Complete one-loop corrections to decays of charged and CP-even neutral Higgs bosons into sfermions

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Abstract
We present the full one-loop corrections to charged and CP-even neutral Higgs boson decays into sfermions including also the crossed channels. The calculation was carried out in the minimal supersymmetric extension of the Standard Model and we use the on-shell renormalization scheme. For the down-type sfermions, we use DR running fermion masses and the trilinear coupling $A_f$ as input. Furthermore, we present the first numerical analysis for decays according to the Supersymmetric Parameter Analysis project. This requires the renormalization of the whole MSSM. The corrections are found to be numerically stable and not negligible.
1 Introduction

The Higgs boson is the last not discovered particle of the Standard Model (SM) and so the search for the Higgs boson is the prime objective of the LHC and other future colliders. Apart from the SM, the Higgs boson is also predicted by its minimal supersymmetric extension - the Minimal Supersymmetric Standard Model (MSSM). As opposed to the SM, the MSSM has not only one neutral Higgs boson but it predicts the existence of two neutral CP-even Higgs bosons \( (h^0, H^0) \), one neutral CP-odd Higgs boson \( (A^0) \) and two charged Higgs bosons \( (H^\pm) \). The existence of a charged Higgs boson or a CP-odd neutral one would be clear evidence for physics beyond the SM.

A further difference in the MSSM is the possibility for the Higgs bosons to decay not only into SM particles. In case the supersymmetric (SUSY) partners are not too heavy, the Higgs bosons can decay into SUSY particles as well (neutralinos \( \tilde{\chi}^0_i \), charginos \( \tilde{\chi}^\pm_k \) and sfermions \( \tilde{f}_m \)). The new decay channels might substantially influence the branching ratios of the MSSM Higgs bosons.

At tree-level the decays into SUSY particles were studied in [1, 2] and one-loop effects of the decays into charginos and neutralinos were analyzed in [3, 4] and were found not to be negligible. For the case of the CP-odd Higgs boson also the full one-loop corrections to the decay into sfermions were analyzed in [5, 6].

This paper is the continuation of the effort in [5, 6] and includes the decays of the remaining Higgs bosons of the MSSM into sfermions (including crossed channels \( \tilde{f}_2 \to \tilde{f}_1 h^0 \)). It also extends the SUSY-QCD one-loop analysis of [7] by including all SUSY-QCD and electroweak effects. The emphasis is put on the decay into 3rd generation sfermions as their masses can be light due to large mixings. Nevertheless, analytical and numerical results are presented for all generations of sfermions (i.e. \( h^0_k \to \tilde{f}_i \tilde{f}_j \) and \( H^\pm \to \tilde{f}_i \tilde{f}_j^\prime \) where \( h^0_k = (h^0, H^0) \) and \( \tilde{f} = (\tilde{u}, \tilde{d}, \tilde{c}, \tilde{b}, \tilde{t}, \tilde{\mu}, \tilde{\tau}) \).

The full electroweak corrections are calculated in the on-shell scheme [8] in the MSSM with real parameters. Due to the known problems of the on-shell scheme as demonstrated in [6], the artificially large on-shell parameters are replaced by the corresponding \( \overline{\text{DR}} \) counterparts. The numerical analysis is made using the \( \overline{\text{DR}} \) input defined by the Supersymmetric Parameter Analysis Project (SPA) [9]. In contrast to [6], the actual calculation uses an on-shell input set fully consistent with the SPA convention. In order to obtain such an input set, the renormalization of the whole MSSM is required.

The paper is organized as follows. In section 2 the tree-level formulae are given for all decays. Section 3 and 4 show the full electroweak corrections including the bremsstrahlung using the analytic formulae from the appendices A and B. The numerical analysis is presented in section 5 and section 6 summarizes our conclusions.
2 Tree-level result

The tree-level widths for a neutral Higgs $h_{1,2}^0 = \{h^0, H^0\}$ decaying into two scalar fermions, $h_k^0 \to \tilde{f}_i \tilde{f}_j$ with $i, j = (1, 2)$, are given by

$$\Gamma_{\text{tree}}(h_k^0 \to \tilde{f}_i \tilde{f}_j) = \frac{N_C^f \kappa(m_{h_k^0}^2, m_{\tilde{f}_i}^2, m_{\tilde{f}_j}^2)}{16 \pi m_{h_k^0}^3} |G_{ij}^f|^2$$

with $\kappa(x, y, z) = \sqrt{(x - y - z)^2 - 4yz}$ and the colour factor $N_C^f = 3$ for squarks and $N_C^{f_i} = 1$ for sleptons, respectively. Analogously, the decay width for the charged Higgs boson $H^+$ is given by

$$\Gamma_{\text{tree}}(H^+ \to \tilde{f}_i^\dagger \tilde{f}_j^\dagger) = \frac{N_C^f \kappa(m_{H^+}^2, m_{\tilde{f}_i}^2, m_{\tilde{f}_j}^2)}{16 \pi m_{H^+}^3} |G_{ij}^{\dagger f_i}|^2,$$

where $\tilde{f}_i^{\dagger/\dagger}$ stand for the up-type or down-type sfermions. The sfermion-Higgs boson couplings $G_{ij}^f$ and $G_{ij}^{\dagger f}$, defined by the interaction lagrangian $\mathcal{L}_{\text{int}} = G_{ij}^f h_k^0 \tilde{f}_i^\dagger \tilde{f}_j + G_{ij}^{\dagger f} H^+ \tilde{f}_i^{\dagger f}_j$ as well as all couplings needed in this paper, are given in [6].

The sfermion mass matrix is diagonalized by a real $2 \times 2$ rotation matrix $R_0^f$ with rotation angle $\theta_f$ [10, 11],

$$M_{\tilde{f}}^2 = \begin{pmatrix} m_{\tilde{f}_L}^2 & m_{\tilde{f}_R}^2 \\ m_{\tilde{f}_R}^2 & m_{\tilde{f}_L}^2 \end{pmatrix} = \begin{pmatrix} m_{\tilde{f}_L}^2 & a_f m_{\tilde{f}_j} \\ a_f m_{\tilde{f}_j} & m_{\tilde{f}_R}^2 \end{pmatrix} = (R^f)^\dagger \begin{pmatrix} m_{\tilde{f}_L}^2 & 0 \\ 0 & m_{\tilde{f}_R}^2 \end{pmatrix} R^f,$$

which relates the mass eigenstates $\tilde{f}_i$, $i = 1, 2$, $(m_{\tilde{f}_i} < m_{\tilde{f}_j})$ to the gauge eigenstates $\tilde{f}_\alpha$, $\alpha = L, R$, by $\tilde{f}_i = R^f_{i\alpha} \tilde{f}_\alpha$. The left- and right-handed and the left-right mixing entries in the mass matrix are given by

$$m_{\tilde{f}_L}^2 = M_{\tilde{Q}, L}^2 + \left(I_{3L}^f - e_f \sin^2 \theta_W\right) \cos 2\beta m_Z^2 + m_{\tilde{f}_j}^2,$$

$$m_{\tilde{f}_R}^2 = M_{\tilde{U}, \tilde{D}, \tilde{E}}^2 + e_f \sin^2 \theta_W \cos 2\beta m_Z^2 + m_{\tilde{f}_j}^2,$$

$$a_f = A_f - \mu (\tan \beta)^{-2} I_{3L}^f.$$

$M_{\tilde{Q}}, M_L, M_{\tilde{U}}, M_{\tilde{D}}$ and $M_{\tilde{E}}$ are soft SUSY breaking masses, $A_f$ is the trilinear scalar coupling parameter, $\mu$ the higgsino mass parameter, $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two neutral Higgs doublet states [10, 11], $I_{3L}^f$ denotes the third component of the weak isospin of the left-handed fermion $f$, $e_f$ the electric charge in terms of the elementary charge $e_0$, and $\theta_W$ is the Weinberg angle.

The mass eigenvalues and the mixing angle in terms of primary parameters are

$$m_{\tilde{f}_i}^2 = \frac{1}{2} \left(m_{\tilde{f}_L}^2 + m_{\tilde{f}_R}^2 \pm \sqrt{(m_{\tilde{f}_L}^2 - m_{\tilde{f}_R}^2)^2 + 4a_f^2 m_f^2} \right),$$

$$\cos \theta_f = \frac{-a_f m_f}{\sqrt{(m_{\tilde{f}_L}^2 - m_{\tilde{f}_1}^2)^2 + a_f^2 m_f^2}} \quad (0 \leq \theta_f < \pi).$$
and the trilinear soft breaking parameter $A_f$ can be written as
\[
A_f = \frac{1}{2m_f} \left( m_{f_{i1}}^2 - m_{f_{i2}}^2 \right) \sin 2\theta_f + \mu (\tan \beta)^{-2f_{jL}}. \tag{9}
\]

The mass of the sneutrino $\tilde{\nu}$ is given by $m_{\tilde{\nu}}^2 = M_L^2 + \frac{1}{2} m_Z^2 \cos 2\beta$.

For the crossed channels, $\tilde{f}_2 \to \tilde{f}_1 h_k^0$ and $\tilde{f}_2^\dagger \to \tilde{f}_1^\dagger H^+$, the decay widths are
\[
\Gamma^{\text{tree}}(\tilde{f}_2 \to \tilde{f}_1 h_k^0) = \frac{\kappa(m_{h_k^0}^2, m_{f_{i1}}^2, m_{f_{i2}}^2)}{16 \pi m_{\tilde{f}_2}^2} |G_{12k}^i|^2, \tag{10}
\]
\[
\Gamma^{\text{tree}}(\tilde{f}_j^\dagger \to \tilde{f}_i^\dagger H^+) = \frac{\kappa(m_{H^+}^2, m_{f_{i1}}^2, m_{f_{i2}}^2)}{16 \pi m_{\tilde{f}_j}^2} |G_{ij1}^{\dagger}|^2. \tag{11}
\]

### 3 One-loop Corrections

The one-loop corrected (renormalized) amplitudes $G_{ijk}^{\tilde{f}_{\text{ren}}}^{\text{ren}}$ and $G_{ij1}^{\dagger\text{ren}}$ can be expressed as
\[
G_{ijk}^{\tilde{f}_{\text{ren}}} = G_{ij1}^{\text{ren}} + \Delta G_{ijk}^{\tilde{f}_{\text{ren}}} = G_{ijk}^{\text{ren}} + \delta G_{ijk}^{\tilde{f}_{\text{ren}}} + \delta G_{ijk}^{\tilde{f}(v)} + \delta G_{ijk}^{\tilde{f}(e)}, \tag{12}
\]
\[
G_{ij1}^{\dagger\text{ren}} = G_{ij1}^{\text{ren}} + \Delta G_{ij1}^{\dagger\text{ren}} = G_{ij1}^{\text{ren}} + \delta G_{ij1}^{\dagger\text{ren}} + \delta G_{ij1}^{\dagger(v)} + \delta G_{ij1}^{\dagger(e)}, \tag{13}
\]
where $\delta G_{ijk}^{\tilde{f}(v)}$, $\delta G_{ijk}^{\tilde{f}(w)}$ and $\delta G_{ijk}^{\tilde{f}(c)}$ and the corresponding terms for the couplings to the charged Higgs boson stand for the vertex corrections, the wave-function corrections and the coupling counter-term corrections due to the shifts from the bare to the on-shell values, respectively.

Throughout the paper we use the SUSY invariant dimensional reduction (DR) as a regularization scheme. For convenience we perform the calculation in the ’t Hooft-Feynman gauge, $\xi = 1$.

