On reverberation and cross-correlation estimates of the size of the broad-line region in active galactic nuclei

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Accepted 2008 June 18. Received 2008 June 18; in original form 2007 May 11

ABSTRACT

It is known that the dependence of the emission-line luminosity of a typical cloud in the active galactic nuclei (AGN) broad-line regions (BLRs) upon the incident flux of ionizing continuum can be non-linear. We study how this non-linearity can be taken into account in estimating the size of the BLR by means of the ‘reverberation’ methods. We show that the BLR size estimates obtained by cross-correlation of emission-line and continuum light curves can be much (up to an order of magnitude) less than the values obtained by reverberation modelling. This is demonstrated by means of numerical cross-correlation and reverberation experiments with model continuum flares and emission-line transfer functions and by means of practical reverberation modelling of the observed optical spectral variability of NGC 4151. The time behaviour of NGC 4151 in the Hα and Hβ lines is modelled on the basis of the observational data by Kaspi et al. and the theoretical BLR model by Shevchenko. The values of the BLR parameters are estimated that allow to judge on the size and physical characteristics of the BLR. The small size of the BLR, as determined by the cross-correlation method from the data of Kaspi et al., is shown to be an artefact of this method. So, the hypothesis that the BLR size varies in time is not necessitated by the observational data.

Key words: galaxies: active – galaxies: individual: NGC 4151 – galaxies: nuclei – galaxies: Seyfert.

1 INTRODUCTION

In the early 1970s, in the course of observations of rapid variability of the optical spectrum of the Seyfert galaxy NGC 4151, the time lag of variations in the Hα line with respect to variations in the optical continuum was discovered (Lyutyi & Cherepashchuk 1971; Cherepashchuk & Lyutyi 1973). The time lag was interpreted by Lyutyi (1977, 1982) as a consequence of the fact that the emission-line clouds are at some distance from the ionizing radiation source. Later on, Antonucci & Cohen (1983) observed much smaller time lags in variations of the Hβ and Hγ lines. Such a difference in the time lag, in Hα greater than in Hβ, is observed in other active galactic nuclei (AGN) as well (see table 6 in Peterson et al. 2004). According to Shevchenko (1984, 1985a), this difference in time lag is due to an essential non-linearity in the dependence of the Hα luminosity of an individual cloud upon the ionizing continuum flux incident on the cloud, the dependence in the higher order Balmer lines being close to linear. This explanation was made with the assumption that the duration of the emission-line flare is much greater than the duration of the flare in the ionizing continuum, and the duration of the latter one allows its description by δ function.

However, continuum variations are not so rapid usually; therefore, in order to extract physical information from the observed emission-line variations, it is necessary, in addition to theoretical estimates, to use numerical modelling taking into account the time-scale of continuum variations.

In the present paper, we study theoretically how the non-linearity in the emission-line luminosity, $L_i$, of the broad-line region (BLR) cloud, in its dependence on the ionizing continuum flux, $F_i$, incident on the cloud, can be taken into account in estimating the BLR sizes by means of the ‘reverberation’ methods. We show that the BLR size estimates obtained by straightforward cross-correlation of emission-line and continuum light curves can be much (up to an order of magnitude) less than those obtained by reverberation modelling. First of all, we demonstrate this by means of abstract representative numerical cross-correlation and reverberation experiments with model continuum flares and emission-line transfer functions. Then we accomplish practical numerical modelling of the light curves of NGC 4151 in the Hα and Hβ lines on the basis of the observational data by Kaspi et al. (1996) and the theoretical model of the BLR by Shevchenko (1984, 1985a). This model is characterized by allowing for thick geometries of the BLR, taking into account the anisotropy of line emission of individual clouds and, most important, taking into account the non-linearity of the ‘$L_i$–$F_i$’ relation. This non-linearity allows one to explain the differences in

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the time lags for different lines. Cross-correlation estimates of the BLR size are also made. They turn out to be small in comparison to the estimates obtained by the direct reverberation modelling.

The values of the parameters of the BLR model are derived directly from the reverberation modelling, and that is why we do not use any specific numeric results of modern photoionization models of the emission-line spectra of AGN. We use only the fact that, according to these models, the emission-line response of an individual cloud (particularly, in Hα and Hβ) can be non-linear. We also allow for a constant emission-line component.

Though the presented theoretical inferences on the time lags can be of general interest, our primary goal is to apply them to explaining the time behaviour of the Hα and Hβ lines in the emission-line spectrum of NGC 4151, in the framework of a simple uniform one-component model. Of course, explaining the time behaviour of the whole emission-line spectrum would require much more complicated multicomponent models.

2 EFFECTIVE STRATIFICATION OF A HOMOGENEOUS BLR

The BLR of an AGN, according to the ‘standard model’, see e.g. Peterson (1988), represents an aggregate of line-emitting clouds under the effect of ionizing radiation of the central source.

The dependence of the emission-line luminosity $L_\lambda$ of an individual cloud upon the value of the incident ionizing flux $F_\gamma$, in accordance with the photoionization models of spectra of AGN (see e.g. Kwan 1984; Mushotzky & Ferland 1984), is described by a power law: $L_\lambda \propto F_\gamma^s$, where $s \approx 0$.

The rate of heat input in a gas cloud optically thick in the ionizing continuum is directly proportional to the value of the ionizing flux incident on this cloud. Kwan (1984) noted that therefore the cloud’s emission-line luminosity should be, in a first approximation, directly proportional to the ionizing flux; different lines, however, behave differently. For example, in the case of Hα the dependence is somewhat weaker than linear. The $L_\alpha$ quanta leaving the cloud are produced in the traditional H II zone. At the high ionization parameters typical of AGN the collisional ionization from the excited levels of hydrogen (in particular, from the second level) are effective even in the H II zone. With increasing ionization parameter their efficiency grows, and this leads to weakening of the specified dependence (Kwan & Krolik 1981; Kwan 1984).

