Abstract

We prove if \( B \) is a tilted or cluster-tilted algebra, then \( B \) is \( \tau \)-tilting finite if and only if \( B \) is representation-finite.

1 Introduction

The theory of \( \tau \)-tilting was introduced by Adachi, Iyama and Reiten in [1] as a far-reaching generalization of classical tilting theory for finite dimensional associative algebras. One of the main classes of objects in the theory is that of \( \tau \)-rigid modules: a module \( M \) over an algebra \( \Lambda \) is \( \tau_\Lambda \)-rigid if \( \text{Hom}_\Lambda(M, \tau_\Lambda M) = 0 \), where \( \tau_\Lambda M \) denotes the Auslander-Reiten translation of \( M \); such a module \( M \) is called \( \tau_\Lambda \)-tilting if the number \( |M| \) of non-isomorphic indecomposable summands of \( M \) equals the number of isomorphism classes of simple \( \Lambda \)-modules. Recently, a new class of algebras were introduced by Demonet, Iyama, Jasso in [8] called \( \tau_\Lambda \)-tilting finite algebras. They are defined as finite dimensional algebras with only a finite number of isomorphism classes of basic \( \tau_\Lambda \)-tilting modules.

An obvious sufficient condition for an algebra to be \( \tau_\Lambda \)-tilting finite is for it to be representation-finite. In general, this condition is not necessary. The aim of this note is to prove that, for tilted and cluster-tilted algebras, this condition is in fact necessary.

Tilted algebras are the endomorphism algebras of tilting modules over hereditary algebras, introduced by Happel and Ringel in [9]. Cluster-tilted algebras are the endomorphism algebras of cluster-tilting objects over cluster categories of hereditary algebras, introduced by Buan, Marsh and Reiten in [6]. The similarity in the two definitions lead to the following precise relation between tilted and cluster-tilted algebras, which was established in [2] by Assem, Brüstle, and Schiffler.

There is a surjective map

\[
\{\text{tilted algebras}\} \twoheadrightarrow \{\text{cluster-tilted algebras}\}
\]

\[
C \twoheadrightarrow B = C \ltimes E
\]
where $E$ denotes the $C$-$C$-bimodule $E = \text{Ext}_{C}^{2}(DC, C)$ and $C \rtimes E$ is the trivial extension.

This result allows one to define cluster-tilted algebras without using the cluster category. Using this construction, we show the following.

**Theorem 1.1.** Let $B$ be a tilted or cluster-tilted algebra. Then $B$ is $\tau_{B}$-tilting finite if and only if $B$ is representation-finite.

## 2 Notation and Preliminaries

We now set the notation for the remainder of this paper. All algebras are assumed to be finite dimensional over an algebraically closed field $k$. If $\Lambda$ is a $k$-algebra then denote by $\text{mod} \Lambda$ the category of finitely generated right $\Lambda$-modules and by $\text{ind} \Lambda$ a set of representatives of each isomorphism class of indecomposable right $\Lambda$-modules. We denote by $\text{add} M$ the smallest additive full subcategory of $\text{mod} \Lambda$ containing $M$, that is, the full subcategory of $\text{mod} \Lambda$ whose objects are the direct sums of direct summands of the module $M$. Given $M \in \text{mod} \Lambda$, the projective dimension of $M$ in is denoted $\text{pd}_{\Lambda} M$ and its injective dimension by $\text{id}_{\Lambda} M$. We let $\tau_{\Lambda}$ and $\tau^{-1}_{\Lambda}$ be the Auslander-Reiten translations in $\text{mod} \Lambda$. We let $D$ be the standard duality functor $\text{Hom}_{k}(-, k)$. Finally, $\Gamma(\text{mod} \Lambda)$ will denote the Auslander-Reiten quiver of $\Lambda$.

### 2.1 Tilted Algebras

Tilting theory is one of the main themes in the study of the representation theory of algebras. Given a $k$-algebra $A$, one can construct a new algebra $B$ in such a way that the corresponding module categories are closely related. The main idea is that of a tilting module.

**Definition 2.1.** Let $A$ be an algebra. An $A$-module $T$ is a partial tilting module if the following two conditions are satisfied:

1. $\text{pd}_{A} T \leq 1$.
2. $\text{Ext}_{A}^{1}(T, T) = 0$.

