Universal Structure of Twist-3 Soft-Gluon-Pole Cross Sections for Single Transverse-Spin Asymmetry

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We prove that twist-3 soft-gluon-pole (SGP) cross section for single spin asymmetries (SSA) is determined by a certain “primordial” twist-2 cross section up to kinematic and color factors in the leading order perturbative QCD. In particular, for the processes in which the partonic hard scattering occurs among massless partons, the invariance of the “primordial” partonic cross section under scale transformation leads to remarkable simplification of the SGP cross section, reproducing compact form that was recently observed for pion production $p^3p \rightarrow \pi X$ and direct-photon production $p^3p \rightarrow \gamma X$.

There has been growing interest in the single transverse spin asymmetry (SSA) in high-energy semi-inclusive reactions (see $^1$ for a review). Due to the “naively T-odd” nature of SSA, it occurs as an interference between the amplitudes which have different phases, with one of the amplitudes causing single helicity-flip in the scattering. In the region of the large transverse momentum of the observed particle in the final state, the approach based on the collinear factorization in perturbative QCD becomes valid, and the SSA can be described as a twist-3 observable in this region $^2, ^3, ^4$. In our recent paper $^4$ we have established the formalism of the twist-3 mechanism for SSA, showing the factorization and gauge invariance of the single spin-dependent cross section in the lowest order perturbative QCD, which has given a solid theoretical basis to the previously obtained cross section formulae for SSA $^5, ^6, ^7, ^8, ^9, ^10, ^11$. The connection of this mechanism to another approach for SSA based on so-called “T-odd” distribution/fragmentation functions with parton’s intrinsic $k_T$ $^{12}$ has also been studied recently $^9, ^10$.

In the twist-3 mechanism for SSA, the interfering phase is provided by the pole of an internal propagator of the partonic hard cross sections, and those poles are classified as soft-gluon-pole (SGP), soft-fermion-pole (SFP) and hard-pole (HP), depending on the parton’s momentum fraction at the poles in the twist-3 quark-gluon correlation function. In our recent paper $^{13}$, we have shown that the hard cross section from SGP is completely determined by the corresponding twist-2 unpolarized cross section, for semi-inclusive deep inelastic scattering (SIDIS), $ep^1 \rightarrow e\pi X$, and Drell-Yan and direct-photon production, $p^3p \rightarrow \gamma^{(*)} X$. In the present paper, we shall extend this study to SSA in “QCD-induced” $pp$ collisions, such as $p^3p \rightarrow \pi X$ and $pp \rightarrow \Lambda^0 X$, and will show that the corresponding SGP contribution also obeys similar but more sophisticated pattern, i.e., can be expressed as a derivative of a certain twist-2 cross section with respect to the parton’s momentum originating from initial- or final-state hadron, up to color factors. In addition, we shall show that the scale invariance of the relevant Born cross sections among massless partons leads to a remarkably compact formula for the SGP cross section as was the case for the direct-photon production $^1$. This clarifies the origin of the observed compact formula for the SGP cross section for $p^3p \rightarrow \pi X$ $^{11}$.

To be specific we consider SSA for $p^3p \rightarrow \pi X$, in particular, the contribution from twist-3 distribution function of the transversely polarized nucleon, defined as $^3, ^5, ^6, ^7, ^8, ^9, ^10, ^11$.

$$M_{Fij}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} \epsilon^{i\lambda x_1} \epsilon^{j\mu(x_2-x_1)} \langle p S_\perp | \bar{\psi}(0) g F^{\mu}_a (\mu n) \gamma_i (l_{\perp}) | p S_\perp \rangle = \frac{M_N}{4} \frac{2}{N_c-1} (gt^a)_{ij} e^{ipnS_\perp \cdot G^a}_{i}(x_1, x_2) + \cdots ,$$

