A NEW CHANNEL FOR THE DETECTION OF PLANETARY SYSTEMS THROUGH MICROLENSING. II. REPEATING EVENTS

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ABSTRACT

In the companion paper we began the task of systematically studying the detection of planets in wide orbits \((a > 1.5R_E)\) via microlensing surveys. In this paper we continue, focusing on repeating events. Repeating events are those in which a distant source is lensed by two or more masses in the planetary system. We find that, if all planetary systems are similar to our own solar system, reasonable extensions of the present observing strategies would allow us to detect \(3\)–\(6\) repeating events per year along the direction to the Bulge. Indeed, if planetary systems with multiple planets are common, then future monitoring programs that lead to the discovery of thousands of stellar-lens events will likely discover events in which several different planets within a single system serve as lenses, with light curves exhibiting multiple repetitions. In this paper we discuss observing strategies to maximize the discovery of all wide-orbit planet-lens events. We also compare the likely detection rates of planets in wide orbits to those of planets located in the zone for resonant lensing. We find that, depending on the values of the planet masses and stellar radii of the lensed sources (which determine whether or not finite-source–size effects are important), and also on the sensitivity of the photometry used by observers, the detection of planets in wide orbits may be the primary route to the discovery of planets via microlensing. We also discuss how the combination of resonant and wide-orbit events can help us to learn about the distribution of planetary system properties. In addition, by determining the fraction of short-duration events due to planets, we indirectly derive information about the fraction of all short-duration events that may be due to low-mass MACHOs.

Subject headings: gravitational lensing — planetary systems

1. SETTING THE STAGE

Even before microlensing events were detected, it was realized that microlensing should provide a route to the discovery of distant planets. The first such route to be considered was through the discovery of “resonant” events (Mao & Paczyński 1991; Gould & Loeb 1992). These are events in which the central star of a planetary system serves as a lens, but the smooth standard form of the point-lens light curve is interrupted, for a time lasting typically from a few hours to a few days, by a distinctive spikelike perturbation due to the presence of a planet. Such perturbations are expected for a fraction of stellar-lens events when the planet orbits in an annulus between \(0.8R_E\) and \(1.5R_E\), where \(R_E\) is the Einstein radius of the central star. The observing teams have designed strategies to discover such resonant planet-lens events. A second important channel has not been systematically considered until recently: this is the channel provided by planets in wider orbits (Di Stefano & Scalzo 1997, 1999; Di Stefano & Keeton 1999). Such planets can also participate in microlensing events, either as single lenses or as one of several lenses within the planetary system that give rise to a detectable “repeating” event.

The companion paper was devoted to considering isolated planet-lens events in which the light curve exhibits a single continuous perturbation from the baseline flux. This paper focuses on repeating events. By “repeating event” we mean an event in which a distant star is lensed, sequentially, in a detectable way by more than one member of the planetary system. Together, the two papers provide a foundation for observing programs to detect and study planet-lens events due to planets in wide orbits. They establish that planets in wide orbits may provide an important discovery channel. In this paper we also begin the task of considering likely relative rates of resonant events and those due to planets in wide orbits.

In §2 we briefly discuss what we can learn from repeating events and also provide basic information about our notation and normalization of event rates. In §3 we derive formulae for the general case of events in which the track of a distant star crosses the lensing region of more than one member of a planetary system. In §4 we consider some specific planetary systems, including our own solar system, to quantify how readily they would be detected as microlenses were they located in the Galactic Bulge. Section 5 is devoted to a discussion of detection strategies that would optimize our ability to detect planets in wide orbits, with special emphasis on repeating events. In §6 we address the likely short- and long-term results of implementing those strategies. Finally, in §7 we summarize our results on the detection of planets in wide orbits.

2. REPEATING EVENTS

The idea behind the definition of “wide orbits” is that, as the orbital separation between a planet and the central star increases, caustic crossing events become rare and events in which the planet acts as a more or less isolated lens become more common. There is no one critical separation beyond which all orbits can be considered “wide.” (See Di Stefano & Scalzo 1999) It is nevertheless useful to have a specific criterion to use in calculations. We have chosen \(a_w\), the inner boundary of the wide-orbit region, to be equal to

\(a_w = \ldots\)
1.5RE. Our results on wide-orbit planets are robust with respect to the choice of a_i.

When the light curve exhibits detectable perturbations due to the presence of more than one mass in the planetary system, we refer to the event as a “repeating event.” The most likely repeating event is one in which the central star and the innermost wide-orbit planet serve as lenses (§ 3). But more exotic events, in which the path the source takes brings it near to several planets, are also possible.

Because a key element of the definition of repeating events is the detectability of light-curve perturbations, it is useful to define a lensing region associated with each lens. Light from every star is, of course, deflected by every intervening mass. When we use the term “lensing region,” however, we refer to that portion of the lens plane through which a source track must pass if lensing by a particular mass is to be detectable. For isolated (or, in the case of wide separations, almost isolated) lenses, the lensing region is a disk surrounding the lens. It is convenient to define the width, w_j, of the lensing region as the radius of this disk. The value of w_j depends on how events are detected, as well as on the properties of the lens and lensed source (see § 2.2.1). The value of the lensing region is important because (1) the duration of the detectable perturbation from baseline due to the ith lens is proportional to w_j; and (2) the rate of detectable events with (k − 1) repetitions scales as \( \prod_{i=1}^{k} w_i \).

2.1. The Planet-Lens Event Menagerie

Consider a star orbited by a planet located a distance a from it. When this system serves as a lens, the likelihood of detecting evidence of the planet’s presence is determined by the value of a. It is convenient to measure a in units of RE, the Einstein radius of the star. The variable w_stars is the width of the star’s lensing region (the region within which the track of the source must pass in order for an event to be detectable); depending on the sensitivity of the photometry used to study the event, w_stars may be larger or smaller than RE.

If a is smaller than some minimum value, a_m, the large majority of lensing events will be indistinguishable from events due to lensing by the star alone. The value of a_m depends on the mass ratio, q, between the planet and star and also depends to some extent on how good we are at detecting deviations from the standard Paczyński light curve. We will take a_m to be equal to 0.8RE, although in some cases it may be significantly smaller.

When the planet is located within an annulus called the zone for resonant lensing, a_i < a < a_m, with a_m ≈ 1.5RE, the planet’s presence is most likely to be detected through short-lived (\( 0.3 \) [hours]) perturbations of events in which the star serves as a lens (see Mao & Paczyński 1991; Gould & Loeb 1992). We will refer to such events as “resonant” events. Thus far, the search for planets has focused on attempts to identify resonant events. When resonant planet-lens events are eventually discovered, it should be possible to determine the values of q and a for many of them, although there are degeneracies (see, e.g., Gaudi & Gould 1997; Gaudi 1997). The rate of detectable resonant events falls sharply when the size of the source star, as projected onto the lens plane, becomes large enough to be comparable to the size of the caustic structure responsible for the perturbation (Bennett & Rhie 1996).

For a > a_m (i.e., for “wide-orbits”) the planet is most likely to act as an independent lens. Repeating events will dominate for \( a < w_stars \), where w_stars is the width of the star’s lensing region (the region within which the track of the lensed source must pass in order for lensing to be detectable). The rate of repeating events falls (as \( 1/a \)), but repeating events may play a significant role for separations out to several times RE. As is the case for resonant events, the values of q and a can, in principle, be determined from study of the light curve. Parameter extraction is generally more straightforward than it is for resonant events. In particular, many light curves can be well fitted by separate point-lens light curves and the others follow a simple generalization (Di Stefano & Mao 1996). Furthermore, blending and finite-source–size effects affect each perturbation in almost exactly the same way they influence point-lens light curves. This, combined with the fact that there are two lensing encounters, each with the same amount of blending, and of a source of the same size, can make it easier to ferret out the physical parameters that characterize the physical system. As a increases, isolated events of short duration dominate; from the light curve alone neither the mass ratio nor the orbital separation can be determined.3 In terms of the complexity of the light curve and the spatial regions giving rise to the events, repeating events provide a bridge between resonant and isolated events.

2.2. Notation

Definitions specific to repeating events are presented. In addition, the notation introduced and discussed in more detail in § 2 of the companion paper (Di Stefano & Scalzo 1999) is presented in abbreviated form, so that the present paper can be read independently.

2.2.1. Photometric Sensitivity, Finite Source Size, and the Size of the Lensing Region

As defined above, the lensing region is that region in the lens plane through which the track of a distant star must pass in order for there to be an observable lensing event. The rate of events involving an individual lens of mass m_i and the time duration of such events is proportional to the geometrical width, w_i, of the lensing region. The value of w_i is related to the value of the Einstein radius of the ith lens:

\[
R_{E,i} = \frac{4Gm_i D_S x(1 - x)}{c^2},
\]

where m_i is the mass of the ith lens, D_S is the distance to the lensed source, and x is the ratio between the distance to the lens, D_L, and D_S. The value of w_i depends on the photometric sensitivity. If finite-source–size effects are negligible, and if the minimum value of A_{min} the peak magnification needed to reliably identify an event, is 1.34 (1.06) then w_i = 1R_E (2R_E). When finite-source–size effects are negligible, w_i = \sqrt{2[1/\sqrt{1 - (1/A)}] - 1}^{1/2}R_E.

Finite-source–size effects alter the width of the lensing region in a manner that depends on the size of the lens's

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3 We note that for all planet-lens events, including isolated events, information about finite-source size and/or blending can help break degeneracies and also help us to learn more about the lensing system than can be derived from the light curve alone (see, e.g., Di Stefano & Keeton 1999). For example, finite-source–size effects can provide further information (about the size of the caustic structure for resonant events and about the size of the planet’s Einstein ring for wide-orbit events). Furthermore, if the planetary system’s central star is shining, detailed study of blending, combined with high-resolution observations carried out after the event, can provide further information about the mass and state of evolution of the central star.
Einstein radius relative to the size of the source as projected onto the lens plane. Generally, if the size of the source is slightly larger than the size of the Einstein ring, the width of the lensing region can be increased. Because, however, the value of the peak magnification is decreased by finite-source-size effects, the event is not observable when the source becomes too much larger than the Einstein radius. 

For example, with \( A_{\text{min}} = 1.06 \) and \( R_E = 4R_E, w_i \sim 4R_E \); if, however, \( R_E = 5R_E \), then the event will not be detectable. More complete discussions can be found in Di Stefano & Scalzo (1999), Di Stefano & Scalzo (1997), and Di Stefano & Keeton 1999.

We define \( n_i \) to be the ratio between the radius of the lensing region of the \( i \)th lens and its Einstein radius. That is, \( n_i \) is the width of the lensing region, expressed in units of the lens’s Einstein radius: \( n_i = w_i/R_{E,i} \). Note that \( n_i \) is not generally an integer.

### 2.2.2. Encounters and Events

When a planetary system with several planets in wide orbits serves as a lens, the track of the source may pass through the region of influence of several lenses. When the source track passes within \( w_i \) of lens \( i \), we will say that an “encounter” is underway. We will use the word “event” to refer to a source track (and the associated light curve) that experiences one or more encounters.

