Probing the reheating phase through primordial magnetic field and CMB

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Abstract

Inflationary magnetogenesis has long been assumed to be the most promising mechanism for production of large scale magnetic field in our universe. However, it is generically shown that such models are plagued with either backreaction or strong coupling problem within the standard framework. In this paper we have shown that reheating phase can play very crucial role in alleviating those problems. Assuming electrical conductivity to be in-effective during the entire period of reheating, the classic Faraday electromagnetic induction changes the dynamics of magnetic field drastically. Our detailed analysis reveals that this physical phenomena not only converts a large class of magnetogenesis model observationally viable without any theoretical problem, but also can uniquely fix the reheating equation of state given the specific values of scalar spectral index and the large scale magnetic field. Our analysis also opens up a new avenue towards constraining the inflationary and magnetogenesis model together via reheating.

Keywords: Primordial magnetic field, Reheating, Inflation,

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I. INTRODUCTION

Reheating is one of the most important early phase of our universe. It essentially links the standard thermal universe with its pre-thermal phase namely the inflationary universe through a complicated non-linear process. Over the years major cosmological observations [1–4] have given us ample evidences in understanding the theoretical as well as observational aspects of both the thermal and the non-thermal inflationary universe cosmology to an unprecedented label. However, the intermediate reheating phase is still at its novice stage in terms of both theory and observation. From the cosmic microwave background (CMB) anisotropy [1], one can estimate the baryon content of the universe which agrees extremely well with the theoretical prediction of big-bang nucleosynthesis (BBN). Furthermore, with the successful standard big-bang model we have a very good understanding over a large time scale of the universe from the present (redshift $z = 0$) to BBN stage ($z \sim 10^9$) at an energy scale $\sim O(1)$ MeV. The tiny fluctuation of CMB anisotropy can be successfully linked with the almost scale-invariant density fluctuation predicted by the inflation in the early Universe [6, 8]. Therefore, precession CMB data has provided us significant insight into how our universe evolves during inflation. However, in this paper, our goal is to understand the intermediate phase which joins the end of inflation and the BBN. This phase is largely ill-understood due to a lack of observational evidences. It is generically described by the coherently oscillating inflaton and its non-linear decay into the radiation field. In the Boltzmann description, the phase is parametrized by the reheating temperature ($T_{re}$) and the reheating equation of state ($\omega_{re}$). Still now both the parameters remain unconstrained except the reheating temperature which is approximately bounded within $10^{16}$ GeV $> T_{re} > T_{BBN} \sim 10$ MeV. However, if one takes into account non-perturbative reheating in the beginning, the upper bound on reheating temperature could be within $10^{10} - 10^{13}$ GeV [79]. Attempts to understand this phase in the literature can be broadly classified into two categories: By studying the background dynamics during reheating reveals useful information about the deep connection among the inflationary scalar spectral index ($n_s$) and the reheating temperature ($T_{re}$), and reheating equation of state ($\omega_{re}$) [43, 51, 56, 58]. Evolution of inflationary stochastic gravitational waves has been shown to encode valuable information when passing through this phase [7], [5] and reference therein.

In this paper we consider present-day Large Scale Magnetic Field (LSMF) combined with the CMB anisotropy to probe the reheating phase of the universe followed by the standard inflationary phase. For LSMF we consider simple model of primordial magnetogeneis [25–27, 51]. While probing the reheating phase through those observables, we observe how the magnetogenisis models itself will be
constrained by the observables as well. Inflationary magnetogenesis models have been studied quite extensively in the literature \[23, 51, 74–77\]. Mechanisms known so far are to introduce the interaction Lagrangian which explicitly breaks the conformal invariance in the electromagnetic sector. However, this mechanism generically suffers from either strong coupling or backreaction problem \[28\] which will be elaborated as we go along. In this regard reheating phase has recently been shown to play a very important \[25\] role. As stated earlier the primary motivation of our present study will be to see in detail how both the problems can be successfully resolved by the reheating phase for various inflationary models, and simultaneously provide constraints on the reheating. Taking into account both the CMB anisotropic constraints on the inflationary power spectrum and the present value of the large scale magnetic field, our analysis reveals an important connection among the reheating parameters \((T_{re}, w_{re})\), magnetogenesis models and inflationary scalar spectral index \((n_s)\).

The universe is observationally proved to be magnetized over a wide range of scales. Zeeman splitting, synchrotron emission, and Faraday rotation are some of the fundamental physical mechanisms by which the existence of a magnetic field can be probed. Various astrophysical and cosmological observations of those quantities tell us that our universe is magnetized over scales starting from our earth, the sun, stars, galaxies, galaxy clusters, and also the intergalactic medium (IGM) in voids. In the galaxies and galaxy clusters of few to hundred-kilo parsecs (kpc) scale, the magnetic fields have been observed to be of order a few micro Gauss \([9, 11, 14]\). Various observational evidence suggests that even the intergalactic medium (IGM) in voids can host a weak \(\sim 10^{-16}\) Gauss magnetic field, with the coherence length as large as Mpc scales \([15]\). At the astrophysical scale, time-evolving plasma helps the magnetic field to persist. The magnetic field at the cosmological scale can evolve and survive today due to the presence of the early plasma state of our universe before the structure formation. The time-evolving plasma, therefore, proves to be an ideal environment to have a sustainable evolution and growth of the magnetic field. However, any physical processes responsible for the successful magnetogenesis inside the time-evolving plasma, it is the tiny seed initial magnetic field which plays a significant role. In the cosmological context, the most popular mechanism in this regard is the primordial inflationary magnetogenesis. Inflation provides us an outstanding mechanism for producing coherent magnetic fields for a wide range of scales. Mpc scale magnetic field can survive until today as a cosmological relic whose magnitude could be \(\sim 10^{-16}\)G. On the other hand at small scales, this tiny inflationary magnetic field can be the seed field which will be further enhanced to galactic scale \(\mu\)G order magnetic field by the well known Galactic dynamo mechanism\([11, 16, 17]\).

We consider a standard scenario of inflationary magnetogenesis where electromagnetic field kinetic
term is conformally coupled with a scalar field. Background inflation dynamics naturally produces a large-scale electromagnetic field which subsequently evolves through the reheating phase. Instead of going into the details of the magnetogenesis mechanism, we concentrate on dynamics during reheating considering various inflationary models. In our analysis we assume the negligible Schwinger effect on the magnetogenesis. The paper is arranged as follows: in section II we discuss the general analysis of the primordial magnetic fields from inflation and also the reheating dynamics used to constraint the parameters in the scenario of inflationary magnetogenesis. In section II we also discuss different inflationary models and their dynamics, which are used to constrain the parameters. Subsequently in section IV we finally show how our analysis constrain the reheating as well as magnetogenesis model considering few observationally viable inflationary scenario.

II. INFLATIONARY MAGNETOGENESIS: GENERAL DISCUSSION

During inflation the large scale magnetic field is generated out of quantum vacuum, and then subsequently evolves though various phases of our universe. Therefore, the evolving magnetic field must encode the valuable information about the reheating. Considering the present value of the large scale magnetic field we, therefore, can place constraints not only on the parameters of the reheating phase, but also on the magnetogenesis model itself. Contrary to the convention, the important point of our present analysis is the assumption of conductivity being negligible until the end of reheating. The reason being the production of the radiation plasma occurs nearly at the end of reheating. From large number of studies [43, 44, 46, 56] so far almost entire reheating phase has been shown to be primarily inflaton energy dominated. In the context of inflationary magnetogenesis scenario, this particular assumption has recently been proposed to be important [25] during reheating. Conventionally after the end of inflation, the magnetic field on the super-horizon scales is assumed to be redshifted with the scale factor $a$ as $B^2 \propto 1/a^4$ provided inflaton energy density transfers into plasma and the universe become good conductor instantly right after the end of inflation. Hence, the electric field ceases to exist. However, in the reference, [25] it has been shown that if the conductivity remains small redshifts of magnetic energy density becomes slower, $B^2 \propto 1/a^6 H^2$, due to electromagnetic Faraday induction. Here $H$ is the Hubble parameter. This helps one to obtain the required value of the present day large scale magnetic field considering large class inflationary model which we describe below,
A. Quantizing the Ratra model: Electromagnetic power spectrum

The simplest magnetogenesis scenario which one can think of is the well-known scalar-gauge field model with the following interaction Lagrangian \( I(\phi)^2 FF \), well known as Ratra model [27]. In this interacting Lagrangian, the conformal symmetry is explicitly broken by the scalar field coupling function \( I(\phi) \) in the gauge field sector. During inflation, the model generally predicts strong primordial electric field than magnetic field, and that can backreact to invalidate the mechanism itself. First we discuss the model in detail and show how the reheating phase can come as our rescue of this backreaction problem. In the frame of a comoving observer having four-velocity \( u^\mu (u^i = 0, u_\mu u^\mu = -1) \), the magnetic and electric fields are defined as

\[
E_\mu = u^\nu F_{\mu\nu} , \quad B_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} u^\sigma F^{\nu\rho} .
\]

(1)

Where, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic field tensor and \( \epsilon_{\mu\nu\rho\sigma} \) is a totally antisymmetric tensor. The background is the well-known FLRW metric with the time dependent scale factor \( a(\tau) \) expressed in conformal coordinate,

\[
ds^2 = a(\tau)^2 (-d\tau^2 + dx^2) .
\]

(2)

As already described before, the gauge field action is taken to be,

\[
S = -\frac{1}{4} \int d^4x \sqrt{-g} I(\tau)^2 F_{\mu\nu} F^{\mu\nu} ,
\]

(3)

At this point let us point out that one can consider other inflationary magnetogenesis models with axion-electromagnetic field coupling [80–82], higher curvature coupling [83, 84] and apply our methodology presented here not only to constrain the reheating phase but also make the models under consideration viable.

