Feasible Path Generation Using Bezier Curves for Car-Like Vehicle

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Abstract. When planning a collision-free path for an autonomous vehicle, the main criteria that have to be considered are the shortest distance, lower computation time and completeness, i.e. a path can be found if one exists. Besides that, a feasible path for the autonomous vehicle is also crucial to guarantee that the vehicle can reach the target destination considering its kinematic constraints such as non-holonomic and minimum turning radius. In order to address these constraints, Bezier curves is applied. In this paper, Bezier curves are modeled and simulated using Matlab software and the feasibility of the resulting path is analyzed. Bezier curve is derived from a piece-wise linear pre-planned path. It is found that the Bezier curves has the capability of making the planned path feasible and could be embedded in a path planning algorithm for an autonomous vehicle with kinematic constraints. It is concluded that the length of segments of the pre-planned path have to be greater than a nominal value, derived from the vehicle wheelbase, maximum steering angle and maximum speed to ensure the path for the autonomous car is feasible.

1. Introduction
Path planning is one of the main requirements for an autonomous vehicle. The first idea of the autonomous car was presented at General Motor Futarama exhibit in 1939. Then in 1953, an experiment by General Motor Cooperation and Radio Cooperation of America on how electronics were able to control the steer was conducted. In 1958 an experiment using a 1958 Chevrolet was performed using a pick-up roles to sense the alternating current of wire implanted in the road. Around 1960 until 1970, a remote control with a long cable and camera was used to control a Stanford cart. In Japan in 1977, the first autonomous car equipped with an analog computer and two cameras for signal processing, with the capability of speeding up to 30km/h was invented[1]. Nowadays, research on autonomous vehicles has been growing rapidly. There are several famous companies such as Audi, Google, Tesla, Mercedes-Benz, and BMW to name a few that are involved in the development of autonomous cars[2–4]. Google has created a functional prototype of a self-driving car [2]. Furthermore, in 2015, Baidu and BMW have launched the first self-driving car on China road [3]. Tesla has also released the software for a self-driving feature to their customer in 2015[2].

There are several methods to plan a collision-free path, such as cell decomposition method, visibility graph and genetic algorithm [5–7]. As research on collision-free path planning techniques advances, the path planning that is capable of producing a feasible path to autonomous vehicles gains more researchers’ attention. The main drawback of the most existing path planning algorithms is they...
are unable to produce a feasible path, which results in the vehicle fails to track such a path because of sharp turns. However, there are several techniques that could be used to smoothen a pre-planned path such as Bezier curves, clothoid, and dubins curves to name a few. Bezier curves can smoothen the path yet easy to develop with less complexity of computation time compared to clothoid [8]. The Bezier curves is created using a starting point, control points and the target point of the pre-planned path.

In this paper, the Bezier curves is implemented and its characteristic is studied. By considering the kinematic constraints of a car-like model, i.e., the minimum turning radius, the Bezier curves is developed from a pre-planned path, which may contain many sharp turns.

A number of researches using Bezier curves have been done to produce a feasible path. Researches by [7][9] using visibility graph (VG) method and Bezier curves developed a path planning algorithm for wheeled mobile robots. The first step of the algorithm was to generate the VG from a starting point to a target point. Then, a linear shortest path was obtained and replaced with piecewise Bezier curves to acquire a smoothing path. In this algorithm, a curvature of fifth order Bezier curves was developed using a Cubic Bezier curves. The algorithm was able to produce the shortest path without any sharp turn.

On the other hand, [10] focused on generating a feasible path for a car-like model vehicle considering the curvature continuity and maximum curvature constraint to develop the cubic Bezier curves. As a result, Bezier turn and Bezier path algorithm were developed to produce a feasible and smooth path for the autonomous vehicles.

A path planning algorithm proposed by[9] for an indoor mobile robot applied Bezier curve to make the planned path feasible. In the proposed algorithm, firstly, a quantic Bezier curves was implemented to plan a continuous curves, then optimization technique was used to approve the plan executed by the quantic Bezier curves. Lastly the velocity profiles were generated to confirm the limit velocity of vehicles.

