A Bayesian Hau-Kashyap Approach for Hepatitis Disease Detection

Andino Maseleno, Rohmah Zahroh Hidayati, Marini Othman, Alicia Y.C. Tang and Moamin A. Mahmoud

Additional information is available at the end of the chapter

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Abstract

World Health Organization reported that viral hepatitis affects 400 million people globally. Every year, 610 million people are newly infected. In this research, we integrate a Bayesian theory and Hau-Kashyap approach for detecting hepatitis and displaying the result of calculation process. The basic idea of the Bayesian theory is using the known prior probability and conditional probability density parameter based on the Bayes theorem to calculate the corresponding posterior probability and then obtain the posterior probability to infer and make decisions. Bayesian methods combine present knowledge, prior probabilities, with additional knowledge derived from new data, the likelihood function. Hau-Kashyap presented an alternative Dempster-Shafer combination rule, and the alternative combination rule is that with the use of this alternative rule, the intersection conflict is put into the union. In this chapter, we get basic possibility assignment value from Bayesian probability. The result reveals that a Bayesian Hau-Kashyap approach has successfully identified the existence of hepatitis.

Keywords: hepatitis, disease diagnosis, Bayesian, Dempster-Shafer theory, Hau-Kashyap approach

1. Introduction

Hepatitis is a medical condition defined by the inflammation of the liver and characterized by the presence of inflammatory cells in the tissue of the organ. The word “hepatitis” comes from the ancient Greek word “hepar,” root word “hepat,” meaning liver [1]. Hepatitis may occur with limited or no symptoms. Hepatitis is acute when it lasts less than 6 months and
chronic when it persists longer. In medical, hepatitis means injury to the liver with inflammation of the liver cells. The liver is the largest glandular organ of the body [2]. It weighs about 1.36 kg. It is reddish brown in color and is divided into four lobes of unequal size and shape. There are six main hepatitis viruses, referred to as types A, B, C, D, E and G. Hepatitis A and E are typically caused if patients eat the contaminated food or water. Hepatitis B, C and D are typically caused by parental contact by infected body fluid, and Hepatitis B also can be infected through sexual contact. Hepatitis B is primarily found in the liver. Researches have been done through methods for diagnosis of hepatitis [3, 4, 5]. Bayesian approaches are successfully applied to a variety of problems [6, 7, 8]; recently, several studies have been conducted and have focused on medical diagnosis. These studies have applied different approaches and have achieved various classification accuracies. Neshat et al. [9] studied an adaptive neural fuzzy system for diagnosing the hepatitis B intensity rate. Neshat et al. [10] describes the combination of two methods of particle swarm optimization, and case-based reasoning has been used to diagnose hepatitis. Mahesh et al. [5] proposed a generalized regression neural network-based expert system for the diagnosis of the hepatitis B virus disease. The system classifies each patient into infected and noninfected. If infected, then how severe it is in terms of intensity rate. Panchal et al. [11] described an artificial intelligence-based expert system for Hepatitis B diagnosis. The main reason for using a Bayesian approach to hepatitis detection is that it facilitates the uncertainties related to models and parameter values. It gives a characteristic and principled method of combining prior information with data, within a solid decision theoretical framework. We can fuse past data about a parameter and form a prior distribution for future analysis. When new observations become available, the previous posterior distribution can be used as a prior. All inferences logically follow from Bayesian Hau-Kashyap approach. The structure of the paper is as follows. Section 2 presents a Bayesian Hau-Kashyap approach. Section 3 presents implementation of Bayesian approach. Bayesian approach results are presented in Section 4. Section 5 presents a Bayesian Hau-Kashyap approach for hepatitis disease detection. Results and discussion are presented in Section 6. Finally, Section 7 presents some concluding remarks.

2. A Bayesian Hau-Kashyap approach

2.1. A Bayesian approach

Let the events $A_1, A_2, \ldots, A_n$ form a partition of the sample space $S$ with $P(A_i) < 0, i = 1, \ldots, n$. For any event $B \subset S$ with $P(B) > 0$, as shown in Eq. (1):

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}, \quad i = 1, \ldots, n. \quad (1)$$

We may rationalize this result as follows. Given $B \subset S = \bigcup_{i=1}^{n} A_i$, it follows that $B = \bigcup_{i=1}^{n} (B \cap A_i)$. If the $A_i$’s are mutually exclusive, then so are the events $B \cap A_i, i = 1, \ldots, n$, and thus, as shown in Eq. (2),
It follows that, as shown in Eqs. (3)–(5).

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$$P(B \cap A_i) = P(A_i) \cdot P(B|A_i), \quad i = 1, \ldots, n$$

Then [12]

$$P(A_i|B) = \frac{P(B \cap A_i)}{P(B)} = \frac{\frac{P(B \cap A_i)}{n} \cdot \frac{P(A_i) \cdot P(B|A_i)}{\sum P(B \cap A_i)}}{n} = \frac{P(A_i) \cdot P(B|A_i)}{\sum P(A_i) \cdot P(B|A_i)}, \quad i = 1, \ldots, n. \quad (5)$$

### 2.2. Dempster-Shafer theory

Belief functions offer a non-Bayesian method for quantifying subjective evaluations by using probability. In the 1970s, it was further developed by Shafer, whose book *Mathematical Theory of Evidence* [13] remains a classic in belief functions or the so-called Theory of Evidence. This theory has been also called the Dempster-Shafer Mathematical Theory of Evidence. In the 1980s, the scientific community working with Artificial Intelligence got involved in using the theory of evidence in applications. The Dempster-Shafer theory or the theory of belief functions is a mathematical theory of evidence, which can be interpreted as a generalization of probability theory [13, 14] in which the elements of the sample space to which nonzero probability mass is attributed are not single points but sets. The sets that get nonzero mass are called focal elements [13]. The sum of these probability masses is 1; however, the basic difference between Dempster-Shafer mathematical theory of evidence and traditional probability theory is that the focal elements of a Dempster-Shafer structure may overlap one another. The Dempster-Shafer mathematical theory of evidence also provides methods to represent and combine weights of evidence.

