Analytical Solution of the Grouting Reinforcement Response of a Circular Cavern in Deeply Buried Fractured Surrounding Rock under a Nonuniform Stress Field

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ABSTRACT: Grouting is widely utilized to reinforce fractured surrounding rock. However, presently, the secondary stress distribution of fractured surrounding rock after grouting is not distinct, which hinders the accurate evaluation of the grouting effect. Therefore, a complex variable function solution is provided for the evolution of the stress field and the displacement field around the reinforced fractured surrounding rock subjected to a nonuniform load. By comparing the evolution of the stress and radial displacement fields around the cavern, the reinforcement effect on the fractured surrounding rock is quantitatively confirmed. The results show that grouting has an obvious modification effect on fractured surrounding rock: its radial stress and circumferential stress fields are affected to different degrees, and the circumferential stress is more obvious. Grouting makes the circumferential stress peak transfer from the wall of the cavern to the roof and floor of the cavern, which is beneficial to the stability of the cavern. The radial displacement around the cavern also decreases. The displacement of the wall decreases by approximately 24.0%, and the radial displacement of the roof and floor decreases by approximately 61.8%.

1. INTRODUCTION

During the construction of underground engineering, we are bound to encounter difficulties, such as fault fracture zones and other undesirable geological bodies. It is easy to induce engineering disasters if they are not properly mitigated. Grouting is widely utilized in engineering as a means of rock modification, which can provide reinforcement for fractured rock and improve its overall strength. Due to the concealed nature of grouting, a quantitative assessment of its reinforcement effect is difficult.

Many experts and scholars have investigated the grouting reinforcement effect on fractured rock masses. For example, certain scholars have systematically analyzed the grouting effect through numerical analysis. This approach makes the migration of grouting in the ideal state more intuitive and reveals that grouting can change the permeability and porosity of rock, change the crack and pore size distribution of rock, and also improve the compactness and strength of fractured rock. Other scholars considered the theoretical basis of fluid mechanics and the distribution state of cracks and established analytical formulas such as grouting diffusion range and diffusion speed. To evaluate the rock strength after grouting, Zhang et al. established a preliminary theoretical model and an empirical formula for rock strength after grouting, which enabled prediction of the rock strength after grouting. In summary, scholars worldwide mainly use theoretical analysis, experimental analysis, and numerical simulation to evaluate the slurry diffusion and the effect of rock mass reinforcement, which greatly enriches and develops grouting theory. However, few reports present the theoretical research on the stress state of fractured surrounding rock after grouting reinforcement, and further research on the quantitative analysis of the effect of rock mass after grouting reinforcement is needed.

As one of the most effective methods for solving plane problems in surrounding rock mechanics, complex variable functions have been favored by many scholars and experts, and many achievements have been made for the problem of a multiconnected domain. The problem analyzed in this paper is the problem of a multiconnected domain. Solving this problem by the complex variable function method can be roughly divided into two categories. The first category is solving double-cavern or multi-cavern problems, and the other category is solving the problems of "one inside another". Therefore, this paper intends to use the existing grouting diffusion formula to establish the characterization formula of the strength of fractured surrounding rock after grouting reinforcement in the ideal state. Based on the "one inside another" solution of the complex variable function theory, we
derived the analytical formulas of the stress and displacement of the fractured surrounding rock after grouting reinforcement and verified the reliability of the solution by a numerical method. Therefore, we can quantitatively explore the effect of grouting reinforcement on fractured surrounding rock. It is expected that the findings of this study can provide theoretical guidance for the grouting engineering practice of deeply buried, fractured surrounding rock.

2. PARAMETER ANALYSIS OF GROUTING REINFORCEMENT

2.1. Basic Assumption and Basic Formula.

(1) In this paper, we assumed the rock mass to be an isotropic material. All the cracks and pores in the rock mass are interconnected and distributed randomly and uniformly.

(2) The slurry is isotropic and incompressible. Its particles are much smaller than the width of the cracks. The grout will only strengthen the cementation between the rock particles and will not cause rock mass compaction.

(3) During single-hole grouting, the slurry diffuses in a hemispherical way. A schematic of the main side view of the final grouting reinforcement region is shown in Figure 1. The assumed grouting reinforcement region is related to the position and number of grouting holes in the outermost circle of the cavern. There are enough grouting holes in the outermost circle, and they are infinitely close to the cavern boundary.

(4) Assuming that the grouting reinforcement effect in the cavern contour is consistent (refer to Figure 1), its strength has been improved to the same degree. The strength of the rock mass from the cavern contour to the equivalent grouting reinforcement region will gradually decrease.

2.2. Derivation of Grouting Reinforcement Formula.

Liu extended Baker’s formula and derived the expression of the diffusion radius of a single-hole grouting slurry for the flow of Newtonian fluid in two-dimensional smooth cracks:

\[ r = \sqrt[3.24]{\frac{0.093(P - P_0)b_0^{0.21}}{\eta}} + r_0 \]  

where \( P \) is the pressure in the grouting hole, \( P_0 \) is the hydrostatic pressure in the fracture at distance \( r \), \( \eta \) is the initial viscosity of the slurry, \( b \) is the crack width, \( r \) is the diffusion radius of the slurry, \( r_0 \) is the grouting hole radius, and \( t \) is the grouting time.

Based on the above basic assumptions, Liu’s formula (eq 1) is better met with our applicable conditions. When grouting is carried out, slurry particles collide with cracks and each other, causing a decrease in grouting pressure, so the slurry retention decreases. When the grouting pressure decreases to zero, the slurry will stop spreading. Logically, the grouting diffusion radius can be infinitely expanded, so the radius of the single-hole grouting reinforcement region can be obtained as

\[ r = \sqrt[3.24]{\frac{0.093P_t b_0^{0.21}}{\eta}} + r_0 \]  

(2)

where \( r_1 \) is the radius from the grouting hole center to the single-hole grouting reinforcement ring in the case of single-hole grouting (refer to Figure 2).

