Topology of supersymmetric $\mathcal{N} = 1, D = 4$ supergravity horizons

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Abstract

All supersymmetric $\mathcal{N} = 1, D = 4$ supergravity horizons have toroidal or spherical topology, irrespective of whether the black hole preserves any supersymmetry.
1 Introduction

Supersymmetric phenomenological models are based on $\mathcal{N} = 1, D = 4$ supergravity. This theory has 4 supersymmetries, and the constraints imposed on the couplings are rather weak, see e.g. [1] and references within. Therefore a large class of models can exist even for a prescribed matter content. Nevertheless, $\mathcal{N} = 1, D = 4$ supergravity exhibits properties which are independent of the particular chosen model. Such properties characterize the theory. One such property which we shall investigate here is the topology of supersymmetric horizons of extreme, but not necessarily supersymmetric, black holes of $\mathcal{N} = 1, D = 4$ supergravity. In particular we show that for any matter content, and under some mild restrictions which we explain later, the associated horizon sections have topology $T^2$ or $S^2$. In addition, the metric in the spherical case is a product $\mathbb{R}^{1,1} \times S^2$.

To prove this, we use the classification of supersymmetric backgrounds of $\mathcal{N} = 1, D = 4$ supergravity [4, 5], and the fact that horizon sections are connected and closed Riemann surfaces which are characterized by their Euler number. An explicit example of a spherical horizon has been given in [6].

Black hole horizons in 4-dimensions have been extensively investigated following the uniqueness theorems in [7] -[12]. More recently, attention has been focused on the near horizon geometries of black holes associated with a potential term. In particular, it can be shown under some mild assumptions that the near horizon geometry of such a black hole has an $O(2,1)$ symmetry. This follows from the results of [13] and [14]. The action of $\mathcal{N} = 1, D = 4$ supergravity differs from the above Einstein-Maxwell system in two respects. First the gauge fields are allowed to be non-abelian and second the scalars are gauged. However, we make the additional assumption that the near horizon geometry is supersymmetric.

Before we proceed with the analysis, we shall first put our result into context. For appropriately chosen couplings and matter content most of the well-known 4-dimensional black holes, like the Schwarzschild, Reissner-Nordström, Kerr, and Kerr-Newman, can be embedded as solutions of $\mathcal{N} = 1, D = 4$ supergravity. From these black holes only those with extreme horizons are of relevance here. Even in this case, near horizon geometries, like that of the Reissner-Nordström black hole, are not supersymmetric in $\mathcal{N} = 1$ supersymmetry, suggesting that supersymmetric horizons may not exist. Nevertheless this argument is not conclusive for restricting the existence of supersymmetric horizons, as it applies only to a particular class of solutions. More generally, one may use energy bounds [15, 16, 17] and observe that in the construction of a Nester tensor the standard gravitino supercovariant connection$^2$ of $\mathcal{N} = 1, D = 4$ supergravity does not include a Maxwell field. As a result no electric or magnetic charge can be detected at asymptotic infinity indicating that there are no supersymmetric charged black holes. However to establish such bounds one needs at least the weak energy condition which does not hold for all matter couplings of $\mathcal{N} = 1, D = 4$ supergravity. Moreover, we do not assume that the black hole spacetime is supersymmetric or put any conditions on the asymptotic geometry allowing

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$^1$To our knowledge the existence of black holes of $\mathcal{N} = 1, D = 4$ supergravity with $T^2$ horizon topology has not been ruled out, however see [2], [3].

$^2$In the energy bound of [17] the supercovariant connection of simple $\mathcal{N} = 2, D = 4$ supergravity was used which includes a Maxwell field.
for example for $AdS_4$ black holes. Thus although our assumption that the near horizon geometry is supersymmetric is rather restrictive, the indirect arguments provided above for the existence of such geometries are not conclusive. The advantage of our approach is that it has a wide range of applicability, which includes any matter content, subject to some assumptions which we now explain. There are three types of restrictions required for the technical proof. First, the kinetic terms of the gauge fields and scalars are canonical, i.e., the gauge group metric and the Kähler metric of the scalars are positive definite. Second, if the conditions imply that the gauge field vanishes, then the gauge potential is always chosen to be the trivial one, i.e., we do not consider the cases for which there are flat but non-trivial connections. Third, at several places we have assumed that fields and tensors are sufficiently smooth and sometimes analytic. This restriction is mentioned as it arises in the proof.

In the considerations that follow, we do not assume that the black hole spacetime is supersymmetric. We only assume that the near horizon geometry is. This is an important distinction as there exist non-supersymmetric black holes with supersymmetric horizons [18, 19]. To implement this in our analysis, we distinguish between the stationary Killing vectors which belong to the equivalence class that characterize the horizon\(^3\) of a black hole and the Killing vector field constructed as Killing spinor bilinear. A similar approach has recently been taken in the context of “pseudo-supersymmetric” extremal near-horizon solutions of the minimal de-Sitter five-dimensional supergravity theory [21]. In this theory, the vector field obtained as a Killing spinor bilinear is not Killing, and so one cannot identify this bilinear with the stationary Killing vector of the black hole. This differs from previous analysis done in the context of 5-dimensional black holes in [22], where the two were identified, and so it is assumed that both the near horizon geometry and the black hole spacetime are supersymmetric.

The plan of the paper is as follows. In section 2, we describe in greater detail the assumptions we make, and the construction of a basis adapted to the Gaussian Null co-ordinates in the near-horizon limit. We also analyse solutions of the Killing spinor equations (KSEs) corresponding to supersymmetric extremal near-horizon geometries in minimal $\mathcal{N} = 1$, $D = 4$ supergravity. In section 3 we use this analysis to prove that the event horizon must have a toroidal topology.

\section{Horizons}

\subsection{Supersymmetric horizons of non-supersymmetric black holes}

A starting point in the analysis of near-horizon geometries in the context of supersymmetry is the identification of the stationary Killing vector field of a black hole with a Killing vector field constructed as a Killing spinor bi-linear [22]. In such an investigation it is assumed that the Killing spinor bilinear near the horizon can be extended to a Killing vector on the spacetime. Both near horizon geometry and black hole spacetime are supersymmetric.

