Nearly Antiferromagnetic Fermi Liquids: A Progress Report

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Abstract

I describe recent theoretical and experimental progress in understanding the physical properties of the two dimensional nearly antiferromagnetic Fermi liquids (NAFL’s) found in the normal state of the cuprate superconductors. In such NAFL’s, the magnetic interaction between planar quasiparticles is strong and peaked at or near the commensurate wave vector, $Q \equiv (\pi, \pi)$. For the optimally doped and underdoped systems, the resulting strong antiferromagnetic correlations produce three distinct magnetic phases in the normal state: mean field above $T_{cr}$, pseudoscaling between $T_{cr}$ and $T_*$, and pseudogap below $T_*$. I present arguments which suggest that the physical origin of the pseudogap found in the quasiparticle spectrum below $T_{cr}$ is the formation of a precursor to a spin-density-wave-state, describe the calculations based on this scenario of the dynamical spin susceptibility, Fermi surface evolution, transport, and Hall effect, and summarize the experimental evidence in its support.
Introduction

A decade of experiments have shown us that nearly all the normal state properties of the cuprate superconductors are anomalous when compared to the Landau Fermi liquids found in the normal state of conventional superconductors. It is generally agreed that the mechanism for high temperature superconductivity must be directly related to this unusual normal state behavior. The proposal that it is the magnetic interaction between planar quasiparticles which is responsible for their anomalous normal state behavior and the transition at high temperatures to a superconducting state with $d_{x^2-y^2}$ pairing was made some seven years ago at a conference on strongly correlated electron systems. It was based on the first generation of nuclear magnetic resonance (NMR) experiments, which showed that the quasiparticles in even optimally doped systems such as YBa$_2$Cu$_3$O$_7$ and La$_{1.85}$Sr$_{0.15}$ displayed almost antiferromagnetic behavior; it led to the ansatz that the normal state is best described as a nearly antiferromagnetic Fermi liquid (NAFL) in which the effective magnetic interaction between planar quasiparticles mirrors the highly anisotropic momentum dependence found in NMR measurements of the spin-spin response function. In this lecture, I review recent theoretical and experimental progress in characterizing the behavior of two-dimensional NAFL’s [a detailed review of work on the NAFL prior to 1995 may be found in Ref. (2)], and in understanding the normal state of the optimally doped and underdoped cuprate superconductors.

Magnetic Behavior and Phase Diagram

Of the various anomalous aspects of normal state behavior of the superconducting cuprates, the low frequency magnetic response is perhaps the most unusual, in that one finds nearly antiferromagnetic behavior and three distinct magnetic phases in all but the highly overdoped systems. A quantitative fit to NMR and INS experiments can be obtained with a phenomenological expression for the dynamical spin susceptibility, $\chi(q,\omega)$, which reflects this close approach to antiferromagnetism. Quite generally, one finds four peaks in $\chi$ at wave vectors, $Q_i$, in the vicinity of the commensurate AF wave vector, $Q = (\pi/a, \pi/a)$, which are located symmetrically about $Q$. In the vicinity of a given peak at $Q_i$, $\chi$ displays considerable structure; it takes the
form proposed by Millis et al. [4],

$$\chi_{\text{NAFL}}(\mathbf{q}, \omega) = \frac{\chi_{Qi}}{1 + (Q_i - \mathbf{q})^2 \xi^2 - i\omega / \omega_{\text{SF}}} \quad (1)$$

where as one approaches the superconducting transition temperature $T_c$, $\chi_{Qi} \equiv \alpha \xi^2$, the static peak susceptibility, is some orders of magnitude larger than the uniform susceptibility, $\chi_0$ and $\xi$, the antiferromagnetic correlation length, is large compared to the lattice spacing, $a$. For example, in La$_{1.86}$Sr$_{0.14}$CuO$_4$, just above $T_c$, neutron scattering experiments [5] show that $\chi_{Qi} = 350$ states/eV and $\xi \approx 7.7a$. The frequency of the relaxational mode, $\omega_{\text{SF}}$, is of order $10 - 20$meV, quite small compared to its value, the Fermi energy or bandwidth, $\sim eV$, in a Landau Fermi liquid. Away from $Q_i$, for $q \xi > 1$, the dynamical susceptibility tends to be Fermi-liquid like, taking the form,

$$\chi_{\text{FL}}(\mathbf{q}, \omega) = \frac{\chi_0}{1 - i\omega / \Gamma_\mathbf{q}} \quad (2)$$

where $\Gamma_\mathbf{q}$ is comparable to the bandwidth, and $\chi_0 \sim \chi_0$.

