Estimating the reliability of forestry machine elements with possibility theory application

I G Skobtsov¹, V N Shilovskiy¹, O L Dobrynina²

¹Institute of Forestry, Mining and Construction Sciences, Petrozavodsk State University, 33 Lenin Street, Petrozavodsk 185910, Russian Federation
²Institute of Foreign Languages, Petrozavodsk State University, 33 Lenin Street, Petrozavodsk 185910, Russian Federation

E-mail: skobtsov@petrsu.ru

Abstract. The working conditions of forestry machines differ from those of agricultural machines. The presence of obstacles during the clearing of forest areas increases loading of machine components and assemblies and, consequently, leads to their failures. This paper deals with an improvement of probabilistic methods of forestry machine design by applying fracture mechanics and possibility theory. The main fracture mechanics expressions linking stress intensity factor with crack-like defect length are presented in the introduction. Fracture toughness and crack-like defect length are viewed as Gaussian random values, maximum applied stress is presented as a fuzzy variable with unknown distribution law in the second part of the paper. Analytical equations for reliability evaluation are obtained by estimation of upper and lower bounds of reliability function. The real value of reliability function is located within this interval. The proposed approach may be applied to give recommendations for engineering of forestry machine and equipment elements in the case of limited statistical information.

1. Introduction

Attaining high-quality performance of forest management works related to forest restoration, forest protection, fire protection and other measures is an important task. The conditions of forestry machine operation are characterized by forest-growing zone, category of forest-cultivated or other area (cuttings, young trees, plantings, etc.), terrain, size of slopes, tractor passability (number of stumps, swampiness, slope) [1].

Working in rough terrain requires significant tractor driving force, good stability, high cross-country ability and maneuverability. The presence of obstacles (stumps, stones, logging waste) causes the need to operate in various speed and load modes, therefore, increasing the load on machine components and parts leads to their failures [2].

Probabilistic methods of estimation of forest machine reliability are sufficiently developed now [3-7]. When using such methods, statistical information on material characteristics, form and dimensions of structural units, actual loads and other parameters that determine the object’s reliability is supposed to be complete. Thus, by applying the well-known random values distribution law, it is possible to determine quite accurately some reliability indices, e.g. the probability of failure-free operation [4]. In practice, the availability of appropriate equipment and experimental data enables establishing laws of mechanical properties distribution of materials, parts dimensions, and defect sizes. However, complete statistical information on the character and level of actual loads is difficult to obtain as the conditions of
forest and agricultural machines performance vary a lot and depend on a multitude of both objective and subjective factors. Supposing that strength and size characteristics have been determined and their distribution laws are known these characteristics may be described by using the probability theory. Actual loads are characterized by incomplete information due to insufficient statistical data; therefore here the method of possibility theory [8-10], evidence theory by Dempster-Shafer [11], Bayesian approach [12] and the interval average method [13] may be applied. These methods have been applied to make conventional calculations based on material resistivity equations [10] or structural theory when deformation criteria are used to account for crack effect but not the principles of fracture mechanics.

According to Irwin’s fundamental concept of a stress intensity factor [14], the condition of operable state is written as

$$\tilde{\sigma}_1 Y(\tilde{l}) \sqrt{\pi \tilde{l}} \leq \tilde{K}_{IC}$$

where $Y(\tilde{l})$ is a dimensionless geometry factor, depending on the machine part shape and crack’s length (semilength); $\tilde{\sigma}_1$ is the maximum applied stress; $\tilde{l}$ is the crack length (semilength); $\tilde{K}_{IC}$ is the critical plane-strain fracture toughness.

2. Materials and methods
We consider the case when:
- $Y(\tilde{l}) = Y = const$, that applies in the case in which the crack length is far less than the machine part dimensions;
- $\tilde{l}$ and $\tilde{K}_{IC}$ are stochastic quantities with $f_\tilde{l}(\tilde{l})$ and $f_{\tilde{K}_{IC}}(\tilde{K}_{IC})$ distribution laws;
- $\tilde{\sigma}_1$ is a fuzzy variable with possibility distribution

$$\pi_\tilde{\sigma}(\tilde{\sigma}_1) = \exp\left\{ -\left( \frac{(\tilde{\sigma}_1 - a_\sigma)^2}{b_\sigma} \right) \right\},$$

where $a_\sigma = 0.5 \cdot (\sigma_{1_{\text{max}}} + \sigma_{1_{\text{min}}});$ $b_\sigma = 0.5 \cdot (\sigma_{1_{\text{max}}} - \sigma_{1_{\text{min}}}) \sqrt{-\ln \alpha}, \alpha \in [0,1].$

The evaluation of parameters $\sigma_{1_{\text{max}}}$ and $\sigma_{1_{\text{min}}}$ is based on the experimental loading process-related data analysis. Obviously, this statistical information is incomplete and, therefore, variable $\tilde{\sigma}_1$ can be viewed as a fuzzy variable.

