Magnetic-field-driven localization of light in a cold-atom gas

S.E. Skipetrov\textsuperscript{1,2} and I.M. Sokolov\textsuperscript{3,*}

\textsuperscript{1}Université Grenoble Alpes, LPMMC, F-38000 Grenoble, France
\textsuperscript{2}CNRS, LPMMC, F-38000 Grenoble, France
\textsuperscript{3}Department of Theoretical Physics, State Polytechnic University, 195251 St. Petersburg, Russia

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We discover a transition from extended to localized quasi-modes for light in a gas of immobile two-level atoms in a magnetic field. The transition takes place either upon increasing the number density of atoms in a strong field or upon increasing the field at a high enough density. It has many characteristic features of a disorder-driven (Anderson) transition but is strongly influenced by near-field interactions between atoms and the anisotropy of the atomic medium induced by the magnetic field.

The transition from extended to localized eigenstates upon increasing disorder in a quantum or wave system is called after Philip Anderson who was the first to predict it for electrons in disordered solids \textsuperscript{11}. More recently, this transition was studied for various types of quantum particles (cold atoms \textsuperscript{2}, Bose-Einstein condensates \textsuperscript{3}) as well as for classical waves (light \textsuperscript{17}, ultrason \textsuperscript{8,9}). In the most common case of time-reversal symmetric systems invariant under spin rotation Anderson transition takes place for a three-dimensional (3D) disorder only, eigenstates of low-dimensional systems being always localized \textsuperscript{10,11}. Anderson localization of light may find applications in the design of future quantum-information devices \textsuperscript{12}, miniature lasers \textsuperscript{13} and solar cells \textsuperscript{14}. However, no undisputable experimental observation of optical Anderson transition in 3D exists to date since alternative explanations were proposed for all published reports of it \textsuperscript{15,16}. Moreover, we have recently shown that the simplest theoretical model in which light is scattered by point scatterers (atoms) does not predict Anderson localization of light at all \textsuperscript{17}.

In the present Letter we show that an external magnetic field may induce a transition between extended and localized states for light in a gas of cold atoms. This result may seem counterintuitive from the perspective of condensed matter physics where a magnetic field leads to a breakdown of the time-reversal invariance and thus suppresses the lowest-order interference effects leading to the weak localization—a precursor of Anderson localization \textsuperscript{18,19}. In atomic systems, however, the main effect of the magnetic field is to lift the degeneracy of states involved in the interaction of an individual atom with light—the Zeeman effect. For light that is resonant with an atomic transition with a degenerate ground state the Zeeman effect can actually amplify interferences leading to an enhancement of the coherent backscattering peak \textsuperscript{20,21}. On the other hand, the Zeeman effect reduces the strength of resonant dipole-dipole interactions between nearby atoms \textsuperscript{22}. These interactions suppress light scattering \textsuperscript{23,24} and prevent Anderson localization \textsuperscript{17}. We show below that the partial suppression of resonant dipole-dipole interactions by the magnetic field is sufficient to induce a transition from extended to localized states in the atomic system. This critical phenomenon stands out from other magneto-optical effects that take place in disordered media (such as, e.g., the photonic Hall effect \textsuperscript{25} or Hanle effect in coherent backscattering \textsuperscript{26}) which only give rise to weak corrections to wave transport.

We consider an ensemble of $N \gg 1$ identical two-level atoms at random position $\{r_i\}$ inside a spherical volume $V$ of radius $R$. The resonant frequency $\omega_0$ of atoms defines the natural length scale $1/k_0 = c/\omega_0$, where $c$ is the vacuum speed of light. The ground state $|g_i\rangle$ of an isolated atom $i$ is nondegenerate with the total angular momentum $J_g = 0$, whereas the excited states $|e_i\rangle$ is three-fold degenerate with $J_e = 1$. The three degenerate substates $|e_{im}\rangle$ correspond to the three possible projections $m = 0, \pm 1$ of the total angular momentum $J_e$ on the quantization axis $z$. The natural lifetime $1/\Gamma_0$ of the excited state sets the time scale of the problem. The atoms are subject to a uniform magnetic field $B \parallel z$ and interact with the free electromagnetic field surrounding them. The system “atoms + field” is described by the following Hamiltonian \textsuperscript{21,27,28}:

$$
\hat{H} = \sum_{i=1}^{N} \sum_{m=-1}^{1} \hbar \omega_0 |e_{im}\rangle \langle e_{im}| + \sum_{i,k} \hbar c k \left( \hat{a}^\dagger_{ks} \hat{a}_{ks} + \frac{1}{2} \right) - \sum_{i=1}^{N} \hat{D}_i \cdot \vec{E}(r_i) + \frac{1}{2\epsilon_0} \sum_{i \neq j}^{N} \hat{D}_i \cdot \hat{D}_j \delta(r_i - r_j) + g_e \mu_B B \cdot J_e. \tag{1}
$$

Here $\hbar$ is the Planck's constant divided by $2\pi$, $k$ and $s$ are the wave and the polarization vectors of the modes of the free electromagnetic field, $\hat{a}_{ks}$ ($\hat{a}^\dagger_{ks}$) are the corresponding creation (annihilation) operators, $\hat{D}_i$ are the atomic dipole operators, $\vec{E}(r_i)$ is the electric displacement vector divided by the vacuum permittivity $\epsilon_0$, $\mu_B$ is the Bohr magneton, and $g_e$ is the Landé factor of the excited state.

Previous work \textsuperscript{29,30} demonstrated that in the absence of magnetic field ($B = 0$), the degrees of freedom corresponding to the electromagnetic field can be traced
out leading to an effective Hamiltonian describing the dynamics of $N$ atoms coupled by the electromagnetic field. This effective Hamiltonian takes the form of a $3N \times 3N$ Green’s matrix $G$ describing the propagation of light between the atoms. The same approach can be used when $B \neq 0$ leading to the following Green’s matrix:

$$G_{eim,ejm'} = (i + 2m\Delta) \delta_{eim,ejm'} - \frac{2}{\hbar \Gamma_0} (1 - \delta_{eim,ejm'})$$

$$\times \sum_{\mu,\nu} d_{e\mu}^\dagger d_{\nu}^{j'} e^{ik_0r_{ij}}$$

$$\times \left\{ \delta_{\mu\nu} \left[ 1 - ik_0r_{ij} - (k_0r_{ij})^2 \right] \right\}$$

$$- \frac{\bar{d}_{\mu}^\dagger r_{ij}^\nu}{r_{ij}} \left[ 3 - 3ik_0r_{ij} - (k_0r_{ij})^2 \right] ,$$

where where $\Delta = g_e \mu_B B / \hbar \Gamma_0$ is the Zeeman shift in units of the natural line width, $d_{eim} = \langle J_e m | \hat{D}_i | J_0 \rangle$, and $r_{ij} = r_i - r_j$.

In the absence of magnetic field ($B = 0$) the eigenvalues of the Green’s matrix $G$ concentrate in a roughly circular domain on the complex plane roughly symmetric with respect to the vertical axis $\text{Re} \Lambda = 0$ and almost touching the horizontal axis $\text{Im} \Lambda = 0$ [17]. The field splits the eigenvalues into three equal groups centered around $\text{Re} \Lambda = 2m\Delta$ ($m = 0, \pm 1$) [32], see Fig. 1. The three groups of eigenvalues become well separated in the limit of strong magnetic field $\Delta \gg 1$ to which we will restrict our consideration in the present Letter. Although at a low density $\rho = N/V$ the three groups of eigenvalues are similar [Fig. 1(a)], the groups corresponding to $m = \pm 1$ start to differ significantly from the $m = 0$ group at higher densities [Fig. 1(b)]. In particular, the $m = \pm 1$ groups of eigenvalues develop “holes” that were previously associated with Anderson localization in the framework of the scalar model of wave scattering [33, 34].

