Transmission and reflection of spin waves in the presence of Néel walls.

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Abstract. Within the framework of the continuum theory of micromagnetism the interaction of spin waves with domain walls is studied. By numerical simulations based on the finite element method a ferromagnetic Co stripe is modelled. The ground state is a homogeneous magnetization state on both sides of the stripe separated by a 180° Néel-type domain wall. On one side of the stripe monochromatic spin waves with well-defined frequency and propagation orientation are excited. At the domain wall the spin wave is reflected and/or transmitted. The corresponding fractions are analyzed. For wavelengths larger than the wall width we observe a full reflection of the spin waves while for smaller wavelengths the spin waves transmit by 100%. Depending on the frequency the phase of the spin waves can shift within a wide range.

1. Introduction

The interaction of spin waves (SWs) with domain walls represents a fundamental topic in magnetism. However there have been only few attempts so far to understand what happens with SWs propagating through a domain-wall.

Glock [1] studied in an analytical calculation the interaction of spin waves with 180° Bloch-type domain walls and analyzed in detail the reflectivity fraction of the SWs at the domain wall as a function of the \( k \)-vector. Furthermore, it was shown that a SW can transfer an impulse to a domain wall resulting in domain wall motion [1, 2]. Hertel et al. [3] demonstrated by micromagnetic numerical simulations that the phase of SWs with \( k \)-vector perpendicular to the domain wall changes by \( \pi/2 \) when the SW propagates in a thin ferromagnetic layer through a transverse wall. Bayer et al. [4] predicted in an analytical calculation the phase shift of a SW propagating through a 180° Bloch-type domain wall with \( k \)-vector perpendicular to the wall. The effect of the phase shift could be used in SW-interferometers, nonvolatile memories and logic gates [3, 5]. An approach based on the time-dependent Schrödinger equation was used to simulate SW propagation through a domain wall in a spin chain [6].

In the following chapters we briefly describe the micromagnetic background and the linearization suitable for calculating SWs. Then the simulation model is presented for the calculation of infinitely expanded SWs. This is supposed to be a good approximation for SWs with a wavefront of a width of a few nm. Results for the phase shift and the reflected and transmitted fractions of the SWs at a 180° Néel wall are given in the case of a Co stripe and compared to the analytical results obtained for 180° Bloch walls. As long as contributions of the dipolar interaction arise from the domain wall an analytical approach cannot be used.
Figure 1.
(a) Draft illustrating the configuration in the infinitely extended thin Co layer with the in-plane equilibrium configuration. The magnetization direction of the Néel wall in region 1 is shown at the position \( x=0 \). (b) Amplitude (\( j_x \) component of the magnetization) for a SW propagating through a 180° Néel wall in the middle of a 6 \( \mu \)m long stripe around 4 ns after excitation. The SW with \( k \)-vector perpendicular to the domain wall is excited at the crossover from region 2 to region 4 by a localized external field. In region 1 the SW is scattered at the Néel wall resulting in a transmitted and a reflected wave. In region 4 and 5 the absorbing boundary due to a damping constant of 0.08 is clearly visible.

2. Micromagnetic background
In the continuum theory of micromagnetism the dynamics of the normalized magnetic polarization \( j = J / J_s \) (\( J_0 \): saturation polarization) can be described by the Landau-Lifshitz-Gilbert equation

\[
\frac{dj}{dt} = -\gamma j \times H_{\text{eff}} + \alpha j \times \frac{dj}{dt}
\]  

(1)

with the gyromagnetic ratio \( \gamma \) and the damping constant \( \alpha \). Under the assumption of small deviations \( \delta j \) of the magnetic polarization from the equilibrium orientation \( j_0 \) eq.(1) can be linearized to

\[
\frac{d\delta j}{dt} = -\gamma j_0 \times (\delta H - \lambda \delta j) + \alpha j_0 \times \frac{d\delta j}{dt}.
\]  

(2)

The parameter \( \lambda = j_0 \cdot H_0 \) denotes the effective field of the equilibrium state \( H_0 \) in the direction of the magnetic polarization. The effective field for the deviation \( \delta j \) is given as

\[
\delta H = \frac{2A}{J_s} \Delta \delta j + \frac{2K_1}{J_s} \delta j_y e_y + \delta H_s
\]  

(3)

where the first term describes the exchange field with the exchange constant \( A \), the second term denotes the crystal anisotropy field in case of uniaxial anisotropy with anisotropy constant \( K_1 \) and the third term includes the stray field resulting from the magnetostatic equations

\[
\text{div} \delta H_s = -J_s \text{div} \delta j \quad \text{and} \quad \text{rot} \delta H_s = 0.
\]  

(4)

3. Numerical treatment
Eqs.(2)-(4) have been numerically treated for a one-dimensional Co stripe of length 6 \( \mu \)m (material parameters: \( J_0 = 1.8 \) T, \( A = 3 \cdot 10^{-11} \) J/m\(^2\), \( K_1 = 4 \cdot 10^5 \) J/m\(^3\)) in \( x \)-direction and of thickness \( d \) in \( z \)-direction. The stripe has been subdivided into five different regions. The first region is the domain wall region in which scattering of the SW takes place. On the left and on the right of the domain wall are the regions two and three with a homogeneous in-plane magnetization \( J = \pm J_0 e_y \) (Fig. 1(a)). To simulate an infinitely extended stripe absorbing boundaries need to be established to avoid reflections at the boundaries. This can be done by
Figure 2. Reflectivity $r$ (a) and phase shift (b) of a SW passing through a 180° Néel wall as a function of the wave vector $k$ for different film thicknesses. SWs with small wavelength pass the domain wall with a phase shift whereas SWs with larger wavelength are partially reflected at the domain wall.

