On Exact and Approximate Policies for Linear Tape Scheduling in Data Centers

Carlos Cardonha*
Department of Operations and Information Management, School of Business, University of Connecticut

Andre A. Cire
Dept. of Management, University of Toronto Scarborough & Rotman School of Management

Lucas C. Villa Real
IBM Research, Brazil

Abstract

This paper investigates scheduling policies for file retrieval in linear storage devices, such as magnetic tapes. Tapes are the technology of choice for long-term storage in data centers due to their low cost per capacity, reliability, and data security. While scheduling problems associated with data retrieval in tapes are classical, existing works focus on more straightforward heuristic approaches due to limited computational times imposed by standard tape specifications. Our first contribution is a theoretical investigation of three standard policies, presenting their worst-case performance and special cases of practical relevance for which they are optimal. Next, we show that the problem is polynomially solvable via two interleaved recursive models, albeit with high computational complexity. We leverage our previous results to develop two new scheduling policies with constant-ratio performance and low computational cost. Finally, we investigate properties associated with the online variant of the problem, presenting a new constant-factor competitive algorithm. Our numerical analysis on synthetic and real-world tapes from an industry partner provides insights into dataset configurations where each policy is more effective, which is of relevance to data center managers. In particular, our new best-performing policy is practical for large datasets and significantly improves upon standard algorithms in the area.

1 Introduction

Data analytics practices have increased the need for efficient, secure, and accessible data storage solutions. Recent studies indicate that the amount of data collected by organizations grows by up to 40% per year on average and resulted, e.g., in the conservative US $35 billion valuation of the data storage market in 2020 (Yezhkova et al. 2020). Accessing data effectively and securely is deemed a strategic priority to firms, with significant investments in both local and cloud-based storage approaches (Reinsel et al. 2018).

In modern data centers, storage solutions comprise a portfolio of random and sequential access technologies. Random access devices, such as non-volatile memories and solid-state drives, host data that is requested frequently, including live documents, working databases, or popular videos in streaming services. For otherwise cold-storage settings where data requests are few and far between, magnetic tapes are the method of choice in industry (Frank et al. 2012, Hope 2019). Examples include high-resolution digital content for media and entertainment platforms (Coughlin 2019); deep-ocean seismic data for the oil industry (Gaul et al. 2007); modeling,
simulation, and analysis of space missions (NASA 2021, Albani et al. 2020); and multi-year research data for projects such as the Large Hadron Collider (Cavalli et al. 2010).

Tapes have compelling cost, space, and security advantages over other storage technologies (Lantz 2018). A modern tape’s dollar per bit ratio can be up to 20 times lower than the alternatives, and almost 400 million compact discs would be necessary to store the same amount of information as a standard robotic tape library with 250 petabytes of data. When compared to hard disk drives, modern magnetic tapes are up to two times and 20% faster for writing and reading operations of large blocks of data, respectively (Fujifilm 2021). Furthermore, tapes have become especially relevant given the increase in ransomware attacks targeting data centers. In particular, tapes are protected by an “air-gap,” i.e., they are not connected directly to the internet and implement a write once, read many (WORM) behavior to prevent tampering by malware (Starr et al. 2020).

Tape-based data archiving systems, in practical settings, are operated in tandem with a hierarchy of faster storage devices. Data can be moved to cold storage to give room to new files in a random access device and, when needed, moved back to the faster device so that users can work on the data. Thus, efficient hierarchical storage management must observe quality-of-service and other user-centric metrics, such as reduced times between file request and data availability.

This paper investigates classical and new scheduling policies to retrieve files from a magnetic tape efficiently. Specifically, we consider a data center that must sequence a batch of read requests for files arranged in a single tape track. Such operations are critical components of tape technologies determined by the linear tape-open specification (LTO 2021), which is currently maintained by Hewlett Packard Enterprise, Quantum, and IBM, the latter one of our partners in this research study. While generally tapes may have other organizational structures, such as a serpentine arrangement, our single-track setting results in a linear positioning of the files, as common and desirable by data center managers to preserve file locality (Oracle 2011).

Current tape storage capacity reaches up to 20 terabytes (e.g., IBM TS1160), and a single track may store up to 400 GB of data, often containing hundreds to thousands of files in real-world settings. Tape file systems currently employ a default scheduling policy to service data retrieval requests (ISO/IEC-20919:2016 2016), often in the form of first-in first-out (FIFO) or greedy strategies as they are interpretable, simple to implement, and scale up to such tape sizes. However, data center managers can modify the tape scheduling policies to best accommodate different dataset profiles. Standard design guidelines impose that no more than a couple of seconds is allotted to scheduling algorithms, e.g., to enable users to run batch analytics directly on archived data (Kathpal and Yasa 2014). Thus, managers must balance the trade-off between the quality of the schedule, measured as a function of the time between when requests are received and when files first become available, and the computational complexity of finding such a schedule.

In this work, we provide a theoretical and numerical study of scheduling policies on single-track magnetic tapes and, more generally, linear storage devices. To the best of our knowledge, existing studies have been restricted to simple heuristics with no performance guarantees (Zhang et al. 2006, Schaeffer and Casanova 2011). Our primary focus is to understand worst-case scenarios of the existing scheduling approaches, settings where policies are optimal, and possible numerical improvements while preserving as much simplicity and interpretability as possible for managerial purposes. In particular, the methodologies investigated here aim at bringing to light theoretical and practical insights that can be leveraged by data center managers in their scheduling process.

The contributions of this paper are as follows. Focusing on the standard quality criterion in the area (minimizing total response times), we first investigate the performance of three low-complexity approximate policies and special cases of practical importance where they are optimal. Next, we close an open complexity question in the field and show that this problem is solvable in polynomial time, albeit with an impractical computational complexity. The existence of an efficient algorithm is unexpected, as the problem is closely related to other notoriously hard combinatorial optimization problems such as dial-a-ride and vehicle routing.
We build upon the ideas of the exact approach and present two new approximate algorithms, one of them a simple-to-implement enhancement of an existing policy based on necessary optimality conditions. Furthermore, we investigate online variants of the problem, showing that the standard scheduling policy (FIFO) is arbitrarily poor and presenting a new approach with a constant-competitive ratio. Finally, we provide a numerical study using artificial and real-world magnetic tapes, the latter containing satellite imagery dataset in agriculture applications. In particular, our best-proposed approximation provides near-optimal sequences and is of low computational time.

The paper is organized as follows. §2 provides a literature review of tape scheduling problems and related work. We formalize the problem and introduce notation in §3. Next, we investigate the standard approximate policies and theoretical performance results in §4. §5 describes an exact polynomial-time algorithm based on dynamic programming to establish the optimal policy. In §6, we present two new approximations and discuss their performance guarantees. We investigate the online variant of the problem in §7. Finally, §8 presents a numerical evaluation of the algorithms studied here. We conclude and describe future work in §9. The proofs for our structural results are included in the electronic appendix.

2 Related Work

Scheduling in sequential storage systems has been a classical research stream in both the operations and computer science literature. To the best of our knowledge, Day (1965) is the first to formalize a request problem over a multiple-file data storage system, proposing an integer programming (IP) formulation and heuristics that incorporate overlapping files. Model-based approaches such as IP, however, do not scale due to the strict time limits imposed by tape hardware.

Recent literature has hence focused on heuristic policies for tapes with varying tape organizational structure. Most notably, Hillyer and Silberschatz (1996a) and Sandstā and Midtstraum (1999) investigated the problem of estimating tape velocity in serpentine tapes, proposing simple scheduling heuristics that orders files based on their physical position in the tape. Hillyer and Silberschatz (1996b) developed a greedy heuristic based on an asymmetric traveling salesperson problem (TSP) reduction for a tertiary storage systems, simulating its performance against a pre-defined weave ordering for a serpentine tape. Similar sorting or greedy heuristics are found in other filesystems, e.g., as discussed in Zhang et al. (2006) and Schaeffer and Casanova (2011).

This paper extends Cardonha and Real (2016), which investigates heuristic strategies to minimize \\text{now time for read/write operations on both an online and an offline setting. Our paper, in turn, focuses on theoretical aspects concerning batch reading requests in cold-storage settings. That is, our reading operations are not interleaved with writing requests and individual batches are addressed separately, thus akin to an offline system.

Our problem is closely related to the traveling repairman problem (TRP), a pervasive model in scheduling that is also known as the traveling deliveryman problem or minimum latency problem (Fischetti et al. 1993, Coene et al. 2011, Pinedo 2016). The TRP asks for a Hamiltonian tour on a graph that minimizes the average vertex waiting time, which may incorporate both travel times between vertices, as well as individual vertex processing times. Of particular relevance to this work is the line-TRP, where vertices are distributed on a straight line. While the TRP is NP-Complete in general (Afrati et al. 1986, Simchi-Levi and Berman 1991), the line-TRP can be solved in polynomial time if processing times are zero (Bock (2015), see also Psaraftis et al. (1990)). Nonetheless, the complexity of the line-TRP with general processing times is still unknown. We discuss in §3.2 that our problem can be formulated as an asymmetric variant of the line-TRP, later exploiting this connection to establish optimal policies for cases of practical importance.

Another related problem is the dial-a-ride on the line (line-DR), where we wish to transport products between pairs of vertices in a graph using one or more capacitated vehicles. de Paepe et al. (2004) present an extensive classification of dial-a-ride problems, often associated with
distinct optimization criteria and constraints. In particular, minimizing the total completion times for the line-DR is NP-hard even if vehicles have capacity one. We also discuss the formal relationship between our problem and the line-TRP in §3.2.

Online variations of the problems above are also connected to our linear sequential tape problem. One example is the online TSP on the line, in which which vertices appear during the tour; see, e.g., Jaillet and Wagner (2006) and Jaillet and Wagner (2008). Bjelde et al. (2017) present a 1.64-competitive algorithm for the online TSP on the line, which is the best-possible competitive factor for the problem (Ausiello et al. 2001).

We also note that, in practice, recent enterprise drives support Recommended Access Order (RAO) tape recalls. Given a list of tape block addresses to visit, RAO outputs an ordered list of addresses that reduces the physical tape movement and wear (Moraru 2017). To date, the algorithms that enable RAO are not disclosed, and are supported by only a few specific combinations of tape drive and cartridges.

3 Problem Description and Notation

In this section, we formalize the linear tape scheduling problem in §3.1 and introduce its operational aspects and the standard quality metric. We also present its connections with classical problems in routing and scheduling in §3.2.

3.1 Tape Scheduling Description

We study the problem of a data centre that wishes to retrieve files stored in a magnetic tape. In our setting, all reading operations concern files located in the same data track, i.e., the tape only moves horizontally with its reading head fixed at a vertical position. Equivalently, the tape is viewed as a set of \( n \) files \( F = \{1, \ldots, n\} \) distributed sequentially and contiguously on a line discretized by bit units, as depicted by Figure 1.

The files in a tape have a left-to-right storage orientation. Specifically, each file \( i \in F \) begins in its left-bit position \( \ell_i \) and has a size of \( s_i \) bits. The right-bit position of file \( i \) is the first bit where the succeeding file starts, i.e., \( r_i = \ell_i + s_i \). We assume without loss of generality that files are in order, in that \( r_i = \ell_{i+1} \) for \( i = 1, \ldots, n - 1 \). Moreover, the first file begins at the initial position of the tape, \( \ell_1 = 0 \), and the last file ends at position \( L = r_n \), which coincides with the logical end of the tape. We refer to \( L \) as the tape length, noting that \( \sum_{i \in F} s_i = L \). For example, Figure 1 depicts a tape with five files, their corresponding sizes, and the right-bit position of file 2, which matches the left-bit position of file 3.

\[
\ell_1 = 0 \quad r_2 = \ell_3 = 4 \quad r_5 = L = 12
\]

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
\ell_1 = 0 & r_2 = \ell_3 = 4 & r_5 = L = 12 \\
\end{array}
\]

\[
\begin{array}{cccccc}
s_1 = 2 & s_2 = 2 & s_3 = 3 & s_4 = 4 & s_5 = 1 \\
\end{array}
\]

Figure 1: Example of a magnetic tape track with five files and length \( L = 12 \).

At the beginning of each planning task, the data centre receives a service request with a subset of \( m \) files that users wish to retrieve from the tape. Reading (or servicing) a file \( i \in F \) consists of positioning the tape head to the left position \( \ell_i \) of \( i \) and traversing its size \( s_i \) to collect the data, whereupon the tape head ends at position \( r_i \). The reading direction is relevant because data cannot be read when the tape track is traversed backwards, i.e., from right to left (ISO/IEC-20919:2016 2016). Thus, to read \( j \) immediately after servicing another file \( i \), the tape head must necessarily traverse a distance of \( d_{i,j} \equiv |\ell_j - r_i| \). Tape hardware also imposes that
most one file can be read at a time, as concurrent coalescing of multiple files is only managed at a software level. We also note that, to position the tape reading head at a particular bit location, the tape medium is either rewinded or fast-forwarded accordingly. We refer to this translational movement directly as “positioning the tape head” for simplicity.

