Modelling clusters of galaxies by $f(R)$ gravity

S. Capozziello, E. De Filippis and V. Salzano

Department of Science, University of Naples Federico II and INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cinthia, Edificio N, 80126 Napoli, Italy

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Abstract

We consider the possibility that masses and gravitational potentials of galaxy cluster, estimated at X-ray wavelengths, could be explained without assuming huge amounts of dark matter, but in the context of $f(R)$ gravity. Specifically, we take into account the weak field limit of such theories and show that the corrected gravitational potential allows to estimate the total mass of a sample of 12 clusters of galaxies. Results show that such a gravitational potential provides a fair fit to the mass of visible matter (i.e. gas + stars) estimated by X-ray observations, without the need of additional dark matter while the size of the clusters, as already observed at different scale for galaxies, strictly depends on the interaction lengths of the corrections to the Newtonian potential.

Key words: galaxies: clusters: general – cosmology: theory – dark matter.

1 INTRODUCTION

Since the pioneering work by Zwicky (1933), the problems of high mass-to-light ratios of galaxy clusters and of the rotation curves of spiral galaxies have been faced by asking for huge amounts of unseen matter in the framework of the Newtonian theory of gravity. It is interesting to stress the fact that Zwicky addressed such an issue dealing with missing matter and not with dark matter.

Later on, several versions of the cold dark matter (CDM) model have been built starting from the assumption that a large amount of non-baryonic matter (i.e. matter non-interacting with the electromagnetic radiation) could account for the observations in the framework of the standard Newtonian dynamics. Besides, by adding a further ingredient, the cosmological constant $\Lambda$ (Carroll, Press & Turner 1992; Sahni & Starobinski 2000), such a model (now $\Lambda$CDM) has become the new cosmological paradigm usually called the concordance model.

In fact, high-quality data coming from the measurements of cluster properties such as the mass, the correlation function and the evolution with redshift of their abundance (Eke et al. 1998; Viana, Nichol & Liddle 2002; Bahcall & Bode 2003; Bahcall et al. 2003), the Hubble diagram of Type Ia supernovae (Riess et al. 2004; Astier et al. 2006; Clocchiati et al. 2006), the optical surveys of large-scale structure (Cole et al. 2005; Eisenstein et al. 2005; Pope et al. 2005), the anisotropies of the cosmic microwave background (de Bernardis et al. 2000; Spergel et al. 2003), the cosmic shear measured from weak lensing surveys (van Waerbeke et al. 2001; Refregier 2003) and the Lyman $\alpha$ forest absorption (Croft, Hu & Dave 1999; McDonald et al. 2005) are evidences towards a spatially flat Universe with a subcritical matter content and undergoing a phase of accelerated expansion. Interpreting all this information in a self-consistent model is the main task of modern cosmology and $\Lambda$CDM model provides a good fit to most of the data (Tegmark et al. 2004; Seljak et al. 2005; Sanchez et al. 2006) giving a reliable snapshot of the today observed Universe.

Nevertheless, it is affected by serious theoretical shortcomings that have motivated the search for alternative candidates generally referred to as dark energy or quintessence. Such models range from scalar fields rolling down self-interaction potentials to phantom fields, from phenomenological unified models of dark energy and dark matter to alternative gravity theories (Padmanabhan 2003; Peebles & Ratra 2003; Copeland, Sami & Tsujikawa 2006).

Essentially, dark energy (or any alternative component) has to act as a negative pressure fluid which gives rise to an overall acceleration of the Hubble fluid. Despite the clear mechanisms generating the observed cosmological dynamics, the nature and the fundamental properties of dark energy remain essentially unknown notwithstanding the great theoretical efforts made till now.

The situation for dark matter is similar: its clustering and distribution properties are fairly well known at every scale but its nature is unknown, till now, at a fundamental level.

On the other hand, the need of unknown components as dark energy and dark matter could be considered nothing else but as a signal of the breakdown of the Einstein general relativity at astrophysical (galactic and extragalactic) and cosmological scales.

In this context, extended theories of gravity could be, in principle, an interesting alternative to explain cosmic acceleration without any dark energy and large-scale structure without any dark matter. In their simplest version, the Ricci curvature scalar $R$, linear in the Hilbert–Einstein action, could be replaced by a generic function $f(R)$ whose true form could be ‘reconstructed’ by the data. In fact, there is no a priori reason to consider the gravitational Lagrangian linear in the Ricci scalar while observations and experiments could
contribute to define and constrain the ‘true’ theory of gravity. For a discussion on this topic, see Capozziello (2002), Nojiri & Odintsov (2007), Capozziello & Francaviglia (2008) and Sotiriou & Faraoni (2008).

In Capozziello, Cardone & Troisi (2007), the Newtonian limit of power-law \( f(R) = f_a R^n \) theories has been investigated, assuming that the metric in the low-energy limit \( (\Phi/c^2 \ll 1) \) may be taken as Schwarzschild like. It turns out that a power-law term \( (r/r_c)^p \) has to be added to the Newtonian 1/r term in order to get the correct gravitational potential. While the parameter \( p \) may be expressed analytically as a function of the slope \( n \) of the \( f(R) \) theory, \( r_c \) sets the scale where the correction term starts being significant. A particular range of values of \( n \) has been investigated so that the corrective term is an increasing function of the radius \( r \) thus causing an increase of interaction length of the problem without \( \theta \) thus causing an increase of interaction length. For a comprehensive review see Bahcall (1996).

