A Simple Model of Hohlraum Power Balance and Mitigation of SRS

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Abstract. A simple energy balance model has been obtained for laser-plasma heating in indirect drive hohlraum plasma that allows rapid temperature scaling and evolution with parameters such as plasma density and composition. This model enables assessment of the effects on plasma temperature of, e.g., adding high-Z dopant to the gas fill or magnetic fields.

1. Introduction
Stimulated Raman Scattering (SRS) is the resonant, three-wave coupling of a light wave into scattered light and electron plasma waves. The scaling of SRS reflectivity $R_{SRS}$ with laser intensity $I$ in a solitary laser speckle in plasma has been measured [1] and found in the electron trapping regime $k\lambda_{De} \gtrsim 0.3$ ($k$ is the EPW wave number and $\lambda_{De} = \sqrt{k_BT_e/4\pi n_e e^2}$ is the Debye length for plasma of electron density $n_e$ and temperature $T_e$) to behave nonlinearly, increasing sharply at a threshold intensity $I_{th}$ and saturating for $I > I_{th}$. The physics in this regime is governed by the growth of large-amplitude EPW that trap “resonant” electrons with speeds along the wave propagation direction matching the wave’s phase speed; this reduces local Landau damping [2] and enhances instability growth. At high intensity, trapping lowers the EPW frequency [3] and introduces variation in EPW phase velocity across the speckle and wave phase fronts bend [4, 5, 6, 7]. As EPW grow, secondary, nonlinear processes break the phase fronts into filaments of small transverse scale [4, 5, 8, 9] that further contribute to nonlinear saturation. In simulations [10], SRS saturation generates hot electrons and back- and side-scattered light waves that propagate obliquely out of hot spots and enhance SRS growth in neighboring speckles. At high gain in multi-dimensions, this coupling enables networks of speckles to self-organize [10] and exhibit emergent behavior where reflectivity exceeds that of the sum of contributions from individual, non-interacting speckles. The nonlinear nature of SRS in this regime is robust, with a threshold at modest laser intensity, $\gtrsim 10^{14}$ W/cm$^2$ for NIF laser conditions where $k\lambda_{De} \approx 0.3$ and the highest levels of backscatter are found [11]. Saturated SRS reflectivity scales with electron temperature $R_{SRS} \sim (k\lambda_{De})^{-4} \sim T_e^{-2}$ [12], so increasing plasma temperature (which remedies inverse bremsstrahlung losses) would likely benefit beam propagation, particularly in the NIF inner beams, which show sizable backscatter.

2. Simplified Model of Beam Energy Balance
Consider a cylindrical laser beam of radius $r$ and length $\ell$ with intensity $I$ propagating through optically thin plasma with electron density $n_e$; we assume the plasma is heated dominantly
by inverse bremsstrahlung absorption of laser light and it cools by thermal conduction, bremsstrahlung, and, if magnetized, cyclotron radiation. For a thin, cylindrical shell of plasma, 

$$\frac{d}{dt} \left[ \frac{3}{2} (2\pi r \, dr) \sum_i k_B (Z_i T_e + T_i) n_i \right] = P_{ib} + P_{br} + P_{cy} + P_{VT},$$

(1)

where \( T_e \) and \( T_i \) are electron and ion temperatures (in keV) and the terms on right are the power gained or lost through inverse bremsstrahlung, bremsstrahlung emission, cyclotron radiation, and thermal conduction, respectively. We denote the thermal gradient heat flux \( q(r) \) at radius \( r \). The summation is over ions \( i \), whose densities and ionization states are \( n_i \) and \( Z_i \), respectively. We shall ignore \( P \, dV \) work done on or by the plasma blob, taking the plasma to be of constant density throughout the evolution, we assume no variation in ionization state, and for simplicity we assume \( T_e \sim T_i \equiv T \). This leads to a simple evolution equation for the temperature \( C_V \, dT/dt = P_{ib} + P_{br} + P_{cy} + P_{VT} \), where \( C_V = \frac{3}{2} k_B (2\pi r \, dr) \sum_i (Z_i + 1) n_i \) is the plasma heat capacity. For light of intensity \( I \) propagating through optically thin plasma, the inverse bremsstrahlung power absorbed [13] is 

$$P_{ib} = 2\pi r \, dr \, I(1 - e^{-\kappa_{ib} \ell}) \approx 2\pi r \, dr \, I(\kappa_{ib} = 2\pi r \, dr \, I(\kappa_{ib}(T/T_0))^{-3/2}$$

