Reanalysis of the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states

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Abstract

In this article, we study the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states which consist of the axial-axial type and the vector-vector type diquark pairs with the QCD sum rules.

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1 Introduction

In 2007, a distinct peak ($Z(4430)$) was observed in the $\pi^\pm\psi'$ invariant mass distribution near 4.43 GeV in the decays $B \rightarrow K\pi^\pm\psi'$ by the Belle collaboration [1]. The fitted Breit-Wigner mass and width are $M_Z = 4433 \pm 4 \pm 2$ MeV and $\Gamma_Z = 45^{+18+30}_{-13-13}$ MeV. The statistical significance of the observed peak is 6.5 $\sigma$. Using the same data sample, the Belle collaboration also performed a full Dalitz plot analysis with a fitted model that takes into account all the known $K\pi$ resonances below 1780 MeV [2]. The significance of the fitted resonance is of 6.4 $\sigma$ and agrees with the previous observation [1], the updated parameters are $M_Z = (4443^{+15+19}_{-12-13})$ MeV and $\Gamma_Z = (109^{+86+74}_{-43-56})$ MeV. However, the BaBar collaboration do not confirm this resonance [3], i.e. they observe no significant evidence for a $Z(4430)$ signal for any of the processes investigated, neither in the total $J/\psi\pi$ or $\psi'\pi$ mass distribution nor in the corresponding distributions for the regions of $K\pi$ mass for which observation of the $Z(4430)$ signal is reported. If the $Z(4430)$ exists indeed, it can’t be a pure $c\bar{c}$ state due to the positive charge, and may be an excellent tetraquark state ($c\bar{c}ud$) candidate [4, 5]. We can distinguish the multiquark states from the hybrids or charmonia with the criterion of non-zero charge.

In 2008, the Belle collaboration reported the first observation of two resonance-like structures ($Z(4050)$ and $Z(4250)$) in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive decays $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ [6]. Their quark contents must be some special combinations of the $c\bar{c}ud$, just like the $Z(4430)$, they can’t be the conventional mesons. The $Z(4050)$ and $Z(4250)$ lie about (0.5 − 0.6) GeV above the $\pi^+\chi_{c1}$ threshold, the decay $Z \rightarrow \pi^+\chi_{c1}$ can take place with the "fall-apart" mechanism and it is OZI super-allowed, which can take into account the large total width naturally. The spins of the $Z(4050)$ and $Z(4250)$ are not determined yet, they can be scalar or vector mesons. If they are scalar mesons, the decays $Z \rightarrow$
π⁺χc1 occur through the relative $P$-wave with the phenomenological lagrangian $\mathcal{L} = g\chi^\alpha(\pi\partial_\alpha Z - Z\partial_\alpha \pi)$. On the other hand, if they are vector mesons, the decays occur through the relative $S$-wave with the phenomenological lagrangian $\mathcal{L} = g\chi^\alpha Z_\alpha \pi$. There have been several interpretations, such as the tetraquark states [7, 8, 9] and the molecular states [10, 11, 12, 13].

In Refs. [7, 8], we assume that the hidden charm mesons $Z(4050)$ and $Z(4250)$ are vector (and scalar) tetraquark states, and study their masses with the QCD sum rules. The numerical results indicate that the mass of the vector hidden charm tetraquark state is about $M_Z = (5.12 \pm 0.15)$ GeV or $M_Z = (5.16 \pm 0.16)$ GeV, and the mass of the scalar hidden charm tetraquark state is about $M_Z = (4.36 \pm 0.18)$ GeV. In Refs. [14, 15], we study the mass spectrum of the scalar and vector hidden charm and hidden bottom tetraquark states using the QCD sum rules, and observe that the scalar hidden charm tetraquark states may have smaller masses than the corresponding vector states. From our previous works, we can draw the conclusion that the hidden charm meson $Z(4250)$ may be a scalar tetraquark state [7, 8, 14, 15], although other possibilities, such as a hadro-charmonium resonance and a $D^+_s \bar{D}^0 + D^+ \bar{D}^+_s$ molecular state are not excluded. We intend to study the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states which consist of diquark pairs differ from our previous works.

The mass is a fundamental parameter in describing a hadron, whether or not there exist those hidden charm and hidden bottom tetraquark configurations is of great importance itself, because it provides a new opportunity for a deeper understanding of the low energy QCD.

In Refs. [16, 17], Ebert et al take the diquarks as bound states of the light and heavy quarks in the color antitriplet channel, and calculate their mass spectrum using a Schrodinger type equation, then take the masses of the diquarks as the basic input parameters, and study the mass spectrum of the heavy tetraquark states as bound states of the diquark-antidiquark system. In Refs. [18, 19, 20], Maiani et al take the diquarks as the basic constituents, examine the rich spectrum of the diquark-antidiquark states with the constituent diquark masses and the spin-spin interactions, and try to accommodate some of the newly observed charmonium-like resonances not fitting a pure $c\bar{c}$ assignment. In Ref. [21], Zouzou et al solve the four-body ($Q\bar{Q}qq$) problem by three different variational methods with a non-relativistic potential considering explicitly virtual meson-meson components in the wave-functions, search for possible bound states below the threshold for the spontaneous dissociation into two mesons, and observe that the exotic bound states $Q\bar{Q}qq$ maybe exist for unequal quark masses (the ratio $m_Q/m_q$ is large enough). The studies using a potential derived from the MIT bag model in the Born-Oppenheimer approximation support this observation [22, 23]. In Ref. [24], Manohar and Wise study systems of two heavy-light mesons interacting through an one-pion exchange potential determined by the heavy meson chiral perturbation theory and observe the long range potential maybe sufficiently attractive to produce a weakly bound two-meson state in the case $Q = b$. In Ref. [25], the $L = 0$ tetraquark states $QQ\bar{Q}\bar{Q}$
(Q denotes both Q and q) are analyzed in a chromo-magnetic model where only a constant hyperfine potential is retained.

In this article, we re-study the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states using the QCD sum rules [26, 27]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [26, 27].

