A model of diffuse Galactic radio emission from 10 MHz to 100 GHz

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ABSTRACT

Understanding diffuse Galactic radio emission is interesting both in its own right and for minimizing foreground contamination of cosmological measurements. Cosmic microwave background experiments have focused on frequencies \( \gtrsim 10 \) GHz, whereas 21-cm tomography of the high-redshift universe will mainly focus on \( \lesssim 0.2 \) GHz, for which less is currently known about Galactic emission. Motivated by this, we present a global sky model derived from all publicly available total power large-area radio surveys, digitized with optical character recognition when necessary and compiled into a uniform format, as well as the new Villa Elisa data extending the 1.42-GHz map to the entire sky. We quantify statistical and systematic uncertainties in these surveys by comparing them with various global multifrequency model fits. We find that a principal component based model with only three components can fit the 11 most accurate data sets (at 10, 22, 45 and 408 MHz and 1.42, 2.326, 23, 33, 41, 61, 94 GHz) to an accuracy around 1–10 per cent depending on frequency and sky region. Both our data compilation and our software returning a predicted all-sky map at any frequency from 10 MHz to 100 GHz are publicly available at http://space.mit.edu/home/angelica/gsm.

Key words: methods: data analysis – astronomical data bases: miscellaneous – ISM: structure – diffuse radiation – radio continuum: ISM.

1 INTRODUCTION

There has been a great deal of interest in mapping diffuse Galactic radio emission, both to better understand our Galaxy and to clean out foreground contamination from cosmic microwave background (CMB) maps (see e.g. Tegmark & Efstathiou 1996; Tegmark et al. 2000; Bennett et al. 2003; Tegmark, de Oliveira-Costa & Hamilton 2003; Hinshaw et al. 2007, and references therein). Because much of this work was both motivated by the CMB and based on maps from CMB experiments, it has focused primarily on frequencies above 10 GHz. Now that neutral hydrogen tomography (NHT) is emerging as a promising cosmological tool where we can map the high-redshift universe three dimensionally via the redshifted 21-cm line (see e.g. Zaldarriaga, Furlanetto & Hernquist 2004; Barkana & Loeb 2007; Loeb 2006), it is timely to extend these efforts down to lower frequencies. The redshift range \( 7 \gtrsim z \gtrsim 50 \) where NHT may be feasible corresponds to the frequency range 30–180 MHz. Indeed, accurate foreground modelling is inarguably even more important for NHT than for CMB: whereas unpolarized CMB fluctuations dominate over foregrounds for the most favourable frequencies and sky directions, and the situation for polarized CMB fluctuations is only one to two orders of magnitude worse, the cosmological neutral hydrogen signal is perhaps four orders of magnitudes smaller than the relevant foregrounds (Morales & Hewitt 2004; Bowman, Morales & Hewitt 2006; Furlanetto, Oh & Briggs 2006; Morales, Bowman & Hewitt 2006; Wang et al. 2006).

In due time, NHT experiments such as LOFAR (Rottgering 2003; Rottgering et al. 2003; Zaroubi & Silk 2005), MWA (Morales & Hewitt 2004; Morales 2005; Bowman et al. 2006) and 21 CMA (Peterson, Pen & Wu 2006) will produce accurate low-frequency maps of Galactic emission much like CMB experiments have above 10 GHz. Even now, however, it is important to have a model for how this emission varies across the sky and across the NHT-relevant frequency band, because this helps optimize experimental design, scan strategy and data analysis pipelines to maximize the scientific return on its investment. Even as new maps are made of small patches of sky, it is valuable to have a pre-existing global sky model (GSM) to be able to quantify and mitigate contamination from distant sidelobes.

As described in Section 2, the radio astronomy community has produced a large number of sky surveys over the years, many of
which are directly relevant to the NHT frequency range. However, most of them have never been used by the cosmology community, because of various challenges: many are not publicly available in electronic form, and they are also hard to combine because they differ in sky coverage, angular resolution, pixelization and quality. Instead, a common approach among cosmologists has been to simply extrapolate the 408-MHz Haslam map (Haslam et al. 1982) to lower frequencies, thus ignoring any spectral variation across the sky. As we will see, significantly better accuracy can be attained by modelling which includes additional data sets. The goal of the present paper is to collect, standardize and model this large body of radio data to make it more useful to the cosmology community.

The rest of this paper is organized as follows. In Section 2, we describe how we compile all publicly available total power large-area radio surveys of which we are aware, digitizing them with optical character recognition when necessary, and converting them into a uniform format. In Section 3, we compare different methods for constructing a GSM from this data that covers the entire sky and the entire frequency range. In Section 4, we present the results of our modelling, quantify the accuracy of our best model, and briefly comment on implications for the physics underlying this emission.

2 DATA SETS

In order to carry out our analysis, we performed a literature search for large-area total power sky surveys in the frequency range 1 MHz to 100 GHz. The result of our search is shown in Fig. 1 and Table 1. Unfortunately, some of the surveys shown in Table 1 are not available in any numerical form. A minority of the surveys are publicly available in digital form (and/or) available on request, while many of the surveys are available only as printed tables which we converted to digital form using optical character recognition (OCR).^1^ Some of the surveys shown in Fig. 1 have a very large angular beam (the 0.0135-, 0.0175-, 0.026-, 0.038-, 0.0815-, 0.176-, 0.400- and 0.404-GHz maps), others are undersampled (the 0.085 and the 0.150 surveys), the 0.820-GHz survey is smoothed to 5° in its anti-centre area and, finally, one of them, the 0.345-GHz map, is missing large-scale structures and has severe striping that makes it unsuitable for use in our analysis. Therefore, all analysis presented in this paper is performed using the 0.010-, 0.022-, 0.045-, 0.408-, 1.42-, 2.326-, 23-, 33-, 41-, 61- and 94-GHz maps – Fig. 2 shows the sky coverage of these different surveys and how they overlap. They were all transformed to Galactic coordinates, pixelized using the HEALPIX RING scheme (Górski et al. 2005) with resolution $n_{side} = 512$ (which corresponds to $12 \times 512^2 = 3145728$ equal-area pixels across the sky), and had the CMB component of 2.725 K subtracted. For the five 5-yr Wilkinson Microwave Anisotropy Probe (WMAP) maps (Hinshaw et al. 2008) used in this analysis, we removed their CMB component as described in (Tegmark et al. 2003; de Oliveira-Costa et al. 2004; de Oliveira-Costa & Tegmark 2006)^2^ and then converted these maps to antenna temperature. Before performing our main analysis, we smooth all maps to a common final angular resolution of 5:1. However, as described below, we also use full-resolution maps for some other purposes.

