Stability of a hot two-temperature accretion disc with advection

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Abstract

The effects of radial advection and thermal diffusion were considered in investigating the linear stability of an optically thin, two-temperature accretion disc. If the disc has only very little advection, we proved that the thermal instability exists when the disc is geometrically thin. But it disperses in a geometrically slim disc if the thermal diffusion was considered. Moreover, if the disc is advection dominated, the thermal instability does not exist. In addition, we found that the instabilities of inertial-acoustic modes exist only in a geometrically thin disc or an advection-dominated disc with low Mach number, whereas the Lightman & Eardley viscous instability always disperses in a two-temperature disc. A simple comparison also showed that an optically thin, bremsstrahlung cooling dominated disc is generally more thermally unstable than a two-temperature disc if it is not advection-dominated.

Keywords: accretion, accretion discs – black hole physics – instabilities
1 Introduction

The optically thin, two-temperature accretion disc model was suggested in 1970s to explain the hard X-ray spectra observed in black hole candidates such as Cyg X-1 (Shapiro, Lightman & Eardley 1976, hereafter SLE). The standard geometrically thin, optically thick accretion disc model, proposed by Shakura & Sunyaev (1973), was unable to account for them. In a two-temperature disc, the temperature of electron is about $10^9$ K and the ion temperature is one or two orders higher. Therefore, the two-temperature disc has much higher temperature than the cool, optically thick disc and can produce the observed hard X-ray spectrum above $\sim$8 keV. In last two decades, SLE model has been widely studied and applied in modeling of X-ray binaries and active galactic nuclei (e.g., Kusunose & Takahara 1988, 1989; White & Lightman 1989; Wandel & Liang 1991; Luo & Liang 1994). However, the early work of Pringle (1976) and Piran (1978) indicated that the two-temperature disc model is still thermally unstable. As in the standard thin disc model, which is both thermally and viscously unstable in the inner region (Shakura & Sunyaev 1976; Lightman & Eardley 1974), the rapid growing of instability may result in the breakdown of the disc equilibrium and make it unlikely to be the real configuration of accretion flow.

In most previous studies we mentioned above, the disc model was relatively simple and many effects were ignored. Recently, the effects of radial advection were extensively studied in both optically thick and optically thin accretion discs (Abramowicz et al. 1988; Kato, Honma & Matsumoto 1988; Narayan & Yi 1994, 1995a,b; Abramowicz et al. 1995; Chen et al. 1995; Chen 1995; Misra & Melia 1996; Nakamura et al. 1996). If the radiative cooling is not efficient to balance the viscous heating in the disc, some energy will be advected inward and the advection will be not negligible. It has been suggested both the disc structure and the stability properties of an accretion disc with advection term included are different from those in previous models where advection was totally neglected. By analyzing the $\dot{M}(\Sigma)$ slope of disc structure and comparing the cooling and heating rates near each equilibrium curve, Chen et al. (1995), Abramowicz et al. (1995) and Narayan & Yi (1995b) have suggested that an advection-dominated disc is both thermally and viscously stable whether the disc is optically thin or optically thick. However, such a stability analysis, as well as Piran's criteria, can only be applied to the case with long-wavelength perturbations (Chen 1996). More recently, Kato, Abramowicz & Chen (1996) performed an analytic stability analysis of the advection-dominated discs by considering the short wavelength perturbations. They indicated that the advection-dominated disc is thermally stable if it is optically thin but thermally unstable if it is optically thick. This result was confirmed by a subsequent study of Wu & Li (1996), who considered not only the stability of thermal mode, but also the stability of viscous mode and inertial-acoustic modes. They also concluded that the thermal diffusion has a significant contribution to stabilize an optically thin, advection-dominated disc but enhance the thermal instability of an optically thick, advection-dominated disc.

Although some authors have mentioned that an optically thin, two-temperature disc may be thermally stable if it is advection-dominated (e.g., Narayan & Yi 1995b), however, no detailed stability analysis has been done so far to consider the effects of advection on the stability of a two-temperature accretion disc. For example, in the recent work of Wu & Li (1996), only the optically thin disc with bremsstrahlung radiative cooling was addressed.
We noted that advection may be quite important in a two-temperature disc as calculated by some authors (for example, see Esin et al. 1996). Moreover, a hot two-temperature disc is probably not geometrically thin and the Piran’s criteria may not be applicable in this case. Therefore, in this paper we performed a detailed linear study to investigate the stability of a hot two-temperature disc, considering both the effects of advection and thermal diffusion. Besides the stability of thermal mode, the stability of viscous mode and that of inertial-acoustic modes have also been investigated.

