Robot Playing Kendama with Model-Based and Model-Free Reinforcement Learning

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Abstract—Several model-based and model-free methods have been proposed for the robot trajectory learning task. Both approaches have their benefits and drawbacks. They can usually complement each other. Many research works are trying to integrate some model-based and model-free methods into one algorithm and perform well in simulators or quasi-static robot tasks. Difficulties still exist when algorithms are used in particular trajectory learning tasks. In this paper, we propose a robot trajectory learning framework for precise tasks with discontinuous dynamics and high speed. The trajectories learned from the human demonstration are optimized by DDP and PoWER successively. The framework is tested on the kendama manipulation task, which can also be difficult for humans to achieve. The results show that our approach can plan the trajectories to successfully complete the task.

I. INTRODUCTION

Reinforcement Learning (RL) has been applied for robot learning complex motor skills in continuous high-dimension space [5]. Many model-based algorithms like Differential Dynamic Programming (DDP) [10], and model-free algorithms like PoWER [6] and PI$^2$ [17] have been developed for such problems. Robots with these algorithms can handle the tasks like parking car [16], playing pool [13], playing hockey [1] and so on. It is recognized by the researchers that model-based algorithms are more reliable in trajectory learning but limited by the accuracy of the system model, whereas the model-free algorithms are good at handling the arbitrary dynamics in physical applications but usually have lower sample efficiency [1]. Naturally many combinations of these two sets of methods have been studied which we will discuss in detail in Section II.

However, most of the existing methods are applicable only in the robot simulators or some quasi-static robot tasks. There are robot tasks with the characters of dynamic discontinuity, high speed, and high precision at the same time, for instance, the kendama as shown in Fig. 1. When we use a typical model-free method to plan the trajectory for kendama, the problems of large acceleration, low sample efficiency [9] and local optimum [8] will occur. On the contrary, a typical model-based method may not perform very precisely for the physical robot tasks due to the model inaccuracy [3].

Therefore, in this paper, we propose a practical framework for the robot manipulator to learn the trajectory for the kendama task, as well as tasks with similar properties. Firstly, an initial trajectory is obtained by the human demonstration. Secondly, a simple pendulum dynamic model [15] is used so as to apply the control-limited DDP [16] to produce a sub-optimal trajectory in the simulator. Thirdly, the modified PoWER [9] together with Dynamic Movement Primitive (DMP) [12] is applied to this sub-optimal trajectory to conduct the model-free exploration directly on the physical robot. It takes the robot only a few iterations to update the sub-optimal trajectory to complete the task. We find that our method can help the robot to complete the precise task which could be difficult for humans to achieve. The proposed framework avoids using complex integrated algorithms or dynamic model by simply applying the economic methods successively. In addition, the resultant trajectories are within the robot capability.

II. RELATED WORK

It is convenient to apply the model-free RL methods to a wide range of robot tasks since they can handle robot tasks with complex dynamics as well as the simple tasks. The model-free methods are also robust to mechanism or measurement errors. PoWER [6] and PI$^2$ [17] are two of the most popular model-free methods in the robot learning field. However, their sample efficiencies are low due to the sampling-based explorations. Furthermore, when they are used in robot tasks where high-speed motions are involved, the explorations may also potentially cause damage to the robot [9]. They are usually plagued by the problem of local optimum [8].

On the other hand, the model-based methods, such as DDP [10], take the robot dynamic model into account when
optimizing the trajectories. Many extensions on DDP have been proposed to make it more reliable for the robot trajectory learning. For example, Tassa [16] presents the control-limited DDP to ensure the generated trajectories to be within the robot dynamic capabilities. Xie [18] makes use of the Lagrangian dual expression of the cost function to consider the arbitrary nonlinear inequality constraints on both the state and the control input. This method can be applied to control the moment when the string loosens in our kendama task.

In this paper, we will like to explore the feasibility of complementary combination the model-free and model-based methods together for the task. We could use any model-based method at the first stage to improve the sample efficiency. Then, the model could be removed or approximated by a model representation method at the second stage. The robot can deal with the constraints and the environment changes better without adding more complex mathematics. Farshidian [3] successively applies DDP and PI \(^2\) to plan the motion for a ballbot Rezero. Our idea is similar to his work. Besides, there are also a number of works directly combining more individual methods [1], [2], [7], [11], [14].

