ABSTRACT For limited-angle computed tomography (CT) image reconstruction, the classical total variation (TV) based algorithms suffer from the limited-angle artifacts, because TV only used the gradient information of the image. The priori image constrained compressed sensing (PICCS) based reconstruction algorithms can reduce the limited-angle artifacts by using a priori image consistent with the target image. However, it is difficult to ensure the consistency of priori image and target image in practice. In order to reconstruct high quality image when the prior image is inconsistent with the target image, we proposed a guided image filter reconstruction based on TV and prior image (TVPI-G). In each iteration phase, our algorithm first performs a TV step (include simultaneous algebra reconstruction technique (SART) and TV) to get initiatory reconstruction image; Then, the results of TV iteration are combined with prior images to form an intermediate result; Finally, we use the guided image filter to modify the intermediate results with the TV result as the guide image. Numerical reconstruction results on simulation phantom with different intensities Poisson noise illustrates that our proposed TVPI-G algorithm is better than other comparison algorithms in both qualitative and quantitative aspects, including TV, PICCS, and SART guided image filtering.

INDEX TERMS Image reconstruction, limited-angle CT, total variation, guided image filtering, prior image.

I. INTRODUCTION X-ray Computed Tomography (CT) has been successfully applied in many aspects, for example, in medical diagnosis [1], industrial nondestructive testing [2], safety inspection [3]. The purpose of the CT is to nondestructively detect the interior of the object. The principle of CT image reconstruction is a typical inverse problem in mathematics. For fan-beam CT scanning, at least 180° plus a fan-angle views are required for accurate reconstruction conditions. This means the rotation angle of the scanning system or object is at least 180° plus fan-angle. In this case, analytic reconstruction method such as filtered back-projection (FBP) algorithm can reconstruct images accurately. However, due to the damage of the X-ray radiation, the scanning environment restrictions and the scanning time requirements, the projection data are insufficient for accurate reconstruction. When the rotation angle is less than 180° plus a fan-angle, the problem is called limited-angle CT problem. Limited-angle CT has a wide range of applications, for example, in medical imaging, the C-arm tomosynthesis [4], and dental CT [5]. In industrial nondestructive testing, CT is used to detect the pipelines in service, and straight-line trajectory CT is used to detect circuit boards and flat object. Due to the limited-angle projection data being severely inadequate, the traditional FBP algorithm cannot reconstruct an accurate image because the FBP algorithm needs complete projection data. J. Frikel presented a modified FBP algorithm that can reduce limited-angle artifacts, but limited-angle artifacts cannot be removed from a large part of the image [6].
CT projection is a Radon transformation that can be discretized into a linear transformation. The target image can be reconstructed by solving the linear equations. In the CT image reconstruction field, the most widely used iterative algorithms are algebraic reconstruction technique (ART) and simultaneous algebra reconstruction technique (SART) [7,8]. Compared with analytical reconstruction algorithms, iterative algorithms can reconstruct higher-quality images. However, when the scanning angle is small, SART algorithm will show strong limited-angle artifacts in reconstructed image.

In recent years, image reconstruction optimization algorithms based on regularization methods have been widely researched. Different regularization terms lead to different models and different solutions. Image reconstruction models based on regularization generally assume that images are sparse in the total variation (TV) transformation domain [9], wavelet transform domain [10] and so on [11]–[14]. Sidky et al. proposed the total variation (TV) minimization reconstruction algorithm in few-view and limited-angle problems to preserve the image edges and reduce the artifacts [9]. The algorithm based on statistical learning has also been applied to CT reconstruction and achieved good reconstruction results, such as dictionary learning method [15], discriminative feature representation method [16], [17] and nonlocal prior Bayesian method [18].

In recent years, prior image information has been used to further constrain CT image reconstruction. Chen et al. proposed a prior image constrained compressed sensing (PICCS) algorithm [19], which is more robust to incomplete projection data reconstruction problems than traditional compressed sensing (CS) based technology [20]. On this basis, Wang et al. introduced prior images and a wavelet compact framework into the problem of limited-angle CT reconstruction with small scanning angles and proposed a new limited-angle CT reconstruction model based on L0 regularization and prior images [21]. They also analyzed the convergence of the algorithm and proved that the bounded sequence generated by the alternating iteration algorithm has a sub-column converging to a stable point. The experimental results also showed that the reconstruction accuracy of the algorithm was higher than that of the PICCS algorithm. Zhang et al. studied an iterative reconstruction method based on the induction and improvement of the similarity between prior image and reconstructed image structures, which used L0 regularization of wavelet compact frame transform coefficients of reconstructed images to deal with limited-angle artifacts [21]. To further suppress noise and limited-angle artifacts, Gong et al. proposed a prior image induced relative total variation reconstruction model that used the structure information of the prior image [22]. For incomplete data, high-quality images can be reconstructed using prior image information. However, when the prior image and reconstructed image are inconsistent, the regularization algorithm based on prior image may reconstruct false edges, because it relies on the structure of prior image.

