Validation of a high resistance calibration method based on a binary voltage divider

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Abstract. A method to measure standard resistors in the range 10 kΩ to 1 TΩ, based in automated bridge using a binary voltage divider is described, tested and validated by comparison with other method based in a Modified Wheatstone Bridge. Evaluation of measurement uncertainty is also validated.

1. Introduction
The aim of this work is to describe a binary voltage divider (BVD) [1][2] bridge system implemented in IPQ, the National Measurement Institute (NMI) of Portugal, and the validation process of its performance through a comparison of its results with a validated Modified Wheatstone Bridge (MWB) method, including an approach to identify the relevant uncertainties sources of the BVD bridge, and the results of the two methods will be compared in order to get an experimental evidence of the uncertainty to the BVD setup.

The validation procedure includes the overall process of uncertainty evaluation, under the GUM uncertainty framework and supplements [3][4]. The validation of the measurement uncertainty evaluation is particularly important in situations where the mathematical model is based on ratios, as it has been shown elsewhere [7]. The GUM conventional approach will be validated using a Monte Carlo method (MCM) [4] in order to propagate the distributions through the functional relationships referred above. The influence of the choice of PDF for the input quantities, based on the experimental data available, will be studied.

2. The BVD Bridge
The simplified diagram of the BVD bridge setup is represented in Figure 1, whereas the MWB is shown in Figure 2 (to be developed in chapter 3). The digital voltmeter (DVM) and the voltage source are independent equipment connected to the acquired commercial bridge from “Measurement International” (MI 6000B).

The bridge works in an automated way to find the balance between the output voltage of the BVD and the test voltages, $V_1$ to $V_4$, using the DVM as null detector. The balance voltages condition detected by the DVM corresponds to the determination of the ratios $r_i$ by the BVD. The ratio, $R$, between the two resistors $R_x$ and $R_s$ calculated by the bridge software, can be expressed in terms of the measured quantities by:

$$R = \frac{R_x}{R_s} = \frac{V_3 - V_2}{V_3 - V_4} = \frac{r_3 - r_2}{r_3 - r_4}$$ (1)
Therefore, the accuracy of the system depends, first of all, on the three independent instruments used. From the measurement equation, the result does not depend on the absolute value of the source voltage, but on its stability along the time interval of the measurements. The DVM is used as null detector and therefore the resolution, zero offset stability and linearity in the mV range are the relevant DVM parameters to be taken into account for the accuracy of the system. The BVD is based on a 13-bit resolution resistive divider made up of resistors with low temperature coefficients. Its auto-calibration feature assures the determination of corrections factors for each bit, with an extended resolution enabled by the resolution of the DVM. Besides the instrumentation, the thermoelectric and leakage effects are also relevant influence quantities. The current leakage effect has significant impact in the final ratio results, due to the higher resistance values involved, as described in [5].

2.1. The bridge MI6000B measurement model

The accompanied software calculates each voltage ratio, $V_i$, by the following relation:

$$V_i = (r_i + c_i) \times E + (V_{ib} - V_{i,off})$$  \hspace{1cm} (2)

where:
- $r_i$ ratio determined by the binary divider for each of the balanced voltages $V_i$.
- $c_i$ correction determined by the bridge auto-calibration feature, to each stage of the BVD.
- $E$ voltage applied to the resistors by the voltage source and measured with the DVM.
- $V_{ib}$ residual voltage measured, in balance condition, between the BVD output and the resistor terminal. This measurement strategy allows an extended resolution of the 13-bit binary divider, limited by the quantity $E/2^{13}$, to the resolution of the DVM.
- $V_{i,off}$ offset voltage of the circuit measured by the DVM, shortcutting the inputs.

After sequential measurements of $E$ followed by the four $V_i$ values, the software calculates ratio $R$:

$$R = \frac{R_x}{R_s} = \frac{V_1 - V_2}{V_3 - V_4}$$  \hspace{1cm} (3); \hspace{1cm} \frac{R_d + R_i^{-1}}{2}$$  \hspace{1cm} (4); \hspace{1cm} \frac{\sum_{j=1}^{n} R_{d,j} + R_{i,j+1}}{2^n}$$  \hspace{1cm} (5)

It is also available an “interchange” option that allows that each $R$ is the mean of two sequential measurements, $R_j$, with the change of the resistors in the circuit to remove the thermoelectric effect error of the circuit, where $R_d$ is the direct ratio $R_x/R_s$ and $R_i$ is the inverse ratio $R_s/R_x$. Finally, it is possible to choose a number ($n +1$) of sequential ratio measurements to produce a mean value $R_m$.