In the inflationary magnetogenesis scenario, the essential idea is to quantize the electromagnetic field in the classical inflationary background. Here \( I(\tau)^2 \equiv I(\phi(\tau)) \), therefore, is the time dependent coupling arising from some classical background scalar field. To maintain generality we do not specify any background dynamics of the scalar field. Through this coupling the electromagnetic field experiences the spatially flat expanding FLRW background. In order to quantize the field components \( A_\mu \) are expressed in terms of irreducible scalar and vector components as follows,

\[
A_\mu = (A_0, \partial_i S + v_i) \quad \text{with} \quad \partial_i V_i = 0 .
\]

(4)
In the standard canonical quantization procedure, one writes $V_i$ in terms of the annihilation ($a_k$) and
the creation operator ($a_{-k}^\dagger$) as

$$V_i(\tau, x) = \sum_{p=1,2} \int \frac{d^3k}{(2\pi)^3} \epsilon_i^{(p)}(k) \left\{ e^{ik \cdot x} a_k^{(p)}(\tau) + e^{-ik \cdot x} a_{-k}^{(p)*}(\tau) \right\},$$  \hspace{1cm} (5)

here the $\epsilon_i^{(p)}(k)$ is the polarization vector corresponding to the two polarization direction $p = 1, 2$, which satisfy the following relations, $\epsilon_i^{(p)}(k)k_i = 0, \epsilon_i^{(p)}(k)\epsilon_i^{(q)}(k) = \delta_{pq}$. The creation and annihilation operators in Eq.(5) namely $a_k^{(p)}$ and $a_{-k}^{(p)*}$ are time independent. They satisfy the commutation relation

$$[a_k^{(p)}, a_{-k}^{(q)*}] = [a_k^{(p)*}, a_h^{(q)}] = 0, \quad [a_k^{(p)}, a_{-k}^{(q)*}] = (2\pi)^3 \delta_{pq} [3](k-h) .$$ \hspace{1cm} (6)

All the dynamics of the field will be encoded into the mode function which satisfy the following equation of motion,

$$u_k^{(p)\prime\prime} + \frac{2I'}{I} u_k^{(p)\prime} + k^2 u_k^{(p)} = 0. $$ \hspace{1cm} (7)

Where the prime denotes derivative with respect to the proper time $\tau$. Conventionally the electromagnetic power spectrum is expressed in terms of those mode functions as follows,

$$P_E(k) = \frac{k^3}{2\pi^2 a^4} \sum_{p=1,2} |u_k^{(p)\prime}|^2; \quad P_B(k) = \frac{k^5}{2\pi^2 a^4} \sum_{p=1,2} |u_k^{(p)}|^2.$$ \hspace{1cm} (8)

However, it would look physically elegant, if we express the power spectrum in terms Bogliubov coefficient which has direct physical interpretation in terms of quantum particle production. In order to do that let us first write down the Hamiltonian in terms of time independent creation and annihilation operator as follows,

$$H = \frac{I'^2(\tau)}{2} \sum_{p=1,2} \left( k|u_k^{(p)}|^2 + \frac{1}{k} |u_k^{(p)\prime}|^2 \right) \left( a_k^{(p)*}\dagger a_k^{(p)} + a_{-k}^{(p)*}\dagger a_{-k}^{(p)} \right) + \left( k|u_k^{(p)}|^2 + \frac{1}{k} |u_k^{(p)\prime}|^2 \right) a_k^{(p)\dagger} a_{-k}^{(p)} \nonumber$$

$$+ \left( k|u_k^{(p)*}\dagger|^2 + \frac{1}{k} |u_k^{(p)*}\dagger'\dagger|^2 \right) a_{-k}^{(p)*}\dagger a_{-k}^{(p)\dagger} + \frac{1}{2} \left( a_k^{(p)*}\dagger a_k^{(p)} - a_{-k}^{(p)*}\dagger a_{-k}^{(p)} \right).$$ \hspace{1cm} (9)

This is clearly not diagonal. Therefore, in order to diagonalize, we employ the Bogoliubov transformation. In this transformation, new set of time dependent creation and annihilation operators ($b_k^{(p)}(\tau), b_k^{(p)\dagger}(\tau)$) are defined in term of old ones. And the new basis of the Hilbert space so constructed diagonalizes the above Hamiltonian,

$$b_k^{(p)}(\tau) = \alpha_k^{(p)}(\tau) a_k^{(p)} + \beta_k^{(p)} a_{-k}^{(p)*}; \quad b_k^{(p)\dagger}(\tau) = \alpha_k^{(p)*}(\tau) a_k^{(p)*\dagger} + \beta_k^{(p)}(\tau) a_{-k}^{(p)*\dagger}.$$ \hspace{1cm} (10)
Where $\alpha_k^{(p)}$ and $\beta_k^{(p)}$ are the Bogoliubov coefficients defined as

$$\alpha_k^{(p)}(\tau) = I \left( \sqrt{\frac{k}{2}} u_k^{(p)} + \frac{i}{\sqrt{2k}} u_k^{(p)} \right)$$

(11)

$$\beta_k^{(p)}(\tau) = I \left( \sqrt{\frac{k}{2}} u_k^{(p)} - \frac{i}{\sqrt{2k}} u_k^{(p)} \right).$$

(12)

The Bogoliubov coefficients follow the following normalization condition

$$|\alpha_k^{(p)}|^2 - |\beta_k^{(p)}|^2 = 1$$

(13)

With all these ingredients one represents the power spectrum in terms of Bogoliubov coefficients, $\alpha_k^{(p)}$ and $\beta_k^{(p)}$ as follows,

$$P_E(k) = \frac{k^4}{4\pi^2 a^4 T^2} \sum_{p=1,2} |\alpha_k^{(p)} - \beta_k^{(p)}|^2$$

$$P_B(k) = \frac{k^4}{4\pi^2 a^4 T^2} \sum_{p=1,2} |\alpha_k^{(p)} + \beta_k^{(p)}|^2$$

(14)

Considering the super-horizon limit, the above expressions for the electromagnetic power spectrum can be further simplified by extracting the amplitude and phase of those coefficients. By using the normalization condition Eq (13) one can write,

$$|\alpha_k^{(p)} \pm \beta_k^{(p)}|^2 = 1 + 2|\beta_k^{(p)}|^2 \pm |\beta_k^{(p)}| \sqrt{1 + |\beta_k^{(p)}|^2 \cos \{ \text{arg}(\alpha_k^{(p)} \beta_k^{(p)}) \}}$$

(15)

Where, the phase factor is expressed as $\text{arg}(\alpha_k^{(p)} \beta_k^{(p)}) \equiv \pi + \theta_k^{(p)}$. In the following discussion, we consider a specific model of the electromagnetic coupling function.

**Magnetogenesis: Modelling the coupling function**

In order to study further we consider the following widely considered power law form of the coupling function [27]

$$I(\tau) = \begin{cases} \left( \frac{a_{\text{end}}}{a} \right)^n & a \leq a_{\text{end}} \\ 1 & a \geq a_{\text{end}}, \end{cases}$$

(16)

where, $a_{\text{end}}$ is the scale factor at the end of inflation. At this stage let us point out that, in all the previous analysis in the literature, $n$ has been considered to be integer. However, for our present discussion we keep the value of $n$ arbitrary. As one of the important focuses of the analysis also is to constrain the magnetogenesis model itself namely the value of $n$ in accord with the present day large scale magnetic field and CMB anisotropy. Further more after the end of inflation the value of the function is so chosen that the usual conformal electrodynamics is restored.
Our main interest is to understand the large scale magnetic field which is assumed to be produced during the initial stage of inflation. Therefore, through out our analysis, we assume the Hubble parameter to be constant. This also helps us to give clear picture in terms of analytic solutions. The perfect di-Sitter background with Hubble parameter $H_{inf}$, one obtains the solution for the mode function as,

$$u_k = \frac{1}{2I} \left( \frac{\pi}{aH_{inf}} \right)^{1/2} H^{(1)}_{-n+\frac{1}{2}} \left( \frac{k}{aH_{inf}} \right),$$  \hspace{1cm} (17)$$

which leads to Bunch-Davis vacuum state at the sub-Horizon scale. $H^{(1)}_{-n+\frac{1}{2}} \left( \frac{k}{aH_{inf}} \right)$ is the Hankel function of first kind. The Hankel functions are defined in terms of the Bessel function of first and second (Neumann) kind.

$$H_{\nu} \left( \frac{k}{aH_{inf}} \right) = J_{\nu} \left( \frac{k}{aH_{inf}} \right) + iY_{\nu} \left( \frac{k}{aH_{inf}} \right)$$  \hspace{1cm} (18)$$

Here $\nu = -n \pm \frac{1}{2}$. Defining a new variable $z = k/aH_{inf}$, the time dependent Bogoliubov coefficient are expressed as,

$$\alpha_k = \left( \frac{\pi z}{8} \right)^{1/2} \left\{ H^{(1)}_{-n+\frac{1}{2}} (z) - iH^{(1)}_{-n-\frac{1}{2}} (z) \right\} \quad ; \quad \beta_k = \left( \frac{\pi z}{8} \right)^{1/2} \left\{ H^{(1)}_{-n+\frac{1}{2}} (z) + iH^{(1)}_{-n-\frac{1}{2}} (z) \right\}$$  \hspace{1cm} (19)$$

Now as we have already described before, our focus is to understand the reheating constraints on inflationary as well as magnetogenesis models considering the CMB anisotropy and LSMF field, which are naturally classified as large scale observables. Our discussion will be mostly in the super-horizon limit. Therefor, in this limit following condition will be satisfied

$$\frac{1}{|\beta_k^{(p)}|^2} \ll \theta_k^{(p)} \ll 1.$$  \hspace{1cm} (20)$$

This essentially suggests that any mode starting from the Bunch-Davis vacuum at the sub-horizon scale transforms into highly squeezed state, parametrized by $|\beta_k^{(p)}|^2 \gg 1$, after its horizon exit during inflation. Using this condition one gets the following simplified form of the electromagnetic power spectrum,

$$P_E(k) \simeq \frac{k^4}{4\pi^2 a^4 T^2} \sum_{p=1,2} 4|\beta_k^{(p)}|^2 ; \quad P_B(k) \simeq \frac{k^4}{4\pi^2 a^4 T^2} \sum_{p=1,2} |\beta_k^{(p)}|^2 \theta_k^{(p)}$$  \hspace{1cm} (21)$$

For any arbitrary value of $n$, the final expression for the spectrum are as follows,

$$P_E(k) \simeq \frac{4k^4}{2\pi^2 a^4 T^2} \left( \frac{\pi z}{8} \right) \left\{ H^{*}_{-n+\frac{1}{2}} (z) H^{(1)}_{-n+\frac{1}{2}} (z) + iH^{*}_{-n+\frac{1}{2}} (z) H^{(1)}_{-n-\frac{1}{2}} (z) \right\} - \frac{4k^4}{2\pi^2 a^4 T^2} \left( \frac{\pi z}{8} \right) \left\{ iH^{*}_{-n-\frac{1}{2}} (z) H^{(1)}_{-n+\frac{1}{2}} (z) - H^{(1)}_{-n-\frac{1}{2}} (z) H^{*}_{-n-\frac{1}{2}} (z) \right\}$$  \hspace{1cm} (22)$$
is conserved. Consequently the magnetic power redshifts as
\[ P_B(k) \simeq \frac{k^4}{2\pi^2a^4I^2} \left( \frac{\pi z}{8} \right) \left\{ H_{-n+\frac{1}{2}}^{*}(z)H_{-n+\frac{1}{2}}^{(1)}(z) + iH_{-n+\frac{1}{2}}^{*}(z)H_{-n-\frac{1}{2}}^{(1)}(z) \right\} \]
\[ - \frac{k^4}{2\pi^2a^4I^2} \left( \frac{\pi z}{8} \right) \left\{ iH_{-n-\frac{1}{2}}^{*}(z)H_{-n+\frac{1}{2}}^{(1)}(z) - H_{-n-\frac{1}{2}}^{(1)}(z)H_{-n+\frac{1}{2}}^{*}(z) \right\} \]
\[ \left\{ \arg \left[ \frac{\pi z}{8} \left( H_{-n+\frac{1}{2}}^{(1)}(z) - iH_{-n-\frac{1}{2}}^{(1)}(z) \right) \left( H_{-n+\frac{1}{2}}^{*}(z) - iH_{-n-\frac{1}{2}}^{*}(z) \right) \right] - \pi \right\}^2 \] (23)

In our final analysis, we will be considering this expression. To this end it is important to remind the reader, the widely studied case for \( n \) being positive integer for which the electromagnetic power spectra assumes the following simple forms in the superhorizon limit \( (k \ll aH_{inf}) \) as [25].

\[ P_E(k) \simeq \frac{8\Gamma(n + \frac{1}{2})^2H_{inf}^4}{I^2\pi^3} \left( \frac{k}{2aH_{inf}} \right)^{-2(n-2)} \]
\[ P_B(k) \simeq \frac{8\Gamma(n - \frac{1}{2})^2H_{inf}^4}{I^2\pi^3} \left( \frac{k}{2aH_{inf}} \right)^{-2(n-3)} \] (24)

With this expression, subsequent magnetic field evolution has been widely studied considering the magnetic energy density decreasing as \( |B|^2 \sim 1/a^4 \). The required value of large scale magnetic field of order \( 10^{-16} \) G can be obtained only if inflation scale \( H_{inf} \) assumes low value which has been proved to be difficult in conventional inflationary model in the effective theory framework. We also have observed this in our numerical analysis. Simultaneously we also observed how this problem can be alleviated by the electromagnetic induction during reheating. Furthermore, we will assume arbitrary value of \( n \).