The hidden charm and hidden bottom tetraquark states (Z) have the symbolic quark structures:

\[
\begin{align*}
Z^+ &= Q \bar{Q} u \bar{d}; \\
Z^0 &= \frac{1}{\sqrt{2}} Q \bar{Q} (u \bar{u} - d \bar{d}); \\
Z^- &= Q \bar{Q} d \bar{u}; \\
Z^+_s &= Q \bar{Q} u \bar{s}; \\
Z^-_s &= Q \bar{Q} s \bar{u}; \\
Z^0_s &= Q \bar{Q} d \bar{s}; \\
Z^0_s &= Q \bar{Q} s \bar{d}; \\
Z_\varphi &= \frac{1}{\sqrt{2}} Q \bar{Q} (u \bar{u} + d \bar{d}); \\
Z_\phi &= Q \bar{Q} s \bar{s},
\end{align*}
\]  

(1)

where the Q denotes the heavy quarks c and b.

We take the diquarks as the basic constituents to study the tetraquark states following Jaffe and Wilczek [28, 29]. The heavy tetraquark system could be described by a double-well potential with two light quarks q'q lying in the two wells respectively. In the heavy quark limit, the c (and b) quark can be taken as a static well potential, which binds the light quark q to form a diquark in the color antitriplet channel. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet \( 3_c \), flavor antitriplet \( 3_f \) and spin singlet \( 1_s \) [30, 31]. The diquarks have five Dirac tensor structures, scalar \( C_{\gamma} \), pseudoscalar \( C \), vector \( C_{\gamma\mu}\gamma_5 \), axial vector \( C_{\gamma\mu} \) and tensor \( C_{\gamma\mu\nu} \). The structures \( C_{\gamma\mu} \) and \( C_{\gamma\mu}\gamma_5 \) are symmetric, the structures \( C_{\gamma} \), \( C \) and \( C_{\gamma\mu\gamma_5} \) are antisymmetric. In Refs. [8, 14], we assume the scalar hidden charm and hidden bottom mesons Z consist of the \( C_{\gamma5} - C_{\gamma5} \) type diquark structures rather than the \( C - C \) type diquark structures, and observe that the \( C_{\gamma5} - C_{\gamma5} \) type tetraquark states have much smaller masses than the corresponding \( C - C \) type tetraquark states; our numerical results of the \( C - C \) type tetraquark states will be presented elsewhere.

In this article, we assume the scalar hidden charm and hidden bottom tetraquark states which consist of the \( C_{\gamma\mu} - C_{\gamma\mu} \) type and the \( C_{\gamma\mu\gamma5} - C_{\gamma\mu\gamma5} \) type diquark pairs and study the mass spectrum. Naively, we expect the \( C_{\gamma\mu} - C_{\gamma\mu} \) type and the \( C_{\gamma\mu\gamma5} - C_{\gamma\mu\gamma5} \) type tetraquark states have larger masses than the corresponding \( C_{\gamma5} - C_{\gamma5} \) type tetraquark states.

The article is arranged as follows: we derive the QCD sum rules for the scalar hidden charm and hidden bottom tetraquark states Z in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.
2 QCD sum rules for the scalar tetraquark states $Z$

In the following, we write down the two-point correlation functions $\Pi(p)$ in the QCD sum rules, 

$$ \Pi(p) = i \int d^4x e^{ipx} \langle 0| T \left\{ J(x) J(0) \right\} |0\rangle, $$

(2)

where the $J(x)$ and $\eta(x)$ denote the interpolating currents $J_{Z^+}(x)$, $J_{Z^0}(x)$, $\eta_{Z^+}(x)$, $\eta_{Z^0}(x)$, $\eta_{Z^+}(x)$, $\eta_{Z^0}(x)$, etc., 

$$ J_{Z^+}(x) = \epsilon^{ijk} \epsilon^{mnn} u_j^T(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{d}_n(x), $$

$$ J_{Z^0}(x) = \epsilon^{ijk} \epsilon^{mnn} u_j^T(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{u}_n(x) - (u \rightarrow d), $$

$$ J_{Z^+}(x) = \epsilon^{ijk} \epsilon^{mnn} u_j^T(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{s}_n^T(x), $$

$$ J_{Z^0}(x) = \epsilon^{ijk} \epsilon^{mnn} d_j^T(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{s}_n^T(x), $$

$$ J_{Z^+}(x) = \epsilon^{ijk} \epsilon^{mnn} \frac{1}{\sqrt{2}} [u_j^T(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{u}_n(x) + (u \rightarrow d)], $$

$$ J_{Z^0}(x) = \epsilon^{ijk} \epsilon^{mnn} \frac{1}{\sqrt{2}} [u_j^T(x) C \gamma_\mu \gamma_5 Q_k(x) \bar{Q}_m(x) \gamma^\mu C \bar{u}_n(x) - (u \rightarrow d)], $$

(3)

where the $i$, $j$, $k$, $\cdots$ are color indexes. In the isospin limit, the interpolating currents result in six distinct expressions for the correlation functions $\Pi(p)$, which are characterized by the number of the $s$ quark they contain. In Refs. [14, 15], we observe that the ground state masses of the scalar and vector tetraquarks are characterized by the number of the $s$ quarks they contain, $M_0 \leq M_s \leq M_{ss}$; the energy gap between $M_0$ and $M_{ss}$ is about $(0.05 - 0.15)$ GeV. In this article, we study the interpolating currents which contains zero and two $s$ quarks for simplicity.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J(x)$ and $\eta(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation [26, 27]. After isolating the ground state contribution from the pole terms of the $Z$, we get the following result, 

$$ \Pi(p) = \frac{\lambda^2_Z}{M_Z^2 - p^2} + \cdots, $$

(4)
where the pole residue (or coupling) $\lambda_Z$ is defined by

$$\lambda_Z = \langle 0| J/\eta(0)| Z(p) \rangle.$$  \hfill (5)