So, the emission-line response of an individual cloud can be non-linear. This fact was recognized already in the first successful photoionization models. According to them, quanta in many lines are produced mainly not in the traditional H II zone, but deeper, in the so-called ‘deep partly ionized zone’. Successful modelling of stationary optical emission-line spectra of AGN requires the following two circumstances to be taken into account (Kwan & Krolik 1979): the power-law shape of the spectrum of ionizing continuum (i.e. the fact that the major fraction of ionizing quanta is in the X-ray part of spectrum) and the big column densities of the clouds emitting in lines. If the X-ray luminosity of the ionizing source is great enough in comparison with the UV one, the ‘deep partly ionized zone’ is formed in the cloud. Taking into account the contribution of this zone increases the luminosity of the cloud in Balmer lines, whereas the luminosity in Hα is stabilized at the level of the luminosity of the H II zone. So, the collisional amplification of Balmer and Paschen lines takes place in the ‘deep partly ionized zone’.

Inside this zone the excitation temperatures of these lines increase with optical depth, but ultimately attain some limiting values. The limiting values are insensitive to variation of the ionizing flux, be-

cause the Balmer and Paschen continua dominate in cooling at such depths (Kwan 1984). According to Kwan & Krolik (1981), when collisional ionization becomes the main source of ionizations and cooling, the rate of cooling increases with increasing electron temperature approximately as $\exp \left( -32 \times 10^4 K/T_e \right)$. In the standard model by Kwan & Krolik (1981), $T_e \approx 8000$ K in ‘the deep zone’; the steep dependence of the rate of cooling on temperature, as Kwan and Krolik noted, provides only weak variation of $T_e$ with depth and insensitivity to variation of the model parameters, in particular, the ionization parameter. Increasing the ionizing flux makes higher levels of hydrogen attain the limiting excitation temperatures; the luminosity of the ‘deep zone’ in the relevant lines then ceases to react to changes of the ionizing flux, i.e. in this limit they are constant. In the Balmer series, the approach to the limiting temperatures affects first of all the Hα line, then Hβ and so on. Thus, according to the photoionization models (Kwan & Krolik 1981; Kwan 1984), the dependence of the cloud’s emission-line luminosity on the incident ionizing flux for the Hα line is weaker than for Hβ, for Hβ is weaker than for Hγ and so on.

Because of the difference between Hα and Hβ in the value of the $s$ parameter, the Balmer decrement increases with increasing distance of clouds away from the central source, therefore a photograpic BLR image (if such an image could be obtained) would be larger in Hα than in Hβ. A formula for an effective BLR radius in a line with an arbitrary $s$ value in the homogeneous model of the cloud aggregate was deduced in Shevchenko (1985a). This effective stratification is explained by differences between emission lines in the degree of non-linearity of the $L(F_\gamma)$ function. Shevchenko (1988) showed that within the framework of the homogeneous model of the cloud aggregate, if one takes into account the results of the photoionization calculations of the emission-line spectra of AGN (Kwan 1984; Mushotzky & Ferland 1984), it is possible to explain the observed time lags and amplitudes of variations in major optical and ultraviolet (UV) emission lines in the spectrum of NGC 4151.

After the first successes of the photoionization computations of the AGN emission-line spectra, significant progress was made in this field; see e.g. reviews by Ferland (2003) and Leigthy & Casebeer (2007). Multicomponent models were proposed and studied (Collin-Souffrin & Lasota 1988; Collin-Souffrin et al. 1988; Korista et al. 1997), which allowed to reproduce the relative fluxes in high-ionization and low-ionization lines simultaneously. Evidence was found for the presence of optically thin line-emitting gas (Ferland, Korista & Peterson 1990; Shields, Ferland & Peterson 1995). This progress promoted much deeper understanding of the AGN emission-line spectra – it turned out that the uniform models are too simple to reproduce the whole spectra. However, in what follows, our study concerns only Balmer lines. We aim to explain the time behaviour of the Hα and Hβ lines in the emission-line spectrum of NGC 4151, in the framework of a simple uniform one-component model. Of course, explaining the behaviour of the emission-line spectrum in total may require much more complicated multicomponent models.

Effective, not stratification, stratification is present in our one-component model, due to the non-linearity in each cloud’s line emission. The alternative to a homogeneous BLR with effective stratification is a physically stratified BLR. Investigating variability of the UV lines of NGC 4151, Ulrich et al. (1984) offered the BLR model consisting of three zones with different physical characteristics (see table 2 in their paper). Gaskell & Sparke (1986) proposed a model consisting of two zones (see table 1 in their paper). These models are not considered henceforth; we adopt the effective
stratification picture as implied by the non-linearity in cloud’s line emission.

3 THE REVERBERATION MODEL

Blandford & McKee (1982) offered a procedure to recover the BLR structure by analysis of line and continuum light curves. This is the so-called method of ‘reverberation mapping’. Its essence consists in the following: the observed light curve in a line is supposed to represent a convolution of two curves: the transfer function describing physical characteristics and the geometry of the BLR and the light curve in ionizing continuum. The emission-line luminosity of an individual cloud was supposed to depend linearly on the incident ionizing flux.

