Synthesis of Asymmetrical Charged Particle Lens Using Analytical Potential Model

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Abstract. In the present work, the optimization by synthesis approach in the field of charged particle optics has been followed, where a modified analytical function has been proposed to represent the axial magnetic scalar potential distribution of the charged particle lens. Thereby, the corresponding axial field distribution can be deduced which is necessary to determine the paraxial ray trajectories and the aberration coefficients. Indeed, the present work deals with asymmetrical distributions of the lens field, accordingly, the first order properties and third order aberrations of the charged particle lens have been evaluated under zero and infinite magnification modes taken into account the effect of some geometrical parameters. The present work aims to reconstruct the shape of the pole piece of the magnetic lens that produced the proposed target function. The results have shown that, the pole piece shape of the lens and the field distributions have new irregular configurations as well as the objective properties of the lens are of acceptable values, according to the constraints that are imposed in the design process.

Keywords: Electron and Ion optics, Objective Lenses, Aberration coefficients, Charged particle lens.

1. Introduction

The origin of charged particle optics (electron optics) date back the first quarter of the twentieth century when it was discovered that magnetic and electric fields could be used as lenses for electron images. However, the main task of charged-particle optics is the manipulation of ensembles of rays each originating from a common point. Important collective properties of optical elements are, e.g., the focusing of homocentric bundles of rays to form an image and the guiding of particles in accelerators or stronger rings [1]. The basic structure in the construction of any electron-optical device is the electron lens which is widely used for an electron or ion beams. However, an axially symmetric electric or magnetic field, or a combination of two, acts in a similar manner on the trajectories of electron travels through the field [2]. Thus, the direction of traveling moving electron will be altered either by applying a magnetic field or an electric field [3]. However, among several types of focusing elements are magnetostatic and electrostatic lenses [4]. It is well-known that when the axial magnetic scalar potential or magnetic field distribution of the magnetic lens is defined, the optical properties of the lens can be completely determined. Thus, the determination of the magnetic scalar potential distribution of asymmetrical double pole piece magnetic lens may be considered being an initial step to start the synthesis procedure of the electron-optical device. Since the invention of the magnetic lenses, different several mathematical models have been introduced to represent the axial magnetic field distribution, in addition, several mathematical target functions have been
used to approximate the distribution of the potential, for more details see [5]. A theoretical computational investigation based on the optimization by synthesis procedure was introduced by[6] to represent the asymmetrical axial magnetic scalar potential distribution by cubic spline functions in the region of the charged particle lens, where the axial interval of the lens was divided into four or six intervals and by solving the linear system of the equations the asymmetrical potential distribution of the charged particle lens evaluated along the axial region of the lens and hence, the first order properties and third order aberrations of asymmetrical charged particle lenses have been evaluated under the effect of the cubic spline coefficients and magnification modes. By approximating the electron beam trajectory of the asymmetrical double pole piece magnetic lens by polynomials of different orders along the optical axis, the asymmetrical objective and projector properties have been evaluated under the effect of the four magnification modes (zero, low, high, and infinite) [7]. The present synthesis approach is concerning with a mathematical model that approximate the asymmetrical potential axial distribution of double pole piece magnetic lens in terms of geometrical parameters instead of physical parameters which are well-commonly used in the design of electron-optical devices. Hence, the profile of the pole piece, i.e., equipotential surfaces as well as the objective properties of the asymmetrical charged particle lens are investigated under zero and infinite magnification modes.

2. Mathematical Remediation

In the present work the magnetic scalar potential distribution along the optical axis of asymmetrical double polepiece magnetic lens may be approximated by the following potential function [8].

\[
V(z) = \frac{NI}{2} \left[ \left(\frac{z-a}{\sqrt{(z-a)^2+a^2}} + \frac{z+b}{\sqrt{(z+b)^2+b^2}} \right) \right]
\]

where NI is the excitation of the lens, \(a\) is a parameter which may relate to the air gap width between the polepieces, \(a, b\) are an optimization parameter which affect the potential distribution of the lens and these two parameters may represent the bore radii of the pole pieces.

