Knowledge Sharing in Coalitions*

Guifei Jiang\textsuperscript{12}, Dongmo Zhang\textsuperscript{1}, and Laurent Perrussel\textsuperscript{2}

\textsuperscript{1} AIRG, Western Sydney University, Penrith, Australia
\textsuperscript{2} IRIT, University of Toulouse 1, Toulouse, France

Abstract. The aim of this paper is to investigate the interplay between knowledge shared by a group of agents and its coalition ability. We characterize this relation in the standard context of imperfect information concurrent game. We assume that whenever a set of agents form a coalition to achieve a goal, they share their knowledge before acting. Based on this assumption, we propose new semantics for alternating-time temporal logic with imperfect information and perfect recall. It turns out that this semantics is sufficient to preserve all the desirable properties of coalition ability in traditional coalition logics. Meanwhile, we investigate how knowledge sharing within a group of agents contributes to its coalition ability through the interplay of epistemic and coalition modalities. This work provides a partial answer to the question: which kind of group knowledge is required for a group to achieve their goals in the context of imperfect information.

1 Introduction

Reasoning about coalitional abilities and strategic interactions is fundamental in analysis of multiagent systems (MAS). Among many others \cite{8,14,15,25,27}, Coalition Logic (CL) \cite{22} and Alternating-time Temporal Logic (ATL) \cite{2} are typical logical frameworks that allow to specify and reason about effects of coalitions \cite{16}. In a nutshell, these logics express coalition ability using a modality in the form, say $\langle\langle G \rangle\rangle \varphi$, to mean coalition $G$ (a set of agents) can achieve a property $\varphi$, regardless what the other agents do. ATL/CL assume that each agent in a multi-agent system has complete information about the system at all states (perfect information). Obviously this is not always true in the real world. Different agents might own different knowledge about their system. To model the systems in which agents have imperfect information, a few attempts have been made in the last few years by extending ATL with epistemic operators \cite{11,17,19,26,28,29}. With the extensions, agents’ abilities are associated with their knowledge. For instance, assuming a few agents are trying to open a safe, only the ones who know the code have the ability to open the safe.

One difficulty of ATL with imperfect information is how to model knowledge sharing among a coalition. In other words, if a group of agents form a coalition, whether their knowledge will be shared and be contributed to the group abilities \cite{13}? To the best of our knowledge, most of the existing epistemic ATL-style logics do not assume

\* This version corrected an error in its previous version published at AI’15. We specially thank Prof. Wojtek Jamroga for pointing out the error.
that members of a coalition share knowledge unless the information is general knowledge \cite{7,24} or common knowledge \cite{11} to a group or a system. However, most of the time when a set of agents form a coalition, their cooperation is not merely limited to acting together, but, more importantly, sharing their knowledge when acting. Safe opening is an example.

This chapter aims to take the challenge of dealing with knowledge sharing among coalitions. By a coalition we mean a set of agents that can not only act together to achieve a goal, but also share their knowledge when acting. We say that a coalition can ensure $\varphi$ if the agents in the coalition distributedly know that they can enforce $\varphi$. Based on this idea, we provide a new semantics for the coalition operator in ATL with imperfect information. It turns out that this semantics is sufficient to preserve desirable properties of coalition ability \cite{12,22}. More importantly, we investigate how knowledge sharing within a group of agents contributes to its coalitional ability through the interplay of distributed knowledge and coalition ability. Our contribution is twofold: firstly, this work can be seen as an attempt towards the difficulty: which kind of group knowledge is required for a group to achieve some goal in the context of imperfect information; secondly, these results show that the fixed-point characterizations of coalition operators which normally fail in the context of imperfect information \cite{3,4} can be partially recovered by the interplay of epistemic and coalitional operators.

The rest of this chapter is structured as follows. Section 2 introduces a motivating example for our new semantics. Section 3 provides the new semantics and investigates its properties. Section 4 explores the interplay of epistemic and coalitional operators. Section 5 discusses related work. Finally we conclude the paper with future work.

## 2 A Motivating Example

Let’s consider the following example which highlights our motivation to study coalition abilities under the assumption of knowledge sharing within coalitions.

**Example 1.** Figure 1 depicts a variant of the shell game \cite{7} with three players: the shuffler $s$, the guessers $g_1$ and $g_2$. Initially the shuffler places a ball in one of the two shells (the left (L) or the right (R)). The guesser $g_1$ can observe which action the shuffler does, while the other guesser $g_2$ can’t. A guesser or a coalition of two wins if she picks up the shell containing the ball. We assume that the guesser $g_1$ takes no action ($n$) and the guesser $g_2$ chooses the shell (the left ($l$) or the right ($r$)).

Clearly, $g_1$ knows the location of the ball but cannot choose. Instead $g_2$ does not know where the ball is, though he has right to choose the shell. It’s easy to see that neither $g_1$ nor $g_2$ can win this game individually. But if $g_1$ and $g_2$ form a coalition, it should follow that by sharing their knowledge they can cooperate to win. However, according to the existing semantics for ATL with imperfect information including the latest one, called truly perfect recall (also referred as no-forgetting semantics) \cite{7}, the coalition of $g_1$ and $g_2$ does not have such ability to win since they claim that coalition abilities require general knowledge or even common knowledge.

Moreover, these semantic variants fail to preserve the coalition monotonicity which is a desirable property for coalition ability in coalitional logics \cite{12,22}, that is, if a coalition can achieve some goal, then its superset can achieve this goal as well. For instance,
The tuple \((\alpha_1, \alpha_2, \alpha_3)\) represents an action profile, i.e., action \(\alpha_1\) of player \(s\), action \(\alpha_2\) of player \(g_1\), and action \(\alpha_3\) of player \(g_2\). The dotted line represents \(g_2\)'s indistinguishability relation: reflexive loops are omitted. State \(q_2\) is labelled with the proposition \(\text{win}\).

The model \(M_2\) in Figure 2 depicts a variant game of Example 1 by just switching the available actions of two guessers. The guesser \(g_1\) chooses the shell and the guesser \(g_2\) takes no action. Then it's clear that the guesser \(g_1\) can win no matter what the others do, as he sees the location of the ball and can pick up the right shell. It should follow that as a group, the guessers \(g_1\) and \(g_2\) can win this game. However, according to most existing semantics, though the guesser \(g_1\) has the ability to win, this ability no longer holds once he forms a coalition with guesser \(g_2\). These counterintuitive phenomena motivate our new semantics for ATL with imperfect information and perfect recall.

### 3 The Framework

In this section, we provide a new semantics for ATL with imperfect information and perfect recall based on the assumption of knowledge sharing in coalitions, and then investigate logical properties of ATL under this semantics.

