Entanglement engineering of one-photon wavepackets using a single-atom source

K.M. Gheri(1), C. Saavedra(1,2), P. Törmä(1), J. I. Cirac(1,3), and P. Zoller(1,3)

(1) Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25/2, A-6020 Innsbruck, Austria
(2) Departamento de Física, Universidad de Concepción, Casilla 4009, Concepción, Chile
(3) ITP, University of California, Santa Barbara, CA 93106-4030

(April 1, 2022)

We propose a cavity-QED scheme for the controlled generation of sequences of entangled single-photon wavepackets. A photon is created inside a cavity via an active medium, such as an atom, and decays into the continuum of radiation modes outside the cavity. Subsequent wavepackets generated in this way behave as independent logical qubits. This and the possibility of producing maximally entangled multi-qubit states suggest many applications in quantum communication.

Pacs number(s): 03.67.Hk, 03.67.-a

 Sources offering a great variety of entangled states are required for the implementation of many quantum communication and computation protocols [13]. With quantum communication in mind the choice of photons as qubits is especially appropriate, since they can be easily transferred over long distances. The standard source presently used in the lab is parametric downconversion in a crystal [14]. It is a reliable source of entangled twin-photons but the process is random and largely untailorable. Moreover, in practice its capability of generating entanglement is limited to states comprising only two photons. In this Letter we propose a scheme for the controlled generation of many entangled photonic qubits. Our source of entanglement produces a train of single-photon wavepackets which are well resolved in time. This permits us to regard them as individual qubits. In its most simple implementation the setup consists of a single multilevel atom inside an optical resonator [15]. The individual wavepackets are generated by applying an external laser pulse to the atom prepared in a superposition state of its internal states. The coupling of the atom to the resonator allows the transfer of a single photon to the resonator and therefrom via cavity decay to the continuum of radiation modes outside the resonator (possibly coupled to an optical fiber). An encoding of quantum information in the one-photon wavepackets could either take place by identifying two orthogonal polarization states of the single photon with logical “0” and “1”, or by regarding the absence of a photon as logical “0” while its presence would correspond to logical “1”.

Our scheme offers a twofold advantage over already existing sources of entangled single-photon wavepackets such as down-conversion. It provides excellent control over the instances in time when a qubit is created as well as over the spectral composition of the wavepacket. The qubits may thus be generated with a well defined "tact frequency" and pulse shape. Moreover, repeated coherent recycling of the state of the atom after the generation of a photon wave packet gives rise to higher order entanglement between subsequent photons [16]. In this regard our scheme generalizes and extends recent work on sources of single photon wavepackets, commonly referred to as photon guns [1] or turnstile devices [10] by allowing the generation of entangled multiphoton states. In particular, states such as the three-particle GHZ state, and more generally n-qubit maximally entangled states (MES) can be generated. Since the individual wavepackets do not overlap in time, each can be sent to a different receiver node using simple classical gating operations. Such high-order entangled qubit states have immediate application in quantum cryptography [11] and teleportation [12], as well as in tests of non-locality and multiparticle interference [13]. Because entangled states of more than two qubits can be generated in a straightforward manner, our scheme is especially useful for quantum communication between many parties [14].

Whilst the theory underlying our proposal can be formulated in a model-independent fashion it is more instructive to illustrate the basic ideas using a specific model: we consider a single atom or ion trapped inside a cavity [15]. For the atom we assume a double three-level Λ structure in the large detuning limit as depicted in Fig. 1. The levels |α⟩ (α = 0, 1) are coupled to the upper levels |r⟩ via classical fields Ω(t)e−i(ωrt+φ(t)), where ωr are the field center frequencies and the subscript refers to the two polarization states. The external control parameters are the real amplitudes Ω(t) and the phases φα(t). The levels |f⟩ are coupled to the upper levels by the cavity modes aα (common frequency ωc, orthogonal polarization), with coupling constants gα. The large detuning (δ) assumption allows us to adiabatically eliminate the upper atomic levels. We are left with two two-level systems describable by generalized spin operators σα = |α⟩⟨fα|). The center frequencies of the external laser pulses fulfill the Raman resonance condition. Note that any offsets can still be accommodated in the phases φα(t). The field outside the resonator is described by a continuum of harmonic oscillators with creation and annihilation operators b†(ω), b(ω), respectively. The reservoir modes satisfy standard bosonic commutation relations: [b(ω), b†(ν)] = δωδ(ω − ν). The coupling of the cavity and reservoir modes is assumed to be flat around the cavity resonance frequency and equal to √κc/π [15].
FIG. 1. A single atom with six internal states interacts with two cavity modes of orthogonal polarization $a_0, a_1$. In a Raman process (step 1) an initial superposition state of levels $|i_0\rangle$ and $|i_1\rangle$ is transformed into an entangled cavity-atom state. Due to cavity leakage the photon will leave the cavity and produce a photon wave packet in the continuum modes outside the resonator. In step 2 the atom is recycled back to $|i_0\rangle$ and $|i_1\rangle$. Between two photon generations levels $|i_0\rangle$ and $|i_1\rangle$ can be coupled (step 3) to tailor the outgoing state.

