Analysis and synthesis of parallel structure mechanism without singularities

G V Rashoyan, A K Aleshin, A V Antonov, I V Gavrilina, V A Glazunov, S A Skvortsov and K A Shalyukhin

Blagonravov Mechanical Engineering Research Institute of the Russian Academy of Sciences, 4, M. Kharitonyevskiy Pereulok, 101000 Moscow, Russian Federation

E-mail: gagik_r@bk.ru

Abstract. Problems of analysis and synthesis of a parallel structure mechanism without singularities are considered in this paper. The spatial l-coordinate parallel structure mechanism, consisting of a base and an output link, connected with each other by six SPS kinematic chain, each of them containing one driven prismatic (P) kinematic pair and two nondriven spherical (S) pairs, is synthesized. The grapho-analytical method was used as a method of analysis and synthesis.

1. Introduction
Parallel structure mechanisms with linear drives are widely used as simulators, test benches, 3D printers, processing machines, measuring and handling technological devices, etc. [1–12]. Due to the design feature of the possibility to resist an external load as a frame, these mechanisms have advantages above the manipulators with a serial structure [13, 14]. The prototype for this class of mechanisms is the well-known Gough platform [15, 16]. Despite the great functionality of these mechanisms, they have a significant drawback associated with singularities, which can lead to loss of controllability (uncontrollable mobility appears) or to a position, from which it is impossible to perform further motion only by means of mechanism’s actuators [17–21]. Thus, the problem of synthesis of mechanisms without singularities in the working space becomes actual.

2. Problem formulation
The article deals with the problems of analysis and synthesis of a spatial l-coordinate parallel structure mechanism [21] with linear drives and without singularities in its working area. The generalized coordinates of the mechanism are the so-called l-coordinates representing the distances between the joints on the base and the output link. Let’s consider the l-coordinate mechanism (figure 1) in which the output link 2 (the triangle $P_1P_2P_3$) is connected to the base 1 (the triangle $B_1B_2B_3$), and the lengths of the triangles are known. The variation limits of the generalized coordinates $I_{imin}$ and $I_{imax}$, $i = 1...6$, are known too and equal for all the kinematic chains.
3. Theory

Singularities in parallel structure mechanism are revealed by a kinematic analysis, in particular while solving the direct kinematics problem and while analyzing the working space. Numerical methods are mainly used for solving the direct kinematics problem. For such a mechanism of a general form it is known that the Plücker coordinates matrix of the mechanism's kinematic chains is degenerate in a singular position [13]. However, for some \( l \)-coordinate mechanisms the direct kinematics problem has an analytical solution.

In considered \( l \)-coordinate parallel structure mechanism with six kinematic chains (figure 1) the number of joints on the base and on the output link are both equal to three. The direct kinematics problem can be solved by successive determination of the \( P_1 \), \( P_2 \) and \( P_3 \) coordinates from the following systems of the quadratic equations:

\[
\begin{align*}
(x_{B1} - x_{P1})^2 + (y_{B1} - y_{P1})^2 + (z_{B1} - z_{P1})^2 &= l_1^2, \\
(x_{B2} - x_{P1})^2 + (y_{B2} - y_{P1})^2 + (z_{B2} - z_{P1})^2 &= l_2^2, \\
(x_{B3} - x_{P1})^2 + (y_{B3} - y_{P1})^2 + (z_{B3} - z_{P1})^2 &= l_3^2, \\
(x_{P1} - x_{P2})^2 + (y_{P1} - y_{P2})^2 + (z_{P1} - z_{P2})^2 &= l_{P1P2}^2, \\
(x_{B2} - x_{P2})^2 + (y_{B2} - y_{P2})^2 + (z_{B2} - z_{P2})^2 &= l_4^2, \\
(x_{B3} - x_{P2})^2 + (y_{B3} - y_{P2})^2 + (z_{B3} - z_{P2})^2 &= l_5^2, \\
(x_{P1} - x_{P3})^2 + (y_{P1} - y_{P3})^2 + (z_{P1} - z_{P3})^2 &= l_{P1P3}^2, \\
(x_{P2} - x_{P3})^2 + (y_{P2} - y_{P3})^2 + (z_{P2} - z_{P3})^2 &= l_{P2P3}^2, \\
(x_{B3} - x_{P3})^2 + (y_{B3} - y_{P3})^2 + (z_{B3} - z_{P3})^2 &= l_6^2,
\end{align*}
\]

