Novel Transformation Methods Among Intuitionistic Fuzzy Models for Mixed Intuitionistic Fuzzy Decision Making Problems

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ABSTRACT

As an extension of classical fuzzy set, intuitionistic fuzzy set (IFS) has been widely utilized in decision making. Based on IFS, intuitionistic fuzzy value (IFV) and linguistic intuitionistic fuzzy set (LIFS) have also been proposed to represent uncertain assessments of alternatives in different intuitionistic fuzzy decision environment. Theoretically, IFV, IFS or LIFS own themselves processing methods in decision making. However, they may be utilized to represent intuitionistic fuzzy assessments of alternatives in the same decision problem, i.e., multi-decision makers provide IFVs, IFSs or LIFSs to represent uncertain assessments of alternatives in the decision problem. In such case, the significant challenge is to unify IFVs, IFSs and LIFSs in the decision making matrices. In the paper, novel transformation methods are presented to unify IFVs, IFSs and LIFSs. To this end, mixed intuitionistic fuzzy decision matrix is presented to represent IFVs, IFSs and LIFSs. To this end, mixed intuitionistic fuzzy decision matrix is presented to represent IFVs, IFSs and LIFSs assessments of alternatives provided by decision makers, then transformation functions are developed to unify IFVs, IFSs and LIFSs in the decision making matrices, their properties are also analysed. The decision making method is presented to solve the intuitionistic fuzzy decision making problems with IFVs, IFSs or LIFSs, which is consisted of six main phases, accordingly the algorithm is designed to carry out the problems. Illustrative examples show that transformation functions are useful in intuitionistic fuzzy decision environment, the proposed decision making method is an effective and alternative tool for the intuitionistic fuzzy decision making problems with IFVs, IFSs or LIFSs.

INDEX TERMS

Mixed intuitionistic fuzzy decision making, intuitionistic fuzzy value, intuitionistic fuzzy set, linguistic intuitionistic fuzzy set.

I. INTRODUCTION

Due to the lack of knowledge or information, there is pervasive uncertainty when the real world physical objects are represented by the human cognition, such as in big data, there is uncertain information when decision makers provide assessments of alternatives, this makes that decision problems become difficult and complex [1]. The concept of linguistic variable [2] is an important and pivotal tool to represent and deal with uncertain information in many real-world applications, formally, a linguistic variable is defined by a quintuple $(L, H, U, G, M)$, in which $L$ is the name of the variable; $H$ denotes the term set of $L$, i.e., the set of names of linguistic values of $L$, with each value being a fuzzy variable denoted generically by $X$ and ranging across a universe of discourse $U$ which is associated with the base variable $u$; $G$ is a syntactic rule (which usually takes the form of a grammar) for generating the names of values of $L$; and $M$ is a semantic rule for associating its meaning with each $L, M(X)$, which is a fuzzy subset of $U$. In a real-world application, fuzzy subsets on the universe $U$ are utilized to quantitatively represent fuzzy information, linguistic values of $L$ to qualitatively express fuzzy information. Based on linguistic variable, many fuzzy decision making methods have been proposed [3]–[6], in which assessments of alternatives provided by decision makers are represented by fuzzy sets or linguistic values.

Theoretically, IFS is an important extension of fuzzy set on the universe $U$, by using degrees of membership and nonmembership, an intuitionistic fuzzy set can represent more fuzzy information than a fuzzy set. Up to now,
many decision making methods with IFVs or IFSs have been widely researched [7]–[9] and applied in investment selection [10], personnel selection [11], credit risk evaluation [12], medical diagnosis [13], supplier selection [14], knowledge management system selection [15], plant location selection [16], outranking problems [17] and so on. In [18], Chen proposed a new fuzzy multi-attributed group decision making method based on IFVs and the evidential reasoning methodology, in which the evidential reasoning methodology is used to aggregate each decision matrix and get the IFV of alternative. In [19], Yao discussed linear orders of IFVs and presented an intuitionistic fuzzy weighted arithmetic average operator, accordingly a new model for intuitionistic fuzzy multi-attributes decision making was proposed. In [20], Tao investigated some operations on IFVs on the basis of Archimedean copulas and corresponding co-copulas, then a family of weighted aggregation operators for decision making with IFVs was developed. In [21], Liu developed interval-valued intuitionistic fuzzy power bonferroni aggregation operators to aggregate interval-valued IFSs in decision making problems. In [22], Zhang constructed a novel single-time ranking method on IFVs by designing a new intuitionistic fuzzy entropy and the closeness degree. In [23], Wan proposed a new risk attitudinal ranking method on IFSs according to the geometrical representation of IFS. In [24], Hashemi extended the ELECTRE method into ELECTRE III in interval-valued intuitionistic fuzzy environment, which is used to carry out intuitionistic fuzzy decision problems with a high degree of uncertainty. In [25], Wang provided TOPSIS method with n-intuitionistic polygonal fuzzy sets to solve corresponding multi-attribute decision making problems. In [26], Wan extended the classic VIKOR method to carry out multi-attribute group decision making problems with triangular intuitionistic fuzzy numbers.

Recently, because linguistic terms provide a more direct way to qualitatively express fuzzy information in decision making, linguistic decision making (LDM, assessments of alternatives provided by decision makers are represented by fuzzy linguistic terms) is closest to human beings’ cognitive process that occurs in daily life, and many researchers have paid attention to LDM problems [1], [5], [27]–[32]. Inspired by IFS, the concept of LIFS has been proposed and applied in LDM problems [33], operations of LIFSs and their fundamental properties are widely researched [34], [35]. In [36]–[38], Meng provided linguistic intuitionistic fuzzy preference relations, multiplicative linguistic intuitionistic fuzzy preference relations, heterogeneous intuitionistic fuzzy preference relations for linguistic intuitionistic fuzzy decision problems, furthermore an additive consistency and its desirable properties of these preference relations have been analyzed. In [39], based on linguistic scale functions, Zhang proposed an extended outranking approach for multi-criteria decision making problems with LIFSs. In [40], [41], Arora and Garg developed some prioritized aggregation operators to aggregate LIFSs in LDM problems. In [42], Garg defined some operational laws, score and accuracy functions of linguistic interval-valued intuitionistic fuzzy sets and proposed several aggregation operators to aggregate linguistic interval-valued intuitionistic fuzzy sets. In [43], Rong developed Muirhead mean operators and their dual operators to fuse LIFSs. In [44], Liu extended the Hamy Mean to aggregate LIFSs and discussed the properties of the two aggregation operators.

In real-world LDM problems, due to different knowledge level, background or experience, decision makers may provide multi-granular linguistic assessments of alternatives with respect to criteria. In multi-granular linguistic decision making methods, transformation functions are utilized to transform multi-granular linguistic terms from one linguistic hierarchy into other linguistic hierarchy [45]–[48], then linguistic aggregation operators or linguistic ordering methods are provided to carry out multi-granular linguistic decision problems. Similarly, in intuitionistic fuzzy decision environment, decision makers maybe provide IFV, IFS or LIFS assessments of alternatives with respect to criteria in the same intuitionistic fuzzy decision problem. In effect, by using membership and non-membership degrees on [0, 1], IFSs are utilized to represent quantitative intuitionistic fuzzy assessments of alternatives. By using membership and non-membership functions on [0, 1], IFSs are utilized to represent more complex quantitative intuitionistic fuzzy assessments of alternatives than IFVs. However, by using positive and negative membership linguistic terms on the primary linguistic term set $H = \{s_0, s_1, \cdots, s_g\}$, LIFSs are utilized to represent qualitative intuitionistic fuzzy assessments of alternatives with respect to criteria. From IFVs to IFSs and LIFSs, different intuitionistic fuzzy assessments of alternatives are provided by decision makers depending on their knowledge level, background or experience, the situation can arises especially in large-scale decision making environment based on big data, in which there are a huge number of decision makers, alternatives and criteria for intuitionistic fuzzy decision problems. The intuitionistic fuzzy decision making with IFVs, IFSs or LIFSs is called as the mixed intuitionistic fuzzy decision making (MIFDM) in the paper. Inspired by transformation functions in multi-granular linguistic decision making methods, in this paper, novel transformation functions are proposed to unify IFVs, IFSs or LIFSs, which are utilized to carry out MIFDM problems, major contributions of the paper are summarized as follows:

1) Intuitionistic fuzzy decision matrices with IFVs, IFSs or LIFSs are analysed, mixed intuitionistic fuzzy decision matrix refers to the decision matrix with IFVs, IFSs and LIFSs;

2) Transformation functions are proposed to unify IFVs, IFSs or LIFSs, their properties are analysed. Formally, the transformation functions can be utilized to unify mixed intuitionistic fuzzy decision matrix, then intuitionistic fuzzy aggregation operators or ordering methods can be utilized to carry out MIFDM problems;

3) The MIFDM method is presented by six main phases, accordingly an algorithm is designed. Examples in the
mixed intuitionistic fuzzy environment are illustrated to show the proposed MIFDM method.

