Absorption-free superluminal light propagation in a Landau-quantized graphene

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1 June 2017

Abstract. Controlling of group velocity has attracted enormous interest in the recent years. One of the main challenges remains the realization of an absorption-free fast or slow light propagation. Here, we report on a gain-assisted superluminal light propagation in a Landau-quantized graphene. Additionally, dynamical behaviour of dispersion and absorption of a weak probe field in a closed-type graphene system is investigated, and we find that the absorption and dispersion can be dramatically affected by the relative phase of the coherent fields and Rabi frequencies in such a way that a large transient gain is achieved and the transient absorption can be completely eliminated just by choosing the proper relative phase.
1. Introduction

Graphene, as the thinnest material known in the universe \cite{1}, consists of carbon atoms in a two-dimensional (2D) hexagonal lattice with unusual Dirac-like electronic excitations. It holds many records relating to mechanical, thermal, electrical and optical properties. In addition, its band-gap structure can be tuned by voltage or chemical doping, through which conductivity and transmission are changed \cite{2}. Subsequently, this feature can endow graphene with a capability of operation in both terahertz and optical frequency ranges. Beside the interest in the fundamental research of optoelectronic and condensed matter physics \cite{1,3}, graphene is gaining increasing attention for the different fields of technological application \cite{4,5,6,7,8}. Moreover, extant literature has reported investigations concerning the optical properties of graphene \cite{9,10,11,12}. Not only do the investigations provide insights into the underlying nature of graphene's excited state, but they also open up interesting perspectives for emerging photonic and optoelectronic applications. Research has shown that magneto-optical properties and thin graphite layer lead to multiple absorption peaks and unique selection rules for the allowed transitions \cite{13,14}. Furthermore, the non-equidistant Landau-levels (LLs) as well as the selection rules makes graphene an excellent candidate for LL laser \cite{15,16}. Moreover, optical nonlinearity features have been exploited in diverse phenomena including four-wave mixing, multi-wave mixing \cite{17,18}, generation of entangled photons \cite{19} and infrared solitons pairs \cite{20}. These nonlinear results demonstrate the feasibility and versatility of graphene systems for nonlinear optical applications such as chip-scale high-speed optical communications, all-optical signal processing as well as in photonics and optoelectronics.

In the past three decades, controlling of the group velocity of optical pulse, i.e. slow and fast light, through different materials has been investigated because of its potential application in information technology. In the meantime, numerous experimental and theoretical works have been devoted to control that in diverse materials including atomic media \cite{21,22,23,24}, alexandrite crystal \cite{25}, optomechanical system \cite{26} and superconducting phase quantum circuit system \cite{27}. Slow light can be used in telecommunications such as developing controllable optical delay lines, optical buffers \cite{28}, true time delay methods for synthetic aperture radar, development of spectrometers with enhanced spectral resolution \cite{29}, optical memories \cite{30} and other interesting applications. On the other hand, the question of wave velocity has been a mooted one, since the advent of Einstein’s special theory of relativity \cite{31,32}; the key issue is whether the speed of the light is an upper limit to the group velocity. Conclusions have been drawn that the group velocity is not limited and there exists various proposals for observing faster than $c$ propagation of optical or electrical wave pulses in absorbing, attenuating or gain materials \cite{33,34,35}. Another interesting scenario in the light propagation is the one where the group velocity of light pulse can even become negative \cite{22,36}. It is perhaps worth mentioning that superluminal light propagation does not violate Einstein’s theory of special relativity since the energy and information flow do not exceed $c$ \cite{37,38}. By using the principle of such propagation, one can improve the speed of information transfer in telecommunications. An ideal condition for practical light propagation is a region in which the light pulse should not attenuate or amplify during passing through the system, primarily due to fact
that pulse propagation does not possible in the presence of a large absorption. Besides, gain may add some noise to the system. As mentioned above, considerable attention was paid in the literature to graphene due to its large optical non-linearity. Superluminal light propagation, however, has received scant study to date. Despite the achievement of superluminal light in graphene systems in a very few studies [39, 40], it is accompanied by considerable absorption which could be a drawback in the practical applications. In this paper, we show that the slope of dispersion can change from positive to negative just with the intensity of the coupling or pumping fields. To reiterate, the advantage of this model over prior attempts is that absorption-free superluminal light propagation is appeared.