A service request is submitted remotely by customers via a specialized software and typically includes from hundreds to a few thousand files. For notation purposes, we denote by $w_i = 1$ if file $i \in F$ is being requested, and $w_i = 0$ otherwise. We assume that the first file is always requested ($w_1 = 1$), adjusting the left-position of the tape appropriately if not the case.

Once a service request is received, the tape driver applies a scheduling policy to decide the sequence at which files will be read. Scheduling policies must observe quality-of-service considerations with respect to the time files are accessible to customers. In particular, each bit becomes available in the system immediately after it is read by the tape, thereby allowing customers to process files even if they are only partially retrieved. Thus, the standard quality criteria in the area (e.g., Hillyer and Silberschatz 1996a) is the sum of response times of requested files. The response time of a file $i$ is defined as the moment when the tape head starts to move from position $t_i$ to $r_i$ in order to read $i$. Formally, let $π = (π_1, \ldots, π_n)$ be a reading sequence of $F$, where $π_i \in F$ is the $t$-th file to be processed. The response time of $π_i$ is defined as

$$R_i(π) \equiv \sum_{r=1}^{t} (s_{π_{r-1}} + d_{π_{r-1}, π_r}) ν,$$

where $s_{π_0} = 0$, $r_{π_0}$ is the position at which the tape head begins, and $ν$ is the velocity of the tape in time units per bit, here assumed to be constant. In particular, $s_{π_{r-1}}$ and $d_{π_{r-1}, π_r}$ account for reading $π_{r-1}$ and repositioning the head to the left-bit position of $π_r$, respectively. We wish to find the sequence with the minimum total response $R^{*}$ incurred by requested files, i.e., a permutation $π^*$ of $F$ that solves model LTS, here referred to as linear tape scheduling (LTS):

$$R^* \equiv \min_π \sum_{i=1}^{n} w_{π_i} R_i(π). \quad \text{(LTS)}$$

We assume without loss of generality that $ν = 1$, i.e., distances and times are indistinguishable, since final results can be scaled accordingly for any other $ν$. We also note in passing that an optimal solution of LTS minimizes the average response time of files, which is a standard measure in scheduling (Pinedo 2016).

One operational challenge when processing linear tape file systems is that the tape head starts at the last bit position of the track, i.e., $r_{π_n} = L$. This is because tapes, being an append-base medium, always store their table of content (TOC) with the list of files, their sizes, and corresponding positions at the last bit of the customer-provided data, i.e., $L$. When loading a tape, the driver must always read the TOC first to know where to locate the remaining files, which positions the tape head at $L$. While that is an ideal location for new data writes, this operation penalizes an eventual demand for data reads. Practical policies must therefore find optimal tape repositionings and reads to ensure files are quickly delivered to customers.

**Example 1.** Figure 2 describes the head movement of the sequence $π = (5, 4, 1, 2, 3)$ for the tape in Figure 1. We assume that all files are being requested, i.e., $m = 5$. The reverse arcs (in red) represent the repositioning of the tape head to the beginning of a file, while the forward arcs (in black) denote file readings. Labels on top of the forward arcs provide the response times of the file that is being read. In particular, file 5 has a response time of 1, since the tape must first move to position $t_5 = 1$ prior to the reading operation. Next, file 4 has a response time of 7, since the tape first finishes reading file 5 at time 2, then requires an additional five time units to reposition the tape to the beginning of file 4. Notice also that no tape rewind movement is required when the tape reaches the left of file 1, i.e., there is a single forward movement to read files 1, 2, and 3. The total response time of $π$ is $1 + 7 + 21 + 23 + 25 = 67$.

Observe that a sequence $π$ in LTS prescribes an ordering over all files $F$ in the tape, including those that are not associated with any requests ($w_i = 0$). This specification does not impact
Figure 2: Tape head movement of the reading sequence \( \pi = (5, 4, 1, 2, 3) \) for the tape in Figure 1.

the final objective and simplifies the development of our structural results. Furthermore, it also represents the reading physical process more accurately, since the tape head is continuously retrieving bits when traversing any tape section with a left-to-right movement orientation. Thus, while only requested files are made available to customers, the tape head eventually reads all files \( F \) when being repositioned to \( L \) after reading file 1.

3.2 Related Problems

The linear tape scheduling problem shares connections with other fundamental optimization models from the scheduling and routing literature. In particular, notice that, for \( t > 1 \), we have \( R_t(\pi) = R_{t-1}(\pi) + \left(s_{\pi_{t-1}} + d_{\pi_{t-1}, \pi_t}\right) \) in (1). We can therefore expand each term in the summation of LTS to obtain the reformulation

\[
R^* = \min_{\pi} \sum_{i=1}^{n} \left(m - \sum_{r=1}^{t-1} w_{\pi_r}\right) \left(s_{\pi_{t-1}} + d_{\pi_{t-1}, \pi_t}\right),
\]

which is akin to a latency-type (or time-dependent) cost function in scheduling. That is, prior to servicing a file at the \( t \)-th position of the sequence, each bit the tape head traverses increases the total response time by the number of requests that are left to be served, i.e., \( m - \sum_{r=1}^{t-1} w_{\pi_r} \).

We describe below the relationship to two closely related problems, the traveling repairman problem and the dial-a-ride problem, which provide insight into the complexity analysis and approximation strategies that we develop in §4-§6 for the LTS.

Traveling Repairman Problem (TRP). Given a set of points \( V \cup \{0\} \) and symmetric distances \( D_{i,j} \) for any pair \( i, j \in V \cup \{0\} \), the TRP asks for a Hamiltonian tour starting at 0 that minimizes the sum of distances traversed from point 0 to each other point in the tour. The line-TRP, where vertices are distributed on a line, is polynomially solvable in the absence of side constraints, such as processing times, time windows, or deadlines (Afrati et al. 1986, Bock 2015).

We reduce the LTS to an asymmetric variant of the line-TRP as follows. The vertex 0 is represented by an artificial zero-sized file \( n+1 \) located at the end of the tape, i.e., \( \ell_{n+1} = r_{n+1} = L \), while the remaining points \( V = \{i \in F : w_i = 1\} \) are mapped to requested files. The distances between any \( i, j \in V \) are \( D_{i,j} = s_i + d_{i,j} \) and \( D_{j,i} = s_j + d_{j,i} \). That is, during a tour, we arrive on the left \( \ell_i \) of a file \( i \), and moving to the next file \( j \) requires us to first traverse the length \( s_i \) of \( i \).

It follows that LTS is polynomially solvable if file sizes are negligible (\( s_i = 0 \) for all \( i \in V \)), since each file would therefore be a point in the line. We generalize this connection in §4.2 and show that the same holds if all files in \( F \) have the same size, which is of practical importance. We
also leverage the objective function from the reduction above in our exact approach developed in §5.

Finally, we note that the TRP and the LTS are special cases of the time-dependent traveling 
salesperson problem (Abeledo et al. 2013), which allows for general distance metrics relative 
to the tour position and also includes a corresponding line version (Blum et al. 1994). The 
complexity of the line versions of such problems are unknown in general in the presence of 
processing times, the variant that is closest to our problem due to file sizes considered here.

Dial-a-ride problem (DARP). Given pickup-and-delivery requests \((o_1, q_1), (o_2, q_2), \ldots\), where each \(o_t\) and \(q_t\) is an origin point and a destination point, respectively, the DARP asks for 
vehicle routes to serve each origin-destination request while observing vehicle capacities and a 
given quality metric associated with the distance between points. Several DARP variations have 
been proposed in the literature; we refer to de Paepe et al. (2004) for an extensive complexity 
analysis.

The LTS can be reduced to a DARP where each \((o_t, q_t)\) is distributed on a line and mapped 
to a requested file \(i_2 f_i 2\): \(w_i = 1\), in that \(o_t = t_i\) and \(q_t = r_i\). The objective is to minimize 
the distances to reach each origin point. Moreover, we assume a single vehicle with unitary 
capacity, i.e., only one request can be served at a time. That is, when reaching an origin point 
\(o_t = t_i\), the vehicle must necessarily move to \(r_i\) prior to serving the next request.

In general, the line-variant of DARP with a single vehicle and capacity one is NP-Complete 
when the objective is to minimize the sum of distances to the destination as opposed to the 
origin. The LTS adds further structure in that \(o_t \leq q_t\) and pickup-and-delivery pairs do not 
intersect in space. It follows from §5 that the single-vehicle line-DARP is polynomially solvable 
in such conditions.

4 Approximate Policies and Performance

In a typical scenario, the number of requests \(m\) is large and commercial software severely limit 
the computational resources for tape scheduling policies. The time limit for solving LTS ranges, 
e.g., between 200 milliseconds and two seconds, depending on service-level agreements.

This section investigates the three fundamental policies in tape scheduling. They are simple 
to implement in tape drivers, interpretable, and of sufficiently low computational complexity to 
be practical, though their theoretical performance was yet unknown. We begin in §4.1 with a 
fundamental property of file sequences in tapes which form the basis of our analysis. In §4.2 we 
present the policies and evaluate their worst-case performance. Finally, we also identify cases 
where they are optimal, hence preferable than more sophisticated approaches.

4.1 Reading Stages

The linear structure of tapes reveals a partition of any sequence \(\pi\) in stages of directional 
movement. More specifically, note that the tape head will always move to the left-most position 
of the track to read the first file, and later performs rightward movements to read the remaining 
files (if any). We state this in Definition 1, which is key to drawing policy intuition.

**Definition 1** (Rewind and Forward Stages). The **rewind stage of a sequence** \(\pi\) is the set of all 
reading operations that are performed before the tape head reaches the bit 0 of the track. The **forward stage** is the set of reading operations performed after the rewind stage.

Given a sequence \(\pi\), we denote by \(R_\pi\) and \(F_\pi \equiv \mathcal{F} \setminus R_\pi\) the set of files read during the rewind and forward stages, respectively. The forward stage always includes the first file and it is a contiguous subsequence of \(\pi\). In the example of Figure 2, \(R_\pi = \{4, 5\}\) and \(F_\pi = \{1, 2, 3\}\).

We next formalize the (intuitive) result that, once the forward stage begins, the tape head 
needs only to perform a single left-to-right movement until the end of the track is reached. As a 
consequence, in any optimal solution, the bit 0 of the track is reached only once, so the rewind 
and the forward stages are well-defined.
Proposition 1. There exists an optimal sequence \( \pi^* \) to the LTS such that the forward-stage files are read in ascending index order, i.e., \( \pi_i^* < \pi_{i+1}^* \) for all \( \pi_i^*, \pi_{i+1}^* \in \Pi^*, i < n \).

Proposition 1 indicates that it suffices to determine the files to read and their ordering in the rewind stage, as the remaining sequence is fully determined once the forward stage begins. The key decision in policies to the LTS is hence to establish which files are serviced in the rewind stage, and their corresponding ordering.

4.2 Policies and Performance

We now discuss and analyze three low-complexity tape policies.

First-in, First-out (FIFO). The requested files arrive with an arbitrary ordering when first processed by the tape driver, as determined by the system in which the requests were submitted from. The FIFO policy, featured in commercial tape management software and drivers, sequences files according to such an input ordering. It is conceptually equivalent to a fully randomized policy, which can be arbitrarily poor if no structure on the request arrivals is assumed. We note that FIFO could still be applicable if a customer wishes to read files in a specific order.

First-File-First (FIFF). The FIFF policy reads files in ascending order of their indices, i.e., it generates a sequence \( \pi \) such that \( \pi_i < \pi_{i+1} \) for all \( i = 1, \ldots, n \). Thus, the policy delays all files to the forward stage so that the read direction changes only once. Such a policy is known to yield strong constant-factor approximations in similar line routing problems (e.g., Bhattacharya et al. 2008). Nonetheless, we show in Proposition 2 that the performance for the LTS degrades at least linearly with the number \( n \) of files in the tape.

Proposition 2. The FIFF policy is an \( \Omega(n) \)-approximation for the LTS.

We next show that FIFF is optimal for scenarios where all the files \( \mathcal{F} \) have the same size and are requested. This result is of theoretical and practical interest, as settings with these features are commonly observed in practice. For example, in tapes storing entertainment and video content, files may be associated with images or videos with the same resolutions or frame rates.

Proposition 3. The FIFF policy is optimal if all files are of the same size and are requested, i.e., \( n = m \) and \( s_i = s \) for all \( i \in \mathcal{F} \).

First-File-Last (FILA). The FILA policy is the opposite of FIFF, in the sense that it services all requested files (except the first) in the rewind stage. That is, it generates a sequence \( \pi \) such that the contiguous subsequence \( (\pi_1, \pi_2, \ldots, \pi_m) \) spans the \( m \) requested files and \( \pi_1 > \pi_2 > \cdots > \pi_m = 1 \), thereby reading files in descending order. Non-requested files, in turn, are read in the forward stage. While this imposes a change in directional movement per file, it resembles a classical myopic policy since files closest to the tape head at any instant are serviced first. We show in Proposition 4 that the policy improves upon FIFF and has a constant worst-case performance.

Proposition 4. The FILA policy is a 3-approximation for the LTS.

The orderings obtained by FILA do not necessarily dominate FIFF in terms of response time, as we depict below in Example 2. However, we can show a best-case result for FILA similar to Proposition 3, but requiring weaker assumptions. This policy is hence preferable for settings where files have equal sizes but few are typically requested.