It should be mentioned that equation (1) represents the asymmetrical axial distribution of the potential which may become symmetric when the bore radii are equal. Since the region of the optical axis is a current free region, thus, the axial magnetic field distribution of the lens may be obtained by using the equation

\[
B_z(z) = -\mu_0 \frac{dV}{dz},
\]

where \(\mu_0\) is the permeability of free space \((4\pi \times 10^{-7} \text{ Hm}^{-1})\).

The reconstruction process of the equipotential surfaces (pole pieces) can be determined easily by using the distribution of the magnetic scalar potential \(V(r,z)\) in \(r-z\) plane which can be evaluated from the axial distribution function \(V(z)\) by using the following power series expansion [9].

\[
V(r, z) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{r}{2}\right)^{2m} \frac{d^{2m}V(z)}{dz^{2m}}
\]

it should be mentioned that \(V(r,z)\) which is given by the above power series expansion represents the analytical solution of Laplace’s equation for the rotationally magnetic flux density distribution, and by using only the first two terms of equation (3), the radial height of the polepiece along the optical axis can be written as following.

\[
B_z(z) = \frac{\mu_0 NI}{2} \left[ \left(\frac{1}{\sqrt{(z-a)^2+a^2}} + \frac{1}{\sqrt{(z+b)^2+b^2}} \right) \right]
\]
where \( V''(z) \) is the second derivative of the axial potential distribution which may be determined from the following formula.

\[
V''(z) = -\frac{B_z}{\mu_0} \quad (5)
\]

In this work spherical and chromatic aberration coefficients \( C_S \) and \( C_C \) respectively are considered to be figures of merit. These two coefficients can be evaluated from the following integrals [10].

\[
c_s = \frac{\eta}{128V_r} \int_{z_0}^{z_1} \left[ \left( \frac{3\eta}{8V_r} \right) B_2^4(z) r_a^4(z) + 8B_2^2(z) r_a^2(z) - 8B_2^2(z) r_a^2(z) r_0^2(z) \right] dz \quad (6)
\]

\[
c_c = \frac{\eta}{8V_r} \int_{z_0}^{z_1} B_2^2(z) r_a^2(z) dz \quad (7)
\]

where \( z_0 \) and \( z_1 \) are the object and image planes positions respectively, \( \eta \) is the charge-to-mass quotient of electron, \( V_r \) is the relativistically corrected accelerating voltage, and \( r(z) \) is the electron beam trajectory that penetrates the electron device. The electron beam trajectory can be determined by solving the following well-known paraxial ray equation [11].

\[
r'' + \frac{\eta}{\gamma V_r} B_2^2(z)r = 0 \quad (8)
\]

the fourth order Runge-Kutta numerical method has been used to solve this ordinary differential equation in the region of the magnetic field under different boundary conditions.

3. Results and Discussion

Synthesis of asymmetrical axially rotational electron magnetic lens depending on the potential field model equation (1) has been followed in the present work. However, according to the target magnetic potential, function there are three different optimization parameters which can affect the design procedure and the optical focal properties of the electron-optical device, these parameters are \( \alpha \), \( a \), and \( b \). From the design point of view it is noted that, these parameters produce different field distributions and consequently different magnetic lenses with various optical properties. Therefore, one of these parameters that affect the design procedure has been chosen. Hence, the parameter \( a \), has been picked out to be the effective one in the synthesis procedure of the magnetic lens under consideration. With aid of the mathematical magnetic
field model, figure 1 shows different axial field distributions that corresponding to different values of the parameter a when the other two parameters b, and $\alpha$ are kept constant at 2mm, and 0.5.

![Graph of asymmetrical axial magnetic field distribution](image)

**Figure 1.** Asymmetrical axial magnetic field distribution for various values of the parameter a around different symmetry z-planes.

It is noted that the field distributions are not asymmetrical about the commonly well-known symmetry plane (i.e., $z=0$ plane), thus, these field distributions have been transformed to be asymmetrical fields about the zero-z plane as shown in figure 2. It is seen that the right field side is different from that on the left side, therefore, fields of two sides are of different half widths and consequently having different focal properties, and as a result the axial magnetic scalar potential distribution of the two field sides will also be asymmetrical as shown in figure 3.
Figure 2. Asymmetrical magnetic field distributions about the zero-z plane.

