Mathematical model of geometric parameters of modular cutting tools

S Lukina1, E Korshunova, I Shinkevich, O Dorozhkina and I Dorozhkin

Institute of Economics and Technology Management, Moscow State Technological University Stankin, 1 Vadkovsky Lane, Moscow, 127994, Russian Federation

1E-mail: lukina_sv@mail.ru

Abstract. The paper presents a computerized method of geometric parameters determination for modular cutting tools. This method allows designing the tools with preset parameters of a cutting point, setting tolerances for machining of locators, and selecting a precision class for cutting inserts and shims depending on precision requirements of a tool. Mathematical and software implementation of this method is based on the principles of analytic geometry and vector-matrix analysis. The general structure of a mathematical model is represented by a graph plotted on multiple local coordinates systems of assembly components of a tool. Communications between coordinate systems are established through transition matrices. Analytic expressions are produced to calculate geometric parameters of a tool assembly depending on geometric parameters of cutting inserts and seat orientation parameters.

1. Introduction

One of the current critical tasks that manufacturers face consists in enhancing the technological level of metal-cutting equipment and cutting tools. Application of modular cutting tools with mechanically clamped cutting components allows for a significant reduction of tool investments and increase in machining performance [1, 2].

At the moment, several thousands of such cutting tool structures with mechanically clamped cutting components are designed and applied. They are equipped with cutting inserts, which are mechanically clamped and made of hard alloys, ceramics, or ultra-hard materials, and have either polycrystalline diamond inserts or diamond coating. The following cutting tools with mechanically clamped components are the most widely used ones: straight turning tools, side-facing tools, parting-off tools, internal boring and free-cutting tools, annular drills, face and end mills, and other tools with standardized principal dimensions [2, 3-6].

The design of modular tools with mechanically clamped cutting components is marked by dependence of cutting point geometric parameters on a cutting insert type and its position inside a tool body. Notably, only some geometric parameters are preset directly, whereas the remaining values are the derivatives of these parameters [7-15].

Current computation methods make it possible to calculate cutting component positioning parameters of certain structures of interlocked side mills, nonetheless, the ratio between orientation angle of a cutting insert and geometric parameters of a tool remain undefined [2, 4, 7-12]. Therefore, a high priority task is the development of a design method that would provide for the evaluation of geometric precision of the tool cutting edge position depending on manufacturing accuracy of structural components and their attachment strength.
2. Research methods
The developed method allows defining a cutting insert position within tool coordinates with regard to
the required geometric parameters of a cutting point, definition of positioning parameters of a mill
body workpiece on a secondary machine for manufacturing an insert pocket, and evaluation of tool
precision upon assembly. Mathematical and software implementation of this method is accomplished
through the principles of analytic geometry and vector-matrix analysis.

At designing modular cutting tools consisting of multiple assembly components the contact
surfaces of these components should be positioned in a way that would ensure the tool preset cutting
properties are achieved [16-19]. To describe relative positioning of the tool assembly components, a
local coordinate system \( XYZ \) should be related to each component [20]. The general structure of a
mathematical model for geometric parameters determination exemplified by an interlocking end mill
is represented as a graph \( \Gamma=(X,E) \).

Each graph node corresponds to a local x-y-z coordinate system of some assembly component of a
cutting mill. Graph edges define transfer matrices between these coordinates. The following
coordinates systems (graph nodes) are defined on the graph \( \Gamma=(X,E) \): \( X_{11}Y_{11}Z_{11} \) – cutting component (mechanically clamped cutting component); \( X_{12}Y_{12}Z_{12} \) – supporting component; \( X_{13}Y_{13}Z_{13} \) – cheap
breaker; \( X_{14}Y_{14}Z_{14} \) – other components of a cutting point; \( X_{21}Y_{21}Z_{21} \) – mill body; \( X_{22}Y_{22}Z_{22} \) – cassette; \( X_{23}Y_{23}Z_{23} \) – other components of a mill body; \( X_{31}Y_{31}Z_{31} \) – cutting point mount;
\( X_{32}Y_{32}Z_{32} \) – supporting component mount; \( X_{33}Y_{33}Z_{33} \) – chip breaker mount; \( X_{4}Y_{4}Z_{34} \) – cassette mount;
\( XYZ_{36} \) – mounts of other cutting components and mill body. For example figure 1a shows
graph model for determining geometric parameters of modular end mill shows on the figure 1b.

