Oblique propagation of longitudinal waves in magnetized spin-1/2 plasmas: Independent evolution of spin-up and spin-down electrons

Pavel A. Andreev and L. S. Kuz’menkov

Faculty of physics, Lomonosov Moscow State University, Moscow, Russian Federation.
(Dated: June 25, 2014)

We consider quantum plasmas of electrons and motionless ions. We describe separate evolution of spin-up and spin-down electrons. We present corresponding set of quantum hydrodynamic equations. We assume that plasmas are placed in an uniform external magnetic field. We account different occupation of spin-up and spin-down quantum states in equilibrium degenerate plasmas. The effect is included via equations of state for pressure of each species of electrons. We study oblique propagation of longitudinal waves. We show that instead of two well-known waves (the Langmuir wave and the Trivelpiece–Gould wave), plasmas reveal four wave solutions. New solutions exist due to both the separate consideration of spin-up and spin-down electrons and different occupation of spin-up and spin-down quantum states in equilibrium state of degenerate plasmas.

PACS numbers: 52.30.Ex, 52.35.Dm
Keywords: quantum plasmas, quantum hydrodynamics, spin evolution, longitudinal wave dispersion

I. INTRODUCTION

Spin evolution in quantum plasmas [1], [2], [3], [4], [5], [6] leads to existence of new waves [7], [8], [9], [10], [11]. Difference of population of spin-up and spin-down quantum states in equilibrium plasmas, which can be accounted by corresponding equation of state, also gives contribution in dispersion of plasma waves [12], [13], [14]. Moreover, consideration of separate evolution of spin-up and spin-down electrons reveal existence of new longitudinal wave [12]. The new longitudinal wave was obtained at consideration of wave propagation parallel and perpendicular to an external magnetic field. Its existence is related to different population of spin-up and spin-down quantum states in equilibrium plasmas.

If we have equal population of spin-up and spin-down quantum states and consider ions as motionless background we have one longitudinal wave. It is the Langmuir wave. Presence of an external magnetic field reveals in anisotropy of the Langmuir wave dispersion. Moreover, if we consider propagation of longitudinal waves parallel or perpendicular to the external magnetic field we have the Langmuir wave only. However, the second wave exists at oblique propagation. It is well-known Trivelpiece–Gould wave. Appearance of the second wave solution at oblique propagation of the longitudinal waves in plasmas encourages us to check existence of new oblique propagating waves in plasmas at different population of spin-up and spin-down quantum states.

In this paper we present further application of separated spin evolution QHD (SSE-QHD). We consider spin-up electrons and spin-down electrons as two different species. Corresponding QHD equations were directly derived from the Pauli equation [12]. Let us mention a couple of papers [15], [16], where attempts to suggest separated spin evolution QHD were made.

