Hall effects on MHD free convective flow and mass transfer over a stretching sheet

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Abstract

Of concern in this paper is an investigation of heat and mass transfer over a stretching sheet under the influence of an applied uniform magnetic field and the effects of Hall current are taken into account. The non-linear boundary layer equations together with the boundary conditions are reduced to a system of non-linear ordinary differential equations by using the similarity transformation. The system of non-linear ordinary differential equations are solved by developing a suitable numerical techniques such as finite difference scheme and Newton’s method of linearization. The numerical results concerned with the velocity, temperature and concentration profiles as well as the skin-friction coefficient, local Nusselt number $Nu$ and the local sherwood number $Sh$ for various values of the non-dimensional parameters presented graphically.

Keywords: Hall current, Magnetohydrodynamic, Stretching sheet, Skin-friction, Heat transfer rate, Chemical reaction

1 Introduction

The study of MHD free-convective flow and mass transfer over a stretching sheet has become a considerable attention to the researchers due to its many engineering and industrial applications

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such as, in polymer processing, electro-chemistry, MHD power generators as well as in flight magnetohydrodynamics. In the extrusion of a polymer sheet from a die, the sheet is sometimes stretched. By drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product of the desired characteristics can be achieved. This problem has also an important bearing on metallurgy where magnetohydrodynamic (MHD) techniques have recently been used.

Crane (Crane 1970) first introduced the study of steady two-dimensional boundary layer flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point in the sheet. Later on several investigators (cf. Gupta and Gupta 1977; Rajagopal et al. 1984; Siddapa and Abel 1985; Chen and Char 1988; Laha et al. 1989; Vajravelu and Nayfeh 1992; Sonth et al. 2002 and Tan et al. 2008) studied various aspects of this problem, such as the heat, mass and momentum transfer in viscous flows with or without suction or blowing. The influence of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching sheet was investigated by Pavlov (Pavlov 1974), Chakraborty and Gupta (Chakraborty and Gupta 1979), Kumari et al. (Kumari et al. 1990), Andersson (Andersson 1992, Andersson et al. 1992) and Char (Char 1994). The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify (Afify 2004) in the presence of transverse magnetic field. In all these investigations the electrical conductivity of the fluid was assumed to be uniform. However, in an ionized fluid where the density is low and/or magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the spiraling of electrons and ions about the magnetic lines of force before collisions take place and a current induced in a direction normal to both the electric and magnetic fields. This phenomenon available in the literature, is known as Hall effect. Thus, the study of magnetohydrodynamic viscous flows, heat and mass transfer with Hall currents has important bearing in the engineering applications.

Hall effects on MHD boundary layer flow over a continuous semi-infinite flat plate moving with a uniform velocity in its own plane in an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field were investigated by Watanabe and
Pop (Watanabe and Pop 1995). Aboeldahab and Elbarbary (Aboeldahab and Elbarbary 2001) studied the Hall current effects on MHD free-convection flow past a semi-infinite vertical plate with mass transfer. The effect of Hall current on the steady magnetohydrodynamics flow of an electrically conducting, incompressible Burger’s fluid between two parallel electrically insulating infinite planes was studied by Rana et al. (Rana et al. 2008).

The aim of this paper is to study the Hall effects on the steady MHD free-convective flow and mass transfer over a stretching sheet in the presence of a uniform transverse magnetic field. The boundary layer equations are transformed by a similarity transformation into a system of non-linear ordinary differential equations and which are solved numerically by using the finite difference technique. Numerical calculations were performed for various values of the magnetic parameter, Hall parameter and the relative effect of chemical diffusion on thermal diffusion parameters. The results are given for the velocity distribution and the coefficients of skin-friction along and transverse to the direction of motion of the stretching surface. Similarity solution for the temperature field in the above flow is also found and the rate of heat transfer at the stretching sheet is computed for various values of magnetic and Hall parameters. Such a study is also applicable to the elongation to the bubbles and in bioengineering where the flexible surfaces of the biological conduits, cells and membranes in living systems are typically lined or surrounded with fluids which are electrically conducting (e.g., blood) and being stretched constantly.