If the cavern radius is \( r_1 \), the radius of the equivalent grouting reinforcement region \( R \) is

\[ R = r_1 + r_2 \]  

(3)

The grouting reinforcement region can be described as shown in Figure 2.

According to the assumptions that the slurry diffusion depth is \( L \), the maximum depth of grout diffusion is \( r_2 \), the grouting retention is \( Q \), and the maximum grouting retention is \( Q_{\text{max}} \) within the cavern contour (\( r \leq r_1 \)), the slurry diffusion depth and grout retention after grouting are shown in Figure 3.

The grouting retention gradually decreases to zero from the cavern contour to the equivalent grouting reinforcement region.
When \( r_1 < r \leq R \), \( L \) and \( r - r_1 \) satisfy the relationship of the 1/4 arc, namely,
\[
(r - r_1)^2 + L^2 = r_1^2
\]
(4)

Since \( r \) is a variable in eq 4, the function \( L(r) \) can be obtained as eq 5:
\[
L(r) = \sqrt{r_1^2 - (r - r_1)^2}
\]
(5)

The grouting retention at the grouting diffusion radius \( r \) can be obtained as
\[
Q = \frac{L(r)}{r_2} Q_{\text{max}}
\]
(6)

where \( Q \) is the grouting retention and \( Q_{\text{max}} \) is the maximum grouting retention. \( Q \) is a function of \( r \), which can be sorted out by substituting eq 5 into eq 6:
\[
Q(r) = \begin{cases} 
Q_{\text{max}} & r \leq r_1 \\
\frac{\sqrt{r_1^2 - (r - r_1)^2}}{r_2} Q_{\text{max}} & r_1 < r \leq R \\
0 & r > R 
\end{cases}
\]
(7)

The elastic modulus \( E \) of rock slurry cement is related to the grouting retention \( Q \), namely, \( E = E(\max) \). \( Q \) is a function of \( r \), so \( E = E(Q(r)) \), namely, \( E = E(r) \). Therefore, when \( r \leq r_1 \), assume that the elastic modulus of rock slurry cement is \( E_{\text{max}} \). When \( r > R \), assume that the elastic modulus of rock slurry cement is \( E_{\text{max}} \). For practical problems, \( E_{\text{max}} \) and \( E_{\text{min}} \) can be measured through indoor experiments. After grouting reinforcement, the elastic modulus of rock slurry cementation will increase accordingly. When \( r_1 < r \leq R \), we can obtain \( E(Q) \) as
\[
E(Q) = \frac{Q(r)}{Q_{\text{max}}} (E_{\text{max}} - E_{\text{min}}) + E_{\text{min}}
\]
(8)

Substituting eq 7 into eq 8 gives
\[
E(r) = \begin{cases} 
E_{\text{max}} & r \leq r_1 \\
\frac{\sqrt{r_1^2 - (r - r_1)^2}}{r_2} (E_{\text{max}} - E_{\text{min}}) & r_1 \leq r \leq R \\
E_{\text{min}} & r > R 
\end{cases}
\]
(9)

After grouting reinforcement, we assume that the fractured surrounding rock is an elastic body.

2.3. Mechanical Model of the Surrounding Rock after Grouting.

(1) The cavern before and after grouting is regarded as a plane strain problem.

(2) The cavern is circular and bears a nonuniform stress on the boundary.

(3) Grouting will not affect the initial stress, and the interface between the grouting wall and the surrounding rock meets the stress continuity condition.

The mechanical model of the secondary stress state after grouting in a nonuniform stress field is shown in Figure 4. [Note: the surrounding rock after grouting consists of the grouting area \( S_2 \) and the surrounding rock \( S_3 \).]

![Figure 3. Schematic of slurry diffusion and retention. (a) Slurry retention. (b) Slurry diffusion depth.](image)

![Figure 4. Mechanical model of the secondary stress state of the surrounding rock in a nonuniform stress field after grouting.](image)
3. SOLUTION

3.1. Complex Function Expression of Stress and Displacement. The stress and displacement components can be expressed by two complex variable functions $\phi(z)$ and $\psi(z)$. The stress and displacement formulas can be described as follows:\(^\ddagger\)

$$\sigma_1 + \sigma_0 = 2[\phi(z) + \bar{\phi} (z)]$$

(10)

$$\sigma_0 - \sigma_1 + 2i\tau_{00} = 2e^{2i\theta}[\tau \phi(z) + \psi(z)]$$

(11)

$$u_r + iu_\theta = \frac{e^{i\theta}}{2G}[\kappa \phi(z) - z\bar{\phi}(z) - \psi(z)]$$

(12)

$$F_x + iF_y = -\frac{1}{k}d[\phi(z) + z\bar{\phi}(z) + \psi(z)]$$

(13)

where $\sigma_1$, $\sigma_0$, and $\tau_{00}$ are the radial stress, circumferential stress, and shear stress, respectively, at any location; $u_r$ and $u_\theta$ are the radial displacement and circumferential displacement, respectively; $z$ is the coordinate of the complex function; $i$ is the imaginary unit; $G$ is the shear modulus, $G = E/(2(1 + \mu))$; $\mu$ is Poisson’s ratio; $k$ is the coefficient, the plane strain model takes $(3-4\mu)$; and $F_x$ and $F_y$ are the horizontal force and vertical force, respectively, on the boundary.

