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Decision Support

Nested dynamic network data envelopment analysis models with infinitely many decision making units for portfolio evaluation

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Portfolio performance evaluation is a major data envelopment analysis (DEA) application in the finance field. Most proposed DEA approaches focus on single-period portfolio performance assessment based on aggregated historical data. However, such an evaluation setting may result in the loss of valuable information in past individual time periods, and violate real-world portfolio managers' and investors' decision making, which generally involves multiple time periods. Furthermore, to our knowledge, all proposed DEA approaches treat the financial assets comprising a portfolio as a "black box": thus there is no information about their individual performance. Moreover, ideal portfolio evaluation models should enable the target portfolio to compare with all possible portfolios, i.e., enabling full diversification of portfolios across all financial assets. Hence, this research aims at developing nested dynamic network DEA models, an additive model being nested within a slacks-based measure (SBM) DEA model, that explicitly utilizes the information in each individual time period to fully and simultaneously measure the multi-period efficiency of a portfolio and its comprised financial assets. The proposed nested dynamic network DEA models, referred to as NDN DEA models, are linear programs with conditional value-at-risk (CVaR) constraints, and infinitely many decision making units (DMUs). In conducting the empirical study, this research applies the NDN DEA models to a real-world case study, in which Markov chain Monte Carlo Bayesian algorithms are used to obtain future performance forecasts in today's highly volatile investment environments.

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1. Introduction

Data envelopment analysis (DEA) techniques have been applied to the important finance field of portfolio performance evaluation (see, e.g., Branda, 2015, 2016; Joro & Na, 2006; Lamb & Tee, 2012; Lin, Chen, Hu & Li, 2017; Liu, Zhou, Liu & Xiao, 2015; Lozano & Gutiérrez, 2008). Target portfolios to be assessed can be obtained through portfolio optimization, which allocates the available capital to the selected financial assets. The allocation rules are based on the portfolio manager's/investor's desired levels of expected return and risk as well as other key investment policy factors (Doumpos & Zopounidis, 2014), which include the goals and constraints that significantly influence portfolio manager/investor investment policy decisions such as regulatory issues, transaction costs, liquidity considerations, diversification constraints and objectives, and the investment’s time horizon (e.g., single-period vs. multi-period investments) (Doumpos & Zopounidis, 2014). It follows that the corresponding portfolio performance evaluation DEA models proposed in the literature are associated with a variety of settings.

Kaffash and Marra (2017) provide an overview of recent DEA applications in portfolio performance evaluation. In addition, Lin et al. (2017) show that earlier studies that apply DEA techniques to portfolio performance evaluation fail to capture the effect of portfolio diversification because they treat the risk of the target portfolio as a linear combination of the risk of the evaluated financial assets. To account for this, subsequent DEA portfolio performance evaluation models all follow Markowitz’s mean-variance framework (Markowitz, 1952) and take the effect of portfolio diversification into account. Hence, Lin et al. (2017) review various DEA models with diversification. As shown in the above two studies, most proposed DEA approaches focus on single-period portfolio performance assessment based on aggregate historical data. In addition, all proposed DEA models treat the financial assets comprising a portfolio as a “black box”. Below, we investigate the key DEA model construction factors of the investment’s time horizon.
(single-period vs. multi-period investments), historical data utilization (aggregate vs. separation), and component performance (black vs. open box).

To date, most proposed DEA approaches focus on single-period portfolio performance assessment based on aggregated historical data (for exceptions, see, e.g., Briec & Kerstens, 2009; Morey & Morey, 1999). However, in practice, portfolio manager and investor decision-making generally involves multiple time periods. Morey and Morey (1999) and Briec and Kerstens (2009), although dealing with multi-period investments, do not account for the connecting activities between two consecutive periods (Lin et al., 2017). Thus, their proposed DEA models cannot be considered dynamic DEA models according to typical DEA terminology (see e.g., Färe & Grosskopf, 1996; Tone & Tsutsui, 2010). Actually, to our knowledge, Lin et al. (2017) is the only research work so far that develops a dynamic DEA model to evaluate multi-period portfolio performance that changes over time. Their DEA model considers portfolio diversification and is based on the directional distance function. However, we note that the dynamic DEA model proposed in Lin et al. (2017) utilizes aggregated historical data to assess portfolio performance, which may result in the loss of valuable information in past individual time periods.

Furthermore, to the best of our knowledge, all proposed DEA approaches in the literature for assessing portfolio performance treat the financial assets comprising a portfolio as a “black box”, there is no information about their individual performance. However, clearly each financial asset’s performance is valuable to portfolio managers and investors to determine their desired portfolios. That is, no proposed DEA models for portfolio performance evaluation have network structures (see, e.g., Färe & Grosskopf, 2000; Tone & Tsutsui, 2009). We note that according to typical DEA terminology, the DEA model proposed in Lin et al. (2017) is simply a dynamic framework, as opposed to a dynamic network framework (see, e.g., Tone & Tsutsui, 2014).

Based on the above analysis of key DEA model construction factors, it is clear that practically promising DEA models for portfolio performance assessment should take into account multi-period investments, historical individual time period information, and component performance. In additional, as indicated in Branda (2013), full diversification is more suitable for investment opportunity comparisons. Hence, this research aims at developing generalized dynamic network DEA models that explicitly utilize the information in each individual time period to fully and simultaneously measure the multi-period efficiency of a portfolio and its comprised assets by allowing full diversification across the financial assets. Generalized dynamic network DEA models have a nested structure, in which an additive DEA model associated with the financial assets is nested within a slacks-based measure (SBM) DEA model associated with the portfolio. Therefore, in this research, generalized dynamic network DEA models with a nested structure are referred to as nested dynamic network (NDN) DEA models. To verify the performance of the proposed DEA models, we conduct a real-world empirical study, in which it is crucial to accurately and completely forecast the future performance of a financial asset (e.g., expected return and risk). Therefore, this research applies Markov chain Monte Carlo Bayesian algorithms to obtain future performance forecasts in modern, highly volatile investment environments.

The remainder of this paper is organized as follows. Section 2 introduces the generalized dynamic network evaluation structure. Section 3 defines portfolios and portfolio diversification, and then describes the considered input and output factors. Section 4 discusses the approaches to determine future possible scenarios and their corresponding transition probabilities and return forecasts. Section 5 constructs the nested DEA models based on the generalized dynamic network evaluation structure shown in Section 2, and then introduces a two-stage approach to construct efficient portfolios. Section 6 conducts a real-world empirical study, and validates the proposed DEA models. Sections 7 concludes based on this research.

2. Generalized dynamic network evaluation structure

Consider an investor who invests her/his wealth among risky financial assets over future multiple periods. Each financial asset is associated with a set of inputs (e.g., transaction costs and CVaR deviations), a set of outputs (e.g., rates of return), and a set of carry-overs (e.g., current and non-current assets). The inputs, outputs, and carry-overs are detailed in the succeeding section. Bascially, the types of the inputs, outputs, and carry-overs remain the same in every period. However, they are represented by realized values in the past-present periods, and random variables in the future periods. Furthermore, some of these, e.g., CVaR deviations, may not exist in the past-present periods. Moreover, in practice, it is much more difficult to accurately forecast carry-overs than input and output values in future periods. Hence, carry-overs in future periods are ignored. Lastly, to help investors evaluate their target portfolios and their comprised financial assets based on their historical and expected data, this section introduces the generalized dynamic network evaluation structure later used to develop the nested DEA models. The term “generalized” for the proposed evaluation structure is used to differentiate it from the conventional dynamic network structure (see, e.g., Tone & Tsutsui, 2014). Note that this research constructs the generalized dynamic network evaluation structure by extending and modifying the generalized dynamic evaluation structures proposed in Chang, Tone and Wu (2016).

First, the proposed evaluation structure is actually a past-present-future intertemporal structure that consists of \((T + k)\) periods \((1, 2, \ldots, T + k)\), where periods \((1, 2, \ldots, T)\), period \(T\), and period \((T + 1, \ldots, T + k)\), respectively, represent the past, present, and future time structures. In addition, each future period \(T + g(l) = 1, \ldots, k\) is comprised of \(h\) sub-periods denoted as \(T + g(l) = 1, \ldots, h\), where \(h\) represents possible scenarios associated with future period \(T + g\). There are transition probabilities from period \(T + g(l) = 1, \ldots, h\) to \(T + g(l + 1)(n)\) that are denoted as \(p_{i,j}^{g,l} = 1\). In addition, there is a transition probability from \(T + g(m)\) to \(T + (g + 1)(n)\) that is denoted as \(p_{m,n}^{g}\), where \(g = 1, \ldots, k - 1\) and \(m, n = 1, \ldots, h\). Note that \(\sum_{m=1}^{h} p_{m,n}^{g} = 1, m = 1, \ldots, h\), and thus future time period \(T + (g + 1)(n)\), is associated with transition probability \(\delta_{g}^{l} = \sum_{m=1}^{h} P_{T + g}^{l,m} P_{m,n}^{g}\).

