d[k] 
- \omega_{r}(k) L_m L_s i_{r}^d[k] + L_s u_{r}^d[k] - L_m u_{r}^d[k] \right] \\
\frac{d}{dt} i_{r}^q[k] &= \frac{1}{\sigma L_s L_r} \left[ -R_s L_s i_{r}^q[k] - (\omega_{sl}(k) L_r L_s - \omega_{s}(k) L_m^2) i_{r}^q[k] + R_s L_m i_{r}^q[k] 
+ \omega_{r}(k) L_m L_s i_{r}^q[k] + L_s u_{r}^q[k] - L_m u_{r}^q[k] \right]
\end{align*}
\]

The FCS-MPC approach uses a discrete-time model for the prediction of the currents at a future sample period. For discretization the forward Euler method with sampling time \( T_s[s] \) is applied to the time-continuous model (22). The discrete model of the DFIG can be written as

\[
\begin{align*}
i_{r}^{d}[k+1] &= i_{r}^{d}[k] + \frac{T_s}{\sigma L_s L_r} \left[ -R_s L_s i_{r}^{d}[k] + (\omega_{sl}(k) L_r L_s - \omega_{s}(k) L_m^2) i_{r}^{d}[k] + R_s L_m i_{r}^{d}[k] 
- \omega_{r}(k) L_m L_s i_{r}^{d}[k] + L_s u_{r}^{d}[k] - L_m u_{r}^{d}[k] \right] \\
i_{r}^{q}[k+1] &= i_{r}^{q}[k] + \frac{T_s}{\sigma L_s L_r} \left[ -R_s L_s i_{r}^{q}[k] - (\omega_{sl}(k) L_r L_s - \omega_{s}(k) L_m^2) i_{r}^{q}[k] + R_s L_m i_{r}^{q}[k] 
+ \omega_{r}(k) L_m L_s i_{r}^{q}[k] + L_s u_{r}^{q}[k] - L_m u_{r}^{q}[k] \right]
\end{align*}
\]

In this paper, for the RSC, the chosen cost function is defined by

\[
g_{RSC} = |i_{r,ref}^{d}[k+1] - i_{r}^{d}[k+1]| + |i_{r,ref}^{q}[k+1] - i_{r}^{q}[k+1]|
\]

where \( i_{r,ref}^{d}[k+1] \) and \( i_{r,ref}^{q}[k+1] \) are the reference values of the \( d- \) & \( q- \) axis currents.

For the prediction algorithm, the cost function (24) (and, hence, (23)) is calculated for each of the seven voltage vectors, producing seven different current predictions. The voltage vector whose current prediction is minimizing the cost function (24) is applied at the next sampling period. However, the future reference current \( i_{r,ref}^{d}[k+1] \) value is unknown. Therefore, it has to be predicted from present and previous values of the current reference using Lagrange extrapolation as follows [5]:

\[
i_{r,ref}^{d}[k+1] = 3i_{r,ref}^{d}[k] - 3i_{r,ref}^{d}[k-1] + i_{r,ref}^{d}[k-2].
\]

The value of the reference current \( i_{r,ref}^{d}[k] \) is calculated from the optimum torque \( m_s^*[k] \) and the value of \( i_{r,ref}^{q}[k] \) is calculated from the reference stator reactive power \( Q_{s,ref}[k] \) as follows [2]

\[
i_{r,ref}^{d}[k] = \frac{2\omega_s[k] L_s}{3n_p L_m u_{r}^{d}[k]} m_s^*[k] \quad \text{and} \quad i_{r,ref}^{q}[k] = \frac{2L_s}{3L_m u_{r}^{q}[k]} Q_{s,ref}[k] - \frac{u_{r}^{d}[k]}{\omega_s[k] L_m}.
\]
4.2. FCS-MPC for GSC

Again, applying the forward Euler method to (16), the discrete model of the grid side filter can be written as follows:

\[
\begin{align*}
    i_{d}^d[k + 1] &= (1 - \frac{T_{r}R_{f}}{L_{f}})i_{d}^d[k] + \omega_{r}T_{r}i_{q}^f[k] + \frac{T_{r}}{L_{f}}(u_{d}^f[k] - u_{d}^d[k]) \\
i_{q}^f[k + 1] &= (1 - \frac{T_{r}R_{f}}{L_{f}})i_{q}^f[k] - \omega_{r}T_{r}i_{d}^f[k] + \frac{T_{r}}{L_{f}}(u_{q}^f[k] - u_{q}^d[k]).
\end{align*}
\]  (27)

