Apochromatic singlets enabled by metasurface-augmented GRIN lenses

J. Nagar, S. D. Campbell, and D. H. Werner

Department of Electrical Engineering, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
*Corresponding author: jun163@psu.edu

Received 6 September 2017; revised 20 November 2017; accepted 13 December 2017 (Doc. ID 306305); published 25 January 2018

Correction of chromatic aberrations is crucial for high-quality imaging systems. Recent advances in nanoscale manufacturing, however, have enabled the creation of metasurfaces: ultra-thin optical components with sub-wavelength features that can locally manipulate the wavefront phase. Meanwhile, there has been renewed interest in Gradient-Index (GRIN) lenses due to the extended degrees of design freedom they offer. When combined, these two technologies can provide unparalleled imaging system performance while realizing drastic reductions in size, weight, and power. Through paraxial theory and full ray tracing we produce a lens singlet that can achieve three-color (apochromatic) correction by employing a metasurface-augmented GRIN. This apochromatic singlet has the potential for application in high-quality optical systems.

OCIS codes: (080.2740) Geometric optical design; (110.2760) Gradient-index lenses; (260.2030) Dispersion; (160.3918) Metamaterials; (050.6624) Subwavelength structures.

https://doi.org/10.1364/OPTICA.5.000099

Chromatic aberrations are a primary limiting factor in thin, high-quality imaging systems. Recent advances in nanoscale manufacturing, however, have enabled the creation of metasurfaces: ultra-thin optical components with sub-wavelength features that can locally manipulate the wavefront phase. Meanwhile, there has been renewed interest in Gradient-Index (GRIN) lenses due to the extended degrees of design freedom they offer. When combined, these two technologies can provide unparalleled imaging system performance while realizing drastic reductions in size, weight, and power. Through paraxial theory and full ray tracing we produce a lens singlet that can achieve three-color (apochromatic) correction by employing a metasurface-augmented GRIN. This apochromatic singlet has the potential for application in high-quality optical systems.

OCIS codes: (080.2740) Geometric optical design; (110.2760) Gradient-index lenses; (260.2030) Dispersion; (160.3918) Metamaterials; (050.6624) Subwavelength structures.

https://doi.org/10.1364/OPTICA.5.000099

Correction of chromatic aberrations is crucial for high-quality imaging applications and has been the study of intense research for centuries [1]. Classical solutions for two- (achromatic) and three-wavelength (apochromatic) correction traditionally require refractive doublets and triplets, respectively. While multiple optical elements provide additional degrees of design freedom and thus higher performance, one seeks to reduce the number of elements in a system to minimize SWaP (size, weight, and power) which is of increasing importance in a wide variety of applications, including IR imaging. Moreover, additional elements may lead to stricter manufacturing constraints and alignment tolerances [1]. Therefore, a color-corrected singlet is highly desirable. To this end, aspherical meniscus elements with a blazed zone plate serving as the back surface were shown to provide achromatic performance comparable to conventional doublets while offering considerable SWaP reduction [2]. In addition, achromatic singlets have been achieved by exploiting the increased degrees of design freedom offered by inhomogeneous GRadient-InDex (GRIN) materials [3,4]. Along with polychromatic aberration correction, GRIN lenses also offer advantages in terms of high performance imaging over an extremely wide field of view [5]. While both conventional and GRIN lenses work by altering the relative phase of an incident wavefront, these devices require an appreciable size in order to deliver their behavior. In contrast, metasurfaces comprised of arrays of subwavelength geometric features can provide an abrupt change of phase in an ultrathin surface [6]. Furthermore, by intelligently arranging the geometric features throughout the metasurface any spatially varying phase change can be achieved, thus enabling the wavefront to be transformed at will. In fact, monochromatic correction over a wide field of view has been achieved with metasurface components [7]. There have also been attempts to correct for chromatic aberration with metasurfaces either through dispersion engineering of the nanostructures [8] or compensating between metallic and structural dispersions [9]. While these techniques have reduced chromatic aberrations present in their designs, their efficacy is heavily dependent on the choice of nanostructure and require numerical optimization in the design process. Nevertheless, these studies showcase the potential for metasurface-augmented lenses to increase optical performance while maintaining a minimal SWaP. However, in order to truly maximize the performance of these hybrid lenses, one should exploit all possible design degrees of freedom available, namely, the pairing of metasurfaces with GRIN materials. This combination has been used for improving the depth of focus of an optical probe [10] and in the design of a high-quality beam splitter [11]. However, no work has been done in combining the advantageous aspects of GRIN lenses and metasurfaces for the purpose of color correction. Such metasurface-augmented GRIN lenses are feasible due to recent breakthroughs in GRIN lens [12] and metasurface [6] manufacturing. This Letter presents a completely general analytical theory for chromatic correction with metasurface-augmented GRIN (MS-GRIN) lenses. Furthermore, it is shown that such hybrid lenses can achieve apochromatic color correction in a thin singlet, suggesting a potential for large realizable SWaP reduction. A theoretical proof is presented for color-corrected MS-GRIN lenses, and its accuracy is validated with numerical ray tracing for several example configurations. To visualize the potential for three-color corrected MS-GRIN lenses, consider the example shown in Fig. 1. The homogeneous plano-convex singlet [Fig. 1(a)] possesses chromatic aberrations, while the plano-convex MS-GRIN lens [Fig. 1(b)] exhibits radically improved chromatic performance, a result of choosing the appropriate front curvature, GRIN distribution, and metasurface phase distribution. Note that these are cartoons.
to illustrate the concept and that the chromatic aberration in Fig. 1(a) is exaggerated for clarity.