2 Mathematical Formulation

We consider the steady free-convective flow and mass transfer of an incompressible, viscous and electrically conducting fluid past a flat surface which is assuming from a horizontal slit on a vertical surface and is stretched with a velocity proportional to distance from a fixed origin O (cf. Fig. 1). We choose a stationary frame of reference \((x, y, z)\) such that \(x\)-axis is along the direction of motion of the stretching surface, \(y\)-axis is normal to this surface and \(z\)-axis is transverse to the \(xy\)-plane. A uniform magnetic field \(B_0\) is imposed along \(y\)-axis and the effect of Hall currents is taken into account. The temperature and the species concentration are maintained
at a prescribed constant values $T_w$, $C_w$ at the sheet and $T_\infty$ and $C_\infty$ are the fixed values far away from the sheet.

Taking Hall effects into account the generalized Ohm’s law (cf. Cowling (Cowling 1957)) may be put in the form:

$$J = \frac{\sigma}{1 + m^2} \left( E + V \times B - \frac{1}{en_e} J \times B \right),$$

in which $V$ represents the velocity vector, $E$ is the intensity vector of the electric field, $B$ is the magnetic induction vector, $H$ the intensity of the magnetic field vector, $\mu_e$ the magnetic permeability, $J$ the electric current density vector, $m = \frac{\sigma B_0}{en_e}$ is the Hall parameter, $\sigma$ the electrical conductivity, $e$ the charge of the electron and $n_e$ is the number density of the electron.

The effect of Hall current gives rise to a force in the $z$-direction which in turn produces a cross flow velocity in this direction and thus the flow becomes three-dimensional. To simplify the analysis, we assume that the flow quantities do not vary along $z$-direction and this will be valid if the surface is of very large width along the $z$-direction.

Under these assumptions the boundary layer free-convection flow with mass transfer and generalized Ohm’s law is governed by the following system of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta (T - T_\infty) + g\beta^* (C - C_\infty) - \frac{\sigma B_0^2}{\rho (1 + m^2)} (u + mw),$$  \hspace{1cm} (2)

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho (1 + m^2)} (mu - w),$$  \hspace{1cm} (3)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2},$$  \hspace{1cm} (4)
\[
  u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_{\infty})^n,
\]

where \((u, v, w)\) are the velocity components along the \((x, y, z)\) directions respectively, \(\nu\) is the kinematic viscosity, \(g\) the acceleration due to gravity, \(\beta\) the coefficient of thermal expansion, \(\beta^*\) the coefficient of expansion with concentration, \(T\) and \(C\) are the temperature and concentration respectively, \(\rho\) the density of fluid, \(\alpha\) the thermal diffusivity, \(D\) the thermal molecular diffusivity, \(k_0\) the reaction rate constant and \(n\) the order of reaction.

The boundary conditions for this problem can be written as

\[
  u = bx, \quad v = w = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \quad (6)
\]

\[
  u = w = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty \quad (7)
\]

where \(b > 0\). The boundary conditions on the velocity in (6) are the no-slip conditions at the surface \(y = 0\), while the boundary conditions on velocity at \(y \to \infty\) follow from the fact that there is no flow far way from the stretching surface.

The temperature and species concentration are maintained at a prescribed constant values \(T_w\) and \(C_w\) at the sheet and are assume to vanish far way from the sheet.

Introducing the similarity transformation:

\[
  u = bx f'(\eta), \quad v = -\sqrt{b\nu} f(\eta), \quad w = bx g(\eta), \quad \eta = \sqrt{\frac{b}{\nu}} y
\]

\[
  \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}
\]

where a prime denotes derivative with respect to \(\eta\). Substituting (8) into equations (2)-(7), yields

\[
  f'''' + f f''' - f'^2 + Gr \theta + Gc \phi - \frac{M}{1 + m^2} (f' + mg) = 0
\]

(9)
\begin{align}
g'' + fg' - \left( f' + \frac{M}{1 + m^2} \right) g + \frac{Mm}{1 + m^2}f' &= 0 \quad (10) \\
\theta'' + Prf\theta' &= 0 \quad (11) \\
\phi'' + Sc(\phi'f - \gamma\phi^n) &= 0 \quad (12)
\end{align}

where \( M = \frac{\sigma B_0^2}{\rho b^2} \) is the magnetic parameter,
\( Gr = \frac{g^3(T_w - T_\infty)}{b^2} \) the local Grashof number,
\( Gc = \frac{g^3(C_w - C_\infty)}{b^2} \) the local modified Grashof number,
\( m = \frac{\sigma B_0}{en_e} \) is the Hall current parameter,
\( Pr = \frac{v}{\alpha} \) the Prandtl number,
\( Sc = \frac{v}{D} \) the Schmidt number,
and \( \gamma = \frac{k}{b}(C_w - C_\infty)^{n-1} \) the non-dimensional chemical reaction parameter.