NMR experiments [6] on quantities dominated by $\chi_{\text{NAFL}}$, the $^{63}$Cu spin-lattice relaxation rate, $^{63}T_1^{-1} \sim (T / \omega_{\text{SF}})$, and the spin-echo decay time, $T_{2G} \sim \xi^{-1}$, show that $\chi_{\text{NAFL}}(\mathbf{q}, \omega)$ varies dramatically with doping and temperature through changes in $\chi_{Qi}$, $\omega_{\text{SF}}$, $\xi$, and the dependence of $\omega_{\text{SF}}$ on $\xi$. [7] Specifically, one finds that above a temperature, $T_{cr}$, $\chi_{\text{NAFL}}$ displays mean field or RPA behavior, with $\omega_{\text{SF}}$, and $\xi^{-2}$, varying linearly with temperature,

$$\omega_{\text{SF}}(T) \sim \xi^{-2}(T) \sim a + bT.$$  

Between $T_{cr}$ and a second crossover temperature, $T_*$, $\chi$ displays $z = 1$ dynamical scaling behavior, with

$$\omega_{\text{SF}}(T) \sim \xi^{-1}(T) \sim c + dT.$$  

The phase between $T_{cr}$, and $T_*$, is called “pseudoscaling” because the scaling behavior found there is not universal. Below $T_*$, one enters the pseudogap phase, in which the antiferromagnetic correlations become frozen, while $\omega_{\text{SF}}$, after reaching a minimum, increases rapidly as the temperature is further decreased. “Pseudogap” denotes the quasiparticle gap-like behavior found
between $T_*$ and $T_c$, a behavior which is not accompanied by the long range order of an antiferromagnet or a superconductor.

The two crossover temperatures, $T_{cr}$, and $T_*$, are, within experimental error, the same as those found in an analysis of the uniform magnetic susceptibility [7], which, unlike that of the familiar Landau Fermi liquid, is highly temperature dependent in all but highly overdoped systems. Specifically, in both optimally doped and underdoped systems, the uniform susceptibility at high temperatures increases with decreasing temperature, displays a maximum at $T_{cr}$, following which it decreases linearly as the temperature is further decreased, until the second crossover temperature, $T_*$, is reached, below which it decreases more rapidly, down to $T_c$. Moreover, as shown in Fig. 1, the crossover behavior found magnetically in probes of both long and short wave length behavior is found as well in transport.

The phase diagram one obtains from an analysis of the NMR and INS experiments is depicted in Fig. 2. Note that from a magnetic perspective, optimally doped systems are in fact underdoped; only when one enters the magnetically overdoped regime at substantively higher doping concentrations, does one find an almost temperature independent uniform susceptibility, and mean field behavior for $\chi_{\text{NAFL}}(q, \omega)$ at all temperatures above $T_c$. The detailed analysis of NMR and INS experiments presented in Refs. (7) and (3) makes it possible to obtain a criterion for $T_{cr}$ in terms of the strength of the antiferromagnetic correlations $\xi$, at $T_{cr}$; one finds

$$\xi(T_{cr}) \approx 2a$$

(3)

**Transport Properties**

It is often stated that for optimally doped systems the longitudinal resistivity, $\rho_{xx}$, is linear in $T$ in the normal state, while the cotangent of the Hall angle, $\cot \theta_H$, displays $T^2$ behavior. However, as discussed in some detail in Ref. (8), experiments using single crystals show that $\rho_{xx}(T)$ displays a downturn from linear in $T$ behavior as $T$ approaches $T_*$, and that such downturns are a common feature of all magnetically underdoped materials. As may be seen in Fig. 3, the character of the departure of $\rho_{xx}$ from linear in $T$ behavior depends on doping level; optimally doped systems display the least departure from $T$-linear behavior, while magnetically overdoped systems display an upturn
at comparatively high temperature.

