ACDM-type cosmological model and observational constraints

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Abstract

In the present work, we have searched the existence of ACDM-type cosmological model in anisotropic Heckmann-Schucking space-time. The matter source that is responsible for the present acceleration of the universe consist of cosmic fluid with \( p = \omega \rho \), where \( \omega \) is the equation of state parameter. The Einstein’s field equations have been solved explicitly under some specific choice of parameters that isotropizes the model under consideration. It has been found that the derived model is in good agreement with recent SN Ia observations. Some physical aspects of the model has been discussed in detail.

Keywords: Heckmann-Schucking metric, dark energy, ACDM model

1 Introduction

The SN Ia observations \([1,2]\) suggest that the observable universe is undergoing an accelerated expansion. This remarkable discovery stands a major break through of the observational cosmology and indicates the presence of unknown fluid - dark energy (DE) that opposes the self attraction of the matter. This acceleration is realized with positive energy density and negative pressure. So, it violate the strong energy condition (SEC). The authors of ref. \([3]\) confirmed that the violation of SEC gives a reverse gravitational effect that provides an elegant description of transition of universe from de-celeration to cosmic acceleration. The cosmological constant cold dark matter (ΛCDM) cosmological model is the simplest model of universe that describes the present acceleration of universe and fits with the present day cosmological data \([4]\). It is based on the Einstein’s theory of general relativity with a spatially flat, isotropic and homogeneous space-time. The observed acceleration of universe has been explained by introducing a positive cosmological constant Λ which is mathematically equivalent to vacuum energy with equation of state (EOS) parameter set equal to \(-1\). It suffers from two problems on the theoretical front, concerning the cosmological constant (Λ). These problems are known as fine tuning and cosmic coincidence problems \([5,6]\). In the contemporary cosmology, the source that derives the present acceleration of universe is still mystery and is discussed under the generic name DE. In the literature, the simplest candidate of dark energy is a positive Λ besides some scalar field DE models, namely the phantom, quintessence and k-essence \([6,7]\). In the physical cosmology, the dynamical form of DE with an effective equation of state (EOS), \( \omega < -\frac{1}{3} \), were proposed instead of constant vacuum energy. The current cosmological data from large scale-structure \([8]\), Supernovae Legacy survey, Gold Sample of Hubble Space Telescope \([9,10]\) do not support the possibility of \( \omega << -1 \). However, \( \omega = -1 \) is a favorable candidate for DE that crossing the phantom divide line (PDL). Setare and Saridakis \([11,12]\) have studied the quintom model that described the nature of DE with \( \omega \) across -1 and give the concrete theoretical justification for existence of quintom model
We notice that after publication of WMAP data, today there is considerable evidence in support of anisotropic model of universe. On the theoretical front, Misner \[13\] has investigated an anisotropic phase of universe, which turns into isotropic one. The authors of ref. \[14, 15\] have investigated the accelerating model of universe with anisotropic EOS parameter and have also shown that the present SN Ia data permits large anisotropy. Recently DE models with variable EOS parameter in anisotropic space-time have been studied by Yadav and Yadav \[16\], Yadav et al \[17, 18\], Akarsu and Kilinc \[19\], Yadav \[20\], Saha and Yadav \[21\] and Pradhan \[22\]. In the present work, however, we present ΛCDM-type cosmological model in spatially homogeneous and anisotropic Heckmann-Schucking space-time.

The outline of paper is as follows: in section 2, the field equation and it’s solution are described. Section 3 deals with dust filled universe and Hubble’s parameter. Section 4 covers the study of observational parameters for the model under consideration. The deceleration parameter (DP) and certain physical properties of the universe are presented in section 5 and 6 respectively. Finally conclusions are summarized in section 7.

2 Field equations

We consider a general Heckmann-Schucking metric
\[ ds^2 = c^2 dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \] (1)

where A, B and C are functions of time only. we consider energy momentum tensor for a perfect fluid i.e.
\[ T_{ij} = (p + \rho) u_i u_j - p g_{ij} \] (2)

where \( g_{ij} u^i u^j = 1 \) and \( u^i \) is the 4-velocity vector.

In co-moving co-ordinates
\[ u^\alpha = 0, \quad \alpha = 1, 2, 3. \] (3)

The Einstein field equations are
\[ R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4} T_{ij} \] (4)

Choosing co-moving coordinates, the field equations (4) in terms of line element (1) can be write down as
\[ \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -\frac{8\pi G}{c^2} p + \Lambda c^2 \] (5)

\[ \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -\frac{8\pi G}{c^2} p + \Lambda c^2 \] (6)

\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -\frac{8\pi G}{c^2} p + \Lambda c^2 \] (7)

\[ \frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{AC} = \frac{8\pi G}{c^2} \rho + \Lambda c^2 \] (8)

where \( A_4, B_4 \) and \( C_4 \) stand for time derivatives of A, B, and C respectively.