A partial tilting module $T$ is called a tilting module if it also satisfies the following additional condition:

3. There exists a short exact sequence $0 \to A \to T' \to T'' \to 0$ in $\text{mod} A$ with $T'$ and $T'' \in \text{add} T$.

Tilting modules induce torsion pairs in a natural way. We consider the restriction to a subcategory $C$ of a functor $F$ defined originally on a module category, and we denote it by $F|_{C}$. Also, let $S$ be a subcategory of a category $C$. We say $S$ is a full subcategory of $C$ if, for each pair of objects $X$ and $Y$ of $S$, $\text{Hom}_{S}(X, Y) = \text{Hom}_{C}(X, Y)$.

**Definition 2.2.** A pair of full subcategories $(\mathcal{T}, \mathcal{F})$ of $\text{mod} A$ is called a torsion pair if the following conditions are satisfied:

1. $\text{Hom}_{\mathcal{T}}(M, N) = 0$ for all $M \in \mathcal{T}, N \in \mathcal{F}$.
(b) \( \text{Hom}_A(M, -)|_{\mathcal{F}} = 0 \) implies \( M \in \mathcal{T} \).
(c) \( \text{Hom}_A(-, N)|_{\mathcal{T}} = 0 \) implies \( N \in \mathcal{F} \).

Consider the following full subcategories of \( \text{mod} A \) where \( T \) is a tilting \( A \)-module.

\[
\mathcal{T}(T) = \{ M \in \text{mod} A \mid \text{Ext}_A^1(T, M) = 0 \}
\]

\[
\mathcal{F}(T) = \{ M \in \text{mod} A \mid \text{Hom}_A(T, M) = 0 \}
\]

Then \((\mathcal{T}(T), \mathcal{F}(T))\) is a torsion pair in \( \text{mod} A \) called the induced torsion pair of \( T \). Considering the endomorphism algebra \( B = \text{End}_A T \), there is an induced torsion pair, \((X(T), Y(T))\), in \( \text{mod} B \).

\[
X(T) = \{ M \in \text{mod} B \mid M \otimes_B T = 0 \}
\]

\[
Y(T) = \{ M \in \text{mod} B \mid \text{Tor}_1^B(M, T) = 0 \}
\]

We now state the definition of a tilted algebra.

**Definition 2.3.** Let \( A \) be a hereditary algebra with \( T \) a tilting \( A \)-module. Then the algebra \( B = \text{End}_A T \) is called a tilted algebra.

The following proposition describes several facts about tilted algebras. Let \( A \) be an algebra and \( M, N \) be two indecomposable \( A \)-modules. A path in \( \text{mod} A \) from \( M \) to \( N \) is a sequence

\[
M = M_0 \xrightarrow{f_1} M_1 \xrightarrow{f_2} M_2 \rightarrow \ldots \xrightarrow{f_s} M_s = N
\]

where \( s \geq 0 \), all the \( M_i \) are indecomposable, and all the \( f_i \) are non-zero non-isomorphisms. In this case, \( M \) is called a predecessor of \( N \) in \( \text{mod} A \) and \( N \) is called a successor of \( M \) in \( \text{mod} A \). A path in \( \text{mod} A \) is called a cycle if its source module \( M_0 \) is isomorphic with its target. Also, we say a torsion pair \((\mathcal{T}, \mathcal{F})\) is split if every indecomposable \( A \)-module belongs to either \( \mathcal{T} \) or \( \mathcal{F} \).

**Proposition 2.4.** [4, VIII, Lemma 3.2.]. Let \( A \) be a hereditary algebra, \( T \) a tilting \( A \)-module, and \( B = \text{End}_A T \) the corresponding tilted algebra. Then

(a) \( \text{gl.dim} B \leq 2 \).
(b) For all \( M \in \text{ind} B \), \( \text{id}_B M \leq 1 \) or \( \text{pd}_B M \leq 1 \).
(c) For all \( M \in X(T) \), \( \text{id}_B M \leq 1 \).
(d) For all \( M \in Y(T) \), \( \text{pd}_B M \leq 1 \).
(e) \((X(T), Y(T))\) is split.
(f) \( Y(T) \) is closed under predecessors and \( X(T) \) is closed under successors.

