Investigation of Radiative Properties of a Multi-particle Cloud with Non-uniform Particle Size Distribution

Z M Cheng¹,², F Q Wang¹,³, D Y Gong¹, H X Liang¹,², Y Shuai¹ and G Q Li³

¹School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, China
²School of New Energy, Harbin Institute of Technology at Weihai, Weihai 264209, China
³School of Engineering, University of Hull, Hull HU6 7RX, UK

Email: Wangfuqiang@hitwh.edu.cn

Abstract. Particle systems involve numerous fields such as combustion, oceanography, photocatalysis, and atmospheric science, among others. In fact, most of these systems were in the form of multi-particle systems composed of different types of particles. In most previous studies, the influence of the type of element on radiative characteristic parameters was not considered, or the effective medium theory was adopted, which led to large calculation errors. In this paper, the differential-integration algorithm developed based on the Mie theory was used to calculate the radiative characteristics (extinction coefficient, absorption coefficient, scattering phase function, asymmetry factor) of a multi-particle system with non-uniform size distribution, and the Monte Carlo ray-tracing (MCRT) method was used to solve the radiative transfer equation (RTE). SiO₂ particles and polystyrene particles were investigated and subsequently divided into two categories: submicron particles ($\mathcal{X} > 1$) and nano particles ($\mathcal{X} < 1$) according to the size parameter. The radiative characteristics and spectral transmittance of particles with different volume fraction ratios and different particle sizes in the multi-particle system were also investigated. The results of the proposed method were compared to those of experimental tests to verify the accuracy of the algorithm. The maximum relative transmittance error decreased from 30.1% to 0.77%, and the average error decreased from 18.78% to 0.67%.

1. Introduction

Particle systems involve several fields such as chemical industry [1], combustion [2, 3], ocean [4], photocatalysis [5], atmospheric science [6], and metamaterial [7, 8], among others [9, 10]. In practical applications, particle systems were usually composed of multicomponent and polydisperse particles [11, 12]. It was very important that the radiative characteristics of the particle system were accurately calculated, for instance: the calculation of radiative heat transfer of carbon particles in boiler furnaces [13], measurement of absorbance of multiple photocatalyst [14, 15], the calculation of cooling power of radiative cooling materials containing various particles [16, 17], target detection in haze weather conditions [18], various microalgae in the ocean [19]. Therefore, the accurate radiative characteristics and radiative transfer of the multicomponent and polydisperse particle system are crucial and need to be researched [20].

The radiative characteristics of each component particle in the multi-particle system cannot be ignored, otherwise it may cause large errors [21, 22]. Marakis et al. [23] studied the effect of different types of particles containing carbon particles, carbon black and fly ash on radiation heat transfer in the
furnace. The results showed that radiative intensity of different particles in the furnace combustion was different. The large heat transfer error was resulted if any type of particles was ignored. Chen et al. [24] investigated the performance of solar photothermal conversion of blended Au and Ag nanoparticles. The experimental results showed that the solar photothermal conversion efficiency of the blended Au-Ag nanoparticles was 31.41%, which was a value that lies between that of pure Au and Ag nanoparticles, under the same nanoparticles volume fraction. They attributed this phenomenon to the scattering of light. Jin et al. [25] found that the proportion of carbon black and dust was different in different seasons, which affected the optical characteristics and associated radiative effects of the aerosol. The results showed that the iron-containing dust in the aerosol in summer had a strong ultraviolet absorption capacity. In winter, carbon black greatly affected the radiative characteristics of aerosols.

