Observation of Partial $U_A(1)$ Restoration from Two-Pion Bose-Einstein Correlations

S.E. Vance\textsuperscript{1}, T. Csörgö\textsuperscript{1,2} and D. Kharzeev\textsuperscript{3}
\textsuperscript{1}Physics Department, Columbia University, New York, NY 10027
\textsuperscript{2}MTA KFKI RMKI, H-1525 Budapest 144, P.O. Box 49, Hungary
\textsuperscript{3}RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973
(24th February 1998)

Abstract

The effective intercept parameter of the two-pion Bose-Einstein correlation function, $\lambda_*$, is found to be sensitive to the partial restoration of $U_A(1)$ symmetry in ultra-relativistic nuclear collisions. An increase in the yield of the $\eta'$ meson, proposed earlier as a signal of partial $U_A(1)$ restoration, is shown to create a “hole” in the low $p_t$ region of $\lambda_*$. A comparison with NA44 data from central S+Pb collisions at 200 AGeV is made and implications for future heavy ion experiments are discussed.

Typeset using REVTEX
Introduction: Intensity interferometry is a useful method for studying the space-time geometry of high energy nucleus-nucleus collisions and elementary particle reactions (for recent reviews, see Ref. [1,2]). In particular, pion interferometry has proved useful in studying the space-time dependence of pion emission as was first shown experimentally by Goldhaber, Goldhaber, Lee and Pais [3]. The method of intensity interferometry, known also as Hanbury-Brown-Twiss (HBT) correlations, was introduced by Hanbury-Brown and Twiss [4] for measuring the angular diameters of main sequence distant stars. The purpose of this Letter is to show that pion interferometry can be used to detect the axial $U_A(1)$ restoration and the related increase of the $\eta'$ production.

As was shown in several papers [5,6], at incident beam energies of 200 AGeV at the CERN SPS, the space-time structure of pion emission in high energy nucleus-nucleus collisions can be separated into two regions: the core and the halo. The pions which are emitted from the core or central region consist of two types. The first type is produced from a direct production mechanism such as the hadronization of wounded stringlike nucleons in the collision region. These pions rescatter as they flow outward with a rescattering time on the order of 1 fm/c. The second type is produced from the decays of short-lived hadronic resonances such as the $\rho$, $N^*$, $\Delta$ and $K^*$, whose decay time is also on the order of 1-2 fm/c. This core region is resolvable by Bose-Einstein correlation (BEC) measurements. The halo region, however, consists of the decay of long-lived hadronic resonances such as the $\omega$, $\eta$, $\eta'$ and $K^0_S$ whose lifetime is greater than 20 fm/c. This halo region is not resolvable by BEC measurements. However, as will be summarized below, this region still affects the Bose-Einstein correlation function.

In recent papers [7,8], it was argued that the partial restoration of $U_A(1)$ symmetry of QCD and related decrease of the $\eta'$ mass [9–12] in regions of sufficiently hot and dense matter should manifest itself in the increased production of $\eta'$ mesons. Estimates of Ref. [7] show that the corresponding production cross section of the $\eta'$ should be enhanced by a factor of 3 up to 50 relative to that for $p+p$ collisions.

The effective intercept parameter, $\lambda_*$, can be written in terms of the one-particle invariant momentum distributions [1,2] of the core and halo pions and thus is sensitive to the abundance of the long-lived hadronic resonances such as the $\eta'$. To see this, consider the two-particle Bose-Einstein correlation function which is defined as:

$$C(\Delta k, K) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)}, \quad (1)$$

where the inclusive $n$-particle invariant momentum distribution is given as

$$N_n(p_1, ..., p_n) = \frac{1}{\sigma_{in}}E_1...E_n \frac{d\sigma}{dp_1...dp_n}, \quad (2)$$

the relative and the mean four-momenta are given by

$$\Delta k = p_1 - p_2, \quad K = \frac{p_1 + p_2}{2}, \quad (3)$$

and $p = (E_p, p)$.

