Inverse analysis of hydraulic parameters of rock mass based on the unsteady seepage calculation theory

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Abstract. The way to obtain the seepage parameters of rock mass includes the experiment method and the inversion method. When the fractures in rock mass are significant developed in the engineering construction site, the stability of both the water injection test and the water pressure test cannot meet the code requirement. It is difficult to obtain the correct seepage parameters. In this paper, the calculation principle of unsteady seepage was applied to doing the inversion analysis of rock hydraulic parameters. The three dimensional finite element model of the comparison-hole was established. Based on this finite element model, the seepage parameters of rock mass was back calculated using the measured data. The results of the inverse analysis coincide with that of the water pressure test. It shows that the proposed inverse analysis model of the hydraulic parameters of the fractures developed rock mass is reasonable.

1. Introduction

In the hydraulic engineering construction area, the earth-rockfill dam is widely used for its strong adaptability to environment, mature design techniques and good economic benefit. The seepage finite element analysis which need accurate hydraulic parameters of the rock mass is one of the most important part of the earth-rockfill dam design [1]. The two ways to obtain the hydraulic parameters of rock mass are the test method and the inversion method. Specifically, the water pressure test is a kind of in-situ test, which presses the water into the hole with high pressure, then analysis the development of rock mass fissures and water permeability based on the water absorption of rock mass. However, in the natural dam area contained much more rock faults, joints and cracks, it is always too difficult to get the accurate seepage parameters through the test method. While the inversion analysis is an optimization method, which provides a new solution for the determination of seepage flow parameters. It calculates the permeability parameters based on known water head data obtained from the groundwater observations or the water pressure test [2]. The inversion analysis fall into three categories: the graphic analysis, the analytical methods and the numerical methods [3]. Then the flexible tolerance method [4] which used in this paper belongs to the normal operation method in
numerical method. In the flexible tolerance method, the head value is one of the main data being observed. Based on the head value, the basic physical mechanics parameters which needed during the structure design are back-calculated.

2. The transient seepage calculation principles and its finite element analysis

2.1. The basic equation of the unsteady seepage flow
The basic characterization quantities change over time at the unsteady seepage field [5]. The matrix form of finite element equation for unsteady seepage flow reads.

\[
[K][h] + [S]\left(\frac{\partial h}{\partial t}\right) + [P]\left(\frac{\partial h^*}{\partial t}\right) = \{F\}
\]  

(1)

Where \([K]\) is the global infiltration parameter matrix, which consist of the water head vector of each nodes. \([S]\) is the collected water parameter matrix of the compressed solid internal element and \([F]\) is the known constant term, determined by the known node water head.

2.2. The boundary conditions of the basic unsteady seepage equation
The starting condition

\[
h|_{t=0} = h_0(x, z, t)
\]  

(2)

The boundary condition

\[
h|_{\Gamma_i} = f_i(x, z, t)
\]  

(3)

The flow boundary condition

\[
k_n \frac{\partial h}{\partial n}|_{\Gamma_i} = f_2(x, z, t)
\]  

(4)

Where \(x, z\) are the coordinates, \(t\) is the time step. \(h\) is the water head.

2.3. The finite element analysis
The body is divides into a number of eight nodes isoparametric hexahedron elements. And the shape function which we choose should meet

\[
h(x, t) = N_m(x_i) h_{iem}(t), \quad i = 1, 2, 3
\]  

(5)

Where \(N_m(x_i)\) is the element shape function and \(h_{iem}(t)\) is the pressure head value of the element node.

The essence of the Galerkin method involves taking the weak form of the governing equations. Here the shape function \(N_m(x_i)\) is taken as the weight function. Let the trial function \(h(x, t)\) approximate the exact solution of the partial differntial equations to divide the whole domain. The unit governing equation reads
\[
\left[K\right]\left[h\right]^{e} + \left[S\right]\frac{\partial h}{\partial t}^{e} = \left[F\right]^{e}
\]

(6)

\[
K_{ab}^{e} = \int_{\Omega} K_{ij}(h)\left(N_{a,i} K_{j}^{e} N_{b,i}\right) d\Omega
\]

(7)

\[
S_{ab}^{e} = \int_{\Omega} \left[C(h) + \beta S_{ij}\right] N_{a,i} N_{b,i} d\Omega
\]

(8)

\[
F_{(a)}^{e} = -\int_{\Omega} K_{ij}(h)\left(N_{a,i} K_{j}^{e} Z_{j}\right) d\Omega + \int_{\Gamma} q_{N} d\Gamma
\]

(9)

Where \(a, b = 1 \sim 8\), \(i, j = 1, 3\), \(N_{a,i}, N_{b,i}\) is the shape function and \(h_{c}\) is the pressure head.

