The distributing law on product measuring dimension and its application in reliability analysis

J F Chen¹, Q P Liu², Q P Hu³ and J Y Shen¹

¹Institute of Computer Software, Xi’an Jiaotong University, Xi’an 710049, CHINA
²Kyoto university, Japan
³Hunan science and Technology Office of National Defence, Changsha 410001, CHINA

E-mail: jfchen@mail.xjtu.edu.cn

Abstract. As some measuring errors exist generally, there are mis-acceptance and mis-rejection that come from random measuring error. A new method which lowers mis-acceptance rate and mis-rejection rate of products is presented in this paper. By analyzing the relationship between measuring error and measuring value, the distributing law on measuring value of over-tolerance and in-tolerance products is obtained under the condition of steady process and measurement. The calculating formulae and the data table of mis-acceptance rate and mis-rejection rate are given also. Finally, the reliability of manufacture products is analyzed based on the data table for actual application.

1. Introduction

When a product is manufactured, a quality guarantee inspection shall be conducted to check whether the product is compliance to the technical specifications, that is, we must find a way, a reliable way, to judge that error tolerance of the product shall be met. As the measurement error is inevitable so far, the measurement value we discuss here is actually the composition of product true value and measurement error. The two problems we are facing now are: 1) regarding the product outside error tolerance as qualified in-tolerance one is called mis-acceptance and 2) regarding the product falling within the error tolerance as unqualified over-tolerance one is so called mis-rejection. So how to analyze quantitatively the adverse effect of measurement error on product inspection and how to settle these problems in product design, manufacturing and measurement become imperative and always top of the agenda. In this article, the effects of measurement error on product inspection in one-dimensional normal condition and steady manufacturing process are described.

2. Basic relationship

Let us look at the formula: \( z = x + y \). Where \( x \) means the product true value, \( y \) stands for measurement error, \( z \) represents the measurement value, And, \( x, y \) and \( z \) are random variables, of which \( x \) is independent to \( y \). During the steady manufacturing process and measurement process, \( x \) and \( y \) are subject to truncated normal distribution, and the center point of error tolerance falls upon coincidently the center point of product dimension distribution. The distribution densities of \( x \) and \( y \) are respectively shown below:
\[ \phi_i(x) = \begin{cases} \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{(x-a)^2}{2\sigma_i^2} \right), & |x-a| \leq \mu \\ 0 & |x-a| > \mu \end{cases} \]  
(1)

\[ \phi_j(y) = \begin{cases} \frac{1}{\sqrt{2\pi} \sigma_j} \exp \left( -\frac{y^2}{2\sigma_j^2} \right), & |y| \leq \nu \\ 0 & |y| > \nu \end{cases} \]  
(2)

Where \( a \) stands for nominal product dimension, \( \sigma_i \) is the product dimension distribution precision, and \( \sigma_j \) is the precision of measurement gauge. \( \nu \leq \mu \), \( k_\mu \) and \( k_\nu \) are all constants.

\[ k_\mu = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{(x-a)^2}{2\sigma_i^2} \right) dx; \quad k_\nu = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_j} \exp \left( -\frac{y^2}{2\sigma_j^2} \right) dy \]

let

\[ \phi_i^*(x) = \frac{1}{k_\mu} \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{(x-a)^2}{2\sigma_i^2} \right); \quad \phi_j^*(y) = \frac{1}{k_\nu} \frac{1}{\sqrt{2\pi} \sigma_j} \exp \left( -\frac{y^2}{2\sigma_j^2} \right) \]

then

\[ \phi_i(x) = \begin{cases} \phi_i^*(x), & |x-a| \leq \mu \\ 0 & |x-a| > \mu \end{cases}; \quad \phi_j(y) = \begin{cases} \phi_j^*(y), & |y| \leq \nu \\ 0 & |y| > \nu \end{cases} \]

If \( Q_1 \) and \( Q_2 \) stand for mis-acceptance rate and mis-rejection rate respectively, then the expressions are:

\[ Q_1 = P \left( |x-a| > \frac{T}{2}, |x-a| \leq \frac{T}{2} \right); \quad Q_2 = P \left( |x-a| \leq \frac{T}{2}, |x-a| > \frac{T}{2} \right) \]

In order to calculate the results of \( Q_1 \) and \( Q_2 \), we hereby define:

\( x_1, z_1 \) stands respectively for the true value and measurement value of unqualified product while \( x_2, z_2 \) stands respectively for the true value and measurement value of qualified product.

