ΛCDM periodic cosmology

Stéphane Fay *
Palais de la Découverte
Astronomy Department
Avenue Franklin Roosevelt
75008 Paris
France

ABSTRACT
We examine the possibility that Universe expansion be made of some ΛCDM expansions repeating periodically, separated by some inflation and radiation dominated phases. This so-called ΛCDM periodic cosmology is motivated by the possibility that inflation and the present phase of accelerated expansion be due to the same dark energy. Then, in a phase space showing the variation of matter density parameter Ωm with respect to this of the radiation Ωr, the curve Ωm(Ωr) looks like a closed trajectory that Universe could run through forever. In this case, the end of the expansion acceleration of the ΛCDM phase is the beginning of a new inflation phase. We show that such a scenario implies the coupling of matter and/or radiation to dark energy. We consider the simplest of these ΛCDM periodic models i.e. a vacuum energy coupled to radiation. From matter domination phase to today, it behaves like a ΛCDM model, then followed by an inflation phase. But a sudden and fast decay of the dark energy into radiation periodically ends the expansion acceleration. This leads to a radiation dominated Universe preceding a new ΛCDM type expansion. The model is constrained with Markov Chain Monte Carlo simulations using supernovae, Hubble expansion, BAO and CMB data and fits the data as well as the ΛCDM one.

Key words: cosmology: theory – dark energy

1 INTRODUCTION
Universe expansion would have accelerated at least two times, a first time during the inflation epoch (Guth 1981) and a second time during the present epoch (Riess 1998; Perlmutter 1999). There are various methods to build a cosmological model with such an expansion history. Hence, in Saez-Gomez (2016), a modified unimodular gravity version of General Relativity is used that is equivalent to General Relativity at the classical level but also provides a quantum description. In Li (2012), a tachyon condensation on an unstable three-brane gives rise to a tachyon inflation followed by a Chaplygin gas dark matter and dark energy universe. In Panigrahi & Chatterjee (2011), a Chaplygin gas in a spherically symmetric inhomogeneous model is considered for which, at early times, Universe behaves as an Einstein-de Sitter solution generalised to an inhomogeneous spacetime and, at late time, as a ΛCDM model. In Perico (2013), the whole Universe history is described by a vacuum decay. In this paper, a class of cosmological models able to describe the whole Universe expansion history as a succession of ΛCDM expansions is found. Let us explain why we consider such a possibility. In a classical way, we assume that Universe is filled by matter, radiation and dark energy with respectively positive densities ρm, ρr and ρd (an assumption that could break down near the singularity). Defining their energy density parameters Ωm, Ωr and Ωd (see their definitions in the next section) such as Ωm + Ωr + Ωd = 1, Universe evolution can then be represented as a trajectory in the space phase (Ωm, Ωr). From today to radiation dominated phases, our Universe is well described by a ΛCDM model. Its trajectory in agreement with observations (Hinshaw 2013) is shown in bold on the phase space of figure 1. One often describes its different epochs by the domination of radiation, matter and then dark energy. But its trajectory also encourages us to describe it as a two phases model, the first one with Ωd ≃ 0 and the second one with Ωr ≃ 0. Moreover, at early time, we should have an inflation phase with a very small matter density parameter Ωm ≃ 0. Inflation is not predicted by the ΛCDM model but it could be due to a flat potential (effective, from a scalar field, etc), behaving like a cosmological constant with a value different from the one at our present time (solving the cosmological constant problem (Weinberg 1989)). An inflation phase could start with Ωd ≃ 1 and Universe should evolve to reach the radiation dominated phase Ωr ≃ 1 (solving the grace-
ful exit problem (Albrecht & Steinhardt 1982). This inflation to radiation phase corresponds to the dashed trajectory on figure 1. Considering then this last trajectory associated to the bold $\Lambda CDM$ one, we observe that Universe evolution could thus be described as a close trajectory in the phase space ($\Omega_m, \Omega_r$), what is named an homoclinic orbit in dynamical system theory. Such a trajectory corresponds to some $\Lambda CDM$ expansions repeating periodically, separated by some inflation and radiation dominated phases (different from the standard $\Lambda CDM$ radiation phase). We call such a periodic behaviour of the expansion a "$\Lambda CDM$ periodic cosmology" and look for the families of cosmological models able to reproduce this scenario of the whole Universe history. Note that we choose to employ the word "periodic" instead of "cyclic" such that there is no confusion with the cyclic cosmological models (Steinhardt & Turok 2005; Baum & Frampton 2007; Penrose 2006; Ashtekar 2009) that avoid the beginning of time thanks to some bounces of the scale factor. This is generally not the case of a $\Lambda CDM$ periodic cosmology that has an initial singularity.

