Research Article

Study on Interactive Damping of Primary-Secondary Coupled System

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Received 9 December 2018; Revised 31 January 2019; Accepted 13 February 2019; Published 28 February 2019

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In order to study the interactive damping of a primary-secondary coupled system, three damping strategies are presented. Based on it, three dynamic models were established: the 2-DOF series model, spatial dynamic model, and 3-DOF series model. According to the concept and calculation methods of random vibration, the displacement, velocity, and acceleration variances of each model were derived as response functions of the coupled system. Using MATLAB to analyze the response functions, we studied the manner in which the system response is affected by the frequency and mass ratios of the primary structure and secondary structures, the plane layout of the secondary structure, and the quality, frequency, and damping of the connection structure, respectively. Thereafter, combined with the multiobjective optimization method, optimal parameters were selected to minimize the coupled system response in order to achieve interactive damping.

1. Introduction

An important feature in most industrial buildings and lifelines is expensive equipment of a large volume and mass. The equipment is usually connected to the structure with high-strength support and connectors to ensure that equipment operates normally under seismic action. But the ultimate seismic behavior was unsatisfactory. According to the statistics, about 1/3 of the losses caused by modern earthquakes are directly related to equipment [1].

In theoretical research, the main structure of a building is usually referred to as the primary structure, while the equipment is known as the secondary structure, and the system formed by these two is called the primary-secondary coupled system [2]. Under external excitation, there is a complex dynamic interaction between the primary structure and the secondary structure, and the inertia of the secondary structure will change the original dynamic characteristics of the primary structure. At the same time, there is a dynamic energy transformation between the primary structure and the secondary structure. Following decades of research, Suarez and Singh [3], Hernried and Sackman [4], Igusa and Der Kiureghian [5, 6], Singh et al. [7–10], Lai and Soong [11, 12], and Villaverde [13, 14] achieved remarkable results in the modeling and dynamic analysis method of the coupled system, and these results are of great significance in understanding the dynamic interaction of the coupled system. Moreover, engineers have used these results extensively in nuclear power plant equipment and its industrial piping, among others. Villaverde [15], Chen and Soong [2], and Soong [16] conducted systematic summaries of the related research work prior to the 1990s. It can be observed from the above research methods and results that the so-called damping system takes the minimum single response of the primary or secondary structure as the damping target. In the current primary-secondary coupled system, the secondary structure appears to be of high quality and cost. Therefore, the damping target of the coupled system should be the response of both the primary and secondary structures being at a small value.

Experiments on the coupled system can be divided into two types. The first is experiments using special equipment, which are commonly used in nuclear engineering, the chemical industry, and others, as reported by Nims and Kelly [17] and Zhu et al. [18]. The other is an experimental study considering a simplified model, which generally simplifies
the secondary structure into a SDOF structure, as reported by Kelly and Tsai [19], Juhn et al. [20], Adam et al. [21, 22], and Lim et al. [23]. The above experiments have obtained numerous beneficial rules, but they exhibit limitations, which demonstrate that the dynamic interaction of the coupled system is complex. Moreover, theoretical analysis is of great importance to studying the coupled system.

In the existing seismic design methods, the dynamic interaction between the primary structure and the secondary structure is not fully understood. Instead of passively avoiding the disaster of the coupled system, it is better to understand the characteristics and rules of the dynamic interaction between the primary structure and the secondary structure. The energy absorption and distribution of the coupled system under seismic action can be changed through reasonable selection of design parameters, so as to make the coupled system become an interactive damping system. For the primary structure, the secondary structure can be used as tuned mass damper to reduce the response of the primary structure under seismic action. For the secondary structure, the primary structure can filter the seismic action so that the significant frequency of the filtered seismic wave is far away from the natural frequency of the secondary structure. Of course, these damping effects can only be obtained when appropriate design parameters are selected.

In order to discover potential methods of achieving interactive damping of the coupled system, three interactive damping strategies were proposed. For each strategy, a dynamic model was established. Moreover, according to the analysis of the three models, beneficial rules were summarized, which will provide a certain theoretical basis for the future seismic design or structural control [24] of the coupled system.

