The proof checker DRAT-trim [2] can be used to check whether a propositional formula in the DIMACS format is unsatisfiable. Given a propositional formula and a clausal proof, DRAT-trim validates that the proof is a certificate of unsatisfiability of the formula. Clausal proofs should be in the DRAT format which is used to validate the results of the SAT competitions. The format is defined below.

The DRAT format and the corresponding checker DRAT-trim are designed based on the following goals: 1) It should be easy to emit clausal proofs to ensure that many SAT solvers will support it; 2) proofs should be compact to reduce the overhead of writing the proof and to make sure that the proof can be stored; 3) proof validation should be efficient; and 4) all techniques used in state-of-the-art SAT solvers should be expressible in the format. Regarding the last point, several techniques used in state-of-the-art SAT solvers cannot be expressed using resolution. The DRAT format is therefore a generalization of Extended Resolution.

In short, each step in a clausal proof is either the addition or the deletion of a clause. Each clause addition step should preserve satisfiability, which should be computable in polynomial time. The polynomial time checking procedure is described in detail below. Clause deletion steps are not checked, because they trivially preserve satisfiability. The main reason to include clause deletion steps in proofs is to reduce the computational costs to validate proofs.

Syntax of Input Files

The DRAT-trim checker requires two input files: a CNF formula in the DIMACS format and a clausal proof in the DRAT format. Both formats are defined below.

The syntax of a CNF formula in DIMACS format is:

```
<formula> = { <comment> } "p cnf " <var-max> " " <num-cls> "\n" <cnf>
<cnf> = { <comment> | <clause> } 
<comment> = "c " <anything> "\n"
<clause> = { <blank> }{ <lit> } "0"
<lit> = <pos> <blank> | <neg> <blank>
<pos> = "1" | "2" | ... | <max-idx>
<neg> = "-" <pos>
<blank> = " " | "\n" | "\t"
```
where | means choice, \{ \ldots \} is equivalent to the Kleene star operation (that is a finite number of repetitions including 0) and <max-idx> is $2^{31} - 1$.

In the first line of a DIMCAS formula, <var-max> should be at least as high as the largest <pos> used in the formula, while <num-cla> should equal the number of clauses in the formula. In case there exists a literal <pos> or "-" <pos>, such that \(|pos| is larger than \(<var-max|>, then the formula is invalid.
The formula is also invalid if the number of clauses is not equal to \(<num-cla|>.
The DIMCAS format is the default format used in SAT solvers and has been used as the input format by all SAT Competitions.

The syntax of a clausal proof in DRAT format is as follows:

\[
\begin{align*}
<\text{proof}> & \quad = \{ \text{<comment>} \mid \text{<clause>} | \text{"d" <blank> <clause>} \} \\
<\text{comment}> & \quad = \"c \ " \text{<anything>} \"n\" \\
<\text{clause}> & \quad = \{ \text{<blank>} \}\{ \text{<lit>} \} \"0\" \\
<\text{lit}> & \quad = \text{<pos>} \text{<blank>} \mid \text{<neg>} \text{<blank>} \\
<\text{pos}> & \quad = \"1\" \mid \"2\" \mid \ldots \mid \text{<max-idx>} \\
<\text{neg}> & \quad = \"-\" \text{<pos>} \\
<\text{blank}> & \quad = \"\mid \"\n\" \mid \"t\"
\end{align*}
\]

Notice that the syntax of the DIMACS and DRAT formats is equivalent for <clause>, <lit>, <pos>, and <neg>.

However, DIMACS and DRAT files differ in their interpretation. Formulas in the DIMACS files are multi-sets of clauses. Hence, swapping clauses in a DIMACS file preserves the interpretation. On the other hand, DRAT files are a sequence of clause addition and deletion steps. As a consequence, the order of the clauses influences the proof.

A clausal proof is valid with respect to a given formula, if 1) all clause addition steps are valid (details are below); and 2) if the proof contains the empty clause ("0").

The empty clause typically is the last clause of the proof file, because all lines after the empty clause are redundant and ignored.