#### 2.2.3. Repeating and Overlap Events

When more than one encounter occurs, we dub the event a “repeating” event. For many repeating events, the orbital separation, \( a \), is larger than the combined width of the lensing regions, \( w_1 + w_2 \). The light curves associated with such events exhibit genuine “repeats”; i.e., between encounters, the magnification falls below the level at which a perturbation from baseline can be detected. As \( a \) decreases below \( w_1 + w_2 \), the lensing regions of the two lenses begin to overlap. Many events that involve encounters with both lenses will give rise to light curves in which the magnification does not fall below the baseline between encounters. The light curves associated with these “overlap” events are similar to others exhibiting repetitions, except that the perturbation due to the first encounter is still noticeable when the second encounter begins. We are almost guaranteed to discover more than \( 1/2 \) of all repeating overlap events, even with little or no change in detection strategy. As the value of \( a \) decreases further, a larger fraction of the encounters with the smaller lens are part of events in which the larger lens is also encountered; for \( a < w_1 - w_2 \), this is true of all encounters involving the smaller lens. Most of the associated “overlap” light curves will exhibit a single connected perturbation, with the structure in the wings largely determined by the most massive lens and the structure near one peak largely determined by the less massive lens. Although this class of “overlap” light curves may appear to be highly anomalous, we should be able to fit them to the appropriate lensing model. Since the duration of the single detectable perturbation from baseline is governed by the size of the star’s Einstein radius, these events are long lived (relative to encounters with an isolated planet) and we might be able to detect and identify 100% of such overlap events (see Fig. 1).

If \( N \) is the number of planets, then in general we expect at most \( N + 1 \) encounters and \( N \) repetitions. In principle, if the source travels slowly and the planets orbit rapidly, a single planet could give rise to more than one encounter. In practice this is extremely rare. Repetitions, however, are not rare. As we will see, if most stars have planetary systems similar to our own, a data set containing the number of events already observed along the direction to the Bulge should contain one or more events in which a repetition is due to lensing by a wide-orbit planetary system. As we will also discuss, the detection of such events can be optimized by frequent sampling with sensitive photometry.

#### 2.2.4. Normalization of Event Rates

All other things (such as the relative velocities) being equal, the rate of events is proportional to the width of the lensing region. We therefore normalize the event rates by normalizing the width of the lensing regions. We define \( w_{0,0} \) to be equal to \( R_E \), the Einstein radius of the central star. The width of every lensing region is measured in units of \( w_{0,0} \). (Note that if the photometry is good, \( w_{0} \), the width of the lensing region of the central star may be larger than \( w_{0,0} \); i.e., the normalized width of even the central star’s lensing region can be different from unity.) At present, the rate of discovery of stellar-lens events (using a criterion that restricts the lensing region to \( 0.75R_E \)) is roughly \( 50 \text{ yr}^{-1} \). It is therefore reasonable to assume 75–100 events of the type we use for our normalization per year along the direction to the Bulge. This means that when we find that a particular detection strategy leads to a rate of detectable events of a certain type (e.g., events with two repetitions) equal to \( p\% \), between 0.75 and \( p \), such events could be discovered per
year along the direction to the Bulge. This normalization is the same as that used in the companion paper.

2.2.5. Power-of-k Planetary Systems

A “power-of-k” planetary system is one in which the distance of the ith planet from the central star is k times the separation between the central star and the (i − 1)th planet. For every planet in the zone for resonant lensing in the power-of-2 (3) model, we expect ~ 10 (7) planets in wide orbits.*

2.2.6. Ongoing Observations

Monitoring teams have found convincing evidence of over 200 microlensing events. (See, e.g., Alcock et al. 1997a, 1997b and numerous references therein; Udalski, Kubiak, & Szymanski 1997a; Udalski et al. 1994a, 1994c, 1995; Ansari et al. 1996; Alard, Mao, & Guillobert 1995.) These teams study tens of millions of stars every night, looking for any deviations from baseline in those stars not known to be intrinsically variable. Once a microlensing event is thought to have started, an alert is called, giving other observers around the world an opportunity to carry out more frequent (~ hourly) monitoring. “Follow-up” teams have formed to take systematic advantage of the microlensing alerts (see, e.g., Albrow et al. 1996; Alcock et al. 1997c; Pratt et al. 1996a, 1996b; Udalski et al. 1994b). The original monitoring and follow-up studies were of individual stars, each included on a template. More recently, the benefits of using a differencing approach that is sensitive to variations of stars not bright enough to appear on the templates have been studied and are beginning to be implemented (see, e.g., Ansari et al. 1997; Crotts 1996; Crotts & Tomaney 1996; Tomaney & Crotts 1996; Han 1996; as well as the brief discussion in § 5 of the companion paper Di Stefano & Scalzo 1999).

3. THE RATE OF REPEATING EVENTS

3.1. One Repetition

Let w1 (w2) represent the width associated with lensing by the more (less) massive lens. We assume that the separation between the two lenses is wide, i.e., the isomagnification contours associated with A = 1.34 are distinct. As we discussed in the previous section, this means that if one lens is the central star and the other is a planet, a is generally larger than ~ 1.5R_E.5 The widths, w1 and w2, however, can be smaller or larger than the orbital separation, since their values are tied to issues of photometric sensitivity.

With w_{0,0} defined as above, the detection rate for repetitions involving lenses 1 and 2 is given by

\[ R_{1,2} = \frac{2}{\pi} \frac{\theta_{\text{max}}(w_1 + w_2) + \theta_{\text{min}}(w_2 - w_1)}{2w_{0,0}} + \frac{a}{2 \pi} \left[ \frac{\cos(\theta_{\text{max}}) - \cos(\theta_{\text{min}})}{a} \right]. \]  

In this expression, the values of \( \theta_{\text{max}} \) and \( \theta_{\text{min}} \) depend on the value of the orbital separation, a, as compared with the widths, w_1 and w_2.

For \( a > w_1 + w_2 \),

\[ \theta_{\text{max}} = \sin^{-1} \left( \frac{w_1 + w_2}{a} \right); \quad \theta_{\text{min}} = \sin^{-1} \left( \frac{w_1 - w_2}{a} \right). \]  

For \( w_1 + w_2 > a > w_1 - w_2 \),

\[ \theta_{\text{max}} = \frac{\pi}{2}; \quad \theta_{\text{min}} = \sin^{-1} \left( \frac{w_1 - w_2}{a} \right). \]  

For \( a < w_1 - w_2 \),

\[ \theta_{\text{max}} = \theta_{\text{min}} = \frac{\pi}{2}. \]  

The limit \( w_1 + w_2 \ll a \) corresponds to the case in which the separations are extremely wide. In this case,

\[ R_{1,2} \approx \frac{2}{\pi} \frac{w_1 w_2}{a w_{0,0}} = n_1 n_2 \left( \frac{2}{\pi} \frac{R_{E,1}}{R_{E,2}} \right). \]  

The rate of repetitions is inversely proportional to the separation. Another key feature of this expression is the quadratic dependence of the width of the lensing region, which is directly related to the photometric sensitivity.

The second extreme limit corresponds to the case when the separation between the lenses is smaller than \( w_1 \). In this case,

\[ R_{1,2} \approx \frac{w_2}{w_{0,0}} = n_2 \left( \frac{R_{E,2}}{w_{0,0}} \right). \]  

That is, the rate of repeats involving lenses 1 and 2 is the same as the rate of events involving lens 2; all source tracks passing through the lensing region associated with lens 2 necessarily pass through the lensing region associated with lens 1.

3.2. Estimates

For the purposes of microlensing, the most important difference between a stellar binary and a planetary system is that the planetary system may contain several planets. The presence of multiple planets provides more chances for planet-lens events to occur. In addition, a small fraction of planet-lens events in the ecliptic of the planetary system may exhibit clear evidence of the presence of multiple planets, as the source travels close to the ecliptic of the planetary system. A naive generalization of equation (6) leads to the following expression for the average probability of events exhibiting a single repetition:

\[ P_2 = \frac{2n^2}{\pi T_{\text{sys}}} \int_0^{T_{\text{sys}}} \left[ \sum_{i=1}^{N} \sqrt{q_i} \left| a(t) \right| + \sum_{j=1}^{N-1} \sum_{j'=j}^{N-1} \sqrt{q_i q_j} \left| a(t) - a(t') \right| \right] dt, \]  

where \( T_{\text{sys}} \) is the time taken for the configuration of the planetary system to approximately repeat and \( q(t) \) is the position vector of the ith planet, expressed in units of the stellar Einstein radius. Note that we are considering the regime \( a \gg w_1 + w_2 \); thus, in deriving this expression, we have implicitly assumed that none of the planets is close enough to the central star for an overlap event to occur. The first term corresponds to the rate of repeats involving the central star and one planet. The second term corresponds to the rate of repeats involving two planets.

To develop a feeling for the numbers, we will consider a simple model, a power-of-k model, in which the plane of the planetary system is coincident with the lens plane and each
planet has the same mass ratio $q$ with the central star. Using $\int_0^\infty dt [a(t) - a(t)]^{-1} (a(t)^2 + a(t)^{-1})^{-1/2}$ allows us to simplify equation (8) in a way that does not overestimate the rate of wide-orbit lensing events:

$$P_2 = \frac{2n^2 \sqrt{q}}{\pi a_1} \left[ \sum_{i=0}^{N-1} \frac{1}{k^2} + \sum_{i=0}^{N-2} \sum_{j=1}^{k^2} \frac{\sqrt{q}}{k^2(k^2(j-i)+1)} \right], \quad (9)$$

where $a_1$ is the distance between the star and the first wide planet. For the power-of-2 model the value of the first term in the above equation is approximately 2 and that of the second term is roughly $1.86/\sqrt{q}$. The rate of repeating events in which the star and one planet each serve as a lens is $[(4n^2/\sqrt{q})(\pi a_1)]$. If $q = 0.001$ and $a_1 = 2$, this becomes $0.02n^2$. The observed ratio between the rate of repeats involving both the star and one planet to the rate ofstellar-lens encounters is $0.02n$. The ratio of such repeats to isolated planet-lens encounters is roughly $[(2n)/(\pi a N)]$, where $N$ is the number of planets in wide orbits. Note that the contribution of repeating events saturates at moderate values of $N$, while the rate of isolated short-duration events is proportional to $N$. The ratio of repeating events in which both encounters are due to planet lenses to those in which one component is due to the central star is suppressed by a factor $\mathcal{O}(\sqrt{q})$.

### 3.3. Multiple Repetitions

Multiple repetitions can occur when the stellar system contains three or more masses. To derive analytic expressions, we first consider lensing by a static planetary system whose orbital plane is aligned with the lens plane. The effects associated with the planets' velocities and with changing the orientation of the orbital plane are then briefly considered. The analytic expressions allow one to make intuitive predictions. In §4 we turn to numerical simulations to derive detailed results that can be checked against the predictions.