Here, we address a different situation, where one has an extremal black hole which

\footnote{This class includes the stationary Killing vector field of a black hole, see e.g. [20] for details.}
does not necessarily preserve any supersymmetry outside the horizon, but whose near-
horizon geometry is supersymmetric. In particular, we do not assume that the Killing
spinor bilinears of the near horizon geometry can be extended into the bulk spacetime.
Moreover, it no longer follows that the horizon is a Killing horizon of a Killing spinor
bilinear vector field. We remark that for a number of theories, with various asymptotic
conditions, it has been shown that black hole event horizons are Killing horizons, [2],
[23], [24], [25], [26], [14]. However, this result has not yet been established for the generic
\( \mathcal{N} = 1, D = 4 \) supergravity theory we consider here, so we shall simply assume that the
event horizons of the black holes we consider are Killing horizons.

Adapting Gaussian null coordinates [27] with respect to a stationary Killing vector
field of an extreme black hole and taking the near horizon limit, the bosonic fields of the
\( \mathcal{N} = 1, D = 4 \) supergravity can be written as

\[
ds^2 = 2du(dr + rh_i dy^i) - \frac{1}{2}r^2 \Delta du + \tilde{g}_{ij} dy^i dy^j,
\]
\[A^a = r\Phi^a du + B^a_i dy^i, \quad \phi^\alpha = \phi^\alpha(y),
\]

where the components of the 4-dimensional spacetime metric \( ds^2, \Delta, h \) and \( \tilde{g} \) are inde-
dependent of \( u, r \), and similarly for the components \( \Phi \) and \( B \) of the gauge potential \( A \). The scalar fields \( \phi \) depend only on \( y \). The horizon section \( S \) is given by the co-dimension two
subspace \( r = 0, u = \text{const} \) and it is required to be oriented, connected, compact without
boundary. The metric on \( S \) is \( \tilde{g} \). Observe that the horizon is Killing with respect to \( \partial_u \).

However, in the analysis that will follow \( \partial_u \) will not be identified with a Killing spinor
bi-linear.

It is most convenient to introduce a particular basis adapted to the Gaussian null
co-ordinates. This also enables one to simplify the solution of the KSEs and to make
optimal use of compactness of spatial cross sections of the horizon. The latter requirement
significantly constrains the spacetime geometry. The basis we shall use is given by

\[
e^+ = du, \quad e^- = dr + rh - \frac{1}{2}r^2 \Delta du, \quad e^i = e^i_j dy^j, \quad i, j = 1, 3,
\]

and the metric and gauge potential can be rewritten as

\[
ds^2 = 2e^+ e^- + \delta_{ij} e^i e^j
\]
\[A^a = r\Phi^a e^+ + B^a_i e^i, \quad \phi^\alpha = \phi^\alpha(y).
\]

The components of the spin connection associated with this basis are presented in Ap-
pendix A.

### 2.2 Killing spinor equations

Since we assume that the near horizon geometries are supersymmetric, the horizons must
be solutions of the KSEs of \( \mathcal{N} = 1, D = 4 \) supergravity. These equations have been solved
in all generality [4, 5]. However, these results are not directly applicable in our case as
the natural frame associated with near horizon geometries is different from that adapted
to supersymmetric solutions [4]. So to distinguish which of the supersymmetric solutions
are near horizon geometries some of the analysis must be repeated. Moreover, we have 
to impose that $S$ is compact and this condition is not included in the investigation of 
supersymmetric solutions.

The action and supersymmetry transformations of $\mathcal{N} = 1, D = 4$ supergravity that 
we shall use can be found in [1] and references within. Our notation follows that of [4] 
and [1], with some minor changes. In particular, the gravitino KSE is

\[ \nabla_\mu \epsilon_L + V_\mu \epsilon_L + \frac{i}{2} \epsilon \gamma^\mu W \gamma_\mu \epsilon_R = 0 \, , \tag{2.4} \]

the gaugino KSE is

\[ F^a_\mu \gamma^{\mu\nu} \epsilon_L - 2i \mu^a \epsilon_L = 0 \, , \tag{2.5} \]

and the KSE associated with the chiral multiplets is

\[ i \mathcal{D}_\mu \phi^\alpha \gamma_\mu \epsilon_R - e^{\frac{K}{2}} G^{\alpha\beta} \mathcal{D}_\beta \bar{W} \epsilon_L = 0 \, . \tag{2.6} \]

Here $\nabla$ is the spin connection of 4-dimensional spacetime, $K = K(\phi^\alpha, \phi^{\bar{\beta}})$ and $G_{\alpha\beta} = \partial_\alpha \partial_\beta K$ are the Kähler potential and Kähler metric of the sigma model manifold $S$, 
respectively, and $W = W(\phi^\alpha)$ is a holomorphic potential. Moreover,

\[ D_\alpha W = \partial_\alpha W + \partial_\alpha KW, \quad \mathcal{D}_\mu \phi^\alpha = \partial_\mu \phi^\alpha - A^a_\mu \xi_a \tag{2.7} \]

where $\xi_a$ are holomorphic Killing vector fields on $S$.

\[ F^a = dA^a - f^a_{\ bc} A^b \wedge A^c \, , \tag{2.8} \]

is the field strength of the gauge potential $A$ which arises from gauging isometries in $S$, 
where $f^a_{\ bc}$ are the structure constants. $\mu_a$ is the moment map defined by

\[ G_{\alpha\beta} \xi^\beta_a = i \partial_\alpha \mu_a \ . \tag{2.9} \]

The 1-form $V$ which enters into the gravitino KSE (2.4) is given by

\[ V_\mu = \frac{1}{4} (\partial_\alpha KW \mathcal{D}_\mu \phi^\alpha - \partial_\alpha KW \mathcal{D}_\mu \phi^{\bar{\alpha}}) \, . \tag{2.10} \]