2 \( f(R) \) GRAVITY AND EXTENDED GRAVITATIONAL SYSTEMS

Let us consider the general action:

\[
A = \int d^4x \sqrt{-g} \left[ f(R) + \mathcal{L}_m \right],
\]

where \( f(R) \) is an analytic function of the Ricci scalar \( R, g \) is the determinant of the metric \( g_{\mu\nu}, \mathcal{L}_m = [(16\pi G)/c^4] \) is the coupling constant and \( \mathcal{L}_m \) is the standard perfect-fluid matter Lagrangian. Such an action is the straightforward generalization of the Hilbert–Einstein action of \( GR \) obtained for \( f(R) = R \). Since we are considering the metric approach, field equations are obtained by varying (1) with respect to the metric:

\[
f' R_{\mu\nu} - \frac{1}{2} f R_{\mu\nu} - f'_{\mu\nu} + g_{\mu\nu} \Box f' = \frac{\chi}{2} T_{\mu\nu},
\]

where \( T_{\mu\nu} = -(2/\sqrt{-g}) [\delta(\sqrt{-g} \mathcal{L}_m)]/\delta g^{\mu\nu} \) is the energy momentum tensor of matter, the prime indicates the derivative with respect to \( R \) and \( \Box = \Box^{\vec{\nabla}} \).

As discussed in details in Capozziello, Stabile & Troisi (2008), we deal with the Newtonian and the post-Newtonian limit of \( f(R) \) gravity on an spherically symmetric background. Solutions for the field equations can be obtained by imposing the spherical symmetry and in terms of gravitational potential.

Among the possible analytic \( f(R) \) models, we can consider the Taylor expansion where the cosmological term and terms higher than second can be discarded. The Lagrangian is then

\[
f(R) \sim a_1 R + a_2 R^2 + \ldots,
\]

and the gravitational potential, generated by a point-like matter distribution, is (Capozziello et al. 2008)

\[
\phi(r) = -\frac{3GM}{4\alpha_1 r} \left[ 1 + \frac{1}{3} e^{-\alpha_1/L} \right],
\]

where

\[
L \equiv L(a_1, a_2) = \left( \frac{6a_2}{a_1} \right)^{1/2}.
\]

L can be defined as the interaction length of the problem\(^1\) due to the correction to the Newtonian potential.

The gravitational potential (4) is a point-like one. We have to generalize this solution to extended systems in order to deal with clusters of galaxies.

Let us describe galaxy clusters as spherically symmetric systems and then we have to extend the above considerations to this geometrical configuration. We simply consider the system composed by many infinitesimal mass elements \( d m \) each one contributing with a point-like gravitational potential. Then, summing up all terms, namely integrating them on a spherical volume, we obtain a suitable potential. Specifically, we have to solve the integral

\[
\Phi(r) = \int_0^\infty r'^2 \, dr' \int_0^\pi \sin \theta' \, d\theta' \int_0^{2\pi} \cos \phi(r').
\]

The point-like potential (4) can be split in two terms. The Newtonian component is

\[
\phi_N(r) = -\frac{3GM}{4\alpha_1 r},
\]

\(^1\) Such a length is function of the series coefficients, \( a_1 \) and \( a_2 \), and it is not a free independent parameter in the following fit procedure.
The extended integral of such a part is well-known [apart from
the numerical constant (3/4 a1)] expression. It is
\[ \Phi_N(r) = -\frac{3}{4a_1} \frac{GM(<r)}{r}, \]  
where \( M(<r) \) is the mass enclosed in a sphere with radius \( r \). The correction term
\[ \Phi_C(r) = -\frac{GM}{4a_1} e^{-dr/L}, \]
considering some analytical steps in the integration of the angular part gives the expression
\[ \Phi_C(r) = -\frac{2\pi G}{4} L \int_0^\infty \frac{dr'}{r'} \rho(r') e^{-dr'/L} - e^{-dr'/L}. \]
The radial integral is numerically estimated once the mass density \( \rho \) is given. We underline a fundamental difference between such a term and the Newtonian one; while in the latter, the matter outside the spherical shell of radius \( r \) does not contribute to the potential, in the former, the external matter, integral function of the form
\[ \Phi(r) = \Phi_N(r) + \Phi_{C,\text{int}}(r) + \Phi_{C,\text{ext}}(r). \]
As we will be shown below, for our purpose, we need the gravitational potential derivative with respect to the variable \( r \); the two derivatives may not be evaluated analytically and so we estimate them numerically, once we have given an expression for the total mass density \( \rho(r) \). While the Newtonian term gives the simple expression
\[ -\frac{d\Phi_N}{dr}(r) = -\frac{3}{4a_1} \frac{GM(<r)}{r^2}, \]
the internal and external derivatives of the corrective potential terms are more involved. We do not give them explicitly for the sake of brevity, but they are integral functions of the form
\[ \mathcal{F}(r, r') = \int_{a(r)}^{b(r)} dr' f(r, r') \]
from which one has
\[ \frac{d\mathcal{F}(r, r')}{dr} = \int_{a(r)}^{b(r)} \frac{dr}{dr'} f(r, r') \]
\[ -f[r, a(r)] \frac{d\beta}{dr}(r) + f[r, b(r)] \frac{d\beta}{dr}(r). \]
Such an expression is numerically derived once the integration extremes are given. A general consideration is in order at this point. Clearly, the Gauss theorem holds only for the Newtonian part since, for this term, the force law scales as 1/r^2. For the total potential, this does not hold anymore due to the correction. From a physical point of view, this is not a problem because the full conservation laws are determined, for \( f(R) \) gravity, by the contracted Bianchi identities which assure the self-consistency. For a detailed discussion, see Capozziello et al. (2007), Capozziello & Francaviglia (2008) and Sotiriou & Faraoni (2008).

3 CLUSTER MASS PROFILES

Clusters of galaxies are generally considered self-bound gravitational systems with spherical symmetry and in hydrostatic equilibrium if virialized. The last two hypotheses are still widely used, despite the fact that it has been widely proved that most clusters show more complex morphologies and/or signs of strong interactions or dynamical activity, especially in their innermost regions (De Filippis et al. 2005; Chakrabarty, de Filippis & Russell 2008).

