Lorentz-invariant Bohmian description of inelastic scattering in QFT

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Abstract

The Lorentz-invariant S-matrix elements in interacting quantum field theory (QFT) are used to represent the QFT state by a Lorentz-invariant many-time wave function. Such a wave function can be used to describe inelastic scattering processes (involving particle creation and destruction) by Bohmian particle trajectories satisfying relativistic-covariant equations of motion.

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1 Introduction

The Bohmian formulation of quantum mechanics (QM) [1, 2, 3, 4, 5, 6] allows to visualize quantum processes in terms of continuous deterministic pointlike-particle trajectories guided by wave functions. In this formulation, all quantum uncertainties emerge from ignorance of the actual initial particle positions. Such a formulation not only offers a possible answer to deep conceptual and interpretational puzzles of QM, but in some cases provides also a powerful practically useful computational tool [3, 7].

A challenge for the Bohmian formulation of quantum theory is to reconcile continuous particle trajectories with phenomena of particle creation and destruction in quantum field theory (QFT). One possibility is to explicitly break the rule of continuous deterministic evolution, by adding an additional equation that specifies stochastic breaking of the trajectories [8, 9]. Another possibility is to introduce an additional continuously and deterministically evolving hidden variable that specifies effectivity of each particle trajectory [10, 11]. However, both possibilities seem rather artificial and contrived. In addition, the explicit constructions in [8, 9, 10, 11] do not obey Lorentz invariance.
Recently, a much simpler approach has been introduced [12], in which the particle trajectories are continuous, but the appearance of particle creation and destruction results from quantum theory of measurements describing entanglement with particle detectors. The QFT states are represented by many-time wave functions (which, as a byproduct, offers a new resolution of the black-hole information paradox [13]), implying that the corresponding Bohmian equations of motion for particle trajectories are relativistic-covariant. The equations determining the many-time wave functions are explicitly Lorentz-covariant for free fields. In the interacting case, however, the many-time evolution of wave functions in [12] is described by the time-evolution operators $\hat{U}(t)$ in the interaction picture, which lack manifest Lorentz invariance for interacting QFT.

In this paper we further develop some of the ideas introduced in [12]. In particular, we find a new method for calculation of the many-time wave function in interacting QFT, which turns out to be (i) simpler than that in [12] and (ii) manifestly Lorentz-invariant. In addition, we propose a new guiding equation for Bohmian trajectories which turns out to be much simpler than that in [12]. We also clarify some conceptual issues in a somewhat different (and hopefully more illuminating) way than in [12], and discuss some limitations of the present approach that represent a challenge for the future research.

The paper is organized as follows. In Sec. 2 we present a brief review of the Bohmian formulation of relativistic QM, as well as a general qualitative description of particle creation and destruction, with emphasis on conceptual (rather than technical) aspects. In Sec. 3 we describe in detail how states in free and interacting QFT can be represented by Lorentz-invariant many-time wave functions. In Sec. 4 we give a probabilistic interpretation of these wave functions and develop the corresponding Bohmian interpretation in terms of particle trajectories. The conclusions are drawn in Sec. 5.

In the paper we use units $\hbar = c = 1$ and the metric signature $(+, -, -, -)$.

## 2 Conceptual preliminaries

### 2.1 Outline of relativistic Bohmian mechanics

Since Bohmian mechanics (BM) is a nonlocal theory, it is frequently objected that it is not compatible with relativity. Yet, as reviewed in [14], all objections of that type can be circumvented. Relativistic-covariant nonlocal Bohmian equations of motion for particle trajectories have been introduced in [15] and further studied in [16, 17, 18]. Using the idea that, in relativistic QM, space probability density should be generalized to spacetime probability density (see, e.g., [19, 20]), a relativistic-invariant probabilistic interpretation (associated with relativistic particle trajectories) has been introduced in [20, 21]. As indicated in [12] and generally shown in [14], this makes the measurable statistical predictions of relativistic BM compatible with all measurable statistical predictions of the “ordinary” purely probabilistic interpretation of QM.