For calculating the radiative characteristics of the multi-particle system, some researchers used approximate theory to calculate them [26-28]. Yin et al. [29] used Bruggeman effective medium theory (EMT) to calculate the effective optical constants and radiative characteristics of synthetic ash, and identified that the influence of multi-components on the optical constants of coal slag cannot be ignored. Computational results showed that the maximum error between the theoretical value of the average absorption efficiency factor and the experimental value was 12.15%. However, Zhang et al. [30] also used the Bruggeman EMT to calculate the radiative characteristics of the aerosol particles mixed with carbon black and sulfate for size parameters ranging from 0.1 to 2.5. The validity of EMT in the light scattering of compact internal mixture was performed based on the comparison of both the aforementioned methods. The EMT was effective in the Rayleigh scattering region. However, it did not perform well in the range of Mie scattering. The maximum relative deviations of extinction factor, absorption factor, and scattering factor were 25%, 88% and 66%, respectively. Du et al. [31] employed the Mie theory to analyze the radiative characteristics of char and ash particles in different parts of the pulverized coal furnace at a wavelength of 560 nm. The results showed that in the burner exit, char had a significant effect on the value of the particles’ radiative coefficient, which accounted for 90%. While in the upper portion of the furnace, ash dominates the particle radiative coefficient.

The literature survey indicated that: 1) The radiative characteristics of various types of particles in a multi-particle system cannot be ignored, and failing to do so leads to large calculation errors. 2) Currently, most studies on the radiative characteristics of multi-particle system used EMT to calculate the optical constants. However, some scholars had identified that this theory was only applicable to the Rayleigh scattering region, and the deviation in the calculated values of the Mie scattering region was significant. In addition, the application of EMT was mostly in the combustion of pulverized coal in the boiler. Additionally, studies focusing EMT were rare in other fields.

In this paper, the radiative characteristics of a multi-particle system including SiO$_2$ and polystyrene was investigated. The differential-integral algorithm, which was developed by Mie theory, was used to calculate the radiative characteristics of the multi-particle system with non-uniform size distribution. The MCRT method was used to solve the RTE. The scattering effect of the particles were considered during the calculation. The effects of different volume ratios and particle sizes in the multi-particle system on spectral extinction coefficient, absorption coefficient, scattering phase function and asymmetry factor were studied.

2. Methodology

2.1. Differential-integration algorithm
Figure 1. Flow chart of the estimation procedure

The proposed model was developed to calculate the radiative characteristics of multi-particles and solve the RTE of a semitransparent medium with multi-particles, as presented in Figure 1. There were three main components constituting this model to solve such a problem, including Mie theory, DIA and MCRT. Additionally, DIA was used for solving the radiative characteristics of the particles on the premise of a sparse particle system, which satisfies independent scattering condition. The simplified calculation process was described as following.

The Mie theory determine the radiative characteristics of a single spherical particle with a diameter $D_{j,i}$ with the use of the particle refractive complex index $n_{p,j} + i k_{p,j}$ and the matrix refractive index $n_t$. The radiative factors included the extinction factor $Q_{e,j,i}$, absorption factor $Q_{a,j,i}$, scattering factor $Q_{s,j,i}$, and scattering phase function $\Phi_{j,i} (\Theta)$. Additionally, the asymmetry factor $g_{j,i}$ of a single spherical particle can be calculated using the scattering phase function, $\Phi_{j,i} (\Theta)$ [32].

The DIA was used to calculate the radiative characteristics of the semitransparent medium with multi-particles. First, each single type of a particle size distribution of the multi-particle system can be obtained through measurement using a laser granularity gauge. The volume fraction, $f_{v,j,i}$, could be calculated by a differential method and the total volume fraction $f_{v,i}$ of each particle type. Second, the radiative characteristics of the particles with the same type and uniform size were calculated using the volume fraction $f_{v,j,i}$ and the radiative characteristics of a single spherical particle with size $D_{j,i}$. Subsequently, the radiative characteristics of the particle systems with the same particle type were calculated by integrating the radiative characteristics of particles of same type and uniform diameter.
The radiative characteristics included the extinction coefficient $\beta_j$, scattering coefficient $\sigma_{s,j}$, absorption coefficient $\kappa_j$ and asymmetry factor $g_j$. Finally, the radiative characteristics of a semitransparent medium with multi-particles were calculated by integrating the radiative characteristics of same type of particles. The radiative characteristics obtained include the scattering coefficient $\sigma$, absorption coefficient $\kappa$, extinction coefficient $\beta$ and asymmetry factor $g$. The MCRT method was used to solve the RTE of a semitransparent medium with multi-particles. Some important radiative characteristics were obtained from the RTE, such as normal transmittance and reflectivity \cite{33, 34}.

