Mass from a third star: transformations of close compact-object binaries within hierarchical triples

R. Di Stefano

1 Harvard-Smithsonian Center for Astrophysics, 60 Garden St, Cambridge, MA 02138, US

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Close-orbit binaries consisting of two compact objects are a center of attention because of the detection of gravitational-radiation-induced mergers. The creation of close, compact-object binaries involves physical processes that are not yet well understood; there are open questions about the manner in which two compact objects come to be close enough to merge within a Hubble time. Here we explore an important, and likely common physical process: mass transfer from a third star in a wider, hierarchical orbit. Mass added to the close binary’s components can reduce the time to merger and can even change the nature of an accretor, transforming a white dwarf to a neutron star and/or a neutron star to a black hole. Some accreting WDs in close binaries may even explode as Type Ia supernovae. Given the ubiquity of higher-order multiples, the evolutionary channels we lay out may be important pathways to gravitational mergers, including Type Ia supernovae. Fortunately, these pathways also lead to testable predictions.

Key words: keyword1 – keyword2 – keyword3

1 WHY MASS TRANSFER TO COMPACT BINARIES IS IMPORTANT

The discovery of gravitational radiation from the mergers of black holes (BHs) and neutron stars (NSs) has made it important to understand how two stellar remnants can come to be in a close-enough orbit that they will merge within a Hubble time (Abbott et al. 2016b, 2017b,a, 2016a). Here we consider the effects of mass transfer from a third star in a wider orbit. Mass gained by components of the inner binary can change white dwarfs (WDs) into NSs, or NSs into BHs. Whether or not the natures of its components are altered, modifications of the total mass and angular momentum of the inner binary changes the time to gravitational merger. For wide ranges of physically reasonable parameters, the times to merger are decreased, although times to merger can also increase. This has implications for the rates of formation and the rates of mergers of binaries producing gravitational radiation, as well as for the rates of Type Ia supernovae (SNe Ia) and the accretion-induced collapse (AIC) of WDs to NSs and NSs to BHs. Mass transfer from a companion in a wider orbit also produces directly detectable signatures. These systems can be bright, with X-ray luminosities on the order of the Eddington luminosity. Modulations of the X-ray luminosity on times governed by the orbital period of the compact binary, and possibly even binary self-lensing, can lead to definitive identifications.

The components of the inner binary have already evolved, yet they are in a close orbit. They therefore have likely experienced at least two epochs of prior interaction involving the transfer of mass and/or episodes during which the binary was engulfed by a common envelope. The wide-orbit companion star has not yet become a stellar remnant. As it evolves, it will begin to transfer mass to the inner binary during an epoch that may be as short as $10^5$ years, or may be longer than $\sim 10^8$ years.

In §2 we develop the model and sketch key elements of the basic science. In §3 we present a set of examples. We find that the times to mergers and the masses of the compact objects are increased under a broad range of physically-motivated input assumptions. In §4 we focus on the transformations that are possible [e.g., through accretion-induced collapse (AIC) or Type Ia supernova (SN Ia)] when mass is added to a WD or NS. A broader range of possibilities for the underlying physical processes is discussed in §5. In §6 we focus on the implications for gravitational mergers, collapsing WDs and NSs, and for exploding WDs. We find that hierarchical triples may contribute to the rates of NS-NS, NS-BH, and BH-BH mergers, as well as to the rate of Type Ia supernovae (SNe Ia).
2 THE MODEL

2.1 Overview

Our model starts with a hierarchical triple. The triple contains a binary composed of two stellar remnants in a close orbit, and a third, unevolved star in a wider orbit. Although we are interested in gravitational mergers, the two stellar remnants need not have an orbital separation small enough to allow them to merge within a Hubble time. This is because interaction with mass from the third star can decrease the time to merger. If one or both of the compact objects is a NS, the eventual merger could nevertheless be a BH-BH merger, since a NS may gain enough mass to collapse. Similarly, WDs may be transformed into NSs or, with more significant mass increase, even to BHs. We therefore consider inner binaries with the full range of compact-object combinations.

The inner binary has evolved into two compact objects. We concentrate on the epoch during which the third star is able to lose mass which comes under the gravitational influence of the inner binary.\(^\text{1}\) Note that, if star 3 is massive, it could start transferring mass during its giant phase very soon after the formation of the inner binary. If, on the other hand, it is a solar-mass star, the wait time could be billions of years.

Mass transfer takes place over a time \(t_{\text{mt}}\). The value of \(t_{\text{mt}}\) also depends on the mass of star 3. It can range from \(10^5\) years for massive donors to tens of millions of years for donors of lower mass. During the epoch of mass transfer, the characteristics of the inner binary can change. Specifically, the masses and orbital separation of the components can be altered, thereby changing the time to merger. As matter accretes onto the components of the inner binary, X-rays will be emitted. When the accretion rates are large, the system can be highly luminous. This means that these systems can be detected as X-ray sources even in external galaxies. Furthermore, if the count rate is high enough, subtle effects, such as modulation of the X-ray flux at harmonics of the inner orbit, may be detectable.

2.2 Orbital dimensions

We consider a binary composed of two compact objects with masses \(M_1\) and \(M_2\), orbiting each other with semimajor axis \(a_{\text{in}}\). If the orbit is circular, the time to merger is

\[
\tau_{\text{merge}} = \left(\frac{1.5 \times 10^8 \text{yr}}{M_1 M_2 (M_1 + M_2)}\right) \left[ a_{\text{in}}^4 - a_{\text{min}}^4 \right],
\]

where \(a_{\text{min}}\) is the separation at the time of merger, and masses and distances are expressed in solar units. In the cases we consider, \(a_{\text{min}}\) is small enough relative to \(a_{\text{in}}\) that it can be neglected in the calculations described below.

A third body, a non-degenerate star with mass \(M_3\), is in orbit with the binary. Dynamical stability requires that the closest approach between \(M_3\) and either component of the binary be much larger than \(a_{\text{in}}\). We define \(q = M_3/M_1\), with \(M_2 < M_1\), and \(q_{\text{out}} = (M_1 + M_2)/M_3\).

\(^{1}\) There may have been an earlier epoch during which three-body dynamics played a role.

(1995) have derived an expression for the minimum possible radius, \(a_{\text{min}}^\text{out}\), of the outer orbit.

\[
a_{\text{min}}^\text{out} = a_{\text{in}} \times \left[ \frac{3.7}{q_{\text{out}}} - \frac{2.2}{1 + q_{\text{out}}} + \left( \frac{1.4}{q_{\text{out}}} \right) \left( \frac{5}{q_{\text{out}} - 1} \right) \right]
\]

In principle, \(a_{\text{min}}^\text{out}\) could correspond to the periapsis of a wide elliptical outer orbit. Here, for the sake of simplicity and also because many mass transfer systems have been tidally circularized, we consider circular orbits.

The finite size of the star in the outer orbit places additional restrictions on the outer orbit. As we discuss in the appendix, the Roche-lobe picture of channelled mass transfer can be applied when mass is transferred to a compact inner binary from a donor in a much wider orbit. If we assume that the triple system is in dynamical equilibrium sometime before mass transfer from the outer star begins, then the outer star must have fit inside its Roche lobe. Thus,

\[
a_{\text{out}} > \frac{R_3}{f(1/q_{\text{out}})},
\]

where \(f(x) = 0.49 x^{0.67}/(0.6 x^{0.67} + \log(1 + x^{0.33}))\). The true value of the minimum separation between star 3 and the center of mass of the inner binary is therefore

\[
a_{\text{min}}^\text{out} = \text{max}\left[ a_{\text{min}}^\text{out}, \frac{R_3}{f(1/q_{\text{out}})} \right].
\]

On the giant branch, the radius, \(R\), and luminosity, \(L\), of the star are strong functions of the instantaneous value of the core mass, \(C(t)\).

\[
R = 0.85 M(0)^{0.85} + \frac{3700 C(t)^4}{1 + C(t)^4 + 1.75 C(t)^4}
\]

\[
L = M(0)^3 + \frac{10^{5.5} C(t)^6}{1 + 10^{6.5} C(t)^3 + 10^{6.5} C(t)^3}
\]

The expressions for \(R\) and \(L\) each consist of a first term meant to correspond to the value of the radius and luminosity, respectively, of a main sequence star. Depending on the specific value of the initial mass, expressions which differ from those above may be more appropriate. For giants, however, these terms are dwarfed by the second terms, which depends only on the core mass. Thus, the radius and luminosity of giants depends only weakly on the initial mass.

2.3 The Flow of Mass

Mass flows from star 3. A fraction, \(\gamma\), of \(M_3\) falls toward the inner binary and the rest, \((1 - \gamma)\), exits the system. The paths taken by mass falling toward the binary may be complex. The upshot is simply that a fraction of the incoming mass is retained by one star and another fraction is retained by the second star. Let \(\beta_1\) and \(\beta_2\) be the fraction of the mass retained by stars 1 and 2, respectively. Then \(\dot{M}_1 = \beta_1 \gamma \dot{M}_3\), \(\dot{M}_2 = \beta_2 \gamma \dot{M}_3\), \(\dot{M}_{\text{in}} = (\beta_1 + \beta_2) \gamma \dot{M}_3\). The remainder of the mass exits the system.

