DERIVED METRICS FOR THE GAME OF GO – INTRINSIC NETWORK STRENGTH ASSESSMENT AND CHEAT-DETECTION

ATTILA EGRI-NAGY¹, ANTTI TÖRMÄNEN²

Abstract. The widespread availability of superhuman AI engines is changing how we play the ancient game of Go. The open-source software packages developed after the AlphaGo series shifted focus from producing strong playing entities to providing tools for analyzing games. Here we describe two ways of how the innovations of the second generation engines (e.g. score estimates, variable komi) can be used for defining new metrics that help deepen our understanding of the game. First, we study how much information the search component contributes in addition to the raw neural network policy output. This gives an intrinsic strength measurement for the neural network. Second, we define the effect of a move by the difference in score estimates. This gives a fine-grained, move-by-move performance evaluation of a player. We use this in combating the new challenge of detecting online cheating.

1. Introduction

The game of Go is an ancient board game with simple rules and enormous complexity. It was the last grand challenge for artificial intelligence (AI) in abstract board games. The challenge is understood as beating the best human player, not as solving the game.

AlphaGo (AG) [14] made history by being the first superhuman Go AI engine. By using deep neural networks, AlphaGo had a way to integrate expertise of master players, further enhanced by reinforcement learning and self-plays. AlphaGo Zero (AGZ) [15] improved the results by removing human expertise from the training process. These developments are revolutionary in AI. However, the real revolution came afterwards, when the technology became available to all players.

Several new implementations followed the success of AG and AGZ [3, 8, 9, 16, 17]. Given some computational resources, now anyone can build a deep learning Go engine [12]. Moreover, just a standard gaming PC is capable of providing superhuman play and analysis.

The new implementations did not just recreate the same architecture, but several of them went beyond it in terms of providing more information about the game. We call second-generation engines those that give information about the expected score, not just the probability of winning. This fixes the problem of ‘slack’ moves, which AG was famous for. These were interpreted as mistakes first, but then it was realized that once a win is secured, the neural network has no preference for choosing efficient moves.

After AG, the focus shifted from creating a superhuman Go-playing entity to developing tools that help in understanding the game and in the learning process of human players. Now, the main usage of superhuman AIs is game analysis.
The structure of the paper. First we will review the basic measures used in deep learning Go AIs, followed by the suggested derived measures. Then we will describe two applications, one for measuring network strength intrinsically, and one for online cheat detection. Next, we describe the developed software tool and close the paper with discussion.

2. Basic Measures

The AGZ-like systems are based on deep reinforcement learning. Therefore we can describe their functioning in games and in analysis (we are not considering training here) in terms of neural networks and Monte-Carlo tree searches. Here we describe three important measures: the visit count, the winrate, and the scoremean.

We denote a state of the game (the board position) by \( s \), specifying the turn number as an index when needed. This way, \( s_0 \) denotes the empty board. We denote a move (action) on turn \( i \) by \( a_i \); this takes the board position \( s_{i-1} \) to \( s_i \). In particular, the first move \( a_1 \) takes \( s_0 \) to \( s_1 \). The action can be a pass.

2.1. Visit count: \( N(s, a) \). For move \( a \) at board position \( s \) the visit count \( N(s, a) \) is the number of times the search algorithm examined a variation starting with \( a \). The Monte-Carlo tree search methods keep track of how many times a node in the search tree gets visited. In AGZ [15], the move selection is solely based on the visit count, since the search algorithm keeps visiting the promising moves. Roughly speaking, the number of visits measures how many times a particular candidate move is considered, how ‘interesting’ it is. Another way to look at the visit count is to use it as a reliability measure. A move may look very promising with a high chance of winning, but with just a few visits we cannot trust its value. While Analysis GUIs expose this value, it may be less used by the end users.

2.2. Value function: \( V(s) \), winrate. The value function \( V(s) \) gives the probability of winning the game at a board position \( s \). For the sake of simplicity, unless otherwise stated we consider the value function from the perspective of Black.

In AG [14], a dedicated network was trained for estimating the value function. In AGZ [15] it became another head of the same network shared by the policy head. It was realized that the same neural computation can be used both for predicting moves and for deciding who is winning.