#### 3.4. Multiple Repetitions in the Static, Face-on Approximation

If a source track crosses through the orbit of a planet, the probability that it will cross within $w_i$ of the planet itself is approximately $[(2w_i)/(\pi a_i)]$. (We assume that the planet's motion during the time the source crosses its orbit can be ignored and that curvature effects are also unimportant.) Define $P_X$ to be the probability that the source track will encounter a specific subset, $X = \{\beta_1, \beta_2, \ldots, \beta_n\}$, of the system's masses, producing an event with $k - 1$ repetitions. Considering the regime $a \gg w_i + w_p$, the probability of encountering these objects, and only these objects, is

$$P_X \approx \frac{w_0}{w_{0,0}} \prod_{i \in X, i \neq 0} \left( \frac{2w_i}{\pi a_i} \right), \quad (10)$$

if the central star is one of the masses encountered. If the central star is not encountered, then a similar expression is derived, but the contribution is suppressed by a factor on the order of $\sqrt{q}$. We note that the curvature of the innermost planetary orbit in the set $X$ may be significant and can increase the probability that a track will encounter this planet. The probability of an event with $k - 1$ repetitions is obtained by summing over all distinct sets, $X$.

The above expression is useful because it clarifies the functional form of the probability of observing a particular combination of $k$ objects: generally, a rough proportionality to the ratio between the width of each planet's lensing region and its distance from the central star. This makes it simple to estimate the effects of varying the parameters of the source-lens system.

#### 3.5. Orbital Inclination

In general, the plane of the orbit will be inclined relative to the line of sight. Let $\alpha$ represent the angle between the normal to the lens plane and the normal to the orbital plane. An approach analogous to the one sketched above for the face-on case can be used. The polar symmetry of the system is broken, however, since the projection of an inclined circular orbit on the sky is an ellipse; the probability of multiple objects acting as lenses depends on the direction from which the source approaches the lensing system. Some directions of approach, the range of impact parameters leading to detectable events is now smaller, but a larger fraction of events will involve encounters with more than one object and so will appear to repeat. The effect can be intuited by multiplying each factor of $a_i$ in the above equations by the geometrical factor

$$G \propto \left( \frac{1}{\sin^2 \theta + \cos^2 \theta \cos^2 \alpha} \right), \quad (11)$$

where $\theta$ is the angle the track of the source makes with the semimajor axis of the elliptical projection of any planet's orbit. Averaging over all possible angles of approach, there is a net increase in the number of repeating events.

As $\alpha$ tends toward $\pi/2$ and we view the planetary system edge-on, most directions from which the source can approach lead only to isolated events. For a small swath of tracks, however, the probability that the source will pass through the Einstein radii of several planets approaches unity. If the planets are of equal mass and events can be detected when the distance of closest approach is $w_i = nR_{E,i}$, then the probability of an event in which the central star and all of the wide-orbit planets serve as lenses is roughly $nR_{E,i}/D$, where $R_{E,i}$ is the Einstein radius of a planet and $D$ is the distance between the two most widely separated objects in the planetary system. The probability of encounters with all but one or all but two of the masses in the planetary system is of roughly the same size, typically $\mathcal{O}(0.001)$. Thus, when the microlensing teams have carefully followed the progress of thousands of events, they may have found several in which the source track traces the global structure of a planetary system that gives rise to several detectable encounters.

It is interesting to note that if the planetary system has a well-defined ecliptic plane, then events in which more than one wide planet is detected will typically be those in which there is a fairly close approach to the central star. Griest & Safizadeh (1997) have pointed out that when there happens to be a planet with projected position located in the zone for resonant lensing, then for approaches in which the distance of closest approach is less than $\sim 0.1R_e$, there is a near certainty of detecting the planet's presence. Gaudi, Naber, & Sackett (1998) have built on this result to show that if the physical separation between the central star and any planet

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6 We note that if $n$ approaches 2, then a planet in an orbit with $a = 2$ would be detected through overlap events; $a$ would need to be larger for the approximation $a \gg w_i + w_p$ to apply.
is within the boundaries set by the values of the zone for resonant lensing, there is still a very high probability of planet detection; furthermore, more than one planet can occupy this zone. Thus, we may imagine that close approaches to the central star that happen to follow the ecliptic will discover several planets in wide orbits and all planets with true positions within ~0.6–1.6 \( R_E \) of the central star.

The orbital inclination also influences the wait times between encounters. Averaging over all possible directions of approach should cause a net decrease in the wait times between encounters for repeating events and should also produce a dispersion in the distribution of wait times.

3.6. Velocity Effects

The planets that can be detected via microlensing are generally far enough from the central star that their orbital speed, \( v_{\text{orb}} \), is low. If \( a = \mu R_E \), then

\[
v_{\text{orb}} = 7 \frac{\text{km}}{s} \left( \frac{2}{\mu} \right)^{1/2} \left\{ \frac{M}{M_\odot} \left( \frac{10 \text{ kpc}}{D_s} \right) \right\}^{1/4} \left( \frac{1}{(1-x)} \right) \]

(12)

This shows that the planets that can be discovered through microlensing and, especially wide planets, tend to be orbiting with fairly low speed. Hence, the transverse speed \( v_t \) of the source (relative to the central star of the lensing system) is likely to be large enough compared with \( v_{\text{orb}} \) so that the static approximation used above can provide a good guide to the event probabilities. Nevertheless, there will be a small number of events in which the relative magnitudes and orientations of the transverse source velocity and a planet’s orbital velocity will influence the encounter probability or the characteristics of an observed event. This can occur when \( v_t \) is drawn from the low-velocity end of the velocity distribution, particularly if \( x \) is close to unity (or zero) and the central star is massive. For example, if the transverse velocity of the source is small enough that \( R_{E,i}/v_t \) is a significant fraction of the orbital period of planet \( i \), the event probability increases. This is because the source spends a nontrivial amount of time within \( R_{E,i} \) of the planet’s orbit; if the planet does not lie in the source’s path at the beginning of this interval, it may move to cross the source’s path during the interval, resulting in a detection. In addition, event durations can be significantly decreased or increased, depending on the angle between the directions of \( v_t \) and \( v_{\text{orb}} \).

4. SIMULATIONS OF LENSING BY A PLANETARY SYSTEM

The analytic formulae and discussions of the previous section allow one to carry out back-of-the-envelope calculations to determine the salient characteristics of the results we might expect observations to yield for single-planet systems and even for more complex systems. (See Table 1.) Given orbital separations, the stellar and planet masses, the orbital eccentricities, and the orientation of the orbital plane with respect to the lens plane (or averaging over possible orientations), simple calculations can determine whether the rate of isolated and/or repeating events is large enough to expect that any specific planetary system could be detected. It is worthwhile, however, to carry out simulations to test the detectability of some specific model planetary systems. These simulations allow us to (1) include all of the relevant effects without making approximations, (2) study subtle features of the distributions of event durations and wait times due to the presence of multiple planets, and (3) systematically study the effects on the probability of detection of different detection strategies and a variety of planetary system properties.

### Table 1

**Predictions: Wide-Orbit Lensing Events for Known and Model Systems**

| System | \( P_1 - P_0 \) | \( p_{\text{2next}}^{\text{model}} \) | \( p_{\text{2next}}^{\text{known}} \) | \( p^{\text{model}} \) | \( p^{\text{known}} \) |
|--------|----------------|-------------------------------|-------------------------------|----------------|----------------|
| **Known Systems** | | | | | |
| Gl 229 | 19.9 | 0.7 | 0.0 | 0.0 | 0.0 |
| PSR B1620–26 | 10.0 | 0.4 | 0.0 | 0.0 | 0.0 |
| **Model Systems** | | | | | |
| Power-of-2 | 26.0 | 2.2 | 0.037 | 0.017 | 8.2 \( \times 10^{-5} \) |
| Power-of-3 | 14.7 | 1.7 | 0.013 | 7.1 \( \times 10^{-3} \) | 1.2 \( \times 10^{-4} \) |
| Power-of-4 | 12.6 | 1.5 | 7.1 \( \times 10^{-3} \) | 4.5 \( \times 10^{-3} \) | 3.7 \( \times 10^{-6} \) |

*The computations that produced the results in this table were based on the analytic approximations discussed in §3. Only encounters with wide planets were considered; the results were averaged over all inclinations of the system to the line of sight and over all angles of approach for the source track. The detectability threshold is \( A_{\text{min}} = 1.34 \) (separation of 1.0 \( R_E \)). No minimum event duration was required for detectability.

*The only systems included are those that fall into the regime for wide lensing for \( D_s = 10 \) kpc and \( x = 0.9 \).

*Rate (per year) of isolated events in which a planet is encountered. (See §2.2.4 for normalization information.)

*Rate of events in which two encounters occur and in which the central star is one of the objects encountered.

*Rate of events in which two encounters occur and in which neither object is the central star.

*Rate of events in which three encounters occur and in which the central star is one of the objects encountered.

*Rate of events in which three encounters occur and in which none of the three objects is the central star.

*In each of these systems, the planet closest to the Sun is in an orbit of radius \( a = 4.8 \) AU. The system is cut off at a maximum possible separation of \( 10^4 \) AU. Thus, the power-of-2 system has 12 planets, the power-of-3 system has seven planets, and the power-of-4 system has six planets.
To carry out these simulations we must choose model planetary systems, even though very little is known about planetary systems with multiple planets. The first system we have chosen is the one we know best, our own solar system. There are two features of the solar system that are important for the simulations and that can be more or less studied separately even in the context of the simulation of this single system. The first feature is that our solar system contains a planet, Jupiter, that is most likely to be detected through overlap events. The influence of Jupiter can therefore, to a good approximation, be measured separately through the rate and characteristics of overlap events. The second feature is that there are several planets in wider orbits, with a range of masses extending down from that of Jupiter. Because of its proximity to the Sun (and, to a lesser extent, because of the fact that its mass is larger than that of the other planets beyond its orbit), Saturn provides the dominant contribution to repeating events in which the magnification falls below baseline between encounters. There are also a significant number of events in which Jupiter is detected through an overlap encounter and Saturn is detected through a subsequent repeat. Jupiter and all of the outer planets contribute to the rate of isolated events, but those beyond Saturn are almost exclusively discovered through such events. Because their masses are smaller than those of Saturn and Jupiter, however, the short time duration of these events provides a challenge to their detection. The solar system model allows us to determine what we would see if all lenses were planetary systems like our own—or if all planetary systems have (1) one planet likely to be detected through overlap encounters and (2) one located roughly twice as far from the central star.

Model systems that are more extreme in that they posit the existence of a relatively large number of massive planets are (1) a power-of-3 model with seven Jupiter-mass planets and (2) a power-of-2 model with 12 Jupiter-mass planets. The power-of-2 model was introduced (Bennett & Rhie 1996) to ensure that each planetary system, regardless of the orientation of its orbital plane with respect to the lens plane, would be likely to harbor a planet in the zone for resonant lensing. Our simulations of these systems share the salient features of the solar system simulations but are designed to mimic the conditions explored by Bennett & Rhie (1996) to maximize the probability of detecting a planet in the zone for resonant lensing. Thus, our simulations of lensing by these systems defines the necessarily large role that planets in wide orbits will have if most planetary systems do indeed have planets in the zone for resonant lensing.