Note also that we found such a cosmological model in Fay (2015) but is was then just a special case for illustration purpose of a subject different from the one of this paper. Here, we want to look for the generic conditions leading to a $\Lambda CDM$ periodic cosmology for various families of cosmological models. This is what is done in the second section of this paper where we examine the case of General Relativity with a dark energy, $F(R)$ or scalar-tensor theories and General Relativity with some viscous or coupled fluids. Among these classes of models, only the last one is able to produce the homoclinic orbits required for a $\Lambda CDM$ periodic cosmology. In a third section, we then study the simplest of these cosmological models consisting in a radiation fluid coupled to a vacuum energy. We constrain it with Markov Chain Monte Carlo simulations applied to supernovae, Hubble expansion, BAO and CMB data. We conclude in the last section by discussing this model from the viewpoints of an effective scalar field potential triggering a warm inflation and an effective fluid unifying dark energy with radiation.

2 COSMOLOGICAL MODELS COMPATIBLE WITH A $\Lambda CDM$ PERIODIC COSMOLOGY

In this section we look for the cosmological classes of models compatible with the presence of some homoclinic orbits in a phase space ($\Omega_m, \Omega_r$). This requires the existence of a center equilibrium point in $\Omega_m \neq 0$ and $\Omega_r \neq 0$ having pure imaginary eigenvalues (Brannan & Boyce 2010). We assume that energy densities of all the species (matter, radiation, dark energy) are positive.

2.1 General Relativity and extended theories of gravity

We first examine General Relativity with non coupled matter, radiation and dark energy. The field equations are

$$ H^2 = \frac{k}{3}(\rho_m + \rho_r + \rho) $$

$$ \frac{d\rho_m}{dt} + 3H\rho_m = 0 $$

Figure 1. Phase space for the energy density parameters ($\Omega_m, \Omega_r$) with the constraint $\Omega_m + \Omega_r + \Omega_\Lambda = 1$. The thick line shows the trajectory of the $\Lambda CDM$ model in agreement with observations and inside the triangular area where the densities are positive. The dashed line is a possible trajectory describing an inflation phase beginning close to the point $\Omega_d \simeq 1$ and followed by a radiation dominated phase going close to the point $\Omega_r \simeq 1$ and different from the $\Lambda CDM$ one. It is then tempting to assume that the whole Universe history could be described by a closed trajectory, i.e. an homoclinic orbit in this phase space.

$$ \frac{d\rho_r}{dt} + 4H\rho_r = 0 $$

$$ \frac{d\rho_\Lambda}{dt} + 3H(w + 1)\rho_\Lambda = 0 $$

$w = p_\Lambda/\rho_\Lambda$ is the dark energy equation of state. We define the energy density parameters

$$ \Omega_m = \frac{k}{3} \frac{\rho_m}{H^2} $$

$$ \Omega_r = \frac{k}{3} \frac{\rho_r}{H^2} $$

$$ \Omega_\Lambda = \frac{k}{3} \frac{\rho_\Lambda}{H^2} $$

The Friedmann equation shows that they are normalised since $\Omega_m + \Omega_r + \Omega_\Lambda = 1$ and the $\Omega_i$ are positive. We then get the following dynamical system

$$ \Omega_m ' = \Omega_m (\Omega_r + 3w(1 - \Omega_m - \Omega_r)) $$

$$ \Omega_r ' = \Omega_r (-1 + \Omega_r + 3w(1 - \Omega_m - \Omega_r)) $$

A prime means a derivative with respect to $N = \ln a$, with $a$ the scale factor. This system is studied in Fay (2013) where it is shown that all its equilibrium points are such as $\Omega_m = 0$ and $\Omega_r = 0$. Hence General Relativity with non coupled matter, radiation and dark energy cannot produce a homoclinic orbit that needs a center equilibrium point with $\Omega_i \neq 0$. It is thus not compatible with a $\Lambda CDM$ periodic.
cosmology. This is also true for any extended theories of gravity that can be rewritten as General Relativity with an effective dark energy (Capozziello & De Laurentis 2011). For instance, let us consider an $f(R)$ theory in the Palatini formalism. Using Fay, Tavakol & Tsujikawa (2007), we define the following normalised variables