2.3. Scoremean: \( \mu_s \). Convolutional neural networks can have different heads, giving other values beyond a probability distribution for the next move. They can be trained to predict the score lead, the score difference at the end of the game [8,9,17]. The score value head combined with the Monte-Carlo search methods give statistical information about the outcome of the game: the scoremean value. It can be interpreted as the estimated score difference between the players at the end of the game.

How reliable is the scoremean? It is part of the loss function for the neural network’s training [17], therefore the reliability of the estimate should increase with the strength of the network. Searching for an indicator, we tried several handcrafted self-plays with KataGo’s final 40 blocks network, starting the game with a balanced integer komi. Handcrafted means that the move is selected by a human operator after extensive analysis, to make sure that the choice is the best possible by the network with no time control. These games reliably produce draws, indicating the stability of \( \mu_s \).
There is an analogy for scoremean in chess, where the advantage is measured by centipawns (\(\frac{1}{100}\)) of the value of the pawn. Scoremean has a similar role in Go, with the added benefit that it fully captures the goal of the game. In chess one may need to consider distance from checkmates as well.

2.4. **Scoremean vs. winrate, the human perspective.** Beyond the obvious relationship (positive scoremean means higher than 50% winning chance) the connection between them is not straightforward. This can be demonstrated with two simple examples: a high-handicap game and a general consideration of the dynamics of scoremean throughout a game.

In a high-handicap game, Black’s advantage might be eroding steadily (reflected in the gradual decline of scoremean), while the winrate stays flat above 90%. Then, suddenly the winrate switches when Black’s scoremean becomes negative. Analysing this situation without the scoremean could mislead us to search for a special meaning for the last little mistake, while it is just one of many.

Every move played in a game reduces the number of its future possibilities. As a game proceeds, its score estimate becomes more likely to be realized; so, while a game’s scoremean might remain constant and close to even, the game’s winrate will eventually drift to an extreme.

Therefore, while winrate is a useful measure for the AI, it is often unintuitive for human players and it can be misleading. A relatively small mistake can cause a big shift in winrate. This effect is further amplified if a game is nearing its end.

Scoremean is a useful measure for human players for two reasons. Firstly, strong human players themselves tend to estimate the values of moves in points, so the scoremean values can be easily understood. Secondly, unlike the winrate, the scoremean is not affected by the stage of the game. For example, a move that loses one point in terms of the score might cause a winrate shift of 50% in the late game, but only 5% in the early game. A human player cannot visualise this winrate shift, but the one-point loss is easy to understand.

3. Derived Measures

Based on the inner measures of deep learning Go AI engines, we define new measures to increase their usability and explainability. These can be viewed as new perspectives, from which we can understand the games and their analyses better.

3.1. **The effect of a move: \(\delta(a)\).** The effect \(\delta(a)\) is the difference between the scoremean after and before a move \(a\): \(\delta(a) = \mu_{s_{i+1}} - \mu_s\), when \(a\) takes board position \(s_i\) to \(s_{i+1}\). The difference in the corresponding winrates is used in Go GUIs, but as discussed before, the scoremean is more stable and more informative.

By gathering statistical information of the effects throughout a game (average of the effects, deviations from the mean, cumulative moving average of the effects) we can characterize the playing skill of a player. However, this alone cannot give a rating to a player, as the effects also depend on the type of the game played.

3.2. **Search gaps: hitrate and KL-divergence.** \(P(s, a)\), the prior probability of move \(a\) at board position \(s\), is provided by the raw network output. This probability distribution \(p\) is called the policy. The tree search guided by this policy then produces an updated policy \(\pi\), the probability distribution of good moves after search. \(\pi\) can be simply defined by the visit counts \([12][15]\), as it is a good measure of the value of a candidate moves, given that enough simulations were made. The
disparity between $p$ and $\pi$ is the search gap, which can be measured in different ways.

3.2.1. Hitrate. How many times does the search select the same move as the top move in the raw policy? Clearly, this depends on the length of the search. If we just allow a couple of simulations, then this number will be high. So hitrate is relative to number of simulations.