For the GSC, the cost function is defined by

\[
g_{\text{GSC}} = |i_{d,\text{ref}}^d[k + 1] - i_{d}^d[k + 1]| + |i_{q,\text{ref}}^q[k + 1] - i_{q}^q[k + 1]|  \tag{28}
\]

where \(i_{d,\text{ref}}^d[k + 1]\) and \(i_{q,\text{ref}}^q[k + 1]\) are the reference values of the \(d\)- & \(q\)-axis currents.

Again, (27) is calculated for each of the seven voltage vectors, yielding seven different current predictions. The voltage vector whose current prediction is minimizing the cost function (28) is applied at the next sampling interval. The future reference current \(i_{d,\text{ref}}^d[k + 1]\) value is calculated also using Lagrange extrapolation as explained before.

The value of the \(d\)-axis reference current \(i_{d,\text{ref}}^d[k]\) is obtained from an outer DC-link voltage control loop. The measured DC-link voltage \(u_{dc}\) is compared with a constant reference value \(u_{dc,\text{ref}}\) and the error is processed by a PI controller producing the \(d\)-axis reference current \(i_{d,\text{ref}}^d[k]\), see Fig. 1.

5. MRAS observer

The MRAS observer is consist of two models [8]: a reference model and an adaptive model, see Fig. 3. In this paper, the reference model (see left part in Fig. 3) is fed by the measured stator current \(i_{s}^s(t)\) and the measured stator (grid) voltage \(u_{s}^s(t)\). From the reference model (based on (8)) the rotor current \(\hat{i}_{r}^s(t)\) is estimated via

\[
\hat{i}_{r}^s(t) = \frac{1}{L_{m}}(\psi_{r}^s(t) - L_{s}i_{s}^s(t)) \quad \text{where} \quad \psi_{r}^s(t) = \int_{0}^{t} (u_{s}^s(\tau) - R_{s}i_{s}^s(\tau))d\tau + \psi_{r}^s(0).  \tag{29}
\]

The adaptive model (see right part in Fig. 3) is fed by the estimated rotor current \(\hat{i}_{r}^s(t)\) and the measured rotor current \(i_{r}^s(t)\) in the rotor reference frame. The objective of the adaptive model is to estimate rotor position \(\hat{\phi}_{r}(t)\) and rotor speed \(\hat{\omega}_{r}(t)\). To achieve that the estimated and the measured rotor current must be compared; to do so, the estimated rotor current \(\hat{i}_{r}^s(t)\) (in the stator reference frame) must be expressed in the rotor reference frame, i.e. \(\hat{i}_{r}^s(t) = T_{r}(\hat{\phi}_{r}(t))^{-1}\hat{\omega}_{r}(t)\). The “error” between estimated \(\hat{i}_{r}^s(t)\) and measured rotor current \(i_{r}^s(t)\) is defined as

\[
e(t) := \hat{i}_{r}^s(t)J\hat{i}_{r}^s(t) = \|\hat{i}_{r}^s(t)\| \|\hat{i}_{r}^s(t)\| \sin(\angle(\hat{i}_{r}^s(t), i_{r}^s(t))).
\]

The PI controller forces this error to zero by adjusting \(\hat{\omega}_{r}(t)\). Its output is the estimated speed \(\hat{\omega}_{r}(t)\) which is integrated to obtain the estimated rotor angle \(\hat{\phi}_{r}(t)\). For more details see [8].

6. Simulation Results and Discussion

A simulation model of a 50kW WECS with DFIG is implemented in Matlab/Simulink. The system parameters are listed in Table 1. The implementation is shown in Fig. 1. The simulation results are shown in Figs. 4–8. The estimation performances of MRAS observer are compared with the actual values for different wind speeds and parameter uncertainties in \(R_{s}\), \(R_{r}\) and \(L_{m}\).
Table 1: Parameters of the WECS with DFIG.