From the four assumptions made in the core-halo model [13], the Bose-Einstein correlation function is found to be
\[ C(\Delta k, K) \simeq 1 + \lambda_s(K = p; Q_{\text{min}})R_c(\Delta k, K), \]  

where the effective intercept parameter and the correlator of the core are defined, respectively, as

\[ \lambda^*_s = \frac{N_c(p)}{N_c(p) + N_h(p)} \]  

and

\[ R_c(\Delta k, K) = \frac{|\tilde{S}_c(\Delta k, K)|^2}{|\tilde{S}_c(\Delta k = 0, K = p)|^2}. \]  

Here, \( \tilde{S}_c(\Delta k, K) \) is the Fourier transform of the one-boson emission function, \( S_c(x, p) \), and the subscripts \( c \) and \( h \) indicate the contributions from the core and the halo, respectively.

Axial Symmetry Restoration and the \( \eta' \) production: In the chiral limit \( (m_u = m_d = m_s = 0) \), QCD possesses a \( U(3) \) chiral symmetry. When broken spontaneously, \( U(3) \) implies the existence of nine massless Goldstone bosons. In nature, however, there are only eight light pseudoscalar mesons, a discrepancy which is resolved by the Adler-Bell-Jackiw \( U_A(1) \) anomaly; the ninth would-be Goldstone boson gets a mass as a consequence of the nonzero density of topological charges in the QCD vacuum. In Refs. [14,15], it is argued that the ninth (“prodigal” [7]) Goldstone boson, the \( \eta' \), would be abundantly produced if sufficiently hot and dense hadronic matter is formed in nucleus-nucleus collisions.

It was also observed, however, that the \( \eta' \) decays are characterized by a small signal-to-background ratio in the direct two-photon decay mode. This may make the observation of \( \eta' \) in this mode difficult, especially at small transverse momenta, where the increase is predicted to be the strongest. However, we now show that the momentum dependence of \( \lambda^*_s \) from pion correlations provides a good observable for partial \( U_A(1) \) restoration.

If the \( \eta' \) mass is decreased, a large fraction of the \( \eta' \)'s will not be able to leave the hot and dense region through thermal fluctuation since they need to compensate for the missing mass by large momentum \[ \tilde{S}_c(\Delta k, K) \] centered around \( p_t \approx 138 \text{ MeV} \). The \( \eta' \)'s then decay to pions via

\[ \eta' \to \eta + \pi^+ + \pi^- \to (\pi^0 + \pi^+ + \pi^-) + \pi^+ + \pi^- \]  

Assuming a symmetric decay configuration \( (|p_t|_{\pi^+} \simeq |p_t|_{\pi^-} \simeq |p_t|_{\eta}) \) and letting \( m_{\eta'} = 958 \text{ MeV}, m_{\eta} = 547 \text{ MeV} \) and \( m_{\pi^+} = 140 \text{ MeV} \), the average \( p_t \) of the pions from the \( \eta' \) decay is found to be \( p_t \approx 138 \text{ MeV} \). In the core-halo picture the \( \eta', \eta \) decays contribute to the halo due to their large decay time \( (1/\Gamma_{\eta',\eta} > 20 \text{ fm}/c) \). Thus, we expect a hole in the low \( p_t \) region of the effective intercept parameter, \( \lambda^*_s = (N_{\text{core}}(p)/N_{\text{total}}(p))^2 \), centered around \( p_t \approx 138 \text{ MeV} \).

We note that the shape of \( \lambda^*_s \) will also be effected if the masses of the \( \omega \) and \( \eta \) mesons decrease. However, due to the large inelastic cross sections for \( \omega \) - meson scattering, the
ω are expected to rapidly reach chemical equilibrium when the hadronic fireball cools and their mass returns to its “normal” value. In this case, we do not expect a sizeable increase in the overall production of the ω mesons. In addition, any enhanced production of the η mesons should only increase the depth of the hole primarily in the $p_t \simeq 117$ MeV region. In the case of equal production of the η and η′, there will be on the order of twice as many $\pi^+$ coming from the decay of the η′’s than from the decay of the ηs. Thus, we concentrate on the dynamics of the η′ mesons giving an estimated lower bound on the depth of the produced hole.