3 METHODS

A wide variety of models of Galactic emission have been used in the literature (Bouchet et al. 1996; Tegmark & Efstathiou 1996; Bouchet & Gispert 1999; Tegmark et al. 2000; Giardino et al. 2002), and there are many additional popular fitting techniques that are purely statistical in nature and do not assume anything about the underlying physics. To maximize the utility of the available data sets and all the hard work that observers have invested into obtaining them, we explored a wide range of modelling methods before selecting one. Below we first present the criterion we will use for choosing the best modelling technique, then explore a range of methods to select the one that is most useful for our goal. The models we compare include physics-inspired fitting functions, power laws, polynomials and splines as well as principal components.

3.1 Criteria: accuracy and simplicity

In this paper, our main criterion for choosing a method is accuracy. In other words, we wish to find the method that most accurately predicts the Galactic emission in any arbitrary sky direction and at any frequency between 0.010 and 94 GHz, independently of whether it is based on physical assumptions or is ‘blind’ and purely statistical. In practice, we implement this criterion as follows: for each of the 11 frequencies where a high-quality sky map is available, we quantify how accurately a method can predict this map by using only information from the other 10 maps.

Since the map used as the ‘truth’ in each test may itself have noise and systematic errors, this procedure can overestimate the true errors. Moreover, our final GSM uses all 11 input maps jointly, not merely 10 at a time, and it is therefore more accurate than the model used in the test. For both of these reasons, the accuracy numbers we quote later on should be interpreted as conservative worst case bounds on the actual accuracy.

In addition to accuracy, we also desire simplicity. Specifically, it is valuable if the modelling method is simple and transparent enough to allow an analytical understanding of how the input determines the output, especially if this makes it possible to characterize how noise and systematic errors propagate and affect the statistical properties of the resulting model.

3.2 Method comparison

3.2.1 Single-component models

The Galactic interstellar medium is a highly complex medium with many different constituents interacting through a multitude of physical processes. Free electrons spiraling around the Galactic magnetic field lines emit synchrotron radiation (Rybicki & Lightman 1979). For the lower frequencies where synchrotron radiation is expected to dominate the Galactic emission, a common approach in the literature has been to simply scale the Haslam map (Haslam et al. 1982) in frequency, usually with a power law

$$T(\vec{r}, \nu) = T(\vec{r}, \nu_s) \left( \frac{\nu}{\nu_s} \right)^{\beta},$$

where $T(\vec{r}, \nu)$ is the brightness temperature at position $\vec{r}$ and frequency $\nu$, $T(\vec{r}, \nu_s)$ is the brightness temperature at position $\vec{r}$ and reference frequency $\nu_s$, and $\beta$ is the spectral index.
where $\mathbf{r}_i$ is the unit vector pointing towards the $i$th pixel in the map, $\nu$ is the frequency which this map is being scaled to, $\nu_\text{\textcolor{green}{$*$}} = 408$ MHz is the Haslam frequency, $\beta$ is the spectral index, and $T$ is the brightness temperature. However, the frequency dependence is known not to be a perfect power law: at higher frequencies, the slope of the synchrotron spectrum typically steepens, and other Galactic components such as free–free and dust emission begin to dominate. This suggests the use of a more general scaling of the type
\begin{equation}
T(\mathbf{r}_i, \nu) = T(\mathbf{r}_i, \nu_\text{\textcolor{green}{$*$}}) f(\nu),
\end{equation}
where the spectrum $f(\nu)$ is optimized by fitting to maps at other frequencies. We will quantify the accuracy of this approach in Section 4. The main problem with it is that the foreground frequency dependence is known to vary across the sky. This occurs both because the synchrotron spectral index $\beta$ depends on the energy distribution of relativistic electrons (Rybicki & Lightman 1979), which varies somewhat across the sky, and also because the ratio of synchrotron to dust and other emission components can vary from place to place. In contrast, equation (2) assumes that a map of Galactic emission looks the same at all frequencies except for an overall change in amplitude.