The rest of this paper is structured as follows: In Section II, IFV, IFS, LIFS and intuitionistic fuzzy decision matrix are briefly reviewed. In Section III, transformation functions are proposed to unify IFVs, IFSs or LIFSs, their properties are analysed, these transformation functions can be used to unify mixed intuitionistic fuzzy decision matrix. In Section IV, the MIFDM method and an algorithm are proposed to carry out the MIFDM problems. Illustrative examples are in Section V to show transformation functions and the proposed MIFDM method. Section VI concludes the paper.

II. PRELIMINARIES

In the section, IFV, IFS, LIFS and intuitionistic fuzzy decision matrix are briefly reviewed.

A. INTUITIONISTIC FUZZY SET

The intuitionistic fuzzy set is proposed to describe membership and nonmembership uncertainty of object.

**Definition 1:** [49] Let a set \( U \) be fixed. An intuitionistic fuzzy set in \( U \) is an object having the form

\[
A = \{ u, (\mu_A(u), \nu_A(u)) \mid u \in U \},
\]

where functions \( \mu_A : U \rightarrow [0, 1] \) and \( \nu_A : U \rightarrow [0, 1] \) define the degree of membership and the degree of nonmembership of the element \( u \in U \) to \( A \subseteq U \), respectively. \( \mu_A \) and \( \nu_A \) should satisfy the condition:

\[
\forall u \in U, \quad 0 \leq \mu_A(u) + \nu_A(u) \leq 1.
\]

Theoretically, the IFS \( A \) is reduced to a fuzzy set \( A \) on \( U \) if \( \forall u \in U, \nu_A(u) = 1 - \mu_A(u) \), hence the notion of intuitionistic fuzzy set is an extension of the notion of fuzzy set. In addition, \( \pi_A(u) = 1 - \mu_A(u) - \nu_A(u) \) is called as the indeterminacy degree or the hesitancy degree of \( u \) to \( A \). Furthermore, for any fixed \( u \in U \), \( (\mu_A(u), \nu_A(u)) \) is called as an IFV [9], more general, for any \( v_1, v_2 \in [0, 1] \), \( (v_1, v_2) \) is an IFV if \( v_1 + v_2 \leq 1 \). For any two intuitionistic fuzzy set \( A = \{ u, (\mu_A(u), \nu_A(u)) \mid u \in U \} \) and \( B = \{ u, (\mu_B(u), \nu_B(u)) \mid u \in U \} \), the following operations and relations are valid [9], [49]:

1. \( A \subseteq B \) if and only if \( \forall u \in U, \mu_A(u) \leq \mu_B(u) \) and \( \nu_A(u) \geq \nu_B(u) \);
2. \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \);
3. \( \overline{A} = \{ u, (\mu_A(u), \nu_A(u)) \mid u \in U \} \);
4. \( A \cap B = \{ u, \min \{ \mu_A(u), \mu_B(u) \}, \max \{ \nu_A(u), \nu_B(u) \} \mid u \in U \} \);
5. \( A \cup B = \{ u, \max \{ \mu_A(u), \mu_B(u) \}, \min \{ \nu_A(u), \nu_B(u) \} \mid u \in U \} \);
6. \( A + B = \{ u, (\mu_A(u) + \mu_B(u) - \mu_A(u) \cdot \mu_B(u), \nu_A(u) + \nu_B(u) - \nu_A(u) \cdot \nu_B(u) \mid u \in U \} \);
7. \( A \cdot B = \{ u, (\mu_A(u) \cdot \mu_B(u), \nu_A(u) + \nu_B(u) - \nu_A(u) \cdot \nu_B(u) \mid u \in U \} \);
8. \( \lambda A = \{ u, (\lambda, \nu_A(u)) \mid u \in U \} \) for \( \lambda > 0 \);
9. \( A^\lambda = \{ u, (\mu_A(u))^\lambda, (1 - \mu_A(u))^\lambda \mid u \in U \} \) for \( \lambda > 0 \).

B. LINGUISTIC INTUITIONISTIC FUZZY SET

The linguistic intuitionistic fuzzy set is proposed to describe the qualitative intuitionistic uncertainty of object according to the primary linguistic term set \( H = \{ s_0, s_1, \ldots, s_9 \} \), such as \( H = \{ s_0 \) (extremely poor), \( s_1 \) (very poor), \( s_2 \) (poor), \( s_3 \) (slightly poor), \( s_4 \) (fair), \( s_5 \) (slightly good), \( s_6 \) (good), \( s_7 \) (very good), \( s_8 \) (extremely good)\}. The 2-tuple fuzzy linguistic representation model is proposed to describe continuous linguistic terms on \( H \), which has the form \((s_j, \alpha)\), where the symbolic translation \( \alpha \) of linguistic term \( s_j \) supports “the difference of information” between a counting of information \( \beta \) obtained from a symbolic aggregation operator and the closest linguistic term \( s_j \in S(j = \text{round}(\beta)) \). Theoretically, 2-tuple fuzzy linguistic representation model provides transformation functions between \([0, g]\) and 2-tuple linguistic terms on \( H \) [27]:

\[
\Delta : [0, g] \rightarrow H \times [-0.5, 0.5],
\]

\[
\beta \mapsto (s_j, \alpha),
\]

\[
\Delta^{-1} : H \times [-0.5, 0.5] \rightarrow [0, g],
\]

\[
(s_j, \alpha) \mapsto \text{round}(\beta) = j + \alpha.
\]

where \( j = \text{round}(\beta) \), \( \alpha = \beta - j \in [-0.5, 0.5] \) and \( \text{round}(\cdot) \) is the usual rounding operation, \( s_j \in H \) is the linguistic term which is mostly close to \( \beta \). In the paper, let all 2-tuple fuzzy linguistic terms on \( H \) be defined as follows.

**Definition 2:** Let a fixed set \( U \) and primary linguistic term set \( H = \{ s_0, s_1, \ldots, s_9 \} \). A linguistic intuitionistic fuzzy set on \( U \) is the form

\[
A_H = \{ u, (s_i, \alpha_i)(u), (s_j, \alpha_j)(u) \mid u \in U \},
\]

where 2-tuple linguistic terms \((s_i, \alpha_i)\) and \((s_j, \alpha_j)\) define positive membership linguistic and negative membership linguistic of the element \( u \in U \), \((s_i, \alpha_i)\) and \((s_j, \alpha_j)\) satisfy the condition:

\[
\forall u \in U, \quad 0 \leq \alpha_i + \alpha_j + j + \alpha_j \leq g.
\]

Compared LIFS \( A_H \) with IFS \( A \), there exists the following difference between them:

1. In Definition 1, \( U \) is the domain of membership and nonmembership functions \( \mu_A \) and \( \nu_A \), for any \( u \in U \), \( \mu_A(u) \) and \( \nu_A(u) \) are membership and nonmembership degrees of \( u \) in IFS \( A \). However, in Definition 2, \( \mu_A(u) \) and \( \nu_A(u) \) are replaced by positive and negative membership 2-tuple linguistic terms \((s_i, \alpha_i)\) and \((s_j, \alpha_j)\), i.e., \( \mu_A(u) \) and \( \nu_A(u) \) are described by 2-tuple linguistic terms \((s_i, \alpha_i)\) and \((s_j, \alpha_j)\), in which the domain of
generally, the alternatives-criteria decision matrix provided by decision maker is utilized to represent assessments of alternatives with respect to criteria, where rows of the matrix are alternatives and columns of the matrix are criteria; elements of the matrix are assessments provided by decision makers. Formally, a decision making problem is described as: decision makers $M = \{d_1, \cdots, d_m\}$ are asked to assess alternatives $A = \{a_1, \cdots, a_n\}$ with respect to criteria $C = \{c_1, \cdots, c_r\}$, where assessments provided by decision maker $d_i (i = 1, \cdots, m)$ are represented in the decision matrix

\[
D_i = (e_{jk}^i)_{n \times r} = \begin{pmatrix}
    e_{11}^i & \cdots & e_{1r}^i \\
    \vdots & \ddots & \vdots \\
    e_{n1}^i & \cdots & e_{nr}^i
\end{pmatrix}
\]

where $e_{jk}^i$ is the assessment of alternative $a_j$ with respect to criterion $c_k$. Based on the alternatives-criteria decision matrix, IFV, IFS and LIFS decision matrix can be distinguished as follows:

1) **IFV decision matrix:** In the matrix, each $e_{jk}^i$ is an IFV, i.e., decision maker $d_i$ provides IFVs $(\mu_{jk}^i, \nu_{jk}^i)$ to assess alternative $a_j$ with respect to criterion $c_k$, and $e_{jk}^i = (\mu_{jk}^i, \nu_{jk}^i)$ such that $\mu_{jk}^i \in [0, 1]$, $\nu_{jk}^i \in [0, 1]$ and $0 < \mu_{jk}^i + \nu_{jk}^i < 1$. In real-world decision making problems, $\mu_{jk}^i$ and $\nu_{jk}^i$ can be understood by numerical favour and against of alternative $a_j$ with respect to criterion $c_k$ provided by decision maker $d_i$, respectively.

2) **IFS decision matrix:** In the matrix, each $e_{jk}^i$ is an IFS, i.e., decision maker $d_i$ provides IFSs $(\mu_{jk}^i(x), \nu_{jk}^i(x))$ to assess alternative $a_j$ with respect to criterion $c_k$, and $e_{jk}^i = (\mu_{jk}^i(x), \nu_{jk}^i(x))$ such that $\mu_{jk}^i : [0, 1] \rightarrow [0, 1]$ and $\nu_{jk}^i : [0, 1] \rightarrow [0, 1]$ are membership and nonmembership functions and $0 < \mu_{jk}^i(x) + \nu_{jk}^i(x) \leq 1$ for any $x \in [0, 1]$. In real-world decision making problems, $\mu_{jk}^i(x)$ and $\nu_{jk}^i(x)$ on $[0, 1]$ can be understood by fuzzy set favour and against of alternative $a_j$ with respect to criterion $c_k$ provided by decision maker $d_i$, respectively. In addition, from $\mu_{jk}^i$ and $\nu_{jk}^i$ to “about $\mu_{jk}^i$ (i.e., fuzzy set $\mu_{jk}^i(x)$)” and “about $\nu_{jk}^i$ (i.e., fuzzy set $\nu_{jk}^i(x)$)”, IFV decision matrix can be transformed into IFS decision matrix;

3) **LIFS decision matrix:** In the matrix, each $e_{jk}^i$ is a LIFS, i.e., decision maker $d_i$ provides LIFSs $((\mu_{jk}^i, \lambda_{jk}^i), (\nu_{jk}^i, \rho_{jk}^i))$ to assess alternative $a_j$ with respect to criterion $c_k$, and $e_{jk}^i = ((\mu_{jk}^i, \lambda_{jk}^i), (\nu_{jk}^i, \rho_{jk}^i))$ such that $(\mu_{jk}^i, \lambda_{jk}^i) \in \overline{\Pi}$ and $(\nu_{jk}^i, \rho_{jk}^i) \in \overline{\Pi}$ defined on the primary linguistic term set $H = \{s_0, \cdots, s_k\}$ and $0 \leq i + j + \lambda_{jk}^i \leq g$. In real-world decision making problems, $(\mu_{jk}^i, \lambda_{jk}^i)$ and $(\nu_{jk}^i, \rho_{jk}^i)$ can be understood by linguistic favour and against of alternative $a_j$ with respect to criterion $c_k$ provided by decision maker $d_i$, respectively.

Theoretically, IFV decision matrix and IFS decision matrix are utilized to represent quantitative intuitionistic fuzzy assessments of alternatives, LIFS decision matrix is utilized to represent qualitative intuitionistic fuzzy assessments of alternatives. In big data or social computing environments, due to the lack of knowledge or familiarity with alternative and criteria, assessments of alternatives may be IFVs, IFSs or LIFSs depending on decision maker’s knowledge level, background or experience, on the other words, IFVs, IFSs and
LIFSs may be placed in the same intuitionistic fuzzy decision matrix, i.e., in Eq. (3),
\[ e_{jk}^{i} = (\mu_{jk}^{i}, v_{jk}^{i}) \]
\[ \text{if and only if } b \leq a' \text{ and } c \leq b' \text{ when } b \leq b' . \]

In the paper, the intuitionistic fuzzy decision matrix \( D \), with IFVs, IFSs and LIFSs also called as mixed intuitionistic fuzzy decision matrix.

**Example 1:** A customer evaluates jacket (\( J \)), trousers (\( T \)) and shirt (\( S \)) by online product reviews, where criteria are \( C = \{ \text{quality (c}_1\text{), price (c}_2\text{), fashion (c}_3\text{)} \} \), the primary linguistic term set is \( H = \{ s_0 \} \) extremely poor, \( s_1 \) very poor, \( s_2 \) (slightly poor), \( s_3 \) (fair), \( s_4 \) (slightly good), \( s_5 \) (good), \( s_6 \) (very good), \( s_8 \) (extremely good), their meanings are triangular fuzzy sets on [0, 1] shown in Figure 1. According to life habit and his/her familiarity with clothing, the customer may provide the following mixed intuitionistic fuzzy decision matrix:

\[
 D = \begin{pmatrix}
 (c_1) & (c_2) & (c_3) \\
 (\mu_1, v_1) & (\mu_2, v_2) & (\mu_3, v_3) \\
 (s_1, s_2) & (s_3, s_4) & (s_6, s_7)
\end{pmatrix}
\]

where \( (\mu_1, v_1) \), \( (\mu_2, v_2) \) and \( (\mu_3, v_3) \) are intuitionistic fuzzy sets on [0, 1], i.e., \( \mu_1 = (0.6, 0.7, 0.8) \), \( v_1 = (0.2, 0.3, 0.4) \), \( \mu_2 = (0.7, 0.8, 1) \), \( v_2 = (0.1, 0.2, 0.3) \), \( \mu_3 = (0.5, 0.6, 0.7) \) and \( v_3 = (0.2, 0.4, 0.6) \), which are shown in Figure 2. Intuitively, such as IFS \( (\mu_1, v_1) \) can also be understood as (about 0.7, about 0.3) provided by the customer.

Example 1 shows that the customer is not familiar to shirt because he/she provides LIFSs to assess shirt, the situation always occurs in our daily life. In effect, when it is the lack of knowledge or familiarity with alternative or criteria, linguistic terms are always utilized to represent uncertainty.

## III. THE TRANSFORMATION METHOD AMONG INTUITIONISTIC FUZZY MODELS

In the section, transformation functions are proposed to unify IFVs, IFSs and LIFSs in mixed intuitionistic fuzzy decision matrix.