3.2.2. KL-divergence. The Kullback-Leibler divergence [6] is a fundamental tool for comparing two discrete probability distributions, $P$ and $Q$.

$$D_{KL}(P \parallel Q) = \sum P(x) \ln \frac{P(x)}{Q(x)}$$
It is a measure of the disparity of the two distribution, although it is not a distance metric. It is the measure of how much information we gain if we use the distribution $Q$ instead of $P$. It is a positive number, and it is zero when the distributions are the same.

We want to measure $D_{KL}(p ∥ π)$, but there are a couple of issues. Both $p$ and $π$ can have zero entries. There are illegal moves (a stone is already there, suicide move, or a ko situation), and the search will also visit only a subset of the possible moves, so in general we do not have visit counts for all legal moves. Therefore, we take the actually visited moves in the search tree, and define $π'$ by their visit counts and using normalization. So $π'$ is the probability distribution of the moves considered by the network. Note that this is now well-defined, while we used $π$ informally before. Then we take the set of moves included in $π'$ and find the corresponding probabilities in $π$, and restricting to those moves, we normalize and get $p'$.

As a rough but useful analogy, we can say that the output of the neural network corresponds to human intuition, while the search algorithm resembles step-by-step logical thinking. Just as humans mix these two types of thinking, the computer combines the deep neural networks with tree search. We want to measure the strength of intuition of the deep neural networks. This can be done by comparing the policy with or without tree search.

4. **Application: Intrinsic Strength of Networks**

*How far are the deep neural networks from perfect minimax play?* For now, the universally agreed answer to this almost philosophical question is that they are very far. We stop training a network due to external reasons (e.g. the cost of computational resources), not because we reached a theoretical limit for improvement. If a network played perfectly, the reported winrates could be more polarized, tending to one of the values 0, 0.5, and 1.0. Also, in that case, we would not need the tree search.

*How long shall we run a game analysis?* This is a more practical, but related question. Can we simply use the raw network output policy? As a calibration test, we analyzed a game position with different visit counts. Since the tree search is probabilistic, we repeated the analyses several times. Fig. 1 shows the results of 7 batches. A low number of visits gives a rather different policy, since it takes a few simulations for the Monte-Carlo algorithm to balance the exploitation/exploration ratio. After that we see an increasing KL-divergence value. Due to practical considerations, we chose 100,000 visits for further experiments.

We analyzed four full games with respect to the hitrates of four different networks (Table 1). The games are chosen to be different in style and strength. The first
is a historical game from 1846, the famous ‘ear-redening’ game [11]. The second game is from the Alphago vs. Lee Sedol match in 2016. The second game of the match contains the famous move 37, an example that a computer can also have creative ideas. The third is taken from the 44th Meijin title match, as an example of post-AG professional play. And the fourth is an amateur game. The results show the tendency of higher hitrates for stronger networks, both in terms of structure and length of training. Interestingly, the amateur game has the opposite tendency.

The percentages are reminiscent of the success rate of supervised learning for predicting human expert moves used in the first version of AlphaGo [14]. In the self-play based reinforcement learning the network is trying to predict the outcome of the tree search indirectly. So one might wonder whether it would be possible to improve the networks without any more self-play games; after all, the tree search is a short-circuited self-play. Of course, this could only work for fine-tuning of networks that are already strong, since the external reward signal is not available. We invite the deep learning community to test this hypothesis.

The above analysis has the problem that moves in a game are correlated. Therefore, we also measured KL-divergence over 915 games from the KGS server. All the games are between players of 4 dan or better, so they represent strong amateur play. We picked a random game from each and did a 100,000 visit analysis. Fig. 2 compares the KL-divergence in the early 20 block and the late 40 block Katago networks. We can observe that the stronger network has smaller KL-divergence values on average; also, the maximal values are more extreme.

5. Application: Cheat Detection

Using an AI engine for finding best moves and variations in a game is called analysis after the game is finished; cheating when the game is still ongoing.

