The signal processing algorithm of automotive FMCW radars with an extended range of speed estimation

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Abstract. The paper considers the algorithm for estimating the speed and distance to the target as applied to the mm wave automotive radar with linear FMCW. To extend the range of target speed estimation, a signal in the form of a Chirp Sequence Waveform is used with a shift of the initial frequency in the adjacent chirps of the frame. A mathematical model of the received microwave signal is proposed, taking into account a wide range of velocity changes of the target, as well as the effect of additive white Gaussian noise on the radar receiver. It is shown that the signal-to-noise ratio of 10 dB is necessary for the algorithm to work, and that the unambiguity and consistency of the speed estimate is limited by the first two Nyquist zones for the Doppler frequency.

1. Introduction
The urgency and the need for remote monitoring of the location and speed of surrounding objects in real time for modern cars is explained by the requirement to ensure the most comfortable and safe control of vehicle movement, especially in autonomous (unmanned) mode. For the technical implementation of the tasks of remote monitoring, active locating systems of the microwave [1-7] and optical [8] wavelength ranges are widely used. Today the most effective and the most commonly used scheme in automotive radar is linear FMCW (Frequency Modulated Continuous Wave). In these microwave systems, the radiated signal from the radar transmitter has frequency changing linearly with time (i.e., chirp) [1-5]. The used Chirp Sequence Radar Waveform allows, as a rule, to estimate the radial velocity $V_R$ and target range $R$ simultaneously, with high accuracy and resolution. Locational microwave systems using this type of probing signal meet the high requirements imposed on the characteristics of automotive radar sensors. However, at large ranges of the measured speed $V_R$ and distance to the target $R$, high spatial resolution $\delta R$, the estimate of $V_R$ becomes ambiguous due to the specificity of the received signal processing algorithm. In [4, 5], the use of a so-called multiple frequency shift keying (MFSK) transmit signal is proposed to solve the problem of the ambiguity of velocity estimation. In this case, the algorithm for processing the signal received by the radar includes an additional phase analysis of two adjacent periods of the MFSK signal, which allows an unambiguous estimate of the radial velocity component.

In the above-mentioned works [4, 5] during the synthesis of the input data processing algorithm, the authors use a simplified mathematical model of the echo signal of the radar, which becomes inadequate at high target speeds. Also, the authors did not analyze the statistical characteristics of the assessment of $V_R$ when exposed to the unavoidable noise microwave signal generated by internal and external sources. These reasons did not allow the [4, 5] authors to determine the boundaries of the $V_R$ estimate when using the proposed MFSK signal and their processing strategy. In this paper, on the basis of a refined mathematical model of the radar echo signal, the limits of the use of
the [4, 5] processing algorithm are analyzed to unambiguously estimate the speed \( V_R \) and the target range \( R \). The effect of additive white noise on the velocity estimate \( V_R \) is also considered.

2. Model derivation

Figure 1 shows the Chirp sequence waveform [4, 5] probe signal of a radar transmitter. One frame of a transmitter signal with a duration of \( T = 2N\Delta f \) contains a sequence of \( N \) pairs of periodically repeated signals with linear frequency modulation. The duration of one chirp \( T \), the frequency deviation \( \Delta f \), its sequence number \( n \) in the sequence of chirp pairs is determined by the variable \( k \) \((k=0,1\ldots N-1)\). The initial frequency of the signal for chirps with odd numbers \( n \) \((n=2k+1)\) is \( f_1 \), for chirps with even numbers \( n \) \((n=2k)\) it is equal to \( f_2 = f_1 - \Delta f \).

The waveform of the radar receiver input (the echo waveform) is delayed with respect to the transmitter's probe signal for a time equal to \( \tau \). For practically important radar applications, the delay \( \tau \) can be calculated by the formula

\[
\tau(t) = 2R(t)/c = 2R(0)/c - 2V_R\delta t/c, \quad t \in [0, T],
\]  

where \( c \) is the speed of light, \( R(0) \) is the initial coordinate of the target, and the radial velocity component is related to the Doppler frequency shifts \( f_{1D} \) and \( f_{2D} \) by the following relations

\[
2V_R/c = f_{1D}/f_1 = f_{2D}/f_2.
\]  

In figure 1, the waveform of an echo is represented by a dashed line. At the radar receiver, the analogue microwave input signal is converted to an intermediate frequency (IF) signal and then converted to the digital form. The sampling frequency \( f_d \) must satisfy the Nyquist conditions. In this case, the duration of the chirp \( T \) must satisfy the inequality

\[
T \geq sR_{\text{MAX}}/(f_d\delta R),
\]
here – s is the parameter that determines the number of samples in the period of the maximum frequency of the input signal of the radar receiver (s ≥ 2). The duration of the processing of a single chirp $T_R$ is less than $T$ by the delay time $\tau$ and the duration of transition processes. In real operation modes of the radar $T_R \approx T$.