3. The inverse analysis model for hydraulic parameters of rock mass

The flow rate was applied as the measurement for the inverse analysis. The core is to find a set of determined hydraulic parameters, which make the error percentage between the measured water leakage of boreholes and the calculated seepage flow in the drilling Area less than 10%. The normal operation method [6] solves unknowns is based on the principle of non-stable seepage calculation theory. The steps are as follows:

Step one. Drill at the zone of rocks to be measured. Seal the bottom of the borehole, which makes the water can only leak through the wall of the hole. Then inject the water into the drill for a certain time, and let the liquid level free fall in the drill hole. Make a record of the liquid level elevation \(h_{l}\) at different time \(t_{i}\) and used as the definite condition for solving the basic unsteady seepage equations. The measured leakage \(u_{i}^{l}\) in the drill hole of each period can be calculated through \(u_{i}^{l} = \left(h_{l} - h_{l-1}\right) \times A\).

Step two. To establish the three dimensional finite element model of the seepage flow and determine its boundary conditions. Give the initial seepage parameters, then adopt the unsteady seepage calculation principle to calculate the seepage field of the drill-hole zone at different time. Using the mid-section method to calculate the flow rate of the borehole model \(q_{s}^{e}\) in different time.

Then use the formula \(w_{s}^{l} = \frac{q_{s}^{e} + q_{s}^{e-1}}{2} \times \left(t_{i} - t_{i-1}\right)\) to approximately calculate the seepage quantity \(w_{s}^{l}\) of the drill-hole area at each time step.

Step three. Adjust the seepage parameters. At each step, to ensure that the error percentage \(\eta = \frac{\left(u_{i}^{l} - w_{s}^{l}\right)}{u_{i}^{l}} \times 100\%\) between the calculated seepage flow of the drill zone \(w_{s}^{l}\) and the measured leakage of the drill \(u_{i}^{l}\) less than 10%.

4. The calculation example analysis

The Shangmo Reservoir is located on the Jinjiahe River in the west of Tianshui in Gansu Province, China. The rock masses under the dam and reservoir area have a good permeability as there is a deep riverbed fault fracture zone. The inverse analysis model established before is applied to obtain the hydraulic parameters of rock mass for the Shangmo Reservoir project. In order to verify the accuracy of the inverse analysis model, the inverted seepage of the rock mass will be compared with that of the water compress test.
4.1. The water pressure test
The water pressure test uses a special sealing device to isolate a certain length of the drill test section. Then a fixed head is applied on this section for injecting water, which makes the water permeates through the cracks of the pore wall into the rock. The seepage will increase to tend a stable value. According to the pressure water head, the length of the test section and the steady leakage, the strength of rock permeability can be determined. The permeability of rock for the water pressure test is usually expressed as the permeability rate in units of lugeon. The borehole locations are selected based on the rock mass properties. Then the length of the specimen is determined by the results of core drill sampling. The lithological characters of the rock mass located at the left bank of the downstream is better than that of other region. Hence, the comparison-hole denoted as ZK1 was chosen at the geodetic coordinate (3823233, 18532788). And its ground elevation is 1637.10m. The core drill sampling is shown in figure 1. The change trend of the water level at the compared hole is shown in figure 2.

![Figure 1. The core drill sampling](image1)

![Figure 2. The change trend of the water level at the compared hole](image2)

The starting and ending time of each period is illustrated in the table 1.

| Time       | Pressure (s) | Maximum Pressure (s) | Leakage | Lugeon value |
|------------|--------------|-----------------------|---------|--------------|
| 0:00:00    | 0.47         | 0.52                  | 72.80   | 108.00       |
| 0:02:02    | 0.48         | 0.58                  | 71.00   | 98           |
| 0:05:09    | 0.49         | 0.63                  | 71.00   | 92           |
| 0:09:12    | 0.46         | 0.55                  | 71.00   | 104          |
| 0:15:35    | 0.51         | 0.59                  | 72.00   | 94           |
| 0:21:55    | 0.49         | 0.59                  | 71.00   | 93           |
4.2. *The computational model*

The calculation model is a cylinder with 16m in height. And its radius is 1m. The top elevation of the model is 1637.10m. Due to the symmetric geometry and the set of boundary conditions the resulting seepage fields are expected to be symmetric as well and, therefore, only 1/4 of the cylinder is considered. A 1/4 round hole with 0.12m aperture and 16m in depth is set at the axis of the model. After meshing, the three-dimensional finite element model has 19056 nodes and 18000 hexahedron elements in total. The finite element mesh generation is shown in figure 1.

