Abstract
The purpose of this work is to reduce the weight of the structure and conclude the most appropriate topology configuration proper for the embedded and co-cured damping composite structure. A topological optimization model for the embedded co-cured damping composite structure is established. For maximizing modal loss factor under the constraint of total amount of experimental substance, the work investigates the damping performance of the fiber-reinforced layer and deduces the sensitivity of modal loss factor by Modal Strain Energy (MSE) method. The best distribution of viscoelastic materials is obtained via the evolution structural optimization (ESO) method. The deleted elements in the structure correspond to the lower sensitivity elements in the sensitivity cloud published in existing document. Considerable related studies have been presented to demonstrate the relationship of structural topological configuration accompanied by the changes of structural parameters.

1. Introduction
The impacts of vibration and extreme noise cannot be overlooked in the development of new satellites, aircraft, high-speed trains, and nuclear submarines. These adverse factors lead to lower accuracy of operational control, structural fatigue damage, shortened safety life, and other consequences. It is an effective way to reduce and control the vibration response of lightweight, flexible structures that viscoelastic damping layers are embedded into the composites [1–7]. However, the addition of viscoelastic materials increases the weight of the structure and shortens the maximum range of the aircraft. How to realize the reasonable layout of viscoelastic damping material and reduce the weight of the structure under the premise of ensuring the damping performance of the structure have become research hotspots.

The embedded and co-cured damping composite structure, short for ECCDS, belongs to a novel pre-processing damping structure which introduces a composite viscoelastic damping layer into the structure. Structural design ascertains the location and distribution of soft material layer. Compared with the free damping structure and constrained damping structure, ECCDS damping layer is co-cured so as to isolate the structure from the exterior. As to its composition, the structure is made up of matrix phase like resin, reinforcing phase like carbon fiber, as well as viscoelastic damping material. Therefore, the work fabricates a new multi-phase composite solid structure that comprises three phases of material either physically or chemically, and enjoys edges in excellent rigidity, specific modulus, aging resistance and damping performance [8–13]. The embedded and co-cured damping composite structure is shown in figure 1.

Topology optimization refers to a mathematical method for optimizing material distribution in a given area according to a given load situation, constraint conditions and performance indicators. It has been universally acknowledged that topology optimization pertains to a useful instrument proficient in creative structural design which comes into play in practice either by optimizing the whole domain during viscoelastic processing or optimizing the unit cell during periodic processing. In order to decrease the vibration response of cylindrical
shells to the uttermost under broadband transverse force motivation conditions, Zheng et al [14] presented a layout optimization method applicable to passive constrained layer damping (PCLD) processing. Hau et al [15] put forward the multi-objective genetic algorithm (MOGA) so as to optimize the flexible beam shape control during active constrained layer damping (ACLD) processing. From the perspective of minimum density increase and insert location, Boucher et al [16] investigated the full damping performance of honeycomb cells. Chen et al [17] raised a two-scale optimization method, with the aim of seeking the optimal microscopic structure, namely the optimal validity, of viscoelastic materials, amid the macroscopic structure containing maximum modal loss factors. Delgado et al [18] came up with a level-set solution helpful in deriving the topology optimization of viscoelastic materials. Yun et al [19] proposed the instant response of the non-viscously damped dynamic system throughout analysis on design sensitivity. The practice was also found to be suitable for the multi-dof system. As indicated by finite difference results, the method embodied great sensitivity. Elsabbagh et al [20] designed a finite element model for composite plates containing a finite element model for composite plates under viscoelastic processing. The optimum distribution of viscoelastic treatment was obtained. Yun et al [21] attempted to realize the topology optimization for viscoelastic damping layers attached to shells, thus lowering the instant response amplitude under load effects. Aiming at the multi-objective optimization issues based on simulation, Delgarm et al [22] conceived a means to solve limitations in building energy consumption optimization in an effective manner. On the grounds of the bi-directional evolutionary structural optimization (BESO) method, Liu et al [23] figured out a topology optimization algorithm for viscoelastic structures. As verified by related numerical instances, this algorithm indeed optimized the viscoelastic cellular of microstructures or composite materials in 2D and 3D structure. Based on the principle of topology optimization, Ma et al [24] proposed a method which could be universally used in the layout optimization of ABH plates with coated surface. Madeira et al [25] drew a finite element model for the sandwich plates containing viscoelastic core and laminated face layers that could minimize material weight and maximizing model damping strength. Multi-material topology optimization method was taken in the research of Der et al [26] to optimize the damping performance of structures. Xu et al [27] presented a topology optimization method based on the Evolutionary Structural Optimization (ESO). The optimization results indicate that the added weight of damping material decreases by 50%; meanwhile the first two orders of modal loss factor decrease by less than 23.5% compared to the original structure. Yi et al [28] presented an inverse homogenization problem for two-phase viscoelastic composites as a topology optimization problem. Yun et al [21, 29] presents microstructural topology optimization of viscoelastic materials for damped structures. The topology optimization of viscoelastic damping layers attached to shell structures for attenuating the amplitude of transient response under dynamic loads was presented. Zhang et al [30] proposed a concurrent topology optimization method for the design of the multi-scale free-layer damping structures with damping composite materials.