The spinors $\epsilon_R, \epsilon_L$ are chiral spinors satisfying

\[ \gamma_5 \epsilon_L = \epsilon_L, \quad \gamma_5 \epsilon_R = -\epsilon_R \tag{2.11} \]

where $\gamma_5 = i\gamma_{0123}$. $\epsilon_R, \epsilon_L$ are related by

\[ \epsilon_R = C \ast \epsilon_L \tag{2.12} \]

where $C = -\gamma_{012}$ is the charge conjugation matrix. We will find it convenient to decom- 
pose spinors as

\[ \epsilon_L = \epsilon_{L^+} + \epsilon_{L^-}, \quad \epsilon_R = \epsilon_{R^+} + \epsilon_{R^-} \ , \tag{2.13} \]

where

\[ \gamma_+ \epsilon_{L^+} = \gamma_+ \epsilon_{R^+} = \gamma_- \epsilon_{L^-} = \gamma_- \epsilon_{R^-} = 0 \ , \quad \gamma_\pm = \frac{\pm \gamma_0 + \gamma_2}{\sqrt{2}} \ . \tag{2.14} \]
2.3 Solution of KSEs

To solve the KSEs we first decompose them along the light-cone and transverse directions and use the analyticity of the fields in the \( r \) and \( u \) coordinates. As a general rule the conditions which arise from the light-cone directions can be solved directly. This together with elements from spinorial geometry \[28\] as well as the compactness of \( S \) allows us to show that \( S \) is topologically \( T^2 \).

2.3.1 Gaugino

The components of the gauge field strength are

\[
F^a_{+} = -\Phi^a, \quad F^a_{+i} = r\left(-\partial_i \Phi^a + \Phi^a h_i - 2f^a_{bc} \Phi^b B^c_i\right),
\]

\[
F^a_{ij} = (dB^a)_{ij} - 2f^a_{bc} B^b_i B^c_j.
\]

(2.15)

On substituting into the gaugino KSE (2.5), one obtains

\[
(\Phi^a - iF^a_{12} - i\mu^a)\epsilon_{L+} = 0,
\]

(2.16)

and

\[
(-\Phi^a + iF^a_{12} - i\mu^a)\epsilon_{L-} + r\left(-\partial_i \Phi^a + h_i \Phi^a - 2f^a_{bc} \Phi^b B^c_i\right)\gamma_{-\gamma}^i \epsilon_{L+} = 0
\]

(2.17)

It is clear that in order for these equations to admit solutions other than \( \epsilon_L = 0 \), one must have

\[
\Phi^a = 0.
\]

(2.18)

Thus the remaining equations are

\[
(\mp F^a_{12} - \mu^a)\epsilon_{L\pm} = 0.
\]

(2.19)

If both \( \epsilon_{L\pm} \neq 0 \), then

\[
F^a = 0, \quad \mu^a = 0.
\]

(2.20)

Note that this does not mean that \( \mu^a \) vanishes identically. It simply vanishes on the space of solutions.

2.3.2 Gravitino

Using (2.18), we first integrate the + and − components of (2.4) to find

\[
\epsilon_{L+} = \eta_{L+}, \quad \epsilon_{L-} = r\left(\frac{1}{4} h_i \gamma_{-\gamma}^i \eta_{L+} - \frac{i}{2} e^\frac{K}{2} W \gamma_{-} C * \eta_{L+}\right) + \eta_{L-},
\]

(2.21)

where \( \eta_{L\pm} \) do not depend on \( r \). These in turn are given by

\[
\eta_{L-} = \tau_{L-}, \quad \eta_{L+} = u\left(\frac{1}{4} h_i \gamma_{+} \gamma_{+} \tau_{L-} - \frac{i}{2} e^\frac{K}{2} W \gamma_{+} C * \tau_{L-}\right) + \tau_{L+}.
\]

(2.22)
where \( \tau_{L\pm} \) do not depend on \( r \) and \( u \). One also finds the conditions

\[
\Delta + \frac{1}{4} h^2 - e^K |W|^2 = 0 ,
\]

(2.23)

\[
dh = 0 ,
\]

(2.24)

and

\[
\Delta h_i - \partial_i \Delta = 0 .
\]

(2.25)

The latter is a parallel transport equation for \( \Delta \). As a result, \( \Delta \) is either positive or negative. Moreover if \( \Delta \) vanishes at a point, it vanishes everywhere on \( S \).

Next consider the remaining components of (2.4), these imply that

\[
\tilde{\nabla}_i \eta_{L\pm} + \frac{1}{4} h_i \eta_{L\pm} + V_i \eta_{L\pm} + \frac{i}{2} e^{\frac{K}{2}} W \gamma_i C * \eta_{L\pm} = 0 ,
\]

(2.26)

where \( \tilde{\nabla}_i \) denotes the spin connection on the horizon section \( S \). One also obtains

\[
\left( \frac{1}{4} \tilde{\nabla}_j h_i \gamma^j - \frac{1}{8} h_i h_j \gamma^j + \frac{1}{2} e^K |W|^2 \gamma_i \right) \eta_{L+} - \left( \frac{i}{2} \tilde{\nabla}_i (e^{\frac{K}{2}} W) + i V_i e^{\frac{K}{2}} W \right) C * \eta_{L+} = 0 .
\]

(2.27)

Writing the above conditions in terms of \( \tau_{L\pm} \), one finds

\[
\tilde{\nabla}_i \tau_{L\pm} + \frac{1}{4} h_i \tau_{L\pm} + V_i \tau_{L\pm} + \frac{i}{2} e^{\frac{K}{2}} W \gamma_i C * \tau_{L\pm} = 0 ,
\]

(2.28)

and

\[
\left( \frac{1}{4} \tilde{\nabla}_j h_i \gamma^j - \frac{1}{8} h_i h_j \gamma^j + \frac{1}{2} e^K |W|^2 \gamma_i \right) \tau_{L\pm} - \left( \frac{i}{2} \tilde{\nabla}_i (e^{\frac{K}{2}} W) + i V_i e^{\frac{K}{2}} W \right) C * \tau_{L\pm} = 0 .
\]

(2.29)

The integrability conditions of either (2.26) or (2.28) imply that

\[
\pm \tilde{R}_S \tau_{L\mp} = 4 i e^{ij} \tilde{\nabla}_i V_j \tau_{L\mp} \pm 2 \Delta \tau_{L\mp} \pm \tilde{\nabla}^k h_k \tau_{L\mp} ,
\]

(2.30)

where \( \tilde{R}_S \) is the Ricci scalar of the horizon section \( S \).