2.4 Retention of Mass

Many factors, including the geometry and dynamics of the mass flow, and the action of magnetic fields determine how
much infalling matter can be retained by a compact object. For each type of compact object we select a formula with which to compute $M_{\text{min}}$, the minimum accretion rate for which all mass is retained ($\beta = 1$) and $M_{\text{max}}$, the maximum accretion rate for which $\beta = 1$. For rates lower than $M_{\text{min}}$, we set $\beta = 0$. For rates larger than $M_{\text{max}}$ we use: $\beta = \frac{M_{\text{ret}}}{M_{\text{in}}}$. For NSs and BHs we chose: $M_{\text{min}} = 0.1 \times M_{\text{Eddington}}$, and $M_{\text{max}} = 10 \times M_{\text{Eddington}}$. Although mass can likely be retained for even smaller values of $M_{\text{min}}$, our prescription leads to a conservative estimate of mass gain and also focuses on intervals when mass transfer is most likely to produce high luminosities, and therefore to be detectable. The upper limit reflects the fact that super-Eddington accretion has been observed in X-ray binaries: e.g., Israel et al. (2017); Bachetti et al. (2014); Fürst et al. (2016). If we assume that the the accretion luminosity is $L_{\text{acc}} = 0.1 \times M_{\text{acc}} c^2$, then the infall rate onto a NS or BH for Eddington-limited accretion is $\approx M \times 2.4 \times 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$, where we use $L_{\text{Edd}} \approx 1.3 \times 10^{38} \text{ erg s}^{-1}$ and where $M$ is the mass of the accretor.

For WDs, there is a narrow range of infall rates for which mass can undergo nuclear burning as it accretes, and can therefore be retained. [See, e.g., Iben (1982); Nomoto (1982); Shen & Bildsten (2007).] The upper and lower bounds of this range depend on the value of the WD mass, but at high masses, $M_{\text{min}}^{\text{WD}}$ may be a few times $10^{-7} \text{ M}_\odot \text{ yr}^{-1}$, and $M_{\text{max}}^{\text{WD}}$ is $\sim 10^{-6} \text{ M}_\odot \text{ yr}^{-1}$. At very low rates of accretion, classical nova-like occur at times corresponding to neutron stars are associated with a binary star orbit, then the mass is added to the accretor's mass. Donors must therefore have the high mass loss rates associated with either giants or massive main-sequence stars, if the rate of mass infall to be adequate to lead to genuine mass gain by the compact objects comprising the inner binary. Other stars can, however, influence the orbital angular momentum and time-to-merger of the inner binary.

2.6 Flow of Angular Momentum

Although three-body motion can be complex, we will focus on intervals during which the inner and outer orbits are each well defined. The two compact objects occupy the inner orbit, and the much larger outer orbit is defined by star 3 in orbit with the binary's center of mass. The orbital angular momentum are $L_{\text{in}} = M_1 M_2 \sqrt{a_{\text{in}}/M_T}$, where $M_T = M_1 + M_2$, and $L_{\text{out}} = M_3 M_T \sqrt{a_{\text{out}}/M_{\text{lat}}}$, with $M_{\text{lat}} = M_2 + M_T$. The total orbital angular momentum is $L = L_{\text{in}} + L_{\text{out}}$.

Gravitational radiation drains angular momentum from each orbit. Mass flowing from the system also carries angular momentum. Thus the net flow of angular momentum from the 3-body system is negative, with the ejection of mass potentially removing more angular momentum per unit time than does gravitational radiation. If some of the exiting angular momentum is drawn from the inner orbit, then the time to merger decreases.

The flow of angular momentum within the system depends on a variety of factors. In many respects the mass flow configurations should be similar to those observed in systems in which a single compact object accretes matter from a companion. The rotating dipole component of the gravitational potential should play a significant role only when the incoming mass approaches the inner binary. It is therefore likely that, in many cases, an accretion disk will be formed and that, as is found in for supermassive BH-BH binaries, the inner binary clears a region just around it. In this case,
the less massive compact object will come closer to the edge of the circumbinary disk; a minidisk may then form around it. This mode of accretion would eventually mean the components of the inner binary could come to have nearly equal masses, even if they had started with very different masses.

A circumbinary disk can play an active role in angular momentum transport and loss. In the calculations performed for this paper, we did not incorporate direct effects produced by such a disk. We note, however, that both tidal interactions between the disk and binary, and the release of even small amounts of mass from the outer disk, may tend to shrink the inner binary.

In addition to situations in which there is a circumbinary disk, it is likely that there are cases in which mass falls almost radially inward toward the inner binary’s center of mass. In this case, the more massive component may be more likely to be the first to capture incoming mass. Alternatively, the mass flow may be well modeled by considering accretion from a dense medium within which the inner binary moves as it orbits its distance companion. Furthermore, if one or both accretors are NSs or WDs, magnetic effects may play important roles in channeling mass toward them; or else they could produce a propeller effect, shooting mass from the system. In addition, the compact objects can act as sinks of angular momentum when accreted mass spins them up, or else as sources of angular momentum, if ejected mass spins them down. In our evolutionary calculations we will not consider spin.

The considerations above indicate that the flow of angular momentum may be complicated and that different processes may play dominant roles in different systems. Nevertheless, there are important commonalities that we try to capture. First, of course is that angular momentum is carried by ejected mass. Second, that a good portion of the angular momentum carried away is drawn from the orbits. Since the orbital angular momentum of the outer orbit is generally much larger than that of the inner orbit, most of the angular momentum carried away will be at the expense of the outer orbit. Nevertheless, even a small decrease in angular momentum of the inner orbit can decrease its time to merger. In our calculations mass ejected from the vicinity of one of the three stellar components carries away an amount of angular momentum that is proportional to the specific angular momentum of that star.

2.7 Mechanisms for Mass Transfer and Internal Loss of Angular Momentum

2.7.1 Winds

To model $\dot{M}_{\text{wind}}$, the rate of mass loss due to winds, we implement an approach well suited to donors which leave WD remnants, using a version of the Reimer’s wind which we have modified so that the envelope of the star is exhausted when the core mass of $M_1$ has reached its final value (i.e., when star 3 has evolved to become a WD). To compute the final mass of a WD-producing star, we use an observationally established initial-mass/final-mass relationship.

This formalism allows the instantaneous value of the stellar mass, $M_*(t)$, to be expressed in terms of the instantaneous value of the star’s core mass, $C(t)$, the initial value $M_*(0)$ of the stellar mass, and a parameter $C_0$, which is set to 0.2 $M_\odot$.

$$M_*(t) = \left[ M_*(0)^2 + \frac{\left(C_{\text{max}}^2 - M_*(0)^2\right) \left(C^2 - C_0^2\right)}{C_{\text{max}}^2 - C_0^2}\right]^{\frac{1}{2}} \quad (7)$$

Wind mass loss in this model increases dramatically toward the end of the giant’s life, consistent with observations.

The stellar wind, $\dot{M}_*(t)$, is just the time derivative of the mass; this includes terms involving the $C(t)$, which is proportional to the stellar luminosity. Because the donor is a giant, releasing mass in many directions, only a fraction $f$ of the ejected mass can be captured by the inner binary. We use the following formula.

$$\gamma = \kappa \left(\frac{R_s}{R_L}\right) \left(\frac{L}{L_\odot}\right)$$

where $R_s$ and $R_L$ are the instantaneous values of the donor’s physical radius and Roche-lobe radius, respectively, and $\kappa$ is a constant, whose value we have taken to be 0.5 in the calculations described here. The functional form above ensures that the rate of wind capture is roughly equal to $\kappa$ when the donor fills or nearly fills its Roche lobe. Because $R_L$ is proportional to $a_{\text{out}}$, the capture fraction falls off as $1/distance$ for systems close to Roche-lobe filling. On the other hand, gravitational focusing plays a smaller role at larger separations, so the capture fraction should fall off as roughly $(1/distance)^2$. The exponential ensures that this is the case at larger distances.

The rate of mass infall to the binary is $\gamma \dot{M}_s$. As we will show in §3, winds alone can produce important changes in the inner binary, decreasing its time to merger and sometimes transforming the nature of its components.

2.7.2 Roche-lobe-filling

The donor star expands with age so that, if the initial size of the outer orbit, $a_{\text{out}}(0)$, is small enough, the donor may come to fill its Roche lobe. Once the Roche lobe is filled, there will either be an instability that leads to a common envelope (see below) or else there can be a relatively long epoch during which mass loss from the donor increases, with a fraction of its mass channeled through the region around the L1 point. When the donor star is a subgiant or giant, this epoch ends when the stellar envelope is exhausted by the combination of mass loss and the growth of the stellar core.

2.7.3 Common envelope

When a star in a binary fills its Roche lobe, the transfer of mass to its companion will change the dimensions of the Roche lobe. At the same time, the loss of mass from the donor may alter its radius. If the Roche lobe shrinks at a rate faster than the star can adjust, mass transfer will proceed on a dynamical time scale and a common envelope will encompass both the core of the donor and the accretor (Webbink 1977; Paczynski 1976). Generally, the envelope will be ejected over an interval of $10^4 - 10^5$ years. Unless super-Eddington accretion occurs during this short-lived phase (a process that is invoked
to form double NSs, for example), little or no mass may be gained by the donor’s companion. The common envelope can, however, have a pronounced effect on the orbital angular momentum of a binary Nelemans & Tout (2005). By imparting angular momentum to mass in the envelope, the components of the binary spiral closer to each other while ejecting the envelope. Angular momentum considerations can be used to express the final orbital separation in terms of the initial system parameters.