Shevchenko (1984, 1985a) found necessary and sufficient conditions for the existence of a time lag of a maximum of an emission-line flare in relation to a (short duration) continuum flare when the BLR structure is isotropic with respect to the central source; these conditions are: the typical cloud should emit in the line mainly from the side facing the central source, and, either a central cavity should be effectively present in BLR, or the $s$ parameter in the formula $L_0 \propto F_s r$, should be less than one. These conditions set useful refer-
sed to $\theta_0 \propto 2$. General formula (3) in Shevchenko (1984) for the

One should make a reservation that the pancake-shaped cloud, as well as a uniform cloud aggregate model itself, is a physical idealization that can be used only as an approximation for the real arrangement of line-emitting material in the BLR. The real structure might be closer to a combination of a disc and an outflowing wind (Emmering, Blandford & Shlosman 1992; Murray et al. 1995; Chiang & Murray 1996; Bottorf et al. 1997; Elvis 2000). We adopt the pancake shape for the BLR cloud exclusively for convenience in mathematical modelling; indeed, according to Shevchenko (1985b), the phase function of the pancake cloud with the plane orthogonal to the ionizing source direction provides the maximum anisotropy of line emission, i.e. this is a physical limit worth theoretical examination, while the phase function of a spherical cloud (or, equivalently, randomly oriented pancakes) gives an approximation for the phase function of randomly oriented optically thick line-emitting material, i.e. it describes a situation that is expected to be closer to reality.

If the ‘pancakes’ are orthogonal to the central source direction, the transfer function representing the dependence of the observed integrated emission-line flux $f(t)$ on time $t$ counted from the moment of the $\delta(t)$ flare of the central source in continuum, is as follows (Shevchenko 1984, 1985a):

$$f(t) \propto \begin{cases} 0, & 0 \leq t \leq R_0, \\ R^{-1} \int_{R_0}^t g(r,t) dr, & R_0 \leq t \leq 2R_0, \\ R^{-1} \int_{t/2}^t g(r,t) dr, & t \geq 2R_0, \end{cases}$$

where

$$g(r,t) = \left( \frac{t}{r} - 1 \right) r^{1-s} e^{-s/r},$$

and $r, R, R_0$ are measured in the light-travel time units.

In the case when the planes of clouds are oriented randomly, their mean phase function coincides with the phase function of a spherical cloud. This function is as follows (Shevchenko 1985b):

$$j(\theta) \propto (1 + \cos \theta) \left( 1 + \frac{s}{2} \cos \theta \right),$$

where $\theta$ is the ‘ionizing source – cloud – observer’ angle, $0 \leq \theta \leq \pi$, $0 \leq s \leq 2$. General formula (3) in Shevchenko (1984) for the transfer function, after substitution of phase function (2), becomes

$$f(t) \propto \begin{cases} \frac{t}{R} \int_{R_0}^\infty g(r,t) dr, & 0 \leq t \leq 2R_0, \\ \frac{t}{R} \int_{t/2}^\infty g(r,t) dr, & t \geq 2R_0, \end{cases}$$

where

$$g(r,t) = \left[ 1 + \frac{s}{2} \left( \frac{t}{r} - 1 \right) \right] r^{-2s} e^{-s/r}.$$

Transfer functions (1) and (3) can be expressed through incomplete $\gamma$ functions. The behaviour of the transfer functions with different values of the $s$ parameter [while $R_0 = 0, R = 15$ lt-day (light-day)] is demonstrated in Fig. 1. The qualitative difference in the behaviour of the functions with $s$ less and greater than unity is clearly seen. In particular, $f(t)$ peaks at $t = 0$ for $s \geq 1$ and at $t > 0$ for $s < 1$.

The model emission-line light curve is determined by the convolution formula

$$F_l(t) = a \int_0^\infty f(\tau) F_l^*(t-\tau) d\tau,$$

where}

$$F_l(\tau) = \int_{4\pi} \left\{ \begin{array}{ll} 0, & 0 \leq \tau \leq \tau_0, \\ R^{-1} \int_{\tau_0}^\infty \tilde{g}(r,\tau) dr, & \tau_0 \leq \tau \leq 2\tau_0, \\ R^{-1} \int_{\tau/2}^{\tau} \tilde{g}(r,\tau) dr, & \tau \geq 2\tau_0, \end{array} \right.$$
Transfer functions (1) and (3) (a and b, respectively) for three parameter values in relation to unity (see relation 5), $R_s = 3.8 \text{ d}$. Note that the exponential-like decay of the resulting $T(t)$ and the computed emission-line light curve are 478–488 $F_0$, $s$ $T(0, s)$ is the observed flux in $R$ parameter value in relation to unity, $R = t_1 \text{ d}$) and the computed emission-line light curve are presented. The latter curve has been obtained by means of convolution of the light curve in the continuum and the transfer function (1) with $R = 15 \text{ lt-day}$, $R_0 = 0, s = 1$. In the lower part of Fig. 2, the normalized CCF of these curves is plotted. The shift of the peak of the CCF is clearly visible; as determined numerically, $\Delta \tau_{\text{peak}} \approx 3.8 \text{ d}$. Note that the exponential-like decay of the resulting curves in both plots reflects the radial structure of the line-emitting region, and the rise in the CCF reflects the shape of the continuum flare.

One may argue that the linear response model is just a linearization of the non-linear response model, and that any BLR radius estimate made in the linear response model is therefore an approximation that might be not far from reality. However, one should take into account, first, that the ‘equal time-travel’ paraboloidal surface inside the BLR covers a whole range of distances from the ionizing source just after the ionizing flare, secondly, that with increasing time after the flare this surface retreats from the source to larger distances. The slopes of the linearized dependences for individual clouds on the surface vary significantly in both space and time. The response slopes might be averaged on the surface, but then the change of the averaged slope with time should be taken into account; the latter is never done in practice. So, it is not surprising that the non-linear response model, as compared to the linear one, can give very different quantitative results on the BLR radius. This directly follows from the qualitative differences in the response function for different values of the $s$ parameter, as seen in Fig. 1. A vivid manifestation of the insufficiency of the linearized response model is that increasing the time lag value in the non-linear response model can be achieved either by increasing the BLR radius or by specific increasing the response non-linearity, namely, by decreasing the $s$ parameter value in relation to unity (see relation 5), while in the linear response model only the BLR radius can be varied.