From formula (1), we know that \( x_1, x_2, z_1 \) and \( z_2 \) are all random variables. \( x_1 \) and \( x_2 \) are independent of variable \( y \), and \( z_1 = x_1 + y \), \( z_2 = x_2 + y \). The distribution densities of \( x_1 \) and \( x_2 \) are expressed as follows:

\[ \phi_i^*(x) = \frac{1}{1-k} \phi_i^*(x); \quad \phi_j^*(x) = \frac{1}{k} \phi_j^*(x) \]

and \( k \) is a constant, \( k = \int_{-\infty}^{T/2} \phi_i^*(x)dx = \int_{-\infty}^{T/2} \phi_j^*(x)dx \)

We call the distribution to which \( x_1 \) is subject “Double Bobtail” normal distribution, utilizing \( z_1, z_2 \), we get:

\[ Q_1 = (1-k)P \left( |z_1 - a| \leq \frac{T}{2} \right); \quad Q_2 = kP \left( |z_2 - a| > \frac{T}{2} \right) \]

As we have known that \( Q_1 \) and \( Q_2 \) have no relationship with \( a \), so for convenience, we assume \( a=0 \), based upon this:

\[ Q_1 = (1-k)P \left( |z_1| \leq \frac{T}{2} \right) \]  
(3)

\[ Q_2 = kP \left( |z_2| > \frac{T}{2} \right) \]  
(4)

3. The distribution law and mis-acceptance rate of over-tolerance products

As \( z_1 = x_1 + y \) and that, \( x_1 \) is independent to \( y \), the measurement distribution density of unqualified over-tolerance products is expressed as:

\[ \phi_i(z) = \int_{-\infty}^{\infty} \phi_i^*(x) \phi_j(z-x)dx + \int_{-\infty}^{\infty} \phi_i^*(x) \phi_j(z-x)dx \]
This is an even function. In accordance with formula (3), we can calculate the mis-acceptance rate as per:

\[ Q_1 = (1 - k) \int_{-\infty}^{\infty} \varphi_{Z_1}(z) \, dz = 2 (1 - k) \int_{0}^{\infty} \varphi_{Z_1}(z) \, dz \]  

After introducing two ratios: \( f_1 = \frac{T}{6\sigma_M} \) and \( f_2 = \frac{\sigma_M}{\sigma_L} \), which respectively represent the process capability index and the ratio of machine tools precision to measurement gauges precision. \( Q_1 \) is the function to \( f_1 \) and \( f_2 \), expressing as \( Q_1(f_1, f_2) \).

Please refer to Table 1 for the results of \( Q_1 \), which are calculated on the formula (5), we find that the fundamental way to lower \( Q_1 \) lies in the promotion of \( f_1 \); furthermore, the promotion of \( f_2 \) by a higher measurement precision is the effective measure to lower \( Q_1 \) and meanwhile control the product quality. When \( f_1 \geq 1.20 \), even though the further promotion of \( f_1 \), we find little response in lowering \( Q_1 \).

| \( f_2 \) | \( f_1 \) | 1.00  | 1.05  | 1.10  | 1.15  | 1.20  | 1.25  | 1.30  | 1.33  |
|---------|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.0     | 0.00106 | 0.00065 | 0.00039 | 0.00023 | 0.00013 | 0.00007 | 0.00004 | 0.00003 | 0.00003 |
| 1.5     | 0.00094 | 0.00057 | 0.00034 | 0.00019 | 0.00011 | 0.00006 | 0.00003 | 0.00002 | 0.00002 |
| 2.0     | 0.00084 | 0.00052 | 0.00031 | 0.00018 | 0.00011 | 0.00006 | 0.00003 | 0.00002 | 0.00002 |
| 2.5     | 0.00076 | 0.00047 | 0.00028 | 0.00017 | 0.00010 | 0.00005 | 0.00003 | 0.00002 | 0.00002 |
| 3.0     | 0.00069 | 0.00043 | 0.00026 | 0.00015 | 0.00009 | 0.00005 | 0.00003 | 0.00002 | 0.00002 |
| 3.5     | 0.00063 | 0.00039 | 0.00024 | 0.00014 | 0.00008 | 0.00005 | 0.00003 | 0.00002 | 0.00002 |
| 4.0     | 0.00058 | 0.00036 | 0.00022 | 0.00013 | 0.00007 | 0.00004 | 0.00003 | 0.00002 | 0.00002 |