**After inflation dynamics: reheating**

In order to associate the observed current magnetic field with the magnetic field produced during inflation, it is essential to study the subsequent evolution. Most of the studies so far considered the fact that when \( I^2 \) becomes constant at the end of the inflation, the co-moving photon density \( |\beta_k^{(p)}|^2 \) is conserved. Consequently the magnetic power redshifts as \( P_B(k) \propto a^{-4} \) until today. Before we embark on our original analysis, for the sake of completeness let us briefly discuss this widely studied case. Thus at the end of the inflation, the phase parameter \( \theta_k^{end} \) and the photon density \( (|\beta_k^{end}|^2) \) is identified as

\[ \theta_k^{end} = \{ \arg [\alpha_k(z_{end})\beta_k^*(z_{end})] - \pi \} \] , (25)

\[ |\beta_k^{end}|^2 = \left( \frac{\pi z_{end}}{8} \right) \left\{ H_{-n+\frac{1}{2}}^{*}(z_{end})H_{-n+\frac{1}{2}}^{(1)}(z_{end}) + iH_{-n+\frac{1}{2}}^{*}(z_{end})H_{-n-\frac{1}{2}}^{(1)}(z_{end}) \right\} \]
\[ - \left( \frac{\pi z_{end}}{8} \right) \left\{ iH_{-n-\frac{1}{2}}^{*}(z_{end})H_{-n+\frac{1}{2}}^{(1)}(z_{end}) - H_{-n-\frac{1}{2}}^{(1)}(z_{end})H_{-n-\frac{1}{2}}^{*}(z_{end}) \right\} \] . (26)
Here we define $z_{\text{end}} \equiv k/a_{\text{end}}H_{\text{inf}}$. For any arbitrary values of $n$, the magnetic power spectrum is

$$P^\text{end}_B(k) \simeq \frac{k^4}{2\pi^2 a_{\text{end}}^4} \left(\vartheta_{k}^{\text{end}}\right)^2 |\beta_k^{\text{end}}|^2.$$  

(27)

For positive integer $n$, the expression above (27) boils down to

$$P^\text{end}_B(k) \simeq \frac{8\Gamma(n - \frac{1}{2})^2}{\pi^3 H_{\text{inf}}} \left(\frac{k}{2a_{\text{end}}H_{\text{inf}}}\right)^{-2(n-3)},$$  

(28)

at the super-horizon scale $k \ll aH_{\text{inf}}$. Considering nontrivial dynamics, the magnetic power spectrum at the present universe can be correlated with the magnetic power at the end of the inflation through the following standard equation

$$P_{B0}(k) \simeq P^\text{end}_B(k) \left(\frac{a_{\text{end}}}{a_0}\right)^4.$$  

(29)

Where $a_{\text{end}}/a_0$ can be expressed in terms of inflationary e-folding number ($N_k$) as

$$\frac{a_{\text{end}}}{a_0} = \frac{k'}{a_0} = \frac{k'}{a_k H_{\text{inf}}} = \left(\frac{k'}{a_0}\right) e^{N_k} H_{\text{inf}}.$$  

(30)

Here $k'/a_0 = 0.05M_{\text{pc}}^{-1}$ is taken as pivot scale set by Planck observation. By combining equations (28), (29), and (30), the expression for the magnetic power considering integer values of $n$, follow the equation

$$P_{B0} \simeq \frac{\Gamma(n - \frac{1}{2})^2}{2^{3-2n}\pi^3} \left(\frac{k}{a_0}\right)^{6-2n} \left(\frac{k'}{a_0}\right)^{2n-2} e^{2(n-1)N_k}.$$  

(31)

It has already been observed and also shown in Fig.1 the well known fact that magnetic strength of order $10^{-15} - 10^{-22}$ Gauss on 1 Mpc scales [51, 54] can not be obtained by the above magnetic power spectrum for high-scale inflation model such as well studied Higgs-Starobinsky inflation. Therefore, number of models have been constructed just to elevate this problem without much success with regard to the theoretical issues which we discuss in the next section. Form here itself we will advocate the need for re-looking into the reheating effect more seriously than model building. In the recent paper [25], authors have showed that considering negligible electrical conductivity during reheating the Faraday’s law of electromagnetic induction plays an interesting role in modifying the magnetic field evolution. Therefore, we want to see how this induction effect can safely generate presently observed magnetic field for different high scale inflationary models. Interestingly incorporating the CMB anisotropy into the analysis will further reveal an intricate interconnection among various apparently disconnected cosmological parameters such as $(n_s, P_{B0}, T_{re}, w_{re})$. Our analysis, therefor, opens up an interesting new possibility of probing the reheating dynamics, which is otherwise difficult, and simultaneously
FIG. 1: We plot the variation of the present magnetic field’s strength as a function of the spectral index within 2$\sigma$ range of $n_s$ from Planck [1] for Higgs-Starobinsky inflationary model. It is clear that the required magnetic field strength ($10^{-9}$ to $10^{-22}$) G is difficult to achieve within the conventional framework, unless one introduces slow decreasing rate of magnetic energy density in some early state of universe evolution.

constraining the inflationary as well as magnetogenesis model parameters through the evolution of the primordial magnetic field.

After the end of inflation the gauge kinetic function $I(\phi)$ is assumed to be unity, rendering the fact that the post inflationary evolution of the electromagnetic field is essentially standard Maxwellian. Therefore, the conformal invariance is restored, and consequently the gauge field production from the quantum vacuum ceases to exist. The electromagnetic field produced during inflation will cross the horizon and turned into the classical one which will subsequently evolve during this phase. In the Fourier space, the mode function solution of the free Maxwell equation is,

$$ u^{(p)}_k = \frac{1}{\sqrt{2k}} \left\{ \alpha^{(p)}_k(z_{\text{end}})e^{-ik(\tau-\tau_{\text{end}})} + \beta^{(p)}_k(z_{\text{end}})e^{-ik(\tau-\tau_{\text{end}})} \right\} , $$

(32)

here $\tau_{\text{end}}$ represents the end of inflationary era. For arbitrary value of $n$, the phase factor is calculated to be

$$ \theta_k^{(p)} = \left\{ \text{Arg} \left[ \alpha^{(p)}_k(z_{\text{end}}) \beta_k^{(p)*}(z_{\text{end}}) \right] - \pi \right\} - 2k(\tau-\tau_{\text{end}}) . $$

(33)

Where the elapsed conformal time is obtained as

$$ \tau - \tau_{\text{end}} = \int_{a_{\text{end}}}^{a} \frac{da}{a^2H} . $$

(34)

Since there is no further production of gauge field, the photon number density $|\beta_k|^2$ becomes independent of time and follows the same equation (26) as before. However, as emphasized already, the
Faraday induction will come into play during this phase. In order to see this let us first express the magnetic power spectra at the end of reheating parametrized by the scale factor \((a_{re})\) as

\[
P_{B_{re}}(k) \simeq \frac{k^4}{2\pi^2 a_{re}^4} (\theta^e_{k})^2 |\beta^e_{k}|^2 ,
\]

where \(\theta^e_{k}\) is the phase parameter at the end of the reheating era, which is defined as,

\[
\theta^e_{k} = \text{Arg} \left[ \alpha_k (z_{end}) \beta_k^* (z_{end}) \right] - \pi - 2k \int_{a_{end}}^{a_{re}} \frac{da}{a^2 H} .
\]

Notable term of the above expression is the second one which leads to the non-conventional dynamics of the magnetic field. Assuming the constant equation of state during reheating dynamics the special term boils down to the following simple from,

\[
2k(\tau - \tau_{end}) \rightarrow \frac{4k}{3\omega + 1} \left( \frac{1}{aH} - \frac{1}{a_{end}H_{inf}} \right) .
\]

The phase term contributes to the dynamics of the magnetic power spectrum in two different ways. The conventional one will be associated with the redshift factor \(\propto a^{-4}\) emerging from the first term in the right-hand side of the Eq.(33). Important one is associated with the redshift factor \(\propto a^{-6} H^{-2}\) emerged out from \(k/aH\) term. As the expansion of the universe is decelerating after inflation, leading contribution to the evolution of the magnetic power would be controlled by the latter one Eq.(33). However, the electric field energy dilutes following the conventional form,

\[
P_{E}(k) \simeq \frac{2k^4}{\pi^2 a^4} |\beta^e_{k}|^2 .
\]

For large scale \((k \ll aH)\), the above expression boils down to,

\[
P_{E}(k) \simeq \frac{8\Gamma(n + \frac{1}{2})^2}{\pi^3} H_{inf}^4 \left( \frac{k}{2a_{end}H_{inf}} \right)^{-2(n-2)} \left( \frac{a_{end}}{a_0} \right)^4 .
\]

for integer values of \(n\). After the end of reheating, inflaton energy is converted into highly conducting plasma containing all the standard model particles. Due to large electrical conductivity primordial electric field decays to zero and comoving magnetic energy density freezes to a constant value until today. Therefore, final general expression of our interest is the present day magnetic field strength given as

\[
P_{B_0}(k) = P_{B_{re}}(k) \left( \frac{a_{re}}{a_0} \right)^4 \simeq \frac{k^4}{2\pi^2 (\theta^e_{k})^2 |\beta^e_{k}|^2} \frac{1}{a_0^4} .
\]

If we take integer value of \(n\), at super horizon scale, the above expression will transforms into [25]

\[
P_{B_0}(k) \simeq \frac{8\Gamma(n - \frac{1}{2})^2}{\pi^3} H_{inf}^4 \left( \frac{k}{2a_{end}H_{inf}} \right)^{-2(n-3)} \left( \frac{a_{end}}{a_0} \right)^4 \\
\left\{ 1 + (2n - 1) \int_{a_{end}}^{a_{re}} \frac{da}{a} \frac{a_{end}H_{inf}}{aH} \right\}^2 ,
\]
where $a_0$ is the scale factor at the present time. In the next section we will briefly discuss about standard theoretical issues related to the magnetogenesis model.

**Discussion on strong coupling and backreaction problem**

In order to generate large scale magnetic field of required strength, conventional magnetogenesis models either face strong coupling problem or destroy the inflation background itself. In order to ameliorate these issue several attempts [26, 28–42] have been made either by advocating different forms of the gauge coupling function or taking non-trivial dynamics of the coupling fields during reheating. In the context of simplest magnetogenesis model proposed in [27], the gauge kinetic function $I(\tau)$ can essentially be interpreted as time dependent effective electromagnetic coupling. By considering field theoretic argument and experimental observations the reference [28] generically argued that the model either suffers from strong coupling problem or backreaction problem.