3.2. Model Solving. 3.2.1. Stress Solution. A complex function is applied to solve the model, and the surface forces on the inner and outer sides of the grouting wall are expressed by a Fourier series as follows:\(^\ddagger\)

$$\begin{align*}
\sigma_1 - i\tau_{r0} &= 0 \\
\sigma_0 - i\tau_{00} &= \sum_{k=-\infty}^{+\infty} \tilde{A}_k e^{ik\theta}
\end{align*}$$

(14)

where $\sigma_1$ and $\tau_{r0}$ are the radial stress and shear stress at the inner boundary of the grouting wall, respectively; $\sigma_0$ and $\tau_{00}$ are the radial stress and shear stress at the outer boundary of the grouting wall, respectively; and $\tilde{A}_k$ is a coefficient. The basic equation for solving the complex function can be obtained as follows:

$$\phi(z) + \bar{\phi}(z) - [\tau \phi(z) + \psi(z)] e^{2i\theta} = 0$$

$$z = r_1 e^{i\theta}$$

$$= \sum_{k=-\infty}^{+\infty} \tilde{A}_k e^{ik\theta} z = R e^{i\theta}$$

(15)

Considering the boundary condition of infinite stress, the complex stress function in the grouting wall is expressed by $\phi_1(z)$ and $\psi_1(z)$. The complex stress function in the surrounding rock is expressed by $\phi_2(z)$ and $\psi_2(z)$, which can be determined as follows:\(^\ddagger\)

$$\begin{align*}
\phi_1(z) &= \sum_{k=-\infty}^{+\infty} a_k z^k, \quad \psi_1(z) = \sum_{k=-\infty}^{+\infty} a'_k z^k \\
\phi_2(z) &= \sum_{k=2}^{+\infty} b_k z^{-k}, \quad \psi_2(z) = \sum_{k=2}^{+\infty} b'_k z^{-k}
\end{align*}$$

(16)

According to the method in refs 26–28, eq 16 can be expressed as follows:

$$\begin{align*}
\phi_1(z) &= \sum_{k=-\infty}^{+\infty} a_k z^k, \quad \psi_1(z) = \sum_{k=-\infty}^{+\infty} a'_k z^k \\
\phi_2(z) &= \sum_{k=2}^{+\infty} b_k z^{-k}, \quad \psi_2(z) = \sum_{k=2}^{+\infty} b'_k z^{-k}
\end{align*}$$

(17)

$$\begin{align*}
\phi_1(z) &= \sum_{k=-\infty}^{+\infty} a_k z^k, \quad \psi_1(z) = \sum_{k=-\infty}^{+\infty} a'_k z^k \\
\phi_2(z) &= \sum_{k=2}^{+\infty} b_k z^{-k}, \quad \psi_2(z) = \sum_{k=2}^{+\infty} b'_k z^{-k}
\end{align*}$$

(17)

Among them,

$$\begin{align*}
\Gamma' &= \frac{1 + \lambda}{4} p \\
\Gamma'' &= \frac{1 - \lambda}{2} p
\end{align*}$$

(18)

The boundary conditions and contact conditions are expressed as follows:

$$(\sigma_1 - i\tau_{00})|_{r=r_1} = 0$$

(19)

$$(\sigma_0 - i\tau_{00})|_{r=R} = (\sigma_0' - i\tau_{00})|_{r=R}$$

(20)

Substituting $z = re^{i\theta}$ into eq 20 and combining this equation with the stress boundary condition of the inner grouting wall, we can obtain

$$\begin{align*}
\sum_{k=-\infty}^{+\infty} (1 - k) a_k r^k e^{ik\theta} + \sum_{k=-\infty}^{+\infty} a'_k r^{-k} e^{ik\theta} \\
- \sum_{k=-\infty}^{+\infty} a'_k r^{-k-2} e^{ik\theta}
\end{align*}$$

(21)

$$= \begin{cases} 0 & r = r_1 \\ \sum_{k=-\infty}^{+\infty} \tilde{A}_k e^{ik\theta} & r = R \end{cases}$$

The coefficients of the same power terms of $e^{i\theta}$ are compared on both sides of the equation. According to the single-value condition of displacement \(^3\)

$$(3 - 4\mu)a_{-1} + \tilde{a}_{-1} = 0$$

we can obtain

$$\begin{align*}
0 &= k = -1 \\
\frac{A_{-1} R^3}{2(R^2 - r_1^2)} &= k = 0 \\
\frac{A_{+1} R^3}{R^4 - r_1^4} &= k = 1 \\
\frac{[(1 + k)(R^2 - r_1^2)R^2-k\tilde{A}_k]}{[k-(R^2-2k)-r_1^2-2k]} &= k = \text{other}
\end{align*}$$

(22)

For the solution of $\tilde{a}_{-k}$, when $k \neq 0, 1$, and $-1$, $k$ is taken as $-k$ in eq 22 and then as the conjugate. Inserting $a_k$ and $\tilde{a}_{-k}$ into eq 21, $a_k$ is obtained. According to eq 20, we can obtain
\[ \phi'(z) + \overline{\phi'(z)} - [\pi \phi'(z) + \psi'(z)] e^{2i\theta} = \sum_{k=-\infty}^{+\infty} A_k e^{ik\theta} \]
\[ z = R e^{i\theta} \]  
(23)

Substituting eqs 16 and 17 and \( z = Re^{i\theta} \) into eq 23 and simplifying yields
\[ \sum_{k=-\infty}^{+\infty} (1 - k)a_k R^k e^{ik\theta} + \sum_{k=-\infty}^{+\infty} \alpha_{k-2} R^{k-2} e^{ik\theta} - \sum_{k=-\infty}^{+\infty} \alpha_k R^k e^{ik\theta} = 0 \]
\[ \sum_{k=-\infty}^{+\infty} (1 - k)b_k R^k e^{ik\theta} = \sum_{k=-\infty}^{+\infty} \beta_k R^k e^{ik\theta} = \sum_{k=-\infty}^{+\infty} \gamma_k R^k e^{ik\theta} = \sum_{k=-\infty}^{+\infty} \lambda_k e^{ik\theta} = 0 \]  
(24)