Additionally, many works have focused on the transient properties of probe field absorption, gain and enhancement of dispersion in both atomic media and solid-state systems [41, 42, 43, 44, 45, 46]. Although the importance of the transient behaviour for its application, there is a little study on the phenomenon in context of graphene [47, 48, 49], especially no previous study on transient optical properties of closed-type graphene system so far as we know. In present paper, we turn our attention to the role of the relative phase of the fields on the transient optical properties of a graphene monolayer system. Motivated by a recent study on the phase sensitivity of the optical bistability and multistability [50], we investigate the transient optical properties of the Landau-quantized graphene monolayer system interacting with three laser fields. The effects of Rabi frequencies and relative phase of the coherent fields on the probe field absorption and dispersion are considered. It is shown that the absorption and dispersion can be dramatically altered by the relative phase and the Rabi frequencies so that the transient absorption can be completely eliminated and large transient gain can be achieved just by choosing the proper relative phase between the laser fields.

2. Model and Equations

Graphene is a one-atom thick allotrope of carbon, which has an honeycomb hexagonal lattice structure [9, 51]. Unlike the usual case $\varepsilon_k = k^2/(2m)$, graphene has a linear dispersion relation in the nearest-neighbour approximation close to the Dirac points. In the presence of a perpendicular strong magnetic field, $B\hat{z}$, the LL energies are given by $\varepsilon_n = \text{sgn}(n)\hbar\omega_c\sqrt{|n|}$, where the integer $n$ is energy quantum number and $\omega_c = \sqrt{2\mu/F}/c$ with $\mu_F \sim 10^6$ m/s and $l_c = \sqrt{\hbar/(eB)}$ being the Fermi velocity and the magnetic length [52, 53]. These energy levels can be expressed as $\lambda_c[\mu\text{m}] = 34(B[\text{Tesla}])^{-1/2}$, in the wavelength scale. Note that transition between the LLs obeys the selection rule of a graphene monolayer system: $\Delta|n| = \pm 1$ [14]. It may be noted that these unique selection rules enable transitions with change in $n$ greater than 1, as opposed to selection rule $\Delta n = \pm 1$ for electrons; for instance, transition from $n = -1$ to $n = 2$ is allowed which leads to an efficient resonant nonlinear mixing. For the external magnetic field, the field condenses the original continuous states in the Dirac cone into discrete LLs which are proportional to $\sqrt{B}$. Note that LLs for the graphene are unequally spaced, unlike LLs in a conventional electron-hole system ($E_n = (n + 1/2)\hbar eB/m$).

It is assumed that inter-Landau level transitions $|4\rangle \leftrightarrow |1\rangle$ is driven by a right-hand circular polarization field with the amplitude $E_1$ and with the carrier frequency $\omega_1$, as another field with the amplitude $E_2$ and the
carrier frequency $\omega_2$ interacts with transition $|1\rangle \leftrightarrow |2\rangle$. Such a configuration has been found to be advantageous in that appearance or disappearance of optical bistability tends to be controlled by adjusting the relative phase of three coherent fields [50]. Electric field strength of the third field can be written as $\vec{E}_3 = (E_3^+ \sigma^+ + E_3^- \sigma^-) \exp(-i\omega_3 t + i\vec{k}_3 \cdot \vec{r})$. Here, $\vec{k}_3$, $\sigma^-$ and $\sigma^+$ are the wave vector, the unit vectors of the left- and right-hand circular polarization, respectively. So, the linearly polarized field with carrier frequency $\omega_3$, drives the transition $|3\rangle \leftrightarrow |2\rangle$ via $\sigma^+$ component and $|4\rangle \leftrightarrow |3\rangle$ via the $\sigma^-$ component. Rabi frequencies of the corresponding fields are denoted by $\Omega_{41} = (E_{41} \mu_{41}.e_1)/(2\hbar)$, $\Omega_{21} = (E_{21} \mu_{21}.e_1)/(2\hbar)$, $\Omega_{32} = (E_{32} \mu_{32}.e_2)/(2\hbar)$ and $\Omega_{43} = (E_{43} \mu_{43}.e_3)/(2\hbar)$, with $\mu_{ij} = e.\langle i|\vec{r}|j\rangle$ being as the electric dipole moment of the relevant transition. Noting that the dipole moment reaches a large magnitude in the mid-far infrared range: $|\mu_{ij}|/e \sim 18 \text{ nm}$ in the field of $1T$.

