LDP-FPMiner: FP-Tree Based Frequent Itemset Mining with Local Differential Privacy

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ABSTRACT
Data aggregation in the setting of local differential privacy (LDP) guarantees strong privacy by providing plausible deniability of sensitive data. Existing works on this issue mostly focused on discovering heavy hitters, leaving the task of frequent itemset mining (FIM) as an open problem. To the best of our knowledge, the state-of-the-art LDP solution to FIM is the SVSM protocol proposed recently. The SVSM protocol is mainly based on the padding and sampling based frequency oracle (PSFO) protocol, and regarded an itemset as an independent item without considering the frequency consistency among itemsets.

In this paper, we propose a novel LDP approach to FIM called LDP-FPMiner based on frequent pattern tree (FP-tree). Our proposal exploits frequency consistency among itemsets by constructing and optimizing a noisy FP-tree with LDP. Specifically, it works as follows. First, the most frequent items are identified, and the item domain is cut down accordingly. Second, the maximum level of the FP-tree is estimated. Third, a noisy FP-tree is constructed and optimized by using itemset frequency consistency, and then mined to obtain the k most frequent itemsets. Experimental results show that the LDP-FPMiner significantly improves over the state-of-the-art approach, SVSM, especially in the case of a high privacy level.

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1 INTRODUCTION
Differential privacy (DP) has become the de facto standard for privacy protection. It was named one of the ten breakthrough technologies in 2020 by the MIT technology review [25]. Generally, there are two types of differential privacy - centralized differential privacy (CDP) [5] and local differential privacy (LDP) [20], and the focus of this work is the local setting. The most typical LDP protocols [5, 16, 18, 28, 33–35, 37, 39, 42, 43] enable users to randomly perturb their inputs. This guarantees strong privacy without relying on a trusted third party by providing plausible deniability of sensitive data. In practice, LDP has many compelling applications deployed by Apple [30, 31], Google [10, 11], Microsoft [7] and Alibaba [38], and so on.

As the development of data analysis, privacy issues have drawn more and more attention. Over the past 30 years, data in various fields have increased on a large scale. Such massive amounts of data might have potential correlations (or patterns), which can be extracted or mined for more interesting knowledge [13]. As a core data mining task, frequent itemset (or pattern) mining (FIM) plays an essential role in mining association rules [1, 2]. However, it also poses a threat to user privacy [19]. An attacker with strong background information may learn private information from the unprotected itemsets discovered. For this reason, extensive studies have been conducted for the task of privacy-preserving frequent itemset mining (PPFIM) [21, 26, 32]. Especially, differentially private schemes for frequent itemset mining have come to the fore [3, 22, 23, 39, 44].

In this paper, we study the task of discovering top-k itemsets over sensitive transactional (or set-valued) data in the context of LDP. Specifically, there are n users, whose transaction t is a subset of d distinct items, denoted by the item domain X = {x₁, x₂, ..., x₃d}. An untrusted analyst (or aggregator) wants to discover the k most frequent itemsets with a given privacy budget ε, which measures the scale of privacy provided. This is more challenging even when one just tries to find heavy hitters, and one alternative to address this problem is to encode each transaction as an input, i.e., a value in the power set p(X), and then apply existing frequency oracle protocols, such as RAPPOR [10] and OLH [37], to privately collect estimations. However, in this particular case, the exponential size of the domain p(X) may inject considerable noise that results in a poor accuracy.

Meanwhile, the heterogeneous number of items that users hold in the transactional data setting makes the task more complicated. To deal with this, the padding and sampling (PS) technique is widely used in the literature, i.e., the user pads her transaction t with dummy items to a specified size l and randomly selects one item as her input, denoted by PS_l(t). However, the optimal selection of l is non-trivial task. For instance, in [12, 28, 36], they suppose each user has at most predefined L items. Ye et al. [43] convert the key-value set of each user to its length d binary form, which does not work well for a large domain d [12]. The baseline strategy of setting suitable l [39] is to use the 90th percentile of the length of inputs collected privately in the context of LDP.
To the best of our knowledge, the state-of-the-art solution is the Set-Value itemSet Mining (SVSM) protocol [39], which discovers top-k itemsets based on the padding and sampling based frequency oracle (PSFO) protocol. The core idea of SVSM is to construct a pony-size domain set \(|S| = 2k| |\) of potential itemsets likely to be frequent according to their guessing frequencies, then encode each itemset as one singleton and apply the PSFO protocol with the domain \(S\) to privately collect estimations. However, the SVSM does not consider the consistency among itemsets, leaving considerable room for the performance improvement.

Inspiringly, we introduce the structure of frequent-pattern tree (FP-tree)[14] to the solution of FIM problem with LDP for the first time. FP-tree can be used to effectively discover frequent patterns in the traditional non-privacy setting with mild computational cost. In our context, we use an FP-tree constructed with LDP to exploit the frequency consistency among itemsets to improve the data utility. The post-processing property guarantees that itemset mining over the noisy FP-tree do not disclose the privacy as well. Specifically, the noisy FP-tree is constructed in breadth-first (BF) order, and instead of dealing with the heterogeneous number of items, each transaction can be converted into one prefix of the FP-tree. And, to allocate the privacy budget, we approximate the maximum level \(M\) of the tree by setting it as the 80th percentile of length distribution of users. Although the large size of the domain at each level increases the noise as well as the cost, we propose a pruning algorithm to effectively cut down the domain into a small fixed size for accuracy improvement.

Summarily, our main contributions are as follows.

• We propose a novel approach called LDP-FPMiner to discover \(k\) most frequent itemsets in LDP setting based on FP-tree for the first time.
• We design an algorithm that constructs an FP-tree with LDP in breadth-first order and optimize the noisy FP-tree by exploring frequency consistency.
• Experimental results on both synthetic and real-world datasets show a significant performance improvement over the state-of-the-art SVSM.