Proposition 5. The FILA policy is optimal if all files in the tape are of the same size, i.e., \( s_i = s \) for all \( i \in \mathcal{F} \).
Example 2. Figure 3 depicts the tape head movement for the FILA ordering $\pi = (5, 4, 3, 2, 1)$ for the tape in Example 1, where all files are requested. The total response time is $1+7+21+29+35 = 93$, which is higher than the one obtained by the ordering in Figure 2 (67). The FIFF ordering $\pi = (1, 2, 3, 4, 5)$ yields a response time of 12, 14, 18, 21, and 24 for the files 1, 2, 3, 4, and 5, respectively, and a total response time of 89.

5 A Polynomial-time Exact Approach

In this section, we present a dynamic programming (DP) algorithm to solve the LTS in polynomial time in the number of files $n$. The approach exploits the linear tape structure and provides insights to special cases of related scheduling problems discussed in §3.2. While the algorithm is technically involved and of high computational complexity for the application — $O(n^4)$ —, it shows that the problem is tractable and can be useful, e.g., as a benchmark to approximate solutions (see §8).

For ease of notation and without loss of generality, we incorporate an artificial, zero-sized file $n+1$ located on the right-most position of the tape with $s_{n+1} = w_{n+1} = 0$ and $\ell_{n+1} = r_{n+1} = L$. We assume further that this file is always the last to be read in all considered sequences, so that the forward stage includes at least files 1 and $n+1$. We note that this assumption is consistent with practice, since the tape head must end at $L$ to properly eject the cartridge from the tape drive.

The section first investigates the structure of the optimal solution in §5.1. Our dynamic programming model is then developed in §5.2.

5.1 Optimality Structure

Our exact approach is a dynamic program that leverages a decomposable structure that the rewind stage exhibits in any optimal sequence $\pi^*$. Specifically, given two rewind-stage files $i, j \in R_{\pi^*}$ such that $i < j$ (i.e., $i$ is positioned on the left of $j$), we must have that $j$ is read prior to $i$ if they are separated by a forward-stage file in $\pi^*$, as shown in Proposition 6.

Proposition 6. Let $\pi^*$ be an optimal sequence to the LTS and consider any file $i \in \mathcal{F}_{\pi^*}$. If $\pi^*_t < i < \pi^*_t'$ for any two rewind-stage files $\pi^*_t, \pi^*_t' \in R_{\pi^*}$, then $t < t'$, i.e., $\pi^*_t$ is read prior to $\pi^*_t'$. 

Figure 3: Tape head movement associated with the FILA ordering $\pi = (5, 4, 3, 2, 1)$ for the tape in Figure 1.
Figure 4: Example depicting the block structure of an optimal sequence $\pi^*$. Forward-stage files are $F_{\pi^*} = \{1, 5, 6, 9\}$ (colored) and rewind-stage files are $R_{\pi^*} = \{2, 3, 4, 7, 8\}$.

Proposition 6 reveals the existence of file “blocks” in the rewind stage that are separated by forward-stage files. In particular, all files in a block are serviced before any files on the left of the block. This is illustrated in Figure 4, where files $\{7, 8\}$ are read prior to $\{2, 3, 4\}$ in an optimal sequencing $\pi^*$. We formalize the concept of block in Definition 2.

**Definition 2 (Block).** Given a sequence $\pi$, a subset $\mathcal{B}_{i,j} \equiv \{i, i+1, \ldots, j\} \subseteq \mathcal{F} \setminus \{1, n+1\}$ of consecutive files from $i$ to $j$ (inclusive) is a block if $\{i-1, j+1\} \subseteq F_{\pi}$ and $\mathcal{B}_{i,j} \subseteq R_{\pi}$.

We say that $\mathcal{B}_{i,j}$ is on the left of $\mathcal{B}_{i',j'}$ if $i < i'$, and on the right otherwise. We omit the dependency of a block on $\pi$ when it is clear from context. Figure 4 depicts the blocks $\mathcal{B}_{2,4} = \{2, 3, 4\}$ and $\mathcal{B}_{7,8} = \{7, 8\}$ that result from the forward-stage files $F_{\pi^*} = \{1, 5, 6, 9\}$. A block can also be composed of a single file.

It follows from Proposition 6 that blocks induce a partial ordering in an optimal sequence $\pi^*$. In particular, each block is necessarily associated with a contiguous subsequence of $\pi^*$. For example, in Figure 4 the files in $\mathcal{B}_{2,4}$ relate to the subsequence $(\pi^*_3, \pi^*_4, \pi^*_5)$. Finally, we note that each rewind-stage file belongs to some block of $\pi^*$, i.e., the union of the blocks spans $R_{\pi^*}$.

We next show our main result concerning the optimality structure of solutions to LTS. Proposition 7 states that the optimal subsequence associated with a block can be obtained by solving a separate optimization problem, itself independent of the other files in the tape except for the number of requests that have been served prior to the block. It also indicates that the objective function of the problem represents the increase in response time due to a particular tape head movement.

**Proposition 7.** Let $\pi^*$ be an optimal solution to LTS and $\mathcal{B}_{i,j}$ a block of $\pi^*$, with $b \equiv |\mathcal{B}_{i,j}|$. The subsequence $\rho^*$ of $\pi^*$ associated with $\{i, i+1, \ldots, j\}$ solves

$$V(\mathcal{B}_{i,j}, m - \sum_{t=1}^{f-1} w_{\pi^*_t})$$

where

$$V(\mathcal{B}_{i,j}, k) = \max_{\rho} \ k d_{i, \rho_1} + \sum_{t=2}^{b} \left( k - \sum_{\ell=1}^{t-1} w_{\rho_\ell} \right) \left( s_{\rho_{\ell-1}} + d_{\rho_{\ell-1}, \rho_\ell} \right) + \left( k - \sum_{\ell=1}^{b} w_{\rho_\ell} \right) \left( s_{\rho_b} + d_{\rho_{b-1}, \rho_b} \right)$$

s.t. $\rho = (\rho_1, \ldots, \rho_b)$ is a permutation of $\{i, \ldots, j\}$.

The function $V(\mathcal{B}_{i,j}, k)$ evaluates to the total increase in response time when servicing the files in the block $\mathcal{B}_{i,j}$, considering that

(i) there are $k$ requests yet to be completed, including those in $\mathcal{B}_{i,j}$;

(ii) the tape head is positioned at $r_j$ prior to servicing files in $\mathcal{B}_{i,j}$; and

(iii) the tape head is positioned at $\ell_i$ after servicing files in $\mathcal{B}_{i,j}$.

We note that the optimization problem (3) associated with $V(\mathcal{B}_{i,j}, k)$ only depends on files $\{i, \ldots, j\}$ and the parameter $k$, here representing the number of requests that are yet to be completed prior to servicing $\mathcal{B}_{i,j}$. We will show later in Proposition 9 that problem (3) can also
be solved in polynomial time in \( n \) and \( k \). If the forward-stage files \( F^* \) of an optimal solution \( \pi^* \) are known, we can determine the sequence by solving (3) for each block and then reading the forward-stage files in ascending index order, as discussed in Proposition 1.

5.2 Recursive formulation

We now derive a polynomial-time algorithm for the LTS. The principle is to enumerate promising sets of forward-stage files using a recursive model with a compact state space. Once any such set is fixed, the problem can be decomposed into smaller subproblems using Proposition 7.

We begin by defining a function \( \mathcal{R}(i, j, k) \) that evaluates to the total increase in response time when servicing files \( \{i, i+1, \ldots, j\}, i \leq j \), considering (a)-(c) below:

(a) the tape head starts and ends at \( r_j \), the right-bit position of file \( j \).

(b) there are \( k \) requests yet to be completed, considering also files not in \( \{i, i+1, \ldots, j\} \).

(c) \( j \) is delayed to the forward stage and hence it is the last file read from \( \{i, i+1, \ldots, j\} \).

Note that the optimal solution value of the LTS is given by \( \mathcal{R}(1, n+1, m) \).

Let \( q_{k, i, j} \equiv k - \sum_{i=1}^{j} w_i \) be the number of requests left after servicing files \( \{i, \ldots, j\} \), assuming we had \( k \) initial requests, which also include files that are not in \( \{i, \ldots, j\} \). We show a recursive reformulation of \( \mathcal{R}(i, j, k) \) in Proposition 8.

Proposition 8. For any \( i, j \in \{1, \ldots, n\}, i \leq j \), and \( k \in \{0, \ldots, m\} \),

\[
\mathcal{R}(i, j, k) = k s_j + \min_{i \leq j' < j} \left\{ \mathcal{V}(\mathcal{R}_{i+j'-1, j}) + \mathcal{R}(i, j', q_{k, j', j-1}) + q_{k, i, j-1} d_{j', j} \right\} + q_{k, i, j} s_j. \tag{4}
\]

In the recursion (4) above, the first term corresponds to the increase in response time when first moving the tape head from the right of \( j \) to the left of \( j \). The second term delays file \( j' < j \) to the forward stage. Thus, by Proposition 7, the optimal response time of the set \( \{j'+1, j'+2, \ldots, j-1\} \) is given by \( \mathcal{V}(\mathcal{R}_{j'+1, j-1}) \). We recursively invoke \( \mathcal{R}(\cdot) \) for the remaining \( q_{k, j'+1, j-1} \) requests in the interval \( \{i, i+1, \ldots, j'\} \), which are serviced after all the requests in \( \{j'+1, \ldots, j-1\} \); observe that \( \mathcal{R}(\cdot) \) is invoked to include files in the forward stage, whereas \( \mathcal{V}(\cdot) \) is used to schedule a sequence of files in the rewind stage. The quantity \( q_{k, i, j-1} d_{j', j} \) is the total response time increase when positioning the tape head from the right of \( j' \) to the left of \( j \), considering that files \( \{i, i+1, \ldots, j-1\} \) have already been serviced. Finally, the last term corresponds to the increase in response time when repositioning the tape to the right of \( j \), taking into account that there are \( q_{k, i, j} \) requests remaining.

Next, we show in Proposition 9 that \( \mathcal{V}(\mathcal{R}_{i, j, k}) \) in (3) is also tractable by leveraging an analogous recursion and \( \mathcal{R}(\cdot) \) as defined earlier.

Proposition 9. For any \( i, j \in \{2, \ldots, n-1\}, i \leq j \), and \( k \in \{0, \ldots, m\} \),

\[
\mathcal{V}(\mathcal{R}_{i, j, k}) = \begin{cases} 
\min_{i < j} \left( \min_{i < j' < j} \left\{ \mathcal{V}(\mathcal{R}_{j' - 1, j}) + \mathcal{V}(\mathcal{R}_{i, j', q_{k, j', j}}) \right\} \right), & \text{if } i < j \, \text{and } k s_j + 2(k - w_i) s_i, \\
\mathcal{R}(i, j, k) + q_{k, i, j} d_{j, j} & \text{if } i = j.
\end{cases}
\]

As in \( \mathcal{R}(\cdot) \), we provide the intuition of the recursion solving \( \mathcal{V}(\cdot, \cdot) \). If \( i = j \), there is a single file \( i \) and it suffices for the tape head to first move to \( \ell_i \) and service \( i \) by moving to \( r_i \). Otherwise, if \( i < j \), two different cases are considered when solving \( \mathcal{V}(\mathcal{R}_{i, j, k}) \):

Case 1. The first case, given by the inner-most "min" expression, decomposes \( \{i, i+1, \ldots, j\} \) into two sub-problems based on a file \( j' \) that is positioned between \( i+1 \) and \( j \). Given \( j' \), we recursively invoke \( \mathcal{V}(\mathcal{R}_{j', j, k}) \) to read the requests in \( \{j', j'+1, \ldots, j\} \) first, and next we service the other requests in \( \mathcal{R}_{i, j} \) by solving \( \mathcal{V}(\mathcal{R}_{i, j-1, q_{k, j', j}}) \). Since \( \mathcal{R}_{i, j-1} \) is on the left of \( \mathcal{R}_{j, j} \), in the second recursive call we consider that the files in \( \mathcal{R}_{j, j} \) have already been read.
Case 2. The second case solves $V(R_{i,j})$ as $R(i,j,k)$ and finalizes with a movement to $l_i$ (recall that $R(i,j,k)$ ends at $r_j$). In this solution, $j$ is the last file of $R_{i,j}$ to be read, thus complementing the set of sequences considered by the first strategy.

The state space size for both the recursions investigated in this section is $O(n^3)$. Since each value function evaluation requires $O(n)$ due to the “min” operator, it follows that our value-iteration algorithm solves the LTS in polynomial time.

Theorem 1. The LTS can be solved in $O(n^4)$.

6 Alternative Approximate Policies

The policies from §4.2 are simple to interpret and scale for larger tape settings, making them preferable over the exact polynomial-time approach in §5.2. However, our complexity analyses suggest that the optimality gap of these approximations is significant in certain problem classes, which we also observe in our numerical analysis in §8.

In this section, we leverage the insights from §4 and §5 to propose two alternative approximate policies. Albeit inheriting the same constant worst-case performance as the best algorithms from §4.2, we observed that they significantly improve numerical performance on both artificial and real data. We begin in §6.1 with an enhancement of the FILA strategy, which preserves interpretability and low complexity. Next, we present an approximate dynamic programming approach in §6.2 that simplifies the recursive model (8) for scalability purposes.

