UNQUENCHING AND UNITARISING MESONS IN QUARK MODELS AND ON THE LATTICE*

GEORGE RUPP

Centro de Física e Engenharia de Materiais Avançados
Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Portugal

EEF VAN BEVEREN

Centro de Física da UC, Departamento de Física
Universidade de Coimbra, 3004-516, Portugal

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Mesons with masses below their lowest OZI-allowed strong-decay thresholds have very small widths. Thus, it is usually believed that they can be safely treated as pure quark-antiquark bound states in spectroscopy models. However, unitarised and coupled-channel models from decades ago already indicated that this may not be the case, owing to significant virtual meson-loop contributions. Recent unquenched lattice calculations that include two-meson interpolators besides the usual $q\bar{q}$ ones confirm the latter conclusion, in particular for the enigmatic narrow $D_{s0}^*(2317)$, $D_{s1}(2460)$, and $X(3872)$ states. Here, we briefly review several predictions of some old and new quark models that go beyond the static description of mesons, also in comparison with up-to-date lattice results.

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1. Introduction

Knowledge of low-energy QCD is encoded in the observable properties of hadrons, that is, mesons and baryons. Most importantly, hadronic mass spectra should provide detailed information on the forces that keep the (anti)quarks in such systems permanently confined, inhibiting their observation as free particles. However, the prominent factor that makes it very difficult to extract this information is the lightness of current (anti)quarks as compared to the QCD scale $\Lambda_{\text{QCD}}$. This complicates the simple model

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of mesons and baryons as pure quark–antiquark \((q\bar{q})\) and three-quark \((qqq)\) states, respectively, endowing them with a flurry of \(q\bar{q}\) pairs being constantly created and annihilated in the strong QCD fields.

Now, in so-called unquenched lattice QCD, effects of \(q\bar{q}\) loops are fully taken into account by including dynamical quark degrees of freedom via a fermion determinant. Nevertheless, allowing for virtual \(q\bar{q}\) pairs does not paint a complete picture, as the created quark and antiquark may recombine with the original (anti)quarks so as to form two new colourless hadrons. Even if the mass of the initial hadron is smaller than the sum of the new hadrons’ masses, so that no real decay can take place, the corresponding virtual processes via meson–meson or meson–baryon loops will contribute to the total mass. This is expected to be all the more significant according as the decay-threshold mass lies closer to the original hadron’s mass. On the other hand, if the latter mass is above threshold, the hadron actually becomes a resonance, whose properties are determined by S-matrix analyticity and unitarity.

In very recent years, different lattice groups have managed to extract scattering phase shifts and resonance properties from unquenched finite-volume simulations including meson–meson or meson–baryon interpolating fields, besides the usual \(q\bar{q}\) or \(qqq\) ones, respectively (see Ref. [1] for a review hot off the press). Some of these works on mesonic resonances [2] show that large mass shifts may result from unitarisation, even when analytically continued to underneath the lowest strong-decay threshold. Typical examples are the \(D^{*}_{s0}(2317)\) [3] and \(D_{s1}(2460)\) [4] open-charm mesons, as well as the mysterious \(X(3872)\) [5] charmonium-like state (see Sec. 4).

Most quark models have been paying little or no attention to these lattice results, despite their far-reaching implications for meson spectroscopy. However, almost four decades ago pioneering work was already published on coupled-channel and fully unitarised models of mesons with predictions some of which only now are starting to be supported in lattice QCD computations. In the present short paper, a selection of such predictions will be reviewed, alongside more recent model results and lattice confirmations. The organisation is as follows: in Sec. 2, we briefly discuss the concepts of unquenching and unitarisation in the context of quark models as well as on the lattice. Section 3 reviews our old model predictions for the light scalar mesons \(\text{vis-à-vis}\) very recent lattice results. In Sec. 4, we compare our much more recent model descriptions of a few puzzling mesonic states with the corresponding lattice calculations. Conclusions are drawn in Sec. 5.
2. Unitarisation and unquenching

The notion that the instability of most hadrons must have implications for their spectra dates back to the late 1970s and early 1980s. The point is that in many cases, hadronic decay widths are of the same order of magnitude as the average level splittings [2]. The corresponding baryon and meson resonances are characterised by poles in the complex-energy plane, whose locations are governed by S-matrix properties such as unitarity and analyticity. To pretend that the real parts of these pole positions should correspond to the real energy levels of a pure confinement spectrum without the possibility of decay is not only naive but even a denial of elementary scattering theory. This was realised by the unitarised meson models of the Cornell, Helsinki, and Nijmegen groups, with first applications to charmonium [6], light pseudoscalar and vector mesons [7,8], and heavy quarkonia [9] as well as pseudoscalar and vector mesons [10], respectively. Although the quantitative predictions for especially mass shifts turned out to be quite varied [11], dependent on details of wave functions as well as decay mechanisms and included channels, these early results unmistakably showed very significant deformations of pure confinement spectra. Even more dramatic was the description of light scalar mesons [2] as dynamical resonances [12] in the model of Ref. [10], appearing as extra, low-lying states alongside the regular scalar mesons with masses above 1.3 GeV. We shall come back to this work in Sec. 3.

