Modelling of inboard stall delay due to rotation

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Abstract. This paper investigates the boundary-layer characteristics on a wind turbine blade of small chord length. The three-dimensional form of momentum integral equations is derived and used to predict the boundary-layer growth and limiting streamline angles on the blade surface for both attached and separating flow. The chordwise skin friction coefficient is used to identify boundary layer separation and shear layer reattachment locations. The nature of flow near the axis of rotation is discussed and the physical mechanism associated with 3-D and rotational effects is identified. A semiempirical correction law for the lift coefficient based on 2-D airfoil data is established. Comparing calculated and measured lift curves of a stall controlled wind turbine, it is shown that the proposed correction law may improve significantly the accuracy of the predictions.

1. Introduction

Flow fields generated by horizontal axis wind turbines are highly complex due to the simultaneous presence and interaction of three-dimensionality unsteadiness separation, reattachment and rotational influences. Both previous theoretical computations and experiments established that stall delay routinely occurs on rotor blades producing beneficial vortex structures responsible for significant force and moment amplifications [1-5]. However, rotational influences remain incompletely characterized and understood, particularly at inboard locations.

Therefore, the boundary layer that develops on the blades of rotating fluid machinery can not be two-dimensional. There are centrifugal and Coriolis forces, which, in addition to pressure and viscous forces, make the direction of the flow inside the boundary layer different from the flow outside, thus forming a three-dimensional flow configuration. The laminar boundary layers on a rotating blade of small chord length were previously studied by Fogarty [6], Harlock and Wordsworth [7], Banks and Gadd [8], Miyake and Fujita [9], Snell [10], Dumitrescu and Cardos[11]. Then, based on viscid/inviscid interaction calculations Snel [12] proposed a simple engineering correction formula for lift coefficient, and in Ref. [13]. Du and Seling investigated the effect of rotation on the laminar and turbulent boundary layer, but for large values of the radius to local chord ratio ($r/c \geq 2$). Lack of information about the boundary-layer behaviour on rotating blade after separation and very close to the rotation centre ($r/c < 1$) prompted this investigation. The boundary layers with separated flow are subject to centrifugal and Coriolis forces, which can: (1) contribute directly to the development of secondary flows and (2) indirectly influence the behaviour of boundary layers by delay and/or suppression of flow separation. Both these rotation-induced phenomena are particularly important in the better understand of flow physics and augmentation of blade aerodynamic response. But, even though the present analysis deals with the laminar boundary layer, the results on boundary-layer separation/reattachment topology are also applicable to large Reynolds numbers. The study provides
valuable insights for a theoretical understanding of the effects of the various secondary terms on the primary flow over the rotor blade.

2. Three-dimensional boundary-layer model

A simple 3-D boundary-layer model has been devised which is computationally reasonable at the same time as it is capable of predicting leading 3-D effects on a rotating blade in attached as well as in stalled conditions. The main aim is not the numerical accuracy but the identification of the boundary-layer state and flow field structure underlying rotation effects and for simplicity to define the separation/reattachment location the equations are used here in their laminar form. The governing equations are derived using the following steps:

Step 1: The 3-D incompressible steady boundary-layer equations are written in the cylindrical coordinate system \((\theta,r,z)\) which rotate with the blade with a constant rotational speed \(\Omega\) (fig. 1) [5]. \(\theta\) denotes the peripheral, \(z\) the axial and \(r\) the radial (blade spanwise) direction. The infinitesimal length in the peripheral direction is \(dx = r d\theta\). The equations are used here in their conservative laminar form:

**Continuity**

\[
\frac{\partial u}{\partial \theta} + \frac{1}{r} \frac{\partial rv}{\partial r} + \frac{\partial (rw)}{\partial z} = 0, \tag{1}
\]