2. Methodology

2.1. Kinematic representation for car-like vehicle
The kinematic constraint for a car-like vehicle is shown in Equations (1) – (3). In this equation, $L$ represents the distance between the rear wheels and axle wheels (wheelbase) in meters, $\theta$ is the car heading angle with respect to the $xy$-frame, and $\phi$ is the orientation or steering angle with respect to the front wheels [11–14].

$$\dot{x} = v \cos \theta$$  \hspace{1cm} (1)
$$\dot{y} = v \sin \theta$$  \hspace{1cm} (2)
$$\dot{\theta} = \frac{v}{L} \tan \phi$$  \hspace{1cm} (3)

The minimum turning radius $r_{min}$ for the car like vehicle is shown Equation (4), which is determined by the velocity of the vehicle $v$, the wheelbase $L$, and steering angle $\phi$.

$$r_{min} = \frac{L}{\tan \phi}$$  \hspace{1cm} (4)
2.2. Bezier curves derivation
A general equation of Bezier is shown in Equations (5) - (6).

\[ P(t) = \sum_{i=0}^{n} B_i, n(t)q_i \quad 0 \leq t \leq 1 \]  

(5)

\[ B_i, n(t) = C_n^i (1 - t)^{n-i} t^i \]  

(6)

\( B_i \) shown in Equation (6) is a Bernstein basis polynomial. \( n \) is the degree of Bernstein basis and \( q_i \) is the control points [8,15].

When a polygon is drawn through these central points, it is known as Bezier polygon. The starting and the target points on the curve are coincident with the first and the last control points. The derivative of the starting point is \( t = 0 \) and for the target, \( t = 1 \).

The Bezier curves is defined by four points; two are the starting point and target point and the other two, \( P_1 \) and \( P_2 \) represent the control points of the curves [16]. Note that the control points will define the radius of the curves.

2.3. Control points determination
A piece-wise linear path is unfeasible due to the sharp turns at waypoints, which result in the car-like vehicle fails to accomplished a given mission. Sharp turns consequently make the vehicle changing its heading abruptly causing the traversed path unfeasible as it could not satisfy the \( r_{\text{min}} \) requirement of the vehicle. To address that issue, we propose a technique in which each waypoint of a path is given an offset called variable waypoint offset (VWO). This offset value is then set as the control points of the Bezier curves. VWO coupled with Bezier curves will solve the problem kinematics of the car-like vehicle. The offset value is proportional to the heading angle and vehicle’s speed. Consider two adjacent line segments of a path. The offset value, \( f \) can be calculated from a simple trigonometry relationship,

\[ f = r_{\text{min}} \tan\left(\frac{\Delta \theta}{2}\right) \]  

(7)

Equation (7) suggests that the offset value is proportional to both \( r_{\text{min}} \) and the change of heading angle. The conceptual representation of the VWO is illustrated in Figure 1(a). In the figure, the piece-wise linear path is made of three linear segments and the circles have radiuses of \( f_1 \) and \( f_2 \), respectively. The control points \( P_1 \) and \( P_2 \) near the first waypoint are then determined from the \( f_1 \). Similarly, the control points at the vicinity of the second waypoint are determined from \( f_2 \). Figure 1(b) illustrates the simulated VWOs, in which the intersection points between the circles and path segments are the control points of the respective Bezier curves.
2.4. Bezier curves simulation
In order to illustrate how a Bezier curve is generated, consider a piece-wise linear pre-planned path as illustrated in Figure 2 (a). The vehicle that will be traversing the path is a car-like vehicle, which has a maximum speed of 15 m/s, a wheelbase of 1 m, and 45° maximum heading angle, which result in the minimum turning radius $r_{\text{min}}$ is 1.73 m. In the figure, the angle $\Psi$ of the two consecutive segments is 90°, which is assumed to be the smallest possible angle for the planned path. The control points, $P_1$ and $P_2$ are calculated using Equations (7) and also depicted in Figure 2(a). Figure 2(b) shows the resulting Bezier curves with 1.73m turning radius, which satisfies the kinematic constraint of the car-like vehicle.
3. Result and Discussion
To demonstrate the application of Bezier curves in path planning, consider a scenario with a number of obstacles, a starting point and a target point as shown in Figure 3. The obstacles are represented by the rectangles. A pre-planned path is then generated, shown in solid red line. Note that there are unfeasible sharp turns along the path that have to be traversed by the car-like vehicle. Using equations (5) & (6), a feasible path is then generated as depicted in solid blue line in the figure.