The boundary conditions (6) and (7) are now obtained from (8) as
\begin{align}
f'(0) &= 1, \quad f(0) = g(0) = 0, \quad \theta(0) = \phi(0) = 1 \quad (13) \\
\phi'(\infty) &= \theta'(\infty) = \phi(\infty) = 0 \quad (14)
\end{align}

The major physical quantities of interest are the skin-friction coefficient \( c_f \), local Nusselt number \( Nu \) and the local Sherwood number \( Sh \) are defined respectively by,
\begin{align}
C_f &= \frac{\tau_w}{\mu f''(0)}, \quad \text{where} \quad \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} = \mu \sqrt{\frac{\alpha}{\nu}} \frac{\partial f''(0)}{\partial x}, \quad (15) \\
Nu &= \frac{q_w}{k\sqrt{\frac{\alpha}{\nu}}(T_w - T_\infty)} = -\theta'(0), \quad \text{where} \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} = -k \sqrt{\frac{\alpha}{\nu}}(T_w - T_\infty)\theta'(0), \quad (16) \\
Sh &= \frac{m_w}{D\sqrt{\frac{\alpha}{\nu}}(C_w - C_\infty)} = -\phi'(0), \quad \text{where} \quad m_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} = -D \sqrt{\frac{\alpha}{\nu}}(C_w - C_\infty)\phi'(0), \quad (17)
\end{align}
When $M = m = 0$ and $Gr = Gc = 0$, present flow problem becomes hydrodynamic boundary layer flow past a stretching sheet whose analytical solution put forwarded by Crane (Crane 1970) as follows:

$$f(\eta) = 1 - e^{-\eta}, \quad i.e. \quad f'(\eta) = e^{-\eta}$$

An attempt has been made to validate our results for the axial velocity $f'(\eta)$, we compared our results with this analytical solution.

3 Numerical Methods

Several authors Andersson et al. (Andersson et al. 1992), Afify (Afify 2004) and Abo-Eldahab (Abo-Eldahab 2001) used numerical techniques for the solution of such two-point boundary value problems is the Runge-Kutta integration scheme along with the shooting method. Although this method provides satisfactory results, it may fail when applied to problems in which the differential equations are very sensitive to the choice of its missing initial conditions. Moreover, difficulty arises in the case in which one end of the range of integration is at infinity. The end point of integration is usually approximated by replacing a finite representation to this point and it is obtained by estimating a value at which the solution will reach its asymptotic state.

On the contrary to the above mentioned numerical method, we used in the present paper has better stability, simple, accurate and more efficient numerical technique. The essential features of this technique is that it is based on a finite difference scheme with central differencing and based on the iterative procedure.

We substitute $F = f'$ and equations (9) and (10) are then transformed as

$$F'' + fF' - F^2 + Gr\theta + Gc\phi - \frac{M}{1 + m^2} (F + mg) = 0 \quad (18)$$

$$g'' + fg' - \left( F + \frac{M}{1 + m^2} \right) g + \frac{Mm}{1 + m^2} F = 0 \quad (19)$$

with the boundary conditions

$$F(0) = 1, \quad F(\infty) = 0, \quad f(0) = 0, \quad g(0) = 0, \quad g(\infty) = 0 \quad (20)$$
Central difference scheme is used for derivatives with respect to $\eta$:

\[
(w_{\eta})_i = \frac{w_{i+1} - w_{i-1}}{2\delta\eta} + O((\delta\eta)^2)
\]

\[
(w_{\eta\eta})_i = \frac{w_{i+1} - 2w_{i} + w_{i-1}}{\delta\eta^2} + O((\delta\eta)^2)
\]

where $w$ stands for $F$, $g$, $\theta$ and $\phi$; $i$ is the grid index in the $\eta$-direction with

\[
\eta_i = i \cdot \delta\eta; \quad i = 0, 1, 2, \ldots,
\]

$\delta\eta$ being the increment along $\eta$-direction.