In the absence of the optical field, the effective-mass Hamiltonian for a graphene monolayer in a magnetic field and in the nearest-neighbour tight-binding model can be written as a $4 \times 4$ matrix [53]:

$$\hat{H}_0 = \nu_F \begin{bmatrix} 0 & \hat{\pi}_x - i\hat{\pi}_y & 0 & 0 \\ \hat{\pi}_x + i\hat{\pi}_y & 0 & 0 & 0 \\ 0 & 0 & \hat{\pi}_x - i\hat{\pi}_y & 0 \\ 0 & 0 & \hat{\pi}_x - i\hat{\pi}_y & 0 \end{bmatrix},$$

(1)

where generalized momentum operator is denoted by $\hat{\pi} = \hat{P} + e\hat{A}/c$ with $\hat{P}$, $e$ and $\hat{A}$ being the electron momentum operator, electron charge and the vector potential (which is equal $(0,Bx)$ for a constant magnetic field), respectively.

In the presence of the incident optical fields, we add vector potential of the optical field, $\vec{A}_{opt} = ic\vec{E}\omega$, to the vector potential of the magnetic field in the generalized momentum operator $\hat{\pi}$. Here, $\vec{E}$ is the sum
of the incident optical fields: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. After adding the interaction term into $H_0$, we can write $H_{\text{int}} = (v_F e/c) \vec{A}_{\text{opt}}$, where $\vec{A}_{\text{opt}} = ic\vec{E}/\omega$ is the vector potential of the incident optical fields. Unlike interaction Hamiltonian for an electron with a parabolic dispersion, there are no higher order terms such as $\pi^2$ near the Dirac point and the Hamiltonian is an exact linear in the vector potential $\vec{A}_{\text{opt}}$, even for strong optical fields. Moreover, the interaction Hamiltonian is not determined by the momentum operator and its matrix elements are only decided by Pauli matrix vector $\vec{\sigma}$.

Now, a standard time-evolution equation for the density matrix of Dirac electrons in the graphene coupled to the laser fields can be written: $d\hat{\rho}/dt = (1/i\hbar)[H_{\text{int}}, \hat{\rho}] + \hat{R}(\hat{\rho})$. It would be worth mention that $\hat{R}(\hat{\rho})$ describes incoherent relation due to disorder, interaction of Dirac electrons with photons and carrier-carrier interactions. The density matrix equations of motion for the system can be written as follows [50]