### Clause Syntax Restrictions

There are two restrictions regarding clauses in both the DIMACS and the DRAT formats. The first restriction is that no clause can be a tautology, i.e., contains a complementary set of literals. Hence if a clause contains <lit> it cannot contain "-" <lit> as well. The second restriction is that clauses cannot contain duplicate literals. On the other hand, formulas are allowed to have duplicate clauses. Hence formulas are multi-sets of clauses.

### Clause Addition Preconditions

A clause with only one literal is called a unit clause. Checking whether a clause is redundant with respect to a CNF formula is computed via Unit Clause Propagation (UCP). UCP works as follows: For each unit clause (I) all literal
occurrences of \( \bar{l} \) are removed from the formula. Notice that this can result in new unit clauses. UCP terminate when either no literals can be removed or when it results in a conflict, i.e., all literals in a clause have been removed.

Let \( C \) be a clause. \( \overline{C} \) denotes the negation of a clause, which is a conjunction of all negated literals in \( C \). A clause \( C \) has the redundancy property Asymmetric Tautology (AT) with respect to a CNF formula \( F \) iff UCP on \( F \land (\overline{C}) \) results in a conflict. The core redundancy property used in the DRAT format is Resolution Asymmetric Tautology (RAT). A clause \( C \) has the RAT property with respect to a CNF formula \( F \) if there exists a literal \( l \in C \) such that for all clauses \( D \) in \( F \) with \( \neg l \in D \), the clause \( C \lor (D \setminus \{\neg l\}) \) has the property AT with respect to \( F \). Notice that RAT property is a generalization of the AT property.

The DRAT proof checking works as follows. Let \( F \) be the input formula and \( P \) be the clausal proof. At each step \( i \), the formula is modified. The initial state is: \( F_0 = F \). At step \( i > 0 \), the \( i^{th} \) line of \( P \) is read. If the line has the prefix \( d \), then the clause \( C \) described on that line is removed: \( F_i = F_{i-1} \setminus \{C\} \). Otherwise, if there is no prefix, then \( C \) must have the RAT property with respect to formula \( F_{i-1} \). This must be validated. Recall that the RAT property requires a pivot literal \( l \). In the DRAT formula it is expected that the first literal in \( C \) is the pivot. If the RAT property can be validated, then the clause is added to the formula: \( F_i = F_{i-1} \land C \). If the validation fails, then the proof is invalid.

The empty clause at the end of the proof should have the AT property as it does not have a first literal.

**Clause Deletion Details**

The only expectation of a clause deletion step is that the to-be-deleted clause is present in the current formula. Clause deletion steps are ignored if the clause is not present in the current formula. DRAT-trim will print a warning to notify the user about such unexpected steps.

A clause deletion step removes a single clause (if the clause is present). Since formulas are multi-sets of clauses, duplicate clauses may exist. In order to remove all \( k \) copies of a clause, \( k \) clause deletion steps are required in a clausal proof. If case a duplicate clause is deleted, then DRAT-trim removes an arbitrary copy.

DRAT-trim ignores deletion steps of unit clauses by design. This decision facilities a more efficient implementation and reduces the complexity of the checker. The main reason to add deletion information to a clausal proof is to reduce the computation costs to validate that proof. However, deletion of unit clauses has the opposite effect. Notice that ignoring deletion steps of unit clauses can turn a valid DRAT proof into an invalid one — and the other way around. DRAT-trim therefore prints a warning statement to inform the user about such a modification.
Example

Consider the following CNF formula in DIMACS format (with spacing to increase readability):

```
p cnf 4 8
  1 2 -3 0
-1 -2 3 0
  2 3 -4 0
-2 -3 4 0
-1 -3 -4 0
  1 3 4 0
-1 2 4 0
  1 -2 -4 0
```

A valid DRAT proof for the above formula is:

```
-1 0
0
0
0
```

The first step is to validate that the first clause in the proof \(-1\ 0\) has the RAT property with respect to \(F_0\) (the input formula). This RAT check can be partitioned into three AT checks: there are three clauses containing the literal \(1\): \(1 2 -3 0\), \(1 3 4 0\) and \(1 -2 -4 0\). Following the description above, the RAT checks requires checking the AT check for the clauses \(-1 2 -3 0\), \(-1 3 4 0\) and \(-1 -2 -4 0\). For the first AT check, UCP on the formula \(F_0 \land (1) \land (\neg 2) \land (3)\) should result the empty clause. First, clause \(-1 -3 -4 0\) is reduced to the unit \((\neg 4)\). Now the clause \(-1 2 4 0\) is reduced to the empty clause.