These structures have been completely investigated in former studies. Whereas, most studies available are confined to the topology optimization of the sandwich structure with viscoelastic core and isotropic constraining layer. However, considering the damping performance of the anisotropic layer itself, few studies have explored the topology optimization of anisotropic sandwich plates with viscoelastic core and the relationship of structural topological configuration as a result of the variation of structural parameters. In the present study, Modal Strain Energy (MSE) method is preferred in the analysis on the damping performance and the sensitivity of the first loss factor to consolidate the damping performance at the fiber-reinforced layer. Optimal distribution of viscoelastic materials was obtained via the evolution structural optimization (ESO).
method. Numerical instances will be cited to illustrate the relationship of structural topological configuration with the change of structural parameters.

2. Methods of modeling and sensitivity analysis

Dynamic analysis on composite damping structures is usually performed by complex eigenvalue, direct frequency response and MSE, in which the former two methods are in the complex domain. As a consequence, it takes a lot of spending in the computing of complex structures. From the perspective of energy, MSE defines structural loss factor as the ratio between dissipation energy and total deformation energy. Loss factor increases concurrently with the damping performance. In finite element analysis, MSE can also conveniently probe into and optimize parameters in the damping structure.

The strain energy of the upper skin $E^u_{ck}$, the viscoelastic layer $E_{ck}$, the lower skin $E^l_{ck}$ and composite structure $E$ are, respectively, expressed as

\[ E^u_{ck} = \frac{1}{2} \{ \varphi \}^T [K]_u \{ \varphi \} \]  
(1)

\[ E_{ck} = \frac{1}{2} \{ \varphi \}^T [K]_v \{ \varphi \} \]  
(2)

\[ E^l_{ck} = \frac{1}{2} \{ \varphi \}^T [K]_l \{ \varphi \} \]  
(3)

\[ E_{ck} = \frac{1}{2} \{ \varphi \}^T [K] \{ \varphi \} \]  
(4)

\[ E_{ck} = E^u_{ck} + E^l_{ck} \]  
(5)

where $\{ \varphi \}$ suggests the real vector of related undamped system and $[K]_u$, $[K]_v$, $[K]_l$, and $[K]$ pertain to stiffness matrices of upper skin, viscoelastic layer, lower skin, as well as the composite structure. Relation among the the parameters can be expressed by the formula as below:

\[ [K] = [K]_u + [K]_v + [K]_l \]  
(6)

$W$, cyclic energy consumption of damping materials, may be computed by the following formula:

\[ W = \eta_h E_{ck} + \eta_v E_{ck} \]  
(7)

Modal loss factor should be computed as

\[ \eta_h = \frac{\eta_v E_{ck} + \eta_v E_{ck}}{E_{ck}} \]  
(8)

where $\eta_v$ is loss factor of viscoelastic materials; $E_{ck}$ is modal strain energy at the viscoelastic damping layer and $E_{ck}$ is gross modal strain energy. In condition that the modal loss factor of the structure is measured by the finite element method, viscoelastic material will be treated as the pure elastomer containing rigid modulus. It is practical to immediately infer the prime vibration type of the whole structure with no damping involved. For fear of complex non-linear eigenvalue considerations, the modal loss factor of the structure under specific variation category can be derived as per strain energy method.

