Determination of polarised parton distributions in the nucleon - next to leading order QCD analysis

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Abstract
We have made next to leading order QCD fit to the deep inelastic spin asymmetries on nucleons and we determined polarised quark and gluon densities. The functional form for such distributions was inspired by the Martin, Roberts and Stirling fit for unpolarised case. In addition to usually used data points (averaged over $x$ and $Q^2$) we have also considered the sample containing points with similar $x$ and different $Q^2$. It seems that splitting of quark densities into valence and sea contribution is strongly model dependent and only their sum (i.e., $\Delta u$ and $\Delta d$) can be precisely determined from the data. Integrated polarised gluon contribution, contrary to some expectations, is relatively small and the sign of it depends on the fact which sample of data points is used.
The final analysis of data on polarised deep inelastic scattering taken in E143 experiment at SLAC and SMC at CERN on proton and deuteron targets were published recently [1, 2]. Together with the older data from SLAC [3, 4, 5, 6, 7, 8, 9], CERN [10, 11, 12, 13, 14] and DESY [15] one has quite a lot data on spin asymmetries. However, the newest data has smaller statistical errors and hence dominates in \( \chi^2 \) fits. Study of polarised deep inelastic scattering were for the first time performed by SLAC-Yale group [3, 4]. After the results of EMC group from CERN [10] leading to the so called spin crisis there was enormous interest in studying polarised structure functions. It was suggested [16] that polarised gluons may be responsible for the little spin carried by quarks. Experimental groups have measured spin asymmetries on proton, neutron (\(^3\)He) and deuteron targets. On the other hand after calculation of two loop polarised splitting functions [17] several next to leading (NLO) order QCD analysis were performed [18, 19, 20, 21] making fits to the actually existing data and trying to determine polarised parton distributions. Determination of polarised gluon distribution was particularly interesting in this context. The aim of this paper is similar. We want to perform next to leading order fit to the data taking into account the recently published data from the new analysis (SLAC E143 and CERN SMC experiments). We will divide the data into two groups. Many experimental groups published data [1, 2] sets for the close values of \( x \) and different \( Q^2 \) in addition to the ”averaged” data where one averages over \( x \) and \( Q^2 \) (the errors are smaller and \( Q^2 \) dependence is smeared out). In principle when we take into account \( Q^2 \) evolution of polarised and unpolarised functions (in NLO analysis) the first group of data points, i.e. non averaged, should give better fit. In most of the fits to experimental data only second group of data namely with averaged \( x \) and \( Q^2 \) dependence was used. We will make fits to the both sets of data (the first group contains 374 points and the second 130 points) and compare them with the fits without \( Q^2 \) evolution taken into account (in other words assuming that asymmetries do not depend on \( Q^2 \)). One should add that many experimental groups have not succeeded in finding \( Q^2 \) dependence for approximately the same value of \( x \) and different \( Q^2 \) [4, 13, 14]. The open question is if we really see the \( Q^2 \) evolution of structure functions from the existing experimental data. In our analysis we limit ourselves, as one usually does, to the data with \( Q^2 \geq 1 \text{ GeV}^2 \). As was already mentioned in our earlier papers [22] and was later stressed by other authors [18] making a fit to spin asymmetries and not directly to \( g_1(x, Q^2) \)
we expect that the higher twist contributions are probably less important in such case. Experiments on unpolarised DIS provide information on the unpolarised quark densities \( q(x, Q^2) \) and \( G(x, Q^2) \) inside the nucleon. These densities can be expressed in term of \( q^\pm(x, Q^2) \) and \( G^\pm(x, Q^2) \), i.e. densities of quarks and gluons with helicity along or opposite to the helicity of the parent nucleon. The unpolarised quark densities are given by the sum of \( q^+ \), \( q^- \) and \( G^+ \), \( G^- \), namely:

\[
q = q^+ + q^- , \quad G = G^+ + G^- . \tag{1}
\]

The polarised DIS experiments give also information about the so called polarised parton density, the difference of \( q^+ \), \( q^- \) and \( G^+ \), \( G^- \):

\[
\Delta q = q^+ - q^- , \quad \Delta G = G^+ - G^- . \tag{2}
\]