The RAT check of \(-1\ 0\) succeeds. Now \(-1\ 0\) will be added to the formula: \(F_1 = F_0 \land (\neg 1)\).

The second step is the removal of the clause \(d -1 2 4 0\). Deletion steps are not checked. This results in \(F_2 = F_1 \setminus (\neg 1 \lor 2 \lor 4)\).

Remarks on Efficient Proof Validation

In order to efficiently validate a DRAT proof, several optimizations to the checking algorithm are required. Examples of such optimizations are backward checking and core first unit propagation. Details about these techniques are described in [1].

1 Binary DRAT Format

This section describes the "binary format" for input proof files.
Mapping DIMACS Literals to Unsigned Integers

The first step of the binary encoding is mapping literals in the DIMACS format to unsigned integers. The following mapping function is used:

\[ \text{map}(l) := (l > 0) ? 2 \cdot l : -2 \cdot l + 1. \]

The mapping for some DIMACS literals are shown below.

| DIMACS literals | unsigned integers |
|-----------------|-------------------|
| -63             | 127               |
| 129             | 258               |
| -8191           | 16383             |
| -8193           | 16387             |

Variable-Byte Encoding of Unsigned Integers

Assume that \( w_0, \ldots, w_i \) are 7-bit words, \( w_1 \) to \( w_i \) all non zero and the unsigned number \( x \) can be represented as

\[ x = w_0 + 2^7w_1 + 2^14w_2 + 2^{7i}w_i \]

The variable-byte encoding of DRAT (also used in AIGER) is the sequence of \( i \) bytes \( b_0, \ldots, b_i \):

\[ 1w_0, 1w_1, 1w_2, \ldots, \text{0wi} \]

The MSB of a byte in this sequence signals whether this byte is the last byte in the sequence, or whether there are still more bytes to follow. Here are some examples:

| unsigned integer | byte sequence of encoding (in hexadecimal) |
|------------------|------------------------------------------|
| \( 2^{7} - 1 \)  | 00 00                                   |
| \( 2^{7} \)      | 01 01                                   |
| \( 2^{8} + 2 \)  | 80 01                                   |
| \( 2^{14} - 1 \) | ff 7f                                   |
| \( 2^{14} + 3 \) | 80 01                                   |
| \( 2^{28} - 1 \) | ff ff ff 7f                             |
| \( 2^{28} + 7 \) | 80 80 80 80 01                          |
Bringing it together

In the binary DRAT format, each clause consists of at least two bytes. The first byte expresses whether the lemma is added (character a or 61 in hexadecimal) or deleted (character d or 64 in hexadecimal). The last byte of each lemma is the zero byte (00 in hexadecimal). In between these two bytes, the literals of the lemma are shown as unsigned integers in the variable-byte encoding.

In the example below, the plain DRAT format requires 26 bytes (including the new line characters and excluding the redundant spaces in the second lemma), while the binary DRAT format of the same proof requires only 12 bytes. Emitting proofs in the binary format reduces the size on disk by approximately a factor of three compared to the conventional (plain) DRAT format.

| plain DRAT | binary DRAT (in hexadecimal) |
|------------|------------------------------|
| d -63 -8193 0 | 64 7f 83 80 01 00 61 82 02 ff 7f 00 |
| 129 -8191 0 |

References:

[1] Marijn J. H. Heule, Warren A. Hunt, Jr., and Nathan Wetzler. Trimming while checking clausal proofs. In Proc. of the 16th Int. Conf. on Formal Methods in Computer-Aided Design (FMCAD 2013), pages 181–188. IEEE, 2013.

[2] Nathan Wetzler, Marijn J. H. Heule, and Jr. Hunt, Warren A. Drat-trim: Efficient checking and trimming using expressive clausal proofs. In Carsten Sinz and Uwe Egly, editors, SAT 2014, volume 8561 of LNCS, pages 422–429. Springer, 2014.