Excitation of surface plasmon polaritons in photonic crystal waveguides that involve dispersive metamaterial

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Abstract. Plasmonics is an area of research that deals mainly with the study of the properties of surface plasmon (SPs) that are collective oscillations of the electron gas in a metal. That is, when the light waves couple with the electronic oscillations, they form a new quasi-particle called the surface plasmon polariton (SPP) that propagates through the surface of the nanometer-sized structure. In this work, we present a numerical study of a photonic crystal waveguide (PCW) composed by an array of cylindrical inclusions with smooth surfaces of dispersive metamaterial. The numerical technique we have used to perform the calculations is known as the “Integral Equation Method”. First, the numerical results are presented of a PCW of infinite length formed by an array of inclusions of dispersive metamaterial (LHM), showing that there is the presence of an SP mode at the frequency $\omega r = 0.7519$. Subsequently, some numerical results of the optical response are presented when the PCW is of finite length, showing the presence of the same surface mode around of $\omega r = 0.7510$. This excitation of the surface plasmon polariton in the proposed waveguide can be another alternative for the development of innumerable applications in several fields of science and technology ranging from biomedicine to telecommunications.

1. Introduction
In recent decades, many researchers have done several theoretical studies on how to control the electromagnetic properties of materials and the behavior of light through them. That is, there is interest in designing materials that are capable of controlling the propagation of electromagnetic waves with a specific wavelength, or that allow these waves trapped or localized in a particular region of space. Within such materials, we have the photonic crystals (PhCs) that constitute periodic arrays of different materials with a size of the unit cell is about the wavelength. These photonic structure have a periodic and ordered modulation of the dielectric constant (or index of refraction), making them have the potential to develop a new integrated optical circuit technology [1]. Other structured materials that have interesting properties in the behavior of light when interacting with these are called “Metamaterials” or “Left-Handed Materials” (LHMs) and owe their name to the fact that the electric field $E$, the magnetic field $H$ and the $k$ wave vector form a system of orthogonal vectors with a left orientation for a wave propagating through these media [2].

Numerical simulations play an important role in the design of systems like those outlined above, particularly the modeling of photonic crystal fibers. To date, various modeling methods in which not only a full-vector model but also an approximate scalar model is used have been developed such as effective index approach [3], plane-wave expansion (PWE) method [4], localized-function method [5],
multipole method (MM) [6]. In this work, we study a photonic crystal waveguide formed with two parallel plates, that includes an array of cylindrical inclusions with periodicity in the x-direction. The material in these plates or inclusions is either a conductor or a dispersive metamaterial. Subsequently, we show the presence of a surface plasmon mode on the vacuum-LHM interface of the PCW that involves an LHM based on ideas presented in recently-published work [7].

This article is organized as follows. In section 2 we present the system under study and introduce an integral method which is used for calculating the electromagnetic modes, and the optical response of our system [8]. In Sec. 3 we present the numerical analysis of the optical response of a PCW which consists of two flat surfaces containing cylindrical inclusions with smooth surfaces of dispersive metamaterial. Finally, we present our main conclusions in Sec. 4.

2. Theoretical approach
The interest of this work is to obtain the electromagnetic response of a PCW of flat surfaces with cylindrical inclusions of dispersive metamaterial with periodicity in the x-direction. This system is shown in Fig 1.

Figure 1. (a) Square unit cell of length $D$ is composed of two different materials with dielectric constants $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$. (b) Schematic of a photonic crystal waveguide, formed with two PEC flat surfaces of width $l$ and length $d$ and a periodic array of circular inclusions. The length of the system in y-direction is $L_y=2l+b$. The 1/e half-width of the modulus of the incident Gaussian beam projected on the plane $x=d$ is $g$. The angles of incidence $\theta_0$ and scattering $\theta_s$ (for transmission and reflection) are also shown; they are defined as positive in the sense indicated in the figure.