Hence the total system consists of three building blocks: the continuum outside the resonator, the cavity modes and the internal degrees of freedom of the atom inside the resonator. We switch to an interaction picture with respect to the free dynamics of the compound system. This eliminates the fast optical timescales from the dynamics and leaves us with a simpler Hamiltonian:

$$H(t) = \sum_{\alpha} \left( i \sqrt{\frac{\kappa_c}{\pi}} \int d\omega \left( a_\alpha b_\alpha^\dagger (\tilde{\omega}) e^{i\omega t} - H. c. \right) \right) + V(t),$$

$$V(t) = \sum_{\alpha} \left( \tilde{\Omega}_\alpha(t) \sigma_{i_{\alpha} i_{\alpha}} + |g_{\alpha}|^2 a_\alpha^\dagger a_\alpha \sigma_{f_{\alpha} f_{\alpha}} + i r_{\alpha}(t) \left( e^{-i\phi_{\alpha}(t)} a_\alpha^\dagger \sigma_{f_{\alpha} f_{\alpha}} - e^{i\phi_{\alpha}(t)} \sigma_{i_{\alpha} i_{\alpha}} a_\alpha \right) \right).$$

We have introduced the following abbreviations $\tilde{\Omega}_\alpha(t) = \Omega_\alpha(t)/2\sqrt{\delta}$, $g_{\alpha} = g_{\alpha}/\sqrt{\delta}$, $r_{\alpha}(t) = g_{\alpha}\tilde{\Omega}_\alpha(t)$ and $\tilde{\omega} \equiv \omega + \omega_c$. The time and intensity dependent terms in $V(t)$ correspond to ac-Stark shifts arising from the adiabatic elimination of the upper atomic levels.

We assume an atom initially prepared in a superposition state $|\psi(0)\rangle_\alpha = c_0|0_\alpha\rangle + c_1|1_\alpha\rangle$ which we wish to map onto a superposition of continuum (reservoir) excitations. The cavity and the reservoir modes are in their vacuum states. Since the dynamics contain no polarization mixing terms we may independently consider these degrees of freedom corresponding to a single index $\alpha$. We may thus work with a smaller system and intermittently drop the index $\alpha$. The final state of the total system can be obtained using the superposition principle and issuing each partial solution with the appropriate probability amplitude $c_\alpha$. As a starting point we will discuss briefly the generation of single photon wavepackets entangled with the atom. Since there can be at most a single excitation transferred to the continuum we are led to the following ansatz for the state of the total system:

$$|\psi(t)\rangle = |\varphi(t)\rangle_{ac}|0\rangle_t + \int d\omega |\varphi_\omega(t)\rangle_{ac}|1_\omega\rangle_t. \quad (1)$$

Here $|\varphi_\omega(t)\rangle_{ac}$ and $|\varphi(t)\rangle_{ac}$ denote atom-cavity states with and without the transfer of a photon having taken place into the reservoir mode with frequency $\tilde{\omega}$, respectively. Note that $|\varphi(t)\rangle_{ac}$ describes the atom-cavity state before the photon has been lost to the reservoir:

$$|\varphi(t)\rangle_{ac} = C_f(t)e^{-i\theta(t)}|\bar{i}\rangle|0\rangle_c + C_f(t)e^{-i|\bar{j}|^2t}|f\rangle|1\rangle_c,$$

where $\theta(t) = \int_0^t dt' \tilde{\Omega}^2(t')$. Applying $a$ to this state projects the coupled atom-cavity system into the state $|f\rangle|0\rangle_c$ which is not coupled by $V$. We thus find:

$$|\varphi_\omega(t)\rangle_{ac} = \sqrt{\frac{\kappa_c}{\pi}} \int_0^t dt' e^{i\omega t'} a|\varphi(t')\rangle_{ac}. \quad (2)$$