The contributions from the two-particle and many-particle reducible states are supposed to be small enough to be neglected safely, for example, the scattering state $\chi_{c1}\pi^+$ in the $\bar{c}c\bar{d}u$ channel,

$$\Pi(p) = i\lambda_{\chi_{c1}\pi^+} \int \frac{d^4q}{(2\pi)^4} \frac{p^\mu p'^\nu}{[q^2 - m_{\chi_{c1}}^2][(p - q)^2 - m_{\pi}^2]} \left[ -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{\chi_{c1}}^2} \right] + \cdots, \hfill (6)$$

where

$$\langle 0| J_{Z^+}(0) | \chi_{c1}\pi^+ \rangle = \lambda_{\chi_{c1}\pi^+} p_\mu \epsilon^\mu,$$  \hfill (7)

the $\epsilon_\mu$ is the polarization vector of the axial-vector meson $\chi_{c1}$. We can estimate the coupling $\lambda_{\chi_{c1}\pi^+}$ with the soft $\pi$ theorem,

$$\langle 0| J_{Z^+}(0) | \chi_{c1}\pi^+ \rangle = -\frac{i}{f_\pi} \langle 0| [Q_5, J_{Z^+}(0)] | \chi_{c1} \rangle, \hfill (8)$$

$$= \frac{i}{f_\pi} \langle 0| J_P(0) | \chi_{c1} \rangle = -\frac{\lambda_P p_\mu \epsilon^\mu}{f_\pi},$$

where

$$Q_5 = \int d^3x u^+(x) i\gamma_5 d(x),$$

$$J_P(x) = \epsilon^{ijk} \epsilon^{imn} [u_j^T(x) C \gamma_\mu \gamma_5 c_k(x) \bar{c}_m(x) \gamma^\mu C \bar{u}_n^T(x)$$

$$+ d_j^T(x) C \gamma_\mu c_k(x) \bar{c}_m(x) \gamma_5^\mu C d_n^T(x)]. \hfill (9)$$

As the main Fock states of the charmonia are the $\bar{c}c$ components, the coupling $\lambda_P$ between the pseudoscalar tetraquark current $J_P(x)$ and the axial-vector meson $\chi_{c1}$ should be very small.

We can perform Fierz re-ordering in both the Dirac spin space and the color space to express the tetraquark current $J_{Z^+}(x)$ in the following form,

$$J_{Z^+}(x) = \frac{3}{8} \bar{d}(x) u(x) \bar{c}(x) c(x) + \frac{3}{8} \bar{d}(x) i\gamma_5 u(x) \bar{c}(x) i\gamma_5 c(x) + \frac{3}{16} \bar{d}(x) \gamma_\alpha u(x) \bar{c}(x) \gamma^\alpha c(x)$$

$$- \frac{3}{16} \bar{d}(x) \gamma_\alpha \gamma_5 u(x) \bar{c}(x) \gamma^\alpha \gamma_5 c(x) - \frac{1}{2} \bar{d}(x) \frac{\lambda_i}{2} u(x) \bar{c}(x) \frac{\lambda_i}{2} c(x)$$

$$- \frac{1}{2} \bar{d}(x) i\gamma_5 \frac{\lambda_i}{2} u(x) \bar{c}(x) i\gamma_5 \frac{\lambda_i}{2} c(x) - \frac{1}{4} \bar{d}(x) \gamma_\alpha \frac{\lambda_i}{2} u(x) \bar{c}(x) \gamma^\alpha \frac{\lambda_i}{2} c(x)$$

$$+ \frac{1}{4} \bar{d}(x) \gamma_\alpha \gamma_5 \frac{\lambda_i}{2} u(x) \bar{c}(x) \gamma^\alpha \gamma_5 \frac{\lambda_i}{2} c(x), \hfill (10)$$

where the $\lambda_i$ are the matrix elements of the $SU(2)$ group in adjoint representation, the $i = 1, 2, 3$ are the color indexes. The scalar tetraquark current which consists
of a axial-vector diquark pair is a special composition of the $S - S$, $P - P$, $V - V$, $A - A$, $S^i - S^i$, $P^i - P^i$, $V^i - V^i$ and $A^i - A^i$ color-singlet and color-triplet meson-meson type currents, the $S$, $P$, $V$ and $A$ denote the scalar, pseudoscalar, vector and axial-vector respectively. The color-singlet meson-meson type currents, for example, $\bar{d}(x)\gamma_\mu\gamma_5 u(x)\bar{c}(x)\gamma^\alpha\gamma_5 c(x)$, $\bar{d}(x)i\gamma_\mu u(x)\bar{c}(x)i\gamma_5 c(x)$, have very small two-particle reducible contributions \[^{[32]}\].

After performing the standard procedure of the QCD sum rules, we obtain the following four sum rules for the interpolating currents contain two $s$ quarks:

\[
\lambda_Z^2 e^{-\frac{M^2}{M^2}} = \int_\Delta^{s_0} ds \rho_\pm(s)e^{-\frac{s}{M^2}},
\]

(11)

the explicit expressions of the spectral densities $\rho_\pm(s)$ are presented in the appendix, the $+$ and $−$ denote the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type interpolating currents respectively; the $s_0$ is the continuum threshold parameter and the $M^2$ is the Borel parameter. We can obtain four sum rules in the $c\bar{c}q\bar{q}$ and $b\bar{b}q\bar{q}$ channels with a simple replacement $m_s \rightarrow m_q$, $(s\bar{s}) \rightarrow \langle \bar{q}q \rangle$ and $(s\bar{g}_s\sigma Gs) \rightarrow \langle \bar{q}g_q\sigma Gq \rangle$.

We carry out the operator product expansion to the vacuum condensates adding up to dimension-10. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in large $N_c$ limit. In this article, we take into account the contributions from the quark condensates, mixed condensates, and neglect the contributions from the gluon condensate. The contributions from the gluon condensates are suppressed by large denominators and would not play any significant roles for the light tetraquark states \[^{[33, 34]}\], the heavy tetraquark state \[^{[8]}\] and the heavy molecular state \[^{[35]}\]. There are many terms involving the gluon condensates for the heavy tetraquark states and heavy molecular states in the operator product expansion (one can consult Refs.\[^{[8, 33, 35]}\] for example), we neglect the gluon condensates for simplicity.