Some results on spinless quantum plasmas and quantum plasmas of spin-1/2 particles were reviewed in Refs. [17], [18]. We would like to mention several results obtained in the field of spin-1/2 quantum plasmas in past years. Contribution of the Coulomb exchange and spin–spin exchange interactions in spectrums of magnetized spin-1/2 quantum plasmas was described in Ref. [3]. Propagation of neutron beam via magnetized spin-1/2 quantum plasmas and generation of waves by neutron beams were also considered in Ref. [1]. Explicit account of the spin-current interaction by means of the many-particle quantum hydrodynamic (MPQHD) method in spin-1/2 quantum plasmas was performed in Ref. [6]. The spin-current interaction is the interaction between magnetic moments related to spins and electric current of moving charges in plasmas occurring by means the magnetic field. The spin-orbit interaction and its influence on spectrums of plasma waves, spin-plasma waves and processes of neutron beam–magnetized spin-1/2 quantum plasmas interaction [11]. Consistent consideration of the quantum Bohm potential in system of spinning particles in terms of the MPQHD was done in Ref. [20]. Development of general problems in modeling of collective behavior of spinning particle quantum plasmas has been performed. For instance, the gauge-free Hamiltonian structure of an extended kinetic theory, for which the intrinsic spin of the particles is taken into account was developed in Ref. [21]. Model the neutron fluid as a spin quantum plasma where the electromagnetic interaction is trough the magnetic moment of the neutron is presented in Ref. [22], as an excellent application of QHD to systems of neutral particles with spin. An extended vorticity evolution equation for the quantum spinning plasma was considered in Refs. [22] and [23]. The influence of the intrinsic spin of electrons on whistler mode was also investigated in Ref. [23]. Effects of the spin and the Bohm potential in the oblique propagation of magne-
tosonic waves were considered in Ref. 24. From QHD description with intrinsic magnetization, a new plasma instability was obtained in Ref. 22. It was shown that the instability develops in a nonuniform plasma when the electron concentration and temperature vary along an externally applied magnetic field. Authors obtained that Alfven waves play an important role in the instability. Linear and nonlinear relations for slow and fast magneto-sonic modes were derived in Refs. 24 and 27, where spin effects are incorporated via spin force and macroscopic spin magnetization current. Their solution shows a general shock wave profile superposed by a perturba-
tive solitary-wave contribution 24. Magneto-sonic waves were studied in magnetized degenerate electron-positron- ion plasmas with spin effects 28. It was demonstrated that the effect of quantum corrections in the presence of positron concentration significantly modifies the disper-
spersive properties of these modes. The magneto-sonic waves and their interactions in spin-1/2 degenerate quantum plasmas were investigated in Ref. 29. Electron spin -1/2 effects on the parametric decay instability of oblique Langmuir wave into low-frequency electromagnetic shear Alfven wave and left-handed circularly polarized wave was considered in Ref. 30. The effect of spin induced magnetization on Jeans instability of quantum plasmas was studied in Ref. 31. Effects of electron spin on the kinetic Alfven waves in the presence of uniform static magnetic field were studied in Ref. 32. It was demonstrated that the kinetic Alfven wave frequency decreases due to the electron spin contribution in the kinetic limit while in the inertial limit they are almost unaffected in a hot magnetized plasma.

This paper is organized as follows. In Sec. II model of separated spin evolution QHD is presented and described. In Sec. III dispersion of longitudinal waves in quantum plasmas with different population of spin-up and spin-down quantum states. We show existence of four wave solutions instead of two well-known solutions. In Sec. IV brief summary of obtained results is presented.

II. MODEL

We should start derivation of the SSE-QHD equations from many-particle Pauli equation with explicit account of interparticle interactions 22, 23, 24, 26, 33, 34. However essential part can be found from the single particle Pauli equation 13, 35, 36, 37.

The Pauli equation

\[ i\hbar \partial_t \psi = \left( \frac{\hat{p} - q_e A}{2m} + q_e \varphi - \gamma_e \hat{\sigma} B \right) \psi \]  

(1)

governs evolution of spinor wave function \( \psi(r, t) \), where \( \varphi = \varphi_{\text{ext}} \), \( A = A_{\text{ext}} \) are the scalar and vector potentials of external electromagnetic fields, \( B = B_{\text{ext}} \) is the external magnetic field, \( q_e = -e \) is the charge of electron, \( m \) is the mass of the particle under consideration, \( \gamma_e \) is the gyromagnetic ratio, \( \hat{p} = -i\hbar \nabla \) is the momentum operator, \( \nabla \) is the gradient operator, \( \hat{\sigma} \) is the vector of Pauli matrixes, \( \hbar \) is the reduced Planck constant, \( c \) is the speed of light, \( \hat{\sigma} \) is the vector constructed of the Pauli matrixes

\[ \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]  

(2)



The spinor wave function \( \psi \) can be presented as

\[ \psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}. \]  

(3)

Applying wave functions describing spin-up \( \psi_\uparrow \) and spin-down \( \psi_\downarrow \) states we can write probability density to find the particle in a point \( r \) with spin-up \( \rho_\uparrow = |\psi_\uparrow|^2 \) or spin-down \( \rho_\downarrow = |\psi_\downarrow|^2 \). We also see \( \rho = \rho_\uparrow + \rho_\downarrow \). Directions \( \uparrow \) (down \( \downarrow \)) corresponds to spins having same (opposite) direction as (to) the external magnetic field. While magnetic moments have opposite to spin directions.