He et al. proposed guided image filtering (GIF) for image restoration [23]. GIF is a filter derived from the local linear model in which the guide image can be the image to be processed or other images. GIF is an image smoothing operator that maintains edges, and the processing near edges is better than that of bilateral filtering. Due to its fast guiding filtering speed and good performance, GIF is widely used in image processing and computer vision [24], [25].

In image reconstruction, there are many image reconstruction algorithms based on GIF [26]–[28]. Yu et al. developed a GIF reconstruction algorithm combined with a TV algorithm [26]. In the TV iteration phase, the results of TV are taken as the guided image and the results of the intermediate SART steps are taken as the input image. Their algorithm can remove noise and sparse-angle artifacts in sparse-angle and low-dose CT image reconstruction. However, for the limited-angle problem, the lack of projection data results in severe ill-posed problem. Their algorithm is not suitable for limited-angle reconstruction. Ref. [27] proposed a SART reconstruction algorithm based on guided image filtering (SART-G). It used guided image filtering to maintain edge features from a few projected data. For CT images with prior images and simple structure, the SART-G algorithm can reconstruct high-quality images from sparse angle projection data [27]. SART-G algorithm iteration consists of two steps: The first is SART step, which uses the SART algorithm to solve the CT discrete linear system; then, a guided image filtering step is used to reduce noise and retain the detailed structure of reconstructed image. Compared with SART algorithm, SART-G algorithm has two advantages. First, the initial priori image is used to constrain the image reconstruction process to accelerate the convergence of the reconstructed image. Second, the guide image is updated in the process of image reconstruction. Although SART-G can achieve satisfactory results in sparse angle CT reconstruction, it cannot completely eliminate the limited-angle artifacts in the reconstruction of limited-angle CT.

Inspired by their work [27], in this paper, we propose a guided image filter reconstruction algorithm based on total variation and prior image (TVPI-G) for limited-angle CT. Our algorithm first performs a TV iteration in each iteration phase; then, we combine the prior image and TV result to form the intermediate result. The introduction of prior images may lead to inconsistency between reconstructed image and projected data. In the third step, we use the guided image filter to modify the intermediate results with the TV result as the guide image. The goal of this step is to force the intermediate result to be consistent with the projected data. Compared with SART-G algorithm, our algorithm has two advantages. To reduce the number of parameters and reduce the influence of the prior image as early as possible in the iterative process, we use a more efficient combination equation shown in (16). To eliminate the limited-angle artifacts, we use the TV algorithm during the iteration.

The remainder of this paper is organized as follows. Related work and the traditional reconstruction model are
introduced in Section II, and then the proposed TVPI-G reconstruction model and the parameter selection are introduced. In Section III, numerical phantom experiments with different intensities of Poisson noise are implemented using TVPI-G algorithm and contrast experiments. Section IV presents the discussion and conclusions of this paper.

II. METHOD

The proposed TV-guided algorithm is mainly based on the TV algorithm and the guided image filter. In this section, the traditional reconstruction models including SART, TV and PICCS are introduced. Then, guided image filtering is illustrated. Finally, the TVPI-G algorithm is proposed and the algorithm flow is given at the end of this section.

A. SART ALGORITHM

The CT scanning process can be discretized into the following linear system:

$$ Af = p $$  \hspace{1cm} (1)

where $ A \in \mathbb{R}^{M \times N} $ is the projection matrix and $ A_{i,j} = a_{i,j} $ denotes the length of the $ i^{th} $ X-ray through the $ j^{th} $ image pixel. $ p = (p_1, p_2, \cdots, p_M)^T $ is the vectorized form of the projection data that is collected by the detector. $ f = (f_1, f_2, \cdots, f_N)^T $ is the vectorized form of the linear attenuation coefficient of the object. The goal of CT reconstruction is to construct the attenuation coefficient of $ f $ from the projection data $ p $ by solving equation (1).

The least-squares method (LSM) is an effective means to solve the CT reconstruction problem that is shown in equation (1). The LSM is as follows:

$$ f^* = \arg \min_f \| Af - p \|_2^2 $$  \hspace{1cm} (2)

In the CT reconstruction field, the matrix dimensions of $ A $ are too large to be solved by general algorithms. A widely used iterative algorithm is SART algorithm [8]. Wang et al. proved that SART algorithm converges to the optimal solution of least-squares. The iterative formula of SART is as follows:

$$ f_j^{(n+1)} = f_j^{(n)} + \lambda_n \frac{1}{\sum_{i \in \phi_j} a_{i,j}} \sum_{k=1}^N a_{i,k} \frac{(p_i - \sum_{k=1}^N f_k^{(n)} a_{i,k})}{\sum_{k=1}^N a_{i,k}^2}, $$  \hspace{1cm} (3)

where $ a_{i,j} = A_{i,j} $, $ 0 < \lambda_n < 2 $ is the relation parameter, and $ n $ is the iteration index. $ \phi_j $ represents the index set of x-rays under the $ i^{th} $ projection. $ J $ is the number of projection angles.

