Dynamic control of tug-debris tethered system after the capturing of the debris

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Abstract. This paper focuses on the problem of control of the tug-debris space tether system for debris De-orbiting. The motion of the tether system is considered when the space tug and tethered debris have an initial relative velocity, which can affect the tether tension. We investigate two stages of the space tether motion: tether elongation stage and tether retraction stage. Two control laws for the tether length are proposed for each stage. The control law for the first stage is based on the measurement of the tether tension and is intended to decrease the relative velocity between the space tug and debris object. The control law for the second stage retracts the tether and decreases the distance between the space tug and debris assisting to the process of the docking of the space tug with debris. The proposed control laws are demonstrated by numerical simulations.

1. Introduction

Intensive exploration of the near-Earth space environment since the launch of the Sputnik 1 leads to the fact that now many thousands of space debris have accumulated in near-Earth orbits [1]. Large debris objects are tracked from the Earth and do not carry an immediate threat to functional spacecraft that can adjust its orbit [2,3]. The collisions of large space debris with other small or large pieces can significantly increase the number of small debris in near-Earth orbits. Such events should cause that some orbits become inaccessible [4, 5]. The space debris problem is recognized internationally. The countries that have space programs share the common view on the necessity to reduce space debris generation and mitigate risks posed by space objects. A set of International debris mitigation guidelines has been proposed [6, 7]. In addition, methods of active debris removal are developing [8–16] many of which involve the use of space tugs for De-orbiting large space debris objects.

The classical docking technique [17] can’t be applied to space debris objects. These objects are not controlled and do not cooperate with the space tug for the docking. Space debris objects can rotate at a significant angular velocity, which also makes it impossible for the space tug to dock. Tethered system can be used by the space tug for the capturing rotating debris objects [15]. The tether reduces the impact of the debris motion on the tug when the tether connection is forming. In this case, the debris can be captured using an autonomous docking module [18], net [14] or harpoon [16]. The active debris removal using tethered space tug is one of the promising techniques.
The control of the space tether system after its formation requires design of new algorithms for the space tug and tether. The quality of this control determines the safety of the active debris removal mission. The control of the tether system during the active stage of a debris removal mission are considered in [19–21]. These works consider the stage of the motion after the space tug burns-up its thruster. These authors investigate the longitudinal oscillations of the tether due to changes of the acceleration of the space tug. These oscillations should be damped to decrease the disturbance to the motion of the space tug and to prevent possible collision of the space tug with the tethered debris.

The control of the tether also is essential before the active stage of the debris removal. When the tethered autonomous module that separates from the space tug is used to capture the debris [18] (figure 1), the capture event may occurs in the presence of relative velocity between the space debris and space tug [22, 23]. This relative velocity can produce a large tension to the tether and to the tether deployment mechanism. So, the tether tension should be controlled after the capture of the space debris.

This work is devoted to the development of the control law for the post-capture stage of the active debris removal mission when the tethered autonomous module is used to capture the debris object that has velocity relative to the space tug.

![Figure 1. Capturing space debris using the autonomous module](image)

2. Statement of the problem

In this work we investigate the motion of the space tether system consisting of a space tug and tethered space debris with the attached autonomous module. Here and after the term space debris refers to the debris with the connected autonomous module [18, 22, 23]. It is supposed that the debris flyby the space tug with some relative velocity. We consider two stages of the motion of the system:

- tether deployment stage when the tether length is increased with decreasing the relative velocity of the space debris;
- tether retraction stage when the length of the tether is decreased assisting to the docking of the autonomous module and attached debris object with the space tug [24].

At the start of the first stage the tether already has some initial length, which is determined by the relative motion of the autonomous module, space tug and space debris object in the previous stage, when the autonomous docking module separated from the space tug and intercepted the debris object [18].

The space tether system is presented in figure 2. The figure shows the space tug ($C_1$), space debris with the autonomous module ($C_2$) and the elastic tether. The space tug has a tether deployment device, which can change length of the tether. The strength of the tether must be
taken into account during the tether length control. The tether length is also limited by the capacity of the tether deployment device.