For simplicity, membership and nonmembership functions of IFS \( e_{jk}^{i} = (\mu_{jk}^{i}(u), v_{jk}^{i}(u)) \) are limited by triangular fuzzy sets on [0, 1] in the paper, i.e., \( \mu_{jk}(u) \) (or \( v_{jk}(u) \)) is a 3-tuple \( (a, b, c) \) (or \( (a', b', c') \)), where \( a' \) (or \( a' \)) and \( c' \) (or \( c' \)) are the left and right boundaries of \( \mu_{jk}(u) \) (or \( v_{jk}(u) \)) such that \( \mu_{jk}(u) = 0 \) (or \( v_{jk}(u) = 0 \)) if \( b' \) is the kernel of \( \mu_{jk}(u) \) (or \( v_{jk}(u) \)) such that \( \mu_{jk}(u) = 1 \) (or \( v_{jk}(u) = 1 \)). Linguistic term \( s_i \in H = \{ s_0, s_1, \ldots, s_q \} \) has a triangular fuzzy set on [0, 1] as it’s meaning. For IFS \( e_{jk}^{i} = (\mu_{jk}^{i}(u), v_{jk}^{i}(u)) \), the following property can be obtained when \( \mu_{jk}^{i}(u) \) and \( v_{jk}^{i}(u) \) are triangular fuzzy sets on [0, 1].

**Property 1:** For any triangular fuzzy sets \( \mu_{jk}^{i}(u) = (a, b, c) \) and \( v_{jk}^{i}(u) = (a', b', c') \) on [0, 1], \( (\mu_{jk}^{i}(u), v_{jk}^{i}(u)) = ((a, b, c), (a', b', c')) \) is an intuitionistic fuzzy set if and only if \( b \leq a' \) and \( c \leq b' \).

**Proof:** 1) if \( (\mu_{jk}^{i}(u), v_{jk}^{i}(u)) = ((a, b, c), (a', b', c')) \) is an IFS, then for any \( u \in [0, 1] \), \( 0 \leq \mu_{jk}^{i}(u) + v_{jk}^{i}(u) \leq 1 \).

When \( b \leq b' \), suppose \( a' < b \), then for triangular fuzzy set \( \mu_{jk}^{i}(u) = (a', b', c') \), \( v_{jk}^{i}(u) > 0 \) due to \( a' < b \leq b' \), and \( \mu_{jk}^{i}(u) + v_{jk}^{i}(u) > 1 \), this is contradiction. Hence, \( b \leq a' \).

Similarly, suppose \( b' < c \), then for triangular fuzzy set \( \mu_{jk}^{i}(u) = (a, b, c) \) and \( v_{jk}^{i}(u) = (a', b', c') \) on [0, 1], if \( b \leq b' \) and \( c \leq b' \), then for any \( u \in [a', c] \), \( \mu_{jk}^{i}(u) + v_{jk}^{i}(u) \leq 1 \), this is contradiction. Hence, \( c \leq b' \).

2) For any triangular fuzzy sets \( \mu_{jk}^{i}(u) = (a, b, c) \) and \( v_{jk}^{i}(u) = (a', b', c') \) on [0, 1], if \( b \leq a' \) and \( c \leq b' \) when \( b \leq b' \), then for any \( u \in [a', c] \), \( v_{jk}^{i}(u) = 0 \), hence \( 0 \leq \mu_{jk}^{i}(u) + v_{jk}^{i}(u) \leq 1 \).

In IFS decision matrix, triangular fuzzy sets \( \mu_{jk}^{i}(u) = (a, b, c) \) and \( v_{jk}^{i}(u) = (a', b', c') \) of the IFS are understood by...
favour or against of alternative $a_j$ with respect to criterion $c_k$, in the paper, any IFS $(\mu_{jk}(u), v_{jk}(u)) = ((a, b, c), (a', b', c'))$ is specially limited by for any $u \in [0, 1]$,
\[ 0 \leq \mu_{jk}(u) + v_{jk}(u) \leq 1 \text{ and } 0 \leq b + b' \leq 1. \quad (4) \]

**A. THE TRANSFORMATION METHOD BETWEEN INTUITIONISTIC FUZZY VALUES AND INTUITIONISTIC FUZZY SETS**

Let all IFVs on $[0, 1]$ be $I' = \{ (\alpha, \beta) | \alpha \in [0, 1], \beta \in [0, 1] \}$ and $0 \leq \alpha + \beta \leq 1$ and all IFSs with triangular membership and nonmembership functions on $[0, 1]$ be $I_S = \{ (\mu(u), v(u)) = ((a, b, c), (a', b', c')) | \forall u \in [0, 1], 0 \leq \mu(u) + v(u) \leq 1, 0 \leq b + b' \leq 1 \}$. According to the maximum membership degree of fuzzy set theory [2], each IFS can be transformed into an IFV as follows:

\[ T_{SV} : I_S \longrightarrow I_V, (\mu(u), v(u)) \longmapsto (\alpha, \beta) = (b, b'), \quad (5) \]

According to Eq.(4), it is obvious that $T_{SV}((\mu(u), v(u)) = (b, b') = (\alpha, \beta)$ is an IFV on $[0, 1]$. In addition, there exist many triangular fuzzy sets on $[0, 1]$ such that their kernels are $b$ and $b'$, this means that transformation function $T_{SV}$ is many-to-one from $I_S$ into $I_V$. Such as in Example 1, IFS $(\mu_3(u), v_3(u))$ is transformed into IFV $(0.6, 0.4)$, i.e., $T_{SV}(\mu_3(u), v_3(u)) = (0.6, 0.4)$.

Conversely, for indeterminacy degree or hesitancy degree $\pi(u) = 1 - \mu(u) - v(u)$ of IFS $(\mu(u), v(u))$, based on the minimum $\pi(u)$ between $\alpha$ and $\beta$, each IFV in $I_V$ can be transformed into an IFS in $I_S$ as follows:

\[ T_{VS} : I_V \longrightarrow I_S, (\alpha, \beta) \longmapsto (\mu(u), v(u)), \quad (6) \]

where $(\mu(u), v(u)) = ((2\alpha - \beta, \alpha, \beta), (\alpha, 2\beta - \alpha))$ if $\alpha \leq \beta$, otherwise, $(\mu(u), v(u)) = ((\beta, \alpha, 2\alpha - \beta), (2\beta - \alpha, \beta, \alpha))$.

**Theorem 1**: For any $(\alpha, \beta) \in I_V$, $T_{SV}(\alpha, \beta)$ is an IFS of $I_S$.

If $\alpha \leq \beta$, then the indeterminacy degree of $T_{SV}(\alpha, \beta)$ is

\[ \pi(u) = 1 - \mu(u) - v(u) = \begin{cases} 1 - \mu(u), & u \in [0, \alpha], \\ 0, & u \in [\alpha, \beta], \\ 1 - v(u), & u \in [\beta, 1]. \end{cases} \quad (7) \]

If $\alpha > \beta$, then the indeterminacy degree of $T_{SV}(\alpha, \beta)$ is

\[ \pi(u) = 1 - \mu(u) - v(u) = \begin{cases} 1 - v(u), & u \in [0, \beta], \\ 0, & u \in [\beta, \alpha], \\ 1 - \mu(u), & u \in [\alpha, 1]. \end{cases} \quad (8) \]

**Proof**: 1) According to $(\alpha, \beta) \in I_V$, we have $0 \leq \alpha + \beta \leq 1$. According to Eq.(6), if $\alpha \leq \beta$, then $T_{SV}(\alpha, \beta) = (\mu(u), v(u)) = ((2\alpha - \beta, \alpha, \beta), (\alpha, 2\beta - \alpha))$. If $\alpha > \beta$, then $T_{SV}(\alpha, \beta) = (\mu(u), v(u)) = ((\beta, \alpha, 2\alpha - \beta), (2\beta - \alpha, \beta, \alpha))$. According to Property 1, for any $u \in [0, 1]$, we always have $0 \leq \mu(u) + v(u) \leq 1$, hence $T_{SV}(\alpha, \beta) \in I_S$, i.e., $T_{SV}(\alpha, \beta)$ is an IFS of $I_S$.