Under the hypothesis of spherical symmetry in hydrostatic equilibrium, the structure equation can be derived from the collisionless Boltzmann equation:
\[ \frac{d}{dr} \left[ \rho_{\text{gas}}(r) \sigma_i^2 \right] + \frac{2}{r} \rho_{\text{gas}}(r) \left( \sigma_i^2 - \sigma_{\text{int}}^2 \right) = -\rho_{\text{gas}}(r) \frac{d\Phi(r)}{dr}, \]
where \( \Phi \) is the gravitational potential of the cluster, \( \sigma_i \) and \( \sigma_{\text{int}} \) are the mass-weighted velocity dispersions in the radial and tangential directions, respectively, and \( \rho \) is gas mass density. For an isotropic system, it is
\[ \sigma_i = \sigma_{\text{int}}. \]
The pressure profile can be related to these quantities by
\[ P(r) = \sigma_i^2 \rho_{\text{gas}}(r). \]
Substituting equations (16) and (17) into equation (15), we have, for an isotropic sphere,
\[ \frac{dP(r)}{dr} = -\rho_{\text{gas}}(r) \frac{d\Phi(r)}{dr}. \]
For a gas sphere with temperature profile \( T(r) \), the velocity dispersion becomes
\[ \sigma_i^2 = \frac{kT(r)}{\mu m_p}. \]
where \( k \) is the Boltzmann constant, \( \mu \approx 0.609 \) is the mean mass particle and \( m_p \) is the proton mass. Substituting equations (17) and (19) into equation (18), we obtain
\[ \frac{d}{dr} \left[ \frac{kT(r)}{\mu m_p} \rho_{\text{gas}}(r) \right] = -\rho_{\text{gas}}(r) \frac{d\Phi}{dr} \]
or, equivalently,
\[ -\frac{d\Phi}{dr} = \frac{kT(r)}{\mu m_p r} \left[ \frac{d\ln \rho_{\text{gas}}(r)}{dr} + \frac{d\ln T(r)}{dr} \right]. \]
Now the total gravitational potential of the cluster is
\[ \Phi(r) = \Phi_N(r) + \Phi_C(r), \]
with
\[ \Phi_C(r) = \Phi_{C,\text{int}}(r) + \Phi_{C,\text{ext}}(r). \]
It is worth underlining that if we consider only the standard Newtonian potential, the total cluster mass \( M_{\text{cl, N}}(r) \) is composed by gas mass + mass of galaxies + eV galaxy mass + dark matter, and it is given by the expression
\[ M_{\text{cl, N}}(r) = M_{\text{gas}}(r) + M_{\text{gal}}(r) + M_{\text{DM}}(r) = -\frac{kT(r)}{\mu m_p G} \left[ \frac{d\ln \rho_{\text{gas}}(r)}{dr} + \frac{d\ln T(r)}{dr} \right]. \]
\[ M_{\text{cl},N}(r) \approx M_{\text{gas}}(r) + M_{\text{DM}}(r) \]

\[
\approx \frac{kT(r)}{\mu m_p} \left( \frac{\ln \rho_{\text{gas}}(r)}{\ln r} + \frac{\ln T(r)}{\ln r} \right) .
\]

Since the gas-mass estimates are provided by X-ray observations, the equilibrium equation can be used to derive the amount of dark matter present in a cluster of galaxies and its spatial distribution.

Inserting the previously defined extended-corrected potential of equation (21) into equation (20), we obtain

\[
\frac{d\Phi_N}{dr} = \frac{d\Phi_C}{dr} = \frac{kT(r)}{\mu m_p G} \left( \frac{\ln \rho_{\text{gas}}(r)}{\ln r} + \frac{\ln T(r)}{\ln r} \right)
\]

from which the extended-corrected mass estimate follows:

\[
M_{\text{cl,EC}}(r) + \frac{4\alpha t_i}{3G} \frac{d\Phi_C}{dr} = \frac{4\alpha t_i}{3} \left( \frac{kT(r)}{\mu m_p G} \left( \frac{\ln \rho_{\text{gas}}(r)}{\ln r} + \frac{\ln T(r)}{\ln r} \right) \right).
\]

Since the use of a corrected potential avoids, in principle, the additional requirement of dark matter, the total cluster mass, in this case, is given by

\[
M_{\text{cl,EC}}(r) = M_{\text{gas}}(r) + M_{\text{gal}}(r) + M_{\text{DM}}(r),
\]

and the mass density in the \( \Phi_C \) term is

\[
\rho_{\text{EC}}(r) = \rho_{\text{gas}}(r) + \rho_{\text{gal}}(r) + \rho_{\text{DM}}(r),
\]

with the density components derived from observations.

In this work, we will use equation (25) to compare the baryonic mass profile \( M_{\text{EC}}(r) \), estimated from observations, with the theoretical deviation from the Newtonian gravitational potential, given by the expression \(-\ln[(4\alpha t_i)/(3G)r^2][(d\Phi_C)/(dr)](r)\). Our goal is to reproduce the observed mass profiles for a sample of galaxy clusters.

4 GALAXY CLUSTER SAMPLE

The formalism described in Section 3 can be applied to a sample of 12 galaxy clusters. We shall use the cluster sample studied in Vikhlinin et al. (2005, 2006) which consists of 13 low-redshift clusters spanning a temperature range 0.7–9.0 keV derived from high-quality Chandra archival data. In all these clusters, the surface brightness and the gas temperature profiles are measured out to large radii, so that the mass estimates can be extended up to \( r_{\text{gas}} \) or beyond.