$$
\Omega_m = \frac{k}{3} \frac{p_m}{fR} \frac{1}{H^2}
$$

$$
\Omega_r = \frac{k}{3} \frac{\rho_r}{fR} \frac{1}{H^2}
$$

$$
\Omega_f = \frac{F}{fR - f} \frac{1}{6f\xi H^2}
$$

with $R$ the usual curvature scalar, $F = df/dR$ and $\xi = [1 - \frac{3}{2} \frac{dF}{dR}(F - 2F)}{F(1 - 2F)}]^2$. Then the field equations rewrite

$$
\Omega_m' = \Omega_m(\Omega_r - (C - 3)(1 - \Omega_m - \Omega_r))
$$

$$
\Omega_r' = \Omega_r(-(1 + \Omega_r - (C - 3)(1 - \Omega_m - \Omega_r))
$$

with $C(R) = -3 \frac{fR - 2F}{F(1 - 2F)} df/dR - 3$. In this phase space, a $\Lambda$CDM periodic cosmology corresponds to the same homoclinic trajectory as described above but with $f \to 1$ when the expansion mimics a $\Lambda$CDM model. Identifying $C - 3 = 3w$, we recover the same dynamical system as previously and, once again, we cannot have a center equilibrium point since there is no equilibrium points with $\Omega_r \neq 0$. The same could be shown for $f(R)$ theory in metric formalism or scalar-tensor theories, that also reduce to General Relativity with a non coupled dark fluid.

### 2.2 Coupled and viscous fluids

We first consider a dark energy coupled to both radiation and matter with coupling functions $q_m$ and $q_r$. We define

$$
q_m = \frac{k}{3} \frac{Q_m}{H^2}
$$

$$
q_r = \frac{k}{3} \frac{Q_r}{H^2}
$$

Then, as shown in Fay (2015), a dynamical system for General Relativity with matter and radiation coupled to dark energy writes

$$
\Omega_m' = \Omega_m [\Omega_r + 3w(1 - \Omega_m - \Omega_r)] + q_m
$$

$$
\Omega_r' = \Omega_r[-1 + \Omega_r + 3w(1 - \Omega_m - \Omega_r)] + q_r
$$

For sake of simplicity, we chose to consider the case $w = -1$ (but our conclusions are the same for any form of $w$). The previous equations system simplifies to

$$
\Omega_m' = \Omega_m(3\Omega_m + 4\Omega_r - 3) + q_m
$$

$$
\Omega_r' = \Omega_r(3\Omega_m + 4\Omega_r - 4) + q_r
$$

We now look for center equilibrium points $\Omega_m(eq), \Omega_r(eq)$, $\Omega_m(eq), \Omega_r(eq)$ do not have to be pure imaginary numbers take the form

$$
\Omega_m(eq) > 0 \text{ and } \Omega_r(eq) > 0
$$

2.2.1 $q_m = 0$

Here, we consider any forms for $w$. There is only one possible center equilibrium point that writes

$$
(\Omega_m(eq), \Omega_r(eq)) = \left( \frac{q_r + 3w - 3q_r w}{3w}, q_r \right)
$$

Then $\Omega_m(eq) > 0, \Omega_r(eq) > 0$ and $\Omega_m(eq) + \Omega_r(eq) < 1$ if $w < 0$

and

$$
0 < q_r < \frac{3w}{3w - 1}
$$

The conditions such as $\lambda_{\pm}$ be pure imaginary numbers are that

$$
\frac{dq_r}{d\Omega_m} = \frac{3w^2(1 + 3w)}{3w^2} + 3q_r^2 w \frac{dw}{d\Omega_m} + q_r(q_r + 3w - 3q_r w) \frac{dw}{d\Omega_m}
$$

and $\frac{dq_r}{d\Omega_m} > (<) 0$ to

$$
\left(3w^2 + q_r \frac{dw}{d\Omega_m}\right) \left(9w^3 + 3q_r^2 w \frac{dw}{d\Omega_m} + q_r(q_r + 3w - 3q_r w) \frac{dw}{d\Omega_m}\right)
$$