The spin density \( S_z \) of electrons is the difference between concentrations of electrons with different projection of spin \( S_z = \psi^+ \sigma_z \psi - \rho_\uparrow - \rho_\downarrow \). We have that the \( z \)-projection of the spin density \( S_z \) is not an independent variable in this representation of the quantum hydrodynamics.

In many-particle systems we have concentration of particles \( n_i(r, t) \), which are proportional to the probability density to find each particle in the point \( r \), hence we have \( n_\uparrow = \langle \rho_\uparrow \rangle, n_\downarrow = \langle \rho_\downarrow \rangle \), and \( n = n_\uparrow + n_\downarrow \).

Applying the explicit form of the Pauli matrixes we can rewrite the Pauli equation (1) in more explicit form, in terms of \( \psi_\uparrow \) and \( \psi_\downarrow \) (see equations (4) and (5) in Ref. 12). These equations allow to derive equations for \( n_\uparrow, v_\uparrow, n_\downarrow, v_\downarrow \). These equations were obtained in Ref. 12. Here we present and apply these equations.

The continuity equations appear for each species of
electrons
\[ \partial_t n_\uparrow + \nabla (n_\uparrow \mathbf{v}_\uparrow) = \frac{\gamma}{\hbar} (B_y S_x - B_x S_y) \]  
(4)
for spin-up electrons, and
\[ \partial_t n_\downarrow + \nabla (n_\downarrow \mathbf{v}_\downarrow) = \frac{\gamma}{\hbar} (B_x S_y - B_y S_x) \]  
(5)
for spin-down electrons.

In the continuity equations we have the following physical quantities: \( n_\uparrow \) (\( n_\downarrow \)) is the concentration of electrons baring spin-up (spin-down), \( \mathbf{v}_\uparrow \) (\( \mathbf{v}_\downarrow \)) is the velocity field of electrons baring spin-up (spin-down), \( S_x \) and \( S_y \) are projections of the spin density vector.

The right-hand side of the continuity equations exist due to the spin-spin interaction between electrons. Numbers of spin-up and spin-down electrons do not conserve due to the spin-spin interaction. Total number of electrons conserves only.

We also have the couple of vector Euler equations. These equations describe evolution of the momentum density in each species of electrons.

\[ m n_\uparrow (\partial_t + \mathbf{v}_\uparrow \nabla) \mathbf{v}_\uparrow + \nabla p_\uparrow - \frac{\hbar^2}{4m} n_\uparrow \nabla \left( \frac{\Delta n_\uparrow}{n_\uparrow} - \frac{(\nabla n_\uparrow)^2}{2n_\uparrow^2} \right) = q_e n_\uparrow \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}_\uparrow, \mathbf{B}] \right) + \frac{\gamma e}{m} n_\uparrow \nabla B_z \]
\[ + \frac{\gamma e}{2m} (S_x \nabla B_x + S_y \nabla B_y) + \frac{\gamma e}{\hbar} (\mathbf{J}_{(M)x} B_y - \mathbf{J}_{(M)y} B_x), \]  
(6)
and
\[ m n_\downarrow (\partial_t + \mathbf{v}_\downarrow \nabla) \mathbf{v}_\downarrow + \nabla p_\downarrow - \frac{\hbar^2}{4m} n_\downarrow \nabla \left( \frac{\Delta n_\downarrow}{n_\downarrow} - \frac{(\nabla n_\downarrow)^2}{2n_\downarrow^2} \right) = q_e n_\downarrow \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}_\downarrow, \mathbf{B}] \right) - \frac{\gamma e}{m} n_\downarrow \nabla B_z \]
\[ + \frac{\gamma e}{2m} (S_x \nabla B_x + S_y \nabla B_y) + \frac{\gamma e}{\hbar} (\mathbf{J}_{(M)y} B_x - \mathbf{J}_{(M)x} B_y), \]  
(7)
with
\[ \mathbf{J}_{(M)x} = \frac{1}{2} (\mathbf{v}_\uparrow + \mathbf{v}_\downarrow) S_x - \frac{\hbar}{4m} \left( \frac{\nabla n_\uparrow}{n_\uparrow} + \frac{\nabla n_\downarrow}{n_\downarrow} \right) S_y, \]
(8)
and
\[ \mathbf{J}_{(M)y} = \frac{1}{2} (\mathbf{v}_\uparrow + \mathbf{v}_\downarrow) S_y + \frac{\hbar}{4m} \left( \frac{\nabla n_\uparrow}{n_\uparrow} + \frac{\nabla n_\downarrow}{n_\downarrow} \right) S_x, \]
(9)
where \( q_e = -e \), \( \gamma_e = -\frac{\hbar}{2mc} \) is the gyromagnetic ratio for electrons, and \( g = 1 + \alpha/(2\pi) = 1.00116 \), where \( \alpha = 1/137 \) is the fine structure constant, gets into account the anomalous magnetic moment of electron. \( \mathbf{J}_{(M)x} \) and \( \mathbf{J}_{(M)y} \) are elements of the spin current tensor \( J^{\alpha\beta} \).

