Propagation of chirped sinh-Gaussian beams in uniaxial crystals orthogonal to the optical axis

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Abstract
This article includes intensity evolution and phase change of chirped sinh-Gaussian (CSG) beam propagating in uniaxial crystal orthogonal to optical axis. Received field expression is calculated utilizing Huygens-Fresnel integration. We see that chirped parameter brings decentered intensity distribution. By adjusting the decay factors, higher peak intensity can be obtained at the receiver side. Broken phase at near field is corrected at phase field. Phase change and number of circles can be controlled by decay factor and chirped parameter. We hope that our results are helpful for scientist working on optical tracking.

Keywords  Chirped sinh-Gaussian beam · Uniaxial crystal · Propagation

1 Introduction
Propagation of nonconventional beams in uniaxial crystal orthogonal to optical axis is studied by scientists due to the effect of crystal on received field. Regarding with this, we show that Airyprme beam has a flat topped shape after propagation (Bayraktar 2020). Authors investigate that extraordinary refractive index of crystal strongly influences the autofocusing of Pearcey beam (Xu et al. 2019). Change in polarization of radially polarized Pearcey beam is discussed in Xu et al. (2018). Received field is obtained benefiting from convolution property of Fourier transform when Airy beam is used initially (Zhou et al. 2012). It is shown in Zhou et al. (2015) that maximum intensity of Airy-Gaussian beam propagating in uniaxial crystal is inversely proportional to ratio of extraordinary refractive index to ordinary. In addition to intensity, phase information of Airy Gaussian vortex beam is lost at long distance in crystal having large refractive index ratio (Yu et al. 2016). Influence of distribution factor of Airy-Gaussian vortex beam is studied in nonparaxial analysis (Li et al. 2017). Chirped parameter of chirped Airy vortex beam controls the intensity and phase information during propagation (Wang et al. 2018). In another nonparaxial study, authors state that chirped parameter of chirped Airy vortex beam has an effect on angular momentum at the receiver side (Zhang et al. 2018). As compared to linear and quadratic chirped parameter, Gaussian factor of chirped Airy Gaussian vortex beam affects intensity...
more (Chen et al. 2018). Change in slope of radially polarized chirped Airy beam is slower in y-direction than in x-direction (Chen et al. 2021). Initial angle of radially polarized Airy-Gaussian beams brings off-axis intensity at the receiver plane (Sun et al. 2019). Super Lorentz-Gauss $SLG_{01}$ mode evolves into elliptic shape lying along x-axis when ratio of refractive index is greater than one (Zhou 2012). Intensity components of Hermite-Laguerre-Gaussian beam propagating in uniaxial crystal can be listed from low to high as $y$, $x$, and $z$ (Xu et al. 2013). Furthermore, inverse relation between longitudinal gradient force of rotating elliptical chirped Gaussian beam and propagation distance is mentioned in Ye et al. (2020).

Untraditional beams which contain trigonometric functions in the source field expression take place in the literature several times. Regarding with this, we show intensity evolution of hyperbolic sinusoidal beam under strong turbulence and it is concluded as astigmatic beam protects their shape at longer distances (Bayraktar 2021a). Besides this, sinh-Gaussian vortex beam provides higher signal to noise ratio than sinh-Gaussian beam since its scintillation is less (Zhang et al. 2019). Root-mean-square width of partially coherent sinh-Gaussian beam is directly proportional with zenith angle (Wu et al. 2020). In strongly nonlocal nonlinear media, hollow sinh-Gaussian beam has a periodic evolution in its intensity (Hricha et al. 2021). The same beam is focused onto a nano sized sphere and it is found that deeper potential than Gauss beam can be obtained (Liu et al. 2019). In addition, circular outer rings occur during the propagation of astigmatic hyperbolic sinusoidal Gaussian beam under oceanic turbulence (Bayraktar 2021b). By controlling the spiral parameter, intensity of spirally polarized sinh-Gaussian beam in the focal region is decreased (Senthilkumar et al. 2019).