In the magnetically underdoped cuprates, the transverse conductivity, $\sigma_{xy}$ is $\sim T^{-3}$ at high temperatures, with significant deviations occurring at temperatures $\lesssim T_\ast$. For example, in $\text{YBa}_2\text{Cu}_3\text{O}_{6.63}$, one finds $\sigma_{xy} \sim T^{-4}$ below 200K ($\sim T_\ast$ for this material). As a result, in the pseudoscaling regime, the Hall resistivity, $\rho_{xy} \cong (\sigma_{xy}/\sigma_{xx}^2)$, is temperature dependent ($\sim T^{-1}$), while $\cotn\theta_H \equiv (\rho_{xx}/\rho_{xy})$ displays $T^2$ behavior. Departures from this behavior are found in the pseudogap regime and may occur as well in the mean field regime. For example, Hwang et al. [9] find that in the $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ system, for $x > \sim 0.15$, the Hall resistivity becomes temperature dependent below a characteristic temperature $\sim T_{cr}$. Thus the crossovers at $T_{cr}$ and $T_\ast$ seen in the spin response possess direct counterparts in the charge response of the planar quasiparticles in the cuprate superconductors.

**Quasiparticle Properties and Fermi Surface Evolution**

Recent specific heat [1] and angle resolved photoemission (ARPES) experiments [1] show that in underdoped systems the quasiparticle spectrum in the normal state is also anomalous. A consistent account of specific heat experiments can be obtained if one assumes that below $T_{cr}$ the energy spectrum of quasiparticles near the Fermi surface becomes temperature dependent in such a way that the effective quasiparticle density of states, $N_0(T)$, mimics the temperature dependence found for $\chi_0(T)$ [1]. Still more detailed information concerning the temperature dependent quasiparticle spectrum comes from ARPES experiments on the BSCCO system, which show that a distinct evolution in the Fermi surface takes place at temperatures $\lesssim 200K$ in the underdoped systems, while for overdoped BSCCO, where one finds a large hole Fermi surface consistent with Luttinger’s theorem, no such evolution is observed [1]. As Z. X. Shen has told us at this meeting, in underdoped quasiparticles on the Fermi surface which are located in the vicinity of $(\pi, 0)$ become gapped, with a leading edge gap $\sim 20\text{meV}$ and a spectral function which has a broad maximum at $100 - 200\text{meV}$. 

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The Nearly Antiferromagnetic Fermi Liquid (NAFL) Model

How can a magnetic interaction between quasiparticles bring about this remarkable normal state behavior? In the NAFL description of the normal state [2], the dominant contribution to the magnetic interaction between planar quasiparticles is assumed to come from spin-fluctuation exchange, and so will be proportional to $\chi$, where $\chi$, wherever possible, is taken from experiment. The quasiparticle spectrum is assumed to take a tight-binding form,

$$\varepsilon_k = -2t (\cos k_x a + \cos k_y y) - 4t' \cos k_x a \cos k_y y - 2t'' [\cos 2k_x a + \cos 2k_y a] \quad (4)$$

where $t$, $t'$, and $t''$ are the appropriate hopping integrals, chosen to obtain agreement with ARPES experiments. Thus the effective magnetic interaction between quasiparticles for momentum transfers in the vicinity of $Q$ is assumed to be

$$V_{eff}^{NAFL}(q, \omega) = g_1^2 \chi_{NAFL}(q, \omega) \quad (5)$$

and elsewhere to be

$$V_{eff}^{FL}(q, \omega) = g_2^2 \chi_{FL}(q, \omega). \quad (6)$$

Here $\chi_{NAFL}$ is specified by Eq. (1) with parameters taken from fits to INS and NMR experiments, while $\chi_{FL}$ takes the general form, Eq. (2), with the band parameters specified by Eq. (4). The system resembles a Fermi liquid in that it possesses a well-defined Fermi surface which for doping levels near or beyond optimal satisfies Luttinger’s theorem, and spin and charge are not separated; both magnetic and transport properties derive from the interaction, Eqs. (5) and (6). However, because the interaction is very strong and sharply peaked near $Q$, none of the other quasiparticle properties resemble those of a conventional Landau Fermi liquid; hence the proposal that the system is, instead, a quite new kind of Fermi liquid, with anomalous spin and charge properties which are doping and temperature dependent.