The last group of terms in the Euler equations exist due to nonconservation of numbers of spin-up and spin-down electrons. Hence these terms are related to the spin-spin interaction.

Equations (13) - (19) contain \( S_x \) and \( S_y \). Equations of evolution of \( S_x \) and \( S_y \) were derived in Ref. [2]. We also present them here to have closed set of the SSE-QHD. \( S_x = \psi^* \sigma_x \psi = \psi^* \psi \uparrow + \psi \uparrow \psi \downarrow, S_y = \psi^* \sigma_y \psi = i(\psi^* \psi \downarrow - \psi \uparrow \psi^* \downarrow). \)

\[ \partial_t S_x + \frac{1}{2} \nabla [S_x (\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)] \]
\[ = \frac{\hbar}{4m} \nabla \left( S_y \left( \frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow} \right) \right) = \frac{2\gamma e}{\hbar} \left( B_x S_y - B_y (n_\uparrow - n_\downarrow) \right), \]  
(10)
and
\[ \partial_t S_y + \frac{1}{2} \nabla [S_y (\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)] \]
\[ + \frac{\hbar}{4m} \nabla \left( S_x \left( \frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow} \right) \right) = \frac{2\gamma e}{\hbar} \left( B_x (n_\uparrow - n_\downarrow) - B_y S_x \right). \]  
(11)

In this paper we are focused on the longitudinal waves. Hence we present quasi-electrostatic set of the Maxwell equations
\[ \nabla E = 4\pi \left( e n_i - e n_{e\uparrow} - e n_{e\downarrow} \right), \]  
(12)
and
\[ \nabla \times \mathbf{E} = 0. \]  
(13)
To get closed set of QHD equations we apply the following equation of state for each species of electrons

\[ p_s = \left( \frac{6\pi^2}{5} \right)^{2/3} \frac{n_s}{m^{5/3}}, \]

where \( s = \uparrow \) or \( \downarrow \).

We show below that difference between \( p_\uparrow \) and \( p_\downarrow \) due to difference of \( n_\uparrow \) and \( n_\downarrow \) leads to new effects in quantum plasmas.

A longitudinal wave propagating parallel to the external magnetic field was discovered in Ref. [12]. A longitudinal wave propagating perpendicular to the external magnetic field was also obtained in Ref. [12]. Their dispersion dependencies differ from each other by a constant

\[ \omega_\perp^2(k) = \omega_\parallel^2(k) + \Omega^2. \]

In paper [12] these solutions were interpreted as part of one dispersion surface \( \omega(k, \theta) \) existing at \( \theta = 0 \) and \( \theta = \pi/2 \). From figures (5) and (6) we see that these two solutions are related to different dispersion surfaces. Figure (5) (figure (6)) shows dispersion surface of wave existing at \( \theta = 0 \) (\( \theta = \pi/2 \)).

### III. DISPERSION OF LONGITUDINAL WAVES

Equilibrium condition is described by the non-zero concentrations \( n_{0\uparrow}, n_{0\downarrow}, n_0 = n_{0\uparrow} + n_{0\downarrow} \), and external magnetic field \( B_{\text{ext}} = B_0 \hat{e}_z \). Other quantities equal to zero \( v_0 = v_{0\downarrow} = 0, E_0 = 0, S_{0x} = S_{0y} = 0 \).