When the accretor is a compact binary, it too can impart angular momentum to the common envelope. This helps the envelope to escape, while at the same time bringing the binary’s components closer to each other. The question of how much angular momentum is lost by the inner binary is difficult to answer, if only because the analogous question has proved challenging even for the simpler two-body systems. A formalism well suited to computing the effects of the common envelope on the separation of the inner binary focuses on the role of angular momentum.

\[ \frac{\Delta L}{L} = g \frac{\Delta M}{M} \]  

The value of \( g \) is uncertain. Nelemans and Tout (2005) estimated that its value is \( \approx 1.6 \) for the common envelope phase that produced a set of double WDs.

### 2.8 Detectability

Mass transfer is potentially detectable. During the epoch in which the accretors are able to retain mass, their X-ray luminosities, in our model (§2.4) are within a factor of 10 of the Eddington luminosity. Many of these systems would have X-ray luminosity above \( 10^{38} \) erg s\(^{-1}\) or \( 10^{39} \) erg s\(^{-1}\) during this interval and would be detectable even in external galaxies. The slower accretion that would take place over longer times prior to the high-accretion phase would be dimmer, but nevertheless detectable in the Milky Way, Magellanic Clouds and, during some intervals, even in M31. The X-ray emission could also show signs of a short-period component, due to the motion of the inner binary. The inner orbital period decreases as mass transfer proceeds. For nearly edge-on orientations, emission from the active accretor(s) could be lensed, producing a distinctive periodic signature. In most cases the donor star would be a giant, and the system would be identified as a symbiotic binary. Any short-period signature associated with the motion of the inner binary would be the tip-off that the accreting system is a binary. The duration of interval when the X-ray emission is detectable depends on the flux (hence the distance to the source), and on the lifetime of high-wind phase of the donor, which is longer for less massive donors.

### 2.9 Calculation of the Evolution

Our calculations start at the time when the core mass of the donor, star 3, is \( 0.2 \) \( M_\odot \), and continue until the donor’s envelope is exhausted. We increment the core mass of star 3 by \( dc \), compute the time \( dt \) it would take for the core to have grown by this amount, and determine the star’s mass, radius, and rate of wind mass loss at the new time.

The donor ejects mass from the system at the rate: \( (1 - \gamma) \dot{M}_{\text{winds}} \). The ejected mass carries specific orbital angular equal to \( \dot{v}_3 \) times the orbital angular momentum of star 3. \( \dot{v}_3 \) is one of the model’s adjustable parameters. The ejected angular momentum comes entirely at the expense of \( L_{\text{out}} \), the angular momentum of the outer orbit. The remainder of the mass, \( \gamma \dot{M}_{\text{winds}} \), flows toward the less massive star, star 2. We use the considerations described in §2.3 to compute \( \beta_2 \), and consider that the rest of the mass, \((1 - \beta_2) \gamma \dot{M}_{\text{winds}} \), is incident on star 1. We compute \( \beta_1 \) and assume that any mass that cannot be retained by star 1 is ejected from the system, carrying \( v_1 \) times the specific angular momentum of star 1; \( v_1 \) is another adjustable parameter. Because star 1 is part of both the inner and outer binary, we have to subtract angular momentum from both \( L_{\text{in}} \) and \( L_{\text{out}} \). To do this we subtract from \( L_{\text{in}} \) (\( L_{\text{out}} \)) the (rate of mass loss from the inner binary) multiplied by \( v_1 \) times the (specific angular momentum associated with the inner [outer] orbit of star 1).

Independently draining angular momentum from the inner and outer orbits is well suited to cases in which the orbits are orthogonal. It is also appropriate for any system without a direct link between the orbital angular momentum of the inner and outer orbits: for example, when accretion onto the compact objects is approximately spherical or else when angular momentum is dissipated within an accretion disk. We discuss cases in which there is a link between the inner and outer orbits in §4.

Note that the above approach paints the evolution with a broad brush. It includes the key processes determining the fates of these hierarchical triples, but does not attempt to track these processes in detail. In fact, the true physical processes are complex and not yet well understood. For example, the rate of mass loss due to winds in evolving stars is not likely to be steady, and both first principles calculations and inferences from observations are challenging, especially for massive stars and for stars in the end stages of stellar evolution. The focusing of winds is suggested by observations, but exactly how this depends on the mass loss rate, the speed of exiting mass, and irradiation from the accretors still needs to be understood. There are also significant uncertainties about the infall of mass to the inner binary, the ability of this mass to reach the components of this binary, and the ability of these components to retain matter. Finally, an important question is: how much angular momentum is carried by matter exiting the system?

Any calculations that attempt to model all of these processes would have to include the above-mentioned significant and difficult-to-quantify uncertainties. Our approach captures the important features of the evolution; and the extent of the associated uncertainties can be gauged by conducting a range of simulations with different values of a small number of input parameters. As we will see, the character of the results depend on only a few key assumptions.

### 3 RESULTS

#### 3.1 Individual Systems

In Figures 1 and 2 we show the results of evolving 16 individual systems. Figure 1 shows, side by side, two sets of evolutions, each starting with a single value of \( M_1 (6 \) \( M_\odot \); corresponding to a stellar-mass BH) and \( M_3 (4 \) \( M_\odot \); corresponding to a donor star of relatively modest mass). On the left,
Figure 1. Sample evolutions: In both cases $M_1 = 6 M_\odot$ and $M_3 = 4 M_\odot$. Left: $M_2 = 0.9 M_\odot$. Right: $M_2 = 1.8 M_\odot$. The top panels show $M_{\text{wind}}$ (red), and $M_{\text{infall}}$, the rate of mass infall to the binary (set of dark blue curves). The next panel down shows the values of $M_2$ (blue, lower), and $M_3$ (red, higher). The straight gold line on the left marks the $M_{\text{Ch}}$ and the gold line on the right corresponds to a maximum NS mass of $2.2 M_\odot$. Proceeding downward, the next panel shows the orbital period of the inner binary. The final panel shows the ratio of the final value of $t_{\text{merge}}$ to $|t - t_{\text{merge, initial}}|$ versus $M_3$ (left) and $a_{\text{out}}(0)$ (right).

Figure 2. The same as Figure 1, except that the donor masses on both the left and right are $14 M_\odot$. All other system parameters are identical.

star 2 is a $0.9 M_\odot$ CO-WD, and on the right it is a $1.6 M_\odot$ NS. For both the WD and NS, we show the evolutions of 4 systems which differ from each other in the initial radius of the outer orbit, which ranges from 10 AU to 23 AU. For all of the initial orbital separations, the WD (left) achieves the Chandrasekhar mass, exploding as an SN Ia. Similarly, for all the initial separations considered, the NS (right) reaches the critical mass and undergoes an AIC to become a BH. In all cases the time-to-merger decreases from above the Hubble time to a value on the order of a few billion years. The orbital period of the inner orbit also significantly decreases. In all cases the time interval during which the most significant changes occurred lasts for a few times $10^6$ years. Only for the two smallest initial values of $a_{\text{out}}$ does star 3 fill its Roche lobe. Roche-lobe filling leads to a slightly larger accretor final mass (a few tenths of a solar mass). The results for wider separations, where the donor never fills its Roche lobe, show that significant changes can be effected in the inner binary through the agency of winds alone.

From the perspective of the processes at work, the key item of note is that, even though the donor star is a subgiant or giant throughout the time shown, there are essentially no changes in the properties of the inner orbits until the rate of wind mass loss is high. This is because, in our models, the accretors gain mass only for large infall rates. The infall rate depends on the donor’s mass loss rate, and also on the size of the outer orbit relative to the donor’s radius. The donor’s winds and radius increase with time. In addition to winds, Roche-lobe filling provides an effective way for the inner binary to gain mass. Even for Roche-lobe-filling systems, winds are important because an epoch of heavy winds precedes Roche-lobe filling. Thus, there is not a dramatic change at the time of Roche-lobe filling unless the donor is so massive that a common envelope forms.

Figure 2 differs from Figure 1 only in the mass of the outer star, which was taken to be $14 M_\odot$ for both the WD (left) and NS (right) cases. The same set of 4 orbital separations were chosen. In this case, two of the evolutions terminate at relatively early times. These correspond to systems in which star 3 fills its Roche lobe when it is more massive than the inner binary, so that a common envelope forms. During the very short duration of the common envelope, angular momentum continues to be lost by the inner orbit, but (in our model) no mass is gained by the compact objects in the inner orbit.

The most obvious difference between the cases with $M_3 = 14 M_\odot$ (instead of $4 M_\odot$, as considered in Figure 1)
Figure 3. Results of calculations for 100,000 hierarchical triples; the model parameters are given in the figure’s top label. Top panel: the logarithm to the base ten of the final time to merger ($\tau_f$, the time to merger as measured after mass transfer is finished) is plotted against the the logarithm to the base ten of $\tau_{f,\text{expected}}$, the time to merger that would have been expected, had no mass transfer occurred. Bottom panel: The logarithm to the base ten of the ratio $\tau_f/\tau_{f,\text{expected}}$ is plotted versus the ratio of the final total mass of the inner binary to its initial total mass. Points in cyan (lightest points) are systems experiencing only wind mass transfer; larger and somewhat darker (blue) points are systems that have experienced wind mass transfer and then stable mass transfer while the donor fills its Roche lobe; darkest points (red) experienced wind mass transfer and then a common envelope after Roche-lobe filling.

is the availability of more mass. There is, however, another difference that plays an important role: the more massive star evolves on a shorter time scale, so that the transitions take place over shorter times.