Roadmap. The remainder of this paper is organized as follows. Section 2 gives the preliminaries. Section 3 introduces the problem setting and the state-of-the-art approach. We present our approach and conduct theoretical analysis in Section 4. In Section 5, we optimize the proposed scheme in several ways. The experimental results are presented in Section 6. Section 7 is the related work and finally Section 8 concludes our work.

2 PRELIMINARIES

2.1 Local Differential Privacy (LDP)

In local differential privacy setting, each user randomly perturbs its raw data and then sends the resulted data to the analyst. The untrusted analyst can only access the perturbed data other than the raw ones, which guarantees the privacy. Formally, let \(\mathcal{T}\) denote the domain of a sensitive value, \(\epsilon\)-local differential privacy (or local privacy) is defined as follows.

Definition 1. (\(\epsilon\)-Local Differential Privacy, \(\epsilon\)-LDP). A randomized algorithm \(\mathcal{A}\) satisfies \(\epsilon\)-local differential privacy (or local privacy), if and only if for (1) any pair of input \(t_i, t_j \in \mathcal{T}\), and (2) any possible output \(O\) of \(\mathcal{A}\), we have:

\[
\Pr[\mathcal{A}(t_i) = O] \leq e^\epsilon \cdot \Pr[\mathcal{A}(t_j) = O].
\]

Two vitally important properties of differential privacy are sequential composability [24] and post-processing [9].

Lemma 2.1. (Sequential composability). Given \(m\) randomized algorithms \(\mathcal{A}_i(1 \leq i \leq m)\), each of which satisfies \(\epsilon_i\)-LDP. Then the sequential application of \(\mathcal{A}_i\) collectively provides \((\sum_{i=1}^{m} \epsilon_i)\)-LDP.

Lemma 2.2. (Post-processing). For any method \(\phi\) which works on the output of an algorithm \(\mathcal{A}\) that satisfies \(\epsilon\)-LDP without accessing the raw data, the procedure \(\phi(\mathcal{A}(\cdot))\) remains \(\epsilon\)-LDP.

2.2 Frequency Oracles with LDP

A frequency oracle (FO) protocol in the local setting enables the analyst to estimate frequency of any given value \(x \in \mathcal{X}\) from all sanitized data received from users. In [37], two effective protocols, generalized random response (GRR) and optimized local hash (OLH) were proposed to estimate frequencies with a large domain size \(d = |\mathcal{X}|\).

Generalized Random Response (GRR) [37]: The GRR protocol makes each user answer correctly \(y = x\) with probability \(p = \frac{e^\epsilon}{e^\epsilon + d - 1}\), and answer wrongly \(y \neq x\) with probability \(q = 1 - p = \frac{d - e^\epsilon}{e^\epsilon + d - 1}\). Specially, it turns out that the fundamental randomized response (RR)[40] protocol is the special case when \(d = 2\) and achieves the best performance [16, 37]. The shortage of GRR is that the estimated variance is linear with \(d\):

\[
\text{Var}[\tilde{f}_{\text{grr}}(t)] = n \cdot \frac{d - 2 + e^\epsilon}{(e^\epsilon - 1)^2}.
\]

Optimized Local Hashing (OLH) [37]: The OLH protocol applies a hash function to map each input value into a value in \([g]\), where \(g \geq 2\) and \(g \ll d\). Then the GRR protocol is used to perturb the hash values in the domain \([g]\). In [37], the optimal choice of the parameter \(g\) is shown to be \([e^\epsilon + 1]\), which leads to the minimum variance.

Typically, let \(\mathbb{H}\) be a universal hash function family, and \(H\) be a function randomly chosen from \(\mathbb{H}\) that outputs a value \(x = H(v) \in [g]\) for every \(v \in [d]\). The perturbing process is formalized as \(\text{Perturb}_{\text{OLH}}(\langle H, x \rangle) = \langle H, y \rangle\), where

\[
\forall i \in [g]\Pr[y = i] = \begin{cases} \frac{p}{e^\epsilon + g - 1}, & \text{if } x = i \\ \frac{q}{e^\epsilon + g - 1}, & \text{if } x \neq i. \end{cases}
\]

Then, the analyst counts the number of perturbed values that “supports” the input \(t\), denoted by \(\mathbb{1}_t\), and transforms it to the unbiased estimation

\[
\tilde{f}_{\text{OLH}}(t) = \frac{\mathbb{1}_t - n/g}{p - 1/g}.
\]

Accordingly, the variance of this estimation is

\[
\text{Var}[\tilde{f}_{\text{OLH}}(t)] = n \cdot \frac{4e^\epsilon}{(e^\epsilon - 1)^2}.
\]
We describe two aspects of it, namely, FP-tree construction and with such a tree. The FP-growth identifies long frequent patterns. This is challenging due to the complicated transactional data as the number of transactions containing \( X \) \( \subseteq X \). The frequency of any itemset \( f(\cdot, \hat{f}(\cdot) \) the actual and estimated frequency \( \mathcal{O}(X) \) the guessing frequency of \( X \) \( S' \) the frequent items set \( \hat{N} \) the noisy FP-tree \( M \) the maximum level of the FP-tree \( \dagger \) the dummy value \( \mathcal{P} \) the set of frequent itemsets identified frequent itemsets. In this paper, we focus on discovering the top-\( k \) itemsets with the highest frequencies. Table 1 lists the main notations used in this paper.

### 3.2 The SVSM Solution

As far as we know, the state-of-the-art work to address the FIM task under LDP is the SVSM (Set-Value itemSet Mining) protocol[39]. Particularly, SVSM mined top-\( k \) itemsets based on the \( k \) most frequent items obtained by the SVIM protocol[39]. More details are as follows.

**Set-Value Item Mining (SVIM)**: The SVIM protocol focuses on discovering the \( k \) most frequent items. It is in fact the PSFO protocol with the same problem setting as the LDPMiner[28]. SVIM divides all users into three mutually disjoint groups \( G_1, G_2 \) and \( G_3 \), and has three steps as follows.