Difference of spin-up and spin-down concentrations of electrons \( \Delta n = n_{0\uparrow} - n_{0\downarrow} \) is caused by external magnetic field. Since electrons are negative their spins get preferable direction opposite to the external magnetic field \( \Delta n \approx \tanh \left( \frac{\gamma_e B_0}{\varepsilon_F} \right) = -\tanh \left( \frac{\gamma_e |B_0|}{\varepsilon_F} \right) \), where \( \varepsilon_F = \frac{3\pi^2}{2m} n_0^2 \bar{\hbar}^2 \) is the Fermi energy.

Assuming that perturbations \( \delta n_\uparrow, \delta n_\downarrow, \delta v_\uparrow, \delta v_\downarrow, \delta E, \delta B, \delta S_x, \delta S_y \) are monochromatic

\[ \delta f = F_A e^{-\omega t + ikr}, \]

FIG. 3: (Color online) The figure shows anisotropic dispersion dependence of the Langmuir wave.

FIG. 4: (Color online) Trivelpiece–Gould wave dispersion is presented on the figure.

FIG. 5: (Color online) This figure shows dispersion surface of the longitudinal wave existing at \( \theta = 0 \).

FIG. 6: (Color online) This figure shows dispersion surface of the longitudinal wave existing at \( \theta = \pi/2 \).
where $\delta f$ presents perturbations of physical quantities, and $F$ is corresponding amplitude. We get a set of linear algebraic equations relatively to amplitudes of perturbations. Condition of existence of nonzero solutions for amplitudes of perturbations gives us a dispersion equation.

We assume that $k = \{k_x, 0, k_z\}$ and $k_x = k \sin \theta$, $k_z = k \cos \theta$, where $k = \sqrt{k_x^2 + k_z^2}$, and $\theta$ is the angle between direction of wave propagation and direction of the external magnetic field.

For longitudinal waves we have that perturbations of magnetic field equal to zero $\delta B = 0$.

$$U_s^2 = \frac{(6\pi^2)^3}{3} \frac{\hbar^2}{m^2 \pi^2} \frac{1}{n_{0s}} \frac{2}{3} v_{Fe(s)}^2,$$  \hspace{1cm} (16)

with $s = \uparrow$ or $\downarrow$.

Equations (10) and (11) describe precession of spins around the external magnetic field. Frequency of precession is $\omega_{pr} = \frac{2\pi}{\gamma} B_0$ It does not affect matter waves described by the continuity and Euler equations.

The longitudinal waves are described by the continuity (4), (5) and Euler (6), (7) equation of material fields and equations of the electric field (12), (13). These equations lead to the following dispersion equation

$$1 - \left( \frac{\sin^2 \theta}{\omega^2 - \Omega^2} + \frac{\cos^2 \theta}{\omega^2} \right) \times$$

$$\left[ \frac{\omega_{Le\uparrow}^2}{1 - \left( \frac{\sin^2 \theta}{\omega^2 - \Omega^2} + \frac{\cos^2 \theta}{\omega^2} \right)(U_{\uparrow}^2 + \frac{\hbar^2 k_x^2}{4m^2})k^2} \right]$$

$$+ \frac{\omega_{Le\downarrow}^2}{1 - \left( \frac{\sin^2 \theta}{\omega^2 - \Omega^2} + \frac{\cos^2 \theta}{\omega^2} \right)(U_{\downarrow}^2 + \frac{\hbar^2 k_x^2}{4m^2})k^2} = 0,$$ \hspace{1cm} (17)

where $\omega_{Le\uparrow} = \frac{4\pi^3 n_{0s}}{m}$, and $\omega_{Le\downarrow} = \frac{4\pi^3 n_{0s}}{m}$ are the Langmuir frequencies for spin-up and spin-down electrons. $\omega_{Le\downarrow}$ and $\omega_{Le\uparrow}$ are partial Langmuir frequencies. Their sum $\omega_{Le}^2 = \omega_{Le\uparrow}^2 + \omega_{Le\downarrow}^2$ gives full Langmuir frequency of the system.

Dispersion equation (17) is an equation of fourth degree on the frequency square $\omega^2$. Hence we can expect existence of four waves, whereas there is two well-known longitudinal waves in magnetized three dimensional electron gas. They are the Langmuir and the Trivelpiece–Gould wave.

