Investigation of image charge induced vertical electric fields inside an asymmetric electrostatic quadrupole

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Abstract: This paper presents studies with regards to an asymmetric electrostatic quadrupole profile, which helps suppressing image beam induced vertical electric fields. These fields need to remain orders of magnitude smaller than the radial electric field in storage ring electric dipole moment experiments. Otherwise, the coupling with the magnetic dipole moment dominates the spin precession, eventually leading to a false signal. Suppression performance of an optimized quadrupole profile is investigated for various beam offset and size configurations. With the optimum profile, the vertical electric field can be reduced by two orders of magnitude. Several centimeter range beam size and position offsets have been observed to have small effects on the field reduction. Besides the quadrupole elements, using non-magnetic conductor materials, this kind of a profile could be implemented at other sections of accelerators as well.

Key words: Electrostatic quadrupoles, particle accelerator, image beams, proton EDM experiment

1. Introduction

In high precision spin physics experiments, electric and magnetic fields require a high-precision control to avoid systematic errors. Let’s consider an electric dipole moment (EDM) experiment in a storage ring. While a charged particle beam (proton [1] or deuteron [2]) travels inside a storage ring in the presence of electric and magnetic fields, its EDM couples with the main field, leading to a spin precession. However, even tiny fields (as low as atto-Tesla level magnetic field) can dominate the EDM signal [3]. In some cases, it may not be feasible to measure and compensate these unwanted fields. On the other hand, even if they can be compensated by means of measurement and feedback, an inhibitory design is more preferable.

The proton EDM experiment [1] aims to measure the electric dipole moment of proton with a $d_p \approx 10^{-29} \text{e} \cdot \text{cm}$ sensitivity through precession of its spin inside a storage ring. Longitudinally polarized proton beams, composed of approximately 100 bunches of $10^9$ particles are to experience a radial electric field for $10^3$ seconds.

From the T-BMT equation [4, 5], in the presence of only electric field $E_z$, the spin precession rate is given as

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\[
\frac{d\vec{s}}{dt} = \frac{e}{m} \vec{s} \times \left( \left( \frac{1}{\gamma^2 - 1} - G \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2} \left( \frac{\vec{E}}{c} - \frac{\gamma}{\gamma + 1} \frac{\vec{\beta} \cdot \vec{E}}{c} \vec{\beta} \right) \right),
\]

(1)

where \(\vec{s}\) is the spin vector, \(c\) is the speed of light, \(e\) and \(m\) are the electric charge and the mass of the particle, \(\vec{\beta} = \vec{v}/c\) and \(\gamma = 1/\sqrt{1 - \beta^2}\) are the relativistic velocity and Lorentz factor, respectively. \(G\) is the magnetic anomaly, a unique parameter proportional to the magnetic dipole moment of the particle. \(\eta\) is the EDM coupling term, similarly a unique parameter that is proportional to the electric dipole moment of the particle. Coupling between \(\eta\) and the electric field in the second term determines the EDM signal, while the first term is a source of a false EDM signal.

For particles with a positive \(G\), the first term can ideally be cancelled by means of the “frozen spin method” [6, 7], which require every particle in the beam to have a specific (magic) momentum. For protons, which have \(G \approx 1.8\), that momentum is \(0.7 \text{ GeV}/c\) and it corresponds to \(\gamma = 1.248\) and \(\|\vec{\beta}\| = 0.59\). Even in the presence of a momentum spread, to first order, every particle can be forced to have magic momentum by means of RF bunching [8]. However, second order effects can still cause a non-negligible spin precession [9]. Assuming an ideal scenario, which includes a mainly longitudinal velocity (\(\|\vec{\beta}\| \approx \beta_L\)) and polarization (\(\|\vec{s}\| \approx s_L\)), a mainly radial electric field (\(\|\vec{E}\| \approx E_r\)) and magic momentum by means of the RF bunching technique, Equation (1) simplifies to the following equation by utilizing the identity \(\beta = \sqrt{1 - 1/\gamma^2}\).