Although these early meson models require the creation of \( q\bar{q} \) pairs in order to allow decay, they do not treat the quark degrees of freedom dynamically, using instead constituent quark masses and a phenomenological decay mechanism, based on e.g. the \( ^3P_0 \) model (see Ref. [10] and references therein). The first prominent approaches employing light current quarks with dynamical chiral-symmetry breaking were developed by the Orsay [13] and Lisbon [14] groups. In the latter work, vacuum condensation of \( ^3P_0 \) light Dirac \( q\bar{q} \) pairs due to a pure vector-like confining potential gives rise to dynamical chiral-symmetry breaking. Through the subsequent yet consistent solution of a single-quark mass-gap equation, a \( q\bar{q} \) Salpeter equation, and a meson–meson scattering equation using the Resonating Group Method, reasonable masses and widths were obtained for the \( \rho \) and \( \phi \) resonances. This was the first — and to our knowledge so far the only — quark model to consider both unitarisation and unquenching, in the sense that the quarks were treated dynamically. Besides accounting for a massless pion in the chiral limit and a correct low-energy \( \pi\pi \) amplitude, a very important result for the present discussion was a negative mass shift of the order of 300 MeV for the \( \rho \) meson with respect to its confinement-only mass, owing to the coupling to the \( \pi\pi \) decay channel. This shift is comparable to the one found before in Ref. [10]. However, no further spectroscopy calculations were done in the
Let us first discuss S-wave ππ scattering. The lowest bound state of our confining potential for \( JPC = 0^{++} \) do not show up in P- and higher-wave scattering, which explains quite naturally why they are not found there.

The term “unquenched” first appeared in the title of a lattice paper due to Montvay [15] back in 1984, describing a Monte Carlo calculation with virtual quark loops through a quark determinant for the Wilson fermions. Progressively, many other lattice groups started to carry out QCD simulations with dynamical quarks. However, only much later hadrons were allowed to decay on the lattice, by employing Lüscher’s finite-volume method [16] and generalisations thereof [1] to extract scattering phase shifts from discrete energy levels for different lattice sizes. Also in quark models the term “unquenching” is being used more and more often, although it usually is a sloppy way of referring to some unitarisation or coupled-channel approach. Two examples are a meson toy model [17] and the baryon model of Ref. [18].

### 3. Light scalar mesons

In Ref. [12], a complete light scalar-meson nonet emerged, as additional and dynamical resonances, by employing the unitarised multichannel model of Ref. [10] with unaltered parameters. Now, more than 30 years later, those predictions for masses, widths, and pole positions are still fully compatible with experiment [2]. In Fig. 1, we show the computed S-wave ππ phase shift, obtained without any fit, together with the then available data.

![S-wave ππ phase shift](image)

Fig. 1. S-wave ππ phases predicted in Ref. [12] (see the text and reference for data).

Very recent lattice calculations [19–21] confirm our original [12] interpretation of the light scalars as P-wave \( q\bar{q} \) states with large meson–meson components. In particular, Ref. [19] described the lightest isoscalar–scalar meson in a lattice calculation with \( q\bar{q} \) and ππ interpolating fields, though
with still too large pion masses, viz. of 236 and 391 MeV. Nevertheless, in
the former case, a broad $\sigma$-like ($f_0(500)$ [2]) resonance shows up, which
becomes a bound state for the larger $\pi$ mass. In the isodoublet case, the
same lattice group studied [20] — among others — a light scalar state, us-
ing $q_1\bar{q}_2$ (light-strange combination), $\pi K$, and $\eta K$ interpolators, and found
a virtual bound state for a pion mass of 391 MeV. This is expected [20] to
evolve into a broad resonance for a more physical pion mass, thus being a
strong candidate for $K^*\bar{0}(800)$ [2] ("$\kappa$"). Finally, again the same collaboration
found [21] a scalar isovector resonance in a calculation with $q\bar{q}$, $\pi\eta$, and $K\bar{K}$
interpolating fields, representing most likely $a_0(980)$ [2].

4. Very narrow $D^*_s(2317)$, $D_{s1}(2460)$, and $X(3872)$ mesons

In Ref. [22], we applied a very simple momentum-space version of the
unitarised model employed in Refs. [9, 10, 12] to a scalar $c\bar{s}$ state strongly
coupled to the $S$-wave $D K$ channel. This allowed to explain the just discov-
ered puzzling and very narrow $D^*_s(2317)$ [2]. In Ref. [3], a lattice calculation
with $c\bar{s}$ and $D K$ interpolating fields confirmed our interpretation. The same
lattice group also studied [4] the narrow axial-vector charm–strange meson
$D_{s1}(2460)$ [2] besides other $D_s$ states, finding a bound state below the $D^*K$
threshold, thus confirming our findings [23] in a multichannel generalisation
of the momentum-space model used in Ref. [22]. Last but not least, again
the same lattice collaboration investigated the controversial $J^{PC} = 1^{++}$
charmonium-like meson $X(3872)$ in various simulations with $c\bar{c}$, tetraquark,
and $D^*D$ interpolating fields. This study was particularly interesting, be-
cause it also searched for signals of tetraquark charmonium-like states up
to 4.2 GeV, not finding any. As for $X(3872)$, this state only survived with
both $c\bar{c}$ and $D^*D$ interpolators included, the tetraquark ones being largely
irrelevant. This lends support to our unitarised description [24] of $X(3872)$
with a $c\bar{c}$ component coupled to several open-charm meson–meson channels.

5. Conclusions

In the present short review, we have tried to convey the message that
modern meson spectroscopy must take into account both real and virtual
strong decay in order to arrive at minimally reliable predictions. In par-
ticular, when a meson is narrow, one cannot automatically conclude that
coupled-channel effects will also be small, since a (quasi-)bound state may
result from a negative mass shift due to meson–meson loops, driving a bare
$q\bar{q}$ state to below its lowest threshold. Also in such cases, S-matrix unitarity
and analyticity should serve as a guidance. Several recent lattice calculations
leave no doubt that the unitarisation issue should finally be taken seriously.
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