Influence of body size on rebound height after shock interaction

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Abstract. The mechanorheological model was used to study the relationship of the body size and the rebound height after impact interaction. With an increase in the diameter of a spherical body, the body mass and the radius of surface curvature increase. Therefore, effects of both factors and each separate factor were evaluated. The study identified that with a simultaneous increase in the body mass and the radius of curvature, the rebound height decreases. An increase in the body mass has a slight effect on the rebound height. For the elastic plastic model, an increase in the body mass causes an increase in plastic deformations and a decrease in the rebound height. For the viscoelastic model, an increase in the body mass causes a slight increase in the rebound height. An increase in model plasticity decreases the rebound height at various values of viscoelastic plastic parameters of the model due to the fact that a lot of impact energy is spent on plastic deformations rather than on the elastic rebound of the body. The research results showed that the rebound height depends on the size of the impacting body.

1. Introduction
Machines and various mechanical systems operate under dynamic loading conditions. These can be shock or re-variable loads (e.g., vibration loads). It is necessary to improve reliability, efficiency and vibration protection of equipment and structures. These tasks can be solved by studying dynamic processes and identifying main patterns.

The mechanical shock and dynamic interaction of bodies are characteristic of machine building equipment. Many studies deal with these issues [1,2,3,4]. Even at the design stage, it is important to evaluate constructive solutions by modeling shock and other dynamic processes [1].

Among the priority machine building tasks are improvement of reliability and durability of equipment. Stress-strain states of interacting bodies are analyzed using mathematical modeling and numerical methods for solving contact problems of mechanics [5]. These problems can be solved through development of operation models and algorithms.

Important and relevant tasks are vibration protection and vibration isolation [3,6,7,8,9,10]. Development of mathematical models of the dynamic processes is an important part of the problems to be solved.

One more area of application of the mathematical modeling method is identification of rational parameters and operating modes of equipment, solution of automatic control issues [11]. This is especially important for analyzing the dynamics of interaction of mechanical systems.
The behavior of various machine parts and equipment (pump rotors, cutters, grinding tools) is modelled and studied under dynamic interaction [12,13,14]. Thus, engineering technologies can be developed by modeling and studying dynamic processes in the mechanical systems.

Energy losses during body deformations influence the dynamics of impact interaction of bodies. High deformation speed rates cause energy dissipation. Significant losses of impact energy occur during the plastic deformation of bodies. As a result, the amount of energy spent on the elastic rebound of the body significantly decreases. Mechanical properties of materials, impact speed, and sizes of the interacting bodies influence energy losses. This phenomenon can have both positive and negative effects.

Thus, the issue of modeling and studying energy dissipation processes is crucial. The mechanorheological approach is widely used for developing mathematical models. It is applied to develop a mechanical structure of the model. A large number of researchers deal with this issue [15-19].

The present paper studies the effect of the size (diameter) of an impacting spherical body on the rebound height. For the research purpose, the viscoelastic plastic mechanorheological model was used.

Similar researches have been already carried out [20,21]. The experimental studies aimed at identifying the influence of the rheological properties of the material of the supporting surface, sizes of the spherical body and initial parameters of the impact interaction (drop height) on the dynamics of impact process. In those studies, a simpler and less accurate viscoelastic model was used. For the
purpose of the present research, a more advanced elastic-viscous-plastic model was used [22]. The present model has a plastic element (a shear element) describing plastic deformations. Based on the research results, the model control method was developed [23,24].

The model consists of two units: viscoelastic unit $K_1 - C$ and elastic plastic unit $K_2 - f_2$ (figure 1).

The mass of the spherical body is concentrated in inertia element $m_1$. The mass of element $m_2$ is negligibly small ($m_2 \rightarrow 0$). This element was introduced for developing the mathematical model with two second order differential equations.

At the initial interaction stage, only elastic deformations occur. Therefore, only the viscoelastic unit $K_1 - C$ becomes deformed. It describes elastic deformations and energy losses caused by energy dissipation. When the dynamic force $F_{ST}$ has targeted values, the viscoelastic model is transformed into the viscoelastic plastic model. The unit $K_2 - f_2$ comes into operation. At this stage, plastic (residual) deformations occur. By modifying the values of $F_{ST}$, one can create various relations between elastic and plastic deformations of the model.

The maximum force of impact interaction $N_{MAX}$ corresponds to the unloading stage. The unloading process is modeled by the viscoelastic unit describing elastic deformations.

2. Purpose statement
When carrying out previous studies [20,21] by modelling the real impact process, the influence of the rheological properties of the material of the supporting surface and the size of the impacting spherical body on the rebound height was studied. It was identified that an increase in the diameter of a spherical body increases the rebound height.

For the research purpose, we used an advanced viscoelastic plastic mechanorheological model taking into account the influence of plasticity on the dynamics of impact interaction. The dynamics of the model movement was controlled using the developed methods [14,15].

The aim of the research was to study effects of various factors on the rebound height of a spherical body. When carrying out the research, viscoelastic plastic parameters of the model were changed in order to assess their influence on the dynamics of impact interaction. Spherical bodies of different diameters (9.5; 14.25; 19 mm) were modeled. An increase in the body size increases the body mass and the radius of surface curvature. Both factors influence the dynamics of impact interaction. Therefore, the influence of two factors at the same time and the influence of each factor separately were studied.