**Step 1. Prune domain \( G_1 \).** Each user applies the OLH protocol to perturb one item randomly sampled from its input, i.e., \( PS_{i=1}(t) \). Then, the analyst estimates the frequency of each value in the original domain \( X \) and selects \( 2k \) items with the highest estimated frequencies as pruned domain \( S \). The analyst broadcasts \( S \) to all users, who prune their transactions by intersecting them with domain \( S \).

**Step 2. Size estimation \( G_2 \).** Since each user possesses at most \( L \) items when using padding and sampling (PS) protocol [28], the selection of an appropriate \( L \) is crucial. The basic strategy is to collect the length distribution of users and select a suitable \( L \) in a private way. Specifically, each user \( i \)'s input is the number of items in its pruned transaction, i.e., \( |t_i \cap S| \), and then given a fraction \( \tau \), \( L \) is computed as the smallest \( l \in \{1, 2, \ldots, 2k\} \) that satisfies

\[
\sum_{j=1}^{2k} f(j) < \tau
\]

For example, the 90th percentile length is the \( l \) value with \( \tau = 0.9 \).

**Step 3. Frequency estimation \( G_3 \).** Once given \( S \) and \( L \), the PSFO protocol is applied to precisely estimate the frequencies of items in the pruned domain \( S \). Firstly, each user \( i \) inputs one item randomly sampled from its pruned transaction padded to length \( L \), i.e., \( PS_{i=L}(t_i \cap S) \). Then, the item frequencies are estimated according to PSFO, and the \( k \) most frequent items are selected.

SVIM cannot be used directly to mine itemsets, since the exponential growth of potential itemsets would incur to much noise in

### 3.3 Problem Definition

In this paper, we focus on the task of discovering top-\( k \) frequent itemsets over transactional (or set-valued) data in the context of LDP, where each user’s input is a set of items, i.e., a transaction. This is challenging due to the complicated transactional data as well as the exponentially growing potential itemsets. Formally, let \( X = \{x_1, x_2, \ldots, x_d\} \) be the domain of \( d \) distinct items. An itemset \( X \) is a subset of \( X \), i.e., \( X \subseteq X \). Suppose there are \( n \) users, the transaction of \( i \)th user is \( t_i \) (\( i \in \{1, n\} \)) and \( T = \{t_1, t_2, \ldots, t_n\} \) denotes the whole transactional database. The frequency of any itemset \( X \) is the number of transactions containing \( X \) in \( T \). That is,

\[
f(X) = |\{t_i | X \subseteq t_i; t_i \in T\}|
\]

Specifically, the untrusted analyst wants to discover the itemsets that occur most frequently. To control the size of output, it gives a minimum frequency threshold \( \delta \) to output all itemsets whose frequency exceeds \( \delta \) or a positive number \( k \) to output the \( k \) most
The overview of LDP-FPMiner is depicted by Figure 2. It has three steps: First, a set of frequent items is identified with the SVIM protocol. Second, OLH protocol is applied to approximate the maximum number of frequent items that users hold, that is the height of the FP-tree. Third, it constructs a noisy FP-tree with LDP in breadth-first order, and mines it for frequent itemsets by the FP-growth algorithm [14]. The overall procedure is presented in Algorithm 1.

| Step | Description |
|------|-------------|
| 1    | SVIM protocol to find frequent items |
| 2    | OLH protocol to approximate the maximum number of frequent items |
| 3    | LDP-FPMiner to mine frequent itemsets |

Algorithm 1: LDP-FPMiner ($T$, $X$, $k$, $\epsilon$)

1. Randomly divide $T$ into three groups $G_1$, $G_2$, $G_3$.
2. Find Frequent Items:
   a. Collect the items set $S' \leftarrow$ SVIM($G_1$, $k$, $\epsilon$);
3. Find Tree Height:
   a. Each user in $G_2$ perturbs the number of frequent items it holds with OLH($\epsilon$);
4. Compute the 80th percentile length $M$ by (4);
5. Find Frequent Itemsets:
   a. $\mathcal{N} \leftarrow$ ConstructNoisyTree($G_3$, $S'$, $M$, $\epsilon$);
6. Mine $\mathcal{N}$ and release the $k$ itemsets set $\mathcal{P}$;
7. return $\mathcal{P}$
As described in Section 3.2, the SVSM protocol uses the PSFO protocol as a building block and constructs exponentially growing candidate itemsets. In this section, by leveraging the FP-tree approach, we aim to mine frequent itemsets under LDP without a costly candidate generation process. Moreover, by making use of the FP-tree structure, we want to reduce the scale of noise added, and efficiently identify frequent itemsets with a high accuracy. However, there are three challenges as follows.

The first challenge is that in LDP setting the analyst must construct an FP-tree over sanitized data. There is no raw user data available to calculate accurate frequencies. Particularly, the original FP-growth [14] constructs a non-privacy FP-tree by scanning raw transactions and updating nodes in the depth-first order. But this cannot be implemented under LDP since the frequencies of patterns are not directly available. To overcome this, we construct a noisy FP-tree in the breadth-first order based on the fundamental FO protocol (e.g. OLH). Given a maximum level \( M \) collected privately (explained in Section 4.2), our construction algorithm queries the users about pattern frequencies for each level, and then constructs the noisy FP-tree level by level.

The second challenge is that the number of patterns generated at an FP-tree level is exponentially explosive. Querying the users about such explosive number of pattern frequencies will soon become infeasible, and severely degrade the accuracy due to a large amount of noise injected. To address this issue, we cut down the number of candidate patterns at each level to no more than \( \varepsilon k \), and expand only these patterns in the FP-tree. Therefore, we design the function CutDownCandidate (Algorithm 3 in Section 4.3) to construct a pony-size candidate set of nodes that is likely to be frequent.

Finally, how to optimize a noisy FP-tree based on the tree structure is challenging. There are frequency consistency constraints between FP-tree nodes that can be used. For example, the sum of the counts of children nodes should be equal to or less than the count of the parent node. However, since we cut down a part of nodes during construction and thus the resulted FP-tree is not complete, whether these constraints can be used should be carefully examined. Additionally, we also optimize an noisy FP-tree by using guessing frequencies to adjust the frequency evaluation in FP-tree construction and mining.