2 Basic equations

The simplified equations which describe the equilibrium structure of a two-temperature disc has been given by SLE. However, in order to study the stability properties, we must involve the detailed time-dependent hydrodynamic equations. In addition, because we will investigate the influences of some terms such as the radial viscous force, advection and thermal diffusion on the disc stability, we should also included them in the equations. Ignoring the self-gravitation and adopting the pseudo-Newtonian potential (Paczyński & Wiita 1980), $\Psi = -GM/(R - r_g)$, we can write the vertical integrated time-dependent equations in a cylindrical system of coordinates as follows:

\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(r \Sigma V_r) = 0, \tag{1}
\]

\[
\Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma (\Omega^2 - \Omega_k^2) r = -2 \frac{\partial (H \rho)}{\partial r} + F_v, \tag{2}
\]

\[
\Sigma r^3 \frac{\partial \Omega}{\partial t} + \Sigma r V_r \frac{\partial (r^2 \Omega)}{\partial r} = \frac{\partial}{\partial r} \left( \Sigma \nu r^3 \frac{\partial \Omega}{\partial r} \right), \tag{3}
\]

\[
C_v \left[ \frac{\partial T}{\partial t} + \Sigma V_r \frac{\partial T}{\partial r} - (\Gamma_3 - 1) T \left( \frac{\partial \Sigma}{\partial t} + V_r \frac{\partial \Sigma}{\partial r} \right) \right] = \Sigma \nu (r \frac{\partial \Omega}{\partial r})^2 - Q_- + Q_t, \tag{4}
\]

where $V_r$, $\Omega$ are the radial velocity and angular velocity, $\Omega_k$ is the Keplerian angular velocity, $\Omega_k^2 = \left(\frac{\partial \Psi}{\partial r}\right)_{z=0}$. $\rho$, $T$, $\Sigma$ and $H$ are the total pressure, temperature, surface density and the disc height, $C_v$ and $\Gamma_3$ are the heat capacity per unit mass and a quantity associated with $\beta$, the ratio of gas to total pressure. For an optically thin, two-temperature disc, $\beta = 1$, $C_v = 3p/(2\rho T)$, $\Gamma_3 = 5/3$ and $p = \rho k(T_i + T_e)/m_p$, where $T_i$, $T_e$ and $m_p$ are the ion, electron temperatures and proton mass. $F_v$ is the radial viscous force, which is often neglected in geometrically thin accretion discs but is perhaps not negligible and should be considered in accretion discs with advection (Narayan & Yi 1995a). It is given by (Papaloizou & Stanley 1986)

\[
F_v = \frac{\partial}{\partial r} \left[ \frac{1}{2} \frac{\nu_r \Sigma \partial (r V_r)}{r} \right] - 2 V_r \frac{\partial (\nu_r \Sigma)}{\partial r}, \tag{5}
\]

where $\nu_r$ is the kinematic viscosity acting in the radial direction. In this paper we take $\nu_r = \nu$, where $\nu$ is the viscosity acting in the azimuthal direction and is expressed as the standard $\alpha$ prescription (Shakura & Sunyaev 1973), $\nu = \alpha c_s H$. $c_s$ is the local sound speed defined by $c_s = p/\rho$. The disc height is given by $H = c_s/\Omega_k$. $Q_-$ at the right side of Eq. (4) represents the radiative cooling. For an optically thin, one-temperature disc, the
The radiative cooling mechanism is usually taken as thermal bremsstrahlung. However, for a two-temperature disc, the cooling mechanism is unsaturated Comptonization through the loss of energy of electrons. In this case, the cooling rate can be expressed as (see also SLE)

\[ Q_\perp = \frac{4kT_e}{m_e c^2} \rho H \kappa_{es} U_e c \]

where \( m_e \) is the electron mass, \( \kappa_{es} = 0.4 cm^2 g^{-1} \) is the electron scattering opacity, \( U_e \) is the radiation energy density of soft photons, which we assume, for simplicity, does not change on timescale short compared to \( \Omega^{-1} \) (Pringle 1976). Because the ions and electrons are coupled by collisional energy exchange, the loss of energy of electrons can be balanced by the energy capture from ions. The exchange rate is