2) If $\alpha \leq \beta$, according to Property 1, $\mu(u) + v(u) = \mu(u)$ when $u \in [0, \alpha]$, hence $\pi(u) = 1 - \mu(u) - v(u) = 1 - \mu(u)$.

\[ \mu(u) + v(u) = v(u) \text{ when } u \in [\beta, 1], \text{ hence } \pi(u) = 1 - \mu(u) - v(u) = v(u). \text{ When } u \in [\alpha, \beta], \]

\[ \mu(u) = \frac{u - \beta}{\beta - \alpha}, \quad v(u) = \frac{u - \alpha}{\beta - \alpha}, \]

\[ \mu(u) + v(u) = 1 - \frac{u - \beta}{\beta - \alpha} + \frac{u - \alpha}{\beta - \alpha} = 1, \]

hence $\pi(u) = 1 - \mu(u) - v(u) = 0$.

If $\alpha > \beta$, according to Property 1, $\mu(u) + v(u) = v(u) \text{ when } u \in [0, \beta]$, hence $\pi(u) = 1 - \mu(u) - v(u) = 1 - v(u)$. $\mu(u) + v(u) = \mu(u) \text{ when } u \in [\alpha, 1]$, hence $\pi(u) = 1 - \mu(u) - v(u) = 1 - \mu(u)$. When $u \in [\beta, \alpha]$, \n
\[ \mu(u) = \frac{u - \beta}{\alpha - \beta}, \quad v(u) = \frac{u - \alpha}{\alpha - \beta}, \]

\[ \mu(u) + v(u) = 1 - \frac{u - \beta}{\alpha - \beta} - \frac{u - \alpha}{\alpha - \beta} = 1, \]

hence $\pi(u) = 1 - \mu(u) - v(u) = 0$.

Theorem 1 means that transformation function $T_{SV}$ can be used to transform IFVs into IFSs, in which $T_{SV}(\alpha, \beta)$ is an IFS of $I_S$ with the minimum indeterminacy degree or hesitancy degree between $\alpha$ and $\beta$.

Summary, functions $T_{SV}$ and $T_{VS}$ defined in Eqs.(5) and (6) provide transformation methods between IFVs and IFSs. However, functions $T_{SV}$ and $T_{VS}$ are not inverse each other, in effect, $T_{SV}$ is based on the maximum membership degree of fuzzy set theory, however $T_{VS}$ is depended on the minimum indeterminacy degree or hesitancy degree of IFS. For example, $T_{SV}(\mu_3(u), v_3(u)) = (0.6, 0.4)$ in Example 1, but $T_{VS}(0.6, 0.4) = ((0.4, 0.6, 0.8), (0.2, 0.4, 0.6)) \neq (\mu_3(u), v_3(u))$. Theoretically, the relationship between IFSs and IFVs is many-to-one, i.e., many IFSs can be transformed into the same IFV via the function $T_{SV}$, this coincides with representation of fuzzy set: different decision maker provides different shape of fuzzy set to represent the same fuzzy information [2].

**B. THE TRANSFORMATION METHOD BETWEEN INTUITIONISTIC FUZZY SET AND LINGUISTIC INTUITIONISTIC FUZZY SET**

Let the primary linguistic term set $H = \{ s_0, \cdots, s_k \}$ with triangular fuzzy sets on $[0, 1]$ as their meanings, i.e., $F_H = \{ \mu_0(u) = ((a_0, b_0, c_0), \mu_1(u) = (a_1, b_1, c_1), \cdots, \mu_k(u) = (a_k, b_k, c_k)) | u \in [0, 1] \}$ such that $0 \leq b_0 < b_1 < \cdots < b_k \leq 1$. $I_L = \{ (s_0, s_0), (s_1, s_0), \cdots, (s_k, s_0) \} \subset H$, $(s_i, s_j) \in H$ is the set of all LIFs on $H$. Inspired by the constructing and managing multi-granular linguistic values method proposed in [48], any IFS $(\mu(u), v(u)) = ((a, b, c), (a', b', c'))$ on $[0, 1]$ can be transformed into a LIFS on $H$ as follows:

\[ T_{SL} : I_S \longrightarrow I_L, (\mu(u), v(u)) \longmapsto ((s_i, s_i), (s_i, s_j)), \quad (9) \]

where $(s_i, s_i)$ and $(s_i, s_j)$ are decided by

\[ |b - b_i| = \min \{ |b - b_0|, |b - b_1|, \cdots, |b - b_k| \}, \]

\[ (s_i, s_i) = \begin{cases} \Delta((b - b_i) + b_i), & |b - b_i| \leq 0, \\ \Delta((b - b_i)(1+i) + b_i - b_{i+1}), & |b - b_i| > 0. \end{cases} \]
\[ |b' - b_j| = \min[|b' - b_0|, |b' - b_1|, \ldots, |b' - b_{g_j}|], \]

\[ (s_j, a_j) = \begin{cases} \Delta \left( \frac{(b_j - b_{j-1})}{b_{j-1} - b_{j-2}} + \frac{(b_j - b_{j+1})}{b_{j+1} - b_{j+2}} \right), & \text{if } b' - b_j \leq 0, \\
\Delta \left( \frac{(b_{j-1} - b_0)}{b_{j-1} - b_{j-2}} + \frac{(b_{j+2} - b_{j+1})}{b_{j+2} - b_{j+1}} \right), & \text{if } b' - b_j > 0. \end{cases} \]

Such as in Example 1, for IFS \((\mu_3, v_3) = ((0.5, 0.6, 0.7), 0.2, 0.4, 0.6))\), due to \([0.6 - b_3] = [0.6 - 0.625] = \min[0.6 - b_0], \ldots, 0.6 - b_8\] and \([0.4 - b_3] = [0.4 - 0.375] = \min[0.4 - b_0], \ldots, 0.4 - b_8\], hence

\[ \Delta \left( \frac{4(b_3 - 0.6)}{b_3 - b_4} + \frac{5(0.6 - b_4)}{b_4 - b_5} \right) = \Delta(4.8) = (s_5, 0.2), \]

\[ \Delta \left( \frac{4(0.4 - b_3)}{b_4 - b_3} + \frac{3(0.4 - 0.4)}{b_4 - b_3} \right) = \Delta(3.2) = (s_3, 0.2), \]

\(i.e., T_{SL}(\mu_3, v_3) = ((s_5, -0.2), (s_3, 0.2))\).

**Property 2:** Let the primary linguistic term set \(H = \{s_0, s_1, \ldots, s_g\}\) with triangular fuzzy set \(F_H = \{\mu_0(u) = (a_0, b_0, c_0), \ldots, \mu_g(u) = (a_g, b_g, c_g)\}(u \in [0, 1])\). For any IFS \((\mu(u), v(u))\) with triangular membership and nonmembership functions on [0, 1], if \((\mu(u), v(u)) = ((a, b, c), (a', b', c'))\), then \(T_{SL}(\mu(u), v(u)) = (s_i, s_j)\).

**Proof:** In Eq.(9), if \(b = b_1 \) and \(b' = b_1\), then \(|b - b_1| = 0 = \min(|b - b_0|, \ldots, |b - b_{g_j}|)\) and \(|b' - b_1| = 0 = \min(|b' - b_0|, \ldots, |b' - b_{g_j}|)\). In addition, \(\Delta \left( \frac{(b_1 - b_{g_j})}{b_1 - b_{g_j}} + \frac{(b_{g_j} - b_1)}{b_{g_j} - b_1} \right) = \Delta(i) = s_i\) and \(\Delta \left( \frac{(b_1 - b_{g_j})}{b_1 - b_{g_j}} + \frac{(b_{g_j} - b_1)}{b_{g_j} - b_1} \right) = \Delta(j) = s_j\), hence \(T_{SL}(\mu(u), v(u)) = (s_i, s_j)\).

