Aerodynamic characteristics of a rotating sphere

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Abstract. Modern ballistic models, both internal and external, are amazingly accurate. This is due in no small part to the advances in technology that reduced tolerances in the guns and provided testing equipment that provided consistent and valid results. There are few variables. This cannot be said for the era of smooth-bore cannon using black powder and firing spherical projectiles. The number of variables, both internal and external, prohibits the use of modern ballistic models. Therefore, research into those weapon systems must be focused on one variable at a time to build a firm understanding for the creation of a suitable model. The discrepancy in the coefficient of aerodynamic drag of the sphere can be explained by the rotation of the ball and the Magnus effect. The maximum variance is ±5.6%. This means that when internal ballistic calculations of smooth-bore guns must determine the speed of rotation of the ball for use in the external ballistics calculations.

1. Introduction
A few years ago, a Database of Naval Artillery was published, the result of decades of research, to reconstruct the external ballistic parameters of rifled artillery. The compiled database now has over 5,000 guns [6]. However, a similar project regarding muzzle-loading smooth-bore artillery stumbled upon unexpected difficulty. It turned out that at present there is no reliable data on both the external and internal ballistics of these systems. For rifled artillery it was possible to use existing mathematical models, but for the smooth-bore systems existing models do not apply. In addition, it turned out that the historical data on internal ballistics available in the literature do not correspond to reality. They were uncritically transferred based on rifled artillery data; as a result, for example, the maximum pressure during firing was overestimated by an order of magnitude. However, the development and verification of models of muzzle-loaded rifled guns is confusing in the scientific community. Historians do not understand the mathematical calculations, and mathematical reasons for the development of models of technical systems two hundred years old. Therefore, we have tested individual elements of these models using the example of actual contemporary data. Thus, the external ballistics of the cannonball was derived from the problems of anti-terrorism protection [7].
In recent years, the use of Improvised Explosive Devices has provided an interesting exterior ballistics problem, to wit, the type of ‘shrapnel’ being projected by the explosive charge relates to its effectiveness. Spherical projectiles are the most dangerous filling of improvised explosive devices. Although the penetration properties of other fillings, such as nails and bolts, may be higher due to greater velocity, these parameters are unstable in flight and depend on the orientation of the axis relative to the obstacle encountered. And since standard nails and bolts do not have stabilization elements on the trajectory, the probability of perpendicularity of their axis of rotation and obstacle is extremely small. In contrast, spherical projectiles, such as ball bearings, have stable properties.

When calculating military and civilian objects for the impact of ‘shrapnel’ from improvised explosive devices, it is necessary to know the aerodynamic properties of these elements. An analysis of the literature has shown that, despite its simple form, the aerodynamics of a sphere differs according to different sources. One explanation for the difference in experiments may be the Magnus effect.

2. Existing calculation methods
The traditional method of assessing the penetration properties of ‘shrapnel’ is described in the work of A.V. Babkin, et al. [1]. It involves solving the equation of motion of the spherical projectiles:

\[ m \frac{dv}{dt} = -X = -c_x \frac{\rho v^2 S_m}{2}, \]

where \( X \) is the force of aerodynamic drag, \( m \) is the mass of the striking element, \( v \) is its speed, \( \rho \) is the density of air, \( S_m \) is the area of the mid-section, \( c_x \) is the coefficient of aerodynamic drag. Since the greatest danger is represented by striking elements having a flat trajectory, the influence of gravity is neglected. The vector of gravitational acceleration and the velocity vector in this case are mutually perpendicular.

As a rule, for simplicity, it is assumed that the density of the medium and the \( c_x \) of the fragment are constant, which makes it possible to separate the variables and solve the equation of motion analytically:

\[ v = v_0 e^{-Ax}, \]

where \( v_0 \) is the initial speed of the projectile; \( x \) is the distance from the epicenter of the explosion, \( A \) is the resistance function.

Since the penetration properties of fragments are determined by their mass and velocity at the instant of impact, this formula makes it possible to determine safe distances from the epicenter of the explosion or to find the characteristics of the barrier, which allows reliably protection from the ‘shrapnel’.

But the experimental data recommended by Babkin et al. [1] on the aerodynamic drag coefficient of a sphere are not the only ones. There are several alternative laws in the literature.
The first experiments to measure the aerodynamic resistance of the sphere were made in England in 1742 by Robins, who determined the air resistance using a ballistic pendulum. He used the same method in 1787–91. Hetton, making experiments with cannonballs. Since 1860, similar experiments with nuclei of different caliber have been carried out in different countries, using electrical devices for registration. Their results are compiled by the French ballistic Elie. Almost simultaneously, published his results of Journet of experiments with round bullets of 18.5 mm caliber [2]. On the other hand, in the USA, the law of the form “Sphere” was adopted, the data on the blowing of which are given, for example, in the work of Jurens [3].

From fig. 1 shows that the maximum degree of difference in the laws of resistance is observed in the range of small numbers M and reaches 20–30%. The most natural explanation is seen in the influence of the Magnus effect, since there is no information about measuring the speed of rotation of a sphere in the works of Elie and Journet. And in the data of Babkin et al. And in the “Sphere” law, the rotation is probably absent, since at present the least laborious method of measuring $c_x$ is to blow, where the rotation can be reduced to zero. Therefore, the difference between these data is not more than 4%, which is comparable with engineering accuracy.

3. Model to determine the effect of the Magnus effect

The standard method of carrying out a ballistic calculation assumes the absence of lift. In this case, the system of equations is as follows:

$$\frac{dv}{dt} = -\frac{X}{m} - g \sin \theta,$$

$$\frac{d\theta}{dt} = -\frac{g}{v} \cos \theta.$$ 

Here $\theta$ is the pitch angle, $g$ is the acceleration of gravity.

