1. Introduction

The $\alpha$-decays of nuclei-chronometers from the only ground states are usually taken into account by the practical applications of the standard methods of nuclear chronometry for the stellar and terrestrial processes. The nuclear cosmic nucleo-synthesis includes also the analysis of formation of the initial long-living isotopes during $s$- and $r$-processes. In these processes, inside stars and supernova not only the ground states but also all possible excited states of synthesized nuclei are being formed as a result of nucleon radiation captures $(n,\gamma)$ and $(p,\gamma)$ (see, for instance, [1-3]).

And from the Geiger and Nutall $\alpha$-decay law it follows directly that the lifetime $\tau_{\text{exc}}$ of the $\alpha$-decaying nucleus does very strongly depend on the $\alpha$-particle kinetic energy. In many cases the lifetime $\tau_{\text{exc}}$ is diminished by several orders with the increasing of the $\alpha$-particle energy by 1-2 MeV! But up to now no systematic experimental study of the excited radioactive nuclei relative to the $\alpha$-decays on their excitation energy had been undertaken in the analysis of lifetimes because of the much more rapid (within $10^{-13}-10^{-9}$ sec) and so strong $\gamma$-decays of the excited nuclei. Previously it had usually assumed that there is no practical reason to take into account the much more slow and so weak processes of the $\alpha$-decays from the excited states. But if there are chains of subsequent emissions and quasi-resonance absorptions of $\gamma$-quanta by the $\alpha$-radioactive nuclei inside stellar masses, then the influence of the excited $\alpha$-decaying nuclei can be much stronger.

Of course, there are the emissions-absorptions chains of excited nuclei $\gamma$-decays and also $\beta$-decays of $\beta$-radioactive chronometers in stars. It had been firstly supposed in [4,5] that inside large masses of stellar substance a part of radioactive nuclei could be supported in the excited states during a long time due to the chains of subsequent emissions and quasi-resonance absorptions of $\gamma$-quanta by nuclei-chronometers if the energy losses, caused by recoils during
emitting and absorbing, are compensated by the nucleus thermal kinetic energy. Further in [6, 7] this supposition it had been continued for modification of the nuclear chronometry in stars and it had been expanded for planets. The formation of the excited states of the $\alpha$-radioactive nuclei in the terrestrial surface layers and in the meteorites is also present under the influence of the weak but constant cosmic radiation. And it has been supposed that the cosmic radiation can also accelerate the $\alpha$-decays.

As to the $\beta$-radioactive chronometers, we shall say later (at the end of the section **Nuclear chronometry in astrophysics**) that sometimes the lifetimes of them become near $10^9$ lesser in a bare (without electrons) atom than in the usual atom with filled electronic shell. And inside the stars these nuclei partially or totally are being deprived their electronic shells.

2. To the standard nuclear chronometry

Here we shall deal with $\alpha$-radioactive nuclei-chronometers. In the standard nucleo-chronometry techniques (see, for instance, [1]) one uses the abundances $P$ and $D$ of parent and daughter nuclei. They are connected with the decay function $L(t-t_0)=\exp(-\Gamma_0(t-t_0)/\hbar)$ ($\Gamma_0$ being the $\alpha$-decay width of the ground state, $t_0$ being an initial time) and the surviving function $W(t)=1-L(t)$, defined and utilized in [4,5], by the following evident relations:

$$L(t-t_0)=P(t)/P(t_0)\quad W(t-t_0)=[P(t)-P(t_0)]/P(t_0)$$

and

$$P(t)+D(t)=P(t_0)+D(t_0),$$

or

$$D(t)-D(t_0)=P(t_0)W(t-t_0)=P(t)[\exp(\Gamma_0(t-t_0)/\hbar)-1].$$

or

$$D(t)-D(t_0)=P(t_0)-P(t)=P(t_0)[1-\exp(\Gamma_0(t-t_0)/\hbar)-1].$$

Usually equality (3) is divided by the abundance of another stable isotope $D_x$ which does not obtain any contribution from the decay of the parent nuclei (i.e. $D_x$ does not depend from time). As a result, (3) acquires the following form

$$P(t)[\exp(\Gamma_0(t-t_0)/\hbar)-1]-d(t)+d(t_0)=0$$

(5)
where \( p = P/D_x \) and \( d = D/D_x \). Measuring \( p = P/D_x \) and \( d = D/D_x \) in different samples (or in different separate parts of the same sample), we obtain the plot (2b) at the plane \( p = P/D_x, d = D/D_x \) which has the form of a straight line, the slope of which with respect to axis \( p \) does simply permit to define age \( t - t_0 \) (see, for instance, [1] and Fig. 1 here).