ESO method is used to study the topological optimization of co-curing damping composite structures. The modal loss factor of the structure is taken as the optimization goal. The amount of damping substance serves as constraint conditions. The following formula shows the algorithm for the modal loss factor of the structure.

\[ \text{find} \quad \beta = [\beta_1, \ldots, \beta_N] \]  
(9)

\[ \text{max} \quad \eta_h \]  
(9)

\[ s.t. \quad W = \sum_{i=1}^{N} \beta_i m_i \leq W^* \]  
(9)

\[ \beta_i = \{0, 1\} \]  
(9)

Where, $N$ represents element number of damping material; $\beta_i$ represents existence status of damping material element; 0 represents deleted damping material element; 1 represents undisclosed damping material
element; \( m_i \) represents the mass of damping material element; \( W^* \) represents the maximum mass of damping material.

According to equation (8), when the viscoelastic damping element is deleted, changes related to the modal loss factor of the structure should be expressed as follows:

\[
\Delta \eta_k = \eta_k \left( \frac{\Delta E_{gk}}{E_{gk}} - \frac{E_{gk} \Delta E_{isk}}{E_{isk}^2} \right) + \eta_k \left( \frac{\Delta E_{sk}}{E_{sk}} - \frac{E_{sk} \Delta E_{isk}}{E_{isk}^2} \right)
\]

Where, \( \Delta E_{gk} \) denotes the change of modal strain energy of viscoelastic damping material after the damping element is deleted; \( \Delta E_{isk} \) presents the change of modal strain energy of the whole structure after the damping element is deleted;

In the progressive optimization method, since the number of deleted elements is limited and the structural changes are small in each iteration, the following approximation is given, as follows:

\[
\Delta E_{gk} \approx -E_{gki}; \Delta E_{isk} \approx -E_{gki}
\]

Equation (10) may be re-expressed through introducing equation (11) into equation (10) as below:

\[
\Delta \eta_k = \eta_k \left( \frac{E_{gki} E_{gki}}{E_{gki}} - \frac{E_{gki}}{E_{gki}} \right) + \eta_k \left( \frac{E_{gki} E_{gki}}{E_{gki}} - 1 \right) + \eta_k \frac{E_{gki} E_{gki}}{E_{gki}^2}
\]

Then the sensitivity of damping material element \( i \) for modal loss factor should be expressed as follows:

\[
\alpha_{i\eta}^k = \eta_k \left( \frac{E_{gki}}{E_{gki}} - 1 \right) + \eta_k \frac{E_{gki} E_{gki}}{E_{gki}^2}
\]

For multi-mode modal loss factors, the sensitivity is the weighted sum of the normalized results. The sensitivity of damping material element \( i \) for modal loss factor can be expressed as follows:

\[
\alpha_{i\eta}^k = \sum_{k=1}^{M} w_k \frac{\alpha_{i\eta}^k}{\alpha_{i\eta}^{k_{\text{max}}}}
\]

Where, \( w_k \) represents weight coefficient; \( w_k \) satisfies both \( \sum_{k=1}^{M} w_k = 1 \) and \( w_k > 0 \) (\( k = 1, 2, \ldots, M \)); \( \alpha_{i\eta}^{k_{\text{max}}} \) represents the maximum value of the absolute value of modal loss factor sensitivity.

3. Steps of topology optimization

The modal loss factor of the structure following optimization is the optimization goal and the existence status of damping material element is taken as the design variable. Whether the element should be deleted is determined by the sensitivity of the element modal loss factor. The optimal topology optimization structure under the constraint of the specified deletion rate is finally obtained. The design process of structural topology optimization is as shown in figure 2.

(1) The finite element model of co-curing damping composite structure was established.

(2) The initial deletion rate \( R_{Ro} \) and the evolution rate \( R_{E} \) are set, and the total material deletion rate is set.

(3) The initial modal analysis is carried out, the modal strain energy of the corresponding element is extracted in the post-processing stage, and modal loss factor sensitivity will be calculated.

(4) The elements with smaller sensitivity absolute values are deleted until there are no cells that can be deleted. The structure evolves a steady state at the current deletion rate.
(5) If there is no unit that can be deleted under the current deletion rate, the processes of (3) to (5) are repeated after updating the current deletion rate according to $RR_{j+1} = RR_j + ER$.

(6) If the constraint is satisfied, the iteration is stopped and the subsequent result is processed. If not, the process of (3) ~ (6) is repeated till the end.