The vertex corrections $\delta G_{ijk}^{\tilde{f}(v)}$ and $\delta G_{ij1}^{\dagger\tilde{f}(v)}$ come from the diagrams listed in Figs. 15 and 16. The analytic formulae are given in Appendix B. The wave-function corrections $\delta G_{ijk}^{\tilde{f}(w)}$ can be written as
\[
\delta G_{ijk}^{\tilde{f}(w)} = \frac{1}{2} \left[ \delta Z_{ij}^{\tilde{f}} G_{ijk}^{\tilde{f}} + \delta Z_{ij}^{\tilde{f}} G_{ijk}^{\tilde{f}} + \delta Z_{ik}^{\tilde{f}} G_{ijk}^{\tilde{f}} + \delta Z_{ik}^{\tilde{f}} G_{ijk}^{\tilde{f}} \right], \tag{14}
\]
with the implicit summations over $i', j', l = 1, 2$. The wave-function renormalization constants are determined by imposing the on-shell renormalization conditions [8]
\[
\delta Z_{ii}^{\tilde{f}} = - \text{Re} \hat{\Pi}_{ii}^{\tilde{f}}(m_{f_{i1}}^2), \quad i = 1, 2, \tag{15}
\]
\[
\delta Z_{ij}^{\tilde{f}} = \frac{2}{m_{f_{i1}}^2 - m_{f_{j1}}^2} \text{Re} \Pi_{ij}^{\tilde{f}}(m_{f_{j1}}^2), \quad i, j = (1, 2), \quad i \neq j, \quad \tilde{f} \neq \nu_{e, \mu, \tau} \tag{15}
\]
\[
\delta Z_{kk}^{H} = - \text{Re} \hat{\Pi}_{kk}^{H}(m_{h_k^0}^2), \quad k = 1, 2, \tag{16}
\]
\[
\delta Z_{kl}^{H} = \frac{2}{m_{h_k^0}^2 - m_{h_l^0}^2} \text{Re} \Pi_{kl}^{H}(m_{h_l^0}^2), \quad k, l = (1, 2), \quad k \neq l. \tag{16}
\]
The explicit forms of the off-diagonal Higgs boson and sfermion self-energies and their derivatives, \( \Pi^H_{kl}, \tilde{\Pi}^H_{kk} \) and \( \Pi^{\tilde{f}}_{ij}, \tilde{\Pi}^{\tilde{f}}_{ii} \) are given in Appendix A and in [6].

The coupling counter-term corrections which come from the shifting of the parameters in the lagrangian can be expressed as

\[
\delta G^{(c)}_{ijk} = \left[ \delta R^i \cdot G^{\tilde{f}}_{LR,k} \cdot (R^i)^T + R^i \cdot \delta G^{\tilde{f}}_{LR,k} \cdot (R^i)^T + R^i \cdot G^{\tilde{f}}_{LR,k} \cdot (\delta R^i)^T \right]_{ij}. \tag{17}
\]

The counter term for the sfermion mixing angle, \( \delta \theta_f \), is determined such as to cancel the anti-hermitian part of the sfermion wave-function corrections [12, 13]. Analogously we fix the Higgs boson mixing angle \( \alpha \) by means of

\[
\delta \alpha = \frac{1}{4} \left( \delta Z^H_{21} - \delta Z^H_{12} \right) = \frac{1}{2(m^2_{H^0} - m^2_{h^0})} \text{Re} \left( \Pi^H_{12}(m^2_{H^0}) + \Pi^H_{21}(m^2_{h^0}) \right). \tag{18}
\]

Using the relations

\[
\frac{\delta G^{\tilde{f}}_{ij1}}{\delta \alpha} = -G^{\tilde{f}}_{ij2}, \quad \frac{\delta G^{\tilde{f}}_{ij2}}{\delta \alpha} = G^{\tilde{f}}_{ij1}, \tag{19}
\]

and absorbing the counter terms for the mixing angles of the outer particles, \( \delta \alpha \) and \( \delta \theta_f \), into \( \delta G^{(w)}_{ijk} \) yields the symmetric wave-function corrections

\[
\delta G^{(w, \text{symm.})}_{ijk} = \frac{1}{4} \left( \delta Z^f_{ii} + \delta Z^f_{ii} \right) G^{\tilde{f}}_{ijk} + \frac{1}{4} \left( \delta Z^f_{jj} + \delta Z^f_{jj} \right) G^{\tilde{f}}_{ijk} + \frac{1}{4} \left( \delta Z^H_{kl} + \delta Z^H_{lk} \right) G^{\tilde{f}}_{ijkl}. \tag{20}
\]

Note that in this symmetrized form momentum-independent contributions from four-scalar couplings and tadpole shifts cancel out.

The sum of wave-function and counter-term corrections then reads

\[
\delta G^{(w+c)}_{ijk} = \delta G^{(w, \text{symm.})}_{ijk} + \left[ R^i \cdot \hat{\delta} G^{\tilde{f}}_{LR,k} \cdot (R^i)^T \right]_{ij}. \tag{21}
\]

The explicit forms of the counter terms \( \hat{\delta} G^{\tilde{f}}_{LR,k} \) for \( k = 1, 2 \) are given by

\[
\delta G^{\tilde{f}}_{LR,1} = -\sqrt{2} h_f m_f c_\alpha \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} \right) - g_Z m_Z e_f s^2_W s_{\alpha + \beta} + g_Z m_Z (t^2_f - e_f s^2_W) s_{\alpha + \beta} \left( \frac{\delta g_Z}{g_Z} + \frac{\delta m_Z}{m_Z} + \frac{\delta \beta}{t_{\alpha + \beta}} \right), \tag{22}
\]

\[
\delta G^{\tilde{f}}_{LR,2} = \frac{\delta h_f}{h_f} (G^{\tilde{f}}_{LR,12} - \frac{h_f}{\sqrt{2}} (\delta A_f c_\alpha + \delta \mu s_\alpha)), \tag{23}
\]

\[
\delta G^{\tilde{f}}_{LR,22} = -\sqrt{2} h_f m_f c_\alpha \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} \right) + g_Z m_Z e_f s^2_W s_{\alpha + \beta} \left( \frac{\delta g_Z}{g_Z} + \frac{\delta m_Z}{m_Z} + \frac{\delta s^2_W}{s_W^2} + \frac{\delta \beta}{t_{\alpha + \beta}} \right) \tag{24}
\]
for the sfermion couplings to the Higgs boson $h^0$ and

$$
(\hat{G}_{LR,2}^{f})_{11} = -\sqrt{2} h_f m_f s_\alpha \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} \right) + g_Z m_Z e_f \delta^2_{W} c_{\alpha+\beta}
$$

$$
- g_Z m_Z (I^3_f - e_f s_w^2) c_{\alpha+\beta} \left( \frac{\delta g_Z}{g_Z} + \frac{\delta m_Z}{m_Z} - t_{\alpha+\beta} \delta \beta \right),
$$

$$
(\hat{G}_{LR,2}^{f})_{12} = \frac{\delta h_f}{h_f} \left( G_{LR,2}^{f} \right)_{12} - \frac{h_f}{\sqrt{2}} (\delta A_f s_\alpha - \delta \mu c_\alpha),
$$

$$
(\hat{G}_{LR,2}^{f})_{22} = -\sqrt{2} h_f m_f s_\alpha \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} \right)
$$

$$
- g_Z m_Z e_f s_w^2 c_{\alpha+\beta} \left( \frac{\delta g_Z}{g_Z} + \frac{\delta m_Z}{m_Z} + \frac{\delta s_w^2}{s_w^2} - t_{\alpha+\beta} \delta \beta \right)
$$

for the couplings to $H^0$.

Analogously to the decays of the CP-even Higgs bosons, the sum of the wave-function and counter-term corrections of the charged Higgs boson can be expressed as

$$
\delta G_{ij}^{\uparrow \downarrow (w+c)} = \delta G_{ij1}^{\uparrow \downarrow (w, \text{symm.})} + \left[ R^{i\uparrow} \cdot \delta G_{LR,1}^{\uparrow \downarrow (w)} \cdot (R^{i\downarrow})^T \right]_{ij} + \delta G_{ij1}^{\uparrow \downarrow (w, HW+HG)}
$$

with the symmetrized wave-function corrections

$$
\delta G_{ij1}^{\uparrow \downarrow (w, \text{symm.})} = \frac{1}{4} (\delta Z_{i\uparrow}^{T_f \uparrow} + \delta Z_{i\downarrow}^{T_f \downarrow}) G_{ij1}^{\uparrow \downarrow (w)} + \frac{1}{4} (\delta Z_{j\uparrow}^{T_f \uparrow} + \delta Z_{j\downarrow}^{T_f \downarrow}) G_{ij1}^{\uparrow \downarrow (w, HW+HG)} + \frac{1}{2} \delta Z_{11}^{H^\uparrow} G_{ij1}^{\uparrow \downarrow (w)}.
$$

The single elements of the matrix corresponding to the variation with respect to the couplings, $\delta G_{LR,1}^{\uparrow \downarrow}$, are given explicitly as follows:

$$
(\delta G_{LR,1}^{\uparrow \downarrow})_{11} = h_f m_f s_\beta \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} + \frac{\delta s_w}{s_w} \right) + h_f m_f c_\beta \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} + \frac{\delta c_w}{c_w} \right)
$$

$$
- \frac{g m_w}{\sqrt{2}} \sin 2\beta \left( \frac{\delta g}{g} + \frac{\delta m_w}{m_w} + \cos 2\beta \frac{\delta \tan \beta}{\tan \beta} \right)
$$

$$
(\delta G_{LR,1}^{\uparrow \downarrow})_{12} = \frac{\delta h_f}{h_f} \left( G_{LR,1}^{\uparrow \downarrow} \right)_{12} + h_f (\delta A_f s_\beta + A_f \delta s_\beta + \delta \mu c_\beta + \mu \delta c_\beta)
$$

$$
(\delta G_{LR,1}^{\uparrow \downarrow})_{21} = \frac{\delta h_f}{h_f} \left( G_{LR,1}^{\uparrow \downarrow} \right)_{21} + h_f (\delta A_f c_\beta + A_f \delta c_\beta + \delta \mu s_\beta + \mu \delta s_\beta)
$$

$$
(\delta G_{LR,1}^{\uparrow \downarrow})_{22} = h_f m_f c_\beta \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} + \frac{\delta c_w}{c_w} \right) + h_f m_f s_\beta \left( \frac{\delta h_f}{h_f} + \frac{\delta m_f}{m_f} + \frac{\delta s_w}{s_w} \right)
$$

The counter terms appearing in eqs. (22-33) can be fixed in the following manner. Some of them can be decomposed further as in the case for $\delta h_f$ and $\delta g$

$$
\frac{\delta h_f}{h_f} = \frac{\delta g}{g} + \frac{\delta m_f}{m_f} - \frac{\delta m_w}{m_w} + \left\{ - \frac{\cos^2 \beta}{\sin^2 \beta} \right\} \frac{\delta \tan \beta}{\tan \beta},
$$

$$
\frac{\delta g}{g} = \frac{\delta e}{e} - \frac{\delta \sin \theta_W}{\sin \theta_W}.
$$
for \{u^+ \} -type sfermions.

For the remaining counter terms we use the standard renormalization conditions. The fixing of the angle $\beta$ is performed using the condition that the renormalized $A^0 - Z^0$ transition vanishes at $p^2 = m_{A^0}^2$ as in [14], which gives the counter term

$$\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{m_Z \sin 2\beta} \text{Im} \Pi_{A^0 Z^0}(m_{A^0}^2).$$

(35)

The higgsino mass parameter $\mu$ is fixed in the chargino sector by the chargino mass matrix, $\delta \mu \equiv \delta X_{22}$, as explained in detail in [15, 16].

The counter term to the Standard Model parameter $\sin \theta_W$ is determined using the on-shell masses of the gauge bosons as in [17]. To avoid the problems with light quarks in the fine structure constant $\alpha$, we use the $\overline{\text{MS}}$ value at the $Z$-pole with the counter term given in [5, 21].

The on-shell counter term that has the biggest influence and also poses a serious problem is the counter term to the trilinear scalar coupling parameter $A_f$. The explicit form of the counter term was already given in [5] and it was shown in [5, 6] that this counter term becomes very large for large values of $\tan \beta$. One of the aims of this paper is to show that this problem is present in all Higgs decays into sfermions. The solution takes advantage of the fact that the SPA convention which we use here, defines the SUSY parameters in the DR scheme. Therefore, the trilinear scalar coupling parameters $A_f$ are taken DR without the use of the large on-shell counter term.