We detail the approaches used to determine the values of \(p_{i,j}^{g,l} = 1, \ldots, h\) and \(p_{m,n}^{g} = 1, \ldots, k - 1\); \(m, n = 1, \ldots, h\) in Section 4. In multi-period portfolio management, at the end of an investment period, the investors must dispose of the financial assets in their portfolios before following the next investment period. That is, there are intermediate activities and thus carry-overs between two consecutive periods. As indicated in Lin et al. (2017), ignoring intermediate performance can lead to wrong investment decisions. Hence, we assume that there are carry-overs at the end of each period \(t = 1, \ldots, T\). Note that including carry-overs from the initial period are usually unknown and are thus omitted (see Tone & Tsutsui, 2010). In this portfolio evaluation research, we assume that there may be both discretionary (free) carry-overs that can be freely handled, and non-discretionary (fixed) carry-overs that cannot be controlled by decision makers. Fig. 1 graphically demonstrates this intertemporal structure. As shown in the figure, past and present periods \((1, 2, \ldots, T)\) exhibit a typical dynamic structure. On the other hand, future periods \((T + 1, \ldots, T + k)\), consisting of \(T + g(l) = 1, \ldots, k; l = 1, \ldots, h\) sub-periods, show a non-typical dynamic structure. In addition, it is noted that each time period is actually associated with a network internal structure that
is not shown in Fig. 1 for clarity but is detailed below. That is, as a whole, this past-present-future intertemporal evaluation structure presents a generalized (asymmetric) structure, and is thus referred to as the generalized dynamic network structure.

As mentioned above, the proposed evaluation structure presents not only the past-present-future intertemporal structure that is shown in Fig. 1, but also a network structure. More precisely, each of the \( T + kh \) periods is associated with a network internal structure, which consists of \( B \) financial assets that comprise target portfolios. Figs. 2(a) and (b) demonstrate the network internal structures corresponding to consecutive period pairs \( (t, t + 1), t = 1, T \) and consecutive period pairs \( (t(m), t + 1(n)), t = T + 1, \ldots, T + (k - 1); m, n = 1, \ldots, h \), respectively. Recall that there is no carry-over information with respect to every consecutive future period pair because of the difficulty of forecasting the values. However, two consecutive periods are still connected with transition probability as shown in Fig. 2(b). It is noted that carry-over corresponds to each single financial asset, but not a whole portfolio.

In summary, Figs. 1 and 2 jointly demonstrate our developed past-present-future intertemporal network evaluation structure that is referred to as the generalized dynamic network evaluation structure in this research. Our proposed nested DEA models (to be detailed later) build on this evaluation structure, and thus enable decision makers to more accurately evaluate a portfolio's and its comprised financial assets' performance by explicitly taking into account not only its past and present but also its forecasted future performance. To help fully understand the construction process of the whole complex evaluation structure, we break down the generalized dynamic network evaluation structure into five different single-period structures that correspond to period 1, period \( t (t = 2, \ldots, T) \), period \( T + 1 \), period \( T + k \), and period \( T + k \), respectively. Hence, in what follows, we detail the five single-period evaluation structures.

Each single-period \( t (t = 2, \ldots, T - 1) \) associates an internal network structure that consists of \( B \) financial assets such that each financial asset \( b (b = 1, \ldots, B) \) is associated with input set \( t_b \) and output set \( t_b \), incoming carry-over \( (t - 1)h \), and outgoing carry-over \( t_b h \). Compared to the single-period evaluation structures corresponding to \( t (t = 2, \ldots, T - 1) \), that corresponding to period 1 lacks the incoming carry-overs from the initial period that are usually unknown and are thus omitted, and that corresponding to period \( T \) lacks the outgoing carry-overs (see Tone & Tsutsui, 2010). The other three types of single-period evaluation structures correspond to future period \( T + 1 \), future periods \( T + 2, \ldots, T + k - 1 \), and future period \( T + k \), which are introduced below in that order. Recall that each future period is associated with \( h \) possible sub-periods. Indeed, these three types of evaluation structures differ only slightly from each other. Each future sub-period \( T + 1(l) (l = 1, \ldots, h) \) associates an internal network structure that consists of \( B \) financial assets such that each financial asset \( b (b = 1, \ldots, B) \) is associated with input set \( T + 1(l)(b) \), output set \( T + 1(l)(b) \), incoming transition probability \( p_{lmb}^{T + 1(l)(b)} \) from financial asset \( b \) in period \( T \), and outgoing transition probabilities \( p_{lmn}^{T + 1(l)(b)}, n = 1, \ldots, h \) to financial asset \( b \) in future sub-periods \( T + 2(n), n = 1, \ldots, h \). Furthermore, each financial asset \( b (b = 1, \ldots, B) \) in each future period \( T + g(l) (g = 2, \ldots, k - 1; l = 1, \ldots, h) \) is associated with input set \( T + g(l)(b) \), output set \( T + g(l)(b) \), and incoming transition probability \( p_{lm}^{T + g(l)(b)} \), that is, \( b \) in period \( T \), and outgoing transition probabilities \( p_{lmn}^{T + g(l)(b)}, n = 1, \ldots, h \), respectively, corresponding to \( T + (g - 1)(m), m = 1, \ldots, h \) and outgoings transition probabilities \( p_{lmn}^{T + g(l)(b)}, n = 1, \ldots, h \), respectively, corresponding to financial asset \( b \) in future sub-periods \( T + (g + 1)(n), n = 1, \ldots, h \). Moreover, each future sub-period \( T + k(l) (l = 1, \ldots, h) \) associates an internal network structure that consists of \( B \) financial assets such that each financial asset \( b (b = 1, \ldots, B) \) is associated with input set \( T + k(l)(b) \), output set \( T + k(l)(b) \), and incoming transition probabilities \( p_{lm}^{T + k(l)(b)} \), that is, \( b \) in period \( T \), and outgoing transition probabilities \( p_{lmn}^{T + k(l)(b)}, n = 1, \ldots, h \) with \( T + (k - 1)(m), m = 1, \ldots, h \), respectively. It follows that each sub-period \( T + g(l) (g = 2, \ldots, k; l = 1, \ldots, h) \) has a total incoming transition probability \( \frac{1}{2} \sum_{b = 1}^{B} p_{lmn}^{T + g(l)(b)} \) from its preceding period, and each sub-period \( T + 1(l) (l = 1, \ldots, h) \) has a total incoming transition probability \( p_{lm}^{T + 1(l)} \) from period \( T \).

3. Portfolios and input-output factors

This section first defines portfolios (DMUs), and constructs infinitely many portfolios (DMUs). To the best of our knowledge, in the DEA literature, there are very few works that consider an infinite number of DMUs (see, e.g., Branda, 2013, 2015; Charnes & Tone, 2017). Then, this section introduces the considered inputs, outputs, and carry-overs for developing the proposed DEA models.

3.1. Portfolio definition and infinitely many portfolios

Assume that the target portfolio consists of \( B \) risky financial assets, and consider \( k \) investment periods. Denote \( \lambda_{tb}^{T} \) as the weight corresponding to financial asset \( b (b = 1, \ldots, B) \) at period \( t (t = 1, \ldots, T + k) \) in the target portfolio with \( \sum_{b = 1}^{B} \lambda_{tb}^{T} = 1 \). The target portfolio can thus be represented as

---

**Fig. 1.** Generalized dynamic network evaluation structure.
Fig. 2. Network internal structures associated with consecutive periods.

\[
\tilde{\Lambda} = (\tilde{\Lambda}^1, \tilde{\Lambda}^2, ..., \tilde{\Lambda}^{T+k}), \quad \text{where} \quad \tilde{\Lambda}^t = (\tilde{\lambda}^t_1, ..., \tilde{\lambda}^t_B), t = 1, ..., T + k.
\]

Likewise, let \( \Lambda = (\Lambda^1, \Lambda^2, ..., \Lambda^{T+k}) \) denote a non-target portfolio with the same setting as the target portfolio, where \( \Lambda^t = (\lambda^t_1, ..., \lambda^t_B), t = 1, ..., T + k \), and \( \sum_{b=1}^{B} \lambda^t_b = 1 \). We note that \( \tilde{\Lambda} = (\tilde{\Lambda}^1, \tilde{\Lambda}^2, ..., \tilde{\Lambda}^{T+k}) \) are known values, whereas the values of \( \Lambda = (\Lambda^1, \Lambda^2, ..., \Lambda^{T+k}) \) are to be determined. Indeed, it is almost impossible to select the best target portfolio \( \tilde{\Lambda} \). However, portfolio managers and investors can determine \( \tilde{\Lambda} \) based on their experience, historical data and so on. For example, in the Empirical Study section, we apply historical data to obtain the values of the four indicators of market value, ROA, ROE and rate of return that are then used to help investors prepare their target portfolio \( \tilde{\Lambda} \).

It is evident that ideally, the target portfolio should benchmark against all possible portfolios. That is, ideal portfolio evaluation models should enable the target portfolio to compare with all possible portfolios, i.e., enabling full diversification of portfolios across all financial assets. In addition, investors apply diversification to reduce volatility of portfolio performance by assuring that not all financial asset quotations move up or down at the same time or at the same rate. It is believed that compared to a single financial asset or an undiversified portfolio, a well diversified portfolio, which consists of a variety of financial assets, can have less risk (Branda, 2015). However, so far there is no common definition of diversification. Hence, there exist various portfolio diversification measures.
Diversification has also been taken into account in DEA models for portfolio evaluation due to its importance (see, e.g., Branda, 2013; Lamb & Tee, 2012). Considering B financial assets with rates of return $V_{b}^{h}, b = 1, ..., B$, Branda (2013) defines three choices of the set of investment opportunities: pairwise efficiency, full diversification, and limited diversification. Among these, the set of investment opportunities corresponding to full diversification is defined as $\{\sum_{b=1}^{B} \lambda_{b} V_{b} : \sum_{b=1}^{B} \lambda_{b} = 1, \lambda_{b} \geq 0\}$. That is, full diversification allows infinitely many possible portfolios. Therefore, in this research, any convex combinations of the $B$ financial assets (i.e., $\sum_{b=1}^{B} \lambda_{b} V_{b} = 1, t = 1, ..., T + k$) are allowed to form a non-target portfolio $\Lambda = (\Lambda^{1}, \Lambda^{2}, ..., \Lambda^{T+k})$. That is, in the proposed DEA models, the target portfolio $\Lambda = (\Lambda^{1}, \Lambda^{2}, ..., \Lambda^{T+k})$ must benchmark against infinitely many non-target portfolios (DMUs). In Section 6, we conduct a real-world case study to show that target portfolio $\Lambda$ is not easy to have high efficiency scores when we benchmark it against all other possible (infinitely many) portfolios. Nonetheless, this also means that if we can find a target portfolio that can obtain high efficiency scores when benchmarking against all other possible portfolios, then the target portfolio can be considered as a good portfolio. Hence, we propose an approach in Section 5.3 to suggest a good portfolio for investors.