Returning to the gaugino KSE (2.5) gives, in addition to (2.18), the following algebraic conditions

\[
(\mu^a \pm F^a_{12}) \tau_{L\pm} = 0 ,
\]

(2.31)

and

\[
\mu^a \left( \frac{1}{4} h_i \gamma^j \eta_{L+} - \frac{i}{2} e^{\frac{K}{2}} WC * \eta_{L+} \right) = 0 .
\]

(2.32)

Observe that if \( \tau_{L\pm} \neq 0 \), then \( F^a = \mu^a = 0 \) as in (2.20).
2.3.3 Chiral

For completeness, the chiral KSEs (2.6) imply

\[ iD_j \phi^\alpha \gamma^j C * \eta_{L\pm} - e^K G^\alpha\beta \bar{D}_\beta \bar{W} \eta_{L\pm} = 0 , \]  
(2.33)

and

\[ e^{\frac{K}{2}} \bar{W} D_j \phi^\alpha \gamma^j \eta_{L+} + ( - i h^j D_j \phi^\alpha + i e^K W G^\alpha\beta \bar{D}_\beta \bar{W} ) C * \eta_{L+} = 0 . \]  
(2.34)

2.3.4 Killing vector bi-linear

The Killing spinor \( \epsilon \) of the near horizon geometry is associated with a null 1-form \( Z \) which in turn gives rise to a Killing vector. From the results of [4], it is known that

\[ i_Z F^a = 0 , \quad i_Z \phi^\alpha = 0 , \]  
(2.35)

where we have used \( Z \) to denote both the 1-form and the associated vector. To continue, it is useful to compute \( Z \). For this, we set

\[ \tau_{L+} = a 1 , \quad \tau_{L-} = b e_{12} \]  
(2.36)

where \( a, b \) are \( r, u \)-independent complex functions. We also adopt a Hermitian basis \( e^1, e^{\dagger} \) for \( \mathcal{S} \), with respect to which

\[ \gamma_1 = \sqrt{2} e_1 \wedge , \quad \gamma_1 = \sqrt{2} e_1 \downarrow . \]  
(2.37)

The Killing spinor can be rewritten as

\[ \epsilon_{L+} = ( a + u ( \frac{1}{2} b h_1 + \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W \bar{b} ) ) 1 \]  
\[ \epsilon_{L-} = ( (1 + \frac{1}{2} \Delta ur) b + r ( - \frac{1}{2} a h_1 + \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W \bar{a} ) ) e_{12} . \]  
(2.38)

The 1-form bilinear can be computed using

\[ Z = \langle \Gamma_{12} e^A , \gamma_A \epsilon \rangle e^A , \]  
(2.39)

where \( \langle \cdot , \cdot \rangle \) is the standard Hermitian inner product. An explicit expression is given in appendix A. We have assumed that the near horizon geometry is supersymmetric. We furthermore assume that all gauge-invariant spinor bilinears constructed from the Killing spinor \( \epsilon \) are smooth and well-defined everywhere on the near-horizon geometry.

2.3.5 Einstein Equation

The Einstein equation is

\[ R_{MN} - 2G_{\alpha\beta} D_{(M} \phi^\alpha D_{N)} \phi^\beta - \text{Re} ( H_{ab} ) F^a_{M L} F^b_{N L} + 2g_{MN} \left( \frac{1}{8} \text{Re} ( H_{ab} ) F^a_{L_1 L_2} F^b_{L_1 L_2} \right) \]
\[-\frac{1}{4} \mu_a \mu^a - \frac{1}{2} e^K (G^{\alpha \beta} D_\alpha W D_\beta \bar{W} - 3|W|^2) = 0. \tag{2.40}\]

From the "+" component, one has
\[
\frac{1}{4} \tilde{\nabla}_i h_i - \frac{1}{4} h^2 - \frac{1}{2} \Delta + \frac{3}{2} e^K |W|^2 - \frac{1}{2} e^K G^{\alpha \beta} D_\alpha W D_\beta \bar{W} = 0. \tag{2.41}\]

If \(\Delta \neq 0\), then on using (2.25) and (2.23), one finds that (2.41) is equivalent to
\[
\frac{1}{4} \tilde{\nabla}^2 \Delta - \frac{1}{8} \Delta^{-1} \tilde{\nabla}_i \Delta \tilde{\nabla}^i \Delta = -\Delta^2 + \frac{1}{2} \Delta e^K G^{\alpha \beta} D_\alpha W D_\beta \bar{W}. \tag{2.42}\]

It then follows, as a consequence of the maximum principle, and compactness of \(S\) that one cannot have \(\Delta < 0\). So one must have
\[
\Delta \geq 0. \tag{2.43}\]

The \(ij\) components of the Einstein equations imply that
\[
\tilde{R}_S - 2 \mu_a \mu^a - 2 G_{\alpha \beta} D_i \phi^\alpha D^i \phi^\beta + 2 e^K |W|^2 = 0, \tag{2.44}\]

and
\[
\tilde{\nabla}_i h_j - \frac{1}{2} h_i h_j - 2 G_{\alpha \beta} D_i (\phi^\alpha D_j) \phi^\beta + 2 \left( \frac{1}{2} G_{\alpha \beta} D_i \phi^\alpha D_j \phi^\beta - \frac{1}{3} e^K G^{\alpha \beta} D_\alpha W D_\beta \bar{W} \right.
\]
\[+ e^K |W|^2) \delta_{ij} = 0. \tag{2.45}\]

In what follows, we shall use the conditions implied by the KSE and the Einstein equation, and the compactness of \(S\) to show that \(S\) is topologically \(T^2\).