2.2. Radiative characteristics of a single particle
The complex index of refraction was $m = n - ik$, where the symbols $n$ represented the index of refraction and the symbol $k$ represented the index of absorption \cite{32}. Additionally, the size parameter was $\chi = \frac{\pi D}{\lambda}$, where the symbol $D$ represented the diameter of the particles, and $\lambda$ represented the incoming wavelength. The spectral complex index of refraction of the SiO$_2$ and Polystyrene material for wavelengths between 400 nm and 850 nm were presented in Figure 2 \cite{35}.

![Figure 2. Spectral complex refractive of Polystyrene and SiO$_2$ in the wavelength range of 400 nm-850 nm [35].](image)

When a single spherical particle with a constant diameter $D_i$ interacts with an electromagnetic wave of wavelength $\lambda$, the extinction efficiency factor $Q_{ext}$, scattering efficiency factor $Q_{sca}$, and scattering phase function $\Phi(\Theta)$ were governed by the Maxwell’s equations and calculated using the Mie theory \cite{32}.

When the nanoparticles were dispersed in the matrix, the relative size parameter, and the normalized index of refraction of the particles in the matrix were calculated using the method used in \cite{32}.

In Eqs. (1) and (2), the symbol $n_f$ represented the index of refraction of the matrix, and the symbols $n_p$ and $k_p$ represented the indexes of refraction and absorption of the particle, respectively.

$$\chi_{rel} = \frac{\pi D n_f}{\lambda}$$  \hspace{1cm} (1)
\[ m_{\text{rel}} = \frac{n_p - ik_p}{n_i} \]  

In Eq. (3), the symbol \( g \) represented the asymmetry factor, it was used to describe the directional scattering behavior and was related to the phase function as follows.

\[ g_p = \cos \Theta = \frac{1}{4\pi} \int_{4\pi} \Phi_p(\Theta) \cos \Theta d\Omega \]  

2.3. Radiative characteristics of multi-particle system

Firstly, for determining the radiative characteristics of same type of particles, the following formulas Eq. (4) and Eq. (5) describing the radiative characteristics of a single type particle system with particle of different sizes were expressed as:

\[ \beta = \sum_{i=1}^{n_i} N_i C_{e,i} = \frac{\pi}{4} \sum_{i=1}^{n_i} D_i^2 N_i Q_{e,i} = 1.5 \sum_{i=1}^{n_i} \frac{Q_{e,i} f_{e,i}}{D_i} \]  

\[ \sigma_s = \sum_{i=1}^{n_i} N_i C_{s,i} = \frac{\pi}{4} \sum_{i=1}^{n_i} D_i^2 N_i Q_{s,i} = 1.5 \sum_{i=1}^{n_i} \frac{Q_{s,i} f_{s,i}}{D_i} \]  

To calculate the radiative characteristics of a multi-particle system with same type particles of uniform diameter, the following formulas Eq. (6) and Eq. (7) for were provided for reference [32].

\[ \beta = \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} N_{j,i} C_{e,j,i} = \frac{\pi}{4} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} D_j^2 N_{j,i} Q_{e,j,i} = 1.5 \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \frac{Q_{e,j,i} f_{e,j,i}}{D_j} \]  

\[ \sigma_s = \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} N_{j,i} C_{s,j,i} = \frac{\pi}{4} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} D_j^2 N_{j,i} Q_{s,j,i} = 1.5 \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \frac{Q_{s,j,i} f_{s,j,i}}{D_j} \]  