Shevchenko (1985a, 1994) obtained an approximate theoretical relation of the time lag of the maximum of the emission-line light curve to the $s$ parameter in the homogeneous model of the cloud aggregate with or without a central cavity ($R_0 \geq 0$).

4 THE TIME LAG AND THE CROSS-CORRELATION METHOD

Techniques for cross-correlation analysis of AGN emission-line variability have demonstrated remarkable progress during the last decade. The methods of calculation of the basic properties of the cross-correlation function (CCF), namely, the lags of the CCF peak and CCF centroid and their uncertainties, were greatly improved (e.g. White & Peterson 1994; Peterson et al. 1995, 1998; Welsh 1999). In particular, it was realized that the CCF peaks and centroids underestimate the BLR size (Pérez, Robinson & de la Fuente 1992; Welsh 1999), and that taking into account the continuum variability time-scale is important for correct estimation of the BLR size (e.g. Edelson & Krolik 1988).

Let us consider the time lag as determined by means of cross-correlation analysis in the case of non-linear emission-line response of an individual cloud. In this section, we measure the time lag in model numerical experiments and study the dependence of the time lag on the parameters of a model transfer function and duration of the continuum flares. We consider the case of a single flare of various durations. As the model transfer function we take equation (1) corresponding to the case of the ‘pancake’ clouds orthogonal to the central source direction. The central cavity in the BLR is set to be absent: $R_0 = 0$.

The model light curve in the continuum is assumed to have the form of the bell-like function $F_c(t) = \text{sech}(t - t_0)/T$, where $t_0 = 50 \text{ d}$ and $T$ is effective duration of the flare, $t$ is time in days. The model emission-line light curves are computed on the time interval of 500 d with the step of 0.05 d.

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Figure 1. Transfer functions (1) and (3) (a and b, respectively) for three values of the $s$ parameter, while $R_0 = 0, R = 15 \text{ lt-day}$.

Figure 2. The upper plot: the model light curve in the continuum (the continuous line; $R = 15 \text{ lt-day}$, $R_0 = 0, T = 1 \text{ d}, s = 1$) and the computed emission-line light curve (the dashed line). The lower plot: the normalized CCF.

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The dependence of the time lag on the parameter \( s; R_0 = 0, T = 1 \text{ d} \). The straight-line segments represent theoretical relations (5).

\[
\Delta t = \begin{cases} 
W(1-s)R, & 0 \leq s \leq 1 - 2R_0/(WR), \\
2R_0, & s \geq 1 - 2R_0/(WR).
\end{cases}
\]

(5)

where the constant \( W \) depends on the choice of phase function; \( W = 3.19 \) in the considered case of clouds with regular orientation. In the case of phase function (2) one has \( W = 2 \).

By examining the shifts of peaks of CCFs at various values of parameters we can assess how well relation (5) works when the ionizing flare has a finite duration. Consider first how the time lag varies with \( s \) at fixed \( R \) (Fig. 3). The time lag, as defined here, is the value of the distance along the time axis from \( t = 0 \) up to the first maximum of the CCF; i.e. it is \( \Delta t_{\text{peak}} \). We do not examine the shift of the centroid of the CCF here. The parameter \( s \) is varied from 0.1 to 2.0 with the step of 0.01. The ionizing flare duration is fixed at \( T = 1 \text{ d} \). The curves for \( R = 5, 15 \) and 25 lt-day are plotted. Theoretical dependences (5) for \( R = 5, 15 \) and 25 lt-day and \( R_0 = 0 \) are plotted as straight-line segments. It is clear that the theoretical and numerical estimates of the time lag at such a relatively small duration of the continuum flare are in a good agreement.

Now we consider the dependence of the time lag on the BLR radius \( R \) (Fig. 4). The ionizing flare duration is the same, \( T = 1 \text{ d} \).

The dependences for \( s = 0.5 \) and 1 are plotted. We see that the linear character of theoretical relation (5), valid for the \( \delta(t) \) ionizing flare, is preserved in the case of \( s = 0.5 \). In what concerns the \( s = 1 \) case, it is completely different. From the viewpoint of relation (5), this case is degenerate, and the predicted value of \( \Delta t_{\text{peak}} \) is constant (zero). In reality we observe the ‘\( \Delta t_{\text{peak}}-R \)’ relation similar to a logarithmic one.

So, as follows from Figs 3 and 4, for \( s \geq 1 \) the CCF \( \Delta t_{\text{peak}} \) value depends only weakly on \( R \). The cross-correlation peak time lag for \( s \geq 1 \) is small in comparison with \( R \) expressed in the light-travel time units. The difference can reach an order of magnitude. For \( s = 1 \), a similar phenomenon was observed by Pérez et al. (1992) for the CCF centroid estimates of the BLR size in an isotropic (with respect to the ionizing source) model of the line-emitting cloud distribution. They found that such estimates can be less than the real size of the BLR, the difference reaching two times. This is in accord with our findings.

What is the role of the ionizing flare duration in the degenerate case \( s = 1 ? \) The dependences of the time lag on \( R \) in this case for various fixed values of \( T \) are plotted in Fig. 5. We see that this role is far greater than that of the BLR radius. The observed ‘\( \Delta t_{\text{peak}}-R \)’ dependences seem to be logarithmic indeed; one can verify that, unlike the rational and power-law functions, the functions of the form \( a + b \ln(R+c) \), where \( a, b, c \) are fitting parameters, provide ideal visual description of the observed curves. Note that further work is required to explore how the logarithmic dependence found here is vulnerable to the choice of the ionizing flare shape.