Differentiate the Eq.(11) with respect to $\frac{1}{M^2}$, then eliminate the pole residues $\lambda_Z$, we can obtain the sum rules for the masses of the tetraquark quark states $Z$,

\[
M_Z^2 = \frac{\int_\Delta^{s_0} ds \frac{d}{d(-1/M^2)}\rho_\pm(s)e^{-\frac{s}{M^2}}}{\int_\Delta^{s_0} ds \rho_\pm(s)e^{-\frac{s}{M^2}}},
\]

(12)

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24\pm 0.01$ GeV)$^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2)\langle \bar{q}q \rangle$, $\langle \bar{g}_s\sigma Gq \rangle = m_0^2\langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.2)$ GeV$^2$, $m_s = (0.14 \pm 0.01)$ GeV, $m_u = m_d \approx 0$, $m_c = (1.35 \pm 0.10)$ GeV and $m_b = (4.8 \pm 0.1)$ GeV at the energy scale $\mu = 1$ GeV \[^{[26, 27]}\].

In the conventional QCD sum rules \[^{[26, 27]}\], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel
parameter $M^2$ and threshold parameter $s_0$. We impose the two criteria on the heavy
tetraquark states to choose the Borel parameter $M^2$ and threshold parameter $s_0$.

In Refs. [8, 14], we assume that the resonance-like structures $Z(4050)$ and $Z(4250)$
are scalar tetraquark states which consist of the scalar diquark pairs, and the threshold parameter tentatively as
are scalar tetraquark states which consist of the scalar diquark pairs, and take the
threshold parameter tentatively as $s_0 = (4.248 + 0.5)^2 \text{GeV}^2 \approx 23 \text{GeV}^2$ to take
into account all possible contributions from the ground states, where the energy gap
between the ground states and the first radial excited states is chosen to be 0.5 GeV.
Then we take into account the $SU(3)$ symmetry of the light flavor quarks and the
mass difference between the heavy quarks, choose other threshold parameters
tentatively, and use those values as a guide to determine the threshold parameters
$s_0$ with the QCD sum rules.

In this article, we study the scalar hidden charm and hidden bottom tetraquark
states which consist of the axial-axial type and the vector-vector type diquark
pairs, and search for other possible tetraquark structures of the resonance-like states
$Z(4050)$ and $Z(4250)$. Naively, we expect the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type and the $C\gamma_\mu - C\gamma^\mu$
type tetraquark states have larger masses than the corresponding $C\gamma_5 - C\gamma_5$ type
tetraquark states, and use the threshold parameters in Ref. [14] as a guide to deter-
mine the threshold parameters $s_0$ with the QCD sum rules.

The contributions from the high dimension vacuum condensates in the operator
product expansion are shown in Figs.1-2, where (and thereafter) we use the $\langle \bar{q}q \rangle$
to denote the quark condensates $\langle \bar{q}q \rangle$, $\langle ss \rangle$ and the $\langle \bar{q}g_s \sigma Gq \rangle$ to denote the mixed
condensates $\langle \bar{q}g_s \sigma Gq \rangle$, $\langle sg_s \sigma Gs \rangle$. From the figures, we can see that the contributions
from the high dimension condensates change quickly with variation of the Borel
parameter at the values $M^2 \leq 2.6 \text{GeV}^2 (2.8 \text{GeV}^2)$ and $M^2 \leq 7.2 \text{GeV}^2 (7.6 \text{GeV}^2)$
in the hidden charm and hidden bottom channels respectively for the $C\gamma_\mu - C\gamma^\mu$
($C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$) type interpolating currents, such an unstable behavior cannot lead
to stable sum rules, our numerical results confirm this conjecture, see Fig.4.

At the values $M^2 \geq 2.6 \text{GeV}^2 (2.8 \text{GeV}^2)$ and $s_0 \geq 23 \text{GeV}^2 (27 \text{GeV}^2)$, the contributions
from the $\langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle$ term are less than 14% (23.5%) in the
$\bar{c}\bar{c}q\bar{q}$ channel, the corresponding contributions are less than 4% (13%) in the $c\bar{c}s\bar{s}$
channels; the contributions from the vacuum condensate of the highest dimension
$\langle \bar{q}g_s \sigma Gq \rangle^2$ are less than 2.5% (2.5%) and 1.5% (3%) in the $c\bar{c}q\bar{q}$ and $c\bar{c}s\bar{s}$ channels
respectively; we expect the operator product expansion is convergent for the $C\gamma_\mu - C\gamma^\mu$
($C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$) type interpolating currents in the hidden charm channels.

At the values $M^2 \geq 7.2 \text{GeV}^2 (7.6 \text{GeV}^2)$ and $s_0 \geq 136 \text{GeV}^2 (146 \text{GeV}^2)$, the contributions
from the $\langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle$ term are less than 10.5% (19%) in the
$\bar{b}\bar{b}q\bar{q}$ channel, the corresponding contributions are less than 3.5% (8.5%) in the $b\bar{b}s\bar{s}$
channels; the contributions from the vacuum condensate of the highest dimension
$\langle \bar{q}g_s \sigma Gq \rangle^2$ are less than 5% (6%) and 3% (6%) in the $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels
respectively; we also expect the operator product expansion is convergent for the $C\gamma_\mu - C\gamma^\mu$
($C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$) type interpolating currents in the hidden bottom channels.

In this article, we take the uniform Borel parameter $M^2_{min}$, i.e. $M^2_{min} \geq 2.6 \text{GeV}^2$
(2.8 GeV$^2$) and $M^2_{min} \geq 7.2 \text{GeV}^2 (7.6 \text{GeV}^2)$ in the hidden charm and hidden bottom
channels respectively for the $C_{\gamma\mu} - C_{\gamma\mu}$ ($C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$) type interpolating currents.