**Description of the Simulation:** In the following calculation of $\lambda_\star$, we suppress the rapidity dependence by considering the central rapidity region, ($-0.2 < y < 0.2$). As a function of $m_t = \sqrt{p_t^2 + m^2}$, $\lambda_\star(m_t)$ is given by

$$\lambda_\star(m_t) = \left[ \frac{N^{\pi^+}_{\text{core}}(m_t)}{N^{\pi^+}_{\text{total}}(m_t)} \right]^2,$$

where the numerator represents the invariant $m_t$ distribution of $\pi^+$ emitted from the core and where the denominator represents the invariant $m_t$ distribution of the total number of $\pi^+$ emitted. The denominator may be explicitly written as

$$N_{\text{total}}^{\pi^+}(m_t) = N_{\text{core}}^{\pi^+}(m_t) + N_{\omega \to \pi^+}(m_t) + N_{\eta' \to \pi^+}(m_t) + N_{\eta \to \pi^+}(m_t) + N_{K^0_S \to \pi^+}(m_t).$$

A detailed analysis [6] has shown that the ω does not contribute to the core in the S+Pb reaction and in the NA44 acceptance.

To calculate the $\pi^+$ contribution from the halo region, the bosons (ω, η′, η and $K^0_S$) are given both a rapidity ($-1.0 < y < 1.0$) and an $m_t$ and then are decayed using Jetset 7.4 [16]. The $m_t$ distribution [5,17] of the bosons is given by

$$N(m_t) = C m_\alpha^\alpha e^{-m_t/T_{\text{eff}}},$$

where $C$ is a normalization constant, where $\alpha = 1 - d/2$ and where $T_{\text{eff}} = T_{fo} + m\langle u_t \rangle^2$.

In the above expression, $d = 3$ is the dimension of expansion, $T_{fo} = 140$ MeV is the freeze-out temperature and $\langle u_t \rangle$ is the average transverse flow velocity. The $m_t$ distribution of the core pions is also obtained from Eqs. (10) and (11). The contributions from the decay products of the different regions (halo and core) are then added together according to their respective fractions, allowing for the determination of $\lambda_\star(m_t)$. The respective fractions of pions are estimated from both the Fritiof [19] and the Relativistic Quantum Molecular Dynamics (RQMD) [20] models as summarized in Ref. [21]. The calculation using Fritiof abundances is shown in Fig 2 (solid line). A similar $m_t$ dependence but with a slightly higher value of $\lambda_\star(m_t)$ is obtained when using RQMD abundances.

Simulating the presence of the hot and dense region involves including an additional relative fraction of η′ with a medium modified $p_t$ spectrum. The $p_t$ spectrum of these η′
is obtained by assuming energy conservation and zero longitudinal motion at the boundary between the two phases. This conservation of transverse mass at the boundary implies,

\[ m_{\eta'}^* + p_{t\eta'}^2 = m_{\eta'}^2 + p_{t\eta'}^2, \]  

(12)

where the (*) denotes the \( \eta' \) in the hot dense region. The \( p_t \) distribution then becomes a twofold distribution. The first part of the distribution is from the \( \eta' \) which have \( p_t^* \leq \sqrt{m_{\eta'}^2 - m_{\eta'}^*} \). These particles are given a \( p_t = 0 \). The second part of the distribution comes from the rest of the \( \eta' \)'s which have big enough \( p_t \) to leave the hot and dense region. These have the same, flow-motivated \( p_t \) distribution as the other produced resonances and are given a \( p_t \) according to the \( m_t \) distribution

\[ N_{\eta'}(m^*_t) = C m_t^* e^{-m_t^*/T'}, \]

(13)

where \( C \) is a normalization constant and where \( T' = 200 \text{ MeV} \) and \( m_{\eta'}^* \) is the effective temperature and mass, respectively, of the hot and dense region.

Assuming \( m_{\eta'}^* = 500 \text{ MeV} \) in the above scenario, the \( m_t \) distribution of the \( \pi^+ \) from the decay of these \( \eta' \) (\( \eta' \rightarrow \eta + \pi^+ + \pi^- \)) is shown in Fig. 1. Also shown is the \( m_t \) distribution of the \( \pi^+ \) from \( \eta' \) assuming no hot and dense matter (Eqs. (14) and (15) with \( T_{fo} = 140 \text{ MeV}, \langle u_t \rangle = 0.5 \) and \( m_{\eta'} = 958 \text{ MeV} \)). Comparison of the two distributions shows the enhancement of the \( \pi^+ \) in the low \( m_t \) region which results from the presence of the hot and dense region.