(1)

where $| p S_\perp \rangle$ is the nucleon state with momentum $p^\perp$ ($p^2 = M_N^2$) and spin vector $S_\perp$ ($S_\perp^2 = -1$), $F^{\mu}_a$ is the gluon field strength tensor with octet color index $a$, and spinor indices $i,j$ associated with both Dirac and color structure of the quark field $\psi$ are shown explicitly. We suppressed the path-ordered gauge-links that connect the fields on the lightcone and make $^1$ gauge invariant. In the twist-3 accuracy, $p$ (and momenta of all other hadrons as well) can be regarded as lightlike $p^2 = 0$ and $n$ is another lightlike vector satisfying $p \cdot n = 1$. Here we take $p$ in the $+z$ direction as $p^z = p_\perp = 0$ and $n^+ = n_\perp = 0$. The twist-3 quark-gluon correlation function $G^a_{i}(x_1, x_2)$ is dimensionless and satisfies $G^a_{i}(x_1, x_2) = G^a_{i}(x_2, x_1)$. The ellipses in (1) stand for Lorentz structures associated with another twist-3 correlation function, antisymmetric under $x_1 \leftrightarrow x_2$ $^3$, and with twist higher than three; the function vanishing at $x_1 = x_2$ is irrelevant (see below).

We first recall the structure of the familiar twist-2 unpolarized cross section for $p(p) + p(p') \rightarrow \pi(H) + X$:

$$E_h \frac{d^4\sigma^{tw2}}{d^3p_h} = \frac{\alpha^2}{S} \sum_{a,b,c=q,q,g} \int \frac{dz \, d^3x}{z^2} \frac{dx'}{x'} \frac{dx}{x} \times f_a(x) f_{a'}(x') D(c) H_U^{ahc} ,$$

(2)

where $E_h = P_h^0$ and $S = (p+p')^2$. $f_a(x)$ and $D_q(z)$ ($f_{a'}(x)$ and $D_s(z)$) are the quark (gluon) distribution and frag-
mentation functions for the proton and the pion, respectively, and these are convoluted with the corresponding partonic hard-scattering functions $H_U^{q,g}$, over the relevant momentum fractions $x, x'$ and $z$. We consider the contribution from quark-quark scattering (Fig. 1) in detail; $q(xp) + q(x'p') → q(P_h/z) + q(xp + x'p' - P_h/z)$. The relevant hard-scattering function can be written as

$$H^{q,g}_U = \frac{1}{2} \epsilon_2^q \text{Tr} \left[ \mathcal{H}^{(0)}(xp, x'p', P_h/z) \frac{1}{2} x \right], \quad (3)$$

where $\mathcal{H}^{(0)}(xp, x'p', P_h/z)$ stands for the $qq → qq$ hard scattering part whose spinor indices $i, j$ are associated with the initial quark with momentum $xp$ and are traced (Tr[· · ·]) over both Dirac and color indices with the insertion $xq^2/2$ to project onto the unpolarized distribution $f_q(x)$. The factor $C_q \equiv 1/N_c$ is the color-averaging factor for the initial quarks; $C_q → C_q \equiv 1/(N_c^2 - 1)$ when we consider the gluon initiated processes associated with the gluon distribution $f_g(x)$.

The hard part (3) can be explicitly written as (here and below, we frequently use the shorthand notation for the partonic momenta, $p \equiv xp, p' \equiv x'p$ and $P_h \equiv P_h/z$)

$$\mathcal{H}^{(0)}_{j_1}(\hat{p}, p', \hat{P}_h) = \sum_{m, m', r, r', s, s'} F_{jm', s'r'}(\hat{p}, p', \hat{P}_h) \left( \hat{P}_h \right)_{m' m} \times \mathcal{D}_{r'r}(\hat{p} + p' - \hat{P}_h) F_{m, rs}(\hat{p}, p', \hat{P}_h) \left( \hat{p} \right)_{s' s}, \quad (4)$$

using the Born amplitude $F_{m, rs}(\hat{p}, p', \hat{P}_h)$ for 2-to-2 scattering, “$q_i(\hat{p}) + q_s(p') → q_m(\hat{P}_h) + q_r(\hat{p} + p' - \hat{P}_h)$”, with the factors for its external lines amputated:

$$F_{m, rs}(\hat{p}, p', \hat{P}_h) = (i\gamma^\mu b^h)_{mi} (i\gamma^\mu b^t)_{rs} (-i)_{\hat{p} - P_h}^2 + i\epsilon \frac{-i}{(\hat{p} - P_h)^2 + i\epsilon}$$

$$+ (i\gamma^\mu b^h)_{ri} (i\gamma^\mu b^t)_{ms} (-i)_{\hat{p} - P_h}^2 + i\epsilon, \quad (5)$$

where $i\gamma^\mu b^h$ comes from each quark-gluon vertex, and we employ the Feynman gauge for the gluon propagator. In (4), the amplitude in the RHS of the cut is obtained by $\overline{F}(\hat{p}, p', \hat{P}_h) = (\gamma^0 \otimes \gamma^0) F(\hat{p}, p', \hat{P}_h) (\gamma^0 \otimes \gamma^0)$, where each $\gamma$-matrix structure in $F(\hat{p}, p', \hat{P}_h)$ is sandwiched by $\gamma^0$, after taking the hermitian conjugate denoted by “$\dagger$”.

$\mathcal{D}_{r'r}(\hat{p} + p' - \hat{P}_h)$ represents the cut quark-line for the unobserved final parton carrying the large transverse-momentum as $\mathcal{D}_{r'r}(k) = \sum_{\text{spins}} u_{r'}(k) \bar{u}_{r}(k) \delta (k^2) = \langle \bar{\psi}' r' | \delta (k^2) | \psi r \rangle$, where $u(k)$ is the spinor for a quark with momentum $k$, and the Dirac structures $\bar{\psi}'$ and $\hat{P}_h$ are associated with $f_g(x')$ and $D_g(z)$ of (2) with $b = q, c = q$.

With this convention for the twist-2 cross section, one can proceed to derive the SGP contributions. The formalism for the twist-3 calculation has been established in [4] in the context of SSA for SIDIS, and it is straightforward to extend it to $p^3p → \pi X$. The relevant hard part is derived from the collinear expansion of a set of cut Feynman diagrams of the type of Fig. 2 which are obtained by attaching the additional gluon, generated by twist-3 effect of (1), to the 2-to-2 partonic Born subprocess in Fig. 1. We denote the sum of the partonic subprocess of those diagrams as $H_{j_1}^{D}(k_1, k_2, p', \hat{P}_h)$ analogously to $H_{j_1}^{(0)}(\hat{p}, p', \hat{P}_h)$ of (3), where $\sigma$ and $a$ are the Lorentz and color indices associated with the additional gluon. Applying the collinear expansion to these diagrams (4), one obtains the general formula for the twist-3 contribution to the spin-dependent cross section as

$$E_h \frac{d^2 \sigma}{d^2 p} = \frac{\alpha_s^2}{28} \int \frac{dz}{z^2} \frac{dx dx'}{x'} \frac{dz}{z^2} \mathcal{C}_q f_q(x') D_q(z) \times \text{Tr} \left[ i\omega^a_{\beta} \frac{\partial H_{j_1}^{(1)\sigma,a}(k_1, k_2, p', \hat{P}_h) \sigma}{\partial k_{2\perp}^a} M^2_{P_F}(x_1, x_2) \right]_{c.l.} \quad (6)$$

where $M^2_{P_F}(x_1, x_2)$ is defined in (1), $\omega^a_{\beta} = \sigma^a \beta - \rho^a n_\beta$, and “c.l.” indicates to take the collinear limit $k_{1,2} → x_{1,2} p$. We note that all other complicated terms arising in the collinear expansion vanish due to Ward identities [4]. Our task is to evaluate and study the SGP contribution in $\partial H_{j_1}^{(1)\sigma,a}(k_1, k_2, p', \hat{P}_h) \sigma / \partial k_{2\perp}^a |_{c.l.}$.