B. TV MODEL

Compared with analytic reconstruction algorithms such as FBP, SART reconstruction algorithm has better noise resistance. However, due to the insufficient projection data obtained from limited-angle CT and the high correlation between the rows of projection data, the above problem is an ill-posed inverse problem mathematically. Therefore, SART reconstruction results will have serious artifacts and noise.

According to the prior knowledge that images may be sparse in the transform domain, much prior knowledge of images is included in the optimization problem to improve the image quality. The generalized regularization model based on prior knowledge can be formulated as follows [9]:

$$ \min_f \psi(f) \text{ s.t. } \| Af - p \|_2^2 < \varepsilon $$  \hspace{1cm} (4)

where $ \psi(f) $ is the regular term and $ \varepsilon $ is the upper error of the data fidelity $ \| Af - p \|_2^2 $. In addition, equation (4) is equivalent to the following:

$$ \min_f \psi(f) + \tilde{\lambda} \| Af - p \|_2^2 $$  \hspace{1cm} (5)

The regularization parameter $ \tilde{\lambda} > 0 $ is used to balance the importance of $ \| Af - p \|_2^2 $ and $ \psi(f) $. The term $ \| Af - p \|_2^2 $ is the data fidelity term. It is used to ensure that the reconstructed image is consistent with the projected data. The image transform $ \psi(f) $ is used to control the sparsity of the solution $ f $. There are many image transforms, such as image total variation (TV) transformation [9], wavelet transform [10] and so on. The TV-based regularization algorithm is widely used in CT image reconstruction. The TV of the image $ f $ can be defined as follows.

$$ TV(f) = \| \nabla f \|_1 = \sum_{i=1}^N \sqrt{(\partial_x f_i)^2 + (\partial_y f_i)^2} $$  \hspace{1cm} (6)

The term $ \| \nabla f \|_1 $ is the $ L_1 $ norm of $ \nabla f $. The term $ \nabla $ is matrix gradient operation. $ \partial_x f_i $ and $ \partial_y f_i $ are the partial derivatives of the matrix $ f $ along horizontal and vertical dimensions. In addition, the TV-based regularization algorithm is summarized as following optimization problem:

$$ \min_f TV(f) \text{ s.t. } \| Af - p \|_2 < \varepsilon $$  \hspace{1cm} (7)

C. PICCS MODEL

According to the definition of image total variation, TV minimization forces the image gradient to be sparse. As a result, the reconstructed image may be piecewise constant, which can easily lead to block artifacts. For the limited-angle CT image reconstruction problem, in addition to block artifacts, there are limited-angle artifacts due to the serious shortage of projected data.

For incomplete projection data, in addition to using prior knowledge of images, high-quality prior images can be included in the reconstruction algorithm. Chen et al. proposed a PICCS algorithm combined with prior images [19]. The PICCS reconstruction model can be formulated as follows:

$$ \min_f [\beta TV(f) + (1 - \beta) TV(f - \bar{f}) \text{ s.t. } \| Af - p \|_2 < \varepsilon ] $$  \hspace{1cm} (8)

where $ \bar{f} $ is the target image and $ \bar{f} $ is the prior image. The regularization consists of two terms, $ TV(f) $ and $ TV(f - \bar{f}) $. $ TV(f) $ enables the total variation of the constructed image to be sparse and the $ TV(f - \bar{f}) $ term forces the reconstructed image...
to be structurally similar to the prior image; this ignores the differences between the reconstructed image and the prior image. The parameter \( \beta \in [0, 1] \) adjusts the weight of the \( TV(f) \) term and \( TV(f - \tilde{f}) \) term. Using the prior image, PICCS can effectively reduce artifacts and improve the image quality. However, the quality of the reconstructed image is closely related to the prior image. When the difference between the prior image and the image to be reconstructed is small, the value of the second term \( TV(f - \tilde{f}) \) is small. Therefore, the minimization of optimization problems (8) tends to reduce the \( TV(f) \) term, which is a smooth term of the reconstructed image. When the prior image contains obvious noise or the prior image is significantly different from the image to be reconstructed, the term \( TV(f - \tilde{f}) \) becomes relatively larger; then, the optimization problem is minimized, and the image to be reconstructed tends to transition to the prior image while smoothing the image. This leads to degradation of the reconstructed image quality.