To see whether localized states indeed appear at high densities of atoms, we analyze the inverse participation ratios (IPRs) of eigenvectors $\psi_n$ of the Green’s matrix $G$, IPR$_n = \sum_{i=1}^N |\psi_{ne_i}|^4 / (\sum_{i=1}^N |\psi_{ne_i}|^2)^2$, where $|\psi_{ne_i}|^2 = \sum_{m=-1}^1 (\psi_{ne_i})^2_m$ is the square of the length of the vector $\psi_{ne_i} = \{ (\psi_{ne_i})_m \}$. Low IPR $\sim 1/N$ corresponds to an extended state whereas IPR $\sim 1/M > 1/N$ signals a state localized on $M < N$ atoms. Figure 2(a) shows that at low density of atoms most of the eigenvectors have low IPRs with the eigenvectors localized on pairs of closely located atoms being an exception. These “subradiant” states exist at any density and should be distinguished from localized states that are due to the multiple scattering of light on many atoms and that appear at higher densities in relatively narrow bands of frequencies $\text{Re} \Lambda$ on the left from the resonances $\text{Re} \Lambda = \pm 2\Delta$ [see Fig. 2(b)]. These states may have smaller IPRs than the subradiant states but they have significantly longer lifetimes (i.e. smaller $\text{Im} \Lambda$).

The appearance of states localized on large clusters of atoms in a magnetic field is due to the removal of degeneracy of the excited states $|e_i\rangle$ by the field. As a result, the transitions $|g_i\rangle \rightarrow |e_im\rangle$ effectively decouple for different $m$ since photons scattered on these transitions have frequencies discrepant by $\approx 2g_e \mu_B B / \hbar > \Gamma_0$. As a consequence, a behavior similar to the scalar case may be expected for a given $m$ with, in particular, localized states appearing at high densities of atoms as found in the scalar model [17]. However, as follows from Fig. 2, this...
naive picture is largely oversimplified because it does not explain the absence of localized states near \( \operatorname{Re}\Lambda = 0 \) corresponding to \( m = 0 \). A more detailed study shows that indeed, the full vector problem can be reduced to an effective scalar one in the limit of strong magnetic field, but the effective Green’s matrix following from this analysis is different from the one corresponding to scalar waves. We have found that for \( \Delta > 1 \), the group of eigenvalues corresponding to a given \( m \) can be approximately found by diagonalizing the effective \( N \times N \) Green’s matrix

\[
G_{ij} = (1 + 2m\Delta) \delta_{ij} + (1 - \delta_{ij}) \frac{e^{ik_0|\mathbf{r}_i - \mathbf{r}_j|}}{k_0|\mathbf{r}_i - \mathbf{r}_j|}
\]

\[
\times \left\{ c_m \left[ 1 - (-1)^m \cos^2 \theta \right] + c_m (-1)^m \left[ \frac{i}{k_0r} - \frac{1}{(k_0r)^2} \right] (1 - 3 \cos^2 \theta) \right\},
\]

where \( c_m = (3/8)[3 + (-1)^m] \) and \( \theta \) is the angle between \( \mathbf{r}_i - \mathbf{r}_j \) and the \( z \) axis.

Equation (3) explains the differences between \( m = 0 \) and \( m = \pm 1 \) seen in Figs. 1 and 2. First, the far-field contribution to \( G_{ij} \) given by the second line of Eq. (3) varies from 0 to 1 for \( m = 0 \) and from \( 1/2 \) to 1 for \( m = \pm 1 \) as a function of \( \theta \). It is thus closer to its scalar-wave value of 1 in the former case, suggesting that the case of \( m = \pm 1 \) may be better approximated by the scalar model than the case of \( m = 0 \). Second, the near-field term [the third line of Eq. (3)] is a factor of two smaller for \( m = \pm 1 \) than for \( m = 0 \). Because near-field terms responsible for resonant dipole-dipole interactions between nearby atoms were shown to suppress the localization transition \([17]\), the weakness of these terms for \( m = \pm 1 \) is an advantage. We see therefore that both far- and near-field features of Eq. (3) are closer to its scalar approximation for \( m = \pm 1 \) than for \( m = 0 \). This explains the appearance of localized states for \( m = \pm 1 \) rather than for \( m = 0 \) transitions.