4.1 Gas density model

The gas density distribution of the clusters in the sample is described by the analytic model proposed in Vikhlinin et al. (2006). Such a model modifies the classical \( \beta \)-model to represent the characteristic properties of the observed X-ray surface brightness profiles, i.e. the power-law-type cusps of gas density in the cluster centre, instead of a flat core and the steepening of the brightness profiles at large radii. Eventually, a second \( \beta \)-model, with a small core radius, is added to improve the model close to the cluster cores. The analytical form for the particle emission is given by

\[
n_p n_e = \frac{n_{\text{e}}^2}{1 + (r/r_c)^{3\gamma}} - \frac{1}{1 + (r/r_i')^{3\gamma}}
\]

\[
+ \frac{n_{\text{e}}^2}{1 + (r/r_i')^{3\gamma}}
\]

which can be easily converted to a mass density using the relation

\[
\rho_{\text{gas}} = n_\text{T} m_p = \frac{1.4}{1.2} n_e m_p,
\]

where \( n_e \) is the total number density of particles in the gas. The resulting model has a large number of parameters, some of which do not have a direct physical interpretation. While this can often be inappropriate and computationally inconvenient, it suits well our case, where the main requirement is a detailed qualitative description of the cluster profiles.

In Vikhlinin et al. (2006), equation (28) is applied to a restricted range of distances from the cluster centre, i.e. between an inner cut-off \( r_{\text{min}} \), chosen to exclude the central temperature bin (\( \approx 10-20 \) kpc), where the intracluster medium (ICM) is likely to be multiphase, and a \( r_{\text{gal}} \), where the X-ray surface brightness is at least 3\( \sigma \) significant. We have extrapolated the above function to values outside this restricted range using the following criteria.

(i) For \( r < r_{\text{min}} \), we have performed a linear extrapolation of the first three terms out to \( r = 0 \) kpc.

(ii) For \( r > r_{\text{gal}} \), we have performed a linear extrapolation of the last three terms out to a distance \( r \) for which \( \rho_{\text{gas}}(r) = \rho_{\text{c}}, \rho_{\text{c}} \) being the critical density of the Universe at the cluster redshift: \( \rho_c = \rho_{\text{c}}(1+z)^3 \). For radii larger than \( r \), the gas density is assumed constant at \( \rho_{\text{gas}}(r) \).

We point out that, in Table 1, the radius limit \( r_{\text{min}} \) is almost the same as given in the previous definition. When the value given by Vikhlinin et al. (2006) is less than the CDD galaxy radius, which is defined in the next section, we choose this last one as the lower limit. On the contrary, \( r_{\text{max}} \) is quite different from \( r_{\text{gal}} \); it is fixed by considering the higher value of temperature profile and not by imaging methods.

We then compute the gas mass \( M_{\text{gas}}(r) \) and the total mass \( M_{\text{cl,EC}}(r) \), respectively, for all clusters in our sample, substituting equation (28) into equations (29) and (23), respectively; the gas temperature profile \( \{\text{E}\} \) described in details in Section 4.2. The resulting mass values, estimated at \( r = r_{\text{max}} \), are listed in Table 1.

4.2 Temperature profiles

As stressed in Section 4.1, for the purpose of this work, we need an accurate qualitative description of the radial behaviour of the gas properties. Standard isothermal or polytropic models, or even the more complex one proposed in Vikhlinin et al. (2006), do not provide a good description of the data at all radii and for all clusters in the present sample. We hence describe the gas temperature profiles using the straightforward X-ray spectral analysis results, without the introduction of any analytic model.

X-ray spectral values have been provided by Vikhlinin (private communication). A detailed description of the relative spectral analysis can be found in Vikhlinin et al. (2005).

4.3 Galaxy distribution model

The galaxy density can be modelled as proposed by Bahcall (1996). Even if the galaxy distribution is a point distribution instead of a
Table 1. Column 1: cluster name; column 2: richness; column 3: cluster total mass; column 4: gas mass; column 5: galaxy mass; column 6: cD galaxy mass. Gas and total mass values are estimated at \( r = r_{max} \). Column 7: ratio of total galaxy mass to gas mass; column 8: minimum radius; column 9: maximum radius.

| Name    | \( R \) | \( M_{\text{gal,N}} \) (\( M_\odot \)) | \( M_{\text{gas}} \) (\( M_\odot \)) | \( M_{\text{gal}} \) (\( M_\odot \)) | \( M_{\text{Dgal}} \) (\( M_\odot \)) | Gal/gas | \( r_{\text{min}} \) (kpc) | \( r_{\text{max}} \) (kpc) |
|---------|-------|-----------------|-----------------|-----------------|-----------------|--------|----------------|----------------|
| A133    | 0     | 4.35874 \times 10^{14} | 2.73866 \times 10^{13} | 5.20269 \times 10^{12} | 1.0568 \times 10^{12} | 0.23   | 86             | 1060         |
| A262    | 0     | 4.45081 \times 10^{13} | 2.76659 \times 10^{12} | 1.71305 \times 10^{11} | 5.16382 \times 10^{12} | 0.25   | 61             | 316          |
| A383    | 2     | 2.79785 \times 10^{14} | 2.82467 \times 10^{13} | 5.88048 \times 10^{12} | 1.09217 \times 10^{12} | 0.25   | 52             | 751          |
| A478    | 2     | 8.51382 \times 10^{14} | 1.05583 \times 10^{14} | 2.15567 \times 10^{13} | 1.67513 \times 10^{12} | 0.22   | 59             | 1580         |
| A907    | 1     | 4.87657 \times 10^{14} | 6.38070 \times 10^{13} | 1.34129 \times 10^{13} | 1.66533 \times 10^{12} | 0.24   | 563            | 1226         |
| A1413   | 3     | 1.09598 \times 10^{15} | 9.32466 \times 10^{13} | 2.30728 \times 10^{13} | 1.67345 \times 10^{12} | 0.26   | 57             | 1506         |
| A1795   | 2     | 5.44761 \times 10^{14} | 5.56245 \times 10^{13} | 4.23211 \times 10^{12} | 1.93957 \times 10^{12} | 0.11   | 79             | 1151         |
| A1991   | 1     | 1.24313 \times 10^{14} | 1.00530 \times 10^{13} | 1.24608 \times 10^{12} | 1.08241 \times 10^{12} | 0.23   | 55             | 618          |
| A2029   | 2     | 8.92392 \times 10^{14} | 1.24129 \times 10^{13} | 3.21543 \times 10^{13} | 1.11921 \times 10^{12} | 0.27   | 62             | 1771         |
| A2390   | 1     | 2.09710 \times 10^{15} | 2.15726 \times 10^{13} | 4.9180 \times 10^{13}  | 1.12141 \times 10^{12} | 0.23   | 83             | 1984         |
| MKW-4   | 0     | 4.69503 \times 10^{13} | 2.83207 \times 10^{12} | 1.71153 \times 10^{11} | 5.29855 \times 10^{11} | 0.25   | 60             | 434          |
| RXJ1159 | 2     | 8.97997 \times 10^{13} | 4.33256 \times 10^{12} | 7.34414 \times 10^{11} | 5.38799 \times 10^{11} | 0.29   | 64             | 568          |