### 2.3.6 Gauge Field equations

The gauge field equations are
\[
\nabla^M \left( \text{Re}(H_{ab}) F^b_{\ MN} \right) + f_{abc} F^b_{\ MN} A^c_{\ MN} + G_{\alpha \beta} \xi^\alpha_{\ a} D_N \phi^\beta + G_{\alpha \beta} \xi^\beta_{\ a} D_N \phi^\alpha = 0, \tag{2.46}\]

where \(f_{abc} = \text{Re}(H_{ad} f^{d}_{\ bc})\), and we have neglected the contribution from the Chern-Simons term, which vanishes for the magnetically charged near-horizon solutions.

As the sign of the flux \(F^a\) depends on whether \(\tau_{L \pm} \neq 0\), as a consequence of (2.31), one finds that if \(\tau_{L \pm} \neq 0\) then on substituting the remaining conditions on the near-horizon geometry into the gauge field equations, one obtains
\[
\mu_a h_i - \partial_{\alpha} \mu_a D_i \phi^\alpha - \partial_{\alpha} \mu_a D_i \phi^\alpha \pm i \epsilon_i^a \left( \partial_{\alpha} \mu_a D_j \phi^\alpha - \partial_{\alpha} \mu_a D_j \phi^\alpha \right) = 0 \tag{2.47}\]

where we note that in the holomorphic basis, \(\epsilon_{11} = -i\).
2.3.7 Scalar Field Equations

The scalar field equations are

$$G_{\alpha\bar{\beta}}(\nabla_M D^M \phi^\beta - (\partial_{\beta} \xi^\beta) A^a M D^M \phi^\beta) + \partial_{\bar{\beta}} G_{\alpha\bar{\beta}} D_M \phi^\beta D^M \phi^\beta$$

$$- \frac{1}{4} (\partial_{\alpha} \text{Re}(H_{ab})) F^a_{MN} F^{bMN} - \frac{1}{2} \partial_{\alpha} (\text{Re}(H^{ab}) \mu_a \mu_b)$$

$$- \partial_{\alpha} \left( e^K (G^{\sigma\bar{\beta}} D_\sigma W D_{\bar{\beta}} \bar{W} - 3|W|^2) \right) = 0$$  \hspace{1cm} (2.48)

where we have neglected the contribution from the Chern-Simons term, which vanishes for the magnetically charged near-horizon solutions under consideration here. On substituting the remaining conditions on the near-horizon geometry into the scalar field equations, one obtains

$$G_{\alpha\bar{\beta}} \left( \tilde{\nabla}^i D_i \phi^\beta - h^i D_i \phi^\beta - \partial_{\beta} \xi^\beta A^a_i D^i \phi^\beta \right) + \partial_{\bar{\beta}} G_{\alpha\bar{\beta}} D_i \phi^\beta D^i \phi^\beta - \mu^a \partial_{\alpha} \mu_a$$

$$- \partial_{\alpha} \left( e^K (G^{\sigma\bar{\beta}} D_\sigma W D_{\bar{\beta}} \bar{W} - 3|W|^2) \right) = 0$$  \hspace{1cm} (2.49)

3 Horizon topology

To establish that the topology of $S$ is $T^2$ several special cases have to be considered. We shall begin by assuming that $\tau_{L+} \neq 0$ and $\tau_{L-} \neq 0$.

3.1 Solutions with $\tau_{L+} \neq 0$ and $\tau_{L-} \neq 0$

If $\tau_{L+} \neq 0$ and $\tau_{L-} \neq 0$, then the gaugino KSE and the integrability conditions (2.30) imply that

$$F^a = \mu^a = dV = 0 \hspace{1cm} (3.1)$$

Moreover, (2.30) can be written as

$$\tilde{R}_S = 2\Delta + \tilde{\nabla}^k h_k \hspace{1cm} (3.2)$$

Since $\Delta \geq 0$, the Euler number of $S$ is not negative and so $S$ is topologically either $T^2$ or $S^2$.

Now if $\Delta = 0$, the Euler number of $S$ vanishes and so $S$ is topologically $T^2$. It remains to investigate the case $\Delta > 0$. Since the field strength $F^a = 0$ and $S$ is topologically $S^2$, the gauge connection is trivial and so we set $A = 0$. As we have mentioned the Euler number is positive. So to avoid contradiction with (2.44), some of the scalars $\phi$ must have a non-trivial dependence on the coordinates of $S$.

The dependence of $\phi$ on the coordinates of $S$ is restricted by supersymmetry. In particular from the results of [4],

$$i_Z d\phi = 0 \hspace{1cm} (3.3)$$
where $Z$ is the 1-form bilinear, see appendix A. $\phi$ depends only on the coordinates of $S$ but the components of $Z$ have $u$ and $r$ polynomial dependence. As a result (3.3) gives rise to a system of conditions on $\phi$, one for every polynomial $u, r$-component of $Z$. If two such conditions are linearly independent, (3.3) would imply that $\phi$ is constant leading to a contradiction. So to maintain that some of the scalars have a non-trivial dependence on the coordinates of $S$, all $u, r$ components of $Z$ along $S$ must be linearly dependent. A straightforward calculation reveals that this is the case provided that
\begin{equation}
\alpha \left( \frac{1}{2} b h_1 - \frac{i}{\sqrt{2}} e^{K} W b \right) \tag{3.4}
\end{equation}
is a real valued function. Since $\alpha \neq 0$ and $b \neq 0$, (3.4) together with (2.26) imply that
\begin{equation}
\alpha^{-1} \left( \frac{1}{2} b h_1 + \frac{i}{\sqrt{2}} e^{K} W b \right) \tag{3.5}
\end{equation}
is a real constant. As
\begin{equation}
\eta_{L+} = \left( u \left( \frac{1}{2} b h_1 + \frac{i}{\sqrt{2}} e^{K} W b \right) + \alpha \right) \tag{3.6}
\end{equation}
it follows that we may, without loss of generality, set $\alpha = 0$, i.e. $\gamma_{L+} = 0$, by making a co-ordinate transformation of the form $u = u' + c$ for an appropriately chosen real constant $c$. Note that this transformation preserves the form of the near-horizon metric. Therefore for $\Delta \neq 0$, it suffices to consider that either $\gamma_{L+}$ or $\gamma_{L-}$ vanishes.