It is known that only the diagrams in Figs. 2(a) and (b), corresponding to the final-state interaction (FSI) and the initial-state interaction (ISI), respectively, survive as the SGP contributions, while the other diagrams cancel out combined with the corresponding “mirror” diagrams [5, 6, 11]. The contribution of these diagrams to $H_{j_1}^{(1)\sigma,a}(k_1, k_2, p', \hat{P}_h) \sigma$ can be easily obtained from (4) by a slight modification in each factor. Noting that the attachment of the extra gluon to the final parton can be implemented by modifying the factor $\hat{P}_h$ in (4), the contribution from Fig. 2(a) is given by

$$H^{L_{P_F}}(k_1, k_2, p', \hat{P}_h) = \overline{F}(k_2, p', \hat{P}_h) \times \left( \hat{P}_h + i \gamma^\mu t^a \frac{i}{\gamma_1 - \gamma_2 + \hat{P}_h + i\epsilon} \right) \mathcal{D}_{r'r}(k_2 + p' - \hat{P}_h) \mathcal{F}(k_1, p', k_1 - k_2 + \hat{P}_h) (\bar{\psi}') \, (7)$$

where the role of each element is defined as in (4), and we have suppressed the spinor indices as well as the summation over those. The contribution from the mirror diagram of Fig. 2(a) is obtained similarly and is expressed.

**FIG. 1:** Generic Feynman diagram contributing to twist-2 unpolarized cross section for $pp → \pi X$, through the “$q(xp) + q(x'p') → q(P_h/z) + q(xp + x'p' - P_h/z)$” scattering channel. White circles denote the hard scattering between partons.
as \( \mathcal{H}_{F}^{M}(k_{2}, k_{1}, \hat{p}', \hat{P}_{h}) = \gamma_{0} \mathcal{H}_{F}^{M}(k_{2}, k_{1}, \hat{p}', \hat{P}_{h}) \gamma_{0} \), so that the total contribution from FSI is \( \mathcal{H}_{F}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) + \mathcal{H}_{F}^{M}(k_{2}, k_{1}, \hat{p}', \hat{P}_{h}) = \mathcal{H}_{F}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) \). Likewise, ISI diagrams in Fig. 2(b) gives

\[
\mathcal{H}_{I}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) = \mathcal{F}(k_{2}, \hat{p}', \hat{P}_{h}) \left( \hat{P}_{h} \right) \times \mathcal{D}(k_{2} + \hat{p}' - \hat{P}_{h})\mathcal{F}(k_{1} - k_{2} - \hat{p}', \hat{P}_{h}) \times \left( \frac{i}{\ell_{Y}^{2} - \ell_{Y} + i\epsilon} \right). \quad (8)
\]

Combining this with its mirror contribution, we denote the total contribution to \( \mathcal{H}^{(1)}_{F,I}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h})p_{\sigma} \) from ISI as \( \mathcal{H}_{F}^{I}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) \). Note that the quark propagators appearing in (7) and (8) have a pole at \( k_{2} = k_{1} \), corresponding to the FSI and ISI SGPs, respectively, as indicated by a cross in Figs. 2(a) and (b).

To obtain the SGP cross section, we substitute into (9), \( \mathcal{H}^{(1)}_{F,I}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h})p_{\sigma} = \mathcal{H}_{F}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) + \mathcal{H}_{I}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h}) \). In calculating the derivative, \( \partial/\partial k_{2\perp} \), we note the important feature of \( \mathcal{H}_{F,I}^{M} \) in (11) and (8). In \( \mathcal{H}_{F,I}^{M} \), the additional gluon couples to an “external parton” associated with a final- or initial-state hadron, while the other part of \( \mathcal{H}_{F,I}^{M} \) is almost the same as \( \mathcal{H}^{(0)}_{F,I} \), except for the trivial “momentum shift” in the arguments of each component, \( \mathcal{F}, \mathcal{F}, \mathcal{D} \). (This is in contrast to the cases for the HP and SFP contributions where the extra gluon attaches to an internal parton line as well.) Because of this structure, the additional gluon in \( \partial\mathcal{H}_{F,I}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h})/\partial k_{2\perp} \) can be systematically disentangled by keeping the other factors in (7) and (8) intact, as was demonstrated in [13] for SIDIS and Drell-Yan processes. It is straightforward to obtain the derivative as