**D. GUIDED IMAGE FILTERING**

Guided image filtering (GIF) has been widely used in noise reduction, artifact removal, image enhancement and image reconstruction [23]–[27]. In this paper, we used GIF to reduce noise, remove artifacts and force the intermediate result to be consistent with the projected data for limited-angle CT reconstruction. GIF is an adaptive weight filter that can smooth the image and maintain edges at the same time [23]. The key assumption of GIF is the local linear model between the guide image and the output image. In every local window, the output image is seen as a linear transformation of the guide image, as shown below:

\[
f_{\text{guide}} : f_{\text{out}} = a_k f_{\text{guide}}^k + b_k, \quad \forall j \in w_k
\]

where \( f_{\text{guide}}^k \) is the guide image, \( f_{\text{out}} \) is the output image, \( w_k \) is a square window that is centered around \( k \) and the radius of \( k \) is \( r \). The coefficients \( (a_k, b_k) \) are the optimal solution of the following function:

\[
\min_{a_k, b_k} \sum_{i \in w_k} \left[(a_k f_i^\text{guide} + b_k - f_i^\text{in})^2 + \alpha a_k^2\right]
\]

Solving the above optimization function, we can obtain \((a_k, b_k)\) as follows:

\[
a_k = \frac{1}{|w|} \sum_{j \in w_k} f_{\text{guide}}^j \frac{f_{\text{in}}^j - \eta_k \tilde{f}_k}{\sigma_k^2 + \alpha}
\]

\[
b_k = \tilde{f}_k - a_k \eta_k
\]

where \( f_{\text{in}}^j \) is the value of the input image \( f_{\text{in}} \) at the point \( j \), \( \tilde{f}_k = \frac{1}{|w|} \sum_{j \in w_k} f_{\text{in}}^j \) is the average of the input image in the square window \( w_k \). \( \eta_k, \sigma_k^2 \) are the mean and variance, respectively, of the guided image \( f_{\text{guide}}^k \) in the square window \( w_k \). \( w \) is the number of elements of \( w_k \), and \( r \) is the radius of \( w_k \). The larger \( r \) is, the stronger the ability of the GIF to remove noise and artifacts, but the image becomes smoother and blurry. \( \alpha \) is the regularization parameter penalizing a large \( a_k \). According to formula (11), the larger \( \alpha \) is, the smaller \( a_k \) will be. This will also result in a smoother and blurrier image.

The square windows are overlap, so the simple solution is to average the output value \( f_{\text{out}}^{\text{avg}} \).

\[
f_{\text{out}}^{\text{avg}} = \frac{1}{w} \sum_{k,j \in w_k} (\tilde{a}_j f_{\text{guide}}^j + b_j)
\]

where

\[
\tilde{a}_j = \frac{1}{w} \sum_{k,j \in w_k} a_k; \quad \tilde{b}_j = \frac{1}{w} \sum_{k,j \in w_k} b_k.
\]

**E. PROPOSED TVPI-G ALGORITHM**

From the above description, it can be seen that TV reconstruction algorithm can suppress noise but cannot effectively suppress limited-angle artifacts. The PICCS reconstruction algorithm can remove limited-angle artifacts by introducing prior images, but it requires the target images to be consistent with prior images. Ref. [26] developed a GIF reconstruction algorithm combined with TV algorithm, and ref. [27] proposed a SART reconstruction algorithm based on guided image filtering and prior image.

In this paper, we propose a guided image filter reconstruction based on TV and prior image (TVPI-G) for limited-angle CT. Each iteration consists of three substeps: the TV step, the guided image update step and the guided image filtering step.

To solve the TV minimization step, we first use the projections on a convex set (POCS) method to calculate the data fidelity term, which consists of two steps: a SART step in equation (3) and a nonnegative constraint step as follows:

\[
f = \max(f, 0)
\]

Then, we use the gradient descent algorithm to minimize the total variation (TV) of the image:

\[
f^{(n)} = f^{(n)} - \frac{\lambda}{f^{(n)}} \frac{\partial \|\nabla f^{(n)}\|_1}{f^{(n)}}
\]

where \( \lambda > 0 \) is the search step of the gradient direction.

For the second main step, ref. [26] used linear combinations to update the guided image, which introduces four additional parameters, and the selection of these parameters is a difficult problem. As seen from the formula, with the increase in the number of iteration phases, the weight of prior images is smaller, and the weight of TV results is larger. To reduce the number of parameters and reduce the influence of the prior image as early as possible in the iterative process, we use a more efficient combination; the image update formula is as follows:

\[
f_{\text{intermediate}} = \frac{f_{\text{prior}}^{\beta_1} + f_{\text{TV}}^{\beta_2} \times n^{\beta_2}}{n^{\beta_1} + n^{\beta_2}}
\]

where \( \beta_2 > \beta_1 > 0 \). In this paper, we manually set \( \beta_1 = 2, \beta_2 = 3 \). \( n \) is the number of iterations, and it is a positive integer starting at 1. For each iteration of the algorithm, it increases by 1. In this way, the number of parameters can be reduced and the weight of prior images can be reduced.
faster. When the prior image and the target image are inconsistent, the rapid reduction of the weight of prior image will accelerate the convergence rate of reconstructed image.