(2)

for inverse bremsstrahlung absorption coefficient [13]

$$\kappa_{ib0} = 9.8 \times 10^{-12} \frac{\sum n_i Z_i (\sum n_i Z_i^2) \omega_0^{-2} \left( 1 - \frac{n_e}{n_{cr}} \right)^{-1/2} T_0^{-3/2} \log \Lambda \, \text{cm}^{-1}.$$

(3)

Here \( \omega_0 = 2\pi c/\lambda_0 \), \( n_{cr} \) is the critical density, \( \log \Lambda \) is the Coulomb logarithm, and \( T_0 \) is the temperature on axis. The plasma radiates power away as x-rays at a rate \( P_{br} = 5.3 \times 10^{-31} (\sum n_i Z_i) (\sum n_i Z_i^2) T(r)^{1/2} 2\pi r \, dr \, T_0 \) Watts. Magnetized plasma also emits cyclotron radiation \( P_{cy} = 6.21 \times 10^{-17} (\pi r^2 \, \text{beam} \, \ell) B^2 n_e T \) Watts, where \( B \) is the magnetic field in Tesla. Generally, one can show that for indirect drive ignition hohlraum plasma, we may omit \( P_{br} \) and \( P_{cy} \) relative to absorbed inverse bremsstrahlung power.

The dominant loss term is thermal conduction. The heat flux is [14]

$$q(r) = -k_0 \alpha \nabla T$$

$$= -\frac{2}{\pi} k_0 \alpha \hat{r} \frac{\partial}{\partial r} \left( \frac{T(r)}{T_0} \right)^{7/2}$$

(4)

where

$$k_0 = (9.4 \times 10^{12} \, \text{Watts}) \overline{S(Z)} \left( \sum n_i Z_i \right) T_0^{5/2} \left( \sum n_i Z_i^2 \right)^{-1} (\log \Lambda)^{-1},$$

(5)

with \( \overline{S(Z)} \) representing the “Spitzer function,” of order unity, and \( \alpha \leq 1 \) is a constant that models the inhibition of thermal conductivity from magnetic insulation [15]; unmagnetized plasma has \( \alpha = 1 \). In general, \( \alpha \) is a complicated function of temperature, magnetic field orientation and strength, and plasma density (e.g., a model interpolating between the magnetized and unmagnetized results from classical transport theory \( \alpha = [1 + (\omega_{ce} \tau_e)^2]^{-1} \), where \( \omega_{ce} = eB/m_e c \) and \( \tau_e \) is the electron collision time, has been applied successfully in magnetized laser-plasma settings [16]). However, for our purposes here we shall assume \( \alpha \) is a constant. Taylor-expanding the heat flux on the outer and inner sides of a thin, cylindrical shell of plasma of thickness \( dr \) and dropping \( \mathcal{O} (dr^2) \) terms results in the net conductive heat loss of the shell

$$P_{VT} = -2\pi \ell \left[ (r + dr) \hat{r} \cdot q(r + dr) - r \hat{r} \cdot q(r) \right]$$
Figure 1. (Left) Solution to radial temperature profile equation (8) with $\eta = \eta_0 \approx 3.45$. (Center) Temperature at the center of the beam as a function of neon ion number density fraction (in percent). (Right) Temperature at the center of the beam for different values of $\alpha$, modeling thermal conductivity inhibition by magnetic fields. In the rightmost two panels we see that increasing beam power, adding neon dopant and suppressing thermal conductivity (e.g., through the introduction of magnetic fields into the hohlraum plasma) all indicate that substantial increases in plasma temperature may be possible.