Let us present a brief overview of the main ideas of relativistic BM. Denoting a
point in spacetime by $x = \{x^\mu\}$, an $n$-particle quantum state is represented by a many-
time wave function $\psi(x_1, \ldots, x_n)$. This wave function lives in the $4n$-dimensional
configuration space with coordinates $x_0^a$, $a = 1, \ldots, n$, referred to as relativistic configu-
ration space. This is a many-time wave function [22], because each particle has
its own time-coordinate $x_0^a$. The quantity $\psi^* \psi$ is interpreted as probability density
in the relativistic configuration space, in the sense that the infinitesimal probability
d$P$ of finding $n$ particles in an infinitesimal $4n$-dimensional volume around the points
$x_1, \ldots, x_n$ is given by
\[
    dP = \psi^* (x_1, \ldots, x_n) \psi(x_1, \ldots, x_n) d^4 x_1 \cdots d^4 x_n.
\] (1)

The wave function satisfies the $n$-particle Klein-Gordon equation
\[
    \sum_{a=1}^n \left[ \partial_\mu \partial^\mu + m_a^2 \right] \psi = 0,
\] (2)

which implies that the $n$-particle Klein-Gordon current
\[
    J^\mu_a = \frac{i}{2} \psi^* \partial^\mu \psi
\] (3)
is conserved:
\[
    \sum_{a=1}^n \partial_\mu J^\mu_a = 0.
\] (4)

The Bohmian particle trajectories are integral curves of the vector field $V^\mu_a(x_1, \ldots, x_n)$ calculated from the wave function as
\[
    V^\mu_a = \frac{J^\mu_a}{\psi^* \psi}.
\] (5)

In the parameterized form, the integral curves are represented by functions $X^\mu_a(s)$ satisfying
\[
    \frac{dX^\mu_a(s)}{ds} = V^\mu_a(X_1(s), \ldots, X_n(s)).
\] (6)

From (6) and the fact that $\psi$ and does not have an explicit dependence on $s$, one finds the equivariance equation
\[
    \frac{\partial (\psi^* \psi)}{\partial s} + \sum_{a=1}^n \partial_\mu (\psi^* \psi V^\mu_a) = 0.
\] (7)

Formally, Eq. (7) shows that (6) is compatible with (1) in the following sense: If a statistical ensemble of particles has the probabilistic distribution (1) for some initial $s$, then it has the probabilistic distribution (1) for any $s$.

To understand the physical meaning of the formal equivariance equation (7), one needs to understand the physical meaning of the parameter $s$. For that purpose, it is useful to exploit the analogy with nonrelativistic Newtonian mechanics [23]. For nonrelativistic particle systems with conserved energy, the forces do not have an explicit dependence on time $t$. The only quantities that have a dependence on $t$ are
particle trajectories $X_i^j(t)$, $i = 1, 2, 3$. Thus, the parameter $t$ has a physical meaning only along trajectories; time without trajectories does not exist! In this sense, $t$ is only an auxiliary parameter that serves to parameterize the trajectories in 3-dimensional space, not a fundamental physical quantity by its own. Yet, a “clock” can measure time indirectly. Namely, a “clock” is nothing but a physical process described by a function $X_i^j(t)$ periodic in $t$. One actually observes the number of periods, and then interprets it as a measure of elapsed time.

The theory of relativity revolutionized the concept of time by replacing the parameter $t$ with a coordinate $x^0$ treated as a 4th dimension not much different from other 3 space dimensions. Yet, it does not mean that an auxiliary Newton-like time parameter is completely eliminated from relativistic mechanics. Such a parameter can still be introduced to parameterize relativistic spacetime particle trajectories in a manifestly covariant manner. This parameter, denoted by $s$, can be identified with a generalized proper time defined along particle trajectories of many-particle systems. The parameter $s$ can even be measured indirectly by a “clock” corresponding to a physical process periodic in $s$, in complete analogy with measurement of $t$ in nonrelativistic mechanics. As discussed in more detail in [23], this makes the parameter $s$ appearing in (6) a physical quantity, very much analogous to Newton time $t$. With this physical insight, the relativistic-covariant equation (7) is to be interpreted as physical probability conservation during the evolution parameterized by the evolution-parameter $s$.

