Diamagnetism of quasi-2D “charged” Bose gases under confinements

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Abstract. A charged Bose gas under confinements ensures a phase transition, in contrast with the homogeneous system in an external magnetic field. The present investigation concerns charged Bose gases trapped in a two-dimensional potential as well as neutral Bose gases rotating around the $z$ axis. It is found that the systems exhibit quite different magnetic properties. A diamagnetization is determined in the charged system while the rotating system shows up an uncommon magnetic response. However, both models possess finite magnetic susceptibilities at vanishing field or at stopping rotation.

1. Introduction

Originally, ideal charged bosons have been used to understand the superconductivity. Schafroth, Blatt and Butler showed that an ideal charged Bose gas (CBG) could exhibit essential equilibrium features of the superconductor [1]. Although the CBG does no longer condense at any finite temperature in a fixed homogeneous magnetic field, it displays extremely large Landau diamagnetism at low temperatures, and this can be used to account for the Meissner effect. Recently, the normal state diamagnetism of high $T_c$ superconductors is explained using the diamagnetism of charged bosons based on the preformed real-space pairs scenario [2]. This stimulates renewed research interest in properties of CBGs.

Moreover, neutral Bose gases have recently attracted much attention in the context of trapped ultracold atomic gases. Rotating neutral particles behave somewhat like charged particles in magnetic field. There exist some techniques to spin up neutral atoms. The direct way is to confine neutral atoms in a trap with a driven rotating deformation [3]. An alternative method is to create “effective” field for atoms by dressing them in a spatially dependent manner with optical field that couples different atomic internal states [4]. Both approaches can generate the vortex state similar to that of a superconductor in magnetic field.

Motivated by recent progress in these two distinct but related research fields, this paper considers charged Bose gases with a confining harmonic potential in the $xy$ plane. To make some connection to high $T_c$ superconductors, the system is supposed to exhibit quasi-2D features by assuming that the boson effective mass in the $z$ direction is larger than that in the $xy$ plane. Moreover, a neutral Bose gas confined in a rotating frame is also discussed.
2. The model
The model Hamiltonian is taken to be

\[ H = \frac{p_x^2 + p_y^2}{2m^*} + \frac{p_z^2}{2\eta m^*} + \frac{1}{2}m^*\omega^2(x^2 + y^2) - \omega l L_z \]  

(1)

with \( m^* \) being the effective mass in the \( xy \) plane and \( \eta \geq 1 \). \( L_z \) is the classical \( z \) component angular momentum. For a charged boson in magnetic field, we have \( \omega^2 = \omega_0^2 + \omega_l^2 \), where \( \omega_0 \) is the trap frequency and \( \omega_l = qB/(2m^*c) \) is the gyromagnetic frequency with \( q \) the particle charge. For a neutral boson in rotating frame, \( \omega \) just amounts to \( \omega_0 \) and \( \omega_l \) refers to the rotational frequency around the \( z \) axis. The quantized energy spectrum can be expressed in terms of the second-quantization representation [5],

\[ \epsilon = \frac{\hbar^2 k_z^2}{2\eta m^*} + n_+\hbar\omega_+ + n_-\hbar\omega_- + \hbar\omega, \]

(2)

where \( \omega_+ = \omega - \omega_l \) and \( \omega_- = \omega + \omega_l \) denote two vibration frequencies for particles situated in the horizontal plane, which depend explicitly on the rotating frequency \( \omega_l \).

We start from the thermodynamic potential for Bose systems which reads

\[ \Omega = \frac{1}{\beta} \sum_{n'} \ln \left( 1 - e^{-\beta(\epsilon_{n'} - \mu)} \right), \]

(3)

where \( \beta = 1/k_BT \) and \( n' \) represents the set of all quantum numbers and all continuous variables implied in the energy levels. The resulting thermodynamic potential is found to be

\[ \Omega = k_BT \ln(1 - u) - \left( \frac{\eta m^* L^2}{2\pi} \right)^{1/2} \frac{(k_BT)^{5/2}L_{15/2}(u)}{\hbar^3(\omega^2 - \omega_l^2)}, \]

(4)

where \( u = e^{\beta(\mu - \hbar\omega)} \). \( L \) is the characteristic length in the \( z \) direction. The polylogarithm function \( \text{Li}_r(x) \), defined by

\[ \text{Li}_r(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^r} \quad ; \quad r > 1 \]

(5)

reduces to Riemann zeta function \( \zeta(r) \) at \( x = 1 \) and diverges for all \( r \leq 1 \) at \( x \to 1 \).

3. Results and discussions
3.1. Charged bosons in homogeneous magnetic field
As announced above, this situation corresponds to \( \omega^2 - \omega_l^2 = \omega_0^2 \). It implies that \( u = e^{\beta(\mu - \hbar\omega)} \) is an explicit function of the external homogeneous magnetic field. The particle number is derived from the thermodynamic potential (4) by \( N = -\partial\Omega/\partial\mu \) at fixed temperature,

\[ N = N_0 + \left( \frac{\eta m^* L^2}{2\pi} \right)^{1/2} \frac{(k_BT)^{5/2}L_{15/2}(u)}{\hbar^3\omega_0^2}, \]

(6)

where \( N_0 \) refers to the condensed particles. The constraints \( N_0 = 0 \) and \( \mu - \hbar\omega = 0 \) (i.e. \( u = 1 \)) define a transition temperature \( T_c \) obtained as

\[ k_BT_c = \left[ \frac{2\pi}{\zeta(5/2)} \frac{\hbar^3\omega_0^2 N}{(\eta m^* L^2)^{1/2}} \right]^{2/5}. \]

(7)
Unlike the magnetized homogeneous three-dimensional bosons, charged bosons in magnetic field may condense under harmonic confinement. Although this critical temperature $T_c$ is irrelevant to the magnetic field strength $B$, one may notice that the transition occurs only at a definite field-dependent value of chemical potential $\mu$.