$$
3w^2 \left(9w^3 + 3q_r^2 w \frac{dw}{d\Omega_m} + q_r(q_r + 3w - 3q_r w) \frac{dw}{d\Omega_m}\right)
$$

when $q_r \frac{dw}{d\Omega_m} > w(1 - 3w)$ (respectively $< w(1 - 3w)$) in $(\Omega_m(eq), \Omega_r(eq))$, Note that if we set $w = -1$, there should thus be some homoclinic trajectories corresponding to a $\Lambda$CDM periodic model. However in Fay (2014), we show
that when radiation is coupled to vacuum energy, an inflation phase needs a negative radiation density at early time whereas here, we assume \( \rho_r > 0 \). This apparent contradiction comes from the fact that in Fay (2014), we assumed a source point to represent the beginning of the inflation phase. But this is no more required in the present paper where we consider homoclinic orbits and we can thus have an inflation phase with \( q_m = 0 \) in the case of a periodic \( \Lambda CDM \) model with positive densities.

### 2.2.2 \( q_r = 0 \)

Here again, we do not specify \( w \). Then the only possible center equilibrium point is located in

\[
(\Omega_m(q_r), \Omega_r(q_r)) = (-q_m \frac{-1 + 3w + 3q_m w}{1 + 3w}, -1 + 3w)
\]

Then \( (\Omega_m(q_r) > 0, \Omega_r(q_r) > 0) \) and \( \Omega_m(q_r) + \Omega_r(q_r) < 1 \) if

\[
w > 1/3
\]

that discards the special constant case \( w = -1 \) (Fay 2014), and

\[
q_1 > \frac{1}{3w} - 1
\]

\( q_1 \) is thus negative, implying an energy transfer from dark matter to dark energy. The eigenvalues \( \lambda_{\pm} \) are pure imaginary numbers if

\[
\frac{dq_m}{dm} = (3q_m (-1 + 3(1 + q_m) w) \frac{dw}{d\Omega_m} + (3w - 1) \times (2 - 9w + 9w^2 - 3q_m \frac{dw}{d\Omega_m})(1 - 3w)^{-2}
\]

and \( \frac{dw}{d\Omega_m} < \) or >

\[
((1 - 3w)^2 + 3q_m \frac{dw}{d\Omega_m}) (3q_m (-1 + 3(1 + q_m) w) \frac{dw}{d\Omega_m} + (1 - 3w)((1 - 3w)^2 - 3q_m \frac{dw}{d\Omega_m}))
\]

\[
(3(1 - 3w)^2(w(-1 + 3w) + q_m \frac{dw}{d\Omega_m}))^{-1}
\]

when \( q_m \frac{dw}{d\Omega_m} > w(1 - 3w) \) (respectively < \( w(1 - 3w) \)) in \((\Omega_m(q_r), \Omega_r(q_r))\).

### 2.2.3 Viscous fluid

A viscous fluid can be defined by adding to the usual definition of pressure a term \( B_i \) containing a viscosity coefficient (see for instance Li & Barrow (2009)). For instance, if dark energy is a viscous fluid, we then have \( p_d = w_d \rho_d - B_d \).

In the extreme case where all the fluids are viscous, then defining \( q = k(B_m + B_r + B_d)\), the dynamical system for viscous fluid is found by putting in equations (4-5) \( q_m = -q \Omega_m \) and \( q_r = -q \Omega_r \) then the equilibrium points are

\[
((0, -1 + 3w), \frac{-q - 3w}{3w}, (0, 0))
\]

Since all of them have a vanishing \( \Omega_m \) or \( \Omega_r \), there is thus no homoclinic orbit for General Relativity with viscous fluids.

Hence, if we assume positive energy densities, a \( \Lambda CDM \) periodic model can only be reproduced by a dark energy coupled to radiation or/and matter. In the next section, we examine the simplest \( \Lambda CDM \) periodic model and constrain it with some observations.

### 3 THE SIMPLEST \( \Lambda CDM \) PERIODIC MODEL

The simplest \( \Lambda CDM \) periodic model is defined by

\[
w = -1
\]

\[
q_m = 0
\]

\[
q_r = \alpha \Omega_m \Omega_d
\]

with \( \alpha \), a positive constant. A dark energy coupled to radiation is not new. It is used in warm inflation paradigm (Berrera 1998) for instance. In what follows, we examine the dynamics of this model (expansion, densities, coupling function, etc) and how observations constrain it.