4 Infrared divergences

To cancel infrared divergences we introduce a small photon mass $\lambda$ and include the real photon emission processes $h_0^0 \to \tilde{f}_i \tilde{f}_j \gamma$ ($h_0^0 = \{h^0, H^0\}$) and $H^+ \to \tilde{f}^+_i \tilde{f}^-_j \gamma$. The decay width of $H^+(p) \to \tilde{f}^+_i(k_1) + \tilde{f}^-_j(-k_2) + \gamma(k_3)$ (Fig. 1) is given by

\begin{equation}
\Gamma(H^+ \to \tilde{f}^+_i \tilde{f}^-_j \gamma) = \frac{N_C}{16 \pi^3 m_{H^+}} \left| G_{ij}^{\pm} \right|^2 (-e_0)^2 \left[ m_{H^+}^2 I_{00} + e_i^2 m_i^2 I_{11} + e_b^2 m_b^2 I_{22} \right]
\end{equation}

Figure 1: Real Bremsstrahlung diagrams relevant to cancel the IR-divergences in $H^+(p) \to \tilde{f}^+_i(k_1) + \tilde{f}^-_j(-k_2) + \gamma(k_3)$. The diagrams for the neutral Higgs decays are analogous.
\[-\epsilon_l \epsilon_b \left( (m_{H^+}^2 - m_j^2 - m_j^2) I_{12} - I_2 - I_1 \right) - \epsilon_l \left( (m_j^2 - m_{H^+}^2 - m_j^2) I_{01} - I_1 - I_0 \right) + \epsilon_b \left( (m_j^2 - m_{H^+}^2 - m_j^2) I_{02} - I_2 - I_0 \right) \]

with the phase-space integrals \( I_n \) and \( I_{mn} \) defined as \([18]\)

\[
I_{1\ldots n} = \frac{1}{\pi^2} \int \frac{d^3 k_1}{2E_1} \frac{d^3 k_2}{2E_2} \frac{d^3 k_3}{2E_3} \delta^4(p - k_1 - k_2 - k_3) \frac{1}{(2k_3 k_{i1} + \lambda^2) \ldots (2k_3 k_{in} + \lambda^2)}. \tag{36}
\]

The full IR-finite one-loop corrected decay width for the physical value \( \lambda = 0 \) is then given by

\[
\Gamma^{\text{corr}}(H^+ \to \tilde{f}_i \tilde{f}_j) = \Gamma(H^+ \to \tilde{f}_i \tilde{f}_j) + \Gamma(H^+ \to \tilde{f}_i \tilde{f}_j \gamma) \tag{37}
\]

Analogously, for the neutral Higgs boson decays and the crossed channels the photon emission processes yield

\[
\Gamma(h_k^0 \to \tilde{f}_i \tilde{f}_j \gamma) = N_C \frac{(\epsilon_0 \epsilon_f)^2 |G_{ijk}|^2}{16 \pi^3 m_{h_k^0}} \left[ (m_{h_k^0}^2 - m_{f_i}^2 - m_{f_j}^2) I_{12} + m_{f_i}^2 I_{11} + m_{f_j}^2 I_{22} - I_1 - I_2 \right] , \tag{38}
\]

\[
\Gamma(f_j^0 \to \tilde{f}_i h_k^0 \gamma) = \frac{(\epsilon_0 \epsilon_f)^2 |G_{ijk}|^2}{16 \pi^3 m_{f_j^0}} \left[ (m_{f_j^0}^2 - m_{f_i}^2 - m_{h_k^0}^2) I_{01} - m_{f_i}^2 I_{11} - m_{f_j}^2 I_{00} - I_0 - I_1 \right] ,
\]

where the IR-finite decay widths are

\[
\Gamma^{\text{corr}}(h_k^0 \to \tilde{f}_i \tilde{f}_j) = \Gamma(h_k^0 \to \tilde{f}_i \tilde{f}_j) + \Gamma(h_k^0 \to \tilde{f}_i \tilde{f}_j \gamma) , \tag{39}
\]

\[
\Gamma^{\text{corr}}(f_j^0 \to \tilde{f}_i h_k^0) = \Gamma(f_j^0 \to \tilde{f}_i h_k^0) + \Gamma(f_j^0 \to \tilde{f}_i h_k^0 \gamma) . \tag{40}
\]

5 Numerical analysis

The numerical results presented in this section are based on the SPS1a benchmark point as proposed by the Supersymmetric Parameter Analysis Project (SPA) \([9]\). A consistent implementation of the SPA convention into the calculation of a decay width and the numerical analysis is a non-trivial endeavor. As the electroweak one-loop calculations are carried out in the on-shell scheme and the SPA project proposes the SUSY input set in the \( \overline{\text{DR}} \) scheme at the scale of 1 TeV, a conversion of the input values is necessary. This conversion requires the renormalization of the whole MSSM in order to transform the input parameters correctly. Moreover, the numerical analysis of a decay makes varying fundamental SUSY parameters necessary. That is why the above mentioned transformation of parameters has to be carried out for every single parameter point. In our case this is provided by the not-yet-public routine DRbar2OS which couples to the spectrum calculator SPheno \([19]\). The transformation is performed in the following two steps:
1. The SPA input, i.e. the on-shell electroweak SM parameters, the strong coupling constant and the masses of the light quarks defined in the \( \overline{\text{MS}} \) scheme and the masses of the leptons and the top quark defined as pole masses and the SUSY parameters defined in the \( \overline{\text{DR}} \) scheme at 1 TeV, is given to SPheno which transforms it to a pure \( \overline{\text{DR}} \) input set including also higher loop corrections.

2. The pure \( \overline{\text{DR}} \) set is taken as input for the DRbar2OS routine which yields as output the complete set in the on-shell scheme. An example of different sets of parameters for the SPS1a’ benchmark point can be seen in Table 1.

All plots below show the dependence of the decay width on a \( \overline{\text{DR}} \) parameter. By varying a single \( \overline{\text{DR}} \) parameter and transforming subsequently to the on-shell scheme, almost all parameters are influenced. That means, not only the corresponding on-shell parameter changes, but also the other parameters through loop effects.

| \( \overline{\text{DR}} \) SUSY Parameters | On-shell SUSY Parameters |
|-----------------|-----------------|
| \( g' \) | 0.36354 | 0.35565 |
| \( g \) | 0.64804 | 0.66547 |
| \( g_s \) | 1.08412 | 1.08419 |
| \( Y_\tau \) | 0.10349 | 0.10771 |
| \( Y_t \) | 0.89840 | 1.04638 |
| \( Y_b \) | 0.13548 | 0.20481 |
| \( \mu \) | 401.63 | 398.71 |
| \( \tan \beta \) | 10.0 | 10.31 |
| \( M_{L_1} \) | 1.8121 \( \cdot 10^2 \) | 1.8394 \( \cdot 10^2 \) |
| \( M_{E_1} \) | 1.1572 \( \cdot 10^2 \) | 1.1784 \( \cdot 10^2 \) |
| \( M_{Q_1} \) | 5.2628 \( \cdot 10^2 \) | 5.6390 \( \cdot 10^2 \) |
| \( M_{U_1} \) | 5.0767 \( \cdot 10^2 \) | 5.4540 \( \cdot 10^2 \) |
| \( M_{D_1} \) | 5.0549 \( \cdot 10^2 \) | 5.4352 \( \cdot 10^2 \) |
| \( M_{H_1}^2 \) | 2.5605 \( \cdot 10^4 \) | 2.7220 \( \cdot 10^4 \) |

Table 1: The input parameters for the SPS1a’ point according to the SPA project. Left: \( \overline{\text{DR}} \) input values at \( Q = 1 \) TeV, Right: on-shell values used in the calculation.

Comparing the parameter sets in Table 1 one can easily see that the on-shell counter term for \( A_b \) is large as there is a huge difference between the \( \overline{\text{DR}} \) and the on-shell value of the trilinear scalar coupling parameter. This is caused by fixing the of \( A_b \) parameter in the sfermion sector [5]. This fixing was shown to lead to a numerically large counter term which should be avoided. The decays of Higgs bosons into sfermions (and the corresponding crossed channels) are the only 2-body decays that are affected directly as the
trilinear coupling parameter appears at tree-level. In this case, a very large counter term makes the perturbative expansion unreliable. Here we make use of the fact that the input parameters are given in the \( \overline{\text{DR}} \) scheme. That means our calculation uses on-shell parameters except for the \( A_b \) and \( m_b \) which take the original \( \overline{\text{DR}} \) values. Although not shown in the Table 1, this behaviour is common to all down-type trilinear scalar coupling parameters, and the same strategy as described for the \( A_b \) is applied for them as well. A distinct feature of all decay modes involving down-type sfermions is a large difference between the on-shell and the SPA tree-level. The origin of this difference is again the large counter term for the trilinear scalar couplings.

Keeping the numerical analysis strictly confined to the SPS1a’ benchmark scenario would mean that most of the possible decays are kinematically not allowed. The vertical red line denotes the position of the SPS1a’ parameter point for the kinematically allowed decay modes. For the other decays we slightly deviate from the SPS1a’ point adjusting mainly \( m_{A^0} \) and the relevant soft supersymmetry breaking terms \( M_{\{\tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}\}} \). These parameters only influence the kinematics and have usually no effect on the couplings.

In general, we always show the results using the on-shell parameters (dotted curve for on-shell tree-level and red dashed curve for on-shell full one loop decay width) as well as the improved decay widths where the parameters \( A_f \) and \( m_f \) are taken \( \overline{\text{DR}} \) (dash-dotted curve denotes SPA tree-level and blue solid curve stands for the full one loop decay width). This convention does not apply in cases where there is no down-type trilinear scalar coupling entering the tree-level. There we show only the on-shell and SPA tree-level together with the final one-loop decay width. For comparison with other calculations, the SPA tree-level is shown as defined in [9] taking all parameters in the couplings in the \( \overline{\text{DR}} \) scheme and using the proper masses for the kinematics.

\[
\{M_{\tilde{D}_3} = 150 \text{ GeV}, m_{A^0}^2 = 10^6 \text{ GeV}\}
\]

![Graph](image.png)

Figure 2: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of \( H^0 \to \tilde{b}_1\tilde{b}_1 \) as a function of \( \tan \beta \) according to the SPA convention.
Figs. 2, 3 and 4 show the dependence of the decay width on \( \tan \beta \) and clearly demonstrate the known fact \([7, 5]\) that the counter term for \( A_b \) grows with \( \tan \beta \). As mentioned above such a large counter term causes the perturbation series to break down. The full one-loop results in the on-shell scheme (red dashed curve) differ from those where \( \overline{\text{DR}} \) parameters are used (blue solid) only by higher orders. Nevertheless, due to the perturbation series breakdown, the higher order corrections are no longer suppressed by the coupling constant. This is the reason why the results using the \( \overline{\text{DR}} \) parameters should be viewed as the final one-loop corrected decay widths for the processes calculated in this paper.

\[
\{ M_{\tilde{D}_1} = 150 \text{ GeV}, \ m_{A_0}^2 = 10^6 \text{ GeV} \}
\]

Figure 3: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of \( H^0 \to \tilde{d}_1\tilde{d}_2 \) as a function of \( \tan \beta \) according to the SPA convention.

Figure 4: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of \( H^0 \to \tilde{\tau}_1\tilde{\tau}_1 \) as a function of \( \tan \beta \) according to the SPA convention.
The next class of plots shows the dependence on the superpotential parameter $\mu$ (see Figs. 6, 7, 8 and 5). The typical behaviour of the tree-level is governed by the square of the coupling $G^f_{ijk}$. It implies that the tree-level is a quadratic function of $\mu$ and as all one-loop corrections in this case are factorizable this dependence is preserved in the full one-loop result. In case down-type sfermions are involved, the on-shell curves are deformed by the large difference in the $A_f$ parameter. The corrections can reach up to $40\%$ for some areas of parameter space and are therefore not negligible.

\[ \{M_{\tilde{D}_3} = 150 \text{ GeV}, \ m_{A^0}^2 = 10^6 \text{ GeV}\} \]

Figure 5: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $H^+ \to \tilde{t}_1 \tilde{b}_2$ as a function of $\tan\beta$ according to the SPA convention.

\[ \{M_{\tilde{D}_2} = 150 \text{ GeV}, \ m_{A^0}^2 = 10^6 \text{ GeV}\} \]

Figure 6: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $H^0 \to \tilde{s}_1 \tilde{s}_2$ as a function of $\mu$ according to the SPA convention.
Furthermore, the pseudothreshold in Fig. 7 comes from the sbottom contribution to the Higgs wave-function correction.