We use Newton’s linearization method to linearize the discretised equations as follows. We assume that the values of the dependent variables at the $k$th iteration are known. Then the values of the variables at the next iteration are obtained from the following equation

\[
w_{i}^{k+1} = w_{i}^{k} + \delta w_{i}^{k}
\]  

(21)

where $\delta w_{i}^{k}$ represents the error at the $k$-th iteration. Using (21) in (18) and (19) and dropping quadratic terms in $\delta w_{i}^{k}$, we get a system of linear algebraic equations for $\delta w_{i}^{k}$. The resulting system of block tri-diagonal equations is then solved by Thomas algorithm. With an aim to test the accuracy of this numerical method we have compared the values of $f'(\eta)$ for $M = m = Gr = Gc = 0$ with those of analytical studies of Crane (Crane 1970) and have found excellent agreement presented through the Fig. 2.

4 Results and Discussion

The system of ordinary differential equations (9)-(12) subject to the boundary conditions (13) and (14) are solved numerically by employing a finite difference scheme with Newton’s linearization method described in the previous section. For numerical computations the following values of the physical parameters have been considered according to the data used in (Afify 2004):

\[
Pr = 0.7, \quad Gr = 0.5, \quad Gc = 0.5, \quad Sc = 0.5, \quad n = 1, 2, 3; \quad M = 0 \text{ to } 5.0; \quad m = 0 \text{ to } 3;
\]
\[ \gamma = 0.1, \ 0.5, \ 1.0 \]

Figs. 3-5 depicted the variation of velocity profiles \( f'(\eta), f(\eta) \) and \( g(\eta) \) corresponds to the axial velocity, transverse velocity and the velocity along z-direction called cross flow velocity for different values of \( M \). From these three figures we see that the velocity decreases with the increase in the magnetic parameter \( M \). This is to be expected from the physical consideration since as \( M \) increases, Lorentz force which opposes the flow and leads to deceleration of the fluid motion. It is interesting to note that for a fixed values of \( M \) and \( m \), \( f(\eta) \) reaches a uniform value asymptotically at a certain height \( \eta \) above the sheet. By contrast, the cross flow velocity component induced due to Hall effects and shows a anomalous behaviour with variation of \( M \). Fig. 5 shows that for fixed values of \( M \) and \( m \), \( g(\eta) \) reaches a maximum value at a certain height \( \eta \) beyond which it decreases gradually in asymptotic nature. It can be noted from Fig. 5 that in the absence of magnetic parameter \( (M = 0) \), cross flow velocity \( g(\eta) \) vanishes.

Figs. 6 and 7 illustrated distribution of non-dimensional temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) for different values of magnetic parameter \( M \) with a fixed values \( m = 0.1, \ Pr = 0.7, \ Gr = Gc = 0.5 \). These two figures shows that the temperature as well as concentration increases with with the increase of magnetic parameter \( M \). It may conclude that in the presence of magnetic field temperature and species concentration can be increased.

Effects of Hall current parameter \( m \) on the velocity profiles are presented in Figs. 8-10. For a fixed value of \( M \) the velocity components \( f'(\eta) \) and \( f(\eta) \) at a given height increases monotonically with the increase in \( m \), while the trend is reverse in the case of cross-flow velocity \( g(\eta) \) induced by Hall effects. It is interesting to observe from Figs. 8 and 9 that velocity greatly affected for \( m < 3 \), beyond which no significant change occur. Figs. 11 and 12 noticed that the temperature and concentration profiles also affected by the Hall current parameter \( m \). It is shown from Figs. 11 and 12 that the temperature as well as concentration decreases with the increase in Hall parameter \( m \).

The effects of chemical reaction on free-convective flow and mass transfer of an electrically conducting fluid over a stretching sheet have also been studied in the presence of magnetic field.
Figs. 13 and 14 shows that velocity and temperature slightly decreases with the increase of the chemical reaction parameter $\gamma$, whereas, the concentration significantly diminishes with an increase in $\gamma$.

Fig. 16 shows the variation of skin-friction with $M$ for different values of Hall parameter $m$. The skin-friction increases with the increase of both magnetic parameter $M$ and Hall parameter $m$. The rate of heat transfer at the sheet is increases with an increase in $m$ but reduces with increasing $M$ shown in Fig. 17. Variation of the rate of mass transfer at the wall is presented in Fig. 18, which shows that the mass transfer rate decreases with $M$ but increases with the increase of $m$. It is interesting to note that the skin-friction coefficient, rate of heat transfer and mass transfer at the sheet vary linearly with $M$.