\[ Q_{i\perp} = \frac{3}{2} \rho H \nu_E T_i (T_i - T_e) / m_p \]

where \( \nu_E \) is the electron-ion coupling rate, and can be approximated by \( \nu_E = 2.4 \times 10^{21} ln \Lambda \rho T_e^{-3/2} \) (Spitzer 1962) where the Coulomb logarithm \( ln \Lambda \) is about 15 (SLE). Taking \( Q_\perp = Q_{i\perp} \) and \( T_i >> T_e \), we can get \( Q_\perp \propto \Sigma^{7/5} T_i^{1/5} \). \( Q_t \) at the right side of Eq. (4) represents the thermal diffusion defined as \( Q_t = \nabla \cdot (K \nabla T) \), where \( K \) is the vertical integrated thermal conductivity given by \( K = \Sigma C_v \nu = \alpha f \Omega_k H^3 p / T \), where \( f = 3(8 - 7\beta) f_* \) and \( f_* \) is of the order of unity (Kato et al. 1996; Wu & Li 1996). In this paper, we take \( f = 3 \) since a two-temperature disc is usually gas pressure dominated and \( \beta \approx 1 \).

The equilibria of a two-temperature disc can be calculated by numerically solving above equations with \( \partial / \partial t = 0 \). Ignoring the influences of radial viscous force and thermal diffusion, the disc equilibria have been calculated by many authors (e.g., Narayan & Yi 1995b; Chen et al. 1995; Nakamura et al. 1996). In this paper, we assume the influences of radial viscous force and thermal diffusion on the disc equilibria are small, and refer to the solution of the disc structure obtained by Narayan & Yi (1995b). We also restrict our stability analyses within the validity of local approximation and vertical integrated equations, which requires that \( \lambda < r \) and \( k V_r < \Omega_k \). These two inequalities can be summerized as \( \frac{T}{H} > \frac{\lambda}{H} > 2\pi \alpha \frac{H}{r} \), which is well satisfied for a geometrically thin accretion disc in a wide range of perturbation wavelength but for a geometrically slim disc only when the perturbation wavelength is short and viscosity efficient \( \alpha \) is sufficiently small (Kato et al., 1996; Wu & Li 1996). The vertical hydrostatic balance can be realized if the time scale associated with the perturbations is longer than the dynamical time scale. Although for a geometrically thick disc, the perturbation wavelength can be smaller than the disc height \( \lambda < H \), which means that the perturbations are also local in the vertical direction (Kato et al. 1997), the consideration of the local vertical perturbations for a two temperature disc does not change the stability properties very much (Yamasaki 1997).

### 3 Linear perturbations and dispersion relation

In this section, we consider the linear perturbations to the two-temperature disc. The radial perturbations of \( V_r, \Omega, \Sigma \) and \( T \) are assumed of the form \( \delta V_r, \delta \Omega, \delta \Sigma, \delta T \sim e^{i(kr - \omega t)} \), where \( k \) is the perturbation wavenumber defined by \( k = 2\pi / \lambda \), \( \lambda \) is the perturbation wavelength. Taking the perturbed quantities to the basic equations given in last section...
and considering the local approximation, we can obtain following perturbed equations:

\[ \tilde{\sigma} \frac{\delta \Sigma}{\Sigma} - i \epsilon \frac{\delta V_r}{\Omega_k r} = 0, \]  

\[ - i \epsilon \tilde{H} \frac{\delta \Sigma}{\Sigma} + (\tilde{\sigma} + \frac{4}{3} \alpha^2) \frac{\delta V_r}{\Omega_k r} - 2 \tilde{\Omega} \frac{\delta \Omega}{\Omega_k} - i \epsilon \tilde{H} \frac{\delta T}{T} = 0, \]  

\[ i \alpha \epsilon \tilde{H} \frac{\delta \Sigma}{\Sigma} + \chi^2 \frac{\delta V_r}{\Omega_k r} + (\tilde{\sigma} + \alpha^2) \frac{\delta \Omega}{\Omega_k} + i \alpha \epsilon \tilde{H} \frac{\delta T}{T} = 0, \]  

\[ -(\tilde{\sigma} + \alpha(-\frac{2}{5} + \frac{7}{5}q)g^2) \frac{\delta \Sigma}{\Sigma} - \frac{\alpha \epsilon g^2}{m \tilde{H}} \frac{\delta V_r}{\Omega_k r} + \frac{2i \alpha \epsilon \phi \Omega}{H} \frac{\delta \Omega}{\Omega_k} + \frac{3}{2} \tilde{\sigma} + \alpha(\frac{4}{5} + \frac{1}{5}q)g^2 + \alpha \epsilon^2 \frac{\delta T}{T} = 0, \]  