continuous function, assuming that galaxies are in equilibrium with gas, we can use a \( \beta \)-model, \( \propto r^{-\beta} \), for \( r < R_c \) from the cluster centre, and a steeper one, \( \propto r^{-\beta-2} \), for \( r > R_c \), where \( R_c \) is the cluster core radius (its value is taken from Vikhlinin et al. 2006). Its final expression is

\[
\rho_{\text{gal}}(r) = \begin{cases} 
\rho_{\text{gal},1} \left[ 1 + \left( \frac{r}{R_c} \right)^2 \right]^{-3/2} & r < R_c, \\
\rho_{\text{gal},2} \left[ 1 + \left( \frac{r}{R_c} \right)^2 \right]^{-2.6/2} & r > R_c, 
\end{cases}
\]

where the constants \( \rho_{\text{gal,1}} \) and \( \rho_{\text{gal,2}} \) are chosen in the following way.

(i) Bahcall (1996) provides the central number density of galaxies in rich compact clusters for galaxies located within a 1.5 \( h^{-1} \) Mpc radius from the cluster centre and brighter than \( m_1 + 2m \) (where \( m_1 \) is the magnitude of the third brightest galaxy): \( n_{\text{gal,0}} \sim 10^{3} h^{-1} \) galaxies Mpc\(^{-3} \). Then, we fix \( \rho_{\text{gal},1} \) in the range \( \sim 10^{-15} \sim 10^{-9} \) kg kpc\(^{-3} \). For any cluster obeying the condition chosen for the mass ratio gal-to-gas, we assume a typical elliptical and cD galaxy mass in the range \( 10^{12} \sim 10^{13} M_\odot \).

(ii) The constant \( \rho_{\text{gal,2}} \) has been fixed with the only requirement that the galaxy density function has to be continuous at \( R_c \).

We have tested the effect of varying galaxy density in the above range \( \sim 10^{-15} \sim 10^{-9} \) kg kpc\(^{-3} \) on the cluster with the lowest mass, namely A262. In this case, we would expect great variations with respect to other clusters; the result is that the contribution due to galaxies and cD galaxy gives a variation \( \leq 1 \) per cent to the final estimate of fit parameters.

The cD galaxy density has been modelled as described in Schmidt & Allen (2007); they use a Jaffe model of the form

\[
\rho_{\text{Dgal}} = \frac{\rho_{\text{Dgal}}}{(r/r_c)^3[1 + (r/r_c)^2]^2},
\]

where \( r_c \) is the core radius while the central density is obtained from \( M_f = (4/3)\pi R_c^3 \rho_{\text{Dgal}} \). The mass of the cD galaxy has been fixed at \( 1.14 \times 10^{12} M_\odot \), with \( r_c = R_c/0.76 \), with \( R_c = 25 \) kpc being the effective radius of the galaxy. The central galaxy for each cluster in the sample is assumed to have approximately this stellar mass.

We have assumed that the total galaxy component mass plus cD galaxy masses is \( \sim 20 \sim 25 \) per cent of the gas mass: in Schindler (2004), the mean fraction of gas versus the total mass (with dark matter) for a cluster is estimated to be 15 to 20 per cent, while the same quantity for galaxies is 3 to 5 per cent. This means that the relative mean mass ratio gal-to-gas in a cluster is \( \approx 20 \sim 25 \) per cent. We have varied the parameters \( \rho_{\text{gal,1}}, \rho_{\text{gal,2}} \) and \( M_f \) in their previous defined ranges to obtain a mass ratio between total galaxy mass and total gas mass which lies in this range. Resulting galaxy mass values and ratios gal/gas, estimated at \( r = r_{\text{max}} \), are listed in Table 1.

In Fig. 1, we show how each component is spatially distributed. The cD galaxy is dominant with respect to the other galaxies only in the inner region (below 100 kpc). As already stated in Section 4.1, cluster innermost regions have been excluded from our analysis and so the contribution due to the cD galaxy is practically negligible in our analysis. The gas is, as a consequence, clearly the dominant visible component, starting from the innermost regions out to large radii, being galaxy mass only 20 to 25 per cent of gas mass. A similar behaviour is shown by all the clusters considered in our sample.

4.4 Uncertainties on mass profiles

Uncertainties on the cluster total mass profiles have been estimated performing Monte Carlo simulations (Neumann & Böhringer 1995). We proceed to simulate temperature profiles and choose random radius–temperature values couples for each bin which we have in our temperature data given by Vikhlinin et al. (2005). Random temperature values have been extracted from a Gaussian distribution centred on the spectral values, and with a dispersion fixed to its 68 per cent confidence level. For the radius, we choose a random value inside each bin. We have performed 2000 simulations for each cluster and two cuts on the simulated profile. First, we exclude...
those profiles that give an unphysical negative estimate of the mass: this is possible when our simulated couples of quantities give rise to too high temperature gradient. After this cut, we have ≈1500 simulations for any cluster. Then, we have ordered the resulting mass values for increasing radius values. Extreme mass estimates (outside the 10–90 per cent range) are excluded from the obtained distribution, in order to avoid other high mass gradients which give rise to masses too different from real data. The resulting limits provide the errors on the total mass. Uncertainties on the electron-density profiles have not been included in the simulations, they being negligible with respect to those of the gas-temperature profiles.