3.2 Large Numbers of Systems

To identify the sets of initial parameters (donor masses, orbital separations) that lead to significant increases in the mass of the inner binary and decreases in the time to merger, we conducted a set of simulations, each starting with tens of thousands of hierarchical triples. To generate each hierarchical triple, we started by generating the value of $M_1$, selecting uniformly between 0.6 $M_\odot$ (corresponding to a CO WD) and 7 $M_\odot$ [corresponding to a stellar-mass BH with mass typical of those discovered in nearby X-ray binaries; Corral-Santana et al. (2016); Remillard & McClintock (2006)]. We allowed $M_2$ to have any mass smaller than $M_1$ but larger than 0.6 $M_\odot$. The value of $M_3$ was chosen to be in the range from 0.8 $M_\odot$ (roughly corresponding to the minimum mass of a star expected to evolve within a Hubble time) and 20 $M_\odot$. The next step was to choose the time-to-merger for the inner binary: $\tau_{\text{merge}}$ was selected to be in the range between $(0.1 \times$ the main-sequence lifetime of star 3) to $(10^{12}$ years), with the exponent chosen from a uniform distribution. We then used Equation 1 to compute the radius of the inner orbit. Equation 4 defines the minimum radius of the outer orbit. We selected the maximum orbital radius to be as large as $10^5$ times the maximum radius of star 3, selecting the exponent from a uniform distribution. We then computed the evolution of each individual system, ending

2 Note, however, that in the evolutions shown, the availability of additional mass did not lead to significantly larger increases in the masses of the accretors. This is because of the interplay in our model between the rate of mass infall and the ability of the accretors to accept and retain mass.

3 With this formulation, many outer stars are in orbits so wide that they cannot send a significant amount of mass to the neighborhood of the inner binary. Our goal, however, is to use the calculations to explore the outer limits, beyond which mass transfer does not significantly change the inner binary.
8  R. Di Stefano

Table 1. Numbers of events per 33,333 triples

| $v_1$ | $v_3$ | $\kappa$ | $N_{<0.5\tau(0)}$ | $N_{<0.1\tau(0)}$ | X-WD  | X-WD  | X-WD  | X-NS  | NS-NS  | WD-WD  | WD-WD  |
|-------|-------|----------|-------------------|-------------------|--------|--------|--------|--------|--------|--------|--------|
| 0.00  | 0.00  | 0.50     | 5282              | 2790              | 618    | 361    | 93     | 889    | 98     | 18     | 36     |
| 0.50  | 0.50  | 0.50     | 5911              | 3000              | 721    | 400    | 82     | 1000   | 102    | 21     | 36     |
| 1.00  | 1.00  | 0.35     | 6321              | 2742              | 709    | 475    | 52     | 1033   | 82     | 22     | 27     |
| 1.00  | 1.00  | 0.50     | 6853              | 3538              | 801    | 498    | 101    | 1217   | 109    | 16     | 37     |
| 1.00  | 1.00  | 0.75     | 7408              | 4368              | 929    | 490    | 108    | 1287   | 137    | 25     | 43     |
| 2.00  | 2.00  | 0.50     | 9024              | 4469              | 912    | 718    | 68     | 1419   | 110    | 30     | 20     |
| 2.00  | 0.00  | 0.50     | 5369              | 2798              | 662    | 389    | 77     | 930    | 82     | 22     | 28     |
| 0.00  | 2.00  | 0.50     | 8923              | 4444              | 873    | 693    | 91     | 1430   | 123    | 35     | 35     |

| Figure 5. Results of the same calculation shown in Figure 3. The change in $M_2$ is shown along the vertical axes in the bottom panels. Right: $M_3$ is plotted along the horizontal axis; Left: $a_{\text{out}}(0)$ (in AU) is plotted along the horizontal axis. In the top panels the quantity $\tau_f/|\tau_0 - t|$ is plotted along the vertical axis. Color (gray scale) coding of points is described in §3.2.1 and also in the caption to Figure 3. |

Points in cyan (lightest color) correspond to systems in which the donor never filled its Roche lobe; all mass transferred through winds. Points in red (same size as cyan points, but darker) correspond to systems in which mass transfer occurred through winds, but the donor filled its Roche lobe at a time when it was more massive than the binary. During the ensuing common envelope phase, no additional mass was gained by the binary, but the inner binary did lose orbital angular momentum as it helped to eject the common envelope. Points in blue (dark and larger than the others) are systems in which the donor filled its Roche lobe |

3.2.1 Times to Merger and Mass Increases

Figures 3 and 4 illustrate and quantify (1) decreases in the time to merger and (2) mass gains by the inner binary.
and was able to continue giving mass to the inner binary in a stable manner.

In the top panel of Figure 3 we consider the time-to-merger as measured at the end of mass transfer, \( \tau_f \). The logarithm to the base 10 of \( \tau_f \) is plotted along the vertical axis. Along the horizontal axis is the logarithm of the “expected” time-to-merger. The definition of \( \tau_f,\text{expected} \) is: the original value of the time to merger (i.e., as calculated prior to mass transfer), minus the time duration of mass transfer. Thus, the value of \( \tau_f,\text{expected} \) would have been the remaining time to merger had no mass transfer occurred. The diagonal green line corresponds to the case in which mass transfer has no effect on the time to merger. The factor by which the time-to-merger decreases is often as small as 0.01, with some systems exhibiting even more dramatic decreases.

The bottom panel explores the relationship between the decrease in the time to merger and the increase in the masses of the components of the inner binary. Along the horizontal axis is the logarithm of the total mass of the inner binary after mass transfer, to its value prior to mass transfer. Along the vertical axis is the logarithm to the base 10 of \( \tau_f/\tau_f,\text{expected} \). The panel shows that the time to merger can decrease significantly, even if the inner components of the binary gain very little mass. As expected, common envelope evolution can lead to significant shortening of the time to merger while the mass of the inner binary experiences only a marginal increase. On the other hand, Roche lobe filling and even winds alone or winds followed by a common envelope phase, can lead to both significant mass increases and to significant decreases in the time to merger.

The same quantities are plotted in Figure 4, but the parameters that control angular momentum loss are larger (\( v_1 = v_2 = v_3 = 2 \)), indicating that more angular momentum is carried away by mass leaving the system. In particular, the outer orbit is more likely to shrink, so that larger numbers of donors will fill their Roche lobes. A larger fraction of common-envelope and Roche-lobe-filling systems inhabit the panels of Figure 4. As a consequence, typical times to merger decrease even more than in Figure 3, and the fractional change in mass of the inner binaries is larger. Two messages emerging from these simulations are the following.

1. There are significant effects even with low angular momentum loss.
2. Increasing the amount of angular momentum lost increases the magnitude of the effects and the numbers of systems experiencing them.

### 3.2.2 What characteristics of the outer binary produce the most pronounced changes?

Figures 5 and 6 relate the changes in the inner binary to the initial characteristics of the outer binary. In the right-hand panels we consider the effect of varying \( a_{\text{out}}(0) \), the initial radius of the outer orbit. By this we mean its radius just prior to the epoch of mass transfer from star 3. In the left-hand panels we explore the influence of the value of \( M_{2}(0) \), the initial mass of the donor.

The bottom panels plot \( (M_{2}(f) - M_{2}(0)) \), the change in the mass of star that was initially the least massive stellar remnant in the inner binary. The top panels plot the quantity \( \tau_f/|\tau_0 - t| \).

Figures 5 and 6 share several common features. First, with regard to the influence of the donor mass: while more massive donors can increase the secondary’s mass by the most, something just over \( 3M_{\odot} \) for \( M_{3}(0) = 20M_{\odot} \), even low mass donors can be responsible for significant increases. A star of \( 2M_{\odot} \) (3\( M_{\odot} \)) can increase the secondary mass by nearly 1\( M_{\odot} \) (2\( M_{\odot} \)). Furthermore, these large changes can be made through the action of winds. (This is more obvious in the left-hand panels, which include fewer of the somewhat larger points associated with Roche-lobe filling.4)

Another commonality is that the initial orbital separation makes a big difference to both the inner-binary mass increase and the decrease in the time to merger. Furthermore, there is a range of values of \( a_{\text{out}}(0) \) over which the changes in mass and merger times are sharply peaked. For \( v_3 = 1 \), the peak lies in the range between about 10 AU and 20 AU. This peak moves out to larger values of \( a_{\text{out}}(0) \) when there is more angular momentum loss. This is because the loss of angular momentum from the outer orbit decreases the value of \( a_{\text{out}} \); thus, the initial value of \( a_{\text{out}} \) is much larger than the values at which most of the mass transfer occurs.

#### 3.2.3 Comparing results across simulations

Table 1 shows the results for a set of 8 simulations. The first two columns show the values used for \( v_1 \) and \( v_3 \). These are the constants of proportionality between the angular momentum per unit time carried away from the outer orbit by mass exiting from star 1 and star 3, respectively, and the specific angular momentum of these stars. Matter incident on the binary first travels to \( M_2 \). Any mass that cannot be accreted by \( M_2 \) then travels to \( M_1 \), and mass that cannot be accreted by \( M_2 \) exits the system. Since matter does not exit directly from \( M_2 \), the value of \( v_2 \) does not directly influence the evolution.