The one-loop width of $H^+ \rightarrow \tilde{\nu}_\tau \tilde{\tau}_2$ (see Fig. 8) is unexpectedly sensitive to the large difference of the on-shell and DR $A_b$ parameters above $\mu = 600$ GeV although there is no such parameter at tree-level. It is caused by the enhanced contribution of the vertex diagram with a 4-sfermion coupling. This diagram contains the coupling of the charged Higgs boson and a stop-sbottom pair where the $A_b$ parameter appears.

$$\{M_{\tilde{\nu}_3} = 150 \text{ GeV, } m_{A^0}^2 = 10^6 \text{ GeV}\}$$

![Graph](image)

Figure 7: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $H^0 \rightarrow \tilde{t}_1 \tilde{t}_2$ as a function of $\mu$ according to the SPA convention.

![Graph](image)

Figure 8: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $H^+ \rightarrow \tilde{\nu}_\tau \tilde{\tau}_2$ as a function of $\mu$ according to the SPA convention.
Fig. 9 illustrates the aforementioned problems of the perturbation series in case of using the on-shell $A_b$ parameter. As one can see there is no obvious divergence of the decay width in the on-shell scheme. Nevertheless, the full one-loop widths in the on-shell scheme and the one with $A_b$ taken OPE are far apart. This separation is a pure two loop effect coming from using different $A_b$ values when calculating the $\delta A_b$ counter term.

In this particular case the electroweak corrections interfere destructively with the QCD corrections reducing them by half.

\[
\{ M_{\tilde{b}_3} = 150 \text{ GeV} \}
\]

Figure 9: On-shell tree-level (dotted line), full on-shell one-loop decay width (dashed line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $\tilde{b}_2 \rightarrow \tilde{b}_1 h^0$ as a function of $\mu$ according to the SPA convention.

Figure 10: On-shell tree-level (dotted line), tree-level (dash-dotted line) and full one-loop corrected width (solid line) of $\tilde{t}_2 \rightarrow \tilde{t}_1 h^0$ as a function of $A_t$ according to the SPA convention.
The only plot over the trilinear coupling $A_f$ is shown in Fig. 10 which is for the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 h^0$. The region $A_t = (-120 \text{ GeV}, 320 \text{ GeV})$ is kinematically forbidden. Although at some regions of parameter space (e.g. $A_t = (700 \text{ GeV}, 900 \text{ GeV})$) the correction to the SPA tree-level is large, one can see there is also a large difference between SPA tree-level and the tree-level used in the perturbation expansion denoted by the dotted line.

6 Conclusions

We have calculated the full electroweak one-loop corrections to the charged and CP-even neutral Higgs boson decays to sfermions including the crossed channels. We have also included the SUSY-QCD corrections which were calculated in [7]. Similar to [5] and [6], the on-shell parameters $A_b$ and $m_b$ (and the corresponding down-type parameters for the first two generations) were replaced by their DR values to avoid the numerically large counter term. Furthermore, we have presented the first consistent numerical analysis for a one-loop decay width based on the Supersymmetric Parameter Analysis project [9]. This required the renormalization of the whole MSSM in a way that allows to carry out a transformation between the on-shell and the DR scheme for every single parameter point. The corrections were shown not to be negligible and were comparable in the magnitude to the QCD in some regions of parameter space.

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A Self-energies and counter terms

Here we give the explicit form of the self-energies needed for the computation of the one-loop decay widths \( \{ h^0, H^0 \} \rightarrow f_i f_j \) and \( H^+ \rightarrow f_i^+ f_j^+ \).

For the neutral and charged Higgs fields we use the notation \( h_k^0 = \{ h^0, H^0 \} \), \( H_k^0 = \{ h^0, H^0, A^0, G^0 \} \), \( H_k^+ = \{ H^+, G^+, H^-, G^- \} \) and \( H_k^- \equiv (H_k^0)^* = \{ H^-, G^-, H^+, G^+ \} \).

The sfermion self-energies and their derivatives as well as the vector boson self-energies and the scalar-vector mixing of \( A^0 Z^0 \) have already been published and can be found in [6]. Also most of the couplings used in this paper are given therein.

In the following we use the standard two- and three-point functions \( B_i \) and \( C_i \) from [20] in the conventions of [18].

A.1 Diagonal Wave-function corrections — derivatives of Higgs boson self-energies

The conventional on-shell renormalization conditions for the diagonal wave-function renormalization constants are given in terms of the derivatives of the corresponding self-energies,

\[
\delta Z^{H_0}_{kk} = - \text{Re} \hat{\Pi}^{H_0}_{kk}(m_{h_k^0}^2),
\]

where the dot in \( \hat{\Pi}_{kk}(k^2) \) denotes the derivative with respect to \( k^2 \). In the following we list the single contributions of the Higgs wave-function corrections. The derivatives of the CP-even Higgs bosons \( h^0 \) and \( H^0 \) depicted in Fig. 11 are given as follows:

\[
\hat{\Pi}^{H_0 f}_{kk} = -\frac{2}{(4\pi)^2} \sum_f N^f C(s_k^f)^2 \left[ (4m_f^2 - m_{h_k^0}^2) \tilde{B}_0(m_{h_k^0}^2, m_f^2, m_f^2) - B_0(m_{h_k^0}^2, m_f^2, m_f^2) \right]
\]

(42)

\[
\hat{\Pi}^{H_0 f}_{kk} = \frac{1}{(4\pi)^2} \sum_f \sum_{m,n=1}^{2} N^f C(G_m^{f,n})^2 \tilde{B}_0(m_{h_k^0}^2, m_{f_m}^2, m_{f_n}^2)
\]

(43)

\[
\hat{\Pi}^{H_0,\tilde{\chi}^0}_{kk} = -\frac{1}{(4\pi)^2} g^2 \sum_{m,n=1}^{2} (F_{mnk}^0)^2 \left[ (m_{\tilde{\chi}^0_m}^2 + m_{\tilde{\chi}^0_n}^2 - m_{h_k^0}^2) \tilde{B}_0 - B_0 \right] (m_{h_k^0}^2, m_{\tilde{\chi}^0_m}^2, m_{\tilde{\chi}^0_n}^2)
\]

(44)

\[
\hat{\Pi}^{H_0,\tilde{\chi}^+}_{kk} = -\frac{1}{(4\pi)^2} g^2 \sum_{m,n=1}^{2} \left[ (F_{mnk}^+)^2 + (F_{mnk}^-)^2 \right] \left( (m_{\tilde{\chi}^+_m}^2 + m_{\tilde{\chi}^+_n}^2 - m_{h_k^0}^2) \tilde{B}_0 - B_0 \right)
\]

\[+ 4m_{\tilde{\chi}^+_m}m_{\tilde{\chi}^+_n}F_{mnk}^+ F_{mnk}^- \tilde{B}_0 \left( m_{h_k^0}^2, m_{\tilde{\chi}^+_m}^2, m_{\tilde{\chi}^+_n}^2 \right) \]

(45)

\[
\hat{\Pi}^{H_0 f}_{kk} = \frac{1}{(4\pi)^2} \frac{1}{2} \left( \frac{gz m_Z}{4} \right)^2 \left[ \sum_{m,n=1}^{2} \left( 2 + \delta_{kn}^m \delta_{mn}^l \right) \left( \cos 2\alpha \tilde{A}_{mn}^{(k)} - 2 \sin 2\alpha \tilde{B}_{mn}^{(k)} \right) \tilde{B}_0 \right]
\]

\[+ 4 \sum_{m,n=3}^{4} \sin^2 \left( \alpha + \beta - \frac{\pi}{2} (k-1) \right) \left( \tilde{C}_{m-2,n-2} \right) \left( \tilde{B}_0 \right) \left( m_{h_k^0}^2, m_{\tilde{\chi}^+_m}^2, m_{\tilde{\chi}^+_n}^2 \right) \]

\[+ \frac{1}{(4\pi)^2} \sum_{m,n=1}^{2} \left( -1 \right)^{mn} \frac{g m_W}{2} \left( 1 - \delta_{mn}^l \delta_{mn}^l \right) \left( 1 + \delta_{mn} \right) \tilde{A}_{mn}^{(k)} - \frac{g m_Z}{2} \]

(46)
Figure 11: Diagonal self-energies of CP-even Higgs bosons $h^0$ and $H^0$

\[
\times \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] \tilde{C}_{mn} \right] \hat{B}_0(m^2_{h_k}, m^2_{H^0}, m^2_{H^0}) \quad (46)
\]

\[
\hat{\Pi}_{k_k}^{H^0,V} = - \frac{1}{(4\pi)^2} \frac{g^2}{2} \sum_{l=1}^{2} (R_{lk}(\alpha-\beta))^2 \\
\times \left[ (2m^2_{h_k} + 2m^2_{H^0} - m^2_{W}) \hat{B}_0 + 2\hat{B}_0 \right] \left( m^2_{h_k}, m^2_{H^0}, m^2_{W} \right) \\
- \frac{1}{(4\pi)^2} \frac{g_Z^2}{4} \sum_{l=1}^{2} (R_{lk}(\alpha-\beta))^2 \\
\times \left[ (2m^2_{h_k} + 2m^2_{H^0} - m^2_{Z}) \hat{B}_0 + 2\hat{B}_0 \right] \left( m^2_{h_k}, m^2_{H^0}, m^2_{Z} \right) \quad (47)
\]

\[
\hat{\Pi}_{k_k}^{H^0,WW} = \frac{1}{(4\pi)^2} \left( R_{2k}(\alpha-\beta) \right)^2 \left[ 4g^2m^2_{W} \hat{B}_0(m^2_{h_k}, m^2_{W}, m^2_{W}) + 2g^2m^2_{Z} \hat{B}_0(m^2_{h_k}, m^2_{Z}, m^2_{Z}) \right] \\
+ \frac{g^2}{4} m^2_{Z} \hat{B}_0(m^2_{h_k}, m^2_{W}, m^2_{W}) + \frac{g^2}{4} m^2_{Z} \hat{B}_0(m^2_{h_k}, m^2_{Z}, m^2_{Z}), \quad (48)
\]

\[
\hat{\Pi}_{k_k}^{H^0,\text{ghost}} = - \frac{1}{(4\pi)^2} \left( R_{2k}(\alpha-\beta) \right)^2 \left[ \frac{g^2}{2} m^2_{W} \hat{B}_0(m^2_{h_k}, m^2_{W}, m^2_{W}) + \frac{g^2}{4} m^2_{Z} \hat{B}_0(m^2_{h_k}, m^2_{Z}, m^2_{Z}) \right], \quad (49)
\]

where we have used

\[
\tilde{A}_{mn}^{(k)} = \begin{pmatrix}
-\sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] & \cos[\alpha + \beta - \frac{\pi}{2}(k - 1)] \\
\cos[\alpha + \beta - \frac{\pi}{2}(k - 1)] & \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)]
\end{pmatrix},
\]

17
\[ \tilde{A}_{mn}^{(k)} = \tilde{A}_{mn}(\beta \rightarrow -\beta), \]
\[ \tilde{B}_{mn}^{(k)} = \begin{pmatrix} (k - 1) \sin(\alpha + \beta) & \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] \\ \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] & (k - 2) \cos(\alpha + \beta) \end{pmatrix}, \]
\[ \tilde{C}_{mn} = \begin{pmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{pmatrix}. \tag{50} \]

The diagrams in Fig. 12 show the diagonal charged Higgs boson self-energies entering in the wave-function corrections.