An interesting feature of the effective interaction, $V_{eff}^{NAFL}(q, \omega)$, is that it is temperature dependent; since $\chi$ scales with $\xi^2$, in both the mean field and
pseudoscaling regimes the effective quasiparticle interaction becomes stronger as the temperature decreases. \( V_{eff}^{NAFL}(\mathbf{q}, \omega) \) depends on the properties of the quasiparticles through \( \omega_{SF} \) as well. Moreover, as the interaction becomes stronger, the behavior of the quasiparticles is modified substantially; both quasiparticle energies and the Fermi surface can develop a substantial temperature dependence. One thus has a system in which, since the effective interaction both modifies quasiparticle behavior and is itself altered by that changed quasiparticle behavior, non-linear feedback, either negative or positive, can play a significant role.

To the extent that one is able to determine both \( \chi^{NAFL}(\mathbf{q}, \omega) \) and the quasiparticle spectrum from experiment (as is the case for \( \text{YBa}_2\text{Cu}_3\text{O}_7 \)), in calculations of system properties based on the model interaction, Eq. (5) and the quasiparticle spectrum, Eq. (4) one is left with only one free parameter. One test, then, of the correctness or utility of this proposed magnetic interaction, is whether the resulting calculations yield agreement with experiment for a number of system properties as one varies temperature and doping. We shall see that this is indeed the case: starting from Eqs. (4) and (5), it is now possible to explain the temperature and doping dependence of quasiparticle lifetimes, the longitudinal resistivity, the Hall conductivity, \( \sigma_{xy} \), the optical properties, the general features of Fermi surface evolution in underdoped cuprates, and the transition at \( T_c \) to a \( d_{x^2-y^2} \) pairing state.

A second test would be the derivation of the model interaction, Eq. (5), from first principles. Here one is, at present, somewhat less successful. As discussed in Refs. (12) and (13), one can, under some conditions, obtain a dynamic susceptibility of the form, Eq. (5), from the effective 2D one-band Hubbard model; moreover one can, at high temperature and for some doping levels, show that the effective interaction obtained in a Hubbard-based Monte Carlo calculation is equivalent to the model interaction specified by Eqs. (4) and (5) \[14\]. However, quite generally, a major source of difficulty for microscopic calculations stems from the fact that significant contributions to the real part of \( \chi \) come from the incoherent part of the quasiparticle spectral density, a region for which the spectral density is imperfectly known.
**Hot and Cold Quasiparticles**

Because the effective interaction in a NAFL is, by definition, highly anisotropic, the resulting quasiparticle behavior as one moves around the Fermi surface necessarily reflects this anisotropy. As shown in Fig. 4, there will, in general, be two distinct groups of quasiparticles on the Fermi surface: hot quasiparticles are those located in the vicinity of hot spots, regions of the Fermi surface where the magnetic interaction is determined by $V_{\text{eff}}^{\text{NAFL}}(\mathbf{q}, \omega)$ and is anomalously large. Cold quasiparticles are found in the remaining parts of the Fermi surface, where the interaction is “normal,” i.e. comparable to that found in conventional Landau liquids. Both the Eliashberg calculations of Monthoux and Pines [15] and the perturbation theoretic analytic calculations reported in Stojkovic and Pines (SP) [8] show that the lifetimes of hot and cold quasiparticles differ significantly, as do their contributions to the transport coefficients in the normal state. Explicit calculations, subsequently borne out by an analysis of experiments on $\sigma_{xx}$ and $\sigma_{xy}$, show that in the vicinity of $T_*$, $\tau_{\text{hot}}^{-1}$ crosses over from being nearly independent of $T$ (above $T_*$) to becoming linear in $T$, while over much of the temperature range of interest, $\tau_{\text{cold}}^{-1} \sim T^2$; it is this latter quantity which is responsible for the measured temperature dependence of $\text{ctn} \, \theta_H$. Stojkovic and I found that the changes with doping and temperature in $V_{\text{eff}}^{\text{NAFL}}(\mathbf{q}, \omega)$ combine with the changes in the quasiparticle spectrum (measured in ARPES experiments) to produce the measured crossovers in $\sigma_{xx}$, and $\sigma_{xy}$, as one varies doping and temperature. I refer the interested reader to Ref. (8) for a detailed account of the resulting theory of the longitudinal and Hall conductivities, a theory which appears capable of accounting for the rich morphology found experimentally in the superconducting cuprates.

