Eccentric discs in binaries with intermediate mass ratios: Superhumps in the VY Sculptoris stars

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ABSTRACT
We investigate the role of the eccentric disc resonance in systems with mass ratios \(q \geq 1/4\), and demonstrate the effects that changes in the mass flux from the secondary star have upon the disc radius and structure. The addition of material with low specific angular momentum to its outer edge restricts a disc radially. Should the mass flux from the secondary be reduced, it is possible for the disc in a system with mass ratio as large as 1/3 to expand to the 3:1 eccentric inner Lindblad resonance and for superhumps to be excited.

Key words: accretion, accretion discs — instabilities — hydrodynamics — methods: numerical — binaries: close — novae, cataclysmic variables.

1 SUPERHUMPS IN NOVA-LIKE VARIABLES

The superhump phenomenon was first observed in superoutbursts of dwarf novae of short orbital period (\(P_{\text{orb}} \leq 2.8\) h: see Warner 1995b). It can be explained as an effect of an eccentric disc that precesses with respect to the tidal field of the secondary star (Whitehurst 1988). In low mass ratio binaries \((q \leq 1/4)\), disc eccentricity can be excited via tidal resonance (Whitehurst 1988, Hirose & Osaki 1990, Lubow 1991). In systems with larger \(q\) however, the disc is truncated at too small a radius for the resonance to be effective.

Observationally there is evidence that discs can be tidally resonant for \(q\) significantly greater than 1/4. Superhumps appear in high mass transfer discs (those of nova-like variables and of nova remnants) in some systems with orbital periods up to 3.5 h (Skillman et al. 1998). At this period the mass of the secondary star is 0.31 M\(_\odot\) (equation 2.100 of Warner 1995b). Unfortunately there are no reliable determinations of \(q\) in the superhumping systems with \(P_{\text{orb}} > 3h\). The mean mass of the primaries of the novae-likes is \(\sim 0.74\) M\(_\odot\) (Warner 1995b) and the range is likely to be at least 0.5 – 1.0 M\(_\odot\). If the primary mass is at the upper end of this range, it is therefore possible that a system with \(P_{\text{orb}} \sim 3.5h\) would have a mass ratio approaching that needed for the eccentric resonance to lie within the accretion disc.

There can be little doubt that a mechanism is required to extend the range of mass ratios for which tidal resonance occurs, beyond the \(q \leq 1/4\) established in equilibrium disc calculations. One possibility is suggested by the VY Scl stars, systems with orbital periods in the range 3.0 < \(P_{\text{orb}} < 4.0\) h (Warner 1995b) in which mass transfer is liable to reduce significantly at irregular intervals. Such a drop in \(M\) would allow a tidally stable, equilibrium disc that just marginally fails to reach the eccentric resonance to expand radially and become eccentrically unstable.

In this paper we demonstrate that the discs in high \(M\) systems can be excited into an eccentric state when the mass transfer from the secondary drops below its equilibrium value. In the next section we discuss the observational peculiarities of the VY Scl stars. In section three we outline our model for superhumps in these systems. We present numerical results that support our case in section four, and make our conclusions in section five.

2 MASS TRANSFER IN NOVA-LIKE VARIABLES

The long-term light curves of nova-like variables show brightness variations on a range of time-scales. Quasi-periodic modulations with ranges \(\sim 0.7\) mag and time-scales of 1 – 2 months may be intrinsic to the disc and in effect suppress dwarf nova outbursts in which the thermal instability of the disc is modified by irradiation from the white dwarf primary (Warner 1995a). Larger brightness variations on time-scales from days to years are also seen, in which the star shows a “bright state” before falling from 1 to 5 magnitudes in brightness. This behaviour is named after the type

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star, VY Sculptoris, and is generally believed to be the result of significant and relatively rapid drops in mass transfer from the secondary.

