The Dynamical Significance of Triple Star Systems in Star Clusters

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ABSTRACT
Over the last few decades, observational surveys have revealed that high-order multiple-star systems (e.g. triples, quadruples, etc.), and triples in particular, are common in our Galaxy. In this paper, we consider the dynamical significance of this transformation in our understanding of stellar multiplicity. Using empirically constrained binary and triple fractions in those star clusters for which these values are available in the literature, we compare analytic rates for encounters involving single, binary, and triple stars. Our results show that, even for relatively low triple fractions, dynamical interactions involving triples occur roughly as often as encounters involving either single or binary stars alone, particularly in low-mass star clusters. More generally, using empirically-derived multiple star catalogues for the young star-forming association Taurus-Auriga and the Galactic field, we show that the data are consistent with the gravitationally-focused cross section for encounters increasing with increasing multiplicity. Consequently, triple stars, and even higher-order multiples, could be more important than previously realized for a number of astrophysical phenomena, including the formation and destruction of compact binaries and various types of stellar exotica, and the dynamical evolution of star clusters.

Key words: gravitation – stellar dynamics – binaries (including multiple): close – stars: formation – globular clusters: general – open clusters and associations: general.

1 INTRODUCTION
Over the last few decades, observations have revealed that multiple-star systems (MSSs), including binary stars and high-order multiples (e.g. triples, quadruples, etc.), are common in our Galaxy. This is the case for the Galactic field (e.g. Duquennoy & Mayor 1991, Tokovinin 1997, Raghavan et al. 2010), young star-forming associations (e.g. Kraus et al. 2011, 2012), old moderately-dense open clusters (e.g. Latham 2007, Talamantes et al. 2010, Geller & Mathieu 2012), and even ancient globular clusters (GCs) (e.g. Prodan & Murray 2012, Milone et al. 2013). The most complete survey is that of Raghavan et al. (2010), who recently updated the seminal work of Duquennoy & Mayor (1991) using a volume-limited sample of field solar-type primary stars in the solar neighborhood selected from the Hipparcos catalogue. They reported that the observed fractions of objects that are single, double, triple, and high-order systems are $56 \pm 2\%$, $33 \pm 2\%$, $8 \pm 1\%$, and $3 \pm 1\%$, respectively. Even more recently, Kraus et al. (2014) performed a high-resolution imaging study of the young star-forming region Taurus-Auriga to characterize its multiple star population. They found that $2/3 - 3/4$ of all targets are MSSs composed of two or more stars, while only the remaining $1/4 - 1/3$ are single stars.

The importance of dynamical encounters involving MSSs, even during the epoch of star formation, is becoming increasingly evident. Bate (2012) recently presented the results of a statistical analysis of MSSs formed during the largest radiation hydrodynamical simulation of star cluster formation conducted to date. These models reproduce reasonably well many of the characteristics of Galactic star-forming regions. Interestingly, the author finds that protostellar accretion is often terminated by dynamical interactions, and therefore that dynamics plays an important role in shaping the stellar initial mass function (IMF) (Podsiadlowski & Price 1992), at least in the dense clusters considered in this study. The importance of dynamics and multiplicity during the star formation process has also been confirmed observationally. Based on a multi-epoch search for wide, low-mass tertiary companions in a volume-limited

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sample of 118 known spectroscopic binaries within 30 pc of the Sun, Allen et al. (2012) find a wide tertiary fraction of 19.5+5.2%. This is consistent with the predictions of star formation simulations, which suggest that the fraction of wide, low-mass companions to spectroscopic binaries is >10%, and roughly twice the wide companion rate of single stars. This trend is thought to arise through three-body interactions during the star formation process, which transfers angular momentum away from a close pair of objects, hardening them further.

Our theoretical understanding of how dynamical encounters involving high-order multiples impact the evolution of older star clusters is limited (e.g. Moeckel & Bonnell 2013). The vast majority of the numerical simulations of star cluster evolution performed to date include only single stars and binaries in the initial population, and often these models do not allow for the dynamical formation of high-order multiples, thus entirely neglecting the dynamical input from this important stellar population. Some authors have recently acknowledged the need to include high-order multiples in their simulations, and list this as a future addition (e.g. Hypki & Giersz 2012). Geller, Hurley & Mathieu (2013) investigate the dynamical impact of both primordial and dynamically formed triples on blue straggler formation in their N-body models, and find that nearly half of the blue stragglers formed through collisions (between 6 and 7.5 Gyr) resulted from stellar encounters involving hierarchical triples. We suggest here that the inclusion of high-order multiples in such models may be critical for accurately reproducing the dynamical evolution of many star clusters as well as the production rates of compact binaries and stellar exotica.

