Reservoir Control and Identification: Motivated by Water Coning

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Abstract. In underground hydrocarbon reservoirs, one of the famous problems which requires to be controlled is water coning. Normally, there are some unknown parameters due to physical nature of petroleum reservoirs that make the control design challenging. However, Lyapunove-based design of adaptive control for the production rate has been proposed to solve the problem, but this kind of design is merely able to follow the dynamic adaptation of the parameter to pursue the zero stability of the system state rather than estimating the unknown parameters. In reality, there is a need to have a good estimate of reservoir horizontal permeability to make a plan for the future of the reservoir. Therefore, a new approach to do system identification, beside its control, is needed. In this paper, Passivity-Based Identifier (PBI) controller is presented to not only control the water coning, but also estimate the reservoir permeability, simultaneously. For this reason, at first, a controller is presented for a case where reservoir permeability is known. Next an identifier is proposed to estimate both system state and parameter. Lastly, some numerical simulations are provided to show the effectiveness and the performance of the proposed adaptive control and adaptation laws. The numerical simulation results showed that the new control law, controls the system such that as time elapses, the whole oil column is depleted without any water production. In addition, the new adaptation law estimates the unknown parameter with a good accuracy.

Keywords: Reservoir identification, Undesirable water, Control, Smart well, Coning

1. Introduction
Dynamic behavior of water-oil interface is described by Partial Differential Equations (PDEs). Normally, because of the physical nature of an underground reservoir, there are several unknown parameters in these equations. For example fluid or medium properties can be unknown. The systems containing unknown parameters should be controlled by adaptive approaches. In other words, in addition to the control law, another equation is needed to adapt the unknown parameter.

Water Coning is defined as a mechanism of the upward movement of water-oil interface into the perforations of a producing well in a cone shape [1]. Water coning leads to water production and consequently early shutdown of the production wells because of increasing operational costs and decreasing profits. Usually water coning is responsible for high amount of water production in the oil
fields [2]. The schematic of the water coning problem is shown in Figure 1. It leads to significant reduction of oil production rate and increase in the water cut. Konieczek [3] proposed a mathematical model based on some assumptions to describe the dynamic behavior of the gas-oil interface for a gas-oil system. The model is presented by a nonlinear partial differential equation (PDE). The production rate is the only controllable factor affecting water coning. Therefore, it has been proposed as an idea for control of water coning development. Definitely in order to prevent the water coning, the flow rate should be decreased as an inverse function of oil-water interface height around the well. So far, several models have been introduced to counter this phenomenon, however all of them require the amount of reservoir permeability [4-8]. Reservoir permeability can be estimated in a variety of ways. Generally, these methods are time-consuming and costly which makes these methods not applicable to many practical applications of flow control of production wells.

Thus, we need a technique to control the system and adapt the unknown parameters simultaneously. This method is called adaptive control.

2. Adaptive control of Nonlinear PDEs

In reality, some physical parameters of process plants are often unknown. Thus a need exists for a “parameter-adaptive” approach which should be able to estimate such unknown parameters, continuously re-compute the controller gains, and apply the resulting controller to stabilize the system. The area of adaptive control design for nonlinear PDEs is still in its infancy. This is an extremely challenging problem, especially for nonlinear PDEs. Smyshlyaev and Krstic [9] differentiate two major classes for adaptive control of PDEs; Lyapunov Scheme (LS) and Certainty Equivalence Schemes (CES).

In the CES, the controller and the identifier are designed separately. In other words, the controller of CES is designed in a form parameterized by the unknown parameters as if they were known. Moreover, the parameter identifier is designed separately, without taking closed-loop stability into account but only with the objective that the parameter estimation error be bounded and the output estimation error and the derivative of the parameter estimate be square integrable in time. The identifier of CES is designed in two ways; Passivity-Based Identifiers (PBI) and Swapping Identifiers (SI). The PBI method uses a copy of the plant to generate a model which is passive from the parameter estimation error, while the SI method employs several filters to convert a dynamic parameterization of the problem into a static parameterization. As they ranked these approaches, Lyapunov is the lowest one as it incorporates only the dynamics of the parameter update, and the passivity-based approach is better than swapping because it uses only one filter, while the swapping approach uses one filter per unknown parameter [9].