In Fig.3, we show the contributions from the pole terms with variation of the Borel parameters and the threshold parameters. The pole contributions are larger than (or equal) 50% (52%) at the value $M^2 \leq 3.2$ GeV$^2$ and $s_0 \geq 23$ GeV$^2$ (27 GeV$^2$), 24 GeV$^2$ (28 GeV$^2$) in the $c\bar{c}q\bar{q}$, $c\bar{s}s$ channels respectively, and larger than (or equal) 51% (52%) at the value $M^2 \leq 8.2$ GeV$^2$ and $s_0 \geq 136$ GeV$^2$ (146 GeV$^2$), 138 GeV$^2$ (148 GeV$^2$) in the $b\bar{b}q\bar{q}$ and $b\bar{s}s\bar{s}$ channels respectively for the $C_{\gamma\mu} - C_{\gamma\mu}$ ($C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$) type interpolating currents. Again we take the uniform Borel parameter $M^2_{\text{max}}$, i.e. $M^2_{\text{max}} \leq 3.2$ GeV$^2$ and $M^2_{\text{max}} \leq 8.2$ GeV$^2$ in the hidden charm and hidden bottom channels respectively.

For the $C_{\gamma\mu} - C_{\gamma\mu}$ type interpolating currents, the threshold parameters are taken as $s_0 = (24 \pm 1)$ GeV$^2$, $(25 \pm 1)$ GeV$^2$, $(138 \pm 2)$ GeV$^2$, and $(140 \pm 2)$ GeV$^2$ in the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{s}s\bar{s}$ channels respectively; the Borel parameters are taken as $M^2 = (2.6 - 3.2)$ GeV$^2$ and $(7.2 - 8.2)$ GeV$^2$ in the hidden charm and hidden bottom channels respectively.

For the $C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$ type interpolating currents, the threshold parameters are taken as $s_0 = (28 \pm 1)$ GeV$^2$, $(29 \pm 1)$ GeV$^2$, $(148 \pm 2)$ GeV$^2$, and $(150 \pm 2)$ GeV$^2$ for the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{s}s\bar{s}$ channels, respectively; the Borel parameters are taken as $M^2 = (2.8 - 3.2)$ GeV$^2$ and $(7.6 - 8.2)$ GeV$^2$ in the hidden charm and hidden bottom channels respectively.

In those regions, the pole contributions are about (47−75)%, (51−78)%, (51−70)% and (53−72)% in the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{s}s\bar{s}$ channels respectively for the $C_{\gamma\mu} - C_{\gamma\mu}$ type interpolating currents; while the pole contributions are about (52−75)%, (52−74)%, (52−68)% and (52−67)% in the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{s}s\bar{s}$ channels respectively for $C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$ type interpolating currents; the two criteria of the QCD sum rules are fully satisfied [26, 27].

If we take uniform pole contributions, the interpolating current with more $s$ quarks requires slightly larger threshold parameter due to the $SU(3)$ breaking effects, see Fig.3. The threshold parameters in the $c\bar{c}q\bar{q}$ and $b\bar{b}q\bar{q}$ channels are slightly smaller than the corresponding ones in the $c\bar{s}s\bar{s}$ and $b\bar{s}s\bar{s}$ channels respectively. Naively, we expect the tetraquark state with more $s$ quarks will have larger mass, our numerical calculations confirm this conjecture, see Fig.4. In that figure we plot the tetraquark state masses $M_Z$ with variation of the Borel parameters and the threshold parameters.

The Borel windows $M^2_{\text{max}} - M^2_{\text{min}}$ change with variations of the threshold parameters $s_0$, see Fig.3. In this article, the Borel windows are taken as 0.6 GeV$^2$ (0.4 GeV$^2$) and 1.0 GeV$^2$ (0.6 GeV$^2$) in the hidden charm and hidden bottom channels respectively for the $C_{\gamma\mu} - C_{\gamma\mu}$ ($C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$) type interpolating currents; they are small enough. Furthermore, we take uniform Borel windows and smear the dependence on the threshold parameters $s_0$ in each channel. If we take larger threshold parameters, the Borel windows are larger and the resulting masses are larger, see Fig.4. In this article, we intend to calculate the possibly lowest masses which are supposed to be the ground state masses by imposing the two criteria of the QCD sum rules.
Table 1: The masses (in unit of GeV) of the scalar tetraquark states, the values for the $C\gamma_5 - C\gamma_5$ type scalar tetraquark states are taken from Ref.\cite{14}.

| tetraquark states | $C\gamma_\mu \gamma_5 - C\gamma_\mu^\ast \gamma_5$ | $C\gamma_\mu - C\gamma_\mu^\ast$ | $C\gamma_5 - C\gamma_5$ | Refs.\cite{16,17} |
|-------------------|-----------------------------------|---------------------------------|-------------------|-------------------|
| $c\bar{c}ss\bar{s}$ | $4.82 \pm 0.14$ | $4.45 \pm 0.16$ | $4.44 \pm 0.16$ | $4.110$ |
| $c\bar{c}q\bar{q}$  | $4.56 \pm 0.14$ | $4.36 \pm 0.18$ | $4.37 \pm 0.18$ | $3.852$ |
| $b\bar{b}ss\bar{s}$ | $11.70 \pm 0.18$ | $11.23 \pm 0.16$ | $11.31 \pm 0.16$ | $11.133$ |
| $b\bar{b}q\bar{q}$  | $11.38 \pm 0.13$ | $11.14 \pm 0.19$ | $11.27 \pm 0.20$ | $10.942$ |

Table 2: The pole residues (in unit of $10^{-2}$ GeV$^5$ and $10^{-1}$ GeV$^5$ for the hidden charm and hidden bottom channels respectively) of the scalar tetraquark states.

| tetraquark states | $C\gamma_\mu \gamma_5 - C\gamma_\mu^\ast \gamma_5$ | $C\gamma_\mu - C\gamma_\mu^\ast$ |
|-------------------|-----------------------------------|---------------------------------|
| $c\bar{c}ss\bar{s}$ | $7.92 \pm 1.95$ | $7.05 \pm 1.45$ |
| $c\bar{c}q\bar{q}$  | $6.32 \pm 2.30$ | $5.85 \pm 1.30$ |
| $b\bar{b}ss\bar{s}$ | $4.46 \pm 1.04$ | $3.68 \pm 0.80$ |
| $b\bar{b}q\bar{q}$  | $3.35 \pm 1.00$ | $3.06 \pm 0.66$ |

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole resides of the scalar tetraquark states $Z$, which are shown in Figs.5-6 and Tables 1-2. In this article, we calculate the uncertainties $\delta$ with the formula

$$
\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 \bigg|_{x_i=\bar{x}_i} \left( x_i - \bar{x}_i \right)^2}, \quad (13)
$$

where the $f$ denote the hadron mass $M_Z$ and the pole residue $\lambda_Z$, the $x_i$ denote the input QCD parameters $m_c, m_b, \langle \bar{q}q \rangle, \langle \bar{s}s \rangle, \cdots$. As the partial derivatives $\frac{\partial f}{\partial x_i}$ are difficult to carry out analytically, we take the approximation $\left( \frac{\partial f}{\partial x_i} \right)^2 \left( x_i - \bar{x}_i \right)^2 \approx \left[ f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i) \right]^2$ in the numerical calculations.