#### 3.1 Syntax of ATL

Let \(\Phi\) be a countable set of atomic propositions and \(N\) be a finite nonempty set of agents. The language of ATL, denoted by \(L\), is defined by the following grammar:

\[
\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \langle\langle G \rangle\rangle \varphi \mid \langle\langle G \rangle\rangle \varphi U \varphi \mid \langle\langle G \rangle\rangle \varphi D \varphi
\]

where \(p \in \Phi\) and \(\emptyset \neq G \subseteq N\).

A coalition operator \(\langle\langle G \rangle\rangle\varphi\) intuitively expresses that the group \(G\) can cooperate to ensure that \(\varphi\). The temporal operator \(\square\) means “from now on (always)” and other temporal connectives in ATL are \(U\) (“until”) and \(\Box\) (“in the next state”). The dual operator \(\diamond\) of \(\Box\) (“either now or at some point in the future”) is defined as \(\diamond \varphi =_{\text{def}} \top U \varphi\). Moreover, the standard epistemic operators can be defined as follows:

\[
K_i \varphi =_{\text{def}} \langle\langle i \rangle\rangle \varphi U \varphi
\]

and

\[
D_G \varphi =_{\text{def}} \langle\langle G \rangle\rangle \varphi U \varphi.
\]

As we will show in the semantics, these abbreviations capture their standard intuitions, i.e., “\(K_i \varphi\)” says the agent \(i\) knows \(\varphi\), and “\(D_G \varphi\)” means it is distributed knowledge among the group \(G\) that \(\varphi\). The dual operators of \(K\) and \(D\) are defined as follows:

\[
\tilde{K}_i \varphi =_{\text{def}} \neg K_i \neg \varphi, \quad \tilde{D}_G \varphi =_{\text{def}} \neg D_G \neg \varphi.
\]
3.2 Semantics of ATL

The semantics is built upon the imperfect information concurrent game structure (iCGS) \cite{17, 24}.

**Definition 1.** An iCGS is a tuple $M = (N, \Phi, W, A, \pi, d, \delta, \{R_i\}_{i \in N})$ where

- $N = \{1, 2, \cdots, k\}$ is a nonempty finite set of players;
- $\Phi$ is a set of atomic propositions;
- $W$ is a nonempty finite set of states;
- $\pi: \Phi \mapsto \varphi(W)$ is a valuation function;
- $A$ is a nonempty finite set of actions;
- $d: N \times W \mapsto \varphi(A)$ is a mapping specifying nonempty sets of actions available to agents at each state. We will write $d_i(w)$ rather $d(i, w)$. The set of joint actions at $w$ for $N$ is denoted as $D(w) = d_1(w) \times \cdots \times d_k(w)$;
- $\delta: W \times D(W) \mapsto W$ is the transition function from every pair $(w \in W, \alpha \in D(w))$ to an outcome state $\delta(w, \alpha) \in W$;
- $R_i \subseteq W \times W$ is an equivalence relation for agent $i$ indicating the states that are indistinguishable from her viewpoint. For consistency, we assume that each agent knows which actions are available for her, i.e., $d_i(w) = d_i(w')$ whenever $wR_iw'$.

A path $\lambda$ is an infinite sequence of states and actions $w_0 \xrightarrow{\alpha_1} w_1 \xrightarrow{\alpha_2} w_2 \cdots$, where for each $j \geq 1$, $\alpha_j \in D(w_{j-1})$ and $\delta(w_{j-1}, \alpha_j) = w_j$. Any finite segment $w_k \xrightarrow{\alpha_k+1} w_{k+1} \xrightarrow{\alpha_{k+2}} \cdots \xrightarrow{\alpha_l} w_l$ of a path is called a history. The set of all histories for $M$ is denoted by $H$. We use $\lambda[j]$ to denote the $j$-th state on path $\lambda$, $\lambda[j, k]$ ($0 \leq j \leq k$) to denote the segment of $\lambda$ from the $j$-th state to the $k$-th state, and $\lambda[j, \infty]$ to denote the subpath of $\lambda$ starting from $j$. The length of history $h$, denoted by $|h|$, is defined as the number of actions.

The following definition specifies what a player with perfect reasoning capabilities can in principle know at a special stage of an imperfect information game.

**Definition 2.** Two histories $h = w_0 \xrightarrow{\alpha_1} w_1 \xrightarrow{\alpha_2} \cdots \xrightarrow{\alpha_m} w_m$ and $h' = w'_0 \xrightarrow{\alpha'_1} w'_1 \xrightarrow{\alpha'_2} \cdots \xrightarrow{\alpha'_n} w'_n$ are equivalent for agent $i \in N$, denoted by $h \approx_i h'$, iff

1. $m = n$,
2. $w_j R_i w'_j$ for any $0 \leq j \leq m$, and
3. $\alpha_k(i) = \alpha'_k(i)$ for any $1 \leq k \leq m$.

where $\alpha_k(i)$ is the $i$-th component of $\alpha_k$.

Intuitively, two histories are indistinguishable for an agent if (1) they have the same length, (2) their corresponding states are indistinguishable for this agent, and (3) the agent takes the same action at the each corresponding stage. Our notion of perfect recall is more like GDL perfect recall \cite{11, 24} as well as perfect recall in extensive games \cite{21} by requiring that an agent remember the past states as well as her own actions. This is stronger than the one in most epistemic ATL-style logics which often use the state-based equivalence without taking the actions into consideration, that is, a (truly) perfect
recall agent just remembers the past states. Our version has the advantage to deal with situations where different actions may have the same effects. For instance, consider two histories \( q_0 \xrightarrow{a} q_1 \) and \( q_0 \xrightarrow{b} q_1 \) with a single agent. According to the state-based equivalence, the agent cannot distinguish the two histories, but actually they are different from her view since she takes different actions at state \( q_0 \).

Note that the perfect recall agent does not observe or remember other agents’ actions.

In particular, we say two paths \( \lambda \) and \( \lambda' \) are equivalent up to stage \( j \geq 0 \) for agent \( i \in N \), denoted by \( \lambda \approx^j_i \lambda' \), iff \( \lambda[0,j] \approx^j_i \lambda'[0,j] \). As mentioned before, we assume that whenever a set of agents form a coalition to achieve their goals, the agents share their own knowledge before acting. To make this idea precise, we extend the indistinguishability relation \( \approx^j \) to groups as the intersection of all its members’ individual equivalence relation, i.e., \( \approx^j_G = \bigcap_{i \in G} \approx^j_i \). Let \( \approx^j_G (\lambda) \) denote the set of all paths that are indistinguishable from \( \lambda \) up to stage \( j \) for coalition \( G \), i.e., \( \approx^j_G (\lambda) = \{ \lambda' \mid \lambda \approx^j_G \lambda' \} \).