By comparing the coefficients of the same power terms of \( e^{ik\theta} \) on both sides of eq 24, we can obtain
\[ \begin{align*}
(1 - k)a_k R^k + \alpha_{k-2} R^{k-2} - \alpha_k R^k &= 0 \\
(1 - k)\beta_k R^k + \gamma_k R^k &= 0 \\
(1 - k)\gamma_k R^k + \lambda_k &= 0
\end{align*} \]
\[ k \leq -2 \]
\[ = k = 1 \]
\[ \geq 3 \]  
(25)

According to another stress continuity condition of eq 20, we substitute eq 10 into eq 11 and then into eq 20 to obtain
\[ \phi'(z) + \overline{\phi'(z)} - [\pi \phi'(z) + \psi'(z)] e^{2i\theta} = \phi'_1(z) + \overline{\phi'_1(z)} + [\pi \phi'_1(z) + \psi'_1(z)] e^{2i\theta} \]  
(26)

Substituting eqs 16 and 17 and \( z = Re^{i\theta} \) into eq 26 and simplifying, we can obtain
\[ \sum_{k=-\infty}^{+\infty} (1 + k)a_k R^k e^{ik\theta} + \sum_{k=-\infty}^{+\infty} \alpha_{k-2} R^{k-2} e^{ik\theta} = 0 \]
\[ \sum_{k=-\infty}^{+\infty} (1 - k)b_k R^k e^{ik\theta} + \sum_{k=2}^{+\infty} \gamma_k R^k e^{ik\theta} = \frac{1}{2} \lambda R e^{2i\theta} + \sum_{k=2}^{+\infty} \beta_k R^k e^{ik\theta} \]  
(27)

By comparing the coefficients of the same power terms of \( e^{ik\theta} \) on both sides of the equation, we can obtain
\[ (1 + k)a_k R^k + \alpha_{k-2} R^{k-2} + \alpha'_k R^{k-2} = 0 \]
\[ k \leq -2 \]
\[ = (1 + k)b_k R^k + \beta_k R^k = 0 \]
\[ k = 0 \]
\[ = (1 + k)c_k R^k + \gamma_k R^k = 0 \]
\[ k = 1 \]
\[ \geq 3 \]  
(28)

According to eq 19, we obtain
\[ (1 - k)a_k R^k + \alpha_{k-2} R^{k-2} = 0 \]  
(29)

By analyzing and calculating eqs 22, 25, and 29, we find that only a few coefficients in the complex stress function are not zero, as described in eq 30, and that the remaining coefficients are zero (the detailed solution and proof process are shown in Appendix A).

\[ a_{-1} = -1 - \frac{1}{2} \frac{2}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2}, \quad a_0 = \frac{1}{4} \]  
\[ a_1 = \frac{1}{2} \frac{1}{4} \frac{R^2}{R^2 - r_i^2}, \quad a'_0 = \frac{1}{2} \frac{3}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} \]  
\[ a'_1 = \frac{1}{2} \frac{1}{2} \frac{R e^{2i\theta}}{R^2 - r_i^2}, \quad a'_2 = \frac{1}{2} \frac{3}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} + \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} \]  
\[ a'_2 = \frac{1}{2} \frac{1}{2} \frac{R e^{2i\theta}}{R^2 - r_i^2}, \quad a'_4 = \frac{1}{2} \frac{3}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} \]  
\[ b_2 = \frac{1}{2} \frac{1}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} \]  
\[ b'_2 = \frac{1}{2} \frac{1}{2} \frac{R e^{2i\theta}}{R^2 - r_i^2}, \quad b'_4 = \frac{1}{2} \frac{3}{4} \frac{R e^{2i\theta}}{R^2 - r_i^2} + \frac{R^2}{R^2 - r_i^2} \]  
(30)

Thus, the first derivative of the complex stress function of the grouting wall and surrounding rock can be obtained as shown in eqs 31 and 32:
\[
\phi_1(z) = -\frac{1 - \lambda}{2} \frac{2R^4 + 3R^2r_1^2 + r_1^4}{4(R^2 - r_1^2)} z^{-1} + \frac{1 + \lambda}{4} z + \frac{1 - \lambda}{2} \frac{3R^4 + R^2r_1^2}{4r_1^2(R^2 - r_1^2)} z^{-2}
+ \frac{1 + \lambda}{2} p \frac{3R^4 + R^2r_1^2}{4r_1^2(R^2 - r_1^2)} z^2
+ \frac{1 + \lambda}{4} + \frac{1 - \lambda}{2} p \frac{3R^4 + R^2r_1^2 + R^2}{4r_1^2(R^2 - r_1^2)} z^3.
\]

\[
\psi_1(z) = -\frac{1 - \lambda}{2} \frac{2R^4 + 3R^2r_1^2 + r_1^4}{4(R^2 - r_1^2)} z^{-1} + \frac{1 + \lambda}{4} z + \frac{1 - \lambda}{2} \frac{3R^4 + R^2r_1^2}{4r_1^2(R^2 - r_1^2)} z^{-2}
+ \frac{1 + \lambda}{2} p \frac{3R^4 + R^2r_1^2 + R^2}{4r_1^2(R^2 - r_1^2)} z^2
+ \frac{1 + \lambda}{4} + \frac{1 - \lambda}{2} p \frac{3R^4 + R^2r_1^2 + R^4}{2r_1^2(R^2 - r_1^2)} z^3.
\]

