Progressive deformation and annealing of quartz inclusion in porphyroblastic feldspar during synmetamorphic non-coaxial deformation

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A single crystalline inclusion in a single crystalline porphyroblast of metamorphic tectonites is deformed to an ellipsoid. The rate of shape change of ellipsoidal inclusion is controlled by the rounding process driven by reduction of the interfacial free energy and the deformation process of the ellipsoidal inclusion caused by deformation of host porphyroblast during synmetamorphic progressive deformation. In this paper, the model for shape change of inclusion mineral during contemporaneous deformation and annealing is proposed in the two dimensional case. It is concluded that the critical grain size of spherical inclusion is possibly defined in a function of strain rate in this model. The application to the Sambagawa and Ryoke metamorphic rocks clarifies that the strain rate of the former is much higher than that of the latter by two orders of magnitude.

Introduction

The distributions of flow stresses and strain rates characterize the regional metamorphic belts and ductile shear zones as well as the spatial distribution of metamorphic temperatures and pressures. Dislocation density, sub-grain size, and the grain size of quartz and calcite are potential piezometers of metamorphic process (Twiss, 1977; Mercier et al., 1977). However, post-tectonic recovery and post-metamorphic higher stress events make it difficult to directly infer the in-situ stresses operating during the synmetamorphic flow of rocks using the dislocation density and the grain size piezometers. Furthermore, the grain size piezometer can be applied to natural deformation only where dynamic recrystallization occurs.

The purpose of this paper is, therefore, to propose another type of potential strain rate meter of metamorphic tectonites using the shape change of inclusion minerals during annealing and/or deformation. This model describes a kinetics of microstructure development during synmetamorphic shear flow of rocks.

Model

Toriumi (1979) proposed a model for the rounding process of an initially polyhedral quartz inclusions driven by the reduction of interfacial free energy between porphyroblastic albite and quartz inclusion. This simple model is based on the nearly isotropic interfacial free energy between them. The assumption of nearly isotropic boundary energy seems to be satisfied for the interface between crystals having very different crystal structure. Vernon (1968) has already found that the boundary energy of quartz-albite is nearly isotropic judging from the similar intersect angles at the triple points of quartz-plagioclase, and others. The diffusion path for rounding process of
quartz in albite single crystal seems to be the grain boundary (Toriumi, 1979). The rounding is not the only process in the case of highly deformed tectonites, because of the concurrent deformation. So in this paper, I will propose a rate equation of the shape change in the contemporaneous deformation and annealing.

Let us consider the annealing of a single crystalline elliptical inclusion embedded in the single crystalline host mineral as shown in Fig. 1. If we assume the nearly isotropic interfacial free energy between inclusion and host as discussed in the following sections, the aspect ratio, \( L = \frac{b}{a} \), increases with time due to reduction of interfacial free energy although the shape change takes place concurrently. The annealing process is governed by grain boundary diffusion of the inclusion. In this case, the boundary between host and inclusion minerals can be freely accommodated by boundary migration of the host because of assumed faster diffusion rate of the host.

After a little advance of time, \( dt \), the long axial length, \( a \) and the short length, \( b \), of the elliptical inclusion changes to \( a + da \) and \( b + db \), respectively. If volume of the inclusion, \( V_0 \), remains constant during the process, we have

\[
a \cdot b = (a + da)(b + db) = V_0 / \pi
\]

Ignoring the second order small values, we obtain

\[
da/a = -db/b
\]

The curvatures at the crests are

\[
r_1 = b^2/a, \quad r_2 = a^2/b
\]

in which \( r_1 \) and \( r_2 \) are at the long and short axial crests, respectively. Flux of vacancy of the inclusion driven by gradient of interfacial tension, which is caused by difference of curvatures, is estimated as follows: the concentrations of vacancy of the inclusion at the crests of the long \((C_{v,a})\) and the short \((C_{v,b})\) axes are given by

\[
C_{v,a} = C_{v,0} \exp \left( \frac{\gamma_s Q}{r_1 kT} \right)
\]
\[
C_{v,b} = C_{v,0} \exp \left( \frac{\gamma_s Q}{r_2 kT} \right)
\]

where \( C_{v,0} \) is the vacancy concentration in the inclusion at the flat surface at temperature, \( T \) and \( \gamma_s \) and \( Q \) are the interfacial tension and volume of a vacancy, respectively, and \( k \) is the Boltzmann’s constant. Then, the mean gradient of the vacancy concentration between the long and short axes is expressed by