For the description of the \( \alpha \)-decay evolution inside stars and under the cosmic radiation on the earth surface one usually uses the Krylov-Fock theorem [8] (see also its application during the generalization in [4,5-7,9]):

\[
L \ (t - t_0) = < \Psi(t - t_0) \mid \Psi(t_0) > \mid^2 = \mid f(t - t_0) \mid^2 / \mid f(t_0) \mid^2 
\]

(6)

where

\[
f(t - t_0) = < \Psi(t - t_0) \mid \Psi(t_0) > = \int_{E_0}^{E_f} \mid G(E) \mid^2 \exp \left( -iE(t - t_0)/\hbar \right) dE
\]

(7)

is the characteristic function for the energy \( E \) distribution in a decaying state with the weight amplitude \( G(E) \), \( t_0 \) is the chosen initial time moment and \( \Psi(t_0) \) is normalized by the condition \( \mid \Psi(t_0) \mid = 1 \).
3. A short previous history of my investigations on the nuclear chronometry

We use in time analysis of nuclear processes elaborated on time as a quantum observable, canonically conjugate to energy, the general expression for the duration $<\tau_f>$ like [10,11]

$$<\tau_f> = <t_f> - <t_i> = \int tw_f(z_f, t) dt - t_{in}$$

where $w_f = j_f(z_f, t)/\int j_f(z_f, t) dt, j_f(z_f, t) = \text{Re}(\psi_f(-i\hbar /2\mu_f \partial /\partial z_f)\psi_f)$ is the $z_f$ the component of the probability flux; the brackets $(...)_f$ define the integration over all coordinates, entered in the wave functions $\psi_f$, besides $z_f$; $z$ is the axis, directed along the mean velocity $<\nu>$ of the relative motion of the decay pair (for instance, $\alpha$-particle and the daughter nucleus) with the reduced mass $\mu$ and their wave function of the internal motion $|\nu>$ with energy $e_\nu$; $\nu = \hbar k_\nu /\mu \epsilon_\nu \hbar^2 /2\mu \epsilon_\nu = E - e_\nu$; $t_{in}$ is defined as $<t_i>$ where $i$ is the initial channel of forming the decaying nucleus or simply as an initial time moment $t_{in}$

The wave packet of the relative motion of the decay pair in one-dimensional radial asymptotic limit is described as

$$\Psi_f(r_f, t) = A \int_0^\infty dEg(E)F_{if}(E)\exp[ikr_f - iEt /\hbar].$$

where

$$F_{if}(E) = \frac{C_{if}}{E-E_r+i\Gamma /2}.$$  

with $C_f$ being a constant or smooth function of final-particle kinetic energy $E$ in the region $(E_r - \Gamma /2, E_r + \Gamma /2)$, $E_r$ and $\Gamma$ are the resonance energy and width, respectively. For $z_f \geq R_f$ where $R_f$ is the radius of interaction in the final channel, and under condition $\Gamma \ll \Delta E$ it is possible to re-write (9) in the following simplified form

$$\Psi_f(R_f, t) = A \int_0^\infty dE \frac{\exp[-iEt /\hbar]}{E-E_r+i\Gamma /2}.$$ 

where $A$ is a constant. For $\Gamma =$ constant we obtain
\[
\psi_{\beta}(R_{\beta}, t) = \begin{cases} 
B \exp[-iE_r t / \hbar -(\Gamma / 2\hbar)t], & \text{for } t > 0 \\
0, & \text{for } t < 0 
\end{cases} 
\] (12)

(shifts, for very small \( \Gamma \), the lower limit of the integration in (11) from 0 till \(-\infty\) and utilizing the residue theorem). Here \( B \) is constant and more precisely here will be \( t - t_{\text{in}} \) instead of \( t \).