![Diagram of a Tug-debris tether system](image)

**Figure 2.** Tug-debris tether system

### 3. Motion of the tether system and tether control

#### 3.1. Tether deployment stage

The space tug and the space debris are considered as mass points, which both have constant masses $m_1$ and $m_2$ respectively. We assume that the time of the considered tether deployment stage is small compared to the orbital period of the system, so the motion of the bodies $C_1$ and $C_2$ is considered in a gravityless space. The thrusters of the space tug are not operating. For the motion of two mass points connected by a tether we can write the angular momentum conservation law relative to the center of mass of the system:

$$m_1\omega l_1^2 + m_2\omega l_2^2 = \text{const.}$$  \hspace{1cm} (1)

Expressing the length $l_1$ and $l_2$ in terms of the tether length $l = l_1 + l_2$ (figure 2) we get:

$$l_1 = \frac{m_2}{m_1 + m_2} l, \quad l_2 = \frac{m_1}{m_1 + m_2} l,$$

and rewrite the equation (1) as:

$$\omega l^2 = \text{const.}$$  \hspace{1cm} (3)

Equation (3) allows to determine the angular velocity of the tether system as a function of the tether length, initial angular velocity $\omega_0$, and initial length $l_0$ of the tether system:

$$\omega_0 l_0^2 = \omega l^2.$$  \hspace{1cm} (4)

Let us consider the motion of the space debris relative to the center of mass of the system. The equation of the motion can be obtained using D’Alembert’s principle (figure 2):

$$\omega^2 l_2 m_2 - \ddot{l}_2 m_2 - T = 0,$$  \hspace{1cm} (5)

where $T$ is the tether tension force. Substituting (2) into (5) we get:

$$\omega^2 l m - \ddot{l} m - T = 0,$$  \hspace{1cm} (6)
where \( m \) is the reduced mass of the system:

\[
m = \frac{m_1 m_2}{m_1 + m_2}.
\]

The angular velocity of the system \( (\omega) \) is determined by the expression (4), so the equation (6) takes the form:

\[
\ddot{l} - \frac{\omega_0^2 l^4}{l^3} + \frac{T}{m} = 0
\]

The tether tension force \( T \) for its deformation \( \delta = l - L \) is determined by the tether stiffness \( c \) and damping properties:

\[
T = \begin{cases} 
  c\delta + k\dot{\delta}, & l - L > 0, \\
  0, & l - L \leq 0, 
\end{cases}
\]

where \( L \) is the tether free length. The stiffness of the tether depends on the modulus of elasticity of the material of the tether \( (E) \), cross sectional area of the tether \( (F) \) and its free length \( L \):

\[
c = \frac{EF}{L}.
\]

The damping coefficient \( k \) is defined as follows:

\[
k = 2\zeta\sqrt{cm},
\]

where \( \zeta \) is a damping ratio and \( 2\sqrt{cm} \) is the critical damping of the considered system.

The free length \( L \) of the tether can be changed by the tether deployment device installed on the space tug. Let us describe the dynamic of this mechanism as (figure 3):

\[
J_r \ddot{L} = M_c + Tr,
\]

where \( J_r \) is the moment of inertia of the reel with the tether of radius \( r \), \( M_c \) is the control torque. The equation (12) can be rewritten as

\[
M\ddot{L} = F_c + T,
\]

where \( M = J_r/r^2 \) and \( F_c = M_c/r \).

\[ \text{Figure 3. The tether free length (L) control mechanism} \]

Let us consider the following control law for the free length of the tether which depends on the deviation of the tether tension \( T \) from some normal operating load \( T_p \):

\[
F_c = (T - T_p) k_T,
\]
where $k_T$ is a positive parameter, $T_p$ is the normal operating tether tension force. The control law (14) ensures that tether tension $T$ is around a certain value $T_p$, which will lead to the decreasing of the relative velocity of the tug and space debris. The equations for the controlled tether system now can be written as:

$$\begin{align*}
\ddot{l} - \frac{\omega_0^2 t^4}{t^3} + \frac{T}{m} &= 0, \\
M \ddot{L} &= (T - T_p)k_T + T.
\end{align*} \tag{15}$$

Equations (15) are to be integrated with the following initial conditions

$$l(0) = l_0, \quad \dot{l}(0) = \dot{l}_0, \quad L(0) = l_0, \quad \dot{L}(0) = \dot{l}_0.$$ 