Similarly to §5, we also assume throughout this section that there exists an empty, non-requested file $n+1$ that is the last to be read in any sequence.

6.1 Large-Files-Last Policy

We recall that the FILA policy is a myopic strategy that reads all requested files in the rewind stage. Our worst-case analysis from Proposition 4 indicates that FILA’s performance is poor when certain files are relatively larger than others. Servicing such large files in the rewind stage delays all subsequent read operations, thus increasing overall response times.

The intuition of the large-files-last (LFL) policy is to consider a necessary optimality condition to postpone a “large” file to a later position in the sequence. Specifically, let $\pi$ be any reading sequence and $\pi_t \in \mathcal{R}_\pi$ a rewind-stage file for some $t$. For a tape with $n \geq 2$ files, we refer to

$$\eta_{\pi_t}(t) \equiv \arg \min_{t' > t} \left\{ \ell_{\pi_{t'}} < \ell_{\pi_t} < \ell_{\pi_{t'+1}} \right\}$$

as the second-visit position of $\pi_t$, in that $\pi_t$ is read for the second time when servicing the consecutive-file pair $(\pi_{\eta_{\pi_t}(t)}, \pi_{\eta_{\pi_t}(t)+1})$. We omit the dependency on $\pi$ when clear from context. Notice that such a position always exist since $\pi_t$ is a rewind-stage file and $\{1, n+1\} \in \mathcal{F}_\pi$, i.e., all files are traversed from left to right when the tape head moves from 0 to $L$.

Proposition 10. A sequence $\pi$ with $n > 3$ is optimal to the LTS only if, for any $t \in \{2, \ldots, n-2\}$ such that $\pi_t \in \mathcal{R}_\pi$, the following inequality holds:

$$d_{\pi_{t-1}, \pi_{t+1}} + \sum_{t' = t+1}^{\eta_{\pi_t}(t)-1} (s_{\pi_{t'}, \pi_{\eta_{\pi_t}(t)}}) + s_{\pi_{\eta_{\pi_t}(t)}, \pi_t} + d_{\pi_{\eta_{\pi_t}(t)}, \pi_t} \geq 2 s_{\pi_t} \left( m - \sum_{t' = 1}^{t} w_{\pi_{t'}} \right). \quad (5)$$

The idea of the LFL policy algorithm is to re-position files in a sequence whenever they violate inequality (5), starting from the solution obtained from FILA. The algorithm is as follows:

LFL Policy:

1. Generate an initial solution $\pi$ using FILA.
2. While there exists some $t \in \{2, \ldots, n\}$ such that inequality (5) is violated:
That is, LFL is an enhancement of FILA that delays a file to its second-visit position according to Proposition 10. Since inequality (5) is a necessary optimality condition, the worst-case performance of this policy is the same as FILA. Additionally, LFL is quadratic in the number of files.

**Proposition 11.** LFL terminates and has a computational complexity of $O(n^2)$. Furthermore, it is a 3-approximation to the LTS.

### 6.2 Approximate Dynamic Programming

We propose a simplification of the recursion $\mathcal{V}(\mathcal{B}_{i,j}, k)$ concerning rewind-stage blocks from Proposition 9. For any two files $i, j \in \{2, \ldots, n-1\}$, and $k \in \{0,\ldots,m\}$, we now write

$$\mathcal{V}(\mathcal{B}_{i,j}, k) = \begin{cases} \min_{i < j' \leq j} \left\{ \mathcal{V}(\mathcal{B}_{i,j'}, k) + \mathcal{V}(\mathcal{B}_{i,j'-1}, q_{k,i,j'}) \right\}, & i < j \\ k_s i + 2(k - w_i)s_i, & i = j. \end{cases} \quad (6)$$

That is, we remove the dependency on the value function $\tilde{\mathcal{R}}(i, j, k)$ in the recursion of $\mathcal{V}(\mathcal{B}_{i,j}, k)$, initially associated with the term $\tilde{\mathcal{R}}(i, j, k) + q_{k,i,j}$ in the first case $i < j$. Conceptually, this imposes that files are traversed at most once in the rewind stage, which reduces an expensive constant factor that we observed in practice. It is, however, still more flexible than FILA and LFL, as it allows for two consecutive rewind-stage files to be read in ascending index order, that is, $\pi_{t+1} = \pi_t + 1$ for some $t < n$ such that $\pi_t, \pi_{t+1} \in \mathcal{R}_n$.

The new response value function replaces $\mathcal{V}(\mathcal{B}_{i,j}, k)$ by $\tilde{\mathcal{V}}(\mathcal{B}_{i,j}, k)$ and becomes

$$\tilde{\mathcal{R}}(i, j, k) = k_s i + \min_{i < j' \leq j} \left\{ \mathcal{V}(\mathcal{B}_{i,j'-1}, k) + \tilde{\mathcal{R}}(i, j', q_{k,i,j'-1}) + q_{k,i,j'-1}d_{i,j'} + q_{k,i,j}\delta_{f,j}. \right\} \quad (7)$$

We refer to this method as approximate dynamic programming (ADP) given that it does not consider all the possible cases when solving the recursion (see, e.g., Powell (2007)). ADP solves $\tilde{\mathcal{R}}(1, n+1, m)$ to identify a solution to the LTS.

**Proposition 12.** ADP is a 3-approximation to the LTS.

### 7 Online Extensions

In a typical cold-storage setting, customers' requests arrive in batches and occur relatively far off from others, motivating the problem formulation investigated in this paper. One natural extension of this setting is to consider an online variant of the LTS which assumes new file requests become available during read operations of a previous batch. While such a dynamic scenario would be more suitable to random-access devices (e.g., solid-state drives), it provides relevant insights on the connection with other related classical online problems, such as the online TSP on the line.

For our online setting, we assume that the response time is measured as in §3, i.e., it represents the total distance the tape traversed from the beginning of operations until the file was read. We first show that the first-in, first-out policy (FIFO), which sequences the files according to their arrival orders, achieves an arbitrarily poor competitive ratio.

**Proposition 13.** The FIFO policy is not $c$-competitive for any constant $c$ in the online LTS.

We now introduce a policy with constant competitive ration for the online LTS that combines ideas from Ausiello et al. (2001) for the online dial-a-ride problem and Baeza-Yates et al. (1993) for point search in a plane. The algorithm partitions the tape into equally-sized contiguous intervals, based on the greatest common denominator of the file sizes, and read such intervals through incrementally larger right-to-left traversals. We denote this policy by augmenting reading interval (ARI), which is described as follows:

**ARI Policy:**

(a) Switch files $\pi_t$ and $\pi_{\eta(t)}$ for the minimum $t$ associated with a violation.
1. Find length $\delta$ of each interval: $\delta \equiv \gcd \{s_i : i \in \mathcal{F}\}$

2. Let $L' \equiv L/\delta$ be the number of intervals

3. For $i = 0, 1, \ldots, \lfloor \log(L') \rfloor$:
   - (a) Move the tape head $\min(2^i, L')$ intervals to the left of $L$.
   - (b) Move the tape head back to $L$, reading all files fully contained in the last $\min(2^i, L')$ intervals.

**Theorem 2.** ARI is 7-competitive for the online LTS:

Finally, we also discuss an online version of LTS with an alternative objective criteria. Specifically, we consider the setting where the response time is adjusted to incorporate the time at which a request arrived. For example, if a request is released at time 7 and serviced at time 10, its adjusted response time is 3, while the actual response time (used in Proposition 13 and Theorem 2) is 10. This relates to the concept of flow time (or time-in-system) objective criteria in the machine scheduling literature (Kanet 1981). We next show that there is no policy with bounded-factor guarantees for the online LTS if the goal is to minimize the sum of the adjusted response times.

**Proposition 14.** There is no $f$-competitive policy that minimizes the sum of adjusted response times in the online LTS for any bounded function $f$.

### 8 Numerical Study

We now evaluate the numerical behaviour of the exact and approximate policies investigated in this work. First, we study in §8.1 the relative policy performance on artificially generated tapes using file size distributions based on empirical studies. In §8.2, we provide a study on real tapes storing large-scale satellite imagery for precision agriculture. Table 1 presents a summary of the policies and their corresponding theoretical time complexity and approximation ratios.

| Name | Policy Description | Time Complexity | Appr. Ratio |
|------|--------------------|-----------------|-------------|
| First-in, first-out (FIFO) | Sequence based on the arrival ordering | $O(1)$ | $\Omega(n)$ |
| First-files-first (FIFF) | All files read in the forward stage | $O(1)$ | $\Omega(n)$ |
| First-files-last (FILA) | All files read in the rewind stage | $O(n^2)$ | 3 |
| Large-files-last (LFL) | Improvement of FILA based on Proposition 10 | $O(n^3)$ | 3 |
| Approximate DP (ADP) | Restricted recursive model (7) | $O(n^4)$ | 3 |
| Dynamic Programming (DP) | Exact recursive model (4) | $O(n^4)$ | 1 |

The calculation of the response times consider a Linear Tape-Open 8 (LTO-8) magnetic tape storage technology with an average read speed of 360 MB/s for uncompressed data (Quantum.com 2021), which we use as basis when converting tape distance traversed to time. Numerical tests were run on an Intel(R) Skylake CPU at 2.40GHz. More details about the data sets and our results can be found in the Appendix B.

#### 8.1 Policy Performance

We start by comparing the performance of each policy in terms of solution quality and time. For our tape configurations, we consider four possible number of files $n \in \{100, 200, 300, 400\}$, where the limit of 400 is due to large memory requirements of the exact approach DP. A file is requested or not based on a Bernoulli distribution with probability $p \in \{0.25, 0.50, 0.75, 1.0\}$. File sizes are based on the study by Douceur and Bolosky (1999), which at the time reported that sizes were distributed according to a log-normal with parameters $\mu = 8.46$ and $\sigma = 2.38$, i.e., an average of $\sim 80$ KB per file and standard deviation of $\sim 1.4$ KB. After incorporating trends in storage requirements (e.g., Agrawal et al. 2007), we draw file sizes from a log-normal
distribution with parameters \( \mu = 13.04 \) and \( \sigma \in \{1.50, 2.00, 2.38, 2.5, 3.0\} \); in particular, the pair \( \mu = 13.04 \) and \( \sigma = 2.38 \) corresponds to an average of \( \approx 7.8 \) megabytes (MB) and standard deviation of \( \approx 13.2 \) MB. We observe that the relative performance discussed here remains the same for other experimented values of \( \mu \).

We generate 10 instances per configuration \((n, p, \mu, \sigma)\), which leads to 800 instances in total. To control for numerical issues, we truncate file sizes to the 90% quantile of the corresponding distribution and divide them by 1,000, i.e., our unit of reference is kilobyte. For \textsc{FIFO}, each run corresponds to the average of 1,000 sequences generated uniformly at random, since the file ordering is arbitrary. Furthermore, \textsc{DP} and \textsc{ADP} were limited to 3,600 seconds and 8 GB of RAM.

Figure 5 depicts the average response time per file, i.e., the total response time divided by the number of requests \( m \). The shaded areas correspond to the 95% confidence interval. The results suggest that \textsc{FIFO}, currently the standard method in practice, performs poorly as \( n \) increases. In particular, the average time to retrieve a requested file reached up to 4 seconds for \( n = 400 \), which is approximately twice the optimal response time proved by \textsc{DP}. \textsc{FIFF} improves upon \textsc{FIFO} but underperforms with respect to \textsc{FILA}, which is consistent with their relative theoretical performance. Finally, the remaining approximate policies were near optimal in this data set.

![Graph](image)

(a) Average response time per file.  
(b) Solution time.

Figure 5: General performance on random tape configurations (in color).

Figure 5-(b) compares the time to generate the sequence once the requests arrive. The performance of the approximate policies (except \textsc{ADP}) are negligible, taking on average less than 0.0001 seconds on all instances. This was also the case for \textsc{LFL}; albeit of relatively high theoretical time complexity, the algorithm quickly converged to solutions that satisfy (5). The times of \textsc{DP} and \textsc{ADP} were several orders of magnitude higher, requiring on average 107 and 59 seconds, respectively, over all instances tested. For reference, commercial tape drives use low-power embedded processors (such as PowerPC and ARM) which are approximately two orders of magnitude slower than the tested processor. Thus, \textsc{DP} and \textsc{ADP} times tend to be prohibitively large except for benchmarking purposes.

The empirical quality performance was strongly correlated with the number of requested files \( m \) and the file size variance. Figures 6-(a) and 6-(b) depict the average approximation ratio as a function of the request probability \( p \) and deviation parameter \( \sigma \) of the log-normal distribution, respectively, with shaded areas representing the 95% confidence interval. The values are relative to the optimal solutions obtained by \textsc{DP}. In particular, \textsc{FIFF} performs worse on instances with fewer requests, producing sequences having twice the optimal total response time for \( p \leq 0.4 \). This is due to the fact that \textsc{FIFF} postpones files to the forward stage and, thus, it may take a significant amount of time to read them after reaching position 0; in contrast, many files are read earlier in the rewind stage in the other policies. \textsc{FILA}, in turn, has the opposite behaviour, and is preferable when there are few requests. The approximation ratio of all policies also increase with the file deviation \( \sigma \), which is consistent with the optimal cases of \textsc{FILA} and \textsc{LFL} from Propositions 3 and 4, respectively. We also note that \textsc{FILA}, \textsc{LFL}, and \textsc{ADP} presented significantly better empirical performance than their theoretical worst case. \textsc{LFL}
and ADP, in particular, had a maximum approximation ratio of approximately 1.029 and 1.021, respectively, over all instances in this dataset.