Of course, \( w_{\text{f}} \) for wave function (12) is equal

\[
L(t) = (\Gamma / \hbar) \exp(-\Gamma t / \hbar). 
\] (13)

The function \( L(t) \), generally speaking, defined by (6) and concretely by (13), sometimes are named by the decay function (see, for instance, [4-10]). The more detailed study (see, for instance, [12-14]) shows that even for an isolated resonance the exponential form of the decay law is approximately performed in the time interval, limited by below by the quantity \( t_1 \sim t_0 \Gamma / E_r \), and from above by the quantity \( t_2 \sim t_0 \ln (\Gamma / E_r) \), where \( t_0 \sim \hbar / \Gamma \), and the accuracy of describing by the exponential function is the better, the lesser is the relation \( \Gamma / E_r \) [14]. The functions (6) and (13) describe the essence of the Krylov-Fock theorem which was derived by another way (see the section before) and states that the decay law of the meta-stable state is totally defined by the energy spectrum of the initial state [8].

The author (V.S.O.) had been extended the contents of the Krylov-Fock theorem, considering the formation of the \( \alpha \)-radioactive chronometers in stars and also the emissions, the successive absorptions etc of \( \gamma \)-quanta by the nuclei-chronometers in [4] on the base of his earlier article [9].

### 4. Nuclear chronometry in astrophysics

This section is based on [4-7] and in principle on the preceding author articles, later generalized in [10,11] and described here in the preceding section.

The decay rate per a time unit is

\[
\rho(t-t_0) = \frac{d[1-L(t-t_0)]}{d(t-t_0)}. 
\] (14)

When the decay is going on by several channels (for example, by \( \alpha \)-and \( \gamma \)-decays from the first excited state of the parent nucleus) we have

\[
\rho_{\text{1}}(t-t_0) = \frac{(\Gamma_1^\text{1} / \Gamma_1) d[1-L_1(t-t_0)]}{d(t-t_0)} = \frac{(\Gamma_1^\text{1} / \hbar) \exp (-\Gamma_1^\text{1}(t-t_0) / \hbar)}{d(t-t_0)} 
\] (15)

instead of (14).
The decay of an ensemble of radioactive nuclei can go on simultaneously with its preparation (in particularly, with the nucleo-synthesis or with decays from the previous state). One can use the following expression for the decay rate per a time unit:

\[ I(t-t_0) = \int_{t_0}^{t} \rho_1^{(0)}(t') \rho_0^{(1)}(t-t') dt', \] (16)

where \( \rho_m^{(n)}(t) \) are defined by the decay functions \( L_m^{(n)}(t) \) \((m\neq n=0,1)\) with relevant spectral distributions \( G_m^{(n)} \) for a “preparative” decay from the initial (first excited) state and the subsequent (ground) state, formed after the \( \gamma \)-decay of that initial one, respectively; \( \Gamma_1, \Gamma_1^a + \Gamma_1^\gamma, \Gamma_1^a \) and \( \Gamma_1^\gamma \) being the \( \alpha \)-decay and \( \gamma \)-decay widths of the excited parent \( \alpha \)-radioactive nucleus (with the first excited state of the internal \( \alpha \)-particle).

In the approximation of the single (one-step) absorptions of the emitted \( \gamma \)-quanta after the \( \gamma \)-decay one can use the expression

\[ M(t) = q M_0 \exp \left(-\frac{ct}{m}\right) \] (17)

(with \( c, \mu \) and \( q \) being the light velocity, the inverted \( \gamma \)-quanta absorption coefficient inside the matter and the dimensionless quantity, which depends on small loss of the \( \gamma \)-quantum energy due to the \( \gamma \)-scattering by nuclei and by electrons, respectively) for the \( \gamma \)-quanta propagation inside the matter sample [4,5]. After the simplifications at the approximations of small recoil nucleus kinetic energy \( \epsilon_{\text{recoil}} \sim (E_1 - E_0)^2 / 2A m_n c^2 \) (\( A \) and \( m_n \) being the atomic number of parent nucleus and the mean nucleon mass, respectively) after \( \gamma \)-quantum emission or absorption and the Doppler width for the resonant \( \gamma \)-emission and absorption \( D \sim 2|\epsilon_{\text{recoil}}| kT |^{1/2} \) (\( k \) and \( T \) being the Boltzman constant and the sample temperature, respectively), then \( \Gamma_0 \ll \Gamma_1, c \ h / \mu \) and \( t - t_0 \gg \ h / \Gamma_1, \mu / c \) and also considering the diminution of directly decaying parent nuclei due to the \( \gamma \)-absorption, one obtains the following expression:

\[ D(t) = D(t_0) + [P_0(t_0) + P_1(t_0) \left[ 1 - \exp(-\Gamma_0(t - t_0) / h) \right] + \] 
\[ + P_1(t_0) Q_l \left[ 1 - \exp(-\Gamma_0(t - t_0) / h) \right] \] (18)

with

\[ Q_l = (\Gamma_1^a / \Gamma_1) q M_0 [P_0(t_0) + \] 
\[ + P_1(t_0) (2 \Gamma_1^a + c h / l) / (2(\Gamma_1^a + c h / l))], \]

or

\[ D(t) = D(t_0) + [P_0(t_0) + (1 - Q_l) P_1(t_0)] \times \] 
\[ \times \left[ 1 - \exp(-\Gamma_0(t - t_0) / h) \right] + Q_l P_1(t_0), \] (19)
$P_0$ and $P_1$ being the abundances of the ground and the excited states of the parent $\alpha$-radioactive nuclei, respectively. The tentative estimations of $q$ give values which are approximately within $(1/2, 1)$.

Taking into account not only single (one-step) $\gamma$-absorptions but all possible multiple $\gamma$-absorptions, one can evaluate an every step of $\gamma$-absorptions by an equal contribution with $M \cong qM_0 \left(2\Gamma_1 + c \hbar / \lambda \right) / 2(\Gamma_1 + c \hbar / \lambda)$ and easily sum up them in (18) or (19). Then, if $\Gamma_1/I \Gamma_1$ is not very small (let us say, $\geq 0.1$), one can also roughly take $N \Gamma_1/I \Gamma_1 \cong 1$ in (19) ($N$ being an effective number of the considered $\gamma$-absorptions steps). We can confirm such valuations with the help of the simple evident reasoning. For the lifetimes $\tau_\gamma = \hbar / \Gamma_\gamma$ which are larger than $10^{-13}$ sec and $\Gamma_\gamma \ll D$ and, moreover, for $A \cong 250$, $\epsilon_\gamma \cong 50$keV and $N \cong 10$, the quantity $N\epsilon_{\text{recoil}}$ satisfies the following expressions:

$$N\epsilon_{\text{recoil}} < D \text{ when } T > 300^\circ \text{K (for the terrestrial lumps)}$$

and

$$N\epsilon_{\text{recoil}} < D \text{ when } T \cong 10^9^\circ \text{K (inside the stars)}.$$

Although up to now the partial $\alpha$-decay widths for excited states experimentally are not studied, we can expect that the condition $N<10$ is rather realistic on account of the Geiger and Nutall law, at least for the high-energy excited states. If the values of $\Gamma_\gamma$ and $N\epsilon_{\text{recoil}}$ get into such spreads D we can generalize (19) for the multiple $\gamma$-absorptions and write (with $Q_\lambda \rightarrow q$)

$$D(t) = D(t_0) + \left[ P_0(t_0) + P_1(t_0)(1-q) \left(1 - \exp(-\Gamma_0(t - t_0) / \hbar) + qP_1(t_0) \right) \right]$$

(20)

and hence

$$D(t) = D(t_0) + P_0(t_0)[1 - \exp(-\Gamma_0(t - t_0) / \hbar)] + P_1(t_0)$$

(21)

at the approximation of $q \rightarrow 1$.

The results (20) and (21) are valid at the approximations of infinitely large medium volumes, and sufficiently large times $t-t_0$ (which are much larger than mean lifetimes of excited states and mean times of the free flight of $\gamma$-quanta inside the medium and also than times of quantum oscillations caused by different interference processes described by applying the Krylov-Fock theorem [6], generalized in [4,5](see also [7]) and at the condition $(\hbar \Gamma_0/ \Gamma_\gamma)^N \geq 1$.