In our previous discussion, we have chosen the effective electromagnetic coupling function as a monomial function of the scale factor,

$$I = \left(\frac{a_{\text{end}}}{a}\right)^n = e^{N_k n}, \quad (42)$$

where $N_k = \ln(a/a_{\text{end}})$ associated with a particular scale $k$ is identified as the e-folding number during inflation. It is clear from the above expression that once the value of $n$ is chosen to be negative, the gauge kinetic function increases during inflation. Now the behavior of $I(\tau)$ is so chosen that it boils down to unity after inflation. Therefore, during inflation, its magnitude must be less than unity. This is where the origin of the strong coupling problem in the electromagnetic sector lies. Under the field redefinition $A_\mu \rightarrow \sqrt{I} A_\mu$, the effective electromagnetic coupling $\alpha_{\text{eff}}$, defined through the fermion-gauge field interaction $L_{\text{int}} = e A_\mu J^\mu$ modified as

$$\alpha_{\text{eff}} = \frac{e^2}{4\pi I} = \frac{\alpha}{I}, \quad (43)$$

e is the charge of the fermionic field contributing to the electric current. $\alpha$ is the standard electromagnetic coupling. Nonetheless, as one goes early in the inflationary phase, the effective electromagnetic coupling necessarily becomes very large, which turns the theory non-perturbative. Moreover, the perturbative computation of magnetogenesis, discussed in our previous section, will no longer be valid. Keeping this problem aside, if we still do our magnetogenesis computation as before, due to large effective electromagnetic coupling $\alpha_{\text{eff}}$, the production of electromagnetic energy density may be sup-
pressed compared to the background energy density \(3M_p^2H_{In}^2\). Therefore, backreaction problem will not come into play.

On the other hand, if one considers positive values of \(n \geq 0\), the whole reasoning expressed just now will be reversed or, more precisely, throughout the inflationary period, the gauge kinetic function \(I(\tau)\) will be way larger than unity. Based on our previous discussions, the perturbative magnetogenesis analysis appears to be still valid. However, considerable reduction of the effective electromagnetic coupling \(\alpha_{eff} \propto 1/I(\tau)\), specifically during the early inflationary stage of our interest, significantly enhances the production of electromagnetic energy. In such a scenario the quantum production of electromagnetic energy may take over the background energy density. This is precisely the backreaction problem, which jeopardizes the fixed background magnetogenesis analysis instead. To avoid the backreaction problem, the energy density of the gauge field must be smaller than the total background energy density \(\rho_{tot} = 3M_p^2H_{In}^2\). The parameter which quantifies the amount of background energy density during inflation against the gauge field energy density \(\rho_A\) is the ratio,

\[
\frac{\rho_A}{\rho_{tot}} \leq \zeta
\]  

(44)

Where \(\zeta\) is identified with another physical quantity associated with the amplitude of the curvature perturbation measured from CMB anisotropy. From the CMB analysis the amplitude of the curvature perturbation is measured to be, \(\zeta = 4.58 \times 10^{-5}\) from Planck [1]. Total gauge field energy density is defined as

\[
\rho_A = \rho_E + \rho_B = \frac{I^2}{2} \int \frac{dk}{k} \{ P_E(k) + P_B(k) \}
\]

\[
\approx \int_{k_{IR}}^{k_{UV}} \frac{dk}{k} \frac{k^4}{4\pi^2a^4} \left( 4|\beta_k|^2 + |\beta_k|^2(\theta_k)^2 \right)
\]  

(45)

In the above expression, for carrying out k-integral, we consider the limit \(k\) from \(k_{IR}\) to \(k_{UV}\). Associated with our observable universe, the IR cut-off \(k_{IR}\) corresponds to the highest mode that exits the horizon at the beginning of the inflation, and the UV cut-off \(k_{UV}\), the lowest scale set at the end of the inflation. Afterwards scaling \(k\) during inflation as \(k \approx aH_{end}\), one can find

\[
\rho_A \approx \int_{a_k}^{a_{end}} \frac{da}{a} \frac{H_{end}^4}{4\pi^2} \left[ 4|\beta_k(1)|^2 + |\beta_k(1)|^2(\theta_k(1))^2 \right]
\]

\[
= N_k \frac{H_{end}^4}{4\pi^2} \left[ 4|\beta_k(1)|^2 + |\beta_k(1)|^2(\theta_k(1))^2 \right].
\]  

(46)

Based on the above expression and the inequality relation (44), the upper bound on the total EM energy density associated with a particular value of coupling parameter \(n_{max}\) follow the following
relation
\[
N_k H_{\text{end}}^2 \frac{4|\beta_k(1)|^2 + |\beta_k(1)|^2 (\theta_k(1))^2}{12\pi^2 M_p^2} \leq \zeta .
\] (47)

Moreover, considering integer values of \(n\), at super horizon limit, the preceding expression turns out as
\[
N_k \frac{8H_{\text{end}}^2}{6\pi^3 M_p^2} \frac{\Gamma(n + \frac{1}{2})^2}{2^{2(n-2)}(2n-1)^2} \leq \zeta .
\] (48)

From the constraint Eqs.(47) and (48), we can set a limit on the coupling parameter as \(n_{\text{max}}\). Thus considering the coupling parameter within the aforementioned range, we are able to take care of both strong coupling and backreaction problems.

For example the scale-invariant electric power spectrum which corresponds to the power \(n = 2\), the total gauge field energy density during inflation roughly of the order of
\[
\rho_A \sim \frac{5}{2} N_k H_{\text{end}}^4 \frac{\pi^2}{\pi^2}.
\] (49)

Here, the ratio between the gauge field and the total energy density is always less than the amplitude of the curvature perturbation \((\rho_A/\rho_{\text{tot}} \sim 10^{-10})\) for all allowed values of the spectral index. In addition to that, the model with power \(n = 3\), which produce a scale-invariant magnetic power spectrum, the ratio \(\rho_A/\rho_{\text{tot}}\) turns out as
\[
\frac{\rho_A}{\rho_{\text{tot}}} \sim \frac{117}{24} \frac{N_k H_{\text{end}}^2}{\pi^2 M_p^2} \sim 10^{-9} ,
\] (50)

which is much smaller than the amplitude of the curvature perturbation. So for both cases, the scale-invariant electric and magnetic field power spectrum, the back reaction as well as strong coupling problem can be avoided.

### B. Reheating dynamics: Connecting Reheating and Primordial magnetic field via CMB

By now it becomes clear that the primary importance of the reheating phase is to enhance the strength of the large scale magnetic field to the required order. We understood the fact that Faraday’s law of electromagnetic induction plays a crucial role in this regard. Therefore, to obtain the correct order of the current magnetic field, understanding the reheating dynamics as well as the evolution of the magnetic field during this period will be of utmost importance. In order to do that, we will consider two possible reheating models and compare the result.
Case-I: For this we follow the effective one fluid description of reheating dynamics proposed in [43], where inflaton energy is assumed to converted into radiation instantaneously at the end of reheating. The dynamics is parametrized by an effective equation of state \( \omega_{\text{eff}} \), reheating temperature \( T_{\text{re}} \) and duration \( N_{\text{re}} \) (e-folding number during reheating era). In this reheating model, following the approximation as mentioned earlier, one can easily derive the expression for \( T_{\text{re}} \) and \( N_{\text{re}} \) in terms of some inflationary parameters as [46]

\[
T_{\text{re}} = \left( \frac{43}{11g_{s,\text{re}}} \right)^{\frac{1}{3}} \left( \frac{a_0T_0}{k'} \right) H_k e^{-N_k e^{-N_{\text{re}}}},
\]

(51)

\[
N_{\text{re}} = \frac{4}{(1 - 3\omega_{\text{eff}})} \left[ -\frac{1}{4} \ln \left( \frac{45}{\pi^2 g_{\text{re}}} \right) - \frac{1}{3} \ln \left( \frac{11g_{s,\text{re}}}{43} \right) - \ln \left( \frac{k'}{a_0T_0} \right) - \ln \left( \frac{V_{\text{end}}^{1/4}}{H_k} \right) - N_k \right],
\]

(52)

where the present CMB temperature \( T_0 = 2.725 \text{ K} \), the pivot scale \( k'/a_0 = 0.05 \text{ Mpc}^{-1} \). \( a_0 \) is the present cosmological scale factor. Here for simplicity we have taken both the values of the degrees of freedom for entropy at reheating \( (g_{s,\text{re}}) \), and the effective number of relativistic species upon thermalization \( (g_{\text{re}}) \) is same \( g_{s,\text{re}} = g_{\text{re}} \approx 100 \). From the above expressions (51) and (52), we can clearly see that the inflationary parameters put constraints on the reheating parameters \( T_{\text{re}} \) and \( N_{\text{re}} \). Considering simple canonical inflation potential \( V(\phi) \), the inflation model-dependent input parameters, the inflationary e-folding number \( (N_k) \), and the inflationary Hubble constant \( (H_k) \) for a particular CMB scale \( k \) are known to be written as

\[
N_k = \log \left( \frac{a_{\text{end}}}{a_k} \right) = \int_{\phi_k}^{\phi_{\text{end}}} \frac{|d\phi|}{\sqrt{2\epsilon_v M_p^2}}, \quad H_k = \frac{\pi M_p \sqrt{r_k A_k}}{\sqrt{2}},
\]

(53)

here the inflaton field \( \phi_{\text{end}} \) defined at the end of the inflation. Furthermore, the scalar spectral index \( n_s^k \), and tensor to scalar ratio \( r_k \) can be related with the above inflationary parameters \( (H_k, N_k) \) through the following equations

\[
n_s^k = 1 - 6\epsilon(\phi_k) + 2\eta(\phi_k), \quad r_k = 16\epsilon(\phi_k).
\]

(54)

Here the slow-roll parameters expressed as

\[
\epsilon_v = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2; \quad |\eta_v| = M_p^2 \left| \frac{V''}{V} \right|.
\]

(55)

The above expressions manage to write reheating parameters in terms of the scalar spectral index \( (n_s^k) \) for a given CMB scale \( k \). After identifying all the required parameters, we will be able to connect CMB and reheating through inflation.

**Reheating parameters and primordial magnetic field**: Now in the context of inflationary
magnetogenesis, as we mentioned earlier, the electric field continues to exist after the post inflationary era until the universe becomes perfect conductor. This is the non-zero electric field during reheating whose dynamics will significantly change the dynamics of magnetic field and produce the strong magnetic field today.