The Dempster-Shafer theory assumes that there is a fixed set of mutually exclusive and exhaustive elements called hypotheses or propositions and symbolized by the Greek letter $\Theta$, represented as $\Theta = \{h_1, h_2, \ldots, h_n\}$, where $h_i$ is called a hypothesis or proposition. A hypothesis can be any subset of the frame, in example, to singletons in the frame or to combinations of elements in the frame. $\Theta$ is also called frame of discernment. A basic probability assignment (bpa) is represented by a mass function $m : 2^\Theta \to [0, 1]$. Where $2^\Theta$ is the power set of $\Theta$.

### 2.3. Integrating Bayesian and Hau-Kashyap approach

Hau and Kashyap [15] presented an alternative Dempser-Shafer rule of combination, denoted by $\odot$. Method to integrate Bayesian theory and Hau-Kashyap approach as follows:

1. **Step 1**: Assume $m_1$ and $m_2$ are two mass functions on the frame of discernment $m(\Theta)$. 

$$P(B) = P\left(\bigcup_{i=1}^n (B \cap A_i)\right) = \sum_{i=1}^n P(B \cap A_i) \quad (2)$$
From Eq. (5),
\[ P(A_i | B) = \frac{P(B \cap A_i)}{P(B)} = \frac{\sum_{i=1}^{n} P(A_i | B) P(B)}{\sum_{i=1}^{n} P(A_i | B) P(B)} , \quad i = 1, \ldots, n. \]

We can get \( m \) from the result of Eq. (5). \( m(P) \) is called basic possibility assignment value, which presents the level of trust to proposition \( P \). Let \( R_i, Z_j \) be their sets of focal elements. \((m_1 \circledcirc m_2)(\emptyset) = 0\).

2. Step 2:
If \( R_i \cap Z_j \neq \emptyset \) then let \( X = R_i \cap Z_j \) and \((m_1 \circledcirc m_2)(X) = \sum_{R_i \cap Z_j} X m_1(R_i) m_2(Z_j) \) (6)

3. Step 3:
If \( R_i \cap Z_j = \emptyset \) then let \( X = R_i \cup Z_j \) and \((m_1 \circledcirc m_2)(X) = \sum_{R_i \cup Z_j} X m_1(R_i) m_2(Z_j) \) (7)

The fundamental distinction between the Dempster-Shafer combination rule and the Hau-Kashyap combination rule is that with the use of Hau-Kashyap rule, the conflict \( m_1(R_i) m_2(Z_j) \) for \( R_i \cap Z_j = \emptyset \) is put into the union \( R_i \cup Z_j \).

3. A Bayesian approach for hepatitis disease detection

Everyday medical practice contains many examples of probability. Medical doctor often uses words such as probably, unlikely, certainly, or almost certainly in all conversations with patients. Medical doctor only rarely attach numbers to these terms, but computerized systems must use some numerical representation of likelihood in order to combine statements into conclusions. Probability is represented numerically by a number between 0 and 1. This study conducts experiments on hepatitis dataset. The main goal of the dataset is to forecast the presence or absence of hepatitis virus. The dataset contains probability of the initial symptoms of hepatitis, which are often similar to other diseases.

The initial symptoms of hepatitis include malaise, fever and headache. The probability of malaise given the presence for hepatitis, malaria, influenza and gastroenteritis. The probability of fever given the presence for hepatitis, malaria, influenza and gastroenteritis. The probability of headache given the presence for hepatitis, malaria, influenza and gastroenteritis. The probability was obtained by studying a series of patients with proven hepatitis by looking up diagnosis codes in the medical records department, and computing the percentage of these patients who present with malaise, fever and headache.

3.1. Probability of hepatitis given the symptom of malaise

Malaise is a feeling of general discomfort, uneasiness or pain, often the first indication of an infection. Table 1 shows the probability of malaise (Ma) given the presence for hepatitis (H), malaria (M), influenza (I), and gastroenteritis (G).

\[ P(\text{Hepatitis} | \text{Malaise}) \], which is read as the probability of hepatitis given the symptom of malaise. \( P(\text{Malaise} (\text{Ma}) | \text{Hepatitis} (\text{H})) \), which is the probability of malaise given the
presence of hepatitis. Bayes rule allows us to compute the probability we really want \( P(\text{Hepatitis} \mid \text{Malaise}) \) with the help of the more readily available number \( P(\text{Malaise} \mid \text{Hepatitis}) \). Bayes’s theorem is a formula with conditioned probabilities. Calculating the probability of hepatitis given the symptom of malaise, which is calculated as follows:

\[
P(\text{Hepatitis} \mid \text{Malaise}) = \frac{0.85 \times 0.45}{(0.85 \times 0.45) + (0.65 \times 0.55) + (0.20 \times 0.50) + (0.60 \times 0.30)} = 0.375
\]

There is about a 37.5% chance that the probability of hepatitis given the symptom of malaise actually has the attribute given that it tested positively for it.

Calculating the probability of malaria given the symptom of malaise, which is calculated as follows:

\[
P(\text{Malaria} \mid \text{Malaise}) = \frac{0.65 \times 0.55}{(0.85 \times 0.45) + (0.65 \times 0.55) + (0.20 \times 0.50) + (0.60 \times 0.30)} = 0.350
\]

There is about a 35% chance that the probability of malaria given the symptom of malaise actually has the attribute given that it tested positively for it.

Calculating the probability of influenza given the symptom of malaise, which is calculated as follows:

\[
P(\text{Influenza} \mid \text{Malaise}) = \frac{0.20 \times 0.50}{(0.85 \times 0.45) + (0.65 \times 0.55) + (0.20 \times 0.50) + (0.60 \times 0.30)} = 0.098
\]

There is about a 9.8% chance that the probability of influenza given the symptom of malaise actually has the attribute given that it tested positively for it.