3.2. Input-output factors and carry-overs

Investors commonly determine their capital allocation among investment alternatives based on their return-risk trade-off (Markowitz, 1952). Hence, DEA models for portfolio performance assessment mainly use risk measures as inputs and return measures as outputs. Markowitz sees investors as risk-averse, and uses the statistical measures of expected return and standard deviation to quantify the return and risk of a portfolio of multiple financial assets. More precisely, a portfolio’s expected return is simply calculated as a weighted sum of the individual financial assets’ returns. In addition, the risk of the portfolio is calculated as a function of the variances of each financial asset and the correlations of each pair of financial assets.

In the portfolio literature, the expected portfolio return is the dominant measure of return performance. Note that the expected rate of return may be negative. By contrast, there are a variety of risk measures other than the standard deviation for portfolio construction. Conditional value-at-risk (CVaR) turns out to be one of the most popular coherent risk measures (Kolm, Tütüncü & Fabozzi, 2014), and has been adopted in DEA research as an input to assess the efficiency of investment opportunities on financial markets (e.g., Branda, 2015). CVaR deviation, another risk measure closely related to CVaR risk, is also applied in DEA research to evaluate the efficiency of investment opportunities, e.g., portfolios and mutual funds (e.g., Branda, 2013; Lin et al., 2017). Note that CVaR risk and CVaR deviation are actually two very different risk management concepts. As noted in Sarykalin, Serraino and Uryasev (2008), CVaR risk evaluates outcomes versus zero, and thus may be positive or negative; however, as CVaR deviation estimates the wideness of a distribution, it is always positive. Note that negative CVaRs are considered as natural hedges that offset some of the risk (Dowd, K., 2005). Therefore, simply replacing negative risk values with zero (see, e.g., Lamb & Tee, 2012) can result in significant loss of the information in the distribution of random parts that is contained in the risk measures (Branda, 2015). In addition, it has been shown that ignoring transaction costs will lead to suboptimal portfolios (Kolm et al., 2014). A sale or a purchase transaction on a financial asset incurs a transaction cost. Hence, transaction cost is one of the most important features of the portfolio management (Mansini, Ogryczak & Speranza, 2015), and transaction costs are also considered as inputs in portfolio evaluation DEA models (e.g., Murthi, Choi & Desai, 1997; Lin et al., 2017).

In this research, we consider the expected rate of return as the single output, and transaction costs and CVaR deviation as the inputs to develop DEA portfolio evaluation models. The reason for selecting CVaR deviation instead of CVaR risk as one of the outputs is due to the difficulty of constructing DEA models, which is explained in Section 5. We note that the proposed DEA portfolio evaluation models assess a target portfolio based not only on future but also on past-present performance. Since the target portfolio’s past-present performance is realized, the input-output factors associated with past-present periods are different from those with future periods that are described above. More precisely, in case of past-present periods, realized rate of return and realized transaction costs are considered as outputs and inputs, respectively. CVaR deviations are excluded from the consideration of inputs because they do not exist in realized investment decisions. Below we mathematically define the considered inputs and outputs.

Let $\bar{V}_{b}^{T}(b = 1, ..., B; t = T + 1, ..., T + k)$ be the random variable of the rate of return with respect to financial asset $b$ at future period $t(l)$ (as defined in Section 4). The realized rates of return with respect to portfolio $\bar{V}_{b}^{T}(b = 1, ..., B; t = T + 1, ..., T + k; l = 1, ..., h)$ is the resulting G investigations. Note that for each $t(l)$, we separately and simultaneously sample $\bar{V}_{b}^{T}(b = 1, ..., B; t = T + 1, ..., T + k; l = 1, ..., h)$ to get a collection of $G$ scenarios with equal probabilities, which is detailed in Section 4. Denote $\bar{V}_{b}^{T} = g = 1, ..., G; g(b = 1, ..., B; t = T + 1, ..., T + k; l = 1, ..., h)$ as the resulting $G$ investigations. Note that for each $t(l)$, we get a set of $G$ scenarios, which we separately and simultaneously sample $\bar{V}_{b}^{T}(b = 1, ..., B; t = T + 1, ..., T + k; l = 1, ..., h) = \bar{V}_{b}^{T}(b = 1, ..., B; t = T + 1, ..., T + k; l = 1, ..., h)$.

The realized rate of return with respect to portfolio $\Lambda$ at the end of period $t(l)$ is $\bar{V}_{b}^{T} = \bar{V}_{b}^{T}(1, ..., T)$. The realized rates of returns of target portfolio $\Lambda$ corresponding to future periods $T + 1$ and $t(l) = T + 2, ..., T + k$ are, respectively, as follows:

$$\mathbb{E}\left[\sum_{n=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right] = \frac{1}{G} \sum_{g=1}^{G} \left(\sum_{n=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right) \bar{V}_{b}^{T}$$

$$\mathbb{E}\left[\sum_{n=1}^{h} \left(\sum_{m=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right) \bar{V}_{b}^{T} \right] = \frac{1}{G} \sum_{g=1}^{G} \left(\sum_{n=1}^{h} \left(\sum_{m=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right) \bar{V}_{b}^{T} \right) \bar{V}_{b}^{T}$$

$$\mathbb{E}\left[\sum_{n=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right] = \frac{1}{G} \sum_{g=1}^{G} \left(\sum_{n=1}^{h} P_{mn}^{T + 1} V_{b}^{T + 1} m\right) \bar{V}_{b}^{T}$$

The realized and expected rates of returns corresponding to non-target portfolio $\Lambda$ can be obtained using the same procedures. Note that rate of return can take both positive and negative values.
Furthermore, the mathematical formulations of CVaR risk and CVaR measurement corresponding to the rate of return are defined below. First, note that CVaR risk is defined as a weighted average between the value at risk and losses exceeding the value at risk (Rockafellar & Uryasev, 2000). That is, CVaR assesses at a specific confidence level the likelihood that a specific loss will occur with the value at risk. Thus, the smaller the value of the CVaR, the better. Given any specified tolerance level \( \alpha \) in (0, 1), CVaR risk of rate of return \( V \) is defined as follows (Rockafellar & Uryasev, 2000):

\[
\text{CVaR}_\alpha (V) = \min_{\xi \in \mathcal{X}} \{ \xi + \frac{1}{1-\alpha} \mathbb{E}[ -V - \xi ] \},
\]

where \( \mathbb{E}[ -V - \xi ] = \max\{0, -V - \xi\} \) denotes the positive part, and \( \xi \) is a real auxiliary variable. Note that since we discretize \( V \) by generating a collection of \( G \) scenarios, i.e., \([v_1, v_2, \ldots, v_G]\), with equal probabilities, problem (5) can become the following linear program (Uryasev, 1999; Rockafellar & Uryasev, 2001):

\[
\text{CVaR}_\alpha (V) = \min_{\xi \in \mathcal{X}} \{ \xi + \frac{1}{1-\alpha} \mathbb{E}[ -V - \xi ] \},
\]

subject to:

\[
\left\{ \begin{array}{l}
v_g \geq -\xi - \gamma, \; g = 1, \ldots, G \\
\bar{v}_g \geq 0, \; g = 1, \ldots, G
\end{array} \right.
\]

Actually, as indicated in Mansini et al. (2015), if every scenario has the same probability, and the specified tolerance level \( \alpha = K/G \), then the CVaR measure \( \text{CVaR}_\alpha (V) \) can be simply defined as average of the \( K \) worst realizations:

\[
\text{CVaR}_\alpha (V) = \frac{1}{K} \sum_{b=1}^{K} (-v_{b\bar{g}}),
\]

where \( (-v_{b\bar{g}}, -v_{b\bar{g}}, \ldots, -v_{b\bar{g}}) \) are the \( K \) worst realizations of the loss \(-V\). In addition, Pflug (2000) shows that CVaR is a coherent risk measurement (Artzner, Delbaen, Eber & Heath, 1999). Thus, let \( s_1 \) represent the number of CVaR (e.g. \( s_1 = 2 \), and \( \alpha_1 = 0.75 \) and \( \alpha_2 = 0.95 \)) such that \( i = 1, \ldots, s_1 \). Then, the CVaRs of financial asset \( b \) with tolerance level \( \alpha_i \) corresponding to future periods \( T + 1 \) and \( t (t = T + 2, \ldots, T + k) \) are, respectively, as follows:

\[
\text{CVaR}^{\text{CVaR}}_{\alpha_i} \left( \sum_{b=1}^{K} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \right) = \frac{1}{K \alpha_i} \sum_{b=1}^{K} \sum_{n=1}^{\beta} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) i (1 = 1, \ldots, s_1; b = 1, \ldots, B),
\]

where \( (-u_{1b}, -u_{2b}, \ldots, -u_{s_1b}) \) are the \( K \) worst realizations of the loss \( \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \), and

\[
\text{CVaR}^{\text{CVaR}}_{\alpha_i} \left( \sum_{b=1}^{K} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \right) = \frac{1}{K \alpha_i} \sum_{b=1}^{K} \sum_{n=1}^{\beta} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) i (1 = 1, \ldots, s_1; b = 1, \ldots, B; t = T + 2, \ldots, T + k),
\]

where \( (-u_{1b}, -u_{2b}, \ldots, -u_{s_1b}) \) are the \( K \) worst realizations of the loss \( \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \).