Using three different effective masses for the \( \eta' \) in the hot and dense region, calculations of \( \lambda_s(m_t) \) including the hot and dense regions are compared to those assuming the standard abundances in Fig 2. The effective mass, \( m_{\eta'}^* = 738 \text{ MeV} \), corresponds to an enhancement of the production cross section of the \( \eta' \) by a factor of 3, while \( m_{\eta'}^* = 403 \text{ MeV} \) and \( m_{\eta'}^* = 176 \text{ MeV} \) correspond to factors of 16 and 50, respectively. The two data points shown are taken from NA44 data on central \( S + Pb \) reactions at the CERN SPS with incident beam energy of 200 AGeV [22]. The lowering of the \( \eta' \) mass and the partial chiral restoration result in a hole in the effective intercept parameter at low \( m_t \). This happens even for a modest enhancement of a factor of 3 in the \( \eta' \) production. Similar results are obtained when using RQMD abundances.

In addition, \( \lambda_s(m_t) \) is calculated using Fritiof abundances with different average flow velocities in Fig 3. Here it is shown that \( \lambda_s(m_t) \) can also be a measure of the average collective flow. In our calculations, an average flow velocity of \( \langle u_t \rangle = 0.50 \) results in an approximately flat, \( m_t \)-independent shape for the effective intercept parameter \( \lambda_s(m_t) \), if the value of \( \alpha = 1 - d/2 = -1/2 \) is kept fixed in Eq. (14). Calculations using RQMD abundances result in a similar dependence on \( \langle u_t \rangle \), but with slightly higher values of \( \lambda_s(m_t) \).

A limitation of our study is that we did not include the effects of possible partial coherence in the \( \lambda_s(m_t) \) function. This is motivated by the success of completely chaotic Monte Carlo simulations in describing the measured two-particle correlation functions at the CERN SPS. However, a recent study [23] indicates that higher order BE symmetrization effects may also result in a decrease of \( \lambda_s(m_t) \) at low \( p_t \). For the present system, this effect seems to be negligible, about a 1 % decrease, where the typical momentum scale of this effect is \( m_t - m = T_{eff} \) and where the typical decrease is estimated [23] from the measured radius and slope parameters. The flat shape of our \( \lambda_s(m_t) \) distribution results from the inclusion of
the flow motivated temperature, $T_{eff}$, along with the effective, $m_t$ dependent volume factor $V_\ast \propto m_t^{-d/2}$, in Eq. (10). This flat shape reproduces the published NA44 data and differs from earlier theoretical calculations where $\lambda_\ast$ is found only to increase with increasing $m_t$.

**Summary:** Our results reveal an important relationship between partial $U_A(1)$ symmetry restoration and the shape (hole) of the $\lambda_\ast(m_t)$ parameter of the Bose-Einstein correlation function. We stress that this proposed signal is observed from the transverse mass dependence of the strength of the two-particle correlations, correlations which are presently being measured for fixed target Pb+Pb collisions at the CERN SPS. Measurements of two-particle correlations are also being planned for nuclear collisions at the Relativistic Heavy Ion Collider (RHIC) at BNL as well as at the CERN Large Hadron Collider (LHC).

A qualitative analysis of NA44 S+Pb data suggests no visible sign of $U_A(1)$ restoration at SPS energies. In addition, we deduce a mean transverse flow of $\langle u_t \rangle \approx 0.50$ in S+Pb reactions. Let us note that the suggested $\lambda_\ast$-hole signal of partial $U_A(1)$ restoration cannot be faked in a conventional thermalized hadron gas scenario, as it is not possible to create significant fraction of the $\eta$ and $\eta'$ mesons with $p_t \approx 0$ in such a case.

**Acknowledgments:** One of the authors, T. Csörgő, would like to express his thanks to Miklós and Györgyi Gyulassy for their kind hospitality while at Columbia University. D. Kharzeev is grateful to J. Kapusta and L. McLerran for sharing their ideas with him and an enjoyable collaboration, and X.-N. Wang for useful discussions. S.E. Vance would like to thank Urs Wiedemann and Ulrich Heinz for useful discussions.