Calculating the probability of gastroenteritis given the symptom of malaise, which is calculated as follows:

\[
P(\text{Gastroenteritis} \mid \text{Malaise}) = \frac{0.60 \times 0.30}{(0.85 \times 0.45) + (0.65 \times 0.55) + (0.20 \times 0.50) + (0.60 \times 0.30)} = 0.177
\]

| Action                  | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|-------------------------|-------------|-------------|-------------|-------------|-------------|
| Malaise | Hepatitis    | 0.85        | 0.70        | 0.80        | 0.75        | 0.60        |
| Hepatitis          | 0.45        | 0.30        | 0.35        | 0.40        | 0.50        |
| Malaise | Malaria      | 0.65        | 0.55        | 0.75        | 0.45        | 0.85        |
| Malaria           | 0.55        | 0.40        | 0.50        | 0.35        | 0.45        |
| Malaise | Influenza    | 0.20        | 0.25        | 0.30        | 0.35        | 0.40        |
| Influenza         | 0.50        | 0.30        | 0.45        | 0.40        | 0.35        |
| Malaise | Gastroenteritis | 0.60    | 0.50        | 0.65        | 0.70        | 0.75        |
| Gastroenteritis  | 0.30        | 0.35        | 0.40        | 0.50        | 0.60        |

Table 1. Hepatitis | malaise.
There is about a 17.7% chance that the probability of gastroenteritis given the symptom of malaise actually has the attribute given that it tested positively for it.

### 3.2. Probability of hepatitis given the symptom of fever

Fever is defined as having a temperature above the normal range due to an increase in the body’s temperature set point. Table 2 shows the probability of fever (Fe) given the presence for hepatitis (H), malaria (M), influenza (I) and gastroenteritis (G).

Calculating the probability of hepatitis given the symptom of fever, which is calculated as follows:

\[
P(\text{Hepatitis}|\text{Fever}) = \frac{0.75 \times 0.40}{(0.75 \times 0.40) + (0.60 \times 0.50) + (0.65 \times 0.45) + (0.50 \times 0.30)} = 0.288
\]

There is about a 28.8% chance that the probability of hepatitis given the symptom of fever actually has the attribute given that it tested positively for it.

Calculating the probability of malaria given the symptom of fever, which is calculated as follows:

\[
P(\text{Malaria}|\text{Fever}) = \frac{0.60 \times 0.50}{(0.75 \times 0.40) + (0.60 \times 0.50) + (0.65 \times 0.45) + (0.50 \times 0.30)} = 0.288
\]

There is about a 28.8% chance that the probability of malaria given the symptom of fever actually has the attribute given that it tested positively for it.

Calculating the probability of influenza given the symptom of fever, which is calculated as follows:

\[
P(\text{Influenza}|\text{Fever}) = \frac{0.65 \times 0.45}{(0.75 \times 0.40) + (0.60 \times 0.50) + (0.65 \times 0.45) + (0.50 \times 0.30)} = 0.280
\]

There is about a 28% chance that the probability of influenza given the symptom of fever actually has the attribute given that it tested positively for it.

| Action         | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|----------------|-------------|-------------|-------------|-------------|-------------|
| Fever | Hepatitis   | 0.75        | 0.70        | 0.80        | 0.60        | 0.65        |
| Hepatitis      | 0.40        | 0.45        | 0.50        | 0.55        | 0.60        |
| Fever | Malaria     | 0.60        | 0.80        | 0.70        | 0.75        | 0.65        |
| Malaria        | 0.50        | 0.40        | 0.45        | 0.55        | 0.35        |
| Fever | Influenza   | 0.65        | 0.70        | 0.75        | 0.60        | 0.80        |
| Influenza      | 0.45        | 0.50        | 0.35        | 0.55        | 0.40        |
| Fever | Gastroenteritis | 0.50    | 0.40        | 0.45        | 0.55        | 0.35        |
| Gastroenteritis | 0.30        | 0.45        | 0.30        | 0.35        | 0.30        |

Table 2. Hepatitis | fever.
Calculating the probability of gastroenteritis given the symptom of fever, which is calculated as follows:

\[
P(\text{Gastroenteritis}|\text{Fever}) = \frac{0.50 \times 0.30}{(0.75 \times 0.40) + (0.60 \times 0.50) + (0.65 \times 0.45) + (0.50 \times 0.30)} = 0.144
\]

There is about a 14.4% chance that the probability of gastroenteritis given the symptom of fever actually has the attribute given that it tested positively for it.

### 3.3. Probability of hepatitis given the symptom of headache

Headache is pain in any region of the head. Headaches may occur on one or both sides of the head, be isolated to a certain location, radiate across the head from one point or have a viselike quality. **Table 3** shows the probability of headache (He) given the presence for hepatitis (H), malaria (M), influenza (I), and gastroenteritis (G).

Calculating the probability of hepatitis given the symptom of headache, which is calculated as follows:

\[
P(\text{Hepatitis}|\text{Headache}) = \frac{0.80 \times 0.45}{(0.80 \times 0.45) + (0.75 \times 0.30) + (0.55 \times 0.50) + (0.60 \times 0.45)} = 0.318
\]

There is about a 31.8% chance that the probability of hepatitis given the symptom of headache actually has the attribute given that it tested positively for it.