Originality of this research is that it is the first study which introduces chirped sinh-Gaussian beam and analyzes its propagation aspects in uniaxial crystal.

As it is mentioned above, intensity, phase, and angular momentum can be manipulated by adjusting chirped parameter. In addition, sinh factor brings some advantages in atmosphere, underwater, and uniaxial crystal. Bearing this in mind, we combine chirped factor and sinh-Gaussian beam and introduce chirped sinh-Gaussian beam to benefit from advantages of both sides.

In this report, propagation properties of chirped sinh-Gaussian beam are studied in detail. We compute received field applying Huygens-Fresnel integration. Effects of crystal structure, attenuation, chirped parameter are plotted at different propagation distances. Variations in intensity and phase are investigated in detail. We hope that outcomes of this study will be used in optical applications.

### 2 Derivation of received field in uniaxial crystal

We define the source field expression of chirped sinh-Gaussian beam as

$$u_{s}(s_{x}, s_{y}) = \sinh(as_{x})\sinh(bs_{y})\exp\left(-\frac{s_{x}^{2} + s_{y}^{2}}{w^{2}}\right)\exp\left(c\frac{s_{x}}{w_{x}} + ds_{y}\right) + j\beta\left(\frac{s_{x}}{w_{x}} + \frac{s_{y}}{w_{y}}\right)$$

(1)

where $c$ and $d$ are the decay factors, $\beta$ being the chirped parameter, $a$ and $b$ are argument of sinh function, and $w = \sqrt{w_{x}^{2} + w_{y}^{2}}$ refers to Gaussian source size which is equal to
where \( n_e \) and \( n_0 \) correspond to extraordinary and ordinary refractive indices of crystal. We set \( n_0 \) to 2.62. Extraordinary refractive index, \( n_e \), calculated as \( en_0 \). Then, received field is in uniaxial crystal is calculated as (Yariv and Yeh 1984)

\[
u_r(r_x, r_y) = \frac{kn_0}{2\pi jz} \exp(-jkn_e z)
\]

\[
x \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_s(s_x, s_y) \exp\left(\frac{jk}{2zn_e} \left[ n_e^2(r_x - s_x)^2 + n_e^2(r_y - s_y)^2 \right]\right) ds_x ds_y
\]

(3)

Here, \( k \) denote wave number and it is calculated using operation wavelength as \( 2\pi / \lambda \). Operation wavelength is taken as 0.53 \( \mu m \) in this study. Additionally, propagation distance is indicated as \( z \) and intensity and phase plots are obtained considering the multiples of Rayleigh distance \( z_R = kw^2 \). Lastly, \( (r_x, r_y) \) refer to transverse receiver or observation plane coordinates. Based on calculations in Gradsteyn and Ryzhik (2015), we evaluate received field as

\[
u_r(r_x, r_y) = -\frac{kn_0}{2\pi jz} \exp(jkn_e z) \frac{\pi}{\sqrt{\left(\frac{jkw^2n_e^2 - 2n_e}{2zn_e w^2}\right)\left(\frac{jkw^2n_e^2 - 2z}{2zn_e w^2}\right)}}
\]

\[
x \exp\left(\frac{jkn_0^2 n_e}{2zn_e} r_x^2 + \frac{jkn_e r_y}{2z} r_y^2\right)
\]

(4)

In order to simplify Eq. 4, we define new variables such as...
By grouping the terms, received field is simplified as

\[ A = \left( -\frac{jkn_0^2r_x}{zn_x} + a + \frac{c}{w_y} + j\beta \right)^2 \left( \frac{jk\omega^2n_0^2-2zn_x}{zn_xw^2} \right) \]

\[ B = \left( -\frac{jkn_0^2r_y}{zn_y} + b + \frac{d}{w_x} + j\beta \right)^2 \left( \frac{jk\omega^2n_0^2-2zn_y}{zn_yw^2} \right) \]

\[ C = \frac{1}{2}\left( \frac{jk\omega^2n_0^2-2zn_x}{zn_xw^2} \right) \]

\[ D = \left( -\frac{jkn_0^2r_y}{zn_y} - a + \frac{c}{w_y} + j\beta \right)^2 \left( \frac{jk\omega^2n_0^2-2zn_y}{zn_yw^2} \right) \]

