UV conformal window for asymptotic safety

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Motivation

• Asymptotic safety: interactive fixed point; residual interactions in the UV
• Critical phenomena: Wilson-Fisher fixed point (\(\phi^4\) theory in \(d = 4 - \epsilon\) dimensions)
• A quantum theory of gravity may be non-perturbatively renormalizable provided there is an interactive UV fixed point.
• Necessary conditions and no-go theorems for asymptotic safety in gauge theories have been derived at weak coupling.
• There are exact proofs of existence in simple and semi-simple gauge groups, as well as SUSY.
• Higher dimensional scalar operators are ok. Fixed points are available away from 4d.
• Asymptotically safe model building now available.
• IR conformal windows in QCD-like theories have been extensively studied, and are known to extend past the domain of perturbation theory.
• What is the size of the conformal window of asymptotically safe theories? How many fields are required for asymptotic safety?

(Wilson & Fisher, 1972)
(Weinberg, 1979)
(Bond & Litim, 2017a)
(Litim & Sanino, 2014)
(Bond & Litim, 2017b)
(Bond & Litim, 2018)
(Buyukbese & Litim, 2017)
(Codello, Langæble, Litim, & Sannino, 2016)
(Bond, Hiller, Kowalska & Litim, 2017)
(Banks & Zaks, 1982)
(Appelquist et al., 2008)
(Del Debbio, 2011)
Interacting fixed points

• Gauge renormalization group (RG) running at two loop in perturbation theory

\[ \beta = \frac{d\alpha}{d\ln\mu} = -B\alpha^2 + C\alpha^3 \]

• The RG flow vanishes at the fixed point

\[ \beta^* = 0 \quad \alpha^* = \frac{B}{C} \]

• Physical coupling: \( B \) and \( C \) must have the same sign

• Weak coupling: \( B \) must be much smaller than \( C \)
Interacting fixed points

• Give-up asymptotic freedom: \( B < 0; \quad C < 0 \)

• \( C > 0 \) for any simple or semi simple gauge group, for any matter fields multiplicities and representation

• \( C < 0 \) can only be achieved with Yukawa interactions

• Necessary ingredients:
  • Gauge group (simple or semi-simple)
  • Fermions
  • Scalars
  • Yukawa interactions

(Bond & Litim, 2017a)
Asymptotically safe theory

\[ L = L_{YM} + L_{\text{kin.}} + L_{\text{Yuk.}} + L_{\text{pot.}} \]

\[ L_{YM} = -\frac{1}{2} \text{Tr} \, F^{\mu\nu} F_{\mu\nu} \]

\[ L_{\text{kin.}} = \text{Tr} \left( \bar{Q} i\gamma^5 Q \right) + \text{Tr} \left( \partial_\mu H^\dagger \partial^\mu H \right) \]

\[ L_{\text{Yuk.}} = -y \, \text{Tr} \left( \bar{Q}_L H Q_R \right) + \text{h.c.} \]

\[ L_{\text{pot.}} = -u \, \text{Tr} \left( H^\dagger H \, H^\dagger H \right) - v \left( \text{Tr} \, H^\dagger H \right)^2 \]

• 4d gauge theory
• NF fermions in the fundamental representation of SU(NC)
• Meson-like complex scalar NF x NF matrix, uncharged
• Yukawa & scalar quartic interactions

(Litim & Sanino, 2014)
Asymptotically safe theory

\[ \begin{align*}
\alpha_g &= \frac{g^2 N_C}{(4\pi)^2}; & \alpha_y &= \frac{y^2 N_C}{(4\pi)^2}; & \alpha_u &= \frac{u N_F}{(4\pi)^2}; & \alpha_v &= \frac{v N_F^2}{(4\pi)^2}.
\end{align*} \]

- Take the Veneziano limit; field multiplicities are taken to infinity, keeping their ratio fixed
- One free parameter left, which can be taken to be perturbatively small
- The theory has a weakly coupled fixed point

\[ N_C \to \infty; \quad N_F \to \infty \]

\[ \epsilon = \frac{N_F}{N_C} \frac{11}{2} \]

\[ 0 < \epsilon \ll 1 \]
Asymptotically safe theory

- Weak coupling: series expansion in a small parameter is justified

\[ \alpha_i^* = \lambda_1 \epsilon + \lambda_2 \epsilon^2 + \lambda_3 \epsilon^3 + O(\epsilon^n) \]

- The beta functions can then be expanded in epsilon. Exactly at the fixed point, all orders must vanish individually. The resulting equations determine the \( \lambda \) coefficients.