2.1 Porous media equation

An adaptive controller for a case of water coning problem has been designed based on Lyapunov approach by Safari et al. [10]. The Governing equation of coning problem which can be described by the porous media equation \( u_t = (u^2)_x = 2(uu_x)_x \).

This adaptation law was merely able to follow the dynamic adaptation of the parameter and in order to pursue the zero stability goal of the problem not estimating the permeability. In other words, the objective of the proposed output feedback adaptive control design was only a regulation not a system identification. It is often of interest to identify the physical parameters of a system from the existing data. The problem with this design is that in reality there is a need to know the amount of parameters to make a plan for the future of the reservoir.

This research uses PBI approach to design a new adaptation law to estimate the reservoir permeability meanwhile controlling the water coning problem in an oil reservoir. In other words, this design not only guarantees the asymptotically stability of the system, but also is able to estimate the unknown parameters. For this reason, an observer is employed in the form of a copy of the plant, plus a stabilizing error term.

The governing equation of the intended system is a nonlinear PDE with no simplifications or linearization. For this reason, at first, a controller is designed as if the reservoir permeability is known. Next, the same controller is used in case of unknown permeability which only \( k \) and system state are replaced by their estimations. The system state is estimated by the proposed identifier. After that, the
zero equilibrium of the system contains; a control law, an identifier and an adaptation law is proved such that the system to be globally asymptotically stable. Finally, some numerical simulations are provided to support the results and to show the effectiveness and performance of the proposed adaptive control and adaptation laws.

To the best of our knowledge, this is the first study that reported the use of PBI type of adaptive control design to control the water coning and estimate the reservoir permeability simultaneously. So far, this kind of adaptive control design (the identifier, control and adaptation laws) for porous media equation has never been presented.

2.2 Problem formulation

Equation 1 describes dynamic behavior of water-oil interface under two boundary conditions (Equations 2 and 3) [3, 5, 7, 10]. Schematic of the phenomena is shown in Figure 1.

Therefore, the governing equation and boundary conditions of the closed-loop system are given as follow

\[ \bar{h}_i = kh\bar{h}_i + k\bar{h}_i^3 \]  \hspace{1cm} (1)

\[ h(1,t) \frac{\partial \bar{h}}{\partial x}(1,\bar{T}) = -\frac{1}{k} \bar{G}(\bar{T}) \]  \hspace{1cm} (2)

\[ \frac{\partial \bar{h}}{\partial x}(0,\bar{T}) = a\bar{h}(0,\bar{T}) \]  \hspace{1cm} (3)

where the model variables are defined as follows.

\[ \bar{h} = \frac{h}{L} \geq 0 \]  \hspace{1cm} (4)

\[ \bar{x} = \frac{x}{L} \geq 0 \hspace{1cm} \bar{x} = 0 \]  \hspace{1cm} (5)

\[ \bar{x} = \frac{x}{L} \geq 0 \hspace{1cm} \bar{x} = 1 \]  \hspace{1cm} (6)

\[ \bar{h}(x,\bar{T}) \geq 0 \]  \hspace{1cm} (7)

\[ h(x,t) \] is the oil column thickness (distance between water-oil interface to the top of the petroleum reservoir) at a spatial point \( x \in [0, L] \) at time \( t \). \( a \) is a known constant or simply thickness of oil column. This mathematical model is a nonlinear parabolic differential equation with the following initial condition:

\[ h(x,0) = \overline{h}_0 \hspace{1cm} \forall x \in [0,1] \]

The objective is to stabilize the system such that \( \lim_{t\to\infty} h(x,t) = 0 \hspace{1cm} \forall \bar{x} \in [0,1] \), it means that the entire oil column is expected to be produced.
Figure 1. Schematic of the water coning phenomenon

As this figure shows, there are two injection wells at outer boundaries and one production well in the middle of the model. Since the model is symmetric, therefore, just half of the model is investigated [11]. Thus, one production and one injection wells are considered in \( x = L \) and \( x = 0 \), respectively. Also, the reservoir aquifer is assumed as a strong one with constant pressure in time and space. There is no flow boundary at the top (cap rock) and bottom of the model.

The reservoir thickness is assumed to be constant, also the model is isotropic and homogeneous with respect to porosity and permeability. A well-defined Water-Oil contact (WOC) and segregated flow is assumed (capillary pressure is neglected). Consequently, the residual oil saturation and connate water saturation is neglected.