Let us analyze the new relation (21) in comparison with the previously known equation (4). They are basic for the determination of the age of a pattern but under the different conditions: with and without taking into account the intermediate $\gamma$-absorptions inside the sample matter. From the simple comparison of (4) and (21) one can see that

i. for the same $D(t_0)$, $P_0(t_0)$, $P_1(t_0)$, $t_0$ and $\Gamma_0$ at any moment $t$ the value of $D(t)$ in (21) is larger than in (4) by the quantity $P_1(t_0)$;
ii. the same value $D(t)$ is obtained in (21) at an earlier moment $t$ than in (4);

iii. the larger is the contribution of $P_i$ into the sum $P_0 + P_i$, the earlier is the moment $t$ in (21) in comparison with (4) at which the same value of $D(t)$ is obtained.

Now we illustrate the inference (ii) by the following instance:

if $P_0(t_0) = P_1(t_0) = (\frac{1}{2})P(t_0)$, then the same value of $D(t)$ is obtained for the values of $t = t_{\text{standard}}$ in (5) and $t = t_{\text{real}}$ in (20) which are connected by the following striking relation

$$t_{\text{real}} = t_{\text{standard}} - (\Gamma_0 / \hbar) \ln 2$$  \hspace{1cm} (22)

(of course, we imply that $t_{\text{standard}} - t_0 > (\Gamma_0 / \hbar) \ln 2$).

The table 1 represents some impressive calculation results for the $\alpha$-decay of the nucleus-chronometer $^{238}U$ with the half-life $T_{1/2} = \ln 2 / (\Gamma_0 / \hbar) = 4.5 \times 10^9$ years on the base of (22).

| $t_{\text{real}}$ years | $2 \times 10^9$ | $4 \times 10^9$ | $6 \times 10^9$ | $8 \times 10^9$ | $1 \times 10^6$ | $1 \times 10^9$ | $0.8 \times 10^9$ | $1.8 \times 10^9$
|------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $t_{\text{standard}}$ years | $3.32002 \times 10^9$ | $3.32004 \times 10^9$ | $3.32006 \times 10^9$ | $3.32008 \times 10^9$ | $3.3201 \times 10^9$ | $3.321 \times 10^9$ | $3.4 \times 10^9$ | $3.5 \times 10^9$ |

Table 1. Comparison of the calculation results for $t_{\text{real}}$ and $t_{\text{standard}}$ the base of formula (22).

In Fig. 2 there is represented qualitative behavior of the corrected chronometry in astrophysics in comparison with the standard chronometry.

So, sometimes billions years obtained by the usual nuclear-chronometry method can correspond to several thousands years and always there is no exponential decay law of the alpha-radioactive chronometer at all.

Of course, the results (i) and (ii) will be even stronger, if one will consider more than one excited state of the parent nuclei with the excited internal $\alpha$-particles. It is possible to state that the usual (non-corrected) “nuclear clocks” do really indicate to the upper limits of the durations of real $\alpha$-decay processes. So, sometimes billions years obtained by the usual nuclear-chronometry method can correspond to several thousands years. Of course, the results (i) and (ii) will be even stronger if one will consider more than one excited state of the parent nuclei with the excited internal $\alpha$-particles. It is possible to state that the usual (non-corrected) “nuclear clocks” do really indicate to the upper limits of the durations of real $\alpha$-decay processes.

Further, also it is experimentally shown in [15] that the lifetime of the $\beta$-radioactive isotope $^{187}\text{Re}$ in a bare (without electrons) atom is less $10^9$ times than the lifetime of the usual atom with totally filled electronic shell (it becomes - 30 years instead of 3 $10^{10}$ years!). And it is known that the stellar matter is in the plasma state with the free nuclei and electrons. So, the nuclei partially or totally are being deprived their electronic shells inside the stars.
5. Nuclear chronometry in geophysics

Now we consider the revision of the methods of the terrestrial nuclear chronometry taking into account the relatively weak but constant cosmic radiation on the upper earth layers up to the depth of ~ 5 meters. Under the influence of the cosmic radiation, the following processes are going on: (1) the constant formation of the excited nuclei-chronometers with the smaller life-times than for the nuclei-chronometers in the ground state, (2) the effective acceleration of the $\alpha$-decay through knocking-out the $\alpha$-particles by cosmic protons and (3) the constant nonzero removal of nuclei-chronometers through the channels of inevitable rearrangement nuclear reactions. These processes lead to the real diminishment of the results of measurements of the decay times.

