Impact of Channel Estimation Error on Multiuser Detection via the Replica Method

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Abstract—For practical wireless DS-CDMA systems, channel estimation is imperfect due to noise and interference. In this paper, the impact of channel estimation error on multiuser detection (MUD) is analyzed under the framework of replica method. System performance is obtained in large system limit for optimal MUD, linear MUD and turbo MUD, and is validated by the numerical results of finite systems.

I. INTRODUCTION

In many practical wireless communication systems, the signal is transmitted through fading channels, whose channel state information (CSI) is unknown to the receiver. For coherent communication systems, channel estimation is necessary; however, such estimation usually produces imperfect CSI due to noise and interference, thus undermining the system performance. It is of both theoretical and practical interest to study the impact of the imperfect channel estimation since the understanding of this issue helps to determine the portion of channel use that should be allocated to training.

This paper considers the impact of imperfect channel estimation on multiuser detection (MUD), which mitigates multiple access interference (MAI), of direct sequence code division multiple access (DS-CDMA) systems. For linear minimum mean square error (LMMSE) MUD, the impact of the channel estimation error on the detection has been studied in [2] and [4] using the theory of large random matrices [6]. However, the corresponding research on other MUD techniques still remains inadequate, particularly on optimal MUD [9], due to the lack of appropriate analytical tools. In recent years, the replica method, which was developed in the theory of spin glasses, provides a framework to analyze the performance of both optimal and linear MUD in the large system limit [3][5][7]. In [1], the performance of both turbo MUD and LMMSE filter based parallel interference cancellation (PIC) [10] is considered using the replica method. Here, we adopt this technique to analyze the corresponding performance of optimal MUD, linear MUD and turbo MUD with imperfect channel estimation.

The remainder of this paper is organized as follows. In Section II, the signal model is explained. Analysis of optimal MUD is discussed in Section III and the results are extended to linear MUD and turbo MUD in Section IV. Numerical results and conclusions are given in Section V and Section VI, respectively.

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II. SIGNAL MODEL

A. Signal Model

We consider a synchronous uplink CDMA system with binary phase-shift keying (BPSK) modulation, which experiences a frequency selective channel of order $P$. Let $K$ denote the number of active users, $N$ the spreading gain, and $\beta \equiv K/N$ the system load. We base our analysis on the large system limit, where $K, N, P \to \infty$ while keeping $\beta$ fixed.

We model the frequency selective channel as a discrete finite-impulse-response (FIR) filter. The $z$-transform of the channel response of user $k$ is thus given by

$$h_k(z) = \sum_{p=0}^{P-1} g_k(p) z^p$$

where \{g_k(p)\}_{p=0,\ldots,P-1} are mutually independent channel coefficients having variances $\frac{1}{P}$. For simplicity, we consider only the case that $\frac{P}{N} \ll 1$. Thus we can ignore the intersymbol interference (ISI) and deal with only the portion uncontaminated by ISI. For simplicity, we assume that the channel coefficients are real, although the results can be extended easily to complex signals.

The received signal at the $l$-th chip period can be written as

$$r(l) = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} b_k h_k(l) + n(l), \quad l = P, P + 1, \ldots, N,$$

where $b_k$ denotes the channel symbol of user $k$, $n(l)$ is white Gaussian noise with variance $E[|n(l)|^2] = \sigma_n^2$, and $h_k(l)$ is the convolution of the spreading codes with the channel coefficients:

$$h_k(l) = \sum_{p=0}^{P-1} h_k(p) \delta_l(p),$$

where $s_k(l)$ is the $l$-th chip of the spreading code of user $k$, which takes value on 1 and -1 equiprobably. We call the $(N - P + 1) \times 1$ vector $h_k = (h_k(P-1), \ldots, h_k(N-1))^T$ the equivalent spreading code of user $k$. Due to the assumption that $\frac{P}{N} \ll 1$, we can approximate $N - P + 1$ by $N$ for notational simplicity. Then the received signal can be written in a vector form:

$$\mathbf{r} = \frac{1}{\sqrt{N}} \mathbf{H} \mathbf{b} + \mathbf{n},$$

where $\mathbf{r} = (r(P-1), \ldots, r(N-1))^T$ and $\mathbf{b} = (b_1, \ldots, b_K)^T$.