Communications between the listed coordinates systems are established through transition
matrices. Transition from one ("old") coordinate system into another ("new") is accomplished through
multiplication of three relative rotation matrices of the "old" coordinates system about the axes of a
"new" one by the respective angles and translation matrices of the coordinate origin:

\[
[M]_{xyz} = [M_x][M_y][M_z][w_x(\phi_y)][w_y(\phi_y)][w_z(\phi_y)][a],
\]

where \([M_x],[M_y],[M_z]\) are matrices of rotation of the “new” coordinates system \( XYZ \) about \( X, Y, Z \)
axes of the “old” coordinates system \( XYZ_{i-1} \); \([M_{i-1}]\) is a matrix of the \( XYZ \) coordinate origin translation
about \( XYZ_{i-1} \) coordinate origin [8]:

![Graph model for determining modular (a) end mil geometric parameters (b): X_{11}Y_{11}Z_{11} – cutting component (mechanically clamped cutting component); X_{21}Y_{21}Z_{21} – mill body; X_{31}Y_{31}Z_{31} – cutting point mount.](image-url)
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \psi_x & \sin \psi_x & 0 \\
0 & -\sin \psi_x & \cos \psi_x & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos \psi_y & 0 & -\sin \psi_y & 0 \\
0 & 1 & 0 & 0 \\
\sin \psi_y & -\sin \psi_y & \cos \psi_y & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

\[
\begin{bmatrix}
\cos \psi_z & \sin \psi_z & 0 & 0 \\
-\sin \psi_z & \cos \psi_z & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(3)

where \( \psi_x, \psi_y, \psi_z \) – are the angles of \( XYZ_{i+1} \) coordinates system rotation in relation to the respective axes of \( XYZ \) coordinates system; \( U_x, U_y, U_z \) are values of \( XYZ_{i+1} \) coordinate center offset in relation to \( XYZ \) system in \( X, Y, Z \) directions.

The implementation of the developed model is reduced to qualifying functional relationships between relative rotations and translations of coordinates systems in order to determine relative positioning of contact surfaces of all components that would ensure the preset cutting properties of the mill.

3. Results

Consider method implementation by the example of geometric parameters obtaining for an interlocking end mill. Define local coordinates systems of a tool: \( XYZ_t \) is a mill body coordinates system, \( XinYinZin \) is a cutting insert coordinates system. The mill rotation axis coincides with the \( Xin \) axis (figure 2a).

![Figure 2. Computational Model for Identification of Parameters of a Cutting Insert Position within Mill Body: a) local coordinates systems of a mill, b) a point F on the insert cutting edge that forms a processed surface.](image)

The following positions are defined on the figure 2: \( XYZ_t \) – a mill body coordinates system, \( XinYinZin \) – a cutting insert coordinates system; \( F \) – a point on the insert cutting edge that forms a processed surface; \( \varphi \) – major entering angle; \( E \) – a unit vector, which is normal to a pocket plane; \( D \) – a mill diameter; \( \omega_x, \omega_y, \omega_z \) – angles between a unit vector \( E \) and its projections on \( ZmYm \) plane and on \( Zm \) axis of the machine coordinates system.

Positioning of the cutting insert is achieved through a series of consecutive rotations from a base point where a front plane of the cutting insert is in the \( XinYin \) plane, a cutting edge is on the \( Yin \) axis, and its origin coincides with the insert coordinate origin about \( XZXY \) axes for angles \( \omega_z, \omega_x, \omega_y \). When a
coordinates system and cutting insert rotation sequence are customary, the cutting edge angles λ (cutting edge tilt angle) and φ (major entering angle), which equal ωx and ωy respectively, are set directly. Cross points of adjacent sides were selected as cutting insert reference points Pi0, the coordinates of which are used as reference parameters. First, we define \( F\{x_F, y_F, z_F\} \), which is a point on the insert cutting edge that forms a processed surface (figure 2b). For turning tools and end mills, the point F position is on the radial section of the cutting edge fixed by angle φ. For cylindrical, grooving, and formed mills, the processed surface is formed directly by the cutting edge, and F is set in the center of a cutting edge, making it possible to keep to a minimum or avoid errors associated with misalignment of the insert cutting edge in relation to the initial tool surface. The point F is in the center of \( X_1Y_1Z_1 \) coordinates system, which is used as reference for cutting insert orientation. The correction of reference points coordinates for \( P_i \) cutting insert after positioning in the \( XYZ_{in} \) system is achieved using equation (1).