![Figure 1.](image1)

**Figure 3.** The three-dimensional finite element mesh of the inverse analysis model.

In the unsteady seepage period, the boundary types of seepage analysis mainly include known head boundary, infiltration boundary and impervious boundary [7]. The known head boundary includes the perimeter of the borehole and the cut-off boundary for a given groundwater level where the perimeter of the borehole is the upstream boundary and the outer side of the model is the downstream boundary; The infiltration boundary is the surface which below the upstream water level and above the downstream of the model; The impermeable boundary includes symmetry plane of the model, that is, the interception boundary (x = 0 and y = 0), the boundary between the water levels of upstream and downstream, the part of the boundary beyond the given groundwater table and the bottom surface of the model; For unsteady seepage, the water level of borehole changes with time. And the head of the upstream boundary around the borehole also changes with time.

4.3. *The inversion analysis*

In order to obtain the parameters of hydraulic conductivity of the rock mass. The seepage discharge is adopted as the measurement for the rock mass hydraulic parameters inversion analysis. Then the error percentage between the measured value of seepage discharge in the borehole and the calculated seepage discharge in the drilling area are controlled in a certain tolerance range. The starting and ending time of each period is illustrated in table 2. The initial unsteady seepage parameter as shown in table 3 was selected according to similar projects and experiments, as the seepage parameters of the rock body did not measure in the water pressure test.

| Period | Initial time (s) | Terminal time (s) |
|--------|-----------------|-------------------|
| 1      | 0               | 18                |
| 2      | 18              | 34                |
| 3      | 34              | 52                |
| 4      | 52              | 72                |
| 5      | 72              | 102               |
| 6      | 102             | 114               |
During the inversion analysis, for the seepage discharge through the hole-wall, its error percentage between the inversion values and the measured value was controlled within 10%. The comparison of the calculated seepage and measured seepage is shown in figure 4.

![Figure 4. The comparison of the calculated seepage and measured seepage in the drill hole.](image)

| Period | seepage discharge (m³/s) |
|--------|--------------------------|
| 1      | 0.0014                   |
| 2      | 0.0016                   |
| 3      | 0.0018                   |
| 4      | 0.0020                   |
| 5      | 0.0022                   |
| 6      | 0.0024                   |

**Figure 4.** The comparison of the calculated seepage and measured seepage in the drill hole.

Based on the three-dimensional finite element seepage inverse analysis model, in the range of 16m of the comparison-hole, the seepage parameter of rock mass is 1.1e-5m/s as shown in table 3. Through the results of the water pressure test as shown in the table 1, the results of the inversion analysis identify with that of the water pressure test.

| Rock mass | Upper and lower limits (m/s) | Inversion value (m/s) |
|-----------|------------------------------|-----------------------|
| Hole depth 16m | 1.0E-7~1.0E-3 | 1.1E-5                |

**Table 3.** The initial value and inversion value of the seepage parameters

5. Conclusion
It can be seen from the water pressure test that for the rock mass with extremely developed cracks or fault fracture zones, the stability of the water pressure test is poor and the water pressure in the hole cannot meet the code, resulting in the calculated water permeability lugeon value distortion. Therefore, for such rock masses, the permeability coefficient of rock mass obtained by unsteady seepage inversion method based on the unsteady seepage principle is proposed. Specifically, the basic equations of unsteady seepage are studied. The governing equations of the seepage free surface are derived and the boundary conditions are determined. The implementation steps of the inverse analysis model for hydraulic parameters of rock mass were illustrated. The inverse analysis model is applied to obtain the hydraulic parameters of rock mass for the Shangmo Reservoir project. Comparing the result of the inversion analysis with that of the water pressure test, it shows that the proposed inversion method of hydraulic parameters of rock mass is reasonable.

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