2.2. An approach to evaluate the measurement uncertainty

In this section, a simplified approach will be presented to evaluate the measurement uncertainty taking as a starting point the known functional relationship implemented by the bridge software. It will not take into account the correlation between the input quantities based on voltage measurements and
of the relation (5). The experimental standard deviation of the measurements is estimated from the repeatability of the output quantity, \( R \). The reproducibility is taken from \( R \) relates to different sequential cycles. The relevant uncertainty about the residual leakage currents in the system is formulated as an input quantity in the model equation, but since there is no immediate way to estimate this source of uncertainty, it will be a target to estimate indirectly by the results comparison obtained with the two distinct methods addressed in the work. Equation (5) may be rewritten as:

\[
R = R_m + \delta R_{rep} + \delta R_{reprod} + \delta R_{leak}
\]  

(6)

The evidence of correction factors, \( \delta \), to each input quantity in expression (2) above results in the following expression:

\[
V_i = (r_i + c_i + \delta c_i) \times (E + \delta E) + (V_{i,b} + \delta V_{i,b} - V_{i,off} - \delta V_{i,off})
\]  

(7)

The terms \( \delta \) above are estimated as a zero-expected value. 
\( \delta c_i \) error correction in the \( c_i \) determination including stability along calibrations. 
\( \delta E \) drift in the \( E \) value during each measurement cycle: \( V_i \) to \( V_4 \). 
\( \delta V_{i,b} \) error correction in the \( V_{i,b} \) due to linearity and finite resolution of the DVM. 
\( \delta V_{i,off} \) error correction in the \( V_{i,off} \) due to the finite resolution of the DVM. 
\( \delta R_{rep} \) auxiliary quantity to represent the global repeatability of the setup (one cycle). 
\( \delta R_{reprod} \) auxiliary quantity to represent the global reproducibility of the setup (sequential cycles). 
\( \delta R_{leak} \) error due to the imperfect correction of leakage current effects of the system.

It is known that bridge software measures and corrects the leakage effect, using a “null virtual configuration of the circuit”, and offers the availability of “an isolated guard source that follows the voltage being measured and keeps the guard circuit at the same potential. The resulting leakage current is then practically zero”, but how such feature is implemented is not addressed by the supplier. [5]

The variance of \( R \), associated with the GUM uncertainty framework [3], is expressed in a compact way, as a function of its covariance and partial derivatives by the matrix relation:

\[
u^2(R) = d^T COV_R d
\]  

(8)

This simplified approach assumes the linearized model entailed by the GUM approach, i.e., linear model, Gaussian input quantities and Central limit theorem, which requires validation for any model not fulfilling those conditions. With most input quantities having uniform distribution in the ratio models, a validation with a Monte Carlo method [4] will be attempted.

2.2.1. Uncertainty of the \( V_i \) terms

The uncertainty of \( V_i \) can be estimated based on the measurement model of equation (7) (for simplicity we will omit the subscript \( i \)):

\[
u(V)^2 = E^2 \sigma^2 + (r + c)^2 \sigma^2 + u(\delta E)^2 + u(\delta V_b)^2 + u(\delta V_{off})^2
\]  

(9)

From the experience in working with the binary divider, the accuracy for each binary factor correction determination, by the software bridge, is taken as less than a few parts in \( 10^{-8} \). From the observation of the factor correction values change, between periodic auto-calibrations made by the bridge, we can also assume a few parts in \( 10^{-8} \) to its stability. To each bit activated in the determination of \( r_i \) ratios, we will consider a value of \( 5 \times 10^{-8} \) to the \( \delta c \) interval limit. The uncertainty related to the Voltage Source, \( E \), is due to its drift during each measurement cycle of \( V_1 \) to \( V_4 \) and was confirmed experimentally to the used source as being limited to \( 10^{-7} \). Moreover, the range of \( V \) values measured by the DVM depends on the resolution of the binary divider applied to the \( E \) value (for \( E \) equal to 100 V, the quantity \( V \) could reach 12 mV); for the DVM used the linearity of the 100 mV.
range was verified to be less than 1 ppm and its resolution is 10 nV. Finally, the uncertainty of $\delta V_{\text{off}}$ is affected by the finite resolution of the DVM and could be estimated, for a resolution of 10 nV.