For this present reheating model, the phase parameter at the end of the reheating $\theta^r_k$ in equation (36) can be defined as,

$$\theta^r_k = \{ \text{Arg} [\alpha_k(z_{end}) \beta^*_k(z_{end})] - \pi \} - \frac{4}{3} \omega_{eff} + 1 \left( \frac{k}{a_{re}H_{re}} - \frac{k}{a_{end}H_{Inf}} \right), \quad (56)$$

where $H_{re}$ is the Hubble rate at the end point of reheating. Using evolution of effective density $\rho = \rho_0 a^{-3(1+\omega_{eff})}$, one can extract the information about the Hubble parameter at the end of reheating ($H_{re}$) as

$$H_{re} = H_{inf} A_{re}^{-\frac{1}{2}(1+\omega_{eff})}, \quad (57)$$

with the normalized scale factor $A_{re} = a_{re}/a_{end} = e^{N_{re}}$. For integer values of $n$, the earlier expression of the phase parameter turns out as

$$\theta^r_k \simeq \frac{2}{2n-1} \frac{k}{a_{end}H_{inf}} \left\{ 1 + \frac{4n-2}{3\omega_{eff} + 1} \left( \frac{a_{end}H_{inf}}{a_{re}H_{re}} - 1 \right) \right\}. \quad (58)$$

Furthermore, utilizing equations (58) and (35), one can obtain the magnetic power spectrum during the reheating epoch as,

$$P_{B, re}(k) \simeq \frac{8\Gamma(n - \frac{1}{2})^2}{\pi^3 H_{Inf}^{-4}} \frac{k}{2a_{end}H_{Inf}}^{-2(n-3)} \left( \frac{a_{end}}{a} \right)^4 \left\{ 1 + \left( \frac{4n-2}{3\omega_{eff} + 1} \right) \left( \frac{a_{end}H_{inf}}{a_{re}H_{re}} - 1 \right) \right\}^2. \quad (59)$$

After reheating, the conductivity of the universe becomes sufficiently large. In consequence of that, the electric field dies out very fast, and the magnetic field redshifts as $P_B \propto a^{-4}$ till today. Since the comoving magnetic power spectrum is conserved after reheating, the present-day magnetic field obeys the following relation [25]

$$P_{B_0}(k) \simeq \frac{8\Gamma(n - \frac{1}{2})^2}{\pi^3 H_{Inf}^{-4}} \frac{k}{2a_{end}H_{Inf}}^{-2(n-3)} \left( \frac{a_{end}}{a_0} \right)^4 \left\{ 1 + \left( \frac{4n-2}{3\omega_{eff} + 1} \right) \left( \frac{a_{end}H_{inf}}{a_{re}H_{re}} - 1 \right) \right\}^2, \quad (60)$$

where the ratio between the scale factor $a_0$ and $a_{re}$, considering entropy conservation can be expressed as

$$\frac{a_0}{a_{re}} = \left( \frac{11g_{s, re}}{43} \right)^{\frac{1}{2}} \frac{T_{re}}{T_0}. \quad (61)$$

From the preceding expression, we can see that the magnetic field’s present strength explicitly depends on the reheating parameters as well as some inflationary parameters. Therefore, this opens up the
window for probing the early stage of the universe, particularly the reheating phase through the current observational amplitude of the magnetic field. Basically, for a specific value of the spectral index, any present value of the magnetic field $P_{B0}$ has a direct one to one correspondence with the effective equation of state of reheating $\omega_{eff}$ and reheating temperature $T_{re}$. Therefore, important conclusion we arrived at that the effective equation of state is no longer a free parameter rather it can be fixed by the present value of $P_{B0}$ via CMB.

Case-II: For the previous case we did not take into account explicit decay of the inflaton field and additionally the effective equation of state was assumed to be constant. In this case we consider perturbative reheating scenario [56] constrained by CMB anisotropy. Therefore, as opposed to the previous case, the effective equation of state is time-dependent. However, average inflaton equation of state is taken to be constant. For example inflaton potential $V(\phi) \propto \phi^p$ gives rise to the value of average equation of state $\omega_{\phi} = (p - 2)/(p + 2)$ considering virial theorem [55]. Through out our discussion we consider $p = 2$. Furthermore, for simplicity we assume inflaton decays into radiation only. The Boltzmann equation for the inflaton energy density ($\rho_{\phi}$) and radiation energy density ($\rho_r$) are,

$$\Phi' + \frac{\sqrt{3} M_p \Gamma_\phi}{m_\phi^2} (1 + w_\phi) \frac{A^{1/2} \Phi}{A^{w_\phi} + \frac{R}{A}} = 0,$$

$$R' - \frac{\sqrt{3} M_p \Gamma_\phi}{m_\phi^2} (1 + w_\phi) \frac{A^{3(1-2w_\phi)}}{2} \frac{\Phi}{A^{w_\phi} + \frac{R}{A}} = 0.$$  \tag{62}

Where the comoving densities in terms of the dimensionless variable are used,

$$\Phi = \frac{\rho_\phi}{m_\phi^4} A^{3(1+w_\phi)}; \ R(t) = \frac{\rho_R A^4}{m_\phi^4}.$$  \tag{63}

In the above equations to increase the stability of the numerical solution, we use the inflaton mass ($m_\phi$) to define the dimensionless scale factor as $A = a/a_{end} = a m_\phi$. For solving the Boltzmann equation, the natural initial conditions will be set at the end of inflation as follows, .

$$\Phi(A = 1) = \frac{3 V_{end}(\phi)}{2 m_\phi^4}; \ R(A = 1) = 0.$$  \tag{64}

For this mechanism the reheating temperature is identified from the radiation temperature $T_{rad}$ at the point of $H(t_{re}) = \Gamma_\phi$, when maximum inflaton energy density transfer into radiation.

$$T_{re} = T_{rad}^{end} = \left( \frac{30}{\pi^2 g_{re}} \right)^{1/4} \rho_R(\Gamma_\phi, A_{re}, n_s)\frac{1}{4}.$$  \tag{65}

To establish one to one correspondence between $T_{re}$ and $\Gamma_\phi$, we combine the equations (65) and (51). To fixed the values of decay width $\Gamma_\phi$ in terms of spectral index ($n_s$), we use one further condition at
the end of the reheating

\[ H(A_{\text{re}})^2 = \left( \frac{\dot{A}_{\text{re}}}{A_{\text{re}}} \right)^2 = \frac{\rho_\phi(\Gamma_\phi, A_{\text{re}}, n^k_e) + \rho_R(\Gamma_\phi, A_{\text{re}}, n^k_e)}{3M_p^2} = \Gamma_\phi^2. \] (66)

Connecting reheating and primordial magnetic field: In order to connect the reheating and primordial magnetic field through the CMB, it is necessary to understand the cosmological evolution of the electromagnetic field during the post inflationary epoch, especially during reheating, which modifies the present strength of the magnetic power spectrum. As we consider the perturbative decay of the inflaton field during reheating, the phase parameter (36) now explicitly depends on the evolution of the two energy components, \( \rho_\phi \) and \( \rho_R \) with time

\[ \theta^r_k \approx \frac{2}{2n - 1} \frac{k}{a_{\text{end}}H_{\text{Inf}}} \left[ 1 + \frac{2(n - 1)}{a_{\text{end}}} \int_{a_{\text{end}}}^{a_{\text{re}}} \frac{\sqrt{3}M_p}{\rho_\phi(a) + \rho_R(a)} \frac{da}{a^2} \right]. \] (67)

Furthermore, for integer values of coupling parameters at the super horizon scale, one can find [25]

\[ \theta^r_k \approx \frac{2}{2n - 1} \frac{k}{a_{\text{end}}H_{\text{Inf}}} \left[ 1 + \frac{2(n - 1)}{a_{\text{end}}} \int_{a_{\text{end}}}^{a_{\text{re}}} \frac{\sqrt{3}M_p}{\rho_\phi(a) + \rho_R(a)} \frac{a_{\text{end}}H_{\text{inf}}}{a \sqrt{\rho_\phi(a) + \rho_R(a)}} \frac{da}{a^2} \right], \] (68)

where the time-dependent density components will be followed from Eq.(62). Thus, after combining equations (41), (61), and (68), one can obtain the magnetic power spectrum in the present universe as

\[ P_{B_0}(k) \approx \frac{\Gamma(n - \frac{1}{2})^2 2^{2n-3} (2.6 \times 10^{39})}{(6.4 \times 10^{-39})^{2n-6}} \left( \frac{k}{a_0 Mpc} \right)^{-2(n-3)} \left( \frac{11g_{s,\text{re}}}{43} \right)^{2-2n} \left( \frac{T_{\text{re}}}{T_0} \right)^{2-2n} \left( \frac{H_{\text{inf}}}{\text{GeV}} \right)^{1/(2n-1)} \left( \frac{A_{\text{re}}}{1} \right)^{2(n-1)} \frac{1}{ \sqrt{\rho_\phi(a) + \rho_R(a)}} \left( \frac{a_{\text{end}}H_{\text{inf}}}{A^2} \right)^2 \right]^{2} G^2. \] (69)

In the preceding expression, we can clearly see the appearance of the reheating parameters, which are the function of inflationary observables. Therefore, we definitely will be able to put indirect bounds on the reheating parameters which in turn can constrain the inflation model. As we emphasized before, from the measurement of the CMB anisotropy, our goal of this paper would be to constraints the reheating dynamics through inflationary parameters considering the present strength of the magnetic field \( P_{B_0}^{1/2} \).

III. MAGNETIC POWER SPECTRUM IN THE PRESENT UNIVERSE IN TERMS OF REHEATING PARAMETERS: AN ANALYTIC STUDY

Before employing the numerical analysis, in this section we present approximate calculation for the present value of the magnetic power spectrum and estimate the reheating parameters \( (T_{\text{re}}, N_{\text{re}}) \) in
terms that. Following our previous work \cite{79}, the radiation temperature assumes the following form when considering Case-II reheating scenario,

\[ T_{rad} \simeq \left( \frac{2\Gamma_\phi}{5} \frac{3M_p^2 H_{inf}}{\beta A^4} \right) \left( A^{\frac{5}{2}} - 1 \right)^\frac{1}{4}, \tag{70} \]

where \( \beta = \pi^2 g_{re}/30 \). Further, utilizing the above expression of the radiation temperature, we can calculate the approximate expression of the reheating temperature. Subsequently, at the point of \( A_{re} \), where the condition \( H(A_{re}) = \Gamma_\phi \) is satisfied, one can define the reheating temperature as

\[ T_{re} \simeq \left( \frac{2\Gamma_\phi}{5} \frac{3M_p^2 H_{inf}}{\beta A_{re}^3} \right)^\frac{1}{4}. \tag{71} \]

Here the decay width \( \Gamma_\phi \) and the normalized scale factor at the end of reheating \( (A_{re}) \) can be approximated as,

\[ \Gamma_\phi \simeq \frac{5\beta G^4}{6M_p^2 H_{inf}} A_{re}^3, \quad G = \left( \frac{43}{11g_{s,re}} \right)^\frac{1}{2} \left( \frac{a_0 T_0}{k'} \right) H_k e^{-N_k}, \quad A_{re} \simeq \frac{25\beta G^4}{12M_p^2 H_{inf}^2}. \tag{72} \]

From the Eq.35 it is obvious that during reheating phase the electromagnetic power spectrum is crucially dependent upon the evolution of the phase \( \theta_k \) which is giving rise to induced magnetic field. Importantly after the reheating phase ends large scale magnetic field freezes inside the plasma. Therefore, it is the hierarchy between the inflationary and reheating scale which sets the current strength of the magnetic field today after the inflation. Hence naturally one can obtain interesting constraints on reheating evolution parameters such as \((T_{re}, N_{re}, w_{eff})\) though the current value of the large scale magnetic field. As the inflaton energy density dominates large part of the duration of the reheating phase, we approximate the solution of the inflaton energy density as

\[ \rho_\phi(A) = \rho_{\phi_{end}} A^{-3} e^{-\Gamma_\phi(t-t_i)} \simeq \rho_{\phi_{end}} A^{-3}. \tag{73} \]

This helps us to further obtaining the approximate Hubble parameter as

\[ H(A) = \sqrt{\frac{\rho_\phi(A) + \rho_R(A)}{3M_p^2}} \simeq H_{inf} \sqrt{\frac{e^{-\Gamma_\phi(t-t_i)}}{A^3}} + \frac{2\Gamma_\phi}{5H_{inf}} A^{-3/2} \simeq \frac{H_{inf}}{A^{3/2}} \sqrt{1 + \frac{2\Gamma_\phi}{5H_{inf}} A^{3/2}}. \tag{74} \]