However, most of these combinations are tested on either simulators or quasi-static tasks for the physical robots. Some algorithms have high computational expense because they are seeking a universal representation of the robot policy or task. In this paper, we propose a simple but practical structure to complete the robot trajectory learning for the kendama, a robot task with discontinuous dynamics, high-speed trajectory and high precision requirement.

### III. Methodology

This section describes the methodology involved in the framework to learn the robot trajectory for the kendama task. The short reviews on DDP and PoWER with smooth and efficient exploration will be provided. We also derive a simple dynamic model for the kendama manipulation, which is required in the DDP optimization and not used in the later PoWER learning.

#### A. Differential dynamic programming

DDP is an optimal control algorithm to solve a control sequence for a nonlinear system iteratively [10]. Each iteration consists of a backward pass and a forward pass. The backward pass computes an open-loop term \( \mathbf{k} \) and a feedback gain term \( \mathbf{K} \) based on a given cost function. The forward pass computes a new trajectory once a backward pass is completed. Consider the discrete-time dynamic model

\[
\mathbf{x}_{i+1} = \mathbf{f} (\mathbf{x}_i, \mathbf{u}_i),
\]

where \( \mathbf{x}_i \in \mathbb{R}^n \) is the state while \( \mathbf{u}_i \in \mathbb{R}^m \) is the control input at time step \( i \). The generic function \( \mathbf{f} \) governs the state transition given the current state and control input.

A complete trajectory \( \{ \mathbf{X}, \mathbf{U} \} \) is made from the input sequence \( \mathbf{U} = \{ \mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{N-1} \} \) and the corresponding state sequence \( \mathbf{X} = \{ \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N \} \) satisfying the dynamics \( \mathbf{f} \). The cost for the whole trajectory can be defined by

\[
J (\mathbf{X}, \mathbf{U}) = \sum_{i=1}^{N-1} \ell (\mathbf{x}_i, \mathbf{u}_i) + \ell_f (\mathbf{x}_N),
\]

which is the sum up of the running cost \( \ell \) and final cost \( \ell_f \).

The target of DDP is to find an optimal control input sequence \( \mathbf{U}^\ast \) which minimizes the above cost function. This optimization problem can be solved by dynamic programming since the optimality of the future input does not depend on the past states or control inputs. Therefore, we define the value function for the particular state at time step \( i \) by applying the optimal control input minimizing the cost-to-go

\[
V_i (\mathbf{x}) = \min_{\mathbf{u}} \sum_{j=i}^{N-1} \ell (\mathbf{x}_j, \mathbf{u}_j) + \ell_f (\mathbf{x}_N).
\]

The value of the final time is defined as \( V_N (\mathbf{x}) = \ell_f (\mathbf{x}) \).

To find out the optimal control input at the current time step, we define the one-step value function for any time step as

\[
V (\mathbf{x}) = \min_{\mathbf{u}} \left[ \ell (\mathbf{x}, \mathbf{u}) + V' (\mathbf{f} (\mathbf{x}, \mathbf{u})) \right],
\]

which is the Bellman equation. For the rest parts of the backward pass, we omit the time index \( i \) and use \( V' \) to express the value for the next time step. We then use \( Q (\delta \mathbf{x}, \delta \mathbf{u}) \) to denote \( \ell (\mathbf{x} + \delta \mathbf{x}, \mathbf{u} + \delta \mathbf{u}) + V' (\mathbf{f} (\mathbf{x} + \delta \mathbf{x}, \mathbf{u} + \delta \mathbf{u})) - \ell (\mathbf{x}, \mathbf{u}) - V' (\mathbf{f} (\mathbf{x}, \mathbf{u})) \) and further Taylor-expand it about zero until the second-order:

\[
Q (\delta \mathbf{x}, \delta \mathbf{u}) \approx \frac{1}{2} \begin{bmatrix} Q_x & Q_u & Q_{xx} & Q_{ux} & Q_{uu} \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}^T \delta \mathbf{u} \end{bmatrix}.
\]

The subscript denotes the derivative.