**The NAFL Description of the Dynamic Susceptibility**

In the NAFL description of $\chi(\mathbf{q}, \omega)$, since the interaction is of electronic origin, the crossovers at $T_{c_r}$ and $T_*$ reflect changes in quasiparticle behavior; specifically, in the behavior of quasiparticles located in hot regions of the Fermi surface. Consider the behavior of $\chi(\mathbf{Q}, \omega)$, the susceptibility at one of the (generally) incommensurate peaks. On writing
\[ \chi(Q, \omega) = \frac{\chi_{Q_i}}{1 - i\omega/\omega_{SF}} = \frac{\tilde{\chi}_{Q_i}}{1 - J_{Q_i} \tilde{\chi}_{Q_i}(\omega)} \]  

(7)

where \( \tilde{\chi}_{Q_i} \) is the irreducible particle hole susceptibility associated with quasiparticle transitions from one hot spot to another, and \( J_{Q_i} \), the effective restoring force in the particle-hole channel, may safely be assumed to be temperature independent, it is evident that the measured temperature dependence of \( \xi \) and \( \omega_{SF} \) originate in the temperature dependence of \( \tilde{\chi}_{Q_i}(T) \). A closer examination shows that \( \tilde{\chi}'' \) is only weakly temperature-dependent; it is \( \tilde{\chi}''(Q_i, \omega) \), or, what is equivalent, the quasiparticle damping of spin motion, which changes character dramatically as the temperature is decreased. Thus, as discussed in Monthoux and Pines [16], and in Chubukov, Pines and Stojkovic (CPS) [12], on writing

\[ \tilde{\chi}''(Q_i, \omega) \approx \tilde{\chi}'(Q_i, \omega) N_{Q_i}(T) \omega \]  

(8)

one sees that above \( T_{cr} \), in the mean field regime, \( N_{Q_i}(T) \sim 1/\tilde{\Gamma}_Q \sim N_Q(\Gamma_Q \text{ is an effective band width}) \), is essentially independent of \( T \), while between \( T_{cr} \) and \( T_{*r} \), the damping becomes strongly \( T \) dependent in such a way that one has \( N_{Q_i}(T) \sim 1/\xi(T) \sim a + bT \) so that \( \omega_{SF} \sim 1/N_{Q_i}(T)\xi^2 \sim c/\xi(T) \), corresponding to non-universal \( z = 1 \) pseudoscaling. Below \( T_{*r} \), in the pseudogap regime, \( \xi \rightarrow \text{constant} \), while one finds a more rapid fall off with decreasing temperature of the quasiparticle damping, as seen in the “effective” density of states \( N_{Q_i}(T) \); it is this which is responsible for the corresponding “pseudogap behavior” seen in \( \omega_{SF} \). The behavior of \( N_{Q_i}(T) \) is similar, but not identical, to that one infers for the quasiparticle state density, \( N_T(0) \), from the measured behavior of \( \chi_0(T) \).

