Charged Kaons semi-leptonic form factors from NA48/2

G Lamanna
Università di Pisa, Largo Pontecorvo, 3, 56127 Pisa (Italy)
E-mail: gianluca.lamanna@cern.ch

Abstract. The NA48/2 experiment collected unprecedent statistics of charged kaon decays in 2003 and 2004. The main purpose of the experiment was to measure direct CP violation in kaon decays to three pions while an extensive physics program was carried out together with the main goal. The huge statistics collected allowed to test with a high accuracy the predictions of low energy QCD theories in several charged kaon decay processes. The precise measurement of the charged kaon semileptonic form factors are reported here, based on data collected in 2004 with a dedicated minimum bias trigger.

1. Introduction
Kaon decays played an important role in the understanding of the flavor structure of the Standard Model (SM). The precise measurement of well-predicted observables is a powerful tool to address questions related to the presence of new physics beyond SM, alternative to direct search and with the potential to probe higher energy scale effects. The NA48/2 collaboration carried out a large program of measurements in the charged kaon sector to study direct CP violation, rare decays and the dynamics of the SM in a low energy regime.

2. The NA48/2 experiment
The beamline of the NA48/2 experiment has been designed to allow the measurement of the CP-violating charged asymmetry through the study of the Dalitz Plot in the $K \rightarrow 3\pi$ decay [1]. Two simultaneous $K^+$ and $K^-$ beams were produced by 400 GeV/c protons extracted...
from the CERN/SPS and impinging on a beryllium target. Secondary unseparated beams with central momenta of \((60 \pm 3)\) GeV/c were selected and brought to a common beam axis, 200 m downstream of the target, by a system of magnetic beam line elements, collimators and muon sweepers. A fraction of the beam kaons decayed in a 114 m long evacuated tank. Charged particles from charged kaon decay were measured by a spectrometer, housed in a helium vessel downstream of the vacuum tank. The spectrometer was composed by four drift chambers (DCH) and a dipole magnet. The momentum resolution achieved was \(\sigma_p / p = (1.02 \pm 0.044)\%\) (where \(p\) is in GeV/c). The spectrometer was followed by two planes of scintillator slabs (CHOD) for triggering and timing purposes. A quasi-homogeneous liquid Krypton calorimeter (LKr), 27 \(X_0\) deep, was used to detect and measure the energy of photons and electrons. The LKr energy resolution was \(\sigma(E) / E = 3.2\% / \sqrt{E + 9\% / E + 0.42\%}\) with \(E\) in GeV. Muon counters, used for particle identification, were located further downstream. In two years of data taking (2003-2004) NA48/2 collected a total of about \(3 \times 10^9\) \(K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp\), about \(9 \times 10^7\) \(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0\) and other decays for rare processes studies. The normal trigger configuration was optimized for both three charged pions and two neutral pions and a single charged pion in the final state. A dedicated data taking period (52 hours in 2004) with a minimum bias trigger (one track in the CHOD and at least 10 GeV in the LKr) collected 4.8 \times 10^8 events used in the analysis presented here. A detailed description of the detector can be found in [2].

3. Measurement of the \(K^\pm_{13}\) form factors

The precise measurement of both \(K_{13} (K^\pm \rightarrow e^\pm \pi^0 \nu)\) and \(K_{\mu3} (K^\pm \rightarrow \mu^\pm \pi^0 \nu)\) form factors enters, together with the determination of their branching ratio, to the determination of the CKM matrix element \(|V_{us}|\). The decay width of \(K_{13}\) decays are described in the Dalitz plot as a function of the lepton and pion energies in kaon rest frame, \(E^*_l\) and \(E^*_\pi\). In absence of electromagnetic effects the density in the Dalitz plot can be defined by:

\[
\frac{d^2 \Gamma_0(K_l)}{dE^*_l dE^*_\pi} = N(A f^2_+(t) + B f_+(t) f_-(t) + C f^2_-(t))
\]

where \(N\) is a normalization factor. The dimensionless \(f_\pm(t)\) are the vector form factors, described as a function of squared four-momentum transferred to the lepton system \(t = (P_K - P_{\pi^0})^2 = (m_{K}^2 + m_{\pi^0}^2 - 2m_K E_{\pi^0})\) (where \(m_K\) and \(m_{\pi^0}\) are the kaon and neutral pion masses respectively).

