The Parton Structure of the Nucleon and Precision Determination of the Weinberg Angle in Neutrino Scattering

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A recently completed next-to-leading-order program to calculate neutrino cross sections, including power-suppressed mass correction terms, has been applied to evaluate the Paschos-Wolfenstein relation, in order to quantitatively assess the validity and significance of the NuTeV anomaly. In particular, we study the shift of sin^2θW obtained in calculations with a new generation of PDF sets that allow s(x) ≠ ˉs(x), enabled by recent neutrino dimuon data from CCFR and NuTeV, as compared to the previous s = ˉs parton distribution functions like CTEQ6M. The extracted value of sin^2θW is closely correlated with the strangeness asymmetry momentum integral \( \int_0^1 x[s(x)− ˉs(x)]dx \). We also consider isospin violating effects that have recently been explored by the MRST group. The results of our study suggest that the new dimuon data, the Weinberg angle measurement, and other data sets used in global QCD parton structure analysis can all be consistent within the Standard Model.

PACS numbers: 13.15.+g, 13.60.Hb

Introduction: An important open question in particle physics in recent years has been the significance of the “NuTeV anomaly”—a 3σ deviation of the measurement of sin^2θW (0.2277 ± 0.0013 ± 0.0009) reported in Ref. [1], from the world average of other measurements (0.2277 ± 0.0004). Possible sources of the NuTeV anomaly, both within and beyond the standard model, have been examined in [2]. No consistent picture has yet emerged in spite of extensive literature [3, 4, 5, 6, 7, 8] on this subject. The measurement in Ref. [1] was based on a correlated fit to the ratios of charged and neutral current (CC & NC) interactions in sign-selected neutrino and anti-neutrino scattering events on a (primarily) iron target at Fermilab. This procedure is closely related to measuring the Paschos-Wolfenstein ratio \( R^- \), which provides the theoretical underpinning of the analysis. Specifically, the Paschos-Wolfenstein ratio \( R^- \) is related to the Weinberg angle \( θ_W \) by

\[
R^- ≡ \frac{\sigma_{NC}^s - \sigma_{NC}^d}{\sigma_{CC}^s - \sigma_{CC}^d} \approx \frac{1}{2} - \sin^2θ_W + δR^-_A + δR^-_{QCD} + δR^-_{EW}
\]

where the three correction terms are due to the non-isoscalarity of the target (δR^-_A), next-to-leading-order (NLO) and nonperturbative QCD effects (δR^-_{QCD}), and higher-order electroweak effects (δR^-_{EW}). Since \( R^- \) is a ratio of differences of cross sections, the correction terms are expected to be rather small. But at the accuracy required to test the consistency of the Standard Model, all the corrections need to be quantified as precisely as possible—similar to some previous combined perturbative/nonperturbative re-analyses [9,10] of challenges in QCD.

In this paper, we focus on QCD corrections, which are generally recognized [3, 4, 5, 6, 7, 8] to be the least well known. Let us write

\[
δR^-_{QCD} = δR^-_s + δR^-_t + δR^-_{NLO}
\]

where the three terms on the right-hand side are due to possible strangeness asymmetry (\( s^- = s - ˉs ≠ 0 \)) and isospin violation (\( u_{p,n} ≠ d_{n,p} \)) effects in the parton structure of the nucleon, and NLO (\( O(α_s) \)) corrections, respectively [24]. The original NuTeV analysis was carried out at LO in QCD and assumed \( δR^-_s = 0 = δR^-_t \). Our analysis is based the recent NLO calculation of [12], together with new parton analyses that explicitly allow strangeness asymmetry (\( δR^-_s ≠ 0 \)) [13] and isospin violation (\( δR^-_t ≠ 0 \)) [14]. The actual calculation is carried out at the cross section level, i.e., using the first line of Eq. [1] rather than using the schematic linearized form given in the second line of Eq. [1] and Eq. [2]. Our results provide more realistic estimates of the sizes and uncertainties of the QCD corrections, and a new look at the significance
of the “anomaly.” (Cf. also a recent re-evaluation of the electroweak correction to the calculation of $R^-$ [12].)

**NLO Calculation:** At sufficiently high neutrino energy, the total neutrino cross section

$$\sigma^\nu \equiv \sigma^{\nu N \rightarrow lX} = \int d^3p_t \frac{d^3\sigma^{\nu N \rightarrow lX}}{d^3p_t}$$

(3)

can be calculated in QCD perturbation theory—in contrast to charged lepton scattering, where the massless photon propagator leads to dominance of nonperturbative photoproduction events over deep inelastic scattering. The differential cross section in Eq. (3) factorizes into a sum of convolutions of parton distribution functions and partonic cross sections

$$d^3\sigma^{\nu N \rightarrow lX} = \sum_{f=q,g} f \otimes d^3\sigma^{f \rightarrow lX}.$$  

(4)

This calculation has been performed at NLO accuracy in Ref. [23]. The analysis included target and charm mass effects. These corrections are needed to obtain reliable results because there are non-negligible contributions from low $Q$ values to the integral in Eq. (3)—e.g., about 5% from $Q^2 < 1$ GeV$^2$ for $\sigma^c_{CC}$ and from $Q^2 < 2$ GeV$^2$ for $\sigma^g_{CC}$. (For NC neutrino events, it is not possible to exclude the low-$Q$ region by experimental kinematic cuts.)