To understand the performance of such MS-GRIN lenses, one must first review the properties of these three components. A metasurface is a thin two-dimensional structure (of potentially arbitrary shape) comprised of subwavelength features that can be engineered to provide local changes in the relative phase of an incident wavefront [6]. Therefore, a metasurface can be thought of as an infinitesimally thin phase-gradient surface that can be analytically described by the generalized Snell’s law [13]

\[ n_i \sin(\theta_i) = n_f \sin(\theta_f) + \frac{1}{k} \frac{d\phi}{d\rho}, \]

(1)

where \( \phi \) is the phase profile, \( \rho \) is the radial distance, and \( n_i, n_f, \) and \( \theta_i, \theta_f \) are the refractive indices and angles relative to the optical axis in the incident and transmitted regions, respectively. For a circular lens of diameter \( D \), to achieve a focal length \( f_m \) at the nominal wavelength \( \lambda_m \), the metasurface must impart the following phase profile [6]:

\[ \phi = -\frac{2\pi}{\lambda_m} \sqrt{\rho^2 + f_m^2 - f_m^2}. \]

(2)

Then, assuming the source is an on-axis object point at infinity and an \( f/\# \) greater than or equal to one, paraxial approximations may be employed to find the power of the metasurface

\[ \phi_m(\lambda) \approx \frac{\lambda}{2mf_m}, \]

(3)

where the subscript \( m \) denotes metasurface. The metasurface Abbe number \( \nu_m \) and partial dispersion \( \nu_f \) are given by

\[ \nu_m = \frac{\phi_2}{\phi_1 - \phi_3} = \frac{\lambda_2}{\lambda_1 - \lambda_3}, \quad \nu_f = \frac{\phi_1 - \phi_2}{\phi_1 - \phi_3} = \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_3}, \]

(4)

where the subscripts 1, 2, and 3 refer to the short, center, and long wavelengths in the band of interest, respectively. In the visible regime, the three wavelengths typically chosen are the hydrogen F line (486.1 nm), helium d line (587.6 nm), and hydrogen C line (656.3 nm). Using these wavelengths, it follows that \( \nu_m = -3.45 \) and \( \nu_f = 0.5962 \), leaving the optical power as the only degree of freedom (DOF) associated with the metasurface. The negative Abbe number of the metasurface immediately stands out here and is desirable not only for chromatic aberration correction but for reducing monochromatic aberrations as well.