We will actually try to determine \( q^\pm(x, Q^2) \) and \( G^\pm(x, Q^2) \), in other words, we will have in some sense a simultaneous fit to unpolarised and polarised data. In principle the asymptotic \( x \) behaviour of \( q^+ \) and \( q^- \) will be taken from the unpolarised data. We will use fits of MRS (called R2) \(^\text{[23]}\) taking into account the behaviour at small \( x \) of quarks and gluon distributions obtained from experiments in Hera. It is of course very restrictive assumption that \( \Delta q \) and \( \Delta G \) have the same behaviour (when the integrals of \( \Delta q \) and \( \Delta G \) exist) as \( q \) and \( G \). On the other hand the small \( x \) behaviour of unpolarised structure functions is determined from the \( x \) values of Hera much smaller than in the polarised case. We will also consider the integrals over the region measured in the experiments with polarised particles with the hope that in this case the behaviour of \( q^\pm \) and \( G^\pm \) could be more plausible. The values of integrals in the whole region \((0 \leq x \leq 1)\) involving asymptotic behaviour taken from the unpolarised structure functions may be not as reliable as for the measured region. But it is an alternative to use of Regge type behaviour.

It is known \(^\text{[18]}\) that the behaviour of the quark and gluon distributions in small \( x \) region is extremely important in extrapolation of calculation of integrals over whole \( 0 \leq x \leq 1 \) range. It could happen that in \( \Delta q = q^+ - q^- \) (when we assume that \( q^+ \), \( q^- \) and \( q = q^+ + q^- \) have similar \( x \) dependence) most singular \( x \) terms cancel (that is especially important in case of d valence quark, sea and gluon where the \( x \) behaviour is relatively singular). We will see how such description infers the fits and calculated parameters. With the less singular distributions for \( \Delta d_{v}, \Delta M \) (total sea polarisation) and
\(\Delta G\) (gluon polarisation) there is no strong dependence of calculated quantities in an unmeasured region but the fits have higher \(\chi^2\). One of the main tasks of considering NLO evolution in \(Q^2\) is the determination of the gluon contribution \(\Delta G\). In \(\overline{MS}\) scheme \(\Delta G(x, Q^2)\) comes in through the higher order corrections. In our fits we obtain \(\Delta G\) relatively small contrary to some expectations. In addition when we use the averaged sample of data \(\Delta G\) contribution is opposite to that in a set without averaging over \(x\) and \(Q^2\). In our fits the averaging over \(x\) and \(Q^2\) (that have nothing to do with physics) changes the sign of the gluon contribution with rather small changes in quark region. It means that gluon contribution is extremely sensitive and cannot be reliably determined.

Let us start with the formulas for unpolarised quark parton distributions given (at \(Q^2 = 1\) GeV\(^2\)) in one of the recent fits performed by Martin, Roberts and Stirling [23]. In this fit \(\Lambda_{\overline{MS}}^{n_f=4} = 0.344\) GeV and \(\alpha_s(M_Z^2) = 0.120\). We have for the valence quarks:

\[
\begin{align*}
    u_v(x) &= 2.251x^{-0.39}(1 - x)^{3.54}(1 - 0.98\sqrt{x} + 6.51x), \\
    d_v(x) &= 0.114x^{-0.76}(1 - x)^{4.21}(1 + 7.37\sqrt{x} + 29.9x),
\end{align*}
\]

(3)

and for the antiquarks from the sea:

\[
\begin{align*}
    2\bar{u}(x) &= 0.392M(x) - \delta(x), \\
    2\bar{d}(x) &= 0.392M(x) + \delta(x), \\
    2\bar{s}(x) &= 0.196M(x), \\
    2\bar{c}(x) &= 0.020M(x).
\end{align*}
\]

(4)

In eq.(4) the singlet contribution \(M = 2[\bar{u} + \bar{d} + \bar{s} + \bar{c}]\) is:

\[
M(x) = 0.37x^{-1.15}(1 - x)^{8.27}(1 + 1.13\sqrt{x} + 14.4x),
\]

(5)

whereas the isovector \(\delta = \bar{d} - \bar{u}\) part:

\[
\delta(x) = 0.036x^{-1.15}(1 - x)^{8.27}(1 + 64.9x).
\]

(6)

For the unpolarised gluon distribution we get:

\[
G(x) = 14.4x^{-0.49}(1 - x)^{5.51}(1 - 4.20\sqrt{x} + 6.47x).
\]