Finally, combine the previous expression Eq. (4)-(7), for calculating the radiative characteristics of a multi-particle system, the following formulas Eq. (8) and Eq. (9) were expressed:

\[ \beta = \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} N_{j,i} C_{e,j,i} = \frac{\pi}{4} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} D_j^2 N_{j,i} Q_{e,j,i} = 1.5 \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \frac{Q_{e,j,i} f_{e,j,i}}{D_j} \]  

\[ \sigma_s = \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} N_{j,i} C_{s,j,i} = \frac{\pi}{4} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} D_j^2 N_{j,i} Q_{s,j,i} = 1.5 \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \frac{Q_{s,j,i} f_{s,j,i}}{D_j} \]  

Where the subscript \( j \) represented the type of particles, the subscript \( i \) represented the size of the particle, the \( n \) represented the number of particle sizes, the symbol \( N \) represented the number density of particles, \( C_{e,j,i} \) and \( C_{s,j,i} \) represented the extinction area and scattering area of the particle with the \( j \)-th type and \( i \)-th size. Furthermore, \( Q_{e,j,i} \) and \( Q_{s,j,i} \) were the extinction factor and scattering factor of the particle with the \( j \)-th type and \( i \)-th size. The symbol \( f_{e,j,i} \) was the volume fraction of the particle with the \( j \)-th type and \( i \)-th size.

The scattering phase function \( \Phi(\Theta) \) of a multi-particle system can be obtained by integrating the diameter \( D_{j,i} \) and expressed as:

\[ \Phi(\Theta) = \frac{1}{\sigma_s} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \sigma_{s,j,i} \cdot \Phi_{j,i}(\Theta) = \frac{1}{\sigma_s} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \frac{Q_{s,j,i} f_{s,j,i}}{D_{j,i}} \cdot \Phi_{j,i}(\Theta) \]  

The asymmetry factor \( g \) of the multi-particle system with different diameter and spherical particles can be modified as [32]:
\[
g = \frac{1}{2} \int_{0}^{\pi} \Phi(\theta) \cos \theta \sin \theta d\theta = \frac{\pi}{2} \sum_{h=1}^{n_{h}} \Phi(\theta_{h}) \cos \theta_{h} \sin \theta_{h} \tag{11}
\]

\[
g_{j,i} = \frac{1}{2} \int_{0}^{\pi} \Phi_{j,i}(\theta) \cos \theta \sin \theta d\theta = \frac{\pi}{2} \sum_{h=1}^{n_{h}} \Phi_{j,i}(\theta_{h}) \cos \theta_{h} \sin \theta_{h} \tag{12}
\]

Where \( n_{h} \) represented the number of discrete angle shares, \( g_{j,i} \) represented the asymmetry factor of spherical particles of \( j \)-th type and \( i \)-th diameter.

From Eq. (10)-(12), the asymmetry factor of the multi-particle system can be expressed as follows:

\[
g = \frac{1}{\sigma_{s}} \sum_{j=1}^{n_{s}} \sum_{i=1}^{n_{i}} \sigma_{s,j,i} \cdot g_{j,i} \tag{13}
\]

2.4. Radiative characteristics of semi-transparent medium with multi-particles

For a semi-transparent medium (i.e., water, glass, and air) with particles, the spectral absorption coefficient can be calculated using Eq. (14):

\[
\kappa_{\lambda,j} = \frac{4\pi k_{i}}{\lambda} \tag{14}
\]

Where \( k_{i} \) represented the index of absorption of the semitransparent medium.