Finally, we provide some further insight on the structure of optimal sequences. Figures 7-(a) and 7-(b) are box plots depicting the number of rewind-stage files with respect to the probability \( p \) and file size deviation parameter \( \sigma \), respectively. The percentages are relative to the number of requested files \( m \) and are calculated according to the solutions obtained by DP. For low-request probabilities, i.e., small values of \( m \), the large majority of the files are read during the rewind stage, which is consistent with the performance of FILA and LFL observed in Figure 6. This behavior inverts as \( m \) increases, since there are marginally less gains in reading files during early tape movement. Finally, Figure 7-(b) suggests that distinct values of \( \sigma \) impact mostly the variance of rewind-stage sizes; in particular, if \( \sigma \) is high, there is a higher chance that the tape stores relatively larger files, which should be preferably read in the rewind-stage.

8.2 Landsat Instances

Landsat is a space mission which images the Earth surface every 16 days. Landsat 8, developed as a collaboration between NASA and the U.S. Geological Survey (USGS), is the eighth of the series of satellites launched by the Landsat mission. As the satellite orbits the Earth, it measures different ranges of frequencies (called “bands”) along the electromagnetic spectrum. Because the sensing process generates massive amounts of data, Landsat 8 data sets are split into a
collection of tiles (or satellite scenes). Each tile is stored in a single file with approximately 3.5 GB of data and features 12 bands – represented by numerical pixel matrices. Of those, 11 bands represent different ranges of the spectrum, and one additional band (particular to Landsat 8), known as QA (Quality Assessment), classifies pixels with an accuracy of up to 80% as water, snow, ice, cloud, or as invalid due to sensor errors during the image acquisition process (Roy et al. 2014).

In our setting, file requests are generated by algorithms that retrieve files in order to classify vineyards’ health conditions for precision viticulture purposes. We use data from three zones of interest in our experiments, corresponding to vineyards in the Atacama desert in Chile, the Serra Gaucha region in Brazil, and the Manduria region in Italy (courtesy of the U.S. Geological Survey). Our benchmark contains 100 instances with different configurations of the Landsat dataset. Every instance configuration consists of 15 Landsat tiles, each composed of 12 files (and hence one file per band). The average size of a file is approximately 280 MB, with small standard deviation (less than 1 MB).

![Figure 8: Performance on different instances of the Landsat dataset.](image)

Figures 8-(a) and 8-(b) depict a cumulative performance profile of the approximation algorithms and exact method on the Landsat dataset. For Figure 8-(a), the performance plot of FILA, ADP, and DP are omitted as they are similar to LFL. In terms of the average response time, the policies can be divided into three different categories. FIFO delivers the worst results, with an average response time of 230 seconds (standard deviation of 19). FIFF is approximately 13% better than FIFO, with average response time of 204 seconds (standard deviation of 13). The remaining policies, however, are considerably better and reduce the objective values by almost 50%, with average response times of 118 seconds (standard deviation of 18). DP improves upon the other algorithms only by hundredths of seconds. The solution time curves in Figure 8-(b) suggest that almost all policies are fast enough to be used in practice and provide sequences under 0.02 seconds, with the exception of DP and ADP which may require up to three seconds.

9 Conclusions

In this paper, we have investigated policies to retrieve files organized in a magnetic tape to minimize the response time to service data requests. We have evaluated the theoretical performance of three interpretable and simple algorithms, showing special cases for which they are optimal. Furthermore, we have also presented the first polynomial-time algorithm based on dynamic programming that, while not practical due to its high complexity, can be applied to benchmark alternative policies numerically. We have also exploited insights from our structural results to derive two new approximate policies with constant-factor approximation guarantees.
Finally, we have also presented the first constant-factor competitive algorithm for the online extension of the problem.

Our numerical analysis on both synthetic and real-world tape settings suggests that the managerial decision as to what policies to apply is highly dependent on the variance of file sizes stored in the tape. However, the default policy in typical tape hardware, FIFO, underperforms when compared to other low-complexity methods. Our results also suggest that simple modifications can be made into existing algorithms that preserve their approximation guarantees, improve the quality of solutions, and incur little computational cost.

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A Proofs

Proof. Proof of Proposition 1. Let \( \pi^* \) be an optimal solution to the LTS that violates the condition of the statement, i.e., there exist indices \( t < t' \) such that \( \pi_i^*, \pi_{i+1}^* \in F_{\pi^*} \) and \( \pi_i^* > \pi_{i+1}^* \). By definition, \( 1 \in F_{\pi^*} \), which implies that the tape head must traverse \( \pi_i^* \) from left to right at some point prior to reading \( \pi_{i+1}^* \). Suppose that such a traversal happens first between two forward-stage files \( \pi_i^* \) and \( \pi_{i+1}^* \) that are read consecutively in \( \pi^* \). Notice that we must have \( i, i+1 \leq t \) and \( \pi_i^* < \pi_{i+1}^* \). If we replace the contiguous subsequence \( \ldots, \pi_i^*, \pi_{i+1}^* \ldots \) in \( \pi^* \) by

\[
\ldots, \pi_i^*, \pi_{i+1}^*, \ldots
\]

that is, we read \( \pi_i^* \), the first time it is traversed from left to right, the response time of \( \pi_i^* \) decreases if \( w_{\pi_i^*} = 1 \). Moreover, the response times of the files that precede \( \pi_i^* \) in \( \pi^* \) do not change, while the response times for the files that succeed \( \pi_i^* \) in \( \pi^* \) can only decrease since there is no need to rewind the tape to \( \ell_{\pi_i^*} \). Thus, the total response time either remains the same or decreases.

\[\square\]

Proof. Proof of Proposition 2: Consider the following family of instances with \( L = n^2 \):

- File 1: \( \ell_1 = 0, w_1 = 1, \) and \( s_1 = 1 \);
- File 2: \( \ell_2 = 1, w_2 = 0, \) and \( s_2 = n^2 - n + 1 \); and
- Files \( 3, \ldots, n \): \( \ell_i = n^2 - n + i - 1, w_i = 1, \) and \( s_i = 1 \) for \( i = 3, \ldots, n \).

The sum of the response times of the sequence \( \pi \) delivered by FIFF is in \( O(n^3) \), since files \( 3, 4, \ldots, n \) are read after the tape head traverses the second file twice, an operation that consumes \( O(n^2) \). Conversely, the sum of the response times for the sequence \( \pi' = (3, 4, \ldots, n, 1, 2) \) is in \( O(n^3) \), and the result follows.

We require the following lemma for the proofs of Propositions 3 and 5.

Lemma 1. Suppose all files have the same size, i.e., \( s_i = s \) for all \( i \in F \). Given an arbitrary set of requested files, there exists an optimal solution \( \pi^* \) such that \( \pi_i^* > \pi_{i+1}^* \) for any rewind-stage file \( \pi_i^* \in R_{\pi^*}, t < n \). That is, files in the rewind stage are read in descending order of their indices.

Proof. Proof of Lemma 1.

Let \( \pi \) be a solution to the LTS with a (non-necessarily contiguous) subsequence \( (\pi_t, \ldots, \pi_{t+k}) \), \( k \geq 1 \), of rewind-stage files such that \( \pi_t < \ldots < \pi_{t+k} \), i.e., they are read in ascending order in \( \pi \). We can assume \( w_{\pi_t} = w_{\pi_{t+k}} = 1 \); otherwise, both files could be delayed to the forward stage without impacting the objective of the LTS.

Since the tape head starts at position \( L \), the file \( \pi_{t+k} \) is eventually traversed from right to left at some moment prior to reading \( \pi_t \). Create a new sequence \( \pi' \) where \( \pi_{t+k} \) is read immediately after the last-serviced file \( \pi_t \) prior to reaching \( \ell_{\pi_t} \) for the first time, where \( t' < t \); observe that such a file must exist, as we would have \( \pi_t = \pi_t \), otherwise, thus contradicting the assumption that \( \pi_t \) is a rewind-stage file. In this new sequence, the response time of each of the \( k - 1 \) files \( (\pi_t, \ldots, \pi_{t+k-1}) \) increases by \( 2s \) with respect to its original value. Thus, the increase in the response time for reading these \( k - 1 \) files is at most \( 2s(k - 1) \) in \( \pi' \), or potentially less if any of such files is not requested.

Conversely, the response time of \( \pi_{t+k} \) in \( \pi' \) decreases by at least \( 2s(k - 1) \), or more if the subsequence \( (\pi_t, \ldots, \pi_{t+k}) \) is not contiguous. Since the response times of the files read after before \( \pi_t \) or after \( \pi_{t+k} \) do not change, the objective value of \( \pi' \) is smaller than or equal to \( \pi \). The result follows by picking \( \pi = \pi^* \) and repeating this argument iteratively.

\[\square\]
Proof. Proof of Proposition 3. Let \( \pi' \) be an optimal sequence such that \( \mathcal{R}_{\pi'} \neq \emptyset \). By Lemma 1 and Proposition 1, we can assume that the forward- and rewind-stage files are ordered, i.e., \( \pi'_i < \pi'_{i+1} \) for all \( \pi'_i, \pi'_{i+1} \in \mathcal{F}_{\pi'} \), and \( \pi'_i > \pi'_{i+1} \) for all \( \pi'_i, \pi'_{i+1} \in \mathcal{R}_{\pi'} \), \( i < n \). Moreover, there exists some \( i > 1 \) such that \( \pi'_i = \pi'_i - 1 \in \mathcal{F}_{\pi'} \) and \( \pi'_i \in \mathcal{R}_{\pi'} \), which follows from \( 1 \in \mathcal{F}_{\pi'} \) and the ascending ordering of files in the tape. Let \( \pi'_i \) be such a file with smallest \( \ell_{\pi'_i} \), i.e., \( \{1, \ldots, \pi'_i\} \in \mathcal{F}_{\pi'} \).

Create a new ordering \( \pi'' \) such that \( \pi''_i \) is read immediately after \( \pi'_i \) in the forward stage of \( \pi'' \). Since all files have the same size, the response time of \( \pi''_i \) in \( \pi'' \) increases by \( 2s(\pi'_i) \) with respect to its response time in \( \pi'' \), as the files positioned to the left of \( \pi''_i \) in the tape, all belonging to the forward stage, are now serviced beforehand. However, the response time of the files \( 1, \ldots, \pi'_i \) reduce each by \( 2s \) in \( \pi'' \), as the reading of \( \pi'_i \) is delayed. Thus, the objective value of LTS is not impacted, since all files are requested and the response time of all remaining files do not change in \( \pi'' \). Thus, \( \pi'' \) is optimal and also preserves the forward-stage ascending ordering. The result follows by repeating this argument iteratively.

\[ \square \]

Proof. Proof of Proposition 4: The response time of every file \( i \in \mathcal{F} \), \( w_i = 1 \), in the sequence \( \pi \) generated by \( \text{FILA} \) is upper-bounded by \( 3(L - \ell_i) \), which is achieved if all files positioned on the right of \( i \) in the tape belong to \( \mathcal{R}_{\pi} \) and are requested. As the response time of \( i \) is never smaller than \( L - \ell_i \), i.e., the distance between the beginning of the tape and \( i \), \( \text{FILA} \) is a 3-approximation for LTS.

\[ \square \]

Proof. Proof of Proposition 5:

Let \( \pi^* \) be an optimal sequence such that there exists at least one file \( 1 \neq i \in \mathcal{F}_{\pi'} \) with \( w_i = 1 \). By Lemma 1 and Proposition 1, we can assume that \( \pi^*_i < \pi^*_{i+1} \) for all \( \pi^*_i, \pi^*_{i+1} \in \mathcal{F}_{\pi'} \), and \( \pi^*_i > \pi^*_{i+1} \) for all \( \pi^*_i, \pi^*_{i+1} \in \mathcal{R}_{\pi'} \), \( i < n \). Furthermore, \( w_i = 1 \) for all \( i \in \mathcal{R}_{\pi'} \), otherwise postponing such files to the forward stage would not increase the total response time.

Next, choose the forward-stage file \( 1 \neq i \in \mathcal{F}_{\pi'} \), \( w_i = 1 \), with the largest \( \ell_i \). Let \( j \in \mathcal{R}_{\pi'} \) be the rewind-stage file that is closest and on the right of \( i \) in the tape, and create a new sequence \( \pi'' \) where \( i \) is read in the rewind stage immediately after \( j \) (if such a file \( j \) does not exist, then \( \pi''_i = i \)).