The model parameter \( \kappa \) appears in the third column. The fourth column shows the number of systems, \( N_{<0.5\tau(0)} \), for which the times to merger were reduced by more than a factor of 2. \( N_{<0.5\tau(0)} \) is a rough estimate of the numbers of systems in each simulation that are significantly influenced by mass flowing from the outer star toward the inner binary; its value ranges from \( \sim 5300 \) to \( \sim 9000 \), representing between 5% and almost 10% of the initial set of triples. Because most members of the initial set did not start with values of \( a_{\text{out}} \) small enough to allow significant mass flow to the inner binary, the value of \( N_{<0.5\tau(0)} \) provides a guide to

\[ 4 \] This functional form yields values almost equal to the ratio \( \tau_f/\tau_0 \) when the evolutionary time is shorter than the initial time to merger, but gives large values when the merger time is already so short that the binary may merge event before star 3 is fully evolved. Such systems are interesting because there would be mass in the vicinity of the binary as it merges, potentially producing detectable electromagnetic signatures to accompany the emission of gravity waves. From the perspective of altering the time to merger, however, the effects are likely to be slight and we therefore designed the functional form \( \tau_f/|\tau_0 - t| \) so that small values would allow us to easily identify the systems in which the time-to-merger was most altered by mass transfer.
the numbers of sets of initial conditions that lead to significant changes.

There are some clear trends in the values of $N_{<0.5\tau(0)}$. For example, larger changes are effected for larger values of $\kappa$. The third, fourth, and fifth rows are for systems with $v_1 = v_3 = 1$, but for $\kappa$ equal to 0.35, 0.50, and 0.75, respectively. The increase in the effectiveness of mass transfer with increasing $\kappa$ is expected, unless incoming mass is processed much less efficiently by the inner binary when the rate of mass infall is large.

Second, more angular momentum loss, particularly from the outer orbit, increases the numbers of systems experiencing significant effects. This can be seen by comparing the first, fourth, sixth, and eighth rows. The more angular momentum lost from the outer orbit, the larger the numbers of outer stars that can be pulled in close enough to the inner binary to donate significant amounts of mass.

The fifth column shows the number of systems, $N_{<0.1\tau(0)}$, for which the times to merger were reduced by more than a factor of 10. We find that, across simulations, $N_{<0.1\tau(0)} \approx 0.5N_{<0.5\tau(0)}$. This includes binaries which started having times-to-merger larger than $\tau_H$.

The sum of the sixth and seventh columns provides the numbers of WD-containing close binaries in which a WD made a transition to an SN Ia or to an NS, while the nature of the companion stayed the same. The numbers of transitions to NSs are smaller than the numbers of SNe Ia in these simulations because the initial WDs masses needed to transition to a NS span only the narrow range between $1.15 M_\odot$ and $1.38 M_\odot$, while SNe Ia occur in our simulations from $0.6 M_\odot$ to $1.15 M_\odot$. Although the ratio of the lengths of the starting ranges is only $0.41$, the ratio of AICs to SNe Ia is larger because less mass is generally needed to effect an AIC.

The sum of the sixth and seventh columns is roughly equal to $0.3 N_{<0.1\tau(0)}$. The numbers of NSs that make transitions to BHs, while their companion does not make transition is also roughly equal to $0.3 N_{<0.1\tau(0)}$. The numbers of binaries that experience double transitions (e.g., NS/NS to BH/BH) is roughly $10\%$ the number of single transitions (e.g., NS/X to BH/X).

The upshot of these comparisons is that, although the amount of mass channeled to the inner binary, and the loss of orbital angular momentum from the outer binary, both play significant roles, changes in the time-to-merger and transitions of compact objects should both be common.

### 3.2.4 X-Ray Hierarchical Triples

Mass approaching close to or accreting onto one of the compact objects is expected to emit X-rays. For each particle, the accretion luminosity is typically a significant fraction of its rest mass. Compact binaries receiving mass emitted by winds or through Roche-lobe-filling can therefore be very bright. With intrinsic and/or lensed luminosities that may be near or above $10^{39}$ erg s$^{-1}$, they would be detectable in external galaxies.

An interesting feature of these systems is that, although they would appear to be X-ray binaries, they are actually X-ray hierarchical triples. They could exhibit periodic or quasiperiodic signatures related to the orbital period of the inner binary. At the same time, they would exhibit features characteristic of symbiotic binaries.

If hierarchical triples are common, a significant fraction of bright variable X-ray sources are likely to actually be accreting binaries with wide-orbit companions. Archival studies searching for variability among bright X-ray sources, and also identifying counterparts across wavebands could identify these systems. The population would consist of premerger binaries, but also many others which cannot merge in a Hubble time. In addition, some short-orbital period binaries of many types might be accreting from low-mass companions. An example of a system in which one of the components of the inner binary may not be a stellar remnant is a cataclysmic variable, in which the companion may be a very low mass star, perhaps itself degenerate. The orbital period would be on the order of a few hours. It is important to conduct archival searches for X-ray triples.
Figure 8. Each diamond represents a close-orbit binary, and is divided into two parts, each representing one of the binary’s components. The links among these diamonds represent evolutionary pathways in which one of the binary’s components gains enough mass to make a transition that changes its nature. Because the addition of only a relatively small amount of mass can spark a transition from a WD to a NS or from a NS to a BH, all of these transitions are possible.
Figure 9. Initial masses (in orchid) and final masses (in blue) of binaries in which a WD undergoes an AIC (top panels), an SN Ia (bottom panels). Each point represents a pair of masses, with the initial masses on the left (orchid points) and the final masses on the right (blue points). The WD mass, $M_2$, is plotted along the horizontal axis, and its companion’s mass is plotted along the vertical axis.
Figure 10. Initial masses (in orchid) and final masses (in blue) of binaries in which NSs undergo AICs to become BHs. In the top panel, the NS starts with a BH companion. In the bottom panel, the NS starts with an NS companion and both NSs collapse. Each point represents a pair of masses, with the initial masses on the left (orchid points) and the final masses on the right (blue points). An NS mass, $M_2$, is plotted along the horizontal axis, and its companion’s mass is plotted along the vertical axis.

4 TRANSFORMATIONS OF THE NATURES OF THE ACCRETOR(S)

Figure 8 illustrates the full set of transformations made possible through the accretion of mass by one or both compact objects comprising a binary. Each diamond in the figure represents a compact-object binary. The binary’s components are labeled: WD; NS; BH. The least massive combination, WD-WD appears at the top of the structure, and the most massive, BH-BH, appears at the bottom. This structure provides a simple visualization of the possible transformative effects of mass infall.

Imagine mass accreting onto the WD-WD pair. It may influence the orbit, shrinking the size of the diamond (which represents the orbit) or in some cases, expanding it. If mass stops falling before either WD reaches the critical mass, then no transitions are made. If, however, one of the WDs is an O-Ne WD that achieves the critical mass, that WD becomes a NS. A red line connects the WD-WD diamond to the diamond below and to the left of it, which represents an NS-WD pair.

We note, however, that the eventual merger of the WDs would produce an SN Ia via the double-degenerate channel.

Note that there are also two red lines connecting the WD-WD binary to a double-NS binary. Each line represents the AIC of a WD; two such connections side-by-side correspond to two transitions that occur during a time interval short compared with the time scale of mass transfer. While we don’t expect two AICs to occur at exactly the same moment, the near equality of the mass-gaining WDs in our accretion scenario suggests that two AICs could occur very close in time to each other. Thus, the after effects of the first event may still be detectable at the time of the second event.

Every red line connecting a diamond on an upper level to a diamond on the level below represents the AIC of a WD. Such transitions are expected if O-Ne WDs are in close binaries accreting from a hierarchical companion. This is because the amount of mass that is needed to effect the transition is at most $\sim 0.25 M_\odot$. Thus, if O-Ne WDs are in close binaries, and also have donor stars in wide orbits, the only condition that must be satisfied in order to produce an AIC is that, for an interval of a few times $10^5$ years, mass from the donor would have to be incident on the WD at a
high enough rate ($\sim 10^{-6} M_{\odot} \text{ yr}^{-1}$) for nuclear burning to occur, or else at the slightly lower rates that would produce recurrent novae. This is achievable for giant donors at orbital separations $O(AU)$.

Mass changes that occur when a compact object makes a transition are illustrated in Figures 9 and 10. The points in Figure 9 and in Figure 10 were generated by the full set of binaries evolved to produce Table 1. In red (blue) are the masses of the WD $[M_2]$ and its companion $[M_1]$ prior to (after) mass transfer.

Figure 9 considers transitions of WDs. The top left-hand panel of Figure 9 shows cases in which a WD collapses into a NS when its initial companion was either a NS or a BH. In the systems shown, the companion did not transition to a compact object of another type. One can see that, in general, both compact objects gain mass. The small gaps that appear around $M_1 = 2$ correspond to systems in which the WD’s NS companion transitioned to a BH. On the right are binaries which started with two WDs and ended with two NSs. Note that the masses of the WDs track each other, staying almost equal. This feature is related to our mass-transfer scenario, in which mass flows first to the least massive component.

The two bottom panels show transitions in which one or both WDs undergo SNe Ia. Each of these panels is analogous to the panel just above it. The gaps in the “before” and “after” masses of $M_2$ reflect the fact that the upper mass limit for C-O WDs is below the Chandrasekhar mass. Otherwise the characteristics are very similar to those illustrated above in the WD to NS transitions.

Note that we did not terminate the evolutions at the time a WD achieves $M_{Ch}$. In principle, a Type Ia supernova could occur at this point. In practice, there will be a simmering phase prior to explosion that could last $\sim 1000$ years (Piro 2011). An even longer delay is possible if the WD has spun up and needs to spin down to explode (Di Stefano et al. 2011). Because we did not assume that a WD explodes immediately upon reaching the Chandrasekhar mass, the masses of the WDs in the final state exhibit a range of values, extending to above $2 M_{\odot}$. If the WDs are spun up by accretion, super-Chandrasekhar explosions are expected (Di Stefano et al. 2011). In the right-hand panel, cases in which both WDs explode are shown. Such a scenario is possible if accretion can continue even after one WD has achieved $M_{Ch}$ (i.e., if that WD does not explode on a short time scale), or else if the orbital parameters are such that, even after a WD undergoes supernova, its companion WD can stay bound to the donor and can continue to accrete.