A strategy is a plan telling one agent what to do at each stage of a given game. With knowledge sharing among members of a coalition \( G \subseteq N \), we say a strategy of agent \( i \in G \) is uniform if the strategy specifies the same action for \( i \) at all histories which are indistinguishable for \( G \). Formally,

**Definition 3.** Given \( i \in G \subseteq N \), a uniform perfect recall strategy for agent \( i \) w.r.t \( G \) is a function \( f_i : H \rightarrow A \) such that for any history \( h, h' \in H \),

1. \( f_i(h) \in d_i(\text{last}(h)) \), and
2. if \( h \approx_G h' \), then \( f_i(h) = f_i(h') \).

where \( \text{last}(h) \) denotes the last state of \( h \).

Intuitively, a uniform perfect recall strategy for an agent in a group tells one of her legal actions to take at each history and specifies the same action for indistinguishable histories of the group. In particular, the standard notion of uniform strategies with respect to individual knowledge can be viewed as a special case. In the rest of paper, we simply call a uniform perfect recall strategy a strategy.

A joint strategy for group \( \emptyset \neq G \subseteq N \), denoted by \( F_G \), is a vector of its members’ individual strategies, i.e., \( \langle f_i \rangle_{i \in G} \). Function \( \mathcal{P}(h, f_i) \) returns the set of all paths that can occur when agent \( i \)’s strategy \( f_i \) executes after an initial history \( h \). Formally, \( \lambda \in \mathcal{P}(h, f_i) \) iff \( \lambda[0,|h|] = h \) and for any \( j \geq |h| \), \( f_i(\lambda[0,j]) = \theta_i(\lambda, j) \) where \( \theta_i(\lambda, j) \) is the action of agent \( i \) taken at stage \( j \) on path \( \lambda \). Obviously, the set of all paths complying with joint strategy \( F_G \) after \( h \) is defined as \( \mathcal{P}(h, F_G) = \bigcap_{i \in G} \mathcal{P}(h, f_i) \). It should be noted that if a group is characterized by full coordination both on the level of strategies and knowledge, we may view the group as a single agent whose abilities and knowledge are the sum of those of all the members [23].

We are now in the position to introduce the new semantics for ATL. Formulae are interpreted over triples consisting of a model, a path and an index which indicates the current stage on the path.

It is worth to mention that [23] proposed a way to embed actions to a state so that the state-based equivalence can achieve the same meaning.
Definition 4. Let $M$ be an iCGS. Given a path $\lambda$ of $M$ and a stage $j \in \mathbb{N}$ on $\lambda$, the satisfiability of a formula $\varphi$ wrt. $M$, $\lambda$ and $j$, denoted by $M, \lambda, j \models \varphi$, is defined as follows:

- $M, \lambda, j \models p$ iff $p \in \pi(\lambda[j])$
- $M, \lambda, j \models \neg \varphi$ iff $M, \lambda, j \not\models \varphi$
- $M, \lambda, j \models \varphi_1 \land \varphi_2$ iff $M, \lambda, j \models \varphi_1$ and $M, \lambda, j \models \varphi_2$
- $M, \lambda, j \models \langle\langle G \rangle\rangle \circ \varphi$ iff $\exists F_G \forall \lambda' \approx_{i_G}^j (\lambda) \forall \lambda'' \in \mathcal{P}(\lambda'[0, j], F_G)$ $M, \lambda', j + 1 \models \varphi$
- $M, \lambda, j \models \langle\langle G \rangle\rangle \triangleright \varphi$ iff $\exists F_G \forall \lambda' \approx_{i_G}^j (\lambda) \forall \lambda'' \in \mathcal{P}(\lambda'[0, j], F_G)$ $\forall k \geq j M, \lambda'' k \models \varphi$, and $\forall j \leq t < k, M, \lambda'' t \models \varphi_1$

The interpretation for the coalition operator $\langle\langle G \rangle\rangle \varphi$ captures its precise meaning that the coalition $G$ by sharing knowledge can cooperate to enforce that $\varphi$. Alternatively, the agents in $G$ distributedly know that they can enforce that $\varphi$. A formula $\varphi$ is valid in an iCGS $M$, written as $M \models \varphi$, if $M, \lambda, j \models \varphi$ for all paths $\lambda \in M$ and every stage $j$ on $\lambda$. A formula $\varphi$ is valid, denoted by $\models \varphi$, if it is valid in every iCGS $M$.

We first show that, as we claimed before, the abbreviations capture the intended meanings of the epistemic operators.

Proposition 1. Given an iCGS $M$, a path $\lambda$ of $M$ and a stage $j \in \mathbb{N}$ on $\lambda$,

- $M, \lambda, j \models K_i \varphi$ iff for all $\lambda' \approx_{i_G}^j \lambda$, $M, \lambda', j \models \varphi$.
- $M, \lambda, j \models D_G \varphi$ iff for all $\lambda' \approx_{i_G}^j (\lambda)$, $M, \lambda', j \models \varphi$.

Proof. We just prove the first clause, and the second one is proved in a similar way. It suffices to show that $M, \lambda, j \models \langle\langle i \rangle\rangle \varphi \triangleright \varphi$ iff for all $\lambda' \approx_{i_G}^j \lambda$, $M, \lambda', j \models \varphi$. The direction from the right to the left is straightforward according to the truth condition for $\triangleright$. We next show the other direction. Suppose $M, \lambda, j \models \langle\langle i \rangle\rangle \varphi \triangleright \varphi$ and for all $\lambda' \approx_{i_G}^j \lambda$, then there is $f_i$ such that for any $\lambda'' \in \mathcal{P}(\lambda'[0, j], f_i)$, $M, \lambda'', j \models \varphi$. And $\lambda'[0, j] = \lambda''[0, j]$, so $M, \lambda', j \models \varphi$.

We demonstrate with the variant shell game that the new semantics justifies our intuitions that the coalition of two guessers by sharing their knowledge can win the game.