\[
\frac{ds_x}{dt} = \frac{es_L}{mc} \left[ \frac{G(3\gamma^4 - 5\gamma^2 + 2) + 6\gamma^4 - 5\gamma^2 + 2}{\gamma^3(\gamma^2 - 1)^{5/2}} E_v(\Delta \gamma)^2 + \frac{\eta E_r}{2} \right].
\]

(2)

Here \(v, l,\) and \(r\) represent the vertical, longitudinal and radial directions, respectively. For a practical hadronic beam, one should expect a \(\Delta \gamma = 10^{-4}\gamma^2\) energy spread. Then, inserting \(G = 1.8\) and \(\gamma = 1.25\), one can see that an average \(E_v = 25 \text{ mV/m}\) can produce a false EDM signal comparable to the real one, which is generated by the coupling between \(E_r = 5 \text{ MV/m}\) and \(\eta \approx 2 \times 10^{-15}\) (corresponding to EDM of \(d_p \approx 10^{-29} \text{ e \cdot cm}\)). This can be a problem if \(E_v\) is not compensated properly, both globally and locally.

Besides the external sources, vertical electric fields originate from image beams on the walls of the surrounding material as well. A significant effect\(^\dagger\) can be observed from the vacuum chamber, quadrupoles, sextupoles, and so on. For most of the ring elements, including magnetic quadrupoles and sextupoles, vertically oriented, grounded parallel plates\(^\ddagger\) can shield the electric fields from the vertical image charges. However, limited apertures of electrostatic quadrupoles and sextupoles do not allow such a setup. This numerical study is dedicated to an alternative method, namely an asymmetric quadrupole profile, which suppresses such vertical electric fields significantly. This method was first pointed out in Ref. [1]. However, no quantitative estimation was provided with regards to the field suppression performance and potential limitations of the method. In this paper, after demonstrating the optimization of a quadrupole profile, we investigate its performance quantitatively under several experimental conditions.

\(^\dagger\) First estimated by William M. Morse.

\(^\ddagger\) Proposed by Yannis K. Semertzidis.
2. Estimation of the electric field inside a grounded quadrupole profile

Figure 1 shows the cross section of a vertically off-center, 1 mm radius beam inside a grounded quadrupole-like profile, and the electric potential around it as calculated by solving the Poisson’s equation. White and black colors represent the ground and maximum potential, respectively.

The square shaped surface that includes the profile and the inner area (vacuum) is divided into 151×151 meshes. Each mesh has a 1mm x 1mm dimension. The shapes of the top/bottom and right/left edges of the quadrupole electrodes are determined by $Y = \pm (a_0 x + a_2 x^2)$ and $X = \pm (b_0 y + b_2 y^2)$, respectively, where $(x, y)$ is the mesh position with respect to the center of the square, and $a_i$ and $b_i$ are constants that define the curvature and the aperture. $X$ and $Y$ represent the points on the quadrupole plates that correspond to $y$ and $x$, respectively. In all of the simulations, $b_0 = 4$ cm ($a_0$ and $b_0$ are shown in Figure 2).

Quadrupole plates, which are fixed at zero potential, cover the region of interest. The edges of the square are also fixed at zero potential to avoid floating boundaries. The electrode shapes are determined by $a_i$ and $b_i$ in such a way that the electric potential on them produces quadrupole electric fields at the region of interest. Despite this specific choice, this method works for a wide range of $a_i$ and $b_i$ values as long as the profile does not have too flat or sharp curves. The beam is defined in all simulations by fixed potential meshes (shown in black) with a total of $10^9$ protons. Regardless of the beam size, this region contains as small number of meshes as possible to obtain image charge induced potential distribution at (almost) every point.
Figure 2 Depiction of the simulated setup. The beam is shown by a black circle. The red circles at every electrode, whose locations are not necessarily accurate in this drawing, represent the image beams. They apply a net vertical electric field/force on the beam. As will be shown below, the vertical field can be cancelled by a correct choice of $a_0/b_0$.