| Table 1. Uncertainty values associated with quantity $V$ in equation (11) |
|---|---|---|---|
| Source | $u(\delta c)$ | $u(\delta E)$ | $u(\delta V_b)$ | $u(\delta V_{\text{off}})$ |
| Value | $\frac{\sqrt{n} \times 5 \times 10^{-8}}{\sqrt{3}}$ | $10^{-7} \times E$ | $\sqrt{\left(\frac{1 \times 10^{-6} \times V}{\sqrt{3}}\right)^2 + \left(\frac{5 \times 10^{-9}}{\sqrt{3}}\right)^2}$ | $5 \times 10^{-9}$ |

where $n$ is the number of bits activated to determine each $r_i$

2.3. Tests to 100 MΩ : 10 MΩ ratio

Experimental measurements at 100 V were made to explore the system behavior in different connection configurations of the bridge, the DVM (Agilent 3458A) and the two Guildline 9330 resistors, with nominal values of 10 MΩ as $R_s$ and 100 MΩ as $R_x$. Each measurement followed the configuration proposed in the bridge manual [1] for this range of resistance: 30 repeated measurements to each $V_i$; 6 (n+1) sequential measurements for a mean $R$ value determination and with the interchange of the resistors as formulated in equation (5). The uncertainty associated with the leakage current correction in its value is one of the open questions in this paper, but an estimation of its value will be attempted based on the results obtained with another experimental method taken as reference and comparing the results of both methods.

2.3.1. Time stability, repeatability of measurements and guarding

From several series of continuous measurements, it was established the need for long periods of stabilization. Figure 3 shows that several hours are needed to obtain an experimental standard-deviation, $s(R)$, of about 0.13 ppm. Figure 4, show two different configurations: active guard ON: a standard deviation of 0.34 ppm was observed; active guard OFF: 1.37 ppm. The difference in the mean values is 0.10 ppm less than the standard deviation, which shows that, for this particular range, the leakage current in the DVM connection does not play a relevant role as systematic effect but increase the noise in the measurement results. For measurements taken with active guard ON an interval due to reproducibility, $(R_{\text{max}} − R_{\text{min}})$ of 1.1 ppm was observed. This means that reproducibility is about one order of magnitude larger than the repeatability and should be taken into account in the configuration of a measurement setup to produce meaningful results.

To observe the relevance of the active guard, a measurement was made floating the screen of the connections (between the resistors and the bridge). A change in the mean value of the ratio from 20.9 ppm to 31.9 ppm was observed. It can be concluded that the leakage currents corrections plays an essential role in this setup and particular attention must be paid to the way how guard is implemented.

2.3.2. Results for the ratio 100 MΩ : 100 MΩ and uncertainties

With the input values indicated in Table 1, the results are illustrated in Figure 5 and Table 2 for the GUM (line) and MCM (bar) approaches. It can be seen that assuming a Gaussian distributions for the measurand, despite the fact that all input quantities are taken as having a uniform distribution, except
for the repeatability, produces a result that is significantly higher than the value obtained by propagating the true uniform distributions. Vertical lines represent 95% confidence limits. This is an important result to alert those that think that only complicated models can produce erroneous results when using the GUM. In this case the GUM significantly overestimates the measurement uncertainty.

### Table 2. Uncertainty values with GUM and MCM methods

| Quantity | $R_d$  | $u(R_d)$ | $R_i$  | $u(R_i)$ | $R_m$  | $U(R_m)$ |
|----------|--------|----------|--------|----------|--------|----------|
| GUM      | 10.000235 | 0.8 ppm  | 0.999998 | 1.7 ppm  | 10.000209 | 2.0 ppm  |
| MCM      | 10.000234 | 0.5 ppm  | 0.999998 | 1.0 ppm  | 10.000209 | 1.2 ppm  |

![Figure 5. Uniform and Gaussian distributions for $V_i$.](image)

### 3. The Modified Wheatstone Bridge

The simplified diagram of the MWB setup implemented in IPQ [6] is represented in Figure 2. This measurement method evaluates the ratio of two high value standard resistors ($R_s$ and $R_x$) based on a well-known ratio of two dc voltages. The guard resistors $R_g$ are chosen to have a value 10 times the value of $R_s$ and $R_x$, respectively.