At $\theta = 0$ equation (17) simplifies to

$$1 - \frac{\omega_{Le\uparrow}^2}{\omega^2 - (U_{\uparrow}^2 + \frac{\hbar^2 k_x^2}{4m^2})k^2} - \frac{\omega_{Le\downarrow}^2}{\omega^2 - (U_{\downarrow}^2 + \frac{\hbar^2 k_x^2}{4m^2})k^2} = 0.$$ \hspace{1cm} (18)

If occupations of states equal to each other then we have $n_{0\uparrow} = n_{0\downarrow}$. As consequence we get $U_{\uparrow}^2 = U_{\downarrow}^2 = \frac{3}{4} v_{Fe}^2$, $v_{Fe} = (3\pi^3 n_0)^{\frac{1}{3}} h/m$, and $\omega_{Le\uparrow} = \omega_{Le\downarrow} = \frac{\omega_{Le}}{2}$. In this limit new solutions do not appear.

Equation (18) has two solutions. One of them is the Langmuir wave. The second branch was discovered in Ref. [12]. Analytical analysis of spectrum of new wave was presented in Ref. [12]. Difference in occupation of spin-up and spin-down quantum states by electrons gives a contribution in the dispersion of Langmuir waves. Analytical expressions for this contribution is obtained in Ref. [12]. In this paper we present numerical analysis of this effect at oblique propagation of the Langmuir wave.

A. Numerical analysis

In this subsection we present numerical analysis of dispersion equation (17). We assume that the equilibrium particle concentration has the following value $n_0 = 10^{22}$ cm$^{-3}$. We also assume that the Langmuir frequency and the cyclotron frequency are related as $\omega_{Le}^2 = 10\Omega^2$. Consequently we have the following magnitude of the external magnetic field $B_0 = 3\sqrt{4\pi} \times 10^7$ G.

During numerical analysis of the dispersion equation we apply dimensionless wave vector module $\kappa = \frac{k U_{Le}}{\hbar} = \frac{(3\pi^3)^{\frac{1}{3}} n_0^{\frac{1}{3}} h/eB_0}{\sqrt{3}} k$ and dimensionless frequency square $\xi = \frac{\omega^2}{\Omega^2}$.

This particle concentration corresponds to electrons in metals. The quantum Bohm potential is essential at much higher densities existing in astrophysical objects. Hence, at numerical analysis we do not include contribution of the quantum Bohm potential.

Figs. 3–6 show anisotropic dispersion dependencies of all four longitudinal waves appearing from equation (17).

Fig. 7 allows to compare behavior of two new branches of dispersion dependencies. From Figs. 6–7
we see that frequencies of both new branches increase with the increasing of the wave vector module.

Lower branch (see Figs. (5), (7)) exists at the parallel propagation of waves, but it does not exist at the perpendicular propagation. Frequency of this wave monotonically decreases to $\omega = 0$ with increasing of angle $\theta$ from $0$ to $\pi/2$. At $\theta = \pi/2$ structure of equation (17) changes. It simplifies to an equation having two solutions only, instead of four solutions of equation (17).

The second (upper) new branch has minimal frequency $\omega_{\text{min}} = \Omega$, which is the electron cyclotron frequency (see Figs. (6) and (7)).

The upper branch shows different behavior than the lower branch. Its frequency also increases with increasing of the wave vector module. However, it reaches maximal value at $\theta = \pi/2$. Upper branch frequency decreases with decreasing of angle $\theta$. The upper branch disappears at $\theta = 0$. At $\theta = 0$ structure of general dispersion equation (17) changes and we obtain equation (18), which has two solutions only. The upper branch has no trace in equation (18).

In paper [12] it was shown that separated spin evolution and different Fermi pressure for two species of electrons lead to extra term in dispersion dependence of the Langmuir wave propagating parallel and perpendicular to an external magnetic field.