Figure 3. The asymmetrical axial magnetic scalar potential distribution of the magnetic lens for various values of the parameter \( a \).
Really, as the field distributions are asymmetrical it is expected that the polepieces of the lens are also asymmetrical, thus, figure 4 shows the polepiece shape of the double polepiece magnetic lens whose field distribution plotted in figure 2.

It is noted that the poles of the charged particle lens on the two sides of the symmetry plane are of different reconstruction unless the symmetry one which produced when the parameters \(a\) and \(b\) are equal to each other (i.e., \(a=b=2\text{mm}\)). However, symmetric pole pieces having the same bore radius, \(D_1=D_2\) which approximately equals to 3mm. On the other hand, pole pieces of the right hand side are of more affected by the parameter \(a\), from those on the left side and the right side pole pieces are of greater bore radii than those on the left side. Also, it is noted that the behavior of the curves of the pole piece profile on the two sides is different, and the resulting shapes of pole pieces compared with the conventional ones are of irregular profile especially on the large bore radius side, the irregular behavior of the equipotential surfaces can be considered as a result to the number of the inversion points in the field distributions.

As it is mentioned in the previous section, the first order properties and third-order aberrations of the charged particle lens under consideration will be evaluated under zero and infinite magnification modes, thus, the electron beam trajectories in sense of these two modes that penetrate the lens fields are of different refraction and focusing as shown in figure 5, where the paths of electron are determined by solving the paraxial ray equation at specified excitation parameter which is \(20 \text{ Ampere−turn/√Volt} \).

![Image](image_url)

*Figure 4. Pole piece profile of the asymmetrical lens for different values of the parameter \(a\).*
Figure 5. Electron beam trajectories that penetrate electron lens corresponding to fields in figure (2), a) zero and b) infinite magnification mode.

Variation of the objective focal length $f_o$, spherical aberration coefficient $C_s$, and chromatic aberration coefficient $C_c$ with the design parameter $a$, under zero and infinite magnification modes at $NI/V_{r1/2}=20$ is shown in figure 6.
Figure 6. Variation of a) objective focal length $f_o$, b) spherical aberration coefficient $C_s$, and c) chromatic aberration coefficient $C_c$ with the parameter $a$ at $NI/V_{r1/2} = 20$.

It is seen that each two curves for any coefficient are concourse at the same point (i.e., $a=b=2\text{mm}$) that is corresponding to the symmetry field point. After that the two curves are departure from each other, where the properties $f_o$, $C_s$, and $C_c$ of the image side (i.e., under infinite magnification mode) are greater than those of the object side (zero magnification mode) see table 1, however, the behavior of the curves is as it is expected since the right hand side fields are of greatest half widths and slowly decreasing fields while fields of the left hand side are of smallest half widths and sharply decreasing fields.

Table 1. The objective properties $f_o$, $C_s$, and $C_c$ under zero and infinite magnification mode at $NI/V_{r1/2} = 20$.

| $a$(mm) | Objective focal length (mm) | Spherical aberration (mm) | Chromatic aberration (mm) |
|---------|-----------------------------|---------------------------|---------------------------|
|         | Zero            | infinite               | zero            | infinite               | Zero            | Infinite               |
| 2       | 1.7560          | 1.7560                 | 0.9899          | 0.9899                 | 1.2413          | 1.2413                 |
| 4       | 2.0081          | 2.9420                 | 1.2771          | 2.7565                 | 1.4431          | 2.1517                 |
| 6       | 2.1521          | 3.7198                 | 1.4260          | 4.1638                 | 1.5482          | 2.7567                 |
| 8       | 2.2334          | 4.1908                 | 1.5140          | 4.9070                 | 1.6029          | 3.0557                 |
| 10      | 2.2767          | 4.4806                 | 1.5659          | 5.2761                 | 1.6296          | 3.1970                 |

4. Conclusions
From the previous results, one can conclude several remarks. The most important of them is that, asymmetrical magnetic lens field in sense of the suggested mathematical potential model can be obtained only when the two effective parameters $a$, and $b$ are not equal to each other. In the sense one may replacing the conventional investigation of the magnetic lens by a carful choice of mathematical expression to approximate the potential distribution. The results have also shown that, the pole piece shape of the resulting lens and its field distribution have new irregular configuration as well as the objective properties of the lens are of acceptable values according to the constraints that are imposed in the design process.
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