The third main step is the guidance image filtering mentioned above. The results of TV iteration are combined with prior images to form the input of the guided image filtering, and TV results are used as the guide image. The description of the proposed TVPI-G algorithm is shown in Table 1.

| TABLE 1. Main steps of TVPI-G. |
|--------------------------------|
| **Parameter initialization**: SART relaxation parameter \( \lambda = 1 \), TV step parameter \( \beta \), linear combination parameter \( \beta \), Input: projection data \( p \), max iteration number \( N_{\text{iter}} \). |
| **Step 1**: TV step: |
| SART: Update \( f^{n+1} \) using (3) |
| Nonnegative constraint: (14) |
| TV gradient descent: (15) |
| **Step 2**: Update the intermediate image using (16) |
| **Step 3**: Guided image filtering |
| Update the parameter \( (a_i, b_i) \) using (11-12) |
| Update the image using (13) |
| \( n \leftarrow n + 1 \) |
| **End** |
| **Output**: The reconstruction result \( f \) |

### III. EXPERIMENTS

In the experiments, a simulated NCAT phantom (Figure 1(a)) is used to prove the effectiveness of the proposed TVPI-G algorithm in limited-angle CT. To illustrate the superiority of proposed TVPI-G algorithm, the state-of-the-art CT reconstruction methods such as TV, PICCS, and SART-G are used in comparison experiments.

In the experience, a simulated digital NCAT phantom is used to evaluate the effectiveness of the TV, PICCS, SART-G and TVPI-G algorithms. In addition, these CT reconstruction algorithms are compared under different noise intensity and different projection views. The experimental equipment is an Intel processor (Inter(R) Xeon(R) E3-1231-v3) with 16G memory. The program is compiled using C++.

In this paper, the quantitative indexes RMSE, PSNR and structural similarity (SSIM) [29] are used to evaluate the quality of reconstructed images. They are defined as follows:

\[
\text{RMSE}(f, f^*) = \sqrt{\frac{\sum_{i,j} (f_{i,j} - f^*_{i,j})^2}{N_{\text{pixel}}}}
\]

\[
\text{PSNR}(f, f^*) = 10 \log_{10} \frac{(\max(f_{i,j}))^2}{\sum_{i,j} (f_{i,j} - f^*_{i,j})^2/N_{\text{pixel}}}
\]

\[
\text{SSIM}(f, f^*) = \frac{(2u_f u_{f^*} + C_1)(2\sigma_{f,f^*} + C_2)}{(u_f^2 + u_{f^*}^2 + C_1)(\sigma_f^2 + \sigma_{f^*}^2 + C_2)}
\]

where \( f \) and \( f^* \) are the reconstructed image and ground truth image. \( f_{i,j} \) and \( f^*_{i,j} \) are the pixel values of \( f \) and \( f^* \) respectively. \( N_{\text{pixel}} \) is the number pixels of the image \( f \). \( u_f \), \( u_{f^*} \) and \( \sigma_f^2 \), \( \sigma_{f^*}^2 \) are the mean value and the variance of \( f \) and \( f^* \). \( \sigma_{f,f^*} \) is the covariance of \( f \) and \( f^* \). \( C_1 \), \( C_2 \) are the parameters of SSIM.

As seen from formula (17-19), the closer the two images are, the smaller the RMSE value is. In addition, the larger the PSNR and the SSIM value are. In comparison experiments, the closer the two images are, the smaller the RMSE value is.

### A. NCAT EXPERIMENTS WITHOUT NOISE

First, a simulated digital NCAT phantom (Figure 1(a)) is used to prove the effectiveness of the TVPI-G algorithm. The prior image is another NCAT image shown in Figure 1(b). The sizes of the images are 256×256. Figure 1(c) is the difference image between the prior image and the target image. It mainly contains the edge position of the image. This means that the prior image is quite different from target image in edge position, but they are similar in structure and gray value for other positions. The regions ROI-1 and ROI-2 are the areas of interest, and they are magnified in the Figure 2-4.