The known VY Scl stars (Table 4.1 in Warner 1995b) have orbital periods in the range \(3.2 \leq P_{\text{orb}} \leq 4.0\) h. Two apparently discrepant systems, V794 Aql and V745 Cyg have since had their orbits re-determined. Thorstensen (private communication) found the orbital period of V745 Cyg to be 4.05 h, and Honeycutt & Robertson (1998) found the period of V794 Aql to be 3.68 h. Furthermore, every nova-like variable in the 3.0 to 4.0 h \(P_{\text{orb}}\) range that has a well observed long-term light curve has been found to be of VY Scl type (Warner 1995b). Unstable mass transfer in these systems may be due to irradiation of the secondary by the primary (Wu, Wickramasinghe & Warner, 1995).

As a consequence, in the range \(3.0 \leq P_{\text{orb}} \leq 4.0\) h, high \(M_{\ast}\) stable accretion discs are subject to occasional reductions of received mass at their outer edge. In the following sections we numerically investigate the disc response.

### 3 TIDAL INSTABILITY IN SYSTEMS WITH INTERMEDIATE MASS RATIOS

Eccentricity can be excited in a close binary accretion disc if it overlaps or very nearly overlaps the 3:1 eccentric inner Lindblad resonance (Lubow 1991). With a characteristic radius almost half the interstellar separation, this resonance is only accessible to discs in low mass ratio systems. Tidal forces prevent discs in high mass ratio systems from reaching the resonance.

Paczyński (1977) approximated the streamlines in a pressureless, inviscid disc with ballistic particle trajectories. He then suggested that such a disc could not be larger than the largest ballistic orbit that doesn’t intersect orbits interior to it. Using Paczyński’s estimate, Hirose & Osaki (1990), and Lubow (1991) found that the 3:1 resonance could only be reached if \(q \leq 4 / 3\).

But the tides that truncate a close binary disc also remove angular momentum from it. Papaloizou & Pringle (1977) calculated the tidal removal of angular momentum from a disc comprised of gas in linearly perturbed circular orbits. They estimated that the disc would need to extend out to approximately 0.85 – 0.9 times the mean Roche lobe radius for tidal torques to be effective (slightly beyond the Paczyński radius). However the assumption of linearity made by Papaloizou & Pringle lead to an underestimate of the dissipation in the neighbourhood of Paczyński’s orbit crossing radius and thus to a larger estimate for the tidal radius. Thus when Whitehurst & King (1991) used the Papaloizou & Pringle value for maximum disc size, they estimated that the 3:1 resonance could be reached in systems with \(q\) as high as 1/3.

For near equilibrium discs subjected to a fixed rate of mass addition from the secondary, the numerical results are more in keeping with the lower mass ratio limit of 1/4. Hirose & Osaki (1990) obtained eccentric discs in simulations with \(0.10 \leq q \leq 0.20\), and Murray (1998) for mass ratios \(\leq 0.25\). Whitehurst (1994) found eccentricity in simulations at mass ratios as high as 1/3. However his discs were initialised with a rapid burst of mass transfer and so were not close to an equilibrium when they encountered the 3:1 resonance.

As discussed in the previous section, the observations are not consistent with a mass ratio limit as low as 1/4. We propose then that higher mass ratio systems can become resonant during periods when the disc is not in equilibrium with the mass transfer stream.

Gas passing through the inner Lagrangian point has a specific angular momentum with respect to the white dwarf equivalent to that of a circular Keplerian orbit of radius \(r_{\text{circ}}\). This circularisation radius is of the order of a tenth to one fifth the interstellar separation and is very much inside the disc’s tidal truncation limit (Lubow & Shu, 1975). Hence, mixing with the mass transfer stream reduces specific angular momentum in the outer disc and restricts the disc radially. Of course the principal confinement of the disc is provided by tidal torques, however the additional confinement due to the stream can be significant. If, as is suspected to happen in the VY Scl stars, \(M_{\ast}\) is suddenly reduced, then the radial restriction on the disc will be relaxed, and the disc will expand. The expansion can be only temporary, as the disc will subsequently adjust to the new \(M_{\ast}\) on its viscous time-scale, and return to its initial size. However, we will show in the next section that the period of readjustment can be sufficient to allow the disc to encounter the 3:1 resonance and become eccentric.