(7)
We assume, in an analogy to the unpolarised case, that the polarised quark distributions are of the form: 

\[ x^\alpha (1 - x)^\beta P_2(\sqrt{x}) \]

where \( P_2(\sqrt{x}) \) is a second order polynomial in \( \sqrt{x} \) and the asymptotic behaviour for \( x \to 0 \) and \( x \to 1 \) (i.e. the values of \( \alpha \) and \( \beta \)) are the same (except for \( \Delta M \), see a discussion below) as in unpolarised case. Our idea is to split the numerical constants (coefficients of \( P_2 \) polynomial) in eqs.(3, 5, 6 and 7) in two parts in such a manner that the distributions are positive defined. Our expressions for \( \Delta q(x) = q^+(x) - q^-(x) \) (\( q(x) = q^+(x) + q^-(x) \)) are:

\[
\begin{align*}
\Delta u_v(x) &= x^{-0.39}(1 - x)^{3.54}(a_1 + a_2\sqrt{x} + a_3 x), \\
\Delta d_v(x) &= x^{-0.76}(1 - x)^{4.21}(b_1 + b_2\sqrt{x} + b_3 x), \\
\Delta M(x) &= x^{-0.65}(1 - x)^{8.27}(c_1 + c_2\sqrt{x}), \\
\Delta \delta(x) &= x^{-0.65}(1 - x)^{8.27}c_3(1 + 64.9x), \\
\Delta G(x) &= x^{-0.49}(1 - x)^{5.51}c_3(d_1 + d_2\sqrt{x} + d_3 x).
\end{align*}
\] (8)

We will not consider \( \Delta \delta \), the parameter that breaks the isospin SU(2) symmetry of a sea, (we assume \( \Delta \bar{s} = 0 \)) because one gets that the determination of such parameters is not reliable. It is very important what assumptions one makes about the sea contribution. From the MRS fit for unpolarised structure functions the natural assumption would be: \( \Delta \bar{s} = \Delta \bar{d}/2 = \Delta \bar{u}/2 \). If we add the condition that \( a_8 \) value should be equal to the value determined from the semileptonic hyperon decays, \( \Delta s \) is practically determined and though also nonstrange sea is determined. For comparison we will also consider such model. We assume that \( \Delta s \) (strange sea) is described by additional parameters namely

\[
\Delta M_i = x^{-0.65}(1 - x)^{8.27}(c_{i1} + c_{2i}\sqrt{x})
\] (9)

where \( i = u, d, s \) and \( c_{i1} \) and \( c_{2i} \) for \( u \) and \( d \) are equal. For the strange quarks we have additional independent parameters. Comparing with the expression (5) we see that in \( \Delta M_i \) there is no term behaving like \( x^{-1.15} \) at small \( x \) (we assume that \( \Delta M_i \) and hence all sea distributions are integrable) which means that in \( \Delta M_i \) coefficient in front of this term have to be splitted into equal parts in \( \Delta M_i^+ \) and \( \Delta M_i^- \). We see that we have relatively strong singular behaviour of \( \Delta d_v \), \( \Delta M \) and \( \Delta G \) for small \( x \) values. For comparison we will also consider the model in which leading most singular terms are put equal to zero namely \( b_1 = c_{1i} = d_1 = 0 \), that means that plus and minus components

5
have the same value for this powers of $x$ that means that we investigate the
dependence of the model on the leading $x$ behaviour of $\Delta q$ and $\Delta G$ (we know
from the ref [19] that such dependence is strong). In the less singular models
the dependence of calculated parameters in the unobserved region (below
$x \leq 0.003$) is weak. In the earlier papers we considered the extrapolation of
various calculated integrals below $x = 0.003$ up to 0 assuming Regge type
of behaviour for small $x$ values. As will be discussed later the less singular
models give slightly higher $\chi^2$.

In order to get the unknown parameters in the expressions for polarised
quark and gluon distributions we calculate the spin asymmetries starting
from initial $Q^2 = 1$ GeV$^2$ for measured values of $Q^2$ and make a fit to the
experimental data on spin asymmetries for proton, neutron and deuteron tar-
gets. The asymmetry $A_1^N(x, Q^2)$ can be expressed via the polarised structure
function $A_1^N(x, Q^2)$ as

$$A_1^N(x, Q^2) = \frac{g_1^N(x, Q^2)}{F_1^N(x, Q^2)} = \frac{g_1^N(x, Q^2)}{F_2^N(x, Q^2)}[2x(1 + R^N(x, Q^2))]$$

(10)

where $R^N = (F_2^N - 2xF_1^N)/2xF_1^N$ and $F_1^N$ and $F_2^N$ are the unpolarised structure functions. We will take the experimental $R_N$ from the [24].