Recently, Leigh & Sills (2011) argued that encounters involving triple stars should be common in the old open clusters NGC 188 and M67, and that this could also be the case for other open clusters. Importantly, however, these results were based on the inferred properties of the binary and triple populations in these clusters. In this paper, we put the prediction of Leigh & Sills (2011) to the test using empirically-constrained binary and triple fractions in those open clusters for which these data are available in the literature. We show in Section 2 that encounters involving triples occur roughly as often as, or even more often than, encounters involving single and binary stars alone in every cluster in our sample. More generally, using empirically-derived multiple star catalogues for the young star-forming association Taurus-Auriga and the Galactic field, we show in Section 3 that the gravitationally-focused cross section for encounters increases with increasing multiplicity. This reduces the average time between encounters and, in general, contributes to increasing the dynamical significance of high-order multiple star systems. The implications of our results to various astrophysical phenomena are discussed in Section 4, and we summarize the key points of this paper in Section 5.

2 IDENTIFYING THE DOMINANT ENCOUNTER TYPE

In this section, we compare the rates of the different encounter types involving singles, binaries, and triples using empirically-measured binary and triple fractions taken from the literature. Leigh & Sills (2011) derive encounter rates for single-single (1+1), single-binary (1+2), binary-binary (2+2), single-triple (1+3), binary-triple (2+3), and triple-triple (3+3) encounters. Importantly, the ratio of any two of these encounter rates only depends on the binary and triple fractions, and the average geometric cross sections for singles, binaries and triples (e.g., the mean stellar radius, $R$, and the mean binary and tertiary semi-major axes, $a_b$ and $a_t$). Therefore, the regions of parameter space where each of these different encounter rates dominate can easily be compared across these three different stellar populations, as we do in Figure 1. Each labeled polygon in this figure denotes the region of parameter space where the frequency of the given encounter type is higher than all others, and dividing lines show where the encounter frequencies are equal. Here we show the result using the value of $a_t/a_b$ for Taurus-Auriga. Perhaps the most striking result from this analysis is that encounters involving triples dominate over encounters with binaries for the majority of parameter space. This is the case on a per star basis, even if the fraction of systems composed of 3 or more stars is low.

In Figure 2 we show, for lines of constant binary fraction, the critical triple fraction at which encounters involving triples begin to dominate over encounters involving single and binary stars alone. To obtain each of these lines, we fix $f_b$ and calculate the number of encounters involving triples (i.e. the sum of the numbers of 1+3, 2+3, and 3+3 encounters) over a fixed time-scale. For the same time-scale, we then compare this to the total number of encounters involving binaries and singles alone (i.e. 1+1, 1+2, and 2+2). The critical triple fraction is shown as a function of the ratio $a_t/a_b$. Importantly, if $f_t \gtrsim 0.3$, encounters involving triples will dominate over encounters involving either single or binary stars, regardless of $f_b$ and the ratio $a_t/a_b$. Equivalently, provided $a_t/a_b \gtrsim 5$ and $f_t \gtrsim 0.16$, triples will be undergoing encounters more often than either single or binary stars alone. Furthermore, for $f_t \gtrsim 0.1$, if triples are not the dominant type of interacting object, they are at least roughly as dynamically-active as are single and/or binary stars.