\[
\text{grad } C_v = (C_{v,a} - C_{v,b})/l
\]

in which \( l \) is the mean distance between the crests of the inclusion ellipse. As \( l \) is replaced by \( \sqrt{(1 + L^2)} \), where \( L = b/a \), consequently, this vacancy flux, \( J_v \), from the long axis crest to the short axis crest becomes
in which \( D_v \) is the diffusion constant of vacancy of the inclusion along the grain boundary between the inclusion and host. Using eqn. (2), we obtain

\[
-J_v = \frac{(D_v r_0 \Omega / kT)(a^3 - b^3)}{a^3 b^3 \sqrt{1 + L^2}} (6)
\]

where \( D_v \) is the boundary diffusion constant of the slowest atom of the inclusion and we use the relation of \( D_v = C_v,0 \cdot D_v \). The volume of material transported by interfacial diffusion as shown in Fig. 1 is given by

\[
dV = - \left( \frac{V_0}{4} \right) \left( \frac{db}{b} \right) (7)
\]

in which we use the following approximations:

\[
\arcsin \left( \frac{(1+2db/b)}{\sqrt{2}} \right) = \left( 1 + 2 \frac{db}{b} \right) \frac{\pi}{4}
\]

On the other hand, the volume decrease due to influx of vacancy governed by eqn. (6) is given by

\[
dV = -J_v \Omega S dt (8)
\]

in which \( S \) is the total surface through which vacancies flow in. Combining eqn. (6) with eqn. (7) and eqn. (8), we have

\[
db = \left( \frac{4J_v dt}{V_0} \right) \Omega S
\]

\[
= \left( \frac{4(D_v r_0 \Omega \delta / V_0 kT)(a^3 - b^3)}{a^3 b^3 \sqrt{1 + L^2}} \right) dt
\]

using \( S = \delta \)

in which \( \delta \) is the thickness of interface through which the vacancies flow in. Considering \( dL = d(b/a) = 2db/a \), equation (9) can be rewritten

\[
dL = K_1(1 - L^2) / \sqrt{(1 + L^2)} dt (10)
\]

with \( K_1 = 2D_v r_0 \Omega \delta / R_0^4 kT \)

and \( R_0 = \sqrt{ab} \).

Next, we must estimate the change of \( L \) due to an incremental shear of ellipse as shown in Fig. 1. The ellipse having the long axis to be \( a \) and the short axis to be \( b \) and the orientation angle, \( \theta \) (Fig. 1) can be written by the matrix formula of

\[
'x A x = 1
\]

with

\[
A = \begin{pmatrix} f, & h \\ h, & g \end{pmatrix}
\]

and

\[
2f = (1/a^2 + 1/b^2) + (1/a^2 - 1/b^2) \cos 2\theta \\
2g = (1/a^2 + 1/b^2) - (1/a^2 - 1/b^2) \cos 2\theta \\
2h = (1/a^2 - 1/b^2) \sin 2\theta
\]

in which \( f \) at the shoulder indicates the transposed matrix. This ellipse changes the shape by operation of deformation gradient tensor of simple shear, \( x' = Dx \), as follows (Shimamoto and Ikeda, 1976):

\[
'x (Dx) x = 'x A x = 1
\]

with

\[
D = \begin{pmatrix} 1, & -dy \\ 0, & 1 \end{pmatrix}
\]

and

\[
A' = \begin{pmatrix} f, & -fdy + h \\ -fdy + h, & g-2hd\gamma \end{pmatrix}
\]

in which \( dy \) is the increment of shear of inclusion. If the incremental shear represents \( dy dt \), change of the aspect ratio and the orientation angle of long axis of ellipse can be estimated from invariants of the matrix \( A' \). The orientation angle, \( \theta' \), of the ellipse after shear deformation can be written by

\[
\tan 2\theta' = 2(-fdy + h) / \left( f - g + 2hd\gamma \right) (11)
\]

in which \( f \), \( g \) and \( h \) are the components of \( A \) matrix. Equation (11) yields

\[
\theta' - \theta = \frac{dy}{2} \left( \frac{1 + L^2}{1 - L^2} \sin^{-1} 2\theta - \tan^{-1} 2\theta - \tan 2\theta \right) / \left( \tan^{-1} 2\theta + \tan 2\theta \right)
\]

(12)