2. Three Interactive Damping Strategies

Based on the existing theoretical results and engineering design methods, we divided the interactive damping strategies of the primary-secondary coupled system into three categories:

Strategy 1: when the properties of the secondary structure remain unchanged, the primary structure parameters can be reasonably selected to reduce the dynamic response of the system

Strategy 2: when the properties of the primary structure and the secondary structure are not suitable to change, the location of the secondary structure can be reasonably selected to reduce the dynamic response of the system

Strategy 3: when the location of the secondary structure is fixed and the properties of the primary structure and the secondary structure are not suitable to change, the appropriate connection mode between them can be selected (to optimize connection structure parameters) to reduce the dynamic response of the system.

If the coupled system is designed as an interactive damping system, it is necessary to understand the dynamic interaction between the primary structure and the secondary structure, so as to grasp the influence rule of design parameters on the system response. The premise of these works is to establish efficient dynamic models that can reflect the actual characteristics of the coupled system. These models should have the following two points:

1. Capable of reflecting the essential characteristics of the system
2. The calculation process is not complex, but the calculation results of the interesting part should meet the precision

According to the characteristics of the research object and target, three dynamic models corresponding to three interactive damping strategies are established in this paper: 2-DOF series model (Figure 1), spatial dynamic model (Figure 2), and 3-DOF series model (Figure 3).

3. Analysis of Interactive Damping of Coupled System Based on Change in Primary Structure Parameters

3.1. Dynamic Model. A 2-DOF series model is selected. The schematic and force analysis of the model are illustrated in Figure 1.

3.2. Parameter Analysis under Random Earthquake Action. The intensity and spectrum characteristics of the stochastic ground motion are described by the power spectral density of the ground acceleration. The stationary-stochastic model proposed by Ou [25] is adopted, and its formula is as follows:

\[
S(w) = \frac{1}{(1 - w^2)/w_g^2 + (4\zeta_s^2 w^2/w_g^2) + (1 + w^2)/w_i^2} \frac{1}{S_0},
\]

where \(w_g\) is the characteristic circular frequency of the soil layer; \(\zeta_s\) is the characteristic damping ratio of the soil layer and is related to the soil hardness; and \(w_i = 8\pi; \) for hard surface soil, \(\xi_s = 0.63, w_i = 5\pi\) [26], and \(S_0 = 1\).

Frequency domain analysis is employed. By introducing \(t_2 = 2\zeta_s^2 w_i^2 w_i^2 + w_i^2\), the displacement, velocity, and acceleration variances of primary and secondary structures are derived. The results are as follows:

\[
H_1(w) = \frac{h_1 + \mu h_1 h_2 t_2}{\mu h_1 h_2 t_2 w^2 - 1},
\]

\[
H_2(w) = \frac{h_2 + h_1 h_2 w^2}{\mu h_1 h_2 t_2 w^2 - 1},
\]

\[
H'_1(w) = \frac{h'_1 + \mu h'_1 h_2 t_2}{\mu h'_1 h_2 t_2 w^2 - 1},
\]

\[
H'_2(w) = \frac{h'_2 + h'_1 h_2 w^2}{\mu h'_1 h_2 t_2 w^2 - 1},
\]

\[
h'_1(w) = \frac{1}{w_i^2 - w^2 + 2\xi_1 w_1 w_i + 2\mu \xi_3 w_2 w_i + \mu w_2^2},
\]
\[ h_n(w) = \frac{1}{w^2 - w^2 + 2\xi_n w \omega_n}, \quad (3b) \]

\[ \sigma_{u_n}^2 = \int_{-\infty}^{\infty} |H_n(w)|^2 S(w) \, dw, \quad (4a) \]

\[ \sigma_{u_n}^2 = \int_{-\infty}^{\infty} w^2 |H_n(w)|^2 S(w) \, dw, \quad (4b) \]

\[ \sigma_{u_n}^2 = \int_{-\infty}^{\infty} w^4 |H_n(w)|^2 S(w) \, dw, \quad (4c) \]

where \( n = 1, 2 \); \( \sigma_{u_n}^2 \) and \( \sigma_{u_n}^2 \) are the stationary displacement variances of the primary structure relative to the ground and the secondary structure relative to the primary structure, respectively; \( \sigma_{u_n}^2 \) and \( \sigma_{u_n}^2 \) are the stationary velocity variances of the primary and secondary structures relative to the ground, respectively; and \( \sigma_{u_n}^2 \) and \( \sigma_{u_n}^2 \) are the stationary acceleration variances of the primary structure and the secondary structures relative to the ground, respectively.