It follows that letting \( s_1 \) represent the number of CVaR such that \( i = 1, \ldots, s_1 \), the CVaRs of target portfolio \( \Lambda \) with tolerance level \( \alpha_i \) corresponding to future periods \( T + 1 \) and \( t (t = T + 2, \ldots, T + k) \) are, respectively, as follows:

\[
\text{CVaR}^{\text{CVaR}}_{\alpha_i} \left( \sum_{b=1}^{K} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \right) = \frac{1}{K \alpha_i} \sum_{b=1}^{K} \sum_{n=1}^{\beta} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) i (1 = 1, \ldots, s_1),
\]

where \( (-U_{1b}, -U_{2b}, \ldots, -U_{s_1b}) \) are the \( K \) worst realizations of the loss \( \sum_{b=1}^{K} \left( - \sum_{n=1}^{\beta} p_{t+1}^{(T+1)n} v_{b}\right) \).
4. Future scenarios, transition probabilities, and return forecasts

Effectively identifying future possible scenarios and their corresponding probabilities, and then accurately estimating the values of the rate of return of each financial asset under each scenario greatly contribute to the effectiveness of the proposed DEA portfolio evaluation models.

4.1. Future scenarios and corresponding transition probabilities

It is quite challenging to determine representative future scenarios and their corresponding probabilities. Chang et al. (2016) consider the high degree of forecast difficulty and limited data availability, and thus suggest using the renowned maximum-entropy principle (see, e.g., Fang, Rajasekera & Tao, 1997; Kapur, 1989) to determine the transition probabilities in future periods. In portfolio, more data are available, and thus more advanced and sophisticated techniques are called for. For example, this research uses the monthly statistics of stock rates of return from the Taiwan Economic Journal (TEJ) database to determine annual transition probabilities associated with optimistic, pessimistic, and neutral market scenarios. More specifically, we first fit a normal distribution to the yearly historical data. Then, we apply the lower limit (L) and upper limit (U) of the 40% equal-tailed credible intervals, and \( \bar{V} - 3\sigma \) and \( \bar{V} + 3\sigma \) to construct optimistic, pessimistic, and neutral intervals, in which \( \bar{V} \) and \( \sigma \) are expected rates of return and standard deviation, respectively. That is, \([\bar{V} - 3\sigma, \bar{V} + 3\sigma]\) represent the pessimistic, neutral, and optimistic intervals, respectively. Finally, we count the number of stock rates of return in each interval to calculate annual transition probabilities associated with the three market scenarios.

4.2. Return forecasts

As indicated in Campbell, Lo and MacKinlay (1997), whether financial asset prices can be forecasted is one of the earliest and most enduring financial questions. Indeed, a variety of forecasting approaches have been used to perform this forecasting task. However, none can be considered to be superior to the others in every respect. Some are well-known and established in practice, e.g., the moving average method (Armstrong, 2001). Currently, approaches such as Markov chain Monte Carlo (MCMC) Bayesian methods have been identified as promising for forecasting financial securities prices (see, e.g., Jacquier & Polson, 2011). Hence, we here argue that the MCMC Bayesian algorithms are better suited for estimating future performance forecasts.

Bayesian forecasting has been considered as a powerful approach for providing distributional estimates for random parameters (see, e.g., Carlin & Louis, 2009; Gelman et al., 2014). Compared to classical methods, Bayesian forecasting rapidly captures changes in nonstationary systems by using limited historical data. Portfolio performance assessment involves a multivariate random variable representing the rates of return of all the financial assets. As indicated in Mansini et al. (2015), a multivariate standard normal distribution with zero mean and unit variance-covariance matrix is the most frequently adopted distribution function. However, this distribution does not consider the “fat-tails” or “heavy-tails” effect that is commonly used to characterize rates of return. Hence, the multivariate t-Student distribution is used to replace the above distribution in the case that the effect is considered to be crucial. To tackle such complex multivariate conditions, Markov chain Monte Carlo (MCMC) can be used to facilitate Bayesian forecasting methods to make predictions. Today, it is well-known that combining Bayesian methods with Markov chain Monte Carlo techniques can produce powerful algorithms for dealing with complex conditions by providing highly effective estimation. MCMC is a general simulation approach that first draws values of \( \theta \) from approximate distributions, and then improves the approximation of the target posterior distribution, \( p(\theta|y) \), by correcting the draws (Gelman et al., 2014). In addition, the samples form a Markov chain because the sampling is performed sequentially, and each draw depends only on the state of the previous draw. However, the Markov property is not the key to MCMC’s success. As noted in Gelman et al. (2014), the success of MCMC lies in improving the approximate distributions at each simulation step so that they tend to converge to the target distribution. MCMC algorithms include the Metropolis-Hastings algorithm (Hastings, 1970), Gibbs sampling (Geman & Geman, 1984), and differential evolution (ter Braak, 2006). The first two are the most basic and widely used algorithms.

5. Nested dynamic network DEA models

This section builds on the generalized dynamic network evaluation structure detailed in Section 3 to develop nested DEA models that can simultaneously measure the efficiency of a portfolio and its comprised financial assets over the past-present-future time span. The nested dynamic network DEA models are also constructed in accordance with the input-output factors and carry-overs described in Section 3, as well as the future scenarios with corresponding transition probabilities determined in Section 4.

To begin with, assume that the target portfolio consists of \( B \) financial assets. In each past-present period \( t \in \{1, ..., T\} \), financial asset \( b(b = 1, ..., B) \) has \( s_b^1 \) realized transaction costs (inputs). Also, in each future period \( t \in \{T + g(t) : g = 1, ..., k = 1, ..., h\} \), financial asset \( b(b = 1, ..., B) \) has \( s_b \) CVaR risks (inputs) with respect to tolerance level \( \alpha_i; i = 1, ..., \alpha \), and \( s_b \) expected transaction costs (inputs). On the other hand, financial asset \( b(b = 1, ..., B) \) has a single realized rate of return (output), and a single expected rate of return (output) in each past-present period \( t \in \{1, ..., T\} \), and in each future period \( t \in \{T + g(t) : g = 1, ..., k = 1, ..., h\} \), respectively. We emphasize that it is easy to add other inputs and outputs into the developed nested DEA models. Lastly, denote the free and fixed carry-overs corresponding to the end of periods, respectively, as \( \bar{V}_{\theta(i)} \), \( i = 1, ..., n_{\text{free}} \); \( b = 1, ..., B; t = 1, ..., T \) and \( \bar{V}_{\theta(i)} \), \( i = 1, ..., n_{\text{fix}} \); \( b = 1, ..., B; t = 1, ..., T \), where \( n_{\text{free}} \) and \( n_{\text{fix}} \) are the number of free and fixed links, respectively.

These proposed portfolio DEA models are all non-radial SBM models (Tone, 2001). That is, these models consider the excesses associated with inputs and/or the shortfalls associated with outputs as the main targets of the evaluation. Except for the dynamic network structure, the specific features of the proposed SBM DEA models involve CVaR risk (CVaR deviation) constraints, infinitely many possible portfolios (DMUs), and negative data. In addition, the proposed DEA models avoid the problem of risk overestimation that is indicated in Branda (2015). That is, conventional DEA models overestimate portfolio risk in the case that a coherent or deviation measure is used to quantify the risk because

\[
\text{CVaR} \left( -\sum_{b=1}^{B} V_b \lambda_b - E \left[ -\sum_{b=1}^{B} V_b \lambda_b \right] \right) \leq \sum_{b=1}^{B} \text{CVaR}(-V_b - E[-V_b]) \lambda_b,
\]

where the left-hand side corresponds to the proposed DEA models, and the right-hand side to conventional ones. Finally, it is noted that all of the proposed DEA models are linear programming models, which can thus be solved by commercial optimization solvers such as Gurobi Optimizer.

5.1. Portfolio DEA models over past-present-future periods

Basically, the proposed nested DEA models integrate and extend the generalized dynamic DEA models proposed in
Chang et al. (2016), the SBM dynamic DEA models with network structure proposed in Tone and Tsutsui (2014), and the additive DEA models proposed in Charnes, Cooper, Golany, Seiford and Stutz (1985). The non-oriented nested dynamic network DEA model corresponding to target portfolio \( \bar{\lambda} = (\bar{\lambda}^1, \bar{\lambda}^2, \ldots, \bar{\lambda}^{T+k}) \), in which \( \bar{\lambda} = (\lambda_1^1, \ldots, \lambda_1^B) \), \( t = 1, \ldots , T + k \), is expressed in the following subsections due to that the DEA model has lots of constraints.