### 3.2.2 Polynomial and spline fitting

Now that so much data are available, it is tempting to allow much more general fitting functions such as polynomials or cubic splines. We tested both of these approaches here and found that they gave their most accurate results when fitting in log–log (when fitting log $T$ as a function of log $\nu$ rather than using $T$ and/or $\nu$ directly), since the function to be fit is then rather smooth – see Fig. 3 (top panel). For instance, the quadratic polynomial fit
\begin{equation}
\ln T(\mathbf{r}_i, \nu) = a(\mathbf{r}_i) + \beta(\mathbf{r}_i) \ln \frac{\nu}{\nu_\text{\textcolor{green}{$*$}}} + \gamma(\mathbf{r}_i) \left( \ln \frac{\nu}{\nu_\text{\textcolor{green}{$*$}}} \right)^2
\end{equation}
generalizes equation (1) to a position-dependent ‘running’ $\gamma$ of the spectral index $\beta$. For a given pixel $i$, let $m_i$ denote the number of surveys that have observed it ($6 \leq m_i \leq 11$). Rewriting equation (3) in a matrix form, we obtain
\begin{equation}
y = Ax + n,
\end{equation}
where $\mathbf{y}$ is an $m$-dimensional vector that contains (the logarithm of) the temperatures at the $i$th pixel at the $m$ survey frequencies, $\mathbf{A}$ is an $m \times 3$ matrix that encodes the frequency dependence, and $\mathbf{x}$ is a three-dimensional vector that contains the $\alpha \beta$ and $\gamma$ values at the $i$th pixel. The extra term $\mathbf{n}$ denotes noise in the broadest sense of the word, i.e. receiver noise, uncorrected offsets and calibration errors, and any other systematic effects or other non-sky signals present in the survey maps. This is an overdetermined system of linear equations since we always have $m > 3$ input maps available, and assuming that the noise has zero mean, i.e. $\mathbf{E}[\mathbf{n}]=\mathbf{0}$, the minimum-variance estimator for $\mathbf{x}$ is

$$\hat{\mathbf{x}} = (\mathbf{A}'\mathbf{N}'^{-1}\mathbf{A})^{-1}\mathbf{A}'\mathbf{N}'^{-1}\mathbf{y},$$  

with covariance matrix

$$\mathbf{\Sigma} = (\mathbf{A}'\mathbf{N}'^{-1}\mathbf{A})^{-1},$$  

where $\mathbf{N}$ is the noise covariance matrix ($\mathbf{n}\mathbf{n}'$). In Fig. 3, we have simply approximated $\mathbf{N}$ by the diagonal matrix with $N_{ij}$ given by the rms of the $i$th map (we find the recovered maps to be rather insensitive to the choice of $\mathbf{N}$). By performing this calculation for all the pixels in the sky, we obtain all-sky maps of the quantities $\alpha \beta$ and $\gamma$. Finally, to obtain a sky map at any frequency $\nu$, we simply use these values of $\alpha \beta$ and $\gamma$ in equation (3).

We also tried the approach of fitting the (log–log) frequency dependence in each pixel to a separate cubic spline. This involves even more fitting parameters (between 6 and 11), as the resulting curve is

$$\alpha = \text{Publicly available in digital form. B = Available on request. C = Available as printed table (which we OCR'd). D = Not available in any numerical form. This work: http://space.mit.edu/home/angelica/gsm.}$$
is seen to perform poorly when forced to extrapolate, for instance below 22 GHz in the Fig. 3 example. The ability to extrapolate reliably is crucial to our sky model because many of our input maps have only partial sky coverage. A related drawback of the spline approach is that if one of the input maps has more noise or systematic errors than others, this will fully propagate into the model rather than getting ‘voted down’ by the other input maps.

A final problem, seem most clearly in the bottom panel, is caused by fitting the log of the temperature rather than the temperature itself: a relatively modest error in the predicted log-temperature translates into an exponentially amplified error in the temperature itself. The logarithmic fitting also complicates the modelling of measurement errors: if they are symmetrically distributed around zero and uncorrelated with the sky signal in the raw input maps, this is no longer true for the log-maps. In contrast, a linear combination of the linear input maps would preserve such desirable statistical properties of the noise.

3.2.3 Principal component analysis

The above examples suggest that we should try a method that (1) can fit the spectral behaviour of the data with as few parameters as possible and (2) is linear (takes some linear combination of the raw input maps). In other words, we want a linear fitting method where the data itself is allowed to select the optimal parametrized form for the frequency dependence. Fortunately, the standard tool known as principal component analysis (PCA) does exactly this (Press et al. 1992). Indeed, we find that PCA performs better than all the approaches tried above when we implement it as described below.

We begin by estimating the $11 \times 11$ matrix of second moments

$$C = \frac{1}{n_{\text{pix}}} \sum_{i=1}^{n_{\text{pix}}} y_i y_i^\dagger$$

by averaging over all of the $n_{\text{pix}}$ pixels $i$ that were observed in all 11 frequencies (the sky region marked as ‘123456’ in Fig. 2).\(^3\) Therefore, the quantities

$$\sigma_j \equiv C_{jj}^{1/2}$$

simply give the rms fluctuations at each frequency, and the correlation matrix

$$R_{jk} = \frac{C_{jk}}{\sigma_j \sigma_k}$$

corresponds to the dimensionless correlation coefficients between all pairs of frequencies; $-1 \leq R_{jk} \leq 1$ and $R_{jj} = 1$. Defining the normalized maps $\zeta_i$ as the input maps rescaled to have rms fluctuations of unity at each frequency, $R$ is simply the matrix of second moments of these normalized maps.

We then diagonalize the matrix $R$, performing a standard eigenvalue decomposition

$$R = P \Lambda P^\dagger,$$

where $P$ is an orthogonal matrix ($P^\dagger P = PP^\dagger = I$) whose columns are the eigenvectors (principal components) and $\Lambda_{ij} = \delta_{ij} \lambda_i$ is a diagonal matrix containing the corresponding eigenvalues, sorted in decreasing order. The eigenvalues $\lambda_i$ are plotted in Fig. 4,

\(^3\) If we had also removed the mean of each map in this region (an issue to which we return latter), $C$ would simply be the covariance matrix between the 11 frequencies.
and the first three principal components are listed in Table 2 and shown in Fig. 5. In this same table we also show the rms of each of the frequency maps calculated in the region 123456 (second column).