From Tables 1-2, we can see that the uncertainties of the masses $M_Z$ are rather small (about $(3-4)\%$ in the hidden charm channels and $(1-2)\%$ in the hidden bottom channels), while the uncertainties of the pole residues $\lambda_Z$ are rather large (about $(20-40)\%$). The uncertainties of the input parameters ($\langle \bar{q}q \rangle, \langle \bar{s}s \rangle, \langle \bar{s}g_s \sigma G_s \rangle, \langle \bar{q}g_s \sigma G_q \rangle, m_s, m_c$ and $m_b$) vary in the range $(2-25)\%$, the uncertainties of the pole residues $\lambda_Z$ are reasonable. We obtain the squared masses $M_Z^2$ through a fraction, see Eq.(12), the uncertainties in the numerator and denominator which origin from a given input parameter (for example, $\langle \bar{s}s \rangle, \langle \bar{s}g_s \sigma G_s \rangle$) cancel out with each other, and result in small net uncertainty.

The $SU(3)$ breaking effects for the masses of the hidden charm and hidden
bottom tetraquark states are buried in the uncertainties. The $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_5 - C\gamma_5$ type interpolating currents result in almost the same ground state masses, while the ground masses of the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type tetraquark states are larger than the corresponding ones of the $C\gamma_\mu - C\gamma^\mu$ type tetraquark states about $(0.2-0.5)$ GeV. Naively, we expect the axial and vector diquarks have larger masses than the corresponding scalar diquarks, and the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type scalar tetraquark states have larger masses than the corresponding $C\gamma_5 - C\gamma_5$ type scalar tetraquark states, because the attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet $\bar{3}_c$, flavor antitriplet $\bar{3}_f$ and spin singlet $1_s$ [30, 31].

The meson $Z(4250)$ may be a scalar tetraquark state ($c\bar{c}u\bar{d}$), irrespective of the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_5 - C\gamma_5$ type diquark structures [8, 14], the decay $Z(4250) \rightarrow \pi^+\chi_{c1}$ can take place with the OZI super-allowed "fall-apart" mechanism, which can take into account the large total width naturally. Other possibilities, such as a hadro-charmonium resonance and a $D_1^+\bar{D}^0 + D^+\bar{D}_1^0$ molecular state are not excluded; more experimental data are still needed to identify it. It is difficult to identify the $Z(4050)$ as the scalar tetraquark state as the lower bound of the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_5 - C\gamma_5$ type scalar tetraquark states are larger than the $Z(4050)$ about 130 MeV.

In this article, we calculate the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states consist of the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_5 - C\gamma_5$ type diquark pairs by imposing the two criteria of the QCD sum rules. In fact, we usually consult the experimental data in choosing the Borel parameter $M^2$ and the threshold parameter $s_0$ [38]. There lack experimental data for the phenomenological hadronic spectral densities of the tetraquark states, the present predictions can’t be confronted with the experimental data. The nonet scalar mesons below 1 GeV (the $f_0(980)$ and $a_0(980)$ especially) are good candidates for the tetraquark states [29, 39, 40]. However, they can’t satisfy the two criteria of the QCD sum rules, and result in a reasonable Borel window, although it is not an indication non-existence of the light tetraquark states (For detailed discussions about this subject, one can consult Refs.[8, 41]). The QCD sum rules is just a QCD model.

For the conventional mesons and baryons, the Borel window $M^2_{\text{max}} - M^2_{\text{min}}$ is rather large and reliable QCD sum rules can be obtained. However, for the multiquark states i.e. tetraquark states, pentaquark states, hexaquark states, etc, the spectral densities $\rho \sim s^n$ with $n$ is larger than the ones for the conventional hadrons, integral $\int_0^\infty s^n \exp\{-\frac{s}{M^2}\} ds$ converges more slowly, which results in some sensitivities to the threshold parameters inevitably. We select the threshold parameters and Borel parameters by imposing the two criteria of the QCD sum rules, and intend to select the possibly lowest threshold parameter which corresponds to the ground state.

In Table 1, we also present the results for the $C\gamma_\mu - C\gamma^\mu$ type scalar tetraquark states from a relativistic quark model based on a quasipotential approach in QCD.
[16, 17], the central values of our predictions are larger than the corresponding ones from the quasipotential model about \((0.1 - 0.5)\) GeV. The predications based on constituent diquark model \(M_{ccq\bar{q}} = 3723\) MeV [18] and \(M_{c\bar{c}q\bar{q}} = 3834\) MeV [20] for the tetraquark states \(c\bar{c}q\bar{q}\) and \(c\bar{c}s\bar{s}\) respectively) are about 0.6 GeV smaller than the corresponding ones in the present work.

The predictions of Refs. [18, 19, 20] depend heavily on the assumption that the light scalar mesons \(a_0(980)\) and \(f_0(980)\) are tetraquark states, the basic parameters (constituent diquark masses) are estimated thereafter. In the conventional quark models, the constituent quark masses are taken as the basic input parameters, and fitted to reproduce the mass spectra of the well known mesons and baryons. However, the present experimental knowledge about the phenomenological hadronic spectral densities of the tetraquark states is rather vague, whether or not there exist tetraquark states is not confirmed with confidence. The predicted constituent diquark masses cannot be confronted with the experimental data.