The mass-energy conservation equation \( T_{ij}^{\alpha\beta} = 0 \) gives
\[ \rho_4 + (p + \rho) \left( \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \] (9)

Subtracting eqs.(5) from (6), (6) from (7) and (7) from (6), we obtain
\[ \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{A_4 C_4}{AC} - \frac{B_4 C_4}{BC} = 0 \] (10)
\[
\begin{align*}
\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4 B_4}{A B} - \frac{A_4 C_4}{A C} &= 0 & (11) \\
\frac{C_{44}}{C} - \frac{A_{44}}{A} + \frac{B_4 C_4}{B C} - \frac{A_4 B_4}{A B} &= 0 & (12)
\end{align*}
\]

Subtracting (12) from (10), we get
\[
\frac{B_{44}}{B} + \frac{C_{44}}{C} + 2 \frac{B_4 C_4}{B C} = 2 \frac{A_4}{A} + \frac{A_4 C_4}{A C} + \frac{A_4 B_4}{A B} & (13)
\]

This equation can be re-written in the following form
\[
\left( \frac{(BC)_4}{BC} \right)^4 + \left( \frac{(BC)_4}{BC} \right)^2 = 2 \left( \frac{A_4}{A} \right)^4 + 2 \frac{A^2_4}{A^2} + \frac{A_4 (BC)_4}{ABC} & (14)
\]

Integrating this equation, we get the following first integral
\[
\left( \frac{(BC)_4}{BC} - \frac{2 A_4}{A} \right) A B C = L & (15)
\]

where L is constant of integration.

The exact solution of eq. (15), in general, is not possible however one can solve eq. (15) explicitly, by choosing \( L = 0 \) that reveals \( A^2 = BC \). The present day observations suggest that the initial anisotropy dissipated out for large value of time and the directional scale factors have same values in all direction i.e. \( A = B = C \) which is easily obtained by putting \( L = 0 \) in eq. (15). That is why \( L = 0 \) has physical meaning.

Now we can assume
\[
B = AD & (16) \\
C = AD^{-1} & (17)
\]

where
\[
D = D(t) & (18)
\]

Further integrating equation (11), we get the first integral
\[
\frac{D_4}{D} = \frac{K}{A^3} & (19)
\]

where \( K \) is an arbitrary constant of integration.

With help of equation (15), eq (9) simplifies as
\[
\rho_4 + 3 A_4 \left( p + \rho \right) = 0 & (20)
\]

Thus the Hubble’s parameter in this model is
\[
H = \nu_i^i = \frac{1}{3} \left( \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = \frac{A_4}{A} & (21)
\]

Equations (5)-(8) are simplified as
\[
2 \frac{A_{44}}{A} + \frac{A^2_4}{A^2} + \frac{K^2}{A^6} = - \frac{8 \pi G}{c^2} p + \Lambda c^2 & (22)
\]

Taking \( K = 0 \), we obtain the Einstein’s field equation for spatially homogeneous and isotropic flat FRW model as
\[
\frac{A^2_4}{A^2} - \frac{K^2}{3 A^6} = \frac{8 \pi G}{3 c^2} p + \frac{\Lambda c^2}{3} & (23)
\]
\[ 2 \frac{A_{44}}{A} + \frac{A_2^4}{A^2} = -\frac{8\pi G}{c^2} p + \Lambda c^2 \]  
(24)

\[ \frac{A_2^4}{A^2} = \frac{8\pi G}{3c^2} \rho + \frac{\Lambda c^2}{3} \]  
(25)

where \( A \) is expansion scale factor.

Thus equations (21)-(23) may be regarded as counterpart of FRW Equations in our anisotropic model. Equations (22) and (23) may be re-written as

\[ 2 \frac{A_{44}}{A} + \frac{A_2^4}{A^2} = -\frac{8\pi G}{3c^2} \left( p - \frac{\Lambda c^4}{8\pi G} + \frac{K^2 c^2}{8\pi GA^6} \right) \]  
(26)

\[ H^2 = \frac{A_2^2}{A^2} = \frac{8\pi G}{3c^2} \left( \rho + \frac{\Lambda c^4}{8\pi G} + \frac{K^2 c^2}{8\pi GA^6} \right) \]  
(27)