Considering that the secondary structure parameters are fixed, the frequency ratio \( \beta = \omega_2 / \omega_1 \) and mass ratio \( \mu = m_2 / m_1 \) of the primary and secondary structures are introduced. Moreover, the parameters affecting the response of the primary and secondary structures can be determined: frequency ratio \( \beta \), mass ratio \( \mu \), and damping ratio \( \xi_1 \) (difficult to control; not analyzed).

Using MATLAB, the calculation of the parameter variances of the primary and secondary structures is carried out, and the results are illustrated in Figure 4. During the calculation, the fixed parameters are as follows: \( m_2 = 200 \text{ kg}, \ k_2 = 1 \times 10^5 \text{ kN/m}, \ \omega_2 = 22.36 \text{ rad/s}, \ \xi_2 = 0.02, \)

Figure 1: Schematic and force analysis of 2-DOF series model.

Figure 2: Establishment of spatial dynamic model of primary-secondary coupled system.

Figure 3: Three-DOF series model.
\[ \xi_1 = 0.05. \] The change parameters are as follows: \( \beta \) (0–1.4), \( \mu \) (0.01–0.14), and frequency of power spectrum (0–50 rad/s).

It can be observed from the figure that the effect of the frequency and mass ratios on the displacement variance of the system and acceleration variance of the secondary structure is more regular. For the primary structure response, the mass ratio has little effect and its regularity is poor, while the frequency ratio has a greater effect and is more regular. However, the primary structure response is less robust [27] at a low frequency (approximately 0.5 to the limit). Compared to the primary structure, the effect of the frequency and mass ratios on the secondary structure is greater, and the effect of the mass ratio is more regular. A lower mass of the primary structure results in a smaller
response of the secondary structure. Avoiding tuning of the primary and secondary structures also reduces the secondary structure response. Therefore, optimizing the frequency ratio is more efficient for realizing the interactive damping coupled system (considering robustness in the low-frequency ratio simultaneously), and appropriate adjustment of the mass ratio will further control the secondary structure response.

3.3. Parameter Optimization. The analysis of Section 3.2 demonstrates that when the parameters (mass and frequency ratios) change, the change trend of the primary and secondary structure responses is not consistent. Based on the multiobjective optimization and combining the advantages of the weighted combination method and perfect point arithmetic, the selected objective function is

$$F(x) = \sum_{i=1}^{N} \omega_{i} \left[ \frac{f_{i}(x) - f_{i}^{*}}{f_{i}^{*}} \right]^{2}.$$

(5)

Considering that the equipment frequency is usually larger than that of the structure in practical engineering combined with the natural vibration period formula of the frame structure, which is $T_{i} = (0.08 - 0.10)n$ ($n$ is the layers of structure), the range of the frequency ratio is 0.2–0.8 and that of the mass ratio is 0.01–0.1. The weight coefficients are divided into three groups: 0.6 and 0.4, 0.5 and 0.5, and 0.4 and 0.6. The optimized mathematical model is

$$\text{min } F(x) = \sum_{i=1}^{2} \omega_{i} \left[ \frac{\sigma_{n_{i}}(x) - \sigma_{n_{i}}^{*}}{\sigma_{n_{i}}^{*}} \right]^{2}.$$  

(6)

By using MATLAB, the parameter optimization results are illustrated in Figure 5 (different colors represent different values of min $F(x)$, and the values for each color are listed at the right of each figure).