Simultaneously, the complex stress function eqs 31, 32, and 34 are expressed as follows:

\[
\sigma = 2 \text{Re} \phi(z) - \text{Re} \left[ \bar{\Sigma} \phi^*(z) + \psi^*(z) \right] \cos \theta + \text{Im} \left[ \bar{\Sigma} \phi^*(z) + \psi^*(z) \right] \sin \theta
\]

\[
\sigma_\theta = 2 \text{Re} \phi(z) - \text{Re} \left[ \bar{\Sigma} \phi^*(z) + \psi^*(z) \right] \cos \theta + \text{Im} \left[ \bar{\Sigma} \phi^*(z) + \psi^*(z) \right] \sin \theta
\]

\[
\sigma_\phi = -2 \text{Re} \phi^*(z) - \psi^*(z) \cos \theta - \text{Im} \bar{\Sigma} \phi^*(z) + \psi^*(z) \sin \theta
\]

Therefore, the stress component \((\sigma_r, \sigma_\theta, \tau_{r\theta})\) of the grouting wall can be obtained as shown in eq 36:

\[
\sigma = \frac{1 + \lambda}{2} p \left[ 1 - \frac{1}{r_1^2} \right] + \frac{(1 - \lambda) p \cos \theta}{2r_1^4(R^2 - r_1^2)} \left\{ 2r_1^4 + R^2r_1^2 + R^4 + \frac{3R^4 + R^2r_1^2 + R^2}{r^4} \right\}
- \frac{4R^4r_1^2 + 2R^2r_1^2 + 2r_1^6}{r^2}
\]

\[
\sigma_\theta = \frac{1 + \lambda}{2} p \left[ 1 + \frac{1}{r_1^2} \right] + \frac{(1 - \lambda) p \cos \theta}{2r_1^4(R^2 - r_1^2)} \left\{ 6r_1^6 + 2R^2r_1^2 - \frac{3R^4 + R^2r_1^2 + R^6}{r^4} - 2r_1^4 - R^2r_1^2 - R^4 \right\}
- \frac{3R^4r_1^2 + R^2r_1^6 + R^4 + 2R^2r_1^2 + 2r_1^4 + 2r_1^6}{r^4}
\]

The stress component \((\sigma_r', \sigma_\theta', \tau_{r\theta}')\) of the surrounding rock can be obtained as shown in eq 37:

\[
\sigma' = \frac{1 + \lambda}{2} p \left[ 1 - \frac{1}{r_1^2} \right] + \frac{(1 - \lambda) p \cos \theta}{2r_1^4(R^2 - r_1^2)} \left\{ 3(R^2 + r_1^2)^2 + \frac{R^6 + 4R^2r_1^2 + R^4}{2r_1^4} \right\}
\]

\[
\sigma_\theta' = \frac{1 + \lambda}{2} p \left[ 1 + \frac{1}{r_1^2} \right] + \frac{(1 - \lambda) p \cos \theta}{2r_1^4(R^2 - r_1^2)} \left\{ (R^2 + r_1^2)^2 + \frac{R^6 + 4R^2r_1^2 + R^4}{2r_1^4} + 1 \right\}
\]

\[
\tau_{r\theta}' = \frac{1 - \lambda}{2} p \sin \theta \left\{ \frac{R^6 + 4R^2r_1^2 + R^4}{2r_1^4} - \frac{(R^2 + r_1^2)^2}{2r_1^4} - 1 \right\}
\]
3.2.2. Displacement Solution. The general displacement can be obtained by substituting eqs 31 and 32 into 12, as shown in eqs 38 and 39:

\[
\begin{align*}
    u_r &= \frac{P}{4G} \left[ r(1 - 2\mu)(1 + \lambda) + \frac{(1 + \lambda)r_1^2}{r} \right] \\
    &\quad + \frac{P}{4G} \left( \frac{1 - \lambda}{2} \right) \cos 2\theta \\
    &= \frac{P}{4G} \left[ (1 - \lambda) \frac{p \sin 2\theta (2R^4 + R^2r_1^2 + r_1^4)(1 - \mu)}{4G(R^2 - r_1^2)} \right] \\
    &\quad + \frac{P}{4G} \left( \frac{1 - \lambda}{2} \right) \frac{(3 - 2\mu)(3r_1^2 + R^2)r^3}{6r_1^2} + \frac{3r_1^4 + R^4r_1^4}{6r^3} \\
    &\quad - \frac{2r^4 + R^2r_1^2 + R^4}{2r_1^2} \\
    \end{align*}
\]

\[ (39) \]

According to eq 10, in the grouting wall \((r_1 < r \leq R)\), the shear modulus \(G\) can be expressed as

\[
G = \frac{\sqrt{r^2 - (r_c)^2} (E_{\text{mat}} - E_{\text{mix}}) + E_{\text{mix}}}{2(1 + \mu)}
\]

\[ (40) \]
Figure 6. Radial and circumferential stress of the rock mass on the side of the cavern ($\theta = 0^\circ$). (a) Radial stress. (b) Circumferential stress.

Substituting eq 40 into eqs 38 and 39, the expressions for the radial and circumferential displacements related to the grouting parameters are

\[ u_r = \frac{(1 + \mu)p\left(r(1 - 2\mu)(1 + \lambda) + \frac{(1 + \lambda)r_1^3}{r}\right)}{2\sqrt{r^2 - (r - r_1)^2}(E_{\text{max}} - E_{\text{min}}) + 2E_{\text{min}}} + \frac{(1 + \mu)p\cos 2\theta}{2r_1^2(R^2 - r_1^2)}\]

\[ \left[\frac{2R^4 + R^2r_1^4 + r_1^6(2\mu - 1)}{r} + \frac{(3R^4r_1^4 + R^2r_1^6)}{3r^3}\right] + \frac{r(2r_1^4 + R^2r_1^4 + R^4) - r_1^3\mu(6r_1^2 + 2R^2)}{3} \]

\[ (41) \]

\[ u_\theta = \frac{(1 - \lambda)(1 + \mu)p\sin 2\theta}{2\sqrt{r^2 - (r - r_1)^2}(E_{\text{max}} - E_{\text{min}}) + 2E_{\text{min}}} \]

\[ \left[\frac{2R^4 + R^2r_1^4 + r_1^6(1 - \mu)}{r} + \frac{(3 - 2\mu)(3r_1^2 + R^2)r^3}{6r_1^2}\right] + \frac{3R^4r_1^4 + R^2r_1^6 - 2r_1^4 + R^2r_1^4 + R^4}{2r_1^2} \]

\[ (42) \]