\begin{align}
\dot{\rho}_{44} &= i\tilde{\Omega}_{41}\rho_{14} + i\tilde{\Omega}_{43}\rho_{34} - i(\tilde{\Omega}_{41})^*\rho_{41}, \\
\dot{\rho}_{33} &= i\tilde{\Omega}_{32}\rho_{23} + i(\tilde{\Omega}_{32})^*\rho_{32} - i\tilde{\Omega}_{34}\rho_{43} \\
\dot{\rho}_{22} &= i\tilde{\Omega}_{21}\rho_{12} + i(\tilde{\Omega}_{21})^*\rho_{21} - i\tilde{\Omega}_{23}\rho_{32} \\
\dot{\rho}_{11} &= i\tilde{\Omega}_{14}\rho_{14} + i\tilde{\Omega}_{13}\rho_{13} - i\tilde{\Omega}_{12}\rho_{12} \\
\dot{\rho}_{42} &= i\tilde{\Omega}_{42}\rho_{21} + i\tilde{\Omega}_{43}\rho_{32} - i\tilde{\Omega}_{44}\rho_{43} \\
\dot{\rho}_{43} &= i\tilde{\Omega}_{42}\rho_{13} + i\tilde{\Omega}_{43}\rho_{22} - i\tilde{\Omega}_{44}\rho_{31} \\
\dot{\rho}_{43} &= i(\tilde{\Omega}_{43})^*\rho_{41} + i\tilde{\Omega}_{42}\rho_{21} - i\tilde{\Omega}_{41}\rho_{32} \\
\dot{\rho}_{34} &= i(\tilde{\Omega}_{34})^*\rho_{31} + i\tilde{\Omega}_{32}\rho_{14} - i\tilde{\Omega}_{31}\rho_{41} \\
\dot{\rho}_{44} &= i(\tilde{\Omega}_{41})^*\rho_{41} + i\tilde{\Omega}_{43}\rho_{34} - i\tilde{\Omega}_{42}\rho_{24} - i\tilde{\Omega}_{43}\rho_{32}.
\end{align}

Here, overdots stand for the time derivations and the remaining equations follow from the constraints $\rho_{lm} = \rho_{ml}^*$ with $l, m \in \{1, ..., 4\}$ and $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$. Also, the parameters $\Delta_{41} = (\epsilon_1 - \epsilon_{-1})/\hbar$, $\Delta_{21} = (\epsilon_2 - \epsilon_{-2})/\hbar$ and $\Delta_{32} = (\epsilon_3 - \epsilon_{-3})/\hbar$ and represent the corresponding detunings. Moreover, the parameter $\gamma_k(k = 2, 3, 4)$ corresponds to the decay rate of the state $|k\rangle$.

3. Results and discussion

3.1. Transient optical properties of a closed-type graphene monolayer system

In this section, we assume that the transitions $|4\rangle \leftrightarrow |1\rangle$ and $|2\rangle \leftrightarrow |1\rangle$ are driven by the probe and coupling fields: $\tilde{\Omega}_{41} = \tilde{\Omega}_p$, $\tilde{\Omega}_{21} = \tilde{\Omega}_c$ with the corresponding detunings $\Delta_{41} = \Delta_p$, $\Delta_{21} = \Delta_c$. Further, the transitions $|4\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |2\rangle$ are driven simultaneously by a control field: $\tilde{\Omega}_{43} = \tilde{\Omega}_p^-$, $\tilde{\Omega}_{32} = \tilde{\Omega}_p^+$, with $\Delta_{43} = \Delta_{32} = \Delta_3$. Then, we proceed to rewrite the density matrix equations for the case of the closed-loop configuration, in which the system becomes quite sensitive to the phases of the laser fields. Taking $\varphi_p$, $\varphi_c$ and $\varphi_3$ as phases of the probe, the coupling and the control fields and treating the Rabi frequencies as complex-valued parameters: $\tilde{\Omega}_p = \Omega_p e^{-i\varphi_p}$, $\tilde{\Omega}_c = \Omega_c e^{-i\varphi_c}$, $\tilde{\Omega}_p^+ = \Omega_p^+ e^{-i\varphi_3}$ and $\tilde{\Omega}_p^- = \Omega_p^- e^{-i\varphi_3}$, and therefore redefining the density matrix elements: $\hat{\rho}_{41} = \rho_{11} e^{-i\varphi_p}$, $\hat{\rho}_{42} = \rho_{11} e^{-i\varphi_c}$, $\hat{\rho}_{43} = \rho_{11} e^{-i\varphi_3}$, $\hat{\rho}_{31} = \rho_{31} e^{-i(\varphi_p - \varphi_3)}$, $\hat{\rho}_{32} = \rho_{32} e^{-i(\varphi_c - \varphi_3)}$, $\hat{\rho}_{33} = \rho_{33} e^{-i(\varphi_c - \varphi_3)}$, $\hat{\rho}_{34} = \rho_{34} e^{-i(\varphi_p - \varphi_3)}$, $\hat{\rho}_{44} = \rho_{44} e^{-i(\varphi_p - \varphi_3)}$, $\hat{\rho}_{22} = \rho_{22} e^{-i(\varphi_c - \varphi_3)}$, $\hat{\rho}_{23} = \rho_{23} e^{-i(\varphi_c - \varphi_3)}$, $\hat{\rho}_{24} = \rho_{24} e^{-i(\varphi_p - \varphi_3)}$, $\hat{\rho}_{11} = \rho_{11} e^{-i\varphi_p}$, $\hat{\rho}_{12} = \rho_{12} e^{-i\varphi_c}$, $\hat{\rho}_{13} = \rho_{13} e^{-i\varphi_3}$, $\hat{\rho}_{14} = \rho_{14} e^{-i\varphi_3}$.
\( \hat{\rho}_{32} = \rho_{32} e^{-i\varphi_3} \) and \( \hat{\rho}_{21} = \rho_{21} e^{-i(\varphi_p - 2\varphi_3)} \), we can obtain equations for the redefined density matrix elements:

\[
\begin{align*}
\dot{\rho}_{44} &= i\Omega_p \rho_{14} + i\Omega_2 \rho_{34} - i\Omega_p^* \rho_{41} \\
&\quad - i(\Omega_2^*)^* \rho_{43} - \gamma_4 \rho_{44}, \\
\dot{\rho}_{33} &= i\Omega_2^+ \rho_{23} + i(\Omega_2^-)^* \rho_{43} - i(\Omega_2^+)^* \rho_{32} \\
&\quad - i\Omega_2^- \rho_{34} - \gamma_3 \rho_{33}, \\
\dot{\rho}_{22} &= i\Omega_1 e^{i\varphi} \rho_{12} + i(\Omega_2^+)^* \rho_{32} \\
&\quad - i(\Omega_1)^* e^{-i\varphi} \rho_{21} - i\Omega_2^+ \rho_{23} - \gamma_2 \rho_{22}, \\
\dot{\rho}_{41} &= i\Omega_p (\rho_{11} - \rho_{44}) + i\Omega_2^- \rho_{31} - i\Omega_1 e^{i\varphi} \rho_{42} \\
&\quad + (i\Delta_p - \frac{\gamma_4}{2}) \rho_{22}, \\
\dot{\rho}_{42} &= i\Omega_p \rho_{12} + i\Omega_2^- \rho_{32} - i(\Omega_1)^* e^{-i\varphi} \rho_{41} \\
&\quad - i\Omega_2^- \rho_{43} + (2i\Delta_3 - \frac{\gamma_1 + \gamma_2}{2}) \rho_{42}, \\
\dot{\rho}_{43} &= i\Omega_p \rho_{13} + i\Omega_2^- (\rho_{33} - \rho_{44}) - i(\Omega_2^+)^* \rho_{42} \\
&\quad + (i\Delta_3 - \frac{\gamma_1 + \gamma_2}{2}) \rho_{43}, \\
\dot{\rho}_{31} &= i(\Omega_2^* \rho_{41} + i\Omega_2^+ \rho_{21} - i\Omega_1 e^{i\varphi} \rho_{32} \\
&\quad - i\Omega_p \rho_{34} + (i\Delta_p + i\Delta_3 - \frac{\gamma_3}{2}) \rho_{31}, \\
\dot{\rho}_{32} &= i\Omega_2^+ (\hat{\rho}_{22} - \hat{\rho}_{33}) - i(\Omega_1)^* e^{-i\varphi} \rho_{31} \\
&\quad + i(\Omega_2^* \rho_{42} + (i\Delta_3 - \frac{\gamma_2 + \gamma_3}{2}) \rho_{32}, \\
\dot{\rho}_{21} &= i\Omega_1 e^{i\varphi} \rho_{11} + i(\Omega_2^+)^* \rho_{31} - i\Omega_1 e^{i\varphi} \rho_{22} \\
&\quad - i\Omega_p \rho_{24} + (i\Delta_c - \frac{\gamma_2}{2}) \rho_{21},
\end{align*}
\]