By grouping the terms, received field is simplified as

\[ u_r(r_x, r_y) = \frac{-kn_0}{2\pi jz} \exp(jkn_0z) \frac{\pi}{\sqrt{\left( \frac{jk\omega^2n_0^2-2zn_x}{2zn_xw^2} \right)}} \]

\[ x\exp\left( \frac{jkn_0^2r_x^2}{2zn_xr_x^2} + \frac{jkn_0^2r_y^2}{2zn_yr_y^2} \right) (\exp(A) - \exp(B)) (\exp(C) - \exp(D)) \]

We see from Eq. 9 that received field is in form of product of sum of Gaussian exponentials. This will bring us received field will be in form of symmetric or asymmetric Gaussian.

Besides this, effective beam width of chirped sinh-Gaussian beam can be calculated via (Ni et al. 2017; Zhou et al. 2019)

\[ \alpha_m = \sqrt{2} \left\{ \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m^2 |u_r(r_x, r_y)|^2 dr_x dr_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_r(r_x, r_y)^2 dr_x dr_y} - \left( \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m |u_r(r_x, r_y)|^2 dr_x dr_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_r(r_x, r_y)^2 dr_x dr_y} \right)^2 \right\} \]

where \( m \) refers to \( r_x \) or \( r_y \). It is numerically evaluated using MATLAB.

### 3 Results and discussions

In this part of the manuscript, comments on numerical plots for derivations above take place. In order to see the influence of decay factors and chirped parameter, we set \( a = b = 1m^{-1} \). First, we analyze beam profile on the source plane. Absolute field
distribution is given in Fig. 1. We see in a of this figure, there is and asymmetric Gaussian shape placed nearly at $x = 1.5$ and $y = 1.5$ for chirped sinh-Gaussian beam with $a = 1$, $d = 1$, $\beta = 0.6$. Edge of main lobe seems like being cut. Additionally, there are two slight intensities on the left and bottom of main lobe. Similar view is observed for $\beta = 0.1$ as it is in b in the same figure. For chirped sinh-Gaussian beam with $c = 3$, $d = 1$, $\beta = 0.6$, it is seen from c that main lobe take place at higher $x$ values and slight lobe on the left side vanishes. As opposed to this, beam center is shifted to higher $y$ values by increasing $d$ as it is seen from d in Fig. 1. Furthermore, side lobe on the bottom disappears. Considering phase, we investigate that phase profile becomes squared when $\beta$ is low. On the other hand, increase in $\beta$ brings diagonal phase change. At the receiver side, we see from Fig. 2 that CSG beam with $a = b = 1m^{-1}$ and $\beta = 0.6$ has an off-axis elliptical distribution. Amount of shifting along $y$-axis is greater than the one along $x$-axis. Additionally, center of elliptic intensity shifts more and beam enlarges as propagation distance increases. In Fig. 3, chirped parameter is selected as 0.1. We investigate that shift in center of elliptic intensity is less as compared to Fig. 2. Figures 4 and 5 involve received intensity plots for the beams in Figs. 2 and 3 respectively where $e = 1.7$. Beam size in Figs. 4 and 5 is less than the ones in Figs. 2 and 3 in order. Moreover, peak intensity in related plots is higher than in Figs. 2 and 3. Figure 6 is plotted to see the effect of $c$. In the comparison of Figs. 4 and 6, we investigate that peak intensity raises significantly when $c$ increases. Furthermore, similar decentered positions are observed. By comparing Figs. 4 and 7, we see that approximately 100 times higher peak intensity can be measured at longer propagation distances. Beam still protects its decentered position. Significant amount of intensity decay is observed for all intensity plots when propagation distance reaches to $2z_R$.

