A Five-Dimensional Grid Multi-wing Butterfly Chaotic System and Its Circuit Simulation

Yun Huang
Asset Management division, Chongqing University of Posts and Telecommunications,
Chongqing 400065, P. R. China
huangyun@cqupt.edu.cn

Abstract. Based on the two-wing butterfly chaotic system, through adding two first-order
differential equations with the piecewise linear function, a five-dimensional grid multi-wing
butterfly chaotic system is presented. Through numerical simulation, it is studied for the gird
multi-wing butterfly chaotic attractors, the Lyapunov exponent spectrum and the bifurcation
diagram of the system. At the end, an electronic circuit is designed to carry out the system. The
circuit experimental results are consistent with the results of numerical simulation, which
confirm the availability and feasibility of this approach.

1. Introduction
In recent years, chaos has been found to be of great use and has gigantic potential in many
technological disciplines, for instance, secret communication [1], power system [2] and bioengineering
[3] etc. At present, it is research hotspot to construct the chaotic system with complex dynamic
behavior.

Chua’s circuit [4] was found and has been widely studied as a paradigm. Based on Chua’s circuit, a
large number of multi-scroll and grid multi-scroll chaotic system [5,6] were constructed via adding
different piecewise linear functions. In 1963, Lorenz system [7] was presented, and it can generate
two-wing butterfly chaotic attractors. To enhance the dynamic behavior of the two-wing butterfly
Lorenz system, the four-wing butterfly chaotic systems [8,9] were constructed via modifying the
structure of the original systems. Furthermore, References [10] presented multi-wing butterfly chaotic
system, and its dynamic behavior is more complex than two- and four-wing butterfly chaotic systems.
Recently, the grid multi-wing butterfly chaotic systems are concerned. As the dynamic behavior of the
grid multi-wing butterfly chaotic systems is more complicated than the multi-wing butterfly chaotic
systems, it is significance to research the grid multi-wing butterfly chaotic system.

The rest of this paper is organized as follows. In section 2, we constructed a novel five-dimensional
grid multi-wing butterfly chaotic system. In the section 3, the Lyapunov exponent spectrum and the
bifurcation diagram of the new system is discussed by numerical simulations. In the section 4, the
electronic circuit of the system is constructed, and the circuit simulation experiment is carried out.
Finally, a summary and conclusions are drawn in section 5.

2. Constructing Five-Dimensional Grid Multi-Wing Butterfly Chaotic System
Reference presented a five-term simple chaotic system as

\[
\begin{align*}
\dot{x}_1 &= b(x_2 - x_1), \\
\dot{x}_2 &= -x_1x_3, \\
\dot{x}_3 &= -c + x_1x_2,
\end{align*}
\]  

(1)
Where \( b = 5 \) and \( c = 90 \).

Based on the system (1), through adding two first-order differential equations about state variables \( x_4 \) and \( x_5 \) respectively, and modifying the system parameter appropriately, a five-dimensional autonomous system is obtained as

\[
\begin{align*}
\dot{x}_1 &= b_1 x_2 - b_2 x_1, \\
\dot{x}_2 &= -b_3 x_1 x_5, \\
\dot{x}_3 &= -b_4 + b_5 x_5, \\
\dot{x}_4 &= b_6 x_1 + b_7 x_2 - b_8 f(x_4), \\
\dot{x}_5 &= b_9 x_2 - b_{10} x_1 x_3 - b_{11} f(x_5),
\end{align*}
\tag{2}
\]

Where \( b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10} \) and \( b_{11} \) are the system parameter. \( f(x_4) \) and \( f(x_5) \) are piecewise linear functions as

\[
\begin{align*}
f(x_4) &= x_4 - \sum_{n=-N_1}^{M_1} \text{sgn}(x_4 + 2n + 1) + (M_1 - N_1 + 1) \\
f(x_5) &= x_5 - \sum_{n=-N_2}^{M_2} \text{sgn}(x_5 + 2n + 1) + (M_2 - N_2 + 1)
\end{align*}
\tag{3, 4}
\]

Where \( N_1, M_1, N_2, M_2 \in \{0, 1, 2, \ldots\} \).