The LHCb is a dedicated \(b\) and \(c\)-physics precision experiment at the LHC (large hadron collider). The LHC will be the world’s most copious source of the \(b\) hadrons, and a complete spectrum of the \(b\) hadrons will be available through gluon fusion. In proton-proton collisions at \(\sqrt{s} = 14\) TeV, the \(b\bar{b}\) cross section is expected to be \(\sim 500\mu b\) producing \(10^{12}\) \(b\bar{b}\) pairs in a standard year of running at the LHCb operational luminosity of \(2 \times 10^{32}\) cm\(^{-2}\) sec\(^{-1}\) [42]. The scalar tetraquark states (irrespective of the \(C\gamma_{\mu} - C\gamma_{\mu}\) type, the \(C\gamma_{\mu}\gamma_5 - C\gamma_{\mu}\gamma_5\) type and the \(C\gamma_5 - C\gamma_5\) type diquark structures) may be observed at the LHCb, if they exist indeed. We can search for the scalar hidden charm tetraquark states in the \(D\bar{D}, D^*\bar{D}^*, D_s\bar{D}_s, D_s^*\bar{D}_s^*, J/\psi\rho, J/\psi\phi, J/\psi\omega, \eta_c\pi, \eta_c\eta, \cdots\) invariant mass distributions and search for the scalar hidden bottom tetraquark states in the \(B\bar{B}, B^*\bar{B}^*, B_s\bar{B}_s, B_s^*\bar{B}_s^*, \Upsilon\rho, \Upsilon\phi, \Upsilon\omega, \eta_b\pi, \eta_b\eta, \cdots\) invariant mass distributions.

4 Conclusion

In this article, we study the mass spectrum of the scalar hidden charm and hidden bottom tetraquark states which consist of the axial-axial type and the vector-vector type diquark pairs with the QCD sum rules, and observe that the scalar-scalar type and the axial-axial type tetraquark states have almost the same ground state masses while the vector-vector type tetraquark states have slightly larger ground state masses. Furthermore, we compare the present predictions with the corresponding ones from a relativistic quark model based on a quasipotential approach in QCD, and discuss the values from the constituent diquark model based on the constituent diquark masses and the spin-spin interactions. We can search for the scalar hidden charm and bottom tetraquark states at the LHCb.

We can perform Fierz re-ordering in both the Dirac spin space and the color space to express the tetraquark currents \(J(x)\) and \(\eta(x)\) into a series of \(S - S, P - P, V - V, A - A, S^i - S^i, P^i - P^i, V^i - V^i\) and \(A^i - A^i\) color-singlet and color-triplet
Cγ variation of the Borel parameter $\langle$ from the $23 \text{ GeV}$ $\alpha_s$ while in the hidden bottom channels they correspond to the threshold parameters $D = 132 \text{ GeV}$, $\beta_c$, $\gamma_c$, $C\gamma$, $\mu qg \bar{s}$ and $\sigma Gq \bar{G}$ terms respectively.

The (I) and (II) denote the contributions from the $\langle \bar{q}g_s\sigma G q \rangle^2$ and $\langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{q}g_s\sigma G q \rangle$ terms respectively. The $A$, $B$, $C$ and $D$ correspond to the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21 \text{ GeV}^2$, $22 \text{ GeV}^2$, $23 \text{ GeV}^2$, $24 \text{ GeV}^2$, $25 \text{ GeV}^2$ and $26 \text{ GeV}^2$ respectively in the hidden charm channels; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 132 \text{ GeV}^2$, $134 \text{ GeV}^2$, $136 \text{ GeV}^2$, $138 \text{ GeV}^2$, $140 \text{ GeV}^2$ and $142 \text{ GeV}^2$ respectively.

Figure 1: The contributions from the high dimension vacuum condensates with variation of the Borel parameter $M^2$ in the operator product expansion for the $C\gamma_{\mu} - C\gamma^{\mu}$ type interpolating currents. The (I) and (II) denote the contributions from the $\langle \bar{q}g_s\sigma G q \rangle^2$ and $\langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{q}g_s\sigma G q \rangle$ terms respectively. The $A$, $B$, $C$ and $D$ correspond to the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21 \text{ GeV}^2$, $22 \text{ GeV}^2$, $23 \text{ GeV}^2$, $24 \text{ GeV}^2$, $25 \text{ GeV}^2$ and $26 \text{ GeV}^2$ respectively in the hidden charm channels; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 132 \text{ GeV}^2$, $134 \text{ GeV}^2$, $136 \text{ GeV}^2$, $138 \text{ GeV}^2$, $140 \text{ GeV}^2$ and $142 \text{ GeV}^2$ respectively.
Figure 2: The contributions from the high dimension vacuum condensates with variation of the Borel parameter $M^2$ in the operator product expansion for the $C\gamma_\mu\gamma_5-C\gamma_\mu\gamma_5$ type interpolating currents. The (I) and (II) denote the contributions from the $\langle q\bar{q}\sigma Gq \rangle^2$ and $\langle q\bar{q}\rangle^2+\langle q\bar{q}\rangle\langle q\bar{q}\sigma Gq \rangle$ terms respectively. The $A$, $B$, $C$ and $D$ correspond to the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 25 \text{ GeV}^2$, $26 \text{ GeV}^2$, $27 \text{ GeV}^2$, $28 \text{ GeV}^2$, $29 \text{ GeV}^2$ and $30 \text{ GeV}^2$ respectively in the hidden charm channels; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 142 \text{ GeV}^2$, $144 \text{ GeV}^2$, $146 \text{ GeV}^2$, $148 \text{ GeV}^2$, $150 \text{ GeV}^2$ and $152 \text{ GeV}^2$ respectively.
Figure 3: The contributions from the pole terms with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{b}s\bar{s}$ channels respectively. The (I) and (II) denote the $C_{\gamma\mu} - C_{\gamma\mu}$ type and the $C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$ type interpolating currents respectively. For the $C_{\gamma\mu} - C_{\gamma\mu}$ ($C_{\gamma\mu\gamma_5} - C_{\gamma\mu\gamma_5}$) type interpolating currents, in the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21 \text{ GeV}^2 (25 \text{ GeV}^2)$, $22 \text{ GeV}^2 (26 \text{ GeV}^2)$, $23 \text{ GeV}^2 (27 \text{ GeV}^2)$, $24 \text{ GeV}^2 (28 \text{ GeV}^2)$, $25 \text{ GeV}^2 (29 \text{ GeV}^2)$ and $26 \text{ GeV}^2 (30 \text{ GeV}^2)$, respectively; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 132 \text{ GeV}^2 (142 \text{ GeV}^2)$, $134 \text{ GeV}^2 (144 \text{ GeV}^2)$, $136 \text{ GeV}^2 (146 \text{ GeV}^2)$, $138 \text{ GeV}^2 (148 \text{ GeV}^2)$, $140 \text{ GeV}^2 (150 \text{ GeV}^2)$ and $142 \text{ GeV}^2 (152 \text{ GeV}^2)$, respectively.
Figure 4: The masses of the scalar tetraquark states with variation of the Borel parameter $M^2$ and threshold parameter $s_0$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{b}s\bar{s}$ channels respectively. The (I) and (II) denote the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type interpolating currents respectively. For the $C\gamma_\mu - C\gamma^\mu$ ($C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$) type interpolating currents, in the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21$ GeV$^2$ (25 GeV$^2$), 22 GeV$^2$ (26 GeV$^2$), 23 GeV$^2$ (27 GeV$^2$), 24 GeV$^2$ (28 GeV$^2$), 25 GeV$^2$ (29 GeV$^2$) and 26 GeV$^2$ (30 GeV$^2$), respectively; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 132$ GeV$^2$ (142 GeV$^2$), 134 GeV$^2$ (144 GeV$^2$), 136 GeV$^2$ (146 GeV$^2$), 138 GeV$^2$ (148 GeV$^2$), 140 GeV$^2$ (150 GeV$^2$) and 142 GeV$^2$ (152 GeV$^2$), respectively.
Figure 5: The masses of the scalar tetraquark states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{b}s\bar{s}$ channels respectively. The (I) and (II) denote the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type interpolating currents respectively.
Figure 6: The pole residues of the scalar tetraquark states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}q\bar{q}$, $c\bar{s}s\bar{s}$, $b\bar{b}q\bar{q}$, and $b\bar{b}s\bar{s}$ channels respectively. The (I) and (II) denote the $C\gamma_\mu - C\gamma^\mu$ type and the $C\gamma_\mu\gamma_5 - C\gamma^\mu\gamma_5$ type interpolating currents respectively.
meson-meson type currents, there are contributions from the two-particle and many-particle reducible states, those contaminations are supposed to be small enough to be neglected safely. In fact, those contributions maybe considerable (and even out of control) and impair the predictive ability. In this article, we take the single pole approximation for the hadronic spectral densities, our predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules; the numerical results are rather good.