### 3.2 Solutions for either $\tau_{L+} = 0$ or $\tau_{L-} = 0$ and $\Delta \neq 0$

To investigate this case define
\begin{equation}
\kappa = \begin{cases} 
\frac{-1}{2} |b|^2 h - \frac{i}{\sqrt{2}} e^{K} W b^2 e^1 + \frac{i}{\sqrt{2}} e^{K} W h^1, & \text{if } \gamma_{L-} \neq 0 \\
\Delta^{-1} \left( \frac{1}{2} |a|^2 h + \frac{i}{\sqrt{2}} e^{K} W a^2 e^1 - \frac{i}{\sqrt{2}} e^{K} W a^1 e^1 \right), & \text{otherwise} .
\end{cases} \tag{3.7}
\end{equation}

Observe that the components of $\kappa$ correspond to the $u^1 r^0$ component of $Z_1, Z_1$ from (A.2) if $\gamma_{L-} \neq 0$, and an appropriately chosen re-scaling of the $u^0 r^1$ component of $Z_1, Z_1$ if $\gamma_{L-} = 0$, respectively. Moreover, by construction, $\kappa$ is not identically zero. As $Z$ is a Killing vector field on the near horizon geometry, one can show that $\kappa$ is an isometry of the horizon section $S$.

To continue consider first the case $\gamma_{L+} = 0, \gamma_{L-} \neq 0$. Note in particular that
\begin{equation}
\frac{i}{\sqrt{2}} \alpha \frac{Z}{\alpha} = 2 \sqrt{2} (1 + \frac{1}{2} \Delta ur)^2 |b|^2 - \sqrt{2} r^2 u^2 \Delta (|b|^2 (-\frac{1}{8} h^2 - \frac{i}{2} e^{K} |W|^2)) \\
+ \frac{i}{2 \sqrt{2}} b^2 e^{K} W h_1 - \frac{i}{2 \sqrt{2}} \bar{b}^2 e^{K} W h_1 \tag{3.8}
\end{equation}

As $Z$ is a smooth 1-form and $\frac{\partial}{\partial u}$ is a smooth Killing vector field in the near-horizon spacetime, it follows that this scalar is also smooth. On evaluating it on the horizon section, $r = u = 0$, it follows that $|b|^2$ is a smooth function on $S$. In addition, as $\kappa$ is
obtained from the pull-back of \( \mathcal{L}_{\frac{\partial}{\partial u}} Z \) on \( S \), it follows that \( \kappa \) is a smooth 1-form on \( S \). Moreover, (2.26) implies that
\[
d(\Delta |b|^2) = -\Delta \kappa . \tag{3.9}
\]
As \( \kappa \) is Killing, we further find
\[
\hat{\nabla}_i \hat{\nabla}_j (\Delta |b|^2) = \Delta^{-1} \hat{\nabla}_{(i} \Delta \hat{\nabla}_{j)} (\Delta |b|^2) . \tag{3.10}
\]
Assuming that \( S \) is not topologically \( T^2 \), and so \( S \) has a non-vanishing Euler number, it follows that \( \kappa \) must vanish at some point \( P \in S \). Using (2.23), this implies that \( b = 0 \) at \( P \). Furthermore (3.9) and (3.10) then imply that all covariant derivatives of \( \Delta |b|^2 \) must also vanish at \( P \). Assuming that \( \Delta |b|^2 \) is analytic\(^4\) on \( S \), it follows that \( b = 0 \) everywhere. However, this leads immediately to a contradiction, as it implies that the Killing spinor must vanish everywhere.

Next, consider the case \( \tau_{L+} \neq 0, \tau_{L-} = 0 \). In this case, we have
\[
i \partial_\varphi Z = -2\sqrt{2}|a|^2 . \tag{3.11}
\]
As \( Z \) is a smooth 1-form and \( \partial_\varphi \) is a smooth vector field in the near-horizon spacetime, it follows that \( |a|^2 \) is a smooth function on \( S \). In addition, as \( \kappa \) is obtained from a linear combination of the pull-back of \( \mathcal{L}_{\frac{\partial}{\partial u}} Z \) to \( u = r = 0 \) and \( |a|^2 h \), it follows that \( \kappa \) is a smooth 1-form on \( S \). Moreover, (2.26) implies that
\[
d(|a|^2) = \Delta \kappa . \tag{3.12}
\]
As \( \kappa \) is Killing, we further find
\[
\hat{\nabla}_i \hat{\nabla}_j (|a|^2) = \Delta^{-1} \hat{\nabla}_{(i} \Delta \hat{\nabla}_{j)} (|a|^2) . \tag{3.13}
\]
Repeating a similar argument to the one we have used for the previous case, one finds that, unless \( S \) is topologically \( T^2 \), \( \tau_{L+} = 0 \). Thus if \( \Delta \neq 0 \), then one concludes that \( S \) is topologically \( T^2 \).

### 3.3 Solutions for either \( \tau_{L+} = 0 \) or \( \tau_{L-} = 0 \) and \( \Delta = 0 \)
There are two separate cases to consider depending on whether the components of \( Z \) along \( S \) vanish or not. To continue denote the components of \( Z \) along \( S \) with \( \tilde{Z} \).

#### 3.3.1 \( \tilde{Z} \neq 0 \)
In this case observe that the supersymmetry conditions (2.35) can be rewritten as
\[
i_Z F^a = i_{\tilde{Z}} F^a = 0 , \quad i_Z D\phi^a = i_{\tilde{Z}} D\phi^a = 0 . \tag{3.14}
\]
Since \( F^a \) has one non-vanishing spacetime component, if \( \tilde{Z} \neq 0 \), then one concludes that
\[
F^a = 0 . \tag{3.15}
\]
\(^4\)This result may hold for \( \Delta |b|^2 \) smooth but we have not been able to extend the proof.
Similarly, if \( \tilde{Z} \neq 0 \), one concludes that the scalars depend on only one coordinate and so
\[
dV = 0 . \tag{3.16}
\]
Using the integrability conditions (2.30) for either \( \tau_{L+} \) or \( \tau_{L-} \), one finds that
\[
\tilde{R}_S = \tilde{\nabla}^k h_{kk} . \tag{3.17}
\]
Thus the Euler number of \( S \) vanishes and so \( S \) is topologically \( T^2 \).