Example 1 (continued.) Consider the model $M_1$ in Figure 7. It's easy to check that at the stage 1 on the left path $\lambda_1 := q_0 q_1 q_2 \cdots$, neither guesser $g_1$ nor guesser $g_2$ has the ability to win at the next stage, i.e., $M_1, \lambda_1, 1 \not\models \langle\langle g_1 \rangle\rangle \circ \text{win}$ and $M_1, \lambda_1, 1 \not\models \langle\langle g_2 \rangle\rangle \circ \text{win}$. Instead when $g_1$ and $g_2$ form a coalition, after sharing knowledge, the guesser $g_2$ is able to distinguish the history $q_0 q_1$ from the history $q_0 q_2$, then they can cooperate to win, i.e., $M_1, \lambda_1, 1 \models \langle\langle\langle g_1, g_2 \rangle\rangle \circ \text{win}$. For the coalition monotonicity property, consider the model $M_2$ in Figure 2. It's easy to check that at the stage 1 on the left path $\lambda_1 := q_0 q_1 q_2 \cdots$, guesser $g_1$ has the ability to win at the next stage by choosing the left shell, i.e., $M_2, \lambda_1, 1 \models \langle\langle g_1 \rangle\rangle \circ \text{win}$.
win. Moreover, when \( g_1 \) and \( g_2 \) form a coalition, then they can cooperate to win, i.e., \( M_2, \lambda_1, 1 \models \langle\{g_1, g_2\}\rangle \circ \text{win} \).

It should be noted that the reason why alternative semantics [7,11,24] fail to keep the coalition monotonicity property is that their interpretations of coalition operators \( \langle\langle G\rangle\rangle \varphi \) are given with respect to either the union of each member’s equivalence relation or its transitive reflexive closure. This means that the coalition ability implicitly requires general knowledge or common knowledge of the group, while neither of them is coalitionally monotonic. Instead distributed knowledge is sufficient for coalition ability to preserve the coalition monotonicity property.

3.3 Properties of the New Semantics

We first show that the new semantics satisfied the desirable properties of coalition ability in traditional coalitional logics [12,22].

Proposition 2. For any \( G, G_1, G_2 \subseteq N \) and any \( \varphi, \psi \in \mathcal{L} \),

1. \( \models \neg \langle \langle G \rangle \rangle \circ \bot \)
2. \( \models \langle \langle G \rangle \rangle \circ \top \)
3. \( \models \langle \langle G \rangle \rangle \circ (\varphi \land \psi) \rightarrow \langle \langle G \rangle \rangle \circ \varphi \)
4. \( \models \langle \langle G_1 \rangle \rangle \circ \varphi \rightarrow \langle \langle G_2 \rangle \rangle \circ \varphi \) where \( G_1 \subseteq G_2 \)
5. \( \models \langle \langle G_1 \rangle \rangle \circ \varphi \land \langle \langle G_2 \rangle \rangle \circ \psi \rightarrow \langle \langle G_1 \cup G_2 \rangle \rangle \circ (\varphi \land \psi) \) where \( G_1 \cap G_2 = \emptyset \)
6. \( \models \langle \langle G \rangle \rangle \circ \varphi \rightarrow \neg \langle \langle N \setminus G \rangle \rangle \circ \neg \varphi \)

Similarly for the \( \boxdot \) and \( \mathcal{U} \) operators.

Clause 1 says that no coalition \( G \) can enforce the falsity while 2 states every coalition \( G \) can enforce the truth. 3 and 4 specify the outcome-monotonicity and the coalition-monotonicity, respectively. 5 is the superadditivity property specifying disjoint coalitions can combine their strategies to achieve more. 6 is called \( G \)-regularity specifying that it is impossible for a coalition and its complementary set to enforce inconsistency.

The next proposition provides interesting validities about epistemic and coalitional operators.

Proposition 3. For any \( G \subseteq N \) and any \( \varphi, \psi \in \mathcal{L} \),

1. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ D_G \varphi \)
2. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ D_G \langle \langle G \rangle \rangle \circ \varphi \)
3. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ D_G \varphi \)
4. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ \varphi \)
5. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ \varphi \)
6. \( \models \langle \langle G \rangle \rangle \circ \varphi \leftrightarrow \langle \langle G \rangle \rangle \circ \varphi \)

Proof. We only give proof for the first two clauses and the proof for \( \boxdot, \mathcal{U} \) is similar.

[1] For every iCGS \( M \), every path \( \lambda \) of \( M \) and every stage \( j \in \mathbb{N} \) on \( \lambda \), assume \( M, \lambda, j \models \langle \langle G \rangle \rangle \circ \varphi \), then there is \( F_G = \{f_i\}_{i \in G} \) such that for all \( \lambda' \in \approx_G^j (\lambda) \), for all \( \lambda'' \in \mathcal{P}(F_G, \lambda' [0, j]) \), \( M, \lambda'', j + 1 \models \varphi \). We next show that \( F_G \) is the joint strategy to verify \( \langle \langle G \rangle \rangle \circ D_G \varphi \). Suppose for a contradiction that there is \( \lambda_1 \in \approx_G^j (\lambda) \), there
is $\lambda_2 \in \mathcal{P}(F_G, \lambda_1[0, j])$, there is $\lambda_3 \in \approx_G^{j+1} (\lambda_2)$ such that $M, \lambda_3, j + 1 \not\models \varphi$. Then $\lambda_3 \in \approx_G^j (\lambda)$ and $\theta_i(\lambda_3, j) = \theta_i(\lambda_2, j) = f_i(\lambda_2[0, j])$ for every $i \in G$, so there is some $\lambda^* \in \bigcup_{\lambda' \in \approx_G(\lambda)} \mathcal{P}(F_G, \lambda'[0, j])$ such that $\lambda^*[0, j + 1] = \lambda_3[0, j + 1]$. And by assumption we have $M, \lambda^*, j + 1 \models \varphi$. It follows that $M, \lambda_3, j + 1 \models \varphi$: contradiction. Thus, $M, \lambda, j \models \langle\langle G\rangle\rangle \circ D_G \varphi$. The other direction is straightforward.

For every iCGS $M$, every path $\lambda$ of $M$ and every stage $j \in \mathbb{N}$ on $\lambda$, assume $M, \lambda, j \models \langle\langle G\rangle\rangle \circ \varphi$, then there is $F_G = \langle f_i \rangle_{i \in G}$ such that for all $\lambda' \in \approx_G^j (\lambda)$, for all $\lambda'' \in \mathcal{P}(F_G, \lambda'[0, j])$, $M, \lambda'', j + 1 \models \varphi$. We next prove that for any $\lambda^* \in \approx_G (\lambda)$, $M, \lambda^*, j \models \langle\langle G\rangle\rangle \circ \varphi$. We consider the strategy $F_G$ and it is easy to check that for all $\lambda_1 \in \approx_G (\lambda^*)$, for all $\lambda_2 \in \mathcal{P}(F_G, \lambda_1[0, j])$, $M, \lambda_2, j + 1 \models \varphi$ as $\approx_G (\lambda^*) = \approx_G (\lambda)$. Thus, $M, \lambda, j \models D_G \langle\langle G\rangle\rangle \circ \varphi$. The other direction is straightforward.