When \( r = r_1 \), the displacement around the cavern can be obtained as follows:

\[ u_{r|r=r_1} = \frac{(1 - \mu^2)(1 + \lambda)p \sqrt{r_1} + (1 + \mu)p(1 - \lambda)\cos 2\theta}{E_{\text{max}}} \]

\[ \left[\frac{4\mu R^2r_1 + r_1^5 + \frac{(1 + 4\mu)R^2r_1^3}{3}\right] \]

\[ (43) \]

4. RESULTS

In this section, the derived solution is applied to an example, and a comparison is provided with ABAQUS finite element code to verify the solution. To further discuss the evolution of the stress field and the displacement field after cavern grouting reinforcement, we take the overlying load \( p = 5 \) MPa, lateral pressure coefficient \( \lambda = 0.8 \), cavern radius \( r_1 = 2 \) m, grouting wall outer diameter \( R = 4 \) m, Poisson’s ratio \( \mu = 0.4 \), elastic modulus \( (E_{\text{max}}) \) of the rock mass before grouting as 2000 MPa, and maximum elastic modulus \( (E_{\text{min}}) \) of the grouting rock mass-grout cement as 4000 MPa.

4.1. Comparison of the Analytical Solution with ABAQUS Finite Element Code. We select ABAQUS numerical analysis software to establish a 2D plane model. Thereafter, the correctness of the analytical solution is verified by comparing the rock mass stress at \( \theta = 0^\circ \) and \( 90^\circ \). As shown in Figure 5, there are certain differences between the analytical solution and the numerical solution. The finite element method of meshing is not sufficient enough to cause certain discrepancies in the results. In addition, the assignment of the finite element method will also cause certain differences in the simulation results. However, the general trend between the analytical solution and the numerical solution is generally consistent. Overall, the analytical and numerical solutions show agreement, indicating the correctness of the analytical solution.

4.2. Stress Characteristics of the Rock Mass before and after Grouting. 4.2.1. Stress Evolution of the Rock Mass around the Cavern. Substituting the assumed data into eqs 36 and 37, we can obtain the stress solutions of the grouting wall and surrounding rock after grouting.
The stress solutions under a conventional nonuniform stress field are shown in eq 47:

\[
\sigma_a = [(1 + \lambda)(1 - r^2/r^2) - (1 - \lambda)] (1 - 4r^2/r^2 + 3r^4/r^4) \cos 2\theta \rho/2
\]

\[
\sigma_0 = [(1 + \lambda)(1 + r^2/r^2) + (1 - \lambda)(1 + 3r^4/r^4)] \cos 2\theta \rho/2
\]

\[
\tau_{\theta} = -[(1 - \lambda)(1 + 2r^2/r^2 - 3r^4/r^4) \sin 2\theta] \rho/2
\]

Substituting the assumed data into eq 47 yields

\[
\sigma_a = [1.8(1 - 4r^2) - 0.2(1 - 16r^2 + 48r^4)] \cos 2\theta0.75
\]

\[
\sigma_0 = [1.8(1 - 4r^2) - 0.2(1 + 48r^4)] \cos 2\theta0.75
\]

\[
\sigma_\theta = -[0.2(1 + 8r^2 - 48r^4) \sin 2\theta]0.75
\]

The radial and circumferential stress curves before and after grouting on the wall of the cavern \((\theta = 0^\circ)\) are shown in Figure 6.

As shown in Figure 6, the peak radial stress after grouting is approximately 4.48 MPa, which is approximately 12.0% higher than that before grouting. The peak circumferential stress before grouting is 11.00 MPa, and the peak circumferential stress after grouting appearing at the interface between the grouting wall and the surrounding rock is approximately 8.19 MPa, which is approximately 25.5% lower than that before grouting. The peak radial stress and circumferential stress are listed in Table 1.

On the roof of the cavern \((\theta = 90^\circ)\), the radial and circumferential stress curves before and after grouting are shown in Figure 7.

As shown in Figure 7, the peak circumferential stress after grouting appears at the cavern boundary and is 7.00 MPa. The peak circumferential stress after grouting appears at the cavern boundary and is 14.00 MPa, which is twice as high as that before grouting. The minimum circumferential stress after grouting appears at the interface between the grouting wall and the surrounding rock and is approximately 3.06 MPa. The peak radial stress and circumferential stress are listed in Table 2.

In summary, the stress curve before grouting is a smooth curve. After grouting, the stress curve of the rock mass at the interface between the grouting wall and the surrounding rock has an "inflection point". The "inflection point" of the radial stress curve is relatively smooth, while the "inflection point" of the circumferential stress curve is relatively sharp. Grouting has an obvious modification effect on the rock mass, and the influence on the circumferential stress is greater than that on radial stress.

4.2.2. Circumferential Stress of the Rock Mass around the Cavern. According to eqs 45 and 48, the circumferential stress curves around the cavern before and after grouting are shown in Figure 8.

When \(\theta \in (0^\circ, 360^\circ)\), the circumferential stress before grouting is greater than that after grouting. When \(\theta \in (45^\circ, 135^\circ)\) or \((225^\circ, 315^\circ)\), the circumferential stress before grouting is less than that after grouting. Before grouting, the peak circumferential stress appears on the wall (\(\theta = 0^\circ\) and 180°, i.e., \(\theta = 360^\circ\), where the positions of 0° and 360° below are the same and will not be repeated here). The peak circumferential stress is 11.00 MPa, and the minimum circumferential stress appears at 90° and 270° around the cavern, which is the roof and floor of the cavern. The minimum value is 7.00 MPa, and the difference between the peak value and the minimum value is 4.00 MPa. After grouting, the peak value of circumferential stress appears at 90° and 270° around the cavern, and the peak value is 14.00 MPa. The minimum value appears at 0° and 180° around the cavern and is 4.00 MPa. The difference between the peak value and the minimum value is 10.00 MPa, and the stress curve fluctuates greatly. Circumferential stress parameters around the cavern are shown in Table 3.