In any real galaxy it seems likely that planetary systems will exhibit a variety of properties—planet masses, orbital separations. The observing teams will therefore detect ensembles of events that are superpositions of those we derive here and, very likely, of even simpler planetary systems.

In each of the simulations we carried out, one model planetary system served as the lens. The projections onto the lens plane of a large number of randomly selected source tracks were followed. In all cases we neglected “inner” planets, i.e., those whose physical spatial separation from the central star was less than \(1.5R_{E}\), where \(R_{E}\) is the Einstein radius of the star. All of the orbits have been taken to be circular, with the motion governed by Newton’s laws. For each model planetary-lens system we have placed the source population in the Galactic Bulge (\(D_{S} = 10 \text{ kpc}\)) and the lens at \(xD_{S} = 9 \text{ kpc}\). The central star in each of these systems was chosen to be of solar mass.

At the time when the source started moving along the track with some velocity, \(v_{s}\), we started the planets in motion, each at a randomly chosen orbital phase. As time progressed, we tracked the position of the source and of each of the planets. We wanted to determine when the source track passed close enough to any mass in order for there to be a potentially observable “encounter,” roughly how long each such encounter would last and how many encounters there would be as the source traveled along a specific track. Because the details of the magnification as a function of time are not needed to derive this information, we used only the value of the projected separation between the source and each planetary-system mass to determine whether an encounter was in progress. Specifically, we asked whether the projected separation was smaller than some preselected value, \(w_{i} = nR_{E, i}\). As before, \(i\) labels the masses in the planetary system, with \(i = 0\) corresponding to the star and \(i\) ranging from 1 to \(N\) for the planets. The values of \(n\) to be used were selected at the beginning of each simulation. \([n = 1 \text{ (} n = 2 \text{) corresponds to a magnification of 1.34 (1.06).}]\]

A total of \(10^{7}\) tracks were used to sample the events expected when the solar system served as a lens, and \(3 \times 10^{7}\) tracks were used to sample the power-of-3 and power-of-2 models. Because even the larger numbers of tracks used for the power-of-3 and power-of-2 models do not provide sampling equivalent to that used for the solar system, we have normalized the results so that the effective linear density of source tracks was the same for each planetary system. We have also smoothed the power-of-2 and power-of-3 distributions to remove effects due to poor sampling. We note, however, that the finer features of the statistical distributions, i.e., those that become apparent only as the number of events increases, are best seen in the plots of the solar system distributions. We have used our simulations to derive in detail what happens for each planetary system and also to test the effects of varying some of the event parameters and detectability criteria.

4.1. Event Parameters

The characteristics of the light curves are determined by the characteristics of the event. In particular, we must consider (1) the orientation angle, \(z\), between the plane of the orbit and the lens plane and (2) the transverse speed, \(v_{e}\), of the source with respect to the central star of the planetary-system lens. We have therefore carried out some simulations in which we have varied \(z\) and \(v_{e}\), in order to test their influence on the results. In the most realistic of our simulations, \(z\) was chosen uniformly over the interval \(0–2\pi\) and \(v_{e}\) was chosen from a Gaussian distribution centered at 150 km s\(^{-1}\), with width equal to 50 km s\(^{-1}\).

4.2. Detectability Criteria

There are two key elements of detectability. First, does an encounter last long enough to be detected? Even in principle, an encounter cannot be detected unless it is caught in progress during at least one observation. Reliable detection generally requires the event to last long enough to span at least the time interval between two or more consecutive observations. In some of our simulations we have assumed that a minimum event duration of 1 day is needed in order to reliably detect the first encounter between a source track.
and the lensing region of a lens. (This assumes that the
transverse velocity is such that the time needed to cross the
Einstein diameter of a solar mass lens is ∼ 94 days. See § 2.1
of Di Stefano & Scalzo 1999.) This criterion can be achieved
by the monitoring teams in some fields, using their present
observing strategy. In others simulations we have dropped
the requirement of a minimum duration for the first event.
This more relaxed condition is appropriate to the follow-up
teams; since they can achieve hourly monitoring, even an
event lasting 8 hours (such as one likely to be due to an
Earth-mass planet) can be readily identified if the follow-up
teams attempt to discover new events.

The second key element of detectability is provided by
the value of the peak magnification: what is the minimum
peak magnification, $A_{\min}$, required in order to reliably
determine that an event occurred? When the observing
teams started, they tentatively chose $A_{\min} = 1.34$,
corresponding to a distance of closest approach equal to $R_E$. It
turned out, however, that some apparent events with mag-
nification above 1.34 but less than ∼ 1.58 were due to stellar
variability; this has led the MACHO team, for example, to
use $A_{\min} = 1.58$. It is likely, however, that this condition can
be relaxed as the continued study of the same fields over
time will allow for better identification and tracking of
stellar variability and decrease its possible contamination of
our count of true microlensing events. In principle, the
value of $A_{\min}$ is set by the photometric precision of the
monitoring system. If the photometry is good to the
1%–2% level, then smaller values of $A_{\min}$ are achievable.
$A_{\min} = 1.06$ corresponds to a distance of closest approach
approximately equal to $2.0R_E$, and $A_{\min} = 1.02$ corresponds
to a distance of closest approach approximately equal to
$3.0R_E$. (To achieve this latter value would require better
photometry than is typical of even the present-day follow-
up teams.) We have carried out two types of simulations. In
the first, we have assumed that event identification was
being done by the monitoring teams; in these we assumed that
the distance of closest approach needed for the identifi-
cation of the first encounter is at least as small as $R_E$. In
the second, we have assumed that event identification was being
done by the follow-up teams; in these we assumed that the
distance of closest approach needed to be at least as small as
$2R_E$. We note that finite source size effects can make events
detectable when the distance of closest approach is even
larger. We return to this point at the end of § 4 and again in
§ 6; the results presented in this section were derived under
the assumption that the size of the lensed source could be
neglected.

We label the three sets of detectability criteria used in our
simulations “A,” “B,” and “C.”

Criteria A.—The first encounter must exhibit magnifi-
cation greater than $A = 1.34$ (source-lens separation less
than $1.0R_E$) for at least 1 day in order for the lensing event
to be detected. After the detection of a first encounter, sub-
sequent encounters can be detected when $A > 1.06$ (source-
lens separation less than $2.0R_E$) and are not subject to
minimum duration requirements.

Criteria B.—All lensing encounters are detected when the
magnification becomes larger than 1.06 (i.e., the source-lens
separation becomes smaller than $2.0R_E$). The first encounter
must have a duration of at least 1 day in order to be
detected, but after one encounter has been detected, there is
no minimum duration required for the detection of sub-
sequent encounters.

Criteria C.—All lenses are detected when the magnifi-
cation becomes larger than 1.06 (i.e., the source-lens separa-
tion becomes smaller than $2.0R_E$). No minimum duration is
required in order to detect any encounter.

Because we want to make contact with observations, we
have defined encounter durations and wait times to reflect
the actual required monitoring times during and between
events. The duration of an encounter is always defined as
the interval of time during which the source was within $2R_E$ of a lens.7 Wait times are defined as the time intervals
between encounters.

We have used the normalization of § 2. $P_1$ is the rate (per
year) of source tracks that cross through the Einstein ring of a
single object. All rates reflect the percentage of events of
the given type relative to the rate of isolated stellar-lens
events (with $A_{\text{peak}} > 1.34$) in our simulations.8

$P_1 - P_\odot$ is the rate (per year) of source tracks passing
through just one object, excluding cases in which the central
star is encountered. $P_{1,\text{overlap}}$ is the rate (per year) of source
tracks in which the influence of encounters with two lenses
is clearly visible in the light curve but the light curve
exhibits just one continuous perturbation (i.e., once the
magnification falls below the detectability limit, it does not
again rise above it). $P_i$ is the rate (per year) of source tracks
crossing through the lensing region of $i$ objects. $P_{i,\text{overlap}}$ is
the rate (per year) of source tracks passing through $i + 1$
ilensing regions, but in which the presence of two of the
lenses (the central star and the innermost planet in a wide
orbit) is detected in an overlap encounter; the magnification
rises above and falls below the detectability limit only $i$
times.

4.3. Results

The simulations allowed us to compute the rates of both
isolated and repeating events. The rates of all events are
provided in Table 2. Figures 3–6 show the distributions of
properties for those events with light curves exhibiting two
separate deviations from the baseline flux.

The primary focus of this section is on repeating events in
which the light curve displays more than one deviation from
baseline. It is important to note that overlap events involv-
ing just two lenses produce only a single deviation from baseline;
graphs showing the distribution of durations for
such events are therefore included in the companion paper
(Di Stefano & Scalzo 1999). Although, from the phenomen-
ological perspective, such events are “isolated,” concep-
tually they are “repeating,” in that the track of the source
does pass through the lensing regions of two lenses. Their
importance can be assessed in two ways. Judging by the
rates, they are important because, depending on the spatial
and mass distribution of planets, their frequency is generally

Note that if a magnification of 1.34 was required for event detection,
the duration can nevertheless be (and for us, is) defined as the time during
which the magnification was above 1.06, since this information can readily
be extracted from the light curve. This is because the sensitivity of the
photometry is always significantly better than the fractional increase in
magnification required in order that the event as a whole be reliably associ-
ated with microlensing.

Note that the exact normalization described in § 2 requires dividing by
the number of encounters, rather than by the number of isolated events in
which the central star serves as a lens. Because these two different ways of
computing the rates lead to similar results (using the number of events
yields rates that are ∼ 2%–10% higher), and because the number of events
provides a more straightforward comparison, we have chosen to divide by
the number of isolated stellar-lens events.
comparable to that of other repeating events. Judging by their detectability, they are, in the short term, almost certain to play a larger role than other repeating events. This is because they should be observable primarily as apparently perturbed isolated stellar-lens events, with no "wait time" between encounters. In the long term, observations of all repeating events will be important. The relative numbers of overlap events and repeating events exhibiting multiple rises from the baseline flux will help us to learn about the typical spatial distributions of planets within planetary systems. In the very long term, the observation of a small number of light curves with multiple repetitions will provide exciting sketches of distant planetary systems.

4.4. The Effects of Systematic Variation of \( v_t \)

We used a truncated (five planet) power-of-2 system to explore the influence of changing \( v_t \) on the rate of events and characteristics of events. In the simulations whose results are shown in Figure 2, we took \( \alpha = 0 \); i.e., the orbital plane coincided with the lens plane. Holding \( \alpha \) fixed, we systematically varied \( v_t \). The distribution of encounter durations for \( v_t = 10 \) and \( 80 \) km s\(^{-1} \) are shown in Figure 2. A comparison between the two cases clearly illustrates that, in keeping with the predictions, the overall rate of detected events is larger for all types of events when \( v_t \) is smaller. This effect is most pronounced for repeating events that involve several planets, since the probability of a repeating event behaves like a product of detection probabilities for each planet separately. The rate (per year) \( P_2,\text{overlap} \) of overlap doubles demonstrates this well, since an overlap double involves at least three objects, two of which must be planets.