Note that it is not generally the case that $M, \lambda_1, 1 \models \langle\langle G\rangle\rangle \varphi$ as $M, \lambda_1, 1 \not\models \langle\langle G\rangle\rangle \varphi$. Here is a counter-example. Consider the model $M_3$ in Figure 3 with two agents 1 and 2 and states $\{q_0, q_1, q_1', q_2, q_2'\}$, where $q_1 R_1 q_1'$, but not for 2, and all the other states can be distinguished by both agents. There are two propositions $p, q$, and $\pi(p) = \{q_1\}, \pi(q) = \{q_1, q_2\}$. The transitions are depicted in Figure 3. Consider the left path $\lambda_1 := q_0 q_1 q_2 \cdots$. It is easy to check that $M_3, \lambda_1, 1 \models \langle\langle 1\rangle\rangle p \varphi$, but $M_3, \lambda_1, 1 \not\models \langle\langle 1\rangle\rangle K_1 p \varphi$.

**Fig. 3.** the counter-model of $M_3$

**Fig. 4.** the counter-model of $M_4$

It follows from Proposition 3 that the distributed knowledge operator and the coalition operator are interchangeable w.r.t temporal operators $\circ$ and $\square$.

**Corollary 1.** For any $G \subseteq \mathbb{N}$ and any $\varphi \in \mathcal{L}$,

- $\models (\langle\langle G\rangle\rangle \circ D_G \varphi) \iff D_G (\langle\langle G\rangle\rangle \circ \varphi)$
- $\models (\langle\langle G\rangle\rangle \square D_G \varphi) \iff D_G (\langle\langle G\rangle\rangle \square \varphi)$

### 4 The Fixed-point Characterization

In this section, we will investigate the interplay between knowledge shared by a group of agents and its coalition ability in ATL with imperfect information and perfect recall. We first show that, similar to [3, 4], the standard fixed-point characterizations of coalition operators for ATL [12] fail under our new semantics.
Proposition 4. For any $G \subseteq N$ and any $\varphi, \psi \in \mathcal{L}$,

- $\not\models \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi \rightarrow \langle\langle G \rangle\rangle \Box \varphi$
- $\not\models \varphi \lor \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi \rightarrow \langle\langle G \rangle\rangle \Box \varphi$
- $\not\models \langle\langle G \rangle\rangle \Box \varphi \rightarrow \varphi \lor \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$
- $\not\models \psi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \phi \land \langle\langle G \rangle\rangle \Box \psi \rightarrow \langle\langle G \rangle\rangle \phi \land \langle\langle G \rangle\rangle \Box \psi$
- $\not\models \langle\langle G \rangle\rangle \phi \land \langle\langle G \rangle\rangle \Box \psi \rightarrow \psi \lor \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \phi \land \langle\langle G \rangle\rangle \Box \psi$

Here is a counter-example for the first one. Consider the model $M_4$ in Figure[4] which is obtained from $M_3$ by just changing the valuations. There is one proposition $p$, and $\pi(p) = \{q_1, q_2, q_3\}$. Consider $\varphi := p$ and the left path $\lambda_1 := q_0 q_1 q_2 \cdots$. Then it is easy to check that $M_4, \lambda_1, 1 \models p$ and $M_4, \lambda_1, 1 \models \langle\langle 1 \rangle\rangle \circ \langle\langle 1 \rangle\rangle \Box p$, but $M_4, \lambda_1, 1 \not\models \langle\langle 1 \rangle\rangle \Box p$. Thus, $M_4, \lambda_1, 1 \not\models \langle\langle 1 \rangle\rangle \circ \langle\langle 1 \rangle\rangle \Box p \rightarrow \langle\langle 1 \rangle\rangle \Box p$.

On the other hand, we have the following proposition showing that the converse direction for $\Box$ holds under the new semantics.

Proposition 5. For any $G \subseteq N$ and any $\varphi \in \mathcal{L}$, $\models \langle\langle G \rangle\rangle \Box \varphi \rightarrow \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$

Proof. For every iCGS $M$, every path $\lambda$ of $M$ and every stage $j \in \mathbb{N}$ on $\lambda$, assume $M, \lambda, t \models \langle\langle G \rangle\rangle \Box \varphi$, then there is $F_{G} = \langle f_{G} \rangle_{i \in G}$ such that for all $\lambda' \in \approx_{G}^{1} (\lambda)$, for all $\lambda'' \in \mathcal{P}(F_{G}, \lambda'[0, j])$, for all $k \geq j M, \lambda'' \circ k \models \varphi$. In particular, $\lambda \in \approx_{G}^{1} (\lambda)$, then for all $\lambda''' \in \mathcal{P}(F_{G}, \lambda[0, j])$, $M, \lambda'' \circ j \models \varphi$. And by $\lambda[0, j] = \lambda'[0, j]$, so $M, \lambda, j \models \varphi$.

We next prove that $M, \lambda, j \models \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$. It suffices to show that $F_{G}$ is just the joint strategy for the both coalition operators. That is, for all $\lambda_1 \in \approx_{G}^{1} (\lambda)$, for all $\lambda_2 \in \mathcal{P}(F_{G}, \lambda_1[0, j])$, for all $\lambda_3 \in \approx_{G}^{j+1} (\lambda_2)$, for all $\lambda_4 \in \mathcal{P}(F_{G}, \lambda_3[0, j + 1])$, we want to prove that for all $r \geq j + 1 M, \lambda_4, r \models \varphi$. As $\lambda_4 \in \mathcal{P}(F_{G}, \lambda_3[0, j + 1])$, then $\lambda_4[0, j + 1] = \lambda_3[0, j + 1]$, then $\lambda_4 \in \approx_{G}^{j+1} (\lambda_2)$, then $\lambda_4 \in \approx_{G}^{j+1} (\lambda_2)$ and $\lambda_4 \in \bigcup_{\lambda_1 \in \approx_{G}^{1} (\lambda)} \mathcal{P}(F_{G}, \lambda_1[0, j])$. So by the assumption we have that $M, \lambda_4, t \models \varphi$, so $M, \lambda, j \models \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$.

Thus, $M, \lambda, j \models \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$.

We now present the main result about the interactions of group knowledge and coalition ability for ATL with imperfect information and perfect recall. Recall that $\overline{K}$ and $\overline{D}$ be the dual operators of $K$ and $D$, respectively.