The response time of \( i \) in \( \pi'' \) reduces by exactly \( 2s(i - 1) \) with respect to its response time in \( \pi'' \), which corresponds to the time to reach 0 from \( \ell_i \) and return to \( \ell_i \). The response times of files \( 1, 2, \ldots, i - 1 \), however, increase by exactly \( 2s \) each. Since not all files \( 1, 2, \ldots, i - 1 \) have requests, the objective value of \( \pi'' \) in LTS does not increase. Moreover, the rewind-stage file ordering is maintained since \( i \) has the largest \( \ell_i \) across all such files, and the argument can be repeated iteratively.

\[ \square \]

Proof. Proof of Proposition 6. For the purposes of contradiction, suppose the assumptions of the proposition hold but \( t > t' \), i.e., \( \pi^*_t \) succeeds \( \pi^*_{t'} \) in \( \pi^* \). This implies that the tape head will eventually move from \( \pi^*_{t'} \) to \( \pi^*_t \) during the rewind stage. This movement necessarily traverses the forward-stage file \( i \) since \( \ell_{\pi^*_i} < \ell_i < \ell_{\pi^*_i} \) by assumption. That is, there exist two rewind-stage files \( \ldots, \pi^*_{\eta}, \pi^*_{\eta+1}, \ldots \) that are consecutive in \( \pi^* \) such that \( \ell_{\pi^*_\eta} < \ell_i < \ell_{\pi^*_{\eta+1}} \), i.e., the tape head traverses \( i \) from its left to its right after reading \( \pi^*_{\eta} \) and before reading \( \pi^*_{\eta+1} \) (notice that possibly \( \eta = t' \) and \( \eta + 1 = t \)). We can therefore modify \( \pi^* \) to read \( i \) during that movement, i.e., \( \ldots, \pi^*_{\eta}, i, \pi^*_{\eta+1}, \ldots \), reducing the response time of \( i \) without impacting the response time of any other file, thus leading to a sequence with lower response time. This contradicts the optimality of \( \pi^* \).

\[ \square \]
Proof. Proof of Proposition 7.

To simplify notation, we write \( m_t \equiv m - \sum_{t'=1}^{t} w_{t'} \) to denote the number of pending requests after servicing the first \( t \) files of a sequence \( \pi_1, \pi_2, \ldots, \pi_t \), and

\[
\Theta(l, u) \equiv \sum_{i=l}^{u} m_{t-1} \left( s_{\pi_{i-1}} + d_{\pi_{i-1}, \pi_i} \right)
\]

to denote the sum of distances weighted by the pending requests between two sequence positions \( l, u \in \{2, \ldots, n\} \). The referred sequence will always be clear from context.

Let \( (\pi_v, \pi_{v+1}, \ldots, \pi_{\eta}) \) be the continuous subsequence associated with the files \( \{i, \ldots, j\} \) in \( \pi \). The sub-term in the objective function reformulation (2) that depends on this subsequence is

\[ m_{v-1} d_{\pi_{v-1}, \pi_v} + \Theta(v, \eta) + m_{\eta}(s_{\pi_{\eta}} + d_{\pi_{\eta}, \pi_{\eta+1}}). \]  \hfill (8)

From Proposition 6, if \( \pi \) is optimal, file \( \pi_v \leq j \) is positioned on the left of its preceding file \( \pi_{v-1} \) in the sequence \( \pi \). Therefore, we have

\[
d_{\pi_{v-1}, \pi_v} = r_{\pi_{v-1}} - \ell_{\pi_v}
= r_{\pi_{v-1}} - l_{j+1} + r_{j} - \ell_{\pi_v}
= d_{\pi_{v-1}, j+1} + d_{j, \pi_v}.
\]

For instance, in Figure 4 we have \( d_{8.3, 8.4} = d_{8.3} + d_{8.4} = d_{8.3} + d_{8.4} \). Thus, if we optimize (8) based on \((\pi_v, \pi_{v+1}, \ldots, \pi_{\eta})\), whereas all other parameters are fixed, the term \( d_{\pi_{v-1}, j+1} \) in the evaluation of \( d_{\pi_{v-1}, \pi_v} \) is a constant, regardless of the file in \( \{i, \ldots, j\} \) assigned to \( \pi_\eta \). Similarly, the optimal solution also minimizes

\[
m_{\eta-1} d_{j, \pi_v} + \Theta(v, \eta) + m_{\eta}(s_{\pi_{\eta}} + d_{\pi_{\eta}, \pi_{\eta+1}}),
\]

and, in particular,

\[
d_{\pi_\eta, \pi_{\eta+1}} = r_{\pi_\eta} - \ell_{\pi_{\eta+1}} = d_{\pi_\eta, i} + d_{i-1, \pi_{\eta+1}},
\]

where \( d_{\pi_{\eta}, i} \) is constant for every solution to the optimization problem (3) defined by \( \rho_1 = \pi_\eta \) and \( \rho_B = \pi_v \). The result follows from the optimality of \( \pi \).

Finally, for statements (i)-(iii), we partition \( \mathcal{V}(\mathcal{B}_{i,j}, k) \) into three parts:

\[
\mathcal{V}(\mathcal{B}_{i,j}, k) = kd_{j, \rho_1} + \sum_{t=2}^{b} \left( k - \sum_{t'=1}^{t} w_{t'} \right) (s_{\rho_{t-1}} + d_{\rho_{t-1}, \rho_t}) + \left( k - \sum_{t=1}^{b} w_{t} \right) (s_{\rho_B} + d_{\rho_B, i}).
\]

(a) \( (a) \) moving the tape head from the left of \( j \) to \( \rho_1 \);
(b) \( (b) \) servicing the files in \( \mathcal{B}_{i,j} \); and
(c) \( (c) \) moving the tape head from the right of \( \rho_B \) to the left of \( i \).

That is, such reading operations equivalently represent (i)-(iii).
Proof. Proof of Proposition 8.

For the case \( i = j \), equation (4) reduces to
\[
\mathcal{R}(i, i, k) = ks_i + (k - w_i)s_i.
\]
By definition of \( \mathcal{R}() \), the tape head starts on \( r_i \) and must read \( i \). There is an increase of \( ks_i \) in the total response time when moving from \( r_i \) to \( \ell_i \), since such a distance will be summed in the remaining \( k \) file requests. Afterwards, there are \( (k - w_i) \) requests left to be serviced, and the second term above corresponds to the increase in response times when returning to \( r_i \) after reading \( i \).

Consider now any \( i \) and \( j \) such that \( 1 < n' = |\{i, i + 1, \ldots, j\}| \). For the subsequent analysis, we extend the notation of the proof of Proposition 7 and write \( m_{k,t} = k - \sum_{t'=1}^t w_{\pi_{t'}} \), with \( m_{k,0} = k \), and
\[
\Theta_k(l, u) = \sum_{t=l+1}^u m_{k,t-1} (s_{\pi_{t-1}} + d_{\pi_{t-1},\pi_t})
\]
for any sequence positions \( l, u \in \{2, \ldots, n-1\} \). Recall that, by definition, \( j \) is the last file read in this subsequence, i.e., \( \pi_{n'} = j \). Using the reformulation (2), the function \( \mathcal{R}(\cdot) \) can be equivalently rewritten as
\[
\mathcal{R}(i, j, k) = \min_{\pi = (\pi_1, \ldots, \pi_{n'})} m_{k,0}d_{j,\pi_1} + \Theta_k(1, n') + m_{k,n'}s_{\pi_{n'}}
\]
\[
\text{s.t. } \pi_1, \pi_2, \ldots, \pi_{n'} \text{ is a permutation of } R_{i,j},
\]
\[
\pi_{n'} = j.
\]

We remark that the first term in the objective of FW accounts for the response time increase for moving the tape head from \( r_j \) to \( \ell_{\pi_1} \), the second term accounts for the response time of intermediate files (ending at \( \ell_j \)), and the last term accounts for moving the tape from \( \ell_j \) to \( r_j \). By decoupling the iterate \( t = n' \) from the sum of the second term and observing that \( d_{j,\pi_1} = s_j + d_{j-1,\pi_1} \), we rewrite the objective as
\[
\mathcal{R}(i, j, k) = m_{k,0}(s_j + d_{j-1,\pi_1}) + \Theta_k(1, n'-1) + m_{k,n'-1}(s_{\pi_{n'-1}} + d_{\pi_{n'-1},\pi_{n'}}) + m_{k,n'}s_j.
\]

Fix now \( \pi_{n'-1} = j' \) for some \( j' \in \{1, \ldots, j-1\} \) in any solution \( (\pi_1, \ldots, \pi_{n'-1}) \), i.e., \( j' \) immediately precedes \( j \). File \( j' \) is on the left of \( j \), so it must also be a forward-stage file in any feasible solution \( \pi \) to FW, since the tape must necessarily move from \( \ell_{j'} \) to \( r_j \) when reading the subsequence \( (j', j) \).

From Proposition 6, the files located at the block \( R_{j'+1, j-1} \) must be read before those located on the left of \( j' \), so \( \pi_1 \) must be a file of \( R_{j'+1, j-1} \). Letting \( (\pi_1, \pi_2, \ldots, \pi_{n'}) \) be a subsequence associated with \( R_{j'+1, j-1} \), we can decouple the related terms in the summation term for \( t = 1 \) to \( t = \eta \) in (9) to obtain the following summation:
\[
\mathcal{R}(i, j, k) = m_{k,0}s_j + \left[ m_{k,0}d_{j-1,\pi_1} + \Theta_k(1, \eta) + m_{k,\eta}(s_{\pi_\eta} + d_{\pi_\eta,j'+1}) \right]
\]
\[
+ \left[ m_{k,\eta}d_{j',\pi_{\eta+1}} + \Theta_k(\eta + 1, n' - 1) + m_{k,n'-1}s_{j'} \right]
\]
\[
+ q_{k,i,j-1}(s_{j'} + d_{j',j}) + q_{k,i,j}s_j.
\]

In an optimal sequence to FW, the first term in brackets is \( V(R_{j'+1, j-1}, k) \) by Proposition 7. The second term in brackets is \( \mathcal{R}(i, j', q_k, j'+1, j-1) \) by definition: note the tape head starts and ends on the right of \( \pi_{n-1} = j' \), after all requests between \( j'+1 \) and \( j-1 \) have already been read and serviced (so these requests are discounted from the original parameter \( k \)), thereby satisfying (a)-(c). Thus, it suffices to optimize over \( j' \) to establish the optimal sequencing to FW, which yields (4).
Proof. Proof of Proposition 9. The case \( i = j \) follows directly by evaluating the objective of (3) when both terms are the same. For \( i < j \), let \( \pi = (\pi_v, \pi_{v+1}, \ldots, \pi_{\eta}) \) be an optimal solution to (3). If the last file read is \( \pi_{\eta} = j \), then the objective of (3) becomes

\[
\mathcal{V} = kd_j,\pi_v + \Theta_k(\eta, v) + q_{k,i,j}(s_{\pi_{\eta}} + d_{\pi_{\eta},i})
\]

\[
= \left( kd_j,\pi_v + \Theta_k(\eta, v) + q_{k,i,j} s_{\pi_{\eta}} \right) + q_{k,i,j} d_{\pi_{\eta},i}
\]

\[
= \mathcal{A}(i, j, k) + q_{k,i,j} d_{j,i}.
\]

Suppose now that \( \pi_{\eta} \neq j = \pi_o \) for some \( o \in \{v, v+1, \ldots, \eta-1\} \). Let \( j' \) be the left-most file (i.e., the one with the smallest index) among \( \pi_v, \pi_{v+1}, \ldots, \pi_{o-1} \); note that \( j' \neq i \), otherwise \( j \) would be the last to be serviced and we would have \( \mathcal{V} \). By construction, \( (\pi_v, \pi_{v+1}, \ldots, \pi_o) \) is a permutation of \( \{j', \ldots, j\} \), with \( \pi_o = j \). Moreover, the term \( \Theta_k(v, \eta) \) can be rewritten as follows.

\[
\Theta_k(v, \eta) = \Theta_k(v, o) + \Theta_k(o, \eta)
\]

\[
= \Theta_k(v, o) + m_{k,o}(s_{\pi_o} + d_{\pi_o,\pi_{o+1}}) + \Theta_k(o+1, \eta)
\]

where the last equality is obtained by decoupling the term \( t = o+1 \) from the second summation of the second equality. Since \( \ell_{j'} = r_{j-1} \), the value of \( \mathcal{V}(i, j, k) \) for a given \( j' \) is

\[
\mathcal{V} = \left[ kd_j,\pi_v + \Theta_k(v, o) + m_{k,o}(s_{\pi_o} + d_{\pi_o,j'}) \right] + \left[ m_{k,o}d_{j-1,\pi_{o+1}} + \Theta_k(o+1, \eta) + q_{k,i,j}(s_{\pi_{\eta}} + d_{\pi_{\eta},i}) \right]
\]

\[
= \mathcal{V}(i, j', k) + \mathcal{V}(i, j-1, k, j')
\]

Selecting the minimum between \( \mathcal{V} \) and \( \mathcal{V} \) gives the desired result.

---

Proof. Proof of Proposition 10.