In summary, according to our numerical–experimental findings, the BLR radius is only weakly related to the measured \( \Delta t_{\text{peak}} \) value in the mathematically degenerate but observationally most common case of \( s = 1 \). The role of the time-scale of variability is far greater. Therefore, the lines with \( s \approx 1 \) are of little help in determining the BLR size by means of cross-correlation techniques; instead, the lines with \( s \) essentially less than 1, such as H\( \alpha \), should be used for this purpose. This conclusion has been obtained in a model framework and thus may be model dependent in some way. However, it makes clear that there are no general theoretical grounds to believe that \( \Delta t_{\text{peak}} \) is mostly determined by the BLR size. Note that solely the CCF peak offset has been examined. The CCF centroid offset should be examined as well in a future study to check whether it exhibits the same behaviour.
5 REVERBERATION MODELLING OF THE EMISSION-LINE LIGHT CURVES OF NGC 4151

In this section we examine the effect of taking into account the non-linearity in a cloud’s line emission in practical modelling of emission-line variability of an AGN. We model the emission-line variability of the Seyfert galaxy NGC 4151. The observational data of Kaspi et al. (1996) on variability of the nucleus of this galaxy in Hα, Hβ and optical continuum are used. The observations of Kaspi et al. (1996), performed in the framework the AGN Watch programme, cover the time interval of approximately three months, from 1993 November until 1994 February. While we use the light curve continuum data for the whole time-span of the observations, the data on the fluxes in the lines during the first 5d and during the last 5d of the observational time-span are excluded, following the usual practice of eliminating the border effects (see Maoz et al. 1991). The error bars of the individual observations, defining the weights of the observations, are taken into account in the modelling.

We optimize the model parameters by using a non-linear least-squares method (Levenberg 1944; Marquardt 1963) to minimize χ², thereby finding best-fitting parameter values and their standard errors (see Press et al. 1997). We find that the iterations converge to the same solution for random starting values in ranges specified below, with the deepest minimum of χ² in all cases, though several other local minima exist, as demonstrated below. We expand the standard errors by the square root of the reduced χ², because the best fit reduced χ² is greater than unity in all best-fitting models found.

As the transfer functions, we use equations (1) and (3). The best approximation of the computed model light curve to the observed one is found by means of the modelling. We vary six parameters: the BLR radius R, the non-linearity parameters s_opt for Hα and Hβ, the normalizing coefficients a for Hα and Hβ, the flux F_opt in the narrow component of Hα (on the F_opt value for Hβ see below).

The radius R0 of the central cavity (a zone free from line-emitting clouds) has not been varied because it is already known to be most probably small. A comparison of the known values of time lags in different Balmer lines of NGC 4515 allowed (Shevchenko 1985a) to conclude that the upper bound of the cavity radius R0 is four–five times less than the effective BLR radius. The deduction that the central cavity is small is in agreement with conclusions by Maoz et al. (1991) and Xue & Cheng (1998). Note that the radius of the accretion disc in the centre of the nuclear region is estimated to be equal to 0.6–2 light-day (Luyten 1955; Sergeev et al. 2005, 2006). Taking into account the uncertainty of this estimate and the uncertainty of the R0 estimates, the probable existence of this component of the nuclear region does not at all contradict the conclusions on the small relative size of the central cavity in the BLR.

For the continuum flux we take the flux at the wavelengths of 4560–4640 Å (the ‘4600 Å’ region), because this region corresponds to the shortest wavelengths at which Kaspi et al. (1996) measured the continuum flux. The series of the observed values of the continuum flux are recalculated for presentation on the uniform time grid by means of cubic spline interpolation.

The constant contribution to the integrated flux in Hα and Hβ due to the narrow components of the lines has been taken into account in the following way. We set Fα(t) = Fα,n(t) + Fα,α and Fβ(t) = Fβ,n(t) + Fβ,α. The contributions of the narrow components Fα,n and Fβ,n are connected to each other via the constant Balmer decrement D0 = Fα,n/Fβ,n. Therefore, in the course of searching for the best model it is enough to satisfy the value of Fα,n; the value of Fβ,n is determined via the Balmer decrement. For each of the considered cases of orientation of the planes of clouds we have accomplished the modelling twice, namely, for the two reported values of the Balmer decrement in the narrow components: D0 = 4.47 as given in table 1 in Ferland & Mushotzky (1982) and D0 = 7.55 as given in table 1 in Sergeev, Pronik & Sergeeva (2001). The difference in the observed values of the decrement D0 may reflect either the difficulty in its evaluation or its probable long-term variability.

The contribution of the stellar component to the observed continuum flux has been taken into account by its subtraction from the observed flux prior to modelling. According to Peterson et al. (1995) and Kaspi et al. (1996), the stellar component contribution at the wavelength of 4600 Å and the aperture used at their observations is approximately equal to 2.2 × 10⁻¹⁴ erg cm⁻² s⁻¹ Å⁻¹.

As it is known from observations (see e.g. Crenshaw et al. 1996; Peterson et al. 2002), the light curves of an AGN in the optical and UV continua can be rather different, and the relation between the continua is non-linear. For the Seyfert galaxy NGC 5548, most studied in this respect, the slow components of variability in the optical and UV continua are connected by the power law F_opt ∝ F核电, where γ ≈ 0.56 (Peterson et al. 2002). Basing on this relation, we find the real values of the s parameter from the values obtained in our modelling of the optical light curves (we designate these values by s_opt) by means of the formula s = γs_opt, where we set γ = 0.6.

The value of R does not depend on the line choice. The values of s_opt for different lines are generally different, and the same is true for a. The initial data for the iterations of the Levenberg–Marquardt algorithm have been taken randomly in the following limits: R – from 1 to 30 lt-day; s_opt in the both lines – from 0.2 to 2.0; a – from 0.01 to 2.0; F_opt – from 0 to 30 × 10⁻¹² erg cm⁻² s⁻¹.