The total scattering coefficient \( \sigma_{\lambda} \), absorption coefficient \( \kappa_{\lambda} \), and extinction coefficient \( \beta_{\lambda} \) of the semi-transparent medium with particles were given by:

\[
\sigma_{\lambda} = \sigma_{s,p,N} \tag{15}
\]

\[
\kappa_{\lambda} = \kappa_{t,j} + \kappa_{p,N} \tag{16}
\]

\[
\beta_{\lambda} = \sigma_{\lambda} + \kappa_{\lambda} \tag{17}
\]

When the radiative characteristics of the semi-transparent medium with particles were obtained, the RTE can be solved by the MCRT method. When the radiative characteristics of a cooling coating were obtained, the calculation model of the radiative transfer can be used. The RTE to describe the spectral radiative transfer of particles in a semitransparent medium can be expressed as follows [32].

\[
\frac{dI_{\lambda}(s)}{ds} = -k_{\lambda}I_{\lambda}(s) - \sigma_{\lambda}I_{\lambda}(s) + k_{\lambda}I_{\beta,\lambda}(s) + \frac{\sigma_{\lambda}}{4\pi} \int_{4\pi} I_{\lambda}(s,\omega')\Phi_{\lambda}(\Omega) d\Omega' \tag{18}
\]

Where \( I_{\lambda} \) was the spectral radiative intensity in the direction of \( \Omega \) along path \( s \); \( I_{\beta,\lambda} \) represented the spectral blackbody intensity, while \( \Omega \) was the solid angle.

3. Model validation

To validate the accuracy of the DIA, the spectral transmittivity of the multi-particle system calculated by it was compared with that obtained by experimental test. The multi-particles included \( \text{SiO}_{2} \) particles and polystyrene (PS) particles for this experiment. Figure 3 showed that the particle size distribution was measured by a laser granulometer (Jnwinner-802). The average diameters of \( \text{SiO}_{2} \) particles and PS particles were \( D_{\text{SiO}_{2}}=298 \text{ nm} \) and \( D_{\text{PS}}=294 \text{ nm} \). The \( \text{SiO}_{2} \) particles and PS particles were diluted with deionized water. And the \( f_{\text{c, \text{SiO}_{2}}} \) and \( f_{\text{c, PS}} \) were 0.062\% and 0.111\%, respectively. The diluted solution was placed in a cuvette with an optical path of 1 mm. The spectral transmittivity of the diluent with different volume ratios of \( \text{SiO}_{2} \) particles and PS particles was measured by a spectrophotometer (TU-1901). The theoretical calculation parameters were obtained based on the above experimental data. The complex refractive index of water and cuvette glass were taken from Ref. [36, 37].

The comparison of experimental verification and theoretical calculation results was shown in Figure 4. The blue line represented the theoretical result calculated by the differential-integration algorithm,
while the black and red lines represented the theoretical results without considering the entity calculated by the traditional Mie theory, and the black spot refers to the experimental transmittance test result. The relative transmittance error $\tau_r$, defined as $\tau_r = \frac{|\tau_T - \tau_C|}{\tau_T} \times 100\%$, was adopted to evaluate the difference between the calculated results and the experimental data. As seen in the figure, the maximum relative transmittance error was 30.1% at 436 nm and the average relative transmittance error was 18.78% when polystyrene was ignored. However, the maximum relative transmittance error decreased from 30.1% to 0.77% and the average error decreased from 18.78% to 0.67% when the radiative characteristics of the particles were calculated by the differential-integration algorithm.

![Volume and cumulative volume distributions of SiO$_2$ and polystyrene particles measured using the laser particle analyzer.](image)

**Figure 3.** Volume and cumulative volume distributions of SiO$_2$ and polystyrene particles measured using the laser particle analyzer.

![Comparisons of directional-directional spectral transmittance of multi-particle system between the calculated results and experimental data.](image)

**Figure 4.** Comparisons of directional-directional spectral transmittance of multi-particle system between the calculated results and experimental data.

### 4. Results and discussion

The calculation wavelength was 400-850 nm considered, the total volume fraction of the particles was 0.1%, and the ratios of the two types of particles, $f_{SiO2} : f_{PS}$, were 0:4, 1:3, 2:2, 3:1, and 4:0, as shown in Tab. 1. According to the scale parameters, particles were divided into two categories: nano particles ($\chi$ <1) and submicron particles ($\chi$ >1). The particle size distribution ratio was based on the measured value, as depicted in Figure 3, the average particle sizes of the nano particles were $D=20$ nm, 40 nm, 60 nm, and 80 nm, the average particle sizes of the submicron particles were $D=200$ nm, 400 nm, 600 nm, and
800 nm. The refraction index of the substrate was equal to the spectral refractive index of water.