Conversely, based on transformation functions \(T_{LS}^\prime\) defined in Eq.(10) and \(T_{SV}^\prime\) defined in Eq.(5), transformation function \(T_{LV}^\prime\) from LIFSs on \([0, 1]\) into IFVs on \([0, 1]\) can be obtained as follows.

\[ T_{LV} : I_L \rightarrow I_L, ((s_i, a_i), (s_j, a_j)) \mapsto (\alpha, \beta), \]

where \((\alpha, \beta, \gamma) = T_{SV}(T_{LS}((s_i, a_i), (s_j, a_j)))\). Such as in Example 1, LIFS \(((s_7, 0.2), (s_1, -0.3))\) can be transformed into IFV

\[ T_{SV}(T_{LS}((s_6, 0.2), (s_2, -0.3))) = T_{SV}((0.65, 0.775, 0.9), (0.0875, 0.2125, 0.3375)) = (0.775, 0.2125). \]

The relationships of transformation functions \(T_{SV}, T_{VS}, T_{SL}, T_{LS}, T_{VL}\) and \(T_{LV}\) is shown in Figure.3, which can be utilized to carry out mixed intuitionistic fuzzy decision making problems in big data or social computing environment.

**IV. THE MIXED INTUITIONISTIC FUZZY DECISION MAKING METHOD**

Generally, ones utilize affirmation, negation and hesitation to express uncertainty in real world decision making. The fuzzy set \([2]\) is an useful tool to model uncertain decision information by using membership degrees (affirmation). To express negation and hesitation, the IFS \([49]\) had been proposed by using membership degree, non-membership degree (negation) and indeterminacy (hesitation). Up to now, the IFSs, including such as IFVs, interval-valued IFSs or LIFSs, have been widely applied in decision making process to express human’s uncertainty from the aspects of affirmation, negation and hesitation \([9]\). Due to different knowledge level, back-ground or experience, decision makers maybe utilize IFVs, IFSs or LIFSs to assess alternatives with respect to criterion. In the section, the mixed intuitionistic fuzzy decision making method is proposed to carry out mixed intuitionistic fuzzy decision making problems.

| TABLE 1. The triangular fuzzy sets on [0, 1] of primary linguistic terms. |
|-------------------|-------------------|-------------------|-------------------|
| \(s_0\)       | \(s_1\)       | \(s_2\)       |
| (0, 0.125)     | (0, 0.125, 0.25) | (0.125, 0.25, 0.375) |
| \(s_3\)       | (0.25, 0.375, 0.5) | (0.375, 0.5, 0.625) |
| \(s_4\)       | (0.625, 0.75, 0.875) | (0.75, 0.875, 1) |
| \(s_5\)       | (0.875, 1, 1) | (0.875, 1, 1) |
Formally, classical decision making process are consisted of five main phases [1]: a) Intelligence: It observes reality to identify the problem, alternatives and objectives to be achieved by its solving process; b) Modelling: It builds a model that defines a framework establishing the problem structure, preferences, uncertainty, and so on; c) Information gathering: It obtains the information, knowledge, and preferences provided by decision makers according to the model previously defined; d) Analysis: It analyses and aggregates the information gathered according to the objectives and constraints, and reports results to be considered in the selection phase; e) Selection: According to the results obtained in the analysis step, an exploitation process is carried out, in which decision makers can choose the solution alternatives for the decision problem.

Inspired by the five phases, the mixed intuitionistic fuzzy decision making method can be described as the follow six phases:

1) **Intelligence**: It observes reality and provides alternatives criteria decision matrix to identify the decision making problem, alternatives and objectives to be achieved by its solving process;

2) **Modelling**: It builds IFVs on [0, 1], IFSs with triangular membership and nonmembership functions on [0, 1] and LIFSs on the primary linguistic term set $H$, which are used to establish the decision making problem structure, preferences, uncertainty, and so on;

3) **Information gathering**: It obtains the information, knowledge, and preferences provided by decision makers according to the decision matrix and intuitionistic fuzzy models previously defined;

4) **Normalization**: It provides transformation functions to unify IFV, IFS or LIFS assessments provided by decision makers, the transformed results are utilized in the analysis phase;

5) **Analysis**: It analyses or aggregates the intuitionistic fuzzy information according to the objectives and constraints, and reports results to be considered in the selection phase;

6) **Selection**: According to the intuitionistic fuzzy results obtained in the analysis step, an exploitation process is carried out, in which decision makers can choose the solution alternatives for the mixed intuitionistic fuzzy decision problem.

Compared with the five main phases of classical decision making process, **Normalization** of the mixed intuitionistic fuzzy decision making method is new phase. In effect, decision makers own themselves knowledge level, background or experience in big data or social computing environment, hence IFVs, IFSs or LIFSs may be utilized to represent intuitionistic fuzzy assessments of alternatives in the same decision problem. Theoretically, the normalization phase can transform a mixed intuitionistic fuzzy decision matrix $D_i = (e^{j}_{ik})_{n \times r}$ into the unified intuitionistic fuzzy decision matrix, i.e.,

$$
D_i' = (v^{j}_{ik})_{n \times r},
$$

where

$$
v^{j}_{ik} = \begin{cases} 
\mu^{j}_{ik}, & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}, \nu^{j}_{ik}), \\
T_{SV}(e^{j}_{ik}), & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}(u), \nu^{j}_{ik}(u)).
\end{cases}
$$

(13)

$$
D_i'' = (v^{j}_{ik})_{n \times r},
$$

where

$$
v^{j}_{ik} = \begin{cases} 
T_{VS}(e^{j}_{ik}), & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}, \nu^{j}_{ik}), \\
e^{j}_{ik}, & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}(u), \nu^{j}_{ik}(u)), \\
T_{LS}(e^{j}_{ik}), & \text{if } e^{j}_{ik} = (\alpha_i, \beta_i).
\end{cases}
$$

(14)

$$
D_i''' = (v^{j}_{ik})_{n \times r},
$$

where

$$
v^{j}_{ik} = \begin{cases} 
T_{SL}(T_{VS}(e^{j}_{ik})), & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}, \nu^{j}_{ik}), \\
T_{SL} e^{j}_{ik}, & \text{if } e^{j}_{ik} = (\mu^{j}_{ik}(u), \nu^{j}_{ik}(u)), \\
e^{j}_{ik}, & \text{if } e^{j}_{ik} = (\alpha_i, \beta_i).
\end{cases}
$$

(15)

According to the six phases of the mixed intuitionistic fuzzy decision making method and transformed decision matrices Eqs.(13)-(15), Algorithm 1 can be designed to carry out a mixed intuitionistic fuzzy decision making problem.

**V. ILLUSTRATIVE EXAMPLE**

In the section, two examples of the mixed intuitionistic fuzzy decision making are considered to show the proposed decision making method.

**Example 2**: A company is to plan the development of large projects for the following five years, there are three possible projects $a_j (j = 1, 2, 3)$ assessed by $c_1$ (financial perspective), $c_2$ (customer satisfaction), $c_3$ (internal business process perspective) and $c_4$ (learning and growth perspective), and their weights are 0.2, 0.3, 0.1 and 0.4, respectively. The primary linguistic term set is $H = \{s_0\}$ (extremely poor), $s_1$ (very poor), $s_2$ (poor), $s_3$ (slightly poor), $s_4$ (fair), $s_5$ (slightly good), $s_6$ (good), $s_7$ (very good), $s_8$ (extremely good), their triangular fuzzy sets on [0, 1] are shown in Figure.1 or Table 1.