The pair of distributions (upper and lower probability distribution functions) are known as a probability box (p-box) in the possibility theory [11]. The unknown “true” distribution $F_\tilde{\sigma}(\sigma_1)$ is located within the p-box [10] (Figure 1)

$$F_\tilde{\sigma}(\sigma_1) \leq F_\tilde{\sigma}(\sigma_1) \leq F_\tilde{\sigma}(\sigma_1),$$

where $F_\tilde{\sigma}(\sigma_1)$ and $F_\tilde{\sigma}(\sigma_1)$ are lower and upper probability distribution functions.

The relationship between p-box and possibility distribution law is as follows

$$F_\tilde{\sigma}(\sigma_1) = \begin{cases} 0, & \sigma_1 \leq a_\sigma \\ 1 - \pi_\tilde{\sigma}(\sigma_1), & \sigma_1 > a_\sigma \end{cases}$$

$$F_\tilde{\sigma}(\sigma_1) = \begin{cases} \pi_\tilde{\sigma}(\sigma_1), & \sigma_1 \leq a_\sigma \\ 1, & \sigma_1 > a_\sigma \end{cases}$$ (1)
It is necessary to determine the probability of failure occurrence $Q = P\{Y\sigma_i \sqrt{l} > K_{IC}\}$ in describing $\tilde{\sigma}_i$ by means of the possibility theory, $\tilde{l}$ and $\tilde{K}_{IC}$—by probabilistic methods. In this case ($\tilde{\sigma}_i$ is a fuzzy variable, $\tilde{l}$ and $\tilde{K}_{IC}$ are random variables) required probability can be expressed as

$$P\{Y\sigma_i \sqrt{l} > \tilde{K}_{IC}\} = \int_{Y\sigma_i \sqrt{l} > \tilde{K}_{IC}} \int f_{\tilde{\sigma}_i}(\sigma) f_{\tilde{l}}(l) f_{\tilde{K}_{IC}}(K_{IC}) d\sigma_i dl dK_{IC},$$

where $f_{\tilde{\sigma}_i}(\sigma)$, $f_{\tilde{l}}(l)$, $f_{\tilde{K}_{IC}}(K_{IC})$ are probability density functions of random variables $\tilde{\sigma}_i$, $\tilde{l}$ and $\tilde{K}_{IC}$.

Substituting the limits of integration, we can receive the following equation

$$P\{Y\sigma_i \sqrt{l} > \tilde{K}_{IC}\} = \int_0^\infty \int_0^\infty \int f_{\tilde{\sigma}_i}(\sigma) f_{\tilde{l}}(l) f_{\tilde{K}_{IC}}(K_{IC}) d\sigma_i dl dK_{IC} =$$

$$= \int_0^\infty \int_0^\infty f_{\tilde{\sigma}_i}(\sigma) f_{\tilde{l}}(l) \left[ \int_{Y\sigma_i \sqrt{l}}^{\tilde{K}_{IC}} f_{\tilde{K}_{IC}}(K_{IC}) dK_{IC} \right] d\sigma_i dl =$$

$$= \int_0^\infty \int_0^\infty f_{\tilde{\sigma}_i}(\sigma) f_{\tilde{l}}(l) \cdot F_{\tilde{K}_{IC}}(Y\sigma_i \sqrt{l}) d\sigma_i dl ,$$

where $F_{\tilde{K}_{IC}}(Y\sigma_i \sqrt{l}) = \int_0^{Y\sigma_i \sqrt{l}} f_{\tilde{K}_{IC}}(K_{IC}) dK_{IC}$ is probability distribution function of the random variable $K_{IC}$.