The signs of angles \( ω_x, ω_y, ω_z \) should be taken into account during computation. An angle is considered positive if rotation is executed counter-clockwise from the point of view of the axis node about which the rotation is executed. Therefore, reference points coordinates of the oriented cutting insert (pocket) are considered defined within coordinates system of the tool.

To ensure preset parameters for the cutting edge of the tool, we need to qualify dependences between angles γ (front clearance angle), \( α \) (side rake angle), \( α_1 \) (end rake angle) \( φ_1 \) (minor entering angle), and orientation angles \( ω_x, ω_y, ω_z \). The angles \( α \) and \( γ \) are formed at rotation through angle \( ω_y \), and angle \( ω_x \) adjusts their values to a certain extent. The angle \( α \) is influenced by the cutting insert rake angle \( α_m \). The value of angles \( α \) and \( γ \) is determined as projected \( ω_y \) angle on a plane perpendicular to the main cutting edge projection, which is \( ω_x \) inclined to angle \( ω_y \) plane. The dependences are as follows:

\[
tg(α + α_m) = tg ω_y \cdot cos ω_x
\]

(4)

\[
tg γ = tg ω_y \cdot cos ω_z
\]

(5)

From (2) and (3) follows that

\[
γ = α + α_m
\]

(6)

The angle \( α_1 \) is formed with respect to \( ω_y=λ \) angle, and depends also on the angle \( ω_x \), cutting insert back angle \( α_{in} \), and the angle between proximate planes of the cutting insert \( \eta \). If a square cutting insert is used (\( \eta=90^o \)) the value \( α_1 \) is determined through a straightforward dependence defined using a formula of projected angles:

\[
α_1 = ω_x + actg\left(tg α_{in} \cdot cos ω_y\right)
\]

(7)

If \( \eta \) differs from \( 90^o \), angle \( α_1 \) is determined the following way:

\[
α_1 = arctg\left(\frac{T}{cos ω_y - T \cdot tg ω_y \cdot cos φ_1}\right) + arctg\left(tg ω_x \cdot sin φ_1\right)
\]

(8)

where \( T = cos φ_1 \cdot sin ω_y + \frac{tg α_{in} \cdot sin φ_1}{2 \cdot sin η} \). The angle \( φ_1 \) is determined with respect to \( η \) angle and depends on all three orientation angles of the cutting insert. The formula is written as:

\[
φ_1 = arctg\left(\frac{sin η \cdot cos ω_y}{-cos η - tg ω_x \cdot sin η \cdot sin ω_y \cdot cos ω_z}\right) - ω_z
\]

(9)
Therefore, through setting the parameters $\varphi$, $\alpha$, $\alpha_{in}$, $\lambda$, $\eta$ and using the formulas (4)-(11) we can calculate the values of orientation angles $\omega_x$, $\omega_y$, $\omega_z$ and parameters $\varphi_1$, $\alpha_1$, $\gamma$.

While a seat within the tool body is somewhat adjusted to the cutting insert positioned within the tool body, positioning parameters of the seat and the cutting insert do not coincide. To process a pocket for the cutting insert, a body workpiece should be installed in such a way that the pocket (insert) plane is parallel to longitudinal and transverse feed axes $X_m$ and $Y_m$ of the machine coordinates system. This means that vector $E$, which is normal to a pocket plane (Figure 2), should be parallel to penetration feed ($Z_m$ axis of the machine coordinates system). In practice, it is accomplished by two relative turns of the mill body workpiece by angles, which equal the angle between a unit vector $E$ and its projection on $Z_mY_m$ plane ($\omega_1$), and that between the unit vector projection and $Z_m$ axis ($\omega_2$) of the machine coordinates system. Moreover, a division motion required to process all pockets should be ensured for multipoint-cutting tools. The mill body $X_m$ axis-turning is regarded as such kind of a motion. To reduce treatment errors and simplify fixture structure or in order to exclude a special fixture from the manufacturing system, a number of relative orientation turns should be kept to a minimum through combining one of the positioning motions with the motion of division. With regard to the above-stated, a computational model was developed to determine a position of a mill body workpiece on the secondary machine, where $X_m$ axis (mill body axis), in relation to which one of the positioning rotations and division motion are executed, is parallel to $X_m$ axis of the machine coordinates system.