Let us analyze the diminishment caused by the first and the second kinds of processes.

In our case, taking into account the cosmic radiation (supposing its flux to be constant in time) and using the same method of the generalization of the Krylov-Fock theorem as in [2-4], we, instead of $L(t-t_0)$ (with the consequences (1)-(2)), obtain:

$$L_{p}(t-t_0) = \left[1 - a \times (t - t_0) \right] L(t-t_0)$$

(23)

for a unit (1cm$^3$) chronometer volume, where $a=j_{\text{cosm}} \alpha \nu n$, $j_{\text{cosm}}$ is the cosmic (mainly proton) radiation flux (in cm$^{-2}$ sec$^{-1}$), $\sigma$ is the total cross section of all reaction proton-chronometer processes with the removal of the nuclei-chronometers, $\nu$ is the number of multiple collisions in...
the medium after the first proton-chronometer collision and \( n \) is the mean nucleus-chronometer number along the 1 cm-depth and \( L_p (t \rightarrow t_0) \) includes all parent-nucleus diminutions through collisions of chronometers with the cosmic protons. Here for the simplicity we neglect the elastic and inelastic scattering. We see that for \( t - t_0 \geq 1/a \) \( L_p \rightarrow D(t - t_0) = 0 \) and \( P(t - t_0) = 0 \).

If we select only the reactions \((p, p' \alpha)\) with the general cross section \( \sigma_D \) which strongly accelerate the emission of the \( \alpha \)-particles with the formation of daughter nuclei, neglecting all other processes, then we obtain the particular relation

\[
L_{p \rightarrow D}(t - t_0) = \left[ 1 - a_D \times (t - t_0) \right] L_{(t - t_0)}
\]  

(24)

where \( a_D = j_{\cosm} \sigma_D \nu n \), and \( L_{p \rightarrow D}(t - t_0) \) denotes the decay function in this case. And in this case relation (2b)4 passes into

\[
p(t) \left[ \left[ 1 - a_D \times (t - t_0) \right] \exp(\Gamma_0(t - t_0) / \hbar) - 1 \right] - d(t) + d(t_0) = 0.
\]  

(25)

From the comparison of (25) with (4) one can see that when \( t - t_0 \rightarrow 1/a \) the time duration defined by the old method (without taking the cosmic radiation into account) becomes very large – much more than \( \Gamma_0 / \hbar \).

So, one can see that that qualitatively in the terrestrial layers of geophysics the main equation (25) is rather similar to the standard chronometry but with one modification – the insertion of a term \( a_D(t - t_0) \) which can noticeably diminish the age of the earth and also distort the exponential law of \( \alpha \)-decay of the \( \alpha \)-radioactive chronometers.

For qualitative evaluations we have taken \( \sigma_D = 3 \times 10^{-25} \text{ cm}^2 \) and \( \nu \) between \( 10^2 \text{ } \cdot \text{ } 10^3 \) for the mean proton energy \( \sim 10^9 \text{ eV} \), the flux \( j_{\cosm} = 0.85 \text{ (cm}^2 \text{sec})^{-1} \) in the top atmosphere layer or \( 1.75 \times 10^2 \text{ (cm}^2 \text{sec})^{-1} \) on the sea level [16] and \( n = 1 \text{cm}^3 / 3 \times 10^8 \text{cm}^3 = 0.33 \times 10^8 \). Of course, practically it is impossible now to calculate \( \nu \), because the effective value of \( \nu \) is defined not only by mean proton energy and by nucleon, cluster and fragment binding energies but also by usually unknown cross sections of all possible reactions for wide energy region – and hence we have used very simplified evaluations. Then we obtain values of \( a \) between \((1/1.5) \times 10^8 \text{ years} \) and \((1/2.7) \times 10^5 \text{ years} \). From the result we can see that when \( P(t) \to 0 \), for both values of \( a \) and also in both cases (with valid and invalid relation (25)) the real time duration is essentially less than without taking the cosmic radiation into account.