Calculating the probability of malaria given the symptom of headache, which is calculated as follows:

\[
P(\text{Malaria}|\text{Headache}) = \frac{0.75 \times 0.30}{(0.80 \times 0.45) + (0.75 \times 0.30) + (0.55 \times 0.50) + (0.60 \times 0.45)} = 0.199
\]

There is about a 19.9% chance that the probability of malaria given the symptom of headache actually has the attribute given that it tested positively for it.

| Action | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|--------|-------------|-------------|-------------|-------------|-------------|
| Headache | 0.80       | 0.75       | 0.70       | 0.65       | 0.60       |
| Hepatitis | 0.45       | 0.35       | 0.40       | 0.50       | 0.55       |
| Headache | 0.75       | 0.70       | 0.60       | 0.80       | 0.65       |
| Malaria | 0.30       | 0.40       | 0.45       | 0.35       | 0.50       |
| Headache | 0.55       | 0.50       | 0.40       | 0.45       | 0.60       |
| Influenza | 0.50      | 0.55       | 0.45       | 0.60       | 0.65       |
| Headache | 0.60       | 0.65       | 0.55       | 0.40       | 0.45       |
| Gastroenteritis | 0.45       | 0.50       | 0.40       | 0.55       | 0.60       |

**Table 3.** Hepatitis | headache.
Calculating the probability of influenza given the symptom of headache, which is calculated as follows:

\[
P(\text{Influenza}|\text{Headache}) = \frac{0.55 \times 0.50}{(0.80 \times 0.45) + (0.75 \times 0.30) + (0.55 \times 0.50) + (0.60 \times 0.45)} = 0.243
\]

There is about a 24.3% chance that the probability of influenza given the symptom of headache actually has the attribute given that it tested positively for it.

Calculating the probability of gastroenteritis given the symptom of headache, which is calculated as follows:

\[
P(\text{Gastroenteritis}|\text{Headache}) = \frac{0.60 \times 0.45}{(0.80 \times 0.45) + (0.75 \times 0.30) + (0.55 \times 0.50) + (0.60 \times 0.45)} = 0.240
\]

There is about a 24% chance that the probability of gastroenteritis given the symptom of headache actually has the attribute given that it tested positively for it.

4. A Bayesian approach for hepatitis disease detection results

Table 4 shows probability of diseases given the symptom of malaise. These probabilities are probability of hepatitis given the symptom of malaise, probability of malaria given the symptom of malaise, probability of influenza given the symptom of malaise and probability of gastroenteritis given the symptom of malaise.

Figure 1 shows graphic of probability of disease given the symptom of malaise. Probability of hepatitis given the symptom of malaise obtained value 0.375 for condition 1, 0.310 for condition 2, 0.267 for condition 3, 0.317 for condition 4 and 0.236 for condition 5. Probability of malaria given the symptom of malaise obtained value 0.350 for condition 1, 0.323 for condition 2, 0.357 for condition 3, 0.166 for condition 4 and 0.300 for condition 5. Probability of influenza given the symptom of malaise obtained value 0.098 for condition 1, 0.110 for condition 2, 0.128 for condition 3, 0.148 for condition 4 and 0.110 for condition 5. Probability of gastroenteritis given the symptom of malaise obtained value 0.177 for condition 1, 0.257 for condition 2, 0.248 for condition 3, 0.369 for condition 4 and 0.354 for condition 5.

| Action         | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|----------------|-------------|-------------|-------------|-------------|-------------|
| Hepatitis | Malaise     | 0.375       | 0.310       | 0.267       | 0.317       | 0.236       |
| Malaria     | Malaise     | 0.350       | 0.323       | 0.357       | 0.166       | 0.300       |
| Influenza   | Malaise     | 0.098       | 0.110       | 0.128       | 0.148       | 0.110       |
| Gastroenteritis | Malaise | 0.177       | 0.257       | 0.248       | 0.369       | 0.354       |

Table 4. Hepatitis | malaise.
Table 5 shows probability of diseases given the symptom of fever. These probabilities are probability of hepatitis given the symptom of fever, probability of malaria given the symptom of fever, probability of influenza given the symptom of fever and probability of gastroenteritis given the symptom of fever.

Figure 2 shows graphic of probability of disease given the symptom of fever. Probability of hepatitis given the symptom of fever obtained value 0.288 for condition 1, 0.270 for condition 2, 0.360 for condition 3, 0.261 for condition 4 and 0.351 for condition 5. Probability of malaria given the symptom of fever obtained value 0.288 for condition 1, 0.275 for condition 2, 0.283 for condition 3, 0.326 for condition 4 and 0.204 for condition 5. Probability of influenza given the symptom of fever obtained value 0.280 for condition 1, 0.300 for condition 2, 0.236 for condition 3, 0.261 for condition 4 and 0.288 for condition 5. Probability of gastroenteritis given the symptom of fever obtained value 0.144 for condition 1, 0.155 for condition 2, 0.121 for condition 3, 0.152 for condition 4 and 0.157 for condition 5.

Table 6 shows probability of diseases given the symptom of headache. These probabilities are probability of hepatitis given the symptom of headache, probability of malaria given the symptom of headache, probability of influenza given the symptom of headache and probability of gastroenteritis given the symptom of headache.

| Action    | Condition | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|-----------|-----------|-------------|-------------|-------------|-------------|-------------|
| Hepatitis | Fever     | 0.288       | 0.270       | 0.360       | 0.261       | 0.351       |
| Malaria   | Fever     | 0.288       | 0.275       | 0.283       | 0.326       | 0.204       |
| Influenza | Fever     | 0.280       | 0.300       | 0.236       | 0.261       | 0.288       |
| Gastroenteritis | Fever | 0.144       | 0.155       | 0.121       | 0.152       | 0.157       |

Table 5. Hepatitis | fever.
Figure 3 shows graphic of probability of disease given the symptom of headache. Probability of hepatitis given the symptom of headache obtained value 0.318 for condition 1, 0.230 for condition 2, 0.295 for condition 3, 0.296 for condition 4 and 0.251 for condition 5. Probability of malaria given the symptom of headache obtained value 0.199 for condition 1, 0.245 for condition 2, 0.284 for condition 3, 0.256 for condition 4 and 0.247 for condition 5. Probability of influenza given the symptom of headache obtained value 0.243 for condition 1, 0.241 for condition 2, 0.189 for condition 3, 0.247 for condition 4 and 0.297 for condition 5. Probability of gastroenteritis given the symptom of headache obtained value 0.240 for condition 1, 0.284 for condition 2, 0.232 for condition 3, 0.201 for condition 4 and 0.205 for condition 5.