Assessments of three projects are evaluated by IFVs on [0, 1],

### Table 2. Assessments of three projects evaluated by $c_1$, $c_2$, $c_3$ and $c_4$.

| $a_1$   | $c_1\{0.2\}$ | $c_2\{0.3\}$ | $c_3\{0.1\}$ | $c_4\{0.4\}$ |
|--------|----------------|----------------|----------------|----------------|
| $(0.2, 0.3, 0.4), (0.4, 0.5, 0.6)$ | $(0.3, 0.4, 0.5)$ | $(0.1, 0.4, 0.5)$ | $(0.3, 0.45, 0.6), (0.4, 0.55, 0.8)$ |
| $(0.35, 0.3, 0.4), (0.4, 0.5, 0.6)$ | $(0.35, 0.4, 0.5)$ | $(0.1, 0.4, 0.5)$ | $(0.3, 0.45, 0.6), (0.4, 0.55, 0.8)$ |
| $(0.4, 0.4, 0.5)$ | $(0.6, 0.5)$ | $(0.6, 0.4)$ | $(0.6, 0.4)$ |

### Table 3. The IFV decision matrix.

| $a_1$   | $c_1\{0.2\}$ | $c_2\{0.3\}$ | $c_3\{0.1\}$ | $c_4\{0.4\}$ |
|--------|----------------|----------------|----------------|----------------|
| $(0.3, 0.5)$ | $(0.425, 0.55)$ | $(0.7, 0.2)$ | $(0.45, 0.55)$ |
| $(0.537, 0.45)$ | $(0.66, 0.3)$ | $(0.4, 0.6)$ | $(0.6, 0.4)$ |
| $(0.4, 0.4, 0.5)$ | $(0.6, 0.3)$ | $(0.8375, 0.15)$ | $(0.675, 0.3)$ |

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TABLE 4. The IFS decision matrix.

|     | $a_1$ | $a_2$ | $a_3$ |
|-----|-------|-------|-------|
| $c_1$ | $(0.2,0.3,0.4),(0.4,0.5,0.6)$ | $(0.4125,0.5375,0.6625),(0.325,0.45,0.575)$ | $(0.4,0.4,0.5),(0.4,0.5,0.6)$ |
| $c_2$ | $(0.2875,0.4125,0.5375),(0.425,0.55,0.675)$ | $(0.4,0.65,0.7),(0.2,0.3,0.5)$ | $(0.4,0.6,0.9),(0.0,0.3,0.6)$ |
| $c_3$ | $(0.2,0.7,1),(0.0,0.2,0.7)$ | $(0.1,0.4,0.6),(0.0,0.6,0.8)$ | $(0.7125,0.8375,0.9625),(0.25,0.15,0.275)$ |
| $c_4$ | $(0.3,0.45,0.6),(0.4,0.55,0.8)$ | $(0.4,0.6,0.8),(0.2,0.4,0.6)$ | $(0.55,0.675,0.8),(0.175,0.3,0.425)$ |

TABLE 5. The LIFS decision matrix.

|     | $c_1(0.2)$ | $c_2(0.3)$ | $c_3(0.1)$ | $c_4(0.4)$ |
|-----|-------------|-------------|-------------|-------------|
| $a_1$ | $((-2,0.4),(0.4,0.4))$ | $((-1,0.3),(0.4,0.4))$ | $((-0.6,0.4),(0.2,0.4))$ | $((-1,0.4),(0.4,0.4))$ |
| $a_2$ | $((-1,0.3),(0.4,0.4))$ | $((-1,0.2),(0.4,0.4))$ | $((-0.6,0.2),(0.2,0.4))$ | $((-1,0.4),(0.4,0.4))$ |
| $a_3$ | $((-1,0.2),(0.4,0.4))$ | $((-0.6,0.2),(0.2,0.4))$ | $((-0.3,0.2),(0.2,0.4))$ | $((-1,0.4),(0.4,0.4))$ |

TABLE 6. Aggregation results of three projects and the best project based on IFV, IFS and LIFS decision methods.

| Intuitionistic fuzzy value method | Intuitionistic fuzzy set method | Linguistic intuitionistic fuzzy method |
|---------------------------------|---------------------------------|-------------------------------------|
| $a_1$ | $(0.4375,0.505)$ | $(0.4375,0.505,0.58125)$ | $(0.3675,0.505,0.7125)$ |
| $a_2$ | $(0.5825,0.35)$ | $(0.3725,0.5825,0.7225)$ | $(0.245,0.4,0.585)$ |
| $a_3$ | $(0.61375,0.325)$ | $(0.44125,0.61375,0.78625)$ | $(0.1525,0.325,0.4975)$ |

Order on projects: $a_1 < a_2 < a_3$

The best project: $a_3$

Algorithm 1 Solving a Mixed Intuitionistic Fuzzy Decision Making Problem

**Input:** All mixed intuitionistic fuzzy decision matrices.

**Output:** Solution set of the best alternatives.

**Method:**

1. Let mixed intuitionistic fuzzy decision matrix $D_i = (e_{jk}^i)_{n \times r}(i = 1, \ldots, m)$.
2. For $i := 1 : m$ do
   3. $D_i^V := \{v_{jk}^i\}_{n \times r}$ by Eq.(13);
   4. $D_i^F := \{f_{jk}^i\}_{n \times r}$ by Eq.(14);
   5. $D_i^L := \{l_{jk}^i\}_{n \times r}$ by Eq.(15).
8. end

9. For $i := 1 : m$ do
   7. analysing or aggregating all $D_i^V, D_i^F$ or $D_i^L$.
11. end

B. USING IFS DECISION METHOD

By utilizing Algorithm 1, the mixed intuitionistic fuzzy decision matrix (Table 2) can be transformed into the IFV decision matrix shown in Table 3.

In the analysis phase, according to weights of $c_1, c_2, c_3$ and $c_4$, the following IFV aggregation operator is adopted to aggregate assessments of each project $a_j (j = 1, 2, 3)$, i.e.,

$$A(((a_k, b_k), w_k)|k = 1, 2, 3, 4)) = \left(\sum_{k=1}^{4} w_k a_k, \sum_{k=1}^{4} w_k b_k\right)$$

such as for project $a_1$, $A(((a_k, b_k), w_k)|k = 1, 2, 3, 4)) = (0.2 \times 0.2 + 0.3 \times 0.2875 + 0.1 \times 0.2 + 0.4 \times 0.3 = 0.26625$.

Results of $a_1, a_2$ and $a_3$ are shown in Table 6.
In the selection phase, ordering on IFSs [49] is adopted to order three projects according to aggregation results, i.e., for any IFSs $((a_1, b_1, c_1), (a_1', b_1', c_1'))$ and $((a_2, b_2, c_2), (a_2', b_2', c_2'))$, $(a_1, b_1, c_1) \preceq (a_2, b_2, c_2)$ and $(a_1', b_1', c_1') \preceq (a_2', b_2', c_2')$ if and only if $(a_1, b_1, c_1) \leq (a_2, b_2, c_2)$ and $(a_1', b_1', c_1') \leq (a_2', b_2', c_2')$, where triangular fuzzy sets are ordered by the centroid of their membership functions on $[0, 1]$, i.e.,

$$C(\mu(u)) = \frac{1}{\int_0^1 \mu(\mu(u))du} \int_0^1 \mu(\mu(u))du.$$  

For triangular fuzzy set $\mu(u) = (a, b, c)$, its centroid is $C(a, b, c) = \frac{a+b+c}{3}$. Hence for any two triangular fuzzy sets $((a_1, b_1, c_1)$ and $(a_2, b_2, c_2)$, $(a_1, b_1, c_1) \preceq (a_2, b_2, c_2)$ if and only if $a_1+b_1+c_1 \leq a_2+b_2+c_2$. The ordering of projects $a_1$, $a_2$ and $a_3$ based on IFSs is shown in Table 6, accordingly, the best project can be selected as $a_3$.

### C. USING LIFS DECISION METHOD

By utilizing Algorithm 1, Table 2 can be transformed into the LIFS decision matrix (shown in Table 5).

In the analysis phase, according to weights of $c_1$, $c_2$, $c_3$ and $c_4$, the following LIFS aggregation operator is adopted to aggregate assessments of each project $a_j$ ($j = 1, 2, 3$),

$$A((s_{i1}^k, \alpha_{i1}^k), (s_{i2}^k, \alpha_{i2}^k), (s_{i3}^k, \alpha_{i3}^k), (s_{i4}^k, \alpha_{i4}^k), w_k)\mid i = 1, 2, 3, 4)) = \left(\sum_{k=1}^{4} w_k(s_{i1}^k, \alpha_{i1}^k), \sum_{k=1}^{4} w_k(s_{i2}^k, \alpha_{i2}^k)\right),$$

such as for project $a_1$, $\sum_{k=1}^{4} w_k(s_{i1}^k, \alpha_{i1}^k) = (0.2 \times s_2 - 0.4) + 0.3 \times (s_3 - 0.3) + 0.1 \times (s_6 - 0.4) + 0.4 \times (s_4 - 0.4) = \Delta(0.2 \times 2.16 + 0.3 \times 3.3 + 0.1 \times 5.6 + 0.4 \times 3.6) = (s_3, 0.47)$. Results of projects $a_1$, $a_2$ and $a_3$ are shown in Table 6.