$^1$Superscript $T$ denotes transposition.
B. Channel Estimation Error

In practical systems, the corresponding channel estimates \( \{ \hat{g}_k(l) \} \) are imprecise due to noise and interference. On denoting the channel estimation error by \( \delta g_k(l) \), we can assume \( \{ \hat{g}_k(l) \} \) to be independent for all values of \( k \) or \( l \). We consider only the following two types of channel estimation techniques:

- Maximum likelihood (ML) channel estimation. It is well known that the ML estimate is asymptotically unbiased and consistent under some regulation conditions. Thus we can assume that the estimation error \( \delta g_k(l) \) has an expectation of zero, thus being uncorrelated with \( g_k(l) \).
- MMSE channel estimation. An interesting property of MMSE estimation, namely the conditional expectation of the channel estimation error. We denote the case of uncorrelated with \( g_k(l) \).

When the channel estimate is used to compute the corresponding equivalent spreading code, the variance of the error of each chip of \( \hat{b}_k \) is equal to that of \( \delta g_k(l) \), which is denoted by \( \Delta g_k(l) \). Due to the central limit theorem, we can show that \( \delta h_k(l) \) is asymptotically Gaussian as \( P \to \infty \). Since \( E(\delta h_k(l)\delta h_k(j)) = 0 \), \( \forall i \neq j \), any two different elements in \( \delta h_k \) are asymptotically independent of each other. We can assume that the elements in \( b_k \) or \( \hat{b}_k \) are mutually independent in the large system limit, which will be validated with numerical results in Section V.

III. IMPACT ON OPTIMAL MUD

In this section, we discuss two types of receivers that are distinguished by whether or not the receiver considers the distribution of the channel estimation error. We denote the case of directly using the channel estimate for MUD by \( D \), and the case of considering the distribution of the channel estimation error to compensate the corresponding impact by \( C \). For optimal MUD based receivers with perfect channel state information, we cite the result in [7] for comparison, which states that the system performance is determined by the following parameters:

\[
\begin{align*}
\{ m = \int_B \tanh (\sqrt{\frac{1}{2}} z + E) Dz, \\
q = \int_B \tanh^2 (\sqrt{\frac{1}{2}} z + E) Dz, \\
E = \frac{\beta^{-1} B}{1 + B(1 - q)^2}, \\
F = \frac{\beta^{-1} B^2 (B^{-1} + 1 - 2m + q)}{(1 + B(1 - q)^2)},
\end{align*}
\]

(3)

where \( D_z = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{z^2}{2}\right) \) and the corresponding bit error rate is given by \( Q \left( \sqrt{E^2 / F} \right) \), where \( Q \) is the complementary Gaussian cumulative distribution function. \( \sigma^2 \) is a control parameter, by which we can obtain individually optimal (IO) MUD (\( \sigma^2 = \sigma_0^2 \)) and jointly optimal (JO) MUD (\( \sigma^2 = 0 \)) [3][7].

A. D-Optimal MUD

In D-optimal MUD, the likelihood ratio (LR) of any binary channel symbol \( b_k \) is given by

\[
P(b_k = 1 | r) = \frac{\sum_b \{ b = 1 \} \exp \left( -\frac{1}{2\sigma^2} \left| r - \frac{1}{\sqrt{N}} \hat{h} b \right|^2 \right)}{\sum_b \{ b = -1 \} \exp \left( -\frac{1}{2\sigma^2} \left| r - \frac{1}{\sqrt{N}} \hat{h} b \right|^2 \right)},
\]

where the hat on \( h \) means that the equivalent spreading codes are obtained from the channel estimates. The key issue in the replica method is the computation of the free energy, which is defined as

\[
F_K(r, \hat{H}, \hat{H}) \triangleq K^{-1} \log Z(r, \hat{H}) = \lim_{K \to \infty} \int_{\bar{r}} P_0(r | H) \log Z(r, \hat{H}) dr,
\]