where \( \tilde{\sigma} = \sigma / \Omega_k \), and \( \sigma = i(\omega - kV_r) \). \( \tilde{\Omega} = \Omega / \Omega_k \), \( \tilde{H} = H / r \), and \( \epsilon = kH \). \( g = \frac{\epsilon^2}{2m} - 2\tilde{\Omega} \), and \( \tilde{\chi} = \chi / \Omega_k \) where \( \chi \) is the epicyclic frequency defined by \( \chi^2 = 2\Omega(2\Omega^2 + \frac{r \alpha^2 g}{m}) \). \( m \) is the Mach number defined by \( m = |V_r| / c_s \). \( q \) is the ratio of advective energy to viscous dissipated energy, namely

\[ C_v[\Sigma V_r \frac{\partial T}{\partial r} - (\Gamma_3 - 1)T (\frac{\partial \Sigma}{\partial t} + V_r \frac{\partial \Sigma}{\partial r})] = q \Sigma \nu (r \frac{\partial \Omega}{\partial r})^2. \]  

If the disc is radiative cooling dominated, \( q \) is nearly zero and if it is advection-dominated, \( q \) is nearly 1.

By setting the determinants of the coefficients in above perturbed equations to zero, we get a dispersion relation:

\[ a_1 \tilde{\sigma}^4 + a_2 \tilde{\sigma}^3 + a_3 \tilde{\sigma}^2 + a_4 \tilde{\sigma} + a_5 = 0, \]  

where \( a_i \) (\( i = 1, ..., 5 \)) is the coefficients given by

\[ a_1 = \frac{3}{2}, \]  

\[ a_2 = \alpha[\epsilon^2(f + \frac{7}{2}) - (\frac{4}{5} + \frac{1}{5}q)g^2], \]  

\[ a_3 = \alpha \epsilon g^2[\alpha \epsilon(\frac{2}{15} - \frac{7}{15}q) - \frac{1}{m}] + \frac{1}{3} (\alpha^2)^2 (6 + 7f) + \frac{5}{2} \tilde{\chi}^2 + \frac{3}{2} \chi^2, \]  

\[ a_4 = 2i \alpha^2 \tilde{\Omega} \epsilon g^3 \frac{q}{m} + \alpha^2 [\frac{4}{5} G + \frac{1}{5} q(\alpha^2 \epsilon^2) - i \alpha \epsilon^3 G] + \frac{3}{m} (\alpha^2)^2 (\frac{4}{5} + \frac{1}{5}q) \tilde{\chi}^2 \]  

\[ + \frac{6}{5}(q - 1)\epsilon^2] + \alpha \epsilon^2 G[\tilde{\chi}^2 \Omega - 5\tilde{\Omega}] + \alpha \epsilon^2 [\frac{4}{5} f(\alpha^2 \epsilon^2)^2 + f \tilde{\chi}^2 + \epsilon^2 (\frac{5}{2} + f)], \]  

\[ a_5 = (\alpha \epsilon)^2 \frac{12}{5}(1 - q) \tilde{\Omega} \epsilon g^3 + \epsilon^2 G^2 \frac{1}{(\frac{4}{5} + \frac{1}{5}q - 2f \tilde{\Omega} + \epsilon^4 f)]}. \]  

The stability properties of two inertial-acoustic modes, thermal and viscous modes can be obtained by analyzing the four kinds of solutions of the dispersion relation. The real parts of these solutions correspond to the growth rates of the perturbation modes and the imaginary parts correspond to their propagating properties. If the growth rate corresponding to certain mode is positive, this mode is unstable. Otherwise, it will be stable. Among
the four kinds of modes in the disc, the stability of thermal and viscous modes have been more extensively studied in the literatures. The instability of inertial-acoustic mode (or called the pulsational instability), was first addressed in viscous accretion discs by Kato (1978) and was detailed studied by Blumenthal, Yang & Lin (1984) later. It has been suggested that the inertial-acoustic instabilities may account for the observed quasi-period oscillations (QPO) in some Galactic black hole candidates (Chen & Taam 1995).