![Figure 3](image)

Figure 3. A scenario with obstacles in which Bezier curves is applied to make a pre-planned path feasible

However, before a feasible path using Bezier curves could be produced from a pre-planned path for the car-like vehicle, it has to be made sure that the each segment length of the path has to be greater than a nominal value, which is determined by the wheelbase, maximum speed and maximum heading angle.

Consider Figure 4 that shows two pre-planned paths (in solid blue lines), which are piece-wise linear, each with three segments. Note that the angle between the two consecutive segments is 90°, which is considered to be the worst case scenario, since it will produce Bezier curves with minimum turning radius. The dimension of each segment and the control points are given. In this paper, it is assumed that the vehicle has the following parameters:

i. The distance between rear wheel and front wheel (L) is 1 m
ii. The steering angle Ø is initialized at 30°
iii. The maximum speed is set to 15m/s.
From the above mentioned parameters, the resulting minimum turning radius \( r_{\text{min}} \) of the car-like vehicle is 1.73 m. In order for the path to be fully feasible, the minimum length \( l \) of each segment of the path has to satisfy the following condition set in (8).

\[
r_{\text{min}} \leq 0.5l
\]  
(8)

Rearranging notation (8), the following inequality is obtained.

\[
l \geq 2r_{\text{min}}
\]  
(9)

Substituting \( r_{\text{min}} = 1.73 \text{m} \), from (8), the segment length \( l \) has to be greater than 3.46\text{m}, to ensure that the resulting path for the car-like vehicle is feasible.

Figure 5(a) shows the resulting Bezier curves which is feasible for the car like vehicle because it satisfies (9). On the other hand, an unfeasible path will be produced if the length of path is smaller than 3.46\text{m} as depicted in Figure 5 (b).

![Figure 4](image.png)

(a) Feasible, (b) unfeasible.

To demonstrate the above idea in a path planning for a car-like vehicle, consider a scenario with random starting, target points and obstacles as depicted in Figure 5. A pre-planned path shown in solid red is then generated. Next, Bezier curves is generated to produce the feasible path illustrated in solid blue lines. However, the resulting path is found unfeasible because the length of certain segment do not satisfy the requirement in (8).
Figure 5. Unfeasible path is produced as the condition of minimum segment length is not fulfilled. Because of the path is unfeasible, the segment whose length smaller than 3.46m has to be removed and this will cause a new path has to be re-planned. Figure 6 shows the piece-wise linear re-planned path shown in solid red and the resulting Bezier curves in solid blue.

Figure 6. The path has been re-planned and a feasible path is generated.
4. Conclusion
As a conclusion, it has been shown that there are several criteria, i.e., minimum turning radius and minimum segment length, need to be considered when applying Bezier curves to a pre-planned piecewise linear path to ensure that the Bezier curves produce a feasible path for an autonomous vehicle. The relationship between the minimum turning radius and the minimum segment length of the path was also given. To produce a feasible path for a car-like vehicle from a pre-planned path, the length \( l \) of each segment has to be greater than twice of \( r_{\text{min}} \). When the minimum segment length \( l \) is fulfilled, the resulting path produced using Bezier curves is feasible for the vehicle and a mission given to the vehicle could be accomplished.

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