| Name                          | Nom. | Value   | Name                  | Nom. | Value |
|-------------------------------|------|---------|-----------------------|------|-------|
| WT rated power               |      | 50 kW   | Rotor inductance      |      |       |
| WT radius                    |      | 4.5 m   | Mutual inductance     |      |       |
| Rated wind speed             |      | 14 m/s  | DFIG moment of inertia|      | 0.1 kg m² |
| Optimal tip speed ratio      |      | 8.036   | DC-link capacitor     |      | 3 mF  |
| DFIG rated power             |      | 50 kW   | DC-link voltage       |      | 700V  |
| DFIG line-line voltage       |      | 400V    | Grid line-line voltage|      | 400V  |
| Number of pair poles         |      | 2       | Grid normal frequency |      | 50 Hz |
| Stator resistance            |      | 0.2448 Ω| Filter resistance     |      | 0.16 Ω|
| Rotor resistance             |      | 0.4847 Ω| Filter inductance     |      | 12 mH |
| Stator inductance            |      | 80.76 mH| Sampling time         |      | 40 μs |

Figure 4: Estimation and control performance of the proposed encoderless FCS-MPC (from top): estimated and actual rotor speed ($\hat{\omega}_r, \omega_r$), estimated and actual rotor position ($\hat{\phi}_r, \phi_r$), actual and reference $d$- & $q$- axis current of the rotor ($i_{rd}^d, i_{rd,ref}^d, i_{rq}^q, i_{rq,ref}^q$), Optimal and actual tip speed ratio ($\lambda^*, \lambda$), optimal and actual power coefficient ($c^*, c_p$).

The estimation performance of the proposed MRAS observer under variable wind speeds is shown in Fig. 4. This wind speed range covers almost the complete speed range of the DFIG (i.e. ±30% around the synchronous speed). Fig. 4 illustrates the tracking ability of the MRAS observer of the rotor speed and position at low and high speeds, and close to synchronous speed. An acceptably high estimation accuracy is achieved as the estimation error is very small. The FCS-MPC performance of the rotor side converter (RSC) under variable wind speed is illustrated in Fig. 4. It is clear that the RSC control system ensures tracking of the maximum power from the wind turbine. The actual rotor currents $i_{r/d/q}$ of the DFIG is following the reference value.
A MRAS observer for estimation of the DFIG rotor speed and position is utilized. The results have shown that the MRAS observer tracks rotor speed and position with high accuracy even under variations of the DFIG parameters. Also, the results show that the proposed FCS-MPC method for variable-speed WECSs with DFIG is still able to track the reference current, see Fig. 6. Moreover, the robustness with respect to changes (due to magnetic saturation) in the mutual inductance $L_m$ and of the reference $d/q$ currents as shown in Fig. 7. Finally, the robustness of the proposed FCS-MPC with respect to changes in the filter resistance $R_f$ and inductance $L_f$ is investigated. Therefore, $R_f$ and $L_f$ are increased by 25% at $t = 0.3s$. Fig. 8 shows the simulation results of the proposed FCS-MPC for this scenario under wind speed $v_w = 12 \frac{m}{s}$. Again, the proposed FCS-MPC for GSC shows a good control performance and is robust against parameter variations of the filter.

7. Conclusion
This paper proposed an encoderless FCS-MPC method for variable-speed WECSs with DFIG. A MRAS observer for estimation of the DFIG rotor speed and position is utilized. The results have shown that the MRAS observer tracks rotor speed and position with high accuracy even under variations of the DFIG parameters. Also, the results show that the proposed FCS-MPC method for variable-speed WECSs with DFIG is still able to track the reference current, see Fig. 6. Moreover, the robustness with respect to changes (due to magnetic saturation) in the mutual inductance $L_m$ and of the reference $d/q$ currents as shown in Fig. 7. Finally, the robustness of the proposed FCS-MPC with respect to changes in the filter resistance $R_f$ and inductance $L_f$ is investigated. Therefore, $R_f$ and $L_f$ are increased by 25% at $t = 0.3s$. Fig. 8 shows the simulation results of the proposed FCS-MPC for this scenario under wind speed $v_w = 12 \frac{m}{s}$. Again, the proposed FCS-MPC for GSC shows a good control performance and is robust against parameter variations of the filter.


for RSC tracks the reference currents for all the operation conditions and is robust against parameter variation of the DFIG. Thus, tracking of the maximum power from the wind turbine is guaranteed. Moreover, the proposed FCS-MPC for the GSC tracks the reference currents for all the operation conditions and is robust against parameter variation of the output filter. Therefore, a constant DC-link voltage is ensured.

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