4 Numerical results

In this section we will numerically solve the dispersion relation Eq. (13). The rotation law of disc is further assumed to be Keplerian. Although Narayan & Yi (1994) obtained a sub-Keplerian rotation law ($\Omega \sim 0.5\Omega_k$ in a hot optically thin disc), such a difference will not affect significantly on our results. According to the different possible structure of a two-temperature disc, we solve the dispersion relation in following three cases: (a) Geometrically thin, cooling dominated two-temperature disc. We take $\tilde{\Omega} = \tilde{\chi} = 1, \alpha = 0.01, m = 0.01$. According to the local restrictions, $\lambda/H$ is set from 1 to 80 for a geometrically thin disc (in this section we take $H/r = 0.01$). By solving the dispersion relation, we get the results shown in Fig.1. For comparison, we also show the stability properties of an optically thin, geometrically thin bremsstrahlung disc, which has been suggested to be thermally unstable (Pringle, Rees & Pacholczyk 1973). Fig. 1(a) shows the case without advection. We can clearly see that the thermal mode is always unstable and the Lightman & Eardley viscous mode is always stable. This result agree well with the previous finding. The inertial-acoustic modes are slightly unstable but stable to very short wavelength perturbations. In addition, we see that the thermal mode in a bremsstrahlung disc is more unstable than that in a two-temperature disc. Fig. 1(b) shows the case with very little advection ($q = 0.01$). It is clear that the inclusion of very little advection has nearly no effects on the thermal and viscous modes, but leads to the departure of two acoustic modes. Comparing to the case without advection, the outward propagating acoustic mode (hereafter O-mode) now becomes more unstable, whereas the inward propagating acoustic mode (hereafter I-mode) becomes more stable. In Fig. 1(c) we show the case with $q = 0.01$ and $m = 0.1$. Comparing with Fig. 1(b), the increase of Mach number leads to the decrease of the departure of two acoustic modes. Actually, as we have noted in a previous work (Wu & Li 1996), such a departure is proportional to the term $q/m$. Both the O-mode and the I-mode are unstable to the longer wavelength perturbation if $q/m$ less than 0.1. The change of $q/m$, however, has nearly no effects on the thermal and viscous modes. Fig. 1(d) shows the case when $q = 0.01, m = 0.01$ but with thermal diffusion included. Comparing with Fig 1.(b), we see that the inclusion of thermal diffusion has nearly no effect on the viscous mode and two acoustic modes, but it tends to stabilize the thermal instability in the short perturbation wavelength case. In general, if the disc is geometrically thin and cooling dominated, the stability properties of a two-temperature disc are quite similar to those of a bremsstrahlung disc. But we still see clearly that the thermal mode in a bremsstrahlung disc is more unstable than that in a two-temperature disc. We noted that these results agree well with those obtained by Luo & Liang (1994), who have pointed out that the thermal mode is always unstable and viscous modes is always stable in a hot optically thin disc. However, they did not consider
the stability of inertial-acoustic modes and the effects of radial viscous force, advection and thermal diffusion.

(b) *Geometrically slim, cooling dominated two-temperature disc.* Some previous works have suggested that a hot optically thin disc may be not geometrically thin but geometrically slim or thick (e.g., SLE) even if the advection term was ignored. Here we show the stability of a geometrically slim, two-temperature disc. In this section we take $H/r = 0.6$. The perturbation wavelength is set from 0.2 to 2 according to the local approximation. Fig. 2(a) shows the case when $m = 0.01, \alpha = 0.001$ and without advection ($q = 0$). We can see that the inertial-acoustic modes and the viscous mode are always stable. Only the thermal mode is unstable when perturbation wavelength is longer than 1.5H. Fig. 2(b) show the similar results as Fig. 2(a) but with very little advection included ($q = 0.01$). In comparison with Fig. 2(a) we see that two acoustic modes now slightly depart from each other. For clarity, Fig. 2(c) compares the stability of thermal modes in two cases above. The stability of thermal mode of an optically thin, bremsstrahlung disc is also shown for comparison. It is quite clear that the inclusion of very little advection has only very slight effects on the thermal mode, which is always unstable in a bremsstrahlung disc but can become stable in a two-temperature disc if the perturbation wavelength is shorter. The case with thermal diffusion considered is shown in Fig. 2(d). We clearly see that all four kinds of modes in a two-temperature disc are always stable if the thermal diffusion is considered. We also noted that an optically thin, geometrically slim bremsstrahlung disc can become stable if the thermal diffusion is considered, even if it is more thermally unstable than a two-temperature disc when the thermal diffusion is ignored.