![Diagram 1](image1)

![Diagram 2](image2)

Figure 12: Self-energy diagrams of the charged Higgs boson \( H^+ \) relevant for the calculation of diagonal wave-function corrections.

\[ \hat{\Pi}_{11}^{H^+,f} = -\frac{1}{(4\pi)^2} \sum_{f = \{f_f\}}^2 N(f)^r \left[ (h_{f_f}^2 \cos^2 \beta + h_{f_f}^2 \sin^2 \beta)((m_{f_f}^2 + m_{f_f^0}^2 - m_{H^+}^2)\hat{B}_0 - B_0) \right. \\
\left. + 4h_{f_f} m_{f_f} h_{f_f} m_{f_f} \sin \beta \cos \beta \hat{B}_0 \right] \left( m_{H^+}, m_{f_f^0}, m_{f_f}^2 \right) \] \tag{51}

\[ \hat{\Pi}_{11}^{H^+}\tilde{\chi} = -\frac{1}{(4\pi)^2} g^2 \sum_{m=1}^2 \sum_{n=1}^2 \left[ \left( (F_{nm1}^L)^2 + (F_{nm1}^R)^2 \right) \left( (m_{\tilde{\chi}_m^+}^2 + m_{\tilde{\chi}_n^0}^2 - m_{H^+}^2) \hat{B}_0 - B_0 \right) \right. \\
\left. + 4m_{\tilde{\chi}_m^+} m_{\tilde{\chi}_n^0} F_{nm1}^L F_{nm1}^R \hat{B}_0 \right] \left( m_{H^+}, m_{\tilde{\chi}_m^+}, m_{\tilde{\chi}_n^0} \right) \tag{52} \]

\[ \hat{\Pi}_{11}^{H^+}\tilde{f} = \frac{1}{(4\pi)^2} \sum_{f = \{f_f\}}^2 N(f)^r \sum_{m,n=1}^2 \left( C_{nm}^{(1)} \hat{B}_0(m_{H^+}^2, m_{f_f^0}^2, m_{f_f^0}^2) \right) \tag{53} \]

\[ \hat{\Pi}_{11}^{H^+,H} = \frac{1}{(4\pi)^2} \sum_{m,n=1}^2 \left[ ((-1)^n g m_{W}/2 (1 + \delta_{2n}) \tilde{A}_{nm}^{(1)} + g z m_{Z}/2 R_{2m}(\alpha + \beta) \tilde{C}_{nm} \right]^2 \\
\times \hat{B}_0(m_{H^+}^2, m_{H^0}, m_{H^0}^2) \right] \left( m_{H^+}, m_{H^0}, m_{H^0}^2 \right) \\
+ \frac{1}{(4\pi)^2} \left( g m_{W}/2 \right)^2 \hat{B}_0(m_{H^+}^2, m_{A^0}, m_{W}^2) \tag{54} \]
\[
\Gamma_{11}^{H^+, H^-} = - \frac{1}{(4\pi)^2} c_0 \left[ (4m_{H^+}^2 - \lambda^2) \tilde{B}_0 + 2B_0 \right] (m_{H^+}^2, m_{H^+}^2, \lambda^2) \\
- \frac{1}{(4\pi)^2} g_Z^2 \frac{1}{2} - s_W^2 \left[ (4m_{H^+}^2 - m_Z^2) \tilde{B}_0 + 2B_0 \right] (m_{H^+}^2, m_{H^+}^2, m_Z^2) \\
- \frac{1}{(4\pi)^2} g_Z^2 \frac{2}{4} \left( R_{1k} (\alpha - \beta) \right)^2 \left[ (2m_{H^+}^2 + 2m_{h_k}^2 - m_W^2) \tilde{B}_0 + 2B_0 \right] (m_{H^+}^2, m_{h_k}^2, m_W^2) \\
- \frac{1}{(4\pi)^2} g_Z^2 \left[ (2m_{H^+}^2 + 2m_{A^0} - m_W^2) \tilde{B}_0 + 2B_0 \right] (m_{H^+}^2, m_{A^0}^2, m_W^2)
\]

A.2 Off-diagonal Wave-function corrections — Mixing of CP-even Higgs bosons

According to Section 3, the off-diagonal wave-function renormalization constants of the external Higgs bosons \( h^0_k = \{ h^0, H^0 \} \) are given by

\[
\delta Z_{kl}^{H^0} = \frac{2}{m_{h_k}^2 - m_{h_l}^2} \text{Re} \Pi_{kl}^{H^0} (m_{h_l}^2), \quad k \neq l
\]

The single contributions are as follows:

\[
\Pi_{12}^{H^0, f} (k^2) = - \frac{2}{(4\pi)^2} \sum_f N_C^f s_1^f s_2^f \left[ (4m_f^2 - k^2) B_0 (k^2, m_f^2, m_f^2) + A_0 (m_f^2) \right]
\]

\[
\Pi_{12}^{H^0, f} (k^2) = \frac{1}{(4\pi)^2} \sum_f \sum_{m,n=1}^2 N_C^f G_{nm}^f G_{nm2} B_0 (k^2, m_{f,m}^2, m_{f,n}^2) \\
+ \frac{1}{(4\pi)^2} \sum_f \sum_{m=1}^2 N_C^f (h_f^2 c_{12}^f + g^2 (c_{12}^f - c_{12}^f) c_{mm}^f) A_0 (m_{f,m}^2)
\]

\[
\Pi_{12}^{H^0, \chi^0} (k^2) = - \frac{1}{(4\pi)^2} g_Z^2 \sum_{m,n=1}^4 F_{nm1}^0 F_{nm2}^0 \left[ (m_{\chi^m_n} + m_{\chi^0_n})^2 - k^2 \right] B_0 (k^2, m_{\chi^m_n}^2, m_{\chi^0_n}^2) \\
+ A_0 (m_{\chi^m_n}^2) + A_0 (m_{\chi^0_n}^2)
\]

\[
\Pi_{12}^{H^0, \chi^+} (k^2) = - \frac{1}{(4\pi)^2} g_Z^2 \sum_{m,n=1}^2 \left[ 2m_{\chi^m_n} m_{\chi^+_n} \left( F_{nm1}^+ F_{nm2}^+ + F_{nm1}^+ F_{nm2}^+ \right) B_0 (k^2, m_{\chi^m_n}^2, m_{\chi^+_n}^2) \\
+ \left( F_{nm1}^+ F_{nm2}^+ + F_{nm1}^+ F_{nm2}^+ \right) \left( A_0 (m_{\chi^m_n}^2) + A_0 (m_{\chi^+_n}^2) \\
+ (m_{\chi^m_n}^2 + m_{\chi^+_n}^2 - k^2) B_0 (k^2, m_{\chi^m_n}^2, m_{\chi^+_n}^2) \right) \right]
\]

\[
\Pi_{12}^{H^0, H} (k^2) = \frac{1}{(4\pi)^2} \frac{1}{2} \left( \frac{g_Z m_Z}{4} \right)^2 \left[ \sum_{m,n=1}^2 \left( 2 + \delta_1 m \delta_m n \right) \left( \cos 2\alpha \tilde{A}_{mn}^{(1)} - 2 \sin 2\alpha \tilde{B}_{mn}^{(1)} \right) \\
\times \left( 2 + \delta_2 m \delta_m n \right) \left( \cos 2\alpha \tilde{A}_{ mn}^{(2)} - 2 \sin 2\alpha \tilde{B}_{mn}^{(2)} \right) B_0 \\
- 2 \sum_{m,n=3}^4 \sin (2\alpha + 2\beta) (\tilde{C}_{m-2,n-2})^2 B_0 \right] (k^2, m_{H^0_m}^2, m_{H^0_n}^2)
\]

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+ \frac{1}{(4\pi)^2} \sum_{m,n=1}^2 \left[ (-1)^{mn} \frac{g m_W}{2} (1 - \delta_m \delta_n) \tilde{A}_{mn}^{(1)} (1 + \delta_m) \tilde{A}_{mn}^{(2)} - \frac{g z m_z}{2} s_{\alpha + \beta} \tilde{C}_{mn} \right]
\times \left[ (-1)^{mn} \frac{g m_W}{2} (1 - \delta_m \delta_n) (1 + \delta_m) \tilde{A}_{mn}^{(2)} - \frac{g z m_z}{2} s_{\alpha + \beta - \pi/2} \tilde{C}_{mn} \right]
\times B_0(k^2, m_{H_m}^2, m_{H_n}^2)
+rac{1}{(4\pi)^2} \frac{g_z^2}{8} \sin 2\alpha \left[ 3(A_0(m_{h_0}^2) - A_0(m_{H_0}^2)) \cos 2\alpha 
+ (A_0(m_{A_0}^2) - A_0(m_{Z}^2)) \cos 2\beta \right]
\left[ (A_0(m_{H_0}^2) - A_0(m_{H_0}^2)) \cos 2\beta \right]
\left( A_0(m_{H_0}^2) - A_0(m_{H_0}^2) \right)

\Pi_{12}^{H_0,V}(k^2) = -\frac{1}{(4\pi)^2} \frac{g_z^4}{2} \sum_{l=1}^2 \left[ (2k^2 + 2m_{H_l}^2 - m_{W}^2) B_0(k^2, m_{H_l}^2, m_{W}^2) + 2A_0(m_{W}^2) 
- A_0(m_{H_l}^2) + \frac{1}{2\epsilon_W^2} \left( (2k^2 + 2m_{H_0}^2 - m_{Z}^2) B_0(k^2, m_{H_0}^2, m_{Z}^2) 
+ 2A_0(m_{Z}^2) - A_0(m_{H_0}^2) \right) \right] R_{1l}(\alpha - \beta) R_{l2}(\alpha - \beta)

\Pi_{12}^{H_0,V}(k^2) = -\frac{1}{(4\pi)^2} \sin(2\alpha - 2\beta) \left( 2g_z^2 m_{W}^2 B_0(k^2, m_{W}^2, m_{W}^2) + g_z^2 m_{Z}^2 B_0(k^2, m_{Z}^2, m_{Z}^2) \right)
The scalar-vector mixing self-energy, $\Pi_{HW}(k^2)$ (see Fig. 14), is defined by the two-point function

$$\Pi_{HW}^H(k^2) = \frac{1}{(4\pi)^2} \sin(2\alpha - 2\beta) \left[ \frac{g^2}{2} m_W^2 B_0(k^2, m_W^2, m_W^2) + \frac{g_2^2}{4} m_Z^2 B_0(k^2, m_Z^2, m_Z^2) \right]$$

\(\text{(63)}\)

A.3 $H^+W^+$-mixing

The scalar-vector mixing self-energy, $\Pi_{HW}(k^2)$, is defined by the two-point function

$$H^+ \rightarrow \begin{array}{c} \bullet \end{array} \hspace{1cm} W^\pm \hspace{1cm} \mathcal{M} = -i k^\mu \Pi_{HW}(k^2) \epsilon^*_\mu(k)$$

$$\Pi_{HW}^f = \frac{1}{(4\pi)^2} \sqrt{2} g \sum_{f=\{f_t\}} N_C^f (m_{f_t} y_1^f B_1 (m_{H^+}^2, m_{f_t}^2, m_{f_t}^2) - m_{f_t} y_1^f B_1 (m_{H^+}^2, m_{f_t}^2, m_{f_t}^2))$$

(65)

$$\Pi_{HW}^\tilde{V} = \frac{1}{(4\pi)^2} 2 g^2 \sum_{k=1}^{2} \sum_{l=1}^{4} [m_{\tilde{\chi}^+_l} (F_{kl1}^L O_{lk}^R + F_{kl1}^R O_{lk}^L) (B_0 + B_1) + m_{\tilde{\chi}^+_l} (F_{kl1}^L O_{lk}^R + F_{kl1}^R O_{lk}^L) B_1 ] (m_{H^+}^2, m_{\tilde{\chi}^+_l}^2, m_{\tilde{\chi}^+_l}^2)$$

(66)

$$\Pi_{HW}^\tilde{\chi} = \frac{1}{(4\pi)^2} 2 g^2 \sum_{f=\{f_t\}} N_C^f G_{m1}^R R_{m1} R_{n1} (B_0 + 2B_1) (m_{H^+}^2, m_{f_t}^2, m_{f_t}^2)$$

(67)

$$\Pi_{HW}^H = \frac{1}{(4\pi)^2} \frac{g}{4} \sum_{m,n=1}^2 \left[ (-1)^n g_{mn} (1 + \delta_{1n}) \tilde{A}_n^{(1)} + g_{Z} m_{Z} R_{2m} (\alpha + \beta) \tilde{C}_{1n} \right] R_{mn}(\beta - \alpha) \times (B_0 + 2B_1) (m_{H^+}^2, m_{H^+_n}^2, m_{H^+_n}^2)$$

(68)

$$\Pi_{HW}^V = \frac{1}{(4\pi)^2} \frac{g^2 m_{W}^2}{2} \sum_{k=1}^2 R_{1k} (\alpha - \beta) R_{2k} (\alpha - \beta) (B_0 - B_1) (m_{H^+}^2, m_{H^+}^2, m_{W}^2)$$

(69)

B Vertex corrections

B.1 $h_{k f j}^{\tilde{V}}$ vertex

Here we give the explicit form of the electroweak contributions to the vertex corrections which are depicted in Fig. 15. For SUSY-QCD contributions we refer to [7].

$$\delta G_{ijk}^{(v)} = \delta G_{ijk}^{(v, HH \tilde{f})} + \delta G_{ijk}^{(v, \tilde{f} \tilde{f} \tilde{f})} + \delta G_{ijk}^{(v, \tilde{f} \tilde{f} \tilde{f})} + \delta G_{ijk}^{(v, \tilde{f} \tilde{x})} + \delta G_{ijk}^{(v, V S S)}$$