The widespread availability of AI engines is beneficial in many ways. Most notably, one can improve their playing skills by reviewing their games with an AI. However, there are downsides of the technological progress: many players report cheating on online Go servers. With the availability of superhuman AI engines, online cheating might be rampant; but, besides the rare cases where a player admitted to cheating, there is no direct evidence for this except the gut feelings of strong human players.

Cheating defeats the purpose of online playing, where one wants to have a human opponent. On Asian servers, the top ranks are reportedly infested by cheaters. This has resulted in previously top-ranked humans to drop to lower ranks, starting a snowball effect inside the servers’ ranking systems. If no countermeasures to cheating are found, in the near future it is possible that online ratings will be largely devalued.

Strong players can quickly and reliably assess the opponent’s strength. Consequently, experienced players can recognize superhuman AI opponents. Could this be reproduced or at least helped by software tools?

Players with a rating history are easier to catch from cheating by noticing a sudden increase in their won games. However, clever cheaters that only consult an AI occasionally may be impossible to detect this way. Also, the availability of AI-based training tools may accelerate individual learning.
Figure 3. Game 1: White is likely using an AI.

Figure 4. White 26, 28, 42, and 44 are exactly correct according to KataGo even though a human player could think of many viable plans in these parts of the game.

In this research, we do not consider players’ histories, so we can deal with newly registered users as well. Our aim is therefore to be able to decide whether cheating happened in a single game solely based on the game record.

Prior work. Chess has a longer history of living with superhuman AI engines, thus the integrity of online games has been investigated extensively. However, the conclusion is that fully automated cheat detection is not possible. In [1] it is demonstrated that ‘false positives’ are abundant. This was shown by the existence of historic games that would be classified as cheating, though that clearly could not have happened.

In [2], the theory of complex networks and the PageRank algorithm was used to find distinguishing statistical features of human and computer play. The analysis was based on local information (3×3 squares) and did not use the modern capabilities of superhuman AIs.

5.1. Human ways of recognizing an AI-using cheater. The ways that human players recognize AI-using cheaters, listed in this section, might be of help in designing software tools for automatically catching cheaters.
5.1.1. **Temporal evidence.** When a cheater consults an AI, there is a near-constant time lag created by the cheater inputting their opponent’s last move to the AI program and waiting a moment for the AI to come up with an answer. When done in a straightforward fashion, this results in a cheater always playing their move after for example five seconds no matter if the move is obvious or extremely difficult for a human player to come up with.

![White's scoremean values](image1.png)
![winrate](image2.png)

**Figure 5.** White’s scoremean and winrate graphs for Game 1. Before move 86, when White’s winrate hits 98%, his moves were almost perfect. Afterwards, White’s play becomes less sharp, as indicated by the distance of the AI and choice lines; but the winrate does not change, suggesting an AI’s ‘safe play mode’.

5.1.2. **Playing style.** It did not take long for human players to notice that AI engines have a discernible playing style, emphasising quick exchanges and maintaining a whole-board balance. At first the difference to human players was glaring, but human players have since adopted the AI’s favored techniques, resulting in a human-AI blend.

Still, there are many moments during games when, according to the AI, an ‘obvious’ move by human intuition is wrong, with the correct move being something very unintuitive. When several such moves get played by the same player in a single game, the player is suspicious.

5.1.3. **Safe play when ahead.** As most AI engines choose their moves by the winrate estimate, when a game is deemed practically ‘over’ (at roughly 98% and above), they will start playing moves that are not optimal in terms of the scoremean but that still retain the player’s winrate. This leads to the AI choosing moves that a human player would consider ‘slack’, and often a strong human player can notice when their opponent enters this kind of ‘safe play mode’.
5.1.4. *Seemingly inconsistent play.* Human players and AI engines choose their moves very differently. Strong human players generally:

- (1) analyse and judge the current whole-board situation,
- (2) try to identify the most important or valuable areas of the board,
- (3) create a plan for how to develop the game, and
- (4) finally choose a move that furthers the plan.

This process is then more or less repeated on each move, with adjustments made as necessary depending on what the opponent is doing.