(c) *Geometrically slim, advection-dominated two-temperature disc.* The advection-dominated equilibrium of an optically thin, two-temperature disc has been recently constructed by some authors (e.g., Narayan & Yi 1995b; Chen et al. 1995; Nakamura et al. 1996). Such a disc is usually geometrically slim. In Fig. 3, we show its stability against short wavelength perturbations. Here we take $q = 0.99$ and $H/r = 0.6$. Fig. 3(a) shows the case with $m = 0.01$ and $\alpha = 0.001$. In this case, the thermal, viscous modes are always stable. The inertial-acoustic modes depart from each other. The I-mode is stable but the O-mode can become unstable if the perturbation wavelength is larger. Fig. 3(b) shows the case with different Mach number ($m = 0.1$). Comparing with Fig. 3(a), the departure of two acoustic modes becomes less due to the decrease of $q/m$. The O-mode can become stable if $q/m$ less than 10. However, the change of $q/m$ has no effects on the thermal and viscous modes, which are always stable. In Fig. 3(c), we show the case similar to Fig. 3(a) but with thermal diffusion included. We can clearly see that the inclusion of thermal diffusion has a significant effect to stabilize the thermal mode, but it has nearly no effects on the viscous and acoustic modes. We noted that in an optically thin, advection-dominated disc, the stability properties do not change evidently if the different radiative cooling mechanism was involved. Whether the disc is bremsstrahlung one or two-temperature one, the stability properties are always the same. This is, of course, due to the unimportance of radiative cooling in an advection-dominated disc.
5 Discussions

The stability of a hot optically thin two-temperature disc has been discussed in above sections by numerically solving the dispersion relation. We proved that the thermal instability exists in the disc if it is cooling dominated but disperses if it is advection-dominat ed. The Lightman-Eardley viscous instability is always absent in a hot optically thin disc. These properties agree well with previous qualitative results. We have also investigated the stability of two inertial-acoustic modes, which has not been discussed previously in a hot optically thin, two-temperature disc. In a geometrically thin, radiative cooling dominated disc, we found that the O-mode is always unstable but the I-mode is unstable only if the term $q/m$ less than 0.1. If the disc is cooling dominated but geometrically slim, no inertial-acoustic mode is unstable. However, in a geometrically slim and advection-dominated disc, the O-mode can become unstable if $q/m$ larger than 10, though the I-mode is always stable. These stability properties of acoustic modes may be important when they are involved to explain the QPO phenomena in some systems such as Galactic black hole candidates (Chen & Taam 1995; Manmoto et al. 1996).

A simple comparison shows that the bremsstrahlung disc is more thermally unstable than the two-temperature disc if they are cooling dominated. This can been seen clearly by comparing their different temperature-dependence of the cooling rates. In a bremsstrahlung disc, the cooling rate $Q_b^-$ is proportional to $\Sigma^2$. In a two-temperature disc, the cooling rate $Q_t^-$ is proportional to $\Sigma^{7/5} T^{1/5}$. In these two cases the energy generated rate by viscous dissipation can be both expressed as $Q^+ = \Sigma \nu \Omega_r \frac{\partial \Omega}{\partial r} \propto \Sigma T$. Thus, if we give a positive perturbation $\delta T$ and keep the $\Sigma$ unperturbed, $Q^+$ will grow more rapidly than $Q_b^-$ and $Q_t^-$, which leads to thermal instability (see also Piran 1978). Because $Q^+ - Q_t^-$ is less than $Q^+ - Q_b^-$ with a positive perturbation of temperature, the two-temperature disc is less thermally unstable than the bremsstrahlung disc. If the hot optically thin disc is advection-dominated, however, the disc stability will not depend on the detailed cooling mechanism due to its less importance.

It is quite evident that the advection and thermal diffusion have significant effects on the stability of a hot optically thin disc. These effects are more evident if the disc is not geometrically thin. As having been pointed out by Wu & Li (1996), the thermal diffusion in an accretion disc is in proportional to $(H/r)^2$. Therefore, the thermal diffusion should be seriously considered in the hot optically thin disc which is usually geometrically slim or thick. We think that the inclusion of thermal diffusion will affect not only the stability but also the structure of accretion discs. A future investigation on this point is expected.