\( \text{TABLE 2} \)

SIMULATION RESULTS: WIDE-ORBIT EVENTS FOR KNOWN AND MODEL SYSTEMS

| Detect*        | \( P_1 - P_0 \) | \( P_1 \text{overlap} \) | \( P_2 \) | \( P_2 \text{overlap} \) | \( P_3 \) | \( P_3 \text{overlap} \) |
|----------------|-----------------|---------------------------|---------|---------------------------|---------|---------------------------|
| Solar System, \( V = 150 \) km s\(^{-1} \), \( \alpha = 0^\circ \) |                |                           |         |                           |         |                           |
| A ……           | 0.3             | 2.2                       | 0.5     | 0.01                      | 0.0     | 0.0                       |
| B ……           | 1.8             | 6.0                       | 1.4     | 0.04                      | 3.0 × 10\(^{-4} \) | 0.0     |
| C ……           | 4.4             | 6.0                       | 1.7     | 0.05                      | 2.4 × 10\(^{-3} \) | 0.0     |
| Solar System, \( V = 150 \) km s\(^{-1} \), \( \alpha = 75^\circ \) |                |                           |         |                           |         |                           |
| A ……           | 0.1             | 1.7                       | 0.8     | 0.02                      | 1.8 × 10\(^{-3} \) | 1.5 × 10\(^{-4} \) |
| B ……           | 0.3             | 2.4                       | 1.7     | 0.04                      | 4.2 × 10\(^{-3} \) | 1.5 × 10\(^{-4} \) |
| C ……           | 1.5             | 2.4                       | 2.3     | 0.04                      | 0.01    | 1.5 × 10\(^{-4} \) |
| Solar System, \( V = \text{Gaussian} \), \( \alpha = \text{Uniform} \) |                |                           |         |                           |         |                           |
| A ……           | 0.3             | 2.1                       | 0.7     | 0.02                      | 1.4 × 10\(^{-3} \) | 0.0     |
| B ……           | 1.4             | 4.2                       | 1.8     | 0.04                      | 3.2 × 10\(^{-3} \) | 0.0     |
| C ……           | 3.1             | 4.2                       | 2.1     | 0.05                      | 5.5 × 10\(^{-3} \) | 1.5 × 10\(^{-4} \) |
| Power-of-2 System, \( V = \text{Gaussian} \), \( \alpha = \text{Uniform} \) |                |                           |         |                           |         |                           |
| A ……           | 23.7            | 2.2                       | 3.0     | 0.03                      | 0.03    | 0.0                       |
| B ……           | 53.8            | 4.0                       | 6.6     | 0.08                      | 0.06    | 0.0                       |
| C ……           | 55.9            | 4.0                       | 6.6     | 0.08                      | 0.08    | 0.0                       |
| Power-of-3 System, \( V = \text{Gaussian} \), \( \alpha = \text{Uniform} \) |                |                           |         |                           |         |                           |
| A ……           | 13.0            | 1.9                       | 1.5     | 0.03                      | 0.01    | 0.0                       |
| B ……           | 29.3            | 3.6                       | 3.1     | 0.06                      | 0.02    | 0.0                       |
| C ……           | 30.4            | 3.6                       | 3.1     | 0.06                      | 0.02    | 0.0                       |

* Descriptions of the detection criteria can be found in the text. All probabilities are given as the rate (per year); these correspond to the percentages of the number of events in which the central star was the only lens encountered and in which the magnification reached at least \( A_\text{min} = 1.34 \).

b Rate (per year) of isolated (nonrepeating) events (one peak in the light curve). In this column we include only events in which a single planet-lens was encountered.

c Rate (per year) of nonrepeating events which exhibited evidence of lensing by two masses. That is, in this column we include only overlap events; in these cases the two lenses were almost always the central star and the innermost planet.

d Rate (per year) of events with one repetition. These are not overlap events; two well-separated masses were encountered. In most cases these two masses are the central star and the second planet out, but there are other contributions as well.

e Rate (per year) of single-repetition events in which one component consisted of overlapping encounters; the repetition was due to lensing by a third mass.

f Rate (per year) of events with two repetitions; all lenses were well separated, with no overlap.

g Rate (per year) of double-repetition events in which one component showed evidence of overlapping encounters; the two repetitions were due to lensing by two other well-separated masses. If the linear density of source tracks passing through the power-of-2 and power-of-3 systems had been the same as for the solar system, there would have been events in these categories.
$v_t = 10 \text{ km/s}$

$P_1 - P_{\text{sun}} = 22\%$
$P_{1,\text{overlap}} = 7.8\%$
$P_2 = 5.6\%$
$P_{2,\text{overlap}} = 0.22\%$

$P_{2,\text{overlap}}$ is larger for $v_t = 10 \text{ km s}^{-1}$ by a factor of $\sim 1.8$. When $v_t$ is comparable to the orbital velocity, as it is in the top panel, there is a clear dispersion in the distribution of encounter durations of short-duration encounters in which a planet serves as a lens. This dispersion is due to the influence of the planets’ orbital velocities; encounters can be lengthened or shortened depending on the angle between the planet’s orbital motion and the transverse source velocity. In contrast, for $v_t = 80 \text{ km s}^{-1}$, the peak due to planets is sharp and well defined.

$P_{2,\text{overlap}}$ becomes less pronounced.

$As v_t$ increases, the projection of the orbits onto the lens plane become ellipses. The projected orbital speed along the semimajor axis is the same as before, but the projected orbital speed along the transverse direction is smaller. Thus, along some directions of approach the effects associated with the finite size of $v_t$ become less pronounced.

4.5. Systematic Variation of Orbital Inclination

To systematically test the results of changing $v_t$, we again used the five planet power-of-2 system, this time keeping $v_t$
Fig. 3.—Distributions of wait times between, encounters for events with one repetition. The model planetary system serving as a lens is the five-planet power-of-2 system. Detection criteria B are used, and $v_\text{c} = 80 \text{ km s}^{-1}$.

fixed at $80 \text{ km s}^{-1}$. The results for four simulations ($\alpha = 0^\circ$, $45^\circ$, $75^\circ$, and $90^\circ$) are shown in Figure 3. For $\alpha = 0^\circ$, four peaks in the distribution of wait times between encounters are sharp and clearly visible; these correspond to wait times between encounters with the central star and encounters with one of the four outermost planets. (The innermost planet is so close that it serves as a lens only in overlap events.) The even spacing between the peaks is a signature of the power-of-2 model, becoming about $0.3 \approx \log_{10} 2$ for planets with $a_j > R_\oplus$. As the inclination increases, the dispersion mentioned in §3 appears; note that increasing inclination can only decrease wait times, so that each peak is “smeared” out to the left until at high inclinations a wedgelike shape is achieved. The detection rates for non-repeating events decreased with increasing inclination, but the relative detection rates for repeating events increased, as predicted. For $\alpha = 45^\circ$, the innermost planet’s orbit comes within $0.7R_\oplus$ of the star, placing it in the zone for resonant lensing most of the time. The detection rates for doubles and overlap singles decrease dramatically because of this, but they rise again at $\alpha = 60^\circ$ as the second planet’s projected orbit becomes small enough for overlap events to occur.
At $\alpha = 90^\circ$ (edge-on), the motion of the planets brings all of them into the zone for resonant lensing, or even closer, part of the time. This decreases overall detection rates for planets in wide orbits. The detection rates for doubles decrease dramatically; this is because all planets lie along the same line and events that might have been doubles actually become triples, quadruples, or higher order events. In fact, we found that for $\alpha = 90^\circ$, the rate (per year) $P_3$, $P_4$, and $P_5$ were all approximately 0.3%. Thus, as mentioned in § 3, a relatively large fraction of events can exhibit multiple repetitions, as the track of the source sweeps across the ecliptic, crossing through the lensing regions of several planets and the central star.

4.5.1. The Effects of Changing the Detectability Criteria

To better learn how to optimize the returns from the microlensing observations, we tested the effects of varying the detectability criteria from the most conservative set of criteria (set A), to the most inclusive (set C). The results are shown in Table 2 and in Figures 4–6.

The planetary systems serving as lenses are our solar system, the 12 planet power-of-2 system, and the seven planet power-of-3 system. Figures 4 and 6 show the duration and wait time distributions for events with one repetition when using detection criteria A and C, respectively. Figure 5 shows the cumulative distributions for the graphs plotted in Figure 4. (Note that we have also kept track of the isolated events that would have been detected in these simulations. The results for isolated events are summarized together with those for repeating events in Table 2 and are illustrated in graphs included in the companion paper; Di Stefano & Scalzo 1999.)

The detection criteria of set A are the most restrictive. When our solar system serves as a lens, with $v_\parallel = 150$ km s$^{-1}$, only Jupiter and sometimes Saturn are able to produce events sustaining a magnification of $A_{\text{min}} = 1.34$ for longer
than 1 day. Any other planets would be detected only by the follow-up teams, i.e., only if a larger object (almost always the Sun) were encountered first. Thus, wide-orbit planets slightly less massive than Saturn are difficult to detect using a strategy based on a set of criteria similar to that of set A. The power-of-2 and power-of-3 models we examined contained only Jupiter-mass planets, so each planet was detectable, except for the small fraction of encounters in which the source track just grazes the lensing region of the planet. We note, however, that the criteria of set A are not powerful tools for the discovery of solar systems like our own.

Using the detection criteria of set B (detecting even a first encounter at \( A = 1.06 \), instead of \( A = 1.34 \)) generally more than doubled the detection frequencies in our most realistic simulations, in which \( \varphi \) was chosen from a Gaussian distribution and \( \alpha \) was chosen from a uniform distribution. Planets could be detected at a lower peak magnification, and more planets in the system were able to sustain a magnification of \( A = 1.06 \) for the 1 day minimum duration required of the first encounter. For the solar system, Saturn was often detected as an isolated lens, or as the first lens encountered, and Uranus and Neptune made occasional appearances as isolated lenses. Note that, taken by itself, the effect of decreasing \( A_{\text{min}} \) to 1.06 should increase the rate by a factor of 2 (instead of 4); this is because, even with the set of detection criteria A, \( n \) was already equal to 2 for the second encounter. Deviations from the factor of 2 increase are mostly associated with events that include encounters with a low-mass planet and are therefore seen primarily in the solar system; these deviations are due to our use of a minimum duration for the detection of the first encounter.

Finally, the detection criteria of set C (removing the requirement that the first encounter last for at least a day) made Uranus and Neptune more regularly detectable as isolated lenses, whereas before they were primarily detected as part of repeating events involving a larger mass. These planets produce significant new structure in the duration distribution of isolated events (Di Stefano & Scalzo 1999), as well as fill out the structures found in the distributions of repeating event characteristics. Even Pluto produces its own peak at \( 10^{-2} \) days (~10 minutes), although this is too short to be detected in practice.