Now let us sketch how all statistical predictions of the purely probabilistic interpretation of QM can be reproduced from relativistic BM (for more details see [14]). Let a physical system be described by a wave function

$$\psi(x) = \sum_b c_b \psi_b(x), \quad (8)$$

where $\psi_b(x)$ are eigenstates of some hermitian operator $\hat{B}$ on the Hilbert space of functions of $x$, normalized in a large but finite 4-dimensional box such that $\int d^4x \psi^*_b(x)\psi_b(x) = 1$. The purely probabilistic interpretation asserts that $|c_b|^2$ is the probability that the observable $B$ will take the value $b$. To see how BM reproduces this assertion, one needs to take into account the entanglement with the measuring apparatus. This leads to a wave function [14]

$$\psi(x, y) = \sum_b c_b \psi_b(x)E_b(y), \quad (9)$$

where $E_b(y)$ are detector wave functions normalized in the relativistic configuration space, with a negligible overlap in that space. Since (9) is compatible with (11), the probability that detector particles will have a relativistic configuration $Y$ in the support of $E_b(y)$ is equal to $|c_b|^2$ [14]. This shows that the measurable statistical predictions of relativistic BM coincide with those of purely probabilistic interpretation.

In particular, if $\psi_b(x)$ are (approximate) eigenstates of the space-position operators $x^1, x^2, x^3$ at time $x^0$, then the measurement procedure above (approximately) reproduces the usual space probability distribution $dP_{(3)} = \psi^*(x)\psi(x)d^3x$. 


2.2 The general mechanism of particle creation and destruction

In Sec. 2.1 we have studied relativistic QM with a fixed number $n$ of particles. Now we give a qualitative description of the general mechanism of particle creation and destruction in QFT. Schematically, it can be described in 3 steps.

In the first step, the initial state $|n_{\text{initial}}\rangle$ with a definite initial number of particles $n_{\text{initial}}$ suffers a unitary deterministic evolution in interacting QFT:

$$|n_{\text{initial}}\rangle \rightarrow \sum_n c_n |n\rangle,$$

(10)

where the final state is a superposition of states $|n\rangle$ with different numbers of particles.

In the second step, the quantum state above interacts with the environment (e.g., a particle detector), which causes a unitary evolution that creates entanglement with environment states $|E_n\rangle$:

$$\left[\sum_n c_n |n\rangle\right]|E_{\text{initial}}\rangle \rightarrow \sum_n c_n |n\rangle |E_n\rangle.$$

(11)

Here state $|E_n\rangle$ can be thought of as a macroscopic state describing a detector in a state of saying that $n$ particles are detected. Since different macroscopic states are macroscopically distinguishable, the corresponding wave functions $E_n(y)$ have a negligible overlap:

$$E_n(y)E_{n'}(y) \simeq 0 \text{ for } n \neq n'.$$

(12)

In the third step, one needs a mechanism that will pick up only one term in the superposition (11). Conventionally, it is usually described by the wave-function “collapse”. The role of the Bohmian formulation is to replace this ad hoc collapse with a mathematically better defined physical process. The wave function depending on $y$ guides the detector particles with a trajectory $Y(s)$. Due to (12), the particles enter only one channel $E_{n_{\text{final}}}(y)$ among many channels $E_n(y)$ in (11). This makes all other channels empty, which for all practical purposes is effectively the same as if the state exhibited a collapse:

$$\sum_n c_n |n\rangle |E_n\rangle \rightarrow |n_{\text{final}}\rangle |E_{n_{\text{final}}}\rangle.$$

(13)

(For more details see also [12].)