If we insert this expression into the equation for $|\varphi(t)\rangle_{ac}$ and perform the Markov approximation we arrive at a simple closed equation:

$$|\varphi(t)\rangle_{ac} = -(\kappa_c a^\dagger a + iV(t))|\varphi(t)\rangle_{ac}. \quad (3)$$

This now permits us to specify the sought evolution equations for the coefficients in the ansatz for $|\varphi(t)\rangle_{ac}$:

$$\dot{C}_f(t) = -\dot{r}(t)C_f(t) \exp(i\theta_c(t)), \quad C_f(t) = \int_0^t dt' r(t')C_f(t') \exp(-i\kappa_c t' - i\theta_c(t')),$$

where $\theta_c(t) = \theta(t) + \phi(t) - |\bar{g}|^2t$. In the limit of an overdamped cavity the integral will get a non-zero contribution only from those times $t'$ which are close to $t$ on the scale of the cavity lifetime $\kappa_c^{-1}$. To good approximation it thus holds that

$$C_f(t) \approx \frac{r(t)}{\kappa_c} e^{-\mu(t)-i\theta_c(t)}, \quad C_i(t) = e^{-\mu(t)},$$

where $\dot{\mu}(t) = r^2(t)/\kappa_c$, and $\mu(0) = 0$. The actual object of interest is the state of the continuum of radiation modes outside the cavity. We thus insert the above result for $C_f$ into Eq. (3) and find:

$$|\varphi_\omega(t)\rangle = \sqrt{\frac{\kappa_c}{\pi}} \int_0^t dt' e^{i\omega t'} \frac{r(t')}{\kappa_c} e^{-\mu(t')-i(\theta(t') + \phi(t'))}|f\rangle|0\rangle_c \equiv \mathcal{G}(\omega, t)\sigma_{f_{\bar{i}|\bar{i}}}|0\rangle_c. \quad (4)$$

Eq. (4) indicates a direct mapping of the initial atomic state to the final one accompanied by the creation of a wavepacket with spectral envelope $\mathcal{G}(\omega, t)$, cf. step 1 in Fig. 1. To make the scheme practical two constraints have to be kept in mind. First of all we would like to implement an efficient transfer of the photon to the continuum. Secondly, we are only interested in pulse sequences that terminate after a finite time $T \gg \kappa_c^{-1}$. This would be warranted if at the time $t = T$ the atom is with near certainty in its internal state $|f\rangle$, i.e., iff $\mu(T) \gg 1$. Recalling the definition of $\mu(t)$ this sets for any given pulse duration $T$ a lower bound for the minimum size of the pulse area of the applied laser field. Under this assumption the total system state for times $t > T$ is given by:
\[
|\psi(t)\rangle = \left(\sum_\alpha c_\alpha B_\alpha^\dagger(t) |f_\alpha\rangle\right)|0\rangle_\tau|0\rangle_c|0\rangle_c,
\]
where \(B^\dagger_\alpha(t_j, T)\) is the creation operator of a one-photon wavepacket with logical or polarization state \(\alpha\) within the time window from \(t_j\) to \(t_j + T\):
\[
B^\dagger_\alpha(t_j, T) = \int d\omega e^{i\omega t_j} G_\alpha(\omega, T)b^\dagger_\alpha(\omega).
\]