4.2 Constructing a Noisy FP-tree

In the non-privacy setting, FP-trees are originally constructed in depth-first order [14]. In the LDP setting, since pattern frequencies are not directly available and should be queried privately, the depth-first order requires too many frequency queries, and would quickly consume the privacy budget. Lee et al.[22] also built noisy FP-trees to derive itemset frequencies in the centralized differential privacy (CDP) setting, where raw data is accessible. To the best of our knowledge, we are the first to propose an approach to constructing a FP-tree with LDP.

Specifically, we construct locally differentially private FP-trees in the breadth-first order. Let \( M \) be the maximum level of the tree, and \( x_1 > x_2 > \cdots > x_k \) be \( k \) frequent items in frequency descending order, which consist a set \( S' \). Our construction method has the following two phases:

Phase I: Preprocessing. All users prune their transactions, remain only the frequent items and rearrange them in the frequency descending order. For instance, Fig. 1 shows the five preprocessed transactions when the frequent items are rearranged into the frequency descending order \( c > f > a > b > p \).

Notably, after preprocessing, massive non-frequent items are pruned. This significantly cuts down the number of candidate itemsets, and thus improves the performance. The underlying basis is the Apriori property[2]: only if the length-\( \alpha \) itemset is frequent are its length-(\( \alpha +1 \)) supersets likely to be frequent.

Phase II: Constructing noisy FP-tree. We construct a noisy FP-tree with LDP in breadth-first order as described in Algorithm 2.

First, initialization is done as follows. (1) All users are divided into \( M \) equal groups (line 2), each of which is used for frequency query at a level of the FP-tree. (2) Each user prunes her transaction in term of the set of frequent items \( S' \), and sorts the frequent items properly for the later FP-tree construction (line 3). (3) The noisy FP-tree \( \mathcal{N} \) is initialized with a valid root holding a count \( n_0 \) (line 4), and the root can be regarded the 0-th level of the tree having a prefix candidate set \( C_0 \).

Then, the noisy FP-tree is constructed in the breadth-first order as below. (1) For each level \( l \), the nodes at this level are added and their corresponding prefixes are constructed as the candidate set \( C_l \) (line 8-16). Subsequently, the CutdownCandidate (line 17) is invoked to cut down \( C_l \) into small size \( \varepsilon k \) (explained in Section 4.3). (2) For each level \( l \), the users in the corresponding group perturb their inputs in term of the prefix candidate set \( C_l \) with the OLH protocol, and the analyst computes an estimate count \( \hat{f}(v) \) for each node \( v \) (line 19-22). The nodes with negative estimated counts are updated with 0 counts (line 23). The algorithm repeats level by level until it reaches the maximum level \( M \). Finally, the algorithm returns the noisy FP-tree \( \mathcal{N} \).

In the following, we remark on the main points of the noisy FP-tree construction algorithm ConstructNoisyTree.

- A node \( v \) in the noisy FP-tree has two fields: \( v.item \) and \( v.count \), where \( v.item \) denotes the indicated item of node \( v \) and \( v.count \) represents the count of times its prefix is appears in database, respectively. For example, in Fig. 1, the node \( v_0 \) indicates the item \( f \) (shown in shaded box) and \( v_0.count = 3 \) means the prefix \((c,f)\) appears 3 times, i.e., users T01,T02 and T05 includes this prefix in their transactions.
- Both \( S' \) and \( M \) are obtained with \( \varepsilon \)-LDP. Meanwhile, since we construct \( \mathcal{N} \) in breadth-first order, each level of the tree is completely dependent on the previous level (line 8-16), which is collected with LDP (line 19-22). Therefore, the noisy FP-tree \( \mathcal{N} \) does not disclose any privacy of the specific transaction.
- The input of each user at level \( l \) (\( 1 \leq l \leq M \)) (line 20) is a prefix in \( C_l \) or the dummy value \( \dagger \) (if her first \( l \) items does not exist in \( C_l \)). We apply OLH protocol with the finite domain \( C_l \cup \dagger \) to gather information. For example, when \( l = 3 \) and \( C_3 = \{p_{x_1},p_{x_2},p_{x_3},p_{x_4}\} \) in Fig. 1, the input of user T01 is \( p_{x_1} = (c,f,a) \), which is her first three preprocessed items, while that of T03 is the dummy value \( \dagger \).
Algorithm 2 ConstructNoisyTree($G_2$, $S'$, $M$, $\epsilon$)

1: // Initialize:
2: Randomly divide users into $M$ groups $g_1, g_2, ..., g_M$ of the same size $n_g = \lceil \frac{|G_2|}{M} \rceil$;
3: Each user prune her items not in $S'$ and rearrange left frequent items in frequency descending order;
4: Initialize tree $\hat{N}$ with a root $\hat{v}_r$, and set $\hat{v}_r$.count = $n_g$;
5: Mark $\hat{v}_r$ as valid, and set $C_0 = \{\hat{v}_r\}$;
6: for $l = 1$ to $M$
7: // Generate Candidates:
8: while there is a valid $v \in C_{l-1}$ and $v$.count > 0 do
9: Initialize a candidate prefix set $C_l = \emptyset$;
10: Mark $v$ as invalid;
11: for each item $x \in S'$ and $v$.item > $x$.item do
12: Add a child $\hat{v}_c$ of $v$ with item $x$ and count 0;
13: Mark $\hat{v}_c$ as valid and obtain its prefix $\hat{p}_\hat{v}_c$;
14: $C_l \leftarrow C_l \cup \hat{p}_\hat{v}_c$;
15: end for
16: end while
17: $C_l \leftarrow$ CutdownCandidate($S'$, $C_l$, $\xi$);
18: // Query Frequencies:
19: for each user in group $g_l$ do
20: Perturb her input (i.e., first $l$ items) with OLH;
21: end for
22: Collect the estimated count $\tilde{f}(v)$ of each node $v$ at level $l$ using the domain $C_l \cup \hat{v}$;
23: Update nodes by converting all negative counts to 0;
24: end for
25: return The noisy FP-tree $\hat{N}$.