The effect of these 3 steps can be summarized as a transition

$$|n_{\text{initial}}\rangle |E_{\text{initial}}\rangle \rightarrow |n_{\text{final}}\rangle |E_{n_{\text{final}}}\rangle,$$

(14)

which typically involves the destruction of some initial particles and the creation of some new (final) ones.
3 QFT states represented by Lorentz-invariant wave functions

3.1 Momentum-eigenstates with fixed number of particles

Free QFT is usually formulated in terms of \( n \)-particle states \( |k_1, \ldots, k_n \rangle \) with on-shell 4-momenta \( k_a, a = 1, \ldots, n \). (For simplicity, we consider particles without spin, but spin can also be included as in [12].) Introducing the condensed notation \( \vec{k}^{(n)} = (k_1, \ldots, k_n) \), we denote these states as

\[
|n, \vec{k}^{(n)} \rangle \equiv |k_1, \ldots, k_n \rangle.
\] (15)

The state \( |n, \vec{k}^{(n)} \rangle \) can also be represented by an \( n \)-point wave function [12]

\[
\psi_{n, \vec{k}^{(n)}}(x_1, \ldots, x_n) = \langle x_1, \ldots, x_n | k_1, \ldots, k_n \rangle = b_n S_{\{x_1, \ldots, x_n\}} e^{-i k_1 x_1} \cdots e^{-i k_n x_n},
\] (16)

where \( b_n \) is a normalization factor and \( S_{\{x_1, \ldots, x_n\}} \) denotes symmetrization over all \( x_1, \ldots, x_n \). The normalization factor \( b_n \) is chosen such that [12]

\[
\int \mathcal{D} x^{(n)} |\psi_{n, \vec{k}^{(n)}}(\vec{x}^{(n)})|^2 = 1,
\] (17)

where \( \vec{x}^{(n)} = (x_1, \ldots, x_n) \) and

\[
\mathcal{D} \vec{x}^{(n)} = d^4 x_1 \cdots d^4 x_n.
\] (18)

3.2 General states in free QFT

A general free QFT state has an expansion of the form

\[
|\Psi\rangle = c_0 |0\rangle + \sum_{n=1}^{\infty} \sum_{\vec{k}^{(n)}} c_{n, \vec{k}^{(n)}} |n, \vec{k}^{(n)}\rangle,
\] (19)

where \( |0\rangle \) is the vacuum. We assume that the state is normalized to unity, in the sense that

\[
\langle \Psi | \Psi \rangle = |c_0|^2 + \sum_{n=1}^{\infty} \sum_{\vec{k}^{(n)}} |c_{n, \vec{k}^{(n)}}|^2 = 1.
\] (20)

Since (19) involves a superposition of states with various numbers \( n \) of particles, it is convenient to slightly modify the notation of Sec. 3.1. To distinguish particle positions \( x_a \) coming from sectors of different \( n \), instead of \( \vec{x}^{(n)} = (x_1, \ldots, x_n) \) we write

\[
\vec{x}^{(n)} \equiv (x_{n,1}, \ldots, x_{n,n}).
\] (21)

Then the collection of particle positions from sectors of all possible \( n \) is denoted as

\[
\vec{x} = (\vec{x}^{(1)}, \vec{x}^{(2)}, \ldots) = (x_{1,1}, x_{2,1}, x_{2,2}, \ldots).
\] (22)
The state (19) can be represented by a many-component wave function

$$\Psi(\vec{x}) = \begin{pmatrix} \tilde{\Psi}_0 \\ \tilde{\Psi}_1(\vec{x}^{(1)}) \\ \tilde{\Psi}_2(\vec{x}^{(2)}) \\ \vdots \end{pmatrix}.$$  \hspace{1cm} (23)

Here

$$\tilde{\Psi}_0 = \sqrt{\frac{1}{V}} c_0,$$ \hspace{1cm} (24)

$$\tilde{\Psi}_n(\vec{x}^{(n)}) = \sqrt{\frac{V^{(n)}}{V}} \tilde{\psi}_n(\vec{x}^{(n)}),$$ \hspace{1cm} (25)

are volumes of the corresponding relativistic configuration spaces, and

$$\tilde{\psi}_n(\vec{x}^{(n)}) = \sum_{\vec{k}^{(n)}} c_{n,\vec{k}^{(n)}} \psi_{n,\vec{k}^{(n)}}(\vec{x}^{(n)})$$ \hspace{1cm} (27)

are $n$-particle wave packets with $\psi_{n,\vec{k}^{(n)}}(\vec{x}^{(n)})$ given by (16). The tilde above wave functions denotes wave functions which are not necessarily normalized to unity. The normalization factors in (24)-(25) are chosen such that \[24\]

$$\int D\vec{x} |\tilde{\Psi}_0|^2 = |c_0|^2, \quad \int D\vec{x} |\tilde{\Psi}_n|^2 = \sum_{\vec{k}^{(n)}} |c_{n,\vec{k}^{(n)}}|^2,$$ \hspace{1cm} (28)

where

$$D\vec{x} = \prod_{n=1}^{\infty} D\vec{x}^{(n)}.$$ \hspace{1cm} (29)