#### 3.1 Model dynamics

The equilibrium points with non vanishing coordinates write

\[
(\Omega_m, \Omega_r) = \left(\frac{1}{3}(3 - 4\alpha(1 - \Omega_m - \Omega_r))\Omega_r, \alpha(1 - \Omega_m - \Omega_r)\right)
\]

The eigenvalues take the form

\[
\lambda_{\pm} = \lambda_1 \pm \sqrt{\frac{3}{6}} \sqrt{\lambda_2}
\]

with

\[
\lambda_1 = 1 - \frac{1}{2} \alpha(\Omega_m + 2\Omega_r - 1)
\]

\[
\lambda_2 = 48 + 24\alpha (-1 + \Omega_m - 2\Omega_r + 2\Omega_m\Omega_r + 2\Omega_r^2) + \alpha^2 (3 + 36\Omega_r - 68\Omega_r^2 + 32\Omega_m^2 + \Omega_m(3 + 48\Omega_r) + \Omega_m(-6 - 84\Omega_r + 80\Omega_r^2))
\]

They are pure imaginary numbers if \( \lambda_1 = 0 \). This implies

\[
\alpha = \frac{2}{\Omega_m + 2\Omega_r - 1}
\]

Introducing this value for \( \alpha \) in the above equilibrium points, we derive that the set of \((\Omega_m, \Omega_r)\) equilibrium points in agreement with an homoclinic orbit is on the line

\[
\Omega_r = \frac{3}{4}(1 - \Omega_m)
\]

and thus \( \alpha > 4 \). Introducing this expression for \( \Omega_r \) in the eigenvalues, it comes

\[
\lambda_{\pm} = \pm \sqrt{-3i\Omega_m}
\]

which are thus pure imaginary. A phase space with \( \alpha = 5.42 \) (in agreement with the data, see subsection 3.2) is plotted on figure 2. The density parameters \( \Omega_i \) for the \( \Lambda CDM \) and coupled models are shown on figure 3. Note that we split \( \Omega_m \) in its two components for cold dark matter and baryon, i.e. \( \Omega_m = \Omega_{CDM} + \Omega_b \) since in subsection 3.2 we use some BAO and CMB observations to constrain the model and the sound horizon depends on the baryons density. The density parameters \( \Omega_i \) of the two models evolve very closely.
The past phases of acceleration can thus be considered as an infinite number of inflation phases. We are presently at \( \Omega_m \) thus behaves the same way in the coupled and \( \Lambda \)CDM models. Then, when \( \rho_d \) is nearly a constant whereas matter and radiation densities decreases, dark energy ends up dominating Universe content, accelerating its expansion. The past phases of acceleration can thus be considered as an infinite number of inflation phases. We are presently at the beginning of a new acceleration phase during which radiation density will increase as shown by the peak in the coupling function on figure 5: dark energy will be cast into radiation that will dominate Universe content in the next period, ending the present acceleration phase.

- Matter is not coupled. Its density \( \rho_m \) thus behaves the same way in the coupled and \( \Lambda \)CDM models.

- Dark energy is a decreasing function. Its density \( \rho_d \) periodically becomes nearly a constant, mimicking a cosmological constant. Then, when \( \rho_d \) is nearly a constant whereas matter and radiation densities decreases, dark energy ends up dominating Universe content, accelerating its expansion.

3.2 Observational constraints

We now check if this model is in agreement with some observational data. We use a Markov Chain Monte Carlo (MCMC) sampler (Arjona, Cardona & Nesseris 2019A,B) whose code is available at members.ift.uam-csic.es/savvas.nesseris/. We consider the following data (where a subscript 0 means a value of a parameter today)
For the CMB, we consider the shift parameters $(R, l_s)$ and $\Omega_{b0}h_0^2$, with $H_0 = 100h_0$, based on Planck final release (Aghanim 2018) as derived by Zhai & Wang (2019).