Therefore, the expansion of the coefficients in the expression can be used to conduct the backward pass, which are

\[
\begin{align*}
Q_x &= \ell_x + f'_x \ell' \\
Q_u &= \ell_u + f'_u \ell' \\
Q_{xx} &= \ell_{xx} + f'_{xx} \ell_x + V'_x \cdot f_{xx} \\
Q_{ux} &= \ell_{ux} + f'_{ux} \ell_x + V'_x \cdot f_{ux} \\
Q_{uu} &= \ell_{uu} + f'_{uu} \ell_x + V'_x \cdot f_{uu}.
\end{align*}
\]

The dots in the equations denote the inner product of a vector with a tensor. Then the optimal control modification can be expressed by minimizing the quadratic model:

\[
\delta \mathbf{u}^\ast = \arg \min_{\delta \mathbf{u}} Q (\delta \mathbf{x}, \delta \mathbf{u}) = \alpha \mathbf{k} + \mathbf{K} \delta \mathbf{x}.
\]

This is a locally-linear feedback policy with an open-loop term \( \mathbf{k} = -Q_{uu}^{-1} Q_u \) and a feedback gain \( \mathbf{K} = -Q_{uu}^{-1} Q_{ux} \). The scaling factor \( 0 < \alpha \leq 1 \) is created as the line-search and functions similarly to the learning rate in other model-based learning algorithms. In our paper, \( \alpha \) should be small considering the discontinuous dynamics.

The derivative of the current value function should also be computed for the recursion of the last time step. Plugging the \( \delta \mathbf{u}^\ast \) back to \( Q (\delta \mathbf{x}, \delta \mathbf{u}) \) and simplifying, we will have

\[
\begin{align*}
V_x &= Q_x - Q_{xx} Q_{uu}^{-1} Q_u \\
V_{xx} &= Q_{xx} - Q_{xx} Q_{uu}^{-1} Q_{ux}.
\end{align*}
\]
The value function is initialized with the boundary condition $V_N(x) = \ell_f(x)$ and the sequences of $k$ and $K$ are calculated backward until the beginning state.

After each backward pass, a new trajectory $\{\hat{X}, \hat{U}\}$ will be obtained by a forward pass

\[
\begin{align*}
\hat{x}_1 &= x_1 \\
\hat{u}_1 &= u_1 + \alpha k_i + K_i (\hat{x}_1 - x_i) \\
\hat{x}_{i+1} &= f(\hat{x}_i, \hat{u}_i),
\end{align*}
\]

where $i = 1, 2, ..., N - 1$.

Eventually, the backward and forward passes are repeated iteratively until the trajectory converges to the local optimum around the initial trajectory.

### B. Kendama dynamics

To make use of the model-based methods in trajectory learning for the kendama task, a dynamic model for the system is required. The whole process of the kendama task is discontinuous since it consists of two phases: the swing up and the catch. During the swing up phase, the system can be simply modeled as a pendulum movement \[15\] as the handle is rigidly attached to the end-effector of the robot manipulator while the ball is attached to the handle by an inelastic string. Once the tension on the string reduces to zero, we assume it will remain so far the rest of the motion. Then the handle will still be under the control of the robot but the ball will be under the projectile motion. Its translation is an oblique projectile. Its rotation is uniform with the same angular momentum as the moment when the string loosens.

![Diagram of kendama dynamics](image)

We simplify the problem to 2-d space by forcing the end-effector of the robot to move in the $y$-$z$ plane. The acceleration $u = [a_y, a_z]^T$ of the end-effector, as well as the handle, is taken as the control input, resulting in the translation of the handle. During the swing up phase, we consider the motion of the ball in the Handle frame, which is a non-inertial reference frame as it is accelerated in the Ground frame. Therefore, the $ma_y$ and $ma_z$ as shown in Fig. 2 will be treated as the translation inertial forces on the ball. The $m$ denotes the ball mass, $L$ denotes the string length, and $\theta$ denotes the angle that the string turns counterclockwise relative to its initial position. $\omega$ and $\beta$ denote the angular velocity and acceleration respectively. Besides, the ball is also affected by the string tension $T$ and the gravity $mg$. Since the ball acts in the Handle frame as a non-uniform circular motion, the dynamic equations can be written as

\[
\begin{align*}
T/m &= (a_z + g) \cos \theta - a_y \sin \theta + \omega^2 L \\
\beta L &= -(a_z + g) \sin \theta - a_y \cos \theta.
\end{align*}
\]

On the other hand, six quantities are selected to describe the state of the system, which are written in the column vector $x = [y, z, v_y, v_z, \theta, \omega]^T$. The $y$ and $z$ denote the coordinate of the handle in the Ground frame with their time derivatives $v_y$ and $v_z$. With the help of Equation (3), we can obtain the discrete-time pendulum dynamics by the state-space equation

\[
\begin{align*}
\begin{pmatrix}
X_{i+1} \\
1
\end{pmatrix}
&= A \begin{pmatrix}
X_i \\
1
\end{pmatrix} + Bu,
\end{align*}
\]

with the state-dependent matrices

\[
A = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} dt
\]

\[
B = \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix} dt,
\]

where the $s = -\sin \theta dt/L$ and $c = -\cos \theta dt/L$. The $0 < \gamma < 1$ in the matrices is set to simply simulate the effects of friction at the string knots. The $dt$ should be a short period.