For further construction of the mathematical model of the considered radar signals and the synthesis of the algorithm for processing them it is necessary to distinguish two scales in changing of the current time $t$ – “fast” (local time) $t_L$ and “slow” (discrete time) $nT$

$$t = t_L + nT.$$ (4)

The signal $SI(t_L, 2k+1)$ at the output of the mixer in the in-phase quadrature channel $I$ of the intermediate frequency (IF) of the radar receiver can be written as

$$SI_1(t_L, 2k+1) = U_M \cos \left[ 2\pi \left( \frac{\Delta f}{T} \tau_0 + f_1 \tau_0 + \Delta F(2k+1) \right)t_L + + 2\pi \left( f_1 \tau_0 + \right) + \Delta \phi(2k+1) \right] \cdot \text{rect} \left( \frac{t_L - \tau}{T} - \frac{1}{2} \right) + n_i(t_L, 2k+1).$$ (5)

Formula (5) describes the mathematical model of the IF signal of an odd 2k+1 chirp in a waveform sequence under the conditions of an impact noise $n_i(t_L, 2k+1)$. The rect($x$) function is a symmetric function of a rectangular window. The functions $\Delta F \left( t_L, 2k+1 \right)$ and $\Delta \phi \left( t_L, 2k+1 \right)$ clarify the mathematical model of the IF radar signal and are missing from the model used by the authors of [4, 5] when considering the scheme in automotive radar linear FMCW. The functions $\Delta F \left( t_L, 2k+1 \right)$ and $\Delta \phi \left( t_L, 2k+1 \right)$ have the following form

$$\Delta F(2k+1) = -\frac{2V_R}{c} \left[ \Delta f \cdot (2k+1) + \frac{\Delta f}{T} t_L - \frac{\Delta f}{T} \tau_0 \right].$$ (6)

$$\Delta \phi(2k+1) = 2\pi \left[ \frac{2V_R}{c} \Delta f \cdot \tau_0 \cdot (2k+1) - \frac{\Delta f}{T} \tau_0 \right] + \Delta \phi_0(t_L, 2k+1).$$ (7)

The expression (6) gives the following approximation of the IF frequency of the signal at a large target velocity. It shows that $f_R$ – the frequency of the IF signal depends on the time $t_L$, and therefore the signal of the intermediate frequency of the radar receiver is a signal with linear frequency modulation (chirp). The temporal change in the $\Delta \phi_0$ phase in expression (6) is determined by the phase change in time when the signal is reflected from the target, by changing the phase of the receiver signal relative to the transmitter. In standard operating modes of the radar, the temporary change in the $\Delta \phi_0$ phase during the synthesis of processing algorithms is usually neglected.

The additive noise $n_i(t_L, 2k+1)$ is described by the model of white Gaussian noise with zero mean and variance (power in the receiver frequency band) $\sigma^2$.

The mathematical model of the IF signal of the odd 2k+1 chirp $SQ_1(t_L, 2k+1)$ in the quadrature channel $Q$ is described by the similar expression (5) replacing the function $\cos(x)$ with $\sin(x)$. The correlation of noise in quadrature channels depends on the ratio of levels of external and internal noise. The mathematical model of IF signals of even 2k chirps in quadrature channels $I$ and $Q$ are described by expressions similar to (5) – (7) by replacing the frequency parameters $f_1$ with $f_2$ and the
variable $2k+1$ with $2k$. The model of radar signals allows you to develop a strategy for the simultaneous assessment of the target range $R$ and its speed $V_R$.

3. Algorithm for estimating $R$ and $V_R$ parameters

For the synthesis of the algorithm for estimating the $R$ and $V_R$ parameters, it is necessary to determine the phase difference $\Delta \Phi(t_L, k)$ of the IF signals of two adjacent chirps with an offset frequency of the $\delta f$.

$$\Delta \Phi(k) = 2\pi \left[ \tau_0 \cdot \delta f + f_1D_\text{amb}T + f_1D_\text{amb}T \cdot p \cdot \frac{2k \cdot \delta f}{f_1} \frac{\Delta f}{f_1T \tau_0} \right].$$

(8)

**Figure 2** The dependence of the function $\Delta F(2k+1)$ on $V_R$.

**Figure 3** The dependence of the function $\Delta \Phi(k)$ on $V_R$.

Equation (8) differs from the similar expression in [4, 5] by the presence of a term with factor $p$. For the $V_R$ parameter estimation algorithm with an extended target speed variation range, the approximation $p = 0$ becomes unacceptable. An ambiguous estimate of the Doppler frequency $f_1D_\text{amb}$ can be made using the Fourier transform of discrete time $nT$.