where parameter \( \varphi = \varphi_p - 2\varphi_3 - \varphi_c \) is the relative phase of the three optical fields. It is also assumed that condition \( \omega_p = \omega_c + 2\omega_3 \) for the carrier frequencies of the fields is fulfilled and constraint on the detunings is presented: \( \Delta_p = \Delta_c + 2\Delta_3 \).

Before discussing the transient behaviour of the absorption and dispersion of the graphene monolayer system, we briefly review a prior research on the transient behaviour of graphene systems. Hamedi and Sahrai have investigated temporal absorption behaviours of graphene under Landau-quantization via the effects of intensity and frequency detunings of driving fields \([49]\); but we will show the possibility of controlling these behaviours by the relative phase of the applied fields. Note that the phase controlling of the phenomena is easier than controlling via the intensity or the frequency and the main advantage of this method is its simple implementation compared to other controlling methods. From experimental point of view, the relative phase could be easily changed by electro-optical devices. Moreover, we will show that the absorption and dispersion can be dramatically affected by the relative phase in such a way that a large transient gain can be achieved as well as a larger steady-state absorption, compared to their suggested scheme.

Here, our main observable is the response of the atomic medium to the probe field and as will be discussed, gain or absorption coefficient of the probe laser field coupled to the transition \( |4\rangle \leftrightarrow |1\rangle \) is characterized with \( \text{Im}[\rho_{41}] \), while the dispersion is proportional to \( \text{Re}[\rho_{41}] \). In the theoretical calculation, we take the transition frequency \( \omega_{41} \sim 10^{14} \text{s}^{-1} \) for the graphene at the magnetic field of the value of \( 1 \sim 3T \) and \( \gamma_4 = \gamma_3 = \gamma_2 = \gamma = 3 \times 10^{13} \text{s}^{-1} \). It should be noted that the value assigned to \( \gamma_4 \) is a rather conservative choice considering the latest experimental and numerical results \([19, 54, 55]\). It is also assumed that the system is initially at the ground state. \( \text{Im}[\rho_{41}] < 0 \) in our notation means that the system exhibits gain, while the probe field is attenuated when we have \( \text{Im}[\rho_{41}] > 0 \).

Now we turn on the discussing the transient behaviour of the absorption and dispersion via the numerical results of equations \([3]\). The transient behaviour of the absorption-dispersion for \( \Delta_3 = \Delta_p = 0, \Omega_p = 0.1\gamma \) and \( \Omega_2^+ = \Omega_2^- = \Omega_c = 6\gamma \) and for the different relative phase is shown in Fig. \( \[2\] \). For a weak probe field and the same-order driving fields, the probe absorption is phase dependent. At
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Figure 2. Temporal evolution of atomic response in the monolayer graphene system, as a function of the relative phase: Φ = 0 (solid line), Φ = π/4 (dashed line), Φ = π/2 (dotted line), Φ = 3π/4 (dot-dashed line) and Φ = π (red line). Imaginary (a) and real (b) parts of ρ_{41} are plotted for ∆_{3} = ∆_{p} = 0, Ω_{p} = 0.1γ and Ω_{c}^{+} = Ω_{c}^{-} = Ω_{c} = 6γ.