As can be observed from the figure, the optimal parameters of the three groups are as follows: $\beta = 0.6$ and $\mu = 0.1$, $\beta = 0.58$ and $\mu = 0.1$, $\beta = 0.56$ and $\mu = 0.1$. Moreover, it can be concluded that the mass ratio has less effect on the displacement variance of the primary and secondary structures compared to the frequency ratio. Therefore, in engineering design, it is more efficient and practical to change the primary structure frequency. Furthermore, it can be found that the optimal parameter exhibits an optimal frequency band and basically covers the mass ratio. Moreover, the optimized band will exhibit an uptrend; that is, when the frequency ratio is significantly greater than the optimal frequency ratio, optimization can still be achieved by increasing the mass ratio. At the same time, with an increase in the weight coefficient of the secondary structure, the alternative optimization scheme will be reduced.

In this section, the displacement variance of the primary and secondary structures is the response parameter. When the response parameter is velocity variance, the optimal combination is as follows: the mass ratio is 0.1, the frequency ratio is 0.2, and the mass ratio exhibits effective robustness.

4. Analysis of Interactive Damping of Coupled System Based on Plane Layout of Secondary Structure

4.1. Dynamic Model. Based on the substructure method, the floor and equipment are assembled to form a spatial dynamic model [28, 29] illustrated in Figure 2.

The dynamic equation of the final assembly space model is

$$M\ddot{u} + C\dot{u} + KU = -M\ddot{u}_{g}(t).$$  

(7)

In order to make the equation more general and avoid the calculation error caused by the matrix morbidity, the Suarez solutions are introduced [8], and the complex-modal method based on FOSS transform is used to solve the problem of nonclassical damping [30]. Finally, the frequency domain response of the system is determined as follows:

$$U = \Phi^{*}T \sum_{i=1}^{2N} A_{i}\psi_{i}^{T} \psi_{i} \Phi\psi_{i}^{*},$$  

(8a)

$$\Phi^{*} = \text{diag}(m_{1}^{-1/2}, m_{1}^{-1/2}, f_{1}^{1/2}, \ldots, m_{c1}^{-1/2}, m_{c1}^{-1/2}, \ldots),$$  

(8b)

where $T = [0 \ 1]$ and $\psi_{i}$ and $\psi_{i}^{*}$ are obtained from the complex-modal method based on FOSS transform.

4.2. Analysis of Plane Layout of Secondary Structure. The model is composed of one primary structure and one secondary structure. The fixed parameters of the secondary structure are $m = 1 \times 10^{3}$ kg, $k = 1 \times 10^{6}$ kN/m, $w = 31.6$ rad/s, and $c = 0.02$. The fixed parameters of the primary structure are $m = 2 \times 10^{3}$ kg and $c = 0.05$.

The floor coordinate diagram is illustrated in Figure 6, where $x_{c}$ and $y_{c}$ are the column coordinates and $x_{e}$ and $y_{e}$ are the secondary structure coordinates. The lateral stiffness of the x-axis and y-axis with three columns is $2 \times 10^{6}$ kN/m, and the stiffness of the fourth column (the lower right corner one of the columns) is adjusted to realize misalignment between the mass and stiffness centers of the primary structure.

The specific analysis parameters are primary structure eccentricity (whether eccentric, degree of eccentricity, and unidirectionally or bidirectionally eccentric) and the ground motion input (unidirectional or bidirectional input). The response parameters are the displacement variance of the primary structure relative to the ground and the displacement variance of the secondary structure relative to its placement.

The dimensions of the floor are $16 m \times 10 m$, the coordinate center is the floor geometric center, and the candidate placement area of the secondary structure is $14 m \times 8 m$. The ground motion input is the model used in Section 2. When the input ground motion is bidirectional, the amplitude ratio of the X and Y directions is 1 : 0.85. Using MATLAB, the system response can be obtained as
illustrated in Figures 7–10 ($X_R$ represents the degree of eccentricity of the primary structure, different colors represent different values of displacement variance of the $X$ direction of primary or secondary structure, and the values for each color are listed at the right of each figure).