Note that the spectral envelope now carries a subscript as the system parameters need not be the same for each of the effective two-level systems we use to implement the mapping. In brief, we have shown how one can transform an initial atomic superposition state into an entangled atom-continuum state. Had we used only a single \(\Lambda\) system we would have recovered the photon gun, a tailorable simple single-photon source. The novel aspect here is the residual entanglement between the internal atomic state and the polarization state of the outgoing wavepacket. We may harness this to create a sequence of entangled one-photon wavepackets which are well resolved in time. Let us introduce the following abbreviation for a one-photon wavepacket with polarization \(\alpha\) that has been generated in the \(j\)-th generation sequence: \((\alpha)_j = B^\dagger_\alpha(t_{j-1}, T)|0\rangle_\tau\) (with \(t_0 = 0\)). The state after the first sequence in more compact form reads:
\[
|\psi(t)\rangle = (c_0|0\rangle|0\rangle_\tau + c_1|1\rangle|1\rangle_\tau)|0\rangle_c|0\rangle_c.
\]

Suppose we apply a further pulse sequence which recycles the atom back to its initial state, i.e., \(|f_\alpha\rangle \rightarrow |i_\alpha\rangle\). Then at a time \(t_1 > T\) we reinitiate the same pulse sequence that we have already used previously. It is plausible that the wavepackets already generated have in the meantime propagated far away from the cavity and thus cannot influence the renewed generation sequence. Going through the same procedure again we obtain for \(t > t_1 + T\):
\[
|\psi(t)\rangle = (c_0|0\rangle|0\rangle_\tau|0\rangle_\tau + c_1|1\rangle|1\rangle_\tau|f_1\rangle)|0\rangle_c|0\rangle_c.
\]

The residual entanglement with the generating system can eventually be broken up by making a measurement of the internal atomic state in an appropriate basis, e.g., \(|f_\alpha\rangle \pm |f_\beta\rangle\). For the state in Eq. (5) the resulting reservoir state would be one of two states \(c_0|0\rangle|0\rangle_\tau \pm c_1|1\rangle|1\rangle_\tau\). Repeating the generation process \(n\)-times followed by a final state measurement we produce an \(n\)-photon wave packet. Note that the description of the reservoir state in terms of products of one-photon wavepackets implies that the wavepackets can be regarded as independent quantum entities. It has to be emphasized that such a description is only possible because of the vanishing temporal overlap of the individual outgoing wavepackets. Actually components of the reservoir state are given by products of the operators \(B^\dagger_\alpha(t_j, T)\) applied to the multimode vacuum \(|0\rangle_c\). By construction operators belonging to different sequences, cf. Eq. (5), however, commute to good approximation:
\[
[B_\alpha(t_k, T), B^\dagger_\beta(t_j, T)] = \delta_{\alpha\beta} \int d\omega e^{i\omega (t_j - t_k)}|G_\alpha(\omega, T)|^2 \approx \delta_{\alpha\beta}\delta_{jk}.
\]

Formally, we may thus regard each creation operator as acting on its own vacuum state. Physically, this corresponds to the fact that each wavepacket is contained within its private time window of duration \(T\) or a box of length \(cT\) with no overlap between successively generated packets, cf. Fig. 3. We have numerically checked the factorization assumption for a two-photon wavepacket modeling the reservoir by a discrete set of 1024 “continuum”-modes (amounting to more than \(10^6\) reservoir states) embedded in a frequency window of width \(40\) \(\kappa_c\). We found that both the Markovian approximation used in Eq. (5) and the factorization assumption for the two-photon spectral density are excellent with relative errors of the order of \(10^{-3}\).

For any source of entanglement it is essential to fathom what the accessible class of states is. In general, a state in a basis spanned by \(n\)-qubits is defined by \(N = 2^{n+1} - 2\) independent coefficients. In our specific model the states can be tailored by coupling the levels \(|f_\alpha\rangle\) by a microwave/Raman pulse inbetween the qubit generation sequences. For example, this transforms Eq. (5) into \(|\psi\rangle = (c_0d_0|0\rangle|0\rangle_\tau + c_0d_1|0\rangle|f_1\rangle + c_1d_0|1\rangle|f_1\rangle - c_1d_1|1\rangle|f_0\rangle)|0\rangle_c|0\rangle_c\). The coefficients \(d_\alpha\) can be chosen at will. With each applied pulse two independent parameters are introduced. For \(n\) qubits we have thus \(2n\) free parameters at hand for the purpose of state engineering. Since this is much less than \(N\), only a restricted subclass of states can be created. However, we emphasize that the accessible class of states includes many useful and interesting states. For example MES such as the four Bell states, the GHZ-state \((|000\rangle + |111\rangle)/\sqrt{2}\) and its higher dimensional counterparts \(|\{s_1, s_2, ..., s_n\} = |1 - s_1, 1 - s_2, ..., 1 - s_n\rangle|/\sqrt{2}, s_i = 0, 1\) can easily be produced.