Polarised structure function $g_1^N(x, Q^2)$ in the next to leading order QCD is related to
the polarised quark, antiquark and gluon distributions $\Delta q(x, Q^2), \Delta \bar{q}(x, Q^2), \Delta G(x, Q^2)$, in the following way:

$$
\begin{align*}
g_1^N(x, Q^2) &= \frac{1}{2} \sum_q e_q^2 \left( \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) + \frac{\alpha_s}{2\pi} [\delta c_q * (\Delta q(x, Q^2)) + \frac{1}{f} \delta c_g * \Delta G(x, Q^2)] \right) \\
&= \frac{1}{2} \sum_q e_q^2 \left( \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) + \frac{\alpha_s}{2\pi} [\delta c_q * (\Delta q(x, Q^2)) + \frac{1}{f} \delta c_g * \Delta G(x, Q^2)] \right)
\end{align*}
$$

(11)

with the convolution $*$ defined by:

$$(C * q)(x, Q^2) = \int_x^1 \frac{dz}{z} C \left( \frac{x}{z} \right) q(z, Q^2)$$

(12)

The explicit form of the appropriate spin dependent Wilson coefficient in
the $\overline{MS}$ scheme can be found for example in ref. [17]. The NLO expressions
for the unpolarised (spin averaged) structure function $F^N(x, Q^2)$ is similar
to the one in eq.(11) with $\Delta q(x, Q^2) \to q(x, Q^2)$ and the unpolarised Wilson
coefficients are given for example in [23, 20].
The $Q^2$ evolution of the parton densities is governed by the DGLAP equations [27]. For calculating the NLO evolution of the spin dependent parton distributions $\Delta q(x, Q^2)$ and $\Delta G(x, Q^2)$ and spin averaged $q(x, Q^2)$ and $G(x, Q^2)$ we will follow the method described in [26, 18]. We will calculate Mellin n-th moment of parton distributions $\Delta q(x, Q^2)$ and $\Delta G(x, Q^2)$ according to

$$\Delta q^n(Q^2) = \int_0^1 dx x^{n-1} \Delta q(x, Q^2)$$  \hspace{1cm} (13)

and then use NLO solutions in Mellin n-moment space in order to calculate evolution in $Q^2$ for non-singlet and singlet i.e. of $\Delta \Sigma_n(Q^2) = \sum_q [\Delta q^n(Q^2) + \Delta \bar{q}^n(Q^2)]$ and $\Delta G(Q^2)$.

In calculating evolution of $\Sigma(Q^2)$ and $\Delta G(Q^2)$ with $Q^2$ we have mixing governed by the anomalous dimension 2x2 matrix. We used explicit formulae given in [26]. Having evolved moments one can insert them into the n-th moment of eq.(11).

$$g_1^n(Q^2) = \frac{1}{2} \sum_q c_q^2 \{ \Delta q^n(Q^2) + \Delta \bar{q}^n(Q^2) + \frac{\alpha_s}{2\pi} [\delta c_{q}^n \cdot (\Delta q^n(Q^2)) + \Delta \bar{q}^n(Q^2)] \}$$  \hspace{1cm} (14)

and then numerically Mellin invert the whole expression. In this way we get $g_1(x, Q^2)$. The same procedure is applied taking the appropriate formulas giving the different $Q^2$ dependence and the correction coefficients for the unpolarised structure functions. Having calculated the asymmetries according to equation (10) for the measured in experiments value of $Q^2$ we can make a fit to a measured asymmetries on proton neutron and deuteron targets. We will take into account for proton 7 points of E80 [3] and 16 points of E130 [4] of SLAC experiments, 10 points of EMC [11] and 59 points of SMC [3] from CERN and 82 points of E143 [1] from SLAC. For deuteron we have 65 points from SMC [3] and 82 points from E143 [1] whereas for neutron 33 points of E142 [4] and 11 points of E154 [8] experiments from SLAC and 9 points from DESY Hermes experiment [15]. The last two sets of data from E154 and Hermes are taken in order to have more data from neutron target and to balance huge number of points from proton and deuteron targets. All together we have 374 data points and together with the assumed $g_8 = 0.579 \pm 0.1$ value we have 375 points (174 for proton, 147 for deuteron and 53 for neutron).
We get the following values of parameters from the fit to all existing (above mentioned) data for \( Q^2 \geq 1 \text{GeV}^2 \) for spin asymmetries:

\[
\begin{align*}
  a_1 &= 0.66, & a_2 &= -4.21, & a_3 &= 14.6, \\
  b_1 &= -0.02, & b_2 &= -0.84, & b_3 &= -1.74, \\
  c_{1u} = c_{1d} &= -0.28, & c_{2u} = c_{2d} &= 3.08, \\
  c_{1s} &= -0.42, & c_{2s} &= -1.15, & c_3 &= 0 \text{ (input)}, \\
  d_1 &= 2.201, & d_2 &= -22.47, & d_3 &= 42.20.
\end{align*}
\] (15)

The resulting \( \chi^2 \) per degree of freedom is \( \chi^2/N_{DF} = \frac{308.66}{375-13} = 0.853 \).