Commonly the scalar form factor, \(f_0(t)\), is defined so that \(f_-(t) = (f_+(t) - f_0(t)(m_{K}^2 - m_{\pi^0}^2))\) and \(f_+(0) = f_-(0)\). The kinematic factors in the equation above are functions of the energies in kaon rest frame:

\[
A = m_K(2E^*_l E^*_\pi - m_K(E^*_{\pi,\text{max}} - E^*_\pi)) + m^2_{K}(E^*_{\pi,\text{max}} - E^*_\pi)/4 - E^*_\pi
\]

\[
B = m^2_{\pi}(E^*_\pi - (E^*_{\pi,\text{max}} - E^*_\pi)/2) \quad C = m^2_{\pi}(E^*_{\pi,\text{max}} - E^*_\pi)/4
\]

There are several parameterizations in literature for the scalar \((f_0(t))\) and vector \((f_+(t))\) form factors. In this work we focus on the following three:

- Quadratic expansion (a.k.a. Taylor expansion): The expansion is done in the variable \(t/m^2_{\pi,\text{max}}\). The slopes and curvatures are defined by the fit parameters \(\lambda_+\), \(\lambda_+\) and \(\lambda_0\) [3]:

\[
f_+(t) = 1 + \lambda_+(\frac{t}{m^2_{\pi}}) + \frac{1}{2} \lambda_+^2 (\frac{t}{m^2_{\pi}})^2 \quad f_0(t) = 1 + \lambda_0 (\frac{t}{m^2_{\pi}})
\]

- Pole parameterization: a single resonance is assumed to be relevant in the expansion and the corresponding pole masses \(M_{V,S}\) are the only free parameters in the fit [4]:

\[
f_+(t) = \frac{M^2_v}{M^2_v - t} \quad f_0(t) = \frac{M^2_s}{M^2_s - t}
\]
Figure 1. Neutral decay vertex distribution for $K_{e3}$ and $K_{\mu3}$ modes. Signal and backgrounds are considered in the Montecarlo. The vertical line represents the upstream cut on vertex position.

- Dispersive approach: two dispersive functions are introduced, $G(t)$ and $H(t)$, and the $C$ and $\Lambda_+$ parameters are extracted from the fit [5]:

$$f_+(t) = \exp((\Lambda_+ + H(t))(\frac{t}{m_\pi^2}))$$

$$f_0(t) = (\ln(C) - G(t))(\frac{t}{M_K^2 - M_\pi^2})$$

The Taylor expansion is the most used and simplest, but it is affected by large correlations between the measured parameters. In the pole parameterization, the pole mass $M_V$ can be physically interpreted as mass of the $K^*(892)$ resonance, but there is not an equivalent interpretation for $M_S$. The dispersive approach relies on general chiral symmetry and analyticity constraints, but external inputs from $K^-\pi$ scattering data, via $H(t)$ and $G(t)$ functions, are needed.

3.1. Event selection

The request of one single good track selected in the spectrometer in time with at least two LKr clusters is the starting point for both $K_{e3}$ and $K_{\mu3}$ selections. Electrons are selected requiring the ratio between the energy measured in LKr ($E$) for the cluster geometrically associated with the charged track and the momentum measured in the spectrometer ($p$), to be in the range $0.9 < E/p < 2.0$. The muon identification requires a track with $E/p < 0.9$ and an in-time signal in the first two planes of the muons counter. A cut for the track momentum $p > 5 \text{ GeV}/c$ is used in the electron selection, while $p > 10 \text{ GeV}/c$ is used for muons, to have a proper efficiency in the muons detector. Additional geometrical acceptance cuts are applied both on the electron and muon samples. To be defined as gammas candidates, the clusters in the LKr are required to have energy larger than 3 GeV and separation of, at least, 20 cm. Both in $K_{e3}$ and $K_{\mu3}$ samples, a $\pi^0$ decay candidate is considered if there are no additional photon candidates in a window of 5 ns with respect to the average of the two gammas candidate and the total energy is at least 15 GeV, in order to ensure a good trigger efficiency. The $z$ position for the kaon decay vertex is obtained from the $\pi^0$ decay assuming the nominal $\pi^0$ mass, while the transverse coordinates $(x,y)$ are extrapolated at the $z$ position of the vertex from the measured direction of the lepton track.
additional request to have a vertex at least 2 meters downstream of the final collimator is done to suppress $\pi^0$ production in the material of the collimator (Fig.1). The kaon momentum in the laboratory frame is computed by imposing the energy-momentum conservation and assuming kaon line of flight along the measured beam axis direction and the mass-less neutrino. From the two possible $P_K$ solutions, the closest to the nominal beam momentum is chosen.