Note that a diffractive element such as a kinoform has similar optical properties to a metasurface only when using the empirical Sweatt model [1]. However, metasurfaces, due to their subwavelength unit cell sizes, can truly be modeled as a continuous phase profile. Equations (3), (4) then follow directly using (1), (2) and paraxial approximations. Furthermore, metasurfaces have a number of advantages over traditional diffractive elements including extremely high efficiencies [14], multifunctional capabilities [15], and the ability to explicitly engineer the dispersion [8]. Next, consider a GRIN lens of diameter \( D \) and thickness \( T \) featuring a hybrid radial-axial (i.e., spherical) index distribution represented by

\[ n(r, z) = n_0 + \Delta n_f \left( \frac{2}{D} \right)^2 + \Delta n_r \left( \frac{1}{T} \right) z. \]

(5)

Its power, Abbe number, and partial dispersion are given by [5]

\[ \phi_G = -\Delta n_{f2} \left( \frac{2}{D} \right)^2 T, \quad \nu_G = \frac{\Delta n_{f2}}{\Delta n_{r1} - \Delta n_{r3}}, \]

\[ P_G = \frac{\Delta n_{r1} - \Delta n_{r2}}{\Delta n_{r1} - \Delta n_{r3}}, \]

(6)

where \( \Delta n_{ri} \) is the radial index difference at the \( i \)th wavelength. The expressions for the GRIN Abbe number and partial dispersions are calculated as ratios between powers similar to Eq. (4). It should be noted that a GRIN lens comprised of a binary material composition whose spatially-dependent index is described by simple volume-filling fraction mixing rules possesses an effective Abbe number that is dependent on the dispersion slopes of its two base materials. Interestingly, a GRIN’s Abbe number can range from positive to negative even if the Abbe numbers of its constituent materials are both positive. Finally, the power, Abbe number, and partial dispersion of a plano-convex refractive lens with front radius of curvature \( R_f \) are given by [1]

\[ \phi_f = \frac{n_f - 1}{R_f}, \quad \nu_f = \frac{n_f - 1}{n_f - n_f}, \quad P_f = \frac{n_f - n_f}{n_f - n_f}, \]

(7)

where \( f \) is a subscript indicating respect to the front surface and \( n_f \) is the effective index that must be used when there is an inhomogeneous surface index distribution. It has been found empirically that evaluating Eq. (5) at a radial position \( r = 0.48D/2 \) to serve as the effective surface index \( n_f \) in Eq. (7) leads to accurate results. Interestingly, while comprised of the same base materials as the GRIN, the surface actually possesses a different Abbe number and partial dispersion than the GRIN volume. Combining this with the metasurface gives the designer three components with unique Abbe numbers and partial dispersions, which can then be intelligently combined to achieve apochromatic color-correction, which ensures that the powers at the three design wavelengths are exactly matched. This condition can be satisfied by the following set of equations [1] assuming a metasurface-backed GRIN with a front radius of curvature:

\[ \phi_f + \phi_G + \phi_m = \phi_T, \]

\[ \frac{\phi_f}{\nu_f} + \frac{\phi_G}{\nu_G} + \frac{\phi_m}{\nu_m} = 0, \]

\[ P_f \frac{\phi_f}{\nu_f} + P_G \frac{\phi_G}{\nu_G} + P_m \frac{\phi_m}{\nu_m} = 0. \]

(8)

Next, substituting Eqs. (3), (4), (6), and (7) into (8) leads to explicit expressions for the front radius of curvature, thickness, and metasurface power:

Fig. 1. Cartoon examples of (a) a homogeneous lens showing chromatic aberrations and (b) an MS-GRIN lens exhibiting apochromatic performance.
components for apochromatic correction [1]. A PMMA/PS mixture possesses approximately the same partial dispersions. Mathematically, with a large difference in Abbe number between them, while theyations, it is desirable for two of the points to lie on a horizontal line, of material choice, allowing the apochromatic conditions to be satisfied, these three points cannot reside on a straight line. A GRIN lens with both a front and back radius of curvature offers three degrees of freedom. However, because the GRIN material is a volume fraction mixture of the two materials, the three points will always be on a straight line, regardless of the choice of materials, index distribution, and curvatures. Therefore, apochromatic color correction is theoretically impossible with a binary-blended GRIN singlet. However, the metasurface \((P_m, \nu_m)\) point only depends on the wavelengths chosen and is independent of material choice, allowing the apochromatic conditions to be satisfied. Furthermore, for minimization of monochromatic aberrations, it is desirable for two of the points to lie on a horizontal line, with a large difference in Abbe number between them, while they possess approximately the same partial dispersions. Mathematically, the triangle area is a measure of the quality of the three chosen components for apochromatic correction [1]. A PMMA/PS mixture with \(\nu_G = 9.3\) and \(P_G = 0.71\) has previously been used in GRIN applications [16]; however, examination of Eq. (8) shows that the apochromatic conditions can only be satisfied if the optical powers of the GRIN and front surface are negative and positive, respectively. Unfortunately, these powers fight against each other, which results in larger monochromatic aberrations. Therefore, a better material choice is a H-BAF6/H-ZK5 mixture that possesses a negative GRIN Abbe number \(\nu_G = -1.43\), a partial dispersion \(P_G = 0.7241\), and similar coefficients of thermal expansion.