To help intuitively interpret this decomposition, Fig. 6 shows maps of the first few principal components. Each principal component map $a_i$ is defined as the dot product of the corresponding eigenvector with the normalized multifrequency vector $z_i$ for each pixel. For each pixel $i$, we can therefore transform back and forth between the normalized multifrequency vector $z_i$ and the principal component vector $a_i$ using the relations

$$a_i = P^j z_i, \quad z_i = Pa_i. \quad (11)$$

The principal component maps can be thought of as a division of the information in the 11 input maps into 11 mutually exclusive and collectively exhaustive chunks. They are mutually exclusive in the sense that they are uncorrelated:

$$\langle P^j z_i \rangle = \langle P^j z_i \rangle^T = \lambda_i z_i^j, \quad \lambda_i = \sum_{j=1}^{11} \lambda_j = \text{tr} \Lambda = \text{tr} P \Lambda P^T = \text{tr} R = 11, \quad (12)$$

They are collectively exhaustive in the sense that they together specify the multifrequency information completely through equa-

![Figure 4](https://academic.oup.com/mnras/article-abstract/388/1/247/1011818/1011818)

**Figure 4.** The eigenvalues $\lambda_j/11$ for the 11 principal components, which can be interpreted as the fraction of the total variance at the 11 frequencies that each component explains.

| $\nu$ (GHz) | rms       | Comp1 | Comp2 | Comp3 |
|-------------|-----------|-------|-------|-------|
| 0.010       | 262 344 K | 0.286 | −0.358| −0.121|
| 0.022       | 33 693 K  | 0.304 | −0.297| 0.086 |
| 0.045       | 6506 K    | 0.306 | −0.291| 0.010 |
| 0.408       | 25.4 K    | 0.315 | −0.251| 0.020 |
| 1.420       | 0.862 K   | 0.314 | −0.242| −0.008|
| 2.326       | 0.204 K   | 0.331 | −0.147| 0.035 |
| 23          | 0.541 mK  | 0.301 | 0.306 | −0.199|
| 33          | 0.230 mK  | 0.294 | 0.331 | −0.327|
| 41          | 0.140 mK  | 0.291 | 0.341 | −0.335|
| 61          | 0.065 mK  | 0.286 | 0.361 | 0.003 |
| 94          | 0.053 mK  | 0.287 | 0.326 | 0.847 |

**Table 2.** The first three principal components for the region 123456.

![Figure 5](https://academic.oup.com/mnras/article-abstract/388/1/247/1011818/1011818)

**Figure 5.** The frequency dependence is plotted for the first three principal components, labelled by black dots, blue squares and red triangles, respectively. The top panel is in units of the total rms fluctuations at each frequency, whereas the middle panel shows the sky brightness temperature divided by $\nu/2.6\,\text{GHz}^{-2.5}$ to keep all frequencies on roughly the same scale. The bottom panel shows typical spectra of various physical components (Tegmark et al. 2000): synchrotron $\propto \nu^{-2.5}$ (long-dashed black), free–free $\propto \nu^{-2.15}$ (dotted magenta), spinning dust (long-dashed green) and thermal dust (long-dashed red). It also shows half the sum (black dots) and difference (blue squares) of the first two components, which are seen to behave roughly as synchrotron (with a spectral index that steepens towards higher frequency) and a sum of free–free, spinning and thermal dust (blue curve), respectively.

since the diagonal elements of the correlation matrix are all unity. In other words, the total variance to be explained in the normalized multifrequency data is 11, with a contribution of unity from each of the 11 normalized input maps, and equation (12) shows that the $j$th principal component map explains a variance $\lambda_j$, i.e. a fraction $\lambda_j/11$ of the total.

Fig. 4 shows that the first component (top panel of Fig. 6) explains 80 per cent of this total variance, the second component explains another 19 per cent, the third explains another 0.6 per cent, and all the remaining eight components combined explain merely the last 0.3 per cent. This is very convenient: we set out searching for a way to accurately parametrize the frequency dependence of the radio sky with as few parameters as possible, and have found that as few as two parameters capture more than 99 per cent of the information.

Although PCA is quite a standard data analysis technique (Press et al. 1992), our analysis also includes some non-standard procedures, tailored for the particular challenges that our global sky modelling problem poses.

(i) We diagonalize $R$ rather than $C$. 

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This usage of the normalized maps also has the advantage that the spectra of the dominant physical components become rather gently varying functions of frequency, which makes them much easier to linearly fit (see Fig. 5). This eliminates the need for logarithmic fits and their above-mentioned problems.

In a standard PCA, one diagonalizes the covariance matrix \((\mathbf{z} - \langle \mathbf{z} \rangle)(\mathbf{z} - \langle \mathbf{z} \rangle)^T\). We instead diagonalize the matrix \((\mathbf{z}\mathbf{z}^T)\), i.e. do not subtract off the mean from the normalized maps before computing their second moment matrix. This is because, as quantified in Section 4, this procedure makes the method more accurate in regions with incomplete data: whereas the principal components from the region with 11 frequency data work well across the entire sky (basically, because they reflect underlying physical emission mechanisms which are the same everywhere), the 11 mean values from this region are not at all representative for other regions, as they depend strongly on Galactic latitude. By not removing the mean, we also exploit the physical property that none of the dominant foreground components can ever contribute a negative intensity.