3.2. Background suppression

The $K^\pm \rightarrow \pi^+\pi^-\pi^0$ decays where the charged pion either decays in flight or is misidentified and one $\pi^0$ is not detected, is the main background both for $K_{\mu3}$ and $K_{e3}$. The energy-momentum conservation is exploited to build a discriminant to distinguish between signal and noise. The $K^\pm \rightarrow \pi^\pm\pi^0$ is a background for $K^{e3}$ due to the $\pi^\pm$ misidentification as electron and it is suppressed requiring the transverse momentum of the neutrino to be $p_{\nu,t} > 30$ MeV. The two pions decay is also background for the $K_{\mu3}$ both for pion decay in flight and pion misidentification. Both this two processes are rejected requiring $m(\mu\nu) > 0.16$ GeV/$c^2$ and $m(\pi^+\pi^-) + p_{\pi^0,t}/c < 0.6$ GeV/$c^2$, where $p_{\pi^0,t}$ is the $\pi^0$ transverse momentum with respect to the beam axis, $m(\pi^+\pi^-)$ is the track-$\pi^0$ invariant mass in the charged pion hypothesis and $m(\mu\nu)$ is the leptonic system invariant mass. In Fig.2 the effect of the cuts is shown (on Montecarlo): 99.5% of the background is rejected with about 17% of signal loss. Other background sources are summarized in table 1, where the background to signal ratio $r_e$ and $r_\mu$ in the selected $K^{e3}$ and $K^{\mu3}$ are evaluated on Montecarlo simulation.

3.3. Form factors fit procedure

A total of $4.4 \times 10^6$ $K^{e3}$ and $2.3 \times 10^6$ $K^{\mu3}$ are selected candidates for the form factors (FF) fit. For each parameterization considered, a set of FF parameters is measured by minimizing the estimator

$$\chi^2(\bar{\lambda}, N) = \sum \frac{(\omega_i^{\text{data}} - \omega_i^{\text{bkg}}(\bar{\lambda}) - N \cdot \omega_i^{\text{sig}}(\bar{\lambda}))^2}{\sigma_{\omega_i^{\text{data}}}^2 + \sigma_{\omega_i^{\text{bkg}}(\bar{\lambda})}^2 + N^2 \cdot \sigma_{\omega_i^{\text{sig}}(\bar{\lambda})}^2}$$

The sum is done in bins of $5 \times 5$ MeV$^2$ of the Dalitz plot distribution in the kinematically allowed region, with at least 20 events per bin. The $\omega_i^{\text{data}}$ is the number of candidates in the bin, while $\omega_i^{\text{sig}}$ and $\omega_i^{\text{bkg}}$ are the expected signal and background events from a simulation in which the radiative effects are taken in to account [6]. The FF are measured independently for each decay mode and each parameterization. A joint analysis is also performed by fitting simultaneously the $K^{e3}$ and $K^{\mu3}$ Dalitz plot distributions with the same set of parameters.

Table 1. Simulated background processes and estimated fraction for $K^{e3}$ and $K^{\mu3}$ (in $10^{-3}$).

| Decay               | BR(%) | $r_e$ | $r_\mu$ |
|---------------------|-------|-------|---------|
| $K^+ \rightarrow \pi^+\pi^0$ | 20.66 | 0.270 | 0.264 |
| $K^+ \rightarrow \pi^+\pi^0\pi^0$ | 1.761 | 0.286 | 1.833 |
| $K^+ \rightarrow \pi^+\pi^0\pi^0_D$ | 1.174 | 0.049 | 0.000 |
| $K^+ \rightarrow \pi^+\pi^0\gamma$ | 0.0275 | 0.004 | 0.044 |
| $K^+ \rightarrow \pi^0\mu^+\nu$ | 0.0335 | 0.004 | 0.000 |
3.4. Systematic Uncertainties

Several sources of systematics uncertainties are considered, including beam modeling, LKr energy scale and non-linearity, residual background, particle identification, event pile-up, acceptance, trigger efficiency, neutrino momentum resolution, Dalitz plot binning and resolution. The main sources of systematics uncertainties come from the imperfect simulation of the divergent component of the beam and the variation of the energy of reconstructed clusters within the uncertainty on the energy scale (0.1%). More details on the systematic uncertainties can be found in [7].