With all the above approximate expressions, one can arrive the following expression of the present magnetic power spectrum

\[ P_{B0}(k) \simeq \frac{\Gamma(n - \frac{1}{2})^2 2^{2n-3} \left( 2.6 \times 10^{39} \right) \left( \frac{k}{a_0 M_p} \right)^{-2(n-3)} \left( \frac{11g_{s,re}}{43} \right)^{\frac{2-2n}{3}} \left( \frac{T_{re}}{T_0} \right)^{2-2n}}{\left( \frac{H_{inf}}{GeV} \right)^{2(n-1)} \left( \frac{1}{A_{re}} \right)^{2(n-1)}} \left\{ 1 + (2n-1) \left[ 2 \left( \eta^{1/3} - 1 \right) - 4\eta^{1/2} \left( A_{re}^{-1/4} - \eta^{-1/6} \right) \right] \right\}^2 G^2. \tag{75} \]

Where \( \eta = \frac{5H_{inf}}{2\Gamma_\phi} \). Interestingly, our analytical expression of the magnetic power spectrum roughly matches with the numerical values.
IV. INFLATION MODELS AND NUMERICAL RESULTS

The main aim of our study is to see how the present observational limit on the magnitude of large scale magnetic field as well as the CMB anisotropy can be used to probe the reheating dynamics and put combined constraints on the reheating and inflationary parameter space. We consider different inflationary models and study how the present limits on the magnetic field strength constrain the effective reheating equation of state \( \omega_{\text{eff}} \) in terms of inflationary scalar spectral index \( n_s \). Further, the allowed range of the current magnetic field can be shown to impose an upper limit on the values of the spectral index \( n_s^{\text{max}} \) and effective equation of state \( \omega_{\text{eff}}^{\text{max}} \), which in turn provides bound on the maximum possible reheating temperature \( T_{\text{re}}^{\text{max}} \). Moreover, to connect reheating parameters such as \((T_{\text{re}}, \omega_{\text{eff}})\) with the present magnetic field power spectrum \( P_{1/2}^{B_0} \), we take into account two different power law form of the gauge kinetic function with power \( n = (2,3) \). It is well known that inflationary magnetogenesis model with \( n = 2 \) produces scale invariant electric power spectrum and \( n = 3 \) gives rise to scale invariant magnetic power spectrum. Furthermore, we also analyze the the maximum possible value of \( n = n_{\text{max}} \) that will be allowed for different inflationary model within 2\( \sigma \) range of \( n_s \) from Planck [1]. In the following discussion, we consider different inflationary models and discuss the constraints on the reheating parameters.

A. Model independent results

In our present paper we have considered a specific magnetogenesis model with dilatonic gauge kinetic function, \( I \propto \phi^{-n} \). As we have described before there exists two special values of \( n = (2,3) \) for which we have scale invariant electric field and magnetic field configuration respectively. Our analysis shows that irrespective of the inflation models, \( n = 2 \) gauge coupling function can not produce enough magnetic field strength within the observable limits \( 10^{-9} \text{G} > P_{1/2}^{B_0} > 10^{-22} \text{G} \) if reheating is considered to be perturbative (case-II scenario) which are clearly shown in the second panel of the figs.(2) and (3). For all other considerations we observed that magnetic fields are being produced within the observable range. Two values of \( n = (2,3) \) have their distinct reheating temperature regime in which they produce required value of the field strength. For scale invariant electric field \((n=2)\), one would always require lower value of reheating temperature \( \sim (10^{-2}, 10^3) \text{GeV} \) as clear form the first and third panels of figs.(2). On the other hand for scale invariant magnetic power spectrum \((n = 3)\), the observable range of magnetic field can be obtained with the reheating temperature as high as \( 10^{12} \sim 10^{14} \text{GeV} \) (see first and third panel in figs.(3)). In the conventional study the magnetic energy
density is assumed to be diluted adiabatically with the expansion of the universe as $1/a^4$ starting from the end of inflation. This framework never gives rise to enough present day magnetic field strength for high reheating temperature, which is believed to be generic prediction of inflation in the effective field theory framework. Therefore, in the inflationary magnetogenesis scenario non-trivial magnetic field dynamics during reheating should be an important physical phenomena that has to be understood quite well. Furthermore, during this phase itself the inflaton decay should happen in such a manner that the electric field survives for Faraday’s effect to play the role.

Most important point of our study is the constraints set on the effective reheating equation of state $\omega_{\text{eff}}$. A particular present day value of $P_{B_0}$, the CMB anisotropic constraint and after reheating entropy conservation law automatically fixes the reheating equation of state uniquely. Most interestingly, this would place severe restriction on the inflaton potential which we will study in our later paper. As has already been discussed, most of our discussions will be centered around two different forms of the dilatonic gauge kinetic functions.

**Scale-invariant electric power spectra :** Even though we are considering scale invariant electric field, important to remember that it will survive only until the end of reheating. The generated large scale magnetic field will be constrained by this condition. In this case how the reheating and inflationary parameters and the large scale magnetic field are intertwined each other are clearly depicted in fig.(2). From the first panel of fig.(2), we can clearly predict a unique value of $\omega_{\text{eff}}$ associated with a specific choice of the present magnetic field once we fixed scalar spectral index $n_s$. Additionally, for a given $\omega_{\text{eff}}$, the reheating temperature is also determined uniquely. Therefore, taking into account CMB constraints the reheating phase can be uniquely probed by the evolution of the primordial magnetic field. In the Table- I, III, V for different sample values of the current magnetic field, we have provided corresponding values of $\omega_{\text{eff}}$ and $T_{re}$ for the central value of the spectral index $n_{s_{\text{central}}} = 0.9649$. This observations essentially narrow down the possible value of the scalar spectral index within the $1\sigma$ range of $n_s = 0.9649 \pm 0.0042$ (68% CL, Planck TT,TE,EE+lowE+lensing) from Planck [1]. Furthermore for any specific choice of the current magnetic field strength within the observational limit, there exits maximum allowed values of the spectral index $n_{s_{\text{max}}}^m$ and associated $\omega_{\text{eff}}^m$ and that naturally leads to the maximum permissible value of the reheating temperature which are provided in the tables II, IV, VI. For $n = 2$ case we consider three distinct values of $P_{B_0}^{1/2} = (10^{-18}, 10^{-20}, 10^{-22})$ G through out the discussion.

For case-I reheating scenario, irrespective of models under consideration large scale magnetic field constrains the effective equation of state within $0.15 < \omega_{\text{eff}} < 0.33$, and consequently predicts low
value of reheating temperature. For the minimum observational limit of the present-day magnetic field strength \((P_{B_0}^{1/2} = 10^{-22} \text{ G})\), set the maximum allowed value of the reheating temperature around \(\sim 1 \text{ TeV}\), which turns out to be inflationary model-independent. As has already been pointed out for pertrubative reheating (case-II), since the inflaton equation of state is taken to be zero, we found it hard to get the strong magnetic field within the observable limit.

**Scale-invariant magnetic power spectra :** Here we will discuss another special case \(n = 3\) when magnetic power spectra itself is scale invariant. Unlike \(n = 2\), inflationary magnetogenesis model with scale invariant magnetic field turned out to be viable for a wide range of reheating temperature. The reheating temperature can be as high as \(\sim 10^{14} \text{ GeV}\) without any backreaction problem which was the main concern of all the previously studied magnetogenesis scenario. Furthermore for case-I reheating scenario the effective reheating equation state now ranges from \(-\frac{1}{3} < \omega_{eff} < 0.3\) irrespective of models under consideration. Negative reheating equation of state during reheating generically indicates the unconventional matter field production from inflaton which has not been studied in the literature extensively. Hence for the time being if we exclude this possibility scale invariant magnetic field generation is very tightly constraints.

As opposed to the scale invariant electric field model, for the present case the observable value of large scale magnetic field can be successfully generated within the perturbative reheating framework shown in the second panel of the Fig.(3). However, as can be observed, the the model is viable only within a very narrow range of magnetic field within \(10^{-10} \text{G} \sim 10^{-12} \text{G}\). Similar to the previous case for any specific choice of the current magnetic field strength within the observational limit, there exists maximum allowed values of the spectral index \(n_s^{max}\) and associated \(\omega_{eff}^{max}\) and that naturally leads to the maximum permissible value of the reheating temperature which are provided in the tables II, IV, VI. Through out our discussion, for \(n = 3\) case we consider three distinct values of \(P_{B_0}^{1/2} = (10^{-9}, 10^{-12}, 10^{-15}) \text{ G}\).

With this general discussion, in the following sections we consider various inflationary models and discuss about their quantitative predictions.

**B. Natural inflation \([49]\)**

In natural inflationary model, the potential is defined by

\[
V(\phi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right],
\]

(76)

here \(\Lambda\) is the height of the potential, and \(f\) is the width, also known as the axion decay width. To fit this model with Planck data, this model needs a super-Planckian value of axion decay constant.
FIG. 2: We plot the magnetic field variation on the upper left side as a function of the effective equation of state for a fixed value of the spectral index. The light pink region indicates the not allowed region as the strength of the magnetic field in that region is less than $10^{-22}$ G (observed value of the magnetic field on scales $\approx 1$ Mpc must be $B_{\text{cosmic}}^0 > 10^{-22}$ G [51]). The allowed value of the present magnetic field range from $10^{-22}$ G to the point where the reheating temperature reaches $10^{-2}$ GeV (comes from BBN constraints), marked as solid and dashed red line. On the upper right side, we plot the variation of the magnetic field’s magnitude as a function of the spectral index within the minimum and maximum values of the reheating temperature for purely perturbative reheating dynamics with the inflaton equation of state $\omega_0 = 0$. On the lower left side, we have plotted the variation of reheating temperature as a function of the spectral index for three different values of the current magnetic field $P_{B0}^{1/2} = (10^{-18}, 10^{-20}, 10^{-22})$ G. The light brown region indicates the reheating temperature ($T_{re}$) below $10^{-2}$ GeV, which would ruin the predictions of big bang nucleosynthesis (BBN). In the last plot, we have shown the variation of the effective equation of state as a function of the spectral index ($n_s$) with two different values of $P_{B0}^{1/2} = (10^{-20}, 10^{-22})$ G, within the minimum and maximum values of reheating temperature. The deep and light green band indicates $1\sigma$ and $2\sigma$ range of $n_s$ from Planck [1]. Here we have plotted the results for three different inflationary models (Natural inflation, Higgs-Starobinsky inflation, minimal plateau model) for the $k$-independent electric power spectrum ($n = 2$).

This is why we have taken $f = 50M_p$ for our numerical analysis purpose. Here the overall scale of the inflation $\Lambda$ fixes by the CMB normalization.