**A Magnetic Scenario for Pseudogap Behavior**

Now that a near-consensus on the nature of the pairing state has been reached [17], explaining pseudogap behavior, the sequence of crossovers seen in the normal state of optimally doped and underdoped cuprate superconductors, is arguably the major challenge facing the high-\( T_c \) community. Two scenarios for pseudogap behavior have been fleshed out in some detail: a d-wave superconductivity precursor scenario, in which a quasiparticle gap with \( \text{d}_{x^2-y^2} \) symmetry appears above \( T_c \) due to superconducting fluctuations [18], and a
magnetic scenario, in which the hot quasiparticles become gapped by the formation of a precursor to a spin density wave [12]. In the (pre-formed pair) SC scenario, the quasiparticle gap near \((0, \pi)\) observed in ARPES experiments is due to BCS pairing, while in the magnetic scenario, it is produced by an SDW precursor which acts primarily on the hot quasiparticles. As noted in CPS, both scenarios imply that the quasiparticle gap near \((0, \pi)\) should not change as the system becomes superconducting, in agreement with the data. There would, however, seem to be at least three reasons for preferring the magnetic scenario:

- The doping dependence of \(T_{cr}\), which marks the onset of pseudogap behavior (recall that for \(T < T_{cr}\), \(\chi_0\) starts to decrease with decreasing temperature) is markedly different from that found for \(T_c\), posing a significant hurdle for a preformed pair scenario. On the other hand, in the magnetic scenario, there is no reason for \(T_{cr}\) and \(T_c\) to be related. Moreover, since \(\xi(T_{cr})\) has been shown to be \(\sim 2a\), in both the 1-2-3 and 2-1-4 systems, the doping dependence of \(T_{cr}\) is a natural consequence of the increasing strength of the AF correlations as one goes from overdoped systems toward the AF insulator.

- As noted by CPS, the magnetic scenario correctly describes the entire sequence of crossovers in the normal state, including the crossover at \(T_{cr}\) to \(z = 1\) pseudoscaling, a crossover which is difficult to explain as an SC precursor.

- It appears extremely difficult, if not impossible, to use a d-wave precursor scenario to explain the onset of pseudogap behavior in YBa\(_2\)Cu\(_4\)O\(_8\) which Curro et al. [5] find takes place at a temperature, \(T_{cr}, \sim 6T_c\).

Let us examine the magnetic scenario in more detail. At temperatures large compared to \(T_{cr}\), strong coupling effects dominate; the scattering of quasiparticles against each other reduces the uniform susceptibility, so as \(T\) decreases (and the quasiparticle lifetime increases) \(\chi_0\) will increase, an effect seen in the Eliashberg calculations of Monthoux and Pines [17]. However, this tendency is opposed by the increasing strength of the AF correlations, which may be expected to suppress \(\chi_0\) (a quasiparticle whose motion is strongly correlated with that of its neighbors is less able to respond to an external magnetic field). \(T_{cr}\), where \(\xi \sim 2a\), is then the temperature at which these two competing effects are roughly in balance; below \(T_{cr}\), the AF correlations dominate. Put another way, when \(\xi \sim 2a\), in a configuration space description this means that a quasiparticle (the hybridized Cu\(^{2+}\) spin and hole) must
now move in such a way that its motion is correlated on average with some twenty-four of its neighbors. Such motion is best described as the precursor to a spin-density wave state.

In a momentum space description, as discussed in CPS, it is the hot spot quasiparticles which are primarily affected by the spin-density wave precursor formation; below $T_{cr}$, the coherent part of the hot quasiparticle Green’s function becomes progressively weakened, until at $T_*$, the end point of the crossover, the system actually begins to lose pieces of the Fermi surface. This Fermi surface surface evolution is clearly seen in the ARPES experiments of the Stanford and Argonne groups. Its equivalent appears in the $T = 0$ studies of the NAFL by Chubukov and his collaborators [12, 19] who find two crossovers in an NAFL as one changes doping and the coupling between quasiparticles increases, crossovers which it is natural to associate with those at $T_{cr}$ and $T_*$ observed at fixed doping and finite temperatures, as discussed by CPS. Very recent finite temperature NAFL calculations by Vladimir Roubtsov show this onset of the Fermi surface evolution with temperature for sufficiently strong coupling.