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The single contributions correspond to the diagrams with three scalar particles ($\delta G_{ijk}^{f(v, H \tilde{f})}$ and $\delta G_{ijk}^{f(v, HH \tilde{f})}$), three fermions ($\delta G_{ijk}^{f(v, \tilde{f} \tilde{f})}$ and $\delta G_{ijk}^{f(v, f \tilde{f})}$), three particles with one or two vector bosons ($\delta G_{ijk}^{f(v, V SS)}$ and $\delta G_{ijk}^{f(v, V VS)}$) and two scalar or two vector particles ($\delta G_{ijk}^{f(v, SS)}$ and $\delta G_{ijk}^{f(v, V V)}$) in the loop. The vertex corrections due to the mixing of the outer particles, i.e. Higgs and sfermion mixing terms $\delta G_{ijk}^{f(v, hH \tilde{f})}$ and $\delta G_{ijk}^{f(v, \tilde{f} \tilde{f})}$ will be combined with the counter terms of the Higgs and sfermion mixing angles, $\delta \alpha$ and $\delta \theta$, see eq. (20). The vertex corrections from the exchange of one Higgs boson and two sfermions are

$$
\delta G_{ijk}^{f(v, H \tilde{f})} = -\frac{1}{(4\pi)^2} \sum_{m,n=1}^{2} \sum_{l=1}^{4} G_{mnk}^{f} G_{iml}^{f} G_{njl}^{f} C_0(m_{l}^2, m_{k}^2, m_{j}^2, m_{H_l}^2, m_{H_k}^2, m_{H_j}^2)
$$

with the standard two-point function $C_0$ [20] for which we follow the conventions of [18]. The graph with 2 Higgs particles and one sfermion in the loop leads to

$$
\delta G_{ijk}^{f(v, HH \tilde{f})} = -\frac{1}{(4\pi)^2} \sum_{m,n=1}^{2} \sum_{l=1}^{2} g_Z m_Z^2 (2 + \delta_{km} \delta_{mn})! \left( \cos 2\alpha \bar{A}_{mn}^{(k)} - 2 \sin 2\alpha \bar{B}_{mn}^{(k)} \right) \times
$$
For the gaugino exchange contributions we get

\[ G^f_{i|lm} G^f_{ij} C_0 \left( m_{f_i}^2, m_{h_k}^2, m_{f_j}^2, m_{f_i}^2, m_{H_m}^2, m_{H_n}^2 \right) \]

\[ + \frac{1}{(4\pi)^2} \frac{g_Z m_Z}{2} \sum_{m,n=3}^4 \sum_{l=1}^{2} \sin[\alpha + \beta - \frac{\pi}{2} (k - 1)] \triangleq_{m-2,n-2} \times \]

\[ G^f_{i|lm} G^f_{ij} C_0 \left( m_{f_i}^2, m_{h_k}^2, m_{f_j}^2, m_{f_i}^2, m_{H_m}^2, m_{H_n}^2 \right) \]

\[ - \frac{1}{(4\pi)^2} \sum_{m,n=1}^2 \sum_{l=1}^{2} \left[ (-1)^{mn} \frac{g_{MW}}{2} (1 - \delta_{m2} \delta_{n2}) (1 + \delta_{mn}) \tilde{A}^{l(k)}_{mn} \right] \times \]

\[ G^f_{i|lm} G^f_{ij} C_0 \left( m_{f_i}^2, m_{h_k}^2, m_{f_j}^2, m_{f_i}^2, m_{H_m}^2, m_{H_n}^2 \right) \right) \quad (72) \]

For the gaugino exchange contributions we get

\[ \delta G^f_{ijk}^{(v,\tilde{c}ff)} = \frac{1}{(4\pi)^2} \sum_{l=1}^{4} F \left( m_{f_i}^2, m_{h_k}^2, m_{f_j}^2, m_{f_i}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2 \right) \]

\[ + \frac{1}{(4\pi)^2} \sum_{l=1}^{2} F \left( m_{f_i}^2, m_{h_k}^2, m_{f_j}^2, m_{f_i}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2 \right) \]

\[ a_{ijl}^{f, b_{ijl}^{f}} \right) + \frac{1}{(4\pi)^2} \sum_{l,m=1}^{2} F \left( m_{f_i}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2, m_{f_j}^2 \right) \]

\[ k_{lml}^{f, l_{ml}^{f}, l_{ml}^{f}, l_{ml}^{f}} \right) \quad (73) \]

where \( F(\ldots) \) shortly stands for

\[ F \left( m_{1}^2, m_{0}^2, m_{2}^2, M_{0}, M_{1}, M_{2}, g_{0}^R, g_{0}^L, g_{1}^R, g_{1}^L, g_{2}^R, g_{2}^L \right) = \left( h_{1} M_{1} + h_{2} M_{2} \right) B_{0} \left( m_{0}^2, M_{1}^2, M_{2}^2 \right) \]

\[ + \left( h_{0} M_{0} + h_{1} M_{1} \right) B_{0} \left( m_{1}^2, M_{0}^2, M_{1}^2 \right) + \left( h_{0} M_{0} + h_{2} M_{2} \right) B_{0} \left( m_{2}^2, M_{0}^2, M_{2}^2 \right) \]

\[ + \left( g_{0}^R g_{1}^R g_{2}^R + g_{0}^R g_{1}^L g_{2}^L \right) M_{0} M_{0} M_{2} + h_{0} M_{0} \left( M_{0}^2 + M_{2}^2 - m_{0}^2 \right) + h_{1} M_{1} \left( M_{0}^2 + M_{2}^2 - m_{0}^2 \right) \]

\[ + h_{2} M_{2} \left( M_{0}^2 + M_{2}^2 - m_{0}^2 \right) \right) C_{0} \left( m_{0}^2, m_{2}^2, M_{0}^2, M_{2}^2 \right) \quad (74) \]

with the abbreviations \( h_{0} = \left( g_{0}^R g_{1}^R g_{2}^R + g_{0}^R g_{1}^L g_{2}^L \right), h_{1} = \left( g_{0}^R g_{1}^R g_{2}^R + g_{0}^R g_{1}^L g_{2}^L \right) \) and \( h_{2} = \left( g_{0}^R g_{1}^R g_{2}^R + g_{0}^R g_{1}^L g_{2}^L \right) \). For up-type sfermions \( F_{lmk}^{+} = F_{lmk}^{+} \) and for down-type sfermions \( F_{lmk} = F_{lmk} \). We split the irreducible vertex graphs with one vector particle in the loop into the single contributions of the photon, the Z-boson and the W-boson,

\[ \delta C^f_{ijk}^{(v,\nu)} = \delta C^f_{ijk}^{(v,\gamma)} + \delta C^f_{ijk}^{(v,\nu)} + \delta C^f_{ijk}^{(v,\nu)} \quad (75) \]

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In order to regularize the infrared divergences we introduce a photon mass \( \lambda \). Thus we have

\[
\delta G_{ijk}^{\hat{\gamma}(v,\gamma SS)} = \frac{1}{(4\pi)^2} \left( e_0 e_j \right)^2 \langle 0 | J_j^\gamma | 0 \rangle V \left( m^2_{h_k}, m^2_{h_k}, m^2_{f_i}, \lambda^2, m^2_{f_i}, m^2_{f_j}, m^2_{f_j}, m^2_{f_j}, m^2_{f_j} \right), \tag{76}
\]

\[
\delta G_{ijk}^{\hat{\gamma}(v,ZZ)} = \frac{1}{(4\pi)^2} \frac{g^2}{2} \sum_{m,n=1}^{2} G^\gamma_{mnk} \delta z_{im} \delta z_{nj} \delta \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) - \frac{i}{(4\pi)^2} \frac{g^2}{2} \sum_{l=3}^{2} \sum_{m=1}^{2} G^\gamma_{mnl} \delta z_{im} \delta \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right), \tag{77}
\]

\[
\delta G_{ijk}^{\hat{\gamma}(v,WSS)} = \frac{1}{(4\pi)^2} \frac{g^2}{2} \sum_{m,n=1}^{2} G^\gamma_{mnk} R_{1i} R_{1j} R_{1m} R_{1n} \delta \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right)
- \frac{I_{j}^{3L}}{(4\pi)^2} \frac{g^2}{2} \sum_{m,l=1}^{2} G^\gamma_{jmkl} R_{1i} R_{1m} R_{1l} \delta \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right)
- \frac{I_{j}^{3L}}{(4\pi)^2} \frac{g^2}{2} \sum_{m,l=1}^{2} G^\gamma_{jmkl} R_{1i} R_{1m} R_{1l} \delta \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right), \tag{78}
\]

where we have used the vector vertex function

\[
V \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) = -B_0 \left( m^2_{f_i}, M^2_1, M^2_2 \right) + B_0 \left( m^2_{f_i}, M^2_1, M^2_1 \right) + B_0 \left( m^2_{f_i}, M^2_1, M^2_2 \right) + \left( -2m^2_{f_i} + m^2_{f_i} + m^2_{f_i} + 2m^2_{f_i} + 2m^2_{f_i} \right) C_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right). \tag{79}
\]

The vertex corrections coming from loops with two vector bosons and one fermion are given by

\[
\delta G_{ijk}^{\hat{\gamma}(v,WW)} = \delta G_{ijk}^{\hat{\gamma}(v,ZZ)} + \delta G_{ijk}^{\hat{\gamma}(v,WW)}, \tag{80}
\]

with

\[
\delta G_{ijk}^{\hat{\gamma}(v,ZZ)} = -\frac{1}{(4\pi)^2} \frac{g^2 m_Z}{2} R_{2k} \delta \left( \alpha - \beta \right) \sum_{m=1}^{2} \left[ 4B_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) - B_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) - B_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) \right] \left( m^2_{f_i} - m^2_{f_i} - 2m^2_{f_i} - 2m^2_{f_i} \right) \times C_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) \delta \left( z_{im} \delta \left( z_{mj} \right) \right) \tag{81}
\]

and

\[
\delta G_{ijk}^{\hat{\gamma}(v,WW)} = -\frac{1}{(4\pi)^2} \frac{g^3 m_W}{4} R_{2k} \delta \left( \alpha - \beta \right) \sum_{m=1}^{2} \left[ 4B_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) - B_0 \left( m^2_{f_i}, m^2_{f_i}, m^2_{f_i} \right) \right] \tag{82}
\]
\[ -B_0 \left( m_{f_j}^2, m_{f_m}^2, m_W^2 \right) - \left( m_{h_k}^2 - 2m_{f_j}^2 - 2m_{f_j}^2 - 4m_{f_m}^2 + 2m_W^2 \right) \times \\
C_0 \left( m_{f_j}^2, m_{h_k}^2, m_{f_j}^2, m_{f_m}^2, m_W^2 \right) R^f_{i_1} R^f_{i_2} \left( R^f_{m_1} \right)^2. \]

We split the irreducible vertex graphs with two scalar particles into the contributions from two Higgs bosons, two sfermions and the corrections stemming from Higgs-sfermion loops. For better reading we introduce the abbreviations

\[
\begin{align*}
R_{ijkl}^D &= R_{i_1}^f R_{j_1}^f R_{k_2}^f R_{l_2}^f, \\
R_{ijkl}^\bar{f} &= R_{i_1}^f R_{j_1}^\bar{f} R_{k_2}^f R_{l_2}^f, \\
R_{ijkl}^\bar{f} &= R_{i_1}^f R_{j_1}^f R_{k_2}^\bar{f} R_{l_2}^f, \\
R_{ijkl}^{\bar{f}f} &= R_{i_1}^f R_{j_2}^\bar{f} R_{k_1}^f R_{l_2}^f, \\
R_{ijkl}^{\bar{f}f} &= R_{i_1}^f R_{j_2}^f R_{k_1}^\bar{f} R_{l_2}^f. \\
\end{align*}
\]

All other combinations occurring are obvious.