(4)

where

\[
P_0(r | H) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2\sigma^2} \left| r - \frac{1}{\sqrt{N}} \hat{H} b \right|^2 \right),
\]

and

\[
Z(r, H) \triangleq \sum_b P(b) \exp \left( -\frac{1}{2\sigma^2} \left| r - \frac{1}{\sqrt{N}} \hat{H} b \right|^2 \right),
\]

and the overbar denotes the average over the randomness of the equivalent spreading codes. On applying Assumption 2 in [7], equation (4) is simplified to

\[
F_K(r, H, \hat{H}) = \lim_{K \to \infty} \left( \lim_{n \to 0} \log Z_n \right),
\]

(5)

where

\[
\Xi_n = \int_{b_0, \ldots, b_n} \prod_{a=0}^n P(b_a),
\]

\[
\times \left\{ \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) \right\}^N \prod_{a=0}^n Q_a \left( \sqrt{\frac{E^2}{F} / \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) dr \right),
\]

(6)

where the index \( l \) in \( h_k(l) \) is dropped for notational simplicity. For ML channel estimation, \( \delta h_k \) is uncorrelated with \( \hat{h}_k \), thus \( E\{ h_k \hat{h}_k \} = 1 \) and \( E\{ \delta h_k \hat{h}_k \} = 1 + \Delta h_k^2 \). Then

\[
\Xi_n = \int_{b_0, \ldots, b_n} \prod_{a=0}^n P(b_a),
\]

\[
\times \left\{ \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) \right\}^N \prod_{a=0}^n Q_a \left( \sqrt{\frac{E^2}{F} / \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) dr \right),
\]

(6)

For MMSE channel estimation, \( \delta h_k \) is uncorrelated with \( \hat{h}_k \), thus \( E\{ h_k \hat{h}_k \} = E\{ h_k^2 \} = 1 - \Delta h_k^2 \). Then we have

\[
\Xi_n = \int_{b_0, \ldots, b_n} \prod_{a=0}^n P(b_a),
\]

\[
\times \left\{ \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) \right\}^N \prod_{a=0}^n Q_a \left( \sqrt{\frac{E^2}{F} / \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\bar{r}} \exp \left( -\frac{1}{2\sigma^2} \left( r - \frac{1}{\sqrt{N}} \hat{v}_a(Q) \right)^2 \right) dr \right),
\]

(6)
As in the replica symmetry assumption in [7], we assume that $Q_{ba} = m$ and $Q_{ab} = q$, $\forall a < b$, $a \neq 0$. Due to the central limit theorem, $v_a$ is asymptotically Gaussian. Thus, we can construct $\{v_a\}$ in a way similar to [7] with a set of independent Gaussian variables.

Applying the same procedure as in [7], we have the following asymptotic expression for (5)

$$\lim_{K \to \infty} K^{-1} \log \Xi_n = \sup_{Q} \left\{ \beta^{-1} g(Q) - I(Q) \right\},$$

where $\exp \left\{ g(Q) \right\}$ equals the expression within the brackets in (6) and $I(Q)$ is the rate function of the probability measure of $\{Q\}$, determined by parameters $E$ and $F$ when the replica symmetry assumption holds.

