Students mathematical representation of Hatyaiwittayalaisomboonkulkanya School Thailand based on SOLO Taxonomy in solving PISA problem

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Abstract. This research aims to describe mathematical representation ability of Hatyaiwittayalaisomboonkulkanya School students based on SOLO taxonomy in solving PISA problem especially change and relationship content. Data were analyzed through descriptive method with qualitative approach. Data were collected through a test using PISA problem and semi-structural interview. Test was given to Mattayom 4 students with 15-year-old average age. The result of this study showed that students with multi-structural and relational level tend to use verbal representation and students with extended abstract level tend to use verbal, visual, and mathematical expression representation in solving PISA problem on change and relationship content. Verbal representation are generally showed by writing argument, possibilities, and reasons of the final answer. Visual representation are generally showed by making diagram and table. Mathematical expression representation are generally showed by writing the reasons of final answer by using addition expression.

1. Introduction
Mathematics is all around us, in everything we do. Mathematics includes the study of such topics as quantity, structure, space, and change [1],[2]. Mathematics is also given in every level of school. Because mathematics is considered as the mother of all sciences, therefore it is necessary to master mathematical concept [3]. In this era, mathematics has big role in science and technology. Learning process especially mathematics also trains students to think logically, creatively, scientifically, critically, and systematically [4]-[6]. In mathematics learning, thinking process is an important thing for students. In order to get a solution, students must do a thinking process, understand the problem, and then represent it. Representation can be interpreted as an expression, idea, or concept how students itself find the solution of a problem. Representation related with communication. To communicate something, students need to have a good representation ability especially in understanding picture, graph, diagram, and other kind of representation. But the ability to represent a problem into mathematics form is a crucial thing and it still becomes a problem for students. Mathematics learning at school is considered too theoretic, rigid, and lack of contextual [7]. From those reasons, it leads students interest of mathematics decreased. Mathematics should not be only related with calculations, but mathematics should develop our ability to apply its knowledge into daily life problem [8].
Representation ability is one of seven mathematical abilities which used in mathematics assessment process for PISA. Those seven abilities are communication; mathematizing; representation; reasoning and argument; devising strategies for solving problem; using symbolic, formal, and technical language, and operations; using mathematical tools [9]-[13]. PISA is an abbreviation of Programme for International Students Assessment which shaded by OECD (Organisation for Economic Co-operation and Development). PISA conducts an evaluation to some countries. 15-year-old students are chosen randomly every 3 years. PISA conducts a test for several main subjects such as science, reading, and mathematics. Questions are given based on real life. So, one of PISA goals is to measure students ability and mathematical knowledge especially in solving daily life problems. The PISA study also aims to provide an evaluation to the education system in a country around the world by testing students abilities (science, reading, and math) [14]. PISA results said that from 70 countries reviewed in 2015, Thailand was ranked at 54th with 415 average score. While the whole average score for mathematics for all countries is 490. This means that Thailand is still in low position or rank [15].

There are four contents for mathematics in PISA (Programme for International Students Assessment), those are change and relationship, space and shape, quantity, uncertainty and data [13],[15]. Change and relationship is used in this research. PISA problems, especially for change and relationship content, have focus on quantification [16]. It means that change and relationship content related with algebra in mathematics learning process.

In order to know how far the students mathematical representation ability is, it is needed a tool to classify students ability into several certain levels. A way that can be used is SOLO (Structured of the Observed Learning Outcome) taxonomy. There are five levels of SOLO taxonomy which designed as a tool to evaluate students response. Those are pre-structural, uni-structural, multi-structural, relational, and extended abstract [17],[18]. Some characteristics of each levels of SOLO taxonomy according to Biggs and Collis are: (1) pre-structural students are rejected to give answer, answer quickly based on observation and emotion but without logical reason, and repeat the question, (2) uni-structural students can make a conclusion based on one suitable data, (3) multi-structural students can make a conclusion based on more than one suitable data, (4) relational students can think inductively and make a conclusion based on suitable data, also make a connection between those data, (5) extended abstract students can think inductively and deductively, see the connection, make hypothesis, make a conclusion, and apply them to different situation [19],[20].

According to several relevant researches, mathematical representation of every student is different. It can be based on mathematical disposition, gender, learning interest, and so on [21]-[24]. For instance is research by Hijriani, research subjects solved PISA problem by using visual and symbolic representation. Due to lack of accuracy caused inability of student to construct visual and symbolic representation correctly [25].

Based on the explanation above and the importance of mathematical representation ability for students, also Thailand rank in PISA, this article will discuss mathematical representation ability of Thailand students based on SOLO taxonomy in solving PISA problem especially for change and relationship content. The objective is to describe mathematical representation ability of Hatyaiwittayalaisomboonkulkanya School students based on SOLO taxonomy in solving PISA problem especially for change and relationship content.