4.5 Fitting the mass profiles

In the above sections, we have shown that, with the aid of X-ray observations, modelling theoretically the galaxy distribution and using equation (25), we obtain an estimate of the baryonic content of clusters.

We have hence performed a best-fitting analysis of the theoretical equation (25),

$$M_{\text{bar,obs}}(r) = \frac{4a_1}{3} \left\{ -\frac{kT(r)}{\mu m_p G} \left[ \frac{\ln \rho_{\text{gas}}(r)}{\ln r} + \frac{\ln T(r)}{\ln r} \right] \right\} - \frac{4a_2}{3G} \frac{\partial \Phi_{\text{gas}}(r)}{\partial r},$$

(32)

versus the observed mass contributions,

$$M_{\text{bar,obs}}(r) = M_{\text{gas}}(r) + M_{\text{gal}}(r) + M_{\text{cgal}}(r).$$

(33)

Since not all the data involved in the above estimate have measurable errors, we cannot perform an exact $\chi^2$ minimization: actually, we can minimize the quantity

$$\chi^2 = \frac{1}{N-n_p-1} \sum_{i=1}^{N} \left( \frac{M_{\text{bar,obs}}(r) - M_{\text{bar,tho}}(r)}{M_{\text{bar,tho}}(r)} \right)^2,$$

(34)

where $N$ is the number of data and $n_p = 2$ the free parameters of the model. We minimize the $\chi^2$ using the Markov Chain Monte Carlo (MCMC) method. For each cluster, we have run various chains to set the best parameters of the used algorithm, the Metropolis–Hastings one: starting from an initial parameter vector $p$ [in our case $p = (a_1, a_2)$], we generate a new trial point $p'$ from a tested proposal density $q(p, p')$, which represents the conditional probability to get $p'$, given $p$. This new point is accepted with probability

$$\alpha(p, p') = \min \left\{ 1, \frac{L(d|p')P(p'|q(p', p))}{L(d|p)P(p|q(p, p'))} \right\},$$

(35)

d where $d$ are the data, $L(d|\theta) \propto \exp (-\chi^2/2)$ is the likelihood function, $P(p)$ is the prior on the parameters. In our case, the prior on the fit parameters is related to equation (5): $L$ being a length, we need to force the ratio $a_1/a_2$ to be positive. The proposal density is Gaussian symmetric with respect of the two vectors $p$ and $p'$, namely $q(p, p') \propto \exp (-\Delta p^2/2\sigma^2)$, with $\Delta p = p - p'$; we decide to fix the dispersion $\sigma$ of any trial distribution of parameters equal to 20 per cent of trial $a_1$ and $a_2$ at any step. This means that the parameter $\sigma$ reduces to the ratio between the likelihood functions. We have run one chain of $10^5$ points for every cluster; the convergence of the chains has been tested using the power spectrum analysis from Dunkley et al. (2005). The key idea of this method is, at the same time, simple and powerful: if we take the power spectra of the MCMC samples, we will have a great correlation on small scales but, when the chain reaches convergence, the spectrum becomes flat (like a white noise spectrum); so that, checking the spectrum of just one chain (instead of many parallel chains as in Gelmann–Rubin test) will be sufficient to assess the reached convergence. Rend-
for each cluster above which our model works really well (typical relative differences are less than 5 per cent), while for lower scale there is a great difference. It is possible to see, by a rapid inspection, that this turning-point is located at a radius \( \approx 150 \text{kpc} \). Except for extremely rich clusters, it is clear that this value is independent of the cluster, being similar for all clusters in our sample.

There are two main independent explanations that could justify this trend: limits due to a break in the state of hydrostatic equilibrium, or limits in the series expansion of the \( f(R) \) models.

If the hypothesis of hydrostatic equilibrium is not correct, then we are in a regime where the fundamental relations (equations 15–20) are not working. As discussed in Vikhlinin et al. (2005), the central (70 kpc) region of most clusters is strongly affected by radiative cooling, and thus its physical properties cannot be directly related to the depth of the cluster potential well. This means that, in this region, the gas is not in hydrostatic equilibrium but in a multiphase state. In this case, the gas temperature cannot be used as a good standard tracer.
Table 2. Column 1: cluster name; column 2: first derivative coefficient, \( a_1 \), of \( f(R) \) series; column 3: 1\( \sigma \) confidence interval for \( a_1 \); column 4: second derivative coefficient, \( a_2 \), of \( f(R) \) series; column 5: 1\( \sigma \) confidence interval for \( a_2 \); column 6: characteristic length, \( L \), of the modified gravitational potential, derived from \( a_1 \) and \( a_2 \); column 7: 1\( \sigma \) confidence interval for \( L \).