To proceed, we calculate the magnetization induced by the applied field, which defined as $M = -\left( \partial \Omega / \partial B \right)_{\beta,\mu}$, where $qB = 2m^* c \omega_l$ characterizes the magnetic field strength. We recall that, specifically $\left( \partial \Omega / \partial B \right) \propto -\left( \partial \Omega / \partial \mu \right) \left( \partial \omega / \partial b \right)$ when $\omega^2 - \omega_l^2 = \omega_0^2$. Therefore, we deduce

$$M = -\frac{Nq\hbar}{2m^*c} \frac{\omega_l}{\sqrt{\omega_0^2 + \omega_l^2}}.$$ (8)

This is the expression for Landau diamagnetization. This magnetization increases monotonously with the field and reaches a limit value at strong field strength

$$M = -\frac{Nq\hbar}{2m^*c}.$$ (9)

The magnetic susceptibility can be obtained by $\chi = \left( \partial M / \partial B \right)$ immediately,

$$\chi = -N\hbar \left( \frac{q}{2m^*c} \right)^2 \frac{\omega_0^2}{\sqrt{\omega_0^2 + \omega_l^2}^{3/2}},$$ (10)

which increases with the field strength from a finite zero field value estimated at

$$\chi = -\left( \frac{q}{2m^*c} \right)^2 \frac{N\hbar}{\omega_0}.$$ (11)

It is worth noting that both the magnetization and the magnetic susceptibility do not depend on the temperature. This is consistent with the charged Bose gas in 3-dimensional harmonic trap[6].

3.2. Neutral bosons in rotating frame
In this case, $\omega = \omega_0$ and $u = e^{\beta(-\mu - \hbar\omega)}$ does not rely on the field, explicitly. The particles number is expressed by

$$N = N_0 + \left( \frac{\eta m^* L^2}{2\pi} \right)^{1/2} \frac{(k_BT)^{5/2} \cdot Li_{5/2}(u)}{h^3(\omega_0^2 - \omega_l^2)}.$$ (12)

And the Bose-Einstein condensation temperature is

$$k_BT_c = \left[ \left( \frac{2\pi}{\eta m^* L^2} \right)^{1/2} \frac{h^3(\omega_0^2 - \omega_l^2)N}{\zeta(5/2)} \right]^{2/5}. $$ (13)

Here $T_c$ is not fixed. It decreases with the rotation speed and tends to zero as $\omega_l \to \omega_0$. This avoids the system to condense in the rapid rotation limit ($\omega_l = \omega_0$).

Similarly, we can examine “magnetic” effects created by the rotation. The “effective” magnetic field is taken to be $b = 2m^* \omega_l$. Thus, the “magnetization” $M$, only provided by thermal particles, is given by $M = -\left( \partial \Omega / \partial b \right)_{\beta,\mu}$, which yields

$$M = \left( \frac{\eta m^* L^2}{2\pi} \right)^{1/2} \frac{(k_BT)^{7/2}Li_{7/2}(u)}{m^*h^3} \frac{\omega_l}{(\omega_0^2 - \omega_l^2)^2}.$$ (14)
Unlike the charged bosons, the total “magnetization” of rotating neutral bosons is positive. It varies with the temperature and when all the particles occupied the ground state energy. Moreover, the speed of rotation affects the magnetization; it is zero in absence of rotation, increases monotonously with the speed of rotation and diverges at \( \omega_l = \omega_0 \). It is very important to remark that as long as the system lies below \( T_c \), the total magnetization is governed by a power function law of temperature.

It results the following magnetic susceptibility

\[
\chi = \left( \frac{\eta m^* L^2}{2\pi} \right)^{1/2} \frac{(k_B T)^{7/2} \text{Li}_{7/2}(u)}{2 m^* \hbar^2} \frac{1 + 3 \omega^2_l}{(\omega_0^2 - \omega^2_l)^3}. \tag{15}
\]

It vanishes only at absolute zero temperature but not at stopping rotation. The magnetic susceptibility also diverges as \( \omega_l \to \omega_0 \).

Finally, we propose the dependency of the magnetization on the number of particles present in the excited states

\[
M = N - N_0 \frac{k_B T \text{Li}_{7/2}(u)}{m^* \text{Li}_{5/2}(u)} \frac{\omega_l}{\omega_0^2 - \omega^2_l}. \tag{16}
\]

4. Summary

Bose gases confined in a transverse harmonic potential may condense whether they are in magnetic field or they are rotating around the vertical direction. Rotation of neutral particles is expected to produce an “effective field” that covers the system in the similar way immersion of charged particles in homogeneous magnetic field. However, their magnetic responses to the field are quite different. A diamagnetization is determined in the charged system while a positive “magnetization” is obtained in the rotating neutral bosons. It is also found that both the magnetization and susceptibility of charged bosons tend to finite values at strong field strength. Nevertheless, rapid rotation destroys the confinement of neutral bosons and results in divergency of the magnetization and susceptibility.

Acknowledgments

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