2. Method
Data collection were conducted in Hatyaiwittayalaisomboonkulkanya School Thailand with purposive area method. Research subject is one class of Mathayom 4 students (15-year-old average age) with purposive and snowball sampling method. Data collection were conducted by giving one PISA problem. Based on test result, students are classified based on SOLO taxonomy.

The following table is operational forms of mathematical representation [26],[27]:
Table 1. Indicators of Mathematical Representation

| Representation | Operational Forms (Indicator) |
|----------------|--------------------------------|
| Visual, such as: | - Representing or restating data or information from certain representation into diagram, graph, or table |
| diagram, graph, table, | - Using visual representation in solving problem |
| picture | - Making picture or geometric patterns |
| | - Making picture of geometric shapes |
| Mathematical expression or equation | - Making equation or mathematical model from given representation |
| | - Making conjecture of number pattern |
| | - Solving problem by involving mathematical expression |
| Verbal | - Making problem situation based on data or given representation |
| | - Writing interpretation of any representation |
| | - Writing steps for solving mathematical problem with words |
| | - Answering problem by using words or verbal representation |

Table 2. Indicators of SOLO taxonomy

| Level                  | Description                                                                 |
|-----------------------|-----------------------------------------------------------------------------|
| Pre-structural        | Students are not able to respond or wrong in giving answer of all questions |
| Uni-structural        | Students are able to answer the questions correctly based on one given data information |
| Multi-structural      | Students are able to answer the questions correctly based on two suitable data or concepts |
| Relational            | Students are able to answer the questions correctly based on suitable data or concepts and make connection between those data and concepts. |
| Extended abstract     | Students are able to answer the questions correctly based on informations or data by generalising situation then applying them into another situation |

3. Result and Discussion
This research conducted by using PISA problem especially change and relationship content. Figure 1 shows PISA problem which given to students. Based on test result, it was chosen 5 research subjects. It consists of 2 students of multi-structural level, 1 student of relational level, and 2 students of extended abstract level. In solving the problem, students wrote the answer including every steps of question. It was given 30 minutes of time. The following explanation is the description of students mathematical representation for each levels of SOLO taxonomy.
3.1. Multi-structural Level

Figure 2 shows test result of 1st subject (S1) of multi-structural level. S1 is able to answer 1 of 3 questions. For the first question, S1 uses verbal (words) representation. S1 writes the maximum and minimum price of self-assembled skateboard in Skaters Shop. S1 also writes the detail of each prices. For the second and third questions, S1 is not able and cannot answer the questions correctly. But S1 also tends to use verbal representation in solving problem. This is indicated by giving reasons of second question and writing the price details of third question.

Based on interview result, S1 has successfully understood the first problem so question can be answered correctly. The comprehension is indicated by S1 has already understood the information and what is being asked. While for the second question, S1 said that he has not understood the problem clearly so S1 is not able to answer the question. For the third question, S1 has been succeeded to understand the problem well but he is not able to combine prices to get the correct one. S1 has not checked other possible combinations.

Figure 1. PISA test (change and relationship content)
Figure 3 shows test result of 2nd subject (S₂) of multi-structural level. S₂ is able to answer 1 of 3 questions. For the first question, S₂ also uses verbal (words) representation. S₂ writes the maximum and minimum price of self-assembled skateboard in Skaters Shop and writes the price details of each answer. For the second and third questions, S₂ is not able and cannot answer the questions correctly. But S₂ also tends to use verbal representation in solving problem by giving details and price combinations on second question and giving price details of each part of skateboard on third question.

Based on interview result, S₂ has successfully understood the first and second problems even though he just could answer the first question correctly. The comprehension is indicated by S₂ has already understood the information and what is being asked. S₂ has already written 5 price combinations correctly, but S₂ does not really know that there will be 7 other combinations. For third question, S₂ said that he could not understand the question well.

Students with multi-structural level could only answer 1 of 3 questions. Students thought that they were not able and difficult to analyzed and answer questions. This is appropriate with Laisouw’s research of multi-structural level students. It shows that the algebraic thinking process which includes the ability to conduct investigations, representations and generalizations, can be done correctly but the ability of interpretation and the ability to analyze of finding the result for new situations cannot be done correctly [23].