The potential distribution around the beam is determined by solving the Poisson’s equation numerically, by means of finite difference method. At every point within the boundaries, it is given by the average potential of the surrounding points:

$$V_{i,j} = \frac{1}{4} (V_{i-1,j} + V_{i+1,j} + V_{i,j+1} + V_{i,j-1}),$$  \hspace{1cm} (3)

where $i$ and $j$ are the mesh indices that are mapped from $x$ and $y$, respectively. It is calculated iteratively on every mesh until the average change between iterations becomes insignificant (<1 ppm). Then, the average vertical electric field on the beam is calculated by integrating the gradient of the electric potential within the beam coverage.

A beam does not experience a vertical electric field from a symmetric quadrupole profile if it is at the geometrical center. On the other hand, the top/bottom electrodes cause vertical, and right/left plates cause both horizontal and vertical forces on the beam, which could be attributed to the asymmetrically distributed image beams. Figure 2 shows the image beams and the corresponding electric forces. It will be shown below that the vertical electric field/force can be cancelled significantly with the correct choice of the aspect ratio $a_0/b_0$.

Figure 3 Average vertical electric field ($E_v$) on the beam inside a quadrupole profile with $a_0/b_0 = 1$, as a function of vertical offset. The vertical electric field that originates from the image beams on the quadrupole electrodes grows linearly with the vertical position.
Finally, let’s assume that as in the proton EDM experiment proposal, the charged particle beams and the quadrupole elements (with $2a_0 \approx 2b_0 = 8$ cm aperture) have comparable longitudinal lengths (at the order of 50 cm). In such a configuration, the potential due to the image beams on a plate is approximately 1.5-2 times the 2-dimensional (2D) solution. As the difference is insignificant, the conclusions of these 2D simulations can be generalized to a realistic 3D scenario.

3. Simulation results

Figure 3 shows the vertical electric field on the beam due to the image beams as a function of its vertical offset. The simulations are made for a beam of 5 mm radius and $10^9$ particles. The aspect ratio of the profile is $a_0/b_0 = 1$. The slope of the line indicates that the image beams apply a 180 V/m net vertical electric field per 1 mm vertical beam offset. As the quadrupoles cover roughly 0.5 percent of the proton EDM storage ring [1], the average vertical electric field due to a 1 mm vertical offset inside a quadrupole becomes at the order of 1 V/m. Then, following the conclusions of Equation 2, the vertical beam offset should be kept smaller than 25 µm to avoid related systematic errors. Clearly, the effect becomes more significant for larger beam currents, eventually requiring a beam/quadrupole alignment with a few micrometers of precision.

![Figure 4](image)

Figure 4 Net vertical electric field as a function of the aspect ratio. The beam is located 1 mm above the geometrical center. $E_v$ passes through zero at $a_0/b_0 \approx 1.7$. The blue curve aims to lead the eye.

As mentioned above, a different quadrupole profile produces a different vertical electric field. Figure 4 shows the net vertical electric field on the beam as a function of the aspect ratio ($a_0/b_0$). The beam is located 1 mm above the geometrical center. As $a_0/b_0$ approaches to 1.7, the vertical electric field components due to the image charges cancel each other out and $E_v$ passes through zero. Then, it starts growing in the opposite direction.

In the proton EDM experiment proposal, the proton beam oscillates around the design orbit with approximately 1 cm amplitude. Figure 5 shows the vertical electric field on the beam as a function of vertical beam offset $y$ within $\pm 1$ cm range. The beam radius is 5 mm, and the aspect ratio is kept at 1.7 in all simulations. As expected, $E_v \to 0$ as $y \to 0$. 


irrespective of the aspect ratio. For nonzero $y$ values, $E_v$ remains roughly two orders of magnitude smaller than the $a_0/b_0 = 1$ case of Figure 3. Unsurprisingly, similar to the $a_0/b_0 = 1$ case, $E_v$ has opposite sign at positive and negative $y$ positions. The zero crossings at around $\pm 7$ mm could be due to numerical error. Despite being less clear because of the scale, the same wavy behavior is visible in Figure 3, as well.