- We cut down the size of the domain set (line 17) as well as filter out the nodes with negative counts (line 23) on each iteration to improve accuracy, which will be explained in Section 4.3.
- We randomly divide users into $M$ equal-sized groups and users in each group use the full privacy budget $\epsilon$. Meanwhile, the estimated count of node $v$ should multiply $M$ to correct the underestimation. It has turned out that the overall process achieves better accuracy and satisfies $\epsilon$-LDP as well.

4.3 Cutting Down Candidate Prefix Set

Recall that, during the noisy tree construction, the size of initial candidate prefixes set $C_l$ at each level $l$ soon becomes very large (e.g., thousands or more). This would degrade the accuracy greatly. According to Algorithm 2, this size is expanded once at each level, with a maximal factor $k$. Our idea is to cut down the size at each level by pruning candidate prefixes when the size exceeds a certain value (i.e., $k \xi$). Although this cutdown may cause information loss and lead to underestimation, it overcomes the size expansion issue and works experimentally well.

Another reason to cutdown candidate prefixes is that there are many redundant prefixes. For example, at the level-3 in Fig. 1, the set $C_3$ is initially $\{\hat{p}_{x_1}, \hat{p}_{x_2}, \hat{p}_{x_3}, \hat{p}_{x_4}, \hat{p}_{x_5}, \hat{p}_{x_6}, \hat{p}_{x_7}, \hat{p}_{x_8}\}$. However, as the counts are collected, half of the prefixes (i.e., $\hat{p}_{x_5}, \hat{p}_{x_6}, \hat{p}_{x_7}, \hat{p}_{x_8}$) should be pruned. Thus, if we can prune these meaningless nodes in advance, then we can effectively reduce the noise added.

Specifically, inspired by SVSM[39], we limit the candidate prefix set within a fixed size $\xi \cdot k$ in term of temporal guessing frequencies, as shown in Algorithm 3. Here, $\xi$ is an adjustable parameter. The temporal guessing frequency $T(\hat{p}_v)$ of each candidate prefix $\hat{p}_v \in C$ is computed as follows.

$$T(\hat{p}_v) = \hat{f}(\hat{p}_{\text{Parent}(v)}) \cdot \hat{f}(v)$$ (6)

where $\hat{f}(\hat{p}_{\text{Parent}(v)})$ denotes the estimated frequency of the prefix of $v$’s parent node, and $\hat{f}(v)$ is a probability computed based on the normalized estimated frequencies of frequent items. Assuming that the items of node $v$ and its parent node are the $(i+j)$th and $i$th frequent items $x_{i+j}$ and $x_i$, respectively, then $\hat{f}(v)$ can be computed by Eq. (7).

$$\hat{f}(v) = \hat{f}(x_{i+j}) \prod_{t=1}^{j} (1 - \hat{f}(x_{i+t}))$$ (7)

Note that the computation of Eq. (7) is due to the FP-tree structure. A node $v$ appears as a child of its parent only if all frequent items ranked between its parent node and itself do not appear.

Then the $\xi k$ candidate prefixes with highest temporal guessing frequencies are selected to form the pony-size set $C'$. The intuition is that a prefix with a high estimated frequency is more likely to be split with frequent items. Note that the temporal frequency of each candidate prefix depends only on the frequencies queried previously and can be computed within $O(1)$ time.

4.4 Mining a Noisy FP-tree

So far, we have obtained a noisy FP-tree satisfying LDP. The final step for this scheme is to mine this noisy FP-tree for frequent itemsets. This mining procedure is completely a post-process, and it is the same as that of no-privacy setting. Namely, it follows the original FP-growth algorithm, which has been outlined in Section 2.3, and more details can be found in [14].

4.5 Theoretical Analysis

4.5.1 Computational Complexity. The computational complexity of constructing a noisy FP-tree is given in Theorem 4.1.

Theorem 4.1. The computational complexity of constructing a noisy FP-tree for LDP-FPMiner is $O(k^3)$. 

Algorithm 3 CutdownCandidate($S'$, $C$, $\xi$)

1: Initialize $C'$;
2: if $|C| > \xi \cdot k$ then
3: for each candidate prefix $\hat{p}_v \in C$ do
4: Compute the temporal guessing frequency $T(\hat{p}_v)$;
5: end for
6: Construct $C'$ by selecting the $\xi k$ prefixes with highest temporal guessing frequencies;
7: else
8: $C' \leftarrow C$;
9: end if
10: return $C'$. 

4.5.2 Estimation Accuracy. The estimation accuracy of a prefix frequency is given in Theorem 4.2.

**Theorem 4.2.** For any node \( v \) in the FP-tree, let \( \tilde{f}(v) \) be the estimated count of its prefix \( p_v \), collected privately in Algorithm 2, and \( f(v) \) be the actual count. When the number of users participating at each level is \( n_g \), and there are \( M \) levels (excluding the 0-level, i.e., root node), with at least 1 - \( \beta \) probability, we have

\[
\max |\tilde{f}(v) - f(v)| = O\left(\frac{\sqrt{n_g \log(1/\beta)}}{\epsilon}\right)
\]  

Proof. According to (2),

\[
|\tilde{f}(v) - f(v)| = \left|\sum_{j=1}^{n_g} \frac{\tilde{g}^j - g^j}{p - 1/g} - \sum_{j=1}^{n_g} \frac{g^j - 1/g}{p - 1/g}\right|
\]

\[
= \sum_{j=1}^{n_g} \frac{\tilde{g}^j - g^j}{p - 1/g}.
\]

where \( g = e^{\epsilon} + 1 \) and \( p = \frac{e^{\epsilon} - 1}{e^{\epsilon} + 1} = 1/2 \) are the optimal probability setting in [37]. Random variables \( \tilde{g}^j \) and \( g^j \) denote the estimated and actual count of the \( j \)th user, respectively. Accordingly, we have

\[
Var[\tilde{g}^j - g^j] = O\left(\frac{1}{\epsilon^2}\right).
\]