Further, change of the short axial lengths can be estimated as follows;

\[
1/a'^2 + 1/b'^2 = f + g - 2hd\gamma (13)
\]

in which \( a' \) and \( b' \) are the long and short axial.
lengths of ellipse after an incremental deformation. Then, using \( db/b = -da/a \), the equation (13) becomes

\[
2db/b = -d\gamma \sin 2\theta \tag{14}
\]

Therefore, we obtain

\[
dL = -(\gamma L/2) \sin 2\theta \, dt \tag{15}
\]

The net change due to counteracting annealing and shear deformation as shown in Fig. 1 can be expressed by coupling the eqns. (10), (12) and (15) as follows

\[
\frac{dL}{d\gamma} = K \left(1 - L^3\right) \frac{1}{\sqrt{1 + L^2}} - \left(\frac{L}{2}\right) \sin 2\theta
\]

\[
\frac{d\theta}{d\gamma} = \left(\frac{1}{2}\right) \left(\frac{1}{1 + L^2} \frac{1}{\sin^2 2\theta} - \tan^{-1} 2\theta - \tan 2\theta\right) \left(\tan^{-1} 2\theta + \tan 2\theta\right)
\tag{16}
\]

with \( K = K_1 / \dot{\gamma} \).

Above set of non-linear differential equations governs the shape change due to simultaneous annealing and deformation of inclusion mineral during synmetamorphic progressive deformation.

As the set of eqns. (16) is non-linear, the numerical solutions for various initial aspect ratios and orientation angles were simulated by Runge-Kutta method using a microcomputer. The simulated results are shown in Figs. 2 to 5. It is obvious that equations have a set of stationary solutions at large finite strains for every set of the initial conditions. The locus of \((\theta, L)\) sets of the stationary solutions according to change of \(K\)-values is shown in Fig. 2. The finite strains at which the aspect ratio attains stationary values (see Fig. 3) decrease with increasing \(K\)-values as shown in Fig. 4. The numerical relations of \(\log K\) and the aspect ratio of the inclusion, \(L\) at the very large strain are shown in Fig. 5, indicating the strong sigmoidal curve. Considering that \(K\)-values strongly depend on the mean grain size of the inclusion, we may define the critical \(K_c\) equal to 10 when single crystalline inclusion becomes spherical grain. Therefore, we obtain the relations between strain rate and the critical grain size of the spherical inclusion as follows

\[
\dot{\gamma} = 0.4(\gamma_0\Delta \delta Db/kT)R_c^{-4} \quad \tag{17}
\]

in which \(R_c\) is the critical grain size of the spherical inclusion. The equation (17) is a potential strain rate meter assuming two dimensional flow and viscosity ratio between host and inclusion is about unity. If the viscosity ratio is not unity, the strain rate of host differs from that of inclusion mineral as indicated by Gay (1968). In this case, the relations obtained here are applicable for progressive deformation of inclusion minerals. Further, the above potential strain rate meter depending on \(R_c^{-4}\) is also correct in the three dimensional flow as shown by easy calculation.

**Application to synmetamorphic deformation**

In this study, samples of the Ryoke high temperature type and the Sambagawa high pressure type metamorphic rocks are chosen. The Sambagawa metamorphic belt runs along the Median Tectonic Line in SW Japan. The metamorphism occurred at late Jurassic to late Cretaceous times. The Sambagawa metamorphic belts consists of the pumpellyite-prehnite zone, pumpellyite-actinolite zone, glaucoephane-epidote zone, and epidote-hornblende zone (eg., Banno, 1986). The metamorphic history of the higher grade zones is characterized by the prograde stage where temperature rose under approximately constant pressure, and by the retrograde stage where pressure dropped keeping nearly constant temperature (Banno et al., 1986). Hornblende and glaucoephane formed at the prograde stage and they are commonly pulled apart and their interstices are filled with actinolite formed during retrograde metamorphism.

Synmetamorphic deformation of the Sam-
Fig. 2. Simulated trajectories of progressive shape change of elliptical mineral inclusion in simple shear deformation and annealing of host mineral at various K-values, showing the stationary set of orientation angle (θ) and aspect ratio (L) at given K-value for different initial shapes. The arrow indicates the trend of shape change of ellipse with progressive shear deformation.

bagawa belt is characterized by deformation path of progressive shear with strong constriction producing prolate strain ellipsoids in the lower grade zones (Toriumi and Noda, 1986). In higher grade zones, there seems to be some variations in shape of strain ellipsoids, judging from shapes of porphyroblastic albite and pressure shadows of garnet.