As to the more deep terrestrial layers and the core of the earth, we have to take into account the history of the earth formation. Now there is no unique generally accepted theory or even model of the planet origin. There are two known groups of such models: (1) models where one considers the planet origin from the final cooled pieces of the exploded star or super-nova, (2) models where one considers planets as results of the cosmic dust condensation.

Relative to any model from the first group, if one takes any piece of the terrestrial mass, it is impossible to distinguish the genetically real parent and daughter nuclei from the admixture of the same kinds of nuclei formed in other parts of the cooled and transformed parent stellar
piece. Therefore one can approximately suppose that the earth age is the sum of the parent star (or the preceding super-nova) age before exploding and of the consequent age of the formed earth. The last age can be determined also by the methods of the nuclear chronometry but in different ways inside the earth and on the surface of the earth. Deeply inside the earth one has to consider consequences of the formation and decay of the excited nuclei-chronometers during the preceding stellar (and super-nova) nucleo-synthesis and during subsequent planet cooling in the melted magma (inside the earth). And in the surface layers of the earth (up to the depth of ~ 5 meters) we can consider the influence of the cosmic radiation which was presented above. For both cases it is also necessary to take into account the unknown now initial nonzero quantity of the daughter nuclei in the earth (in the examined earth pieces) from the previous stellar (nucleo-synthesis and chronometer-decay-chain) processes.

Relative to any model from the second group, from the very beginning it is necessary to take into account the cosmic dust origin. Hypothetically the cosmic dust was born partially simultaneously with first stars after Big Bang and partially during the star evolution – from the cooled micro-rejections out of stars and super-nova during their perturbations and explosions. And now there is neither a systematic theory of the dust origin, independent from the star origin, nor a systematic theory of the dust condensation → clotting into a planet. We can approximately evaluate the mean existence time of the earth beginning from the hypothetical mean instant of the conventional dense clotting of the condensed dust into the planet by the methods of nuclear chronometry if we know the real initial quantities of the parent and the daughter nuclei just in this mean instant. And a nonzero initial quantity of the daughter nuclei always leads to a real diminishment of the evaluation of the decay time – a larger diminishment for a larger initial quantity of the daughter nuclei. Moreover, we have to take into account the constant excitation of radioactive nuclei-chronometers by the cosmic radiation and then, in the case of large masses, also the formation of γ-emission-absorption chains with accompanied multiple excitations of nuclei-chronometers.

6. A short previous history of my investigations on the nuclear chronometry

We use in time analysis of nuclear processes elaborated on time as a quantum observable, canonically conjugate to energy, the general expression for the duration \( \langle \tau_f \rangle \) like [10,11]

\[
\langle \tau_f \rangle = \langle t_f \rangle_f - \langle t_i \rangle_i = \int_{-\infty}^{\infty} t \, w_f (z_f, t) \, dt - t_{ih}
\]

where \( w_f = \int_{-\infty}^{\infty} j_f (z_f, t) \, dt \),

\[
j_f (z_f, t) = \text{Re} \left( \psi_f (-i\hbar / 2\mu_f \partial / \partial z_f) \psi_f \right)
\]

is the \( z_f \) the component of the
probability flux; the brackets (…) define the integration over all coordinates, entered in the wave functions \( \psi_f \), besides \( z_f \), is the axis, directed along the mean velocity \(< \vec{v}>_f \) of the relative motion of the decay pair (for instance, \( \alpha \)-particle and the daughter nucleus) with the reduced mass \( \mu \) and their wave function of the internal motion \( |\nu> \) with energy \( e_\nu \); \( \vec{v}_\nu=\hbar k_\nu / \mu_\nu \epsilon_\nu \). 

\( k_\nu^2 / 2\mu_\nu = E - e_\nu \); \( t_m \) is defined as \( t; \tau \) where \( t \) is the initial channel of forming the decaying nucleus or simply as an initial time moment \( t_m \).