The obtained quark and gluon distributions lead for \( Q^2 = 1 \text{ GeV}^2 \) to the following integrated quantities: \( \Delta u = 0.77 \) (\( \Delta u_e = 0.70, 2\Delta \bar{u} = 0.07 \)), \( \Delta d = -0.49 \) (\( \Delta d_v = -0.56, 2\Delta \bar{d} = 0.07 \)), \( \Delta s = -0.15 \). These numbers give the following predictions: \( \Delta \Sigma = 0.13 \), \( \Delta M = 0.00 \), \( \Delta G = 0.22 \), \( \Gamma_p^1 = 0.113 \), \( \Gamma_{1d}^d = -0.062 \), \( \Gamma_{1d}^d = 0.024 \), \( g_A = \Delta u - \Delta d = 1.26 \).

We have relatively small positively polarised sea for up and down quarks and stronger negatively polarised sea for strange quarks. The gluon polarisation is positive but very small. The value of \( g_A \) was not assumed as an input in the fit and comes out correctly.

As was already stressed in [18], the asymptotic behaviour at small \( x \) of our polarised quark distributions is determined by the unpolarised ones and these do not have the expected theoretically Regge type behaviour or pQCD which is also used by experimental groups, to extrapolate results to small values of \( x \). Some of the quantities change rapidly for \( x \leq 0.003 \).

Now we will present quantities integrated over the region from \( x=0.003 \) to \( x=1 \) (it is practically integration over the region which is covered by the experimental data, except of non controversial extrapolation for highest \( x \)). The corresponding quantities are \( \Delta u = 0.78 \) (\( \Delta u_e = 0.67, 2\Delta \bar{u} = 0.11 \)), \( \Delta d = -0.42 \) (\( \Delta d_v = -0.53, 2\Delta \bar{d} = 0.11 \)), \( \Delta s = -0.12, \Delta \Sigma = 0.24, \Delta M = 0.11, \Delta G = 0.06 \). We can also calculate \( \Gamma_p \), \( \Gamma_n \) and \( \Gamma_d \) in the measured region for \( Q^2 = 5 \text{ GeV}^2 \) and compare them with the quantities given by the experimental groups.

We get \( \Gamma_p^1 = 0.119 \), \( \Gamma_n^1 = -0.078 \) and \( \Gamma_d^1 = 0.019 \) in the whole region, whereas in the region between \( x = 0.003 \) and \( x = 0.8 \) (covered by the data) we have \( \Gamma_p^1 = 0.125 \), \( \Gamma_n^1 = -0.051 \) and \( \Gamma_d^1 = 0.034 \). The experimental group SMC present [20] the following values in the measured region (for \( Q^2 = 5 \text{ GeV}^2 \)):

\[
\Gamma_p^1 = 0.130 \pm 0.007,
\]
\[ \Gamma^p_1 = 0.121 \pm 0.018, \]
\[ \Gamma^n_1 = -0.075 \pm 0.021, \]
\[ \Gamma^d_1 = 0.021 \pm 0.017. \]

Our results are in good agreement with given experimental values. For comparison we have also made fits using formulas of the simple parton model (as in our papers before [22]) neglecting evolution of parton densities with \( Q^2 \). More detailed result of these fits (integrated densities and so on) will be given later.

Figure 1: Plots of proton spin asymmetry predicted by our basic fit (for experimental \( Q^2 \)). The data points from different experiments (E143, SMC) with total errors are also shown.

In figs.1, 2 and 3 we present the comparison of our basic fit with measured asymmetries for proton (1), deuteron (2) and neutron (3) targets. The curves are obtained by joining the calculated values of asymmetries corresponding to actual values of \( x \) and \( Q^2 \) for measured data points. For the same value of \( x \) we have experimental points corresponding to different \( Q^2 \) values. We see that distribution of experimental points is much bigger than the lengths of vertical lines measuring the changes of influence of evolution in \( Q^2 \) for different values of \( Q^2 \) and the same value of \( x \). It seems that with such errors it is difficult to see \( Q^2 \) dependence of asymmetries.
Figure 2: Plots of deuteron spin asymmetry predicted by our basic fit (for experimental $Q^2$). The data points from different experiments (E143, SMC) with total errors are also shown.

Figure 3: Plots of neutron spin asymmetry predicted by our basic fit (for experimental $Q^2$). The data points from different experiments (E142, E154) with total errors are also shown.