To have a more quantitative characterization of the localization transition demonstrated in Fig. 2, we compute the Thouless parameter \( g \) that we define as a ratio of the inverse of the average lifetime of eigenstates \( \delta \omega = (1/\operatorname{Im}\Lambda)^{-1} \) to the average eigenvalue spacing along the horizontal axis \( \Delta \omega = (\operatorname{Re}\Lambda_n - \operatorname{Re}\Lambda_{n-1}) \), \( g = \delta \omega / \Delta \omega \) \([10, 17, 35]\). This quantity is calculated as a function of \( \operatorname{Re}\Lambda \) with the averaging performed over all eigenvalues in a unit interval around \( \operatorname{Re}\Lambda \). As can be seen from Fig. 3, \( g \) reaches small values \( g < 1 \) expected for localized states only at large densities corresponding to \( k_0\ell_0 = k_0^3/6\pi \rho < 1 \) and only for the values \( \operatorname{Re}\Lambda \) cor-

FIG. 2. Map of the average IPR at low (a) and high (b) densities of atoms in a strong magnetic field \( \Delta = 10^3 \). Dashed lines show lines along which the eigenvalues of a two-atom system would be situated for atoms placed along the direction of magnetic field (\( \theta = 0 \)) or perpendicular to it (\( \theta = \pi/2 \)). For \( 0 < \theta < \pi/2 \) the corresponding eigenvalues are in between the two lines.
FIG. 3. Thouless number $g$ as a function of the bare Ioffe-Regel parameter $k_0\ell_0 = k_0^2/6\pi \rho$ for a strong magnetic field $\Delta = 10^3$. The curves are obtained by averaging over a unit interval of $\text{Re}\Lambda$ around their positions. Different curves at the same value of $\text{Re}\Lambda$ correspond to different numbers of atoms $N$. The gray plane corresponds to $g = 1$.

FIG. 4. Same as Fig. 3 but for selected values of $\text{Re}\Lambda = -2002$ (a), $-2$ (b) and 1998 (c). The insets show the scaling function $\beta(g)$ estimated from the numerical data of the main plots.

responding to $m = \pm 1$, in full agreement with Fig. 2.

The simple fact that $g$ becomes smaller than 1 does not necessarily signals a transition from extended to localized eigenstates. It is much more important that curves corresponding to different $N$ (and thus to different sample sizes at a given $\rho$) cross in Figs. 3(a) and (c). We reproduce $g(k_0\ell_0)$ for $\text{Re}\Lambda$ slightly shifted to the left of the single-atom resonant frequencies $\text{Re}\Lambda = 2m\Delta$ in Fig. 4.

The independence of $g$ from the sample size at the point where curves corresponding to different $N$ cross is a hallmark of critical point and confirms localization transition for light at frequencies $\omega = \omega_0 \pm g_{\alpha}\mu_B B/\hbar + \alpha(\Gamma_0/2)$ with $\alpha \sim 1$. This transition is also evidenced by the scaling function $\beta(g) = \partial \ln g/\partial \ln k_0\ell_0$ shown in the insets of Fig. 4. $\beta(g)$ changes sign for $\text{Re}\Lambda = -2002$ and 1998 but not for $\text{Re}\Lambda = -2$ proving that the localization transition takes place at large frequency shift $\text{Re}\Lambda \approx \pm 2\Delta$ but not around the fundamental resonance $\text{Re}\Lambda = 0$.

In conclusion, we have found that the magnetic field can induce a transition from extended to localized states for light in an ensemble of identical immobile two-level atoms. This is due to the removal of degeneracy of the excited atomic state by the field and the resulting partial suppression of resonant dipole-dipole interactions between nearby atoms. However, the discovered localization transition is different from that in the scalar model with a nondegenerate excited state. In particular, higher atomic density is required to reach localization and the values of Thouless number $g$ that can be realized in the localized regime are not as small as in the scalar case. These differences will be the subject of further studies.

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