### 5.1.1. Objective function

Eqs. (15)–(17) constitute the non-oriented nested DEA objective function, where the \( M \) in (15) is a very large positive number, and the \( W \) in (15) and (18) is an artificial variable. It is evident that the DEA model is a fractional (nonlinear) program. How to handle this fractional issue is described below.

\[ \alpha^*_o = \min \frac{P}{Q} - MW \quad \text{s.t.} \]

\[ P = \frac{1}{\sum_{t=1}^{T+k}} \left[ \sum_{t=1}^{T} \sigma^t \left[ 1 - \frac{1}{S_1} \left( \sum_{i=1}^{B} \lambda_{1}^{i} \xi_{1}^{t} - \sum_{i=1}^{B} \bar{\lambda}_{1}^{i} \bar{\xi}_{1}^{t} \right) \right] \right. \]

\[ + \sum_{t=T+1}^{T+k} \sigma^t \left[ 1 - \frac{1}{S_1} \sum_{i=1}^{B} \lambda_{1}^{i} \xi_{1}^{t} \right] \left( \frac{1}{S_1} \sum_{i=1}^{B} \bar{\lambda}_{1}^{i} \bar{\xi}_{1}^{t} \right) \left] \right] \]

\[ Q = \frac{1}{\sum_{t=1}^{T+k}} \left[ \sum_{t=1}^{T} \sigma^t \left[ 1 + \left( \frac{\bar{\mu} + \bar{\sigma}^2}{\bar{\gamma}^{t} } \right) \right] + \sigma^t \left[ 1 + \left( \frac{\bar{\mu} + \bar{\sigma}^2}{\bar{\gamma}^{t} } \right) \right] \right]\]

\[ W \leq \left( \sum_{b=1}^{B} \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} + \sum_{b=1}^{B} \sum_{t=1}^{T} \delta^{t}_{b} \bar{\xi}^{t}_{b} + \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{i=1}^{T+k} \delta^{t-1}_{b} \right) \]

\[ + \sum_{b=1}^{B} \sum_{t=1}^{T} \delta^{t+1}_{b} + \sum_{b=1}^{B} \sum_{t=1}^{T} \delta^{t}_{b} \quad \text{(18)} \]

### 5.1.2. Transaction cost (input 1) related constraints

Constraints (19)–(20) and (22)–(23) are related to the past-present-future financial asset and portfolio transaction costs, respectively. It is assumed that in a future period \( t (t = T + 1, \ldots , T + k) \), the future transaction of financial asset associated with its sub-periods \( (l) \), \( l = 1, \ldots , h \) are the same. Constraints (21) and (24) ensure that if the values of the slack variables corresponding to the transaction costs of a financial asset are greater than zero, then a portfolio including the financial asset cannot have zero value of the slack variables corresponding to the transaction costs of the portfolio. Note that the \( M \) in (21) and (24) is a very large positive number.

\[ c_{b} = \frac{1}{\sum_{t=1}^{T+k}} \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} + \sum_{t=1}^{T} \delta^{t}_{b} \bar{\xi}^{t}_{b} \quad (i = 1, \ldots , s_2; b = 1, \ldots , B; t = 1, \ldots , T) \]

\[ M_{\delta} = \frac{1}{\sum_{t=1}^{T+k}} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} + \sum_{t=1}^{T} \delta^{t}_{b} \bar{\xi}^{t}_{b} \right) \quad (i = 1, \ldots , s_2; b = 1, \ldots , B; t = 1, \ldots , T) \]

\[ c_{b} = \frac{1}{\sum_{t=1}^{T+k}} \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} + \sum_{t=1}^{T} \delta^{t}_{b} \bar{\xi}^{t}_{b} \quad (i = 1, \ldots , s_2; b = 1, \ldots , B; t = 1, \ldots , T) \]

\[ M_{\delta} = \frac{1}{\sum_{t=1}^{T+k}} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} + \sum_{t=1}^{T} \delta^{t}_{b} \bar{\xi}^{t}_{b} \right) \quad (i = 1, \ldots , s_2; b = 1, \ldots , B; t = 1, \ldots , T) \]

\[ \text{5.1.3. CVaR deviation (input 2) related constraints} \]

Constraints (25)–(26) and (27)–(32) correspond to the CVaR deviations of financial assets and portfolios, respectively. Recall that \( \text{CVaR}_b(\bar{\alpha}) = \text{CVaR}_b(\bar{\alpha}) \) and \( \text{CVaR}_b(\bar{\alpha}) = \text{CVaR}_b(\bar{\alpha}) \). Note that objective function (15) maximizes \( \bar{\delta}^{t}_{b} \), and thus minimizes \( s_{b}^{t+1} + \frac{1}{(T_{h} + 1) \sum_{b=1}^{B} \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b}} \) in constraints (27) and (28). It follows that constraints (27)–(32) define CVaR risks and thus CVaR deviations. Constraints (33) play a similar role to that of constraints (21) and (24).

\[ \text{CVaR}_{b}^{(T+1)} = \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ = \sum_{b=1}^{B} \left( \text{CVaR}_{b}^{(T+1)} \right) = \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \text{CVaR}_{b}^{(T+1)} \left( \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \right) \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]

\[ \sum_{b=1}^{B} \left( \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right) - \text{E} \left[ \sum_{t=1}^{T} \delta^{t+1}_{b} \xi^{t+1}_{b} \right] \]
5.14. Rates of return (output) related constraints

Constraints (34), (37), (38), and (35), (39), (40) correspond to the past-present-future financial asset and portfolio rates of return, respectively. Constraints (36) and (41) play a similar role to that of constraints (21), (24) and (33).

\[ \tilde{V}_b = \sum_{r=1}^{B} \tilde{V}_r \delta_{br} - \tilde{d}^+ (b = 1, ..., B; t = 1, ..., T) \]  
\[ \sum_{b=1}^{B} \tilde{V}_b \hat{\lambda}_b = \sum_{b=1}^{B} \tilde{V}_b \lambda_b - \tilde{d}^+ (t = 1, ..., T) \]  
\[ M \tilde{d}^+ \geq \tilde{d}^+ \lambda_b (b = 1, ..., B; t = 1, ..., T) \]  

\[ \sum_{b=1}^{B} \tilde{z}_{b(t+1)} \hat{\lambda}_b = \sum_{b=1}^{B} \tilde{z}_{b(t+1)} \lambda_b (\forall i = 1, ..., nfix; t = 1, ..., T - 1) \]  
\[ \sum_{b=1}^{B} \tilde{z}_{b(t+t+1)} = \sum_{b=1}^{B} \tilde{z}_{b(t+t+1)} \lambda_b + s^{\text{free}}_{bt} (\forall b = 1, ..., B; i = 1, ..., nfix; t = 1, ..., T - 1) \]  
\[ \sum_{b=1}^{B} \tilde{z}_{b(t+t+1)} = \sum_{b=1}^{B} \tilde{z}_{b(t+t+1)} \lambda_b + s^{\text{free}}_{bt} (\forall b = 1, ..., B; i = 1, ..., nfix; t = 1, ..., T - 1) \]  

5.15. Other constraints

Constraints (48) and (50), and (49) and (51) define intensity variables corresponding to financial asset and portfolio, respectively. Constraints (52)–(53) enforce non-negative restrictions on the input-output slack variables. Constraints (54)–(55) define slack variables related to free carry-overs.

\[ \sum_{b=1}^{B} \lambda_b = 1 (t = 1, ..., T + k) \]  
\[ \sum_{b=1}^{B} \tilde{\delta}_{br} = 1 (\forall b = 1, ..., B; t = 1, ..., T + k) \]  
\[ \lambda^t_b \geq 0 (\forall b = 1, ..., B; t = 1, ..., T + k) \]  
\[ \delta^t_b \geq 0 (\forall b = 1, ..., B; r = 1, ..., B; t = 1, ..., T + k) \]  

5.14. Carry-over related constraints

Constraints (42)–(45) are basic carry-over related constraints. Constraints (46)–(47) ensure the continuity of carry-overs between two consecutive periods. These constraints are critical for dynamic type models because they connect two consecutive periods (Tone & Tsutsui, 2010). Note that continuity constraints correspond to each single financial asset, but not a whole portfolio (see Fig. 2). Note also that these continuity constraints exert a direct effect on the values of asset-related intensity variables, and thus an indirect effect on the values of portfolio-related intensity variables.

\[ \tilde{z}_{b(t+1)} = \sum_{t=1}^{T} \tilde{z}_{b(t+1)} \delta_{br} (\forall b = 1, ..., B; i = 1, ..., nfix; t = 1, ..., T - 1) \]  
\[ \tilde{s}_{bt} \text{ free: unrestricted in sign} (\forall i = 1, ..., nfix; t = 1, ..., T - 1) \]  
\[ \tilde{s}_{bt} \text{ free: unrestricted in sign} (\forall i = 1, ..., nfix; t = 1, ..., T - 1) \]
5.16. NDN DEA models

Model (15)–(55) is exactly the proposed NDN DEA model. It is noted that constraint (18) and the second term in objection function (15) ensure that model (15)–(55) determines the efficiency of the first financial assets. Therefore, we transform the model into a linear program by using Charnes-Coopier transformation (see e.g., Cooper, Seiford & Tone, 2007). More precisely, to do so, we remove W in (15) and constraint (18), transform the resulting model into a linear program by using the Charnes-Cooper transformation, and then add -W to the linear objective function of the transformed linear program and add constraint (18) with variables being modified by using the transformation procedure to the constraints of the transformed model. The validity of this three-stage method is easy to check.

5.2. Nested DEA model structure

It is important to note that the second term in (15) and constraints (18), and constraints (19), (22), (25), (26), (34), (37), (38), (42), (44), (46), (47), (49), (51), and (52) comprise an additive dynamic model for simultaneously evaluating the efficiency of all of the financial assets. That is, the additive model is nested within the SBM DEA model (15)–(55), which is thus referred to as the nested DEA model. Note that financial asset and portfolio related constraints are the associated δb and λb intensity variables, respectively. Namely, this model separately determines the efficiency of the financial assets and the financial portfolio. However, the big M in (18) with objective function (15) ensures that the efficiency of the financial assets dominates that of the target portfolio; the rationale is clear. In addition, constraints (21), (24), (33), (36), and (41) ensure that if the target portfolio is non-oriented portfolio efficient, then its constituent financial assets are efficient. For brevity, nested dynamic network DEA model (15)–(55) is referred to as the NDN DEA model in this research.