In order to connect the primordial magnetic field with those observed in the present universe through reheating dynamics, we need to define inflationary e-folding number $N_k$ and tensor-to-scaler ratio $r_k$. In the framework of the natural inflation model, the inflationary parameters $N_k$ and $r_k$ in terms of $n_s$
are defined as

$$N_k = \frac{f^2}{M_p^2} \ln \left( \frac{2f^2(f^2(1-n_s) + M_p^2)}{(2f^2 + M_p^2)(f^2(1-n_s) - M_p^2)} \right), \quad r_k = 4 \left( \frac{f^2(1-n_s) - M_p^2}{f^2} \right).$$  \hspace{1cm} (77)

In addition with that for the perturbative reheating model, the initial conditions to solve the Boltzmann equations for two density component are set at the end of the inflation to be

$$\Phi(A = 1) = \frac{3}{2} \frac{2\Lambda^4 M_p^2}{(2f^2 + M_p^2)m_\phi^2}, \quad R(A = 1) = \frac{3}{1 - 3\omega_{eff}} \frac{\omega_{eff} - \omega_\phi}{\Phi(A = 1)},$$  \hspace{1cm} (78)

where

$$\Lambda = \left( \frac{3\pi^2 M_p^2 A_s(f^4(1-n_s)^2 - M_p^4)}{2f^2} \right)^{\frac{1}{4}}, \quad m_\phi = \frac{\Lambda^2}{f}. \hspace{1cm} (79)$$

Here, the primordial scalar amplitude of the inflationary scalar fluctuation is $A_{\delta \phi} \sim 2.19 \times 10^{-9}$ from Planck [1]. As we have the connection relation (Eqs. 60, 69) between the reheating parameters and the parameters of magnetogenesis model, in the following, we discuss their implications and various
constraints for two distinct scenarios which we have discussed before.

**Scale-invariant electric power spectra**: In the first panel of the Fig.2, for PLANCK central value of $n_s = 0.9649$, the reheating equation of state $\omega_{eff}$ has to be bounded within a very small window $(0.294, 0.304)$ which is close to the radiation equation of state. The upper bound is associated with the BBN limit of $T_{re} \sim 10^{-2}$ GeV. From the third panel of Fig.2, one observes that with the decreasing magnetic field, maximum allowed reheating temperature is increasing which seems to be intuitively obvious as reheating e-folding is accordingly decreasing. All the above results are for the case-I when the reheating is governed by an effective equation of state. For the perturbative reheating (case-II) we can see from the second panel of the Fig.2 that the maximum strength of the large scale magnetic field is $\sim 10^{-28}$ G which much smaller compared to the observational limit. But this is resolved if we consider scale invariant magnetic field.

**Scale-invariant magnetic power spectra**: As has already been pointed out, scale invariant magnetic field is generically comes out to be stronger than the $n = 2$ case for all the scenarios we have considered. In the first plot of fig.2, for the central value of the spectral index $n_{scentral} = 0.9649$ from planck [1], unlike the inflationary magnetogenesis model with $n=2$, the $n=3$ model set the effective equation of state with a wide range $(-\frac{1}{3}, 0.172)$. The upper limit of the effective equation of state associated with the present magnetic field strength $P_{B0}^{\frac{1}{2}} = 10^{-9}$ G. From the third and fourth plot of fig.3, where we have plotted the variation of reheating temperature and effective equation of state as a function of the spectral index, we can see that the reheating temperature now ranges from $(10^{-2}, 10^{15})$ GeV and the effective equation of state varies from $(-\frac{1}{3}, 0.31)$. So here gauge kinetic function with power $n = 3$ after a certain value of the present-day magnetic field strength indicates a negative effective equation of state during reheating. All the preceding results are for the case-I reheating model. Further, for the perturbative reheating model (shown in the second plot of fig.2), interestingly, the scale-invariant magnetic power spectrum model within the minimum and maximum values of $n_s$ produces the magnetic field’s strength lies within the present magnetic field’s observational range.

C. α–attractor model [50]

In this section, we will consider the α–attractor model, which is a class of theoretical models that unifies a large number of the existing inflationary models proposed in [50]. Conformal transformation of a large class of non-canonical inflaton field lagrangian, a canonical exponential potential obtained
TABLE I: Probing reheating phase (fixing effective equation of state and reheating temperature)

Natural inflation model ($f = 50M_p$)

| Parameters | Current observed value of the magnetic field ($P_{B0}^{1/2}$) measured in unit of Gauss |
|------------|--------------------------------------------------------------------------------------|
| $n_s$      | Scale invariant electric field ($n = 2$)                                             |
|            | Scale invariant magnetic field ($n = 3$)                                             |
|            | 10^{-18} G   | 10^{-20} G   | 10^{-22} G   | 10^{-9} G   | 5.4 \times 10^{-12} G |
| $\omega_{eff}$ | 0.9649 | 0.303 | 0.299 | 0.294 | 0.172 | -0.33 |
| $T_{re}$ (GeV) | 0.033 | 3.36 | 339.30 | 4.10 \times 10^{12} | 1.15 \times 10^{15} |

TABLE II: Constraining reheating and inflationary parameters through inflationary magnetogenesis

Natural inflation model ($f = 50M_p$)

| Parameters | Current observed value of the magnetic field ($P_{B0}^{1/2}$) measured in unit of Gauss |
|------------|--------------------------------------------------------------------------------------|
| $n_s$      | Scale invariant electric field ($n = 2$)                                             |
|            | Scale invariant magnetic field ($n = 3$)                                             |
|            | 10^{-18} G   | 10^{-20} G   | 10^{-22} G   | 10^{-9} G   | 10^{-12} G | 10^{-15} G |
| $n_{s,\text{min}}$ | 0.9645 | 0.9630 | 0.9612 | 0.9561 | 0.9537 & 0.9510 |
| $n_{s,\text{max}}$ | 0.9654 | 0.9654 | 0.9654 | 0.9654 | 0.9641 | 0.96135 |
| $\omega_{eff,\text{min}}$ | 0.2842 | 0.2171 | 0.1523 | 0.0207 | -\frac{1}{3} | -\frac{1}{3} |
| $\omega_{eff,\text{max}}$ | 0.3307 | 0.3303 | 0.3298 | 0.3109 | -0.0233 | -0.0636 |
| $T_{re,\text{min}}$ (GeV) | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} |
| $T_{re,\text{max}}$ (GeV) | 0.16 | 16.42 | 1.6 \times 10^3 | 3.7 \times 10^{13} | 3.2 \times 10^{14} | 7.1 \times 10^{12} |

TABLE III: Probing reheating phase (fixing effective equation of state and reheating temperature)

Higgs-Starobinsky inflation model

| Parameters | Current observed value of the magnetic field ($P_{B0}^{1/2}$) measured in unit of Gauss |
|------------|--------------------------------------------------------------------------------------|
| $n_s$      | Scale invariant electric field ($n = 2$)                                             |
|            | Scale invariant magnetic field ($n = 3$)                                             |
|            | 10^{-18} G   | 10^{-20} G   | 10^{-22} G   | 10^{-9} G   | 5.8 \times 10^{-12} G |
| $\omega_{eff}$ | 0.9649 | 0.3093 | 0.3061 | 0.3019 | 0.1944 | -0.33 |
| $T_{re}$ (GeV) | 0.044 | 4.45 | 447.62 | 6.06 \times 10^{12} | 1.33 \times 10^{15} |

in the following form

$$ V(\phi) = \Lambda^4 \left[ 1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_P}} \right]^{2n} . $$

(80)

The canonical property of this class of model predicts inflationary observable ($n_s, r$), compatible with Planck observation [1, 66]. Here the mass scale $\Lambda$ of the E-model fixed from CMB normalization. However, this class of models includes the Higgs-Starobinsky model [67, 68] for $n = 1, \alpha = 1$. For numerical study, we have taken Higgs-Starobinsky model.
### TABLE IV: Constraining reheating and inflationary parameters through inflationary magnetogenesis

**Higgs-Starobinsky inflation model**

| Parameters | Current observed value of the magnetic field \( (P_{B0}^{1/2}) \) measured in unit of Gauss |
|------------|---------------------------------------------|
|            | Scale invariant electric field \( (n = 2) \) | Scale invariant magnetic field \( (n = 3) \) |
|            | \( 10^{-18} \) G | \( 10^{-20} \) G | \( 10^{-22} \) G | \( 10^{-9} \) G | \( 10^{-12} \) G | \( 10^{-15} \) G |
| \( n_s^{\text{min}} \) | 0.9644 | 0.9629 | 0.9611 | 0.9559 | 0.9535 | 0.9507 |
| \( n_s^{\text{max}} \) | 0.9653 | 0.9653 | 0.9653 | 0.9653 | 0.96402 | 0.9612 |
| \( \omega_s^{\text{min}} \) | 0.2852 | 0.2185 | 0.1538 | 0.0212 | \( -\frac{1}{3} \) | \( -\frac{1}{3} \) |
| \( \omega_s^{\text{max}} \) | 0.3314 | 0.3311 | 0.3307 | 0.3167 | \(-0.0228 \) | \(-0.0630 \) |
| \( T_{re}^{\text{min}} \) (GeV) | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} | 10^{-2} |
| \( T_{re}^{\text{max}} \) (GeV) | 0.16 | 15.7 | \( 1.6 \times 10^3 \) | \( 3.4 \times 10^{13} \) | \( 3.4 \times 10^{14} \) | \( 6.1 \times 10^{12} \) |

Before we jump into our analysis details, let us first determine the relationship between the inflationary parameters with potential parameters. The inflationary e-folding number, \( N_k \), and tensor to scalar ratio, \( r_k \), can be expressed in terms of \( n_s \) and model parameter as

\[
N_k = \frac{3\alpha}{4n} \left[ e^{\frac{\sqrt{2} \Phi_k}{3\alpha M_p}} - e^{\frac{\sqrt{2} \Phi_{\text{end}}}{3\alpha M_p}} - \frac{2}{3\alpha} \left( \Phi_k - \Phi_{\text{end}} \right) M_p \right] , \quad r_k = \frac{64n^2}{3\alpha \left( e^{\frac{\sqrt{2} \Phi_k}{3\alpha M_p}} - 1 \right)^2} . \tag{81}
\]

In addition to that, the initial conditions to solve the Boltzmann equations for different energy components, considering the perturbative reheating model in the context of the present scenario, can be expressed as

\[
\Phi(A = 1) = \frac{3}{2} \Lambda^4 \left( \frac{2n}{2n + \sqrt{3\alpha}} \right)^{2n} , \quad R(A = 1) = 0 , \tag{82}
\]

where

\[
\Lambda = M_p \left( \frac{3\pi^2 r A_s}{2} \right)^{\frac{1}{2}} \left[ \frac{2n(1 + 2n) + \sqrt{4n^2 + 6\alpha(1 + n)(1 - n_s)}}{4n(1 + n)} \right]^{\frac{1}{2}} . \tag{83}
\]

Since we want to study all reasonable constraints on the reheating parameter space through inflationary magnetogenesis, we use equations (60), (69) with the details of the potential parameters, as we have done in the previous inflationary model. This section will briefly review the essential outcomes concerning the Higgs-Starobinsky model for two different invariant magnetogenesis models considering gauge kinetic function with power \( n = (2, 3) \). All important results of the Higgs-Starobinsky model are depicted in Figs.(2), (3) and tables III, IV.

**Scale-invariant electric power spectra**: In our present analysis, for case-I reheating model, once
TABLE V: Probing reheating phase (fixing effective equation of state and reheating temperature)

| Parameters | Current observed value of the magnetic field \( (P_{B_0}^{1/2}) \) measured in unit of Gauss |
|------------|-----------------------------------------------------------------------------------------|
|            | Scale invariant electric field \( (n = 2) \)                                          |
|            | Scale invariant magnetic field \( (n = 3) \)                                          |
| \( n_s \)  | \( 10^{-18} \) G                                                                      | \( 10^{-20} \) G | \( 10^{-22} \) G | \( 10^{-9} \) G | \( 1.05 \times 10^{-12} \) G |
| \( \omega_{eff} \) | 0.9722 | 0.32725 | 0.3264 | 0.3253 | 0.2975 | -0.33 |
| \( T_{re} \) (GeV) | 0.023 | 2.28 | 228.6 | 1.96 \times 10^{12} | 9.73 \times 10^{14} |

again we can settle the effective equation of state as well as reheating temperature from the observed strength of the magnetic field through the evolution of the primordial magnetic field along with the cosmological history. As an example for Planck central value of \( n_s = 0.9649 \), the effective equation of state set within a very narrow range \((0.302, 0.310)\). As already predicted for the previous inflationary models, the present inflation model also prognosticates low reheating temperature, high effective reheating equation of state (close to \( \frac{1}{3} \)), and set the upper limit of the reheating temperature associated with the lower limit of the magnetic field strength \( (P_{B_0}^{1/2} \sim 10^{-22} \) G) about \( \sim 1 \) TeV. In addition to that, over again, the perturbative reheating model results are not fit with the strength of the magnetic field we observe today.