The maximum number \(n\) of entangled photon wavepackets (qubits) our scheme can generate is limited by decoherence. Relevant sources of decoherence are: (i) laser phase and amplitude fluctuations; (ii) spontaneous emission during the atomic transfer; (iii) absorption in the cavity mirrors; (iv) atomic motion. The chosen configuration miminizes these effects. Stabilization of laser phase fluctuations below 1kHz represents a technical challenge. In the present scheme, the state produced after each cycle only depends on the phase difference between the laser beams driving both transitions in Fig. 1. When these two laser beams are derived from the same source the fluctuations in the phase difference are effectively suppressed. Amplitude fluctuations cause a distortion of the pulse form and lead to incomplete population transfer. An estimate gives that \(n \ll I/\Delta I \sim 10^4\), where \(\Delta I/I\) are the relative intensity fluctuations. Spontaneous emission from the auxiliary levels \(|r\rangle\) at rate \(\Gamma\) is quenched by choosing a large detuning \(|\delta\rangle\) which leads to an effective decay rate \(\Gamma_{\text{eff}} = \Gamma\Omega^2/4\delta^2\) with
\( \Omega = \max(\Omega_a, \kappa_a) \). For a peak frequency \( \Omega_0 = 55 \text{ MHz} \) and a Gaussian pulse shape for the classical field \( \Omega(t) \), \( \delta = 1.5 \text{ GHz} \), \( g = 55 \text{ MHz} \), and \( \kappa_0 = 50 \text{ MHz one-photon pulse durations of around 10 cavity lifetimes are possible. Recycli} 

The error probability of spontaneous emission per cycle is \( \approx 10^{-3} \). Photon absorption in the cavity mirrors is an essential effect for high–Q optical cavities. In general, it leads to two types of errors: Photon absorption and concomitant destruction of the entanglement. These errors are evaded by postselection through discarding sequences with a number of detected photons smaller than \( n \). State distortion \( \epsilon \) can occur even in the absence of loss of a photon according to:

\[
|\Psi\rangle = \sum_{x \in \{0,1\}^n} q_x |x\rangle \rightarrow \sum_{x \in \{0,1\}^n} q_x e^{-\langle \epsilon_{x1} - \kappa_0 \rangle T n_1(x)} |x\rangle \tag{10}
\]

where the \( x \) are binary representations of different photon states, \( n_1(x) \) is the number of ones contained in \( x \), \( \kappa_{0,1} \) are the loss rate for modes 0 and 1, respectively. Errors as in Eq. (11) vanish for a cavity with equal absorption rates for both polarizations, i.e., \( \kappa_0 \approx \kappa_1 \). The optimal way to suppress fluctuations induced by the motion of the atom is to place the atom at an antinode of both the cavity modes and the laser beam (which have to be in standing wave configuration), where the effect of spatial variations is minimum and operate in the Lamb–Dicke regime [17]. Finally, there might be systematic and random errors in the adjustment of the laser pulses used in the recycling reinitialization.

![FIG. 2. Ensemble averaged fidelity as a function of the number of qubits \( n \). Curves (a),(c), and (e) assume \( \delta_m = 0 \) and \( \epsilon_m = 0.0125, 0.025, 0.1 \), respectively. Curves (b) and (d) are the same as (a) and (c) with \( \delta_m = 0.05 \).](image)

To assess these effects we assume the following imperfect mapping in each of the generation sequences: \( |f_{\alpha}\rangle \rightarrow A_{\alpha} |\alpha\rangle + B_{\alpha} |f_{\alpha}\rangle \), where \( A_{\alpha} = (1 - \epsilon_{\alpha}) \exp(i \delta_{\alpha}) \). In Fig. 2 we plot the fidelity \( F(n) \) of an \( n \)-qubit MES produced by a source which is ideal except that the magnitude and phase of \( \epsilon_{\alpha} \) are evenly distributed over a range of \([0, \epsilon_m] \) and \([-\epsilon_m, \epsilon_m] \), respectively, and the dephasing angle is evenly distributed over \([-\delta_m, \delta_m] \). Fig. 2 shows that the process is rather robust against global dephasing \( \delta_{\alpha} \) but that the correct timing of the \( \pi \)-pulses is critical [13]. From curve (a) we gather that for errors in the 2% range approximately 10 qubits can be created with a fidelity of 90%.