When the system starts from the initial state, the Equation (4) will be used to compute the state sequence for the system while the string tension over the ball mass $T/m$ will be checked every time step by Equation (2). When the $T/m$ is still positive, the kinematic information of both the handle and the ball can be obtained from the state $x$. Once the $T/m$ decreases from the positive value to zero, the ball will be thrown as an oblique projectile and no longer controlled by the input $u$. The translational and angular velocities of the ball can be calculated by $x$ at the string loosing moment. Even though, the input $u$ will continue to be offered to guide the handle go to the desired position to catch the falling ball.

The dynamics introduced in this sub-section is able to model and simulate the robot task with a minimal complexity requirement. On the other hand, there are several error sources due to the problem assumptions: (1) We have to use a factor $\gamma$ in $A$ and $B$ to simulate the effects of friction. (2) The mass distribution of the ball is not considered in this model since we treat it as a point of mass. (3) There should be energy loss at the string loosing moment that we are also not able to evaluate. Therefore, these are why we prefer a model-free method after the robot trajectory is brought from the simulator to the physical robot.

### C. PoWER with smooth and efficient exploration

In PoWER, we use a set of parameters $w$ to represent the parameterized policy. In this paper, the policy representation is selected as DMP \[12\]. The resultant trajectory of DDP (the first two rows of $X$) will be taken as the desired trajectory
for DMP to calculate the initial policy $w$. The agent then explores by sampling the stochastic policy with $w$ as the mean. A trajectory $\tau$ will be produced with a probability $p(\tau|w)$ and the performance of this trajectory $J(\tau)$ will be evaluated by the sensor. The expected return of the current policy $w$ can be written as $J(w) = \int p(\tau|w)J(\tau)d\tau$. The goal of policy search is to adjust $w$ to maximize $J(w)$.

It has been shown that if we select the new policy $w'$ that maximizes the lower bound $L_w(w')$, the expected return will be improved [6]. The lower bound is expressed as

$$L_w(w') = \int p(\tau|w)J(\tau)\log \frac{p(\tau|w')}{p(\tau|w)}d\tau.$$ 

Therefore, to find the updated policy $w'$ in each episodic iteration, we solve the equation

$$\lim_{w' \to w} \frac{\partial}{\partial w'} L_w(w') = 0,$$ 

with the stochastic policy adopting a multi-dimensional Gaussian distribution with a covariance matrix $\Sigma$. Solving the Equation (5) results in the policy updating rule of

$$w' = w + E \left\{ \sum_{t=1}^{T} W_t Q^T_t \right\}^{-1} \left\{ \sum_{t=1}^{T} W_t \epsilon Q^T_t \right\},$$ 

with $W_t = \phi(t)\phi(t)^T \left( \phi(t)^T \Sigma \phi(t) \right)^{-1}$, where the $\phi(t)$ denotes a set of state-independent basis functions for the movement primitive and the $\epsilon$ is the policy exploration. In this paper, we simply make the state-action value $Q^T_t$ equal to the reward function defined by the human. The standard expression and more details should be referred to [6].

When we apply the PoWER on a physical robot to do the trajectory learning, it is important to tune the covariance matrix $\Sigma$ considering the trade-off between the sample efficiency and the smoothness of the resultant trajectories [9]. We use the $\Sigma = \beta^2 R^{-1}$ as the initial exploration rate, where the $\beta^2$ is a scaling factor and the matrix $R$ is defined [4] by $R = A_2^T A_2$, with the second-order differential matrix

$$A_2 = \begin{pmatrix}
  1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
  -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\
  1 & -2 & 1 & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & 1 & -2 & 1 \\
  0 & 0 & 0 & \cdots & 0 & 1 & -2 \\
  0 & 0 & 0 & \cdots & 0 & 0 & 1 
\end{pmatrix}.$$ 

At the meanwhile, we also expect a better sample efficiency so we apply the Equation (5) with respect to the exploration rate $\Sigma$ then we have $\frac{\partial}{\partial \Sigma} L_\Sigma(\Sigma') = 0$. Note that although the equation has more than one solution, if we consider the eigendecomposition $\Sigma = \Psi \Lambda \Psi^T$, where $\Lambda$ is a diagonal matrix while $\Psi$ is an orthogonal matrix, the equation yields

$$\Lambda' = \frac{E \left\{ \sum_{t=1}^{T} \text{diag} \left\{ \Psi^T \epsilon \right\}^2 Q^T_t \right\}}{E \left\{ \sum_{t=1}^{T} Q^T_t \right\}}.$$ 