It is well known that the Auslander-Reiten quiver of a tilted algebra has an acyclic component containing a finite section. Here, a path in \( \text{mod} B \) is called a cycle if its source module \( M_0 \) is isomorphic with its target \( M_t \).
Definition 2.5. Let $B$ be an algebra. A connected full subquiver $\Sigma$ of $\Gamma(\text{mod } B)$ is a section if the following conditions are satisfied:

1. $\Sigma$ contains no oriented cycles.
2. $\Sigma$ intersects each $\tau_B$-orbit exactly once.
3. If $X_0 \to X_1 \to \cdots \to X_t$ is a path in $\Gamma(\text{mod } B)$ with $X_0, X_t \in \Sigma$, then $X_i \in \Sigma$ for all $i$ such that $0 \leq i \leq t$.

Theorem 2.6. [4, VIII, Theorem 3.5.] Let $A$ be a hereditary algebra, $T$ a tilting $A$-module, and $B = \text{End}_A(T)$. Then the class $\Sigma$ of all $B$-modules of the form $\text{Hom}_A(T, I)$, where $I$ is an indecomposable injective $A$-module, forms a section lying in an acyclic component $C_T$ of $\Gamma(\text{mod } B)$. Any predecessor of $\Sigma$ in $C_T$ lies in $\mathcal{Y}(T)$, and any proper successor of $\Sigma$ lies in $\mathcal{X}(T)$.

One may think of the connected component $C_T$ of $\Gamma(\text{mod } B)$ as connecting the torsion free part $\mathcal{Y}(T)$ with the torsion part $\mathcal{X}(T)$ along the section $\Sigma$. For this reason, the connected component $C_T$ is called the connecting component of $\Gamma(\text{mod } B)$ determined by $T$. See [4] for more details.

We say an indecomposable $B$-module $M$ is a directing module if $M$ lies on no cycle in mod $B$. A nice property of the connected component $C_T$ is that every indecomposable module $M$ in $C_T$ is directing.

Lemma 2.7. [4, IX, Lemma 1.1(b).] Let $A$ be a hereditary algebra, $T$ a tilting $A$-module, $B = \text{End}_A(T)$, and $C_T$ be the connecting component of $\Gamma(\text{mod } B)$ determined by $T$. Then every indecomposable $B$-module in $C_T$ is directing.

2.2 Cluster categories and cluster-tilted algebras

Let $C = kQ$ be the path algebra of the quiver $Q$ and let $\mathcal{D}(\text{mod } C)$ denote the derived category of bounded complexes of $C$-modules. The cluster category $\mathcal{C}_C$ is defined as the orbit category of the derived category with respect to the functor $\tau^{[1]}_C$, where $\tau_C$ is the Auslander-Reiten translation in the derived category and $[1]$ is the shift. Cluster categories were introduced in [5], and in [7] for type $A$.

An object $T$ in $\mathcal{C}_C$ is called cluster-tilting if $\text{Ext}^1_{\mathcal{C}_C}(T, T) = 0$ and $T$ has $|Q_0|$ non-isomorphic indecomposable direct summands where $|Q_0|$ is the number of vertices of $Q$. The endomorphism algebra $\text{End}_{\mathcal{C}_C} T$ of a cluster-tilting object is called a cluster-tilted algebra [6].

2.3 Relation extensions

Let $C$ be an algebra of global dimension at most 2 and let $E$ be the $C$-$C$-bimodule $E = \text{Ext}^2_C(DC, C)$.

Definition 2.8. The relation extension of $C$ is the trivial extension $B = C \ltimes E$, whose underlying $C$-module structure is $C \oplus E$, and multiplication is given by $(c, e)(c', e') = (cc', ce' + ec')$. 
Relation extensions were introduced in [2]. In the special case where \( C \) is a tilted algebra, we have the following result.

**Theorem 2.9.** [2, Theorem 3.4]. *Let \( C \) be a tilted algebra. Then \( B = C \ltimes \text{Ext}^2_C(DC, C) \) is a cluster-tilted algebra. Moreover all cluster-tilted algebras are of this form.*

### 2.4 Induction, coinduction, and \( \tau \)-rigidity

A fruitful way to study cluster-tilted algebras is via induction and coinduction functors. Recall, \( D \) denotes the standard duality functor.

**Definition 2.10.** Let \( C \) be a subalgebra of \( B \), then

\[
- \otimes_C B : \text{mod } C \rightarrow \text{mod } B
\]

is called the *induction functor*, and dually

\[
D(B \otimes_C D-) : \text{mod } C \rightarrow \text{mod } B
\]

is called the *coinduction functor*. Moreover, given \( M \in \text{mod } C \), the corresponding induced module is defined to be \( M \otimes_C B \), and the coinduced module is defined to be \( D(B \otimes_C DM) \).

