“GLUEBALLS”: RESULTS AND PERSPECTIVES FROM THE LATTICE

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I review the present status of lattice calculations of properties of “gluon-rich” hadrons and comment on future prospects, in view of planned experiments.

1 Review of lattice results

The gluons of QCD should not only manifest themselves in deep inelastic scattering but also affect the hadron spectrum. In the sector of pseudoscalar mesons this is indeed the case: gluodynamics results in the axial anomaly which in turn implies a big mass gap between the singlet $\eta$ and the octet of SU(3) Goldstone pions. In addition to such indirect effects, QCD in principle offers the possibility of bound states made entirely out of glue.

In lattice simulations of the so-called quenched approximation (or quenched model) to QCD, i.e. QCD without sea quarks, a rich spectrum of glueballs has been established in the past decade. The scalar ($J^{PC} = 0^{++}$) turns out to be lightest with a mass between 1.4 and 1.8 GeV, followed by a tensor of mass between 1.9 and 2.3 GeV and a pseudoscalar that is heavier by another 150 MeV. All but five states turn out to be heavier than 3 GeV, overlapping with charmonia states, a mass region that future experiments might shed more light onto. The other striking features are the somewhat counter-intuitive spin ordering of the spectrum, e.g. $1, 3, 2, 0$ in the $PC = +-$ sector but $0, 2, 3, 1$ in the $++$ sector as well as the fact that the lightest spin-exotic state is well above 4 GeV.

The scalar glueball is of particular phenomenological interest. While all raw lattice data agree with each other within statistical errors of about 40 MeV, rather different values are quoted in the literature: in QCD an experimental input is required to set the mass scale. However, in the quenched model ratios of light hadronic masses can easily deviate from real world experiment by as much as 10%. Hence, to some degree the translation into physical units is a matter of personal preference. This uncertainty is accounted for in the mass ranges quoted above.

In real QCD with sea quarks, it is not entirely obvious in how far e.g. a vector glueball that contains $c\bar{c}$ sea quarks can be distinguished from a $J/\psi$...
that contains sea gluons: no pure glueballs exist but then neither do pure quark model mesons and yet the $J/\psi$ is distinctively different from a $\phi$ that shares the same quantum numbers. We can interpret the former as being close to a quenched $c\bar{c}$ state and the latter as a dominantly $s\bar{s}$ state. In QCD some almost pure glueballs might exist. It might also be that some QCD states can be understood in terms of mixing between glueballs and mesons of the quenched model. In some sectors it might even happen that an interpretation in terms of mixing breaks completely down and the gluons merely result in extra states that are hard to distinguish from radial excitations. So-called spin-exotic quantum numbers like $J^{PC} = 0^{--}, 0^{+-}, 1^{+-}, 2^{+-}, \cdots$ are of particular interest in the search for gluon-rich states, i.e. “glueballs” and (quark-gluon) “hybrid mesons”, however, even in this sector exotic four-quark “molecules” and hybrid mesons can have very similar signatures.

On the theoretical side two directions of research are being pursued: quenched and “un-quenched”. The quenched model is a natural extension of the quark model and provides the language required to speak about mixing between quark model states and glueballs. In order to make the connection to phenomenology glueballs are not enough but the corresponding flavour singlet meson states have to be studied too. Lattice simulations indicate that the quenched $s\bar{s}$ isoscalar meson is about 200 MeV lighter than the scalar glueball. Another important question is that of molecules. Despite of some attempts in this direction, this possibility is vastly unexplored at present. The $f_0(980), a_0(980)$ and the $f_0(400 - 1200)$ are widely believed to be $K\pi$ and $\pi\pi$ resonances, however, this view which is important for the interpretation of the $f_0(1370), f_0(1500)$ and $f_0(1710)$ as mixtures between a scalar glueball and the two lightest isoscalar quark model mesons, is not completely un-debated. Molecules might also be required to explain the difference between the spin-exotic $1^{--}$ mesons observed around 1.4 and 1.6 GeV in experiment but predicted around 1.9 GeV in lattice studies. The next step would be to look into mixing. In addition to the first exploratory lattice investigation, several models have been proposed. In some references the $f_0(1500)$ receives the dominant gluonic contribution in others it is the $f_0(1710)$. Finally, production and decays reveal information about the quark content of a given resonance, provided one knows what to expect from a glueball. Lattice methods are only of limited use here although an exploratory study does exist.

The second, cleanest path is to compute the spectrum of QCD as is. One can then compare with experiment and hopefully find agreement. Unfortunately, the direct approach does at present not only turn out to be prohibitively expensive computationally but it does not really tell us what we
Figure 1. The scalar “glueball” with $n_f = 2$ vs. the lattice spacing $a$ ($r_0^{-1} \approx 400$ MeV).

want to know either: just like in real experiment we would only be able to determine masses and, by varying the lattice volume, decay widths within a given channel but little would be revealed about the nature of the states. Mixing cannot be studied because the resonances are just out there. Fortunately, in our virtual computer experiment we can gradually reduce the quark mass, starting from the quenched approximation, and trace any changes, in particular in the neighbourhood of decay or mixing thresholds.

We are still in the position that the combined “world data” on the scalar $n_f = 2$ “glueball” fits into Fig. 1. The quenched case is included for reference. The un-quenched results have been obtained by use of three different lattice discretisations of the Dirac action: staggered (HEMCGC), Wilson (SESAM) and clover (UKQCD). The quarks are all heavier than $m_s/3$, the scalar meson is still stable and the wave function turns out to be very close to that of the quenched glueball. Most $n_f = 2$ points clearly lie below the quenched line, however, there is certainly a slope in the results, such that the mass in the physical $a = 0$ limit appears consistent with the quenched result. Within the SESAM data set there is an apparent discontinuity because different points have been obtained at different quark masses; the “glueball” becomes lighter as the quark mass is reduced. Whether this effect weakens as the continuum limit is approached is a question as open as whether anything will substantially change once the quarks have become realistically light.
2 Outlook

The quenched glueball spectrum is “solved” and first promising $n_f = 2$ results exist. Future studies of mixing in the quenched set-up are important. Flavour singlet mesons and molecules as well as standard charmonium spectroscopy has been neglected in the past for various reasons but lattice methods and computers have sufficiently matured to allow for a fast quenched relief. More challenging but ultimately necessary is an analysis of the quark mass, volume and lattice spacing dependence with $n_f = 2 + 1$ sea quarks.

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