In the case of this material combination, both the GRIN and surface powers are positive and the resulting relative monochromatic aberrations are extremely small. While the proposed material mixture is hypothetical, another goal of this study is to shed light on what material properties are desirable and inspire new fabrication techniques. To this end, recent advancements in GRIN manufacturing, including 3D printing [12, 17], have made more material combinations accessible for the manufacture of GRIN lenses. For comparison purposes, a refractive apochromatic triplet consisting of Pilkington-Optiwhite, S-FPL53, and L-BBH1 will be considered. These materials were chosen because they have the largest triangle area in \(P\) versus \(\nu\) space for conventional materials in the visible regime using material specifications from [18]. A graphical examination of the relative partial dispersions versus Abbe number for the MS-GRIN and refractive triplet is shown in Fig. 2. One can see that the MS-GRIN is a better choice for achieving apochromatic performance due to its larger triangle area.

The validity of the theory can be tested through the application of ray tracing. To this end, we employ reTORT (transformation optics ray tracer), an arbitrary-GRIN-capable ray tracer developed in-house [19] that has been rigorously validated against Code V and OSLO for a wide variety of examples. In particular, reTORT is utilized here to evaluate the performance of a MS-GRIN comprised of H-BAF6/H-ZK5 whose index variation is given by Eq. (5) and restricted to a purely radial configuration. An infinite conjugate on-axis ray bundle is considered. When using the GRIN and metasurface parameters given by the paraxial theory directly, one finds that there is a slight discrepancy between the theoretical prediction and the ray trace results. This is due to the paraxial nature of the theory, which does not include aberrations or principle plane shifts. Thus, to ensure the apochromatic performance of the MS-GRIN lens was maximized, a global optimizer based on the covariance matrix adaptation evolutionary strategy (CMA-ES) [20] was employed. Nevertheless, the lens parameters predicted by the paraxial theory agree quite well with the optimized parameters. Figure 3 shows the ratio of the lens parameters predicted by Eq. (9) and those found by the optimizer for various \(f/\#\) configurations. As can be seen, the error for all lens parameters is quite small above around \(f/2\), and all three ratios approach unity as the \(f/\#\) increases. Again, these results suggest the theory provides an extremely accurate estimate of the lens parameters, which can then be used to drastically expedite optimizations. Next, the observed focal drift versus wavelength is...
plotted in Fig. 4 for a standard GRIN, the MS-GRIN, the refractive doublet, and the refractive triplet.

The refractive doublet and GRIN are both achromatic, with $f/\delta f$ values of $-1e3$ and $2.8e3$, respectively, highlighting the superior performance of the GRIN. Meanwhile, the refractive triplet and MS-GRIN are both apochromatic, with $f/\delta f$ values of $5.5e3$ and $2e5$, respectively. Furthermore, the triplet suffers from extreme spherochromatism while the MS-GRIN has a much smaller amount. To minimize these aberrations, an r\textsuperscript{4} GRIN term and two aspherical perturbations to the front surface were added to the MS-GRIN. Moreover, air spacing between elements and two aspherical perturbations to both the front and back surfaces were added to the refractive triplet. Both configurations feature a fully illuminated 12.7 mm lens diameter and are extremely fast operating at $f/2$. The performances of these lenses are compared in Fig. 5. Specifically, Fig. 5(a) shows the modulation transfer function (MTF) for both designs compared to the diffraction limit at $\lambda_d$. As can be seen, the MS-GRIN is nearly diffraction limited while the triplet suffers from poor performance. Finally, Fig. 5(b) shows the Strehl ratio (which takes into account all aberrations of the system) [16] for the two designs versus wavelength, highlighting the broadband diffraction-limited performance of the MS-GRIN. We envision that the extremely high-performing devices enabled by the MS-GRIN will motivate research in new manufacturing techniques.