3. Theoretical background
The mathematical model is a system of equations connected by special conditions. The differential equation of the motion of an elastic-viscous model can be written as:

$$m_1 \ddot{y}_1 + C_1 \dot{y}_1 \ddot{y}_1 + K_1 y_1^{3/2} = -m_1 g$$

The motion of viscoelastic plastic model is described by two differential equations:

$$m_1 \ddot{y}_1 + C_1 (y_1 - y_2) (\dot{y}_1 - \dot{y}_2) + K_1 (y_1 - y_2)^{3/2} = -m_1 g$$

$$m_2 \ddot{y}_2 + K_2 y_2^{3/2} + f_2 \dot{y}_2 + C_1 (y_2 - y_1) (\dot{y}_2 - \dot{y}_1) + K_2 (y_2 - y_1)^{3/2} = -m_2 g + F_{ST}$$

where $y_1, y_2, \dot{y}_1, \dot{y}_2$ are the shift and speed of mass $m_1$ and mass $m_2$; $K_1$ is the ratio of stiffness of the elastic element of the viscoelastic model unit; $C$ is the ratio of viscosity of a viscous element of the viscoelastic model unit; $F_{ST}$ is the force corresponding to plastic deformations; $K_2$ is the ratio of stiffness of an elastic element of the elastic-plastic unit; $f_2$ is the ratio of shift of the elastic plastic unit or the ratio of ductility.
The shift from the viscoelastic model to the viscoelastic plastic model occurs when the force of normal response \( N = C_1 y_1 y_1^{1/2} \) has the target value of \( F_{ST} \).

The elastic element \( K_1 \) of the model has non-linear characteristics [16-18]. Viscous resistances \( F_{DIS} \) were assumed to be proportional to speed and elastic deformation values: \( F_{DIS} = C_1 y_1 y_1 \) [23]. This is an advanced model. In previous studies [20,21], viscous resistances \( F_{DIS} \) were assumed proportional to the elastic deformation value: \( F_{DIS} = C_1 y_1 \).

Equations (1) - (3) can be solved by the Runge-Kutta method. Based on the mathematical model, a special research program was developed.

4. Analysis of research results
The impact interaction process was modelled using steel spherical bodies of three diameters: 9.5; 14.25; 19 mm. The radii of curvature of the spherical surface were 4.75; 7.125; 9.5 mm respectively. The body masses were 3.5; 11.82; 28.01 g respectively. Several calculation variants were analyzed. Variant 1: \( F_{ST} = N_{\max} \) – only elastic deformations occur. Variant 2: \( F_{ST} = 2 N_{\max} / 3 \) Variant 3: \( F_{ST} = N_{\max} / 3 \). Variant 4: \( F_{ST} = 0 \) – plastic and elastic deformations occur since the beginning of the impact interaction. The relative height of rebound of the impacting spherical body \( dh = h_1 / h_2 \) (\( h_1 \) - rebound height, \( h_2 \) - drop height) was used as an example of the impact interaction. Model parameters \( K_1, K_2, f_2, C \) changed.

Figures 2 and 3 show the dependence of the rebound height on the diameter of the spherical body at various values of \( F_{ST} \). The simultaneous influence of two factors (body mass \( m_1 \) and radius of curvature \( R \)) were taken into account.

![Figure 2](image)

**Figure 2.** Dependence of the rebound height \( dh \) on the body diameter \( d \) at \( C - \text{min} \).

Figure 2 shows that the viscous parameter of the model \( \dot{C}_1 \) has a minimum value. Figure 3 shows that the viscous parameter of the model \( \dot{C}_2 \) has a maximum value. Other model parameters changed in a similar way.
Figures 4 and 5 show the graphs describing separate effects of body mass $m_1$ and radius of curvature $R$ on the rebound height. The value of the viscous model parameter $C$ was average.

The analysis of the results allows for the following conclusions. With a simultaneous increase in two factors (body mass and radius of curvature), the rebound height decreases (figures 2 and 3). However, the results of the previous studies [6, 7] showed that an increase in the body size increases the rebound height. Modification of the other impact interaction parameters (time, force) is identical in both models. Differences might be due to the fact that the elastic-viscous-plastic model is more accurate and reliable.

Figure 3. Dependence of the rebound height $dh$ on the body diameter $d$ at $C – \text{max}$.  

Figure 4. Dependence of the rebound height $dh$ on the body diameter $d$ at $R – \text{const}$. 
An increase in one of the factors (the body mass at $R-\text{const}$ (figure 4) or the radius of surface curvature at $m_1-\text{const}$ (figure 5)) has different effects on the rebound height. In general, an increase in the body mass has a slight effect on the rebound height. For the elastic plastic model ($F_{ST}=0$), an increase in the body mass increases plastic deformations and decreases the rebound height. For the viscoelastic model ($F_{ST}=N_{max}$), an increase in the body mass causes an insignificant increase in the rebound height.

**Figure 5.** Dependence of the rebound height $dh$ on the body diameter $d$ at $m_1-\text{const}$.

**Figure 6.** The rebound height of the body of different diameters
An increase in the radius of curvature decreases the rebound height. An increase in plasticity of the model ($F_{ST}=N_{max} \rightarrow F_{ST}=0$) decreases the rebound height at various values of model parameters $K_1$, $K_2$, $f_2$, $C$ due to the increasing volume of energy spent on plastic deformations of the bodies and the decreasing volume of energy spent on the elastic rebound of the impacting body.

Figure 6 presents experimental data confirming the results of theoretical studies.

5. Conclusion

Thus, the study on impact interaction of a spherical body with a surface carried out using a mechanorheological viscoelastic plastic model shows that the rebound height depends on the interacting body size. This is due to the influence of the body mass and the radius of curvature on the dynamics of body movement and deformation. These factors have different effects on the rebound height. Proper consideration of the factors influencing the dynamics of impact processes will improve modelling and research accuracy and reliability for various dynamic processes.

6. References

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