The following conclusions can be drawn from the above figures. The input of the seismic motion determines whether the secondary structure plane layout is symmetrical. The primary structure eccentricity has little effect on the secondary structure plane layout, while the eccentric direction has a significant effect (particularly when the damping object is the primary structure). The seismic response of the primary and secondary structures has a complicated relationship with the primary structure eccentricity and the seismic motion input, particularly for the secondary structure seismic response. Therefore, optimizing the secondary structure plane layout is necessary for reducing the coupled system response.

4.3. Parameter Optimization. The secondary structure plane layout is optimized according to the multiobjective optimization method discussed in Section 3.3. Considering that the $y$-direction seismic response of the primary and secondary structure and the rotation of the primary structure of the coupled system are small under the action of unidirectional earthquake, the seismic response in the $x$-direction of the system is only considered when the objective function is set. Table 1 displays the objective function weight under different combinations. Optimization results under different combinations are illustrated in Figure 11 (different colors

![Figure 5: Optimization results of primary-secondary coupled interactive damping system under different weights. (a) Weight coefficients are 0.6 + 0.4. (b) Weight coefficients are 0.5 + 0.5. (c) Weight coefficients are 0.4 + 0.6.](image)

![Figure 6: Floor coordinate diagram.](image)
Figure 7: System response under bidirectional ground motion when mass center and stiffness of primary structure are coincident. (a) Displacement variance of the X direction of primary structure. (b) Displacement variance of the X direction of secondary structure.

Figure 8: System response under bidirectional ground motion when primary structure is unidirectionally eccentric. (a) Displacement variance of the X direction of primary structure ($X_R = 1.6$ m). (b) Displacement variance of the X direction of primary structure ($X_R = 2.7$ m). (c) Displacement variance of the X direction of secondary structure ($X_R = 1.6$ m). (d) Displacement variance of the X direction of secondary structure ($X_R = 2.7$ m).
Figure 9: System response under unidirectional ground motion when primary structure is unidirectionally eccentric. (a) Displacement variance of the $X$ direction of primary structure ($X_R = 1.6$ m). (b) Displacement variance of the $X$ direction of primary structure ($X_R = 2.7$ m). (c) Displacement variance of the $X$ direction of secondary structure ($X_R = 1.6$ m). (d) Displacement variance of the $X$ direction of secondary structure ($X_R = 2.7$ m).

Figure 10: System response under bidirectional ground motion when primary structure is bidirectionally eccentric. (a) Displacement variance of the $X$ direction of primary structure. (b) Displacement variance of the $X$ direction of secondary structure.
Table 1: Weight values of the coupled system’s seismic response under different combinations.

| Group | 1  | 2  | 3  | 4  | 5  | 6  |
|-------|----|----|----|----|----|----|
| Primary structure \((X + Y)\) | 0.5 | 0.3 | 0.3 | 0.3 | 0.5 | 0.5 |
| Primary structure \((\theta)\) | 0   | 0.2 | 0.2 | 0.2 | 0   | 0   |
| Secondary structure \((X + Y)\) | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

Figure 11: Optimization results under different combinations. (a) Group 1. (b) Group 2. (c) Group 3. (d) Group 4. (e) Group 5. (f) Group 6.
Figure 12: Continued.
represent different values of $\min F(x)$, and the values for each color are listed at the right of each figure).

The first group is bidirectional seismic input and not eccentric; the second and third groups are bidirectional seismic input and unidirectionally eccentric. The fifth and sixth groups are unidirectional seismic input and unidirectionally eccentric. The fourth group is bidirectional seismic input and unidirectionally eccentric.

The results demonstrate that bidirectional seismic input causes the optimization of the secondary structure plane layout to exhibit poor robustness, while unidirectional seismic input causes the secondary structure plane layout to have additional selection. The degree of eccentricity of the primary structure has little effect on the optimization of the secondary structure plane layout, and the eccentricity direction of the primary structure has a significant effect on the optimization of the secondary structure plane layout. The optimal location of the secondary structure is always towards the location of the primary structure stiffness center.