Figure 1 (Left) Solution to radial temperature profile equation (8) with $\eta = \eta_0 \approx 3.45$. (Center) Temperature at the center of the beam for different values of $\alpha$, modeling thermal conductivity inhibition by magnetic fields. In the rightmost two panels we see that increasing beam power, adding neon dopant and suppressing thermal conductivity (e.g., through the introduction of magnetic fields into the hohlraum plasma) all indicate that substantial increases in plasma temperature may be possible.

The plasma temperature profile responds rapidly (of order 10s of ps) to changing plasma or laser conditions, so the inverse bremsstrahlung heating and thermal conductive cooling nearly balance and we may solve for the radial temperature profile as a function of plasma and laser parameters. Let us define $\rho \equiv r/r_{\text{beam}}$, $\tau \equiv (T/T_0)^{7/2}$, and

$$\eta \equiv \frac{7r_{\text{beam}} I_{\kappa i0}}{2\alpha k_0}.$$  

Equating (2) and (6), we arrive at the nonlinear ordinary differential equation (ODE)

$$\rho \tau'' + \tau' + \eta \rho^{3/7} = 0,$$

where primes denote $d/d\rho$. The boundary conditions are that $\tau(0) = 1$ and $\tau'(0) = 0$. This ODE has a singular point as $\rho \downarrow 0$, so numerical integration requires that we match to a boundary layer solution near the origin, which can be shown to be of the form $\tau(\rho) \sim 1 - (\eta\rho^2/4) + O(\rho^4\eta^2)$. Numerical integration in $\rho$ obtains that for the specific value of the eigenvalue $\eta = \eta_0 \approx 3.45$, the boundary condition $\tau(1) = 0$ is satisfied (a radial power-balance boundary condition analogous to the one in Lindl’s model of an ignition hot spot [14]). The resulting temperature profile as a function of radius is shown in the left panel of Fig. 1.

Setting $\eta = \eta_0$ in (7) allows us to solve for the core temperature $T_0$ and determine its dependence on physics parameters within our model. Let $P_{12}$ be the beam power in units of TW, $n_{21}$, the ion density in units of $10^{21}$ cm$^{-3}$, $\lambda_{\mu m}$, the wavelength in microns, and $Z^2 = \sum_i n_i Z_i^2 / \sum_i n_i$, and let us take $S(Z) \approx 1.5$ for a dominantly helium gas fill. Then,

$$T_0 \approx (0.67 \text{ keV}) \left[ P_{12} n_{21} \lambda_{\mu m}^2 (\log \Lambda)^2 Z^2 \alpha^{-1} \right]^{1/5}.$$  

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3. SRS Mitigation
Let us examine (9) and consider the various ways we might increase $T_0$ and thus decrease SRS reflectivity. Let us consider nominal laser and hohlraum plasma parameters $\lambda_{\text{em}} = 0.351, n_{21} = 0.5$, and $\log \Lambda \approx 5$, typical NIF hohlraum parameters during high SRS. An obvious way would be to increase beam power, though this is limited when near the maximum rated power on an ICF facility. A second way would be through introducing trace amounts of high-Z dopant, noting that up to around 0.5% number density fraction of neon can be added without freezing out into the cryo environment of an ignition hot spot [17]. The center panel of Fig. 1 shows the resulting increase in $T_0$ as a function of neon gas fill number density fraction. A third way would be through decreasing thermal conductivity through addition of a magnetic field, as illustrated in the rightmost panel of Fig. 1. Finally, one could consider going to green light.

Note that because we’ve assumed cold surrounding plasma to provide our boundary condition on the ODE, this leads to a numerically larger eigenvalue $\eta_0$ than if we had finite temperature surrounding plasma. Physically, this implies enhanced thermal conductive losses and a predicted $T_0$ is somewhat lower than in experiment. Nevertheless, we believe the power balance model a reasonable sense of the trade-space for increasing hohlraum plasma temperature. Based on the results of this model, it appears that a significant increase in $T_0$ may be obtained from the application of one or a combination of these approaches and that they may be fruitful avenues of further exploration.

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