We can say more in the situation when \( B \) is a split extension of \( C \).

**Definition 2.11.** Let \( B \) and \( C \) be two algebras. We say \( B \) is a *split extension* of \( C \) by a nilpotent bimodule \( E \) if there exists a short exact sequence of \( B \)-modules

\[
0 \rightarrow E \rightarrow B \xrightarrow{\pi} C \rightarrow 0
\]

where \( \pi \) and \( \sigma \) are algebra morphisms, such that \( \pi \circ \sigma = 1_C \), and \( E = \ker \pi \) is nilpotent.

In particular, relation extensions are split extensions. Following [1] we state the following definition.

**Definition 2.12.** A \( C \)-module \( M \) is *\( \tau_C \)-rigid* if \( \text{Hom}_C(M, \tau_C M) = 0 \). A *\( \tau_C \)-rigid module* \( M \) is *\( \tau_C \)-tilting* if the number of pairwise, non-isomorphic, indecomposable summands of \( M \) equals the number of isomorphism classes of simple \( C \)-modules.

It follows from the Auslander-Reiten formulas that any \( \tau_C \)-rigid module is rigid and the converse holds if the projective dimension is at most 1. In particular, any partial tilting module is a \( \tau_C \)-rigid module, and any tilting module is a \( \tau_C \)-tilting module. Thus, we can regard \( \tau_C \)-tilting theory as a generalization of classic tilting theory.

Given a \( \tau_C \)-rigid module \( M \), we are interested when \( M \) or \( M \otimes_C B \) is \( \tau_B \)-rigid. The next two results provide sufficient conditions. We assume \( C \) is tilted and \( B = C \ltimes E \) is the corresponding cluster-tilted algebra with \( E = \text{Ext}^2_C(DC, C) \).

**Proposition 2.13.** [12, Proposition 3.2] *Let \( M \) be a \( \tau_C \)-rigid \( C \)-module. If \( \text{id}_C M \leq 1 \), then \( M \) is \( \tau_B \)-rigid.*

**Proposition 2.14.** [12, Proposition 3.3] *Let \( M \) be a \( \tau_C \)-rigid \( C \)-module. If \( \text{pd}_C \tau_C M \leq 1 \), then the induced module \( M \otimes_C B \) is \( \tau_B \)-rigid.*
2.5 Slices and local slices

**Definition 2.15.** A slice $\Sigma$ in $\Gamma(\text{mod } A)$ is a set of indecomposable $A$-modules such that

1. $\Sigma$ is sincere.
2. Any path in $\text{mod } A$ with source and target in $\Sigma$ consists entirely of modules in $\Sigma$.
3. If $M$ is an indecomposable non-projective $A$-module then at most one of $M$, $\tau_A M$ belongs to $\Sigma$.
4. If $M \to S$ is an irreducible morphism with $M, S \in \text{ind } A$ and $S \in \Sigma$, then either $M$ belongs to $\Sigma$ or $M$ is non-injective and $\tau^{-1}_A M$ belongs to $\Sigma$.

The existence of slices is used to characterize tilted algebras in the following way.

**Theorem 2.16.** ([10]) Let $B = \text{End}_A T$ be a tilted algebra. Then the class of $A$-modules $\text{Hom}_A(T, DA)$ forms a slice in $\text{mod } B$. Conversely, any slice in any module category is obtained in this way.

The following notion of local slices was introduced in [3] in the context of cluster-tilted algebras. We say a path $X = X_0 \to X_1 \to X_2 \to \cdots \to X_s = Y$ in $\Gamma(\text{mod } A)$ is sectional if, for each $i$ with $0 < i < s$, we have $\tau_A X_{i+1} \neq X_{i-1}$.

**Definition 2.17.** A local slice $\Sigma$ in $\Gamma(\text{mod } A)$ is a set of indecomposable $A$-modules inducing a connected full subquiver of $\Gamma(\text{mod } A)$ such that

1. If $X \in \Sigma$ and $X \to Y$ is an arrow in $\Gamma(\text{mod } A)$, then either $Y$ or $\tau_A Y \in \Sigma$.
2. If $Y \in \Sigma$ and $X \to Y$ is an arrow in $\Gamma(\text{mod } A)$, then either $X$ or $\tau^{-1}_A X \in \Sigma$.
3. For every sectional path $X = X_0 \to X_1 \to X_2 \to \cdots \to X_s = Y$ in $\Gamma(\text{mod } A)$ with $X, Y \in \Sigma$, we have $X_i \in \Sigma$, for $i = 0, 1, \ldots, s$.
4. The number of indecomposable $A$-modules in $\Sigma$ equals the number of non-isomorphic summands of $T$, where $T$ is a tilting $A$-module.