In the selection phase, ordering on LIFSs is adopted to order three projects according to linguistic aggregation results, i.e., for any two LIFSs $((s_{i1}, \alpha_{i1}),(s_{i2}, \alpha_{i2}))$ and $((s_{i2}, \alpha_{i2}),(s_{i3}, \alpha_{i3}),(s_{i4}, \alpha_{i4})) \preceq ((s_{i3}, \alpha_{i3}),(s_{i4}, \alpha_{i4}),(s_{i5}, \alpha_{i5}))$ and $((s_{i4}, \alpha_{i4}),(s_{i5}, \alpha_{i5}),(s_{i6}, \alpha_{i6}),(s_{i7}, \alpha_{i7}),(s_{i8}, \alpha_{i8})) \preceq ((s_{i5}, \alpha_{i5}),(s_{i6}, \alpha_{i6}),(s_{i7}, \alpha_{i7}),(s_{i8}, \alpha_{i8}),(s_{i9}, \alpha_{i9}))$ if and only if $s_1 + s_2 \leq s_5 + s_6$ and $s_3 + s_4 \leq s_7 + s_8$ and $s_4 + s_5 \leq s_8$. The ordering of projects $a_1$, $a_2$ and $a_3$ based on LIFSs is shown in Table 6, accordingly, the best project can be selected as $a_3$.

It can be seen from Table 7 that the ordering on three projects and the best project are same for IFV, IFS and LIFS decision methods, i.e., $a_1 \prec a_2 \prec a_3$ and the best project is $a_3$. This means that the proposed transformation methods can not change the ordering on alternatives and the best alternative of the mixed intuitionistic fuzzy decision making problem. In other words, the proposed transformation functions are consistent in mixed intuitionistic fuzzy environment.

**Example 3**: Shared bikes are useful and effective schemes to solve short trips in Chinese, many shared bikes are rapidly appeared over the past several years, such as “mobike” $(a_1)$, “hellowbike” $(a_2)$ and “green orange” $(a_3)$. User experience of shared bike is important for development of shared bike, in the example, “Fund Security” $(c_1)$, “Riding Safety” $(c_2)$, “Riding Convenience” $(c_3)$ and “Riding Comfort” $(c_4)$ as criteria are utilized to assess user experience of shared bike, an university student $(d_1)$, a citizen $(d_2)$ and an office worker $(d_3)$ are asked to provide their experiences of the four shared bikes with respect to the four criteria. The primary linguistic term set is $H = \{s_0 \text{ (extremely poor)}, s_1 \text{ (very poor)}, s_2 \text{ (poor)}, s_3 \text{ (slightly poor)}, s_4 \text{ (fair)}, s_5 \text{ (slightly good)}, s_6 \text{ (good)}, s_7 \text{ (very good)}, s_8 \text{ (extremely good)}\}$, their triangular fuzzy sets on $[0, 1]$ is shown in Figure.1 or Table 1. Assessments of user experiences provided by $d_1$, $d_2$ and $d_3$ are shown in Tables 7-9, in which $d_1$ likes to provide IFV assessments of user experiences, $d_2$ likes to provide LIFS assessments of user experiences and $d_3$ likes to provide IFS assessments of user experiences. Suppose that weights of the four criteria are $0.3, 0.3, 0.2$ and $0.2$, respectively. According to IFV, IFS and LIFS aggregation operators in Example 2, aggregation results of user experiences provided by $d_1$, $d_2$ or

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**TABLE 9.** Aggregation results of user experiences provided by $d_1$, $d_2$ or $d_3$.

| $a_1$ | $a_2$ | $a_3$ |
|-------|-------|-------|
| $c_1(0.3)$ | $c_2(0.3)$ | $c_3(0.3)$ |
| $(0.5, 0.7, 0.8), (0.1, 0.3, 0.4)$ | $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ | $(0.6, 0.8, 1), (0.2, 0.3, 0.4)$ |
| $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ | $(0.6, 0.7, 0.8), (0.2, 0.3, 0.4)$ | $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ |
| $(0.6, 0.8, 0.9), (0.2, 0.3, 0.4)$ | $(0.6, 0.7, 0.8), (0.2, 0.3, 0.4)$ | $(0.6, 0.8, 1), (0.2, 0.3, 0.4)$ |
| $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ | $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ | $(0.6, 0.8, 1), (0.1, 0.2, 0.4)$ |

**TABLE 10.** The final aggregation results of user experiences provided by $d_1$, $d_2$ or $d_3$.

| $a_1$ | $a_2$ | $a_3$ |
|-------|-------|-------|
| $d_1$ | $d_2$ | $d_3$ |
| $(0.75, 0.22)$ | $(0.63, 0.32)$ | $(0.64, 0.33)$ |
| $(0.57, 0.75, 0.99)$ | $(0.6, 0.7, 0.79)$ | $(0.54, 0.64, 0.81)$ |
| $(0.12, 0.25, 0.42)$ | $(0.25, 0.42)$ | $(0.33, 0.53)$ |

**TABLE 11.** The final aggregation results of user experiences provided by $d_1$, $d_2$ and $d_3$ with respect to the four criterias.

| $a_1$ | $a_2$ | $a_3$ |
|-------|-------|-------|
| IFV results | IFS results | LIFS results |
| $a_1$ | $a_2$ | $a_3$ |
| $(0.77, 0.21)$ | $(0.69, 0.29)$ | $(0.72, 0.25)$ |
| $(0.49, 0.77, 0.95)$ | $(0.49, 0.69, 0.91)$ | $(0.51, 0.72, 0.94)$ |
| $(0.0525, 0.21, 0.48)$ | $(0.13, 0.29, 0.51)$ | $(0.07, 0.25, 0.46)$ |

The best selection $a_1 > a_2 > a_3$
$d_3$ can be obtained, which are shown in Table 10, accordingly, ordering on shared bikes and the best selection can be obtained. It can be noticed from Table 10 that the best selection of $d_1$ is “mobike”, the best selection of $d_2$ is “green orange” and the best selection of $d_3$ is “hellobike”.

When we want to aggregate results provided by $d_1$, $d_2$ and $d_3$ based on Table 10, we can do nothing because there is no any aggregation operator to aggregate IFVs, IFSs and LIFVs at the same time. However, based on transformation functions shown in Figure.3, IFVs, IFSs and LIFVs in Table 10 can be unified, if weights or important degrees of $d_1$, $d_2$ and $d_3$ are 1/3, respectively, then the aggregation results of user experiences provided by $d_1$, $d_2$ and $d_3$ with respect to the four criteria can be obtained and the best shared bike can be selected according to user experiences, i.e., the best shared bike is “mobike”, all of these are shown in Table 11.

VI. CONCLUSION

Due to knowledge level, background or experience of decision makers, IFVs, IFSs or LIFSs may be utilized by decision makers to assess alternatives with respect to criteria in the mixed intuitionistic fuzzy decision making. To analyse and aggregate mixed intuitionistic fuzzy assessments in intuitionistic fuzzy decision matrices, transformation functions are proposed to unify IFVs, IFSs and LIFSs, properties of transformation functions are analysed. Inspired by classical decision making process, the mixed intuitionistic fuzzy decision making method is presented, and an algorithm is designed to carry out a mixed intuitionistic fuzzy decision making problem. Illustrative examples show that the proposed transformation functions are consistent in mixed intuitionistic fuzzy environment, which is suitable for solving the mixed intuitionistic fuzzy decision making problems.

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