### 4 RESULTS

#### 4.1 Numerical Method

In this section we present numerical results obtained using a two dimensional, fixed spatial resolution, smooth particle hydrodynamics (SPH) code. Previous calculations and detailed descriptions of the code are to be found in Murray (1996, 1998). Although the code has been improved to allow for variable spatial resolution (see e.g. Murray, de Kool & Li, 1998) and for the three dimensional modelling of accretion discs (Murray & Armitage, 1998), we found that by restricting ourselves to two dimensions and fixed resolution, we could more thoroughly explore parameter space with a reasonable expenditure of computer time. Our ability to capture the essential features of the eccentric instability is not compromised, and previous results have been consistent with those of three dimensional calculations (see e.g. Simpson & Wood, 1998).

The simulations described here demonstrate that, in systems with intermediate mass ratio, a sudden reduction in mass transfer from the secondary star can allow an equilibrium disc to expand so as to come into contact with the eccentric resonance.

Two sequences of simulations were completed. In the first series we subjected near-equilibrium discs to sudden reductions in \(M_{\ast}\), the aim being to demonstrate the viability of the proposed mechanism.

We then completed a second set of simulations in which we replaced the mass transfer stream with steady mass addition at the circularisation radius, \(r_{\text{circ}}\). By removing the additional constraining effect of the stream, and allowing the disc to reach a fully extended steady state, we made it as easy as possible for the disc to interact with the eccentric resonance. We ran such simulations for several values of \(q\) and thus determined the range of mass ratios for which a reduction in mass transfer might result in superhumps.
Figure 1. Energy dissipation “light curve” from the outer disc ($r > 0.2$) of the $q = 0.29$ simulation with mass transfer from the secondary shut off at time $t = 1652\Omega_{\text{orb}}^{-1}$. The contribution of the inner disc to the dissipation was not included for clarity. The units for the luminosity are $M^2 d^3 \Omega_{\text{orb}}^3$.

Figure 2. The response of the disc eccentricity and the $l = m = 2$ tidal mode to a complete cessation of mass transfer from the secondary. See figure 1 for the corresponding light curve.

As in Murray (1996) and Murray (1998), we use the interstellar separation $d$, the binary mass $M$, and the inverse of the orbital angular velocity $\Omega_{\text{orb}}^{-1}$, as the length, mass and time scalings for our simulations.

4.2 Variable $M_\ast$ Calculations

In Murray (1998) we described a calculation for mass ratio $q = 0.29$ that failed to excite the eccentric resonance (simulation 1 of that paper). In that calculation mass was introduced at a constant rate of one particle per $\Delta t = 0.01 \Omega_{\text{orb}}^{-1}$ at the inner Lagrangian point. An isothermal equation of state was used with $c = 0.02 d \Omega_{\text{orb}}$. The spatial resolution was fixed with the smoothing length $h = 0.01 d$. The high rate of viscous dissipation that occurs in CV discs in outburst was modelled using an artificial viscosity term (Murray 1996, 1998). The artificial viscosity parameter $\zeta = 10$. The gave an effective Shakura-Sunyaev parameter $\alpha \simeq 1.9$ at the resonance radius. This was somewhat on the high side but made the calculations more manageable. At the end of the simulation ($t = 1652.00 \Omega_{\text{orb}}^{-1}$) the non-resonant disc had an equilibrium mass of 19950 particles.

For this paper we used the final state of the above calculation as the initial state for a new simulation with $M_\ast = 0$ (no mass addition). All other system parameters were left unchanged. The disc evolution was then followed to $t = 2000.00 \Omega_{\text{orb}}^{-1}$, by which time there were only 5031 particles remaining in the disc. By plotting the energy viscously dissipated in the disc as a function of time we obtained a “light curve” for the simulation (figure 1). The stream-disc impact is responsible for a significant fraction of the energy dissipation in the outer disc. Hence when the accretion stream was shut off (at $t = 1652.00 \Omega_{\text{orb}}$) there was a sudden drop in disc luminosity. Almost immediately after mass transfer ceased the accretion disc became resonant and eccentric. The subsequent motion of the eccentric mode relative to the binary frame introduced a cyclic contribution to the energy dissipation in the outer disc (this is discussed in more depth in Murray, 1998) which we identify as a superhump signal. Figure 2 shows the superhumps reaching maximum amplitude somewhere between $t = 1800$ and $1900 \Omega_{\text{orb}}^{-1}$. The subsequent decay of both the superhump amplitude and of the background luminosity was simply due to the decline in disc mass.