More recently, we noted that Kato et al. (1997) pointed out that the perturbations could be also local in the vertical direction for a geometrically thick disc. This is true but the consideration of the vertical local perturbations seems to have only minor contribution to the growth rate of unstable mode. For a two temperature disc, the independent analytic stability analysis of Yamasaki (1997) found that there are two kinds of modes in the disc. One is thermal mode which is slightly unstable and the other is viscous Lightman & Eardley mode which is always stable. The inclusion of thermal diffusion will stabilize the thermal mode. These result agree quite well as our quantitative analyses. However, we noted that the radial viscous force was not included in the analysis of Yamasaki (1997). Consiration of this will contribute a term in proportional to $(kH)^2$ in the perturbed equation of radial momentum conservation, which can not be neglected for a geometrically thick disc. It will
result in the absence of the weak thermal instability of a hot optically thin disc when the short wavelength perturbations are considered (Wu, Yang & Yang 1994; Wu & Li 1996).

We should noted that the radiative cooling rate of a two-temperature disc may be not as simple as we assumed in Section 2. For example, we have assumed that the radiation energy density of soft photons, $U_r$, is unchanged on dynamical timescale, which may be not always the case. Some recent works on the hot optically thin discs suggested that the cooling mechanism is rather complicated (See e.g., Esin et al. 1996). The disc structure also depends on the radius significantly. This implies that the disc stability may be quite different from one radius to another. Thus, the global stability analyses of a hot optically thin disc are still needed. In addition, the pair production and annihilation in a hot accretion disc are totally neglected in this paper for simplicity because the inclusion of pair process will make the linear stability analysis very complicated. Although this process may be important especially if the disc temperature is higher enough, the pair density is believed to be not too high and their influence is limited (Bjornsson et al. 1996; Kusunose & Mineshige 1996). However, we still expect a future detailed work would be done to investigate their influences on the structure and stability of a hot accretion disc.

Together with some previous detailed stability analyses, our study show that there are perhaps only two stable thermal equilibria of accretion discs. One is optically thin, advection-dominated and the other is optically thick, gas-pressure and radiative cooling dominated. These equilibria are probably related with some stable, inactive astronomical systems. For example, the optically thin, advection-dominated accretion discs have been suggested to exist at the center of our Galaxy (Narayan, Yi & Mahadevan 1995), nearby elliptical galaxies (Fabian & Rees 1995), fainter AGNs (Lasota et al. 1996) and soft X-ray transient sources in the quiescent state (Narayan, McClintock & Yi 1996). On the other hand, the unstable thermal equilibria of accretion discs, such as the optically thin, radiative cooling dominated one, the optically thick, radiation pressure dominated one and the optically thick, advection-dominated one, probably exist in some unstable systems such as the inner region of AGNs, X-ray binaries and cataclysmic variables. In order to understand the light variability of these systems more clearly, the detailed time-dependent nonlinear studies on the evolution of their disc structures are still expected.

Acknowledgments

I am very grateful to Professor Ramesh Narayan for mentioning me a few important references and some valuable suggestions. I also thank Professor Qibin Li and Dr. Yongheng Zhao for many helpful discussions. The work was partially supported by the Postdoc Science Foundation of China.