Letting \( b_1 = 2.25, b_2 = 20, b_3 = 20, b_4 = 10, b_5 = 10, b_6 = 10, b_7 = 5, b_8 = 5, b_9 = 5, b_{10} = 20 \) and \( b_{11} = 5 \), the system (2) can generate \( 2(N_1 + M_1 + 2) \times (N_2 + M_2 + 2) \)-wing butterfly chaotic attractors which are shown in Fig. 1. When \( N_1 = N_2 = M_1 = M_2 = 0 \), the system (2) creates \( 4 \times 2 \)-wing butterfly chaotic attractors as is shown in Fig. 1(a). When \( N_1 = M_1 = M_2 = 0 \) and \( N_2 = 1 \), the system (2) generates \( 4 \times 3 \)-wing butterfly chaotic attractors as shown in Fig. 1(b). When \( N_1 = N_2 = 1 \) and \( M_1 = M_2 = 0 \), demonstrated in Fig. 1(c), the system (2) creates \( 6 \times 3 \)-wing butterfly chaotic attractors. When \( N_1 = N_2 = M_1 = M_2 = 1 \), the system (2) generates \( 8 \times 4 \)-wing butterfly chaotic attractors as is demonstrated in Fig. 1(d).
Figure 1. Grid multi-wing butterfly chaotic attractors of the system (2). (a) $4 \times 2$; (b) $4 \times 3$; (c) $6 \times 3$; (d) $8 \times 4$

3. Lyapunov Exponent Spectrum and Bifurcation Diagram

Let $N_1 = N_2 = 1$ and $M_1 = M_2 = 0$. The Lyapunov exponent spectrum of system (2) and its bifurcation diagram which varies with the coefficient $b_2$ are shown in Fig. 2. The range of values for the coefficient $b_2$ is 1 to 6. From the Fig. 2(a), we can see that the maximum Lyapunov exponent of the system (2) is greater than zero when $b_2 \in [4.48, 4.65] \cup [5.74, 6]$. Therefore, the system (2) is in the chaotic state. When $b_2 \in (1, 4.48] \cup (4.65, 5.74)$, the maximum Lyapunov exponent of the system (2) is equal to zero. The system (2) is in the periodic state. From the Fig. 2(b), we can see that the bifurcation diagram is in line with the corresponding Lyapunov exponent.

Figure 2. Lyapunov exponent spectrum of the system (2) and its bifurcation diagram. (a) Lyapunov exponent spectrum; (b) bifurcation diagram
4. Circuit Simulation of the Five-Dimensional Grid Multi-Wing Butterfly Chaotic System

Let \( N_1 = N_2 = M_1 = M_2 = 1 \), according to the system (2) with (3) and (4), an electronic circuit is constructed, which is shown in Fig. 3.

![Circuit Diagram](image)

Figure 3. Circuit diagram of the system (2) with (3) and (4).

In Fig. 3, all the operational amplifiers are picked as UA741CN. The supply voltage \( E = \pm 15V \) and saturated voltage \( V_{sat} \approx \pm 13.5V \). All the multipliers' type is AD633JN and the gain is 0.1. \( E^+ \) is the positive pole of the supply voltage \( E \), i.e., \( E^+ = 15V \). \( E^- \) is the negative pole of the supply voltage \( E \), i.e., \( E^- = -15V \).

By the Fig. 3, the circuit equation can be obtained as

\[
\begin{align*}
\frac{dx_1}{d\tau} &= \frac{1}{R_0C_0} \left( \frac{R}{R_6} x_1 - \frac{R}{R_2} x_3 \right), \\
\frac{dx_2}{d\tau} &= \frac{1}{R_0C_0} \left( R \frac{x_2}{10R_4} x_3 \right), \\
\frac{dx_3}{d\tau} &= \frac{1}{R_0C_0} \left( \frac{R}{10R_4} x_1 x_2 + \frac{R_5}{R_5 + R_6} E^- \right), \\
\frac{dx_4}{d\tau} &= \frac{1}{R_0C_0} \left( \frac{R}{R_7} x_2 + \frac{R}{R_9} x_1 - \frac{R}{R_9} f(x_4) \right), \\
\frac{dx_5}{d\tau} &= \frac{1}{R_0C_0} \left( \frac{R}{R_{10}} x_2 - \frac{R}{R_{12}} x_1 x_3 - \frac{R}{R_{12}} f(x_5) \right).
\end{align*}
\]
\[-f(x_4) = -x_4 + \frac{R}{R_{v1}} \left| V_{ss} \right| \text{sgn} \left( x_4 - \left( \frac{R_{14} + R_{15}}{R_{13} + R_{14} + R_{15}} \right) E \right) - \frac{R_{16}}{R_{16} + R_{17}} E^+ + \frac{R}{R_{v3}} \left| V_{ss} \right| \text{sgn} \left( x_4 - \frac{R_{16}}{R_{16} + R_{17}} E^+ \right) \frac{R_{15}}{R_{13} + R_{14} + R_{15}} E^- + \frac{R}{R_{v1}} \left| V_{ss} \right| \text{sgn} \left( x_4 - \frac{R_{15}}{R_{13} + R_{14} + R_{15}} E^- \right) - \frac{R_{16}}{R_{16} + R_{17}} E^+ \right]\]