Table 2 shows the geometric parameters of the simulated CT system for the simulation experiments.

| TABLE 2. The geometric parameters of the simulated CT system. |
|-------------------------------------------------------------|
| **System parameter**                                 | **Parameter value** |
| The distance from X-ray source to center              | 900.0 mm           |
| The distance from center to detector                  | 500.0 mm           |
| Interval angle between two views                      | 1°                 |
| The number of detector units                          | 372                |
| The length of the detector                            | 372 mm             |
| Image size                                            | 256 × 256          |
| Pixel size                                            | 1.0 × 1.0 mm²      |

First, the NCAT phantom in the limited projection angle of 60°, 90°, and 120° is used to evaluate the proposed TVPI-G in eliminating limited-angle artifacts. The geometric structure of the projection is fan-beam projection. The interval angle between each pair of views is 1°, so the 60°, 90° and 120° projection angles represent that the projection views of the CT are 60, 90 and 120, respectively. This means there are 22320, 33480 and 44640 underdetermined equations for the 60°, 90° and 120° limited-angle projection, respectively. The number of equations is less than the number of unknowns.
FIGURE 2. The limited-angle reconstruction results of the NCAT phantom on 60 views (the first row), 90 views (the second row) and 120 views (the third row) using TV, PICCS, SART-G and TVPI-G: (a)-(c) TV; (d)-(f) PICCS; (g)-(i) SART-G; (j)-(l) TVPI-G.

FIGURE 3. The zoomed image of Figure 2 in ROI-1.

The iteration number $N_{iter} = 300$, and TV step parameter $\lambda = 0.15$. The key parameters for TVPI-G are set as follows. The GIF parameters are $r = 2, \alpha = 0.00001$ for the 60° projection data, and $r = 1, \alpha = 0.0000025$ for the 90° and 120° projection data.

The reconstructed images are shown in Figure 2. Each row from top to bottom has 60°, 90° and 120° projection angles. Clearly, for each reconstruction algorithm, the quality of the reconstructed image improves with more views. When the projection angles are 120° (the first rows in Figure 2), these reconstruction algorithms can reconstruct high-quality images, and the entire edge of the image can be reconstructed. When the projection angles are 90° (the second rows in Figure 2), the results of TV and PICCS have a blurry edge in the upper left corner (the place indicated by the red arrow in (b) and (e)). The edges of SART-G and our method are almost clear. However, it can be seen that the grayscale is uneven in the middle of (h) in Figure 2, and the artifact appears in the place where the grayscale changes sharply. When the projection angles are 60°, the artifacts are more serious, when there is no projected data for the result of TV algorithm (Figure 2 (b)). And there are serious limited-angle artifacts at the edges (Figure 2 (a)). PICCS can suppress the limited-angle artifacts, but there are slight artifacts when there is no projected data (Figure 2 (d)). We can see that SART-G and the proposed TVPI-G can effectively suppress the shading artifacts when the number of views is 60. By comparing g and j in Figure 2, we can see at the red arrow that the result of SART-G algorithm is better than that of our proposed algorithm. In image reconstruction, we are more concerned with the internal details of the image, for example in lung CT we are more concerned with the internal pulmonary nodules. We can see from Figure 3, the proposed TVPI-G has a better reconstruction effect in ROI-1 (Figure 3 (g) and (j)) and ROI-2 (Figure 4 (g) and (j)). We can also find that SART-G results in ROI-1 are similar to the prior image in 60, 90, and 120 scan views. The results of the proposed TVPI-G are similar to the target image in ROI-1. By comparing the ROI-2 images (Figure 4), the images reconstructed by our proposed algorithm have accurate boundary and low noise.

FIGURE 4. The zoomed image of Figure 2 in ROI-2.

In Figure 5, the reconstruction error of our algorithm is the smallest. The gray values of Figure 5 (a) and (b) mainly appear in the direction where the projection angle is missing, and the reconstruction result of PICCS is somewhat better than that of TV. The residual graph of SART-G (Figure 5 (c)) is similar to Figure 1 (c), which indicates that the SART-G relies excessively on the information of the prior image. The
distribution of residuals in our method is similar to Figure 5 (a) and (b), but the gray value is obviously lower. This indicates that our reconstruction results are closer to the target image.

![Figure 5](image_url)

**FIGURE 5.** The first row are the reconstructed images on 90 views (Same as the second row in Figure 2). The second row are the difference images between the desired image and the reconstructed images on 90 views.