Table 6.

| Action             | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|--------------------|-------------|-------------|-------------|-------------|-------------|
| Hepatitis | Headache    | 0.318       | 0.230       | 0.295       | 0.296       | 0.251       |
| Malaria | Headache    | 0.199       | 0.245       | 0.284       | 0.256       | 0.247       |
| Influenza | Headache    | 0.243       | 0.241       | 0.189       | 0.247       | 0.297       |
| Gastroenteritis | Headache    | 0.240       | 0.284       | 0.232       | 0.201       | 0.205       |

Figure 4 shows overall malaria disease diagnosis. Condition 1 of malaria disease diagnosis obtained value 35% for probability of malaria given the symptom of malaise, 28.8% for probability of malaria given the symptom of fever and 19.9% for probability of malaria given the symptom of headache. Condition 2 of malaria disease diagnosis obtained value 32.3% for probability of malaria given the symptom of malaise, 27.5% for probability of malaria given the symptom of fever and 24.5% for probability of malaria given the symptom of headache. Condition 3 of malaria disease diagnosis obtained value 35.7% for probability of malaria given the symptom of malaise, 28.3% for probability of malaria given the symptom of fever and 28.4% for probability of malaria given the symptom of headache. Condition 4 of malaria
disease diagnosis obtained value 16.6% for probability of malaria given the symptom of malaise, 32.6% for probability of malaria given the symptom of fever and 25.6% for probability of malaria given the symptom of headache. Condition 5 of malaria disease diagnosis obtained value 30% for probability of malaria given the symptom of malaise, 20.4% for probability of malaria given the symptom of fever and 24.7% for probability of malaria given the symptom of headache.

Figure 5 shows overall influenza disease diagnosis. Condition 1 of influenza disease diagnosis obtained value 9.8% for probability of influenza given the symptom of malaise, 28% for probability of influenza given the symptom of fever and 24.3% for probability of influenza given the symptom of headache. Condition 2 of influenza disease diagnosis obtained value 11% for probability of influenza given the symptom of malaise, 30% for probability of influenza given the symptom of fever and 24.1% for probability of influenza given the symptom of headache. Condition 3 of influenza disease diagnosis obtained value 12.8% for probability of influenza given the symptom of malaise, 23.6% for probability of influenza given the symptom
of fever and 18.9% for probability of influenza given the symptom of headache. Condition 4 of influenza disease diagnosis obtained value 14.8% for probability of influenza given the symptom of malaise, 26.1% for probability of influenza given the symptom of fever and 24.7% for probability of influenza given the symptom of headache. Condition 5 of influenza disease diagnosis obtained value 11% for probability of influenza given the symptom of malaise, 28.8% for probability of influenza given the symptom of fever and 29.7% for probability of influenza given the symptom of headache.

Figure 6 shows overall gastroenteritis disease diagnosis. Condition 1 of gastroenteritis disease diagnosis obtained value 17.7% for probability of gastroenteritis given the symptom of malaise, 14.4% for probability of gastroenteritis given the symptom of fever and 24% for probability of gastroenteritis given the symptom of headache. Condition 2 of gastroenteritis disease diagnosis obtained value 25.7% for probability of gastroenteritis given the symptom of malaise, 15.5% for probability of gastroenteritis given the symptom of fever
and 28.4% for probability of gastroenteritis given the symptom of headache. Condition 3 of gastroenteritis disease diagnosis obtained value 24.8% for probability of gastroenteritis given the symptom of malaise, 12.1% for probability of gastroenteritis given the symptom of fever and 23.2% for probability of gastroenteritis given the symptom of headache. Condition 4 of gastroenteritis disease diagnosis obtained value 36.9% for probability of gastroenteritis given the symptom of malaise, 15.2% for probability of gastroenteritis given the symptom of fever and 20.1% for probability of gastroenteritis given the symptom of headache. Condition 5 of gastroenteritis disease diagnosis obtained value 35.4% for probability of gastroenteritis given the symptom of malaise, 15.7% for probability of gastroenteritis given the symptom of fever and 20.5% for probability of gastroenteritis given the symptom of headache.

**Figure 7** shows overall hepatitis diagnosis. Condition 1 of hepatitis diagnosis obtained value 37.5% for probability of hepatitis given the symptom of malaise, 28.8% for probability of hepatitis given the symptom of fever and 31.8% for probability of hepatitis given the symptom of headache. Condition 2 of hepatitis diagnosis obtained value 31% for probability of hepatitis given the symptom of malaise, 27% for probability of hepatitis given the symptom of fever and 23% for probability of hepatitis given the symptom of headache. Condition 3 of hepatitis diagnosis obtained value 26.7% for probability of hepatitis given the symptom of malaise, 36% for probability of hepatitis given the symptom of fever and 29.5% for probability of hepatitis given the symptom of headache. Condition 4 of hepatitis diagnosis obtained value 31.7% for probability of hepatitis given the symptom of malaise, 26.1% for probability of hepatitis given the symptom of fever and 29.6% for probability of hepatitis given the symptom of headache. Condition 5 of hepatitis diagnosis obtained value 23.6% for probability of hepatitis given the symptom of malaise, 35.1% for probability of hepatitis given the symptom of fever and 25.1% for probability of hepatitis given the symptom of headache.

![Figure 7](http://dx.doi.org/10.5772/intechopen.74638)
5. A Bayesian Hau-Kashyap approach for hepatitis disease detection

5.1. Probability of hepatitis given the symptom of malaise

1. There is about 37.5% chance that the probability of hepatitis given the symptom of malaise

\[ m_1\{H\} = 0.375, m_1\{\theta\} = 1 - 0.375 = 0.625 \]

2. There is about 35% chance that the probability of malaria given the symptom of malaise

\[ m_2\{M\} = 0.35, m_2\{\theta\} = 1 - 0.35 = 0.65 \]

The calculation of the combined \( m_1 \) and \( m_2 \) is shown in Table 7. Each cell of the table contains the intersection of the corresponding propositions from \( m_1 \) and \( m_2 \) along with the product of their individual belief.