Other corrections included are the non-isoscalarity of the target material (iron), i.e., $\delta R^\alpha$ in (4); energy averaging over the neutrino and anti-neutrino flux spectra; and cuts in hadronic energy ($20$ GeV $< y_{E\nu} < 180$ GeV for lepton inelasticity $y$) as used in the experimental analysis [4].

Ref. [23] used previously available parton distributions [16, 17], all of which assume isospin symmetry and $s = \bar{s}$ symmetry within the nucleon. The study confirmed the smallness [20] of the higher order corrections to $R^-$ in general. (The same conclusion is reached by the NLO and NNLO moment analyses of [8, 9, 13].) It was also shown that the non-monochromatic neutrino and anti-neutrino beams, with different profiles, and typical cuts in the hadronic event energy do not alter $\delta R^\alpha_{NLO}$ substantially.

In the next two sections, we will examine shifts of the parton distributions that allow strangeness asymmetry and isospin violation.

In principle, the parton distribution functions in Eq. (4) should be those of nuclear targets. Our calculation is done as an incoherent sum of contributions from parton densities of unbound nucleons. This approximation is reasonable in that we only calculate relative shifts between $[S^-] = 0$ and $[S^-] = 0$ PDFs (where $[S^-]$ is defined in Eq. (7)); similarly for isospin. In fact, experimental information on nuclear PDFs is relatively scarce, and nuclear PDFs only account for leading twist $2$ ($\tau = 2$) effects. Higher twists, whether they relate to nuclear modifications or not, are generally difficult to handle consistently. By limiting ourselves to $\tau = 2$, our error estimates may be underestimates.

**Strangeness Asymmetry:** Because the strange quark mass $m_s$ is comparable to $\Lambda_{QCD}$, the strange quark PDF is a nonperturbative component of the nucleon bound state. Except for the strangeness number sum rule,

$$\int [s(x) - \bar{s}(x)] \, dx = 0,$$  

(5)

there is no fundamental or approximate symmetry that relates the strange quark PDF $s(x)$ to the antiquark PDF $\bar{s}(x)$. Limits on $s^- = s(x) - \bar{s}(x)$ can, therefore, only be derived from data (or perhaps eventually from a lattice QCD calculation). Until recently, $s^-$ has been largely unknown and usually assumed to vanish. However, the recently published CCFR-NuTeV data on dimuon cross sections in $\nu N$ and $\bar{\nu} N$ scattering yield a direct handle on $s(x)$ and $\bar{s}(x)$, and hence on $s^- [10]$, because the dimuon data reflect semileptonic decays of the charm quark in $W^+ s \rightarrow c$ and $W^- \bar{s} \rightarrow \bar{c}$ events.

An asymmetric strange sea in the nucleon ($s^- \neq 0$) contributes to a correction term to $R^-$ at LO [8]. If the scale dependence of the parton distributions is neglected, i.e. $f(x, Q) \simeq f(x)$, and in the approximation of overlooking experimental cuts, the total cross section in Eq. (6) is sensitive to the second Mellin moment integrals $\int dx f(x)$ of the PDFs [6, 7]. Making the further approximation of an isoscalar target, and in the limit of a negligible charm quark mass, a strange sea asymmetry contributes at LO as

$$\delta R_s^- \simeq - \left( \frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \frac{[S^-]}{[Q^-]},$$  

(6)

where the strangeness asymmetry is quantified by

$$[S^-] = \int x [s(x) - \bar{s}(x)] \, dx ;$$  

(7)

and $[Q^-] = \int x[q(x)-\bar{q}(x)]dx$ with $q(x) = (u(x)+d(x))/2$ represents the isoscalar up and down quark combination.

By including the dimuon data, and by exploring the full allowed parameter space in a global QCD analysis, Ref. [10] presents a general picture of the strangeness sector of nucleon structure. The strong interplay between the existing experimental constraints and the global theoretical constraints, especially the sum rule [8], places useful limits on acceptable values of the strangeness asymmetry momentum integral $[S^-]$. The limit quoted in [10] is $-0.001 < [S^-] < +0.004$. A large negative $[S^-]$ is strongly disfavored by both dimuon and other inclusive data. The strict sum rule [13] implies that a non-zero $s^-(x)$ function must change sign at least once. Studies in [13] demonstrate that the exact value of $[S^-]$ is a volatile quantity. The best fit “B” is a solution where negative $s^-(x)$ at low $x$ is compensated by positive $s^-(x)$ at large $x$; this leads to positivity of the second moment integral in Eq. (7). The same trend had previously been observed in a fit to inclusive neutrino scattering [4]. Also, this behavior was anticipated by a dynamical model [13] based
TABLE I: Shifts in $R^-$, calculated with PDF sets of Ref. [13] (with non-zero $[S^-]$) compared to the value with the CTEQ6M set ($[S^-] = 0$), are given in the last column. The quality of these new fits is gauged by the relative $\chi^2$ values (normalized to that of the reference set “B”) for the dimuon data set [24] and for the subset of global data set which have some sensitivity to $s^-(x)$ (labeled “inclusive I”). See [13] for details.