In the case that materials that make up the PCW is a conductive medium, the behavior of when interacting with light will be modeled by means of Drude Model (DM). In this model, we have that the dielectric function for conductive media is given by

$$\varepsilon(\omega) = \varepsilon_R(\omega) + i\varepsilon_i(\omega) = n^2(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2} + i \frac{\omega_p^2 n}{\omega^2 + \gamma^2}\right)$$

(1)

where $\omega_p = \sqrt{\frac{N e^2}{m \varepsilon_0}}$ is the plasma frequency, below which the electrical permittivity is negative and consequently the propagation of electromagnetic waves is prohibited and above, the permittivity is positive, so the medium is transparent and allows the propagation of electromagnetic waves. On the other hand, when there is a dispersive LHM, the optical properties of the metamaterial are given by $\varepsilon(\omega)$ and $\mu(\omega)$ which are expressed in the form [9]

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \ \text{y} \ \mu(\omega) = 1 - \frac{F \omega^2}{\omega^2 - \omega_0^2}$$

(2)

with the parameters $\omega_p = 10c/D$, $\omega_0 = 4c/D$, and $F = 0.56$. The region where this LHM presents a negative refractive index is within the frequency range $\omega_0 < \omega < \omega_{LM}$ with $\omega_{LM} = \omega_0/\sqrt{1-F}$. 
2.1. Integral Equation Method

The numerical technique used is briefly described and is known as the Integral Equation Method that has been developed by A. Mendoza and his collaborators [7, 8].

Assuming the time dependence $e^{-i\omega t}$ for electromagnetic fields, the wave equation can be transformed to the Helmholtz equation

$$\nabla^2 \Psi_j(\mathbf{r}) + k_j^2 \Psi_j(\mathbf{r}) = 0$$

(3)

In this equation, $\Psi_j(\mathbf{r})$ represents the electric field $E_z$ in the case of polarization TE in the $j$-th medium (Fig. 1) and $\mathbf{r} = x\hat{i} + y\hat{j}$ is the position vector in the $X$-$Y$ plane. The magnitude of the wave vector is given by $k_j = n_j(\omega)\omega/c$ being $n_j(\omega) = \pm \sqrt{\mu_j(\omega)\varepsilon_j(\omega)}$ the refractive index involves the material properties, which is given in terms of the magnetic permeability $\mu_j(\omega)$, and the electric permittivity is given by $\varepsilon_j(\omega)$, both functions depend on the frequency $\omega$. The speed of light is indicated by $c$. The sign appearing in the expression of the refractive index must be taken as negative when considering an LHM and positive when the medium is a vacuum or a dielectric material. The electromagnetic field $\Psi_j(\mathbf{r})$ satisfies the boundary conditions and periodicity conditions of the PCW in the $x$-direction by means of the Bloch theorem. Now we introduce a Green’s function $G(\mathbf{r}, \mathbf{r}') = i\pi H_0^0(\omega |\mathbf{r} - \mathbf{r}'|/c)$ where $H_0^0(z)$ is the Hankel function of the first kind and zero order. Taking into consideration the geometry of the system shown in Fig. 1(a) and applying the two-dimensional Green’s second identity for the functions $\Psi$ and $G$, we obtain

$$\frac{1}{4\pi} \int_C \left[ \frac{\partial \Psi_j(\mathbf{r})}{\partial n} G(\mathbf{r}, \mathbf{r}') - \frac{\partial G(\mathbf{r})}{\partial n} \Psi_j(\mathbf{r}') \right] ds' = \theta(\mathbf{r}) \Psi_j(\mathbf{r}),$$

(4)

being $\theta(\mathbf{r}) = 1$ if $\mathbf{r}$ is inside the unit cell and $\theta(\mathbf{r}) = 0$ otherwise. $ds'$ is the differential arc’s length, $\hat{n}$ is the outward normal vector to $C$, and the observation point $\mathbf{r}$ is infinitesimally separated of contour $C$ outer to the unit cell.