As the simulation ran we Fourier analysed the disc’s density distribution using the technique described in Lubow (1991). Each Fourier mode’s variation in azimuth and time is given by $10 - m \Omega_{\text{orb}} t$ where $\theta$ is azimuth measured in the inertial frame, $\Omega_{\text{orb}}$ is the angular velocity of the binary, and $l$ and $m$ are integers. Hence the disc’s eccentricity is measured by the $l = 1, m = 0$ mode, and the largest amplitude non-resonant tidal response has $l = m = 2$. In some sense this latter mode (which appears in density maps as a two armed spiral pattern fixed in the binary frame) indicates the radial extent of the disc. The larger the tidal response, the more disc material there is at large radii.

The strengths of the eccentric and spiral modes are plotted in figure 2. The tidal mode strength jumped sharply once mass addition ceased, strongly indicating a redistribution of matter radially outward through the disc. We confirmed this interpretation by checking the actual particle distributions. Immediately prior to the demise of the mass transfer stream, 18.2% of the disc mass was at radii $r > 0.4 d$. By $t = 1700.00 \Omega_{\text{orb}}^{-1}$, this figure had increased to 25.6%. There was also an increase in absolute terms from 3636 to 4046 particles.

The eccentric mode also became immediately stronger
upon cessation of mass transfer, and then continued to grow as the resonance took effect. The maximum in the eccentric mode strength coincided with the largest amplitude superhumps.

We have produced a sequence of grey scale density maps of the disc (figure 3), from which it is clear that the mass transfer stream did indeed restrict the disc’s radial extent. In its original equilibrium state (top left panel), the disc immediately downstream of the stream impact region lay entirely within the Roche lobe of the primary. Compare that with the disc shortly after mass transfer had ceased (top right) which extended beyond the Roche lobe.

Having determined the mechanism to be viable, we then investigated whether disc eccentricity could be excited with a less drastic reduction in $\dot{M}_s$. We completed a second calculation with identical initial conditions, but with the mass transfer rate reduced to half its initial value instead of being extinguished completely. Figures 4 and 5 follow the evolution of the disc to $t = 3000\Omega_{\text{orb}}^{-1}$, by which time the disc mass had stabilised at approximately 10500 particles. As in the first simulation, the disc immediately came into contact with the resonance and became more eccentric. This time however the superhump amplitude was much reduced. The eccentricity reached a maximum at $t \approx 2300\Omega_{\text{orb}}^{-1}$ and then declined. Note that despite the reduction in superhump amplitude, the amplitude of the $(1,0)$ mode was larger in the second calculation (compare figures 4 and 5). Whereas the eccentric mode strength is a measure of the eccentricity as a whole, it does not directly measure the eccentricity of individual particle orbits. 

Figure 3. Grey scale density maps for the first $q = 0.29$ simulation, covering the time interval shown in figures 1 and 2. The same absolute density scale is used in each frame. $\dot{M}_s$ was set to zero at $t = 1652.00\Omega_{\text{orb}}^{-1}$. The Roche lobe of the primary and the 3:1 resonance are shown as solid lines.
is generated by those particles on the most eccentric orbits. In the second simulation, the continued mass transfer restricted the maximum eccentricity which could be acquired by gas in the outer disc, and hence limited the superhump amplitude.