3. Results and discussion
If $\tilde{\sigma}_i$, $\tilde{l}$ and $\tilde{K}_{IC}$ are independent quantities, then we can obtain lower and upper values of the failure probability in the fracture mechanics terms by using equations (1) and limits of integration (Figure 1).
\[
Q = \int_{a_{\sigma}}^{\infty} \int_{a_{\sigma}}^{\infty} f_{\bar{\sigma}_1}(\sigma_1) \left[ f_p(l) \cdot F_{\tilde{K}_{IC}}(\sqrt{\pi l} \sigma_1) \right] d\sigma_1 \]
\[
\overline{Q} = \int_{a_{\sigma}}^{\infty} \int_{a_{\sigma}}^{\infty} f_{\overline{\sigma}_1}(\sigma_1) \left[ f_p(l) \cdot F_{\tilde{K}_{IC}}(\sqrt{\pi l} \sigma_1) \right] d\sigma_1 \]

or
\[
Q = \int_{0}^{a_{\sigma}} \int_{0}^{a_{\sigma}} f_{\bar{\sigma}_1}(\sigma_1) \left[ f_p(l) \cdot F_{\tilde{K}_{IC}}(\sqrt{\pi l} \sigma_1) \right] d\sigma_1 \]
\[
\overline{Q} = \int_{0}^{a_{\sigma}} \int_{0}^{a_{\sigma}} f_{\overline{\sigma}_1}(\sigma_1) \left[ f_p(l) \cdot F_{\tilde{K}_{IC}}(\sqrt{\pi l} \sigma_1) \right] d\sigma_1 \]

where
\[
f_{\bar{\sigma}_1}(\sigma_1) = \frac{dF_{\bar{\sigma}}(\sigma_1)}{d\sigma_1}, \quad f_{\overline{\sigma}_1}(\sigma_1) = \frac{dF_{\overline{\sigma}}(\sigma_1)}{d\sigma_1}
\]
ar lower and upper probability density functions;
\[
dF_{\bar{\sigma}}(\sigma_1), \quad dF_{\overline{\sigma}}(\sigma_1)
\]
ar lower and upper probability distribution functions,
\[
f_{\bar{\sigma}_1}(\sigma_1) = \frac{d(1-\pi_{\bar{\sigma}}(\sigma_1))}{d\sigma_1} = 2 \frac{\sigma_1 - a_{\sigma}}{b_{\sigma}} \cdot \frac{1}{b_{\sigma}} e^{-\left(\frac{\sigma_1 - a_{\sigma}}{b_{\sigma}}\right)^2}
\]
(3)
\[
f_{\overline{\sigma}_1}(\sigma_1) = \frac{d\pi_{\overline{\sigma}}(\sigma_1)}{d\sigma_1} = -2 \left(\frac{\sigma_1 - a_{\sigma}}{b_{\sigma}}\right) \cdot \frac{1}{b_{\sigma}} \cdot e^{-\left(\frac{\sigma_1 - a_{\sigma}}{b_{\sigma}}\right)^2}
\]
(4)
\[
f_{\bar{\sigma}_1}(\sigma_1) = 0 \text{ if } \sigma_1 < a_{\sigma} \text{ and } f_{\overline{\sigma}_1}(\sigma_1) = 0 \text{ if } \sigma_1 > a_{\sigma}.
\]

If \(\tilde{l}\) and \(\tilde{K}_{IC}\) are Gaussian random quantities with mean values \(m_{\tilde{l}}, m_{K_{IC}}\) and dispersions \(S_{\tilde{l}}^2, S_{K_{IC}}^2\), then the crack length probability density function is
\[
f_{\tilde{l}}(l) = \frac{1}{S_{\tilde{l}}\sqrt{2\pi}} e^{-\frac{(l-m_{\tilde{l}})^2}{2S_{\tilde{l}}^2}},
\]
(5)
and the fracture toughness probability distribution function is
\[
F_{\tilde{K}_{IC}}(\sqrt{\pi l} \sigma_1) = \int_{0}^{\infty} 1 \cdot e^{-\left(\frac{K_{IC} - m_{K_{IC}}}{S_{K_{IC}}\sqrt{2\pi}}\right)^2} dK_{IC}.
\]
(6)

The estimation of the failure probability interval \([Q, \overline{Q}]\) can be calculated by equations (2) according to (3) – (6). Lower \(R\) and upper \(\overline{R}\) bounds of the reliability function are
\[
R = 1 - \overline{Q}
\]
and
\[
\overline{R} = 1 - Q.
\]

The real value of the reliability function is located within the interval \([R, \overline{R}]\).
4. Conclusion

It is of importance to improve the probabilistic and statistical methods of fracture mechanics, which allow accounting for the influence of crack-like defects on the level of reliability of forestry and agricultural machine parts and structural elements. However, in some cases there is not enough statistical information, which determines the direction of further research.

The method of probability function estimation of forestry machine parts and structural elements under the influence of extreme load using force criteria of fracture mechanics with random and fuzzy parameters is developed. It is recommended to consider the maximum applied stress as a fuzzy variable described by the possibility distribution law. The crack length and fracture toughness were considered as random variables with known distribution laws. The suggested method may be applied to give recommendations for engineering of forestry machine and equipment elements in the case of limited statistical information.

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