There is a relationship between tilted and cluster-tilted algebras given in terms of slices and local slices.

**Theorem 2.18.** ([3] Corollary 20) Let $C$ be a tilted algebra and $B$ the corresponding cluster-tilted algebra. Then any slice in $\text{mod } C$ embeds as a local slice in $\text{mod } B$ and any local slice $\Sigma$ in $\text{mod } B$ arises in this way.

The existence of local slices in a cluster-tilted algebra gives rise to the following definition. The unique connected component of $\Gamma(\text{mod } B)$ that contains local slices is called the transjective component.
2.6 Induced and coinduced modules in cluster-tilted algebras

Following [11] we have the following definition.

**Definition 2.19.** Let $B$ be a cluster-tilted algebra and $M$ a $B$-module.

1. $M$ is **induced from some tilted algebra** if there exists a tilted algebra $C$ and a $C$-module $X$ such that $B$ is the relation extension of $C$ and $M = X \otimes_C B$.

2. $M$ is **coinduced from some tilted algebra** if there exists a tilted algebra $C$ and a $C$-module $X$ such that $B$ is the relation extension of $C$ and $M = D(B \otimes_C DX)$.

**Theorem 2.20.** [11, Theorem 6.4.] Let $B$ be a cluster-tilted algebra. Then for every transjective indecomposable $B$-module $M$, there exists a tilted algebra $C$, such that $B$ is the relation extension of $C$, and $M$ is an indecomposable $C$-module. In particular, every transjective $B$-module is induced or coinduced from $C$.

2.7 $\tau$-tilting finite algebras

Following [8], we have the following definition.

**Definition 2.21.** Let $A$ be a finite dimensional algebra. We say that $A$ is **$\tau$-tilting finite** if there are only finitely many isomorphism classes of basic $\tau$-tilting $A$-modules.

The authors provide several equivalent conditions for an algebra $A$ to be $\tau$-tilting finite. In particular, we need the following.

**Lemma 2.22.** [8, Corollary 2.9.] $A$ is $\tau$-tilting finite if and only if there are only finitely many isomorphism classes of indecomposable $\tau$-rigid $A$-modules.

2.8 A criterion for representation-finiteness

The following result is critical in the proofs of our main results.

**Theorem 2.23.** [4, IV Theorem 5.4.] Assume $A$ is a basic and connected finite dimensional algebra. If $\Gamma(\text{mod} A)$ admits a finite connected component $C$, then $C = \Gamma(\text{mod} A)$. In particular, $A$ is representation-finite.

3 Main Results

We begin with tilted algebras.

**Theorem 3.1.** Let $A$ be hereditary, $T$ a tilting $A$-module, and $B = \text{End}_A T$ a tilted algebra. Then $B$ is $\tau_B$-tilting finite if and only if $B$ is representation-finite.

**Proof.** The sufficiency is obvious so we prove the necessity. Assume $B$ is $\tau_B$-tilting finite but representation-infinite. Consider the connecting component $C_T$ of $\Gamma(\text{mod} B)$. Since $B$ is representation-infinite, Theorem 2.23 implies that $C_T$ is infinite. Let $M$ be an indecomposable module in $C_T$. Lemma 2.7 says $M$ is directing. Assume $M$ is
not $\tau_B$-rigid. Then there exists a non-zero homomorphism $M \to \tau_B M$, and hence a cycle $M \to \tau_B M \to \ast \to M$, and we have a contradiction. Thus, $M$ must be $\tau_B$-rigid. Since $M$ was arbitrary, we conclude every indecomposable module in $C_T$ is $\tau_B$-rigid. Since $C_T$ is infinite, we have an infinite number of isomorphism classes of indecomposable $\tau_B$-rigid modules. By Lemma 2.22, we have a contradiction. Hence, $B$ must be representation-finite.

We are now ready to prove the corresponding result for cluster-tilted algebras.

**Theorem 3.2.** Let $B$ be a cluster-tilted algebra. Then $B$ is $\tau_B$-tilting finite if and only if $B$ is representation-finite.