4. Results and discussion

The finite element numerical model is 200 mm wide and 200 mm long. The thickness of viscoelastic damping in single layer continuous damping structure is 0.24 mm, the thickness of upper and lower skin is 1 mm, and the total thickness is 2.04 mm. The four sides of the numerical model are fixed. Please refer to table 1 and table 2 for specific statistics of material parameters.

For promoting the effectiveness of topology optimization and ensuring algorithm accuracy, the length and width of the structure are divided into 30 units, the thickness direction is divided into 9 units, the total number of
units is 8100, and the unit type is selected as Solid185. Considering damping material dosage as one of the constraint conditions, the first-order mode modal loss factor is maximized as the target function. The optimization program has been compiled in APDL language. The optimal topology configuration of the structure for different removal rate is shown in figure 3, and the configuration boundary is clear and has strong practicability.

In the literature [31], sensitivity map of first order loss factor is shown in figure 4. Thus it can be seen that factors with low sensitivity in figure 3 are basically the same as the elements deleted in figure 4. The correctness of the topology optimization result is verified.

The change history of first-order modal loss factor is shown in table 3. With the gradual deletion of the damping material, the first-order modal loss factor of the constrained damping structure slowly decreases. When 50% of the damping material is deleted, the first-order modal loss from 7.759% in initial structure to 7.6746% and the reduction is only 1.088% of the initial structure.

Damping topology optimization technology can reduce the loss factor along the slowest path under the constraints of the specified amount of damping materials. The final damping topology can enable the remaining damping materials to exert maximum energy dissipation capabilities, while saving materials and reducing the loss factor to a minimum.

### 4.1. Influence of length to width ratio on topological configuration of the structure

Taking the above example as a research object, the effects of different length to width ratios of the structure, shear modulus and loss factor of damping material on topological configuration of ECCDS are further discussed. The term \(a\) represents the length of the thin plate; the term \(b\) represents the width of the thin plate; \(a^*\) represents the length of the largest rectangle formed by the internal deleted elements; \(b^*\) represents the width of the largest rectangle formed by the internal deleted elements; and \(G\) represents the shear modulus of viscoelastic materials.

The first step is to consider the effect of structural aspect ratio on the topological configuration of thin plates. Keep the area of the sheet surface the same, and when the aspect ratios are 10:10, 11:10, 12:10, and 13:10, calculate the topological configuration under different aspect ratios. The calculation results are shown in figure 5 below.

It can be seen from table 4 that when the material is not deleted, the length-to-width ratios of the thin plates are 10:10, 11:10, 12:10, and 13:10, and the corresponding first-order loss factors are 7.759%, 7.8733%, 7.8981%, and 7.981% respectively. Whenever damping material is removed by 50%, the loss factor drops to 7.6746%, 7.7914%, 7.8007%, and 7.8986%, with minor changes. The topological configuration of the structure with 50% damping material deleted is shown in figure 5. When the length-to-width ratios \(a/b\) are 1, 1.1, 1.2 and 1.3 respectively, \(a^*/b^*\) are 1, 1.14, 1.5 and 2.2 respectively. As the aspect ratio increases, the topological configuration of the structure changes accordingly. In the central area of the structure, the number of cells deleted along the x direction (length direction) increases, and the number of cells deleted along the y direction (width direction) decreases.

| Table 1. T300/QY8911 material parameters. |
|-------------------------------------------|
| Name | Value       |
|------|-------------|
| Density kg\(^{-1}\) m\(^{-3}\) | 1900         |
| Elastic modulus/Gpa Ex Ey | 26.226      |
| Shear modulus/Gpa Gxy Gyz Gxz | 3.9 3.7 3.7 |
| Poisson’s ratio/PRxy PRyz PRxz | 0.33 0.4 0.4 |
| Loss factor | 0.02         |

| Table 2. Viscoelastic material parameters. |
|------------------------------------------|
| Name | Value  |
|------|--------|
| Density kg\(^{-1}\) m\(^{-3}\) | 1080   |
| Elastic modulus/Mpa Ex | 15.5   |
| Poisson’s ratio/PRxy PRyz PRxz | 0.498  |
| Loss factor | 0.5    |
4.2. Influence of damping layer shear modulus on topological configuration

The influence of damped shear modulus on the topological configuration of thin plates is discussed. With other parameters unchanged, when damping shear modulus is 0.1736 MPa, 1.1736 MPa, 2.1736 MPa, 3.1736 MPa, 4.1736 MPa and 5.1736 MPa respectively, the topological configuration of the structure under different damping shear moduli is computed. Please refer to figure 6 for more details about the computation results. The relationship between structure loss factor and shear modulus in damping material at different deletion rates is shown in table 5.