As the opponent generally acts on a similar modus operandi, it becomes valuable for a strong player to try to infer what the opponent is planning and to adjust their own plan accordingly. For strong human players, this generates a kind of non-verbal discussion or give-and-take that takes place on the go board. For this reason, Go is sometimes referred to as ‘hand talk’ in Asian countries.

As the AI does not form plans in a similar way as humans, it is not possible for a human to create this kind of a higher-level discussion with an AI engine. The AI will constantly play moves that, to a human, seem to betray its plan possibly only because the human player is unable to grasp it.

5.2. *Case studies.* In this section, we have analysed four games, three of which (most likely) involve cheaters. All four games were played online and analysed by a professional Go player. As we have no input from the other player, ultimately there is no hard evidence on whether they were cheating or not.

The difficulty of identifying a cheater depends greatly on whether the cheater is trying to cover their cheating or not. A clever cheater will vary the time they use for their moves, playing ‘obvious’ moves quickly and taking more time for difficult moves; and they will also not always play the AI’s best recommended move. Additionally, as AI engines rarely make big mistakes (especially early on in the game), a clever cheater would optimally try to include a few larger mistakes in their play.

The analysis has been performed as follows: first, the winrate graph of the game is checked. Since a cheater is using the AI to win the game, the winrate graph will generally tend to be one-sided, steadily rising to 99%; large shifts should not take place, as even a strong AI engine might not be able to beat a strong human if it falls too much behind. An exception is if both players are cheating, in which case the winrate usually progresses evenly for the most of the game.

Secondly, the development of the player’s average effect during the game is checked. Of particular interest are the final average effect for the whole game, which is a general indicator of the player’s skill, and if the players’ average effects develop in similar stages. Also, a player’s moves after their winrate reached 98% can be indicative of AI involvement, as an AI will start playing score-mean-inefficient moves after this point.

Thirdly, we check how the player performed in comparison to KataGo’s move recommendations. If the player played moves that are roughly as good as KataGo’s first recommendations, the player is suspect; whereas, if the player does considerably worse than KataGo, that is evidence of either human play or at least the player avoiding the AI’s best recommended moves.

5.3. *Game 1.* The white player in Fig. 3 is most likely consulting an AI.
Firstly, an experienced human player can already find White’s opening suspicious when comparing White’s choices with the AI’s suggestions. 26 and 28, shown in Fig. 4, are non-obvious moves to a human but first options for the AI. A bit later, 42 and 44 are another combination that looks made-up on the go, but exactly matches the AI’s recommendation. For a third example, 58 and its follow-up are very rarely seen in human play and, while not KataGo’s first recommendation, perform just about as well.

Secondly, as shown in Fig. 5 White basically makes no mistakes up until 86, even though Black is a professional player. This is difficult to accomplish even for a top human player.

Thirdly, after White reaches 98% winrate at move 86 as shown in Fig. 5, White’s play gets sloppy in terms of the scoremean. After this point, the white average effect starts decreasing, but the winrate is firmly stuck at 99%.

All three pieces of evidence put together, it is very likely that an AI engine was involved.

5.4. **Game 2.** Both players in Fig. 6 are most likely consulting an AI.

Most of the moves in this game are among KataGo’s top picks. Furthermore, the players’ average effects are extremely small ($-0.25$ and $-0.20$) even though there is a large variance in the scoremeans of KataGo’s considered moves, as shown in Fig. 7. Even the world champion of Go would find it difficult to play this well.

5.5. **Game 3.** Most likely neither player in Fig. 8 consulted an AI—this is a game by strong human players.

As shown in Fig. 8 the average effect for the players peaks at around $-1.0$ and finally settles to around $-0.65$ for each, which are reasonable numbers for strong human players. Comparing the players’ chosen moves with KataGo’s recommended alternatives, we see that both players generally perform better than the average choice but worse than the best choice, with plenty of exceptions to both directions.
Figure 7. The two players’ average effects and scoremeans for Game 2. Both players’ average effects are considerably small when taking into amount the ‘volatility’ of the game, indicated by the distance of the AI, average, and median lines in the scoremean graphs.