5.3. Construction of an efficient portfolio

Note that the goal of the NDN DEA model is to evaluate the performance of the target portfolio. However, we can actually go one step further by suggesting an efficient portfolio. More precisely, we first modify the NDN DEA model to obtain the following additive dynamic DEA model for evaluating financial asset b ∈ {1, ..., B}:

\[ \sigma_b^* = \max \sum_{i=1}^{T} \sum_{t=1}^{T} \sigma_{i}^{T} + \sum_{i=1}^{T-1} \sum_{t=1}^{T-1} \sigma_{i}^{T-1} + \sum_{i=1}^{T} \sum_{t=1}^{T} \sigma_{i}^{T} + \sum_{i=1}^{T-1} \sum_{t=1}^{T-1} \sigma_{i}^{T-1} \quad (59) \]

\[ \sigma_{ib}^* = \sum_{r=1}^{B} \sigma_{ib}^* + \sigma_{ib}^- \quad (i = 1, ..., s_i; t = 1, ..., T) \quad (60) \]

\[ \sigma_{ib}^- = \sum_{r=1}^{B} \sigma_{ib}^* + \sigma_{ib}^- \quad (i = 1, ..., s_i; t = T + 1, ..., T + k) \quad (61) \]

\[ \text{CVar}_{i}^{T-1} = \left( - \sum_{n=1}^{h} p_{n}^{(T+1)\text{V}_{T}^{(T+1)}} \right) - E \left[ - \sum_{n=1}^{h} p_{n}^{(T+1)\text{V}_{T}^{(T+1)}} \right] \quad (i = 1, ..., s_i) \quad (62) \]

Lastly, it is noted that the non-oriented nested dynamic network DEA model (15)–(55) is a fractional (nonlinear) program. Note also that the DEA model with expected rate of return as the output, and transaction costs and CVAR risks as the inputs retains linear constraints. Hence, we transform the model into a linear program by using the Charnes-Cooper transformation (see e.g., Cooper, Seiford & Tone, 2007). More precisely, to do so, we remove W in (15) and constraint (18), transform the resulting model into a linear program by using the Charnes-Cooper transformation, and then add -W to the linear objective function of the transformed linear program and add constraint (18) with variables being modified by using the transformation procedure to the constraints of the transformed model. The validity of this three-stage method is easy to check.
\[ \text{CVaR}_{\mu_b} \left( - \sum_{n=1}^{h} \frac{1}{h} \sum_{m=1}^{n} p_{mn}^B \right) \nu_{b}^{T} - \left[ - \sum_{n=1}^{h} \frac{1}{h} \sum_{m=1}^{n} p_{mn}^B \right] \nu_{b}^{T} \] 

\[ + \epsilon_t^B (i = 1, \ldots, s; t = T + 2, \ldots, T + k) \] 

(63)

\[ \tilde{V}_{b}^T = \sum_{r=1}^{B} \tilde{V}_{b}^T \delta_t^T - s_b^T \] 

(64)

\[ E \left[ \sum_{n=1}^{h} \frac{1}{h} \sum_{m=1}^{n} p_{mn}^B \right] \nu_{b}^{T} \] 

(65)

\[ \tilde{z}_{ir(fix)}^{T} = \sum_{r=1}^{B} \tilde{z}_{ir(fix)} \delta_t^T \] 

(67)

\[ \tilde{z}_{ir(free)}^{T} = \sum_{r=1}^{B} \tilde{z}_{ir(free)}^{T} + s_{it}^{free} \] 

(68)

\[ \sum_{r=1}^{B} \tilde{z}_{ir(fix)} \delta_t^T = \sum_{r=1}^{B} \tilde{z}_{ir(fix)} \delta_t^T + s_{it}^{free} \] 

(69)

\[ \sum_{r=1}^{B} \tilde{z}_{ir(free)}^{T} = \sum_{r=1}^{B} \tilde{z}_{ir(free)}^{T} + s_{it}^{free} \] 

(70)

\[ \delta_t^T = 1 \] 

(71)

\[ \delta_t^T \geq 0 \] 

(72)

\[ s_{it}^{free}, s_{it}^{free}, \epsilon_t^B \geq 0 \] 

(73)

\[ s_{it}^{free}, \text{unrestricted in sign} \] 

(74)

**Definition 2** (financial asset efficient). Financial asset \( b \in \{1, \ldots, B\} \) is financial asset efficient if and only if \( \sigma_b^2 = 0 \).

**Theorem 1.** If target portfolio \( \tilde{\Lambda} = (\tilde{\Lambda}^1, \tilde{\Lambda}^2, \ldots, \tilde{\Lambda}^{T+k}) \) is non-oriented portfolio efficient, then all its constituent financial assets are efficient, but not vice versa.

**Proof.** According to Definition 1, target portfolio \( \tilde{\Lambda} \) is non-oriented portfolio efficient if and only if \( \sigma_b^2 = 0 \). Note that this condition is equivalent to \( \tilde{z}_{ir}^{T}, \tilde{z}_{ir}^{T}, \tilde{z}_{it}^{T}, \tilde{z}_{it}^{T}, \epsilon_t^T, \epsilon_t^T = 0 \) (\( \forall i, b, t \)) and \( z_{ir}^{T}, z_{ir}^{T}, z_{it}^{T}, z_{it}^{T}, z_{it}^{T}, \epsilon_t^T = 0 \) (\( \forall i, t \)). Likewise, according to Definition 2, financial asset \( b \in \{1, \ldots, B\} \) is financial asset efficient if and only if \( \sigma_b^2 = 0 \). Notice that this condition is equivalent to \( \tilde{z}_{ir}^{T}, \tilde{z}_{ir}^{T}, \tilde{z}_{it}^{T}, \tilde{z}_{it}^{T}, \epsilon_t^T, \epsilon_t^T = 0 \) (\( \forall i, b, t \)). However, \( \tilde{z}_{ir}^{T}, \tilde{z}_{ir}^{T}, \tilde{z}_{it}^{T}, \tilde{z}_{it}^{T}, \epsilon_t^T \geq 0 \) (\( \forall i, t \)). Therefore, even though all target portfolio \( \tilde{\Lambda} \)’s constituent financial assets are efficient, target portfolio \( \tilde{\Lambda} \) may not be non-oriented portfolio efficient. □

For brevity, additive dynamic DEA model (59)–(74) is referred to the AD DEA mode hereafter. Let \( \Omega \) be the set of resulting efficient financial assets. Then, we construct a new target portfolio that consists of only the efficient financial assets in \( \Omega \). We validate this two-stage approach in the Empirical Study section.

6. **Empirical study**

This research proposes new NDN DEA models within the DEA literature for evaluating portfolio performance. Hence, we use actual data to conduct an empirical study to analyze and evaluate this new system of DEA models. The actual data, extracted from the Taiwan Economic Journal (TEJ) database from years 2011 to 2018, contain 44 textile companies in Taiwan. The textile industry, considered a mature industry, seems to be affected by the vicissitudes of the global economy, the tariffs protection of each country, and the demand or the supply of materials such that the overall textile industry’s rate of return is highly volatile from 2011 to 2018. Taiwan’s textile industry was once extremely dependent on China’s population dividend before the US-China trade war, but textile firms have since diversified their customer bases and spread out their production centers. The coronavirus epidemic that occurred during 2019–2020 has accelerated manufacturing hubs to move out of China (Lu et al., 2020). The actual data are used to prepare the output (realized and expected rates of return), the inputs (transaction costs and CVaR deviation), and fixed and free carry-overs.

To justify the efficacy of the proposed DEA models, this study uses the cross-validation technique to benchmark their performance against realized outcomes. That is, we separate the 2011–2018 data into training and testing sets, and consider three cases in which the 2011–2015, 2011–2016, and 2011–2017 data are used for training, and the 2016, 2017, and 2018 data are used for testing, respectively. That is, in this study, we consider only one future period because short-period forecasts are generally more accurate than medium- and long-term ones under highly volatile portfolio flows. Take the first case as an example: 2011–2014 (periods 1–4) are past periods, 2015 (period 5) is the present period, and 2016 (period 6) represents the future period. The other two cases are processed likewise.

6.1. **Data analysis**

In this section, we describe the selected input, outputs, and carry-overs for the illustrated textile companies. First, as indicated, this research applies the MCMC Bayesian algorithm to determine the forecast rate of return associated with each financial asset \( b = 1, \ldots, B \) in \( T + g(l)(g = 1, \ldots, k; l = 1, \ldots, h) \). More precisely, this research considers the normal-normal conjugate pair. That is, if the sampling distribution for \( V \) (rate of return) is a normal distribution \( N(\mu, \sigma^2) \) with known variance \( \sigma^2 \) but unknown
mean $\mu$, and the prior distribution on $\mu$ is a normal distribution $N(\mu_0, \sigma_0^2)$, then the posterior distribution on $\mu$ is also normal. We use the commercial software STATA (version 15) that adapts the Metropolis–Hastings algorithm (an MCMC algorithm) to obtain the posterior distribution on the mean of the rate of return $\hat{V}$. In addition, we also obtain the posterior summary statistics of 40% equal-tailed credible intervals with lower limit ($L$) and upper limit ($U$) of the intervals from STATA. The known standard deviation $\sigma$, derived expected rates of return $\hat{V}$, and their corresponding lower ($L$) and upper ($U$) limits are shown in Appendix 1. Second, recall that to derive CVar (CVar deviation), we discretize $V$ by generating a collection of $G$ scenarios, i.e., $\{v_1, v_2, ..., v_G\}$, with equal probabilities. In addition, as described in Section 4, we consider three future scenarios of optimistic, pessimistic, and neutral market scenarios with annual transition probabilities. In the section, we also propose an approach that first fits a normal distribution to the yearly historical data, then constructs a pessimistic interval $[\hat{V} - 3\sigma, L]$, a neutral interval $[L, U]$ and an optimistic interval $[U, \hat{V} + 3\sigma]$, and finally counts the number of stock rates of return in each interval to calculate annual transition probabilities associated with the three market scenarios. The resulting annual transition probabilities are shown in Table 1. We then independently sample $G$ future rates of return with respect to each interval to derive CVar as described in Section 3.1. Recall that we consider only one future period due to volatile portfolio flows. Let the number of CVar $S_1 = 2$, and $a_1 = 0.75$ and $a_2 = 0.95$. The resulting CVar deviations with respect to single future year 2016, 2017, and 2018 cases are shown in Appendix 2.