**Momentum, \(\theta\) component:**

\[
\frac{1}{r} \frac{\partial}{\partial \theta} \left(u^2\right) + \frac{\partial (uv)}{\partial r} + \frac{\partial (uw)}{\partial z} + \frac{2v}{r} (u - \Omega z) = -\frac{1}{\rho} \frac{\partial P}{\partial \theta} + \frac{1}{\rho} \frac{\partial \tau_r}{\partial r} + \frac{1}{\rho} \frac{\partial \tau_z}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2}, \tag{2}
\]

**Momentum, \(r\) component:**

\[
\frac{1}{r} \frac{\partial}{\partial \theta} (uv) + \frac{\partial (v^2)}{\partial r} + \frac{\partial (vw)}{\partial z} + \frac{v^2}{r} (u - 2\Omega z) = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{1}{\rho} \frac{\partial \tau_r}{\partial r} + \frac{1}{\rho} \frac{\partial \tau_z}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2}, \tag{3}
\]

where \(u, v\) and \(w\) stand for velocity components in \(\theta, r\) and \(z\) directions, respectively, \(r\) for the local radius measured from the centre of rotation, \(\rho\) for the fluid density, \(\nu\) for the kinematic viscosity, and \(P\) is a pressure like term including the centrifugal effect

\[
P = \frac{P}{\rho} \left(\frac{1}{2} (\Omega r)^2\right), \tag{4}
\]

with \(P\) denoting the static pressure.

Step 2: The equations are integrated with respect to \(z\) (normal to blade) from 0 to \(\delta\) (boundary-layer thickness) and the integral forms of equations (2) and (3) are obtained. They may be written...
\[
\frac{\partial \delta_{2x}}{\partial x} + \frac{\partial \delta_{2y}}{\partial r} + \frac{1}{U_e} \frac{\partial U}{\partial r} \left(2\delta_{2x} + \delta_{1x}\right) - \frac{\zeta}{U_e} \left(2\delta_{2y} + \delta_{1y}\right) - 2 \frac{\Omega}{U_e} \delta_{ix} = \frac{\tau_{ix}}{\rho U^2_e},
\]

\[
\frac{\partial \delta_{2y}}{\partial x} + \frac{\partial \delta_{2y}}{\partial r} + \frac{2}{U_e} \frac{\partial U}{\partial r} \left(2\delta_{2y} + \delta_{1y}\right) + \frac{1}{U_e} \frac{\partial U}{\partial r} \left(\delta_{2x} + \delta_{1x} + \delta_{2i} + 2 \frac{\Omega}{U_e} \delta_{ix} - \frac{\zeta}{U_e} \left(\delta_{2y} - \delta_{1x} - \delta_{2i}\right) = \frac{\tau_{iy}}{\rho U^2_e},\right.
\]

where \( U_e \) is the peripheral velocity outside the boundary layer. The direction of \( x \) is chosen to coincide with an external streamline (assumption used in blade element theory), so that \( V_e = 0 \), and outside the boundary layer the flow with respect to some inertial system is assumed irrotational so that \( \zeta = (\nabla \times \mathbf{W}_e)_z = -\frac{1}{r} \frac{\partial}{\partial r} (r U_e) = -2\Omega e \). The various thicknesses are defined as

\[
\delta_{1x} = \int_0^\delta \left(1 - \frac{U}{U_e}\right) dz, \quad \delta_{1y} = \int_0^\delta \frac{v}{U_e} dz; \quad \delta_{2x} = \int_0^\delta \left(1 - \frac{U}{U_e}\right) \frac{u}{U_e} dz, \quad \delta_{2y} = \int_0^\delta \frac{v}{U_e} dz, \quad \delta_{2y} = \int_0^\delta \left(1 - \frac{U}{U_e}\right) \frac{v}{U_e} dz,
\]

An order of magnitude analysis shows that \( \frac{\partial \delta_{2x}}{\partial r} = O(\varepsilon \delta_{2x}) \) and \( \frac{\partial \delta_{2y}}{\partial r} = O(\varepsilon^2 \delta_{2y}) \). As a consequence these terms can be neglected in a first approximation.

