Localized magnetic polaritons in antiferromagnetic superlattice with impurity

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Abstract. The dispersion relation for magnetic polaritons localized at the antiferromagnetic impurity film in the antiferromagnetic superlattice (antiferromagnetic/nonmagnetic or antiferromagnetic/antiferromagnetic) are derived in the effective –medium approximation and calculations are performed for the properties of long-wavelength electromagnetic modes. In such systems one finds both surface polaritons which are localized near the surface and guided modes where excitations have a standing-wave –like character and the impurity region acts as a waveguide, because of the magnetic polaritons propagating freely over the impurity layer and dampen in the perpendicular direction on either sides of this region. We assume an external magnetic field parallel to the magnetization and the film interfaces. The dispersion curves and frequency region of the existence of the surface-guided modes of the magnetic polaritons localized at the impurity layer of the antiferromagnetic superlattice are investigated for two values of the external magnetic field: H=0 and H=0.3T.

1. Introduction
The properties of the collective excitations such as polaritons (coupled-mode excitations originating from dipole-active elementary excitations such as phonons, plasmons, magnons, interacting with photons) which propagate in the various magnetic (ferromagnetic, antiferromagnetic) superstructures (superlattices, thin film) have been the subject of increasing interest in recent years [1-8]. A number of different antiferromagnetic-based superstructures and properties of the bulk and surface excitations propagating in such systems have been considered theoretically in the literature [9-12]. Superlattices constructed from antiferromagnetic materials are of interest to communications and signal processing technologies for devices that work at wavelengths in the infrared [13,14]. In this paper we derive the general dispersion relation for magnetic polaritons, which appear at the impurity layer of the superlattice composed of alternating antiferromagnetic or antiferromagnetic and non-magnetic layers. This problem is considered within the framework of a macroscopic theory in the Voigt configuration in the presence of an external magnetic field.

2. Theory
It is known that the effective-medium approximation can be applied in the regions of high frequency dispersion, where wavelength is much greater than the superlattice period L (L=a+b) and the wavevector k appearing in the dispersion relation is small compared to \( L^{-1} \). In this case the antiferromagnetic superlattice behaves like an anisotropic bulk medium, because of antiferromagnetic resonance frequencies are in the far infrared region.
The effective medium is described by the effective-medium permeability tensor with the following components [3]:

\[ \langle \mu_{xx} \rangle = \frac{(a+b)^2 \mu_{11}^{(i)} \mu_{22}^{(i)} + ab(\mu_{11}^{(i)} - \mu_{22}^{(i)})^2 - (\mu_{11}^{(i)} - \mu_{22}^{(i)})^2}{(a+b)(a+b') + b'b} \]  
(1)

\[ \langle \mu_{yy} \rangle = \frac{\mu_{11}^{(i)} \mu_{22}^{(i)}}{a+b} \]  
(2)

\[ \langle \mu_{yz} \rangle = \frac{(a+b)\mu_{11}^{(i)} \mu_{22}^{(i)}}{a+b + b'} \]  
(3)

For the TE mode, the relevant component of the dielectric tensor in the effective-medium description is \( \varepsilon_{xx}^{SL} \):

\[ \varepsilon_{xx}^{SL} = \frac{\varepsilon_1 + \varepsilon_2}{a+b} \]  
(4)

where \( \varepsilon_{i=1,2} \) is the dielectric constant of the i-th component of the superlattice.

Here \( \mu^{(i)}_1 \) and \( \mu^{(i)}_2 \) are non-vanishing components of the frequency-dependent magnetic permeability tensor:

\[ \mu^{(i)}(\omega) = \begin{bmatrix} \mu^{(i)}_1 & \mu^{(i)}_2 & 0 \\ -\mu^{(i)}_2 & \mu^{(i)}_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(5)

and for an antiferromagnet the elements of \( \mu^{(i)}(\omega) \) are

\[ \mu_1(\omega) = 1 + \frac{\alpha^2}{\Omega_1^2 - \omega^2}, \]  
(6)

\[ \mu_2(\omega) = \frac{\Omega_2^2 - \omega^2 - \Omega_0^2}{\Omega_1^2 - \omega^2}, \]  
(7)

where \( \Omega_0 = \gamma M_0 \) (\( M_0 \) is the sublattice magnetization), \( \Omega_0 = \gamma H_0 \) and \( \omega_c = \omega \pm \Omega_0 \). The antiferromagnetic resonance frequency in zero applied field \( \Omega_1 \) is given by the anisotropy \( H_{an} \) and the exchange \( H_{ex} \) fields as

\[ \Omega_1 = \gamma \left[H_{an}(2H_{ex} + H_{an})\right]^{\frac{1}{2}}, \]  
(8)