By Bernstein’s inequality,

\[
Pr\left[|\tilde{f}(v) - f(v)| \geq \lambda\right] = Pr\left[\lambda \leq \sum_{j=1}^{n_g} \frac{\tilde{g}^j - g^j}{p - 1/g}\right]
\]

\[
\leq 2 \times \exp\left\{-\frac{\lambda^2}{\sum_{j=1}^{n_g} Var[\frac{\tilde{g}^j - g^j}{p - 1/g}] + \frac{3}{2} \cdot 2(e^{\epsilon} + 1)^2}\right\}
\]

\[
\leq 2 \times \exp\left\{-\frac{\lambda^2}{2} \cdot \frac{1}{n_g} \cdot \frac{O\left(\frac{1}{\epsilon^2}\right)}{\frac{1}{\epsilon^2} + 2}\right\}
\]

Therefore, \( |\tilde{f}(v) - f(v)| < \lambda \) holds with at least 1 - \( \beta \) probability while \( \lambda = O\left(\frac{\sqrt{n_g \log(1/\beta)}}{\epsilon}\right)\). □

4.5.3 Local Differential Privacy. The local differential privacy of LDP-FPMiner is given in Theorem 4.3 and Theorem 4.4.

**Theorem 4.3.** The LDP-FPMiner (i.e., Algorithm 1) satisfies \( \epsilon \)-LDP.

Proof. LDP-FPMiner divides all users into three mutually disjoint groups, that is, group \( G_1 \) for finding frequent items \( S' \), \( G_2 \) for computing maximum length \( M \) and \( G_3 \) for constructing a noisy FP-tree \( \tilde{N} \). LDP-FPMiner provides \( \epsilon \)-LDP protection for the first two groups of users obviously, and for the third group due to Theorem 4.4. The mining process over the noisy FP-tree does not consume any privacy budget due to the post-processing property. Thus the whole process of LDP-FPMiner satisfies \( \epsilon \)-LDP. □

5 OPTIMIZATIONS

We optimize LDP-FPMiner using frequency consistency between estimated frequencies and guessing frequencies, and also using frequency consistency in term of the FP-tree structure. These optimizations satisfy the post-processing property of local differential privacy (Lemma 2.2), and thus the local differential privacy is certainly preserved.

5.1 Guessing Probability

We use the estimated frequencies of frequent items to compute guessing probabilities for all candidate prefixes in a noisy FP-tree. As previous, let \( x_1 > x_2 > \cdots > x_k \) be \( k \) frequent items in frequency descending order. For a candidate prefix \( p = x_1 x_2 \cdots x_i \), with \( 1 \leq i_1 < i_2 < \cdots < i_k \leq k \), we can compute its guessing probability by Eq. (9).

\[
\tilde{f}(p) = \tilde{f}(x_{i_1}) \prod_{u=i_1+1}^{i_2} \tilde{f}(x_u) \prod_{a=1}^{u-1} \left(1 - \tilde{f}(x_0)\right)
\]

The guessing probability of a prefix can be regarded as the estimated occurrence probability of the prefix. For example, in Figure 3, we can compute \( \tilde{f}(x_1) = \tilde{f}(x_1), \tilde{f}(x_2) = (1 - \tilde{f}(x_1))\tilde{f}(x_2), \tilde{f}(x_3) = (1 - \tilde{f}(x_1))\tilde{f}(x_3), \tilde{f}(x_1; x_2) = \tilde{f}(x_1)\tilde{f}(x_2), \) and \( \tilde{f}(x_1; x_3) = \tilde{f}(x_1)\tilde{f}(x_3)\) (excluding the root node). For each iteration, algorithm 2 allocates \( n_g \) users to gather information of \( V_l \). Due to the fact that each user applies OLH to perturb her input only once on the iteration participated with full privacy budget \( \epsilon \), thus every user is protected by \( \epsilon \)-LDP and the overall process of constructing a noisy FP-tree satisfies \( \epsilon \)-LDP. □

5.2 Two-folded Weighted Combination

Since the space of itemsets is large, most of the itemsets have low frequencies. When these frequencies are queried with LDP and their
We optimize a noisy FP-tree by the tree structure constraint that weighted frequency we call prefix weighted combination (PWC), alleviates the bias of weighted frequency with their maximum. Let \((IWC) greatly improves the performance of our scheme.

\[ \omega \]

The noisy counts by constrained inference [15, 27] with appropriate all children of node \(N\) children nodes may be cut down. Specifically, given a node \(v\) in \(N\), denoting \(\hat{f}(\cdot)\) as the noisy count function, \(\text{succ}(v)\) as the set of all children of node \(v\), and \(b\) as the number of children, we correct the noisy counts by constrained inference [15, 27] with appropriate ratio adjustments by Eqs. (10) and (11).

\[
\begin{align*}
\hat{f}^*(v) &= \frac{b^2 - b}{b^2 - 1} \hat{f}(v) + \frac{b - 1}{b^2 - 1} \cdot \frac{1}{\theta} \sum_{v_c \in \text{succ}(v)} \hat{f}(v_c) \\
\hat{f}^*(v_c) &= \hat{f}(v_c) + \frac{1}{b} (\hat{f}^*(v) \cdot \theta - \sum_{v_c \in \text{succ}(v)} \hat{f}(v_c))
\end{align*}
\]

Here, \(\theta\) is the ratio of the sum of guessing probabilities of the remaining children nodes to the guessing probability of the parent node. \(\theta\) is computed by Eq. (12).

\[
\theta = \frac{1}{\hat{f}(p(v))} \sum_{v_c \in \text{succ}(v)} \hat{f}(p(v_c))
\]

where \(p(v), p(v_c)\) represent the prefixes of the parent node \(v\) and the children nodes \(v_c\). Note that, when

\[
\hat{f}(v) < \sum_{v_c \in \text{succ}(v)} \hat{f}(v_c),
\]

we simply set \(\theta = 1\) due to the probable large amount of noise in children nodes. Since when \(\theta\) is small, the noise may impact the corrector results greatly, in the experiments, we set a threshold value \(\theta_0\) (e.g., \(\theta_0 = 0.3\)) and do constrained inferences conditionally only when \(\theta \geq \theta_0\). This process can be repeated several times to get a better result. The times of repetition can be determined experimentally, and in our experiments, we find 5-10 times of repetition seem sufficient.

