Developing a Hydrology Coupled 2D Cellular Automata Model for Efficient Urban Flood Simulation

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Abstract. To achieve fast and efficient flood modelling for urban area, a two-dimensional cellular automata based model was developed and further coupled with a hydrological model. The model was implemented in parallel environments and tested by an analytical solution and a real world case. The analytical case results showed that the model is capable of simulating water-depth under a reasonable slope threshold and grid resolution condition using only a short computational time. Slope threshold and grid resolution are the two key factors affecting the computational efficiency and model accuracy. The introduction of the state update interval significantly improved the computational efficiency. In the case of the real world example, the results were also in reasonable range. In general, this model can be applied to rapid flood analysis for large urban areas.

1. Introduction

As a powerful tool for urban flood control and disaster mitigation, the urban flood modelling has become an important research topic over the last few years [1]. Recently, 2D urban flood model has become popular in urban flood simulation since the traditional 1D model cannot simulate the evolution of surface water accumulation. However, the typical 2D model requires high computational resources to solve the Shallow Water Equations (SWEs). In order to improve the efficiency of the 2D model as much as possible without significantly reducing the calculation accuracy, some scholars have made improvements by simplifying the SWEs and some models have benefited from parallelised computation [2]. But these simplified physical based models are still computationally intensive due to the complex mathematical formulae and inherently sequential computations.

Cellular Automata (CA) technique offers a versatile method for modelling the spatiotemporal evolution process of complex physical systems using simple operations with well suited parallel computation capacity. Recently, the CA technique has been used in developing simple and efficient 2D urban flood models by many studies. Dottori and Todini [3] developed a 2D CA model which employed
Manning’s equation for the computation of interfacial discharges between computing cells. Ghimire et al. [4] developed a model that used a ranking system to evaluate the volume transferred between cells.

The main idea of this work is to present a high-precision and efficient 2D urban flood model by coupling a 2D CA model followed the Guidolin [5] with a precipitation-runoff model. The new model was applied to an analytical solution proposed by Hunter et al. [6] and a real flood event in the area of Wuhan, China.

2. Methodology

2.1. 2D CA model

The proposed 2D CA model was developed following the work of Guidolin et al. [5]. The CA 2D model is a diffusive-like model which ignores inertia terms and momentum conservation. The model was designed to work with five rectangular grid cells of the von-Neumann (VN) neighbourhood. The major feature of our new model is that it adopted a simplified weighting system which has four steps:

1) Identify the downstream neighbour cells by using water level differences between the central cell and the neighbour cells (Eq. (1));
2) Compute the specific weight of each downstream cell based on the available storage volume (Eq. (2) – Eq. (6));
3) Compute the total amount of volume that will leave the central cell (Eq. (7) – Eq. (10));
4) Compute the eventual intercellular-volume for each downstream cell (Eq. (10)).

\[
\Delta Z_i = Z_0 - Z_i \quad \forall i \in \{1,2,3,4\} \quad (1)
\]

\[
w_i = \frac{A v_i}{A v_T + A v_{\min}} \quad (6)
\]

\[
A v_i = A \max \{\Delta Z_i, 0\} \quad \forall i \in \{1,2,3,4\} \quad (2)
\]

\[
w_0 = \frac{A v_{\min}}{A v_T + A v_{\min}} \quad (7)
\]

\[
A v_{\min} = \max \{A v_i \mid \Delta Z_i > \tau \} \quad \forall i \in \{1,2,3,4\} \quad (3)
\]

\[
v_M = \min \left\{ \sqrt{\frac{1}{g} \cdot h^{2/3} \frac{\Delta Z_{0,M}}{\Delta x_{0,M}}} \right\} \quad (8)
\]

\[
A v_{\max} = \max \{A v_i \mid i = 1,2,3,4\} \quad (4)
\]

\[
I_M = v_M h_0 \Delta e_M \quad (9)
\]

\[
A v_T = \sum_{i=1}^{4} A v_i \quad (5)
\]

\[
I_i = w_i I_T \quad (10)
\]

Where \( \theta \) is the index of the central cell, \( i \) is the index of the neighbour cell, \( Z(\text{m}) \) is the water level, \( A(\text{m}^2) \) is the cell area, \( A v_i (\text{m}^3) \) is the available storage volume between the central cell and the \( i \)th neighbour cell, \( A v_{\min} (\text{m}^3) \) is the minimum available storage volume, \( A v_{\max} (\text{m}^3) \) is the maximum available storage volume, and \( A v_T (\text{m}^3) \) is the total available storage volume, \( w_i \) is the weight of the \( i \)th cell, \( h(\text{m}) \) is the water depth, \( g (\text{m/s}^2) \) is the gravitational acceleration, \( n (\text{m}^{1/3}\text{s}) \) is the Manning’s roughness, \( M \) is the index of the neighbour cell with the largest weight, \( v_M (\text{m/s}) \) is the maximum permissible intercellular velocity, \( \Delta x_{0,M} (\text{m}) \) is the distance between the central cell and the cell with the largest weight, \( I_M (\text{m}^3) \) is the maximum intercellular-volume achievable into the neighbour cell with the largest weight, \( I_T (\text{s}) \) is the time step, \( e_M (\text{m}) \) is the length of a cell edge with the largest weight.