In the case when the planes of clouds are orthogonal to the direction to the central source, the best-fitting model light curves are presented in Fig. 6 for two different values of D0. The circles designate the observed values of the flux in optical continuum and the observed integrated emission-line fluxes according to the data in Kaspi et al. (1996, table 2). In both parts of the figure, the continuous curve in the upper plot is the spline interpolation of the continuum flux. The flux is given in the units of 10⁻¹² erg cm⁻² s⁻¹ Å⁻¹. In the lower two plots, the best model light curves in Hα and Hβ are presented as continuous curves; here the integrated emission-line flux is given in the units of 10⁻¹² erg cm⁻² s⁻¹ Å⁻¹.

In Fig. 7, the best model emission-line light curves are presented for the case when the planes of clouds are oriented randomly.

To demonstrate the effect of variation of different parameters in determining the best-fitting models, χ²/N (where N is the number of degrees of freedom) is shown in Figs 8–10 in dependence on the main parameters. Fig. 8 gives the dependence of the reduced χ² on the BLR radius R with all other parameters assigned to their best-fitting values. Figs 9 and 10 show the reduced χ² in dependence on the s parameters for Hα and Hβ with all other parameters assigned to their best-fitting values. (Note that these graphs are presented here for illustrative purposes solely; for quantitative analysis, when offsetting one parameter, one should re-optimize all the other parameters while holding the one parameter fixed at the offset value.) In the presented graphs one can see that the χ² dependence on R is characterized by a single well-defined minimum. The dependences of χ² on the s parameters have four minima each. The Levenberg–Marquardt algorithm finds the deepest one.

In Tables 1 and 2, the obtained values of the model parameters, corresponding to the model emission-line light curves in Figs 6 and 7, are presented together with the reduced χ² values of the models. In total, the results of our modelling give the following values of the radius of the BLR of NGC 4151: R = 11–14 lt-day,
Figure 6. The best model light curves of NGC 4151 in Hα and Hβ in the case when the planes of clouds are orthogonal to the direction to the central source. \( D_n = 4.47 \) (a), \( D_n = 7.55 \) (b). The continuum flux is given in the units of \( 10^{-14} \text{erg cm}^{-2} \text{s}^{-1} \text{Å}^{-1} \). The emission-line fluxes are given in the units of \( 10^{-12} \text{erg cm}^{-2} \text{s}^{-1} \).

with the uncertainty of 5–8 lt-day. At \( D_n = 4.47 \), the best-fitting values of the \( s \) parameter are \( s \approx 0.6 \) for Hα and \( s \approx 0.85 \) for Hβ. From comparison of Tables 1 and 2 one can see that at increasing decrement \( D_n \) from 4.47 up to 7.55 the difference in the computed values of the \( s \) parameter for Hα and Hβ becomes less, though does not seem to disappear completely.

Mushotzky & Ferland (1984) elaborated photoionization models of stationary AGN optical spectra. In the framework of the photoionization modelling they carried out calculations, in our equivalent terms representing the calculations of the functions \( L_l(F_i) \) for the BLR clouds. According to the results of their calculations, \( s \approx 0.6 \) for Hα, and \( s \approx 0.8 \) for Hβ. Thus, there exists a satisfactory agreement of the results of our modelling with the data of Mushotzky & Ferland (1984), especially in the case of \( D_n = 4.47 \). Let us remark that the values of \( s \) can be non-constant inside the BLR, varying from cloud to cloud, because they depend on physical characteristics of the clouds. As a result of our modelling we obtain some ‘effective’ values of \( s \).

The model of a homogeneous distribution of clouds implies that the covering factor is close to one, but, as it has been noted above, an interpretation of the adopted model is possible as a model with an exponential decrease of the cloud concentration with distance away from the centre; then the covering factor can be small.

Figure 7. The same as in Fig. 6, but the planes of clouds are oriented randomly.

Figure 8. The dependence of the reduced \( \chi^2 \) on the BLR radius \( R \) with all other parameters assigned to their best-fitting values.

6 DISCUSSION

There are several ways of estimating the BLR size. Besides the reverberation and cross-correlation methods, discussed above, there exists a technique based on estimating the ionization parameter from modelling the stationary emission-line spectra of AGN. Using this technique, Mushotzky & Ferland (1984) obtained an estimate of the radius of the BLR of NGC 4151, equal to approximately 16 lt-day.
By a similar argument, Cassidy & Raine (1997) found the inner and outer radii equal to 6 and 40 lt-day in their theoretical model of the BLR of this galaxy. At the same time when the photoionization estimate was made by Mushotzky & Ferland (1984), the reverberation estimate $R \approx 15$ lt-day was obtained independently by Shevchenko (1984), in agreement with the photoionization estimate by Mushotzky & Ferland (1984). This reverberation estimation was performed within the framework of the model of a homogeneous isotropic distribution of line-emitting matter around the central ionizing source, on the basis of the observational data of Lyutyi & Cherepashchuk (1971) and Cherepashchuk & Lyutyi (1973) on the time lags in the $H\alpha$ line variations.