The wave packet of the relative motion of the decay pair in one-dimensional radial asymptotic limit is described as

\[
\Psi_f(r_f, t) = n\int_0^\infty dE g(E) F_{if}(E) \exp[ikr_f - iEt / \hbar] 
\]

where

\[
F_{if}(E) = \frac{C_{if}}{E - E_r + i\Gamma / 2}, \tag{28}
\]

with \( C_f \) being a constant or smooth function of final-particle kinetic energy \( E \) in the region \( (E_r - \Gamma/2, E_r + \Gamma/2) \), \( E_r \) and \( \Gamma \) are the resonance energy and width, respectively. For \( z_f \geq R_\beta \), where \( R_\beta \) is the radius of interaction in the final channel, and under condition \( \Gamma << \Delta E << E_r \) it is possible to re-write (27) in the following simplified form

\[
\Psi_f(R_f, t) = A\int_0^\infty dE \frac{\exp[-iEt / \hbar]}{E - E_r + i\Gamma / 2}, \tag{29}
\]

where \( A \) is a constant. For \( \Gamma=constant \) we obtain

\[
\Psi_f(R_\beta, t) = \begin{cases} 
B \exp[-iEt / \hbar - (\Gamma / 2\hbar)t], & \text{for } t > 0 \\
0, & \text{for } t < 0 
\end{cases} \tag{30}
\]

(shifting, for very small \( \Gamma \), the lower limit of the integration in (29) from 0 till \(-\infty \) and utilizing the residue theorem). Here \( B \) is constant and more precisely here will be \( t-t_m \) instead of \( t \).

Of course, \( w_f \) for wave function (30) is equal

\[
L(t) = (\Gamma / \hbar) \exp(-\Gamma t / \hbar). \tag{31}
\]

The function \( L(t) \), generally speaking, defined by (6) and concretely by (31), sometimes are named by the decay function (see, for instance, [4-10]). The more detailed study (see, for instance, [12-14]) shows that even for an isolated resonance the exponential form of the decay law is
approximately performed in the time interval, limited by below by the quantity \( t_1 \sim t_0 \Gamma / E \), and from above by the quantity \( t_2 \sim t_0 \ln (\Gamma / E) \), where \( t_0 \sim \hbar / \Gamma \), and the accuracy of describing by the exponential function is the better, the lesser is the relation \( \Gamma / E \) [14]. The functions (6) and (31) describe the essence of the Krylov-Fock theorem which was derived by another way (see the section before) and states that the decay law of the meta-stable state is totally defined by the energy spectrum of the initial state [8].

The author (V.S.O.) had been extended the contents of the Krylov-Fock theorem, considering the formation of the \( \alpha \)-radioactive chronometers in stars and also the emissions, the successive absorptions etc of \( \gamma \)-quanta by the nuclei-chronometers in [4] on the base of his earlier article [9].

7. Several words on measurements of \( P \) and \( D \) for astrophysical and geophysical processes

Of course, it should be noted that is impossible for us to measure directly with usual radioactive methods \( P \) and \( D \) in the sun or any astrophysical \( \alpha \)-and \( \gamma \)-radioactive decay processes. And the only way for such measurements is to adapt the photon (\( \gamma \)-ray) spectroscopic method for defining \( P \) and \( D \) for the \( \alpha \)-radioactive chronometer with correspondent \( \gamma \)-transitions, analyzing the correspondent energy peaks of \( \alpha \)-and \( \gamma \)-lines with the application of \( \alpha \)-and \( \gamma \)-sensors of the sun or star photon radiation.

As to the geophysical processes, it is possible to use the usual for earth measurements of \( P \) and \( D \), which are usual for earth. As to moon or some other planet or satellite, it is possible to use the remote satellite sensors with the broadcast of their measurements for the earth through the earth atmosphere and in cosmos.

8. Conclusions and perspectives

The presented simplified estimations brings to the conclusion that the usual (non-corrected) “nuclear clocks” do really indicate not to the realistic values but to the upper limits of the durations of the \( \alpha \)-decay stellar and planet processes and also that the realistic durations of these processes have to be noticeably (at least several orders) smaller. And the results of this paper represent what physical processes can strongly influence on estimations of the sun and earth ages, which has to be much smaller than usual estimations.

As a continuation and expansion of the exposed results, it is necessary to propose the following program for future investigations, taking into account:

1. the chains of the successive decays of every star and terrestrial chronometer,
2. all the initial excited states of every star and terrestrial chronometer,
3. joint considering the both \( \alpha \)-and \( \beta \)-radioactive decays and finally,
4. the possible modifications of processes of the stellar and cosmic nucleo-synthesis of all nuclei-chronometers.

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