Theorem 1. For any $G \subseteq N$ and for any $\varphi, \psi \in \mathcal{L}$,

1. $\models \langle\langle G \rangle\rangle \Box \varphi \rightarrow D_{G} \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$
2. $\models \langle\langle G \rangle\rangle \Box \varphi \rightarrow D_{G} \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$
3. $\models D_{G} \varphi \lor \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi \rightarrow \langle\langle G \rangle\rangle \Box \varphi$
4. $\models \langle\langle G \rangle\rangle \varphi \land \langle\langle G \rangle\rangle \Box \psi \rightarrow \langle\langle G \rangle\rangle \varphi \land \langle\langle G \rangle\rangle \Box \psi$
5. $\models \langle\langle G \rangle\rangle \varphi \land \langle\langle G \rangle\rangle \Box \psi \rightarrow \langle\langle G \rangle\rangle \varphi \land \langle\langle G \rangle\rangle \Box \psi$

Proof. For every iCGS $M$, every path $\lambda$ of $M$ and every stage $j \in \mathbb{N}$ on $\lambda$,

[1] assume $M, \lambda, j \models D_{G} \varphi \land \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$, then $M, \lambda, j \models D_{G} \varphi$ and $M, \lambda, j \models \langle\langle G \rangle\rangle \circ \langle\langle G \rangle\rangle \Box \varphi$. By the latter, we get that there is $F_{G} = \langle f_{G} \rangle_{i \in G}$ such that for all $\lambda_1 \in \approx_{G}^{1} (\lambda)$, for all $\lambda_2 \in \mathcal{P}(F_{G}, \lambda_1[0, j])$, $M, \lambda_2, j + 1 \models \langle\langle G \rangle\rangle \Box \varphi$. It follows that there is $F_{G}^{x} = \langle f_{G}^{x} \rangle_{i \in G}$ where $x = \lambda_2[0, j + 1]$ such that for all $\lambda_3 \in \approx_{G}^{j+1} (\lambda_2)$, for
all $\lambda_t \in \mathcal{P}(F_G^{x_2}, \lambda^*_t[0, j + 1])$, for all $t \geq j + 1$, $M, \lambda_t, t \models \varphi$. We next construct a
new joint strategy $F_G = \langle f_t \rangle_{t \in G}$ based on $F_G^1$ and $F_G^{x_2}$. In order to define $F_G$, we first
need the following notation. Let

$$X = \{ \lambda'[0, j] \to w | \lambda' \in \mathcal{P}_G(\lambda), \forall i \in G, \alpha(i) = f^*_i(\lambda'[0, j]) \text{ and } w = \delta(\lambda'[j], \alpha) \}$$

Intuitively, $X$ is the set of all possible outcomes generated by the agents in $G$ taking
the next actions specified by $F_G^1$ from a history that is indistinguishable from history
$\lambda[0, j]$. We now define the strategy $F_G = \langle f_t \rangle_{t \in G}$ as follows: For all $h \in H(M)$ and
for all $i \in G$,

$$f_i(h) = \begin{cases} f^2_t(h) & \text{if } l \in X \text{ such that } l \text{ is a segment of } h \\ f^*_i(h) & \text{otherwise} \end{cases}$$

Note that this strategy is well defined, because if a history $h$ has a segment in $X$, there
is only one such segment due to the fact that all histories in $X$ has the same length
according to the definition for equivalence relation.

We next show that $F_G$ is just the joint strategy we need to verify $\langle \langle G \rangle \rangle \models \varphi$. That is,
for all $\lambda' \in \mathcal{P}_G(\lambda)$, for all $\lambda'' \in \mathcal{P}(F_G, \lambda'[0, j])$, we want to prove that for all $s \geq j,
M, \lambda'', s \models \varphi$. As for any $l \in X$, $|l| > |\lambda'[0, j]|$, then there is no $l \in X$ such that $l$
is a segment of $\lambda'[0, j]$; then by the definition of $F_G$, $F_G(\lambda'[0, j]) = F_G^1(\lambda'[0, j])$, so
$\lambda''[0, j + 1] \in X$ and $M, \lambda'', j + 1 \models \langle \langle G \rangle \rangle \models \varphi$. From the later, we get that for all
$\lambda^* \in \mathcal{P}(\lambda', \lambda''[0, j + 1])$, for all $\lambda' \in \mathcal{P}(F_G^1, \lambda^*[0, j + 1])$ where $y = \lambda''[0, j + 1]$, for all $t \geq
j + 1, M, \lambda', t \models \varphi$. Since $\lambda''[0, j + 1] \in X$, then $\lambda^*[0, j + 1] \in X$. And by the definition
of $F_G$ and the assumption $\lambda'' \in \mathcal{P}(F_G, \lambda'[0, j])$, we get $\lambda'' \in \mathcal{P}(F_G^1, \lambda^*[0, j + 1])$, so
for all $s \geq j + 1, M, \lambda'', s \models \varphi$. And by the assumption $M, \lambda, j \models D_G \varphi$, we get
$M, \lambda'', j \models \varphi$. So for all $s \geq j, M, \lambda'', s \models \varphi$, so $M, \lambda, j \models \langle \langle G \rangle \rangle \models \varphi$.

The other direction is proved by a similar method in Proposition 5.
Note that this strategy is well defined, because if a history \( h \) is only one such segment due to the fact that all histories in \( \lambda \) then there is no \( l \) \( \lambda \) for all \( i \) and for all \( F \) the next actions specified by \( \text{Intuitively}, X \) \( M,\lambda \) \( y \lambda \) \( F \) \( M,\lambda \) \( M,\lambda, j \). And by the assumption there is maintain by a similar method of clause 2. states that a coalition by sharing their knowledge can eventually achieve strategy for this coalition to possess this ability at the next stage. The second statement \( \phi \) \( \phi \) to possess this ability at the next stage, while the third statement provides a sufficient \( \phi \) the coalition considers it is possible that \( \phi \) \( \phi \) similar to above two. In particular, we have the following result for a single agent.

\[
X = \{ \lambda'[0, j] \overset{\delta}{\rightarrow} w \mid \lambda' \in \mathcal{P}_G(\lambda), \forall i \in G, \alpha(i) = f_1^i(\lambda'[0, j]) \text{ and } w = \delta(\lambda'[j], \alpha) \}
\]

Intuitively, \( X \) is the set of all possible outcomes generated by the agents in \( G \) taking the next actions specified by \( F_G \) from a history that is indistinguishable from from history \( \lambda[0, j] \). We can now define the strategy \( F_G = \{ f_i \}_{i \in G} \) as follows: For all \( h \in H(M) \) and for all \( i \in G \),

\[
f_i(h) = \begin{cases} f_i^2(h) & \text{if } \exists l \in X \text{ such that } l \text{ is a segment of } h \\ f_i^1(h) & \text{otherwise} \end{cases}
\]

Note that this strategy is well defined, because if a history \( h \) has a segment in \( X \), there is only one such segment due to the fact that all histories in \( X \) have the same length according to the definition for equivalence relation.