To assess the relative benefits of the three detection strategies used, we note that the largest increase in the rates of
detectable planet-lens events was realized by switching from the set of criteria A, in which $A_{\text{min}}$ for the first encounter is 1.34, to set B, in which $A_{\text{min}}$ for both encounters is 1.06. The gains in switching from B to C, which eliminated the requirement that the first detectable encounter have a duration of 1 day, could be substantial only if, like our own solar system, the lens planetary system contains wide-orbit planets less massive than Jupiter.

In general, the smaller the value of $A_{\text{min}}$, the longer the duration of the observed event anyway. Thus, while frequent monitoring is desirable, and more frequent monitoring than is presently achieved by the monitoring teams would be very valuable, the key issue is sensitivity to encounters in which the peak magnification may be smaller than 1.34. We note that this result is likely to remain valid, even when other effects are considered. For example, both blending and finite-source-size effects decrease the peak magnification. Thus, detection rates for events subject to these effects may also be improved if encounters that lead to lower magnification peaks can be detected.

4.5.2. General Features

Perhaps the most interesting feature of the results is that they clearly indicate the feasibility of the search for planetary systems containing planets in wide orbits. For the most optimistic model, the 12 planet power-of-2 model, even the strictest detection criteria yielded a rate of 5% for events showing some evidence of two or more objects in the system and 24% for isolated short-duration events.\(^\text{10}\) For the detection criteria of set C, 11% was the rate of events showing some evidence of two or more objects in the system, and $\sim 56\%$ was the rate of isolated short-duration events. It is clear that if the power-of-2 model is realized in nature with any great frequency, we will be able to observe many planet-lens events even during the next few years.

\(^{10}\) These rates are computed using the normalization described in § 2. That is, they are 100 times the number of events of the type described, divided by the number of events in which the central star served as the lens in an isolated event, with peak magnification of the stellar event achieving a value greater than 1.34.
Conversely, an absence of large numbers of interesting planet-lens events, particularly if we use the detection criteria of set C, would allow us to definitively falsify the hypothesis that most stars are accompanied by power-of-2 planetary systems.

While it may not be surprising that such a radical model is verifiable or falsifiable, the relative ease with which planetary systems such as our own can be discovered is indeed worthy of note. The rates of wide-orbit events range from a significant subset (1%--4%) of all events for all of the models (solar system through to power-of-2) and for all detection criteria (sets A through C). This is simply due to the fact that each model system has a planet located just outside the zone for resonant lensing. Note that if overlap events are not seen at roughly this level, there are likely to be few resonant events, since the planets that serve as lenses in overlap events are the ones that can be brought into the zone for resonant lensing as the orientation of the planetary system changes.

4.6. Finite-Source–Size Effects

To better quantify the influence of finite source size, we have carried out simulations using the solar system model and a power-of-3 model to study the detectability of Earth-mass planets. The placement of the planets was exactly as in our previous simulations of the solar system and the power-of-3 model, but the mass of every planet was the same as that of the Earth. We assumed that the source radii are distributed uniformly from $R_S$ to $10R_S$. In addition, we assume that lensing by Earth-mass planets cannot be detected if $R_S > 7R_E$. Thus, roughly 43% of the sources could not produce identifiable lensing events. For the remainder of the sources, we used the approximation that $D_{min} \sim R_S$. (See Fig. 2 of the companion paper; Di Stefano & Scalzo 1999.) Note that because the model planetary systems in these simulations are in the Bulge and are lensing more distant Bulge stars, the Einstein radius of an Earth-mass planet is approximately $1R_\odot$.

Thus, because we have assumed $R_S > 3R_E \sim 3R_\odot$, we are not considering the contributions of the lensing of dim stars; because such stars are likely to be numerous, their
contributions could be important, particularly when difference techniques are used to analyze the data. This is because such techniques allow the lensing of stars not bright enough to appear on a template to be detected. Furthermore, our assumption that $R_S < 10R_\odot \sim 10 R_\odot$ means that we are not considering the contribution of giants. In fact, giants cannot contribute to the lensing by Earth-mass planets since their large radii can allow only very small values of the peak magnification to be achieved. Their contributions can be important, however, for the detection of more massive planets.

We used $v_i =$ Gaussian, $\alpha =$ uniform, detect $= C$. These results are summarized in Table 3 and are illustrated for the solar system model in Figure 7.

Because their Einstein rings are smaller, Earth-mass planets are expected to yield a smaller event rate. The rate would be smaller than that due to Jupiter-mass planets by a factor of roughly 18. Yet, in the power-of-3 model, where all of the planets in our original model had a mass equal to that of Jupiter, the computed attenuation of the Earth-mass system relative to the Jupiter-mass system is only a factor of $\sim 3$. Even though finite-source–size effects made event detection impossible when the Earth-mass planets lensed roughly 43% of the sources, the fact that the detection rate was higher for the remainder of the sources provided a net increase in the detection rate over what might otherwise have been expected. The relative increase is even more pronounced for the solar system model, since Saturn, Uranus, Neptune, and Pluto are responsible for many of the isolated events when a model of the solar system serves as a lens, and these each have a smaller Einstein radius than Jupiter.

It is also interesting to make the comparison to the lensing by this mock solar system, with only Earth-mass planets, when the sources it lenses are all pointlike. We find that finite-source–size effects are associated with a significant overall increase in the detection probability. Single-perturbation events (top panel of Fig. 7) would occur at a rate of less than 0.5%, were the sources pointlike, whereas
doubles would occur at a rate of less than 0.2%. Thus, finite-source–size effects have increased the detection probability by a factor of ~ 5. (This occurs despite the fact that some sources were either too small or too large to be detectably lensed in the finite-source–size case.)

Although more realistic stellar surface brightness profiles and stellar population models should be used, the basic result that Earth-mass planets are detectable should be robust. Also robust is the increased event duration for a range of values of \( R_S \). For a fixed value of \( A_{\text{min}} \), this corresponds to a larger value of the effective width, \( w_E \), and therefore a larger value of \( n_i = w_i/R_{E,i} \). As predicted and as demonstrated by the simulations, this can increase the detection probabilities above what is expected if the source is pointlike.

5. DETECTION STRATEGIES

5.1. Optimizing Our Ability to Detect Planets in Wide Orbits

Three steps can improve our sensitivity to repeating events: (1) improving the photometric sensitivity; (2) instituting reasonably frequent monitoring; (3) being alert to repeats and to the oddball light curves associated with overlap events. See Sackett (1997) for an analysis of the strategies best suited to the discovery of planets in the zone for resonant lensing.

5.1.1. Photometric Sensitivity

Photometric sensitivity is the key issue for the detection of planets in wide orbits. (See also § 5 of the companion paper, Di Stefano & Scalzo 1999.) The better the photometry, the smaller the value of \( A_{\text{min}} \) (the minimum peak magnification needed for reliable event detection), the larger the value of \( n_i = w_i/R_{E,i} \) for each lens and the larger the number of detectable events. The increase is largest for repeating events, since the rate of such events scales as \( \prod_{i=1}^{k} n_i \), where \( k \) is the number of repeats.

5.1.2. Frequent Monitoring

Not only are more events discovered when the \( n_i \) are greater than unity, but the duration of detected events is also longer, making less frequent monitoring possible. The optimal monitoring frequency therefore depends on \( A_{\text{min}} \).

5.1.3. Oddball Events

The early folklore on repeating events said that they should be discarded: the microlensing of light from a specific star is such a rare event that if a microlensing-like signal is seen twice when monitoring the same star, the repetition is almost surely a sign that the observed variation is due to something other than microlensing. Although it has been pointed out that binary sources (Griest & Hu 1992) and binary lenses (Di Stefano & Mao 1996) can each lead to microlensing events that repeat, it is nevertheless the case that, until recently, one of the cuts used to eliminate lensing candidates from further consideration is based on whether the event appears to repeat.11 As a result, the monitoring teams may not have systematically pursued the implications of evidence of repeating events in their data sets. The oddball events most likely to have been missed are the following:

1. Events that were not recognized in real time and for which no alert was called. Software packages that sort through the data post facto might have eliminated such events from further consideration. This is not a serious problem because (a) it can be corrected by updating the software if necessary and (b) there do not seem to have been many events eliminated by this criterion (T. S. Axelrod 1997, private communication).

2. Events in which either encounter led to a peak with a small maximum magnification. These events can also be discovered by reexamining the data, unless the low-magnification peak was so small that the photometric sensitivity of the monitoring team does not allow it to be distinguished from baseline. In other cases, there would simply be one encounter leading to an observable event and a second perturbation, too small to be reliably identified as due to microlensing. The frequency of these situations can be determined post facto.

3. Events in which both encounters led to peaks with small peak magnification. Neither perturbation would have been identified as a microlensing event. As above, however, such events can be found by sifting through the data with a software program designed to look for them. It may, however, be difficult to reliably attribute them to microlensing. The statistics of such events can be interesting, however, because the numbers of those that are due to microlensing should be closely related to the numbers of other repeating events with larger peak magnifications and smaller distances of closest approach.

4. Events in which one or more deviation from baseline is short lived. Such events present the most serious challenge to event detection, because this description fits all repeating events (except for overlap events) in which a planet serves as a lens. This makes it clear that overlap events will be the easiest to detect at present. Repeats in which the first rise from baseline is due to lensing by the central star will be the next easiest to detect, because their detection requires only that the follow-up teams continue to carefully monitor a stellar-lens event that has apparently ceased. Repeats in which the first rise from baseline is due to lensing by a planet will be the most difficult to detect with present strategies; the changes needed to detect such events are the same as those needed to detect isolated events of short duration and are discussed in Di Stefano & Scalzo (1999).

In the companion paper we noted that the Bulge data collected by the MACHO team already contains enough short-duration events to demonstrate the monitoring teams’ capability to systematically detect such events, especially with some modification in their observing strategy. Here we note that repeating events are also well within the realm of presently detectable events. The first “repeater” mentioned in the MACHO alert pages is 96-BLG-4. The first encounter of that event was apparently a relatively low magnification Paczyński light curve (\( A_{\text{peak}} = 1.9 \)) of long duration (~111 days), and the second perturbation occurred in the autumn of 1997. There is no information posted about the duration of magnification of the second deviation from baseline. The second “repeater” is the pair 97-BLG-45 (which exhibited a peak magnification of 4.8

11 Of course the complication of ensuring that a repetition is not due to stellar variability is serious, since variability is far more common than microlensing. Nevertheless, if a star that has never been observed to vary from baseline produces two deviations well fitted by microlensing models, it is certainly worth exploring the possibility that the deviations are indeed due to microlensing. In some cases blending or finite-source–size effects may make it possible to carry out further tests of the conjecture that multiple disturbances are due to microlensing.
and a duration of $\sim 107$ days) and 97-BLG-47. Although no information about the second deviation is provided, its place in the sequence of 1997 Bulge events indicates that there was likely not a long wait between the two encounters, if indeed both perturbations are due to microlensing.