### 5.4 Negative-positive Balance

When querying prefix frequencies from users in LDP setting during the noisy FP-tree construction, the estimated frequencies may be positive or negative. We propose a post-processing method called negative-positive balance (NPB) to reduce the noise added effectively.

The NPB method works as follows. First, the analyst calculates the sum of absolute values of all negative frequencies, and set all negative frequencies to 0; Then, the analyst randomly picks a positive frequency and subtracts 1 from it, and this repeats until the value subtracted from positive frequencies is equal to the absolute sum of negative frequencies.

The intuition of this method is that frequencies are originally non-negative, and if an estimated frequency is negative, it is certainly underestimated. Thus, setting negative frequencies to 0 is on the right way to reduce the noise. However, since we add some value to the overall frequencies, and we need to subtract the same value from them to maintain unbiasedness. Moreover, subtracting values from positive frequencies makes about half of them reduce the noise, since roughly half of the positive frequencies should be overestimated in probability. Therefore, most of frequencies are on their right way to reduce the noise, and the NP balance would reduce the noise overall.

### 6 EXPERIMENTS

In this section, we experimentally evaluate the performance of LDP-FPMiner, and compare it with the state-of-the-art protocol SVSM. All experiments are performed on an Intel Core i5-7500 3.4GHz CPU with 16GB RAM.
We implement both LDP-FPMiner and SVSM in Python 3.8. Specifically, LDP-FPMiner is implemented in a way to be 50% more memory efficient. Specifically, we evaluate the NCR and Var metrics of discovering length-$\alpha$ itemsets (where $\alpha \geq 2$) over three datasets when only $\epsilon$ varies and when only $k$ varies (the length-1 itemsets, that is, the $k$ frequent items, have been collected privately through the same protocol). Besides, for LDP-FPMiner, we present the performance of a key optimization, i.e., the itemset weighted combination.

### 6.2 Overall Results

Now we compare the performance between LDP-FPMiner and SVSM. Specifically, we evaluate the NCR and Var metrics of discovering length-$\alpha$ itemsets (where $\alpha \geq 2$) over three datasets when only $\epsilon$ varies and when only $k$ varies (the length-1 itemsets, that is, the $k$ frequent items, have been collected privately through the same protocol). The results on three datasets when only $\epsilon$ varies are presented in Figure 4 (the NCR metric on the first line, and the Var metric on the second line). In almost all settings, the LDP-FPMiner has the significantly higher NCR values than SVSM. This means that LDP-FPMiner is more accurate than SVSM. The advantage is more obvious when the $\epsilon$ values are small. As $\epsilon$ increases, the advantage gradually decreases. Similarly, the LDP-FPMiner has much smaller Var values than SVSM when the $\epsilon$ is small, which means that the noise added by LDP-FPMiner is effectively reduced, and the advantage decreases as the $\epsilon$ becomes large. Surprisingly, even the curves of LDP-FPMiner when $\epsilon = 100$ perform better than those of SVSM when $\epsilon = 50$. For different datasets, it appears that both schemes work best for the Synthetic dataset. Despite of this, LDP-FPMiner has still an obvious advantage over SVSM over this dataset. In short, LDP-FPMiner introduces much less noise, and thus outperforms SVSM significantly when $\epsilon$ is small.

#### 6.2.1 The impact of $\epsilon$

The results on three datasets when only $\epsilon$ varies are presented in Figure 4 (the NCR metric on the first line, and the Var metric on the second line). In almost all settings, the LDP-FPMiner has the significantly higher NCR values than SVSM. This means that LDP-FPMiner is more accurate than SVSM. The advantage is more obvious when the $\epsilon$ values are small. As $\epsilon$ increases, the advantage gradually decreases. Similarly, the LDP-FPMiner has much smaller Var values than SVSM when the $\epsilon$ is small, which means that the noise added by LDP-FPMiner is effectively reduced, and the advantage decreases as the $\epsilon$ becomes large. Surprisingly, even the curves of LDP-FPMiner when $\epsilon = 100$ perform better than those of SVSM when $\epsilon = 50$. For different datasets, it appears that both schemes work best for the Synthetic dataset. Despite of this, LDP-FPMiner has still an obvious advantage over SVSM over this dataset. In short, LDP-FPMiner introduces much less noise, and thus outperforms SVSM significantly when $\epsilon$ is small.

#### 6.2.2 The impact of $k$

Both the results of NCR (on the first line) and Var (on the second line) are presented in Figure 5 when setting $\epsilon = 1$ and 2 separately (the privacy settings in deployed Apple protocol[30]). In all the settings where $\epsilon = 1$, LDP-FPMiner has the significantly higher NCR values than SVSM. In the setting where $\epsilon = 2$, LDP-FPMiner has higher NCR values than SVSM for both Synthetic and Kosarak datasets, but has slightly lower NCR values for BMS-POS dataset at some points. It seems that when $\epsilon$ is big enough, the SVSM scheme is still very effective, and comparable to the LDP-FPMiner. However, when $\epsilon = 1$ for all the datasets, LDP-FPMiner greatly outperforms SVSM. In the view of metric Var, LDP-FPMiner has lower Var values than SVSM in almost all the cases. It is surprising that even when LDP-FPMiner has slightly lower NCR values than SVSM, the corresponding Var values are still lower than SVSM. This again indicates that LDP-FPMiner is very effective for reducing the noise.

In conclusion, LDP-FPMiner outperforms the SVSM in the top-$k$ task of FIM in the context of LDP. More specifically, in the case when $\epsilon$ is small (e.g., smaller than 2), it achieves higher score of itemsets identified as well lower noise injected on large domain datasets.