The system (15) is nonlinear and its solution can be found numerically but if the initial angular velocity of the system and damping ratio are equal to zero and if the tether stiffness changes slightly during the considered stage of the motion and can be replaced by the constant value, the equation (15) can be integrated analytically. For that case, the equations (15) take the following form:

$$\begin{align*}
\ddot{l} + \frac{c_0 \delta}{m} &= 0, \\
M \ddot{L} &= (c_0 \delta - T_p)k_T + c_0 \delta,
\end{align*} \tag{16}$$

where $c_0 = EF/l_0$ is the initial stiffness of the tether. The equations (16) has the following solution:

$$\begin{align*}
l &= l_0 + \dot{l}_0 t - \frac{c_0 k_T m^2 M^2 T_p}{A^4} \left( mM \cos \left( \frac{At}{mM} \right) + c_0 \left( M + (1 + k_T) m \right) \frac{t^2}{2} - mM \right), \\
L &= l_0 + \dot{l}_0 t + \frac{k_T m^2 M^2 T_p}{A^4} \left( c_d m^2 \cos \left( \frac{At}{mM} \right) - c_0 \left( c_0 M + c_d m \right) \frac{t^2}{2} - c_d m^2 \right),
\end{align*} \tag{17}$$

where

$$A = \sqrt{mM \left( c_0 M + c_d m \right)} \quad \text{and} \quad c_d = c_0 \left( 1 + k_T \right). \tag{18}$$

The tether tension is

$$T = c_0 \left( l - L \right) = \frac{1}{A^4} k_T m^3 M^2 T_p \left( c_0 M + c_d m \right) \left( 1 - \cos \left( \frac{At}{mM} \right) \right). \tag{19}$$

The tether tension has the maximum when $t = mM/A$:

$$T_{\text{max}} = \frac{2c_0 k_T T_p (k_T m + m + M)}{mM^2 \left( c_0 M + c_d m \right)^4}. \tag{20}$$

The expressions (17) and (20) allow to estimate the time needed for the deployment the tether to the maximum length, maximum length of the tether, and maximum tether tension during the deployment stage.

### 3.2. Tether retraction stage

Let us consider the tether retraction stage. The stage starts with zero velocity of the debris relative to the space tug. We assume that after the tether deployment stage the system is stabilized, and its angular velocity is equal to zero. The tether deployment device in this stage reduces the length of tether assisting to the space tug for docking with the space debris. To
avoid collisions of the debris with the space tug and to ensure that the tether is tensioned, the space tug produce a constant thrust force $P$. The motion equations of the system for the second stage are:

\[
\begin{align*}
m\ddot{l} &= T + P \frac{m}{m_1}, \\
M\ddot{L} &= F_c + T.
\end{align*}
\]  

(21)

We propose the following tether length control law [24]:

\[
L = L(t) = l_b + \frac{(l_b - l_e)}{2} (1 + \cos \varphi t),
\]

(22)

where $\varphi = \pi/t_e$, $l_b$ is the initial length of the tether, $t_e$ is the duration of the retraction stage, $l_e$ is the final length of the tether (we suppose that the space tug docks with the space debris when the tether length reaches $l_e$). The control law (22) is the modification of the law, considered in [24]. For the tether length control (22) the control force $F_c$ has the form:

\[
F_c = \frac{\pi^2 M (l_b - l_e)}{2t_e^2} \cos \left( \frac{\pi t}{t_e} \right).
\]

(23)

4. Simulation

In this section we illustrate the proposed control laws by numerical examples. The parameters of the system are presented in Table 1.

We consider three simulation cases differing in the initial angular velocity of the system to investigate the effect of this velocity on the solution of the system (15) and to compare the results with the estimations (17) and (20). In the first simulation case initial angular velocity of the system is equal to zero, in the second case $\omega_0 = 0.1$ rad/s and in the third case $\omega_0 = 0.1$ rad/s.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $m$, kg   | 500   | $T_p$, N  | 1000  |
| $M$, kg   | 1     | $EF$, N   | $1.35 \cdot 10^5$ |
| $l_0 = L_0$, m | 100 | $\zeta$   | 0.005 |
| $\dot{l}_0 = \dot{L}_0$, m/s | 20 | $t_e$, s  | 100   |
| $k_T$     | 10    | $l_e$, m  | 1     |
| $P$, N    | 100   |           |       |

4.1. Tether deployment stage

Figure 4 shows the tether tension (figure 4(a)) and the tether length (figure 4(b)) as functions of time for the first simulation case. These results obtained by numerical integration of the system (15).