In another point of view, we analyze the phase variations during propagation. CSG beam with $c = 1$ and $\beta = 0.6$ has a circular non-uniform and off-axis phase distribution at near field in crystal with $e = 0.7$. When $z = z_R$, broken in phase is recovered and phase distribution becomes uniform by locating on-axis. Phase circles place compactly when we go through the edge of the observation plane. When chirped parameter is set as 0.1 as it is

![Fig. 1 Absolute field and phase distribution of chirped sinh-Gaussian beam on transverse source plane](image-url)
in Fig. 3, strict phase break is observed along y axis at near field. Beam recovers itself at longer distances and becomes smooth. As compared to Fig. 8, we change chirped parameter in Fig. 9 and the crystal setting as $e = 1.7$ in Fig. 10. We see that number of circles in the observation plane is inversely proportional to refractive index ratios. Similar with higher ratio, phase is broken at close distance and then it recovers itself in longer distances. Comparing Figs. 9 and 11, we see that effect of strict phase break gets better at $z = 2z_R$. Similar with the previous investigation, circles are located frequently in this setting of
crystal. In the comparison of Figs. 10 and 12, we investigate that raise in parameter $c$ brings us a slight phase change at near field but phase seems still close to uniform. In other words, phase is not affected by the variations in parameter $c$. On the other hand, off-axis localization of phase distribution along x-axis attracts the attention after $z = 2z_R$. Lastly, change in parameter $d$ changes the origin of the phase circles along y-axis. Directly proportion to parameter $c$, important phase break is observed at near field and beam enhances itself at longer distances as it is in Fig. 13. As a general investigation, peak intensity reduces by
amount of $10^{15}$ which is nearly equal to infinity. It means that beam loses most of its power during propagation. In other words, beam disappears as propagation distance increases. Moreover, asymmetric shape is generated due to the anisotropic nature of crystal.

In order to make an investigation about effective beam width along x axis, Fig. 14 is given. Effective beam width in positive crystal is extremely high as compared to the one in negative crystal. Additionally, beam width is not affected by $\beta$ both in positive and
negative crystal. On the other hand, increase in $c$ brings higher effective beam width along x axis. However, variation in $d$, does not have an influence along x direction. In general, beams have a critical point at $z/z_R = 2$. At this distance, slope of effective beam width increases sufficiently. In addition, beam width decreases until this distance in negative crystal. In other words, beam shows focusing behavior when $e < 1$.

Fig. 8 Phase distribution of chirped sinh-Gaussian beam having $c = d = 1$ and $\beta = 0.6$ in crystal with $e = 0.7$

Fig. 9 Phase distribution of chirped sinh-Gaussian beam having $c = d = 1$ and $\beta = 0.1$ in crystal with $e = 0.7$
4 Conclusion

We derive the received field expression of CSG beam propagating in uniaxial crystal by solving the Huygens-Fresnel integral. We investigate that decentered beam can be obtained by adjusting chirped parameter. Peak intensity is strongly dependent on the decay factor. Higher intensity is measured when decay factor is increased. Chirped parameter shifts phase distribution to off-axis position. While beam has non-uniform
Phase at close distance, uniform phase distribution is observed at longer distances. Decay factors are also effects the center of the phase distribution. We anticipate that these results will be beneficial in optical applications.

Fig. 12 Phase distribution of chirped sinh-Gaussian beam having $c = 3$, $d = 1$ and $\beta = 0.6$ in crystal with $e = 1.7$

Fig. 13 Phase distribution of chirped sinh-Gaussian beam having $c = 1$, $d = 3$ and $\beta = 0.6$ in crystal with $e = 1.7$
Conflict of interest  The authors declare no conflicts of interests.

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