For asymmetries the curves with $Q^2$ evolution taken into account and evolution completely neglected do not differ very much so we do not present them. We see that in the case of $g_1$ function for proton (fig.4) the dashed line corresponding to the fit with no evolution in $Q^2$ taken into account (parton model) that for small $x$ is continuation of E143 data (small errors) but lays within experimental errors of SMC results. May be it could be considered as some tendency in evidence for seeing $Q^2$ dependence in the data but certainly
not a very strong one. In the deuteron and neutron data the effect is even less pronounced.

Figure 4: Plots of structure function $g_1(x, Q^2)$ obtained in our fit (for experimental values of $Q^2$) and compared with data points for proton, deuteron and neutron targets. The solid curves are predictions for SMC or E142 experimental points, whereas dot-dashed ones are for E143 or E154 data. The dashed curves are predictions from the fit with no $Q^2$ evolution of considered structure functions.

Polarised quark distributions for up and down valence quarks as well as non strange, strange quarks and gluons for $Q^2 = 1$ GeV$^2$ are presented in figure 5. They are compared with the distributions when $Q^2$ evolution is not taken into account and with the corresponding unpolarised quark and gluon distributions. We see that especially in the case of polarised gluon distribution function does not resemble the distribution function for unpolarised
case. This function is also quite different from the gluon distribution (given in [28]) used to estimate $\Delta G/G$ in COMPASS experiment planned at CERN [29].

Fixing the value of $g_8$ is very important for the fit. When we relax the condition for $g_8 = 0.579$, $\chi^2$ goes a little bit down to the value 308.65. We get the fit with the parameters not very different from our basic fit but with very small $g_8$ (0.03) and bigger $\Delta \Sigma = 0.27$, ($\Delta \Sigma = 0.37$ for $0.003 \leq x \leq 1$) and positive $\Delta s = 0.08$, $\Delta \bar{u} = \Delta \bar{d} = 0.03$. It means that fixing the value of $g_8$ equal to experimental value (gotten from hyperon $\beta$-decays data) enforces the negative value of $\Delta s$ between -0.15 and -0.12. The obtained solution without fixing $g_8$ value is somehow dual to our fit, $g_8$ very small and $\Delta \Sigma$ relatively large comparing with $g_8$ close to its experimental value and $\Delta \Sigma$ rather small.

In order to check how the fit depends on the assumptions made about the sea contribution we have also made fit with $\Delta \bar{u} = \Delta \bar{d} = 2\Delta \bar{s}$, the assumption that follows directly from MRS unpolarised fit. The $\chi^2$ value per degree of freedom $\chi^2/N_{DF} = 311.50/375-11 = 0.856$ is a little bit worse. In this case we have $\Delta u = 0.76$ ($\Delta u_v = 0.98$, $2\Delta \bar{u} = -0.21$), $\Delta d = -0.48$ ($\Delta d_v = -0.26$, $2\Delta \bar{d} = -0.21$), $\Delta s = -0.11$, $\Delta \Sigma = 0.18$, $\Delta M = -0.53$, $\Delta G = 0.28$. Hence, we see that with the different assumption about sea behaviour the overall sea contribution changes quite drastically. The quantity $\Delta s$ must be negative in order to get experimental value for $g_8$ and because of our assumption ($\Delta \bar{u} = \Delta \bar{d} = 2\Delta \bar{s}$) we obtain relatively big negative values of non strange sea for up and down quarks. We see that the values for sea polarisation depend very strongly on the taken assumptions (in many papers [18, 19, 28] SU(3) symmetric sea is assumed that also together with fixing of $g_8$ value gives negative non strange sea). It means that the sea contribution is not very well determined. On the other hand $\Delta u = \Delta u_v + 2\Delta \bar{u}$ and $\Delta d = \Delta d_v + 2\Delta \bar{d}$ practically do not change (however, $\Delta u_v$ and $\Delta d_v$ also change). Also $\Delta G$ does not change.