For the CMB, we also consider its temperature that constrains $\Omega_\gamma$, and avoids its degeneracy with the $\alpha$ parameter. $\Omega_\gamma$ can be separated in two components, one for the photons $\Omega_\gamma$ and one for the relativistic neutrinos $\Omega_\nu$. The photons density $\Omega_{b0}h_0^2$ is precisely measured by the CMB temperature as $\Omega_{b0}h_0^2 = 2.47282 \times 10^{-5}$. $\Omega_\gamma$ is related to $\Omega_\nu$ by (see Komatsu (2009) for instance) $\Omega_\gamma = 0.227\Omega_{b0}N_{eff}$ where $N_{eff}$ is the effective number of neutrino species. Then, the values $h_0 = 0.674 \pm 0.005$ and $N_{eff} = 2.99 \pm 0.17$ by Planck (Aghanim 2018) lead to the $1\sigma$ constraint $\Omega_\gamma = (9.13 \pm 0.25) \times 10^{-5}$ that we introduce as a prior in the $\chi^2$ defined below.

For the supernovae, we consider the 1048 supernovae from the last Pantheon data (Scalzo 2018).

For the BAO measurements, we consider the Lyα forest from BOSS DR11 (Delubac 2013), BOSS DR12 (Gil-Marín 2016), WiggleZ (Blake 2011), 6dFGS (Beutler 2011) and DES Year 1 (Abbott 2018). This corresponds to 10 measurements.

For the Hubble function, we consider 36 data points from MoreSCO (2012B, A, 2015, 2016) and Guo & Zhang (2016).

The total $\chi^2$ is defined as $\chi^2 = \chi^2_{\text{CMB}} + \chi^2_{SN} + \chi^2_{\text{BAO}} + \chi^2_{\text{H}} + (\Omega_{\alpha} - 9.13 \times 10^{-5})/0.25 \times 10^{-5})^2$. The priors used for the MCMC analysis are $\Omega_{\alpha0} \in [0.2, 0.4]$, $\Omega_\alpha \in [10^{-5}, 10^{-4}]$, $\Omega_{b0}h_0 \in [0.001, 0.06]$ and $h_0 \in [0.55, 0.80]$. Figure 6 shows the 68.3%, 95.4% and 99.7% confidence contours for the coupled model and the one-dimensional marginalized likelihoods with a sample of 500000 MCMC points (the black points are the mean MCMC values). At 68.3%, we have $\Omega_{\alpha0} = 0.298 \pm 0.006$, $10^4\Omega_{\nu0} = 0.091 \pm 0.002$, $\Omega_{b0}h_0^2 = 0.0224 \pm 0.0001$, $\alpha = 5.42 \pm 0.13$ and $H_0 = 67.74 \pm 1.17$, the best $\chi^2$ being 1068.07. The best $\chi^2$ and coupled models $\chi^2$ with their components are shown in Table 1. Using the Akaike information criterion (AIC)/(Akaike 1974) as done in Arjona, Cardona & Nesseris (2019A) and Arjona, Cardona & Nesseris (2019B), we can compare the two models. Both of them being constrained by the same 1098 data and the coupled model having one more free parameter than the $\Lambda$CDM one, we find that

$$AIC_{\text{coupled}} - AIC_{\Lambda\text{CDM}} \approx 2.7$$
4 DISCUSSION AND CONCLUSION

This paper starts from the idea that Universe expansion is well described by a $\Lambda CDM$ model, preceded by an inflation phase at early time that could also be due to a vacuum energy or a very flat potential when matter and radiation were negligible. In the phase space $(\Omega_m, \Omega_r)$, such a description looks like the bold and dashed trajectories on figure 1. This suggests that such a scenario could be described by a homoclinic orbit. The behaviours of the density parameters $\Omega$ would thus repeat periodically as well as a $\Lambda CDM$ type expansion with different values of $\Lambda$ (see below), hence the name $\Lambda CDM$ periodic cosmology. Among the classes of models we consider in section 2, the only one in agreement with homoclinic orbits is General Relativity with a dark energy coupled to matter and/or radiation. We then defined the conditions on the dark energy equation of state $w$ and the coupling functions $q_m$ and $q_r$ to get some homoclinic orbits. Then, we considered the simplest of these models defined by $w = -1$, $q_m = 0$ and $q_r = \alpha \Omega_m$. The conditions for homoclinic orbits impose $\alpha > 4$, meaning that the model cannot be reduced to a $\Lambda CDM$ model with $\alpha = 0$. This corresponds to a vacuum energy coupled to the radiation, the former decaying into the latter.

We constrained this model with Markov Chain Monte Carlo simulations using supernovae, Hubble expansion, BAO and CMB data. Then, we found at 68.3\% confidence  that $\Omega_{m0} = 0.298 \pm 0.006$, $10^3 \Omega_{r0} = 0.091 \pm 0.002$, $\Omega_{m0}h_0^2 = 0.0224 \pm 0.0001$, $\alpha = 5.42 \pm 0.13$ and $H_0 = 67.74 \pm 1.17$.