The Ryoke metamorphic belt runs along the Sambagawa metamorphic belt in the continental side of SW Japan, and is composed of the biotite zone and cordierite zone in the southern and northern areas, and of the sillimanite zone in the central area (Shimizu, 1984; Toriumi and Masui, 1986). The metamorphic P–T paths have not been clarified yet, because of the lack of the zonal structure of metamorphic minerals. Deformation of the Ryoke metamorphic rocks has been studied by Toriumi and Masui (1986) and is characterized by uniaxial extension strain ellipsoids and progressive nature of strain intensity due to increasing metamorphic temperature. Judging from asymmetric pressure shadow around garnet and sigmoidal shape of cordierite, the synmetamorphic deformation is non-coaxial with strong constriction similar to the Sambagawa deformation.

Samples studied here were collected from
the higher grade zones of the Sambagawa (81307, 81313, 81314, 81312; biotite zone of the Asemigawa River section of central Shikoku, T-24; garnet zone of the Kanto Mountains) and of the Ryoke (KM03; sillimanite zone of the Yanai district, SW Japan) metamorphic belts of Japan. The plane of thin section is perpendicular to schistosity or gneissosity plane and parallel to the mineral lineation. This plane is parallel to the maximum and the minimum elongation axes of strain ellipsoid of metamorphic rocks. Therefore, the shear direction is in the thin section plane. The aspect ratio and the orientation angle of the
Progressive deformation and annealing of quartz inclusion are measured under the thin section.

The albite porphyroblasts in the epidote-hornblende zone of the Sambagawa belt contain abundant quartz, epidote, garnet, rutile and amphibole inclusions. These inclusions are commonly single crystalline. The shapes of garnet and amphibole inclusions display often polyhedral outline with sharp corner, while quartz, epidote and rutile inclusions show the elongated outline with rounded corner and edges (Fig. 6). Porphyroblastic K-feldspar in the sillimanite zone pelitic gneisses contains commonly quartz and biotite inclusions. These inclusions are often rounded and slightly elongated. The orientations of ellipsoidal quartz inclusions are almost always parallel to the sigmoidal arrays of inclusions (Fig. 6c).

The aspect ratios and the mean grain size of the single crystalline quartz inclusions were measured in the samples listed above. The data are plotted in Fig. 7, suggesting that the aspect ratios decrease with increasing mean grain size in the single rock specimen. The critical mean grain sizes of the Sambagawa pelitic schists are approximated to be 2 µm. In contrast, the critical size of a Ryoke gneiss is estimated to be about 20 µm. It seems that the strain rate of synmetamorphic deformation of the Ryoke metamorphic rocks is much less than that of the Sambagawa schists, considering the difference of metamorphic temperatures between above two metamorphic rocks studied here.

The relation between the strain rate and temperature in eqn. (17) is strongly dependent on the activation energy for boundary diffusion of Si or oxygen of quartz along the albite-quartz and K-feldspar-quartz interface. Assuming \( Q_d = 30-60 \) kcal/mol of the activation energy (Toriumi, 1979), the ratio of strain rates of the Sambagawa (\( \dot{\gamma}_s \)) and Ryoke (\( \dot{\gamma}_r \)) metamorphic rocks yielded by difference of the critical mean grain sizes between them is estimated to be

\[
\frac{\dot{\gamma}_s}{\dot{\gamma}_r} = \left( \frac{R_{cs}}{R_{cr}} \right)^4 \exp \left( \frac{-Q_d}{2} \left( \frac{1}{T_r} - \frac{1}{T_s} \right) \right)
\]  

(18)

Fig. 6. Microstructures of the quartz inclusions in porphyroblastic albite of the Sambagawa metamorphic rocks (A), and in porphyroblastic K-feldspar of the Ryoke metamorphic rocks (B). C: sigmoidal array of mineral inclusions in a porphyroblastic albite in the Sambagawa metamorphic rocks. Abbreviations are as follows; qz, quartz; ab, albite; Kf, K-feldspar. Bar represents 10 µm in length.
in which $T_s$ and $T_r$ are deformation temperatures of the Sambagawa and the Ryoke metamorphic rocks studied here. Assuming $T_s=500^\circ$C (Enami, 1983), and $T_r=700^\circ$C (Shimizu, 1984), we have the above ratio of 0.006 to 0.3. It follows that the shear strain rates of the Sambagawa synmetamorphic deformation are much larger than those of the Ryoke deformation, although metamorphic temperatures of the Ryoke are much larger than those of the Sambagawa schists.