An important observation is that the channel estimation error does not affect $I$ in (7); thus the expressions for $m$ and $q$ are unchanged. Thus we focus on only the evaluation of $G$. With some algebra, we can obtain the corresponding free energy for ML channel estimation based D-optimal MUD, from which we obtain the following expressions for $E$ and $F$:

$$E = \frac{\beta^{-1} B}{1+B(1-q)(1+\Delta_2^a)},$$

$$F = \frac{\beta^{-1} B^2(1-\Delta_2^a)(1+\Delta_2^a)(2m-q)}{(1+B(1-q)(1+\Delta_2^a))^2}.$$

And for MMSE channel estimation based D-optimal MUD, we have

$$E = \frac{\beta^{-1} B(1-\Delta_2^a)}{1+B(1-q)(1+\Delta_2^a)},$$

$$F = \frac{\beta^{-1} B^2(1-\Delta_2^a)(1+\Delta_2^a)(2m-q)}{(1+B(1-q)(1+\Delta_2^a))^2}.$$

The corresponding output signal-to-interference-plus-noise-ratios (SINRs) are given by

$$SINR_{ML} = \frac{1}{(1+\Delta_2^a)(\sigma_n^2 + \beta(1-2m+1+\Delta_2^a)(q))},$$

and

$$SINR_{MMSE} = \frac{1}{(\sigma_n^2 + \beta(1-1-(1+\Delta_2^a)(2m-q)))}.$$

Thus, we can summarize the impact of the channel estimation error on the D-optimal MUD as follows:

- The factors $\frac{1}{1+\Delta_2^a}$ in (8) and $1-\Delta_2^a$ in the numerator of (9) represent the effect of the error of the desired user’s equivalent spreading code, which is equivalent to deceasing the received power.
- The imperfect channel estimation also increases the variance of the residual MAI, which equals to $\beta(1-2m+(1+\Delta_2^a)(q))$ for ML channel estimation based systems and $\beta(1-(1+\Delta_2^a)(2m-q))$ for MMSE channel estimation based systems.
- The replica symmetry that $m = q$ and $E = F$ when $\sigma^2 = \sigma_n^2$ is broken. Thus there is no simple analytical expression for obtaining the multiuser efficiency similar to the Tse-Hanly equation [8].

### B. C-IO MUD

1) **ML channel estimation based MUD:** We consider the ML channel estimation based MUD first. When the distribution of the channel estimation error is considered at the receiver, the conditional probability $p(h|\hat{h})$ needs to be taken into account. Thus the *a posteriori* probability of the received signal $r$ at some chip period, conditioned on the channel estimate $\hat{h}$ and the transmitted symbols $\{b\}$, is given by

$$p(r|\{\hat{h}\}, \{b\}) \propto \int_r p(r|\{h\}, \{b\}) p(\{h\} | \{\hat{h}\}) dh,$$

where $p(\{h\} | \{\hat{h}\}) = \prod_{k=1}^{K} p(h_k|\hat{h}_k)$ and

$$p(h_k|\hat{h}_k) \propto \exp \left( -\frac{(h_k - \hat{h}_k)^2}{2\Delta_h^2} \right).$$

Applying the ML criterion, we can obtain the output LR of the C-IO MUD:

$$P(h_k = 1|r) = \frac{\sum_{b_k=1} p(b_k=1) \exp \left( -\frac{1}{\gamma} \left| r - \frac{1}{N(1+\Delta_2^a)} \hat{H} b \right|^2 \right)}{P(h_k = -1|r)} = \frac{\sum_{b_k=-1} p(b_k=-1) \exp \left( -\frac{1}{\gamma} \left| r - \frac{1}{N(1+\Delta_2^a)} \hat{H} b \right|^2 \right)}{\sum_{b_k=1} p(b_k=1) \exp \left( -\frac{1}{\gamma} \left| r - \frac{1}{N(1+\Delta_2^a)} \hat{H} b \right|^2 \right)},$$

where $\sigma^2 = \sigma_n^2 + \frac{\beta \Delta_2^2}{2\Delta_h^2}$. Compared with the expression for D-IO MUD, the C-IO MUD merely scales the predicted signal with $\frac{\Delta_h^2}{\Delta_2^2}$ and incorporates $\Delta_2^2$ into $\sigma^2$. Applying the same procedure as in the D-optimal MUD, we can obtain

$$E = \frac{\beta^{-1} B}{1+B(1-q)(1+\Delta_2^a)},$$

$$F = \frac{\beta^{-1} B^2(1-\Delta_2^a)(1+\Delta_2^a)(2m-q)}{(1+B(1-q)(1+\Delta_2^a))^2}.$$