Table 3 shows the quantitative indexes for the results of the TV, PICCS, SART-G and proposed TVPI-G algorithm. It shows that with the increase in projection number, the RMSE index decreases, the PSNR and the SSIM indexes increase. The proposed TVPI-G obtains the best quantitative evaluation in all three evaluation indexes.

| Views | Method | PSNR | RMSE | SSIM  |
|-------|--------|------|------|-------|
| 60    | TV     | 19.1969 | 31.3039 | 0.7342 |
|       | PICCS  | 21.3774 | 24.194 | 0.8531 |
|       | SART-G | 29.9286 | 8.8416 | 0.9415 |
|       | TVPI-G | **31.8463** | **6.5877** | **0.9508** |
| 90    | TV     | 29.8724 | 9.0054 | 0.9570 |
|       | PICCS  | 34.5509 | 4.8617 | 0.9818 |
|       | SART-G | 32.0180 | 6.9665 | 0.9544 |
|       | TVPI-G | **37.7825** | **3.4000** | **0.9875** |
| 120   | TV     | 35.8024 | 4.2563 | 0.9887 |
|       | PICCS  | 40.0234 | 2.5769 | 0.9956 |
|       | SART-G | 33.5193 | 5.7390 | 0.9602 |
|       | TVPI-G | **41.7354** | **2.1473** | **0.9961** |

**TABLE 3.** Quantitative results for the NCAT image on 60, 90 and 120 views.

### B. NCAT EXPERIMENTS WITH POISSON NOISE

In practice, real CT projection data usually contain noise. The noise is mainly caused by electronic noise, X-ray scattering and so on. The main factor is the electronic noise that is distributed in Poisson form. In this study, we add Poisson noise to the simulated projection data. We set the Poisson noise intensity as $5 \times 10^3$, $1 \times 10^5$ and $1 \times 10^6$. These numbers represent the number of photons. The more photons there are, the less noise intensity there is in the projected data. We set the projection angles to 90°. For the above three different noise intensities, we set different parameters as follows: $r = 1$, $\alpha = 0.0001$; $r = 1$, $\alpha = 0.0001$ and $r = 2$, $\alpha = 0.001$.

In Figure 6, each row of images from top to bottom has a Poisson noise intensity of $1 \times 10^3$, $1 \times 10^5$ and $5 \times 10^5$. In addition, each column of images, from left to right, represents the reconstruction results of the TV, PICCS, SART-G and TVPI-G algorithms. Figure 6 shows that TV cannot suppress the noise, and the more serious the noise, the more blurred the edge of the reconstruction results. SART-G also cannot suppress the noise. PICCS and the proposed TVPI-G can suppress the Poisson noise when the noise level is relatively low. When the level of Poisson noise is $1 \times 10^5$, our proposed TVPI-G cannot suppress the Poisson noise, but our proposed algorithm is better than other algorithms in terms of the image structure (Figure 7 and 8). Figure 9 shows the difference in images between the desired image and the reconstructed images. (e) and (f) indicate that the artifact of TV and PICCS are more serious along the missing projection angle. (h) and (k) show that the SART-G result is similar to the reconstruction result of our method and that none of the results showed significant limited-angle artifacts, but the zoomed images in Figures 7 and 8 show that our results are more accurate for edge reconstruction.

![Figure 6](image_url)

**FIGURE 6.** Limited-angle CT reconstruction results from the NCAT image on 90 views. The rows, from top to bottom, have a Poisson noise level of $1 \times 10^4$, $1 \times 10^5$, $5 \times 10^5$, respectively. Each column is the result of a different reconstruction algorithm. (a)-(c): TV; (d)-(f): PICCS; (g)-(i): SART-G; (j)-(l): proposed TVPI-G.

To quantitatively analyze the results, Table 4 shows the quantitative assessment for the TV, PICCS, SART-G and proposed TVPI-G algorithms. It can be found in Table 5 that proposed TVPI-G algorithm performs best in quantitative evaluation at different noise levels. In addition, the results are much better than those of the SART-G and PICCS algorithms. The quantitative results of TV algorithm are the worst. This is consistent with the reconstruction results shown in Figures 6-8.
FIGURE 7. The magnified image of Figure 6 in ROI-1.

FIGURE 8. The magnified image of Figure 6 in ROI-2.

C. ADVANTAGES AND DISADVANTAGES OF EACH ALGORITHM
In the experiment, we used three algorithms as comparison algorithms, namely TV algorithm, PICCS algorithm and SART-G algorithm. Since most CT images can be approximated by the sharp constant function, the gradient transformation of images with the property of sharp constant is sparse. Based on such prior knowledge, TV algorithm can achieve good reconstruction quality for CT reconstruction. However, when the missing projection angle is too large or the projected data noise is large, TV reconstruction algorithm cannot achieve good reconstruction effect due to the TV algorithm only uses the prior knowledge of image gradient sparsity, as shown in Figure 2 (a) and (b) and Figure 6 (a) and (b). PICCS reconstruction algorithm introduces a prior image on the basis of TV reconstruction algorithm. Compared with TV, PICCS reconstruction results are better than TV reconstruction results when the missing projection angle is too large or the projected data is noisy. But due to the inconsistency of the prior image and target image, the reconstruction results may bring in some feature information of the prior image, as shown in Figure 2 (e) and figure 3 (e). The SART-G algorithm is effective for sparse angle CT image reconstruction, and this is mainly because it uses the guided image filtering and introduces prior images. But for limited-angle CT reconstruction, the reconstruction of SART results can lead to excessive limited-angle of artifacts, thus unable to get satisfactory results. The limited-angle artifacts is most evident in (g,h,i) of Figure 3 and Figure 7. The proposed algorithm is based on the improvement of the SART-G reconstruction algorithm. In the first step, the TV reconstruction algorithm is used for image reconstruction. TV algorithm can partially eliminate the limited-angle artifacts and reconstruct the relatively smooth image. Then the prior image and the fidelity term are used to modify the reconstruction results.