| Name   | \( a_1 \)  | \([ a_1 - 1\sigma, a_1 + 1\sigma ]\) | \( a_2 \) \( (kpc^2) \) | \([ a_2 - 1\sigma, a_2 + 1\sigma ]\) \( (kpc^2) \) | \( L \) \( (kpc) \) | \([ L - 1\sigma, L + 1\sigma ]\) \( (kpc) \) |
|--------|------------|-----------------------------------------|------------------------|-----------------------------------------|-----------------|-----------------------------------------|
| A133   | 0.085      | [0.078, 0.091]                          | -4.98 \( \times 10^3 \) | [-2.38 \( \times 10^4 \), -1.38 \( \times 10^4 \)] | 591.78         | [323.34, 1259.50]                       |
| A262   | 0.065      | [0.061, 0.071]                          | -10.63                 | [-5.76, -3.17]                          | 31.40           | [17.28, 71.71]                          |
| A383   | 0.099      | [0.093, 0.108]                          | -9.01 \( \times 10^2 \) | [-4.10 \( \times 10^3 \), -3.14 \( \times 10^3 \)] | 234.13         | [142.10, 478.06]                       |
| A478   | 0.117      | [0.114, 0.122]                          | -4.61 \( \times 10^3 \) | [-1.01 \( \times 10^5 \), -2.51 \( \times 10^3 \)] | 484.83         | [363.29, 707.73]                       |
| A907   | 0.129      | [0.125, 0.136]                          | -5.77 \( \times 10^3 \) | [-1.54 \( \times 10^5 \), -2.83 \( \times 10^3 \)] | 517.30         | [368.84, 825.00]                       |
| A1413  | 0.115      | [0.110, 0.119]                          | -9.45 \( \times 10^4 \) | [-4.26 \( \times 10^5 \), -3.46 \( \times 10^4 \)] | 2224.57        | [1365.40, 4681.21]                     |
| A1795  | 0.093      | [0.084, 0.103]                          | -1.54 \( \times 10^3 \) | [-1.01 \( \times 10^4 \), -2.49 \( \times 10^2 \)] | 315.44         | [133.31, 769.17]                       |
| A1991  | 0.074      | [0.072, 0.081]                          | -50.69                 | [-3.42 \( \times 10^2 \), -13]           | 64.00          | [32.63, 159.40]                        |
| A2029  | 0.129      | [0.123, 0.134]                          | -2.10 \( \times 10^4 \) | [-7.95 \( \times 10^4 \), -8.44 \( \times 10^3 \)] | 988.85         | [637.71, 1890.07]                      |
| A2390  | 0.149      | [0.146, 0.152]                          | -1.40 \( \times 10^6 \) | [-5.71 \( \times 10^6 \), -4.46 \( \times 10^5 \)] | 7490.80        | [4245.74, 15715.60]                    |
| MKW 4  | 0.054      | [0.049, 0.060]                          | -23.63                 | [-1.15 \( \times 10^2 \), -8.13]         | 51.31          | [30.44, 110.68]                        |
| RX J1159 | 0.048      | [0.047, 0.052]                          | -18.33                 | [-1.35 \( \times 10^2 \), -4.18]         | 47.72          | [22.86, 125.96]                        |

Figure 5. Baryonic mass versus radii for Abell A133. Dashed line is the experimental-observed estimation equation (33) of baryonic matter component (i.e. gas, galaxies and cD galaxy); solid line is the theoretical estimation equation (32) for baryonic matter component. Dotted lines are the 1\( \sigma \) confidence levels given by errors on fitting parameters in the left-hand panel; and from fitting parameter plus statistical errors on mass profiles as discussed in Section 4.4 in the right-hand panel.

Figure 6. Same as Fig. 5 but for cluster Abell 262.

We have also to consider another limit of our modelling: the requirement that the \( f(R) \) function is Taylor expandable. The corrected gravitational potential which we have considered is derived in the weak field limit, which means

\[ R - R_0 \ll \frac{a_1}{a_2}, \tag{39} \]

where \( R_0 \) is the background value of the curvature. If this condition is not satisfied, the approach does not work (see Capozziello et al. 2008 for a detailed discussion of this point). Considering that \( a_1/a_2 \) has the dimension of \( \text{length}^{-2} \), this condition defines the length scale where our series approximation can work. In other words, this indicates the limit in which the model can be compared with data.

For the considered sample, the fit of the parameters \( a_1 \) and \( a_2 \) spans the length range (19–200 kpc) (except for the richest clusters). It is evident that every galaxy cluster has a proper gravitational length scale. It is worth noticing that a similar situation, but at
Figure 7. Same as Fig. 5 but for cluster Abell 383.

Figure 8. Same as Fig. 5 but for cluster Abell 478.

Figure 9. Same as Fig. 5 but for cluster Abell 907.

Figure 10. Same as Fig. 5 but for cluster Abell 1413.
Figure 11. Same of Fig. 5 but for cluster Abell 1795.

Figure 12. Same of Fig. 5 but for cluster Abell 1991.

Figure 13. Same as Fig. 5 but for cluster Abell 2029.

Figure 14. Same as Fig. 5 but for cluster Abell 2390.
completely different scales, has been found out for LSB galaxies modelled by $f(R)$ gravity (Capozziello et al. 2007).

Considering the data at our disposal and the analysis which we have performed, it is not possible to quantify exactly the quantitative amount of these two different phenomena (i.e. the radiative cooling and the validity of the weak field limit). However, they are not mutually exclusive but should be considered in detail in view of a more refined modelling.\footnote{Other secondary phenomena as cooling flows, merger and asymmetric shapes have to be considered in view of a detailed modelling of clusters. However, in this work, we are only interested to show that extended gravity could be a valid alternative to dark matter in order to explain the cluster dynamics.}

Similar issues are present also in Brownstein & Moffat (2006): they use the metric-skew-tensor gravity (MSTG) as a generalization of the Einstein general relativity and derive the gas mass profile of a sample of clusters with gas being the only baryonic component of the clusters. They consider some clusters included in our sample (in particular, A133, A262, A478, A1413, A1795, A2029, MKW 4) and they find the same different trend for $r \leq 200$ kpc, even if with a different behaviour with respect to us: our model gives lower values than X-ray gas mass data while their model gives higher values with respect to X-ray gas mass data. This stresses the need for a more accurate modelling of the gravitational potential.

However, our goal is to show that potential (4) is suitable to fit the mass profile of galaxy clusters and that it comes from a self-consistent theory.