We had expected a priori that the disc would return to an axisymmetric state in equilibrium with the reduced mass flux from the secondary. However we followed this calculation to $t = 5700 \Omega_{\text{orb}}^{-1}$ and were surprised to find that the eccentricity did not decline to zero as expected, but instead stabilised with $0.05 < S_{(1,0)} < 0.07$. The plot of the tidal (2,2) mode (figure 5) and density maps (not shown) indicate that the disc returned to its original radial extent by time $t \approx 2100 \Omega_{\text{orb}}^{-1}$. There is thus the intriguing possibility that this eccentric state, once reached, was quasi-stable.

To complete this section we performed a third simulation, in which we took the previous simulation at time $t = 2400.00 \Omega_{\text{orb}}^{-1}$ and restored the mass transfer rate to its original value. Figures 6 and 7 show that the disc was initially forced rapidly inwards and circularised by the increased mass flux from the secondary. Further adjustment then occurred on the viscous time-scale as the disc re-expanded to its equilibrium radius and mass flux, and the eccentricity decayed away. By the end of the calculation ($t = 2400.00 \Omega_{\text{orb}}^{-1}$) the disc had returned to its original axisymmetric state.

4.3 Stable $M_s$ Calculations

In this section we describe four simulations of discs built up to steady state from zero initial mass via steady mass addition at the circularisation radius. Although one would be hard-pressed to find such a disc in nature, it is perhaps the most favourable disc configuration for the excitation of the eccentric resonance. Whereas in the simulations of the previous section the interaction with the resonance was limited to the viscous time-scale, there is no such time limitation in these calculations. Thus if, for a given mass ratio, eccentricity cannot be excited with this artificially favourable setup, then it is hard to see how it could be excited with more realistic mass input. Hence we can obtain a limiting mass ratio for which mass transfer reductions can inspire superhumps.

As in the previous section we took an isothermal equa-
The mass flux from the secondary was halved at time $t = 1652\Omega^{-1}_{\text{orb}}$, and then restored to its original value at $t = 2400\Omega^{-1}_{\text{orb}}$.

![Figure 7.](image)

**Figure 7.** Eccentric and tidal mode strengths corresponding to figure 6 light curve. The mass flux from the secondary was halved at time $t = 1652\Omega^{-1}_{\text{orb}}$, and then restored to its original value at $t = 2400\Omega^{-1}_{\text{orb}}$.

| $q$  | final $S_{(1,0)}$ | $P_0/P_{\text{orb}}$ |
|------|-------------------|---------------------|
| 0.29 | $4.5 \times 10^{-2}$ | 1.153 ± 0.008 |
| 0.30 | $3.5 \times 10^{-2}$ | 1.163 ± 0.005 |
| 0.31 | $5.9 \times 10^{-3}$ | 1.173 ± 0.007 |
| $1/3$ | $2.7 \times 10^{-3}$ | – |
| 0.29 $^*$ | $6.3 \times 10^{-2}$ | 1.1583 ± 0.0001 |

$^*$simulation from section 4.2 with $M_0$ half initial value, added at $L_1$.

A mass ratio of approximately $1/3$ would appear to be the upper limit beyond which the eccentric resonance cannot be excited even temporarily. Our results correspond well with those of Whitehurst (1994) who found superhumps in a simulation with $q = 1/3$ but not in a simulation with $q = 0.34$. He constructed his discs with a massive initial mass transfer burst, followed by a much reduced but constant $M_0$. The mass distribution in Whitehurst’s discs would have been therefore more akin to a disc built up via mass addition at $r_{\text{circ}}$ than to a disc subject to steady mass addition from $L_1$.

### 5 CONCLUSIONS

Under conditions of steady mass transfer, a mass ratio $\lesssim 1/4$ is required for a close binary accretion disc to encounter the $3 : 1$ eccentric inner Lindblad resonance. However, it is possible for eccentricity to be excited in the disc of a high mass transfer system with $q \lesssim 1/3$ if $M_0$ is reduced, as is thought to occur in the VY Sculptoris systems. Our simulations suggest that the disc will remain eccentric as long as $M_0$ remains at the lower value.

Thus, a precessing, eccentric disc remains the best explanation of superhumps, even for systems with $3.0 < P_{\text{orb}} < 4.0$ h.

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