FIGURE 9. The first row are the reconstructed images on 90 views with $1 \times 10^5$ Poisson noise. The second row are the difference images between the desired image and the reconstructed images.

| Noise level | Method | PSNR | RMSE | SSIM |
|-------------|--------|------|------|------|
| $5 \times 10^5$ | TV | 28.0775 | 10.6798 | 0.9322 |
| | PICCS | 31.3984 | 7.0122 | 0.955 |
| | SART-G | 31.7991 | 7.1234 | 0.9392 |
| | TVPI-G | 35.1116 | 4.6369 | 0.9727 |
| | TV | 25.5768 | 14.5303 | 0.8815 |
| | PICCS | 29.1770 | 9.3742 | 0.9133 |
| | SART-G | 31.4659 | 7.7635 | 0.8949 |
| | TVPI-G | 33.3676 | 5.9212 | 0.9422 |
| | TV | 22.5540 | 20.296 | 0.7458 |
| | PICCS | 27.7772 | 10.7243 | 0.8121 |
| | SART-G | 29.7816 | 9.2862 | 0.8826 |
| | TVPI-G | 32.0450 | 8.5746 | 0.8301 |
so that better results can be obtained in the limited-angle reconstruction. Although the visual effect of our reconstruction results is not as good as that of SART-G when there are only 60 projection angles (g and j in Figure 2 and Figure 6), our reconstruction results are better in the enlarged image of the region of interest (g and j in Figure 3 and Figure 7). In addition, our algorithm results in quantitative index analysis are also good, as shown in Table 3 and Table 4.

### IV. DISCUSS AND CONCLUSION

We propose a guided image filter reconstruction based on TV and prior image (TVPI-G) for limited-angle CT reconstruction. Differing from classical TV algorithm and PICCS algorithm, the proposed method uses TV and prior images successively in each iteration phase. As illustrated in the experiments, the limited-angle artifacts and noise caused by incomplete projection data can be better eliminated by our proposed TVPI-G algorithm. In an NCAT phantom with different intensities of Poisson noise, the proposed TVPI-G algorithm outperforms better than other comparison algorithms, including TV, PICCS, and SART guided image filtering, in both qualitative and quantitative aspects. And compared with these algorithms, our proposed TVPI-G algorithm has greater computational validity.

There are several parameters that need to be determined in the TVPI-G algorithm, and the parameters were selected experimentally in this study. Future work will be devoted to the self-adaptation of these parameters, especially the selection of the combination parameters in formula (15) and the selection of parameters for guiding image filtering. The algorithm based on statistical learning has also been applied to CT reconstruction and achieved satisfactory reconstruction results, such as deep CNN reconstruction methods [30]–[32] and GAN based reconstruction methods [33], [34] and so on. The reconstruction algorithm based on deep learning is also the method we will research.

### APPENDIX

The following table 5 shows the explanations of abbreviations in the article.

| Abbreviation | Full spelling                      |
|--------------|-----------------------------------|
| CT           | Computed Tomography               |
| FBP          | Filtered BackProjection           |
| TV           | Total Variation                   |
| PICCS        | Prior image constrained compressed sensing |
| ART          | Algebraic reconstruction technique |
| SART         | Simultaneous algebra reconstruction technique |
| SART-G       | SART reconstruction algorithm based on guided image filtering |
| TVPI-G       | Guided image filter reconstruction based on TV and prior image |
| PSNR         | Peak signal to noise ratio        |
| SSIM         | Structural similarity             |

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| Abbreviation | Full spelling                      |
|--------------|-----------------------------------|
| CT           | Computed Tomography               |
| FBP          | Filtered BackProjection           |
| TV           | Total Variation                   |
| PICCS        | Prior image constrained compressed sensing |
| ART          | Algebraic reconstruction technique |
| SART         | Simultaneous algebra reconstruction technique |
| SART-G       | SART reconstruction algorithm based on guided image filtering |
| TVPI-G       | Guided image filter reconstruction based on TV and prior image |
| PSNR         | Peak signal to noise ratio        |
| SSIM         | Structural similarity             |
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