This Letter presented, for the first time, a lens singlet that is capable of apochromatic color correction. Such performance was achieved through the addition of a metasurface coating on a plano-convex GRIN lens. Analytical theory was presented for these metasurface-augmented GRIN singlets, which was then tested against ray tracing to prove its efficacy. Future studies include off-axis monochromatic performance and lateral color correction.

**Funding.** Defense Advanced Research Projects Agency (DARPA) (HR00111720032).

†These authors contributed equally to this work.

**REFERENCES**

1. W. J. Smith, Modern Optical Engineering (Tata McGraw-Hill Education, 2000).
2. A. P. Wood, Appl. Opt. 31, 2523 (1992).
3. S. D. Campbell, D. Brocker, J. Nagar, and D. H. Werner, Appl. Opt. 55, 3594 (2016).
4. R. A. Flynn, E. F. Fleet, G. Beadie, and J. S. Shirk, Opt. Express 21, 4970 (2013).
5. E. W. Marchand, Gradient Index Optics (Academic, 1978).
6. C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O’Hara, J. Booth, and D. R. Smith, IEEE Trans. Antennas Propag. Mag. 54(2), 10 (2012).
7. A. Arbabi, A. Arbabi, S. M. Kamali, Y. Horie, S. Han, and A. Farahon, Nat. Commun. 7, 13682 (2016).
8. M. Khorasaninejad, Z. Shi, W. T. Chen, V. Sanjeev, A. Zaidi, and F. Capasso, Nano Lett. 17, 1819 (2017).
9. Y. Li, M. Pu, Z. Zhao, X. Ma, Y. Wang, and X. Luo, Sci. Rep. 6, 19885 (2016).
10. J. Xing, J. Kim, and H. Yoo, Opt. Express 24, 1037 (2016).
11. M. R. Gordova, A. A. Lipovski, V. V. Zhurzhina, D. K. Tagantsev, B. V. Tatarnitsev, and J. Turunen, Opt. Eng. 40, 1507 (2001).
12. S. D. Campbell, D. E. Brocker, D. H. Werner, C. Dupuy, S. K. Park, and P. Harmon, IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting (2015), 605.
13. N. Yu, P. Genevet, M. A. Kats, F. Aieta, J. P. Tetienne, F. Capasso, and Z. Gaburro, Science 334, 333 (2011).
14. R. C. Devlin, M. Khorasaninejad, W. T. Chen, J. Oh, and F. Capasso, Proc. Natl. Acad. Sci. USA 113, 10473 (2016).
15. Z. L. Deng, X. Li, and G. P. Wang, “A multifunctional metasurface: from extraordinary optical transmission to extraordinary optical diffraction in a single structure,” arXiv:1705.10171 (2017).
16. P. McCarthy and D. T. Moore, Optical Fabrication and Testing (2012), OTu4D-2.
17. D. T. Nguyen, C. Meyers, T. D. Yee, N. A. Dudukovic, J. F. Destino, C. Zhu, E. B. Duoss, E. F. Baumann, T. Suratwala, and J. E. Smay, Adv. Mater. 29, 1701181 (2017).
18. M. N. Polyanskiy, “Refractive index database,” 2017, [https://refractiveindex.info](https://refractiveindex.info).
19. J. P. Turpin, D. Brocker, and D. H. Werner, “Optimization of quasi-conformal transformation optics lenses with an arbitrary GRIN-capable ray tracer,” in IEEE AP-S Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting, Orlando, Florida (IEEE, 2013), pp. 1898–1899.
20. N. Hansen, S. D. Müller, and P. Koumoutsakos, Evol. Comput. 11, 1 (2003).