5.2. Observing Programs to Discover Planets

The design of the observing programs of the future should take the above requirements seriously. There are likely to be several different ways to proceed effectively. For example, a system of monitoring could achieve a detection rate $\sim 10$ times as high as the present system does by (1) using photometry as good as or better than the present follow-up teams, (2) using difference techniques to identify lensing of stars that are otherwise (i.e., at baseline) below the detection limit, and (3) monitoring frequently. This would mean discovering $\mathcal{O}(1000)$ Bulge events per year. Not only would the numbers of events be high, allowing us to detect some very low probability events (such as multiple repetitions), but the quality of the data would be so high that we would understand each event and the population of lenses and sources much better than we do at present. Another approach might be to have a worldwide network of telescopes, each taking deep images of a few fields once or twice per night. The detection rate per field and the quality of the data would be high, but the number of events discovered per year would be smaller, since fewer stars are being monitored. The total number of events discovered per year might be comparable to the discovery rate of the present MACHO team.

It is worthwhile to consider what the teams using the wide-planet search strategies will learn, even if nature has been so unkind as to neglect providing most stars with planets. Planet-motivated investigations will yield interesting fruit regardless of the size of the population of planets. First, they will increase the detection rates of all events, particularly short-duration events. Second, frequent monitoring with good photometry will allow us to learn more about each event detected. Common astronomical effects, such as stellar binarity, blending, and finite-source size, are expected to significantly affect the shape of lensing light curves, introduce time dependence into the spectra, and even produce apparent repetitions (see, e.g., Griest & Hu 1992; Mao & Paczynski 1991; Mao & Di Stefano 1995; Di Stefano & Esin 1995; Kamionkowski 1995; Loeb & Sasselov 1995; Simmons, Willis, & Newsam 1995; Di Stefano & Mao 1996; Sasselov 1997; Di Stefano & Perna 1997). Studying the manifestations of these effects in the data sets can (1) break the degeneracy of individual light curves and (2) allow us to learn more about the populations of sources and lenses. Along the direction to the Bulge, we will be able to learn a good deal about the stellar luminosity and mass functions and the binary fraction, as well as the distribution of binary properties. The information we collect can inform our design of the next generation of microlensing observations. For example, satellite projects have been proposed and preliminary calculations indicate that they are likely to be productive (see, e.g., Boutreux & Gould 1996 and references therein). The detailed observations that would be made as part of the search for planets in wide orbits would provide solid input, useful for the detailed planning needed for such space-based projects.

5.3. Useful Modifications of Existing Programs

5.3.1. The Monitoring Teams

The present strategy of the monitoring teams would allow them to detect reliably the first encounter of a repeating event only if the first mass serving as a lens is the central star and if the peak magnification is large enough. There would be some chance of detecting the first encounter if the first mass serving as a lens is a planet with a mass comparable to or greater than that of Jupiter. It is possible, however, that even the present data sets may contain evidence of repetitions in addition to those listed on the alert web pages.\footnote{http://darkstar.astro.washington.edu.} If this is so, such evidence might best be discovered through searching the existing data sets for repetitions, perhaps in conjunction with a spike analysis. Particularly in fields visited every day, a spike analysis can provide valuable information. The ratio of spikes expected to be part of a repeating event to those expected to be isolated depends on the number of planets and on the spatial and mass distribution among the planets. It should mirror the overall ratio of repeating events to isolated events: for systems similar to the solar system this ratio might be on the order of unity, while for power-of-$n$ type systems it could be as low as 0.1.

The present-day monitoring teams may also be able to detect some repeating events when the first mass to serve as a lens is a planet less massive than Jupiter. But to do this reliably would require some modifications of their observing strategy. Because the modifications likely to be useful are exactly those that would allow the monitoring teams to have a better chance of detecting isolated events of short duration, they are discussed in the companion paper (Di Stefano & Scalzo 1999). Here we simply mention the possibility of selecting a subset of their monitored fields for more frequent monitoring.

5.3.2. The Follow-up Teams

The follow-up teams need to continue to monitor all events after the flux has apparently fallen back to baseline. It should be possible for the follow-up teams to discover repetitions in most cases in which a star is the first lens encountered. For the planetary systems we have simulated, roughly $\frac{1}{2}$ of all other repetitions could be detected if monitoring were continued for 100 days. In addition, overlap events would not require a significant wait time, since the second encounter would start (i.e., be associated with $A > 1.06$) even before the first encounter ended.

Note that continued monitoring may be useful even when the first event observed is a resonant-lensing event, since (1) if the observed event was due to a planet, there may well be a second planet and (2) if the observed event was due to a binary lens, one or both members of the binary (or even the combination) may support a planetary system.

We note that, even when the monitoring teams have identified a short-duration event that appears to have ceased before more frequent monitoring could begin, it is worthwhile for the follow-up teams to continue to monitor the flux. If the short-timescale event was due to lensing by a planet, the result of frequent monitoring could be the detection of a repetition due to a moon revolving about the planet (Di Stefano & Keeton 1999). If a moon exists and if we were fortunate in the orientation of the source track,
then a repetition could occur and should be detected within a matter of days. A more delayed repetition could be due to later lensing by the central star or by another planet in the system. If they can detect events with $A_{\text{max}}$ as small as 1.06, the follow-up teams will be at least twice as likely to discover such events as are the monitoring teams.

6. EXPECTATIONS

What results are likely to be derived if the strategies sketched in the previous section are utilized? It is difficult to answer this question, because we know so little about how common planetary systems are and about the distributions of planetary masses and orbital periods. Because, however, planetary systems have begun to be discovered in the Sun’s local neighborhood, it is beginning to seem likely that many, perhaps most, stars support planetary systems.

If this is so, then the preliminary results represented by the known planetary systems provide encouragement that microlensing will play an important role in the discovery of planets. Systems with confirmed planets in orbits that would be perceived by us to be either resonant or wide, were the system to be placed in the Bulge, make up roughly of all the confirmed planetary and binary brown dwarf systems. PSR B1620–26 and the brown dwarf system Gl 229 would produce larger numbers of isolated events than our solar system, but fewer repeating events. PSR 1257 + 12 would also lead to more isolated than repeating events. In addition, 55 Cnc and HD 29587 are excellent candidates for detection. Thus, considering only confirmed systems, we find that both of the pulsars with planets have wide-orbit planets, two of the 11 brown dwarf systems have either a resonant or wide-orbit planet, and two of the nine planetary systems also have either a resonant or wide-orbit planet. We note that since planets in wide orbits, and even those in the zone for resonant lensing, are certain to be under-represented in our present census of planets, inferences based on the systems discovered so far may be conservative. Thus, our present knowledge of planetary systems makes it seem likely that the observing teams will observe some events over the next few years.

More important than the detection of any individual event, however, is our ability to extract information about the population of planetary systems in the regions surveyed by the microlensing teams. We would like to know the answers to basic questions: what fraction of stars have planetary systems? What are typical numbers of planets in a single planetary system? What are the distributions of planet masses and orbital periods?

If the teams begin to discover planets, they will likely be able to establish the statistics of and distributions of properties among planetary systems in the Bulge. Section 6.2 explores this issue in more detail.

6.1. The Relative Numbers of “Resonant” and “Wide” Events

Without knowing more about the distributions of planet properties—which is exactly what we are trying to learn about through the proposed microlensing studies—it is not possible to make definitive predictions for the relative numbers of events due to planets in the zone for resonant lensing and events due to planets in wide orbits. We do know enough, however, to understand the issues that determine the relative event rates.

If we place a Jupiter-mass planet in a resonant orbit, the chance of detecting evidence of the planet’s presence is close to 20%. The present observing setup is optimized to discover this type of event. If we place the same planet in a wide orbit, the probability of detecting an isolated short-duration event due to the planet is smaller, approximately $(3n)\%$ and the probability of detecting a repeating event in which the central star serves as the other lens is $(6n^2/5)\%(\pi a)$. Thus, with the present observing strategy, individual examples of each type of wide-orbit event are much less likely to be observed than a resonant event. Even with the present setup, however, planets in wide orbits may be detected at a rate comparable to or even larger than the rate of detecting planets in resonant orbits, simply because on average there may be on the order of 10 times as many of them as there are planets in resonant orbits.

Two factors can enhance the relative probability of detecting planets in wide orbits. The first is that improved sensitivity to short-duration events and better photometric sensitivity can significantly increase the detection rates for all wide-orbit events, particularly of repeating events. The second is the influence of finite-source–size effects, which can also effectively increase the value of $n$, even for a fixed value of $A_{\text{min}}$.

Table 3 illustrates the situation for our solar system and for a solar system composed of Earth-mass planets. (See also Fig. 7.) The first three lines pertain to a planetary system identical to our own solar system, placed in the Bulge. Since, averaging over angles, Jupiter has a 20%–25% chance of being viewed in the zone for resonant lensing, and since there is a ~20% chance of detecting a Jupiter-mass in the zone for resonant lensing, we have estimated that there is a ~5% chance of detecting evidence of the solar system through a resonant-lensing event. For the detection criteria of set A, the resonant-lensing signature would be the dominant mode of detection. Changing the detection strategy to allow $A_{\text{min}} = 1.06 (n = 2)$ allows wide-orbit events to dominate, with overlap and repeating events having a combined detection rate of 6.3% (7.3% if we eliminate the requirement of a 1 day duration for the first encounter). Thus, overlap and repeating events are competitive with resonant-lensing events, even if we assume that we may catch most of the overlap events but only $1/4$ of the repeating events. Isolated planet-lens events would be found at a rate of 1.4% and 3.1% for detection criteria B and C, respectively. These results indicate that the conjecture that all stars have planetary systems similar to our solar system predicts the detection of resonant, repeating, and short-duration isolated events in roughly equal numbers. If, however, all of the planets in the model solar system were of Earth mass, the detection rate would fall. The fall is much more precipitous for resonant events, which are 5 times less likely than the combined rate of repeating and overlap events and 8 times less likely than isolated short-duration events.

For the power-of-3 model, we have assumed that the probability of finding a planet in the zone for resonant lensing ranges from 50% to 100%. The general pattern of relative rates is similar to that for the solar system. For Jupiter-mass planets, resonant events are competitive and can even be dominant when the detection criteria of set A are used. The detection criteria B and C increase the detection rate of overlap and repeating events. Isolated short-
duration events can also be important and can occur at a rate higher than resonant events. Finite-source–size effects decrease the overall detection rates, making the repeating and overlap events as common as resonant events and increasing the relative importance of isolated planet-lens events.