The Sambagawa and Ryoke metamorphic belts are typically paired belts in SW Japan and the strain patterns of the synmetamorphic deformation resemble with each other (Toriumi and Masui, 1986). Therefore, difference in strain rate between the Sambagawa high pressure type and the Ryoke high temperature type metamorphisms is due to difference of mechanical coupling within the accretionary wedges between two plates. Thus, a strong drag stress in the ocean side and weak drag one in the continental side of the accretionary wedge is suggested.

**Conclusion**

The isolated crystalline mineral inclusion in the single crystalline porphyroblastic host changes the shape due to the competing two rate processes: one is the rounding process due to reduction of nearly isotropic interfacial free energy, and the other is the deformation process. When the non-coaxial shear flow is assumed, the orientation and the aspect ratio of the elliptical inclusion are governed by nonlinear differential equations having the single characteristic coefficient. Numerical solutions indicate that the differential equations have a set of stationary solutions which are independent of the initial orientation and aspect ratio of the ellipse.

From application of the strain rate meter studied to the Sambagawa and the Ryoke metamorphic belts, it is inferred that the strain rates of the former are larger than those of the latter by one to two orders of magnitude.

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**References**

Banno, S. (1986), The high pressure metamorphic belts in Japan. *Geol. Soc. Amer., Memoir*, 164.

Banno, S., Sakai, C. and Higashino, T. (1986), Pressure-temperature trajectory of the Sambagawa metamorphism deduced from garnet zoning. *Lithos*, 19, 51–63.

Enami, M. (1983), Petrology of pelitic schists in the oligoclase-biotite zone of the Sanbagawa metamorphic terrain, Japan: phase equilibria in the highest grade zone of a high pressure
intermediate type of metamorphic belt. *J. Metamorphic Geol.*, 1, 141-161.

Gay, N.C. (1968), Pure shear and simple shear deformation of inhomogeneous viscous fluids. 1. Theory. *Tectonophysics*, 5, 211-234.

Mercier, J.C., Anderson, D.A. and Carter, N.L. (1977), Stress in lithosphere: inference from rheomorphic petrology. *J. Geophys. Res.*, 85, 6293-6303.

Shimamoto, T. and Ikeda, Y. (1976), A simple algebraic method for strain estimation from deformed ellipsoidal objects. 1. basic theory. *Tectonophysics*, 36, 315-337.

Shimizu, Y. (1984), Petrology of Ryoke metamorphic rocks in the Yanai-Iwakuni district, Japan. Ms thesis, Ehime University, 1984.

Toriumi, M. (1979), A mechanism of shape-transformation of quartz inclusions in albite of regional metamorphic rocks. *Lithos*, 12, 325-333.

Toriumi, M. (1985), Two types of ductile deformation/regional metamorphic belt. *Tectonophysics*, 113, 307-326.

Toriumi, M. and Masui, M. (1986), Strain patterns in paired metamorphic belts in Japan. *Geol. Soc. Amer., Memoir*, 164.

Toriumi, M. and Noda, H. (1986), The origin of strain patterns resulting from contemporaneous deformation and metamorphism in the Sanbagawa metamorphic belt. *J. Metamorphic Geol.*, 4, 409-420.

Twiss, R.J. (1977), Theory and applicability of a recrystallized grain size paleopiezometer. *Pure Appl. Geophys.*, 115, 227-244.

Vernon, R.H. (1968), Microstructures of high-grade metamorphic rocks at Broken Hill, Australia. *J. Petrol.*, 9, 1-22.

非共軸的変形における長石変晶中の石英の累進的変形と焼結

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変成岩に含まれている変晶中の単結晶包有物は一般に楕円体に変形する。包有結晶の形の変化速度は表面自由エネルギー減少に伴う球粒化と変晶の高温塑性変形による変形によって支配される。この論文では、変形と焼結プロセスが同時に起る場合の変化のモデルを論じる。球状化が包有結晶の粒径に強く依存するので、本研究においては、その変化の変化を定量することが出来る。変成岩変形の歪み速度と温度に依存する。三波川変成岩と隆起変成岩に応用した所、前者の変成作用のときの歪み速度は後者より2ケタ以上大きいことが推定される。