5. **Analysis of Interactive Damping of Coupled System Based on Connection Structure Parameters**

5.1. **Dynamic Model.** This section is intended to provide a preliminary understanding of the effect of the connection parameters on the coupled system response; thus, the 3-DOF series model illustrated in Figure 3 is used to represent the coupled system with the connection structure. Thereafter, with reference to Section 3 can be obtained three transfer functions of the primary structure relative to the ground, connection structure relative to the primary structure, and secondary structure relative to the connection structure [$h_p(w)$, $h_c(w)$, and $h_e(w)$].

5.2. **Parameter Analysis of Connection Structure.** The parameters of the connection structure usually refer to its mass, stiffness (frequency), and damping coefficient. In this
For the primary structure, \( m = 2 \times 10^4 \) kg, \( k = 8 \times 10^6 \) kN/m, \( \omega = 20 \) rad/s, and \( u = 0.05 \). For the secondary structure, \( m = 1 \times 10^3 \) kg, \( k = 1 \times 10^6 \) kN/m, \( \omega = 31.6 \) rad/s, and \( u = 0.02 \). For the connection structure, when the three parameters are fixed, the values are \( m_1 = 5 \times 10^2 \) kg, \( c_1 = 0.1 \), and \( \omega_1 = 18 \) rad/s. When the three parameters are variables, the values are \( m_2 = 2 \times 10^2 \) kg, \( 5 \times 10^2 \) kg, \( 1 \times 10^3 \) kg, \( 2 \times 10^3 \) kg, \( c_1 = 0.02, 0.05, 0.1, 0.2 \), and \( \omega_1 = 14 \) rad/s to 35 rad/s. Then, MATLAB is used to calculate the transfer functions, as illustrated in Figure 12.

Based on the analysis of nine function graphs, the following conclusions can be obtained. The increase in the connection structure frequency is advantageous to itself and not effective for the secondary structure. For the primary structure, there are both advantages and disadvantages, depending on the spectrum characteristics of the input seismic motion (the site condition). Increasing the damping coefficient of the connection structure is the preferred strategy for realizing interactive damping. The increase in the connection structure mass is advantageous to itself. For the primary structure, the effects of the connection structure mass and frequency are similar. For the secondary structure, an optimal connection structure mass exists, which can minimize the peak value of the transfer function of the secondary structure.

### 6. Conclusion

In this paper, three dynamic models are established under three damping strategies to investigate the dynamic interaction between the primary structure and the secondary structure, and the effects of system parameters on the system seismic response are analyzed. The conclusions are as follows:

1. For damping strategy 1, the mass ratio has little effect on the seismic response of the primary structure, and the larger the frequency ratio is, the smaller the displacement variance and velocity variance of the primary structure will be. The frequency ratio and mass ratio have great effect on the seismic response of the secondary structure. When the mass ratio is larger and the frequency ratio is far away from 1, the seismic response of the secondary structure is relatively small. The parameter combination to achieve the optimal system seismic response is the following: the frequency ratio is 0.55–0.6 and the mass of the primary structure is as small as possible.

2. For damping strategy 2, the input of seismic motion determines whether the optimal plane layout of the secondary structure is symmetrical. The bidirectional seismic input makes the optimal plane layout of the secondary structure less robust, while the unidirectional seismic input makes the plane layout of the secondary structure have more choices. The degree of eccentricity of the primary structure does not have great effect on the optimal plane layout of the secondary structure, while the eccentricity direction of the primary structure has great effect on it, and the optimal position of the secondary structure always tends to the rigid center of the primary structure.

3. For damping strategy 3, increasing the damping coefficient of the connection structure can effectively reduce the peak of the transfer function of the couple system. The increase of frequency and mass of the connection structure is beneficial to the connection structure itself; for the primary structure, there are both advantages and disadvantages, depending on the spectral characteristics of the input seismic motion (the site conditions); for the secondary structure, there is an optimal mass of the connection structure.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

This work was funded by the project of the National Natural Science Foundation of China (51875852) and Shaanxi Province Key Research and Development Program on Industry Innovation Chain (2018ZDCXL-SF-03-03-01).

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