It is easy to check that $E = F$ when $m = q$, which means the corresponding replica symmetry is recovered. Thus we can obtain the multiuser efficiency $\eta$ by solving the following equation:

$$\frac{1}{\eta} + \frac{\beta}{\sigma_2^2} \int_{\mathbb{R}} \tanh^2 \left( \sqrt{\frac{\eta}{\sigma_2^2}} z + \frac{\eta}{\sigma_2^2} \right) Dz = \left( 1 + \Delta_2^2 \right) \left( 1 + \frac{\beta}{\sigma_2^2} \right).$$

2) **MMSE channel estimation based MUD:** For MMSE channel estimation, the channel estimation error $\delta h$ is independent of the estimate $h$. Thus we have

$$p(h_k|\hat{h}_k) \propto \exp \left( -\frac{(h_k - \hat{h}_k)^2}{2\Delta_h^2} \right).$$

With some algebra, we can obtain the same expression for the LR as the D-IO MUD except that $\sigma^2 = \sigma_n^2 + \beta \Delta_2^2$. And the corresponding multiuser efficiency is given by solving the following equation:

$$\frac{1}{\eta} + \frac{\beta}{\sigma_2^2} \int_{\mathbb{R}} \tanh^2 \left( \sqrt{\frac{\eta}{\sigma_2^2}} z + \frac{\eta}{\sigma_2^2} \right) Dz = \left( 1 + \Delta_2^2 \right) \left( 1 + \frac{\beta}{\sigma_2^2} \right).$$

On comparing (10) and (11), an immediate conclusion is that the C-IO MUD is more susceptible to the error incurred by MMSE channel estimation than to that incurred by ML channel estimation.
IV. IMPACTS ON LINEAR MUD AND ITERATIVE MUD

For simplicity, we discuss only ML channel estimation based systems in this section. The MMSE channel estimation based systems can be analyzed in a similar way.

A. Linear MUD

The analysis of linear MUD can be incorporated into the framework of the replica method by merely regarding the channel symbols as Gaussian distributed random variables. The system performance is determined by parameters \( m, q, p, E, F \) and \( G \) (for LMMSE MUD, \( \sigma^2 = \sigma_n^2 \); for the decorrelator, \( \sigma^2 \to 0 \)) with a group of saddle-point equations [7].

1) D-Linear MUD: Since the channel estimation error does not affect \( I \), the parameters \( m, q \) and \( p \) are unchanged. With the same manipulation on \( G \) as in the preceding section, we can obtain the parameters \( E, F \) and \( G \) as follows:

\[
E = \frac{\eta - p}{1 + \beta(P - G)(1 + \Delta^2_h)}, \quad F = \frac{(1 + \Delta^2_h) \beta^{-1} B (1 + \Delta^2_h)^2}{(1 + \beta(P - G)(1 + \Delta^2_h))^2}, \quad G = F - (1 + \Delta^2_h) E.
\]

2) C-LMMSE MUD: Similar to the preceding section, the LMMSE detector with the consideration of the distribution of the channel estimation error is given by merely scaling \( H \) with a factor of \( \frac{1}{1 + \Delta^2_h} \), which recovers the replica symmetry. The corresponding multiuser efficiency is given by

\[
(1 + \Delta^2_h) \eta + \frac{\beta \eta}{\sigma_n^2 + \eta} = 1 - \frac{\Delta^2_h}{\sigma_n^2}.
\]

B. Iterative MUD

Here, we consider only the case of directly using the channel estimation.

1) Turbo MUD: For turbo MUD, since the channel estimation error does not affect \( I(Q) \) in evaluating the free energy, the impact of channel estimation error is similar to that for non-iterative MUD.