**Proof.** Again, the sufficiency is obvious so we prove the necessity. Assume $B$ is $\tau_B$-tilting finite but representation-infinite. By Theorems 2.16 and 2.18, we know the transjective component of $\Gamma(\text{mod } B)$ exists. Since $B$ is representation-infinite, Theorem 2.23 guarantees the transjective component must be infinite. Theorem 2.20 says there exists a tilted algebra $C$ such that $B$ is the relation extension of $C$ and every transjective $B$-module is induced or coinduced from $C$. Since $B$ is representation-infinite, we must have $C$ is representation-infinite.

Since $C$ is tilted, by definition, there exists a hereditary algebra $A$ and a tilting $A$-module $T$ such that $C = \text{End}_A T$. Consider the connecting component $C_T$ of $\Gamma(\text{mod } C)$. Since $C$ is representation-infinite, Theorem 2.23 says $C_T$ is infinite. As was shown in the proof of Theorem 3.1, we know every indecomposable module $M$ in $C_T$ is $\tau_C$-rigid.

By Proposition 2.3(e), the induced torsion pair $(X(T), Y(T))$ of $T$ splits. Suppose $M \in X(T)$. Then $\text{id}_C M \leq 1$ by Proposition 2.3(c). By Proposition 2.13, $M$ is $\tau_B$-rigid. Assume $M \in Y(T)$. By Proposition 2.3(f), $Y(T)$ is closed under predecessors. Thus, $\tau_C M \in Y(T)$. By Proposition 2.3(d), pd$_C \tau_C M \leq 1$. This implies the induced module, $M \otimes_C B$, is $\tau_B$-rigid by Proposition 2.14. We have shown that, for every indecomposable module $M$ in $C_T$, either $M$ or $M \otimes_C B$ is $\tau_B$-rigid. Since $C_T$ is infinite, we conclude there exists an infinite number of isomorphism classes of indecomposable $\tau_B$-rigid modules in $\text{mod } B$. By Lemma 2.22, we have a contradiction. Hence, $B$ must be representation-finite. □

**References**

[1] T. Adachi, O. Iyama and I. Reiten, $\tau$-tilting theory, *Compos. Math.* **150** (2014), no. 3, 415–452.

[2] I. Assem, T. Brüstle and R. Schiffler, Cluster-tilted algebras as trivial extensions, *Bull. Lond. Math. Soc.* **40** (2008), 151–162.

[3] I. Assem, T. Brüstle and R. Schiffler, Cluster-tilted algebras and slices, *J. Algebra* **319** (2008), 3464–3479.

[4] I. Assem, D. Simson and A. Skowronski, *Elements of the Representation Theory of Associative Algebras, 1: Techniques of Representation Theory*, London Mathematical Society Student Texts 65, Cambridge University Press, 2006
[5] A. B. Buan, R. Marsh, M. Reineke, I. Reiten and G. Todorov, Tilting theory and cluster combinatorics, *Adv. Math.* **204** (2006), no. 2, 572–618.

[6] A. B. Buan, R. Marsh and I. Reiten, Cluster-tilted algebras, *Trans. Amer. Math. Soc.* **359** (2007), no. 1, 323–332.

[7] P. Caldero, F. Chapoton and R. Schiffler, Quivers with relations arising from clusters (A_n case), *Trans. Amer. Math. Soc.* **358** (2006), no. 4, 359–376.

[8] L. Demonet, O. Iyama, and G. Jasso. τ-tilting finite algebras, bricks, and g-vectors. *International Mathematics Research Notices*, page rnx135, 2017

[9] D. Happel and C. M. Ringel, Tilted algebras, *Trans. Amer. Math. Soc.* **274** (1982), no. 2, 399–443

[10] C.M. Ringel, Tame algebras and integral quadratic forms, *Lecture Notes in Math.*, vol. 1099, Springer-Verlag, 1984.

[11] R. Schiffler and K. Serhiyenko, Induced and coinduced modules in cluster-tilted algebras, *J. Algebra* **472** (2017), 226–258.

[12] S. Zito, τ-rigid modules from tilted to cluster-tilted algebras, preprint (2017), [arXiv:1608.02418v2](https://arxiv.org/abs/1608.02418), to appear in *Comm. Algebra.*

Department of Mathematics, University of Connecticut-Waterbury, Waterbury, CT 06702, USA
*E-mail address:* stephen.zito@uconn.edu