According to proposed in [4, 5] algorithm the expression (8) allows, in the approximation of $p = 0$, to calculate the estimation of the parameter $R$, and further to calculate the unambiguous Doppler frequency estimation $f_1D_\text{amb}$.

$$f_1D_\text{amb} = f_{IF} - \left[ \frac{\Delta \Phi(t_L, k)}{2\pi} + f_1D_\text{amb}T + W\left( \frac{\Delta \Phi(t_L, k)}{2\pi} + f_1D_\text{amb}T \right) \right] \frac{\Delta f}{\delta f \cdot T},$$

(9)

where the function $W(x)$ takes into account the requirement that the value of the parameter $R$ is nonnegative as the distance to the observed target, and the ambiguity of the phase estimate $\Delta \Phi(t_L, k)$. Depending on the argument $x$, the function $W(x)$ takes discrete values from the sequence $[-1, -1/2, 0, 1/2, 1, 3/2]$. The authors of [4, 5] do not use the $W(x)$ function, which leads to an incorrect result when using their parameter estimation algorithm directly. An analysis of formula (9) shows that with a large value of velocity, the estimation of the parameters $\Delta \Phi(t_L, k), f_1D_\text{amb}$ and $f_{IF}$ should be carried out with a small error. The operation of adding virtual zero samples to the signal before the Fourier transform leads to the error reduction. However, such an operation inevitably reduces the signal-to-noise ratio (SNR) at which the parameter estimate is calculated with a given error.
4. Simulation results
The above mathematical model of the input signal of the radar receiver and the algorithm for estimating the parameters \( R \) and \( V_R \) allow us to analyze the limits of applicability of the algorithm described in [4, 5]. In these papers, the function \( \Delta F(2k+1) \) and the coefficient \( p \) of the function \( \Delta \Phi(k) \), defined by equations (6) and (8), were set equal to zero. In a computer simulation of the echo processing algorithm, we used the following initial parameters: \( R_{\text{MAX}} = 250 \text{ m} \), \( R_{\text{MIN}} = 10 \text{ m} \), initial signal frequency \( f_1 = 76.5 \text{ GHz} \), frequency offset \( \delta f = 300 \text{ kHz} \), chirp frequency deviation \( 300 \text{ MHz} \), sampling rate of the IF signal – 5 MHz, Chirp duration \( T = 1.64 \mu\text{s} \), the number of pairs of chirps in the frame is \( N = 16 \). These parameters correspond to the specifications of the serially supplied to the market mmWave FMCW Radars for Automotive and Industrial Applications by various manufacturers [9].

Figure 2 shows the dependence of the function \( \Delta F(2\cdot16+1) \) on the radial velocity \( V_R \). Relation (6) allows us to calculate the frequency \( f_{\text{IF}} \) in the next approximation by the small parameter \( V_R/c \). It also contains the boundary that determines the operability of the [4, 5] algorithm for the case when the calculation of the frequency \( f_{\text{IF}} \) is performed in the zero approximation. Similarly, figure 3 shows the dependence of the function \( \Delta \Phi(k) \) on the radial velocity \( V_R \) and the boundary that determines the operability of the [4, 5] algorithm for the case when the phase is calculated in the zero approximation \( (p = 0) \). In general, the maximum target speed at which the [4, 5] algorithm is applicable for an unambiguous estimate of the \( R \) and \( V_R \) parameters is determined by the boundary of the second Nyquist zone.

In the technical implementation of the [4, 5] algorithm, in order to unambiguously estimate the \( R \) and \( V_R \) parameters, it is necessary to take into account the effect of noises of various nature. We analyzed the influence of additive white Gaussian noise of different power levels on a unique estimate of the \( R \) and \( V_R \) parameters. In [10 - 12], it was shown that the asymptotic dispersion of the phase \( \sigma_{p}^2 \) estimate has the form

\[
\sigma_{p}^2 = \frac{1}{\sigma_{f}N \cdot \text{SNR}},
\]

where \( \text{SNR} \) is the power signal to noise power ratio in the working frequency band and \( \sigma_{f} \) – number of samples per period \( f_{\text{IF}} \) of the radar receiver input frequency. Thus, the maximum effect of noise with a fixed number of \( 2N \) chirps will be at the maximum target range \( R_{\text{MAX}} \). For the considered parameters, the operability of the [4, 5] algorithm under the effect of noise on the useful signal for the case when the phase and frequency are calculated in the zero approximation is determined by the conditions \( \text{SNR} > 10 \text{ dB} \) and \( V_R < c/2f_1T \).

5. Conclusion
The authors of [4, 5] proposed the New Chirp Sequence Waveform with the offset of the initial frequency in the adjacent chirps of the frame for simultaneous and unambiguous evaluation of the \( R \) and \( V_R \) parameters. An incorrect conclusion is made about the possibility of an unambiguous assessment of the Doppler frequency and speed \( V_R \) in the Nyquist zones with a large number. We have considered a strict mathematical model of the input microwave signal that takes into account a wide range of velocity changes of the target (relation (1)–(3)). Based on this model, the operability limits of the algorithm for unambiguous estimation of speed and range proposed in [4, 5] have been analyzed.

Computer modelling showed that for reasonably selected parameters of the radar - target system the operability of the algorithm for unambiguous evaluation of the \( V_R \) is determined by the boundary of the second Nyquist zone for the Doppler frequency. Additive noise also significantly affects the unambiguous estimate of the \( V_R \) parameter. The condition for the operability of the speed estimation algorithm in the specified Nyquist zones is that the \( \text{SNR} \) parameter exceeds the 10 dB level.

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