Table 2. Form factor results for the parameterization considered. The joint fit results are indicated with $K_{l3}$.

| Quadratic  | $\lambda_+ (10^{-3})$ | $\lambda_+'' (10^{-3})$ | $\lambda_0 (10^{-3})$ |
|-----------|------------------------|--------------------------|------------------------|
| $K_{\mu3}$ | 24.27 ± 2.88 $\text{stat}$ ± 2.89 $\text{syst}$ | 1.83 ± 1.05 $\text{stat}$ ± 1.09 $\text{syst}$ | 14.20 ± 1.14 $\text{stat}$ ± 1.07 $\text{syst}$ |
| $K_e3$     | 24.26 ± 0.78 $\text{stat}$ ± 1.30 $\text{syst}$ | 1.64 ± 0.30 $\text{stat}$ ± 0.39 $\text{syst}$ | |
| $K_{l3}$   | 24.24 ± 0.75 $\text{stat}$ ± 1.30 $\text{syst}$ | 1.67 ± 0.29 $\text{stat}$ ± 0.41 $\text{syst}$ | 14.47 ± 0.63 $\text{stat}$ ± 1.17 $\text{syst}$ |

| Pole       | $M_V$ (MeV/c²) | $M_S$ (MeV/c²) |
|-----------|----------------|----------------|
| $K_{\mu3}$| 878.4 ± 8.8 $\text{stat}$ ± 8.3 $\text{syst}$ | 1214.8 ± 23.5 $\text{stat}$ ± 49.2 $\text{syst}$ |
| $K_e3$    | 885.2 ± 3.3 $\text{stat}$ ± 7.2 $\text{syst}$ | |
| $K_{l3}$  | 884.4 ± 3.1 $\text{stat}$ ± 6.7 $\text{syst}$ | 1208.3 ± 21.2 $\text{stat}$ ± 47.5 $\text{syst}$ |

| Dispersive | $\Lambda_+ (10^{-3})$ | $\ln[C] (10^{-3})$ |
|-----------|------------------------|---------------------|
| $K_{\mu3}$| 25.36 ± 0.58 $\text{stat}$ ± 0.72 $\text{syst}$ | 182.17 ± 6.31 $\text{stat}$ ± 14.45 $\text{syst}$ |
| $K_e3$    | 24.94 ± 0.21 $\text{stat}$ ± 0.64 $\text{syst}$ | |
| $K_{l3}$  | 24.99 ± 0.2 $\text{stat}$ ± 0.62 $\text{syst}$ | 183.65 ± 5.92 $\text{stat}$ ± 14.25 $\text{syst}$ |

Figure 2. Montecarlo distributions of kinematical variables (defined in the text) to reject the $K^\pm \rightarrow \pi^\pm \pi^0$ background (right) and effect on the signal (left).
4. Results
In table 2 the form factors result for $K_{e3}$, $K_{\mu3}$ and the joint fit for all parameterizations proposed, are shown. The results of the present analysis for the quadratic expansion are compared with the previous experiments in Fig.3. The 68% contours are shown for KTeV [8], KLOE [9, 10], NA48 [11, 12] and ISTRA+ [13, 14] experiments. Only results validated by FLAVIANET [15] collaboration are quoted, the OKA recent result [16] is not included yet. The NA48/2 results presented here [7] are in good agreement with the previous results, with similar precision for the $K_{\mu3}$ mode and with an improved precision for the $K_{e3}$ mode.

References
[1] Batley J R et al. [NA48/2 Collaboration] 2007 Eur. Phys. J. C 52 875
[2] Fanti V et al. [NA48 Collaboration] 2007 Nucl. Instrum. Meth. A 574 433
[3] Patrignani C et al. [Particle Data Group] 2016 Chin. Phys. C 40
[4] Lichard P 1997 Phys. Rev. D 55 5385
[5] Bernard V, Oertel M, Passemar E and Stern J 2009 Phys. Rev. D 80 034034
[6] Gatti C, Eur. Phys. J. C 45 (2006) 417
[7] Batley J R et al. [NA48/2 Collaboration], JHEP 1810 (2018) 150
[8] Alexopoulos T et al. [KTeV Collaboration], Phys. Rev. D 70 (2004) 092007
[9] Ambrosino F et al. [KLOE Collaboration], Phys. Lett. B 636 (2006) 166
[10] Ambrosino F et al. [KLOE Collaboration], JHEP 0712 (2007) 105
[11] Lai A et al. [NA48 Collaboration], Phys. Lett. B 604 (2004) 1
[12] Lai A et al. [NA48 Collaboration], Phys. Lett. B 647 (2007) 341
[13] Yushchenko O P et al., Phys. Lett. B 581 (2004) 31
[14] Yushchenko O P et al., Phys. Lett. B 589 (2004) 111
[15] Antonelli M et al. [FlaviaNet Working Group on Kaon Decays], Eur. Phys. J. C 69 (2010) 399
[16] Yushchenko O P et al. [OKA Collaboration], JETP Lett. 107 (2018) no.3, 139 [Pisma Zh. Eksp. Teor. Fiz. 107 (2018) no.3, 147]