#### 6.2.3 The impact of $\omega$

The effectiveness of the itemset weighted combination (explained in Section 5) over three datasets when fixing $k = 100$ are presented in Figure 6. We use the version with all optimizations to illustrate the effectiveness. It turns out that this optimization effectively improves the performance of LDP-FPMiner, where the original result is when $\omega = 1$. As shown in Figure 6, the selection of $\omega$ should balance between accuracy and error, and we give the reference selection that used in this paper as shown in Table 2.

---

### Table 2: Dataset description. The numbers of transactions $|T|$ and the dimensions of transactions $|X|$ of the three datasets are listed. The weighted parameters $\omega'$ and $\omega$ values for the datasets used in the experiments are given.

| Dataset       | $|T|$   | $|X|$   | $\omega'$ | $\omega$ |
|---------------|--------|--------|-----------|----------|
| Synthetic     | 969,223| 4,411  | 0.9       | 0.7      |
| Kosarak       | 990,002| 41,270 | 0.9       | 0.7      |
| BMS-POS       | 515,597| 1,658  | 0.7       | 0.5      |

---
6.3 Optimizations

In this subsection, we first compare LDP-FPMiner only applying a single optimization with the original version (without any optimizations). Then, we compare LDP-FPMiner simultaneously applying two or more optimizations with the original version. These experiments illustrate the effectiveness of single or combined optimizations.

The optimizations are introduced in Section 7?, and for convenience we list both their abbreviations and full names as below:

- PWC: prefix weighted combination,
- CCI: conditional constrained inference,
- NPB: negative-positive balance,
- IWC: itemset weighted combination.

These optimizations will be applied separately or simultaneously to LDP-FPMiner, and the BMS-POS dataset is used for evaluation.

6.3.1 Single Optimizations. We apply a single optimization to the original LDP-FPMiner each time, and show the effectiveness of each optimization separately. Figure 7 traces both NCR and Var values for each single optimization. We can see that both PWC and CCI optimizations are separately effective to improve the NCR values, namely the accuracy to identify frequent itemsets, but they cause higher Var values, namely the square error. On the contrary, the NPB causes slightly lower NCR values than the original, but it reduces the noise very effectively. The last optimization IWC works rather well, and it raises NCR values and reduces Var values, improving the both metrics greatly. In a word, these optimizations tend to be combined together to get a good performance with high NCR and low Var values.

6.3.2 Combined Optimizations. We gradually combine two or more optimizations together and apply them to LDP-FPMiner. Figure 8 illustrates the effectiveness of combined optimizations. We can see that the combination of PWC and CCI optimizations increases the NCR values, but also increases the Var values. However, when we combine PWC, CCI and NPB optimizations, it is surprising that the NCR values are further increased and the Var values are greatly reduced and become lower than those of the original version. Finally, we further combine all optimizations PWC, CCI, NPB and IWC together, and obtain the final result with even higher NCR values and lower Var values. It appears that the combination of optimizations magnifies the improvements. The underlying reason may be that all optimizations are on the right way to identify frequent itemsets and reduce frequency noise added, and they are complementary to each other. Additionally, for optimizations PWC and CCI, the improvement over NCR values gradually become more significant as \(k\) increases, while for optimizations NPB and IWC, the Var values are reduced more greatly as \(k\) increases. It may be because there are more room to improve when \(k\) is large.

7 RELATED WORK

Local differential privacy (LDP) has become more and more popular for data privacy preservation, and most of existing works focus on basic statistics (e.g., [10, 16–18, 37, 40]) to estimate mean values over numeric attributes or frequencies over categories. Besides, in
recent years, there are many more complicated statistical analyses (e.g. heavy hitters\cite{4, 6, 18, 34}, key-value collection\cite{12, 43}, multidimensional data\cite{35, 38, 41} and set-valued data\cite{28, 36, 39} analysis) are proposed using frequency estimation as a building block.

Due to the set property of the data, the transactional (or set-valued) data setting is more challenging even when one just tries to find heavy hitters, not mention discovering itemsets. In the particular LDP setting, Qin et al.\cite{28} propose the LDPminer that discover heavy hitters in two phases and leave FIM problem as an open problem. In \cite{36}, the set-valued data aggregation mechanism PrivSet is proposed with low computational overhead but does not work well when the domain is large. To the best of our knowledge, the state-of-the-art solution \cite{39} to FIM identified itemsets based on the PSFO protocol, and did not consider frequency consistency among itemsets. In this paper, we propose and optimize the FP-tree based approach by exploiting frequency consistency, and identify frequent itemsets effectively with high accuracy and low noise.

Besides, in the centralized differential privacy (CDP) setting, Bhaskar et al.\cite{3} propose an approach with the exponential mechanism as well as the Laplace mechanism to release top-$k$ itemsets of length not greater than predefined factor $m$. Li et al.\cite{23} define the $\theta$-basis set to improve the utility. The concurrent approach\cite{22} improves the trade-off between privacy and utility with smart truncating as well as double standards. Lee et al.\cite{44} identify top-$k$ itemsets and then construct a compact, differentially private FP-tree to
derive frequencies of itemsets. These works are quite different from ours for the raw data from users are available in the CDP setting.

8 CONCLUSION

In this paper, we study the problem of privacy-preserving frequent itemset mining, and discover $k$ most frequent itemsets from sensitive transactions with LDP. The state-of-the-art protocol SVSM mainly applies the idea of guessing frequencies to find the candidate itemsets and then further identifies the top-$k$ itemsets with frequency oracle protocol without considering frequency consistency. Different from this, we combine frequent pattern tree (FP-tree) method, frequency oracle protocol, and guessing frequencies to build and optimize a noisy FP-tree with LDP by exploiting frequency consistency among itemsets, and then mining this FP-tree to find the top-$k$ frequent itemsets. To the best of our knowledge, this is the first time that FP-tree is applied in LDP setting to mine frequent itemsets. The experimental results show that the proposed approach LDP-FPMiner outperforms the SVSM significantly.

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