### 3.3.2 Solutions with \( \tau_{L+} = 0 \) and \( \tilde{Z} = 0 \)

Using \( \tilde{Z} = 0 \), one finds that the vector field associated to the 1-form bilinear (A.2) is
\[
Z = 2\sqrt{2}|b|^2 \frac{\partial}{\partial r} . \tag{3.18}
\]
Note that as \( g(Z, \frac{\partial}{\partial u}) = 2\sqrt{2}|b|^2 \), it follows that \( |b|^2 \) is a smooth function on \( S \). Furthermore, the requirement that \( Z \) is Killing gives
\[
|b|^2 h + d|b|^2 = 0 . \tag{3.19}
\]
This condition implies that if \( b = 0 \) at any point in \( S \), then \( b = 0 \) everywhere. Hence, for the solution to be supersymmetric, we take \( b \neq 0 \) everywhere. Next, note that (2.41) can be rewritten as
\[
\tilde{\nabla}^i (|b|^{-1}) = |b|^{-1} e^K G^{\alpha\beta} D_\alpha W D_\beta \tilde{W} . \tag{3.20}
\]
On integrating this expression over \( S \), the contribution from the LHS vanishes, and one obtains the conditions
\[
h = 0, \quad D_a W = 0, \quad W = 0 , \tag{3.21}
\]
and \( |b| \) is a nonzero constant.

Using (3.21), (2.33) implies that
\[
D_1 \phi^\alpha = 0 , \tag{3.22}
\]
ie the scalar fields up to a gauge transformation are holomorphic. In turn, the gauge equation (2.47) implies that
\[
\partial_\alpha \mu_a D_1 \phi^\alpha = 0 , \tag{3.23}
\]
and the scalar equation (2.49) implies
\[
\mu^a \partial_\alpha \mu_a = 0 . \tag{3.24}
\]
Furthermore (2.44) implies
\[
\tilde{R}_S - 2 \mu_a \mu^a - 2 G_{\alpha\beta} D_1 \phi^\alpha D_1 \phi^\beta = 0 . \tag{3.25}
\]
Thus provided that inner product Re\( H \), (2.46), of the gauge group and the Kähler metric are positive definite, the Euler number of \( S \) is non-negative. This is a mild assumption on the couplings. Therefore \( S \) is topologically either \( T^2 \) or \( S^2 \). In particular \( S \) is topologically \( T^2 \) if, and only if, \( \mu^a = 0 \) and \( D_1 \phi^\alpha = 0 \), ie if and only if the gauge field vanishes and the scalars are constant. In turn this implies that \( S \) is isometric to \( T^2 \). For all these horizons \( \Delta = h = 0 \), hence the near horizon geometry is \( \mathbb{R}^{1,1} \times T^2 \) or \( \mathbb{R}^{1,1} \times S^2 \) though the metric on \( S^2 \) may not be the round one.
3.3.3 Solutions with $\tau_{L_+} = 0$ and $\tilde{Z} = 0$

Using $\tilde{Z} = 0$, one finds that the vector field associated to the 1-form bilinear (A.2) is

$$Z = -2\sqrt{2}|a|^2 \frac{\partial}{\partial u}. \quad (3.26)$$

Moreover $|a|^2$ is constant because $Z$ is Killing. Thus $a \neq 0$ everywhere on the spacetime. Note that the condition $\tilde{Z} = 0$ implies that

$$\frac{1}{2}|a|^2 h_1 + \frac{i}{\sqrt{2}} \xi^\alpha \bar{W} a^2 = 0, \quad (3.27)$$

and the chiral KSE conditions (2.33) and (2.34) are equivalent to

$$\sqrt{2}i\bar{a} D_1 \phi^\alpha - ae^{K/2} G^{a\bar{b}} D_{\bar{b}} \bar{W} = 0. \quad (3.28)$$

In addition, the gauge field equation (2.47) gives

$$\mu_a h_1 - 2 \partial_\alpha \mu_a D_1 \phi^\alpha = 0. \quad (3.29)$$

On comparing (2.27) with (2.45) and after some computation, one obtains the condition

$$WA_i^a (\xi_a \partial_\alpha K + \xi_\alpha \partial_a K) = 0. \quad (3.30)$$

Observe that the above equation is satisfied provided that the Kähler potential is invariant under the isometries generated by $\xi$.

Next observe that (3.27) and (3.28) imply that

$$\bar{D}_1 \phi^\alpha - \frac{1}{2} h_1 W^{-1} G^{a\bar{b}} D_{\bar{b}} W = 0. \quad (3.31)$$

Using this and other conditions derived from the KSEs, one can show after some computation that the scalar equation (2.49) can be rewritten as

$$\mu^a \partial_\alpha \mu_a = 0. \quad (3.32)$$

On combining (3.29) with (3.32), one obtains

$$\mu^a \mu_a h = 0. \quad (3.33)$$

Since the kinetic term of the gauge fields is canonical, i.e., the inner product $\text{Re}H$ in (2.46) is positive definite, either $\mu^a = 0$ for all $a$, or $h = 0$.

Suppose first that there exists some $\mu^a \neq 0$. Then $h = 0$ implies the conditions

$$W = 0, \quad D_a W = 0, \quad (3.34)$$

and the conditions from the chiral, gauge and scalar field and KSEs simplify to

$$\bar{D}_1 \phi^\alpha = 0, \quad \partial_\alpha \mu_a \bar{D}_1 \phi^\alpha = 0, \quad \mu^a \partial_\alpha \mu_a = 0. \quad (3.35)$$
These conditions are identical to those derived in the previous section with the only difference that here the scalar fields, up to a gauge transformation, are antiholomorphic instead of holomorphic. As a result (2.44) implies

$$\tilde{R}_S - 2\mu_a\mu^a - 2G_{\alpha\beta}D_1\phi^\alpha D_1\phi^{\bar{\beta}} = 0.$$  

(3.36)

As in the previous case provided that inner product Re$H$, (2.46), of the gauge group and the Kähler metric are positive definite, the Euler number of $S$ is non-negative. Therefore $S$ is topologically either $T^2$ or $S^2$. In particular $S$ is topologically $T^2$ if, and only if, $F = \mu = 0$ and the scalars are constant. In turn this implies that $S$ is isometric to $T^2$. Again for all these horizons $\Delta = h = 0$, and so the near horizon geometry is $\mathbb{R}^{1,1} \times T^2$ or $\mathbb{R}^{1,1} \times S^2$ though the metric on $S^2$ may not be the round one.