It seems natural, therefore, to associate the “leading edge” gap found in ARPES experiments with the influence of the precursor SDW on the “hot” quasiparticles, and to designate that gap as $\Delta_{hot}$. Although brought about by a near approach to AF behavior (with $\xi$, in general, large compared to $2a$), it resembles a “precursor” pairing gap, in that a substantial build-up of a peak in $\chi_Q$ automatically produces strong “pairing” correlations between adjacent, barely itinerant, Cu$^{2+}$ spins. The mathematical consequences of this pairing are, however, different from conventional BCS pairing, since the fall-off in $\chi_0(T)$ between $T_*$ and $T_c$ is concave downwards, while that produced by BCS pairing between $T = 0$ and $T_c$ is concave upwards. The pairing is $d_{x^2-y^2}$-like, since the SDW-induced gap is maximum for hot quasiparticles, and is zero for cold quasiparticles maximally distant from the hot regions. Both $\Delta_{hot}$ and the strength of the AF correlations are essentially fully formed at temperatures a little below $T_*$, since non-linear feedback effects associated with the appearance of $\Delta_{hot}$ will prevent the real part of the static irreducible particle-hole susceptibility from increasing appreciably beyond its value at $T_*$ (recall that $\chi_Q = \bar{\chi}_Q/1 - J_Q\bar{\chi}_Q$, so a freezing of $\bar{\chi}_Q$ via $\Delta_{hot}$ acts to freeze $\xi$). Note that between $T_*$ and $T_c$, while the hot quasiparticles change their character, the cold quasiparticles are relatively unaffected until $T_c$ is reached.

On the above picture, the existence of two classes of quasiparticles in the
normal state, hot and cold, translates directly into the existence of two kinds of quasiparticle energy gaps. The first to appear, and largest in magnitude, is $\Delta_{\text{hot}}$, which may be related to $T_\star$, but which is almost certainly not related to $T_c$. Superconductivity comes about only when one gets BCS pairing of the “cold” quasiparticles; the pairing state will definitely be $d_{x^2-y^2}$, with a maximum gap, $\Delta_{\text{cold}}$, which is proportional to $T_c$, and can easily be of order $3\sim3.5kT_c$. The symmetry of $\Delta_{\text{cold}}(T)$, as well as its temperature dependence (it is expected to reach its maximum magnitude at $T \sim T_c/2$), means that below $T_c/2$ all thermally excited quasiparticles are sitting near the nodes of $\Delta_{\text{cold}}$; hence gap values deduced from NMR or $\lambda(T)$ measurements refer to $\Delta_{\text{cold}}$.

On this scenario, the superfluid density, $\rho_s$, refers only to the “cold” quasiparticles, so that it reflects not the total density of quasiparticles, but only those quasiparticles which have not been “gapped” by the precursor to an SDW. It will be smaller for low hole density superconductors in part because these contain a higher percentage of hot quasiparticles which cannot participate in the s.c. behavior, tied up as they are by $\Delta_{\text{hot}}$.

Finally, this scenario explains the observations that below $T_c$, the position of the leading edge does not change (it can’t, being set already by SDW physics well above $T_c$), while it does sharpen (because quasiparticles away from the hot spot no longer scatter freely against the spin fluctuations). It also provides alternative explanations for the appearance of a new high energy scale, $\lesssim 200\text{meV}$, in the underdoped systems. One is that it is the spin gap, a collective mode with the energy $\lesssim 2J$ - required to flip the spin of one of the hot SDW-paired quasi-particles; a second, suggested by Branko Stojkovic (private communication) is that it reflects the appearance of new van Hove singularities associated with the evolution of the Fermi surface.

Since the magnetic scenario predicts two distinct energy gaps for underdoped systems, with the larger ($\Delta_{\text{hot}}$) varying little with doping, the smaller ($\Delta_{\text{cold}}$) scaling with $T_c$, a systematic identification of these gaps and study of their doping dependence would provide an important test of its applicability.