In Figure 10, the top panel shows mass changes during transitions in which a BH-NS pair becomes a BH-BH pair when the NS collapses. The bottom panel shows systems in which two NSs collapse to become two BHs. Again the near-equality of the final BH-BH masses is exhibited. Note that there are clear deviations from exact equality. This is because the mode of mass transfer in our simulations allows mass that could not be retained by the lower-mass accretor to flow to its companion, which may then retain some of incident mass.

Observable Signatures: The question arises: how can we know if some of the compact objects we detect are the result of transformations from less massive compact objects? Of particular relevance: how can we know that the mass that led to collapse was donated by a third star in a hierarchical triple?

The first answer is to study the distribution of masses of the compact objects we detect through the gravitational radiation they emit during merger. Analysis of the gravitational wave signatures allows us to measure the masses of the compact objects that merged. While may mergers are likely to occur in systems that did not undergo accretion from a third star, those that have followed the evolutionary pathways we outline here could help to shape the form of the mass distribution. For example, the accretion-induced collapses of NSs produced BHs that, at least at the time of collapse, have masses lower than those of the BHs we have been studying within X-ray binaries. Furthermore, if mass is channeled toward the least massive component, then the masses of the compact objects that merge should tend to be similar. Thus, our model is likely to produce pairs of merging BHs, with the two members of the pair having similar masses, and those masses may each be less than $\sim 4 - 5 M_{\odot}$, with some hovering right near the maximum NS mass. In addition, we may NS-BH mergers in which the masses of the NS and BH are nearly the same.

WD-NS mergers will, with present-generation instruments, be detected primarily through electromagnetic signals. If, however, the mass of the WD can be established, our model predicts that some of the WDs should have masses very close to the critical mass. AICs of WDs to NSs and NSs to BHs should also be electromagnetically luminous. In our model, roughly half of them should take place within an accreting hierarchical triple. Furthermore, if the electromagnetic signature

In addition, pulsar searches are discovering many more systems in which a NS is in orbit with another compact companion

5 WIDER RANGE OF CHARACTERISTICS AND POSSIBILITIES

Mass transfer for a star in a wide orbit can influence the masses and orbits of compact objects in a close binary. We have illustrated this with examples of mass transfer from an evolved star, because this process can be followed in a simple way, closely connected to an analytic formalism. We have focused on cases in which angular momentum from the outer orbit is carried away from the triple. Here we consider elements of our model that extend the results beyond those derived in §3 and §4.

5.1 Mass Transfer from a Main Sequence Star

Main sequence stars can serve as donors in hierarchical triples. There are several important differences between systems with main-sequence and giant donors. Main-sequence stars have wind mass-loss rates much higher than evolved stars. If a main-sequence star is not filling its Roche lobe, the mass inflow rate to the inner binary is likely to be low. We therefore consider only cases in which the main-sequence donor fills its Roche lobe. The mass, $M_3$, of the main-sequence star determines its equilibrium radius, and therefore the radius, $R_L$, of its Roche lobe. The value of the orbital separation at which the donor fills its Roche lobe,
Figure 12. Flow chart in which all inner binaries start on the left-hand side and evolve toward the right. The green rectangle of length $Y(b)$ corresponds to binaries that would merge within $\tau_H$ in binary-only scenarios. In binary-only scenarios, all other systems end at the red oval. In the model in which there is a mass-donating star in an outer orbit, all evolutions continue to the right. Note that some systems that would have merged in the binary-only scenarios may now fail to merge within $\tau_H$. The flattened red oval represents such failures. The upper green oval on the right, of length $Y(b,t)$, corresponds to mergers that could have happened, even in binary-only scenarios, and the lower green oval of length $Y(t,t)$ corresponds to mergers that are added through the effects of mass from the third star. The sum of the lengths of these two ovals is $Y(b,t) + Y(t,t)$ and is expected to be larger than $Y(b)$.

$a_{\text{out}}$, is determined by the combination of $R_L$, $M_3$, and the total mass, $M_1 + M_2$ of the inner binary.\footnote{At the time the donor first comes to fill its Roche lobe, its radius is likely to be the same as the equilibrium radius. As the star loses mass, however, it can either shrink or expand. The difference from the equilibrium radius is not typically large, so here we use it as a guide.}

Once we know the value of $a_{\text{out}}$, we can employ the condition for orbital stability to determine $a_{\text{in max}}$, the maximum possible value of $a_{\text{in}}$ for which the triple is dynamically stable. The ratio $a_{\text{out}}/a_{\text{in}}$ provides a basic guideline to whether the orbits are stable. If the effects of mass flow drive the ratio to values that are too low, the hierarchical triple will no longer be dynamically stable, and mergers or ejections may occur. If, on the other hand, the triple is dynamically stable, then a large fraction of the mass lost by the donor will come under the gravitational influence of the inner binary.

The bottom panel of Figure 11 plots $a_{\text{in max}}$, the maximum value of $a_{\text{in}}$ consistent with orbital stability, for three values of the total mass, $M_1 + M_2$, of the inner binary, and for a range of donor masses extending to $20 M_\odot$. The upper panel plots values of the logarithm to the base 10 of the time to merger when the separation is $a_{\text{in max}}$.

Figure 11 demonstrates that the times to merger in all of these cases is short. For example if the total mass of the inner binary is $15 M_\odot$, the time to merger ranges from tens of thousands of years for an M-dwarf donor star to just under $10^8$ years for a donor of $20 M_\odot$.\footnote{We used a mass-radius relationship appropriate for stars below roughly $9 M_\odot$. The results would not be qualitatively different with a mass-dependent formulation.} We can also consider cases in which the inner orbits are smaller than $a_{\text{in max}}$, since these will also be stable with respect to the dynamical evolution. The times to merger would then be even shorter; in most cases shorter than the main-sequence lifetime of the donor.

Main-sequence donors are therefore likely to be present at the time of merger. Mass from the donor can provide a luminous electromagnetic signature before, during, and after merger. The prior signal would be strong at X-ray wavelengths if one or both of the compact objects in the inner binary accretes. The orbital period of the inner binary may be measurable through variations in the X-ray flux. Furthermore, the signal from one accretor can be enhanced through gravitational lensing by its compact companion D’Orazio & Di Stefano (2018b,a). If the merger result and donor star are able to remain in orbit, the long-term result after merger will be an X-ray binary.

5.2 Angular Momentum: General Considerations

If the angular momentum flow is more complex than in the simple examples we have considered, there can be interesting consequences.
Possible increases in the angular momentum of the inner orbit: The angular momentum of the inner orbit can be significantly smaller than the angular momentum associated with the outer orbit, yielding small values of the following ratio.

\[
\frac{L_{\text{in}}}{L_{\text{out}}} = \left( \frac{M_1 M_2}{M_3 (M_1 + M_2)} \right) \left( \frac{M_1 + M_2 + M_3}{M_1 + M_2} \right)^{\frac{1}{2}} \left( \frac{a_{\text{in}}}{a_{\text{out}}} \right)^{\frac{1}{2}}
\]

(10)

Thus, if even a small fraction of angular momentum from the outer orbit is transferred to the inner orbit, the results can be dramatic.

Consider, for example, the case of perfectly aligned orbits in which mass from the outer star is accreted by one or both of the inner stars without any mediation by, e.g., a disk. The binary can be “spun up”. If the triple remains stable, the time to merger will increase. If, however, \(a_{\text{in}}\) increases and \(a_{\text{out}}\) either decreases or increases at a rate slower than \(a_{\text{in}}\), the triple may become unstable, calling its final fate into question. The ejection of one of the stars is a possibility, and so is a prompt merger. In the latter case, the merger would take place in a mass-containing region, and the donor star would also be present at the time of merger, potentially producing an electromagnetic signature. It is interesting to note that, if the outer orbit is not too big, expansion of the inner orbit can trigger a dynamical instability, producing a prompt merger.

Three-dimensional rotation: The inner and outer orbits may not be in exact alignment. In such cases, mass incident on the inner binary from star 3 can induce rotation in the orbital plane of the inner binary. Were the inner binary isolated, the force of gravity acting on its two components would define a two dimensional plane. In our case, the inner orbit defines one plane, with an angular momentum vector \(\vec{L}_{\text{in}}\) perpendicular to that plane. The same is true of the outer orbit. The two angular momenta, \(\vec{L}_{\text{in}}\) and \(\vec{L}_{\text{out}}\) may point in different directions. Mass flow from the outer star influences not only the motion of the compact objects within the plane of the inner orbit, but can also serve to rotate the orbital plane, setting it “spinning”, as the two compact masses continue to orbit each other. The complex dynamics has much in common with situations in which mass is not flowing through the system, but instead the dynamics is dominated by three-body interactions (second appendix).

5.3 Massive Donors

Massive donors introduce additional features. When such donors have close stellar companions, binary interactions can strip them of their hydrogen envelopes Uomoto (1986). Energy provided by the remaining nuclear-burning core can produce pre-supernova outbursts, consistent with observational evidence for precursor events [Fuller & Ro (2018) and references therein]. Heavy winds, and precursor events may inject mass into the orbit of the inner binary. While such mass injections may or may not increase the mass of the inner binary, they are likely to decrease the time to merger. Furthermore, any alterations made close to the time of supernova would have been preceded by an epoch of sustained winds more likely to produce genuine mass increases and to decrease the orbital angular momentum. Thus, even prior to supernova, the close binary may be more massive and closer to merger than it would have been had the companion not been in orbit with it.