**Scale-invariant magnetic power spectra:** Here again from the table-IV and fig.3, for case-I reheating model one can conclude that if the inflationary magnetogenesis model produces the scale-invariant magnetic power spectra, the reheating temperature must lie within \((10^{-2} \sim 10^{14}) \) GeV and \( \omega_{eff} \) bounded within \((-\frac{1}{3}, 0.3)\). Moreover, the central value of the spectral index from PLANCK \( n_s = 0.9649 \) set the effective equation of state in a wide range of \((-\frac{1}{3}, 0.19)\). In addition to that, the perturbative analysis of reheating is efficient to produce the observational strength of the magnetic field of the order \((10^{-10} \sim 10^{-11}) \) GeV.

### D. Minimal plateau model

In this section, we will introduce a special class of the inflationary model, the minimal plateau models proposed in [64]. The potential of this type of model is a non-polynomial modification to the simple power-law potential \( \phi^n \) and is given by

\[
V_{min} = \Lambda \frac{m^{1-p} \phi^p}{1 + \left( \frac{\phi}{\phi_*} \right)^p},
\]  

(84)
where \( m \) and \( \phi_* \) are two mass scales. The parameter \( \lambda \) and the scale \( m \) are fixed from WMAP normalization \([65]\). Only even values of the index \( p \) are taken, as was the case for the chaotic inflation model. The new scale \( \phi_* \) can be shown to fix the scalar spectral index and the scalar-to tensor ratio within the observational limit from Planck \([1, 66]\). For the numerical purpose, we choose \( \phi_* = 0.01 M_p \) with \( n = 2 \).

Before we go to the quantitative discussion to acquire one to one correspondence between the reheating parameters \( (\omega_{\text{eff}}, T_{\text{re}}) \) with \( P_{E_0}^{1/2} \), let us point out the usual inflationary parameters for this class of minimal models we discussed. The inflationary parameters \( N_k \) and \( r_k \) can be written as,

\[
\begin{align*}
\omega_{\text{eff}} & = \frac{\phi}{\phi^p} \left( \frac{1}{2} + \left( \frac{\phi}{\phi_*} \right)^p \right) \,, \quad N_k = \int_{\phi_k}^{\phi_{\text{end}}} \frac{-\phi (\phi^p + \phi^p)}{p M_p^2 \phi^p} d\phi \, .
\end{align*}
\]

Moreover, similar to the other inflation model, the initial conditions to solve the differential equation in the case of the perturbative reheating model are set as

\[
\Phi(A = 1) = \frac{3 V_{\text{end}}}{2 m_{\phi}^4} \,, \quad R(A = 1) = 0 \, ,
\]

where

\[
V_{\text{end}} = \frac{m^4 \phi_{\text{end}}^p}{1 + \left( \frac{\phi_{\text{end}}}{\phi_*} \right)^p} \,, \quad m = \left( \frac{3 \pi^2 M_p^4 r_k A_*}{2 \Lambda \phi_{\text{end}}^p} \left( 1 + \left( \frac{\phi}{\phi_*} \right)^p \right) \right)^{\frac{1}{p}} \, .
\]

We set \( \Lambda = 1 \) for \( p = 2 \).

Details constraints on the reheating parameter space can be read from Figures (2), (3) and tables (V),(VI). Let us first consider the usual reheating model (case-I). Besides, again in this inflationary

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Parameters} & \text{Current observed value of the magnetic field \( (P_{E_0}^{1/2}) \) measured in unit of Gauss} & \text{Scale invariant electric field \( (n = 2) \)} & \text{Scale invariant magnetic field \( (n = 3) \)} \\
\hline
\text{\( n_*^{\text{min}} \)} & 0.972 & 0.9707 & 0.9693 & 0.965 & 0.9629 & 0.9607 \\
\text{\( n_*^{\text{max}} \)} & 0.9722 & 0.9722 & 0.9722 & 0.9722 & 0.97188 & 0.9694 \\
\text{\( \omega_{\text{eff}}^{\text{min}} \)} & 0.3144 & 0.239 & 0.1743 & 0.033 & -\frac{1}{3} & -\frac{1}{3} \\
\text{\( \omega_{\text{eff}}^{\text{max}} \)} & 0.32725 & 0.3264 & 0.3253 & 0.2975 & -0.0122 & -0.0544 \\
\text{\( T_{\text{re}}^{\text{min}} \) (GeV)} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} \\
\text{\( T_{\text{re}}^{\text{max}} \) (GeV)} & 0.023 & 2.3 & 228.6 & 1.9 \times 10^{12} & 5.2 \times 10^{14} & 7.3 \times 10^{12} \\
\hline
\end{array}
\]
model, once we fixed \( n_s \), we can identify a particular value of the effective reheating equation of state for each specific choice of \( P_{B_0}^{1/2} \), which in turn specified reheating temperature. As an example for \( n_s = 0.9722 \), the \( n = 2 \) inflationary magnetogenesis model restricts effective equation of state within a very tight range \((0.325, 0.327)\) whereas \( n = 3 \) model set \( \omega_{eff} \) in a wider range \((-\frac{1}{3}, 0.297)\). For two scale-invariant inflationary magnetogenesis scenario \( n = (2, 3) \), the numerical results of reheating parameters \((\omega_{eff}, T_{re})\) concerning different sample values of \( P_{B_0}^{1/2} \) are provided in table-VI. Further, in this model, these exist a maximum allowed value of the reheating temperature, which is roughly equal, as has been pointed out for other inflationary models. However, this maximum possible value of reheating temperature depends on which magnetogenesis model parameter \( n \) we acknowledge. The main outcomes for the usual perturbative reheating model will be the same as the previous inflationary model has given in the table-VII, shown in the second panel of both Fig.(2) and (3).

### E. Constraining magnetogenesis model: maximum possible value of \( n \)

We discussed earlier in section-II.A that any values of the coupling parameter \( n \) in the conventional inflationary magnetogenesis model are not permissible. To overcome either strong coupling or the backreaction problem coupling parameter must lie between \( 0 \leq n \leq n_{\text{max}} \), where \( n_{\text{max}} \) can be determined from equation (47). In Figure.4, we show the numerical value of the maximum allowed coupling parameter as a function of the scalar spectral index for various inflationary models. From Fig.4, we can clearly see that the permissible range of \( n_{\text{max}} \) for various inflationary models nearby to 5 except for the minimal plateau model where \( n_{\text{max}} \) is predicted to lie around 6.
V. SUMMARY AND DISCUSSION:

Among all magnetogenesis models, the inflationary magnetogenesis models are well-motivated to explain the origin of the large scale magnetic field in our universe. Allowing the exchange of the power between two fields as a simple consequence of Maxwell’s law of induction, the primordial magnetic field does not decrease as radiation during the decelerating era of the universe, especially during reheating. Moreover, we have given that during inflation, the electric field is much stronger than the magnetic field, so the electric field continues to be the source of the magnetic field and can secure the magnetic field to slow redshift. Therefore the magnetic field power no longer scales as $\propto a^{-4}$ during the post inflationary era till the end of the reheating. After the end of the reheating electric field dies out fast due to the large electric conductivity and magnetic power decay as radiation until today. Considering this effect during the reheating epoch, the present-day amplitude of the magnetic field arising from inflationary magnetogenesis can match with the observational limit of the intergalactic magnetic field on the Mpc scale.

So far, we have discussed mainly understanding the connection between reheating parameters and the parameters of the inflationary magnetogenesis in consideration of the two different reheating scenarios, the conventional reheating model and usual perturbative reheating. We show that the large-scale magnetic field gives non-trivial constrain on both the reheating parameters ($\omega_{\text{eff}}, T_{\text{re}}$) and the inflationary parameter $n_s$. This bound can provide stringent limits on reheating parameters.
with those already derived from the cosmic microwave background by assuming a high energy scale inflation model. We also derive the possible constraints on the coupling parameter by analyzing the gauge field’s gravitational backreaction and the strong coupling problem in the electromagnetic sector. The maximum allowed coupling parameter is nearly 5 for the natural and Higgs-Starobinsky model. However, for the minimum plateau model \( n_{\text{max}} \) lies around 6.

Interestingly, once we fixed the scalar spectral index, our analysis clearly predicts a unique value of the effective equation of state associated with the specific choice of the present-day magnetic field. Once we fix \( \omega_{\text{eff}} \), the reheating temperature can be determined uniquely. The corresponding results for different inflationary models are shown tables I,II and III. As reheating and inflationary parameters are connected, the present-day strength of the magnetic field also provides possible limits on the inflationary scalar spectral index. This extra bound from inflationary magnetogenesis essentially narrows down the possible value of \( n_s \) within the 1\( \sigma \) range of \( n_s = 0.9649 \pm 0.0042 \) (68\% CL, Planck TT,TE,EE+lowE+lensing) from Planck [1]. (shown in tables II, IV, and VI for different inflationary models).

Furthermore, irrespective of the inflationary model under consideration for conventional reheating scenario (case-I) with scale-invariant electric power spectra model set the effective equation of state within a narrow range of \( (0.15 < \omega_{\text{eff}} < \frac{1}{3}) \). However for scale invariant magnetic power spectra the \( \omega_{\text{eff}} \) can be any value between \( -\frac{1}{3} < \omega_{\text{eff}} < 0.3 \). So we can conclude that the inflationary magnetogenesis model with scale-invariant electric power spectra imposes very tight constraints on the reheating parameter space \( (\omega_{\text{eff}}, T_{\text{re}}) \) than the magnetogenesis model with scale-invariant magnetic power spectra.

On the other hand, for the perturbative reheating model, we found that in the \( n = 2 \) inflationary magnetogenesis model, it is hard to get the strength of the magnetic field within the observational limit. Though, \( n = 3 \) magnetogenesis model is viable to produce the magnetic field of the order \( 10^{-10} \sim 10^{-12} \) G, which is within the observational limit. The main outcomes of the perturbative analysis for the \( n = 3 \) model shown in table VII. Compared and combined the inflationary magnetogenesis with CMB data, we have shown that for the case-I reheating model, there exists an inflationary model-independent maximum value of the reheating temperature \( T_{\text{re}}^{\text{max}} \). However, \( T_{\text{re}}^{\text{max}} \) depends on the magnetogenesis model parameter \( n \). In all inflationary scenario discussed and proposed so far, the scale-invariant electric power spectra predict \( T_{\text{re}}^{\text{max}} \) about 1 TeV. Whereas for scale-invariant magnetic power spectra, \( T_{\text{re}}^{\text{max}} \) turns out to be \( 10^{15} \) GeV. In addition to that, in all the inflationary models we discussed and proposed so far, BBN constraints set the minimum value of the reheating temperature.
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