To ensure the calculation stability, the time step \( \Delta t \) is calculated by computing on each cell of the grid the adaptive time step using the formula provided by Hunter et al. [6]:

\[
\Delta t = \frac{\Delta x^2}{4} \min \left( \frac{2n}{h^{5/3}} S^{1/2} \right), \quad S > \sigma \quad (11)
\]

where \( \sigma \) is the slope tolerance which used to prevent the simulation time become too small.
To reduce the calculation time of the model, the state update interval \( \Delta t_u \) is used. The time step is recalculated every \( \Delta t_u \) instead of each time step.

2.2. Precipitation-runoff model

In this paper, the runoff simulation was carried out over each cell which was further divided into different hydrological response units according to the land use. Each permeable unit were formed by a vegetation canopy, a surface reservoir layer, a soil layer, and a groundwater layer from top to bottom. A physical equation describing the movement of the water flow was established for each hydrological response unit. The Richards equation was used to describe the water movement, and the interflow along the slope direction was considered. The mass conservation equation and Darcy’s law were used to evaluate the water exchange between groundwater and river channel. The detailed calculation principle can be found in the reference [7].

3. Case and study areas

3.1. Analytical solution

Hunter et al.[6] developed a one-dimensional analytical solution where the full Saint-Venant equations can be simplified over a horizontal plane with a constant flow velocity. The solution can be written as:

\[
h(x, t) = \left[ \frac{7}{3} (C - n^2 u^2 (x - ut)) \right]^{7/3}
\]

Where \( h \) (m) is the water depth, \( u \) (m/s) is the component of the depth-averaged velocity in the \( x \) direction, \( C \) is a constant of integration, which can be fixed by referring to the initial conditions of the problem.

Eq. (1) can be used to provide an analytical solution against which the model can be tested with boundary conditions and initial conditions. In this study the model was implemented using parameter values of \( u=1 \) m/s, \( n=0.01 \) m\(^{1/3}\)/s for a simulation of duration 3600s.

3.2. Real-world case

The Fruit Lake Gate area (Figure 1), in the middle of the Wuhan city, China, is a challenging case due to the high level of urbanization. The catchment area is the seat of the provincial government of Hubei Province, and is also an intensive residential gathering place. The research area is 2.48 km\(^2\) with 27.38% of building, 8.29% of green space, 18.96% of road and 45.36% of paved area. During June 30 to July 6, 2016, Wuhan suffered from a heavy rainfall. The precipitation at several rainfall stations were exceeded the historical extreme value, reaching 565.7~719.1 mm. In this study, the 10 minutes interval precipitation with a total number of 249 mm during July 5 and July 6 were used for the simulation.

4. Results and discussion

4.1. Analytical solution

The computation grid was 5 km length in the flow direction and 1 km width. Figure 2 and Table 1 showed the analytical solution using three different grid resolutions (10m, 25m and 50m). As it’s showed in Figure 2, the model results were in good agreement with the theoretical solution, with the RMSE are 0.0042, 0.0072, and 0.0104m for three dx. As expected, the model is sensitive to the grid resolution. Therefore, in practical applications, it is necessary to select an appropriate spatial step size to ensure the accuracy of the simulation.
Figure 1. (a) DEM (b) Landuse Study area

Figure 2. Comparison between analytical solution and the model at 3600 s for different grid resolutions

Table 1. RMSE, minimum time step and total runtime for different $dx$ and $\sigma$

| $dx$ (m) | $\sigma$ (%) | RMSE (m) | Min$\Delta$t (s) | Runtime (s) |
|----------|---------------|-----------|------------------|-------------|
| 10       | 1             | 0.2671    | 0.0206           | 522.59      |
| 10       | 0.5           | 0.1994    | 0.0143           | 835.31      |
| 10       | 0.1           | 0.0042    | 0.0061           | 2916.14     |
| 10       | 0.01          | 0.0042    | 0.0061           | 2886.78     |
| 25       | 1             | 0.2981    | 0.1000           | 70.09       |
| 25       | 0.5           | 0.1823    | 0.0890           | 135.62      |
| 25       | 0.1           | 0.0072    | 0.0380           | 460.24      |
| 25       | 0.01          | 0.0072    | 0.0380           | 462.96      |
| 50       | 1             | 0.2396    | 0.1000           | 23.63       |
| 50       | 0.5           | 0.1389    | 0.1000           | 33.59       |
| 50       | 0.1           | 0.0104    | 0.1000           | 117.16      |
| 50       | 0.01          | 0.0104    | 0.1000           | 113.28      |
The grid resolution and slope threshold were expected as two important parameters that affect the computational accuracy and computational efficiency. The model was applied with a combination of different slope thresholds (1‰, 0.5‰, 0.1‰, 0.01‰) and different grid resolutions (10m, 25m and 50m). The RMSE, minimum time step (MinΔt) and total runtime are showed in Table 1 and Figure 3. The results showed that the calculation accuracy of the model decreased with the increase of the grid resolution and the slope threshold. When the slope threshold was less than 0.1‰, the RMSE was no longer reduced with the decrease of slope threshold (Figure 3a). The relationship between RMSE and spatial grid resolution varied between different slope thresholds.