Next we compare these analytic results to observed multiple-star populations from the literature. The points in Figure 3 show the observed binary and triple fractions for Taurus-Auriga (Kraus et al. 2011), the Pleiades (Mermilliod et al. 1992, Bouvier, Briand & Nadeau 1997, Prasoppe Mermilliod & Mavromatakis 1999, Bouvier et al. 2001), and the Hyades (Patience et al. 1998). We also place a point on Figure 4 representing a star cluster with the binary and triple fractions observed in the Galactic field (Raghavan et al. 2010). We plot these empirical results for illustrative purposes, but note that each survey has a unique level of completeness and sample selection criteria, which we do not attempt to correct for here. The raw numbers of single, binary, and triple stars are also shown for each surveyed
Table 1. The numbers of single, binary, and triple stars in our samples.

| Cluster Name | Singles | Binaries | Triples | Total |
|--------------|---------|----------|---------|-------|
| Taurus-Auriga | 48      | 50       | 12      | 110   |
| Galactic Field | 56      | 33       | 8       | 97    |
| Hyades       | 98      | 59       | 10      | 167   |
| Praesepe     | 43      | 32       | 5       | 80    |
| Pleiades     | 54      | 29       | 3       | 86$^a$|

$^a$The original Mermilliod et al. (1992) sample contains 88 stars, but four of these stars are associated as visual binaries, respectively.

The most complete surveys shown here are likely those of the Galactic field, Taurus-Auriga, and the Hyades. The Raghavan et al. (2010) Galactic field survey covers a complete sample of solar-type dwarfs within 25 pc of the Sun having $0.5 \leq (B-V) \leq 1.0$ (which roughly corresponds to spectral types F6 - K3). This study combines multiple stars detected through spectroscopy, eclipsing binary surveys, separated fringe packets (using the Center for High Angular Resolution Astronomy, CHARA, Array), speckle interferometry, adaptive optics, and multiple astrometric techniques (e.g., astrometric orbits, common-proper motion pairs, etc.). This study is nearly complete for orbital separations $\lesssim 10^4$ AU and mass ratios $\gtrsim 1$. The Kraus et al. (2011) study contains all known members of Taurus-Auriga with spectral types between G0 and M4 and masses between 2.5 $M_\odot$ and 0.25 $M_\odot$. Kraus et al. combine visual companions detected in 2MASS images with companions detected through aperture masking with adaptive optics imagers to compile a fairly complete sample of multiple stars with separations between 3 and 5000 AU and mass ratios $\gtrsim 0.1$. The Patience et al. (1998) Hyades speckle imaging survey covers a nearly complete sample of stars with $K < 8.5$ mag (including dwarf stars of spectral type between about A0 and K5, and some evolved stars), stellar separations of about 5-50 AU and mass ratios $\gtrsim 0.2$. The Hyades $f_1$ and $f_2$ values shown here also include multiple stars detected from spectroscopy and direct imaging (see references in Patience et al. 1998).

The Pleiades and Praesepe samples are somewhat less complete. For both clusters, we take the samples from the radial-velocity surveys of Mermilliod et al. (1992) and Mermilliod & Mayor (1999), respectively, and add additional companions detected in the adaptive optics imaging surveys of Bouvier, Rigaut & Nadeau (1997) and Bouvier et al. (2001), respectively. For both clusters, the radial-velocity and adaptive optics numbers do not completely overlap, and the coverage in moderate orbital separation is incomplete. The Mermilliod et al. (1992) Pleiades survey covers stars with $0.4 \leq (B-V) \leq 0.9$ (about F5-K0 spectral types), while the Bouvier, Rigaut & Nadeau (1997) survey includes stars with $0.56 \leq (B-V) \leq 0.6$ (note, $E(B-V) = 0.03$) with known rotational velocities, in the central part of the Pleiades. The combined Pleiades radial-velocity and adaptive optics numbers shown in Table 1 are not sensitive to systems with separations between about 5 and 10-15 AU. The Mermilliod & Mayor (1999) Praesepe survey covers stars with $0.4 \leq (B-V) \leq 0.8$ (about F5-K0 spectral types), while the Bouvier et al. (2001) survey includes proper-motion members with $0.52 \leq (B-V) \leq 1.4$. The combined Praesepe radial-velocity and adaptive optics numbers shown in Table 1 are not sensitive to systems with separations between about 4 and 15 AU. For both clusters we also include photometric binaries identified by Mermilliod et al. as being in the binary region on the color-magnitude diagram (brighter and redder than the single-star main sequence), which may alleviate some of the incompleteness in orbital separation.

We note that the Pleiades, Praesepe and Hyades are all OCs with ages on the order of several tens to hundreds of Myrs, and densities on the order of 1 - 10 $M_\odot$ pc$^{-3}$. The ages of these clusters are comparable to the typical time between dynamical encounters, so that a large fraction of the MSSs in these clusters (and those in Taurus-Auriga) must be primordial.