In summary, compared with the peak circumferential stress before and after grouting, the minimum value decreases. The peak value before grouting appears at the wall of the cavern. However, the peak value after grouting appears at the roof and floor. Grouting reinforcement significantly affects the secondary stress state of the surrounding rock. The increased circumferential stress at the roof and floor is beneficial to the stability of the cavern. The transfer of circumferential stress to the roof and floor also benefits to the stability of the cavern.

4.3. Radial Displacement around Cavern. Substituting the assumed data into eq 43, we can obtain the radial displacement around the cavern after grouting in this example.

\[
u_{r=\theta} = 0.00378 + 0.0017539 \cos 2\theta
\]

The radial displacement around the cavern before grouting is shown in eq 50:
Substituting the assumed value into eq 50, we get

\[ u_{r=1} = \frac{(1 + \mu) p_{n}}{2E_{\min}} - \{(1 + \lambda) + (1 - \lambda)(3 - 4\mu) \cos 2\theta\} \]

(50)

Substituting the assumed value into eq 50, we get

\[ u_{r=1} = 0.0063 + 0.00098 \cos 2\theta \]

(51)

According to eqs 49 and 51, the radial displacement curves around the cavern before and after grouting are shown in Figure 9.

Table 2. Stress Comparison (\( \theta = 90^\circ \))

|               | peak radial stress (MPa) | PIOPRSAG (%) | peak circumferential stress (MPa) | PIOCSAG (%) |
|---------------|--------------------------|--------------|-----------------------------------|--------------|
| nongrouted    | 5.00                     | 0            | 7.00                              | 100          |
| grouted       | 5.00                     | 14.00        |                                   |              |

Table 3. Related Parameters of Circumferential Stress around the Cavern

|                  | peak value (MPa) | position where the peak value occurs | minimum value (MPa) | position where the minimum value occurs | difference between the peak value and the minimum value (MPa) |
|------------------|-----------------|-------------------------------------|---------------------|-----------------------------------------|-------------------------------------------------------------|
| nongrouted       | 11.00           | 0°, 180°                             | 7.00                | 90°, 270°                               | 4.00                                                        |
| grouted          | 14.00           | 90°, 270°                            | 4.00                | 0°, 180°                               | 10.00                                                       |

Figure 7. Radial and circumferential stress of the rock mass on the side of the cavern (\( \theta = 90^\circ \)). (a) Radial stress. (b) Circumferential stress.

Figure 8. Circumferential stress before and after grouting. (a) Rectangular coordinate diagram. (b) Polar coordinate diagram.
Figure 9 shows that the radial displacement around the cavern after grouting is smaller than that before grouting. On the wall of the cavern ($\theta = 0^\circ$ and $180^\circ$), the maximum radial displacement is approximately 5.53 mm after grouting and approximately 7.28 mm before grouting, which is a reduction of approximately 24.0%. At the roof and floor of the cavern ($\theta = 90^\circ$ and $270^\circ$), the minimum radial displacement is generated, which is about 2.03 mm after grouting and approximately 5.32 mm before grouting, which is a reduction of approximately 61.8%. Grouting has a significant effect on the radial displacement around the cavern, especially the deformation of the roof and floor of the cavern.

5. CONCLUSIONS

(1) This paper presents an analytical solution of a fractured surrounding rock under a nonuniform stress field after grouting reinforcement. Based on the complex function theory, we solved the general solutions of stress and displacement in the polar coordinate system of the deeply buried circular cavern. The reliability is verified by numerical methods. The research results of this paper provide a theoretical basis for grouting modification to improve the overall strength of fractured surrounding rock from a quantitative point of view and provide support for the design of grouting reinforcement schemes for deep fractured rock mass engineering.

(2) Through comparative analysis of examples, the radial and circumferential stress curves have “inflection points” at the interface between the grouting wall and the surrounding rock. The inflection points of the radial stress curves are relatively smooth, and the inflection points of the circumferential stress curves are relatively sharp. The effect of grouting reinforcement on circumferential stress is greater than that on radial stress. Taking the circumferential stress around the cavern as an example, after grouting, the peak value increases and the minimum value decreases.

(3) A comparison of the radial displacement curves around the cavern reveals that the displacement curves fluctuate greatly after grouting, that the displacement of the wall is reduced by 24.0%, and that of the roof and floor is reduced by approximately 61.8%. After grouting, the displacement around the cavern is obviously decreased due to the modification in the stress state and the improvement in the surrounding rock strength. The influence of grouting on the deformation of the roof and floor is greater than that of the wall.