Using the Maxwell's equations we derive the dispersion relation for localized magnetic polaritons. The dynamic magnetic field satisfies the following equation:

\[ \nabla^2 \vec{h} - \nabla (\nabla \cdot \vec{h}) - \frac{\varepsilon}{c^2} \frac{\partial^2}{\partial t^2} (\vec{h} + 4\pi \vec{m}) = 0, \]  
(9)

The general solution of equation (9) can be written in the form:

\[ h^{(s)}(t) = \left\{ \begin{array}{ll} H^{(s)}_x & \varepsilon \mathbf{B}_y \exp(i(kx - \omega t)) \\ H^{(s)}_y & \varepsilon \mathbf{B}_x \exp(i(ky - \omega t)) \end{array} \right\} \]  
(10)

for the superlattice and
\[ h^{(0)} = [(H_{1x}^{(0)}, H_{1y}^{(0)}) e^{-\alpha_0 y} + (H_{2x}^{(0)}, H_{2y}^{(0)}) e^{-\alpha_0 y}] e^{i(kx - \omega t)} \]  

(11)

for the impurity film occupying the region 0<y<d. Here \( \langle H_{x,y}^{(d)} \rangle \) and \( \langle B_{x,y}^{(d)} \rangle \) are the average values of \( H_{x,y}^{(d)}, B_{x,y}^{(d)} \) in the superlattice. The parameter \( \alpha_0 \) is defined as \( \alpha_0^2 = k^2 - \frac{\omega^2}{\epsilon c} \mu_0^{(0)} \). Here \( \mu_0^{(0)} = \mu_1^{(0)} - \frac{\epsilon_{\text{eff}}^{(0)}}{\mu_0^{(0)}} \) is the effective magnetic permeability of the antiferromagnetic impurity layer in the Voigt geometry. The parameter \( \alpha_0 \) can now be either purely real (this case corresponds to the surface modes) or purely imaginary (for guided modes). The parameter \( \beta = -k_1^2 \) is the decay parameter of the magnetic polaritons in the antiferromagnetic superlattice and \( (\text{Re}\beta)^{1/2} > 0 \) is the penetration depth of the magnetic polaritons into the antiferromagnetic superlattice. The expression for the parameter \( \beta \) can be obtained from the dispersion relation for bulk magnetic polaritons in the effective medium [3]:

\[ \beta^2 = k^2 f_a^2 + f_b^2 + f_a f_b \frac{\mu_1^{(0)} + \mu_2^{(0)} - 2\mu_1^{(0)} - 2\mu_2^{(0)}}{\mu_4^{(0)}} - \frac{\omega^2}{\epsilon c} (e_1 + e_2)(f_1 + f_2)(f_1 + f_2) \]  

(12)

Here \( L = a + b \) is the superlattice period.

In order to derive the dispersion relation for magnetic polaritons which are localized at the impurity film we apply the electromagnetic boundary conditions to the left \( (y=0) \) and right \( (y=d) \) surfaces of the impurity layer, namely, the continuity of the tangential component of the magnetic field \( \hat{h} \) and normal component of \( \hat{b} \). After some algebra, we obtain:

\[ \frac{1}{A} \left( k_1 \epsilon_0 E_{\parallel}^{(0)}(\mu_1^{(0)} - \mu_2^{(0)})(e_1 - e_2) \frac{\omega^2}{\epsilon c} (\mu_2^{(0)} - k^2) - (k_1 \epsilon_0 E_{\parallel}^{(0)}(\mu_1^{(0)} - \mu_2^{(0)})(e_1 - e_2) \frac{\omega^2}{\epsilon c} (\mu_2^{(0)} - k^2)) + k_2 \epsilon_0 E_{\parallel}^{(0)}(\mu_1^{(0)} - \mu_2^{(0)})(e_1 - e_2) \frac{\omega^2}{\epsilon c} (\mu_2^{(0)} - k^2) - (k_2 \epsilon_0 E_{\parallel}^{(0)}(\mu_1^{(0)} - \mu_2^{(0)})(e_1 - e_2) \frac{\omega^2}{\epsilon c} (\mu_2^{(0)} - k^2)) \right) = 0, \]

(13)

Equation (13) together with equation (12) determines the frequencies of localized the magnetic polaritons, which appear in the antiferromagnetic superlattice with antiferromagnetic impurity film in the effective-medium description. This equation is the general dispersion relation for surface-guided localized magnetic polaritons propagating parallel to the impurity film and can be applied to both ferromagnetic and antiferromagnetic systems. Only those solutions of equation (13) for which the condition \( \beta > 0 \) are fulfilled describe physical localized magnetic polaritons.