5 Conclusions

The effects of Hall current parameter on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet have been studied in the presence of magnetic field. The non-linear boundary value problem is solved numerically by finite difference scheme with Newton’s linearization method.

From the present investigation, it may be concluded that all the instantaneous flow characteristics are affected by the Hall current parameter $m$. In the presence of an external magnetic field of an electrically conducting fluid, Hall current induced which in turn produces a cross flow velocity in the direction perpendicular to the directions of both axial and transverse velocity. The velocity profiles $f'$, $f$, $g$ decreases with the increase in $M$ whereas the temperature as well as concentration increases with the increase of magnetic field strength. The flow velocities $f'$ and $f$ gradually increases with the increase of Hall current parameter $m$ while, in the case of cross flow velocity $g$ and the temperature as well as the concentration a reversal trend is observed.

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Fig. 1 Physical sketch of the problem

Fig. 2 Variation of $f'(\eta)$ with $\eta$ for $M = m = 0$ and $Gr = Gc = 0$
(Comparison of the present study with the analytical solution of Crane (1970)
Fig. 3 Variation of $f'(\eta)$ with $\eta$ for different values of $M$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $m = 1$, $\gamma = 0.1$.

Fig. 4 Variation of $f(\eta)$ with $\eta$ for different values of $M$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $m = 1$, $\gamma = 0.1$. 
Fig. 5 Variation of $g(\eta)$ with $\eta$ for different values of $M$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $m = 1$, $\gamma = 0.1$.

Fig. 6 Distribution of non-dimensional temperature $\theta(\eta)$ for different $M$ with $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $m = 1$, $\gamma = 0.1$. 

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Fig. 7 The concentration profile $\phi(\eta)$ for different $M$ with $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $m = 1$, $\gamma = 0.1$

Fig. 8 Effect of Hall parameter $m$ on axial velocity $f'(\eta)$ for $M = 1.0$, $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $\gamma = 0.1$, $n = 1$
Fig. 9 Effect of Hall parameter $m$ on axial velocity $f(\eta)$ for $M = 1.0, Pr = 0.7, Gr = Gc = 0.5, Sc = 0.5, \gamma = 0.1, n = 1$

Fig. 10 Effect of Hall parameter $m$ on the $z$-direction velocity $g(\eta)$ for $M = 1.0, Pr = 0.7, Gr = Gc = 0.5, Sc = 0.5, \gamma = 0.1, n = 1$
Fig. 11 Effect of Hall parameter $m$ on non-dimensional temperature $\theta(\eta)$ for $M = 1.0$, $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $\gamma = 0.1$, $n = 1$

Fig. 12 Effect of Hall parameter $m$ on the concentration profile $\phi(\eta)$ for $M = 1.0$, $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $\gamma = 0.1$, $n = 1$
Fig. 13 Variation of axial velocity profile $f'(\eta)$ with $\eta$ for different $\gamma$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $M = 0.5$, $m = 0.1$, $n = 1$

Fig. 14 Variation of the temperature profile $\theta(\eta)$ with $\eta$ for different $\gamma$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $M = 0.5$, $m = 0.1$, $n = 1$
Fig. 15 Variation of the concentration profile $\phi(\eta)$ for different $\gamma$ and $Pr = 0.7$, $Gr = G_c = 0.5$, $Sc = 0.5$, $M = 0.5$, $m = 0.1$, $n = 1$

Fig. 16 Variation of skin-friction for different values of Hall parameter $m$ and $Pr = 0.7$, $Gr = G_c = 0.5$, $Sc = 0.5$, $n = 1$, $\gamma = 0.1$
Fig. 17 Distribution of heat transfer rate at the channel wall for different values of Hall parameter $m$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $\gamma = 0.1$

Fig. 18 Distribution of the rate of concentration at the channel wall for different values of Hall parameter $m$ and $Pr = 0.7$, $Gr = Gc = 0.5$, $Sc = 0.5$, $n = 1$, $\gamma = 0.1$
$g(\eta)$ vs $\eta$ for $\gamma = 0.1, 0.5, 1.0$