APPENDIX A

According to eqs 23, 26, 29, and 30 in the text, when $k = 0$, eq A.1 can be obtained as follows:

$$
\begin{align*}
 a_0 + \pi_0 - a_{-2}R^2 &= \frac{1 + \lambda}{2} p - b_2R^{-2} = \bar{A}_0 \\
 a_0 + \pi_0 + a_{-2}R^2 &= \frac{1 + \lambda}{2} p + b_2R^{-2} \\
 a_0 + \pi_0 - a_{-2}r_2^{-2} &= 0 \\
 a_0 &= \frac{\bar{A}_0 R^2}{2(R^2 - r_1^2)}
\end{align*}
$$

(A.1)

According to eq A.1,

$$
a_0 = p\frac{1 + \lambda}{4}, \quad a_{-2} = p\frac{(1 + \lambda)r_1^2}{2}
$$

(A.2)

Similarly, we can obtain eq A.3 when $k = 1$:

$$
\begin{align*}
 \bar{\pi}_{-1}R^{-1} - a_{-1}R^{-1} &= 0 \\
 2a_1R + \bar{\pi}_{-1}R^{-1} - a_{-1}R^{-1} &= 0 \\
 a_{-1} &= 0
\end{align*}
$$

(A.3)

According to eq A.3, we can obtain

$$
a_{-1} = a_1 = 0 \Rightarrow \bar{A}_{-1} = 0
$$

(A.4)

When $k = -1$, we can obtain
According to eqs A.4 and A.5, eq A.6 can be simultaneously obtained:

\[
\begin{align*}
\alpha_{-2}' &= b_2' = 0 \\
\alpha_\pm &= b_\pm = 0 \\
\alpha_2 &= \frac{1 - \lambda}{2} p - \frac{3R^2\tau_2^2 + R^4\tau_2^4}{(R^2 - \tau_2^2)} - \frac{1 - \lambda}{2} p = \tilde{A}_2
\end{align*}
\]

(A.8)

According to eq A.8, all of the coefficients in eq A.7 and \( \tilde{A}_2 \) are obtained as

\[
\begin{align*}
\alpha_{-2}' &= -\frac{1 - \lambda}{2} p - \frac{2R^4 + R^2\tau_1^2 + \tau_1^4}{4(R^2 - \tau_1^2)} \\
\alpha_2 &= \frac{1 - \lambda}{2} p - \frac{3R^2\tau_2^2 + R^4}{2R^2(R^2 - \tau_2^2)} \\
\alpha_0' &= \frac{1 - \lambda}{2} p - \frac{2R^4 + R^2\tau_1^2 + \tau_1^4}{2R^2(R^2 - \tau_1^2)} \\
b_2 &= \frac{1 - \lambda}{2} p - \frac{R^2 + \tau_1^4 - R^2\tau_1^4 - \tau_1^6}{4R^2(R^2 - \tau_1^2)}
\end{align*}
\]

(A.9)

When \( k = -2 \), we can obtain

\[
\begin{align*}
3a_{-2}R^2 + \alpha_2 R^2 - \alpha_{-2}' R^4 &= 3b_2 R^2 - b_\pm R^4 = \tilde{A}_{-2} \\
- \alpha_{-2}'R^2 + \alpha_2 R^2 + \alpha_{-2}' R^4 &= -b_\pm R^4 + b_\pm R^4
\end{align*}
\]

(A.10)

Combining eq A.10 with eq A.9 yields

\[
\begin{align*}
\alpha_{-2}' &= -\frac{1 - \lambda}{2} p - \frac{3R^2\tau_1^2 + \tau_1^4}{2(R^2 - \tau_1^2)} \\
b_2' &= \frac{1 - \lambda}{2} p - \frac{3R^4\tau_1^6 - 3R^4\tau_1^4 - R^2\tau_1^6}{2R^2(R^2 - \tau_1^2)}
\end{align*}
\]

(A.11)

The other coefficients are 0, which is proven in detail in what follows.

When \( k \geq 3 \), we can obtain

\[
\begin{align*}
(1 - k)a_k R^k + \alpha_2 R^k - \alpha_{-2}' R^{k - 2} &= \tilde{b}_k R^k = \tilde{A}_k \\
(1 - k)a_k \tau_1^k + \alpha_2 \tau_1^k &= \alpha_{-2}' \tau_1^{k - 2} \\
(1 + k)a_k R^k + \alpha_2 R^k + \alpha_{-2}' R^{k - 2} &= \tilde{b}_k R^k
\end{align*}
\]

(A.12)
pressure in the grouting hole
P

hydrostatic pressure in the fracture at distance r
P_0

grouting hole radius
r_0

initial viscosity of slurry
η

radius of cavern
r_1

crack width
b

radius of single-hole grouting reinforcement region
r_2

grouting time
\tau

radius of equivalent grouting reinforcement region
R

slurry diffusion depth
L

grouting retention
Q

maximum grouting retention
Q_{max}

radial displacement of cavern before grouting
\delta u_r

radial stress
\sigma_r

modulus of rock slurry cement
E_f

circumferential stress
\sigma_\theta

nongrouted rock mass
E_m

shear stress
\sigma_\tau

radial displacement
u_r

coordinate of complex function
z

circumferential displacement
u_\theta

shear modulus
G

Poisson’s ratio
\kappa

coefficient, takes (3 – 4\mu)
\mu

horizontal force
F_x

vertical force
F_y

natural logarithm
\ln

imaginary unit
i

radial stress at the outer boundary of the grouting wall
\sigma_r'

radial stress at the inner boundary of the grouting wall
\sigma_r

shear stress at the outer boundary of the grouting wall
\tau_{\theta 0}

shear stress at the inner boundary of the grouting wall
\tau_{\theta 0}'

coefficient of Fourier series
\lambda_k

serial number of correlation coefficient
k

coefficient of \phi_1(z)
\alpha_k

coefficient of \psi_1(z)
b_k

coefficient of \phi_2(z)
\beta_k

coefficient of \psi_2(z)
\delta_k

angle from the positive half axis of x
\theta

lateral pressure coefficient
\lambda

overlying load
\rho

real part of complex number
Re

imaginary part of complex number
Im

stress component of grouting wall after grouting
\sigma_{r'}(z), \sigma_{\theta'}(z)

stress component of surrounding rock after grouting
\sigma_{r''}(z), \sigma_{\theta''}(z)

stress component before grouting
\sigma_{r''}(z)

complex variable functions
\phi_1(z), \psi_1(z)

complex stress function in grouting wall
\phi_2(z), \psi_2(z)

complex stress function in surrounding rock

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