Alternatively, suppose that $\mu^a = 0$ for all $a$. Then $F^a = 0$ for all $a$, and under the assumption we have made one can without loss of generality work locally in a gauge for which $A^a = 0$. Note that in such a case, the metric and scalars satisfy the field equations obtained from coupling gravity to scalar matter with a scalar potential. In particular, this type of Lagrangian was considered in the analysis of [14], in which it was shown that if one assumes that the spacetime metric and the scalars are analytic, then the horizon section $S$ admits a rotational isometry $\kappa$. If we assume sufficient conditions on the metric and scalars such that the rigidity theorem of [14] holds here \footnote{It is not a priori clear that the scalars are globally well-defined and analytic. However, in the analysis of [14], it appears that only analyticity of the metric is required in order to construct a preferred set of Gaussian Null co-ordinates.}, then one finds that the Lie derivative of the scalars with respect to $\kappa$ vanishes; i.e. the scalars depend only on one co-ordinate. This in turn implies that $dV = 0$, and hence from (2.30) it follows that the Euler number of $S$ vanishes, so $S$ is topologically $T^2$.

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Appendix A  Conventions and bilinear

The non-vanishing components of the spin connection associated with the basis (2.2) are

$$\begin{align*}
\Omega_{-,+i} &= -\frac{1}{2}h_i, & \Omega_{+,--} &= -r\Delta, & \Omega_{+,++} &= r^2\left(\frac{1}{2}\Delta h_i - \frac{1}{2}\partial_i\Delta\right), \\
\Omega_{+,--} &= -\frac{1}{2}h_i, & \Omega_{+,ij} &= -\frac{1}{2}rdh_{ij}, & \Omega_{i,++} &= \frac{1}{2}h_i, \\
\Omega_{i,++} &= -\frac{1}{2}rh_{ij}, & \Omega_{i,jk} &= \tilde{\Omega}_{i,jk},
\end{align*}$$

(A.1)

where $\tilde{\Omega}$ denotes the spin-connection of the horizon section $S$ in the $e^i$ basis.
To find the Killing spinor bilinear form $Z$, we use $\Gamma_{12}1 = e_{12}$, $\Gamma_{12}e_1 = -e_2$ and $\Gamma_{12}e_2 = e_1$, and (2.36). Then, one obtains

$$Z_+ = \frac{1}{\sqrt{2}} r^2 (|a|^2 \left( \frac{1}{2} h^2 + 2 e^K |W|^2 \right) + \sqrt{2} i a^2 h_1 e^{\frac{K}{2}} W - \sqrt{2} i a^2 h_1 e^{\frac{K}{2}} W)$$

$$+ \sqrt{2} r \left( b(-\bar{a} h_1 - \sqrt{2} i e^{\frac{K}{2}} Wa) + \bar{b}(-a h_1 + \sqrt{2} i e^{\frac{K}{2}} W\bar{a}) \right) + 2 \sqrt{2} |b|^2$$

$$+ 2 \sqrt{2} u \left( \frac{1}{4} r^2 \Delta \left( b(-\bar{a} h_1 - \sqrt{2} i e^{\frac{K}{2}} Wa) + \bar{b}(-a h_1 + \sqrt{2} i e^{\frac{K}{2}} W\bar{a}) \right) + r \Delta |b|^2 \right)$$

$$+ \frac{1}{\sqrt{2}} r^2 \Delta |b|^2 u^2$$

$$Z_- = -2 \sqrt{2} |a|^2 + 2 \sqrt{2} u \left( -\frac{1}{2} ab h_1 - \frac{1}{2} \bar{a} b h_1 + \frac{i}{\sqrt{2}} a e^{\frac{K}{2}} W b - \frac{i}{\sqrt{2}} \bar{a} e^{\frac{K}{2}} W \bar{b} \right)$$

$$+ 2 \sqrt{2} u^2 \left( |b|^2 \left( -\frac{1}{8} h^2 - \frac{1}{2} e^K |W|^2 \right) + \frac{i}{2 \sqrt{2}} b^2 e^{\frac{K}{2}} W h_1 - \frac{i}{2 \sqrt{2}} \bar{b}^2 e^{\frac{K}{2}} W \bar{h}_1 \right)$$

$$Z_1 = -2 \sqrt{2} a \bar{b} + 2 \sqrt{2} r \left( \frac{1}{2} |a|^2 h_1 + \frac{i}{\sqrt{2}} e^{\frac{K}{2}} Wa^2 \right)$$

$$+ 2 \sqrt{2} u \left( -\frac{1}{2} |b|^2 h_1 - \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W b^2 \right)$$

$$+ r \left( -\frac{1}{2} \Delta a \bar{b} + \left( \frac{1}{2} a h_1 + \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W a \right) \left( -\bar{a} h_1 + \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W \bar{a} \right) \right)$$

$$+ \sqrt{2} r \Delta u^2 \left( -\frac{1}{2} |b|^2 h_1 - \frac{i}{\sqrt{2}} e^{\frac{K}{2}} W b^2 \right)$$

$$+ 2 \sqrt{2} \Delta |b|^2 u^2$$

$$+ \sqrt{2} r^2 |b|^2 u^2$$

$$+ \sqrt{2} r^2 \Delta |b|^2 u^2$$

$$+ \frac{1}{\sqrt{2}} r^2 \Delta |b|^2 u^2$$

and $Z_1 = Z_1$.

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