Besides, S₁ and S₂ have different gender. But the representation displayed by each students is not quite different. This is appropriate with previous research which states that male and female students both use verbal representation so their mathematical representation is not quite different in solving mathematics PISA problem [22]. Verbal representation shown by S₁ and S₂ are also similar. S₁ tends to write the answers in a sentence form and mention the details. S₂ writes the answers by listing all possible answers. This is appropriate with previous research which states that the representation ability of 10 grade students (equivalent with Mattayom 4), especially in verbal representation, showed a uniform pattern [29].
3.2. Relational Level

Figure 4 shows test result of 3rd subject (S3) of relational level. S3 is able to answer 2 of 3 questions. For the first question, S3 uses verbal (words) representation. S3 writes the maximum and minimum price of self-assembled skateboard in Skaters Shop and writes the price details of each answer. For the second question, S3 also uses verbal representation. S3 writes all possible price combinations correctly. There are 12 price combinations. S3 cannot answer the third questions correctly. But S3 also tends to use verbal representation in solving problem by giving price details.

Based on interview result, S3 has successfully understood all questions. The comprehension is indicated by S3 has already understood the information and what is being asked. The cause of S3 is wrong to answer last question is because S3 combined the prices randomly and S3 was not really sure with her answer.

Students with relational level have not been able to answer all questions correctly. When students are given new situations, they have not been able to understand the problem well. This is appropriate with Laisouw’s research. It shows that students with relational level, the algebraic thinking process which includes the ability to conduct investigations, representations and generalizations, and interpretation to find final results can be done properly and correctly, but the ability to analyze of finding result for new situation cannot be done correctly [23].

Figure 3. 2nd subject answer (S2)
3.3. Extended Abstract Level

Figure 5 shows test result of 4th subject (S₄) of extended abstract level. S₄ is able to answer all questions. For the first question, S₄ uses visual representation by making diagram. S₄ writes the maximum and minimum price of self-assembled skateboard in Skaters Shop and writes the price details of each answers by using lines to make a diagram. For the second question, S₄ uses mathematical expression representation. S₄ writes all possible price combinations by using conjecture for some numbers which is “+” symbol. For the third question, S₄ uses visual representation by drawing a table which consist of price details of final answer.

Based on interview result, S₄ has successfully understood all questions correctly. The comprehension is indicated by S₄ has already understood the information and what is being asked. Although S₄ has already answered all question correctly, but S₄ said that she is not really sure with the answer she has written. For instance is the second question. The reason why S₄ just wrote the final answer is because S₄ was not really sure. S₄ wrote steps of second question on the other paper. But S₄ is able to write the unwritten answer correctly when interview section was conducted.
Figure 5. 4th subject answer (S₄)

Figure 6 shows test result of 5th subject (S₅) of extended abstract level. S₅ is able to answer all questions. For the first question, S₅ uses verbal and mathematical expression representation by writing the maximum and minimum price of self-assembled skateboard in Skaters Shop and writes the price details of each answers. S₅ also writes the mathematical expression by using addition operation. For the second question, S₅ uses mathematical expression representation. S₅ writes all possible price combinations by using conjecture for some numbers which is “+” symbol. For the third question, S₅ uses verbal representation by writing final answer then S₅ gave the price details of it.

Based on interview result, S₅ has successfully understood all questions correctly. The comprehension is indicated by S₅ has already understood the information and what is being asked. There is a little bit difference between S₄ and S₅. S₅ felt more sure with the final answers.

Students with extended abstract level have more complex and structured answers. This is appropriate with Laisouw’s research. It shows that students with extended abstract level have high mathematical analysis skill. So students with high mathematics learning interest will have the highest rate in solving algebraic problems [23]. Moreover, students with the highest ability, which is extended abstract level, are able to display all the representation, verbal, visual, and mathematical expression. This is appropriate with previous research which states that students with high ability can display all the mathematical representation ability, namely presenting data or information from problem to table representation, solving problems involving mathematical expression, and writing steps to solve mathematical problems with words [24].
Figure 6. 5th subject answer (S₅)

From all research subjects, all students tend to use and show verbal representation in solving PISA problem. Previous relevant researches said that based on test answers analysis of PISA problem which is categorized as verbal representation, it can be concluded that verbal ability of most students is good enough. Students have been capable to write steps correctly [30].

4. Conclusion
According to research and data analysis of students mathematical representation based on SOLO taxonomy, it can be concluded that students with multi-structural and relational level tend to use verbal representation and students with extended abstract level tend to use verbal, visual, and mathematical expression representation in solving PISA problem on change and relationship content. Verbal representation are generally showed by writing argument, possibilities, and reasons of the final answers. Visual representation are generally showed by making diagram and table. Mathematical expression representation are generally showed by writing the reasons of final answers by using addition expression. The lack of accuracy, comprehension, and students confidence make students fail or wrong in answering the question.

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