Looking at the dependence of unpolarised quark and gluon densities we see that the most singular behaviour for small $x$ we have for $d_v(x)$, $M(x)$ and $G(x)$. For comparison we have investigated the model when in polarised densities these most singular contributions are absent. In this case $\Delta d_v$, $\Delta M$ and $\Delta G$ are $\sqrt{x}$ less singular than in our basic fit. For such a fit we get $\chi^2/N_{DF} = 314.63/375-9 = 0.864$ i.e only slightly higher than in our basic fit. We get in this case: $\Delta u = 0.78$ ($\Delta u_v = 0.71$, $2\Delta \bar{u} = 0.07$), $\Delta d = -0.41$ ($\Delta d_v = -0.48$, $2\Delta \bar{d} = -0.49$, $\Delta \bar{s} = -0.11$, $\Delta \Sigma = 0.18$, $\Delta \bar{M} = -0.53$, $\Delta \bar{G} = 0.28$. Hence, we see that with the different assumption about sea behaviour the overall sea contribution changes quite drastically. The quantity $\Delta s$ must be negative in order to get experimental value for $g_8$ and because of our assumption ($\Delta \bar{u} = \Delta \bar{d} = 2\Delta \bar{s}$) we obtain relatively big negative values of non strange sea for up and down quarks. We see that the values for sea polarisation depend very strongly on the taken assumptions (in many papers [18, 19, 28] SU(3) symmetric sea is assumed that also together with fixing of $g_8$ value gives negative non strange sea). It means that the sea contribution is not very well determined. On the other hand $\Delta u = \Delta u_v + 2\Delta \bar{u}$ and $\Delta d = \Delta d_v + 2\Delta \bar{d}$ practically do not change (however, $\Delta u_v$ and $\Delta d_v$ also change). Also $\Delta G$ does not change.

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Figure 5: Polarised quark \((x\Delta u, x\Delta d)\), antiquark \((x\Delta \bar{u}, x\Delta \bar{s})\) and gluon distributions \((x\Delta G)\) predicted by our basic fit at \(Q^2 = 1\text{GeV}^2\) (solid curve). For comparison prediction for such quantities for the fit without \(Q^2\) evolution taken into account (long-dashed curves) as well as nonpolarised quark, antiquark and gluon distributions from \([23]\) (dashed ones) are also shown.
2∆d = 0.07), ∆s = −0.10, ∆Σ = 0.27, ∆M = 0.05, ∆G = −0.40. If we do not modify ∆G(x, Q^2) omitting the most singular term ∆G remains positive. In this fit integrated quantities taken over the whole range of 0 ≤ x ≤ 1 and in the truncated one (0.003 ≤ x ≤ 1) differ very little. ∆G is negative opposite to the basic fit. So it is possible to get the fit of comparable quality to our basic fit with practically no change of integrated quantities in the region between x = 0 and x = 0.003. For Q^2=1 GeV^2 we have Γ^u_1 = 0.121 and Γ^n_1 = −0.044 (to be compared with Γ^n_1 = −0.062) and relatively big ∆Σ = 0.27.

The obtained results can be compared with the fit when instead of 374 points for different x and Q^2 values we take spin asymmetries for only 130 data points with the averaged values for the same x, averaged Q^2 and smaller errors. We have then for proton target points obtained at CERN (10 from EMC, 12 for SMC) and at SLAC (4 from E80, 8 from E130 and 28 for E143). For deuteron we take into account 12 points from SMC and 28 from E143 whereas for neutron target data points from SLAC (8 from E142 and 11 from E154) and DESY(9 from Hermes). In this fit first of all the errors are smaller than in our basic fit and the ratio of number of neutron to number of deuteron and proton data points are different. It seems that the influence of neutron points is stronger than in basic fit χ^2/N_{DF} = 99.55/131−13 = 0.844 is a little bit better than in our basic fit. The integrated values for quark and gluon densities are: ∆u = 0.80 (∆u_v = 0.57, 2∆u = 0.23), ∆d = −0.54 (∆d_v = −0.78, 2∆d = 0.23), ∆s = −0.16, ∆Σ = 0.10, ∆M = 0.31, ∆G = −0.69 and g_A = 1.35. We see that averaging over x and Q^2 and different numbers of data points leads to not a very different fit (for example χ^2 = 101.2 when the parameters of the basic fit are used) but the values for integrated valence densities and nonstrange sea contribution are different (∆u and ∆d do not differ very much and the difference is smaller for integrated quantities in the region 0.003 ≤ x ≤ 1). Integrated gluon density is negative. The fits become more similar when in the average fit we fix the g_A value, i.e. the condition for valence contribution.