- In perturbation theory only the first few loop orders of the beta functions are known.

- Approximations have to be consistent.
Asymptotically safe theory

• The approximation is denoted by the number of loop orders retained:

\[
\begin{align*}
\text{n loop gauge} & \quad \equiv (n, 0, 0) \\
\text{m loop Yukawa} & \quad \equiv (0, m, 0) \\
\text{l loop scalar} & \quad \equiv (0, 0, l)
\end{align*}
\]

• The approximation \((n + 1, n, n)\) completely determines the coefficient of order \(O(\epsilon^n)\) in the series expansion of the fixed point.

\[
\alpha_i^* = \lambda_{1i}\epsilon + \lambda_{2i}\epsilon^2 + \lambda_{3i}\epsilon^3 + O(\epsilon^n)
\]

(Bond, Litim, Medina Vazquez & Steudtner, 2018)
Fixed points and vacuum stability

• We have computed the running of the scalar self-interactions to two loop. This means our most advanced approximation is (322), an improvement over the original analysis (321).

• The fixed point can be determined analytically to order $\epsilon^2$. Numerically, we find:

$$\begin{align*}
\alpha_g^* &= 0.4561\epsilon + 0.7808\epsilon^2 + O(\epsilon^3) \\
\alpha_y^* &= 0.2105\epsilon + 0.5082\epsilon^2 + O(\epsilon^3) \\
\alpha_u^* &= 0.1998\epsilon + 0.4403\epsilon^2 + O(\epsilon^3) \\
\alpha_v^* &= -0.1373\epsilon - 0.6318\epsilon^2 + O(\epsilon^3)
\end{align*}$$

(Bond et al., 2018)
In order to have a stable vacuum state, we require for the scalar potential to be bounded from below. In the present setting, this means:

\[ \alpha_u^* > 0, \quad \alpha_u^* + \alpha_v^* > 0 \]  
(Litim, Mojaza & Sannino, 2016)

Comparing with the previous approximations, we detect a change of sign in the subleading term. This implies that the vacuum becomes unstable at a finite value of epsilon.

\[
\begin{align*}
\alpha_u^* + \alpha_v^*|_{(211)} &= 0.0625\epsilon + O(\epsilon^3) \\
\alpha_u^* + \alpha_v^*|_{(321)} &= 0.0625\epsilon + 0.1535\epsilon^2 + O(\epsilon^3) \\
\alpha_u^* + \alpha_v^*|_{(322)} &= 0.0625\epsilon - 0.1915\epsilon^2 + O(\epsilon^3)
\end{align*}
\]
Fixed points and vacuum stability

- Vacuum instability imposes a physical constraint.
  \[ 0 < \epsilon < 0.326 \]

- This effect is driven by the two loop running of the scalar self-interactions.

- The plot shows the UV conformal window. Blue dots correspond to smallest field multiplicities:

  \((NC,NF) = (3,17), (4,23), (5,28), (5,29), (6,34), (7,39), \ldots\)

(Bond et al., 2018)
The size of the conformal window can be understood as a competition of fluctuations.

Consider the following qualitative indicator:

\[ \beta_g\big|_{(322)} = 10.24\epsilon^5, \quad \beta_y\big|_{(322)} = -1.71\epsilon^4, \quad \beta_u\big|_{(322)} = 1.70\epsilon^4, \quad \beta_v\big|_{(322)} = 7.24\epsilon^4 \]

This is obtained by inserting the fixed point at order \( \epsilon^2 \) back into the beta functions.

Positive shifts to the flow equations destabilize the fixed point, while negative ones stabilize it.

The running of the gauge and scalar interactions tend to close the conformal window, while the Yukawa interaction tends to open it.
• We now consider the (incomplete) subleading terms in the beta functions

• The plot shows the conformal window of approximation (321) bounded by a fixed point merger (light yellow), and (322), bounded by vacuum instability (dark yellow).

• Vacuum instability still poses the tightest constraint at a numerically smaller value

\[ \epsilon_{max} \approx 0.09 \ldots 0.13 \]

• Blue dots are:

\( (N_C, N_F) = (7,39), (9,50), (11,61), (12,67), \ldots \)

(Bond et al., 2018)
• Vacuum instability imposes the tightest constraint in the theory. This is driven by the two loop running of the scalar self-interactions.

• Two pictures: Which one is right? Both are approximations that we use to understand the trend at higher loop orders.
Conclusions

• The UV conformal window remains roughly within the region of perturbation theory.

• It would be interesting to obtain the next approximation (433), then we can start to say something about the convergence of the boundary of the conformal window.
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