This provides that the total wave function \[23\] is normalized to unity, in the sense that

$$\int D\vec{x} \Psi^\dagger(\vec{x})\Psi(\vec{x}) = \int D\vec{x} |\tilde{\Psi}_0|^2 + \sum_{n=1}^{\infty} \int D\vec{x} |\tilde{\Psi}_n|^2 = 1.$$ \hspace{1cm} (30)

Thus we see that the many-time wave function in free QFT is uniquely determined by a set of expansion coefficients $c_0$, $c_{n,\vec{k}^{(n)}}$.

### 3.3 Scattering wave function

In interacting QFT, particles may be created and destructed. The information on dynamics of this creation and destruction is encoded in the scattering matrix (shortly, $S$-matrix) with matrix elements in the momentum space (see, e.g., [24, 25, 26])

$$\langle k_1, \ldots, k_n|\hat{S}|p_1, \ldots, p_m \rangle \equiv S(n, \vec{k}^{(n)}|m, \vec{p}^{(m)}).$$ \hspace{1cm} (31)
These matrix elements are Lorentz-invariant. In general, the initial state before scattering is a superposition of the form (19) with expansion coefficients $c_{\text{in}}^{0}$, $c_{\text{in}}^{n,\vec{k}(n)}$. Then, the final state after scattering is also a superposition of the form (19), but with different expansion coefficients $c_{\text{out}}^{0}$, $c_{\text{out}}^{n,\vec{k}(n)}$ given by

$$c_{\text{out}}^{n,\vec{k}(n)} = \sum_{m} \sum_{\vec{p}(m)} S_{\text{in}}(n, \vec{k}(n)|m, \vec{p}(m)) c_{\text{in}}^{m,\vec{p}(m)}.$$ 

(Eq. (32) can be thought of as including the vacuum terms with $n = 0$ and $m = 0$, but they are not written explicitly because the S-matrix elements involving vacuum are usually trivial.) Thus, the total wave function can be written as

$$\Psi(\vec{x}) \simeq \begin{cases} \sqrt{V_{\text{in}}} \Psi_{\text{in}}(\vec{x}) & \text{for } \vec{x} \in \mathcal{R}_{\text{in}}, \\ \sqrt{V_{\text{out}}} \Psi_{\text{out}}(\vec{x}) & \text{for } \vec{x} \in \mathcal{R}_{\text{out}}, \end{cases}$$

(33)

where $\mathcal{R}_{\text{in}}$ and $\mathcal{R}_{\text{out}}$ are in and out regions of the relativistic configuration space with volumes $V_{\text{in}}$ and $V_{\text{out}}$, respectively. The wave functions $\Psi_{\text{in}}(\vec{x})$ and $\Psi_{\text{out}}(\vec{x})$ are defined as in Sec. 3.2 with coefficients $c_{\text{in}}^{0}$, $c_{\text{in}}^{n,\vec{k}(n)}$ and $c_{\text{out}}^{0}$, $c_{\text{out}}^{n,\vec{k}(n)}$, respectively. The normalization factors in (33) are chosen such that $\int \mathcal{D}\vec{x} \Psi^\dagger(\vec{x})\Psi(\vec{x}) = 1$.