The Pairing Potential and Vertex Corrections

A key question concerning the NAFL model is the role played by vertex corrections to the strong coupling Eliashashberg calculations of normal state prop-
erties and the superconducting instability by Monthoux and Pines [15]. Early estimates by Monthoux (private communication) showed that for optimally-doped YBCO, where $\xi(T_c) \sim 2a$, these would not be large ($\lesssim 10 - 20\%$), while Schrieffer [20] has argued that for the underdoped systems, where the correlation length is long, one is so close to SDW behavior that vertex corrections will dramatically reduce the magnetic interaction between quasiparticles and the pairing potential for the transition to the superconducting state. The NAFL model calculations at $T = 0$ by Chubukov and his collaborators [19, 13] show that as one reduces the doping concentration, and hence increases the coupling strength of the magnetic interaction, vertex corrections initially act to increase this interaction and hence enhance the pairing potential; however in the limit of very strong coupling, where a preformed SDW has brought about substantial changes in the quasiparticle Fermi surface, vertex corrections do bring about the substantial reduction in the quasiparticle coupling to spin excitations proposed by Schrieffer. Quite recently, Monthoux [21] has carried out a systematic study at finite temperatures of the “two-loop” corrections to his earlier Eliashberg results. He finds, in agreement with his earlier work, and with Chubukov et al. that for systems near optimal doping, vertex corrections are not large, and act to enhance the pairing potential, but that as the magnetic correlation length increases, the magnitude of the vertex correction to the effective interaction and pairing potential is quite different for hot and cold quasiparticles; it is substantially larger for the hot quasiparticles, and, for these, to first approximation, scales with $\xi$.

An interesting question, then, is whether in underdoped systems it is Monthoux’s vertex correction enhancement of the hot quasiparticle interaction which brings about their transition to an SDW-paired state at temperatures $\sim T_s$, and whether, below $T_s$, one then finds a hot quasiparticle gap, $\Delta_{\text{hot}}$, of the magnitude ($\sim 20\text{meV}$) required to explain the ARPES experiments. If so, one would have not only a “proof of concept” for the magnetic scenario for the underdoped systems, but would also be able to understand the change in character of the vertex corrections as the temperature decreases. Thus for $T \gtrsim T_s$, vertex corrections enhance the magnetic interaction between hot quasiparticles, while below $T_s$, the coupling of these “SDW-gapped” quasiparticles to spin excitations is indeed quite weak, being proportional to $(Q_i - q)$, in agreement with the Ward identity arguments of Schrieffer. On the other hand, for optimally doped YBCO and for the higher
T_c, T_\ell and Hg systems, where \( \xi(T_c) \sim 2a \), and, as well, for cold quasiparticles in the 2-1-4 and other underdoped systems, vertex corrections will not play a large role; Eliashberg calculations of the normal state properties, the pairing potential and T_c should provide a quite reasonable approximation.

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Figure Captions

Fig. (1) A schematic depiction of the temperature crossovers measured for various observable quantities in the underdoped cuprates.

Fig. (2) A representative phase diagram for the cuprate superconductors. Note that the transition from underdoped to overdoped magnetic behavior occurs at a concentration, $x_{\text{over}}$, which is $> x_{\text{opt}}$, the concentration for which, within a given system, the superconducting transition temperature, $T_c$, is maximal.

Fig. (3) The measured deviation from linear in T behavior of $\rho_{xx}(T)$ for underdoped, overdoped, and optimally doped superconducting cuprates. From top to bottom, results are shown for a representative overdoped system, a $T_c = 15$K sample of $T\ell 2201$, an optimally-doped system, “$\text{YBa}_2\text{Cu}_3\text{O}_7$,”
and an underdoped system, YBa$_2$Cu$_3$O$_{6.63}$. The quantity plotted is $[(\rho_{xx}(T) - ρ_0)/αT]$ where $ρ_0$ and $α$ are obtained from a fit to the high temperature, linear in $T$, part of the resistivity [from Ref. (8)].

Fig. (4) A model of a Fermi surface in cuprates (solid line) and the magnetic Brillouin zone boundary (dashed line). The intercept of the two lines marks the center of the hot spots on the Fermi surface, regions near $(π,0)$ which, because they can be connected by the wave vector $Q_i$, are most strongly scattered into each other.
Pseudogap (PG)

Mean Field (MF)

Pseudoscaling (PS)
\[ \xi \geq 2a \]
\[ \omega_{sf} = \frac{c}{\xi} \]

\[ \omega_{sf} \sim \xi^{-2} \]
\[ \xi \leq 2a \]
\( \frac{(\rho_{xx} - \rho_0)}{\alpha T} \) vs. \( T \) (K)