By using similar arguments to those employed in the proof of Proposition 6, it follows that a requested file is serviced the first time the tape head traverses it from left to right. Thus, the left-hand side expression of inequality (5) is equal to the amount of time \( \Delta_t \) elapsed between the first and the second visit of \( \pi_t \), i.e.,

\[
\Delta_t = d_{\pi_{t+1}, \pi_{t+1}} + \sum_{\ell_{t'}, \ell_{t+1}} \left( s_{\pi_{t'}} + d_{\pi_{t'}, \pi_{t'+1}} \right) + s_{\pi_{\tau(t)}} + d_{\pi_{\tau(t)}, \pi_t},
\]

where \( \tau(t) \) is the index of the last file read by \( \pi_t \).

The right-hand side of inequality (5) corresponds to the increase \( \delta_t \) in the objective value from the response times of \( \pi_{t+1}, \ldots, \pi_n \) that incur when reading \( \pi_t \), i.e.,

\[
\delta_t = 2s_{\pi_t} \cdot \left( m - \sum_{t'=1}^{t} w_{\pi_{t'}} \right).
\]

By postponing \( \pi_t \) to position \( \eta_{\pi}(t)+1 \) in \( \pi \), we obtain a solution \( \pi' \) that increases the response time of \( \pi_t \) by \( \Delta_t \) and decreases the response times of \( \pi_{t+1}, \ldots, \pi_n \) by \( \delta_t \). Thus, if inequality (5) is violated and \( \Delta_t < \delta_t \), then \( \pi \) is not optimal, as its objective value is greater than that of \( \pi' \).
Proof. Proof of Proposition 11. Since inequality (5) is a necessary condition for optimality, the sequence produced by the algorithm has either the same or lower objective value than the one produced by FILA, and hence it is a 3-approximation according to Proposition 4.

It remains to show that the algorithm terminates in $O(n^2)$ steps. First, based on Proposition 10, a file $\pi_r$ that violates inequality (5) is in the rewind stage and is postponed to the forward stage in Step 2-a. Thus, $\pi_r$ is not selected again and the algorithm must necessarily terminate in $O(n)$ iterations. The computational complexity follows because a violation of (5) can be identified in $O(n)$ by simulating the reading sequence and evaluating the corresponding times of the first and the second visit for each rewind-stage file. 

Proof. Proof of Proposition 12. It suffices to show that the sequence produced by FILA, which is a 3-approximation according to Proposition 4, is a feasible trajectory in the recursion specified by ADP. To this end, let

$$f_{i,j} \equiv \max_{j'} (i \leq j' : w_{j'} = 0),$$

i.e., $f_{i,j}$ is the right-most non-requested file positioned between files $i$ and $j$. Notice that such a file always exists (e.g., $n + 1$). Next, we set

$$\hat{R}(i, j, k) = ks_j + \hat{V}(\hat{R}_{f_{i,j+1},j-1,k}) + \hat{R}(i, f_{i,j}, q_k, f_{i,j+1}) + q_k(i,j+1) + q_k,i,js_j,$$

i.e., the forward-stage files are exactly the ones without requests, and

$$\hat{V}(\hat{R}_{i,j,k}) = \begin{cases} 
\hat{V}(\hat{R}_{j,j,k}) + \hat{V}(\hat{R}_{i,j-1,k} - 1), & i < j; \\
ks_i + 2(k - w_j)s_i, & i = j, 
\end{cases}$$

i.e., we service requested files in the rewind stage sequentially. The recursion described above therefore delivers the same sequence (and objective value) as FILA.

Proof. Proof of Proposition 13: Let us consider the family of instances where all files have the same size and all requests are released within the first time-step and induce the sequence $\pi = (n/2, n/2 + 1, n/2 - 1, n/2 + 2, n/2 - 2, \ldots, n, 1)$ according to the FIFO policy. Observe that the movements of the tape reader defined by $\pi$ follow a zig-zag pattern, starting from the middle of the tape.

Let us consider the response times of the files $1, 2, \ldots, n/2$. For convenience, we define $v(k)$ to denote the response time of file $n/2 - k$ for $k \in \{0, 1, \ldots, n - 1\}$; observe that file $n - k$ is the $(k + 1)$-th to be read among files $1, 2, \ldots, n/2$. By construction, we have $v(0) = n/2$, and for $k \in \{1, \ldots, n - 1\}$ we have

$$v(k) = v(k - 1) + \frac{2k}{\text{Time to move from } f_{n/2-(k-1)} \text{ to } f_{n/2+k}} + \frac{2k + 1}{\text{Time to move from } f_{n/2+k} \text{ to } f_{n/2-k}}$$

$$= v(k - 1) + 4k + 1.$$

We show that $v(k) = 2k^2 + 3k$ for $k \in \{1, \ldots, n - 1\}$. Direct verification shows that the result holds for $k = 1$; assuming that the result holds for $k - 1$, we have

$$v(k) = v(k - 1) + 4k + 1$$

$$= 2(k - 1)^2 + 3(k - 1) + 4k + 1$$

Induction hypothesis

$$= 2k^2 - 4k + 2 + 3k - 3 + 4k + 1$$

$$= 2k^2 + 3k,$$
Proof. Proof of Theorem 2: We assume w.l.o.g. that 

In order to simplify the notation used in this proof, we use the FIFO rightwards for the first time. First, for every position \(2^k\), \(k \in \mathbb{N}\), we show that \(p(2^k) = 2^{k+1} + 2^k - 2\) by induction in \(k\). Direct substitution shows that the result holds for \(k = 0\), as 

\[
p(2^0) = 2^1 + 2^0 - 2 = 1,
\]

so let us assume that it holds for \(p(2^{k-1})\). Position \(2^k\) is visited for the first time at time 

\[
p(2^k) = p(2^{k-1}) + 2^{k-1} - 2 + 2^{k-1} + 2^k - 2 = 2^{k+1} + 2^k - 2,
\]

so the result holds. Next, observe that the first visit to files \(2^k - 1, 2^k - 2, \ldots, 2^k - 1 + 1\) take place after the first visit to \(2^k\). In particular, observe that position \(2^{k-1} + 1\) is the last among \(1, 2, \ldots, 2^k\) to be traversed leftwards, an operations that takes place at time 

\[
p(2^{k-1} + 1) = p(2^k) + 2^{k-1} - 1 = 2^{k+1} + 2^k + 2^{k-1} - 3.
\]

Additionally, observe that the position of the file defines a lower bound on its response time, i.e., if a file starts at position \(2^k + i\), the minimum response time is also \(2^k + i\). Thus, the ratio \(c\) between the response time given by our policy for any file starting from \(1, 2, \ldots, 2^k\) and the minimum response time for any request for \(i\) released within the first \(2^k\) time steps is maximum at position \(2^{k-1} + 1\) and is given by 

\[
c \leq \frac{p(2^{k-1} + 1)}{2^{k-1} + 1} = \frac{2^{k+1} + 2^k + 2^{k-1} - 3}{2^{k-1} + 1}
\]

\[
\leq \frac{2^{k+1} - 1}{2^{k-1} + 1} + \frac{2^k - 1}{2^{k-1} + 1} + \frac{2^{k-1} - 1}{2^{k-1} + 1}
\]

\[
< 4 + 2 + 1 = 7,
\]

i.e., ARI takes at most 7 times longer to reach and traverse any file leftwards for the first time than any other policy. Observe that this ratio holds for any request associated with any file starting at positioned \(2^k + i\) released between time steps 0 and \(2^k + i\), i.e., between the beginning of the planning horizon and the minimum amount of time the tape would need to reach \(2^k + i\) for the first time.

\[
\sum_{k=0}^{n/2} v(k) = \sum_{k=0}^{n/2} \frac{n}{2} + 2k^2 + 3k
\]

\[
= O(n^2) + 2 \sum_{k=0}^{n/2} k^2
\]

\[
= O(n^2) + 2 \cdot \frac{n(n+1)(n+1)}{6} = O(n^3)
\]

Finally, the sum of the response times for solution \(\pi' = (1, 2, \ldots, n)\) is \(O(n^2)\), so it follows that the FIFO policy is not \(c\)-competitive for any constant \(c\). \(\blacksquare\)
Let us consider now the \(v\)-th read operation of each file. First, observe that after reading \(2^k\) for the first time, the policy moves rightwards to \(L\) first and then left to \(2^{k+1}\), from where it moves back to read \(2^k\) for the second time. The same procedure is repeated for the other visits; we show that the \(v\)-th visit of position \(2^k\) takes place at time \(p(2^k, v) = \sum_{q=0}^{v} 2^{k+q} - 2\) by induction in \(v\). For \(v = 1\), we have \(p(2^k, 1) = p(2^k)\), so the base case holds. If we assume that the result holds for the \((v-1)\)-th visit, we have

\[
p(2^k, v) = p(2^k, v-1) + 2^k \quad \text{(\((v-1)\)-th visit)} + 2^{k+v-1} + 2^{k+v-1} - 2^k \quad \text{Moving from } 2^k \text{ to } 0 \quad \text{Moving from } 0 \text{ to } 2^{k+v-1} \quad \text{Moving from } 2^{k+v-1} \text{ to } 2^k
\]

\[
= \sum_{q=0}^{v-1} 2^{k+q} - 2 + 2^{k+v} = \sum_{q=0}^{v} 2^{k+q} - 2.
\]

For arbitrary files \(2^k - q, 0 \leq q < 2^{k-1}\), the \(v\)-th visit takes place at time

\[
p(2^k - q, v) = p(2^k, v) + q = \sum_{q=0}^{v} 2^{k+q} - 2 + q,
\]

and the difference between the \((v+1)\)-th and the \(v\)-th visit for \(v \geq 1\) is given by

\[
p(2^k - q, v+1) - p(2^k - q, v) = 2^k - q + 2^{k+v+1} + 2^{k+v+1} - (2^k - q) \quad \text{Moving from } 2^k - q \text{ to } 0 \quad \text{Moving from } 0 \text{ to } 2^{k+v+1} \quad \text{Moving from } 2^{k+v+1} \text{ to } 2^k
\]

\[
= 2^{k+v+2}.
\]

Therefore, the competitive ratio \(c'\) for the \(v\)-th visit of an arbitrary position \(2^k - q, v \geq 2\), which covers requests released after the \((v-1)\)-th visit, is

\[
c' = \frac{p(2^k - q, v+1)}{p(2^k - q, v)} = 1 + \frac{2^{k+v+2}}{\sum_{i=0}^{v} 2^{k+i} - 2 + q}
\]

\[
\leq 1 + \frac{2^{k+v+2}}{2^{k+v+1} - 1} \leq 4,
\]

so it follows that ARI is 7-competitive for the online LTS.

\[\blacksquare\]

**Proof.** Proof of Proposition 14: We present an adversarial strategy to derive the result. First, let us assume w.l.o.g. that \(n = 3\), all files have size 1, and that the tape head is in \(\ell_2\) at a certain instant of time \(t\). Let \(\mathcal{A}_0\) be the family of policies that move the tape head to the right and reach \(\ell_3\) at time \(t+1\). In this case, it suffices for the adversarial environment to requests file 2 at time \(t+1\). In this case, an optimal strategy would only start moving the tape head to the right at time \(t+1\), thus achieving an adjusted response time of zero, whereas any policy in \(\mathcal{A}_0\) would deliver an adjusted response time of at least 2. Thus, the competitive ratio of any algorithm in \(\mathcal{A}_0\) is unbounded. The same adversarial strategy shows that the competitive ratio of any policy that moves the reader to the left and reaches \(\ell_1\) at time \(t + 1\) also has an unbounded competitive ratio.

Finally, for policies that maintains the tape reader positioned at \(\ell_2\) until time \(t+1\), it suffices for the adversarial environment to release a request for file 1 at time \(t + 1\), thus yielding an unbounded competitive ratio. Thus, there is no policy that delivers a bounded competitive ratio for the online LTS when considering the minimization of the sum of adjusted response times.

\[\blacksquare\]

28
B Computational Experiments

In this section we present details and complementary results for our experiments.

B.1 File System Instances

Tables 2, 3, 4, and 5 present the details of our experiments for the file system instances with 100, 200, 300, and 400 files, respectively. An entry corresponds to the average response time (in seconds) based on a configuration \((p, \sigma)\) as described in the main text, i.e., \(p\) is the probability that a file in the tape is requested and \(\sigma\) is the parameter of the log-normal distribution for file sizes (with \(\mu = 13.04\) for all cases). Each value is the mean of the 10 instances for the associated configuration.