Step 3: The associated closure relations are introduced to solve the 3-D laminar boundary layer equations. It is assumed that the streamwise and cross-flow velocities can be adequately represented by Pohlhausen and Mager [14] velocity profiles, respectively

\[
\frac{u}{U_e} = \eta + \eta (1 - \eta) + \eta^2 (1 - \eta)^2 + \Lambda \frac{\eta}{6} (1 - \eta)^3;
\]

\[
\frac{v}{u} = (1 - \eta)^2 \varepsilon, \quad \varepsilon = \tan \beta_w
\]

where \( \eta = z/\delta \) is the non-dimensional boundary-layer height, \( \Lambda = \left(\delta^2 / v\right) \frac{\partial U_e}{\partial x} \) is the Pohlhausen shape parameter, and \( \beta_w \) is the limiting streamline angle, (the angle between the surface streamline and the projection of external streamline upon the surface).

Using these profiles, the resulting laminar boundary-layer equations can be expressed as

\[
A_{0,i} \frac{d}{dx} \left(\frac{\delta^2}{\nu}\right) + A_{1,i} \frac{d\Lambda}{dx} + A_{2,i} \frac{d\varepsilon}{dx} = B_i + F_i, \quad (i = 1, 2, 3),
\]

where the coefficients \( A_{ij} \), the right-hand sides \( B_i \) and the various thicknesses \( \delta_{1i}, \delta_{2i} \) have the expressions
The functions $F_i$ stand for the derivatives with respect to the cross wise direction.

Step 4: An inviscid flow distribution along the blade is needed to complete the above boundary-layer description. By considering the flow around an infinite cylinder of arbitrary cross-section rotating steadily about the $z$--axis, it was shown by Sears [15] that the inviscid velocity components may be expressed as

$$U = \Omega r \frac{\partial \phi}{\partial x}, \quad V = \Omega(\phi - 2x), \quad W = \Omega r \frac{\partial \phi}{\partial z}$$

(11)

where $\phi = \phi(x,z)$ denotes the equivalent 2-D velocity potential due to a blade in the translational movement with unit speed in the negative $x$--direction. Thus the inviscid chordwise velocity distribution, is found to be the same at all spanwise positions. This is a plausible approximation that allows us to include the important effects of Coriolis and centrifugal forces only in the three-dimensional boundary-layer equations as additional terms.

To derive an extended 3-D boundary-layer approach, capable of predicting leading 3-D effects on a rotating blade in attached as well as in stalled conditions, a hypothetical zero skin friction flow (hereafter called Stratford flow) is assumed to describe the inviscid external flow after separation. In fact, this approach is a sort of inverse method that obviously introduces deviations, as compared to a fully separated flow, but they are not significant as long as this approximation allows us to increase the understanding degree of the three-dimensional separation topological structures on a rotating blade. Therefore, Eqs. (9,10) are solved for a particular external flow represented by a two flow fields:

- an attached flow field given by

$$U_e = U_u \left(1 - k \frac{x}{c}\right) = \sqrt{V_w^2 + (\Omega r)^2 \left(1 - k \frac{x}{c}\right)} \text{ for } \frac{x}{c} \leq \left(\frac{x}{c}\right)_\text{wp}$$

(12)

where $c$ is the chord length and $V_w$ the wind velocity.

- a separating (zero skin friction) flow field given by
\[ U_e \delta_{2}^{1/(2+H)} = \text{const, for } \frac{x}{c} \geq \left( \frac{x}{c_{sep}} \right) \]

where \( H = \delta_{2} / \delta_{2} \) is the boundary layer shape parameter.

**Figure 2** External velocity distributions for various gradients, \( k \).

Figures 2 and 3 show chordwise inviscid velocity distributions according to Eqs. (12), (13) and the corresponding variations of peripheral skin-friction coefficient, calculated from Eq. (5) for various values of velocity gradient parameter, \( k \), and \( r/c = \infty \) (2-D).