Let us make a few comments on validity of the approximation (33). Strictly speaking, the S-matrix elements (31) refer only to in states $|p_1,\ldots,p_m\rangle$ at $x_0^a \to -\infty$ and out states $|k_1,\ldots,k_m\rangle$ at $x_0^a \to \infty$. Nevertheless, they also represent a good approximation for large but finite in and out times. After all, the predictions obtained from these matrix elements are in excellent agreement with experiments, and no experiments are performed at infinity. In fact, the S-matrix elements represent a good approximation wherever particles can be well approximated by free particles. But this means that the approximation (33) is good almost everywhere, except in a very small region of spacetime at which the collision of localized wave packets actually happens. For relativistic collisions, the size of this small region of collision is typically of the order $1/E$, where $E$ is a typical energy of the colliding particles. (For decay processes, $E$ is the decay width, which is large for short-living particles.) Thus, we can conclude that, in a typical situation, (33) is a good approximation almost everywhere, i.e., that $\mathcal{R}_{\text{in}} \cup \mathcal{R}_{\text{out}}$ covers almost the whole relativistic configuration space.

Let us compare it with the results of [12]. In principle, the method developed in [12] allows to calculate $\Psi(\vec{x})$ everywhere, but is not manifestly Lorentz-invariant. By contrast, the method developed here is manifestly Lorentz-invariant, but is not valid everywhere. It seems that it should be possible to develop a method that is both valid everywhere and manifestly Lorentz-invariant, but we leave the development of such a method as a program for the future research.

4 Probability and particle trajectories in QFT

In this section we develop the physical interpretation of the wave function $\Psi(\vec{x})$, as a natural generalization of the results in Sec. 2.1. The probabilistic interpretation (11)
generalizes to
\[ \mathcal{D} P = \Psi^\dagger(\vec{x}) \Psi(\vec{x}) \, \mathcal{D} \vec{x}. \] (34)

Due to (23), the probability density \( \Psi^\dagger \Psi \) decomposes as
\[ \Psi^\dagger(\vec{x}) \Psi(\vec{x}) = |\tilde{\Psi}_0|^2 + \sum_{n=1}^{\infty} |\tilde{\Psi}_n(\vec{x}^{(n)})|^2. \] (35)

Each \( n \)-particle wave function \( \tilde{\Psi}_n(\vec{x}^{(n)}) \) satisfies the \( n \)-particle Klein-Gordon equation of the form of (2).

For further analysis, it is convenient to introduce a condensed label \( A = (n, a_n) \), such that (22) can be written as
\[ \vec{x} = \{ x_A \} = (x_1, x_2, x_3, \ldots). \] (36)

With this notation, we introduce the current
\[ J_A^\mu = \frac{i}{2} \Psi^\dagger \partial_A^\mu \Psi, \] (37)

which generalizes (3). Due to (35), it can be decomposed into a collection of \( n \)-particle currents, each of which is conserved due to the \( n \)-particle Klein-Gordon equation. This implies that (37) is also conserved
\[ \sum_{A=1}^{\infty} \partial_A^\mu J_A^\mu = 0, \] (38)

which implies a generalization of (7)
\[ \frac{\partial(\Psi^\dagger \Psi)}{\partial s} + \sum_{A=1}^{\infty} \partial_A^\mu (\Psi^\dagger \Psi V_A^\mu) = 0, \] (39)

where
\[ V_A^\mu = \frac{J_A^\mu}{\Psi^\dagger \Psi} \] (40)

generalizes (5). The Bohmian particle trajectories are given by the generalization of (6)
\[ \frac{dX_A^\mu(s)}{ds} = V_A^\mu(X_1(s), X_2(s), X_3(s), \ldots), \] (41)

which are compatible with (34) due to the equivariance equation (39).

It is straightforward to apply this to the scattering wave function (33), which, together with physical insights from Sec. 2.2, provides a Bohmian description of inelastic scattering processes involving particle creation and destruction.
5 Conclusion

The Bohmian formulation of quantum theory describes all quantum processes in terms of continuous deterministic particle trajectories guided by wave functions. The results of the present paper show that it can be formulated in a form which (i) obeys manifest Lorentz invariance and (ii) includes a description of particle creation and destruction in QFT. (For simplicity, in this paper we have discussed only particles without spin, but, by using the results presented in [12], the generalization to particles with spin is relatively simple.) These results reinforce the view that the Bohmian formulation is a viable formulation of quantum theory.

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