The machine mechanisms, which execute consecutive rotations of the body workpiece by angles $\omega_1$ (in relation to $Y_m$ axis) and $\omega_2$ (in relation to $X_m$ axis), perform tool body orientation after which the pocket plane becomes parallel to the penetration feed. The calculation of coordinates for the pocket reference points in the machine coordinates system is done in the following sequence. If a fixture is used, coordinates of the pocket reference points at initial body workpiece position are recalculated from $X_tY_tZ_t$ coordinates system of the tool into $X_fY_fZ_f$ coordinates system of the fixture. Further, once the body is positioned, the coordinates of the pocket reference points are calculated in $X_fY_fZ_f$ system using a formula:

$$\{P_{io}\} = [M_{\omega_2}] [M_{\omega_1}] \{P_0\},$$

where $[M_{\omega_1}]$, $[M_{\omega_2}]$ are rotation matrices of the mill body workpiece.

Coordinates of $P_{io}$ points are then recalculated in the machine coordinates system, after which a pocket machining code is created for the CNC. If there is no fixture, the calculation is performed directly in the machine coordinates system.

The precision of pocket manufacture depends on the accuracy of the initial positioning of the body workpiece on the machine (in the fixture), precision of body orientation, precision of secondary cutting tool positioning, and manufacturing system stiffness. The extent, to which pocket and standard structural components manufacturing precision influences the resulting accuracy of the cutting edge position, depends on the cutting insert orientation parameters, namely, $\omega_x$, $\omega_y$, $\omega_z$ angles, body dimensions, and body positioning parameters $\omega_1$, $\omega_2$.

The estimation of relationships between tolerances should be done for point $F$ of the cutting inserts. It has been established that $X$-direction tolerance has the most influence on machining accuracy of the turning tools and end mills, whereas accuracy of cylindrical, grooving, and formed mills depends on $Y$-direction tolerance. The coordinates of the point $F$ with regard to the sum of error magnitudes $\Delta x_{in}$, $\Delta y_{in}$, $\Delta z_{in}$ that includes all the above-mentioned types of linear errors in the corresponding coordinate axes direction in the system $X_{in}Y_{in}Z_{in}$ are determined in the tool coordinates system $X_tY_tZ_t$ using an equation (1):
A total of error magnitudes $\Delta x_{i,n}$, $\Delta y_{i,n}$, $\Delta z_{i,n}$ includes linear errors of the initial positioning of the mill body workpiece, errors of pocket surface milling, errors of cutting insert machining in the corresponding axes direction, and error of shim machining in $Z_s$ axis direction. The errors of the shim in the directions of $X_{i,n}$ and $Y_{i,n}$ axes have no effect on the insert cutting edge position. The errors of pocket surfaces milling include errors of the tool positioning and errors resulting from elastic strain of the manufacturing system.

Errors in the tool body orientation made for each insert machining operation also alternate pocket position and, as a result, the cutting insert position within the body. The actual coordinates of a reference point position of the cutting insert in the tool coordinates system, if linear errors are not included, should be determined using the equation:

$$\{\Delta l_i\} = [M_y, [M_z, [M_z, [M_{1,z}, [M_x, [M_{1,x}, [F, \Delta x_{i,n}\] \Delta y_{i,n}\] \Delta z_{i,n}\}]. \quad (11)$$

where, the rotation matrices $M_{1,x}$ and $M_{1,z}$ include values of the angles $\omega_1$, $\omega_2$ with regard to angular errors.

Dependence $\Delta \omega = f(\Delta \omega_1, \Delta \omega_2)$ is non-linear, and all combinations of extreme deviations of body orientation angles should be analyzed to determine the maximum deviation of the cutting edge reference point. The most extreme position deviations of the cutting edge reference point with regard to machining and assembly errors are an algebraic sum of the maximum and minimum values of linear $\Delta l$ and angular $\Delta \omega$ errors, as well as elastic deformation errors of the cutting edge components after the cutting insert clamping.

Therefore, if we know the extreme deviation values of initial positioning and orientation of a mill body workpiece, pocket surfaces milling, and machining of the cutting insert and shim, then the extreme position deviations of the cutting edge reference point in the finished tool could be calculated, and regulations of operation cycle accuracy could be tightened if necessary.

Delphi was used to implement the developed models (1) - (12). The research results allow calculating of geometric parameters of modular cutting tools with identification of the confidence bound for their intended use and reference operating conditions, revelation of interrelation between the structural component layout factors and the machining accuracy output parameters at the technical production preparation stage. The developed models were used to calculate Sandvik Coromant modular end mills.