In many cases, the linear adverse velocity gradient assumption (Eq. (12)) is a satisfactory approximation to the real operating condition and can be used for simulating the velocity distribution on the upper surface of wind turbine blade. The extension of this flow type after separation with distributions (Eq. (13)) that maintain a separating flow state with continuously zero skin friction allows us further to use advantageously the boundary layer model for the study of the 3-D separation process. Although different patterns may appear on the same body caused by minor changes in the parameter space of Reynolds number, Mach number, angle of attack, etc., however the qualitative accuracy of results is not affected by these.
Figure 3 Chordwise skin-friction coefficients for various external velocity distributions and $r/c = \infty$.

Step 5: The outcome of the analysis is the set of equations (9) including Eqs (12,13) for computing of the external velocity derivatives. The numerical solution begins from the leading edge of blade by integrating Eqs. (9) directly, using the boundary layer quantities (10). A Runge-Kutta fourth-order scheme is used to integrate the boundary layer equations. The shape factor $H = 3.5$ (or $\Lambda = -12$) is used to determine the separation point.

3. Results and discussion
An exact theoretical description of the flow in the vicinity of a singular point on a separation line leads to difficulties which are not yet entirely solved. Only few results have been obtained by series expansion solution techniques of the full Navier-Stokes equations [16], [17]. In these conditions, the study of the 3-D separation process can be advantageously tackled by calling upon a simpler approach based on the solution of boundary-layer equations according to an inverse procedure. A simple skin-friction distribution ($C_f = 0$) is used to derive analytical expressions for the inviscid external velocity. The results obtained for the chordwise skin-friction coefficients, at different small values of non-dimensional spanwise distance, $r/c$, are used to detect and track boundary layer separation and reattachment. The pattern of separation and attachment lines suggests the presence of a conical bubble trapping a vortex in the inner portion of the blade and a free-share layer in the outer part of the blade.

Distributions of the chordwise skin-friction coefficient, $\frac{1}{2} \text{Re}_{\text{b}_2} C_f$, are in Fig. 4 shown at various spanwise distance, $r/c$ for $k = 1$. A decrease is seen in the extent of separated region (the distance between the separation and reattachment locations) for decreasing $r/c$. Since the skin-friction coefficients predicted by the present model after separation are continuously increasing the reattachment locations are identified in conjunction with a given value of the skin-friction coefficient, say 0.01. Also, it is worth noting on the separation and reattachment kinematics for individual chordwise sections of the blade; while the separation is only slightly affected by ratio $r/c$ (i.e., the separation remains at the leading edge), the reattachment point location moves upstream with decreasing $r/c$. An incipient reattachment appears at the trailing edge and then for decreasing $r/c$ moves forward up to a complex structure singular point (a focus-saddle point combination) is reached near the leading edge.
In Fig. 5 where variations of cross-flow angle $\beta_{w}$ are shown at varying distances from the rotational axis, $r/c$, an increase of the spanwise velocities (and implicitly of Coriolis force) is seen in the separated flow region for decreasing $r/c$. Also, it may be remarked that after separation the terms containing derivatives with respect to crosswise direction are significant and they have a favorable effects on pressure gradients. At the same conclusion lead the variations of chordwise and spanwise boundary layer shape parameters, $H_x = \delta_{1x}/\delta_{2x}$ and $H_r = -\delta_{1r}/\delta_{2x}$, shown in Fig. 6 a) and b).
Figure 7 shows the effect of increasing of the ratio $r/c$ on development of skin-friction coefficient and cross-flow angle. A limiting value, $r/c = 0.39755$, for which the solution is non-singular is found with the present model.

![Graph showing skin-friction coefficient and cross-flow angle](image)

**Figure 6** Development of shape parameters: a) of streamwise velocity profiles, b) of spanwise velocity profiles

We can interpret the above results in terms of limiting wall streamlines [17], that may come from a focus-saddle point combination. There is a small separation near the leading edge of the chordwise section and after this reattachment takes place. Thus, a focus is trapped from where a vortex starts spiraling off into the flow.
Figure 7 Development of chordwise component of skin-friction a) and cross-flow angle b) at extreme inboard sections.