If these clusters have a similar ratio of $a_1/a_0$ as in Taurus-Auriga, Figure 1 shows that all of these clusters lie in the region where encounters involving triples will dominate over encounters involving both single and binary stars alone (not accounting for any incompleteness in the respective observational surveys).

For reference, the Geller, Hurley & Mathieu (2013) N-body open cluster model predicts that, for dynamically formed solar-type triples in a rich open cluster like NGC 188, the ratio of $a_1/a_0$ will remain roughly constant over the cluster lifetime (7 Gyr for their model), with a mean value of 4.2 and a standard deviation of 0.7. Geller, Hurley & Mathieu (2013) did not include any triples in the initial population. Therefore their predictions are for triples formed during stellar encounters. In the Pleiades, the ratio $a_1/a_0$ is only a factor of $\sim 2 - 3$. However the 1$\sigma$ uncertainty exceeds this value and, as discussed above, the Pleiades data are incomplete for moderate semi-major axes. As also discussed above, the Taurus-Auriga data are the most complete of all of the open clusters in our sample, and we note that the observed value in Taurus-Auriga of $a_1/a_0 = 13.6$ is somewhat larger than the theoretical prediction of Geller, Hurley & Mathieu (2013) or the empirical value of the Pleiades. (We were unable to perform a similar comparison of the $a_1/a_0$ values for the other open clusters in our sample as these data were not readily available in the literature.) We conclude that the available data are consistent with a typical ratio $a_1/a_0$ of at least a few, and possibly more. Even if $a_0 = a_1$, encounters involving triples occur roughly as frequently as encounters involving single stars, and encounters involving binaries occur at most $\sim 4$ times as often as those involving triples.

Thus, we conclude that encounters involving triples should currently be as common as encounters involving only single or binary stars alone in every cluster shown in Figure 1 to within a factor of at most a few. In fact, the data...
are consistent with triples being the dominant interacting objects in these clusters. Furthermore, given that in general triples are harder to detect than binaries (e.g., one may often require both spectroscopy and direct imaging for a given system to detect all three components) the true locations of these clusters on Figure 1 may lie even further into the triple dominated regime. This can be properly tested when completeness-corrected period distributions become available for both binaries and triples in these clusters. This will also allow for the results presented in this paper, which are calculated using average cross-sections and hence average encounter timescales, to be tested using timescales derived by integrating the full period and velocity distributions.

3 EXTENSION TO HIGHER-ORDER MULTIPlicity

In this section, we extend our discussion to higher-order multiples using the MSS samples of Kraus et al. (2011) and Tokovinin (1997). These are representative of two different environments, namely a young (few Myrs) very low-density star-forming region and an older Galactic field population. The key point we will make is that the data are consistent with a trend in which the mean cross-section for stellar encounters increases with increasing multiplicity. As with triples, this reduces the average time between encounters, and contributes to increasing the dynamical significance of higher-order multiplicity. To show this trend, we calculate for each multiplicity the gravitationally-focused cross-section or, more specifically, the product of the total system mass and the geometric cross-section. All calculations performed in this section correspond to the arithmetic mean, however we confirm our results using the geometric mean as well. All uncertainties correspond to one standard error of the mean.

First, as the number of stars increases, the geometric cross-section (i.e. the diameter of the object) increases, same spectral range as the Taurus-Auriga sample. We therefore choose to use the complete field sample for Figure 2.

4 We note that some of the distributions in our samples, and particularly those of the semi-major axes, are not symmetric about the mean. Therefore their uncertainties about the mean are asymmetric. However, the sample sizes are too small to properly calculate asymmetric error bars, and therefore we simply provide the standard errors of the means as our best estimates of the uncertainties.
and therefore the encounter time decreases. We inspect the MSS sample for the star-forming association Taurus-Auriga (Kraus et al. 2011), and compute means and standard deviations of the mean for the maximum orbital separations (i.e. within a given system) for all binaries, triples, and quadruples. These are $(1.3 \pm 0.4) \times 10^3$ AU, $(1.8 \pm 0.5) \times 10^3$ AU, and $(1.4 \pm 0.3) \times 10^3$ AU, respectively. There are also two additional multiple star systems in this sample, namely a quintuple and a sextuple, with maximum orbital separations of $3.1 \times 10^3$ AU, and $3.8 \times 10^4$ AU, respectively. A similar analysis of the Multiple Star Catalogue (MSC; Tokovinin 1997) yields mean maximum (i.e. highest-order) orbital separations for triples, quadruples, quintuples, sextuples, and septuples of $(1.8 \pm 0.1) \times 10^3$ AU, $(2.3 \pm 0.5) \times 10^3$ AU, $(9.0 \pm 1.7) \times 10^3$ AU, $(1.1 \pm 0.2) \times 10^4$ AU, and $(9.4 \pm 3.9) \times 10^4$ AU, respectively.