From Table 7, we get:

\[ m_3\{M, H\} = 0.131, m_3\{H\} = 0.244, m_3\{M\} = 0.219, m_3\{\theta\} = 0.406. \]

3. There is about 9.8% chance that the probability of influenza given the symptom of malaise

\[ m_4\{I\} = 0.098, m_4\{\theta\} = 1 - 0.098 = 0.902 \]

The calculation of the combined \( m_3 \) and \( m_4 \) is shown in Table 8. Each cell of the table contains the intersection of the corresponding propositions from \( m_3 \) and \( m_4 \) along with the product of their individual belief.

|          | \( M \) | \( \theta \) | \( \theta \) |
|----------|---------|-------------|-------------|
| \( H \)  | 0.375   | \( M, H \)  | 0.131       |
| \( \theta \) | 0.625   | \( M \)     | 0.219       |

Table 7. The first combination of probability of hepatitis given the symptom of malaise.

|          | \( I \) | \( \theta \) | \( \theta \) |
|----------|---------|-------------|-------------|
| \( M, H \) | 0.131   | \( M, H, I \) | 0.013       | \( M, H \) | 0.118   |
| \( H \)  | 0.244   | \( H, I \)  | 0.024       | \( H \)    | 0.220   |
| \( M \)  | 0.219   | \( M, I \)  | 0.021       | \( M \)    | 0.197   |
| \( \theta \) | 0.406   | \( I \)     | 0.04        | \( \theta \) | 0.366   |

Table 8. The second combination of probability of hepatitis given the symptom of malaise.
From Table 8, we get:
\[
\begin{align*}
m_5\{M,H,I\} &= 0.013, m_5\{M,H\} = 0.118, m_5\{H,I\} = 0.024, m_5\{H\} = 0.220, \\
m_5\{M,I\} &= 0.021, m_5\{M\} = 0.197, m_5\{I\} = 0.04, m_5 \{\theta\} = 0.366. 
\end{align*}
\]

4. There is about 17.7% chance that the probability of gastroenteritis given the symptom of malaise

\[
m_6\{G\} = 0.177, m_6\{\theta\} = 1 - 0.177 = 0.823
\]

The calculation of the combined \(m_5\) and \(m_6\) is shown in Table 9. Each cell of the table contains the intersection of the corresponding propositions from \(m_5\) and \(m_6\) along with the product of their individual belief.

From Table 9, we get:
\[
\begin{align*}
m_7\{M,H,I,G\} &= 0.02, m_7\{M,H,I\} = 0.01, m_7\{M,H,G\} = 0.021, m_7\{M,H\} = 0.097, \\
m_7\{H,I,G\} &= 0.004, m_7\{H,I\} = 0.02, m_7\{H,G\} = 0.039, m_7\{H\} = 0.181, \\
m_7\{M,I,G\} &= 0.004, m_7\{M,I\} = 0.017, m_7\{M,G\} = 0.035, m_7\{M\} = 0.102, \\
m_7\{I,G\} &= 0.007, m_7\{I\} = 0.033, m_7\{G\} = 0.06, m_7 \{\theta\} = 0.301.
\end{align*}
\]

5.2. Probability of hepatitis given the symptom of fever

1. There is about 28.8% chance that the probability of hepatitis given the symptom of fever

\[
m_1\{H\} = 0.288, m_1\{\theta\} = 1 - 0.288 = 0.712
\]

2. There is about 28.8% chance that the probability of malaria given the symptom of fever

\[
m_2\{M\} = 0.288, m_2\{\theta\} = 1 - 0.288 = 0.712
\]

|        | \{G\} | \{G\} & \theta & \{G\} & \theta |
|--------|--------|--------|--------|--------|
| \{M,H,I\} | 0.013  | 0.02   | \{M,H,I\} | 0.01  |
| \{M,H\}   | 0.118  | 0.021  | \{M,H\}   | 0.097 |
| \{H,I\}   | 0.024  | 0.004  | \{H,I\}   | 0.02  |
| \{H\}     | 0.220  | 0.039  | \{H\}     | 0.181 |
| \{M,I\}   | 0.021  | 0.004  | \{M,I\}   | 0.017 |
| \{M\}     | 0.197  | 0.035  | \{M\}     | 0.102 |
| \{I\}     | 0.04   | 0.007  | \{I\}     | 0.033 |
| \theta    | 0.366  | 0.06   | \theta    | 0.301 |

Table 9. The third combination of probability of hepatitis given the symptom of malaise.
The calculation of the combined $m_1$ and $m_2$ is shown in Table 10. Each cell of the table contains the intersection of the corresponding propositions from $m_1$ and $m_2$ along with the product of their individual belief.

From Table 10 we get:

\[
m_3 \{M, H\} = 0.083, \quad m_3 \{H\} = 0.205, \quad m_3 \{M\} = 0.205, \quad m_3 \{\theta\} = 0.507.
\]

3. There is about 28% chance that the probability of influenza given the symptom of fever

\[
m_4 \{I\} = 0.28, \quad m_4 \{\theta\} = 1 - 0.28 = 0.72
\]

The calculation of the combined $m_3$ and $m_4$ is shown in Table 11. Each cell of the table contains the intersection of the corresponding propositions from $m_3$ and $m_4$ along with the product of their individual belief.