Figure 8 gives an impression of the three-dimensional vortical flow separation on a rotating blade. The separation structure connecting the saddle with the focus contains a line of strong wall streamline convergence, on the surface called separation line (SL) and a line of flow attachment called reattachment line (RL) near which divergence of wall streamlines occurs. All the streamlines emanating from the saddle point form under the Coriolis force action the singular stream surface enclosing a vortex (called trapped-vortex bubble (VB). At same critical spanwise location, where the strength of Coriolis forces decreases, the vortex bubble starts to burst and moves away from the wall forming a free-shear layer type separation [5].
It follows that the inboard stall delay due to rotation is characterized by the formation of a spanwise vortical disturbance near leading edge of the blade which starts to burst and separates from the surface at some critical radial location.

The idea of the correction law for the lift coefficient is based on the appearance of such an inboard standing vortex on the upper surface of blade, at high angles of attack. The onset of this vortex, which it was found to be in the \((r/c)\) range of 0.5 to 1.0 set at the given location of 1.0. If the blade has at root a larger value of the \(r/c\) ratio, then the blade twist and taper distributions are extrapolated to \(r/c=1\). The initial strength of the vortex is defined as the difference \(\Delta C_{l,1}\) between the potential value \(C_{l,POT}\) and the corresponding 2-D value \(C_{l,2D}\) at its origin. The angle of attack at the origin of vortex is computed according to

\[
\alpha_1 = \tan^{-1} \left( \frac{2 \ V_w}{3 \ \Omega c_1} \right) - \beta_1, \tag{14}
\]

where \(\Omega\) is the angular velocity of the rotor, \(V_w\) is the wind velocity, and \(c_1, \beta_1\) are the chord and twist angle at the fixed origin.

In view of above the initial condition for the vortex lift is

\[
\Delta C_{l,1} = 2 \pi \sin \left( \alpha_1 - \alpha_0 \right) - C_{l,2D} \left( \alpha_1 \right). \tag{15}
\]

Now, assuming a viscous decay of the vortex lift in spanwise direction the following correction law is proposed

\[
C_{l,3D} = C_{l,2D} + \Delta C_{l,1} \left[ 1 - \exp \left( - \frac{\gamma}{r/c - 1} \right) \right]. \tag{16}
\]

Finally, to test the correction formula (16) a BEM computation has been performed on a stall regulated wind turbine, known as the NREL Unsteady Aerodynamic Experiment [18]. The lift curves resulting from the above model with \(\gamma = 1.25\) are shown in Fig. 9 and, in general, compare well with the experimental results.
4. Conclusions

In this paper the 3-D incompressible steady momentum integral boundary-layer equations have been developed to study the effect of rotation on the boundary layer of a wind turbine blade in attached as well as in stalled conditions. The following main conclusions are obtained:

1. The separation structure is mainly affected by the non-dimensional spanwise distance, $r/c$. At the inner portion of blade ($r/c < 3$) an energetic vortex bubble is formed that can be responsible for aerodynamic force augmentation on stalled rotating wind turbine blades. At outboard locations conventional 2-D separation structures are found.

2. The wall streamline pattern in the $(r/c)$ range of 0.5 to 1.0, depending on the pressure gradient, involves a singular point with complex structure, namely a focus-saddle point combination. Physically, this point represents the onset of the standing vortical structure formed under the Coriolis action.

3. Accurate predictions of post-stall airfoil performance at inboard locations must include the induced effects from the spanwise distribution of proper additional circulation.

4. Probably the most serious deficiency of the study is the inviscid flowfield used, which did not have any full separation and wake effects included. In a real flow situation the structure of the separation and the wake will influence the entire flowfield around blade. This influence will take the form of pressure gradients imposed on the boundary layer which could be considerably different that the one used in the present theoretical investigation. However, it has been shown in the investigation that at inboard locations the cordwise pressure gradients are beyond doubt alleviated/suppressed by Coriolis force and cross flow derivative terms. The test of this hypothetical flow structure will come only after considerably more numerical and experimental data become available.

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