As expected, the computational efficiency significantly dropped as the grid resolution increased, and decreased as the slope threshold increased. However, the total runtime only increased slightly when slope threshold is less than 0.1‰ or greater than 0.5‰ (Figure 3b). Table 2 gave the correlation coefficient matrix of grid resolution, slope threshold, RMSE, minimum time step and total running time. The correlation coefficient between RMSE and slope threshold is 0.9710 which indicated that simulation accuracy is primarily controlled by the slope threshold. The minimum time step is mainly related to the grid resolution with a correlation coefficient of 0.8726. There is a significant negative correlation between computational efficiency (run time) and grid resolution or slope threshold. In general, an appropriate slope threshold can be selected to ensure the calculation accuracy, and a larger space step can be selected to improve the calculation efficiency of the model in simulation.

### Table 2. Correlation coefficient matrix of grid resolution, slope threshold, RMSE, minimum time step and total running time.

| dx (m) | σ (‰) | RMSE (m) | minΔt (s) | Runtime (s) |
|--------|--------|----------|-----------|-------------|
| 1 | 0.0000 | -0.0758 | 0.8726 | -0.6464 |
| 1 | 0.9710 | 0.2738 | -0.4127 |
| 1 | 0.2512 | 0.4261 |
| 1 | 0.7609 | 1 |

### Table 3. RMSE, minimum time step and total runtime for different state update intervals

| Updatetime(s) | RMSE(m) | minΔt(s) | Runtime(s) |
|---------------|---------|----------|------------|
| 5 | 0.0072 | 0.0374 | 700.79 |
| 15 | 0.0072 | 0.0375 | 446.71 |
| 30 | 0.0072 | 0.0377 | 423.05 |
| 60 | 0.0072 | 0.0380 | 408.71 |
| 90 | 0.0072 | 0.0384 | 402.20 |
| 120 | 0.0072 | 0.0387 | 393.43 |
In order to improve the computational efficiency, a state update interval ($\Delta t_u$) is used. The time step is recalculated at each $\Delta t_u$ time interval instead of each time step. Table 3 showed the results of different state update intervals (5s, 15s, 30s, 60s, 90s, 120s, and 360s) with a 10m grid resolution and a 0.1% slope threshold. As it’s showed in Table 3, the accuracy of the model did not decrease as the state update interval increased. The runtime decreased as the calculation update interval increased.

4.2. Real-word case

In the realistic case, the model was performed using DEMs with a spatial resolution of 5m. A Constant Manning roughness of 0.028 ($m^{-1/3}s$) was applied to road and 0.08($m^{-1/3}s$) and 0.04($m^{-1/3}s$), 0.05 ($m^{-1/3}s$) for green space, pavement and building.

There is no official measured data on the flooding during the event. Therefore, this study did a field survey for one point specifically. The water depth change process simulated at the point is shown in Figure 4(a). The maximum water depth is 0.51m which is close to the result of field survey (0.6m) and the process is also in line with the description of the residents. Figure 4(b) shows the submerged area change process during the stormwater. The area of flooding increased as the precipitation increased and reach their peaks after the rain peak 50 minutes. The peaks of flooding area for 0.4m and 0.15m are 23.16 ha and 47.91 ha. Since the model does not consider the drainage pipe, the lower area was still flooded after the rain stopped. Figure 5 shows the water depth distribution at the peak rainfall time (Figure 5a) and the maximum water depth during simulation (Figure 5b). The lower part of the study area is severely suffered from the water lag which is realistic and reasonable.
5. Conclusions
In this paper, we presented an efficient two-dimensional flood model for urban area based on CA technology and a hydrology model. It was applied to the analytical solution and a real case simulation. The main conclusions are as follows:

1. The model simulation results are in good agreement with the theoretical solution, with a minimum RMSE of 0.0042m.

2. The calculation accuracy of the model is mainly affected by the slope threshold. The calculation efficiency of the model is mainly affected by both the grid resolution and the slope threshold. An appropriate slope threshold can be selected to ensure the calculation accuracy, and a larger grid resolution can be selected to improve the calculation efficiency of the model.

3. The results showed that the introduction of the state update interval in this model significantly improved the computational efficiency and did not reduce the accuracy of the model.

4. The results of the Wuhan case showed that the model can simulate the flood process of the urban area well and indicated that the model has a good application prospect.

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