Figure 3. Temporal evolution of atomic response in a monolayer graphene system for different relative phase: Φ = 0 (solid line), Φ = π/4 (dashed line), Φ = π/2 (dotted line), Φ = 3π/4 (dot-dashed line) and Φ = π (red line). Imaginary (a) and real (b) parts of ρ_{41} are plotted for Ω_{c}^{+} = Ω_{c}^{-} = 6γ and Ω_{c} = γ. Other parameters are the same as in Fig. 2.

time t = 0, the imaginary part of ρ_{41} is zero, once time increases this term oscillates with a fast-damped oscillating amplitude, however, it will finally reach a steady-state value. For Φ = 0 and Φ = π/4, these steady-state values are negative, while for other relative phases being positive. The dispersion properties of the weak probe field is also phase dependent, as can be seen in Fig. 2(b). For Φ = 0 and Φ = π, dispersion response is zero; as for Φ = π/4, Φ = π/2 and Φ = 3π/4, we can see an oscillation signal in dispersion with a very small positive steady-state value.

Fig. 3 shows the imaginary and the real parts of ρ_{41} for Ω_{c}^{+} = Ω_{c}^{-} = 6γ and Ω_{c} = γ. As can be seen, transient properties are greatly changed in such a way that they exhibit different features with different values of the relative phase. In the presence of a strong control field, the probe one no longer exhibits periodic amplification and absorption. For Φ = 0 and Φ = π/4, the transient absorption completely disappear, leaving the large transient gain. However, for other values of the relative phase, the absorption oscillates above the zero-absorption line and reaches to the positive values at the steady-state. So we can achieve much larger gain just by decreasing the coupling field and choosing the proper values of the relative phase.

In Fig. 4 we plot temporal evolution of atomic response in the graphene system for a weak control field, Ω_{c}^{+} = Ω_{c}^{-} = γ. It can be seen that the transient properties under
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3.2. Gain-assisted superluminal light propagation

In this section, we put very few restrictions on the scheme presented in Sec. (3.1): Ω_{21} = 0, Ω_{43}, Ω_{41} and Ω_{32} are assumed to be the probe, coupling and pumping fields, respectively. Our main observable, i.e. the response of the atomic medium to the probe field and the susceptibility of the weak probe field \( \chi(\omega_p) \) can be written as

\[
\chi(\omega_p) = \frac{N \mu_{34}^2 \epsilon_r}{\epsilon_r \hbar \Omega_{43}} \rho_{43},
\]

where \( N \) and \( \epsilon_r \) are the sheet electron density of graphene and the substrate dielectric constant, respectively.

We then introduce the group velocity, \( n_g = c/v_g \), where the group velocity of the probe field is then given by \[56\]

\[
v_g = \frac{c}{1 + \frac{1}{2} \left[ Re[\chi(\omega_p)] + \omega_p \partial Re[\chi(\omega_p)]/\partial \omega_p \right]}.
\]

As can be seen, for a negligible absorption the group velocity can be significantly reduced via a steep positive dispersion, as the strong negative dispersion can increase the group velocity to establish even a negative group velocity. For the present system, the electron density is assumed to be \( N \approx 55 \times 10^{13} \text{cm}^{-2} \), the dielectric constant turns out to be \( \epsilon_r = 4.5 \) and the dipole moment between the transition \( |4\rangle \leftrightarrow |3\rangle \) has a magnitude of the order of \( 1/\sqrt{B} \) \[57-58\].