In policy search, right after each iteration of updating the policy parameters $w$ with Equation (6), we apply Equation (7) to update the eigenvalues of the exploration rate. We then use the updated $\Lambda'$ to generate the uncorrelated random vector $\lambda' \sim N(0, \Lambda')$. Thus, the $\epsilon' = \Psi \lambda'$ can be taken as the correlated exploration sample for the next iteration of policy search. Therefore, the exploration rate is updated automatically. The whole procedure goes iteratively until the policy converges to the ideal optimum for the robot to complete the tasks in the real world while $\Lambda$ converges to zero. In addition, importance sampling and rollout reusing are required when using both Equation (6) and (7).

IV. Evaluation

In this section, we develop the simulator in MATLAB to simulate the kendama system in a 2-d space. We then compare the results when we use the individual PoWER and DDP to the same initial trajectory learned from human demonstration. Eventually, we use the KUKA manipulator to carry out the kendama task. The PoWER is applied and initialized with the resultant trajectory of DDP.

A. Learning from demonstration

The initial trajectory can be acquired by learning from demonstration [9]. It is necessary to conduct either PoWER or DDP simulations. The trajectory is represented as the sequences $\{y_1, y_2, \ldots, y_{350}\}$ and $\{z_1, z_2, \ldots, z_{350}\}$ to express the desired Cartesian coordinates of the intermediate points for the robot end-effector. The frequency of these points is 100Hz, which means the duration of a trajectory is 3.5 seconds. The initial control input $u$ can be computed with the inverse dynamics $f^{-1}$ from Equation (1). Applying the $u$ back to Equation (1), we will receive the trajectories for both handle and ball for the system as shown in Fig. 3.

![Simulation of the initial trajectory](image)

Fig. 3. This figure illustrates the movement of the kendama system in the y-z plane. The green points demonstrate the positions of the handle, as well as the ball, at each 10ms. The red points demonstrate the positions of the ball during the swing-up phase while the cyan points for the catching phase. The orientations where the hole on the ball is facing are expressed as the blue arrows.
The sequences \(\{y_1, y_2, \ldots, y_{350}\}\) and \(\{z_1, z_2, \ldots, z_{350}\}\) are also sent to KUKA robot. The comparison between the execution of the physical robot and the simulation result can be used to calibrate the factor \(\gamma\) in matrices \(A\) and \(B\). In our experiments, the \(\gamma\) is selected as 0.825.

To discuss the experiment results easily, we quantify the performance of the execution of one trajectory with Fig. 4. The figure displays the moment when the ball is falling and passes by the handle. There are three elements we care about. The horizontal distance between the handle and the ball is denoted by \(d\). The orientation of the ball and the direction of the ball movement are described by \(\theta\) and \(\varphi\). To successfully complete the robot task, the trajectory should result in that \(d = 0\), \(\theta = \zeta + 90^\circ\) and \(\varphi = \zeta\), where the \(\zeta\) depends on the pose of the handle at that moment.

It should be pointed out that we design a modified ball with a 3-d printer for our experiments instead of the traditional one. The modified ball has the hole facing up. Also, the initial trajectory is unable to complete the kendama task. The pose of the ball does not match the orientation of the movement while the ball is far away from the handle. The three criteria for the task are \(d = 108.6\text{mm}, \theta = 193^\circ\) and \(\varphi = 76^\circ\). The initial trajectory should be further optimized to complete the task.

Thus, from Fig. 3 we can see the hole of the ball is initially facing up. Also, the initial trajectory is unable to complete the kendama task. The pose of the ball does not match the orientation of the movement while the ball is far away from the handle. The three criteria for the task are \(d = 108.6\text{mm}, \theta = 193^\circ\) and \(\varphi = 76^\circ\). The initial trajectory should be further optimized to complete the task.