where \( l_i \) are the generalized coordinate (figure 1); \( l_{P1P2}, l_{P2P3} \) and \( l_{P1P3} \) are the sizes of the output link; \( (x_{Bj} \ y_{Bj} \ z_{Bj})^T \) and \( (x_{Pj} \ y_{Pj} \ z_{Pj})^T, j = 1...3, \) are the coordinates of the points \( B_j \) and \( P_j \) respectively.
Each kinematic chain is represented as a quadratic equation of a sphere connecting the unknown coordinates of the output link joints with the known coordinates of the base joints through the generalized \( l \)-coordinates. Without going into details of solving systems of quadratic equations, we note that the above equations have the closed-form solution.

Geometrically, solving these systems of equations can be identified as determining the vertices coordinates of three tetrahedrons: with a base \( B_1B_2B_3 \) and an apex in \( P_1 \), with a base \( B_2B_3P_1 \) and an apex in \( P_2 \), with a base \( P_1P_2P_3 \) and an apex in \( B_3 \). The analysis of the direct kinematics problem solution shows that these tetrahedrons degenerate into their base planes in singular positions. This means that we will have from one to three flattened tetrahedrons instead of the spatial geometrical figures.

Let’s consider the tetrahedron with a base \( B_1B_2B_3 \) and an apex in \( P_1 \) (figure 1), for which \( l_1, l_2 \) and \( l_3 \) are the lateral edges. Let’s carry out some elementary graphical constructions on the tetrahedron base plane (figure 2): we can plot two concentric circles with the radii \( l_2 \text{min} \) and \( l_2 \text{max} \) and the center in \( B_2 \) and two other concentric circles with the radii \( l_3 \text{min} \) and \( l_3 \text{max} \) and the center in \( B_3 \).

As a result, we will have two regions on the base plane, \( S_L \) and \( S_R \), where \( l_2 \) and \( l_3 \) exist. Next, we can find the coordinates of intersection points of circles with the radii \( l_2 \text{min}, l_2 \text{max}, l_3 \text{min} \) and \( l_3 \text{max} \) at the points \( S_j, j = 1…8 \) (figure 2). One can write the following systems of the circles equations:

\[
\begin{align*}
(x_{B_2} - x_{S_1})^2 + (y_{B_2} - y_{S_1})^2 &= l_{2 \text{min}}^2, \\
(x_{B_3} - x_{S_1})^2 + (y_{B_3} - y_{S_1})^2 &= l_{2 \text{min}}^2, \\
(x_{B_2} - x_{S_2})^2 + (y_{B_2} - y_{S_2})^2 &= l_{2 \text{max}}^2, \\
(x_{B_3} - x_{S_2})^2 + (y_{B_3} - y_{S_2})^2 &= l_{2 \text{max}}^2, \\
(x_{B_2} - x_{S_3})^2 + (y_{B_2} - y_{S_3})^2 &= l_{3 \text{min}}^2, \\
(x_{B_3} - x_{S_3})^2 + (y_{B_3} - y_{S_3})^2 &= l_{3 \text{min}}^2, \\
(x_{B_2} - x_{S_4})^2 + (y_{B_2} - y_{S_4})^2 &= l_{3 \text{max}}^2, \\
(x_{B_3} - x_{S_4})^2 + (y_{B_3} - y_{S_4})^2 &= l_{3 \text{max}}^2,
\end{align*}
\]

where \((x_{S_j}, y_{S_j}, z_{S_j})^T\) are the coordinates of the points \( S_j \).

Because of the solutions ambiguity of the equations systems above, we will obtain the coordinates of all the intersection points \( S_1…S_8 \).

The next step is to calculate the distances \( l_{S_j} \) from all the points \( S_j \) to the point \( B_1 \):
\[ I_{ij} = \left[ (x_{B1} - x_j)^2 + (y_{B1} - y_j)^2 \right]^{1/2}. \]

We compare the obtained results with the limits of the \( l_1 \) generalized coordinate \( (l_{imin} \text{ and } l_{imax}) \). If the results are within these limits, then there is a singular position. One should mention another approach to singularity analysis: we can plot two circles with the radii \( l_{imin} \text{ and } l_{imax} \) and the center in \( B_1 \), and there will be a singular position, if these circles surround or intersect \( S_L \) and \( S_R \) areas.