2) LMMSE filter based PIC: The situation is more complicated for the case of LMMSE filter based PIC. The corresponding LMMSE filter is used to suppress the residual MAI and is constructed with the estimate of the power of the residual MAI. However, the power estimate is different from the actual value \( |\hat{h}|^2 \) since \( b \) is unknown to the receiver, thus making the filter unmatched for the residual MAI. Hence, the analysis in [11] might have overestimated the system performance since such power estimation error is not considered. Thus we need to take into account the corresponding power mismatch.

Denote the actual value of the residual MAI power of user \( k \) by \( P_k \) and the estimated value by \( \hat{P}_k \). Confining our discussion to unbiased power estimations, we normalize the power of the residual MAI such that \( E\{P_k\} = E\{\hat{P}_k\} = 1 \). The equivalent noise variance is given by \( \sigma^2 = \frac{\sigma_n^2}{\hat{P}_k} \), where \( \Delta^2_h \) is the mean square error of decision feedback. The bit error rate is given by \( Q \left( \sqrt{\frac{E^2}{P \sigma^2}} \right) \) since the power of the desired user is 1. We observe that the power mismatch has no influence on \( G \) in (7); thus the parameters \( E, F \) and \( G \) are identical to those in (12). By reevaluating \( I \), we can obtain the updated parameters \( m, q, \) and \( p \) by

\[
\begin{align*}
\tilde{m} &= E \left\{ \frac{p P E}{1 + P (F - G)} \right\}, \\
\tilde{q} &= E \left\{ \frac{P^2 (PE + F)}{(1 + P (F - G))^2} \right\}, \\
\tilde{r} &= E \left\{ \frac{P (PE + 2 P F + 1 - P G)}{(1 + P (F - G))^2} \right\},
\end{align*}
\]

A. Numerical Results for D-IO MUD

Figure 1 shows the bit error rate due to different variances of the channel estimation error in a D-IO MUD system with \( K = 10, N = 150, P = 50 \) and \( \sigma_n^2 = 0.2 \), where the dark curves represent the results for MMSE channel estimation and the light curves represent those for ML channel estimation. In this figure, “independent” represents the case of equivalent spreading codes with mutually independent elements and “convolution” represents the case in which the equivalent spreading codes are the convolution of binary spreading codes and channel gains. From this figure, we can see that the assumption of independent elements in the equivalent spreading codes is valid and the asymptotic results can predict the performance of finite systems reasonably well. This figure also shows that the D-IO MUD is more susceptible to the error in MMSE channel estimation than that in ML estimation.

Figure 2 shows the bit error rate in D-IO and C-IO MUD system with \( \beta = 0.5 \) and \( \sigma_n^2 = 0.2 \), where the dark curves...
represent the results of the C-IO MUD systems and the light curves represent those of the D-IO MUD systems. For ML channel estimation, the C-IO MUD can achieve better performance than the D-IO MUD. For MMSE channel estimation, the two IO MUD schemes attain almost the same performance.

Figure 3 shows the bit error rate in LMMSE MUD systems with the same configuration as in Fig.2. Both the simulation and asymptotic results are given for D-LMMSE MUD, and match fairly well. Note that C-LMMSE MUD achieves marginally better performance than D-LMMSE MUD.

Figure 4 shows the bit error rate of LMMSE filter based PIC systems with the same configurations as in Fig.2, convolutional code \((23,33,37)\) and \(\Delta^2 h = 0.0488\). We can observe that the optimal scheme, which assumes the decision feedback error is known, achieves only marginally better performance.

VI. CONCLUSIONS

In this paper, we have discussed the impact of channel estimation error on various types of MUD algorithms in DS-CDMA systems by obtaining the asymptotic analytical expressions of the system performance in terms of the error variance. The analysis is unified under the framework of the replica method. The following conclusions are of particular interest:

- The performance of MUD is more susceptible to MMSE channel estimation errors.
- The MUD schemes that consider the distribution of channel estimation errors show improved the system performance.
- When the LMMSE MUD treats the different users as being transmitted with equal power, it attains the multiuser efficiency of the corresponding equal-power systems.

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