4. Conclusion
The developed model allows designing modular cutting tools of any complexity with set geometric parameters of the cutting point, definition of machining tolerances for cutting point locators, choosing the accuracy class of the cutting inserts and shims depending on the required tool precision and, therefore, improvement of the design concept quality, all other conditions being equal.

References
[1] Hagglund S 2003 New procedure for optimizing cutting data for general turning. Proceedings of the Institution of Mechanical Engineers 217 349
[2] Grigorev S N, Gribkov A A and Zakharchenko D V 2013 Global trends in machine-tool design Russian Engineering Research 33(8) 468
[3] Cemal CM, Ensarioglu C and Demiryak I 2009 Mathematical modeling of surface roughness for evaluating the effects of cutting parameters and coating material Journal Materials Processing Technology 209 102
[4] Chang S and Tseng H Design of a novel cutter for manufacturing helical cutting tools 2004 J. Mech. Eng. Sci. 219 395
[5] Sharma K, Mahto D and Sen S S 2013 In metal turning, effect of various parameters on cutting tool: A Review International Journal of Application or Innovation in Engineering & Management 2 32
[6] Fang N and Fang G 2007 Theoretical and experimental investigations of finish machining with a rounded edge tool Journal of Materials Processing Technology 191 331
[7] Kilic Z M and Altintas Y 2016 Generalized Modelling of Cutting Tool Geometries for Unified Process Simulation International Journal of Machine Tools and Manufacture 104 14
[8] Fulemova J and Janda Z 2014 Influence of the Cutting Edge Radius and the Cutting Edge Preparation on Tool Life and Cutting Forces at Inserts with Wiper Geometry Procedia Eng. 69 565 https://doi.org/10.1016/j.proeng.2014.03.027
[9] Tandon P, Gupta Ph and Dhande S G 2017 Geometric Modeling of End Mills Computer-Aided Design and Applications 2(1) 57 https://doi.org/10.1080/16864360.2005.1073853
[10] Engin S and Altintas Y 2017 Mechanics and dynamics of general milling cutters. Part I: Helical end mills International Journal of Machine Tools and Manufacture 41(15) 2195 https://doi.org/10.1016/S0890-6955(01)00045-1
[11] Jixiang Y J, Aslan D and Altintas Y 2018 A feedrate scheduling algorithm to constrain tool tip position and tool orientation errors of five-axis CNC machining under cutting load disturbances CIRP Journal of Manufacturing Science and Technology 23 78 https://doi.org/10.1016/j.cirpj.2018.08.005
[12] Jixiang Y and Alexander Y 2017 An analytical local corner smoothing algorithm for five-axis CNC machining International Journal of Machine Tools and Manufacture 23 22
[13] Aslan D and Altintas Y 2018 Prediction of Cutting Forces in Five-Axis Milling Using Feed Drive Current Measurements IEEE/ASME Transactions on Mechatronics 23 386
[14] Habibi M, Tuysuz O and Altintas Y 2019 Modification of Tool Orientation and Position to Compensate Tool and Part Deflections in Five-Axis Ball End Milling Operations Journal of Manufacturing Science and Engineering 141 (3) https://doi.org/10.1115/1.4042019
[15] Altintas Y, Tuysuz O, Habibi M and Li Z L 2018 Virtual Compensation of Deflection Errors in Ball End Milling of Flexible Blades CIRP Annals 67(1) 365
[16] Grechishnikov V A, Lukina S V, Veselov A I and Makarov D V 2001 The forming processes modeling for geometric parameters of inserted cutting tools with regard to their assembly technology Automation and Modern Technology [in Russian] 4 32
[17] Lukina S V 2011 Automating procedures for formation and choice of structural component layout of modular cutting tools in step of technical preparation production Vestnik Saratov State Technical University 1(57) 214
[18] Lukina S V 2009 Modeling procedures for formation and choice of structural component layout of modular cutting tools using network graph-models Obrabotka Metallov 2(43) 28
[19] Lukina S and Krutyakova M 2017 Analytical study of modular cutting tools dynamic properties MATEC Web of Conferences 129 01061 https://doi.org/10.1051/mateconf/201712901061
[20] Lukina S, Korshunova E and Dorozhkin I 2018 Methods of automated control over composition and structure of metalworking equipment MATEC Web of Conferences 224 01095 https://doi.org/10.1051/mateconf/201822401095