The equilibrium condition, measured by an electrometer is described by the expression (10). The ratio between the two voltages is equal to the ratio of the two resistors, and one can say that the uncertainty with which the ratios are determined is the same. In a first measurement, there is an imbalance situation translated by a current $\delta I_1$ (11). To cancel this current, assuming that the two branches of the bridge are practically independent of each other, we can simply change the $E_{\text{var}}$ voltage of a value corresponding to (12).

$$R = \frac{E_{\text{var}}}{E_{\text{ref}}} = \frac{R_s}{R_x} \quad (10); \quad E_{\text{var}1} = \frac{R_s \times E_{\text{ref}}}{R_g} + \delta I_1 \times R_x \quad (11); \quad \delta E_1 = \delta I_1 \times R_x \quad (12)$$

With this new voltage value, a second measurement is carried out, obtaining a current already very close to zero. In the implemented procedure, a third measurement is performed to test the linearity. For this, a 20% increase over the previously calculated $\delta E$ is introduced. Zero compensation is applied to each set of three measurements by subtracting the mean of the two consecutive zero readings. From equations above and for each of determinations, we can assume:

$$\frac{E_{\text{var}}}{E_{\text{ref}}} = R + \delta I \times R_x \quad (13)$$

The theoretical value at which $\delta I$ is cancelled, $R$, is calculated by a linear adjustment of the three points previously determined ($\frac{E_{\text{var}}}{E_{\text{ref}}}$, $\delta I$) and corresponds to the intercept of the line fit. The mean square error value of this adjustment gives us a measure of the uncertainty of this value. The other uncertainty sources are due to the accuracy of the voltage ratio determination and to the electrometer accuracy in the $\delta I$ measurements. This entire process is then repeated by inverting the polarity of the connections with a low noise switch so as to override thermal effects in the measuring circuit. The mean of the two theoretical $R$ values is then considered to best describe the ratio of the resistors under
evaluation, \( R_m \). Neglecting differences in the resistors thermal behaviours (as they were kept under a bath controlled temperature within \( \pm 0.01 \) °C), considering a global repeatability and reproducibility estimated for \( n \) sequential measurements of \( R_m \), \( \delta R_{\text{rep}} \), and assuming the correction error due to current leakage effects on the system, \( \delta R_{\text{leak}} \), a final equation for the output quantity can be formulated:

\[
R = R_m + \delta R_{\text{rep}} + \delta R_{\text{leak}}
\]  

(14)

4. Methods comparison and conclusions

Measurements were made to determine the 10:1 ratio resistors allowing comparing the results of both systems setups presented, with the following final results:

| System | Ratio measured ppm | Expanded Uncertainty ppm | Uncertainty method approach |
|--------|--------------------|--------------------------|----------------------------|
| BVD    | 20.9               | 2.0                      | GUM                        |
| MWB    | 17.1               | 1.2                      | MCM                        |
|        |                    | 3.7                      | GUM                        |

An agreement of about 3.8 ppm has been determined that is in accordance with the combined uncertainty of the comparison 4.2 ppm for the GUM method approach and 3.9 for the MCM method approach. As the uncertainty values presented for each system setups (BVD and MWM) don’t take into account any specific contribution due to leakage currents, we can conclude that, in those particular setups, the automated corrections guaranteed by the BVD software seems to be effective. For the resistors type used, it was seen that leakage currents could reach 11 ppm in the difference ratios. The use of the guarded voltage in DVM and resistors connections has a determinant influence in the correction of leakage currents and in the noise measurements.

The simplified uncertainty approach here presented, based in the model equation of the measurements implemented by the BDV Bridge, conducts to values that are in accordance with other proven method (MWD). The uncertainty assumed to the factor correction values of the BVD have a dominant weight in the combined uncertainty resulting from the model equation derived from the way as the bridge effectively works.

Further measurements will be needed to consolidate this preliminary result and extended to higher values of resistance where the effects of leakage currents will be more relevant.

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