From Table 11, we get:

\[
m_5 \{M, H, I\} = 0.023, \quad m_5 \{M, H\} = 0.06, \quad m_5 \{H, I\} = 0.057,
\]

\[
m_5 \{H\} = 0.148, \quad m_5 \{M, I\} = 0.057, \quad m_5 \{M\} = 0.148, \quad m_5 \{I\} = 0.142, \quad m_5 \{\theta\} = 0.365.
\]

4. There is about 14.4% chance that the probability of gastroenteritis given the symptom of fever

\[
m_6 \{G\} = 0.144, \quad m_6 \{\theta\} = 1 - 0.144 = 0.856
\]

The calculation of the combined $m_5$ and $m_6$ is shown in Table 12. Each cell of the table contains the intersection of the corresponding propositions from $m_5$ and $m_6$ along with the product of their individual belief.

|       | (M)  | 0.288 | θ     | 0.712 |
|-------|------|-------|-------|-------|
| [H]   | 0.288| 0.083 | [H]   | 0.205 |
| θ     | 0.712| 0.205 | θ     | 0.507 |

Table 10. The first combination of probability of hepatitis given the symptom of fever.

|       | [I]  | 0.28 | θ     | 0.72  |
|-------|------|------|-------|-------|
| [M,H] | 0.083| 0.023| [M,H] | 0.06  |
| [H]   | 0.205| 0.057| [H]   | 0.148 |
| [M]   | 0.205| 0.057| [M]   | 0.148 |
| θ     | 0.507| 0.142| θ     | 0.365 |

Table 11. The second combination of probability of hepatitis given the symptom of fever.
From Table 12, we get:

\[
\begin{align*}
&m_7 \{M, H, I, G\} = 0.003, \\
&m_7 \{M, H, I\} = 0.02, \\
&m_7 \{M, H, G\} = 0.009, \\
&m_7 \{M, H\} = 0.05, \\
&m_7 \{H, I, G\} = 0.008, \\
&m_7 \{H, I\} = 0.049, \\
&m_7 \{H, G\} = 0.02, \\
&m_7 \{H\} = 0.05, \\
&m_7 \{I, G\} = 0.008, \\
&m_7 \{I, H\} = 0.049, \\
&m_7 \{I, G\} = 0.02, \\
&m_7 \{I\} = 0.049, \\
&m_7 \{G\} = 0.003.
\end{align*}
\]

Table 12. The third combination of probability of hepatitis given the symptom of fever.

From Table 12, we get:

\[
\begin{align*}
&m_7 \{M, H, I, G\} = 0.003, \\
&m_7 \{M, H, I\} = 0.02, \\
&m_7 \{M, H, G\} = 0.009, \\
&m_7 \{M, H\} = 0.05, \\
&m_7 \{H, I, G\} = 0.008, \\
&m_7 \{H, I\} = 0.049, \\
&m_7 \{H, G\} = 0.02, \\
&m_7 \{H\} = 0.05, \\
&m_7 \{I, G\} = 0.008, \\
&m_7 \{I, H\} = 0.049, \\
&m_7 \{I, G\} = 0.02, \\
&m_7 \{I\} = 0.049, \\
&m_7 \{G\} = 0.003.
\end{align*}
\]

Table 12. The third combination of probability of hepatitis given the symptom of fever.

| \{M, H, I\} | 0.023 | \{M, H, I, G\} | 0.003 | \{M, H, I\} | 0.02 |
| \{M, H\} | 0.06 | \{M, H, G\} | 0.009 | \{M, H\} | 0.05 |
| \{H, I\} | 0.057 | \{H, I, G\} | 0.008 | \{H, I\} | 0.049 |
| \{H\} | 0.148 | \{H, G\} | 0.02 | \{H\} | 0.127 |
| \{M, I\} | 0.057 | \{M, I, G\} | 0.008 | \{M, I\} | 0.049 |
| \{M\} | 0.148 | \{M, G\} | 0.02 | \{M\} | 0.127 |
| \{I\} | 0.142 | \{I, G\} | 0.02 | \{I\} | 0.121 |
| \theta | 0.365 | \{G\} | 0.052 | \theta | 0.312 |

5.3. Probability of hepatitis given the symptom of headache

1. There is about 31.8% chance that the probability of hepatitis given the symptom of headache

\[
m_1\{H\} = 0.318, m_1\{\theta\} = 1 - 0.318 = 0.682
\]

2. There is about 19.9% chance that the probability of malaria given the symptom of headache

\[
m_2\{M\} = 0.199, m_2\{\theta\} = 1 - 0.199 = 0.801
\]

The calculation of the combined \(m_1\) and \(m_2\) is shown in Table 13. Each cell of the table contains the intersection of the corresponding propositions from \(m_1\) and \(m_2\) along with the product of their individual belief.

| \{M\} | 0.199 | \theta | 0.801 |
| \{H\} | 0.318 | \{M, H\} | 0.063 | \{H\} | 0.255 |
| \theta | 0.682 | \{M\} | 0.136 | \theta | 0.546 |

Table 13. The first combination of probability of hepatitis given the symptom of headache.
From Table 13, we get:

\[ m_3\{M, H\} = 0.063, m_3\{H\} = 0.255, m_3\{M\} = 0.136, m_3 \{\theta\} = 0.546. \]

3. There is about 24.3% chance that the probability of influenza given the symptom of headache

\[ m_4\{I\} = 0.243, m_4\{\theta\} = 1 - 0.243 = 0.757 \]

The calculation of the combined \( m_3 \) and \( m_4 \) is shown in Table 14. Each cell of the table contains the intersection of the corresponding propositions from \( m_3 \) and \( m_4 \) along with the product of their individual belief.

From Table 14, we get:

\[ m_5\{M, H, I\} = 0.015, m_5\{M, H\} = 0.047, m_5\{H, I\} = 0.062, \]
\[ m_5\{H\} = 0.193, m_5\{M, I\} = 0.033, m_5\{M\} = 0.103, m_5\{I\} = 0.133, m_5 \{\theta\} = 0.413. \]

4. There is about 24% chance that the probability of gastroenteritis given the symptom of headache

\[ m_6\{G\} = 0.24, m_6\{\theta\} = 1 - 0.24 = 0.76 \]

The calculation of the combined \( m_5 \) and \( m_6 \) is shown in Table 15. Each cell of the table contains the intersection of the corresponding propositions from \( m_5 \) and \( m_6 \) along with the product of their individual belief.