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Appendix

The spectral densities at the level of the quark-gluon degrees of freedom:

\[
\rho_\pm(s) = \frac{1}{256\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2 (s - \bar{m}_Q^2)^2 (7s^2 - 6s\bar{m}_Q^2 + \bar{m}_Q^4) + \frac{1}{256\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2 (s - \bar{m}_Q^2)^3 (3s - \bar{m}_Q^2) \\
\pm \frac{m_s m_Q}{128\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) (1 - \alpha - \beta)^2 (s - \bar{m}_Q^2)^2 (5s - 2\bar{m}_Q^2) \\
+ \frac{m_s \langle \bar{s}s \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) (1 - \alpha - \beta)(10s^2 - 12s\bar{m}_Q^2 + 3\bar{m}_Q^4) \\
+ \frac{m_s \langle \bar{s}s \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) (1 - \alpha - \beta)(2s - \bar{m}_Q^2) \\
\pm \frac{m_Q \langle \bar{s}s \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) (1 - \alpha - \beta)(s - \bar{m}_Q^2)(2s - \bar{m}_Q^2) \\
\pm \frac{m_Q \langle \bar{s}s \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(3s - 2\bar{m}_Q^2) \\
\frac{m_s \langle \bar{s}s \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta \bigg[ 2s - \bar{m}_Q^2 + \frac{s^2}{6} \delta(s - \bar{m}_Q^2) \bigg] \\
+ \frac{m_s \langle \bar{s}s \rangle}{2\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s - \bar{m}_Q^2) \\
+ \frac{m_s \langle \bar{s}s \rangle}{48\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha (1 - \alpha)(3s - 2\bar{m}_Q^2) \\
+ \frac{m_Q \langle \bar{s}s \rangle^2}{3\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha + \frac{m_s m_Q^2 \langle \bar{s}s \rangle \langle \bar{s}s \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \\
\pm \frac{m_s m_Q \langle \bar{s}s \rangle}{12\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \bigg[ 2 + s\delta(s - \bar{m}_Q^2) \bigg] \\
- \frac{m_Q \langle \bar{s}s \rangle \langle \bar{s}s \rangle}{6\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \bigg[ 1 + \frac{s}{M^2} \bigg] \delta(s - \bar{m}_Q^2) \\
\pm \frac{5m_s m_Q \langle \bar{s}s \rangle \langle \bar{s}s \rangle}{72\pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \bigg[ 1 + \frac{s}{M^2} + \frac{s^2}{2M^4} \bigg] \delta(s - \bar{m}_Q^2) \\
+ \frac{m_Q \langle \bar{s}s \rangle \langle \bar{s}s \rangle}{48\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha \bigg[ 2 + s\delta(s - \bar{m}_Q^2) \bigg] \\
\pm \frac{m_s m_Q \langle \bar{s}s \rangle \langle \bar{s}s \rangle}{288\pi^2 M^8} \int_{\alpha_i}^{\alpha_f} d\alpha \bigg[ 2 + s\delta(s - \bar{m}_Q^2) \bigg],
\]
where $\alpha_f = \frac{1+\sqrt{1-4m_Q^2/s}}{2}$, $\alpha_i = \frac{1-\sqrt{1-4m_Q^2/s}}{2}$, $\beta_i = \frac{am_Q^2}{as-m_Q^2}$, $\tilde{m}_Q = \frac{(\alpha+\beta)m_Q^2}{\alpha\beta}$, $\tilde{\tilde{m}}_Q = \frac{m_Q^2}{\alpha(1-\alpha)}$, and $\Delta = 4(m_Q + m_s)^2$.

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