Table 1. Calculation parameter table

| Parameters | Wavelength | $f_v$ | $f_{SiO_2}/f_{PS}$ | $D_{average}$ ($\chi <1$) | $D_{average}$ ($\chi >1$) |
|------------|------------|-------|-------------------|--------------------------|--------------------------|
| Values     | 400-850nm  | 0.1%  | 0:4, 1:3, 2:2, 3:1, 4:0 | 20 nm, 40 nm, 60 nm, 80 nm, 200 nm, 400 nm | 600 nm, 800 nm |

4.1. Spectral transmittivity

Figure 5 presented the spectral transmittivity of a multi-particle system with nano particles ($\chi <1$) at different volume ratios. As shown in the figure, the spectral transmittance increased with the increase in wavelength. Figure 5 (a) showed that the spectral transmittance increased with the increase in the $f_{SiO_2}$: $f_{PS}$ Ratio; there was no cross point in the wavelength range of 400-850 nm. Figure 5 (b)-(d) showed that the spectral transmittance decreased with the increase in the $f_{SiO_2}$: $f_{PS}$ ratio when the wavelength was less than intersection wavelength. However, the spectral transmittance increased with the increase in the ratio of $f_{SiO_2}$: $f_{PS}$ when the wavelength was greater than intersection wavelength. Additionally, the wavelength of the transmittance cross points of $D=40$ nm, 60 nm, and 80 nm were 546 nm, 676 nm, and 779 nm, respectively. With increasing particle size, the spectral transmittance range increased and the spectral transmittance gradually decreased. In addition, the cross points of the spectral transmittance gradually moved backward with increasing particle size.

![Figure 5](#)

Figure 5. Directional-directional transmittivity of multi-particle system with two kinds of nano-
Figure 6 presented the spectral transmittivity of a multi-particle system with submicron particle sizes \( (\chi > 1) \) at different volume ratios. As shown in Figure 6 (a)-(d), the transmittance of the hybrid submicron particles system increased with the increase in the wavelength, in a way similar to that of mixed nano particles. The spectral transmittance of the mixed submicron particles gradually increased with the increase in the \( f_{SiO_2}:f_{PS} \) ratio over the entire wavelength range of 400-850 nm.

![Figure 6](image_url)

**Figure 6.** Directional-directional spectral transmittance of multi-particle system with two types of submicron particles.

4.2. Scattering phase function

Figure 7 presented the scattering phase function of a multi-particle system with nano particles \( (\chi < 1) \) at different volume ratios. As shown in Figure 7 (a)-(d), the scattering phase function changed only slightly with the change in the \( f_{SiO_2}:f_{PS} \) ratio, when the particle size parameter was small. The scattering phase function gradually showed a forward dominant trend with increasing particle size.

Figure 8 presented the scattering phase function of a multi-particle system with submicron particles \( (\chi > 1) \) at different volume ratios. As shown in Figure 8 (a)(b), when \( D \) was 200 and 400 nm, the scattering phase function did not significantly change with the increase in the volume ratio of \( f_{SiO_2}:f_{PS} \). As shown in Figure 8 (c)(d), when \( D \) was 600 nm, and 800 nm, the forward scattering effect increased with the increase in the volume ratio of \( f_{SiO_2}:f_{PS} \). At the same volume fraction, the forward scattering effect of particle system also increased with the increase in the particle size. This was consistent with the fact that the spectral transmittance decreased as the ratio of \( f_{SiO_2}:f_{PS} \) increased.
Figure 7. Scattering phase function of multi-particle system with two types of nano particles at 404 nm.