\[-f(x_5) = -x_5 + \frac{R}{R_{v1}} \left| V_{ss} \right| \text{sgn} \left( x_5 - \left( \frac{R_{14} + R_{16}}{R_{13} + R_{14} + R_{16}} \right) E \right) - \frac{R_{16}}{R_{16} + R_{17}} E^+ + \frac{R}{R_{v3}} \left| V_{ss} \right| \text{sgn} \left( x_5 - \frac{R_{16}}{R_{16} + R_{17}} E^+ \right) \frac{R_{15}}{R_{13} + R_{14} + R_{16}} E^- + \frac{R}{R_{v1}} \left| V_{ss} \right| \text{sgn} \left( x_5 - \frac{R_{15}}{R_{13} + R_{14} + R_{16}} E^- \right) - \frac{R_{16}}{R_{16} + R_{17}} E^+ \right]\]

To observe the output wave experimentally, \( \tau \) needs to take timescale transformation, that is, letting \( \tau = \tau_0/\tau_0 \), the Eq. (5) can be changed as

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{10^{-3}}{R_0 C_0} \left( \frac{R}{R_1} x_2 - \frac{R}{R_2} x_1 \right), \\
\frac{dx_2}{dt} &= \frac{10^{-3}}{R_0 C_0} \left( -\frac{R}{10R_1} x_1 x_3 \right), \\
\frac{dx_3}{dt} &= \frac{10^{-3}}{R_0 C_0} \left( \frac{R}{10R_1} x_1 x_2 + \frac{R}{R_5} \right), \\
\frac{dx_4}{dt} &= \frac{10^{-3}}{R_0 C_0} \left( \frac{R}{R_7} x_2 + \frac{R}{R_4} x_1 - \frac{R}{R_5} f(x_3) \right), \\
\frac{dx_5}{dt} &= \frac{10^{-3}}{R_0 C_0} \left( \frac{R}{10R_1} x_2 - \frac{R}{10R_1} x_2 x_3 - \frac{R}{R_2} f(x_5) \right). \\
\end{align*}
\]

Let \( R_0 = 10k\Omega \), \( C_0 = 10nF \), \( R = 10k\Omega \), \( R_5 = 3k\Omega \), \( R_4 = 2k\Omega \), \( R_{15} = 1k\Omega \), \( R_{16} = 1k\Omega \), according to (2), (3), (4), (6), (7) and (8), we can obtain \( R_1 = 10k\Omega \), \( R_2 = 20k\Omega \), \( R_3 = 0.5k\Omega \), \( R_4 = 0.5k\Omega \), \( R_5 = 197k\Omega \), \( R_6 = 10k\Omega \), \( R_7 = 20k\Omega \), \( R_8 = 10k\Omega \), \( R_{10} = 20k\Omega \), \( R_{11} = 0.5k\Omega \), \( R_{12} = 20k\Omega \), \( R_{13} = 12k\Omega \), \( R_{17} = 14k\Omega \), \( R_{18} = 135k\Omega \).

According to Fig. 3, the circuit simulation is carried out in the software Multisim 10.0, as is shown in Fig. 4. As \( k_1 \) and \( k_8 \) are turned on and \( k_1, k_2, k_4 \) and \( k_5 \) are turned off, the circuit generate 4×2 -wing butterfly chaotic attractors as shown in Figure 4(a). When \( k_3, k_5 \) and \( k_6 \) are turned on and \( k_1, k_2, k_4 \) are turned off, the circuit create 4×3 -wing butterfly chaotic attractors as is shown in Figure 4(b). When \( k_2, k_3, k_5 \) and \( k_6 \) are turned on and \( k_1 \) and \( k_4 \) are switched off, the circuit generate 6×3 -wing butterfly chaotic attractors as shown in Figure 4(c). When \( k_1, k_2, k_3, k_4, k_5 \) and \( k_6 \) are turned on, the circuit create 8×4 -wing butterfly chaotic attractors, which shown in Figure 4(d).
From the Figures 1 and 4, we can obtain that the circuit experimental results are in agreement with numerical simulation results, which prove that the system (2) is realizable.

5. Conclusion
In this paper, a new five dimensional grid multi Wing Butterfly chaotic system is proposed. Through numerical simulation and model analysis, the chaotic characteristics of the system are revealed. Moreover, the system has been performed by designing an electronic circuit. And the $4 \times 2$, $4 \times 3$, $6 \times 3$ and $8 \times 4$ -wing butterfly chaotic attractors are obtained.

Because the dynamic behavior of grid multi Wing Butterfly chaotic system is more complex than two wing, four wing and multi Wing Butterfly chaotic system, and has more advantages, it has great application value in the engineering field.

6. References
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