\[
\frac{\partial \mathcal{H}_{F}^{M}(k_{1}, k_{2}, \hat{p}', \hat{P}_{h})}{\partial k_{2\perp}} \bigg|_{c.l.} = \frac{1}{x_{1} - x_{2} + i\epsilon} \frac{\partial}{\partial Y_{1\perp}} \left[ \mathcal{H}^{(0)}(x_{1}, p, \hat{p}', \hat{P}_{h}) \right]_{Y_{1\perp} \rightarrow t_{Y}} \left( t_{Y} \right) \quad (9)
\]

with \( Y = F, I \), and we set \( \ell_{Y} = \hat{P}_{h} \) and \( \ell_{1} = -\hat{p}' \). In the RHS of (9), the form \( \ell_{Y} = (\ell_{Y}^{2}/2\ell_{Y} \cdot p) + (\ell_{Y} \cdot p)n + \ell_{Y\perp} \) should be used for the on-shell momentum \( \ell_{Y} \), i.e., \( \ell_{Y\perp} \) should be treated as a dependent variable. The replacement \( \ell_{Y} \rightarrow t \ell_{Y} \) before taking this derivative, indicates that we keep \( t^{2} \) in (7) and (8) intact, and such different insertion of color matrix results in different color factors in (8) for the FSI and ISI contributions.

Inserting (9) into (9) and evaluating the pole contribution, only \( G_{t}^{q}(x_{1}, x_{2}) \) in (11) contributes. The result gives the SGP cross section from the interference between \( qgq \rightarrow qq \) and \( qq \rightarrow qq \), but its hard cross section is now written as the response of \( qq \rightarrow qq \) scattering to the change of the transverse momentum carried by the external parton, to which the extra gluon had coupled. Also for other channels where antiquarks and/or gluons participate as initial- or final-state partons, it is straightforward to see that the results similar to (9) hold with appropriate substitutions of each factor in (11), \( \mathcal{F}, \mathcal{D}, \hat{P}_{h} \) and \( \hat{p}' \), as was done for SIDIS and Drell-Yan processes in [13]. Namely all these SGP contributions to (11) can be expressed as the derivative of the corresponding 2-to-2 partonic Born subprocess, e.g., \( qg \rightarrow q\bar{q}, qg \rightarrow qg \), etc., which participates in the twist-2 unpolarized cross section (2). The only difference is the associated color structure, e.g., when the “extra gluon” couples to initial- or final-state gluon, \( t^{2} \) in the RHS of (9) is simply replaced by its adjoint representation, \( (t^{a})_{bc} = -if^{abc} \).

A comment is in order regarding the gauge invariance of the SGP hard cross section in (11). Since (9) exhibits only the single pole at \( x_{1} = x_{2} \), the color gauge invariance with respect to the external gluon line is guaranteed by \( (x_{2} - x_{1})\delta(x_{1} - x_{2}) = 0 \). To make clear the gauge invariance regarding the internal gluon lines, we note that, in the twist-3 accuracy, one can replace the factor \( p \) from \( M_{t}^{\alpha t}(x_{1}, x_{2}) \) in (6) by \( \delta_{Y_{1\perp}2}(2x_{1}x_{2}) \) with the on-shell condition \( k_{t}^{2} = 0 \) and perform the derivative \( \partial/\partial k_{2\perp} \) from the outside of \( Tr[\cdots] \), without changing the result (4). With this modification, all quark-gluon vertices in (7) and (8) are sandwitched by the on-shell quark lines, and thus the color gauge invariance is guaranteed for \( Tr[\cdots] \) in (11) before taking the derivative by \( \partial/\partial k_{2\perp} \). The same argument applies to other channels as well.