An AI-using smart cheater might attempt to play bad moves from time to time, but not so much that it should threaten their win. The winrate graph in Fig. 9 shows that this is not the case, as there are large shifts in the winrate in the first third of the game: first Black got a considerable lead, then White turned the game around, after which Black caught up again, after which White took off to a decisive lead. For further evidence, White’s winrate wavers even after first hitting 99%, which is common to human games.

While it is impossible to prove that neither player used the AI at any point during the game, it does not look like an AI was consulted to decide the outcome of the game.

5.6. Game 4. The black player in Fig. 10 is most likely consulting an AI.
Figure 8. Game 3: Neither player is likely using an AI.

Figure 9. The two players’ average effects, scoremeans, and the winrate for Game 3. The fairly low size of the players’ average effects, the variance in the scoremean graphs, and the up-and-down in the winrate graph suggest that this was a game by strong human players.

This case is possibly the most obvious to a strong human player. First, black 31 and 35 in Fig. 11 are moves that few human players could consider. Then, Black’s play from 39 to 51, after which Black lives comfortably in the centre, would also
Figure 10. Game 4: Black is likely using an AI.

Figure 11. Most of these black moves would be difficult for a human to come up with, but they align with KataGo’s recommendations.

be unthinkable to most but all of these black moves are KataGo’s first recommendations. A bit later, black 63 also looks mistimed in human terms, but is among KataGo’s top choices.

Secondly, looking at the winrate graph in Fig. 12, Black’s winrate is headed directly to 99% with practically no drops. This is evidence of a vast difference of skill between the players even though White is a professional player who did not play particularly badly in this game, according to KataGo.

Thirdly, looking at the size of Black’s average effect in Fig. 12 we see that Black manages an impressive −0.16 until move 61, at which point Black’s winrate has reached 98%. After this, Black’s moves get sloppier in terms of the scoremean, which further suggests an AI.

All three pieces of evidence put together, it is very likely that an AI engine was involved.

6. Software Implementation

We developed a dedicated software package for the described computations. The source code is available at https://github.com/egri-nagy/lambdago.
Figure 12. Black’s average effect, both players’ scoremeans, and the winrate for Game 4. The straightforwardness of the winrate graph as well as Black’s small average effect suggest AI involvement.

The core system (including a game engine) is written in the Clojure language https://www.clojure.org. Due to its dynamic nature, this functional language is particularly suited for data-driven experimentation [4]. It is hosted on the JVM, therefore it also has convenient access to the whole JAVA ecosystem.

For parsing the game record SGF files, in order to avoid writing yet another parser, we use a parser generator, Instaparse https://github.com/Engelberg/instaparse. This library is based on the idea of parsing with derivatives [7].

The visualization of the graphs is done by the Vega-lite library [13]. It is a high-level grammar of graphics that allowed us automate the task of diagram generation. The Go diagrams are made with GOWrite 2 [10], a high-quality Go publishing tool.

The workflow of the system evolved through the cheat-detection application, and it has two steps: analysis and visualization.
Analysis. The analysis can be done by the Lizzie GUI application[https://github.com/featurecat/lizzie]. This was designed as an interface to Leela Zero[3], but later it was adapted to work with other engines as well. It produces SGF files with the analysis information added. The KataGo engine[17] also has a direct interface to its analysis engine, which accepts and emits information in JSON format. The analysis is a GPU-intensive and time consuming computation, so for practical reasons we need to limit the visit counts.

Visualization. The output of the analysis can be quickly processed to generate the diagrams. They can be generated in batch mode as well. We expect that these visualization features will appear in other tools as well, as the analysis needs of the users will reach more sophisticated levels.

7. Discussion

Building upon the advances in artificial intelligence, and the developments in open-source software projects, we suggested novel measures for evaluating and understanding AI game analyses. Measuring the search gap (the added value of the tree search to the raw output of the neural network) allows us to measure the strength of the network intrinsically, without playing other networks. The effect of a move can be used for assessing a player’s performance with high resolution (move by move). We showed that an investigation of the effect can be helpful in detecting online cheating. Although automated cheat-detection may never be feasible due to the danger of false positives, we used these tools in a real online tournament and could catch a cheating player, who admitted the misconduct. This is an example of a successful collaboration of a human arbiter and an AI engine, according to the human-plus-machine paradigm envisioned by former chess world champion Garry Kasparov[5]. What happens in the world of the game of Go will happen in other aspects of our life, and therefore it is valuable to understand the effects of AI technologies on the game.