In this paper we study oblique propagation of the Langmuir wave. We also consider properties of the Trivelpiece–Gould wave existing at oblique propagation. We are interested in consideration of described effects in dispersion of these waves. Figs. (8) and (9) show contribution of these effects in dispersion surfaces of the Langmuir wave and the Trivelpiece–Gould wave. The lower surface on Fig. (8) presents usual dispersion dependence of the Langmuir wave $\omega_{\text{0}(\perp)} = \sqrt{\frac{1}{2} \left( \omega_{\text{Le}}^2 + \Omega^2 + \frac{1}{3} v_{\text{Fe}}^2 k^2 \right)} - \frac{\Omega^2}{\omega_{\text{Le}}^2} \cos^2 \theta$ , where $\omega_{\text{0}(\parallel)}$ is the frequency of the Langmuir wave propagating parallel to the external field $\theta = 0$, and $\omega_{\text{0}(\perp)}$ is the frequency of the oblique propagating Langmuir wave. The upper surface describes dispersion of Langmuir wave obtained in this paper, which is also presented on Fig. (3). Upper surface gives dispersion of the Langmuir wave at separated spin evolution and different Fermi pressure for two species of electrons.

The effects under discussion give a small contribution in dispersion of the Trivelpiece–Gould wave revealing in increasing of the frequency. The shift of dispersion surface $\Delta \xi = (\omega_{\text{new}}^2 - \omega_{\text{old}}^2)/\Omega^2$ is depicted on Fig. (9). We see that maximal shift appears at $\theta = \pi/4$. This shift increases with increasing of the wave vector module. The shift disappears at $\theta \to 0$ and $\theta \to \pi/2$.

In this section we have numerically described behavior of the four longitudinal waves existing in magnetized degenerate spin-1/2 plasmas. Some analytical results for limit cases of waves propagating parallel and perpendicular to the external magnetic field can be found in Ref. [12].
IV. CONCLUSIONS

We have presented the QHD model of spin-1/2 quantum plasmas, where spin-up and spin-down electrons are considered as two different species. This model contains the continuity and Euler equations for each species. Structure of these equations differs from structure of similar equations in QHD with electrons considered as a single species. Particularly we should mention that extra non-linear terms appear in the SSE-QHD equations related to un-conservation of numbers of spin-up and spin-down electrons.

The SSE-QHD also contains equations for evolution of the spin density projections $S_x$ and $S_y$ on directions perpendicular to the external magnetic field. Projection of the spin density on the direction of the external magnetic field $S_z$ is not an independent variable. It appears as difference of concentrations of spin-up and spin-down electrons $S_z = n_\uparrow - n_\downarrow$.

All projections of the spin density $S$ are simultaneously related to both species of electrons. The concentrations $n$ and velocity fields $v$ wear subindexes $\uparrow$ (for spin-up) and $\downarrow$ (for spin-down), but the spin density does not wear them.

The SSE-QHD model arises as a rigorous consequence of the Pauli equation.

Being placed in an external magnetic field a system of degenerate electrons (ions are considered to be motionless, they create positively charged background) has different distributions of spin-up and spin-down electrons. Consequently the Fermi pressure is different for each species.

Account of this effect in the SSE-QHD reveals in existence of two new longitudinal waves in magnetized plasmas.

At consideration of limit cases of wave propagation parallel and perpendicular to the external field we have only one new longitudinal solution existing along with the Langmuir wave. One of two new waves reveals at parallel propagation, and another one exists at perpendicular propagation. Considering oblique propagation we have both new waves existing together with the Langmuir and the Trivelpiece–Gould waves.

We have described described dispersion properties of new waves at oblique propagation. We have studied changes of dispersion of the Langmuir and the Trivelpiece–Gould waves appearing due to different distributions of degenerate spin-up and spin-down electrons in the external magnetic field.

Separated spin evolution QHD shows itself as an useful tool for research of quantum plasmas in magnetic fields. It allows to discover new phenomenon in linear regime of small amplitude perturbations. It also opens possibilities for discovering of new non-linear phenomenon.