Cross-correlation estimates are usually less than the ‘photoionization’ values. Cross-correlation analysis by Peterson & Cota (1988) (see also discussion by Peterson 1988), accomplished on the basis of their own observational data and the data of Antonucci & Cohen (1983) on variability in the lines $H\beta$ and $He\,II\,\lambda4686$, gave $\sim6$ lt-day as the estimate for the radius of the BLR of NGC 4151. Similar cross-correlation estimates of the BLR size were recovered by Clavel et al. (1990) on the basis of the $IUE$ (International Ultraviolet Explorer) data on variability of the major UV lines: $R = 4 \pm 3$ lt-day. These values correspond to the peak CCF time lags; the centroid ones are greater by about 2 d. Wandel et al. (1999) find similar centroid CCF time lags, $4 \pm 3$ d, for the $H\beta$ line. Clavel et al. (1990) note that their cross-correlation estimates of the BLR size for NGC 4151 are an order of magnitude less than the typical ‘photoionization’ estimates for Seyfert galaxies.

By means of cross-correlation analysis of their own data, Kaspi et al. (1996) found that the time lag of variations in the $H\alpha$ and $H\beta$
lines in relation to continuum is 0–3 d; thus the cross-correlation estimate of the BLR radius is 0–3 lt-day. According to the modern analysis of these data accomplished by Bentz et al. (2006) and Metzroth et al. (2006), the value of the cross-correlation time lag for the data of Kaspi et al. (1996) has no clear-cut statistical bounds.

In total, the cross-correlation estimates of the BLR radius of NGC 4151 are all in the range of 0–6 lt-day. Direct reverberation modelling, in comparison with the cross-correlation analysis, gives very different values of $R$ similar to the given above ‘photoionization’ estimates. According to the results of Maoz et al. (1991), who carried out reverberation modelling of light curves of NGC 4151 in H$\alpha$ and H$\beta$, the weighted-mean (by the local emission-line luminosity of clouds) BLR radius $\approx 15$–18 lt-day for the best found model, and the central cavity radius $R_0 \approx 2$ lt-day. The linear character of the $L_\gamma(F_\gamma)$ dependence was assumed, as in practically all modern research on this subject. Xue & Cheng (1998) numerically recovered the BLR transfer functions on the basis of the data of Maoz et al. (1991) and Kaspi et al. (1996). They obtained the following estimates: $R \approx 10$ lt-day, $R_0 \leq 1$ lt-day. As mentioned above, the reverberation estimate $R \approx 15$ lt-day was obtained in Shevchenko (1984). All these reverberation estimates are in agreement with our reverberation modelling results presented in Tables 1 and 2.

So, the known reverberation estimates of the BLR size of NGC 4151 are in agreement with ‘photoionization’ estimates, and they all are much greater than the cross-correlation estimates. The strong difference between the BLR radii found by reverberation modelling, on one side, and its estimates following from cross-correlation analysis, on the other side (10–18 versus 0–6 lt-day), underlines the conditional character of the cross-correlation estimates. Such a difference is no surprise: the size identified as the value of the observed time lag can be much (an order of magnitude) less than the true size of the BLR in lt-day (Section 4). For example, if the cloud aggregate is uniform, the time lag of variation of a line with $s \approx 1$ with respect to an ionizing flare is small compared to the BLR radius $R$ in light-travel time units, and depends on $R$ only weakly. The ultimate cause of this phenomenon is the degeneracy of relation (5) at $s \geq 1$. This degeneracy means that in practice there are no rigorous theoretical grounds to believe that the $\Delta t_{\text{peak}}$ value is mostly determined by the BLR size, if $\Delta t_{\text{peak}}$ is calculated for a typical line (i.e. a line with $s \approx 1$).

However, cross-correlation analysis by Kaspi et al. (1996) of their observational data indicated that the cross-correlation time lag was small not only for H$\beta$ (the line with $s \approx 1$ presumably), but for H$\alpha$ as well (the line with $s$ definitely less than 1). To clarify this point, we have examined cross-correlations between the splined curve in continuum and our theoretical model light curves. The case when the planes of clouds are orthogonal to the direction to the central source (a) and the case when they are oriented randomly (b).

![Figure 11](https://example.com/figure11.png)

**Figure 11.** The computed cross-correlations between the splined curve in continuum and our theoretical model light curves. The case when the planes of clouds are orthogonal to the direction to the central source (a) and the case when they are oriented randomly (b).
et al. (1996) gives the value of the BLR radius matching the majority of the BLR size estimates of other authors. This removes necessity in any special physical interpretation of the small value of the cross-correlation time lag in Hz for these light-curve data. In particular, the hypothesis by Kaspi et al. (1996) that the physical size of the BLR at the moment of their observations was an order of magnitude less than usually is not necessitated.

7 CONCLUSIONS

We have studied how the non-linearity in the ‘$L_i – F_i$’ relation (the emission-line luminosity, $L_i$, of the BLR cloud in dependence on the ionizing continuum flux, $F_i$, incident on the cloud) can be taken into account in estimating the size of the BLR in AGN by means of ‘reverberation’ methods. We have shown that the BLR size estimates obtained by cross-correlation peaks of emission-line and continuum light curves can be much (up to an order of magnitude) less than the values obtained by reverberation modelling. This has been demonstrated by means of abstract representative numerical cross-correlation and reverberation experiments with model continuum flares and emission-line transfer functions and by means of practical reverberation modelling of the observed emission-line variability of NGC 4151. The modelling of the observed light curves of NGC 4151 in Hz and H$\beta$ has been accomplished on the basis of the observational data by Kaspi et al. (1996) and the theoretical BLR model by Shevchenko (1984, 1985).

In the abstract representative numerical cross-correlation and reverberation experiments with model continuum flares and emission-line transfer functions, we have found that the value of the cross-correlation peak time lag $\Delta t_{\text{peak}}$ for $s \geq 1$ is small in comparison with the BLR size $R$ expressed in the light-travel time units and depends on $R$ only weakly. We have shown that in the case of $s \geq 1$ the effect of the ionizing flare duration on the $\Delta t_{\text{peak}}$ value is far greater than that of the BLR radius. In other words, the BLR radius has little effect on the measured value of the $\Delta t_{\text{peak}}$ value in the mathematically degenerate but observationally most common case of $s = 1$; the role of the time-scale of variability is far greater. Therefore, the lines with $s \approx 1$ seem to be of little help in determining the size of the BLR by means of estimating the cross-correlation peak time lag.