Figure 5. Subluminal and superluminal regions via the Rabi frequency of pumping and coupling fields, around zero detuning. The superluminal region is shown by dark color.
Before presenting our results, we drive an analytical expression and the necessary parameters for switching the group velocity of the probe field from subluminal to superluminal. In the case of $\Delta_{41} = 0$, $\Delta_{43} = \Delta_{32} = \Delta_p$ and $\Omega_{41} < \Omega_{32}$, the expression of the steady-state coherence $\rho_{43}$ yields:

$$\rho_{43} = -\frac{8i\Omega_{43} |\Omega_{41}|^2 a_0}{d [d_0 - d_1 \Delta_p - d_2 \Delta_p^2 - 36i\Delta_p^3 + 16\Delta_p^4]}$$

where we define $a_0 = \gamma^2 - 5i\gamma \Delta_p - 4\Delta_p^2 + 5|\Omega_{32}|^2$, $d = \gamma_2 + 8|\Omega_{41}|^2$, $d_0 = \gamma^4 + \gamma^2 (5|\Omega_{32}|^2 + 2|\Omega_{41}|^2) + 4|\Omega_{32}|^2$, $d_1 = 9i\gamma^3 + 6i\gamma (3|\Omega_{32}|^2 + 2|\Omega_{41}|^2)$ and $d_2 = 28\gamma^2 + 16(|\Omega_{32}|^2 + |\Omega_{41}|^2)$ in order to obtain compact forms in the following expression.

Critical value of the pumping field to change the slope of dispersion from positive to negative is $\Omega_{cr} = (1 - \Omega_{32}^2)^{1/6} \sqrt{2}$. For $\Omega_{32} < \Omega_{cr}$, the slope of dispersion around zero probe detuning is positive, while for $\Omega_{32} > \Omega_{cr}$ it becomes negative. In Fig. 5 we display the subluminal and superluminal regions via the Rabi frequency of coherent pumping and coupling rate fields. The superluminal region is shown by dark color.

As the linear susceptibility of the weak probe field is determined by the probe transition coherence $\rho_{43}$, we therefore proceed by solving equations (2) in the steady-state. In Fig. 6 we show the imaginary (a) and the real (b) parts of the $\rho_{43}$ versus the probe detuning $\Delta_p$ for various intensity of the pumping fields ($\Omega_{32}$). In this figure, $\Omega_{21}$ is switched off, $\Omega_{43} = 0.01\gamma$, $\Omega_{41} = 0.5\gamma$ and the other parameters are the same as in Fig. 2. As Fig. 6(a) shows, for all values of the pumping fields, the system shows a gain structure and with increasing the intensity of coherent pumping field, the initial gain separates into two dips. In Fig. 6(b), we
see how the superluminal can be established for a specific value of the pumping field. It can be seen that when the intensity of the pumping field is sufficiently small, the slope of dispersion is positive corresponding to the subluminal light propagation. By increasing the intensity of the pumping field, when the $\Omega$ reaches a critical value $\Omega_{cr}$, the nature of the curve changes so that it has a zero slope at $\Delta_p = 0$; however, for $\Omega > \Omega_{cr}$ the slope of dispersion changes to the negative.

In Fig. 6(c), we display the group index, $c/v_g - 1$, versus probe field detuning. It can be realized that for $\Omega_{32} = 0.2\gamma$, the group index around the zero detuning is positive, corresponding to the subluminal light propagation. For $\Omega_{32} = 0.7\gamma$, the group index around $\Delta_p$ is zero and the group velocity is equal to the speed of light in vacuum. By increasing the intensity of pumping field, the group index become negative; that is, the superluminal light propagation can be achieved. Noting that the superluminal light accompanied with a reduced absorption to ensure that the light pulse does not attenuate as it passes through the medium.

4. Conclusion

In summary, we investigate the dispersion and the absorption properties of a weak probe field in the Landau-quantized graphene monolayer system. We found that the slope of dispersion can be changed from positive to negative just with the intensity of the pumping field, and showed that this configuration allows for the gain-assisted subluminal and superluminal light propagation. In addition, the transient optical properties of a weak probe field in a closed-type graphene system was investigated, the effects of Rabi frequencies as well as the relative phase of the coherent fields on the probe field absorption and dispersion was also considered. It was shown that the dispersion and absorption are dramatically changed with the relative phase and in the case of a weak coupling field, a large transient gain can be achieved and the transient absorption can be completely eliminated just by choosing the proper relative phase.

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