### 3. Numerical calculations

For numerical calculations we have introduced the following dimensionless parameters. We discuss our analytical results for antiferromagnetic superlattice \( \text{SL}(\text{MnF}_2/\text{ZnF}_2) \) with antiferromagnetic...
impurity film FeF$_2$. Here we assume $f_a = f_b = 0.5$ and the thickness of impurity film $d = 0.0005$ cm. In figure 1 we present the magnetic polariton spectra through a plot of the reduced frequency $\omega^* = \omega^{(\text{FeF}_2)} / \Omega_{\omega a}^{(\text{FeF}_2)}$ against the wavevector $k^* = ck^{(\text{FeF}_2)} / \Omega_{\omega a}^{(\text{FeF}_2)}$ in zero external magnetic field $H_0 = 0$. The different symbols (open and solid squares, triangles, dots) denote the surface (SM) and guided (GM) modes spectral branches. We observe two frequency regions which represent the bulk excitations and four branches of the localized magnetic polariton modes (three surface and one guided) for each direction of $k^*$. Two high-frequency surface modes exist between the bulk bands and start at $\omega^{(\text{FeF}_2)}_{m1} = 15.6576$ (where $\omega^{(\text{FeF}_2)}_{m1}$ is the antiferromagnetic resonance frequency in $H_0 = 0$ and determined by equation (8)). In the MnF$_2$–resonance region the low-frequency branch starts as guided mode for small value of the wavevector but with increasing $|k^*|$ transforms to the surface mode and tend to $\omega^{(\text{MnF}_2)}_{m2} = 12.4261$, where $\omega^{(\text{MnF}_2)}_{m2}$ is the magnitostatic frequency in MnF$_2$, as well as in the semi-infinite antiferromagnetic superlattice.

Figure 1. Dispersion relation for bulk and surface (guided) polaritons in FeF$_2$ antiferromagnetic film in zero field.
The frequency of the surface (guided) mode does not depend on the sign of the wavevector, i.e. the surface modes are reciprocal.

Figures 2-5 show the surface - guided modes for the same structure with H=0.3T. In contrast to the above, there are now three bulk bands and in the presence of the external magnetic field the new localized modes appear making the spectra more complex.
Figure 4. Dispersion relation for bulk and surface (guided) modes in a superlattices SL(MnF₂/ZnF₂).

Figure 5. Same as in figure 4, but in other scale

As one can see, there are two types of localized modes: those, which separate from and lies below a bulk band (denoted as 1 and 3 on figures 2,3), these branches are pure surface modes and the other one, which start as guided modes and then transforms to surface modes at $|k^*| \to \infty$ (branches 2 and 4 on figures). The frequencies $\omega_{1m}^* = 15.796$ and $\omega_{1}^* = 15.5966$ on figures 2 and 3 are the limiting frequencies for high and low bulk branches, respectively. Generally, we observe seven surface-guided modes of localized magnetic polaritons for both directions of the wavevector, respectively: four curves in the FeF₂-resonance region (see figures 2,3) and three surface modes in the MnF₂–resonance region (see figures 4,5). It is essential that the surface mode denoted as 1 and 2 on the figure 5 exists for the restricted values of the frequency $\omega^*$ and the wavevector $k^*$ ($k^* = 75-100$).

4. Conclusions
In summary, we have derived the general effective-medium expression for the surface-guided magnetic polaritons, which propagate in the antiferromagnetic superlattice with antiferromagnetic impurity film and investigate the influence of the external magnetic field on the energy of localized magnetic polaritons. It is essential that the spectrum of magnetic polaritons in the presence of an external magnetic field is weakly non-reciprocal, in contrast to ferromagnetic superlattice. In the system under consideration one finds both surface polaritons which are localized near the surface and guided modes where excitations have a standing-wave –like character and the impurity region acts as a waveguide, because of the magnetic polaritons propagating freely over the impurity layer and dampen in the perpendicular direction on either side of this region. We hope that our theoretical predictions will motivation further experimental work.

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