By doing a discretization of the contours $\Gamma_j$ we can numerically represent Eq. (4), through an algebraic linear system $M(\omega)F(\omega) = 0$, which has an associated representative matrix, $M$, has a nontrivial solution if the determinant function $D(k, \omega) = \ln(|\det(M)|)$ is zero. This expression will give us a numerical dispersion relation $\omega = \omega(k)$ that determines the band structure corresponding to a PCW of infinite length. In Fig. 2(a) the real determinant function $D(0, \omega_r)$ as a function of the frequency is shown. The position of the extreme minimum is identified as the frequency of the mode with the value $\omega_r = 0.7519$. Moreover, the intensity of the electric field within of the unit cell is shown in Fig. 2(b).

![Figure 2](image.png)

**Figure 2.** (a) Function $D(0, \omega_r)$ for a photonic crystal waveguide, formed with two PEC flat surfaces and a periodic array of circular inclusions of dispersive LHM. (b) Electric field distribution at the frequency $\omega_r = 0.7519$.

For a surface LHM-vacuum, there exist a plasmonic surface mode with a frequency $\omega_r^{PSW} = \omega_0\sqrt{2/(2 - F)} = 0.7502$ [10]. We found a mode with a frequency very close to $\omega_r^{PSW}$ and whose corresponding intensity distribution is highly localized in the vicinity of the interface vacuum-LHM.
For these reasons we think that exists a plasmonic surface mode for the considered system in this work.

However, in reality a waveguide has a finite length, so we will verify the existence of modes by modeling the reflectivity with the integral method. Let us consider the problem of calculating the reflectance of a photonic crystal waveguide with finite length that illuminated with an incident field, \( \Psi_{\text{inc}}(r,t) = \Psi_{\text{inc}}(r)e^{-i\omega t} \), as sketched in Fig. 1. The system formed by two parallel plates and a periodic array of cylindrical inclusions is considered as a system of \( M \) bodies. Region 0 is characterized by a (real) index refraction \( n_0 = \sqrt{\varepsilon_0(\omega)} \), and region 1 to \( M \) are defined by the curves \( \Gamma_j \) and is characterized by the corresponding refractive indices \( n_j \) or, alternatively, by the dielectric constants \( \varepsilon_j(\omega) \). The curves describing the profiles can be written in terms of a single parameter \( t_j \) as \( r_j = [\xi_j(t), \eta_j(t)] \). Similary, employing Green’s integral theorem, the field in region 0 can be expressed as

\[
\Psi^{(0)}(r) = \Psi^{(0)}_{\text{inc}}(r) + \frac{1}{4\pi} \sum_{j=1}^{M} \int_{\Gamma_j} \left[ \frac{\partial G_0(r,t_j)}{\partial n_j} \Psi^{(0)}(t_j) - G_0(r,t_j) \frac{\partial \Psi^{(0)}(t_j)}{\partial n_j} \right] dt_j
\]

and for region \( j \)

\[
\theta_j(r)\Psi^{(j)}(r) = -\frac{1}{4\pi} \int_{\Gamma_j} \left[ \frac{\partial G_j(r,t_j)}{\partial n_j} \Psi^{(j)}(t_j) - G_j(r,t_j) \frac{\partial \Psi^{(j)}(t_j)}{\partial n_j} \right] dt_j
\]

where \( \theta_j(r) \) is unity for points inside the \( j \)-th medium and zero otherwise, and \( G_j(r,t_j) \) is Green’s function for the \( j \)-th medium. By making a discretization of the contours that make up the PCW of finite size we find the electromagnetic response through the reflectance [7],

\[
R(k_y) = \frac{P_{\text{ref}}(k_y)}{P_{\text{inc}}(k_y)} = \frac{1}{2\pi F(k_y)} \int_{-\alpha_0(\omega/c)}^{\alpha_0(\omega/c)} S(q,k_y) \left| S(q,k_y) \right|^2 dq
\]

being

\[
S(q,k_y) = -\frac{i}{2\alpha_0(q)} \sum_{j=1}^{M} \left[ \int_{\Gamma_j} \frac{\partial \Psi^{(0)}(r)}{\partial n_j} \exp\left\{ -i[q\xi(t_j) + \alpha_0(q)\eta(t_j)] \right\} dt_j \right]
\]

and

\[
F(k_y) = \sqrt{\frac{\pi}{2q\alpha_0(k_y)}}
\]

For propagating waves (i.e. when \( q < n_0\omega/c \)), we can identify the components of the wave vector as \( q = \omega/c \sin \theta_s \) and \( \alpha_0(q) = \omega/c \cos \theta_s \), where \( \theta_s \) is the scattering angle (see Fig. 1).