From table 5, structural loss factor first increases and then decreases together with the growth of shear modulus of the damping layer. Before the damping material is deleted, the first-order loss factors corresponding to the shear modulus of the damping layer are 0.1736 MPa, 1.1736 MPa, 2.1736 MPa, 3.1736 MPa, 4.1736 MPa.
and 5.1736 MPa, respectively are 3.3492%, 7.0718%, 8.0303%, 8.1623%, 8.099% and 7.759%. In case that damping material is removed by 50%, the loss factor drops to 3.1744%, 6.6443%, 7.6895%, 7.9292%, 7.8549%, and 7.6746%, with minor changes. It can be seen from figure 6 that when the shear modulus of the damping layer is 0.1736 MPa, the number of elements deleted in the central area of the structure is 44, and the number of elements deleted around the four sides of the rectangular plate is 405. In case that damping material decreases by 50% together with the growth of shear modulus of the damping layer, the number of elements deleted in the central area of the structure increases, and the number of elements deleted around the four sides of the rectangular plate decreases. When the shear modulus increases to 5.1736 MPa, the number of elements deleted in the central area of the structure is 64, and the number of elements deleted around the four sides of the rectangular plate is 386.

4.3. Influence of damping layer loss factor on topological configuration
Discuss the impact of loss factor of the damping layer on the topological configuration of the thin plate. With other parameters unchanged, when the damping layer loss factors are 0.35, 0.4, 0.45 MPa and 0.5, calculate the topological configuration of the structure under different damping layer loss factors. Please refer to figure 7 and table 6 for more details about the computation results.

As shown in table 6, in case that the loss factor of the viscoelastic material ascends, the loss factor of the entire structure also goes up accordingly. The viscoelastic damping loss factor has a greater influence on the loss factor of the entire structure. As shown in figure 7, for structures with different loss factors of viscoelastic materials, in case that damping material decreases by 50%, and the change range of structural loss factor is small. The number

| Ply angle θ(°) | Length a (mm) | Width b (mm) | Delete rate (%) | First loss factor η(%) |
|---------------|---------------|--------------|-----------------|-----------------------|
| 0             | 200           | 200          | 0               | 7.759                 |
| 0             | 200           | 200          | 5               | 7.7581                |
| 0             | 200           | 200          | 10              | 7.7572                |
| 0             | 200           | 200          | 15              | 7.7533                |
| 0             | 200           | 200          | 20              | 7.7482                |
| 0             | 200           | 200          | 25              | 7.7485                |
| 0             | 200           | 200          | 30              | 7.7477                |
| 0             | 200           | 200          | 35              | 7.7352                |
| 0             | 200           | 200          | 40              | 7.7048                |
| 0             | 200           | 200          | 45              | 7.698                 |
| 0             | 200           | 200          | 50              | 7.6746                |
of elements deleted in the central area of the structure is 64, and the number of elements deleted around the four sides of the rectangular plate is 386. With different loss factors of damping layer, the topological configuration of the structure is basically the same. It can be seen that the viscoelastic loss factor has little effect on the topological configuration of the structure.

![Figure 5. Effects of aspect ratio on topological configuration of the structure.](image)

![Table 4. Relationship between loss factor at different deletion rates and aspect ratio of the structure.](table)

| Deletion rate (%) | 10:10 | 11:10 | 12:10 | 13:10 |
|-------------------|-------|-------|-------|-------|
| Loss factor (%)   |       |       |       |       |
| 0                 | 7.759 | 7.8733| 7.8981| 7.981 |
| 5                 | 7.7581| 7.8724| 7.8973| 7.9802|
| 10                | 7.7572| 7.8715| 7.8964| 7.9794|
| 15                | 7.7533| 7.8676| 7.8933| 7.9763|
| 20                | 7.7482| 7.8652| 7.8896| 7.9737|
| 25                | 7.7485| 7.8612| 7.8838| 7.9674|
| 30                | 7.7477| 7.8604| 7.8845| 7.9606|
| 35                | 7.7352| 7.8453| 7.8693| 7.9505|
| 40                | 7.7048| 7.8312| 7.8537| 7.9518|
| 45                | 7.698 | 7.8113| 7.8379| 7.9264|
| 50                | 7.6746| 7.7914| 7.8007| 7.8986|
5. Conclusions