[1] L. S. Kuz’menkov, S. G. Maksimov, and V. V. Fedoseev, Vestn. Mosk. Univ., Ser. 3: Fiz., Astron., No. 5, 3 (2000) [Moscow Univ. Phys. Bull., No. 5, 1 (2000)].
[2] L. S. Kuz’menkov, S. G. Maksimov, and V. V. Fedoseev, Russian Phys. Jour. 43, 718 (2000).
[3] L. S. Kuz’menkov, S. G. Maksimov, and V. V. Fedoseev, Theor. Math. Fiz. 126 136 (2001) [Theoretical and Mathematical Physics, 126 110 (2001)].
[4] M. Marklund and G. Brodin, Phys. Rev. Lett. 98, 025001 (2007).
[5] G. Brodin and M. Marklund, New J. Phys. 9, 277 (2007).
[6] P. A. Andreev and L. S. Kuz’menkov, Russian Phys. Jour. 50, 1251 (2007).
[7] D. V. Vagin, N. E. Kim, P. A. Polyakov, A. E. Rusakov, Izvestiya RAN (Proceedings of Russian Academy of science) 70, 443 (2006).
[8] P. A. Andreev, L.S. Kuz’menkov, Moscow University Physics Bulletin 62, N.5, 271 (2007).
[9] P. A. Andreev, L. S. Kuz’menkov, Physics of Atomic Nuclei 71, N.10, 1724 (2008).
[10] A. P. Misra, G. Brodin, M. Marklund and P. K. Shukla, J. Plasma Physics 76, 857 (2008).
[11] P. A. Andreev, L. S. Kuz’menkov, Int. J. Mod. Phys. B 26 1250186 (2012).
[12] P. A. Andreev, arXiv:1405.0719.
[13] P. A. Andreev, arXiv:1403.6075.
[14] Mariya Iv. Trukhanova and Pavel A. Andreev, arXiv:1405.6294.
[15] Mubashar Iqbal, J. Plasma Physics, 79, 19 (2013).
[16] M. Shahid, D. B. Melrose, M. Jamil, and G. Murtaza, Phys. Plasmas, 19, 112114 (2012).
[17] P. K. Shukla, B. Eliasson, Phys. Usp. 53, 51 (2010).
[18] P. K. Shukla, B. Eliasson, Rev. Mod. Phys. 83, 885 (2011).
[19] D. A. Uzdensky and S. Rightley, Reports on Progress in Physics, 77, Issue 3, 036902 (2014).
[20] P. A. Andreev, F. A. Asenjo, and S. M. Mahajan, arXiv: 1304.5780.
[21] M. Marklund, P. J. Morrison, Physics Letters A 375, 2362 (2011).
[22] S. M. Mahajan, F. A. Asenjo, Phys. Lett. A 377, 1430 (2013).
[23] M. I. Trukhanova, Prog. Theor. Exp. Phys., 111101 (2013).
[24] F. A. Asenjo, Phys. Lett. A 376, 2496 (2012).
[25] Vitaly Bychkov, Mikhail Modestov, and Mattias Marklund, Phys. Plasmas 17, 112107 (2010).
[26] A. Mushtaq, and S. V. Vladimirov, Phys. Plasmas 17, 102310 (2010).
[27] A. Mushtaq, and S. V. Vladimirov, Eur. Phys. J. D 64, 419 (2011).
[28] A. Mushtaq, R. Maroof, Zulfiaqr Ahmad, and A. Qamar, Phys. Plasmas 19, 052101 (2012).
[29] Sheng-Chang Li and Jiu-Ning Han, Phys. Plasmas 21, 032105 (2014).
[30] M. Shahid and G. Murtaza, Phys. Plasmas 20, 082124 (2013).
[31] Prerana Sharma and R. K. Chhajlani, Phys. Plasmas 21, 032101 (2014).
[32] A. Hussain, Z. Iqbal, G. Brodin, G. Murtaza, Phys. Lett.
[33] L. S. Kuz'menkov and S. G. Maksimov, Teor. i Mat. Fiz., 118, 287 (1999) [Theoretical and Mathematical Physics 118, 227 (1999)].

[34] P. A. Andreev, L. S. Kuzmenkov, M. I. Trukhanova, Phys. Rev. B 84, 245401 (2011).

[35] T. Takabayasi, Prog. Theor. Phys. 13, 222 (1955).

[36] T. Takabayasi, Prog. Theor. Phys. 12, 810 (1954).

[37] T. Takabayasi, Prog. Theor. Phys. 14, 283 (1955).