The supernova emits high-power matter which flows past the binary, interacting with it, possibly torquing it. Depending on the initial configuration of the triple, its evolution, and the response of the binary to the supernova, the binary may merge near the time of the supernova or at least while the supernova remnant is still detectable. We would then detect a gravitational wave (GW) source within a supernova remnant. The supernova would not be associated with either of the merging compact objects, however. It is therefore important to search for tell-tale signatures, such as the location of the gravity-wave emitter away from the center of the supernova remnant. Depending on the total change in the mass of the triple-star system, the remnant of star 3 could become unbound from the compact-object binary.

5.4 Uncertain or New Physics

Common Envelope: Many elements of the common envelope are not yet well understood Iben & Livio (1993); MacLeod et al. (2018), and references therein. These include the initiation of the common envelope; its evolution, and whether there is mass gain by the stars engulfed by it; the evolution of the stellar orbit; and the end state left behind. In our calculations we have assumed that the compact objects in the inner binary gain no mass during the common envelope phase. Some evolutionary calculations allow hyper-critical accretion. (See Abadie et al. (2010) for an overview for binary evolution of systems leading to compact binary coalescences.) Mass gain during such a phase would likely be minimal. Nevertheless, it could be enough to change the nature of one or both components of the compact binary.

In our conservative approach, the common envelope doesn’t produce prompt mergers and no mass is gained during the common envelope phase. It is likely, however, that in some real systems the consequences of the formation of a common envelope are more dramatic than those allowed in our calculations. This will have the effect of driving more close binaries to an earlier time of merger. Also, in combination with mass gained by the components of the close binary prior to the formation of the common envelope, any mass gain during the common envelope could change the natures of some accretors.

Circumbinary Disks: Circumbinary disks have been studied in the context of supermassive black hole binaries. The least massive component of the inner binary passes closest to the accretion disk, and simulations show that a mini-disk can therefore form around it, allowing this least-massive compact object to gain mass [Tang et al. (2018)]. Should its mass become equal to that of its companion, then both stars may gain mass, with one of them accreting, and then the other. In this scenario, the two masses stay nearly equal to each other, so that mass from star 3 tends to equalize the masses of the inner binary.

There are other possibilities, both for supermassive and stellar mass BHs [Maureira-Fredes et al. (2018); Khan et al. (2018)]. These possibilities include the accelerated growth
of the more massive component. It is also worth noting that the compact objects in the inner binary are close enough to each other that if incoming material forms a structure, such as a corona, some of the mass forming that structure could be transferred to the other star. This reasoning produced the flow of mass we incorporated into the calculations of §3, which gave the least massive component of the inner binary a chance to accrete incoming mass; mass which could not be retained by the least massive component was then passed on to the more massive component.

Studies of circumbinary disks show that they can play active roles in, for example, extracting angular momentum from the binary. Because the possibilities are not yet well understood, we have not explicitly incorporated disks into our calculations. Generally, a disk promotes dissipative effects. These tend to erase the effects of the detailed mass flow history. Our approach of considering only the local flow of matter in the vicinity of each accretor is therefore likely to be valid in many cases.

**Gravitational Lensing Within the Compact Binary:** Consider inner binaries whose components are NSs and/or BHs. When mass is incident on one of these compact objects, it emits electromagnetic radiation, primarily at X-ray wavelengths.

Roughly 10% of the inner binaries have orientations favorable for the detection of gravitational lensing. That is, the projected distance between the luminous accretor and its companion (during a time interval in which the companion passes in front of the accretor) is small enough that radiation from the accretor is sufficiently lensed. (See DoDi:2018inprep, 2018MNRAS.474.2975D.) When this happens, the X-ray count rate is temporarily increased. This would occur once, or (if both compact objects are accreting) twice per orbital period.

Furthermore, the resultant X-ray flux, produced by lensing from a baseline that can be $10^{18} - 10^{19}$ erg s$^{-1}$, can be high. If lensing occurs, the number of photons we receive may be large enough to permit detection in external galaxies. Even binaries that will require more than $\tau_H$ to merge may be detectable through such lensing.

In addition to this unique effect, which can allow the masses of the compact objects to be determined, there are other reasons that the periodicity of the inner orbit may be imprinted on the X-ray signal. A definitive identification of lensing would, however, nail the case for the presence of an accreting inner binary composed of NSs and/or BHs. The reason we have explicitly discussed NSs and BHs in this subsection, is that the relatively larger size of WDs means that finite-lens-size effects decrease the probability of binary-self-lensing and also diminish the magnitude of the any effects that do occur.

### 6 SUMMARY AND IMPLICATIONS

#### 6.1 Summary

We have considered triple-star systems consisting of two compact objects in a close orbit and an unevolved star in a wider orbit. We have shown that, for a large range of plausible initial conditions, mass from the outer star can influence the evolution of the inner binary. This is possible simply through the agency of winds if a small fraction of the donor’s mass comes close enough to the inner binary to interact with it. Roche-lobe filling can also play a role. If the subsequent mass transfer is stable, it contributes to the mass of the components of the inner binary as well as possibly altering the orbital angular momentum. If it is unstable, then a common envelope is likely to drain angular momentum from the inner orbit. The primary effects are the following:

- **Increases in the mass of one or both components of the inner binary.** It is of interest that even donor stars with masses similar to or a few times larger than that of the Sun can provide significant mass to the components of an inner compact binary.
- **Mass increase can transform one type of compact object into another,** with O-Mg-Ne WDs becoming NSs, and NSs becoming BHs. Such transformations substantially increase the pool of NS-NS, NS-BH, and BH-BH binaries that merge within a Hubble time. Only a modest amount of added mass is required to effect these transformations.
- **Changes in the time to merger.** These can occur if the inner binary’s components gain mass, even if its orbital angular momentum is unchanged. Decreases in time to merger can be more dramatic when mass from the donor drains angular momentum from the inner orbit.
- **A new model of Type Ia supernovae.** When WDs that would not have merged in a Hubble time can do so because of mass provided by a companion in a hierarchical orbit, the rates of SNe Ia generated through the double-degenerate channel can increase. In addition, mass gain by C-O WDs during mass transfer from the third star can produce SNe Ia through an analog of the single-degenerate model. The first-formed WD would already have accreted matter during the evolution of its original stellar companion that eventually produced the second WD. Mass from the third star provides another chance for it (and a first chance for its WD companion) to increase its mass to the Chandrasekhar mass. Thus, the rate of SNe Ia through the accretion channel can also be increased because of mass provided by the outer star.

Our calculations have been conservative. It therefore seems likely that gains in mass and losses of orbital angular momentum much larger than those we have considered are possible. For example, (a) the third star could be more massive than the stars we considered, (b) the ejection of the common envelope almost certainly drains more angular momentum than we have assumed, (c) there may be more than one outer-orbit star that can come to donate mass.

#### 6.2 Implications

Our model has several points of connection to observations. Key elements of it can therefore be tested.

##### 6.2.1 Gravitational Mergers

Triples in which the outer star sends mass toward the inner binary can increase the rate of gravitational mergers. $^8$

---

$^8$ In an appendix we briefly discuss the role of three-body dynamics, even when mass is not transferred from the outer star.
Consider, for example, BH-BH mergers. Binary interactions provide a baseline prediction. Binary-only calculations apply to true binaries and to hierarchical triples in which the third star is too distant to influence the fate of the inner binary. We have shown that interactions with mass provided by the outer star can change the time-to-merger from values larger than $\tau_H$ to values smaller than $\tau_H$. This adds to the total reservoir of binaries that can merge within a Hubble time. Note that the number of binaries added to this reservoir is almost certainly much larger than the numbers that leave it by being pushed to longer merger times by the effects of mass infall from an outer star. In addition, the conversion of NSs to BHs, and even of WDs to NSs and then to BHs provides a brand new reservoir of binaries that will experience BH-BH mergers. If triple systems are not rare, such transformations have a good chance of contributing significantly because binaries containing the less massive stars that produce NSs and WDs are more common than the binary systems producing only BHs. There are caveats, including the effects of supernovae. Nevertheless it is worth noting that binaries undergoing transformations should be considered as potentially important sources of BH-BH mergers. A parallel set of arguments applies to mergers involving NSs.

In addition to influencing the merger rates, mass from a third star can change the distribution of merger properties. In particular, the merging components are likely to have nearly equal mass, at least if the principles we have used to trace the path of incoming mass are correct. The values of the masses depend on how much mass is available from the third star and on the efficiency of accretion. If the total mass added to the inner binary is small, then we expect there to be merging NSs with masses near the Chandrasekhar mass, and low-mass merging BHs, which masses less than about $5 M_\odot$. If the outer star is more massive, and/or of there is a sequence of outer stars, mergers of nearly-equal high-mass BHs are expected.

### 6.2.2 X-ray emission

Continuing accretion is also a signature expected for a subset of our post-merger systems. If the compact object formed through the merger continues to accrete material provided by the donor star, it will emit X-rays. Whether we would be able to detect the X-rays depends on the accretion luminosity and the distance from us. Consider, for example, an X-ray source with X-ray luminosity $L_X = 10^{40}$ erg s$^{-1}$ and a power-law spectrum with $\Gamma = 1.7$ and intervening gas with $N_H = 1 \times 10^{23}$ cm$^{-1}$. If this X-ray source (XRS) were 50 Mpc away, Chandra’s ACIS-S detector would record roughly 2 counts per ks, so that the source would be detectable. The proposed Lynx mission could record $\sim 50$ times as many counts, potentially allowing short-time-scale variations to be traced. Thus, post-mergers at intermediate distances may be detected as X-ray binaries.