Whereas a standard PCA can be performed in the region shown in Fig. 2 where all 11 frequencies have been observed, we wish to build a GSM covering the entire sky. Fortunately, we have \(m \geq 6\) measured frequencies available everywhere, and have found that much fewer than six parameters are required for an excellent fit. We therefore take the best \(m\) principal components determined in the region with complete data and fit them to the data available. In Section 4 we will explore what is the best choice of \(m\), by quantifying the accuracy attained using \(1 \leq m \leq 5\) components. We perform this fitting by modelling the observed data in a pixel with \(m\) observed frequencies as

\[
\mathbf{z}_i = \tilde{\mathbf{P}} \mathbf{\hat{a}}_i + \mathbf{n}_i, 
\]

where the tildes indicate that we are truncating to only \(m\) components: \(\tilde{\mathbf{P}}\) is the \(m \times m\)-dimensional matrix containing the \(m\) first principal components as its columns, \(\mathbf{\hat{a}}_i\) is the \(m\)-dimensional vector corresponding to the first \(m\) principal component maps (see Fig. 5), \(\mathbf{z}_i\) contains the \(m\) normalized input maps that have data for this pixel, and the residual \(\mathbf{n}_i\) models random noise from both measurement errors and additional principal components not included in the fit. We perform this fitting separately for each pixel \(i\) by minimizing

\[
\chi^2 \equiv (\mathbf{z}_i - \tilde{\mathbf{P}} \mathbf{\hat{a}}_i)^T \mathbf{N}_i^{-1} (\mathbf{z}_i - \tilde{\mathbf{P}} \mathbf{\hat{a}}_i),
\]

which gives the solution

\[
\mathbf{\hat{a}}_i = [\tilde{\mathbf{P}}^T \mathbf{N}_i^{-1} \tilde{\mathbf{P}}]^{-1} \tilde{\mathbf{P}}^T \mathbf{N}_i^{-1} \mathbf{z}_i.
\]

We describe our choice for the ‘noise’ covariance matrix \(\mathbf{N}_i \equiv (\mathbf{m} \mathbf{m}^T)\) in Section 4.

Let us summarize the above steps: we first find the principal components using the sky region with data at all 11 frequencies, then use the frequency dependence of these best \(m\) components to fit for maps of their amplitudes across the entire sky. This leaves us with an all-sky model predicting the emission at the 11 frequencies. However, we also wish to predict the emission at any frequency.
in the range \(10 \text{ MHz} \lesssim \nu \lesssim 100 \text{ GHz}\). We do this by fitting the frequency dependence of both \(\log \sigma_j\) and each of the \(m_i\) principal components with a cubic spline as a function of \(\log \nu\). This works well only because, as seen in Fig. 5, these are smooth, slowly varying functions. In contrast, it is notoriously difficult to perform useful interpolation of matrices, e.g. \(\mathbf{R}\), without wreaking havoc with their eigenvalues and physical behaviour.

4 RESULTS

Above we have described how we construct our GSM. However, how accurate is it, and what does it teach us?

4.1 Accuracy of our GSM

4.1.1 Accuracy in the fully mapped region

Table 3 shows the accuracy of our GSM in the sky region where we have data at all 11 frequencies. As described in Section 3.1, we measure how accurately each map can be predicted by the other maps. Specifically, for each of the 11 frequencies, we compute the difference between this map and the map predicted by using only information from the other 10 maps, then tabulate the rms of this difference map divided by the rms of the observed map. Fig. 7 illustrates this procedure for the more challenging all-sky case to which we return below: for these two examples, the relative rms error is the rms of the bottom panel divided by the rms of the corresponding top panel.

Not surprisingly, Table 3 shows that using two components is much more accurate than using only one, reducing errors by almost an order of magnitude at some frequencies. Adding a third component is seen to further improve the accuracy, although not by as much, and mainly in the 20–40 GHz range. Adding a fourth component produces only minor gains, and reduces the 94 GHz accuracy ever so slightly, and adding a fifth component makes the accuracy noticeably worse at three frequencies, suggesting that we are beginning to overfit the data just as with the above-mentioned cubic spline approach.

It is interesting to compare these accuracy numbers with what information theory tells us is the best possible case. The most accurate prediction for a given map using a linear combination of the others is easily computed using a standard linear regression analysis (Press et al. 1992), and these optimal errors are listed in the second column of Table 3. A popular statistical measure of how useful something is for predicting something else is the fraction of the variance that it explains. Under the heading of ‘unexplained fraction’, we therefore also tabulate the fraction \(f_j\) of the map variance that is not explained by the other maps; this is simply the square of the rms residual, since the maps are normalized to have total variance of unity. Note that there is no need to actually perform a linear regression to compute these optimal numbers, as a simple derivation shows that they can be computed directly from the correlation matrix:

\[
f_j = \frac{1}{(R^{-1})_{jj} R_{jj}}.
\]  

The results of this analysis are very encouraging. Table 3 shows that the residuals achieved by our GSM with three components are very close to these smallest possible ones, which means that we need not worry about having overlooked some alternative modelling method that does much better. The results also raise an important question: if linear regression is so good, why do we not use it instead of our GSM? The answer is that we cannot: regression only works when the matrix \(\mathbf{R}\) is known, and we can only compute \(\mathbf{R}\) when we already have data at the frequency that we are trying to model. In other words, whereas we can use regression for accuracy testing, where we already know the answer, it does not help us with modelling all the unobserved frequencies between 10 MHz and 100 GHz. We did explore the idea of predicting the \(\mathbf{R}\)-matrix entries corresponding to new frequencies using interpolation, but were unable to obtain useful results. In contrast, our GSM is straightforward to interpolate to other frequencies, because we simply need to interpolate the spectra plotted in Fig. 5.

When we presented our GSM method described in Section 3, there were two details that we never specified: the choice of noise covariance matrix \(\mathbf{N}\) in equation (16) and the choice of \(m_i\), the number of principal components to use. Let us now discuss these two choices in turn.

4.1.2 The noise covariance matrix

The ‘noise’ is simply the residual signal in a map that we are unable to predict using the other maps, so it will contain contributions from both measurement errors in the input maps and sky emission mechanisms modelled with inadequate precision. Both of these contributions are captured by the remaining principal components not included in the fit, which according to equation (10) make a contribution to \(\mathbf{R}\) that is \(\mathbf{PAP}\) except with all eigenvalues from the included components set to zero. However, it is easy to show that adding noise for the included components has no effect on the
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Figure 7. Accuracy of our model at 408 MHz (left-hand column) and 33 GHz (right-hand column). The top row shows the observed data (the Haslam and WMAP Ka-band maps), the middle row shows the maps predicted by our three-component GSM without using the observed map above it, and the bottom row shows the observation minus the prediction (which is visually indistinguishable from zero for the 33-GHz case, because the residuals are less than 1 per cent in and around the Galactic plane).