B. Simulation

In this subsection, we individually apply the DDP and PoWER algorithms to optimize the above initial trajectory in the simulator.

a) PoWER reward function: In our experiments, the \(\zeta\) in Fig. 4 is roughly \(67^\circ\) and can change slightly with the ball orientation. Therefore, the desired criteria are \(d = 0\), \(\theta = 150^\circ\) and \(\varphi = \zeta\). A total reward function can be defined as

\[
    r = \frac{\alpha_1 r_1 + \alpha_2 r_2 + \alpha_3 r_3}{\alpha_1 + \alpha_2 + \alpha_3},
\]

with \(r_1 = e^{1 \times 10^{-4} d^2}, r_2 = e^{5 \times 10^{-3}(\theta - 150)^2}\) and \(r_3 = e^{5 \times 10^{-3}(\varphi - \zeta)^2}\). The resultant trajectory after 200 iterations can be seen in Fig. 5(a).

b) DDP cost function: We find that if the robot goes through the state \(x^b\) at time \(t_1\), then the handle will theoretically catch the ball at \(x^c\) and at \(t_2\). The \(t_1\) is the moment when string tension reaches zero while the \(t_2\) is the moment as shown in Fig. 4. The final cost \(\ell_f(X)\) can be actually defined as

\[
    \frac{1}{2}(x_{i_1} - x^b)^T \hat{Q}_1(x_{i_1} - x^b) + \frac{1}{2}(x_{i_2} - x^c)^T \hat{Q}_2(x_{i_2} - x^c).
\]

While the running cost \(\ell(x_i, u_i)\) is defined as

\[
    \frac{1}{2}(x_i - x_i^{init})^T \hat{Q}(x_i - x_i^{init}) + \frac{1}{2}(u_i - u_i^{init})^T R(u_i - u_i^{init})
\]

in order to constrain the resultant trajectory around the initial one learned from human demonstration. The resultant trajectory after 200 iterations can be seen in Fig. 5(b).

In Fig. 5 we can investigate how the three criteria change with the iterations. Since PoWER is a sampling-based method, we should test it with different random number series and calculate the average. Fig. 6(a) uses the PoWER reward function to calculate the rewards for both algorithms. It shows that the PoWER is stuck in a poor local optimum while the DDP can find a trajectory with higher rewards. Fig. 6(b) shows that with PoWER the robot can easily control the performance of distance. On the contrary, the robot with DDP knows to sacrifice the distance to obtain a better performance of \(\theta\) and \(\varphi\). From Fig. 5 and 6(c) we can see the DDP is able to control the \(\theta\) while the PoWER is not. When the iteration number goes over 50, due to being stuck in the local optimum, the PoWER starts generating unstable explorations. It causes vibrations in the learning curves and is also dangerous for the physical robot.

In addition, if we try to increase the \(\alpha_2\) in Equation (8), the performance of \(\theta\) may be reduced by jumping over the local optimum with the large \(det(\Sigma)\) to explore. Thus, the shape of the resultant trajectory will become strange and the required acceleration will surpass the robot capability.

C. Physical robot

Note that the reward of the resultant trajectory of DDP is just 0.96, due to the existence of running cost \(\ell(x_i, u_i)\).
The running cost is necessary to guarantee a safe trajectory exploration for the physical robots. It is possible that we continue to apply PoWER to the result of DDP in the simulator just similarly to [7], which can actually make the reward to 1. However, the practical approach is to directly apply PoWER on the physical robot so as to compensate the model errors and mechanical errors at one time.

The resultant trajectory of DDP is tested on the KUKA manipulator. A camera is used to evaluate the performance for the real-world system. The performance is 

\[ d = 27 \text{ mm}, \quad \theta = 153^\circ \quad \text{and} \quad \varphi = 66^\circ. \]

Then the trajectory is optimized by PoWER. With about 20 iterations, the robot can complete the task for the first time. The screenshots of a successful execution can be seen in Fig. 7. With round 60 iterations, the robot can succeed in the continuous 5 iterations.

V. Conclusion

In this paper, we propose a framework for robot trajectory learning the precise task with discontinuous dynamics and high speed requirement. We apply the model-based methods in the simulator then the model-free ones on the physical robot. This framework considers the trade-off among the sample efficiency, precision and safety. It is tested by successfully planning a trajectory for the kendama task.

One drawback of our approach is that the cost function of DDP requires manually tuning, for example, the \( x^b, x^c, Q \) and \( R \). Some universal approaches are required to further reduce the human efforts while to constrain the required acceleration of trajectories, which will be the future work.

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Fig. 7. Screen shots of a successful kendama catch by KUKA LBR iiwa 7 R800 robot manipulator with 7 degrees of freedom. The duration for one trial of execution is 3.5 seconds. The maximum acceleration required to execute the trajectory is limited in 5500 mm/s².