Figure 5 shows graphs for the same quantities obtained by the analytical solutions (17) and (19). The maximum tension of the tether obtained by using the expression (20) is equal to 1.8 kN. The numerical solution of the system (15) gives us value of $T_{max} = 1.7$ kN. If the tether length $L$ did not controlled by the tug, the maximum tether tension would be:

\[
T_{max} \approx \dot{l}_0 \sqrt{c_0 m} \approx 16.4 \text{ kN},
\]
Figure 4. Time history of the tether tension and length for the first case (numerical solution) which illustrates necessity of the tether control.

The maximum length of the tether, obtained by the numerical solution of the system (15), and by the solution (20) is 210 m.

Figure 5. Time history of the tether tension and length for the first case (analytical solution)

Figure 6 shows the elongation of the tether \((l - L)\) and the tether length for the second simulation case \((\omega_0 = 0.1 \text{ rad/s})\). The graph for the tether tension is not presented here because for the tether tension for the second case does not differ much from the first case (figure 4(a)).

The maximum magnitude of the tether tension is primarily depending on the initial linear relative velocity of the tug and the space debris, which is not changed. In the second simulation case the maximum tether length is about of 232 m (figure 6(a)), which is 22 meters greater than the value obtained by using the approximate solution (17) (figure 5(b)). The tether deployment times also differ from each other.

Figure 7 shows the results for the third case. Figure 7(a) presents the tether elongation and figure 7(b) shows time history of the tether length for the third simulation case. In this case the maximum of length of the tether is about of 308 m, which is 98 meters greater than the value obtained by using the solution presented in figure 5(b). The second and third cases illustrate that when the tether system has initial angular velocity the numerical solution of the system (15) should be used to estimate the final length of the tether.
4.2. Tether retraction stage

Let us consider the second stage of the motion (tether retraction stage). As previously noted, the stage begins after the stabilization of the tether system, so the angular velocity of the system is equal to zero.

Figure 8 shows how the tether tension and tether length change with time. The results show that the influence of the tug’s thrust leads to the longitudinal oscillations of the tether. The amplitude of the oscillations of the tether tension can be increased if the damping ratio is small. This case is illustrated by the figure 8(a). The increase of the amplitude of oscillations of the tension leads to the slacking of the tether and uncontrolled relative motion of the space debris. To reduce these oscillations the tether deployment device should damp longitudinal oscillations of the tether.

Figure 9 shows graph for the same quantities with the damping ratio increased by 10 times to a value of 0.05. In this case, the tension amplitude is reduced and at the end of the retraction stage the tether oscillation is absent.

Figure 10 shows the tether retraction velocity. The figure demonstrate that at the end of the tether retraction stage the relative velocity of the tug and the space debris decreases to zero.
5. Conclusion
In this paper we propose the control laws for the elongation and retraction stages of the space tether system after the capturing of the space debris object by the autonomous module that was separated from the space tug, assuming that the capture event occurs when the space debris
have some velocity relative to the space tug.

For the first stage the control law for the tether free length is proposed, which reduce the relative velocity of the bodies to zero and also controls the tension of the tether. For the case of the system motion with the angular velocity equal to zero, the analytical solutions for the tether length and tether tension are obtained. These solutions can help to select the parameters of the proposed control \((k_T, T_p)\) and estimate tether tension and maximum tether length (final length) at the end of the stage. When the tether system has initial angular velocity the numerical solution of the system (15) should be used to estimate the final length of the tether.

In contrast to the tether length control law considered in [22], which depends only on time and initial conditions of the motion of the tether system, the tether length control law proposed in this paper takes into account the tension force of the tether, providing control of this value.

Tether retraction stage should begin after the stabilization of the system for zeroing the initial angular velocity. During this stage the longitudinal oscillations of the tether should be substantially damped to eliminate oscillations of the tether at the end of the stage. The tether length control law for the second stage assists to the space tug for docking with the debris and provides zero relative velocity of the tug and space debris during their rendezvous at the distance \(l_e\). Proposed control requires a constant thrust force acting on the space tug.

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