We next show that \( F_G \) is just the joint strategy we need to verify \( \langle (G) \rangle \varphi \cup \psi \). That is, for all \( \lambda' \in \mathcal{P}_G(\lambda) \), for all \( \lambda'' \in \mathcal{P}(F_G, \lambda[0, j]) \), we want to prove that there is \( r \geq j \), \( M, \lambda'', r \models \psi \) for all \( j < r < s < r, M, \lambda'', s \models \varphi \). As for any \( l \in X, |l| > |\lambda'[0, j]| \), then there is no \( l \in X \) such that \( l \) is a segment of \( \lambda'[0, j] \), then by the definition of \( F_G \),

\[
F_G(\lambda'[0, j]) = F_G(\lambda'[0, j]), \quad \text{so } \lambda''[0, j+1] \in X \text{ and } M, \lambda'', j+1 \models \langle (G) \rangle \varphi \cup \psi.
\]

From the later, we get that for all \( \lambda^* \in \approx_{F_G}^j(\lambda''), \text{ for all } \lambda' \in \mathcal{P}(F_G, \lambda''[0, j+1]) \) where \( y = \lambda''[0, j+1], \text{ there is } k \geq j + 1 \text{ such that } M, \lambda^*, k \models \psi \) and for all \( j + 1 < t < k, M, \lambda^*, t \models \varphi \). Since \( \lambda''[0, j+1] \in X \), then \( \lambda^*[0, j+1] \in X \). And by the definition of \( F_G \) and the assumption \( \lambda'' \in \mathcal{P}(F_G, \lambda'[0, j]) \), we get \( \lambda'' \in \mathcal{P}(F_G, \lambda'[0, j+1]) \), so there is \( r \geq j + 1 \) such that \( M, \lambda'', r \models \psi \) and for all \( j < r < s < r, M, \lambda'', s \models \varphi \). And by the assumption \( M, \lambda, j \models D_G \varphi \), we get \( M, \lambda', j \models \varphi \). So there is \( r \geq j \), \( M, \lambda'', r \models \psi \) and for all \( j \leq s < r, M, \lambda'', s \models \varphi \), so \( M, \lambda, j \models \langle (G) \rangle \varphi \cup \psi \).

Thus, in both cases \( M, \lambda, j \models \langle (G) \rangle \varphi \cup \psi \).

The clause 3 is proved by a similar method of clause 5 while the clause 4 is proved by a similar method of clause 2.

The first statement says that a coalition by sharing their knowledge can cooperate to maintain \( \varphi \) if the coalition distributedly knows \( \varphi \) at the current stage and there is a joint strategy for this coalition to possess this ability at the next stage. The second statement states that a coalition by sharing their knowledge can eventually achieve \( \varphi \) only if either the coalition considers it is possible that \( \varphi \) at the current stage or it has a joint strategy to possess this ability at the next stage, while the third statement provides a sufficient condition that a coalition by sharing their knowledge can eventually achieve \( \varphi \) if either it is distributed knowledge among the coalition that \( \varphi \) or the coalition can cooperate to achieve this ability at the next stage. The intuitions behind the last two statements are similar to above two. In particular, we have the following result for a single agent.

**Corollary 2.** For any \( i \in N \) and any \( \varphi, \psi \in \mathcal{L} \),

1. \( \models \langle \langle i \rangle \rangle \Box \varphi \leftrightarrow K_i \varphi \land \langle \langle i \rangle \rangle \bigcirc \langle \langle i \rangle \rangle \Box \varphi \)
2. \( \models \langle \langle i \rangle \rangle \Diamond \varphi \rightarrow \overline{K}_i \varphi \lor \langle \langle i \rangle \rangle \bigcirc \langle \langle i \rangle \rangle \Diamond \varphi \)
3. \( \models K_i \varphi \lor \langle \langle i \rangle \rangle \bigcirc \langle \langle i \rangle \rangle \Diamond \varphi \rightarrow \langle \langle i \rangle \rangle \Diamond \varphi \)
4. \( \models \langle \langle i \rangle \rangle \varphi \cup \psi \rightarrow \overline{K}_i \psi \lor (K_i \varphi \land \langle \langle i \rangle \rangle \bigcirc \langle \langle i \rangle \rangle \varphi \cup \psi) \)
5. \( \models K_i \psi \lor (K_i \varphi \land \langle \langle i \rangle \rangle \bigcirc \langle \langle i \rangle \rangle \varphi \cup \psi) \rightarrow \langle \langle i \rangle \rangle \varphi \cup \psi \)
5 Related Work

In recent years, there are many logical formalisms for reasoning about coalition abilities and strategic interactions in MAS. [10,13] provide a latest survey of this topic. In this following, we will review several works which are most related to ATL with imperfect information and perfect recall.

In the context of imperfect information, several semantic variants have been proposed for ATL based on different interpretations of agents’ ability [1,19,24,17]. In particular, [6,18] provide formal comparisons of validity sets for semantic variants of ATL. Similar to Bulling et al.’s no forgetting semantics [7], our semantics is also history-based w.r.t a path and an index on the path, but there are fundamental differences. First of all, we consider a finer notion of perfect recall by taking both past states and actions into considerations to deal with situations where different actions may have the same effects. Secondly, our notion of group uniform strategies is defined in terms of distributed knowledge instead of general knowledge as we assume that when a set of agents form a coalition, they are able to share their knowledge before cooperating to ensure a goal.

Several epistemic-ATL style logics have been proposed to investigate the interaction of group knowledge and coalition ability [6,11,24,?]. In particular, the most relevant works are [11,?]. Specifically, [?] presents a variant of ATL with knowledge, perfect recall and past. Different from our motivation, they use the distributed knowledge of coalitions so as to have a decidable model-checking problem. [11] proposes three types of coalition operators to specify different cases of how all agents in the coalition cooperate to enforce a goal. Among them, the communication strategy operator $\langle\langle G \rangle\rangle_c$ captures the intuition behind our coalition operator. Specifically, we have the following correspondence.