Here after, the bar signs of the variables are removed for simplicity.

flow rate is chosen to be as follows:

\[
Q_{\text{in}} = ah(0,t)^n \geq 0 \quad n \geq 1
\]  

(8)

It means that the water injection to the outer boundary decreases, while oil column decreases.

2.3 Adaptive Control

In this section, a control law for well production rate \( Q(t) \), an identifier to estimate the system state and an adaptation law to estimate the unknown parameter (reservoir permeability) are presented using the PBI method.

**Definition 1.** Second norm of a function on a domain \( \Omega \subset R \) is defined as \( \| \phi \| = \left( \int_\Omega \phi^2 \, dx \right)^{1/2} \).
By certainty equivalence principle, the controller in case of unknown $k$ will be given by same known controller [10] such that $\hat{k}$ is replaced by its estimate $\hat{\hat{k}}$. In other words, the unknown case is controlled by designing parameter identifiers and substituting the estimated parameter into the control law. Therefore, another law is needed to estimate the unknown parameter based on the measurement from the plant.

For this reason, an identifier is proposed in the following form

$$\dot{\hat{h}} = \ddot{\hat{k}} \hat{h}_{\text{ss}} + \dot{\hat{k}} \dot{\hat{h}} + \gamma^2 (h - \hat{h}) \| h \|$$

(9)

As it is clear, the identifier involves first and second derivatives of $\hat{h}$ with respect to spatial.

After rearranging

$$\dot{\hat{h}} = \ddot{\hat{k}}(\hat{h}_{\text{ss}}) + \gamma^2 (h - \hat{h}) \| h \|$$

(10)

As shown, this identifier employs an observer in the form of a copy of the plant, plus an additional nonlinear term. The term $\gamma (h - \hat{h}) \| h \|$ is named stabilizing error term. This term slows down the adaptation process by damping the suddenly changes whose task is to ensure square integrability over infinite time of the terms uses in adaptation law [12]. The present authors suggest to select the norm part of the stabilization term in a same form of adaptation law. This is a nonlinear partial differential equation with the following initial condition.

$$\forall x \in [0,L] \Rightarrow \hat{h}(x,0) = \frac{h_0}{L}$$

(10)

Also, the following boundary equations

$$\hat{h}_s(1,t) = h_s(1,t)$$
$$\hat{h}_s(0,t) = h_s(0,t)$$

(11)

The error signal $e$ and parameter estimation error $\tilde{k}$ are defined as:

$$e = h - \hat{h} \Rightarrow e_s = h_s - \hat{h}_s \text{ and } e_{\text{ss}} = h_{ss} - \hat{h}_{ss}$$

(12)

$$\tilde{k} = k - \hat{k}$$

(13)

Hence, the error system (subtraction of Equation (9) from Equation (1)) is achieved as:

$$e_s = \ddot{\hat{h}}_{\text{ss}} + \dot{\hat{k}} h_{\text{ss}} + \ddot{\hat{k}} e_s + \dot{\hat{k}} h e - \gamma^2 e \| h \|$$

(14)

After rearranging

$$e_s = \ddot{\hat{k}} (h_{\text{ss}}) + \gamma^2 e \| h \|$$

(15)

Also, the following inner and outer boundary conditions

$$e_s(1,t) = 0$$
$$e_s(0,t) = 0$$

(16)

Controller

Consider the system Equations (1)-(3) in case of unknown $k$, if the closed loop system consists of the following boundary control law...
\[ Q(t) = -\frac{\dot{k}h(0,t)h_x(0,t)}{h(1,t)} + ax^\beta(1,t) \]  
(18)

(where \( \alpha \) and \( 0 \leq \beta \leq 1 \) are the control constant gains)

and the identifier Equation (9) and update law

\[ \dot{k}(t) = \gamma \text{Proj}_{L^2} \{ Q_{\text{in}}e(1,t) + Q_{\text{out}}e(0,t) \} \]  
(19)

has a classical solution \((\dot{k}, h, \dot{h})\), then for any \( \dot{k}(0) \) and any initial conditions \( h(0,x), \dot{h}(0,x) \in L^2(0,1) \), the signals \( \dot{k}, h, \dot{h} \) are bounded and \( h \) is regulated to zero for all \( x \in [0, 1] \).

\[ \lim_{t \to \infty} h(x,t) = 0 \]  
(20)

It means that the production rate of a smart well should be controlled such that the water oil interface grows uniformly and the oil column thickness in entire domain reaches zero at the end \( (h \to 0) \).