The general pattern of relative rates illustrated by these examples is likely to be reflected in our data sets. That is, when finite-source–size effects are not important, the wide-orbit discovery channel we have studied is competitive with the resonant-event channel. Repeating and overlap events tend to occur at a lower rate than either resonant or isolated short-duration events, but the rates of the latter two types of events can be comparable. When finite-source–size effects are important and/or when a strategy to optimize the discovery of planets in wide orbits is implemented, discovery of wide-orbit planets may dominate. Isolated short-duration events should be the planet-lens events most frequently detected, and the combined rate of repeating and overlap events may be comparable to or, depending on the influence of finite-source–size, somewhat larger than the rate of resonant-zone lensing events.

6.2. The Populations of Planetary Systems and Low-Mass MACHOs

6.2.1. Planetary Systems: General Considerations

We found in § 4 that we could hope to test simple hypotheses about specific types of planetary systems, even within the next few years. For example, do most stars have planetary systems similar to the solar system? or similar to the power-of-2 or power-of-3 models? Of course it is most likely that planetary systems come in several different varieties. We can use the data to systematically extract information about lenses with a possibly complicated distribution of planetary system properties as follows.

Repeating and resonant events each allow us to determine specific features of the planetary system that served as a lens. Each provides the value of the projected separation between one planet and another object (either the Sun for resonant events [and most repeating events] or a second planet for some repeating events) in the planetary system. Each allows the derivation of a mass ratio. If more information is available, derived from evidence of blending or finite-source–size effects, for example, it can even be possible to place reasonably tight limits on the spectral type of the central star and/or the mass of the planet-lens (Di Stefano & Keeton 1999). If we are able, therefore, to discover and analyze a number of repeating and/or resonant planetary-system lens events, we will, by also including the influence of observational selection effects, be able to derive some of the characteristics of the population of planetary systems in the Galactic Bulge and elsewhere.

Consider a population of planetary systems that act as lenses. Let \( \mathcal{P}_{\text{res}}(m) \) be the probability that a planet of mass \( m \) is in the zone for resonant lensing and \( \mathcal{P}_{\omega}(m, a) \) be the probability that a planet of mass \( m \) is in a wide orbit, with separation \( a \) from the central star. As described above, discovering and measuring the rates of repeating and resonant events constrains the form of these probability functions. Furthermore, there is a consistency check, since the values of \( \mathcal{P}_{\text{res}}(m) \) and \( \mathcal{P}_{\omega}(m, a) \) must be mutually consistent. In particular, there is a close relationship between the rate of resonant events and the rate of overlap events. Among wide-orbit events, there is a similar close relationship between the rate of overlap events and the rate of repeating events. Observations of resonant, overlap, and repeating events can help to constrain the functional form of \( \mathcal{P}_{\omega}(m, a) \). The integral \( \int_{a_{\min}}^{a_{\max}} da \mathcal{P}_{\omega}(m, a) \) then predicts the rate of isolated short-duration events that should be due to lensing by planetary systems. We note, in addition, that the aggressive and systematic study of blending and finite-source–size effects in all isolated events of short duration can also be helpful. Indeed, such a study can provide direct confirmation that some such events, those for which the central star of the planetary system provides a significant fraction of the baseline flux, are due to lensing by planets.

6.2.2. Low-Mass MACHOs

Should the predicted rates of detectable short-duration events be larger than the numbers actually observed, then the analysis of planetary system lenses must be carefully reconsidered. We expect, however, that if there are deviations from the predictions, they should be because the observed rate is higher than the predicted rate. This is because planets in planetary systems are not the only lenses expected to produce short-timescale events. For example, orbital dynamics may lead some planets to be ejected from their home planetary system. If even 10% of massive planets are so ejected, then, considering that the microlensing observations are sensitive to the planet debris of many generations of stars, some of these ejected planets could be detected. In addition, it is certainly possible that a significant fraction of Galactic dark matter exists in the form of low-mass objects. It is an interesting fact that, without the information provided by repeating and resonant events, it will not be possible to determine whether there is a low-mass MACHO component in directions in which lensing by stellar systems contributes significantly to the rate of microlensing. Thus, when stars contribute significantly to the rate of lensing, the detection of repeating and resonant events is important in helping us to learn about any Galactic component of low-mass MACHOs.

In summary, a statistical analysis of the data on different types of planet-lens events can provide useful information. The planetary systems in each galaxy can be characterized by a distribution function of physical planetary separations. Because of the orbital inclination, the distribution function will be mapped to another function. Events may then be observed in resonant, wide, overlap, and isolated configurations. Although these different configurations will be detected by different methods, the fact that they are all drawn from the same distribution means that they all must be mutually consistent. Thus, when a sufficient number of one type of events is measured, one can derive the general distribution function by calculating the detection efficiency and then convolving with the orbital inclination distribu-

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13 The extraction of planet parameters, the degeneracies in those parameters, and the degeneracies between the planet-lens interpretation and other effects have been studied for planets in the zone for resonant lensing by Gaudi & Gould (1997) and Gaudi (1997). Parameter extraction should generally be more straightforward for planet lenses in wide orbits. If the lens separations are very large, then the light curves are much like the standard point-lens light curves; if the separations are smaller, the fit given by Di Stefano & Mao (1996) can be applied. For repeating events, it may be possible to derive some constraints from the conditions that both encounters were due to lensing of one star of fixed radius and that the light contributed by the lens system was essentially the same for both encounters.
tion. Then one can use this “recovered” distribution function to predict the number of repeating (overlap, isolated) events that should have been detected and compare this with the number that were actually detected (convolved with detection efficiency) to check that they are mutually consistent. Subtracting the number of expected isolated events from the observed distribution of timescales recovers the true distribution of MACHOs (objects not associated with stars).

7. SUMMARY

7.1. Repeating Events

Repeating events are a necessary part of the menagerie of planet-lens events. These events form a sort of transition or bridge between resonant and isolated planet-lens events.

Individual repeating events tell us (1) that the lens was very likely to be a planetary system; (2) the projected separation (in units of the Einstein radius of the central star); (3) the mass ratio between the planet and the central star; (4) that if the central star is luminous, we may be able to learn the mass of the central star. The ensemble of repeating events, in combination with resonant events and isolated events of short duration, can allow us to extract information about the population of planetary systems among the lens population.

7.2. Planets in Wide Orbits

In this paper and its companion (Di Stefano & Scalzo 1999) we have introduced the tools needed to systematically use microlensing to search for planets in wide orbits around distant stars. Because these searches can be conducted as part of the ongoing microlensing observations, we have carried out detailed simulations to determine what signatures should be expected if our solar system or other known or model systems serve as lenses. We have also studied how different detection strategies influence detection efficiencies. Our results are encouraging in that they clearly indicate that a systematic search for planets in wide orbits is not only feasible, but that, even over the short term, it can yield interesting results about the population of planetary systems in our own and other galaxies.

Until now, microlensing searches for planets have concentrated on searching for planets that might be located in the zone for resonant lensing. Resonant-lensing events are expected and will play an important role in our quest to learn more about planets through microlensing. Planetary systems, however, seem likely to exhibit enough structure, in the form of multiple planets, moons revolving about planets, and even belts of compact debris, that it is important to explore all the ways microlensing can help us to study them (see also Di Stefano & Scalzo 1997 and Di Stefano & Keeton 1999). We point out that planets in wide orbits \( (a > 1.5R_\odot) \) (1) should exist in larger numbers than planets in the zone for resonant lensing, (2) can also yield distinctive signatures, and (3) may yield larger numbers of detectable events, particularly if finite-source-size effects are important. Thus, previous estimates of the microlensing detection rate (see, e.g., Peale 1997) need to be revised upward.

The key elements of detectability are to pick up evidence of microlensing events as early as possible and to be sensitive to events even if the peak magnification is smaller than 1.34. The detection rate of isolated planet-lens events is proportional to \( n_i \), where \( n_i = w_i/R_{E,i} \), and \( w_i \) represents the distance of closest approach needed for reliable detection of lens \( i \). The rate for repeating events scales as \( \prod_{k=1}^{\infty} n_i \), where \( k \) is the number of repetitions. (Note, however, that there is also a dependence on relative separations.) Just as in the case for the detection of planets in the zone for resonant lensing, frequent monitoring of ongoing events is important, although hourly monitoring may not be necessary.

The features that will improve the rate of planet detection should be considered when designing the next generation of microlensing observations. In the meantime, the MACHO results to date indicate that the teams have the necessary capabilities to detect wide-orbit planets (Di Stefano & Scalzo 1999). We have suggested a set of modifications in the present detection strategy that could, in the short term, significantly improve detection rates for isolated and repeating planet-lens events. One important component is to be alert for the possibility that repeating events may be microlensing events and to continue to carry out frequent follow-up monitoring, even after an apparently isolated event has ceased. It is true that the likelihood of finding a repetition may be on the order of a few percent, but the relative importance of the events makes the study worthwhile. After all, microlensing itself is a low-probability phenomenon, but has been well worth looking for. In fact, repeating events must be present for a number of reasons in addition to the wide-planet connection (Griest & Hu 1992; Di Stefano & Mao 1996). Nevertheless, until recently, the search for them has remained something of a taboo.

An important consequence of the likely presence of planets in a stellar population being studied for signs of microlensing is that isolated short events are very likely to be present at a level that can be as high (compared with single stellar-lens events) as \( \sigma(10\%) \). This means that any signature due to low-mass MACHOs cannot be unambiguously identified unless the contribution due to planets can first be quantified. (There are exceptions when, for example, it is known that the majority of lensing events cannot be due to ordinary stellar systems or when the rate of short-timescale events is so high that the associated optical depth is larger than could possibly be because of planetary systems.) Fortunately, it is possible to detect planet-lens events that exhibit clear signatures of the fact that the lensing is due to a planetary system. These distinctive events are repeating and resonant events and any short-duration events subject to the blending of light from the central star with that of light from the lensed star. Measuring the rates of such events should allow the contribution of planets to short-duration, apparently isolated events to be quantified. This contribution can then be subtracted from the total to derive the magnitude of any contribution due to dark matter existing outside the realm of ordinary stellar systems.

Another point we have emphasized (see, especially, Di Stefano & Keeton 1999 and Di Stefano & Scalzo 1999) is that, even though the planetary systems discovered via microlensing will be far away, some may nevertheless be the subjects of fruitful further study. Indeed, when the central star is luminous, we can hope to determine its spectral type. In some cases this can help to set the mass scale for the system and can therefore help us to determine the mass of the planets that served as lenses. Finite-source-size effects can also put constraints on the lens mass. Thus, although we will not image beach front property on the planets dis-
covered via microlensing, we should not give up on the possibility of learning more about individual planetary systems that serve as microlenses. We have also pointed out (Di Stefano 1999) that it is precisely in those systems in which a planet is discovered via microlensing that it is most likely to have Earth-like conditions and that the central star may be luminous enough to permit further study.

Searches for planets using microlensing should be able to extend the reach of local planetary searches by discovering planets in distant parts of our own and other galaxies and by discovering even low-mass planets orbiting at low speeds. The search for planets in wide orbits represents a significant extension of the ongoing microlensing searches. Indeed, it seems likely that planets in wide orbits will provide an important, and possibly even the dominant, mode for the detection of planetary systems via microlensing, particularly Earth-mass planets.

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