Second, the total stellar mass tends to increase with increasing multiplicity, which increases the gravitationally-focused cross section, and thereby further decreases the encounter time. The mean system masses for the Taurus-Auriga sample (derived using the component masses given in Table 5 of Kraus et al. 2011) for the binaries, triples, quadruples, quintuple, and sextuple, respectively, are $4.5 \pm 0.1 M_\odot$, $5.0 \pm 0.6 M_\odot$, $5.6 \pm 0.6 M_\odot$, and $13.1 \pm 1.9 M_\odot$ (excluding systems with only minimum mass measurements). There is also a single septuple in the MSC sample with firm mass estimates for all components, yielding a total system mass of $28 M_\odot$.

The key conclusion from these results is that the gravitationally-focused cross-section for encounters systematically increases with increasing multiplicity. Figure 3 shows the product of the mean total system mass and the mean maximum orbital separation, which is directly proportional to the gravitationally-focused cross-section, as a function of multiplicity for the Galactic field (top inset) and Taurus-Auriga (middle inset). For comparison, we also show the results from the hydrodynamical simulations of Bate (2012) using the values provided in their Table 3. The Spearman rank correlation coefficients for these samples are 1.0 in all cases. Thus, our proxy for the mean gravitationally-focused cross-section increases monotonically with increasing multiplicity in both of these observed MSS samples, which is also consistent with the theoretical predictions of Bate (2012).

To better quantify the statistical significance of these correlations, we also performed weighted lines of best-fit to the log($m_{sys}$) values shown in Figure 3. For the Galactic field, Taurus-Auriga, and Bate (2012) samples we find slopes of, respectively, $0.47 \pm 0.07$, $0.63 \pm 0.09$, and $0.40 \pm 0.10$. These are all inconsistent with zero at greater than the 3σ confidence level. To confirm this, we also calculated chi-squared values by comparing the log($m_{sys}$) results to a flat distribution at the respective mean for each sample. In all three cases, we rule out the hypothesis that the data are consistent with a flat distribution at very high confidence (>99.99%). In summary, these data show that the gravitationally-focused cross-section increases with increasing multiplicity at a statistically significant level.

The gravitationally-focused cross-section typically dominates over the geometric cross-section, especially for single stars and in low-velocity dispersion environments. However, with high-order multiples, the orbital separations can be so large that both cross-sections can be equally important. As discussed above, the data for these high-order multiples are also consistent with the geometric cross-section increasing with increased multiplicity, particularly for the MSC sample, where the data extend from triples to a septuple. We note that the binding energies corresponding to the external orbits of the MSSs in our sample are typically below the hard-soft boundary in their host cluster. Thus, we expect that most of these systems are indeed bound objects.

The combination of the results presented in this section and the dynamical significance of triple stars shown in Section 3 suggests that high-order multiples may be very important for many stellar dynamical processes in star clusters that have generally been attributed only to binary stars. We discuss the implications of our result below.

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\[ \sum \]

\[ \text{We note that our results remain unchanged with or without excluding these systems.} \]
4 DISCUSSION

Multiplicity is more likely to be preserved during dynamical interactions in low-velocity-dispersion environments. This is because the hard-soft boundary is located at very long orbital periods in such star clusters, so that even wide high-order multiples are typically classified as dynamically hard. We have verified this using a suite of numerical scattering experiments involving single, binary, and triple stars (see Leigh & Geller 2012 for the details of the experiments). By keeping the orbital energies fixed and varying the relative velocity at infinity, we find that decreasing the total encounter energy on average increases the fraction of encounter outcomes in which high-order multiples are produced or preserved. This higher likelihood to produce and preserve MSSs suggests that high-order multiples should be more common and act as more efficient heat sources in low-velocity-dispersion environments such as open clusters (OCs).