Choosing the best fit value $\alpha = 5.42$, a period of this constrained cosmological model, defined as the interval of time during which a trajectory leaves and comes back near the phase space point $(\Omega_m, \Omega_r)$, lasts $\Delta N = 38.1$ and can be described as follows. It begins when density parameters of the matter and radiation are very small. Dark energy behaves then like a cosmological constant. The coupling between dark energy and radiation is stronger and stronger. More and more dark energy is cast into radiation that comes to dominate. The expansion acceleration ends whereas Universe becomes dominated by radiation. The densities of all the species then decrease. The density of dark energy reaches a new constant value and the expansion dominated by matter begins to behave as a $\Lambda CDM$ expansion. Consequently, dark energy dominates again and a new phase of accelerated expansion takes place whereas the radiation density starts to increase, beginning a new (colder) period. Note that our period is the first one long enough for life and structures to appear. It begins 13.9 billions years ago (nearly the Universe age) and will end in 35 billions years.

We conclude this paper by examining the periodic $\Lambda CDM$ model from the viewpoints of an effective scalar field potential triggering a warm inflation and an effective fluid unifying dark energy with radiation. From the viewpoint of inflation, periodic $\Lambda CDM$ models are examples of warm inflation (Berera 1995) since radiation is produced during the inflation phase by the decay of dark energy. Hence the inflationary phase ends into a radiation dominated phase, avoiding the graceful exit problem (Guth & Weinberg 1983). For the specific model of section 3, it is interesting to look for an effective scalar field $\phi$ with a density $\rho_{eff} = \rho_\phi + \rho_r$. Its effective potential $V_{eff}$ is plotted on the first graph of figure 7 as a function of $N$. When expansion is accelerated, the scalar field potential $V_{eff}$ behaves as a cosmological constant $\Lambda_{eff}$ with smaller and smaller value at each period as we go to the future. Comparing the scalar field potential on figure 7 with the radiation density parameter on figure 3, we note that each time a slow roll (Linde 1982, Albrecht & Steinhardt 1982) ends, a reheating starts. With $\alpha = 5.42$, we have from one period $i$ to the next one $i + 1$, $\Lambda_{eff}(i+1) = \Lambda_{eff}(i)e^{-114.66}$. When $V_{eff}$ is not a constant, $\ln(V_{eff})$ can be approximated by a straight line with a slope $-\alpha$. Note that one gets a similar figure for $V_{eff}(\phi)$, but with slightly curved instead of straight lines.

The meaning of this slope is clarified when considering unification of dark energy and radiation from the viewpoint of an effective fluid with equation of state $w_{eff} = p_{eff}/\rho_{eff}$. It is plotted on the second graph on figure 7. Since $w_{eff} \geq -1$, the problem of a ghost-like dark energy (Caldwell, Kamionkowski & Weinberg 2003) is avoided. The effective fluid alternatively behaves like a vacuum energy ($w = -1$ and the scalar field potential $V_{eff}$ is constant) or a radiation fluid ($w = 1/3$ and the potential decreases as $e^{-4N}$, i.e. $\alpha^{-1}$ like during a standard radiation dominated phase). The transition between these two values for $w_{eff}$ corresponds to the rise of the radiation density parameter (transition from $w_{eff} = -1$ to $1/3$) and the rise of the dark energy density parameter (transition from $w_{eff} = 1/3$ to $-1$).

The specific $\Lambda CDM$ periodic model we studied in this paper is the simplest of this class of models. It does not pretend to solve all the cosmological problems. Hence,

|       | $\chi^2_{CDM}$ | $\chi^2_{N}$ | $\chi^2_{BAO}$ | $\chi^2_{H}$ | $\chi^2$ |
|-------|----------------|--------------|----------------|--------------|----------|
| $\Lambda CDM$ | 0.59         | 1036.42      | 13.50          | 22.34        | 1072.86  |
| Coupled model  | 0.43         | 1035.97      | 10.56          | 21.10        | 1068.07  |

Table 1. $\chi^2$ for the $\Lambda CDM$ and best fit coupled model with $\alpha = 5.42$.
one could wish longer phases of inflation to get more e-fold at each period (for instance by choosing different forms of the coupling functions) or avoid the Big Bang singularity (which possibly could be done by reversing the sign of $H$ on an homoclinic orbit). It would also be interesting to look for a specific signature of such a periodic expansion, maybe in the CMB.