This work was supported by the OTKA Grants T016206, T024094, T026435, by the NWO - OTKA Grant N25487, by an Advanced Research Award of the Fulbright Foundation and by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-FG02-93ER40764.
REFERENCES

[1] W. A. Zajc in “Particle Production in Highly Excited Matter”, ed. by H. Gutbrod and J. Rafelski, NATO ASI series B303 (Plenum Press, 1993) p.435.
[2] B. Lörstad, Int. J. Mod. Phys. A12 (1989) 2861.
[3] G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, Phys. Rev. 120 (1960) 300.
[4] R. Hanbury-Brown and R. Q. Twiss, Phyl. Mag. 45 663 (1954); R. Hanbury-Brown and R. Q. Twiss, Nature (London) 177 (1956) 27 and 178 (1956) 1046.
[5] T. Csörgő, B. Lörstad and J. Zimányi, Z. Phys. C71 491 (1996), [hep-ph/9411307], and references therein, J. Bolz et al., Phys Rev. D47 (1993) 3860.
[6] S. Nickerson, T. Csörgő and D. Kiang, Phys Rev. C57 (1998) 3251.
[7] J. Kapusta, D. Kharzeev and L. McLerran, Phys Rev. D53 (1996) 5028.
[8] Z. Huang and X.-N. Wang, Phys Rev. D53 (1996) 5034.
[9] R.D. Pisarski and F. Wilczek, Phys Rev. D29 (1984) 338.
[10] T. Kunihiro, Phys Lett. B219 (1989), 363; ibid. B245 (1990) 687 (E).
[11] E. Shuryak, Comm. Nucl. Part. Phys. 21 (1994) 235, and references therein.
[12] T. Hatsuda and T. Kunihiro, Phys Rep. 247 (1994) 221, and references therein.
[13] T. Csörgő, Phys Lett. B 409 (1997) 11; [hep-ph/9705422].
[14] E. Witten, Nucl. Phys. B156 (1979) 269.
[15] G. Veneziano, Nucl. Phys. B159 (1979) 213.
[16] T. Sjostrand, Computer Physics Commun. 82 (1994) 74.
[17] T. Csörgő and B. Lörstad, Phys Rev. C54 (1996) 1390; Nucl. Phys. A590 (1995) 465c.
[18] I. G. Bearden et al., (NA44 Collaboration), Phys Rev. Lett. 78 (1997) 2080.
[19] B. Anderson et al., Nucl.Phys. B281 (1987) 289.
[20] J. P. Sullivan et al., Phys Rev. Lett. 70 (1993) 3000.
[21] H. Heiselberg, Phys. Lett. B 379 (1996) 27.
[22] H. Beker et al. (NA44 Collaboration), Phys Rev. Lett. 74 (1995) 3340.
[23] T. Csörgő and J. Zimányi, Phys Rev. Lett. 80 (1998) 916.
FIG. 1. Invariant $m_t$ distributions are shown for the $\pi^+$ from the decay $\eta' \rightarrow \eta + \pi^+ + \pi^-$. The solid line assumes the $\eta'$s come from a hot and dense region where the $\eta'$ have a twofold $p_t$ distribution in which $m_{\eta'}^* = 500$ MeV and $T' = 200$ MeV. The dashed line assumes the $\eta'$ come from nucleus-nucleus collisions at SPS energies where their $m_t$ spectrum is given by Eqs. (10) and (11), with $T_{fo} = 140$ MeV, $\langle u_t \rangle = 0.5$ and $m_{\eta'} = 958$ MeV.
FIG. 2. Using the estimates of pion abundances given by Fritiof, the solid line represents $\lambda_*(m_t)$ assuming normal $\eta'$ abundances while the other lines represent the inclusion of hot and dense regions, where $T' = 200$ MeV and the decreased mass of the $\eta'$ is $m_{\eta'}^* = 738$ MeV (dashed line), $m_{\eta'}^* = 403$ MeV (dotted line) and $m_{\eta'}^* = 176$ MeV (dot-dashed line). These curves are calculated for $\langle u_t \rangle = 0.5$. The data shown are from S+Pb reactions at 200 AGeV from the NA44 collaboration.
**FIG. 3.** Using the estimates of pion abundances given by Fritiof, $\lambda_*(m_t)$ is calculated using different average flow velocities in the $m_t$ distribution. It is shown for $\langle u_t \rangle = 0.00$ by the solid line, for $\langle u_t \rangle = 0.25$ by the dashed line, for $\langle u_t \rangle = 0.50$ by the dotted line and for $\langle u_t \rangle = 0.75$ by the dashed-dotted line.