3. Model description

Finite difference method in cylindrical coordinate was used to solve the presented mathematical model and apply the aforementioned control and adaptive laws. The properties of the model is shown in Table 1.

### Table 1. Properties of the used model

| Parameter      | Value       | Unit   |
|----------------|-------------|--------|
| Permeability   | 0.25 \times 10^{-12} | m²     |
| Porosity       | 0.1         | fraction |
| Oil viscosity  | 2.3 \times 10^{-3} | Kg/m.s |
| Oil density    | 927         | Kg/m³  |
| Water density  | 1149        | Kg/m³  |
| Reservoir      | 300         | m      |
| Initial column | 90          | m      |
| Grid size      | 10          | -      |
| Time step      | 50          | Day    |

4. Simulation study

Figures 2 to 8 present a cross section of cone grow up along the length of the model (WOC distribution) after implementing the adaptive controller and adaptation law.
Figure 2. Cone shape time lapse during production by PBI adaptive control

Figure 2 shows Cone shape time lapse during production by PBI adaptive control. The controller tends to reduce the cone size and cone tangent value around the well. It successfully prevents the cone development and the cone shape is not developed during the production life cycle. This is occurred because when the cone height starts to increase, the controller commences to close the chock valve in order to reduce oil production rate. It means that the gravity forces could overcome the viscous forces and pulled the cone downward.

Figure 3. Controlled flow rate versus time.

Figure 3 depicts that how the control laws reduces the well flow rate as time elapses. It shows that the controlled rate started at 0.28 bbl/day, and then it decreased slightly.

Figure 4 shows the time evaluation of the system state. It shows that using the control law, the entire domain approaches zero.
Figure 4. System state $h(x,t)$ versus time (a) constant flow rate (b) control mode.

Figure 5 presents the oil column thickness at the inner and outer boundaries and at maximum value. It demonstrates that using the control and adaptation law the system state approaches zero in the whole spatial domain as time goes to infinity.

Figure 5. Thickness of oil column at the boundaries and the maximum value versus time (a) constant flow rate (b) control mode.

The second objective of this design, tending the unknown parameter to its actual value, is also obtained as shown in Figure 6. In fact, the designed controller not only is able to stabilize the system, but also estimates the unknown parameter value with great accuracy. This estimate has been made shortly after the start of the control process, which indicates the high convergence rate of the estimated parameter to its actual value. Also, this convergence is independent of the initial value of the estimate.
Figure 6. Parameter adaptation versus time in different initial estimated permeability ($\hat{k}_0$), $\Delta t = 100$ days.

Figure 7 displays that for various controller gain, the convergence conditions of the estimated parameter is different. In other words, the convergence condition depends on the controller gain.

Figure 7. Parameter adaptation versus time in different gains, $\Delta t = 100$ days, $\hat{k}_0 = 0.3$ Darcy.
Figure 8. Integral of state error during production life cycle in different gains, $\Delta t = 100$.

Also, Figure 8 shows that the system state error varies with controller gain.

Figure 9. Parameter adaptation versus time in different sampling frequency ($\Delta t$), Gama=8, $\hat{k}_0 = 0.3$ Darcy.

Figure 9 shows the frequency of sampling that affects on convergence of the unknown parameter. As the sampling increases, better convergence is achieved.

5. Conclusion
Control of the phenomenon has been suggested as one of the best approach to prevent the water production. In this study, a new adaptive control law, a parameter estimation update law and an identifier are proposed for a nonlinear PDE by PBI approach. Numerical results show that using the mentioned laws as time elapses, the whole oil column is depleted without any water production and the cone breakthrough.

Nomenclature
Porosity $\varphi$
Width of model \( w \)
Viscosity \( \mu \)
Density \( \rho \)
Gravity acceleration \( g \)
Oil column thickness \( h \)
Spatial parameter \( x \)
Time \( t \)
Permeability \( k \)
Flow rate \( Q \)
External boundary constant \( a \)
Initial thickness of oil column \( h_0 \)
Estimated permeability \( \hat{k}(t) \)
Controller gains \( \alpha \) & \( \beta \)
Constant \( \gamma \)
Minimum of \( h \) \( h_w \)

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