The presence of a noticeble time lag of variations of NGC 4151 in Hz (Lyutyi & Cherepashchuk 1971; Cherepashchuk & Lyutyi 1973) and significantly shorter time lags in other Balmer lines (Antonucci & Cohen 1983) with respect to variations in the optical continuum has been attributed, in agreement with conclusions by Shevchenko (1984, 1985a), to the effect of essential non-linearity in the ‘$L_i – F_i$’ relation for Hz. The low value of the power-law index, $s \approx 0.6$, distinguishes this line from the other Balmer lines.

The values of the model parameters of the BLR of NGC 4151 have been estimated. In particular, estimates of the BLR radius have been made. Our reverberation modelling of the emission-line variability based on the observational data by Kaspi et al. (1996) gives values of the BLR radius agreeing with the majority of its known ‘reverberation’ and ‘photoionization’ estimates. Much smaller $R$ values obtained by means of the cross-correlation method have been shown to be an artefact of this method. The hypothesis by Kaspi et al. (1996) that the size of the BLR of NGC 4151 at the time interval of their observations was an order of magnitude less than usually is not necessitated.

Concluding, a power-law emission-line response model and simple spherically symmetric thick geometries of the BLR cloud distribution, taken here as a basis for modelling the emission-line AGN variability, give the size of the BLR of NGC 4151 equal to 11–14 lt-day, with the uncertainty of 5–8 lt-day. This agrees satisfactorily with BLR size estimates in photoionization models fitting emission-line strengths in the mean spectrum of NGC 4151. Much shorter time lags found in the cross-correlation analysis of the emission-line and continuum light curves of NGC 4151 correspond to the size of a smaller emission-line region that is reverberating.

ACKNOWLEDGMENTS

The authors thank anonymous referees whose advice and remarks led to a significant improvement of the manuscript. We express appreciation to S. G. Sergeev for extremely valuable comments. We are deeply grateful to V. V. Kouprianov for programming assistance and discussions. We thank E. Yu. Aleshkina for technical help. This work was supported by the Programme of Fundamental Research of the Russian Academy of Sciences ‘Origin and Evolution of Stars and Galaxies’. AVM is grateful to the Russian Science Support Foundation for support. The computations were partially carried out on the computers of the St. Petersburg Branch of the Supercomputer Centre of the Russian Academy of Sciences.

REFERENCES

Antonucci R. J., Cohen R. D., 1983, ApJ, 271, 564
Bentz M. C. et al., 2006, ApJ, 651, 775
Blandford R. D., McKee C. F., 1982, ApJ, 255, 419
Bottorff M., Korista K. T., Shlosman I., Blandford R. D., 1997, ApJ, 479, 200
Cassidy I., Raine D. J., 1997, A&A, 322, 400
Cherepashchuk A. M., Lyutyi V. M., 1973, Astrophys. Lett., 13, 165
Chiang J., Murray N., 1996, ApJ, 466, 704
Clavel J. et al., 1990, MNras, 246, 668
Collin-Souffrin S., Lasota J.-P., 1988, PASP, 100, 1041
Collin-Souffrin S., Dyson J. E., McDowell J. C., Perry J. J., 1988, MNras, 232, 539
Crenshaw D. M. et al., 1996, ApJ, 470, 322
Edelson R. A., Kirolik J. H., 1988, ApJ, 333, 646
Elvis M., 2000, ApJ, 545, 63
Emmering R. T., Blandford R. D., Shlosman I., 1992, ApJ, 385, 460
Ferland G. J., 2003, ARA&A, 41, 517
Ferland G. J., Mushotzky R. F., 1982, ApJ, 262, 564
Ferland G. J., Korista K. T., Peterson B. M., 1990, ApJ, 363, L21
Gaskell C. M., Sparke L. S., 1986, ApJ, 305, 175
Horne K., Peterson B. M., Collier S. J., Netzer H., 2004, PASP, 116, 465
Kaspi S. et al., 1996, ApJ, 470, 336
Korista K., Baldwin J., Ferland G., Verner D., 1997, ApJS, 108, 401
Kwan J., 1984, ApJ, 283, 70
Kwan J., Krich L. H., 1979, ApJ, 233, L91
Kwan J., Krich L. H., 1981, ApJ, 250, 478
Leighly K. M., Casebeer D., 2007, in Ho L. C., Wang J.-M., eds., ASP Conf. Ser. Vol. 373, The Central Engine of Active Galactic Nuclei. Astron. Soc. Pac. San Francisco, p. 365
Levenberg K., 1944, Q. Appl. Math., 2, 164
Lyutyi V. M., 1977, AZh, 54, 1153
Lyutyi V. M., 1882, in Sunyaev R. A., ed., Astrophysics and Cosmic Physics. Izdatel’stvo Nauka, Moscow, p. 66 (in Russian)
Lyutyi V. M., 2005, Astron. Lett., 31, 723
Lyutyi V. M., Cherepashchuk A. M., 1981, Astron. Circ., 633, 3
Maoz D. et al., 1991, ApJ, 367, 493
Marquardt D. W., 1963, SIAM J. Appl. Math., 11, 431
Metzroth K. G., Onken C. A., Peterson B. M., 2006, ApJ, 647, 497
Murray N., Chiang J., Grossman S. A., Voit G. M., 1995, ApJ, 451, 498
Mushotzky R., Ferland G. J., 1984, ApJ, 278, 558
O’Brien F. T., Goad M. R., Gondhalekar P. M., 1994, MNras, 268, 845
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