In the presence of rotation of the striking element, if the vector of angular velocity is perpendicular to the plane of fire, due to the Magnus effect a lifting force arises, the direction of which, depending on the direction of rotation, can be directed up or down. In this case, the equations of external ballistics for pitch are converted to the form:

$$\frac{d\theta}{dt} = \frac{Y}{mv} - \frac{g}{v} \cos \theta,$$

where $Y$ is the lift.

![Figure 2](image-url) Changing the flow pattern of a rotating sphere: speed 20 m/s, rotation 5 sec$^{-1}$ (a) speed 50 m/s, rotation 15 sec$^{-1}$ (b), speed 40 m/s, rotation 5 sec$^{-1}$ (c), speed 40 m/s, rotation 15 sec$^{-1}$ (d).
Therefore, the calculation scheme for determining the degree of influence of the Magnus effect will be as follows. The calculation is made taking into account the lifting force, after which the form factor is selected in the standard method, so that the range coincides. The degree of difference between the two form factors gives the desired effect.

To determine the aerodynamic characteristics of the rotating sphere, a series of calculations were performed in the ANSYS CFX package by a post-graduate Yu.V. Ganzy [4]; a typical calculation is shown in Fig. 2. The calculation results are summarized in fig. 3. As can be seen from fig. 3 (a), at a speed of 50 m/s, the difference in speed between 0 and 15 sec⁻¹ leads to a change in the aerodynamic drag force of 0.02 Newtons. It is clear that this value does not greatly affect the ballistics. But it is the emergence of lift that needs to be evaluated.

![Figure 3. Variation of the drag force on the speed and angular velocity of the sphere (a); change in lift from speed and angular velocity (b): 1 – 0 sec⁻¹, 2 – 5 sec⁻¹, 3 – 10 sec⁻¹, 4 – 15 sec⁻¹.](image)

4. Evaluation of lift effect
A sphere with a diameter of 40 mm was taken as the object of modeling in ANSYS. Taking into account the density of steel, it can be determined that the mass of the sphere is 0.26 kg. The calculation of the distance from these data was made by a student, B.A. Timirzyanov during the implementation of his thesis [5]. The calculation results are summarized in table. 1. Based on these data, a series of reverse ballistic calculations was performed to determine the shape factor that would correspond to the range.

The obtained data on the deviation of the form factor are summarized in Table. 2. As can be seen, the maximum deviation is 5.6%. This means that the rotation speed of the cannonball was greater than the calculated.

| $v$, m/s | $\omega$, 1/sec | $\theta_0$ | $\theta_0$ |
|----------|----------------|-----------|-----------|
|          | $10^\circ$     | $30^\circ$| $50^\circ$|
| 30       | 0  | 59.86 | 100.28 | 92.85 |
|          | 5  | 59.86 | 100.30 | 92.85 |
|          | 10 | 59.86 | 100.30 | 92.85 |
|          | 15 | 59.87 | 100.30 | 92.85 |
| 50       | 0  | 134.55| 278.71 | 293.28 |
|          | 5  | 134.62| 278.82 | 293.32 |
|          | 10 | 134.71| 278.95 | 293.33 |
|          | 15 | 134.77| 279.06 | 293.35 |
| 70       | 0  | 247.22| 605.11 | 715.60 |
|          | 5  | 247.45| 605.83 | 716.06 |
|          | 10 | 247.82| 607.02 | 716.74 |
|          | 15 | 248.10| 607.90 | 717.25 |
Table 2. The deviation of the shape factor due to the influence of the Magnus effect.

| $v$, m/s | $\omega$, 1/sec | $\theta_0$ | $\theta_0$ | $\theta_0$ |
|----------|-----------------|------------|------------|------------|
|          |                 | 10$^\circ$ | 30$^\circ$ | 50$^\circ$ |
| 30       | 10              | 0%         | 0,6%       | 0%         |
|          | 15              | 2,0%       | 0,6%       | 0%         |
| 50       | 10              | 5,0%       | 0,8%       | 0,1%       |
|          | 15              | 7,0%       | 1,1%       | 0,1%       |
| 70       | 10              | 3,8%       | 1,1%       | 0,3%       |
|          | 15              | 5,6%       | 1,5%       | 0,4%       |

5. Conclusions

1. Calculations showed that the deviation of the results of Elie and Journet can be explained by the influence of the Magnus effect. Probably, in them there were inaccuracies in measurements due to the imperfection of the measuring base too.

2. In the range of velocities up to 70 m/s, the maximum impact of the Magnus effect corresponds to maximum speeds and increases with increasing angular velocities. Most of all, the Magnus effect affects flat trajectories, and to a lesser extent on trajectories from a higher gun elevation. For a speed of 70 m/s, an angular speed of 15 sec$^{-1}$, and an initial pitch of 10$^\circ$, the change in the shape factor reaches 5.6%.

The indicated percentages are the maximum deviation of the shape factor, which can be either positive (the vector of angular velocity directed to the right of the plane of firing) or negative (the vector of angular velocity is directed to the left). In fact, these are two extreme cases, the rotation of the outgoing ball can occur in any plane. When departing from the muzzle, a cone of equiprobable nuclear positions arises, which was one of the reasons for the low accuracy.

It should be noted that this exploration of the Magnus effect on spherical ‘shrapnel’ also applies, at a different level, to the behavior of spherical projectiles fired from smooth bore artillery. However, due to the sheer number of variables involved, both interior and exterior, modern ballistics cannot reliably describe the ballistics of smooth bores.

At present, an approach is used in which the output of the internal ballistic calculation is used as input data for external ballistics. The results of the work show that the angular velocity of rotation of the cannonball should be an obligatory additional condition for smooth-bore guns.

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