**Hierarchical X-Ray Triples** Finally, many of the types of systems we consider should be detectable prior to merger as X-ray hierarchical triples. In our model of mass impinging on an inner binary, the compact accretors may be detected as powerful X-ray emitters. X-ray emission therefore provides a way to identify systems within which a compact-object binary is accreting. The signature for which in order to identify hierarchical triples is periodic or quasiperiodic modulation of the X-rays, where the repetition times are harmonics of the inner orbital period. There are many possible reasons for X-ray from binaries to exhibit a range of periodic and/or quasiperiodic signatures: for example, complex accretion flows and warped disks can introduce periodicity. It is therefore important to model the signatures in order to test whether they are consistent with accretion onto an inner binary, or whether another explanation is equally good or better.

Typical galaxies house dozens of bright X-ray binaries. If accretion from a wide-orbit star is common, then a fraction of X-ray sources that we have assumed to be binaries may actually be hierarchical triples. Even if there are only a handful of X-ray triples among the hundreds of bright X-ray sources in nearby galaxies, it may be possible to identify them in archived data. We note that only a fraction of the triples may include close binaries that will merge in a Hubble time. Thus, the numbers of hierarchical triple accretors could be relatively large compared with the lifetime-scaled merger rate. The discovery of such systems would be a powerful indication that our model of mass transfer to compact inner binaries may be important.

### 6.3 Conclusions

Close binary systems form one of the most important classes of astrophysical objects. Their importance is highlighted by the fact that aLIGO has begun to discover the gravitational-wave signatures of their mergers. This makes them the first observed multimessenger emitters. We have introduced triple-star pathways that have the potential to contribute substantially to the rates of gravitational mergers. This reduces the pressure on other channels to produce all events. The contributions of triples however, have still to be quantified relative to the contributions of isolated binaries. This will be an important next step. The ubiquity of triples involving massive stars tells us that it is an important channel to explore. It remains to quantify the relative contributions of hierarchical triples to several important processes: Type Ia supernovae, merging NS-NSs, NS-BHs, and BH-BHs. Do triples contribute only a small fraction? Or do they make significant and measurable differences to the event rates?

These questions can be answered in part through continuing observations of the events themselves. For example, are merging low-mass BHs more common than expected based on binary interactions alone? Other observations, for example improved measurements of the properties of primordial triples, will also be important.

Theoretical work is also needed to determine the role that 3-body dynamical interactions have in helping to determine the initial characteristics of inner binaries: what are the initial conditions needed as input to calculations designed to realistically model mass transfer from a third star? Other questions have to do with developing more detailed simulations of mass transfer from a third star. For example, how are winds focused? What is the role of irradiation? What is the geometry of accretion?

We have shown that the conditions needed for components of an inner binary to interact with mass from a star in a hierarchical orbit should be common in that they extend over a wide range of orbital separations, donor masses, and characteristics of the inner binary. Accretion from an
outer star is, therefore, a process that is certain to occur. Fortunately there are many links between our model and a range of observational signatures that may be detected pre-merger, during merger, or even in the epoch after merger. In addition there are connections to Type Ia supernovae.

7 APPENDIX: ROCHE-LOBE APPROXIMATION

Mass from star 3 can be channelled directly to the inner binary in a process analogous to what happens when a donor star in an isolated binary system fills its Roche lobe.

The definition of the Roche lobe of a star in a binary is based on the existence of an equipotential surface that takes into account both the gravitational attraction of both the donor and its companion, and the rotation of the donor (whose spin period is equal to the orbital period). This equipotential surface is smooth and constant in a frame that rotates with the outer binary. Our hierarchical triple, however, introduces a crucial new feature: a time-dependent gravitational potential, $\Phi(t)$, whose instantaneous value at any point in space depends of the positions of the individual components of the inner binary.

Fortunately, the fact that the outer orbit is significantly wider than the inner orbit means that a good approximation to the gravitational potential of its donor can be written as a sum of (a) a time-independent monopole term, corresponding to the concentrating the total mass of the inner binary at its center of mass; and (b) a time dependent dipole term associated with the orbital motion of the inner binary. The dominant term is the monopole term, which is the same as it would be for a single star with mass $M_T = M_1 + M_2$. In addition, the donor star in the hierarchical triple would have a rotational period roughly equal to its orbital period, although this too is an approximation, since the motion of the individual stars in the close binary can introduce time-dependent tidal interactions. Thus, the Roche-lobe formalism can be applied to the case of mass transfer from a wide-orbit star onto a close-orbit binary, although there may be observable signatures associated with the short-orbital-period binary.

8 APPENDIX: DYNAMICAL THREE-BODY INTERACTIONS

The presence of the third body may have played a role in the earlier evolution of the hierarchical triple. For example, the Lidov-Kozai mechanism creates an interplay between the eccentricity of the inner orbit and its orientation relative to the outer orbit Kozai (1962); Lidov (1962); Naoz (2016). A change in eccentricity can promote mass transfer, influencing the evolution of the inner binary. Thus, the initial conditions for mass transfer from the outer star may have been influenced by prior 3-body interactions. These dynamical interactions could have been active before the components of the inner binary interacted; during the binary interactions that produced the close-orbit compact-object binary$^9$.

and/or after the close binary was formed. It will be important to consider the full range of prior interactions in order to develop the profile of realistic configurations for the start of mass transfer from the third star.

Furthermore, if the inner binary has not yet merged by the time mass transfer has ceased, 3-body dynamical interactions continue to influence the characteristics of the inner orbit. These interactions have the potential to influence the time at which the eventual merger will occur, and should be carefully considered.

ACKNOWLEDGEMENTS

I would like to thank Daniel D’Orazio for discussions, and he, Morgan McCleod, and Amber Medina for their careful reading and comments on the manuscript.

REFERENCES

Abadie J., et al., 2010, Classical and Quantum Gravity, 27, 173001
Abbott B. P., et al., 2016a, Physical Review X, 6, 041015
Abbott B. P., et al., 2016b, Physical Review Letters, 116, 061102
Abbott B. P., et al., 2017a, Physical Review Letters, 119, 141101
Abbott B. P., et al., 2017b, Physical Review Letters, 119, 161101
Bachetti M., et al., 2014, Nature, 514, 202
Corral-Santana J. M., Casares J., Muñoz-Darias T., Bauer F. E., Martínez-Pais I. G., Russell D. M., 2016, A&A, 587, A61
D’Orazio D. J., Di Stefano R., 2018a, in preparation
D’Orazio D. J., Di Stefano R., 2018b, MNRAS, 474, 2975
Di Stefano R., Voss R., Claes J. S. W., 2011, ApJ, 738, L1
Eggleton P., Kiseleva L., 1995, ApJ, 455, 640
Farr W. M., Srinav N., Cantrell A., Kreidberg L., Bailyn C. D., Mandel I., Kalogera V., 2011, ApJ, 744, 103
Fuller J., Ro S., 2018, MNRAS, 476, 1853
Furst F., et al., 2016, ApJ, 831, L14
Iben I. Jr., 1982, ApJ, 259, 244
Iben I. J., Livio M., 1993, PASP, 105, 1373
Israel G. L., et al., 2017, Science, 355, 817
Khan A., Paschalidis V., Ruiz M., Shapiro S. L., 2018, preprint, (arXiv:1801.02624)
Kozai Y., 1962, AJ, 67, 591
Lidov M. L., 1962, Planet. Space Sci., 9, 719
MacLeod M., Ostriker E. C., Stone J. M., 2018, preprint, (arXiv:1803.03261)
Margalit R., Metzger B. D., 2017, ApJ, 850, L19
Maureira-Fredes C., Goicovic F. G., Amaro-Seoane P., Sesana A., 2018, preprint, (arXiv:1801.06179)
Naoz S., 2016, ARA&A, 54, 441
Nelemans G., Tout C. A., 2005, MNRAS, 356, 753
Nomoto K., 1982, ApJ, 253, 798
Paczynski B., 1976, in Eggleton P., Mitton S., Whelan J., eds, IAU Symposium Vol. 73, Structure and Evolution of Close Binary Systems, p. 75
Piro A. L., 2011, ApJ, 738, L5
Remillard R. A., McClintock J. E., 2006, ARA&A, 44, 49
Shen K. J., Bildsten L., 2007, ApJ, 660, 1444
Tang Y., Haiman Z., MacFadyen A., 2018, preprint, (arXiv:1801.02266)
Uomoto A., 1986, ApJ, 310, L35

$^9$ Dynamical interactions with the third body are not likely to play a dominant role during the intervals of most active mass transfer. They could, however, serve to enhance mass transfer. In addition, there are intervals, e.g., after the more massive of the two stars star has evolved and before its close companion evolves, whereby the donor star in the hierarchical triple would have a rotational period roughly equal to its orbital period, although this too is an approximation, since the motion of the individual stars in the close binary can introduce time-dependent tidal interactions. Thus, the Roche-lobe formalism can be applied to the case of mass transfer from a wide-orbit star onto a close-orbit binary, although there may be observable signatures associated with the short-orbital-period binary.
Webbink R. F., 1977, *ApJ*, 215, 851