Acknowledgment. We are thankful to David Wu, the developer of the KataGo system for the useful conversation in GitHub issues.

References

[1] David J. Barnes and Julio Hernandez-Castro. On the limits of engine analysis for cheating detection in chess. Computers & Security, 48:58 − 73, 2015.
[2] C. Coquidé, B. Georgeot, and O. Giraud. Distinguishing humans from computers in the game of go: A complex network approach. EPL (Europhysics Letters), 119(4):48001, aug 2017.
[3] Gian-Carlo Pascutto et al. Leela Zero – Go engine with no human-provided knowledge, modeled after the AlphaGo Zero paper. https://zero.sjeng.org 2019. https://github.com/leela-zero/leela-zero
[4] Rich Hickey. A history of Clojure. Proc. ACM Program. Lang., 4(HOPL), June 2020.
[5] G. Kasparov. Deep Thinking: Where Machine Intelligence Ends and Human Creativity Begins. Millennium series. Hodder & Stoughton, 2017.
[6] S. Kullback and R. A. Leibler. On information and sufficiency. The Annals of Mathematical Statistics, 22(1):79–86, 1951.
[7] Matthew Might, David Darais, and Daniel Spiewak. Parsing with derivatives: A functional pearl. SIGPLAN Not., 46(9):189195, September 2011.
[8] Francesco Morandin, Gianluca Amato, Marco Fantozzi, Rosa Gini, Carlo Metta, and Maurizio Parton. SAI: a sensible artificial intelligence that plays with handicap and targets high scores in 9x9 Go (extended version), 2019.
[9] Francesco Morandin, Gianluca Amato, Rosa Gini, Carlo Metta, Maurizio Parton, and Gian-
   Carlo Pascutto. SAI: a sensible artificial intelligence that plays Go. 2019 International Joint
   Conference on Neural Networks (IJCNN), Jul 2019.
[10] Lauri Paatero. Gowrite, [https://gowrite.net/](https://gowrite.net/) 2009.
[11] J. Power. Invincible, the Game of Shusaku. Game Collections Series. Kiseido Publishing
    Company, 1998.
[12] M. Pumperla and K. Ferguson. Deep Learning and the Game of Go. Manning Publications,
    2019.
[13] Arvind Satyanarayan, Dominik Moritz, Kanit Wongsuphasawat, and Jeffrey Heer. Vega-Lite:
    A Grammar of Interactive Graphics. IEEE Trans. Visualization & Comp. Graphics (Proc.
    InfoVis), 2017.
[14] David Silver, Aja Huang, Chris J. Maddison, Arthur Guez, Laurent Sifre, George van den
    Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot,
    Sander Dieleman, Dominik Grewe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy
    Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel, and Demis Hassabis. Mas-
    tering the game of Go with deep neural networks and tree search. Nature, 529(7587):484–489,
    January 2016.
[15] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur
    Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, Yutian Chen, Timothy
    Lillicrap, Fan Hui, Laurent Sifre, George van den Driessche, Thore Graepel, and Demis
    Hassabis. Mastering the game of go without human knowledge. Nature, 550:354–359, October
    2017.
[16] Yuandong Tian, Jerry Ma, Qucheng Gong, Shubho Sengupta, Zhuoyuan Chen, James Pinker-
    ton, and C. Lawrence Zitnick. ELF OpenGo: An analysis and open reimplementation of
    AlphaZero, 2019.
[17] David J. Wu. Accelerating self-play learning in go, 2019.

1Akita International University, Department of Mathematics and Natural Sciences,
Yuwa, Akita-City 010-1292, Japan

2Nihon Ki-in – Japan Go Association, 7-2 Gobancho, Chiyoda City., Tokyo 102-0076,
Japan

E-mail address: egri-nagy@aiu.ac.jp, tormanen.antti@gmail.com