References

Abramowicz, M.A., Czerny, B., Lasota, J.P., Szuszkiewicz, E. 1988, ApJ, 332, 646
Abramowicz, M.A., Chen, X., Kato, S., Lasota, J.-P., Ragev, O. 1995, ApJ, 438, L37
Bjornsson, G., Abramowicz, M., Chen, X., Lasota, J.-P., 1996, ApJ, 467, 99
Blumenthal, G.R., Yang, L.T., Lin, D.N.C. 1984, ApJ, 287, 774
Chen, X. 1995, MNRAS, 275, 641
Chen, X. 1996, in “Basic Physics of Accretion Disks” ed. by Kato S. et al., Gondon and Breach Science Publishers, in press
Chen, X., Abramowicz, M.A., Lasota, J.-P., Narayan, R., Yi, I. 1995, ApJ, 443, L61
Chen, X., Taam, R.E. 1995, ApJ, 441, 354
Esin, A.A., Narayan, R., Ostriker, E., Yi, I., 1996, APJ, 465, 312
Fabian, A.C., Rees, M.J., 1995, MNRAS, 277, L55
Kato, S. 1978, MNRAS, 185, 629
Kato, S., Abramowicz, M.A., Chen, X. 1996, PASJ, 48, 67
Kato, S., Honma, F., Matsumoto, R. 1988, MNRAS, 231, 37
Kato, S., Yamasaki, T., Abramowicz, M.A., Chen, X., 1997, PASJ, 49, 221
Kusunose, M. Mineshige, S., 1996, ApJ, 468, 330
Kusunose, M., Takahara, F., 1988, PASJ, 40, 709
Kusunose, M., Takahara, F., 1990, PASJ, 1, 263
Lasota, J.P., Abramowicz, M., Chen, X., Krolik, J., Narayan, R., Yi, I., 1996, ApJ, 462, 142
Lightman, A., Eardley, D. 1974, ApJ, 187, L1
Luo, C., Liang, E.P., 1994, MNRAS, 266, 386
Manmoto, T., Takeuchi, M., Mineshige, S., Matsumoto, R., Negoro, H., 1996, ApJ, 464, L135
Misra, R., Melia, Fulvio., 1996, ApJ, 465, 869
Nakamura, K.E., Matsumoto, R., Kusunose, M., Kato, S., 1996, PASJ, 48, 761
Narayan, R., McClintock, J.E., Yi, I., 1996, ApJ, 457, 821
Narayan, R., Yi, I. 1994, ApJ, 428, L13
Narayan, R., Yi, I. 1995a, ApJ, 444, 231
Narayan, R., Yi, I. 1995b, ApJ, 452, 710
Narayan, R., Yi, I., Mahadevan, R., 1995, Nature, 374, 623
Paczyński, B., Wiita, P.J., 1980, A&A, 88, 23
Papaloizou, J.C.B., Stanley, G.Q.G. 1986, MNRAS, 220, 593
Piran, T. 1978, ApJ, 221, 652
Pringle J.E., 1976, MNRAS, 177, 65
Pringle, J.E., Rees, M.J., Paczolczyk, A.G. 1973, A&A, 29, 179
Shakura, N.I., Sunyaev, R.A. 1973, A&A, 24, 337
Shakura, N.I., Sunyaev, R.A. 1976, MNRAS, 175, 613
Shapiro, S.L., Lightman, A.P., Eardley, D.N. 1976, ApJ, 204, 187
Spitzer, L., 1962, Physics of fully ionized gases, Wiley, New York.
Wandel, A., Liang, A.P., 1991, ApJ, 380, 84
Write, T.R., Lightman, A.P., 1989, APJ, 340, 1024
Wu, X.B., Li, Q.B., ApJ, 469, 776
Wu, X.B., Yang, L.T., Yang, P.B., 1994, MNRAS, 270, 465
Yamasaki, T., 1997, PASJ, 49, 227
**FIGURE CAPTIONS**

**Figure 1.** Stability of a geometrically thin, optically thin disc. Heavy lines represent the case of a two-temperature disc and light lines represent the case of bremsstrahlung disc. In (a) no advection is included, the solid, long-dash and short-dash lines correspond to the acoustic modes, thermal mode and viscous mode respectively. (b), (c) and (d) show the cases with very little advection (q=0.01), parameters (m, f) are taken as (0.01, 0) in (b), (0.1, 0) in (c) and (0.01, 3) in (d). The long-dashed and short-dashed lines in (b), (c) and (d) have the same meaning as in (a) but the solid and dotted lines correspond to the O-mode and the I-mode.

**Figure 2.** Stability of a geometrically slim, hot two temperature disc. In (a) no advection is included, the solid, long-dashed and short-dashed lines correspond to the acoustic modes, thermal mode and viscous mode respectively. (b) shows the case with very little advection but without thermal diffusion (q=0.01, f=0). The solid and dotted lines correspond to the O-mode and the I-mode. In (c) the differences of thermal instabilities in two kinds of optically thin discs are indicated. The heavy lines represent the case of a two-temperature disc and light lines represent the case of bremsstrahlung disc. The dot long-dashed lines correspond to the thermal modes when little advection is included (q=0.01). (d) shows the case similar with (b) but with thermal diffusion included (f=3). The thermal and viscous modes are mixed together in this case.

**Figure 3.** Stability of a geometrically slim, advection-dominated two temperature disc. The solid, dotted, long-dashed and short-dashed lines correspond to the O-mode, I-mode, thermal and viscous mode respectively. The parameters (m, f) are (0.01, 0) in (a), (0.1, 0) in (b) and (0.01, 3) in (c). The thermal and viscous modes are mixed together in (c) where the thermal diffusion is included.
Figure 1
Figure 2
Figure 3