Figure 5 Net vertical electric field that is induced by the image beams, as a function of vertical beam offset $y$. The aspect ratio $a_0/b_0$ of the quadrupole aperture is 1.7, and the beam radius is 5 mm. The improvement with respect to the $a_0/b_0 = 1$ case is roughly two orders of magnitude.

Figure 6 shows $E_v$ as a function of radius for a beam located at $y = 1$ mm. The simulations are done with two different aspect ratios. In both cases, the vertical electric field is almost flat over the simulated beam radii. Comparison of the two plots implies that the method works best at $a_0/b_0 \approx 1.67$ if the vertical beam offset is around 1 mm. It is worth noting that for a quadrupole with $b_0 \approx 4$ cm, $a_0$ differs by roughly 0.7 mm between $a_0/b_0 = 1.67$ and $a_0/b_0 = 1.7$ profiles.

Figure 6 The vertical electric field on the beam is almost flat as a function of the beam radius. Comparison of the two plots implies that the best performance can be obtained with the aspect ratio of $a_0/b_0 \approx 1.67$. 

\[ E_v \approx 1.65 \] 
\[ a_0/b_0 = 1.70 \]
Finally, horizontal beam offset is also investigated. These simulations are conducted with a beam of 5 mm radius and \( y = 1 \) mm vertical offset inside a quadrupole of \( a_0/b_0 = 1.7 \) aspect ratio. The \( x \) range is within \( \pm 8 \) mm, which is comparable to the oscillation amplitude of the beam in the proton EDM experiment proposal. As shown in Figure 7, the vertical electric field for this range is growing as the beam moves away from the origin. The left and right sides do not cancel each other because the horizontal offset does not change the direction of the vertical forces (See Figure 2). This causes \( E_v \) increase within a factor of two on average, compared to a non-oscillating beam. Overall, the two orders of magnitude reduction gets slightly worse as a result of horizontal oscillations at practical amplitudes. Note that this limited dependence is consistent with the slight shift on the beam radius study (Figure 6).

![Figure 7 Vertical electric field as a function of horizontal offset. The simulations are conducted with \( a_0/b_0 = 1.7 \) and a vertical beam offset \( y = 1 \) mm. The increase in \( E_v \) is insignificant, even though the effect does not cancel on opposite sides of \( x = 0 \).](image)

### 4. Summary and conclusions

Image charge induced vertical electric fields cause a systematic error in storage ring EDM experiments, which can be reduced significantly by means of an asymmetric quadrupole profile. Quantitative estimations regarding this method, and its limitations were not presented in the literature before. This paper aims to fill that gap.

Finite difference simulations show that these vertical electric fields can be suppressed by two orders of magnitude with the optimized profile. While beam radius and transverse oscillations affect the suppression performance, the simulations have shown that the vertical electric field does not increase by more than a factor of two as long as the beam diameter and position are kept within a centimeter. That is, the two orders of magnitude suppression of the systematic error is achievable in a practical storage ring experiment. The simulation results also indicate that a millimeter level accuracy is sufficient for machining the needed quadrupole profile.

It is worth noting that usage of the asymmetric profile is not limited to electric quadrupoles. A similar approach can be applied to electric sextupoles, as well. Moreover, in case installation of vertical plates is not feasible, a non-magnetic grounded quadrupole profile can be installed inside other accelerator elements such as magnetic quadrupoles, magnetic sextupoles, magnetic deflectors, straight sections, and so on. In conclusion, the
asymmetric quadrupole profile is shown to be a useful tool to significantly suppress the image charge induced vertical electric fields with no additional cost.

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