In consideration of the energy radiation in fiber-reinforced composite material, the loss factor sensitivity of ECCDS is derived. Using ESO, the topological optimization of ECCDS is completed to achieve the maximization of modal loss factor under the constraint of total amount of damping materials. Reasonable layout of viscoelastic damping material is realized. The impact of overall parameters on topological configuration of ECCDS is subsequently studied. Different aspect ratios affect the number of deleted elements in the x and y directions. In

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Figure 6. Topological configuration of the structure with different shear Modulus of damping Layer.

(a) $G = 0.1736$ MPa  
(b) $G = 1.1736$ MPa  
(c) $G = 2.1736$ MPa  
(d) $G = 3.1736$ MPa  
(e) $G = 4.1736$ MPa  
(f) $G = 5.1736$ MPa
condition that the elastic modulus of viscoelastic material increases, the number of deleted elements in structure center increases. The viscoelastic loss factor has little effect on the topological configuration of the structure. Therefore, a conclusion could be fitly judged that topology optimization is a useful instrument in the optimization design of ECCDS damping attributes.

Table 5. Relationship between structural loss factor and shear modulus in damping material at different deletion rates.

| Deletion rate (%) | Shear modulus of viscoelastic material (MPa) |
|-------------------|---------------------------------------------|
|                   | 0.1736 | 1.1736 | 2.1736 | 3.1736 | 4.1736 | 5.1736 |
| Loss factor (%)   |        |        |        |        |        |        |
| 0                 | 3.3492 | 7.0718 | 8.0303 | 8.1623 | 8.0099 | 7.759  |
| 5                 | 3.3491 | 7.0718 | 8.0303 | 8.162  | 8.0092 | 7.7581 |
| 10                | 3.3484 | 7.0705 | 8.0294 | 8.1608 | 8.0086 | 7.7572 |
| 15                | 3.3461 | 7.0647 | 8.0255 | 8.1576 | 8.0048 | 7.7533 |
| 20                | 3.3424 | 7.0558 | 8.0148 | 8.1504 | 7.9989 | 7.7482 |
| 25                | 3.3343 | 7.0383 | 8.0047 | 8.142  | 7.9926 | 7.7485 |
| 30                | 3.3194 | 7.0047 | 7.9777 | 8.1247 | 7.9819 | 7.7477 |
| 35                | 3.2973 | 6.9479 | 7.9317 | 8.0973 | 7.9725 | 7.7352 |
| 40                | 3.2677 | 6.8756 | 7.872  | 8.0455 | 7.9253 | 7.7048 |
| 45                | 3.2273 | 6.7767 | 7.7945 | 7.9954 | 7.9109 | 7.698  |
| 50                | 3.1744 | 6.6443 | 7.6895 | 7.9292 | 7.8549 | 7.6746 |

Figure 7. Topological configuration in structures with different loss factors of damping Layer.
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Conflict of interests and data availability statement

The authors hereby confirm that no conflict of interest exists for this article. The datasets used or analysed during the current study are available from the corresponding author on reasonable request.

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| Deletion rate (%) | 0.35 | 0.4 | 0.45 | 0.5 |
|-------------------|------|-----|------|-----|
| Loss factor (%)   | 5.9593 | 6.5392 | 7.1591 | 7.7591 |
| 3                 | 5.9587 | 6.5385 | 7.1583 | 7.7581 |
| 10                | 5.9558 | 6.5357 | 7.1575 | 7.7572 |
| 15                | 5.9554 | 6.5347 | 7.1534 | 7.7533 |
| 20                | 5.9519 | 6.5307 | 7.1494 | 7.7482 |
| 25                | 5.9521 | 6.5309 | 7.1497 | 7.7485 |
| 30                | 5.9515 | 6.5302 | 7.1490 | 7.7477 |
| 35                | 5.9429 | 6.5404 | 7.1378 | 7.7332 |
| 40                | 5.9221 | 6.5163 | 7.1106 | 7.7048 |
| 45                | 5.9174 | 6.5109 | 7.1044 | 7.6982 |
| 50                | 5.9013 | 6.4924 | 7.0835 | 7.6746 |

Table 6. Relationship between structural loss factor and loss factor of damping layer at different deletion rates.
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