(a) $D=20$ nm    
(b) $D=40$ nm    
(c) $D=60$ nm    
(d) $D=80$ nm
4.3. Asymmetry factor

In this section, the asymmetry factor was investigated in order to further explore the scale of forward and backward scattering of particles. Figure 9 presented the asymmetry factor of a multi-particle system with nano particles (\( \chi <1 \)) at different volume ratios. As shown in Figure 9 (a)-(d), the asymmetry factor decreased with the increase of wavelength, which was mainly due to the decrease in the particle size parameter. In other words, there was a decrease in the forward scattering effect and an increase in the backward scattering effect. For nano particles, the asymmetry factor slightly increased with the increase in the volume ratio of \( f_{SiO_2}: f_{PS} \). From Figure 9 (a) to (d), it can be observed that the value of the asymmetric factor increased rapidly with the increase in the particle size. The forward scattering effect of particles also appears to be enhanced.

Figure 8 Scattering phase function of multi-particle system with two types of submicron particles at 404 nm.
Figure 9. Asymmetry factor of multi-particle system with two types of nano particles

Figure 10 presented the asymmetry factor of a multi-particle system with submicron particles ($\chi > 1$) at different volume ratios. As shown in Figure 10 (a) (b), the asymmetry factor decreased with the increase in the wavelength. Figure 10 (c) (d) showed that the asymmetry factor first increased and subsequently decreased with the increase in the wavelength. Figure 10 (a) showed that the asymmetry factor increased with the increase in the volume ratio $f_{SiO_2}/f_{PS}$. Notably, the law presented in Figure 10 (b)-(d), the asymmetry factor decreased with the increase in the volume ratio $f_{SiO_2}/f_{PS}$. From Figure 10 (a)-(d), it can be observed that the asymmetry factor increased with the increase in the particle size.
4.4. Spectral extinction coefficient

Figure 11 presented the spectral extinction coefficient of a multi-particle system with nano particles ($\chi <1$) at different volume ratios. As shown in Figure 11 (a)-(d), the spectral extinction coefficient decreased with the increase in the wavelength. Figure 11 (a) showed that the spectral extinction coefficient decreased with the increase in the $f_{\text{SiO}_2}:f_{\text{PS}}$ volume ratio. Figure 11 (b)-(d) showed that the spectral extinction coefficient increased with the increase in the volume ratio $f_{\text{SiO}_2}:f_{\text{PS}}$ when the wavelength less than wavelength of intersection. And the spectral extinction coefficient decreased with the increase in the $f_{\text{SiO}_2}:f_{\text{PS}}$ ratio when the wavelength greater than wavelength of intersection. At the same volume fraction, the spectral extinction coefficient increased with the increase in the particle size. At $D=40$ nm, 60 nm, and 80 nm, the wavelength of the cross point of the spectral extinction coefficient gradually moves backward.
Figure 11. Spectral extinction coefficient of multi-particle system with two kinds of nano particles.

Figure 12 presented the spectral extinction coefficient of a multi-particle system with submicron particles (χ > 1) at different volume ratios. As shown in Figure 12 (a)-(d), the spectral extinction coefficient also decreased with the increase in the wavelength. Unlike mixed nano particles, the spectral extinction coefficient of mixed submicron particles increased with the increasing the \( f_{SiO_2} : f_{PS} \) ratio at 400-850 nm. Furthermore, the spectral extinction coefficient first increased and subsequently decreased with the increase in the particle size.
4.5. Spectral absorption coefficient

Figure 13 and Figure 14 presented the spectral transmittance of a multi-particle system at different volume ratios. Unlike other spectral characteristic parameters, it can be seen that the variation of the submicron particles and nano particles were consistent. The spectral absorption coefficient decreased with increasing the volume ratio of $f_{SiO2}:f_{PS}$. This was attributed to the absorption index of these two particles being small. As shown in Figure 13 and Figure 14, the spectral absorption coefficient did not change significantly with the increase in the particle size. However, combined with the spectral extinction coefficient, the spectral absorption coefficient accounted for a small proportion of the spectral scattering effect, which indicated that as the particle size parameter increased, the absorption effect changed only negligibly, and the scattering effect gradually increased.