Combining all the above results, one obtains the SGP cross section for SSA in \( p^{\dagger}p \rightarrow \pi X \), including all relevant channels. One should note that the result (11) holds in any frame with \( p_{\perp} = 0 \); in particular, one can move to a frame even with \( \ell_{Y\perp} = 0 \) after taking the derivative. In terms of the usual three partonic-invariants, \( \bar{s} = (xp + x'p')^{2} \), \( \bar{t} = (xp - \hat{P}_{h}/z)^{2} \), \( \bar{u} = (x'p' - \hat{P}_{h}/z)^{2} \), this implies that the derivative \( \partial/\partial \ell_{Y\perp} \) can be performed through that for
\[ E_h \frac{d^3 \sigma_{\text{SGP}}}{d^3 P_h} = \frac{\pi M N c^2}{S} \sum_{b,c=q,q,g} \int \frac{dz dx' dx}{z^2 x' x} f_b(x') D_c(z) \]
\[ \times G_p^q(x, x) \left\{ \left( \frac{\hat{s}}{z} \epsilon^S_{\perp} \rho_{b,p} + x' \epsilon^S_{\perp} p' \rho_{n} \right) \frac{\partial H^{b,c}_{F,I}(s, \hat{t}, \hat{u})}{\partial \hat{u}} \right\} - \left( \frac{1}{z} \epsilon^S_{\perp} \rho_{b,p} + x' \epsilon^S_{\perp} p' \rho_{n} \right) \frac{\partial H^{b,c}_{F,I}(s, \hat{t}, \hat{u})}{\partial \hat{u}} \right\}, \]

where
\[ H^{q,a}_{q,d}(\hat{s}, \hat{t}, \hat{u}) = \text{Tr} \left\{ \frac{C_q x \phi^a}{2(N_c^2 - 1)} H^{(0)}(\hat{p}, \hat{p}', \hat{P}_h) \right\}_{\phi \rightarrow e^\phi}, \] (11)

with \( \phi^a/[2(N_c^2 - 1)] \) from (11), and similarly for other channels. The relation between (11) and (3) is apparent, with
\[ \frac{\hat{s}}{z} \epsilon^S_{\perp} \rho_{b,p} + x' \epsilon^S_{\perp} p' \rho_{n} \]

appearing as is obvious from the formula of the SGP cross section. The relevant twist-2 partonic cross sections are obtained as
\[ \sigma_{W}(W = U, F, I) \]

as
\[ H^{q,b,c}(s, \hat{t}, \hat{u}) = \sigma_{W}^{q,b,c}(s, \hat{t}, \hat{u}) \delta(s + \hat{t} + \hat{u}). \] (12)

Explicit form of \( \sigma_{W}^{q,b,c} \) is obtained as the sum of the contributions of 2-to-2 Feynman diagrams for the corresponding partonic subprocess, and they can be written as
\[ \sigma_{W}^{q,b,c} = \sum_{i} C_{W(i)}^{q,b,c} \sigma_{i}^{q,b,c}, \]

where \( \sigma_{i}^{q,b,c} \) are dimensionless functions expressed solely by \( \hat{s}, \hat{t}, \hat{u}, \) and \( C_{W(i)}^{q,b,c} \) denote the color factor associated with the diagram \( i \). The difference among \( \sigma_{W}^{q,b,c} \) is represented only through the difference of \( C_{W(i)}^{q,b,c} \) among \( W = U, F, I \).

Upon substitution of (12) into (10), the derivative \( \partial/\partial \hat{u} \) acts both on \( \sigma_{F,I}^{q,b,c} \) and \( \delta(s + \hat{t} + \hat{u}) \), the latter contribution being handled by partial integration with respect to \( x \). One thus obtains the SGP cross section as
\[ E_h \frac{d^3 \sigma_{\text{SGP}}}{d^3 P_h} = \frac{\pi M N c^2}{S} \sum_{b,c=q,q,g} \int \frac{dz dx' dx}{z^2 x' x} f_b(x') D_c(z) \]
\[ \times \left[ \frac{dG_p^q(x, x)}{dx} - G_p^q(x, x) \left\{ \left( \frac{1}{z} \epsilon^S_{\perp} \rho_{b,p} + x' \epsilon^S_{\perp} p' \rho_{n} \right) \frac{\partial H^{b,c}_{F,I}(s, \hat{t}, \hat{u})}{\partial \hat{u}} \right\} \right] \]
\[ \times \left( \frac{\hat{s}}{z} \epsilon^S_{\perp} \rho_{b,p} + x' \epsilon^S_{\perp} p' \rho_{n} \right) \frac{\partial H^{b,c}_{F,I}(s, \hat{t}, \hat{u})}{\partial \hat{u}} \right\} \delta(s + \hat{t} + \hat{u}). \] (13)