Third, the inclusion of transaction costs in the portfolio selection problem may present a challenge to the portfolio manager, but is an important practical consideration. Kolm et al. (2014) posit that extensions to the classical mean-variance framework may consider the inclusion of transaction costs, such as market impact costs and tax effects. Individual investors in Taiwan are tax-exempt from capital gains for trading stocks listed in the Securities Exchange Markets. Prior research uses the inverse of the stock price as a proxy of transaction costs toward market impact costs, which represents frictions to trade. These costs erode trading profits and thus reduce trading incentives (Dhalwil & Li, 2006). We follow the definition of transaction costs used in Dhalwil and Li (2006) as the inverse of the stock price, because it is found that a higher level of the inverse of the stock price results in a higher percentage of brokerage costs and bid-ask spreads (Naranjo, Nimaldenland & Rynagert, 2000). Hence, we use the measure of $1/(\text{Share Price})$ in a year to calculate the realized transaction costs in the year. In addition, we use the median value of monthly share prices from 2011 to 2015, from 2011 to 2016, and from 2011 to 2017 as the forecasts of the transaction costs in 2016, 2017, and 2018, respectively. Let $s_1 = 1$. The realized and expected transaction costs are shown in Appendix 3. Fourth, following Chang, Tone and Wu (2015), we treat a textile company's total assets as carry-over activities that connect two consecutive periods. Total assets can be classified into free carry-over and fixed carry-over, in which free carry-over represents the current assets that a company expects to convert to cash or use up within one year. In contrast, fixed carry-over represents the non-current assets that are not easily converted into cash or used up within one year. Here, each of the past and present periods is associated with a free and a fixed carry-over, i.e., $n_{\text{free}} = n_{\text{fix}} = 1$. The free and fixed carry-overs with respect to years 2011 to 2017 are shown in Appendixes 4–5.

Fifth, it is surely difficult, if not impossible, to select the best portfolio (i.e., financial asset distribution). Investors preparing their financial portfolios commonly use indicators such as market value, ROA, ROE, and rate of return. Market value is frequently used as a proxy variable of company size. Hence, it is considered as a measure of the total dollar market value of a company's outstanding shares of stock, which is also referred to as market capitalization. In addition, market value can be also treated as the representative of the degree of investors' attention/interest in this company. ROA is used as an indicator of how efficient the firm's managers utilize their assets to generate earnings. This paper measures the ROA by the earnings before interests, taxes, and depreciation expenses divided by the firm's total assets. ROE is considered as an indicator

| Table 1 | Estimated annual transition probabilities. |
|---------|-----------------------------------------|
| Year    | Pessimistic | Neutral | Optimistic |
| 2016    | 0.54        | 0.23    | 0.23       |
| 2017    | 0.48        | 0.13    | 0.39       |
| 2018    | 0.48        | 0.09    | 0.43       |

| Table 2 | Portfolio efficiency scores w.r.t. market value. |
|---------|--------------------------------------------------|
|         | Forecasted | Realized | Difference |
| 2016    | Three scenarios 0.446779 0.288195 0.158584 |
|         | One scenario 0.433921 0.288195 0.145726 |
| 2017    | Three scenarios 0.435633 0.140859 0.294774 |
|         | One scenario 0.417062 0.140859 0.276203 |
| 2018    | Three scenarios 0.414927 0.170268 0.244639 |
|         | One scenario 0.385705 0.170268 0.215437 |
| Average | 0.422338 0.199774 0.222564 |
| Max     | 0.446779 0.288195 0.294774 |
| Min     | 0.385705 0.140859 0.145726 |
| St dev  | 0.021601 0.069742 0.061038 |

| Table 3 | Portfolio efficiency scores w.r.t. ROA. |
|---------|--------------------------------------|
|         | Forecasted | Realized | Difference |
| 2016    | Three scenarios 0.179870 0.239966 0.060096 |
|         | One scenario 0.269854 0.239966 0.029888 |
| 2017    | Three scenarios 0.217852 0.091561 0.126291 |
|         | One scenario 0.295601 0.091561 0.204040 |
| 2018    | Three scenarios 0.181911 0.100741 0.081170 |
|         | One scenario 0.221223 0.100741 0.120482 |
| Average | 0.227719 0.144089 0.103661 |
| Max     | 0.295601 0.239966 0.204040 |
| Min     | 0.179870 0.091561 0.029888 |
| St dev  | 0.046714 0.074379 0.061184 |

| Table 4 | Portfolio efficiency scores w.r.t. ROE. |
|---------|---------------------------------------|
|         | Forecasted | Realized | Difference |
| 2016    | Three scenarios 0.193784 0.205539 0.011755 |
|         | One scenario 0.251349 0.205539 0.047610 |
| 2017    | Three scenarios 0.218614 0.086747 0.131867 |
|         | One scenario 0.275656 0.086747 0.189909 |
| 2018    | Three scenarios 0.192159 0.082239 0.109920 |
|         | One scenario 0.235117 0.082239 0.152878 |
| Average | 0.228080 0.124842 0.107157 |
| Max     | 0.275656 0.205539 0.189909 |
| Min     | 0.192159 0.082239 0.011755 |
| St dev  | 0.033159 0.062540 0.066392 |

| Table 5 | Portfolio efficiency scores w.r.t. rate of return. |
|---------|--------------------------------------------------|
|         | Forecasted | Realized | Difference |
| 2016    | Three scenarios 0.158954 0.256682 0.097730 |
|         | One scenario 0.168352 0.256682 0.088330 |
| 2017    | Three scenarios 0.206184 0.111343 0.094841 |
|         | One scenario 0.212742 0.111343 0.010399 |
| 2018    | Three scenarios 0.191819 0.170268 0.021551 |
|         | One scenario 0.186898 0.170268 0.016630 |
| Average | 0.187492 0.179413 0.070050 |
| Max     | 0.212742 0.256682 0.010399 |
| Min     | 0.158954 0.111343 0.016630 |
| St dev  | 0.020913 0.065384 0.039758 |
Table 6
Performance comparison between three-scenario and one-scenario cases.

| Asset | 2016 (forecasted) | 2016 (realized) | Difference | 2017 (forecasted) | 2017 (realized) | Difference | 2018 (forecasted) | 2018 (realized) | Difference |
|-------|------------------|-----------------|------------|------------------|-----------------|------------|------------------|-----------------|------------|
| 1     | 0                | 0.319490        | 0.319490   | 0.319490         | 0.319490        | 0.319490   | 0.319490         | 0.319490        | 0.319490   |
| 2     | 0.322672         | 0.212455        | 0.110217   | 0.322672         | 0.212455        | 0.110217   | 0.322672         | 0.212455        | 0.110217   |
| 3     | 0.441504         | 0.448339        | 0.037448   | 0.441504         | 0.448339        | 0.037448   | 0.441504         | 0.448339        | 0.037448   |
| 4     | 0.378343         | 0.883711        | 0.144330   | 0.378343         | 0.883711        | 0.144330   | 0.378343         | 0.883711        | 0.144330   |
| 5     | 0.004103         | 0.004031        | 0.000071   | 0.004103         | 0.004031        | 0.000071   | 0.004103         | 0.004031        | 0.000071   |
| 6     | 5.284841         | 5.284841        | 0.000000   | 5.284841         | 5.284841        | 0.000000   | 5.284841         | 5.284841        | 0.000000   |
| 7     | 0.004031         | 0.004031        | 0.000000   | 0.004031         | 0.004031        | 0.000000   | 0.004031         | 0.004031        | 0.000000   |

Table 7
Objective function values of AD DEA model.

| Asset | 2016 (forecasted) | 2016 (realized) | Difference | 2017 (forecasted) | 2017 (realized) | Difference | 2018 (forecasted) | 2018 (realized) | Difference |
|-------|------------------|-----------------|------------|------------------|-----------------|------------|------------------|-----------------|------------|
| 1     | 0                | 0.124063        | 0.124063   | 0.124063         | 0.124063        | 0.124063   | 0.124063         | 0.124063        | 0.124063   |
| 2     | 0.322672         | 0.187507        | 0.130167   | 0.322672         | 0.187507        | 0.130167   | 0.322672         | 0.187507        | 0.130167   |
| 3     | 0.441504         | 0.358784        | 0.082720   | 0.441504         | 0.358784        | 0.082720   | 0.441504         | 0.358784        | 0.082720   |
| 4     | 0.378343         | 0.468339        | 0.090000   | 0.378343         | 0.468339        | 0.090000   | 0.378343         | 0.468339        | 0.090000   |
| 5     | 0.004103         | 0.004031        | 0.000071   | 0.004103         | 0.004031        | 0.000071   | 0.004103         | 0.004031        | 0.000071   |
| 6     | 5.284841         | 5.284841        | 0.000000   | 5.284841         | 5.284841        | 0.000000   | 5.284841         | 5.284841        | 0.000000   |
| 7     | 0.004031         | 0.004031        | 0.000000   | 0.004031         | 0.004031        | 0.000000   | 0.004031         | 0.004031        | 0.000000   |

of the profitability of the firm relative to the equity. This paper measures the ROE by the net income divided by total shareholders of equity. In addition, rate of return can be a straightforward albeit myopic indicator to prepare the target portfolio.