One can perform similar calculations and graphical constructions with the tetrahedron with a base \( P_1P_2P_3 \) and an apex in \( B_3 \) (figure 3).

**Figure 3.** Graphical constructions in the tetrahedron \( P_1P_2P_3B_3 \) base plane.

Next, let’s consider the third tetrahedron with a base \( B_2B_3P_1 \) and an apex in \( P_2 \). We propose two approaches for the graphical solution of the problem of degeneration this tetrahedron into a plane figure.

The first approach is convenient when the limits \( l_{imin} \text{ and } l_{imax} \) are equal for all the kinematic chains. We can plot the following concentric circles:

- with the radii \( l_{imin} \text{ and } l_{imax} \) and the center in \( B_2 \);
- with the radii \( l_{imin} \text{ and } l_{imax} \) and the center in \( B_3 \);

As a result of intersections, there will be a region of the points \( P_1 \) and \( P_2 \) existence on the tetrahedron possible flattening plane (figure 4).
Figure 4. First approach for the tetrahedron $B_2 B_3 P_1 P_2$ singularity analysis.

We can calculate the maximum distance $d_{\text{max}}$ between the intersection points of the circles inside this region as:

$$d_{\text{max}} = \left[ (x_{S1} - x_{S3})^2 + (y_{S1} - y_{S3})^2 \right]^{1/2},$$

compare it with the length $l_{P1P2}$, and if the latter is greater than $d_{\text{max}}$, then it follows that the tetrahedron cannot degenerate into a plane and, hence, there is no singularity. Otherwise a singular position is possible.

The second approach for singularity analysis is also graphical. This approach is convenient then variation limits of lengths $l_2...l_5$ are not equal. The constructions should be started from plotting concentric circles with the centers at points $P_1$ and $P_2$. The regions of the points $B_2$ and $B_3$ position on the tetrahedron possible flattening plane are formed as a result of these circles intersections (figure 5).

Figure 5. Second approach for the tetrahedron $B_2 B_3 P_1 P_2$ singularity analysis.

Next, one should check whether it is possible to place the segment $|B_2 B_3|$ with a point $B_2$ inside the region $S_{11}$ and a point $B_3$ in the region $S_{12}$ at the same time (figure 5). If such an arrangement is not possible, then there is no singularity.

4. Experiment
Let’s consider the $l$-coordinate mechanism (figure 1) with the following parameters:

- output link sizes:
  $$l_{P1P2} = l_{P1P2} = l_{P1P2} = 28 \text{ mm};$$
• base sizes:
  $$|B_1B_2| = |B_2B_3| = |B_3B_1| = 38 \text{ mm};$$

• for all the kinematic chains:
  $$l_{\text{min}} = 50 \text{ mm}, l_{\text{max}} = 60 \text{ mm}.$$ 

Graphical constructions showed that the tetrahedron with a base $$B_2B_3P_1$$ and an apex in $$P_2$$ can degenerate, since $$|S_1S_3|$$ is equal to 29 mm, that is greater than $$|P_1P_2|$$, and, therefore, a singular position is possible. To eliminate the singularity, it is proposed to increase the length of the segment $$|P_1P_2|$$ in such a way that it will be greater than 29 mm.

5. Conclusion

Thus, there was developed a grapho-analytical method for analysis of the $$l$$-coordinate parallel structure mechanism singularities, in which the output link is connected to the base by means of six kinematic chains of a variable length with three joints both on the base and the output link. This method also allows one to solve the problem of synthesizing the $$l$$-coordinate mechanism without singularities in its working space. In this case, different construction parameters of the mechanism can serve as the optimization variables depending on the given task. One should also mention that the absence of singular positions is important in the following kinematic or dynamic control of the $$l$$-coordinate mechanism.

As a conclusion, we note that this paper presented the grapho-analytical method for analyzing and synthesizing the parallel structure $$l$$-coordinate mechanism, in which the direct kinematic problem can be solved in a closed-form. This approach can be also extended on the other schemes of the $$l$$-coordinate mechanisms.