The fact that the averaged and non averaged samples of data points results for valence quark densities and sea contributions are different and in the case of integrated gluon density even sign is different means that these quantities are not very precisely determined in the fits. After assuming g_8 value from experiment and fixing ∆s contribution we can determine ∆u and ∆d values (the splitting of ∆q in ∆q_v and 2∆q depends on data sample and
assumptions about sea contribution). It could be that differences that come out in comparing fits to the averaged and non averaged data are connected with the fact that rather singular polarised parton distributions are able to pick up differences in two experimental data samples, due to different number of neutron to proton and deuteron data points. The fit with less singular densities ($\Delta d_v$, $\Delta \bar{u}$, $\Delta \bar{d}$ and $\Delta G$) with $\chi^2$ a little bit higher 104.87 ($\chi^2/N_{DF} = 104.87/131-9 = 0.860$) is for averaged data points nearly identical to that for non averaged one. $\Delta G$ is like in previous case small and negative $\Delta G = -0.4$.

As was already mentioned before we have also made for comparison fits neglecting evolution of parton densities with $Q^2$ (formulas from the simple parton model). We get for non averaged data sample $\chi^2/N_{DF} = 318.25/375-10 = 0.872$ (higher than in our basic fit where: $\chi^2/N_{DF} = 308.32/375-13 = 0.853$): $\Delta u = 0.68$ ($\Delta u_v = 0.46, 2\Delta \bar{u} = 0.22$), $\Delta d = -0.41$ ($\Delta d_v = -0.63, 2\Delta \bar{d} = 0.22$), $\Delta s = -0.16$, $\Delta \Sigma = 0.11$, $\Delta M = 0.27$, $\Gamma^p_1 = 0.120$, $\Gamma^n_1 = -0.062$. For averaged data points we get $\chi^2/N_{DF} = 103.79/131-10 = 0.858$ (this number should be compared with $\chi^2/N_{DF} = 103.79/131-10 = 0.858$, the corresponding quantity from NLO fit) and we have: $\Delta u = 0.69$ ($\Delta u_v = 0.40, 2\Delta \bar{u} = 0.29$), $\Delta d = -0.42$ ($\Delta d_v = -0.72, 2\Delta \bar{d} = 0.29$), $\Delta s = -0.16$, $\Delta \Sigma = 0.11$, $\Delta M = 0.42$. Hence, $\chi^2$ is smaller in the case of averaged sample. We see that also in this case the value $\Delta u$ and $\Delta d$ and $\Delta s$ are practically the same and there are some shifts in valence values and non strange sea contribution (the differences are smaller in the $0.003 \leq x \leq 1$ region). Similarly to the case with evolution taken into account the fits are nearly identical when we make them with less singular $\Delta d_v$, $\Delta \bar{u} = \Delta \bar{d}$ and $\Delta G$ densities ($\chi^2$ is in this case higher than in the basic fit for non averaged data sample or for averaged one).

It has been pointed out [13] that the positivity conditions could be restrictive and influence the contribution of polarised gluons. We have also made a fit to experimental data without such assumption for polarised partons. The $\chi^2$ value does not changed much $\chi^2/N_{DF} = 308.32/375-13 = 0.852$ and we get $\Delta u = 0.79$ ($\Delta u_v = 0.35, 2\Delta \bar{u} = 0.44$), $\Delta d = -0.50$ ($\Delta d_v = -0.94, 2\Delta \bar{d} = 0.44$), $\Delta s = -0.12$, $\Delta \Sigma = 0.17$, $\Delta M = 0.76$, $\Delta G = 0.12$. Hence, sea contribution for non strange quarks became big and there are shifts between valence and sea contribution. The gluon contribution does not change staying close to zero for non averaged data and small and negative for averaged data. The most important restriction is for the valence $d$ quark (in the case without positivity assumption it becomes big close to -1). Relaxing the positivity of other quantities does not change the fit. It seems that
without positivity the values of parton distributions are strongly pushed in the direction of small contribution of valence $u$ quark and big (and negative) valence $d$ quark. The gluon contribution is not modified very much. Hence, we decided to use the positivity assumptions in our fit.

In summary we have made fits to two samples of data with averaged $x$ and $Q^2$ values and nonaveraged ones (adding averaged neutron data from E154 and Hermes experiments). To check the influence of different model dependent assumptions we consider fits without fixing $g_8$ value, with modified sea contribution and less singular behaviour for valence $d$ quark, sea contribution and gluon densities. For comparison we have also considered fits to the simple parton model neglecting $Q^2$ dependence of parton densities. It seems that splitting of integrated densities $\Delta u$, $\Delta d$ in valence and sea contribution is model dependent ($\Delta u$ and $\Delta d$ were more or less the same). The integrated gluon contribution comes out relatively small and of different sign in averaged and nonaveraged data sample. The comparison of $g_1$ for fits without evolution and with $Q^2$ evolution of parton densities taken into account is given. It seems that experimental accuracy is still not enough to make precise statements about polarised quark and gluon densities.
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