\[
\begin{align*}
\delta G_{ijk}^{(v,SS)} &= \delta G_{ijk}^{(v,HH)} + \delta G_{ijk}^{(v,\bar{f}f)} + \delta G_{ijk}^{(v,\bar{f}\bar{f})} + \delta G_{ijk}^{(v,f\bar{f})} + \delta G_{ijk}^{(v,f\bar{f})} \\
&\quad + \delta G_{ijk}^{(v,\bar{f}\bar{f})} + \delta G_{ijk}^{(v,\bar{f}\bar{f})} + \delta G_{ijk}^{(v,\bar{f}\bar{f})} + \delta G_{ijk}^{(v,\bar{f}\bar{f})}
\end{align*}
\]

with

\[
\begin{align*}
\delta G_{ijk}^{(v,HH)} &= -\frac{1}{(4\pi)^2} \frac{g_z m_Z}{2} \sum_{m,n=1}^2 (2 + \delta_{km} \delta_{mn}) (\cos 2\alpha \tilde{A}^{(k)} - 2 \sin 2\alpha \tilde{B}^{(k)}) \times \\
&\quad \left( h^2 c_{mn}^i \delta_{ij} + g^2 (c_{mn}^i - c_{mn}^f) e_i^f \right) B_0 \left( m_{h_k}^2, m_{h_0}^2, m_{H}^2 \right) \\
&\quad + \frac{1}{(4\pi)^2} \frac{g_z m_Z}{4} \sum_{m,n=3}^4 \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] \tilde{C}_{m-2,n-2} \times \\
&\quad \left( h^2 c_{mn}^i \delta_{ij} + g^2 (c_{mn}^i - c_{mn}^f) e_i^f \right) B_0 \left( m_{h_k}^2, m_{H_0}^2, m_{H}^2 \right) \\
&\quad - \frac{1}{(4\pi)^2} \frac{g_z m_Z}{2} \sum_{m,n=1}^2 \left( h^2 q_{mn}^i R_{i_1}^f R_{j_1}^f + h^2 q_{mn}^i R_{i_2}^f R_{j_2}^f + g^2 (d_{mn}^i - d_{mn}^f) f_{ij} \right) \times \\
&\quad \left[ (-1)^{mn} \frac{g m_W}{2} (1 - \delta_{m2} \delta_{n2}) (1 + \delta_{mn}) \tilde{A}^{(k)}_m \right.
\end{align*}
\]

\[
\begin{align*}
&\quad - \frac{g z m_Z}{2} \sin[\alpha + \beta - \frac{\pi}{2}(k - 1)] \tilde{C}_{m_0} \right] B_0 \left( m_{h_k}^2, m_{H_0}^2, m_{H}^2 \right), \\
\delta G_{ijk}^{(v,\bar{f}f)} &= -\frac{h^2}{(4\pi)^2} \sum_{m,n=1}^2 G_{mnk}^{\bar{f}} \left[ R_{i_1}^{\bar{f}mn} + R_{nij}^{\bar{f}m} + N_C^{\bar{f}} \left( R_{inmj}^{\bar{f}} + R_{mijn}^{\bar{f}} \right) \right] B_0 \left( m_{h_k}^2, m_{f_m}^2, m_{f_n}^2 \right) \\
&\quad - \frac{g^2}{(4\pi)^2} \sum_{m,n=1}^2 G_{mnk}^{\bar{f}} \left\{ \left[ \left( \frac{1}{4} - (2I_3^L - e_f) e_f s_W^2 \right) R_{ijmn}^{\bar{f}} + e_f^2 s_W^2 R_{i_2}^{\bar{f}m} \right] N_C^{\bar{f}} + 1 \right. \\
&\quad + (I_3^L - e_f) e_f s_W^2 \left[ N_C^{\bar{f}} (R_{ijnm}^{\bar{f}} + R_{mijn}^{\bar{f}}) + R_{inmj}^{\bar{f}} + R_{mjin}^{\bar{f}} \right] \right\} \\
&\quad \times B_0 \left( m_{h_k}^2, m_{f_m}^2, m_{f_n}^2 \right),
\end{align*}
\]
\[ \delta G_{ij}^{(\tilde{v}, \tilde{v}')} = -\frac{1}{(4\pi)^2} \sum_{m,n=1}^{2} G_{nmk}^{\tilde{f}} \left\{ \frac{h_f}{2} R_{mnij}^f + \frac{h_f}{2} R_{ijmn}^f + \frac{g^2}{4} \left[ N_C^f \left( (t_W^2 Y_L^f Y_L^f - 1) R_{ijmn}^f + t_Y^2 Y_R^f Y_R^f - t_Y^2 Y_L^f R_{ijmn}^f - t_Y^2 Y_L^f R_{ijmn}^f \right) + 2 R_{ijmn}^\tilde{f} \right] \right\} B_0 \left( m_{h_k}^2, m_{\tilde{f}}^2, m_f^2 \right), \]  

(87)

\[ \delta G_{ij}^{(\tilde{v}, \tilde{f})} = -\frac{N_C^f}{(4\pi)^2} \sum_{m,n=1}^{2} G_{nmk}^{\tilde{f}} \left\{ \frac{h_f}{2} h_f \left( R_{ijmn}^f + R_{ijnm}^f \right) + \frac{g^2}{4} \left[ \left( t_W^2 Y_L^f Y_L^f + 1 \right) R_{ijmn}^f \right. \right. 

+ t_Y^2 Y_R^f Y_R^f - t_Y^2 Y_L^f R_{ijmn}^f - t_Y^2 Y_L^f Y_R^f R_{mnij}^f \left. \left. \right] \right\} B_0 \left( m_{h_k}^2, m_{\tilde{f}}^2, m_f^2 \right), \]  

(88)

and

\[ \delta G_{ij}^{(\tilde{v}, \tilde{f}')} = -\frac{N_C^f}{(4\pi)^2} \sum_{m,n=1}^{2} G_{nmk}^{\tilde{f}} \left\{ (t_W^2 Y_L^f Y_L^f - 1) R_{ijmn}^f + t_Y^2 Y_R^f Y_R^f \right. 

\left. - t_Y^2 Y_L^f R_{ijmn}^f - t_Y^2 Y_L^f Y_R^f R_{mnij}^f \right\} B_0 \left( m_{h_k}^2, m_{\tilde{f}}^2, m_f^2 \right). \]  

(89)

The contributions due to the exchange of sfermions from the other two generations, \( \tilde{F}_m \), are given by

\[ \delta G_{ij}^{(\tilde{v}, \tilde{F})} = \delta G_{ij}^{(\tilde{v}, \tilde{F})} (\tilde{f} \rightarrow F), \quad \delta G_{ij}^{(\tilde{v}, \tilde{F})} = \delta G_{ij}^{(\tilde{v}, \tilde{F})} (\tilde{f} \rightarrow \tilde{F}), \]

\[ \delta G_{ij}^{(\tilde{v}, \tilde{F}'), \tilde{F}''} = \delta G_{ij}^{(\tilde{v}, \tilde{F}'), \tilde{F}''} (\tilde{f}' \rightarrow F'), \quad \delta G_{ij}^{(\tilde{v}, \tilde{F}'), \tilde{F}''} = \delta G_{ij}^{(\tilde{v}, \tilde{F}'), \tilde{F}''} (\tilde{f}' \rightarrow \tilde{F}''), \]  

(90)

where \( \tilde{F} \) denotes values belonging to the scalar fermions with the same isospin as \( \tilde{f} \), but from the other two generations (e.g. \( \tilde{F}_1 = \{ \tilde{u}_1, \tilde{c}_1 \} \) for the stop case, \ldots), and \( \tilde{F}' \) sfermions with different isospin etc.

With the abbreviations

\[ \tilde{t}_{0+} = \begin{pmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & 0 & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & 0 & 0 \\ -i \cos^2 \beta & -\frac{i}{2} \sin 2 \beta & 0 & 0 \\ -i \frac{1}{2} \sin 2 \beta & -i \sin^2 \beta & 0 & 0 \end{pmatrix}, \]

\[ \tilde{h}_{0+} = \begin{pmatrix} 0 & 0 & -\sin \alpha \sin \beta & \sin \alpha \cos \beta \\ 0 & 0 & \cos \alpha \sin \beta & -\cos \alpha \cos \beta \\ 0 & 0 & -i \sin^2 \beta & \frac{i}{2} \sin 2 \beta \\ 0 & 0 & i \frac{1}{2} \sin 2 \beta & -i \cos^2 \beta \end{pmatrix}, \]

26
the diagrams with one Higgs boson and one sfermion in the loop lead to

\[
\delta G^{(i, Hf)}_{ijk} = -\frac{1}{(4\pi)^2} \sum_{l,m=1}^2 G^{f}_{ilm}(h^2 f c^f_{kl} \delta_{mj} + g^2 (c^f_{kl} - c^f_{kl})' e^f_{mj}) B_0(m^2_{f_l}, m^2_{h^0}, m^2_{f_m})
\]

\[
+ \frac{1}{(4\pi)^2} \frac{1}{\sqrt{2}} \sum_{l,m=1}^2 G^{f'}_{ilm} \left( (h^2 f c^f_{kl} + h^2 c^f_{k,l+2}) R^f_{l1} R^f_{m1} + h^1 h^1 (c^f_{kl} + c^f_{k,l+2}) \right)
\]

\[
\times R^f_{l2} R^f_{m2} - \frac{g^2}{2} (c_{kl} + c_{k,l+2}) R^f_{l1} R^f_{m1} B_0(m^2_{f_l}, m^2_{H^+_l}, m^2_{f_m})
\]

\[
+ i \leftrightarrow j .
\]

with \( h^1 = \{ h_f, 0 \} \) and \( h^1 = \{ h_{f_i}, h_{f_j} \} \) for the decay into \{squarks, sleptons\}, respectively. \( i \leftrightarrow j \) stands for both terms before with \( i \) and \( j \) interchanged.

Finally, for the vertex graphs with two vector bosons we obtain

\[
\delta G^{(i,\bar{V}V)}_{ijk} = \frac{4}{(4\pi)^2} g_Z^2 m_Z R_{2k} (\alpha - \beta) \left[ (C^f_L) R^f_{l1} R^f_{m1} + (C^f_R) R^f_{l2} R^f_{m2} \right] B_0(m^2_{h^0}, m^2_Z, m^2_Z)
\]

\[
+ \frac{2}{(4\pi)^2} g^3 m_W R_{2k} (\alpha - \beta) R^f_{l1} R^f_{m1} B_0(m^2_{h^0}, m^2_W, m^2_W) .
\]

\[ \text{B.2 } H^+ f^+_i \tilde{f}^+_j \text{ vertex} \]

In this chapter we list the various contributions to the vertex corrections of the charged Higgs boson decays \( H^+ \rightarrow \tilde{f}^+_i \tilde{f}^+_j \). For simplicity the formulae are given for third generation sfermions as the substitution for the first and second generation sfermions are obvious.