Acknowledgment

This work was supported by Russian Foundation for Basic Research, grant no. 16-29-04273, oﬁ-m.

References

[1] Hunt K 1983 Structural kinematics of in-parallel-actuated robot-arms J. Mech., Trans. and Automation 105 705–12
[2] Gogu G 2004 Structural synthesis of fully-isotropic translational parallel robots via theory of linear transformations Eur. J. Mech. A 23 1021–39
[3] Glazunov V and Kheylo S 2016 Dynamics and control of planar, translational and spherical parallel manipulators Dynamic Balancing of Mechanisms and Synthesizing of Parallel Robots ed D Zhang and B Wei (Cham: Springer) pp 365–402
[4] Aleshin A K, Glazunov V A, Shai O, Rashoyan G V, Skvortsov S A and Lastochkin A B 2016Infinitesimal displacement analysis of a parallel manipulator with a circular guide via the differentiation of constraint equations J. Mach. Manuf. and Rel. 45 398–402
[5] Chunikhin A Yu and Glazunov V A 2017 Developing the mechanisms of parallel structure with five degrees of freedom designed for technological robots J. Mach. Manuf. and Rel. 46 313–21
[6] Glazunov V A and Borisov V A 2017 Development of parallel-structure mechanisms with four degrees of freedom and four kinematic chains J. Mach. Manuf. and Rel. 46 417–25
[7] Ganiev R F, Glazunov V A and Filippov G S 2018 Urgent problems of machine science and ways of solving them: wave and additive technologies, the machine tool industry, and robot surgery J. Mach. Manuf. and Rel. 47 399–406
[8] Glazunov V A 2018 Mechanisms of parallel structure and their application: robotic, technological, medicine, teaching systems (Moscow-Izhevsk: Izhevsk Institute of Computer Science)
[9] Rashoyan G V, Shalyukhin K A and Gaponenko E V 2018 Development of structural schemes of parallel structure manipulators using screw calculus IOP Conf. Ser.: Mat. Sc. and Eng. 327 042090
[10] Vorobyev E I et al 2018 New mechanisms in modern robotics (Moscow: Technosphere)
[11] Antonov A V, Glazunov V A, Aleshin A K, Rashoyan G V and Laktionova M M 2018 Kinematic analysis of a parallel structure mechanism for work in extreme environments J. Mach. Manuf. and Rel. 47 121–27
[12] Glazunov V, Nosova N, Kheylo S and Tsarkov A 2018 Design and analysis of the 6-DOF decoupled parallel kinematics mechanism Dynamic Decoupling of Robot Manipulators ed V Arakelian (Cham: Springer) pp 125–70
[13] Merlet J-P 2000 Parallel Robots (Dordrecht: Springer)
[14] 2016 Springer Handbook of Robotics ed B Siciliano and O Khatib (Cham: Springer)
[15] Gough V E 1957 Contribution to discussion of papers on research in automobile stability, control and tire performance Proc. Auto Div. Inst. Mech. Eng. 171 392–95
[16] Stewart D A 1965 A platform with six degrees of freedom Proc. Inst. Mech. Eng. 180 371–86
[17] Merlet J-P 1989 Singular configurations of parallel manipulators and Grassmann geometry Intern. J. Rob. Res. 8 45–56
[18] Gosselin C and Angeles J 1990 Singularity analysis of closed-loop kinematic chains IEEE Trans. on Rob. and Automation 6 281–90
[19] Aleshin A K, Glazunov V A, Rashoyan G V and Shai O 2016 Analysis of kinematic screws that determine the topology of singular zones of parallel-structure robots J. Mach. Manuf. and Rel. 45 291–96
[20] Shalyukhin K A, Rashoyan G V, Aleshin A K, Skvortsov S A, Levin S V and Antonov A V 2018 Problems of kinematic analysis and special positions of mechanisms of robots with parallel structure J. Mach. Manuf. and Rel. 47 310–16
[21] Arzumanyan K S and Koliskor A Sh 1988 Synthesis of 1-coordinate structure systems for research and diagnostics of industrial robots Testing, Monitoring and Diagnosis of the Flexible Manufacturing Systems ed I M Makarov and E G Nahapetyan (Moscow: Science) pp 70–81