4.1.3 Accuracy across the entire sky

How many principal components should we include to maximize the GSM accuracy? To determine this, we must quantify the accuracy not only in the best case sky region where we have complete data, but also over the rest of the sky as well, since we ultimately care about the whole sky. Table 4 shows how this sky-averaged accuracy depends on the number of components used. Specifically, we have computed the relative rms error just as in Table 3, but separately for each of the 10 sky regions show in Fig. 2, then computed their average weighting by sky area. The numbers show that the two best choices are 3 and 4 components. However, whereas these two choices were essentially tied in the fully observed region, \( m_* = 3 \) comes out slightly ahead in the all-sky average because it is twice as accurate at 1.42 GHz. This means that, although the three-component model is slightly less sophisticated and therefore typically slightly less accurate, it is also more robust and less likely to go badly wrong in unusual parts of the sky. For this reason, we focus on the three-component model in the rest of this paper.

Table 4. Relative error averaged over the entire sky.

| \( v \) (GHz) | 1       | 2       | 3       | 4       | 5       |
|--------------|--------|--------|--------|--------|--------|
| 0.010        | 0.438  | 0.091  | 0.098  | 0.088  | 0.168  |
| 0.022        | 0.690  | 0.164  | 0.144  | 0.142  | 0.138  |
| 0.045        | 0.712  | 0.110  | 0.109  | 0.111  | 0.100  |
| 0.408        | 0.436  | 0.112  | 0.115  | 0.127  | 0.134  |
| 1.420        | 0.546  | 0.143  | 0.144  | 0.698  | 0.875  |
| 2.326        | 0.216  | 0.148  | 0.155  | 0.158  | 0.503  |
| 23.0         | 0.423  | 0.082  | 0.062  | 0.062  | 0.064  |
| 33.0         | 0.453  | 0.077  | 0.013  | 0.013  | 0.014  |
| 41.0         | 0.458  | 0.071  | 0.032  | 0.032  | 0.031  |
| 61.0         | 0.444  | 0.069  | 0.068  | 0.068  | 0.070  |
| 94.0         | 0.385  | 0.223  | 0.121  | 0.121  | 0.160  |

Table 4 shows that the typical accuracy is around 10 per cent for the frequencies below WMAP, and noticeably better for the four lowest WMAP-frequencies. It is striking that the 33-GHz accuracy is as good as 1.3 per cent, which means that if WMAP had not made this particular map, as much as 99.97 per cent of the sky variance at this frequency could have been predicted by the other maps (and this is not even counting the CMB signal, which we subtracted off at the outset). Also, as shown in the results for the WMAP 94-GHz
map, the amount of noise in a map clearly worsens the accuracy to which we can reconstruct it.

4.1.4 Accuracy at different Galactic latitudes

It is difficult to quantify the accuracy of our GSM in a meaningful way with a single number, since the sky signal varies so dramatically with Galactic latitude: any measure of absolute error (in Kelvin) will therefore be dominated by the inner Galactic plane, while any measure of relative error will tend to be dominated by the cleanest regions where the signal-to-noise ratio is the poorest. To get a more nuanced picture of how accurate our GSM is, let us therefore quantify the relative errors separately for the regions shown in Fig. 8, which subdivide the sky into six parts of increasing Galactic emission. They were defined in (Tegmark et al. 2003) by computing the four differences of WMAP maps at neighbouring frequencies (to subtract out the CMB), computing the largest absolute value at each pixel, and making a contour plot of the resulting ‘junk map’. From outside in, the regions correspond to junk map temperatures $T < 100, 100–300\mu K, 300\mu K–1 mK, 1–3, 3–10 mK, and T > 10 mK, respectively, so the typical sky signal differs by about half an order of magnitude between neighbouring regions. Table 5 shows that this scaling is roughly valid at all the WMAP frequencies, and that the differences between dirty and clean regions become less extreme towards lower frequencies. Although we of course do not have complete frequency coverage across the entire sky, comparing Fig. 2 with Fig. 8 shows that we are lucky to have coverage at all frequencies somewhere within each of the six sky regions of Fig. 8, with the only exception that the very dirtiest region is not observed at 10 MHz.

Table 6 summarizes how the accuracy of our 3-component GSM depends on both frequency and Galactic signal level. At the sub-GHz frequencies relevant to 21cm tomography, we see that the accuracy is typically $\sim 10$ per cent or better in the cleanest parts of the sky, and degrades in the inner Galactic plane. For the higher frequencies relevant to CMB research, the situation is the opposite: the accuracy is best in the dirtiest parts of the sky (as good as 1 per cent at 33 and 41 GHz), but degrades in the cleanest regions. This is clearly due to the fact that detector noise is non-negligible at the higher WMAP frequencies, so that the lower the signal is, the lower the signal-to-noise ratio level and the accuracy. Future WMAP data releases are therefore likely to further improve the accuracy of our GSM at CMB frequencies.

Finally, it is important to remember that the errors in our downloadable GSM are likely to be even smaller than the tables above suggest, because a map used as ‘truth’ in a test may itself have noise and systematic errors, and also because it uses all 11 input maps jointly, not merely 10 at a time. For example, one can clearly make vastly better predictions near 408 MHz than Table 6 suggests if the Haslam map is included in the modelling.