The values of $\delta R^-_s$ in Table I are also approximately unchanged when the cut on $q y E_\nu$ is eliminated. These findings suggest that the incorporation of other detector effects [12,21], which make the analysis in Ref. [13] somewhat more involved than a direct measurement of $R^-$, will not significantly impact the importance of the $[S^-]$ contribution to $\sin^2 \theta_W$ [23].

The shift in $\sin^2 \theta_W$ corresponding to the central fit B bridges a substantial part of the original $3 \sigma$ discrepancy between the NuTeV result and the world average of other measurements of $\sin^2 \theta_W$. For PDF sets with a shift toward the negative end, such as $-0.004$, the discrepancy is reduced to less than 1 $\sigma$. On the other hand, for PDF sets with a shift toward the positive end, such as $+0.001$, the discrepancy remains.

More input on $s^-(x) = s(x) - \bar{s}(x)$ would, of course, be helpful in pinning down the contribution of strangeness asymmetry to $\delta R^-$. Measurements of associated production of charmed jets and $W^\pm$ bosons at the Tevatron, at RHIC, or at the future LHC, would increase our knowledge of $s(x)$ and $\bar{s}(x)$ (cf. [22]). It will help that the “valence” density $s^-(x)$ is more easily accessible than the predominantly singlet $s(x) + \bar{s}(x)$, which is concentrated at small $x$; however, the low expected statistics will make this measurement extremely challenging. In principle it seems also feasible to study $s(x) - \bar{s}(x)$ on the lattice [23]. Unfortunately, the most relevant moment $[S^-]$ does not correspond to a local operator and cannot be calculated on the lattice.

Possible Isospin Violation: Isospin symmetry holds to a good approximation in low energy hadron spectroscopy and scattering, but it is not an exact symmetry. The level of accuracy of the usual assumption of isospin symmetry at the parton level, e.g. $u_\nu = d_\nu$ and $\bar{u}_\nu = \bar{d}_\nu$, is largely unknown. Isospin symmetry violation effects at the parton level contribute a shift of the P-W ratio $R^-$ by

$$\delta R^-_I \sim -\left( \frac{1}{2} - \frac{7}{6} \sin^2 \theta_W \right) \frac{[D_N^F - U_N^F]}{[Q^-]}$$  \hspace{1cm} (10)
the determination of $\sin^2 \theta_W$ via the measurement of $R^-$ is thus subject to a non-negligible uncertainty due to isospin violation.

To make this point more concrete, we have applied the candidate PDFs from [14] to our NLO calculation, in the same spirit as the study of strangeness asymmetry discussed above. We find that the range of allowed $\kappa$ parameter given in [14], $-0.7 < \kappa < 0.7$, implies

$$-0.007 \lesssim \delta_R \lesssim 0.007,$$

and the best fit value of $\kappa = -0.2$ corresponds to a shift of $\delta_R = -0.0022$. A one-parameter functional form may not be general enough to pin down the true isospin violations of the parton structure. Nevertheless, the large range of $\delta_R$ in Eq. (11) indicates that a reasonable theoretical uncertainty due to isospin violation needs to be assigned to the determination of $\sin^2 \theta_W$.

Conclusion: The uncertainties in the parton structure of the nucleon that relate to $R^-$ will not decrease substantially at any time soon. The uncertainties in the theory that relates $R^-$ to $\sin^2 \theta_W$ are substantial on the scale of precision of the high statistics NuTeV data [1]. Within their bounds, the results of this study suggest that the new dimuon data, the Weinberg angle measurement, and other global data sets used in QCD parton structure analysis can all be consistent within the standard model of particle physics.

Acknowledgments: We thank members of NuTeV collaboration, particularly K. McFarland, for interesting discussions; P. Gambino for discussions and useful comments; and R. Thorne for discussions and for providing grids of the PDFs in [14]. This research was supported by the National Science Foundation (grant No. 0100677), the U.S. Department of Energy (Contracts No. DE-FG03-95ER40908 and No. FG02-91ER40664), and by the Lightner-Sams Foundation. S. K. is grateful to RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy (contract No. DE-AC02-98CH10886) for providing the facilities essential for the completion of this work.

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[26] Note, however that NLO effects may be more important for the analysis that does not measure $R^-$ directly.
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[28] To estimate the size of detector-dependent effects, we have calculated $\delta \sin^2 \theta_W$ using the prescription of [21], summarized in the functional $\int F(\sin^2 \theta_W, s - s; x)dx$. The shifts are within 30% of those presented above.