In this research, we use all of the above four indicators to determine the asset distribution of the target portfolio. More precisely, for each indicator, we use its values for the considered companies in a year to estimate the asset distribution, i.e., the target portfolio \( \lambda = (\lambda_1, \lambda_2, ..., \lambda_{i+k}) \), for the year. In addition, we use the median values of their monthly values from 2011 to 2015, from 2011 to 2016, and from 2011 to 2017 as the forecasts of the asset distributions in 2016, 2017, and 2018, respectively. The realized and expected portfolios corresponding to the market value, ROA, ROE, and rate of return are shown in Appendices 6–9, respectively.

Lastly, it is assumed that the importance of the information decreases away from the present time period. That is, let the period weights \( \alpha^t, t = 1, ..., T + k \) be set as \( \alpha^t > \alpha^{t+1} > ... > \alpha^{T+k} = 0 \), in which \( \alpha^t > \alpha^{t+1} = 0.2, t = 2, ..., T \) and \( \alpha^t - \alpha^{t-1} = 0.2, t = T + 1, ..., T + k \). In addition, assume that all input and output weights are the same, and each input/output weight is equal to 1. That is,

\[
\mu^t_i = \bar{\mu}^t_i = 1, \quad \sum_{i=1}^{s_1} \mu^t_i = s_1, \quad \sum_{i=1}^{s_2} \mu^t_i = s_2, \quad \text{and} \quad \sum_{i=1}^{s_3} \mu^t_i = s_3.
\]
In this section, we first benchmark the outcomes obtained from the proposed NDN DEA models with the realized (true) ones to justify the efficacy of these new proposed DEA models. Then, we validate the two-stage approach proposed in Section 5.3. Recall that the two-stage approach proceeds as follows. We first apply the AD DEA model to obtain efficient financial assets, and then construct the target portfolio that consists of only the efficient financial assets.

### 6.2.1. Performance benchmarking

In this section, we analyze the empirical results obtained by implementing the proposed NDN DEA model for the 44 textile companies in Taiwan with both forecasted and realized data. Specifically, there are four superset of efficiency scores with respect to the indicators of market value, ROA, ROE, and rate of return, respectively. Each superset contains three sets of efficiency scores that are obtained based on 2016 realized and forecasted data (using 2011–2015 historical data), 2017 realized and forecasted data (using 2011–2016 historical data) and 2018 realized and forecasted data (using 2011–2017 historical data), respectively. Further, each set contains two subsets of efficiency scores that consider three scenarios (optimistic, pessimistic, and neutral market scenarios) and one scenario, respectively. The four superset of efficiency scores corresponding to market value, ROA, ROE, and rate of return are shown in Tables 2–5, respectively.

According to Tables 2–5, the return indicator yields both the smallest absolute difference of 0.07008 and the smallest standard deviation of 0.039758 between the forecasted and realized portfolio efficiency scores. However, the market value and ROE perform the worst in terms of the average difference (0.222564) and standard deviation of the difference (0.066392), respectively. In addition, as shown in Table 6, the three-scenario design performs better than the one-scenario design with respect to ROA and ROE, whereas the one-scenario design performs better than the three-scenario design with respect to market value. In terms of the rate of return, the one-scenario design has a smaller average difference but larger standard deviation of the difference than the three-scenario design. Surprisingly, none of the four indicators are appropriate proxies to determine the combination of financial assets in the target portfolio because of the resulting low efficiency scores with respect to all indicators. More precisely, all of the efficiency scores are within [0.082239, 0.446779], in which the market value indicator offers the highest efficiency score. However, we emphasize that it is extremely difficult for any target portfolio to benchmark against all other possible (infinitely many) portfolios.

### 6.2.2. Validation of the two-stage approach

As indicated in the preceding section, it is difficult if not impossible for investors to select their optimal portfolios. Therefore, in Section 5.3, we propose a two-stage approach to help investors construct their desired portfolios. In this section, we validate this approach under the following comparison setting: we consider three sets of objective function values of the AD DEA model that are obtained based on 2016 realized and forecasted data (using 2011–2015 historical data), 2017 realized and forecasted data (using 2011–2016 historical data) and 2018 realized and forecasted data (using 2011–2017 historical data), respectively. The resulting three-set objective function values of the AD DEA model, with each based on both forecasted and realized data, are shown in Table 7.

Table 7 shows that 2 (assets 1 and 41), 4 (assets 6, 21, 30, and 41), 2 (assets 1 and 41), 3 (assets 37, 40, and 41), 2 (assets 41 and 42), and 3 (assets 12, 14, and 41) out of the 44 financial assets are financial asset efficient according to Definition 2 with respect to 2016 (forecasted), 2016 (realized), 2017 (forecasted), 2017 (realized), 2018 (forecasted), and 2018 (realized) cases, respectively. Note that the number of efficient financial assets in any of these cases is much less than the 44 available financial assets. In addition, the absolute average differences between the forecasted and realized objective function values of the AD DEA model are, respectively, 0.603675, 0.742891, and 0.988105 with respect to 2016, 2017, and 2018 cases, which verify the AD DEA model.

Therefore, we construct new target portfolios that consist of only the efficient financial assets as shown in Table 7. Consider the same comparison setting described in Section 6.2.1. The four superset of efficiency scores based on the efficient assets corresponding to market value, ROA, ROE, and rate of return are shown.
in Tables 8–11, respectively. As indicated in the tables, the market value indicator gives the smallest absolute difference of 0.13818, whereas ROA indicator gives the smallest standard deviation of 0.11531 between the forecasted and realized portfolio efficiency scores. The market value indicator is associated with a deviation of 0.13169, which is closest to the smallest one. Also, the market value indicator provides the smallest maximum difference and minimum difference. In addition, all of the efficiency scores are within [0.592738, 1]. The highest forecasted efficiency scores with respect to the four indicators are all equal to 1, while the market value indicator is associated with the highest realized efficiency score of 0.991185. It is emphasized that the realized efficiency scores with respect to market value in all cases are greater than 0.94 with an average of 0.958399. However, the average realized efficiency scores with respect to ROA, ROE, and rate of return are only 0.794512, 0.778117, and 0.801798, respectively. Note that if a target portfolio contains only a single efficient financial asset, then the forecasted efficient score of the portfolio is equal to 1. Based on the above analysis, the market value indicator can be considered as an appropriate proxy for the investors to determine their combinations of financial assets in their target portfolios. Moreover, as shown in Table 12, the three-scenario design performs better than the one-scenario design with respect to ROA, ROE, and rate of return. In terms of market value, the one-scenario design has a smaller average difference but a larger standard deviation of the difference than the three-scenario design.

7. Conclusions

This study proposes new NDN DEA models to help investors evaluate the performance of their target portfolios. The proposed DEA models can handle multi-period portfolio performance assessment and open the “black box”, and thus can measure the multi-period efficiency of a portfolio and its comprised financial assets, in contrast to the portfolio evaluation DEA models in the literature. In addition, the NDN DEA models enable investors to fully assess their target portfolios by benchmarking them against all other possible (infinitely many) portfolios.

To our knowledge, this study proposes the first nested DEA models, in which an additive model is nested within a SBM DEA model. Such a nested structure ensures that if a portfolio is non-oriented portfolio efficient, then all of its comprised financial assets must be financial asset efficient. In addition, it is worth mentioning that the NDN DEA model (a linear programming model) is associated with CVaR (CVaR deviation) constraints, and thus involves a CVaR optimization problem. We show that CVaR risk may take negative values, and thus cannot be used as a risk measure in the SBM DEA models with infinitely many DMUs (full diversification), which is ignored in the studies in the DEA literature.

The empirical study that implements the two-stage approach to the real data concerning textile companies in Taiwan shows that market value can be considered as an appropriate measure for investors to prepare their desired portfolios. In addition, the empirical study also validates the suggested two-stage approach. That is, first use the AD DEA to select efficient financial assets, and then apply the NDN DEA model to evaluate the portfolios comprised of the selected efficient financial assets. The portfolios obtained by using the two-stage approach are associated with significantly higher efficiency scores than those obtained by simply using the NDN DEA model. We note that the number of efficient financial assets selected by the AD DEA model in any of the cases is much less than the available financial assets. Prior finance literature has favored the diversification of portfolio investments. However, this may not be the case in practice. For example, anecdotal evidence reveals that Warren Buffett’s Berkshire Hathaway owns Apple’s shares more than 43% of its entire portfolio. Indeed, Warren Buffett once said that “Diversification is protection against ignorance. It makes little sense if you know what you are doing.” (Langlois, 2020).

Supplementary materials

Supplementary material associated with this article can be found in the online version, at doi:10.1016/j.ejor.2020.09.044.

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Table 12

| Market value | ROA | ROE | Return |
|--------------|-----|-----|--------|
| Three scenarios | One scenario | Three scenarios | One scenario | Three scenarios | One scenario |
| Average diff.  | 0.122185 | 0.154181 | 0.167023 | 0.109971 | 0.203886 | 0.153719 | 0.181197 | 0.135264 |
| Max diff. | 0.301600 | 0.301600 | 0.329178 | 0.228096 | 0.347262 | 0.300010 | 0.364673 | 0.261763 |
| Min diff. | 0.000815 | 0.004004 | 0.036940 | 0.032407 | 0.093171 | 0.049922 | 0.076884 | 0.041994 |
| St dev (diff.) | 0.157169 | 0.133749 | 0.141400 | 0.103960 | 0.17636 | 0.130347 | 0.159391 | 0.113590 |
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