Using the definitions for various generic vertex functions and products of couplings from the previous chapters we get for the vertex corrections,

\[
\delta G^{\tilde{b}(v)}_{ij1} = \delta G^{\tilde{b}(v, Hf)}_{ij1} + \delta G^{\tilde{b}(v, HHf)}_{ij1} + \delta G^{\tilde{b}(v, \tilde{f}f)}_{ij1} + \delta G^{\tilde{b}(v, \tilde{f} \tilde{f})}_{ij1} + \delta G^{\tilde{b}(v, \gamma SS)}_{ij1}
\]

\[
+ \delta G^{\tilde{b}(v, ZSS)}_{ij1} + \delta G^{\tilde{b}(v, WSS)}_{ij1} + \delta G^{\tilde{b}(v, HH)}_{ij1} + \delta G^{\tilde{b}(v, \tilde{f} f)}_{ij1} + \delta G^{\tilde{b}(v, \tilde{f} \tilde{f})}_{ij1} + \delta G^{\tilde{b}(v, \tilde{F} \tilde{F})}_{ij1}
\]

\[
+ \delta G^{\tilde{b}(v, Hf)}_{ij1} .
\]
The single contributions are given as follows:

\[
\delta G_{ij_1}^{\bar{b}(v, Hf)} = -\frac{1}{(4\pi)^2} \sum_{m,n=1}^{4} \sum_{k=1}^{4} G_{m1}^{\bar{b}} G_{n1}^{\bar{f}} G_{m1}^{\bar{f}} C_0 \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, m_{t_m}^2, m_{b_n}^2 \right)
\]

(94)

\[
\delta G_{ij_1}^{\bar{b}(v, Hf)} = -\frac{1}{(4\pi)^2} \frac{1}{2} \sum_{k,l,m=1}^{2} \left[ (-1)^k g m_W (1 + \delta_{1l}) \tilde{A}_{ik}^{(1)} + g_Z m_Z R_{2k} (\alpha + \beta) \tilde{C}_{1l} \right] \times \\
G_{n1}^{\bar{f}} G_{m1}^{\bar{f}} C_0 \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, m_{t_m}^2, m_{b_n}^2 \right)
\]

(95)

\[
\delta G_{ij_1}^{\bar{b}(v, \tilde{f}f)} = \frac{1}{(4\pi)^2} \sum_{k=1}^{4} F \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_i^+}, m_{t_1}^2, m_{b_2}^2, \right)
\]

(96)

\[
\delta G_{ij_1}^{\bar{f}(v, \tilde{f}\tilde{\chi}^0)} = -\frac{1}{(4\pi)^2} \sum_{k=1}^{4} \sum_{l=1}^{2} F \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, m_{t_1}^2, m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_i^+}, m_{t_1}^2, m_{b_2}^2 \right)
\]

(97)

\[
\delta G_{ij_1}^{\bar{b}(v, \gamma SS)} = \frac{1}{(4\pi)^2} e_0^2 e_t e_b G_{ij_1}^{\bar{b}} V \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, \lambda^2, m_{t_1}^2, m_{b_j}^2 \right)
\]

\[
+ \frac{1}{(4\pi)^2} e_0^2 e_t G_{ij_1}^{\bar{b}} V \left( m_{H^+}^2, m_{b_j}^2, m_{t_1}^2, \lambda^2, m_{H^+}^2 \right)
\]

\[
- \frac{1}{(4\pi)^2} e_0 e_b G_{ij_1}^{\bar{b}} V \left( m_{b_j}^2, m_{t_1}^2, m_{H^+}^2, \lambda^2, m_{b_j}^2 \right)
\]

(98)

\[
\delta G_{ij_1}^{\bar{b}(v, ZSS)} = \frac{1}{(4\pi)^2} g_Z^2 \sum_{m,n=1}^{2} G_{m1}^{\bar{b}} z_{im}^{\bar{b}} z_{nj}^{\bar{b}} V \left( m_{i_1}^2, m_{H^+}^2, m_{b_j}^2, m_{Z^2}, m_{t_m}^2, m_{b_n}^2 \right)
\]

\[
+ \frac{1}{(4\pi)^2} g_Z^2 \left( \frac{1}{2} - s_W^2 \right) \sum_{m=1}^{2} G_{m1}^{\bar{b}} z_{jm}^{\bar{b}} V \left( m_{H^+}^2, m_{b_j}^2, m_{b_j}^2, m_{Z^2}, m_{H^+}, m_{t_m}^2 \right)
\]

\[
- \frac{1}{(4\pi)^2} g_Z^2 \left( \frac{1}{2} - s_W^2 \right) \sum_{m=1}^{2} G_{m1}^{\bar{b}} z_{jm}^{\bar{b}} V \left( m_{b_j}^2, m_{t_1}^2, m_{H^+}^2, m_{Z^2}, m_{b_m}^2, m_{b_n}^2 \right)
\]

(99)
\[ \delta G_{ij1}^{\tilde{b}(v,WSS)} = \frac{1}{(4\pi)^2} \frac{g^2}{2\sqrt{2}} \sum_{m=1}^{2} \sum_{k=1}^{3} G_{mjk}^{\tilde{b}} R_{ij1}^{\tilde{b}} R_{m1}^{\tilde{b}} w_k V \left( m_{H^+}^2, m_{b_j}^2, m_{t_i}^2, m_{W}^2, m_{H^0}^2, m_{b_m}^2 \right) \]
\[ - \frac{1}{(4\pi)^2} \frac{g^2}{2\sqrt{2}} \sum_{m=1}^{2} \sum_{k=1}^{3} G_{imk}^{\tilde{t}} R_{m1}^{\tilde{t}} R_{j1}^{\tilde{t}} w_k V \left( m_{b_j}^2, m_{t_i}^2, m_{H^+}^2, m_{W}^2, m_{t_m}^2, m_{H^0}^2 \right), \]

with \( w_k = \{ \cos(\alpha - \beta), \sin(\alpha - \beta), -i \} \).

\[ \delta G_{ij1}^{\tilde{b}(v,HH)} = -\frac{1}{(4\pi)^2} \frac{1}{2\sqrt{2}} \sum_{k,l=1}^{2} \left[ (1 - \delta_{1l}) \tilde{A}_{lk}^{(1)} + g_Z m_Z R_{2k} (\alpha + \beta) \tilde{C}_{1l} \right] \times \]
\[ \left[ \left( h_t^2 - \frac{g^2}{2} \right) c_{kl}^{0+} + \frac{g^2}{2} (c_{kl}^{0+})^* \right] R_{ij1}^{\tilde{b}} \]
\[ + h_t h_b \left( c_{kl}^{0+} + (c_{kl}^{0+})^* \right) R_{12}^{\tilde{t}} B_0 \left( m_{H^+}, m_{b_k}^2, m_{H^0}^2 \right) \]
\[ - \frac{1}{(4\pi)^2} \frac{g m_W}{2\sqrt{2}} \left[ (h_t^2 + h_b^2 - g^2) \sin 2\beta R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} + h_t h_b R_{11}^{\tilde{t}} R_{11}^{\tilde{b}} B_0 \left( m_{H^+}, m_{A_0}, m_W^2 \right) \right]. \]

\[ \delta G_{ij1}^{\tilde{b}(v,\bar{f}f')} = -\frac{1}{(4\pi)^2} \sum_{m,n=1}^{2} G_{nm1}^{\tilde{b}} \left\{ N_C \left( h_t^2 R_{mjin}^{\tilde{b}} + h_b^2 R_{rimnj}^{\tilde{b}} \right) + \frac{g^2}{4} \left[ (2N_C - 1) R_{rinmj}^{\tilde{b}} \right. \right. \]
\[ \left. \left. + \frac{1}{2} Y_{L}^{\tilde{t}} Y_{L}^{\tilde{b}} R_{mijn}^{\tilde{d}} + Y_{R}^{\tilde{t}} Y_{R}^{\tilde{b}} R_{rinmj}^{\tilde{d}} \right] B_0 \left( m_{H^+}, m_{b_m}^2, m_{e_n}^2 \right) \right\} \]
\[ - \frac{1}{(4\pi)^2} \sum_{m=1}^{2} G_{1m1}^{\tilde{b}} \left( h_{bR} R_{m2}^{\tilde{b}} R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} + \frac{g^2}{2} R_{m1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{11}^{\tilde{b}} B_0 \left( m_{H^+}, m_{\tau_m}^2, m_{\nu_e}^2 \right) \right). \]

\[ \delta G_{ij1}^{\tilde{b}(v,\bar{F}F')} = -\frac{N_C}{(4\pi)^2} \sum_{m,n=1}^{2} G_{nm1}^{\tilde{b}} \left( \frac{g^2}{2} R_{ij1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{m1}^{\tilde{b}} + h_t h_u R_{t2}^{\tilde{b}} R_{11}^{\tilde{b}} R_{11}^{\tilde{b}} \right. \]
\[ \left. + h_b h_d R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} R_{m2}^{\tilde{b}} B_0 \left( m_{H^+}, m_{d_m}^2, m_{\tilde{u}_n}^2 \right) \right) \]
\[ - \frac{1}{(4\pi)^2} \sum_{m=1}^{2} G_{1m1}^{\tilde{b}} \left( \frac{g^2}{2} R_{ij1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{m1}^{\tilde{b}} + h_b h_e R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} R_{m2}^{\tilde{b}} B_0 \left( m_{H^+}, m_{e_m}^2, m_{\nu_e}^2 \right) \right) \]
\[ - N_C^2 \sum_{m,n=1}^{2} G_{nm1}^{\tilde{b}} \left( \frac{g^2}{2} R_{ij1}^{\tilde{b}} R_{t1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{n2}^{\tilde{b}} R_{m1}^{\tilde{b}} \right. \]
\[ \left. + h_b h_d R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} R_{m2}^{\tilde{b}} B_0 \left( m_{H^+}, m_{d_m}^2, m_{\tilde{u}_n}^2 \right) \right) \]
\[ - \frac{1}{(4\pi)^2} \sum_{m=1}^{2} G_{1m1}^{\tilde{b}} \left( \frac{g^2}{2} R_{ij1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{m1}^{\tilde{b}} + h_b h_e R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} R_{m2}^{\tilde{b}} B_0 \left( m_{H^+}, m_{e_m}^2, m_{\nu_e}^2 \right) \right) \]
\[ - \frac{1}{(4\pi)^2} \sum_{m=1}^{2} G_{1m1}^{\tilde{b}} \left( \frac{g^2}{2} R_{ij1}^{\tilde{b}} R_{11}^{\tilde{b}} R_{m1}^{\tilde{b}} + h_b h_e R_{11}^{\tilde{b}} R_{12}^{\tilde{b}} R_{m2}^{\tilde{b}} B_0 \left( m_{H^+}, m_{e_m}^2, m_{\nu_e}^2 \right) \right). \]
\[ \delta G_{ij1} = - \frac{1}{(4\pi)^2} \frac{1}{\sqrt{2}} \sum_{k=1}^{4} \sum_{m=1}^{2} G^i_{imk} \left[ R^i_{m1} R^b_{j1} \left( h_{i}^2 c_{k1}^{0+} + h_{b}^2 (\tilde{c}_{k1}^{0+})^* \right) + R^i_{m2} R^b_{j2} h_{i} h_{b} \times \right] \left( \tilde{c}_{k1}^{0+} + (\tilde{c}_{k3}^{0+})^* \right) - \frac{g^2}{2} R^i_{m1} R^b_{j1} \left( c_{k1}^{0+} + (\tilde{c}_{k3}^{0+})^* \right) \right] B_0 \left( m_{i1}^2, m_{H_2}, m_{t_m}^2 \right) \]

\[ \delta G_{ij1} = - \frac{1}{(4\pi)^2} \frac{1}{\sqrt{2}} \sum_{k=1}^{4} \sum_{m=1}^{2} G^b_{mjk} \left[ R^b_{m1} R^i_{j1} \left( h_{i}^2 c_{k1}^{0+} + h_{b}^2 (\tilde{c}_{k1}^{0+})^* \right) + R^b_{m2} R^i_{j2} h_{i} h_{b} \times \right] \left( \tilde{c}_{k1}^{0+} + (\tilde{c}_{k3}^{0+})^* \right) - \frac{g^2}{2} R^b_{m1} R^i_{j1} \left( c_{k1}^{0+} + (\tilde{c}_{k3}^{0+})^* \right) \right] B_0 \left( m_{b1}^2, m_{H_2}, m_{b_m}^2 \right) \]

\[ \delta G_{ij1} = - \frac{1}{(4\pi)^2} \frac{1}{\sqrt{2}} \sum_{k,m=1}^{2} G^{i0}_{mjk} \left[ h_{i}^2 d_{1k}^{0+} R^i_{m1} R^i_{j1} + h_{b}^2 d_{1k}^{0+} R^b_{m2} R^i_{j2} + g^2 f_{im} \left( d_{1k}^{0+} \right) \right] \times \]

\[ B_0 \left( m_{b1}^2, m_{H_2}^2, m_{t_m}^2 \right) \]

\[ \delta G_{ij1} = - \frac{1}{(4\pi)^2} \frac{1}{\sqrt{2}} \sum_{k,m=1}^{2} G^{b0}_{imk} \left[ h_{i}^2 d_{1k}^{0+} R^i_{m1} R^b_{j1} + h_{b}^2 d_{1k}^{0+} R^b_{m2} R^b_{j2} + g^2 f_{mj} \left( d_{1k}^{0+} \right) \right] \times \]

\[ B_0 \left( m_{i1}^2, m_{H_2}^2, m_{b_m}^2 \right) \]

(104)
Figure 15: Vertex diagrams relevant to the calculation of the virtual electroweak corrections to the decay width $h_k^0 \rightarrow \tilde{f}_i \tilde{f}_j$ with $h_k^0 = \{h^0, H^0\}$. 
Figure 16: Vertex diagrams relevant to the calculation of the virtual electroweak corrections to the decay width $H^+ \rightarrow \tilde{f}_i \tilde{f}_j$. In the fourth row, $\tilde{f}_m^\uparrow$ and $\tilde{f}_m^\downarrow$ denote up- and down-type sfermions of all three generations, respectively.
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