In our case, the parameters $\alpha_1, \alpha_2$, which determine the gravitational correction and the gravitational coupling, come out ‘directly’ from a field theory with the only requirement that the effective action of gravity could be more general than the Hilbert–Einstein theory $f(R) = R$. This main hypothesis comes from fundamental physics motivations due to the fact that any unification scheme or quantum field theory on curved space has to take into account higher order terms in curvature invariants (Birrell & Davies 1982). Besides, several recent results point out that such corrections have a main role also at astrophysical and cosmological scales. For a detailed discussion see Nojiri & Odintsov (2007), Capozziello & Francaviglia (2008) and Sotiriou & Faraoni (2008).

With this philosophy in mind, we have plotted the trend of $\alpha_1$ as a function of the density in Fig. 17. As one can see, its values are strongly constrained in a narrow region of the parameter space, so that $\alpha_1$ can be considered a ‘tracer’ for the size of gravitational structures. The value of $\alpha_1$ range between 0.8 and 0.12 for larger clusters and 0.4 and 0.6 for poorer structures (i.e. galaxy groups like MKW 4 and RX J1159). We expect a particular trend when applying the model to different gravitational structures. In Fig. 17, we give characteristic values of density which range from the biggest structure, the observed Universe (large dashed vertical line), to the smallest one, the Sun (vertical dotted line), through intermediate steps like clusters (vertical short dashed line) and galaxies (vertical dot–dashed line). The bold black horizontal line represents the Newtonian limit $\alpha_1 = 3/4$ and the boxes indicate the possible values of $\alpha_1$ that we obtain by applying our theoretical model to different structures.

Similar considerations hold also for the characteristic gravitational length $L$ directly related to both $\alpha_1$ and $\alpha_2$. The parameter $\alpha_2$ shows a very large range of variation ($-10^4$ to $-10$) with respect to the density (and the mass) of the clusters. The value of $L$ changes with the sizes of gravitational structure (see Fig. 18), so it can be considered, beside the Schwarzschild radius, a sort of additional gravitational radius. Particular care must be taken when considering

\[ L = \frac{\alpha_2}{\alpha_1} R. \]
6 DISCUSSION AND CONCLUSIONS

In this paper, we have investigated the possibility that the high observational mass-to-light ratio of galaxy clusters could be addressed by \( f(R) \) gravity without assuming huge amounts of dark matter. We point out that this proposal comes out from the fact that, up to now, no definitive candidate for dark matter has been observed at fundamental level and then alternative solutions to the problem should be viable. Furthermore, several results in \( f(R) \) gravity seem to confirm that valid alternatives to \( \Lambda \)CDM can be achieved in cosmology. Besides, as discussed in the Introduction, the rotation curves of spiral galaxies can be explained in the weak field limit of \( f(R) \) gravity. Results of our analysis go in this direction.

We have chosen a sample of relaxed galaxy clusters for which accurate spectroscopic temperature measurements and gas mass profiles are available. For the sake of simplicity, and considering the sample at our disposal, every cluster has been modelled as a self-bound gravitational system with spherical symmetry and in hydrostatic equilibrium. The mass distribution has been described by a corrected gravitational potential obtained from a generic analytic \( f(R) \) theory. In fact, as soon as \( f(R) \neq R \), Yukawa-like exponential corrections emerge in the weak field limit while the standard Newtonian potential is recovered only for \( f(R) = R \), the Hilbert–Einstein theory.

Our goal has been to analyse if the dark matter content of clusters can be addressed by these correction potential terms. As discussed in detail in the previous section and how it is possible to see by a rapid inspection of Figs 5–16, all the clusters of the sample are consistent with the proposed model at 1\( \sigma \) confidence level. This shows, at least qualitatively, that the high mass-to-light ratio of clusters can be explained by using a modified gravitational potential. The good agreement is achieved on distance scales starting from 150 to 1000 kpc. The differences observed at smaller scales can be ascribed to non-gravitational phenomena, such as cooling flows, or to the fact that the gas mass is not a good tracer at these scales. The remarkable result is that we have obtained a consistent agreement with data only by using the corrected gravitational potential in a large range of radii. In order to put in the evidence for this trend, we have plotted the baryonic mass versus radii considering, for each cluster, the scale where the trend is clearly evident.

In our knowledge, the fact that \( f(R) \) gravity could work at these scales has been only supposed but never achieved by a direct fitting with data (see Boehmer et al. 2008; Lobo 2008 for a discussion). Starting from the series coefficients \( a_1 \) and \( a_2 \), it is possible to state that, at cluster scales, two characteristic sizes emerge from the weak field limit of the theory. However, at smaller scales, e.g. Solar...
system scales, standard Newtonian gravity has to be dominant in agreement with observations. Another issue is related to the radial velocity dispersion relation which is an important parameter for galaxy clusters. As discussed above, such a quantity is sensible to the radial mass profile through the Jeans and Boltzmann equations (Bahcall 1996). By confronting the observed velocity profiles \( \sigma_r \), up to large distances from the cluster centre to the theoretical gravitational potential, derived from \( f(R) \) models, one could obtain another independent test bed for the theory. This will be the argument of another paper.

In conclusion, if our considerations are right, gravitational interaction depends on the scale and the infrared limit is led by the series coefficient of the considered effective gravitational Lagrangian. Roughly speaking, we expect that starting from cluster scale to galaxy scale, and then down to smaller scales as Solar system or Earth, the terms of the series lead the clustering of self-gravitating systems beside other non-gravitational phenomena. In our case, the Newtonian limit is recovered for \( \alpha_1 \rightarrow 3/4 \) and \( L(\alpha_1, \alpha_2) \gg r \) at small scales and for \( L(\alpha_1, \alpha_2) \ll r \) at large scales. In the first case, the gravitational coupling has to be redefined, in the second \( G_\infty \simeq G \). In these limits, the linear Ricci term is dominant in the gravitational Lagrangian and the Newtonian gravity is restored (Quandt & Schmidt 1991). Reversing the argument, this could be the starting point to achieve a theory capable of explaining the strong segregation in masses and sizes of gravitationally bound systems.

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