We now assume that the cosmological constant \( \Lambda \) and the term due to anisotropy also act like energies with densities and pressures as

\[ \rho_\Lambda = \frac{\Lambda c^4}{8\pi G} \]  

\[ \rho_\sigma = \frac{K^2 c^2}{8\pi GA^6} \]  

\[ p_\Lambda = -\frac{\Lambda c^4}{8\pi G} \]  

\[ p_\sigma = \frac{K^2 c^2}{8\pi GA^6} \]  
(28)

It can be easily verified that energy conservation law (20) holds separately for \( \rho_\Lambda \) and \( \rho_\sigma \) i.e.

\[ (\rho_\Lambda)_4 + 3H(p_\Lambda + \rho_\Lambda) = 0 \]

\[ (\rho_\sigma)_4 + 3H(p_\sigma + \rho_\sigma) = 0 \]

The equations of state for matter, \( \sigma \) and \( \Lambda \) energies are as follows

\[ p_m = \omega_m \rho_m \]  
(29)

where \( \omega_m = 0 \) for matter in form of dust, \( \omega_m = \frac{1}{3} \) for matter in form of radiation. There are certain more values of \( \omega_m \) for matter in different forms during the course of evolution of the universe.

\[ p_\Lambda = \omega_\Lambda \rho_\Lambda \]  
(30)

Since

\[ p_\Lambda + \rho_\Lambda = 0 \]

Therefore

\[ \omega_\Lambda = -1 \]

Similarly

\[ p_\sigma = \rho_\sigma \]

So,

\[ \omega_\sigma = 1 \]

Now we use the following relation between scale factor \( A \) and red shift \( z \)

\[ \frac{A_0}{A} = 1 + z \]  
(31)
The suffix(0) is meant for the value at present time.  

The energy density $\rho$ comprises of following components

$$\rho = (\rho_m + \rho_\Lambda + \rho_\sigma)$$  \hfill (32)

Equations (20) and (31) yield

$$\rho = \sum_i (\rho_i)_0 (1 + z)^{3(1 + \omega_i)}$$  \hfill (33)

Equations (26) and (27) take the form

$$2 \frac{A_{44}}{A} + H^2 = \frac{-8 \pi G}{c^2} (p_m + p_\Lambda + p_\sigma)$$  \hfill (34)

$$H^2 = \frac{8 \pi G}{3c^2} (\rho_m + \rho_\Lambda + \rho_\sigma)$$  \hfill (35)

Figure 1: Hubble constant vs redshift & scale function.

3 Dust filled universe

For dust filled universe, we have $p_m = 0$ and $\rho_m = \frac{3c^2 H^2}{8 \pi G}$  

Then eq. (35) gives

$$\Omega_m + \Omega_\Lambda + \Omega_\sigma = 1$$  \hfill (36)

where $\Omega_m = \frac{\rho_m}{\rho_c} = \frac{(\Omega_m)_0 H_0^2 (1+z)^3}{H^2}$, $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{(\Omega_\Lambda)_0 H_0^2}{H^2}$ and $\Omega_\sigma = \frac{\rho_\sigma}{\rho_c} = \frac{(\Omega_\sigma)_0 H_0^2 (1+z)^6}{H^2}$

3.1 Expression for Hubble’s Constant

Equations (31) and (35) yields

$$H^2 = H_0^2 \left[ (\Omega_m)_0 (1+z)^3 + (\Omega_\sigma)_0 (1+z)^6 + (\Omega_\Lambda)_0 \right] = H_0^2 \left[ (\Omega_m)_0 (\frac{A_0}{A})^3 + (\Omega_\sigma)_0 (\frac{A_0}{A})^6 + (\Omega_\Lambda)_0 \right]$$  \hfill (37)

The behaviour of Hubble’s parameter versus redshift and scale function have been depicted in Figure 1. We notice from Hubble’s parameter versus redshift that it increases with redshift while it decreases with scale function. From the right panel of figure (1), it is also clear that for $\Lambda$ dominated universe, the Hubble’s parameter is either almost stationary or it is decreasing slowly with scale function i.e. time.
4 Some observational constraints

The luminosity distance which determines flux of the source is given by

\[ D_L = A_0 x (1 + z) \]  

(38)

where \( x \) is the spatial co-ordinate distance of a source.

The Geodesic for metric (1) ensures that if in the beginning \( \frac{dy}{ds} = 0; \frac{dz}{ds} = 0 \) then \( \frac{d^2y}{dx^2} = 0; \frac{d^2z}{dx^2} = 0 \).