Equations (5) and (6) constitute a set of \( 2M \) coupled integral equations that can be solved numerically to obtain the boundary values for the field and its normal derivative on the surface of the dispersion bodies. Details of the discretization of these equations and their conversion into matrix equations can be found in Ref. [7].

3. Plasmonic surface mode in a finite PCW that include LHM

Since our objective focuses on studying the excitation of SPPs in finite-sized photonic crystal waveguides containing dispersive metamaterial, we proceed to consider a truncated PCW that is formed with two planar surfaces and a periodic array of cylindrical inclusions of dispersive LHM in the \( x \)-direction (see Fig. 1(b)).

Considering PCW with the distance between the flat surfaces \( b = \pi \) and finite waveguide length is \( d = 12\pi \). In Fig. 3(a) shows the results that under normal incidence (\( \theta_0 = 0^\circ \)) we have obtained for \( R \) with different size of the cylindrical inclusions; that is, for different filling fractions \( f \). In Fig. 3(b), we show the reflectances \( R \) obtained at different angles of incidence (\( \theta_0 \)) for the PCW with cylindrical inclusions of filling fraction of \( f = 0.005 \).
Figure 3. Reflectance under polarization TE of the PCWs of parameters $b = \pi$ and $d = 12\pi$, corresponding (a) different filling fractions $f$ of the cylindrical inclusion, and (d) for the PCW with $f = 0.005$ at different angles of incidence $\theta_0$.

From Fig. 3 we observe the presence of a minimum in the reflectance. The position of the minimum in reflectance is not affected when considering different sizes of cylindrical inclusions as shown in Fig. 3(a). Finally, the results presented in Fig. 3(b) show us the presence of a very marked minimum in the reflectance at the frequency $\omega_r = 0.7510$, which was found for an angle of incidence of $\theta_0 = 25^\circ$.

In order to corroborate that the minimum in the reflectance presents indicates the presence of an SPP mode, we will analyze the phase $\varphi$ of the reflected field. The phase is given as [11]

$$
\varphi = \tan^{-1} \left( \frac{\text{Re}(\text{Reflection coefficient})}{\text{Im}(\text{Reflection coefficient})} \right)
$$

In Fig. 4, show the phases $\varphi$ as a function of frequency $\omega_r$ for different angles of incidence $\theta_0$, which correspond to the reflectances presented in Fig. 3(b).

Figure 4. Phases $\varphi$ as a function of the frequency $\omega_r$ for different angles of incidence $\theta_0$.

This analysis is similar to the work developed by R. Depine [12], which shows the propagation characteristics of surface plasmon polaritons (surface eigenmodes) in ATR systems (configuration of Kretschmann) with metamaterials.

Thus, we have found a minimum with a frequency very close to that reported in Refs. [8] and [10]. We also note that the slope change in the phase occurs at the frequency in which the minimum occurs in reflectance (see Fig. 4). For this reason we think that exists a plasmonic surface mode for the considered system in this work.
4. Conclusions
We have developed a numerical integral method to study a PCW formed by two parallel plates and an array of circular cylindrical inclusions, that involves an LHM. The numerical results obtained show that there is a existence of an SPP mode in the studied waveguide, since at an angle of incidence of around $\theta_0 = 25^\circ$ we obtained that the minimum of the reflectance $R$ is given a frequency of $\omega_r = 0.7510$, under polarization TE. This mode has a great correspondence with the frequency of the mode found in the literature. These surface waves can be used as another alternative in the development of applications in different fields of science and technology ranging from biomedicine to telecommunications.

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