We have presented a CQED-based source for the controlled generation of entangled \( n \)-qubit states where the individual qubits are nonoverlapping one-photon wavepackets. Our model seems experimentally feasible with state-of-the-art equipment and could form the experimental basis for multi-party communication in future quantum networks. The theory presented can be easily adapted to other implementations (e.g. quantum dots or single atoms embedded in a host material [10]) which may emerge in the course of time as quantum systems with long coherence times.

Acknowledgements This work was supported by the Austrian Research Foundation under grant no. S06514-TEC and the European TMR network ERB-FMRX-CT96-0087. C.S. thanks Fundacion Andes for support.

\[ \text{[1]} \ D. \ P. \ DiVincenzo, \ Science 270, 255 \ (1995). \]
\[ \text{[2]} \ C. H. Bennett, \ Phys. \ Today 24 \ (October 1995) \text{ and references cited}; A. \ K. \ Ekert, \ Phys. \ Rev. \ Lett. 67, 661 \ (1991). \]
\[ \text{[3]} \ J. I. Cirac et al., \ Phys. \ Rev. \ Lett. 78, 3221 \ (1997); T. Pellizzari, ibid. 79, 5242 \ (1997). \]
\[ \text{[4]} \ D. Bouwmeester et al., Nature 390, 575 \ (1997). \]
\[ \text{[5]} \ D. Boschi et al., \ Phys. \ Rev. \ Lett. 80, 1121 \ (1998). \]
\[ \text{[6]} \ Q. A. Turchette et al., \ Phys. \ Rev. \ Lett. 75 4710 \ (1995). \]
\[ \text{[7]} \ G. M. Meyer, H.-J. Briegel, and H. Walther, Europhys. Lett. 35, 317 \ (1997). \]
\[ \text{[8]} \ E. Hagley et al., \ Phys. \ Rev. \ Lett. 79, 1 \ (1997); J. A. Bergou and M. Hillary, \ Phys. \ Rev. \ A 55 4585 \ (1997). \]
\[ \text{[9]} \ C. K. Law and H. J. Kimble, J. Mod. Opt. 44 \ (1997) 2067. \]
\[ \text{[10]} \ S. N. Molotkov and S. S. Nazin, \ JETP Lett. 63, 687 \ (1996); A. Imamoglu and Y. Yamamoto, \ Phys. \ Rev. \ Lett. 72, 210 \ (1994); F. De Martini et al., ibid 76, 900 \ (1996). \]
\[ \text{[11]} \ C. H. Bennett et al., \ Sci. \ Am. 267 (4), 50 \ (1992); W. Tittel et al., quant-ph/9707042. \]
\[ \text{[12]} \ C. H. Bennett et al., \ Phys. \ Rev. \ Lett. 70, 1895 \ (1993). \]
\[ \text{[13]} \ D. M. Greenberger et al., \ Am. \ J. \ Phys. 58, 1131 \ (1990); D. A. Rice et al., \ Phys. \ Lett. A 186, 21 \ (1994). \]
\[ \text{[14]} \ S. N. Molotkov and S. S. Nazin, \ JETP Lett. 62, 956 \ (1995). \]
\[ \text{[15]} \ C. W. Gardiner, \ Quantum \ Noiseurrer, Berlin, 1991). \]
\[ \text{[16]} \ T. Pellizzari et al, \ Phys. \ Rev. \ Lett. 75, 3788 \ (1995); H. Mabuchi and P. Zoller, ibid 76, 3108 \ (1996). \]
\[ \text{[17]} \ Here the size of the atomic wavepacket \( L \) is smaller than the optical wavelength \( \eta = L/\lambda \ll 1 \) and the population left behind in the wrong level is \( \eta^4 \), which puts a limit on the number of photon pulses \( n \ll 4/\eta^4 \). \]
\[ \text{[18]} \ Assuming Gaussian-distributed error \( |\epsilon_a| \) a simple analytical estimate yields the exponential decay in Fig.2. \]