So if a particle moves along \( x \)- direction, it continues to move along \( x \)- direction always. If we assume that line of sight of a vantage galaxy from us is along \( x \)- direction then path of photons traveling through it satisfies

\[ ds^2 = c^2 dt^2 - A^2 dx^2 = 0 \]  

(39)

From this we obtain

\[ x = \int_0^x dx = \int_t^{t_0} \frac{dt}{A(t)} = \frac{1}{A_0 H_0} \int_0^z \frac{dz}{h(z)} \]

(40)

where we have used \( dt = dz/\dot{z} \) and from eqs. (21) and (31)

\[ \dot{z} = -H(1 + z) \]

\[ h(z) = \frac{H}{H_0} \]

So, the luminosity distance is given by

\[ D_L = \frac{c(1 + z)}{H_0} \int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0 (1 + z)^3 + (\Omega_\sigma)_0 (1 + z)^6 + (\Omega_\Lambda)_0]}} \]  

(41)

4.1 Apparent Magnitude and Red Shift relation:

The absolute magnitude \((M)\) and apparent magnitude \((m)\) are related to the redshift by following relation

\[ m - M = 5 \log_{10} \left( \frac{D_L}{Mpc} \right) + 25 \]  

(42)

For low redshift, one can easily obtain the luminosity distance from eq. (41)

\[ D_L = \frac{cz}{H_0} \]  

(43)

Combining equations (41), (42) and (43), one can easily obtain the expression for the apparent magnitude in terms of redshift parameter \((z)\) as follows

\[ m = M + 5 \log_{10} \left( \frac{c(1 + z)}{H_0 Mpc} \right) \int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0 (1 + z)^3 + (\Omega_\sigma)_0 (1 + z)^6 + (\Omega_\Lambda)_0]}} \]  

(44)

Figure 2 shows the behaviour of luminosity distance \((D_L)\) and apparent magnitude \((m)\) with redshift for some certain values of \(\Omega_m, \Omega_\Lambda\) and \(\Omega_\sigma\).

In the present analysis, we use 60 data set of SN Ia for the low red shift \((z < 0.5)\) as reported by Perlmutter et al. [2]. In this case \(\chi^2_{SN} \) has been computed according to the following relation
\[ \chi^2_{SN} = A - \frac{B^2}{C} + \log_{10}(\frac{C}{2\pi}) \]

\[ A = \sum_{i=1}^{60} \frac{[(D_{L})_{ob} - (D_{L})_{th}]^2}{\sigma_i^2} \]

\[ B = \sum_{i=1}^{60} \frac{[(D_{L})_{ob} - (D_{L})_{th}]^2}{\sigma_i^2} \]

\[ C = \sum_{i=1}^{60} \frac{1}{\sigma_i^2} \]

| $(\Omega_m)_0$ | $(\Omega_\Lambda)_0$ | $(\Omega_\sigma)_0$ | $\chi^2_{SN}$ | $\chi^2_{SN}/dof$ |
|----------------|---------------------|---------------------|--------------|-----------------|
| .29            | .69                 | .02                 | 7.41417      | 0.1257          |
| .98            | 0                   | .02                 | 8.9135       | 0.1511          |
| 0              | .98                 | .02                 | 7.8437       | 0.1329          |

Table: 1

Here, dof stands for degree of freedom. From the table 1, we note that the best fit values of $(\Omega_\Lambda)_0 = 0.69$ with $\chi^2_{SN} = 7.41417$ and the reduced $\chi^2$ value is 0.1257.

### 4.2 Age of the Universe

The present age of the universe is obtained as follows

\[ t_0 = \int_0^{t_0} dt = \int_0^{\infty} \frac{dz}{H_0(1+z)\sqrt{[((\Omega_m)_0(1+z)^3 + (\Omega_\sigma)_0(1+z)^6 + (\Omega_\Lambda)_0]}} \]

(45)

where we have used $dt = dz/\dot{z}$ and $\dot{z} = -H(1+z)$

The left panel of figure 3 shows the variation of time with redshift. It is also observed that the $\Lambda$ dominated universe gives the age of the universe as $H_0 t_0 \approx 0.9$

Since, $H_0^{-1} = 1.3574e + 010$

⇒ $t_0 = 0.9(1.3574e + 010) Gyrs = 12.30891 Gyrs$.

From WMAP data, the empirical value of present age of universe is $13.73^{+1.13}_{-1.17}$ which is very close to present age of universe, estimated in the derived model.
5 Deceleration Parameter

The deceleration parameter is given by

\[ q = -\frac{A_{44}}{4AH^2} = -\frac{AA_{44}}{A^2_4} \]  

(46)

From equation (34)

\[ -2q + 1 = -3\sum_i \omega_i \Omega_i \]

\[ -2q = 3\Omega_\Lambda - 3\Omega_\sigma - 1 \]

This equation clearly shows that without presence of \( \Lambda \) term in the Einstein’s field equation (41), one can’t imagine of accelerating universe. This equation also expresses the fact that anisotropy raises the lower limit value of \( \Lambda \) required for acceleration. This may be seen in the following way.