![Graphs showing spectral absorption coefficient](image)

**Figure 13.** Spectral absorption coefficient of multi-particle system with two types of nano particles
The corresponding effects can be improved

During practical applications, the particles in the semitransparent medium were generally in the form of multi-particle systems. The differential-integration algorithm developed by Mie theory was used to calculate the radiative characteristics of a multi-particle system with non-uniform size distribution. SiO₂ particles and polystyrene particles were investigated and divided into submicron particles (χ > 1) and nano particles (χ < 1) according to particle size. The effect of volume fraction ratios and particle sizes on radiative characteristics and spectral transmittance of multi-particle system were also studied. The following conclusions can be drawn:

1. The maximum relative transmittance error decreased from 30.1% to 0.77%, and the average error decreased from 18.78% to 0.67%, when the radiative characteristics of the multi-particle system were calculated using the differential-integration method.

2. For a multi-particle system with nano particle, with the increase in the ratio of fvSiO₂: fvPS, the spectral transmittance decreased in the short wavelength range, and increased in the long wavelength range. With increasing the particle size, the spectral transmittance gradually decreased, while the forward scattering intensity and the asymmetry factor increased.

3. For a multi-particle system with submicron particle, with the increase of the fvSiO₂: fvPS ratio and the decrease of the particle size, the asymmetry factor and spectral transmittance gradually decreased, while the spectral extinction coefficient increased.

4. In the fields of radiative cooling and photocatalysis, the corresponding effects can be improved by adjusting the volume fraction and size of various kinds of particles in the multi-particle system.

Figure 14. Spectral absorption coefficient of multi-particle system with two types of submicron particles

5. Conclusion
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Nomenclature

| Symbol | Description |
|--------|-------------|
| $D$    | diameter of the particle, nm |
| $f_r$  | particle volume fraction |
| $g$    | asymmetry factor |
| $G$    | projected area, m$^2$ |
| $I_m$  | imaginary parts |
| $I$    | radiative intensity |
| $k$    | index of absorption |
| $m$    | complex index of refraction |
| $n$    | index of refraction |
| $N$    | number density of particles, m$^{-3}$ |
| $Q$    | efficiency factor |
| $S_1$  | amplitude function |
| $S_2$  | amplitude function |
| $\omega$ | scattering albedo |
| $\phi$ | azimuth angle |
| $\Theta$ | scattering angle |
| $\kappa$ | absorption coefficient, m$^{-1}$ |
| $\beta$ | extinction coefficient, m$^{-1}$ |
| $\xi$  | random number |
| $\Omega'$ | solid angle, sr |
| $\mu$  | index of refraction |
| $\mu_{abs}$ | absorption |
| $\mu_{ave}$ | average |
| $\mu_{abs}$ | absorption |
| $\mu_{b}$ | blackbody |
| $\mu_{c}$ | calculated |
| $\mu_{D-D}$ | direct-direct |
| $\chi$ | size parameter |
| $\pi_n$ | directional dependent function |
| $\tau$ | Transmittance |
| $\lambda$ | wavelength, nm |
| $\Phi$ | scattering phase function |
| $\sigma$ | scattering coefficient, m$^{-1}$ |

Subscripts

- $n$ | index of refraction |
- $b$ | blackbody |
- $c$ | calculated |
- $b$ | blackbody |
- $D$- | direct-direct |

Greek symbols

- $\chi$ | size parameter |
- $\pi_n$ | directional dependent function |
- $\tau$ | Transmittance |
- $\lambda$ | wavelength, nm |
- $\Phi$ | scattering phase function |
- $\sigma$ | scattering coefficient, m$^{-1}$ |

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