From Table 15, we get:

\[ m_7\{M, H, I, G\} = 0.004, m_7\{M, H, I\} = 0.011, m_7\{M, H, G\} = 0.011, m_7\{M, H\} = 0.036, \]
\[ m_7\{H, I, G\} = 0.015, m_7\{H, I\} = 0.047, m_7\{H, G\} = 0.046, m_7\{H\} = 0.147, \]
\[ m_7\{M, I, G\} = 0.008, m_7\{M, I\} = 0.025, m_7\{M, G\} = 0.025, m_7\{M\} = 0.078, \]
\[ m_7\{I, G\} = 0.032, m_7\{I\} = 0.101, m_7\{G\} = 0.099, m_7 \{\theta\} = 0.314. \]

|       | \{I\} | \theta | \{\theta\} |
|-------|-------|-------|------------|
| \{M,H\} | 0.063 | 0.015 | (M,H) 0.047 |
| \{H\} | 0.255 | 0.062 | (H) 0.193  |
| \{M\} | 0.136 | 0.033 | (M) 0.103  |
| \{\theta\} | 0.546 | 0.133 | \theta 0.413 |

Table 14. The second combination of probability of hepatitis given the symptom of headache.

|       | \{G\} | \theta | \{\theta\} |
|-------|-------|-------|------------|
| \{M,H,I,G\} | 0.004 | (M,H,I,G) 0.011 |
| \{M,H,G\} | 0.011 | (M,H,G) 0.036 |
| \{H,I,G\} | 0.015 | (H,I,G) 0.047 |
| \{H,G\} | 0.046 | (H,G) 0.147 |
6. Results and discussions

Figure 8 shows probability of hepatitis given the symptom of malaise using the Bayesian Hau-Kashyap approach. Probability of hepatitis given the symptom of malaise obtained value 0.181 for condition 1, 0.139 for condition 2, 0.113 for condition 3, 0.142 for condition 4, 0.095 for condition 5.

Figure 9 shows probability of hepatitis given the symptom of fever using the Bayesian Hau-Kashyap approach. Probability of hepatitis given the symptom of fever obtained value 0.127 for condition 1, 0.116 for condition 2, 0.173 for condition 3, 0.110 for condition 4, 0.168 for condition 5.

Table 15. The third combination of probability of hepatitis given the symptom of headache.

| Condition | (G) | θ | (I,G) |
|-----------|-----|----|-------|
| (M,I)     | 0.033 | 0.008 | (M,I) |
|          | 0.103 | 0.025 | (M)   |
| (I)       | 0.133 | 0.052 | (I)   |
| θ         | 0.413 | 0.099 | θ     | 0.314 |

Figure 8. Probability of hepatitis given the symptom of malaise using the Bayesian Hau-Kashyap approach.

Figure 9. Probability of hepatitis given the symptom of fever using the Bayesian Hau-Kashyap approach.
Figure 10 shows probability of hepatitis given the symptom of headache using the Bayesian Hau-Kashyap approach. Probability of hepatitis given the symptom of headache obtained value 0.147 for condition 1, 0.094 for condition 2, 0.131 for condition 3, 0.133 for condition 4, 0.106 for condition 5.

We compare the Bayesian approach and Bayesian Hau-Kashyap approach, where the comparison results are shown in Table 16. As shown in Table 16, it is obvious that the Bayesian Hau-Kashyap approach has minimum probability, so it can minimize the hepatitis disease level.

| Approach                | Symptom | Condition     | Condition 1 | Condition 2 | Condition 3 | Condition 4 | Condition 5 |
|-------------------------|---------|---------------|-------------|-------------|-------------|-------------|-------------|
| Bayesian                | Malaise | 0.375         | 0.310       | 0.267       | 0.317       | 0.236       |
|                         | Fever   | 0.288         | 0.270       | 0.36        | 0.261       | 0.351       |
|                         | Headache| 0.318         | 0.230       | 0.295       | 0.296       | 0.251       |
| Bayesian Hau-Kashyap    | Malaise | 0.181         | 0.139       | 0.113       | 0.142       | 0.095       |
|                         | Fever   | 0.127         | 0.116       | 0.173       | 0.110       | 0.168       |
|                         | Headache| 0.147         | 0.094       | 0.131       | 0.133       | 0.106       |

Table 16. Probability of hepatitis comparison between the Bayesian approach and Bayesian Hau-Kashyap approach.

7. Conclusion

The initial symptoms of hepatitis are often similar to other diseases. A Bayesian approach has been proposed and implemented in order to diagnosis hepatitis. The hepatitis is a serious disease, its treatment is expensive and severe side effects can appear very often. Therefore, it is important to set a correct diagnosis and to identify those patients who most probably have hepatitis. That is for what the use of such a system can support the medical doctor decisions.
The most highest probability of hepatitis given the presence of disease in this work which include condition 1 of hepatitis diagnosis obtained value 37.5% for probability of hepatitis given the presence of malaise, condition 2 of hepatitis diagnosis obtained value 31% for probability of hepatitis given the presence of malaise, condition 3 of hepatitis diagnosis obtained value 36% for probability of hepatitis given the presence of fever, condition 4 of hepatitis diagnosis obtained value 31.7% for probability of hepatitis given the presence of malaise, condition 5 of hepatitis diagnosis obtained value 35.1% for probability of hepatitis given the presence of fever. Using the Bayesian Hau Kashyap approach, the most highest probability of hepatitis given the presence of malaise obtained value 14.2% in condition 4, probability of hepatitis given the presence of fever obtained value 17.3% in condition 3 and probability of hepatitis given the presence of headache obtained value 14.7% in condition 1. A numerical example was illustrated that the Bayesian Hau-Kashyap approach was efficient and feasible.

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Author details

Andino Maseleno1*, Rohmah Zahroh Hidayati2, Marini Othman1, Alicia Y.C. Tang1 and Moamin A. Mahmoud1

*Address all correspondence to: andimaseleno@gmail.com

1 Institute of Informatics and Computing Energy, Universiti Tenaga Nasional, Malaysia
2 Moyudan Public Health Centre, Yogyakarta, Indonesia

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