For FRW model, acceleration requires

\[ \Omega_\Lambda \geq 0.33 \]  

(47)

where as for anisotropic model

\[ \Omega_\Lambda \geq 0.33 + \Omega_\sigma \]  

(48)

Combining equations (34), (35), (36) and (37), the expression for DP in terms of redshift \( z \) is given by

\[ q = \frac{3}{2} \left( \frac{(\Omega_m)_0(1+z)^3 + 2(\Omega_\sigma)_0(1+z)^6}{(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0 + (\Omega_\sigma)_0(1+z)^6} \right) - 1 \]  

(49)

Since in the derived model, the best fit values of \( (\Omega_m)_0, (\Omega_\Lambda)_0 \) and \( (\Omega_\sigma)_0 \) are 0.29, 0.69 and 0.02 respectively hence we compute the present value of DP for derived ΛCDM universe by putting \( z = 0 \) in eq. (49). The present value of DP is given by

\[ q_0 = -0.505 \]  

(50)

6 Some Physical Properties of the Model

6.1 The energy density in the universe

The energy density \( \rho \) is given by

\[ \rho = \sum_i (\rho_i)_0(1+z)^{3(1+\omega_i)} \]  

(51)

where

\[ (\rho_i)_0 = \frac{3c^2H_0^2}{8\pi G}(\Omega_i)_0 \]  

(52)

Here \( (\rho_i)_0 \) are the present energy density of various components. Taking,

\( (\Omega_m)_0 \approx 0.3, (\Omega_\Lambda)_0 \approx 0.7, H_0 = 72 \text{ km/sec./Mpc} \)

Therefore, the present value of dust energy density \( (\rho_m)_0 \) and dark energy density \( (\rho_\Lambda)_0 \) are obtained as

\[ (\rho_m)_0 = 2.8727 \times 10^{-19} \text{ gm/cm}^3 \]  

(53)

\[ (\rho_\Lambda)_0 = \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}(\rho_m)_0 = 6.7030 \times 10^{-19} \text{ gm/cm}^3 \]  

(54)
6.2 Shear Scalar

The shear scalar is given by

\[ \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \]  

(55)

where

\[ \sigma_{ij} = u_{i;j} - \Theta(g_{ij} - u_i u_j) \]  

(56)

In our model

\[ \sigma^2 = \frac{D_2^2}{D^2} = \frac{K^2}{A^6} = 3(\Omega_\sigma)H_0^2(1 + z)^6 \]  

(57)

From eq. (57), it is clear that shear scalar vanishes as \( A \to \infty \).

6.3 Relative Anisotropy

The relative anisotropy is given by

\[ \frac{\sigma^2}{\rho_m} = \frac{3(\Omega_\sigma)H_0^2(1 + z)^3}{(\rho_c)_0(\Omega_m)_0} \]  

(58)

This follows the same pattern as shear scalar. This means that relative anisotropy decreases over scale factor i.e. time.

6.4 Evolution of the scale factor \((A)\)

We begin with the integral

\[ t = \int_0^t dt = \int_0^A \frac{dA}{AH} \]  

(59)

Equations (37) and (59) lead to

\[ t = \int_0^A \frac{dA}{AH_0 \sqrt{[((\Omega_m)_0(\frac{4}{3})^3 + (\Omega_\sigma)_0(\frac{4}{3})^6 + (\Omega_\Lambda)_0]}} \]  

(60)

The right panel of figure 3 shows the variation of time with scale function for the derived model.

Figure 3: Time vs redshift & scale function.
7 Final remarks

In this paper, we have investigated the ΛCDM-type cosmological model in Heckmann-Shucking space-time. Under some specific choice of parameters, the model under consideration isotropizes and have consistency with recent SN Ia observation. We have estimated some physical parameters at present epoch for derived model which is summarized as

| Parameter | Value |
|-----------|-------|
| $t_0$     | 12.30891 Gyrs |
| $q_0$     | -0.505 |
| $(\rho_m)_0$ | $2.8727 \times 10^{-19}$ gm/cm$^3$ |
| $(\rho_\Lambda)_0$ | $6.7030 \times 10^{-19}$ gm/cm$^3$ |

Table: 2

We observe from the result displayed in table 2 that the derived model is observationally indistinguishable in the vicinity of present epoch of universe. Thus the ΛCDM model fits better with the observational data.

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