Introducing a finite damping constant in regions 4 and 5 (e.g. $\alpha = 0.08$) where the SWs are then annihilated.

Eqs. (3)-(4) are treated quasi-one-dimensional except of the stray field calculation. The stripe is infinitely expanded in $y$-direction leading to an ideal Néel wall. A homogeneous magnetization profile within the stripe along the $z$-coordinate is assumed. Therefore the mesh can be discretized using two-dimensional finite elements. In this simulation quadrilaterals with first order polynomials with an element size of $\Delta x = 1.5$ nm and $\Delta z = d$ are used. A hybrid finite element/boundary element method [7] adapted to the stray field calculation of this sample configuration was implemented. The time domain was discretized using the stable implicit midpoint rule presented in [8]. Further details about the simulation code will be presented in a forthcoming publication. The equilibrium state $j_0(x)$ was achieved by dynamic micromagnetic simulation using a finite damping constant of $\alpha=0.5$.

On the left side of the absorbing boundary monochromatic plane waves are excited by a localized sinusoidal external field $H_x = H_0 \cdot \sin(\omega t)$. We assume that the damping constant $\alpha$ in regions 1, 2 and 3 is zero. Otherwise the propagating SWs may disappear before they reach the domain wall. As scattering takes place only in region 1, a small damping constant in this region may only have an insignificant effect. Dynamic simulation of eq. (2)-(4) leads after a few ns to the state shown in Fig. 1. (b) Then in regions 2 and 3 the conditions

$$j = I \cdot e^{ikx+i\omega t} + R \cdot e^{-ikx+i\omega t} \quad \text{(region 2)}$$
$$j = T \cdot e^{ikx+i\omega t} \quad \text{(region 3)}$$

are fulfilled. $I$ is the amplitude of the incoming wave, $R$ the amplitude of the reflected wave and $T$ the amplitude of the transmitted wave.

4. Results
All three amplitudes are obtained by Fourier transformation of the spin waves in regions 2 and 3 separately in time- and space-domain. Physical relevant properties are the reflectivity defined
as
\[ r = \frac{|R|^2}{|I|^2} = 1 - \frac{|T|^2}{|I|^2} \]  
and the phase shift \( \Delta \phi \) of the transmitted SW.

Fig. 2 (a) and (b) show the reflectivity and the phase shift of SWs, respectively, passing through a 180° Néel wall. Parameters are the wave vector \( k \) and the thickness \( d \) of the stripe. The finite stripe length of 6 \( \mu m \) limits the wavelength range in which SWs are adequately simulated to a wavelength smaller than 400 nm.

For wavelengths larger than the domain wall width (long-wave range) the SWs are partially reflected at the wall depending on the thickness of the stripe. For thin stripes the SWs show only a slight increase of the reflectivity with decreasing wave vector whereas for thick stripes the SWs can be totally reflected at the wall. The reason for the different reflectivity behavior is the increasing influence of the stray field with increasing thickness of the stripe.

Due to the complex domain wall barrier a resonance effect inside the domain wall is clearly visible for the 30 nm thick stripe leading to a second minimum in the reflectivity. SWs with a large wave vector \( k \gtrsim 0.1 \text{ nm}^{-1} \) are transmitted but show a phase shift after transmission. For the chosen sample the phase shift is nearly independent of the stripe thickness but tends to increase with increasing wave vector. In the intermediate range \((0.04 \text{ nm}^{-1} < k < 0.08 \text{ nm}^{-1})\) the phase changes drastically over a range of nearly \( 2\pi \) which is correlated with the change in the reflectivity \( r \).

In case of SWs propagating perpendicularly through a 180° Bloch wall (i. e. infinitely thick stripe) with the same boundary conditions as for Néel walls the reflectivity and the phase shift can be analytically calculated [1, 4] and leading to a reflectivity of \( r = 0 \) independent of the wave vector and a phase shift of
\[ \Delta \phi = 2 \cdot \arctan \sqrt{\frac{K_1/A}{k}}. \]  
Comparing the phase shift occurring for a a 180° Bloch wall with that of the 180° Néel wall for the short-wave range (Fig. 2), the slope of the phase shift is the same but the absolute phase shift differs by a factor of \( \pi \).

In conclusion, we found a significant influence of the thickness of a Co stripe on the reflectivity of SWs at 180° Néel walls. The fundamental differences between SWs propagating through Bloch and Néel walls were investigated. Within the theory of micromagnetism we have shown that the simulation model is suitable to calculate quantitatively the scattering of SWs at domain walls. The simulation model is well-suited for application to all kinds of domain walls.

5. References
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