4.2 Implications for our input maps

An interesting byproduct of our modelling effort is an independent quality assessment of the 11 input maps. If two maps are highly correlated, this implies that none of them can be afflicted by large noise or systematic errors, which would have spoiled the

**Table 5.** rms sky signal in K for regions of different cleanliness.

| $\nu$ (GHz) | 1   | 2   | 3   | 4   | 5   | 6   |
|------------|-----|-----|-----|-----|-----|-----|
| 0.010      | 203.740 | 272.337 | 304.115 | 328.310 | 281.838 |
| 0.022      | 27.336  | 41.972  | 63.535  | 98.153  | 118.713 | 130.600 |
| 0.045      | 548.6   | 834.7   | 13.019  | 21.285  | 31.287  | 35.926  |
| 0.408      | 20.0    | 30.0    | 52.0    | 103.3   | 182.9   | 230.4   |
| 1.420      | 0.744   | 1.021   | 1.614   | 3.016   | 5.356   | 6.839   |
| 2.326      | 0.150   | 0.238   | 0.487   | 1.184   | 2.196   | 2.760   |
| 23         | 0.000098 | 0.000260 | 0.001106 | 0.004140 | 0.010357 | 0.015078 |
| 33         | 0.000036 | 0.000097 | 0.000435 | 0.001693 | 0.004343 | 0.006444 |
| 41         | 0.000021 | 0.000056 | 0.000255 | 0.000996 | 0.002569 | 0.003851 |
| 61         | 0.000007 | 0.000024 | 0.000117 | 0.000433 | 0.001091 | 0.001639 |
| 94         | 0.000006 | 0.000022 | 0.000106 | 0.000332 | 0.000758 | 0.001078 |
correlation. More quantitatively, the unexplained variance fraction listed in Table 3 places an upper bound on the total contribution from detector noise and systematic errors in a map. If we focus on its square root, the optimal rms column in the same table, we see that the lowest frequency WMAP maps give the lowest residuals. This is not surprising, considering that in order to meet its CMB science goals, WMAP had to be designed with significantly stricter systematic error control than typical in radio astronomy. The low WMAP residuals also place a bound on any noise or other errors introduced by our above-mentioned removal of the CMB component. As mentioned above, the WMAP increase in residuals with frequency reflects the drop in foreground signal while detector noise remains important and roughly constant.

At the lower frequencies relevant to 21-cm tomography, we see that the 10–408 MHz map errors can be at most at the 10 per cent level in the cleaner parts of the sky (see Table 6), and no more than 6 per cent in the region where we have full frequency coverage (see Table 3, column 2). The remaining radio maps (at 1.42 and 2.326GHz) have error bounds about a factor of 2 higher.

An interesting future application of our approach could be to use it for systematic error mitigation. If one has reason to believe that a certain systematic error can be parametrized (say by a constant additive offset in a given map), one could find the parameter that minimizes the unexplained variance.

Finally, there is one kind of systematic error that our modelling cannot detect: an overall position-independent calibration error in a map. Because this would not affect the dimensionless correlation coefficients with other maps, it would not affect our goodness-of-fit either, merely cause corresponding calibration errors in the predictions.

4.3 Physical interpretation of our GSM

The goal of this paper is simply to model the Galactic emission, not to understand it physically. However, since our statistical results automatically encode interesting physical information, let us briefly comment on possible interpretations.

4.3.1 Component interpretation

A number of physical components of Galactic emission in our frequency range have been thoroughly discussed in the literature, notably synchrotron radiation, free–free emission, spinning dust and thermal dust. However, we should not expect these physical components, which tend to be highly correlated, to match our principal components, which are by definition uncorrelated. We should instead expect our first principal component (top panel in Fig. 6) to trace the total amount of ‘stuff’, and the remaining principal components to describe how the ratios of different physical components vary across the sky. The frequency dependence seen in Fig. 5 confirms this. The first component is shown to contribute an essentially constant fraction of the rms at all frequencies, corresponding to \( \lambda_1/11 \approx 80 \) per cent of the total variance.

The second component, which explains another \( \lambda_2/11 \approx 19 \) per cent of the total variance, is seen to have the negative of a synchrotron-like spectrum below a few GHz, and a spectrum at higher frequencies that is suggestive of a sum of free–free emission, spinning dust and thermal dust. This suggests that this component encodes what fraction of the total emission is due to synchrotron radiation. Sure enough, the second panel in Fig. 6 is seen to be negative in the north polar spur region which is known to be dominated by synchrotron emission, and positive in regions like the inner Galactic plane and the Large Magellanic Cloud where one expects higher fractions of non-synchrotron emission.

The third component, which explains two thirds of the remaining variance (and \( \lambda_3/11 \approx 0.6 \) per cent of the total variance), is seen in Fig. 5 to have a spectrum that looks like thermal dust at the high end, goes negative below that, and essentially vanishes below a few GHz where synchrotron radiation becomes dominant. This suggests crudely interpreting it as encoding what fraction of the non-synchrotron signal is due to thermal (vibrational) dust emission. It is unclear whether the 10-MHz blip in its spectrum is a fluke or reflects a physical correlation between dust properties and low-frequency synchrotron properties like self-absorption (Peterson & Webber 2002).

4.3.2 Synchrotron and non-synchrotron templates

As we discussed before, we do not expect our principal components to correspond directly to physical components, because the former are by definition uncorrelated while the latter are not (‘stuff traces stuff’, and there tends to be more of everything at low Galactic latitudes). However, it is interesting to ask whether we can form linear combinations of our principal components that have a simple physical interpretation.