**ACKNOWLEDGEMENT**

I thank Professor Savvas Nesseris for his Mathematica codes for the Markov Chain Monte Carlo simulations and the referee who suggests to do such an analysis.

**References**

Abbott T. M. C. et al, 2018, MNRAS, 483, 4, 4866.

Ade P. A. R. et al, 2016, A&A, 594.

Akaike H., 1974, IEEE Transactions on Automatic Control 19, 716.

Arjona R., Cardona W., Nesseris S., 2019, Phys. Rev. D 99, 043516.

Arjona R., Cardona W., Nesseris S., 2019, Phys. Rev. D 100, 063526.

Aghanim N. et al, 2018, Planck Collaboration, arXiv:1807.06209.

Albrecht A., Steinhardt P. J., 1982, Phys. Rev. Lett. 48 (17): 1220-1223.

Ashtekar A., 2009, Gen. Relativ. Grav. 41:707-741.

Baum L., Frampton P. H., 2007, Phys.Rev.Lett. 98 (7): 071301.

Berera A., 1998, Phys. Rev. Lett. 75, 3218.

Beutler F. et al, 2011, MNRAS 416, 3017.

Blake C. et al., 2012, MNRAS Soc. 425, 405.

Bond J. R., Efstathiou G., Tegmark M., 1997, MNRAS , 291, L33.

Brannan J. R., Boyd W. E., 2010, Differential Equations: An Introduction to Modern Methods and Applications, Wiley, 2nd edition, chapter 3.4, p177.

Caldwell R. R., Kamionkowski M., Weinberg N. N., 2003, Phys. Rev. Lett., 91, 7.

Capozziello S., De Laurentis M., 2011, Phys. Rept. 509, 167.

Delubac T. et al, 2015, A&A 574, A59.

Elgaroy O., Multamaki T., 2007, A&A, 471, 1, 65-70.
Figure 7. Effective potential (the dashed lines approximate the potential) and equation of state of the effective fluid \(\rho_{\text{eff}} = \rho_d + \rho_r\).

Fay S., Tavakol R., Tsujikawa S., 2007, Phys.Rev.D75:063509.
Fay S., 2013, JCAP.09,023.
Fay S., 2014, Phys.Rev.D 89, 063514.
Fay S., 2017, Phys.Rev.D 95, 8, 083504.
Gil-Marín H. et al., 2016, MNRAS, 460, 4210.
Guo R.-Y., Zhang X., 2016, EPJ. C 76, 163.
Guth A., 1981, Phys. Rev. D 23, 347.
Guth A., Weinberg E., 1983, Nucl. Phys., B212, 321.
Hinshaw G. et al., 2013, WMAP Collaboration, ApJS. 208, 19.
Li B., Barrow J. D., 2009, Phys. Rev. D79:103521.
Li H., 2012, JCAP, Vol 2012.
Linde A., 1982, Phys. Lett., 108B, 389.
Moresco M. et al., 2012, JCAP, 07, 053.
Moresco M. et al., 2012, JCAP, 08, 006.
Moresco M., 2015, MNRAS Letter, 450, 16.
Moresco M. et al., 2016, JCAP, 014.
Nesseris S., J. Garcia-Bellido, 2013, JCAP 1308, 036.
Panigrahi D., Chatterjee S., 2011, JCAP 1110 002.
Penrose R., 2006, Proc. of the EPAC, Edinburgh, Scotland, 2759-2762.
Perico E. L. D. et al, 2013, Phys. Rev. D 88, 063531.
Perlmutter S. et al, 1999, ApJ. 517 (2): 565-86.
Riess A. et al, 1998, AJ. 116 (3): 1009-38.
Riess A. et al, 2019, AJ. 876 (1).
Saez-Gomez D., 2016, Phys. Rev. D 93, 124040.
Scolnic D. M., 2018, ApJ, 859, 101.
Steinhardt P. J., Turok N., 2005, New Astron. Rev. 49:43-57.
Weinberg S., 1989, Rev. of Modern Phys. 61, 1-23.
Zhai Z., Wang Y., 2019, JCAP, Vol 2019.
Komatsu E. et al, 2009, ApJS. 180:330-376.