| \( f_2 \) | \( f_1 \) | 1.00  | 1.05  | 1.10  | 1.15  | 1.20  | 1.25  | 1.30  | 1.33  |
|---------|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.0     | 0.03226 | 0.02494 | 0.01904 | 0.01437 | 0.01072 | 0.00791 | 0.00576 | 0.00474 | 0.00474 |
| 1.5     | 0.01808 | 0.00771 | 0.00542 | 0.00374 | 0.00254 | 0.00170 | 0.00111 | 0.00086 | 0.00086 |
| 2.0     | 0.00544 | 0.00373 | 0.00251 | 0.00165 | 0.00107 | 0.00068 | 0.00042 | 0.00032 | 0.00032 |
| 2.5     | 0.00341 | 0.00228 | 0.00150 | 0.00097 | 0.00061 | 0.00038 | 0.00023 | 0.00017 | 0.00017 |
| 3.0     | 0.00242 | 0.00160 | 0.00104 | 0.00066 | 0.00041 | 0.00025 | 0.00015 | 0.00011 | 0.00011 |
| 3.5     | 0.00185 | 0.00121 | 0.00078 | 0.00049 | 0.00030 | 0.00018 | 0.00011 | 0.00008 | 0.00008 |
| 4.0     | 0.00149 | 0.00097 | 0.00062 | 0.00039 | 0.00024 | 0.00014 | 0.00008 | 0.00006 | 0.00006 |

4. The distribution law and mis-rejection rate of in-tolerance products
We have known that \( z_2 = x^2 + y \) and \( x_2 \) is independent of \( y \), so the distribution density of qualified product measurement value \( z_2 \) is expressed as:

\[ \varphi_{Z_2}(z) = \int_{-\infty}^{\infty} \varphi_{x_2}(x) \varphi_{y}(z - x) \, dx \]

This is an even function. From formula (4), we can calculate the mis-rejection rate through the following equation:

\[ Q_2 = kP\left[z_2 > \frac{T}{2}\right] = 2kP\left[z_2 > \frac{T}{2}\right] = 2k \int_{\frac{T}{2}}^{\infty} \varphi_{Z_2}(z) \, dz \]  

Here we still use the ratios \( f_1 \) and \( f_2 \), and then expressed accordingly \( Q_2(f_1, f_2) \). Based upon formula (6), we reflect the results of \( Q_2 \) in Table 1, and find that the variable zone (1.0, 3.0) of \( f_2 \) is the optimal zone for the reduction of \( Q_2 \). When \( f_2 \) moves from 1.0 to 3.0, we witness a dramatic decrease of \( Q_2 \). Therefore, we summarize that the adoption of higher precision machine tool will lower \( Q_2 \). Meanwhile, proper promotion of process capability index \( f_1 \) (not exceeding 1.20) is another effective way to lower \( Q_2 \).

5. Application
Now let us integrate the misjudgement probability function table with the reliability study in production preparation.
Party A wants to order an important component of space vehicles from Party B, the component, expensive and of high profit, shall meet some, for example of \( n \) in number, key physical parameters which are the bases for product reliability. The error tolerance are expressed as \( T_i \) \((i=1,2,...,n)\) respectively. Party A requires that non-reliability of the component from Party B shall be \( 1-P<10^{-6} \). Party B, from the view point of economic efficiency, will only prepare the raw material for one piece. Providing that Party B prepares \( n \) machine tools \( M_i \) for \( n \) physical parameters, the precisions of machine tools are \( \sigma_{Mi} \) and Party B also prepares \( n \) measurement gauges \( I_i \) with precision \( \sigma_{Li} \) for product measurement. Now the question is that whether Party B is capable of undertaking such a task?

As the error tolerance of the component, the manufacturing precision and measurement precision are all available now, we can directly find out mis-acceptance rate \( Q_{bi} \) in the misjudgment function table. The mis-acceptance rate \( Q_{bi} \) is also known as reliability risk rate, which means mis-acceptance and mis-rejection. Using \( z \) distribution table we can get probability \( S_i \) within error tolerance, which is also called reliability of reliability risk.

As \( P_i = (S_i - Q_{bi}) \), the product reliability is calculated as the following:

\[
P = \prod_{i=1}^{n} P_i = \prod_{i=1}^{n} (S_i - Q_{bi})
\]

In case that \( 1-P<10^{-6} \), Party B may accept the order from Party A and schedule his manufacturing process, otherwise Party B may rearrange its machine tools with higher precision and improve its manufacturing and measurement ways or methods, or seek help from other manufactures, the last option is to give up the order.

6. Conclusions

This paper puts forward a theory and method to lower mis-acceptance rate and mis-rejection rate. The first step is the analysis on distributive law of qualified and unqualified products and upon which a mathematical model on misjudgement probability function is established and then is the misjudgement probability function table. We can utilize the misjudgement probability function table, an easy and practical way and of bright future, to realize the effective control on misjudgement, to reasonably arrange machine tools and measurement gauges and to conduct quality analysis on products.

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