Interestingly, we can. In Fig. 5, we see that taking the sum and difference of the first two principal components (from the second panel) gives components whose spectra look distinctly like what is theoretically expected for synchrotron and a combination of the other emission components, respectively (as seen in the bottom panel). First of all, Fig. 5 (bottom) shows that the two new template spectra are approximately non-negative at all 11 frequencies. This is a non-trivial result, since generic 11-dimensional eigenvectors or combinations of them will have both significantly negative and significantly positive components – in contrast, we know that neither synchrotron, free–free nor dust emission can be negative. Second, the same figure shows that first template, which we will hereafter refer to as our synchrotron template, has a spectral index \( \beta \approx -2.5 \) at low frequencies, gradually steepening towards higher frequencies just as expected for synchrotron radiation (Banday & Wolfendale 1991; Jonas 1999). In contrast, the second template, which we will refer to simply as our non-synchrotron template, is seen to have a spectrum such that \( \nu^{f/2} S(\nu) \) rises towards higher frequencies, and can be fit by a sum of free–free, spinning dust (Draine & Lazarian 1998) and thermal dust emission. The corresponding sky maps (the sums and differences of the first two principal components) are shown in Fig. 9.

In other words, our spectral results are consistent with the interpretation that the top panel of Fig. 9 is a diffuse synchrotron template while the bottom one is a template of diffuse non-synchrotron emission. Indeed, the sky regions where the non-synchrotron template dominates tend to have an increasing spectral index at 5 GHz (see \( \gamma \)-map in lower right-hand panel of Fig. 10), whereas one expects the synchrotron spectral index to fall faster towards higher frequencies. As expected, known supernova remnants subtending large angles (Cas A, North Polar Spur and Loop III) are prominent in the synchrotron template, while diffuse dusty sources like the Cygnus Region stand out in the other template. However, we cannot make any such interpretations for the point sources that appear in the paper. This is because some point sources were removed from some of the low-frequency radio maps we used, which can fool our analysis into removing them from the synchrotron template. For example, in the 22-MHz map that we used, areas
Figure 9. Our synchrotron (top) and non-synchrotron (bottom) templates are the sum and difference of our first two principal components, where the colour scales corresponds to $\log(T/1\text{ K})$. Labels indicate bright objects in our Galaxy such as supernova remnants (Cas A, North Polar Spur, Loop III), an emission nebula (Gum nebula), giant molecular clouds (Orion A, R Corona Australis, the Ophiuchus Complex, W3) and an active star-forming region (Cygnus region) as well as bright extragalactic sources like giant elliptical galaxies (Virgo A, Fornax A), radio galaxies (Centaurus A, 3C84) and quasars (3C273, 3C279).

4.4 Angular resolution options

To be able to use all 11 of our input maps, our spectral modelling has been performed at 5:1, the lowest common denominator. If we make the approximation that the spectral shape, but not its amplitude, varies only slowly across the sky, then we can create a higher resolution GSM by locking the amplitude to a higher resolution input map. For example, for each pixel, we can rescale all three principal components used by the same constant, chosen such that the prediction at 408 MHz matches the full resolution Haslam map. This procedure is illustrated in Fig. 9: the top panel locks to the 1° Haslam map (recommended for applications below 1 GHz where synchrotron dominates) while the bottom panel locks to the WMAP 23-GHz map smoothed to 2° to suppress detector noise.

around the strong point sources Tau A, Cas A, Cyg A and Vir A. have been blanked. At 1420 MHz, only Cas A was blanked. Although it would be useful to repeat our analysis with new versions of the input maps where point sources have not been removed (or where they have been reinserted using measured fluxes), the present paper of course has the opposite focus: our key purpose is to model the diffuse emission for use in the cleanest parts of the sky, which are the ones most relevant to 21-cm tomography and CMB observations.

It is worth emphasizing the blind nature of our analysis: by simply forming those two unique linear combinations of our two dominant principal components for which the spectra were non-negative, our approach discovered the synchrotron and non-synchrotron spectra in the data using no physics input whatsoever.
A model of diffuse Galactic radio emission

Figure 10. Sky maps (from top to bottom) of the temperature, spectral index $\beta$, and ‘the running’ (or variation) of spectral index $\gamma$ at 150 MHz (shown on the left-hand side) and 5 GHz (shown on the right). Whereas the 150-MHz emission is dominated by synchrotron radiation with a spectrum that is both falling ($\beta \sim -2.5$) and steepening ($\gamma < 0$), the 5-GHz emission has a much broader range of spectral indices that are mostly getting less negative towards higher frequency ($\gamma > 0$).

(Recommended for applications at CMB frequencies). These 1°, 2°, and 5.1 versions of our GSM are all available on the above-mentioned web site. Fig. 10 shows examples of our output maps at 5.1 resolution.

5 CONCLUSIONS

We have presented a GSM for 10 MHz to 100 GHz Galactic emission derived from all publicly available total power large-area radio surveys, digitized with optical character recognition when necessary and compiled into a uniform format. Both our data compilation and software for returning a predicted all-sky map at any frequency from 10 MHz to 100 GHz are available at http://space.mit.edu/home/angelica/gsm.

We found that a PCA-based model with only three components can fit the 11 most accurate data sets (at 10, 22, 45 and 408 MHz and 1.42, 2.326, 23, 33, 41, 61, 94 GHz) to an accuracy around 1–10 per cent depending on frequency and sky region. We found that using these three principal components comes very close to the maximal accuracy allowed by information theory, with the added advantage of allowing robust frequency interpolation and some physical interpretation. The fact that our model has so few fitting parameters in a given spatial direction also makes it rather robust to the input data: a map with lots of noise or systematic errors will have smaller correlations with other maps, and therefore get ‘voted down’ by the other maps and given less weight.

Strong correlations between different physical emission mechanisms would explain why such accurate fits are possible with fewer principal components than known physical components: one rapidly counts beyond three when including free–free emission, spatial variations of the synchrotron and dust spectra, etc.

We have focused entirely on unpolarized Galactic emission. To help maximize the future scientific impact of 21-cm tomography experiments, it will be important to extend this work to both extragalactic point sources and polarized emission. Since these experiments will provide a gold mine of cosmological information buried
by under ~10^4 times larger foreground signals, this should be well worth the effort!

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