Table 2: File system instances for \(n = 100\)

| Instance \(p\) (%) | \(\sigma\) | Algorithm |
|--------------------|-----------|-----------|
| FIFO | FIFF | FILA | LFL | ADP | DP |
| 25 | 1.50 | 0.92 | 0.84 | 0.43 | 0.42 | 0.42 | 0.42 |
| 25 | 2.00 | 0.92 | 0.84 | 0.43 | 0.42 | 0.42 | 0.42 |
| 25 | 2.38 | 0.92 | 0.84 | 0.43 | 0.42 | 0.42 | 0.42 |
| 25 | 2.50 | 0.92 | 0.84 | 0.43 | 0.42 | 0.42 | 0.42 |
| 25 | 3.00 | 0.92 | 0.84 | 0.43 | 0.42 | 0.42 | 0.42 |
| 50 | 1.50 | 0.98 | 0.85 | 0.54 | 0.46 | 0.46 | 0.46 |
| 50 | 2.00 | 0.98 | 0.85 | 0.54 | 0.46 | 0.46 | 0.46 |
| 50 | 2.38 | 0.98 | 0.85 | 0.54 | 0.46 | 0.46 | 0.46 |
| 50 | 2.50 | 0.98 | 0.85 | 0.54 | 0.46 | 0.46 | 0.46 |
| 50 | 3.00 | 0.98 | 0.85 | 0.54 | 0.46 | 0.46 | 0.46 |
| 75 | 1.50 | 1.03 | 0.88 | 0.72 | 0.52 | 0.52 | 0.52 |
| 75 | 2.00 | 1.03 | 0.88 | 0.72 | 0.52 | 0.52 | 0.52 |
| 75 | 2.38 | 1.03 | 0.88 | 0.72 | 0.52 | 0.52 | 0.52 |
| 75 | 2.50 | 1.03 | 0.88 | 0.72 | 0.52 | 0.52 | 0.52 |
| 75 | 3.00 | 1.03 | 0.88 | 0.72 | 0.52 | 0.52 | 0.52 |
| 100 | 1.50 | 1.9 | 1.61 | 1.49 | 0.98 | 0.98 | 0.98 |
| 100 | 2.00 | 1.9 | 1.61 | 1.49 | 0.98 | 0.98 | 0.98 |
| 100 | 2.38 | 1.9 | 1.61 | 1.49 | 0.98 | 0.98 | 0.98 |
| 100 | 2.50 | 1.9 | 1.61 | 1.49 | 0.98 | 0.98 | 0.98 |
| 100 | 3.00 | 1.9 | 1.61 | 1.49 | 0.98 | 0.98 | 0.98 |

B.2 Landsat Instances

For the real-world tape settings, we consider a case study on the use of remote sensing for precision viticulture in 3 zones of interest: the Atacama desert in Chile, the Serra Gaucha region in Brazil, and the Manduria region in Italy. The Atacama region is represented by a cluster of 4 Landsat-8 tiles, and it is known for its extremely dry weather and a mostly cloud-free sky all year round, which translates into crisp satellite imagery. The software accessing these tiles computes the Normalized Difference Vegetation Index (NDVI) of the region by reading Landsat bands number 4 and 5, which represent the reflective radiation in visible Red and Near-Infrared wavelengths, respectively. Differently from the Atacama region, Serra Gaucha and Manduria are frequently covered with clouds that, at times, invalidate whole satellite scenes (NASA 2017). The Serra Gaucha region is covered by 6 Landsat-8 tiles that, once processed, are converted into false-color images that emphasize vegetation; the image composition procedure used in this application requires the use of bands number 4, 5, and 6. Last, Manduria is composed of 5 tiles.
The associated application is a deep neural network that processes the assembled data in order to classify the vineyards’ health conditions.

Each aforementioned region is analysed by a different software, but in all cases read requests are contingent on the analysis of the QA band. If an image has a cloud cover above a certain threshold, then no bands other than the QA are read; the missing data is then interpolated or extrapolated at a later time using data from past and future scenes. This aspect is important, as cloud cover thresholds have a direct impact on read requests. Our experiments involve only the second stage of this pipeline, i.e., we assume that the QA bands have been inspected already and that all read requests involve spectrum bands of images that satisfy the cloud cover conditions.

The results of our experiments involving the Landsat instances are presented in Table 6. Each instance is associated with a configuration \(\alpha–\beta–\gamma\); numbers \(\alpha, \beta, \gamma\) belong to the interval \([0, 100]\) and indicate the threshold for cloud covers adopted by the applications using data associated with the Atacama, Serra Gaucha, and Manduria region, respectively. The data set has 10 instances for each of the following configurations of \((\alpha, \beta, \gamma)\): (2,15,22), (17,15,32), (7,21,22), (9,18,38), (17,21,32), (9,15,22), (2,26,38), (17,21,22), (2,21,22), and (17,26,38). The first column in the table indicates the configuration \(\alpha – \beta – \gamma\), and each entry reports the average response times obtained by each algorithm over the 10 instances.

| Instance \(p\) (%) | \(\alpha\) | Algorithm | FIFO | FIFF | FILA | LFL | ADP | DP |
|-------------------|--------|-----------|------|------|------|-----|-----|-----|
| 25                | 1.50   | 1.99      | 1.74 | 0.89 | 0.86 | 0.86| 0.86| 0.86|
| 25                | 2.00   | 1.99      | 1.74 | 0.89 | 0.86 | 0.86| 0.86| 0.86|
| 25                | 2.38   | 1.99      | 1.74 | 0.89 | 0.86 | 0.86| 0.86| 0.86|
| 25                | 2.50   | 1.99      | 1.74 | 0.89 | 0.86 | 0.86| 0.86| 0.86|
| 25                | 3.00   | 1.99      | 1.74 | 0.89 | 0.86 | 0.86| 0.86| 0.86|
| 50                | 1.50   | 2.02      | 1.74 | 1.16 | 0.96 | 0.96| 0.96| 0.96|
| 50                | 2.00   | 2.02      | 1.74 | 1.16 | 0.96 | 0.96| 0.96| 0.96|
| 50                | 2.38   | 2.02      | 1.74 | 1.16 | 0.96 | 0.96| 0.96| 0.96|
| 50                | 2.50   | 2.02      | 1.74 | 1.16 | 0.96 | 0.96| 0.96| 0.96|
| 50                | 3.00   | 2.02      | 1.74 | 1.16 | 0.96 | 0.96| 0.96| 0.96|
| 75                | 1.50   | 2.14      | 1.82 | 1.5  | 1.07 | 1.07| 1.07| 1.07|
| 75                | 2.00   | 2.14      | 1.82 | 1.5  | 1.07 | 1.07| 1.07| 1.07|
| 75                | 2.38   | 2.14      | 1.82 | 1.5  | 1.07 | 1.07| 1.07| 1.07|
| 75                | 2.50   | 2.14      | 1.82 | 1.5  | 1.07 | 1.07| 1.07| 1.07|
| 75                | 3.00   | 2.14      | 1.82 | 1.5  | 1.07 | 1.07| 1.07| 1.07|
| 100               | 1.50   | 2.1       | 1.77 | 1.76 | 1.1  | 1.1 | 1.1 | 1.1 |
| 100               | 2.00   | 2.1       | 1.77 | 1.76 | 1.1  | 1.1 | 1.1 | 1.1 |
| 100               | 2.38   | 2.1       | 1.77 | 1.76 | 1.1  | 1.1 | 1.1 | 1.1 |
| 100               | 2.50   | 2.1       | 1.77 | 1.76 | 1.1  | 1.1 | 1.1 | 1.1 |
| 100               | 3.00   | 2.1       | 1.77 | 1.76 | 1.1  | 1.1 | 1.1 | 1.1 |
Table 4: File system instances for $n = 300$

| Instance $p$ (%) | Algorithm | FIFO | FIFF | FILA | LFL | ADP | DP |
|-----------------|-----------|------|------|------|-----|-----|----|
| 25 1.50         | 2.98      | 2.62 | 1.34 | 1.3  | 1.3 | 1.3 | 1.3|
| 25 2.00         | 2.98      | 2.62 | 1.34 | 1.3  | 1.3 | 1.3 | 1.3|
| 25 2.38         | 2.98      | 2.62 | 1.34 | 1.3  | 1.3 | 1.3 | 1.3|
| 25 2.50         | 2.98      | 2.62 | 1.34 | 1.3  | 1.3 | 1.3 | 1.3|
| 25 3.00         | 2.98      | 2.62 | 1.34 | 1.3  | 1.3 | 1.3 | 1.3|
| 50 1.50         | 3.05      | 2.6  | 1.77 | 1.46 | 1.46| 1.46| 1.46|
| 50 2.00         | 3.05      | 2.6  | 1.77 | 1.46 | 1.46| 1.46| 1.46|
| 50 2.38         | 3.05      | 2.6  | 1.77 | 1.46 | 1.46| 1.46| 1.46|
| 50 2.50         | 3.05      | 2.6  | 1.77 | 1.46 | 1.46| 1.46| 1.46|
| 50 3.00         | 3.05      | 2.6  | 1.77 | 1.46 | 1.46| 1.46| 1.46|
| 75 1.50         | 3.08      | 2.57 | 2.19 | 1.55 | 1.55| 1.55| 1.55|
| 75 2.00         | 3.08      | 2.57 | 2.19 | 1.55 | 1.55| 1.55| 1.55|
| 75 2.38         | 3.08      | 2.57 | 2.19 | 1.55 | 1.55| 1.55| 1.55|
| 75 2.50         | 3.08      | 2.57 | 2.19 | 1.55 | 1.55| 1.55| 1.55|
| 75 3.00         | 3.08      | 2.57 | 2.19 | 1.55 | 1.55| 1.55| 1.55|
| 100 1.50        | 3.1       | 2.59 | 2.59 | 1.61 | 1.61| 1.61| 1.61|
| 100 2.00        | 3.1       | 2.59 | 2.59 | 1.61 | 1.61| 1.61| 1.61|
| 100 2.38        | 3.1       | 2.59 | 2.59 | 1.61 | 1.61| 1.61| 1.61|
| 100 2.50        | 3.1       | 2.59 | 2.59 | 1.61 | 1.61| 1.61| 1.61|
| 100 3.00        | 3.1       | 2.59 | 2.59 | 1.61 | 1.61| 1.61| 1.61|

Table 5: File system instances for $n = 400$

| Instance $p$ (%) | Algorithm | FIFO | FIFF | FILA | LFL | ADP | DP |
|-----------------|-----------|------|------|------|-----|-----|----|
| 25 1.50         | 4.02      | 3.46 | 1.78 | 1.73 | 1.73| 1.73| 1.73|
| 25 2.00         | 4.02      | 3.46 | 1.78 | 1.73 | 1.73| 1.73| 1.73|
| 25 2.38         | 4.02      | 3.46 | 1.78 | 1.73 | 1.73| 1.73| 1.73|
| 25 2.50         | 4.02      | 3.46 | 1.78 | 1.73 | 1.73| 1.73| 1.73|
| 25 3.00         | 4.02      | 3.46 | 1.78 | 1.73 | 1.73| 1.73| 1.73|
| 50 1.50         | 4.14      | 3.51 | 2.36 | 1.95 | 1.95| 1.95| 1.95|
| 50 2.00         | 4.14      | 3.51 | 2.36 | 1.95 | 1.95| 1.95| 1.95|
| 50 2.38         | 4.14      | 3.51 | 2.36 | 1.95 | 1.95| 1.95| 1.95|
| 50 2.50         | 4.14      | 3.51 | 2.36 | 1.95 | 1.95| 1.95| 1.95|
| 50 3.00         | 4.14      | 3.51 | 2.36 | 1.95 | 1.95| 1.95| 1.95|
| 75 1.50         | 4.28      | 3.58 | 3.04 | 2.15 | 2.15| 2.15| 2.15|
| 75 2.00         | 4.28      | 3.58 | 3.04 | 2.15 | 2.15| 2.15| 2.15|
| 75 2.38         | 4.28      | 3.58 | 3.04 | 2.15 | 2.15| 2.15| 2.15|
| 75 2.50         | 4.28      | 3.58 | 3.04 | 2.15 | 2.15| 2.15| 2.15|
| 75 3.00         | 4.28      | 3.58 | 3.04 | 2.15 | 2.15| 2.15| 2.15|
| 100 1.50        | 4.17      | 3.5  | 3.49 | 2.19 | 2.19| 2.19| 2.19|
| 100 2.00        | 4.17      | 3.5  | 3.49 | 2.19 | 2.19| 2.19| 2.19|
| 100 2.38        | 4.17      | 3.5  | 3.49 | 2.19 | 2.19| 2.19| 2.19|
| 100 2.50        | 4.17      | 3.5  | 3.49 | 2.19 | 2.19| 2.19| 2.19|
| 100 3.00        | 4.17      | 3.5  | 3.49 | 2.19 | 2.19| 2.19| 2.19|
Table 6: Landsat instances

| Instance  | $\alpha$ | $\beta$ | $\gamma$ | FIFO | FIFF | FILA | LFL | ADP | DP |
|-----------|----------|---------|----------|------|------|------|-----|-----|----|
| 02-15-22  | 238      | 208     | 127      | 127  | 127  | 127  |     |     |    |
| 16-15-32  | 221      | 212     | 103      | 103  | 103  | 103  |     |     |    |
| 7-21-22   | 222      | 205     | 116      | 116  | 116  | 116  |     |     |    |
| 9-18-38   | 236      | 205     | 116      | 116  | 116  | 116  |     |     |    |
| 17-21-32  | 228      | 202     | 119      | 119  | 119  | 119  |     |     |    |
| 9-15-22   | 229      | 201     | 131      | 131  | 131  | 131  |     |     |    |
| 2-26-38   | 224      | 200     | 108      | 108  | 108  | 108  |     |     |    |
| 17-21-22  | 236      | 209     | 117      | 117  | 117  | 117  |     |     |    |
| 2-21-22   | 233      | 199     | 123      | 123  | 123  | 123  |     |     |    |
| 17-26-38  | 230      | 198     | 118      | 118  | 118  | 118  |     |     |    |