In a frame in which \( p \) and \( p' \) are collinear, \( \epsilon^S_{\perp} p' \rho_{n} = 0 \). We note that a straightforward calculation would produce additional terms in the integrand, which are proportional to \( G_p^q(x, x) (\hat{s} \partial / \partial \hat{s} + \hat{t} \partial / \partial \hat{t} + \hat{u} \partial / \partial \hat{u}) \sigma_{F,I}^{q,b,c} \). However, such terms vanish, because of the scale-invariant property \( \sigma_{F,I}^{q,b,c}(s, \hat{t}, \hat{u}) = \sigma_{F,I}^{q,b,c}(s, \hat{t}, \hat{u}) \). As is obvious from the form of \( \sigma_{F,I}^{q,b,c} \) discussed below (12). As a remarkable result, in (13), both “derivative” and “non-derivative” terms of the SGP function \( G_p^q(x, x) \) appear with the common partonic cross section expressed by \( \sigma_{F,I}^{q,b,c} \), which are identical to the twist-2 cross section up to color factors. Explicit forms of \( \sigma_{F,I}^{q,b,c} \) obtained by “color insertion” to \( \sigma_{F,I}^{q,b,c} \) coincide with the corresponding hard cross sections derived in (i) and (ii), and (13) gives the complete SGP cross section for \( p' p \rightarrow \pi X \). The above derivation demonstrates that the simplification as in (13), which was recently observed in [11], arises from the quite general origin common to the SGP contribution with twist-3 distributions: (i) the general structure (9) obeyed by the hard part; (ii) scale invariance of the 2-to-2 partonic Born cross section among massless partons. Because (i) and (ii) are independent of specific initial- or final-state, the SGP contributions to SSAs in other processes with twist-3 distributions follow the same pattern as (10) and (13). For example, consider the hyperon polarization in the unpolarized \( pp \) collision, \( pp \rightarrow \Lambda^* X \), in particular, the contribution from the chiral-odd twist-3 unpolarized distribution \( E_p^F(x_1, x_2) \) combined with the transversity fragmentation function \( \delta q(x) \) for \( \Lambda^* \), \( E_p^F(x, x) \otimes f_2(x') \otimes \delta g(x) \otimes \sigma \). The corresponding cross section is obtained by the replacement \( P_h \rightarrow \gamma_s \bar{s}_L P_h \) in the above relevant formulae to project onto \( \delta q(x) \), where \( S_L \) is the transverse spin vector for \( \Lambda^* \), and by \( \gamma_s \bar{s}_L P_h \rightarrow \gamma_s \bar{s}_L P_h e^{i \beta np} \) in (1) for the contribution of \( E_p^F \). The relevant twist-2 cross section is that for \( p' p \rightarrow \pi^* X \), i.e., involves (13) with \( \phi \rightarrow \gamma_s \beta \). It is clear that the similar relation between the SGP and the twist-2 cross sections holds for this case as well.

The simplification of the SGP cross section due to the above (i) and (ii) can be also extended to the processes in which more number, \( n \geq 3 \), of final-state (massless) partons are involved. This is because, for any such processes, the hard part for the SGP cross section is obtained by attaching the extra gluon from the twist-3 distribution function to the external parton lines in the “primordial” twist-2 hard part, i.e., the 2-to-n partonic Born subprocess. Owing to this property, the relation similar to (9) holds, with \( \epsilon_Y \) corresponding to the momenta of the relevant external partons. As the result, the hard part for the twist-3 SGP cross section associated with the \( n \) final-state partons is also completely determined by the primordial twist-2 partonic cross section up to color factors, and the scale invariance of the twist-2 partonic cross section among massless partons leads to further reduction of the formula of the SGP cross section.

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