Leigh & Geller (2012) recently showed that the probability that a direct collision will occur between any two stars during an encounter scales as $N^2$, where $N$ is the number of interacting stars. Therefore, on a per encounter basis, a higher multiplicity translates into an increased probability of direct stellar collisions, which may be observed as stellar exotica such as BSs. Several peculiar examples of MSSs containing BSs have recently been identified in OCs. Their existence is difficult to explain without at least some involvement from dynamical interactions. For example, the suspected triple system S1082 in the old OC M67 appears to contain two BSs (van den Berg et al. 2003; Sandquist et al. 2003). A triple system in which the outer companion is a BS was recently found in the OC NGC 6819 by Talamantes et al. (2010). A fascinating binary containing two BSs was also reported by Mathieu & Geller (2009) in the old OC NGC 188. In all of these cases, mass transfer alone could not have produced the systems containing these BSs, whereas a collateral origin or at least some subsequent stellar encounter(s) involving high-order multiples may potentially account for their existence.

In fact, the multiple-star population may play a large role in governing both the creation and destruction rates of stellar exotica, in particular those that rely on mass transfer in compact binaries. The results of Leigh & Geller (2012) also show that physical collisions during dynamical encounters, which become more likely with increasing multiplicity, offer a mechanism for destroying compact binaries. This stems from the fact that, if a collision occurs during an encounter involving three or more stars, it is the most compact orbit going into the interaction that will typically be destroyed (e.g. Valtosen & Karttunen 2006). The details of the balance between creation and destruction of compact binaries resulting from encounters with MSSs in clusters is an important question that can be answered by the next generation of $N$-body models that include significant populations of high-order multiples (e.g. Geller, Hurley & Mathieu 2013).

On a more global scale, future $N$-body models will also show how high-order MSSs influence the overall dynamical evolution of star clusters. For example, $N$-body simulations have shown that the hardening of compact binaries through successive dynamical encounters in the dense cores of GCs can halt core collapse (e.g. Hut 1983). This evolutionary phase is often called “binary burning” (e.g. Fregeau, Ivanova & Rasio 2009), since these hardening interactions act to prevent any further increase in the central cluster density. However, the collisional cross-section for a compact binary is very small, so that the time-scale for it to undergo a direct dynamical interaction with a single star can be very long. Consequently, the onset of the binary burning phase is expected to exceed a Hubble time for most of the GCs in the Milky Way (Fregeau, Ivanova & Rasio 2009). However, if most compact binaries are members of high-order multiples, then the time-scale for them to undergo direct encounters can be quite short due to the much larger cross-section of their parent MSS. The wide outer orbit of a triple effectively acts as a “net”, drawing stars in where they can be scattered and interact resonantly with the close inner binary of the triple (Leigh & Sills 2011; Moeckel & Bonnell 2013). In this way, the binding energy of the compact inner binary of the triple can be tapped, and re-distributed throughout the cluster via relaxation processes (once the higher-order resonantly interacting multiple system has decayed into stable components). The role played by high-order multiples as heat sources in clusters should be the most pronounced in low-velocity-dispersion environments, like OCs. As discussed above, in such clusters even very wide orbits, which are a prerequisite for dynamical stability within high-order multiples, are often classified as dynamically hard. Our results suggest that triples may undergo dynamical interactions roughly as frequently as binary stars, and hence could be relevant for the overall evolution of certain clusters. Future detailed $N$-body modeling will be necessary in order to quantify the role of triples as heat sources and their significance for the dynamical evolution of star clusters.

5 SUMMARY

In this paper, we address the question: How important are high-order MSSs to the dynamics of star clusters and the production/destruction of compact binaries and stellar exotica? The key conclusion is that dynamical encounters involving triple stars should be roughly as common as encounters involving only single and binary stars in low- to moderate-density star clusters. Furthermore, the available data on higher-order multiples are consistent with a trend in which the gravitationally-focused cross-section for encounters increases with increasing multiplicity. These results suggest that simulations of star cluster evolution should include high-order multiples in the initial conditions and allow such systems to evolve dynamically, and be created and destroyed, throughout the cluster evolution. This will be a crucial next step in the development of realistic star cluster simulations.

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