**Proposition 6.** Given an iCGS $M$, a path $\lambda$ of $M$ and a stage $j \in \mathbb{N}$ on $\lambda$, let $\varphi$ be any formula of the form $\Box \psi$, $\square \psi$ or $\psi_1 U \psi_2$, $M, \lambda, j \models \langle\langle G \rangle\rangle_c \varphi$ iff $M, \lambda[0, j] \models_{euATL} \langle\langle G \rangle\rangle_c \varphi$.

However, their work is different from ours in the following aspects: firstly, they propose two epistemic versions of ATL, namely uATL and euATL, to address the issue of uniformity of strategies in the combination of strategic and epistemic systems, while we introduce a new semantics without adding new operators to the language to explore the interplay of epistemic and coalitional operators; secondly, their results mainly focus on the relations and logical properties of three coalition ability operators, while we investigate fixed-pointed characterizations for the interplay of distributed knowledge and coalition operators which is not involved in [11]; thirdly, their meaning of coalition is more subtle than ours. Except the communication strategy operator, the comparison with the other two strategy operators is less straightforward since they are based on assumptions of coalitions without sharing knowledge. We hope to understand them better in the future.

Finally, it is also worth mentioning that [15] adopts a similar meaning of coalition so as to capture the notion of “knowing how to play”. Besides the different motivations, that work is based on STIT framework and just considers one-step uniform strategies without investigating the interplay of epistemic and coalitional operators.
6 Conclusion

In this paper, we have proposed new semantics for ATL with imperfect information and perfect recall to explore the interplay of the knowledge shared by a group of agents and its coalition abilities. Compared to existing alternative semantics, we have showed that our semantics can not only preserve the desirable properties of coalition ability in traditional coalitional logics, but also provide a finer notion of perfect recall requiring an agent remembers the past states as well as the past actions. More importantly, we have investigated the interplay of epistemic and coalitional operators.

In the future we intend to study the computational complexity of ATL with this new semantics, such as the model-checking problem. In this paper, we have investigated how knowledge sharing within a group of agents contributes to its coalitional ability. This work can be seen as an attempt towards the question: which kind of group knowledge is required for a group to achieve some goal in the context of imperfect information. We believe that it is an interesting question for further investigation by considering other cases such as group without knowledge sharing or with partial knowledge sharing [?].

Acknowledgments

We are grateful to Heng Zhang for his valuable help, and special thanks are due to three anonymous referees for their insightful comments. This research was partially supported by A key project of National Science of China titled with A study on dynamic logics for games (15AZX020).

References

1. Ågotnes, T., Goranko, V., Jamroga, W.: Alternating-time temporal logics with irrevocable strategies. In: TARK’07. pp. 15–24. ACM (2007).
2. Alur, R., Henzinger, T.A., Kupferman, O.: Alternating-time temporal logic. Journal of the ACM 49(5), 672–713 (2002).
3. Belardinelli, F.: Reasoning about knowledge and strategies: Epistemic strategy logic. In: SR’14. pp. 27–33 (2014).
4. Belardinelli, F.: A logic of knowledge and strategies with imperfect information. In: LAMAS’15 (2015).
5. Bolander, T., Bratüer, T.: Tableau-based decision procedures for hybrid logic. Journal of Logic and Computation 16(6), 737–763 (2006).
6. Bulling, N., Jamroga, W.: Comparing variants of strategic ability: how uncertainty and memory influence general properties of games. Autonomous agents and multi-agent systems 28(3), 474–518 (2014).
7. Bulling, N., Jamroga, W., Popovici, M.: Agents with truly perfect recall in alternating-time temporal logic. In: Proceedings of AAMAS’14. pp. 1561–1562 (2014).
8. Chatterjee, K., Henzinger, T.A., Piterman, N.: Strategy logic. Information and Computation 208(6), 677–693 (2010).
9. Diaconu, R., Dimu, C.: Model-checking alternating-time temporal logic with strategies based on common knowledge is undecidable. Applied Artificial Intelligence 26(4), 331–348 (2012).
10. van Ditmarsch, H., Halpern, J.Y., van der Hoek, W., Kooi, B.P.: Handbook of epistemic logic. College Publications (2015).
11. van Ditmarsch, H., Knight, S.: Partial information and uniform strategies. In: Computational Logic in Multi-Agent Systems, pp. 183–198. Springer International Publishing (2014).
12. Goranko, V., Van Drimmelen, G.: Complete axiomatization and decidability of alternating-time temporal logic. Theoretical Computer Science 353(1), 93–117 (2006).
13. Herzig, A.: Logics of knowledge and action: critical analysis and challenges. Autonomous Agents and Multi-Agent Systems pp. 1–35 (2014).
14. Herzig, A., Lorini, E.: A dynamic logic of agency I: STIT, capabilities and powers. Journal of Logic, Language and Information 19(1), 89–121 (2010).
15. Herzig, A., Troquard, N.: Knowing how to play: uniform choices in logics of agency. In: AAMAS’06. pp. 209–216. ACM (2006).
16. van der Hoek, W., Pauly, M.: Modal logic for games and information. Handbook of modal logic 3, 1077–1148 (2006).
17. van der Hoek, W., Wooldridge, M.: Cooperation, knowledge, and time: Alternating-time temporal epistemic logic and its applications. Studia Logica 75(1), 125–157 (2003).
18. Jamroga, W., Bulling, N.: Comparing variants of strategic ability. In: IJCAI’11. pp. 252–257 (2011).
19. Jamroga, W., van der Hoek, W.: Agents that know how to play. Fundamenta Informaticae 63(2), 185–219 (2004).
20. Jamroga, W.: Some remarks on alternating temporal epistemic logic. In: FAMAS’03. pp. 133–140. Citeseer (2003).
21. Kuhn, H.W.: Extensive games and the problem of information. Contributions to the Theory of Games 2(28), 193–216 (1953).
22. Pauly, M.: A modal logic for coalitional power in games. Journal of logic and computation 12(1), 149–166 (2002).
23. Ruan, J., Thielscher, M.: Strategic and epistemic reasoning for the game description language GDL-II. In: ECAI. pp. 696–701 (2012).
24. Schobbens, P.Y.: Alternating-time logic with imperfect recall. Electronic Notes in Theoretical Computer Science 85(2), 82–93 (2004).
25. Thielscher, M.: A general game description language for incomplete information games. In: AAAI 10. pp. 994–999 (2010).
26. Van Otterloo, S., Jonker, G.: On epistemic temporal strategic logic. Electronic Notes in Theoretical Computer Science 126, 77–92 (2005).
27. Zhang, D., Thielscher, M.: A logic for reasoning about game strategies. In: AAAI’15. pp. 1671–1677 (2015).